Prediction of Convergence Deformation and Stability Analysis of Deep Buried Tunnel Based on Grey Theory

Qiwen Zheng 1,2,3,a, Haiyun Wei 1,2,3,b, Shang Gao 1,2,3,c, Qiang Peng 1,2,3,d, Zhenbo Zhao 1,2,3,e and Li Lu 1,2,3,f

1 Zhejiang Institute of Hydraulics & Estuary, Hangzhou, Zhejiang, 310020, China
2 Zhejiang Key Laboratory of Water Conservancy Disaster Prevention and Mitigation, Hangzhou, Zhejiang, 310020, China
3 Zhejiang Guangchuan Engineering Consulting Corporation, Hangzhou, Zhejiang, 310020, China
a zqw_yu@163.com, b 35851220@qq.com, c 834525125@qq.com,
d 576038228@qq.com, e 68146157@qq.com, f 409588361@qq.com

Abstract. The monitoring, analysis and prediction of surrounding rock deformation during underground tunnel construction is the crucial points of NATM construction. The convergence deformation and vault settlement of surrounding rock comprehensively reflect the dynamic adjustment process of surrounding rock stress and the supporting effect of supporting structure after tunnel excavation. The observed values can serve as the basis for judging the initial supporting stability of surrounding rock and selecting a reasonable time for secondary supporting. In this paper, the grey theory is implemented to establish the prediction model of the convergence deformation and vault settlement of tunnel surrounding rock. According to field monitoring and grey prediction results, the deformation stability of surrounding rock of underground tunnel is analyzed from two aspects of relative displacement and deformation rate, and the secondary support time is reasonably determined. The analysis results can provide reference for the design, construction and monitoring of similar projects.

1. Introduction

The new Austrian method is a tunnel excavation construction method proposed by Austrian professor L.V.Rabcwice based on rock mechanics theory, which is currently widely used in underground tunnel excavation construction[1]. The advantages of NATM are based on real-time monitoring data, taking full advantage of the self-bearing capacity of surrounding rock and establishing a more equitable supporting system. New Austrian Tunneling Method requires real-time observation of surrounding rock dynamics during construction, comprehensive analysis of monitoring index observation values, revision and improvement of the original design scheme, and guidance for the next stage of construction. The monitoring, analysis and prediction of surrounding rock deformation during underground tunnel construction is the major points of NATM construction.

The main influencing factors of tunnel surrounding rock deformation include surrounding rock type, rock mass structure, rock properties, burial depth, excavation method, support measures, support timing, section shape, distance between monitoring section and working face, etc. [2], and its mechanical process is relatively complex. At present, the prediction methods for tunnel surrounding rock
deformation monitoring indexes include empirical analogy method, BP neural network, time series analysis, regression analysis, gray prediction, etc. [3-6]. Among them, gray prediction has been widely applied in tunnel surrounding rock deformation prediction research due to the characteristics of small amount of measured data, short time and no special requirements [7-9].

Based on the on-site monitoring results of the water conveyance tunnel of Qiandao Lake Water Distribution Project in Hangzhou City, the grey theory is applied to establish a prediction model for the convergence deformation of tunnel surrounding rock and the settlement of the vault. According to the field monitoring and grey prediction results, the deformation stability of surrounding rock of underground tunnel is analyzed from two aspects of relative displacement and deformation rate, and the secondary support time is reasonably determined. The analysis results can provide reference for the design, construction and monitoring of similar projects.

2. Grey prediction theory model
Grey system theory is a new theory founded by Chinese scholar professor Deng Julong in 1982, which takes "small data, poor information" uncertainty system as the research object. Grey prediction modeling has four major characteristics: first, small data modeling; second, modeling based on the "accumulation generation" of the original sequence; third, using the "buffer operator" data transformation technology to ensure the validity and rationality of the model prediction results; fourth, the modeling process is relatively simple without considering the probability distribution law and statistical characteristics of sample data [10].

Conventional prediction models such as regression analysis prediction model, autoregressive moving average model and BP neural network prediction model are all based on large sample data, while grey prediction modeling takes "small data" uncertainty system as the research object. The samples of observation data of surrounding rock deformation during underground tunnel construction are small. New Austrian Tunneling Method requires short-term prediction of surrounding rock deformation dynamics. At the same time, the space-time influencing factors of convergence deformation of tunnel surrounding rock are complex and have great uncertainty. Therefore, the application of grey system theory to predict surrounding rock deformation during underground tunnel construction has obvious advantages over conventional prediction methods.

**DNGM (1,1) Model.** DNGM (1,1) model is an approximate nonhomogeneous exponential sequence grey prediction model based on whitening differential equation parameter direct estimation method, which can better reflect the approximate nonhomogeneous exponential growth characteristics of complex system behavior sequence. The DNGM (1,1) model directly starts from the time response function of the differential equation instead of the difference equation, solves the estimated values of the model parameters \(a, b\) and \(c\), substitutes the model parameters into the time response function formula, and obtains the prediction expression of the approximate nonhomogeneous exponential sequence through cumulative reduction [11].

Assuming the original observation data sequence is

\[
X^{(0)} = \left( x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n) \right), \quad x^{(0)}(k) \geq 0, \quad k = 1, 2, \cdots, n. \tag{1}
\]

**Definition 1:** sequence \(X^{(1)}\) is an accumulation sequence of sequence \(X^{(0)}\).

\[
X^{(1)} = \left( x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n) \right), \quad x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(k), k = 1, 2, \cdots, n. \tag{2}
\]

**Definition 2:** \(Z^{(1)}\) is the nearest mean generation sequence of \(X^{(1)}\).

\[
Z^{(1)} = \left( z^{(1)}(1), \cdots, z^{(1)}(n) \right), z^{(1)}(k) = 0.5 \times \left( x^{(1)}(k) + x^{(1)}(k-1) \right), k = 2, \cdots, n. \tag{3}
\]

**Definition 3:** The whitening differential equation of DNGM (1,1) model is
\[
\frac{dx^{(1)}}{dt} + ax^{(1)} = bt + c
\]  
(4)

Where \(a\), \(b\) and \(c\) are undetermined parameters.

The time response formula of DNGM (1,1) model can be solved by formula (4) as follows:

\[
x^{(1)}(t) = \left( x^{(1)}(1) - \frac{b}{a} + \frac{b}{a^2} + \frac{c}{a} \right) e^{-a(t-1)} + \frac{b}{a} \frac{1}{a^2} + \frac{c}{a}
\]  
(5)

The final reduction formula of DNGM (1,1) model can be obtained from formula (5) as follows:

\[
\tilde{x}^{(0)}(t) = \left( 1 - e^{-a} \right) \left( x^{(0)}(1) - \frac{b}{a} + \frac{b}{a^2} + \frac{c}{a} \right) e^{-a(t-1)} + \frac{b}{a}
\]  
(6)

A simple transformation of equation (6) yields the following equation

\[
x^{(1)}(t+1) = \alpha x^{(1)}(t) + \beta t + \gamma
\]  
(7)

\[
\alpha = e^{-a}
\]  
(8)

\[
\beta = \frac{b}{a} \left( 1 - e^{-a} \right)
\]  
(9)

\[
\gamma = \left( 1 - e^{-a} \right) \left( \frac{c}{a} - \frac{b}{a^2} \right) + \frac{b}{a}
\]  
(10)

Where \(a\), \(\beta\) and \(\gamma\) are undetermined parameters.

According to the principle of least square method, it is obtained from formula (7)

\[
S = \sum_{t=1}^{n-1} \left[ x^{(1)}(t+1) - \hat{\alpha} x^{(1)}(t) - \hat{\beta} t - \hat{\gamma} \right]^2
\]  
(11)

\[
\frac{\partial S}{\partial \hat{\alpha}} = 0, \quad \frac{\partial S}{\partial \hat{\beta}} = 0, \quad \frac{\partial S}{\partial \hat{\gamma}} = 0
\]  
(12)

The estimated values of parameters \(\alpha\), \(\beta\) and \(\gamma\) obtained by simultaneous solutions of equations (11) and (12) are

\[
\hat{\alpha} = \frac{B_1}{B}, \quad \hat{\beta} = \frac{B_2}{B}, \quad \hat{\gamma} = \frac{B_3}{B}
\]  
(13)

\[
B = \begin{vmatrix}
\sum_{t=1}^{n-1} x^{(1)}(t)^2 & \sum_{t=1}^{n-1} t x^{(1)}(t) & \sum_{t=1}^{n-1} x^{(1)}(t) \\
\sum_{t=1}^{n-1} t x^{(1)}(t) & \sum_{t=1}^{n-1} t^2 & \sum_{t=1}^{n-1} t \\
\sum_{t=1}^{n-1} x^{(1)}(t) & \sum_{t=1}^{n-1} t & n-1
\end{vmatrix}
\]  
(14)
Substituting the estimated values of the undetermined parameters $\alpha$, $\beta$ and $\gamma$ into equations (8), (9) and (10) can obtain the estimated values of parameters $a$, $b$ and $c$ as follows:

$$\hat{a} = -\ln \hat{\alpha}$$  \hspace{1cm} (18)

$$\hat{b} = \frac{\hat{\alpha} \hat{\beta}}{1 - \hat{\alpha}}$$  \hspace{1cm} (19)

$$\hat{c} = \frac{\hat{\alpha} \hat{\gamma} + \hat{\beta}}{1 - \hat{\alpha}}$$  \hspace{1cm} (20)

Finally, by substituting the estimated values of parameters $a$, $b$ and $c$ into equation (6), the simulated and predicted values of the original sequence can be obtained.

**Model accuracy test.** Assuming the original observation data sequence is

$$X^{(0)} = \left( x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n), x^{(0)}(n+1), \ldots, x^{(0)}(n+t) \right)$$  \hspace{1cm} (21)

The grey prediction model is established for the sequence consisting of the first $n$ elements of sequence $X(0)$, and the corresponding simulation sequence is

$$\hat{S}^{(0)} = \left( \hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(n) \right)$$  \hspace{1cm} (22)

The prediction sequence composed of the last $t$-step data predicted based on the grey model is
\[ \hat{F}^{(0)} = \left( \hat{x}^{(0)}(n+1), \hat{x}^{(0)}(n+2), \ldots \hat{x}^{(0)}(n+t) \right) \]  

The residual sequence of analog sequence \( \hat{S}^{(0)} \) is

\[ \varepsilon_{\hat{S}} = (\varepsilon_{\hat{S}}(1), \varepsilon_{\hat{S}}(2), \ldots, \varepsilon_{\hat{S}}(n)) \]  

\( \varepsilon_{\hat{S}}(u) = x^{(0)}(u) - \hat{x}^{(0)}(u), \quad u = 1, 2, \ldots, n \)  

The residual sequence of prediction sequence \( \hat{F}^{(0)} \) is

\[ \varepsilon_{\hat{F}} = (\varepsilon_{\hat{F}}(n+1), \varepsilon_{\hat{F}}(n+2), \ldots, \varepsilon_{\hat{F}}(n+t)) \]  

\( \varepsilon_{\hat{F}}(v) = x^{(0)}(v) - \hat{x}^{(0)}(v), \quad v = n+1, n+2, \ldots, n+t \)  

The relative analog percent error sequence of analog sequence \( \hat{S}^{(0)} \) is recorded as

\[ \Delta s = (\Delta s(1), \Delta s(2), \ldots, \Delta s(n)) \]  

\( \Delta s(u) = \left| \frac{\varepsilon_{\hat{S}}(u)}{x^{(0)}(u)} \right| \times 100\% \quad u = 1, 2, \ldots, n \)  

Then the average relative simulation percentage error of simulation sequence \( \hat{S}^{(0)} \) is

\[ \overline{\Delta s} = \frac{1}{n} \sum_{u=1}^{n} \Delta s(u) \]  

The relative prediction percentage error sequence of prediction sequence \( \hat{F}^{(0)} \) is recorded as

\[ \Delta F = (\Delta F(n+1), \Delta F(n+2), \ldots, \Delta F(n+t)) \]  

\( \Delta F(v) = \left| \frac{\varepsilon_{\hat{F}}(v)}{x^{(0)}(v)} \right| \times 100\% \quad v = n+1, n+2, \ldots, n+t \)  

Then the average relative simulation percentage error of prediction sequence \( \hat{F}^{(0)} \) is

\[ \overline{\Delta F} = \frac{1}{t} \sum_{v=n+1}^{n+t} \Delta F(v) \]  

For a given \( \alpha \) and \( \beta \), when \( \overline{\Delta s} < \alpha \) and \( \overline{\Delta F} < \beta \) hold, the grey model simulation and prediction residual are qualified.

3. Engineering Case Analysis

**Engineering situation.** The Qiandao Lake Water Distribution Project in Hangzhou draws water from Chun’an County of Qiandao Lake and leads water to Xianlin Reservoir in Yuhang District of Hangzhou City through a water conveyance tunnel. The total length of the water conveyance tunnel is 112.34km, of which the concrete lining length of the water conveyance tunnel is about 102.39km. The axis length of the water conveyance tunnel is long, and the tunnel excavation will inevitably encounter unfavorable geological conditions such as fault fracture zone, karst, water-rich soft rock stratum, etc. During the
excavation of cavern in Qiandao Lake Water Distribution Project, Class IV and V weak surrounding rocks and karst unfavorable geological conditions are common.

In this paper, a tunnel surrounding rock convergence deformation monitoring section with a depth of 290m and surrounding rock category IV is selected as the research object for analysis. The excavation of Class IV surrounding rock tunnel is carried out by full-face smooth blasting. Prior to tunnel blasting excavation, advance supporting hollow grouting anchor rods are arranged along the direction of the tunnel axis. After blasting is completed, stone slag is quickly cleaned, then steel arch frame is erected, and steel mesh is bound and fixed on the surface of fresh surrounding rock. Meanwhile, hollow grouting anchor rods of arch crown, systematic mortar anchor rods of arch waist and mortar anchor rods of arch foot are carried out. Finally, plain shotcrete is carried out to complete the whole initial supporting process of tunnel excavation. In order to effectively protect the convergence pile from flying rock damage caused by blasting, the convergence pile is usually installed when the convergence deformation monitoring section of surrounding rock is 1m away from the excavation face, and then continuous monitoring is carried out. The convergence pile is mainly composed of expansion bolts, cross reflectors and round hooks, with a length of 30cm. The depth of convergence pile embedded in the surrounding rock is more than 5cm. Surrounding rock convergence deformation monitoring instrument is GK-1610 steel rule convergence electronic reading instrument.

The arrangement of convergent piles is shown in fig. 1. X1~X5 are the design numbers of convergent piles, X1 is arranged at the arch crown, X2 and X3 are arranged at the arch shoulder and at approximately the same elevation, X4 and X5 are arranged at the arch foot and at approximately the same elevation. There are a total of 8 survey lines to be observed, namely X1-X4, X1-X5, X2-X4, X2-X5, X3-X4, X3-X5 and X4-X5. The monitoring frequency is once a day when the distance between the monitoring section and the excavated face is not more than 2 times the hole diameter, otherwise once a week.

**Grey Prediction of Tunnel Surrounding Rock Deformation.** 61 groups of observation data of surrounding rock convergence deformation have been obtained after the tunnel K4+537m section convergence pile is installed. The DNGM (1,1) grey prediction model is established according to equations (1) to (20). The estimated values of parameters and average relative simulation errors of the DNGM (1,1) model are shown in Table 1. The correlation curves of measured and predicted values of

![Fig. 1 layout for monitoring convergence deformation of initial support of class IV surrounding rock of underground tunnel](attachment:fig1.png)
surrounding rock convergence deformation with time are shown in fig. 2, and the correlation curves of measured and predicted values of surrounding rock vault settlement with time are shown in fig. 3.

Table 1. Parameter Estimation and Average Relative Simulation Error of DNGM (1,1) Model.

| Line number |  a        |  b        |  c        | The average relative simulation error (%) |
|-------------|-----------|-----------|-----------|------------------------------------------|
| x1-x4       | 0.034485  | 1.825157  | 4.460130  | 5.0                                      |
| x1-x5       | 0.001021  | 0.325072  | 1.743166  | 15.5                                     |
| x2-x4       | 0.036651  | -0.599073 | -0.218153 | 5.8                                      |
| x2-x5       | 0.096723  | 0.618205  | 1.211322  | 8.7                                      |
| x3-x4       | 0.030011  | 1.935716  | 7.085069  | 4.4                                      |
| x3-x5       | 0.002598  | 0.184469  | -0.200941 | 9.3                                      |
| x4-x5       | 0.025610  | 1.878296  | 13.529859 | 5.1                                      |
| x1          | 0.020012  | 0.583511  | -0.012408 | 19.7                                     |

![Full-section steel arch reinforcement support](image)

Fig. 2 correlation curves of measured and predicted values of surrounding rock convergence deformation with time
From Table 1, it can be seen that the average relative simulation errors of DNGM (1,1) model for X1~X5 survey lines and X1 convergent pile vault settlement are 15.5% and 19.7% respectively, which are relatively large. The average relative simulation error of DNGM (1,1) model of other survey lines is less than 10%. Comparing the measured values with the fitting values, it is found that the relative simulation errors of the 2~3 fitting values in the initial stage of the sequence are relatively large, which makes the average relative simulation errors relatively large, and the relative simulation errors of the fitting values in the middle and later stages of the sequence are relatively small. On the whole, DNGM (1,1) model can effectively predict the deformation of surrounding rock based on real-time monitoring data, and its prediction results can be used to evaluate the stability of surrounding rock support system, and can also be used as the basis for dynamic design and construction of informatization.

**Stability Analysis of Surrounding Rock and Reasonable Selection of Secondary Lining Time.**

The stress of surrounding rock is released after tunnel excavation. After the primary lining and other supporting structures play their roles, the surrounding rock and supporting structures will have a dynamic equilibrium process in a long period of time. The convergence deformation and vault settlement of surrounding rock during this period reflect this process, and the measurement results can serve as a basis for judging the stability of initial support of surrounding rock and selecting a reasonable time for secondary support.

The Code for Design of Hydraulic Tunnels [12] stipulates that the allowable relative displacement of tunnel surrounding rock shall be determined according to the type of surrounding rock, the excavation span of the cavern and the buried depth of the cavern according to Table 2. When there are a large number of cracks on the surface of surrounding rock or the measured relative displacement reaches 70% of the allowable relative displacement and the convergence deformation rate is still not significantly decreased, immediate measures must be taken to strengthen the support and modify the original design.
Table 2 Allowable Relative Displacement Value of Tunnel Surrounding Rock

| Buried depth (m) | Surrounding rock type |
|-----------------|----------------------|
|                 | III      | IV      | V       |
| <50             | 0.10     | 0.15    | 0.40    |
| 50~300          | 0.20     | 0.40    | 0.80    |
| >300            | 0.40     | 0.60    | 1.00    |

Note: 1) Relative displacement refers to the ratio between the cumulative convergence displacement and the distance between two measuring points; 2) This table is applicable to situations where the height-span ratio is 0.8~1.2, the excavation span of Class III surrounding rock is not more than 20m, the excavation span of Class IV surrounding rock is not more than 15m, and the excavation span of Class V surrounding rock is not more than 10m.

Technical Regulation for Monitoring and Measurement of Railway Tunnels [13] stipulates that the allowable relative displacement value of the initial supporting vault settlement of the tunnel shall be determined according to Table 3.

Table 3 Allowable Relative Displacement Value of Tunnel Arch Settlement

| Buried depth (m) | Surrounding rock type |
|-----------------|----------------------|
|                 | II       | III      | IV      | V       |
| <50             | /        | 0.03~0.06| 0.06~0.10| 0.08~0.16|
| 50~300          | 0.03~0.06| 0.04~0.15| 0.08~0.40| 0.14~1.10|
| >300            | 0.05~0.12| 0.12~0.30| 0.30~0.80| 0.80~1.40|

Note: 1) The hard surrounding rock tunnel takes the smaller value in the table and the soft surrounding rock tunnel takes the larger value in the table; 2) The vault settlement relative displacement value refers to the ratio of the vault settlement value minus the tunnel settlement value to the height from the original vault to the tunnel ground.

When the horizontal convergence rate of the surrounding rock is less than 0.2mm/d, the vault subsidence rate is less than 0.1mm/d, the horizontal convergence rate of the surrounding rock and the vault subsidence rate are obviously reduced, and the relative displacement value of the surrounding rock has reached more than 90% of the total relative displacement, it is considered that the deformation of the surrounding rock of the tunnel is basically stable, and secondary lining operation can be carried out [14].

The correlation curves between convergence deformation rate of surrounding rock and time are shown in fig. 4, and the correlation curves between settlement rate of surrounding rock dome and time are shown in fig. 5.

Fig. 4 Curves of Convergence Deformation Rate of Surrounding Rock Versus Time
The excavation span of the tunnel K4+537m section cavern is 7.6m, and the height-span ratio is 1.04, which meets the usual conditions in Table 2. The burial depth of the cavern is 290m, and the surrounding rock type is Class IV. The allowable value of relative displacement of surrounding rock of the tunnel is 0.4 according to Table 2, and the allowable value of vault subsidence is 0.4 according to Table 3.

As can be seen from figs. 2 to 5, the convergence displacement of X4-X5 line is the largest, the convergence displacement of X3-X4 line and X1-X4 line is the second, and the arch foot shows the trend of extrusion deformation into the tunnel. The convergence displacement of X1-X5 line is larger than that of X3-X5 line and X2-X5 line, but much smaller than that of X4-X5 line, X3-X4 line and X1-X4 line. The convergence displacement of X2-X4 survey line is negative, and the X4 convergence pile at the arch foot may have partial settlement deformation. On the 7th day after the convergence pile was buried, the relative displacement value of X4-X5 measuring line at the arch foot reached 70% of the allowable relative displacement and the convergence deformation rate still did not drop significantly. At this time, the convergence deformation rate of surrounding rock was 2.3 mm/d. It is suggested to immediately adopt a secondary reinforcement and support scheme of adding transverse support beams at the bottom of the existing steel arch frame to form full-section steel arch frame support. After the scheme is constructed, the convergence deformation rate of surrounding rock is obviously reduced. On the 30th, the convergence deformation rate of the surrounding rock decreased to about 0.5 mm/d, and on the 43rd, the convergence deformation rate of the surrounding rock decreased to below 0.2 mm/d, the vault subsidence rate was less than 0.1 mm/d. When the relative displacement of the surrounding rock is expected to reach more than 90% of the total relative displacement, the deformation of the tunnel surrounding rock is basically stable and secondary lining operation can be carried out.

4. Conclusion
(1) Based on the on-site monitoring results of the diversion tunnel of Qiandao Lake Water Distribution Project in Hangzhou City, the DNGM (1,1) model for predicting the convergence deformation of tunnel surrounding rock and vault settlement is established by applying grey theory. The results show that DNGM (1,1) model can effectively predict the deformation of surrounding rock based on real-time monitoring data. The prediction results can be used to evaluate the stability of surrounding rock support system, and can also be used as the basis for dynamic design and construction of informatization, which has high theoretical value and engineering practice significance.

(2) According to the field measured values and grey prediction results of DNGM (1,1) model, the deformation stability of surrounding rock of underground tunnel is analyzed from two aspects of relative displacement and deformation rate, and the secondary support time is reasonably determined. The analysis results can provide reference for the design, construction and monitoring of similar projects.
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