Servo-controlling structure of five-axis CNC system for real-time NURBS interpolating

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Abstract. NURBS (Non-Uniform Rational B-Spline) is widely used in CAD/CAM (Computer-Aided Design / Computer-Aided Manufacturing) to represent sculptured curves or surfaces. In this paper, we develop a 5-axis NURBS real-time interpolator and realize it in our developing CNC (Computer Numerical Control) system. At first, we use two NURBS curves to represent tool-tip and tool-axis path respectively. According to feedrate and Taylor series extension, servo-controlling signals of 5 axes are obtained for each interpolating cycle. Then, generation procedure of NC (Numerical Control) code with the presented method is introduced and the method how to integrate the interpolator into our developing CNC system is given. And also, the servo-controlling structure of the CNC system is introduced. Through the illustration, it has been indicated that the proposed method can enhance the machining accuracy and the spline interpolator is feasible for 5-axis CNC system.

1. Introduction

Many commercial CAD systems currently provide their correspondents the capability of defining sculptured surfaces. Among the mathematical techniques used for designing specific surfaces, the NURBS is one that attracts a lot of attention[1~3]. NURBS method offers a common mathematical form for describing both standard analytical shapes and free-form surfaces[3].

In order to get a part with sculptured surfaces, a CAD system is firstly chosen to design the part with NURBS method, and then a CAM system is used to calculate tool paths for cutting the part[4]. The tool paths are typically approximated with piecewise line or circular segments. As a result, the corresponding NC codes are generated and the motion controller can be applied to guide the CNC machines to execute accurately cutting[5].

However, the above approximation approach may result in several problems such as the large size of the CNC program, discontinuity of velocity and acceleration and so on[6,7].

In order to overcome the disadvantages of conventional CNC machining methods and also keep the advantages of CAD systems (use NURBS to represent curves), the need to develop general real-time NURBS command generators for CNC machines cannot be overlooked[8]. In addition, the high-speed and high-accuracy controlled machining of parts with NURBS surfaces is generally finished in 5-axis coordinated CNC machine tools[9]. Consequently, the CNC system should be provided with the ability to directly real-time generate 5-axis servo-controlling signals to drive the cutting tool along the NURBS curve relative the part.
The rest of the paper is organized in the following way. Section 2 describes how a NURBS curve is represented in a CAD system. Section 3 introduces the generating process of NC code. Section 4 shows the data structures of NC translators. Section 5 presents the realization method of 5-axis spline interpolator. Section 6 demonstrates a real example to validate the proposed method. Section 7 concludes the paper.

2. NURBS curve representation
Most CAD/CAM systems use parametric forms to represent curves or surfaces. The general form of a parametric curve in three-dimensional (3D) space can be expressed, in terms of a free parameter \( 0 \leq u \leq 1 \), as
\[
C(u) = x(u)i + y(u)j + z(u)k
\]
where \( u \) is an arbitrary parameter, \( i, j, k \) is the unit vector of axis orientation of \( x, y, z \) respectively.
A \( p \)-degree NURBS curve is defined as follows
\[
C(u) = \left[ x(u) \ y(u) \ z(u) \right]^T = \frac{\sum_{i=0}^{n} N_{i,p}(u)WP_i}{\sum_{i=0}^{n} \sum_{i=0}^{n} N_{i,p}(u)Wi} \quad 0 \leq u \leq 1 \tag{2}
\]
Among which \( P_i \) is the \( i \)-th 3-D control point; \( W_i \) is the corresponding weight factor of \( P_i \); \( (n+1) \) is the number of control points. \( N_{i,p}(u) \) is the so-called normalized B-spline basis function, where \( p \) is its degree, and can be calculated by the following recursive formula
\[
N_{i,p}(u) = \begin{cases} 
1 & (u_i \leq u < u_{i+1}) \\
0 & \text{(otherwise)} 
\end{cases} \]
\[
N_{i,p}(u) = \frac{u-u_i}{u_{i+p}-u_{i}} N_{i,p-1}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}} N_{i+1,p-1}(u) \\
0 = 0 \quad \text{(prescribed)}
\]

3. Tool paths and NC code generation
General CAD/CAM software such as Unigraphics cannot generate NC code including the introduced 5-axis coordinated double NURBS spline interpolation format, thus the CAD/CAM module needs to be developed by ourselves. The flow chart of generating NC code by the CAD/CAM module is shown as Figure 1.
The CAD part in the module supplies interfaces connecting other CAD/CAM software to make full use of the geometry modeling functions of excellent CAD/CAM softwares. After pre-process procedure such as tool path generation, interference checking, the CAM part in the module calculates the cutter center point coordinates and the corresponding tool-axis unit vectors, which are then fitted into cutter center point spline and orientation spline in the post-process procedure and the NC code including the double NURBS spline curve interpolation format is generated and output.

4. Realization of the translator

The modular structure of the developing CNC system is shown as Figure 2, among which, TaskGenerator module is realized in Microsoft’s COM technology and is responsible for the translation of NC programs, and the AxisGroup module is realized in VenturCom’s RTDLL (real-time dynamical link library) technology and is responsible for the real-time interpolation and other real-time tasks.

It is to be integrated a 5-axis NURBS translator to the CNC system. Modules need to be changed are only TaskGenerator module and AxisGroup module. Data structures (described in C++) used in TaskGenerator module is given as follows:

```cpp
typedef enum {LINE, DNURBS} SingleStep_TYPE;
typedef struct dnurbs_struct{
    double start_rate, traverse_rate, end_rate;
    int p;
    double r[20], kv[26];
    double clx[20], cly[20], clz[20];
} dnurbs;
typedef struct singleStep_struct{
    SingleStep_TYPE singleStep_type;
    line line_struct;
    dnurbs dnurbs_struct;
} singleStep;
typedef std::deque<singleStep> singleStep_deque;
```

The dnurbs struct is added in the module for the translation of 5-axis NURBS code. The information extracted from each motion segment after translation is put into singleStep_deque and is transferred into the AxisGroup module for interpolation.

5. Real-time 5-axis spline interpolation

Footnotes should be avoided whenever possible. If required they should be used only for brief notes that do not fit conveniently into the text.

The task of real-time NURBS spline interpolation is to calculate next interpolation period’s CL data that including tool-tip location coordination and tool-axis orientation and then transfer the CL data into machine tool’s motion commands such as X,Y,Z,A,C through post-processing.

5.1. Real-time calculation of CL data

![Figure 3. Principle of 5-axis NURBS interpolation.](image-url)
As shown in Figure 3, $C_0(u)$ and $C_1(u)$ is the cutter center point spline and the orientation spline, respectively. The two splines are constructed in the same knot vector. Assumed that parameter $u$ is function of time $t$, that is, $u = u(t)$. By using Taylor’s expansion of the parameter $u$ with respect to time $t$ to obtain the first order approximation interpolation algorithm, the first order approximation up to the first derivatives is

$$u_{k+1} = u_k + T_s \frac{du}{dt} \bigg|_{t=t_k}$$  \hspace{1cm} (3)

where $T_s$ is interpolation period, $u_k$ and $u_{k+1}$ are corresponding parameters of current and next time $t_k$ and $t_{k+1}$. The feedrate of tool-tip point along $C_0(u)$ is defined by

$$V(u) = \frac{dC_0(u)}{dt} = \left( \frac{dC_0(u)}{du} \right) \left( \frac{du}{dt} \right)$$  \hspace{1cm} (4)

and the curve speed at the parameter $u_k$ is

$$V(u_k) = \left\| \frac{dC_0(u)}{dt} \right\|_{u=u_k}$$  \hspace{1cm} (5)

According to equations (4) and (5), the first derivative of $u$ with $t$ is obtained as

$$\left. \frac{du}{dt} \right|_{t=t_k} = \frac{V(u_k)}{\left\| \frac{dC_0(u)}{du} \right\|_{u=u_k}}$$  \hspace{1cm} (6)

Therefore, the first-order interpolation algorithm is obtained by substituting equation (6) into equation (3), and the latter can be expressed as follows

$$u_{k+1} = u_k + TV(u_k)$$  \hspace{1cm} (7)

The first derivative of $C_0(u)$ with $u$ is obtained as

$$\frac{dC_0(u)}{du} = \sum_{i=0}^{n} N_i^p(u)W_iP_i - \sum_{i=0}^{n} N_i^p(u)W_i \sum_{j=0}^{n} N_j^p(u)W_jP_i$$  \hspace{1cm} (8)

where the general algorithm for $1^{st}$ order derivative of $N_i^p(u)$ is

$$N_i^p(u) = p \left[ \frac{N_{i,p-1}(u)}{u_{r,p-1} - u_i} - \frac{N_{i+1,p-1}(u)}{u_{r+1,p-1} - u_{i+1}} \right]$$  \hspace{1cm} (9)

$C_0(u_{k+1})$ and $C_1(u_{k+1})$ are obtained by substituting the calculated $u_{k+1}$ into $C_0(u)$ and $C_1(u)$.

Suppose the tool-axis unit vector is $T_{k+1}$, so

$$T_{k+1} = \frac{C_1(u_{k+1}) - C_0(u_{k+1})}{\left\| C_1(u_{k+1}) - C_0(u_{k+1}) \right\|}$$  \hspace{1cm} (10)

5.2. Real-time post-procession of CL data

To here, we have the following CL data:

$$C_0(u_{k+1}) = (\alpha_x, \alpha_y, \alpha_z) \hspace{1cm} T_{k+1} = (t_x, t_y, t_z)$$

For 5-axis machine of table tilting/rotating type (Figure 4), the following post-processing method is presented to transfer the CL data into the machine’s motion commands in real-time.
Internal telegram buffer
SoftSERCANS card
(shared memory)

Double NURBS
curve Interpolator
Shared memory
Double NURBS
curve translator

CAD/CAM data

TaskGenerator
module

AxisGroup
module

Axis
module

Driver X
CNC
Driver Y
Driver Z
Driver A

IO module
Driver C
Motor
Motor
Motor
Motor
Motor

Sercos fiber
loop

Figure 4. A type of 5-axis machine tool.

Figure 5. Servo-controlling structure of CNC system.

(a) if \( t_x = t_y = 0 \), \( t_z = 1 \), then \( A' = 0 \), \( C' = 0 \)

(b) else if \( t_x = 0 \), \( t_y \neq 0 \), then \( A' = \arcsin(-t_y) \), \( C' = 0 \)

(c) else if \( t_y = 0 \), \( t_z \neq 0 \), then \( A' = \arcsin(t_z) \), \( C' = \pi / 2 \)

(d) else

\[
A' = \arccos(t_z)
\]

\[
C' = \begin{cases} 
\arccos(-t_y / \sin A') & \text{if } t_z > 0 \\
2\pi - \arccos(-t_y / \sin A') & \text{if } t_z < 0
\end{cases}
\]

5.3. Servo-controlling structure

The proposed format’s double NURBS NC code is generated in CAD/CAM and disposed by the translator. Then the obtained data is sent to the interpolator in which it is disposed according to the given interpolation period and thereafter, the calculated target position is treated with control law. For the CNC system adopts SoftSERCANS technology (IEC61491-SERCOS technology’s software realization, as shown in Figure 2), the final data is put into an internal telegram buffer (a block of shared memory), and the SoftSERCANS card exchanges data with it. Then the data telegram containing moving instructions is sent to drivers by SoftSERCANS through fiber optic ring and thus motors are driven. The controlling structure is shown as Figure 5.

6. Example and realization

The 5-axis NURBS NC code is shown as follows:

\[
\begin{align*}
N30 & \text{ G06.5 P3} \\
X & 47.7773 Y 71.0202 Z 58.8006 U 47.8066 V 71.9791 W 59.0828 R 1 K 0 F 300 \\
X & 44.3229 Y 75.6066 Z 54.4778 U 44.3651 V 76.5494 W 54.8138 \\
X & 37.2164 Y 84.6443 Z 44.5755 U 37.2554 V 85.5421 W 45.8138 \\
X & 28.7567 Y 101.5653 Z 29.7151 U 28.7081 V 102.3535 W 30.3361 \\
X & 1.6963 Y 163.6796 Z 5.8369 U 1.8572 V 163.9906 W 6.7762 K 0.5046 K 1
\end{align*}
\]

It’s obvious the proposed method can greatly reduce the size of NC files; especially when in high precision machining, the size change will be more obvious. Meanwhile, the CNC system’s pre-reading one part program segment equals pre-reading tens or even hundreds of linear interpolation segments; thus, the “prospective property” of CNC system can be solved easily.
In Figure 6, the curve inflection point is near the parameter 0.42, where its curvature is the smallest. Compared with traditional linear approximation machining mode, the proposed method can greatly ameliorate the machining performance, improve machining accuracy and the machining accuracy is far less influenced by the curve curvature change.

![Double NURBS Interpolation vs 5-Axis Linear Interpolation](image)

**Figure 6.** Error comparison of two methods.

The proposed method’s real-time property is tested in a computer (Pentium IV 2.4GHz, RAM 512M) and the result is shown in Figure 7, the calculation is based on the second order approximation algorithm and the beginning of a time segment is the calculation of the basis function and the end is the finish of each axis’ position calculation. It can be found the average run time in a double NURBS interpolation period is near 28μs and the time expense occupies a small proportion in a CNC system of which the interpolation period is 1 ms or even 500μs.

**Figure 7.** Run time of the proposed method.

7. Conclusions
On the basis of analyzing the defects of the existing 5-axis linear interpolation method used in the machining of sculptured surface, 3-axis coordinated NURBS interpolation format is extended and a spline expressing the tool-axis orientation is added in the paper, and double NURBS curve interpolation method is put forward, the NC code generating method is introduced and the method to realize the interpolation function in an open CNC system developed by ourselves is given. Therefore, the proposed double NURBS interpolation method can not only be used in the real CNC system, but also have prospective future.

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