Quantization of $N = 1$ chiral/nonminimal (CNM) scalar multiplets and supersymmetric Yang–Mills theories

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ABSTRACT

We give the superfield quantization of chiral/nonminimal (CNM) scalar multiplets defined by pairs of $N = 1$ chiral and complex linear scalar superfields kinematically coupled. In the pure massive case we develop the covariant quantization when CNM multiplets are coupled to background gauge superfields. Furthermore, we study some properties of $N = 1$ supersymmetric Yang–Mills theories constructed using CNM scalar matter superfields. In particular, we compute the one–loop contribution to the effective action for the matter superfields, we study the analogue of the Konishi anomaly and discuss some properties of the glueball superpotential.

PACS: 11.15.-q, 11.30.Pb, 12.60.Jv
Keywords: Supersymmetry, Nonminimal scalar multiplets, Complex linear superfield, Supersymmetric gauge theories.
1 Introduction

Generally, the $N = 1$ irreducible scalar chiral multiplet, defined by the superfield $\Phi$ satisfying $D_\alpha \Phi = 0$, is used to construct four-dimensional supersymmetric models having matter contents given by $N = 1$ scalar multiplets. However, other less studied nonminimal off-shell representations of the $N = 1$ scalar multiplet can be found in the literature [1, 2, 3]. While all these representations describe the dynamics of spin $(0,1/2)$ physical fields [3, 4], they differ from the chiral scalar multiplet in the auxiliary fields content. In particular, among these nonminimal multiplets the complex linear superfield presents a number of interesting properties.

The complex linear superfield, defined by the kinematic constraint $\bar{D}^2 \Sigma = 0$, appears in various contexts in the superspace description of supersymmetric field theories. It is present, for example, as a conformal compensator in different formulations of supergravity [5], and naturally appears in the context of $N=2$ off-shell supersymmetric sigma models [6]. Furthermore, in contrast with other nonminimal scalar multiplet representations, it can be easily coupled to Yang–Mills fields [3, 9]. The massless complex linear superfield also possesses interesting properties of duality with the massless chiral superfield [1, 3, 8, 9].

In the literature, the Dirac spinor is usually embedded in $N = 1$ SUSY theories using a pair of chiral multiplets. However, an alternative realization of a Dirac spinor in $N = 1$ SUSY theories makes use of one chiral and one complex linear superfield [3, 4], and this kind of construction differs from the pure chiral case in some interesting aspects. For example, it was observed in [4] that the formulation of the Dirac spinors using chiral and complex linear multiplets gives gauge group transformation properties for the Dirac spinors which are holomorphic vector–like. Moreover, it was also seen that the chiral/nonminimal Dirac spinor could provide a solution to the propagation of auxiliary fields in $N = 1$ supersymmetric extensions of the low energy effective QCD actions, a problem which arises naturally in the formulation in terms of chiral multiplets. This kind of models also provide a new way of realizing parity violation [4]. Therefore, the complex linear superfield could be a relevant tool in the formulation of phenomenological $N = 1$ supersymmetric models [4].

To define a consistent supersymmetric mass term it is possible to build a SUSY model for a chiral superfield ($\Phi$) and a complex linear superfield ($\Sigma$) coupled through a modification of the complex linear superfield kinematic definition: $\bar{D}^2 \Sigma = Q(\Phi) = m \Phi + \Phi \bar{P}(\Phi)$ [3]. This coupling gives the same mass $m$ to the two multiplets and produces a nontrivial interaction. Therefore, the complex linear multiplet can acquire a mass $m$ “in tandem” with a chiral multiplet through the previous definition [3]. Moreover, this kinematic constraint does not break the natural Dirac spinor construction of the two superfields [3]. Following [4], we call these models chiral/nonminimal (CNM) models.

In order to study some properties of CNM models in this letter we quantize the CNM superfields in superspace with generic $Q(\Phi)$. To do this we generalize to the present case the known quantization techniques for the massless chiral [1] and complex linear [7, 8] cases using unconstrained superfields solving the kinematic constraints. When the mass parameter is strictly $m \neq 0$ and the CNM multiplets are coupled to background gauge fields, we build the covariant quantization.

Once given the quantization, we then construct $N = 1$ Super–Yang–Mills theories with CNM matter superfields. In particular, for the CNM multiplet we consider the simplest case $\bar{D}^2 \Sigma = m \Phi$. Taking into account the propagation of both the matter and SYM vector superfields, we compute the one–loop contribution to the effective action for the matter superfields $\Sigma$ and $\Phi$.

\[2\text{A different way to give mass to the complex linear superfield without introducing chirals is discussed in [10].}\]
Using the covariant formalism we then derive the analogue of the Konishi anomaly [13] in CNM theories finding, as argued in [4], that the CNM theory is anomaly free.

The letter is organized as follow: In section 2 we give a brief description of CNM models. In section 3 we develop the quantization of CNM theories, while in section 4 we calculate the one–loop contribution to the effective action for the matter fields $\Phi$ and $\Sigma$ in SYM theories with CNM matter superfields. In section 5 we first discuss the covariant quantization of CNM multiplet coupled to background gauge fields and then apply the covariant formalism to the study of the Konishi anomaly. In section 6 we discuss some properties of the glueball superpotential in CNM SYM theories. In section 7 we present some final remarks concerning the dynamics of CNM models.

For the conventions adopted see reference [1].

2 Chiral/nonminimal (CNM) scalar models

In this section we introduce models built through $N = 1$ chiral and nonminimal scalar multiplets [2, 1] and we consider the possibility of coupling these multiplets in accordance with [3]. In particular, using the $N = 1$ superspace formalism [1], we consider a chiral superfield $\Phi$ satisfying

$$\bar{D}^{\dot{\alpha}} \Phi = 0$$

and, for the nonminimal scalar multiplet, take a complex linear superfield $\Sigma$ satisfying

$$\bar{D}^2 \Sigma = 0$$

Separately the two multiplets have the kinetic actions

$$S_C = \int d^4x d^4\theta \, \bar{\Phi} \Phi , \quad S_{NM} = - \int d^4x d^4\theta \, \Sigma \Sigma . \quad (1)$$

In components these have the form [2, 1]

$$S_C = \int d^4x [\bar{A} \square A - \bar{\psi}^{\dot{\alpha}} i \partial_{\dot{a}\dot{\alpha}} \psi^\alpha + \bar{F} F ] , \quad (2)$$

$$S_{NM} = \int d^4x [\bar{B} \square B - \bar{\zeta}^{\dot{\alpha}} i \partial_{\dot{a}\dot{\alpha}} \zeta^\alpha - \bar{H} H + \beta^\alpha \rho_\alpha + \bar{\rho}^{\dot{\alpha}} \bar{\beta}_{\dot{\alpha}} - \bar{\rho}^{\dot{\alpha}} \rho_{\dot{a} \dot{\alpha}} ] . \quad (3)$$

From these expressions it is possible to see that both actions describe the free dynamics of two $N = 1$ scalar multiplets with physical fields given by $(A, \psi_\alpha)$ for the chiral scalar multiplet, and by $(B, \zeta_\alpha)$ for the complex linear multiplet. Clearly, the two multiplets have different auxiliary field contents.

It is possible to introduce interaction terms between these multiplets described by

$$S_{int} = \int d^4x d^4\theta \, K(\Phi, \bar{\Phi}, \Sigma, \bar{\Sigma}) + \left\{ \int d^4x d^2\theta \, W(\Phi) + h.c. \right\} , \quad (4)$$

where $K$ is the Kähler potential (at least cubic) and $W$ is a holomorphic function of the chiral superfield $\Phi$ only.

There is also the possibility to introduce a mass term and a nontrivial interaction between the two multiplets $\Phi$ and $\Sigma$ by modifying the kinematic constraints for the superfields as [3]

$$\bar{D}^{\dot{\alpha}} \Phi = 0 \quad , \quad \bar{D}^2 \Sigma = Q(\Phi) \quad ,$$

$$D_\alpha \bar{\Phi} = 0 \quad , \quad D^2 \bar{\Sigma} = \bar{Q}(\bar{\Phi}) \quad , \quad (5)$$

where $Q(\Phi)$ is a holomorphic function of the chiral superfield $\Phi$. In this letter we consider $Q(\Phi)$ of the form

$$Q(\Phi) \equiv \Phi \left[ m + \bar{P}(\Phi) \right] , \quad (6)$$
where $\bar{P}(\Phi)$ is polynomial in $\Phi$ at least linear.

The simplest action for the CNM models [3] is then

$$S_{\text{CNM}} = \int d^4x d^4\theta \left[ \Phi \Phi - \Sigma \Sigma \right].$$

(7)

Due to the constraints (5), in components (7) takes the nontrivial form [3]

$$S_{\text{CNM}} = \int d^4x \left[ \bar{B} \Box B + \bar{A} \Box A - \bar{\zeta}^\alpha i \partial_{\alpha \dot{\alpha}} \zeta^\alpha - \bar{\psi}^\alpha i \partial_{\alpha \dot{\alpha}} \psi^\alpha + \bar{F} F - \bar{H} H + \beta^\alpha \rho_\alpha + \bar{\rho}^\alpha \bar{\beta}_\alpha + -\bar{p}^\alpha p_{\alpha \dot{\alpha}} - Q(\bar{A})Q(A) - \left\{ [B(Q(\bar{A})\bar{F} + \frac{1}{2} \bar{Q}''(\bar{A}) \bar{\psi}^\alpha \bar{\psi}_{\dot{\alpha}}) + Q'(A)\zeta^\alpha \psi_\alpha] + \text{h.c.} \right\} \right],$$

(8)

with $Q'(A) = \frac{\partial Q}{\partial A}$, $Q''(A) = \frac{\partial^2 Q}{\partial A^2}$. In the particular case $Q(\Phi) = m\Phi$ we have

$$S_{\text{CNM}}^0 = \int d^4x [\bar{B} \Box B + \bar{A} \Box A - \bar{\zeta}^\alpha i \partial_{\alpha \dot{\alpha}} \zeta^\alpha - \bar{\psi}^\alpha i \partial_{\alpha \dot{\alpha}} \psi^\alpha + \bar{F} F - \bar{H} H + \beta^\alpha \rho_\alpha + \bar{\rho}^\alpha \bar{\beta}_\alpha + -\bar{p}^\alpha p_{\alpha \dot{\alpha}} - |m|^2 \bar{A} A - \left\{ (\sqrt{m}B \bar{F} + m\zeta^\alpha \psi_\alpha) + \text{h.c.} \right\},$$

(9)

which, after integration of the auxiliary fields, gives

$$S_{\text{CNM}}^0 = \int d^4x [\bar{B}(\Box - |m|^2)B + \bar{A}(\Box - |m|^2)A + \frac{1}{2} \bar{\Psi}_{\text{CNM}}(i\gamma^\mu \partial_\mu - m)\Psi_{\text{CNM}}].$$

(10)

Here $\Psi_{\text{CNM}}$ is the Dirac spinor

$$\Psi_{\text{CNM}} \equiv \left( \frac{\psi_\alpha}{\zeta^\alpha} \right) = \left( \frac{D_\alpha \Phi}{\bar{D}^\dot{\alpha} \Sigma} \right).$$

(11)

From (10) it follows that the chiral/nonminimal multiplets acquire the same mass. Thus they naturally define Dirac spinors in $N = 1$ supersymmetric theories [3]. This interesting property was considered in [4] in order to study $N = 1$ supersymmetric extensions of QCD effective actions, and it is one of the main reasons to consider Super–Yang–Mills theories with CNM matter multiplets. In fact, it is not difficult to extend the previous construction to CNM scalar multiplets minimally coupled to gauge multiplets; we need only to replace the superspace covariant derivatives with derivatives $\nabla_A \equiv (\nabla_\alpha, \nabla_\dot{\alpha}, \nabla_{\alpha \dot{\alpha}})$ covariant under supersymmetry and gauge transformations [1, 3].

To conclude this section we consider the possibility of also introducing in (9) the quadratic chiral mass term $-\frac{m'}{2} \int d^4x d^4\theta \Phi^2 + \text{h.c.} = -\int d^4x \{ m'(A\Phi + \psi^2) + \text{h.c.} \}$, typical of pure chiral theories. Integrating out the auxiliary fields the resulting action is

$$\int d^4x [\bar{B} \Box B + \bar{A} \Box A - \bar{\zeta}^\alpha i \partial_{\alpha \dot{\alpha}} \zeta^\alpha - \bar{\psi}^\alpha i \partial_{\alpha \dot{\alpha}} \psi^\alpha - (|m|^2 + |m'|^2)\bar{A} A - |m|^2 \bar{B} B - m \bar{m} AB + -\bar{m} m \bar{A} B - m\zeta^\alpha \psi_\alpha - m' \psi^2 - \bar{m} \bar{\zeta}^\alpha \bar{\psi}_\alpha - \bar{m'} \bar{\psi}^2].$$

(12)

The bosonic mass matrix is not diagonal in this case. It is possible to diagonalize the bosonic field equations by a unitary constant bosonic field redefinition

$$\begin{pmatrix} B \\ \bar{A} \end{pmatrix} = \begin{pmatrix} \frac{|m|}{\sqrt{|m|^2 + |m'|^2}} & \frac{|m|}{\sqrt{|m|^2 + |m'|^2}} \\ \frac{|\bar{m}|(\bar{m}_1 - |m|^2)}{\sqrt{|m|^2 + |m'|^2}} & \frac{|\bar{m}|(\bar{m}_2 - |m|^2)}{\sqrt{|m|^2 + |m'|^2}} \end{pmatrix} \begin{pmatrix} \bar{B} \\ A \end{pmatrix},$$

$$|\bar{m}_{1,2}|^2 = |m|^2 + \frac{|m'|}{2} \left( |m'| \pm \sqrt{|m'|^2 + 4|m|^2} \right),$$

(13)
\[|m_1,2|^2 \text{ being the eigenvalues of the bosonic mass matrix. Therefore (12) can be written as} \]
\[
\int d^4x \left[ \tilde{B}(\Box - |m_1|^2) \tilde{B} + \tilde{A}(\Box - |m_2|^2) \tilde{A} + \frac{1}{2} \Psi_{CNM} (i \gamma^\mu \partial_{\mu} - m) \Psi_{CNM} + \left[ - \frac{m'}{2} \Psi_{CNM} \left( \frac{1 + \gamma^5}{2} \right) \Psi_{CNM} + \text{h.c.} \right] \right], \tag{14}\]

where \( \Psi_{CNM} = \left( \begin{array}{c} \zeta_{\alpha} \\ \bar{\phi}_{\dot{\alpha}} \end{array} \right) = \mathcal{C} \Psi_{CNM}^c \) is the charge conjugate spinor of \( \Psi_{CNM} \) (11) being \( \mathcal{C} \) the charge conjugation matrix [1, 10]. From (14) we observe that the resulting fermion mass matrix breaks the Dirac spinor construction. Therefore, in this letter we assume \( m' = 0 \) and focus only on actions where the chiral interaction potential \( [\int d^4z \mathcal{W}(\Phi) + \text{h.c.}] \) is at least cubic in the chiral superfields.

3 Quantization of CNM models with generic coupling

We consider the action \( (7) \) with a pair of superfields \( \Phi \) and \( \Sigma \) satisfying the kinematic constraints \( (5) \). We also assume in the action an interaction term that is local and at least cubic in the fields \( \Phi, \bar{\Phi}, \Sigma \) and \( \bar{\Sigma} \) as given in \( (4) \).

Our goal is to develop the superspace quantization of the model for a generic potential \( Q(\Phi) \) as in \( (6) \) generalizing the quantization procedure of the massless complex linear superfield \([7, 8]\).

Since, unlike the chiral scalar superfield [1], an explicit formulation of the functional differentiation and integration for the superfields \( \Sigma \) and \( \bar{\Sigma} \) is not known, we solve the kinematic constraints which define \( \Phi \) and \( \Sigma \) through two unconstrained superfields \( \chi \) and \( \sigma_\alpha \)

\[
\Phi \equiv \bar{D}^2 \chi, \quad \Sigma = \bar{D}^\alpha \sigma_\alpha + m \chi + \bar{\chi} \bar{P}(\bar{D}^2 \chi), \tag{15}\]
\[
\bar{\Phi} \equiv D^2 \bar{\chi}, \quad \bar{\Sigma} = D^\alpha \sigma_\alpha + m \bar{\chi} + \chi \bar{P}(D^2 \bar{\chi}). \tag{16}\]

In terms of \( \chi, \bar{\chi}, \sigma_\alpha \) and \( \bar{\sigma}_{\dot{\alpha}} \) superfields action \( (7) \) reads

\[
S_{CNM} = \int d^4xd^4\theta \left[ (D^2 \bar{\chi})(\bar{D}^2 \chi) - \left( D^\alpha \sigma_\alpha + m \bar{\chi} + \chi \bar{P}(D^2 \bar{\chi}) \right) \left( \bar{D}^\dot{\alpha} \bar{\sigma}_{\dot{\alpha}} + m \chi + \bar{\chi} \bar{P}(D^2 \bar{\chi}) \right) \right]. \tag{17}\]

Once the kinematic constraints have been solved, varying the superfields \( \chi, \bar{\chi}, \sigma_\alpha \) and \( \bar{\sigma}_{\dot{\alpha}} \) in the action \( S_{CNM} + S_{\text{int}} \), we obtain the classical equations of motion

\[
\bar{D}_{\dot{\alpha}} \left[ -\Sigma + \frac{\partial K(\Phi, \bar{\Phi}, \Sigma, \bar{\Sigma})}{\partial \Sigma} \right] = 0, \quad D_\alpha \left[ -\Sigma + \frac{\partial K(\Phi, \bar{\Phi}, \Sigma, \bar{\Sigma})}{\partial \Sigma} \right] = 0, \tag{18}\]

From these equations it follows that on–shell \( -\Sigma + \frac{\partial K(\Phi, \bar{\Phi}, \Sigma, \bar{\Sigma})}{\partial \Sigma} \) defines a class of composite chiral operators. In particular, if \( \frac{\partial K(\Phi, \bar{\Phi}, \Sigma, \bar{\Sigma})}{\partial \Sigma} \equiv 0 \), the \( \Sigma \) (\( \Sigma \)) superfield becomes on–shell chiral (antichiral).
The quadratic part of action (17) is

\[ S_{CNM}^0 = \int d^8 z \begin{pmatrix} \bar{\chi}, \sigma_\alpha \end{pmatrix} \begin{pmatrix} \begin{pmatrix} D^2 \bar{D}^2 - |m|^2 \\ -m D^\alpha \bar{D}^\dot{\alpha} \end{pmatrix} \begin{pmatrix} \chi \\ \sigma_{\dot{\alpha}} \end{pmatrix} \end{pmatrix}. \] (19)

The kinetic operator is not invertible, since \( \chi, \bar{\chi}, \sigma_\alpha \) and \( \bar{\sigma}_{\dot{\alpha}} \) in (15, 16) are defined up to two sets of gauge transformations which leave \( \Phi, \bar{\Phi}, \Sigma, \bar{\Sigma} \) invariant. The first set of invariances is associated with the solution of the constraint \( \bar{D}^\dot{\alpha} \dot{\chi} = 0 \). It is given by

\[ \delta \chi = \bar{D}^\dot{\alpha} \bar{\chi} , \quad \delta \bar{\sigma}_{\dot{\alpha}} = -\bar{\chi} [m + \bar{P}(\Phi)] . \] (20)

The second set of invariances is

\[ \begin{align*}
\delta \chi &= 0 , \\
\delta \sigma_\alpha &= D_\beta \sigma^{(\beta \alpha)} , \\
\delta \bar{\sigma}^{(\beta \dot{\alpha})} &= D_\gamma \sigma^{(\gamma \beta \dot{\alpha})} , \\
\delta \sigma^{(\gamma \beta \dot{\alpha})} &= D_\delta \sigma^{(\delta \gamma \beta \dot{\alpha})} , \\
&\vdots \\
\delta \sigma^{(\alpha \alpha_{n-1} \cdots \alpha_1)} &= D_{\alpha_{n+1}} \sigma^{(\alpha_{n+1} \alpha_n \alpha_{n-1} \cdots \alpha_1)} , \\
&\vdots 
\end{align*} \] (21)

The \( \sigma^\alpha \) part of (21) was studied in [7, 8] where the quantization of the massless complex linear superfield was performed.

We can gauge–fix these invariances in two steps. First we consider the transformations (20). We use the well known gauge–fixing procedure used for the massless scalar chiral superfield [1]. This amounts to adding a gauge–fixing term [1] which brings the operator \( D^2 \bar{D}^2 \) to \( \Box \), and then, in our case, \( (D^2 \bar{D}^2 - |m|^2) \) to \( (\Box - |m|^2) \) invertible also for \( m = 0 \). Furthermore, the ghost fields introduced by this gauge–fixing completely decouple from the physical fields [1].

As a second step we consider the transformations (21). Since \( \chi \) does not transform, we can use the gauge–fixing procedure described in [7, 8] for the case of a pure complex linear superfield. This procedure makes the operator \( D^\alpha \bar{D}^\dot{\alpha} \) invertible. The gauge–fixing is developed by introducing an infinite tower of ghosts [7] according to a superspace version of the Batalin–Vilkovisky formalism. Furthermore, in [8] it was proved that the tower of ghosts can be completely decoupled from the \( \sigma_\alpha \) and \( \bar{\sigma}_{\dot{\alpha}} \) fields by a finite number of ghost fields redefinitions. The same procedure can be applied without modifications to our case. The net result is the conversion of the operator \( D^\alpha \bar{D}^\dot{\alpha} \) into the invertible operator \( W^{\alpha \dot{\alpha}} \), the explicit expression of which was given in [8].

What is important is that at the end of the two gauge–fixing procedures the ghosts introduced completely decouple from \( \sigma_\alpha, \bar{\sigma}_{\dot{\alpha}}, \chi \) and \( \bar{\chi} \), while the interaction terms are not modified. The gauge–fixed kinetic action is then

\[ S_{CNM}^0 + S_{GF}^{tot} = \int d^4 x d^4 \theta \begin{pmatrix} \bar{\chi}, \sigma_\alpha \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \Box - |m|^2 \\ -m D^\alpha \end{pmatrix} \begin{pmatrix} \chi \\ \sigma_{\dot{\alpha}} \end{pmatrix} \end{pmatrix}. \] (22)
Since the inverse of $W^a\bar{a}$ is [8]
\[
W^{-1}_{a\bar{a}} = -i\partial_{a\bar{a}} \frac{\Box}{\Box} + \frac{3(kk')^2 + 4 - 2k'_i}{4(kk')^2} i\partial_{a\bar{a}} \frac{\bar{D}^2D^2}{\Box^2} + \\
\quad + \frac{3k^2 - 2i\partial_{a\bar{a}} D_\beta \bar{D}^2D^\beta}{\Box^2} + \frac{2 - k^2}{4k^2} i\partial_{a\bar{a}} \bar{D} \frac{D^\beta}{\Box^2},
\]
where $k$ and $k'_i$ are two gauge-fixing parameters, it is possible to invert the kinetic operator in (22) and find the following propagators

\[
\begin{pmatrix}
<\chi\bar{\chi}> & <\chi\sigma_\alpha>

<\bar{\sigma}_{\alpha}\bar{\chi}> & <\bar{\sigma}_{\alpha}\sigma_\alpha>
\end{pmatrix} = \begin{pmatrix}
-\frac{1}{\partial} \left(1 + \frac{|m|^2}{\Box} \frac{D_\alpha \bar{D}^2}{\Box}\right) & \frac{m}{\Box} \left(D_\alpha \bar{D}^2 - \frac{1}{2} \bar{D}^2 D_\alpha\right) \\
-\frac{m}{\Box} \left(D_\alpha D_{\bar{\alpha}} - \frac{1}{2} \bar{D}^2 D_\alpha\right) & W^{-1}_{a\bar{a}} - \frac{|m|^2}{\Box} W^{-1}_{a\bar{a}} \bar{D} \frac{D^\beta}{\Box^2} W^{-1}_{\alpha\bar{\beta}}
\end{pmatrix}.
\]

We observe that, in the limit $m = 0$, the resulting $<\chi\bar{\chi}>$ and $<\bar{\sigma}_{\alpha}\sigma_\alpha>$ propagators are exactly those known for the massless chiral and complex linear superfields, as expected.

4 One–loop effective potential for CNM SYM theories

We now look at the $N = 1$ Super–Yang–Mills model described by the classical action
\[
S = \int d^8z \left[\Phi^V \Phi - \Sigma e^V \Sigma\right] + \frac{1}{4} \int d^8z \ Tr W^aW_a + \frac{1}{4} \int d^6\bar{z} \ Tr \bar{W}^{\bar{a}}\bar{W}_{\bar{a}},
\]
which is the CNM generalization of the SYM model with massless complex linear matter superfields studied in [9]. The CNM matter superfields $\Phi^i$ and $\Sigma^i$ satisfy $D_\alpha \Phi = 0$ and $D^2\Sigma = m\Phi$, where $m$ is a gauge singlet. Thus both $\Phi$ and $\Sigma$ belong to the same representation of a gauge group. The vector superfield is in the adjoint representation $(V)_{ij} = V^a(T_a)_{ij}$ with $(T_a)_{ij}$ the Lie algebra generators in the representation of $\Phi$ and $\Sigma$.

Our aim is to compute the one–loop Kähler effective potential for the matter superfields. The one–loop divergent terms come from the contributions which have external $\Phi$, $\Phi$, $\Sigma$ and $\Sigma$ fields without any spinorial and space–time derivatives acting on them. We focus on this kind of diagram.

In order to proceed we perform the quantum–background splitting $\Sigma \rightarrow \Sigma_Q + \Sigma_B$, $\Phi \rightarrow \Phi_Q + \Phi_B$ and require that $D^2\Sigma_Q = m\Phi_Q$ and $D^2\Sigma_B = m\Phi_B$, even if the latter condition is not strictly necessary for the computation we are going to perform.

Inserting the splitting into the action (25), in addition to the ordinary kinetic terms for $\Phi$, $\Sigma$ and the gauge fields, we find the one–loop relevant interaction terms
\[
\int d^8z \left[\Phi_B V\Phi_Q + \Phi_Q V\Phi_B + \frac{1}{2} \Phi_B V^2\Phi_B\right] + \left(-\Sigma_B V\Sigma_Q - \Sigma_Q V\Sigma_B - \frac{1}{2} \Sigma_B V^2\Sigma_B\right) + \cdots.
\]

Since we are considering $D^2\Sigma_Q = m\Phi_Q$, using the quantization procedures of the previous section, it is possible to find the effective propagators for the physical superfields $\Phi_Q$, $\Phi_Q$, $\Sigma_Q$ and $\Sigma_Q$

\[
<\Phi^i_Q\Phi^i_Q> = -\delta^i_j \frac{\bar{D}^2D^2}{\Box - |m|^2} \delta^8(z - z') + <\Sigma^i_Q\Sigma^i_Q> = -\delta^i_j \frac{m\bar{D}^2}{\Box - |m|^2} \delta^8(z - z') + <\Phi^i_Q\Sigma^i_Q> = -\delta^i_j \frac{mD^2}{\Box - |m|^2} \delta^8(z - z') .
\]

\[
<\Phi^i_Q\Sigma^i_Q> = -\delta^i_j \frac{m\bar{D}^2}{\Box - |m|^2} \delta^8(z - z') + <\Sigma^i_Q\Sigma^i_Q> = \delta^i_j \left[1 - \frac{D^2\bar{D}^2}{\Box - |m|^2}\right] \delta^8(z - z').
\]
Furthermore, we have the propagator for the vector superfield which, in the Landau gauge, reads [1, 9, 12]

\[ < V^a V^b > = \delta^{ab} \frac{D_\alpha D^2 D^\alpha}{\Box^2} \delta^8 (z - z') . \] (28)

Now, we calculate only one-loop amplitudes without derivatives on the external fields, using the above propagators.

It is convenient to consider the effective Yang–Mills propagators obtained by inserting the vertices \( \frac{1}{2} \Phi_B V^a \Phi_B \) and \( -\frac{1}{2} \Sigma_B V^a \Sigma_B \). Summing on 1-PI diagrams we find

\[ << V^a V^b >> = \left( \Box - \bar{V}^{(4)} \right)^{-1} \frac{D_\alpha D^2 D^\alpha}{\Box^2} \delta^8 (z - z') , \] (29)

with

\[ \bar{V}^{(4)} = \frac{1}{2} \left( (T_a T_b + T_b T_a)_{ij} (\Phi_B^i \Phi_B^j - \Sigma_B^i \Sigma_B^j) \right) = \frac{1}{2} \left[ [\Phi_B \Phi_B]_{ab} - [\Sigma_B \Sigma_B]_{ab} \right] , \] (30)

having defined \( [AB]_{ab} \equiv (T_a T_b + T_b T_a)_{ij} (A^i B^j) \). The one–loop amplitudes can now be calculated considering the cubic vertices in (26) and using \( << V^a V^b >> \) as the vector propagator.

For the calculation we are performing the matter propagators with at least one superfield \( \Phi_Q \) or \( \Phi_Q \) in (27) are orthogonal to the YM–propagator (29) and then terms built with the cubic vertices \( \Phi_B(T_a)_{ij} V^a \Phi_B^j + \Phi_B(T_a)_{ij} V^a \Phi_B^j \) in (26) are zero. Consequently, only diagrams constructed using the vertices \( V^{(3)} = \Sigma_B(T_a)_{ij} V^{a \Sigma_B} + \Sigma_B(T_a)_{ij} V^{a \Sigma_B} \) connected by the propagators \( << V^a V^b >> \), \(< \Sigma_Q \Sigma_Q >\) contribute. After Fourier transforming, and summing up all the diagrams, we obtain

\[ \Gamma^{(3)} = - \int \frac{d^4p}{(2\pi)^4} d^4 \theta \frac{2}{p^2} \text{Tr} \left\{ \ln \left[ 1 + \frac{[\Phi_B \Phi_B]_{ab} + [\Sigma_B \Sigma_B]_{ab}}{2p^2} \right] - \ln \left[ 1 + \frac{[\Phi_B \Phi_B]_{ab} - [\Sigma_B \Sigma_B]_{ab}}{2p^2} \right] \right\} . \] (31)

Beyond this contribution, we find another term obtained with only the vertices \( \frac{1}{2} \Phi_B(T_a T_b)_{ij} V^a V^b \Phi_B^j \) and \( -\frac{1}{2} \Sigma_B(T_a T_b)_{ij} V^a V^b \Sigma_B^j \) considered

\[ \Gamma^{(4)} = - \int \frac{d^4p}{(2\pi)^4} d^4 \theta \frac{2}{p^2} \text{Tr} \left\{ \ln \left[ 1 + \frac{[\Phi_B \Phi_B]_{ab} - [\Sigma_B \Sigma_B]_{ab}}{2p^2} \right] \right\} . \] (32)

Summing \( \Gamma^{(3)} \) and \( \Gamma^{(4)} \) we find the following divergent contribution to the effective potential

\[ - \int \frac{d^4p}{(2\pi)^4} d^4 \theta \frac{2}{p^2} \text{Tr} \left\{ \ln \left[ 1 + \frac{[\Phi_B \Phi_B]_{ab} + [\Sigma_B \Sigma_B]_{ab}}{2p^2} \right] \right\} . \] (33)

To cancel this divergence we introduce a counterterm which renormalizes the original matter fields action. Evaluating the momentum integrals, having introduced a renormalization mass \( \mu \), we find for the renormalized effective potential

\[ \Gamma_{eff} = \frac{-1}{(4\pi)^2} \text{Tr} \left[ (\Phi_B \Phi_B) + [\Sigma_B \Sigma_B] \right] \ln \left( \frac{[\Phi_B \Phi_B] + [\Sigma_B \Sigma_B]}{2 \mu^2} \right) . \] (34)

Some comments are now in order. We note that although we are working with a massive theory, our result does not depend on mass \( m \). This is due to the fact that we have focused only on amplitudes without space–time and spinor derivatives acting on the external fields. Furthermore, the mass term is not explicit in the action as it is defined using the kinematic constraints.
The next, convergent, contributions to the one–loop Kähler effective action will result from terms having derivatives also on the external fields. For this kind of terms all the massive propagators would be relevant, and since \( \bar{D}^2 \Sigma_B = m \Phi_B \) (and in general \( \bar{D}^2 \Sigma_B = Q(\Phi_B) \)), these contributions depend explicitly on \( m \) (and in general on the parameters of the potential \( Q \)).

It would be interesting to further pursue this investigation by considering the nontrivial interactions \( \tilde{P}(\Phi), W(\Phi), K(\Phi, \bar{\Phi}, \Sigma, \bar{\Sigma}) \), studying in detail the structure of the classical vacua around which the saddle point approximation should be developed, and extending the analysis to more than one–loop.

5 Covariant quantization of CNM superfields and the Konishi anomaly

In section 3 we developed a superspace formalism for quantizing the CNM multiplet with a generic potential \( Q(\Phi) \). Now we study the coupling with background gauge fields. This can be done by introducing a completely covariant formalism with respect to both SUSY and gauge transformations. Given the set of covariant derivatives \( \nabla_A = (\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}, \nabla_\alpha \bar{\nabla}_{\dot{\alpha}}) \) \([1, 11]\), we define covariant chiral and covariant complex linear superfields as

\[
\bar{\nabla}_{\dot{\alpha}} \Phi = 0 , \quad \bar{\nabla}^2 \Sigma = Q(\Phi) .
\]

with \( Q(\Phi) \) as in (6).

We study the particular case \( Q(\Phi) \equiv m \Phi, m \neq 0 \). We proceed here by choosing slightly different solutions of the kinematic constraints compared with (15, 16)

\[
\Phi = \bar{\nabla}^2 \chi , \quad \Sigma = m \chi , \quad \bar{\Phi} = \nabla^2 \bar{\chi} , \quad \bar{\Sigma} = \bar{m} \bar{\chi} .
\]

This is a natural consequence of the fact that, apart from the constant \( m \), the present constraint \( \bar{\nabla}^2 \Sigma = m \Phi \) simply identifies \( \Sigma \) with the generic superfield \( \chi \) which solves the chiral constraint \( \nabla_\alpha \Phi = 0 \) \([10]\). In other words, since \( \tilde{P}(\Phi) \equiv 0 \) and \( m \neq 0 \), we can absorb the \( \bar{\sigma}_{\dot{\alpha}} (\sigma_\alpha) \) superfield in (15, 16) by redefining the \( \chi (\bar{\chi}) \) superfield as \( \chi + \frac{1}{m} \bar{\nabla}_{\dot{\alpha}} \bar{\sigma}_{\dot{\alpha}} (\bar{\chi} + \frac{1}{m} \nabla_\alpha \sigma_\alpha) \) obtaining (36, 37).

Using (36, 37), the action \( \int d^8 z \left[ \tilde{\Phi} \Phi - \Sigma \bar{\Sigma} \right] \) gives a kinetic quadratic term

\[
S_{\text{kin}}(m \neq 0) = \int d^8 z \, \bar{\chi} (\nabla^2 \nabla^2 - |m|^2) \chi ,
\]

which has an invertible kinetic operator; no gauge–fixing procedure is now necessary. The \( < \chi \bar{\chi} > \) covariant propagator is then

\[
< \chi \bar{\chi} > = \frac{1}{|m|^2} \left[ 1 - \nabla^2 \frac{1}{\Box_+ - |m|^2} \nabla^2 \right] ,
\]

where

\[
\Box_+ \equiv \nabla^2 \nabla^2 + \nabla^2 \nabla^2 + \nabla_{\dot{\alpha}} \nabla^2 \nabla_{\dot{\alpha}} = \frac{1}{2} \nabla^{\alpha \dot{\alpha}} \nabla_{\alpha \dot{\alpha}} - i W^\alpha \nabla_\alpha - \frac{i}{2} (\nabla^\alpha W_\alpha)
\]

and \( W_\alpha \equiv i \frac{1}{2} [\nabla^\dot{\alpha}, \{\nabla_\alpha, \nabla_{\dot{\alpha}}\}] \) is the gauge field strength \([1, 11]\).
We note that (39) is not defined when $m \equiv 0$. This is a natural consequence of the fact that, as previously said, solutions (36, 37) are suitable for treatment of the kinetic term associated with the CNM multiplets only when $m \neq 0$ and $Q(\Phi) \equiv m\Phi$. In the case of generic $Q(\Phi)$ a covariant generalization of solutions (15, 16) should be used to develop the quantization. This would require the construction of an explicitly covariant generalization of the quantization procedure of section 3. In particular, a suitable procedure would have to be developed to treat the ghosts in a manifestly covariant way.

It is possible to explicitly study the quantization of the present CNM model using the physical covariant superfields $\Phi$, $\bar{\Phi}$, $\Sigma$ and $\bar{\Sigma}$. In fact, the effective propagators are

$$
<\bar{\Phi}\Phi> = \nabla^2 <\bar{\chi}\chi> = -\nabla^2 \frac{1}{\Box_+ - |m|^2} \nabla^2 , \quad <\Sigma\bar{\Phi}> = m <\chi\bar{\chi}> \nabla^2 = -\frac{m\nabla^2}{\Box_- - |m|^2} ,
$$

$$
<S\Sigma> = m <\chi\bar{\chi}> \overline{m} = 1 - \nabla^2 \frac{1}{\Box_+ - |m|^2} \nabla^2 , \quad <\Phi\bar{\Sigma}> = \overline{\nabla^2 <\chi\bar{\chi}>} \overline{m} = -\frac{m\nabla^2}{\Box_+ - |m|^2} .
$$

(41)

We observe from (41) that, in this case, the physical propagators are also well defined when $m \equiv 0$. This leads to the conjecture that the covariant $<S\Sigma>$ propagator for the massless complex linear superfield should be $<S\Sigma> = 1 - \nabla^2 \frac{1}{\Box_+} \nabla^2$.

We now apply the covariant quantization formalism to study the Konishi anomaly in CNM SYM theories. We consider a pair of covariantly CNM superfields $\Phi$ and $\Sigma$ satisfying (35) with $Q(\Phi) \equiv m\Phi$. The kinetic action for the CNM theory has the form of (7). Beyond the gauge invariance, action (7), together with the constraint $\bar{\nabla}^2 \Sigma = m\Phi$, has the global invariance

$$(\Phi, \Sigma) \to \exp[ia](\Phi, \Sigma) , \quad (\bar{\Phi}, \bar{\Sigma}) \to \exp[-ia](\bar{\Phi}, \bar{\Sigma}) .$$

(42)

It is important to note that, due to the kinematic constraints, we are forced to choose the same charges for $\Phi$ and $\Sigma$. The resulting current $J_0 = (\Phi\Phi - \Sigma\Sigma)$ satisfies the classical conservation equation

$$\bar{D}^2 (\Phi\Phi - \Sigma\Sigma) = 0 .$$

(43)

We observe that, in the case of SYM theories with pure chiral matter multiplets, the $U(1)$ symmetry $\Phi \to \exp[ia]\Phi$ presents the chiral anomaly known as the Konishi anomaly [13]. In [4] it was observed that the phase trasformation (42) acts on the Dirac spinor (11) as a pure vector transformation $\Psi_{CNM} \to \exp[ia]\Psi_{CNM}$, and so it was argued that the CNM theory should be anomaly free.

Now, as a simple application of the covariant quantization formalism previously developed, we derive this property explicitly. As in the pure chiral case [13], we are led to study potential anomalies for the composite invariant gauge operators $\Sigma\Sigma$ and $\Phi\Phi$. We separately study the expectation values of the operators $\bar{D}^2 (\Sigma\Sigma)$ and $\bar{D}^2 (\Phi\Phi)$. In particular, we expect

$$\langle \bar{D}^2 (\Sigma\Sigma)(z) \rangle = m \langle (\Sigma\Phi)(z) \rangle + \text{NM.Anomaly} , \quad \langle \bar{D}^2 (\Phi\Phi)(z) \rangle = m \langle (\Sigma\Phi)(z) \rangle + \text{C.Anomaly} .$$

(44)

where the anomaly terms are quantum corrections to the classical equations for the composite operators. We follow [13, 1] and regularize the composite operators $\Sigma\Sigma$ and $\Phi\Phi$, using the Pauli–Villars regularization. In particular, as for the pure chiral case, it is possible to regularize the theory by introducing pairs of CNM covariant superfields connected by the kinematic constraints.
\[ \nabla_\alpha \Phi_M = 0 \quad \text{and} \quad \nabla^2 \Sigma_M = M \Phi_M, \] with the parameter \( M \) satisfying \( M \gg m \). At this point, we consider the regularized amplitudes

\[ < \Phi(z) > - < \Phi_M \Phi_M(z) > \quad \text{and} \quad < \Sigma \Sigma(z) > - < \Sigma_M \Sigma_M(z) >, \tag{45} \]

and compute

\[ \lim_{M \to \infty} \nabla^2( < \Sigma \Sigma(z) > - < \Sigma_M \Sigma_M(z) >) \quad \text{and} \quad \lim_{M \to \infty} \nabla^2( < \Phi(z) > - < \Phi_M \Phi_M(z) >). \tag{46} \]

Using the effective massive covariant propagators (41), we find that these terms are both equal to

\[ \lim_{M \to \infty} \frac{d}{dM} \int d^8 z' \delta^8(z' - z) \left\{ \frac{-1}{\Box - m^2} \nabla^2 \delta^8(z - z') + \frac{1}{\Box - M^2} \nabla^2 \delta^8(z - z') \right\} = \lim_{M \to \infty} \frac{d}{dM} \int d^8 z' \delta^8(z' - z) \left\{ \frac{-|m|^2}{\Box - |m|^2} \nabla^2 \delta^8(z - z') + \frac{M^2}{\Box - M^2} \nabla^2 \delta^8(z - z') \right\} = m < \Phi \Sigma(z) > + \lim_{M \to \infty} \frac{d}{dM} \int d^8 z' \delta^8(z' - z) \left\{ \frac{M^2}{\Box - M^2} \nabla^2 \delta^8(z - z') \right\}. \tag{47} \]

Then, the anomaly term is

\[ \lim_{M \to \infty} \left\{ M^2 \frac{d}{dM} \int d^8 z' \delta^8(z' - z) \frac{\nabla^2}{\Box - M^2} \delta^8(z - z') \right\}. \tag{48} \]

Once the \( \nabla \)-algebra and the integral have been performed, we find that the anomaly term (48) is exactly \( -\frac{1}{32\pi^2} \frac{d}{dM} \int \Sigma_\alpha W_\alpha \), the same as the Konishi anomaly. The anomalous equations for the gauge invariant composite operators \( \Sigma \Sigma(z) \) and \( \Phi \Phi(z) \) are then

\[ D^2 \langle \Sigma \Sigma(z) \rangle = m \langle \Sigma \Phi(z) \rangle - \frac{1}{32\pi^2} \frac{d}{dM} \int \Sigma_\alpha W_\alpha \], \tag{49} \]

\[ D^2 \langle \Phi \Phi(z) \rangle = m \langle \Sigma \Phi(z) \rangle - \frac{1}{32\pi^2} \frac{d}{dM} \int \Sigma_\alpha W_\alpha \]. \tag{50} \]

Therefore, there is a complete cancellation between the chiral and nonminimal anomalies in the conservation equation for the current \( J_0 \) (43) which is found to be anomaly free, as expected.

### 6 Glueball superpotential in N=1 CNM SYM

Recently, some new insights have been obtained into the non-perturbative dynamics of \( N = 1 \) supersymmetric gauge theories constructed by chiral matter superfields. In particular Dijkgraaf and Vafa have argued the existence of a connection between the 't Hooft limit sector of zero dimensional bosonic matrix models and the effective Wilsonian holomorphic superpotential for a wide class of \( N = 1 \) supersymmetric gauge theories. The relation has been originally conjectured by using a chain of dualities in string theories [15]. Subsequently it has been proved by using covariant supergraph techniques [16]. In [17], the connection has been discussed by using generalized Konishi anomaly equations having the same structure of the loop equations of a matrix model [15, 17]. Both these approaches determine the superpotential up to a term given by the Veneziano–Yianikelowicz superpotential which takes care of pure gauge dynamics.

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3For references on the covariant formalism in chiral theories and an explicit calculation of (48) see [11] and [1, 14].
Inspired by these interesting results it is natural to ask if and how $N = 1$ supersymmetric gauge theories defined by CNM matter multiplets present low–energy dynamical properties similar to the pure chiral case studied on the grounds of the Dijkgraaf–Vafa conjecture. In this section we discuss some simple properties of the glueball superpotential in $N = 1$ CNM SYM theories. In particular, we concentrate on the simplest class of CNM multiplet defined by $\nabla^2 \Sigma = Q(\Phi) \equiv m \Phi$, $m \neq 0$. Furthermore, we consider the following interaction term

$$\int d^4x d^2\theta \ W(\Phi) + \text{h.c.}, \quad (51)$$

where $W(\Phi)$ is a gauge invariant superpotential at least cubic in the chiral superfields $\Phi$. For these class of theories, in the previous section, we have constructed the covariant quantization. We discuss a perturbative approach to the calculation of the glueball superpotential by using covariant supergraph techniques as in [16]. We want to integrate out the massive degrees of freedom and find the effective superpotential for the gauge superfields. We consider the simplest case with unbroken gauge group. Then, the perturbative part of the effective superpotential $\int d^2\theta W_{\text{pert}}$ is obtained from vacuum amplitudes where only CNM matter fields propagate considering the gauge superfields as a background. Using the analysis of [16, 17], we can argue that the perturbative part of the superpotential have the form

$$\int d^2\theta \ W_{\text{pert}}[S, w_\alpha, (\cdots)] \quad \text{with} \quad S = \frac{1}{32\pi^2} \text{Tr} \ W^\alpha W_\alpha, \quad w_\alpha = \frac{1}{4\pi} \text{Tr} \ W_\alpha, \quad (52)$$

where $(\cdots)$ indicate the dependence of $W_{\text{pert}}$ from the matter coupling constants. The form of (52) follows from the fact that $W_{\text{eff}}$, by definition, is an element of the chiral ring [17] which in this case is generated by $S$ and $w_\alpha$. We remember that for pure chiral theories it is well known that the superpotential is holomorphic in the matter coupling constants [18, 16, 17]. At the moment we have not imposed any restriction, as holomorphicity, in the CNM case. We now discuss this point.

The covariant propagators for the CNM matter superfields are give in (41). Considering that the interactions (51) are defined by the superfields $\Phi$ and $\bar{\Phi}$ alone, it is clear that all the perturbative contributions involve only the $<\bar{\Phi}\Phi>$ covariant propagator in (41). Therefore, from the perturbative point of view the considered CNM theories has the same structure of pure massless chiral theories with superpotential $W(\Phi)$ [1]. Using this observation we can deduce that holomorphicity perturbatively works also in these CNM models being a property of the pure chiral theories. Furthermore, since there are no holomorphic propagators $<\Phi\Phi>$ in our case and all the perturbative contribution will be necessary non–holomorphic, it is then simple to observe that there are no perturbative contributions to the glueball superpotential in these CNM theories.

It is interesting to note that we have not explicited the representation in which the CNM matter multiplets are. Anyway, the above arguments, being associated to the covariant perturbative structure of CNM models, should not depend on the matter fields representation.

Therefore, it seems that from the point of view of the low–energy gauge dynamics these CNM theories look very different from the pure chiral theories studied on the grounds of the Dijkgraaf–Vafa conjecture [15, 16, 17].

We observe that in order to derive the above result we have assumed unbroken gauge group and $Q(\Phi) \equiv m \Phi$. It would be very interesting to extend the analysis of this section to the general case. To this respect it should be found a generalization of the covariant quantization of section 5.
7 Conclusions and further issues

In this letter we have studied some quantum properties of $N = 1$ supersymmetric field theories with chiral/nonminimal scalar multiplets defined by a chiral ($\Phi$) superfield and a complex linear ($\Sigma$) superfield kinematically coupled by $D^2 \Sigma = Q(\Phi)$.

In particular, generalizing the quantization techniques typical of massless chiral and complex linear superfields, we have developed the superspace quantization for CNM models with a general potential $Q(\Phi)$. When $Q(\Phi) \equiv m\Phi$ and the CNM scalar multiplets are coupled to background gauge fields, we have also constructed the covariant quantization formalism.

As preliminary applications, in sections 4, 5 and 6 we have studied some quantum properties of supersymmetric gauge theories built using CNM scalar matter superfields. In the particular case $Q(\Phi) = m\Phi$ we have computed the one–loop contribution to the effective action for the matter superfields, proved the anomaly free nature of these models, and discussed some properties of the glueball superpotential.

There are several classical and quantum issues in the study of CNM models that we hope to clarify in the future. In particular, the dynamical properties of CNM theories with nontrivial kinematic interaction $\bar{P}(\Phi)$ (6) call for further investigation. For example, we would like to clarify the precise structure of the chiral sector of CNM theories and see exactly how the fact that on–shell $\Sigma$ is a chiral superfield influences the quantum properties of these theories. Looking in this direction we should understand whether the amplitudes in the chiral ring ($F$–terms) [16, 17] are holomorphic in the tree–level coupling constants of the chiral potentials $Q(\Phi)$ (6) and $W(\Phi)$ (4). Holomorphicity seems to be valid in the CNM case as in the pure chiral, since the potentials $Q(\Phi)$ and $W(\Phi)$ are only defined in terms of chiral superfields and it is then possible to argue holomorphicity using arguments of naturalness widely used in pure chiral theories [18].

Furthermore, we would like to find a generalization of the covariant quantization formalism of section 5 for CNM multiplet defined with generic potential $Q(\Phi)$. Then, it will be interesting to extend the analysis of sections 5 and 6.

Finally, once the properties of CNM models constructed giving mass only kinematically are clearer, it will be interesting to study models with both CNM and chiral mass terms. The quantization procedure of this letter can be easily generalized to this case [19].

Acknowledgements

The author would like to thank Silvia Penati for suggesting the problem and for helpful discussions and suggestions.

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