Counterfactual Full-Duplex Communication

Fakhar Zaman\textsuperscript{1}, Hyundong Shin\textsuperscript{1}, and Moe Z. Win\textsuperscript{2}

\textsuperscript{1}Department of Electronics and Information Convergence Engineering, Kyung Hee University, 1732 Deogyeg-do, Yongin-si, Gyeonggi-do 17104, Korea  
\textsuperscript{2}Laboratory for Information and Decision Systems (LIDS), Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139 USA.

This paper proposes two new full-duplex quantum communication protocols to exchange classical or quantum information between two remote parties simultaneously without transferring a physical particle over the quantum channel. The first protocol, called \textit{quantum duplex coding}, enables to exchange a classical bit using a preshared maximally entangled pair of qubits by means of counterfactual disentanglement. The second protocol, called \textit{quantum telexchanging}, enables to exchange an arbitrary unknown qubit without using preshared entanglement by means of counterfactual entanglement and disentanglement. We demonstrate the quantum duplex coding and quantum telexchanging by exploiting counterfactual electron-photon interaction gates and show that these quantum duplex communication protocols form full-duplex binary erasure channel and quantum erasure channels, respectively.

1 Introduction

A duplex communication system is to convey information between remote parties in both directions, whereas simplex communication allows sending the information in one direction only. In a half-duplex system, remote parties can transfer information in both directions but not at the same time. Full-duplex communication is the way to transfer information in both directions simultaneously. In classical (radio frequency) communication, the full-duplex capability is typically achieved by channelization and/or transceiver configurations [1]. In quantum mechanics, full-duplex communication can be achieved by means of quantum dialogue [2, 3, 4, 5] and quantum state exchange protocols [6, 7, 8, 9] based on the laws of quantum physics such as quantum entanglement [10, 11, 12].

Quantum dialogue [2, 3, 4, 5] provides the novel way of exchanging classical information by utilizing the preshared entanglement, which is unattainable in classical communication. Although the quantum dialogue allows to achieve full-duplex communication, it requires multiple entangled pairs to exchange even one bit of classical information. To date, it is not possible to transfer classical information in both directions by using only one entangled pair. This paper proposes a full-duplex protocol which allows remote parties to exchange one bit of classical information by using only one entangled pair but no physical particle is transmitted over the quantum channel. In contrast, quantum state exchange allows remote parties to exchange arbitrary quantum states by means of \textit{preshared entanglement}, local operations and classical announcements [6, 7, 8]. However, the amount of

\begin{flushleft}
Hyundong Shin: \texttt{hshin@khu.ac.kr}, (corresponding author)
\end{flushleft}
information exchanged between remote parties relies on the amount of preshared entanglement. This paper proposes a full-duplex quantum communication protocol to exchange one qubit quantum information in both directions simultaneously without using preshared entanglement by exploiting the counterfactual quantum communication.

Counterfactual quantum communication is a revolutionary achievement in quantum informatics, which enables remote parties to transmit information under a probabilistic model of sending a physical particle over the channel [13, 14, 15, 16, 17]. At the time of successful transmission of information, no physical particle is found in the transmission channel. The counterfactuality was first introduced in quantum protocols as the counterfactual quantum computation [18, 19, 20, 21] followed by the counterfactual quantum cryptography [22, 23, 24, 25, 26]. The basic concept is originated from the interaction-free measurement (IFM) to ascertain the presence or absence of an absorptive object (AO) in an interferometer without physically interrogating it [27, 28].

The direct counterfactual quantum communication is based on the chained QZ (CQZ) effect where a classical bit is encoded as the presence or absence of the AO in the interferometer [13, 14, 29]. In contrast, transferring the quantum information requires a quantum AO in the superposition of presence and absence states. This idea has been first demonstrated in [30] to transfer the quantum information counterfactually along with a one-bit classical announcement. This was extended in [31] to transfer quantum information without transmitting any physical particle over a neither quantum nor classical channel by means of controlled disentanglement. A more efficient protocol for counterfactual communication of quantum information was presented in [32] exploiting the dual CQZ (DCQZ) effect. In recent years, the quantum protocols have been also designed for counterfactual entanglement distribution [33, 34, 35], counterfactual Bell-state analysis [36, 37], and counterfactual cloning [38].

This paper proposes two new full-duplex quantum communication protocols to exchange information between Alice and Bob by exploiting the inherent property of counterfactual quantum communication. The first protocol, called *quantum duplex coding*, enables to exchange classical information without transferring any physical particle over a neither quantum nor classical channel. The second protocol, called *quantum telexchanging*, enables to exchange an arbitrary unknown one-qubit quantum information simultaneously without using preshared entanglement and without transferring any physical particle over the quantum channel. The protocols are accomplished by designing nonlocal operations to transfer classical as well as quantum information in both directions at the same time counterfactually. The key contributions of this paper are as follows:

- **Quantum duplex coding**: A bit exchange protocol is proposed by developing the distributed controlled NOT (D-CNOT) operation. The protocol enables each party to exchange a one-bit classical information in each direction simultaneously using a single preshared Bell pair of qubits by means of counterfactual disentanglement. The protocol is demonstrated by devising the nonlocal D-CNOT operation for Bell-type states based on the modified QZ (MQZ) gate. In contrast to the CQZ gate [36], the protocol is designed by using only the blocking event (presence of the absorptive object) for full counterfactuality. It is shown that the quantum duplex coding forms a full-duplex binary erasure channel (BEC).

- **Quantum telexchanging**: A quantum state exchange protocol is proposed by generalizing the idea of D-CNOT operation. The protocol enables remote parties to exchange an arbitrary unknown one-qubit quantum information simultaneously without using preshared entanglement by means of counterfactual entanglement and disentangle-
ment of the dual D-CNOT (DD-CNOT) operation. The protocol is demonstrated by devising the DD-CNOT operation for general input states based on the dual MQZ (D-MQZ) and CQZ gates [36]. It is shown that the quantum telexchanging protocol creates a full-duplex form of the quantum erasure channel (QEC).

The remaining sections are organized as follows: Section 2 briefly explain the preliminaries to design the protocols for quantum duplex coding and telexchanging. In Section 3, the quantum duplex coding protocol is proposed to transfer classical information in both directions simultaneously. In Section 4, the quantum telexchanging protocol is demonstrated by using a combination of the CQZ and D-MQZ gates for quantum state exchange in a counterfactual way. In Section 5, the brief comparison is drawn between the quantum duplex coding (telexchanging) and quantum superdense coding (teleportation). Finally, Section 6 gives our conclusions and final remarks.

Notations: Random variables are displayed in sans serif, upright fonts; their realizations in serif, italic fonts. Vectors and matrices are denoted by bold lowercase and uppercase letters, respectively.

2 Counterfactual Quantum Communication

The counterfactual quantum communication is based on the single-particle nonlocality and quantum measurement theory. A quantum state usually collapses back to its initial state if the time between repeated measurements is short enough [39]. This QZ effect has been demonstrated to achieve IFM where the the state of a photon acts as an unstable quantum state corresponding to the presence of the absorptive object [28]. This section begin by introducing a brief review on the overall actions of the quantum Zeno (QZ) and CQZ gates [36, 40] that are invoked to formulate the D-CNOT and dual D-CNOT operations to transfer classical and quantum information in both directions simultaneously.

2.1 QZ Gates

Fig. 1 shows the Michelson version of the QZ gate [40] to perform IFM. The QZ gate is to ascertain the classical behavior of an absorptive object, i.e., to infer the absence state $|0\rangle_{AO}$ or the presence state $|1\rangle_{AO}$ of AO without interacting with it. The H(V)-QZ$_N$ gate takes an H (V) polarized photon as input. The switchable mirror $SM_N$ is initially turned off to allow passing the photon and is turned on for $N$ cycles once the photon is passed.
Table 1 shows the overall action of the QZ gate. Note that the H(V)-QZ$_N$ gate has the output $|H(V)\rangle_p$ in the presence state $|1\rangle_{AO}$ if the photon has not traveled over the control terminal (quantum channel). Hence, the QZ gate is counterfactual only for this measurement outcome.

2.2 CQZ Gates

Fig. 2 shows the nested version of QZ gates with $M$ outer and $N$ inner cycles [36]. The CQZ gate enables to ascertain the absence or presence of the absorptive object counterfactually
for both the outcomes. The $H(V)$-CQZ$_{M,N}$ gate also takes an $H$ ($V$) polarized photon as input. In each outer cycle, the $V$ ($H$) component of the photon enters the inner $V(H)$-QZ$_N$ gate.

- **AO = $|0\rangle_{AO}$:** In the absence of the absorptive object, the inner $V(H)$-QZ$_N$ gate transforms the photon state $|V(H)\rangle_p$ into $|H(V)\rangle_p$ after $N$ cycles. This component ends up at the detector D after PBS. Hence, the inner QZ gate acts as an absorptive object for the outer QZ gate in the absence state $|0\rangle_{AO}$, where D serves to detect the event that the photon is found in the control terminal. In each outer cycle, unless the photon is discarded, the photon state collapses back to the initial state $|H(V)\rangle_p$ with probability $\cos^2 \theta_M$. After $M$ outer cycles, the photon is not discarded at the detector D and ends up in the initial state $|H(V)\rangle_p$ with probability

$$\lambda_0 = \cos^{2M} \theta_M.$$  \hspace{1cm} (4)

tending to one as $M \to \infty$.

- **AO = $|1\rangle_{AO}$:** In case the absorptive object is present, the $V$ ($H$) component of the photon recombines with the $H$ ($V$) component and the photon state remains unchanged for the next outer cycle, unless the photon is absorbed by AO. Hence, the inner QZ gate acts as a mirror for the outer QZ gate in the presence state $|1\rangle_{AO}$. After $i (< M)$ outer cycles, unless the photon is absorbed, the photon state is given by (3), which is again not absorbed by AO for the next outer cycle with probability

$$\left[1 - \sin^2 (i\theta_M) \sin^2 \theta_N \right]^N.$$  \hspace{1cm} (5)

Hence, unless the photon is absorbed by AO, the $H(V)$-CQZ$_{M,N}$ gate transforms the input state $|H(V)\rangle_p$ into $|V(H)\rangle_p$ with probability

$$\lambda_1 = \prod_{i=1}^{M} \left[1 - \sin^2 (i\theta_M) \sin^2 \theta_N \right]^N$$  \hspace{1cm} (6)

tending to one as $M, N \to \infty$.

Note that the CQZ gate is counterfactual for both the outcomes and infers the absence or presence of the absorptive object (with probability $\lambda_0$ or $\lambda_1$) but no physical particle (photon) is found in the control terminal (see Table 1).
Figure 3: H(V)-CQZ$_{M,N}$ counterfactual communication where Alice encodes her classical message $b$ in the state $|b\rangle$$_{AO}$ of AO and Bob throws his H (V) polarized photon towards the H(V)-CQZ$_{M,N}$ gate to decode this message corresponding to the detector $D_b$ clicks. This CQZ counterfactual communication forms a classical asymmetric BEC with the erasure probability $1 - \lambda_b$ for the message $b$.

### 2.3 Counterfactual Communication

A communication task can be achieved in a counterfactual way by using the QZ or CQZ gate where the sender (Alice) has an absorptive object and the receiver (Bob) equips the (C)QZ gate [14, 13]. To transfer a classical bit $b \in \{0, 1\}$, Alice encodes this information as

$$\text{AO} = |b\rangle_{AO}. \quad (7)$$

The communication with the QZ gate is counterfactual only for the one classical bit—i.e., semi-counterfactual [13, 40]. The photon is found in the transmission channel with probability one for $b = 0$. To communicate both 0 and 1 without transmitting any physical particle over the transmission (quantum) channel, Bob uses the CQZ gate as shown in Fig. 3. Bob starts the protocol for decoding the information by throwing his H (V) polarized photon towards the H(V)-CQZ$_{M,N}$ gate and decides that the message 0 or 1 was transmitted if it ends up in the state $|H(V)\rangle_p$ or $|V(H)\rangle_p$. That is, the CQZ receiver decides the decoded message as $b$ if $D_b$ clicks. Otherwise Bob declares that the photon is erased (discarded or absorbed).

In case the photon is found in the transmission channel, it is either discarded by the detector in the CQZ gate (when $b = 0$ with probability $1 - \lambda_0$)$^1$ or absorbed by AO (when $b = 1$ with probability $1 - \lambda_1$). Hence, this CQZ counterfactual communication forms a classical (but not symmetric) BEC [41]. Let $p = \Pr [b = 1]$. Then, the mutual information $I(A; B)$ between Alice (A) and Bob (B) is given by

$$I(A; B) = h(p) - q h\left(\frac{p(1 - \lambda_1)}{q}\right), \quad (8)$$

where $h(p) = -p \log_2(p) - (1 - p) \log_2(1 - p)$ is the binary entropy function and

$$q = \Pr [b \text{ is erased}] = (1 - p)(1 - \lambda_0) + p(1 - \lambda_1). \quad (9)$$

By optimizing the message distribution $p$ such that $\left[\frac{\partial I(A; B)}{\partial p}\right]_{p=p^*} = 0$, we obtain the capacity $C$ in bits/photon for the CQZ counterfactual communication as follows:

$$C = \left[I(A; B)\right]_{p=p^*} \quad (10)$$

$^1$In this case, Bob knows that $b = 0$ but the photon is discarded by the protocol for counterfactuality.
taking the minimum value of 0.1515 bits/photon with $p_\star = 0.606$ when $N = M = 2$ and
tending to 1 bit/photon with $p_\star = 1/2$ as $M, N \to \infty$ (see Fig. 4).

Using the dual CQZ (DCQZ) gate, the counterfactual Bell-state analysis has been pro-
posed in [36] to achieve the distinguishability task of four Bell states without transmitting
physical particle over the transmission channel. In this DCQZ Bell-state analyzer, one
entangled particle (electron) of the Bell pair acts as a quantum absorptive object and
the other entangled particle (photon) is input to the DCQZ gate to perform the counter-
factual CNOT operation. To improve the efficiency of quantum superdense coding, the
semi-counterfactual Bell-state analyzer has been also proposed in [40] using the dual QZ
(DQZ) gate (instead of the DCQZ gate) with the sacrifice of full counterfactuality. This
DQZ superdense coding achieves the 90% efficiency (1.8 bits/qubit) when $N = 12$.

3 Quantum Duplex Coding

In this section, we develop a full-duplex quantum protocol to transfer classical information
in both directions simultaneously and counterfactually.

3.1 Protocol

Consider that Alice and Bob have a preshared maximally entangled pair (Bell state):

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}} |00\rangle_{AB} + \frac{1}{\sqrt{2}} |11\rangle_{AB},$$  \hspace{1cm} (11)

where the subscripts A and B denote Alice and Bob, respectively. Alice and Bob encode
the classical message $b_1,b_2$ in $|\psi_1\rangle_{AB}$ where $b_1$ is the classical bit Alice wants to send to
Bob and $b_2$ is vice versa as follows (see Fig. 5):

$$
|\psi_1\rangle_{AB} : \begin{cases} 
00 \rightarrow (I \otimes I) |\Phi^+\rangle_{AB} = |\Phi^+\rangle_{AB}, \\
01 \rightarrow (I \otimes X) |\Phi^+\rangle_{AB} = |\Psi^+\rangle_{AB}, \\
10 \rightarrow (Z \otimes I) |\Phi^+\rangle_{AB} = |\Phi^-\rangle_{AB}, \\
11 \rightarrow (Z \otimes X) |\Phi^+\rangle_{AB} = |\Psi^-\rangle_{AB},
\end{cases}
$$

where $I$ is the identity operator; $X$ and $Z$ represent Pauli $x$ and $z$ operators, respectively.

The duplex encoding transforms the initial Bell state $|\Phi^+\rangle_{AB}$ to $|\psi_1\rangle_{AB}$, one of the four Bell states $|\Phi^\pm\rangle_{AB}$ and $|\Psi^\pm\rangle_{AB}$.

To transfer the classical information in both directions simultaneously, Alice and Bob perform the D-CNOT operation in a counterfactual way where Alice’s qubit acts as a target bit and Bob’s qubit acts as a control bit. The D-CNOT operation disentangles the encoded Bell state $|\psi_1\rangle_{AB}$ to produce $|\psi_2\rangle_{AB}$ as follows:

$$
|\psi_2\rangle_{AB} : \begin{cases} 
|\Phi^\pm\rangle_{AB} \rightarrow |0\rangle_A |\pm\rangle_B, \\
|\Psi^\pm\rangle_{AB} \rightarrow |1\rangle_A |\pm\rangle_B,
\end{cases}
$$

where $|\pm\rangle = (|0\rangle \pm |1\rangle) / \sqrt{2}$ is the Hadamard basis. To decode the classical information, Alice directly measures her qubit and decides the one-bit message $b_2$, whereas Bob first applies the Hadamard gate $H$ followed by measuring his qubit in computational basis and decodes the one-bit message $b_1$. Alice and Bob decide the decoded messages as $b_2$ and $b_1$ from their post-measurement states $|b_2\rangle_A$ and $|b_1\rangle_B$, respectively. Here it is important to note that whenever a physical particle is found in the quantum channel during the implementation of D-CNOT operation, the protocol discards it and declares an erasure of the classical information $b_1b_2$.  

Figure 5: Quantum duplex coding for classical information $b_1b_2$. For the D-CNOT operation, Alice’s qubit acts as a target bit and Bob’s qubit acts as a control bit in a counterfactual way. Here, $H$ is the Hadamard gate and $Z$ represents the Pauli $z$ operator, respectively; $b_1$ (or $b_2$) is the classical bit Alice (or Bob) wants to transmit to Bob (or Alice); $|\psi_1\rangle_{AB}$ is the encoded Bell state; and $|\psi_2\rangle_{AB}$ is the disentangled state by the D-CNOT operation (viewed as counterfactual full-duplex transmission) for decoding the message.
Figure 6: A quantum absorptive object (electron) for (a) the QZ gate (type I) and (b) the CQZ gate (type II). The electron takes the superposition of two paths $|\uparrow\rangle_e$ and $|\downarrow\rangle_e$. In type I, the electron states $|\uparrow\rangle_e$ and $|\downarrow\rangle_e$ act as the presence (absence) state $|1\rangle_{AO}$ and the absence (presence) state $|0\rangle_{AO}$ of the absorptive object for the H(V)-QZ gate, respectively. In type II, the electron states simply act as $|\uparrow\rangle_e = |0\rangle_{AO}$ and $|\downarrow\rangle_e = |1\rangle_{AO}$ for the CQZ gate. If the photon is absorbed by the electron, the electron state is in an erasure state orthogonal to $|\uparrow\rangle_e$ and $|\downarrow\rangle_e$.

Figure 7: A H(V)-MQZ$_N$ interaction where the superposition state $|\text{electron}\rangle_e = \alpha |\uparrow\rangle_e + \beta |\downarrow\rangle_e$ of the quantum absorptive object (electron) is collapsed to $|\uparrow (\downarrow)\rangle_e$ (dequantumization) using the H(V)-QZ$_N$ gate unless the photon is absorbed by the electron. If the photon is found in the quantum channel, the pair of photon and electron is discarded in transforming $|\phi_1\rangle_{ep}$ to $|\phi_2\rangle_{ep}$ where the photon that has travelled over the channel is diverted again to the quantum absorptive object and absorbed by the electron. This electron-photon interaction is designed to output the photon and electron by using the presence state (blocking event) only. Hence, the protocol is fully counterfactual.

3.2 MQZ Duplex Coding

In this section, the quantum duplex coding protocol is demonstrated using the H(V)-QZ$_N$ gate. As shown in Fig. 6, an electron as a quantum absorptive object for duplexing coding takes the superposition of two paths $|\uparrow\rangle_e$ and $|\downarrow\rangle_e$ where the subscript $e$ denotes the electron. In type I (Fig. 6(a)), the electron state $|\uparrow (\downarrow)\rangle_e$ or $|\downarrow (\uparrow)\rangle_e$ acts as the presence state $|1\rangle_{AO}$ or the absence state $|0\rangle_{AO}$ of the absorptive object for the H(V)-QZ$_N$ gate. For counterfactuality, we setup the electron-photon interaction H(V)-MQZ$_N$ shown in Fig. 7 where the quantum absorptive object is in the superposition state

$$|\text{electron}\rangle_e = \alpha |\uparrow\rangle_e + \beta |\downarrow\rangle_e$$  \hspace{1cm} (16)
Figure 8: A MQZ D-CNOT operation for Bell states when $b_2 = 0 (1)$. Here, $R(\theta_K)$ is a rotation operator of rotation angle $\theta_K$ where $K$ is the number of H(V)-MQZ gates. Initially, Alice and Bob have the maximally entangled state $|\psi_1\rangle_{AB}$, which is transformed by $K$ sets of the $\theta_K$ rotation and MQZ gates successively to the separable state $|\psi_{2K}\rangle_{AB}$ in a controlled manner. Finally, the $U_2$ operator is performed on the recombined photon to complete the MQZ D-CNOT operation.

with $|\alpha|^2 + |\beta|^2 = 1$. The H(V)-MQZ interaction collapses this quantum state by entangling and disentangling the electron-photon pair

$$|\phi_0\rangle_{ep} = |\text{electron}\rangle_e |H(V)\rangle_p$$

as follows:

$$|\phi_0\rangle_{ep} \rightarrow |\phi_1\rangle_{ep} = \alpha |\uparrow H\rangle_{ep} + \beta |\downarrow V\rangle_{ep}$$

$$|\phi_2\rangle_{ep} = |\uparrow (\downarrow)\rangle_e |H(V)\rangle_p$$

unless the photon is absorbed by the electron, with probability

$$\left(1 - \Delta_0 \sin^2 \theta_N\right)^N \Delta_0$$

where $\Delta_0 = |\alpha|^2 (|\beta|^2)$ is the probability that the electron is in the presence state for the H(V)-QZ gate.

The second (first) term of $|\phi_1\rangle_{ep}$ is the outcome corresponding to the electron in the absence state for the H(V)-QZ gate. Since this outcome is not counterfactual, it is discarded (absorbed) by the electron using the polarizing beam splitter PBS$^H(V)$ and the $X$ operator. Hence, whenever the photon is found in the quantum channel, the electron absorbs it and becomes in an erasure state, leading the MQZ gate to output no photon and electron (e.g., particles in the erasure state). To discard the factual (non-counterfactual) outcome $|V(H)\rangle_p$ of the H(V)-QZ gate, we can simply use a photon detector after PBS$^H(V)$. Instead, we redirect this photon component to the quantum absorptive object (followed by the $\sigma_x$ operator) to be absorbed by the electron. This enables the protocol to abort non-locally by discarding both the photon and the electron whenever its counterfactuality is broken.

To implement the D-CNOT operation for MQZ duplex coding, we cascade $K$ H(V)-MQZ gates, where Alice is equipped with the electron and Bob has the MQZ gates (see Fig. 8). Consider that

$$|0\rangle_A = |0\rangle_e,$$

$$|1\rangle_A = |1\rangle_e,$$

$$|0\rangle_B = |H\rangle_p,$$

$$|1\rangle_B = |V\rangle_p.$$
Figure 9: Capacity $C$ [bits/Bell-pair] of MQZ duplex coding as a function of $N$ and $K$. Since the success probability $ζ_c$ in (30) is concave in $K > 1$ for any positive integer $N$ (see the left plot), there exists the optimal value (positive integer) of $K$ that maximizes the capacity for a given $N$. The blue solid line is the trajectory of capacity as a function of $N$ for the optimal values of $K$. The left plot depicts the capacity as a function of $K$ when $N = 100$. We also plot the trajectory of $(N, K)$ achieving the capacity of 1.8 bits/Bell-pair (white dashed line).

The MQZ duplex coding protocol takes the following steps to implement D-CNOT operation after encoding the classical information $b_1 b_2$.

1. Bob starts the protocol by separating the H and V components of the photon into two paths $|0\rangle_C$ and $|1\rangle_C$, respectively by using PBS$^H$. Bob locally applies the unitary operation $U_1 = X^{b_2}$ on the component of the photon in path state $|0\rangle_C$.

2. Alice applies the rotation operation $R(\theta_K)$ on her qubit:

$$
R(\theta_K) = \begin{bmatrix}
\cos \theta_K & -\sin \theta_K \\
\sin \theta_K & \cos \theta_K
\end{bmatrix},
$$

where $i = \sqrt{-1}$ and $\theta_K = \pi/(2K)$. The rotation gate $R(\theta_K)$ transforms $|0\rangle_A$ and $|1\rangle_A$ as follows:

$|0\rangle_A \rightarrow \cos \theta_K |0\rangle_A + \sin \theta_K |1\rangle_A$

$|1\rangle_A \rightarrow \cos \theta_K |1\rangle_A - \sin \theta_K |0\rangle_A$.  

3. Bob inputs the component of the photon in path $|0\rangle_C$ of the photon to H(V)-MQZ$_N$ gate for $b_2 = 0 (1)$. Unless the photon is absorbed by the electron, it transforms the composite state of the Alice and Bob to $|\psi_{21}\rangle_{AB}$

$$
|\psi_{21}\rangle_{ABC} : \begin{cases}
|\Phi^\pm\rangle_{ABC} \rightarrow \frac{1}{\sqrt{2}}(|000\rangle_{ABC} \pm \cos \theta_K |111\rangle_{ABC} \mp \sin \theta_K |011\rangle_{ABC}), \\
|\Psi^\pm\rangle_{ABC} \rightarrow \frac{1}{\sqrt{2}}(|\pm010\rangle_{ABC} \pm \cos \theta_K |011\rangle_{ABC} \mp \sin \theta_K |111\rangle_{ABC}).
\end{cases}
$$
Table 2: Decoding the classical message $b_1b_2$ for the MQZ dyplex coding.

| Alice  | Bob  |
|--------|------|
| Electron | $b_2$ | Photon | $b_1$ |
| $|\uparrow_e\rangle$ | 0     | $|H\rangle_p$ | 0     |
| $|\downarrow_e\rangle$ | 1     | $|V\rangle_p$ | 1     |

with probability

$$\lambda_2 = \left(1 - \frac{1}{2}\cos^2 \theta_N \sin^2 \theta_K\right)^N \left(1 - \frac{1}{2}\sin^2 \theta_K\right),$$

which tends to one as $N, K \to \infty$. Whenever the physical particle is traveled over the quantum channel between Alice and Bob, it is absorbed by the electron and the protocol declares an erasure.

4. Alice and Bob keep recurring the step 2) and step 3) for remaining $K - 1$ H(V)-MQZ$_N$ gates unless the protocol declares the erasure with probability $1 - \zeta_c$ where

$$\zeta_c = \lambda_2^K.$$ (30)

5. Bob applies $U_1$ and recombines the H and V components of the photon. The encoded Bell pair $|\psi_1\rangle_{AB}$ is disentangled to $|\psi_{2K}\rangle_{ABC}$ as follows:

$$|\psi_{2K}\rangle_{ABC} : \begin{cases} 
|\Phi^\pm\rangle_{ABC} \to |0\rangle_A |\mp\rangle_B |0\rangle_C, \\
|\Psi^\pm\rangle_{ABC} \to |1\rangle_A |\pm\rangle_B |0\rangle_C.
\end{cases}$$ (31)

6. Bob finally performs the $U_2 = Z^{1-b_2}$ operation on the component of the photon in path state $|0\rangle_C$ to complete the MQZ D-CNOT operation

Alice measures the path of the electron to decode the classical information $b_2$. Bob first applies the Hadamard gate $H$ to the photon, which transforms its polarization as

$$H |+\rangle_B \to |H\rangle_p,$$ (32)
$$H |-\rangle_B \to |V\rangle_p.$$ (33)

Bob measures the polarization of the existing photon to decode the classical information $b_1$. Table 2 shows the decoded messages corresponding to the measurement outcomes. The MQZ duplex coding creates a full-duplex form of the classical BEC with the erasure probability $1 - \zeta_c$ (see (30)). The bidirectional capacity $C$ in bits/Bell-pair of the MQZ duplex coding is given by²

$$C = 2\zeta_c$$ (34)

which tends to 2 bits/Bell-pair as $N, K \to \infty$ (see Fig. 9).

²This protocol enables each party to achieve unidirectional capacity of $\zeta_c$ bits/Bell-pair in each direction simultaneously by using only one Bell-pair.
4 Quantum Telexchanging

In this section, we develop a full-duplex quantum protocol to transfer quantum information in both directions simultaneously and counterfactually.

4.1 Protocol

Consider that Alice and Bob want to exchange their quantum states $|\eta_1\rangle_A$ and $|\eta_2\rangle_B$ simultaneously where

$$|\eta_1\rangle_A = \alpha |0\rangle_A + \beta |1\rangle_A,$$

$$|\eta_2\rangle_B = \gamma |0\rangle_B + \delta |1\rangle_B.$$  

To transfer the quantum information in both directions at the same time, Alice and Bob perform the DD-CNOT operation on their message qubits to entangle and disentangle them counterfactually. Bob starts the protocol by entangling his qubit $|\eta_2\rangle_B$ with his ancillary qubit $|0\rangle_C$ by performing the CNOT operation locally as shown in Fig. 10. Then, Alice and Bob have the separable composite state $|\psi_1\rangle_{ABC}$ as follows:

$$|\psi_1\rangle_{ABC} = |\eta_1\rangle_A (\gamma |00\rangle_{BC} + \delta |11\rangle_{BC}).$$  

Alice and Bob perform the two nonlocal CNOT operations on their qubits. In the first CNOT operation, Alice’s message qubit acts as a control qubit and Bob’s message qubit acts as a target qubit. In the second CNOT operation, Alice’s message qubit acts as a target qubit and Bob’s message qubit acts as a control qubit. These nonlocal operations transform the composite state $|\psi_1\rangle_{ABC}$ as in follows

$$|\psi_1\rangle_{ABC} \rightarrow |\psi_2\rangle_{ABC} = \alpha \gamma |000\rangle_{ABC} + \alpha \delta |011\rangle_{ABC} + \beta \gamma |110\rangle_{ABC} + \beta \delta |101\rangle_{ABC}$$  

$$\rightarrow |\psi_3\rangle_{ABC} = \alpha \gamma |000\rangle_{ABC} + \alpha \delta |111\rangle_{ABC} + \beta \gamma |010\rangle_{ABC} + \beta \delta |101\rangle_{ABC}$$  

$$= \gamma |00\rangle_{AC} (\alpha |0\rangle_B + \beta |1\rangle_B) + \delta |11\rangle_{AC} (\alpha |1\rangle_B + \beta |0\rangle_B).$$
Bob then applies the CNOT gate locally on his message and ancilla qubits to decode Alice’s message state. It transforms $\ket{\psi_3}_{ABC}$ as follows:

$$\ket{\psi_4}_{ABC} = (\gamma \ket{00}_A + \delta \ket{11}_A) (\alpha \ket{0}_B + \beta \ket{1}_B). \quad (41)$$

To further disentangle Bob’s ancilla and Alice’s qubit, Bob applies the Hadamard gate $H$ on his ancilla followed by measuring it in computational basis. Bob announces this measurement outcome $\mu \in \{0, 1\}$ to Alice by classical communication and Alice finally performs the $Z^\mu$ operation on her qubit to decode Bob’s message state as follows:

$$\ket{\psi_5}_{AB} = (\gamma \ket{0}_A + \delta \ket{1}_A) (\alpha \ket{0}_B + \beta \ket{1}_B) = \ket{\eta_2}_A \ket{\eta_1}_B. \quad (42)$$

Whenever a physical particle is found in the quantum channel for the DD-CNOT operation, the protocol discards it and declares an erasure of the quantum information $\ket{\eta_1 \eta_2}_{AB}$.

### 4.2 MQZ–CQZ Telexchanging

In this section, the quantum telexchanging protocol is demonstrated using the H(V)-QZ$_N$ and H(V)-CQZ$_{M,N}$ gates. Fig. 11 shows the dual form of the MQZ gate in Fig. 7. For counterfactuality, this D-MQZ$_N$ gate works similarly to the H(V)-MQZ$_N$ gate. The only difference is that the superposition polarization state $\ket{\text{photon}}_p = \gamma \ket{H0}_p + \delta \ket{V1}_p$ of the input photon is entangled with the ancillary path state in the D-MQZ$_N$ gate as follows:

$$\ket{\text{photon}}_pa = \gamma \ket{H0}_pa + \delta \ket{V1}_pa \quad (43)$$

where the ancilla states $\ket{0}_a$ and $\ket{1}_a$ show the paths for the H- and V-QZ gates, respectively. The D-MQZ$_N$ gate transforms the electron-photon pair

$$\ket{\phi_0}_{e p a} = \ket{\text{electron}}_e \ket{\text{photon}}_p \quad (44)$$

as follows:

$$\ket{\phi_0}_{e p a} \rightarrow \ket{\phi_1}_{e p a} = \alpha \gamma \ket{\uparrow H0}_{e p a} + \beta \gamma \ket{\downarrow V0}_{e p a} + \alpha \delta \ket{\uparrow H1}_{e p a} + \beta \delta \ket{\downarrow V1}_{e p a} \quad (45)$$

$$\rightarrow \ket{\phi_2}_{e p a} = \gamma \ket{\uparrow H0}_{e p a} + \delta \ket{\downarrow V1}_{e p a} \quad (46)$$
Figure 12: A MQZ-CQZ dual D-CNOT (DD-CNOT) operation for an unknown pair of quantum states. Initially, Alice and Bob have an untangled pair $|\eta_1\eta_2\rangle_{AB}$ of the electron and photon. This message pair is entangled by the DCQZ$_{M,N}$ gate and disentangled by $K$ rounds of the $\theta_K$ rotation and D-MQZ$_N$ gates counterfactually in a controlled manner. Finally, Bob applies the $X$ on the photon component in path $|1\rangle_C$ and recombines the respective components of the photon to complete the MQZ-CQZ DD-CNOT operation.

unless the photon is absorbed by the electron, with probability

$$1 - \Delta_1 \sin^2 \theta_N^N \Delta_1$$

where $\Delta_1 = |\alpha\gamma|^2 + |\beta\delta|^2$ is the probability that the electron is in the presence state for the QZ gates in both paths.

For MQZ-CQZ quantum telexchanging, Alice and Bob initially have an untangled pair of quantum information $|\eta_1\rangle_A$ and $|\eta_2\rangle_B$ prepared in the electron and photon, respectively, where we consider (21)–(24) again. Bob starts the protocol by throwing his photon towards PBS$^H$ to entangle the polarization state $|\text{photon}\rangle_B$ with the ancillary path state $|0\rangle_C$ as shown in Fig. 12. Then, Alice and Bob have the composite state $|\psi_1\rangle_{ABC}$ in (37). To implement the DD-CNOT operation in Fig. 10, Alice and Bob first entangle their qubits counterfactually by using the DCQZ$_{M,N}$ gate and then perform $K$ D-MQZ$_N$ gates for controlled disentanglement (see Fig. 12). The MQZ-CQZ telexchanging takes the following steps to device the DD-CNOT operation after preparing the message states.

1. Alice and Bob start the dual D-CNOT protocol by entangling their message states $|\text{electron}\rangle_A$ and $|\text{photon}\rangle_B$ counterfactually where Alice’s message state $|\text{electron}\rangle_A$ acts as a quantum AO and Bob is equipped with the DCQZ gate. Unless the photon is absorbed by the electron or discarded at the detector in the DCQZ$_{M,N}$ gate, this counterfactual entanglement transforms the encoded state $|\psi_1\rangle_{ABC}$ to $|\psi_2\rangle_{ABC}$ in (38), with probability

$$\lambda_3 = \left(1 - |\alpha|^2 \sin^2 \theta_M\right)^M \prod_{m=1}^M \left[1 - |\beta|^2 \sin^2 (m\theta_M) \sin^2 \theta_N\right]^N,$$

which tends to one as $M, N \to \infty$ [36].

2. Bob applies PBS$^H$ in each path of the photon to separate the H and V components of the photon and performs the $U_3$ operation locally where

$$U_3 = I \otimes (|0\rangle_C\langle 0| + |3\rangle_C\langle 3|) + X \otimes |1\rangle_C\langle 1| + Z \otimes |2\rangle_C\langle 2|.$$
Figure 13: Transfer efficiency (fidelity) $\zeta_q$ for the MQZ-CQZ telexchaning as a function of $|\alpha|^2$ and $|\gamma|^2$ when $N = 100$ and $M^* = K^* = 10$ where $M^*$ and $K^*$ are the optimal values that maximize $\zeta_q$ for given $N$ such that $M^* = \arg \max_M \zeta_3$ and $K^* = \arg \max_K \zeta_K^4$. When $|\alpha|^2 = 1/2$, the transfer efficiency is equal to $\zeta_q = 0.659$ independent of the message states (black dashed line). We can see that $\zeta_q$ increases as $\Delta_1 \to 0$ (the message states are collapsing to the classical information). When $N = 100$, the maximum efficiency is equal to $\zeta_q = 0.903$ for $|\alpha|^2 = 0$ and $|\gamma|^2 = 1$.

3. Alice applies the rotation operation $R(\theta_K)$ on her qubit and Bob applies the D-MQZ gate on the components of the photon in path state $|0\rangle_C$ and $|1\rangle_C$. Unless the photon is discarded in the D-MQZ gate, the D-MQZ gate transforms $|\psi_2\rangle_{ABC}$ as follows

$$
|\psi_{21}\rangle_{ABC} = \alpha \gamma |000\rangle_{ABC} + \beta \delta |111\rangle_{ABC} + \beta \gamma \sin \theta_K |012\rangle_{ABC} - \beta \gamma \cos \theta_K |112\rangle_{ABC} + \alpha \delta \cos \theta_K |013\rangle_{ABC} + \alpha \delta \sin \theta_K |113\rangle_{ABC},
$$

with probability

$$
\lambda_4 = \left(1 - \Delta_1 \cos^2 \theta_K \sin^2 \theta_N\right)^N \left(1 - \Delta_1 \sin^2 \theta_K\right),
$$

which tends to one as $N, K \to \infty$.

4. Alice and Bob keep repeating the step 3) for remaining $K - 1$ D-MQZ gates. The remaining $K - 1$ D-MQZ gates transform the composite state $|\psi_{21}\rangle$ to $|\psi_{2K}\rangle$ as follows:

$$
|\psi_{2K}\rangle_{ABC} = \alpha \gamma |000\rangle_{ABC} + \beta \delta |111\rangle_{ABC} + \beta \gamma |012\rangle_{ABC} + \alpha \delta |113\rangle_{ABC},
$$

unless the photon is discarded in the DD-CNOT operation with probability $1 - \zeta_q$

where

$$
\zeta_q = \lambda_3 \lambda_4^K.
$$

Whenever the physical particle is transmitted over the quantum channel between Alice and Bob, the protocol declares an erasure of the quantum message $|\eta_1\eta_2\rangle_{AB}$.

5. Bob applies $U_4$ on his photon and recombines the respective components of the photon to complete the DD-CNOT operation $|\psi_3\rangle_{ABC}$ where

$$
U_4 = I \otimes (|0\rangle_C\langle 0| + |2\rangle_C\langle 2| + |3\rangle_C\langle 3|) + X \otimes |1\rangle_C\langle 1|.
$$
Figure 14: Quantum capacity $Q$ [qubits/electron-photon], $M_*$, and $K_*$ for the MQZ-CQZ telexchanging as a function of $N$ when $|\alpha|^2 = |\beta|^2 = 1/2$ where $M_*$ and $K_*$ are the optimal values of $M$ and $K$ that maximize the quantum capacity $Q$ or equivalently the transfer efficiency $\zeta_q$ for given $N$ as in Fig. 13. The 50% efficiency ($Q = 1$ qubit/electron-photon) is attained when $N = 218$ with $M_* = 21$ and $K_* = 15$.

This MQZ-CQZ telexchanging for quantum information creates a full-duplex form of the QEC [42] with the erasure probability $1 - \zeta_q$ as follows:

$$|\eta_1\eta_2\rangle_{AB} \rightarrow \mathcal{N} (|\eta_1\eta_2\rangle_{AB}) = \zeta_q |\eta_1\eta_2\rangle_{BA} + (1 - \zeta_q) |\varepsilon\rangle_{BA}$$  \hspace{1cm} (55)

where $\mathcal{N}$ denotes the full-duplex QEC formed by the protocol and $|\varepsilon\rangle_{BA}$ is the erasure state orthogonal and independent to the message state $|\eta_1\eta_2\rangle_{AB}$. The transfer efficiency $\zeta_q$ can be also viewed as the fidelity

$$F = \langle \eta_1\eta_2 | \mathcal{N} |\eta_1\eta_2\rangle_{AB},$$  \hspace{1cm} (56)

which depends on the message state $|\eta_1\eta_2\rangle_{AB}$ in general. Since $\Delta_1 = 1/2$ if $|\alpha|^2 = |\beta|^2 = 1/2$, this dependence vanishes when Alice’s message $|\eta_1\rangle_A$ is in the superposition state of equiprobable $|0\rangle_A$ and $|1\rangle_A$ (see Fig. 13). The quantum capacity $Q$ in qubits/electron-photon for the MQZ-CQZ telexchanging is given by

$$Q = 2 \max \{0, 2\zeta_q - 1\}$$  \hspace{1cm} (57)

tending to 2 qubits/electron-photon as $K, M, N \rightarrow \infty$ (see Fig. 14).

5 Discussion

The quantum duplex coding and telexchanging are full-duplex counterparts to the quantum superdense coding and teleportation, respectively. The quantum superdense coding utilizes a striking nonclassical property of Bell states, which sends two bits of classical information in one qubit whereas quantum teleportation enables remote parties to transfer quantum information with two-bit classical information by using the preshared quantum entanglement. The quantum duplex coding and telexchanging have the following advantages over these quantum communication protocols.

- Quantum superdense coding is a simplex protocol to allow the communication of a two-bit classical message in one direction only. In contrast, quantum duplex coding
allows the full-duplex communication by means of the nonlocal D-CNOT operation. Although both the protocols transmit 2 bits/Bell-pair, the main advantage of the quantum duplex coding is to transfer classical information in both directions simultaneously. In quantum superdense coding, a sender needs to transmit a qubit over the quantum channel to enable a receiver to decode two bits of classical information. In contrast, the quantum duplex coding enables remote parties to communicate without transmitting any physical particle over the channel at the cost of the erasure probability $1 - \zeta_c$. As $N$ and $K$ increase, the capacity of the quantum duplex coding approaches to 2 bits/Bell-pair same as the capacity of quantum superdense coding.

- Quantum teleportation utilizes a prior entanglement to send one qubit of a quantum message. In contrast, quantum telexchanging enables communicating parties to transmit quantum information without preshared entanglement. To exchange a qubit of quantum information by using quantum teleportation, remote parties need two Bell pairs at the cost of four qubits and four bits of classical information. In contrast, the quantum telexchanging enables remote parties to exchange one qubit quantum information at the cost of one ancilla qubit and one bit of classical information. To ensure the counterfactuality of the protocol, the quantum capacity of quantum telexchanging is limited to $Q$ in (57) due to the erasure probability $1 - \zeta_q$.

Recently, the counterfactual quantum state exchange has been demonstrated by using the CQZ gates along with time-bin device [9]. Fig. 15 shows the comparison between quantum telexchanging (our scheme) and the protocol proposed in [9] in terms of quantum capacity and optimal value of $M$ as a function of $N$ when $|\alpha|^2 = |\gamma|^2 = 1/2$. Here $Q$ is the quantum capacity achieved by MQZ-CQZ telexchanging with $M = K = M*$ and $\hat{Q}$ is the quantum capacity achieved by the protocol in [9] with $M = \hat{M}$. It can be clearly seen that $Q > \hat{Q}$ whereas $M* < \hat{M}$ for all values of $N$, showing that our protocol gives better performance while using less resources (number of outer cycles).

Figure 15: Quantum capacity and the optimal value of $M$ for the quantum telexchanging and the counterfactual state exchange protocol presented in [9] where $K$ is simply set to $M*$ for the MQZ-CQZ telexchanging protocol. Here $Q$ and $\hat{M}$ denote the quantum capacity and the optimal value of $M$ for the protocol presented in [9].
6 Conclusion

We have put forth the new quantum communication protocols that achieve both full duplexity and counterfactuality for the classical as well as quantum information. Using the preshared entanglement and the nonlocal D-CNOT operation (counterfactual disentanglement), this unique quantum protocol allows remote parties to swap a one-bit of classical information simultaneously without transmitting any physical particle over the channel. We have generalized this counterfactual duplex communication framework for the quantum information by devising the dual D-CNOT operation (counterfactual entanglement followed by disentanglement in a distributed way) along with local operations and one-bit announcement of classical information. The communication without transmitting any physical particle over the channel can further provide inherent security advantages over the most of eavesdropping attacks such as the photon-number splitting attack and the intercept-and-resend attack. The future work can be done to extend the quantum communication protocols to transfer the information in both directions simultaneously, counterfactually—and securely.

Acknowledgment

This work was supported in part by the the National Research Foundation of Korea under Grant 2019R1A2C2007037, ITRC (Information Technology Research Center) support program (IITP-2021-0-02046) supervised by the IITP (Institute for Information & Communications Technology Planning & Evaluation) and in part by the Office of Naval Research under Grant N00014-19-1-2724.

References

[1] Zhongshan Zhang, Keping Long, Athanasios V Vasilakos, and Lajos Hanzo. Full-duplex wireless communications: Challenges, solutions, and future research directions. Proc. IEEE, 104(7):1369–1409, July 2016. DOI: 10.1109/JPROC.2015.2497203.
[2] He Wang, Yu Qing Zhang, Xue Feng Liu, and Yu Pu Hu. Efficient quantum dialogue using entangled states and entanglement swapping without information leakage. Quantum Inf Process, 15(6):2593–2603, March 2016. DOI: 10.1007/s11128-016-1294-z.
[3] Arpita Maitra. Measurement device-independent quantum dialogue. Quantum Inf Process, 16(12):1–15, November 2017. DOI: 10.1007/s11128-017-1757-x.
[4] Lan Zhou, Yu-Bo Sheng, and Gui-Lu Long. Device-independent quantum secure direct communication against collective attacks. Sci. Bull., 65(1):12–20, January 2020. DOI: 10.1016/j.scib.2019.10.025.
[5] Wei Zhang, Dong-Sheng Ding, Yu-Bo Sheng, Lan Zhou, Bao-Sen Shi, and Guang-Can Guo. Quantum secure direct communication with quantum memory. Phys. Rev. Lett., 118(22):220501, May 2017. DOI: 10.1103/PhysRevLett.118.220501.
[6] Yonghae Lee, Ryuji Takagi, Hayata Yamasaki, Gerardo Adesso, and Soojoon Lee. State exchange with quantum side information. Phys. Rev. Lett., 122:010502, January 2019. DOI: 10.1103/PhysRevLett.122.010502.
[7] Yonghae Lee, Hayata Yamasaki, Gerardo Adesso, and Soojoon Lee. One-shot quantum state exchange. Phys. Rev. A, 100:042306, October 2019. DOI: 10.1103/PhysRevA.100.042306.
[8] Jonathan Oppenheim and Andreas Winter. Uncommon information (the cost of exchanging a quantum state). arXiv:0511082, 2005.
[9] Zheng-Hong Li, M. Al-Amri, Xi-Hua Yang, and M. Suhail Zubairy. Counterfactual exchange of unknown quantum states. *Phys. Rev. A*, 100:022110, August 2019. DOI: 10.1103/PhysRevA.100.022110.

[10] Ahmad Farooq, Junaid ur Rehman, Youngmin Jeong, Jeong San Kim, and Hyundong Shin. Tightening monogamy and polygamy inequalities of multiqubit entanglement. *Sci. Rep.*, 9:3314, March 2019. DOI: 10.1038/s41598-018-37731-z.

[11] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki. Quantum entanglement. *Rev. Mod. Phys.*, 81(2):865–942, June 2009. DOI: 10.1103/RevModPhys.81.865.

[12] Julio T Barreiro, Nathan K Langford, Nicholas A Peters, and Paul G Kwiat. Generation of hyperentangled photon pairs. *Phys. Rev. Lett.*, 95(26):260501, December 2005. DOI: 10.1103/PhysRevLett.95.260501.

[13] Hatim Salih, Zheng-Hong Li, M Al-Amri, and M Suhail Zubairy. Protocol for direct counterfactual quantum communication. *Phys. Rev. Lett.*, 110(17):170502, April 2013. DOI: 10.1103/PhysRevLett.110.170502.

[14] Yakir Aharonov and Lev Vaidman. Modification of counterfactual communication protocols that eliminates weak particle traces. *Phys. Rev. A*, 99(1):010103, January 2019. DOI: 10.1103/PhysRevA.99.010103.

[15] Zheng-Hong Li, M. Al-Amri, and M. Suhail Zubairy. Direct quantum communication with almost invisible photons. *Phys. Rev. A*, 89:052334, May 2014. DOI: 10.1103/PhysRevA.89.052334.

[16] Yuan Cao, Yu-Huai Li, Zhu Cao, Juan Yin, Yu-Ao Chen, Hua-Lei Yin, Teng-Yun Chen, Xiongfeng Ma, Cheng-Zhi Peng, and Jian-Wei Pan. Direct counterfactual communication via quantum Zeno effect. *Proc Natl Acad Sci USA*, 114(19):4920–4924, May 2017. DOI: 10.1073/pnas.1614560114.

[17] Yakir Aharonov, Eliahu Cohen, and Sandu Popescu. A dynamical quantum cheshire cat effect and implications for counterfactual communication. *Nature Communications*, 12(1):1–8, August 2021. DOI: 10.1038/s41467-021-24933-9.

[18] Onur Hosten, Matthew T Rakher, Julio T Barreiro, Nicholas A Peters, and Paul G Kwiat. Counterfactual quantum computation through quantum interrogation. *Nature*, 439(7079):949, February 2006. DOI: 10.1038/nature04523.

[19] Fei Kong, Chenyong Ju, Pu Huang, Pengfei Wang, Xi Kong, Fazhan Shi, Liang Jiang, and Jianguo Du. Experimental realization of high-efficiency counterfactual computation. *Phys. Rev. Lett.*, 115(8):080501, August 2015. DOI: 10.1103/PhysRevLett.115.080501.

[20] Graeme Mitchison, Richard Jozsa, and Sandu Popescu. Sequential weak measurement. *Phys. Rev. A*, 76:062105, December 2007. DOI: 10.1103/PhysRevA.76.062105.

[21] Fakhar Zaman, Hyundong Shin, and Moe Z. Win. Counterfactual concealed telecommunication. *arXiv:2012.04974*, 2020.

[22] Tae-Gon Noh. Counterfactual quantum cryptography. *Phys. Rev. Lett.*, 103(23):230501, December 2009. DOI: 10.1103/PhysRevLett.103.230501.

[23] Zhen-Qiang Yin, Hong-Wei Li, Yao Yao, Chun-Mei Zhang, Shuang Wang, Wei Chen, Guang-Can Guo, and Zheng-Fu Han. Counterfactual quantum cryptography based on weak coherent states. *Phys. Rev. A*, 86(2):022313, August 2012. DOI: 10.1103/PhysRevA.86.022313.

[24] Hatim Salih. Tripartite counterfactual quantum cryptography. *Phys. Rev. A*, 90(1):012333, July 2014. DOI: 10.1103/PhysRevA.90.012333.

[25] Ying Sun and Qiao-Yan Wen. Counterfactual quantum key distribution with high efficiency. *Phys. Rev. A*, 82(5):052318, November 2010. DOI: 10.1103/PhysRevA.82.052318.
Yang Liu, Lei Ju, Xiao-Lei Liang, Shi-Biao Tang, Guo-Liang Shen Tu, Lei Zhou, Cheng-Zhi Peng, Kai Chen, Teng-Yun Chen, Zeng-Bing Chen, and Jian-Wei Pan. Experimental demonstration of counterfactual quantum communication. *Phys. Rev. Lett.*, 109:030501, July 2012. DOI: 10.1103/PhysRevLett.109.030501.

Avshalom Elitzur and Lev Vaidman. Quantum mechanical interaction-free measurement. *Found. Phys.*, 23(76):987–997, July 1993. DOI: 10.1007/BF00736012.

Paul Kwiat, Harald Weinfurter, Thomas Herzog, Anton Zeilinger, and Mark A Kasevich. Interaction-free measurement. *Phys. Rev. Lett.*, 74(24):4763, November 1995. DOI: 10.1103/PhysRevLett.74.4763.

I Alonso Calafell, T Strömberg, DRM Arvidsson-Shukur, LA Rozema, V Saggio, C Greganti, NC Harris, M Prabhu, J Carolan, M Hochberg, et al. Trace-free counterfactual communication with a nanophotonic processor. *npj Quantum Information*, 5(1):1–5, 2019. DOI: 10.1038/s41534-019-0179-2.

Qi Guo, Liu-Yong Cheng, Li Chen, Hong-Fu Wang, and Shou Zhang. Counterfactual quantum-information transfer without transmitting any physical particles. *Sci. Rep.*, 5:8416, February 2015. DOI: 10.1038/srep08416.

Zheng-Hong Li, M Al-Amri, and M Suhail Zubairy. Direct counterfactual transmission of a quantum state. *Phys. Rev. A*, 92(5):052315, November 2015. DOI: 10.1103/PhysRevA.92.052315.

Hatim Salih. Protocol for counterfactually transporting an unknown qubit. *Front. Phys.*, 3:94, January 2016. DOI: 10.3389/fphy.2015.00094.

Qi Guo, Liu-Yong Cheng, Li Chen, Hong-Fu Wang, and Shou Zhang. Counterfactual entanglement distribution without transmitting any particles. *Opt. Express*, 22(8):8970–8984, April 2014. DOI: 10.1364/OE.22.008970.

Yuanyuan Chen, Xuemei Gu, Dong Jiang, Ling Xie, and Lijun Chen. Tripartite counterfactual entanglement distribution. *Opt. Express*, 23(16):21193–21203, August 2015. DOI: 10.1364/OE.23.021193.

Yuanyuan Chen, Dong Jian, Xuemei Gu, Ling Xie, and Lijun Chen. Counterfactual entanglement distribution using quantum dot spins. *JOSA B*, 33(4):663–669, January 2016. DOI: 10.1364/JOSAB.33.000663.

Fakhar Zaman, Youngmin Jeong, and Hyundong Shin. Counterfactual Bell-state analysis. *Sci. Rep.*, 8(1):14641, October 2018. DOI: 10.1038/s41598-018-32928-8.

Fakhar Zaman, Een-Kee Hong, and Hyundong Shin. Local distinguishability of Bell-type states. *Quantum Inf Process*, 20(5):1–12, May 2021. DOI: 10.1007/s11128-021-03114-z.

Qi Guo, Shuqin Zhai, Liu-Yong Cheng, Hong-Fu Wang, and Shou Zhang. Counterfactual quantum cloning without transmitting any physical particles. *Phys. Rev. A*, 96:052335, November 2017. DOI: 10.1103/PhysRevA.96.052335.

Wayne M Itano, Daniel J Heinzen, J J Bollinger, and DJ Wineland. Quantum Zeno effect. *Phys. Rev. A*, 41(5):2295, March 1990. DOI: 10.1103/PhysRevA.41.2295.

Fakhar Zaman, Youngmin Jeong, and Hyundong Shin. Dual quantum Zeno superdense coding. *Sci. Rep.*, 9(1):11193, August 2019. DOI: 10.1038/s41598-019-47667-7.

Thomas M Cover and Joy A Thomas. *Elements of Information Theory*. Wiley, New York, NY, USA, 2012. DOI: 10.1002/04714882X.

Charles H Bennett, David P DiVincenzo, and John A Smolin. Capacities of quantum erasure channels. *Phys. Rev. Lett.*, 78(16):3217, April 1997. DOI: 10.1103/PhysRevLett.78.3217.