A New Study of the Transition to Uniform Nuclear Matter in Neutron Stars and Supernovae

W. G. Newton

† University of Oxford, Condensed Matter Dept., Clarendon Laboratory, Parks Rd., Oxford, OX1 3PU, U.K.

Abstract

A comprehensive microscopic study of the properties of bulk matter at densities just below nuclear saturation $\rho_s = 2.5 \sim 10^{14}$ g cm$^{-3}$, zero and finite temperature and high neutron fraction, is outlined, and preliminary results presented. Such matter is expected to exist in the inner crust of neutron stars and during the core collapse of massive stars with $M \gtrsim 8 M_\odot$.

1 Introduction

Understanding the phase transition from inhomogeneous to uniform nuclear matter is important in the study of a number of astrophysical phenomena. In the region of density $0.05 \rho_s \lesssim \rho \lesssim 0.16 \rho_s$, temperature $0 < T \lesssim 10$ MeV and proton fraction $0.01 \lesssim y_p \lesssim 0.3$, which is expected at the bottom of neutron star crusts and during core collapse supernovae, heavy nuclei immersed in a fluid of neutrons are expected to become distorted into a series of exotic structures known as ‘nuclear pasta’ [1] in order to minimize the sum of their surface tension energy and Coulomb repulsion with adjacent nuclei. These structures may extend over distances many times ‘normal’ nuclear radii $\approx 10$ fm.

Such matter could have a significant impact on the dynamics of core collapse supernovae. It exists in the outer regions of the collapsed core where neutrinos are expected to be trapped. The nuclear pasta phases may affect the neutrino opacity of the matter as excitation of collective modes in the pasta offers another channel for the transfer of energy from neutrino flux to the nuclear medium [2].

In the inner crust of neutron stars the pasta phases are expected to be a form of ‘soft’ condensed matter, that is, in a state between liquid and solid [3]. In addition, the external neutron gas is expected to be in a superfluid state, and as such its bulk flow will be quantized into vortices, whose cores are comparable to the extent of the pasta shapes in size [4].

Our theoretical studies are required for the explanation of observations. Thermal emission has been observed from seven young neutron stars [5]; to constrain models of the cooling of the neutron stars’ cores, we must know the relevant mechanisms of heat transport in the crust and their efficiency. Quasi-periodic oscillations have recently been observed in the tails of flares from soft gamma-ray repeaters [6] and have been interpreted as being produced by seismic waves in the crust as it relaxes after the significant readjustment that triggered the flare. Glitches in the spin down rates of pulsars tell us something about the

---

1 Participant Contribution at the “Dense Matter in Heavy Ion Collisions and Astrophysics” Summer School, JINR, Dubna, Aug. 21 - Sept. 1, 2006.
coupling of the crust to the core [7]. Later in a neutron stars’ life, accretion may lead to the formation of ‘mountains’ on the neutron star surface, leading to a quadrupole moment and gravitational wave emission: whether the gravitational waves can be detected on Earth depends on how big a mountain the crust can support [8]. Accretion can also reheat the crust, melting it in layers [9] and will also compress the innermost layers of the crust.

2 Motivation for a New Study of the Pasta Phases

In order to study the structure of the inner crust in an unbiased way, one must perform the calculations self-consistently in three dimensions, making no assumptions about the shape of the nuclear distribution or lattice type (i.e. free of the Wigner-Seitz approximation) and no distinction between the nuclear clusters and external neutron gas. Because of the computationally intense nature of such calculations, they have only recently been attempted, by two methods in particular; the semi-classical Quantum Molecular Dynamics (QMD) [11] and the fully quantum mechanical Hartree-Fock (HF) method [12]. QMD allows for a large volume (of order 100fm) to be simulated, and thus longer range (lower energy) effects explored. HF is more intensive computationally, so the simulation volumes are smaller (of order 20-40fm), but effects arising from the shell structure of the nuclear clusters and the unbound nucleons which scatter off them are automatically included. Both the above studies also neglected the band structure of the unbound nucleons, for which one requires general Bloch boundary conditions.

We would like to extend the Hartree-Fock study above to a far more comprehensive range of density and proton fraction space, as well as extending it to finite temperature thus exploring the properties of matter in core collapse supernovae. In doing this we would survey the effects of shell structure on the energy and ordering of the shapes, paying particular attention to separating physical effects from numerical artifacts, and implementing the general Bloch boundary conditions so we can examine the band structure of the neutron fluid.

3 3d Hartree-Fock Simulation of Bulk, Inhomogeneous Nuclear Matter

The Hartree-Fock approximation assumes a definite set of quasi-particle states with energies $\epsilon_1, ..., \epsilon_A$ that are occupied with a probability given by the Fermi-Dirac distribution

$$n_{i,q} = \frac{1}{e^{(\epsilon_{i,q} - \mu_q)/k_B T} + 1}$$

where $q = p, n$ labels the isospin states, $i$ the single particle states and $\mu$ is the chemical potential. Physically we are making the assumption that the nucleons in a nucleus or a nuclear configuration move independently of each
Figure 1: Right: The total free energy density, at temperature, density and proton fraction as given, as a function of number of nucleons in the cell (or, equivalently, the size of the cell). Left: some nuclear configurations obtained: top left - $A=460$, top right - $A=2200$, bottom left - $A=280$, bottom right - $A=1300$.

other in an average potential created by all the other nucleons. The variational principle
\[
\delta E[\Phi] = \delta \langle \Phi|\hat{H}|\Phi \rangle = 0
\]
is used to obtain our approximation to the ground state given a Hamiltonian containing a two-body nuclear interaction. We use the Skyrme interaction, a zero-range effective nuclear force particularly suited to Hartree-Fock calculations. Carrying out the variational procedure with respect to the single particle wavefunctions of a single Slater determinant, the two body Skyrme potential becomes a one body density-dependent potential and the $A$-body Schrödinger equation for the $A$-body wavefunction becomes $A$ single body Schrödinger equations for the single particle wavefunctions:

\[
h_{HF} \phi_{i,q} = \left[ -\nabla^{2} + \frac{\hbar^{2}}{2m^{*}_{q}} \nabla + u_{q}(r) + w_{q}(r) \left( \frac{\nabla \times \sigma}{i} \right) \right] \phi_{i,q} = \epsilon_{i,q} \phi_{i,q}
\]

Here, $w_{q}$ is the spin-orbit potential (which we currently set to zero), $u_{q}$ is the single particle potential, and $m^{*}_{q}$ is the effective mass. See [10] for the form of the Skyrme interaction and derived one body potentials.

We also impose a constraint that the neutron quadrupole moment of the nuclear configuration be given by an input value $\langle Q \rangle_{p}$. If we didn’t do this, we would have no control over which of the local minima in deformation space the simulation would fall into, destroying self-consistency.

We make the assumption that at a given temperature and density the matter is arranged locally in a periodic structure with an identifiable unit cell. We then take our computational volume to be that unit cell. We impose Bloch
boundary conditions:

\[ \phi_{i,q}(\vec{r} + \vec{T}) = e^{iK \cdot \vec{T}} \phi_{i,q}(\vec{r}) \]  

(3)

where \( \vec{T} \) is the translation from the position \( \vec{r} \) to the equivalent positions in the adjacent cells, and \( \vec{K} \) is the Bloch momentum covector. We must perform one simulation for each value of \( \vec{K} \) within the first Brillouin zone.

Each unit cell will contain a certain number of neutrons \( N \) and protons \( Z \), making a total baryon number of \( A = N + Z \). We have freedom to increase the cell volume \( V \) and number of nucleons \( A \) and still describe the same density.

Thus at a constant density, temperature and proton fraction we must scan across different cell sizes, neutron quadrupole moments and Bloch covectors. In order to reduce the numerical work, we will actually scan across cell size and quadrupole moment with simple periodic boundary conditions (\( \vec{K} = 0 \)), select the configuration that gives the minimum energy, and then perform the calculation for that configuration with \( \vec{K} \neq 0 \).

We are in the process of performing the calculations outlined above on the Cray XT3 super computer Jaguar at Oak Ridge National Lab. A sample of preliminary results is given in figure (1).

References

[1] Ravenhall D.G., Petthick C.J. and Wilson J.R. // Phys. Rev. Lett. 1983. V.50(26). P.2066.

Hashimoto M., Seki H. and Yamada M. // Prog. Th. Phys. 1984. V.71(2). P.320.

[2] Horowitz C.J., Perez-Garcia M.A. and Piekarewicz, J. // Phys. Rev. C. 2004. V.70. P.065806

[3] Watanabe G. and Sonoda H. // cond-mat/0502515

[4] Langlois D. // astro-ph/0008161

[5] Trumper J.E. // astro-ph/0502457

[6] Israel G.L. et al // Ap.J. L 2005. V.628. P.L53

[7] Horvath J.E. // Int. J. Mod. Phys. D 2004. V.13(7). P. 1327.

[8] Haskell B. // astro-ph/0609438

[9] Haensel P. and Zdunik J.L. A&A 2003. V.404. P. L33.

[10] Greiner W. and Maruhn J.A. // Nuclear Models Springer-Verlag

[11] Watanabe G., Sonoda H. // nucl-th/0512020

[12] Magierski P. and Heenen P.-H.// Phys. Rev. C. V.65(4). P.045804