How to Analyze the Students’ Thinking Levels Based on SOLO Taxonomy?

U H Putri1*, M Mardiyana1 and D R S Saputro2

1Departement of Mathematics Education, Universitas Sebelas Maret, Jl. Ir. Sutami No.36 A, Kentingan, 57126,Jebres, Surakarta, Indonesia
2Departement of Mathematics, Universitas Sebelas Maret, Jl. Ir. Sutami No.36 A, Kentingan, 57126, Jebres, Surakarta, Indonesia

*uninghapsari@student.uns.ac.id

Abstract. This study aims to determine the extent to which the students’ thinking level based on SOLO taxonomy in geometry learning viewed from students’ self efficacy (SE). This research utilizes descriptive research with qualitative approach. The data was collected by giving SE questionnaires to 32 students of grade VIII Junior High School which were then categorized into three (high, medium, and low). The subjects of the study were taken by one student in each SE category (high, medium, and low) which then given the problem solving test and the result was triangulated by interview. From this research, it is found that students with high SE can achieve uniructural-relational thinking level, subject with medium SE can be reach the level of unistructural-multistructural thinking, and subject with low SE able to reach unistructural-pristructural level. By paying attention to SE in learning, especially in problem solving process, it is expected that the educator can prepare the learning as well as possible so as to obtain optimal learning achievement.

1. Introduction

In the learning process, mathematics is closely related to the problem solving process. Problem solving is an emerging situation where there is a difference between fact and desire [1]. Centering mathematics instruction around to problem solving can help all students learn key concepts and skills within motivating contexts. In contrast with previous emphases on problem solving as an end in itself (e.g., focusing on heuristics), the current emphasis is on problem solving as a means to learn mathematical content and processes [2][3].

Since the 1980s, mathematics educators have agreed that developing problem solving skills should be the focus of mathematics learning [4]. In the other words, it has long been known that mathematical problem solving is very important something in education. Mathematical problem solving is defined as a process whereby the student faces a problem in the form of a question that requires a certain algorithm to solve it, which usually involves several repetitive cycles to refine the concept group either from within or outside of mathematics [4][5]. Problem solving has a prominent role in a curriculum for several reasons: (1) to build new mathematical knowledge, (2) to solve problems that arise in mathematics or in other context, (3) to implement and adapt various steps or problem solving strategies, and (4) to monitor and reflect on the problem solving process [6][7][8]. Based on the description of the definition of mathematical problem solving, can be drawn a definition that solving
the problem of mathematics is a process in which students face problems in the form of problems involving the concepts either originating from within or outside mathematics where the solution requires a certain algorithm.

Mathematics is a hierarchically organized subject [1] because every concept in mathematics is continuous and consecutive. In the other words, the more complex a given problem, the more connected concepts are needed. To solve a given problem, a student must be able to select and determine the elements that can be used in problem solving [9]. The problem solving process is related to the cognitive domain of the students. Therefore, problem solving is also related to students’ mathematics learning achievement. Each problem of mathematical problem solving has different characteristics, so the response given by each student to the problem solving can also different.Biggs and Collis (1982) explain that each stage of cognitive response is the same and increasing from the simple to the abstract. This theory is known as Structure of the Observed Learning Outcome (SOLO) which is the structure of the observed learning outcomes. The SOLO taxonomy is used to classify students’ ability to respond to a problem to five different levels and to be hierarchical, i.e., pre structural, uni structural, multi structural, relational, and extended abstract [10][11]. Several previous studies have shown that applying SOLO taxonomy in learning will help the students to study and prepare for the best answer [12].

A person’s problem solving ability is also influenced by his attitude toward mathematics. One’s beliefs and attitudes toward mathematics have an important role to one’s learning achievement [13][14]. How certain he can solve a problem will determine the extent to which he/she is able to solve the problem. Self efficacy (SE) is defined as an assessment of one’s self-beliefs to unleash their best ability to accomplish any given task or problem [14][15]. A person with strong SE, then he will complete every task given to him optimally, as he can see the opportunities that exist and use them to solve any obstacles that exist [2]. SE results in differences in how people think, act, and feel [3][16]. Therefore, with a person’s SE can be seen how will solve the problem with the belief that he has then can be seen also how far the level of one’s thinking. The material of the two-circles communion tangent line is the material taught to the Junior High School students. This material is one part of the realm of geometry. In this material, can be made various forms of problems that spur self efficacy students to be issued with the truth so that it can be known to what extent the level of thinking students to solve the problem.

Based on the description, it can be seen that SE has an important role to the problem solving ability of students. Furthermore, will be analyzed to know the extent to which level of student thinking on the material tangent alliance of two circles can use SOLO taxonomy reviewed from SE.

2. Experimental Method

2.1. The SOLO Taxonomy

The SOLO Taxonomy is a taxonomy that classifies how students’ thinking levels fall into five categories: pre-structural, uni structural, multi structural, relational, and extended abstract levels [10][11]. Criteria for each level based on SOLO taxonomy can be seen in Table 1 [17][18].

| No | The Level Thinking of SOLO Taksonomi | Criteria |
|----|------------------------------------|----------|
| 1. | SOLO 1: “The Pre-Structural Level” | Here the student does not have any kind of understanding but uses irrelevant information and/or misses the point altogether. |
|    |                                    | - Scattered pieces of information may have been acquired, but they are unorganized, unstructured, and essentially void of actual content or relation to a topic or problem. |
| 2. | SOLO 2: “The Uni-Structural         | The student can deal with one single aspect and make obvious connections. |
Table 1. Cont.

| Level | Description |
|-------|-------------|
| 3. SOLO 3: “The Multi-Structural Level” | At this level the student can deal with several aspects but these are considered independently and not in connection. - He is able to enumerate, describe, classify, combine, apply methods, structure, execute procedures, etc. |
| 4. SOLO 4: “The Relational Level” | At level four, the student may understand relations between several aspects and how they might fit together to form a whole. - A student may thus have the competence to compare, relate, analyze, apply theory, explain in terms of cause and effect, etc. |
| 5. SOLO 5: “The Extended Abstract Level” | At this level, which is the highest, a student may generalize structure beyond what was given, may perceive structure from many different perspectives, and transfer ideas to new areas. - The students may have the competence to generalize, hypothesize, criticize, theorize, etc. |

Table 1 shows the level of thinking criteria held by students based on SOLO taxonomy. Criteria et al. [12] used to determine and describe the extent to which the quality of thinking levels of students in solving the problem. The SOLO taxonomy can also illustrate how the structure of cognitive complexity or students’ response from the existing level of thinking. SOLO taxonomy is a classification of students in solving/solving problem by taking into account the characteristics of the five levels of ability.

2.2. Method
This study used grounded theory, which is suitable to analyze large quantities of unstructured or semi-structured data is qualitative [19]. The instruments of data collection are questionnaires and interviews. The questionnaire given is a SE questionnaire which consist of 40 items to 32 students of grade VIII in 2 Jaten Junior High School. Furthermore, the questionnaire results are analyzed and grouped under SE categories (high, medium, and low). From each SE category, randomly assigned one subject representing it, i.e., HSE, MSE, and LSE subjects. The three subjects were given a problem-solving test consisting of 2 questions and the results were analyzed based on the SOLO taxonomy guide. The given problem-solving test has been tested for validity and reliability with cronbach’s alpha and obtained reliability of 0.79. Content validity test is performed by three mathematics material experts and obtained valid results. Furthermore, to check the validity of the results of the analysis, triangulate the method by conducting interviews to each subject.

3. Result and Discussion
The instrument used to determine subject selection based on students’ SE is the SE questionnaire. The results of the questionnaire analysis were used to classify subjects with high, medium, and low SE. The subjects were chosen for 3 students to be given the problem solving test and its clarification. The three subjects are HSE, MSE, and LSE. The results of the analysis show that the HSE subject meets three levels of thinking based on SOLO taxonomy, i.e., starting from the unistructural, multistructural, and relational phase. MSE is able to meet the SOLO taxonomy thinking level from the unistructural and multistructural stages. Then, the LSE is only able to reach the level of pre structural and uni structural level.

3.1 Subjects with High SE (HSE)
For subjects with high SE (HSE), the subjects able to reach the level of thinking starting from the unistructural level, multistructural level, and relational level. For more detail, note the Figure 1 and Figure 2.

**Figure 1.** Photograph of HSE subject’s answer for number 1

**Figure 2.** Photograph of HSE subject’s answer for number 2

In Figure 1, it is seen that the HSE subject can illustrate the sketches of the question and can put the desired size of the problem on the picture correctly. This means, the subject can represent the matter into the appropriate image form. Thus, the HSE subject is capable of passing the unistructural level. At the multi structural level, the HSE subject can use two pieces of information or more of the given problem, so that it can be understand that the question asked is a tangent to the circle. Since the sketches have been drawn and their size, then to determine the length of the tangent line formed is to apply the phytagoras theorem. Then, the relational level, the HSE subject can think by using two pieces of information or more of the given question. The subject can determine the initial idea by determining the length of the tangent line formed which means it can determine half of the length of the seesaw. Thus, the subject is able to determine the length of the seesaw as a whole. At the extended abstract level, the subject can link the information as well as draw conclusions to build new concepts and apply them for possible application on the other issues, but at this level, HSE subject can’t reach this level.

In Figure 2, it is seen that the HSE subject has been able to reach the unistructural level because it can be use a piece of clear and direct information from the problem. Then, at the multi structural level, the HSE subject can use two pieces of information or more of the given question. Furthermore, the subject can understand that the question asked is a tangent to the circle. Since the drawing sketch is already known and its size, then to determine the length of the outside tangent outline of two circles, the subject can write down the formula used correctly. Furthermore at the relational level, the HSE subject can think by using two pieces of information or more of the given problem so that from the information the subject can determine the length of the tangent outer alliance of two circles. The length of the tangent can be understood by the students as the height of the formed wake. Then, the subject is asked to determine the area of the image formed, ie trapezoid. The HSE subject can determine the trapezoidal area exactly according to its size, wherein the two parallel lines are the radius of the two circles and the height of the trapezium is the outline of the outer alliance of two circles. Then at the extended abstract level, the HSE subject can not relate the information and draw conclusions to build new concepts and apply them for possible application to other issues.

Based on the result of student’s answer analysis and interview, it is concluded that based on SOLO taxonomy thinking level, students with high SE (HSE) are able to reach the level of thinking until relational phase, where it can use two or more pieces of information to be used according to the given problem [17][18].

### 3.2 Subject with Medium SE (MSE)
For subjects with medium SE (MSE), subjects are able to reach thinking levels ranging from unistructural and multistructural levels. For more details, note the Figure 3 and Figure 4.

In Figure 3, it can be seen that the MSE subject is able to reach the thinking level from the unistructural to multi structural stages. At the unistructural stage, the MSE subject can use a clear and straightforward piece of information from the problem. The subject of MSE can describe the sketches of the question and can put the desired size of the problem on the picture correctly. This means, the subject can represent the matter into the appropriate image form. Then, at the multistructural level, the SES can use two or more pieces of information from the given question. Furthermore, the subject can understand that the question asked is the tangent to the two-circle alliance. Since the sketches of the drawing and its size have been determined, the length of the tangent line is calculated using the phytagoras theorem.

Furthermore at the relational level, MSE subject can think by using two pieces of information or more of the given problem. Then the MSE can relate the information that the subject can determine the initial idea by being able to determine the length of the tangent line formed which means it can determine half of the length of the seesaw. At this stage, the MSE subject has been able to reach the multistructural level. However, the MSE subject completes its work only to find the length of the tangent to the circle with the phytagoras theorem, which means that it has only discovered half of the length of the seesaw in question. This means MSE has not been able to reach the relational level. Then at the extended abstract level, the MSE subject can not link the information as well as draw conclusions to build new concepts and apply them for possible application to other issues.

In Figure 4, the MSE subject can use a clear and direct piece of information from the problem so that it is able to reach the unistructural level. The subject of MSE can put the desired size on the sketch of the picture correctly. Then, at the multistructural level, the MSE subject can use two pieces of information or more of the given question. Furthermore, the subject can understand that the question asked is the outline of the two-circle outdoor partnership. Since the drawing sketch is already known and its size, then to determine the length of the outside tangent outline of two circles, the subject can write down the formula used correctly.

Next at the relational level, the MSE subject can think by using two pieces of information or more of the given problem so that the subject can determine the length of the outside tangent outline of the two circles. Then to determine the area of ABCD, the subject understands that the area in question is trapezoidal, so he uses the trapezium wide formula to determine the area of his area. However, after the SES determines the length of the tangent line, they do not understand that the length of the outer crossword of the two circles (DC) is the height of the trapezium. The MSE subject assumes that DC and AB are parallel lines on trapezium and use the AD radius as trapezium height. Thus, the subject of HSE subject has not been able to use a fragment of information that has been owned properly so that the subject has not been able to reach the relational level. At the extended abstract level, since the
MSE subject can not use the exact piece of information they have, they can not link the information and draw conclusions to build new concepts and apply them for possible application to other issues. Thus, the MSE subject also can not reach the extended abstract level.

Based on the results of the analysis and interviews, it can be concluded that based on the SOLO taxonomy thinking level, students with moderate SE (MSE) is broadly able to reach the level of thinking until the multi structural stage, where he understands two or more pieces of information but has not been able to use it [17][18] in accordance with the given problem.

3.3 Subject with Low SE (LSE)
For subject with Low SE (LSE), subjects able to reach the level of thinking range from prastructural to unistructural levels. For more details, note the Figure 5 and Figure 6.

In Figure 5, the LSE subject is able to use a piece of clear and direct information from the problem. He can describe the sketch of the question and can put the appropriate size of the problem on the picture exactly. This means that the subject can represent the matter in the form of a suitable image, which means it is capable of reaching the unistructural stage. After the subject sketches the drawing and size, then he understands what the problem means, ie determines the length of the circle tangent. However, the LSE of the subject is wrong in applying the pythagoras theorem where it should determine the length of the vertical side, but the LSE subject assumes that the tangent line formed is the oblique side of the right triangle formed. Thus, the LSE subject is incapable of using two or more pieces of known information properly. In other words, the LSE subject is only able to achieve auni structural level where the subject is only able to determine a piece of information obtained directly and clearly.

In Figure 6, the LSE subject is able to use a piece of information provided directly and clearly. The LSE subject understands that what is meant to determine the length of the outside tangent outline of two circles. He was able to determine the elements of the matter correctly and could write the exact formula of the tangent line of the match. However, the LSE subject is wrong in the calculation process where the square of the difference of the radius of the two circles is understood as the square difference of the radius of the circle. This results in incorrect calculations. In this process, the LSE subject has been able to reach the unistructural level. He was able to use a fragment of information given about, but not yet accurate.

Based on the above analysis, it can be concluded that based on the SOLO taxonomy level of thinking, students with low SE (LSE) are able to reach the level of pre-structural thinking to the unistructural level only, where it is able to use a piece of information to be used in accordance with the given problem [17][18] but not yet accurate.

4. Conclusion
Based on the research objectives and data analysis that has been done by the researcher about the level of student thinking in solving the mathematics problem of the tangent material of the two-circle alliance based on the SOLO taxonomy, it can be concluded that students with high SE able to meet
three levels of thinking, ranging from uni structural, multi structural, and relational. Then students with medium SE, able to meet the level of uni structural to multi structural thinking, then students with low SE only able to meet the level of pre-structural and uni structural thinking.

Because each student has a different level of thinking, the student’s ability to solve problems is also different. With regard to SE as one of the factors that play an important role to the students' mathematics learning achievement, especially in problem solving, it is expected that teachers or educators do not neglect SE owned by students. Thus, if teachers make optimal learning preparation with attention to SE and students’ level of thinking, it is expected to obtain optimal learning achievement as well.

Acknowledgements
The authors thank to Universitas Sebelas Maret and local government in Karanganyar regency due to gave permission for this study.

References
[1] Rahman A and Ahmar A S 2016 exploration of mathematics problem solving process based on the thinking level of students in junior high school Int. J. Environmental & Science Edu. 11 7278-85
[2] Pajares F and Miller MD 1994 Role of self-efficacy and self concept beliefs in mathematical problem solving: a path analysis J. Educ. Psych. 86 193-203
[3] Bandura A and Adams NE 1997 Analysis of self efficacy theory of behavioral change J. Cog. Theory and Research 1287-310
[4] Kuzle A 2013 Patterns of metacognitive behavior during mathematics problem-solving in a dynamic geometry environment Int. Electronic J. Math. Edu. 8 20-40
[5] Tripathi, PN 2009 Problem Solving in Mathematics: A Tool for Cognitive Development (New York: State University of New York) p 168
[6] National Council for Teachers of Mathematics 2000 Principle and Standards for School Mathematics. Reston, VA: NCTM.
[7] National Council for Teachers of Mathematics 1989 Principle and Standards for School Mathematics. Reston, VA: NCTM.
[8] National Council for Teachers of Mathematics 1997 Principle and Standards for School Mathematics. Reston, VA: NCTM.
[9] Nor NM and Idris N 2009 Assessing students’ informal inferential reasoning using solo taxonomy based framework Procedia Social and Behavioral Sciences 2 4805-09.
[10] Jimoyiannis 2011 Using SOLO taxonomy to explore students’ mental models of the programming variable and the assignment statement Themes in Science & Tech. Edu. 4 53-74.
[11] Brabrand Cand Dahl B Using the SOLO Taxonomy to Analyze Competence Progression of University Science Curricula (Denmark: IT University of Copenhagen)
[12] Lister R, Simon B, Whalley Jand Thompson E 2006 ITICSE '06 06118-122(http://www/researchgate.net/publication/22080137)
[13] Kislenko K, Grevholm Band Lepik M Mathematics is Important but Boring: Students’ Beliefs and Attitudes Towards Mathematics (Norway: Adger University Collage)
[14] Peker M 2016 Mathematics teaching anxiety and self efficacy beliefs toward mathematics teaching: a path analysis self efficacy in mathematics: affective, cognitive, and conative domains of functioning J. Edu. Research and Reviews 11 97-104
[15] Tait-McCutcheon S L 2008 Self efficacy in mathematics: affective, cognitive, and conative domains of functioning Proc. 31st Ann. Conf. Math. Edu. Research Group of Australia, 507-513.
[16] Hahkioniemi M, Lepaaho H and Fransisco J 2012 Model for Teacher Assisted Technology Enriched Open Problem Solving Proc. 13th ProMath Conf. 30-43.
[17] Zarch MK and Kadivar P 2006 The role of mathematics self-efficacy and mathematics ability in the structural model of mathematics performance Proc. 9th WSEAS Int. Conf. Appl. Math. 9 242-
[18] Lawrence J and Tar U J Travel & Tourism Marketing, 31 417–42 (https://doi.org/10.1080/10548408.883346)
[19] Parakash E S, Narayan K A, and Sethuraman 2010 Adv. Physiol. Educ. 34 145-149