Cherenkov radiation by Josephson vortex travelling in the long sandwich

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Abstract. Vortex motion in the long Josephson sandwich embedded in dielectric media is described. It is shown that vortices traveling with velocities greater than the speed of light in the dielectric generate electromagnetic waves. Appearance of radiation is due to Cherenkov phenomenon. Radiation appearing at rather high vortex velocities has high enough frequencies. For typical sandwiches radiation frequencies fall on THz domain.

Emission of electromagnetic waves by Josephson junctions attract attention of researchers for a long time (see, e.g. the recent review [1] and references therein). In connection with the problem of THz radiation generation the significant attention was paid to experimental study of radiation emission by high-temperature superconductors, which can treated as multilayered Josephson structure (see [2] and references therein). Theoretical conceptions on radiation generation by single Josephson junction are based on the possibility of Swihart waves emission through edges of junction with finite length in the direction of vortex motion [3, 4]. Another possibility of radiation generation by moving vortex is presented in this contribution. This possibility is due to Cherenkov effect and realizes in the Josephson sandwich embedded into dielectric media. It is accepted that junction size is unlimited in the direction of vortex motion (z-axis). The possibility of Cherenkov radiation generation from electrodes into surrounding media appears in the case of vortex velocity is greater than the speed of light in the dielectric. Cherenkov radiation is emitted from the surfaces of superconducting electrodes of the sandwich. Such conditions are realized in the annular Josephson junctions with large radius.

The system under consideration consists of plane Josephson sandwich placed into media with dielectric constant $\epsilon_m$ (Fig. 1). Sandwich is formed by two identical superconducting electrodes with thickness $L$ separated by thin nonsuperconducting layer. We derived the following equation for phase difference $\varphi$ of superconducting order parameters of electrodes at different sides of tunnel layer:

$$\omega_j^2 \sin \varphi(z,t) + \frac{\partial^2 \varphi(z,t)}{\partial z^2} = \frac{\partial}{\partial z} \int \int dz\,dt' Q(z - z', t - t') \frac{\partial \varphi(z',t')}{\partial z'},$$

(1)

where $\omega_j$ is the Josephson plasma frequency. Fourier transform of the kernel $Q(z,t)$ is given by:

$$Q(k, \omega) \equiv v_s^2 \text{th} \left( \frac{L}{\lambda} \right) \frac{c_m^2 \kappa - \lambda \omega^2 \text{cth} \left( \frac{L}{\lambda} \right)}{c_m^2 \kappa - \lambda \omega^2 \text{th} \left( \frac{L}{\lambda} \right)},$$

(2)
Figure 1. Section of the Josephson sandwich in the $xOz$ plane. Vortex is travelling along $z$-axis, ellipses are level lines of magnetic field, which is oriented along $y$-axis. Inclined dashed lines are front lines of the Cherenkov radiation, which is emitted from side surfaces of the sandwich. Arrows, that are perpendicular to front lines, show direction of energy flux $S$.  

where $v_s$ is the Swihart velocity in the case of bulk electrodes, $\lambda$ is the London penetration depth, $c_m \equiv c/\sqrt{\epsilon_m}$ is the speed of light in dielectric media with susceptibility $\epsilon_m$, 

$$
\kappa \equiv \sqrt{\left[k^2 - \omega^2/c_m^2\right]} \left[\Theta \left(c_m^2 k^2 - \omega^2\right) - i\Theta \left(\omega^2 - c_m^2 k^2\right) \text{sign}\omega\right],
$$

(3)

where $\Theta(x)$ is the Heaviside step function. Fourier transforms of electric and magnetic fields outside the sandwich depend on coordinate as $\propto \exp(-\kappa x)$, so the sign of $\kappa$ imaginary part corresponds to radiation propagation from sandwich deep into nonconducting media.

Let us first consider the radiation of vortex travelling in Josephson sandwich in the case of characteristic frequencies and wave numbers satisfy the following condition:

$$
\sqrt{\omega^2 - c_m^2 k^2} \epsilon_m/\omega^2 \lambda \ll \text{th}(L/\lambda).
$$

(4)

In that case the Eq. (1) for phase difference takes the form:

$$
\omega_j^2 \sin \varphi + \frac{\partial^2 \varphi}{\partial t^2} = U_s^2 \frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial}{\partial z} \int \int dq'dt' Q_{rad}(z - q', t - t') \frac{\partial \varphi(q', t')}{\partial q'}.
$$

(5)

where $U_s \equiv v_s \text{cth}^{1/2}(L/\lambda)$, 

$$
Q_{rad}(k, \omega) \equiv -2i \frac{c_m U_s^2}{\lambda \text{sh}(2L/\lambda)} \sqrt{\omega^2 - c_m^2 k^2} \omega |\omega|.
$$

(6)

The Swihart velocity $U_s$ differs from the well-known Swihart velocity $V_s \equiv v_s \text{th}^{1/2}(L/\lambda)$ of sandwich in the quasimagnetostatic limit [5, 6]. The Swihart velocity modification is due to the strong interaction between Swihart wave and electromagnetic wave in the dielectric surrounding sandwich and especially important in the case of thin electrodes when $L \ll \lambda$.

Integral term in the right-hand side of Eq. (5) describes the phase difference dependence on the radiation field inside the dielectric surrounding sandwich. It leads to radiation losses of the vortex structure energy per unit time:

$$
\left(W\right)_{rad} = \frac{\epsilon \phi_0^2}{32 \pi^3 c^2 d} \int dz^2 \frac{\partial \varphi(z, t)}{\partial t} \frac{\partial \varphi(z, t)}{\partial z} \int \int dq'dt' Q_{rad}(z - q', t - t') \frac{\partial \varphi(q', t')}{\partial q'},
$$

(7)
where $\phi_0$ is magnetic flux quanta, $\epsilon$ and $2d$ are dielectric constant and width of tunnel layer.

In the case of uniform motion of elementary vortex $\phi_0 = 4\pi \text{rtg} \left[ \exp(-k_j z) \right]$, $\zeta = z - vt$, with constant velocity $v > 0$, where $k_j \equiv \omega_j/\sqrt{U_s^2 - v^2}$, small power losses due to electromagnetic waves radiation from sandwich surface ($x = \pm (d + L)$) into dielectric media $(|x| > d + L)$ are equal to

$$\left(W\right)_{rad} = -\frac{\phi_0^2}{4\pi^3 \lambda^2 \text{sh}^2(L/\lambda)} \frac{k_j}{v} \frac{c_m}{v^2 - c_m^2}. \quad (8)$$

The Poynting vector in dielectric media (see Fig. 1), corresponding to radiation energy losses, is given by formula:

$$S = \frac{\phi_0^2}{64\pi^3 \lambda^2 \text{sh}^2(L/\lambda)} \frac{c_m^2}{v} \times \left[ \frac{\varphi_0}{\zeta + \sqrt{(v/c_m)^2 - 1(|x| - d - L)}} \right]^2 \left( \sqrt{(v/c_m)^2 - 1} \text{sign} \cdot e_x + e_z \right). \quad (9)$$

From Eq. (9) one can see that for $\sqrt{v^2 - c_m^2} \ll c_m$ radiation flux propagates mainly along the sandwich electrodes.

According to Eq. (8) Cherenkov losses due to electromagnetic waves emission from sandwich into dielectric media are possible if vortex velocity is greater than the light speed in the dielectric: $v > c_m$. At the same time, velocity of the elementary vortex is limited by the renormalized Swihart velocity of the sandwich. Thereby, one can discuss radiation losses only in the systems for which $U_s > c_m$, or $\text{cth}(L/\lambda) > \epsilon \lambda / \epsilon_m d$. Assuming $d \ll L \sim \lambda$, we find that dielectric constant of surrounding media must be much greater than dielectric constant of tunnel layer $\epsilon_m > \epsilon \lambda / \lambda \text{th}(L/\lambda) \sim \epsilon \lambda / d \gg \epsilon$. For example, if $\epsilon = 4$, $L/d = 25$, than $\epsilon_m > 100$.

Cyclic frequencies of Cherenkov radiation are given by formula $\omega = kv$, where $k \sim k_j$. Assuming $L \sim \lambda \sim 100\text{nm}$, $\lambda_j \sim 10^4\text{nm}$ and $v \sim c_m \sim 3 \cdot 10^8\text{cm/sec}$, we obtain that frequency $\nu \equiv \omega/2\pi$ belongs to the THz range of frequencies.

The next point for consideration is a radiation from Josephson sandwich with characteristic frequencies and wave numbers satisfying the condition reverse to Eq. (4). In this case interaction between Swihart and electromagnetic wave in dielectric is rather small. As above, the power losses of vortex due to radiation can be find:

$$\left(W\right)_{rad} = -\frac{\phi_0^2}{12\pi^3 \text{ch}^2(L/\lambda)} \frac{v^3 k_j^3}{c_m \sqrt{v^2 - c_m^2}}. \quad (10)$$

Corresponding to this emission Poynting vector in dielectric media is equal to:

$$S = \frac{\phi_0^2}{64\pi^3 \lambda^2 \text{ch}^2(L/\lambda)} \frac{v^3}{c_m \sqrt{v^2 - c_m^2}} \times \left[ \frac{\varphi_0}{\zeta + \sqrt{(v/c_m)^2 - 1(|x| - d - L)}} \right]^2 \left( \sqrt{(v/c_m)^2 - 1} \text{sign} \cdot e_x + e_z \right). \quad (11)$$

According to Eq. (11) at $v \gg c_m$ the radiation diagram is wide. For $\sqrt{v^2 - c_m^2} \ll c_m$ radiation propagates mainly along electrodes surface. Distribution of the magnetic field inside sandwich and in the dielectric is shown at the Fig. 2.

Spectral composition of Cherenkov radiation from the sandwich is described by formula $\omega = kv$, where $k \sim \omega_j/\sqrt{U_s^2 - v^2}$. For parameters of the system, that were accepted above, frequencies of radiation $\nu$ again belong to THz range.

In summary, we have shown that vortices, travelling in the Josephson sandwich with velocities greater than the speed of light in the external dielectric, generate the THz electromagnetic radiation. This radiation is due to the Cherenkov effect and can be realized in the case of rather great dielectric constant of external media.
Figure 2. Magnetic field of the elementary vortex travelling in the Josephson sandwich. Dependence $H(x, \zeta)$ is obtained for $v = 0.7v_s$, $L = \lambda$, $v_s = 2c_m$. Field is mainly localized inside sandwich. Outside the sandwich one can see electromagnetic wave, which is goes away from the surface.

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