Learning efficient structured dictionary for image classification

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Abstract. Recent years have witnessed the success of dictionary learning (DL) based approaches in the domain of pattern classification. In this paper, we present an efficient structured dictionary learning (ESDL) method which takes both the diversity and label information of training samples into account. Specifically, ESDL introduces alternative training samples into the process of dictionary learning. To increase the discriminative capability of representation coefficients for classification, an ideal regularization term is incorporated into the objective function of ESDL. Moreover, in contrast with conventional DL approaches which impose computationally expensive $\ell_1$-norm constraint on the coefficient matrix, ESDL employs $\ell_2$-norm regularization term. Experimental results on benchmark databases (including four face databases and one scene dataset) demonstrate that ESDL outperforms previous DL approaches. More importantly, ESDL can be applied in a wide range of pattern classification tasks. The demo code of our proposed ESDL will be available at https://github.com/li-zi-qi/ESDL.

Keywords: pattern classification; dictionary learning; ideal regularization term; label information.

1 Introduction

Dictionary learning (DL) has aroused considerable interest in recent years, and it has been successfully applied in various tasks, such as face recognition, image fusion and person re-identification. According to the way of encoding the input data, DL can be divided into two different categories, i.e. synthesis dictionary learning (SDL) and analysis dictionary learning (ADL). SDL aims to learn a dictionary by which the input data can be well approximated by the dictionary, while ADL tries to produce the sparse representation by employing the dictionary as a transformation matrix. An illustration of SDL and ADL is presented in Fig. 1.

The most famous SDL method is the K-SVD algorithm which has been widely used in image compression and denoising. Nevertheless, K-SVD mainly focuses on the representational power of the dictionary without considering its capability for classification. To tackle this problem, Zhang et al. presented a discriminative K-SVD (D-KSVD) method by introducing the classification error.
into the framework of K-SVD. Jiang et al.\textsuperscript{6} further incorporated a label consistency constraint into K-SVD and proposed a label consistent K-SVD (LC-KSVD) algorithm. Kviatkovsky \textit{et al.}\textsuperscript{7} mathematically proved the equivalence of the LC-KSVD and the D-KSVD algorithms up to a proper choice of regularization parameters. Zheng \textit{et al.}\textsuperscript{8} developed a Fisher discriminative K-SVD (FD-KSVD) method which imposes Fisher discrimination criterion on the sparse coding coefficients. Similarly, by restricting the within-class scatter of a dictionary’s representation coefficients, Xu \textit{et al.}\textsuperscript{9} explored a supervised within-class-similar discriminative DL (SCDDL) algorithm. Motivated by the fact that kernel trick can capture the nonlinear similarity of features, Song \textit{et al.}\textsuperscript{10} proposed an Euler label consistent K-SVD (ELC-KSVD) approach for image classification. By jointly learning a multi-class support vector machine (SVM) classifier, Cai \textit{et al.}\textsuperscript{11} presented a support vector guided dictionary learning (SVGDL) model. To fully exploit the locality information of atoms in the learned dictionary, Yin \textit{et al.}\textsuperscript{12} proposed a locality constraint dictionary learning with support vector discriminative term (LCDL-SV) algorithm for pattern classification. Readers can refer to Ref. \textsuperscript{13} for a survey of SDL approaches.

Although SDL achieves encouraging results in classification tasks, it is time-consuming to learn the synthesis dictionary. Recently ADL has attracted increasing attention due to its efficacy and efficiency. Rubinstein \textit{et al.}\textsuperscript{14} presented analysis K-SVD which is parallel to the synthesis K-SVD. To enhance the classification performance of ADL, Guo \textit{et al.}\textsuperscript{15} proposed discriminative ADL (DADL) method. By introducing a synthesis-linear-classifier-based error term into the basic ADL model, Wang \textit{et al.}\textsuperscript{16} presented a synthesis linear classifier based ADL (SLC-ADL) algorithm. Inspired by LC-KSVD,\textsuperscript{6} Tang \textit{et al.}\textsuperscript{17} incorporated the label consistency term and classification error term into the framework of ADL and developed a structured ADL (SADL) approach. It should be noted that transform learning\textsuperscript{18,19} and ADL have similar formulation. To adapt transform
learning to classification tasks, Maggu et al.\textsuperscript{20} proposed discriminative transform learning (DTL) for hyperspectral image classification. Yin et al.\textsuperscript{21} presented a label consistent transform learning (LCTL) method for pattern classification.

However, conventional dictionary learning approaches do not fully exploit the diversity of training samples, especially when there are insufficient training samples. Moreover, the $\ell_0$ or $\ell_1$-norm constraint is often introduced to promote the sparsity of representation matrix, which is computationally expensive. To alleviate the above two problems, Xu et al.\textsuperscript{22} proposed a new DL method in which the alternative training samples are introduced. The alternative training samples can be derived through the following two schemes, when providing insufficient training samples, virtual training samples can be generated and used as the alternative training samples. When we have large-scale training data, the whole training data can be divided into two parts with the same size,
then one part is utilized as original training samples and the other part is the alternative training samples. DL method proposed in Ref. 22 outperforms many conventional DL approaches. Nevertheless, the label information of training samples is not exploited in Ref. 22, which undermines its classification performance. To incorporate the label information of training samples into the formulation of DL, we introduce an ideal regularization term into the objective function of our proposed method. Through this term, representations of the training samples belonging to the same class are encouraged to be similar, which is beneficial for the subsequent classification stage.

Our main contributions can be summarized as follows.

- We take both the diversity and label information of training samples into account, and the introduced ideal regularization term associates the label information of training samples with that of atoms in the dictionary.

- In a departure from conventional DL approaches which impose $\ell_1$-norm on the coefficient matrix, ESDL employs the $\ell_2$-norm constraint which is computationally efficient.

- Our proposed ESDL is a general framework which can be applied in a wide range of pattern classification tasks.

The remainder of this paper is arranged as follows: Section 2 reviews the related work. Section 3 presents our proposed dictionary learning approach. Experimental results and analysis are presented in Section 4. Finally, Section 5 concludes this paper.

2 Related work

In this section, we briefly review the basic K-SVD$^4$ and its two discriminative extensions, i.e., D-KSVD$^5$ and LC-KSVD.$^6$ Additionally, the dictionary learning method proposed by Xu et al.$^{22}$
is also introduced. We first give an introduction to the notations used throughout this paper. Let \( Y = [y_1, y_2, \ldots, y_N] \in \mathbb{R}^{n \times N} \) be the data matrix of \( N \) training samples belonging to \( C \) classes, where \( n \) is the dimension of vectorized data and \( N \) is the total number of training samples, \( D = [d_1, d_2, \ldots, d_K] \in \mathbb{R}^{n \times K} \) is the learned dictionary which has \( K \) atoms, \( X = [x_1, x_2, \ldots, x_N] \in \mathbb{R}^{K \times N} \) is the coding coefficients matrix of \( Y \) on the dictionary \( D \).

2.1 K-SVD and its extensions

By generalizing the K-means clustering process, Aharon et al.\(^4\) proposed K-SVD to learn an overcomplete dictionary that best approximates the given data. The objective function of K-SVD is formulated as follows,

$$
\min_{D, X} \| Y - DX \|_{F}^2, \text{ s.t. } \| x_i \|_0 \leq T_0
$$

where \( D \) is the dictionary that is to be learned, \( X \) is the coding coefficient matrix, and \( T_0 \) is a given sparsity level. Equation (1) can be solved by alternatively updating \( D \) and \( X \).

Although K-SVD achieves superb results in image denoising and restoration, its performance for classification is limited. To adapt K-SVD to classification tasks, Zhang et al.\(^5\) developed D-KSVD algorithm by introducing the classification error term into the framework of K-SVD,

$$
\min_{D, W, X} \| Y - DX \|_{F}^2 + \beta \| H - WX \|_{F}^2 + \lambda \| W \|_{F}^2, \text{ s.t. } \| x_i \|_0 \leq T_0
$$

where \( H = [h_1, h_2, \ldots, h_N] \in \mathbb{R}^{C \times N} \) is the label matrix of training data, \( h_i = [0, 0, \ldots, 1, \ldots, 0, 0]^T \in \mathbb{R}^{C \times 1} \) is the label vector of \( y_i \), and \( W \) is the parameters for a linear classifier. As can be seen from (2), dictionary and a linear classifier are jointly learned in D-KSVD. To further promote the discriminative capability of K-SVD, Jiang et al.\(^6\) presented LC-KSVD by solving the following
optimization problem,

$$\min_{D,W,A,X} \left\| Y - DX \right\|^2_F + \alpha \left\| Q - AX \right\|^2_F + \beta \left\| H - WX \right\|^2_F, \quad \text{s.t.} \quad \left\| x_i \right\|_0 \leq T_0$$

(3)

where $Q = [q_1, q_2, \ldots, q_N] \in \mathbb{R}^{K \times N}$ is an ideal representation matrix and $A$ is a linear transformation matrix.

2.2 Dictionary learning method proposed by Xu et al.

In order to promote the robustness of the learned dictionary to variations in the original training samples, such as illumination and expression changes in face recognition, Xu et al. proposed a dictionary learning framework which takes the diversity of training samples into account, and the objective function is formulated as follows,

$$\min_{D,X} \left\| Y - DX \right\|^2_F + \alpha \left\| Y_{alter} - DX \right\|^2_F + \beta \left\| X \right\|^2_F, \quad \text{s.t.} \quad \left\| d_i \right\|^2 = 1, \ i = 1, 2, \ldots, K$$

(4)

where $Y_{alter}$ is the data matrix for the alternative training data. For the scenario of insufficient training data, $Y_{alter}$ can be obtained by generating virtual samples of the training samples. For instance, we can employ the mirror face images of the training data to form $Y_{alter}$, and Fig. 2 presents an original face image and its mirror face image, these two images belong to the same individual but they have different poses. Therefore, by introducing the mirror face images, diversity of training samples can be promoted to some extent. For large-scale training data, we can simply divide it into two parts with the same size and treat the first part and the second part as the original and virtual training samples, respectively.
3 Proposed approach

From the formulation of Eq. (4), we can see that there is no supervised information involved in the process of dictionary learning, which leads to limited performance for pattern classification. For classification tasks, utilization of label information of training data can bring improved results. Therefore, to enhance the performance of dictionary learning approach presented in Ref. 22, we propose an efficient structured dictionary learning (ESDL) algorithm which incorporates an ideal regularization term. By introducing this term, label information of training data and dictionary atoms are associated. The objective function of our proposed ESDL is formulated as,

$$\begin{align*}
\min_{D,X} & \|Y - DX\|_F^2 + \alpha \|Y_{alter} - DX\|_F^2 + \beta \|X\|_F^2 + \gamma \|X - Q\|_F^2, \\
\text{s.t.} & \|d_i\|^2 = 1, i = 1, 2, \ldots, K
\end{align*}$$

(5)

where $Q = [q_1, q_2, \ldots, q_N] \in \mathbb{R}^{K \times N}$ is an ideal representation matrix formed by the label information of training data and dictionary atoms, $q_i = [0, 0, \ldots, 1, 1, \ldots, 0, 0]^T \in \mathbb{R}^{K \times 1}$. The entries in $q_i$ are 1 when the training samples and the dictionary atoms have the same class label. An illustration of $Q$ is shown in Fig. 3, suppose $Y = [y_1, y_2, \ldots, y_{10}]$ and $D = [d_1, d_2, \ldots, d_6]$, where $y_1, y_2$ and $y_3$ belong to the first class, $y_4, y_5, y_6$ and $y_7$ belong to the second class, and $y_8, y_9$ and
\(y_{10}\) belong to the third class. \(D\) has 3 sub-dictionaries and each has 2 atoms. As we can see from Fig. 3, \(Q\) exhibits a block-diagonal structure.

![Fig 3 The ideal representation matrix \(Q\) for the training data \(Y\).](image)

3.1 Optimization

We employ alternative strategy to optimize Eq. (5), i.e., update one variable when the other is fixed. The detailed updating procedures are presented as follows.

Update \(X\): when \(D\) is fixed, Eq. (5) degenerates into the following problem,

\[
\min_{X} \|Y - DX\|_F^2 + \alpha \|Y_{\text{alter}} - DX\|_F^2 + \beta \|X\|_F^2 + \gamma \|X - Q\|_F^2
\]

Eq. (6) has the following closed-form solution,

\[
X = (D^T D + \alpha D^T D + \beta I + \gamma I)^{-1} (D^T Y + \alpha D^T Y_{\text{alter}} + \gamma Q)
\]

Update \(D\): when \(X\) is fixed, \(D\) can be updated by solving the following problem,

\[
\min_{D} \|Y - DX\|_F^2 + \alpha \|Y_{\text{alter}} - DX\|_F^2
\]
Eq. (8) has the following closed-form solution,

$$D = \left( YX^T + \alpha Y_{\text{alter}}X^T \right) \left( XX^T + \alpha XX^T \right)^{-1}$$  \hspace{2cm} (9)

At the beginning of optimization of Eq. (5), dictionary $D$ is initialized via K-SVD, i.e., K-SVD is performed on the training data of each class to obtain a sub-dictionary, then all the sub-dictionaries are concatenated to form the whole dictionary. Based on the label information of training samples and atoms in the dictionary, the ideal representation matrix $Q$ can be constructed. Then Eq. (5) can be optimized by iteratively updating $X$ and $D$. The complete optimization process of Eq. (5) is outlined in Algorithm 1.

Algorithm 1 Optimization process of Equation (5)

**Input:** Training data matrix $Y$, alternative training data $Y_{\text{alter}}$, parameters $\alpha$, $\beta$, and $\gamma$.

1. Initialize $D$ via K-SVD, construct the ideal representation matrix $Q$;
2. while not converged do
3. Update $X$ by Eq. (7);
4. Update $D$ by Eq. (9);
5. end while

**Output:** The learned dictionary $D$ and the coefficient matrix $X$ of training data.

### 3.2 Classification Scheme

When the dictionary learning process is completed, the learned dictionary $D$ and representation matrix $X$ of training data are obtained. Based on the representation matrix $X$ and label matrix $H$ of training data, a linear classifier can be learned by solving the following problem,

$$W = \arg \min_W \|H - WX\|_F^2 + \lambda \|W\|_F^2$$  \hspace{2cm} (10)
Eq. (10) has closed-form solution, which is formulated as,

\[ W = H X^T (X X^T + \lambda I)^{-1} \]  

(11)

To classify a test sample \( y \), first we obtain its coefficient vector \( x \) via orthogonal matching pursuit (OMP) algorithm,\(^{23}\) then the label for \( y \) is given by,

\[ \text{identity}(y) = \arg \max_k (g_k), \text{ where } g = Wx \]  

(12)

The classification procedures of our proposed method are summarized in Algorithm 2.

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**Algorithm 2** Classification process of our proposed method

**Input:** The learned dictionary \( D \), the coefficient matrix \( X \) of training data, label matrix \( H \) of training data and test sample \( y \).

1: Obtain the linear classifier \( W \) via (11);
2: Compute the coding vector \( x \) of test sample \( y \) via OMP;
3: Calculate \( g = Wx \);

**Output:** identity\((y) = \arg \max_k (g_k)\).

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**4 Experimental results and analysis**

In this section, we evaluate the classification performance of our proposed ESDL on five benchmark datasets: the Extended Yale B database, the AR database, the PIE database, the LFW database, and the Scene 15 dataset. To illustrate the superiority of ESDL, we compare ESDL with the following approaches: SRC,\(^{24}\) CRC,\(^{25}\) K-SVD,\(^{4}\) D-KSVD,\(^{5}\) LC-KSVD,\(^{6}\) SVDGL\(^{11}\) and the method in Ref. 22. There are three parameters in our proposed ESDL, on the four face databases, \( \alpha, \beta \) and \( \gamma \) are set to 1e-4, 1e-3 and 1e-3, respectively, while on the Scene 15 dataset, \( \alpha, \beta \) and \( \gamma \) are set to 0.1, 1e-4 and 1e-4, respectively. Apart from the recognition accuracy, we also present
the training time and testing time (in seconds) of all the competing methods. All experiments are run with MATLAB R2019b under Windows 10 on a PC equipped with Intel i9-8950HK 2.90 GHz CPU and 32 GB RAM.

4.1 Experiments on the Extended Yale B Database

There are 2414 face images of 38 subjects in the Extended Yale B database, and these images have variations in illumination, some example images are shown in Fig. 4. Each individual contains 59-64 images. In our experiments, all images are resized to 32×32 pixels. Twenty images per person (the first five images per person are always selected, and the other fifteen images per person are randomly selected from the remaining of the images) are used as training samples and the rest as test samples. We repeat the experiments ten times and record the average recognition accuracy. Experimental results are summarized in Table 4.1. It can be seen that the proposed algorithm achieves a higher average recognition accuracy than its competing approaches. Moreover, ESDL is very efficient in terms of training and testing time.

Fig 4 Example images from the Extended Yale B database.
### Table 1  Recognition results on the Extended Yale B database.

| Methods   | Accuracy (%) | Training time (s) | Testing time (s) |
|-----------|--------------|-------------------|------------------|
| SRC²⁴     | 95.3         | No Need           | 1.56             |
| CRC²⁵     | 95.0         | No Need           | 0.82             |
| K-SVD⁴    | 94.0         | 1.82              | 0.35             |
| D-KSVD⁵   | 94.3         | 22.80             | 0.42             |
| LC-KSVD⁶  | 92.7         | 38.08             | 0.43             |
| SVGDL¹¹   | 93.8         | 43.82             | 0.12             |
| Xu’s DL²² | 95.6         | 2.87              | 0.56             |
| ESDL      | **95.9**     | 2.72              | 0.44             |

#### 4.2 Experiments on the AR Database

The AR face database includes over 4000 images of 126 subjects, each subject has 26 images collected in two separate sessions which vary in expression, illumination and disguise (wearing sunglasses or scarves), example images are shown in Fig. 5. As the experimental setting in [], we use a subset of the AR face database containing 3120 images for 120 subjects (65 men and 55 women). The size of the images is $40 \times 50$ pixels.

In our experiments, seven undisguised images in session 1 and one occluded image per subject are used as training samples (the first face image per subject with sunglasses in sessions 1 and 2 are not used as training samples). Therefore, sixteen images per subject (seven undisguised images in session 2 and the rest nine occluded images) are used for testing. Experiments are repeated for ten times and the average recognition results are shown in Table 4.2. One can see that ESDL outperforms the others in accuracy, and its training and testing time are comparable to those of Xu’s method.²² ESDL is 437 times faster than SVGDL for the training phase, about 24 times faster than LC-KSVD.
The AR database\cite{18} consists of over 4000 face images of 126 subjects. For each subject, 26 images are taken during two different sessions with large variations in terms of facial disguise, illumination and expressions. Fig. 5a shows example images from the database. For our experiments, a 165×120 face image was projected onto a 540-dimensional vector using a random projection matrix. Thus, the used samples are the Random-Face features. We followed a common experimental protocol by selecting a subset of 2600 images of 50 male and 50 female subjects from the database. For each subject, 10 random images were chosen to create the training data and the remaining images were used for testing. The error tolerance of SRC is 0.05, the balancing parameter of CRC is 0.0014. The sparsity level and number of atoms for D-KSVD and LC-KSVD are 50 and 600, respectively. 7 label-particular atoms for each class and 5 common atoms in COPAR. Sparsity level and of SA-CRC are set to be 50 and 0.002, respectively. 23% reduction in the error rate of WCRC.

### Table 2 Recognition results on the AR database.

| Methods   | Accuracy (%) | Training time (s) | Testing time (s) |
|-----------|--------------|-------------------|------------------|
| SRC\textsuperscript{24} | 72.2         | No Need           | 65.19            |
| CRC\textsuperscript{25} | 71.4         | No Need           | 53.36            |
| K-SVD\textsuperscript{4} | 78.8         | 3.40              | 0.44             |
| D-KSVD\textsuperscript{5} | 74.1         | 69.03             | 0.70             |
| LC-KSVD\textsuperscript{6} | 74.2         | 131.76            | 0.64             |
| SVGDL\textsuperscript{11} | 78.0         | 2403.67           | 0.22             |
| Xu’s DL\textsuperscript{22} | 79.8         | 5.74              | 0.77             |
| ESDL     | 80.2         | 5.50              | 0.64             |

### 4.3 Experiments on the PIE Database

The PIE database contains 41,368 front-face images of 68 subjects, and the images of each subject are captured under 13 different poses, 43 different illumination conditions, and 4 different facial expressions, example images from this database are depicted in Fig. 6.

Following the common experimental settings, we choose the five near-frontal poses (C05, C07, C09, C27, C29) of each subject and use all the images under different illumination conditions and facial expressions. Thus we obtain 170 images for each subject. Each image is normalized to the size of 32×32 pixels. Ten images per subject (including the first five images) are randomly selected.
as training samples and the remaining as test samples. Experiments are repeated for ten times the average results are listed in Table 4.3. It can be observed that ESDL achieves the highest accuracy. Specifically, it outperforms Xu’s method and SVGDL by 0.6% and 1.2%, respectively. Meanwhile, ESDL is 426 times faster than SVGDL.

### 4.4 Experiments on the LFW Database

We use a subset of the LFW database which contains 1215 images of 86 individuals. Example images from this database are shown in Fig. 7. In our experiments, six images per person are randomly selected as training samples and the remaining as test samples. Experiments are repeated for ten times and the average recognition accuracy is summarized in Table 4.4. It can be seen from Table 4.4 that the performance gains of our proposed ESDL is significant on this dataset. It
Table 4 Recognition results on the LFW database.

| Methods       | Accuracy (%) | Training time (s) | Testing time (s) |
|---------------|--------------|-------------------|------------------|
| SRC$^{24}$    | 30.3         | No Need           | 3.28             |
| CRC$^{25}$    | 24.8         | No Need           | 0.49             |
| K-SVD$^4$     | 29.7         | 1.65              | 0.10             |
| D-KSVD$^5$    | 19.6         | 8.51              | 0.14             |
| LC-KSVD$^6$   | 19.4         | 11.03             | 0.14             |
| SVGDL$^{11}$  | 29.1         | 606.48            | 0.02             |
| Xu’s DL$^{22}$| 33.1         | 0.93              | 0.14             |
| ESDL          | **35.4**     | 0.93              | 0.13             |

outperforms Xu’s method by 2.3%. And it is very efficient in terms of training time.

4.5 Experiments on the Scene 15 Dataset

Scene 15 dataset has 15 natural scene categories, which comprises a wide range of indoor and outdoor scenes, such as bedroom, office and mountain, example images from this dataset are shown in Fig. 8. Each category has 200-400 images, and the average image size is about 250×300 pixels. For fair comparison, we employ the 3000-dimensional SIFT-based features used in LC-KSVD.$^6$ According to the experimental settings in Ref. 22, 100 images per category are randomly selected as training data and the rest as test data. For the training samples, the first 50 images per category are used as original training samples and the other 50 images as alternative training samples. The number of atoms in the learned dictionary is 450. Experimental results are listed in Table 4.5. We
Scene 15 dataset has 15 natural scene categories, which comprises a wide range of indoor and outdoor scenes, such as bedroom, office, suburb, street, and livingroom. Experiments on the Scene 15 dataset demonstrate that ESDL is not only suitable for face recognition, but for scene categorization as well. Actually, our proposed ESDL is a general framework, which can be applied to other pattern classification tasks.

| Methods       | Accuracy (%) | Training time (s) | Testing time (s) |
|---------------|--------------|-------------------|-----------------|
| SRC\(^{24}\)  | 91.8         | No Need           | 52.68           |
| CRC\(^{25}\)  | 95.8         | No Need           | 45.26           |
| K-SVD\(^{4}\) | 86.7         | 9.05              | 0.49            |
| D-KSVD\(^{5}\) | 89.1         | 63.44             | 0.52            |
| LC-KSVD\(^{6}\) | 92.9         | 77.86             | 0.53            |
| SVGDL\(^{11}\) | 95.4         | 226.07            | 0.08            |
| Xu’s DL\(^{22}\) | 97.5         | 1.86              | 0.56            |
| ESDL          | 97.7         | 1.74              | 0.57            |

Table 5 Recognition results on the Scene 15 dataset.

We also plot the confusion matrix for ESDL in Fig. 9, in which diagonal elements are well-marked. It can be seen that ESDL obtains 100% recognition accuracy for categories of suburb, street, and livingroom. Experiments on the Scene 15 dataset demonstrate that ESDL is not only suitable for face recognition, but for scene categorization as well. Actually, our proposed ESDL is a general framework, which can be applied to other pattern classification tasks.

There are three parameters in the formulation of our proposed ESDL, i.e., $\alpha$, $\beta$ and $\gamma$ in Eq. 5, $\beta$ is usually set to a relatively small value ($1e-4$ or $1e-3$) in our experiments. To examine how the remaining parameters $\alpha$ and $\gamma$ influence the performance of ESDL, we conduct experiments on

![Example images from the Scene 15 dataset.](image)

**Fig 8** Example images from the Scene 15 dataset.

4.6 Parameter Analysis

There are three parameters in the formulation of our proposed ESDL, i.e., $\alpha$, $\beta$ and $\gamma$ in Eq. 5, $\beta$ is usually set to a relatively small value ($1e-4$ or $1e-3$) in our experiments. To examine how the remaining parameters $\alpha$ and $\gamma$ influence the performance of ESDL, we conduct experiments on
the LFW database. Experimental setting is the same as in Section 4.4 and the number of training samples per subject is 6. Fig. 10 illustrates the effect of parameter selection. One can see that the recognition performance of ESDL is stable when the value of parameter $\alpha$ varies in quite a wide range, i.e., $[10^{-6}, 0.1]$. Meanwhile, ESDL achieves better accuracy when $\gamma$ has relatively small value, i.e., $[10^{-6}, 10^{-3}]$. According to the above experimental results, we set $\alpha = 10^{-4}$ and $\gamma = 0.001$ on the LFW database.

5 Conclusions

We proposed an efficient structured dictionary learning (ESDL) method in which both the diversity and label information of training samples are considered. By introducing the ideal regularization term, label information of training data and the dictionary atoms are associated. Moreover, ESDL imposes $\ell_2$-norm constraint instead of the $\ell_1$-norm on the coefficient matrix which makes ESDL
computationally efficient. It is worth noting that our proposed ESDL can be applied in a variety of pattern classification tasks. By generating virtual training samples, ESDL can tackle the problem of insufficient training samples. When providing large scale training samples, we can simply divide the original training set into two parts and feed them into the framework of ESDL. Experimental results on five well known datasets demonstrate the superiority of ESDL over some state-of-the-art DL approaches.

In this paper, we did not explicitly consider the situation that both the training and test samples are contaminated due to occlusion or corruption, thus in future, we will introduce low rank matrix recovery (LRMR) technique into ESDL to tackle the above scenarios.
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