Errata Note: Discovering Order Dependencies through Order Compatibility

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ABSTRACT

A number of extensions to the classical notion of functional dependencies have been proposed to express and enforce application semantics. One of these extensions is that of order dependencies (ODs), which express rules involving order. The article entitled “Discovering Order Dependencies through Order Compatibility” by Consonni et al., published in the EDBT conference proceedings in March 2019, investigates the OD discovery problem. They claim to prove that their OD discovery algorithm, OCDDISCOVER, is complete, as well as being significantly more efficient in practice than the state-of-the-art. They further claim that the implementation of the existing FASTOD algorithm (ours)—we shared our code base with the authors—which they benchmark against is flawed, as OCDDISCOVER and FASTOD report different sets of ODs over the same data sets.

In this rebuttal, we show that their claim of completeness is, in fact, not true. Built upon their incorrect claim, OCDDISCOVER’s pruning rules are overly aggressive, and prune parts of the search space that contain legitimate ODs. This is the reason their approach appears to be “faster” in practice. Finally, we show that Consonni et al. misinterpret our set-based canonical form for ODs, leading to an incorrect claim that our FASTOD implementation has an error.

1. INTRODUCTION

Integrity constraints specify the intended semantics of dataset attributes. They are commonly used in a number of application areas, such as schema design, data integration, data cleaning, and query optimization [2]. Past work focused primarily on functional dependencies (FDs). In recent years, several extensions to the notion of an FD have been studied, including that of order dependencies (ODs) [3][4][5][6][7][8][9]. FDs cannot capture relationships among attributes with naturally ordered domains, such as over timestamps, numbers, and strings, which are common in business data [9]. For example, consider Table 1, which shows employee tax records in which tax is calculated as a percentage of salary. Both tax and percentage increase with salary.

Order dependencies naturally express such semantics. For a second example from Table 1, the OD salary orders group, subgroup holds. When the table is sorted by salary, it is then sorted by group (with ties broken by subgroup). However, salary orders subgroup, group does not hold. This illustrates that the order of attributes matters.

The theory of order dependency subsumes that of functional dependency. Any FD can be mapped to an equivalent OD by prefixing the left-hand-side attributes onto the right-hand side [8][10]. For example, if salary functionally determines tax, then salary orders salary, tax.

The purpose of this article is to refute the following claims in Consonni et al. [3].

1. The authors present a definition of minimality for order compatibility dependencies (OCDs). An OCD is a more specific form of order dependency in which two lists of attributes order each other, when taken together [8]. Consonni et al. [3] claim that their definition of minimality is complete; that is, from it, one can discover all valid OCDs that hold over a given table.

2. Given their definition of minimal OCDs, Consonni et al. [3] propose an algorithm to discover ODs via OCDs, which has factorial complexity in the number of attributes. They claim to prove that their algorithm produces a canonically complete set of ODs. (That is, a minimal set of ODs with respect to their definition, from which all the ODs which hold over the data could purportedly be inferred.)

3. The authors claim that their experimental evaluation illustrates an implementation error in our implementation of our OD discovery algorithm (FASTOD) [6][7], which leads to ours discovering many additional—and, purportedly, incorrect—dependencies. In spite of this claim of an “implementation error” in the FASTOD implementation that we provided them, they support via benchmark experiments that their algorithm, OCDDISCOVER, outperforms our algorithm, FASTOD.

We show that each of these three claims is incorrect, in turn.

1. The definition of minimality in Consonni et al. [3]—insofar as its intended purpose is a canonical form—is incorrect. Their “canonical” form does not allow for the inference of all OCDs. It misses an important subclass of OCDs (and, respectively, ODs), any dependency which has a common prefix on the left and right (that is, repeated attributes at the beginning of the dependency).

2. The claim of completeness of the OD discovery algorithm in Consonni et al. [3] is incorrect, as it relies upon their incorrect notion of “minimal” OCDs. Their conjecture that their algorithm is complete is incorrect; it is incomplete.

3. Consonni et al. [3] misinterpret our set-based canonical form for ODs [5][7] (which is equivalent to the list-based canonical form for ODs). This leads the authors to confuse set-based OCDs with ODs. Their claim that our implementation has an error arises from this, and their belief that their approach is complete. Consonni et al. [3] conclude that their algorithm is faster in practice, despite being significantly worse in asymptotic complexity.
This arises in their benchmark experiments, however, due to the fact that their algorithm is incomplete.

In Section 2, we provide basic definitions and canonical forms for ODs. In Section 3, we analyze the completeness of OD discovery. In Section 4, we discuss the experimental evaluation conducted by Consonni et al. [3]. We conclude in Section 5.

2. FOUNDATIONS

2.1 Background

We use the notational conventions in Table 2. Next, we provide a summary of the relevant definitions. The operator \( \preceq \) defines a weak total order over any set of tuples, where \( X \) denotes a list of attributes. Unless otherwise specified, numbers are ordered numerically, strings are ordered lexicographically and dates are ordered chronologically.

**Definition 1.** [6] Let \( X \) be a list of attributes. For two tuples \( t \) and \( s \), \( X \subseteq Y \) if:

- \( X = [] \); or
- \( X = [A | T] \) and \( I_A < S_A \); or
- \( X = [A | T] \), \( I_A = S_A \), and \( t \preceq_T s \).

Let \( t \sim_X s \) if \( t \preceq_X s \) but \( s \not\preceq_X t \).

Next, we define order dependencies.

**Definition 2.** [3] Let \( X \) and \( Y \) be lists of attributes over a relation schema \( R \). A table \( t \) over \( R \) satisfies an OD \( X \rightarrow Y \) \((r \models X \rightarrow Y)\), read as \( X \) orders \( Y \), if for all \( t, s \in r, t \sim_X s \) implies \( t \sim_Y s \). \( X \rightarrow Y \) is said to hold \( r \models X \rightarrow Y \) if, for each admissible table instance \( r \) of \( R \), table \( r \) satisfies \( X \rightarrow Y \). \( X \rightarrow Y \) is trivial if, for all \( r, t \models X \rightarrow Y, X \rightarrow Y, \) read as \( X \) and \( Y \) are order equivalent, if \( X \rightarrow Y \) and \( Y \rightarrow X \).

The OD \( X \rightarrow Y \) means that \( Y \) values are monotonically non-decreasing \( r \), but not necessarily vice versa.

**Example 1.** Consider Table 1 in which tax is calculated as a percentage of salary, and tax groups and subgroups are based on salary. Tax, percentage, and group are not decreasing with salary. Furthermore, within the same group, subgroup are not decreasing with salary. Finally, within the same year, bin increases with salary. Thus, the following order dependencies hold in that table: \( \{\text{salary} \} \rightarrow \{\text{tax} \}, \{\text{salary} \} \rightarrow \{\text{percentage} \}, \{\text{salary} \} \rightarrow \{\text{group, subgroup} \}, \text{and} \{\text{year, salary} \} \rightarrow \{\text{year, bin} \} \).

**Definition 3.** [3] Two order specifications \( X \) and \( Y \) are order compatible, denoted as \( X \sim \prec \sim Y \), if \( XY \rightarrow YY \). ODs in the form of \( X \rightarrow Y \) are called order compatible dependencies (OCDs)

### Table 1: Table with employee information.

| # | ID | yr | posit | bin | sal | perc | tax | grp | subg |
|---|----|----|-------|-----|-----|------|-----|-----|------|
| 1 | 10 | 19 | secr  | 1   | 5K  | 20%  | 1K  | A   | III  |
| 2 | 11 | 19 | mmgr | 2   | 8K  | 25%  | 2K  | C   | II   |
| 3 | 12 | 19 | direct | 3   | 10K | 30%  | 3K  | D   | I    |
| 4 | 10 | 18 | secr  | 1   | 4.5K| 20%  | 0.9K| A   | III  |
| 5 | 11 | 18 | mmgr | 2   | 6K  | 25%  | 1.5K| C   | I    |
| 6 | 12 | 18 | direct | 3   | 8K  | 25%  | 2K  | C   | II   |

### Table 2: Notational conventions.

- **Relations.** \( R \) denotes a relation schema and \( r \) denotes a specific table instance. Letters from the beginning of the alphabet, \( A, B, \) and \( C \), denote single attributes. Additionally, \( t \) and \( s \) denote tuples, and \( I_A \) denotes the value of an attribute \( A \) in a tuple \( t \).

- **Sets.** Letters from the end of the alphabet, \( X, Y, \) and \( Z \), denote sets of attributes. Also, \( TX \) denotes the projection of a tuple \( t \) on \( X \). \( XY \) is shorthand for \( X \cup Y \). The empty set is denoted as \( \{ \} \).

- **Lists.** \( X, Y, \) and \( Z \) denote lists. The empty list is represented as \( [ \] \). List \( [A, B, C] \) denotes an explicit list. \([A | T] \) denotes a list with the head \( A \) and the tail \( T \). \( XY \) is shorthand for \( X \) concatenate \( Y \). Set \( X \) denotes the set of elements in list \( X \) \( X' \) denotes an arbitrary permutation of list \( X \) or set \( X \). Given a set of attributes \( X \), for brevity, we state \( \forall i, X_i \) to indicate indices \( [1, \ldots, i] \) that have valid ranges \( (i < |X|) \).

The empty list of attributes (i.e., \( [ \] \)) is order compatible with any list of attributes. There is a strong relationship between ODs and FDs. Any OD implies an FD, modulo lists and sets, however, not vice versa.

**Lemma 1.** [8,10] If \( R \models X \rightarrow Y \) (OD), then \( R \models X \rightarrow Y \) (FD).

Also, there is a correspondence between FDs and ODs.

**Theorem 1.** [8,10] \( R \models X \rightarrow Y \) iff \( X \rightarrow XY \), for any list \( X \) over the attributes of \( X \) and any list \( Y \) over the attributes of \( Y \).

ODs can be violated in two ways.

**Theorem 2.** [8,10] \( R \models X \rightarrow Y \) (OD) iff \( R \models X \rightarrow XY \) (FD) and \( X \rightarrow Y \) (OCD).

We are now ready to explain the two sources of OD violations: splits and swaps [3, 10]. An OD \( X \rightarrow Y \) can be violated in two ways, as per Theorem 2.

**Definition 4.** [8,10] A split wrt an OD \( X \rightarrow XY \) (FD) is a pair of tuples \( s \) and \( t \) such that \( sx = tx \) but \( sy \neq ty \).

**Definition 5.** [8,10] A swap wrt \( X \rightarrow Y \) (OCD) is a pair of tuples \( s \) and \( t \) such that \( s \sim_X t \), but \( t \not\sim_X s \).

**Example 2.** In Table 1 there are three splits with respect to the OD \( \{\text{position} \} \rightarrow \{\text{position, salary} \} \) because position does not functionally determine salary. The violating tuple pairs are \( t1 \) and \( t4 \), \( t2 \) and \( t5 \), and \( t3 \) and \( t6 \). There is a swap wrt \( \{\text{salary} \} \sim \{\text{subgroup} \} \), e.g., over pair of tuples \( t1 \) and \( t2 \).

2.2 Canonical Forms

Consonni et al. [3] use a native list-based canonical form, which is based on decomposing an OD into a FD and an OCD [8,10]. Recall that based on Theorem 2, OD = FD + OCD, as \( X \rightarrow YY \) (FD) and \( X \rightarrow Y \) (OCD). The authors exploit this relationship to guide their discovery algorithm through order compatibility. Since they use a list-based representation for ODs, this leads to factorial complexity of OD discovery in the number of attributes.

Expressing ODs in a natural way relies on lists of attributes, as in the SQL order-by statement. One might well wonder whether
lists are inherently necessary. We provide a polynomial mapping of list-based ODs into equivalent set-based canonical ODs [6, 7]. The mapping allows us to develop an OD discovery algorithm that traverses a much smaller set-containment lattice (to identify candidates for ODs) rather than the list-containment lattice used in Consonni et al. [3].

Two tuples, \( t \) and \( s \), are equivalent over a set of attributes \( \mathcal{X} \) if \( t_x = s_x \). An attribute set \( \mathcal{X} \) partitions tuples into equivalence classes [4]. We denote the equivalence class of a tuple \( t \) over a set \( \mathcal{X} \) as \( E(t_x) \), i.e., \( E(t_x) = \{ s \in r | s_x = t_x \} \). A partition of \( r \) over \( \mathcal{X} \) is the set of equivalence classes, \( \Pi_r = \{ E(t_x) | t \in r \} \).

In Table 1, \( E(t_{\{\text{year}\}}) = E(t_{\{\text{year}\}}) = \{ t_{\{\text{year}\}}, t_{\{\text{id}, \text{year}\}}, \{ t_{\{\text{id}, \text{year}\}}, t_{\{\text{t}, \text{year}\}} \} \).

We present a set-based canonical form for ODs.

**Definition 6.** [6, 7] An attribute \( A \) is a constant within each equivalence class over \( \mathcal{X} \), denoted as \( \mathcal{X}: [ \] \( A \rightarrow \mathcal{A} \), if \( A \rightarrow \mathcal{X}A \)\). Furthermore, two attributes, \( A \) and \( B \), are order-compatible within each equivalence class wrt \( \mathcal{X} \), denoted as \( \mathcal{A}: A \sim B \), if \( A \rightarrow \mathcal{X}A \sim \mathcal{X}B \). ODs of the form of \( \mathcal{A}: \[ \] \rightarrow \mathcal{A} \) and \( \mathcal{A}: A \sim B \) are called (set-based) canonical ODs, and the set \( \mathcal{X} \) is called a context.

**Example 3.** In Table 1, a bin is a constant in the context of position \( \text{position} \), written as \( [ \] \( A \rightarrow \mathcal{A} \), bin]. This is because \( E(t_{\{\text{position}\}}) = \{ [ ] \rightarrow \text{bin} \}, E(t_{\{\text{age}\}}) = \{ [ ] \rightarrow \text{bin} \} \). Also, there is no swap between bin and salary in the context of year, i.e., \( \{ \text{year} \} \): bin \( \sim \) salary. This is because \( E(t_{\{\text{year}\}}) = [ ] \rightarrow \text{salary} \) and \( E(t_{\{\text{age}\}}) = [ ] \rightarrow \text{salary} \).

Based on Theorem 3 and Theorem 4, list-based ODs in the form of FDs and OCDS, respectively, can be mapped into equivalent set-based ODs.

**Theorem 3.** [6, 7] \( \mathcal{R} \models X \rightarrow Y \) \( \forall A \in Y, \mathcal{R} \models \mathcal{X}: [ ] \rightarrow A \).

**Theorem 4.** [6, 7] \( \mathcal{R} \models X \sim Y \) \( \forall \langle \text{between left and right (repeated attributes) can hold over a table}, \mathcal{R} \models \langle X \sim Y \rangle \) \( \forall A \in X, \mathcal{R} \models \mathcal{X}: [ ] \rightarrow A \) and \( \forall \langle \text{between left and right (repeated attributes) can hold over a table}, \mathcal{R} \models \langle X \sim Y \rangle \).

**Example 4.** An OD \( [AB] \rightarrow [CD] \) can be mapped into the following equivalent canonical ODs: \( \{A, B\}: [ ] \rightarrow C \), \( \{A, B\}: [ ] \rightarrow D \), \( \{A, C\}: B \sim C \), \( \{A, C\}: A \sim D \), \( \{A, C\}: B \sim D \).

3. **Completeness Analysis**

While the theoretical search space for FASTOD [6] is \( O(2^{|\mathcal{R}|}) \), the search space for ODDISCOVER [3] is \( O(|\mathcal{R}|^3) \), which is much larger as it traverses a lattice of attribute permutations (where \( |\mathcal{R}| \) denotes the number of attributes over a relational schema \( \mathcal{R} \)). To mitigate the factorial complexity, the list-based algorithm in Consonni et al. [3] uses pruning rules. We show that, despite the authors’ claim that their approach discovers a canonically complete set of ODs, their pruning rules lead to incompleteness.

Section 3 in Consonni et al. [3] addresses their completeness “proof” for their OD discovery algorithm. The authors introduce a notion of minimality of a set of dependencies which is incorrect. Herein, a set of dependencies is called minimal if all dependencies that logically hold over a relation schema \( \mathcal{R} \) can be inferred from this minimal (canonical) set of dependencies [3]. That is, a set of dependencies \( \mathcal{M} \) is minimal over a table \( \mathcal{R} \) if \( \{X \rightarrow Y \} \models \mathcal{M} \models \{X \rightarrow Y \} \) is equivalent to \( \{X \rightarrow Y \} \models \{X \rightarrow Y \} \).

Thus, one should be able to infer from a minimal set of dependencies via the inference rules (axioms), \( \mathcal{T} \), all the dependencies that are valid over the given instance of the table. That is, \( \{X \rightarrow Y \} \models \mathcal{M} \models \{X \rightarrow Y \} \) is equal to \( \{X \rightarrow Y \} \models \{X \rightarrow Y \} \). Consonni et al. [3] use the set of sound and complete OD inference rules, \( \mathcal{T} \), from [9, 10].

Pruning applied by a dependency discovery algorithm, thus, must respect minimality. This allows for the implicit discovery of the full set of valid dependencies, and thus be deemed complete.

**Example 5.** \( \{A, B, C\} \) is not minimal if \( \{A, B, C\} \models \{A, B\} \).

It follows then that an OCD is minimal in [3] if and only if there are no repeated attributes in the OCD. That is, there are no repeated attributes within the left or within the right list of the minimal OCD, as each is a minimal attribute list, and there is no repeated attribute between left and right.

**Definition 8.** [3] An OCD \( \mathcal{X} \sim Y \) is minimal if
- \( \mathcal{X} \) and \( Y \) are minimal attribute lists and
- \( \mathcal{X} \cap Y = \emptyset \).

Definition 8 of minimality with no permitted repeated attributes is at the heart of their incompleteness problem, as it does not allow for the inference of all the dependencies that are valid over the given table. Theorem 5 states this, that an OCD with a common prefix between left and right (repeated attributes) can hold over a table, while no OCD without repeated attributes holds. Our proof of Theorem 5 is by example, offering a simple counterexample to the completeness premise in [3].

**Theorem 6.** \( \mathcal{R} \not\models \langle Y \sim Z, \mathcal{R} \not\models \langle XY \sim Z \rangle \) and \( \mathcal{R} \not\models \langle Y \sim XZ \rangle \) do not imply \( \mathcal{R} \not\models \langle XY \sim XZ \rangle \).

**Proof.** It suffices to construct a table in which the OCD of the form
- \( \mathcal{X} \sim \mathcal{Z} \)
holds, but OCDs
- \( \mathcal{Y} \sim \mathcal{Z} \)
- \( \mathcal{XY} \sim \mathcal{Z} \)
- \( \mathcal{Y} \sim \mathcal{XZ} \)
do not.

Consider Table 1, constructed over attributes \( \mathcal{A}, \mathcal{B} \) and \( \mathcal{C} \). In Table 1, an OCD \( \mathcal{A}, \mathcal{B} \not\models \mathcal{A}, \mathcal{C} \) holds, but \( \mathcal{B} \sim \mathcal{C} \), \( \mathcal{AB} \not\models \mathcal{AC} \), and \( \mathcal{B} \not\models \mathcal{AC} \) do not.

In [3], the authors only show—as is stated in Theorem 7 below—that OCDs of the form \( \mathcal{XY} \sim \mathcal{XZ} \) can be derived from \( \mathcal{Y} \sim \mathcal{Z} \) (Theorem 3.5 via Theorem 3.10 in [3]).

**Theorem 7.** [3] If \( \mathcal{R} \models \langle Y \sim Z \rangle \) then \( \mathcal{R} \models \langle XY \sim XZ \rangle \)

2In some previous work [1], minimal dependencies \( \mathcal{M} \) also satisfy an additional condition that no proper subset of \( \mathcal{M} \) can be used to infer all dependencies.
Table 3: Showing incompleteness.

| # | A | B | C |
|---|---|---|---|
| t1 | 0 | 0 | 1 |
| t2 | 1 | 1 | 0 |
| t3 | 2 | 3 | 2 |
| t4 | 3 | 2 | 3 |

Figure 1: Lattice permutation tree.

Table 4: Verifying correctness of implementation.

| # | A | B | C | D |
|---|---|---|---|---|
| t1 | 1 | 3 | 1 | 1 |
| t2 | 2 | 3 | 3 | 2 |
| t3 | 2 | 3 | 2 | 2 |
| t4 | 2 | 5 | 2 | 2 |
| t5 | 3 | 1 | 2 | 3 |
| t6 | 4 | 4 | 4 | 2 |
| t7 | 4 | 5 | 3 | 2 |

Theorem[7][3] is true. The flaw in their logic is that this theorem proves only one direction (the “if” of an intended “if and only if”). The “only if” (not proved by the theorem) is implicitly assumed to be true, though (while it assuredly is not). It follows that their claim of canonical completeness for their definition of minimal OCDs is incorrect (Section 3.3 in [3]). OCDs with common prefixes between its left and right attribute lists are not redundant, by Theorem[6]

This leads to an incomplete approach for OD discovery, as the recovery of the full set of valid dependencies is not possible.

Details of the OD discovery algorithm, OCDDISCOVER, by Consonni et al. [3] are presented in their Section 4. Let \( U \) be a set of attributes over a relation schema \( R \). In the first level of the lattice, they generate candidates of the form \( A \sim B \), where \( A, B \in U \) and \( A \neq B \). (An OCD \( B \sim A \) is not generated as it is equivalent to \( A \sim B \).) At each level of the lattice (Fig. 1), if the candidate \( X \sim Y \) is order compatible, they generate dependencies for the next level of the lattice. For each attribute not already present in the OCD, for each attribute \( A \in U \setminus \{ X \cup Y \} \), they add it to the right of each attribute list; i.e., \( XA \sim Y \) and \( X \sim YA \). Thus, important OCDs with repeated attributes in a common prefix are never considered (as is consistent with their incorrect definition of minimality for OCDs). For example, an OCD \([\text{year, month}] \sim [\text{year, week}] \) would be missed. As a consequence, the authors do not discover ODs with repeated attributes, such as \([\text{year, salary}] \Rightarrow [\text{year, bin}] \) (recall Table 1).

In contrast, our FASTOD algorithm[6][7] is complete for OD discovery. It does not miss dependencies with common prefixes. This is because the algorithm considers as candidates dependencies of the set-based form: \( \text{OCD} \{ A \}: A \sim B \). Thus, dependencies with common prefixes are considered. (This is built into the context, set-based notation used in [6][7], and cannot be missed when using this representation.)

4. EXPERIMENTAL ANALYSIS

We demonstrate that the experimental analysis in Consonni et al. [3] that compares their OD discovery algorithm, OCDDISCOVER, with ours, FASTOD [6][7], is incorrect. The authors misinterpret the set-based canonical representation for ODs as introduced in [6][7] and as used in FASTOD. They conflate OCDs and ODs as we report them when reporting the results. In [6][7], we report the numbers of found FDs and OCDs. In [3], they incorrectly report these as the FDs and ODs, respectively, that we found. This occurs in their Table 6, where, for instance, they report 400 ODs and 89,571 FDs found by FASTOD, whereas this should be 400 OCDs and 89,571 FDs, respectively.

As a consequence of this misunderstanding of the set-based canonical representation for ODs [6][7], the authors in [3] claim that the implementation of FASTOD finds ODs that are not present in the data. As an example of this, they provide the OD \([B] \Rightarrow [A,C] \) over Table 4 (Table 4 is not present. Therefore, the authors confuse the OCD \([B]: A \sim C \) with the OD \([B] \Rightarrow [A,C] \). Consequently, they falsely assert that the reason the number of ODs found by OCDDISCOVER and FASTOD differ is due to an implementation error in the implementation of FASTOD that we provided them. The real reason that the number of reported dependencies differ, however, is, of course, that OCDDISCOVER [3] is incomplete. The claim that they outperform the state-of-art despite a much worst asymptotic complexity, when tested in practice on real datasets, is invalid.

The authors in Consonni et al. [3] also state that FASTOD considers all columns to be of type string, while their code also considers real and integer numbers. While a minor point, we wish to clarify that the implementation we sent the authors does discover ODs over data types including real and integer numbers. The dependencies 1–10 reported in Table 4 remain the same, regardless of using numerical or string data type, given that the values are in the range of 1 to 5.

While Consonni et al. [3] state that they were not able to isolate and resolve the root cause of what they felt was incorrect behavior in the implementation of FASTOD (which we had provided to them at their request for “ensuring fairness and reproducibility”), they never contacted us to help resolve it.
5. CONCLUSIONS

In this article, we have conducted a detailed analysis of the correctness of the results in the recent article by Consonni et al. [3] concerning the order dependency discovery problem. We have shown that, for the main claimed results related to the OD discovery problem, there are fundamental errors and omissions in the proof or experiments.

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