EXTRACTION OF $\gamma$

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Abstract

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SETTING THE STAGE

The recent observation of CP violation in the $B$ system by the BaBar and Belle collaborations [1] manifests the beginning of a new era in the exploration of particle physics. One of the central goals of the $B$-factories is to overconstrain the unitarity triangle of the Cabibbo–Kobayashi–Maskawa (CKM) matrix as much as possible through independent measurements both of its sides and of its angles $\alpha$, $\beta$ and $\gamma$ [2].

A particularly important element in this stringent test of the Kobayashi–Maskawa mechanism of CP violation is the direct determination of the angle $\gamma$. The corresponding experimental values may well be in conflict with the indirect results provided by the fits of the unitarity triangle, yielding at present $\gamma \sim 60^\circ$ [3]. Moreover, we may encounter discrepancies between various different approaches to determine $\gamma$ directly. In such exciting cases, the data may shed light on the physics lying beyond the Standard Model.

Key Problem in the Determination of $\gamma$

At leading order of the well-known Wolfenstein expansion of the CKM matrix, all matrix elements are real, apart from

$$V_{td} = |V_{td}|e^{-i\beta} \quad \text{and} \quad V_{ub} = |V_{ub}|e^{-i\gamma}.$$

(1)

In non-leptonic $B$-meson decays, we may obtain sensitivity on $\gamma$ through interference effects between different CKM amplitudes. Making use of the unitarity of the CKM matrix, it can be shown that at most two different weak amplitudes contribute to a given non-leptonic decay $B \to f$, so that we may write

$$A(B \to f) = |A_1|e^{i\delta_1} + e^{+i\gamma}|A_2|e^{i\delta_2}, \quad A(B \to \bar{f}) = |A_1|e^{i\delta_1} + e^{-i\gamma}|A_2|e^{i\delta_2},$$

(2)
where $\gamma$ enters through $V_{ub}$ and the $|A_{1,2}|e^{i\delta_{1,2}}$ denote CP-conserving strong amplitudes. Consequently, the corresponding direct CP asymmetry takes the following form:

$$A_{\text{CP}} = \frac{|A(B \to f)|^2 - |A(B \to \bar{f})|^2}{|A(B \to f)|^2 + |A(B \to \bar{f})|^2} = \frac{2|A_1||A_2|\sin(\delta_1 - \delta_2)\sin\gamma}{|A_1|^2 + 2|A_1||A_2|\cos(\delta_1 - \delta_2)\cos\gamma + |A_2|^2}. \quad (3)$$

Measuring such a CP asymmetry, we may in principle extract $\gamma$. However, due to hadronic uncertainties, which affect the strong amplitudes

$$|A|e^{i\delta} \sim \sum_k C_k(\mu) \times \frac{\langle f | Q_k(\mu) | B \rangle}{\text{pert. QCD}} \quad \text{“unknown”}, \quad (4)$$

a reliable determination of $\gamma$ is actually a challenge.

**Major Approaches to Determine $\gamma$**

In order to deal with the problems arising from the hadronic matrix elements, we may employ one of the following approaches:

- The most obvious one – and, unfortunately, also the most challenging from a theoretical point of view – is to try to calculate the $\langle f | Q_k(\mu) | B \rangle$. In this context, interesting progress has recently been made through the development of the “QCD factorization” [4, 5] and perturbative hard-scattering (or “PQCD”) [6] approaches, as discussed by Neubert and Keum, respectively, at this symposium. As far as the determination of $\gamma$ is concerned, $B \to \pi K$, $\pi\pi$ modes play a key rôle.

- Another avenue we may follow is to use decays of neutral $B_{d,-}$ or $B_s$-mesons, where interference effects between $B^0_d$-$\bar{B}^0_d$ mixing ($q \in \{d,s\}$) and decay processes arise. There are fortunate cases, where hadronic matrix elements cancel:
  - Decays of the kind $B_d \to D^{(*) \pm} \pi^\mp$, allowing a clean extraction of $\phi_d + \gamma$ [7], where the $B^0_d$-$\bar{B}^0_d$ mixing phase $\phi_d = 2\beta$ can be fixed through $B_d \to J/\psi K_S$.
  - Decays of the kind $B_s \to D^{(*) \pm}_s K^{(*) \mp}$, allowing a clean extraction of $\phi_s + \gamma$ [8], where the $B^0_s$-$\bar{B}^0_s$ mixing phase $\phi_s$ is negligibly small in the Standard Model, and can be probed through $B_s \to J/\psi\phi$ modes.

- An important tool to eliminate hadronic uncertainties in the extraction of $\gamma$ is also provided by certain amplitude relations. There are two kinds of such relations:
  - Exact relations between $B^\pm \to K^\pm \{D^0, \bar{D}^0, D^0_{\pm}\}$ amplitudes [9], where $D^0_{\pm}$ denotes the CP eigenstates of the neutral $D$ system. This approach is realized in an ideal way in the $B^\pm_c \to D^\pm \{D^0, \bar{D}^0, D^0_{\pm}\}$ system [10]. Unfortunately, $B_c$-mesons are not as accessible as “ordinary” $B_{u,d}$- or $B_d$-mesons.
  - Amplitude relations, which are implied by the flavor symmetries of strong interactions, i.e. isospin or $SU(3)$ [11, 12]. In the corresponding strategies to determine $\gamma$, we have to deal with $B_s \to \pi\pi, \pi K, KK$ modes.
In the following discussion, we shall focus on the latter kind of strategies, involving the $B \to \pi K$ system, and the $U$-spin-related $B_d \to \pi^+ \pi^-$, $B_s \to K^+ K^-$ and $B_d \to \pi^+ K^\pm$, $B_s \to \pi^\pm K^\mp$. These approaches are particularly promising from a practical point of view: BaBar, Belle and CLEO-III can probe $\gamma$ nicely through $B \to \pi K$ modes, whereas the $U$-spin strategies, requiring also the measurement of $B_s$-meson decays, are interesting for Tevatron-II and can be fully exploited at BTeV and the LHC experiments.

**EXTRACTION OF $\gamma$ FROM $B \to \pi K$ DECAYS**

Let us first point out some interesting features of the $B \to \pi K$ system. Because of the small ratio $|V_{u_s} V_{u_d}^\ast/(V_{t_s} V_{t_d}^\ast)| \approx 0.02$, these decays are dominated by QCD penguin topologies, despite their loop suppression. Due to the large top-quark mass, we have also to care about electroweak (EW) penguins. In the case of $B^+ \to \pi^+ K^0$ and $B_d^0 \to \pi^- K^+$, these topologies contribute only in color-suppressed form and are hence expected to play a minor rôle, whereas they contribute also in color-allowed form to $B_d^0 \to \pi^0 K^0$ and $B^+ \to \pi^0 K^+$ and may here even compete with tree-diagram-like topologies.

Using the isospin flavor symmetry of strong interactions, we may derive the following amplitude relations:

$$\sqrt{2} A(B^+ \to \pi^0 K^+) + A(B^+ \to \pi^+ K^0) = \sqrt{2} A(B_d^0 \to \pi^0 K^0) + A(B_d^0 \to \pi^- K^+)$$

$$= -\left[|T + C|e^{i\delta_T + C} e^{-i\gamma} + P_{\text{ew}}\right] \propto [e^{-i\gamma} + q_{\text{ew}}] ,$$

where $T$ and $C$ denote the strong amplitudes of color-allowed and color-suppressed tree-diagram-like topologies, respectively, $P_{\text{ew}}$ is due to color-allowed and color-suppressed EW penguins, $\delta_{T+C}$ is a CP-conserving strong phase, and $q_{\text{ew}}$ denotes the ratio of the EW to tree-diagram-like topologies. A relation with an analogous phase structure holds also for the “mixed” $B^+ \to \pi^+ K^0$, $B_d^0 \to \pi^- K^+$ system. Because of these relations, the following combinations of $B \to \pi K$ decays were considered in the literature to probe $\gamma$:

- The “mixed” $B^\pm \to \pi^\pm K$, $B_d \to \pi^\mp K^\pm$ system [13]–[16].
- The “charged” $B^\pm \to \pi^\pm K$, $B_d \to \pi^0 K^\pm$ system [17]–[19].
- The “neutral” $B_d \to \pi^0 K$, $B_d \to \pi^\mp K^\pm$ system [19, 20].

Interestingly, already CP-averaged $B \to \pi K$ branching ratios may lead to non-trivial constraints on $\gamma$ [14, 17]. In order to determine this angle, also CP-violating rate differences have to be measured. To this end, we introduce the following observables [19]:

$$\left\{ \begin{array}{c}
R \\
A_0
\end{array} \right\} \equiv \frac{\text{BR}(B_d^0 \to \pi^- K^+) \pm \text{BR}(B_d^0 \to \pi^+ K^-)}{\text{BR}(B^+ \to \pi^+ K^0) + \text{BR}(B^- \to \pi^- K^0)} \frac{\tau_{B^+}}{\tau_{B_d^0}}$$ (6)

$$\left\{ \begin{array}{c}
R_c \\
A_0^c
\end{array} \right\} \equiv 2 \left[ \frac{\text{BR}(B^+ \to \pi^0 K^+) \pm \text{BR}(B^- \to \pi^0 K^-)}{\text{BR}(B^+ \to \pi^+ K^0) + \text{BR}(B^- \to \pi^- K^0)} \right]$$ (7)

---

1 $U$ spin is an $SU(2)$ subgroup of $SU(3)_C$, relating down and strange quarks to each other.
\[
\begin{align*}
&\left\{ \frac{R_n}{A_0^{}(c,n)} \right\} = \frac{1}{2} \left[ \frac{\text{BR}(B_d^0 \to \pi^- K^+) \pm \text{BR}(B_d^0 \to \pi^+ K^-)}{\text{BR}(B_d^0 \to \pi^0 K^0) + \text{BR}(B_d^0 \to \pi^0 K^0)} \right], \\
&\text{where the } R_{(c,n)} \text{ are ratios of CP-averaged branching ratios and the } A_0^{(c,n)} \text{ represent CP-violating observables.}
\end{align*}
\]

If we employ the \( SU(2) \) flavor symmetry, which implies (5), and make plausible dynamical assumptions, concerning mainly the smallness of certain rescattering processes, we obtain parametrizations of the following kind [16, 19] (for alternative ones, see [18]):

\[
R_{(c,n)}, A_0^{(c,n)} = \text{functions } (q_{(c,n)}, r_{(c,n)}, \delta_{(c,n)}, \gamma). 
\]

Here \( q_{(c,n)} \) denotes the ratio of EW penguins to “trees”, \( r_{(c,n)} \) is the ratio of “trees” to QCD penguins, and \( \delta_{(c,n)} \) the strong phase between “trees” and QCD penguins for the mixed, charged and neutral \( B \to \pi K \) systems, respectively. The EW penguin parameters \( q_{(c,n)} \) can be fixed through theoretical arguments: in the mixed system [13]–[15], we have \( q \approx 0 \), as EW penguins contribute only in color-suppressed form; in the charged and neutral \( B \to \pi K \) systems, \( q_c \) and \( q_d \) can be fixed through the \( SU(3) \) flavor symmetry without dynamical assumptions [17]–[20]. The \( r_{(c,n)} \) can be determined with the help of additional experimental information: in the mixed system, \( r \) can be fixed through arguments based on factorization [4, 13, 15] or \( U \)-spin, as we will see below, whereas \( r_c \) and \( r_n \) can be determined from the CP-averaged \( B^\pm \to \pi^\pm \pi^0 \) branching ratio by using only the \( SU(3) \) flavor symmetry [11, 17]. The uncertainties arising in this program from \( SU(3) \)-breaking effects can be reduced through the QCD factorization approach [4, 5], which is moreover in favour of small rescattering processes. For simplicity, we shall neglect such FSI effects below; more detailed discussions can be found in [21].

Since we are in a position to fix the parameters \( q_{(c,n)} \) and \( r_{(c,n)} \), we may determine \( \delta_{(c,n)} \) and \( \gamma \) from the observables given in (9). This can be done separately for the mixed, charged and neutral \( B \to \pi K \) systems. It should be emphasized that also CP-violating rate differences have to be measured to this end. Using just the CP-conserving observables \( R_{(c,n)} \), we may obtain interesting constraints on \( \gamma \). In contrast to \( q_{(c,n)} \) and \( r_{(c,n)} \), the strong phase \( \delta_{(c,n)} \) suffers from large hadronic uncertainties. However, we can get rid of \( \delta_{(c,n)} \) by keeping it as a “free” variable, yielding minimal and maximal values for \( R_{(c,n)} \):

\[
R_{(c,n)}^{\text{ext}} \bigg|_{\delta_{(c,n)}} = \text{function } (q_{(c,n)}, r_{(c,n)}, \gamma). 
\]

Keeping in addition \( r_{(c,n)} \) as a free variable, we obtain another – less restrictive – minimal value for \( R_{(c,n)} \):

\[
R_{(c,n)}^{\text{min}} \bigg|_{r_{(c,n)}, \delta_{(c,n)}} = \text{function } (q_{(c,n)}, \gamma) \sin^2 \gamma. 
\]

These extremal values of \( R_{(c,n)} \) imply constraints on \( \gamma \), since the cases corresponding to \( R_{(c,n)}^{\text{exp}} < R_{(c,n)}^{\text{min}} \) and \( R_{(c,n)}^{\text{exp}} > R_{(c,n)}^{\text{max}} \) are excluded. The present experimental status is summarized in Table 1. We observe that both the CLEO and the Belle data point towards \( R_c > 1 \) and \( R_n < 1 \), whereas the central values of the BaBar collaboration are close to one, with a small preference of \( R_c > 1 \).
TABLE 1. Present experimental status of the observables $R_{(c,n)}$. For the evaluation of $R$, we have used $\tau_{B^+/\tau_B^0} = 1.060 \pm 0.029$.

|       | CLEO [22]  | BaBar [23] | Belle [24] |
|-------|------------|------------|------------|
| $R$   | 1.00 ± 0.30| 0.97 ± 0.23| 1.50 ± 0.66|
| $R_c$ | 1.27 ± 0.47| 1.19 ± 0.35| 2.38 ± 1.12|
| $R_n$ | 0.59 ± 0.27| 1.02 ± 0.40| 0.60 ± 0.29|

In Fig. 1, we show the dependence of (10) and (11) on $\gamma$ for the neutral $B \to \pi K$ system; the charged $B \to \pi K$ curves look very similar [20]. Here the crossed region below the $R_{\text{min}}$ curve, which is described by (11), is excluded. On the other hand, the shaded region is the allowed range (10) for $R_n$, arising in the case of $r_n = 0.17$. This figure allows us to read off immediately the allowed range for $\gamma$ for a given value of $R_n$. In order to illustrate this feature, let us assume that $R_n = 0.6$ has been measured, which would be in accordance with the central values of CLEO and Belle in Table 1. In this case, the $R_{\text{min}}$ curve implies $0^\circ \leq \gamma \leq 19^\circ$ or $97^\circ \leq \gamma \leq 180^\circ$. If we use additional information on $r_n$, we may put even stronger constraints on $\gamma$. For $r_n = 0.17$, we obtain, for instance, the allowed range $134^\circ \leq \gamma \leq 180^\circ$. It is interesting to note that the $R_{\text{min}}$ curve is only effective for $R_n < 1$. Assuming $R_c = 1.3$ to illustrate implications of the CP-averaged charged $B \to \pi K$ branching ratios, (11) is not effective and $r_c$ has to be fixed in order to constrain $\gamma$. Using $r_c = 0.21$, we obtain $84^\circ \leq \gamma \leq 180^\circ$. Although it is too early to draw definite conclusions, it is important to emphasize that the second quadrant for $\gamma$, i.e. $\gamma \geq 90^\circ$, would be preferred in these cases. Interestingly, such a situation would be in conflict with the standard analysis of the unitarity triangle [3], yielding $\gamma \approx 60^\circ$. Here the stringent present experimental lower bound on $\Delta M_s$ implies $\gamma < 90^\circ$.

![FIGURE 1](image-url)  
**FIGURE 1.** The dependence of the extremal values of $R_n$ on $\gamma$ for $r_n = 0.68$.  

Another “puzzle” may arise from CP-conserving strong phases, which can also be constrained through the observables $R_{(c,n)}$ [20]. Interestingly, the CLEO and Belle data point towards $\cos \delta_c > 0$ and $\cos \delta_n < 0$, whereas we would expect equal signs for these quantities. Moreover, $\cos \delta_n < 0$ would be in conflict with factorization.

Consequently, the present CLEO and Belle data point towards a “puzzling” situation, whereas no such discrepancies are indicated by the results of the BaBar collaboration. It is of course too early to draw any definite conclusions. However, if future data should confirm such a picture, it may be an indication for new physics or large flavor-symmetry-breaking effects [20]. Further studies are desirable to distinguish between these cases. Since $B \rightarrow \pi K$ modes are governed by penguin processes, they actually represent sensitive probes for new physics [25].

Due to the recent theoretical progress in the description of $B \rightarrow \pi K, \pi\pi$ decays, the theoretical uncertainties of $r_{c,n}$ and $q_{c,n}$ can be reduced to the level of [5] $O\left(\frac{1}{N_C} \times \frac{m_s - m_d}{\Lambda_{QCD}} \times \frac{\Lambda_{QCD}}{m_b}\right) = O\left(\frac{1}{N_C} \times \frac{m_s - m_d}{m_b}\right)$, (12)

and confidence into dynamical assumptions related to rescattering effects can be gained. Making more extensive use of QCD factorization, approaches complementary to the ones discussed above, which rely on a “minimal” input from theory, are provided. As a first step, we may use that the CP-conserving strong phase $\delta_c$ is predicted to be very small in QCD factorization, so that $\cos \delta_c$ governing $R_c$ is close to one. As a second step, information on $\gamma$ can be obtained from the predictions for the branching ratios and the observables $R_{(c,n)}$. Finally, the information from all CP-averaged $B \rightarrow \pi K, \pi\pi$ branching ratios can be combined into a single global fit for the allowed region in the $\rho-\eta$ plane [5, 26]. For these approaches, it is of course crucial that the corrections entering in the QCD factorization formalism at the $\Lambda_{QCD}/m_b$ level can be controlled reliably. As argued in a recent paper [27], non-perturbative contributions with the same quantum numbers as penguin topologies with internal charm- and up-quark exchanges may play an important rôle in this context. The issue of $\Lambda_{QCD}/m_b$ corrections in phenomenological analyses will certainly continue to be a hot topic in the future.

**U-SPIN STRATEGIES**

Let us now focus on strategies to extract $\gamma$ from pairs of $B$-meson decays, which are related to each other through the $U$-spin flavor symmetry of strong interactions. In order to deal with non-leptonic $B$ decays, $U$-spin offers an important tool, and first approaches to extract CKM phases were already pointed out in 1993 [28]. However, the great power of the $U$-spin symmtery to determine weak phases and hadronic parameters was noticed just recently in the strategies proposed in [29]–[32]. Since these methods involve also decays of $B_s$-mesons, $B$ experiments at hadron colliders are required to implement them in practice. At Tevatron-II, we will have first access to the corresponding modes and interesting results are expected [33]. In the era of BTeV and the LHC, the $U$-spin strategies can then be fully exploited [34], as emphasized by Stone at this symposium.
In the following discussion, we shall focus on two particularly promising approaches, using the $B_d \rightarrow \pi^+\pi^-$, $B_s \rightarrow K^+K^-$ [30] and $B_d \rightarrow \pi^\pm K^\mp$, $B_s \rightarrow \pi^\pm K^\mp$ [31] systems.

**Extraction of $\beta$ and $\gamma$ from $B_d \rightarrow \pi^+\pi^-$, $B_s \rightarrow K^+K^-$ Decays**

Looking at the corresponding Feynman diagrams, we observe that $B_s \rightarrow K^+K^-$ is obtained from $B_d \rightarrow \pi^+\pi^-$ by interchanging all down and strange quarks. The structure of the corresponding decay amplitudes is given as follows [30]:

\[
A(B_d^0 \rightarrow \pi^+\pi^-) = C \left[ e^{i\gamma} - de^{i\theta} \right]
\]  

(13)

\[
A(B_s^0 \rightarrow K^+K^-) = \lambda C' \left[ e^{i\gamma} + \left( \frac{1 - \lambda^2}{\lambda^2} \right) d'e^{i\theta'} \right],
\]  

(14)

where $C$, $C'$ are CP-conserving strong amplitudes, and $de^{i\theta}$, $d'e^{i\theta'}$ measure, sloppily speaking, ratios of penguin to tree amplitudes. Using these general parametrizations, we obtain the following expressions for the direct and mixing-induced CP asymmetries:

\[
\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+\pi^-) = \text{function}(d, \theta, \gamma)
\]  

(15)

\[
\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+\pi^-) = \text{function}(d, \theta, \gamma, \phi_d = 2\beta)
\]  

(16)

\[
\mathcal{A}_{\text{CP}}^{\text{dir}}(B_s \rightarrow K^+K^-) = \text{function}(d', \theta', \gamma)
\]  

(17)

\[
\mathcal{A}_{\text{CP}}^{\text{mix}}(B_s \rightarrow K^+K^-) = \text{function}(d', \theta', \gamma, \phi_s \approx 0).
\]  

(18)

Consequently, we have four observables, depending on six “unknowns”. However, since $B_d \rightarrow \pi^+\pi^-$ and $B_s \rightarrow K^+K^-$ are related to each other by interchanging all down and strange quarks, the $U$-spin flavor symmetry of strong interactions implies

\[
d'e^{i\theta'} = de^{i\theta}.
\]  

(19)

Using this relation, the four observables (15)–(18) depend on the four quantities $d$, $\theta$, $\phi_d = 2\beta$ and $\gamma$, which can hence be determined. It should be emphasized that no dynamical assumptions about rescattering processes have to be made in this approach, which is an important conceptual advantage in comparison with the $B \rightarrow \pi K$ strategies discussed above. The theoretical accuracy is hence only limited by $U$-spin-breaking effects. Theoretical considerations allow us to gain confidence into (19), which does not receive $U$-spin-breaking corrections in factorization [30]. Moreover, there are general relations between observables of $U$-spin-related decays, allowing experimental insights into $U$-spin breaking [29, 30, 35, 36].

The $U$-spin arguments can be minimized, if we employ the $B_d^0 - \overline{B_d^0}$ mixing phase $\phi_d = 2\beta$ as an input, which can be determined straightforwardly through $B_d \rightarrow J/\psi K_S$. The observables $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+\pi^-)$ and $\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+\pi^-)$ allow us then to eliminate the strong phase $\theta$ and to determine $d$ as a function of $\gamma$. Analogously, $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_s \rightarrow K^+K^-)$ and $\mathcal{A}_{\text{CP}}^{\text{mix}}(B_s \rightarrow K^+K^-)$ allow us to eliminate the strong phase $\theta'$ and to determine $d'$ as
a function of $\gamma$. The corresponding contours in the $\gamma$–$d$ and $\gamma$–$d'$ planes can be fixed in a theoretically clean way. Using now the $U$-spin relation $d' = d$, these contours allow the determination both of the CKM angle $\gamma$ and of the hadronic quantities $d$, $\theta$, $\theta'$; for a detailed illustration, see [30].

This approach is very promising for Tevatron-II and the LHC era, where experimental accuracies for $\gamma$ of $O(10^\circ)$ [33] and $O(1^\circ)$ [34] may be achieved, respectively. It should be emphasized that not only $\gamma$, but also the hadronic parameters $d$, $\theta$, $\theta'$ are of particular interest, as they can be compared with theoretical predictions, thereby allowing valuable insights into hadron dynamics. For strategies to probe $\gamma$ and constrain hadronic penguin parameters using a variant of the $B_d \to \pi^+ \pi^-$, $B_s \to K^+ K^-$ approach, where the latter decay is replaced through $B_d \to \pi^\mp K^\pm$, the reader is referred to [37].

**Extraction of $\gamma$ from $B_{(s)} \to \pi K$ Decays**

Another interesting pair of decays, which are related to each other by interchanging all down and strange quarks, is the $B^0_d \to \pi^- K^+$, $B^0_s \to \pi^+ K^-$ system [31]. In the strict $U$-spin limit, the corresponding decay amplitudes can be parametrized as follows:

$$A(B^0_d \to \pi^- K^+) = -P \left(1 - re^{i\delta} e^{i\gamma}\right),$$

$$A(B^0_s \to \pi^+ K^-) = P \sqrt{\varepsilon} \left(1 + \frac{1}{\varepsilon} re^{i\delta} e^{i\gamma}\right),$$

where $P$ denotes a CP-conserving complex amplitude, $\varepsilon \equiv \lambda^2/(1 - \lambda^2)$, $r$ is a real parameter, and $\delta$ a CP-conserving strong phase. At first sight, it appears as if $\gamma$, $r$, and $\delta$ could be determined from the ratio of the CP-averaged rates and the two CP asymmetries provided by these modes. However, because of the relation

$$|A(B^0_d \to \pi^- K^+)|^2 - |A(B^0_s \to \pi^+ K^-)|^2 = 4r \sin \delta \sin \gamma = -\left[|A(B^0_s \to \pi^+ K^-)|^2 - |A(B^0_d \to \pi^- K^+)|^2\right],$$

we have actually only two independent observables, so that the three parameters $\gamma$, $r$, and $\delta$ cannot be determined. To this end, the overall normalization $P$ has to be fixed, requiring a further input. Assuming that rescattering processes play a minor rôle and that color-suppressed EW penguins can be neglected as well, the isospin symmetry implies

$$P = A(B^+ \to \pi^+ K^0).$$

In order to extract $\gamma$ and the hadronic parameters, it is useful to introduce observables $R_s$ and $A_s$ by replacing $B_d \to \pi^\pm K^\mp$ through $B_s \to \pi^\pm K^\mp$ in (6). Using (20), (21) and (23), we then obtain

$$R_s = \varepsilon + 2r \cos \delta \cos \gamma + \frac{r^2}{\varepsilon}$$

Note that these observables are independent of $P$.  

---

2 Note that these observables are independent of $P$.  

\[ A_s = -2r \sin \delta \sin \gamma = -A_0. \]  

(25)

Together with the parametrization for \( R \) as sketched in (9), these observables allow the determination of all relevant parameters. The extraction of \( \gamma \) and \( \delta \) is analogous to the “mixed” \( B_d \to \pi^\mp K^\pm, B^\pm \to \pi^\pm K \) approach discussed above. However, now the advantage is that the \( U \)-spin counterparts \( B_s \to \pi^\pm K^\mp \) of \( B_d \to \pi^\mp K^\pm \) allow us to determine also the parameter \( r \) without using arguments related to factorization [31]:

\[ r = \sqrt{\frac{\epsilon [R + R_s - 1 - \epsilon]}{1 + \epsilon}}. \]  

(26)

On the other hand, we still have to make dynamical assumptions concerning rescattering and color-suppressed EW penguin effects. The theoretical accuracy is further limited by \( SU(3) \)-breaking effects. An interesting consistency check is provided by the relation \( A_s = -A_0 \), which is due to (22). A variant of this approach using the CKM angle \( \beta \) as an additional input was proposed in [38].

**CONCLUSIONS**

There are many strategies to determine \( \gamma \), suffering unfortunately in several cases from experimental problems. The approaches discussed above, employing penguin processes, are on the other hand very promising from a practical point of view and exhibit further interesting features. As a by-product, they also allow us to determine strong phases and other hadronic parameters, allowing comparisons with theoretical predictions. Moreover, these strategies are sensitive probes for the physics lying beyond the Standard Model, which may lead to discrepancies in the extraction of \( \gamma \) or the hadronic quantities. Let us hope that signals for new physics will actually emerge this way.

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