Flux-driven gyrokinetic simulations of ion turbulent transport at low magnetic shear

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Abstract.

Ion Temperature Gradient driven turbulence is investigated with the global full-$f$ gyrokinetic code GYSELA for different magnetic equilibria. Reversed shear and monotonous $q$ profile cases do not exhibit dramatic changes nor in the dynamics nor in the level of turbulence, leading to similar mean profiles. Especially, no transport barrier is observed in the vicinity of $s = 0$ in the general case, although the radial extent of the gap without resonant modes is larger than the typical turbulence correlation length. Conversely, a transport barrier is found to develop in the gap region if non resonant modes are artificially suppressed from the simulation. Such simulations tend to reconcile previously published contradictory results, while extending the analysis to more realistic flux-driven gyrokinetic regimes.

1. Introduction

Turbulence and transport studies in tokamak plasmas primarily aim at investigating routes towards improved confinement regimes. In the core of fusion plasmas, internal transport barriers (ITBs) are commonly reported in magnetically confined fusion plasmas, provided the heating power exceeds some critical value (see [1] for a review on experimental results). Once the barrier is triggered, the following positive feed back loop is supposed to play a key role in its sustainment: the strong pressure gradient (and possibly its curvature as well) at the barrier position drives strongly sheared poloidal flows via the radial force balance; this shear has now long proven efficient in stabilizing the turbulence through decorrelation processes of convective cells [2]. Two main ingredients are invoked to explain the empirical threshold for the onset of ITBs (see e.g. [3] for a review): velocity shear and magnetic topology. The former can be either intrinsic, i.e. self-generated by the turbulence itself, or extrinsic in the presence of sources of momentum. This issue has been extensively studied in the literature and will not be addressed here (see [4] for an overview). The later can affect plasma turbulence at least in two ways: either through large (negative) magnetic shear $s = \frac{d \log q}{d \log r}$, with $q$ the safety factor profile, which is linearly stabilizing, or due to the rarefaction of resonant surfaces close to low order rational $q$ surfaces and/or low shear regions $s \approx 0$ [5]. As a matter of fact, the formation of ITBs has been experimentally associated with the presence of rational $q$ surfaces in both JET and ASDEX Upgrade plasmas [8, 9].
From the non-linear theoretical side, the possibility to trigger ITBs in the vicinity of low magnetic shear and/or low order rational $q$ values remains an open issue. Indeed, contradictory results have been obtained with first principle numerical simulations of core plasma turbulence. Fluid simulations retaining resonant modes only have reported impressive agreements with experimental observations, especially regarding the splitting of the ITB when two $q = 2$ magnetic surfaces were present in the plasma [5]. However, such results were not recovered with both local and global gyrokinetic simulations [6]. Non-resonant modes were found to play a critical role in driving finite turbulent heat flux in the vicinity of $s \approx 0$. Later on, similar simulations with fully kinetic electrons, including trapped electron modes, reported fine scale corrugations on the electron temperature and density profiles close to low order $q$ rational values, resulting from the local increase of self generated zonal flows [7].

The role of a large gap region without resonant modes in the case of reversed shear profiles is re-investigated in this paper and extended to flux-driven global gyrokinetic simulations of ion temperature gradient driven turbulence. Section 2 highlights the main equations which are solved. Two cases are then compared in section 3, with (hollow $q$ profile) and without (monotonous $q$ profile) any large radial gap in resonant modes. Finally, the case with an even larger gap is considered in section 4, when Fourier filtering only retains those modes which are resonant within the simulation domain.

2. The model
The numerical investigation is performed with the global gyro-kinetic code GYSELA, which models the self consistent time evolution of the full ion distribution function $f$ in the electrostatic limit. The electron response is taken adiabatic. The semi-Lagrangian numerical scheme is detailed in reference [10]. Most of the simulations reported in this paper are performed with the set of equations detailed in reference [11]. Such a system neglects small order terms (of order $\beta \ll 1$, the ratio of kinetic over magnetic pressure) which are proportional to the parallel current. The simulations reported in section 4 make use of the recently upgraded set of equations, which accounts for the conservative formulation consistent with modern gyro-kinetic theory [12]. They read:

$$B^*_{\parallel} \frac{\partial f}{\partial t} + \nabla \cdot \left( B^*_{\parallel} \frac{d\mathbf{x}_G}{dt} \right) f + \frac{\partial}{\partial v_{G\parallel}} \left( B^*_{\parallel} \frac{d\mathbf{v}_{G\parallel}}{dt} \right) f = \mathcal{C}(f) + S$$  \hspace{1cm} (1)

where the equations of motion of the guiding centers are given below:

$$B^*_{\parallel} \frac{d\mathbf{x}_G}{dt} = v_{G\parallel} \mathbf{B}^* + \frac{b}{e_s} \times \nabla \Xi$$  \hspace{1cm} (2)

$$B^*_{\parallel} \frac{d\mathbf{v}_{G\parallel}}{dt} = -\frac{\mathbf{B}^*}{m_s} \cdot \nabla \Xi$$  \hspace{1cm} (3)

with $\nabla \Xi = \mu_e \nabla B + e_s \nabla \bar{\phi}$

$$\mathbf{B}^* = \mathbf{B} + \frac{v_{G\parallel}}{v_{G\parallel}} \nabla \times \mathbf{b}$$

These modified equations only differ from the old ones by small terms, proportional to $(m_s v_{G\parallel}/e_s B^2)\mu_0\mathbf{\bar{\phi}}$. This ratio is indeed very small, of the order of $\rho_\star (a/q R) \ll 1$ for thermal particles ($\rho_\star = \rho_\alpha / a$, with $\rho_\alpha$ the thermal gyro-radius and $a$ the minor radius of the torus). Therefore, no significant impact on the results presented here is to be expected. Here, $\bar{\phi}$ denotes the gyro-averaged electric potential, where the Bessel function is replaced by a Padé approximation. The scalar $B^*_{\parallel}$ is simply $B^*_{\parallel} = \mathbf{B}^* \cdot \mathbf{b}$, with $\mathbf{b} = \mathbf{B}/B$ the unit vector along the magnetic field line at the guiding-center position. The magnetic equilibrium is made of tori with concentric and circular cross-sections. The Fokker-Planck type collision operator $\mathcal{C}(f)$ acts on $v_{G\parallel}$ only [13], and has been modified to preserve mass, momentum and energy. The source term
Figure 1. (a) Profiles of $q$ (bold lines) and magnetic shear $s$ (thin lines) used in the simulation runs. The triangles on the hollow $q$ profile refer to the locations of the resonant modes present in the simulation; (b) distance between resonant modes for the hollow $q$ case, normalized by the thermal gyro-radius $\rho_0$.

$S$ provides constant heat in the system. It is given a double tanh shape, localized close to the inner radial boundary of the simulation domain (see Fig. 2a). Such flux-driven simulations are particularly time-consuming since the equilibrium state typically develops on time scales of the order of the energy confinement time $\tau_E$. Since $\omega_c\tau_E$ scales like $\rho^3$ in the gyro-Bohm regime, small $\rho_*$ relevant simulations remain extremely challenging. Still, as evident in the following, turbulence reaches statistical steady state on much smaller time scales, which then becomes accessible on nowadays supercomputers.

3. Comparative dynamics with monotonous and hollow $q$ profiles

Two main simulations are reported here. They are characterized by the following set of parameters. The relatively small value of $\rho_* = 1/128$ already requires about $8.6 \times 10^9$ grid points in the 5D phase space: $N_r \times N_\theta \times N_\varphi \times N_v \times N_\mu = 256 \times 256 \times 128 \times 128 \times 8$. At the considered collisionality $\nu_*$, neoclassical transport is mainly governed by trapped particles (banana regime). Finally, the source magnitude is $S_0 = 10^{-2}$, and vanishes for $\rho > 0.3$, while the simulation domain is $0.2 < \rho < 0.8$. Similar results were obtained with smaller (factor 1/2) and larger (factor 2) forcings as well. As detailed in reference [11], the corresponding additional power $P_{\text{add}}$ that is injected in the system reads, in physical units and for a purely Deuterium plasma:

$$P_{\text{add}} \approx 2.89 \frac{R_0}{a} \frac{S_0}{\rho_*} \frac{n_0[10^{20} m^{-3}] T_0[\text{keV}]}{B_0[T]} \quad [\text{MW}]$$

(4)

where $n_0$, $T_0$ and $B_0$ are the arbitrary normalizing density, temperature and magnetic field, respectively. Considering a Deuterium plasma with $n_0 = 2.10^{19} m^{-3}$, $T_0 = 4keV$ and $B_0 = 3T$, and given the parameters of the simulations, namely $A = R_0/a = 3.2$ and $\rho_* = 1/128$, one finds that $S_0 = 10^{-2}$ would correspond to $P_{\text{add}} \approx 3.2$ MW of injected heating power.

These two simulations only differ by their prescribed $q$ profile: monotonous and hollow $q$ profiles. They are depicted on Fig. 1a. The absolute magnitude of the magnetic shear is comparable in each case. The hollow $q$ profile case, which we will hereafter refer to as "RS case" (for "reversed shear"), is characterized by a large gap of resonant modes in the vicinity of the low order rational $q_{\text{min}} 1.5$, at $\rho = 0.5$. The resonant $(m, n)$ (poloidal and toroidal wave numbers) modes are such that there exists a radial location $\rho_{mn}$ in the simulation domain where
\[ q(\rho_{mn}) = \frac{m}{n}. \] Indeed, as visible on Fig.1b, while most of the resonant modes are distant from fractions of one thermal Larmor radius \( \rho_c0 \), this normalized distance reaches about \( \Delta_{\text{gap}}/\rho_c0 \approx 9 \) at \( \rho = 0.5 \). We shall refer to this region as the ”gap” in the following. Simulations with even larger gaps have been performed (up to \( \Delta_{\text{gap}}/\rho_c0 \approx 17 \), cf. section 4) at this \( \rho_* \) value, leaving unchanged the main conclusions.

These two cases exhibit roughly the same linear growth rate. As inferred from the early time evolution of those \((m, n)\) modes which exhibit the fastest exponential growth at \( \rho = 0.5 \), they are of the order of \( \gamma_{\text{lin}} \approx 1.5 \times 10^{-3} \omega_c0 \) for the monotonous \( q \) profile, and about \( \gamma_{\text{lin}} \approx 1.17 \times 10^{-3} \omega_c0 \) for the RS case.

The time averaged profiles, averaged over the saturated non linear phase of turbulence (cf. Fig.3), are displayed on Fig.2 for both of these simulations. Neither the temperature gradient length nor the turbulent heat flux \( Q \) or the \( E \times B \) shearing rate exhibit significant differences in the gap region, comprised within \( 0.464 \leq \rho \leq 0.534 \). Conversely, the temperature is larger inside \( \rho \approx 0.7 \) for the RS case, as a consequence of the larger value of \( R/L_T \) in this case at the edge, from \( \rho \approx 0.6 \) to \( \rho \approx 0.75 \). As evident on Fig.2b, the radial shape of \( \langle R/L_T \rangle \) mimics the one of the critical gradient, as derived from [14] (we do not have included the \( \epsilon = r/R \) correction to the calculation of \( R/L_{T,\text{crit}} \) to get a better fit of the non linear results). Especially, the RS case exhibits a significant increase of \( R/L_{T,\text{crit}} \) at the edge, due to the larger values of the ratio \( s/q \) in this region. The larger \( E \times B \) shearing rate in this region (cf. Fig.2d) should contribute to further increase the effective critical gradient. Consistently, the turbulent flux is smaller at
The time evolution of the turbulent heat flux at $\rho = 0.5$ is shown on Fig. 3a. From this graph, it is evident that both simulations have reached statistical steady state on turbulent time scales. Although they share approximately the same magnitude at this location, the fluxes exhibit significant differences with respect to their dynamics. Indeed, they both exhibit skewed probability density functions, characteristic of avalanche-like dynamics and already many times reported in fluid (see e.g. [15]) and more recently gyrokinetic [16, 17, 18, 11] flux-driven simulations of turbulence. However, the RS case is characterized by much less large scale transport events, as evident from the less asymmetric histogram of the flux. Such a property possibly further reinforces the reduction of the transport level in the outer region, as noticed in Fig. 2c.

Still, avalanches are observed to cross the gap region. In previous gyrokinetic local simulations [6], it was reported that non resonant modes can fill in the gap region. Indeed, modes of the electric potential are clearly visible in this region on Fig. 4b. One possible mechanism for their excitation is when the propagation time of an avalanche event into this depleted region, namely $\tau_{aval} \approx \Delta_{gap}/v_{aval}$, becomes comparable or even smaller than the time $\tau_{damp}$ for such modes to be locally damped. On the one hand, the avalanche velocity is of the order of the diamagnetic velocity $v_{aval} \approx \rho_\star v_T$ [11]. On the other hand, the damping time is likely to be dominated by Landau damping for non resonant modes. Landau damping is all the larger since the modes are far from their resonant surface, i.e. basically since $k_{||}v_T$ is large ($\gamma_L$ scales approximately like $\gamma_L \sim -k_{||}v_T \exp\{-\frac{1}{2}(\omega/k_{||}v_T)^2\}$, with $\omega$ the real frequency of the mode, of the order of the diamagnetic frequency $\omega \approx (k_\theta \rho_\star)v_T/\rho_\star$ with $k_\theta = m/r$). In the gap region, $k_{||}v_T/\omega_\theta = \rho_\star(n + m/q)/A$ remains finite, whatever the considered couple of poloidal and toroidal mode numbers $(m, n)$. Therefore, in small $\rho_\star$ simulations, $k_{||}v_T/\omega_\theta$ remains small even in the gap region for two reasons. First because it scales like $\rho_\star$. Second because the maximum $m$ and $n$ mode numbers of the simulation must increase when decreasing $\rho_\star$ (linear modes peak at around $k_{\theta,\text{peak}} \rho_\star \approx 0.3$, which translates into $m_{\text{peak}} \approx 0.3(r/a)\rho_\star^{-1}$). In this case, high order rational $q_{\min}$ values only are likely to satisfy $(n + m/q) \neq 0$ whatever $-m_{\text{max}} \leq m \leq m_{\text{max}}$ and $-n_{\text{max}} \leq n \leq n_{\text{max}}$. In other words, for a given hollow $q$ profile, the gap width decreases with $\rho_\star$, like $\Delta_{gap} \approx \rho_\star^{1/2}(2q_{\min}/n_{\min}q_{\min}^{\prime\prime})^{1/2}$ [5]. The fact that avalanches do cross the gap then suggests that, in this case, the following hierarchy of times prevails: $\tau_{aval} < \tau_{\text{damp}}$.

The turbulent convection cells exhibit similar $k_\theta$ spectrum in both simulations, peaking...
Figure 4. Snapshots of the poloidal cross-section of the electric potential fluctuations during one large outburst in the (a) monotonous and (b) hollow $q$ profile cases.

Figure 5. (a) Time averaged Fourier spectra of the $k_\theta = nq/r$ modes at $\rho = 0.5$ and $\theta = 0$ in both cases; (b) Two point radial auto-correlation function of the electric potential. The radial window slightly exceeds the gap region $0.4 < \rho < 0.6$.

around $k_\theta \rho_c \sim 0.2 - 0.3$, Fig.5a. As far as their radial property is concerned, Figure 4b shows that they can even cross the gap (cf. for instance at $\theta \approx 0$). This striking observation is reinforced by the structure of the two-point correlation function of the electric potential fluctuations, Fig.5b. Indeed, the correlation function is even wider in the RS case than for the monotonous $q$ profile, with a long tail extending approximately to $r_{\log}/\rho_c \approx 15$. Candy et al. have already shown that, in the absence of global effects such as profile variations, the radial mode structure of those modes resonant at the edge of the gap region can significantly overlap [6]. Unfortunately, at such a still moderate $\rho_c$ value, it has not been possible with the global GYSELA code to further investigate this point, and decouple global effects from linear properties.
4. Case without non-resonant modes

In order to sort out the role played by non resonant modes in the triggering of transport barriers in low shear regions, another simulation has been run, making use of the conservative expression of the gyrokinetic equation, eq.1. Also, for the simple reason that these simulations are part of another set of runs, they also differ from the ones of section 3 by two other parameters: the number of $\mu$ values is larger by a factor 2 ($N_{\mu} = 16$), and the collisionality is smaller ($\nu_{*} = 5 \times 10^{-2}$). As far as the hollow $q$ profile is concerned, it has essentially the same shape as the one reported on Fig.1, but with $q_{\text{min}} = 2.008$ (it is slightly above 2 to prevent the existence of any resonant mode in the vanishing shear region), such that the gap region is even larger $\Delta_{\text{gap}}/\rho_{c0} \approx 17$.

The simulation run is divided in two time windows. Up to $\omega_{c0} t_0 = 128,000$, all modes are retained like in standard simulations. From $t_0$ onwards, only those modes resonant within the simulation domain are kept, the other being artificially set to zero. For each toroidal $n$ mode, the retained poloidal $m$ modes satisfy the relation $n q_{\text{min}} \leq m \leq n q_{\text{max}}$, with $q_{\text{min}}$ and $q_{\text{max}}$ the minimum and maximum values of the $q$ profile, such that a conic filter is applied in Fourier space. Notice that non-resonant modes can still exist locally (such that $(n + m/q) \neq 0$, with $m$ and $n$ still satisfying the previous relation). However, their number is reduced. It is even more reduced for the extremal $q$ values, and particularly at $q_{\text{min}}$ for hollow $q$ profiles.

The beginning of the simulation does not depart from what we have learned in the previous section. Especially, the turbulent heat flux does not exhibit any drop in the gap region, although it is significantly larger than in the previous cases, Fig.6a. From $t_0$ however, the dynamics changes dramatically. First of all, the turbulent heat flux rapidly decreases everywhere, as a result of the energy sink introduced by the conic filter. While the flux partly recovers left from the gap, i.e. close to the heat source, it remains at an extremely low value within the gap, about 2 orders of magnitude below. More surprisingly, the heat flux does completely vanish right from the gap. It is worth noticing that such a behavior is specific of this RS case. Indeed, when applying the same filter to a case with monotonous $q$ profile, the turbulent heat flux was marginally and homogeneously reduced in the whole radial domain, with no drastic change on the simulation results. Incidentally, we note in these new simulations the impact on the resonance condition of the Doppler shift $\omega_{E} = k_{\parallel} v_{E}$ induced by any mean $E \times B$ flow $v_{E} = \phi_{00}/B$.

Indeed, the most prominent $(m, n)$ modes are then such that $k_{\parallel} v_{E} + \omega_{E} \approx 0$, or alternatively...
\[ n = -(m/q)\{1 \pm (qR_0/r)v_E/v_T\}. \] Such a time dependent \((v_E)\) relationship is well fulfilled in the course of the simulation.

The impact of the conic filter on the RS simulation can be understood as follows. It introduces an asymmetry between, on the one hand, the radial regions where \(q\) is close to the extremum \(q_{\text{min}}\) and \(q_{\text{max}}\), and, on the other hand, the rest of the radial domain. Neglecting the Doppler shift discussed above, those \((m,n)\) modes satisfying \(m = n(q \pm \delta q)\) (i.e. from one side to another of the resonant condition \(m = nq\)) are excited at each position, either linearly or via nonlinear coupling. Typical values of the parallel wave vector \(k_\parallel qR \approx 1\) suggest that \(\delta q\) remains small, of the order of \(\delta q \approx 1/n\). Then, focussing on the region close to \(q_{\text{min}}\), only half of these non resonant modes are allowed when the filter is present, namely those such that \(m = n(q_{\text{min}} + \delta q)\). In the case of monotonous \(q\) profile, it is not expected to significantly impact the simulation results, since it occurs at the radial boundary of the domain, where fluctuations are usually forced to zero in global codes. Conversely, when \(q_{\text{min}}\) stands in the middle of the considered radial domain, as is the case for hollow \(q\) profiles, such a drastic reduction of the number of accessible modes is likely to become visible.

As a matter of fact, turbulence is almost completely quenched in the gap, as evident in Fig.7a: the turbulent flux stops at the entrance of the gap region. No avalanche succeeds in crossing the gap in this case. Such a behavior characterizes transport barriers. Yet, no strong temperature gradient could be observed in this too short simulation run. Indeed, the temperature profile builds up on energy confinement time scale, which is much larger than the explored time window. Also, the root mean square value of the \(E \times B\) shearing rate is lower in the gap region when the Fourier filter is active. Finally, consistently with the radial structure of the turbulent flux, turbulence eddies do not appear in the gap region. They develop characteristic structures of toroidal ITG modes in the inner region, but vanish from the inner boundary of the gap outwards, Fig.7b.

Two additional observations can be made with respect to the transport barrier. First of all, its inner boundary seems to freeze, in the sense that low frequency oscillations (the period is about \(\omega q \tau_{\text{slow}} \approx 2.10^4\)) are visible on the color plot of the flux, around e.g. \(\rho \approx 0.45\). This slow dynamics was also reported in fluid flux-driven simulations of interchange turbulence in the scrape-off layer of tokamak plasmas, for barriers triggered by strong velocity shear \[19\]. In this case, they were understood as the result of turbulence spreading into otherwise the linearly stable
sheared region. Secondly, fast oscillating outbursts are also visible, with decreasing magnitude from the inner gap boundary toward almost $\rho \approx 0.5$. Surprisingly, the high characteristic frequency is very close to the one expected for Geodesic Acoustic modes, which is equal to $\omega_{GAM} = [(2 + 1/q^2)T_e/T_i + 7/2]^{1/2}c_s/R \approx 6.10^{-3}\omega_{ci0}$ in the present simulation, as evident on Fig. 6b. Such a result is surprising since those specific modes characteristic of the GAM dynamics, namely $(m,n) = (\pm 1,0)$, are absent from the simulation when the filter is applied. As a matter of fact, a detailed analysis (not shown here) reveals that the dominant $(m,n)$ modes with $5 \leq m \leq 20$ do exhibit large frequency oscillations. This frequency increases with $m$, approximately linearly like $\omega/\omega_{ci0} \sim 3.10^{-4}(m+9)$. We do not have any explanation for this observation so far.

5. Conclusions

Flux-driven simulations of ITG turbulence in the cases of hollow and monotonous $q$ profiles have been performed with the global and full-$f$ gyrokinetic GYSELA code. In the vicinity of $q_{min}$ where $s \approx 0$, the reversed shear cases exhibit large radial gap regions without any resonant mode. Still, no transport barrier has been observed so far in these cases, although different heat source magnitudes and increasing gap widths have been explored. Conversely, turbulent transport dramatically drops (by 2 orders of magnitude) in the gap region when a conical Fourier filter is applied, which only retains those modes which are resonant in the simulation domain. Such gyrokinetic simulations tend to reconcile – and extend to the turbulence flux-driven regime – previously published contradictory results on the topic. Still, understanding and reproducing the experimental triggering of internal transport barriers in JET remain an open issue.

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References

[1] Wolf R C 2003 Plasma Phys. Control. Fusion 45 R1-R91
[2] Biglari H, Diamond P H and Terry P 1990 Phys. Fluids B 2 1
[3] Garbet X, Baranov Y, Bateman G et al 2003 Nucl. Fusion 43 975
[4] Terry P W 2000 Rev. Mod. Phys. 72 109
[5] Garbet X, Bourdelle C, Hoang G T et al 2001 Phys. Plasmas 8 2793
[6] Candy J, Waltz R E and Rosenbluth M N 2004 Phys. Plasmas 11 1879
[7] Waltz R E, Austin M E, Burrell K H and Candy J 2006 Phys. Plasmas 13 052301
[8] Challis C D et al 2002 Plasma Phys. Control. Fusion 44 1031
[9] Jolfrin E, Challis C D, Conway G D et al 2003 Nucl. Fusion 43 1167
[10] Grandgirard V, Brunetti M, Bertrand P et al 2006 J. Comput. Phys. 217 395
[11] Sarazin Y, Grandgirard V, Abiteboul A et al 2010 Nucl. Fusion 50 054004
[12] Brizard A J and Hahm T S 2007 Rev. Mod. Phys. 79 421
[13] Dif-Pradalier G, Grandgirard V, Sarazin Y, Garbet X and Ghendrih Ph 2009 Phys. Rev. Lett. 103 065002
[14] Jenko F, Dorland W and Hammett G W 2001 Phys. Plasmas 8 4096
[15] Garbet X, Sarazin Y et al 1999 Nucl. Fusion 39, 2063
[16] Idomura Y, Urano H, Aiba N and Tokuda S 2009 Nucl. Fusion 49 065029
[17] Ku S, Chang C S, Adams M et al 2006 J. Phys.: Conf. Ser. 46 87
[18] McMillan B F, Jollet S, Tran T M, Villard L, Bottino A and Angelino P 2009 Phys. Plasmas 16 022310
[19] Ghendrih Ph, Sarazin Y, Ciricolo G et al 2007 J. Nucl. Mater. 365-365 581