Coherent helicity amplitude for sequential decays

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We present a derivation of coherent helicity amplitudes for a particle decaying into multifinal states with nonzero spins. The results show that the coherent amplitudes introduce additional rotations to transform the helicities into a consistent helicity system, which allows us to add helicity amplitudes for different decay chains coherently. These rotations may have significant effects on the interference between the decay chains in the partial wave analysis.

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I. INTRODUCTION

The amplitude for a particle decaying into final states with nonzero spins can be formalized in different descriptions, such as the covariant description \[1\], the projection operators of arbitrary spin \[2\], and the helicity formalism originally developed by Jacob and Wick \[3\]. The structure of helicity formalism can be decomposed into the angular distribution and kinematic dependence part. This feature facilitates the determination of the parent particle properties, e.g., spin and parity quantum numbers in experiment, by analyzing the angular distribution of daughter particle. The relationships of the helicity formalism to the covariant and operator formalism were developed in Refs. \[4–6\].

The helicity amplitude for a sequential decay can be formalized by multiplying the amplitude of each decay chain together in a straightforward way \[6, 7\]. Then the partial decay rate is calculated by taking the sum of the helicity amplitude squared over the helicities of final states and taking the average over the parent particle spin. However, if there are multidecay chains into the same final states, their helicities may be defined in the different helicity system. This makes the helicities inconsistent when taking the sum of the decay chain amplitudes to calculate the partial decay rate. Such an issue of coherent helicity amplitude has been addressed in recent analyses \[8, 9\].

II. TWO-BODY DECAY

Considering a particle with spin \(J\) decaying into two-body final states with spin and helicity defined in Table \[II\], states of angular momentum \(J\) may be constructed in the center-of-mass (CM) system of the parent particle as \[3, 6\]

\[
|JM\lambda_1\lambda_2\rangle = a\int d\Omega D_{M,\lambda_1-\lambda_2}^{\dagger}(\phi, \theta, 0)|\phi\theta\lambda_1\lambda_2\rangle, \quad (1)
\]

where \(a\) is a normalization constant, \(\Omega(\theta, \phi)\) is the solid angle of final states, \(|\phi\theta\lambda_1\lambda_2\rangle\) is the two-particle state in the helicity basis, defined as

\[
|\theta\phi\lambda_1\lambda_2\rangle = aU[R(\Omega)]|U[L_z(p)]|s_1\lambda_1\rangle \times U[L_{-z}(p)]|s_2-\lambda_2\rangle, \quad (2)
\]

where \(R(\Omega)\) denotes a rotation which carries the \(z\) axis into the direction of momentum \(p\), and \(L_{\pm z}(p)\) is the boost along the \(\pm z\) axis, \(U[...]\) denotes operator.
Tab. I: Variables defined for the two-body decay \( J \to s_1 + s_2 \).

| Spin | J.s | s_1 | s_2 |
|------|-----|-----|-----|
| Spin z projection | M | m_1 | m_2 |
| Helicity | &lambda;_1 | &lambda;_2 |
| Momentum | 0 | p | -p |

Helicity states are related to the canonical states by

\[
|\theta &phi; &lambda;_1 &lambda;_2 \rangle = a U[L(p)|U[R(\Omega)]|s_1 &lambda;_1 \rangle \\
\times U[L(-p)|U[R(\Omega)]|s_2 - &lambda;_2 \rangle \\
= \sum_{m_1 m_2} D_{m_1, &lambda;_1}^{s_1} (\phi, &theta; ,0) \\
\times D_{m_2, -&lambda;_2}^{s_2} (\phi, &theta; ,0)|\theta &phi; m_1 m_2 \rangle.
\]

Using above equations, the element to project helicity states onto canonical states reads

\[
\langle \theta &phi; m_1 m_2 |JM &lambda;_1 &lambda;_2 \rangle = a D_{M, &lambda;_1 - &lambda;_2}^{J*} (\phi , &theta; ,0) \\
\times D_{m_1, &lambda;_1}^{s_1} (\phi , &theta; ,0) D_{m_2, -&lambda;_2}^{s_2} (\phi , &theta; ,0).
\]

The matrix element of amplitude for the two-body decay defined in canonical basis is related to that defined in the helicity basis by

\[
A_{\lambda_1}^{J}(\theta , &phi; m_1 m_2) = \langle \theta &phi; m_1 m_2 |JM|J\lambda_1 \lambda_2 \rangle \\
= a \sum_{\lambda_1 \lambda_2} \langle \theta &phi; m_1 m_2 |JM \lambda_1 \lambda_2 \rangle |JM \lambda_1 \lambda_2 |JM|J\lambda_1 \lambda_2 \rangle \\
= a \sum_{\lambda_1 \lambda_2} D_{M, &lambda;_1 - &lambda;_2}^{J*} (\phi , &theta; ,0) D_{m_1, &lambda;_1}^{s_1} (\phi , &theta; ,0) \\
\times D_{m_2, -&lambda;_2}^{s_2} (\phi , &theta; ,0) F_{\lambda_1, \lambda_2}^{J}(\theta , &phi; m_1 m_2).
\]

The amplitude for sequential decays. The solid angles for each decay are defined in the rest frame of the parent particle. Considering two different decay chains I: \( J \to s_1 + s_2, s_1 \to s_3 + s_4 \) as shown in Fig. I and II: \( J \to s_0 + s_3, s_0 \to s_2 + s_4 \), the solid angles and kinematic variables are defined in Tables II and III.

### A. Amplitude for single decay chain

Let us consider the first decay chain I, where the amplitude in the canonical state, \( A_{\lambda_1}^{a} \), can be expressed with the helicity amplitude as

\[
A_{\lambda_1}^{a}(\theta_1 , &phi_1 ; M, m_1 m_2) = a D_{m_1, &lambda_1}^{s_1} (\phi_1 , &theta_1 ,0) \\
\times D_{m_2, -&lambda_1}^{s_2} (\phi_1 , &theta_1 ,0) D_{M, &lambda_1 - &lambda_2}^{J*} (\phi_1 , &theta_1 ,0) F_{\lambda_1, &lambda_2}^{J}. (8)
\]

The amplitude for the second decay is

\[
A_{\lambda_1}^{b}(\theta_2 , &phi_2 ; m_1 m_3, m_4) = a D_{m_3, &lambda_3}^{s_3} (\phi_2 , &theta_2 ,0) \\
\times D_{m_4, -&lambda_3}^{s_4} (\phi_2 , &theta_2 ,0) D_{M, &lambda_3 - &lambda_4}^{J*} (\phi_2 , &theta_2 ,0) F_{\lambda_3, &lambda_4}^{J}. (9)
\]

The rotation \( D_{m_1, &lambda_1 - &lambda_2}^{s_1} (\phi_2 , &theta_2 ,0) \) carries the z axis into the direction of s_3 momentum, and we decompose it into two successive rotations. We first rotate the z axis into the direction of s_1 in its parent CM system, and then rotate into the direction of s_3 momentum at the s_1 CM system by the helicity angle \( \Omega_2 (\theta_2 , &phi_2) \). It follows the multiplication rule

\[
D_{m_1, &lambda_3 - &lambda_4}^{s_1,s_2} (\phi_2 , &theta_2 ,0) = a D_{m_1, &lambda_3 - &lambda_4}^{s_1,s_2} (\phi_2 , &theta_2 ,0) \\
\times D_{m_2, -&lambda_4}^{s_2} (\phi_2 , &theta_2 ,0). (10)
\]

The amplitude in the canonical state for the sequential decay is

\[
\text{Fig. 1: The orientation of the coordinate systems associated with the sequential decay } J \to s_1 + s_2, s_1 \to s_3 + s_4.
\]
Tab. II: Variables defined for the decay chain I, $J \rightarrow s_1 + s_2$, with $s_1 \rightarrow s_3 + s_4$.  

|             | Parent | Daughter 1 | Daughter 2 |
|-------------|--------|------------|------------|
| Spin        | $J$    | $s_1$      | $s_2$      |
| Spin z projection | $M$    | $m_1$      | $m_2$      |
| Helicity    | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ |
| Helicity amplitude | $F^{s_3}_{\lambda_1, \lambda_2}$ | $F^{s_3}_{\lambda_2, \lambda_3}$ |
| Solid angle | $\Omega_1(\theta_1, \phi_1)$ | $\Omega_2(\theta_2, \phi_2)$ |

The orthogonality relation, 
\[ \sum_{m_1} D^{s_3}_{m_1, \lambda_1} (\phi, \theta_1, 0) D^{s_3*}_{m_1, \lambda_1} (\phi, \theta_1, 0) = \delta_{\lambda_1, \delta}, \]
is used in the above equation. The decay rate for the sequential decay is proportional to 
\[ d\Gamma \propto \sum_{M, m_2, m_3} \left| A_I(\theta_1, \theta_2, \phi_1, \phi_2; J, M, m_2, m_3, m_4) \right|^2. \]

The amplitude in the canonical states for the decay chain II is 
\[ A_{II}(\theta^1, \theta^2, \phi^1, \phi^2; J, M, m_2, m_3, m_4) \]
\[ = \sum_{\lambda_{k_3}, \lambda_{k_2}, \lambda_{k_1}} D^{s_3}_{m_3, k_3} (\phi, \theta_2, 0) D^{s_3}_{m_3, -\lambda_{k_3}, \lambda_{k_2}} (\phi^1, \theta^1, 0) \]
\[ \times D^{s_3*}_{m_2, k_2} (\phi^2, \theta_2, 0) D^{s_3*}_{m_2, -\lambda_{k_2}, \lambda_{k_1}} (\phi, \theta_1, 0) \]
\[ \times F^{s_3}_{k_1, k_2, k_3} F_{k_1, k_2, k_1} \left( \phi_2, \theta_2, \phi_1, \theta_1 \right) \]
\[ = \sum_{M, \lambda_2, \lambda_3} \left| A_I(\theta_1, \theta_2, \phi_1, \phi_2; J, M, m_2, m_3, m_4) \right|^2 d\Omega_1 d\Omega_2, \]
where $\delta_{\lambda_1, \delta}$ is a normalization factor.

In the above equations, no azimuthal rotations are needed to align the $s_i$ helicities in two decay chains, since the decay planes are the same.

The decay rate corresponding to the total amplitude reads 
\[ d\Gamma \propto d\Omega_1 d\Omega_2 \sum_{M, m_2, m_3, m_4} \left| A_I(\ldots) \right|^2 + \left| A_{II}(\ldots) \right|^2 \]
\[ + A_I(\ldots) A_{II}^{*}(\ldots) + A_{II}^{*}(\ldots) A_{II}(\ldots), \]
where $\delta_{\lambda_1, \delta}$ denotes $(\theta_1, \theta_2, \phi_1, \phi_2; J, m_2, m_3, m_4)$, and Eq. (15) has been used to make a replacement in the amplitude $A_{II}$. Calculation of interference terms is straightforward. Using the orthogonal relations of
D-functions and summing over the spin projections $m_2, m_3,$ and $m_4$, one has

$$
\sum_{M, m_2, m_3, m_4} A_1(...)(...)
= \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} D_{M, \lambda_1 - \lambda_2}^{s_1} (\phi, \theta_1, 0) D_{s_2, \lambda_3 - \lambda_4}^{\phi_2} (\bar{\phi}_2, \bar{\theta}_2, 0)
\times F_{\lambda_1, \lambda_2}^{s_1} F_{\lambda_3, \lambda_4}^{s_2}
\times D_{M, \lambda_0 - \lambda_3}^{s_0} (\phi, \theta_1, 0)
\times d_{\lambda_0, \lambda_2 - \lambda_1}^s (\phi^2, \bar{\phi}^2, 0) F_{M, \lambda_3, \lambda_4}^{s_0} F_{M, \lambda_3, \lambda_4}^{s_0} d_{\lambda_0, \lambda_2 - \lambda_1}^s (\theta^2)
\times d_{\lambda_3, \lambda_2}^s (\theta_1) d_{\lambda_3, \lambda_2}^s (\theta_2),
$$

(17)

where the rotations, $d_{\lambda_0, \lambda_2 - \lambda_1}^s (\theta^2)$, $d_{\lambda_3, \lambda_2}^s (\theta_1)$ and $d_{\lambda_3, \lambda_2}^s (\theta_2)$, transform the helicities $\lambda^2$, $\lambda^3$ and $\lambda^4$ defined in chain II, into those defined in chain I, respectively. However, these rotations are canceled in the calculation of the term $\sum_{M, m_2, m_3, m_4} |A_{11}(...)|^2$ due to the orthogonal relations of the Wigner-d function. Hence, the coherent amplitude in helicity bases for the two chains is taken as

$$\mathcal{H}(M, \lambda_2, \lambda_3, \lambda_4) = a \ BW(M_1) \sum_{\lambda_1} D_{M, \lambda_1 - \lambda_2}^{s_1} (\phi, \theta_1, 0)
\times D_{s_2, \lambda_3 - \lambda_4}^{\phi_2} (\bar{\phi}_2, \bar{\theta}_2, 0) F_{\lambda_1, \lambda_2}^{s_1}
\times b \ BW(M_0) \sum_{\lambda_0, \lambda_2 - \lambda_1} d_{\lambda_0, \lambda_2 - \lambda_1}^s (\phi^2, \bar{\phi}^2, 0) F_{M, \lambda_3, \lambda_4}^{s_0} F_{M, \lambda_3, \lambda_4}^{s_0}
\times d_{\lambda_3, \lambda_2}^s (\theta_1) d_{\lambda_3, \lambda_2}^s (\theta_2),
$$

(18)

where $a$ and $b$ are coupling constants, $BW(M_0)$ and $BW(M_1)$ are the Breit-Wigner functions for resonances $s_0$ and $s_1$, respectively. To coherently add the third decay chain to the amplitude, e.g., $J \rightarrow s_4 + s_5$ with $s_5 \rightarrow s_2 + s_3$, the generation of the above formula is straightforward by multiplying its helicity sequential decay amplitude with the rotations to transform the helicities defined in this chain to those defined in chain I.

### IV. ILLUSTRATIVE EXAMPLES

Here we give two examples to illustrate the principle to construct the helicity amplitudes with coherent interference effects. We confine ourselves to the case of three-body decays with two pseudoscalar mesons in the final states.

#### A. $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^- J/\psi$

The charmoniumlike state, $Z_c(3900)^\pm$, was observed for the first time by the BESII Collaboration in the process $e^+e^- \rightarrow \pi^+\pi^- J/\psi$, and confirmed by the Belle and CLEO Collaborations.\footnote{We consider two kinds of decays in this process,}

$I: e^+e^- \rightarrow \gamma^* \rightarrow f_0(980)J/\psi(\lambda_1) (\phi_1, \theta_1)$
with $f_0(980) \rightarrow \pi^+\pi^-$.

$II: e^+e^- \rightarrow \gamma^* \rightarrow \pi^\pm Z_c(3900)^\pm (\lambda_2) (\phi_2, \theta_2)$
with $Z_c(3900)^\pm \rightarrow \pi^\pm J/\psi(\lambda_3) (\phi_3, \theta_3)$.

Here $\lambda_i (i = 1, 2, 3)$ are the helicity values for the corresponding particles, and $\theta_i$ and $\phi_i$ are the polar and azimuthal angles defined in the helicity reference for each decay, respectively. We assume the spin and parity of $Z_c(3900)$ to be $1^+$. The coherent helicity amplitude reads

$$\mathcal{H} = a \ BW(f_0, m_{\pi^+\pi^-}) D_{M, \lambda_1}^{s_1} (\phi_1, \theta_1, 0) F_{\lambda_1, 0}^{\gamma^* \rightarrow J/\psi f_0}
+ b \ BW(Z_c^\pm, m_{\pi^+\pi^-}) \sum_{\lambda_2, \lambda_3} D_{M, \lambda_2}^{s_1} (\phi_2, \theta_2)
\times D_{M, \lambda_3}^{s_1} (\phi_3, \theta_3, 0) d_{\lambda_3, \lambda_2}^s (\phi^2) F_{M, \lambda_3, \lambda_4}^{s_0} F_{M, \lambda_3, \lambda_4}^{s_0} d_{\lambda_3, \lambda_2}^s (\theta^2),
$$

(19)

where $M = \pm 1$ is the $z$ projection of $\gamma^*$ spin, $a$ and $b$ are coupling constants, $\theta^2$ is the angle between the momenta of $J/\psi$ in the $e^+e^-$ CM system and $Z_c^\pm$ rest frame, $BW$ denotes Breit-Wigner function, $F$s are the helicity amplitudes and are reduced by relating them.
to the LS coupling amplitudes as \[46\]

\[F_{1,0}^{\gamma \rightarrow J/\psi} = F_{1,0}^{J/\psi \rightarrow \phi} = \frac{g_0^1}{\sqrt{3}} + \frac{g_2^1 r_2^2}{\sqrt{6}},\]

\[F_{1,0}^{\gamma \rightarrow J/\psi} = \frac{\gamma_0 g_0^1}{\sqrt{3}} - \sqrt{3} \frac{\gamma_2 g_2^1 r_2^2}{\sqrt{6}},\]

\[F_{1,0}^{\gamma \rightarrow Z_\pi} = \frac{g_0^1}{\sqrt{3}} + g_2^1 r_2^2,\]

\[F_{1,0}^{\gamma \rightarrow J/\psi} = \frac{\gamma_0 g_0^1}{\sqrt{3}} - \sqrt{3} \frac{\gamma_2 g_2^1 r_2^2}{\sqrt{6}},\]

\[F_{1,0}^{\gamma \rightarrow J/\psi} = \frac{\gamma_0^2 g_0^1}{\sqrt{3}} - \sqrt{3} \frac{\gamma_2^2 g_2^1 r_2^2}{\sqrt{6}},\] (20)

where \(g_{0,2}^i(i = a, b, c)\) are coupling constants, \(r_i(i = a, b, c)\) is the magnitude of breakup momentum of the two-body decays. \(\gamma_{0,2}^i(i = a, b, c)\) is the ratio of \(J/\psi\) energy to its mass in the decay. In these decays, parity conserves the helicity amplitudes.

**B. \(\Lambda^+_c \rightarrow pK^-\pi^+\)**

The \(\Lambda^+_c\) has a sizable branching fraction \((5.0 \pm 1.3)\%\) decaying into the \(pK^-\pi^+\) final states. Amplitude analysis is desirable to extract the resonance contributions to this decay, such as \(K^*(892)^0\), \(\Delta(1232)^{++}\), excited \(\Lambda\) and \(\Sigma\) states. We consider three types of decay like

- **I:** \(\Lambda^+_c \rightarrow p(\lambda_1)K^*(892)^0(\lambda_2) (\phi_1, \theta_1))\), \(K^*(892)^0 \rightarrow K^-\pi^+ (\phi_2, \theta_2)\);
- **II:** \(\Lambda^+_c \rightarrow \Delta(1232)^{++}(\lambda_3)K^- (\phi_3, \theta_3), \Delta(1232)^{++} \rightarrow p(\lambda_4)\pi^+ (\phi_4, \theta_1)\);
- **III:** \(\Lambda^+_c \rightarrow \Lambda(1520)(\lambda_5)\pi^+ (\phi_5, \theta_5), \Lambda(1520) \rightarrow p(\lambda_6)K^- (\phi_6, \theta_6)\), (21)

where \(\lambda_i(i = 1, ..., 6)\) are helicity values for corresponding particles, and \(\phi_i\) and \(\theta_i\) are the polar and azimuthal angles defined in the helicity reference system for each decay. For the decay I, the helicity amplitude reads

\[\mathcal{H}_I \propto BW(K^*, m_{K^-\pi^+}) \sum_{\lambda_2} D_{M_{\lambda_1\lambda_2}}^{\pi^+} \phi_1(\phi_1, \theta_1, 0) \times D_{\lambda_2,0}^{\phi_2, \theta_2, 0} F_{1,0}^{\Lambda^+_c \rightarrow pK^- \pi^+} F_{0,0}^{\gamma \rightarrow K^* \pi},\] (22)

where \(M_{\lambda_1\lambda_2}\) is the \(z\) projection of \(\Lambda^+_c\) and \(F_s\) are the helicity amplitudes.

For the decay II, the helicity amplitude reads

\[\mathcal{H}_I \propto BW(\Delta^{++}, m_{p\pi^+}) \sum_{\lambda_3, \lambda_4} D_{M_{\lambda_3\lambda_4}}^{\pi^+} \phi_3(\phi_3, \theta_3) \times D_{\lambda_3,0}^{\gamma \rightarrow K^*} F_{1,0}^{\Delta^{++} \rightarrow K} F_{0,0}^{\gamma \rightarrow p\pi^+} d_{\lambda_4,1}^{\Lambda^+_c} (\theta_4),\] (23)

where \(\theta_4^i\) is the angle between the momenta of proton in \(\Delta^{++}\) and \(\Lambda^+_c\) rest frames.

For the decay III, the helicity amplitude reads

\[\mathcal{H}_{III} \propto BW(\Lambda, m_{p\pi^-}) \sum_{\lambda_6, \lambda_5} D_{M_{\lambda_5\lambda_6}}^{\pi^+} \phi_5(\phi_5, \theta_5, 0) d_{\lambda_5,1}^{\Delta^{++}} (\theta_5)^2 \times F_{A_{\lambda_6}^+ \rightarrow \lambda\pi^+} F_{A_{\lambda_5}^+ \rightarrow p\pi^-},\] (24)

where \(\theta_5^i\) is the angle between the momenta of proton calculated in \(\Lambda\) and \(\Lambda^+_c\) rest frames.

The coherent helicity amplitude is taken as

\[\mathcal{H} = c_1 \mathcal{H}_I + c_2 \mathcal{H}_{II} + c_3 \mathcal{H}_{III},\] (25)

where \(c_1, c_2, c_3\) are coupling constants.

The helicity amplitudes of \(F_s\)’s functions are reduced by relating them to the LS coupling amplitudes as

\[F_{1,0}^{\Lambda^+_c \rightarrow pK^*} = -F_{-1,0}^{\Lambda^+_c \rightarrow pK^*} = \frac{W g_0^1}{\sqrt{3}} - g_1 r^1 r_2 / \sqrt{6} - \frac{W g_2 r^2}{\sqrt{6}},\]

\[F_{0,0}^{K^* \rightarrow K\pi} = r g_0^1,\]

\[F_{1,0}^{\Lambda^+_c \rightarrow \Delta K} = \frac{r^2 W \gamma^1 g_2^1}{\sqrt{2}} - \frac{r^2 W g_0^1}{\sqrt{2}},\]

\[F_{-1,0}^{\Lambda^+_c \rightarrow \Delta K} = \frac{-1.5 r^2 W (\gamma^1 + 1) g_2^1}{3 \sqrt{2}} - \frac{r^2 W g_0^1}{3 \sqrt{2}},\]

\[F_{1,0}^{\Lambda^+_c \rightarrow \Lambda\pi^+} = -F_{-1,0}^{\Lambda^+_c \rightarrow \Lambda\pi^+} = \frac{r^2 W g_0^1}{\sqrt{2}},\]

\[F_{1,0}^{\Lambda^+_c \rightarrow \Lambda\pi^+} = \frac{g_0^1}{\sqrt{2}} + \frac{r W g_1^1}{\sqrt{2}},\]

\[F_{-1,0}^{\Lambda^+_c \rightarrow \Lambda\pi^+} = \frac{g_0^1}{\sqrt{2}} + \frac{r W g_1^1}{\sqrt{2}},\]

\[F_{1,0}^{\Lambda^+_c \rightarrow \Lambda\pi^+} = \frac{g_0^1}{\sqrt{2}} + \frac{r W g_1^1}{\sqrt{2}},\]

\[F_{-1,0}^{\Lambda^+_c \rightarrow \Lambda\pi^+} = \frac{g_0^1}{\sqrt{2}} + \frac{r W g_1^1}{\sqrt{2}},\]

\[F_{1,0}^{\Lambda^+_c \rightarrow \Lambda\pi^+} = \frac{g_0^1}{\sqrt{2}} + \frac{r W g_1^1}{\sqrt{2}},\]

\[F_{-1,0}^{\Lambda^+_c \rightarrow \Lambda\pi^+} = \frac{g_0^1}{\sqrt{2}} + \frac{r W g_1^1}{\sqrt{2}},\]

\[F_{1,0}^{\Lambda^+_c \rightarrow \Lambda\pi^+} = \frac{g_0^1}{\sqrt{2}} + \frac{r W g_1^1}{\sqrt{2}},\]

\[F_{-1,0}^{\Lambda^+_c \rightarrow \Lambda\pi^+} = \frac{g_0^1}{\sqrt{2}} + \frac{r W g_1^1}{\sqrt{2}},\]

where \(g_{LS}^1\) is the coupling constant, \(\gamma_2^1\) is the ratio of energy to mass of the \(x\) particle in the decay, and \(r\) is the magnitude of breakup momentum for the two-body decay. In the \(\Lambda^+_c\) decays, the parity doesn’t conserve; therefore all possible waves of orbital momentum are included. For \(K^+, \Delta,\) and \(\Lambda(1520)\) decays, the parity conserves the amplitudes.

**V. CONCLUSION AND DISCUSSION**

The method of helicity amplitude is widely used in the partial wave analysis. From the viewpoint of the experiment side, a few resonances are introduced to
model the production of final states. The helicities of final states are defined along the direction of outgoing particle in the rest frame of their mother particle system. If there are different decay chains involved, this makes the sum of the amplitudes inconsistent since the helicity of the same particle may have different definitions. One has to introduce an additional rotation to transform the helicity into the same reference. We present a deviation of coherent helicity amplitude for the three-body decays. The principle can be generalized to other cases, e.g., four-body decays.

If the amplitudes for multichain decays are constructed in the canonical basis, the spins of particles involved are defined in the same reference. This allows one to add them coherently. We borrow this idea in our derivation by relating the amplitude defined in the helicity base to that defined in the canonical basis. In the examples of three-body decays, we show that the helicity amplitudes need additional rotations to allow them to add coherently [see Eq. (18)]. These rotations may have significant impact on the interference between the different decay chains.

Two examples are shown to construct the coherent helicity amplitudes for the case of three-body decays. For practical purposes, the formulas are further reduced by using the covariant helicity-coupling amplitudes. It is important to note that these additional rotations are unneeded if the intermediate states are introduced with the same decay sequence topology. Since the intermediate states are reconstructed with the same final states, their helicities are defined in the same reference.

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