A Message Passing Based Average Consensus Algorithm for Decentralized Frequency and Phase Synchronization in Distributed Phased Arrays

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Abstract

We consider the problem of decentralized frequency and phase synchronization in distributed phased arrays via local broadcast of the node electrical states. Frequency and phase synchronization between nodes in a distributed array is necessary to support beamforming, but due to the operational dynamics of the local oscillators of the nodes, the frequencies and phases of their output signals undergo the random drift and jitter in between the update intervals. Furthermore, frequency and phase estimation errors contribute to the total phase errors, leading to a residual phase error in the array that degrades coherent operation. Recently, a classical decentralized frequency and phase synchronization algorithm based on consensus averaging was proposed with which the standard deviation of the residual phase errors upon convergence were reduced to $10^{-4}$ degrees for internode update intervals of 0.1 ms, however this was obtained for arrays with at least 400 nodes and a high connectivity ratio of 0.9. In this paper, we propose a message passing based average consensus (MPAC) algorithm to improve the synchronization of the electrical states of the nodes in distributed arrays. Simulation results show that the proposed MPAC algorithm significantly reduces the residual phase errors to about $10^{-11}$ degrees, requiring only 20 moderately connected nodes in an array. Furthermore, MPAC converges faster than the DFPC-based algorithms particularly for the larger arrays with a moderate connectivity.

Index Terms

Average Consensus, Distributed Phased Arrays, Frequency and Phase synchronization, Message Passing Algorithm, Oscillator Frequency Drift and Phase Jitter.
I. INTRODUCTION

Distributed phased arrays (DPAs) are collections of separate antenna systems that are wirelessly coordinated to perform coherent operations such as beamforming. When compared to the large single-platform architecture that uses analog feed networks and a single transceiver chain to drive the antennas, this distributed architecture brings several advantages to wireless applications, including higher signal power at the destination, improved spatial diversity, improved adaptability to changing environments, higher resistance to the overall system failure, and ease in scalability of the system [1], [2]. Each node in a DPA has its own transceiver chain with an independent local oscillator. When free running, the signals produced by each oscillator undergo random drift and jitter over time that introduces a decoherence between the signals emitted by the array [3], [4]. Existing methods that can be used for node synchronization purposes may be classified as either suitable for a closed-loop system or an open-loop system. In a closed-loop system, the nodes use a feedback from the destination, e.g., the received signal strength, to tune their oscillators until a significant coherence level is achieved at the destination [5]–[7]. The benefit of this approach is that little coordination is explicitly required between nodes; however, closed-loop systems cannot arbitrarily beamform, thus operations like radar and sensing are not feasible. On the other hand, in an open-loop system, the nodes do not use any feedback from the destination and synchronize their oscillators by exchanging signals with each other. Therefore, open-loop methods as proposed in [2], [8], [9] can also be used for the radar applications [10], however they require more synchronization than closed-loop systems.

In [11], we proposed a decentralized frequency and phase consensus (DFPC) algorithm for open-loop DPAs in which the nodes share their frequencies and phases with each other through a local broadcast of their signals, and iteratively update these parameters by computing a weighted average of the received values. Simulation results in [11] show that with the DFPC-based algorithms, the standard deviation of the residual phase errors upon convergence can be reduced to about $10^{-4}$ degrees, for a practical update interval of 0.1 ms, that requires at least 400 nodes in an array having a high connectivity ratio of 0.9 (see Fig. 9 in [11]). Essentially, the DFPC algorithm is based on the average consensus algorithm of [12] and thus begins with constructing a Markov chain (MC) with a doubly-stochastic transition matrix (a.k.a the mixing matrix or the weighting matrix). The DFPC algorithm is started with an arbitrary distribution over the frequencies and phases, and it progresses by mixing the frequencies and phases in each iteration.
until convergence where the synchronization is also achieved. However, with the MC-based mixing matrix, the synchronization is achieved only asymptotically, and therefore, DFPC takes a large number of iterations to converge even for larger arrays with a moderate connectivity. To improve its convergence speed, a better mixing matrix can be constructed with a smaller second largest eigenvalue; however, this requires global connectivity information at each node, which is not generally available in dynamic distributed arrays. In general, a large number of convergence iterations introduces a delay in achieving the synchronized state that is intolerable particularly when low-powered nodes are considered. Furthermore, an improved synchronization level between the nodes is also highly desirable to ensure a high gain coherent operation at the destination [2].

Recently, a Gaussian belief propagation [13] based on an average consensus algorithm was proposed in [14], [15] with the motivation that computing the marginals can be equated to solving an average consensus problem [16]. Thus, this algorithm is based on an alternative approach where the messages containing the coarse weighted averages and the weights-sum information are propagated through the network to solve the consensus problem. In this paper, we extend the algorithm in [14], [15] to solve the frequency and phase synchronization problem in a distributed phased array. To this end, we take into account the frequency drifts and phase jitters induced by the oscillators, as well as the frequency and phase estimation errors at the nodes due to the local broadcasting of the signals. A message passing based average consensus (MPAC) algorithm is developed in which the nodes iteratively exchange messages with their neighboring nodes to reach the consensus, i.e., synchronization in frequency and phase. Unlike the previously proposed DFPC and Kalman filtering based DFPC (KF-DFPC) algorithms in [11], the MPAC algorithm does not require the network connectivity information to assign weights to the nodes. Simulation results show that compared to DFPC and KF-DFPC, MPAC significantly reduces the residual phase errors upon convergence to about $10^{-11}$ degrees with only 20 moderately connected nodes in the array and irrespective of the signal to noise ratio (SNR) of the signals. Furthermore, MPAC takes fewer iterations for convergence than the DFPC-based algorithms particularly for larger arrays with a moderate connectivity.

The rest of this article is outlined as follows. Section II formulates the decentralized frequency and phase synchronization problem in a DPA, proposes an MPAC algorithm to synchronize these parameters across the array, and theoretically analyzes the residual phase errors. Simulation results are included in Section III wherein the synchronization performance of MPAC is investigated.
and compared to the DFPC algorithm. Finally, Section IV concludes this work.

II. DECENTRALIZED FREQUENCY AND PHASE SYNCHRONIZATION IN DISTRIBUTED PHASED ARRAYS

Consider a group of $N$ nodes that are connected together with bidirectional communication links to form a distributed phased array. The network of these $N$ nodes can be represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ in which $\mathcal{V} = \{1, 2, \ldots, N\}$ represents the set of vertices, and $\mathcal{E} = \{(m, n): m, n \in \mathcal{V}\}$ denotes the set of all undirected edges in the graph. We assume that the nodes are communicating with each other via a local broadcast of signals to synchronize their frequencies and phases across the array. Let the signal generated by the $n$-th node in iteration $k$ over the time duration $T$ be given by $s_n(t) = e^{j(2\pi f_n(k)t + \theta_n(k))}$, in which $f_n(k)$ and $\theta_n(k)$ represent the frequency and phase of the signal in the $k$-th iteration. In practice, these parameters in each iteration are influenced by the frequency drifts and phase jitters of the oscillators, and thus we model them in the $k$-th iteration as

$$f_n(k) = f_n(k-1) + \delta f_n$$

$$\theta_n(k) = \theta_n(k-1) + \delta \theta_n$$, (1)

in which $\delta f_n$ denotes the frequency drift of the oscillator at the update time, and parameter $\delta \theta_n$ represents the phase due to its temporal variation over the time duration $T$ which is given by $\delta \theta_n = -\pi T \delta f_n$ [3], [11], and $\delta \theta_n$ models the phase jitter of the oscillator at the $n$-th node [3]. The frequency and phase evolutions in (1) start with the initial values $f_n(0)$ and $\theta_n(0)$, respectively. We assume that the initial frequency of the $n$-th node is normally distributed as $f_n(0) \sim \mathcal{N}(f_c, \sigma^2)$ where $f_c$ is the carrier frequency, $\sigma = 10^{-4} f_c$ denotes a crystal clock accuracy of 100 parts per million (ppm), and the initial phase is uniformly distributed as $\theta_n(0) \sim \mathcal{U}(0, 2\pi)$. As the nodes in the array share their frequencies and phases with each other through a local broadcast of their signals, the shared values are also influenced by the estimation errors. Thus, the frequency and phase of the $n$-th node observed in the iteration $k$ are written as

$$\hat{f}_n(k) = f_n(k) + \varepsilon_f$$

$$\hat{\theta}_n(k) = \theta_n(k) + \varepsilon_\theta$$, (2)

where $\varepsilon_f$ and $\varepsilon_\theta$ represent the frequency and phase estimation errors at the node.
Next we describe the statistical modeling of the frequency drift and phase jitter of the oscillators, as well as the frequency and phase estimation errors at the nodes as follows. To begin, the frequency drift of the oscillator at the \(n\)-th node is modeled as \(\delta f_n \sim \mathcal{N}(0, \sigma_f^2)\) in which the standard deviation of the frequency drift \(\sigma_f\) can be set equal to the Allan deviation (ADEV) of the oscillator [3]. The ADEV is defined as the standard deviation of the averaged fractional frequency errors computed over multiple shifted time intervals. We model the ADEV as \(\sigma_f = f_c \sqrt{\beta_1 + \beta_2 T}\) with \(\beta_1\) and \(\beta_2\) depend on the design of the oscillator that we define as \(\beta_1 = \beta_2 = 5 \times 10^{-19}\) [11]. The phase jitter of the oscillator at the \(n\)-th node is modeled as \(\delta \theta_n \sim \mathcal{N}(0, \sigma_\theta)\) where its standard deviation is defined as \(\sigma_\theta = \sqrt{2 \times 10^{-4}/10}\) in which \(A\) represents the integrated phase noise power of an oscillator. The parameter \(A\) is defined as the \(\log_{10}\) of the total area under the entire curve of the phase noise profile of an oscillator. Herein, we set \(A = -53.46\) dB that models a typical high phase noise voltage controlled oscillator [3], [11]. Finally, the frequency and phase estimation errors, i.e., \(\varepsilon_f\) and \(\varepsilon_\theta\), are also modeled as normally distributed with zero mean and standard deviations \(\sigma_f^m\) and \(\sigma_\theta^m\), respectively. As the focus here is on the synchronization problem, we set these standard deviations equal to the Cramer-Rao lower bounds (CRLBs) that are derived in [17]. Thus we set \(\sigma_f^m = \sqrt{\frac{6}{(2\pi)^2 L^3 \text{SNR}}}\) and \(\sigma_\theta^m = \frac{2 L^{-1} \text{SNR}}{\text{SNR}}\) in which the SNR denotes the signal to noise ratio of the received signals, and \(L = T f_s\) represents the number of samples collected over the observation window of length \(T\) with sampling frequency \(f_s\). Note that these CRLBs can be achieved by an unbiased and efficient estimators, for e.g., the FFT-based maximum likelihood estimators described in [18], assuming a large number of samples are available for the estimation purposes.

For an ideal synchronization of nodes, the total phase error defined as \(\delta \phi_n = 2\pi \delta f_n T + 2\pi \varepsilon_f T + \delta \theta_n + \varepsilon_\theta\) must be zero for all the nodes across the array. However, in practice the residual error will not converge to zero due to propagation delays of the signals and the continual drift of the oscillators; thus we define synchronization of the frequency and phase of the nodes in the array when the standard deviation of the total phase errors \(\delta \phi_n\) satisfies:

\[
\sigma_\phi = \sqrt{\frac{1}{N-1} \sum_{n=1}^{N} | \delta \phi_n - \bar{\phi} |^2} \leq \eta, \quad (3)
\]

in which \(\eta\) represents some pre-defined threshold, and \(\bar{\phi}\) denotes the average value of the total phase errors. It is established in Fig. 4 in [2], that at least 90% of the ideal coherent gain can be achieved at the destination if the standard deviation \(\sigma_\phi\) is below the threshold \(\eta = 18^\circ\). In
other words, any $\eta$ below $18^\circ$ guarantees high coherent gain at the destination.

A. Message Passing Based Average Consensus Algorithm

We assume that each node iteratively exchanges its frequency and phase only with its neighboring nodes and updates these parameters in each iteration by computing a weighted average of the shared values. This local sharing of the information between the nodes enables the use of a fully decentralized (distributed) algorithm that is easily scalable as the required resources per node for its implementation – for instance, the memory storage, the computational power, and the bandwidth – are mainly controlled by the average number of neighbors per node in a network.

Each node $n$ in the array has a weight $w_n$ assigned to it, and let $f_n(k)$ and $\theta_n(k)$ represent its updated frequency and phase in the $k$-th iteration. As shown in Fig. 1, we assume that, for each $m \in N_n$, $\mu_f^{m\rightarrow n}(k - 1)$ and $\mu_\theta^{m\rightarrow n}(k - 1)$ denote the messages sent from node $m$ to node $n$ in the $(k - 1)$-st iteration, that represents the coarse frequency and phase weighted averages, respectively, computed at node $m$ by using all the values of its neighboring nodes from the previous iteration except the shared values from node $n$. Likewise, let $s_m\rightarrow n(k - 1)$ denote the sum of the weights of the neighboring nodes of node $m$, computed in the $(k - 1)$-st iteration, excluding the weight $w_n$ of node $n$. Node $m$ uses $s_m\rightarrow n(k - 1)$ to compute the coarse weighted averages $\mu_f^{m\rightarrow n}(k - 1)$ and $\mu_\theta^{m\rightarrow n}(k - 1)$ in iteration $k - 1$ and thus passes that scalar to node $n$ as well. Node $n$ receives these messages from all its neighboring nodes, then it updates its frequency and phase values in the $k$-th iteration by combining all the received values as follows

$$f_n(k) = \frac{w_n \hat{f}_n(k) + \sum_{m \in N_n} s_m\rightarrow n(k - 1) \mu_f^{m\rightarrow n}(k - 1)}{s_n(k)},$$

(4)

$$\theta_n(k) = \frac{w_n \hat{\theta}_n(k) + \sum_{m \in N_n} s_m\rightarrow n(k - 1) \mu_\theta^{m\rightarrow n}(k - 1)}{s_n(k)},$$

(5)

in which $s_n(k) = w_n + \sum_{m \in N_n} s_m\rightarrow n(k - 1)$ and $N_n$ is the set of neighboring nodes of node $n$.

Next node $n$ sends out the updated messages (the coarse frequency and phase weighted averages) to each of its $m$ neighbors for all $m \in N_n$. These messages are computed as

$$\mu_f^{n\rightarrow m}(k) = \frac{w_n \hat{f}_n(k) + \sum_{m \in \{N_n\setminus m\}} s_m\rightarrow n(k - 1) \mu_f^{m\rightarrow n}(k - 1)}{w_n + \sum_{m \in \{N_n\setminus m\}} s_m\rightarrow n(k - 1)},$$

(6)

and

$$\mu_\theta^{n\rightarrow m}(k) = \frac{w_n \hat{\theta}_n(k) + \sum_{m \in \{N_n\setminus m\}} s_m\rightarrow n(k - 1) \mu_\theta^{m\rightarrow n}(k - 1)}{w_n + \sum_{m \in \{N_n\setminus m\}} s_m\rightarrow n(k - 1)},$$

(7)
Fig. 1. A portion of an undirected graph depicting the flow of messages needed to update the frequency and phase of node \( n \) in the \( k \)-th iteration. The red-colored arrows between nodes \( l \) and \( m \) (for each \( l \in \{N_m \setminus n\} \)) represent the messages \((\mu_{l \rightarrow m}(k-2), \mu_{l \rightarrow m}^\theta(k-2), s_{l \rightarrow m}(k-2))\) computed in \((k-2)\)-th iteration. These messages are used by node \( m \) to send out the messages \((\mu_{m \rightarrow n}(k-1), \mu_{m \rightarrow n}^\theta(k-1), s_{m \rightarrow n}(k-1))\) to node \( n \) in the \((k-1)\)-st iteration (following (6) and (7)) which is shown with the blue-colored arrows. Similarly, the nodes in set \( \{N_n \setminus m\} \) also follow the same procedure to compute these blue-colored messages in \((k-1)\)-st iteration using the messages from the previous iteration. Finally, node \( n \) uses (4) and (5) to update its frequency and phase by combining all these blue-colored messages from its neighboring nodes in the set \( N_n \). It then sends out the new messages \((\mu_{n \rightarrow m}(k), \mu_{n \rightarrow m}^\theta(k), s_{n \rightarrow m}(k))\) to all \( m \in N_n \) as shown by the green-colored arrows.

where \( \{N_n \setminus m\} \) is the set of neighboring nodes of node \( n \) excluding node \( m \). Note that the message \( s_{n \rightarrow m}(k) \) is defined as \( s_{n \rightarrow m}(k) = f_{\gamma} \left( w_n + \sum_{m \in \{N_n \setminus m\}} s_{m \rightarrow n}(k-1) \right) \) where the function \( f_{\gamma}(x) = \gamma x / (\gamma + x) \) with \( \gamma \gg 1 \) is used to ensure that the proposed MPAC algorithm converges in case of both acyclic as well as cyclic networks [14]. The updated coarse weighted averages from (6) and (7) are then used in the next iteration by the \( m \)-th node for updating its frequency and phase values following (4) and (5). The above steps are repeated at each node in the graph in every iteration until the convergence is achieved. This message passing based average consensus (MPAC) algorithm is described in detail in Algorithm 1.

B. Residual Phase Error Analysis

In this subsection, we theoretically examine the residual phase error of the proposed MPAC algorithm in the presence of the frequency and phase offset errors introduced at the nodes. To begin, the MPAC algorithm tends to solve the following optimization problem [14].

\[
\arg\min_x \sum_{n=1}^{N} w_n |x_n - z_n(I)|^2 + \gamma \sum_{(m,n) \in \mathcal{E}} |x_m - x_n|^2,
\]  

(8)
When \( diag \) in which \( L \) \( x \) function in (8) is strictly convex and its global minimum is given by \( G \) \( x \) in which \( x \) is the penalty parameter added to enforce the convergence where all \( x \) define \( e \) \( z \) define \( e \) \( z \) \( N \) \( x \) \( z \) \( N \) \( 1 \) \( x \) \( w \) \( \sum_{n=1}^{N} w_n x_n \) is offset error vector with \( \epsilon_n(I) = \{ f_n(I), \hat{\theta}_n(I) \} \) at iteration \( I \) and thus correspondingly \( x_n \) either represents a frequency consensus or phase consensus. \( \gamma > 0 \) is the penalty parameter added to enforce the convergence where all \( x_n \)s become similar for a connected graph \( G \). Following Lemma 2 in [14], it can be easily shown that the objective function in (8) is strictly convex and its global minimum is given by \( x^* = (\gamma L + W)^{-1} Wz(I) \) in which \( L \) is the \( N \times N \) Laplacian matrix of \( G \), \( W \) is a diagonal matrix defined as \( W = \text{diag}\{w_1, w_2, \ldots, w_N\} \), and \( z(I) = [z_1(I), z_2(I), \ldots, z_N(I)]^T \). Note that using (1) and (2), we define \( z(I) = z(I-1) + \epsilon(t) \) where \( \epsilon(t) = [\epsilon_1(t), \epsilon_2(t), \ldots, \epsilon_N(t)]^T \) is offset error vector with \( \epsilon_n(I) \sim N(0, \sigma^2_e) \) in which \( \sigma^2_e = \sigma^2_f + (\sigma^2_m)^2 \) when \( z_n(I) = f_n(I) \), and \( \sigma^2_e = (\pi T \sigma_f)^2 + (\sigma^2_m)^2 + \sigma^2_\theta \) when \( z_n(I) = \hat{\theta}_n(I) \). Using the backward recursion, the consensus vector \( x^* \) can be written as \( x^* = (\gamma L + W)^{-1} Wz(0) + \sum_{i=0}^{I-1} (\gamma L + W)^{-1} W \epsilon_{I-i}(0) \). The first term in this solution gives an average of the initial vector values and likewise the second terms gives the accumulated averaged errors. Essentially, the accumulated averaged errors becomes negligible for the large connected networks when larger \( \gamma \) is used. Specifically, when \( \gamma \to \infty \) then (8) reduces to \( \arg \min_{x_n} \sum_{n=1}^{N} w_n |x_n - z_n(I)|^2 \) as all the \( x_n \) become similar. Then by taking the derivative of this new objective function with respect to \( x_n \) and setting it to zero, we get \( x_n^* = \frac{\sum_{n=1}^{N} w_n z_n(I)}{\sum_{n=1}^{N} w_n} \).

In terms of the offset errors, \( x_n^* \) can be written as \( x_n^* = \frac{\sum_{n=1}^{N} w_n z_n(0)}{\sum_{n=1}^{N} w_n} + \frac{\sum_{n=1}^{N} w_n \sum_{i=0}^{I-1} \epsilon_n(I-i)}{\sum_{n=1}^{N} w_n} \).

When \( w_n = 1 \), the two summands compute the statistical means and because the sum of the

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**Algorithm 1: MPAC Algorithm**

**Input:** \( k = 0 \), define \( w_n \). Next for each node \( n \), and for each \( m \in \mathcal{N}_n \), set \( \mu_{m \to n}(0) = f_c \), \( \theta_{m \to n}(0) = \pi \), and \( s_{m \to n}(0) = f_s(w_m) \).

while convergence criterion is not met

\( k = k + 1 \)

For each node \( n \):

a) Update the frequency \( f_n(k) \) and phase \( \theta_n(k) \) using (4) and (5), respectively.

then for each node \( n \) and its each neighbor \( m \in \mathcal{N}_n \):

b) Update the scale \( s_{m \to n}(k) \) using:

\[
  s_{m \to n}(k) = f_s \left( w_n + \sum_{m \in \{\mathcal{N}_n \setminus m\}} s_{m \to n}(k-1) \right)
\]

c) Compute the new messages \( \mu_{m \to n}(k) \) and \( \theta_{m \to n}(k) \) using (6) and (7), respectively, and send them out to node \( m \).

end

**Output:** \( f_n(k) \) and \( \theta_n(k) \) for all \( n = 1, 2, \ldots, N \)
offset errors $e_n(.)$ is normally distributed with zero mean, the errors are averaged out for larger networks when $\gamma \to \infty$. This results in an improved synchronization performance of the MPAC algorithm.

III. Simulation Results

In this section, we investigate the frequency and phase synchronization performance of the proposed MPAC algorithm through simulations. To this end, we consider a network of $N$ nodes randomly generated with a connectivity $c$ which is defined as the ratio of the number of active edges in the network to the number of all possible edges $(N(N-1)/2)$. Thus $c \in [0,1]$ and a higher value of $c$ implies a densely connected network, whereas a smaller value of $c$ means a sparsely connected network. Furthermore, the average number of connections per node are given by $D = c(N-1)$. We assume that the nodes transmit at a carrier frequency of $f_c = 1$ GHz and use the sampling frequency of $f_s = 10$ MHz to sample the received signals over $T = 0.1$ ms interval. The weight of the $n$-th node in MPAC is set as $w_n = 1$ and $\gamma = 10^{12}$ to ensure improved synchronization between the nodes. To generate the figures in this section, the array network was randomly generated in each trial and the results were averaged over $10^3$ independent trials.

In Fig. 2, we compare the standard deviation of the total phase errors $\delta \phi_n$ of the MPAC algorithm upon convergence to that of the DFPC and KF-DFPC algorithms proposed in [11] by varying the number of nodes in the array, the connectivity $c$ between the nodes, and the SNR of the received signals. Note that the minimum possible connectivity for $N = 5$ nodes is $c = 0.4$, whereas for $N \geq 10$ nodes we can set either $c = 0.2$ or $0.5$. Furthermore, the performance of KF-DFPC is independent of the SNR values as shown in [11], and thus we illustrate here its performances for $c = 0.2$ and $0.5$ values for the comparison purposes. This figure shows that with the increase in the number of nodes $N$ in the array, the standard deviation of the total phase errors decreases for all the algorithms. This is due to the increase in $D$ which assists in computing more accurate local averages at the nodes. However, the decrease is more rapid and significant for the MPAC algorithm as compared to the other algorithms for the larger $N$ values. Moreover, it is observed that while the performance of the DFPC algorithm improves with the increase in SNR due to the decrease in the estimation errors, on the other hand, the MPAC's performance is consistent at higher $N$ values irrespective of the SNR values. The KF-DFPC algorithm, which uses a Kalman filter, reduces the total phase errors for larger $N$ values but not as much as the MPAC algorithm. This improvement in the performance of MPAC is due to its
averaging out of the errors as explained in Section II-B where the averages are more unbiased for the larger $D$ values. In contrast, the residual phase errors of DFPC and KF-DFPC depend on the modulus of the second largest eigenvalue of the weighting matrix that is controlled by the $c$ value (as derived in Section III-A in [11]). Thus, the KF-DFPC’s residual phase errors decreases with the increase in $c$, but not as much as the MPAC algorithm.

Finally, in Fig. 3 we compare the convergence speeds of the three algorithms for different number of nodes in the array by varying the connectivity $c$ between the nodes for SNR = 0 dB. This figure shows the average value and standard deviation of the $10^3$ samples using the errorbar plot. The threshold $\eta$ for convergence was set to $1^\circ$ which ensures high coherent gain operation at the destination [2]. As expected, it is observed that the convergence rate of both algorithms improves with the increase in the connectivity $c$ between the nodes or the number of nodes $N$ in the array. However, the proposed MPAC algorithm takes considerably smaller number of convergence iterations for the moderately connected arrays with $c$ in $[0.05, 0.6]$. For e.g., for $N = 20$ and $c = 0.2$, DFPC takes 14 iterations, KF-DFPC takes 9 iterations, and MPAC takes 3 iterations, whereas for $N = 100$ and $c = 0.05$, DFPC takes 17 iterations, KF-DFPC takes 8 iterations, and MPAC takes 2 iterations.
IV. CONCLUSIONS

The frequency and phase synchronization problem is an essential bottleneck for leveraging the benefits of the distributed phased array, particularly when the frequency and phase offset errors are introduced at the nodes. We developed a decentralized MPAC algorithm that synchronizes these parameters across the array through a local propagation of messages between the nodes. Simulation results show that our proposed MPAC algorithm significantly reduces the residual phase errors upon convergence as compared to the DFPC-based algorithms. In particular, MPAC reduces the standard deviation of the total phase errors to about $10^{-11}$ degrees with only 20 moderately connected nodes in the array and irrespective of the SNR of the received signals. Moreover, it converges in a fewer iterations as compared to the DFPC-based algorithms.

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