A Comparative Study of Quantum Entanglement and Resonance Energy Transfer Mediated by Epsilon-Near-Zero and Other Plasmonic Waveguide Systems

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The entanglement and resonance energy transfer between two-level quantum emitters are typically limited to sub-wavelength distances due to the inherently short-range nature of the dipole-dipole interactions. Moreover, the entanglement of quantum systems is hard to preserve for a long period of time due to decoherence and dephasing mainly caused by radiative and nonradiative losses. Recently, nanoscale plasmonic waveguide systems have been proposed to tackle this problem to a certain degree. In this work, we thoroughly investigate the efficient inter-emitter entanglement and large enhancement of resonance energy transfer between two optical qubits mediated by epsilon-near-zero (ENZ) plasmonic waveguides. We calculate the concurrence and the resonance energy transfer rate for the ENZ waveguide and compare them to similar metrics for two other commonly used types of plasmonic waveguides, i.e., V-shaped groove milled in a flat metallic surface and cylindrical nanorod. We demonstrate, that the ENZ waveguide drastically outperforms the other two waveguides when it comes to improving the
entanglement and the resonant energy transfer. What is even more important, the efficiency of the entanglement between the emitters does not depend on their position within the ENZ waveguide, as the ENZ plasmonic mode has an infinite phase velocity combined with strong and homogeneous electric field. Moreover, we demonstrate that efficient steady-state entanglement can be achieved by using coherent external pumping and report a practical way to detect it by computing the second-order correlation function. The presented results are expected to be useful for the future quantum communication and information plasmonic nanodevices.

I. INTRODUCTION

One of the main limitations of the current quantum photonic systems is the rapid loss of spatial and temporal coherence [1,2]. For instance, Förster resonance energy transfer [3], a well-known dipole-dipole interaction between quantum emitters important to light sources, biomedical imaging, and photovoltaic applications, is limited to subwavelength ranges [4,5]. In addition, quantum entanglement [6], which is important for a variety of emerging applications in quantum communication and computing [7], usually takes place at extremely short distances and for very short time periods, due to the decoherence associated with unavoidable coupling between the system and the surrounding environment [8].

During the last years, considerable research efforts in the area of quantum plasmonic metamaterials have been dedicated to significantly improve coherence. These artificially engineered nanostructures can serve as a novel platform to trigger, harness, and enhance coherent light-matter interactions at the nanoscale [9,10]. Both long-range energy transfer [11–14] and quantum entanglement [15–18] can be achieved by coupling the quantum emitters to the plasmonic modes that can propagate over long distances in nanoscale waveguides. However, the
emitters’ positions in these plasmonic systems are crucial to the efficiency of the aforementioned quantum processes leading to rapidly varying concurrence and energy transfer with the emitters’ separation distance. Therefore, creating and preserving over extended time periods and long distances the efficient entanglement and strong energy transfer between quantum dipole emitters randomly distributed in a nanophotonic system remains a key challenge.

Currently, metamaterials exhibiting epsilon-near-zero (ENZ) behavior have attracted increased attention due to the large field enhancement in combination with a uniform distribution, leading to various applications in enhancing fluorescence, squeezing and tunneling light in bended waveguides, coherent perfect absorption, and boosting optical nonlinear effects [19–21]. Recently, some interesting works have been proposed that connect ENZ metamaterials, even zero index metamaterials (ZIM), with qubit entanglement [22,23]. In these configurations, the effective near zero permittivity has been designed by utilizing bulk materials operating at their plasma frequencies, complex layered hyperbolic metamaterial structures [23] or all-dielectric photonic crystal waveguides [22]. However, all of these approaches to achieve an ENZ response suffer from fabrication limitations, limited tunability, and very narrowband responses, which make them impractical for realistic applications. In addition, and more importantly, the field enhancement within these ENZ materials is relatively low and has limited spatial extent, leading to relatively weak light-matter interactions.

In this work, we prove that a specific ENZ plasmonic system can substantially improve the quantum entanglement and the resonance energy transfer compared to the other commonly used plasmonic waveguide configurations, such as V-shaped grooves milled in a flat metallic surface and cylindrical nanorods. The proposed quantum ENZ metamaterial system can support efficient
long-range resonance energy transfer (RET) and entanglement between quantum dipole emitters independent of their positions within the waveguide, over extended time periods and long separation distances. The plasmonic system is comprised of an array of metallic waveguides that exhibit an effective ENZ response at their cutoff frequency in combination with enhanced and homogeneous electromagnetic fields inside their nanochannels [24–29]. These interesting features, combined with the strong omnidirectional resonant coupling at the ENZ frequency, are ideal conditions to boost coherent light-matter interactions along elongated regions and can increase the temporal and spatial coherence between different emitters. For example, in the past, it was shown that long-range superradiance can be sustained by using this system [27]. Here, we will demonstrate quantum entanglement that persists over extended time periods and long distances which is achieved by the ENZ modes inside the plasmonic nanowaveguide channels. Enhanced RET will also be obtained with the proposed ENZ plasmonic system for large donor-acceptor separation distances and independent of their positions. Hence, the proposed plasmonic device constitutes an ideal platform for supporting efficient quantum electrodynamic effects. Moreover, the efficiencies of the RET and the quantum entanglement mediated by the proposed ENZ metamaterial are substantially enhanced compared to the finite metallic V-shaped and cylindrical plasmonic waveguides. The potential of these two particular waveguide types to improve the quantum effects have been extensively studied in the literature [11,16,17] and successful subwavelength waveguiding [30] and single plasmon generation [31] were experimentally demonstrated based on these systems.
II. ENHANCED RESONANCE ENERGY TRANSFER

We investigate the enhancement of the RET between a donor (D) and acceptor (A) pair mediated by the proposed ENZ and the other plasmonic waveguide types. The geometry of one unit cell of the proposed ENZ plasmonic waveguide is illustrated in Fig. 1(a). It is composed of a rectangular slit carved in a silver screen and loaded with a dielectric material. The slits (cyan) are loaded with glass with a relative permittivity equal to $\varepsilon = 2.2$, and the silver permittivity dispersion values are taken from previously obtained experimental data [32]. This free-standing waveguide geometry was originally introduced in [24] and can sustain an ENZ resonance at the cutoff wavelength of its dominant quasi-TE$_{10}$ mode. The extraordinary optical transmission combined with large field enhancement and uniform phase distribution inside the nanoslits was demonstrated at the ENZ operation wavelength [27,28]. This is due to an anomalous impedance matching phenomenon that depends only on the interface properties, i.e., on the aperture to period ratio of the array, and is therefore independent of the grating periodicity and the waveguide channel thickness [27,28]. Here, the slit dimensions have width $w = 200\,\text{nm}$, height $t = 40\,\text{nm}$ ($t \ll w$) and length $l = 1\,\mu\text{m}$, respectively, and the grating period is equal to $a = b = 400\,\text{nm}$. To clearly demonstrate the aforementioned ENZ response, we operate close to the cutoff wavelength of the proposed waveguide, which is $\lambda = 1016\,\text{nm}$. At this frequency, the effective electromagnetic properties of the waveguides’ plasmonic channels become equivalent to the properties of a bulk ENZ metamaterial exhibiting uniformly enhanced electric field distribution, as illustrated in the right inset of Fig. 2, plotted along the xz-plane of the unit cell geometry shown in Fig. 1(a) [27,28].
Next, we consider a pair of identical two-level quantum emitters as the D-A pair located at the center of the plasmonic waveguide channel along the x-axis [see Fig. 1(a)]. We assume that the emitter A with dipole moment $\mathbf{\mu}_A$ is located at the arbitrary position $\mathbf{r}_A$ and the emitter D with identical dipole moment $\mathbf{\mu}_D = \mathbf{\mu}_A$ is placed at the position $\mathbf{r}_D$. In order to quantify the RET, we use the following normalized energy transfer rate (nETR) formula [5,11]:

$$
nETR = \frac{\text{Im} \left[ \mathbf{\mu}_A^* \cdot \mathbf{G}(\mathbf{r}_A, \mathbf{r}_D) \cdot \mathbf{\mu}_D \right]}{\text{Im} \left[ \mathbf{\mu}_A^* \cdot \mathbf{G}_{\text{vac}}(\mathbf{r}_A, \mathbf{r}_D) \cdot \mathbf{\mu}_D \right]} = \left| \frac{\mathbf{n} \cdot \mathbf{E}_D(\mathbf{r}_A)}{\mathbf{n} \cdot \mathbf{E}_{D,\text{vac}}(\mathbf{r}_A)} \right|^2
$$

(1)

where $\mathbf{\mu}_A^* = \mathbf{\mu}_D^*$ is the complex conjugate of the transition dipole moment of the emitter and $\mathbf{n}$ is the unit vector in the direction of the dipole moment ($\mathbf{\mu}_A = \mathbf{\mu}_D = \mathbf{\mu} \cdot \mathbf{n}$). $\mathbf{G}(\mathbf{r}_A, \mathbf{r}_D)$ [$\mathbf{G}_{\text{vac}}(\mathbf{r}_A, \mathbf{r}_D)$] is the system’s dyadic Green’s function in the ENZ waveguide (vacuum), while $\mathbf{E}_D(\mathbf{r}_A)$ ($\mathbf{E}_{D,\text{vac}}(\mathbf{r}_A)$) is the electric field induced at the acceptor location $\mathbf{r}_A$ by an electric dipole at the source (donor) point $\mathbf{r}_D$ when both emitters are inside the waveguide (in the vacuum). Therefore, the nETR represents the energy transfer enhancement in the presence of the waveguide system with respect to vacuum.

We also introduce two alternative designs of finite plasmonic waveguides, a V-shaped channel cut in a metal plane and a cylindrical metallic nanowire, as shown in Figs. 1(b) and 1(c), and compare their performance with the presented ENZ nanowaveguide. In the former case [Fig. 1(b)], we choose the depth of the groove to be equal to $L = 235$ nm and the opening angle to be $\theta = 10^\circ$; in the nanowire configuration [Fig. 1(c)], the radius is fixed to $R = 25$ nm. The lower panels (d)-(f) in Fig. 1 display the electric field spatial distribution of the supported fundamental resonant mode for each of the plasmonic waveguides. We want to point out that all the
plasmonic waveguides studied in our work have finite length, which makes a significant difference compared to the infinite-length plasmonic waveguide designs presented in [11]. The finite length leads to stronger emitter-plasmon interactions and a more fair performance comparison between the different plasmonic waveguide systems [17]. The channel lengths are chosen to be \( l = 1\mu m \) for the ENZ and cylindrical rod (nanowire) plasmonic waveguides and \( l = 1.4\mu m \) for the groove plasmonic waveguide. Unlike the cases of the ENZ and the rod waveguides, where the mode electric field is strong at the waveguides ends, the modal field is weak at the groove ends. Thus we choose the groove’s channel length to be longer in order to ensure that the distance between two antinodes located near the groove ends is also 1\( \mu m \). We place the emitters in the area of the strong electric fields in order to achieve a fair performance comparison between the different plasmonic waveguides. Particularly, the D-A pair is embedded in the center of the dielectric slit inside the ENZ waveguide. In the case of the two other waveguides, the emitters are placed at a height equal to \( h = 20\text{nm} \) above them.

We always assume that the dipole moments of the D-A pair are identical and polarized vertically (along the z-direction) in the case of ENZ and rod waveguides and horizontally (along the y-axis) in the groove waveguide in order to achieve optimal coupling with their electromagnetic resonant modes [16]. The dipole transition wavelength of the emitters is 1018 nm, which is near the ENZ cutoff resonant wavelength (\( \lambda = 1016\text{nm} \)). The geometric parameters of the finite groove and rod waveguides are chosen with the goal to have the same propagation length (about 17 \( \mu m \)) at the operating wavelength \( \lambda = 1018\text{nm} \). In our case, the propagation length of the ENZ plasmonic waveguide (approximately 1.7 \( \mu m \)) is shorter due to the higher plasmonic losses compared to the case of the other two waveguides. However, the exceptional field enhancement
properties of the ENZ configuration alleviate this problem and lead to much more efficient energy transfer over longer distances compared to the other two waveguide designs, as it will be demonstrated in the following.

To study the energy transfer rate for each waveguide, we fix the location of emitter A at the edge of each channel for the ENZ and rod waveguides and at the antinode location near the groove end for the groove waveguide case. The emitter D is gradually moved along the x-axis. The nETR is calculated according to Eq. (1) and illustrated in Fig. 2 as a function of the separation distance between the D-A emitter pair. Note that the transfer rate in the case of the ENZ waveguide (black line) gradually grows with increasing separation distances, and can reach up to six orders of magnitude enhancement compared to the free space scenario. Whereas, the transfer rate enhancement for groove and rod waveguides oscillates with the separation distance and cannot exceed four orders of magnitude enhancement. The large enhancement of RET, especially for the ENZ waveguide case, is directly related to the field distribution properties shown in the right inset of Fig. 2, and computed by using full-wave numerical simulations (COMSOL Multiphysics), where an excited donor (D) with a unit electric current dipole moment of 1 A · m is located in the center or slightly to the right of the center (rod) of each waveguide, where the maximum field enhancement of each resonant mode occurs. It is clear that the ENZ plasmonic waveguide shows much larger field enhancement combined with uniform field distribution along the entire channel in contrast to the other two waveguide types, where a standing wave distribution is observed along the finite-length channels due to Fabry-Perot (FP) interference. Therefore, a substantial two orders of magnitude RET enhancement is achieved by using the ENZ waveguide compared to the other two waveguides types, which is also independent to the emitters’ separation distance. Finally, we want to point out that, unlike the infinite waveguide
case [11], the nETR does not continuously increase in the finite groove or rod waveguides. The nETR rapidly oscillates as a function of the separation distance because the plasmon reflections from the waveguides ends can increase or decrease qubit-waveguide coupling due to constructive or destructive interference [17]. Note that the large increase of RET rates shown in Fig. 2 with an increment of the distance between the D-A emitter pair is counterintuitive, since ohmic losses exist in the three plasmonic waveguide cases. This is because the used channel lengths are always smaller than the propagation lengths in all the currently investigated waveguide cases. Hence, the absorption of each waveguide propagating mode is limited and does not cause the RET to decrease for the currently used separation distances [11].

III. ROBUST ENTANGLEMENT MEDIATED BY THE PLASMONIC WAVEGUIDES

In this section, we study the quantum entanglement between a pair of two-level quantum emitters mediated by the ENZ plasmonic waveguide. The response is compared to the other two plasmonic waveguides presented in Figs. 1(b), (c). The theory of both transient and steady-state quantum entanglements, quantified by computing the concurrence metric, is briefly introduced. Then, we numerically calculate the entanglement using the classical electromagnetic Green’s function. Finally, we describe how to achieve the steady-state enhanced entanglement and propose a practical way to detect it by computing the second-order correlation function

A. Entanglement Theory

We consider two identical two-level emitters (also known as qubits) with the same emission frequency \( \omega_0 \) placed inside different plasmonic waveguides, as schematically illustrated in Figs.
The dynamic evolution of quantum systems coupled to lossy plasmonic environments can be fully characterized by the dyadic Green’s function in combination with the formalism of master equations [33,34]. The former is a classical quantity that is widely used to study the spontaneous decay of quantum emitters coupled to lossless or lossy structures [5]. The latter is a quantum equation that is used to describe the dynamics of the density matrix $\rho$ of the two-qubit system in the vicinity of a plasmonic reservoir [4]. Assuming weak excitation (no saturation) and operation in the weak coupling regime, the Born-Markov and rotating wave approximations can be applied to compute the master equation which is given by [33,34]:

$$\frac{\hat{\rho}}{\hat{\rho}} = \frac{1}{\hbar} [H, \rho] - \frac{1}{2} \sum_{i,j=1}^{2} \gamma_{ij} \left( \rho \sigma_i^\dagger \sigma_j + \sigma_j^\dagger \sigma_i \rho - 2 \sigma_i \rho \sigma_j \right),$$  \hspace{1cm} (2)

where the Hamiltonian characterizing the coherent part of the dynamics in Eq. (2) is equal to:

$$H = \sum_i \hbar (\omega_i + g_{ii}) \sigma_i^\dagger \sigma_i + \sum_{i\neq j} \hbar g_{ij} \sigma_i^\dagger \sigma_j.$$  \hspace{1cm} (3)

In the above equations, $\rho$ is the density matrix of the system of two identical qubits; and $\sigma_i$ ($\sigma_i^\dagger$) is the destruction (creation) operator applied to the $i$-th qubit. The Lamb shift $g_{ii}$ is due to the self-interaction of each qubit placed inside the ENZ waveguide and is usually much lower (GHz range) compared to the higher THz optical frequencies involved in our study [35]. Thus, it can be neglected, especially for distances between the emitter and the metallic walls larger than 10 nm [5]. The other part of Eq. (2), given by $g_{ij}$, characterizes the coherent dipole-dipole interactions and can be computed by the formula [16]:

$$g_{ij} = \left( \frac{\omega_i^2}{\varepsilon_e \hbar c^2} \right) \text{Re}[\mathbf{\mu}_i^* \cdot \mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega_e) \cdot \mathbf{\mu}_j],$$  \hspace{1cm} (4)
where the Green’s tensor $G(r_i, r_j, \omega_0)$ satisfies the classical Maxwell equations for a point dipole source located at an arbitrary spatial position $r_j$ [5]. Equation (4) characterizes the dipole-dipole coupling between the qubits placed in spatial points $r_j$ and $r_i$. The parameters $\gamma_{ij}$ in the dissipative and noncoherent term of the master equation (2) are given as a function of the imaginary part of the Green’s tensor, which is also referred to as local density of states (LDOS) [16]:

$$\gamma_{ij} = \left(2\omega_0^2 / e_0 h c^2 \right) \text{Im} [\mathbf{\mu}^* \cdot G(r_i, r_j, \omega_0) \cdot \mathbf{\mu}]. \quad (5)$$

Equation (5) can be used to compute the decay rate induced by self-interactions ($\gamma_{ii}$), also known as spontaneous emission rate, and mutual interactions ($\gamma_{ij}$). Therefore, $\gamma_{12}$ represents the contribution to the decay rate of Emitter 1 at an arbitrary position $r_i$ due to interference caused by Emitter 2 located at an arbitrary position $r_2$ inside the plasmonic waveguides. Both Eqs. (4) and (5) can be computed by solving Maxwell’s equations, either analytically or numerically, through full-wave simulations. Note that the relations $\gamma_{ij} = \gamma_{ji}$ and $g_{ij} = g_{ji}$ are always valid in the currently studied reciprocal waveguides because of the Green’s dyadic symmetry.

The goal is to solve Eq. (2) for the proposed ENZ and the other plasmonic waveguide systems. In order to solve this equation and compute the density matrix, a convenient basis for the vector space of the two-qubit system needs to be defined. As we study identical emitters placed into equivalent positions, i.e. $\gamma_{11} = \gamma_{22} = \gamma$, it is convenient to work in the Dicke basis:

$$|3\rangle = |e_1\rangle \otimes |e_2\rangle = |e_1, e_2\rangle, \quad |0\rangle = |g_1\rangle \otimes |g_2\rangle = |g_1, g_2\rangle, \quad \text{and} \quad |\pm\rangle = 1/\sqrt{2} (|e_1, g_2\rangle \pm |g_1, e_2\rangle), \text{where} \ |e_1\rangle$$
is the excited state of the $i$-th qubit, while $|g_i\rangle$ is the ground state of the $i$-th qubit. This basis is the appropriate way to characterize the response of two-qubit systems since it leads to a diagonalized Hamiltonian [Eq. (3)] [15–17]. A schematic of the collective states of two identical emitters coupled to a dissipative plasmonic system is given in Fig. 3, where it can be seen that the ground state $|0\rangle$ and the excited state $|3\rangle$ are not affected by the dipole-dipole interactions. However, the dipole-dipole interactions lead to a shift at the energies of the symmetric $|+\rangle$ and antisymmetric $|-\rangle$ collective states by $\pm g_{12}$ compared to their energies in the case when the dipole-dipole interactions are negligible. Moreover, the qubit-qubit dissipative coupling induces the modified collective decay rates $\gamma + \gamma_{12}$ and $\gamma - \gamma_{12}$, which correspond to superradiant and subradiant states, respectively [34]. It is shown in the next section that in the case of the ENZ waveguide $\gamma = \gamma_{12}$ and we observe a pure superradiant emission and a zero subradiant decay rate independent to the emitters’ separation distance [27,36]. This is different compared to the collective response of the groove and rod plasmonic waveguides.

The master equation (2) is solved in the two qubit state basis shown in Fig. 3. Only one of the qubits is assumed to be excited initially, thus $\rho_{33}(t) = 0$ and $\rho_{00}(0) = 0$. Hence, the system is prepared in the unentangled state $|e_1, g_2\rangle = 1/\sqrt{2}(|+\rangle + |-\rangle)$. In this case, the only nonzero components of the density matrix at $t = 0$ are $\rho_{++}(0) = \rho_{--}(0) = \rho_{+-}(0) = \rho_{-+}(0) = 1/2$; and the density matrix elements simply become:

$$\rho_{++}(t) = 0.5e^{-(\gamma + \gamma_{12})t}$$ (6)

$$\rho_{--}(t) = 0.5e^{-(\gamma - \gamma_{12})t}$$ (7)
\[ \rho_{++}(t) = 0.5 e^{-(\gamma - 2g_{12})t} \]
\[ \rho_{--}(t) = 0.5 e^{-(\gamma + 2g_{12})t}. \]

In general, the entanglement between two emitters can be quantified by computing the concurrence \( C \) that was originally introduced by Wootters [37]. This quantity is defined as:

\[ C = \max(0, \sqrt{u_1 - u_2 - u_3 - u_4}) \],

where \( u_i \) are the eigenvalues of the matrix \( \rho \tilde{\rho} \) and \( \tilde{\rho} = \sigma_y \otimes \sigma_y \rho \sigma_y \otimes \sigma_y \) is the spin-flip density matrix, with \( \sigma_y \) being the Pauli matrix. The value of concurrence can vary between zero (unentangled state) to one (completely entangled qubits).

If the density matrix of the system is characterized by Eqs. (6)-(9) where only one emitter is initially in the excited state, then the concurrence can be simplified and computed by the formula [17]:

\[ C(t) = \sqrt{(\rho_{++} - \rho_{--})^2 + 4 \text{Im}(\rho_{+-})^2}. \]  

After substituting Eqs. (6)-(9) in Eq. (10), it takes the final form:

\[ C(t) = 0.5 \sqrt{\left[e^{-(\gamma + 2g_{12})t} - e^{-(\gamma - 2g_{12})t}\right]^2 + 4e^{-2\gamma t} \sin^2 \left(2g_{12}t\right)}. \]

The derived transient concurrence [Eq. (11)] is a very useful quantity that can provide physical insights about the transient entanglement process between the two emitters when only one of the emitters is excited. First, it can be seen that \( C(0) = 0 \) which is expected because the system is initially at an unentangled state. As the time progresses, \( t > 0 \), the concurrence becomes larger than zero meaning that the emitters become entangled. However, at some point the concurrence starts to decay with time and becomes zero again, \( C(t) = 0 \), after a long period of time \( (t \to \infty) \).
In addition, the concurrence can strongly oscillate for short time durations \((t < 1/2\gamma)\) if the coherent dipole-dipole interactions are large \((g_{12} \gg \gamma)\). This is usually the case for a photonic-crystal or a microcavity-based entanglement [38,39] that can lead to high concurrence values; but for a very short time duration. Notice that for the ideal case of a lossless and infinite waveguide, \(\gamma = \gamma_{12}\), i.e., the decay rate of the asymmetric state \(\ket{-}\) is zero, so the entanglement can grow with time monotonically and obtain a steady-state with concurrence up to the maximum value of \(C = 0.5 \) [15,16,40]. It is demonstrated in the next section that the proposed ENZ waveguide system satisfies the ideal conditions, i.e., \(g_{12} \ll \gamma\) and \(\gamma = \gamma_{12}\), and achieves long lasting strong concurrence without the onset of oscillations.

In order to prevent the transient concurrence from decaying after some time and obtain a more practical steady-state entanglement, external pumps with the same frequency \((\omega_p)\) are used to individually pump each emitter or an emitter cluster loaded in the plasmonic waveguides. In this case, an additional term \(1/i\hbar[V, \rho]\) needs to be introduced in the right hand side of the master equation (2), where the operator:

\[
V = -\sum_i \hbar \left( \Omega_i e^{-i\Delta_i t} \sigma_i^+ + \Omega_i^* e^{i\Delta_i t} \sigma_i \right),
\]

characterizes the interaction between the pump field and the emitter [17]. The parameter \(\Omega_i = \mu \cdot E_{\omega_i}/\hbar\) is the effective Rabi frequency of the pump that depends on the field \(E_{\omega_i}\) induced at the pumped \(i\)-th emitter inside the ENZ waveguide. The parameter \(\Delta_i = \omega_0 - \omega_p\) is the detuning parameter due to the pump frequency \(\omega_p\). After expressing \(\rho\) in the usual basis \(|e_1, e_2\rangle, |e_1, g_2\rangle, |g_1, e_2\rangle,\) and \(|g_1, g_2\rangle\), we can calculate the steady state concurrence by solving
numerically the modified versions of the previously derived differential equations with density matrix solutions given by Eqs. (6)-(9), where we have included the Rabi frequency and detuning parameter due to the external pumping [17].

**B. Numerical Results**

The theory presented in the previous section is applied to the ENZ, groove, and rod plasmonic waveguide systems. The emission wavelength of the emitters is $\lambda = 1018\text{nm}$, almost equal to the ENZ cutoff wavelength of the ENZ plasmonic waveguide, where the field is strong and homogeneous along the nanochannel (Figs. 1(d) and 2). Note that the cutoff wavelength of this robust ENZ plasmonic system can be tuned to any value, if the width $w$ of the waveguide’s channel is varied [28]. This useful property can be used to accommodate a plethora of different emitters embedded in the proposed ENZ effective medium with various emission frequencies. In the current case, the two identical qubits are either embedded in the middle of the ENZ waveguide, or placed along the narrow plasmonic channels of the rod and groove waveguides (Figs. 1(a)-(c)). The qubits are always placed at two equivalent (symmetric) positions along the waveguides to ensure that $\gamma_{11} = \gamma_{22} = \gamma$ and separated by a distance $d$. Full-wave simulations are performed to compute the Green’s function and, as a consequence, the dipole-dipole interaction coefficients $g_{ij}$ and decay rates $\gamma_{ij}$ given by Eqs. (4) and (5), respectively.

To better understand the collective behavior of the qubits, which is independent of the qubit separation distance, in the case of the ENZ waveguide only, we plot in Fig. 4 the computed mutual interaction decay rates $\gamma_{12}$ and dipole-dipole interactions $g_{12}$ as a function of the separation distance between the qubits positioned symmetrically inside the ENZ waveguide. All
the results are normalized to $\gamma$, i.e., the computed spontaneous emission decay rate of each emitter. We demonstrated in [27], by using the nonlocal density of optical states, that a strong and homogeneous enhancement of the collective spontaneous emission can be achieved by using the proposed ENZ structure, leading to a perfect superradiance effect at the ENZ cutoff resonance for large separation distances between the qubits. Interestingly, the results presented in Fig. 4 provide an alternative approach to prove the perfect superradiance decay that does not depend on the qubits separation distance. It can be seen that, as we gradually increase the separation distance of the qubits placed inside the ENZ waveguide channel, the decay ratio $\gamma_{12}/\gamma$ decreases due to the inevitable decoherence induced by the system’s radiation and plasmonic losses but the ratio is still close to 1 along the entire channel. Therefore, a perfect superradiance decay, independent on the qubits separation distance, is achieved as the self-interactions characterized by $\gamma$ are almost equal to the mutual interactions computed by $\gamma_{12}$ along almost the entire channel of the ENZ waveguide. As a result, when both qubits are excited, the qubit-qubit dissipative coupling happens along the superradiant collective decay state $|+\rangle$ only (see Fig. 3); and the collective decay rate along the subradiant state $|-\rangle$ is suppressed and becomes almost equal to zero. In addition, we note that the dissipative decay rate ratio $\gamma_{12}/\gamma$ plotted in Fig. 4 is much larger than the normalized coherent coupling term $g_{12}/\gamma$ along the entire ENZ channel, directly indicating that the proposed ENZ system satisfies the ideal condition to achieve the best entanglement performance, i.e., $g_{12} \ll \gamma$ and $\gamma = \gamma_{12}$ [16]. Hence, strong quantum entanglement is expected, that persists over extended time periods and long distances, independent of the emitters’ positions and distances, by using the proposed ENZ plasmonic waveguide system.
Next, we study the entanglement between the same two qubits placed inside the ENZ waveguide channel by calculating the concurrence given by Eq. (11), where the dissipative and coupling decay rates are shown in Fig. 4. Note that in this case only one qubit is excited, which is different from the superradiance scenario; and the computed transient concurrences can be seen in Fig. 5 (black lines), where the distances between the two qubits are fixed to three values: (a) 100 nm, (b) 300nm, and (c) 700nm, respectively. We also consider the behavior of concurrence for the emitters coupled to several different environments, such as a homogeneous dielectric bulk space with relative permittivity $\varepsilon = 2.2$ (green lines), and the finite groove (red lines) and cylindrical rod (blue lines) plasmonic waveguides. It is clear that all the three plasmonic waveguides demonstrate much more efficient entanglement compared to the case of a pure dielectric medium, where moderately high concurrence values are only obtained for very small inter-qubit distances (near-field) and for very short time durations [Fig. 5(a)]. Strong SPP and ENZ modes are excited along the plasmonic waveguides [Figs. 1(d)-1(f)] that lead to remarkable confinement and enhancement of the electromagnetic fields in stark contrast to the dielectric medium case. We also clearly observe in Fig. 5 that the transient concurrence, and hence entanglement, mediated by the ENZ plasmonic waveguide is superior compared to the other two plasmonic waveguides, even for large emitters’ separation distances (Fig. 5(c)). This is due to the homogeneous electromagnetic field mode (inset of Fig. 2) at the ENZ cutoff wavelength which spreads across the entire ENZ waveguide geometry resulting in the large absolute values of the decay rate $\gamma_{12}$ and very small values of the interaction rate $g_{12}$ along the entire dielectric nanochannel (Fig. 4). On the contrary, the qubits entanglement mediated by the finite groove and rod waveguides depends strongly on the spatial position of both emitters, which is a severe disadvantage for their practical applications, since it is very difficult to accurately position
nanoemitters in the nanoscale. The qubits concurrence in these waveguides depends strongly on
the inter-emitter distance due to the standing wave field pattern of the FP cavity mode obtained
along each finite-length waveguide channel (inset of Fig. 2).

Up to this point, we have clearly demonstrated that the transient entanglement, mediated by the
ENZ plasmonic waveguide, persists over long periods of time and long distances and, in
addition, is independent of the qubits’ positions and separation distances. However, the transient
entanglement between the emitters placed inside the ENZ waveguide still disappears after a long
time and becomes equal to zero at steady-state, as it is shown in Fig. 5. This effect is a direct
consequence of decoherence caused by the depopulation of the emitters’ excited states due to
radiation and plasmonic losses. Thus, in order to achieve a high steady-state entanglement with
concurrence $C(t \to \infty) \neq 0$, an additional pump excitation applied to the qubits inside the ENZ
waveguide is introduced. Figure 6 shows the concurrence of two qubits driven by the external
pump in the presence of the proposed ENZ plasmonic waveguide compared to the finite groove
and rod plasmonic waveguide cases. The emitters’ locations are fixed at each waveguide’s end
with separation distance $d = 700\text{nm}$. In the upper panels of Fig. 6, we use three types of coherent
pumps: (a) asymmetric pumping with Rabi frequencies $\Omega_1 \neq 0$, $\Omega_2 = 0$, (b) symmetric pumping
with $\Omega_1 = -\Omega_2$, and (c) antisymmetric pumping with $\Omega_1 = -\Omega_2$, leading to different relative
phases among the pumping lasers applied to each qubit. In the ENZ waveguide case we can only
excite the emitters at the ENZ or FP resonances, because this is a closed waveguide system that
can couple energy inside its channel only around these particular resonance frequencies. For the
groove and rod waveguides, the emitters’ excitation is not a problem, since we can excite the
emitters at any frequency point from outside these plasmonic waveguides. Therefore, here, we
set the pump frequency $\omega_p$ to be equal to the emission frequency $\omega_0$ of the emitters (corresponding wavelength $\lambda = 1018\text{nm}$), which is close to the ENZ resonance, leading to a detuning parameter equal to $\Delta_i = \omega_0 - \omega_p = 0$. Note, that the time scale (x-axis) is substantially elongated in Fig. 6 compared to the results in Fig. 5. The ENZ waveguide has the highest transient concurrence values compared to the finite groove and rod waveguides for all types of pumping cases. In addition, the concurrence peaks of all waveguide systems are not very sensitive to the different pumping intensities. However, they rapidly decay when symmetric pumping is used (Fig. 6(b)).

Furthermore, we plot the contour maps of steady-state concurrence in Figs. 6(d)-(f) for a wide range of pumping intensities after a long time duration, $\gamma t = 70$, to compare the entanglement efficiency mediated by the different plasmonic waveguides. It is clear that the largest steady-state concurrence is obtained by using the proposed ENZ plasmonic waveguide system, in contrast to the groove and rod cases, due to the relative large values of $\gamma_{12}$ and small values of $g_{12}$ that are independent of the emitters separation distance (Fig. 4, $d = 700\text{nm}$). Considering the different pumping strategies, we find that the antisymmetric pumping leads to much larger steady-state concurrence in the cases of the ENZ and groove waveguides, while the entanglement in the rod waveguide benefits from the symmetric pumping scheme (as it can be seen in Figs. 6(a)-(c)). It is also interesting to note that the pump strength should not be too large, in order to achieve strong steady-state entanglement between the qubits, otherwise strong interactions between the pump and qubits will occur and eventually lead to qubit decoupling and lasing [17]. Therefore, the contour plots presented in Figs. 6(d)-(f) demonstrate the lower and upper threshold values for the
pump intensities in the case of each of the waveguides, that still preserve the entanglement between the qubits.

The panels in Fig. 7 demonstrate the steady-state concurrence after a long period of time ($\gamma t = 70$, as in Fig. 6) as a function of the emitters’ separation distance for the ENZ (left panels), finite groove (middle panels), and cylindrical rod (right panels) waveguides. Note, that for certain separations distances, a strong steady-state entanglement can be achieved by using all types of plasmonic waveguides when the pumping is relatively weak. However, the unique property of the ENZ waveguide is that the high steady-state concurrence does not depend on the inter-emitter separation distance [Fig. 7(a)]. On the contrary, in the case of the finite groove and rod waveguides, the steady-state concurrence can be high or very small depending on the separation distances between the emitters. In addition, the strong and homogeneous entanglement is observed at the ENZ waveguide [Fig. 7(a)] only for the asymmetric or antisymmetric pumping and relatively low pump intensities, which is consistent with the results presented in Fig. 6.

Finally, we calculate the second-order correlation function for the entangled steady-state, which is a metric that can be measured experimentally. In previous works, it was demonstrated that the degree of photon coherence given by the second-order photon correlation function is related to the concurrence that was used to characterize the entanglement [16,41]. This quantity, for a zero time delay, can be calculated using the density matrix elements [4]:

$$g_{12}^{(2)}(0) = \frac{\langle \sigma_1^+ \sigma_2^+ \sigma_2 \sigma_1 \rangle / \langle \sigma_1^+ \sigma_1 \rangle \langle \sigma_2^+ \sigma_2 \rangle}{\langle \rho \sigma_1^+ \sigma_1 \rangle \langle \rho \sigma_2^+ \sigma_2 \rangle}$$

(13)
It has been demonstrated that the antibunching \((g_{12}^{(2)}(0) \to 0)\) at zero time delay corresponds to the high steady-state concurrence and, as a consequence, to the strong qubit entanglement [16]. Therefore, measurements of the two-photon correlation function at zero delay can be used for entanglement detection.

To verify the relationship between \(C_{ss}\) and \(g_{12}^{(2)}(0)\), we plot in Fig. 8 the contour maps of the zero-delay second-order correlation function \(g_{12}^{(2)}(0)\) for a wide range of pumping intensities. The emitters’ separation distance is fixed to the same value used in Fig. 6 \((d = 700\text{nm})\). As can be seen by comparing Figs. 6(d)-(f) and 8(a)-(c), the antibunching signature coincides with the high steady state concurrence values. When \(C_{ss}\) is large, the corresponding \(g_{12}^{(2)}(0)\) is close to zero (antibunching), and when \(C_{ss}\) goes to zero, \(g_{12}^{(2)}(0)\) grows to large values, which is consistent with the results presented in [16] and [41] for groove and cylindrical rod waveguides, respectively. In addition, the plot in Fig. 8 clearly shows that antibunching \((g_{12}^{(2)}(0) \to 0)\) occurs for a much broader pumping range only in the case of the ENZ plasmonic system and not with the other plasmonic waveguides, confirming another practical advantage of the presented ENZ waveguide system.

Finally, we compute and plot in Fig. 9 the second-order correlation function at zero time delay, \(g_{12}^{(2)}(0)\), as a function of the inter-emitter separation distance for the three plasmonic waveguides and for fixed pumping intensity values. Interestingly, the computed \(g_{12}^{(2)}(0)\) value is almost constant for the ENZ waveguide case, when we spatially vary the relative positions of the two emitters, in agreement with the results presented in Fig. 7(a), where the steady-state
entanglement $C_{ss}$ was found to be independent of the emitters’ position and separation distances in the ENZ system. This constitutes the major advantage of the proposed ENZ plasmonic system when it comes to the experimental detection and measurement of the strong entanglement, especially in the common practical scenario of quantum emitters arbitrarily placed inside the resonating plasmonic system, since it is extremely difficult to accurate place emitters in nanoscale regions. On the contrary, the $C_{ss}$ strongly fluctuates as a function of the inter-emitter separation distance for the finite groove and rod waveguides, which means that the emitters need to be accurately placed at particular nanoscale spots along these nanowaveguides to detect the entanglement.

IV. CONCLUSIONS

The dynamic evolution of the qubits coupled to lossy plasmonic environments have been studied using the dyadic Green’s function formalism in combination with the master equation. We have demonstrated efficient entanglement and giant RET enhancement between two quantum emitters (qubits) placed inside the ENZ metamaterial waveguide. What is even more important is that the entanglement and RET are independent of the emitters’ positions and separation distances due to the peculiar properties of the ENZ waveguides operating near their cutoff frequency, e.g., infinite phase velocity of the tunneled electromagnetic wave and a phase that is uniform along the nanochannels. These interesting features, combined with the strong and homogeneous electromagnetic fields inside the ENZ waveguides, are ideal conditions to boost coherent light-matter interactions over long distances and increase the energy transfer rate between a donor-acceptor pair placed inside the nanochannels. In particular, the simulation results show that the
RET can be enhanced by two orders of magnitude by using the ENZ system with respect to the widely studied V-shaped and cylindrical rod plasmonic waveguides. Moreover, both the transient and steady-state entanglement between the quantum emitters can be improved by placing them in arbitrary positions inside the ENZ waveguide nanochannel. Since it is very difficult to control the position of the emitters in nanoscale regions, the proposed ENZ plasmonic waveguide is advantageous for the experimental implementation of the strongly entangled states compared to alternative plasmonic waveguides (groove or rod), where the emitters need to be placed in predefined positions to achieve maximum entanglement. Moreover, the entangled state of the emitter placed inside the ENZ waveguide is robust enough to be observed experimentally by using second-order correlation function measurements because it persists over extended time periods and long distances. We envision that the presented ENZ-based entangled states will find applications in future quantum information and communications integrated systems on a chip [42,43], the design of new low-threshold subwavelength nanolasers [44], and the creation of ultrasensitive quantum metrology devices [45].

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Figure 1 – Geometry of the proposed (a) ENZ plasmonic waveguide and two alternative plasmonic waveguides: (b) V-shaped groove and (c) cylindrical nanorod. All the presented waveguides are made of silver. A pair of two-level emitters [the donor (D) and acceptor (A)] are placed inside the nanochannel or along the different plasmonic waveguide structures. The lower panels (d)-(f) display the normalized fundamental mode distributions in the transverse cross section (yz-plane) at the same wavelength $\lambda = 1018\text{nm}$ and the geometry dimensions of each waveguide.
Figure 2 – The computed nETR values as a function of the separation distance $d$ between the donor-acceptor pair in the cases of the ENZ, finite groove, and cylindrical rod plasmonic waveguides (see Fig. 1(a) for the specifications of the waveguides geometries). We assume the location of the acceptor to be fixed in the beginning of the waveguide channel while the donor position is being continuously moved along the x-axis. Right insets: The static electric field patterns when the donor with the electric dipole moment polarized along the z-axis is placed in the center (ENZ, groove) or slightly to the right off the center (rod) of each waveguide.
Figure 3 – Collective states diagram of two identical emitters. The two identical qubits (emitters) are located at equivalent positions with respect to the plasmonic system and have identical orientations.

Figure 4 – Normalized decay rate $\gamma_{12}/\gamma$ and dipole-dipole interaction $g_{12}/\gamma$ of two qubits placed inside the ENZ plasmonic waveguide as a function of the emitters’ separation distance $d$. The obtained values are normalized to $\gamma$, which is the spontaneous emission decay rate of a single emitter placed in the waveguide.
Figure 5 – Transient concurrence between the two qubits placed in the ENZ (black lines), groove (red lines), and cylindrical rod (blue lines) nanowaveguides for three inter-qubits separation distances: (a) $d = 100\text{nm}$, (b) $d = 300\text{nm}$, and (c) $d = 700\text{nm}$. The green lines refer to the qubits placed in a dielectric bulk medium (no waveguide) with relative permittivity $\varepsilon = 2.2$. 


Figure 6 – Time dependence of the concurrence between two qubits pumped by (a) asymmetric, (b) symmetric, and (c) antisymmetric pumping. Lower panels (d)-(f): Steady state concurrence vs pumping intensities for the cases of (d) ENZ, (e) groove, and (f) cylindrical rod plasmonic waveguides. The emitters’ separation distance is fixed to \( d = 700 \text{nm} \) in these results.
Figure 7 – Steady state concurrence as a function of the qubits’ separation distance monitored at the normalized time $\gamma t = 70$ for the cases of (a) ENZ, (b) groove, and (c) cylindrical rod plasmonic waveguides by using the asymmetric, symmetric, and antisymmetric pumping.

Figure 8 – Zero-delay correlations vs pumping intensities for the cases of (a) ENZ, (b) groove, and (c) cylindrical rod plasmonic waveguides, using the same parameters reported before in Figs. 6(d)-(f). The emitters’ separation distance is fixed to $d = 700\text{nm}$ in these results.
Figure 9 – A potential experimental validation of the qubit-qubit entanglement characterized by the second-order photon correlation function at zero time delay as a function of the emitters’ separation distance for the cases of (a) ENZ, (b) groove, and (c) cylindrical rod plasmonic waveguides by using asymmetric, symmetric, and antisymmetric pumping.