Spin waves in magnetic quantum wells with Coulomb interaction and \textit{sd} exchange coupling

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Abstract

We theoretically describe the spin excitation spectrum of a two dimensional electron gas embedded in a quantum well with localized magnetic impurities. Compared to the previous work, we introduce equations that allow to consider the interplay between the Coulomb interaction of delocalized electrons and the \textit{sd} exchange coupling between electrons and magnetic impurities. Strong qualitative changes are found: mixed waves propagate below the single particle continuum, an anticrossing gap is open at a specific wavevector and the kinetic damping due to the electron motion strongly influences the coupling strength between electrons and impurities spins.

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I. INTRODUCTION

Collective spin dynamics in dilute magnetic semiconductors (DMS) has recently drawn lots of attention. This field provides an insight into the origins of carrier-induced ferromagnetism in semiconductors and particular features in the spin excitation spectrum due to the presence of two spin sub-systems that are dynamically coupled by Coulomb-exchange interaction: that of the itinerant carrier and that of the localized magnetic impurities. The transverse spin excitation spectrum has been theoretically found to be composed of three types of excitations. These are: two collective spin waves corresponding to itinerant and localized spins precessing in phase or out of phase to each other, and single-particle (or Stoner-like) excitations of the itinerant carriers. If the DMS is in the ferromagnetic state, the in-phase spin wave (IPW) becomes the Goldstone-like mode with an acoustic type dispersion responsible for long-range spin order in the ground state. The out of phase spin wave (OPW) develops an optical branch, with a zone-center energy determined by the strength of Coulomb-exchange interaction between carriers and the spins of magnetic impurities.

Experimental evidence of the entire spectrum in a ferromagnetic DMS like GaMnAs is not available. What has been reported so far are features related to the zone-center IPW, dominated by the Mn spin precession, its dynamics and its ferromagnetic resonance. We find no experimental data available for the out of phase mode. Indeed, ferromagnetism in GaMnAs systems requires a high Mn concentration, which destroys the periodicity of the crystal potential and smooths out all optical resonances.

More insight into the DMS spin excitation spectrum has been gained in CdMnTe doped quantum wells (QW), which constitute a clean test-bed system, appropriate to capture general properties of the collective spin dynamics in DMS materials. Evidence for carrier-induced ferromagnetism has been found in CdMnTe quantum wells doped with holes. When doped with electrons, due to the very low Curie temperature, only the paramagnetic phase is available to most experiments. The OPW mode dispersion and single-particle excitations have been probed by Raman measurements in the paramagnetic state. The mixed nature of the IPW and OPW waves has been evidenced in the frequency and time domain. Nevertheless, there is a lack of a full theoretical description of the spin excitation spectrum in CdMnTe QW. Indeed, so far two approaches were followed to describe the spin
The first subband is populated by \( n_{2D} \) electrons per unit surface. The 2DEG-DMS Hamiltonian under the influence of a static magnetic field \( \mathbf{B} = B\mathbf{e}_z \) applied in the plane of the QW writes :
\[
\hat{H} = \hat{H}_{Kin} + \hat{H}_{Coulomb} + \hat{H}_{s-d} + \hat{H}_{Zeeman}
\]

\[
\hat{H}_{s-d} = -\alpha \iiint \hat{S}(r) \cdot \hat{M}(r) \, d^3r
\]

\[
\hat{H}_{Zeeman} = g_e \mu_B \iiint \hat{S}(r) \cdot \hat{B} d^3r + g_{Mn} \mu_B \iiint \hat{M}(r) \cdot \hat{B} d^3r
\]

where \(\alpha\) is the exchange coupling between conduction electrons and Mn spins \((\alpha > 0)\), and \(g_e\) and \(g_{Mn}\) are normal g-factor of, respectively, conduction electrons and Mn electrons. In the convention where \(\mu_B > 0\), we have \(g_e \simeq -1.44\) and \(g_{Mn} \simeq 2.00\). We have introduced two vector operators: \(\hat{S}(r) = \chi^2(y) \sum_i \hat{s}_i \delta(r/ - r_i//)\) is the 3D electron spin density in a splitted coordinates frame \(r = (r//, y)\) with \(r//\), the in-plane position and \(y\) the out of plane coordinate. \(\chi(y)\) is the electron envelope-function of the first subband of the QW. The \(i\) index accounts for the \(i\)-th electron of the 2DEG, its spin \(1/2\) is described by the operator \(\hat{s}_i\) and its position is \(r_i//\). \(\hat{M}(r) = \sum_j \hat{I}_j \delta(r - R_j)\) is the Mn 3D spin density. The \(j\)-th \(5/2\)-spin \(\hat{I}_j\) of a single Mn impurities is localized on the cation site \(R_j\). In the equilibrium state at temperature \(T\), each Mn spin has the average value \(\langle \hat{I}_z \rangle (B, T)\), which is the thermodynamic average over the five occupied states of the Mn atom d-shell, given by the modified Brillouin function\(^{21}\). The 2DEG has the equilibrium spin polarization \(\zeta = (n^+ - n^-)/n_{2D}\).

We, now, rewrite the \(s-d\) Hamiltonian using the electron (and Mn) spin fluctuations operators at in-plane wave-vector \(q\), respectively, \(\hat{S}_q = \iiint \hat{S}(r) e^{-iqr///d^2r//dy} = \sum_i \hat{s}_i e^{-iqr///}\) and \(\hat{M}_q = \sum_j \hat{I}_j e^{-iqR//j}\). Due to the 2D and 3D characters of, respectively, the conduction electron and Mn spins subsystems, the electron spin-degrees of freedom naturally couples to Mn spin profile weighted by the squared electron wave-function. For later convenience, we introduce the following \(n\)-profile Mn spin fluctuations operators:

\[
\hat{M}_{q}^{(n)} = w^n \iiint \chi^{2n}(y) \hat{M}(r) e^{-iqr///d^2r//dy} = w^n \sum_j \chi^{2n}(y_j) \hat{I}_j e^{-iqR//j}
\]

Hence, \(\hat{M}_q = \hat{M}_q^{(0)}\). \(\hat{M}_q^{(n)}\) are vector operators verifying the following commuting relations:

\[
\left[ \hat{M}_{\alpha, q}^{(n)}, \hat{M}_{\beta, q'}^{(p)} \right] = i\epsilon_{\alpha, \beta, \gamma} \hat{M}_{\gamma, q+q'}^{(n+p)}
\]

(2)
where $\alpha, \beta, \gamma = x, y, z$ and $\epsilon_{\alpha,\beta,\gamma}$ is the Levi-Cevita tensor. It follows:

$$
\hat{H}_{s-d} = -\frac{\alpha}{wL^2} \sum_\mathbf{q} \hat{S}_\mathbf{q} \cdot \hat{M}_\mathbf{q}^{(1)}
$$

$$
= -\tilde{\alpha} \left\{ \delta \hat{S}_{z,\mathbf{q}=0} \cdot \langle \hat{M}_{z,\mathbf{q}=0}^{(1)} \rangle_0 + \langle \hat{S}_{z,\mathbf{q}=0} \rangle_0 \cdot \delta \hat{M}_{z,\mathbf{q}=0}^{(1)} + \frac{1}{2} \hat{S}_{+,\mathbf{q}=0} \cdot \hat{M}_{-,\mathbf{q}=0}^{(1)} + \frac{1}{2} \hat{S}_{-,\mathbf{q}=0} \cdot \hat{M}_{+,\mathbf{q}=0}^{(1)} \\
+ \sum_{\mathbf{q}\neq 0} \hat{S}_\mathbf{q} \cdot \hat{M}_\mathbf{q}^{(1)} \right\}
$$

In Eq. (3a), we have defined the exchange coupling constant $\tilde{\alpha} = \alpha/wL^2$, equilibrium averaging $\langle \rangle_0$ and fluctuation operators: $\delta \hat{A} = \hat{A} - \langle \hat{A} \rangle_0$.

Finally, we get:

$$
\hat{H}_{s-d} + H_{\text{Zeeman}} = Z_e (B) \cdot \delta \hat{S}_{z,\mathbf{q}=0} + g_{Mn} \mu_B B \cdot \delta \hat{M}_{z,\mathbf{q}=0}^{(0)} + K \cdot \delta \hat{M}_{z,\mathbf{q}=0}^{(1)}
$$

$$
- \tilde{\alpha} \frac{1}{2} \left( \hat{S}_{+,\mathbf{q}=0} \cdot \hat{M}_{-,\mathbf{q}=0}^{(1)} - \hat{S}_{-,\mathbf{q}=0} \cdot \hat{M}_{+,\mathbf{q}=0}^{(1)} \right)
$$

$$
- \tilde{\alpha} \delta \hat{S}_{z,\mathbf{q}=0} \cdot \delta \hat{M}_{z,\mathbf{q}=0}^{(1)}
$$

$$
- \tilde{\alpha} \sum_{\mathbf{q}\neq 0} \hat{S}_\mathbf{q} \cdot \hat{M}_\mathbf{q}^{(1)}
$$

with the total bare Zeeman energy of conduction electrons given by:

$$
Z_e (B) = \tilde{\alpha} \gamma_1 N_{Mn} \left| \int \langle \hat{I}_z \rangle (B, T) \right| - |g_e| \mu_B B
$$

where $\gamma_1 = \int_0^w x^2 (y) \, dy$ is the probability to find the electron in the QW, $N_{Mn} = x_{eff} N_0 wL^2$ is the number of Mn spins available in the QW. In Eq. (5), we evidence the "Overhauser shift" $\Delta$ and the normal Zeeman contribution, opposite to $\Delta$ (the $sd$ contribution). Indeed, as $g_e < 0, g_{Mn} > 0$ and $\alpha > 0$, the Mn spins are anti-parallel to the field, thus, the $sd$ coupling aligns the electron spins anti-parallel to the field, while the normal Zeeman aligns the electron spin parallel to the field. When the $sd$ coupling dominates over the normal Zeeman contribution and $B > 0$, both Mn and electron spins are $\downarrow: \langle \hat{I}_z \rangle < 0$ and the 2DEG spin polarization degree $\zeta$ is also negative.

The first line, Eq. (4a) gives the mean-field "effective Zeeman" Hamiltonian, where no-dynamical coupling between electrons and Mn spins appears. The mean-field Hamiltonian
naturally introduces the "Knight shift", due to the equilibrium electron spin polarization
\[ \langle S_z, q = 0 \rangle_0 = n_{2D} L^2 \zeta / 2, \]
which shifts the Mn spin precession energy:
\[ K = -\tilde{\alpha} \langle S_z, q = 0 \rangle_0 = \frac{1}{2} \frac{\alpha}{w} n_{2D} |\zeta| \]  
\[ (6) \]

Eq. (4b) and Eq. (4d) give rise to a first order sd—dynamical coupling between transverse spin degrees of freedom and induce spin-mixed electron-Mn modes of precession. They also contain higher orders correlation terms which have been indentified to be responsible for a damping\(^5\). The effects of the correlations contained in Eqs. (4b)-(4d) are out of the scope of the present work and will be neglected when they appear.

III. TRANSVERSE SPIN DYNAMICS

A. SP2DEG dynamics without the s-d dynamical coupling

In this paragraph, we take into account only the first line of Eqs. (4a)-(4d). Hence conduction electron and Mn dynamics are independent. The 2DEG is polarized by the static exchange field of Mn spins, this forms a spin-polarized 2DEG (SP2DEG) as described in Ref\(^{18}\). The Hamiltonian which rules the electron dynamics in the SP2DEG reduces to:
\[ \hat{H}_{SP2DEG} = \hat{H}_{Kin} + \hat{H}_{Coulomb} + Z_e(B) \hat{S}_{z, q = 0} \]  
\[ (7) \]
Introducing the the electron creation-annihilation operators, a spin-flip single particle excitation (SF-SPE) is described by a single electron-hole pair operator \( c^+_{k, q, \uparrow} c_{k, q, \downarrow} \), and electrons spin-wave operators, introduced above, are given by \( \hat{S}_{+, q} = \sum_k c^+_{k-q, \uparrow} c_{k, \downarrow} \). Let’s notice that \( \{ \hat{S}_{+, q, i}, c^+_{k-q', \uparrow} c_{k, \downarrow} \} = 0 \), such that collective and single particle modes are not intrinsically coupled. In the following we will use exact commutation rules to write equation of motions for these normal modes of the SP2DEG. We will note \( \left( d\hat{A}/dt \right)_{2DEG} = \left[ \hat{A}, \hat{H}_{SP2DEG} \right] /i\hbar \) the time derivative of \( \hat{A} \) related to \( \hat{H}_{SP2DEG} \) only. Further, we will make use of linear response theory to derive quantities like:
\[ \langle \langle \hat{A}; \hat{B} \rangle \rangle_\omega = -\frac{i}{\hbar} \lim_{\varepsilon \to 0^+} \int_0^{+\infty} \langle \hat{A}(t), \hat{B} \rangle_0 e^{-i\omega t - \varepsilon t} dt \]  
\[ (8) \]
which gives the linear response of an observable \( \hat{A} \) to a perturbation coupled linearly to \( \hat{B} \) in the considered Hamiltonian\(^{22}\).
\[ \langle\langle \hat{A}; \hat{B} \rangle\rangle_{\omega} \text{ has the following equations of motion:} \]

\[
\begin{align*}
\langle\langle \hat{A}; \hat{B} \rangle\rangle_{\omega} &= -\frac{i}{\omega} \langle\langle \hat{A}; \hat{B} \rangle\rangle_{\omega} - \frac{1}{\hbar \omega} \langle [\hat{A}, \hat{B}] \rangle_0 \\
&= \frac{i}{\omega} \langle\langle \hat{A}; \hat{B} \rangle\rangle_{\omega} - \frac{1}{\hbar \omega} \langle [\hat{A}, \hat{B}] \rangle_0 
\end{align*}
\]

(9a)

(9b)

where \( \dot{\hat{A}} = [\hat{A}, \hat{H}] / \hbar \) is the time derivative of \( \hat{A} \) related to the considered Hamiltonian.

We will note \( \langle\langle \hat{A}; \hat{B} \rangle\rangle_{2DEG} \) when \( \hat{A} \) is replaced by \( \left( \frac{d\hat{A}}{dt} \right)_{2DEG} \).

1. Single particle modes dynamics

The kinetic Hamiltonian \( \hat{H}_{Kin} = \sum_{k, \sigma} E_k c_{k, \sigma}^+ c_{k, \sigma} \) and the mean-field Zeeman Hamiltonian conserve the single-particle modes:

\[
\left[ c_{k-q, \uparrow}^+ c_{k, \downarrow}, \hat{H}_{Kin} + Z_e (B) \hat{S}_z, q = 0 \right] = (E_k - E_{k-q} - Z_e) c_{k-q, \uparrow}^+ c_{k, \downarrow}
\]

But the Coulomb Hamiltonian \( \hat{H}_{Coulomb} = \frac{1}{2} \sum_{q \neq 0, k', \sigma} V_q c_{k+q, \sigma}^+ c_{k', -q, \sigma'}^+ c_{k', \sigma} c_{k, \sigma} \) couples a single particle mode to multi-pair modes having a spin +1:

\[
\left[ c_{k-q, \uparrow}^+ c_{k, \downarrow}, \hat{H}_{Coulomb} \right] =
\]

\[
+ \sum_{q' \neq 0, k', \sigma} V_{q'} \left( c_{k-q+q', \uparrow}^+ c_{k', -q', \sigma}^+ c_{k', \sigma} c_{k, \downarrow} \right) \\
- \sum_{q' \neq 0, k', \sigma} V_{q'} \left( c_{k-q, \uparrow}^+ c_{k', \sigma} c_{k', -q', \sigma}^+ c_{k-q', \downarrow} \right)
\]

where \( V_q \) is the space Fourier transform of the bare Coulomb interaction. It follows from Eq. (10) that \( \hat{H}_{Coulomb} \) does not conserve the SPE motion, but introduces an infinite hierarchy where a single electron-hole pair of the Fermi sea (a SPE) couples to multiple pairs having the same global spin. Approximations can be made: in the rhs terms of Eq. (10), some conserve the SPE motion and renormalize it, others introduce a scattering effect, the so-called spin-Coulomb drag \(^{23}\), and can be described by an electron-electron scattering time \( \tau_{e-e} \). The former consists in making the random phase approximation (RPA) on single
mode dynamics, i.e., keeping in Eq. (10) only terms which can be written as a product of a SF-SPE with an occupation number \( \hat{n}_{k,\sigma} \), and replacing \( \hat{n}_{k,\sigma} \) by its average value \( \langle \hat{n}_{k,\sigma} \rangle_0 \). Then the Coulomb factor \( V_q \) has to be replaced by a local field factor \( G_{xc} \) which accounts for the effective dynamical exchange-field produced by other electrons (a part of what has been suppressed in making the RPA). Adding a damping rate \( \eta = \hbar/\tau_{e-e} \) due to the scattering leads to the SF-SPE equation of motion that we will use in the following:

\[
i\hbar \left( \frac{d}{dt} c_{k-q,\uparrow} c_{k,\downarrow} \right) = \left( E_k - E_{k-q} - Z_e + 2G_{xc} \left( \hat{S}_{z,q=0} \right)_0 \right) c_{k-q,\uparrow}^+ c_{k,\downarrow} \\
+ G_{xc} \left( \langle \hat{n}_{k,\downarrow} \rangle_0 - \langle \hat{n}_{k-q,\uparrow} \rangle_0 \right) \hat{S}_{+,q} - i\eta c_{k-q,\uparrow}^+ c_{k,\downarrow}
\]

Eq. (11) evidences the renormalized Zeeman energy, i.e., the spin-flip energy of single electrons:

\[
Z^* = Z_e - 2G_{xc} \left( \hat{S}_{z,q=0} \right)_0
\]

Compared to the bare Zeeman energy \( Z_e \), \( Z^* \) is enhanced by Coulomb-exchange between spin-polarized electrons, a phenomenon linked to the spin-susceptibility enhancement. Each SF-SPE is characterized by two wavevectors \( k \) and \( q \). At \( q = 0 \), SF-SPE are degenerate to \( Z^* \). When \( q \neq 0 \) the degeneracy is lifted by the kinetic spread of velocities which depend on the initial momentum \( k \).

2. Collective modes dynamics

Along the SF-SPE, the above spin polarized SP2DEG develops collective modes, the so-called spin-flip waves (SFW). SFW dynamics are described by the \( \hat{S}_{+,q} \) operators dynamics. As \( \hat{H}_{Coulomb} \) conserves the macroscopic spin, it follows:

\[
\left[ \hat{S}_{+,q}, \hat{H}_{Coulomb} + Z_e (B) \hat{S}_{z,q=0} \right] = -Z_e (B) \hat{S}_{+,q}
\]

But, the kinetic Hamiltonian couples collective states to the spin current \( \hat{J}_{+,q} = \hbar \sum_k (k - \frac{g}{2}) c_{k-q,\uparrow}^+ c_{k,\downarrow} \) carried by single particle states:

\[
\left[ \hat{S}_{+,q}, \hat{H}_{Kin} \right] = \hbar \vec{q} \cdot \vec{J}_{+,q}
\]
Finally the equation of motion of collective SFW modes writes:

\[
\left( \frac{d}{dt} \hat{S}_{+,q} \right)_{2DEG} = i\omega_e \hat{S}_{+,q} - i\mathbf{q} \cdot \hat{\mathbf{J}}_{+,q}
\] (15)

where \( \omega_e = Z_e (B) / \hbar \) is the frequency of the Larmor’s electron mode.

One is left with evaluating the spin-current dynamics to find the 2DEG electron spin waves. The spin current evolution is dominated by single particle states dynamics as \( \hat{H}_{\text{Coulomb}} \) does not conserve \( \hat{\mathbf{J}}_{+,q} \) and destroys the coherence between the single particle objects composing \( \hat{\mathbf{J}}_{+,q} \). As seen from Eq. (11), the exchange field produced by the spin fluctuation \( \hat{S}_{+,q} \) drives the spin current. The interplay between the spin-current dynamics and the \( \hat{S}_{+,q} \) dynamics then determines both the SFW dispersion and its damping. The relevant spin-current response is the transverse spin-conductivity \( \bar{\sigma}_+ \), which links the spin current to the gradient of the exciting exchange field:

\[
\left\langle \hat{\mathbf{J}}_{+,q} \right\rangle_{\omega} = \mathbf{q} \bar{\sigma}_+ (\mathbf{q}, \omega) G_{xc} \left\langle \hat{S}_{+,q} \right\rangle_{\omega}
\] (16)

where \( \langle \rangle_{\omega} \) is the expectation value at frequency \( \omega \). The spin-conductivity has an imaginary part originating from the damping of SF-SPE, intrinsically due to \( \hat{H}_{\text{Coulomb}} \) or any source of disorder acting on transverse spin degrees of freedom. Consequently, the real part of the spin conductivity determines the SFW dispersion, while the imaginary part determines its damping. It is worth noting, that the spin wave damping originates from the kinetic motion of the conduction electrons and from the topoly of the conduction band, a 2D parabolla. In a Luttinger liquid, the conduction band is linear in \( \mathbf{k} \) and 1D, thus Eq. (14) conserves the macroscopic spin. It breaks the coupling between \( \hat{S}_{+,q} \) and SF-SPEs, which are coupled to charge degrees of freedom by \( \hat{H}_{\text{Coulomb}} \). This property is at the origin of the well known spin-charge separation occuring in Luttinger liquids. Injecting Eq. (16) into Eq. (15) and solving it in the frequency domain for long wavelength \( (q \ll k_F) \) leads to:

\[
\frac{d}{dt} \hat{S}_{+,q} = i\bar{\omega}_q \hat{S}_{+,q}
\] (17)
with $\tilde{\omega}_q$ a complex pulsation:

\[
\text{Re} \tilde{\omega}_q = \omega_e - q^2 G_{xc} \lim_{q \to 0, \omega \to 0} \text{Re} \tilde{\sigma}_+ \tag{18a}
\]
\[
\text{Im} \tilde{\omega}_q = q^2 G_{xc} \lim_{q \to 0, \omega \to 0} \text{Im} \tilde{\sigma}_+ \tag{18b}
\]

In the following we note $\sigma_+ = \lim_{q \to 0, \omega \to 0} \text{Im} \tilde{\sigma}_+$, the imaginary spin-conductivity. It was calculated in Ref.[20] and some corrections were added in Ref.[19] which gave also an experimental evidence of the kinetic damping law found in Eq. (18b). We highlight that these $q^2$ laws are valid in the longwavelength limit when the SFW propagates far from the SF-SPE continuum (see Ref.[18]). When close to this continuum, the stronger coupling with SF-SPEs introduces corrections to the above laws and one should better replace the dispersion law with the pole appearing in the transverse spin susceptibility (see Ref.[18]) which will be derived in the next paragraph.

3. Electron spin-susceptibility

The transverse spin-susceptibility, defined by the ratio of the expectation value $\left\langle \hat{S}_{+q} \right\rangle_\omega$ to the perturbing potential $g_e \mu_B b_{+,q} \omega$, where $b_{+,q} \omega$ is the amplitude at the same pulsation $\omega$ and wavevector $q$ of the exciting magnetic field, is given by:

\[
\chi_+ (q, \omega) = \left\langle \left\langle \hat{S}_{+q}; \hat{S}_{-q} \right\rangle \right\rangle_\omega \tag{19}
\]

Straightforward calculations using the equation of motion Eq. (11) lead to:

\[
\chi_+ (q, \omega) = \left\langle \left\langle \hat{S}_{+q}; \hat{S}_{-q} \right\rangle \right\rangle_\omega = \frac{\Pi_{\downarrow \uparrow} (q, \omega)}{1 + G_{xc} \Pi_{\downarrow \uparrow} (q, \omega)} \tag{20}
\]

where we have introduced the transverse Lindhardt-type response:

\[
\Pi_{\downarrow \uparrow} (q, \omega) = \sum_k \frac{\langle \hat{n}_{k-q \uparrow} \rangle_0 - \langle \hat{n}_{k \downarrow} \rangle_0}{Z^* + E_{k-q} - E_k - \hbar \omega - \eta} \tag{21}
\]

A comparison between the above spin-susceptibility expression and the one given by local spin-density approximation[18], gives the expression of the local field factor $G_{xc}$:

\[
G_{xc} = -\frac{2}{n_{2D} L^2} \frac{1}{\zeta} \frac{\partial E_{xc}}{\partial \zeta} \tag{22}
\]
where $E_{xc}$ is the exchange-correlation part of the ground state energy\cite{30}.

SFW appear as poles of $\chi_+ (q, \omega)$, one finds in the long wavelength limit, another expression for $\text{Re} \tilde{\omega}_q$:

$$
\text{Re} \tilde{\omega}_q = \omega_e - \frac{1}{|\zeta|} \frac{Z_e}{Z^* - Z_e} \frac{\hbar}{2m^*} q^2 \quad (23)
$$

Alternatively, if one uses the approximated equation of motion Eq. (15), one finds the spin susceptibility in the long wavelength limit:

$$
\chi_+ (q, \omega) = -\frac{2 \left< \hat{S}_{z, q=0} \right>_0}{\hbar \omega - \hbar \tilde{\omega}_q} \quad (24)
$$

### B. Transverse spin dynamics equations with s-d dynamical coupling

Now, we keep lines (4b) to (4d) in the s-d Hamiltonian, and we reconsider collective transverse spin dynamics. In the following, the derivative $d\hat{A}/dt = [\hat{A}, \hat{H}] / i\hbar$ takes into account the coherent coupled dynamics due to lines (4b) and (4d) in the s-d Hamiltonian, but reduced to first order terms: higher order correlation terms like $\sum_{q'} \delta \hat{S}_{z, q+q'} \cdot \hat{M}_{+, q}^{(1)}$ have been dropped.

1. **Electron dynamics**

We find:

$$
\frac{d}{dt} \hat{S}_{+, q} = \hat{S}_{+, q} - \frac{i}{\hbar} K \hat{M}_{+, q}^{(1)} \quad (25)
$$

where $\hat{S}_{+, q} = i\omega_q \hat{S}_{+, q}$. Compared to the SP2DEG dynamics, the $sd$-dynamical coupling adds the second term of Eq. (25) which is a coherent coupling with Mn transverse degrees of freedom. One key feature is that the collective electron motion naturally couples with $\hat{M}_{+, q}^{(1)}$ Mn-modes, a Mn precession having a profile, out of the QW plane, following the electron probability distribution. We are left with deriving the equation of motion for these Mn-modes.
2. **Manganese dynamics**

We obtain the first order equation of motion for Mn spins:

\[
\frac{d}{dt} \hat{M}^{(n)}_{+,q} = \frac{i}{\hbar} g_{Mn} \mu_B B \hat{M}^{(n)}_{+,q} + i \frac{K}{\hbar} \hat{M}^{(n+1)}_{+,q} - i \frac{\Delta_{n+1}}{\hbar} \hat{S}_{+,q}
\]

where we have introduced \( n \)-profile Overhauser shifts: \( \Delta_n = \tilde{\alpha} \left\langle \hat{M}^{(n)}_{z,q=0} \right\rangle_0 = \frac{\gamma_n}{\gamma_1} \Delta \)

with \( \gamma_n = w^{n-1} \int_0^w \chi^{2n}(y) dy \). The important features are the second and third terms in Eq. (26). The later couples the Mn-precession with collective electron modes. The former couples a \( n \)-profile Mn mode to a \((n + 1)\)-profile mode, because this coupling is mediated by the 2DEG. Thus, the Mn-dynamics is given by an infinite serie of equations. This is a consequence of the 3D nature of the Mn dynamics. A variable like \( \hat{M}^{(n)}_{+,q} \) describes an oscillation propagating in the plane with a rigid profile in the normal direction, but the out of plane degree of freedom is restored by the possibility for Mn spins to build modes which are combinations of \( \hat{M}^{(n)}_{+,q} \) resulting in different out of plane profiles\(^{10}\). Obviously, the \( \hat{M}^{(n)}_{+,q} \) are not independent variables because they don’t correspond to orthogonal out of plane profiles. Solving the serie of infinite equations requires a projection of \( \hat{M}^{(n)}_{+,q} \) over a set of modes with orthogonal profiles as it was carried out in Ref.\(^{10}\). Along with modes having a strong mixed nature (electron-Mn modes), we then expect to find a high number of modes having essentially a Mn character, but with orthogonal profiles (Mn modes). The number of Mn modes has to be consistent with the initial number of degrees of freedom present in the system. Ref\(^{10}\) found a high number of Mn modes branches which were separated by energies of the order of 0.1\( \Delta \). However, in the experimental data of Ref\(^4\), only one branch of these Mn modes was apparent. It appears then, that the set of modes chosen in Ref\(^{10}\) is not the most appropriate to describe properly all the modes contained in Eqs.(25)-(26), at least in the vicinity of the anticrossing gap (see below). Anyway, this point requires further developments out of the scope of the present study. Indeed, we are particularly interested in discussing mixed electron-Mn modes which are strongly coupled to electrons rather than modes specific to the 3D nature of the Mn dynamics. We can remark that the coupling between \( \hat{M}^{(n)}_{+,q} \) and \( \hat{M}^{(n+1)}_{+,q} \) has a strength given by \( K \), which is very small compared to \( \Delta_n \) due to the ratio \( \left| \left\langle \hat{M}^{(n)}_{z,q=0} \right\rangle_0 / \left\langle \hat{S}_{z,q=0} \right\rangle_0 \right| \gg 1 \). Hence, considering only modes strongly coupled with electron modes is reasonable. As electron modes are naturally coupled to \( \hat{M}^{(1)}_{+,q} \) modes, we will consider the dynamics for these ones only by cutting the infinite serie with
an homothetic approximation:

\[ \hat{M}^{(2)}_{+,q} = (\gamma_2/\gamma_1) \hat{M}^{(1)}_{+,q} \]  \hspace{1cm} (27)

Consequently the set of coupled electron-Mn equations reduces to:

\[ \frac{d}{dt} \hat{S}_{+,q} = i\tilde{\omega}_q \hat{S}_{+,q} - \frac{i}{\hbar} K \hat{M}^{(1)}_{+,q} \] \hspace{1cm} (28a)

\[ \frac{d}{dt} \hat{M}^{(1)}_{+,q} = i\omega_{Mn} \hat{M}^{(1)}_{+,q} - \frac{i}{\hbar} \Delta_2 \hat{S}_{+,q} \] \hspace{1cm} (28b)

where:

\[ \omega_{Mn} = \left( g_{Mn}\mu_B B + K \frac{\gamma_2}{\gamma_1} \right) / \hbar \] \hspace{1cm} (29)

is the natural precession pulsation of the free \( \hat{M}^{(1)}_{+,q} \) mode.

IV. MIXED MN-ELECTRON SPIN WAVES

A. Spin susceptibilities

To find the dynamically coupled modes, we will derive the electron spin susceptibility with help of equations of motion (9a) and (28a)-(28b).

From Eqs.(9a) and (28a), we first get:

\[ \left\langle \left\langle \hat{S}_{+,q}; \hat{S}_{-,q} \right\rangle \right\rangle_\omega = \frac{\tilde{\omega}_q}{\omega} \left\langle \left\langle S_{+,q}; S_{-,q} \right\rangle \right\rangle_\omega - \frac{K}{\hbar \omega} \left\langle \left\langle \hat{M}^{(1)}_{+,q}; \hat{S}_{-,q} \right\rangle \right\rangle_\omega - \frac{2}{\hbar \omega} \left\langle \hat{S}_{+,q=0} \right\rangle_0 \]

then from Eq. (28b), we get:

\[ \left\langle \left\langle \hat{M}^{(1)}_{+,q}; \hat{S}_{-,q} \right\rangle \right\rangle_\omega = \frac{\omega_{Mn}}{\omega} \left\langle \left\langle \hat{M}^{(1)}_{+,q}; \hat{S}_{-,q} \right\rangle \right\rangle_\omega - \frac{\Delta_2}{\hbar \omega} \left\langle \left\langle \hat{S}_{+,q}; \hat{S}_{-,q} \right\rangle \right\rangle_\omega \]

hence,

\[ \left\langle \left\langle \hat{M}^{(1)}_{+,q}; \hat{S}_{-,q} \right\rangle \right\rangle_\omega = -\Delta_2 \frac{\left\langle \left\langle \hat{S}_{+,q}; \hat{S}_{-,q} \right\rangle \right\rangle_\omega}{\hbar \omega - \hbar \omega_{Mn}} \]

which finally leads to:
\[ \langle \langle \hat{S}_{+;q}; \hat{S}_{-;q} \rangle \rangle _{\omega} = \frac{(\hbar \omega - \hbar \omega_{Mn}) \chi_{\pm} (\mathbf{q}, \omega)}{\hbar \omega - \hbar \omega_{Mn} - \frac{\alpha \Delta}{2} \chi_{\pm} (\mathbf{q}, \omega)} \]  

(30)

and

\[ \langle \langle \hat{M}_{+;q}^{(1)}; \hat{M}_{-;q}^{(1)} \rangle \rangle _{\omega} = \frac{-2 \langle \hat{M}_{z; \mathbf{q} = \mathbf{0}}^{(1)} \rangle}{\hbar \omega - \hbar \omega_{Mn} - \frac{\alpha \Delta}{2} \chi_{\pm} (\mathbf{q}, \omega)} \]  

(31)

Consequently, e-Mn mixed spin excitations appear as poles of the above responses, i.e., are zeros of the propagator:

\[ \hbar \omega - \hbar \omega_{Mn} - \frac{\alpha \Delta}{2} \chi_{\pm} (\mathbf{q}, \omega) \]  

(32)

with \( \chi_{\pm} (\mathbf{q}, \omega) \) being the spin-susceptibility of the uncoupled SP2DEG described in Section III B.

We can understand the above equation as follows. Consider the Mn point of view; in the presence of the SP2DEG, the precession frequency of Mn spins is shifted from the normal precession \((g_{Mn}\mu_B B/\hbar)\) by two quantities: a blue shift due to static exchange field with spin polarized electrons \((K/\hbar)\) and an additional shift due to the dynamic change of the electron spin-polarization. The later is induced by the Mn precession itself. Finally Eq. (32) describes a recursive closed loop where: Mn transverse precession induces electron transverse precession proportional to \(\Delta^2 \chi_{\pm} (\mathbf{q}, \omega)\), this dynamically changes the electron spin polarization which in turn shifts the Mn precession frequency by an amount \(\tilde{\alpha} \Delta^2 \chi_{\pm} (\mathbf{q}, \omega)\).

Finally, in dropping correlation terms given by Eqs. (4c) and (4d), one finds a collective behavior where electrons and Mn respond adiabatically to the dynamical perturbation from the opposite spin-subsystem.

Similar expressions for the coupled modes propagator have been obtained in previous works. To our knowledge it was first derived in Ref. 9 for bulk DMS, and more recently using a spin-path integral approach in DMS quantum wells with electrons 6,10 or bulk DMS with holes 8. However, none of these works did include the influence of the Coulomb interaction between carriers. Instead of Eq. (32), they resulted in the following propagator:

\[ \hbar \omega - \hbar \omega_{Mn} - \frac{\tilde{\alpha} \Delta}{2} \Pi^{\uparrow \downarrow} (\mathbf{q}, \omega) \]  

(33)

where the electron spin-susceptibility was replaced by the non-interacting single-particle response introduced in Eq. (21). Introduction of Coulomb interaction results in strong
qualitative changes in the spectrum which have been partially addressed in Ref.\textsuperscript{15} and will be detailed below.

B. Homogeneous modes

![Graphical representation of electron-Mn modes](image)

FIG. 1: (Color online). Zone center electron-Mn modes. Dotted lines are the uncoupled electron (upper curve) and Mn (lower curve) modes. Full lines are the solutions $\omega_{q=0}^±$ of Eq. (34). Out of the resonant field $B_\text{R}$ where the modes anticross, branches have electron or Mn character. Sample parameters are, $x_{\text{eff}} = 0.23\%$, $T = 2$ K, $w = 150\,\AA$ and $n_{2\text{D}} = 3.1 \times 10^{11}\,\text{cm}^{-2}$.

Eq. (33) was used to successfully fit the experiment of Teran et al. (Ref.\textsuperscript{17}) where homogenous modes ($q=0$) were probed and shown to experience an anticrossing at a magnetic field $B_R$ such that $\omega_{\text{Mn}} = \omega_e$. It is a consequence of the Larmor’s theorem that the homogenous electron mode behaves as if electron were not interacting (Eq. (15) for $q = 0$). Indeed, setting $q = 0$ in Eq. (32), leads to the homogenous precession modes equation:

\[
(h\omega - h\omega_{\text{Mn}})(h\omega - h\omega_e) - K\Delta_2 = 0
\]

which solutions are real:
Figure 1 shows the magnetic field dependence of these modes and the gap opening at the resonant field $B_R$. The upper branch $\omega_{q=0}^+ = 0$ (resp $\omega_{q=0}^- = 0$) has an electron character (resp. Mn character) when $B < B_R$ and vice-versa for $B > B_R$. The amplitude of the homogeneous anticrossing gap $\Delta \omega_{q=0} = \sqrt{4K\Delta_2}$ denotes the strength of the dynamical coupling between the two spin subsystems. Detailed discussions on this gap have been given in Ref. 4 and Ref. 6. In particular, Ref. 6 identifies the anticrossing Mn mode as $\hat{M}_{+q}^{(1)}$, consistent with the homothetic approximation of Eq. (27) used here. The anticrossing gap was found to be $\Delta \omega_{q=0} = \sqrt{4K\Delta_2 - (\frac{\hbar}{T_{2e}})^2}$ where $\frac{\hbar}{T_{2e}}$ is the damping rate of the homogeneous uncoupled electron mode, a quantity that we have neglected here in dropping electron-Mn correlation terms contained in Eqs. (4b)-(4d). In Ref. 6, $\frac{\hbar}{T_{2e}}$ was estimated from measurements of the electronic spin wave damping at $q = 0$. A rigorous simultaneous determination of $\Delta_2$ and $T_{2e}$, lead to the extraction of $K$ from the anticrossing gap and furthermore to the spin-polarization degree $\zeta$ of the 2DEG. Data showed that the so-extracted $\zeta$ was slightly exceeding the prediction [18] made for $\zeta$ in contradiction with other determinations of $\zeta$ performed in the same type of samples [31], which showed that the model used to predict $\zeta$ was reliable. A more accurate description of the anticrossing gap taking into account the infinite set of coupled $n$-profile mode $\hat{M}_{+q}^{(n)}$ equation of motion might overcome this discrepancy. One should also mention, that the infinite serie of equations must be cut in order to conserve the initial number of degrees of freedom (number of available spins in the system). But finding the right number of Eq. (26)-like to be taken depends on how the total number of degrees of freedom separates into a number of (quasi-) individual modes and a number of collective electron-Mn modes. Determining this separation is also an important and interesting issue.

C. Spin waves

For $q > 0$, Eqs (32) and (33) give very different qualitative results as illustrated on Fig. 2. Without Coulomb interaction between electrons, uncoupled modes of the electrons are the SF-SPE which are degenerate to $Z_e$ at $q = 0$. The $sd$ dynamical coupling introduces two additional collective mode: the OPW propagating above the SF-SPE domain with a positive dispersion and the IPW propagating below with a negative dispersion. Introducing the
FIG. 2: (Color online). Illustration of the changes introduced by the Coulomb interaction on the mixed modes dispersions for the same sample parameters as in Fig.1. (a) Solutions without Coulomb given by Eq.(33) for $B = 5.8$ T. At $q = 0$ SF-SPE are degenerate to $Z_e$. The $sd$ dynamical shift introduces two propagating waves: above (resp. below) the SF-SPE domain, the OPW (resp. IPW) propagates with a positive (resp. negative) dispersion. The dashed line is the uncoupled Mn mode (degenerate to $\hbar \omega_{Mn}$). (b) and (c) The Coulomb interaction between electrons is included as in Eq.(32) of this work. Dashed lines are the uncoupled electron and Mn modes. At $q = 0$ the SF-SPE energy is shifted to $Z^*$. An anticrossing gap opens at the wavevector $q_R (B)$ if $B \leq B_R$. In usual conditions (see Ref.15) both the OPW and IPW propagate below the SF-SPE continuum with negative dispersions. Dipsersions were calculated after setting to zero the kinetic damping rate.

Coulomb interaction between electrons shifts the SF-SPE to higher energies ($Z^*$) and gives rise to the collective wave SFW propagating below the SF-SPE continuum. The SFW is further coupled to Mn modes through the $sd$ interaction. An evaluation of the coupling between SF-SPE and Mn modes was given in Ref.17 and found to be negligible. Thus the
Coulomb interaction introduces a shift between the SF-SPE and the SFW energies, and the later is further shifted by the sd dynamical coupling. In realistic conditions, it was shown in Ref. 15 that the Coulomb shifts dominates over the sd dynamical shift. Hence, when Coulomb interaction is taken into account, Eq. (32), except under unrealistic conditions, gives rise to two spin wave modes propagating below the SF-SPE continuum, the IPW and OPW. An anticrossing gap opens at a specific wavevector \( q_R(B) \) given by:

\[
q_R(B) = \sqrt{|\zeta| \left( \frac{Z^*/Z_e - 1}{Z_e} \right) \frac{2m^*}{\hbar} (\omega_e - \omega_{Mn})}
\]  

(36)

Note that \( q_R(B_R) = 0 \) and that \( \tilde{\omega}_{q_R} = \omega_{Mn} + i q^2_R G_{xc}\sigma_+ \). If \( q_R(B) > 0 \), compared to the homogenous gap, the anticrossing gap at \( q_R \) is dramatically reduced by the kinetic damping of the electron wave and is given by:

\[
\omega^+_{qn} - \omega^-_{qn} = \sqrt{4K\Delta^2/\hbar^2 - \eta^2_{qn}}
\]  

(37)

where \( \eta_{q_R} = q^2_R G_{xc}\sigma_+ \). We note that the kinetic damping is the only one considered here. Other sources of damping, as e.g., the ones dropped in Eq. (4d), will of course further reduce the amplitude of the gap.

Fig. 3 illustrates the variation of \( q_R(B) \) with the magnetic field. It is always smaller than \( q_m = k_{F \downarrow} - k_{F \uparrow} \), the wavevector delimiting the window where the SFW propagates. Overlaid in Fig. 3 are the anticrossing gap at \( q_R \) and the corresponding damping rate \( \eta_{q_R} \). In the absence of the kinetic damping, the gap would be given by \( \sqrt{4K\Delta^2} \). One sees the dramatic effect of this intrinsic kinetic damping, which kills the gap outside a very narrow range of magnetic fields. As coupling between spin waves of the electron and the Mn spin systems is responsible for the appearance of the carrier induced ferromagnetism, we might conclude that the above disappearance of the gap diminishes the possibilities for ferromagnetic transitions with complex order (out of \( q = 0 \)).

The disappearance of the gap is illustrated in Fig. 4 by comparing the dispersions obtained from the zeros of the propagator in Eq. (32) in the presence or absence of the kinetic damping. In the presence of the damping, the solutions have a non-zero imaginary part for \( q > 0 \). The corresponding damping rate is plotted in the lower insets of Fig. 4. It is well known that when the frequencies of two coupled oscillators anticross each other, their corresponding damping rates cross themselves. Clearly for \( B < 5.7 \) T, the mixed modes do
FIG. 3: (Color online). Left axis: variation of the wavevector $q_R(B)$ and the wavevector $q_m = k_{F\downarrow} - k_{F\uparrow}$. Right axis: variation of the anticrossing gap and the kinetic damping rate $\eta_{qR} = \text{Im} \tilde{\omega}_{qR}$. The anticrossing gap is killed by the damping rate when the later is of the same order of magnitude as $\sqrt{4K\Delta^2}$. A break in the horizontal axis scale has been introduced to zoom the region close to $B \lesssim B_R$. To calculate the damping rate, we have used a typical SF-SPE scattering time $\tau = 2\text{ps}$ instead of $\tau_{e-e} (\sim 150\text{ps})$ to match the experimental conditions of Ref.\textsuperscript{19}.

not anticross at any $q$ and each branch conserves its former character, Mn-like or electron-like. On the contrary, for $5.7 \text{ T} < B < B_R$, the modes anticross at $q_R$, and the OPW transfers the kinetic damping ($q^2$ law) of the SFW to the IPW when $q > q_R$. It is worth to note that this $q^2$ law for the IPW damping rate was also found in GaMnAs compounds in the ferromagnetic state\textsuperscript{19}. 
FIG. 4: (Color online). Illustration of the progressive disappearance of the anticrossing gap when $B$ goes away from $B_R$. Continuous lines (open symbols) are dispersions calculated in absence (presence) of the kinetic damping. Dashed lines are the uncoupled modes. Upper insets: zoom on the anticrossing gap region. Lower insets: variation with $q$ of the damping rate of both the OPW (dashed line) and IPW (straight line).

V. CONCLUSION

In conclusion, we have introduced equations for the spin dynamics in a test-bed diluted magnetic system that allow to take into account the interplay between the Coulomb interaction dynamics and the $sd$ dynamical coupling between the electrons and the localized spins. We have shown how the Coulomb interaction introduces strong qualitative changes: the mixed electron-Mn modes propagate below the SF-SPE continuum and an anticrossing gap is open for a given range of magnetic field. Because of Coulomb interaction, the intrinsic kinetic damping due to the electron motion is always present (the SF-SPE scattering time cannot be longer than $\tau_{e-e}$), this damping kills the anticrossing gap outside a very narrow range.
of magnetic fields. Our calculations illustrate also how this kinetic damping is transfered to the IPW, a phenomenon found in GaMnAs compounds.

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