Simulations of the Local Universe constrained by observational peculiar velocities

Jenny G. Sorce,1,2* Hélène M. Courtois,1 Stefan Gottlöber,2 Yehuda Hoffman3 and R. Brent Tully4

1Institut de Physique Nucléaire, Université Lyon 1, CNRS/IN2P3, F-69622 Villeurbanne, Lyon, France
2Leibniz-Institut für Astrophysik, D-14482 Potsdam, Germany
3Racah Institute of Physics, Hebrew University, Jerusalem 91904, Israel
4Institute for Astronomy, University of Hawaii, 2680 Woodlawn Drive, HI 96822, USA

Accepted 2013 November 5. Received 2013 November 5; in original form 2013 September 3

ABSTRACT
Peculiar velocities, obtained from direct distance measurements, are data of choice to achieve constrained simulations of the Local Universe reliable down to a scale of a few megaparsec. Unlike redshift surveys, peculiar velocities are direct tracers of the underlying gravitational field as they trace both baryonic and dark matter. This paper presents the first attempt to use solely observational peculiar velocities to constrain cosmological simulations of the nearby Universe. In order to set up initial conditions, a Reverse Zel’dovich Approximation (RZA) is used to displace constraints from their positions at $z = 0$ to their precursors’ locations at higher redshifts. An additional new feature replaces original observed radial peculiar velocity vectors by their full 3D reconstructions provided by the Wiener–Filter (WF) estimator. Subsequently, the constrained realization (CR) of Gaussian fields technique is applied to build various realizations of the initial conditions. The WF/RZA/CR method is first tested on realistic mock catalogues built from a reference simulation similar to the Local Universe. These mocks include errors on peculiar velocities, on data point positions and a large continuous zone devoid of data in order to mimic galactic extinction. Large-scale structures are recovered with a typical accuracy of 5 $h^{-1}$ Mpc in position, the best realizations reaching a 2–3 $h^{-1}$ Mpc precision, the limit imposed by the RZA linear theory. Then, the method is applied to the first observational radial peculiar velocity catalogue of the project Cosmicflows. This paper is a proof of concept that the WF/RZA/CR method can be applied to observational peculiar velocities to successfully build constrained initial conditions.

Key words: methods: numerical – techniques: radial velocities – large-scale structure of Universe.

1 INTRODUCTION

Under the assumption of a dark matter only Universe, even the simplest problem of the emergence of structures defies a proper and detailed analytical treatment. As a result, the study of the formation of the large-scale structure of the Universe relies heavily on numerical simulations. Large-scale dark matter simulations (e.g. Klypin, Trujillo-Gomez & Primack 2011; Alimi et al. 2012; Angulo et al. 2012; Prada et al. 2012; Watson et al. 2013) constitute the backbone of the study of structure formation in the Universe.

The standard model of cosmology asserts that the primordial fluctuations are constituted by a Gaussian random field whose statistical properties are determined by its power spectrum. An accurate determination of the power spectrum is enabled by the cosmological parameters. These latter are obtained from observations of the fluctuations in the cosmic microwave background radiation combined with baryonic acoustic oscillations and supernova measurements (Komatsu et al. 2011; Larson et al. 2011; Planck Collaboration et al. 2013). Standard cosmological computations use initial conditions (ICs) drawn from random realizations of the primordial perturbation field.

By contrast, constrained simulations stem from ICs obeying a set of observational constraints in addition to the random component. They provide a different approach to cosmological simulations to better approximate the observed nearby Universe. Constraints can either be peculiar velocities or galaxy distributions. The first constrained ICs were produced by Ganon & Hoffman (1993), using the Mark III catalogue of peculiar velocities (Willick et al. 1996). These ICs were then used to perform the first constrained simulation of the nearby Universe by Kolatt et al. (1996). The Constrained Local Universe Simulations (CLUES) project has been running a variety of pure dark matter and hydrodynamical constrained simulations of...
the Local Universe, aiming mostly at studying a variety of issues concerning the Local Group (for a general review see Gottløber, Hoffman & Yepes 2010, and references therein). Galaxy redshift surveys provide a different source of constraints. This was first pioneered by Bistolas & Hoffman (1998) and followed later on by Mathis et al. (2002) and Lavaux (2010) and very recently by Heß, Kitaura & Gottløber (2013). There is a considerable trade-off between using peculiar velocities and spatial distributions of galaxies from redshift surveys.

Galaxy redshifts are quite easy to measure accurately. Very large and deep surveys are now routinely produced. However, galaxy distributions constitute biased tracers of the underlying density field. The mass-to-light bias has yet to be completely modelled and corrected for. On the other hand, measuring peculiar velocities poses formidable challenges to observational cosmologists. The observations are susceptible to systematic biases, and the resulting catalogues are noisy, sparse and with an incomplete sky coverage. Still, on the theoretical side, peculiar velocities are unbiased tracers of the underlying mass distribution. As long as virial motions inside clusters can be suppressed, the construction of the underlying density and velocity fields can be easily performed.

The procedure of constraining ICs with peculiar velocities is based on the linear Wiener–Filter (WF) and constrained realization (CR) of Gaussian fields algorithms (Hoffman & Ribak 1991; Zaroubi et al. 1995; Hoffman 2009). The main deficiency of the method in these preliminary studies has been not to account for the cosmic displacement field of the data points from their primordial positions. Consequently, constrained simulated haloes were, at $z = 0$, located $10 \, h^{-1} \, \text{Mpc}$ away from the reference objects’ original positions. An effective remedy to the problem is to evaluate the Zel’dovich linear displacement field from the WF reconstructed overdensity field. Then, this displacement is reversed in time to move the constraints from $z = 0$ backwards to their progenitors’ positions at higher redshifts. This Reverse Zel’dovich Approximation (RZA) has been previously devised and tested against simple mock catalogues. For these simple mocks it was shown to recover the concentration of the universe. As a matter of fact, the observational data set used in this paper only reaches once to twice the size of our filament (from Ursa Major to Centaurus clusters, the distance is roughly $40 \, h^{-1} \, \text{Mpc}$). This leads to two biases. First, our position (as observers) is unique (peculiar) and impacts highly the data collections. In other words since we are living in a supercluster bounding a very large void, the observed peculiar velocities are very much dominated by a specific local structure dynamics. Secondly, we are only observing one possible (coherent with the constraints) realization of the universe. Thus we must (1) test the methodology

its inverse. Assuming that the observed peculiar velocities are not strongly affected by non-linear dynamics (curl-free field above the scale of virial motions), and assuming a prior cosmological model (here growth rate constant with time), the ICs are readily calculated. Detailed equations are available in Appendix A.

The major drawback of the CR method is the fact that it is formulated in an Eulerian way: the cosmic displacement field is neglected, although galaxies observed today are at different comoving positions from their progenitors at higher redshifts. A first attempt to improve this shortcoming has been recently suggested by Doumler et al. (2013a,b,c): constraints are re-located at their precursors’ positions before feeding them to the CR method. The progenitor’s positions are computed with the quasi-linear Zel’dovich approximation using the cosmic displacement field provided by the WF estimator (see Appendix B, for the equation, and Doumler et al. 2013b, for a detailed description). However, because observed constraints have uncertainties, the peculiar velocity field is not accurately described by solely one (radial) component. An additional step can be added to the initial technique called RZA-radial from now on. In the refined technique (RZA3D) constraints are not only moved to their progenitors’ positions but also the observed uncertain peculiar velocities are replaced by fully WF-reconstructed three-component vectors. Such resulting constraints have been given by the WF a weight according to their precision. The WF field goes to the null value when there is no coherent signal or when data points have too large errors (see Appendix A). Thus no error should be given to RZA3D derived constraints when input in the CR. ICs are then produced in the standard way, namely a random component and the peculiar velocity constraints are combined to produce a primordial perturbation field assuming a prior power spectrum.

In other words RZA3D differs from the initial RZA-radial on two points: (1) instead of observed radial peculiar velocities, the constraints are now the WF estimated peculiar velocities and (2) ICs are constructed under the assumption of null statistical errors [to prevent the double signal suppression resulting from the successive application of the WF (to obtain 3D velocities) and CR (to produce ICs)]. Fig. 1 provides a schematic presentation of the WF/RZA method which prepares the constraints to be input in the CR algorithm.

In the next section, RZA3D is applied to mock catalogues and is compared with the previous RZA-radial method. In both cases, a $\sigma_{\text{NL}}$ term accounts for the non-linear contributions of the radial peculiar velocities, which are not included in the model. It is added to the theoretical data-data correlated matrix (cf. Appendix A and Doumler et al. 2013b).

3 BUILDING MOCK CATALOGUES

When working on scales of a few tens of megaparsec (enclosing a volume we call the Local Universe), the cosmic variance is a major concern because these scales are far smaller than the scale of homogeneity of the Universe. As a matter of fact, the observational data set used in this paper only reaches once to twice the size of our filament (from Ursa Major to Centaurus clusters, the distance is roughly $40 \, h^{-1} \, \text{Mpc}$). This leads to two biases. First, our position (as observers) is unique (peculiar) and impacts highly the data collections. In other words since we are living in a supercluster bounding a very large void, the observed peculiar velocities are very much dominated by a specific local structure dynamics. Secondly, we are only observing one possible (coherent with the constraints) realization of the universe. Thus we must (1) test the methodology
multiplied by $\sigma$ and masses in $(\text{see Appendix B})$. In the initial RZA-radial technique the observational = to their progenitors’ po-
shows two planes centred on the look-alike of the Milky Mpc region, between the = $d_{\text{Mpc}}$ 60. They are run until = $\pm 0.75$ normalization = $73 \text{ km s}^{-1}$ or 1.7 $3^h z \text{GADGET}$ peculiar velocities particles in a computational box of side length = 0. Haloes are selected in a sphere h $160$ Mpc, peculiar velocities displays the confidence level zones of the reconstruction with $H$ h grid using a clouds-in-cells interpolation scheme. The refinement of the WF/RZA technique. The WF applied to = WMAP = on 29 July 2018 mocks of Doumler et al. (2013a,b,c) are introduced. To simulate a zone without data similar to that produced by the extinction of our galaxy’s disc (Zone of Avoidance), every halo with a latitude in between $\pm 10^\circ$ is removed. Major players in the local dynamics, such as the mock Great Attractor, are thus (partly) masked by this extinction zone. The mock catalogue is also designed to reproduce the current observational limits: a 20 per cent uncertainty on galaxy distances, and thereby on derived radial peculiar velocities. For sim-
= $\Psi_1$ 3D WAq This simulation was computed in the 3-year $\Omega_0 = 0.24, \sigma_b = 0.75$ normalization and $H_0 = 73 \text{ km s}^{-1} \text{Mpc}^{-1}$. BOX160 reproduces many of the key structures of the nearby Universe, such as Virgo, Coma and Centaurus clusters, Perseus-Pisces supercluster and the Great Attractor region. A Local-Group-like structure has been identified in the simulation and a mock observer is attached to that object. The catalogue is built with respect to this observer. We assume that galaxies follow the peculiar velocities of dark matter haloes in which they reside. A mock catalogue of dark matter haloes has been extracted from BOX160 with the AMIGA Halo Finder (AHF; Knollmann & Knebe 2009). The output list of parameters contains halo coordinates in $h^{-1}$ Mpc, peculiar velocities in $\text{km s}^{-1}$ and masses in $h^{-1} \text{M}_{\odot}$. Haloes are selected in a sphere of $30 h^{-1} \text{Mpc}$ radius around the mock observer to mimic as much as possible the extent of the Cosmicflows project’s first catalogue to be used later on in this paper. Two novelties with respect to the mocks of Doumler et al. (2013a,b,c) are introduced. To simulate a zone without data similar to that produced by the extinction of

Figure 1. The refinement of the WF/RZA technique. The WF applied to observed radial peculiar velocities provides full 3D reconstructed peculiar velocity field $\psi_{3D}^{\text{WF}}$ which allows us to derive the cosmic displacement field $\psi_{3D}^{\text{WF}}$ (see Appendix B). In the initial RZA-radial technique the observational (radial) constraints at $z = 0$ are re-located by $-\psi_{3D}^{\text{WF}}$ to their progenitors’ positions at higher redshifts. Since the peculiar velocity field is curl free (above the scale of virial motions) it is supposed to be fully defined by only one component. However, because observed peculiar velocities have uncertainties, RZA-radial is insufficient. The proposed refinement in RZA3D takes care of this flaw by using the full 3D WF reconstructed peculiar velocities $\psi_{3D}^{\text{WF}}$ as constraints.

4 Constrained Simulations With Mock Data

The full machinery to obtain constrained simulations is tested on the mock built in Section 3. The WF/RZA/CR algorithm is first applied to obtain ICs. These latter are then input in $\text{GADGET-2}$ (Springel 2005) $N$-body code to perform dark matter only simulations. The outcomes are compared with the initial BOX160. The box size, $160 h^{-1} \text{Mpc}$ long on each side, is almost three times the extent of the mock. Periodic boundary conditions can be assumed without any risk of spurious phenomena in the central $60 h^{-1} \text{Mpc}$ region to be analysed. The grid size is $N = 256^3$. To avoid shell-crossing, simulations are started at $z = 60$. They are run until $z = 0$.

4.1 Wiener–Filter reconstruction of the mock universe

To facilitate the comparisons between the initial BOX160 and its WF reconstruction from a mock, the BOX160 velocity field is inter-
polated on a $256^3$ grid using a clouds-in-cells interpolation scheme. Fig. 2 displays the confidence level zones of the reconstruction with respect to the original BOX160. The zones result from a cell-to-cell comparison, within the central $60 h^{-1} \text{Mpc}$ region, between the velocity grids of the WF reconstruction and of the reference simulation. The total scatter around the 1:1 relation is $201 \text{ km s}^{-1}$ or 1.7$\sigma$. The WF velocity field is thus a good reconstruction of the BOX160 source.

Fig. 3 shows two planes centred on the look-alike of the Milky Way of the reference simulation and of its reconstruction obtained
with the WF applied to the mock. Velocity (black arrows) and density fields (contours) are plotted. The green contour displays the mean density level. The main features – direction of the cosmic flows and attractors’ positions – are properly reconstructed. The feature in the $XY$ plane is the Great Attractor region look-alike with three density peaks from the reference simulation marked by red crosses in both quadrants. In $YZ$ the red cross locates the density peak of the mock Virgo halo in the reference simulation. These qualitative analyses illustrate the claim that with a sparse and noisy mock similar to cosmicflows-1 (in terms of number of constraints, zone of extinction without data and large errors on peculiar velocities) the WF is an optimal reconstruction tool in the linear regime of the gravitational instability. Structures are not necessarily reconstructed at their exact positions since the intrinsic accuracy is about $2\ h^{-1}\text{Mpc}$. Still, overall, the density field is recovered when considering only the linear theory on all scales.

### 4.2 Constrained simulations: RZA-radial versus RZA3D

Once the continuous fields obtained with the WF technique are extrapolated at the data points’ positions, constraints are displaced from their $z = 0$ location to their progenitors’ position at higher redshifts. In addition, constraints are replaced by their full WF reconstruction in the RZA3D technique. Since simulations are run with periodic boundary conditions, only the divergent part of the velocity field (velocities due to densities inside the box solely) is used to generate ICs. Hence, any tidal motion due to densities outside of the box is removed.

A major objective of the paper is to compare RZA-radial and RZA3D algorithms. However, cosmic variance can affect the comparison between methods. To take care of this effect (1) each RZA-radial derived IC shares the same random component with one of the RZA3D obtained IC. Hence, the simulations resulting from the same random seed ICs are expected to reproduce the same large-scale structure; (2) 10 constrained ICs are built to estimate the confidence level on structure position for each procedure. Resulting simulations are also compared with the reference BOX160 to estimate the average misplacement of simulated structures at $z = 0$ with respect to original locations. The comparison between the constrained simulations and BOX160 is done on a $256^3$ clouds-in-boxes grid after smoothing the density and velocity fields with a Gaussian kernel of $2.0\ h^{-1}\text{Mpc}$. When averaging over an increasing number of constrained simulations, the standard deviation with respect to BOX160 starts at 0.47 in logarithmic unit of density for one simulation and decrease to a plateau value of 0.37 when considering eight or more simulations. Adding more than 10 simulations would not produce on average other high and deep density zones that could be compared between the two methods and with BOX160 (or the 0.37 value would have continued to decrease). The standard deviation of RZA3D simulations around their average is smaller than that of RZA-radial both in terms of velocity and density (0.34 against 0.35 in logarithmic unit of density and 246 against 258 km s$^{-1}$). Although there is a random component, constrained simulations of BOX160 obtained with RZA3D method have stronger features reproduced at very similar positions than RZA-radial constrained simulations. The cosmic variance is reduced with RZA3D because constraints are stronger than in the RZA-radial case as seen in Fig. 4. The $\eta$ components of the data–data correlation vector $\eta$ have higher absolute values with RZA3D than with RZA-radial (see Appendix A for detailed equations).

BOX160 contains some replicas of prominent nearby structures such as Virgo, Hydra and Centaurus. These haloes are named hereafter s-Virgo, s-Hydra and s-Centaurus to distinguish them from the observed ones. BOX160 contains also a halo called s-Cz (in accord with Doumler et al. 2013c). These target objects are used to monitor the quality of the simulations. Fig. 5 shows the density field in the planes containing these objects of the actual simulation BOX160 (top panel), RZA-radial (middle panel) and RZA3D (bottom panel) with simulations averaged on 10 different realizations. The main dark matter haloes from BOX160 used as tracers are marked by red crosses in the six panels. Only in the RZA3D simulations there is a recurrent overdensity at the expected location of s-Virgo (high-density peak in the $YZ$ plot).

For each one of the simulations, dark matter haloes are obtained with the AHF and the s-haloes are identified when recovered. A halo in a constrained simulation is considered to be a replica of a BOX160 halo when the difference in position is smaller than $\sim 6\ h^{-1}\text{Mpc}$ and when masses are of the same order. The search is restricted to a sphere of $\sim 6\ h^{-1}\text{Mpc}$ since the scope of this work is to find a method resulting in an error below $6\ h^{-1}\text{Mpc}$ (3$\sigma$). Blue crosses in Fig. 5 are located at the average position of the look-alikes of s-Virgo, s-Hydra, s-Centaurus and s-Cz haloes in the constrained simulations. The cross sizes are proportional to the number of simulations (out of 10) in which a replica has been found. Table 1 recapitulates the characteristics of the targeted haloes and of their look-alikes: virial masses, positions and standard deviations. RZA-radial fails to recover s-Hydra and s-Centaurus as separate individual objects in 5 out of 10 simulations, thus they are not reported in the table. In these 5 out of 10 simulations, they are collapsed into a single object. The table also records the average of each replicas distance to the genuine halo. The typical difference is about $5\ h^{-1}\text{Mpc}$ for the RZA3D technique against $6\ h^{-1}\text{Mpc}$ for RZA-radial. However, because with RZA-radial more haloes are not found in the $6\ h^{-1}\text{Mpc}$ sphere (they are outside of the sphere so farther away) than with RZA3D, the value for RZA-radial is more biased (lowered) by the restricted search than that of RZA3D. Still, studying the standard
Figure 3. XY and YZ views of the reference simulation BOX160 (left) and its WF reconstruction (right) restricted to the central $60 h^{-1}$ Mpc zone. The reconstruction has been obtained using only radial peculiar velocity data from a realistic mock catalogue containing about the same errors and same number of data points as the observational cosmicflows-1 catalogue. The cosmic flows are represented by black arrows. Overdensity isocontours are delimited by solid black lines. The green contour delimitates the mean density. Red crosses show the positions of the major density peaks in the reference simulation. Even with this sparse and noisy realistic mock, the WF has enough signal to properly recover the original cosmic flows and density peaks, with a precision of about $2–3 h^{-1}$ Mpc in position.

The table proves an enhanced accuracy of the RZA3D method in terms of position errors (when compared with BOX160). The gain is also clear in term of reliability—robustness of the results since more replicas (out of 10 different random seed simulations) are found at a similar location (smaller standard deviations in positions) with RZA3D than with RZA-radial.

We can also consider a comparison between high-density peaks in the WF and in the constrained simulations. The density peak reconstructed by the WF in the bottom right-hand quadrant of Fig. 3 is also present in six RZA3D simulations out of 10 when looking within a $\sim 6 h^{-1}$ Mpc sphere centred on the WF peak. By contrast, there is a peak in only three RZA-radial simulations out of 10 within the same sphere. In both cases, the typical misplacement is $4–5 h^{-1}$ Mpc with a standard deviation about $1 h^{-1}$ Mpc. RZA3D applied to a mock cosmicflows-1 catalogue outperforms RZA-radial applied to the same mock. The stronger the constraints, the more the cosmic variance that exists over 10 constrained simulations because of a different random component is reduced. The number and accuracy of constraints in a cosmicflows-1-like catalogue are adequate to simulate properly a look-alike of the Local Universe with a precision reaching the intrinsic limitation of the technique.
still there is absolutely no density peak 6000 km s \(^{-1}\). Red crosses mark stand for the positions (average positions, in the /Lambda1 30 Mpc. Red crosses mark = = h provides details = h Mpc. Although the supergalactic /Lambda1 30 Mpc) the error in \(\eta\) Mpc volume is dominated 2.5 Mpc sphere centred on the WF density peak. The blue \(\eta\) = h provides details \(\eta\) = h Mpc with a Distribution functions of the component values \(\eta\) = h Mpc 2012b h 0.272 and \(\eta\) = h Mpc), the error in \(\eta\) = h Mpc). Still, there is absolutely no density peak \(\eta\) = h Mpc). Still, there is absolutely no density peak \(\eta\) = h Mpc). Still, there is absolutely no density peak \(\eta\) = h Mpc). Still, there is absolutely no density peak \(\eta\) = h Mpc). Still, there is absolutely no density peak \(\eta\) = h Mpc). Still, there is absolutely no density peak \(\eta\) = h Mpc). Still, there is absolutely no density peak \(\eta\) = h Mpc). Still, there is absolutely no density peak

5.3 Constrained simulations and local cosmography

Since no Virgo cluster is simulated out of 10 RZA-radial simulations constrained by the observational peculiar velocity catalogue, the rest of the paper focuses only on the analysis of RZA3D constrained simulations. This section presents how compatible they are with the observed local cosmography.

Fig. 6 displays the WF reconstruction of the cosmflows-1 catalogue (left), one single RZA3D simulation (middle) and an average of 10 RZA3D constrained simulations (right). Overdensity fields are smoothed by a Gaussian kernel of 2 h \(^{-1}\) Mpc. Red crosses mark the Virgo cluster’s position in the observed Universe (see Table 2 for the exact position). The Local Void and Virgo Void are also indicated. The WF maps of cosmflows-1 are taken as proxies for the actual Universe. They serve as targets for the constrained simulations, with the caveat that the WF provides only the linear overdensity field. The inner \(R = 30 h^{-1}\) Mpc volume is dominated by the Local Supercluster. The general structure of the Local Universe including positions of the voids are quite well reproduced by a constrained simulation. The average over 10 different realizations shows that in general the Virgo cluster region is well simulated at a similar location whatever random component is used. To quantify the reliability of the RZA3D technique in simulating the Virgo cluster and the area surrounding it, (1) the high-density peak of the WF reconstruction is identified in each RZA3D simulation and (2) the \(\text{AHF}\) is used to identify replicas of the cluster in the constrained simulations. We use the same process as with the mock catalogue. There is a density peak at a similar location to the WF peak in 10 out of 10 RZA3D constrained simulations. The typical misplacement with respect to the WF peak is about \(8-9 h^{-1}\) Mpc with a standard deviation about \(2 h^{-1}\) Mpc. Although the supergalactic Y and Z components are very similar in the WF (\(\sim 13\) and \(1 h^{-1}\) Mpc) and in the simulations (\(\sim 13 \pm 1\) and \(3 \pm 2 h^{-1}\) Mpc), the error in position is very high because in the supergalactic Y direction the shift in position with respect to the cosmflows-1 Virgo cluster is negative in the WF (\(\sim -3 h^{-1}\) Mpc) while it is positive in the simulations (\(\sim 4 h^{-1}\) Mpc). Still, there is absolutely no density peak in the RZA-radial constrained simulations even when looking in a \(\sim 10 h^{-1}\) Mpc sphere centred on the WF density peak. The blue crosses in Fig. 6 stand for the positions (average positions, in the right-hand column) of the Virgo-like haloes. Table 2 provides details about masses, positions, error in position and standard deviations. For completeness, the table presents the results obtained in both
Figure 5. Visualization of planes containing main simulated attractors (positioned at $X = 7$ and at $Z = -6 \, h^{-1} \text{Mpc}$). Solid black isocontours delimit overdensities. The green colour stands for the mean density in the box. Top: reference simulation. Middle: average over 10 constrained simulations using RZA-radial on the mock. Bottom: average over 10 constrained simulations applying RZA3D on the mock. Red crosses show original positions of s-Virgo, s-Hydra and s-Centaurus in the reference simulation. Positions of the averaged replicas in the constrained simulations are shown in blue. Crosses’ sizes are proportional to the number of replicas found out of 10 simulations.

WMAP7 and WMAP3 frameworks. Differences are negligible. A Virgo-like halo is present in 8 out of 10 simulations. By comparison, with RZA-radial, no replica of Virgo in $6 \, h^{-1} \text{Mpc}$ spheres centred on the observational position was found. A synthetic Local Universe with a Virgo cluster using only observational peculiar velocities is produced for the first time thanks to the WF/RZA/CR technique described in this paper.

6 CONCLUSION

The main aim of this paper is to perform numerical cosmological simulations constrained for the first time solely by an observational catalogue of peculiar velocities. To build ICs, the WF, the RZA and the CR techniques are successively applied. A refinement is also added to the methodology.
Table 1. Average parameters and standard deviations $\sigma$ for target haloes looked for in a $6 \, h^{-1} \text{Mpc}$ sphere centred on their original positions in the reference simulation. Column (1): simulation in which the haloes are looked for; column (2): dark matter mass in $h^{-1} \, 10^{14}$ solar mass; column (3): average coordinates $X$, $Y$ and $Z$ in $h^{-1} \text{Mpc}$ and standard deviations; column (4): average distance in $h^{-1} \text{Mpc}$ to the genuine halo and standard deviation $\sigma$ and column (5): number of simulations (out of 10 with a different random seed) which contain a replica.

| Simulation case   | Mass       | Average position $X$, $Y$, $Z$ | Average distance to reference halo | Nb of occurrences |
|-------------------|------------|--------------------------------|-----------------------------------|-------------------|
| BOX160 s-Virgo    | 3.3        | 7.12, 10.7, 11.5               | 6.9                               | 1/10              |
| RZA-radial s-Virgo| 0.25       | 3.40, 9.88, 17.2               |                                   |                   |
| RZA3D s-Virgo     | 0.34; $\sigma = 0.05$ | 7.78, 14.4, 11.1; $\sigma = 2.4$ | 5.4; $\sigma = 1.6$               | 5/10              |
| BOX160 s-Centaurus| 6.07       | −13.9, 15.1, −8.81             |                                   |                   |
| RZA-radial s-Centaurus | 3.1; $\sigma = 2.8$ | −16.9, 13.6, −9.62; $\sigma = 3.0$ | 6.0; $\sigma = 0.4$               | 5/10              |
| RZA3D s-Centaurus | 7.9; $\sigma = 4.0$ | −17.2, 13.2, −10.5; $\sigma = 1.9$ | 5.0; $\sigma = 1.5$               | 10/10             |
| BOX160 s-Hydra    | 5.18       | −22.0, 11.8, −3.35             |                                   |                   |
| RZA-radial s-Hydra| 4.7; $\sigma = 2.3$ | −21.2, 9.32, −4.87; $\sigma = 1.9$ | 3.1; $\sigma = 1.5$               | 5/10              |
| RZA3D s-Hydra     | 4.7; $\sigma = 2.0$ | −21.9, 10.1, −4.58; $\sigma = 1.3$ | 3.0; $\sigma = 1.2$               | 10/10             |
| BOX160 s-Cz       | 0.96       | −12.7, 2.68, −6.36             |                                   |                   |
| RZA-radial        | 0.80; $\sigma = 0.44$ | −15.3, 5.56, −10.1; $\sigma = 2.3$ | 6.3; $\sigma = 0.57$               | 3/10              |
| RZA3D             | 0.40; $\sigma = 0.19$ | −12.4, 7.37, −7.20; $\sigma = 2.3$ | 6.0; $\sigma = 1.9$               | 5/10              |

Figure 6. $XY$, $YZ$ supergalactic slices of the WF reconstruction (left), of one constrained simulation (middle) and of the average of 10 constrained simulations of the Local Universe within a $30 \, h^{-1} \text{Mpc}$ radius sphere. The supergalactic slices are located at $X = −2.5$ and $Z = 0 \, h^{-1} \text{Mpc}$ to fit the location of the Virgo cluster. The overdensity at $2 \, h^{-1} \text{Mpc}$ Gaussian smoothing is represented with black isocontours. The green contour stands for the mean density. The flows are shown with black arrows. In the $XY$ supergalactic plane, the Virgo cluster and the Virgo Void are both reconstructed (left-hand column) and simulated (middle column). Virgo is also visible next to the Local Void in the $YZ$ supergalactic slice. In general Virgo is well simulated, at a similar location, whatever random component is used (right-hand column). V8k (catalogue of redshifts) galaxies are shown for reference as grey dots in a $\pm 10 \, h^{-1} \text{Mpc}$ thick slice on the WF reconstruction. The red crosses locate Virgo in cosmicflows-1. The bigger blue crosses represent the (average) location of the Virgo-like haloes.
Since the cosmicflows-1 peculiar velocity catalogue extends only out to about 30 $h^{-1}$ Mpc (radius), derived constrained simulations are subject to considerable cosmic variance. To study this effect, mock catalogues have been drawn from a previous constrained simulation which looks like the Local Universe and an ensemble of 10 ICs has been constructed. The mock catalogues have been designed to mimic the observational catalogue by including distance measurement errors and a large continuous zone without data (Zone of Avoidance due to our Galaxy extinction). The RZA algorithm with its new feature (called RZA3D) has been tested against these mocks and resulting simulations have been compared with the ones obtained with the original RZA version (called RZA-radial). The enhanced precision and reliability of the RZA3D method are validated. The methodology has been subsequently applied to the actual cosmicflows-1 catalogue. An ensemble of 10 constrained simulations has been constructed and analysed.

The methodology succeeds in performing robust constrained simulations using only observational peculiar velocities as constraints. The cosmicflows-1 catalogue is still too shallow to enable constrained simulations that can reproduce all the main attractors and voids of the local dynamics. The recently published cosmicflows-2 catalogue (Tully et al., 2013) contains more than 8000 galaxy distances (1800 in cosmicflows-1) and extends out to about 150 $h^{-1}$ Mpc. The technique reported in this paper is to be applied to this larger data set with the aim of providing ICs for a constrained simulation of the Universe in a $640 h^{-1}$ Mpc box that will more thoroughly mimic observed large-scale structure.

**ACKNOWLEDGEMENTS**

We acknowledge help from and discussions with Timur Doumler. We thank the anonymous referee whose comments have contributed to improve this paper a lot. JGS received support from the Projet d’a venir Lyon-St Etienne and from the Région Rhône-Alpes via the Explora’doc grant. JGS and HMC acknowledge support from the Lyon Institute of Origins under grant ANR-10-LABX-66. RBT acknowledges support from NASA through the Spitzer Science Center Cosmicflows with Spitzer award and through the Astrophysical Data Analysis Program Cosmicflows with WISE award. YH has been partially supported by the Israel Science Foundation (13/08). SG and YH have been partially supported by the Deutsche Forschungsgemeinschaft under the grant GO563/21-1. The simulations have been performed at the Leibniz Rechenzentrum (LRZ) in Munich.

**REFERENCES**

Alimi J.-M. et al., 2012, Supercomputing 2012 Conference, preprint (arXiv:1206.2838)

Angulo R. E., Springel V., White S. D. M., Jenkins A., Baugh C. M., Frenk C. S., 2012, MNRAS, 426, 2046

Bistolas V., Hoffman Y., 1998, ApJ, 492, 439

Courtois H. M., Tully R. B., Makarov D. I., Mitronova S., Koribalski B., Karachentsev I. D., Fisher R. J., 2011a, MNRAS, 414, 2005

Courtois H. M., Tully R. B., Héraudeau P., 2011b, MNRAS, 415, 1935

Doumler T., Hoffman Y., Courtois H., Gottl"ober S., 2013a, MNRAS, 430, 588

Doumler T., Courtois H., Gottl"ober S., Hoffman Y., 2013b, MNRAS, 430, 902

Doumler T., Gottl"ober S., Hoffman Y., Courtois H., 2013c, MNRAS, 430, 912

Ganon G., Hoffman Y., 1993, ApJ, 415, L5

Gottl"ober S., Hoffman Y., Yepes G., 2010, Proc. High Performance Computing in Science and Engineering. Springer-Verlag, Berlin, preprint (arXiv:1005.2687)

Heß S., Kitaura F.-S., Gottl"ober S., 2013, MNRAS, 435, 2065

Hoffman Y., 2009, in Martínez V. J., Saar E., Martínez-González E., Pons-Bordería M.-J., eds, Lecture Notes in Physics, Vol. 665, Data Analysis in Cosmology. Springer-Verlag, Berlin, p. 565

Hoffman Y., Ribak E., 1991, ApJ, 370, L5

Klypin A., Hoffman Y., Kravtsov A. V., Gottl"ober S., 2003, ApJ, 596, 19

Klypin A. A., Trujillo-Gomez S., Primack J., 2011, ApJ, 740, 102

Knollmann S. R., Kaebe A., 2009, ApJS, 182, 608

Koll T., Dekel A., Ganon G., Willick J. A., 1996, ApJ, 458, 419

Komatsu E. et al., 2011, ApJS, 192, 18

Larson D. et al., 2011, ApJS, 192, 16

Lavaux G., 2010, MNRAS, 406, 1007

Mathis H., Lemson G., Springel V., Kauffmann G., White S. D. M., Eldar A., Dekel A., 2002, MNRAS, 333, 739

Planck Collaboration et al., 2013, preprint (arXiv e-prints)

Prada F., Klypin A. A., Cuesta A. J., Betancort-Rijo J. E., Primack J., 2012, MNRAS, 423, 3018

Sorce J. G., Courtois H. M., Tully R. B., 2012a, AJ, 144, 133

Sorce J. G., Tully R. B., Courtois H. M., 2012b, ApJ, 758, L12

Sorce J. G. et al., 2013, ApJ, 765, 94

Springel V., 2005, MNRAS, 364, 1105

Tully R. B., Courtois H. M., 2012, ApJ, 749, 78

Tully R. B., Shaya E. J., Karachentsev I. D., Courtois H. M., Kocevski D. D., Rizzi L., Peel A., 2008, ApJ, 676, 184

Tully R. B., Rizzi L., Shaya E. J., Courtois H. M., Makarov D. I., Jacobs B. A., 2009, AJ, 138, 323

Tully R. B. et al., 2013, AJ, 146, 86

**Table 2.** Average parameters and standard deviations $\sigma$ for the haloes representative of Virgo. Column (1): simulations in which the haloes are looked for; column (2): mass in $h^{-1} 10^{11-14} \times$ solar mass; column (3): average supergalactic coordinates $X$, $Y$ and $Z$ in $h^{-1}$ Mpc and standard deviation; column (4): distance in $h^{-1}$ Mpc from the simulation halo to the observed Virgo location and standard deviation $\sigma$ and column (5): number of occurrences in 10 different simulations (if a halo similar to Virgo was found in a $6 h^{-1}$ Mpc sphere).

| Case                  | Mass | Average supergalactic position | Average distance to the observed Virgo cluster | Nb of occurrences |
|-----------------------|------|--------------------------------|-----------------------------------------------|-------------------|
| Observed Virgo cluster| 4$^a$| $-2.74, 12.0, -0.518$          |                                               |                   |
| **RZA3D**             |      |                                |                                               |                   |
| Virgo (WMAP7)         | 0.7  | $1.23, 13.8, 2.2; \sigma = 1.2$| 5.4; $\sigma = 1.2$                          | 8/10              |
| **RZA3D**             | 0.5  | $1.30, 13.8, 0.7; \sigma = 1.0$| 4.7; $\sigma = 0.8$                          | 8/10              |
| **RZA-radial**        |      |                                |                                               |                   |
| Virgo (WMAP7)         | 0    |                                |                                               | 0/10              |
| Virgo (WMAP3)         | 0    |                                |                                               | 0/10              |

$^a$Estimation of the total (baryonic+dark matter) mass.
Because data sample a typical realization of the prior model, i.e. the power spectrum, \( \chi^2 \) should be close to 1 where \( \chi^2 = \sum_{i=1}^{M} \sum_{j=1}^{M} C_i (C_j)^{-1} C_j \) and dof is the degree of freedom. However, data include non-linearities which are not taken into account in the model. Consequently, a \( \sigma_{NL} \) such that \( (C_i C_j) = (c_i c_j) + \delta_i^k \epsilon_j^k + \delta_j^k \epsilon_i^k \sigma_{NL}^2 \) is required to compensate for the non-linearities to drive \( \chi^2 \) closer to 1.

**A2 The Reverse Zel’dovich Approximation**

In the framework of Lagrangian perturbation theory, the comoving (Eulerian) position \( r \) of a data point can be written as

\[
r(t) = q(t) + \Psi(r, t),
\]

where \( q \) is the initial (Lagrangian) position and \( \Psi \) is the displacement field.

The Zel’dovich approximation consists in writing \( \Psi \) as the product of two independent functions of position \( \Psi_i(r) \) and time \( D_{ij}(t) \), respectively,

\[
r(t) = q(t) + D_{ij}(t) \Psi_i(r),
\]

where \( D_{ij}(t) \) is the growing mode of structures (the decaying mode is assumed to have reached zero).

The comoving (peculiar) velocity \( v = a \frac{df}{dt} \) (with \( a \) the scale factor) is obtained deriving equation (A6):

\[
v = \dot{a} f \Psi,
\]

where \( \dot{a} \) is the time derivative of the scale factor and \( f \) is the growth rate \( f = \frac{d \ln D_{ij}}{dt} \).

At \( z = 0 \), \( a = 1 \) thus \( \dot{a} = H_0 \). It results that nowadays,

\[
v = H_0 f \Psi.
\]

Assuming that the original positions of protohalos \( r_{ZZA}^{init} \) is approximately equal to \( q \), the equation to shift back galaxies at \( r \) at \( z = 0 \) to their precursors’ positions at \( r_{ZZA}^{init} \) at higher redshifts is from equations (A5) and (A8):

\[
r_{ZZA}^{init} = r - \frac{v}{H_0 f}.
\]