Nonequilibrium Mesoscopic Conductors Driven by Reservoirs

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In order to specify a nonequilibrium steady state of a quantum wire (QWR), one must connect reservoirs to it. Since reservoirs should be large 2d or 3d systems, the total system is a large and inhomogeneous 2d or 3d system, in which \( e-e \) interactions have the same strength in all regions. However, most theories of interacting electrons in QWR considered simplified 1d models, in which reservoirs are absent or replaced with noninteracting 1d leads. We first discuss fundamental problems of such theories in view of nonequilibrium statistical mechanics. We then present formulations which are free from such difficulties, and discuss what is going on in mesoscopic systems in nonequilibrium steady state. In particular, we point out important roles of energy corrections and non-mechanical forces, which are induced by a finite current.

I. INTRODUCTION

According to nonequilibrium thermodynamics, one can specify nonequilibrium states of macroscopic systems by specifying local values of thermodynamical quantities, such as the local density and the local temperature, because of the local equilibrium [1]. When one studies transport properties of a mesoscopic conductor (quantum wire (QWR)), however, the local equilibrium is not realized in it, because it is too small. Hence, in order to specify its nonequilibrium state uniquely, one must connect reservoirs to it, and specify their chemical potentials \((\mu_L, \mu_R)\) instead of specifying the local quantities of the conductor (Fig. 1). The reservoirs should be large (macroscopic) 2d or 3d systems. Therefore, to really understand transport properties, we must analyze such a composite system of the QWR and the 2d or 3d reservoirs. Although the QWR itself may be a homogeneous 1d system, the total system is a 2d or 3d inhomogeneous system without the translational symmetry. Moreover, many-body interactions are important both in the conductor and in the reservoirs: If electrons were free in a reservoir, neither could electrons be injected (absorbed) into (from) the conductor, nor could they relax to achieve the local equilibrium. However, most theories considered simplified 1d models, in which reservoirs are absent or replaced with noninteracting 1d leads [3].

![FIG. 1. A two-terminal conductor composed of a QWR and reservoirs.](image)

In this paper, we study transport properties of a composite system of a QWR plus reservoirs, where \( e-e \) interactions are present in all regions. By critically reviewing theories of the conductance, we first point out fundamental problems of the theories in view of nonequilibrium statistical mechanics. We then present formulations which are free from such difficulties, and discuss what is going on in mesoscopic systems in nonequilibrium steady state. In particular, we point out important roles of energy corrections and non-mechanical forces, which are induced by a finite current.
II. A CRITICAL REVIEW OF THEORIES OF THE DC CONDUCTANCE

In this section, we critically review theories of the DC conductance \( G \) of interacting electrons in a QWR. Note that two theories which predict different nonequilibrium states can (be adjusted to) give the same value of \( G \) (to agree with experiment). Hence, the comparison of the values of \( G \) among different theories is not sufficient. For definitiveness, we consider a two-terminal conductor composed of a quantum wire (QWR) and two reservoirs (Fig. 1), which are defined by a confining potential \( u^c \), at zero temperature. Throughout this paper, we assume that \( u^c \) is smooth and slowly-varying, so that electrons are not reflected by \( u^c \) (i.e., the wavefunction evolves adiabatically). We also assume that only the lowest subband of the QWR is occupied by electrons. A finite current \( I \) is induced by applying a finite difference \( \Delta \mu = \mu_L - \mu_R \) of chemical potentials between the two reservoirs, and the DC conductance is defined by \( G \equiv \langle I \rangle / (\Delta \mu / e) \) \([13]\), where \( \langle I \rangle \) is the average value of \( I \).

Let us consider a clean QWR, which has no impurities or defects. For non-interacting electrons the Landauer-Büttiker formula gives \( G = e^2 / \pi h \) \([14]\), whereas \( G \) for interacting electrons has been a subject of controversy \([15]\). Most theories before 1995 \([3–5]\) predicted that \( G \) should be “renormalized” by the e-e interactions as \( G = K_\rho e^2 / \pi h \), where \( K_\rho \) is a parameter characterizing the Tomonaga-Luttinger liquid (TLL) \([16–19]\). However, Tarucha et al. found experimentally that \( G \simeq e^2 / \pi h \) for a QWR of \( K_\rho \simeq 0.7 \) \([20]\). Then, several theoretical papers have been published to explain the absence of the renormalization of \( G \) \([8–12,21]\). Although they concluded the same result, \( G = e^2 / \pi h \), the theoretical frameworks and the physics are very different from each other. Since most theories are based either on the Kubo formula \([22]\) (or, similar ones based on the adiabatic switching of an “external” field), or on the scattering theory, we review these two types of theories critically in this section.

A. Problems and limitations of the Kubo formula when it is applied to mesoscopic conductors

When one considers a physical system, it always interacts with other systems, \( R_1, R_2, \ldots \), which are called heat baths or reservoirs. Nonequilibrium properties of the system can be calculated if one knows the reduced density matrix \( \tilde{\rho} \equiv \text{Tr}_{R_1 \cdots R_2 \cdots} [\tilde{\rho}_{\text{total}}] \). Here, \( \tilde{\rho}_{\text{total}} \) is the density operator of the total system, and \( \text{Tr}_{R_1 \cdots R_2 \cdots} \) denotes the trace operation over reservoirs’ degrees of freedom. To find \( \tilde{\rho} \), Kubo \([22]\) assumed that the system is initially in its equilibrium state. Then an “external field” \( E_{\text{ext}} \) is applied adiabatically (i.e., \( E_{\text{ext}} \propto e^{-i/t} \)), which is a fictitious field because it does not always have its physical correspondence (see below). The time evolution of \( \tilde{\rho} \) was calculated using the von Neumann equation of an isolated system; i.e., it was assumed that the system were isolated from the reservoirs during the time evolution \([2] \). Because of these two assumptions (the fictitious field and isolated system), some conditions are required to get correct results by the Kubo formula. To examine the conditions, we must distinguish between non-dissipative responses (such as the DC magnetic susceptibility) and dissipative responses (such as the DC conductivity \( \sigma \)). The non-dissipative responses are essentially equilibrium properties of the system; in fact, they can be calculated from equilibrium statistical mechanics. For non-dissipative responses, Kubo \([22,23]\) and Suzuki \([24]\) established the conditions for the validity of the Kubo formula, by comparing the formula with the results of equilibrium statistical mechanics: (i) The proper order should be taken in the limiting procedures of \( \omega, q \to 0 \) and \( V \to \infty \), where \( \omega \) and \( q \) are the frequency and wavenumber of the external field, and \( V \) denotes the system volume. (ii) The dynamics of the system should have the following property;

\[
\lim_{t \to \infty} \langle \hat{A}\hat{B}(t) \rangle_{\text{eq}} = \langle \hat{A} \rangle_{\text{eq}} \langle \hat{B} \rangle_{\text{eq}},
\]

where \( \langle \cdots \rangle_{\text{eq}} \) denotes the expectation value in the thermal equilibrium, and \( \hat{A} \) and \( \hat{B} \) are the operators whose correlation is evaluated in the Kubo formula. Any integrable models do not have this property \([24,26,28]\). Hence, the Kubo formula is not applicable to integrable models, such as the Luttinger model, even for (the simple case of) non-dissipative responses \([24]\).
FIG. 2. Schematic plots of the chemical potential $\mu$ and the electrostatic potential $\phi$, for (a) a macroscopic inhomogeneous conductor and (b) a mesoscopic conductor. For case (a), the local equilibrium is established, and thus $\mu$ and $\phi$ can be defined in all regions. The differences $e\Delta \phi$ and $\Delta \mu$ are not equal if one takes the differences between both ends of the conductor, whereas $e\Delta \phi = \Delta \mu$ if the differences are taken between the leads. For case (b), $\mu$ cannot be defined in the QWR and boundary regions (although in some cases $\mu$ could be defined separately for left- and right-going electrons), whereas $\phi$ can be defined in all regions. Similarly to case (a), $e\Delta \phi \neq \Delta \mu$ if one takes the differences between both ends of the QWR, whereas $e\Delta \phi = \Delta \mu$ if the differences are taken between the reservoirs.

For dissipative responses, the conditions for the applicability of the Kubo formula would be stronger. Unfortunately, however, they are not completely clarified, and we here list some of known or suggested conditions for $\sigma$:

(i') Like as condition (i), the proper order should be taken in the limiting procedures. For $\sigma$ the order should be

$$\sigma = \lim_{\omega \to 0} \lim_{q \to 0} \lim_{V \to \infty} \sigma_{\text{formula}}(q, \omega; V).$$

(ii') Concerning condition (ii), a stronger condition seems necessary for dissipative responses: The closed system that is taken in the calculation of the Kubo formula should have the thermodynamical stability, i.e., it approaches the thermal equilibrium when it is initially subject to a macroscopic perturbation. (Otherwise, it would be unlikely for
the system to approach the correct steady state in the presence of an external field.) In classical Hamiltonian systems, this condition is almost equivalent to the “mixing property” \([29, 28]\), which states that Eq. (1) should hold for any \(A\) and \(B\), where \(\{\cdot, \cdot\}_{eq}\) is now taken as the average over the equi-energy surface. It is this condition, rather than the “ergodicity”, that guarantees the thermodynamical stability \([24, 28]\). Although real physical systems should always have this property, some theoretical models do not. In particular, any integrable models do not have this property \([29, 28]\).

(iii') We here suggest that all driving forces, including non-mechanical ones, should be identified \([29]\). In fact, the formula gives the current density in the following form, 

\[
\langle J \rangle = \sigma_{\text{formula}} E_{\text{ext}},
\]

whereas the exact definition of \(\sigma\) is given by nonequilibrium thermodynamics as \([12]\)

\[
\langle J \rangle = -\sigma \nabla(\mu/e) - L_{12} \nabla \beta = \sigma E - \sigma \nabla(\mu_c/e) - L_{12} \nabla \beta.
\]

Here, \(\beta\) denotes the inverse temperature, \(\mu\) is the “chemical potential” which consists of a chemical portion \(\mu_c\) and the electrostatic potential \(\phi\) \([30]\);

\[
\mu = \mu_c + e \phi \quad \text{(hence, } \Delta \mu = \Delta \mu_c + e \Delta \phi \text{ for differences}).
\]

Hence, to evaluate \(\sigma\), one must find the relation between \(E_{\text{ext}}\) and \(E\), \(\nabla \mu_c\) and \(\nabla \beta\). In homogeneous systems, it is expected that \(\nabla \mu_c = \nabla \beta = 0\), hence it is sufficient to find the relation between the fictitious field \(E_{\text{ext}}\) and the real field \(E\) \([10, 13]\). In inhomogeneous systems, however, \(\nabla \mu_c \neq 0\) and/or \(\nabla \beta \neq 0\) in general \([32]\), as shown in Fig. 2 (a).

Therefore, one must find the relation between \(E_{\text{ext}}\) and these “non-mechanical forces” \([24, 33]\). (See section V.)

Unfortunately, these conditions are not satisfied in theories based on simplified models of mesoscopic systems. For example, the Luttinger model \([17]\) used in much literature does not satisfy conditions (i) and (ii) because it is integrable. To get reasonable results, subtle procedures, which have not been justified yet, were taken in actual calculations. Moreover, the non-mechanical forces have not been examined, although they would be important because some theoretical models do not. In particular, any integrable models do not have this property \([29, 39]\).

B. Scattering-theoretical approaches

In view of many problems and limitations of the Kubo formula, it is natural to try to generalize Landauer’s theory \([14]\) to treat conductors with many-body interactions. Namely, the DC conductance may be given in terms of the scattering matrix (S matrix) for interacting electrons \([1, 30, 34]\).

The advantages of the scattering-theoretical approaches may be as follows: (i) Neither the translation of \(\Delta \phi_{\text{ext}}\) into \(\Delta \mu\) nor the subtle limiting procedures of \(\omega, q\) and \(V\) is necessary. (ii) There is no need for the mixing property of the 1d Hamiltonian \(\hat{H}_1\). Hence, \(\hat{H}_1\) can be the Hamiltonian of integrable 1d systems such as the TLL. (iii) In contrast to the Kubo formula, one can calculate the NEN \([29, 33]\). However, to define the S matrix, one must define incoming and outgoing states. Although they can be defined trivially for free electrons, it is nontrivial in the presence of many-body interactions. In high-energy physics, they are defined based on the asymptotic condition, which assumes that particles behave like free (but renormalized) ones as \(t \to \pm \infty\), i.e., before and after the collision \([14]\). For example, an electron (in the vacuum) before or after the collision becomes a localized “cloud” of electrons and positrons, which extend only over the Compton length, and this cloud can be regarded as a renormalized electron. In condensed-matter physics, on the other hand, the asymptotic condition is not satisfied for electrons in metals and doped semiconductors. In fact, elementary excitations (Landau’s quasi particles) are accompanied with the backflow, which extends all over the crystal \([11]\), in contradiction to the asymptotic condition. Because of this fundamental difficulty, the scattering approaches to mesoscopic conductors replaced the reservoirs with 1d leads in which electrons are free \([6, 11, 14]\). Therefore, real reservoirs, in which electrons behave as 2d or 3d interacting electrons, have not been treated by the scattering-theoretical approaches.
III. COMBINED USE OF MICROSCOPIC THEORY AND THERMODYNAMICS

The basic idea of this method [21] is as follows: Since a QWR is a small system, and is most important, it should be treated with a full quantum theory. On the other hand, reservoirs are large systems whose dynamics is complicated, hence it could be treated with thermodynamics (in a wide sense). Utilizing these observations, we shall develop thermodynamical arguments to find the nonequilibrium steady state that is realized when a finite $\Delta \mu$ is applied between the reservoirs. This is the key of this method because when the steady state is found, $G$ (and other observables) can be calculated by straightforward calculations. Although in some cases formal calculations can be performed without finding the steady state [2], we stress that such formal theories are incomplete because another theory is required to relate $\Delta \mu$ of such theories with $\Delta \mu$ of the reservoirs, by which $G$ is defined.

An advantage of the present method is that we do not need to find the relation between $\Delta \phi_{\text{ext}}$ and $\Delta \mu$ because $\langle I \rangle$ is directly calculated as a function of $\Delta \mu$. Another advantage is that it is applicable to NEN and nonlinear responses because nonequilibrium steady state is directly obtained.

A. Conductance of the 1d Fermi liquid

It is generally believed that a 1d interacting electron system is not the Fermi liquid (FL) [41], but the Tomonaga-Luttinger liquid (TLL) [16–19]. For this reason, many papers on 1d systems [3–6,8,9,11,12] use the word FL to indicate non-interacting electrons, i.e., a Fermi gas. However, we do not use such a misleading terminology; by a FL we mean interacting quasi-particles. Since the backflow is induced by the interaction [41], the Landauer’s argument of non-interacting particles [14] cannot be applied to a FL. On the other hand, real systems have finite length and finite intersubband energies, in contradiction to the assumptions of the TLL. Hence, some real systems might be well described as a FL. Therefore, $G$ of a FL is non-trivial and interesting [14]. Furthermore, we will show in section V that the results for the FL suggest very important phenomena that is characteristic to nonequilibrium states of inhomogeneous systems. Note also that the following calculations look similar to the derivation of fundamental observables) can be calculated by straightforward calculations. Although in some cases formal theories are incomplete because another theory is required to relate $\Delta \mu$ of such theories with $\Delta \mu$ of the reservoirs, by which $G$ is defined.

We find the nonequilibrium steady state using a thermodynamical argument as follows: In the reservoirs, electrons behave as a 2d or 3d (depending on the thickness of the reservoir regions) FL. Since we have assumed that $u^c$ is smooth and slowly-varying, a 2d or 3d quasi-particle in a reservoir, together with its backflow, can evolve adiabatically into a 1d quasi-particle and its backflow in the QWR, without reflection. In this adiabatic evolution, the quasi-particle mass $m^*$ and the Landau parameters $f$ also evolve adiabatically, and the energy is conserved. Therefore, quasi-particles with $\varepsilon(k > 0) \leq \mu_L$ are injected from the left reservoir. Here, $\varepsilon$ is the quasi-particle energy;

$$\varepsilon(k) = \frac{\hbar^2 k^2}{2m^*} + \frac{\hbar}{L} \sum_{k'} f(k,k') \delta n(k'),$$

(6)

where $\delta n(k) \equiv n(k) - \Theta(|k| \leq k_F)$, with $n(k)$ being the quasi-particle distribution. The last term of this expression represents energy correction by interactions among quasi-particles [11]. On the other hand, a quasi-hole below $\mu_L$ should not be injected because otherwise the recombination of a quasi-particle with the quasi-hole would produce excess entropy, in contradiction with the principle of minimum entropy production. Similarly, quasi-particles with $\varepsilon(k < 0) \leq \mu_R$ are injected from the right reservoir, with no quasi-holes are injected below $\mu_R$. Therefore, the nonequilibrium steady state under a finite $\Delta \mu = \mu_L - \mu_R$ should be the “shifted Fermi state”, in which quasi-particle states with $\varepsilon(k \geq 0) \leq \mu_L$ and $\varepsilon(k < 0) \leq \mu_R$ are all occupied. Hence, the right- (left-) going quasi-particles have the chemical potential $\mu_+ = \mu_L$ ($\mu_+ = \mu_R$). Considering also the charge neutrality, we can write the distribution function as

$$n(k) = \Theta(|k - q| \leq k_F).$$

(7)

Then, Eq. (6) yields

$$\mu_{\pm} = \frac{\hbar^2 k^2_{F \pm}}{2m^*} \pm \hbar q \left[ \frac{\hbar k_F}{m^*} + \frac{f_{++} - f_{+-}}{2\pi} \right],$$

(8)

where $f_{++} \equiv f(k_F,k_F)$ and $f_{+-} \equiv f(k_F,-k_F) = f(-k_F,k_F)$. Hence,

$$\Delta \mu = 2\hbar q \left[ \frac{\hbar k_F}{m^*} + \frac{f_{++} - f_{+-}}{2\pi} \right].$$

(9)
on the other hand, considering the spin degeneracy, \( \langle I \rangle \) is calculated as

\[
\langle I \rangle = 2e \frac{q}{\pi} \left[ \frac{\hbar k_F}{m^*} + \frac{f_{++} - f_{+-}}{2\pi} \right].
\]

(10)

Here, the \( f_{+\pm} \)-dependent terms represent the backflow. Since the same factor appears in Eq. (9), we find that the conductance is independent of \( m^* \) and \( f_{+\pm} \):

\[
G = \frac{\langle I \rangle}{\Delta \mu/e} = \frac{e^2}{\pi \hbar}.
\]

(11)

Since we have identified the nonequilibrium steady state, we can calculate not only \( G \) but also other nonequilibrium properties such as the NEN [21].

It is instructive to represent Eqs. (8)-(10) in terms of the bare parameters. As in the case of 3d Fermi liquid [41], we can show that

\[
\frac{\hbar k_F}{m} = \frac{\hbar k_F}{m^*} + \frac{f_{++} - f_{+-}}{2\pi}.
\]

(12)

Hence, we can rewrite Eq. (10) as \( \langle I \rangle = 2e(q/\pi)(\hbar k_F/m) \). Therefore, quasi particles (whose group velocity is \( \hbar k_F/m^* \)) plus their backflows carry exactly the same current as the bare particles, for the same \( q \), i.e., for the same shifted Fermi distribution. On the other hand, Eq. (9) is rewritten as \( \mu_{\pm} = \hbar^2 k_F^2/2m^* \pm \hbar^2 q k_F/m \). Although \( \mu_{\pm} \neq |\mu_0| \) of bare particles, \( \Delta \mu = [\Delta \mu \text{ of bare particles}] \) for the same \( q \). These facts result in the independence of \( G \) on the Landau parameters.

### B. Conductance of the Tomonaga-Luttinger liquid

We now consider a clean TLL [21]. The low-energy dynamics of a TLL is described by the charge (\( \rho \)) and spin (\( \sigma \)) excitations (whose quantum numbers are \( N^\rho_q \) and \( N^\sigma_q \), respectively, where \( q \neq 0 \) denotes the wavenumber), and the zero modes (quantum numbers \( N^\rho_0 \), \( N^\sigma_0 \)) [16–19]. The eigenenergy is given by

\[
E = \sum_{\nu=p,\sigma} v^\nu \sum_q \hbar |q| N^\nu_q + \frac{\pi \hbar}{2L} \sum_{\nu=p,\sigma} \frac{(v^\nu N^\nu_+ + N^\nu_0)^2 + v^\nu(N^\nu_+ - N^\nu_0)^2}{2}\]

(13)

where \( v^\nu_N = v^\nu/K_\nu \) and \( v^\nu_j = K_\nu v^\nu \ (\nu = p, \sigma) \). Here, the parameters \( v^\nu \) and \( K_\nu \) are renormalized by the \( e-e \) interactions (except that \( K_\sigma = 1 \) by the SU(2) symmetry). The DC current is given by

\[
\langle I \rangle = 2e v^\rho (N^\rho_+ - N^\rho_0)/L.
\]

(14)

We apply a thermodynamical argument to find the nonequilibrium steady state. Unlike the FL case, there is no adiabatic continuity between the TLL in the QWR and the FL in the reservoirs. We therefore argue differently: In the linear response regime the steady state must be the state with the minimum energy among states which satisfy given external conditions. Otherwise, the system would be unstable and would evolve into a state with lower energy. For our purpose, it is convenient to take the value of \( \langle I \rangle \) as the given external condition. Then, from Eqs. (13) and (14), we find that the steady state should be the state with \( N^\rho_q = N^\sigma_q = 0 \) (for all \( q \)), \( N^\rho_+ + N^\rho_0 = 0 \), \( N^\sigma_+ + N^\sigma_0 = 0 \), and \( N^\rho_+ - N^\rho_0 > 0 \). This state may be called the “shifted Fermi state” of the TLL. Furthermore, in the steady state, electrons in the left reservoir and right-going electrons in the TLL should be in the “chemical equilibrium”, in which electrons in the FL phase are transformed into right-going electrons in the TLL phase at a constant rate. Therefore, their chemical potentials should be equal [13]:

\[
\mu_{L,R} = \mu^\rho_{0,-} = \frac{1}{\hbar} \frac{\partial E}{\partial N^\rho_{0,-}} = \frac{\pi \hbar}{L} \left[ v^\rho_N (N^\rho_+ + N^\rho_0) \pm v^\rho_j (N^\rho_+ - N^\rho_0) \right].
\]

(15)

where we have used Eq. (13). Hence,

\[
\Delta \mu = \mu_L - \mu_R = \frac{2\pi \hbar}{L} v^\rho_j (N^\rho_+ - N^\rho_0).
\]

(16)

By dividing Eq. (14) by this expression, we obtain the same result for \( G \) as Eq. (11), in agreement with experiment [20].

Since we have identified the nonequilibrium steady state, we can calculate not only \( G \) but also other nonequilibrium properties such as the NEN [21].
IV. PROJECTION THEORY

Although we have successfully found the nonequilibrium steady state of interacting electrons in section [11], a possible objection against the formulation may be that the theory is rather intuitive. In this section, we present a full statistical-mechanical theory [12,13], which is free from such an objection. In this theory, we start from the Hamiltonian of 3d interacting electrons confined in the composite system of the QWR and reservoirs, Fig. [4]. This original system is projected onto an effective 1d system, and the equation of motion for the reduced density operator of the 1d system is derived. From this equation, we can find the nonequilibrium steady state as a function of Δμ between the reservoirs. This allows us to evaluate various nonequilibrium properties.

A. Decomposition of the 3d electron field

We start from the 3d electron field \( \hat{\psi}(r) \) subject to a confining potential \( u^c(r) \) (which defines the QWR and two reservoirs connected to it), impurity potential \( u^i(r) \) (whose average \( \bar{u} \) is absorbed in \( u^c(r) \), hence \( \bar{u} = 0 \)), external electrostatic potential \( \phi_{\text{ext}}(r) \), and the e-e interaction of equal strength \( \epsilon \). Namely, \( \hat{\psi}(r) \) can be expanded in terms of \( \phi(r) = e\phi_{\text{ext}}(r) + \int d^3r' \, v(r - r') \langle \hat{\rho}(r') \rangle \), and a c-number \( V_{\text{av}} \). Namely, \( \hat{H} \) is recast in terms of \( \delta \hat{\rho}(r) \equiv \hat{\rho}(r) - \langle \hat{\rho}(r) \rangle \) as

\[
\hat{H} = \int d^3r \, \hat{\psi}^\dagger(r) \left[ -\frac{\hbar^2}{2m} \nabla^2 + u^c(r) + u^i(r) + e\phi(r) \right] \hat{\psi}(r) + \frac{1}{2} \int d^3r \, \int d^3r' \, \delta\hat{\rho}(r)v(r - r')\delta\hat{\rho}(r'),
\]

where \( \hat{\rho}(r) \) is the charge density. We will find the nonequilibrium steady state for \( \Delta\mu > 0 \). For this state, \( \langle \hat{\rho}(r) \rangle \neq 0 \), which gives rise to a long-range force. We extract it as the renormalization of the electrostatic potential \( e\phi(r) = e\phi_{\text{ext}}(r) + \int d^3r' \, v(r - r') \langle \hat{\rho}(r') \rangle \), (17) and a c-number \( V_{\text{av}} \). Namely, \( \hat{H} \) is recast in terms of \( \delta\hat{\rho}(r) \equiv \hat{\rho}(r) - \langle \hat{\rho}(r) \rangle \) as

\[
\hat{H} = \int d^3r \, \hat{\psi}^\dagger(r) \left[ -\frac{\hbar^2}{2m} \nabla^2 + u^c(r) + u^i(r) + e\phi(r) \right] \hat{\psi}(r) + \frac{1}{2} \int d^3r \, \int d^3r' \, \delta\hat{\rho}(r)v(r - r')\delta\hat{\rho}(r') + V_{\text{av}}.
\]

To decompose \( \hat{\psi}(r) \), we consider the single-body part of \( \hat{H} \). Recall that \( u^c(r) \) is assumed to be smooth and slowly-varying, to avoid undesirable reflections at the QWR-reservoir boundaries. In this case, the single-body Schrödinger equation

\[
\left[ -\frac{\hbar^2}{2m} \nabla^2 + u^c(r) + u^i(r) + e\phi(r) \right] \varphi(r) = \epsilon \varphi(r)
\]

(20) has solutions that propagate through the QWR having the energy \( \epsilon \simeq \epsilon_F \) [16];

\[
\varphi_k(r) \simeq \frac{1}{\sqrt{L}} \exp \left[ i \int_0^x K_k(x)dx \right] \varphi^+ (y, z; x).
\]

(21)

Here, \( \varphi^+(y, z; x) \) is the wavefunction of the lowest subband at \( x \), representing the confinement in the lateral (yz) directions, and \( L \) is the normalization length in the \( x \) direction. All the other modes are denoted by \( \varphi\nu(r) \), which includes solutions that are localized in either reservoir, and extended solutions whose \( \epsilon \) are not close to \( \epsilon_F \). Since any function of \( v \) can be expanded in terms of \( \varphi_k(r) \)'s and \( \varphi\nu(r) \)'s, so is the \( r \) dependence of the electron field operator;

\[
\hat{\psi}(r) = \sum_k \hat{c}_k \varphi_k(r) + \sum_{\nu} \hat{d}_\nu \varphi\nu(r) \equiv \varphi^+(y, z; x)\hat{\psi}_1(x) + \hat{\psi}_R(r).
\]

(22)

The 3d electron field has thus been decomposed into the 1d field and the 3d field \( \hat{\psi}_R(r) \), which we call the “reservoir field.” It can be decomposed into the low-energy components \( \hat{\psi}_{R_L} \) and \( \hat{\psi}_{R_R} \) (which are localized in the left and right reservoirs, \( R_L \) and \( R_R \), respectively) and the high-energy component \( \hat{\psi}_{R_H} \) as

\[
\hat{\psi}_R = \hat{\psi}_{R_L} + \hat{\psi}_{R_R} + \hat{\psi}_{R_H}.
\]

(23)

For low-energy phenomena, we can take \( \hat{\psi}_R = \hat{\psi}_{R_L} + \hat{\psi}_{R_R} \).
By expressing $\hat{H}$ in terms of $\hat{\psi}_1$ and $\hat{\psi}_R$, we obtain the single-body part of $\hat{\psi}_1$ (denoted by $\hat{H}_1^0$), the $\hat{\psi}_1$-$\hat{\psi}_1$ interaction ($\hat{V}_{11}^0$), the $\hat{\psi}_1$-$\hat{\psi}_R$ interaction ($\hat{V}_{1R}^0$), the $\hat{\psi}_R$-$\hat{\psi}_R$ interaction ($\hat{V}_{RR}^0$), and the single-body part of $\hat{\psi}_R$ ($\hat{H}_R^0$) [4]. By the screening effect of $\hat{V}_{RR}^0$, $\hat{V}_{1R}^0$ is renormalized as the screened interaction $\hat{V}_{1R}$. Similarly, by the screening effect of $\hat{V}_{11}, \hat{V}_{11}^0$ is renormalized as the screened interaction $\hat{V}_{11}$. We therefore recast $\hat{V}_{11}^0 + \hat{V}_{1R}^0 + \hat{V}_{RR}^0$ as $\hat{V}_{11} + \hat{V}_{1R} + \hat{V}_{RR}$, where $\hat{V}_{1R}$ and $\hat{V}_{RR}$ have no screening effects on $\hat{V}_{11}$ and $\hat{V}_{1R}$. In this way, $\hat{H}$ is decomposed as

$$\hat{H} = \hat{H}_1 + \hat{V}_{1R} + \hat{H}_R,$$

where $\hat{H}_1 = \hat{H}_1^0 + \hat{V}_{11}$ is the Hamiltonian for $\hat{\psi}_1(x)$, $\hat{H}_R = \hat{H}_R^0 + \hat{V}_{RR}$ is the one for $\hat{\psi}_R(r)$, and $\hat{V}_{1R}$ is the interaction between $\hat{\psi}_1(x)$ and $\hat{\psi}_R(r)$. In particular, $\hat{H}_1$ is evaluated as

$$\hat{H}_1 = \int dx \hat{\psi}_1^\dagger(x) \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + u^\perp(x) + u_1^\perp(x) \right] \hat{\psi}_1(x) + \frac{1}{2} \int dx \int dx' \delta \hat{\rho}_1(x,t) v_{11}(x,x') \delta \hat{\rho}_1(x',t).$$

(25)

Here, $u^\perp$ is the subband energy [4], $\delta \hat{\rho}_1(x,t) \equiv \hat{\psi}_1^\dagger(x) \hat{\psi}_1(x) - \langle \hat{\psi}_1^\dagger(x) \hat{\psi}_1(x) \rangle$ is the density fluctuation of the 1d field, and

$$u_1^\perp(x) \equiv \int dy dz |\varphi^\perp(y,z;x)|^2 u_1^\perp(r),$$

(26)

$$v_{11}(x,x') \equiv \int \int \int dy dz dy' dz' |\varphi^\perp(y,z;x)|^2 v_{\text{sc}}(r,r') |\varphi^\perp(y',z';x')|^2$$

(27)

are the impurity and two-body potentials for the 1d field, where $v_{\text{sc}}$ is the screened two-body potential for $\hat{\psi}$.

It is seen that $u_1^\perp(x)$ is the average of the random potential $u_1^\perp(r)$ over the lateral wavefunction $\varphi^\perp$, which, as a function of $y$, is localized in a region of width $\sim W(x)$ for each $x$. Here, $W(x)$ denotes the width of the region in which electrons are confined (Fig. 3). From these observations, we can show that

$$u_1^\perp(x) \sim u^\perp(r) \quad \text{for } x \in \text{QWR},$$

(28)

$$|u_1^\perp(x)| \propto 1/\sqrt{W(x)} \quad \text{for } x \in \text{a reservoir}.\quad (29)$$

In a similar manner, the two-body potential for the 1d field behaves as

$$v_{11}(x,x') \lesssim v_{\text{sc}}(||x-x'|| + W(0)^2)^{1/2} \quad \text{for } x \text{ or } x' \in \text{QWR},$$

(30)

$$v_{11}(x,x') \sim (r_{\text{sc}}/W)^{v_{\text{sc}}} \quad \text{for } x \sim x' \in \text{a reservoir}.\quad (31)$$

where $r_{\text{sc}}$ denotes the range of $v_{\text{sc}}$, and $v_{\text{sc}}$ the average of $v_{\text{sc}}$ in the region $|r-r'| \lesssim r_{\text{sc}}$. We now assume that the width of the reservoirs is very large;

$$W(x) \to \infty \text{ as } x \to \pm \infty.\quad (32)$$

Then, Eqs. (29)-(31) yield $u_1^\perp(x) \to 0$ as $x \to \pm \infty$, and $v_{11}(x,x') \to 0$ as $x$ or $x' \to \pm \infty$. Namely, $\hat{H}_1$ represents interacting $\hat{\psi}_1(x)$ field that gets free as $x \to \pm \infty$. On the other hand, the interaction $\hat{V}_{1R}$ between $\hat{\psi}_1(x)$ and $\hat{\psi}_R(r)$ becomes stronger in the reservoir regions, whereas it is negligible in the QWR because at low energies $\hat{\psi}_R(r)$ does not penetrate into the QWR. Therefore, the 1d field $\hat{\psi}_1(x)$ is subject to different scatterings in different regions of $x$: In the QWR, $\hat{\psi}_1$ is scattered by the $\hat{\psi}_1$-$\hat{\psi}_1$ interaction and the impurity potentials, whereas $\hat{\psi}_1$ is excited and attenuated by the reservoir field in the reservoir regions through the $\hat{\psi}_1$-$\hat{\psi}_R$ interactions (Fig. 3).
C. Equation of motion for the reduced density operator

We have successfully rewritten the Hamiltonian $\hat{H}$ in terms of $\hat{\psi}_1$ and $\hat{\psi}_R$. We must go one step further because $\hat{H} = \hat{H}_1 + \hat{V}_{1R} + \hat{H}_R$ describes very complicated dynamics, and thus the von Neumann equation for the density operator $\hat{\zeta}$,

$$i\hbar \frac{\partial}{\partial t} \hat{\zeta}(t) = \left[\hat{H}_1 + \hat{V}_{1R} + \hat{H}_R, \hat{\zeta}(t)\right]$$

is impossible to solve. This unsolvability guarantees the thermodynamical stability (mixing property) of the total system \cite{20,23}. We turn this fact to our own advantage, and reduce the theory to a tractable one \cite{44,45}. The basic idea is as follows: $\hat{H}_1$ describes the 1d correlated electrons, and is most important. Hence, it should be given a full quantum-mechanical treatment. Concerning $\hat{V}_{1R}$, on the other hand, multiple interactions by $\hat{V}_{1R}$ seem unimportant. Hence, we may treat it by a second-order perturbation theory \cite{47}. For $\hat{H}_R$, it describes the 2d or 3d interacting electrons, for which many properties are well known, and we can utilize the established results. Moreover, since the reservoirs are large, we can assume the local equilibrium: both reservoirs are in their equilibrium states with the chemical potentials $\mu_L$ and $\mu_R$, respectively. We denote the reduced density operator of the reservoir field for this local equilibrium state by $\hat{\zeta}_R$.

From these observations, we may project out the reservoir field $\hat{\psi}_R$ as follows. Consider the reduced density operator for the 1d field: $\hat{\zeta}_1(t) \equiv \text{Tr}_R[\hat{\zeta}(t)]$. Up to the second order in $\hat{V}_{1R}$ \cite{17}, the equation of motion of $\hat{\zeta}_1$ in the interaction picture of $\hat{H}_1 + \hat{H}_R$, is evaluated as

$$\frac{\partial}{\partial t} \hat{\zeta}_1(t) = \frac{-1}{\hbar^2} \int_{-\infty}^{t} dt' \text{Tr}_R \left( \left[\hat{V}_{1R}(t'), \left[\hat{V}_{1R}(t'), \hat{\zeta}_R \hat{\zeta}_1(t)\right]\right]\right),$$

where we have used the fact that $\hat{\zeta}_1(t)$ in the interaction picture varies only slowly, so that $\hat{\zeta}_1(t') \simeq \hat{\zeta}_1(t)$ in the correlation time of $\langle \hat{V}_{1R}(t)\hat{V}_{1R}(t')\rangle$ \cite{18}. This equation represents that $\hat{\zeta}_1$ is driven by two reservoirs, which have different chemical potentials $\mu_L$ and $\mu_R$, through the $\hat{\psi}_1\hat{\psi}_R$ interaction. Since the trace is taken over the reservoir field, Eq. (34) is a closed equation for $\hat{\psi}_1$ and $\hat{\zeta}_1$. Its steady solution represents the nonequilibrium steady state of the 1d field driven by the reservoirs.

D. Current of the 1d field

We now turn to observables. We are most interested in the total current $\hat{I}$ which is given by $\hat{I}(x,t) \equiv \int dydz \hat{J}_x(r,t)$, where $\hat{J}_x$ denotes the $x$ component of the current density. Note that $\hat{I}$ is different from the current of the 1d field defined by \cite{14}.

![FIG. 3. Schematic diagram of the strengths of scatterings of the 1d field.](image)
\[
\hat{I}_1(x, t) \equiv \frac{e}{2m} \left[ \hat{\psi}_1^\dagger(x, t) \left\{ \frac{\hbar}{i} \frac{\partial}{\partial x} \hat{\psi}_1(x, t) \right\} + \text{h.c.} \right].
\] (35)

The expectation values of \( \hat{I} \), \( \hat{I}_1 \), and \( \hat{I}_R \) (the current carried by the reservoir field) are schematically plotted in Fig. 4, which shows that \( \langle I \rangle \) is mainly carried by \( \langle I_1 \rangle \) in the QWR and by \( \langle I_R \rangle \) in the reservoirs, respectively. The transformation between \( \hat{I}_1 \) and \( \hat{I}_R \) is caused by \( \hat{V}_{1R} \), and thus \( \hat{I}_1 \) is not conserved: \( \frac{\partial}{\partial t} \hat{\rho}_1 + \frac{\partial}{\partial x} \hat{\rho}_1 \neq 0 \). At first sight, these facts might seem to cause difficulties in calculating \( \langle I \rangle \) and \( \langle \delta I^2 \rangle \) from \( \zeta_1 \). Fortunately, however, we can show that for any (nonequilibrium) steady state \( \langle I \rangle \) and \( \langle \delta I^2 \rangle \) are independent of \( x \), and that
\[
\langle I \rangle = \langle I_1 \rangle \text{ at } x \simeq 0 \tag{36}
\]
\[
\langle \delta I^2 \rangle \omega \simeq 0 = \langle \delta I^2_1 \rangle \omega \simeq 0 \text{ at } x \simeq 0 \tag{37}
\]
where \( x = 0 \) corresponds to the center of the QWR. Therefore, to calculate \( \langle I \rangle \) and \( \langle \delta I^2 \rangle \), it is sufficient to calculate \( \langle I_1 \rangle \) and \( \langle \delta I^2_1 \rangle \omega \simeq 0 \) at \( x \simeq 0 \), which can be calculated from \( \zeta_1 \). Therefore, we have successfully reduced the 3d problem, Eq. (17), into the effective 1d problem, Eqs. (24), (34) and (35).

![FIG. 4. Schematic plots of the expectation values \( \langle I \rangle \), \( \langle I_1 \rangle \), and \( \langle I_R \rangle \), of the currents carried by \( \hat{\psi}, \hat{\psi}_1 \), and \( \hat{\psi}_R \), respectively.](image)

Actual calculations can be conveniently performed as follows. Although \( \langle I \rangle \) and \( \langle \delta I^2 \rangle \) have both low- and high-frequency components, we are only interested in the low-frequency components, which are denoted by \( \tilde{I}(t) \) and \( \delta \tilde{I}^2(t) \), respectively. They are given by
\[
\tilde{I}(t) = \int_{-\infty}^{\infty} dt' f(t' - t) \text{Tr}[\hat{I}(t')\hat{\zeta}(t')],
\] (38)
and similarly for \( \delta \tilde{I}^2(t) \). Here, \( f(t' - t) \) is a filter function that is finite only in the region \( t - \tau/2 \lesssim t' \lesssim t + \tau/2 \), where \( 1/\tau \simeq \) the highest frequency of interest. From Eqs. (24), (34)-(35), we can construct the equations for \( \tilde{I}(t) \) and \( \delta \tilde{I}^2(t) \). They can be solved more easily than the equation for \( \zeta_1 \), and the solutions fully describe the low-frequency behaviors of \( \langle I \rangle \) and \( \langle \delta I^2 \rangle \).

In the following, we present the results for \( \tilde{I}(t) \) for the case of impurity scatterings and for the the case of e-e interactions.

E. Application of the projection theory to the case where impurity scatterings are present in all regions

When electrons are scattered by impurities (one-body potentials) in all regions including reservoirs, whereas many-body scatterings are negligible, the \( \hat{\psi}_1, \hat{\psi}_R \) interaction is given by
\[
\hat{V}_{1R} = \int d^3r \hat{\psi}_1^\dagger(x) \varphi^{1*}(y, z; x) \hat{u}^\dagger(r) \hat{\psi}_R(r) + \text{h.c.}
\] (39)

If we put \( \hat{Y}_\alpha(r) \equiv \varphi^{1*}(y, z; x) \hat{u}^\dagger(r) \hat{\psi}_R\alpha(r) \) (\( \alpha = L, R \)), then Eq. (34) becomes
injected electrons, and we obtain electron becomes easier or harder as compared with the case of \( \bar{I} \), and the correction terms are proportional to the presence of a finite current \( \mu \)

would be renormalized by the factor \( \langle \cdots \rangle_{\alpha} \equiv \langle \cdots \rangle_{\alpha}^{\text{bare}} + \frac{\hbar}{2} \sum_{\alpha} \int_{-\infty}^{t} dt' \int d^3 r \int d^3 r' \left\{ \left\langle \hat{Y}^{\dagger}_{\alpha}(r,t) \hat{Y}_{\alpha}(r',t') \right\rangle_{\alpha} \left[ \bar{\psi}_{\alpha}(x,t), \bar{\psi}^\dagger_{\alpha}(x',t') \right] + \right. 

\left. \left\langle \hat{Y}_{\alpha}(r,t) \hat{Y}^{\dagger}_{\alpha}(r',t') \right\rangle_{\alpha} \left[ \psi_{\alpha}(x,t), \psi^\dagger_{\alpha}(x',t') \right] \right\} + \text{h.c.} \tag{40} \]

where \( \langle \cdots \rangle_{\alpha} \) denotes the expectation value for the equilibrium state of reservoir \( R_{\alpha} \), which has the chemical potential \( \mu_{\alpha} \). After careful calculations using Eqs. (35)-(38), and considering the spin degeneracy, we find [15]

\[
\frac{d}{dt} \bar{I}(t) = -\gamma \left[ \bar{I}(t) - \bar{I}_{\text{steady}} \right], \tag{41}
\]

where \( \bar{I}_{\text{steady}} \equiv (e/\pi \hbar) T \Delta \mu \), and \( \gamma \sim (2 \pi / \hbar) n_{\text{imp}} |u|^2 D_F \). Here, \( n_{\text{imp}} \) is the impurity density, \( u^d \) denotes the potential of an impurity \( \bar{u} = \sum_{i} u^d(r - r_i) \), and \( D_F \) is the density of states per unit volume, \( D(\mu_L) \simeq D(\mu_R) \equiv D_F \). It is seen that \( \bar{I} \) approaches \( \bar{I}_{\text{steady}} \) as \( t \to \infty \). Therefore, the DC conductance is given by

\[
G = T \frac{e^2}{\pi \hbar}, \tag{42}
\]

in agreement with the Landauer-Büttiker formula [14]. Moreover, we find that the steady state is stable: For any (small) deviation from the steady state, \( \bar{I} \) relaxes to the value \( \bar{I}_{\text{steady}} \), with the relaxation constant \( \gamma \).

F. Application of the projection theory to the case where e-e scatterings are present in all regions

When the e-e interaction is important in all regions including reservoirs, whereas impurity scatterings are negligible, we find that the most relevant term of the \( \psi_1^\dagger \psi_{R} \) interaction is given by [43]

\[
\hat{V}_{IR} = \int d^3 r \int d^3 r' \bar{\psi}^\dagger_1(x) \phi^+ (y, z; x) \psi_R(r) e^{i \sigma} (r, r') \psi^\dagger_R(r') \psi_R(r') + \text{h.c.} \tag{43}
\]

By this interaction, an electron is scattered into the QWR through the collision of two electrons in a reservoir, or, an electron in the QWR is absorbed in a reservoir. By putting \( \hat{Y}_{\alpha}(r) \equiv \int d^3 r' \phi^+ (y, z; x) \psi_{R_{\alpha}}(r) e^{i \sigma} (r, r') \psi^\dagger_{R_{\alpha}}(r') \psi_{R_{\alpha}}(r') \) (for \( \alpha = L, R \)), which differs from \( \hat{Y}_{\alpha} \) of the impurity-scattering case, we obtain the equation of motion for \( \bar{\psi}_1^\dagger \) in the same form as Eq. (39). To derive the equation of motion for \( \bar{I} \) from that equation, we need to calculate the correlation functions in the reservoirs, \( \langle \hat{Y}_{\alpha}(t) \hat{Y}_{\alpha}^\dagger(t') \rangle_{\alpha} \) and \( \langle \hat{Y}_{\alpha}^\dagger(t) \hat{Y}_{\alpha}(t') \rangle_{\alpha} \). We can easily calculate them using well-known results for the 2D or 3D FL because the reservoir electrons are believed to be the 2D or 3D FL. We also need correlation functions of the 1d field. They are quite different depending on the nature (FL or TLL) of the electrons in the QWR.

In the case where \( \bar{\psi}_1 \) behaves as a 1d FL, we obtain the equation for \( \bar{I}(t) \) in the same form as Eq. (41), but now \( \gamma \) is a function of the e-e interaction parameters, and

\[
\bar{I}_{\text{steady}} = 2 \frac{e}{2} \sum_{k > 0} \frac{\hbar k}{m} [\Theta (\mu_L - \varepsilon(k)) - \Theta (\mu_R - \varepsilon(-k))]. \tag{44}
\]

Here, \( m \) is the bare mass, \( \Theta \) is the step function, and \( \varepsilon(\pm k) \) denotes the 1d quasi-particle energy, Eq. (11), in the shifted Fermi state. Note that if we simply took \( \varepsilon(\pm k) = \hbar^2 k^2 / 2m^* \), then \( \bar{I}_{\text{steady}} = (m^*/m)(e/\pi \hbar) \Delta \mu \), hence the conductance would be renormalized by the factor \( m^*/m \). However, the correct expression [16] shows that \( \varepsilon(\pm k) \) are modified in the presence of a finite current, and the correction terms are proportional to \( q \propto I \). As a result, the injection of an electron becomes easier or harder as compared with the case of \( I = 0 \). This automatically “calibrates” the number of injected electrons, and we obtain

\[
\bar{I}_{\text{steady}} = \frac{2 \hbar k_F}{m} \left[ \frac{\hbar k_F}{m^*} + \frac{1}{2\pi} (f_{++} - f_{+-}) \right]^{-1} \frac{e}{2\pi \hbar} \Delta \mu = \frac{e}{\pi \hbar} \Delta \mu, \tag{45}
\]

where we have used Eq. (12). Therefore, \( G = e^2 / \pi \hbar \). Here, the interaction parameters of the 1d field are canceled in \( G \), and those of the reservoir field are absorbed in \( \gamma \). These observations confirm the results of section [15], the shifted Fermi state is realized as the nonequilibrium steady state, and the conductance is quantized.

The application of the projection theory to the case where \( \bar{\psi}_1 \) behaves as a TLL will be a subject of future study.
G. Advantages of the projection theory

A disadvantage of the projection theory is that calculations of $G$ become rather hard as compared with the simple theories that are reviewed in section II. However, the simple theories have many problems and limitations, as discussed there. The projection theory is free from such problems and limitations, and has the following advantages: (i) The value of $(I)$ for the nonequilibrium state is directly calculated as a function of $\Delta \mu$. Hence, neither the translation of $\Delta \phi_{\text{ext}}$ into $\Delta \mu$ nor the subtle limiting procedures of $\omega, q$ and $V$ is necessary. (ii) There is no need for the mixing property of the 1d Hamiltonian $H_1$. Hence, $\hat{H}_1$ can be the Hamiltonian of integrable 1d systems such as the TLL. (iii) In contrast to the Kubo formula, which evaluates transport coefficients from equilibrium fluctuations, the projection theory gives the nonequilibrium steady state. This allows us to discuss what 1d state is realized and how the current is injected from the reservoirs. Moreover, we can calculate the NEN and nonlinear responses. (iv) The projection theory can describe the relaxation to the nonequilibrium steady state. This allows us to study the stability and the relaxation time of the nonequilibrium state.

V. APPEARANCE OF A NON-MECHANICAL FORCE

We here discuss the applicability of the Kubo formula to inhomogeneous systems. The general conclusion of this section is independent of natures (such as a FL or TLL) and the dimensionality of the electron system. Hence, we will use the results for the 1d FL, which are obtained in section III A and confirmed by a full statistical-mechanical theory in section IV I.

The original form of the Kubo formula gives a conductivity that corresponds to the following conductance; $(I)/\Delta \phi_{\text{ext}} \equiv G_{\text{Kubo}}$.[3] Izuyama suggested that the conductance should be $(I)/\Delta \phi \equiv G_{\text{Izuyama}}$, by considering the screening of $\phi_{\text{ext}}$. On the other hand, the exact definition of the conductance is $G \equiv (I)/\Delta \mu$.[1][2]. For macroscopic inhomogeneous conductors, $e\Delta \phi \neq \Delta \mu$ in general if one takes the differences between both ends of the conductor, as sketched in Fig. 2(a). Therefore, $G \neq G_{\text{Kubo}}, G_{\text{Izuyama}}$ in such a case. Hence, to obtain the correct value of $G$ by the Kubo formula, one must find the relation between $\Delta \phi_{\text{ext}}$ and $\Delta \mu$. Unfortunately, no systematic way of doing this has been developed.

The same can be said for mesoscopic conductors, Fig. 2(b), for which $e\Delta \phi \neq \Delta \mu$ in general if one takes the differences between both ends of the QWR. Therefore, $G \neq G_{\text{Kubo}}, G_{\text{Izuyama}}$. It is only for fortunate cases that $G_{\text{Kubo}}$ or $G_{\text{Izuyama}}$ coincides with $G$. For example, Kawabata[49] calculated $G_{\text{Izuyama}}$ for the case where the backward scattering with amplitude $V(2k_F)$ is present, which had been neglected in the previous calculations. He found that

$$G_{\text{Izuyama}} = \frac{e^2}{\pi \hbar} \left[ 1 + \frac{V(2k_F)}{2\pi \hbar v_F} \right].$$

(46)

However, this result disagrees with $G$ obtained in the previous sections. The origin of this discrepancy may be understood as follows. By taking the Fourier transforms of both sides of Eq. (13), we can see that only the $q \approx 0$ component of the two-body potential $\nu$ contributes to the screening of the electrostatic potential. On the other hand, both $q \approx 0$ (forward) and $q \approx 2k_F$ (backward) components of $\nu$ contribute the Landau parameter $f_{++}$, i.e., $f_{++} = f_{\text{forward}} + f_{\text{backward}}^{\text{backward}}$. Therefore, Eq. (13) shows that $\Delta \mu$ has a term (proportional to $f_{\text{backward}}^{\text{backward}}$) which cannot be interpreted as coming from the screening of $\phi_{\text{ext}}$. If we interpret this term in terms of nonequilibrium thermodynamics (although it is not fully applicable because the local equilibrium is not established), the term may be interpreted as a non-mechanical force in Eq. (13):

$$\Delta \mu_c \rightarrow -(hq/\pi) f_{\text{forward}}^{\text{backward}} + \cdots.$$

(47)

Here, $\cdots$ accounts for possible contributions from $f_{++}$ and/or $f_{\text{forward}}^{\text{forward}}$. Since $q \propto (I)$, so is $\Delta \mu_c$. This means that a finite current $I$ induces a finite non-mechanical force $\Delta \mu_c$, and $I$ is driven by both $e\Delta \phi$ and $\Delta \mu_c$ in the steady state. Hence, $G \equiv (I)/(\Delta \mu/e)$ is not equal to either $G_{\text{Kubo}} \equiv (I)/\Delta \phi_{\text{ext}}$ or $G_{\text{Izuyama}} \equiv (I)/\Delta \phi$. Therefore, the Kubo formula cannot give the correct value of $G$ if the $q \approx 2k_F$ component of the two-body potential is non-negligible, even if the screening of $\Delta \phi_{\text{ext}}$ is correctly taken into account, because a non-mechanical force is inevitably induced. Note that this is not the unique problem of the Kubo formula, but a common problem of many microscopic theories which calculate a nonequilibrium state by applying a mechanical force.

A possible way of getting the correct result by the Kubo formula would be to apply the formula to a larger system that includes the homogeneous reservoirs or leads[60]: in that case, $e\Delta \phi = \Delta \mu$, as sketched in Fig. 2, and thus $G = G_{\text{Izuyama}}$. However, this seems very difficult because it is almost equivalent to trying to solve the Schrödinger equation of the total system, including complicated processes that lead to the mixing property and to the equality
Another possible solution may be to apply Zubarev’s method \(^{2}\), which, to the authors’ knowledge, has not been applied to interacting electrons in mesoscopic conductors. However, one must also include (a part of) reservoirs into the Hamiltonian because Zubarev’s method assumes that macroscopic variables (such as \(\mu\)) are well-defined in the nonequilibrium steady state. As compared with these approaches, the formulations presented in sections \(\text{II}A\) and \(\text{IV}\) would be simpler ways of getting correct results which include effects of non-mechanical forces.

VI. DEVIATION FROM THE QUANTIZED CONDUCTANCE

For a clean QWR, we have obtained the quantized value \(G = \frac{e^2}{\pi \hbar}\) in both cases of the FL and the TLL in sections \(\text{III A}, \text{III B}\) and \(\text{IV F}\), using different formulations. The essential assumptions leading to this result are the following. (i) The QWR is clean enough and the temperature is low enough (zero temperature has been assumed for simplicity), so that scatterings by impurities, defects or phonons are negligible and e-e interactions are the only scattering mechanism. (ii) The boundaries between the QWR and reservoirs are smooth and slowly-varying so that reflections at the boundaries are absent. (iii) The reservoirs are large enough, so that they remain at equilibrium even in the presence of a finite current between the reservoirs through the QWR.

When some of these assumptions are not satisfied the observed conductance may deviate from the quantized value. For example, if boundary reflections are non-negligible, the transmittance \(T\) (calculated from the single-body Schrödinger equation) between the reservoirs through the QWR is reduced. This results in the reduction of \(G\) by the factor \(T\) for non-interacting electrons. For interacting electrons, \(G\) will be further reduced for the TLL because the TLL will be “pinned” by the reflection potential at the boundaries. This can be understood simply as follows: Although the TLL of infinite length is a liquid, for which a long-range order is absent, it behaves like a solid at a short distance. Hence, the TLL is pinned by a local potential, like a charge-density wave is. This is the physical origin of the vanishing \(G\) (at zero temperature) for the case where a potential barrier is located in the TLL \(^{4,5}\). Since the pinning occurs irrespective of the position of the local potential, the TTL would be pinned also by the boundary reflections. Note that if one neglects the weakening of \(\nu_{11}\) in reservoirs (due to the broadening of \(W(x)\), as shown in section \(\text{IV B}\)), the TTL would then be pinned also by impurities in reservoirs \(^{5}\).

Another example is dissipation by, say, phonon emission. By the dissipation, the 1d system will lose any correlations over a distance \(L_{\text{rx}}\), where \(L_{\text{rx}}\) is the “maximal energy relaxation length” \(^{[8]}\), which is generally longer than the simple dephasing length (over which an energy correlation may be able to survive). In such a case the 1d system of length \(L\) (\(> L_{\text{rx}}\)) will behave as a series of independent conductors of length \(L_{\text{rx}}\). One will then observe Ohm’s law \(^{[21]}\):

\[
G_{\text{obs}} \simeq \frac{(L_{\text{rx}}/L) \times (e^2/\pi \hbar)}{L}
\]

(48)

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In thermodynamics, \( \mu \) is called an “electrochemical potential”, whereas \( \mu_c \) is called a chemical potential [1]. We here follow the terminology used in the solid state physics, where \( \mu \) is called a chemical potential [1, 12-14, 19]. Note that it is not \( \Delta \phi \) but \( \Delta \mu \) that is applied by a battery.

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For example, the \( \nabla \mu_c \) term is dominant in p-n junctions.

Gradients or differences of density, temperature, and so on, are driving forces that induce a finite current. These driving forces are called “non-mechanical forces” because they cannot be represented as a mechanical term in the Hamiltonian [2]. This point is sometimes disregarded in the literature.

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The only exception is the NEN of (tunnel) junctions with a high barrier. In this case one can take the approximate equilibrium state with \( \Delta \mu > 0 \) as the equilibrium state of the Kubo formula.

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This relation is derived from the identity, \( \langle I \rangle = e\langle P \rangle / mL \), where \( \langle I \rangle \) and \( \langle P \rangle \) denote the DC components of the current and the total momentum, respectively. Remark: This identity does not hold for the LLT if one describes it by the Luttinger model [17] because of its idealized linear dispersion. As a result, the mechanisms leading to the universal value of \( G \) are different between the FL and TLL, as described in A. Shimizu, J. Phys. Soc. Jpn. 65, 3096 (1996).

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This is an adiabatic approximation, which has been widely used in studies of optical waveguides. The equation for the optical field in a waveguide has the same form as the single-body Schrödinger equation of an electron in a quantum wire.

Note that the second-order treatment of \( V_{IR} \) does not limit the accuracy of the theory because, e.g., the result for the steady current does not include \( V_{IR} \). Only the accuracy of the relaxation time \( \gamma \) is limited to the second order.

Precisely speaking, the equality \( \zeta(t') \approx \zeta(t) \) holds not for \( \zeta_1 \) but for expectation values evaluated from \( \zeta_1 \). We therefore apply Eq. (14) to expectation values, as demonstrated in sections IV E and IV F.

A. Kawabata, unpublished.

For an approach similar (but different) to this, see A. Kawabata, J. Phys. Soc. Jpn. 67, 2430 (1998) A. Kawabata, another chapter of this book