On fundamental quality of fission chain reaction to oppose rapid runaways of nuclear reactors

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Abstract. It has been shown that the in-hour equation characterizes the barriers and resistibility of fission chain reaction (FCR) against rapid runaways in nuclear reactors. Traditionally, nuclear reactors are characterized by the presence of barriers based on delayed and prompt neutrons. A new barrier based on the reflector neutrons that can occur when the fast reactor core is surrounded by a weakly absorbing neutron reflector with heavy atomic weight was proposed. It has been shown that the safety of this fast reactor is substantially improved, and considerable elongation of prompt neutron lifetime "devalues" the role of delayed neutron fraction as the maximum permissible reactivity for the reactor safety.

1. The in-hour equation and its physical sense

Within the frames of one-point kinetic model with six groups of delayed neutrons (DN) without accounting for the reactivity feedbacks, the in-hour equation that links the asymptotic period of the runaway $T$ with the inserted positive reactivity $\rho$ may be written in the following form:

$$\rho = \frac{A_{pr}}{T} + \sum_{i=1}^{6} \beta_i \cdot \Lambda_i \cdot \beta_i$$  \hspace{1cm} (1)

where: $\rho$ – the reactor reactivity, i.e. $\rho = (K_{cf} - K_0) \cdot (K_{cf} \cdot K_0)^{-1}$; $K_{cf}, K_0$ – the values of effective neutron multiplication factor in two different reactor states: the unperturbed state and the state perturbed by the reactivity insertion, respectively; $A_{pr}$ – means prompt neutron lifetime in the reactors with traditional neutron reflectors; $\Lambda_i$ – means lifetime of nuclei-emitters of the $i$-th DN group ($\Lambda_i = 1/\lambda_i$, where $\lambda_i$ – decay constant of nuclei-emitters of the $i$-th DN group, $\Lambda_i = 0.3 \div 80$ s); $\beta_i$ – effective fraction of the $i$-th DN group; $\beta$ - effective DN fraction ($\beta = \sum_{i=1}^{6} \beta_i$). The left term of the in-hour equation is the reactivity that should be inserted to provoke the runaway with a certain value of asymptotic period. The right part of the in-hour equation consists of two summands. The first summand depends only on mean prompt neutron lifetime $A_{pr}$, while the second summand depends only on DN parameters $\Lambda_i, \beta_i$. It seems natural to ascribe the contributions given by prompt and delayed neutrons into the reactor reactivity to the first and second summand, respectively. However, this conclusion contradicts with physical sense because of the following simple reason. Infinitely long asymptotic period of the runaway ($T \rightarrow \infty$) corresponds to the stationary mode of the reactor operation. As a consequence, the contribution from the second summand tends to zero, i.e. there is no any DN contribution into the FCR balance. But it is well-known that delayed neutrons mostly
contribute into the reactor reactivity just in the case of the stationary reactor operation mode, and the maximal DN contribution is equal to effective DN fraction \( \beta \). On the other hand, at rapid runaway, i.e. at very short asymptotic period \( (T \to 0) \), the so-called DN contribution tends to its maximal value, namely to effective DN fraction \( \beta \). But this conclusion contradicts with elementary logical consideration. Indeed, at rapid runaway, delayed neutrons have no enough time to contribute into the FCR balance. According to the laws of radioactive decay, DN generation is a time-stretched process. When the most DN fraction appears in the reactor core, the FCR already increases substantially total neutron population. So, DN fraction in total neutron population and DN contribution into FCR balance become negligible. Similar arguments are also correct for prompt fission neutrons.

Thus, it may be concluded that the two summands in the right part of the in-hour equation can define the value of REDUCTION in the contribution fraction (or the contribution deficit) into the reactor reactivity. The reactivity (the left part of the in-hour equation) determines the outside reactivity to be inserted in order to compensate the effects caused by postponed appearance of prompt and DN.

2. Analysis of the in-hour equation for fast and thermal reactors

By using the in-hour equation, dependencies of the reactor reactivity \( \rho \) on the asymptotic period of the runaway were calculated for the following two types of nuclear reactors (figure 1): fast lead-cooled reactor BREST (uranium-plutonium nitride fuel, neutron reflector made of natural lead, 50-cm thick) and thermal heavy-water reactor CANDU (uranium oxide fuel). Mean prompt neutron lifetimes were taken as 0.5 \( \mu \)s for BREST reactor and as 1 ms for CANDU reactor.

![Figure 1. Dependencies of the required reactivity \( \rho \) on the asymptotic period of runaway \( T \) for BREST-type and CANDU-type reactors.](image)

As is seen, both reactors behave themselves similarly at slow runaways \( (T \geq 100 \text{ s}) \). Very small reactivities \( (\rho \leq \beta) \) must be inserted to provide these runaways. It may be said that there are no significant barriers against slow enough runaways.

Rapider runaways \( (T \leq 10 \text{ s}) \) require inserting the larger reactivities \( (\rho \geq 0.5 \cdot \beta) \) because not all delayed neutrons are able to contribute into FCR balance in proper time. So, the missing DN fraction (the DN contribution deficit) must be compensated by insertion of appropriate outside reactivity.

If the runaway rate becomes more large \( (T \leq 1 \text{ s}) \), then different reactivities must be inserted into different reactor types. In fast BREST-type reactor the runaway requires inserting the reactivity slightly lower than DN fraction \( (\rho \leq \beta) \). In thermal CANDU-type reactor the runaway requires inserting slightly more reactivity \( (\rho \approx \beta) \).
This difference may be explained as follows. At relatively short asymptotic period of the runaways \((T \approx 1 \text{ s})\) a significant part of delayed neutrons is not able to contribute into the FCR balance in proper time. The DN contribution deficit appears in both reactor types. In fast reactor, prompt neutron lifetime \((0.5 \mu \text{s})\) is considerably shorter than the asymptotic runaway period, and all prompt neutrons have enough time to contribute into the FCR balance (the deficit of prompt neutron contribution is absent). It is sufficient to insert the outside reactivity only for compensation of the DN contribution deficit \((\rho \leq \beta)\). In thermal CANDU-type reactor with prompt neutron lifetime at the level of \(1 \text{ ms}\) not all prompt neutrons are able to contribute into the FCR balance in proper time. The deficit of prompt neutron contribution appears at the level of \(\approx 0.1 \cdot \beta\).

The difference between fast and thermal reactors becomes especially large at even more rapid runaways \((T = 0.1 \pm 0.001 \text{ s})\). Such the runaways in thermal CANDU-type reactor requires inserting very large outside reactivity to compensate significant deficits of contributions from prompt and delayed neutrons. In fast BREST-type reactor all prompt neutrons, as before, are able to contribute into the FCR balance in proper time. So, it is only sufficient to compensate the DN contribution deficit by small additional reactivity \((\rho = \beta + e, \ e \to 0)\). It may be concluded from these considerations that thermal CANDU-type reactor is more protected against very rapid runaways than fast BREST-type reactor because thermal reactor requires the larger outside reactivity.

The protective properties of fast BREST-type reactor become more evident only under extremely rapid runaways \((T < 0.001 \text{ s})\), when not all prompt neutrons are able to contribute into the FCR balance in proper time. These extremely rapid runaways can be already regarded as explosive events.

How can we enhance operational safety of fast reactors? As it follows from the in-hour equation, it is necessary to elongate mean prompt neutron lifetime. Neutron moderator can slow down the FCR, but in this case the neutron spectrum in the reactor becomes thermal. Way out of the situation is considered in the next section.

3. Fast reactor with weak neutron absorber as a neutron reflector

3.1. The in-hour equation

Mean prompt neutron lifetime can be elongated by using relatively intense neutron leakage from the reactor core. It is proposed to surround the reactor core by a low-absorbing neutron reflector where neutrons could stay as long as possible. Then, neutrons could come back to the reactor core and contribute into the FCR balance with a certain time delay, thus elongating mean lifetime of prompt neutrons. So, it is necessary to select such a reflecting material that allows neutrons to penetrate as deep as possible and then come back, with a high probability, to the reactor core. As is known, mean neutron displacement in the slowing-down process is equal to \(\sqrt{6 \cdot \tau}\), where \(\tau\) – age of neutrons, and mean displacement of thermal neutrons is equal to \(\sqrt{6 \cdot L}\), where \(L\) – length of diffusion. Material for neutron reflector must satisfy the following two conditions. Firstly, the neutron age must be long enough to provide the deepest penetration of the slowing-down neutrons into the neutron reflector (maximal value of \(\tau\)). Secondly, in order to provide high probability for the slowed neutrons to come back to the reactor core, the neutron diffusion length must be much larger than square root of the neutron age \((L \gg \sqrt{\tau})\). Till now, it was found no one suitable material to be used as a neutron reflector in fast reactors that would satisfy the second requirement, including project of fast reactor BREST with natural lead as a coolant and neutron reflector. The required material must be characterized by heavy atomic weight and weak neutron absorption. We propose to use lead enriched with isotope \(^{208}\text{Pb}\) as a quite suitable material. Mean neutron displacement during the slowing-down process in \(^{208}\text{Pb}\) is equal to \(~1.3\) m, mean displacement of thermal neutrons is equal to \(~8\) m. This means that fast leakage neutrons are able, in the process of their slowing-down, penetrate deep enough in the neutron reflector (up to \(~1.3\) m, in average), and then, with a high probability, to come back from deep layers of the neutron reflector into the reactor core. On the one hand, thanks to very low cross-
section of neutron absorption by $^{208}$Pb ($\sigma_e^{208} = 0.23 \text{ mb}$) [1], mean lifetime of thermal neutrons in infinite $^{208}$Pb is equal to 0.6 s (this value may be considered as a typical delay time of neutrons in the $^{208}$Pb-reflector). On the other hand, heavy atomic weight of $^{208}$Pb allows us to hope that neutrons coming back from the neutron reflector to the reactor core are mainly relatively high-energy neutrons, partially slow neutrons and thermal neutrons only as a minor fraction. So, the reactor could remain to be the fast spectrum reactor.

Those neutrons which are coming back from the neutron reflector are prompt neutrons in their physical essence, but they are delayed neutrons for the FCR balance because these neutrons can contribute into the FCR balance only after some time delay they spent on migration from the reactor core, slowing-down in the neutron reflector and backward diffusion to the reactor core. By analogy with delayed neutrons, the neutrons which are coming back from the neutron reflector may be divided into several groups each of them includes the neutrons coming from the $j$-th layer of the neutron reflector. Then, the following modified in-hour equation can be obtained with accounting for neutrons from the neutron reflector:

$$\rho = \frac{A_c}{T} + \sum_{j=1}^{\infty} \frac{A_j \cdot \beta_j}{T + A_j} + \sum_{j=1}^{\infty} \frac{A_j \cdot \rho_{j,R}}{T + A_j}$$

(2)

where: $J$ – the number of spatial layers in the neutron reflector, $A_j^R$ – lifetime of neutrons coming back from the $j$-th layer of the neutron reflector, this value is a sum of neutron lifetime in the reactor core before leakage into the neutron reflector, time needed to migrate from the reactor core to the $j$-th layer of the neutron reflector, time of staying in the $j$-th layer of the neutron reflector, time needed to migrate from the $j$-th layer of the neutron reflector to the reactor core and lifetime in the reactor core after coming back; $\rho_{j,R}^R$ – the reactivity gain caused by surrounding the reactor core with the $j$-th layer of the neutron reflector: $\rho_{j,R}^R = (K_{ef}^j - K_{ef}^{j-1}) \cdot (K_{ef}^{j-1} \cdot K_{ef}^{j-1})^{-1} \cdot K_{ef}^{j-1}$ – effective neutron multiplication factor in the reactor with $j$ layers of the neutron reflector. For brevity, this value may be called as the reflector reactivity.

It is noteworthy here that the first summand in the in-hour equation (1) includes prompt neutron lifetime $\Lambda_{pr}^R$ in the reactor as a whole. As distinct to equation (1), the first summand in the modified in-hour equation (2) includes neutron lifetime in the unreflected reactor core $\Lambda_c$.

If the number of spatial layers tends to infinity, then the discrete form of the in-hour equation (2) transforms into its continuous analog:

$$\rho = \frac{A_c}{T} + \sum_{j=1}^{\infty} \frac{A_j \cdot \beta_j}{T + A_j} + \frac{\int d\rho_{j,R}(r) \cdot A_j(r) \cdot dv}{T + A_j(r)}$$

(3)

where: $V$ – volume of the neutron reflector; $d\rho_{j,R}(r) / dv$ – spatial distribution of the specific reactivity gain caused by the neutron reflector, $A_j(r)$ – spatial lifetime distribution for neutrons from the neutron reflector.

3.2. Lifetime of neutrons from the neutron reflector

Lifetime of neutrons from the neutron reflector can be calculated by using the balance relationship presented in [2]. For the discrete model of the neutron reflector the balance relationship can be written in the following form: $A_{pr}^j = (1 - \rho_{j,R}^R) \cdot A_{pr}^{j-1} + \rho_{j,R}^R \cdot A_{g}^j$, where $A_{pr}^j$ – prompt neutron lifetime in the reactor with $j$ layers of the neutron reflector, if $j = 0$, then $A_{pr}^0 = A_g^j$ – neutron lifetime in the unreflected reactor core. For the continuous model of the neutron reflector the balance relationship can be rewritten as follows: $A_g(r) = A_{pr}(r) + A_{pr}(r) / \rho_{j,R}^R$.

Some dependencies of relevant physical parameters on thickness of $^{208}$Pb-reflector are presented in figure 2 for fast BREST-type reactor. These dependencies were calculated within the frames of spherical continuous model of neutron kinetics. It can be seen that the reactivity gain thanks to the reflector $\rho_{j,R}^R$ grows as the reflector becomes thicker and approaches a saturation level.
Correspondingly, derivative of the reactivity gain decreases and vanishes. Derivative of the reactivity gain \( \frac{d\rho_R(r)}{dr} \) defines the growth rate of the contribution given by neutrons from the neutron reflector to the FCR balance (analog of the DN fraction) as the neutron reflector becomes thicker. So, it is natural that derivative of the reactivity gain decreases as the reflector thickness increases because contribution of neutrons coming back from more distant reflector layers into fission chain reaction in the reactor core vanishes.

![Figure 2](image_url)

**Figure 2.** Characteristics of fast BREST-type reactor depending on thickness of \(^{208}\text{Pb}\)-reflector.

Those neutrons which are coming back to the reactor core from the deeper layers of the neutron reflector are characterized by the longer lifetime. But the number of these neutrons gradually decreases and, as a result, their contribution into the FCR balance decreases too. The product of two values \( \rho_R(r) \cdot \Lambda_r \), denominator in the third summand of equation (3), is shown in figure 2. The product sharply increases within the range of small thickness, approaches the plateau level and then begins decreasing at large thickness of the neutron reflector. This means that there is a certain effective thickness which should not be exceeded. As is seen from figure 2, total contribution of neutrons from the thick (4 m) neutron reflector into the reactor reactivity reaches a significant value (about \( 35 \cdot \beta \)), and their lifetimes cover the range from 0.4 \( \mu \)s to 0.1 s. The reflector with so long neutron lifetime may be regarded as a physically thick medium in contrast to the reflectors with relatively short neutron lifetimes.

### 3.3. Analysis of the in-hour equation

The spherical continuous model of neutron kinetics was used to determine dependencies of the reactor reactivity and its components, namely the components related with prompt neutrons (Prompt), delayed neutrons (DN), their sum (Prompt + DN) and with neutrons from the reflector, on the asymptotic period of the runaway \( T \). These dependencies are presented in figure 3 for fast BREST-type reactor with thick (4 m) \(^{208}\text{Pb}\)-reflector.

Since lifetimes of neutrons from the reflector (from 1 \( \mu \)s to 0.1 s) are between lifetimes of prompt and delayed neutrons, under relatively slow runaways (relatively long asymptotic period \( T \geq 1 \) s), large fraction of neutrons has enough time to come back from the neutron reflector to the reactor core and contribute into the FCR balance. Therefore, neutrons from the reflector can produce only insignificant effect on the outside reactivity to be inserted into the reactor core (see figure 3). So, the only protective barrier against such runaways in fast reactors is the barrier related with delayed neutrons. However, under rapid runaways (\( T = 1 + 0.001 \) s) the situation changes drastically. Within the range of so rapid and the most dangerous runaways the protective barrier caused by DN effects has exhausted completely its capability because delayed neutrons are not able now to contribute into the
FCR balance in proper time. The outside reactivity compensates the DN contribution deficit. Prompt fission neutrons can introduce their full contribution into the FCR balance. The protective barrier related with prompt neutrons is not working yet. Just within this range of the runaways a very small outside reactivity (much lower than DN fraction) inserted into fast BREST-type reactor without $^{208}\text{Pb}$-reflector can accelerate the runaway by three orders of magnitude (the asymptotic period reduces from 1 s to 0.001 s, see sum of prompt and delayed neutrons in figure 3). But in this case a favorable effect produced by neutrons from the reflector on the reactor safety becomes definitive. Under so rapid runaways, not all neutrons have enough time to come back from the neutron reflector, and it becomes necessary to compensate their deficit. For example, it is sufficient to insert roughly the same outside reactivity (about DN fraction $\beta$) for initiation of the runaway in fast BREST-type reactor without $^{208}\text{Pb}$-reflector with the asymptotic periods of 0.1 s, 0.01 s and 0.001 s. Initiation of the same runaways in the reactor core surrounded by $^{208}\text{Pb}$-reflector requires inserting substantially larger values of the outside reactivity ($\sim 2\beta$, $\sim 4\beta$ and $\sim 7\beta$, respectively). The most fraction of these reactivities ($\sim \beta, \sim 3\beta$ and $\sim 6\beta$, respectively) is needed to compensate the contribution deficit related with neutrons from the reflector, Only about one $\beta$ is needed to compensate the DN contribution deficit. If the asymptotic period of the runaway decreases, then the protective barrier related with neutrons from the reflector becomes stronger at the degree as the protective barrier related with prompt neutrons. As a result, the reactor resistibility to the runaways becomes substantially stronger too.

![Figure 3. Reactivity and its components as functions of the asymptotic runaway period for fast BREST-type reactor with thick (4 m) $^{208}\text{Pb}$-reflector.](image)

Fuel rods with time constant about 0.1 s [3, 4] can provide additional protective effect even under so rapid runaways. In this case, heat will be efficiently removed by coolant from fuel rods, and negative reactivity feedback on the coolant temperature can counteract with the runaway.

4. Devaluation of DN fraction $\beta$ as a maximal permissible reactivity for the reactor safety

Let us consider fast BREST-type reactor with $^{208}\text{Pb}$-reflector and thermal CANDU-type reactor with prompt neutron lifetime at the level of 1 ms. Both reactors use such fuel rods for which the minimal asymptotic runaway period of 0.1 s is an acceptable value. If total reactivity margin in these reactors is below $2\beta$, then the runaways with the asymptotic periods shorter than 0.1 s are impossible under any reactivity-induced accidents (see figures 1 and 3). One half of the required reactivity is needed to compensate the DN contribution deficit while the other half is needed to compensate the contribution deficit related with neutrons from the reflector (for fast BREST-type reactor with $^{208}\text{Pb}$-reflector) or with prompt neutrons (for thermal CANDU-type reactor).
Thus, the generally accepted threshold of the maximal permissible reactivity for safe reactor operation, namely effective fraction of delayed neutrons, can be exceeded, and the reactor operation will remain to be safe.

5. Conclusions

As is well-known, the FCR resistibility to rapid runaways in thermal reactors is substantially stronger than that in fast reactors with traditional neutron reflectors. However, a special neutron reflector can upgrade the FCR resistibility in fast reactors up to the level of the FCR resistibility in thermal reactors. This advantage of thermal reactors does not become apparent during their routine operation. The same situation takes place in fast reactors with a special neutron reflector because both reactor types use fuel rods with relative long time constant (~1-3 s). Under rapid runaways with the asymptotic periods shorter than time constant of these fuel rods the released heat has no enough time to flow down into coolant, and fuel rods can be melted. If fuel rods with relatively short time constants (0.01÷0.1 s) [3-4] are used in thermal and fast reactors with sufficiently long mean prompt neutron lifetime, then an opportunity arises to create conditions for timely actuation of various stabilization feedbacks even under the reactivity jumps above DN fraction β [5]. So, strong FCR resistibility to rapid runaways in fast reactors could be practically used in their routine operation. In particular, fast lead-cooled reactor BREST with a special neutron reflector, i.e. with strong FCR resistibility to rapid runaways, could be regarded as a reactor with inherent safety only if the rapidest acceptable runaway would require inserting the reactivity that would exceed full explicit (and implicit) reactivity margins stored in the reactor. **This property may be considered as an additional protective barrier in the reactor safety system that, as is known, includes fuel meat, cladding of fuel rod, and so on.**

The following main conclusions can be made from the obtained results:

1. New interpretation was proposed for the terms of the in-hour equation. The summands in the right part should be considered as the contribution deficits into the FCR balance from prompt and delayed neutrons, respectively. The left part of the in-hour equation is the reactivity needed to compensate the contribution deficits caused by time delays of prompt and delayed neutrons and to provide the runaway with given asymptotic period.
2. The FCR resistibility to rapid runaways in fast reactors can be significantly strengthened by surrounding the reactor core with the neutron reflector made of weak neutron absorber with heavy atomic weight.
3. The modified in-hour equation with accounting for neutrons from the reflector was derived and analyzed.
4. Neutrons from the reflector can play a significant role in upgrading the FCR resistibility to rapid runaways.

References

[1] Shibata K et al 2011 JENDL-4.0: a new library for nuclear science and engineering Nuclear Science and Technology 48 1
[2] Kulikov G G, Shmelev A N and Apse V A 2014 Improving nuclear safety of fast reactors by slowing down fission chain reaction Int. J. Hindawi Publication Corporation Nuclear Energy 2014 373726
[3] Ponomarev-Stepnoj N N et al 1999 Atomic Energy 86 6 pp 443–449
[4] Mallone J, Totenmeier A, Shapiro N and Vaidyanathan S 2012 Lightbridge’s advanced metallic fuel for light water reactors Nuclear Technology 180 pp 437–442
[5] Shmelev A N, Kulikov G G, Apse VA, Ternovoykh M Yu, Chubko N V and Kozhahmet B K 2014 Nuclear Physics and Engineering (Moscow: NRNU MEPhI Press) 5 7–8 pp 578–592

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