Learning how to teach experiments in the school physics laboratory

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Abstract. The focus of this research is in the broader area of physics teacher education. The aim is to investigate physics teachers’ efforts to learn how to teach and assess experiments, and in particular, concepts related to scientific evidence in the school physics laboratory. The study has looked at the participants as both learners and prospective teachers. It has taken place within the context of the course “Secondary Science Lab Applications” within a pre-service teacher education program in a Department of Secondary Science and Mathematics Education. Twenty-four students participated. The participants: a) revised the main concepts related to scientific practices (in particular, experimental validity and reliability of measurement), b) developed lesson plans, teaching and assessment methods and, c) taught school physics experiments. Interviews were conducted with the participants during their preparation for teaching and after teaching. They were observed when teaching and all classes were videotaped. A qualitative and quantitative data analysis identified particular trends among the participants. Students’ difficulties while they designed, carried out experiments and wrote lab reports were identified, as well as the difficulties they experienced when teaching experiments.

1. Introduction

Physics is not only theory, concepts, laws and formulas. It is also an experimental science. Laboratory work is at the heart of physics. In Turkey, although the National Curriculum includes experiments, physics classes often do not include experimentation. Requirements for high-stake exams restrict teachers’ instruction time available to teach lab work.

In addition, in pre-service teacher education, major emphasis is given to conceptual understanding of physics and concepts across the curriculum. In our department, an initiative was taken to prepare our student teachers to teach experiments effectively in the school physics laboratory: to address the knowledge and understandings required to teach scientific skills and methods explicitly. The goal has been to elicit pre-service teachers’ understandings of scientific evidence and, through targeted instruction, to support them to revise such understandings and learn how to write laboratory reports.

2. Research into secondary students’ difficulties with laboratory work

In physics education, many secondary students experience difficulties in the laboratory. The literature provides a range of difficulties that secondary pupils encounter in the school science laboratory [1–5]. Many research studies support the lack of a fundamental understanding of scientific evidence in secondary students. For example, studies on performance on lab work questioned the assumption that
students understand the nature of measurement and experimentation [6,7]. This work has led to the
development and refinement of various laboratory teaching materials [8]. After completing a traditional
laboratory course, the majority of students have ideas about measurement that are inconsistent with the
generally accepted scientific model. For example, as Séré and her team found, a large proportion of
students view the ideal outcome of a single measurement as an “exact” or “point-like” value. Only if a
measurement is considered really “bad” would it be reported in terms of an interval [6].

Boudreaux et al investigated the ability of students to reason on the basis of the control of variables
[9]. The participants were undergraduate physics students and they were asked to decide whether or not
a given variable influences the behaviour of a system. They found out that although most of the students
recognized the need to control variables, many had significant difficulty with the underlying reasoning.
It was argued that teachers should be prepared for how to teach experiments and develop science
practices in secondary school students.

Yet, little is known about pre-service teachers’ understanding of scientific evidence. In physics
departments it is often assumed that the participation of physics students in regular undergraduate
science laboratory courses provides them with the required knowledge and skills to teach in a school
laboratory [10]. This assumption stands in parallel with the belief that the more rigid and difficult
modules one attends in undergraduate programs, the better and well-prepared the physics teacher will be [11]. There is a need to develop ‘special’ courses for physics teachers’ preparation since teachers
need to have deeper knowledge than their students.

The aim of this research study was to investigate physics teachers’ efforts to learn how to teach and
assess experiments, and in particular, concepts related to scientific evidence in the school physics
laboratory. The study looked at the participants as both learners [12,13] and prospective teachers [14,15].

3. Context of the research study and research questions
In Turkey, the secondary science curriculum gives much attention to conceptual knowledge, leaving
little space for teaching of experiments. Although the National Curriculum includes experiments,
physics classes often do not include experimentation. Requirements for high-stake exams restrict
teachers’ instruction time available to teach lab work. In our department, an initiative was taken to
prepare our student teachers for how to teach experiments effectively in the school physics laboratory:
to address the knowledge and understandings required to teach scientific skills and methods explicitly.
The goal has been to develop a good understanding of what makes ‘good scientific evidence’ firstly, in
teachers and secondly, to their secondary school students. Two research questions have guided the study:

1) What are the understandings of scientific evidence that pre-service physics teachers demonstrate
when they conduct experiments?
2) What are the difficulties and challenges that they experience when they teach and assess
laboratory work?

The study has taken place within the context of the course “Secondary Science Lab Applications”
within a pre-service teacher education program in a Department of Secondary Science and Mathematics
Education. Twenty-four pre-service physics teachers participated for one academic year. The
participants firstly revised the main concepts related to scientific practices (experimental validity,
planning and design of experiments, identification of variables, controlled experiment, measurement
and uncertainty, identification of errors and sources of errors, reliability of measurement) taught in
physics lab classes. Secondly, they carried out school experiments (i.e., Hooke’s law, simple pendulum
motion, insulation experiment, experiments with simple electrical circuits, refraction and reflection,
friction, motion on an inclined plane and electromagnetic induction). Then, they learned about effective
teaching and assessment methods (criteria and feedback) in the lab, informed by research. Finally, they
taught a 40-minute lesson including an experiment to their peers.
4. Research methodology
Individual interviews based on tasks specifically designed to look at ideas related to scientific evidence were conducted. Students’ difficulties while they design, carry out experiments and write lab reports have been identified. All experiments and discussion in the lab were videotaped. Mid-term and final exam items are other important sources of data to answer the first research question.

In the second part of the semester, the participants have the opportunity to prepare lesson plans and teach one lesson to their peers. Lessons were videotaped and interviews (during their preparation and after their teaching) were conducted to identify prospective teachers’ difficulties. Their lesson plans and the tasks they developed for their teaching and assessment were collected to be analysed and answer the second research question.

The participants were given samples of actual secondary school students’ work and they were asked to give written feedback to them. Research studies give much importance on teachers’ ability to give constructive feedback so that learners understand what they need to do to improve [16,17]. Teachers should be skillful in assessing lab skills and, more generally, students’ performance in the laboratory [18–21]. In addition, a research study by Waren Little et al reported on the benefits for teachers’ learning and professional development when teachers look at actual student work and think about students’ performance [22]. Also, Crespo found evidence that by examining students’ work, teachers have the opportunity to learn about students’ thinking and practices [23].

4.1. Data collection and analysis
The reliability of the analysis is based on the triangulation of the methods employed. Qualitative data analysis [24] was conducted to identify particular trends among the participants. We generated initial categories from interviews, observation transcripts and exam item responses of each teacher. We constantly compared new data from the interviews, and from the observations and exam items (mid-term and final) with the current categories, and refined them. When clarification was needed, we collected more data by conducting focused conversations with the teachers. The data were analyzed by comparing the responses for each question both across and the interviewees and through each interview in order to identify key categories and features among teachers and possible changes within the same teacher. In the next phase, analysis results of the individuals were compared to identify common themes. Triangulation was also applied by comparing and discussing the interpretations by the main researcher and the two project assistants.

5. Results

5.1. Student understanding of the fair test
Students had difficulties in planning a fair test or a controlled experiment. One of the difficulties was that they could not identify all relevant variables involved and then decide which variables should be independent, constant, or dependent in their design. The ability to control some variables to design a fair test and related understandings was documented by asking the participants to design a controlled experiment and by looking at their reports when experiments were performed. Most importantly, participants were presented with tasks and asked to evaluate other experimenters’ conclusions. Such tasks were used as research instruments. For example, the following task asks students to evaluate conclusions by previous experimenters who performed an experiment. Notice that students were asked to interpret conclusions based on the design of a controlled experiment. Thus, they were not explicitly asked to examine whether the experiment is a fair test; a basic requirement for the evaluation.
Task 1: Fair Test

| Constantan Wire Number | Average Diameter, D_{av}/mm | Cross-sectional area, a/mm² | Current, I/mA | Voltage, V/V | Resistance R/Ω |
|-------------------------|----------------------------|-----------------------------|---------------|--------------|---------------|
|                         |                            |                             | ΔI = ± 2%      | ΔV = ± 2%    | ΔR = ± 4%     |
| 1                       | 0.40                       | 0.13                        | 175.0         | 0.69         | 0.40          |
| 2                       | 0.36                       | 0.10                        | 164.2         | 0.72         | 0.44          |
| 3                       | 0.30                       | 0.07                        | 140.6         | 0.78         | 0.54          |
| 4                       | 0.26                       | 0.05                        | 127.1         | 0.81         | 0.67          |
| 5                       | 0.22                       | 0.04                        | 121.3         | 0.93         | 0.76          |
| 6                       | 0.17                       | 0.02                        | 104.5         | 1.10         | 1.09          |

Ali and Selin wanted to find out the relationship between the resistance and the cross-sectional area of a constantan wire. They performed a series of measurements for six pieces of constantan wire with different cross-sectional areas, and reported these in the table above.

a) What are the variables involved in this experiment?

b) Based on their data, Ali and Selin concluded that the resistance is inversely proportional to the cross-section. Do you agree?

c) What is the total percentage error involved in the measurement of the resistance?

d) How can this experiment be improved?

The respondents gave various answers along with different explanations. The majority of students agree with the conclusion (of Ali and Selin), without examining whether the experiment is controlled or a fair test.

Many students did not mention the idea of the fair test and they did not consider whether the design is that of a fair test along with variables, which are involved in the experiment. In fact, they do not mention that there is no information given about length of the wire, temperature and material type. Thus, most of them replied: “I trust the data, because their conclusion is correct; according to the theory, according to what I know”. Similarly, another student wrote: “I agree with their conclusion, because their conclusion matches with the theory”; “I agree with this conclusion, because I learned it from theory”. Those students do not look at the data and the procedure followed to examine whether the experiment is a controlled one. For improvement, they only recommend that they need to repeat without explaining how repetition of measurements improve the results.

When students are asked to evaluate a claim that is consistent with what they have been taught (theory), they agree without examining the design of the test and in fact, the validity of the experiment (whether it is actually an adequate fair test). Without examining whether the experiment is a controlled experiment, they report that they trust the data. Thus, although one student mentioned that more variables (as length) should be included, she did not examine whether it is a fair test or not, because their conclusion is consistent with theory. “I agree with this claim since I know it from the theory. If someone does not know the theory, he cannot conclude this from data set because it is not a fair test.” “It is not a fair test, but still I agree with the conclusion, because I know from theory”.

Only a few students reported that they do not agree with the conclusion because there is missing information about the length of the wire and the temperature. Such information is crucial when one makes judgements about whether the experiment is a controlled experiment: “We do not have
information about the variable of length. We do not know that it is a fair test. We need information about length and that length is constant (so that it is a controlled experiment)”. “We cannot conclude this because the experiment is not a fair test, they did not report on the length of the wires, which is also an independent variable to determine its resistance …”; “This is not a fair test and it should be a fair test in order to be able to make conclusions”; “We need to have a fair test. Length also needs to be considered”; or, better, “This experiment is not valid because information about length of the wire is missing from our data table. Without it, as a variable, we cannot conclude with any statement like this, since length could be different, and this would affect the conclusion. I do not agree with the two students”. On this basis they suggest that the experiment needs to be controlled.

For the twenty-four students the results are shown in Table 1. These categories are not mutually exclusive.

| Table 1: Student understanding of fair test in the first two weeks of the semester. |
|---------------------------------|------------------|
| categories                        | no of students |
| no mention of the concept of fair test | 22              |
| no mention of all the variables involved (length of wires and temperature) | 19              |
| “We know from theory that the resistance is inversely proportional to cross-sectional area” | 23              |
| This is not a fair test. They should make it a fair test. | 2               |

Student understanding of fair test improved only after targeted instruction and work on more tasks with secondhand data asking them to evaluate the procedure and experimental evidence. In later tasks, they think about the controlled experiment and answer that they need more information about some variables so that they know whether the experiment is a controlled one or not.

5.2. Student difficulties in designing a controlled experiment
Our students demonstrated difficulty in recognizing the variables that contribute to an experimental result. For example, this is the case when they are asked about the variables that are involved in the insulation experiment.

Task 2: The insulation experiment
You, as a physics student, are investigating how the temperature of hot water drops in three similar cans each wrapped with three different insulating materials.

1. What variables do you think affect the temperature of hot water in a can wrapped with an insulating material?
2. How does each variable affect the temperature of hot water while it is cooling down and its rate of change?

I would like you to design an experiment that would allow you to decide which of the three insulating materials is best. After you are finished, you need to describe how you plan your experiment, obtain evidence, analyze and explain your data, as well as how you evaluate your experiment.

Student difficulties in designing the fair test are recalled in their lab reports, too. For example, they did not include all the variables involved and did not control all the variables in the insulation experiment. This is due to the fact that when performing experiments in lab classes in the physics department, students are always provided with guidelines and instructions on how to proceed. In the insulation experiment, they need to include the following variables: amount of water, temperature drop/time, starting temperature, material of the container, room temperature, number of layers, and thickness of insulation layer.
5.3. The participants giving feedback to laboratory reports as prospective teachers

The participants were asked to give feedback to secondary student lab reports. The reports were actual reports taken from secondary schools.

**Task 3: You, giving feedback as a teacher**

Give your written feedback to a secondary school student (grade 11) who submitted the attached laboratory report (the report is attached).

In giving feedback, the participants would give information about what needs to get improved and how:

- “You need to draw a graph and use it”.
- “You need to collect better data / results so that you draw a better graph and be able to look at the pattern”; “Theory is missing”, or, “You need to improve your theory”.
- “You need to describe your method. What are the variables involved in your experiment?”
- “Nice description of the procedure” “The fair test is OK, but more measurements are needed”; “The data table is good enough”; “Why this number of measurements?”
- “Variables (dependent and independent) are missing”; “Points are plotted properly”.
- “You need to draw a graph”; “What does the graph tell you about the whole pattern?”
- “Analysis is missing. What are the different rates of cooling?”
- “You need to write a statement about the relationship between the two variables”; “You need to give the graph a title”.

Giving feedback helps pre-service teachers develop a good understanding of the quality of a lab report and what is included in each section. Also, the process helps them make the transition from undergraduate student to practicing teacher. They reported that they enjoy grading and giving feedback. It is a process through which they learn how to improve the quality of their own lab reports.

The assessment criteria have been developed during the semester. Through practice and by time, they can give detailed feedback.
5.4. Our students as prospective teachers: difficulties in preparing and planning to teach

Preparing to teach in the laboratory presents a great challenge. Our students are not confident to teach laboratory skills and their plans are weak. The difficulty to prepare a lesson plan with learning goals related to the development of particular experimental skills is clear. Tasks like the following were distributed in the class, as homework and in the exam papers. Such tasks were developed in order to develop teaching methods and assessment methods closely related to experimental skills.

Task 4: You, as a teacher

Prepare a lesson plan to teach your students how to plan (planning part) and take measurements (obtaining evidence) in the insulation experiment.

Task 5: You, as a teacher

Write a set of rubrics for assessment of the design an experiment and for making a prediction of the outcome of the experiment.

Task 6: You, as a physics teacher

You need to write a set of rubrics for assessment of the “analysis and explanation of experimental results” in an experiment and the lab report. The set of rubrics should help teachers in their teaching and in the assessment of the “analysis and explanation” part. In addition, secondary students will use the set of rubrics in peer- and self-assessment of analysis and explanation.

When working on the insulation task (Task 2), the participants talked only about the theory of insulation, heat and temperature, heat capacity and Newton’s law of cooling. They also sketched the graph of the cooling down of hot water. Similarly, while they were preparing to teach Ohm’s law experiment, students wrote that the topic is about Ohm’s law (not about the teaching of experimental skills). And the lesson objective is: “By the end of the lesson, students will have understood the relation between voltage and current and the associated Ohm’s law”. Similarly, “Students will be able to identify and determine the difference between heat and temperature”. And, for the insulation and the insulation experiment: “Students will learn what an insulator is”; “Understand how insulators work”. They confuse the context (theory) with the experimental/laboratory skills they want to teach and develop in their secondary students. Instead, we wanted them to talk about the preliminary experiment and the class discussion, in which, students talk in groups about planning, variables and the planning of a controlled experiment. Similarly, in their lesson plans they describe the different parts of an experiment, when they need to write what the teacher and the students will do (teaching methods and strategies) in the laboratory or in the classroom.

Only a few and senior students (not more than 3 from the total 24) answered that they will teach the theory of the experiment (Newton’s cooling law) in the introductory lesson. They also said that they are going to review each part of the experiment and what to include in the lab report. “Use instruments to make measurements of temperature”; “Carry out a controlled experiment”. Other teaching goals and student/learning objectives are: “Students will be able to draw a best fit line/ best fit curve”.

Most students had difficulties in preparing a lesson plan to teach students how to plan and design a fair test or, in preparing a lesson to help them improve the analysis of evidence. Through practice they wrote:

- “Teach to judge the range of measurements they need to take, when and why these need to be repeated, and how to deal with anomalous or discrepant results”
- “Teach students to examine evidence for validity and reliability by considering questions of accuracy, error and discrepancy”
- “Teach students how to draw the best fit line or best fit curve”
- “Teach how to use the graph in the analysis of results”
- “How to design a fair test”; “How to collect and record data”
• “How to analyze data”; “How to make a detailed analysis of results by using the plotted graph and make the calculation of slopes”.

Students experienced difficulties in writing learning goals and objectives and teaching goals related to the development of lab skills. This is mainly because in all education and physics education classes, they have had training and practice in developing lessons plans to teach only concepts of physics (theory). At the end of the semester were they are able to articulate and write some teaching goals:

- “Students learn how to plan the insulation experiment”
- “How to take measurements while performing the experiment”
- “In the preliminary experiment, they will make decisions about...”
- “Students to be able to plan the insulation experiment, to identify the variables involved, to carry out the preliminary experiment and how to design a controlled experiment”

They also developed assessment methods; they wrote assessment goals, were introduced to performance assessment, and developed assessment criteria for feedback and rubrics for grading. When it comes them to prepare a lesson to address specific weaknesses (as demonstrated in a lab report), they do not know how to match the learning goals with laboratory activities. One senior student with considerable teaching experience in internship schools wrote in a reflection journal: “Planning a lesson was difficult for me. I needed some guidance to design and plan my lesson. I keep the theory simple, so that I teach them laboratory skills. I learned about progression in teaching (teaching sequences)”.

One of the important issues in their teaching is that pre-service teachers do not seem to understand the common components of controlled experiments. There is also an interplay between theory (the context of the experiment) and the development of lab skills, which makes them attach more importance to the underlying theory than to the development of experimental skills, when asked to evaluate experiments.

Pre-service teachers have difficulties in connecting what they learn with teaching practice. Our participants were not successful in applying what they learned in the course to prepare their lesson plans and do the actual teaching. They experienced particular difficulties when preparing lessons plans for teaching in the lab, when writing lesson objectives and so on. Only a few of from the total of twenty-four students had prepared adequate lesson plans.

6. Discussion of results and conclusions

This study has provided strong evidence about the difficulties and needs of our participants. The aim was to document pre-service physics teachers’ difficulties in understanding scientific evidence and then, difficulties in teaching and assessing lab skills and experiments in the school laboratory. Tasks were developed to elicit such understandings and challenges. Conversely, the findings guided the development of tasks and teaching materials to explicitly address such deficiencies. In addition, the study confirmed research results from previous studies, but also elicited new findings.

Firstly, the results show that teachers have a limited understanding of experimental design and concepts of validity and reliability. Similar to findings from two studies [6, 25] the participant students have a limited understanding of the uncertainty that is inherent to each measurement. There is strong belief in making one measurement carefully [1]. The participants struggled with many aspects of scientific evidence and with writing laboratory reports. Through discussion in the interviews and feedback by the instructor, students improve the versions of lab reports and submit lab reports of better quality for subsequent experiments. The structure of the reports and what is included in each section improves across subsequent lab experiments for each participant.

Secondly, the results indicate serious shortcomings in the preparation of future physics teachers. The study revealed a lack of knowledge related to experimental skills. Pre-service teachers have difficulties in connecting what they learn with teaching practice. Our participants were not successful in applying what they learned in the course to prepare their lesson plans and do the actual teaching. They experienced particular difficulties when preparing lessons plans for teaching in the lab, when writing lesson
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