CP Violation in $B$ Meson Decays

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Recent CP asymmetry measurements in tree-dominated processes, $B^0 \to \pi^+\pi^-, \rho^+\rho^-, \rho^\pm\pi^\mp, B^+ \to DK^+$, and in penguin-dominated decays, $B \to \pi^0K_S, \eta'K_S, \phi K_S$, are interpreted in the framework of the Kobayashi-Maskawa (KM) mechanism of CP violation. The KM phase emerges as the dominant source of CP violation in tree-dominated decays, which are beginning to constrain the unitarity triangle beyond other constraints. Improving precision of CP asymmetry measurements in penguin-dominated decays may indicate the need for new physics.

1. INTRODUCTION

Measurements of time-dependent CP asymmetries in $b \to c\bar{s}s$ decays including $B^0 \to J/\psi K_S$, carried out in the past few years at the two $e^+e^-$ B factories at SLAC and KEK [1], are interpreted in the Standard Model as $\sin 2\beta \sin \Delta m t$, where $\beta \equiv \arg(-V_{ub}V_{td}^*V_{cb})$. These measurements were proposed in [2] to provide a first test of the Kobayashi-Maskawa mechanism of CP violation [3] in the $B$ meson system. This test is theoretically clean because a single weak phase dominates $B \to J/\psi K_S$ within a fraction of a percent [4,5]. These measurements have not only passed successfully the Standard Model test; they also improve our knowledge of the Cabibbo-Kobayashi-Maskawa (CKM) matrix parametrized in terms of the unitarity triangle. (See Fig. 1 [6].) A recent time and angular analysis of $B^0 \to J/\psi K^{*0}$ [7] seems to resolve the plotted ambiguity, $\beta \to \pi/2 - \beta$.

CP asymmetries in $B$ decays are often related to the three angles of the unitarity triangle, for which currently allowed ranges are, at 95% confidence level (CL) [6]:

$$78^\circ \leq \alpha \leq 122^\circ, \hspace{0.5cm} 21^\circ \leq \beta \leq 27^\circ, \hspace{0.5cm} 38^\circ \leq \gamma \leq 80^\circ$$

(1)

The essence of measuring CP asymmetries in a variety of $B$ and $B_s$ decays is first to see whether the measured asymmetries are consistent with (1). Then one would like to restrict these ranges further, hoping to eventually observe deviations from Standard Model expectations. A class of processes, susceptible to a possible early observation of new physics, mimicking loop effects, consists of $b \to s$ penguin-dominated $B^0$ decays into CP-eigenstates [8], where CP asymmetries may deviate from $\sin 2\beta$.

While the phase $2\beta$ characterizes CP violation in the interference between $B^0 - \bar{B}^0$ mixing and a $b \to c\bar{s}s$ decay amplitude or a $b \to s$ penguin-dominated amplitude, the phase $\gamma \equiv \arg(-V_{ub}V_{td}^*V_{cd}V_{cb})$ is responsible for direct CP violation. The phase $\alpha \equiv \pi - \beta - \gamma$ occurs when

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{CKM constraints [6].}
\end{figure}
$B^0 - \bar{B}^0$ mixing interferes with $b \to u d$, where an additional $b \to d$ penguin amplitude implies also direct CP violation. Since a direct CP asymmetry involves a ratio of two hadronic amplitudes and their relative strong phase, both of which are not reliably calculable, one faces hadronic uncertainties whenever one is relating CP asymmetries to $\gamma$ or $\alpha$.

A model-independent way of resolving or at least reducing these uncertainties is by flavor symmetries, isospin or broken SU(3), relating certain processes to others. SU(3) breaking corrections are introduced using QCD factorization results proven in a framework of a Soft Collinear Effective Theory [9].

An impressive progress is being reported at this conference in measurements of CP asymmetries [10][11][12]. The purpose of my talk is to study theoretical implications of these measurements. In Section 2.1 I discuss tree dominated decays, including $B^0(t) \to \pi^+\pi^-$, $\rho^+\rho^-, \rho^+\pi^-\pi^+$. Section 3 studies pure tree decays $B^\pm \to DK^\pm$, while Section 4 focuses on penguin-dominated processes, $B^0(t) \to \pi^0K_S, \eta'K_S, \phi K_S$. Section 5 concludes.

2. TREE DOMINATING DECAYS

2.1. $B^0 \to \pi^+\pi^-$ and $B \to \rho^+\rho^-$

The amplitude for $B^0 \to \pi^+\pi^-$ contains two terms [15], conventionally denoted “tree” and “penguin” amplitudes, $T$ and $P$, involving a weak phase $\gamma$ and a strong phase $\delta$:

$$A(B^0 \to \pi^+\pi^-) = |T|e^{i\gamma} + |P|e^{i\delta}. \quad (2)$$

The time-dependent decay rates, for an initial $B^0$ or a $\bar{B}^0$, are given by [4]

$$\Gamma(B^0(t)/\bar{B}^0(t) \to \pi^+\pi^-) \propto e^{-\Gamma t} [1 \pm C_{\pi\pi} \cos(\Delta t) \mp S_{\pi\pi} \sin(\Delta t)]. \quad (3)$$

where

$$S_{\pi\pi} = \frac{2Im(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2} = \sqrt{1 - C_{\pi\pi}^2} \sin 2\alpha_{\text{eff}}, \quad (4)$$

$$C_{\pi\pi} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \quad (5)$$

$$\lambda_{\pi\pi} = e^{-2i\beta} \frac{A(B^0 \to \pi^+\pi^-)}{A(B^0 \to \pi^+\pi^-)}. \quad (6)$$

Recent measurements by the BaBar [13] and Belle collaborations, which Belle used to announce evidence for CP violation, are

BaBar \quad $C_{\pi\pi} = -0.19 \pm 0.19 \pm 0.05, \quad S_{\pi\pi} = -0.40 \pm 0.22 \pm 0.03$. \quad (7)

These values imply averages,

$$C_{\pi\pi} = -0.46 \pm 0.13, \quad S_{\pi\pi} = -0.74 \pm 0.16, \quad (7)$$

which also determine a range for $\sin 2\alpha_{\text{eff}}$.

2.1.1 Isospin symmetry in $B \to \pi\pi$

Since the two measurable, $S_{\pi\pi}$ and $C_{\pi\pi}$, depend on $r = |P/T|, \delta, \gamma$ (and $\beta$ which may be assumed to be given), one needs further information to study the weak phase. This information is provided by isospin symmetry [15], which distinguishes between tree and penguin amplitudes, also relating a tiny electroweak penguin term to the tree amplitude [16]. One forms an isospin triangle for $B$ decays and a similar one for $\bar{B}$ decays,

$$A(\pi^+\pi^-) + \sqrt{2}A(\pi^0\pi^0) = \sqrt{2}A(\pi^+\pi^0). \quad (8)$$

A mismatch angle between the two triangles determines $2(\alpha_{\text{eff}} - \alpha)$, which then fixes $\alpha$ from $\sin 2\alpha_{\text{eff}}$ up to a discrete ambiguity.

As long as separate $B^0$ and $\bar{B}^0$ decays into $\pi^0\pi^0$ have not yet been measured, one may try to use the three measured charge averaged decay rates for bounds on $\alpha_{\text{eff}} - \alpha$ [17]. Current branching ratios of the three processes in [8] are, in units of $10^{-6}$ [18], $4.6 \pm 0.4, 1.9 \pm 0.5$ and $5.2 \pm 0.8$, respectively. These values imply $|\alpha_{\text{eff}} - \alpha| < 49^o$ at 90% CL, which is not very useful because of the sizable branching ratio into $\pi^0\pi^0$.

2.1.2 Isospin symmetry in $B \to \rho\rho$

The two $\rho$ mesons in $B \to \rho\rho$ are described by three possible polarization states, longitudinal or transverse to the momentum direction, parallel or orthogonal to each other in the case of transversely polarized states. Angular distributions of the outgoing pions measured by BaBar [14][15] show that the $\rho$ mesons are dominantly longitudinally polarized, $\Gamma_L/T = 0.99 \pm 0.03 \pm 0.01$, describing CP-even states. That is, the case of $B^0 \to \rho^+\rho^-$ is almost identical to the case of
$B^0 \rightarrow \pi^+\pi^-$, albeit a possible small correction from a CP-odd state, and a slight violation of the $B \rightarrow \rho\rho$ isospin relation when the two $\rho$ mesons are observed at different invariant masses [21].

Time-dependent asymmetries in $B^0 \rightarrow \rho^+\rho^-$, analogous to $C_{\pi\pi}$ and $S_{\pi\pi}$ in [3], were measured by BaBar for longitudinally polarized $\rho$'s [20],

\begin{align*}
C_{\rho\rho} &= -0.17 \pm 0.27 \pm 0.14, \\
S_{\rho\rho} &= -0.42 \pm 0.42 \pm 0.06.
\end{align*}

(9)

This implies two possible central values $\alpha_{\text{eff}} = 103^\circ, 167^\circ$ with large experimental errors. A small branching ratio of $B \rightarrow \rho^0\rho^0, B(\rho^0\rho^0) < 2.1 \times 10^{-6}$ [22], much smaller than $B(\rho^+\rho^-) = (25 \pm 9) \times 10^{-6}$ [20] and $B(\rho^+\rho^0) = (26 \pm 6) \times 10^{-6}$ [22,23], implies a 90% CL upper bound $|\alpha_{\text{eff}} - \alpha| < 17^\circ$ [17]. This bound and [9] exclude the range $19^\circ \leq \alpha \leq 71^\circ$ at 90% CL [21] consistent with [1].

2.1.3 Broken SU(3) symmetry for $B \rightarrow \pi^+\pi^-$

As long as separate $B^0$ and $\bar{B}^0$ decays into $\pi^0\pi^0$ have not yet been measured, one may replace isospin symmetry by the less precise flavor SU(3) symmetry relating $B \rightarrow \pi\pi$ to $B \rightarrow K\pi$ [21,25]. This imposes constraints on the ratio of penguin-to-tree amplitudes in $B \rightarrow \pi\pi, r \equiv |P/T|$. To improve the quality of the analysis, one introduces SU(3) breaking in tree amplitudes in terms of a ratio of $K$ and $\pi$ decay constants, as given by factorization [2], neglecting a very small variation of the $B$ to $\pi$ form factor when $q^2$ varies from $m_K^2$ to $m_K^2$. This assumption and the neglect of small annihilation amplitudes may be checked experimentally. Here we follow briefly a study presented recently in [20], which contains further references.

Writing $[\lambda \equiv \lambda/(1 - \lambda^2/2) = 0.230]$

\begin{align*}
A(B^+ \rightarrow K^0\pi^+) &= -\bar{\lambda}^{-1}Pe^{i\delta}, \\
A(B^0 \rightarrow K^+\pi^-) &= -\frac{f_K}{f_\pi}\lambda T e^{i\gamma} + \bar{\lambda}^{-1}Pe^{i\delta},
\end{align*}

(10)

one defines two ratios of charged averaged rates,

\begin{align*}
R_+ &= \frac{\bar{\lambda}^2 \tilde{\Gamma}(B^+ \rightarrow K^0\pi^+)}{\tilde{\Gamma}(B^+ \rightarrow \pi^+\pi^-)} = \frac{r^2}{R_{\pi\pi}}, \\
R_0 &= \frac{\bar{\lambda}^2 \tilde{\Gamma}(B^0 \rightarrow K^+\pi^-)}{\tilde{\Gamma}(B^0 \rightarrow \pi^+\pi^-)} = \frac{r^2 + 2r\bar{\lambda}^2z + \bar{\lambda}^4}{R_{\pi\pi}}
\end{align*}

(11)

where $R_{\pi\pi} \equiv 1 - 2rz + r^2$ and

\begin{equation}
\bar{\lambda} = \sqrt{\frac{f_K}{f_\pi}}, \quad z = \cos \delta \cos(\beta + \alpha).
\end{equation}

(12)

Current experimental values [18].

\begin{align*}
R_+ &= 0.235 \pm 0.026, \quad R_0 = 0.209 \pm 0.020,
\end{align*}

(13)

imply a large penguin amplitude

\begin{equation}
0.51 \leq r \leq 0.85 \quad (\text{assuming } |\delta| < \pi/2).
\end{equation}

(14)

This range is consistent with a recent global SU(3) fit to charmless $B$ decays into two pseudoscalar mesons, $B \rightarrow P P$ [27,28]. However, it is somewhat in conflict with most QCD-based calculations which typically obtain smaller values for $r$ [29].

A study of $\alpha$ using the CP asymmetries $C_{\pi\pi}$ and $S_{\pi\pi}$ proceeds as follows. One expresses the two asymmetries in terms of $r$, $\delta$ and $\alpha$,

\begin{align*}
C_{\pi\pi} &= \frac{2r\sin \delta \sin(\beta + \alpha)}{R_{\pi\pi}}, \\
S_{\pi\pi} &= \frac{\sin 2\alpha + 2r \cos \delta \sin(\beta - \alpha) - r^2 \sin 2\beta}{R_{\pi\pi}}.
\end{align*}

(15)

We now impose in addition the SU(3) constraints

\begin{align*}
\bar{\lambda} &= 0.230, \\
\bar{\lambda}^2 &= 0.053, \\
\lambda T &= 1.22 \pm 0.02.
\end{align*}

(16)

Figure 2. Curves of $C_{\pi\pi}$ vs. $S_{\pi\pi}$.
and \([11]\). The resulting two plots of \(C_{\pi^\pi}\) vs. \(S_{\pi^\pi}\) \([20]\), shown in Fig. 2(a) using \(R_+\) and in Fig. 2(b) using \(R_0\), are rather similar, supporting the SU(3) assumption in these processes. The plotted experimental average \((7)\) implies an allowed range, \(\alpha = (104 \pm 18)\degree\), which overlaps nicely with the range of \(\alpha\) in \([11]\), favoring large values in this range.

SU(3) breaking corrections in penguin amplitudes modify the allowed range of \(\alpha\). The bounds become stronger (weaker) if SU(3) breaking enhances (suppresses) the penguin amplitude in \(B \to K\pi\) relative to \(B \to \pi\pi\). A 22% SU(3) correction, implied for instance by a factor \(f_K/f_\pi\) (or by its inverse), modifies the bounds by \(8\degree\).

2.2. Broken SU(3) symmetry for \(B \to \rho^\pm\pi^\mp\)

Very recently broken flavor SU(3) has also been applied to study \(\alpha\) in \(B^0(t) \to \rho^\pm\pi^\mp\) \([30]\). The number of hadronic parameters and the number of measurable quantities are twice as large as in \(B^0 \to \pi^+\pi^-\) which complicates somewhat the analysis. Here we discuss only a bound on \(\alpha\), while a complete determination of \(\alpha\) will be presented by J. Zupan at this conference \([31]\).

Time-dependent decay rates for initially \(B^0\) decaying into \(\rho^\pm\pi^\mp\) are given by \([32]\).

\[
\Gamma(B^0(t) \to \rho^\pm\pi^\mp) \propto 1 + (C \pm D\Delta) \cos \Delta m t - (S \pm D\Delta) \sin \Delta m t, \quad (16)
\]

while for initially \(\overline{B}^0\) decays the \(\cos \Delta m t\) and \(\sin \Delta m t\) terms have opposite signs. As in \(B \to \pi^+\pi^-\), one defines a measurable phase, \(\alpha_{\text{eff}}\), which equals \(\alpha\) in the limit of vanishing penguin amplitudes,

\[
4\alpha_{\text{eff}} \equiv \arcsin \left[ \frac{(S + D\Delta)}{\sqrt{1 - (C + D\Delta)^2}} \right] + \arcsin \left[ \frac{(S - D\Delta)}{\sqrt{1 - (C - D\Delta)^2}} \right]. \quad (17)
\]

The difference \(|\alpha_{\text{eff}} - \alpha|\), which is governed by penguin contributions, can be shown to be bounded by ratios of strangeness changing decay rates of \(B \to K^+\pi\) and \(B \to K\rho\) (dominated by penguin amplitudes) and decay rates of \(B \to \rho^+\pi^-\) and \(B \to \rho^-\pi^+\) (dominated by tree amplitudes). In particular, using the following two ratios of rates,

\[
R_+^0 \equiv \frac{\lambda^2\Gamma(B^+ \to K^0\pi^0)}{\Gamma(B^0 \to \rho^+\pi^-)},
\]

one may show that

\[
2|\alpha_{\text{eff}} - \alpha| \leq \arcsin \sqrt{R_+^0} + \arcsin \sqrt{R_+^0}. \quad (18)
\]

Using current values \([18]\)

\[
R_+^0 = 0.032 \pm 0.007, \quad R_+^0 = 0.047 \pm 0.015, \quad (20)
\]

one obtains an upper bound at 90% CL,

\[
|\alpha_{\text{eff}} - \alpha| \leq 12\degree. \quad (21)
\]

This bound, which assumes exact SU(3) for penguin amplitudes, becomes stronger (weaker) if SU(3) breaking corrections in penguin amplitudes are enhanced (suppressed) relative to \(\Delta S = 0\) penguin amplitudes. In any event, one expects this bound to change by no more than 30%, implying \(|\alpha_{\text{eff}} - \alpha| \leq 15\degree\) in the presence of SU(3) breaking corrections.

Both BaBar \([6,33]\) and Belle \([11]\) measured time-dependence in \(B^0 \to \rho^+\pi^\pm\):

\[
C = \begin{cases} 0.35 \pm 0.14 \\ 0.25 \pm 0.17 \end{cases} \quad \Delta C = \begin{cases} 0.20 \pm 0.14 \\ 0.38 \pm 0.18 \end{cases} \quad (22)
\]

\[
S = \begin{cases} -0.13 \pm 0.18 \\ -0.28 \pm 0.24 \end{cases} \quad \Delta S = \begin{cases} 0.33 \pm 0.18 \\ -0.30 \pm 0.26 \end{cases} \quad (23)
\]

where the first values are BaBar’s and the second are Belle’s. These values imply single solutions,

\[
\alpha_{\text{eff}} = \begin{cases} (93 \pm 7)\degree & \text{BaBar} \\ (102 \pm 11)\degree & \text{Belle}, \end{cases} \quad (24)
\]

when making a mild and experimentally testable assumption \([30]\) that the two arcsin’s in \([17]\) differ by much less than 180\degree. Combining \([24]\) with the above upper bound on \(|\alpha_{\text{eff}} - \alpha|\) one obtains

\[
\alpha = \begin{cases} (93 \pm 7 \pm 15)\degree = (93 \pm 17)\degree & \text{BaBar} \\ (102 \pm 11 \pm 15)\degree = (102 \pm 19)\degree & \text{Belle} \end{cases} \quad (25)
\]

where experimental and theoretical errors are added in quadrature. These values are in good agreement with the range of \(\alpha\) in \([11]\) and largely overlap with this range.

One may show that the relative contributions of penguin amplitudes in \(B^0 \to \rho^+\pi^\mp\) are much smaller than in \(B^0 \to \pi^+\pi^-\), involving ratios of penguin and tree amplitudes, \(r_+ \sim 0.2\) \([24,29]\) \([30,34]\). Consequently, effects of SU(3) breaking in penguin amplitude lead to an intrinsic uncertainty of only a few degrees in the determination of \(\alpha\) in \(B^0(t) \to \rho^\pm\pi^\mp\) \([30]\).
3. PURE TREE DECAYS $B^\pm \to DK^\pm$

The process $B^+ \to DK^+$ and its charge-conjugate provide a way of determining $\gamma$ in a manner which is pure in principle, avoiding uncertainties in penguin amplitudes. Here we wish to study a scheme proposed in [35], which uses both $D^0$ CP-eigenstates and $D^0$ flavor states. An extensive list of variants of this method is given in [36]. In all variants one makes use of an interference between tree amplitudes in decays of the type $B^\pm \to DK^\pm$, from $b \to c\bar{s}u$ and $\bar{b} \to \bar{u}c\bar{s}$, for which the weak phase difference is $\gamma$. We choose to also discuss briefly one variant [37], in which the Dalitz plot of $D^0 \to K_S\pi^+\pi^-$ is being analyzed in $B^\pm \to DK^\pm$.

We denote by $r$ the magnitude of the ratio of two amplitudes, $A(B^+ \to D^0K^+)$ from $b \to \bar{u}c\bar{s}$ and $A(B^+ \to D^0K^+)$ from $b \to \bar{c}u\bar{s}$, and we denote by $\delta$ the strong phase of this ratio. One then obtains the following expressions for two ratios of rates, for even and odd $D^0$-CP states, and for two corresponding CP-asymmetries:

$$ R_\pm = \frac{\Gamma(D_{CP\pm}^0K^-) + \Gamma(D_{CP\pm}^0K^+)}{\Gamma(D^0K^-)} $$

$$ A_\pm = \frac{\Gamma(D_{CP\pm}^0K^-) - \Gamma(D_{CP\pm}^0K^+)}{\Gamma(D_{CP\pm}^0K^-) + \Gamma(D_{CP\pm}^0K^+)} $$

In principle, these three independent observables determine $r$, $\delta$ and $\gamma$. However, this is difficult in practice since one must be sensitive to an $r^2$ term, where the current 90\% CL upper limit on $r$ is $r < 0.22$ [38]. A crude estimate is $r \sim 0.2$ [39], since this ratio involves a CKM factor, $|V_{ub}^*V_{cb}/V_{ub}V_{cb}| = 0.4 - 0.5$, and probably a comparable color-suppression factor.

Taking averages of Belle and BaBar measurements when both are available, one finds [40]

$$ R_+ = 1.09 \pm 0.16 \text{ (Belle & BaBar)}, $$
$$ R_- = 1.30 \pm 0.25 \text{ (Belle)}, $$
$$ A_+ = 0.07 \pm 0.13 \text{ (Belle & BaBar)}, $$
$$ A_- = -0.19 \pm 0.18 \text{ (Belle)}. $$

This implies

$$ r = 0.44^{+0.14}_{-0.22}, \quad |A_\pm|_{\text{ave}} = 0.11 \pm 0.11. \quad (29) $$

In order to obtain constraints on $\gamma$ we eliminate $\delta$, plotting in Fig. 3 $R_\pm$ versus $\gamma$ for allowed $A_\pm$. We are using 1$\sigma$ bounds on $r$, $R_\pm$ and $|A_\pm|_{\text{ave}}$. The ratios $R_+$ and $R_-$ are described by the lower and upper branches, respectively, corresponding to $\cos \delta \cos \gamma < 0$. This implies a very strong 1$\sigma$ lower bound, $\gamma > 72^\circ$, and requires $\cos \delta < 0$ for allowed values of $\gamma$. More precise measurements of $R_\pm$ are needed for constraints beyond 1$\sigma$.

A recent study by Belle of $B^\pm \to D^{(*)}K^\pm$ [12] [41], analyzed the Dalitz plot for $D \to K_S\pi^+\pi^-$ in terms of a sum of a non-resonant term and a set of resonances given by Breit-Wigner forms. This study, involving some model-dependence, confirmed $\cos \delta < 0$ and obtained 2$\sigma$ bounds, $26^\circ < \gamma < 126^\circ$, considerably wider than and including the range of $\gamma$ in [11].

4. PENGUIN DOMINATED DECAYS

In a class of penguin-dominated $B^0$ decays into CP-eigenstates, including the final states $f =$
$\pi^0 K_S, \eta' K_S, \phi K_S$ and $(K^+ K^-)_{\text{even}} e K_S$, decay amplitudes contain two terms: a penguin amplitude, $p'_f$, involving a dominant CKM factor $V^*_{ub} V_{cs}$, and a color-suppressed tree amplitude, $c'_f$, with a smaller CKM factor $V^*_{ub} V_{us}$. The first amplitude by itself would imply a CP asymmetry of magnitude $\sin 2\beta \sin \Delta m t$. The second amplitude modifies the coefficient of this term, and introduces a $\cos \Delta m t$ term in the asymmetry \[4\]. The coefficients of $\sin \Delta m t$ and $\cos \Delta m t$ for a final state $f$ are denoted by $S_f$ and $C_f$, respectively, as given in Eq. \[9\] for $B \to \pi^+ \pi^-$. The observables, $\Delta S_f \equiv S_f \pm \sin 2\beta$ (where the sign depends on the final state CP) and $C_f$, increase with $|c'_f/p'_f|$, but are functions of unknown strong interaction phases. A search for new physics effects in these processes requires a careful theoretical analysis within the Standard Model of $\Delta S_f$ and $C_f$ and not only of $|c'_f/p'_f|$.

Model-independent studies of the ratios $|c'_f/p'_f|$, providing estimates for $\Delta S_f$ for the above final states, were performed in \[34,42\]. Here we will follow Ref. \[43,44\] in order to obtain correlated bounds directly on $S_f$ and $C_f$ in the two cases of $B^0 \to \pi^0 K_S$ and $B^0 \to \eta' K_S$.

For simplicity of expressions we will expand the two asymmetries up to terms linear in $|c'_f/p'_f|$. We will not study theoretical bounds on asymmetries in $B^0 \to \phi K_S$, where one is awaiting a greater consistency between BaBar and Belle measurements \[45\].

\subsection*{4.1. $B^0 \to \pi^0 K_S$}

Writing

$$A(B^0 \to \pi^0 K^0) = |p'|e^{i\delta'} - |c'|e^{i\gamma},$$

(30)

and denoting $r' \equiv |c'/p'|$, one obtains \[4\]

$$S_{\pi K} \approx \sin 2\beta - 2r' \cos 2\beta \sin \gamma \cos \delta',$$

$$C_{\pi K} \approx -2r' \sin \gamma \sin \delta'.$$

(31)

The allowed region in the $(S_{\pi K}, C_{\pi K})$ plane is confined to an ellipse centered at $(\sin 2\beta, 0)$, with semi-principal axes $2[r' \sin \gamma]_{\text{max}} \cos 2\beta$ and $2[r' \sin \gamma]_{\text{max}}$. An approximate ellipse providing these bounds is obtained by relating $B^0 \to \pi^0 K^0$ within flavor SU(3) to $B^0 \to \pi^0 \pi^0$ and $B^0 \to K^+ K^-$. For simplicity, we will first neglect the second process given by an exchange-type amplitude which is expected to be negligible. It then follows from SU(3) that

$$A(B^0 \to \pi^0 \pi^0) = -\bar{\lambda} |p'| e^{i\delta'} - \bar{\lambda}^{-1} |c'| e^{i\gamma}. \quad (32)$$

Defining a ratio of rates,

$$R_{\pi/K} = \frac{\lambda^2 B(B^0 \to \pi^0 \pi^0)}{B(B^0 \to \pi^0 K^0)},$$

(33)

with $R_{\pi/K} = 0.0084 \pm 0.0023 \ [13]$, one has

$$R_{\pi/K} = \frac{r'^2 + \bar{\lambda}^4 + 2\bar{\lambda}^2 r' \cos \delta' \cos \gamma}{1 + r'^2 - 2r' \cos \delta' \cos \gamma}. \quad (34)$$

A scatter plot of $|C_{\pi K}|$ vs. $S_{\pi K}$, using \[34\] and exact expressions instead of \[31\], is shown in Fig. 4 \[13\]. It describes the allowed region. Also shown is the experimental point given by a BaBar measurement \[10\]. The inner ellipse, which is excluded when neglecting $A(B^0 \to K^+ K^-)$, is still allowed when using the current upper bound on this amplitude. In any event, current errors in the asymmetries are seen to be too large to provide a sensitive test for new physics, and must be improved for such a test.

![Figure 4](image-url)

**Figure 4.** Points in $(S_{\pi K}, |C_{\pi K}|)$ plane satisfying \[34\].

\subsection*{4.2. $B^0 \to \eta' K_S$}

In order to apply SU(3) to asymmetries in $B^0 \to \eta' K_S$, where measurements are \[47\],

$$S_{\eta' K} = 0.27 \pm 0.21, \ C_{\eta' K} = 0.04 \pm 0.13, \quad (35)$$

...
one may optimize bounds over a whole continuum of combinations of corresponding strangeness conserving decays. These include $B^0$ decays involving $\pi^0, \eta$ and $\eta'$ in the final state. Considerable improvements in some of these branching ratios were achieved recently by BaBar [48]. The resulting bounds are shown in Fig. 5 [44]. Regions enclosed by the solid (dashed) curve give current bounds including (neglecting) annihilation-type amplitudes, while earlier bounds are given by the dot-dashed curve. Also shown is a point labeled $x$ denoting the central value predicted in a global SU(3) fit to $B \rightarrow PP$ [27]. With recent improvement in bounds on $\Delta S = 0$ rates, only a mild improvement in the asymmetry measurements is required for achieving sensitivity to new physics.

Figure 5. Allowed regions in ($S_{\eta'K}$, $C_{\eta'K}$) plane.

5. CONCLUSIONS

- Nothing is as pure and easy as $B \rightarrow J/\psi K_S$.
- The evidence for CP violation in $B \rightarrow \pi^+\pi^-$ is consistent with the KM mechanism, and is beginning to exclude low values of $\alpha$ permitted by other CKM bounds.
- CP asymmetries in $B^0 \rightarrow \rho^+\rho^-$ exclude values $\alpha < 71^\circ$, in agreement with CKM constraints. Smaller errors in asymmetry measurements may restrict the range of $\alpha$.
- Bounds on $\alpha$ from $B \rightarrow \rho^+\pi^+$ overlap nicely with the range obtained from other CKM constraints. The intrinsic uncertainty from SU(3) breaking is merely a few degrees.
- Current $B^+ \rightarrow DK^+$ measurements are consistent with CKM constraints on $\gamma$ and require more statistics for stronger bounds.
- All this indicates that the KM phase governs CP violation in tree-dominated decays.
- Some improvement in asymmetry measurements in $B^0 \rightarrow \pi^0K_S, \eta'K_S$, and convergence of BaBar and Belle in $B^0 \rightarrow \phi K_S$, are required for sensitivity to new physics.

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