Traffic modeling on the road of the fly over branch using totally asymmetric exclusion process a macroscopic approach

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Abstract. This research examines a dynamic model, namely the Totally Asymmetric Exclusion Process (TASEP). In addition, it discusses the boundary conditions and dynamic rules used in this modeling. TASEP has been applied in various fields, one of which is the traffic flow modeling. This system is modified into a unidirectional fly-over fork. Furthermore, a vehicle which travels through a road segment is modeled as a particle which jumps from one lattice to another. The continuity term which is to describe a particle dynamic in TASEP is solved by using the level set method. The profile of particle density and current density is affected by input rate and output rate of the particle jump.

1. Introduction

Transportation is an important means of supporting the development of a city. Therefore the need for transportation routes is increasing. However, it needs to be realized that this situation creates new problems surrounding the traffic density of roads. For example the city of Jakarta which has traffic problems (congestion). Currently, traffic flow research from a macroscopic perspective is being discussed. Basically, a macroscopic review looks at traffic globally.

This research offers theoretical analysis and an extension of a mathematical model that can describe the dynamics of vehicle traffic on one-way roads as an effort to find solutions to the problems of traffic congestion and congestion. This mathematical model is known as a totally simple asymmetric exclusion process or better known as TASEP. TASEP is a non-equilibrium model in which particles with hard core interactions can jump left or right (not both) in a grid (path). TASEP is specified by the presence of dynamic rules and boundary conditions. From the ‘traffic’ of these particles, it can be obtained the average value of the position of the particles in occupying a grid or density, as well as the current density of the particles.

Partial differential equations can be considered as an interface so that it can be worked with the level set method. The level set method is especially useful in calculating multivalued solutions. In their article developed a level set method for calculating the solution of a nonlinear equation of the order of one general form in any dimension of space. More specifically developed a level set method for calculating the velocity and multivalued electric fields for the 1-dimensional (1D) Euler-Poisson equation [1].

1.1 Fluid Dynamics (Macroscopic)

Dynamic, which means it can change depending on time. Macroscopic dynamic modeling describes the changes in traffic over time and space using differential equations. Traffic flow is often analogized as liquid or gas. Analytical approaches can still be used when observing a road section. But when the temporal and spatial interactions of traffic flows on the road network need to be evaluated, then the simulation method would be more appropriate[2].

The general term for a traffic flow model simulation is macroscopic simulation. Their use has grown widely, and has been facilitated by the extensive development of traffic measurement systems that have been installed in major cities and highways. An additional factor that helps gain the macroscopic
popularity of the model is the fact that the data required for the model (flow count, velocity) are at the same level of aggregation as the data provided by the measurement.

1.2 Totally Asymmetric Exclusion Process (TASEP)

In this section, we would explain in general the physical model known as totally asymmetric exclusion process (TASEP). TASEP in one dimension (1D) is a particle jump model used to study unbalanced systems. TASEP in a mathematical model in which particles (hard nuclei) filling the 1D grid can jump into the neighboring lattice as long as the neighboring lattice is not filled with other particles. This jump occurs in only one direction, namely to the right, which results in asymmetric motion of the particles on the grid.

This is why the TASEP model has been studied intensively. On the other hand, the periodic boundary condition produces a ring-shaped geometry of the grid so that the number of particles in the grid is fixed. Thus, an equilibrium system is obtained in which the particles randomly jump along the lattice with equal probability. However, if the particle jump rate is made homogeneous, a phase transition from low to high density may occur. This model was originally used to study the bio-polymerization kinetics of nucleic acid. In its development, the TASEP model has been studied more extensively as one of the basic models for one-dimensional transport, where its application is related to intra-cell transport, and traffic and transport in porous media. This model can also be used for:

2. Result and Discussion

2.1 Traffic Flow Theory

This section would explain the flow of vehicles in the macroscopic model approach. The relationship between density, speed and flow can be analogous to road density flow. Based on these parameters, a traffic flow conservation law would be obtained which would be derived into a scalar function of a macroscopic model [3].

Notations and Definitions

Some of the notations that would be used in this paper are as follows.

- Traffic flow ($f$): Number of vehicles that would be passed in a time interval
- Traffic density ($\rho$): Number of vehicles per unit distance (measured in vehicles / meter)
- Traffic speed ($v$): movement of traffic flow rate (measured in meters / second)
- $v_m$: Maximum traffic speed
- $\rho_m$: Maximum traffic density. Based on data on the average length of cars, we set the value of $\rho$ max to be 0.25 vehicles / meter [2]. B.

2.2 Theory Fundamental

There are three important factors if we discuss traffic flow macroscopically, namely vehicle flux, density, and velocity here, flux depends only on 2 independent variables, namely density and velocity. Assume a stable state (flux does not change along the way during the observation) and all vehicle speeds are considered constant. Therefore, the flux function can be rendered simpler because the dependence on distance, time, and interval is no longer valid in the stable state. The observations made by [4] show that one vehicle with another has a different tendency but approaches at a point and depends on the existing density.
Combining all possible currents available to the equilibrium function, therefore the relationship between flux, density $k$, and velocity can be presented in a graphical form known as a fundamental diagram.

Look at Figure 1. Some points that need to be considered in the diagram include:

2.2.1 Free flow of vehicles
When the road conditions are in a free state, the vehicle can increase its speed until it reaches the maximum. In this condition the density would be close to zero.

2.2.2 Jam Point
In this condition the vehicle can no longer accelerate, and the density increases until it reaches the maximum where the vehicle is stopped.

2.2.3 Traffic jam
condition reaches the maximum density and the vehicle comes to a complete stop

2.3 Traffic Flow

Suppose that on a road there is a traffic flow moving at a constant speed ($v_0$), and has a density ($\rho_0$), such that the distance between the vehicles is considered the same as shown in Figure 2 (a). The observer measures the number of vehicles per unit time ($t$). In $t$ units of time each vehicle has traveled the distance $v_0t$, and the number of vehicles passing by the observer in units of time is the number of vehicles in the distance $v_0t$. Figure 2 (b).

Density $\rho_0$ is the number of vehicles in a road segment, then the $v_0t$ current equation is given in the distance,

\[ f = \rho_0v_0 \]

or in general form

\[ f(\rho, v) = \rho(x, t) v(x, t) \quad (1.1) \]

Where $f(\rho, v)$ is a function for flux.
Assume that the number of vehicles passing through \( x_0 \) a very small point in time \((\Delta t)\) has not changed significantly, therefore \( \rho(x, t) \) and \( v(x, t) \) can be obtained by \( x = x_0 \) and \( t = t_0 \) approach. The number of vehicles passing the observer over a short distance, and can be estimated to be equivalent to \( \rho(x, t), v(x, t), \Delta t \), where the vehicle flux is given by (1.1)

### 2.4 Conservation Laws

Traffic modeling, whether consisting of a single equation or a system of equations, is all based on the laws of conservation of physics. It can be said to be constant if the number of particles does not change during the process. By applying this to a mathematical form, the pattern of density and velocity over the next several times would be possible to predict.

In this case, the number of vehicles in road units is \([x_1, x_2]\) kept constant. Suppose the vehicle flow is moving. It is assumed that a straight road would have no road exit obstacles and fork. The number \( N \) of vehicles \([x_1, x_2]\) in that time \( t \) is integral to the current density given in

\[
N = \int_{x_1}^{x_2} \rho(x, t) \, dx
\]

(1.2)

On the basis of the above equation, it is explained that \([x_1, x_2]\) the number of vehicles must have reached its limit \( \rho_m \) when the maximum density is reached. The number of vehicles can change, increase or decrease at certain times due to car displacement on the road. Assuming there are no vehicles affected by accident disturbances, the change in the number of vehicles would only occur on the boundary line. Therefore the number of vehicles per unit time can be written as follows.

\[
\frac{dN}{dt} = f_{in}(\rho, v) - f_{out}(\rho, v)
\]

(1.3)

Hence, from equation (1.2) and (1.3), a law of conservation can be written as follows,

\[
\frac{d}{dt} \left( \int_{x_1}^{x_2} \rho(x, t) \, dx \right) = f_{in}(\rho, v) - f_{out}(\rho, v)
\]

(1.4)

This equation would show the fact that the change in the number of vehicles is due to the flow on the boundary line. If the endpoint is the independent variable, then the total derivative in equation (1.4) can be replaced by a partial derivative to get

\[
\frac{\partial}{\partial t} \int_{x_1}^{x_2} \rho(x, t) \, dx = f_{in}(\rho, v) - f_{out}(\rho, v)
\]

(1.5)

With the number of vehicles would change depending on the distance between entries and out, given

\[
f_{in}(\rho, v) - f_{out}(\rho, v) = - \int_{x_1}^{x_2} \frac{\partial}{\partial x} f(\rho, v) \, dx
\]

(1.6)

Substitute equation (1.5) into (1.6), so that the equation is obtained

\[
\int_{x_1}^{x_2} \left( \frac{\partial}{\partial t} \rho(x, t) + \frac{\partial}{\partial x} f(\rho, v) \right) \, dx = 0
\]

(1.7)

This equation states that the definite integral of some part is always zero for all values of any part definite integral limit variable. The only function that satisfies is function zero. Then we assume \( \rho(x, t) \), and \( f(x, t) \) that both are consistent, then the one-dimensional equation of the conservation law is expressed by the transport equation.

\[
\rho_t + f_x = 0
\]

(1.8)

### 2.5 Density-Velocity relation

Traffic density and vehicle speed are related to one equation, namely

\[
\rho_t(x, t) + (\rho(x, t) v(x, t))_x = 0
\]

(1.9)

If the density and initial speed of the road are known, equation (1.9) can be used to estimate the density of the road that would occur. Thus, the speed of traffic flow depends on the density, it is called \( v(\rho) \), the choice of speed function depending on the nature of the model to be observed.

Several models have been observed by other researchers. The model approach green shield would be used in several flow models of vehicles. This model green shield is simple and widely used. Green shield was able to develop a model of uninterrupted traffic flow that predicts and explains the trends that are observed in real traffic flows. It is assumed that the velocity is directly proportion
Where $v_f$ is the free flow velocity and $\rho_m$ is the maximum density. For zero density, the model represents current velocity $v_f$, so that when the density reaches its maximum $\rho_m$ no vehicles enter or exit.

2.6 Model “exits and entrances”

Assume no cars enter or leave except at the end of the road. This section would discuss the development of a traffic model that would consider incoming and outgoing cars in road segments, as illustrated in Figure 3.

The rate of change in the number of intermediate cars $x = x_1$ and $x = x_2$ is not only determined by the number of cars passing $x = x_1$ and $x = x_2$ through, but also by the number of cars entering or leaving between $x = x_1$ and $x = x_2$. Suppose $\int_{x_1}^{x_2} \beta(x, t) dx$ is the difference between the number of cars entering and leaving the road segment per unit distance $x = x_1$ and $x = x_2$. Thus, the law of the conservation of a car can be expressed as

$$\frac{\partial}{\partial t} \int_{x_1}^{x_2} \rho(x, t) dx = f_{\text{in}}(x, v) - f_{\text{out}}(x, v) + \int_{x_1}^{x_2} \beta(x, t) dx$$

(1.11)

So it applies

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial q(x, t)}{\partial x} = \beta(x, t)$$

(1.12)

If $q = q(\rho)$, then by using the chain rule, (1.10) can be written

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial \rho} \frac{\partial \rho}{\partial x} = \beta$$

By substituting (1.10) to (1.12), the traffic model can be written

$$\frac{\partial \rho(x, t)}{\partial t} + u_{\text{max}} \left( 1 - \frac{2\rho(x, t)}{\rho_{\text{max}}} \right) \frac{\partial \rho(x, t)}{\partial x} = \beta(x, t)$$

(1.13)

2.7 Lighthill Whitham Rihards (LWR)

This section would show four different models of traffic flow in one dimension. The first model is a model with one equation, while the rest is a system with a two equation model. All models would be described by partial differential equations and focused on mass conservation, which seek to capture the dynamic interactions found in traffic flow movements. In addition, this model offers a particular means of achieving velocity and density. As a consequence, the flow is not in harmony as a formula of a single equation. This section is the key distinction between scalar systems and model systems.

In the first model used in explaining problems in vehicle flow, it is known as the LWR model. The LWR model is a scalar model, which is time dependent, non-linear, and includes hyperbolic differential equations. In this model, the density currents are kept in number, so that they are obtained

$$\rho_t(x, t) + \left( \rho(x, t) v(x, t) \right)_x = 0$$

(1.12)

with the flux denoted by

$$f(x, t) = \rho(x, t) V(\rho(x, t))$$

(1.13)
Where $V(\rho(x, t))$ is the speed function given by (1.10).

In the LWR model, speed depends only on density. As a result, the current is balanced when the velocity-density function is used. When the traffic density is small enough, the vehicle speed would be relatively constant, so the model does not show any speed distribution for each vehicle. Therefore, this model cannot explain the properties to be observed in currents that are not so dense.

However, $\nu_f$ the average velocity would probably solve this problem. On this side, the model is anisotropic as the principal of the observed vehicle flow, for example, the nature of the vehicle is largely influenced by the vehicle in front of it. In this case it can be found in the eigenvalue given by

$$f'(\rho(x,t)) = \frac{\partial f}{\partial x}(\rho(x,t)) = V(\rho(x,t)) + \rho V'(\rho(x,t))$$  \hfill (1.14)

Thus, the model allows the information data to travel as fast as the current vehicle, and not more, as it satisfies $0 < f'(\rho) < V(\rho)$, due

$$V'(\rho) = -\frac{\nu_f}{\rho_m}$$  \hfill (1.15)

LWR models given by (1.9) and (1.10) are simple models and cannot produce all too complex interactions on the actual vehicle flow. Obviously in this review modifications to this model have been suggested. One of them is speed and density. The second way is to pair the mass conservation equation with equation (1.10) which would mimic the current motion compared to the velocity-density model.

Road segment to be modeled would select a road length of 10km to determine the transportation waves. For the main line in the road segment and added roads in and out fork in the road segment. Each of the forked roads would be examined for 5 km for in and 5 km for out. This modeling would limit the vehicles traveling on the road segment and divided by three speeds and is illustrated in Figure 4.

### 2.8. Analysis of Transport Waves Using Level Set Method

In the first spot present the density profile figures with the comparative original and 6 minute location of cars with respect to some 10 km highway points. For Figure 4 (a) the maximum velocity is $v_{max} = 30 \text{ km/h}$, while for Figure 4 (b) $v_{max} = 60 \text{ km/h}$ and $v_{max} = 90 \text{ km/h}$ for Figure 4 (c). In each case, we find that the density of cars increases at the 5$^{th}$ km point and decreases at the 8$^{th}$ km point of our 10 km highway. This particular condition occurs due to the influence of continuous inflow and outflow at 5$^{th}$ km and 8$^{th}$ km respectively. Although the overall velocity is 3 (a) It is $v_{max} = 30 \text{ km/h}$, which is very low so that the density of cars at the inflow position exceeds our fixed limit density of $\rho_{max} = 550 \text{ cars/km}$, i.e. $\rho_{max} \rho > \rho_{jam}$. This state brings us to a congested traffic situation at the inflow spot.

So in this situation, we should consider the inflow to be a local interruption in the flow of traffic. In Figure 4 (b) and Figure 4 (c), the median velocity of cars is equal to that of the case discussed in Figure 4 (a). In each of the following cases, the car density increases considerably at the inflow position but does not surpass the jam density, i.e. the density at the inflow position $\rho < \rho_{jam}$. In each of these instances, however the inflow source word has no meaningful impact. Cars are free to drive at their optimal speed. We have also added the sink word at the 8$^{th}$ km point where some of the vehicles will exit our 10 km highway. As a result, the vehicle density steadily decreases at the outflow position. As a result, drivers are able to maintain their own safe driving pace at the outflow position.
Figure 4. Position comparison between the starting point to the next 6 minutes
a) $v_{\text{max}} = 30 \text{ km/h}$, b) $v_{\text{max}} = 60 \text{ km/h}$, c) $v_{\text{max}} = 90 \text{ km/h}$.

According to the wave diagram, it would be compared with the special case to the visual on the incoming and outgoing waves of vehicles on the road segment. In each case give a simulation of 6 minutes. In this case each simulation is given a different speed to see the results of the traffic waves that would be studied. Henceforth, it would be simulated at each current density, average speed, and traffic waves on the road segment that has crossed and would illustrate the solution diagram for the unidirectional forked road segment.
Figure 5 (a) and Figure 5 (b) display density profiles and velocity profiles at three different periods. Figure 5 (a) shows that the density of traffic at inflow location steadily increases the decreased wave height as time passes. But in the event of an outflow located at the 8th km location, the density of cars increases as time goes by. This particular situation is due to the continuous increase in the flow rate on the single-lane highway, but the number of cars on the single-lane highway. Leaving through the sink is constant. In Figure 5 (b), the average velocity of the cars decreases at a high density at the inflow source position at the 5th km and vice versa at the outflow position. Figure 5 (c) displays the flow profile over a period of minutes.

Figure 5. a) density profile on road segments, b) speed profiles of vehicles on road segments, c) transport wave profiles on road segments.
The solution for this road segment $\rho(x, t)$ is on a straight line in the direction of the line $(x, t)$. The lines are not parallel to each other, because to enter different values results in different values. The profile image clearly shows the degradation of this traffic equation. Hence lower traffic capacities would travel faster than higher ones. As a result, the initial profile which is not constant would change over time. According to the flow profile, it is clear that the flow of traffic increases at the 5th km location due to the constant inflow rate and decreases at the 8th km position due to the constant outflow through the sink located at that position.

3. Conclusion
In this article, a modification of the LWR model has been presented to investigate the significant effects of constant inflows and outflows on road segments that are forked. The level set method used shows the various simulations that occur on these roads. In this article add to the road segment and use set level method that does not exist in previous research. This investigation also reveals that the level set conducts finite difference methods in the presence of stationary shock waves. Various numerical calculations carried out during this investigation would obviously make substantial improvements to current models in order to mitigate traffic congestion problems in an efficient manner. Obviously, in this case it would make an important contribution to the existing model so that we can minimize the problem of traffic jams more efficiently.

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