Improving fast-particle confinement in quasi-axisymmetric stellarator optimization

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Abstract

A method to improve fast-particle confinement during quasi-axisymmetric stellarator optimization has been identified. Quasi-axisymmetric (qa) stellarator designs have improved neoclassical transport due to their special symmetry of the magnetic field strength. Previously, it has been shown that, in general, quasi-symmetry can only be obtained on one single flux surface (Garren and Boozer 1991 Phys. Fluids B 3 2805–21). Even though quasi-symmetry can be a crucial property of stellarator design, there is no established convention for choosing the flux surface on which this should be optimized. To address this question, the flux surface on which quasi-axisymmetry is optimized has been varied in a qa configuration. The optimal location was found to lie between half radius and the plasma edge, since this allows for two beneficial features: it increases the number of flux-surfaces with improved quasi-axisymmetry and it increases the volume enclosed by the flux surface with the best qa quality.

Keywords: stellarator, optimization, fast-particle confinement, quasi-axisymmetric stellarator

(Some figures may appear in colour only in the online journal)

1. Introduction

With the successful design, construction and first operation of Wendelstein 7–X [1], optimized stellarators have been shown to be a feasible alternative to tokamaks. However, stellarators do not automatically exhibit good particle confinement and therefore can drastically benefit from optimization.

One way to reduce neoclassical transport in stellarators is to make the magnetic field strength, rather than the magnetic field itself, independent of one of the Boozer coordinate angles [2]. This property implies the conservation of the canonical momentum conjugate to the angle in question since in these coordinates the guiding center Lagrangian only depends on the magnetic field strength and not on its direction. One thus obtains a third invariant, besides the energy and the magnetic moment, which leads to good particle confinement. This type of magnetic field is said to be quasi-symmetric [3, 4]. In this paper, we focus on quasi-axisymmetric (qa) equilibria in which magnetic field strength is symmetric in the toroidal Boozer coordinate [5, 6], but the results can be extended to other symmetries.

Currently, there are two ways to develop quasi-symmetric stellarator designs. The analytic approach is to derive them by large aspect ratio expansion [7]—mainly by employing a near-axis expansion [8–10]. This approach is valuable as it allows physical insight into this class of stellarators and can serve as a starting point for the second approach: numerical optimization. Optimizing stellarators numerically is the standard method for obtaining optimized designs such as Wendelstein 7–X [11], the quasi-helical stellarator HSX [12, 13], or the qa designs NCSX [14], CHS-qa [15], and ESTELL [16]. One reason for the numerical optimization is that the list of desired properties of a stellarator often involve many qualitatively different aspects such as confinement, stability, bootstrap current, which are difficult to combine into a single criterion. One particularly important property for future fusion reactors is the fast-particle confinement. Fast, high-energy (3.5 MeV) alpha particles are created in the fusion reaction and need to be confined for at least a slowing-down time in order that they heat the plasma and keep the fusion process running and therefore this is a crucial property for the future success of stellarator reactors.
One can optimize this property directly by following particle orbits, but this can be numerically expensive. Instead, as mentioned above, this property can be improved by simply enhancing quasi-symmetry.

It has been shown analytically that in general one can only achieve quasi-axisymmetry on one flux-surface, but not necessarily on the neighborng flux-surfaces due to additional constraints on the magnetic field, e.g. the requirement that it should be divergence-free [7, 8]. When optimizing qa configuration one has the freedom to choose on which flux-surface one improves the quasi-axisymmetry [17]. Therefore the following questions arise naturally: does it matter for fast-particle confinement on which flux surface one enhances quasi-axisymmetry? If yes, are there better locations to optimize this property compared to other locations based on fast-particle losses?

To the best of the authors’ knowledge, these questions have not been investigated in depth before. If one could obtain perfect qa-symmetry at only one flux surface, one could assume that it would be preferable at the plasma edge, since all the particles would have to pass this flux surface to leave the plasma. However, perfect quasi-axisymmetry is normally not achieved and deteriorates away from the optimized surface. To maximize the region of improved quasi-axisymmetry, one would expect the best qa-symmetry to be closer to the half radius.

2. Optimization methods

2.1. Optimization

The code ROSE [18] was used to perform the optimization of the ideal-MHD equilibrium properties by varying the plasma boundary used as in input in the fixed boundary equilibrium version of VMEC [19]. We used the standard optimization method of ROSE, which employs Brent’s algorithm [20].

Within the optimization routine, the cost function $f$ is defined as the weighted sum:

$$f = \sum_i w_i (F_i - \bar{F_i})^2,$$

where $w_i$ are weights which are usually adapted for obtaining various optimal configurations. An optimal configuration is characterized by the feature that one cannot improve one criterion without diminishing at least another one. The set of all optimal points is called the Pareto frontier. If the Pareto frontier is non-convex, there are points on it that cannot be found by the weighted sum method. $F_i$ is the value for the criterion, $i$, calculated for each plasma boundary and $\bar{F_i}$ is the corresponding target value.

For this study, we kept the following properties fixed: number of field periods, $N = 2$, the volume-averaged plasma beta, $\beta_v = 2\mu_0 \langle B^2 \rangle / (\mu_0) = 3.5\%$, where $p$ is the plasma pressure and $\mu_0$ is the vacuum permeability, angular brackets denote a volume average, and the aspect ratio $A = \frac{a}{R} \approx 3.4$ where $R = R_{pol}$ and $a = \sqrt{R^2 + a_0}$ with the $r_{pol}$ and $z_{pol}$ given by the VMEC Fourier series describing the plasma boundary:

$$r(u, v) = \sum_{m,n} r_{m,n} \cos(2\pi (mu - nNv)),$$

$$z(u, v) = \sum_{m,n} z_{m,n} \sin(2\pi (mu - nNv)),$$

where $r$ and $z$ are cylindrical coordinates, $u$ the VMEC poloidal and $v$ the VMEC toroidal angle [19]. Both VMEC angles vary from 0 to 1 for one rotation.

Additionally the pressure profile is proportional to

$$p(s) \propto 1 - 0.8s + 1.3s^2 - 1.5s^3,$$

where $s$ is the normalized toroidal flux and the current density profile is given by

$$\frac{dI(s)}{ds} \propto 14s - 54s^2 + 80s^3 - 40s^4,$$

where $l(s)$ is the total toroidal current enclosed by the flux surface $s$.

The targeted equilibrium properties for the optimization are:

- Rotational transform, $\iota$, at the magnetic axis and at the plasma boundary, where the rotational transform is the change of the poloidal flux $\chi$ with respect to the toroidal flux $\psi$:

$$\iota = \frac{d\chi}{d\psi}.$$

- Vacuum rotational transform, $\iota_{vac}$, at the magnetic axis
- Vacuum magnetic well $\frac{\partial}{\partial s} \int \frac{d}{\partial} < 0$, which is numerically evaluated as:

$$2 \frac{\Phi'(0) - \Phi' (\Delta \rho)}{\Phi'(0) + \Phi'(\Delta \rho)},$$

where $\Phi'$ is the specific volume given by $\Phi' = \int \frac{d}{\partial}$ and $\Delta \rho$ is a small distance away from the axis.

- The integrated absolute value of the Gaussian curvature of the plasma boundary, $|K| = |\kappa_1 \kappa_2|$, where $\kappa_1$ and $\kappa_2$ are the two principle curvatures:

$$\int_{\delta P} |K| ds,$$

where the integral is over the plasma boundary $\delta P$.

- And at a particular flux surface $s = s_{qa}$ the qa error

$$E_{qa}(s) = \sqrt{\sum_{m,n} B_{m,n}^2(s)} B_{00}(s = 0),$$

where the magnetic field strength is given by

$$B = \sum_{m,n} B_{m,n} \cos(m\theta + nN\phi).$$

We performed a scan by varying the flux surface $s_{qa}$ on which the qa error was minimized. All the other input parameters of the optimization were fixed.

2.2. Fast-particle losses

The confinement of fast alpha particles with energies of 3.5 MeV was investigated without collisions using the full-f Monte-Carlo code ANTS [21].
First, the designs were scaled to reactor size with a volume of 1900 m$^3$, and with a volume averaged magnetic field of 5 T. Next, one thousand test particles were evenly distributed on each evaluated flux surface and then launched with uniformly distributed pitch angles similar to a fusion-produced fast-particle population. The particles were traced for half a second. Every particle which crosses the plasma boundary is counted as lost.

### 3. Dependence of fast-particle losses on the choice of qa-location

The qa error $E_{qa}$, defined in equation (9), was minimized at different radial locations $s_{qa}$ for eight different optimization runs$^1$. This led to small changes in some of the other targeted quantities, see table 1.

The minimum value of $E_{qa}$ after the optimization is approximately located where it was minimized $s_{qa}$, see figure 1(a) which makes it possible to examine how the location $s_{qa}$ of a minimum value of $E_{qa}$ changes the properties of the design. The qa quality deteriorates away from that location as one would expect from $[7, 8]$. The smallest value for the error ($E_{qa} = 0.102\%$) is found for the smallest flux surface investigated: $s_{qa} = 0.06$. Towards $s_{qa} = 1$ there is a slight increase in the the minimum qa-error: $E_{qa} = 0.388\%$.

$^1$ The VMEC input files are available upon request.

### Table 1. Targeted quantities for all designs.

| $s_{qa}$  | 0.06 | 0.125 | 0.25 | 0.4 | 0.5 | 0.75 | 0.9 | 1.0 |
|-----------|------|-------|------|-----|-----|------|-----|-----|
| $\epsilon (s = 0.0)$ | 0.35 | 0.35 | 0.35 | 0.33 | 0.33 | 0.313 | 0.32 | 0.32 |
| $\epsilon (s = 1.0)$ | 0.43 | 0.43 | 0.43 | 0.42 | 0.41 | 0.39 | 0.38 | 0.39 |
| $\epsilon_{vac} (s = 0.0)$ | 0.34 | 0.34 | 0.34 | 0.32 | 0.31 | 0.28 | 0.29 | 0.29 |
| Vacuum magnetic well | 0.015 | 0.016 | 0.016 | 0.015 | 0.014 | 0.01 | 0.009 | 0.008 |
| Gaussian curvature | 16 | 16 | 16 | 15.8 | 15.6 | 16 | 15.6 | 15.4 |
| $E_{qa}(s_{qa})$ | 0.001 | 0.0011 | 0.002 | 0.0027 | 0.0036 | 0.0043 | 0.0035 | 0.0039 |

![Figure 1. Overview of quasi-axisymmetric-error $E_{qa}$ profile and effective ripple profile for designs with different flux surfaces $s_{qa}$ on which the quasi-axisymmetric error is optimized. (a) The profiles of the quasi-axisymmetric error $E_{qa}$ for the different optimized designs. (b) The profiles of $\epsilon_{eff}$ for the different optimized designs.](image1.png)

First, the designs were scaled to reactor size with a volume of 1900 m$^3$, and with a volume averaged magnetic field of 5 T. Next, one thousand test particles were evenly distributed on each evaluated flux surface and then launched with uniformly distributed pitch angles similar to a fusion-produced fast-particle population. The particles were traced for half a second. Every particle which crosses the plasma boundary is counted as lost.

![Figure 2. Fast-particle loss-fraction rate after 0.5 s of particles launched on different flux surfaces $s_{launched}$ for different qa-optimization locations $s_{qa}$. For smaller flux surfaces the best results are obtained for $s_{qa} = 0.5$ then for $s_{qa} = 0.4$.](image2.png)

This suggests that it is more difficult to optimize for the qa-error at the plasma edge with the additional targets of the optimization presented in this paper.

Interestingly, the location of the minimum values of the effective ripple, which is a measure of neoclassical transport [22], does in general not coincide with $s_{qa}$ see figure 1(b). The
The following integrals were evaluated:

$$\bar{f} \equiv \int_0^1 f \rho d\rho$$

for \( f = E_{qa} \) and \( f = \epsilon_{\text{eff}} \) where \( \rho \equiv \sqrt{\tau} \). This integral is approximately equal to the volume average of \( f \). For both quantities, \( E_{qa}(s_{qa}) \) and \( \epsilon_{\text{eff}}(s_{qa}) \), the minimum is approximately located at \( s_{qa} = 0.5 \). Therefore, the volume averaged values of the qa error and the effective ripple are minimized for \( s_{qa} = 0.5 \).

As is clear from figure 2, for particles launched on flux surfaces on \( s = 0.5 \) or farther inside the designs with \( s_{qa} = 0.4 \) and \( s_{qa} = 0.5 \) are clearly better than the others, but for larger flux surfaces this advantage disappears. For comparison, the volume-average of the fast-particle losses given by equation (11) was evaluated, see figure 3, and the lowest averaged losses are also found for \( s_{qa} = 0.5 \) and \( s_{qa} = 0.4 \).

This result might seem surprising at first. If one could reach perfect qa symmetry at only one flux-surface, then one would perhaps choose the edge \( s_{qa} = 1 \), since all the particles of the confined plasma would have to cross this flux surface to leave the plasma. However, in practice perfect quasi-axisymmetry is not achieved anywhere in general and the quality of qa-symmetry deteriorates in both directions away from its

Figure 3. The effect of varying the location for the QA error optimization: the dependence of the volume-averaged fast-particle loss fraction on the location of quasi-axisymmetric optimization \( s_{qa} \).

Figure 4. Fast-particle loss-fraction rate after 0.5 s of particles launched on different flux surfaces \( s_{\text{launched}} \) with collisions. Dashed lines are particle losses and solid lines are energy losses. Different colors are for different qa-optimization locations \( s_{qa} \). For smaller flux surfaces the best results are obtained for \( s_{qa} = 0.5 \) then for \( s_{qa} = 0.4 \).

Figure 5. The effect of varying the location for the QA error optimization: the dependence of the volume-averaged fast-particle loss fraction with collisions on the location of quasi-axisymmetric optimization \( s_{qa} \).

Figure 6. The poloidal cross-section of the flux surfaces for the toroidal angels \( \phi = 0^\circ, 45^\circ, \) and \( 90^\circ \) for the \( s_{qa} = 0.5 \) (purple, dashed lines) and the \( s_{qa} = 0.06 \) design (blue, solid lines).
minimum value at \( s_{qa} \). Choosing \( s_{qa} \) somewhat between half-radius and the edge requires a large number of particles to pass through this surface and also reduces the maximum value of the qa-error \( E_{qa} \). In addition, while there exists a minimum at only one location (approximately at \( s_{qa} \)), nearby flux surfaces will also have a relatively low value for \( E_{qa} \). By locating \( s_{qa} \) at the edge, one loses around half the nearby, good flux surfaces. Conversely, optimizing at the axis only allows a small fraction of particles to be confined in the highly-optimized region. For these reasons, the \( s_{qa} = 0.5 \) design has the smallest averaged \( E_{qa} \) value of all designs.

3.1. Fast-particle losses with collisions

Up to this point, all fast-particle losses were calculated without considering collisions. There are two reasons why this is a sensible approach. First, if collisions are to be retained, one must assume specific density and temperature profiles, which are uncertain since the exact transport is not known, which creates an certain arbitrariness. Second, the most detrimental losses of alpha particles are the fast losses because those particles cannot transfer their energy to the bulk plasma. To evaluate those fast losses which happen without many collisions, one can neglect the collisions.

To determine whether collisions alter the location of the optimum flux surface \( s_{qa} \), we used the same profiles as in [23]. The temperature and radial electric field profiles are self-consistent with the deuterium and tritium profiles. Ten thousand test particles were followed for each evaluated flux surface.

The smallest losses were again achieved with \( s_{qa} = 0.5 \) and \( s_{qa} = 0.4 \) for flux surfaces near the magnetic axis, see figure 4. Interestingly, the energy losses of the design with \( s_{qa} = 1.0 \) improved compared to the case without collisions but averaged over the entire volume it has more losses than the \( s_{qa} = 0.5 \) case, see figure 5. In general, the volume averaging favors the \( s_{qa} = 1.0 \) case as its best results compared to the other \( s_{qa} \) lies closer to the plasma edge which encompasses the largest volume.

Thus, at least for this specific set of profiles, the approach described above remains valid even when collisions are taken into account. More generally, if reliable plasma density and temperature profiles are available one could try to minimize the energy losses of fast ions but simultaneously maximize their particle losses, in order to alleviate the problem of ash-removal.

4. Difference between the best and worst design

The best design with respect to the fast-particle loss fraction is now briefly compared with the worst design. Only major differences are pointed out since both designs are very similar to that with \( s_{qa} = 0.4 \), which was presented in greater depth in [23]. It is shown there that the design is stable to ideal magnetohydrodynamic instabilities and possesses low fast-particle losses. Additionally, the neoclassical transport coefficients are shown to be almost equivalent to those of a tokamak, with a clear banana regime at half-radius.

Despite the great difference in fast-particle confinement, the shape of flux surfaces only varies slightly, as shown in figure 6. However, since the non-qa components of the magnetic field strength at \( s = s_{qa} \) was the only difference in the optimization procedure, Fourier spectra of the magnetic field strength, already visible in figure 1(a), differ, see figure 7. The greatest difference is in the mirror component \( B_{0}\). Only the components \( B_{mn} \) with \( m \neq 0 \) components are finite on axis. Therefore if quasi-axi-symmetry is optimized near the axis these contributions tend to change the most. Other evident differences are the largest components at the edge: for \( s_{qa} = 0.5 \) there are three qa contributions \( B_{m0} \) out of the largest four components, but only two for the \( s_{qa} = 0.06 \) case.

We next compare the fast-particle losses with respect to time, see figure 8: the biggest relative change is for the

![Figure 7. Comparison of the Fourier spectra of the magnetic field strength normalized to \( B_{0} \) for the designs optimized with \( s_{qa} = 0.5 \) and \( s_{qa} = 0.06 \). (a) Fourier spectrum of the magnetic field for design optimized with \( s_{qa} = 0.5 \). (b) Fourier spectrum of the magnetic field for design optimized with \( s_{qa} = 0.06 \).](image-url)
innermost flux surface \( s = 0.06 \). There are 14 times more losses for the \( s_{qa} = 0.06 \) than for the best design. The fast-particle losses launched on the flux surface \( s = 0.25 \) only differ by a factor of 3.5. In the case with collisions, figure 9, the trends are similar as for the case without collisions. Both the particle and energy losses of the fast ions are lower in the configuration with better quasisymmetry.

It might again be surprising that the largest difference appears to be at \( s = 0.06 \) since the \( s_{qa} = 0.06 \) design has its minimum value of both the \( qa \)-error \( E_{qa} \) and effective ripple \( \epsilon_{eff} \) at this radial location. But this again supports the picture that the \( E_{qa} \) of the entire plasma matters since any particle which is lost has to drift across all flux surfaces outside the one on which it was born.

5. Conclusion

It has been numerically shown that quasi-axisymmetry does not, in general, provide the best fast-particle confinement when
optimized at the magnetic axis or at the edge of the plasma. For the special case presented here, the optimal flux surface for minimizing the qa error has been identified to be $s_{qa} = 0.5$. This can be explained by two beneficial effects: first, the closer this flux surface is to the plasma edge the more volume is enclosed by it and, second, the plasma region close to the flux-surface $s_{qa}$ is maximized, which reduces the maximum qa-error $E_{qa}$ and decreases the average $E_{qa}$ in the equilibrium. These arguments suggest that the qa error should generally be optimized somewhere between $s = 0.25$ and $s = 1$.

This finding also means that designs found through analytic near-axis expansion are not necessarily optimal for maximizing confinement and could probably be improved by optimizing with an $s_{qa}$ other than at the axis.

An additional motivation to optimize qa-symmetry away from the edge is that the neoclassical transport seems not to be the dominant transport process in the outer region in Wendelstein 7–X [24], and therefore the optimization of qa-symmetry there might not have a great impact of the overall transport.

The approach presented here has already led to a qa symmetric, ideal MHD stable design with improved fast-particle loss-fractions, and with neoclassical transport coefficient similar to an equivalent tokamak design at half-radius [23].

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References

[1] Klinger T et al 2019 Overview of first wendelstein 7–x high performance operation Nucl. Fusion 59 112004
[2] Boozer A H 1981 Plasma equilibrium with rational magnetic surfaces Phys. Fluids 24 1999–2003
[3] Nührenberg J and Zille R 1988 Quasi-helically symmetric toroidal stellarators Phys. Lett. A 129 113–7
[4] Boozer A H 1995 Quasi-helical symmetry in stellarators Plasma Phys. Control. Fusion 37 A103
[5] Nührenberg J, Lotz W and Gori S 1994 Quasi-axisymmetric tokamaks Theory of Fusion Plasmas Varenna 1994 (Bologna: Editrice Compositori) p 3
[6] Garabedian P R 1996 Stellarators with the magnetic symmetry of a tokamak Phys. Plasmas 3 2483–5
[7] Plunk G G and Helander P 2018 Quasi-axisymmetric magnetic fields: weakly non-axisymmetric case in a vacuum J. Plasma Phys. 84 905840205
[8] Garren D A and Boozer A H 1991 Magnetic field strength of toroidal plasma equilibria Phys. Fluids B 3 2805–21
[9] Landreman M and Sengupta W 2018 Direct construction of optimized stellarator shapes: I. Theory in cylindrical coordinates J. Plasma Phys. 84 905840616
[10] Landreman M, Sengupta W and Plunk G G 2019 Direct construction of optimized stellarator shapes. II. Numerical quasiisymmetric solutions J. Plasma Phys. 85 905880103
[11] Beidler C et al 1990 Ewald Harmeyer. Physics and engineering design for wendelstein vii–x Fusion Technol. 17 148–68
[12] Canik J, Anderson D T, Anderson F S B, Likin K M, Talmadge J N and Zhai K 2007 Experimental demonstration of improved neoclassical transport with quasihelical symmetry Phys. Rev. Lett. 98 085002
[13] Canik J M, Anderson D T, Anderson F S B, Clark C, Likin K M, Talmadge J N and Zhai K 2007 Reduced particle and heat transport with quasiisymmetry in the helically symmetric experiment Phys. Plasmas 14 056107
[14] Nelson B E et al 2003 Design of the national compact stellarator experiment (NCSX) 22nd Symposium on Fusion Technology; Fusion Eng. Des., 66(Supplement C) 169–74
[15] Okamura S et al 2001 Physics and engineering design of the low aspect ratio quasi-axisymmetric stellarator CHS-qa Nucl. Fusion 41 1865
[16] Drevlak M, Brochard F, Helander P, Kisslinger J, Mikhailov M, Nührenberg C, Nührenberg J and Turkyn Y 2013 ESTELL: a quasi-toroidally symmetric stellarator Contrib. Plasma Phys. 53 459–68
[17] Nührenberg C, Mikhailov M I, Nührenberg J and Shafranov V D 2010 Quasi-helical symmetry at finite aspect ratio Plasma Phys. Rep. 36 558–62
[18] Drevlak M, Beidler C B, Geiger J, Helander P and Turkyn Y 2019 Optimisation of stellarator equilibria with ROSE Nucl. Fusion 59 016010
[19] Hirshman S P and Whitson J C 1983 Steepest descent moment method for three dimensional magnetohydrodynamic equilibrium Phys. Fluids 26 3553–68
[20] Brent R P 1973 Algorithms for Minimization without Derivatives (Englewood Cliffs, NJ: Prentice-Hall) ch 5
[21] Drevlak M, Geiger J, Helander P and Turkyn Y 2014 Fast particle confinement with optimized coil currents in the W7-X stellarator Nucl. Fusion 54 073002
[22] Nemov V V, Kaslov S V, Kernbichler W and Heyn M F 1999 Evaluation of 1/2 neoclassical transport in stellarators Phys. Plasmas 6 4622–32
[23] Henneberg S A, Drevlak M, Nührenberg C, Beidler C D, Turkyn Y, Loizu J and Helander P 2019 Properties of a new quasi-axisymmetric configuration Nucl. Fusion 59 026014
[24] Dinklage A, Beidler C D, Helander P et al 2018 Magnetic configuration effects on the wendelstein 7–x stellarator Nat. Phys. 14 855–60