HALF-INTEGER TOPOLOGICAL CHARGES BELOW AND ABOVE THE DECONFINEMENT TRANSITION?

E.-M. Ilgenfritz\textsuperscript{a}, M. Müller–Preussker\textsuperscript{b}, and A. I. Veselov\textsuperscript{c}

\textsuperscript{a} University of Kanazawa, Institute of Theoretical Physics, Kanazawa 920-1192, Japan
\textsuperscript{b} Humboldt-Universität zu Berlin, Institut für Physik, D–10115 Berlin, Germany
\textsuperscript{c} Institute of Theoretical and Experimental Physics, Moscow, 117259, Russia

Abstract. For pure $SU(2)$ lattice gauge theory at finite $T$, by the help of the cooling method, we search for classical (approximate) solutions having non-trivial holonomy at the spatial boundary. We identify various typical objects and provide their relative frequency of occurrence for the confinement and deconfinement phases. Among the configurations obtained we see also the dissociated BPS monopole pairs recently discussed by van Baal and collaborators.

1 Introduction

During the last two years, the carriers of topological charge in a Yang-Mills field at finite temperature (calorons) have been thoroughly reconsidered by van Baal and co-workers. The progress has been summarized at this conference [1]. It has been demonstrated that completely different caloron solutions appear once a non-trivial holonomy $\mathcal{P}(x)$ at $|x| \to \infty$ is admitted; the Polyakov line $L(x) = \frac{1}{2} \text{tr} \mathcal{P}(x)$ is the trace of the holonomy. Prior to this development, semiclassical models at finite temperature were based exclusively on properties of periodic instantons,

\footnote{Presented by A. I. Veselov at the NATO Advanced Research Workshop Lattice Fermions and Structure of the Vacuum, October 1999, Dubna, Russia.}
being classical solutions with trivial holonomy \( i.e. \mathcal{P}(x) \rightarrow 1 \) for \(|x| \rightarrow \infty \) (‘t Hooft periodic instanton) \[2, 3, 4\].

The outstanding feature of these new calorons is the fact that (within a certain parameter range) monopole constituents of an instanton can become explicit as degrees of freedom \[5\]. They carry magnetic as well as electric charge (in fact, they are Bogomol’ny-Prasad-Sommerfield (BPS) monopoles or ‘dyons’) and \( 1/N_{\text{color}} \) units of topological charge. Being part of classical solutions of the Euclidean field equations, one can hope that the instanton constituents can play an independent role in a semiclassical analysis of \( T \neq 0 \) Yang-Mills theory (and of full QCD).

The variety of selfdual solutions for various topological charge \( Q \) has been discussed as classical solutions on the lattice with nontrivial holonomy from twisted boundary conditions \[6\].

The nontrivial holonomy \( \textit{per se} \) is no obstacle to a semiclassical approach if that is not restricted to the one-instanton approximation. To what extent such a semiclassical description is reliable (and exhaustive), has to be investigated for each phase (confinement and deconfinement) of pure Yang-Mills theory.

In exploratory studies we have searched for characteristic differences between the two phases as far as quasiclassical background fields are concerned. These become visible in the result of cooling.

In a previous study \[7\] we discussed finite temperature \( SU(2) \) lattice gauge theory in a finite spatial box with specific boundary conditions. The latter were chosen such that the finite (not too large) system was put into a definite single-monopole background field. By varying the monopole scale we could study both the situations: the purely magnetic ‘t Hooft-Polyakov (HP) monopole and the self-dual BPS monopole (‘dyon’). We observed a specific influence of these different boundaries on the quantum state inside the box. Whereas the HP monopole has turned out to favour deconfinement, the BPS monopole has been keeping the system in the confinement even at temperatures larger then the usual critical one. Let us note that these boundary conditions are characterized by different holonomy values, too.

In comparison with the previous study in our present work we have employed simpler spatial boundary conditions. We have fixed and have left untouched under cooling only the boundary time-like link variables in order to keep a certain value of \( \mathcal{P}(x) = \mathcal{P}_\infty \) everywhere on the spatial surface of the system while conserving periodicity.

In this study, the influence of the respective phase, that we want to describe, is twofold: (i) the cooling starts from genuine thermal Monte Carlo gauge field configurations, generated on a \( N_s^3 \times N_t \) lattice; (ii) the value of the holonomy \( \mathcal{P}_\infty \) was chosen in accordance with the average of \( L \), which is vanishing in the confinement phase and nonvanishing in deconfinement.
2 Results

We consider SU(2) lattice gauge theory with the standard Wilson action. Our cooled samples are obtained from Monte Carlo ensembles on a $16^3 \times 4$ lattice. For $N_t = 4$, the gauge coupling $\beta = 2.2$ stands for the confinement phase with $\langle L \rangle \simeq 0$, and $\beta = 2.4$ for the deconfinement phase with $\langle L \rangle = 0.27$, respectively. The timelike links $U_{x,\mu=4}$ are frozen at the spatial boundary equal to each other such that $(U_{x,\mu=4})^{N_t} = P_\infty$. For the holonomy itself, an ‘Abelian’ form $P_\infty = a_0 + i a_3 \tau_3$ was chosen, with $a_0 = \langle L \rangle$ and $a_3 = \sqrt{1 - a_0^2}$ in correspondence with the average Polyakov line. The cooling method chosen was the simplest, rapid relaxation method keeping the Wilson action. In order to search exclusively for objects with low action the criterion for the first stopping at some cooling step $n$ was that $S_n < 2 S_{\text{inst}}$, the last change of action $|S_n - S_{n-1}| < 0.01 S_{\text{inst}}$, and that the second derivative $S_n - 2 S_{n-1} + S_{n-2} < 0$ ($S_{\text{inst}}$ denoting the action of a single instanton). For each $\beta$-value we have scanned $O(200)$ configurations obtained by cooling.

The cooled sample taken from the confinement phase has a clearly different composition compared to that from the deconfinement ensemble. This is listed in Table 1 which shows the relative occurrence of the few characteristic types of non-perturbative configurations. In the following let us explain these configurations in some detail.

| Type of solution | $\beta = 2.2$ | $\beta = 2.4$ |
|------------------|--------------|--------------|
| $DD$             | $0.63 \pm 0.08$ | $0.02 \pm 0.01$ |
| $D\bar{D}$      | $0.27 \pm 0.05$ | $0.78 \pm 0.07$ |
| $CAL$            | $0.02 \pm 0.01$ | $0.$          |
| $M, 2M$          | $0.01 \pm 0.01$ | $0.07 \pm 0.02$ |
| trivial vacuum   | $0.07 \pm 0.03$ | $0.13 \pm 0.03$ |

Confinement phase. Here dominate selfdual (or antiselfdual) configurations, and among them ‘dyon-dyon’ pairs ($DD$) which are reminiscent of the new caloroon solutions. In Fig. 1 we show, projected onto the $x_1 - x_2$-plane (i.e. summed over $x_3, x_4$ or $x_3$, resp.), the topological charge and the Polyakov line of such a ‘dyon’ pair. Notice the opposite sign of the Polyakov line near to the two same-sign topological charge bumps.

Other selfdual objects, having a rather $O(4)$ rotationally invariant distribution of action and topological charge, are frozen out relatively infrequently. They
Figure 1: (a) The 2D projected distribution of topological charge for a selfdual \('dyon-dyon\)' pair (DD) discovered by cooling in a $16^3 \times 4$ thermal configuration generated at $\beta = 2.2$; (b) Similarly for the Polyakov line.

Figure 2: (a) The 2D projected distribution of topological charge for a selfdual, rotationally symmetric caloron (CAL) discovered by cooling in a $16^3 \times 4$ thermal configuration generated at $\beta = 2.2$; (b) Similarly for the Polyakov line.

resemble the 't Hooft periodic instanton. We call them caloron (CAL), shown in Fig. 2. Under the specific boundary conditions, however, the structure of the Polyakov line around the caloron is nontrivial in the sense that it has the opposite peaks of the Polyakov line near the center of the action (and topological charge) distribution. Thus, this type of configurations appears as a limiting case of the 'dyon-dyon' pairs.

Mixed configurations with two lumps of opposite topological charge are found in a quarter of the configurations. We call them 'dyon-antidyon' pairs (DD). In fact, they are typical for the deconfined phase and below we discuss an example taken from $\beta = 2.4$. In these configurations the Polyakov line has a same-sign maximum on top of the opposite-sign topological charge lumps. Besides of this, the two sums $Q_+ = \sum_x q(x) \Theta(q(x))$ and $Q_- = \sum_x q(x) \Theta(-q(x))$ are
Figure 3: (a) The 2D projected distribution of topological charge for a mixed ‘dyon-antidyon’ pair ($\overline{DD}$) discovered by cooling in a $16^3 \times 4$ thermal configuration generated at $\beta = 2.4$; (b) Similarly for the Polyakov line.

almost equal to $\pm \frac{1}{2}$ and $\pm \frac{1}{2}$, respectively, which supports an interpretation as half-instanton and half-antiinstanton.

When one allows the holonomy $\mathcal{P}(\mathbf{x})$ freely to relax as in usual cooling with periodic boundary conditions, configurations like $DD$ and $\overline{DD}$ do not survive. Deconfinement phase. It is remarkable that selfdual or antiselfdual $DD$ configurations are very rare in this case. $\overline{DD}$ mixed configurations are typical for the deconfined phase. For one example we show in Fig. 3 the topological charge and Polyakov line (similar to Fig. 1).

In the deconfined phase the next important type of cooled configurations are purely magnetic ones ($S_{\text{magnetic}} >> S_{\text{electric}}$) with quantized action in units of $S_{\text{inst}}/2$. We call these $M$ configurations. With a smaller probability also magnetic configurations with twice as large action ($2M$ type configurations) are found. In all projections, the action is constantly distributed over the lattice with a high precision. A closer look at the different field strength components reveals that the action of $M$ type configurations resides only in a single magnetic field strength component (in one 3D direction), while in the case of $2M$ type configurations two such fluxes, generically orthogonal to each other, are present. After fixing the maximally Abelian gauge (see below) these configurations turn out to be completely Abelian. Therefore, we can identify these configurations as pure equally distributed magnetic fluxes which should be related to world-sheets of Dirac strings (‘Dirac sheet’) on the dual lattice. With some rate they also emerge in the result of further cooling of $\overline{DD}$ configurations. Such configurations are present in the confinement phase as well, but occur with tiny probability only.

Maximizing with respect to gauge transformations the gauge functional $R = \sum_{x\mu} \text{tr} \left( \tau_3 \ U_{x\mu} \tau_3 \ U_{x\mu}^+ \right)$, we have put all these configurations into the maximally Abelian gauge and have measured their ‘Abelianicity’, i.e. $R_{\text{max}}$ per link. The $DD$ and $\overline{DD}$ configurations have an Abelianicity of 99.8% (independent of the
phase where they are found) The rotationally invariant caloron is somewhat more Abelian (99.9%). But for the purely magnetic configurations we found an Abelianicity of exactly 100%.

Employing the Abelian projection, $U(1)$ monopoles can be localized. The $DD$ and $D\bar{D}$ configurations are found to be static to a high precision, and Abelian monopole worldlines were observed to coincide with the ‘dyon’ or ‘antidyon’ position, again irrespective of the phase where these background configurations have been extracted. The non-static ‘caloron’ configuration, however, is typically enclosed by a small Abelian monopole loop (of length 6).

3 Conclusions

We have studied finite temperature Yang-Mills lattice fields with given non-trivial holonomy at the spatial boundary of a finite box. Starting from Monte Carlo equilibrium configurations by cooling we have found quasi-stable solutions in accordance with that (periodic) boundary condition. The ensembles of solutions obtained strongly depend upon whether we are in the confinement or in the deconfinement phase. Most typically we observe ‘dyon-dyon’ pairs within the confinement phase, i.e. selfdual solutions of the type discussed by van Baal and collaborators. However, in the deconfinement phase ‘dyon-antidyon’ solutions dominate. The latter objects have to be understood analytically. We did not find pure magnetic HP-like monopoles as seen in [3], where cooling had been applied to finite temperature fields with purely periodic boundary conditions. The latter monopoles - always accompanied by a spurious opposite charged monopole - have trivial holonomy and, thus, could not show up in the present analysis.

We feel that the development of a semiclassical approach based on solutions with non-trivial holonomy, i.e. in a mean-field-like setting, might have a chance to shed light on the mechanisms of the deconfinement transition.

Acknowledgements

The authors are grateful to P. van Baal, B. V. Martemyanov, S. V. Molodtsov, M. I. Polikarpov, A. van der Sijs, Yu. A. Simonov, and J. Smit for useful discussions. This work was partly supported by RFBR grants N 97-02-17491 and N 99-01-01230 as well as by the joint RFBR-DFG project grant 436 RUS 113/309 (R) and the INTAS grant 96-370.
References

[1] P. van Baal (1999), these Proceedings, e-Print Archive: hep-th/9912035.

[2] G. ’t Hooft (1976), unpublished; see R. Jackiw, C. Nohl, and C. Rebbi (1977), Phys. Rev., D15, p. 1642.

[3] B.J. Harrington and H.K. Shepard (1978), Phys. Rev., D17, p. 2122; (1978), Phys. Rev., D18, p. 2990.

[4] D.J. Gross, R.D. Pisarski, and L.G. Yaffe (1983), Rev. Mod. Phys., 53, p. 43.

[5] T.C. Kraan and P. van Baal (1998), Phys. Lett., B435, p. 389.

[6] M. Garcia Perez, A. Gonzalez-Arroyo, A. Montero, and P. van Baal (1999), JHEP 06, p. 001; e-Print Archive: hep-lat/9903022.

[7] E.-M. Ilgenfritz, S.V. Molodtsov, M. Müller-Preussker, and A.I. Veselov (1999), Eur. Phys. J., C8, p. 335.

[8] T.A. DeGrand and D. Toussaint (1980), Phys. Rev., D22, p. 2478.

[9] M.L. Laursen and G. Schierholz (1988), Z. Phys., C38, p. 501.