Design of Digital Pulse-Position Modulation System

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Abstract. Pulse-Position Modulation (PPM) is a modulation method that only makes every pulse in the carrier pulse sequence change with time but without changing the shape and amplitude of the pulse signal. In this paper, a PPM system is designed. Firstly, an appropriate mathematical model is established to represent PPM transmission, and the shape of the pulse signal is designed. After that, we write the code and add a white Gaussian noise channel. Then the transmission process is simulated and visualized. At last, the error rate of the scheme is analyzed and discussed through MATLAB simulation then compared with other modulation methods. The goal of this paper is to study PPM by designing a PPM system fully. Besides, our method is compared with other modulation methods to understand the advantages and disadvantages of PPM. This may help other scholars to design and research the PPM system.

1. Introduction

Wireless communication technology, in essence, uses a wireless transmission medium to achieve interconnection between terminals. This kind of wireless medium can be an electromagnetic wave or light wave. In order to prevent the signals of multiple terminals from interfering with each other, we need to modulate the electromagnetic wave signal to represent the information to be transmitted. According to the characteristics of electromagnetic waves, signal modulation can be realized in different ways. For example, adjust the electromagnetic wave amplitude to make the amplitude of carrier wave change according to the change law of the required transmission signal, but the frequency remains unchanged. This kind of modulation is called amplitude modulation (AM) [1]. Because of its strong anti-interference ability and slight distortion, AM is widely used in cable or radio communication and broadcasting. Pulse modulation mainly includes pulse amplitude modulation (PAM) [2], pulse intensity modulation (PIM) [3], pulse width modulation (PWM) [4], pulse frequency modulation (PFM) [5], and pulse position modulation (PPM) [6]. Suppose the modulation signal only changes the generation time of each pulse in the carrier pulse series without changing its shape or amplitude. The change in generation time is proportional to the amplitude of the modulation signal voltage and independent of the modulation signal frequency. In that case, this type of modulation is known as pulse position modulation. Using PPM, M message bits are conveyed by sending a single pulse in one of 2M potential time shifts [7]. Since there is little multipath interference in optical communication systems, PPM is widely used in optical communication systems, such as optical label switching [8]. J.R.Pierce of California Institute of technology first proposed pulse position modulation technology. It was initially applied in the field of atmospheric laser communication. Strictly speaking,
PPM modulation technology is a way of amplitude modulation. The traditional amplitude modulation uses the signal amplitude to load information, which can improve the anti-interference ability of the channel. At the same time, compared with the traditional amplitude modulation, the requirement of PPM for laser power is not very high so that it can achieve higher information transmission efficiency with the minimum average optical power [9]. In addition to optical communication, PPM has also been extended in radar. Cooper first proposed the random PPM signal in 1976 and derived the average ambiguity function of the signal. The research shows that the frequency structure of the average ambiguity function of the pulse sequence largely depends on the frequency structure of the characteristic function of the pulse interval, and the characteristic function of the pulse interval depends on the random quantity of the modulation. The random PPM signal is essentially similar to the random interleaved pulse sequence in radar applications [10].

In recent years, scholars have carried out some research on PPM, from which we have learned some knowledge about PPM, such as the difference of EBR between PPM and PAM [11]. In this paper, we search for materials and related literature and research them. Then we finish the square wave signal design, data sampling and quantization, and modulation model design. Lastly, the demodulation model is designed, and we analyze the BER, compare PPM with BPSK.

This paper is organized as follows: The research background and current situation are introduced in the first part. In the second part, the design method and model of this paper are introduced. In the third part, the model is simulated and verified, and the results are analyzed. In the fourth part, this paper is summarized.

2. METHODOLOGY AND MODEL

2.1. Model

![System model](image)

Our modulation model is shown in Figure 1. The source signal (digital signal) is introduced into the PPM modulation system, and the corresponding waveform represents the 0,1 signal to be transmitted. After modulation, it is put into the channel for (Baseband) transmission, and there will be additive Gaussian white noise interference in the transmission process. After being transmitted to the receiving end, demodulate accordingly to restore the actual baseband signal.

2.2. Pulse shape

Considering that the digital signal modulation uses the analogue signal waveform to represent 0 and 1, this paper mainly studies the communication system of PPM modulation. We choose the square wave with a simple model to get the pulse image which is easier to observe and analyze. There are two kinds of square waves, and their expressions are as follows:

\[
g_1(t) = \begin{cases} 
1 & 0 \leq t \leq \frac{T}{2} \\
0 & \frac{T}{2} \leq t \leq T 
\end{cases}
\]  

(1)
2.3. Modulation and signal transmission

2.3.1. Detail od modulation: When the analogue signal is transmitted, it needs to be sampled and quantized to get the discrete digital signal. The original signal is \( \cos 2 \pi ft \), and its frequency is \( f \), the sampling frequency that we select is \( f_s = 10f \). The figure after sampling is as follows:

![Signal waveform after sampling](image)

It can be seen that the expressions of \( g_1(t) \) and \( g_2(t) \) correspond to the waveforms in each cycle, respectively. And the waveforms of the whole transmission process are \( s(t) \), which is represented above.

After the quantizer quantizes the signal, the corresponding binary sequence of 0 and 1 (uniform quantization, quantization level is 8) can be obtained, the quantized discrete data. After PPM modulation, 0 is represented by \( g_1(t) \) and 1 is represented by \( g_2(t) \), the modulation signal \( S_1(t) \) can be written as:

\[
g_2(t) = \begin{cases} 
0 & \text{if } 0 \leq t \leq \frac{T}{2} \\
1 & \text{if } \frac{T}{2} \leq t \leq T 
\end{cases}
\]
The image through the AWGN channel after modulation is shown in Figure 3, and the corresponding formula is:

\[ s(t) = S_n(t) + n(t) = g_{ak}(t - kT) + n(t) = r(t) \]  

(4)

Which is the signal received by the receiving terminal.

Where \( a_k \in \{1,2\} \), \( T \) are pulse periods. Several of these pulses are combined to form \( S_n(t) \). After modulation, the signal figure is shown in Figure 3.

2) BPSK modulation (Binary Phase Shift Keying):

When modulating, we only use one carrier \( \cos \omega_0 t \), and \( s(t) \) can be expressed as:

\[ s(t) = I \cos(\omega_0 t) - Q \sin(\omega_0 t) \]  

(5)

When the input binary signal is 0, \( I = 1 \), \( Q = 0 \), the modulated waveform is:

\[ s(t) = \cos(\omega_0 t) \]  

(6)

When the input binary signal is 1, \( I = -1 \), \( Q = 0 \), the modulated waveform is:

\[ s(t) = -\cos(\omega_0 t) = \cos(\omega_0 t + \pi) \]  

(7)

Thus, we can get the corresponding modulation waveform in each signal cycle.

3) QPSK modulation (Quadrature Phase Shift Keying):

The principle of QPSK is similar to BPSK, except that four different carrier phases are used to represent digital information. Each carrier represents two bits of information, so each quaternary symbol is also called a double-bit symbol. The modulation signal is similar to BPSK in form, expressed as:

\[ s(t) = I \cos(\omega_0 t) - Q \sin(\omega_0 t) \]

\[ = \sqrt{2} \cos(\omega_0 t + \phi_n) \]  

(8)

When the input binary signal is 11, \( I = 1 \), \( Q = 1 \), the modulated waveform is:

\[ \phi_n = \frac{\pi}{4} \]

\[ s(t) = \cos(\omega_0 t) - \sin(\omega_0 t) \]

\[ = \sqrt{2} \cos \left(\omega_0 t + \frac{\pi}{4}\right) \]  

(9)

When the input binary signal is 01, \( I = -1 \), \( Q = 1 \), the modulated waveform is:

Figure 3. The modulated signal with 36 bits transmission
\[ \phi_n = \frac{3\pi}{4} \]

\[ s(t) = \cos(\omega_0 t) - \sin(\omega_0 t) \]

\[ = \sqrt{2} \cos \left( \omega_0 t + \frac{3\pi}{4} \right) \] (10)

Similarly, when the input signal is 00, \( I = -1 \), \( Q = -1 \), \( \phi_n = \frac{5\pi}{4} \). And \( I = 1 \), \( Q = -1 \), \( \phi_n = \frac{7\pi}{4} \), when the input signal is 10. Thus, we can get the corresponding modulation waveform in each signal cycle.

### 2.4. Received signal

After modulation, the image through the AWGN channel is as follows:

And the corresponding formula is:

\[ r(t) = S_n(t) + n(t) = g_{ak}(t - KT) + n(t) \] (11)

Which is also the signal received by our receiver.

Demodulation

The demodulation process mainly focuses on eliminating the problematical influence caused by noise and recovering the original digital signal. According to the characteristic of white noise that the mean value is approximately 0, the influence on a single pulse period is minimal. Therefore, if the mean value of the white noise is ignored, the sum of the first-half points in every single period can be simplified to be the sum of the modulation signal:

\[ S_F = \sum_{first\,\,half} r(t) \]

\[ = \sum_{first\,\,half} S_n(t) + n(t) \]

\[ \approx \sum_{first\,\,half} S_n(t) \] (12)

By comparing the sum of the first half and second-half points, two kinds of square waves \( g_1(t) \) and \( g_2(t) \) can be determined. In other words, the binary sequence of 0 and 1 can be recovered as well.

\[ S_{ fir} > S_{ sec} \rightarrow g_1(t) \rightarrow 0 \]
\[ S_{ fir} < S_{ sec} \rightarrow g_2(t) \rightarrow 1 \]
During demodulation, we multiply the received signal by $e^{-j\omega_0 t}$ and then multiply by 2. Assuming that the currently received waveform is $\cos(\omega_0 t)$, that means signal 0. According to the Euler formula, we have this equation:

$$\cos(\omega_0 t) = \frac{1}{2}(e^{-j\omega_0 t} + e^{j\omega_0 t})$$

multiply by $e^{-j\omega_0 t}$ and 2 then we get:

$$2e^{-j\omega_0 t} \cos(\omega_0 t) = 2e^{-j\omega_0 t} \cdot \frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t}) = 1 + e^{-2j\omega_0 t}$$

For signal 1, the waveform should be $-\cos(\omega_0 t)$, then do the same calculation, we get:

$$-2e^{-j\omega_0 t} \cos(\omega_0 t) = 2e^{-j\omega_0 t} \cdot \frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t}) = -1 + e^{-2j\omega_0 t}$$

It can be seen that the above two equations contain a static component 1 (or - 1) and a high-frequency component (vector) with an angular frequency of - 2w0. Therefore, we can make the signal filter the high-frequency component through the low-pass filter, and the DC static component left is our judgment basis. 1 corresponds to signal 0, and - 1 corresponds to signal 1 to complete demodulation.

In order to evaluate the performance of PPM, another two kinds of modulation methods, Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift Keying (QPSK) are introduced to compare with PPM.

The most critical difference between PPM and BPSK is the carrier wave. Two cosine waves with a phase difference of $\pi$ it is shown in the table below:

Table 1. Carrier wave of BPSK

| Binary signal | Modulation signal |
|---------------|-------------------|
| 0             | $s_0(t) = \cos(\omega_0 t)$ |
| 1             | $s_1(t) = \cos(\omega_0 t + \pi)$ |

During demodulation, two channels of modulated signals are led out:

$s(t) = s_A(t) = s_B(t) = \sqrt{2} \cos(\omega_0 t + \phi_n)$

$s_A(t)$ times $\cos(\omega_0 t)$, we get:

$$y_A(t) = \cos(\omega_0 t + \phi_n) \cos(\omega_0 t) = \frac{1}{2} \cos(2\omega_0 t + \phi_n) + \frac{1}{2} \cos \phi_n$$

$s_B(t)$ times $\sin(\omega_0 t)$, we get:

$$y_A(t) = \cos(\omega_0 t + \phi_n) \sin(\omega_0 t) = -\frac{1}{2} \sin(2\omega_0 t + \phi_n) + \frac{1}{2} \sin \phi_n$$

The two signals pass through a low-pass filter to filter out $2\omega_0$ components to obtain:

$$X_A = \frac{1}{2} \cos \phi_n$$

$$X_B = \frac{1}{2} \sin \phi_n$$

Pass the two signals through their respective sampling decider:

- When $\phi_n = \frac{\pi}{4}$, $X_A$ ($\cos \phi_n$) is positive, $X_B$ ($\sin \phi_n$) is positive, the decider output 11.
- When $\phi_n = \frac{3\pi}{4}$, $X_A$ ($\cos \phi_n$) is negative, $X_B$ ($\sin \phi_n$) is positive, the decider output 01.
- When $\phi_n = \frac{5\pi}{4}$, $X_A$ ($\cos \phi_n$) is negative, $X_B$ ($\sin \phi_n$) is negative, the decider output 00.
- When $\phi_n = \frac{7\pi}{4}$, $X_A$ ($\cos \phi_n$) is positive, $X_B$ ($\sin \phi_n$) is negative, the decider output 10.
Then input the two signals into the serial-parallel switch, and the output is the required original signal.

As for QPSK, the digital signal is sent with a group of 2, so there are 4 different kinds of combinations. Correspondingly, the carrier waves contain 4 different cosine waves with the phase difference of $\frac{\pi}{2}$, in other words, the four waves are orthometric to each other. The carrier waves are shown below:

| Double Bit Code Element | Modulation signal |
|-------------------------|-------------------|
| $a_k = +1, b_k = +1$    | $s_1(t) = \cos(\omega_0 t + \frac{\pi}{4})$ |
| $a_k = -1, b_k = +1$   | $s_2(t) = \cos(\omega_0 t + \frac{3\pi}{4})$ |
| $a_k = -1, b_k = -1$   | $s_3(t) = \cos(\omega_0 t + \frac{5\pi}{4})$ |
| $a_k = +1, b_k = -1$   | $s_4(t) = \cos(\omega_0 t + \frac{7\pi}{4})$ |

The bit error rate (BER) is an important criterion to judge whether the performance of a modulation method is good enough. It is calculated by the equation:

$$\text{BER} = \frac{\text{error bit number}}{\text{sample size}} \times 100\%$$

By comparing the BER of the three methods, the performance of PPM can be evaluated.

Results and Discussion

In order to verify our design method, the following experimental design is carried out.

It can be observed in figure 3 that although the noise has a particular influence on the function of a specific point, it is difficult to change the rising or falling trend in a single pulse period. In addition, it can also be seen from the above two tables that in the demodulation of 36 data, the influence of Gaussian white noise with SNR of 15dB on demodulation is 0, and the bit error rate is also 0. It can be observed from Figure 4 that the anti-noise effect of PPM modulation is better than that of BPSK modulation. However, through literature investigation and reading, we know that the anti-noise performance of BPSK modulation should be better [12]. For the error of the result, the possible reason is that there is a problem in the process of BPSK modulation or PPM modulation is improper.

Table 3. A sample of demodulated signal under the condition of SNR=15

| Modulated | 0 0 0 0 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 |
| Demodulated | 0 0 0 0 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 |

Table 4. A sample of demodulated signal under the condition of SNR=15

| Modulated | 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 |
| Demodulated | 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 |

The sample above shows that when the sampling size is 36 and SNR is 15, there is no error bit, so the BER is 0.

Figure 5 above shows the BER changing with the increase of Signal Noise Ratio (SNR) from 1 to 10. The BER of all three modulation methods has a decreasing trend due to the noise reduction.
compared to the signal. In addition, PPM and QPSK have better anti-noise effects than BPSK in all the intervals. Compared with QPSK, PPM has better performance when SNR is smaller than approximately 6.5.

Figure 5. BER between BPSK and PPM

3. Conclusion And Future Work

3.1 Conclusion
This project first selects and designs two square wave pulses to simulate the signal representing 0 and 1 and using MATLAB to visualize the processed discrete digital signal through sampling and quantization. Then Gaussian white noise is added to observe the modulation results, and it is found that the rising or falling trend of noise to a single pulse period is minimal. Finally, when demodulating, the bit error rate is compared with BPSK modulation by changing the data volume and signal-to-noise ratio. The results of the two modulation methods are observed and analyzed. However, the experimental results are affected and deviate from the theoretical results due to the improper parameter setting of BPSK modulation or PPM modulation.

3.2 Future Work
In this project, we only verify the anti-interference ability of PPM against Gaussian white noise and cannot well prove the robustness of ppm. In future work, we will introduce the impact of fading channel and strive to simulate the natural channel transmission environment to verify the performance of PPM. In addition, compared with BPSK and QPSK, we will add more existing and mature digital modulation methods to compare the anti-noise performance and increase the number of experiments to get more accurate experimental conclusions.

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