Transport Simulations with $\pi$ and $\Delta$ In-Medium Properties$^\dagger$

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July 10, 1998

Abstract:

Transport simulations including in-medium properties derived in a microscopic $\pi + NN^{-1} + \Delta N^{-1}$ model in infinite nuclear matter are presented. In-medium pion dispersion relations, partial $\Delta$ decay widths, pion absorption cross sections and $\Delta$ cross sections are incorporated into the transport description by means of a local-density approximation. Strong modifications of $\pi$ and $\Delta$ production and absorption rates are found, but only small effects on pion observables.

PACS: 25.75.Dw, 25.75.-q, 24.10.Cn, 13.75.Gx

Keywords: Spin-isospin modes, transport simulation, in-medium properties, heavy-ion collisions, Delta-hole model, pion production

$^\dagger$This work was supported by the Training and Mobility through Research (TMR) programme of the European Community under contract ERBFMBICT950086 and by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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1 Introduction

In collisions between two heavy nuclei at bombarding energies from a few hundred MeV up to several GeV per nucleon, hadronic matter at high density and temperature is formed and a large number of energetic particles are produced \cite{1, 2, 3, 4}. The main features in such collisions have been fairly well described by microscopic transport models employing vacuum properties of resonances and mesons, such as most versions of BUU and QMD \cite{2, 3, 5, 6}. However, $\pi$ mesons, nucleons, and $\Delta$ isobars are strongly interacting particles, and in infinite nuclear matter a system of such particles would couple to form spin-isospin modes. It is quite possible that such in-medium effects could play an important role for the transport properties. First attempts to employ such in-medium modifications in transport simulations of nuclear collisions have been carried out. In Refs. \cite{7, 8, 9, 10, 11} the employed in-medium properties were based on a simple two-level $\Delta N^{-1}$ model. A more complete $\pi + NN^{-1} + \Delta N^{-1}$ model was used in Refs. \cite{12} to derive a consistent set of in-medium quantities suitable for transport descriptions, with exploratory transport simulations reported in Refs. \cite{13}.

The previous attempts to include $\pi$ and $\Delta$ in-medium properties have suggested that only small effects are found in pion observables. This is not surprising since most emitted pions are created at the surface where the in-medium effects are small. However, in-medium modifications of pions and $\Delta$ isobars may be important for other observables. For example, presently large efforts are made to study effects of possible chiral restoration at high densities and temperatures, and in particular by investigating kaon and dilepton spectra which may carry signatures of in-medium modifications of vector meson masses due to chiral restoration \cite{4, 14, 15, 16}. Since kaons and dileptons are produced through multistep processes, involving pions and $\Delta$’s, in-medium modifications of $\pi$ and $\Delta$ quantities could implicitly have an important impact.

In this letter we present selected results from transport simulations that include in-medium quantities obtained from the $\pi + NN^{-1} + \Delta N^{-1}$ model of Ref. \cite{12}. The present work constitutes an extension of Ref. \cite{13}, now including a more complete and consistent set of in-medium quantities. Our intention is to present a qualitative picture of some in-medium effects that survive the transport dynamics, while a more complete and systematic presentation will be given in a subsequent paper. While the model presented here contains the main in-medium modifications of pions and $\Delta$ isobars, a number of additional processes might play a role as well. For example, effects originating from a possible partial chiral restoration have not been included. Moreover, there might also be in-medium modifications of cross sections and decay widths originating from the fact that the time and volume available in collisions and decays are finite. We expect that the present transport simulations contain the most important in-medium features. Furthermore, they provide the most consistent tests carried out so far of in-medium effects within $\Delta N^{-1}$ models.
2 The Model

Our approach is to obtain the medium modifications by microscopic calculations in uniform matter at various densities and then incorporate those into a transport treatment by a local-density approximation [12, 13]. For simplicity, all microscopic quantities are approximated by their zero-temperature values and the ∆ width is omitted in the microscopic treatment of the dispersion relations. Detailed discussions of these approximations are given in Refs. [12, 13].

We consider interacting nucleons (N), delta isobars (∆), pion mesons (π), and rho mesons (ρ) in a periodic box for \( T = 0 \). The in-medium properties are obtained by using the Greens function technique, starting from non-interacting hadrons. At pion vertices we use effective \( p \)-wave interactions containing form factors, \( F_{N\pi N}(q) \) and \( F_{N\pi \Delta}(q) \). Since various choices are made in the literature we will in this work utilize two different sets. The first set, denoted FF1, is taken

\[
F_{N\pi \alpha}(q) = \left[ \frac{2m_{\alpha}c^2}{m_{\alpha}c^2 + \sqrt{s}} \right]^{\frac{1}{2}} \frac{\Lambda_{\pi}^2 - (m_{\pi}c^2)^2}{\Lambda_{\pi}^2 - (cq)^2}, \quad \alpha = N, \Delta
\]

while for the second set, FF2, the relativistic corrections are replaced by an off-shell correction at vertices including a ∆ isobar and neglected at other vertices. In addition the monopole form is approximated by an exponential form. Thus for FF2 we take

\[
F_{N\pi N}(q) = e^{-[(cq)^2-(m_\pi c^2)^2]/\Lambda_{\pi}^2}, \quad F_{N\pi \Delta} = \left[ \frac{q_0^2 + \kappa^2}{q_{cm}^2 + \kappa^2} \right]^{\frac{1}{2}} e^{-[(cq)^2-(m_\pi c^2)^2]/\Lambda_{\pi}^2}. \tag{2}
\]

For most quantities of interest here the difference between the monopole and exponential form is practically negligible, but the exponential form yields better convergence properties for calculating the real part of the ∆ self energy. The relativistic correction \( 2m_{\alpha}/[\sqrt{s} + m_{\alpha}] \) is frequently used in the ∆-hole model in connection with pion absorption and scattering on nuclei [13, 14], while the form \( [q_0^2 + \kappa^2]/[q_{cm}^2 + \kappa^2] \) takes into account the ∆ off-shell correction and has been used for calculating vacuum \( N + N \rightarrow \Delta + N \) cross sections [20], and is also frequently included in vacuum ∆ widths incorporated in transport simulations [17, 21]. Since the two form factor choices, FF1 and FF2, lead to somewhat different in-medium properties, we have chosen to test both forms.

At the \( N\rho N \) and \( N\rho \Delta \) vertices interactions corresponding to those used at the pion vertices are used, and at baryon-hole vertices effective short-range interactions are included, moderated by the correlation parameters \( g'_{NN}, g'_{N\Delta}, \) and \( g'_{\Delta\Delta} \).

A set of RPA equations for spin-isospin modes, corresponding to an infinite iteration of non-interacting pion (ρ-meson), nucleon-hole, and ∆-hole states, were derived in Ref. [12]. From these equations eigenvectors and eigenenergies are obtained for the different spin-isospin modes. We also calculate total and partial ∆ widths, \( N\pi \rightarrow \Delta \) cross sections, \( NN \leftrightarrow \Delta N \) cross sections and ∆ spectral functions within the RPA approximation (explicit expressions are given in Ref. [12]).

The results of the ∆N\(^{-1}\) model depend on the model parameters which cannot be determined uniquely by comparison to experimental data. In Ref. [12] was chosen
Table 1: Parameter values used in microscopic calculations (energy units: GeV).

| Parameter | FF1 & FF2 | FF1 & FF2 | FF1 | FF2 |
|-----------|-----------|-----------|-----|-----|
| \( m_N c^2 \) | 0.940 | 1.0 | 6.2 | 0.9 | 0.9 |
| \( m_\Delta c^2 \) | 1.23 | 2.2 | 10.5 | 0.38 | 0.43 |
| \( m_\pi c^2 \) | 0.14 | 0.0 | 0.0 | 0.50 | 0.40 |
| \( m_\rho c^2 \) | 0.77 | 1.0 | 1.5 | 1.5 | 1.5 |
| \( m_\pi^* N \) | \( \rho_0 = 0.153 \text{ fm}^{-3} \) | \( V_\Delta - V_N = 0.025 \rho/\rho_0 \) | \( \rho \leq \rho_0 \) | \( V_\Delta - V_N = 0.025 \) | \( \rho > \rho_0 \) |

In this work we explicitly propagate both the lower and the upper pionic modes, as was also done in Ref. [10]. This differs from Refs. [3, 11] where only a single average pionic mode was incorporated. The importance of treating the two pionic modes equally was discussed in Refs. [12, 23].
Figure 1: Partial $\Delta$ decay widths for the lower pionic mode $\pi_1$ (upper panel) for zero $\Delta$ momentum, and pion absorption cross sections for the process $p\pi_1^+ \rightarrow \Delta^{++}$ (lower panel). The cross sections are obtained for the special case when the $p\pi_1^+\text{c.m.}$ frame is equal to the rest frame of the medium. The curve labeled Kitazoe represents the parameterization in Ref. [21], and the empirical cross section for $\pi^+p$ scattering are estimated from Fig. 2.2 in Ref. [24].

depends explicitly on the $\Delta$ momentum in the medium, in addition to the $\Delta$ energy. This is especially important for the partial widths. To this end we calculate and store in a large table the partial $\Delta$ decay widths for discrete values of invariant mass, momentum, density and angle. The $\Delta$ then decays in the medium frame according to linear interpolation in the table of decay widths (for details see Ref. [13]).

The pionic modes may be reabsorbed through the process $\bar{\pi}N \rightarrow \Delta$. In the standard transport treatment, the vacuum reabsorption cross section may be written as an interaction factor times a Breit-Wigner factor containing the free $\Delta$ width. The in-medium pion absorption cross section has a similar structure. For the transport treatment we make use of the fact that the interaction factor is also used for calculating the partial $\Delta$ width, and we use the full in-medium $\Delta$ width in the Breit-Wigner factor. Examples of $N\pi \rightarrow \Delta$ cross sections for FF1 are presented in Fig. 1.

The transport simulations are based on the hadronic transport model of Li and Bauer [17]. In this model the dynamical evolution of a heavy-ion collision is described by a set of coupled transport equations, explicitly treating nucleons, $\Delta$ isobars and
pions. In this treatment the inelastic $NN$ cross section is parameterized following Ref. [25]. In the medium-modified simulations we will replace the VerWest and Arndt vacuum cross sections for the processes $NN \leftrightarrow \Delta N$ by the density dependent cross sections obtained from the microscopic calculations. These calculated cross sections constitute in vacuum a somewhat cruder approximation than the fit obtained in Ref. [25], where three independent functions $\sigma_{IH}$ were used. Since our aim in this work is to test in-medium effects, we will compare to transport simulations based on the model described in [17], but with the VerWest and Arndt $NN \leftrightarrow \Delta N$ cross sections replaced by our calculated cross sections at zero density. This we will refer to as a standard simulation. The microscopic cross sections are calculated for the case when the c.m. system of the colliding particles coincides with the rest system of the medium. To keep the transport treatment as simple as possible these cross sections are then used also when this c.m. system differs from the local medium frame.

One important medium modification of the $NN \rightarrow \Delta N$ cross section originates from the $\Delta$ spectral function which is modified in the medium. This spectral function contains the full in-medium $\Delta$ width, which is calculated self consistently. The $\Delta$ spectral function is also used in the transport treatment to determine the invariant $\Delta$ mass, once a $NN \rightarrow \Delta N$ collision has occured. The total in-medium $\Delta$ width leads at high densities to a substantial reduction of the $NN \rightarrow \Delta N$ cross section for moderate and large c.m. energies and to an enhancement for low c.m. energies. This differs from the results found in Refs. [8, 26] where an enhancement was found for large densities. However, as noted in Ref. [8], the result is very sensitive to the $g'$ parameter values. In Ref. [26] it was shown that the enhancement found in Ref. [8] is weakened by the inclusion of the full $\Delta$ width in the pion polarization function. Since our calculations employ large values of the $g'$ parameters and in several aspects differ from Refs. [8, 24], we find a net surpression of $\sigma(N \rightarrow \Delta N)$. Most important, we include selfconsistently the full in-medium $\Delta$ width (including the $\Delta \rightarrow N + NN^{-1}$ contribution which is important at low $\Delta$ mass and high densities) and we perform an integration over $\Delta$ masses including the in-medium $\Delta$ spectral function.

The cross section for the process $N\Delta \rightarrow NN$ is obtained and implemented analogously to the $NN \rightarrow \Delta N$ cross section. Note though that the absence of the $\Delta$ spectral function simplifies the treatment. However, another complication arises because the cross section also depends on the invariant mass of the colliding $\Delta$, in addition to the c.m. energy. We have empirically found that $\sigma(\sqrt{s}; m) \approx \sigma(\sqrt{s'}; m_\Delta)$, with $\sqrt{s'} = \sqrt{s} - 0.778(m - m_\Delta)$ yields a good approximation. The $N\Delta \rightarrow NN$ cross sections show only a weak dependence on the nuclear density.

3 Results

In this section we present results from BUU simulations containing the in-medium quantities as discussed in Sect. 2. The purpose of these simulations is to elucidate the effects of the included in-medium properties. Therefore, to make the treatment simple

\footnote{Since the $g'$ parameters are multiplied by form factors, there is not a one-to-one correspondance between the values used in Refs. [8, 26] and those used here.}
and the signals as clean as possible, we have ignored some processes that may be of equal importance when comparing to experiment. We have performed simulations for central $^{197}$Au + $^{197}$Au collisions at 1.0 $A$ GeV and we have obtained results from standard simulations and from three different medium-modified simulations, denoted M1–M3. M1 contains in-medium modifications of only pion dispersion relations and partial $\Delta$ decay widths. M2 contains in addition to M1 also the in-medium pion reabsorption cross section $\sigma(N\pi \rightarrow \Delta)$. Finally, in M3 also the in-medium $NN \leftrightarrow \Delta N$ cross sections are added. All simulations were performed with 250 test particles per nucleon utilizing a mean field $U(\rho) = -0.218(\rho/\rho_0) + 0.164((\rho/\rho_0)^{4/3}$ GeV.

![Graph](image)

**Figure 2:** Distribution of invariant mass for $\Delta$'s produced in $NN$ collisions.

The production rate of $\Delta$ isobars from $NN$ collisions depends on the $NN \leftrightarrow \Delta N$ cross section and the density and energy distributions of nucleons. When the density dependent in-medium $NN \rightarrow \Delta N$ cross sections are incorporated (simulation M3) the $NN \rightarrow \Delta N$ rates are reduced, due to a smaller in-medium cross section at high densities. For FF1 this effect is small, due to a strong enhancement of the cross section at low $\sqrt{s}$-energies, while the reduction for FF2 is substantial.

The in-medium $NN \rightarrow \Delta N$ cross section, together with the $\Delta$ spectral function also lead to strong modifications of the invariant $\Delta$ mass distributions. Figure 2

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3 The standard simulations with FF1 and FF2 yield slightly different results due to slightly different input vacuum $NN \leftrightarrow \Delta N$ cross sections.
shows that for simulation M3, there is a strong reduction of $\Delta$ isobars with invariant mass around $m_\Delta = 1232$ MeV, and an enhancement at lower masses. Note particularly that the invariant mass distribution is non-vanishing also for masses below $m = m_N + m_\pi$. These $\Delta$’s cannot decay to a pionic mode and the only reabsorption channel is $N\Delta \rightarrow NN$. As a result, the net number of pions is somewhat reduced. The effects found with FF1 are similar to FF2 but show more pronounced and peculiar enhancement of low mass $\Delta$’s. This originates from the microscopic calculations which for FF1 show strong effects of pion condensation for densities around twice normal nuclear density (while FF2 does not lead to pion condensation at the densities probed in the applied transport simulations). In the microscopic calculations, this phenomenon manifest itself as large enhancements of the $\Delta$ decay width to low-energy nucleon-hole modes which in turn leads to a strong enhancement of the $\Delta$ spectral function around $m \approx 1$ GeV and corresponding effects in the $NN \rightarrow \Delta N$ cross section. These effects survive the transport treatment and we find strong enhancements of the number of produced $\Delta$’s in the invariant mass region around 1 GeV. The net effect on the number of produced pions is a slight reduction.

Inspecting the rate of produced pions from $\Delta$ decay (upper panel of Fig. 3), we find that for FF1 there is in simulation M1 a slight reduction as compared to the standard treatment, due to a reduction in the partial $\Delta$ width at high densities\textsuperscript{4}. FF2, which for $m \geq m_\Delta$ has only about 60% of the corresponding width obtained for FF1, gives a strong reduction in the pion production rate. The further reduction in the $\Delta$ decay rate for simulations M2 to M3 reflects that there are fewer produced $\Delta$’s in the system. The simulation M2 illustrates this effect due to a reduction of the process $N\bar{\pi} \rightarrow \Delta$ while for simulation M3 the reduction is due to fewer produced $\Delta$’s, in the mass region $m > m_N + m_\pi$, from $NN$ collisions.

The pion reabsorption rate (lower panel of Fig. 3) depends on the density of pions and nucleons and on the reabsorption cross section. The reduction in the rate for simulation M1 is due to the fact that fewer pions are produced from $\Delta$ decay. There is a further strong reduction when including also the in-medium $N\bar{\pi} \rightarrow \Delta$ cross section (M2). This is because this cross section is suppressed at high densities (see Fig. 4). The pion reabsorption rate is reduced even further for simulation M3. This is a secondary effect of the fewer produced $\Delta$’s which then yield fewer pions.

For FF2 the net number of $\Delta$ isobars and pions present in the system at different times varies much less with the in-medium input than the production and absorption rates. This is because different effects balance each other and because the production and absorption depends on particle densities which yields an “automatic” regulation of the net number. Note particularly that the enhancement of pions due to the low reabsorption cross section at high densities is compensated by a low production of $\Delta$ isobars by a likewise low $NN \rightarrow \Delta N$ cross section at high densities. This compensation is not accidental as it depends mainly on the $\Delta$ in-medium spectral function which is included in both cross sections. The net effect is a reduction of the number of emitted pions of only about 3-10% for simulation M3 as compared to the result from

\textsuperscript{4} For the medium-modified simulations we find that about 95% of the emitted pions originate from the lower pionic mode.
the standard simulation. For FF1 the number of present Δ’s varies somewhat more with the different types of medium-modified simulations. These variations originate from the microscopic onset of pion condensation, as briefly discussed above.

In-medium effects show up also in the pion energy spectrum, although to a rather small degree. Transverse momentum spectra \( \frac{d\sigma}{dp_T} \) have been obtained at impact parameter \( b = 1.0 \) fm assuming that all collisions with an impact parameter up to \( b_{\text{max}} = 2 r_0 A_{\text{Au}}^{1/3} \) contribute equally. The effect seen in the simulations incorporating the in-medium properties, as compared to the standard simulations (see Fig. 4), is a modest but significant enhancement at low transverse momenta \( (p_T < 250 \text{ MeV}/c) \) and a reduction at higher momenta \( (p_T > 350 \text{ MeV}/c) \), corresponding to a reduction of the effective transverse temperature. We find the enhancement of low-energy pions encouraging, even though this effect cannot alone explain the enhancement seen in experimental spectra (Fig. 1 of Ref. [27]). It should be emphasized that the obtained spectra are not entirely suitable for quantitative comparison with experimental spectra partly because of the simple impact-parameter averaging employed, and partly because the incorporation of higher nucleon resonances and additional medium effects might be of importance. Note, for example, that higher-lying resonances contribute to both low and high energy pions through the two-pion and one-pion emission channels, respectively [28].
Figure 4: The transverse momentum spectrum for neutral pions in the central rapidity interval $-0.20 < y_{cm} < 0.20$. The error bars represent the statistical errors. The results obtained for FF2 are very similar.

4 Summary

In this letter, we have presented selected results from transport simulations incorporating in-medium properties of pions and $\Delta$ isobars. In particular, we have included in-medium pion dispersion relations, partial $\Delta$ widths, pion reabsorption cross sections, $NN \leftrightarrow \Delta N$ cross sections and $\Delta$ spectral functions. These in-medium quantities have been calculated microscopically from the $\Delta N^{-1}$ model presented in Ref. [12] and incorporated into the transport formalism of Li and Bauer [17] by a local-density approximation. The medium-modified simulations presented in this work show strong effects on properties not directly observable during the collision process, such as pion and $\Delta$ production and reabsorption rates, but only minor effects on spectra of emitted pions. This is rather reasonable since most of the emitted pions are produced at the surface at low densities where the in-medium effects are quite small. However, in an energetic nucleus-nucleus collision also other particles, not studied in this work, are produced in multistep processes, where $\Delta$’s and pions act as intermediate particles. Thus, modified properties of $\Delta$’s and pions in the nuclear medium might be important to consider when studying production of such secondary particles.

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