A Modified Schmidl-Cox OFDM Timing Detector

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1 Abstract

We describe a simple modification of the Schmidl-Cox detector for establishing timing in OFDM transmissions that stabilizes performance in transitions from no-signal to signal, or vice-versa. Moreover, the proposed modification scales the detector’s metric between 0 and 1 for all scenarios, simplifying threshold setting, and improves timing detector SNR.

2 Introduction

Schmidl and Cox [1] proposed a simple two-symbol preamble for establishing frame timing and frequency offset estimation in OFDM. The first symbol provides a time-domain sequence whose second-half samples are identical to those in the first half. This is imposed in OFDM by zeroing alternate frequency-domain variables.\(^1\) Normally placed at the beginning of an OFDM burst, this symbol allows detection of start-of-frame (SOF) as described below. This same detection procedure also provides coarse estimation of the beginning of OFDM symbols to follow.

Though not under consideration here, Schmidl and Cox also showed how frequency estimation can be performed from this first preamble symbol, to within an ambiguous interval corresponding to the OFDM subcarrier spacing. A second preamble symbol is used to resolve ambiguity of frequency—see [1] for details.

In related work, Minn et al [2] published a modification that sharpens the SOF peak, by modifying the S-C symbol construction. Other preamble-based designs are reported in [3, 6, 5]. Some of this work has focused on maximum likelihood estimation of timing and/or frequency offset using the cyclic prefix property or the preamble structure itself, [4, 7]. These could possibly be used to refine over time S-C estimates obtained from the two preamble symbols.

In the following, we denote the FFT size in an OFDM implementation by \(N\), and let \(s_n\) denote the time-domain sequence at the output of the \(N\)-point IFFT in OFDM transmitter. This sequence is acted upon by a multipath channel , then additive white

\(^1\)More broadly, this procedure could be applied to locate any non-OFDM pattern that has such repetition in time.
Gaussian noise to produce the complex baseband sequence at the receiver

\[ r_n = \sum_{m=1}^{D} h_m s_{n-m} + n_n , \]  

where \( D \) is the anticipated multipath channel duration in samples. Typically a cyclic-prefix is added to the IFFT output, having length \( N_{CP} > D \), whose removal at the receiver allows easy frequency-domain equalization of the channel.

Letting \( L = N/2 \) denote half the symbol duration (without CP), the S-C procedure [1] first defines

\[ P(n) = \sum_{m=0}^{L-1} r_{n+m}^* r_{n+m+L} , \]  
a sliding lag-\( L \) correlation of the received sequence computed over \( L \) samples, and

\[ R(n) = \sum_{m=0}^{L-1} |r_{n+m+L}|^2 \]  

which is energy measured over \( L \) contiguous samples. (Both are non-causal as defined.) Then the ratio

\[ M(n) = \frac{|P(n)|^2}{R^2(n)} \]  
is formed whose peak locates a symbol boundary in time and also SOF. \( M(n) \) is normally subjected to peak-finding, which is accepted provided the peak is above some threshold.

Assuming high SNR, the samples are exactly repetitive, even with multipath, and \( M(n) \) has maximum value 1. However, the statistic is not guaranteed to be bounded by 1, and we have found the statistic above to be ill-behaved in transition between signal-present and signal-absent situations. In particular, in a transition from signal-present to no signal, the denominator \( R^2(n) \) quickly drops to zero faster than the numerator, leading to high-amplitude peaks in \( M(n) \) and thus possible false SOF declarations. (Delaying \( R(n) \) by \( L \) samples to mitigate this problem, induces similar difficulties at the beginning of a signal span.)

Though not mentioned in the Schmidl-Cox paper, the procedure is reminiscent of the Cauchy-Schwarz inequality for complex sums. C-S holds that

\[ |\sum_i a_i b_i|^2 \leq \sum_i |a_i|^2 \sum_i |b_i|^2 \]  

with equality iff \( b_i = a_i^* \). With a small change in the definition of \( M(n) \) we can claim the peak of \( M(n) \) will never exceed 1, no matter the signal nature. The proposed modification is

\[ \tilde{M}(n) = \frac{|P(n)|^2}{R(n)R(n-L)} \]  
i.e. we change only the denominator, and with no significant difference in computational complexity. By the C-S inequality, the maximum of \( \tilde{M}(n) \) will be 1 for any signal and
noise scenario, and attain 1 when the first and second sets of $L$ samples are identical. This provides a helpful self-scaling property, again holding in multipath conditions.

The procedure can be made causal by allowing delay in the peak of $M(n)$ relative to the start of the SOF symbol. Defining $M'(n) = M(n - 2L)$ gives

$$M'(n) = \frac{(\sum_{m=0}^{L-1} r_{n-2L+m} r_{n-L+m})^2}{\sum_{m=0}^{L-1} |r_{n-2L+m}|^2 \sum_{m=0}^{L-1} |r_{n-L+m}|^2}$$

Now the ‘current’ value of $M'(n)$ depends only on current and past inputs over a span of $2L$ samples.

A block diagram is sketched below. The $L$-sample moving averages for $P$ and $R$ can be efficiently computed recursively if desired by an accumulator, adding a new sample and subtracting the sample value $L$ samples earlier.

![Block Diagram](image)

Figure 1: Block Diagram

3 Comparison

We show simulation results for a case with $N = 128$ (so $L = 64$) on a Gaussian noise channel with QPSK modulation and $E_b/N_0 = 10$ dB. For illustration, we precede a 16-symbol burst with the two-symbol Schmidl-Cox SOF preamble. Additive noise precedes this and also follows this interval in time. In our study we have removed the cyclic prefix from the first S-C symbol, eliminating the plateau in $M(n)$ without other consequence. The expected peak in the SOF detector is at index 933 in subsequent plots.

Figure 2 shows the traces of $M'(n)$ for the conventional S-C detector, (Mold), as well as the proposed statistic (Mnew). Note the rapid rise in the ratio at the end of the 16-symbol interval for the S-C detector, in addition to the correct placement of the SOF at the location of the S-C symbol. On the other hand, the Mnew trace exhibits no such spurious peaks, while still correctly finding the correct SOF.
A simple two-tap multipath test with multipath delay corresponding to a quarter symbol, and tap weights $[0.8, 0.5e^{j\pi/4}]$ was also done to confirm robustness of the procedure. Figure 5 below repeats the above traces, again at the same SNR. Similar differences are noted, though here multipath actually reduces the spurious peak in Mold.
3.1 Improved SNR at Peak

The proposed timing detector also has better detection statistics at the proper peak of the $M(n)$ signature, which we illustrate for a white Gaussian noise case with SNR=7 dB (energy per subcarrier divided by one-sided noise power density) by showing histograms of the original S-C statistic (Figure 4) and the modified one (Figure 5). Clearly the proposed modification has a more concentrated p.d.f which translates to a better probability of detection versus probability of false alarm tradeoff. The mean of the two statistics is the same; however the variance of the new statistic is smaller.

![Normalized histogram for S-C statistic](image-url)
An intuitive explanation for this result is that while the numerator statistic is identical in both procedures, the modified denominator exhibits less variance as the product of two non-central chi-squared variates, versus the square of a single non-central chi-squared variate. The latter involves a fourth moment of a complex Gaussian random variable, whereas the modified denominator involves the product of two second moments.

4 Conclusion

A simple modification of the Schmidl-Cox detector for SOF in OFDM transmission is presented that eliminates false SOF’s at beginning or end of transmission, while simultaneously retaining an amplitude self-scaling property. Moreover, the detector statistic exhibits improved decision quality at the desired timing peak, and no increase in complexity relative to the original S-C algorithm is needed.

This discussion pertains to producing a clean, reliable SOF trigger signal. A complete detector needs a simple sliding peak-finding scheme, followed by a threshold test to mitigate against false peaks in presence of noise alone.

Finally, the proposed modification is completely compatible with frequency estimation methods earlier proposed.
5 Bibliography

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