Quark Virtuality Distribution in Non-perturbative QCD Vacuum∗

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Abstract

We introduce the quark virtuality distribution $[\lambda^2_q(x)]$ as an extension of the quark virtuality by using nonlocal rather than local quark vacuum condensates. Our results show that the quark virtuality distribution decreases gradually from a finite value to zero as $x$ increases. This clearly indicates that quarks are confined inside hadrons, whereas, as $x$ approaches zero quarks have a finite virtuality, which shows asymptotic freedom of quarks in QCD. The present predictions are important not only for studying the properties of QCD vacuum states, and non-perturbative QCD problems, but also for study of the cosmological constant problem, which we have shown should be treated using nonlocal quark vacuum condensate in the Ghost theory of QCD.

Key Words: quark virtuality distribution, non-perturbative QCD, fully dressed quark propagator.

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1 Introduction

The quark virtuality, an expression for the momentum carried by the quark vacuum condensate, was introduced during the early study of QCD sum rules\cite{1,2,3}, and was calculated\cite{4,5} using the local quark condensate and local mixed quark-gluon condensate calculated using Dyson-Schwinger equations\cite{6,7}. Our present estimate of the quark virtuality distribution is based on our previous calculations of quark virtuality and the nonlocal quark condensate used in our recent work on the cosmological constant\cite{8}.

Studying the quark virtuality distribution in non-perturbative QCD vacuum state is of paramount importance for present day particle and nuclear physics, since it is not only related to the property of QCD vacuum states but also to studying non-perturbative QCD issues, as well as the cosmological constant problem.

In Sect.2 we first review the quark local vacuum condensates and the calculation of the quark virtuality in our previous work, and then calculate the quark virtuality distribution using a nonlocal quark condensate. Finally, concluding remarks are given in Sect.3.

2 The quark virtuality and quark virtuality distribution

In the first subsection we review the quark condensates virtuality, $\lambda^2_q$, which was estimated in Ref.\cite{5}. We then derive the quark virtuality distribution using a nonlocal quark condensate related to that used in Ref.\cite{8}.

2.1 The quark virtuality

The quark virtuality is defined in terms of the local quark condensate and mixed quark-gluon condensate. These are given by the Taylor expansion of the quark propagator, which we now review.

$$S_q(x) = <0| : \hat{T}q^a(x)q_b(0) : |0>$$

where $q^a(x)$ is quark field with color index $a$, and $\hat{T}$ is the time-ordering operator. The fully dressed quark propagator in Eq.(1) can be decomposed into a perturbative part and a non-perturbative part, with only the non-perturbative part needed for the present work. The non-perturbative part of the quark propagator is

$$S_q^{NP}(x) = -\frac{1}{12} <0| : \bar{q}(x)q(0) : |0> + x_\mu <0| : \bar{q}(x)\gamma^\mu q(0) : |0>$$

in configuration space, with a sum over color implied. $\gamma^\mu$ are Dirac matrices. For short distances $x$, the Taylor expansion in $x$ of the scalar part of $S_q^{NP}(x)$, $<0| : \bar{q}(x)q(0) : |0>$, is\cite{6}

$$<0| : \bar{q}(x)q(0) : |0> = <0| : \bar{q}(0)q(0) : |0> - \frac{x^2}{4} <0| : \bar{q}(0)[i\sigma G]q(0) : |0> + \cdots$$

In Eq.(3) the quantities in the expansion are the local quark vacuum condensate, the quark-gluon mixed local vacuum condensate and so forth. The $G$ in the second term of
Eq. (4) is the gluon field strength tensor, where the Dirac and color indexes have not been shown explicitly.

In Ref. [5] the Dyson-Schwinger Equations [6, 7] were used to derive the local quark vacuum condensate $<0|:\bar{q}(0)q(0):|0>$ and the local mixed quark-gluon condensate, $<0|:\bar{q}(0)[ig\sigma G]q(0):|0>$. See Ref. [5] for these local condensates for three sets of quark interaction parameters.

We review the definition of quarks mean squared momentum in non-perturbative QCD vacuum, which is also called the quark virtuality, $\lambda_q^2$. The quark virtuality $\lambda_q^2$ is defined [9, 10] as

$$\lambda_q^2 \equiv \frac{<0|:\bar{q}D^2q:|0>}{<0|:\bar{q}q:|0>},$$

where $D_\mu = \partial_\mu - ig_sA_\mu$, with $A_\mu = A^a_\mu \lambda^a/2$, is the covariant derivative and $\lambda^a$ is a $SU_c(3)$ Gell-Mann matrix. As shown in Refs. [9, 10], $<0|:\bar{q}D^2q:|0>$ can be written

$$<0|:\bar{q}D^2q:|0> = -m_q^2 <0|:\bar{q}(0)q(0):|0> + \frac{1}{2} <0|:\bar{q}(0)[ig\sigma G]q(0):|0>.$$  \hspace{1cm} (5)

Thus, from Eq. (5) the quark virtuality $\lambda_q^2$ can be rewritten as

$$\lambda_q^2 = \frac{1}{2} \frac{<0|:\bar{q}(0)[ig\sigma G]q(0):|0> - m_q^2}{<0|:\bar{q}(0)q(0):|0>}. \hspace{1cm} (6)$$

For light quarks $u, d$ and $s$, the mass $m_q^2$ term in Eq. (6) is so small that it can be neglected. Therefore, we finally arrive at

$$\lambda_q^2 \simeq \frac{1}{2} \frac{<0|:\bar{q}(0)[ig\sigma G]q(0):|0>} {<0|:\bar{q}(0)q(0):|0>}. \hspace{1cm} (7)$$

The quark virtuality for three sets of parameters is given in Ref. [5]

2.2 Quark virtuality distribution in the non-perturbative QCD vacuum

We define the quark virtuality distribution $\lambda_q^2(x)$ in the non-perturbative QCD vacuum as a natural extension of Eq (1) by

$$\lambda_q^2(x) \equiv \frac{<x|:\bar{q}D^2q:|0>}{<0|:\bar{q}q:|0>},$$

similar to the extension of the quark condensate to the nonlocal quark condensate in Refs. [11, 12]. In Ref. [11] $<x|:\bar{q}D^2q:|0>$ was evaluated, giving

$$<x|:\bar{q}D^2q:|0> = \frac{g_\lambda(x^2)}{2} <0|:\bar{q}(0)[ig\sigma G]q(0):|0>,$$

with

$$g_\lambda(x^2) = \frac{1}{1 + \lambda^2x^2/8}. \hspace{1cm} (10)$$

The quantity $\lambda^2$ in Eq. (10) is $\lambda^2 \simeq 0.3 - 0.8\text{GeV}^2$. From Eqs (8, 9, 10, 6)

$$\lambda_q^2(x) \simeq \frac{1}{1 + \lambda^2x^2/8} \lambda_q^2. \hspace{1cm} (11)$$

For the present work we take $\lambda^2 = 0.8\text{GeV}^2$, and $g_\lambda(x^2)$ is shown in the figure.
Figure 1: $x$-dependence of the function $g_{\lambda}(x^2)$.

Note that $g_{\lambda}(x^2)$, and therefore $g_{\lambda}(x^2)$ has decreased by a factor of 0.024 when $x=4.0$ fm.

3 Summary and Concluding remarks

We study the distribution of the nonzero mean squared momentum of quarks (called the quark virtuality) in the non-perturbative QCD vacuum. The quark virtuality $\lambda_q^2$ is given by the ratio of the local quark-gluon mixed condensate $<0|\bar{q}(0)[i\gamma_{\mu}\sigma G]q(0)|0>$ to the local quark vacuum condensate $<0|\bar{q}(0)q(0)|0>$. For our theory of the quark virtuality distribution, $\lambda_q^2(x)$, we replace the local $<0|\bar{q}D^2q|0>$ by the nonlocal $<x|\bar{q}D^2q|0>$.

Our calculation shows that as $x$ approaches zero a quark has a finite nonzero mean squared momentum. In this case, a quark is similar to a free particle. This indicates asymptotic freedom of quarks in the QCD vacuum. When $x$ becomes large, say $x=10$ fm, $\lambda_q^2(x)$ approaches zero, which indicates that a quark cannot move forward. This is a confining phenomena of QCD. Our present prediction of the quark mean squared momentum distribution is an initial calculation in QCD about properties of the QCD vacuum state.

Our results on quark virtuality at small $x$ and various local vacuum condensates are in good agreement with the predictions of other computations such as QCD sum rules, Lattice QCD calculations and Instanton model predictions.
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