The Energy-dependent $\gamma$-Ray Morphology of the Crab Nebula Observed with the Fermi Large Area Telescope

Paul K. H. Yeung and Dieter Horns

Institute for Experimental Physics, Department of Physics, University of Hamburg, Ludver Chaussee 149, D-22671 Hamburg, Germany; kin.hang.yeung@desy.de

Received 2019 January 31; revised 2019 March 5; accepted 2019 March 15; published 2019 April 23

Abstract

The Crab Nebula is a bright emitter of non-thermal radiation across the entire accessible range of wavelengths. The spatial and spectral structures of the synchrotron nebula are well-resolved from radio to hard X-ray emission. The unpulsed emission at GeV–TeV energies is mostly produced via inverse-Compton scattering of energetic electrons with the synchrotron-emitted photons. The spatial structure observed at these energies provides insights into the distribution of electrons and indirectly constrains the so-far unknown structure of the magnetic field in the nebula. Analyzing the Large Area Telescope (LAT) data accumulated over $\sim 9.1$ yr with a properly refined model for the Crab pulsar’s spectrum, we determined the $68\%$ containment radius ($R_{68}$) of the Crab Nebula to be $(0.0330 \pm 0.0025_{\text{stat}} \pm 0.0012_{\text{sys}})''$ $(1.98 \pm 0.15_{\text{stat}} \pm 0.07_{\text{sys}})$ in the $5–500$ GeV band. The estimated systematic uncertainty is based on two factors: (1) different analysis methods, morphological models and event types, and (2) the point-spread function evaluated with observations of Mrk 421. When comparing the Fermi-LAT and High Energy Stereoscopic System results on the spatial extension, we find evidence for an energy-dependent shrinking of the Crab Nebula’s $\gamma$-ray extension ($R_{68} \propto E_\gamma^{\alpha}$, where $\alpha = 0.155 \pm 0.035_{\text{stat}} \pm 0.037_{\text{sys}}$).

Key words: gamma rays: ISM – ISM: individual objects (Crab Nebula)

1. Introduction

Isolated neutron stars are efficient particle accelerators, leading to the formation of pulsar wind nebula (PWN) systems. The extended cloud of non-thermal plasma radiates in a broad energy range, from radio to X-ray and even extends toward the highest gamma-ray energies (Aharonian et al. 2004; Bühler & Blandford 2014; Dubner et al. 2017).

The Crab Nebula is a PWN powered by a $\sim 1$ kyr old pulsar (Hester 2008). It is a part of the core-collapse supernova remnant located in the constellation of Taurus, at a distance of $2$ kpc (Trimble 1968). The exceptionally broad energy range observed from the Crab Nebula enables us to study the processes of particle acceleration occurring at the termination shock (e.g., Spitkovsky & Arons 2004; Fraschetti & Pohl 2017).

The discovery of its intense $\gamma$-ray emission dates back to the observations at MeV–GeV energies with the second NASA Small Astronomy Satellite (Kniffen et al. 1974) and at TeV energies with the Whipple Observatory 10 m reflector (Weekes et al. 1989). The observed $\gamma$-ray spectrum of the Crab Nebula from $1$ GeV to $80$ TeV has been compared to various model calculations that use widely different approaches (de Jager & Harding 1992; Atoyan & Aharonian 1996; Hillas et al. 1998; Volpi et al. 2008; Meyer et al. 2010; Martín et al. 2012). However, all these models are based upon the assumption that the gamma-ray emission in this energy range is predominantly produced via inverse-Compton scattering of relativistic electrons with synchrotron-radiated photons, as initially suggested by Rees (1971) and Gunn & Ostriker (1971). The spatial and spectral properties of the synchrotron nebula from optical to $\gamma$-rays are accurately described by a spherically symmetric magnetohydrodynamic model of the outflow forming the Crab Nebula (Kennel & Coroniti 1984). The thermal dust emission and the cosmic microwave background (CMB) contribute to an additional seed-photon field.

An investigation of the spatial structure of the Crab Nebula in $\gamma$-ray is certainly required, as it will provide additional insights into the concrete mechanisms of the nebula’s $\gamma$-ray emission. Among the different theoretical models, the predicted characteristic size of the $\gamma$-ray nebula varies only a little from $60''$ (Atoyan & Aharonian 1996) to $80''$ (de Jager & Harding 1992), even though the surface brightness shape is quite different.

In a previous study by Ackermann et al. (2018) with the Fermi Large Area Telescope (LAT), an extended morphology seemingly fit the $> 10$ GeV $\gamma$-ray emission from the Crab Nebula better than the point model did, even when taking the systematic uncertainties related to the point-spread function (PSF) into account. On the other hand, the High Energy Stereoscopic System (H.E.S.S.) revealed that the Crab Nebula is extended at TeV $\gamma$-ray energies with an rms width of $52''$ (Holler et al. 2017).

The energy losses of the electrons diffusing outward in the nebula lead to the observed energy-dependent size of the synchrotron nebula. Similarly, the $\gamma$-ray extension of the inverse-Compton nebula may decrease with increasing energy.

In this work, we study the $\gamma$-ray morphology of the Crab Nebula and its energy dependence in detail, with the $> 5$ GeV LAT data accumulated over $\sim 9.1$ yr and a properly refined model for the Crab pulsar’s spectrum. The systematic uncertainties associated with the PSF are evaluated as well. We compare the physics interpreted from the $\gamma$-ray spectrum, the radio extension, and the energy-dependent $\gamma$-ray extension.

2. Observation and Data Reduction

We perform a series of unbinned maximum-likelihood analyses for a region of interest (ROI) of $15''$ radius centered at R.A. = $05^h34^m31^s.94$, decl. = $+22^\circ00'52''2$ (J2000), which is approximately the center of the Crab Nebula (Lobanov et al. 2011). We use the data of $>5$ GeV photon energies, registered with the LAT between 2008 August 4 and 2017 September 25.
The data are reduced and analyzed with the aid of the Fermi Science Tools v11r5p3 package. Considering that the Crab Nebula is quite close to the Galactic plane (with a Galactic latitude of \(-5^\circ.7844\)), we adopt the events classified as Pass8 “Clean” class for the analysis so as to better suppress the background. The corresponding instrument response function (IRF) “P8R2_CLEAN_V6” is used throughout the investigation. We further filter the data by accepting only the good time intervals where the ROI was observed at a zenith angle of less than 90° so as to reduce the contamination from the albedo of Earth.

In order to subtract the background contribution, we include the Galactic diffuse background (gll_iem_v06.fits), the isotropic background (iso_P8R2_CLEAN_V6_v06.txt), as well as all other point sources cataloged in the most updated Fermi/LAT catalog (FL8Y\(^1\)) within 25° from the ROI center in the source model. We set the spectral parameters of the sources within 5° from the ROI center in the analysis. For the sources beyond 5° from the ROI center, their spectral parameters are fixed to the catalog values.

The two point sources located within the nebula are cataloged as FL8Y J0534.5+2200 and FL8Y J0534.5+2201i, which respectively model the Crab pulsar and the Crab Nebula. We leave the point-source morphology of the pulsar component unchanged throughout our work. For the PWN component, we choose a point-source model as well as disk models of different radii, in order to determine the most likely morphology in each energy range we chose.

We fix the spectral parameters of FL8Y J0534.5+2200 (the pulsar component) at certain values so as to avoid degeneracies in the fitting procedure. Since it is the most contaminating “background” source in our work and its spectral fitting of a power law (PL) with a sub-exponential cutoff (PLEC) in the FL8Y catalog is dominated by the <1 GeV data, we refine its spectral model at larger energies based on the phase-folded spectrum of the Crab pulsar in 69–628 GeV measured with MAGIC (Ansoldi et al. 2016). We thereby determine a PL spectrum (see Figure 1): \( \frac{dN}{dE} = 2.19 \times 10^{-10} \left( \frac{E}{\text{GeV}} \right)^{-3.13} \) photons cm\(^{-2}\) s\(^{-1}\) MeV\(^{-1}\). This PL model intersects with the catalog PLEC model at \( \sim 38 \) GeV, below which the PL seriously underpredicts the Crab pulsar’s flux. Below 38 GeV, the more recent PLEC model (FL8Y) is in a good agreement with the binned spectrum reported in the LAT Second Pulsar Catalog (Abdo et al. 2013)—see also Figure 1. Therefore, we keep the FL8Y spectrum of the Crab pulsar for energies below 38 GeV, while we replace the PLEC with the PL for other energies.

As presented in Figure 1 and Section 3.3, such a hybrid model for FL8Y J0534.5+2200 (the pulsar component) yields a >5 GeV spectrum of FL8Y J0534.5+2201i (the PWN component), which essentially matches the off-pulse spectrum reported by Buehler et al. (2012). In particular, above 20 GeV (and 40 GeV), the predicted flux of the Crab pulsar only accounts for \(<21\%\) (and \(<5\%\)) of the Crab system’s total flux. Clearly, the spectral model assigned to the Crab pulsar is not expected to introduce any obvious bias in our analyses for such high energies, due to its minor contribution of flux.

---

\(^1\) Fermi-LAT 8 yr Source List: https://fermi.gsfc.nasa.gov/ssc/data/access/lat/8yr.

---

Figure 1. GeV–TeV spectral energy distribution of the Crab pulsar (in gray, black, and green) and the Crab PWN (in red and blue). The gray dashed curve represents the PLEC pulsar model in FL8Y, which is kept for energies below 38 GeV. The phase-averaged pulsar spectrum as reported in the LAT Second Pulsar Catalog (Abdo et al. 2013) is shown as green open circles for comparison. The black solid line represents the PL model fit to the 69–628 GeV pulsar spectrum measured with MAGIC (open squares), where the data are taken from Ansoldi et al. (2016). It replaces the PLEC for energies above 38 GeV. The red dots are the Fermi-LAT fluxes of the PWN determined in our analyses. The red line represents the maximum-likelihood broken-power-law (BKPL) model we determined for the 5–500 GeV PWN spectrum (see Section 3.3 for more details). The blue solid curve represents the off-pulse model of the PWN reported by Buehler et al. (2012).

3. Data Analysis and Results

3.1. The Centroid of the Nebula Emission

The 5–500 GeV test-statistic (TS) map is shown in Figure 2, where all FL8Y catalog sources except FL8Y J0534.5+2201i (the PWN component of the Crab system) are subtracted. The pixel size of the map is chosen that the PSF is oversampled (0°001 × 0°001). Therefore, the map covers a small field of view (0°014 × 0°014). The TS map demonstrates that the catalog position of FL8Y J0534.5+2201i (marked as a red cross in Figure 2) is comfortably located within the 68% error circle of the centroid for four degrees of freedom (d.o.f.), where the TS value is lower than the maximum by 4.7.\(^2\) This centroid is within 2σ consistent with the centroid position of the radio nebula (marked as red box in Figure 2) but offset from the radio position of the Crab pulsar at a >3σ level.

We divide the entire 5–3000 GeV band into several energy intervals: 5–10, 10–20, 20–40, 40–80, 80–150, 150–300 GeV, and 0.3–3 TeV.\(^3\) For each spectral segment, we repeated creating the TS map with the same pixel size and field of view. The separations of Crab Nebula’s centroids from the radio position of the Crab pulsar are plotted in Figure 3.

---

\(^2\) A \(\chi^2\) Distribution is assumed. There are four d.o.f. because of the four variables: R.A., decl., flux normalization, and photon index.

\(^3\) The spectral coverages of the Galactic diffuse background (gll_iem_v06.fits) and the isotropic background (iso_P8R2_CLEAN_V6_v06.txt) are up to ~0.5 TeV and ~0.9 TeV, respectively. Yet, their contamination becomes negligible above 0.3 TeV. Therefore, we remove them from the source model for the 0.3–3 TeV analyses.
The catalog positions of FL8Y J0534.5+2201i (the Crab PWN) are substracted. The color scale represents the TS value subtracting the maximum. The pixel size (0.001 × 0.001) oversamples the PSF and the map covers a field of view of 0″014 × 0″014. The green and red crosses represent the catalog positions of FL8Y J0534.5+2201 (the Crab pulsar) and FL8Y J0534.5+2201i, respectively. The 68%, 95%, and 99.7% error circles of the γ-ray centroid for four degrees of freedom, where the TS value is lower than the maximum by 4.7, 9.5, and 16.0 respectively. The 68%, 95%, and 99.7% error circles of the γ-ray centroid in all seven segments are consistently offset from the radio position of the Crab pulsar taken from Lobanov et al. (2011).

Because no discrepancy among the centroids in different energy bands and the FL8Y position can be robustly claimed, we consistently leave the position of FL8Y J0534.5+2201i unchanged in subsequent analyses (i.e., there are no additional d.o.f. from the centroid position).

### 3.2. Variability of the Flux

We divide the first ~9.1 yr of Fermi-LAT observation into a number of 15 day segments, and perform an unbinned maximum-likelihood analysis for 5–500 GeV data in each individual temporal segment. Considering that the isotropic background γ-ray emission cannot noticeably change within a short timescale of 10 yr, we fix it at the ~9.1 yr average obtained from the full-timespan analysis, so the statistical fluctuations from the isotropic background model are avoided. The light curve of FL8Y J0534.5+2201i is shown in Figure 4.

A constant flux satisfactorily fits the entire temporal distribution with $\chi^2 \sim 256$ for 221 d.o.f. ($p(>\chi^2) = 0.05$). In order to check for significant deviations where subsequent flux points are either above or below the average, we perform an additional Wald–Wolfowitz run test (where we define two kinds of runs: runs of bins above the average and runs of those below it), the observed number of runs deviates from the expected number by only ~0.6σ. Furthermore, the 5–500 GeV flux of the nebula shows no correlation with the flares that enhanced its >0.1 GeV flux by a factor of >5 (see Figure 3 of Buehler et al. 2012). This is as expected because the γ-ray spectra during the flaring states have their cutoff energies well below 1 GeV (see Figures 6 and 7 of Buehler et al. 2012).

We hereby confirm that the γ-ray flares of the Crab system at lower energies do not perturb the results above 5 GeV and the 5–500 GeV flux is essentially steady. It is therefore appropriate to accept all the good time intervals between 2008 August 4 and 2017 September 25 in subsequent analyses (i.e., no further screening of data is required).

### 3.3. Extension and Its Energy Dependence

In order to examine whether the γ-ray emission from the PWN is spatially extended, we perform a likelihood-ratio test to quantify the significance of extension in the 5–500 GeV band.

---

**Figure 2.** TS map of the field around the Crab system in 5–500 GeV, where all FL8Y catalog sources except FL8Y J0534.5+2201i (the Crab PWN) are substracted. The color scale represents the TS value subtracting the maximum. The pixel size (0.001 × 0.001) oversamples the PSF and the map covers a field of view of 0″014 × 0″014. The green and red crosses represent the catalog positions of FL8Y J0534.5+2201 (the Crab pulsar) and FL8Y J0534.5+2201i, respectively. The 68%, 95%, and 99.7% error circles of the γ-ray centroid for four degrees of freedom, where the TS value is lower than the maximum by 4.7, 9.5, and 16.0 respectively (see footnote 2), are plotted in cyan (from innermost to outermost). The red square indicates the radio centroid of the Crab Nebula, which is determined from a VLA (5.5 GHz) image published in Bietenholz et al. (2004; see Section 4.2 and Figure 7 for more detail). The green diamond indicates the radio position of the Crab pulsar taken from Lobanov et al. (2011).

**Figure 3.** Difference of the Crab Nebula’s centroid position from the radio position of the Crab pulsar taken from Lobanov et al. (2011) in different energy segments, along the axes of R.A. and decl., respectively. On each panel, the black solid line indicates the best-fit constant-value function (i.e., the error-weighted mean), and sandwiched between the black dashed lines is its 1σ error range. The gray dotted lines indicate the position of the Crab Nebula’s radio centroid (determined from Figure 7) relative to the radio position of the Crab pulsar.

Centroids in all seven segments are consistently offset from the pulsar by $\Delta$R.A. $= (3.05 \pm 6.51)''$ ($\chi^2 \sim 1.41$ for 6 d.o.f.) and $\Delta$decl. $= (20.86 \pm 6.51)''$ ($\chi^2 \sim 2.60$ for 6 d.o.f.).
After refinement of the Crab pulsar’s spectrum, we found that a BKPL spectral model is preferred over a PL by $\sim 5.4\sigma$ for the PWN ($\Delta TS \sim 32.7$ for 2 d.o.f.). The spectrum of the PWN softens from $\Gamma_1 = 1.585 \pm 0.067$ to $\Gamma_2 = 2.047 \pm 0.036$ at $E_h = 18.05 \pm 1.60$ GeV, consistent with the spectral model determined for the off-pulse phase by Buehler et al. (2012; see Figure 1). Therefore, we assign it a BKPL spectral model. We attempt uniform-disk morphologies of different sizes as well as a point-source model on it. The $2\Delta \ln(\text{likelihood})$ of different sizes relative to the point-source model are plotted in Figure 5.

The most likely disk radius is determined to be $(0.040 \pm 0.003)^{\circ}$ and this morphology is preferred over a point-source model by $\sim 7.6\sigma$. The corresponding 68% containment radius ($R_{68}$; the uniform-disk radius multiplied by $\sqrt{0.68}$) of $(0.0330 \pm 0.0025)^{\circ}$ resp. $(1.98 \pm 0.15)^{\circ}$ is consistent with that determined in Ackermann et al. (2018) with 1 GeV–1 TeV data and a Gaussian morphology $(0.030 \pm 0.003^{\text{stat}} \pm 0.007^{\text{sys}})$, within the tolerance of statistical uncertainties. Motivated by the uncertainties in the Crab pulsar’s spectrum, we repeated this analysis while altering the flux normalization of the Crab pulsar by $\pm 20\%$. It turns out that the maximum-likelihood radius remains unchanged even though $\Gamma_1$ is altered by $\pm 0.28$ (i.e., the spectral model of the Crab pulsar has no noticeable contribution to the systematic uncertainty).

We further verified the robustness of the 5–500 GeV result by performing binned maximum-likelihood analyses with the aid of the “fermipy” package (Wood et al. 2017). We adopted a bin size of 0°01, which is sufficiently small to sample the PSF as well as the $\gamma$-ray nebula. In addition to the analysis with “FRONT+BACK” data, we also performed a joint analysis with “PSF2” and “PSF3” data, and an analysis with only “PSF3” data (respectively sacrificing the photon statistics by a factor of $\sim 1/2$ and $\sim 1/4$ for better spatial resolution). For each data set we worked on, we examined both uniform-disk and Gaussian morphologies.

As can be seen in Table 1, regardless of the event type and morphological model, the values of $R_{68}$ are all consistent with $0.0330 \pm 0.0025$ (the result of the unbinned maximum-likelihood analysis) within the tolerance of statistical uncertainties. For each event type we attempted, the two morphological models have roughly the same goodness of fit ($\Delta TS_{\text{ext}} \leq 1.4$), and their difference in $R_{68}$ is negligible ($\leq 0.6\sigma$). Also, screening out the data partitions of poorer resolution did not lead to a noticeable drop in $TS_{\text{ext}}$. We hereby compute a systematic uncertainty of $R_{68}$ of $\pm 0.0012$, which stems from the analysis method, morphological model, and event selection.

In order to investigate whether the $\gamma$-ray morphology changes with photon energy, we divide the entire 5–3000 GeV band in the same way as in Section 3.1. We repeat the likelihood-ratio test for each spectral segment, with a PL assigned to the PWN spectrum. The results are tabulated in Table 2. We sum up the differences between $2\ln(L_{\text{ext, max}}/L_{\text{pp}})$ and $2\ln(L_{\text{ext,0.04L_{pp}}})$ over all seven segments, hence we get a $\chi^2$ value of the energy dependence of 18.2 for 7 d.o.f.. In other words, based on our Fermi-LAT results only, an energy-dependent morphology with the nebula size shrinking with increasing energy is preferred over a constant size by $\sim 2.5\sigma$. In addition, the PWN’s flux in each segment is consistent with the off-pulse spectrum reported by Buehler et al. (2012). This confirms that the systematic uncertainties associated with the Crab pulsar’s spectral model are not a serious issue.

| Event Type | Morphological Model | Radius (deg) $^{\circ}$ | $R_{68}$ (deg)$^{\circ}$ | $TS_{\text{ext}}$ |
|------------|---------------------|-------------------------|------------------------|----------------|
| FRONT+BACK | Disk                | $0.040 \pm 0.003$       | $0.0330 \pm 0.0025$    | 57.81          |
|            |                     | Binned maximum-likelihood analysis in “fermipy,” bin size = 0°01 |
| FRONT+BACK | Disk                | $0.0385 \pm 0.0026$     | $0.0317 \pm 0.0028$    | 45.50          |
| PSF2+PSF3 | Disk                | $0.0400 \pm 0.0032$     | $0.0330 \pm 0.0023$    | 62.34          |
| PSF3      | Disk                | $0.0405 \pm 0.0036$     | $0.0334 \pm 0.0027$    | 51.48          |

Notes.

$^{4}$ The 68% containment radius. For a disk model, it is the radius multiplied by $\sqrt{0.68}$.

$^{5}$ The $2\Delta \ln(\text{likelihood})$ between the best-fit morphology and the point-source model.

---

Figure 5. The $2\Delta \ln(\text{likelihood})$ in 5–500 GeV, when uniform disks of different radii replace the point-source model to be the morphology of FL8Y J0534.5+2201i.
Table 2
Morphological Studies for the Crab PWN in Different Energy Segments, with Fermi-LAT and H.E.S.S.

| Energy Range (GeV) | Disk Radius (deg) | $R_{68}$ (deg)$^c$ | $2\ln(L_{\text{ext},\text{max}}/L_{\text{p}})^{b}$ | $2\ln(L_{\text{ext},0.01}/L_{\text{p}})^{b}$ |
|-------------------|------------------|------------------|------------------|------------------|
| 5–500             | 0.040 ± 0.003    | 0.0330 ± 0.0025  | 57.81            | 57.81            |
| 5–10              | 0.073 ±0.002     | 0.0602 ±0.0082   | 15.75            | 8.40             |
| 10–20             | 0.057 ±0.005     | 0.0470 ±0.0041   | 33.80            | 26.27            |
| 20–40             | 0.034 ±0.007     | 0.0280 ±0.0044   | 11.14            | 10.52            |
| 40–80             | 0.038 ±0.006     | 0.0313 ±0.0049   | 13.78            | 13.46            |
| 80–150            | 0.032 ±0.010     | 0.0264 ±0.0068   | 5.06             | 4.40             |
| 150–300           | 0.028 ±0.010     | 0.0231 ±0.0054   | 3.34             | 1.63             |
| 300–1000          | 0.041 ±0.013     | 0.0338 ±0.0107   | 2.78             | 2.76             |

H.E.S.S. result (Holler et al. 2017)

| Energy Range (GeV) | Disk Radius (deg) | $R_{68}$ (deg)$^c$ | $2\ln(L_{\text{ext},\text{max}}/L_{\text{p}})^{b}$ | $2\ln(L_{\text{ext},0.01}/L_{\text{p}})^{b}$ |
|-------------------|------------------|------------------|------------------|------------------|
| 700–10000         | ...              | 0.0219 ±0.0012stat ±0.0033sys | 83               | ...             |

Notes.

$^a$ The 68% containment radius, which is the disk radius multiplied by $\sqrt{0.68}$.

$^b$ The $2\Delta \ln(\text{likelihood})$ between the best-fit uniform-disk morphology and the point-source model.

$^c$ The $2\Delta \ln(\text{likelihood})$ between the uniform-disk morphology of a 0.04 radius and the point-source model.

Thus, the dependence of $R_{68}$ on the photon energy ($E$) can be formulated as $R_{68} = (0.0357 \pm 0.0021)(E/44.0\text{ GeV})^{-0.155\pm 0.035}$ deg without any correlation between the prefactor and index.

The extension of the nebula at energies between 20 and 40 GeV appears to deviate from the PL ($\sim 1.8\sigma$). Interestingly, the energy flux of the nebula levels off to an almost flat peak at the same energy (see Figure 1).

4. Discussion

4.1. Evaluation of the PSF

Because the radius of the most likely uniform-disk morphology of FL8Y J0534.5+2201i is at least two times smaller than the 68% containment radius of the acceptance weighted PSF for all energy bands we investigate (see SLAC$^5$), it is necessary to evaluate the accuracy of the IRF “P8R2_CLEAN_V6” we used. We did this by determining the “apparent” $\gamma$-ray extension of Mkn 421, a GeV- and TeV-bright blazar at high Galactic latitude, through the same procedures.

It turns out that the best-fit uniform-disk morphology of Mkn 421 in 5–500 GeV has a radius of $0.025^{+0.003}_{-0.004}$ with a $T_{S_{\text{ext}}}$ of 10.2, which are significantly smaller than those determined for the Crab Nebula. Also, the most likely extensions of Mkn 421 in the divided energy segments are, as overlaid in Figure 6, collectively smaller than those of the Crab Nebula. Assuming that the extension determined for Mkn 421 is purely an instrumental effect, we estimate a systematic uncertainty of $-0.009$ for the disk radius of the Crab Nebula in 5–500 GeV (corresponding to a systematic uncertainty of $-0.0074$ for $R_{68}$). Combining this with the effects of changing the analysis method, morphological model, event type, and Crab pulsar’s spectrum, we estimate the total systematic uncertainty of $R_{68}$ to be $(0.00012, 0.0075)^{10}$.

$^5$ Fermi-LAT Performance: http://www.slac.stanford.edu/exp/glast/groups/canda/lat_Performance.htm.

Figure 6. Characteristic extensions (defined as $R_{68}$, for a uniform-disk model, it is the disk radius multiplied by $\sqrt{0.68}$) of the Crab PWN in different energy segments, from the Fermi-LAT band to the H.E.S.S. band (in which the data are taken from Holler et al. 2017). For the H.E.S.S. bin, we take its combined uncertainty (where statistical and systematic uncertainties are added in quadrature). The red solid line indicates the characteristic extension of the energy-independent morphology (disk) fit to the 5–500 GeV emission, and sandwiched between the red dashed lines is its 1$\sigma$ error range. The blue line is the power-law function that best describes the relation of characteristic extension to the photon energy, and sandwiched between the blue dashed lines is its 1$\sigma$ error range. The green solid line indicates the characteristic radio extension, based on a VLA (5.5 GHz) image published in Bietenholz et al. (2004; see Section 4.2 and Figure 7 for more details about how the measurement is done). The “apparent” extensions of Mkn 421 determined through the same procedures (for evaluating the PSF) are plotted in gray, and we place upper limits of a 95% confidence level for those segments in which a point model is preferred over any uniform-disk model.

We scale the disk radii determined with LAT to $R_{68}$, so that they can be compared with the H.E.S.S. Gaussian extension reported by Holler et al. (2017), as plotted in Figure 6. The $R_{68}$ in 5–500 GeV observed with Fermi-LAT is larger than that observed at higher energies of 0.7–10 TeV with H.E.S.S. at a $\sim 2.6\sigma$ level. A constant-value function yields a poor fit to the distribution of $R_{68}$ ($\chi^2 \sim 29.4$ for 7 d.o.f.; i.e., it deviates from a uniform distribution at a $\sim 3.8\sigma$ level). When we fit a PL function instead, the goodness of fit greatly improves ($\chi^2 \sim 8.7$ for 6 d.o.f.). An F-test yields a statistic of $\sim 14.3$ for (1, 6) d.o.f., implying a chance probability of $\leq 0.9\%$ for the energy-dependent shrinking.
Array morphology for the Crab Nebula, we retrieved a Very Large radio position of the Crab pulsar taken from Lobanov et al. (2011). The position of the intensity-weighted centroid relative to the Crab pulsar is indicated in Figure 3. The position of the intensity-weighted centroid relative to the Crab pulsar is overlaid in Figure 6.

4.2. Comparison of the γ-Ray Nebula to the Radio Nebula

In order to compare the γ-ray morphology with the radio morphology for the Crab Nebula, we retrieved a Very Large Array (5.5 GHz) image (Figure 7) published in Bietenholz et al. (2004). First of all, we determined the intensity-weighted centroid and overlaid it in Figure 2. The position of this centroid relative to the Crab pulsar is indicated in Figure 3. Then, we determined the 68% containment circle centered at this centroid; its radius is overlaid in Figure 6.

As shown, the radio centroid is on the very edge of the 95% error circle of the 5–500 GeV centroid. Both the γ-ray and radio centroids of the PWN are northward offset from the Crab Pulsar. Neglecting the systematic uncertainties associated with the PSF, the average of 5–20 GeV extensions is larger than the radio extension by ϵ ≈ 4.7σ, while the >20 GeV extensions do not exceed the radio extension. Even after reducing the 5–20 GeV extensions based on the assumption that the LAT extensions determined for Mkn 421 are purely instrumental effects, their average still exceeds the radio extension by ϵ ≈ 2.9σ.

The comparison of the size of the inverse-Compton nebula at 5–20 GeV with the size of the synchrotron nebula at 5 GHz provides a measure of the ratio of seed photon field energy density and magnetic field energy density. The synchrotron emission at 5 GHz is mainly produced by electrons with Lorentz factors γ ≈ 6 × 10^3 (B/120 \mu G)^{-1/2}. The same electrons will produce inverse-Compton emission at energies below a GeV. Therefore, the ratio of r_{IC}/r_{SY} is a measure of the size of the seed photon field r_{seed} and the size of the magnetized nebula r_{B}. In a more detailed modeling approach, it is therefore necessary to include the spatial distribution of additional seed photon fields, including the emission of the dusty plasma in which the synchrotron nebula is embedded.

4.3. Comparison of the Observed Energy Dependence of the γ-Ray Extension to Theoretical Models

We found that, as the photon energy (E_{IC}) increases from 5 GeV to 10 TeV, the spatial extension of the Crab Nebula is shrinking with a PL index such that R_{68} \propto E_{IC}^{0.155 \pm 0.035}. Even after we modified the spectral distribution of the Crab Nebula’s extensions based on the assumption that the LAT extensions determined for Mkn 421 are purely instrumental effects, the size of the Crab Nebula still deviates from a uniform distribution at a ϵ ≈ 2.9σ level, and it still shrinks with a PL index of ϵ ≈ 0.118, which is a reasonable estimate of the systematic lower bound. Such an observed energy dependence of the γ-ray extension is comparable to the energy dependence of the size of the underlying electron distribution, which was found to be r_{e} \propto \gamma^{-0.17} (Meyer et al. 2010) when assuming a homogeneous magnetic field. This approach effectively models the radiative cooling of electrons while expanding into the nebula.

For Thomson-type inverse-Compton scattering with E_{IC} ≈ γ^2\epsilon (where \epsilon is the seed photon energy), provided that the spectral number density of the seed photon field is uniform (e.g., like the CMB), the resulting nebula size should shrink with a harder PL: R_{68} \propto \sqrt{E_{\epsilon}} \propto E_{IC}^{-0.17/2}, which is similar to the energy dependence of the synchrotron nebula size. However, at energies larger than a few 100 GeV, inverse-Compton scattering with the synchrotron seed-photon field starts to be affected by Klein–Nishina effects. In the case of dominating Klein–Nishina effects, the energy dependence of the inverse-Compton nebula will proceed with R_{68} \propto r_{e} \propto E_{IC}^{-0.17}. Even though this is apparently a closer match to the observed energy dependence, Klein–Nishina effects are not expected to dominate in the low-energy part (E < 500 GeV) covered with the measurement presented here.

The stronger energy dependence can be interpreted through a change in the ratio of energy densities \rho_\epsilon(r)/\rho_{B}(r) in the seed-photon field \rho_\epsilon and in the magnetic field \rho_{B}. In turn, this may be an indication of an unknown magnetic field structure in the nebula. Further details on the interpretation of the energy dependence of the spatial extent of the inverse-Compton nebula require a careful modeling of the interplay of the spatial distribution of seed-photon and magnetic fields and of the transition between Thomson and Klein–Nishina scattering, which are beyond the scope of this publication.

5. Summary

With the proper refinement of the spectral model of the Crab pulsar, we unbiasedly determined the 68% containment radius (R_{68}) of the inverse-Compton nebula to be (0.0330 ± 0.0025_{stat} ± 0.0012_{sys}) × (1.98 ± 0.15_{stat} ± 0.05_{sys}) in the 5–500 GeV band. The particularly large 5–20 GeV extensions, compared with the radio size of the synchrotron nebula, imply that additional sources of seed photons (e.g., CMB and dust) must be taken into account in theoretical modeling. The strong energy dependence of its extension from 5 GeV to 10 TeV (R_{68} \propto E_{IC}^{\alpha}, where \alpha = 0.155 \pm 0.035_{stat} − 0.037_{sys}), deviates from the synchrotron nebula, where the size shrinks with E_{SY}^{−0.085}. Possible explanations have been considered (transition from Thomson to Klein–Nishina regime and a non-uniform
magnetic field). While the former explanation appears to be unrealistic, the latter is a well-known feature of the downstream flow, as expected for the Crab Nebula (Kennel & Coroniti 1984).

P.K.H.Y. acknowledges the support of the DFG under the research grant HO 3305/4-1. We thank J. Hahn and M. Holler for useful discussions. We thank the anonymous referee for very useful comments that helped to improve the manuscript.

**Software:** fermipy (Wood et al. 2017) and Fermi Science Tools (v11r5p3).

**ORCID iDs**

Paul K. H. Yeung @ https://orcid.org/0000-0003-3476-022X
Dieter Horns @ https://orcid.org/0000-0003-1945-0119

**References**

Abdo, A. A., Ajello, M., Allafort, A., et al. 2013, *ApJS*, 208, 17
Ackermann, M., Ajello, M., Baldini, L., et al. 2018, *ApJS*, 237, 32
Aharonian, F., Akhperjanian, A., Beilicke, M., et al. 2004, *ApJ*, 614, 897
Ansoldi, S., Antonelli, L. A., Antoranz, P., et al. 2016, *A&A*, 585, A133
Atoyan, A. M., & Aharonian, F. A. 1996, *MNRAS*, 278, 525
Bietenholz, M. F., Hester, J. J., Frail, D. A., & Bartel, N. 2004, *ApJ*, 615, 794
Bühler, R., Scargle, J. D., Blandford, R. D., et al. 2012, *ApJ*, 749, 26
Kennel, C. F., & Coroniti, F. V. 1984, *ApJ*, 283, 710
Holler, M., Berge, D., Hahn, J., et al. 2017, ICRC (Busan), 35, 676
Holler, M., Berge, D., Hahn, J., et al. 2017, ICRC (Busan), 35, 676
Hillas, A. M., Akerlof, C. W., Biller, S. D., et al. 1998, *ApJ*, 503, 744
Holler, M., Berge, D., Hahn, J., et al. 2017, ICRC (Busan), 35, 676
Kennel, C. F., & Coroniti, F. V. 1984, *ApJ*, 283, 710
Kniffen, D. A., Hartman, R. C., Thompson, D. J., Bignami, G. F., & Fichtel, C. E. 1974, *Natur*, 251, 397
Lobanov, A. P., Horns, D., & Maxlow, T. W. B. 2011, *A&A*, 533, A10
Martín, J., Torres, D. F., & Rea, N. 2012, *MNRAS*, 427, 415
Meyer, M., Horns, D., & Zechlin, H.-S. 2010, *A&A*, 523, A2
Rees, M. J. 1971, *NPhS*, 230, 55
Spitkovsky, A., & Arons, J. 2004, *ApJ*, 603, 669
Trimble, V. 1968, *AJ*, 73, 535
Volpi, D., Del Zanna, L., Amato, E., & Bucciantini, N. 2008, *A&A*, 485, 337
Weekes, T. C., Cawley, M. F., Fegan, D. J., et al. 1989, *ApJ*, 342, 379
Wood, M., Caputo, R., Charles, E., et al. 2017, ICRC (Busan), 301, 824

**The Astrophysical Journal, 875:123 (7pp), 2019 April 20**

Yeung & Horns