Influence of Lorentz-violating terms on a two-level system

Manoel M. Ferreira Jr\textsuperscript{a,c}, Adalto R. Gomes\textsuperscript{b}, Rafael C. C. Lopes\textsuperscript{a}

\textsuperscript{a}Departamento de Física, Universidade Federal do Maranhão (UFMA), Campus Universitário do Bacanga, São Luís-MA, 65085-580 - Brazil
\textsuperscript{b}Departamento de Ciências Exatas, Centro Federal de Educação Tecnológica do Maranhão - CEFET-MA São Luís-MA, 65025-001, Brazil
\textsuperscript{c}Grupo de Física Teórica José Leite Lopes, C.P. 91933, CEP 25685-970, Petrópolis, RJ, Brazil

The influence of Lorentz- and CPT-violating terms of the extended Standard Model on a semi-classical two-level system is analyzed. It is shown that the Lorentz-violating background (when coupled with the fermion sector in a vector way) is able to induce modifications on the Rabi oscillation pattern, promoting sensitive modulations on the usual oscillations. As for the term involving the coefficient coupled in an axial vector way, it brings about oscillations both on energy states and on the spin states (implied by the background). It is also seen that such backgrounds are able to yield state oscillations even in the absence of the electromagnetic field. The foreseen effects are used to establish upper bounds on the Lorentz-violating coefficients.

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I. INTRODUCTION

Planck scale physics is an unknown frontier where gravitational and quantum effects are closely entwined. At this scale, it might occur that Lorentz covariance is jeopardized. Such kind of idea has caught much attention mainly after some authors argued the possibility of Lorentz and CPT spontaneous breaking in the context of string theory \cite{1}. The detection of Lorentz violation at a lower energy scale, even minuscule, could be interpreted as a signature of spontaneous Lorentz violation at the underlying theory (defined at a higher energy scale). These remanent Lorentz violating effects, inherited from a high energy theory, would then employed to indicate possible features of a Planck scale physics. The Standard Model Extension (SME) \cite{2} is a broader version of the usual Standard Model that incorporates all Lorentz-violating (LV) coefficients (generated as vacuum expectation values of the underlying theory tensor quantities) that yield Lorentz scalars (as tensor contractions) in the observer frame. Such coefficients govern Lorentz violation in the particle frame, where are seen as sets of independent numbers.

The SME is actually the suitable framework to investigate properties of Lorentz violation on physical systems involving photons \cite{3}, \cite{4}, radiative corrections \cite{5}, fermions \cite{6}, neutrinos \cite{7}, topological defects \cite{8}, topological phases \cite{9}, cosmic rays \cite{10}, supersymmetry \cite{11}, particle decays \cite{12}, and other relevant aspects \cite{13}, \cite{14}. The SME has also been used as framework to propose Lorentz violating \cite{15} and CPT \cite{16} probing experiments, which have amounted to the imposition of stringent bounds on the LV coefficients.

Concerning the fermion sector of the SME, there are two CPT-odd terms, $v_{\mu} \bar{\psi} \gamma^{\mu} \psi$, $b_{\mu} \bar{\psi} \gamma^{5} \gamma^{\mu} \psi$, where $v_{\mu}$, $b_{\mu}$ are the LV backgrounds. The influence of these terms on the Dirac theory has already been examined in literature \cite{17}, passing through its nonrelativistic limit, with close attention on the hydrogen spectrum \cite{18}. A similar study has also been developed for the case of a non-minimal coupling with the background, with new outcomes \cite{19}. Atomic and optical physics is another area in which Lorentz violation has been intensively studied. Indeed, there are several works examining Lorentz violation in electromagnetic cavities and optical systems \cite{20}, \cite{21}, which contributed to establish upper bounds on the LV coefficients.

The present work is devoted to investigating the influence of Lorentz violation induced by the coefficients $v_{\mu}, b_{\mu}$ on the physics of a semi-classical two-level system. As some fundamental concepts of two-level system are

*Electronic address: manojr@pq.cnpq.br, argomes@pq.cnpq.br
also important for the description of laser systems, some results obtained at a semi-classical level may indicate perspectives on the quantum behavior of photons on a resonant cavity. We start from the Lorentz-violating non-relativistic Hamiltonian stemming from the Dirac Lagrangian supplemented with the terms $v_\mu \bar{\psi} \gamma^\mu \psi, b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi$. The LV terms are then considered as perturbations that may modify the dynamics of Rabi oscillations on a two-level system. The first analysis is performed for the term $v_\mu \bar{\psi} \gamma^\mu \psi$. It is seen that it induces modifications on the population inversion function (PIF), that may be partially frustrated in some situations or modulated as a beat for other parameter values. Numerical simulations indicated the absence of LV effects on the system for $v_\mu \leq 10^{-10}eV$, which thus can be taken as an upper bound for this background. In order to examine the effect of the term $b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi$, we have defined a four-state basis, considering the possibility of the electron spin to be up or down. As a consequence, both eigenenergy and spin state oscillations take place. These backgrounds are able to induce Rabi oscillations even in the absence of electromagnetic external field. The non observation of spin oscillation in a real situation was used to set up an upper bound on the $b$ magnitude ($|b_\mu| < 10^{-19}eV$).

This paper is outlined as follows. In Sec. II, it is presented the fermion sector Lagrangian here taken into account, with the associated nonrelativistic Hamiltonian. In Sec. III, some topics of a two-level system are revisited. Further, the Lorentz-violating effects on such a system are discussed and analyzed. In Sec. IV, we finish with our concluding remarks.

II. LORENTZ-VIOLATING DIRAC LAGRANGIAN

We begin considering the presence of the two Lorentz- and CPT-violating terms $(v_\mu \bar{\psi} \gamma^\mu \psi, b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi)$ in the fermion sector,

$$\mathcal{L} = \mathcal{L}_{\text{Dirac}} - v_\mu \bar{\psi} \gamma^\mu \psi - b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi,$$

where $\mathcal{L}_{\text{Dirac}}$ is the usual Dirac Lagrangian ($\mathcal{L}_{\text{Dirac}} = \frac{i}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi - m_\psi \bar{\psi} \psi$), $v_\mu$ and $b_\mu$ are two CPT-odd coefficients that here represent the fixed background responsible for the violation of Lorentz symmetry in the frame of particles. In true, the terms $v_\mu \bar{\psi} \gamma^\mu \psi, b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi$ behave as a scalar and a pseudoscalar only in the observer frame, in which $v_\mu$ and $b_\mu$ are seen as genuine 4-vectors and no Lorentz-violation takes place\[2\]. The Euler-Lagrange equation applied on this Lagrangian provides the modified Dirac equation:

$$(\gamma^\mu \partial_\mu - v_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - m_\psi) \psi = 0,$$

which corresponds to the usual Dirac equation supplemented by the Lorentz-violating terms associated with the background. Such equation is also attainable in the momenta space:

$$(\gamma^\mu p_\mu - v_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - m_\psi) w(p) = 0,$$

with $w(p)$ being the $(4 \times 1)$ spinor in momentum space. It is possible to show that each component of the spinor $w$ satisfies a changed Klein-Gordon equation which represents the dispersion relation of this model, given as follows:

$$\left[ \left( (p - v)^2 - b^2 - m^2 \right)^2 + 4b^2(p - v)^2 - 4b \cdot (p - v) \right] = 0,$$

We now asses the nonrelativistic limit of such modified Dirac equation. To correctly do it, Lagrangian \[\Box\] must be considered in the presence of an external electromagnetic field ($A_\mu$) coupled to the matter field by means of the covariant derivative ($D_\mu = \partial_\mu + ieA_\mu$). Lagrangian \[\Box\] is then rewritten in the form:

$$\mathcal{L} = \frac{1}{2} \bar{\psi} \gamma^\mu D_\mu \psi - m_\psi \bar{\psi} \psi - b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi - v_\mu \bar{\psi} \gamma^\mu \psi,$$

which implies: $(\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - v_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - m_\psi) w(p) = 0$. Writing $w(p)$ in terms of two-component spinors ($w_A, w_B$), there follows:

$$(E - eA_0 - m_\psi - \sigma \cdot b - v_0) w_A = [\sigma \cdot (p - eA - v) - b_0] w_B,$$

$$(E - eA_0 + m_\psi - v_0 - \sigma \cdot b) w_B = [\sigma \cdot (p - eA - v) - b_0] w_A.$$
These two equations yield the free-particle solutions for this model (see refs. [2],[18]). The non-relativistic regime is realized by the well-known conditions \( p^2 \ll m_c^2, \rho A_0 \ll m_c, E = m_e + H \), where \( H \) represents the nonrelativistic Hamiltonian. Now, replacing the small spinor component \( (w_B) \) on the eq. (9), it is attained an equation for the large spinor component \( (w_A) \) that governs the behavior of the system at this regime. Such equation also provides an expression for \( H \) (see ref. [18]):

\[
H = H_{\text{Pauli}} + \left[ -\frac{(p-eA) \cdot v}{m_e} + \sigma \cdot B - \frac{b_0}{m_e} \sigma \cdot (p-eA) \right].
\] (8)

To properly study the influence of this Hamiltonian on a quantum system, the Lorentz-violating terms (into brackets) should be considered into the Schrödinger equation. In the next section, it will be accomplished for a two-level system.

### III. EFFECTS ON A TWO-LEVEL SYSTEM

#### A. Typical description of Rabi oscillation on a two-level system

Consider a two-level system defined by the energy eigenstates \( |a\rangle, |b\rangle \), under the action of a semiclassical electromagnetic field \( (A^\mu) \). The wavefunction for this system is:

\[
|\psi(t)\rangle = A(t) |a\rangle + B(t) |b\rangle,
\] (9)

so that \( |A(t)|^2, |B(t)|^2 \) represent the probability of finding the electron in the states \( |a\rangle, |b\rangle \), respectively. The evolution of this system is given by the Schrödinger equation,

\[
|\psi(t)\rangle = -iH |\psi(t)\rangle,
\] (10)

with \( H \) being the associated Hamiltonian, which may be written in terms of an unperturbed and an interaction part, namely: \( H = H_0 + H_{\text{int}} \), where \( H_0 |a\rangle = \hbar \omega |a\rangle, H_0 |b\rangle = \hbar \omega_0 |b\rangle \), and \( H_{\text{int}} = -e \cdot E(r,t) \), whereas \( r \) concerns to the atom position. For the case when the electric field is polarized along the x-axis, \( E(t) = E_0 \cos \nu t \), we get the result \( H_{\text{int}} = -e E_0 \cos \nu t \). Observe that the electric field modulus \( (E_0) \) is taken as a constant. This is a consequence of the dipole approximation (see ref. [22]).

In order to know how the electric field acts on the system, we should determine the state vector \( |\psi(t)\rangle \), that is, the coefficients \( A(t) \) and \( B(t) \). For this, we write the Hamiltonian in the basis of states \( \{|a\rangle, |b\rangle\} \):

\[
H_0 = \hbar \omega |a\rangle \langle a| + \hbar \omega_0 |b\rangle \langle b|, \quad H_{\text{int}} = -(P_{ab}|a\rangle \langle b| + P_{ba}|b\rangle \langle a|) E(t),
\]

where \( P_{ab} = e \langle a|x|b\rangle \) is the matrix element of the electric dipole moment. Replacing the Hamiltonian and the ket \( |\psi(t)\rangle \) in the Schrödinger equation, two coupled differential equations for \( A(t), B(t) \) arise:

\[
\dot{A}(t) = -iA\omega_a + i\Omega_R B \cos \nu t,
\] (11)

\[
\dot{B}(t) = -iB\omega_b + i\Omega_R A \cos \nu t,
\] (12)

where \( \Omega_R = |P_{ab}|E_0 \) is the Rabi frequency and \( P_{ab} \) is here supposed to be a real function. Equations (11,12) may be easily solved for the slowly varying amplitudes, \( a(t) = A e^{i\omega_at}, b(t) = B e^{i\omega bt} \), with which they are read as:

\[
\dot{a}(t) = i\frac{\Omega_R}{2} b(t) e^{i(\omega - \nu)t},
\] (13)

\[
\dot{b}(t) = i\frac{\Omega_R}{2} a(t) e^{-i(\omega - \nu)t}.
\] (14)

Here \( \omega = (\omega_a - \omega_b) \), and we used the rotating wave approximation (RWA), in which the rapidly oscillating terms, \( \exp[\pm i(\omega + \nu)t] \), were neglected. Under this approximation the equations for \( a(t) \) and \( b(t) \) may be exactly solved.
Considering the system in the state $|a\rangle$ at $t = 0$, we get the results

$$a(t) = \left[ \cos(\Omega t/2) - i(\Delta/\Omega) \sin(\Omega t/2) \right] e^{i\Delta t/2},$$  \hspace{1cm} (15)$$

$$b(t) = \frac{\Omega R_{i}}{\Omega} \sin(\Omega t/2) e^{-i\Delta t/2},$$  \hspace{1cm} (16)$$

with $\Delta = (\omega - \nu), \Omega = \sqrt{\Omega_{R}^{2} + (\omega - \nu)^{2}}$. At resonance, the frequency of the external field coincides with the two-level frequency difference $(\nu = \omega)$, so that $\Delta = 0, \Omega = \Omega_{R}$. For more details, see ref. [22]. The population inversion function (PIF), defined as $W(t) = |a(t)|^{2} - |b(t)|^{2}$, is then equal to

$$W(t) = \cos \Omega_{R} t.$$  \hspace{1cm} (17)$$

It varies from $-1$ and $1$, reflecting the alternation of the particle between the states $|a\rangle, |b\rangle$ along the time.

**B. Lorentz-violating effects due to the vector coupling**

Our first step is to determine the role played by the Hamiltonian terms stemming from $v_{\mu}\overline{\psi} \gamma^{\mu} \psi$ on the two-level system, whose effect is governed by the nonrelativistic terms $eA \cdot \mathbf{v}/m_{e}$ and $p \cdot \mathbf{v}/m_{e}$. We begin by regarding the effect of the term $\mathbf{A} \cdot \mathbf{v}$. This can be done following the procedure of the last section. Taking $\mathbf{E}(t) = E_{0} \cos \nu t \mathbf{i}$, it results $\mathbf{A}(t) = -A_{0} \sin \nu t \mathbf{i}$, with $A_{0} = E_{0}/\nu$. The Hamiltonian reads as $\mathbf{H} = \mathbf{H}_{0} + \mathbf{H}_{int} + \mathbf{H}_{1}$, where $\mathbf{H}_{1} = -eA_{0}v_{x} \sin \nu t/m_{e}$. In the basis $\{|a\rangle, |b\rangle\}$, $\mathbf{H}_{1}$ has the simple form: $\mathbf{H}_{1} = \alpha(t) (|a\rangle\langle a| + |b\rangle\langle b|)$, with: $\alpha(t) = -\alpha_{0} \sin \nu t, \alpha_{0} = eA_{0}v_{x}/m_{e}$. Replacing the modified Hamiltonian in eq. (10), we obtain:

$$A(t) = -iA_{0} \omega_{0} + i\Omega_{R} B \cos \nu t + iA_{0} \sin \nu t,$$  \hspace{1cm} (18)$$

$$B(t) = -iB_{0} \omega_{0} + i\Omega_{R} A \cos \nu t + iB_{0} \sin \nu t.$$  \hspace{1cm} (19)$$

where it was used $P_{ab} = P_{ab}^{*}$. In terms of the slowly varying amplitudes:

$$\dot{a}(t) = i\alpha_{0} a(t) \sin \nu t + i(\Omega_{R}/2)b(t) e^{i(\omega - \nu)t},$$  \hspace{1cm} (20)$$

$$\dot{b}(t) = i\alpha_{0} b(t) \sin \nu t + i(\Omega_{R}/2)a(t) e^{-i(\omega - \nu)t},$$  \hspace{1cm} (21)$$

At resonance $(\omega = \nu)$, and with the initial condition $a(0) = 1$, such differential equations can be exactly solved, yielding

$$a(t) = e^{-i(\alpha_{0} \cos \nu t)/\nu} e^{i\alpha_{0} t/\nu} \cos \left( \frac{\Omega_{R} t}{2} \right),$$  \hspace{1cm} (22)$$

$$b(t) = e^{-i(\alpha_{0} \cos \nu t)/\nu} e^{i\alpha_{0} t/\nu} \sin \left( \frac{\Omega_{R} t}{2} \right).$$  \hspace{1cm} (23)$$

These coefficients are different from the former ones (Eqs. (15) and (16)), but the amplitude probabilities $|A(t)|^{2}, |B(t)|^{2}$ and the population inversion (PI), $W(t) = \cos \Omega_{R} t$, are not altered. This shows that the Lorentz-violating term $\mathbf{A} \cdot \mathbf{v}$ does not modify the Rabi oscillation and the physics of the two-level system, except for phase effects.

We should now investigate the effect of term $p \cdot \mathbf{v}$ on the two-level system. In this case, the Hamiltonian is: $\mathbf{H} = \mathbf{H}_{0} + \mathbf{H}_{int} + \mathbf{H}_{2}$, where $\mathbf{H}_{2} = -(p \cdot \mathbf{v})/m_{e}$. For an x-axis polarized electric field, the electron momentum should also be aligned along the x-axis. In this case: $\mathbf{H}_{2} = -v_{x}p_{x}/m_{e}$. Using the relation $x = -i[x, H_{0}]$, it results: $\mathbf{H}_{2} = iv_{x}[x, H_{0}]$. Representing such operator in the $\{|a\rangle, |b\rangle\}$ basis, we have $H_{2} = -i\beta_{0} |a\rangle\langle b| - |w| \langle b| a\rangle$ with $\beta_{0} = (v_{x}P_{ab}), p_{ab} = \langle a|x|b\rangle$. Replacing all that into the Schrödinger equation, we get the following system of coupled differential equations:

$$\dot{A}(t) = -i\omega_{0} A + iP_{ab} E_{0} B \cos \nu t - \beta_{0} \omega B,$$  \hspace{1cm} (24)$$

$$\dot{B}(t) = -i\omega_{0} B + iP_{ab} E_{0} A \cos \nu t + \beta_{0} \omega A.$$  \hspace{1cm} (25)$$
Writing such equations for the slowing varying amplitudes (in the rotating wave approximation), we attain:

\begin{align}
\dot{a}(t) &= i \frac{P_{ab} E_0}{2} b(t) e^{i(\omega - \nu) t} - \beta \omega b(t) e^{i \omega t}, \\
\dot{b}(t) &= i \frac{P_{ab} E_0}{2} a(t) e^{-i(\omega - \nu) t} + \beta \omega a(t) e^{-i \omega t},
\end{align}

(26) (27)

An interesting preliminary analysis consists in analyzing the behavior of this system under the action only of the background, in a situation where the external electromagnetic field is null. In this case, it is possible to show that the Lorentz-violating background is able to induce Rabi oscillations, once the system has a non-null electric dipole moment. In the absence of the external field, the system of Eqs. (26), (27) takes the form

\[ a(t) = -\beta \omega b(t) e^{i \omega t}, \quad b(t) = \beta \omega a(t) e^{-i \omega t}, \]

which implies the solution:

\[ a(t) = \frac{1}{2 \sqrt{1 + 4 \beta^2_0}} \left[ k_- e^{\frac{1}{2} k_- \omega t} + k_+ e^{-\frac{1}{2} k_+ \omega t} \right], \quad b(t) = \frac{-i \beta_0}{\sqrt{1 + 4 \beta^2_0}} \left[ e^{\frac{1}{2} k_- \omega t} - e^{-\frac{1}{2} k_+ \omega t} \right], \]

(28)

with \( k_\pm = (\sqrt{1 + 4 \beta^2_0} \pm 1) \). The corresponding population inversion is

\[ W(t) = \frac{1}{[1 + 4 \beta^2_0]} \left[ 1 + 4 \beta^2_0 \cos(\sqrt{1 + 4 \beta^2_0} \omega t) \right]. \]

(29)

This result shows that the Lorentz-violating background, by itself, is able to induce state oscillations with a fixed frequency \( \omega \), that approximates to \( \omega = (\omega_a - \omega_b) \), once the background is supposed to be of small magnitude \((\beta^2_0 < 1)\). As the corrections induced are proportional to the factor \( 4 \nu e^2 p_{ab}^2 \), it may then be used to impose a bound on the background magnitude. In fact, considering that effects on the population inversion larger than \( 10^{-10} \) might be experimentally detectable, we shall have \( 4 \nu e^2 p_{ab}^2 < 10^{-10} \). Taking \( p_{ab} = \langle a | x | b \rangle \simeq 1 (eV)^{-1} \), it yields: \( \nu e < 5 \cdot 10^{-9} eV \). This is not a tight upper bound, but may be taken as a preliminary result. A more stringent bound can be attained analyzing the system behavior in the presence of electric field \( (E_0 \neq 0) \).

In the presence of the external field \( E_0 \), Eqs. (26) (27) do not possess analytical solution even at resonance \((\omega = \nu)\). A numerical approach is then employed to provide a graphical solution for the PIF of the system. To do it, it is necessary to establish a set of numerical values compatible with the physics of a typical two-level system. Here, we are working at the natural units \((c = 1, \hbar = 1)\), where the relevant parameters present the following mass dimension: \([\Omega_R] = [\omega] = [\nu^{-1}] = 1, [E_0] = 2, [P_{ab}] = [p_{ab}] = -1\).

As a starting step, we search for the effects of a small magnetic field, \(|\nu| = 10^{-6} - 10^{-8} eV\), on the two-level system. We take values for the electric field corresponding to a typical magnetic field \((B_0 = 10^{-4} - 10^{-3} T \Rightarrow E_0 = 3 \times 10^7 - 3 \times 10^9 eV/\text{m})\). In natural units \([1 \text{ volt/m} = 2.3 \times 10^{-6} (eV)^2]\), so it results: \( E_0 \simeq 0.7 - 7 (eV)^2 \). For the wave frequency, we take a typical electromagnetic value: \( \nu \sim 10^{16} \text{Hz} \) (ultraviolet limit), which in natural units is equivalent to \( \nu = 6.6 eV \) (since \( 1s^{-1} = 6.6 \times 10^{-16} eV \)).

In Fig. 1, the effect of the background on the PIF is shown in detail. It induces alterations (peak deformations) on the perfect harmonic sinusoidal pattern, given by eq. (17). Such modifications appear at the form of peak reversions and nonhomogeneities in the PIF. These alterations are present along with all sinusoidal oscillation pattern. In the absence of an analytical solution, it is necessary to scan the relevant parameters to gain some feeling about the role played by each one. Naturally, Lorentz-violating effects on the PIF increase with the background magnitude. Keeping \( E_0, \nu \), and the external frequency \( (\nu) \) constant, several numerical simulations revealed that the LV effects tend to diminish with \( P_{ab} \), as observed in Fig. 2. The variation of the electric field seems to have a cyclic effect on \( W \). Initially, while \( E_0 \) magnitude is reduced, increasing LV effects are implied. Reducing even more \( E_0 \), such effects diminish, so that in the limit \( E_0 \rightarrow 0 \), the background influence becomes tiny, recovering the weak oscillations described by eq. (29). Concerning the oscillation frequency, it continues to be sensitive to the value of \( P_{ab} E_0 \) (the larger this product, the larger the frequency), but such frequency is also affected by the background magnitude, so that \( \Omega_R = E_0 P_{ab} \) does not hold anymore. Now, it increases with the background, being larger than \( E_0 P_{ab} \). Only when the background tends to vanish, the oscillation frequency recovers the usual value \( E_0 P_{ab} \).

Moreover, for some specific parameter values, the background induces a clear modulation on the PIF, which takes place only for some values of the product \( P_{ab} E_0 \). Such a modulation is obviously associated with a kind of
FIG. 1: Population Inversion versus time plot for $P_{ab} = 3(eV)^{-1}, \nu = 3eV, v = 10^{-7}eV, E_0 = 2(eV)^2$.

FIG. 2: Population Inversion versus time plot for $P_{ab} = 1(eV)^{-1}, \nu = 3eV, v = 10^{-7}eV, E_0 = 2(eV)^2$.

partial inversion frustration (when the inversion is not fully accomplished) at some stages. In fact, the graph of Fig. 3 shows a pattern of modulation very similar to the one of a beat (superposition of close frequencies). The behavior of Fig. 3 occurs for some specific combinations of $P_{ab}$ and $E_0$ which yield $P_{ab}E_0 = 3$, for $\nu = 3eV, v = 10^{-8}eV$. It was also reported for ($P_{ab} = 3/2, E_0 = 2$) and ($P_{ab} = 3/4, E_0 = 4$), keeping the values of $\nu$ and $v$ unchanged. As already commented, a $P_{ab}$ reduction implies an attenuation on the LV effects, except when the resulting value of $P_{ab}E_0$ brings about a beat oscillation pattern.

FIG. 3: Population Inversion versus time plot for $P_{ab} = 1(eV)^{-1}, \nu = 3eV, v = 10^{-8}eV, E_0 = 3(eV)^2$
Finally, an important point also to be analyzed refers to the minimum background magnitude for which modifications on the PIF are still present. Numerical simulations for \( v_x = 10^{-10} \text{eV} \) do not reveal any effect on the usual sinusoidal Rabi oscillations for the following parameter ranges: \( 0 < P_{ab} < 10 \langle \text{eV} \rangle^{-1}, 0 < E_0 < 10 \langle \text{eV} \rangle^2, 0.001 < \nu < 6 \langle \text{eV} \rangle \). Parameter values outside these ranges could also be considered, but violate the dipolar approximation and are not suitable to simulate the physics of a two-level system (except for frequencies smaller than \( 0.001 \langle \text{eV} \rangle \)). Once the Lorentz-violating modifications alluded to here are not observed, we should conclude that the background magnitude can not be larger than \( 10^{-10} \text{eV} \) \( (v_x \leq 10^{-10} \text{eV}) \).

### C. Lorentz-violating effects due to the axial-vector coupling

We now examine the effects that the coupling \( b^\mu \bar{\psi} \gamma_5 \gamma^\mu \psi \) amounts to the two-level system. Initially, we choose a purely spacelike background, \( b^\mu = (0, b) \), for which there corresponds the nonrelativistic term \( \sigma \cdot b \). In this case, we shall adopt a four-state basis: \( \{|a^+, |a^-, |b^+, |b^-\} \), where each element is a tensor product of an eigenstate of energy \( (|a\rangle \) or \( |b\rangle \) with an eigenstate of spin \( (|+\rangle \) or \( |\rangle \)). In this case, the general state vector is

\[
|\psi(t)\rangle = A_1(t)|a^+\rangle + A_2(t)|a^-\rangle + B_1(t)|b^+\rangle + B_2(t)|b^-\rangle,
\]

which formally describes a four-level system. The Hamiltonian is \( H = H_0 + H_{\text{int}} + H_3 \), with \( H_3 = \sigma \cdot b \). In this basis, we write:

\[
H_0 = \hbar \omega_a (|a^+\rangle \langle a^+| + |a^-\rangle \langle a^-|) + \hbar \omega_b (|b^+\rangle \langle b^+| + |b^-\rangle \langle b^-|),
\]

\[
H_{\text{int}} = -(P_{ab^+}|a^+\rangle \langle b^+| + P_{ab^-}|a^-\rangle \langle b^-| + P^{ab^+}_{a^+}|b^+\rangle \langle a^+| + P^{ab^-}_{a^-}|b^-\rangle \langle a^-|) E(t),
\]

\[
H_3 = (b_x |a^+\rangle \langle a^+| + |a^-\rangle \langle a^-| + |b^+\rangle \langle b^+| + |b^-\rangle \langle b^-| + b_z (|a^+\rangle \langle a^+| + |a^-\rangle \langle a^-| + |b^+\rangle \langle b^+| + |b^-\rangle \langle b^-|)
\]

\[
+ |b^+\rangle \langle b^-| - |b^-\rangle \langle b^+| - ib_y (|a^+\rangle \langle a^-| + |a^-\rangle \langle a^+| + |b^+\rangle \langle b^-| + |b^-\rangle \langle b^+|),
\]

where it was taken: \( H_3 = b_x \sigma_x + b_y \sigma_y + b_z \sigma_z \), \( P_{ab^+} = e \langle a^+ | b^+ \rangle \), \( P_{ab^-} = e \langle a^- | b^- \rangle \), and \( \langle a^+ | b^+ \rangle = \langle a^- | b^- \rangle = 0 \), once the operator \( x \) does not act on spin states. The spin operators acts as \( \sigma_x \pm \sigma_y = \pm 1 \), \( \sigma_z \pm \sigma_y = \pm 1 \). Replacing all that in the Schrödinger equation, we obtain four coupled differential equations for the time-dependent coefficients:

\[
A_1 = -i [\omega_a A_1 - E(t) P_{ab^+} B_1 + b_x A_2 + b_z A_1 - ib_y A_2],
\]

\[
A_2 = -i [\omega_a A_2 - E(t) P_{ab^-} B_2 + b_x A_1 - b_z A_2 + ib_y A_1],
\]

\[
B_1 = -i [\omega_b B_1 - E(t) P^{ab^+}_{a^+} A_1 + b_x B_2 + b_z B_1 - ib_y B_2],
\]

\[
B_2 = -i [\omega_b B_2 - E(t) P^{ab^-}_{a^-} A_2 + b_x B_1 - b_z B_2 + ib_y B_1],
\]

Such equations may be written in terms of the slow varying amplitudes \( a_1(t) = A_1 e^{i\omega_a t}, a_2(t) = A_2 e^{i\omega_a t}, b_1(t) = B_1 e^{i\omega_b t}, b_2(t) = B_2 e^{i\omega_b t} \):

\[
\dot{a}_1(t) = -i \left[ -E_0 P_{ab} e^{i(\omega_\nu - \epsilon)t} b_1(t) + b_x a_2(t) + b_z a_1(t) - ib_y a_2 \right],
\]

\[
\dot{a}_2(t) = -i \left[ -E_0 P_{ab} e^{i(\omega_\nu - \epsilon)t} b_2(t) + b_x a_1(t) - b_z a_2(t) + ib_y a_1 \right],
\]

\[
\dot{b}_1(t) = -i \left[ -E_0 P_{ab} e^{-i(\omega_\nu - \epsilon)t} a_1(t) + b_x b_2(t) + b_z b_1(t) - ib_y b_2 \right],
\]

\[
\dot{b}_2(t) = -i \left[ -E_0 P_{ab} e^{-i(\omega_\nu - \epsilon)t} a_2(t) + b_x b_1(t) - b_z b_2(t) + ib_y b_1 \right],
\]

where it was assumed that the dipolar transitions are real quantities and the same between states of different spin polarizations, that is, \( P_{ab^+} = P_{ab^-} = P_{ab} = P_{ab} \). Moreover, the term \( e^{i(\omega_\nu + \epsilon)t} \) was neglected due to the
RWA. Considering a general situation, it may occur both spin state oscillations and energy state oscillations, so that it is necessary to define two population inversion functions - a spin PIF \((W_S)\) and an energy PIF \((W_E)\):

\[
W_S = |a_1(t)|^2 + |b_1(t)|^2 - |a_2(t)|^2 - |b_2(t)|^2, \tag{42}
\]

\[
W_E = |a_1(t)|^2 + |a_2(t)|^2 - |b_1(t)|^2 - |b_2(t)|^2. \tag{43}
\]

As a first insight, this model is to be regarded in the absence of the Lorentz-violating background \((b_x = b_y = b_z = 0)\) and in the presence of the external field \(E_0\). In this case, the solution for the four coupled equations has the form:

\[
a_1(t) = \cos(E_0 P_{ab} t), a_2(t) = 0, b_1(t) = -i \sin(E_0 P_{ab} t), b_2(t) = 0, \tag{44}
\]

which implies the following PIFs:

\[
W_E = |a_1(t)|^2 - |b_1(t)|^2 = \cos(2E_0 P_{ab} t), \quad W_S = 1. \tag{45}
\]

These results indicate an energy eigenstate oscillation \((a_1(t) \neq 0, b_1(t) \neq 0, a_2(t) = b_2(t) = 0)\), generated by the external field, and total absence of spin oscillation (the field does not act on the spin states), denoted by \(W_S = 1\).

On the other hand, we can search for the solution of this system in the absence of external field \((E_0 = 0)\), for the background configuration \(b = (b_x, 0, b_z)\):

\[
a_1(t) = \cos(\sqrt{b_x^2 + b_z^2} t) - i \frac{b_x}{\sqrt{b_x^2 + b_z^2}} \sin(\sqrt{b_x^2 + b_z^2} t), a_2(t) = -i \frac{b_z}{\sqrt{b_x^2 + b_z^2}} \sin(\sqrt{b_x^2 + b_z^2} t), b_1(t) = 0, b_2(t) = 0. \tag{46}
\]

Such relations obviously reflect an oscillation on the spin states, once the system undergoes alternation between \(|a+\rangle, |a-\rangle\) states, and the inexistence of energy state oscillation. The obtained PIFs,

\[
W_S = \cos^2(\sqrt{b_x^2 + b_z^2} t) + \left[\frac{b_x^2 - b_z^2}{b_x^2 + b_z^2}\right]^2 \sin^2(\sqrt{b_x^2 + b_z^2} t), \quad W_E = 1, \tag{47}
\]

confirm that this is really the case. For \((b_x = 0, b_z \neq 0)\), we have \(W_S = 1\) (no spin oscillation). For \((b_x \neq 0, b_z = 0)\), we have \(W_S = \cos(2b_x t)\). This latter outcome shows that the background may itself induce spin oscillation (even in the absence of external field), which may be used to establish an upper bound on \(b_x\).

We should now solve the system of equations \((45-48)\) in the presence both of the Lorentz-violating background and external field \(E_0\). As a starting situation, we regard the background as \(b = (0, 0, b_z)\), which provides:

\[
a_1(t) = \frac{1}{2} e^{-i(b_x - E_0 P_{ab}) t} + \frac{1}{2} e^{-i(b_y + E_0 P_{ab}) t}, a_2(t) = 0, b_1(t) = -\frac{1}{2} e^{-i(b_x - E_0 P_{ab}) t} + \frac{1}{2} e^{-i(b_y + E_0 P_{ab}) t}, b_2(t) = 0. \tag{48}
\]

It is easy to note that the population inversion of the energy states is still equal to \(\cos(2E_0 P_{ab} t)\), showing that the term \(b_z \sigma_z\) does not modify the population inversion of this system. Furthermore, it does not yield spin state oscillation as well \((W_S = 1)\), which is consistent with the fact that the operator \(\sigma_z\) does not flip the spin.

Once the influence of the \(b_z\)-component is understood, the role played by a more general background, \(b = (b_x, b_y, 0)\), should be now analyzed. In this case, the spin inversion is an expected result. The system of Eqs. \((45-48)\) also exhibits an exact solution, namely,

\[
a_1(t) = \frac{1}{2} [\cos Mt + \cos Nt], \quad a_2(t) = -\frac{i(b_x + ib_y)}{2b} [\sin Nt - \sin Mt],
\]

\[
b_1(t) = \frac{i}{2} [\sin Mt + \sin Nt], \quad b_2(t) = -\frac{i(b_x + ib_y)}{2b} [\cos Nt - \cos Mt]. \tag{49}
\]

where \(M = (E_0 P_{ab} - b), N = (E_0 P_{ab} + b), b = \sqrt{b_x^2 + b_y^2}\). Carrying out the spin and energy PIF, defined in Eqs. \((42,43)\), we get
\[ W_E = \cos(2E_0 P_{ab} t), \]  
\[ W_S = \cos(2\beta t). \]  

It is seen that the terms \( b_x \sigma_x, b_y \sigma_y \) do not modify \( W_E \), that is governed only by the external field, whereas the external field does not disturb the spin oscillation, which depends only on the background. The spin population inversion has period \( \pi/b \), so that the smaller the parameter \( b \), the larger is such period. For a tiny \( b \) the period may be so large that the inversion could become unobservable. Here, we take a case where such spin oscillation is undetectable (period bigger than \( 10^4 s \)) to establish an upper bound for the background: \( b < 10^{-19} eV \). This upper bound holds equally as \( b_x < 10^{-19} eV \) or \( b_y < 10^{-19} eV \) for the cases \( b_y = 0 \) or \( b_x = 0 \), respectively. It is clear that the background \( b = (0, b_y, 0) \) induces effects totally similar to the ones of \( b = (b_x, 0, 0) \), thus requiring no special attention.

Another situation of possible interest is \( b = (b_x, 0, b_z) \), for which the energy oscillation might be correlated with the spin oscillation. In this case, the system of Eqs. (58)-(61) provides the following solution:

\[ a_1(t) = \frac{1}{2bMN}[ib_z E_0 P_{ab}(M \sin Nt - N \sin Mt) - ib_x b(N \sin Mt + M \sin Nt) + MNb(\cos Mt + \cos Nt)], \]  
\[ b_1(t) = -\frac{1}{2bMN}[-ib^2(M \sin Nt - N \sin Mt) + iE_0 P_{ab}b(N \sin Mt + M \sin Nt) + b_z MN(\cos Nt - \cos Mt)], \]  
\[ a_2(t) = -\frac{i b_x}{2b} [\sin Mt - \sin Nt], \quad b_2(t) = \frac{b_x}{2b} [\cos Mt - \cos Nt], \]

where \( b = \sqrt{b_x^2 + b_z^2} \). These outcomes allow one to write lengthy expressions for \( W_E \) and \( W_S \) which reveal the influence of \( b_x \) and \( b_z \) on the dynamics of the system. PIF graphs for several values of \( b_x \) and \( b_z \) show that these coefficients do not have much effect on the energy inversion, which remains almost invariant while \( b_x \) and \( b_z \) take on different values. This is not the case for the spin inversion, however. Indeed, \( W_S \) is sensitive to variations of the ratio \( b_z/b_x \), being suppressed for small values of \( b_z/b_x \) (\( b_x/b_z << 1 \)), while increasing with it, becoming total \( (-1 \leq W_S \leq 1) \) for the case \( b_x/b_z >> 1 \). On the other hand, it is also observed that the frequency of the spin oscillation increases with the background modulus, \( b \). Given that the coefficient \( b_y \) plays a role similar to the one of \( b_x \), we should conclude that the case \( b = (0, b_y, b_z) \) presents the same general behavior of this previous case.

As a final investigation, we consider the case of a purely timelike background, \( b^\mu = (b_0, 0) \), in which the corresponding nonrelativistic term is \( \sigma \cdot (p - eA) \). Proceeding as earlier, the Hamiltonian is: \( H = H_0 + H_{int} + H_4 \), with \( H_4 = -(b_0/m_c) \sigma \cdot (p - eA) \). In the basis \( \{|a+\rangle, |a-\rangle, |b+\rangle, |b-\rangle\} \), the interaction \( H_4 \) takes the form:

\[ H_4 = -i b_0 \omega (P_{ab} |b-\rangle \langle a+ | + P_{ab} |b+\rangle \langle a- | - P_{ab} |a-\rangle \langle b+ | - P_{ab} |a+\rangle \langle b- | + \gamma_0 \sin(\nu t) [(|a+\rangle \langle a- | + |a-\rangle \langle a+ | + |b+\rangle \langle b- | + |b-\rangle \langle b+ |], \]

where \( \gamma_0 = eb_0 E_0/(m \nu) \), and \( H_0, H_{int} \) are already written in Eqs. (31)-(32). Replacing the full Hamiltonian in the Schrödinger equation, the following system of coupled equations is obtained:

\[ \dot{a}_1(t) = iP_{ab} \left[ E_0 \cos(\nu t) e^{i\omega t} b_1(t) - ib_0 \omega e^{i\omega t} b_2(t) - \gamma_0 (\sin \nu t) a_2(t) \right], \]  
\[ \dot{a}_2(t) = iP_{ab} \left[ E_0 \cos(\nu t) e^{i\omega t} b_2(t) - ib_0 \omega e^{i\omega t} b_1(t) - \gamma_0 (\sin \nu t) a_1(t) \right], \]  
\[ \dot{b}_1(t) = iP_{ab} \left[ E_0 \cos(\nu t) e^{-i\omega t} a_1(t) + ib_0 \omega e^{-i\omega t} a_2(t) - \gamma_0 (\sin \nu t) b_2(t) \right], \]  
\[ \dot{b}_2(t) = iP_{ab} \left[ E_0 \cos(\nu t) e^{-i\omega t} a_2(t) + ib_0 \omega e^{-i\omega t} a_1(t) - \gamma_0 (\sin \nu t) b_1(t) \right]. \]

Such a system does not provide an analytical solution, so that a numerical approach must be employed. It is important to point out that the constant \( \gamma_0 \) is much smaller than \( E_0, P_{ab} \), in such a way that the term linear in \( \gamma_0 \) turns out negligible in comparison with the others. Following the example of the last section, we solve the system numerically. The graph of Fig. 4 depicts the behavior of the PIF for the energy and spin states (thicker
FIG. 4: Simultaneous plot of spin PIF (black thick line) and energy PIF (thin line) for the following parameter values: $P_{ab} = 0.47 \, (eV)^{-1}$, $b_0 = 0.10eV$, $E_0 = 1.00(eV)^2$, $\nu = 1.00eV$.

FIG. 5: Simultaneous plot of spin PIF (black thick line) and energy PIF (thin line) for the following parameter values: $P_{ab} = 0.5 \, (eV)^{-1}$, $b_0 = 0.1eV$, $E_0 = 1.0(eV)^2$, $\nu = 1.0eV$.

black line). It shows an appreciable modification on the eigenenergy inversion induced by the background - a partial frustration at some moments, whereas the spin inversion is always partially frustrated (the system remains predominantly in the state $|+\rangle$). This effect may be amplified if we take $P_{ab} = 0.5 \, (eV)^{-1}$, as properly shown in Fig. 5, where the spin oscillation becomes total and the energy inversion is entirely frustrated at some moments. Numerical investigations have shown that this scenario takes place only for specific values of the product $P_{ab}E_0$, (in this case, $P_{ab}E_0 = 0.5 \, (eV)^{-1}$). A quite similar behavior was verified for $P_{ab} = 0.25 \, (eV)^{-1}$, $b_0 = 0.10eV$, $E_0 = 2.00(eV)^2$, $\nu = 1.00eV$. For values of $P_{ab}$ slightly different, $P_{ab} = 0.4 \, (eV)^{-1}$ or $P_{ab} = 0.6 \, (eV)^{-1}$, the spin oscillation is nearly annihilated, while the energy oscillation becomes approximately total. This behavior is much similar to the one described in Fig. 4. The picture of Fig. 5 reveals an interesting inversion pattern, observed for some background values. This graph shows that the system is nearly collapsed at the state $|a-\rangle$ at a moment, after undergoing oscillation between states $|a+\rangle, |b+\rangle$, and turning back to the state $|a-\rangle$ in the sequel. This cycle of alternations is repeated along with the time. By Fig. 6, one notes that a $P_{ab}E_0$ reduction implies a lower frequency oscillation. This is confirmed by analyzing several plots for different $E_0, P_{ab}$ values. Lastly, it was observed that the frequency of energy oscillation decreases with $b_0$ magnitude as well.

Numerical simulations have shown that more appreciable LV effects are just attainable for high values of the background, such as $b_0 = 0.1eV$. For smaller background magnitudes, $b_0 \leq 0.01eV$ (see Fig. 6), Lorentz-violating effects tend to vanish. Indeed, while the energy PIF tends to assume the usual sinusoidal form, the spin PIF tends to collapse to 1 (absence of spin oscillation), in a behavior similar to the one of Fig. 6.
IV. CONCLUSION

In this work, the effects of CPT- and Lorentz-violating terms (stemming from the SME model) on a two-level system were investigated. In the first case analyzed, it was reported that the background $v_\mu$ alters the Rabi oscillations, implying partial population inversion frustration and some kinds of modulation. Depending on the parameter values, the induced modulation might be stronger, becoming similar to a beat. It also occurs that $v_\mu$ may induce Rabi oscillations even in the absence of an external electromagnetic field, but it would become a sensitive effect just for large background magnitudes. Once the Lorentz-violating effects here foreseen are supposed to be not observed, the minimum value below which the background yields no modifications on the PIF was identified as an upper bound for this Lorentz-violating coefficient ($v_x \leq 10^{-10} eV$). In order to determine the effects induced by the background $b_\mu$, a four state basis was adopted. It was then shown that it occurs two types of population inversion, one referred to the energy states (determined by the external electromagnetic field), another concerned with the spin states (implied by the background). The supposed non observation of this spin oscillation (relying on the validity of the dipole approximation) was used to impose an upper bound on such a coefficient: $b_x < 10^{-19} eV$.

A point that deserves some attention concerns to the validity of the rotating wave approximation (RWA) in the cases where the Lorentz-violating perturbation contains terms like $\sin(\nu t)$ or $\cos(\nu t)$, as in Eqs. (20, 21, 56-59). In this case, the perturbation term oscillates at a frequency ($\nu$) equal to half the frequency of the RWA-neglected term. The question is to know if the rapidly oscillating term (with frequency equal to $2\nu$) may be dropped out while the perturbation term (with frequency equal to $\nu$) is kept. To answer this issue, some numerical calculations were performed out of the RWA, that is, maintaining the rapidly oscillating terms together. The observed results do not differ qualitatively and appreciably from the ones previously obtained, which leads to the conclusion that RWA is still a good approximation.

Finally, the cases here studied in the semiclassical viewpoint can also be considered in the context of a quantized electromagnetic field in a cavity. In this case, for the $A \cdot v$ term the results depend on the initial state of the electromagnetic field. For an initially excited atom in a coherent state with large number of photons, the term induces only a phase on the wavefunction coefficients. As it does not alter the probability amplitudes, it corroborates the results obtained at Sec. IIIB (at semi-classical level). We are now investigating the effects due to the background for small number of photons in the cavity, such as corrections induced on the population inversion and photon statistics. This work is under development [23].
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