MASS OUTFLOWS FROM DISSIPATIVE SHOCKS IN HOT ACCRETION FLOWS

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ABSTRACT

We consider stationary, axisymmetric hydrodynamic accretion flows in Kerr geometry. As a plausible means of efficiently separating a small population of nonthermal particles from the bulk accretion flows, we investigate the formation of standing dissipative shocks, i.e., shocks at which fraction of the energy, angular momentum, and mass fluxes do not participate in the shock transition of the flow that accretes onto the compact object but are lost into collimated (jets) or uncollimated (winds) outflows. The mass-loss fraction (at a shock front) is found to vary over a wide range (0%–95%), depending on flow’s angular momentum and energy. On the other hand, the associated energy-loss fraction appears to be relatively low (≤1%) for a flow onto a nonrotating black hole case, whereas the fraction could be an order of magnitude higher (≤10%) for a flow onto a rapidly rotating black hole. By estimating the escape velocity of the outflowing particles with a mass-accretion rate relevant for typical active galactic nuclei, we find that nearly 10% of the accreting mass could escape to form an outflow in a disk around a nonrotating black hole, while as much as 50% of the matter may contribute to outflows in a disk around a rapidly rotating black hole. In the context of disk-jet paradigm, our model suggests that shock-driven outflows from accretion can occur in regions not too far from a central engine. Our results imply that a shock front under some conditions could serve as a plausible site where (nonthermal) seed particles of the outflows (jets/winds) are efficiently decoupled from bulk accretion.

Subject headings: accretion, accretion disks — black hole physics — galaxies: jets — hydrodynamics — shock waves

1. INTRODUCTION

It has been established by now that a large body of astrophysical objects hosting supermassive black holes (e.g., quasars, galactic black hole candidates, or microquasars) exhibit collimated, powerful jets/winds (e.g., Begelman et al. 1984; Livio 1999, for review). In particular, strong outflows generally occur in the radioloud active galactic nuclei (AGNs). It is now widely accepted that the outflows observed from a large class of objects as jets or winds have their origin in accretion flows, which at the same time power the radiation emission associated with these objects (e.g., Blandford & Payne 1982; Fender et al. 2004). Many astrophysical systems apparently manage to transfer the energy from inflowing accretion flows to outflowing jets/winds of a small population of nonthermal particles. For instance, Junor et al. (1999), in radio observations of the nearby active galaxy M87, found a remarkably broad jet with strong collimation occurring already at ~30–100 Schwarzschild radii from a central engine. Recently, Kataoka et al. (2007) discussed the disk-jet connection of the radio galaxy 3C 120 observed with Suzaku. Such a population of relativistic outflowing particles is thought to produce a subsequent synchrotron emission that is observed in several sources (e.g., Mirabel & Rodríguez 1999). This issue clearly points to the importance of investigating a fundamental connection between the accreting flows and outflows in regions not too far from the central engines, presumably within ~100 Schwarzschild radii. Because the emission of radiation requires the dissipation of the kinetic energy of the accretion flow, it is not unreasonable to suggest that the two phenomena, i.e., the dissipation/emission of radiation and the presence of outflows, are related to the same generic process, which manifests itself with different guises at the diverse sites that these phenomena are observed.

Given that an accretion flow may very well become turbulent or, at a very minimum hot due to adiabatic compression, it is not surprising to anticipate that such a flow will emit radiation, with luminosity and spectrum depending strongly on the specific character of the accretion process (e.g., quasi-spherical or disk). However, the ubiquitous presence of outflows in the same objects presents a different problem altogether; an outflow requires that a fraction of the accreted matter be endowed with velocity higher than the escape velocity associated with the specific radius at which the outflow is launched. Given that the only free energy available to the accreting gas is that of the gravitational field, hydrodynamic dissipation of the accretion kinetic energy can never produce an outflow, since this process simply converts the kinetic energy of a gravitationally bound flow into thermal, with the energy per unit mass (thermal plus kinetic) never higher than that of the local gravitational potential.

The launch of an outflow such as those observed requires the presence of an “engine,” i.e., a mechanism that expends mechanical work (i.e., low-entropy energy), in order to transfer a fraction of the available energy to an even smaller fraction of the available mass, thereby imparting to it specific energy greater than the local gravitational potential; it is then expected that in its further evolution the excess energy will be converted into directed motion and the fraction of the mass will escape to infinity in a collimated (jet) or uncollimated (wind) outflow.

The well-known models of outflows, usually involving the action of magnetic fields such as the magnetocentrifugal jet models (e.g., Blandford & Payne 1982; Contopoulos & Lovelace 1994; Königl & Kartje 1994; Vlahakis et al. 2000), are specific examples as to what may constitute such an engine. These models assume the presence of a thin Keplerian disk “threaded” by a poloidal magnetic field; the Keplerian rotation of the magnetic field line footpoints and the magnetic tension transfer energy and angular momentum to the disk plasma which can escape to infinity. These models are consistent in that they solve simultaneously for the poloidal field geometry and the flow velocity (under certain simplifying assumptions, i.e., the self-similarity of the solutions). Pelletier & Pudritz (1992) further developed
the general theory of non–self-similar solutions of hydromagnetic disk winds. In these models, the transfer of excess energy to the escaping particles is made at the expense of the rotational energy of the matter in the disk, and it is mediated by the magnetic field.

While this type of model gained popularity, the issue of jet/outflow formation took a different turn with the introduction of advection-dominated accretion flow (ADAF; e.g., Narayan & Yi 1994; Mannoto et al. 1997). These radiatively inefficient accretion flows (RIAFl) were found to have a positive Bernoulli integral of the flow and could therefore fulfill the condition necessary for the launching of jet/wind outflows; as such, they present potentially interesting sites for the origin of such outflows. The positivity of the Bernoulli integral has been discussed and analyzed by Blandford & Begelman (1999), who pointed out that it is due to the combination of energy transfer by the viscous torques from the inner to the outer sections of the flow (the gas of the flow becomes bounded at its inner edge) and the local dissipation of the flow’s azimuthal kinetic energy which is not radiated away but stored in the fluid to increase its internal energy. The latter authors then argued that the excess energy can be carried away to infinity (along with some fraction of the accreting mass and angular momentum) to produce continuous outflows from all radii to infinity, while leaving the remaining flow with a negative Bernoulli constant to naturally accrete onto the compact object: advection-dominated inflow-outflow solution (ADIOS). In this case, while the necessary excess energy is transferred by the viscous torques from the flow’s more highly bound inner section, the necessary separation of mass to components with positive and negative total energy is still left unspecified.

An altogether different model that offers a simplified picture of such a separation was presented by Subramanian et al. (1999), who proposed that in the tenuous, collisionless plasma of an ADAF particles (protons) could be accelerated via a second-order Fermi mechanism by the shear motions of the underlying quasi-Keplerian azimuthal flow. They then argued that if sufficiently large pressure is built in the accelerated particle proton population (the electrons generally lose energy on timescales short compared to their transit time through the system and cannot build an energy density that could be dynamically important) and for favorable geometries of the disk magnetic field (large-scale poloidal loops that open up above the disk), the relativistic particle population could naturally (through the action of the gravitational field) segregate itself from the nonrelativistic one, carrying off to infinity only the accelerated (\(E \gtrsim m_p c^2\)) portion of the disk plasma. In this case the engine is a combination of the particle acceleration and the action of the gravitational field.

Finally, a model along the same lines was proposed by Contopoulos & Kazanas (1995), who suggested that even in the case of a completely turbulent magnetic field, a separation of the relativistic and nonrelativistic particle populations is possible through the production of relativistic neutrons in the collisions of the relativistic protons with the ambient plasma and the ensuing production of relativistic neutrons. The subsequent decay of neutrons back into protons produces then a proton fluid in regions of space devoid of inertia whose energy-to-mass ratio (and hence its asymptotic Lorentz factor) depends only on the ratio \(R/c\), where \(R\) is the size of the system and \(\tau_n\) the neutron lifetime and can lead to highly relativistic flows for black hole masses \(M \gtrsim 10^8 M_\odot\).

In the present note we follow a similar simplified view to study outflows in objects powered by accretion; we consider the presence of (two-dimensional) shocks as a means of dissipation of the accretion kinetic energy in a fashion similar to that considered by Chakrabarti (1990) and collaborators. That is, we consider the transonic accretion of matter with the proper angular momen-

tum to produce a standing shock at a radius close to the horizon, which subsequently accretes onto the black hole after passing through a downstream sonic point. Previous steady state analysis found that it is possible to judiciously choose the specific angular momentum of the flow, so that the outer transonic one could be connected through a shock transition to an inner transonic one which, passing through an inner sonic point, accretes onto the black hole. The location of the shock in this situation is determined by finding a radial position at which the density and velocity of the two flow sections were those demanded by the dissipative Rankine-Hugoniot conditions across a shock. So far, the formation of standing shocks in hydrodynamic accretion has been extensively studied by a number of authors for both inviscid flows (e.g., Chakrabarti 1990, 1996; Sponholz & Molteni 1994; Lu & Yuan 1998; Fukumura & Tsuruta 2004) and viscous flows (e.g., Chakrabarti 1990; Lu et al. 1999; Chakrabarti & Das 2004), considering either dissipative or nondissipative shock jump conditions. Recently, standing shocks in the presence of poloidal magnetic fields (i.e., magnetohydrodynamic [MHD] shocks) around a black hole was also studied for various parameter dependence (see Das & Chakrabarti [2007] for pseudo-Newtonian geometry; Takahashi et al. [2002] and Fukumura et al. [2007] for Kerr geometry). In the context of the particle acceleration via the first-order Fermi mechanism across a shock front, the production of shock-accelerated relativistic protons was discussed in spherical accretion (Protheroe & Kazanas 1983; Kazanas & Ellison 1986), while other authors have explored the relativistic outflows in ADAF with shocks (Le & Becker 2004, 2005). Similar attempts have been made to make physical connections between the shocked accretion and outflows. For instance, mass outflow rate were estimated from adiabatic shocked-flow region in Newtonian gravity (e.g., Chakrabarti 1999; Das 2000). Das & Chakrabarti (1999) took a similar approach to study the shock-generated outflows with little energy dissipation in pseudo-Newtonian geometry. Independently, from general relativistic MHD simulations, the formation of jets (magnetically driven and gas-pressure-driven jets) is found in the high-pressure regions due to the shock/adiabatic compression (e.g., Koide et al. 1999; Nishikawa et al. 2005). They concluded that the jets are mainly produced by the gas-pressure gradient, which is greatly enhanced by the shock front at around \(r \approx 6\) gravitational radii (note that this feature was not seen in the Newtonian calculations). These studies also suggest an essential connection between the shocked accretion flows and the jets, i.e., the shock front may serve as a base of the outflows.

The novelty of our approach lies in considering the possibility of shock formation (i.e., obeying the general relativistic, dissipative Rankine-Hugoniot conditions at a shock front) in which part of the mass, angular momentum, and energy fluxes escape in the \(z\)-direction and do not participate in the shock transition. We then examine the energy per unit mass of the escaping matter, which we compare to the escape velocity at the shock radius; if it is greater than the latter we conclude that this scenario can produce an outflow, with an outflow rate of \(\dot{m}\), and luminosity that are calculable and can be compared to those of the entire accretion to obtain a measure of the efficiency of our “engine” in producing outflows. Some population of energetic nonthermal particles, produced via a shock acceleration, may then be well separated from the equatorial accretion flows (which consist primarily of thermal particles).

More specifically, since we are interested in the formation of outflows through shocks in the inner disk region relatively close to a presumably rotating black hole at a center (say, \(r \lesssim 30\) gravitational radii), it is important to include the strong gravity and frame-dragging effects described by general relativity. To the best of our knowledge, no relevant work in the literature incorporates
such mass and energy loss in the shock jump conditions. In the framework of our model presented here, mass loss is coupled to energy loss via the shock jump conditions, and therefore it must be considered simultaneously. It is this point that motivates us to explore, for the first time, the formation of outflowing particles as a consequence of the formation of dissipative standing shocks in accretion in a fully relativistic treatment. The formalism of our current model is partly based on the previous works (Yang & Kafatos 1995; Lu & Yuan 1998; Fukumura & Tsuruta 2004). Our main objective in this study is therefore to explore in detail a possibility that the formation of outflows (jets/winds) can occur at a dissipative shock front in transonic accreting flows, i.e., a connection between shocked-accreting gas and the outflowing particles.

The structure of this paper is as follows. In § 2 we review and explain our simplified model, which simultaneously considers both shocks and outflows with appropriate jump conditions. The main parameter dependence of the shock-outflow solutions is explored in § 3, where we show the nature of the shock-outflow solutions and the corresponding global accretion solutions. Our primary goal in these analyses is to examine the coupling between the shock-outflow solutions. In § 4 we discuss our results and discuss some observation implications. A brief summary and concluding remarks are given there as well.

2. MODEL ASSUMPTIONS AND BASIC EQUATIONS

In black hole accretion, accreting gas must be transonic. After passing through a first sonic radius, the gas is slowed down, and a shock may develop. For causality, the shocked gas must become supersonic again before crossing the event horizon. Below, we will explain the details of our simplified model.

2.1. Accreting Flows around a Black Hole

We consider steady state, axisymmetric accreting flows in Kerr geometry. The spacetime metric is expressed by the Boyer-Lindquist coordinates as

\[ ds^2 = -\left(1 - \frac{2mr}{\Sigma}\right)dt^2 - \frac{4mra\sin^2\theta}{\Sigma}dt \, d\phi + \frac{A\sin^2\theta}{\Sigma}d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \frac{\Sigma}{\Delta}d\theta^2, \]

where \( \Delta \equiv r^2 - 2mr + a^2, \Sigma \equiv r^2 + a^2\cos^2\theta, A \equiv (r^2 + a^2)^2 - a^2\Delta\sin^2\theta, \) and \( m \) and \( a \) are mass and specific angular momentum (or the Kerr parameter) of a black hole. Following the standard geometrized units, we have taken \( G = c = 1 \) in equation (1). Thus, the length (distance) \( r \) and \( a \) are measured in units of \( m \).

Since we are interested in the equatorial flows, we set \( \theta = \pi/2 \) throughout this paper. The black hole horizon is then expressed by \( r_H \equiv m + (m^2 - a^2)^{1/2} \). The self-gravity of the flow is ignored, and we do not include the effects of magnetic fields for simplicity.

We follow the earlier works on relativistic shock formation as follows: accretion time is assumed to be shorter than that of energy diffusion; thus, we treat the flows as adiabatic except at a shock front, where a fraction of fluid energy, angular momentum, and mass are dissipated. To prescribe thermodynamic quantities we adopt a polytropic form:

\[ P = K\rho_0^{1 + 1/N}, \]

where \( P \) and \( \rho_0 \) are the thermal pressure and rest-mass density of the flow, which are locally measured in the fluid frame. Here, \( N \) denotes the polytropic index. The entropy of the fluid is characterized by \( K \), which is related to the entropy \( S \) by \( S \equiv c_s \log K \), where \( c_s \) denotes a specific volume heat of the flow. Because of stationarity (\( \partial t = 0 \)) and axial symmetry (\( \partial \phi = 0 \)), there exist two conserved quantities along a fluid stream line, namely, specific energy \( E \) and axial angular momentum \( L \) of the fluid:

\[ E \equiv -\mu u_t, \]

\[ L \equiv \mu u_t, \]

where \( \mu = (P + \rho)/\rho_0 \) is the relativistic enthalpy of the fluid, and \( \rho = \rho_0 + NP \) is the net baryon mass-energy density (including internal energy \( NP \)). The number density of the constituent baryon \( n \) is given by \( \rho_0 \equiv nmp_0 \), where \( m_p \) is the baryon mass in the flow. We assume that energy \( E \) and angular momentum \( L \) are both conserved along the flow except at a shock location \( (r = r_0) \), where they are partially dissipated (this will be explained in detail in § 3).

From the four-velocity normalization of the flow \( (u_\alpha u^\alpha = -1) \), we get

\[ 1 + u_\alpha u^\alpha + (u^\alpha) V_{eff}(r, \lambda) = 0, \]

where the effective potential has been introduced by \( V_{eff}(r, \lambda) \equiv g^{00} - 2\lambda g^{0\alpha} + \lambda^2 g^{\alpha\beta} \), with \( g^{0\beta} \) being the inverse metric components for \( g_{0\beta} \) in equation (1). Here, the specific angular momentum of the flow \( \lambda \) is defined by

\[ \lambda \equiv \frac{L}{E} = -\frac{u_\alpha}{u_t}, \]

which is assumed to be conserved along the whole flow in our model. From equation (5) we obtain

\[ u_t(r, \lambda) = \left[ 1 + u_\alpha u^\alpha \right] ^{1/2} V_{eff}(r, \lambda). \]

From the definition of enthalpy and the polytropic relation given by equation (2) we can rewrite the enthalpy as

\[ \mu = 1 + (N + 1)K/\rho_0^{1/N}. \]

Local adiabatic sound speed \( c_s \) is defined as

\[ c_s^2 \equiv \frac{(\partial P)}{(\partial \rho)_{ad}} = \frac{1 + 1/N}{\mu \rho_0}. \]

Combining the equations (2), (8), and (9), we can express \( \mu \) in terms of the sound speed \( c_s \):

\[ \mu = \frac{1}{1 - Ne_s^2}. \]

Accordingly, the energy of the flow \( E \) in equation (3) can be explicitly rewritten as a function of \( c_s \):

\[ E \equiv \left[ \left( 1 + u_\alpha u^\alpha \right) ^{1/2} \right] \sqrt{1 - N c_s^2}, \]

which is conserved along the flow except at a shock front. That is, across a shock front, the energy \( E \) and the angular momentum \( L \) will both decrease in such a way that the ratio \( \lambda \equiv L/E \) is continuous across the shock. Using equations (8) and (10), the baryon rest-mass density can be rewritten as

\[ \rho_0 = \left[ \frac{c_s^2}{(1 + 1/N)(1 - Nc_s^2)} \right]^N \frac{1}{K^N}. \]
We define the mass-accretion rate $\dot{M}$ as

$$\dot{M} = -4\pi r H u' \rho_0,$$  \hspace{1cm} (13)

where $H$ represents the vertical scale height of the flow defined as

$$H \equiv \frac{c_s}{\Omega_K},$$  \hspace{1cm} (14)

from the conventional hydrostatic equilibrium assumption. Here, $\Omega_K(r) \equiv m^{1/2}(r^{3/2} + am^{1/2})$ is the Keplerian angular velocity. Note that for accretion we have $u' < 0$. As is often assumed, we only consider a constant mass-accretion rate in this paper, although a variable accretion rate has been discussed in the literature (e.g., Blandford & Begelman 1999). Eliminating $\rho_0$ from equations (12) and (13), we rewrite $\dot{M}$ as

$$\dot{M} = -4\pi r \left[ 1 + \frac{u'}{\sqrt{\frac{2}{C_0} + \frac{a}{C_1}}} \right] \frac{1}{K^N}.$$  \hspace{1cm} (15)

Similar to $E$ and $L$, the mass-accretion rate $\dot{M}$ is also conserved along the flow, except at a shock location. After defining all the physical quantities necessary to solve for black hole accretion, we will describe below the transonic properties of the physical accretion solutions.

### 2.2. Regularity Conditions

For accretion to continue on to the event horizon, it is required that the accreting flows become supersonic at a sonic radius. After taking the derivatives of equations (11) and (15) with respect to $r$ (note that $E$ and $\dot{M}$ are both constants), $dc_s/dr$ can be eliminated. Finally, we obtain

$$\frac{da'}{dr} = \frac{\mathcal{N}}{\mathcal{D}},$$  \hspace{1cm} (16)

where

$$\mathcal{D} \equiv 2Nc_s^2 + u' \left[ N(3c_s^2 - 2) - 1 \right],$$  \hspace{1cm} (17)

$$\mathcal{N} \equiv \zeta \left[ 1 + u' \right] \left( r^{3/2} + am^{1/2} \right) \frac{dV_{\text{eff}}}{dr}$$

$$- \left\{ a(4Nc_s^2 + \eta) \right.$$  

$$+ r^{3/2} \left[ 2Nc_s^2 + 3g_{rr} + \eta \right] \right\} V_{\text{eff}},$$  \hspace{1cm} (18)

and

$$\zeta \equiv r^2 \left[ 3c_s^2 - 1 \right],$$  \hspace{1cm} (19)

$$\zeta' \equiv r \left[ N(c_s^2 - 2) - 1 \right].$$  \hspace{1cm} (20)

In equation (16), the fact that the velocity gradient $da'/dr$ is finite at $r = r_c$ requires that $D(r = r_c) = 0$ and $N(r = r_c) = 0$ simultaneously (i.e., regularity conditions), which will allow us to find the critical radius $r_c$ for a given parameter set. A physically valid accretion solution, therefore, must pass through a critical point at $r = r_c$ before reaching the horizon, whether or not the shock formation is possible. In the presence of a shock, a global shock-included accretion solution must go through a critical point on both sides of the shock location (i.e., before and after the shock forms). This fact requires multiple critical points. It is widely known that multiple critical points (up to three at most) can exist, in general, for a certain flow parameter space. Many work has been done on examining the topological behaviors (i.e., saddle, node, center, and spiral points) of the critical points (e.g., Chakrabarti 1990); thus, we will not repeat this. Because our main goal of this paper is to explore the possibility of outflows that are coupled to the shocks in the global accretion flows, we will only investigate the upstream flows that pass through the outer critical points $r_c^{in}$ at the same time that the downstream flows are passing through the inner critical points $r_c^{in}$ (the middle one is known to be unphysical).

So far, for a specified flow parameter set, one can obtain accretion solutions. Next, let us impose the shock conditions that connect one solution (i.e., upstream flow) to another (i.e., downstream flow), taking into account energy, angular momentum, and mass loss at a shock location.

### 2.3. Shock Formation with Energy and Mass Loss

Accreting flows around a black hole are generally subject to a number of “invisible” obstacles that can decelerate the flow: (1) a centrifugal barrier due to the fluid’s angular momentum, (2) a gas-pressure gradient, (3) a radiation-pressure gradient, and (4) magnetic forces (i.e., a pressure gradient and/or tension force). Although our model is purely adiabatic and hydrodynamic (thus 3 and 4 are absent), flows are still under the influence of the deceleration mechanisms 1 and 2.

Following the previous works (see Yang & Kafatos 1995; Lu & Yuan 1998), let us assume jump conditions that allow energy, angular momentum, and mass loss at a standing shock front. Figure 1 illustrates a schematic description of our model. From equation (11) we have

$$E_1 = \left[ \frac{1 + g_{rr}(u_r')^2}{-V_{\text{eff}}} \right]^{1/2} \frac{1}{(1 - Nc_{s1})},$$  \hspace{1cm} (21)

$$E_2 = \left[ \frac{1 + g_{rr}(u_r')^2}{-V_{\text{eff}}} \right]^{1/2} \frac{1}{(1 - Nc_{s2})},$$  \hspace{1cm} (22)

where the subscripts “1” and “2” denote the quantities for upstream and downstream flows evaluated at a shock location.
(r = r_{sh}), respectively. We require E_1 > E_2 and define the energy dissipation and its fraction as

\[ \Delta E = E_1 - E_2 \quad \text{and} \quad f_E = \frac{\Delta E}{E_1}, \quad (23) \]

where \( 0 < f_E < 1 \). The associated angular momentum carried by the outflows is then given by \( L = L_1 - L_2 = \frac{\Delta E}{\omega^2}. \)

In the baryon mass conservation across the shock front, we also consider some mass loss (equivalently, the loss of mass-accretion rate) associated with the outflows that are blown away as winds/jets. From equation (15) we have

\[ M_1 = -4\pi r_{sh} \left( \frac{r_{sh}^3}{2} + am_{1/2} \right) u_1 \]
\[ \times \left[ \frac{c_s^{2N+1}}{(1 + 1/N)(1 - Nc_s^2)^N} \right] \frac{1}{K_1^N}, \quad (24) \]
\[ M_2 = -4\pi r_{sh} \left( \frac{r_{sh}^3}{2} + am_{1/2} \right) u_2 \]
\[ \times \left[ \frac{c_s^{2N+1}}{(1 + 1/N)(1 - Nc_s^2)^N} \right] \frac{1}{K_2^N}. \quad (25) \]

Clearly, the mass-accretion rate depends on the measurement of entropy \( K \), which must increase across the shock because of the heat generated (second law of thermodynamics). However, in the presence of both energy dissipation and mass loss at the shock front, a fraction of total energy (including mass and thermal energies) can be released from the flow surface, quickly reducing the rise of \( K \). This hypothesis is justifiable when cooling processes are very efficient, although it is beyond the scope of this work to discuss the details of these mechanisms. Hence, for simplicity, we assume that the entropy will roughly remain unchanged at the shock front as a result from partial heat loss of the shocked flow. Thus, we set \( K_1 = K_2 \equiv K_0 \) and require \( \dot{M}_1 \geq \dot{M}_2 \).

Similar to energy dissipation, let us define the mass loss and its fraction as

\[ \Delta \dot{M} = \dot{M}_1 - \dot{M}_2 \quad \text{and} \quad f_{\dot{M}} = \frac{\Delta \dot{M}}{\dot{M}_1}, \quad (26) \]

where \( 0 \leq f_{\dot{M}} < 1 \). That is, \( f_{\dot{M}} = 0 \) corresponds to no mass outflows from shocks.

The momentum flux density \( T^{\alpha\beta} \) for an ideal fluid is given by

\[ T^{\alpha\beta} = (\rho + P)u^\alpha u^\beta + Pg^{\alpha\beta}. \quad (27) \]

Therefore, the radial component \( T^{rr} \) is

\[ T^{rr} = \rho_0 u^r \left[ \frac{c_s^2}{(1 + 1/N)u_r} \right]. \quad (28) \]

From the mass and momentum conservations in radial direction, we finally obtain

\[ \mu_1 c_{s1} \left[ u_{1r} + \frac{c_s^2}{(1 + 1/N)u_{1r}} \right] \]
\[ = \mu_2 c_{s2} (1 - f_{\dot{M}}) \left[ u_{2r} + \frac{c_s^2}{(1 + 1/N)u_{2r}} \right], \quad (29) \]

which completes a series of our dissipative shock conditions.

The strength of shocks is measured by the local compression ratio of the flow, \( n_2/n_1 \), which is expressed as

\[ \frac{n_2}{n_1} = \frac{u_2^r}{u_1^r} \frac{c_{s2}}{c_{s1}} (1 - f_{\dot{M}}) = \left( \frac{c_{s2}}{c_{s1}} \right) \left( \frac{1 - Nc_{s1}^2}{1 - Nc_{s2}^2} \right)^N. \quad (30) \]

Hence, in the absence of mass loss (\( f_{\dot{M}} = 0 \)), the compression ratio must be greater than unity when shocks occur. However, in
the presence of mass loss ($0 < f_m < 1$), the compression ratio becomes a product of the velocity ratio and the mass-loss fraction. Therefore, there can be a case where rarefaction or decompression with $n_2/n_1 \leq 1$ might take place even if the fluid velocity (and sound speed) abruptly decreases across a very strong shock. To avoid further complications, we will focus our attention on compression shock waves only.

2.4. Dynamical Stability of Shocks

In terms of the radial momentum balance across the shock front, some standing shocks can be dynamically unstable. That is, the shock may decay away (either radially inward or outward) as a result of a small (radial) perturbation of its position. In order to examine the stability of the obtained shock solutions, we perturb the radial momentum flux density $T_{rr}$ (equivalent to pressure) by invoking an infinitesimally small variation of the shock location $\delta r_{sh}$. If a shock front shifts back to its original location to retain the momentum equilibrium there, it is dynamically stable. The criteria is expressed as

$$
(\delta T_{rr} - \delta T_{rr}')_{sh} = \left( \frac{d T_{rr}}{dr} \right)_{sh} \delta r_{sh} = \kappa(r_{sh}) \delta r_{sh},
$$

where $\kappa(r_{sh})$ is a function of $r_{sh}$ alone, and thus can be numerically evaluated. In our calculations we take advantage of the known fact that $\kappa(r_{sh}) < 0$ always guarantees the stable standing shocks (see Yang & Kafatos 1995; Lu & Yuan 1998; Fukumura & Tsuruta 2004).

3. NUMERICAL RESULTS

Following the formalism outlined above we calculate the fractions of energy and mass losses for various model parameters. There are essentially three primary variables ($E_1$, $\lambda$, and $r_{sh}$) that determine the solutions for a given geometry ($a$, $\theta$). In this paper, we restrict ourselves to the flows corotating with the black hole ($a \lambda > 0$) at the equator ($\theta = \pi/2$).

The topology of the accreting solutions is normally classified as “x-type” or “α-type” depending on whether the shock-free solution is global or not (e.g., see Lu & Yuan 1998, for definition). Although this is also one of the important aspects of the studies of standing shocks, we do not consider the distinction here since it is not crucial to our current investigations.

In our calculations we set $N = 3$ and choose several representative values of the upstream flow energy: $E_1 = 1.003$, $1.004$, and $1.005$ for $a/m = 0$ (Schwarzschild case) and $0.99$ (Kerr case), to illustrate the frame-dragging effect.

3.1. Dependence of Energy and Mass Loss

We first present in Figures 2 and 3 the mass-loss fraction $f_m$ as a function of shock location $r_{sh}$ for (a) $E_1 = 1.003$, (b) $1.004$, and (c) $1.005$, with $a/m = 0$ (Fig. 2) and $0.99$ (Fig. 3). Stable shocks are represented by solid curves, while (dynamically) unstable shocks are represented by dotted curves. Stable standing shocks, according to the criterion of equation (31), can form, in most cases, in regions relatively close to the black hole ($r_{sh}/m < 80$). For a rotating black hole case, the shock location can be considerably closer to the hole ($r_{sh}/m \geq 2-3$), results that are similar to the cases with no mass loss (e.g., Sponholz & Molteni 1994; Lu & Yuan 1998), due to the fact that the horizon shifts more inward. There appears to be a smooth transition between the stable and unstable ones. Filled circles denote the maximum stable shock location, also corresponding to the weakest shock (i.e., smallest $n_2/n_1$), while open circles denote the minimum stable shock location, also corresponding to the strongest shocks (i.e., largest $n_2/n_1$). In other words, the stronger dissipative shocks can develop at smaller radii, in agreement with the previous result in the absence of mass loss (e.g., Lu & Yuan 1998; Fukumura & Tsuruta 2004, in which $f_m = 0$). Unstable shocks start to develop from where stable shocks become strongest. At both ends of the solution the curves are restricted by the dissipative shock conditions (eqs. [21], [22], [24], [25], and [29]) and the transonic properties (eq. [16]). It is seen that smaller angular momentum is required for shock (and accretion) to take place when the fluid energy is larger, also consistent with previous studies of standing shocks (Chakrabarti 1996; Lu et al. 1997; Lu & Yuan 1998;
Only stable shocks are allowed when angular momentum becomes sufficiently large (see Figs. 2c and 3). To simplify our discussion, we will focus primarily on stable shocks alone from this point on. As the angular momentum of the flow $k$ increases, the shock location tends to (but not always) shift radially outward for a given energy $E_1$, as expected, because the centrifugal force correspondingly increases at a given radius. Hence, accreting flows subject to more outward force must decelerate at larger distance, forcing the shock location to shift outward, as seen in our results. This trend appears to be more clear for the $a/m = 0.99$ case. It is also noted that in these figures larger angular momentum can allow for a larger mass outflow fraction $f_\dot{M}$.

In our model, the mass-loss fraction $f_\dot{M}$ can vary over a wide range ($0 \% \leq f_\dot{M} \leq 95 \%$), depending on the shock location and angular momentum. That is, mass outflows are not suppressed by relativistic effects and energy dissipation (for a comparison with the pseudo-Newtonian case with no energy dissipation, see, e.g., Das & Chakrabarti [1999]). For a fixed angular momentum, on the other hand, a higher value of $f_\dot{M}$ is expected from stronger shocks occurring in the inner regions, and this seems to be the case more in rotating black hole cases (see Fig. 3). We shall explain the shaded regions in these figures in § 4.

Figures 4 ($a/m = 0$) and 5 ($a/m = 0.99$) display the same mass-loss fraction $f_\dot{M}$ as a function of the energy loss fraction $f_E$, corresponding to the solutions in Figures 2 and 3, respectively. At a first glance, there is an explicit positive correlation between $f_E$ and $f_\dot{M}$. As the angular momentum increases, the maximum value of $f_E$ is clearly reduced. In these figures, the upper left portion corresponds to the outflow solutions with larger $f_\dot{M}$ and smaller $f_E$ (i.e., less energetic outflows), while the lower right portion represents the solutions with smaller $f_\dot{M}$ and larger $f_E$ (i.e., more energetic outflows). Therefore, the results above suggest that more energetic outflowing particles may be separated from a shock front when the upstream flow possesses smaller angular momentum regardless of black hole spin $a$. Mass-loss fraction $f_\dot{M}$ can become as high as $\leq 95 \%$ for both the $a/m = 0$ and 0.99 cases, regardless of energy $E_1$, whereas energy loss fraction is only $f_E < 1 \%$ for $a/m = 0$ case but $f_E \leq 10 \%$ for $a/m = 0.99$ case. By direct comparison of these figures, we note that rotation of a black hole $a$ is also effective, such that $df_E/df_\dot{M}(a/m = 0.99) > df_E/df_\dot{M}(a/m = 0)$, i.e., an increase in mass loss would allow more energy loss for $a/m = 0.99$.

In Fig. 6, we plot $n_2/n_1$ vs. $r_{sh}$ for the ($a$) $a/m = 0$ and ($b$) 0.99 cases. We choose values of $E_1 = 1.003$, 1.004, and 1.005 for the curves from top to bottom. The other notation is the same as in Fig. 2.

Fig. 7.— Mass-loss efficiency $f_\dot{M}$ as a function of black hole spin $a$ and energy loss efficiency $f_E$ for a set of fixed parameters. We choose $\lambda = 3.45$ and $r_{sh}/m = 30$. The solution curve is projected on each plane as shown.
Fig. 8.—Mass-loss efficiency $f_{\text{sh}}$ vs. $r_{\text{sh}}$ for $a/m = 0$. We set $E_1 = 1.000001$ and $\lambda = 3.73$. The notation is the same as in Fig. 2.

The upstream flow energy $E_1$ also affects the mass-loss fraction. It is noted that the energy dependence of $f_{\text{sh}}$ is strong; a higher energy $E_1$ can lead to larger mass outflow fraction for a given angular momentum $\lambda$ and a shock location $r_{\text{sh}}$. For instance, with $\lambda = 3.5$ for the $a/m = 0$ case (in Fig. 4), $f_{\text{sh}} \approx 20\%$ when $E_1 = 1.003$, 20%–30% when $E_1 = 1.004$, and as large as ~40% when $E_1 = 1.005$. With $\lambda = 2.16$ for $a/m = 0.99$ (in Fig. 5), $f_{\text{sh}} \approx 20\%$–85% when $E_1 = 1.003$, 40%–85% when $E_1 = 1.004$, and it is as high as 60%–85% when $E_1 = 1.005$. As a reference, we also examine the energy dependence of the shock strength in Figure 6 where the (local) compression ratio $n_2/n_1$ is plotted against $r_{\text{sh}}$ for different energies: $E_1 = 1.003, 1.004,$ and 1.005, as before. Angular momentum $\lambda$ is fixed in each case to see energy dependence alone (although different values of $\lambda$ for different spin $a$ must be chosen to obtain the solutions). We find that our dissipative shocks (coupled to mass loss) become stronger with decreasing energy, a behavior similar to the typical types of shock formation (e.g., see Lu et al. [1997] for adiabatic shocks; Lü & Yuan [1998] and Fukumura & Tsuruta [2004] for isothermal shocks).

To exclusively illustrate the black hole spin dependence of $a$ the mass-loss efficiency $f_{\text{sh}}$, we fix all other parameters ($E_1, \lambda$, and $r_{\text{sh}}$) except for $a$. Figure 7 shows $f_{\text{sh}}$ against $a$ and $f_{\text{sh}}$ for $\lambda = 3.45$ and $r_{\text{sh}}/m = 30$. As seen in the earlier results, black hole rotation alone can clearly enhance the efficiency of mass outflows $f_{\text{sh}}$ from $\sim 3\%$ (for $a/m = 0$) up to $\sim 95\%$ (for $a/m = 0.35$). On the other hand, the corresponding energy loss efficiency $f_{\text{E}}$ remains as low as $\sim 0.02\%$–0.1%. Note here that $\lambda$ would have to be properly adjusted in order to obtain the solutions for higher black hole spin $a$.

We have chosen above some representative values for the flow energy for parametric purpose. Weakly viscous/inviscid accretion in general is a good model for some limited specific cases, like our Galactic center, for example. For such specific cases, the realistic choice of energy should be very small. From this perspective we examine to see whether low-energy flows can still produce shock-driven outflows. Figure 8 shows mass-loss efficiency $f_{\text{sh}}$ as a function of $r_{\text{sh}}$ for $a/m = 0$. We set $E_1 = 1.000001$ and $\lambda = 3.73$. Mass outflows can indeed be produced with $f_{\text{sh}}$ ranging from $\sim 1\%$ up to $\sim 65\%$. Both unstable and stable shocks are present, as in the earlier cases, but not continuously connected (no shock regions between the two). The range of shock location is much narrower in radius in this case, over which the mass-loss efficiency can significantly change as mentioned above. We will discuss this more in § 4.

The major correlations we find among the primary parameters are summarized in Table 1. Table 2 shows various correlations with shock strength. Compression ratio $n_2/n_1$ is strongly correlated with $r_{\text{sh}}, f_{\text{E}}$, and $f_{\text{sh}}$: stronger shocks are expected in regions very close to the central engine, particularly around a rotating black hole. Strong shocks in principle are accompanied by high energy and mass-loss fractions.

To sum up, our results show that strongest stable shocks generally develop at the smallest radii (closer to the black hole), accompanied by the largest mass loss $f_{\text{sh}}$ (and the largest energy loss $f_{\text{E}}$ as well). The rotation of the black hole apparently amplifies the shock strength by more than a factor of 2, also extending the outflowing site significantly inward (i.e., $r_{\text{sh}}/m \gtrsim 2-3$ when $a/m = 0.99$, while $r_{\text{sh}}/m \gtrsim 12$ when $a/m = 0$). We will address some implications of the obtained shock-outflow solutions in the last section, § 4.

3.2. Global Accreting Flows

Samples of the global run of the physical parameters of accretion flows that include shocks are given in Figures 9 ($a/m = 0$) and 10 ($a/m = 0.99$). In Tables 3 and 4 we provide the specifics of the shocks associated, respectively, with the flows of Figures 9 and 10. These plots have been made assuming that all flows accrete at 1% of the Eddington rate, i.e., $\dot{m} \equiv M/M_{\text{Edd}} = 0.01$ and that the black hole mass is $m = 10^7 M_\odot$, typical of Seyfert nuclei. Each panel shows (a) the radial velocity $|u|$, (b) the angular velocity $\Omega$, (c) the electron scattering optical depth defined by $\tau \equiv n \sigma_T H$, (d) flow temperature $T(K)$, (e) the density $\rho_0$ (g cm$^{-3}$), and (f) the ratio of the vertical scale height to the radius $H/r$. Vertical lines denote the positions of shocks connecting the upstream and downstream values of the corresponding quantities of each flow. The dotted curve in panel b shows the Keplerian angular velocity $\Omega_K$. Note that for clarity purposes we only show three of the representative shock solutions obtained, although the shock can occur at any radius between the outermost and innermost solutions (with different values of $f_{\text{E}}$ and $f_{\text{sh}}$).

We find that in both the nonrotating (Fig. 9) and rotating (Fig. 10) cases the upstream flow density scales as $\rho_0(r) \sim r^{-3/2}$, as in the ADAF self-similar solution. On the other hand, as we explain below, the downstream flow density generally has a slightly steeper power-law slope, $\rho_0(r) \sim r^{-3/2}$ to $r^{-3}$ unless the shock location is too close to the horizon. As seen in Figures 9a and 10a, this is primarily because of the decreasing (radial) downstream flow speed $|u_2(r)|$, whose feature is more obvious around a rotating

| Parameter | $r_{\text{sh}}$ | $f_{\text{E}}$ | $f_{\text{sh}}$ | $n_2/n_1$ |
|-----------|--------------|-------------|-------------|-----------|
| $\lambda$ | +            | -           | +           | +         |
| $E_1$     | -            | +           | -           | -         |

Notes.—Strong positive (negative) correlation is denoted by double plus signs (double minus signs), while relatively weak positive (negative) correlation is denoted by a single plus sign (single minus sign). No significant correlation is shown by a double sided arrow.
black hole. After the shock transition, frame-dragging of the rotating black hole forces the accreting flow to accelerate more in the toroidal direction (i.e., to corotate) rather than in the radial direction. Under adiabatic assumption, at the same time, the flow temperature continues to rise (Figs. 9d and 10d) due to compression of the flow. Since \(c_s/T \approx 2\), sound speed also increases. The resulting (thermal) pressure gradient (\(dP/dr < 0\)) prevents the incoming flow from speeding up and, in fact, decelerates the downstream flow (for a while) until the flow falls deep inside the gravitational potential well to radially speed up again. Consequently, the fluid motion is governed mainly by increasing toroidal velocity rather than a radial one (compare Fig. 9a with Fig. 10a).

In differentially rotating flows, the Keplerian frequency increases with decreasing \(r\) even more rapidly than the sound speed, leading to a decreasing scale height \(H\) (see eq. [14]). The dependence of these two quantities, \(\Omega_K\) and \(c_s\), on the radius in the downstream flow could lead to geometrically slim/thick structure all the way to the horizon (see, e.g., \(H/r \approx 0.2-0.3\) in Figs. 9f and 10f).

Recalling \(\rho_0 \propto 1/\left(rH[u'_r]\right)\) from equation (13), the above fact that both \(|u'_r|\) and \(H\) decrease allows a very steep downstream density profile \(\rho_0 \propto r\) after the shock transition (and this is more so in a rotating black hole case because of larger drop in \(|u'_r|\), as mentioned earlier). The downstream flows become slightly more opaque to electron scattering because of the shock compression, yet they continue to remain optically thin (\(\tau < 1\)).

4. DISCUSSION AND CONCLUSIONS

In this work we have examined the structure of accretion flows that include shocks of a more general character than those discussed so far in the literature (e.g., Chakrabarti 1990). In particular...
we have examined whether it is possible to have flows whose mass
and/or energy fluxes are lost and do not participate in the
shock transition (this may be the case in multidimensional shocks
or in shocks that involve acceleration of particles that escape from
the shock region). Such generalized shocks obey jump conditions
that are more general than those of Rankine-Hugoniot, and it is
not a priori certain if accretion flows can allow for such shocks.
We have also examined the degree of mass and energy loss al-
lowed that is at the same time consistent with the continuation of
the downstream flow through another sonic transition onto the
black hole. In this respect, we have focused our study on shocks in
which the energy per particle for those escaping the shock transi-
tion is greater than that of the local gravitational potential. In such
a case one can argue that these particles can escape to infinity,
producing the jets/winds observed in many accretion-powered
systems. We have found that there are indeed flows with global
parameters (i.e., $\lambda$ and $E$) that allow for such shocks. On the other
hand, we have also obtained solutions at which the energy per par-
ticle of the escaping matter is smaller than the local gravitational
potential, indicating that in all likelihood these particles will stag-
nate and eventually join the rest of the flow onto the compact ob-
ject. However, a deeper understanding of the precise mechanisms
that can lead to the particles that do not participate in the shock
transition, as we conjectured above, requires the knowledge of the
detailed microphysics of the shock (i.e., the acceleration efficiency,
the fraction of energy put into relativistic particles at the shock,
and their transport in and around the shock geometry); these are
beyond the scope of the present paper.

To get a crude estimate of the energetics of these outflow-
ing particles, we compute a threshold energy of the escaping par-
ticles (of mass outflow rate $\dot{m}$) to escape the bulk flow is given
by $4\pi r_{sh} \bar{H} \bar{n} \bar{u}^r \Delta E / c^2$, where $\bar{H} \equiv (H_1 + H_2)/2$, $\bar{n} \equiv (n_1 + n_2)/2$ and $\bar{u}^r \equiv (u_1^r + u_2^r)/2$ are evaluated at the shock location ($r \approx r_{sh}$).
The minimum kinetic energy necessary for the separated parti-
cles (of mass outflow rate $\Delta M$) to escape the bulk flow is given
by $\Delta M v_c^2 / 2$, where $v_c$, determined below, is the required
velocity of the particles. By equating these, we solve for $v_c$ to obtain
\[
\frac{1}{2} v_c^2 \approx \frac{Gm}{r_{sh}},
\]
\[
\Delta E \approx E_c.
\]

where $E_c \equiv Gm \Delta M / (4\pi r_{sh}^2 \bar{H} \bar{n} \bar{u}^r \bar{m}_p c^2)$. Since all of these param-
eters ($\bar{H}$, $\bar{n}$, and $\bar{u}^r$) are functions of shock location $r_{sh}$, which
depends on $\dot{f}_M$, $E_c$ is obtained once we specify $\dot{f}_M$. Now we can
review the shock solutions obtained earlier from this new per-
spective; in Figures 2–5 the allowed shock solutions satisfying
$\Delta E \geq E_c$, which is relevant for outflows that may escape the
shocked flow, are displayed by shaded regions, while the rest of
the solutions correspond to the outflows presumably bound to the
bulk flow. According to these estimates, an outflow that expels
a certain amount of the preshock accreting matter is possible over a
rather broad range of radii in the vicinity of the central object; up
to nearly 10% of matter may participate in producing seed parti-
cles for jets/winds within $\sim 20–40$ gravitational radii around a non-
rotating black hole, while as much as 50% of preshock matter could
contribute to forming jets/winds within $\sim 2–30$ gravitational radii
around a rapidly rotating black hole. More interestingly, our so-
lutions also allow for dissipative shocks ($f_k \neq 0$) with negligible
mass loss ($\dot{f}_M \sim 0$). Such solutions that involve the loss of infini-
tesimal mass and finite amount of energy can lead to relativistic
outflows from the corresponding shocks. Outflows via these types
of shocks may produce kinematically strong jets (with high Lorentz
factor) rather than moderate velocity winds. Thus, shock-driven
outflows may provide clues on the origin of the jets/winds from
the inner accretion regions, for instance, in some active galaxies,
e.g., M87 and 3C120.

In this simple model, shocks in principle could drive axi-
symmetric outflowing matter. Near the base of the outflow, as il-
ustrated in Figure 1, its shape would be that of a hollow cone.
However, as it propagates to longer distances we do not expect
that this shape would be preserved all the way. Within the frame-
work of our current scenario it can be speculated that most of the
outflows discussed here could be in a diffuse form, while some
strong outflows with low $f_k$ and high $f_k$ might possess a relatively
more collimated geometry (perhaps in the poloidal direction)
due to its high Lorentz factor. In the presence of large-scale mag-
netic fields, the dissipated plasma would stream along the field
lines, producing collimated outflows. However, it is beyond the
scope of our model to further discuss the exact geometry of the
outflows.

It should be noted that for a given $\lambda$ there exists the degeneracy
of shock-outflow solutions, i.e., for a single value of angular
momentum $\lambda$ there is a finite range of possible shock locations
allowed ($r_{sh}^{\min} < r_{sh} < r_{sh}^{\max}$) that corresponds, respectively, to out-
flows with $f_{k1}^{\min} \leq f_k \leq f_{k1}^{\max}$, where the indices “$\min$” and
“$\max$” denote minimum and maximum values, respectively. In
the context of our model, one can predict the shock location $r_{sh}$
by the (observational) knowledge of $\lambda$, $E_1$, and $f_{k1}$, although
the obtained parameter space can be altered by additional physical in-
gredients (e.g., viscosity, magnetic fields, for instance).

Another issue to be addressed is the fact that in general critical
points are not necessarily equivalent to sonic points, depending on
the flow geometry and equation of state used (e.g., Das 2007).
That is, there could be a potential danger that a false shock could

\begin{table}[h]
\centering
\caption{Sets of Parameters for the Global Solutions in Fig. 9}
\begin{tabular}{cccc}
\hline
Model & $r_{sh}/m$ & $n_2/n_1$ & $f_k$ (\%) & $f_{\dot{M}}$ (\%) \\
\hline
1 & 15 & 4.2 & 0.319 & 52.4 \\
2 & 31 & 3.8 & 0.136 & 47.3 \\
3 & 54 & 3.1 & 0.00253 & 43.4 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Sets of Parameters for the Global Solutions in Fig. 10}
\begin{tabular}{cccc}
\hline
Model & $r_{sh}/m$ & $n_2/n_1$ & $f_k$ (\%) & $f_{\dot{M}}$ (\%) \\
\hline
4 & 2.4 & 10.2 & 8.3 & 89 \\
5 & 10 & 7.9 & 2.6 & 51.0 \\
6 & 78 & 2.73 & 0.002 & 17.6 \\
\hline
\end{tabular}
\end{table}

Note.—Note that $a/m = 0$, $E_1 = 1.003$, and $\lambda = 3.6$. 

\[ E_c \equiv \frac{Gm \Delta M}{4\pi r_{sh}^2 \bar{H} \bar{n} \bar{u}^r \bar{m}_p c^2} \]
occur in subsonic region between a critical radius and an actual sonic radius, in which case the obtained shock would be unphysical. To ensure that a physically valid shock forms in supersonic regions, we have checked the validity of our shock solutions by computing a three-velocity component of the flow measured by a suitable local observer at the shock location. We calculated the (radial) flow velocity $v^r$ measured by a locally stationary observer in the corotating reference frame (e.g., Lu 1986; Lu & Yuan 1998),

$$v^r = \frac{u_r u^r}{1 + u_r u^r},$$

and compared this velocity to the local sound velocity $c_s$ given by equation (9). All the shock solutions presented here are found to form in supersonic regions (i.e., $|v^r| > c_s$ at $r = r_{sh}$).

We showed that low-energy flows can still produce mass outflows with suitable angular momenta. Although continuous accreting solutions (i.e., shock-free solutions) are persistently present even for smaller energy $E_1$ (as expected), we do not find shocked flow solutions (or perhaps they are present but in much narrower parameter space). It can be speculated that maybe by prohibiting our jump condition in energy (energy dissipation at a shock front), we may obtain shock-driven outflows for smaller flow energy $E_1$, which could be a case similar to Das & Chakrabarti (1999), where they considered little energy loss and found mass outflows in pseudo-Newtonian geometry. However, powerful outflows should carry away a significant amount of (kinetic) energy. Hence, outflow solutions should in principle be coupled to energy dissipation, as treated here.

We have explored a coupling between shock solutions in accretion and mass/energy losses (fractions) under a scenario that the shock-driven outflowing particles may participate in forming a base of jets/winds. For various flow parameters with a given black hole spin, we have shown, by steady state, axisymmetric hydrodynamic calculations, that the dissipative shock front could be a plausible site where a fraction of the accreting matter can be decoupled as jets/winds from the bulk accretion flows.

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