Estimating the Turn-around Radii of Six Isolated Galaxy Groups in the Local Universe

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Abstract

Estimates of the turn-around radii of six isolated galaxy groups in the nearby universe are presented. From the Tenth Data Release of the Sloan Digital Sky Survey, we first select those isolated galaxy groups at redshifts \( z \leq 0.05 \) in the mass range \( [0.3−1] \times 10^{14} \, h^{-1} \, M_\odot \) whose nearest-neighbor groups are located at distances larger than 15 times their virial radii. Then, we search for a gravitationally interacting web-like structure around each isolated group, which appears as an inclined streak pattern in the anisotropic spatial distribution of the neighboring field galaxies. Out of 59 isolated groups, only seven are found to possess such web-like structures in their neighbor zones, but one of them turns out to be NGC 5353/4, whose turn-around radius was already measured in a previous work and was thus excluded from our analysis. Applying the Turn-around Radius Estimator algorithm devised by Lee et al. to the identified web-like structures of the remaining six target groups, we determine their turn-around radii and show that three out of the six targets have larger turn-around radii than the spherical bound limit predicted by Planck cosmology. We discuss possible sources of the apparent violations of the three groups, including the underestimated spherical bound limit due to the approximation of the turn-around mass by the virial mass.

Key words: cosmology: theory – large-scale structure of universe

1. Introduction

When a dark matter (DM) halo forms in the universe, its gravitational struggle against the Hubble expansion in its linear stage leaves behind a unique vestige that is hardly effaced by any nonlinear complications in its subsequent evolution. This vestige, called the turn-around radius, reflects the very moment when the radial velocity of a protohalo begins to change sign as its self-gravity catches up with the Hubble flow. The merit and power of this vestige reside in the fact that even though it is a local quantity, it can be sufficiently well modeled by linear physics. If the turn-around radii of DM halos are directly estimated from observations, then a comparison of the estimated values with the model predictions would put a new constraint on the initial conditions of the universe (Pavlidou & Tomaras 2014; Pavlidou et al. 2014).

Linear physics predicts that in an accelerating phase of the universe the turn-around radii, \( r_t \), of DM halos are bounded by a finite upper limit, \( r_{\text{sa}} \), whose value depends sensitively on the amount and the equation of state of dark energy (DE) (Pavlidou & Tomaras 2014). In the standard picture where DE is given as the cosmological constant \( \Lambda \) and DM is cold (i.e., \( \Lambda \text{CDM} \) cosmology), the upper bound limit of the turn-around radii is given as

\[
 r_{\text{sa}} = f \left( \frac{GM}{\Omega_\Lambda H^2} \right)^{1/3},
\]

where \( H \) is the Hubble parameter, \( \Omega_\Lambda \) is the density parameter of \( \Lambda \), \( M \) is the turn-around mass (i.e., the mass of a spherical region enclosed by the turn-around radius \( r_t \)), and \( f \) is a parameter introduced to account for the effect of asymmetry in the DM distribution of a halo (Pavlidou & Tomaras 2014).

According to the linear physics that precludes an individual halo from having \( r_t \geq r_{\text{sa}} \) (i.e., violation of the bound limit) in the \( \Lambda \text{CDM} \) cosmology, \( f \) has an exact value of unity only if a DM halo forms through spherical collapse (Pavlidou & Tomaras 2014). For the more realistic case of non-spherical DM distributions, the value \( f \) has been estimated to be approximately 1.5 on the whole mass scale (see Figure 1 in Pavlidou & Tomaras 2014). This value of \( f \approx 1.5 \), however, is an empirical value obtained from the numerical simulations, unlike the case of spherical symmetry. Given that the degree of deviation of the DM distribution from spherical symmetry varies from halos to halo, the parameter \( f \) for the non-spherical bound limit should be regarded as a stochastic variable and the empirically obtained value of 1.5 as an average of this stochastic variable. Hence, the stochastic nature of the parameter \( f \) implies that it should be possible for individual halos with non-spherical DM distributions to violate the spherical bound limit on rare occasions since the spherical bound limit is 1.5 times lower on average than its non-spherical counterpart.

Lee & Li (2017) numerically examined whether or not this theoretical prediction derived purely from linear physics is valid in the deeply nonlinear regime. Directly measuring the mean turn-around radii averaged over sample halos on various mass scales from the MultiDark Planck simulations (Klypin et al. 2016), they confirmed that the mean turn-around radius never exceeds both the spherical and the non-spherical bound limits (say, \( r_{\text{sa}}^{(0)} \) and \( r_{\text{sa}}^{(00)} \), respectively) on the whole mass scale. Lee & Li (2017) also explored the probability of finding an individual DM halo with \( r_t \geq r_{\text{sa}}^{(0)} \) and found that 14% of DM halos with masses equal to or larger than \( 10^{13} \, h^{-1} \, M_\odot \) violate the spherical bound limit in a Planck cosmology (Planck Collaboration et al. 2014). Besides, their analysis revealed that in a modified gravity (MG) model where the notion of DE is replaced by the deviation of the gravitational law from general relativity (e.g., Joyce et al. 2015, and references therein), the frequency of occurrence of violations of the spherical bound limit becomes elevated, which indicates that it should be possible in principle to test the gravitational law on the cosmological scale by exploring the rarity of violations of the spherical bound limit.

As mentioned in Pavlidou & Tomaras (2014), the optimal targets for observational inspection of violations of the bound...
limit are not clusters but groups of galaxies, especially those located in a low-density environment. This is because Equation (1) was derived under the assumption that a DM halo has already reached a state of complete relaxation, which can hardly be justified in the case of galaxy clusters. Although the detections of a bound-violating cluster and a bound-violating supercluster were reported in previous works (e.g., Karachentsev et al. 2014; Pearson et al. 2014), the detections did not attract much attention, because it was suspected that the apparent violations of the spherical bound limit by that cluster and supercluster could be ascribed to the deviation of their dynamical states from complete relaxation and/or large uncertainties associated with the estimates of their turn-around radii from the peculiar velocities of their neighboring galaxies.

A practical difficulty in detecting violations of the bound limit on the group scale stems from the following fact. Since galaxy groups have a much weaker gravitational influence on their neighboring galaxies than galaxy clusters, the conventional methodology based on the direct measurements of the peculiar velocity profile (e.g., see Karachentsev et al. 2014) is likely to fail in properly estimating the turn-around radii of galaxy groups. Recently, Lee et al. (2015b) developed an efficient practical algorithm dubbed the Turn-around Radius Estimator (TRE) that does away with measurements of the peculiar velocities, unlike the conventional methodology. The applicability of the TRE, however, is contingent upon the existence of a gravitationally interacting filament or sheet-like structure around a target object.

Lee et al. (2015b) applied the TRE to NGC 5353/4, a nearby isolated galaxy group around which a thin straight filamentary structure had already been detected (Kim et al. 2016), and showed that the turn-around radius of NGC 5353/4 seemed to exceed the spherical bound limit set by the Planck ΛCDM cosmology (Planck Collaboration et al. 2014). This was the first observational detection of a violation of the spherical bound limit on the scale of a galaxy group. A detection of a single group that appears to violate Equation (1) with \( f = 1 \), however, cannot shatter down the ΛCDM cosmology, due to the stochastic nature of the parameter \( f \), as mentioned above. It is necessary to apply the TRE to a larger sample of galaxy groups and to statistically and systematically explore the frequency of occurrence of violations of the spherical bound limit, which we attempt to conduct in this paper.

The subsequent sections will present the following: a concise review of the TRE in Section 2; estimates of the turn-around radii of nearby isolated galaxy groups via the TRE and the frequency of occurrence of violations of the spherical bound limit on the...
group scale in Section 3; a summary of the results and a discussion of the physical implications in Section 4.

2. A Brief Review of the TRE Algorithm

The TRE algorithm is based on the numerical discovery of Falco et al. (2014) that the following universal formula depicts well the radial velocity profile, \( v_r(r) \), of DM particles in the bound zone around a halo with virial radius \( r_v \):

\[
v_r(r) = r - a \frac{V_v}{H} \left( \frac{r}{r_v} \right)^{-b},
\]

(2)

where the bound zone corresponds to radial distances from the halo center, \( r \), in the range \((3-8)r_v\), \( H \) is the Hubble parameter, and \( V_v \) is the central velocity at \( r_v \). Falco et al. (2014) determined empirically the amplitude and slope parameters, \( a \) and \( b \), of Equation (2) with the help of an N-body simulation and suggested that the best-fit values of \( a \) and \( b \) should be universal, independent of mass scales and redshifts. According to their claim, once the amplitude and slope parameters are set at the universal values, it is possible to estimate the value of \( r_v \) (or equivalently, the virial mass \( M_v \)) of a given halo by adjusting Equation (2) to the observed radial velocity profile.

Pointing out that neither \( v_r \) nor \( r \) in Equation (2) is directly observable, Falco et al. (2014) brought up the following heuristic scheme via which Equation (2) could be put into practice. Provided that a target halo is surrounded by a web-like structure (i.e., either a filament-like or a sheet-like structure) of DM in its bound zone, it is possible to express Equation (2) in terms of observables:

\[
l_z = \frac{r_{2d}}{\tan \beta} - a \cos \beta \frac{V_v}{H} \left( \frac{r_{2d}}{r_v} \right)^{-b},
\]

(3)

where \( l_z \equiv cz/H \) with relative redshift \( z \) and speed of light \( c \), \( r_{2d} \) is the radial separation distance from the halo center in the projected plane of the sky orthogonal to the sightline toward the target halo, and \( \beta \) is the angle between the position vector from the halo center and the sightline. Given that both \( r_{2d} \) and \( z \)

### Table 1

Equatorial Coordinates, Redshifts, and Virial Masses of the Target Groups

| Group | R.A. (deg) | Decl. (deg) | \( z \) | \( M_v \) (\( 10^{12} h^{-1} M_\odot \)) |
|-------|------------|-------------|--------|----------------------------------|
| GG1   | 226.08     | 1.64        | 0.007  | 40.85                            |
| GG2   | 144.43     | 17.06       | 0.029  | 60.69                            |
| GG3   | 157.20     | 8.59        | 0.049  | 37.00                            |
| GG4   | 226.81     | 9.59        | 0.045  | 30.13                            |
| GG5   | 254.37     | 27.35       | 0.037  | 48.72                            |
| GG6   | 123.70     | 55.16       | 0.033  | 48.85                            |

Figure 3. Configurations of the field galaxies (filled red dots) located in the overdense sites around an isolated galaxy group, GG1 (blue dots) from the catalog of Tempel et al. (2014). The open black dots indicate those field galaxies belonging to a web-like structure identified around GG1.
are readily observable, it can be said that Equation (3) is a practical version of Equation (2), which has an additional parameter $\beta$ as a trade-off. In other words, when Equation (3) is fitted to the observed radial velocity profile by adjusting the value of $M_v$, the angle $\beta$ becomes a nuisance parameter, whose presence would unavoidably enlarge the associated statistical errors on $M_v$ (see also Lee et al. 2015a).

Having a practical version of Equation (2), however, does not ensure success in its application to real observational data. It was necessary to test whether or not the radial velocity profile obtained not from DM particles but from luminous galaxies in the bound zone through anisotropic averaging would be well described by the same universal formula as Equation (3). In addition, it was also necessary to prove the claim of Falco et al. (2014) about the universality of the slope and amplitude parameters, $a$ and $b$, in Equations (2) and (3).

Several numerical works that were conducted in the light of Falco et al. (2014) consolidated the usefulness of Equations (2) and (3) by using larger samples from higher-resolution $N$-body simulations. For instance, Lee (2016) proved by analyzing the data from the Millennium Run II simulations (Boylan-Kolchin et al. 2009) that even when the radial velocity profile was obtained not from DM particles but from galaxy-size halos, the same analytic formula as Equation (2) still validly described the numerical result. Yet, they noted that the best-fit values of $a$ and $b$ do not show universal constancy but exhibit variance from halo to halo, implying that not only $\beta$ but also $a$ and $b$ should be treated as nuisance parameters in Equation (3). Lee & Yepes (2016) confirmed by analyzing the data from the MultiDark Planck simulations (Klypin et al. 2016) that Equation (2) worked even when the bound-zone radial velocity profile was constructed not through isotropic averaging but through anisotropic averaging over the filaments or sheets. Very recently, Albaek et al. (2017) showed that baryonic processes would not severely alter the functional form of the bound-zone radial velocity profiles.

The key idea of Lee et al. (2015b), who devised the TRE algorithm, is that Equation (2) can be used to mimic the expansion of a protogroup until the turn-around moment. A protogroup expands at a slower rate than the Hubble flow due to its self-gravity before the turn-around moment ($t_t$). Claiming that the radial velocity profile of a protogroup before $t_t$ may be well described by the same formula as Equation (2), Lee et al. (2015b) suggested that Equation (2) should become equal to zero at the turn-around radius ($r_t$):

$$ r_t = \frac{a}{H} \left( \frac{M_v}{M_*} \right)^{-b} . $$

The procedure to estimate the value of $r_t$ of a massive object via the TRE algorithm can be summarized as follows (for a detailed description, see Lee et al. 2015b). (1) For a galaxy group or cluster whose viral mass $M_v$ is already known from priors, search for a filament-like or sheet-like (collectively called web-like) structure in its neighbor zone; (2) construct the

![Figure 4. Same as Figure 3 but for a different galaxy group, GG2.](image-url)
3. Turn-around Radii of the Sloan Galaxy Groups

Tempel et al. (2014) applied a redshift-space adapted version of the friends-of-friends (FoF) algorithm to the galaxy sample from the Tenth Data Release of the Sloan Digital Sky Survey (SDSS DR10) (Ahn et al. 2014) to obtain a catalog of the FoF groups. From the catalog, one can draw out information on various properties of the FoF groups including their redshifts ($z$), equatorial coordinates of their centers (R.A. and decl.), and their virial radii ($r_v$) and masses ($M_v$), which were estimated under the assumption that their DM density profiles follow the Navarro–Frenk–White (NFW) formula (Navarro et al. 1997). They also provided the galaxy catalog from which information on the spectroscopic properties of the member galaxies belonging to each FoF group can be extracted.

Analyzing the group catalog of Tempel et al. (2014), we select those FoF groups with $0.3 \leq M_v/(10^{14} \ h^{-3} \ M_{\odot}) \leq 1$ (typical group scale) and $z \leq 0.05$, which are isolated enough to be separated by their nearest groups of comparable masses by more than $15r_v$. We consider only isolated groups, given the numerical result of Lee & Yepes (2016) that the best agreement between Equation (2) and the reconstructed radial velocity profile is achieved for the case of isolated halos. Furthermore, the TRE algorithm substitutes $M_v$ for the turn-around mass in Equation (1), an approximation that works best for the case of an isolated object.

Selected are a total of 59 isolated galaxy groups, in the neighbor zones of which we attempt to identify web-like structures composed of field galaxies. Although in the previous works of Falco et al. (2014) and Lee (2017), the neighbor zone around a cluster was defined to have a large extent of $|l| \leq 40 \ h^{-1} \ Mpc$ and $4 \leq r_{2d}/(h^{-1}\text{Mpc}) \leq 20$, we confine the neighbor zone to a much smaller extent of $|l| \equiv |cz/H| \leq 20 \ h^{-1} \ Mpc$ and $2 \leq r_{2d}/(h^{-1}\text{Mpc}) \leq 10$, given that the target objects are not the clusters but the less massive groups whose gravitational influence can reach out only this small extent. It is also worth explaining here why we identify a web-like structure from the distributions of only the field galaxies, excluding the wall galaxies. It is because the wall galaxies, unlike their field counterparts, are expected to be heavily influenced by their own hosts even if they are located in the same neighbor zone.

Adopting the methodology suggested by Falco et al. (2014) for the identification of web-like structures, we first look for overdense pixels in the neighbor zone around each isolated group by counting the field galaxies. The neighbor zone around
each isolated group is partitioned into 80 pixels of equal size in two-dimensional space spanned by \( r_{2d} \) and \( l_z \), as illustrated in the left panel of Figure 1. The spherical shell, with inner and outer radii of \( 2h^{-1}\) Mpc and \( 10h^{-1}\) Mpc, respectively, is also partitioned into eight wedges (say, \( \{W_1, \ldots, W_8\} \)), as depicted in the right panel of Figure 1, where \((x, y)\) denotes a two-dimensional position vector from the group center in the equatorial coordinate system, with \( r_{2d} = (x^2 + y^2)^{1/2} \). Each wedge represents a realization of the neighbor zone of a given isolated group, and the eight wedges form an ensemble over which the average residual number densities of the neighboring field galaxies around the group will be evaluated.

From the galaxy sample from the SDSS DR10, we select those field galaxies that belong to the neighbor zone of each isolated group by estimating the values of \( r_{2d} \) and \( l_z \). Then, we investigate to which pixel and to which wedge each of the neighboring field galaxies belongs. Suppose that one wants to find the dimensionless residual number density of the neighboring field galaxies at the \( ij \)th pixel by dividing the difference in the results between the first and third steps by the background number density at the \( ij \)th pixel. The fifth step is to compute the standard deviation, \( \sigma_{ij}^{w} \), of the residual number density in a similar manner. The final step is to see whether or not the condition \( \delta_{ij}^{w} \geq \sigma_{ij}^{w} \) is met at the \( ij \)th pixel of wedge \( W_1 \). If met, the \( ij \)th pixel of wedge \( W_1 \) is selected as a candidate overdense site where a web-like structure composed of the neighboring field galaxies may be found. Follow this procedure repeatedly for the other pixels and wedges to find all the overdense sites in the neighbor zone of each isolated group. See Falco et al. (2014) for a detailed description.

Before proceeding to identify a web-like structure in the overdense pixels of the neighbor zone around each isolated group, it is worth emphasizing that the TRE algorithm would be applicable only to those pixels that would appear altogether as inclined streak lines in the \( r_{2d}-l_z \) configuration space, as explained in Brinckmann et al. (2016). Figure 2 plots the analytic formula of Equation (3) for six different cases of \( \beta \), setting \( M_e \) at \( 5 \times 10^{13} h^{-1} M_\odot \). We look for a web-like structure composed of the field galaxies located in the neighbor zone around a target group, which would appear similar to the inclined lines shown in Figure 2.

Figure 6. Same as Figure 3 but for a different galaxy group, GG4.
We find that only seven out of the 59 isolated groups possess such web-like structures in their neighbor zones. Among the seven groups, one turns out to be NGC 5353/4, whose turnaround radius was already estimated by Lee et al. (2015b) to exceed the spherical bound limit. NGC 5353/4 being excluded, the remaining six groups (say, GG1, GG2, GG3, GG4, GG5, GG6) become our target groups to which the TRE algorithm is going to be applied for the estimation of their turnaround radii. Table 1 presents the equatorial coordinates, redshifts, and virial masses of the six target groups.

Figure 3 shows as red full circles the locations of the neighboring field galaxies belonging to the overdense pixels around GG1 in the two-dimensional configuration space spanned by $r_{2d}$ and $l_z$. The green dotted lines correspond to the locations at which the condition $l_z = r_{2d}$ is met, while the blue full circles represent the configurations of the member galaxies of GG1. Noting the existence of an inclined streak of the neighboring field galaxies in the overdense pixels of the wedge $W_6$, which look similar to the inclined lines shown in Figure 2, a web-like structure is identified around GG1 and shown as black open circles in Figure 3. Figures 4–8 show the same as Figure 3 but for the other five target groups. As can be seen, the web-like structures around GG2, GG3, GG4, GG5, and GG6 are identified in the wedges of $W_3$, $W_4$, $W_5$, $W_6$, and $W_7$, respectively. Figure 9 shows the same as Figures 3–8 but for the case of an isolated group around which no web-like structure is identified, and which is thus not selected as a target.

Suppose that we identify a web-like structure composed of $n_f$ neighboring field galaxies from one of the eight wedges around a target group. Employing the maximum likelihood method as Lee et al. (2015b) and Lee (2017) did, we determine the best-fit values of $a$, $b$, and $\beta$ in Equation (3), which maximize the following posterior distribution:

$$p(a, b, \beta) \propto \exp \left[ -\frac{\chi^2(a, b, \beta)}{2} \right],$$

$$\chi^2(a, b, \beta) = \sum_{k=1}^{n_f} [l_{z,k} - l^2_{2d,k}(a, b, \beta)]^2,$$

$$l^2_{2d,k}(a, b, \beta) = \frac{r^2_{2d,k}}{\tan \beta} - a \cos \beta \frac{V_c}{H} \left( \frac{r_{2d,k}}{\sin \beta r_f} \right)^b,$$

where $(r_{2d,k}, l_{z,k})$ is the observed position vector of the $k$th neighboring field galaxy belonging to an identified web-like structure, while $l^2_{2d,k}$ represents Equation (3) with $r_{2d}$ set at $r_{2d,k}$.

To improve the efficiency of the TRE algorithm in its practical application, we rearrange the terms of Equation (4) to derive the following closed analytic expression for $r_f$ as a function of $a$ and $b$:

$$r_f(a, b) = \exp \left\{ \frac{1}{1 + b} \ln \left[ a \frac{V_c}{H} \right] \right\},$$
Suppose that the posterior function, Equation (5), is found to reach its maximum at \(a = \hat{a}, b = \hat{b}, \beta = \hat{\beta}\). Putting the best-fit values, \(\hat{a}\) and \(\hat{b}\), into Equation (8), one can readily estimate the turn-around radius of a target group as \(\hat{r}(\hat{a}, \hat{b})\).

We estimate the associated errors on \(\hat{r}_t\), \(s_{rt}\), according to the error propagation formula (Wall & Jenkins 2012):

\[
\sigma_{\hat{r}}^2 \approx \left(\frac{\partial \hat{r}}{\partial a}\right)_{\hat{a}, \hat{b}}^2 \sigma_a^2 + \left(\frac{\partial \hat{r}}{\partial b}\right)_{\hat{a}, \hat{b}}^2 \sigma_b^2 + 2 \left(\frac{\partial \hat{r}}{\partial a}\right)_{\hat{a}, \hat{b}} \left(\frac{\partial \hat{r}}{\partial b}\right)_{\hat{a}, \hat{b}} \text{cov}(a, b),
\]

where \(\sigma_a\) and \(\sigma_b\) denote the marginalized errors in the determination of the best-fit values of \(a\) and \(b\), respectively, and \(\text{cov}(a, b)\) is the marginalized covariance between \(a\) and \(b\); these three can be calculated as

\[
\sigma_a^2 = \int d\beta \int da \int db \, (a - \langle a \rangle)^2 \, p(a, b, \beta),
\]

\[
\sigma_b^2 = \int d\beta \int da \int db \, (b - \langle b \rangle)^2 \, p(a, b, \beta),
\]

\[
\text{cov}(a, b) = \int d\beta \int da \int db \, (a - \langle a \rangle)(b - \langle b \rangle) \, p(a, b, \beta),
\]

where \(\langle a \rangle = \int d\beta \int da \int db \, a \, p(a, b, \beta)\) and \(\langle b \rangle = \int d\beta \int da \int db \, b \, p(a, b, \beta)\). The best-fit values of \(a\) and \(b\) determined by the maximum likelihood method along with Equations (5)–(7) as well as the associated errors and covariances estimated from Equations (10)–(12) for the six target groups are presented in Table 2.

We calculate the partial derivatives, \(\partial \hat{r}/\partial a\) and \(\partial \hat{r}/\partial b\) at \(a = \hat{a}\) and \(b = \hat{b}\), in Equation (9) and derive the following expressions:

\[
\left.\frac{\partial \hat{r}}{\partial a}\right|_{\hat{a}, \hat{b}} = \frac{\hat{r}_t}{\hat{a}(1 + \hat{b})}, \quad \left.\frac{\partial \hat{r}}{\partial b}\right|_{\hat{a}, \hat{b}} = \frac{\hat{r}_t}{\hat{a}(1 + \hat{b})} \ln \left(\frac{\hat{r}_t}{\hat{\nu}}\right),
\]

where \(\hat{\nu}\) is the estimated turn-around radius for each target group. It is worth emphasizing here that although the error on the nuisance parameter \(\beta, \sigma_{\beta}\), does not explicitly appear in Equation (9), the variation of \(\beta\) is fully taken into account for the determination of the marginalized error, \(\sigma_{rt}\), since both \(\sigma_a\) and \(\sigma_b\) are determined by the simultaneous marginalization of the posterior distribution, \(p(a, b, \beta)\), over \(a, b,\) and \(\beta\).

Table 3 lists the estimated turn-around radii and the associated errors for the six targets, and compares the values with the spherical and non-spherical bound limits set by the

Figure 8. Same as Figure 3 but for a different galaxy group, GG6.
Planck cosmology. As can be seen, for the cases of GG1, GG2, and GG3, the differences between $\hat{r}$ and $r_{tu}$ are larger than $\sigma_c$, while for the cases of GG4, GG5, and GG6, the differences fall within $\sigma_c$. Given this result, the former three groups could be regarded as candidates for violation of the spherical bound limit. Yet, the comparison of $\hat{r} - r_{tu}^{(ns)}$ with $\sigma_c$ reveals that none of the six targets violates the non-spherical bound limit.

4. Summary and Discussion

Employing the TRE algorithm developed by Lee et al. (2015b), we have estimated the turn-around radii of six isolated galaxy groups with masses in the range $0.3 \leq M/M_\odot \leq 10^{14}$ at redshifts of $z \leq 0.05$ from the SDSS DR10. To ensure the validity and efficacy of the TRE algorithm, our analysis has been restricted to local isolated galaxy groups around which the neighboring field galaxies exhibit anisotropic spatial distributions. For each of the six targets, we have constructed a radial velocity profile along the anisotropic distribution of the neighboring field galaxies (Figures 3–8) and fitted it to the analytic formula derived by Falco et al. (2014). Finally, the turn-around radius of each target has been determined as the radial distance at which the best-fit formula hits zero, and the marginalized errors propagated through the fitting procedure have also been evaluated (Table 3).

The measured turn-around radii of the six targets have been compared with the spherical and non-spherical upper bound limits predicted by the $\Lambda$CDM cosmology. Among the six targets, three have been shown to violate the spherical bound limit, while the other three abide by it. Although no violation of the non-spherical bound limit is found, we have noted that the
observed frequency at which violation of the spherical bound occurs on the galaxy group scale is rather high compared with the numerical result of Lee & Li (2017), who found the frequency to be as low as 14% in a ΛCDM universe. Yet, before rushing to a conclusion that our observational result challenges the ΛCDM cosmology, it is worth inspecting a more mundane source of this rather high frequency of occurrence of violation of the spherical bound limit.

The first suspicion falls on the underestimate of the spherical bound limit caused by substituting the virial mass for the turn-around mass in Equation (1). Although it has been presumed throughout our current analysis that for the case of an isolated galaxy group the virial mass would approximate well the turn-around mass, it has yet to be quantitatively addressed how close the virial mass of each target is to its turn-around mass, how the difference between the two masses would depend on the mass scale, and how significantly the difference would change the value of the spherical bound limit.

Another factor that has not been taken into account but may have contaminated the final result is the uncertainties associated with the measurements of the virial masses of the galaxy groups. Tempel et al. (2014) measured the virial masses of the SDSS groups under the assumption that the DM density profiles are well described by the universal NFW formula (Navarro et al. 1997). However, several numerical experiments have already invalidated the concept of the universality of the NFW density profile (e.g., Navarro et al. 2004). The deviation of the true density profiles from the NFW formula may have caused systematic errors in the measurements of the virial masses of the target groups, which may in turn have contaminated our estimates of their turn-around radii.

The other downside is the small size of our sample, consisting only of six target groups, which prevents a statistically conclusive interpretation of the final result. This small sample size is an inevitable outcome of the generic limitation of the TRE algorithm, which is applicable only to those isolated groups that have web-like structures in their neighbor zones. Furthermore, since a web-like structure had to be identified from the anisotropic spatial distribution of field galaxies to guarantee its gravitational link with the target, each identified web-like structure has turned out have a very low richness, which incurred inaccuracy in the construction of the radial velocity profiles. Our future work will be in the direction of addressing these remaining issues and improving further the statistical analysis as well as the TRE algorithm.

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