Weak universality in the two dimensional randomly disordered three-state Potts ferromagnet

Jae-Kwon Kim

Department of Physics, University of California, Los Angeles, CA 90095-1547

Abstract

For the two dimensional randomly disordered three-state Potts ferromagnet, we find numerically that the critical exponent $\eta$ unchanges with the degree of disorder while those of the correlation length ($\nu$) and magnetic susceptibility ($\gamma$) increase with it continuously. We discuss some consequences of the finding.

PACS numbers: 75.40.Mg, 75.10.Nr, 05.70.Jk
The effect of random quenched disorder to the critical behavior of a ferromagnetic system has been a subject of intensive studies. According to a heuristic argument by Harris [1], that has been widely accepted over two decades, the disorder induces a new universality class only when the critical exponent of the specific heat \( \alpha_p \) in the corresponding undisturbed system is positive.

A more rigorous approach [2] claims that in a D-dimensional disordered system obeys

\[ \nu \geq \frac{2}{D}. \] \hspace{1cm} (1)

The proof of Eq.(1), however, presumes the existence of so-called finite-size scaling correlation length whose relation to the intrinsic correlation length \( \xi \) is by no means obvious, defined in terms of probability. Note that Eq.(1) combined with hyperscaling relation, \( \alpha_p = 2 - D \nu_p \) (with \( \nu_p \) denoting the value of \( \nu \) in the corresponding undisturbed system), implies that for a disordered system with \( \alpha_p > 0 \) \( \nu \) jumps discontinuously from the undisturbed value \( \nu_p \). To be more specific, random disorder is supposed to induce a different scaling region near criticality while the rest of the scaling region is equivalent to that of the undisturbed system (crossover from the undisturbed to disordered).

The 2D random bond disordered Ising ferromagnet is unique in that its continuum limit is found to be a certain tractable field theoretic model, i.e., \( O(N) \) Gross-Neveu model in the limit \( N \rightarrow 0 \). Accordingly it is predicted that the value of \( \eta \) in this system does not vary with the strength of weak disorder [3], which has now been confirmed by various numerical methods even for strongly disordered case [4,5]. Albeit \( \alpha_p = 0 \), recent extensive numerical studies [6-8] find that both \( \nu \) and \( \gamma \) change with disorder, in disaccord with Harris criterion.

Most numerical studies on the 3D random site diluted Ising ferromagnet, where \( \alpha_p \simeq 0.1 \), report evidences for new universality class [9,10,12]. Still is it not clear whether Eq.(1) is indeed satisfied or not [11]; nor is it clarified whether critical exponents change continuously with the strength of disorder or jump to certain fixed values regardless of it. An accurate estimate of \( \eta \) is far more difficult, for it is very sensitive to the choice of the critical point. Generally speaking, current understanding on generic randomly disordered ferromagnet is
rather limited theoretically, numerically, and experimentally.

In this Letter we report new finding of the extensive Monte Carlo studies on the 2D random bond disordered three-state Potts ferromagnet. The technical difficulty is significantly lightened in this system, since the exactly known value of $\alpha_p$ is large enough ($\alpha_p = 1/3$) and the exact critical point can be calculated for certain realizations of disorder. Thus we obtain clean numerical evidence that the value of $\eta$ unchanges with disorder while both $\nu$ and $\gamma$ increase continuously. With the very same results found in the 2D disordered Ising ferromagnet [8], we conjecture that those features are generic in any randomly disordered ferromagnets.

The Hamiltonian of a $q$-state Potts ferromagnet with quenched random interaction (bond) can be written as

$$H = - \sum_{<x,y>} J_{xy} \delta_{\sigma_x \sigma_y},$$

where the spin at site $x$, $\sigma_x$, can take on the values 1,...,$q$ ($q = 3$ in this work), $\delta$ is the Kronecker delta function, and the sum is over all the nearest-neighbor pairs on the lattice system. The coupling $J_{xy}$ for each link $<x,y>$ is selected randomly from two positive (ferromagnetic) values $J$ and $J'$ with probability $p$ and $1-p$ respectively.

For $p = 1/2$ the system is self-dual with its self-dual point given by [13]

$$[\exp(J) - 1][\exp(J') - 1] = q$$

We fix $J = 1$ and $p = 1/2$ without any loss of generality, and consider four different values of $J'$, i.e., $J' = 1.0$ (undisordered case), 0.9, 0.5, and 0.25. Accordingly, the inverse self-dual temperature (critical temperature) is given by $\beta_c = \ln(\sqrt{3} + 1)$, 1.0588..., 2ln(2), and 1.8235..., respectively for $J' = 1.0$, 0.9, 0.5, and 0.25. The values of the critical exponents for the undisordered case are exactly known [14], e.g., $\nu_p = 5/6$ and $\gamma_p = 13/9$ (hence $\eta_p = 4/15$).

Our raw-data for each $J'$ are obtained by choosing a realization of random $J'$, then running Monte Carlo simulations in Wolff’s single cluster algorithm with periodic boundary
conditions imposed on square lattice. Details of our single cluster algorithm for Potts model will be presented in a longer paper. For each realization, measurements were taken over 10 000 configurations each of which is separated by 6-24 one cluster updatings according to autocorrelation time. The mean values over all the different realizations are our final data. The numbers of different realizations are typically 30-60, 150-200, and 250-350 respectively for $J' = 0.9, 0.50$ and $0.25$. Larger number of realizations are required for smaller value of $J'$ due to wilder fluctuations among different realizations.

Our magnetic susceptibility and correlation length on a square lattice of linear size $L$ are defined as

$$\chi_L = \tilde{G}(0),$$
$$\xi_L = \frac{1}{\sin(2\pi/L)} \sqrt{\tilde{G}(0)/\tilde{G}(2\pi/L) - 1},$$

where the two-point correlation function $G(x)$ and its Fourier transformation $\tilde{G}(k)$ are respectively defined as $G(x) \equiv \langle \delta_{\sigma_0 \sigma_x} - 1/q \rangle$ and $\tilde{G}(k) \equiv \sum_x G(x) \exp(kx_1)$ with $x_1$ denoting the first component of $x$.

In order to determine the value of $\eta$ we measured $\chi_L$ at each critical point (self-dual point) by varying $L$ from 20 to 120. According to the standard theory of finite size scaling, $\chi_L$ at criticality satisfies

$$\chi_L \sim L^{2-\eta},$$

where $\eta = 2 - \gamma/\nu$. Our results are summarized in Figure(1) showing undoubtedly that the value of $\gamma/\nu$ remains unchanged irrespective of the value of $J'$. The $\chi^2$ fits yield $\eta=0.266(3), 0.264(3), 0.267(8), 0.268(10)$, respectively for $J'=1.0, 0.9, 0.5,$ and $0.25$, to be compared with the exact value 0.266... ($\chi^2/\text{NDF} < 0.5$ for all the $J'$).

The value of $\gamma$ is determined by the $\chi^2$ fit of bulk $\chi$ data measured at various values of the inverse temperature $\beta$. The data with $\xi(\beta) \geq 5$ are considered in the fit to eliminate possible correction to the scaling

$$\chi \sim t^{-\gamma},$$
where \( t \equiv (\beta_c - \beta)/\beta_c \). We used \( \xi_L \) to monitor the finite-size effect in our measurements of bulk \( \chi \); that is, the condition \( L/\xi_L \geq 8 \) was always imposed for the extractions of our bulk data (see Figure(2)). The largest values of \( \xi \) are roughly 20 \( \sim \) 30 for all the values of \( J' \), using \( L \) up to \( L=240 \).

Our procedure for the undisordered case yields \( \gamma=1.442(3) \) in excellent agreement with the exact value \( \gamma=1.444... \). The data for different \( J' \) are summerized in Figure(3). The value of \( \gamma \) clearly increases with the strength of disorder: \( \gamma=1.44(1), 1.56(4), \) and \( 1.84(1) \) respectively for \( J'=0.9, 0.5, \) and \( 0.25 \) (\( \chi^2/N_{DF} < 1 \)). Also we do not find any indication of crossover, as in the 2D randomly disordered Ising ferromagnets [6,8]. In other word, the presence of random quenched disorder affects the entire scaling regime (Figure(3)).

For the weakly disordered case (\( J' = 0.9 \)) the value of \( \gamma \) thus extracted is virtually the same value as in the undisordered case. We do not take this as an evidence that their values are strictly the same in those two systems; it rather appears that \( \gamma \) increases indistinguishably mildly for a very weakly disordered system.

Eq.(1) combined with our finding that \( \gamma/\nu \) remains a constant implies that \( \gamma \geq 26/15 \) in the two dimensional randomly disordered three-state Potts ferromagnets, in obvious dis-accord with our results for \( J' = 0.9 \) and 0.5.

Our results contrast with some previous numerical results in the 3D disordered Ising ferromagnet [10,13], but are consistent with the results in Ref. [12] where the critical exponent of the magnetization was found to vary continuously. The variant conclusions of the previous numerical studies on the 3D disordered Ising ferromagnet are perhaps due to some technical reasons: Without measuring correlation length, a priori, the finite size effect in the measurement of bulk data can hardly be monitored, nor can there be a criterion to tell whether or not a data point (\( \beta, \chi(\beta) \)) is in the scaling regime. Moreover, without knowing precise critical point one usually needs very broad range of the data for a reliable extraction of the parameters from a fit to Eq.(7). Our current investigation is free of all the drawbacks.

Our results bear striking resemblance to the results of the 2D random-bond disordered Ising ferromagnet [8]: There the data up to \( \xi \simeq 540 \) are analyzed for the similar values of
$J'$ to those here, and it is found that $\nu$ and $\gamma$ for $J' = 0.9$ are virtually the same values as in the corresponding undisordered system while they are clearly increased for $J' = 0.25$ and 0.1. It thus appears that the critical behavior of a disordered system is not characterized by the value of $\alpha_p$.

Some time ago Suzuki proposed [16] the concept of weak universality to comprise the rigorously known critical exponents of the eight-vertex model into the concept of universality. According to it the value of $\nu$ may change with the details of microscopic interaction whereas the value of $\eta$ does not, so that a (weak) universality class is characterized by the value of $\eta$, but not by the value of $\nu$ or $\gamma$. The presence of the random quenched disorder does not change the symmetry of the system although microscopic interaction changes. Thus, in view of the weak universality the value of $\eta$ remains the same value as in the corresponding undisordered system whereas both $\nu$ and $\gamma$ vary.

Some non-trivial examples of the realization of the same weak universality class would be the D-dimensional XY and Ising models, for the Z(2) group in the Ising ferromagnet is the center group of the O(2) symmetry in the XY model. It has recently been proved numerically that the value of $\eta$ in the 2D XY model is indeed $1/4$ [17] (as in the 2D Ising model), while $\eta = 0.031(4)$ and $0.026(3)$ [18] respectively for the 3D XY and Ising models being equal to each other within the statistical errors. We conjecture that the similar pattern of weak universality should manifest itself even in a disordered system with $\alpha_p < 0$: the 3D disordered XY and Ising models should be in the same weak universality class as the corresponding undisordered models, albeit $\alpha_p$ is negative (positive) in the 3D undisordered XY (Ising) models.

To conclude: We have shown that the value of $\eta$ in the random bond-disordered three-state Potts ferromagnet does not change with the strength of disorder, while both $\nu$ and $\gamma$ vary continuously. Our results disprove so-called semi-rigorous inequality, but satisfies the concept of weak universality. The features found in the model seem to be generic in any disordered ferromagnets irrespective of the value of $\alpha_p$, contrary to the conventional wisdom.
REFERENCES

[1] A.B. Harris, J. Phys. C 7, 1671 (1974)

[2] J.T. Chayes, L. Chayes, D.S. Fisher, and T. Spencer, Phys. Rev. Lett. 57, 2999 (1986)

[3] B.N. Shalaev, Sov. Phys. Solid State 26, 1811 (1984); *ibid*, Phys. Rep. 237, 129 (1994); R. Shankar, Phys. Rev. Lett. 58, 2466 (1987); A.W.W. Ludwig, *ibid* 61, 2388 (1988)

[4] J.-S. Wang, W. Selke, V.S. Dotsenko and V.B. Andreichenko, Europhys. Lett. 11, 301 (1990)

[5] S.L.A. de Queiroz and R.B. Stinchcombe, Phys. Rev. B 46, 6635 (1992); *ibid*, 50, 9976 (1994)

[6] J.-K. Kim and A. Patrascioiu, Phys. Rev. Lett. 72, 2785 (1994); *ibid*, Phys. Rev. B 49, 15764 (1994)

[7] R. Kühn, Phys. Rev. Lett. 73, 2268 (1994)

[8] J.-K. Kim, preprint, cond.- mat./9502053 (1995)

[9] M.A. Novotny, Phys. Rev. Lett. 70, 109 (1993)

[10] J.-S. Wang and D. Chowdhury, J. de Phys. (Paris) 50, 2905 (1989)

[11] T. Holey and M. Faehnle, Phys. Rev. B 41, 11709 (1990)

[12] P. Braun and M. Faehnle, J. Stat. Phys. 52, 775 (1988)

[13] W. Kinzel and E. Domany, Phys. Rev. B 23, 3421 (1981)

[14] See, e.g., F.Y. Wu, Rev. Mod. Phys. 54, 235 (1982)

[15] D.P. Landau, Phys. Rev. B 22, 2450 (1980)

[16] M. Suzuki, Prog. Theor. Phys. 51, 1992 (1974)

[17] J.-K. Kim, preprint, [hep-lat/9502002](http://arxiv.org/abs/hep-lat/9502002) (1995)
[18] J.C. Le Guillou and J. Zinn-Justin, Phys. Rev. B 21, 3976 (1980); C.F. Baillie, R. Gupta, K.A. Hawick, and G.S. Pawley, Phys. Rev. B 45, 10438 (1992)

**Figure Captions**

Figure(1): $\ln(\chi_L)$ as a function of $\ln(L)$. The slope of a straight line would correspond to $2 - \eta$. The dotted line represents the case of the exact value of $\eta$ of the undisordered system.

Figure(2): $\chi_L/\chi_\infty$ as a function of $L/\xi_L$ at certain fixed temperatures. For each $J'$, $\chi_L$ becomes equivalent to its bulk value within the statistical errors under the condition $L/\xi_L \geq 5$. Note that this condition is independent of temperature according to the standard theory of finite size scaling, that is, e.g., $\chi_L(\beta)/\chi_\infty(\beta) = q(\xi_L(\beta)/L)$.

Figure(3): $\ln(\chi(t))$ as a function of $|\ln(t)|$. The slope of a straight line is equivalent to the value of $\gamma$. Each dotted line represents the best approximation of the data obtained from the $\chi^2$ fit. It is clear that the value of the slope increases with decreasing $J'$ for $J' \leq 0.9$, and that for each $J'$ it does not vary with $t$. 