Structures for Data Processing in the Quantum Regime

Simon C. Benjamin* and Neil F. Johnson

Physics Department, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, England

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Abstract

We present a novel scheme for data processing which is well-suited for implementation at the nanometer scale. The logic circuits comprise two-state cellular units which are driven by externally applied updates, in contrast to earlier proposals which relied on ground-state relaxation. The present structures can simultaneously process many inputs and are suitable for conventional, dissipative computing in addition to classical reversible computing and quantum computing.
There has been much recent interest in the topics of computation and information processing at the nanometer scale where quantum mechanical effects can play an important role [1,2]. Theoretical design schemes have been reported for the regimes of conventional, classical computation [3–8] and, more recently, quantum computation utilizing wavefunction coherence across the entire structure [9–11]. The proposed structures usually consist of networks containing a large number of identical cellular units or ‘cells’. Various cells have been suggested, from a group of five elementary quantum dots arranged as a square of four with one in the center [3] through to individual quantum dots [4–7,11]. A drawback of such schemes is that a rather complex network of cells may be needed in order to reproduce the logical operation of a given, conventional logic circuit [3].

This letter proposes a new architecture for such cell-based computation which can be easily generalized to reproduce any conventional logic circuit. The architecture has many possible physical realizations, although we will here focus on a specific implementation where the cells consist of coupled semiconductor quantum dots (see Ref. [5] and below). Each cell has two distinct internal states, referred to as ‘0’ and ‘1’, in the energy range of interest. One important requirement of the network design is that it should be possible to change the state of a cell conditional on the cell’s current state and the states of its neighboring cells. The cells in the network are only required to be sensitive to the states of their nearest neighbors: we will take the term ‘neighbors’ to mean ‘nearest neighbors’. The cells are not required to be able to distinguish one neighbor from another. Such a requirement would reduce the range of possible physical realizations. Moreover in the particular case of the double quantum-dot implementation, the requirement that cells distinguish their neighbors effectively prohibits two-dimensional networks. It is a feature of all our designs that a cell and its neighbors are never addressed by the same update.

The general scheme employs a number of different ‘types’ of cell, where ‘type’ denotes a subset of cells with the same energy separation between ‘0’ and ‘1’. With a greater number of types one can produce more compact designs. Conversely, using more complex designs one can perform certain functions with only a single cell type; this is described more fully
Here we present patterns that represent a good ‘trade-off’ between number of cell types and the pattern’s complexity. We will employ two cell types to produce patterns which perform certain elementary functions, namely transportation of data, fanning-out (i.e. copying) of data, and the logical operations XOR and NOR. This set of components suffices to produce the cellular equivalent of any conventional logic circuit. We then proceed to show that a modified version of this cellular scheme is suitable for reversible classical computing \cite{13,14} and quantum computing. The authors have made available on the internet a Java Applet \cite{15} which allows the user to follow the time evolution of the patterns shown here, and to design new patterns.

Figure 1 shows how cells of just two types, $\alpha$ and $\beta$, can be arranged to produce the above-mentioned elementary functions. The cells may have one, two or three neighbors. Those cells having just one neighbor are referred to as ‘terminators’ since they maintain the same state (‘0’ in all cases shown here - see Fig. 1). In order that the patterns carry out their data processing functions, they must be subject to a certain repeating sequence of conditional updates. For the conditional updates we employ the notation $t \rightarrow u$, to denote the following: cells of type $w$ which are presently in state $t$ will change to state $u$ if and only if the ‘field’ is of strength $v$; the ‘field’ is defined as the number of nearest neighbors in state ‘1’ minus the number in state ‘0’. For example, $\beta_{-2} \rightarrow 0$ indicates that those cells of type $\beta$ whose current state is ‘1’ are to change their state to ‘0’ if, and only if, two more neighbors are in state ‘0’ than are in state ‘1’.

Figure 1(a) provides a pattern which acts as a wire. Suppose that at some instant the arrangement of data bits is as shown in Fig. 1(a)(i), i.e. one bit of information is represented by the state of each $\alpha$ cell which has two neighbors. We use symbols such as $x_i$ to label these data bits along the ‘wire’. After application of the update $\alpha_{0} \rightarrow 1$, followed by $\beta_{0} \rightarrow 0$ the bits will all have moved one cell to the right, so that the $\beta$ cells with two neighbors now represent the data bits and the $\alpha$ cells are all in state ‘0’. If the updates $\alpha_{-1} \rightarrow 1$, $\beta_{0} \rightarrow 0$ are now applied, the data bits will move one cell further, onto the $\alpha$ cells with three neighbors. Similarly $\alpha_{-1} \rightarrow 0$, $\alpha_{0} \rightarrow 1$, $\beta_{0} \rightarrow 0$ will move the data onto the set of $\alpha$ cells with two neighbors, so completing the cycle.
By repeatedly applying the sequence

\[
\begin{align*}
0 \rightarrow 1, & \quad 1 \rightarrow 0, & \quad 0 \rightarrow 1, & \quad 1 \rightarrow 0, \\
\beta_0, & \quad \alpha_0, & \quad \alpha_{-1}, & \quad \beta_0, & \quad \alpha_0, & \quad \alpha_{-1}
\end{align*}
\]

the stream of bits is caused to flow along the ‘wire’, progressing three cells with each repetition.

Figure 1(b) shows a pattern which performs the *copy* or *fanout* operation: one stream of data is divided into two identical copies. The patterns shown in Fig. 1(c),(d) and (e) perform the logical operations XOR, NOR and NOT, respectively. The following master sequence of updates suffices for any and all of the patterns shown in Fig. 1:

\[
\begin{align*}
0 \rightarrow 1, & \quad 1 \rightarrow 0, & \quad 0 \rightarrow 1, & \quad 0 \rightarrow 1, & \quad 0 \rightarrow 1, & \quad 0 \rightarrow 1, \\
\beta_0, & \quad \alpha_0, & \quad (\alpha_{-1}, \beta_{-3}), & \quad (\beta_0, \beta_{-2}), & \quad \alpha_0, & \quad (\alpha_{-1}, \alpha_1, \beta_{-1})
\end{align*}
\]

The brackets in the above sequence enclose sub-sequences which can be performed in any order, or simultaneously. Repeating the above sequence will cause data to flow through *any* circuit formed from the components shown in Fig. 1. It is therefore straightforward to produce a complex cellular pattern that performs any function for which a conventional logic diagram is known. In Fig. 2 we provide an example of such a circuit which performs the task of adding binary numbers. Figure 2(a) shows how the minor circuit known as a ‘half-adder’ can be built from NOT, XOR and NOR gates. Figure 2(a) also shows the corresponding cellular half-adder formed from the cellular components of Fig. 1. Note that in Fig. 2 the passive ‘terminator’ cells are drawn with dashed borders.

The upper part of Figure 2(b) shows a novel addition circuit formed from half-adders and XOR gates. In this diagram the symbols \(A_i\) (\(i = 0..2\)) denote the bits of a number \(A\) in binary form, i.e. \(A = \sum_i A_i 2^i\), and similarly for \(B\) and \(T\). When any two three-bit numbers \(A\) and \(B\) are input to this circuit, the sum \(T = A + B\) is output. The ‘depth’ of the circuit is eight gates. The lower part of Fig. 2(b) shows how the novel addition circuit can be directly translated into cellular form by replacing each gate with the appropriate cellular pattern from Fig.1. Note that in this complex pattern, terminators sometimes occur with two neighbors - despite this they still remain in state ‘0’ throughout.

An important feature of our cellular circuitry is the ability of a single circuit to simultaneously process a long sequence of inputs. Consider the XOR gate shown in Fig.
This pattern is shown containing three independent sets of bits, \([x_i, y_i], [x_{i+1}, y_{i+1}],\) XOR\((x_{i+2}, y_{i+2})\); the other patterns shown in Fig. 1 also contain three independent sets of bits. In general a circuit formed from these components with a total ‘depth’ of \(n\) cells can simultaneously process \(n/3\) sets of bits. The cellular addition circuit shown in the lower part of Fig. 2(b) has a depth of 45 cells, and at a given instant it will be processing the addition of 15 separate pairs of numbers. This property may be very advantageous for certain problems such as multi-dimensional numerical integration in which the same function must be applied to a vast number of inputs; provided that the number of inputs is much greater than \(n/3\), then the time required to process each input is just one repetition of the update sequence, regardless of the circuit depth \(n\). This property, which we refer to as ‘simultaneous sequential processing’, is distinct from (but could possibly act in addition to) the phenomenon of ‘quantum parallelism’ mentioned below.

So far we have discussed only conventional, irreversible computation. We have employed two-input, one-output gates and moreover we have used irreversible updates. These updates were of the form \(t \rightarrow u\) where \(t\) denoted the initial state and \(u\) the final state; this ability to address cells of a given state allowed us to irreversibly dissipate information. To see this, consider a simple line of cells \(\ldots \alpha \beta \alpha \beta \alpha \ldots\) in which one \(\alpha\) cell is in state ‘1’ and all the other cells are in state ‘0’. Applying the update \(1 \rightarrow 0\) \(\alpha\) will change the ‘1’ to a ‘0’ without altering any of the other cells, i.e. we will have dissipated information. We now introduce a reversible update using the notation \(w\) which means that cells of type \(w\) must NOT themselves (i.e. invert their state) if and only if the ‘field’ (defined above) is of strength \(v\). It is clear that we may always reverse the effect of such an update merely by repeating it (recall that a cell and its neighbors are never addressed by the same update). More generally we may reverse a sequence of such updates merely by repeating them in reverse order. Figure 3(a) shows a line of cells that can act as a wire when subject to reversible updates. With each single bit of data represented by the states of a pair of adjacent cells (i.e. 00 or 11), the short sequence \(\beta_0, \alpha_0\) is sufficient to move all the bits forward along the wire by two cells. It is interesting to note that this change could not have been accomplished by the dissipitive updates used.
It remains to show that the reversible updates suffice to produce a general reversible
computer. It is known [1] that a reversible computer can be constructed using just one
type of three-input, three-output gate, for example the Toffoli or control-control-not gate
(CCNOT). Therefore, we need only show that a CCNOT gate can be implemented using our
reversible updates. Figure 3(b) shows a CCNOT gate rendered in cellular form. Note that
we have introduced a third cell type $\gamma$. The following update sequence [17] will propagate
information through the gate:

$$\beta_0, \alpha_{-1}, \alpha_0, \alpha_1, \gamma_2, \alpha_0, (\alpha_3, \alpha_1), \alpha_0, \beta_0, \gamma_0, \alpha_0, (\alpha_1, \alpha_3), \beta_0, \gamma_2, \alpha_1, \beta_0, \alpha_{-1}, \alpha_0.$$ 

If the simple wire structure shown in Fig. 3(a) were subject to this sequence, the net effect
would be to propagate the data bits forward by six cells. Thus to operate a reversible
computer formed from CCNOT gates and connecting wires, we need only repeatedly apply
the above sequence of 21 updates.

The reversible classical computer can be seen as mid-point between conventional comput-
ing and ‘quantum computing’ [9,10], a relatively new paradigm that is receiving considerable
attention presently. A quantum computer (QC) would exploit ‘quantum parallelism’; the
input would be a superposition of a vast number of classical inputs. Together with the phe-
nomena of entanglement and interference, this would allow the QC to solve certain problems
[18] such as factorization exponentially faster than any classical computer. It is therefore
interesting to see if the reversible cellular architecture can be generalized to a full quantum
architecture. As noted above, a single three-bit gate such as the CCNOT is sufficient to
implement a general reversible computer. For a general quantum computer, we would re-
quire two additional one-bit gates [19] which rotate a cell’s state by $\pi/2$ about the $x$-axis
and the $z$-axis of the Bloch-sphere. The implementation of the CCNOT gate shown in Fig.
3(b) remains valid when we generalize the bits $x_i$ to ‘qubits’ $x_i = A|0\rangle + B|1\rangle$, which would
be represented by a pair of cells as $A|00\rangle + B|11\rangle$. In Fig. 3(c) we show a trivial way of
implementing one of the $\pi/2$ rotation gates by introducing a further cell-type, $\delta$. Suppose
that at some instant the two cells labeled $x_i$ in Fig. 3(c) are in the state $A|00\rangle + B|11\rangle$. 

earlier.
Now consider the effect of the following update sequence:
\[ \beta_0, \alpha_0, \delta_0, \beta_0, \delta_{-2}^x, \alpha_0, \delta_0, \beta_0, \alpha_0. \]
Immediately after the fourth update is applied, the qubit \( x_i \) will be stored on only the \( \delta \) cell, whose state will be \( A|0\rangle + B|1\rangle \). The update written \( \delta_{-2}^x \) represents a \( \pi/2 \) rotation of the \( \delta \) cell’s state about the \( x \)-axis of the Bloch-sphere. After this update the state of the \( \delta \) cell will be \( \frac{1}{2}\sqrt{2}[(A - B)|0\rangle + (A + B)|1\rangle] \). The remaining updates will move this rotated qubit three cells to the left so that it is represented by a \( \beta, \alpha \) pair in the state \( \frac{1}{2}\sqrt{2}[(A - B)|00\rangle + (A + B)|11\rangle] \). The implementation of the gate which rotates qubits about the \( z \)-axis would be exactly analogous, and would employ a fifth cell type \( \epsilon \). Finally we observe that the sequence
\[ \beta_0, \alpha_{-1}, \alpha_0, \alpha_1, \gamma_{-2}, \alpha_0, (\alpha_3, \alpha_1), \alpha_0, (\delta_0, \epsilon_0), \beta_0, (\gamma_0, \delta_{-2}^z, \epsilon_{-2}^z), \alpha_0, (\delta_0, \epsilon_0), \beta_0, (\alpha_1, \alpha_3), \beta_0, \gamma_{-2}, \alpha_1, \beta_0, \alpha_{-1}, \alpha_0. \]
will operate all of the components shown in Fig. 3 (including the \( z \)-axis rotator analogous to Fig 3(c)); hence this sequence would drive a general QC formed from these components \cite{17}.

Finally we discuss a physical realization of the cell, i.e. a bistable double quantum-dot driven through its internal states by laser pulses \cite{4}. The beam would encompass a broad area and update all cells that respond to its frequency; a single tunable laser could therefore drive an entire computer. The potential for such structures as a realization of a cellular automata has been considered in some detail in Ref. \cite{4} and will only be briefly summarized here. We will consider a cell with two neighbors since the generalization to three neighbors is straightforward. We use the notation \( x-y-z \) to refer to a cell in state \( y \) whose left and right neighbors are in states \( x \) and \( z \) respectively. The cell consists of a coupled pair of quantum dots; the low-lying single particle states are those for which the electron is localized on one or other of the dots. The lowest energy state within each of the two localizations are the physical representations of ‘1’ and ‘0’. One dot is assumed to be slightly smaller than the other, so that the two localizations are non-degenerate. The energy difference and wavefunction overlap are very small so that the rate of spontaneous decay
from one dot to the other is much longer than the total computation time. The circuitry is built up from such cells simply by producing the appropriate pattern of them. There is no tunneling allowed between the cells; they ‘feel’ the states of their neighbors via the Coulomb interaction. The Coulomb repulsion is greater for a pair of cells in the same state than for a pair in opposite states; the energy difference has a $r^{-3}$ dipole-like form where $r$ is the cell-cell separation. If the distance to a cell’s neighbor on the right is equal to the distance to its left neighbor, then the two stable energy levels for an isolated cell split into six levels (see Fig. 4). The idea of having more than one type of cell is realized by using different sizes of double-dot. The size difference shifts the double-dot’s energy levels and thus makes it possible to address one type at a time. In order to produce the desired updates of the cells, a third transient state is employed \cite{5}. This is a single-particle state in which the electron probability distribution is spread over both dots. The transient state spontaneously decays very quickly into one of the stable states. As shown in Fig. 3, an irreversible update is produced by pumping the system with light of a frequency that will excite cells of one size from a given stable state (say 1-0-0) into the transient state. The cell may decay from the transient state into either the original state (1-0-0) or the flip state (1-1-0). However, if the former occurs the electron will be re-excited by the pump, so that we can flip the state with any desired certainty ($<1$) simply by using a pulse of sufficient duration. Note that the frequency width of the pulse must be sufficiently narrow to excite from only one of the levels shown in Fig. 4, yet sufficiently broad to cover the sub-splitting (shown shaded gray) due to non-neighboring cells. For a reversible update, $\pi$-pulses can be used to switch states.

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REFERENCES

[1] R. P. Feynman, *Feynman Lectures on Computation*, Addison-Wesley 1996.

[2] S. Lloyd, Sci. Am. Special Issue *The Solid-State Century*, vol.8 no.1, 1997.

[3] P. D. Tougaw, and C. S. Lent, J. App. Phys. 75, 1818 (1994).

[4] Alexander N. Korotkov, App. Phys. Lett. 67, 2412.

[5] K. Obermayer, W. G. Teich and G. Mahler, Phys. Rev. B 37, 8096 and 8111 (1988).

[6] H. Körner and G. Mahler, Phys. Rev. B 48, 2335 (1993).

[7] S. Fussy, G. Groessing, H. Schwabl, and A. Scrinzi, Phys. Rev. A 48, 3470 (1993).

[8] S. C. Benjamin and N. F. Johnson, App. Phys. Lett. 70, 2321 (1997).

[9] D. Deutsch, Proc. Roy. Soc. Lond. A 400, 97 (1985).

[10] A. M. Steane, *Quantum Computing*, to appear in Rep. Prog. in Phys. Available from xxx.lanl.gov as quant-ph/9708022.

[11] D. Loss and D. P. DiVincenzo, Phys. Rev. A 57, 120 (1998).

[12] S. C. Benjamin and N. F. Johnson, unpublished.

[13] C. H. Bennett, IBM J. Res. Dev. 32, 16 (1988).

[14] R. Landauer, Nature 27, 779 (1988).

[15] The Applet is available at

http://cm-th.physics.ox.ac.uk/SimonB/circuit/simulator.html.

[16] We have designed this circuit to be particularly well suited for rendering in cellular form. We could equally well have exhibited a cellular form of the well known ‘ripple carry’ adder, but that adder would require twice the depth (measured in half-adders). Note that our novel adding circuit can easily be generalized to add two \( n \) bit numbers,
the depth would then be \( n \) half-adders.

[17] There may exist shorter sequences that perform the same function; the quoted sequence is the shortest that the authors have found so far.

[18] see for example, R. Cleve, A. Ekert, C. Macchiavello and M. Mosca, Proc. Roy. Soc. Lond. A, 454, 339 (1998).

[19] P. W. Shor in 37th Symposium on Foundations of Computing, IEEE Computer Society Press, p. 56 (1996).
Figure Captions

Figure 1: A set of cellular circuit components: (a) a wire before (i) and after (ii) the application of the update sequence. (b) Fan-out. (c) XOR gate. (d) NOR gate. (e) NOT gate.

Figure 2: (a) The cellular form of a ‘half-adder’ circuit. (b) Top: a novel addition circuit. Bottom: the addition circuit rendered in cellular form.

Figure 3: (a) A ‘wire’ suitable for reversible updates. (b) A Toffoli or control-control-not gate. (c) A rotation gate required for quantum computing.

Figure 4: The splitting of the energy levels of a two-neighbor cell as the states of its neighbors are specified. The shaded regions on the far right represent splitting due to the states of non-adjacent cells.
Figure 2

(a) XOR

| A | B | C | S |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

(b) NOR

A2 B2 A1 B1 A0 B0

T3 T2 T1 T0
