Split signals for a neural model of Bernoulli memristor

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Abstract. The cascade structure of behavioral models in nonlinear dynamic systems is considered. This structure includes a splitter, implemented in the form of a delay line, and a nonlinear inertialess converter in the form of a feed forward neural network. The splitter provides the unique mapping of the input signals subset into the output signals subset. It adapts the form of a mathematical model to the class of the input signals; as a result, the behavioral model becomes simpler compared with universal models for the given class of inputs. The influence of split operation on the approximation of the Bernoulli memristor operator is demonstrated on the example of the harmonic input signals. The splitter is constructed as a delay line with a unit memory, the nonlinear inertialess converter – as a two-layer neural network.

1. Introduction

The behavioral simulation of nonlinear dynamic systems (NDS) is developing in several directions [1-4]. Firstly, growing requirements for the NDS parameters and characteristics cause the need to achieve high accuracy in signal processing. Under significant nonlinearity power of the investigated objects, polynomial models become non-constructive, so neural approximators come to the fore. Secondly, the emergence of new materials and technologies leads to a variety of the mathematical NDS models. The behavioral approach allows to solve simulation tasks regardless the physical nature of the processes and phenomena being researched. Behavioral models are universal and they are used when there is either the lack of information about research objects or they are very complex, i.e. analytical and numerical methods are not constructive. Thirdly, the development of nanotechnologies leads to mathematical simulation, for instance, multi-purpose memristive devices, which, having a low power consumption, are distinguished by high-energy efficiency and resource conservation.

Memristive devices contain a memristor – the fourth passive circuit element, the resistance of which nonlinearly depends on the history of the current flowing through it. Other passive elements are the resistor, the capacitance, the inductance. The application of memristive devices can be different: neuromorphic systems, image converters, self-tuning analog-digital control devices, non-volatile storage devices, programmable logic controllers, neural networks, etc. [5-8]. In behavioral modeling, the memristive device is represented as a “black or grey box”, and its mathematical model describes the mapping of input signals into the output ones. Among universal behavioral models one can name the Volterra series [1, 2, 9], split signals polynomials [10], regression structures [4], different neural network architectures [3-5, 11]. The combination of these models is effective providing the combination of their advantages.

The paper considers the combination of the signal splitting method and the method of the NDS operator approximation based on neural networks. With the help of the input signals splitting, a
mathematical model adapts to the specified class of inputs and becomes simpler compared to universal models. Moreover, neural models allow to describe the devices of significant nonlinearity, when functional higher degree polynomials become non-constructive. To simulate the Bernoulli memristor, where current dynamic is described by the Bernoulli differential equation [12-14], the class of the harmonic input signals is split and the resulting vector is processed by feedforward neural network. The modeling errors of the Bernoulli memristor are calculated and the effect of the input signals splitting on the modeling accuracy is analyzed.

2. Splitting of input signals
The operator of a nonlinear device is represented as a composition of two operators. The first operator is a splitter, which converts a scalar input signal into a vector signal in the way that the phase portraits of vector signals do not cross, do not touch and do not pass through zero. Vector signals with the mentioned properties are split signals. The second operator, which is a nonlinear inertialess converter (NIC), maps the vector of split signals into the scalar output signal. The NIC has the form of a polynomial, fraction, neural network, etc. [10].

It is known that a splitter can be a linear dynamic circuit (LDC), for example, a delay line with different memory lengths [10]. Let us apply the delay line to split the harmonic signal often used in practice:

\[ x(A, \bar{t}) = A \sin(\omega t) = A \sin(\bar{t}), \]  

where \( A \) is an amplitude; \( \omega_0 = 2\pi / T \) is an angular frequency; \( T \) is the signal period; \( t, \bar{t} \in [0, T] \) is the continuous time variable during the signal period; \( \bar{t} = \omega t, \bar{t} \in [0, 2\pi] \) is the continuous normalized time variable. It should be noted, that when the angular frequency \( \omega_0 \) (period \( T \)) of signal (1) is changed, the normalized time variable \( \bar{t} \) is always within range \( \bar{t} \in [0, 2\pi] \).

On the base of the input signal (1) let us form three vector signals:

\[ S_1(A, \bar{t}) = [x(A, \bar{t})] = [A \sin(\bar{t})], \]  
\[ S_2(A, \bar{t}) = [A \sin(\bar{t}), A \sin(\bar{t} - \bar{t}_0)], \]  
\[ S_3(A, \bar{t}) = [A \sin(\bar{t}), A \sin(\bar{t} - \bar{t}_0), A \sin(\bar{t} - 2\bar{t}_0)]. \]

where \( \bar{t}_0 \) is the normalized delay time. The represented vector signals are the output signals of linear dynamic circuits, which are designed as the delay lines with different memory lengths. In the first case, LDC transmits the input signal (1) to its output. In the second case, LDC is a line with one delay element. In the third case, LDC is a line with two delay elements. The LDC block-schemes are shown in Figure 1, a, b, c.

\[ \begin{align*}
\text{LDC} & \quad x(A, \bar{t}) \quad x(A, \bar{t}) \\
\text{LDC} & \quad \bar{t}_0 \\
\text{LDC} & \quad \bar{t}_0 \quad \bar{t}_0 \\
\end{align*} \]

\( x(A, \bar{t}) \quad x(A, \bar{t}) \quad x(A, \bar{t}) \quad x(A, \bar{t}) \quad x(A, \bar{t}) \)

**Figure 1.** The LDC block-schemes: a – without delay; b – with one delay element; c – with two delay elements.

The described LDC is followed by the NIC. As the NIC structure we use a two-layer feedforward neural network, which is the simplest universal approximator among multi-layer feedforward neural networks. Obviously, adding hidden layers and increasing the number of neurons in layers, we improve the approximation accuracy. However, our goal is to investigate the influence of the splitting property of the input signals on the modeling accuracy, therefore, it is enough to limit ourselves to considering
the simplest approximator (a two-layer neural network) [10]. The NIC block-scheme is shown in Figure 2.

![Figure 2](image)

**Figure 2.** The block-scheme of a two-layer neural network.

The «Input» block contains vector signal in form (2) \((\gamma = 1)\), (3) \((\gamma = 2)\), or (4) \((\gamma = 3)\), i.e. \(S_{\gamma}(A,T)\), \(\gamma = 1, 2, 3\). The «Hidden Layer» block means the internal layer of the network, where the number of neurons can change, for example, from 1 to 5, neuron activation functions are hyperbolic tangents. The «Output Layer» block means the output layer of a neural network with one neuron containing the linear activation function. The obtained scalar output signal \(y_{\gamma}(t)\), relevant to the case \(\gamma = 1, 2, 3\), is saved in the «Output» block.

3. Behavioral modeling of Bernoulli memristor

The cascade connection of the splitter and the NIC is the model structure of the Bernoulli memristor, controlled by charge and excited by voltage (1) [12-14]. Under the input signal (1) and some assigned parameters, the analytical solution of the nonlinear differential Bernoulli equation is of form [13, 14]:

\[
i(t) = \frac{v(t)}{2[1 + A(1 - \cos(T))]^{1/2}},
\]

where \(v(t), i(t)\) are the voltage (input) and current (output) signals of the memristor, respectively.

Let us perform the behavioral modeling of the Bernoulli memristor at the harmonic input voltage (1), \(A = 1\), \(T \in [0, 2\pi]\). Let us construct three behavioral models with the different DLC depicted in Figure 1, a, b, c and with the NIC in the form of two-layer neural networks (Figure 2). When solving the approximation problem:

\[
\|i(T) - \tilde{y}_{\gamma}(T)\| \rightarrow \min_{\tau \in [0, 2\pi]} \text{, } \gamma = 1, 2, 3,
\]

where \(i(T)\) is the memristor output signal from equation (5), \(\tilde{y}_{\gamma}(T)\) is the output signal of the neural model, we train neural networks using the backpropagation method. To analyze the effect of the input signal splitting on the modeling errors, we estimate the uniform error:

\[
\Delta_{\gamma}(T) = i(T) - \tilde{y}_{\gamma}(T), \ n = 1, 2, ..., 628, \ \gamma = 1, 2, 3,
\]

where \(\tau_n\) is the normalized time variable, varying with a step 0.01 in the interval \(\tau_n \in [0, 2\pi]\), and the root-mean-square error:

\[
\varepsilon_{\gamma} = \sqrt{\frac{1}{Q} \sum_{q=1}^{Q} (i(T) - y_{\gamma}(T))^2}, \ Q = 628, \ \gamma = 1, 2, 3.
\]

Table 1 shows the root-mean-square errors \(\varepsilon_{\gamma}, \ \gamma = 1, 2, 3\), obtained by three two-layer neural models with different splitters and number of neurons in a hidden layer. The number \(G_{\gamma}, \ \gamma = 1, 2, 3\) of
parameters in every neural network is indicated in Table 1.

**Table 1.** Root-mean-square errors $\varepsilon_\gamma$, $\gamma = 1, 2, 3$ and number $G_\gamma$, $\gamma = 1, 2, 3$ of parameters in neural networks.

| Number of neurons in model | $S_1(A, \bar{t})$ | $S_2(A, \bar{t})$ | $S_3(A, \bar{t})$ |
|---------------------------|-------------------|-------------------|-------------------|
| 2                         | $G_1 \times 10^{-4}$ | $G_2 \times 10^{-4}$ | $G_3 \times 10^{-4}$ |
| 3                         | 4                 | 5                 | 6                 |
| 4                         | 7                 | 9                 | 11                |
| 5                         | 10                | 13                | 16                |
| 6                         | 13                | 17                | 21                |

Figure 3 demonstrates dependencies $\Delta_\gamma(t_n)$, $n = 1, 2, ..., 628$, $\gamma = 1, 2, 3$, obtained by neural models with different splitters and four neurons in the neural networks (we set this number of neurons, since the largest descent (by approximately 50 times) of the root-mean-square error occurred on turning from three neurons to four ones).
Figure 3. The uniform errors obtained by behavioral models with four neurons on the basis of: a – signal $S_1(A, \bar{T})$, b – signal $S_2(A, \bar{T})$, c – signal $S_3(A, \bar{T})$.

From the analysis of Table 1 and Figure 3, we see that:

– the signal $S_1(A, \bar{T})$ from equation (2) is considered not to be split for the harmonic input signal (1), since the neural network following the splitter is no approximator (a large modeling error does not change when increasing the number of neurons);

– the vector signal $S_2(A, \bar{T})$ from (3) and $S_3(A, \bar{T})$ from (4) are split, since the neural network following the splitter performs approximation (the descent of the root-mean-square error is observed when increasing the number of neurons);

– the splitter complexity of the sinusoidal signal does not affect the NDS modeling error. This property follows from the comparison of the errors $\varepsilon_2$ and $\varepsilon_3$ (Table 1), as well as of Figure 3, a and Figure 3, b. Obviously, to simplify the models, one should use a simpler splitter structure (the splitter with a shorter memory length, Figure 1, b). The split vector (3) and the splitter block-scheme, shown in Figure 1, b, are preferable.

4. Conclusion

The behavioral model based on the splitting method and the approximation of the nonlinear dynamic systems operators by means of neural networks have a number of advantages and greater development potential compared to polynomial models (the Volterra series, the polynomials with split signals), as well as to neural networks without the signal splitting. In fact, the splitting ensures a unique correspondence between the sets of the input and output signals of a dynamic system and adapts a mathematical model to the assigned input signals. As a result, a built model can be simpler compared to other universal behavioral models. Let us note that neural networks are attractive in a situation where increasing the polynomial power has little effect on an increase in the accuracy of the NDS modeling. These factors encourage constructing neural models with split signals.

At harmonic input signals, the cascade structure of the behavioral NDS model based on the splitter (a delay line) and the nonlinear inertialess converter (a feedforward neural network) is considered. On the example of the Bernoulli memristor, it is shown that, a simple splitter for harmonic signals is a delay line with one element. A two-layer neural network was chosen as a simple approximator. The influence of the splitting on the modeling accuracy of the Bernoulli memristor was demonstrated.

References

[1] Jing X and Lang Z 2015 Frequency domain analysis and design of nonlinear systems based on Volterra series expansion. A parametric characteristic approach (New York: Springer
[2] Ogunfunmi T 2007 *Adaptive nonlinear system identification: The Volterra and Wiener model approaches* (New York: Springer Science+Business Media) p 331

[3] Janczak A 2005 *Identification of nonlinear systems using neural networks and polynomial models. A block-oriented approach* (Heidelberg: Springer-Verlag Berlin Heidelberg) p 197

[4] Mathews V J and Sicuranza G L 2000 *Polynomial signal processing* (New York: John Wiley & Sons) p 452

[5] Corinto F and Torcini A 2019 *Nonlinear dynamics in computational neuroscience* (Cham: Springer International Publishing AG) p 152

[6] Vourkas I and Sirkoulis G Ch 2016 *Memristor-based nanoelectronic computing circuits and architectures* (Cham: Springer) p 241

[7] Abunahla H and Mohammad B 2018 *Memristor technology: synthesis and modeling for sensing and security applications* (Cham: Springer) p 106

[8] Radwan A G and Fouda M E 2015 *On the mathematical modeling of memristor, memcapacitor and meminductor* (Cham: Springer) p 231

[9] Solovyeva E 2012 *Radioelectronics and Communications Systems* **55**(8) 375–380

[10] Solovyeva E 2017 *J. of Phys. Conference Series* **803**(1) 0121156

[11] Solovyeva E 2008 *Radioelectronics and Communications Systems* **51**(12) 661–668

[12] Biolek Z, Biolek D and Biolkova V 2015 *Radioengineering* **24**(2) 369–377

[13] Ma C, Xie S, Jia Y and Lin G 2014 *Microelectronics Journal* **45**(3) 325–329

[14] Georgiou P S, Barahona M, Yaliraki S N and Drakakis E M 2012 *Proceedings of the IEEE* **100**(6) 1938–1950