Prediction Method of Electromagnetic Wave Propagation on High Sea State Based on P-M Wave Spectrum

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1. Introduction

The propagation of VHF electromagnetic waves in high sea states is mainly related to the sea surface environment and the frequency of electromagnetic waves [1]. The attenuation process of electromagnetic waves on the sea surface is an important factor affecting the prediction of communication quality and the completion of information transmission. However, due to the complex and changeable environment on the sea surface and the unpredictable wind and wave conditions, it is particularly difficult to predict the propagation process of electromagnetic waves [2]. High sea states block the offshore antennas, thereby interfering with the radiated electromagnetic waves. At the same time, because seawater has a higher dielectric constant than free space, it will absorb the electromagnetic waves grazing on the seawater, and the energy of VHF electromagnetic waves attenuates quickly in high sea states [3]. However, its advantage is that the communication speed can be better guaranteed, and the communication efficiency is greatly increased compared to the electromagnetic waves of lower frequency bands. In addition, for some small surface unmanned boats, in many cases, their communication environment is not stable [4]. When the wind and wave level is very high, whether it can complete data transmission becomes a major research topic. Therefore, it is of great significance to study the prediction process of the propagation attenuation of VHF electromagnetic waves in high sea states [5].

Based on the above analysis, the main points of work to be completed in this article are as follows: (1) Modeling the ocean waves and analyzing their statistical characteristics: using the P-M wave spectrum to model the 5th-level sea states, so that the complex sea surface environment has statistical characteristics; (2) Using the single-peak diffraction model to roughly predict the attenuation of electromagnetic wave propagation in sea state 5. A link-peak diffraction model is proposed; the attenuation value of VHF electromagnetic wave propagation under this model is calculated and compared with the prediction results of the
single-peak diffraction model, and the electromagnetic wave prediction model that is closer to the simulation results is obtained.

2. P-M Wave Spectrum Modeling and Analysis

Modeling of high sea state sea surface is an important point in the study of the electromagnetic wave propagation process. The classic method is based on wave spectrum model modeling. In the establishment of the wave spectrum model, some classic empirical spectra, such as Neumann, P-M (Pierson–Moscowitz), and JONSWAP spectrum, gradually emerged. Among them, the P-M spectrum has the advantages of sufficient growth, stable waveform, and concentrated wave energy. The P-M wave spectrum is obtained by P and M in the North Atlantic with a large amount of measured data and is widely used. It provided the theoretical basis for a series of wave spectra that appeared later. The basic formula of the P-M spectrum in the frequency domain is as follows[6]:

\[
S(\omega) = \frac{\alpha g^2}{\omega^3} \exp\left\{-\beta \left(\frac{g}{U_{19.5} \omega}\right)^4\right\},
\]

(1)

where \( g \) is the acceleration of gravity and \( U_{19.5} \) is the wind speed at 19.5 m above the sea.

By analyzing the spectrum formula of the P-M spectrum, it can be seen that the amplitude of the spectrum changes with the wind speed level, and the amplitude of the spectrum can also represent the height of the waves to a certain extent [7]. It can be seen from formula (1) that the changing trend of the frequency spectrum amplitude and the wind speed is the same. Figure 1 shows the different frequency domain changes corresponding to different wind speeds at 19.5 m on the sea surface. [8].

It can be seen that under different wind speeds, the frequency spectrum of the waves is also different. As the wind speed increases, the frequency with which the energy of the waves is concentrated decreases [9]. This is because the greater the wind speed and the greater the wave height, the greater the wavelength of the waves and the corresponding decrease in frequency.

The modeling of wave spectrum is usually divided into the linear model and nonlinear model. The nonlinear model will show the details of the waves more clearly and can better represent the changes of the waves over a period of time, but the modeling process is extremely complicated [10]. For general electromagnetic wave propagation model analysis, the linear model can also well represent the general situation of ocean waves, but the modeling process is relatively simple. This article uses the bilinear superposition method to establish the P-M spectrum wave model. The ocean wave model can be regarded as a three-dimensional model formed by superimposing many cosine waves with different frequencies [11], amplitude, direction, and other parameters on the coordinate system. It can be expressed as follows:

\[
\zeta(x, y, t) = \sum_{n=1}^{\infty} a_n \cos(k_n x + k_n y - \omega_n t + \epsilon_n),
\]

(2)

where \( \zeta \) represents the height on the sea surface at any coordinates \((x, y)\), which is a stationary uniform normal process; \( a_n = \sqrt{2S(\omega, \theta)} \delta \omega \delta \theta \) represents the amplitude of each single wave; \( k_n \) represents the wave number; \( \omega_n \) represents the frequency; and \( \epsilon_n \) represents the random phase.

It can be seen that with the increase of wind speed, the height of the waves increases, and the wavelength of the waves increases accordingly [13]. A P-M ocean wave model is established to facilitate the calculation and derivation of the propagation attenuation of electromagnetic waves under high sea states [14].

3. Electromagnetic Wave Propagation Prediction Model

Electromagnetic wave propagation prediction in high sea states is an important way to estimate the quality of maritime communication. It mainly tries to simulate the real maritime communication by predicting the attenuation value of electromagnetic wave propagation in high sea states [15]. In this article, a “link-peak diffraction model” is proposed for electromagnetic propagation in high sea states. The main idea is to regard the high sea state sea surface environment as multiple single-edge peaks with different frequencies and different heights and connect each obstacle peak to obtain a relatively accurate propagation loss value. It is based on the single-peak diffraction model, but it is constantly revised to propose the “link-peak diffraction model.” The correction process can be divided into “simple single-edge peak diffraction model,” “multi-edge peak diffraction model,” “multi-
circular peak diffraction model,” and “link-peak diffraction model.”

3.1. Simple Single-Edge Peak Diffraction Model. The single-edge peak diffraction model is a classical model in which the obstacle in the propagation process is regarded as a sharp-edged obstacle, and the equivalent height of the transmitting source, the receiving source, and the obstacle is known so as to obtain the energy loss of electromagnetic waves in the process of diffraction obstacle. However, this method ignores the details of many obstacles, and the predicted propagation loss using the approximate single peak has a large error. At the same time, the appearance of obstacles is random, and it is difficult to apply the approximate single-peak diffraction model to various situations. Figure 3 shows the single-edge peak diffraction propagation model [16].

For a single-edge peak with an ideal surface, the propagation loss is related to the peak height, the distance from the two receiving points to the peak tip, and the wavelength of the electromagnetic wave. For a model shown in Figure 4, according to Recommendation ITU-R P.526-10, the propagation attenuation can be expressed as follows [17]:

\[ J(v) = 6.9 + 20 \log \left( \frac{\sqrt{2/\lambda}}{1/d_1 + 1/d_2} \right) dB, \tag{3} \]

where \( v = h/\sqrt{2/\lambda} (1/d_1 + 1/d_2) \) is a dimensionless parameter, but it gives the relationship between the height of the peak, the wavelength of the electromagnetic wave, and the distance from the point of emission and reception to the peak.

For the high sea state approximate single-peak diffraction model, there are many places that need to be corrected: the details of the ocean waves cannot be clearly represented. To be precise, multiple connected peaks in high sea states become single-edge peaks after being enveloped, and the correlation of electromagnetic wave propagation between multiple ocean waves is lost, and there is a large error in the calculation.

3.2. Multi-Edge Peak Diffraction Model. Because the single-peak diffraction model ignores the details of the waves in the propagation process, resulting in a large error, the
single-peak edge diffraction model is revised. The basic idea is to regard each wave of the ocean wave as an edge-shaped obstacle and calculate the propagation attenuation of each single-edge peak. Combined with the statistical properties of the ocean wave spectrum, propagation attenuation at a certain distance can be obtained. Compared with the single-edge peak diffraction model, this method can better show the attenuation trend of electromagnetic waves, considering every instantaneous wave state. Figure 5 shows the equivalent model of the multi-edge diffraction model [18].

It can be seen that after the first peak, the electromagnetic wave will generate a direct component and a reflected component, and after each wave peak, these two components will be generated. Some electromagnetic waves will be absorbed by seawater, but some will continue to travel. Therefore, the attenuation of electromagnetic waves propagating on the surface of seawater at high sea state at this time will be [19].

\[
J(v) = \sum_{i=1}^{n} J(v_i) + L_{top} dB,
\]

where \(J(v_i), (i = 1, ..., n)\) represents the loss value of electromagnetic wave field strength along the surface of each peak, and \(L_{top}\) is the loss value of air propagation between each peak of electromagnetic wave [20].

Although the multi-edge diffraction model can theoretically improve the accuracy of electromagnetic wave propagation attenuation, it still needs to be pointed out that most of the ocean waves in reality are circular, and it will cause errors to be equivalent to the edge shape. This error may not be obvious in a short distance, but after the error is superimposed, a qualitative change will be produced. Therefore, based on the multi-edge peak diffraction model, Section 3.3 proposes a multi-circular peak diffraction model, which is closer to the real propagation process.

3.3. Multi-Circular Peak Diffraction Model. The multi-circular peak diffraction model is modified based on the multi-edge peak diffraction model, which is more in line with the actual electromagnetic wave propagation in high sea states. The main change is to convert the equivalent edge peak into a round peak along the wave surface. Figure 6 shows the propagation process of the multi-circular peak diffraction model. The correction of the multi-circular peak diffraction should first consider the difference between the single-edge peak and the single-circular peak in the process of diffracting propagation.

It adds the correction term \(T(m,n)\) based on the multi-edge peak diffraction model. \(T(m,n)\) is the additional attenuation due to the curvature of the ocean wave and is calculated as

\[
J(v) = J(v) + T(m,n) dB.
\]

Figure 7 is a schematic diagram of each distance parameter corresponding, where

\[
T(m,n) = 7.2m^{1/2} - (2 - 12.5n)m + 3.6m^{3/2} - 0.8m^2 dB,
\]

where

\[
m = R \left( \frac{d^4 + d^2}{d^4 + d^2} \right) \left( \frac{\pi R}{\lambda} \right)^{1/3}
\]

As \(R\) tends to zero, \(T(m,n)\) also tends to zero. Therefore, for the peak with zero radius, the correction part \(T'(m,n)\) also tends to zero, that is, the multi-edge diffraction model.

The multi-circular peak diffraction model can accurately calculate the field strength attenuation value of electromagnetic waves propagating on the sea surface. As long as the established model is fine enough, the calculation based on the multi-circular peak diffraction model can achieve a good effect. Because the model considers the details of each wave, it predicts electromagnetic wave attenuation very reliably. Similarly, it has a large amount of calculation and needs to consider the direct and reflected components of the electromagnetic wave. After reaching the next peak, these

**Figure 5:** Equivalent model of the multi-edge diffraction model.

**Figure 6:** Propagation process of the multi-circular peak diffraction model.
issues need to be reconsidered. This consumes a lot of time cost and space cost.

3.4. Link-Peak Diffraction Model. The multi-peak diffraction model is theoretically very suitable for both short wave and ultrashort wave frequency bands. However, in the short-range communication, the communication rate is a major factor affecting the completion of the maritime communication task, so the ultrashort wave VHF frequency band communication is still used as a means of communication in the communication process. Considering that the absorption ability of waves to electromagnetic waves in higher frequency bands is stronger, in order to propose a link-peak diffraction model that is easy to calculate, it is necessary to ignore the reflection effect of electromagnetic waves. When the field strength of the reflected electromagnetic wave is not in the same order of magnitude as the field strength of the direct electromagnetic wave, the influence of the reflected electromagnetic wave can be ignored. At this time, the calculation is not only simplified but the relative accuracy of the electromagnetic wave attenuation can still be ensured. Figure 8 shows a schematic diagram of the link-peak diffraction model.

For the applicability of the link-peak diffraction model, several issues need to be considered: (1) Frequency range. The electric field strength values of electromagnetic waves propagating between the peak-to-peak waves and reflected waves must not be not in the same order of magnitude in a certain VHF frequency band. The field strength of the direct wave must be much greater than the field strength of the reflected wave so that the reflected wave can be completely absorbed by the seawater. (2) The attenuation value of the direction conversion of the electromagnetic wave on the seawater surface. Although the above formulas (5)–(7) illustrates the calculation method, it does not elaborate on the attenuation of the electromagnetic wave in the process of propagation. (3) Combining P-M spectrum with five-level sea state link-peak diffraction model (take 1 km as an example). Section 4 will address these issues in detail.

4. Solutions to Problems Existing in Link-Peak Diffraction Model

4.1. Applicable Frequency Range. Figure 9 is a schematic diagram of the propagation between two peaks. Consider the apex of peak A as an emission source. The propagation of VHF electromagnetic waves in the air is equivalent to the peak-to-peak propagation in the link-peak diffraction model, and there are still many sub-paths reflected on peak B through the sea surface.

At this time, the electromagnetic waves received on peak B will be composed of the superposition of the airborne electromagnetic waves between the peaks and the reflected electromagnetic waves from the sea surface. At this time, the calculation and analysis of the received electromagnetic waves will become particularly complicated. However, considering that with the increase of the frequency of electromagnetic waves, the stronger the absorption ability of seawater to electromagnetic waves, the weaker the reflection ability of electromagnetic waves on the sea surface will be. If the frequency of the electromagnetic wave increases to a certain value, the attenuation of the airborne electromagnetic wave between the peaks is much smaller than that of the reflection from the sea surface. At this time, the influence of the reflected waves on the sea surface can be ignored, which greatly reduces the complexity of calculating the attenuation of electromagnetic wave propagation under high sea states and still maintains a high accuracy.

A set of comparative simulation experiments are set up for this purpose. The schematic diagram is shown in Figures 10(a) and 10(b).

Both sets of simulation experiments use a traditional whip antenna as the radiation source of electromagnetic waves and then three waves X, Y, and Z in the shape of three waves to simulate waves. The function of wave X is to resist the electromagnetic wave of the whip antenna directly radiating to wave Y. The peak of wave Y is the electromagnetic wave transmitting a source of the equivalent simulation experiment, and the peak of wave Z is the electromagnetic wave receiving a source of the equivalent simulation experiment. Both experiments were carried out on the sea surface. In the first set of experiments, the wave X, Y, Z are filled with the sea water, its permittivity is 81 F/m, the conductivity is 4 S/m, and the electromagnetic wave field intensity received on the wave Z is the superposition of the direct electromagnetic wave (path 1) and the reflected electromagnetic wave (path 2). In the second group of experiments, the wave X is still the seawater medium, which is to make the electromagnetic wave parameters of the equivalent emission source on the wave Y peak of the two groups of experiments the same. Between waves X, Y is filled with a medium of high permittivity, and its purpose is to
make almost no reflected wave appear when the electromagnetic wave on the wave Y peak is incident on the material. At this time, the received electromagnetic wave on the wave Z peak is only the direct electromagnetic wave. If the received field strength of the two groups of experiments is not much different (here, the reflection loss is set to be greater than 10 times the air loss), it means that the electromagnetic waves are almost completely absorbed in the material.

\[
\frac{E_2}{E_1} > 10
\]

**Table 1: Comparative test results.**

| Frequency (MHz) | The first set (V/m) | The second set (V/m) | Field strength of reflected wave \( E_1 \) (V/m) | Field strength of direct wave \( E_2 \) (V/m) | Applicable frequency \( \frac{E_2}{E_1} > 10 \) |
|----------------|-------------------|-------------------|-----------------|-----------------|-----------------|
| 30             | 95.64             | 82.35             | 13.29           | 82.35           | No              |
| 35             | 94.51             | 81.44             | 13.07           | 81.44           | No              |
| 40             | 93.25             | 80.73             | 12.52           | 80.73           | No              |
| 45             | 91.16             | 79.94             | 11.22           | 79.94           | No              |
| 50             | 89.00             | 79.00             | 10              | 79.00           | No              |
| 55             | 86.51             | 78.02             | 8.49            | 78.02           | No              |
| 60             | 83.34             | 76.97             | 6.37            | 76.97           | No              |
| 65             | 80.82             | 75.74             | 5.08            | 75.74           | Yes             |
| 70             | 77.62             | 73.12             | 4.5             | 73.12           | Yes             |
| 75             | 74.16             | 70.38             | 3.78            | 70.38           | Yes             |
| 80             | 70.32             | 67.35             | 2.97            | 67.35           | Yes             |
| 85             | 66.03             | 64.84             | 1.19            | 64.84           | Yes             |
| 90             | 61.21             | 60.62             | 0.59            | 60.62           | Yes             |
| 95             | 67.13             | 66.33             | 0.8             | 66.33           | Yes             |
| 100            | 63.21             | 62.84             | 0.37            | 62.84           | Yes             |

**Figure 10:** The experimental diagram of the applicable frequency of the link-peak diffraction model. (a) The first set of experiments. (b) The second set of experiments.

**Figure 11:** Invalid ocean waves in the link-peak diffraction model.

**Figure 12:** Schematic diagram of propagation on two circular peaks in the link-peak diffraction model.
seawater. The corresponding frequency is the applicable frequency of the link-peak diffraction model.

In the two groups of experiments, the selected frequency is 30–100 MHz, the step is 5 MHz, the distance between the peaks of wave \(X\), \(Y\), and \(Z\) is 30 m, and the distance between the emission source and wave \(X\) is 20 m. At this point, the simulation results shown in Table 1 can be obtained. The reason why the final value is 100 MHz is that we believe that electromagnetic waves above 100 MHz have poor diffraction ability and the waveform may have a great impact on the results. It is difficult to summarize the results with statistical characteristics.

At this time, it can be seen that as the frequency increases, the field strength of the reflected wave is smaller, and the received field strength of the second group of experiments is closer to the received field strength of the

![Diagram of link-peak diffraction model.](image-url)
first group. This shows that the sea surface’s ability to absorb electromagnetic waves is also increasing. The field strength received on the wave Z peak is mostly a direct wave between peak-to-peak. When the received direct wave field strength component is greater than 10 times the reflected wave field strength, it can be considered that the air propagation loss and the sea surface absorption loss are not of the same order of magnitude, and the influence of the reflected electromagnetic wave on the sea surface can be ignored. So, we can think that in 65 MHz–100 MHz, this frequency range is suitable for the link-peak diffraction model.

4.2. Attenuation Value of Electromagnetic Wave Direction Conversion on the Sea Surface. In fact, in Section 3.3, the multi-circular peak diffraction model mentioned the attenuation of electromagnetic waves with circular peak diffraction. Under the condition that the frequency requirements of the link-peak diffraction model are met, what values should the parameters in the formula take? This section is mainly to explain the field strength attenuation due to the direction change of the electromagnetic wave when it is diffracted when the electromagnetic wave propagates on the two circular peaks.

It should be emphasized that the link-peak diffraction model does not need to be considered for every wave. Its “link-peak” does not mean link-peak propagation, but peaks are linked (e.g., as shown in Figure 11).

The wave X can directly propagate to the Y wave and the Z wave. At this time, the Y wave can be ignored because the electromagnetic wave propagating through the Y wave to the Z wave needs to be reflected once. However, it has been proved in Section 4.1 that the applicable frequencies satisfying the link-peak diffraction model can ignore the influence of reflected electromagnetic waves. So, not every wave is something to consider.

Based on this consideration, Figure 12 is a schematic diagram of electromagnetic wave propagation on two circular peaks.

In the figure, the best way to propagate is along the common tangent between two adjacent waves (adjacent here refers to adjacent waves that satisfy the link-peak diffraction model). The propagation mode of a-b and c-d in the figure is the air propagation mode, and the direction attenuation of the electromagnetic wave mainly studied is the b-c section.

At this time, refer to Figure 7 and formulas (3)–(7). The falloff value along the surface can be obtained.

4.3. Five-Level Sea State Link-Peak Diffraction Model Combined with P-M Spectrum. To study the propagation of electromagnetic waves on the sea surface under high sea states, it is necessary to combine the wave spectrum. The modeling method and statistical analysis of the P-M wave spectrum have been described in detail in Section 2.

In order to calculate the attenuation of the propagation process of electromagnetic waves in high sea states, it is necessary to carry out digital simulation experiments for the process. Since there is no measurement condition for real sea state level 5, this method provides a prediction method for the propagation and attenuation process of electromagnetic waves in high sea state. The program diagram shown in Figure 13 is realized by MATLAB software.

According to the above procedure, the electromagnetic wave is chosen whose frequency is 70 MHz, 80 MHz, 85 MHz, 90 MHz, 95 MHz, and 100 MHz, and the field strength of the electromagnetic wave at the first peak is 100 V/m. The simulation results of electromagnetic wave propagation attenuation based on the P-M spectrum of the 5-level sea state can be obtained as shown in Table 2. For the convenience of comparison, the distance is set to 1 km here, because in a short distance, it is convenient to calculate the attenuation of the multi-edge peak diffraction model and the multi-circular peak diffraction model so as to compare with the link-peak diffraction model. If a longer distance is used, the amount of calculation is too large, and if a short distance is used, it can be seen that the link-peak diffraction model considers the accuracy while still maintaining the computational superiority.

It can be seen that with the increase of frequency, the absorption ability of seawater to electromagnetic waves gradually becomes stronger, the loss on the propagation path becomes larger, and the receiving field strength becomes smaller. However, the decay of several models is not the same. The simple single-peak diffraction model has the smallest attenuation because it ignores too many details, followed by the multi-peak diffraction model with less attenuation because it does not consider the additional attenuation of the circular peak. The attenuation of the link-peak diffraction model is smaller than that of the multi-peak diffraction model because a part of the energy at the

| Frequency (MHz) | Simple single-peak diffraction model (V/m) | Multi-edge diffraction model (V/m) | Multi-circular peak diffraction model (V/m) | Link-peak diffraction model (V/m) |
|----------------|------------------------------------------|----------------------------------|---------------------------------------------|----------------------------------|
| 70             | 15.84                                    | 7.63                             | 6.94                                        | 6.75                             |
| 80             | 14.61                                    | 6.84                             | 5.43                                        | 5.37                             |
| 85             | 13.54                                    | 6.12                             | 4.85                                        | 4.43                             |
| 90             | 12.21                                    | 5.34                             | 4.24                                        | 4.12                             |
| 95             | 10.19                                    | 4.98                             | 3.96                                        | 3.88                             |
| 100            | 8.36                                     | 4.16                             | 3.54                                        | 3.47                             |
receiving point is superimposed by the reflected waves, but the two models are very close. It shows that although theoretically, the multi-peak diffraction model is closer to the real value, the link-peak diffraction model can maintain relatively similar results and greatly reduce the amount of calculation.

5. Conclusions

This article mainly proposes a prediction method suitable for some VHF electromagnetic wave propagation processes in high sea states. Through the process of theoretical derivation and simulation experiments, the P-M wave model was established, and the simple single-edge peak model was continuously revised, which proved the consistency of the link-peak diffraction model. This method can be used in remote sensing technology on the sea surface; can be used in the fields of radar detection, offshore surface communication, etc; and provides a theoretical basis for quantifying the communication quality of high sea states.

However, there are still some points that can be improved in this article. For the link-peak diffraction model, its accuracy is somewhat different from the multi-circular peak diffraction model, so the applicable frequency needs to be very strictly limited. However, the theoretical derivation of the applicable frequency is too cumbersome, so this article uses a simulation experiment to obtain the applicable frequency range. In addition, for the emission source, the ideal electromagnetic wave is a plane wave or the emission source is a power source. In this article, the antenna is used as a radiation source, and the wave after the first peak can be approximately equivalent to a plane wave. It is worth mentioning that the applicability of the multi-peak diffraction model for long distances is not reflected in this article because the calculation amount of the multi-circular peak diffraction model is too large and it cannot be compared with the link-peak diffraction model. However, in short-range communication, the link-peak diffraction model can play a very good role, and, theoretically, it is still consistent in long distances. [21].

Data Availability

The data of the simulation and experimentation that support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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