Common Fixed Point problem for Classes of Nonlinear Maps in Hilbert Space

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Abstract. in this article, we present a definition of k-generalized map independent of non-expansive map and give infinite families of non-expansive and k-generalized maps new iterative algorithms. Such algorithms are also studied in the Hilbert spaces as the potential to exist for asymptotic common fixed point.

1. Introduction
We still presume That's the whole \( \mathcal{H} \) be a (R-H-S) real Hilbert space \( C \subseteq \mathcal{H} \). Fixed point Theory(FPT) takes a great deal of literature, because it provides useful tools to solve a variety of design problems in different fields. In recent years, the researchers used various iterative algorithms to estimate fixed and specific points of contractive form\[ 1\-5, 9, 10, 12\-22\]. Nonetheless, once the presence of a (FP) of some maps is known then it is not a simple task to consider a fixedpoint value, which is why we use algorithms to compute them. By the time a lot of iterative algorithms have been created and all of them can not be guarded. Recall that single-valued \( f: C \rightarrow C \) is non-expansive, if it is contraction with \( k = 1 \). Also, every multi-valued map \( B \) is said to be monotone if: \( \langle z_1 - z_2, w_1 - w_1 \rangle \geq 0 \) \( \forall z_1 \in D(B), \forall w_1 \in B(z_1) \). And it is said to be maximal-monotone(max monotone) if \( \forall (z, h) \in \mathcal{H} \times \mathcal{H} , \langle z - w, h - k \rangle \geq 0 \) and \( \forall (w, k) \in \text{gph}(B) \) then we get, \( h \in B(z) \). Consider a single-valued non-expansive map as follows: \( J_{\tau_n} = (I + \tau_n A^{-1})(x) \), which is called resolvent map where \( \langle \tau_n \rangle \) be a seq in \( R^+ \).

1.1 Lemma [7] Let \( < \alpha_n > \) and \( < \beta_n > \) are seqs of in \( R^+ \) such that \( \alpha_{n+1} \leq \alpha_n + \beta_n \), for each \( n \geq 1 \). If \( \sum_{n=0}^{\infty} \alpha_n \) converge then \( \lim_{n \rightarrow \infty} \alpha_n \) exists.

1.2 Definition: [4]
Let \( T: C \rightarrow C \) a map. Then, every \( p \in C \) is called asymptotic, fixed point(AFP) of \( T \), if \( \exists \langle x_n \rangle \) is seq in \( C \) such that \( x_n \rightarrow p \) and \( \|x_n - T(x_n)\| \rightarrow 0 \).

1.3 Lemma: [5]
If $C$ be a convex closed nonempty subset of $(R-H-S)$ and $T$ is non-expansive multi-valued map such that $\text{Fix}(T) \neq \emptyset$. Then $T$ is demi-closed, i.e., $x_n \to p$ and $\lim_{n \to \infty} d(x_n, T(x_n)) = 0$. Then $p \in T(p)$.

1.4 Lemma: [6]

If $\mathcal{H}$ be a $(H-S)$, $C$ be a convex nonempty closed subset of $\mathcal{H}$ if $(x_n)$ is a seq in $\mathcal{H}$ and $\|x_{n+1} - x\| \leq \|x_n - x\|$ for all $n \in N, x \in C$. Then $(P_C(x_n)) \to q; q \in C$.

In this article, we analyze the convergence of a new algorithm for $k$-generalization to an ACFP.

2. Main Results

We are now implementing the definition of $k$-generalized map as follows:

2.1 Definition: A map $f$ is called $k$-generalized map if for each seq $(z_n)$ in $[0,1]$ converges to 0 there exists $a$ in $\mathbb{Z}^+$:

$$||f(x) - f(w)||^2 \leq (1 - k_n)||x - w||^2 + k|\langle (x - f_x)f_x, (w - f_w)f_w \rangle|,$$

for all $k > 0$ and $x, w \in C$.

The concept of $k$-generalized map is independent of concepts contraction and non-expansive map. As shown by the examples:

2.2 Example: Consider the map $f: (0, \infty) \to (0, \infty)$ s.t $f(x) = 2x$.

the map $f$ is not contraction and not non-expansive but it is $k$-generalized map.

2.3 Example: Consider the map $f: \mathcal{H} \to \mathcal{H}$ s.t $f(x) = x$.

The map $f$ is not $k$-generalized map but it is a contraction and non-expansive at the same time.

2.4 Theorem: Let $A_1, A_2, \ldots, A_m$ be max monotone multivalued maps, $C$ convex closed nonempty in $\mathcal{H}$, $(f_n)$ is a seq of non-expansive maps and $(T_n)$ is bounded seq of $k$-generalized maps on $C$. Let $(a_n), (b_n)$ be seqs in $(0,1]$ converges to 0, such that $a_n + b_n = 1$ and $\sum_{i=1}^{m} y_{n,i} = 1$. Define the iterative algorithm $(x_n)$ as follows:

$$y_n = b_n x_n + (1 - b_n) \sum_{i=1}^{m} y_{n,i} J_{T_n}^{i} x_n$$

$$x_{n+1} = a_n [b_n T_n x_n + (1 - b_n)f_n y_n] + (1 - a_n)f_n x_n$$

If $(\cap_{n=1}^{\infty} \text{Fix}(J_{T_n}^{i})) \cap (\cap_{n=1}^{\infty} \text{Fix}(T_n)) \cap (\cap_{n=1}^{\infty} \text{Fix}(f_n)) \neq \emptyset$. Then $(x_n)$ has converges weakly to an ACFP of $T_n$, for each $n \in N$. Moreover $(P_C(x_n)) \to q; q \in C$.

Proof:

Let, $p \in (\cap_{n=1}^{\infty} \text{Fix}(J_{T_n}^{i})) \cap (\cap_{n=1}^{\infty} \text{Fix}(T_n)) \cap (\cap_{n=1}^{\infty} \text{Fix}(f_n))$
\[\|y_n - p\|^2 = \left\| \sum_{i=1}^{m} y_{n,i} f_{r_{n,i}} (x_n - p) \right\|^2 \]
\[\leq b_n \|x_n - p\|^2 + (1 - b_n) \sum_{i=1}^{m} y_{n,i} \|x_n - p\|^2 \leq b_n \|x_n - p\| + (1 - b_n)\]
\[\|x_n - p\|^2 = \|x_n - p\|^2\]

Now, for each seq \(\langle k_n \rangle\) in \([0,1]\) converges to 0, \(\exists, k\) lies in \(R^+\) s.t

\[\|x_n - p\|^2 \to \|x_n - p\|^2\]

By lemma (1.1), we get \(\lim_{n \to \infty} \|x_n - p\|\) exists and hence \(\langle f_n \rangle\) are also bounded. So by lemma (1.4) we get \(\langle P_C(x_n) \rangle\) converges strongly to the point in \(C\).

\[\|x_n - T_n x_n\| \leq \left\| \frac{b_n}{1 - a_n}(f_n)_n y_n + (1 - b_n) \right\| \frac{f_n}{1 - a_n} x_n - p \right\|^2\]
\[\leq a_n b_n \|T_n x_n - p\|^2 + (1 - b_n) \|f_n y_n - p\|^2 \leq a_n \left\| \frac{b_n}{1 - a_n}(f_n) y_n - p \right\| \|f_n x_n - p\|^2\]
\[\leq a_n b_n \|x_n - p\|^2 + (1 - b_n) \|f_n y_n - p\|^2\]
\[\leq a_n \frac{b_n}{1 - a_n} \|x_n - p\|^2 + (1 - b_n) \|x_n - p\|^2\]
\[\leq a_n \frac{b_n}{1 - a_n} \|x_n - p\|^2 + (1 - b_n) \|x_n - p\|^2\]
\[\leq a_n \|x_n - p\|^2 + (1 - b_n) \|x_n - p\|^2 = \|x_n - p\|^2\]

By lemma (1.1), we get \(\lim_{n \to \infty} \|x_n - p\|\) exists and hence \(\langle f_n \rangle\) are also bounded. So by lemma (1.4) we get \(\langle P_C(x_n) \rangle\) converges strongly to the point in \(C\).

\[\|x_n - T_n x_n\| \leq a_n \frac{b_n}{1 - a_n} \|T_n x_n - x_n - 1\| + (1 - b_n) \|f_n y_n - T_n x_n\|\]
\[\leq a_n \frac{b_n}{1 - a_n} \|T_n x_n - x_n\| + (1 - b_n) \|f_n y_n - T_n x_n\|\]

Since \(\langle f_n \rangle\) and \(\langle T_n \rangle\) are also bounded and \(a_n, b_n\) are seqs in \((0,1]\) converges to 0. As \(n \to \infty\), we get \(\|x_n - T_n x_n\| \to 0\).

Now, since \(\langle x_n \rangle\) is bounded then there exists subseq \(\langle x_{nk} \rangle\) of \(x_n\) such that \(x_{nk} \to z\) and \(\|x_n - T_n x_n\| \to 0\). Then we get,

\(z\) is an ACFP of \(T_n\), for each \(n \in N\).

Then the iterative algorithm \(\langle x_n \rangle\) has converges weakly to an ACFP of \(T_n\), for each \(n \in N\). ■

2.5 Definition: Let \(\langle T_n \rangle\) be a seq of maps. Then, we say that \(\langle T_n \rangle\) has property \(F\) if \(\langle T_n \rangle\) satisfy the condition, \(\|T_n - z\|^2 \leq \|T_n\|^2\), for each \(z \in (\cap_{n=1}^{\infty} \text{Fix}(T_n))\).

In the following theorem we study the convergence strongly of the new iteration process:
\[ y_n = \hat{b}_n \left[ \frac{a_n x_n + (1 - a_n)}{\sum_{i=1}^{m} y_{n,i} f_{r_{n,i}} x_n} \right] + (1 - \hat{b}_n) g_n x_n \]

\[ x_{n+1} = a_n [a_n T_n x_n + b_n f_n x_n + c_n g_n x_n] \\
+ \hat{b}_n [a_n b_n (T_n x_n - f_n x_n) + b_n (f_n x_n - f_n T_n x_n) + c_n a_n (f_n T_n x_n - T_n x_n) + d_n g_n y_n] \] (2.1)

where, \( \langle a_n \rangle \), \( \langle b_n \rangle \), \( \langle c_n \rangle \), \( \langle d_n \rangle \), and \( \langle a_n \rangle \) are sequences in \([0, 1]\) such that \( \langle a_n \rangle \), \( \langle b_n \rangle \) converges to 0, \( a_n \geq b_n \). Such that

1. \( a_n + b_n + c_n = 1 \), \( \sum_{i=1}^{m} y_{n,i} \) and \( d_n = a_n b_n + b_n c_n + c_n a_n = 1 \)

2. \( T_n \) has property \( \mathcal{F} \).

2.6 Theorem: Let \( A_2, A_2, \ldots , A_m \) be max monotone multi-valued maps and \( \emptyset \neq C \) closed convex in \( X \), \( \langle T_n \rangle \) is bounded, seqs. of \( k \) - generalized maps on \( C \) and \( \langle f_n \rangle \), \( \langle g_n \rangle \) are seqs of non-expansive map on \( C \). If the iteration process defined as (2.1) and \( \left( Fix(f_{r_{n,i}}) \right) \cap \left( \bigcap_{n=1}^{\infty} Fix(T_n) \right) \cap \left( \bigcap_{n=1}^{\infty} Fix(f_n) \right) \cap \left( \bigcap_{n=1}^{\infty} Fix(g_n) \right) \neq \emptyset \). Then \( \langle x_n \rangle \) has converges weakly to an ACFP of \( T_n \), for each \( n \in N \). Moreover \( \langle P_C(x_n) \rangle \to q; q \in C \).

Proof: Let \( p \in \left( Fix(f_{r_{n,i}}) \right) \cap \left( \bigcap_{n=1}^{\infty} Fix(T_n) \right) \cap \left( \bigcap_{n=1}^{\infty} Fix(f_n) \right) \cap \left( \bigcap_{n=1}^{\infty} Fix(g_n) \right) \)

\[ \| y_n - p \|^2 \leq \frac{\hat{b}_n \left[ a_n (x_n - p) + (1 - a_n) \right]}{\sum_{i=1}^{m} y_{n,i} f_{r_{n,i}} x_n - p} + (1 - \hat{b}_n) g_n x_n - p \]

\[ \leq \hat{b}_n \left[ a_n (x_n - p) + (1 - a_n) \right] \left( \sum_{i=1}^{m} y_{n,i} f_{r_{n,i}} x_n - p \right) + (1 - \hat{b}_n) \| g_n x_n - p \|^2 \]

\[ \leq b_n \| x_n - p \|^2 + (1 - a_n) \| x_n - p \|^2 + (1 - \hat{b}_n) \| x_n - p \|^2 \]

\[ \| y_n - p \|^2 = \hat{b}_n \| x_n - p \|^2 + (1 - \hat{b}_n) \| x_n - p \|^2 \]

\[ = \| x_n - p \|^2 \]

Hence, \( \| y_n - p \|^2 \leq \| x_n - p \|^2 \)

Now, by (2.1), then we have
\[\|x_{n+1} - p\|^2 \leq \alpha_n\|\alpha_n T_n x_n + b_n f_n x_n + c_n g_n x_n - p\|^2 + b_n \|a_n b_n (T_n x_n - f_n x_n) + b_n c_n (f_n x_n - T_n x_n) + c_n a_n (f_n T_n x_n - T_n x_n) + d_n g_n y_n - p\|^2\]

\[\|x_{n+1} - p\|^2 \leq \alpha_n a_n \|T_n x_n - p\|^2 + \alpha_n b_n \|f_n x_n - p\|^2 + \alpha_n c_n \|g_n x_n - p\|^2 + \alpha_n a_n \|f_n T_n x_n - T_n x_n\|^2 + \alpha_n b_n \|T_n x_n - f_n x_n\|^2 + \alpha_n a_n \|f_n T_n x_n - T_n x_n\|^2 + \alpha_n b_n \|f_n x_n - f_n T_n x_n - p\|^2 + b_n c_n \|f_n x_n - T_n x_n - p\|^2 + b_n c_n \|f_n T_n x_n - T_n x_n - p\|^2\]

\[\leq \alpha_n a_n \|T_n x_n - p\|^2 + \alpha_n b_n \|f_n x_n - p\|^2 + \alpha_n c_n \|g_n x_n - p\|^2 + \alpha_n a_n \|f_n T_n x_n - T_n x_n\|^2 - \alpha_n a_n b_n \|T_n x_n - f_n x_n\|^2 - \alpha_n a_n c_n \|f_n T_n x_n - T_n x_n\|^2 + \alpha_n a_n b_n \|T_n x_n - f_n T_n x_n\|^2 + \alpha_n b_n c_n \|f_n x_n - f_n T_n x_n - p\|^2 + b_n c_n \|f_n x_n - T_n x_n - p\|^2\]

\[\|x_{n+1} - p\|^2 \leq \alpha_n a_n \|T_n x_n - p\|^2 + \alpha_n b_n \|f_n x_n - p\|^2 + \alpha_n c_n \|g_n x_n - p\|^2 + \alpha_n a_n \|f_n T_n x_n - T_n x_n\|^2 - \alpha_n a_n b_n \|T_n x_n - f_n x_n\|^2 - \alpha_n a_n c_n \|f_n T_n x_n - T_n x_n\|^2 + \alpha_n a_n b_n \|T_n x_n - f_n T_n x_n\|^2 + \alpha_n b_n c_n \|f_n x_n - f_n T_n x_n - p\|^2 + b_n c_n \|f_n x_n - T_n x_n - p\|^2\]

By lemma (1.1), we get \(\lim_{n \to \infty} \|x_n - p\|\) exists . Hence, \(\langle x_n \rangle\) is bounded seq , so that \(\langle g_n \rangle\), and \(\langle f_n \rangle\) are also bounded seqs

So, by lemma (1.4) we deduce \(\langle P_C(x_n) \rangle\) converges strongly to the point in \(C\).

\[\|x_n - T_n x_n\| = \|a_n' \|a_n' - T_n - x_n_{n-1} + b_n' f_n x_n - f_n x_n - c_n' g_n x_n - c_n' g_n x_n\|\]

Since \(\alpha_n' \to 0\) and \(\langle T_n \rangle, \langle f_n \rangle\) and \(\langle g_n \rangle\) are bounded , then we get

\[\|x_n - T_n x_n\| \to 0 \text{ as } n \to \infty\]

Now, since \(\langle x_n \rangle\) is bounded seq then there exists subseq \(\langle x_{nk} \rangle\) of \(\langle x_n \rangle\) such that \(x_{nk} \to z\) and since \(\|x_n - T_n x_n\| \to 0\) , then we get,

\(z\) is ACFP of \(T_n\), for all \(n \in N\).

Then the iteration process \(\langle x_n \rangle\) has converges weakly to an ACFP of \(T_n\), for all \(n \in N\). ■

In the following theorem we give a new iterative algorithms and we study the convergence for this algorithms to an asymptotic common fixed point.
2.7 Theorem: If \( \langle f_n \rangle \) be a seq of non-expansive maps on \( C \) and \( \langle T_n \rangle \) be a bounded sequence of \( k \)-generalized maps on \( C \). Define the algorithm \( \langle x_n \rangle \) as follows:

\[
y_n = a_n f_n x_n + (1 - a_n) (b_n^* T_n x_n + (1 - b_n) f_n T_n x_n)
\]

\[
x_{n+1} = b_n \left[ a_n \sum_{i=1}^{m} y_{n,i} f_{n,i} x_n + (1 - a_n) f_n x_n \right] + (1 - b_n) g_n f_n y_n
\]

where \( (a_n), (b_n), (a_n), (b_n) \) are seqs in \([0,1]\) such that \( a_n + b_n \leq 1 \). If \( \bigcap_{n=1}^{\infty} \text{Fix}(f_{n,i}) \cap \bigcap_{n=1}^{\infty} \text{Fix}(T_n) \cap \bigcap_{n=1}^{\infty} \text{Fix}(f_n) \neq \emptyset \). Then the iterative algorithms \( \langle x_n \rangle \) has converges weakly to an ACFP of \( T_n \), for all \( n \in \mathbb{N} \).

Proof: Let \( p \in \left( \text{Fix}(P_c) \right) \cap \left( \bigcap_{n=1}^{\infty} \text{Fix}(T_n) \right) \cap \left( \bigcap_{n=1}^{\infty} \text{Fix}(f_n) \right) \)

Since \( y_n = a_n f_n x_n + (1 - a_n) (b_n^* T_n x_n + (1 - b_n) f_n T_n x_n) \) then we have,

\[
\|y_n - p\|^2 \leq a_n \|T_n x_n - p\|^2 + (1 - a_n) \left[ b_n \|T_n x_n - p\|^2 + (1 - b_n) \right]
\]

\[
\|y_n - p\|^2 \leq a_n \|x_n - p\|^2 + (1 - a_n) \left[ b_n \|T_n x_n - p\|^2 + (1 - b_n) \right]
\]

\[
\leq a_n \|x_n - p\|^2 + (1 - a_n) \left[ b_n \|T_n x_n - p\|^2 + (1 - b_n) \right]
\]

\[
= a_n \|x_n - p\|^2 + (1 - a_n) \|T_n x_n - p\|^2 \text{for each } \langle z_n \rangle \text{ in } [0,1] \text{ converges to } 0 \exists k \text{ lies in } R^+ \text{s.t}
\]

\[
\|y_n - p\|^2 \leq a_n \|x_n - p\|^2 + (1 - a_n) \left( (1 - k_n) \|x_n - p\|^2 + k \right) (\|x_n - f_{n,i} x_n\| (p - f_n) f_n)
\]

\[
\|y_n - p\|^2 \leq a_n \|x_n - p\|^2 + (1 - a_n) \|x_n - p\|^2 = \|x_n - p\|^2
\]

\[
\|x_{n+1} - p\|^2 \leq b_n \left[ a_n \sum_{i=1}^{m} y_{n,i} f_{n,i} x_n - p \right] + (1 - a_n) \|f_n x_n - p\|^2 + (1 - b_n) \|g_n f_n y_n - p\|^2
\]

\[
\|x_{n+1} - p\|^2 \leq b_n \left[ a_n \sum_{i=1}^{m} y_{n,i} \|x_n - p\|^2 + (1 - a_n) \|x_n - p\|^2 \right] + (1 - b_n) \|f_n y_n - p\|^2
\]

\[
\leq b_n \|a_n\| \|x_n - p\|^2 + (1 - a_n) \|x_n - p\|^2 + (1 - b_n) \|y_n - p\|^2
\]

\[
\leq b_n \|x_n - p\|^2 + (1 - b_n) \|x_n - p\|^2 = \|x_n - p\|^2
\]

By lemma (1.1), we get \( \lim_{n \to \infty} \|x_n - p\| \) exists

Hence, the algorithm \( \langle x_n \rangle \) is bounded seq. So \( \langle f_n \rangle \) and \( \langle g_n \rangle \) also bounded sequences. So, by lemma (1.4) we deduce \( \langle P_c(x_n) \rangle \) converges strongly to the point in \( C \). Now,
As we get \( \|x_n - T_n x_n\| \rightarrow 0 \), Since \( \langle x_n \rangle \) bounded seq then there exist \( \langle x_{nk} \rangle \) subseq of \( \langle x_n \rangle \) such that \( x_{nk} \rightarrow z \). Since \( \|x_n - T_n x_n\| \rightarrow 0 \), Then we get, 

\[ z \] is an asymptotic-common fixed of \( T_n \), for each \( n \in N \). And hence, 

the algorithm \( \langle x_n \rangle \) converges weakly to an asymptotic-common fixed point of \( T_n \), for each \( n \in N \).

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