Stabilization of energetic-ion driven toroidal Alfvén eigenmode by energetic electrons in tokamak plasmas

Jialei Wang¹,², Yasushi Todo², Hao Wang², Zheng-Xiong Wang¹ and Malik Idouakass²

¹ Key Laboratory of Materials Modification by Laser, Ion, and Electron Beams (Ministry of Education), School of Physics, Dalian University of Technology, Dalian 116024, China
² National Institute for Fusion Science, National Institutes of Natural Sciences, Toki 509-5292, Japan

E-mail: zxwang@dlut.edu.cn

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Abstract

Energetic electron effects on an energetic-ion driven toroidal Alfvén eigenmode (TAE) are investigated via hybrid simulations of an MHD fluid interacting with energetic particles. Both energetic electrons and energetic ions described by drift-kinetic equations are included in the present work. It is found that the TAE can be effectively stabilized by off-axis peaked energetic electrons which are located near the mode center, while the centrally peaked energetic electrons fail to stabilize the mode. It is confirmed that the spatially localized pressure profile of energetic electrons causes the stabilization of TAE. The stabilized TAE has a more localized mode structure accompanied by a significant reduction in the energetic ion driving rate. The small change of mode frequency and dissipation rate indicate the stabilization mechanism is different from the so-called pressure gradient stabilization that drives the TAE into continuum. The results suggest that the strong plasma non-uniformity induced by the energetic electron beta profile may be responsible for the change of mode structure. It is also found that this stabilizing effect is more effective for a high-\(n\) TAE. Moreover, it is numerically verified that the positive (negative) pressure gradient at the TAE center will increase (decrease) the mode frequency. The wave-particle interactions are also analysed for a case with energetic electrons peaked at the inner side of the TAE center. It is found that the power transfer to a resonant barely trapped energetic electron, which taps energy from the wave, can be comparable to the power transfer from a resonant energetic ion. This suggests that if a sufficient number of resonant barely trapped electrons are present, they might stabilize energetic-ion driven TAE through the wave-particle interaction.

Keywords: energetic electron, energetic ion, toroidal Alfvén eigenmode, Alfvén eigenmode control, wave-particle interaction

(Some figures may appear in colour only in the online journal)
global mode structure and a wide spectrum of unstable toroidal mode numbers [5–9]. In present day experiments, TAE destabilized by energetic beam ions are found that can induce a loss of up to 50% of the injected beam power and the lost beam ions could damage the first wall [10]. It is therefore crucial to mitigate or even fully suppress the dangerous Alfvénic activities.

In recent years, significant efforts have been devoted to the development of AE control techniques both in tokamaks and stellarators. The main AE control techniques based on AE drive and damping mechanisms, including electron cyclotron resonance heating (ECRH) [11–14], electron cyclotron current drive (ECCD) [15, 16], externally applied resonant magnetic perturbation (RMP) [17, 18] and variable neutral beam injection (NBI) [19, 20], are briefly reviewed in a recent paper [18]. Among these techniques, electron cyclotron (EC) wave is a unique and very promising tool, because the localized EC wave can be flexibly and precisely targeted to AEs which can be excited at a wide range of radial positions. However, effects of EC wave on AEs are quite complicated. In recent DIII-D reversed shear discharges [21], more unstable RSAEs with ECRH near the magnetic shear reversal point ($q_{\text{min}}$) were observed, in contrast to the original experiments where the unstable RSAE were significantly suppressed by the similar ECRH [11]. In ASDEX-Upside (AUG) experiments [12], the full suppression of RSAE by ECRH was observed in a reversed shear discharge, while, on the other hand, the TAE was facilitated by off-axis deposited ECRH in a discharge with monotonic $q$-profile. These seemingly contradictory results are due to the fact that both AE drive and damping would be significantly affected by the EC wave [21], including (i) the increase of electron temperature $T_e$, which will enhance radiative damping, thermal ion Landau damping, but also increase the energetic ion slowing-down time; (ii) the increase of plasma density, which will change the value of Alfvén velocity and affect the wave-particle resonance; (iii) modification of the $q$-profile, which will change the continuum damping and particle distribution; (iv) EC wave generated energetic electrons, which can interact with AEs [22, 23]. Moreover, the modification of local pressure gradient and magnetic shear by EC wave will directly affect the existence of some AEs, such as reversed shear Alfvén eigenmode (RSAE) [24–26] and TAE [27–30]. The competition of those effects will determine the final state of AEs. Among various effects, the influence of energetic electrons generated by EC wave is an unexplored area, and one worthy of investigations.

In this work, we numerically investigate the energetic electron effects on an energetic-ion driven TAE with a single toroidal mode number. Simulations were performed with an extended version of MEGA code which is a hybrid simulation code for a magnetohydrodynamic (MHD) fluid interacting with energetic particles. In the extended version of MEGA, both energetic ions and energetic electrons described by drift-kinetic equations [31] are included. Equilibria with both on-axis and off-axis peaked energetic electron beta profiles are considered. It is found that energetic electrons with a centrally peaked beta profile have only a little effect on the stability of energetic-ion driven TAE with a toroidal mode number $n = 4$, while the TAE can be significantly stabilized by off-axis peaked energetic electrons. After checking the pressure profile effects of energetic electrons and power transfer of energetic particles, we confirm that the spatially localized energetic electron pressure profile, which is peaked near the TAE center, causes the mode stabilization. A comparison of energetic electron stabilizing effect on TAEs with high toroidal-mode-number ($n = 12$) and low toroidal-mode-number ($n = 2$) is also conducted. Besides, we analyse the wave-particle resonance between energetic electrons and TAE. The results show that energetic electrons may tap considerable energy from TAE, if sufficient resonant barely trapped energetic electrons can be loaded and the peak position of energetic electrons is at the inner side of the mode center. The remainder of this paper is organized as follows. In section 2, the simulation model is introduced. In section 3, the numerical results are presented. In the final section 4, we summarize and discuss the results.

### 2. Physical model

Several hybrid simulation models have been constructed to study the evolution of AE modes destabilized by energetic particles [6, 32–35]. In the MEGA code, the bulk plasma is described by the non-linear MHD equations and the energetic particles described by drift-kinetic equations are simulated with the particle-in-cell method [35–37]. In this work, the MHD equations with both energetic-ion and energetic-electron effects are given by

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho v) + \nu_\alpha \Delta (\rho - \rho_{eq}), \quad (1)
\]

\[
\rho \frac{\partial v}{\partial t} = -\rho \omega \times v - \rho \nabla \left( \frac{v^2}{2} \right) + (j - j_e - j_w) \times B - \nabla p + 4 \left( \frac{\rho}{3} \nabla \cdot v \right) - \nabla \times (\nu \rho \omega), \quad (2)
\]

\[
\frac{\partial p}{\partial t} = -\nabla \cdot (pv) - (\gamma - 1) p \nabla \cdot v + (\gamma - 1) \left[ \frac{\nu \rho \omega^2}{2} + 4 \frac{\nu p \nabla \cdot v}{3} + \eta \left( j - j_{eq} \right) \right] + \chi \Delta (p - p_{eq}), \quad (3)
\]

\[
\frac{\partial B}{\partial t} = -\nabla \times E, \quad j = \frac{1}{\mu_0} \nabla \times B, \quad (4)
\]

\[
E = -v \times B + \eta \left( j - j_{eq} \right), \quad (5)
\]

\[
\omega = \nabla \times \nu, \quad (6)
\]

where $\mu_0$ is the vacuum magnetic permeability, $\gamma = 5/3$ is the adiabatic constant, $\nu$, $\nu_\alpha$ and $\chi$ are artificial viscosity and diffusion coefficients chosen to maintain numerical stability, and all the other quantities are conventional. The dissipation terms
(viscosity, resistivity, and diffusivity) play a physical role to enhance the damping of AEs in the MHD simulation that includes continuum damping [38–40], but does not include kinetic damping such as radiative damping [41] and thermal ion Landau damping. The subscript ‘eq’, ‘hr’ and ‘hr’ denote the equilibrium variables, the energetic ion and the energetic electron, respectively. In this work, we use the same value of the dissipation coefficients, $1 \times 10^{-6}$ normalized by $v_A R_0$ where $v_A$ is the Alfvén velocity at the plasma center, and $R_0$ is the major radius at the geometrical center of the simulation domain. The energetic particle contribution is included in the MHD momentum equation (equation (2)) as the energetic particle current density without $\mathbf{E} \times \mathbf{B}$ drift due to the quasi-neutrality [35]. This model is accurate under the condition that the total energetic particle density is much less than the bulk plasma density. The MHD equations are solved using standard 4th order Runge-Kutta and finite difference schemes.

The MEGA code participated in the code benchmark of the Energetic Particle Physics Topical Group of the International Tokamak Physics Activity. Good agreements were found in the spatial profile, mode frequency and growth rate of a TAE among nine codes including MEGA [42]. The MEGA code has been applied to and validated on the energetic-particle-driven instabilities in several devices, such as DIII-D, JT-60U and LHCB [43–47].

We employ the tokamak equilibrium and the physical condition used in [23]. MHD fields and kinetic particles are both evolved in right-handed cylindrical coordinates $(R, \varphi, Z)$. Spatial profiles of energetic-ion beta $\beta_{\text{EI}}$, energetic-electron beta $\beta_{\text{EE}}$ and safety factor $q$ are shown in figure 1. Both centrally peaked and off-axis peaked energetic electron beta profiles are investigated. It should be noted that a uniform bulk plasma beta profile with an initial value of 1% is adopted in the present work. The isotropic Maxwellian distribution function, $f_0 = C(m_0 / T_0)^{3/2} e^{-m_0 v^2 / 2 T_0}$, is used as the initial velocity-space distribution of energetic ions and energetic electrons. $C$ is an integration constant. The uniform temperature $T_0 = 0.3$ is adopted for both energetic ions and energetic electrons, which is normalized by $m_D v_A^2$, where $m_D$ is the mass of energetic deuterion. The normalized mass of the energetic particle $m_{\text{h}}$ is set to be 1 for the energetic ion and 1/3672 for the energetic electron. In the present work, we focus on ‘single-n’ simulations, where energetic particles drive only one toroidal mode number $n$. Energetic electron effects on the stability of $n = 4$ TAE which is an exact solution of the equations of a quarter of the tokamak domain, are systematically studied. In the simulation of $n = 4$ mode, only the toroidal angle from $\varphi = 0$ to $\varphi = \pi/2$ is used, while toroidal angle from $\varphi = \pi/2$ to $\varphi = 2\pi$ are obtained by periodic extension. This simplification is made to save computational resources and time compared with a full torus simulation. Energetic electron effects on $n = 2$ and $n = 12$ TAEs are also investigated. When we simulate $n = 2$ and $n = 4$ TAEs, the number of grid points for the cylindrical coordinates $(R, \varphi, Z)$ is $128 \times 16 \times 128$, and the number of computational particles is $2.1 \times 10^6$ for each species. When we simulate $n = 12$ TAE, the number of grid points in $R$ and $Z$ directions is doubled leading to $256 \times 16 \times 256$ grid points for $(R, \varphi, Z)$, and the number of computational particles is increased to $8.4 \times 10^6$.

3. Simulation results

3.1. Weak effects on TAE stability with energetic electrons peaked on-axis

In this subsection, we investigate the effects of on-axis peaked energetic electrons on the stability of an energetic-ion driven
Figure 3. (a) Alfvén continuous spectra for $n = 4$. (b)–(d) Radial velocity fluctuation profiles of energetic-ion driven $n = 4$ mode with energetic electron central beta $\beta_{EE0} = 0\%$, 1.5\% and 2.5\%, respectively. Frequencies and spatial locations of the $n = 4$ modes with different $\beta_{EE0}$ are shown in panel (a) with the horizontal lines.

$\beta_{EI0} = 2.0\%$ for cases (I)–(IV).

Figure 4. Kinetic energy evolution of energetic-ion driven $n = 4$ TAE for cases (I)–(IV). The central value of on-axis peaked energetic ion beta profile is $\beta_{EI0} = 2.0\%$ for cases (I)–(IV).

$\beta_{EE0} = 0\%$, the mode is a TAE driven by energetic ions with two major harmonics $(m, m + 1)/n = (5, 6)/4$ and the mode frequency lies in the TAE gap as shown in figure 3. In figure 2, we see that the frequency

Figure 5. Evolution of phase space integrated power transfer from energetic ions (blue curve) and energetic electrons (red curve) to $n = 4$ TAE in case (II).
of energetic-ion driven mode decreases and the linear growth rate changes slightly with $\beta_{EE}$ increases. When $\beta_{EE}$ exceeds a critical value around 3%, the linearly most unstable mode will be changed from an energetic-ion driven mode to an energetic-electron driven mode which propagates in the electron diamagnetic drift direction with negative sign of mode frequency, as shown in figure 2. As a wide energetic electron beta profile is adopted, the increase of $\beta_{EE}$ will also increase the pressure gradient at the TAE center where $q = (5 + 1/2)/4$, which will decrease the TAE frequency [27]. However, over the whole range of $\beta_{EE}$ considered, there is no obvious stabilization of energetic-ion driven TAE observed.

The continuous spectra and spatial profiles for the unstable modes in figure 2 with $\beta_{EE} = 0\%$, 1.5% and 2.5% are shown in figure 3. The slow sound approximation [48] is adopted for
the Alfvén continua analysis. For the case with $\beta_{\text{EE}_0} = 1.5\%$, the mode frequency decreases from $\omega_0 = 0.363\omega_A$ to $\omega_0 = 0.325\omega_A$ and the linear growth rate increases by about 25%, compared to the case without energetic electrons. The change of wave-particle resonance condition due to the decrease of mode frequency and the extension of radial mode structure of wave-particle resonance condition due to the decrease of compared to the case without energetic electrons. The change

As shown in figure

Scan of
electron beta profile peaked at $r_h/a = 0.54$, accompanied with a rapid variation of the sine part at the place where the mode intersects with the continuum. This rapid phase variation of spatial profiles corresponds to the continuum damping in the resistive MHD model used in MEGA [50].

It should be noted that the critical central beta value of energetic electrons for mode conversion closely depends on the initial distribution of energetic electrons. In our previous work with similar equilibrium and physical conditions, we have shown that the TAE-type mode propagating in the electron diamagnetic drift direction is mainly driven by deeply trapped energetic electrons [23]. Therefore, if an anisotropic Maxwellian distribution with peak pitch angle $\Lambda_{\text{peak}} = 1.0$, where pitch angle variable is defined as $\Lambda = \mu_{\text{B}_\text{D}_E}$ with $B_0$ magnetic field strength at the magnetic axis, is assumed for energetic electrons, the critical central beta value will be significantly reduced compared to the present case using an isotropic Maxwellian distribution function.

3.2. Stabilization of TAE with energetic electrons peaked off-axis

We have run several cases in order to clarify the effects of off-axis peaked energetic electrons on the energetic-ion driven TAE. The energetic electron beta profile at a value of 1.0% denoted by red solid curve in figure 1 is used. Firstly, two cases are carried out: (I) with only centrally peaked energetic ions, and (II) with both centrally peaked energetic ions and off-axis peaked energetic electrons. Case (I) is the baseline case with an energetic-ion driven $n = 4$ TAE. In case (II), the energetic electrons are loaded at $\omega_A t = 200$ and the equilibrium used for simulation is recalculated after the energetic electron loading. Time evolutions of kinetic energy for these two cases are shown in figure 4. A reduction of up to 50% in the mode linear growth rate is observed after the consideration of off-axis peaked energetic electrons. Figure 5 shows the evolution of power transfer $P_w$ from wave to energetic particles in the linear phase. The particles drive the mode when $P_w < 0$, and vice
versa. It is found that the power transfer from energetic electrons oscillates around zero and the net power transfer is much smaller than the energetic-ion drive, which indicates a weak wave-particle interaction. Therefore, the significant reduction of linear growth rate is not due to the energy transfer from TAE to energetic electrons.

Another possibility for mode stabilization is the beta profile of energetic electrons. In order to verify this, only the energetic electron beta profile is retained in the simulation, which is treated as part of the MHD fluid. Then, the other two cases are conducted: (III) with centrally peaked energetic ions and the off-axis peaked energetic electron beta profile shown in figure 1, and (IV) with centrally peaked energetic ions and a uniform energetic electron beta profile at a value of 1%. The corresponding time evolutions of kinetic energy for cases (III) and (IV) are shown in figure 4. Case (III) with off-axis peaked energetic electron beta profile shows a similar reduction of mode linear growth rate as case (II), while the uniform energetic electron beta profile in case (IV) has little effect on the mode stability. Figure 6 shows the linear structures of $n = 4$ mode in cases (I), (II) and (III). The inclusion of drift-kinetic energetic electrons in case (II) and the adoption of energetic electron beta profile in case (III) show almost the same mode structures, where the poloidal harmonics $m \geq 6$ are significantly damped leading to a more spatially localized mode structure. These results indicate that the stabilization of the energetic-ion driven TAE is caused by the off-axis peaked energetic electron beta profile.

Linear results of the unstable $n = 4$ mode in case (I) and (III), including mode frequency $\omega_0$, growth rate $\gamma_L$, energetic-ion driving rate $\gamma_h$, mode damping rate $\gamma_d$ and parallel component of mode kinetic energy $E_{kin}^\parallel$, are listed in table 1. The mode frequency changes little after the consideration of off-axis peaked energetic electron beta profile due to the fact that the peak position $r_h = 0.43$ of energetic electron beta profile with zero pressure gradient is very close to the mode center $r/a = 0.433$. Therefore, the mode stabilization in case (III) is not caused by the so-called pressure gradient stabilization that drives mode into continuum [27, 28]. On the other hand, the increase of local beta value may raise the slow magneto-sonic wave continua and result in the coupling to the nearby shear Alfvén wave continua. Such coupling will give rise to an energy channel that contributes to the damping of Alfvénic instabilities [51]. However, the low level of parallel component of mode kinetic energy given in table 1, as well as the weak effect by a uniform energetic electron beta profile in case

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**Figure 10.** The mode frequency (circle) and linear growth rate (triangle) of $n = 4$ mode versus the peak value of energetic electron beta $\beta_{EE_0}$ (a) and the profile width of energetic electron beta $\xi_\psi$ (c). (b) and (d) Alfvén continuous spectra for $n = 4$. The peak position ($r_h = 0.43$) is held fixed for all scans. $\xi_\psi$ is held fixed at a value of 0.16 in the scan of $\beta_{EE_0}$ in panels (a) and (b), and $\beta_{EE_0}$ is held fixed at a value of 0.5% in the scan of $\xi_\psi$ in panels (c) and (d). $A_{m_6}/A_{m_5}$ in panels (b) and (d) denotes the ratio of $m = 6$ poloidal harmonic amplitude to the $m = 5$ poloidal harmonic amplitude of the $n = 4$ TAE.

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Table 1: Linear results of the unstable $n = 4$ mode in case (I) and (III), including mode frequency $\omega_0$, growth rate $\gamma_L$, energetic-ion driving rate $\gamma_h$, mode damping rate $\gamma_d$ and parallel component of mode kinetic energy $E_{kin}^\parallel$, are listed in table 1. The mode frequency changes little after the consideration of off-axis peaked energetic electron beta profile due to the fact that the peak position $r_h = 0.43$ of energetic electron beta profile with zero pressure gradient is very close to the mode center $r/a = 0.433$. Therefore, the mode stabilization in case (III) is not caused by the so-called pressure gradient stabilization that drives mode into continuum [27, 28]. On the other hand, the increase of local beta value may raise the slow magneto-sonic wave continua and result in the coupling to the nearby shear Alfvén wave continua. Such coupling will give rise to an energy channel that contributes to the damping of Alfvénic instabilities [51]. However, the low level of parallel component of mode kinetic energy given in table 1, as well as the weak effect by a uniform energetic electron beta profile in case
and trapped energetic ions in case (III) are all significantly shrunk this potential well, and then result in a narrow bound state.

The stabilizing effect of energetic electron beta profiles with different peak positions $r_h$ has been analysed. The mode frequency and the growth rate are plotted versus the peak position $r_h$ in figure 8. The peak value ($\beta_{EE} = 1.0\%$) and profile width of energetic electron beta are held fixed in the scan of $r_h$. It is found that the energetic-ion driven TAE is significantly stabilized when $r_h$ is around the mode center. For the case with $r_h/a = 0.47$, two modes with comparable linear growth rate coexist in the linear phase. One is the stabilized mode with dominant poloidal harmonics $m = 5$ and $m = 6$ centred around $r/a = 0.43$, while the other is the newly born TAE with dominant poloidal harmonics $m = 4$ and $m = 5$ centred around $r/a = 0.25$. At $r_h/a = 0.50$ and $r_h/a = 0.54$, the newly born mode becomes the dominant mode in the linear phase. The mode spatial profiles at $r_h/a = 0.54$ are shown in figure 9. For the mode frequency, it is found that the TAE frequency decreases first and then recovers when $r_h$ moves from the plasma center to the TAE center. The change of mode frequency is closely related to the variation of pressure gradient at the TAE center.

Influence of peak value $\beta_{EE}$ and profile width of the off-axis peaked energetic electron beta on the TAE stability is further investigated. The energetic electron beta profile in this work is $\beta_{EE}(r) = \beta_{EE0}e^{-(r-r_0)^2/(\xi_0)^2}$, where $\xi_0$ is a normalized spatial scale length. The peak position $(r_h/a = 0.43)$ is held fixed in the scan of $\beta_{EE0}$ and $\xi_0$. Figure 10 shows the mode frequency and linear growth rate versus $\beta_{EE0}$ and $\xi_0$. The increase of energetic electron peak value $\beta_{EE0}$ or the decrease of profile width will enhance the TAE stabilization. It is found that the $m = 6$ poloidal harmonic is gradually damped for increasing $\beta_{EE0}$ and decreasing $\xi_0$. The mode frequency remains nearly unchanged throughout the entire scan range, as $r_h$ for these cases are very close to the TAE center.

3.3. Comparison of high–n and low–n TAEs

In this subsection, we compare the energetic electron beta profile effects on high–n TAE with $n = 12$ and low–n TAE with $n = 2$. The same kind of energetic electron beta profiles with a peak value 0.5% are considered for these two modes, as shown in figure 11(a). The linear growth rates for the energetic-ion driven $n = 12$ TAE and $n = 2$ TAE in the baseline cases are $\gamma_{\ell} = 0.0305\omega_\Lambda$ and $\gamma_{\ell} = 0.0302\omega_\Lambda$, respectively.

The mode frequency and the linear growth rate of the $n = 12$ TAE versus the peak position $r_h$ of energetic electron beta profile are shown in figure 11(b). An obvious stabilization of the $n = 12$ TAE is observed when $r_h$ is located around the mode center which is $r/a = 0.417$, while the energetic electron effect on the TAE is weak when the energetic electron beta profile lies out of the mode region. At $r_h/a = 0.43$ and $r_h/a = 0.49$, the mode is strongly stabilized with a linear growth

(IV), rules out the possibility of such stabilization mechanism. It can be seen in table 1 that the significant reduction of linear growth rate is more related to the decrease of energetic-ion driving rate $\gamma_0$, rather than the increase of mode damping rate $\gamma_\ell$. The driving rate for energetic ions is shown in figure 7. It should be noted that isotropic velocity space distribution is adopted for energetic ions in the present work. Then co-passing, counter-passing and trapped energetic ions will account for the mode destabilization in both case (I) and case (III). The driving rates for co-passing, counter-passing and trapped energetic ions in case (III) are all significantly decreased by 39%, 22% and 31%, respectively. The decrease of driving rate occurs throughout the region of energetic electron beta profile, as shown in figure 7(b). Therefore, it seems most likely that the strong plasma non-uniformity induced by the spatially localized energetic electron beta profile shrinks the TAE spatial profiles and leads to the TAE stabilization in case (III). Specifically, for TAE, also known as a ‘gap mode’, that resides in an effective potential well provided by the radial non-uniformity of background plasmas and the equilibrium geometry [2, 3, 18], we speculate that the energetic electron beta profile adopted in case (III) would significantly shrink this potential well, and then result in a narrow bound state.

![Figure 10](image_url)

**Figure 10.** Spatial profiles of energetic electron beta with the same peak value $\beta_{EE0}(0.5\%)$, but different peak positions $r_h$. (a) The radial velocity fluctuation profiles of energetic-ion driven $n = 12$ TAE without energetic electron beta profile. The grey dotted line in panel (b) indicates the radial position of $q = (14 + 1/2)/12$ rational surface. The central value of on-axis peaked energetic ion beta profile is $\beta_{EE0} = 2.0\%$.**
rate $\gamma_L = 0.0142 \omega_A$. The mode frequency decreases, as $r_h$ approaches the mode center. A sudden frequency jump from $\omega_0 = 0.311 \omega_A$ at $r_h/a = 0.48$ to $\omega_0 = 0.415 \omega_A$ at $r_h/a = 0.49$ is found. It is also noticed that the mode frequency for the case with $r_h/a < 0.49$ is always smaller than 0.384 $\omega_A$, which is the mode frequency for the baseline case, while the the mode frequency for the case with $r_h/a \geq 0.49$ is higher than 0.384 $\omega_A$.

Spatial profiles and Alfvén continuous spectra for the $n = 12$ mode with different $r_h$ are shown in figure 12. TAE in the baseline case has two major harmonics ($m, m + 1$) $n = (14, 15)/12$, while other poloidal harmonics with $m \geq 16$ also have finite amplitudes, as shown in figure 12(b). It is found that the spatial profiles become localized when $r_h$ is around the mode center. At $r_h/a = 0.40$, the poloidal harmonic $m = 17$ is weakly damped and the mode frequency is significantly decreased due to a strong energetic electron pressure gradient at the mode region. When $r_h$ is located at $r_h/a = 0.43$, which is the TAE center of the baseline case, the mode shows two dominant poloidal harmonics $m = 15$ and $m = 16$ centring at $q = (15 + 1/2)/12$ rational surface, while the other poloidal harmonics, including $m = 14$, are significantly damped. Therefore, the mode frequency at $r_h/a = 0.43$ is still significantly reduced with experiencing a strong energetic electron pressure gradient at the mode center. At $r_h/a = 0.465$, the radial location of the mode further moves outwards. When $r_h$ is located at $r_h/a = 0.49$, which is close to the rational surface $q = (15 + 1/2)/12$, a newly born TAE lies out of the energetic electron beta region with the dominant poloidal harmonics $m = 13$ and $m = 14$ in the linear phase. From $r_h/a = 0.49$ to $r_h/a = 0.575$, the dominant poloidal harmonics of the mode remains $m = 13$ and $m = 14$. If $r_h$ further moves outwards, the mode will gradually recover to the TAE in the baseline case.

Figure 12. (a) Alfvén continuous spectra for $n = 12$. (b) Radial velocity fluctuation profiles of energetic-ion driven $n = 12$ mode without energetic electron beta profile. (c) and (f) Radial velocity fluctuation profiles of energetic-ion driven $n = 12$ mode with different energetic electron peak positions $r_h$. Frequencies and spatial locations of the $n = 12$ modes with different $r_h$ are shown in panel (a) with the horizontal lines.
The central value of on-axis peaked energetic electron beta profile is $\beta_{EE} = 0.5\%$. It should be noted again that the peak value of pitch angle width $\Delta\Lambda$ is located around the passing-trapped boundary, and the pitch angle width $\Delta\Lambda$ is set to be 0.2.

Time evolutions of kinetic energy for energetic-ion driven TAEs before and after including energetic electrons are shown in figure 15. The mode evolutions are very similar for the cases with kinetic energetic electrons and with the energetic electron beta profile, which indicates a weak wave-particle interaction between energetic electrons and the energetic-ion driven TAE. The slight increase of mode growth rate after the inclusion of energetic electrons is related to the energetic electron beta profile. TAE enhancement in figure 15 is very similar to the cases with centrally peaked energetic electrons discussed in section 3.1, where energetic electron beta profile exerts a negative pressure gradient at the TAE center and the mode frequency is decreased. As a result, the mode spatial width and the wave-particle interactions are changed, leading to the increase of TAE growth rate here. Figure 16 shows the resonance and the power transfer between energetic electrons/ions and TAE in the case with kinetic energetic electrons. The resonance condition for a low frequency wave (mode frequency much less than the cyclotron frequency) in a torus is

$$\omega_0 - L\omega_B - n\omega_\varphi = 0,$$

where $\omega_0$ is the mode frequency, and integer $L$ is the resonance number. $\omega_B$ and $\omega_\varphi$ are particle poloidal and toroidal orbit frequency, respectively. The resonance number $L$ of trapped energetic electron is less than 1 due to the large poloidal orbit frequency, and most of trapped energetic electrons have a resonance number $L \geq 0.05$, as shown in figure 16(a). It should be noted that the trapped energetic electrons with the resonance number very close to 0, such as $L = 0.00$ and $L = 0.01$, can resonate with the TAE, and particles with a relatively large resonance number, such as $L = 0.05$, are not resonant particles although $L = 0.05$ seems close to 0. As seen in figure 16(b), the evolutions of $\delta f$ for trapped energetic electrons with $L = 0.05$ and $L = 0.14$ oscillate around 0, while the absolute value of $\delta f$ for trapped energetic electrons with $L = 0.00$ grows continuously. Therefore, the previous results suggest that barely trapped energetic electrons may resonate with an energetic-ion driven TAE which also propagates in the ion diamagnetic drift direction. The detailed resonance and power transfer between energetic electrons and energetic-ion driven TAE are discussed in this subsection.

The equilibrium safety factor remains unchanged, $q = 1 + 2(r/a)^2$. In the baseline case, the $n = 4$ TAE driven by energetic ions with dominant poloidal harmonics $m = 5$ and $m = 6$ is centred at $r/a = 0.433$ and the central value of energetic ion beta profile is $\beta_{EI} = 1.5\%$. An off-axis peaked energetic electron beta profile with a peak value $\beta_{EE} = 0.5\%$ at $r/a = 0.33$ is adopted in this subsection. In order to increase the proportion of barely trapped energetic electrons in the simulation, an anisotropic Maxwellian distribution with a Gaussian type of the peak pitch angle distribution $f(\Lambda) = \exp[(\Lambda - \Lambda_{\text{peak}})^2/\Delta\Lambda^2]$ is adopted for energetic electrons, where the peak value of pitch angle $\Lambda_{\text{peak}}$ is set to be 0.85, which is located around the passing-trapped boundary, and the pitch angle width $\Delta\Lambda$ is set to be 0.2.

Figure 13 shows the mode frequency and the linear growth rate of the $n = 2$ TAE versus the peak position $r_h$ of energetic electron beta profile. Similarly, the energetic-ion driven TAE is stabilized, when $r_h$ is located around the mode center which is $r/a = 0.354$. It should be noted again that the peak value and profile width of the energetic electron beta profiles are the same as those used for investigating $n = 12$ TAE in figure 10. The mode frequency and the linear growth rate of energetic electron beta profile on the $n = 2$ TAE is decreased when peak position is located at the outer side of the mode center, and the negative (positive) pressure gradient will decrease (increase) the TAE frequency.

Figure 13. The mode frequency (circle) and linear growth rate (triangle) of $n = 2$ mode versus $r_h$. The grey dotted line indicates the radial position of $q = (2 + 1/2)/2$ rational surface. The peak value (0.5%) and the profile width of energetic electron beta profiles are the same as those used for investigating $n = 12$ TAE. In our previous work $(positive)$ pressure gradient will decrease (increase) the TAE stabilization occurs at $r_h/a = 0.35$, where the $m = 3$ harmonic is obviously damped leading to a more localized spatial profiles. The grey dotted line indicates the peak value and profile width of the energetic electron beta profile. The equilibrium safety factor remains unchanged, $q = 1 + 2(r/a)^2$. In the baseline case, the $n = 4$ TAE driven by energetic ions with dominant poloidal harmonics $m = 5$ and $m = 6$ is centred at $r/a = 0.433$ and the central value of energetic ion beta profile is $\beta_{EI} = 1.5\%$. An off-axis peaked energetic electron beta profile with a peak value $\beta_{EE} = 0.5\%$ at $r/a = 0.33$ is adopted in this subsection. In order to increase the proportion of barely trapped energetic electrons in the simulation, an anisotropic Maxwellian distribution with a Gaussian type of the peak pitch angle distribution $f(\Lambda) = \exp[(\Lambda - \Lambda_{\text{peak}})^2/\Delta\Lambda^2]$ is adopted for energetic electrons, where the peak value of pitch angle $\Lambda_{\text{peak}}$ is set to be 0.85, which is located around the passing-trapped boundary, and the pitch angle width $\Delta\Lambda$ is set to be 0.2.

Time evolutions of kinetic energy for energetic-ion driven TAEs before and after including energetic electrons are shown in figure 15. The mode evolutions are very similar for the cases with kinetic energetic electrons and with the energetic electron beta profile, which indicates a weak wave-particle interaction between energetic electrons and the energetic-ion driven TAE. The slight increase of mode growth rate after the inclusion of energetic electrons is related to the energetic electron beta profile. TAE enhancement in figure 15 is very similar to the cases with centrally peaked energetic electrons discussed in section 3.1, where energetic electron beta profile exerts a negative pressure gradient at the TAE center and the mode frequency is decreased. As a result, the mode spatial width and the wave-particle interactions are changed, leading to the increase of TAE growth rate here. Figure 16 shows the resonance and the power transfer between energetic electrons/ions and TAE in the case with kinetic energetic electrons. The resonance condition for a low frequency wave (mode frequency much less than the cyclotron frequency) in a torus is

$$\omega_0 - L\omega_B - n\omega_\varphi = 0,$$
Figure 14. (a) Radial velocity fluctuation profiles of energetic-ion driven $n = 2$ mode without energetic electron beta profile. (b)-(d) Radial velocity fluctuation profiles of energetic-ion driven $n = 2$ mode with different energetic electron peak positions $r_h$.

Figure 15. Kinetic energy evolution for energetic-ion driven TAEs before (blue curve) and after (red and yellow curves) including energetic electrons. The red curve represents the case with kinetic energetic electrons, while the yellow curve represents the case with the energetic electron beta profile adopted into the fluid beta profile.

only a small number of trapped energetic electrons can resonate with TAE in the present simulation. The power transfer of trapped energetic electrons is shown versus the resonance number $L$ in figure 16(c). For resonant trapped energetic electrons with $|L| \leq 0.01$, the power transfer is negative, which indicates absorbing energy from TAE. However, the transferred power from TAE to trapped energetic electrons is negligible compared to the power transfer of energetic ions shown in figure 16(d).

In the present simulation, small population of resonant trapped energetic electrons leads to a weak wave-particle interaction between energetic electrons and TAE. However, it is still necessary to compare the power transfer of single resonant energetic electron and energetic ion in the present case. Orbit trajectories and evolutions of wave-to-particle power transfer of two resonant energetic particles with the largest $|\delta f|$ value at $\omega_A t = 376.9$ are shown in figure 17. The resonant trapped energetic electron is a barely trapped particle which experiences toroidal precession reversal $[52, 53]$, as shown in figure 17(a). The power transfer of the barely trapped energetic electron oscillates rapidly and the instantaneous power transfer is nearly all above 0, while the power transfer of the trapped energetic ion is always less than 0. The power transfer of these two particles is in the same level, which suggests that energetic electrons may absorb considerable energy from the energetic ion driven TAE through the wave-particle resonance if a sufficient number of resonant barely trapped energetic electrons can be loaded in the simulation.
Figure 16. Resonance and power transfer between energetic electrons/ions and TAE. Particle number (a) and power transfer (c) of trapped energetic electrons versus resonance number $L$. (b) Evolutions of $\delta f$ for three typical energetic electrons with large $|\delta f|$ values at $\omega_A t = 376.9$. (d) Power transfer of energetic ions versus resonance number $L$. The resonance condition is given by $\omega_0 - L\omega_0 - n\omega_\varphi = 0$, where $\omega_0$ is the mode frequency, $L$ is the resonance number, $\omega_\theta$ and $\omega_\varphi$ are particle poloidal and toroidal orbit frequency, respectively.

Figure 17. Analysis of particles with largest $|\delta f|$ values. Trajectories of resonant trapped energetic electron (blue curve) and resonant trapped energetic ion (red curve) are shown in panel (a), and the corresponding evolutions of wave-to-particle power transfer are shown in panel (b). The particle drift direction is indicated by the arrows on the curve in panel (a). $w_i$ is the particle weight and $dE_i/dt$ is the time derivative of the particle kinetic energy.
4. Summary

In this paper, energetic electron effects on an energetic-ion driven TAE with a single toroidal mode number $n = 4$ are investigated using an extended version of MEGA code, where both energetic ions and energetic electrons described by drift-kinetic equations are included. Both centrally peaked and off-axis peaked energetic electrons are considered. There is no obvious stabilization of TAE observed for the case with centrally peaked energetic electrons, even for a large energetic electron beta at which the most unstable mode is going to be an energetic-electron driven AE. On the contrary, a reduction of up to 50% in the TAE growth rate is observed for off-axis peaked energetic electrons at a value of 1.0% whose deposition position is around the mode center. A similar reduction of growth rate is achieved when we treat the off-axis peaked energetic electrons as part of the MHD fluid, which indicates the stabilizing effect arises from the beta profile of energetic electrons rather than the wave-particle resonance between energetic electrons and the TAE. In addition, a spatially uniform energetic electron beta profile, at the same beta value 1.0%, shows little effect on the TAE stability. The small change of mode frequency and mode damping rate indicates that the stabilization mechanism is different from the so-called pressure gradient stabilization. In the present work, the stabilized TAE has a more localized mode structure accompanied by a significant reduction of energetic ion driving rate. It has been identified that strong plasma non-uniformity can modify the mode structure of RSAE [21]. The results here suggest that the strong plasma non-uniformity induced by the energetic electron beta profile can also change the mode structure of TAE, leading to a decrease of mode growth rate.

A comparison of the stabilizing effect on high-$n$ ($n = 12$) and low-$n$ ($n = 2$) TAEs with the same linear growth rate is also conducted. It is found that the high-$n$ TAE with multiple poloidal harmonics is more easily stabilized by the off-axis peaked energetic electron beta profile. After considering an energetic electron beta profile peaked around the mode center at a value of 0.5%, the linear growth rate of $n = 12$ TAE decreases by 50%, while the linear growth rate of $n = 2$ TAE only decreases by 22%. This result is encouraging for large devices, such as ITER, where the toroidal mode number of the most unstable TAE will be $n \sim 15$ or even higher [54, 55]. Moreover, it is numerically verified that the positive (negative) pressure gradient at the TAE center can increase (decrease) the mode frequency, which is the key mechanism of the so-called pressure gradient stabilization. On the other hand, the opposite influence of pressure gradient on the TAE frequency may enhance the stabilizing effect of the energetic electron beta profile, when the peak position of energetic electron beta profile $r_h$ has a slightly outward shift from the mode center where $q = (m + 1/2)/n$. The strongest stabilization of TAE with multiple poloidal harmonics usually occurs for $r_h$ being located at such a position in the present work (figures 8 and 11).

We have also analysed the wave-particle interaction between energetic electrons and the energetic-ion driven TAE. The peak position of energetic electrons is at the inner side of the TAE center. Unfortunately, limited by the initial distribution of energetic electrons, only a very small population of resonant trapped energetic electrons, which is barely trapped particles, are loaded in the present work. Although these resonant energetic electrons are able to tap the energy from TAE, the net energy transfer of these energetic electrons is negligible compared to the energy transfer of energetic ions that drives the mode. Interestingly, when we compare the evolutions of energy transfer of a typical resonant trapped energetic electron and a resonant trapped energetic ion, the energy transfer magnitudes of these two particles are found in the same order. This suggests the wave-particle interaction might be a potential stabilization mechanism only if we can load a sufficient number of resonant barely trapped energetic electrons. Hence, analysis of energy transfer between energetic electrons and TAE based on a more realistic energetic electron distribution generated from the high-field side ECRH will be the subject of the future work. In addition, the negative magnetic shear can enhance the precession reversal of trapped particles [53], and the efficient energy transfer from an energetic-ion driven AE to energetic electrons requires the deposition location of energetic electrons locating at the inner side of the mode center, which is usually coincident with the negative shear region. Then, the potential stabilization mechanism via wave-particle interaction seems more effective in a reversed shear configuration and it remains to be identified.

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ORCID iDs

Jialei Wang https://orcid.org/0000-0002-8678-8075
Yasushi Todo https://orcid.org/0000-0001-9323-8285
Hao Wang https://orcid.org/0000-0002-9819-7483

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