Lepton Electric Dipole Moments
from Heavy States Yukawa Couplings

Isabella Masina

Service de Physique Théorique *, CEA-Saclay
F-91191 Gif-sur-Yvette, France

Abstract

In supersymmetric theories the radiative corrections due to heavy states could leave their footprints in the flavour structure of the supersymmetry breaking masses. We investigate whether present and future searches for the muon and electron EDMs could be sensitive to the CP violation and flavour misalignment induced on slepton masses by the radiative corrections due to the right-handed neutrinos of the seesaw model and to the heavy Higgs triplets of $SU(5)$ GUT. When this is the case, limits on the relevant combination of neutrino Yukawa couplings are obtained. Explicit analytical expressions are provided which accounts for the dependencies on the supersymmetric mass parameters.

*Laboratoire de la Direction des Sciences de la Matière du Commissariat à l’Énergie Atomique et Unité de Recherche Associée au CNRS (URA 2306).
1 Introduction

In low energy supersymmetric extensions of the Standard Model (SM), unless sparticle masses are considerably increased, present limits on flavour violating (FV) decays and electric dipole moments (EDMs) respectively allow for a quite small amount of fermion-sfermion misalignment in the flavour basis and constrain the phases in the diagonal elements of sfermion masses, involving the parameters $\mu$ and $A$, to be rather small. The bounds on the supersymmetric contribution to lepton (L)FV decays and EDMs have the advantage, as compared to the corresponding squark sector ones, of being not biased by the SM contribution - nor by the non-supersymmetric seesaw [1] contribution [2]. Experimental limits on LFV decays and EDMs are then a direct probe of the flavour and CP pattern of slepton masses - see e.g. Ref. [3] for a recent collection. From a theoretical perspective, understanding why CP phases and deviations from alignment are so strongly suppressed is one of the major problems of low energy supersymmetry, the CP and flavour problem. On the other hand, precisely because FV decays and EDMs provide strong constraints, they can share some light on the features of the (possibly) supersymmetric extension of the SM.

Indeed, it is well known that even if these CP phases and misalignments were absent - or suppressed enough - from the effective broken supersymmetric theory defined at $M_{Pl}$, they would be generated at low energy by the RGE corrections due to other flavour and CP violating sources already present in the theory, in particular from the Yukawa couplings of heavy states like the right-handed neutrinos of the seesaw model [4] and the Higgs triplets of $SU(5)$ grand unified theories (GUT) [5]. Effects of radiative origin have the nice features of being naturally small, exactly calculable once specified a certain theory and - most interestingly - if experiments are sensitive to them, they yield limits on some combination of Yukawa couplings $^1$.

Recently, it has been stressed that the present (planned) limit on $\mu \rightarrow e\gamma$ [6] ($\tau \rightarrow \mu\gamma$ [7]) is sensitive to the lepton-slepton misalignments induced by the radiative corrections in the framework of the supersymmetric seesaw model and that the associated constraints on neutrino Yukawa couplings have a remarkable feedback on neutrino mass model building [8, 9]. However, LFV decays cannot provide any information on the pattern of CP violation of slepton masses, while EDMs are sensitive to both LF and CP violations. Since the present sensitivities to the electron and muon EDMs, $d_e < 10^{-27}$ e cm [10] and $d_\mu < 10^{-18}$ e cm [11], could be lowered by planned experiments by up to three to five [12, 13] and six to eight [14, 15] orders of magnitude respectively, it is natural to wonder whether lepton EDMs would explore the range associated to LF and CP violations of radiative origin.

In this work we analyze the predicted range for $d_e$ and $d_\mu$ when the radiative corrections due to the right-handed neutrinos of the seesaw model and, possibly, the

---

$^1$Of course, several effects of different origin could be simultaneously present, but there would be no reason for a destructive interference among them to occur.
Higgs triplets of $SU(5)$ GUT provide the main source of CP violation and misalignment in slepton masses. This allows to extract many informations. If experimental searches could explore this range, one could obtain limits on the imaginary part of the relevant combination of neutrino Yukawa couplings. Moreover, the eventual discovery of a lepton EDM in this range might be interpreted as indirectly suggesting the existence of such a kind of fundamental particles which are too heavy to be more manifest. On the other hand, finding $d_e$ and/or $d_\mu$ above this predicted range would prove the existence of a source of CP (and likely also LF) violation other than these heavy states.

The pure seesaw case has been considered in Ref.\[16\] - see also the related studies \[17, 18\] on specific seesaw textures -, where it has been pointed out that threshold effects due to hierarchical right-handed neutrino masses enhance the radiatively-induced $\Im(A_{ii})$, $i = e, \mu, \tau$. By extensively reappraising this framework, we find that for $\tan\beta \gtrsim 10$ the amplitude with a LL RR double insertion - proportional to $\tan^3\beta$ - dominates over the one involving $\Im(A_{ii})$ - insensitive $^2$ to $\tan\beta$ -, the exact ratio depending on the particular choice of supersymmetric mass parameters. We provide general expressions from which it is easy to recognize the model dependencies and which complete previous analyses. Lepton EDMs are then strongly enhanced in models with large $\tan\beta$. To see how close experiments are getting to the seesaw induced lepton EDMs range, we compare the upper estimates for the radiatively-induced misalignments and CP phases with the corresponding present and planned experimental limits collected in \[3\].

The range for a seesaw-induced $d_\mu$ turns out to be quite far from present searches and, in particular, its eventual discovery above $10^{-23} \text{ e cm}$ would prove the existence of a source of CP violation other than the neutrino Yukawa couplings. On the contrary, the present sensitivity to $d_e$ explores the seesaw-induced range in models with large $\tan\beta$ and small R-slepton masses, say $m_R$ around $100 - 200$ GeV. The planned improvements for $d_e$ would allow to test also models with $m_R$ up to the TeV region and moderate $\tan\beta$. When present or planned experimental limits turns out to overlap with these allowed ranges, bounds on the imaginary part of the relevant combinations of neutrino Yukawa couplings are obtained and plotted in the plane $(\tilde{M}_1, m_R)$, respectively the bino and the average R-slepton masses at low energy. These plots allow to check whether any particular seesaw model is consistent with present data and, if so, which level of experimental sensitivity would test it. In any case, an experimentally interesting contribution requires hierarchical right-handed neutrino masses.

This feature is no more necessary when, in addition to the seesaw, a stage of $SU(5)$ grand unification is present above the gauge coupling unification scale. It is well known that the main drawback of minimal $SU(5)$ is that the value of the triplet mass, $M_T$, required by gauge coupling unification is sizeably below the lower bound on $M_T$ derived from proton lifetime (see for instance \[19\] for recent reviews and

\[2\] In Ref. \[17\] a dependence on $\tan\beta$ arises due to the particular class of seesaw textures studied.
references). In our analysis we therefore keep the triplet mass as a free parameter. We nevertheless exploit the minimal $SU(5)$ relations between the doublet and triplet Yukawa couplings since in general they are mildly broken in non-minimal versions of $SU(5)$. The simultaneous presence of right-handed neutrinos and heavy triplets turns out to further enhance the amplitude with the LL RR double insertion over the one with $\text{Im}(A_{ii})$, essentially unaffected by triplets. This cannot be derived by the - otherwise elegant - technique based on the allowed invariants [20].

When right-handed neutrinos and Higgs triplets are simultaneously present, the predicted range for the radiatively-induced $d_e$ is already sizeably excluded by the present experimental limit. This in turn is translated into a strong constraint on the imaginary part of the combination of Yukawa couplings which is relevant for the LL RR amplitude. Such a constraint could be hardly evaded even for large values of the triplet mass and unfavorable supersymmetric mass parameters. The radiatively-induced $d_\mu$ in the presence of triplets should not exceed $10^{-23}$ e cm, as was the case for the pure seesaw. However, at difference of the latter case, in the former one $d_\mu$ only mildly depends on the spectrum of right-handed neutrinos. Notice that planned searches for $d_\mu$ could constrain (depending on the triplet mass mass, of course) the imaginary part of the combination of Yukawa couplings whose absolute value could be independently constrained by the LFV decay $\tau \rightarrow \mu \gamma$.

The paper is organized as follows. In Section 2 we introduce our notations, discuss the framework and make some preliminary considerations. Section 3 considers the pure seesaw case by separately analyzing the flavour conserving (FC) and flavour violating (FV) amplitudes contributing to lepton EDMs. In Section 4 the framework of seesaw and $SU(5)$ is discussed along the same lines. Concluding remarks are drawn in Section 5. Finally, in Appendix A and B we collect the RGE in the case of the seesaw, without and with a minimal $SU(5)$ unification respectively.

## 2 Framework and Method

In this section we recall the expression for the lepton EDMs in the mass insertion approximation [21, 22, 23, 25, 3] to display the supersymmetric mass parameters that are constrained by the present and projected searches for $d_e$ and $d_\mu$. We then draw some preliminary considerations to introduce our procedure to calculate the radiative 1-loop contribution to these mass parameters. The relevant RGE can be found in the Appendix.

We adopt here the following conventions for the $6 \times 6$ slepton mass matrix in the lepton flavour (LF) basis where the charged lepton mass matrix, $m_\ell$, is diagonal:

$$
\begin{pmatrix}
\tilde{\ell}_L^\dagger & \tilde{\ell}_R^\dagger
\end{pmatrix}
\begin{pmatrix}
m_{LL}^2 & A_e^\dagger v_d - \mu \tan \beta m_\ell \\
A_e v_d - \mu^* \tan \beta m_\ell & m_{RR}^2
\end{pmatrix}
\begin{pmatrix}
\tilde{\ell}_L \\
\tilde{\ell}_R
\end{pmatrix}
$$

(1)

where $A_e$ is the $3 \times 3$ matrix of the trilinear coupling, the $A$–term. All deviations
from alignment in this mass matrix are gathered in the \( \delta \) matrices, which contain 30 real parameters (including 12 phases) and are defined as:

\[
m^2_{LL} = m^2_L(\mathbb{1} + \delta^{LL}) \quad m^2_{RR} = m^2_R(\mathbb{1} + \delta^{RR})
\]

\[A^4_i v_d - \mu \tan \beta m_\ell = (A^*_{\ell \ell} v_d - \mu \tan \beta m_\ell) + m_L m_R \delta^{LR} (2)\]

where \( m_L, m_R \) are average masses for L and R sleptons respectively and \( A_\ell \sim O(m_{susy} m_\ell / v_d) \) are the diagonal elements of \( A_\ell \), so that \( \delta^{LR} \) has only non-diagonal, flavour violating, elements.

The supersymmetric contributions to \( d_i \), \( (i = e, \mu, \tau) \), can be split in two parts, involving respectively only flavour conserving (FC) or flavour violating (FV) elements of the slepton mass matrix \( (1) \):

\[d_i = d_i^{FC} + d_i^{FV} (3)\]

\[
d_i^{FC} = \frac{e}{2} \frac{\alpha M_1}{4\pi |\mu|^2 \cos^2 \theta_W} \left[m_i \text{Im}(\mu) \tan \beta (I_B + \frac{1}{2} I_L - I_R + I_2) - v_d \text{Im}(A^*_{\ell} I_B)\right] (4)\]

\[
d_i^{FV} = \frac{e}{2} \frac{\alpha M_1}{4\pi |\mu|^2 \cos^2 \theta_W} \left[m_R m_L (\text{Im}(\delta^{LL} \delta^{LR})_{ii} I_{B,L} + \text{Im}(\delta^{LR} \delta^{RR})_{ii} I_{B,R}' + I_{B,R}'') + \tan \beta (\text{Im}(\delta^{LL} \mu \eta \mu \delta^{RR})_{ii} I_{B}' + \text{Im}(\delta^{LR} \mu \eta \delta^{LR})_{ii} I_{B}'')\right] (5)\]

where \( \eta_\ell \equiv i \bar{\nu}_\ell v_d / (m_\ell \mu \tan \beta) \), the functions \( I \) are defined as in \( (1) \) and terms that are less relevant \(^3\) or higher order in the \( \delta \)’s matrix elements are omitted. Notice that \( \eta_\ell \approx 1 \) for relatively large values of \( \tan \beta \), favored in mSUGRA and for which the LF and CP violations are most likely to be detected. The FC and FV contributions could result from different seeds of CP violation but could also be correlated in many different ways in models. Anyway, the experimental limit can be put on both because, due to the different nature of the many parameters involved, an eventual cancellation between these contributions appears unnatural. The distinction between the FC and FV contributions is also phenomenologically relevant: some of the \( |\delta| \)’s in the FV terms are already constrained to be smaller than \( O(1) \) by LFV decays, while \( \text{Im}(\mu) \) and \( \text{Im}(A_{ii}) \) are directly constrained by the EDMs.

Our aim here is to estimate the radiative contribution to the lepton EDMs induced by the seesaw interactions, first alone and subsequently accompanied by a stage of \( SU(5) \) GUT. Since the present experimental bounds on LFV decays and EDMs already point towards family blind soft terms (i.e. sparticles with the same quantum numbers must have the same soft terms) with very small CP phases, at the cut-off \( \Lambda = \Lambda_{\text{Pl}} \) corresponding to the decoupling of gravitational interactions we assume real and flavour blind soft terms, namely in eq. \((1)\):

\[
m^2_{LL} = m^2_{RR} = m^2_{LL} \equiv m^2_{LR} = y_\ell a_0 , \quad A_\ell = y_\ell a_0 \quad , (6)
\]

\(^3\)E.g. a contribution to \( d_i^{FC} \) of the form of \((5)\), with \( \delta^{LR} \rightarrow (A^*_{\ell \ell} v_d - \mu \tan \beta m_\ell) / (m_L m_R) \), is in principle present, but it is negligible with respect to \((1)\).
with real $a_0$, $m_0$ and $\mu$ term. In the next sections we separately study the FC and FV contributions to $d_i$ and obtain explicit approximate expressions for $\mathcal{I}m(A_{ii})$ and the products of $\delta$’s in [5]. The effects of a more general family independence assumption are important but not crucial and can be easily included in our analysis.

By means of these general approximated expressions we will:

i) derive the upper prediction for the radiatively induced leptonic EDMs, stressing the model dependences;

ii) compare it with the experimental limits;

iii) when allowed by the experiment, obtain an upper bound on the imaginary part of the relevant combination of Yukawa couplings.

A couple of preliminary considerations are in order before presenting the results of the next sections.

2.1 On the naive scaling relation

In the limit that all slepton masses are family independent, the FV contribution vanishes and the FC one is proportional to the mass of the $i$–th lepton ($\mathcal{I}m(A_{ii})v_d \approx \mathcal{I}m(a_0)m_i$) leading, except an accidental cancellation $^4$ with the $\mu$-term amplitude, to the ”naive” scaling relation

$$d_i/d_j = m_i/m_j$$

Then, due to the present experimental limit on $d_e$, $d_\mu$ could not exceed $d_e m_\mu/m_e \sim 2 \cdot 10^{-25}$ e cm, which roughly corresponds to the planned sensitivity and, if the limit on $d_e$ were still to be lowered, next generation experiments would have no chance of measuring $d_\mu$.

Such considerations could provide interesting informations because [4] strictly apply only to the FC $\mu$-contribution to the EDM, while in general both $\mathcal{I}m(A_{ii})$ and the FV terms may strongly violate it. This is the case for the radiatively induced $\mathcal{I}m(A_{ii})$. Some of the FV contributions are instead naturally proportional to a different lepton mass, $m_k$, possibly heavier than $m_i$, as discussed in [23, 20] - and, before, for the quark sector, in [20]. In particular, it will turn out in the next sections that the FV contribution can even take over the FC one. Hence, a value of $d_\mu$ above $\sim 2 \cdot 10^{-25}$ e cm is a possibility that deserves experimental tests and, interestingly enough, it would imply the source of lepton EDM being either the FV contribution or a non-universal $\mathcal{I}m(A_{ii})$, so providing a remarkable hint for our understanding of CP violation.

$^4$For dedicated studies on cancellations between amplitudes, also in more general frameworks, see e.g. [23, 24].
2.2 On the relevant combination of Yukawa couplings

Once a theory is specified, it is relatively easy to list the combinations of Yukawa couplings appearing in the slepton mass radiative corrections that may contribute to lepton EDMs. However, the actual calculation is more involved as we now turn to discuss.

As an example, let us look for \( \text{Im}(A_{ii}) \) by studying the evolution of \( A_e \) in the case of degenerate right-handed neutrino masses, \( \tilde{M} \), for simplicity. Let us first consider the case of the pure seesaw and define the hermitian matrices
\[
E \equiv y_e^\dagger y_e, \quad N \equiv y_\nu^\dagger y_\nu.
\]
Notice that \( E \) is real and diagonal in the LF defining basis and \( N \) is diagonalized by a unitary matrix similar to the CKM one with only one phase (even for non-degenerate right-handed neutrinos). By solving the RGE for \( A_e \), eq. (37), linearly in \( t^3 \equiv 1/(4\pi)^2 \ln(M_{Pl}/\tilde{M}) \), \( A_e \) can only be proportional to \( y_e E \) and \( y_\nu N \), whose diagonal elements are real. At \( O(t^2_3) \) only \( y_e E \) and \( y_\nu N \) appear, whose diagonal elements are again real. A potential \( \text{Im}(A_{ii}) \) shows up at \( O(t^2_3) \), through \( \text{Im}(y_e N[N,E]N)_{ii} \). On the other hand, when also Yukawa interaction of the \( SU(5) \) triplets are present, eq. (38) shows that, at first order in \( t_T \equiv 1/(4\pi)^2 \ln(M_{Pl}/M_T) \), \( A_e \) can be proportional to \( y_e E \), \( y_\nu N \) but also to \( U^* y_e \), where \( U \equiv y_u^\dagger y_u \). The latter have real diagonal elements but allow at \( O(t^2_T) \) for a combination with diagonal imaginary part, namely \( (U^* y_e N)_{ii} \).

Accordingly, in order to evaluate the actual coefficients in front of the products of Yukawa coupling matrices which are likely to have phases, we solve the RGE for the soft parameters by a Taylor expansion in the small parameters \( t_{ij} = 1/(4\pi)^2 \ln(Q_i/Q_f) \) associated to the intervals between the successive decoupling thresholds of the various heavy states. For instance, by integrating the RGE for \( A_e \) in the case of \( SU(5) \) plus seesaw, a non vanishing coefficient is obtained for the combination \( \text{Im}(U^* y_e N)_{ii} \) at \( O(t^2_7) \). Now, lepton FV transitions and CP phases are naturally defined in the LF basis where \( y_e \) and the Majorana masses \( M_R \) are diagonal and real. This basis is not invariant under the RGE evolution and one should diagonalize \( y_e \) and \( M_R \) again at the lower scale. Therefore, one has to find out the effect of these final rotations.

We adopt the rotating basis method introduced in Ref. [26], where the RGE are modified to incorporate the fact that the matrices are defined in the LF basis at each scale, so that \( y_e \) and \( M_R \) are always diagonal. It is worth to stress that, even if the rotations needed to diagonalize \( y_e \) are small, their effect could be crucial for \( \text{Im}(A_{ii}) \) and the four products of \( \delta \)'s in eq. (3). For instance, in the case of \( SU(5) \) plus seesaw it turns out that this correction exactly cancels the term proportional to \( t^2_T \text{Im}(U^* y_e N)_{ii} \) in \( A_e \). On the contrary, these rotations can be safely neglected when deriving approximate expressions for the radiatively-induced LFV decays because the latter are only sensitive to absolute values of \( \delta \)'s.

---

5Here and in the following Dirac mass terms are always written as \( \bar{f}_R m_f f_L \).
3 Lepton EDMs and Seesaw

In this section we consider the predictions for lepton EDMs in the context of the supersymmetric extension of the seesaw model, namely we assume the MSSM supplemented with the seesaw Yukawa interactions and Majorana masses for the right-handed neutrinos as the effective theory valid up to the cut-off $\Lambda = M_{Pl}$ where gravitational interactions decouple. Starting with real and universal boundary conditions at $M_{Pl}$, we solve the RGE displayed in Appendix A by expanding in the small parameters defined by the right-handed neutrino thresholds

$$
t_3 = \frac{1}{(4\pi)^2} \ln \frac{\Lambda}{M_3} \quad t_2 = \frac{1}{(4\pi)^2} \ln \frac{M_3}{M_2} \quad t_1 = \frac{1}{(4\pi)^2} \ln \frac{M_2}{M_1} ,
$$

with the ordering $M_3 > M_2 > M_1$. We thus obtain approximate analytic expressions for the radiatively induced $\delta$’s and $\text{Im}(A_{ii})$ which depend on $m_0, a_0$ and the Yukawa couplings defined in the LF basis at the scale $\Lambda$. Of course the latter can be immediately translated into the corresponding ones defined at any scale, e.g. $M_1$ or $m_{\text{susy}}$.

Precisely because of the lepton flavour and CP violating Yukawa couplings, the LF basis is continuously rotated and rephased with the RGE evolution and, as already discussed, we handle this by working in a rotating basis. It turns out that the basis transformation introduces negligible corrections in the FC terms (4) but important ones in the FV terms (5). Notice also that the seesaw effects stop at the decoupling threshold of the lightest right-handed neutrino, $M_1$, and that the RGE evolution of these effects down to the supersymmetric scales where CP and lepton FV transitions are estimated is generically small and can be neglected for our estimates. We now study separately the FC and the FV contributions.

3.1 Flavour conserving contribution

Starting in the LF basis at $M_{Pl}$ from $A_e = y_e a_0$, the seesaw interactions generate a $\text{Im}(A_{ii})$ in the LF basis at $M_1$. At leading order in the $t$’s and defining $y_\nu^a P_a y_\nu \equiv N_a$ ($a = 1, 2, 3$) and the right-handed neutrinos projectors $P_1 = \text{diag}(1, 0, 0)$, $P_2 = \text{diag}(1, 1, 0)$, $P_3 = i3$, the latter reads $^6$:

$$
\text{Im}(A_{ii}) = 8a_0 y_{e_i} \left( t_2t_3\text{Im}(N_2N_3)_{ii} + t_1t_3\text{Im}(N_1N_3)_{ii} + t_1t_2\text{Im}(N_1N_2)_{ii} \right) ,
$$

where the Yukawas in the r.h.s. are evaluated at $\Lambda$. A similar $^7$ formula were previously presented in Refs. $^{[16, 17, 18]}$, but with the various Yukawas involved defined

$^6$Actually, $^6$ is an approximation by excess and improves as the involved Yukawa couplings become small. However, for our estimates it is reliable up to $O(1)$ Yukawa couplings.

$^7$As far as the comparison with the expressions in Refs. $^{[16, 17, 18]}$ is possible, we find agreement with the last papers up to the coefficient of $\text{Im}(N_1N_2)_{ii}$. 
at different scales. Then, one must be more cautious in drawing general conclusions and the authors validate theirs with specific numerical examples. Our approach offers the advantage that the corresponding results become more transparent and allow for an easier estimate of the effects once a pattern of seesaw parameters is assigned. Anyway, the crucial point is that the more right-handed neutrinos are degenerate, the more the FC contribution increases. Indeed, it vanishes in the limit that right-handed neutrinos are degenerate, in which case a contribution to $\text{Im}(A_{ii})$ only appears at fourth order, proportional to $\text{Im}(y_D N [N, E] N)_{ii}$. Notice also that the naive scaling relation is generally violated by (9).

Eq. (9) displays a linear dependence on the unknown parameter $a_0$. By defining $A_{ii} \equiv |A_{ii}|e^{i\phi_{A_{ii}}}$, then $\phi_{A_{ii}} \approx t_2 t_3 \text{Im}(N_2 N_3)_{ii} + t_1 t_2 \text{Im}(N_1 N_3)_{ii} + t_1 t_2 \text{Im}(N_1 N_2)_{ii}$ is completely specified by the seesaw parameters. However, as appears from eq. (11) the experimental limit on $d_i$ doesn’t probe directly $\phi_{A_{ii}}$, rather it gives a bound on $\text{Im}(A_{ii}) \nu_{ii} \equiv m_i \text{Im}(a_i)$ once supersymmetric masses are fixed (and up to unnatural conspiracies between the various amplitudes). Figs. 1 taken from Ref. 3, show the present upper limits on $|\text{Im}(a_{ee})|/m_R$ and the planned ones for $|\text{Im}(a_{ij})|/m_R$ in the plane $(\tilde{M}_1, m_R)$, respectively the bino and R slepton mass at $m_{\text{susy}}$. Notice that these limits are quite model independent because, apart from $m_R$ and $\tilde{M}_1$, there is only a mild dependence on $m_L$, which we have fixed as in mSUGRA for definiteness. Indeed, since the $A$-term amplitude in $d_i^{FC}$ arises from pure bino exchange, it does not involve $\mu$ nor $\tan \beta$. In mSUGRA there is an unphysical region in the plane $(\tilde{M}_1, m_R)$ corresponding to $m_0^2 < 0$ and which has been indicated in light grey in the plots. Anyway, the dark grey region and below is also excluded because $m_R \leq \tilde{M}_1$, in contrast with the requirement of neutrality for the LSP.

To find an upper estimate for the seesaw induced $d_i^{FC}$, let us evaluate $|\text{Im}(a_{ij})|/m_R$ from eq. (9) by considering only its first term. This situation is representative because the terms proportional to $t_1$ are negligible when the lightest right-handed neutrino has smaller Yukawa couplings, as happens in many models and as one would guess from similarity with the charged fermion sectors. Anyway, the following discussion is trivially adapted to other cases. For definiteness, we adopt this set of reference threshold values: $M_2 = 10^{12}$ GeV, $M_3 = 10^{15}$ GeV, $\Lambda = 2 \times 10^{18}$ GeV. Since, as demanded by perturbativity, $(N_2 N_3)_{ii} \leq O(1)$,

$$\frac{\text{Im}(a_{ii})}{m_R} \leq O \left( \frac{0.02 t_2 t_3}{2 \times 10^{-3}} \right) \frac{a_0}{m_R} .$$

This upper estimate is easily adapted to any given model once the relation between $a_0$ and $m_R$ is made explicit. To make the dependence more manifest, let us introduce the following two mSUGRA situations: a) $a_0^2 = 2 m_0^2$; b) $a_0^2 = \tilde{M}_{1/2}^2$, where $\tilde{M}_{1/2}$ is the universal gaugino mass at the gauge coupling unification scale. In the first case

---

8The function $I_B$ in eq. (11) contains a factor $|\mu|^2$ that cancels the one in the overall coefficient. The exact expressions for $I_B$ can be found in 3, as well as approximate ones suitable for various pattern of supersymmetric masses.
Figure 1: Experimental upper bound on $|\text{Im}(a_e)|/m_R$ and $|\text{Im}(a_\mu)|/m_R$ for $d_e < 10^{-27} \text{ e cm}$ and $d_\mu < 10^{-24} \text{ e cm}$ respectively. $r^{-1}$ corresponds to the slope: $r \equiv \tilde{M}_1/m_R$. For the present sensitivity $d_\mu < 10^{-18} \text{ e cm}$, the numbers have to be multiplied by $10^6$. $m_L$ is fixed as in mSUGRA.

Let us now compare the upper estimate (10) with the experimental bound. As appears from eq. (11), the latter improves linearly with the experimental sensitivity to $d_i$. Taking $a_0 \sim m_R$, figs. (11) show that values around 0.02 for $|\text{Im}(a_i)|/m_R$ would require an experimental sensitivity to $d_e$ and $d_\mu$ respectively at the level of $10^{-28} - 10^{-29} \text{ e cm}$ and $2 \times 10^{-26} - 2 \times 10^{-27} \text{ e cm}$, the exact value depending on the particular point of the $(\tilde{M}_1, m_R)$ plane. However, to avoid charge and color breaking, the constraint $a_0/m_R \leq 3$ in general applies. Thus, focusing for instance around $m_R \approx 500 \text{ GeV}$, the FC contribution cannot exceed $\sim 10^{-28} \text{ e cm}$ for $d_e$ and $\sim 2 \times 10^{-26} \text{ e cm}$ for $d_\mu$, even with highly hierarchical right-handed neutrinos. Notice that the former value could be at hand of future experimental searches for $d_e$, while the latter is at the very limit of the planned experimental sensitivity to $d_\mu$. Allowing for smaller $m_R$ values, $m_R \approx 200 \text{ GeV}$, the FC seesaw upper estimate comes close to the present limit for $d_e$ while it cannot be more than $\sim 2 \times 10^{-25} \text{ e cm}$ for $d_\mu$. 

The result for $d_\mu$ is thus a kind of "negative" one, since its eventual future discovery above $\sim 2 \times 10^{-25}$ e cm couldn’t be attributed to the radiative FC contribution of the seesaw. Even if the situation for $d_e^{FC}$ appears more optimistic, it is worth to underline that the previous upper estimate actually applies to models with at least four $O(1)$ neutrino Yukawa couplings and large CP phases, like those in Ref. [18]. However, as we now turn to discuss, the FV contribution, underestimated by previous analyses, could drastically enhance the $d_e$ upper estimate.

### 3.2 Flavour violating contributions

In the following we isolate the potentially most important products of $\delta$’s and give an estimate of their relative magnitude with respect to the FC amplitude. The relevant approximations for the flavour violating elements, $i \neq j$, of the $\delta$’s are:

$$ m_L m_R \delta_{ij}^{LR} = a_0 m_i [-2t_3N_{ij} + \sum_a \frac{t_a^2}{2} F_A(a, a)_{ij} + \sum_{a>b} t_a t_b F_A(a, b)_{ij}] $$

(11)

$$ m_L^2 \delta_{ij}^{RR} = \sum_a \frac{t_a^2}{2} F_R(a, a)_{ij} + \sum_{a>b} t_a t_b F_R(a, b)_{ij} $$

(12)

$$ m_L^2 \delta_{ij}^{LL} = -(6m_0^2 + 2a_0^2)t_3 N_{ij} + \sum_a \frac{t_a^2}{2} F_L(a, a)_{ij} + \sum_{a>b} t_a t_b F_L(a, b)_{ij} $$

(13)

where $a, b = 1, 2, 3$, the matrix $\mathcal{N}$ is defined as $t_3 \mathcal{N} \equiv t_3 N_3 + t_2 N_2 + t_1 N_1$ and

$$ F_A(a, b) = 15\{E, N_a\} - 5\{E, N_b\} + 12\{N_a, N_b\} + 4[N_b, u_E^{(b)}] $$

+ $2((N_b^{(d)} + D_e)N_a + (N_a^{(d)} + D_e)N_b) + 4(D_{\nu}^{(a)}/a_0 + D_{\nu}^{(a)})N_b$

+ $[N_a, E] + 4[N_b, N_a] + 7[E, N_b]$ ,

(14)

$$ F_R(a, b) = 8[(6m_0^2 + 4a_0^2)y_e N_3 y_e + 2a_0^2] \frac{N_3 y_e y_e^T}{N_3 y_e y_e^T} $$

(15)

$$ F_L(a, b) = 2(6m_0^2 + 2a_0^2)(3N_b + E, N_a) + 2D_{\nu}^{(a)}N_b + [N_a, u_E^{(b)}] + 2m_{H_u}^2N_b $$

+ $4a_0^2(3N_b + E, N_a) + E, N_b)) + 2(G_L + 4a_0 \tilde{D}_{\nu}^{(a)})N_b $$(16)

with the Yukawas in the r.h.s. evaluated at the scale $\Lambda$. The subscript $(d)$ indicates to take only the diagonal elements of the matrix, $u_E^{(a)}$ is defined through $[u_E^{(a)}, E] = \{E, 3E + N_a + D_e\}$ and the definition of all the other quantities can be found in Appendix A.

By means of the above expressions, the predictions for the imaginary part of the various product of $\delta$’s can be studied. An imaginary part in the products $(\delta^{LL}\delta^{LR})_{ii}$, $(\delta^{LR}\delta^{RR})_{ii}$, ..., only arises at third order in at least two different $\delta$’s.
3.2.1 The LL RR contribution

Let us firstly discuss the LL RR double insertion, $\Im(\delta^{LL} \mu \eta_m \delta^{RR})_{ii}$, which turns out to be the quantitatively most interesting one. Since in general $\eta_\ell \approx 13$ and the phase of $\mu$ - if any - is extremely small so that it can be safely neglected in the discussion, $\Im(\delta^{LL} \mu \eta_m \delta^{RR})_{ii} \approx \mu \Im(\delta^{LL} \mu \delta^{RR})_{ii}$. From eqs. (12, 13) it appears that $\delta^{RR}_{ij}$, $\delta^{LL}_{ij}$ are respectively of second and first order in the $t$’s. Thus, the lowest order is the cubic and the LL RR contribution reads:

$$\Im(\delta^{LL} \mu \delta^{RR})_{ii} = 8 \mu t_a t_b (t_a + t_b) \Im(N_a EN_b)_{ii} .$$

Notice that $\Im(N_a EN_b)_{ii} = \tan^2 \beta \Im(N_a m_t^2 N_b)_{ii} / m_t^2$, where $m_t$ is the top mass. Due to the hierarchy in $m_\ell$, a potentially important effect could only come from $\Im(N_{a3} N_{b3})$. Notice also that, on the contrary of the FC contribution, (17) doesn’t strongly depend on $a_0$.

The relative importance between this amplitude and the FC one can be easily appreciated by neglecting the terms proportional to $t_1$, which, as already mentioned, is justified when the lightest right-handed neutrino has the smallest Yukawa couplings:

$$d_{LLRR}^{FV} / d_{FC}^{FV} = \frac{\mu \tan^3 \beta (6m_0^2 + 2a_0^2)(6m_0^2 + 3a_0^2)}{m_R^2 m_L^2} \frac{t_2 + t_3}{m_t^2} \frac{\Im(N_3 m_t^2 N_2)_{ii}}{\Im(N_3 N_2)_{ii}} \frac{t_2 + t_3}{m_t^2} \frac{\Im(N_3 m_t^2 N_2)_{ii}}{\Im(N_3 N_2)_{ii}} .$$

For realistic values, the ratio of the two loop functions is slightly smaller than one. To obtain a rule of thumb, let us take $m_0 \sim a_0 \sim m_{L,R}$ and the reference ratio $\Lambda / M_2 = 2 \times 10^6$. Then, unless ad hoc fine-tunings in the structure of $N$, $\Im(N_3 m_t^2 N_2)_{ii} \sim m_t^2 \Im(N_3 N_2)_{ii}$, and one finds

$$\frac{d_{LLRR}^{FV}}{d_{FC}^{FV}} \sim 0.5 \frac{\tan^3 \beta \mu}{10^3} \frac{t_2 + t_3}{\mu_{ew}} \frac{m_t^2}{0.1} \quad \text{(rule of thumb),}$$

where $|\mu_{ew}|^2 \approx 0.5 m_R^2 + 2 \tilde{M}_1^2$ is the value of $\mu$ accounting for radiative electroweak breaking in mSUGRA. Despite being of third order in the $t$’s, the FV amplitude can take over the FC thanks to its $\tan^3 \beta$ dependence. The precise value of this ratio is displayed in figs. 2 in the plane $(\tilde{M}_1, m_R)$. Cases a) and b) are separately displayed so that the behavior for any situation in between can be easily extrapolated. The relevant generalizations are also reminded. The plots show that the rule of thumb is quite reliable and allow to extract, for each point of the plane, the value of $\tan \beta$ for which the FV amplitude takes over the FC one. In both cases a) and b), this happens for $\tan \beta \gtrsim 10$ - the only exception being the region with $r \approx 1$ for case a), where $\tan \beta \gtrsim 20$ is required.
Theory: \( \frac{d_i^{FVLLRR}}{d_i^{FC}} \) for Seesaw

Figure 2: Ratio \( d_i^{FVLLRR}/d_i^{FC} \) when \( \mathcal{I}m(N_3m_\ell^2N_2)_{ii} \approx m_0^2\mathcal{I}m(N_3N_2)_{ii} \). We have assumed as reference: \( \Lambda/M_2 = 2 \times 10^6 \), \( m_L \) as in mSUGRA, \( \mu = \mu_{\text{ew}} \) and \( \tan \beta = 10 \).

3.2.2 The other contributions

The amplitudes with the other products of \( \delta \)'s are less important than the LL RR one. Consider for instance \( \mathcal{I}m(\delta^{LR}\delta^{RR})_{ii} \). The flavour violating elements in \( \delta^{RR} \) and \( \delta^{LR} \) are respectively of second and first order, so that the product appears at third order:

\[
\mathcal{I}m(\delta^{LR}\delta^{RR})_{ii} = 8m_i a_0 \frac{(12m_0^2 + 6a_0^2)}{m_R^3 m_L} \sum_{a>b} t_a t_b (t_a + t_b) \mathcal{I}m(N_a E N_b)_{ii} . \tag{20}
\]

It vanishes in the limit \( a_0 \to 0 \), as the FC contribution. Neglecting the terms proportional to \( t_1 \), the ratio of the LR RR amplitude and the FC one reads

\[
\frac{d_i^{FVLLRR}}{d_i^{FC}} = \tan^2 \beta \frac{12m_0^2 + 6a_0^2}{m_R^2} (t_2 + t_3) \frac{\mathcal{I}m(N_3m_\ell^2N_2)_{ii}}{m_i^2\mathcal{I}m(N_3N_2)_{ii}} \frac{I_B(R)}{I_B} . \tag{21}
\]

It is easy to check that this ratio is smaller than one (unless ad hoc fine-tunings in the structure of \( N \)): the ratio of the two loop functions is slightly smaller than one for realistic values of the supersymmetric parameters and, taking for instance \( m_0 = a_0 \) and \( \Lambda/M_2 = 2 \times 10^6 \), (21) should not exceed \( \sim 10^{-4} \tan^2 \beta \).
For $\Im(\delta^{LL}\delta^{LR})_{ii}$, no imaginary part can arise at second order in $t’s$ because both $\delta^{LL}$ and $\delta^{LR}$ are proportional to $t_3 N$. At third order there are many contributions and it is lengthy but straightforward to check that they are proportional to at least two different $t’s$. This contribution is also proportional to $a_0$ and could be comparable to the FC one but is in any case smaller than the LL RR one. The expression for the double LR insertion is also quite involved. It is proportional to $a_0^2$ and, being also suppressed by a factor $m_\tau^2/m_L^2$ with respect to the LL RR one, it can be safely neglected.

### 3.3 Predicted range and constrains on Yukawas

For values of $\tan \beta \gtrsim 10$, for which the EDMs are enhanced and thus most likely to be observed, the FV amplitude with the LL RR double insertion is generically dominant with respect to all other amplitudes. Then, when $\tan \beta \gtrsim 10$, all the considerations made previously for $d_i^{FC}$ actually apply to $d_i$ when strengthened by the rule of thumb factor (19). To give an example, if $t_2 + t_3 \sim 0.1$ and $\mu \sim \mu_{ew}$, $d_e$ cannot exceed $\sim 0.5 \times 10^{-28(-27)} \tan^3 \beta/10^3$ e cm and $d_\mu \sim 10^{-26(-25)} \tan^3 \beta/10^3$ e cm when $m_R \sim 500(200)$ GeV. Planned experimental sensitivities to $d_\mu$ could then test the seesaw radiative contribution for models with small $m_R$ and/or large $\tan \beta$, in which case the factor (19) could be up to $\sim 50 - 60$ so that $d_\mu$ should not exceed $O(10^{-23})$ e cm. On the contrary, the range of the seesaw induced $d_e$ already overlaps with the present experimental limit for values of $m_R$ up to $1/2$ TeV when $\tan \beta \sim 30$.

Barring unnatural cancellations, planned searches for $d_e$ could thus test each term of the sum in (17), namely each

$$t_at_b(t_a + t_b)\Im(N_a \frac{m_L^2}{m_\tau^2} N_b)_{11} \quad (a > b) \quad .$$

The effect of an eventual two orders of magnitude improvement for $d_e$ on the upper limit on $\Im(N_a m_L^2 N_b)_{11}/m_\tau^2$ is displayed in fig. 3 by taking as reference values $\tan \beta = 30$ and $t_at_b(t_a + t_b) = 2 \times 10^{-4}$. It turns out that planned limits could be severe enough to test models with hierarchical neutrino Yukawa couplings. This cannot be done by present limits. For any given seesaw model, it is straightforward to extrapolate from the plot the level of experimental sensitivity required to test it. Notice also that for $d_{\mu}$, limits on $\Im(N_a m_L^2 N_b)_{22}/m_\tau^2$ as strong as the present ones on $\Im(N_a m_L^2 N_b)_{11}/m_\tau^2$ would require a sensitivity to $d_\mu$ at the level of $2 \times 10^{-25}$ e cm.

---

9Needless to say, this remains true even if the FC were the dominant amplitude, in which case the bound would be stronger.
4 Lepton EDMs, Seesaw and $SU(5)$ Triplets

We now add to the supersymmetric seesaw model a stage of a minimal $SU(5)$ GUT above the gauge couplings unification scale, $M_{GUT} \sim 2 \times 10^{16}$ GeV. Namely, we include the contribution of the higgs triplets Yukawa interactions to the RGE evolution of slepton masses from $M_{Pl}$ down to their threshold decoupling scale $M_T$, which is very likely to be bigger than $M_3$. Notations are defined in Appendix B.

It is not restrictive to work (at any scale) in the basis where $(y^T_d =) y_e$ and $M_R$ are real and diagonal and $y_u = V^T d_u \phi_u V$, where $d_u$ are the (real and positive) eigenvalues of $y_u$, $V$ is the CKM matrix in the standard parameterization (more on this later) and $\phi_u$ is a diagonal $SU(3)$ matrix. The RGE for the radiatively induced misalignments are written in eqs. (17) to (20). At first order in $t_{1,2,3}$ defined in eq. (8) and

$$ t_T \equiv \frac{1}{(4\pi)^2}\ln \frac{M_{Pl}}{M_T}, \qquad \text{(23)} $$

their solutions at the scale $M_1$ reads:

$$ m^2_R \delta^{RR} = -(6m_0^2 + 2a_0^2)(2t_1y_e^2 + 3t_T U^*) $$
$$ m^2_L \delta^{LL} = -(6m_0^2 + 2a_0^2)((3t_T + t_1)y_e^2 + t_3 N) $$
$$ m_L m_R \delta^{RL} = -a_0(6t_T U^* m_\ell + 2m_\ell t_3 N) \quad \text{(24)} $$
where the matrix $N$ is defined as $t_3 N \equiv t_3 N_3 + t_2 N_2 + t_1 N_1$. It is understood that all the quantities in the r.h.s. of (24) are evaluated at $\Lambda$. The small effects of the subsequent evolution from $M_1$ down to $m_{\text{susy}}$ can be neglected in the following discussion. We explicitly write only the first order terms \(^{10}\) in the $t$’s because, contrarily to the seesaw case, they already produce a potential imaginary part for the FV contribution, eq. (5). Instead, for the diagonal part of $A_e$ this is not the case, as we now turn to discuss.

### 4.1 Flavour conserving contribution

As anticipated in the simplified discussion of Section 2, a potential candidate for $\Im(A_{ii})$, proportional to $\Im(U^* y_e N)_{ii}$, could show up at $O(t_T^2)$. It is lengthy but straightforward to see that such term is exactly canceled by the effect of rotating the basis. For the same reason, in the general case with different thresholds $M_{1,2,3}$, $M_T$, the overall coefficient of $\Im(U^* y_e N)_{ii}$ is zero. Therefore, the second order contribution to $\Im(A_{ii})$ is just

$$
\Im(A_{ii}) = 8 a_0 y_e \left[ \left( \frac{3}{2} t_T + t_3 \right) t_2 \Im(N_2 N_3)_{ii} + \left( \frac{3}{2} t_T + t_3 \right) t_1 \Im(N_1 N_3)_{ii} + t_2 t_1 \Im(N_1 N_2)_{ii} \right],
$$

namely the pure seesaw one discussed in the previous section, eq. (10), with the substitution $t_3 \to (3/2 t_T + t_3)$, due to the fact that above $M_T$ also the triplets circulate in the loop renormalizing the wave functions. $t_T$ is naturally expected to be small (triplets will not decouple much below $10^{16}$ GeV) so that eventual higher order contributions involving $t_T$ are expected to be negligible with respect to (25). As a result, a stage of $SU(5)$-like grand unification, cannot enhance by much the FC contribution with respect to the pure seesaw case.

### 4.2 Flavour violating contribution

On the contrary, products of two $\delta$’s have an imaginary part proportional to $t_3 t_T$ $\Im(U^* y_e N)_{ii}$ and the FV contribution to $d_e$ is potentially bigger than the FC one. Most interestingly, the predicted range for the radiatively induced $d_e$ turns out to have been already sizeably excluded by the present experimental bounds. Planned searches for $d_\mu$ would also get close to test the range corresponding to radiatively induced misalignments.

Let us consider in turn the predictions for the imaginary part of the products of $\delta$’s, eq. (5), from their expressions given in eq. (24). As before, the most important contribution comes out from the LL RR double insertion. Since $\eta_\ell \approx i 3$ and the

\(^{10}\)Of course eqs. (24) overestimate the misalignment. However, for our estimates here they are reliable up to $y_t \sim y_{\nu_3} \sim 1$. 

15
phase of $\mu$ - if any - is experimentally small enough to be safely neglected in the present discussion, one has at the lowest relevant order in the $t$’s

$$\Im(\delta^{LL}m_\ell^2\delta^{RR})_{ii} = 3\,t_Tt_3\,\frac{(6m_0^2 + 2a_0^2)^2}{m_R^2m_L^2}\,\Im(Nm_iU^*)_{ii}. \quad (26)$$

This FV contribution is potentially much bigger than the FC one because, apart from Yukawas and numerical coefficients, it is enhanced by a factor $(m_\tau\mu \tan \beta)/(m_\tau a_0)$.

The other FV combinations in (5) are:

$$\Im(\delta^{LR}m_\ell^2\delta^{LR})_{ii} = 12\,t_Tt_3\,\frac{a_0^2}{m_R^2} \frac{\Im(Nm_iU^*)_{ii}}{m_L^2} \quad (27)$$

$$\Im(\delta^{LR}\delta^{RR})_{ii} = 6\,t_Tt_3\,\frac{a_0(6m_0^2 + 2a_0^2)}{m_R^3} \frac{\Im(Nm_iU^*)_{ii}}{m_L} \quad (28)$$

$$\Im(\delta^{LL}\delta^{LR})_{ii} = \frac{m_R^2}{m_L^2}\,\Im(\delta^{LR}\delta^{RR})_{ii} \quad (29)$$

The only contribution which does not vanish in the limit $a_0 \to 0$ is the LL RR one. To compare the amplitudes, (26), (27) and (28), (29) have to be multiplied respectively by $\mu \tan \beta$ and $m_Rm_L$ and also by the appropriate loop functions, which have the same sign and in general are of the same order of magnitude (for more details see [3]). Then, if $\mu > 0$ the four FV amplitudes have the same sign. However, the double LR insertion is always negligible with respect to the LL RR one because of the suppression factor $m_\tau^2/m_L^2$. For the other amplitudes (28), (29) the suppression factor with respect to (26) is $m_{R,L}/(\mu \tan \beta)$ (actually smaller due to the numerical coefficients). Then, even in the case of $\mu < 0$, a reduction of the LL RR amplitude due to accidental cancellations seems unrealistic. Of course, also the contributions due to different thresholds in the right-handed neutrino spectrum are present, in exact analogy to what has been discussed in the previous section.

It is instructive to focus on the magnitude and dependencies of the combination $\Im(Nm_iU^*)_{ii}$. For $d_e$ and $d_\mu$, neglecting subleading terms proportional to $y_e^2$, $y_u^2$ and defining $V_{td} \equiv |V_{td}| e^{i\phi_{td}}$:

$$\Im(Nm_iU^*)_{22} \approx m_\tau y_e^2 V_{ts} \left( \Im(N_{23}) - \frac{m_e}{m_\tau} |V_{td}| \Im(e^{i\phi_{td}}N_{12}) \right) \quad (30)$$

$$\Im(Nm_iU^*)_{11} \approx m_\tau y_\ell^2 |V_{td}| \left( \Im(e^{-i\phi_{td}}N_{13}) + \frac{m_\mu}{m_\tau} V_{ts} \Im(e^{-i\phi_{td}}N_{12}) \right) \quad (31)$$

where we exploited the fact that $V_{ts}$ is real in the standard parameterization. The latter is convenient to stress that the CP phases involved in the above combinations could be naturally large - as is indeed the case for $\phi_{td}$ - but, of course, any other choice must give equivalent results. The contribution proportional to $\Im(N_{12})$

\[\text{[11]}\] Had we exploited the freedom of parameterizing $V$ in such a way that $V_{ti}$ are real numbers, then the CKM phase of $V$ would have been hidden in the redefinition of $N$. 

16
has important suppression factors. Moreover $|N_{12}|$ is independently constrained to be quite small from the present limits on $\mu \rightarrow e\gamma$. A plot of the present upper limit on $|C_{12}|$, with $C \equiv (4\pi)^2 t_3 N$, in the plane $(M_1, m_R)$ can be found in Ref. [9]. The limit were derived for the seesaw but also applies without significant modifications to the case of $SU(5)$ plus seesaw. As a result, once fixed $M_T$, experimental searches for $d_e$ and $d_\mu$ represent a test for $I_m(e^{-i\phi_d}N_{13})$ and $I_m(N_{23})$ respectively. Although present searches for $\tau \rightarrow \mu\gamma$ ($\tau \rightarrow e\gamma$) are not able by now to interestingly constrain $|C_{23}|$ ($|C_{13}|$), eventual experimental improvements would have an impact on $d_\mu$ ($d_e$) [28].

Notice also that the naive scaling relation is violated according to

$$\frac{d_e}{d_\mu} = \frac{|V_{td}|}{V_{ts}} \frac{I_m(e^{-i\phi_d}N_{13})}{I_m(N_{23})}$$

and that the combinations of Yukawas relevant for $d_i$ are independent on the phases of the diagonal $SU(3)$ matrix, $\phi_u$. On the contrary, the latter affects (see for instance Ref. [27]) the proton decay lifetime due to $d=5$ operators, whose most important decay mode in the case of minimal $SU(5)$ is $p \rightarrow K^+ \bar{\nu}$.

### 4.3 Predicted range and constraints on Yukawas

To understand how close to the experimental sensitivity is the radiatively induced EDM range, we plot in figs. 5 and 8 the upper estimate for the most important products of $\delta$'s. We consider a degenerate spectrum of right-handed neutrinos to pick out just the effect of the triplets, substitute the $M_{PL}$-values for $y_t$ ($\approx .7$) and the relevant CKM elements and choose as reference values $\Lambda/M_T = 10^2$ and $\Lambda/M_3 = 2 \times 10^3$. Then, the upper estimate follows by requiring perturbativity, $I_m(N_{23}) \leq 1$, $I_m(e^{-i\phi_d}N_{13}) \leq 1$. Solid and dashed lines refers to cases a) and b) respectively. We don’t show case b) for $I_m(\delta^{LL}m_\ell\delta^{RR})_{ii}/m_\tau$, because the predicted value is essentially flat. The upper estimate for $I_m(\delta^{LL}\delta^{RR})_{ii}$ is not shown, being closely related to that on $I_m(\delta^{LR}\delta^{RR})_{ii}$ (see eq. 29). The bounds on $I_m(\delta^{LR}m_\ell\delta^{LR})_{ii}/m_\tau$ are not displayed because they are too small to be of potential interest.

For an easy comparison, we have taken from the sleptonarium [3] the experimental limits on the same quantities, figs. 4 and 7. The experimental limits on $I_m(\delta^{LR}\delta^{RR})_{ii}$ are close to those on $I_m(\delta^{LL}\delta^{RR})_{ii}$ and mildly depend on $m_\ell$, which has been fixed as in mSUGRA in the plots. On the contrary, the limits on $I_m(\delta^{LL}m_\ell\delta^{RR})_{ii}/m_\tau$ are proportional to $1/(-\mu\tan\beta)$. For definiteness, $\tan\beta = 10$ and $\mu = \mu_{ew}$ have been assumed. The experimental bounds shown in figs. 4 and 7 correspond to the present bound $d_e < 10^{-27}$ e cm and to the planned sensitivity $d_\mu < 10^{-24}$ e cm. Since the experimental bounds are proportional to the bound on $d_\ell$, it is straightforward to extrapolate the sensitivity to $d_\mu$ and $d_e$ required to test the radiatively induced $d_e$ and $d_\mu$. 

17
Figure 4: Experimental upper bounds on various products of $\delta$’s corresponding to the present sensitivity $d_e < 10^{-27}\text{ e cm}$.

Theory: $SU(5)$ with Seesaw

Figure 5: Upper estimate for various products of $\delta$’s in $SU(5)$ with seesaw. The reference values $\Lambda/M_T = 10^2$ and $\Lambda/M_3 = 2 \cdot 10^3$ have been taken. Solid (dashed) lines correspond to case a) (b)).
Upper limit on
\[ \mathcal{I}m(e^{-i\phi_{td}}N_{13}) \]

Figure 6: Present upper bound on \( \mathcal{I}m(e^{-i\phi_{td}}N_{13}) \). The reference values \( \tan \beta = 10, \Lambda/M_T = 10^2 \) and \( \Lambda/M_3 = 2 \cdot 10^3 \) have been taken. Solid (dashed) lines correspond to case a) (b)).

Let us firstly discuss the \( d_e \) range. For our reference values, the upper estimate for \( \mathcal{I}m(\delta^{LR}\delta^{RR})_{11} \) is \( O(10^{-6}) \) and the present bound on \( d_e \) already constrains it to be smaller in a large region of the plane. Although we already know that they are not dominant, it is worth to discuss the LR RR amplitude and the similar LL LR one because, as already mentioned, they are quite model independent. Allowing for higher triplet masses, however, these amplitudes shift below the present experimental sensitivity. This is not the case for the LL RR amplitude. The maximum value allowed for \( \mathcal{I}m(\delta^{LL}m_{\ell}\delta^{RR})_{11}/m_\tau \) is displayed in the right panel of fig. 5 for case a). In case b) it is \( \approx 10^{-3} \) everywhere. Then, the present experimental bound on \( d_e \) has already explored the radiative range for roughly 2 – 3 orders of magnitude.

This can be translated into an upper limit on \( \mathcal{I}m(e^{-i\phi_{td}}N_{13}) \), as in fig. 6. Notice that this limit is indeed very strong and can be hardly evaded even looking for less favorable parameters than those taken as reference. To check this, keep e.g. \( t_3 \) fixed and try to worsen the limit: a reduction of \( \mu \) with respect to \( \mu_{ew} \) is unlikely to reduce it more than one order of magnitude; a suppression by another factor 10 would require a \( \Lambda/M_T \approx 1.6 \); on the other hand, for values of \( \tan \beta \) larger than 10 the limit linearly improves. As a result, in the framework of the seesaw accompanied by \( SU(5) \), the limit on \( \mathcal{I}m(e^{-i\phi_{td}}N_{13}) \) can be considered robust.
Experiment

Figure 7: Same as fig. but for $d_\mu < 10^{-24} \text{ e cm}$.

Theory: $SU(5) \text{ with Seesaw}$

$\Im(\delta^{LR} \delta^{RR})_{22}$

$\Im(\delta^{LL} m_t \delta^{RR})_{22}$

Figure 8: Same as fig. but for $i = 2$. 
Let us now turn to discuss $d_{\mu}$. By comparing figs. 7 and 8, it turns out that the upper bound on the LR RR insertion would require a sensitivity to $d_{\mu}$ at $O(10^{-26})$ e cm, at the very limit of planned experimental improvements. Quantitatively, the upper estimate for $\Im(\delta^{LL}m_{\ell}\delta^{RR})_{22}/m_{\tau}$ which, for case b) is $\approx 4 \times 10^{-3}$, is more promising. The major part of the plane in fig. 8 could be tested with $d_{\mu}$ at the level of $10^{-24}$–$10^{-25}$ e cm and in general $d_{\mu}$ should not exceed $O(10^{-23})$ e cm. As a result, the eventual presence of triplets doesn’t enhance by much the range for $d_{\mu}$ with respect to the pure seesaw case. Nevertheless, here too the possibility of constraining $\Im(N_{23})$ can be envisaged, as shown in fig. 9. Notice that, due to (32), a limit on $\Im(N_{23})$ comparable to the present one on $\Im(e^{-i\phi_{d}}N_{13})$ would require to improve the $d_{\mu}$ sensitivity down to $5 \times 10^{-27}$ e cm.

Figure 9: The upper bound on $\Im(N_{23})$ which would be extracted by improving the present limit by many orders of magnitude [14, 15]. Solid (dashed) lines correspond to case a) (b)).
5 Conclusions

Planned experiments might significantly strengthen the limit on $d_e$ [12,13] and $d_\mu$ [14,15]. Their eventual discovery could be interpreted as an indirect manifestation of supersymmetry but could not reveal which source of CP (and possibly flavour) violation is actually responsible for the measured effect. Clearly, all sources in principle able to give the lepton EDM even at an higher level would be automatically constrained while those which fail in giving the lepton EDM at the desired level would be automatically excluded from the list of possible candidates.

In this work, we have estimated the ranges for the lepton EDMs induced by the Yukawa interactions of the heavy neutrinos, both alone and with the simultaneous presence of the heavy $SU(5)$ triplets. It turns out that the FV LLRR amplitude is in general larger or comparable to the FC one. So, EDMs are enhanced for large values of tan $\beta$ and do not strongly depend on $a_0$.

The pure seesaw, even with large tan $\beta$ and very hierarchical right-handed neutrinos, cannot account for $d_\mu$ above $10^{-23}$ e cm. Its eventual discovery above this level would then signal the presence of some source of CP and LF violation other than the neutrino Yukawa couplings. The heavy triplets Yukawa couplings would be excluded from the list of possible sources because their additional presence do not significantly enhance the predicted range for $d_\mu$. Notice however that in the latter case a hierarchical right-handed neutrino spectrum is no more essential to end up with $d_\mu$ at an interesting level for planned searches. From the theoretical point of view, finding $d_\mu$ above $10^{-23}$ e cm would indeed have a remarkable impact.

Interestingly enough, the present experimental sensitivity to $d_e$ is already testing the simultaneous presence of triplets and right-handed neutrinos. Correspondingly, constrains on $\text{Im}(e^{-i\phi_\nu}N_{13})$ have been derived which are significant even for quite large values of the triplet mass and unfavorable supersymmetric masses. Without the triplets, the radiatively-induced $d_e$ is close to the present experimental sensitivity only in models with large tan $\beta$ and small slepton masses. Therefore, an experimental improvement would eventually provide interesting limits on the imaginary part of the relevant combination of neutrino Yukawa couplings and right-handed neutrino masses.

In the present discussion we have been looking for results as general as possible. Indeed, the specification of any particular seesaw model has been avoided and the attention has rather focused on the dependencies on the supersymmetric masses and heavy thresholds. Although some relevant seesaw models deserve a dedicated analysis [28], figs. 3, 6 and 9 are suitable for a quick check of the status of any given seesaw model with respect to the present and planned experimental limits on lepton EDMs.
Acknowledgements

We thank C.A. Savoy for useful discussions and collaboration in the early stage of this work. I.M. acknowledge the CNRS and the “A. Della Riccia” Foundation for support and the SPhT, CEA-Saclay, for kind hospitality. We also acknowledge M. Peskin and Y. Farzan for pointing out a wrong numerical coefficient in the previous version of eq. (15); the impact of this correction is quantitatively negligible and the subsequent analysis and results stay unaffected.

A Appendix: Seesaw

In the basis where charged fermion and right handed Majorana neutrino masses are diagonal

$$W \ni u^T y_u Q H_u + d^T y_d Q H_d + e^T y_e L H_d + \nu^T y_\nu L H_u + \frac{1}{2} \nu^T M_R \nu$$

(33)

where \( Q = (u \ d)^T \), \( L = (\nu \ e)^T \) and \( \langle H_{d(u)}^0 \rangle = v_{d(u)}. \) Soft scalar masses are defined as

$$L_{soft} \ni \tilde{u}^\dagger \tilde{u} = \sum_{a=1}^{3} \tilde{u}_a \tilde{u}_a, \quad \tilde{d}^\dagger \tilde{d} = \sum_{a=1}^{3} \tilde{d}_a \tilde{d}_a, \quad \tilde{e}^\dagger \tilde{e} = \sum_{a=1}^{3} \tilde{e}_a \tilde{e}_a, \quad \nu^\dagger \nu = \sum_{a=1}^{3} \nu_a \nu_a$$

(34)

Let us also introduce the following notations:

$$y^\dagger_x y_x \equiv X \quad y^\dagger_x y_x \equiv \tilde{X} \quad (x = e, u, d)$$

$$P_a y_\nu \equiv y^{(a)}_\nu \quad y^{(a)}_\nu P_a y_\nu = N_a \quad P_a y_\nu y^\dagger_a P_a = \tilde{N}_a \quad (a = 1, 2, 3)$$

where \( P_2, P_1 \) project out \( M_3 \) and \( M_{3,2} \) respectively, \( P_2 = \text{diag}(1,1,0), \) \( P_1 = \text{diag}(1,0,0) \), and \( P_3 = \text{id}. \)

A.1 Running

Defining \( t = \frac{1}{(4\pi)^2} \ln Q \), the running of the Yukawa coupling constants is governed by:

$$\begin{align*}
\frac{dy^{(a)}_\nu}{dt} &= y^{(a)}_\nu [3N_a + E + D^{(a)}_\nu] \\
\frac{dy_e}{dt} &= y_e [3E + N_a + D_e] \\
\frac{dy_u}{dt} &= y_u [3U + D + D^{(a)}_u] \\
\frac{dy_d}{dt} &= y_d [3D + U + D_d] \\
\frac{d(P_a M_R P_a)}{dt} &= 2[P_a M_R \tilde{N}_a^T + \tilde{N}_a M_R P_a]
\end{align*}$$

(35)
where
\[ D_{\nu}^{(a)} = [Tr(3U + N_u) - (3g_2^2 + \frac{9}{5}g_1^2)]i3 \]
\[ D_e = [Tr(3D + E) - (3g_2^2 + \frac{9}{5}g_1^2)]i3 \]
\[ D_u^{(a)} = [Tr(3U + N_u) - (\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2)]i3 \]
\[ D_d = [Tr(3D + E) - (\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2)]i3 \] (36)

For the trilinear couplings, defining \( P_a A_{\nu} \equiv A_{\nu}^{(a)} \)
\[
\frac{dA_{\nu}^{(a)}}{dt} = 4\tilde{N}_a A_{\nu}^{(a)} + 5A_{\nu}^{(a)}N_u + 2y_{\nu}^{(a)} y_d^\dagger A_e + A_{\nu}^{(a)} E + D_{\nu}^{(a)} A_{\nu}^{(a)} + 2\tilde{D}_{\nu}^{(a)} y_{\nu}^{(a)}
\]
\[
\frac{dA_e}{dt} = 4\tilde{E} A_e + 5A_e E + 2y_{\nu} y_e^{(a)\dagger} A_{\nu}^{(a)} + A_e N_u + D_e A_e + 2\tilde{D}_e y_e
\]
\[
\frac{dA_u}{dt} = 4\tilde{U} A_u + 5A_u U + 2y_u y_d A_d + A_u D + D_u^{(a)} A_u + 2\tilde{D}_u^{(a)} y_u
\]
\[
\frac{dA_d}{dt} = 4\tilde{D}_d A_d + 5A_d D + 2y_d y_u A_u + A_d U + D_d A_d + 2\tilde{D}_d y_d
\]

where
\[
\tilde{D}_{\nu}^{(a)} = [Tr(3y_u^\dagger A_u + y_{\nu}^{(a)\dagger} A_{\nu}^{(a)}) - (3g_3^2 \tilde{M}_2 + \frac{3}{5}g_1^2 \tilde{M}_1)]i3
\]
\[
\tilde{D}_e = [Tr(3y_d^\dagger A_d + y_e^{(a)\dagger} A_{\nu}^{(a)}) - (3g_2^2 \tilde{M}_2 + \frac{9}{5}g_1^2 \tilde{M}_1)]i3
\]
\[
\tilde{D}_u^{(a)} = [Tr(3y_u^\dagger A_u + y_{\nu}^{(a)\dagger} A_{\nu}^{(a)}) - (\frac{16}{3}g_3^2 \tilde{M}_3 + 3g_2^2 \tilde{M}_2 + \frac{13}{15}g_1^2 \tilde{M}_1)]i3
\]
\[
\tilde{D}_d = [Tr(3y_d^\dagger A_d + y_e^{(a)\dagger} A_{\nu}^{(a)}) - (\frac{16}{3}g_3^2 \tilde{M}_3 + 3g_2^2 \tilde{M}_2 + \frac{7}{15}g_1^2 \tilde{M}_1)]i3
\] (38)

For soft scalars, defining \( P_a m_{\nu}^2 P_a \equiv m_{\nu}^{(a)} \)
\[
\frac{dm_{\nu}^2}{dt} = \{m_{L, E}^2, E + N_u\} + 2(y_{\nu} m_{\nu}^2 y_e + m_{H_u}^2 E + A_{\nu}^\dagger A_e) + 2(y_{\nu} m_{\nu}^2 y_{\nu}^{(a)\dagger} y_{\nu}^{(a)} + m_{H_u}^2 N_u + A_{\nu}^{(a)^\dagger} A_{\nu}^{(a)}) + G_L
\]
\[
\frac{dm_e^2}{dt} = 2\{m_{L, E}^2, E\} + 4(y_{\nu} m_{L}^2 y_e + m_{H_u}^2 E + A_{\nu}^\dagger A_e) + G_e
\]
\[
\frac{dm_u^{(a)}}{dt} = 2\{m_{L}^2, N_u\} + 4(y_{\nu} m_{L}^2 y_{\nu}^{(a)}) + m_{H_u}^2 N_u + A_{\nu}^{(a)^\dagger} A_{\nu}^{(a)}\dagger\}
\]
\[
\frac{dm_Q^2}{dt} = \{m_{Q, U}^2, U + D\} + 2(y_{\nu} m_{\nu}^2 y_u + m_{H_u}^2 U + A_{\nu}^\dagger A_u) + 2(y_{\nu} m_{\nu}^2 y_d + m_{H_d}^2 D + A_{\nu}^\dagger A_d) + G_Q
\]
\[
\frac{dm_u^2}{dt} = 2\{m_{Q}^2, \tilde{U}\} + 4(y_{\nu} m_{Q}^2 y_u + m_{H_u}^2 \tilde{U} + A_{\nu}^\dagger A_u) + G_u
\]
\[
\frac{dm_d^2}{dt} = 2\{m_{Q}^2, \tilde{D}\} + 4(y_{\nu} m_{Q}^2 y_d + m_{H_d}^2 \tilde{D} + A_{\nu}^\dagger A_d) + G_d
\]

where
\[ G_L = -\left(\frac{6}{5}g_3^2 \tilde{M}_2^2 + 6g_2^2 \tilde{M}_2^2\right)i3 \]
\[ G_e = -\left(\frac{24}{5}g_1^2 \tilde{M}_1^2\right)i3 \]
\[ G_Q = -\left(\frac{2}{15}g_3^2 \tilde{M}_1^2 + 6g_2^2 \tilde{M}_2^2 + \frac{32}{3}g_1^2 \tilde{M}_1^2\right)i3 \] (40)
\[ G_u = -\left(\frac{32}{15}g_1^2 \tilde{M}_1^2 + \frac{32}{3}g_2^2 \tilde{M}_2^2\right)i3 \]
\[ G_d = -\left(\frac{8}{15}g_1^2 \tilde{M}_1^2 + \frac{32}{3}g_3^2 \tilde{M}_2^2\right)i3 . \]
Finally,
\[
\frac{dm_{H_u}^2}{dt} = 6Tr(y_d m_Q^2 y_u^T + y_d^T m_Q^2 y_d + m_{H_u}^2 D + A_d^T A_d) + 2Tr(y_e m_L^2 y_e^T + y_e^T m_L^2 y_e + m_{H_d}^2 E + A_e^T A_e) + G_L
\]
\[
\frac{dm_{H_d}^2}{dt} = 6Tr(y_u m_Q^2 y_u^T + y_u^T m_Q^2 y_u + m_{H_u}^2 U + A_u^T A_u) + 2Tr(y_v m_L^2 y_v^T + y_v^T m_L^2 y_v + m_{H_d}^2 N_a + A_v^T A_v) + G_L
\]
and we define \(m_{H_{(u,d)}}^2 \equiv \frac{dm_{H_{(u,d)}}^2}{dt}\).

For energy scales between \(\Lambda_a\) and \(M_3\), one has to take \(a = 3\); below \(M_3\) and above \(M_2\), \(a = 2\); while below \(M_2\) and above \(M_1\), \(a = 1\).

### B Appendix: \(SU(5)\) + See-saw

We adopt the following notation to write matter and Higgs superfields:

\[
\psi_{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3 & -u_2 & u_1 & \frac{d_1}{\bar{h}} \\ -u_3^T & 0 & u_3^T & u_2^T & d_2^T \\ u_3 & -u_2 & 0 & u_1 & e^c \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e \\ e^c \end{pmatrix} \phi_5 = \begin{pmatrix} \frac{H_{3d_1}}{\sqrt{2}} \\ \frac{H_{3d_2}}{\sqrt{2}} \\ \frac{H_{3d_3}}{\sqrt{2}} \\ \frac{H_{3d_d}}{\sqrt{2}} \\ \frac{H_{3d_{0}}}{\sqrt{2}} \end{pmatrix} \eta_1 = \nu^c
\]

where \(<H_{2d(2u)}^0> \equiv y_{d(u)}\). The superpotential

\[
W \equiv \frac{1}{4} \psi_{10}^A y_u \psi_{10}^C D \psi_{10}^D E \psi_{10}^{E,F} + \sqrt{2} \psi_{10}^{AB} y_e \phi_A H_B + \eta_1 y_{\nu} \phi_A H^A + \frac{1}{2} \eta_1 M_R \eta_1
\]

where \(A, B, \ldots = 1, \ldots, 5\) and flavour indices are understood, gives rise to \(6\) with \(y_d = y_e^T\) and \(y_u = y_{\nu}^T\). For the soft breaking part of the Lagrangian

\[
\mathcal{L}_{soft} = \bar{\psi}^T m_{\psi}^2 \psi + \bar{\phi}^T m_{\phi}^2 \phi + \bar{\eta}^T m_{\eta}^2 \eta + m_{\psi}^0 h^T h + m_{\phi}^0 \tilde{h}^T \tilde{h} + \left( \frac{1}{2} M_5 \lambda_5 \lambda_5 + \text{h.c.} \right)
\]

\[
+ \left( \bar{u}^c T A_u \bar{v}_u + \bar{c}^c T A_c \bar{v}_c \bar{u}_d + \bar{c}^c T A_c \bar{v}_c \bar{v}_u + \bar{\nu}^c T A_{\nu} \bar{v}_\nu \bar{v}_u + \text{h.c.} \right)
\]

where the scalar fields are \(\tilde{\psi} = (\tilde{u}^c, \tilde{u}, \tilde{d}, \tilde{c})\), \(\tilde{\phi} = (\tilde{e}, \tilde{\nu}, \tilde{\nu})\) and \(\tilde{\eta} = \tilde{\nu}^c\) and gauginos are denoted with \(\lambda_5\). In this way, in the scalar lepton mass matrix, \(m_{LL}^2 = m_{\phi}^2, m_{RR}^2 = m_{\psi}^2\).

### B.1 Running

Setting \(t \equiv \frac{1}{(4\pi)^2} \ln Q, dg_5/dt = -3g_5^3, dM_5/dt = -6g_5^2 M_5\) and

\[
\frac{dy_e}{dt} = 6y_e E + 3\tilde{U} y_e + y_e N + G_e y_e
\]
\[
\frac{dy_u}{dt} = 6y_u U + 2\tilde{E} y_u + 2y_u D + G_u y_u
\]
\[
\frac{dy_{\nu}}{dt} = 6y_{\nu} N + 4y_{\nu} E + G_{\nu} y_{\nu}
\]
where $G_e = -84/5g_5^2 + 4Tr(E)$, $G_u = -96/5g_5^2 + Tr(3U + N)$, $G_\nu = -48/5g_5^2 + Tr(3U + N)$. For scalar masses

$$\frac{dm^2_{\psi}}{dt} = \{ m^2_{\psi}, 2\bar{U} + 3\bar{U} \} + 4(y_e m^2_{\phi} y_e + m^2 E + A_e A_{\nu}^0) + 6(y_u m^2_{\psi} y_u + m^2 E + A_u A_{\nu}^0) + G_{\psi}$$

$$\frac{dm^2_{\phi}}{dt} = \{ m^2_{\phi}, 4E + N \} + 8(y_e m^2_{\psi} y_e + m^2 D + A_e A_{\nu}) + 2(y_u m^2_{\psi} y_u + m^2 N + A_u A_{\nu}) + G_{\phi}$$

$$\frac{dm^2_{\phi}}{dt} = 5\{ m^2_{\psi}, \bar{N} \} + 10(y_e m^2_{\psi} y_e + m^2 \bar{N} + A_u A_{\nu}) \tag{47}$$

where $G_{\psi} = -144/5g_5^2 M_2^2 i\bar{3}$. $G_{\phi} = -96/5g_5^2 M_2^2 i\bar{3}$. For trilinear couplings

$$\frac{dA_{\phi}}{dt} = 10\bar{E} A_e + 3\bar{U} A_e + 8A_e E + 6A_u y_u y_e + A_e N + 2y_e y_e A_{\nu} + G_e A_e + 2\tilde{G}_e y_e \tag{48}$$

$$\frac{dA_{\phi}}{dt} = 9\bar{U} A_u + 2\bar{E} A_u + 9A_u U + 4A_u y_u y_e + 2A_u D + 4y_u y_u A_d + G_u A_u + 2\tilde{G}_u y_u \tag{49}$$

$$\frac{dA_{\phi}}{dt} = 7\bar{N} A_{\nu} + 11A_e N + 4A_e E + 8y_e y_e A_{\nu} + G_{\nu} A_{\nu} + 2\tilde{G}_{\nu} y_e \tag{50}$$

where $\tilde{G}_e = -84/5g_5^2 M_5 + 4Tr(y_l A_e)$, $\tilde{G}_u = -96/5g_5^2 M_5 + Tr(3y_l A_u + y_l A_{\nu})$, $\tilde{G}_{\nu} = -48/5g_5^2 M_5 + Tr(3y_l A_u + y_l A_{\nu})$. Finally, for scalar higgses

$$\frac{dm^2_{h}}{dt} = Tr(6\bar{U} + 2\bar{N}) m^2_{h} + 6Tr(y_e m^2_{\psi} y_e + y_e y_e y_e + A_e A_{\nu})$$

$$+ 2Tr(y_e m^2_{\psi} y_e + y_e y_e y_e + A_e A_{\nu}) - 96 g_5^2 M_5^2 \tag{51}$$

$$\frac{dm^2_{h}}{dt} = 8Tr(E) m^2_{h} + 8Tr(y_e m^2_{\psi} y_e + y_e y_e y_e + A_e A_{\nu}) - 96 g_5^2 M_5^2$$

References

[1] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, 1979; T. Yanagida, in Proceedings of the Workshop on unified theory and baryon asymmetry of the universe, 1979; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.

[2] S.T. Petcov, Sov. J. Nucl. Phys. 25 (1977) 340 [Yad. Fiz. 25 (1977) 641].

[3] I. Masina and C.A. Savoy, to appear on Nucl. Phys. B. [hep-ph/0211283]

[4] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57 (1986) 961.

[5] R. Barbieri and L. Hall, Phys. Lett. B 338 (1994) 212 [hep-ph/9408406] R. Barbieri, L. Hall and A. Strumia, Nucl. Phys. B 445 (1995) 219 [hep-ph/9501334]

[6] R. Barbieri, A. Romanino and A. Strumia, Phys. Lett. B 369 (1996) 283, [hep-ph/9511305] A. Romanino and A. Strumia, Nucl. Phys. B 490 (1997) 3, [hep-ph/9610485]

[6] R. Bolton et al., Phys. Rev. D 38 (1988) 2077; M.L. Brooks et al. [MEGA Collaboration], Phys. Rev. Lett. 83 (1999) 1521, [hep-ex/9905013]
[7] I. Hinchliffe, F.E. Paige, Phys. Rev. D 63 (2001) 115006, hep-ph/0010086; D.F. Carvalho, J.R. Ellis, M.E. Gomez, S. Lola and J.C. Romao, hep-ph/0206148; J. Kalinowski, hep-ph/0207051.

[8] W. Buchmuller, D. Delepine and F. Vissani, Phys. Lett. B 459 (1999) 171, hep-ph/9904219; J. L. Feng, Y. Nir and Y. Shadmi, Phys. Rev. D61 (2000) 113005, hep-ph/9911370; J. Ellis, M.E. Gomez, G.K. Leontaris, S.Lola and D.V. Nanopoulos, Eur. Phys. J. C 14 (2000) 319, hep-ph/9911450; K. S. Babu, B. Dutta and R. N. Mohapatra, Phys. Lett. B458 (1999) 93; W. Buchmuller, D. Delepine and L. T. Handoko, Nucl. Phys. B576 (2000) 445; J. Sato, K. Tobe and T. Yanagida, Phys. Lett. B 498 (2001) 189, hep-ph/0010348; J. Hisano and K. Tobe, Phys. Lett. B 510 (2001) 197, hep-ph/0102315; J.A. Casas and A. Ibarra, Nucl. Phys. B 618 (2001) 171, hep-ph/0103065; D.F. Carvalho, J. Ellis, M.E. Gomez and S. Lola, Phys. Lett. B 515 (2001) 323, hep-ph/0103256; S. Davidson and A. Ibarra, JHEP 0109 (2001) 013, hep-ph/0104076; T. Blazek and S.F. King, Phys. Lett. B 518 (2001) 109, hep-ph/0105005; S. Lavignac, I. Masina and C.A. Savoy, Phys. Lett. B 520 (2001) 269, hep-ph/0106245; E.O. Ilton, Phys. Rev. D 64 (2001) 115005, hep-ph/0107107; S. Lavignac, I. Masina and C.A. Savoy, Nucl. Phys. B 633 (2002) 139, hep-ph/0202086; A. Masiero, S.K. Vempati and O. Vives, Nucl. Phys. B 649 (2003) 189, hep-ph/0209303; K.S. Babu, B. Dutta and R.N. Mohapatra, hep-ph/0211068; S. Pascoli, S.T. Petcov and C.E. Yaguna, hep-ph/0301095; G.C. Branco, D. Delepine and S. Khalil, hep-ph/0304164.

[9] I. Masina, in Proceedings of SUSY '02, DESY, Hamburg, 2002, hep-ph/0210125.

[10] E.D. Commins, S.B. Ross, D. Demille, B.C. Regan, Phys. Rev. A 50 (1994) 2960; B.C. Regan et al., Phys. Rev. Lett 88 (2002) 071805.

[11] CERN-Mainz-Daresbury Collaboration, Nucl. Phys. B 150 (1979) 1.

[12] C. Chin et al., Phys. Rev. A 63 (2001) 033401; J.J. Hudson et al., hep-ex/0202014; B.E. Sauer, talk at Charm, Beauty and CP, 1st Int. Workshop on Frontier Science, October 6-11, 2002, Frascati, Italy.

[13] S.K. Lamoreaux, nucl-ex/0109014.

[14] R. Carey et al., Letter of Intent to BNL (2000); Y.K. Semertzidis et al., hep-ph/0012087.

[15] J. Aysto et al., hep-ph/0109217.

[16] J. Ellis, J. Hisano, S. Lola and M. Raidal, Nucl. Phys. B 621 (2002) 208, hep-ph/0109125.

[17] J. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Lett. B 528 (2002) 86, hep-ph/0111324.
[18] J. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Rev. D 66 (2002) 115013, hep-ph/0206110; J. Ellis and M. Raidal, Nucl. Phys. B 643 (2002) 229, hep-ph/0206174.

[19] I. Masina, Int. J. of Mod. Phys. A, Vol. 16, No. 32 (2001) 5101, hep-ph/0107220; J.C. Pati, hep-ph/0204240; S. Raby, hep-ph/0211024.

[20] A. Romanino and A. Strumia, Nucl. Phys. B 622 (2002) 73, hep-ph/0108275; See also: A. Romanino and A. Strumia, Nucl. Phys. B 490 (1997) 3, hep-ph/9610485; O. Lebedev, Phys. Rev. D 67 (2003) 015013, hep-ph/0209023.

[21] L.J. Hall, V.A. Kostelecky and S. Raby, Nucl. Phys. B 267 (1986) 415; F. Gabbiani and A. Masiero, Nucl. Phys. B 322 (1989) 235.

[22] T. Moroi, Phys. Rev. D 53 (1996) 6565, hep-ph/9512396; Erratum-ibid. D 56 (1997) 4424.

[23] S. Pokorski, J. Rosiek and C.A. Savoy, Nucl. Phys. B 570 (2000) 81, hep-ph/9906200.

[24] T. Ibrahim and P. Nath, Phys. Rev. D 57 (1998) 478, Errata-ibid. D 58 (1998) 019901; Phys. Rev. D 58 (1998) 111301, Errata-ibid. D 60 (1999) 099902; S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B 606 (2001) 151, hep-ph/0103320; T. Ibrahim and P. Nath, Phys. Rev. D 64 (2001) 093002, hep-ph/0105025; hep-ph/0210251.

[25] J.L. Feng, K.T. Matchev and Y. Shadmi, Nucl. Phys. B 613 (2001) 366, hep-ph/0107182.

[26] P. Brax and C.A. Savoy, Nucl. Phys. B 447 (1995) 227, hep-ph/9503306.

[27] T. Goto and T. Nihei, Phys. Rev. D 59 (1999) 115009.

[28] work in progress.