A systematic study for the connection between hadron and its quark component nuclear modification factors

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Abstract. We have systematically studied the connection between hadron and its quark component nuclear modification factors and the flavor (mass) ordering at both parton and hadron levels in the nucleus-nucleus collisions at the LHC energies by the PACIAE model. It turns out that these two physical phenomena explored in our last publication (J. Phys. G 49 065104, 2022) are generally held, irrespective of the rapidity, centrality, reaction energy, and the collision system size. The mass ordering at hadron level in nuclear modification factor seems to be really existed, which should be studied further both theoretically and experimentally.

Keywords: heavy-ion collision, PACIAE model, nuclear modification factor

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1. Introduction

The hot and dense Quark-Gluon Plasma (QGP), a phase of deconfined matter, has been found to be created in ultra-relativistic heavy-ion collisions at both the Relativistic Heavy Ion Collider (RHIC) [1, 2, 3, 4] and the Large Hadron Collider (LHC) [5, 6, 7, 8, 9]. One of the most important signatures for QGP formation is the suppression of hadron production at high transverse momentum ($p_T$) due to the energy loss (jet quenching) [10, 11]. To quantify such suppression, the nuclear modification factor was proposed [12]. It is defined as the ratio of the $p_T$-differential multiplicities $dN/dp_T$ in nucleus-nucleus collisions ($AA$) to that in the nucleon-nucleon collisions ($pp$), scaled by the number of binary nucleon-nucleon collisions per nucleus-nucleus collisions $\langle N_{coll} \rangle$ for a given range of centrality [12, 13]

$$R_{AA}^X(p_T) = \frac{1}{\langle N_{coll} \rangle} \frac{dN_{AA}^X/dp_T}{dN_{pp}^X/dp_T},$$

where $X$ stands for a specific particle, $\langle N_{coll} \rangle$ can be obtained from the optical Glauber model and/or the Monte-Carlo Glauber model [14, 15, 16, 17, 18, 19]. The value of $R_{AA}$ would be unity if an $AA$ collision is just a simple superposition of the $pp$ collisions. Conversely, one could expect a non-unity $R_{AA}$ in the presence of the cold and hot nuclear medium effects. Therefore, $R_{AA}$ could serve as an excellent observable for exploring the jet quenching effect.

The $R_{AA}$ of partons in the final partonic state (FPS) is responsible for the QGP medium-induced parton energy loss during the formation and evolution processes of QGP. On the other hand, the $R_{AA}$ of hadrons in the final hadronic state (FHS), the one actually measurable in experiments, is the sum of partonic jet quenching above and the hadronic energy loss in hadronization and hadronic rescattering stages. Naturally, one would seek the connection between the $R_{AA}$ of partons in FPS and the $R_{AA}$ of hadrons in FHS. One should talk about this connection in specifics rather than in general. Taking $R_{AA}$ of $\Lambda$ as an example, the connection between $R_{AA}$ of $\Lambda$ and that of its constituent quarks could be considered in simple forms as follow:

(a) connect $R_{AA}^\Lambda \rightarrow$ single $R_{AA}^u$ ($R_{AA}^d$, $R_{AA}^s$);

(b) connect $R_{AA}^\Lambda \rightarrow$ sum $R_{AA}^{u+d}$ ($R_{AA}^{u+s}$, $R_{AA}^{d+s}$), calculated by the sum of $u$- and $d$- ($u$- and $s$-, $d$- and $s$-) quark $p_T$-distribution without weight factor;

(c) connect $R_{AA}^\Lambda \rightarrow R_{AA}^{u+d+s}$, calculated by the sum of $u$-, $d$-, and $s$-quark $p_T$-distribution without weight factor.

However, all of them are incomplete:

(a) the $d$- and $s$- ($u$- and $s$-, $u$- and $d$-) constituent quarks are ignored;

(b) the $s$- ($d$-, $u$-) constituent quark is excluded;

(c) the contribution of sea quark $s$ is underestimated.
As far as we know, such a connection is unable to be introduced from the first principle theory, even from the recombination (coalescence) model [20, 21, 22, 23, 24], because of the complication in dealing with the flavor composition of constituent quarks. Recently, such a connection between the hadron $R_{AA}$ (in FHS) and its quark component one (in FPS) has been proposed by us for the first time [25], and the $R_{AA}$ mass ordering at hadron level, initiating from the dead-cone effect [26], is also explored [25]. In this work, we extend the study to the rapidity, centrality, reaction energy, and the collision system size dependences of above two physical phenomena.

The paper is organized as follows. In section 2, the method of physical deduction of the $R_{AA}$ correspondence between hadron and its quark component as well as the numerical framework for event generation are described. In section 3, how the above two physical phenomena depending on the rapidity, centrality, reaction energy and the collision system size is presented. We summarize in section 4.

2. Method

In this work, we follow the formalism of the correspondence between the hadron $R_{AA}^h$ in FHS and its quark component $R_{AA}^{h-q}$ (the script $h-q$ refers to the quark component of the hadron $h$) in FPS established in reference [25]. The brief physical deduction is described as follows:

Considering the hadron normalized $p_T$–differential distribution

$$\frac{1}{N_h}dN_h/dp_T,$$

its corresponding quark component normalized $p_T$–differential distribution is

$$\frac{1}{N_{h-q}}\sum_q \frac{1}{N_q}dN_q/dp_T.$$

In the above expressions, $N_h$ ($N_q$) refers to the multiplicity of the hadron $h$ (quark $q$). $N_{h-q}$ denotes the number of constituent quarks in a hadron $h$ and the sum is taken over all constituent quarks.

Multiplying above two expressions by $N_h$, one can get the hadron un-normalized $p_T$–differential distribution

$$dN_h/dp_T$$

and the corresponding quark component un-normalized $p_T$–differential distribution

$$\frac{1}{N_{h-q}}\sum_q \frac{N_h}{N_q}dN_q/dp_T.$$

Substituting above two un-normalized $p_T$–differential distributions into equation (1), respectively, one obtains the hadron nuclear modification factor

$$R_{AA}^h(p_T) = \frac{1}{\langle N_{coll} \rangle} \frac{dN_{AA}^h/dp_T}{dN_{pp}^h/dp_T},$$

(2)
and its corresponding quark component nuclear modification factor

\[ R_{AA}^{h\rightarrow q}(p_T) = \frac{1}{\langle N_{\text{coll}} \rangle} \frac{\sum_q w_q^AA dN_q^{AA}/dp_T}{\sum_q w_q^{pp} dN_q^{pp}/dp_T}, \tag{3} \]

where \( w_q = N_h/N_q \) is the weight factor.

To investigate the correspondence between hadron and its quark component and the mass ordering at hadron level in \( R_{AA} \), a numerical Monte-Carlo event generator, PACIAE [27], is employed to simulate the \( pp \) and \( AA \) collisions. PACIAE is a microscopic parton and hadron cascade model based on but beyond the PYTHIA6.4 event generator [28].

For nucleon-nucleon (NN) collisions, with respect to PYTHIA, the partonic and hadronic rescatterings are introduced before and after the hadronization, respectively. The final hadronic state is developed from the initial partonic hard scattering and parton showers, followed by parton rescattering, string fragmentation, and hadron rescattering stages. Thus, the PACIAE model provides a multi-stage transport description on the evolution of the collision system.

For AA collisions, the initial positions of nucleons in the colliding nuclei are sampled according to the Woods-Saxon distribution. Together with the initial momentum setup of \( p_x = p_y = 0 \) and \( p_z = p_{\text{beam}} \) for each nucleon, a list containing the initial state of all nucleons in a given AA collision is constructed. A collision happened between two nucleons from different nuclei if their relative transverse distance is less than or equal to the minimum approaching distance: \( D \leq \sqrt{\sigma_{\text{tot}}^{NN}/\pi} \). The collision time is calculated with the assumption of straight-line trajectories. All such nucleon pairs compose an NN collision time list. An NN collision with least collision time is selected from the list and executed by PYTHIA (PYEVNW subroutine) with the hadronization temporarily turned-off, as well as the strings and diquarks broken-up. The nucleon list and NN collision time list are then updated. A new NN collision with least collision time is selected from the updated NN collision time list and executed by PYTHIA. With repeating the aforementioned steps till the NN collision list empty, the initial partonic state is constructed for a AA collision.

Then, the partonic rescatterings are performed, where the LO-pQCD parton-parton cross section [29, 30] is employed. After partonic rescattering, the string is recovered and then hadronized with the Lund string fragmentation scheme resulting in an intermediate hadronic state. Finally, the system proceeds into the hadronic rescattering stage and produces the final hadronic state observed in the experiments.

Thus PACIAE Monte-Carlo simulation provides a complete description of the NN and/or AA collisions, which includes the partonic initialization stage, partonic rescattering stage, hadronization stage, and the hadronic rescattering stage. Meanwhile, the PACIAE model simulation could be selected to stop at any stages desired conveniently. In this work, the simulations are stopped at the final partonic state (FPS) after partonic rescattering or at final hadronic state (FHS) after hadronic rescattering.
A systematic study

for the calculations of $R_{AA}^{h}$ and $R_{AA}^{h-q}$, respectively. More details could be found in the reference [27].

In order to maintain self-consistency, the tuning parameters are kept consistent with those in reference [25]: a factor multiplying on the hard scattering cross-section $K=2.7$ (0.7), the Lund string fragmentation parameters of $\alpha=1.3$ (0.1) and $\beta=0.09$ (0.58), as well as the Gaussian width of the primary hadron transverse momentum distribution $\omega=0.575$ (0.36) are implemented in $AA$ ($pp$) simulations.

3. Results and discussions

In reference [25], we have proposed a method connecting the hadron nuclear modification factor $R_{AA}^{h}$ in FHS to its quark component nuclear modification factor $R_{AA}^{h-q}$ in FPS, and explored the mass ordering in $R_{AA}^{h}$ at hadron level in the 0-5% most central Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV. In this section, we will expand our research discussing their dependences on rapidity, centrality, reaction energy, and the collision system size. Our simulations are all performed in full $\eta$ phase space, except those in section 3.1.

3.1. Rapidity dependence

Figure 1 shows the meson $R_{AA}^{h}$ in FHS (black solid circles) and its quark component $R_{AA}^{h}$ in FPS (red open circles) within different $\eta$ ranges in the 0-5% most central Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV. From top to bottom rows are $R_{AA}^{h}$ of $\pi^{+}(u\bar{d})$, $K^{+}(u\bar{s})$ and $\phi^{0}(s\bar{s})$, while from left to right columns are those in $|\eta| < 0.8$, 2.5 and full $\eta$ phase space. Figure 2 shows the same content, just for the baryon sector. A peak appears at $p_{T}$ around $1 \sim 2$ GeV/c, which is the so-called Cronin effect [31] attributed to the multiple initial-state scattering of partons [32]. In figures 1 and 2, one can see the correspondence between hadron $R_{AA}^{h}$ and its quark component $R_{AA}^{h}$ maintains well in all three rapidity ranges. The meson and baryon $R_{AA}$ are smaller than their quark component $R_{AA}^{h}$ above $p_{T} \sim 2$ GeV/c, even in the midrapidity of $|\eta| < 0.8$, due to more radiation and collision energy losses the hadron suffered than the ones of its quark component. Moreover, this discrepancy becomes more pronounced as the $\eta$ range increases. This is because the energy loss increases with the increasing number of colliding and radiating particles involved in a wider $\eta$ range.

In figure 3, we compare the flavor (mass) ordering of the quarks (in FPS), mesons (in FHS), and the baryons (in FHS) nuclear modification factors among three different $\eta$ phase spaces, as displayed from left to right column, respectively. For quark sector, a nearly flat $R_{AA}$ of the heavy $c$-quark is observed. This could be understood from the fact that the $c$-quark is produced in initial hard processes and transparent in the partonic and hadronic rescatterings. Hence the $p_{T}$ distribution of $c$-quark in Pb+Pb collisions is approximately parallel to that in p+p collisions at $\sqrt{s}=200$ GeV.

‡ Such a graph arrangement manner would also be utilized hereafter, just for different particle sectors and conditions applied.
the same energy. The flavor (mass) ordering seems to be generally held for quarks and mesons. However, for baryons, it seems to be recognizable only in full $\eta$ phase space. This can be explained by the relative mass discrepancy among selected observing particles in the three categories above. The relative mass discrepancy among baryons ($m_p \approx 0.938$ GeV, $m_{\Lambda^0} \approx 1.116$ GeV and $m_{\Xi^-} \approx 1.322$ GeV) is smaller than that of the mesons ($m_{\pi^+} \approx 140$ MeV, $m_{K^+} \approx 494$ MeV and $m_{\phi^0} \approx 1.02$ GeV) and is much smaller than the one of the quarks ($m_u \approx 2.2$ MeV, $m_s \approx 93$ MeV and $m_c \approx 1.27$ GeV).

Figure 1. The nuclear modification factors of mesons (in FHS, black solid circles) and their quark component (in FPS, red open circles) in the 0-5% most central Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV within three pseudo-rapidity ranges.
Figure 2. The nuclear modification factors of baryons (in FHS, black solid squares) and their quark component (in FPS, green open squares) in the 0-5% most central Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV within three pseudo-rapidity ranges.

Figure 3. The nuclear modification factors of quarks (in FPS), mesons (in FHS) and baryons (in FHS) in the 0-5% most central Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV within three pseudo-rapidity ranges.
3.2. Centrality dependence

In figure 4 and figure 5, we show hadron (meson and baryon in FHS) $R_{AA}$ and its quark component $R_{AA}$ (in FPS) in 0-5%, 5-20% and 20-60% centrality classes of Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV. Still, the physical phenomenon of the correspondence between hadron and its quark component nuclear modification factors as well as the hadron $R_{AA}$ less than its quark component $R_{AA}$ is keeping in all of the three centrality classes. One can see approximately a more depressed magnitude of both $R_{AA}$ in more central centrality class stemming from a stronger hot medium effect. Nevertheless, the discrepancy between hadron $R_{AA}$ and its quark component $R_{AA}$ brought about by centrality classes is not so significant.

Meanwhile, in figure 6 we give the simulated $R_{AA}$ of quarks (in FPS), mesons (in FHS) and baryons (in FHS) in 0-5%, 5-20%, and 20-60% centralities of Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV. The good flavor (mass) ordering in the region of $p_T > 2$ GeV/c is generally held. However, the ordering extent changing with centrality is insensitive.

**Figure 4.** The nuclear modification factors of mesons (in FHS, black solid circles) and their quark component (in FPS, red open circles) in the different centrality classes of Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV.
Figure 5. The nuclear modification factors of baryons (in FHS, black solid squares) and their quark component (in FPS, green open squares) in the different centrality classes of Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV.

Figure 6. The nuclear modification factors of quarks (in FPS), mesons (in FHS), and baryons (in FHS) in the different centrality classes of Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV.
3.3. Energy dependence

We now study the energy dependence of the two physical phenomena: the correspondence between hadron $R_{AA}$ and its quark component one, and the flavor (mass) ordering at both the parton and hadron levels. In figure 7 and 8, we give the $R_{AA}$ of both the hadrons (in FHS) and their quark component (in FPS) in 0-5% most central Pb+Pb at $\sqrt{s_{NN}}=0.9$, 2.76 and 5.02 TeV. The physical phenomenon of correspondence is kept well at all three reaction energies. However, the discrepancy of $R_{AA}$ between hadron and its quark component is not varied strongly with the collision energies.

In figure 9 the flavor (mass) ordering at both the parton and hadron levels are given. Here we can see the mass ordering at hadron level appears in all three reaction energies, like the one at parton level. However, it seems also showing the mass ordering at both of parton and hadron levels is more pronounced at the lower energy rather than the higher one. It should be studied further.

Figure 7. The nuclear modification factors of mesons (in FHS, black solid circles) and their quark component (in FPS, red open circles) in the 0-5% most central Pb+Pb collisions at different reaction energies.
Figure 8. The nuclear modification factors of baryons (in FHS, black solid squares) and their quark component (in FPS, green open squares) in the 0-5% most central Pb+Pb collisions at different reaction energies.

Figure 9. The nuclear modification factors of quarks (in FPS), mesons (in FHS), and baryons (in FHS) in the 0-5% most central Pb+Pb collisions at different reaction energies.
3.4. System size dependence

In figure 10 (meson) and figure 11 (baryon), we compare the hadron $R_{AA}$ with its quark component one in the 0-5% most central Cu+Cu, Xe+Xe and Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV. We present the mass ordering at both the parton and hadron levels for the same reaction systems above in figure 12. These figures show again that, the physical phenomena of the correspondence between hadron and its quark component as well as the flavor (mass) ordering are well kept.

As the amount of interacting matter increases with the system size increasing, the particle propagating length is longer in larger collision system size. Hence the larger the system size, the more energy losses there are \[34\]. Consequently, the $R_{AA}$ would decrease with the collision system size increasing, as shown in figure 10, 11 and 12 from left to right.

Figure 10. The nuclear modification factors of mesons (in FHS, black solid circles) and their quark component (in FPS, red open circles) in the 0-5% most central Cu+Cu, Xe+Xe and Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV.
A systematic study

Figure 11. The nuclear modification factors of baryons (in FHS, black solid squares) and their quark component (in FPS, green open squares) in the 0-5% most central Cu+Cu, Xe+Xe and Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \).

Figure 12. The nuclear modification factors of quarks (in FPS), mesons (in FHS), and baryons (in FHS) in the 0-5% most central Cu+Cu, Xe+Xe and Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \).
4. Summary

In summary, via the parton and hadron cascade model PACIAE, we study the correspondence between hadron nuclear modification factor and its quark component one, as well as the flavor (mass) ordering at both parton and hadron levels. Meanwhile, how the above two physical phenomena change with the (pseudo-)rapidity, centrality, reaction energy and the collision system size are all investigated systematically.

Generally speaking, the correspondence between hadron and its quark component and the mass ordering at hadron level in nuclear modification factors are held, irrespective of the rapidity, centrality, reaction energy, and the collision system size.

For the correspondence, the $R_{AA}$ of the hadron is always less than that of its quark component in the $p_T$ region above 2 GeV/c. The discrepancy between them becomes more pronounced in a wider $\eta$ range. However, the centrality, reaction energy and the system size do not show noticeable influence on it. The flavor (mass) ordering is easier to distinguish in the wider $\eta$ and the lower reaction energy, while this observation does not clearly exhibit in the dependence of centrality and/or the collision system size.

The mass ordering in nuclear modification factor at hadron level, like the one at parton level, seems to be existed really. Its clear observation is relevant to the relative discrepancy among the selected candidates in mass. The larger the relative discrepancy among those candidates in mass, the clearer mass ordering is observed.

Of course, the correspondence between hadron and its quark component and the hadronic mass ordering in the nuclear modification factor should be studied further, both theoretically and experimentally. In the next study, we would consider the open-charm and/or the open-bottom heavy-hadron \footnote{\label{fn:35}needs citation} as the candidates.

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References

[1] Adcox K et al. (PHENIX) 2005 Nucl. Phys. A 757 184–283 (Preprint nucl-ex/0410003)
[2] Arsene I et al. (BRAHMS) 2005 Nucl. Phys. A 757 1–27 (Preprint nucl-ex/0410020)
[3] Back B B et al. (PHOBOS) 2005 Nucl. Phys. A 757 28–101 (Preprint nucl-ex/0410022)
[4] Adams J et al. (STAR) 2005 Nucl. Phys. A 757 102–183 (Preprint nucl-ex/0501009)
[5] Aamodt K et al. (ALICE) 2010 Phys. Rev. Lett. 105 252302 (Preprint 1011.3914)
[6] Aad G et al. (ATLAS) 2010 Phys. Rev. Lett. 105 252303 (Preprint 1011.6182)
[7] Aamodt K et al. (ALICE) 2011 Phys. Lett. B 696 30–39 (Preprint 1012.1004)
[8] Chatrchyan S et al. (CMS) 2011 Phys. Rev. C 84 024906 (Preprint 1102.1957)
[9] Abelev B et al. (ALICE) 2012 Phys. Rev. Lett. 109 072301 (Preprint 1202.1383)
A systematic study

[10] Bjorken J D 1982 Energy Loss of Energetic Partons in Quark-Gluon Plasma: Possible Extinction of High $p_T$ Jets in Hadron-Hadron Collisions Tech. Rep. FERMILAB-PUB-82-059-T FERMILAB URL [http://lss.fnal.gov/archive/preprint/fermilab-pub-82-059-t.shtml](http://lss.fnal.gov/archive/preprint/fermilab-pub-82-059-t.shtml)

[11] Gyulassy M and Plunier M 1990 Phys. Lett. B 243 432–438

[12] Wang X N 1998 Phys. Rev. C 58 2321 (Preprint [hep-ph/9804357](http://arxiv.org/abs/hep-ph/9804357))

[13] Klein-Bösing C (ALICE) 2018 PoS LHCP2018 222 (Preprint [1809.04936](http://arxiv.org/abs/1809.04936))

[14] Glauber R J and Matthiae G 1970 Nucl. Phys. B 21 135–157

[15] Miller M L, Reygers K, Sanders S J and Steinberg P 2007 Ann. Rev. Nucl. Part. Sci. 57 205–243 (Preprint [nucl-ex/0701025](http://arxiv.org/abs/nucl-ex/0701025))

[16] Abelev B I et al. (STAR) 2009 Phys. Rev. C 79 034909 (Preprint [0808.2041](http://arxiv.org/abs/0808.2041))

[17] Abelev B et al. (ALICE) 2013 Phys. Rev. C 88 044909 (Preprint [1301.4361](http://arxiv.org/abs/1301.4361))

[18] Loizides C, Nagle J and Steinberg P 2015 SoftwareX 1-2 13–18 (Preprint [1408.2549](http://arxiv.org/abs/1408.2549))

[19] Loizides C, Kamin J and d’Enterria D 2018 Phys. Rev. C 97 054910 [Erratum: Phys.Rev.C 99, 019901 (2019)] (Preprint [1710.07098](http://arxiv.org/abs/1710.07098))

[20] Hwa R C and Yang C B 2003 Phys. Rev. C 67 034902 (Preprint [nucl-th/0211010](http://arxiv.org/abs/nucl-th/0211010))

[21] Greco V, Ko C M and Levai P 2003 Phys. Rev. Lett. 90 202302 (Preprint [nucl-th/0301093](http://arxiv.org/abs/nucl-th/0301093))

[22] Greco V, Ko C M and Levai P 2003 Phys. Rev. C 68 034904 (Preprint [nucl-th/0305024](http://arxiv.org/abs/nucl-th/0305024))

[23] Fries R J, Muller B, Nonaka C and Bass S A 2003 Phys. Rev. Lett. 90 202303 (Preprint [nucl-th/0301087](http://arxiv.org/abs/nucl-th/0301087))

[24] Fries R J, Muller B, Nonaka C and Bass S A 2003 Phys. Rev. C 68 044902 (Preprint [nucl-th/0306027](http://arxiv.org/abs/nucl-th/0306027))

[25] Sa B H, Zhou D M, Yan Y L, Liu W D, Hu S Y, Li X M, Zheng L, Chen G and Cai X 2022 J. Phys. G 49 065104

[26] Dokshitzer Y L, Khoze V A and Troian S I 1991 J. Phys. G 17 1602–1604

[27] Sa B H, Zhou D M, Yan Y L, Li X M, Feng S Q, Dong B G and Cai X 2012 Comput. Phys. Commun. 183 333–346 (Preprint [1104.1238](http://arxiv.org/abs/1104.1238))

[28] Sjostrand T, Mrenna S and Skands P Z 2006 JHEP 05 026 (Preprint [hep-ph/0603175](http://arxiv.org/abs/hep-ph/0603175))

[29] Combridge B L, Kripfganz J and Ranft J 1977 Phys. Lett. B 70 234

[30] Field R D 1989 Applications Of Perturbative QCD (Addison-Wesley Publishing Company, Inc.)

[31] Cronin J W, Frisch H J, Shochet M J, Boymond J P, Mermod R, Piroue P A and Sumer R L (1910) 1975 Phys. Rev. D 11 3105–3123

[32] Wang X N 2000 Phys. Rev. C 61 064910 (Preprint [nucl-th/9812021](http://arxiv.org/abs/nucl-th/9812021))

[33] Zyla P A et al. (Particle Data Group) 2020 PTEP 2020 083C01

[34] Wang X N 2005 Nucl. Phys. A 750 98–120 (Preprint [nucl-th/0405017](http://arxiv.org/abs/nucl-th/0405017))

[35] Andronic A et al. 2016 Eur. Phys. J. C 76 107 (Preprint [1506.03981](http://arxiv.org/abs/1506.03981))