Decaying into the Hidden Sector

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Abstract

The existence of light hidden sectors is an exciting possibility that may be tested in the near future. If DM is allowed to decay into such a hidden sector through GUT suppressed operators, it can accommodate the recent cosmic ray observations without over-producing antiprotons or interfering with the attractive features of the thermal WIMP. Models of this kind are simple to construct, generic and evade all astrophysical bounds. We provide tools for constructing such models and present several distinct examples. The light hidden spectrum and DM couplings can be probed in the near future, by measuring astrophysical photon and neutrino fluxes. These indirect signatures are complimentary to the direct production signals, such as lepton jets, predicted by these models.
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1. Introduction

The existence of a low energy hidden sector, weakly coupled to the Standard Model (SM), is an exciting possibility that will be tested by upcoming experiments. Hidden sector particles can be produced in high energy colliders, as stressed in the context of ‘Hidden Valley’ models [1] and models where gauge kinetic mixing results in ‘lepton jets’ [2, 3]. Such hidden sectors can also be probed with low energy $e^+e^-$ colliders and fixed target experiments [4]. Here, we point out that the existence of a low energy hidden sector, together with weakly interacting DM (WIMP) and gauge coupling unification, implies the generic possibility that DM may decay directly into the hidden sector through operators suppressed by the GUT scale. These decays, followed by decays into SM particles through kinetic mixing, provide the intriguing possibility of using astrophysical observations to study the hidden sector spectrum, complementing direct production experiments. This decaying DM framework provides a simple and natural explanation for the recent cosmic ray (CR) anomalies [5], while avoiding the tensions and pitfalls of many previously proposed models.

A DM explanation of the electronic CR excess requires a DM mass greater than a TeV [6, 7], and predominantly leptonic production [8]. Consequently, the vanilla MSSM WIMP scenario is disfavored, and many new models have been proposed, bifurcating into annihilating models [9, 10] and decaying models [11, 12, 13, 14, 15]. Annihilating models are difficult to reconcile with the FERMI and HESS CR data, because the softening of the spectrum above a few TeV requires an annihilation cross-section $\mathcal{O}(1000)$ times larger than that of the standard thermal WIMP [6, 7]. Such a large cross-section is in tension with constraints from photon and neutrino measurements from the Galactic Center (GC) [6, 16, 17], extragalactic emissions [18], and the CMB [19]. There is also model building tension for achieving such a large cross-section. Possible mechanisms include non-perturbative Sommerfeld enhancements [20, 9], or a resonance [21, 22]. In the latter case, a very narrow resonance and degenerate states are required, while in the former, either large ($\gtrsim 1$) gauge or Yukawa couplings to the light mediator or tuned parameters are necessary [9]. As we discuss below, the required large couplings conflict with a need for Yukawa interactions that generate a DM splitting, necessary in many models to avoid constraints from direct detection [23]. Indeed, the mechanism that generates the splitting typically opens up new annihilation channels that
can parametrically dominate at freeze out. As a consequence, in order to achieve the correct relic abundance, the couplings responsible for the Sommerfeld enhancement are constrained and cannot produce a large enough enhancement.

Decaying models replace the need for a large annihilation cross-section. Since the DM lifetime is much longer than the age of the Universe, its decays do not affect the attractive features of the thermal WIMP and leave no signature on the CMB radiation. Moreover, constraints from the GC or subhalos are easily evaded [12], since the emission rate depends on one power of the DM density, $\rho$, as opposed to the $\rho^2$ dependence in the annihilating case. Interestingly, the correct lifetime to explain the anomalies, $\mathcal{O}(10^{26} \text{ sec})$, is obtained if the decays are induced by dimension-6 operators suppressed by the GUT scale [11]. Still, it is non-trivial to construct a decaying DM model that does not over-produce antiprotons, and many existing models are fine tuned or have small and ad hoc parameters.

In this paper we study a new and natural class of models, where DM decays into a light hidden ‘dark sector’, with gauge group $G_d$. Working in the supersymmetric framework appropriate in the context of GUTs, the dark sector has a stable mass gap at the GeV scale, and communicates with the supersymmetric SM (SSM) through kinetic mixing [24]. The GeV gauge bosons decay into light SM fermions, explaining the lack of antiproton production [25]. The dark sector is close in spirit to the models discussed in [9, 2]. Nonetheless, it is more general in the sense that the DM may or may not be charged under $G_d$ and/or the SM. This opens the door for a wider range of models and is potentially simpler. Dimension-6 decay operators appear naturally, and are expected to be present at low energy unless forbidden by global symmetries. For related work where DM decays into light states, see [14].

Models of the type studied here involve several scales. Physics at the GUT scale, $M_{\text{GUT}}$, is responsible for producing the decay operators. More formally, in the limit $M_{\text{GUT}} \to \infty$, the DM is completely stable due to a preserved global symmetry. Fields at the GUT scale then break that symmetry, inducing the required decays. It is important that dimension-5 operators which would trigger a fast DM decay are not generated. Below we show several mechanisms that prevent such operators from showing up at low energy. The TeV scale generates the DM mass which can be naturally related to the supersymmetry (SUSY) breaking scale, thereby avoiding the usual $\mu$-problem. The GeV scale which controls the branching fractions of the DM decays into SM fields, is generated either by communicating
supersymmetry breaking to the dark sector indirectly through the SM [2] or through D-term mixing [3, 26, 27]. Finally, splittings between DM states may be required to avoid direct detection. Such splittings are naturally of order an MeV, thereby accommodating the inelastic DM (iDM) [28] and eXciting DM (XDM) scenarios [29]. Below we study mechanisms that can appear at each of these scales, stressing the modular nature of such models, which significantly simplifies the model building.

The models studied here predict distinctive signatures in many upcoming experiments, and unique indirect signals which will complement the direct production experiments mentioned above. For instance, if the dark sector is approximately supersymmetric, or if the dark gaugino is lighter than the dark gauge boson, $m_{\tilde{\gamma}d} \lesssim m_{\gamma d}$, it typically decays into a gravitino and a SM photon. Such primary photons will show up as sharp features in the measured flux. If the DM is also charged under the SM, its decays are accompanied by primary neutrinos, again admitting a sharp and hard spectral feature. In the corresponding annihilating models, these decay channels are excluded due to the excess of primary photons or neutrinos produced, for example, at the GC. Both possibilities are studied in [30], where it was shown that current and future experiments will have the ability to measure these signatures and thereby differentiate between the annihilating and decaying DM scenarios. In sections 2 and 3 we provide detailed examples that illustrate the presence of these signatures.

The paper is organized as follows. In section 2 we discuss the tools for constructing decaying DM models. We first list the dangerous pitfalls of these models, and then discuss solutions, organized by energy scale. In section 3 we apply these tools to study four distinct example models. In 3.1 we show the simplest $U(1)$ model, which is UV completed in 3.2. In 3.3 we construct a model where the DM is charged under the SM and decays into primary neutrinos, and in section 3.4 we demonstrate how one can evade direct detection without splitting the DM multiplets. In section 4 we discuss the cosmology of these models. In particular, we show that a supersymmetric dark sector can have long lived gauginos which decay into photons, without violating constraints from big bang nucleosynthesis (BBN). We conclude in section 5. In appendix A we revisit the symmetries of the four models, showing that these forbid the presence of any dangerous operators.
2. Tools for Modeling Decaying Dark Matter

In this section we describe our strategy for building models of hidden sector decaying DM. After briefly introducing our framework and notations, we list several potential dangers for models of this type, which arise from cosmological and experimental constraints. We then introduce a series of model building tools, organized by energy scale, that address these dangers and can be used to build viable models. We stress that these tools are modular, and can be used to construct a variety of models. We demonstrate the use of these tools to build some example models in section 3.

2.1. Framework

We consider models where weak-scale DM, $\chi$, decays into a hidden sector with a gauge group, $G_d$, through a dimension-6 operator suppressed by the GUT scale, $M_{\text{GUT}}$. We take this ‘dark sector’ to be weakly coupled with a GeV mass gap, in resemblance to the annihilating models proposed in Ref. [9]. Throughout this paper we work in the supersymmetric framework which comes naturally with GUT models, and can stabilize the GeV scale. Furthermore, we assume the breaking of supersymmetry to be mediated through gauge interactions, allowing for a low scale of mediation. This assumption can be somewhat relaxed, if the breaking is sequestered from the dark sector [31]. The dark sector consists of massive gauge bosons, $\gamma_i^d$, gauginos, $\tilde{\gamma}_i^d$, Higgses, $h_i$, and Higgsinos, $\tilde{h}_i$. We couple it to the SM through gauge kinetic mixing, and consequently dark sector particles decay through the mixing to SM particles. Due to the low dark gauge boson mass, it decays predominantly into light leptons. Within this framework, DM decays can naturally explain the PAMELA and FERMI measurements. Our notations are summarized in Fig. 1.

2.2. Model Building Dangers

- **Dimension-5 DM Decay**

  As we discuss in section 2.3, dimension-6 decay operators suppressed by the GUT scale induce DM decays with a lifetime of $\tau_6 \approx 10^{26}$ sec, the correct timescale to account for the PAMELA and FERMI signals. Alternatively, dimension-5 operators suppressed by
Figure 1: We summarize our notations, organized by energy scale. $X$ and $Y$ denote GUT scale fields that are integrated out to generate dimension-6 operators that induce DM decays. We use $\langle H \rangle$ to denote a GUT scale VEV, which can partially break the dark gauge group, $G'_d \rightarrow G_d$, as demonstrated in section 3.2. The DM may be composed of multiple species, $\chi_i$, with mass at the TeV scale. This scale is naturally generated through the VEV of a singlet, $S$, that communicates with the SUSY breaking sector. Here, $N_i$ denote electroweak scale fields that participate in the mechanism that generates a DM mass splitting. Such splittings can help evade the bounds from direct detection, as we discuss in section 2.6. The dark gauge group, with gauge bosons $\gamma^i_d$, is entirely broken at the GeV scale by the VEVs of light Higgses. DM decays by dimension-6 operators into these GeV scale states. We use $h_i$ to denote light fields charged under the dark gauge group, at least some of which will receive VEVs, and we use $n$ to denote a light singlet.
the GUT scale correspond to a lifetime of $\tau_5 \simeq 1$ sec, and must be avoided.

- **Sommerfeld Enhancement**

  If DM is directly charged under the light dark sector, the annihilation cross-section is Sommerfeld-enhanced \cite{9}. It is important that this enhancement is not too large, since there are various strong constraints on the annihilation rate. These include constraints from gamma rays and neutrinos from the Galactic Center and Galactic Ridge (GR) \cite{16, 6, 17}, diffuse gammas from extragalactic DM annihilations \cite{18}, and modified CMB radiation from DM annihilation during recombination \cite{19}.

- **Direct Detection**

  There are strong limits from direct detection on models in which a weak-scale DM couples elastically to a light gauge boson that kinetically mixes with the photon. One finds a DM-nucleon cross-section of the order \cite{9}:
  \begin{equation}
  \sigma_0 \simeq 10^{-37} \text{cm}^2 \left( \frac{\epsilon}{10^{-3}} \right)^2 \left( \frac{\alpha_d}{0.01} \right) \left( \frac{m_{\gamma_d}}{1 \text{ GeV}} \right)^{-4},
  \end{equation}

  where $\epsilon$ parametrizes the size of the kinetic mixing. Current measurements rule out a cross-section of this size by 6 orders of magnitude \cite{32}. There are also strong limits from direct detection on DM that couples elastically to the $Z$. For example, models where DM is the neutral component of an $SU(2)_W$ doublet are excluded by 2-3 orders of magnitude \cite{33}.

- **Inelastic Capture in the Sun**

  As we discuss in section 2.6, one way to avoid the above constraints from direct detection is to split the mass between the DM states, $\delta M_{DM} \gtrsim 100$ keV, and couple inelastically to $\gamma_d$ or $Z$ \cite{23}. It was recently demonstrated that if $\delta M_{DM} \simeq 100 - 500$ keV, there are strong constraints on the inelastic capture of DM in the sun which is followed by annihilations into $W^+W^-$, $ZZ$, $\tau^+\tau^-$, or $t\bar{t}$ \cite{34}. This constraint is particularly important if DM is charged under $SU(2)_W$.

- **Long-Lived GeV Scale Fields**

  The dark sector may contain light long-lived fields, and one must make sure that their cosmology is safe. On the one hand, stable particles must not overclose the universe,
$\Omega_\gamma h^2 < 0.1$. On the other hand, the dark sector may contain long-lived particles that decay electromagnetically through the kinetic mixing. For such decays, lifetimes of order $\tau \simeq 10^4 - 10^{12}$ sec are constrained by Big Bang Nucleosynthesis [35] and decays after recombination, $\tau \gtrsim 10^{13}$ sec, are constrained by diffuse gamma rays [36].

- **Long-Lived Colored Particles**

If the DM is charged under the GUT gauge group, then there is a colored component $\chi_3$. There are strong constraints on colored particles with lifetimes $\tau \gtrsim 10^{17}$ sec as they form exotic atoms [37]. $\chi_3$ must therefore have a much shorter lifetime than $\chi$.

### 2.3. GUT Scale: Decay Operators

We consider models where weak-scale DM decays through dimension-6 operators suppressed by the GUT scale, into the dark sector. The GeV-scale dark fields then decay through gauge kinetic mixing to leptons. We focus on two possible scenarios, both of which include multiple, non-degenerate DM states: (i) One of the TeV fields receives a VEV, breaking part of $G_d$ at the weak scale, and (ii) none of the states obtain VEVs and $G_d$ is fully broken at the GeV scale. For scenario (ii), transitions between the TeV fields can be induced by the three body decay operators,

$$
\frac{1}{M^2_{\text{GUT}}} \int d^4\theta \, \chi_1^\dagger \chi_2 h_1^\dagger h_2, \quad \frac{1}{M^2_{\text{GUT}}} \int d^2\theta \, \chi_1 \bar{\chi}_2 W^2_d, \quad \frac{1}{M^2_{\text{GUT}}} \int d^2\theta \, \chi_5 f W^2_d. \quad (2.2)
$$

For the first two operators, $\chi_1$ and $\chi_2$ are both weak-scale with $m_{\chi_2} > m_{\chi_1}$. Consequently, the DM is dominantly composed of $\chi_2$ which generically has a larger density than $\chi_1$. For the third operator $\chi$ is a $5$ of $SU(5)_{\text{SM}}$. We will consider examples that generate each of these operators in section [3]. The decay rate of these operators is given parametrically by:

$$
\tau \simeq \left( \frac{M^5_D}{16\pi^2 M^4_{\text{GUT}}} \right)^{-1} \simeq 10^{26} \text{sec} \left( \frac{M_{\text{DM}}}{1 \text{TeV}} \right)^{-5} \left( \frac{M_{\text{GUT}}}{5 \times 10^{15} \text{GeV}} \right)^4. \quad (2.3)
$$

This is the correct timescale to account for the PAMELA and FERMI signals, as was first noticed by Ref. [11]. For scenario (i), two body decays will typically dominate. An example operator that we will consider in section [3.4] follows from inserting a $\langle \chi_1 \rangle$ VEV into the second operator of Eq. (2.2),

$$
\frac{1}{M^2_{\text{GUT}}} \int d^2\theta \, \langle \chi_1 \rangle \bar{\chi}_2 W^2_d. \quad (2.4)
$$
As mentioned in the introduction, in the $M_{\text{GUT}} \to \infty$ limit, the DM is completely stable. This is typically achieved by a $\mathbb{Z}_2$ discrete symmetry under which $\chi_i$ and $\bar{\chi}_i$ are charged. The superpotential at the GUT scale breaks this symmetry, destabilizing the DM. We demonstrate the existence of these symmetries in the models of section 3. Still, such symmetries do not ensure that the DM is sufficiently long-lived. Indeed, when integrating out GUT fields to generate the above dimension-6 decays, it is important to make sure that no dimension-5 decay operators are generated. This can follow from symmetries at the GUT scale. For each specific model of section 3 we identify these symmetries in appendix A. To demonstrate that this is possible, we now discuss two general mechanisms for generating dimension-6 decays that do not generate dimension-5 decays. One simple possibility is that the hidden sector gauge group is broken at the GUT scale, $G'_d \to G_d$, without breaking supersymmetry. By going to Unitary gauge and integrating out the massive $G'_d/G_d$ vector superfields, it is simple to check that dimension-6 decay operators, of the form of the first operator in Eq. (2.2), are generated in the Kähler potential [38]. Moreover, if the DM and light Higgses have a canonical Kähler potential at the GUT scale, no dimension-5 terms are generated. We will discuss this in more detail for a specific example in section 3.2.

A second way to generate dimension-6 operators without generating dimension-5 ones is by coupling canonical GUT-scale fields to the DM in a chiral manner,

$$W \supset M_{\text{GUT}}X\bar{X} + X\chi h.$$  \hspace{1cm} (2.5)

Integrating out $X$ and $\bar{X}$, and allowing for a weak-scale VEV for the DM, results in the dimension-6 Kähler potential operator in Eq. (2.4). It is straightforward to see from the equations of motion that no dimension-5 operators are generated in the superpotential or Kähler potential. More generally, global symmetries prevent quantum corrections from generating dimension-5 decays in the Kähler potential, as we discuss in appendix A.

When the DM is charged under $SU(5)_{\text{SM}}$, as for the third operator of Eq. (2.2), one must ensure that its colored partner decays on a timescale shorter than the current age of the universe. For example, suppose that the DM is the neutral component of the doublet of a $5 + \bar{5}$. The model is viable if the triplet can decay through a dimension-5 operator that does not induce DM decays. This is straightforward to achieve since the triplet is typically heavier than the doublet at the weak scale, due to the RG evolution of their masses. Example triplet
Figure 2: A sample DM 3-body decay induced by one of the two last operators of Eq. (2.2). The DM decays to a GeV-scale gauge boson, gaugino, and a neutrino or the lighter field, \( \chi_1 \). The gauge boson decays through the kinetic mixing to a lepton pair and the gaugino decays through the kinetic mixing to a photon and gravitino, assuming that the gaugino is lighter than, or degenerate with, the dark photon. The resulting leptons can explain the PAMELA and FERMI excesses while the gamma rays and neutrinos lead to hard and sharp spectral features that can be probed by upcoming experiments \[30\].

Decay operators include:

\[
\frac{1}{M_{\text{GUT}}} \int d^2 \theta \chi 2 \bar{5}_f^2, \quad \frac{1}{M_{\text{GUT}}} \int d^2 \theta \chi 10^f s, \quad \frac{1}{M_{\text{GUT}}} \int d^4 \theta \chi \bar{5}_f^s, \quad (2.6)
\]

where in the first operator the triplet partner decays into the DM while in the other two the triplet decays into a singlet, \( s \), with \( m_{\chi_2} < m_s < m_{\chi_3} \).

2.4. Weak Scale: Dark Matter Mass and Communicating SUSY Breaking

As we discuss in the sections 2.5 and 5, sharp spectral features in the photon flux may exist, depending on the light dark spectrum. As a consequence, the low lying excitations, and indirectly the SUSY-breaking effects in that sector, may be probed in the near future \[30\]. Below, we briefly discuss the possible effects which may influence the spectrum.

In our framework, the DM has a weak-scale mass. A GUT-scale one can be avoided by imposing a PQ or R symmetry that is spontaneously broken at the weak scale by a neutral scalar, \( S \). The DM mass term then takes the form,

\[
y \langle S \rangle \chi \bar{\chi} . \quad (2.7)
\]
This is similar to the well-known $\mu$-problem, and we present no new solution. Instead, we simply assume the coupling above, with a VEV induced by the SUSY-breaking sector. In principle, $S$ may have a soft mass which arises from coupling to the SUSY-breaking sector. We distinguish between two cases,

- $\chi$ is not charged under $G_d$ and couples to the light sector only through GUT suppressed couplings. Examples of such a scenario are given in sections 3.3 and 3.4. In this case, SUSY-breaking effects are primarily communicated to the light sector through the kinetic mixing, as is worked out in [39]. The leading contribution to the soft mass squared of the light Higgses is generated as a threshold effect at the gauge messenger scale and is proportional to $\epsilon^2$,

$$\delta m^2_h \simeq \epsilon^2 \frac{g_d^2}{g_Y^2} M^2_{\tilde{E}} = (100 \text{ MeV})^2 \left( \frac{\epsilon}{5 \times 10^{-4}} \right)^2 \left( \frac{g_d}{g_Y} \right)^2 \left( \frac{M_{\tilde{E}}}{200 \text{ GeV}} \right)^2. \quad (2.8)$$

Here $g_Y$ is the hypercharge gauge coupling and $M_{\tilde{E}}$ is the soft mass of the right-handed selectron. As we show in the next subsection, this is parametrically smaller by one power of $\epsilon$ compared to the supersymmetric mass squared of the dark vector boson. The corresponding contribution to the gaugino soft masses is even smaller [39] and may be neglected. The GeV scale, which we discuss below, is therefore approximately supersymmetric.

- $\chi$ is charged under $G_d$ and may couple directly to the light Higgses. Such examples are given in sections 3.1 and 3.2. Here the SUSY-breaking effects can be communicated either through $S$ or through the kinetic mixing as discussed above. Below, for simplicity we assume the latter. We stress that $S$ can be naturally supersymmetric and still solve the $\mu$-problem. This can be achieved for example through retrofitting [40]. We postpone the details of such a scenario to future work. If, on the other hand, $S$ is accompanied by a soft mass, SUSY breaking effects in the light sector are expected to be of order GeV, and therefore dominate over the kinetic mixing contributions.

In the above discussion we assumed the absence of TeV-scale messengers that couple to both the dark sector and SM. If such states exist, SUSY breaking is mediated as in gauge mediation and the supermultiplets are split at the GeV scale [2]. Finally, we note that
when the DM is approximately supersymmetric, both the fermion and boson components are cosmologically long-lived and constitute order one fractions of the DM relic density. On the other hand, when there is large splitting within the DM supermultiplet, either the fermion or scalar component dominates the relic density, in a model-dependent fashion. The analysis that follows does not depend on the spin of the DM.

2.5. GeV Scale: Breaking the Dark Sector

In correspondence to the discussion above, there are two ways to naturally generate the GeV scale in the dark sector. One is with the use of D-term mixing which results from supersymmetric kinetic mixing [41]. In such a case, the light dark sector is approximately supersymmetric, at the GeV scale. We review this mechanism below. The second way to generate the GeV scale is by communicating weak scale SUSY breaking as mentioned above. For simplicity, below we only consider a $U(1)_d$ model with the D-term mixing mechanism. The approximate supersymmetry in the light dark sector simplifies the analysis, since we do not need to consider GeV-scale soft terms. Nevertheless, we stress that this is only a simplifying assumption, which can easily be relaxed. Indeed, introducing GeV SUSY-breaking may change the low energy spectrum and consequently the astrophysical signatures, but does not affect the discussions below in a significant way.

The GeV scale of our theory resembles that of [9, 2]. We assume that the SM and dark sector interact with each other through gauge kinetic mixing. The kinetic mixing between $U(1)_d$ and hypercharge is given by:

$$-\frac{\epsilon}{2} \int d^2 \theta \, W_d W_Y.$$  

(2.9)

$\epsilon$ is naturally of order $10^{-3} - 10^{-4}$ and arises from integrating out heavy fields charged under both sectors. Supersymmetric kinetic mixing of this size automatically generates the GeV scale in the dark sector [3, 27]. To see this, we expand Eq. (2.9) in components. One finds D-term mixing, $V \supset \epsilon D_{\text{dark}} D_Y$, which upon electroweak symmetry breaking generates a Fayet-Illiopolous (FI) term for $U(1)_d$. Such a term triggers the breaking of the dark sector at the GeV scale:

$$m_{\gamma_d}^2 = \epsilon g_d \langle D_Y \rangle = (1 \text{ GeV})^2 \left( \frac{\epsilon}{5 \times 10^{-4}} \right) \left( \frac{g_d}{0.35} \right) \left( \frac{\sqrt{\langle D_Y \rangle}}{75 \text{ GeV}} \right)^2.$$  

(2.10)

12
As discussed in the previous section, the dark sector spectrum is approximately supersymmetric when kinetic mixing is the only form of low-energy communication between the two sectors.

When produced, the GeV scale particles can decay through the kinetic mixing to SM particles. The dark photon, $\gamma_d$, decays directly through the kinetic mixing to pairs of SM leptons, $l^+l^-$. If the dark Higgs, $h$, is too light to decay to two dark photons, it decays at one loop to lepton pairs. Both of these decays are prompt on galactic scales for typical values of the parameters:

$$\gamma_d \to l^+l^- \quad \tau \simeq (\epsilon^2 \alpha_{\text{EM}} m_{\gamma_d})^{-1} \simeq 10^{-16} \text{ sec}, \quad h \to l^+l^- \quad \tau \simeq 4\pi (\epsilon^4 \alpha_{\text{EM}}^2 m_h)^{-1} \simeq 10^{-6} \text{ sec}, \quad (2.11)$$

where for the last step we have chosen the representative values $m_{\gamma_d}, m_h = 1$ GeV and $\epsilon = 5 \times 10^{-4}$.

The decay of the lightest fermion in the dark sector has important consequences for the astrophysical signals of our model. If the lightest fermion mixes with the dark gaugino, it can always decay through the kinetic mixing to the SM photon and the gravitino $\tilde{\gamma}_d \to \gamma \tilde{G}$. The lifetime is found to be:

$$\tau_{\tilde{\gamma}_d \to \gamma \tilde{G}} \simeq \epsilon^{-2} \left( \frac{m_{\tilde{\gamma}_d}^5}{16\pi F^2} \right)^{-1} 10^4 \text{ sec} \left( \frac{5 \times 10^{-4}}{\epsilon} \right)^2 \left( \frac{1 \text{ GeV}}{m_{\tilde{\gamma}_d}} \right)^5 \left( \frac{\sqrt{F}}{100 \text{ TeV}} \right)^4. \quad (2.12)$$

This decay is prompt on galactic scales for low-scale SUSY breaking, and leads to a hard gamma ray signature. If the lightest fermion is significantly heavier than its bosonic superpartner, it can also decay to its superpartner and the gravitino, $\tilde{\gamma}_d \to \gamma_d \tilde{G}$, or $\tilde{h} \to h \tilde{G}$, with lifetime:

$$\tau_{\tilde{\gamma}_d \to \gamma_d \tilde{G}} \simeq \left( \frac{m_{\tilde{\gamma}_d}^5}{16\pi F^2} \right)^{-1} \left( 1 - \frac{m_{\gamma_d}^2}{m_{\tilde{\gamma}_d}^2} \right)^{-4}$$

$$= 3 \times 10^{-3} \text{ sec} \left( \frac{1 \text{ GeV}}{m_{\tilde{\gamma}_d}} \right)^5 \left( \frac{\sqrt{F}}{100 \text{ TeV}} \right)^4 \left( 1 - \frac{m_{\gamma_d}^2}{m_{\tilde{\gamma}_d}^2} \right)^{-4}. \quad (2.13)$$

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1We only consider models where gravity mediation is not the dominant source of scale generation in the dark sector, such that $m_{\tilde{G}} \simeq F/M_p < \text{GeV}$. This is the case for the general framework of low-scale gauge mediation.
Due to the phase space suppression above, the decay into the dark photon is subdominant when the dark sector is approximately supersymmetric as in our case. Consequently DM decays into $\gamma_d$ and $h$ produce hard leptons, while decays into $\tilde{\gamma}_d$ produce hard gamma rays. Since the DM decays into both bosonic and fermionic states in the dark sector, we are led to the generic conclusion that the hard lepton signals may be correlated with hard gamma ray signals. These signatures are studied in detail in \cite{30}.

The dark spectrum and lifetimes are constrained by the requirement that the GeV scale cosmology is safe. We discuss the dark sector cosmology and the resulting constraints in section \ref{sec:DarkSectorCosmology}.

2.6. MeV Scale: Dark Matter Splitting

It is important for DM to evade the strong constraints on direct detection mentioned in section \ref{sec:DirectDetection}. There are three possible solutions:

1. Very small kinetic mixing, $\epsilon$, between the dark sector and the SM.

2. The DM does not directly couple to $\gamma_d$ or $Z$.

3. The DM multiplets are split.

The first solution applies when DM is charged under the light dark sector. As we can see from equation (2.1), the DM evades direct detection if the kinetic mixing is small enough, $\epsilon \lesssim 10^{-6}$. Interestingly, as discussed in section \ref{sec:DirectDetection}, mixing of this size may be insufficient to keep the dark sector in thermal equilibrium thereby interfering with the usual WIMP cosmology.

The second solution can be realized by keeping the DM neutral under both the SM and the light gauge group. For example, in section \ref{sec:NeutralDM}, we consider a $U(1)_\chi \times U(1)_d$ model where DM is charged only under $U(1)_\chi$, which is broken at the weak scale, while $U(1)_d$ is broken at the GeV scale. Kinetic equilibrium is maintained between DM and the SM through double kinetic mixing, as we discuss in section \ref{sec:KineticEquilibrium}.

The third possibility is to introduce a DM splitting $\delta M_\chi \gtrsim 100$ keV. Indeed in such a case the DM couples inelastically together with an excited state, $\chi'$, to the dark gauge boson,
\( \gamma_d \), or \( Z \), suppressing direct detection \[23\]. This bound is all that is necessary to evade the current constraints, but there are two special values for the splitting that are of experimental significance. If the splitting is of order 100 keV, the DAMA signal \[42\] can be reconciled with the bounds from other experiments through the inelastic DM scenario (iDM) \[28\] (see however \[43\]). If, on the other hand, the splitting is of size \( \delta M_\chi \gtrsim 1 \text{ MeV} \), it can account for the anomalous production of positrons observed by the INTEGRAL satellite close to the Galactic Center \[44\]. This is the eXciting DM (XDM) proposal \[29\] (see however \[45\]). If there are enough DM states, both scenarios can be realized.

Suppose first, that DM is charged under \( G_d \). Splittings with the right parametric size for iDM or XDM are generated by direct couplings between DM and the light Higgses \[27\]:

\[
W \supset S (y_N N^2 + y_\chi \chi \bar{\chi}) + y_{\text{split}} N \chi h.
\]

(2.14)

As discussed above, we assume that \( S \) interacts with the SUSY breaking sector and gets a weak scale VEV. \( N \) is a singlet and stability of DM requires \( N \) to be heavier than \( \chi \), \( |y_N| > |y_\chi| \). In this limit, we integrate out \( N \) and find a DM splitting of size:

\[
\delta m_\chi = \frac{y_{\text{split}}^2 \langle \bar{h} \rangle^2}{4 m_N} = 100 \text{ keV} \left( \frac{y_{\text{split}}}{1 \text{ GeV}} \right)^2 \left( \frac{\langle \bar{h} \rangle}{1 \text{ GeV}} \right)^2 \left( \frac{m_N}{2.5 \text{ TeV}} \right)^{-1}.
\]

(2.15)

If \( \chi \) is charged under the SM, the last term in Eq. (2.14) can be replaced with a coupling to the SM Higgs. In that case the splitting is expected to be larger.

There is an important caveat to the above mechanism. If \( S \) gets a weak-scale \( F \)-term, the \( \chi \) scalars receive a weak-scale splitting and the dark gauge boson couples across the splitting. This SUSY-breaking splitting provides another mechanism for evading the constraint from direct detection, but the splitting is generically too large to account for iDM or XDM. If we wish to include these proposals, \( S \) must receive a weak-scale VEV but should have no \( F \)-term to leading order\(^2\). Consequently, \( S \) cannot be the NMSSM singlet. We note that there are more options other than Eq. (2.14) for generating an MeV size DM splitting, and we will employ a slightly different mechanism in appendix A.

Another possibility is that the DM is charged under a non-Abelian hidden sector. In this case, the splittings among the DM multiplet are generated radiatively after dark sector

\(^2\)An alternative possibility is to introduce another source of SUSY breaking that lifts both \( \chi \) scalars above the fermions so that the fermions constitute DM.
symmetry breaking \cite{9}. In practice, non-Abelian dark sectors are more difficult to construct and often require elaborate Higgs sectors \cite{3}. In the explicit models that we study below, we will instead focus on the simplest possibility of a $U(1)_d$ hidden sector at low energies. This is for illustrative purposes only, and more complicated dark sectors remain a valid possibility.

3. Models

In this section we use the tools described above to construct four explicit models of hidden sector decaying DM. There are many possible models within this framework, and these should be viewed only as illustrative examples. The models are roughly ordered by increasing complexity. We begin with a minimal $U(1)_d$ dark sector which includes all of the main ideas. The second model embeds $U(1)_d$ into $SU(2)_d$ at the GUT scale. The $SU(2)_d$ breaking generates dimension 6 DM decay. For the third model, we consider DM charged under the SM, and find that there is always an associated hard neutrino signal. The reader who is primarily interested in the new correlated signals that we propose may want to skip directly to this model. All four models can produce hard gammas that are correlated with the astrophysical leptons, but only the third model also produces hard neutrinos. Finally, we consider a $U(1)_\chi \times U(1)_d$ model, where no splitting is required to evade direct detection. In section 4 we discuss the constraints that cosmology places on these models. In appendix A we discuss some further technical details for each model.

In the last two models the DM is not charged under the GeV-scale dark sector. Consequently no Sommerfeld enhancement is present at all, so the astrophysical constraints are automatically avoided. Such models are in sharp contrast to the annihilating models of \cite{9}.

3.1. $U(1)_d$: The Minimal Model

We begin by considering the simplest possibility, $G_d = U(1)_d$. This model captures the main ideas of our framework and serves as an example for the models that follow. We assume a kinetic mixing between $U(1)_d$ and hypercharge, as in equation (2.9). The field content is listed in table 1 and the setup is illustrated in Fig. 3. All fields are assumed to have canonical Kähler potential at the GUT scale. In order to stress the modularity of the model, we split
Figure 3: The setup of our minimal model. DM is charged under the hidden sector, $U(1)_d$, and decays through dimension-6 GUT scale suppressed operators into the dark sector gauge multiplet. $U(1)_d$ kinetically mixes with hypercharge, and this kinetic mixing has three important effects: (i) D-term mixing causes $U(1)_d$ to break at the GeV scale, (ii) dark gauge bosons decay through the kinetic mixing to leptons while decays into antiprotons are kinematically forbidden, and (iii) DM stays in kinetic equilibrium with the SM through the kinetic mixing, allowing for the usual ‘WIMP Miracle’ cosmology (see section 4).

Table 1: The matter content of the $U(1)_d$ model, where $i = 1, 2$. We stress the modularity of the model by grouping the fields according to their scales.

| GUT | TeV | GeV |
|-----|-----|-----|
| $X$ | $\chi_i$ | $h$ |
| $\bar{X}$ | $\bar{\chi}_i$ | $\bar{h}$ |
| $Y$ | $S$ | $n$ |
| $\bar{Y}$ | $N_i$ |          |

up the superpotential into three pieces,

$$ W = W_{\text{decay}} + W_{\text{DM}} + W_{\text{split}}. \quad (3.1) $$

The first term leads to DM decay:

$$ W_{\text{decay}} = (M_{\text{GUT}} + X) \bar{Y}Y + M_{\text{GUT}}X\bar{X} + \bar{X}\chi_1\bar{\chi}_2. \quad (3.2) $$

Integrating out the GUT scale fields generates the second operator of equation (2.2), at one loop [13]. Meanwhile, it is straightforward to see from the equations of motion that no dimension-5 decays are generated in the superpotential.
The second term of equation (3.1) determines the DM and dark sector spectrum:

$$W_{\text{DM}} = S (y_1 \chi_1 \bar{\chi}_1 + y_2 \chi_2 \bar{\chi}_2) + n h \bar{h}.$$  \hspace{1cm} (3.3)

We assume that $S$ obtains a weak-scale VEV, possibly through interactions with the SUSY breaking sector. The different Yukawa couplings $y_{1,2}$ generate masses for $\chi_1$ and $\chi_2$ with $m_{\chi_2} > m_{\chi_1}$. Both $\chi_i$ are stable on cosmological timescales and contribute to the relic density, however, DM is mostly composed of $\chi_2$, whose larger mass leads to a smaller annihilation cross-section and therefore to a larger abundance. The dimension-6 decay operator in Eq. (2.2), leads to three body decays of $\chi_2$ into $\chi_1$ and dark sector gauge bosons, $\gamma_d$, and/or gauginos, $\tilde{\gamma}_d$. $\gamma_d$ and $\tilde{\gamma}_d$ then decay to SM leptons and photons through the channels described in section 2.5.

At the GeV scale, this model resembles the low-energy $U(1)$ construction of Ref. [27]. The D-term mixing, described in section 2.5, generates an effective FI term for the $U(1)_d$, which triggers one of the light Higgses to get a VEV at the GeV scale. Without loss of generality, we take $\bar{h}$ to be the one with a non-vanishing VEV. Expanding around $\langle \bar{h} \rangle$, $h$ and $n$ obtain a GeV mass through the last term of Eq. (3.3). Consequently, all fields are lifted, forming a GeV scale mass gap.

The last term of Eq. (3.1) corresponds to two copies of the splitting mechanism described by Eqs. (2.14),(2.15),

$$W_{\text{split}} = \sum_{i=1}^{2} (SN_i^2 + N_i \chi_i \bar{h}).$$ \hspace{1cm} (3.4)

Splittings are generated for both $\chi_i$, evading the constraints from direct detection. The two splittings are of different sizes, and we note that both iDM and XDM can be incorporated in this model if $\delta M_{\chi_1} \sim 100$ keV and $\delta M_{\chi_2} \sim 1$ MeV, or vica versa. It would be interesting to conduct a more detailed study of this multi-species DM model to see if indeed the two scenarios can be accommodated.

That $\chi_1$ and $\chi_2$ are long-lived follows from an unbroken $\mathbb{Z}_2^1 \times \mathbb{Z}_2^2$, as $M_{\text{GUT}} \rightarrow \infty$. $\chi_i$, $\bar{\chi}_i$, and $N_i$ are charged under $\mathbb{Z}_2^2$, respectively. This symmetry is broken by Eq. (3.2), resulting in DM decays. In appendix A we verify that dimension-5 decays are forbidden by a GUT scale symmetry.
Table 2: The matter content for the $SU(2)_d \rightarrow U(1)_d$ model.

We conclude by remarking that with this field content, the absence of Landau poles below the GUT scale places a bound on the dark gauge coupling at the GeV scale, $\alpha_d \lesssim 1/30$.

3.2. $SU(2)_d \rightarrow U(1)_d$: GUT Scale Symmetry Breaking

We now consider a UV completion of the previous model, by embedding $U(1)_d$ into $SU(2)_d$ which is broken at the GUT scale. In the following discussion, we focus on the two new features of this model: (i) heavy gauge bosons generate the dimension-6 DM decay, and (ii) the low-energy theory contains split $SU(2)_d$ multiplets. The field content is summarized in table 2. Again, we assume a canonical Kähler potential and group the terms in the superpotential according to their role,

$$ W = W_{\text{decay}} + W_{\text{GUT}} + W_{\text{DM}}. \quad (3.5) $$

The first term above, triggers the GUT-scale breaking $SU(2)_d \rightarrow U(1)_d$,

$$ W_{\text{decay}} = f(H) + HX^2. \quad (3.6) $$

Here $H = H^a T^a$ is a triplet and $T^a = \sigma^a/2$ are the generators of $SU(2)_d$. In most of what follows we suppress color indices. We take $f(H)$ to be a potential for $H$ with a minimum at $\langle H \rangle = M_{\text{GUT}} T^3$. Consequently, $X$, which is introduced to cancel $SU(2)$ anomalies \cite{46}, obtains a GUT-scale mass and is integrated out.

To see the effect of the breaking, we integrate out the broken $SU(2)_d/U(1)_d$ generators. Going to the Unitary gauge and solving for the massive vector superfields, $V_\pm = V_1 \mp i V_2$, one finds an additional contribution to the Kähler potential \cite{38},

$$ \delta K_{\text{eff}} = - (\varphi_i^\dagger T^+ \varphi_i) \lambda_\pm^{-1} (\varphi_j^\dagger T^- \varphi_j), \quad (3.7) $$
where $\varphi_i$ collectively denote all fields (subject to the Unitary gauge constraint), $T^\pm = T^1 \pm iT^2$ are the broken generators in the corresponding representation and

$$\chi^\pm = \frac{1}{2} H\dagger \{T^+, T^-\} H = M^2_{\text{GUT}}. \quad (3.8)$$

Substituting the DM states, $\chi_\alpha = (\chi_1, \chi_2)$, $\bar{\chi}^\alpha = (\bar{\chi}_1, \bar{\chi}_2)$ and light Higgs, $h_\alpha = (h_1, h_2)$ into Eq. (3.7) one finds the contributions,

$$-\frac{1}{M^2_{\text{GUT}}} \int d^4 \theta \left( \chi_1^\dagger \chi_2^\dagger h_2^\dagger h_1 + \bar{\chi}_1^\dagger \bar{\chi}_2^\dagger h_1^\dagger h_2 \right) + (1 \rightarrow 2). \quad (3.9)$$

These operators are precisely of the form of the first operator in Eq. (2.2). As in the $U(1)_d$ model, we will require $m_{\chi_2} > m_{\chi_1}$. In this case, DM is mostly composed of $\chi_2$ and Eq. (3.9) generates 3-body decay of $\chi_2$ into $\chi_1$ and the lights Higgses $h_1$ and $h_2$. We again assume that there is kinetic mixing between the low-energy $U(1)_d$ and hypercharge, generated by integrating out fields charged under both the dark sector and SM, so that D-term mixing generates a GeV-scale VEV for $h_2$, which is eaten by the gauge multiplet and decays to SM leptons and photons as we describe in section 2.5.

In order to minimize the low-energy field content so that it matches the $U(1)_d$ model of the previous section, and in order to give different masses to $\chi_1$ and $\chi_2$, we work with split $SU(2)_d$ multiplets. We can split the triplets $\Phi$ and $n$ by coupling them to $H$ and GUT scale singlets, denoted by $S_{\Phi}$ and $s_n$:

$$W_{GUT} = \text{Tr} \left[ g(H) \left( \Phi \bar{\Phi} + S_{\Phi} \bar{\Phi} + n' \bar{n} + s_n \bar{n} \right) \right]. \quad (3.10)$$

For generic $g(H)$, the VEV of $H$ generates GUT scale masses for all component fields except for one linear combination of $\Phi_3$ and $S_{\Phi}$, which we denote by $S$, and one linear combination of $n'_3$ and $s_n$, which we denote $n$. The specific linear combinations that remain light depend on $g(H)$. In appendix A, we use a discrete symmetry to prove that $S$ and $n$ remain light and to show that dimension-5 DM decays can be forbidden for generic superpotentials.

The low-energy theory is dictated by the superpotential terms:

$$W_{\text{DM}} = (\Phi + S\Phi)\chi\bar{\chi} + (n' + s_n)h^2. \quad (3.11)$$

3There is in general a different polynomial of $H$ in front of each term of Eq. (3.10), which we have suppressed to keep our notation compact.
Figure 4: A model with DM charged under the SM. DM is the neutral component of a $5 + \bar{5}$ representation of $SU(5)_{SM} \supset SU(3)_C \times SU(2)_W \times U(1)_Y$. DM decays through a dimension-6 operator into the gauge multiplet of $U(1)_d$, which kinetically mixes with hypercharge. The conservation of hypercharge (at the GUT scale) implies that this decay must be accompanied by the associated production of a neutrino. This results in a primary neutrino spectrum that is correlated with the leptonic cosmic rays, and will be tested by upcoming experiments such as IceCube/DeepCore [30].

After the $SU(2)_d$ breaking splits the multiplets, the low-energy effective superpotential is of the form:

$$W_{\text{eff}} = S(y_1 \chi_1 \bar{\chi}_1 + y_2 \chi_2 \bar{\chi}_2) + nh_1 h_2,$$

with $y_i$ couplings of order one that depend on $g(H)$. The projection onto the light state $S$ results in couplings that are not $SU(2)$ invariant, and the TeV scale VEV of $S$ therefore generates different masses for $\chi_1$ and $\chi_2$. As before, the DM is long-lived because as $M_{\text{GUT}} \to \infty$, there is an unbroken $Z_{1/2} \times Z_2$ symmetry, under which $\chi_i$ and $\bar{\chi}_i$ are separately charged for $i = 1, 2$. The third term is the same as the last term of Eq. (3.3), and the low-energy dark sector is thus the same as the $U(1)_d$ model. It is also straightforward to induce small DM splittings, in order to evade the constraints from direct detection and possibly incorporate iDM and XDM. This is shown in appendix A.

3.3. $SU(5)_{SM} \times U(1)_d$: SM Charged-DM and Correlated Neutrinos

We now consider a model where DM is charged under the SSM and decays through a dimension-6 operator into the dark sector. In this model, DM itself is not charged un-
Table 3: The matter content for our model with DM charged under $SU(5)_{SM} \supset SU(3)_C \times SU(2)_W \times U(1)_Y$.

under the GeV sector, avoiding the constraints due to Sommerfeld enhancement discussed in section 2.2. We take $\chi + \bar{\chi}$ to be charged under an $SU(5)$ GUT gauge group, residing in a $5 + \bar{5}$. A schematic description of the model is shown in Fig. 4. By gauge invariance, decay into the dark sector must be accompanied by associated SM particle production. If one SM particle is produced, it must be a neutrino or Higgs. The latter produces antiprotons, which are constrained by PAMELA, and thus we focus on the possibility that DM decays produce hard neutrinos that accompany dark sector production. The discovery of such neutrinos is discussed in [30].

An important requirement for this model is that the colored partner of DM decays faster than the current age of the universe. This is because there are strong constraints on stable colored particles, as discussed in section 2.2. These constraints are evaded if the triplet DM decays through a dimension-5 operator. For this model, we assume that the canonical Kähler potential is supplemented by one irrelevant operator, generated at the GUT scale,

$$K_{DM} = \frac{1}{M_{GUT}} \int d^4 \theta \, \chi 5^\dagger_3 s_1 ,$$

(3.13)

where $s_1$ is a singlet with mass: $m_{\chi_2} < m_{s_1} < m_{\chi_3}$. This mechanism can be easily arranged since the triplet partner is expected to be heavier than the DM, due to the RG evolution of their masses below the GUT scale.

We list the field content in table 3 and again we group the superpotential terms according to their roles:

$$W = W_{\text{decay}} + W_{\text{DM}} + W_{\text{split}} .$$

(3.14)

4We thank N. Arkani-Hamed for drawing our attention to this point.
The first term generates dimension-6 DM decay using the same mechanism as our \( U(1)_d \) model of section 3.1:

\[
W_{\text{decay}} = (M_{\text{GUT}} + X)YY + M_{\text{GUT}}X \bar{X} + \bar{X} \bar{\chi} \bar{5}_f. \tag{3.15}
\]

Integrating out \( X \) and \( Y \) generates the third dimension-6 decay operator of Eq. (2.2) at one-loop:

\[
\frac{1}{M_{\text{GUT}}^2} \int d^2 \theta \frac{\alpha_d}{4\pi} \chi \bar{5}_f W_d^2. \tag{3.16}
\]

This operator results in three-body decay, with DM decaying into one neutrino or sneutrino and two dark gauge bosons or gauginos, which subsequently decay to SM leptons and photons through the operators discussed in section 2.5.

At low energies, this model resembles the constructions above:

\[
W_{\text{DM}} = S(\chi \bar{\chi} + s_1^2) + nh \bar{h} \tag{3.17}
\]

We assume that \( S \), which may be the NMSSM singlet, gets a weak-scale VEV. This generates a mass for the DM and the singlet \( s_1 \), which plays a role in the triplet decay of Eq. (3.13). As in the models above, we take \( U(1)_d \) to kinetically mix with hypercharge, and the D-term mixing generates a GeV-scale VEV for \( \bar{h} \). With no DM splitting, this model would be ruled out because the DM couples strongly to the SM \( Z \) boson. This constraint is evaded by coupling the DM to the Higgs, which generates a small splitting:

\[
W_{\text{split}} = SN^2 + \chi H_d N. \tag{3.18}
\]

Here \( N \) is a singlet that must be heavier than \( \chi \), to ensure its stability. The resulting splitting is too large to account for iDM or XDM. In fact, iDM is already ruled out for this model by the constraints from inelastic capture in the sun, as discussed in section 2.2. Finally, the DM is long-lived due to an unbroken \( Z_2 \) at the renormalizable level, under which \( \chi, \bar{\chi}, \) and \( N \) are charged.

If the DM relic density is only determined by its \( SU(2)_W \) gauge interaction, its mass is fixed to be: \( m_\chi \simeq 1.1 \) TeV. This mass is too small to fit the FERMI excess with DM decays. Fortunately, the second operator of the above splitting mechanism, Eq. (3.18), opens up a new annihilation channel into SM Higgses. This raises the DM annihilation cross-section, allowing for heavier masses which can fit FERMI. We discuss the DM relic density further in section 4.
Figure 5: A model with double kinetic mixing. DM, $\chi_2$, is charged under $U(1)_\chi$, which is broken by a different species, $\chi_1$, at the TeV scale. Decays are induced by a dimension-6 GUT suppressed operator into the $U(1)_d$ gauge multiplet, which kinetically mixes with both hypercharge and $U(1)_\chi$. This double kinetic mixing is sufficient to keep DM in kinetic equilibrium with the SM, preserving the usual WIMP cosmology (see section 4). There is no strong constraint from direct detection because the DM does not couple directly to the $Z$ or $U(1)_d$ gauge boson, and therefore no DM splitting is required.

|        | GUT $X$ $\bar{X}$ $Y$ $\bar{Y}$ | TeV $\chi_i$ $\bar{\chi}_i$ $S_i$ | GeV $h$ $\bar{h}$ $n$ |
|--------|----------------------------------|-----------------------------------|-------------------|
| $U(1)_\chi$ | 0 0 0 0 | 1 -1 0 | 0 0 0 |
| $U(1)_d$    | 0 0 1 -1 | 0 0 0 | 1 -1 0 |

Table 4: The matter content for the $U(1)_\chi \times U(1)_d$ model, where $i = 1, 2$.

3.4. $U(1)_\chi \times U(1)_d$: No Mass Splitting

We now consider a model with a $U(1)_\chi \times U(1)_d$ hidden sector. We illustrate the basic idea in Fig. 5. The DM, $\chi_2$, is charged under $U(1)_\chi$, which is broken at the weak scale by the VEV of a different species $\chi_1$. It decays through a dimension-6 operator into the $U(1)_d$ gauge multiplet. There are two advantages to this setup. First, this model does not have a strong constraint from direct detection, because DM does not couple directly to the $Z$ boson or $\gamma_d$. Therefore, unlike the previous models, no DM splitting is required. Second, there is no constraint from photon or neutrino measurements, as in the model of section 3.3, since...
DM is not charged under \( U(1)_d \). Another unique feature of this model is that 2-body decays dominate over 3-body decays because the decay operator contains a field, \( \chi_1 \), which obtains a weak scale VEV.

The field content of this model is listed in table 4. We assume a canonical Kähler potential, and we group the superpotential terms according to their role,

\[
W = W_{\text{decay}} + W_{\text{DM}}. \tag{3.19}
\]

The first term is identical to the GUT scale interactions of the \( U(1)_d \) model, Eq. (3.2),

\[
W_{\text{decay}} = (M_{\text{GUT}} + X) Y \bar{Y} + M_{\text{GUT}} X \bar{X} + \bar{X} \chi_1 \bar{\chi}_2,
\]

generating the second decay operator of Eq. (2.2). Integrating out bifundamentals generates kinetic mixing between \( U(1)_d \) and \( U(1)_\chi \),

\[
- \frac{\epsilon_d}{2} \int d^2 \theta \ W_\chi W_d, \tag{3.20}
\]

of the same size as the kinetic mixing between \( U(1)_d \) and hypercharge, \( \epsilon_d \sim \epsilon \sim 10^{-3} - 10^{-4} \). A mixing of this size is small enough to keep the \( U(1)_d \) mass gap at a GeV, but large enough to keep the \( U(1)_\chi \) sector in thermal equilibrium with the \( U(1)_d \) sector. The latter guarantees, through the double kinetic mixing, that \( U(1)_\chi \) is in thermal equilibrium with the SM. We discuss the cosmology of this model in more detail in section 4.

The terms in the superpotential relevant at low energies are:

\[
W_{\text{DM}} = S_2 \chi_2 \bar{\chi}_2 + S_1 (\chi_1 \bar{\chi}_1 + S_2^2) + n h \bar{h}, \tag{3.21}
\]

where \( S_2 \) receives a weak scale VEV from communicating with the SUSY breaking sector, giving the DM a mass. Solving for the \( F \)-term of \( S_1 \), one finds VEVs for \( \chi_1 \) and \( \bar{\chi}_1 \) of order \( \langle S_2 \rangle \). This breaks \( U(1)_\chi \) at the weak scale, and the dominant DM decay is 2-body, with a \( \chi_1 \) VEV insertion resulting in the operator of Eq. (2.4). DM is long-lived because as \( M_{\text{GUT}} \to \infty \) there is an unbroken \( \mathbb{Z}_2 \) symmetry, \( (\chi_2, \bar{\chi}_2) \to -(\chi_2, \bar{\chi}_2) \). As in the previous models, \( \bar{h} \) gets a VEV at the GeV scale due to the D-term mixing between hypercharge and \( U(1)_\chi \). The last term of Eq. (3.21) generates a GeV scale mass gap.
4. Cosmology of the Dark Sector

In this section we discuss the cosmology of the dark sector and the resulting constraints on our framework. We find constraints on the size of the kinetic mixing between the dark sector and SM, $\epsilon$, on the DM interactions, and on the spectrum of the GeV scale states. The cosmology of our model resembles the cosmology of the annihilating DM framework of Ref. [9]. For related discussions of the cosmology of GeV scale hidden sectors, see Refs. [29, 2, 27, 47, 39]. Below we include several new observations and a novel emphasis on the aspects of the cosmology that are important for decaying DM. We begin this section by discussing the relic density of DM, and end with a discussion on the cosmology of light dark sector fermions, which can decay to observable gamma rays providing a smoking gun signature of decaying DM [30].

4.1. Thermal DM Abundance

A model of DM must of course reproduce the observed relic density, $\Omega_\chi h^2 \simeq 0.1$. The ‘WIMP Miracle’ implies that the correct abundance is achieved if DM is in kinetic equilibrium with the SM when it freezes out, with a WIMP cross-section, $\langle \sigma_v \rangle \simeq 3 \times 10^{-26}\text{ cm}^3\text{ s}^{-1}$. The same cosmology applies for decaying DM, as mentioned in the introduction, since the decay rate is much longer than the age of the Universe. We now discuss how our model can satisfy these requirements.

DM retains the usual thermal history by interacting with dark gauge bosons which are in kinetic equilibrium with the SM [29, 2, 47]. The kinetic equilibrium is maintained by interacting with the SM thermal bath through the kinetic mixing, $\gamma_d \psi_{SM} \leftrightarrow \gamma \psi_{SM}$, where $\psi_{SM}$ denotes any relativistic SM particle with hypercharge. This reaction remains efficient for temperatures in the range $m_{\gamma_d} \lesssim T_{\text{kin}} \lesssim (\epsilon^2 \alpha_{\text{EM}}^2 / \pi^2 g^*_s) M_{\text{Pl}}$, where $g_*$ is the number of relativistic degrees of freedom at temperature $T_{\text{kin}}$. For the DM to be a thermal relic with a WIMP cross-section, $T_{\text{kin}}$ must be larger than the DM decoupling temperature, $T_{\text{dec}} \simeq m_\chi/20$. The thermal history therefore places a lower-bound on the size of the kinetic mixing:

$$\epsilon \gtrsim 10^{-5} - 10^{-6}. \quad (4.1)$$

There is tension between this constraint, and the constraint on $\epsilon$ from direct detection when
DM couples elastically to the dark photon, Eq. (2.1). One way to evade the constraint of Eq. (4.1) is to introduce weak scale particles charged under both the dark sector and the SM. Particles charged under both sectors can maintain kinetic equilibrium, but they must be very light, $O(100 \text{ GeV})$, in order to do so until $T_{\text{dec}}$. Another way to alleviate this tension is to introduce a DM splitting, which evades the constraint from direct detection.

The introduction of a DM splitting can change the DM annihilation cross-section in an interesting way. A splitting can be generated radiatively or through Yukawa interactions, as we discuss in section 2.6. Radiative splittings are generated after breaking non-Abelian dark sectors with specific matter content, however such models are significantly more complicated to construct [3]. A simpler alternative, when the DM is charged under $U(1)_d$, is to couple it directly to the light Higgses, as in Eq. (2.14). In addition to introducing splittings, these interactions provide the DM a direct annihilation channel into light Higgses. This Yukawa annihilation rate, $\sigma_y$, can be parametrically related to the annihilation rate into dark gauge bosons, $\sigma_g$, as

$$\frac{\sigma_y}{\sigma_g} \sim \left( \frac{m_\chi}{m_{\gamma_d}} \right)^4 \left( \frac{\delta m_\chi}{m_\chi} \right)^2 \left( \frac{0.5 \text{ GeV}}{2.5 \text{ TeV}} \right)^4 \left( \frac{m_\chi}{100 \text{ keV}} \right)^2 \left( \frac{\delta m_\chi}{100 \text{ keV}} \right)^2.$$  

(4.2)

Here $\delta m_\chi$ is the size of the DM splitting, Eq. (2.15). We see that the Yukawa annihilation channel parametrically dominates the DM relic density when $m_{\gamma_d} \lesssim 500 \text{ MeV}$ or when the splitting is sufficiently large. In this regime, the DM gauge coupling must be small in order for the DM to have the correct relic density. This implies that non-perturbative Sommerfeld enhancements to the annihilation cross-section are $\lesssim O(100)$. Decaying DM models in this regime evade the constraints from the Sommerfeld enhancements discussed in section 2.2, and annihilating models of this type cannot achieve a large enough Sommerfeld enhancement to fit FERMI [6].

A similar analysis applies when DM is charged under the SM and couples to the $Z$, as in the model of section 3.3. A splitting is required to evade the constraints from direct detection, which can be introduced by coupling the DM to the SM Higgs. This opens up a new annihilation channel of DM into SM Higgses, which allows for a larger annihilation cross-section and heavier DM masses, as discussed in section 3.3.

An interesting example that has no tension between the thermal history and direct detection, and does not require a DM splitting, is our $U(1)_\chi \times U(1)_d$ model of section 3.4.
Here, the DM is charged under $U(1)_\chi$, which is broken at the weak scale and kinetically mixes with the GeV-scale dark sector, $U(1)_d$, with mixing of order $\epsilon_d \sim 10^{-3}$. There is no strong constraint on this model from direct detection because DM does not couple directly to the $Z$ or the light dark photon. The light dark sector, $U(1)_d$, stays in kinetic equilibrium with the SM through kinetic mixing, as discussed above. The kinetic mixing between $U(1)_\chi$ and $U(1)_d$ keeps $U(1)_\chi$, and therefore the DM, in kinetic equilibrium with $U(1)_d$ through the interaction $\gamma_\chi h \leftrightarrow \gamma_d h$, with $h$ corresponding to any of the light Higgses or Higgsinos charged under the $U(1)_d$. The DM is thus kept in kinetic equilibrium with the SM through double kinetic mixing, yielding the correct relic abundance\(^5\).

4.2. The Lightest Dark Sector Fermion

The dark sector may contain light particles that are long-lived. Such fields are constrained by cosmology, as we discuss now. There is typically no constraint on light scalars and gauge bosons since both can decay through the kinetic mixing with cosmologically fast timescales, as in Eq. (2.11). An exception to this are stable light scalars due to an unbroken discrete symmetry, which we discuss below. The lightest fermion, on the other hand, must decay to the gravitino, if kinematically allowed, which can lead to cosmologically long lifetimes. In what follows, we focus on the situation where the lightest fermion mixes with the gaugino, and we consider separately the cases where it is heavier than, approximately degenerate with, or lighter than the dark gauge boson. We show that the last two scenarios require the lightest fermion to decay to a photon and a gravitino on sub-galactic length scales, leading to observable gamma ray signatures [30].

- $m_{\tilde{\gamma}_d} > m_{\gamma_d}$

This regime applies when there is sizeable SUSY breaking in the dark sector $\gtrsim$ GeV. The fermions can annihilate into dark gauge boson pairs, with cross-section $\sigma \simeq g_d^4/(8\pi GeV^2)$ which leads to an abundance $\Omega_{\tilde{\gamma}_d} h^2 \simeq 10^{-6}$. After freezeout, the fermion can decay to the dark gauge boson and a gravitino, which is kinematically allowed for

\(^5\)For this model, the DM does kinetically decouples from the SM during freezeout at $T_{kin} = m_{\gamma_x} \lesssim m_\chi$. After decoupling, the DM temperature scales as $T = T_{\tilde{\gamma}}^2/T_{kin}$, but this only modifies the relic density by an $O(1)$ amount.
low scale SUSY breaking, $\sqrt{F} \lesssim 10^9$ GeV. The dark gauge boson then decays through the kinetic mixing to leptons. The corresponding fermion lifetime is given by Eq. (2.13). For an abundance this small, there is no constraint from BBN for electromagnetic decays \cite{35}. There are, on the other hand, strong constraints on electromagnetic decays after recombination \cite{36}, however the decay discussed above always proceeds before recombination and hence evades the bound. An analogous discussion applies if the lightest dark fermion is a Higgsino that is heavier than its scalar superpartner. We conclude that the dark sector is not constrained by the lightest fermion when it is heavier than its superpartner.

- $m_{\tilde{\gamma}_d} \sim m_{\gamma_d}$

Let us now consider the regime where the dark gaugino is approximately degenerate with the dark gauge boson. This is the case when the dark sector spectrum is approximately supersymmetric, for instance when D-term mixing dominates as discussed in section 2.5. When the temperature is above $m_{\gamma_d}$, the dark sector is in kinetic equilibrium with the SM and the number density of the dark bosons and fermions are of the same order of magnitude. When the temperature drops below $m_{\gamma_d}$, the dark gauge bosons cannot be created from the thermal bath, and they decay instantly to SM leptons through the kinetic mixing, on a timescale much faster than the Hubble rate, as in Eq. (2.11). The dark gauginos, on the other hand, are long lived with abundance controlled by their available annihilation channels. As long as $m_{\tilde{\gamma}_d} > \mathcal{O}(0.85) m_{\gamma_d}$, the finite temperature allows dark gauginos to annihilate into a dark gauge boson pairs \cite{22}, with cross-section

$$\langle \sigma_{\tilde{\gamma}_d} v \rangle \simeq \mathcal{O}(0.1) \times \frac{g_d^4}{8\pi m_{\tilde{\gamma}_d}^2} \simeq 10^4 \langle \sigma_\chi v \rangle \left( \frac{g_d}{0.35} \right)^4 \left( \frac{1 \text{ GeV}}{m_{\tilde{\gamma}_d}} \right)^2,$$

where the $\mathcal{O}(0.1)$ suppression results from thermal averaging, and $\sigma_\chi$ is the DM annihilation cross-section.

For the above parameters, the resulting relic density is $\Omega_{\tilde{\gamma}_d} h^2 \simeq 10^{-5}$. The gauginos will decay to photons and gravitinos with lifetime given by Eq. (2.12). For an abundance of this size, there is no constraint from BBN on the resulting electromagnetic decays (see Fig. 9 from the first reference of \cite{35}), but the gauginos must decay before
recombination, $\tau < 10^{13}$ sec, to avoid constraints from diffuse gammas [36]. Amusingly, there is a coincidence in which the time of recombination roughly equals the amount of time it takes light to cross our Galaxy. As a consequence, the constraint from recombination guarantees that dark gauginos produced in our galaxy decay to observable gamma rays. The resulting constraint on the size of the kinetic mixing is $\epsilon \gtrsim 10^{-9}$ and fixing $\epsilon \simeq 5 \times 10^{-4}$, the constraint on the SUSY breaking scale is $\sqrt{F} \lesssim 2 \times 10^7$ GeV. A possible caveat in the above argument, is that by the time the fermions decouple, the gauge boson are already kinetically decoupled from the thermal bath. This may alter the final abundance by some (order one) amount. A better understanding requires solving the exact Boltzmann equations, which is beyond the scope of this paper.

- $m_{\tilde{\gamma}} < O(0.85) m_{\gamma_d}$

Lastly, we consider the regime where the dark gaugino is significantly lighter than the dark gauge boson, which as in the first case requires GeV-scale SUSY breaking in the dark sector. If $m_{\tilde{\gamma}} \gtrsim 0.5 m_{\gamma_d}$, a gaugino pair can annihilate into one dark gauge boson, and an $e^+ e^-$ pair, through kinetic mixing. The resulting cross-section is suppressed by $\epsilon^2$,

$$\langle \sigma_{\tilde{\gamma} d} v \rangle \simeq \epsilon^2 \alpha_{\text{EM}} \frac{g_d^4}{8 \pi m_{\tilde{\gamma}}^2} = 10^{-4} \langle \sigma_X v \rangle \left( \frac{\epsilon}{5 \times 10^{-4}} \right)^2 \left( \frac{g_d}{0.35} \right)^4 \left( \frac{1 \text{ GeV}}{m_{\tilde{\gamma}}} \right)^2.$$  \hspace{1cm} (4.4)

The abundance is $\Omega_{\tilde{\gamma}} h^2 \simeq 10^3$, and the BBN constraint now requires $\tau_{\tilde{\gamma}} \lesssim 10^4$ sec. This constraint is rather strong and can be marginally satisfied for the parameters of Eq. (2.12). We see that rather large kinetic mixing and a low SUSY breaking scale are both necessary. Again, the gaugino decays to an observable gamma ray. Finally, we note that when $m_{\tilde{\gamma}} < m_{\gamma_d}/2$, the gauginos must annihilate into $2e^+ 2e^-$, with cross-section suppressed by an additional $\epsilon^2 \alpha_{\text{EM}}$ relative to Eq. (4.4), ruling out models where the lightest fermion is lighter than $m_{\gamma_d}/2$.

To summarize our findings, we see that our model is unconstrained by the lightest fermion when $m_{\tilde{\gamma}} \gtrsim m_{\gamma_d}$, and that otherwise cosmological constraints imply that the dark gaugino must decay to gamma rays with short lifetimes compared to galactic length scales, leading to observable gamma ray signatures. These constraints are manifest as limits on the size of kinetic mixing, $\epsilon$, and the SUSY breaking breaking scale $\sqrt{F}$, as we discuss above.
We conclude this section by noting that there may be light particles that are completely stable. For example, in the model of section 3.1, $h$ and $n$ are stable, which follows from their charges under an unbroken $\mathbb{Z}_2$. If $h$ and $n$ are heavier than the dark gauge boson, they have a large annihilation cross-section which is parametrically similar to the heavy gaugino case discussed above, thus resulting in a small relic density, $\Omega_h \simeq 10^{-6}$. On the other hand, if $h$ and $n$ are lighter than the dark gauge boson, they will have a large abundance and the model is excluded. In general, light fields that are stable due to discrete symmetries must be heavier than, and annihilate into, the unstable and lighter dark sector fields.

5. Discussion

The decaying DM models proposed in this paper predict a number of signals at upcoming experiments. The light dark sector particles can be produced in colliders, resulting in lepton jets, as in the annihilating models of [9, 2, 3]. The dark sector can also be probed at low energy $e^+e^-$ colliders and fixed target experiments [4]. These direct production experiments have the potential to discover the dark sector, but probably cannot tell apart decaying and annihilating models. On the other hand, astrophysical signals can differentiate between the two scenarios and provide a complementary means to probe the dark spectrum [30]. As we discuss above, primary photons are produced when the dark gaugino is degenerate with or lighter than the dark photon. This results in a hard gamma ray spectrum that can be discovered by HESS, AGIS, and CTA and possibly FERMI [30]. Moreover, if DM is charged under the SM, as in the model of section 3.3, decays produce primary neutrinos, resulting in a hard neutrino spectrum that can be measured at upcoming experiments such as IceCube/DeepCore. The situation is distinct from the annihilating models. For those, measurements from the GC exclude the production of primary photons and neutrinos with sizeable branching fractions [6, 7, 16, 17].

We conclude with two further directions that can be explored in these models.

- It would be interesting to construct a model that is more directly related to the SUSY breaking sector. We have taken DM to receive a weak scale mass by coupling it to a singlet. Since the DM is not required to be charged under the SM or under the dark sector, another interesting possibility is for the DM to reside in the SUSY breaking
sector, for example as a pseudomodulus [48].

- The $U(1)_d$ and $SU(2)_d$ models in sections 3.1 and 3.2 respectively, include two species of DM $\chi_1$ and $\chi_2$. The existence of several species has several interesting implications. First, there can be ‘Wimponium’ [49] bound states, $\chi_1\bar{\chi}_2$ and $\chi_2\bar{\chi}_1$, which are cosmologically long-lived. Second, it may be possible to include both the iDM and XDM proposals since we have shown that both species can have MeV-sized DM splittings. The viability of these ideas requires further study.

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A. The Models: Superpotentials and Charges

In each of the models of section 3, the DM can decay through dimension-6 GUT suppressed operators into GeV states in the hidden sector. For these models to work, it is necessary that renormalizable or dimension-5 operators that allow DM decays are absent. Such dangerous decays can be forbidden by global discrete symmetries at the GUT scale. Such symmetries also forbid a GUT scale mass for the DM. In this appendix we verify that the above models are generic and safe, by presenting such global symmetries that forbid both dangerous DM decays and GUT scale masses for light fields. We also collect the full superpotentials of each model, for easy reference.
The superpotential of our minimal $U(1)_d$ model is given by:

$$ W = W_{\text{decay}} + W_{\text{DM}} + W_{\text{split}}, $$

$$ W_{\text{decay}} = (M_{\text{GUT}} + X)Y\bar{Y} + M_{\text{GUT}}X\bar{X} + X\chi_1\bar{\chi}_2, $$

$$ W_{\text{DM}} = S(\chi_1\bar{X}_1 + \chi_2\bar{X}_2) + nh\bar{h}, $$

$$ W_{\text{split}} = \sum_{i=1}^2 (SN_i^2 + N_i\chi_i\bar{h}). \quad (A-1) $$

There are several dangerous operators that are allowed by $U(1)_d$ gauge invariance. These include a GUT scale mass of the form, $\chi_i\bar{\chi}_j$, a TeV scale mass for the light Higgses, $Sh\bar{h}$, renormalizable DM decay, $\chi_2\bar{h}n$, and dimension-5 DM decay operators, $\chi_2h\bar{h}^2$. All dangerous operators of these types are forbidden by the $\mathbb{Z}_4^R \times \mathbb{Z}_4$ symmetry displayed in the upper left of table 5.

$SU(2)_d \rightarrow U(1)_d$

The superpotential of our $SU(2)_d \rightarrow U(1)_d$ model is:

$$ W = W_{\text{decay}} + W_{\text{GUT}} + W_{\text{DM}} + W_{\text{split}}, $$

$$ W_{\text{decay}} = f(H) + HX^2, $$

$$ W_{\text{GUT}} = \text{Tr} [\frac{1}{2}g(H)(\Phi\bar{\Phi} + S\Phi\bar{\Phi} + n^2\bar{n} + s_n\bar{n} + s_N\bar{N} + s_{N}\bar{N})], $$

$$ W_{\text{DM}} = (\Phi + S\Phi)\chi\bar{\chi} + (n^2 + s_n)h^2, $$

$$ W_{\text{split}} = (\Phi + S\Phi)N\bar{N} + N(\chi^2 + \bar{\chi}^2) + N\bar{h}^2. \quad (A-2) $$

$W_{\text{split}}$ is not discussed in section 3.2 and is necessary to generate a DM splitting that evades the constraints from direct detection, as described in section 2.2. We also add the final two terms to $W_{\text{GUT}}$. Once $H$ obtains a VEV, $N_3$ and $\bar{N}_3$ receive GUT scale masses while the charged components remain light. At low energies, $W_{\text{split}}$ takes the form

$$ W_{\text{split}}^{\text{eff}} = S(N_+\bar{N}_+ + N_+\bar{N}_-) + N_-(\chi_1^2 + \chi_2^2) + N_+(\chi_2^2 + \chi_1^2) + N_1\bar{h}_1^2 + N_2\bar{h}_2^2, \quad (A-3) $$

where $S$ is the light linear combination of $\Phi_3$ and $S\Phi$, as in Eq. (3.12). Expanding around the true minimum with $\langle S \rangle \sim \text{TeV}$ and $\langle h_2 \rangle \sim \text{GeV}$, we see that $\bar{N}_-$ has a tadpole term
which induces a VEV for $N_-$ of order GeV$^2$/ TeV. Consequently, $\langle N_- \rangle$ contributes to the mass of $\chi_i$ and $\bar{\chi}_i$, which splits the $\chi_i$ and $\bar{\chi}_i$ multiplets. These splittings allow the model to evade the constraints from direct detection, and to possibly incorporate the iDM and/or XDM proposals. In the upper right of table we display a $Z_{16}$ symmetry which forbids GUT scale masses for light fields and dangerous decays for DM.

In order to avoid a Landau pole below the GUT scale, the field content of this model requires that $\alpha_d \lesssim 1/100$. If DM annihilates only into light gauge fields, a gauge coupling of this size is insufficient to produce the correct DM relic density. Fortunately, $W_{\text{split}}$ introduces DM annihilations into the light Higgses, which can dominate the annihilation cross-section and lead to the correct relic density.

**SM Charged DM: SU(5)$_{\text{SM}} \times U(1)_d$**

The superpotential and Kähler term of our model with DM charged under the SM are given by:

$$
W = W_{\text{decay}} + W_{\text{DM}} + W_{\text{SM}},
$$

$$
W_{\text{decay}} = (M_{\text{GUT}} + X)Y\bar{Y} + M_{\text{GUT}}X\bar{X} + \bar{X}\chi_5 f,
$$

$$
W_{\text{DM}} = S(\chi\bar{X} + N^2 + s_1^2) + \chi H_d N + n\bar{h}h,
$$

$$
W_{\text{SM}} = SH_u H_d + 10_f \bar{5}_f H_d + 10_f^2 H_u + \frac{H_u^2 \bar{5}_f^2}{M_{\text{GUT}}},
$$

$$
K \supset \frac{\bar{X}_{5}^{\dagger} s_1}{M_{\text{GUT}}}. \quad (A-4)
$$

Here $W_{\text{SM}}$ denotes the usual $SU(5)$ GUT superpotential with Majorana neutrino masses and the NMSSM singlet for generating the $\mu$ term. Dangerous decay operators now include renormalizable Yukawa couplings between DM and the SM, such as $10_f \bar{X} H_d$. Such operators must be forbidden, and for this reason DM cannot be a fourth flavor. In table we list the charges under a $Z_2^R \times Z_3^R \times Z_6$ symmetry that forbids all dangerous decays and GUT scale masses, where the $Z_2^R$ extends the usual R-parity to the new fields.
The superpotential of our $U(1)_\chi \times U(1)_d$ model is given by:

\[
W = W_{\text{decay}} + W_{\text{DM}},
\]
\[
W_{\text{decay}} = (M_{\text{GUT}} + X) Y \bar{Y} + M_{\text{GUT}} X \bar{X} + \bar{X} \chi_1 \bar{\chi}_2,
\]
\[
W_{\text{DM}} = S_2 \chi_2 \bar{\chi}_2 + S_1 (\chi_1 \bar{\chi}_1 + S_2^2) + nh\bar{h}.
\] (A-5)

This model is particularly simple since no DM splitting is required to evade the constraints from direct detection. There is a $\mathbb{Z}_9^R$ symmetry, listed in the lower left side of table, that forbids both dangerous DM decays and GUT scale masses for the light fields.
|        | $U(1)_d$ | $Z_4^R$ | $Z_8$ |
|--------|-----------|----------|--------|
| GUT    | $X$       | 0        | 0      |
|        | $\bar{X}$| 0        | 2      |
|        | $Y$       | 1        | 0      |
|        | $\bar{Y}$| -1       | 2      |
| TeV    | $\chi_1$ | 1        | 2      |
|        | $\bar{\chi}_1$| -1| 0      |
|        | $\chi_2$ | 1        | 0      |
|        | $\bar{\chi}_2$| -1| 2      |
|        | $S$       | 0        | 0      |
|        | $N_1$     | 0        | 3      |
|        | $N_2$     | 0        | 1      |
| GeV    | $h$       | 1        | 1      |
|        | $\bar{h}$| -1       | 2      |
|        | $n$       | 0        | 0      |

|        | $SU(2)_d$ | $Z_{16}$ |
|--------|-----------|----------|
| GUT    | $H$       | $\text{Adj}$ | 0      |
|        | $X$       | $\square$   | 8      |
|        | $\bar{\Phi}$| $\text{Adj}$| 2      |
|        | $\bar{n}$ | $\text{Adj}$| 4      |
| TeV    | $\chi$   | $\square$   | 13     |
|        | $\bar{\chi}$| $\square$  | 5      |
|        | $\Phi$   | $\text{Adj}$| 14     |
|        | $S_{\Phi}$| 1        | 14     |
|        | $N$      | $\text{Adj}$| 6      |
|        | $\bar{N}$| $\text{Adj}$| 12     |
| GeV    | $h$      | $\square$   | 2      |
|        | $n'$     | $\text{Adj}$| 12     |
|        | $s_n$    | 1         | 12     |

|        | $SU(5)$ | $U(1)_d$ | $Z_9^R$ | $Z_9^R$ | $Z_6$ |
|--------|---------|----------|----------|----------|--------|
| GUT    | $X$     | 1        | 0        | 0        | 0      |
|        | $\bar{X}$| 1        | 0        | 0        | 2      |
|        | $Y$     | 1        | 1        | 0        | 2      |
|        | $\bar{Y}$| 1        | -1       | 0        | 2      |
| TeV    | $\chi_1$| 1        | 0        | 0        | 5      |
|        | $\bar{\chi}_1$| -1| 0      |
|        | $\chi_2$ | 1        | 0        | 4      |
|        | $\bar{\chi}_2$| -1| 0      |
|        | $S_1$   | 0        | 0        | 6      |
|        | $S_2$   | 0        | 0        | 7      |
| GeV    | $h$     | 0        | 1        | 2      |
|        | $\bar{h}$| 0        | -1       | 6      |
|        | $n$     | 0        | 0        | 3      |

Table 5: The gauge charges and example global charges for each model. Clockwise from the upper left, are the $U(1)_d$, $SU(2)_d \rightarrow U(1)_d$, $SU(5)_{SM} \times U(1)_d$, and $U(1)_\chi \times U(1)_d$ models. For each model, the charges forbid renormalizable and dimension 5 dark matter decays and GUT scale masses for light fields.
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