PlatEMO: A MATLAB Platform for Evolutionary Multi-Objective Optimization

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Abstract

Over the last three decades, a large number of evolutionary algorithms have been developed for solving multi-objective optimization problems. However, there lacks an up-to-date and comprehensive software platform for researchers to properly benchmark existing algorithms and for practitioners to apply selected algorithms to solve their real-world problems. The demand of such a common tool becomes even more urgent, when the source code of many proposed algorithms has not been made publicly available. To address these issues, we have developed a MATLAB platform for evolutionary multi-objective optimization in this paper, called PlatEMO, which includes more than 50 multi-objective evolutionary algorithms and more than 100 multi-objective test problems, along with several widely used performance indicators. With a user-friendly graphical user interface, PlatEMO enables users to easily compare several evolutionary algorithms at one time and collect statistical results in Excel or LaTeX files. More importantly, PlatEMO is completely open source, such that users are able to develop new algorithms on the basis of it. This paper introduces the main features of PlatEMO and illustrates how to use it for performing comparative experiments, embedding new algorithms, creating new test problems, and developing performance indicators. Source code of PlatEMO is now available at: http://bimk.ahu.edu.cn/index.php?s=/Index/Software/index.html.

Keywords

Evolutionary multi-objective optimization, MATLAB, software platform, genetic algorithm, source code, benchmark function, performance indicator

I. INTRODUCTION

Multi-objective optimization problems (MOPs) widely exist in computer science such as data mining \textsuperscript{[1]}, pattern recognition \textsuperscript{[2]}, image processing \textsuperscript{[3]} and neural network \textsuperscript{[4]}, as well as many other application fields \textsuperscript{[5]–[8]}. An MOP consists of two or more conflicting objectives to be optimized, and there often exist a set of optimal solutions trading off between different objectives. Since the vector evaluated genetic algorithm (VEGA) was proposed by Schaffer in 1985 \textsuperscript{[9]}, a number of multi-objective evolutionary algorithms (MOEAs) have been proposed and shown their superiority in tackling MOPs during the last three decades. For example, several MOEAs based on Pareto ranking selection and fitness sharing mechanism including multi-objective genetic algorithm (MOGA) \textsuperscript{[10]}, non-dominated sorting genetic algorithm (NSGA) \textsuperscript{[11]}, and niched Pareto genetic algorithm (NPGA) \textsuperscript{[12]} were proposed in the 1990s. From 1999 to 2002, some MOEAs characterized by the

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elitism strategy were developed, such as non-dominated sorting genetic algorithm II (NSGA-II) [13], strength Pareto evolutionary algorithm 2 (SPEA2) [14], Pareto envelope-based selection algorithm II (PESA-II) [15] and cellular multiobjective genetic algorithm (cellular MOGA) [16]. Afterwards, the evolutionary algorithm based on decomposition (MOEA/D) was proposed in 2007 [17], and some other MOEAs following the basic idea of MOEA/D had also been developed since then [18]–[21].

In spite of the large number of MOEAs in the literature [22], there often exist some difficulties in applying and using these algorithms since the source code of most algorithms had not been provided by the authors. Besides, it is also difficult to make benchmark comparisons between MOEAs due to the lack of a general experimental environment. To address such issues, several MOEA libraries have been proposed [23]–[27] to provide uniform experimental environments for users, which have greatly advanced the multi-objective optimization research and the implementation of new ideas. For example, the C-based multi-objective optimization library PISA [27] separates the implementation into two components, i.e., the problem-specific part containing MOPs and operators, and the problem-independent part containing MOEAs. These two components are connected by a text file-based interface in PISA. jMetal [23] is an object-oriented Java-based multi-objective optimization library consisting of various MOEAs and MOPs. MOEA Framework [24] is another free and open source Java framework for multi-objective optimization, which provides a comprehensive collection of MOEAs and tools necessary to rapidly design, develop, execute and test MOEAs. OTL [25], a C++ template library for multi-objective optimization, is characterized by object-oriented architecture, template technique, ready-to-use modules, automatically performed batch experiments and parallel computing. Besides, a Python-based experimental platform has also been proposed as the supplement of OTL, for improving the development efficiency and performing batch experiments more conveniently.

It is encouraging that there are several MOEA libraries dedicated to the development of evolutionary multi-objective optimization (EMO), but unfortunately, most of them are still far from useful and practical to most researchers. On one hand, the existing MOEA libraries cannot catch up with the development of MOEAs, where most of the MOEAs included in them are outdated and not able to cover the state-of-the-arts. On the other hand, due to the lack of professional GUI for experimental settings and algorithmic configurations, these libraries are difficult to be used or extended, especially for beginners who are not familiar with EMO. In order to collect more modern MOEAs and make the implementation of experiments on MOEAs easier, in this paper, we introduce a MATLAB-based EMO platform called PlatEMO [28]. Compared to existing EMO platforms, PlatEMO has the following main advantages:

- **Rich Library.** PlatEMO now includes 50 existing popular MOEAs as shown in Table II, where most of them are representative algorithms published in top journals, including multi-objective genetic algorithms, multi-objective differential evolution algorithms, multi-objective particle swarm optimization algorithms, multi-objective memetic algorithms, multi-objective estimation of distribution algorithms, and so on. PlatEMO also contains 110 MOPs from 16 popular test suites covering various difficulties, which are listed in Table

1. PISA: [PISA](http://www.tik.ee.ethz.ch/pisa)
2. jMetal: [jMetal](http://jmetal.sourceforge.net/index.html)
3. MOEA Framework: [MOEA Framework](http://moeaframework.org/index.html)
4. OTL: [OTL](http://github.com/O-T-L/OTL)
5. PlatEMO: [PlatEMO](http://bimk.ahu.edu.cn/index.php?s=/Index/Software/index.html)
### Table I
The 50 Multi-Objective Optimization Algorithms Included in the Current Version of PlatEMO.

| Algorithm                  | Year of Publication | Description                                                                 |
|----------------------------|---------------------|-----------------------------------------------------------------------------|
| **Multi-Objective Genetic Algorithms** |                      |                                                                            |
| SPEA2 [14]                 | 2001                | Strength Pareto evolutionary algorithm 2                                    |
| PSEA-II [15]               | 2001                | Pareto envelope-based selection algorithm II                                |
| NSGA-II [13]               | 2002                | Non-dominated sorting genetic algorithm II                                  |
| r-MOEA [28]                | 2003                | Multi-objective evolutionary algorithm based on r-dominance                 |
| IBEA [29]                  | 2004                | Indicator-based evolutionary algorithm                                       |
| MOEA/D [17]                | 2007                | Multi-objective evolutionary algorithm based on decomposition                |
| SMS-EMOA [30]              | 2007                | S metric selection evolutionary multi-objective optimization algorithm      |
| MSOPS-II [31]              | 2007                | Multiple single objective Pareto sampling algorithm II                      |
| MTS [32]                   | 2009                | Multiple trajectory search                                                  |
| AIE-II [33]                | 2013                | Approximation-guided evolutionary algorithm II                              |
| NSLS [34]                  | 2015                | Non-dominated sorting and local search                                      |
| BCE-IBEA [35]              | 2015                | Bi-criterion evolution for IBEA                                             |
| MOEA/IGD-NS [56]           | 2016                | Multi-objective evolutionary algorithm based on an enhanced inverted generational distance metric |
| **Many-Objective Genetic Algorithms** |                      |                                                                            |
| Hype [47]                  | 2011                | Hypervolume-based estimation algorithm                                       |
| PICEA-g [48]               | 2013                | Preference-inspired coevolutionary algorithm with goals                     |
| GtEA [49]                  | 2013                | Grid-based evolutionary algorithm                                           |
| NSGA-III [40]              | 2014                | Non-dominated sorting genetic algorithm III                                |
| A-NSGA-III [41]            | 2014                | Adaptive NSGA-III                                                          |
| SPEA2+SDE [42]             | 2014                | SPEA2 with shift-based density estimation                                   |
| BiGE [43]                  | 2015                | Bi-goal evolution                                                          |
| EFR-RR [49]                | 2015                | Ensemble fitness ranking with ranking restriction                           |
| I-DBEA [44]                | 2015                | Improved decomposition based evolutionary algorithm                         |
| KnEA [45]                  | 2015                | Knee point driven evolutionary algorithm                                    |
| MaOEA-DDFC [56]            | 2015                | Many-objective evolutionary algorithm based on directional diversity and convergence |
| MOEA/DDD [41]              | 2015                | Multi-objective evolutionary algorithm based on dominance and decomposition |
| MOMIB-II [48]              | 2015                | Many-objective metaheuristic based on the R2 indicator II                    |
| Two-Archive [48]           | 2015                | Two-archive algorithm 2                                                    |
| MaOEA-R&D [56]             | 2016                | Many-objective evolutionary algorithm based on objective space reduction and diversity improvement |
| RPEA [51]                  | 2016                | Reference points-based evolutionary algorithm                               |
| RVEA [52]                  | 2016                | Reference vector guided evolutionary algorithm                              |
| RVEA+ [52]                 | 2016                | RVEA embedded with the reference vector regeneration strategy              |
| SPEA2-K [51]               | 2016                | Strength Pareto evolutionary algorithm based on reference direction         |
| PR-DEA [54]                | 2016                | P-dominance based evolutionary algorithm                                   |
| **Multi-objective Genetic Algorithms for Large-Scale Optimization** | 2016 | Multi-objective evolutionary algorithm based on decision variable analyses |
| MOEA/D-VA [55]             | 2016                | Large-scale many-objective evolutionary algorithm                           |
| **Multi-Objective Genetic Algorithms with Preference** | 2016 |                                                                            |
| g-NSGA-II [57]             | 2009                | g-dominance based NSGA-II                                                  |
| r-NSGA-II [58]             | 2010                | r-dominance based NSGA-II                                                  |
| WV-MOEA-P [59]             | 2016                | Weight vector based multi-objective optimization algorithm with preference  |
| **Multi-objective Differential Algorithms** |                      |                                                                            |
| GDE3 [60]                  | 2005                | Generalized differential evolution 3                                       |
| MOEA/D-DE [18]             | 2009                | MOEA/D based on differential evolution                                      |
| **Multi-objective Particle Swarm Optimization Algorithms** | 2002 | Multi-objective particle swarm optimization                                  |
| MOPSO [61]                 | 2002                | Multi-objective particle swarm optimization                                 |
| SMPSO [62]                 | 2009                | Speed-constrained multi-objective particle swarm optimization              |
| aMPSO [63]                 | 2011                | Decomposition-based particle swarm optimization                            |
| **Multi-objective Memetic Algorithms** | 2000 | Memetic algorithm based on Pareto archived evolution strategy               |
| M-PAES [64]                | 2000                |                                                                            |
| **Multi-objective Estimation of Distribution Algorithms** | 2007 | Multi-objective covariance matrix adaptation                               |
| MO-CMA [65]                | 2007                | Multi-objective covariance matrix adaptation                               |
| RM-MEDA [66]               | 2008                | Regularity model-based multi-objective estimation of distribution algorithm |
| IM-MOEAs [67]              | 2015                | Inverse modeling multi-objective evolutionary algorithm                     |
| **Surrogate Model Based Multi-objective Algorithms** | 2005 | Efficient global optimization for Pareto optimization                      |
| ParEGO [68]                | 2005                | S-metric-selection-based efficient global optimization                      |
| SMS-EGO [69]               | 2008                | Kriging assisted RVEA                                                       |
| Problem        | Year of Publication | Description                                                                 |
|---------------|---------------------|-----------------------------------------------------------------------------|
| MOKP [71]     | 1999                | Multi-objective 0/1 knapsack problem and behavior of MOEAs on this problem analyzed in [72] |
| ZDT1–ZDT6 [73] | 2000                | Multi-objective test problems                                               |
| mQAP [74]     | 2003                | Multi-objective quadratic assignment problem                               |
| DTLZ1–DTLZ9 [75] | 2005           | Scalable multi-objective test problems                                      |
| WFG1–WFG9 [76] | 2006                | Scalable multi-objective test problems and degenerate problem WFG3 analyzed in [77] |
| MONRP [78]    | 2007                | Multi-objective next release problem                                       |
| MOTSP [79]    | 2007                | Multi-objective traveling salesperson problem                             |
| Pareto-Box [80] | 2007               | Pareto-Box problem                                                          |
| CF1–CF10 [81] | 2008                | Constrained multi-objective test problems for the CEC 2009 special session and competition |
| F1–F10 for RM-MEDA [82] | 2008 | The test problems designed for RM-MEDA                                     |
| UFI–UF12 [81] | 2008                | Unconstrained multi-objective test problems for the CEC 2009 special session and competition |
| F1–F9 for MOEA/D-DE [83] | 2009 | The test problems extended from [82] designed for MOEA/D-DE               |
| C1, C2, C3, C4 | 2014                | Constrained DTLZ and inverted DTLZ                                         |
| F1–F7 for MOEA/D-M2M [83] | 2014 | The test problems designed for MOEA/D-M2M                                 |
| F1–F10 for IM-MOEA [83] | 2015 | The test problems designed for IM-MOEA                                     |
| BT1–BT9 [83]  | 2016                | Multi-objective test problems with bias                                     |
| LSMOP1–LSMOP9 [83] | 2016       | Large-scale multi-objective test problems                                  |

In addition, there are a lot of performance indicators provided by PlatEMO for experimental studies, including Coverage [71], generational distance (GD) [85], hypervolume (HV) [86], inverted generational distance (IGD) [87], normalized hypervolume (NHV) [37], pure diversity (PD) [88], Spacing [89], and Spread (Δ) [90]. PlatEMO also provides a lot of widely-used operators for different encodings [91–97], which can be used together with all the MOEAs in PlatEMO.

- **Good Usability.** PlatEMO is fully developed in MATLAB language, thus any machines installed with MATLAB can use PlatEMO regardless of the operating system. Besides, users do not need to write any additional code when performing experiments using MOEAs in PlatEMO, as PlatEMO provides a user-friendly GUI, where users can configure all the settings and perform experiments on MOEAs via the GUI, and the experimental results can further be saved as a table in the format of Excel or LaTeX. In other words, with the assistance of PlatEMO, users can directly obtain the statistical experimental results to be used in academic writings by one-click operation via the GUI.

- **Easy Extensibility.** PlatEMO is not only easy to be used, but also easy to be extended. To be specific, the source code of all the MOEAs, MOPs and operators in PlatEMO are completely open source, and the length of the source code is very short due to the advantages of matrix operation in MATLAB, such that users can easily implement their own MOEAs, MOPs and operators on the basis of existing resources in PlatEMO. In addition, all new MOEAs developed on the basis of interfaces provided by PlatEMO can be also included into the platform, such that the library in PlatEMO can be iteratively updated by all users to follow state-of-the-arts.

- **Delicate Considerations.** There are many delicate considerations in the implementation of PlatEMO. For
example, PlatEMO provides different figure demonstrations of experimental results, and it also provides well-designed sampling methods for different shapes of Pareto optimal fronts. Fig. 1 shows the reference points sampled by PlatEMO on the Pareto optimal fronts of some MOPs with 3 objectives, while such reference points have not been provided by any other existing EMO libraries. It is also worth noting that, since the efficiency of most MOEAs is subject to the non-dominated sorting process, PlatEMO employs the efficient non-dominated sort ENS-SS [98] for two-objective optimization and the tree-based ENS termed T-ENS [56] for optimization with more than two objectives as the non-dominated sorting approaches, which have been demonstrated to be much more efficient than the fast non-dominated sort [13] used in other EMO libraries.

The rest of this paper is organized as follows. In the next section, the architecture of PlatEMO is presented on several aspects, i.e., the file structure of PlatEMO, the class diagram of PlatEMO, and the sequence diagram of executing algorithms by PlatEMO. Section III introduces how to use PlatEMO for analyzing the performance of algorithms and performing comparative experiments. The methods of extending PlatEMO with new MOEAs, MOPs, operators and performance indicators are described in Section IV. Finally, conclusion and future work are given in Section V.

II. ARCHITECTURE OF PLATEMO

After opening the root directory of PlatEMO, users can see a lot of .m files organized in the structure shown in Fig. 2 where it is very easy to find the source code of specified MOEAs, MOPs, operators or performance indicators. As shown in Fig. 2 there are six folders and one interface function main.m in the root directory of PlatEMO. The first folder \Algorithms is used to store all the MOEAs in PlatEMO, where each MOEA has an independent subfolder and all the relevant functions are in it. For instance, as shown
Fig. 2. Basic file structure of PlatEMO.

in Fig. 2 the subfolder \Algorithms\NSGA-II contains three functions NSGAII.m, CrowdingDistance.m and EnvironmentalSelection.m, which are used to perform the main loop, calculate the crowding distances, and perform the environmental selection of NSGA-II, respectively. The second folder \Problems contains a lot of subfolders for storing benchmark MOPs. For example, the subfolder \Problems\DTLZ contains 14 DTLZ test problems (i.e., DTLZ1–DTLZ9, C1_DTLZ1, C2_DTLZ2, C3_DTLZ4, IDTLZ1 and IDTLZ2), and the subfolder \Problems\WFG contains 9 WFG test problems (i.e., WFG1–WFG9). The folders \Operators and \Metrics store all the operators and performance indicators, respectively. The next folder \Public is used to store some public classes and functions, such as GLOBAL.m and INDIVIDUAL.m, which are two classes in PlatEMO representing settings of parameters and definitions of individuals, respectively. The last folder \GUI stores all the functions for establishing the GUI of PlatEMO, where users need not read or modify them.

PlatEMO also has a simple architecture, where it only involves two classes, namely GLOBAL and INDIVIDUAL, to store all the parameters and joint all the components (e.g., MOEAs, MOPs and operators). The class diagram of these two classes is presented in Fig. 3 The first class GLOBAL represents all the parameter setting, including the handle of MOEA function algorithm, the handle of MOP function problem, the handle of operator function operator and other parameters about the environment (the population size, the number of objectives, the length of decision variables, the maximum number of fitness evaluations, etc.). Note that all the properties in GLOBAL are read-only, which can only be assigned by users when the object is being instantiated. GLOBAL also provides several methods to be invoked by MOEAs, where MOEAs can achieve some complex operations via these methods. For instance, the method Initialization() can generate a randomly initial population with specified size, and the method Variation() can generate a set of offsprings with specified parents.

The other class in PlatEMO is INDIVIDUAL, where its objects are exactly individuals in MOEAs. An INDIVIDUAL object contains four properties, i.e., dec, obj, con and add, all of which are also read-only. dec is the array of decision variables of the individual, which is assigned when the object is being instantiated. obj and con store the objective values and the constraint values of the individual, respectively, which are calculated
after \textit{dec} has been assigned. The property \textit{add} is used to store additional properties of the individual for some special operators, such as the ‘speed’ property in PSO operator \cite{96}.

In order to better understand the mechanism of PlatEMO, Fig. 4 illustrates the sequence diagram of running an MOEA by PlatEMO without GUI. To begin with, the interface \textit{main.m} first invokes the algorithm function (e.g., \textit{NSGAII.m}), then the algorithm obtains an initial population (i.e., an array of \textit{INDIVIDUAL} objects) from the \textit{GLOBAL} object by invoking its method \texttt{Initialization}(). After that, the algorithm starts the evolution until the termination criterion is fulfilled, where the maximum number of fitness evaluations is used as the termination criterion for all the MOEAs in PlatEMO. In each generation of a general MOEA, it first performs mating pool selection for selecting a number of parents from the current population, and the parents are used to generate offsprings by invoking the method \texttt{Variation}() of \textit{GLOBAL} object. \texttt{Variation}() then passes the parents to the operator function (e.g., \textit{DE.m}), which is used to calculate the decision variables of the offsprings according to the parents. Next, the operator function invokes the \textit{INDIVIDUAL} class to instantiate the offspring objects, where the objective values of offsprings are calculated by invoking the problem function (e.g., \textit{DTLZ1.m}). After obtaining the offsprings, the algorithm performs environmental selection on the current population and the offsprings to select the population for next generation. When the number of instantiated \textit{INDIVIDUAL} objects exceeds the maximum number of fitness evaluations, the algorithm will be terminated and the final population will be output.

As presented by the above procedure, the algorithm function, the problem function and the operator function do not invoke each other directly; instead, they are connected to each other by the \textit{GLOBAL} class and the \textit{INDIVIDUAL} class. This mechanism has two advantages. First, MOEAs, MOPs and operators in PlatEMO are independent mutually, and they can be arbitrarily combined with each other, thus providing high flexibility...
Fig. 4. Sequence diagram of running a general multi-objective optimization algorithm by PlatEMO without GUI.

PlatEMO. Second, users need not consider the details of the MOP or the operator to be involved when developing a new MOEA, thus significantly improving the development efficiency.

III. RUNNING PLATEMO

As mentioned in Section I, PlatEMO provides two ways to run it: first, it can be run with a GUI by invoking the interface main() without input parameter, then users can perform MOEAs on MOPs by simple one-click operations; second, it can be run without GUI, and users can perform one MOEA on an MOP by invoking main() with input parameters. In this section, we elaborate these two ways of running PlatEMO.

A. Running PlatEMO without GUI

The interface main() can be invoked with a set of input parameters by the following form: main('name1', value1, 'name2', value2, ...), where name1, name2, ... denote the names of the parameters and value1, value2, ... denote the values of the parameters. All the acceptable parameters together with their data types and default values are listed in Table III. It is noteworthy that every parameter has a default value such that users need not assign all the parameters. As an example, the command main('-algorithm',@NSGAII,'-problem',@DTLZ2,-N',100,-M',3,-D',10,-evaluation',10000) is used to perform NSGA-II on DTLZ2 with a population size of 100, an objective number of 3, a decision variable length of 10, and a maximum fitness evaluation number of 10000.
TABLE III
ALL THE ACCEPTABLE PARAMETERS FOR THE INTERFACE OF PlatEMO.

| Parameter Name | Type             | Default Value | Description                      |
|----------------|------------------|---------------|----------------------------------|
| -algorithm     | function handle  | @NSGAII       | Algorithm function               |
| -problem       | function handle  | @DTLZ2        | Problem function                 |
| -operator      | function handle  | @EAreal       | Operator function                |
| -N             | positive integer | 100           | Population size                  |
| -M             | positive integer | 3             | Number of objectives             |
| -D             | positive integer | 12            | Number of decision variables     |
| -evaluation    | positive integer | 10000         | Maximum number of fitness        |
| -run           | positive integer | 1             | Run No.                          |
| -mode          | 1 or 2           | 1             | Run mode                         |
| -X parameter   | cell             | N/A           | The parameter values for function X |

Fig. 5. The objective values of the population obtained by NSGA-II on DTLZ2 with 3 objectives by running PlatEMO without GUI.

By invoking main() with parameters, one MOEA can be performed on an MOP with the specified setting, while the GUI will not be displayed. After the MOEA has been terminated, the final population will be displayed or saved, which is determined by the parameter -mode shown in Table III. To be specific, if -mode is set to 1, the objective values or decision variable values of the final population will be displayed in a new figure, and users can also observe the true Pareto front and the evolutionary trajectories of performance indicator values. For example, Fig. 5 shows the objective values of the population obtained by NSGA-II on DTLZ2 with 3 objectives, where users can select the figure to be displayed on the rightmost menu. If -mode is set to 2, the final population will be saved in a .mat file, while no figure will be displayed.
Fig. 6. The test module of PlatEMO.

Generally, there are four parameters to be assigned by users as listed in Table III (i.e., the population size \(-N\), the number of objectives \(-M\), the number of decision variables \(-D\), and the maximum number of fitness evaluations \(-\text{evaluation}\)); however, different MOEAs, MOPs or operators may involve additional parameter settings. For instance, there is a parameter \(rate\) denoting the ratio of selected knee points in KnEA [45], and there are four parameters \(proC\), \(disC\), \(proM\) and \(disM\) in EAreal [91], [92], which denote the crossover probability, the distribution index of simulated binary crossover, the number of bits undergone mutation, and the distribution index of polynomial mutation, respectively. In PlatEMO, such function related parameters can also be assigned by users via assigning the parameter \(-X\text{parameter}\), where \(X\) indicates the name of the function. For example, users can use the command \(\text{main}(...)\text{-KnEA\_parameter}',\{0.5\},...\) to set the value of \(rate\) to 0.5 for KnEA, and use the command \(\text{main}(...)\text{-EAreal\_parameter}',\{1,20,1,20\},...\) to set the values of \(proC\), \(disC\), \(proM\) and \(disM\) to 1, 20, 1 and 20 for EAreal, respectively. Besides, users can find the acceptable parameters of each MOEA, MOP and operator in the comments at the beginning of the related function.

B. Running PlatEMO with GUI

The GUI of PlatEMO currently contains two modules. The first module is used to analyze the performance of each MOEA, where one MOEA on an MOP can be performed in this module each time, and users can observe the result via different figure demonstrations. The second module is designed for statistical experiments, where multiple MOEAs on a batch of MOPs can be performed at the same time, and the statistical experimental results can be saved as Excel table or LaTeX table.

The interface of the first module, i.e., test module, is shown in Fig. 6. As can be seen from the figure, the main panel is divided into four parts. The first subpanel from left provides three popup menus, where users can select the MOEA, MOP and operator to be performed. The second subpanel lists all the parameters to be
assigned, which depends on the selected MOEA, MOP and operator. The third subpanel displays the current population during the optimization, other figures such as the true Pareto front and the evolutionary trajectories of performance indicator values can also be displayed. In addition, users can observe the populations in previous generations by dragging the slider at the bottom. The fourth subpanel stores the detailed information of historical results. As a result, the test module provides similar functions to the PlatEMO without GUI, but users do not need to write any additional command or code when using it.

The other module on the GUI is the experimental module, which is shown in Fig. 7. Similar to the text module, users should first select the MOEAs, MOPs and operators to be performed in the leftmost subpanel. Note that multiple MOEAs and MOPs can be selected in the experimental module. After setting the number of total runs, folder for saving results, and all the relevant parameters, the experiment can be started and the statistical results will be shown in the rightmost subpanel. Users can select any performance indicator to calculate the results to be listed in the table, where the mean and the standard deviation of the performance indicator value are shown in each grid. Furthermore, the best result in each row is shown in blue, and the Wilcoxon rank sum test result is labeled by the signs ‘+’, ‘−’ and ‘≈’, which indicate that the result is significantly better, significantly worst and statistically similar to the result in the control column, respectively. After the experiment is finished, the data shown in the table can be saved as Excel table (.xlsx file) or LaTeX table (.tex file). For example, after obtaining the experimental results shown in the table in Fig. 7, users can press the 'saving' button on the GUI to save the table in the format of LaTeX, where the corresponding LaTeX table is shown in Table IV.

It can be concluded from the above introduction that the functions provided by PlatEMO are modularized, where two modules (i.e., the test module and the experimental module) are included in the current version of PlatEMO. In the future, we also plan to develop more modules to provide more functions for users.
TABLE IV
IGD VALUES OF KnEA AND RVEA ON DTLZ1–DTLZ4. THE LaTeX CODE OF THIS TABLE IS AUTOMATICALLY GENERATED BY PlatEMO.

| Problem | M | D | KnEA | RVEA |
|---------|---|---|------|------|
| DTLZ1   | 2 | 6 | 1.2629e-1 (1.80e-1) + | 5.5304e-1 (2.65e-1) |
|         | 3 | 7 | 1.8853e-1 (1.90e-1) + | 4.9382e-1 (3.91e-1) |
|         | 4 | 8 | 2.7106e-1 (1.74e-1) ≈ | 2.8460e-1 (2.01e-1) |
| DTLZ2   | 2 | 11| 5.5828e-2 (1.50e-2) − | 8.2011e-3 (1.16e-3) |
|         | 3 | 12| 6.9440e-2 (4.43e-3) − | 5.5822e-2 (9.16e-4) |
|         | 4 | 13| 1.5405e-1 (5.07e-3) − | 1.3956e-1 (3.43e-4) |
| DTLZ3   | 5 | 14| 1.6855e+1 (6.53e+0) ≈ | 1.8521e+1 (6.83e+0) |
|         | 6 | 15| 4.0119e+1 (1.39e+1) − | 2.0441e+1 (8.20e+0) |
|         | 7 | 16| 7.6642e+1 (2.15e+1) − | 1.7181e+1 (7.44e+0) |
| DTLZ4   | 8 | 17| 4.1601e-1 (9.60e-3) + | 5.6715e-1 (7.03e-2) |
|         | 9 | 18| 4.5396e-1 (8.92e-3) + | 5.7960e-1 (5.25e-2) |
|         | 10| 19| 4.9322e-1 (6.48e-3) + | 5.8410e-1 (4.66e-2) |

+/−/≈ 5/5/2

IV. EXTENDING PLATEMO

PlatEMO is an open platform for scientific research and applications of EMO, hence it allows users to add their own MOEAs, MOPs, operators and performance indicators to it, where users should save the new MOEA, MOP, operator or performance indicator to be added as a MATLAB function (i.e., a .m file) with the specified interface and form, and put it in the corresponding folder. In the following, the methods of extending PlatEMO with a new MOEA, MOP, operator and performance indicator are illustrated by several cases, respectively.

A. Adding New Algorithms to PlatEMO

All the .m files of MOEA functions are stored in the folder \Algorithms in the root directory of PlatEMO, and all the .m files for the same MOEA should be put in the same subfolder. For example, as shown in the file structure in Fig. 2, the three .m files for NSGA-II (i.e., NSGAII.m, CrowdingDistance.m and EnvironmentalSelection.m) are all in the subfolder \Algorithms\NSGA-II. A case study including the source code of the main function of NSGA-II (i.e. NSGAII.m) is given in Fig. 8 where the logic of the function is completely the same to the one shown in Fig. 4.

To begin with, the main function of an MOEA has one input parameter and zero output parameter, where the only input parameter denotes the GLOBAL object for the current run. Then an initial population Population is generated by invoking Global.Initialization(), and the non-dominated front number and the crowding distance

Fig. 8. The source code of the main function of NSGA-II. The common code required by any MOEA is underlined.
Fig. 9. The source code of DTLZ2. The common code required by any MOP is underlined.

of each individual are calculated (line 2–4). In each generation, the function `Global.NotTermination()` is invoked to check whether the termination criterion is fulfilled, and the variable `Population` is passed to this function to be the final output (line 5). Afterwards, the mating pool selection, generating offsprings, and environmental selection are performed in sequence (line 6–9).

The common code required by any MOEA is underlined in Fig. 8. In addition to the interface of the function, one MOEA needs to perform at least the following three operations: obtaining an initial population via `Global.Initialization()`, checking the optimization state and outputting the final population via `Global.NotTermination()`, and generating offsprings via `Global.Variation()`, where all these three functions are provided by the `GLOBAL` object. Apart from the above three common operations, different MOEAs may have different logics and different functions to be invoked.

B. Adding New Problems to PlatEMO

All the `.m` files of MOP functions are stored in the folder `\Problems`, and one `.m` file usually indicates one MOP. Fig. 9 gives the source code of DTLZ2, where the common code required by any MOP is underlined. It can be seen from the source code that, the interface of DTLZ2 is more complex than the one of NSGA-II, where the function `DTLZ2()` includes three input parameters and one output parameter. The input parameter `Operation` determines the operation to be performed; the parameter `Global` denotes the `GLOBAL` object; and the parameter `input` has different meanings when `Operation` is set to different values, so does the output parameter `varargout`.

Different from the MOEA functions which are invoked only once in each run, an MOP function may be invoked multiple times.
invoked many times for different operations. As shown in Fig. 9 an MOP function contains three independent operations: generating random decision variables (line 3–10), calculating objective values and constraint values (line 11–23), and sampling reference points on the true Pareto front (line 24–27). To be specific, if \textit{Operation} is set to ’init’, the MOP function will return the decision variables of a random population with size \textit{input} (line 9–10). Meanwhile, it sets \textit{Global.M}, \textit{Global.D}, \textit{Global.lower}, \textit{Global.upper} and \textit{Global.operator} to their default values, which denote the number of objectives, number of decision variables, lower boundary of each decision variable, upper boundary of each decision variable, and the operator function, respectively (line 4–8). When \textit{Operation} is set to ’value’, the parameter \textit{input} will denote the decision variables of a population, and the objective values and constraint values of the population will be calculated and returned according to the decision variables (line 14–23). And if \textit{Operation} is set to ’PF’, a number of \textit{input} uniformly distributed reference points will be sampled on the true Pareto front and returned (line 25–27).

\textbf{C. Adding New Operators or Performance Indicators to PlatEMO}

Fig. 10 shows the source code of evolutionary operator based on binary coding (i.e. \textit{EAbinary.m}), where the .m files of the operator functions are all stored in the folder \textbackslash Operators. An operator function has two input parameters, one denoting the \textit{GLOBAL} object (i.e. \textit{Global}) and the other denoting the parent population (i.e. \textit{Parent}), and it also has one output parameter denoting the generated offsprings (i.e. \textit{Offspring}). As can be seen from the source code in Fig. 10 the main task of an operator function is to generate offsprings according to the values of \textit{Parent}, where \textit{EAbinary()} performs the single-point crossover in line 6–11 and the bitwise mutation in line 12–13 of the code. Afterwards, the \textit{INDIVIDUAL} objects of the offsprings are generated and returned (line 14).

Fig. 11 shows the source code of IGD, where all these performance indicator functions are stored in the folder \textbackslash Metrics. The task of a performance indicator is to calculate the indicator value of a population according
1. function Offspring = EAreal(Global,Parent)
2. % <operator> <real>
3. % Simulated binary crossover and polynomial mutation
4. % proC --- 1 --- The probability of doing crossover
5. % disC --- 15 --- The distribution index of SBX
6. % proM --- 1 --- The expectation of number of bits doing mutation
7. % disM --- 15 --- The distribution index of polynomial mutation
8. [proC,disC,proM,disM] = Global.ParameterSet(1,15,1,15);
   ...

Fig. 12. The comments and the source code in the head of the function of evolutionary operator based on real value coding.

to a set of reference points. The input parameters of $IGD()$ consists of two parts: the objective values of the population (i.e. $PopObj$), and the reference points sampled on the true Pareto front (i.e. $PF$). Correspondingly, the output parameter of $IGD()$ is the IGD value (i.e. score). Thanks to the merits of matrix operation in MATLAB, the source code of IGD is quite short as shown in Fig. 11 where the calculation of the mean value of the minimal distance of each point in $PF$ to the points in $PopObj$ can be performed using a built-in function $pdist2()$ provided by MATLAB.

D. Adding Acceptable Parameters for New Functions

All the user-defined functions can have their own parameters as well as the functions provided by PlatEMO, where these parameters can be either assigned by invoking `main(...,'-X_parameter',{...},...)` with $X$ denoting the function name, or displayed on the GUI for assignment. In order to add acceptable parameters for an MOEA, MOP, operator or performance indicator function, the comments in the head of the function should be written in a specified form. To be specific, Fig. 12 shows the comments and the source code in the head of the function of evolutionary operator based on real value coding (i.e. $EAreal.m$).

The comment in line 2 of Fig. 12 gives the two labels of this function, which are used to make sure this function can be identified by the GUI. The comment in line 3 is a brief introduction about this function; for an MOEA or MOP function, such introduction should be the title of the relevant literature. The parameters `proC, disC, proM` and `disM` for this function are given by the comments in line 4–7, where the names of the parameters are in the first column, the default values of the parameters are in the second column, and the introductions about the parameters are given in the third column. The columns in each row are divided by the sign ‘—’.

The comments define the parameters and their default values for the function, and invoking `Global.ParameterSet()` can make these parameters assignable to users. As shown in line 9 of Fig. 12 the function invokes `Global.ParameterSet()` with four inputs denoting the default values of the parameters, and sets the four parameters to the outputs. More specifically, if users have not assigned the parameters, they will equal to their default values (i.e. 1, 15, 1 and 15). Otherwise, if users assign the parameters by invoking `main(...,'-EAreal_parameter',{a,b,c,d},...)`, the parameters `proC, disC, proM` and `disM` will be set to $a$, $b$, $c$ and $d$, respectively.

V. CONCLUSION AND FUTURE WORK

This paper has introduced a MATLAB-based open source platform for evolutionary multi-objective optimization, namely PlatEMO. The current version of PlatEMO includes 50 multi-objective optimization algorithms and
110 multi-objective test problems, having covered the majority of state-of-the-arts. Since PlatEMO is developed on the basis of a light architecture with simple relations between objects, it is very easy to be used and extended. Moreover, PlatEMO provides a user-friendly GUI with a powerful experimental module, where engineers and researchers can use it to quickly perform their experiments without writing any additional code.

This paper has described the architecture of PlatEMO, and it has also introduced the steps of running PlatEMO with and without the GUI. Then, the ways of adding new algorithms, problems, operators and performance indicators to PlatEMO have been elaborated by several cases.

We will continuously maintain and develop PlatEMO in the future. On one hand, we will keep following the state-of-the-arts and adding more effective algorithms and new problems into PlatEMO. On the other hand, more modules will be developed to provide more functions for users, such as preference optimization, dynamic optimization, noisy optimization, etc. We hope that PlatEMO is helpful to the researchers working on evolutionary multi-objective optimization, and we sincerely encourage peers to join us to shape the platform for better functionality and usability.

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REFERENCES

[1] A. Mukhopadhyay, U. Maulik, S. Bandyopadhyay, and C. Coello Coello, “A survey of multiobjective evolutionary algorithms for data mining: Part I,” IEEE Transactions on Evolutionary Computation, vol. 18, no. 1, pp. 4–19, 2014.
[2] J. Handl and J. Knowles, “An evolutionary approach to multiobjective clustering,” IEEE Transactions on Evolutionary Computation, vol. 11, no. 1, pp. 56–76, 2007.
[3] B. Lazzerini, F. Marcelloni, and M. Vecchio, “A multi-objective evolutionary approach to image quality/compression trade-off in JPEG baseline algorithm,” Applied Soft Computing, vol. 10, no. 2, pp. 548–561, 2010.
[4] F. Pettersson, N. Chakraborti, and H. Saxén, “A genetic algorithms based multi-objective neural net applied to noisy blast furnace data,” Applied Soft Computing, vol. 7, no. 1, pp. 387–397, 2007.
[5] S. H. Yeung, K. F. Man, K. M. Luk, and C. H. Chan, “A trapeziform U-slot folded patch feed antenna design optimized with jumping genes evolutionary algorithm,” IEEE Transactions on Antennas and Propagation, vol. 56, no. 2, pp. 571–577, 2008.
[6] A. Ponsich, A. L. Jaimes, and C. A. C. Coello, “A survey on multiobjective evolutionary algorithms for the solution of the portfolio optimization problem and other finance and economics applications,” IEEE Transactions on Evolutionary Computation, vol. 17, no. 3, pp. 321–344, 2013.
[7] J. G. Herrero, A. Berlanga, and J. M. M. López, “Effective evolutionary algorithms for many-specifications attainment: Application to air traffic control tracking filters,” IEEE Transactions on Evolutionary Computation, vol. 13, no. 1, pp. 151–168, 2009.
[8] H. Ishibuchi and T. Murata, “Multiobjective genetic local search algorithm and its application to flowshop scheduling,” IEEE Transactions on Systems, Man, and Cybernetics-Part C, vol. 28, no. 3, pp. 392–403, 1998.
[9] J. D. Schaffer, “Multiple objective optimization with vector evaluated genetic algorithms,” in Proceedings of the 1st International Conference on Genetic Algorithms, 1985, pp. 93–100.
[10] C. M. Fonseca, P. J. Fleming et al., “Genetic algorithms for multiobjective optimization: Formulation, discussion and generalization,” in Proceedings of the Proceedings of the Fifth International Conference, vol. 93, 1993, pp. 416–423.
[11] N. Srinivas and K. Deb, “Multiobjective optimization using nondominated sorting in genetic algorithms,” Evolutionary Computation, vol. 2, no. 3, pp. 221–248, 1995.
Y. Tian, X. Zhang, R. Cheng, and Y. Jin, “A multi-objective evolutionary algorithm based on an enhanced inverted generational
E. J. Hughes, “MSOPS-II: A general-purpose many-objective optimiser,” in Proceedings of the 2007 IEEE Congress on Evolutionary Computation, 2007, pp. 3944–3951.
L.-Y. Tseng and C. Chen, “Multiple trajectory search for unconstrained/constrained multi-objective optimization,” in Proceedings of the 2009 IEEE Congress on Evolutionary Computation, 2009, pp. 1951–1958.
M. Wagner and F. Neumann, “A fast approximation-guided evolutionary multi-objective algorithm,” in Proceedings of the 15th Annual Conference on Genetic and Evolutionary Computation, 2013, pp. 687–694.
B. Chen, W. Zeng, Y. Lin, and D. Zhang, “A new local search-based multiobjective optimization algorithm,” IEEE Transactions on Evolutionary Computation, vol. 19, no. 1, pp. 50–73, 2015.
M. Li, S. Yang, and X. Liu, “Pareto or non-Pareto: Bi-criterion evolution in multi-objective optimization,” IEEE Transactions on Evolutionary Computation, vol. 20, no. 5, pp. 645–665, 2015.
Y. Tian, X. Zhang, R. Cheng, and Y. Jin, “A multi-objective evolutionary algorithm based on an enhanced inverted generational distance metric,” in Proceedings of the 2016 IEEE Congress on Evolutionary Computation, 2016, pp. 5222–5229.
[37] J. Bader and E. Zitzler, “HypE: An algorithm for fast hypervolume-based many-objective optimization,” *Evolutionary Computation*, vol. 19, no. 1, pp. 45–76, 2011.

[38] R. Wang, R. C. Purshouse, and P. J. Fleming, “Preference-inspired coevolutionary algorithms for many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 17, no. 4, pp. 474–494, 2013.

[39] S. Yang, M. Li, X. Liu, and J. Zheng, “A grid-based evolutionary algorithm for many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 17, no. 5, pp. 721–736, 2013.

[40] K. Deb and H. Jain, “An evolutionary many-objective optimization algorithm using reference-point based non-dominated sorting approach, part I: Solving problems with box constraints,” *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 4, pp. 577–601, 2014.

[41] H. Jain and K. Deb, “An evolutionary many-objective optimization algorithm using reference-point based non-dominated sorting approach, part II: Handling constraints and extending to an adaptive approach,” *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 4, pp. 602–622, 2014.

[42] M. Li, S. Yang, and X. Liu, “Shift-based density estimation for pareto-based algorithms in many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 3, pp. 348–365, 2014.

[43] M. Li, S. Yang, and X. L. (2015), “Bi-goal evolution for many-objective optimization problems,” *Artificial Intelligence*, vol. 228, pp. 45–65, 2015.

[44] M. Asafuddoula, T. Ray, and R. Sarker, “A decomposition based evolutionary algorithm for many objective optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 19, no. 3, pp. 445–460, 2015.

[45] X. Zhang, Y. Tian, and Y. Jin, “A knee point driven evolutionary algorithm for many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 19, no. 6, pp. 761–776, 2015.

[46] J. Cheng, G. Yen, and G. Zhang, “A many-objective evolutionary algorithm with enhanced mating and environmental selections,” *IEEE Transactions on Evolutionary Computation*, vol. 19, pp. 592–605, 2015.

[47] K. Li, K. Deb, Q. Zhang, and S. Kwong, “Combining dominance and decomposition in evolutionary many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 19, no. 5, pp. 694–716, 2015.

[48] R. Hernández Gómez and C. A. Coello Coello, “Improved metaheuristic based on the R2 indicator for many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 19, pp. 694–716, 2015.

[49] H. Wang, L. Jiao, and X. Yao, “Two_Arch2: An improved two-archive algorithm for many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 19, no. 4, pp. 524–541, 2015.

[50] Z. He and G. G. Yen, “Many-objective evolutionary algorithm: Objective space reduction and diversity improvement,” *IEEE Transactions on Evolutionary Computation*, vol. 20, no. 1, pp. 145–160, 2016.

[51] Y. Liu, D. Gong, X. Sun, and Z. Yong, “Many-objective evolutionary optimization based on reference points,” *Applied Soft Computing*, 2016, in press.

[52] R. Cheng, Y. Jin, M. Olhofer, and B. Sendhoff, “A reference vector guided evolutionary algorithm for many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, 2016, in press.

[53] S. Jiang and S. Yang, “A strength Pareto evolutionary algorithm based on reference direction for multi-objective and many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, 2016, in press.

[54] Y. Yuan, H. Xu, B. Wang, and X. Yao, “A new dominance relation-based evolutionary algorithm for many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 20, no. 1, pp. 16–37, 2016.

[55] X. Ma, F. Liu, Y. Qi, X. Wang, L. Li, L. Jiao, M. Yin, and M. Gong, “A multiobjective evolutionary algorithm based on decision variable analyses for multiobjective optimization problems with large-scale variables,” *IEEE Transactions on Evolutionary Computation*, vol. 20, no. 2, pp. 275–298, 2016.

[56] X. Zhang, Y. Tian, R. Cheng, and Y. Jin, “A decision variable clustering-based evolutionary algorithm for large-scale many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, 2016, in press.

[57] J. Molina, L. V. Santana, A. G. Hernández-Díaz, C. A. C. Coello, and R. Caballero, “g-dominance: Reference point based dominance for multiobjective metaheuristics,” *European Journal of Operational Research*, vol. 197, no. 2, pp. 685–692, 2009.

[58] L. B. Said, S. Bechikh, and K. Ghédira, “The r-dominance: A new dominance relation for interactive evolutionary multicriteria decision making,” *IEEE Transactions on Evolutionary Computation*, vol. 14, no. 5, pp. 801–818, 2010.

[59] X. Zhang, X. Jiang, and L. Zhang, “A weight vector based multi-objective optimization algorithm with preference,” *Acta Electronica Sinica*, vol. 44, no. 11, pp. 2639–2645, 2016.

[60] S. Kukkonen and J. Lampinen, “GDE3: The third evolution step of generalized differential evolution,” in *Proceedings of the 2005 IEEE Congress on Evolutionary Computation*, vol. 1, 2005, pp. 443–450.
[61] C. C. Coello and M. S. Lechuga, “MOPSO: A proposal for multiple objective particle swarm optimization,” in *Proceedings of the 2002 IEEE Congress on Evolutionary Computation*, vol. 2, 2002, pp. 1051–1056.

[62] A. J. Nebro, J. J. Durillo, J. García-Nieto, C. C. Coello, F. Luna, and E. Alba, “SMPSO: A new PSO-based metaheuristic for multi-objective optimization,” in *Proceedings of the Computational Intelligence in Multi-Criteria Decision-Making*, 2009, pp. 66–73.

[63] S. Zapotecas Martínez and C. A. Coello Coello, “A multi-objective particle swarm optimizer based on decomposition,” in *Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation*, 2011, pp. 69–76.

[64] J. D. Knowles and D. W. Corne, “M-PAES: A memetic algorithm for multiobjective optimization,” in *Proceedings of the 2000 IEEE Congress on Evolutionary Computation*, 2000, pp. 325–332.

[65] C. Igel, N. Hansen, and S. Roth, “Covariance matrix adaptation for multi-objective optimization,” *Evolutionary computation*, vol. 15, no. 1, pp. 1–28, 2007.

[66] R. Cheng, Y. Jin, M. Olhofer, and B. Sendhoff, “Test problems for large-scale multiobjective and many-objective optimization,” in *Proceedings of the International Conference on Parallel Problem Solving from Nature*, 2008, pp. 784–794.

[67] T. Chugh, Y. Jin, K. Miettinen, J. Hakanen, and K. Sindhya, “A surrogate-assisted reference vector guided evolutionary algorithm for computationally expensive many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, 2016, in press.

[68] J. Knowles, “ParEGO: A hybrid algorithm with on-line landscape approximation for expensive multiobjective optimization problems,” *IEEE Transactions on Evolutionary Computation*, vol. 10, no. 1, pp. 50–66, 2006.

[69] W. Ponweiser, T. Wagner, D. Biermann, and M. Vincze, “Multiobjective optimization on a limited budget of evaluations using model-assisted S-metric selection,” in *Proceedings of the International Conference on Parallel Problem Solving from Nature*, 2008, pp. 784–794.

[70] T. Chugh, Y. Jin, K. Miettinen, J. Hakanen, and K. Sindhya, “A surrogate-assisted reference vector guided evolutionary algorithm for computationally expensive many-objective optimization,” *IEEE Transactions on Evolutionary Computation*, 2016, in press.

[71] E. Zitzler and L. Thiele, “Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach,” *IEEE Transactions on Evolutionary Computation*, vol. 3, no. 4, pp. 257–271, 1999.

[72] H. Ishibuchi, N. Akedo, and Y. Nojima, “Behavior of multiobjective evolutionary algorithms on many-objective knapsack problems,” *IEEE Transactions on Evolutionary Computation*, vol. 19, no. 2, pp. 264–283, 2015.

[73] K. D. E. Zitzler and L. Thiele, “Comparison of multiobjective evolutionary algorithms: Empirical results,” *Evolutionary Computation*, vol. 8, no. 2, pp. 173–195, 2000.

[74] J. Knowles and D. Corne, “Instance generators and test suites for the multiobjective quadratic assignment problem,” in *Proceedings of the International Conference on Evolutionary Multi-Criterion Optimization*, 2003, pp. 295–310.

[75] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, *Scalable test problems for evolutionary multiobjective optimization*, 2005.

[76] L. B. S. Huband, P. Hingston and L. While, “A review of multiobjective test problems and a scalable test problem toolkit,” *IEEE Transactions on Evolutionary Computation*, vol. 10, no. 5, pp. 477–506, 2006.

[77] H. Ishibuchi, H. Masuda, and Y. Nojima, “Pareto fronts of many-objective degenerate test problems,” *IEEE Transactions on Evolutionary Computation*, vol. 20, no. 5, pp. 807–813, 2016.

[78] Y. Zhang, M. Harman, and S. A. Mansouri, “The multi-objective next release problem,” in *Proceedings of the 9th Annual Conference on Genetic and Evolutionary Computation*, 2007, pp. 1129–1137.

[79] D. W. Corne and J. D. Knowles, “Techniques for highly multiobjective optimisation: Some non-dominated points are better than others,” in *Proceedings of the 9th Conference on Genetic and Evolutionary Computation*, 2007, pp. 773–780.

[80] M. Köppen and K. Yoshida, “Substitute distance assignments in NSGA-II for handling many-objective optimization problems,” in *Proceedings of the Evolutionary Multi-criterion Optimization*, 2007, pp. 727–741.

[81] Q. Zhang, A. Zhou, S. Zhao, P. N. Suganthan, W. Liu, and S. Tiwari, “Multiobjective optimization test instances for the CEC 2009 special session and competition,” University of Essex, Colchester, UK and Nanyang technological University, Tech. Rep. CES-487, Tech. Rep., 2008.

[82] T. Okabe, Y. Jin, M. Olhofer, and B. Sendhoff, “On test functions for evolutionary multi-objective optimization,” in *Proceedings of the International Conference on Parallel Problem Solving from Nature*, 2004, pp. 792–802.

[83] H. Li, Q. Zhang, and J. Deng, “Biased multiobjective optimization and decomposition algorithm,” *IEEE Transactions on Cybernetics*, 2016, in press.

[84] R. Cheng, Y. Jin, M. Olhofer, and B. Sendhoff, “Test problems for large-scale multiobjective and many-objective optimization,” *IEEE Transactions on Cybernetics*, 2016, in press.

[85] D. A. Veldhuizen and G. B. Lamont, “Multiobjective evolutionary algorithm research: A history and analysis,” Department of Electrical and Computer Engineering. Graduate School of Engineering, Air Force Inst Technol, Wright Patterson, Tech. Rep. TR-98-03, Tech. Rep., 1998.
[86] L. While, P. Hingston, L. Barone, and S. Huband, “A faster algorithm for calculating hypervolume,” *IEEE Transactions on Evolutionary Computation*, vol. 10, no. 1, pp. 29–38, 2006.

[87] A. Zhou, Y. Jin, Q. Zhang, B. Sendhoff, and E. Tsang, “Combining model-based and genetics-based offspring generation for multi-objective optimization using a convergence criterion,” in *Proceedings of the 2006 IEEE Congress on Evolutionary Computation*, 2006, pp. 892–899.

[88] H. Wang, Y. Jin, and X. Yao, “Diversity assessment in many-objective optimization,” *IEEE Transactions on Cybernetics*, 2016, in press.

[89] J. R. Schott, “Fault tolerant design using single and multicriteria genetic algorithm optimization,” Master’s thesis, Cambridge: Massachusetts Institute of Technology, 1995.

[90] Y. Wang, L. Wu, and X. Yuan, “Multi-objective self-adaptive differential evolution with elitist archive and crowding entropy-based diversity measure,” *Soft Computing*, vol. 14, no. 3, pp. 193–209, 2010.

[91] K. Deb, *Multi-Objective Optimization Using Evolutionary Algorithms*. New York: Wiley, 2001.

[92] K. Deb and M. Goyal, “A combined genetic adaptive search (GeneAS) for engineering design,” *Computer Science and Informatics*, vol. 26, no. 4, pp. 30–45, 1996.

[93] L. Davis, “Applying adaptive algorithms to epistatic domains,” in *Proceedings of the International Joint Conference on Artificial Intelligence*, vol. 1, 1985, pp. 162–164.

[94] D. B. Fogel, “An evolutionary approach to the traveling salesman problem,” *Biological Cybernetics*, vol. 60, no. 2, pp. 139–144, 1988.

[95] K. Price, R. M. Storn, and J. A. Lampinen, *Differential evolution: A practical approach to global optimization*. Springer Science & Business Media, 2006.

[96] J. Kennedy, J. F. Kennedy, R. C. Eberhart, and Y. Shi, *Swarm intelligence*. Morgan Kaufmann, 2001.

[97] X. Yao, Y. Liu, and G. Lin, “Evolutionary programming made faster,” *IEEE Transactions on Evolutionary Computation*, vol. 3, no. 2, pp. 82–102, 1999.

[98] X. Zhang, Y. Tian, R. Cheng, and Y. Jin, “An efficient approach to non-dominated sorting for evolutionary multi-objective optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 19, no. 2, pp. 201–213, 2015.