Lepton-Flavor-Dependent Angular Analysis of $B \to K^*\ell^+\ell^-$

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We present a measurement of angular observables and a test of lepton flavor universality in the $B \rightarrow K^{*} \ell^{+} \ell^{-}$ decay, where $\ell$ is either $e$ or $\mu$. The analysis is performed on a data sample corresponding to an integrated luminosity of 711 fb$^{-1}$ containing $772 \times 10^{6}$ $B\bar{B}$ pairs, collected at the $\Upsilon(4S)$ resonance with the Belle detector at the asymmetric-energy $e^{+}e^{-}$ collider KEKB. The result is consistent with Standard Model (SM) expectations, where the largest discrepancy from a SM prediction is observed in the muon modes with a local significance of $2.6\sigma$.

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In this Letter, a measurement of angular observables and a test of lepton flavor universality (LFU) in the \( B \to K^\ast \ell^+ \ell^- \) decay is presented, where \( \ell = e, \mu \). The \( B \to K^\ast \ell^+ \ell^- \) decay involves the quark transition \( b \to s \ell^+ \ell^- \), a flavor-changing neutral current that is forbidden at tree level in the Standard Model (SM). Various extensions to the SM predict contributions from new physics (NP), which can interfere with the SM amplitudes. In recent years, several measurements have shown deviations from the SM in this particular decay [1–3]. Global analyses of \( B \) decays hint at lepton-flavor non-universality, in which case muon modes would have larger contributions from NP than electron modes [4, 5].

The decay can be described kinematically by three angles \( \theta_\ell, \theta_K, \phi \) and the invariant mass squared of the lepton pair \( q^2 \equiv M_\ell^2 c^2 \). The angle \( \theta_\ell \) is defined as the angle between the direction of \( \ell^+ \) (\( \ell^- \)) and the direction opposite the \( B \) (\( \bar{B} \)) in the \( K^\ast \) rest frame. Finally, the angle \( \phi \) is defined as the angle between the plane formed by the \( \ell^+ \ell^- \) system and the \( K^\ast \) decay plane in the \( B \) (\( \bar{B} \)) rest frame. The differential decay rate can be parametrized using definitions presented in Ref. [6] by

\[
\frac{1}{d \Gamma/dq^2} = \frac{d^4 \Gamma}{d \cos \theta_\ell d \cos \theta_K d \phi dq^2} = \frac{9}{32 \pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2 \theta_\ell \right. \\
- F_L \cos^2 \theta_K \cos 2 \theta_\ell + S_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2 \phi + S_1 \sin 2 \theta_K \sin 2 \theta_\ell \cos \phi \\
+ S_3 \sin 2 \theta_K \sin \theta_\ell \cos \phi + S_6 \sin^2 \theta_K \cos \theta_\ell + S_7 \sin 2 \theta_K \sin \theta_\ell \sin \phi \\
\left. + S_8 \sin 2 \theta_K \sin 2 \theta_\ell \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2 \phi \right], \tag{1}
\]

where the observables \( F_L \) and \( S_i \) are functions of \( q^2 \) only. The observables \( P'_i \), introduced in Ref. [7] and defined as

\[
P'_i = \frac{S_j = 4, 5, 6, 8}{\sqrt{P'_L (1 - P'_L)}}, \tag{2}
\]

are considered to be largely free of form-factor uncertainties [8]. Any deviation from zero in the difference \( Q_i = P'_{i\mu} - P'_{i\tau} \) would be a direct hint of new physics [9]; here, \( i = 4, 5 \) and \( P'_4 \) refers to \( P'_{4,5} \) in the corresponding lepton mode. The definition of \( P'_i \) values follows the LHCb convention [1].

In previous measurements of the \( P'_i \) observables only \( B^0 \) decays, followed by \( K^{0}\) decays to \( K^+\pi^- \), were used [1]. This measurement also uses \( B^+ \) decays, where \( K^{+} \to K^{+}\pi^0 \) or \( K^{0}_{\pi}\pi^+ \). In total, the decay modes \( B^0 \to K^{*0}\mu^+\mu^- \), \( B^+ \to K^{*+}\mu^+\mu^- \), \( B^0 \to K^0\tau^{+\tau^-} \), and \( B^+ \to K^{*+}\tau^{+\tau^-} \) are reconstructed, where the inclusion of charge-conjugate states is implied if not explicitly stated. The full \( \Upsilon(4S) \) data sample is used containing \( 772 \times 10^6 B\bar{B} \) pairs recorded with the Belle detector [10] at the asymmetric-energy \( e^+e^- \) collider KEKB [11]. The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector, a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals (ECL) located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect \( K^0_L \) mesons and to identify muons (KLM). The detector is described in detail elsewhere [10]. This analysis is validated and optimized using simulated Monte Carlo (MC) data samples. EvtGen [12] and PYTHIA [13] are used to simulate the particle decays. Final-state radiation is calculated by the PHOTOS package [14]. The detector response is simulated with GEANT3 [15].

For all charged tracks, impact parameter requirements are applied with respect to the nominal interaction point along the beam direction \(|dz| < 5.0 \text{ cm}\) and in the transverse plane \(|dr| < 1.0 \text{ cm}\). For electrons, muons, \( K^+ \), and \( \pi^+ \), a particle identification likelihood is calculated from the energy loss in the CDC \( (dE/dx) \), time-of-flight measurements in the TOF, the response of the ACC, the transverse shape and size of the showers in the ECL and information about hits in the KLM. For electrons, energy loss from bremsstrahlung is recovered by adding to the candidate the momenta of photons in a cone of 0.05 radians around the initial direction of the charged track. \( K_{\pi}^0 \) candidates are reconstructed from pairs of oppositely-charged tracks (treated as pions) and selected based on vertex fit quality. \( \pi^0 \) mesons are reconstructed from photon pairs with the requirement \( E_\gamma > 30 \text{ MeV} \) and \( 115 \text{ MeV}/c^2 < M_{\gamma\gamma} < 153 \text{ MeV}/c^2 \). \( K^+ \) candidates are formed from \( K^{+}\tau^- \), \( K^{+}\pi^0 \) and \( K_{\pi}^{0}\pi^+ \) combinations that satisfy the requirements on invariant mass of 0.6 GeV/c^2 < \( M_{K\pi} \) < 1.4 GeV/c^2 and on vertex fit quality (to suppress background). The \( K^+ \) candidates are combined with oppositely charged lepton pairs to form \( B \).
meson candidates, where the charge of the kaon or pion defines the charge or flavor of the B meson. The particle selection criteria lead to combinatorial background that is suppressed by applying requirements on the beam-energy constrained mass, $M_{bc} = \sqrt{E_{beam}^2 - (p_B)^2}/c^2$, and the energy difference, $\Delta E = E_B - E_{beam}$, where $E_B$ and $p_B$ are the energy and momentum, respectively, of the reconstructed candidate in the $T(4S)$ rest frame and $E_{beam}$ is the beam energy in the center-of-mass frame. Correctly reconstructed candidates are centered at the nominal $B$ mass in $M_{bc}$ and at zero in $\Delta E$. Candidates that satisfy $5.22 \text{ GeV}/c^2 < M_{bc} < 5.30 \text{ GeV}/c^2$ and $-0.10 < \Delta E < 0.05 \text{ GeV}$ for the electron (muon) modes are retained. Large irreducible background contributions arise from charmonium decays $B \rightarrow J/\psi K^*$ and $B \rightarrow \psi(2S)K^*$, in which the $c\bar{c}$ state decays into two leptons. These decays are vetoed with the requirements $-0.25 < M_{\ell\ell - m_{J/\psi}} < 0.08 \text{ GeV}/c^2$ and $-0.20 < \Delta E < 0.08 \text{ GeV}/c^2$ for the electron (muon) modes. In the electron case, the veto is applied twice: with and without the bremsstrahlung-recovery treatment. Di-electron background from photon conversions ($\gamma \rightarrow e^+e^-$) and $\pi^0$ Dalitz decays ($\pi^0 \rightarrow e^+e^-\gamma$) is rejected by requiring $M_{ee} > 0.14 \text{ GeV}/c^2$.

To maximize signal efficiency and purity, neural networks are utilized sequentially from the bottom to the top of the decay chain, transferring the output probability from each step to the subsequent step so that the most effective selection requirements are applied in the last stage based on all information combined. For all particle hypotheses, a neural network is trained to separate signal from background and an output value, $o_{NB}$, is calculated for each candidate. The classifiers for $e^\pm, \mu^\pm, K^\pm, K_S^0, \pi^0$, and $\pi^\pm$ are taken from the neural-network-based full event reconstruction described in Ref. [16]. For $K^*$ selection, a classifier is described on MC samples using kinematic variables and vertex fit information. The final classification is performed with a requirement on $o_{NB}$ for each $B$ decay channel using event-shape variables (i.e., modified Fox-Wolfram moments [17]), vertex fit information, and kinematic variables as input for the classifier. The most important variables for the neural networks are $\Delta E$, the reconstructed mass of the $K^*$, the product of the network outputs of all secondary particles, and the distance between the two leptons along the beam direction $\Delta z_{\ell\ell}$. If multiple candidates are found in an event (less than 2% of the time), the most probable candidate is chosen based on $o_{NB}$. The selection requirements for the neural networks are optimized by maximizing the figure of merit $n_s/\sqrt{n_s + n_b}$ separately for the electron and muon channels, where $n_s$ and $n_b$ are the expected numbers of signal and background candidates, respectively, calculated from MC.

Signal and background yields are extracted by an unbinned extended maximum likelihood fit to the $M_{bc}$ distribution of $B \rightarrow K^*\ell^+\ell^-$ candidates, presented in Fig. 1, where the signal is parametrized by a Crystal Ball function [18] and the background is described by an ARGUS function [19]. The signal shape parameters are determined from a fit to $B \rightarrow J/\psi K^*$ data in the corresponding $q^2$ veto region while the background shape parameters are allowed to float in the fit. In total 127 ± 15 and 185 ± 17 signal candidates are obtained for the electron and muon channels, respectively.

The analysis is performed in four independent bins of $q^2$, as detailed in Table I, with an additional bin in the range $1.0 \text{ GeV}^2/c^2 < q^2 < 6.0 \text{ GeV}^2/c^2$, which is favored for theoretical predictions [6]. To make maximum use of the limited statistics, a data-transformation technique [20, 21] is applied, simplifying the differential decay rate without losing experimental sensitivity. The transformation is applied to specific regions in the three-dimensional angular space, exploiting the symmetries of the cosine and sine functions to cancel terms in Eq. 1. With the following transformations to the dataset, the data are sensitive to the observable of interest:

$$P_4', S_4 : \begin{cases} \phi \rightarrow -\phi & \text{for } \phi < 0 \\ \phi \rightarrow \pi - \phi & \text{for } \theta_\ell > \pi/2 \\ \theta_\ell \rightarrow \pi - \theta_\ell & \text{for } \theta_\ell > \pi/2, \end{cases}$$

$$P_5', S_5 : \begin{cases} \phi \rightarrow -\phi & \text{for } \phi < 0 \\ \theta_\ell \rightarrow \pi - \theta_\ell & \text{for } \theta_\ell > \pi/2. \end{cases}$$

With this procedure, the remaining observables are the $K^*$ longitudinal polarization, $F_L$, the transverse polarization asymmetry, $A_L^{(2)} = 2S_4/(1 - F_L)$, and $P_4'$ or $P_5'$. Two independent maximum likelihood fits for each bin of $q^2$ are performed to the angular distributions to extract the $P_4', S_5$ observables. The fits are performed using the data in the signal region of $M_{bc}$ of all decay channels and separately for the electron and muon mode. The signal (background) region is defined as $M_{bc} \geq 5.27 \text{ GeV}/c^2$.
(\(M_{bc} < 5.27 \text{ GeV}/c^{2}\)). For each measurement in \(q^2\), the signal fraction is derived as a function of \(M_{bc}\). The background angular distribution is described using the direct product of kernel density template histograms \([22]\) for \(\phi, \theta_t\) and \(\theta_K\) while the shape is predetermined from the \(M_{bc}\) sideband. Acceptance and efficiency effects are accounted for in the fit by weighting each event by the inverse of its combined efficiency, which is derived from the direct product of the efficiencies in \(\phi, \theta_t, \theta_K\) and \(q^2\). The individual reconstruction efficiency for each observable is obtained by extracting the ratio between the reconstructed and generated MC distributions.

All methods are tested and evaluated in pseudo-experiments using MC samples for each measurement and the results are compared to the input values. Systematic uncertainties are considered if they introduce an angular- or \(q^2\)-dependent bias to the distributions of signal or background candidates. Small correlations between \(\theta^t\) and \(q^2\) are not considered in the treatment of the reconstruction efficiency. The deviation between a fit based on generator truth and an MC sample after detector simulation and reconstruction reweighted with efficiency corrections is evaluated for a bias. The difference between the two fits (0.045 on average) is taken as the systematic uncertainty for the efficiency correction; this is the largest systematic uncertainty. Peaking backgrounds are estimated for each \(q^2\) bin using MC. In total, fewer than six (one) such background events are expected in the muon (electron) channels. The impact of the peaking component is simulated by performing pseudo-experiments with MC samples for signal and background according to the measured signal yields, replacing six randomly selected events from the signal class with events from simulated peaking background in each measurement. The observed deviation from simulated values (0.02 on average) is taken as the systematic uncertainty. An error on the background parametrization is estimated by repeating all fits with an alternative background description using third-order polynomials and taking the observed deviation (0.028 on average) as the systematic error. Finally, an error on the signal parametrization is considered by repeating the fit with the signal shape parameters adjusted by \(\pm 1\sigma\), leading to systematic uncertainties of order \(10^{-4}\). Signal cross-feed is evaluated for all signal decay channels and found to be insignificant.

The parametrization in Eq. 1 does not include a possible S-wave contribution under the \(K^*\) (892) mass region. With the expected fraction of 5\% \([1, 20]\), we estimate the S-wave contribution for each measurement to be less than one event and the resulting effects to be negligible. Statistically equal numbers of \(B\) and \(\bar{B}\) candidates in the signal window are found; consequently, \(CP\)-asymmetric contributions to the measured \(CP\)-even parameters are neglected. The total systematic uncertainty is calculated as the sum in quadrature of the individual values.

The result of all fits is presented in Table 1 and displayed in Fig. 2 where it is compared to SM predictions by DHMV, which refers to the soft form-factor method of Ref. [23]. Predictions for the \(14.18 \text{ GeV}^2/c^2 < q^2 < 19.00 \text{ GeV}^2/c^2\) bin are calculated using lattice QCD with QCD form factors from Ref. [24]. The predictions include the lepton mass, leading to minor corrections between the SM values for the electron and muon modes. For the electron mode, fits in the region \(10.09 \text{ GeV}^2/c^2 < q^2 < 12.90 \text{ GeV}^2/c^2\) are excluded because it overlaps with the \(\psi(2S)\) veto range, leading to insufficient statistics for stable fit results. In total, all measurements are compatible with SM predictions. The strongest tension of 2.6\(\sigma\) (including systematic uncertainty) is observed in \(P'_5\) of the muon modes for the region \(4 \text{ GeV}^2/c^2 < q^2 < 8 \text{ GeV}^2/c^2\); this is in the same region where LHCb reported the so-called \(P'_5\) anomaly \([1, 20]\). In the same region, the electron modes deviate by 1.3\(\sigma\) and all channels combined
TABLE I. Fit results for $P_4'$ and $P_5'$ for all decay channels and separately for the electron and muon modes. The first uncertainties are statistical and the second systematic.

| $q^2$ in GeV$^2$/c$^2$ (GeV$^2$/c$^2$) | $P_4'$ | $P_5'$ | $P_4''$ | $P_5''$ | $P_4'''$ | $P_5'''$ | $P_4''''$ | $P_5''''$ |
|-------------------------------------|--------|--------|---------|---------|---------|---------|---------|---------|
| [1.00, 6.00]                        | -0.45^{+0.23}_{-0.22} ± 0.09 | -0.72^{+0.26}_{-0.29} ± 0.06 | -0.22^{+0.03}_{-0.02} ± 0.15 | 0.23^{+0.04}_{-0.03} ± 0.07 | -0.22^{+0.03}_{-0.02} ± 0.07 | 0.43^{+0.04}_{-0.03} ± 0.10 |
| [0.10, 4.00]                        | 0.11^{+0.07}_{-0.06} ± 0.05 | 0.34^{+0.06}_{-0.06} ± 0.11 | -0.02^{+0.04}_{-0.03} ± 0.12 | 0.34^{+0.06}_{-0.06} ± 0.05 | -0.02^{+0.04}_{-0.03} ± 0.07 | 0.43^{+0.06}_{-0.05} ± 0.14 |
| [4.00, 8.00]                        | -0.34^{+0.17}_{-0.17} ± 0.03 | -0.32^{+0.19}_{-0.21} ± 0.09 | -0.30^{+0.19}_{-0.21} ± 0.09 | -0.32^{+0.19}_{-0.21} ± 0.09 | -0.30^{+0.19}_{-0.21} ± 0.09 | -0.03^{+0.31}_{-0.30} ± 0.09 |
| [10.00, 12.90]                      | -0.18^{+0.20}_{-0.20} ± 0.06 | -0.40^{+0.32}_{-0.33} ± 0.09 | -0.17^{+0.20}_{-0.20} ± 0.09 | -0.40^{+0.32}_{-0.33} ± 0.09 | -0.17^{+0.20}_{-0.20} ± 0.09 | -0.09^{+0.29}_{-0.28} ± 0.02 |
| [14.18, 19.00]                      | -0.14^{+0.26}_{-0.26} ± 0.05 | -0.15^{+0.41}_{-0.40} ± 0.04 | -0.10^{+0.36}_{-0.35} ± 0.07 | -0.51^{+0.24}_{-0.22} ± 0.01 | -0.91^{+0.30}_{-0.29} ± 0.03 | -0.13^{+0.35}_{-0.35} ± 0.06 |

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