The Casimir frequency spectrum: can it be observed?

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Abstract. The frequency spectrum of the Casimir force between real materials is studied with a view to assess possible ways of by which its dramatic oscillatory behaviour might be observed. Two simple attempts are made in which a model material is perturbed so as to become semitransparent in a band of frequencies. It is found that the real frequency formalism of the Casimir Lifshitz force is extremely volatile upon manipulations of optical properties and produces nonsensical results when extreme care is not taken to ensure the physicality of all perturbations, whereas the imaginary frequency formalism is highly robust and behaves well even under unphysical manipulations. The indication is thus that the general physical requirements of response functions preclude the observation of the frequency spectrum.

1. Introduction

The dependence of the Casimir force on the frequency dependent dielectric response of materials has long been an issue of intense research and at times a source of controversy. Casimir’s original calculation of the fluctuation induced attraction between parallel plates assumed ideal conductors, perfectly reflecting at all field energies [1], and was generalised by Lifshitz a few years later [2] to materials of arbitrary frequency dependent dielectric response. The Lifshitz formula has since come to be recognised as a special case of a much more general formalism of multiple scattering [3,4], yet while much progress has been made in research on the Casimir effect between more general bodies, the role of the inclusion of material dispersion properties in the simple two-plate geometry is still imperfectly understood. This has been most clearly demonstrated in the long ongoing disagreement over the thermal behaviour of the Casimir force and free energy (e.g. [5,6] and references therein), a controversy centered in essence on the exact optical properties of materials at very low frequencies.

Upon incorporating the dispersive properties of materials, the expression for the total Casimir attraction between objects must necessarily include an integral over all frequencies of the electromagnetic field. Since all real materials obey the laws of causality it is possible [7] to shift the integral from the real to the positive imaginary frequency axis, which for the purposes of calculation is most often preferable. While the integrand along the imaginary frequency axis will typically be nicely peaked and exponentially decaying at high frequencies and therefore well suited for numerical evaluation, the real-frequency integrand is the direct opposite. Ford and others investigated the real-frequency spectrum of the Casimir force some time ago and showed how it is wildly oscillating, with the area under one oscillation peak of the frequency integrand much greater than the measurable Casimir force itself [8–10].

Formally, thus, the Casimir effect appears to emerge as the sum of an alternating series of very
large and almost exactly canceling contributions. This inspired the suggestions that the optical properties of materials may be tuned so as to produce a Casimir force of desired magnitude and even sign [10, 11]. A handful of efforts have since elaborated this prospect [12–15], while an analysis by Genet and co-workers concluded that due to general restrictions of causality and analyticity such possibilities could not be realised [16].

Recently, the interpretation of an experiment by Iannuzzi and co-workers [17] was revisited in light of the considerations of the frequency spectrum of the Casimir force [18]. The experiment measured the Casimir attraction between so-called hydrogen switchable mirrors (HSMs). A HSM is a material which is a good metallic reflector in its as-deposited state but becomes transparent in the visible frequency range upon placement in a hydrogen rich atmosphere. While the reflection properties of the HSM are quite drastically different in the two states, the group was unable to detect a significant change in the Casimir attraction from one atmosphere to the other at a precision in the order of 15%. At face value this negative result seems at odds with the predictions of Ford, yet an alternative calculation of the effects of such a change of reflectivity indicates that the expected effects on the force might indeed be rather modest. A more careful analysis was made in [19].

In the following the recent analysis on the effect of perturbations of optical properties on the real frequency axis is reviewed and extended.

2. Frequency spectrum of the Casimir Lifshitz force
The frequency spectrum of Casimir interactions may be taken from the Lifshitz formula for real frequencies or worked out on more general grounds from the stress tensor operator of the quantised electromagnetic field. The Lifshitz formula in the form most useful for this purpose reads [2] (zero temperature is assumed henceforth)

\[
P(a) = -\frac{1}{2\pi^2} \Re \int_{0}^{\infty} d\omega \omega^3 \int_{C} d\rho^2 \sum_{\sigma=s,p} \frac{r_{\sigma}^2 \exp(2i\rho \omega a)}{1 - r_{\sigma}^2 \exp(2i\rho \omega a)}.
\]

(1)

Here \(P(a)\) is the Casimir pressure between the parallel plates separated by a distance \(a\) where the minus sign signifies an attractive force. Natural units \(\hbar = c = k_B = 1\) are used throughout. \(\omega\) is the real-valued frequency and the integral over the Lifshitz variable \(\rho = \sqrt{(k_{\perp}/\omega)^2 - 1}\) (physically the sum over all momentum components parallel to the surfaces) runs from 1 to 0 covering the propagating part of the spectrum, and thence to \(i\infty\) summing up all evanescent field contributions.

The frequency spectrum \(P_\omega(a)\) is simply defined as

\[
P(a) = \int_{0}^{\infty} d\omega P_\omega(a) = \int_{0}^{\infty} d\omega \sum_{\sigma=s,p} P_{\omega,\sigma}(a).
\]

(2)

This spectrum in fact equals that derived by Ford with a more fundamental procedure [8, 10]. With the special modelling assumption that reflection coefficients are constant with respect to \(k_{\perp}\) the \(p\)-integral (1) may be solved explicitly by use of the polylogarithmic function

\[
\text{Li}_n(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^n}
\]

which obeys the useful recursion relation \((A\text{ and }K\text{ are constants})

\[
\int dy \text{Li}_n(Ae^{ym}) = \frac{1}{\eta} \text{Li}_{n+1}(Ae^{ym}) + K
\]

(4)
to yield the following frequency spectrum

\[ P_\omega(a; \{ r_\sigma \}) = \frac{-1}{16\pi^2a^4} \sum \left[ -\xi^23\text{mLi}_1(r_\sigma^2e^{i\xi}) - 2\xi\text{ReLi}_2(r_\sigma^2e^{i\xi}) + 2\text{ImLi}_3(r_\sigma^2e^{i\xi}) \right] \] (5)

where the shorthand dimensionless quantity \( \xi = 2\omega a \) is introduced. The spectrum (5) is plotted in figure 1 assuming \( r_\sigma \) are also constant with respect to frequency.

When the latter assumption is made, the frequency integral is explicitly solvable and the Casimir zero temperature pressure is found as

\[ P(a, \{ r_\sigma \}) = -\frac{3}{16\pi^2a^4} \sum \text{ReLi}_4(r_\sigma^2) \] (6)

Casimir’s result \( P_C(a) = -\pi^2/(240a^4) \) is regained by inserting \( r_p^2 = r_s^2 = 1 \) and \( \text{Li}_4(1) = \zeta(4) = \pi^4/90 \). Exactly the same treatment starting from the Lifshitz formula for the zero temperature free energy rather than pressure yields

\[ \mathcal{F}(a, \{ r_\sigma \}, T = 0) = -\frac{1}{16\pi^2a^4} \sum \text{ReLi}_4(r_\sigma^2) \] (7)

which again simplifies trivially to Casimir’s result in the limit \( r_\sigma = 1 \).

Figure 1. The frequency spectrum (5) normalised by \( a^{-3} \) for different (constant) values of \( r_\sigma = r \), assumed equal for both polarisations.

Figure 2. Casimir pressure (6) as function of constant reflection coefficient \( r \), normalised by the ideal conductor limit \( P_C(a) = -\pi^2/(240a^4) \).

3. Frequency spectrum with real materials

When more realistic material properties are employed, the frequency spectrum of the Casimir force is no longer as regular as that shown in figure 1, but shares the property of wild oscillation of increasing amplitude throughout the frequency range in which optical data are typically available. Given an explicit expression of reflection coefficients as functions of frequency and transverse momentum, it is straightforward to calculate and plot the frequency spectrum, as shown in figure 3. In these calculations the permittivity function for gold is used, represented
by, respectively, the simple plasma model \( \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \) with \( \omega_p = 9.0 \text{eV} \) and interpolation of experimental optical data tabulated in [20]. The graphs share the trait of large oscillations which make them unsuitable for most numerical purposes. It is notable, however, that the two spectra are radically different even though a calculation of the total Casimir force by the standard method of rotating the integral onto the imaginary frequency axis gives almost identical results whether the plasma model or the corresponding data set from [20] is used.

![Figure 3](image-url)

**Figure 3.** Numerically calculated frequency spectrum using respectivly data from [20] and the plasma model, \( \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \). Note that the spectrum is now a function of \( a \) and \( \omega \) individually, not only their product; \( a = 100 \text{nm} \) is used in the figure.

The striking nature of these spectra naturally raises the question whether or not such giant oscillations could be observable, and, if so, might even be used for experimental or technological purposes as proposed in previous publications. On purely qualitative grounds one may conclude from the extreme difference between the graphs in figure 3 that if observation of such a spectrum were indeed possible, quantitative predictions cannot be made using simple models such as the plasma or Drude model (the latter produces a spectrum reminiscent of that of the plasma model in figure 3 with a greater number of sharp peaks), and the direct use of extremely accurate optical data would be required.

4. Effects of a band perturbation of reflectivity
As figure 3 shows clearly, due to its highly oscillating nature the real frequency formalism is not very helpful for calculating the absolute Casimir force [21]. One may think, however, that it could nonetheless be fruitful for calculating a change of optical properties which was limited to a small interval of frequencies, in which case an integral might not need to span the entire frequency axis.

The experiment described above in which HSMs were used for the measurement of Casimir forces would seem a natural candidate for the use of a real-frequency formalism. Upon hydrogenation of the HSM, the material became transparent in the visible frequency range whereas the optical properties in the infrared and ultraviolet remained largely unchanged. The authors of [17, 19] propose a rough model for the description of the alteration of reflectivity in which the metallic permittivity of the mirror in the as-deposited state is \textquotedblleft switched off\textquotedblright at \( \omega_1 \approx 7.5 \cdot 10^{14} \text{rad/s} \) and \textquotedblleft on\textquotedblright again at \( \omega_2 \approx 9.4 \cdot 10^{15} \text{rad/s} \). That is, \( \varepsilon(\omega) = 1 + 4\pi \chi(\omega) \longrightarrow \tilde{\varepsilon}(\omega) = 1 + 4\pi \tilde{\chi}(\omega) \) where

\[
\tilde{\chi}(\omega) = \chi(\omega)[1 - \theta(\omega - \omega_1)\theta(\omega_2 - \omega)].
\]
Here $\theta(x)$ is the unit step function. In fact the transition alters the properties at all frequencies to some extent [19], but as a first approximation one might expect that the “off/on” model would give at least an idea of the magnitude of the effect.

Using the formalism of imaginary frequencies, the effect of such an “off/on” model can be calculated by integration over the imaginary part of $\epsilon(\omega)$ in the relevant Kramers-Kronig relation. Using as an example the Drude model,

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)},$$

with $\nu$ being the electronic relaxation frequency (for gold $\nu = 35\text{meV}$ is often used [22]), one may readily calculate the correction $\epsilon(i\zeta) \rightarrow \epsilon(i\zeta) - \Delta\epsilon(i\zeta)$ explicitly as

$$\Delta\epsilon(i\zeta) = \frac{2}{\pi} \int_0^{\infty} \frac{d\omega [\epsilon''(\omega) - \tilde{\epsilon}''(\omega)]}{\omega^2 + \zeta^2} = \frac{2}{\pi} \int_{\omega_1}^{\omega_2} \frac{d\omega \epsilon''(\omega)}{\omega^2 + \zeta^2}$$

$$= \frac{\omega_p^2}{\zeta^2 - \nu^2} \frac{2}{\pi} \left[ \arctan \frac{\nu \Delta \omega}{\nu^2 + \omega_1 \omega_2} - \frac{\nu}{\zeta} \arctan \frac{\zeta \Delta \omega}{\zeta^2 + \omega_1 \omega_2} \right]$$

with $\Delta \omega = \omega_2 - \omega_1$. Here the real and imaginary parts of $\epsilon(\omega)$ are denoted with a single and double prime respectively. For moderate values of $\Delta \omega$ this gives only a relatively small contribution to $\epsilon(i\zeta)$, on the level of a few percent for the values of $\omega_1$ and $\omega_2$ mentioned, and the consequences for the pressure would be unobservable at the precision of the experiment [17], in line with the negative conclusion of that paper. The “off/on” model is appealing as a first approach due to its simplicity and intuitiveness and when applied this way provides a very simple qualitative explanation of the effects of the introduction of a transparency window.

Using the real-frequency spectrum directly, however, gives very different results. The change in the force is calculated by subtracting a band of frequencies from the spectrum plotted in figure 3. To avoid possible troubles stemming from cutting the spectrum off sharply, an envelope function is used,

$$\varphi(\omega) = 1 - \frac{\Delta}{\pi} \{ \arctan[sa(\omega - \omega_1)] + \arctan[sa(\omega_2 - \omega)] \},$$

so that $\epsilon(\omega) \rightarrow 1 + 4\pi \chi(\omega)\varphi(\omega)$. The parameter $\Delta \in [0, 1]$ is the relative reduction of $\epsilon(\omega)$ in the band and $s$ determines the smoothness of the edges. The limit $s \rightarrow \infty$ giving the unit step behaviour of (8) [18]. A numerical integration over a sufficiently large frequency integral around $\Delta \omega$ with and without $\varphi(\omega)$ yields a rough prediction of the change in the force, which is plotted in figure 4. Clearly the change found this way is absurdly large and, counterintuitively, has opposite sign whether the Drude or plasma model is employed. At separation $a = 100\text{nm}$ as used in this calculation, the absolute Casimir pressure between ideally conducting parallel plates is approximately 13Pa, shown as a grey band in the figure for scale. It is immediately clear that a simple “off/on” model such as this is not compatible with direct real frequency calculation.

5. Causal perturbation on narrow frequency band
For all its intuitive appeal, the “off/on” model, even in its smooth form (11) is not very realistic and suffers from problems which could give rise to the dramatic failure when applied on the real frequency axis. The sharp edges of the frequency band initially employed violate conditions of continuity and analyticity, but the indication is that this is not important since the introduction of smooth boundaries through the envelope function $\varphi(\omega)$ makes for only qualitative adjustments of the results.
More serious is the fact that the way the transparency band is calculated on the real axis does not adhere to requirements of causality, in particular, that $\epsilon(\omega)$ fulfils the Kramers-Kronig relations. In the following an investigation is made of the feasibility of using real-frequency calculation for the assessment of the effect of a narrow-band perturbation of $\epsilon(\omega)$ in a way so that (12) and the so-called f-sum rule are obeyed.

It is a standard procedure in the study of the optical properties of materials to model the absorption spectrum $\epsilon''(\omega)$ of the material and work out $\epsilon'(\omega)$ based on this using

$$
\epsilon'(\omega) = 1 + \frac{2}{\pi} \Im \int_0^\infty d\xi \frac{\xi \epsilon''(\xi)}{\xi^2 - \omega^2}. \tag{12}
$$

Likewise, $\epsilon(i\zeta)$ is calculated using the corresponding Kramers Kronig relation (10). Remarkably informative and useful information about different classes of materials can be extracted from very simple models [23]. This provides motivation to regard the consequences of perturbing $\epsilon''(\omega)$ in a causal way.

The perturbation in the dissipation function is modelled in the following way$^1$

$$
\epsilon''(\omega) \to \tilde{\epsilon}''(\omega) = \epsilon''(\omega)[(1 + \eta) - \Delta \cdot \theta(\omega - \omega_0)\theta(\omega_0 + \delta\omega - \omega)], \quad \omega \geq 0
$$

that is, $\epsilon''$ is approximately unchanged everywhere except in a small frequency band at $\omega_0$ of width $\delta\omega$ in which dissipation is reduced by a constant quantity $\Delta$. The small parameter $\eta$ has the same sign as $\Delta$ and is introduced because the imaginary part of the permittivity must satisfy the f-sum rule (e.g. [24]):

$$
\int_0^\infty d\omega \omega \epsilon''(\omega) = \text{const.} = \frac{\pi}{2} \sum_\alpha \frac{4\pi \kappa_\alpha q_\alpha^2}{m_\alpha^*} = \frac{\pi}{2} \Omega_p^2
$$

where sum is over the types of particles, $\alpha$, causing dissipation, characterised by number density $\kappa_\alpha$, charge $q_\alpha$ and effective mass $m_\alpha^*$. When dissipation is due to conduction electrons only, $\Omega_p = \omega_p$ as in the Drude and plasma models. Insisting that (14) be satisfied both before and after the perturbation implies

$$
\eta = \frac{2}{\pi} \frac{\Delta}{\Omega_p^2} \int d\omega \omega \epsilon''(\omega) \approx 2\Delta \frac{\omega_0}{\Omega_p^2} \epsilon''(\omega_0)\delta\omega
$$

$^1$ Causality requires that the perturbation is an odd function of $\omega$, but only positive frequencies are considered here.
where $\int_{\omega_0} d\omega$ denotes integration from $\omega_0$ to $\omega_0 + \delta \omega$ and the latter form is true when $\delta \omega$ is small compared to all other frequency scales. Assuming $\delta \omega$ small implies that $\eta \ll 1$.

To keep the considerations a little more concrete, consider the special case of the Drude model (9), for which $\eta$ becomes:

$$\eta_D = \frac{2}{\pi} \Delta \cdot \arctan \left( \frac{\nu \cdot \delta \omega}{\nu^2 + \omega_0 (\omega_0 + \delta \omega)} \right) \approx \frac{2 \Delta}{\pi} \frac{\nu \cdot \delta \omega}{\nu^2 + \omega_0^2}. \quad (16)$$

The perturbed real part $\tilde{\epsilon}'(\omega)$ now follows from (12)

$$\tilde{\epsilon}'(\omega) = 1 + \frac{2}{\pi} \nu \partial \left\{ (1 + \eta) \int_0^{\infty} d\xi - \Delta \int_{\delta \omega} d\xi \right\} \frac{\xi \tilde{\epsilon}''(\xi)}{\xi^2 - \omega^2}$$

which gives, to linear order in $\delta \omega$ or $\eta$, the change in $\epsilon'(\omega)$

$$\delta \epsilon'(\omega) \equiv \tilde{\epsilon}'(\omega) - \epsilon'(\omega) \approx \eta \left[ \epsilon'(\omega) - 1 - \frac{\Omega_p^2}{\omega_0^2 - \omega^2} \right] \quad (17)$$

where (15) was used. The case of the Drude model gives

$$\delta \epsilon_D(\omega) = -\eta_D \frac{\omega_0^2 (\omega_0^2 + \nu^2)}{(\omega^2 + \nu^2)(\omega_0^2 - \omega^2)}. \quad (18)$$

In the same fashion, inserting (13) into the corresponding Kramers-Kronig integral for imaginary argument, (10), yields to linear order in $\eta$

$$\delta \epsilon(i\zeta) \equiv \tilde{\epsilon}(i\zeta) - \epsilon(i\zeta) \approx \eta \left[ \epsilon(i\zeta) - 1 - \frac{\Omega_p^2}{\omega_0^2 + \zeta^2} \right]. \quad (19)$$

The important difference between (17) and (19) is that while $\delta \epsilon(i\zeta) \ll \epsilon(i\zeta)$ over the entire imaginary frequency axis, the perturbation $\delta \epsilon'(\omega)$ grows large close to its pole at $\omega_0$. While the perturbation (13) is negligible when calculated for imaginary frequencies, thus, it may be worth taking a closer look at what happens in the real frequency setting, equation (1).

The permittivity $\epsilon(\omega)$ for a Casimir cavity of two dielectric half-spaces (assumed to be made of the same material for simplicity) enters into the Lifshitz formula through the Fresnel reflection coefficients

$$r_s = \frac{p - s}{p + s}; \quad r_p = \frac{ep - s}{ep + s}, \quad (20)$$

where $s \equiv \sqrt{p^2 + \epsilon - 1}$. The reflection coefficients stay approximately unchanged by $\delta \epsilon'$ except when $\omega$ is in the neighbourhood of $\omega_0$ where $\tilde{\epsilon}'(\omega)$ becomes dominated by the otherwise negligible correction $\delta \epsilon'$. Here, thus, $\tilde{s} \sim \sqrt{\delta \epsilon'(\omega)}$ and $\tilde{s}$ therefore rises towards $+\infty$, skips to $+i\infty$ and decreases quickly thence to its unperturbed value once more. The effect for $r_p$ and $r_s$ is, roughly, that over a frequency interval of order $\delta \omega$ near $\omega_0$ they rise to unity and return to their unperturbed value once more. Since $\delta \omega$ is assumed smaller than all other frequency scales involved, all other quantities can be seen as approximately constant over this interval from which it follows that the change in Casimir pressure from (1) behaves as

$$\delta P(a) \sim \delta \omega \cdot P_{\omega_0}(a; \{r_\sigma\}) |_{r_s=r_p=1} \quad (21)$$

where $P_{\omega_0}(a; \{r_\sigma\})$ is the discontinuous spectrum function (5) at $\omega = \omega_0$ taken with unity reflection coefficients just as worked out by Ford [10] and shown in figure 1.
The result (21) is again counterintuitive in that it depends sensitively on the frequency $\omega_0$ and could be either positive or negative depending on the quantity $\omega_0a$. Indeed the spectrum $P_{\omega_0}(a)|_{\sigma=1}$ is discontinuous at $\omega_0a = n\pi$, $n \in \mathbb{N}$, meaning $\delta P$ would make a leap were $\omega_0a$ to be shifted slightly past such a value, e.g. by changing the separation slightly close to $a = n\pi/\omega_0$.

In conclusion it seems that the fact that the perturbation is made causal by use of (12) and made to fulfil the f-sum rule (14) does not in itself salvage the apparent paradox that a calculation using the real frequency integral yields apparently nonsensical results while a calculation along the imaginary axis appears robust. If the perturbation $\Delta\epsilon''(\omega)$ fulfilled all the requirements of analyticity and causality and were taken into account by an integral (12) spanning the entire positive frequency axis, the real-frequency and imaginary frequency formalisms must necessarily give the same results. The observation of the vast fluctuations of the Casimir frequency spectrum, however, requires that access be somehow gained to finite intervals of the spectrum, and although a somewhat coarse analysis, the above considerations strongly indicate a pessimistic conclusion with regards to the feasibility of such an enterprise.

6. Conclusions and outlook

The frequency spectrum of the Casimir force between real materials is studied with a view to assess whether it may be possible to observe the dramatic oscillatory behaviour of the real frequency Casimir energy spectrum. A generalisation of Ford’s result for ideal conductors [10] to a model of constant subunity reflection coefficients reveals a smoothening of the spectrum, but the oscillatory behaviour remains unchanged.

Upon inserting more realistic optical data for real materials, the frequency spectra obtain an even more wild and erratic behaviour. Observation of the dependence on real frequencies requires that access be somehow gained to finite intervals of real frequencies as opposed to the absolute force itself, which depends only on the integral over all frequencies.

Two simple attempts are made to investigate the question whether a perturbation of the optical properties of materials which is restricted to a finite band of real frequencies could reveal a way to observe the large oscillations described. These have not been concerned with whether and when such perturbations may be made in practice, but have focussed on a problem of theoretical nature which occurs upon attempting to calculate the effects of such perturbations on the Casimir force: the predictions are radically different whether it is performed using the real frequency or imaginary frequency formulation of the Lifshitz formula. This paradox was first reported in [18].

In the first and coarsest approach the materials involved are assumed to be made transparent in a band of frequencies but remain unchanged outside this band. At a qualitative level this mimics the situation created in a recent experiment by Iannuzzi and co-workers [17] in which a hydrogen-switchable mirror is used which becomes transparent in the optical region upon hydrogenation. Using an imaginary time formalism the change in the force predicted is quite modest, in accordance with the experiment, whereas an exclusion of a part of the real frequency axis leads to nonsensical predictions of enormous amplitude and apparently arbitrary sign.

In a second attempt the perturbation is made in the imaginary part of permittivity $\epsilon''(\omega)$ over a very small frequency range and causality is ensured by the invocation of the Kramers Kronig relations to calculate $\epsilon$ for imaginary frequencies and the real part of $\epsilon$ for real frequencies. Again a real frequency calculation reveals highly counterintuitive results whereas the imaginary frequency calculation appears reasonable in sign and magnitude and in accordance with intuition.

It is clear that the real frequency formalism of the Casimir Lifshitz force is very volatile upon manipulations of optical properties whereas the imaginary frequency formalism is robust and behaves well even with permittivities violating criteria of analyticity and causality. While it may be shown that the two formalisms must yield the same result for measurable quantities when the entire frequency integral is evaluated, both attempts to access the dependency on a
finite frequency band made herein have yielded very different results in the two formalisms, of which there are good reasons to believe the result in the imaginary frequency domain to be the physical one. While this study is far from exhaustive, it indicates that the peculiar real-frequency spectrum of the Casimir force is not observable.

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