The Crab Pulsar: Origin of the Crab Nebula’s Radio Pairs

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Abstract

Previously, we constructed a model—essentially a plausibility argument—in which the Crab Pulsar produces a spatially separated ion dominated and pair plasma dominated, magnetically striped relativistic wind, with the ion wind’s kinetic energy and electromagnetic Poynting fluxes being comparable. In this paper, the polar cap ion–photon pair production of that model is replaced with pair production by ion curvature synchrotron photons. The first primary ion curvature photons, and, contrary to conventional wisdom, also the first primary electron curvature photons, do not immediately convert into pairs. The primary beam particles continue to accelerate, and the actual photons that convert into pairs, which then short out the parallel electric field and terminate the acceleration, are produced by the further accelerated, higher energy particles. Simple estimates of the ensuing pair production cascade give pair multiplicities—the number of pairs per primary beam particle—of $M_p \approx 6-8 \times 10^4$, comparable to standard calculations, but much less than the $3 \times 10^6$ value deduced by Rees and Gunn in order to sustain the Crab Nebula’s $N_{e,\gamma} \approx 10^{31}$ radio-emitting pairs against adiabatic expansion energy losses. Using a simple spin-down evolution model for the pulsar’s rotation frequency, the time-integrated pair cascade production driven by the primary ion beam can produce the $N_{e,\gamma} \approx 10^{31}$ radio pairs, whereas the primary electron beam produces about an order of magnitude fewer pairs.

Key words: ISM: supernova remnants – pulsars: general – stars: neutron

1. Introduction

Many years ago, Arons (1983) suggested that the Crab Pulsar might emit an energetically significant flux of ions. Gallant & Arons (1994) subsequently showed that termination shocked ions might produce the Crab Nebula’s wisps, and that the ion kinetic energy flux might transport a significant fraction of the Crab Pulsar’s spin-down luminosity (Spitkovsky & Arons 2004). Recently, for the simple case of the orthogonal rotator, we presented a series of plausibility arguments for an ion dominated Crab Pulsar relativistic wind in which a super Goldreich & Julian (1969) flux of ions (assumed to be hydrogen) are strongly accelerated through a thin polar cap sheath (Coroniti 2017, hereafter CP). The accelerating parallel electric field was shorted out by the onset of ion–photon pair production, with the ions reaching final Lorentz factors of $\gamma_i \approx 5-10 \times 10^7$. A set of general magnetic geometry relativistic wind equations was derived, and the ion (and electron) field-aligned current was determined by the wind’s Alfvén point condition to be about eight times the Goldreich–Julian value. The asymptotic ratio of the Crab wind’s Poynting flux to ion kinetic energy flux was found to be $\sigma_{\infty,\gamma} \approx 1/2$. Thus, the ion dominated wind might resolve the so-called $\sigma$—problem (Arons 2009), and provide wind parameters that are roughly consistent with the Gallant & Arons (1994) model of the wisps.

The objectives of this paper are two-fold. First, we replace the ion–photon pair production process of CP by the conversion of photons produced by ion curvature synchrotron radiation, the same process that dominates pair production from the primary electron beam in fast pulsars (Muslimov & Harding 1997; Hibschman & Arons 2001). Ion curvature photons pair produce, and therefore short out the accelerating parallel electric field, at a lower altitude than was found in CP for the ion–photon pair production process, which results in a somewhat lower final ion Lorentz factor. Thus, the ion curvature process is likely to dominate over the ion–photon process; however, a detailed computation that included the acceleration and pair production of both ion curvature and ion–photon processes would be needed to determine the fraction that each process contributed to the total production of pairs (and is well beyond the scope of this paper and the competence of the author).

We analyze the initial pair production for both the primary ion and electron curvature photons, finding that their pair formation fronts (PFF’s) occur, not at the location where the first above pair energy threshold curvature photon is emitted, but at a distance along the field line that exceeds the local acceleration scale length ($l_p^{-1} = \gamma^{-1}d\gamma/ds$) in the exponentially accelerating sheath of CP. Thus, after the first curvature photon is emitted, the primary beam particles gain additional energy, and continue to radiate higher energy curvature photons until the local optical depth to pair production rises to about unity on a spatial scale much less than $l_p$. At the PFF, the ion (electron) Lorentz factor $\gamma_i (\gamma_e)$ reaches a final value of about $\gamma_i \approx 3.5 \times 10^7 (\gamma_e \approx 7.5 \times 10^7)$. A revised set of relativistic wind parameters is given, with the asymptotic ratio of Poynting to ion kinetic energy fluxes $\sigma_{\infty,\gamma} \approx 1.35$ still being of order unity. A “back-of-the-envelope” pair cascade calculation estimates the pair multiplicity—the number of pairs per primary beam particle—for the primary ion (electron) beam to be $M_p \approx 8.6 \times 10^4 (M_e \approx 6.9 \times 10^3)$, somewhat high, but not untypical values (Harding 2013).

The second objective is to show that the modified CP model can, over the history of the Crab Nebula, produce the number of radio gyro-synchrotron emitting pairs that are inferred to exist in the Nebula today. The total radio luminosity between the ionospheric cutoff at frequency $\nu_l = 3 \times 10^7$ Hz and the

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transition to optical at frequency \( \nu_2 = 10^{14} \) Hz is \( L_{\text{rad}} = 1.2 \times 10^{37} \) erg s\(^{-1}\) (Baldwin 1971; Baars 1972). The spectral luminosity (erg s\(^{-1}\)Hz\(^{-1}\)) over that frequency range varies as \( L_\nu \propto \nu^{-\alpha} \) with a spectral index of approximately \( \alpha \approx 1/4 \).

The radio-emitting pairs have a power-law energy distribution function \( f(\gamma) \propto \gamma^{-(2+\alpha)} \) between \( \gamma_1 < \gamma < \gamma_2 \) where \( \nu_1 = 3/2 \gamma_1^2 \nu_B \), \( \nu_2 = 3/2 \gamma_2^2 \nu_B \), and \( 2\pi\nu_B = eB/m_e c \) is the electron gyro-frequency; \( m_e \) is the electron/proton mass, \( c \) is the speed of light, and \( e \) is the electronic charge. For the Crab Nebula’s average magnetic field strength of \( B \approx 2 \times 10^{-4} \) Gauss, \( \gamma_1 = 1.9 \times 10^2 \) and \( \gamma_2 = 3.4 \times 10^4 \). For \( \alpha = 1/4 \), the average energy of the radio pairs \( \langle \gamma \rangle = \int_{\gamma_1}^{\gamma_2} \gamma f(\gamma) d\gamma / \int_{\gamma_1}^{\gamma_2} f(\gamma) d\gamma \) turns out to be the geometric mean \( \langle \gamma \rangle = (\gamma_1\gamma_2)^{1/2} = 8 \times 10^3 \).

In the delta function approximation, the synchrotron power per Hz emitted by a single electron/positron is

\[
P_\nu = \frac{2\sigma_T c B^2}{4\pi} \gamma^2 \sin^2 \psi (\nu - \nu_c),
\]

where \( \sigma_T \) is the Thompson cross-section, \( \nu_c = 3/2 \gamma^2 \nu_B \sin \psi \) is the characteristic synchrotron frequency, and \( \psi \) is the pitch-angle. Since the radio emission is approximately uniform within the Crab Nebula, the total radio luminosity is readily obtained by integrating over the frequency range and the pair distribution function to obtain

\[
L_\gamma = \frac{4}{9} N_\pm \frac{\alpha}{1 - \alpha} \left( \frac{\sigma_T c B^2}{4\pi} \right) \left( \frac{\gamma_1}{\gamma_2} \right)^{3\alpha} \gamma_2^2,
\]

where \( N_\pm \) is the total number of radio-emitting electrons and positrons. Setting \( L_\gamma = L_{\text{rad}} \), we obtain \( N_\pm \approx 10^{51} \). Since the synchrotron lifetime of the radio pairs exceeds the age of the Crab Nebula, the usual assumption is that during the evolution of the Nebula, at least \( N_\pm \approx 10^{51} \) pairs must have been injected by the Crab Pulsar and/or accelerated from electrons produced in the filaments.

Rees & Gunn (1974) estimated the pair injection rate \( \dot{N}_\pm \) by balancing the energy loss from the adiabatic expansion of the Nebula and the energy input from the newly injected pairs

\[
dE_{\text{rad}}/dt = -\dot{R} \frac{R}{E_{\text{rad}}} + \frac{\dot{N}_\pm}{N_\pm} E_{\text{rad}} \approx 0,
\]

where \( \dot{R} (\dot{R}) \) is the “spherical” radius (expansion speed) of the Nebula. The expansion timescale \( \tau_{\text{exp}} = \dot{R}/\dot{R} = 3 \times 10^{10} \) s yields \( \dot{N}_\pm = 3 \times 10^{40} \) s\(^{-1}\). A long-standing conundrum is that the Rees & Gunn (1974) estimate for the injection rate of radio pairs is about 300 times higher than the injection rate of the high-energy pairs needed to produce the present optical—X-ray—gamma-ray luminosities (Kennel & Coroniti 1984a, 1984b; Arons 2009).

The high-energy pair injection rate is conventionally explained by arguing that the polar cap field-aligned currents scale with the Goldreich–Julian current \( \eta_{\gamma} c / \Omega \cdot B/2\pi \), where \( \Omega \) is the angular rotation frequency of the Crab Pulsar. The Goldreich–Julian number flux per polar cap for the aligned rotator is

\[
\dot{N}_\pm = \frac{\Omega}{2\pi e} B \pi a^2 \theta_1^2 \approx 10^{34} \text{ s}^{-1},
\]

where \( B = 2\mu/a^3 \) is the polar cap magnetic field strength, \( \mu \approx 4 \times 10^{30} \) G cm\(^3\) is the dipole magnetic moment, \( a \approx 10^6 \) cm

is the neutron star radius, and \( \theta_1 = \Omega a/c \) is the colatitude of the polar cap’s last closed field line—the boundary of the open magnetic flux. Multiplying \( \dot{N}_\pm \) by the typical pair multiplicity \( M_\pm \approx 10^4 \) obtained in most analyses of the pair production cascade (Arons & Scharlemann 1979; Muslimov & Harding 1997; Hibschen & Arons 2001) yields the \( 10^{38} \) s\(^{-1}\) injection rate of high-energy pairs. The Rees & Gunn (1974) estimate for the radio pair injection rate would apparently imply that the pair multiplicity should be \( M_\pm \approx 3 \times 10^6 \), which considerably exceeds the multiplicities found by typical pair cascade calculations.

We will argue that the Rees and Gunn radio pair injection rate \( \dot{N}_\pm = 3 \times 10^{40} \) s\(^{-1}\) and the implied multiplicity \( M_\pm \approx 3 \times 10^6 \) actually represent “historical averages,” and not necessarily the current values. The values of \( \dot{N}_\pm \) and \( M_\pm \) that we will obtain for the modified CP model are slightly larger than those found in most pair production cascade calculations, but still well below the Rees and Gunn radio pair estimate. However, we find that a simple model for the temporal evolution of the Crab Pulsar’s rotation frequency results in a time-integrated number of injected pairs that is able to account for the \( \dot{N}_\pm \approx 10^{51} \) inferred to exist in the Nebula today, with pair production by the primary ion beam exceeding that of the primary electron beam by about an order of magnitude.

From a different perspective, Atoyan (1999) argued that the existence of the Crab Nebula’s radio electrons (pairs) might imply that the Crab Pulsar’s rotation frequency at birth was much higher (6–10 times today’s value, rather than the conventional estimate of about 2); his argument is based on assuming that the radio electrons are relic, and an attempt to fit the radio-optical-X-ray spectrum of the Nebula by analyzing the energy losses due to adiabatic expansion and synchrotron radiation. However, recent MHD modeling of the Crab Nebula by Olmi et al. (2014, 2016) found that the distribution of the radio electrons in the Nebula is inconsistent with their being a relic population.

Section 2 determines the height of the electron and ion PFFs and the final Lorentz factors of the primary beams, and states the new asymptotic relativistic ion dominated wind parameters. Section 3 presents estimates of the ion and electron pair multiplicities from a very simple pair cascade model, and states the parameters for the relativistic pair plasma wind. Section 4 develops the temporal evolution of the pair injection rate into the Crab Nebula, and obtains the total number of pairs by integrating the injection rate over the age of the Nebula. Throughout this paper, we maintain the philosophy of CP—to develop simple estimates in order to determine whether a relativistic ion dominated outflow model for the Crab Pulsar is plausible; clearly, far more sophisticated calculations will be required to raise the certainty level from being merely plausible. Finally, the stated and calculated values of all physical parameters are based on the physical conditions that exist in the Crab Pulsar and Crab Nebula.

### 2. Pair Formation Front

#### 2.1. Threshold for Pair Production

In CP, we found that the polar cap parallel electric field accelerates primary beam ions and electrons to Lorentz factor

\[
\gamma_i,c(s) = \gamma_i,c \Delta [\cosh (k_i s) - 1],
\]

where \( \gamma_i,c \) is the Lorentz factor of the ions in the primary beam, and \( \Delta \) is the Δ function.
where  \( \tau_c = 1.57 \times 10^6 \),  \( \tau_e = (m_i/m_e)\tau_c = 2.88 \times 10^6 \);  \( k_1 = 3.83/\theta_c = 75.67 \),  \( \theta_c = 0.62 \theta \) is the colatitude of the polar cap boundary for the orthogonal rotator at the longitude of the magnetic moment; the spherical radius  \( r = a(1 + s) \) , and the dimensionless height  \( s \) is used in all subsequent formulas;  \( \Delta \equiv \cos \phi \delta J_1(k_1\theta_\phi)/0.58 \) with  \( \theta_a, \phi \) being the colatitude and longitude in the polar cap and  \( J_1(k_1\theta_\phi) \) being a Bessel function; 0.58 is the maximum value of  \( J_1(k_1\theta_\phi) \) that occurs at  \( k_1\theta_\phi = 1.8 \) or  \( \theta_a/\theta_c = 0.47 \); the ion (electron) outflow region is  \( \pi/2 < \phi < 3\pi/2 \) ( \( -\pi/2 < \phi < \pi/2 \) ); the plane containing the rotation angular frequency and the magnetic moment is at  \( \phi = 0 \). In CP the polar cap was approximated as a circle with radius  \( a\theta_\phi \) , where the above value of  \( \theta_\phi \) was taken at  \( \phi = 0 \). This assumption allowed a simple analytic solution for the polar cap electric potential in terms of Bessel functions, and with radius  \( s \) dimensionless height  \( s \) is used in all subsequent formulas.

The production rate of curvature photons is

\[ \frac{\Delta s}{\theta_c} = \frac{256}{81 \ln \Lambda} \frac{B_C}{B} \left( \frac{a}{\lambda} \right)^{-1} \left( \frac{\theta_c}{\gamma} \right)^3 \frac{(\theta_c)^2}{(\gamma/\theta_c)^3} = \frac{2.04 \times 10^{16}}{(\gamma/\theta_c)^3} \frac{(\theta_c)^2}{(\gamma/\theta_c)^3} \]

(10)

for  \( B_s/B_C \approx 1/5 \); note that  \( a/\theta_c \) is the radius of the polar cap. For the above values of  \( \gamma(s_0) \),  \( \gamma(s_0) \) , we obtain  \( \Delta s/\theta_c = 3.21 \times 10^{-2} (k_1\Delta s = 0.122) \),  \( \Delta s/\theta_c = 40.16 (k_1\Delta s = 153.8) \). The usual interpretation of  \( \Delta s/\theta_c \ll 1 \) is that the curvature photons produced by the primary electron beam convert “on-the-spot,” and that the PFF essentially occurs at the location where the first above threshold curvature photon is emitted. We will show below that, for the rapid accelerating electric potential of CP, this assumption is not quite correct. However,  \( \Delta s/\theta_c \gg 1 \) indicates that the first above threshold curvature photons produced by the primary electron beam do not convert locally, and may not convert at all. In order to determine the final Lorentz factor of the both the primary electron and ion beams we have to construct a more accurate model of the PFF.

2.3. Pair Production

The opacity  \( \kappa_E \) for the magnetic conversion of photons into pairs is given by Erber (1966)

\[ \kappa_E = 0.23 \frac{\alpha_f}{\theta_c} \frac{B}{B_C} \sin \psi \exp \left( \frac{-8}{3\gamma} \right) \]

(9)

\[ \chi = \frac{B \sin \psi}{B_C} \frac{\hbar \omega_c}{m_e c^2} \sin \psi = \frac{a}{\rho_c} \Delta s, \]

where  \( B_C = 4.4 \times 10^{13} \) Gauss is the quantum critical magnetic field strength  \( (\hbar B_C/m_e c = m_e e^2) \), and  \( \lambda = \hbar/m_e c \) is the Compton wavelength of the electron. A curvature photon that is emitted in the direction of the magnetic field will acquire a pitch-angle  \( \psi \approx a\Delta s/\rho_c \) after it travels a distance  \( \Delta s \). Since photon conversion is a super-exponential process, the conventional assumption is that conversion occurs when  \( \chi \approx 8/(3 \ln \Lambda) \) with  \( \ln \Lambda \approx 20 \) (Arons & Scharlemann 1979).

Substituting for  \( \hbar \omega \) from Equation (6), we find that  \( \chi \approx 8/(3 \ln \Lambda) \) yields

\[ \frac{\Delta s}{\theta_c} = \frac{256}{81 \ln \Lambda} \frac{B_C}{B} \left( \frac{a}{\lambda} \right)^{-1} \left( \frac{\theta_c}{\gamma} \right)^3 \frac{(\theta_c)^2}{(\gamma/\theta_c)^3} = \frac{2.04 \times 10^{16}}{(\gamma/\theta_c)^3} \frac{(\theta_c)^2}{(\gamma/\theta_c)^3} \]

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2.4. The PFF

The complete equation for the production and conversion of curvature photons is

\[ \frac{\partial N_c}{\partial s} = \frac{4}{9} \alpha_f \frac{a}{\rho_c} \gamma(s) \mathrm{N}_c - a \kappa_E \mathrm{N}_c. \]

(11)

Integrating from  \( s_0, s \) where the first curvature photon is produced \( (\Delta N_c(s_0, s_0) = 1) \) yields

\[ \mathrm{N}_c(s) = \exp \left( -\int_{s_0}^{s} a N_c ds' \right) + \int_{s_0}^{s} ds' \left( \frac{4}{9} \alpha_f \frac{a}{\rho_c} \gamma(s') \right) \times \exp \left( -\int_{s_0}^{s} a N_c ds'' \right), \]

(12)

where the exponential factors just involve  \( \tau_E(s, s_0) \), the optical depth for the first emitted photons, and  \( \tau_E(s, s') \), the optical depth for photons that are emitted later at  \( s' > s_0 \).
The production of pairs $N_{\pm}(s)$ is given by

$$\frac{\partial N_{\pm}}{\partial s} = \kappa_E N(s). \quad (13)$$

Substituting from Equation (12) and integrating Equation (13) from $s_0$ to $s$ yields two terms (Hibschman & Arons 2001)

$$N_{\pm}(s) = N_{\pm}^{(1)}(s) + N_{\pm}^{(2)}(s)$$

$$N_{\pm}^{(1)}(s) = 1 - \exp(-\tau_E(s, s_0)) \quad (14)$$

$$N_{\pm}^{(2)}(s) = \frac{4}{9} \frac{a_{\alpha_f}}{\rho_C} \int_{s_0}^{s} \gamma(s') [1 - \exp(-\tau_E(s, s'))] ds'. \quad (15)$$

For the relatively low energy of the first curvature photon, the optical depth remains very small ($\tau_E(s, s_0) \ll 1$) until $s$ considerably exceeds the location of the PFF. Thus, the PFF, the production of pairs considerably exceeds the location of the PFF. Therefore, the PFF, which is just above the calculated value for the fiducial parameters at $\gamma_i = 3.5 \times 10^7$, which is just above the calculated value for the fiducial parameters. In developing the solution of the wind equations, the ion Lorentz factor is scaled to the parallel electric potential $\gamma_e = e \Phi_0/m_i c^2 = 5.68 \times 10^7 f_i$, where $\Phi_0 = \Omega^2 \mu / c^2$ is the open voltage along the polar cap field lines; thus, $f_i \approx 0.6$. (In CP, we chose $f_i = 1.5$.)

For the new estimates, the ratio of the electromagnetic Poynting flux to the ion kinetic energy flux at the star is $\sigma_{\text{ion}} = L_{\gamma} / L_{\gamma'}$ the total ion luminosity $L_{\gamma'}$ (in the notation of CP)

$$\sigma_{\text{ion}} = \frac{L_{\gamma}}{L_{\gamma'}} = \frac{3 \pi}{8 f_i} \frac{\theta_C}{\theta_{\text{C}}^2} = \frac{8.94}{f_i} = 14.9. \quad (21)$$

Hence, the ion wind starts mildly dominated by the Poynting flux. The ion wind energy parameter—the sum of the kinetic energy flux and the Poynting flux normalized to the kinetic energy flux at the star (Kennel et al. 1983; CP)—is

$$E = 1 + \frac{1}{4 f_i} g \theta_{\text{C}} \approx 8.46, \quad (22)$$

where $g = 4/3 \pi$ is a polar cap areal factor that was retained in CP in case the actual polar cap either differs from circular geometry and/or in the angular location of the last closed field lines (Arons 2012). The field-aligned current parameter $J_0$, which scales the current density to the Goldreich–Julian current $\eta_{\text{C}}$, is determined by the Alfven point condition to be

$$J_0 = \frac{8}{3} \left( \frac{E}{E - 1} \right)^{1/2} \left( \frac{\theta_C}{\theta_{\text{C}}} \right)^2 \approx 7.4, \quad (23)$$
which is slightly below the value of $J_0 = 8$ found in CP. For the simple model of the wind region that was constructed in CP, the asymptotic ratio of Poynting flux to ion kinetic energy flux—the standard pulsar wind $\sigma$—parameter—is

$$\sigma_{\infty}^i = \frac{4(E - 1)}{3 \Omega_w / J_0} = 1.35,$$

where $\Omega_w$ is the fraction of $2\pi$ steradians that is filled with field lines from one polar cap, and is taken to be unity in the above estimate. In CP, the higher initial ion Lorentz factor/larger factor $f_i$ resulted in a smaller value of $E$, which gave $\sigma_{\infty}^i = 0.5$. However, to the limited accuracy of all of these estimates, $\sigma_{\infty}^i$ should probably just be considered to be of order unity.

### 3. Production of Radio Pairs—Today

#### 3.1. Pair Production Cascade

Following Hibschman & Arons (2001), after acceleration stops at the PFF, the primary ion and electron beams will continue to radiate curvature photons. Rewriting Equation (6) in terms of energy normalized to $m_e c^2$ ($\varepsilon = \hbar \omega / m_e c^2$) in order to conform with the Hibschman & Arons (2001) notation, the curvature photon dimensionless energy is

$$\varepsilon_0 = \frac{9 \lambda}{8 \alpha} \gamma^3 \theta^3_\gamma.$$

Upon reaching the pitch-angle $\sin \psi \approx (8/3 \ln \Lambda)(B_c/(B \varepsilon_0))$, the photons convert into pairs with pair Lorentz factors of $\gamma_\pm \approx \varepsilon_0/2$. The pairs, which are created in a high Landau level, then gyro-synchrotron radiate their perpendicular momentum $p_i / m_e c = \gamma_\pm \sin \psi$ into first generation photons with energy (Hibschman & Arons 2001)

$$\varepsilon^{(1)}_i = \frac{3}{2} \frac{B}{B_c} \gamma_\pm^2 \sin \psi = \frac{3}{2} \frac{B}{B_c} \gamma_\pm \left( \frac{8}{3 \ln \Lambda} \frac{B_c}{B} \frac{\gamma_\pm}{\varepsilon_0} \right) = \frac{2}{3} \frac{\gamma_\pm}{\ln \Lambda} \approx \frac{\varepsilon_0}{\ln \Lambda}. \tag{26}$$

The first generation synchrotron photons can convert into additional generations of pairs with the photon energy of each subsequent generation reduced by $1/\ln \Lambda$ from the immediately prior generation (i.e., $\varepsilon_i^{(2)} = \varepsilon_i/\ln(\Lambda)^2$). Hibschman & Arons (2001) find that the minimum photon energy to produce pairs is (in their notation)

$$\varepsilon_{\min} = \frac{64}{27} \varepsilon_0 = \frac{64}{27} \left( \frac{128}{9} \frac{B_c}{B \ln \Lambda \theta_c} \right) \left( \frac{\theta_c}{\theta_\gamma} \right) = 166.56 \left( \frac{\theta_c}{\theta_\gamma} \right), \tag{27}$$

where the numerical factor refers to present Crab Pulsar values. When the $(n + 1)$th generation photon energy $\varepsilon_i^{(n+1)}$ falls below $\varepsilon_{\min}$, the cascade ends, and the energy of the final pairs is $\gamma_{\pm}^{(n)} \approx \varepsilon_i^{(n)} / 2$.

#### 3.2. Pair Multiplicity

After the PFF, the total energy emitted by the primary beams as curvature photons is

$$E_R = \int_{\gamma_i}^{\gamma_f} \left( \frac{2}{3} \frac{e^2 \varepsilon^2}{\rho_C} \gamma^4(s') \right) ds'. \tag{28}$$

The primary beam electrons lose energy at the rate $m_e c^2 d\varepsilon / ds = -dE_R / ds$, so that $E_R^e$ is most conveniently written as

$$E_R^e = \frac{\gamma_e}{\gamma} m_e c^2 \left( 1 - \frac{\gamma_i}{\gamma} \right) \gamma(s) = \frac{\gamma}{\gamma_+ \gamma_-} \ln(1 + \gamma)^2 \gamma_+ \gamma_- \ln(1 + \gamma). \tag{29}$$

In Equation (29) and below, since we have not solved for the spatial extent $s$ of the pair production cascade, which may occur over a significant fraction of the star’s radius (Hibschman & Arons 2001), the factor $\ln(1 + s)$ is retained for completeness, where $s$ may be of order unity; in what follows, we will take the logarithm factor to be of order one. The energy of the primary beam ions is essentially unchanged by the radiation of curvature photons. With $\mu_C = (4a/3 \theta_\theta) (1 + s)^{1/2}$, the total radiated energy from the ion beam is

$$E_R^i = \frac{3}{8} \frac{e^2}{\alpha} \left( \frac{\theta_\theta}{\theta_c} \right)^2 \left( \frac{\theta_c}{\gamma_i} \right)^2 \ln \left( \frac{1 + s}{1 + s_i} \right). \tag{30}$$

The multiplicity $M_{1,e}$—the number of pairs per primary beam particle—at the end of the cascade (the $n$th generation) is

$$M_{1,e} = \frac{E_R^e}{2 \gamma_{(n)} m_e c^2} = \ln(\Lambda)^n \frac{E_R^e}{\epsilon_0 m_e c^2}. \tag{31}$$

For $\theta_\theta / \theta_c = 0.47$ and $\gamma_i = 7.5 \times 10^7$, the primary electrons emit curvature photons with $\varepsilon_0 = 4.4 \times 10^5$ that produce two cascade generations of pairs. We obtain the electron multiplicity

$$M_{1,e}^e = 6.9 \times 10^4 \left( 1 - \frac{\gamma(s)}{\gamma_e} \right). \tag{32}$$

where, for the purpose of making estimates, we will assume the bracket term is of order unity. For $\theta_\theta / \theta_c = 0.47$ and $\gamma_i = 3.5 \times 10^7$, the primary ions produce only one gyro-synchrotron generation of pairs. Substituting Equation (25) into (31), the ion pair multiplicity can be conveniently written as

$$M_{1,i}^i = \frac{\ln \Lambda}{3} \alpha f_i \gamma_i \left( \frac{\theta_\theta}{\theta_c} \right) \ln \left( \frac{1 + s}{1 + s_i} \right) = 8.6 \times 10^4 \ln \left( \frac{1 + s}{1 + s_i} \right). \tag{33}$$

for $\theta_\theta / \theta_c = 0.47$; in making estimates, we will take the logarithmic factor to be of order unity.

Thus, both the primary electron and ion beams produce comparable pair multiplicities; both fall short of the Rees & Gunn (1974) estimate $M_{lee} \approx 3 \times 10^9$ in order to replace the energy lost by the Crab Nebula’s radio pairs to adiabatic expansion. However, the $N_{ee} / R / R = 3 \times 10^{10} \mathrm{s}^{-1}$ injection rate and the estimated multiplicity from the Goldreich–
Julian number flux (Equation (4)) \( M_\perp = \frac{N_\perp}{N_{R\perp}} = 3 \times 10^6 \) values do not need to apply today, but should actually be interpreted as “historical mean” values. The fundamental number whose origin needs to be understood is the \( N_\perp \approx 10^{51} \) radio-emitting pairs that exist in the Nebula today.

3.3. Asymptotic Relativistic Pair Plasma Wind Parameters

For an initial electron beam Lorentz factor of \( \gamma_e = 7.5 \times 10^7 \) \( f_s = e \Phi_i/m_e c^2 \), we find \( f_s \approx 7.19 \times 10^{-4} \). For the electron beam multiplicity (Equation (32)), the average Lorentz factor of the pair plasma wind is \( \gamma_p = \gamma_e M_\perp = 1.09 \times 10^3 \). The primary electron beam luminosity from the polar caps is \( L_e = (f_s/f_i) L_i \approx 1.2 \times 10^{-3} L_i \), which implies that \( e^* = L_e/L_e = 1.24 \times 10^4 \), the classic estimate that the pair plasma wind is Poynting flux dominated at the star. The pair plasma wind’s energy parameter (Equation (22) with \( f_s \)) is \( E = 6.22 \times 10^7 \), and the field-aligned current parameter is \( J_0 = 6.94 \), slightly less than for the ion wind. Finally, for the asymptotic pair plasma wind, the ratio of the Poynting flux to pair plasma kinetic energy flux in the asymptotic wind is \( \sigma_{p,e}^0 \approx 1.2 \times 10^3 \), which is a factor of about seven times lower than the classic \( \sigma_{p,e} \approx 10^4 \) estimate due to the higher final electron beam Lorentz factor; in CP, we found \( \gamma_{e*} \approx 10^7 \).

4. Production of Radio Pairs—Historical

4.1. Simple Spin-down Model

A simple (the simplest?) model for the spin-down evolution of the Crab Pulsar assumes that the magnetic moment is constant in magnitude and in orientation relative to the spin axis; although probably not correct, the evolution of the dipole and higher order moments is essentially unknown. Following Lyne et al. (1993), the model assumes that the evolution of the angular rotation frequency is given by \( d\Omega/dt = -k_\Omega \Omega^{5/2} \), where, for simplicity, the braking index has been approximated as \( 5/2 \), which is close to the braking index that is measured between glitches (Lyne et al. 2015). The braking index and \( k_\Omega \) are assumed to be a constant in time. The spin-down model yields the result

\[
\Omega(t) = \frac{\Omega_0}{(1 + t/t_0)\Omega_0^{5/2}},
\tag{34}
\]

where \( k_\Omega = -\Omega(t_0)/\Omega^{5/2}(t_0) \approx 4.9 \times 10^{-15} \) s\(^{-1/2} \) is determined from the present \( t_0 \) (actually 1975 data (Lyne et al. 1993); \( t_0 = 2.9 \times 10^{10} \) s) time rate of change of the period and the current rotational angular frequency \( \Omega(t_0) = 189.9 \) rad s\(^{-1} \approx 200 \) rad s\(^{-1} \) (herein); the initial angular rotation frequency is \( \Omega_0 = 1.72 \Omega(t_0) \). The characteristic spin-down time \( \tau_0 = 2.32 \times 10^{10} \) s, corresponding to 735 yr.

4.2. Goldreich–Julian Number

The classic Goldreich–Julian particle injection rate per polar cap for the aligned rotator is

\[
\dot{N}_p(t) = \frac{2\mu}{2\pi e a^3} \frac{\pi a^2 \theta_t^2}{\mu} \frac{\Omega^2}{ee^2} = \frac{c \Phi_i}{e}.
\tag{35}
\]

Substituting for \( \Omega(t) \) from Equation (34) and integrating over the age of the Crab Pulsar, we find that the total number of particles that have been injected at the Goldreich–Julian rate since the birth of the Crab Pulsar is \( \dot{N}_{R}(t_0) = 4.9 \times 10^{54} \). If the field-aligned current is \( J_0 \approx 7 \) times the Goldreich–Julian current, combining both polar caps yields the total primary beam particle number \( \dot{N}_{R}(t_0) = 2 J_0 N_{R} = 6.8 \times 10^{45} \). The inferred number of radio-emitting pairs \( N_\perp \approx 10^{51} \) would then imply a pair multiplicity of \( M_\perp = 1.5 \times 10^3 \), significantly below the Rees & Gunn (1974) estimate, but above the current multiplicity estimate in our model, and in most other models, as well (Bucciantini et al. 2011; Arons 2012; Harding 2013).

4.3. Temporal Evolution of Characteristic Energies

The primary beam Lorentz factors at the PFF \( (s_*, s_e) \) are

\[
\gamma_{i,e} = \pm \frac{e \Phi(s_*)}{m_i c^2} \Phi(s) = \frac{\pm 5 (0.58) \Omega_\mu}{\theta_C} \frac{\theta_i}{(J_0 - 1)} \frac{1}{k_i^4} \Delta \left[ \cosh(k_i s) - 1 \right].
\tag{36}
\]

where the \( +/-(+) \) sign holds for the electron (ion) polar cap acceleration region. Since \( k_i \propto 1/\theta_C, \) \( \theta_C \propto \theta_L = (\Omega a/c)^{1/2}, \) \( \gamma_{i,e} \) (and \( \Phi(s) \)) scale as

\[
\gamma_i = 3.5 \times 10^7 \left( \frac{\Omega(t)}{\Omega(t_0)} \right)^{5/2} \gamma_e = 7.5 \times 10^7 \left( \frac{\Omega(t)}{\Omega(t_0)} \right)^{5/2}.
\tag{37}
\]

At \( t = 0 \), the increase of \( \gamma_{i,e} \) above their current values is slightly less than four. Using the scaling relations from Equation (37), the energy of the primary beam curvature photons (Equation (25)) scales as

\[
\epsilon_0(t) = \epsilon_0(t_0) \left( \frac{\Omega(t)}{\Omega(t_0)} \right)^8.
\tag{38}
\]

and the minimum cascade photon energy for gyro-synchrotron pair production (Equation (27)) scales as

\[
\epsilon_{\min}(t) = \epsilon_{\min}(t_0) \left( \frac{\Omega(t)}{\Omega(t_0)} \right)^{1/2}.
\tag{39}
\]

In Equations (38) and (39), \( \epsilon_0(t) \) and \( \epsilon_{\min}(t) \) will be evaluated at the fiducial parameters and today’s values of the ion and electron Lorentz factors in Equation (37).

4.4. Ion Cascade Pair Production

At \( t = 0 \), the increased energy of the primary ions results in higher energy curvature photons that produce three generations of gyro-synchrotron photons with energies

\[
\begin{align*}
\epsilon_s^{(1)} &= 3.6 \times 10^{45} \left( \frac{\theta_k}{\theta_C} \right) \\
\epsilon_s^{(2)} &= 1.8 \times 10^{49} \left( \frac{\theta_k}{\theta_C} \right) \\
\epsilon_s^{(3)} &= 8.95 \times 10^{52} \left( \frac{\theta_k}{\theta_C} \right),
\end{align*}
\tag{40}
\]

where the latitude \( \theta_k \) has been left as a variable, but will be evaluated at \( \theta_k = 0.47 \theta_C \) below. As the pulsar spins down, \( \gamma_i(t) \) decreases, and, from Equations (39) and (40), setting \( \epsilon_s^{(3)} = \epsilon_0(t_3)/(\ln \Lambda)^3 = \epsilon_{\min}(t_3) \), we find that the third generation is no longer produced after a time \( t_3/\tau_0 = 0.08 \) (\( t_3 \approx 60 \) yr). A similar
calculation yields that the second generation is not produced after
time $t_2 / \gamma_0 = 0.83$ ($t_2 = 618$ yr).

The ion pair multiplicity for the $n$th generation
(Equation (33)) is

$$M_{\pm}^{(n)}(t) = \left( \frac{\ln \Lambda}{3} \right)^n \alpha_\gamma \gamma (t_0) \theta_C (t_0) \left( \frac{\Omega(t)}{\Omega(t_0)} \right)^{1/2} \left( \frac{\theta_v}{\theta_C} \right) \ln(1 + s),$$

where, since the spatial extent of the cascade has not been
determined by our simple analysis, the $\ln(1 + s)$ factor has
been retained for completeness, and is assumed to be of order
unity. The production rate of pairs during the $n$th generation is then

$$\dot{N}_{\pm}^{(n)}(t) = M_{\pm}^{(n)}(t) J_0 \dot{N}_R(t),$$

where we have added the two ion-halfes of the two polar caps.
Integrating Equation (42) with Equation (34) yields the total
number of low energy pairs produced from pulsar birth to the present
time

$$N_{\pm} = \frac{J_0 \mu}{3 e_c} \frac{\Omega^2(t_0)}{\tau_0 (1 + t_0/\tau_0)^{10/3}} \left[ \ln(1 + s) \right] \times \left[ \ln(1 + s) \right] \times \left( \frac{\ln \Lambda}{3} \right)^n \int_0^{t_0/\tau_0} \frac{dx}{(1 + x)^{10/3}}$$

$$+ \left( \frac{\ln \Lambda}{3} \right)^n \int_0^{t_0/\tau_0} \frac{dx}{(1 + x)^{10/3}} + \left( \frac{\ln \Lambda}{3} \right)^n \int_0^{t_0/\tau_0} \frac{dx}{(1 + x)^{10/3}}$$

The values of the three integrals in Equation (43) are 567.8,
104.1, and 0.79. Thus, the pair production by the primary ion
beam would be dominated by the brief early epoch when three
generations of gyro-synchrotron cascade photons were energetically possible. For $\theta_v / \theta_C = 0.47$, the coefficient of the
integral terms within the brackets in Equation (43) is
$4.92 \times 10^{19}$, so that the total number of primary ion produced
pairs is $N_{\text{Tot}}^e = 3.3 \times 10^{52}$.

If correct, since $N_{\text{Tot}}^e$ exceeds the $10^{51}$ radio-emitting pairs that are inferred to exist in the Crab Nebula today, the implication
would be that significant fluxes of the radio pairs must have escaped over the lifetime of the Crab Nebula. Recent MHD
simulations (Porth et al. 2014) find that the magnetic field in the
outer Nebula becomes dissipative and highly turbulent, which
might account for the implied losses. However, an alternate
possibility is that during the early epoch, the physics of the
pulsar magnetosphere may be considerably different from
today’s highly relativistic response to essentially vacuum external
conditions, which favor the production of pairs. For example, if the outflowing wind were to be choked by the Nebula’s debris,
polar cap pair production might be inhibited or prevented. Perhaps future observations and modeling of SN 1987A might shed some light on the early phase of pulsar-nebula evolution.

If pair production is stymied during the early ion third
generation epoch, the total number of ion pairs produced by
the second and first generation cascades would then be
$N_{\text{Tot}}^e = 5 \times 10^{51}$. Finally, as done above, the pair production rate is conventionally scaled to the Goldreich–Julian flux $\dot{N}_R$ for
the aligned rotator. For moderately oblique inclination angle $i$ of
the magnetic moment to the rotation axis, the Goldreich–Julian flux is reduced by $\cos i$; for the Crab Pulsar $\cos i \approx 1/2$. (For the orthogonal rotator, the Goldreich–Julian flux is even
smaller $\dot{N}_R = \mu \Omega^2 / (\pi a e)$; however, in CP, reasonable
arguments were presented that the basic physics of the primary
ion and electron beams in the orthogonal rotator should also
occur at least out to the range of inclination angles $i \approx 45^\circ–60^\circ
$ thought to encompass the Crab Pulsar’s inclination Harding
et al. 2008.) Thus, reducing the aligned rotator $\dot{N}_R$ by
$\cos i \approx 1/2$ may better estimate the pair production by
the primary ion beam to be $N_{\text{Tot}}^e = 2.5 \times 10^{51}$, which should probably be viewed as a fairly generous estimate since we evaluated all of the $(\theta_v, \phi_\alpha)$ angular dependences at the maximal location.

4.5. Electron Cascade Pair Production

The curvature radiation from the initial primary electron beam
produces three generations of gyro-synchrotron pairs, and the
third generation continues until $t_3 / \gamma_0 = 0.633$ ($t_3 \approx 465$ yr).
From Equation (31), the electron pair multiplicity for the $n$th
generation scales as

$$M_{\pm}^{(n)}(t) = \left( \frac{\ln \Lambda}{3} \right)^n \frac{\gamma (t_0)}{\gamma_0} \left( \frac{\Omega(t)}{\Omega(t_0)} \right)^{11/2} \left( 1 - \frac{\gamma(t)}{\gamma_0} \right).$$

The total number of pairs produced by the primary electron
beam is

$$N_{\pm}^e = \frac{J_0 \mu}{3 e_c} \frac{\Omega^2(t_0)}{\tau_0 (1 + t_0/\tau_0)^{10/3}} \left[ \ln(1 + s) \right] \times \left[ \ln(1 + s) \right] \times \left( \frac{\ln \Lambda}{3} \right)^n \int_0^{t_0/\tau_0} \frac{dx}{(1 + x)^{10/3}}$$

The values of the two terms within the brackets are 9.92 $\times 10^3$,$1.18 \times 10^3$. The coefficient of the bracketed
integral terms is $8 \times 10^{46}$, so that the total electron cascade
pair production is $N_{\text{Tot}}^e = 4.4 \times 10^{50}$, which should be reduced by $\cos i$ to $N_{\text{Tot}}^e = 2.2 \times 10^{50}$.

5. Conclusion

Although today’s pair production multiplicity is below the
classical Rees & Gunn (1974) estimate of $3 \times 10^6$ deduced
from adiabatic expansion energy losses, a simple spin-down
model for the Crab Pulsar’s rotational angular frequency
indicates that higher multiplicities occurred in the past, and that
the primary ion (electron) beam could produce somewhat more
(less) than the $10^{51}$ radio-emitting pairs that are inferred to exist
today in the Crab Nebula. Clearly, the rough model estimates
developed here at best indicate that the origin of the Nebula’s
radio pairs could be the Crab Pulsar’s primary ion and electron beams. Far more sophisticated pair production calculations
will be needed in order to confirm this conclusion.

6. Discussion

In the same “spirit” as adopted in CP, we have used “back-
of-the-envelope” calculations of the Crab Pulsar’s primary ion
electron beam-emitted curvature synchrotron photons to
determine the location of the PFFs, the final beam Lorentz
factors, and to obtain a very rough estimate of the gyro-
synchrotron cascade’s pair production multiplicity. We also
addressed the Crab Nebula’s radio electron/positron con-
undrum that the present cascade pair production rate falls short of
the Rees & Gunn (1974) estimate of the injection rate needed to
balance the radio pair’s adiabatic expansion energy loss rate.
Using a very simple spin-down model of the Crab Pulsar’s
rotation frequency, we found that the cascade pair production
rate was considerably higher in the past, and that the time-
integrated number of generated pairs could plausibly account
for the Nebula’s $10^{31}$ radio pairs that exist in the Nebula today.
We also found that the cascade driven by the primary ion beam produced about a factor of $10$
more pairs than did the electron-driven cascade. Thus, a
tempting conclusion is that the existence of the $10^{31}$ radio-
emitting pairs is evidence that the Crab Pulsar’s relativistic
wind is dominated by ions; however, this conclusion should
undoubtedly await confirmation by more sophisticated pair
production calculations.

We have not attempted to explain the radio frequency spectral
luminosity of the Crab Nebula ($L_{\nu} \propto \nu^{-\alpha}$ with $\alpha \approx 1/4$) or the corresponding distribution function ($f(\gamma) \propto \gamma^{-(2\alpha+1)}$ between $\gamma_1 < \gamma < \gamma_2$) of the radio pairs. The final Lorentz factors at the end of the gyro-synchrotron pair production cascade are (1) today
— with the initial ion (electron) curvature photon energy $\epsilon_0^i = 4.4 \times 10^4 (\epsilon_0^e = 4.4 \times 10^5)$ at the start of the cascade, $\gamma_1^i = \epsilon_0^i/2\ln \Lambda \approx 1.1 \times 10^3 (\gamma_1^e = \epsilon_0^e/2\ln \Lambda^3 \approx 5 \times 10^2)$; and (2) at pulsar birth—with the initial ion (electron) curvature photon energy $\epsilon_0^i = 3.36 \times 10^6 (\epsilon_0^e = 3.3 \times 10^7)$, the second generation ion $\gamma_2^i = \epsilon_0^i/2(\ln \Lambda)^3 \approx 4.2 \times 10^2$ (third generation electron $\gamma_2^e = \epsilon_0^e/2(\ln \Lambda)^3 \approx 2 \times 10^3$). Thus, the final pair cascade Lorentz factors are comparable to the average Lorentz factor of the Crab Nebula’s radio pairs ($\langle \gamma \rangle = (\gamma_1 \gamma_2)^{1/2} = 8 \times 10^3$, with $\gamma_1$ being uncertain since it is derived from the ionospheric cutoff frequency). Clearly, the actual distribution function of the radio pairs will depend on the microscopic details of the cascade pair production process, the interaction of the pair plasma wind with the ion wind and with the termination shock, and the possibly turbulent acceleration dynamics within the Nebula. In any case, the proposed origin of the radio pairs—the Crab Pulsar—is consistent with the 2D and 3D MHD simulation results of Olmi et al. (2014, 2016) that the radio pairs are continuously injected into the Nebula, and are not a relic population.

For the orthogonal and moderately oblique rotators, the relativistic wind in the vicinity of the rotational equator contains magnetic stripes (Michel 1971) with alternating azimuthal magnetic field directions; one polarity (the other polarity) contains the ion dominated (the pair plasma dominated) wind. The two stripes have different Lorentz factors ($\gamma_i, \gamma_p \approx 10^3$), and therefore have slightly different radial flow speeds ($\beta = v/c$

$$\Delta \beta = \beta_i - \beta_p \approx \frac{1}{2} \left( \frac{1}{\gamma_{p*}} - \frac{1}{\gamma_i} \right) \approx \frac{1}{2} \gamma_i^2 / \gamma_{p*}. \quad (46)$$

The ion stripe will overtake the pair plasma stripe in a time $\Delta t \approx (1/\Delta \beta) (\pi /3)$, during which time the wind will travel a radial distance $\Delta r = c \Delta t \approx 7.5 \times 10^5 c/\Omega$, far smaller than the distance to the termination shock ($\approx 2 \times 10^6 c/\Omega$). Thus, the ion and electron magnetic stripes are very likely to undergo significant magnetic field annihilation, and the ion and pair plasmas to mix, long before the wind reaches the shock.

The frequently quoted (Arons 2012) argument by Lyubarsky & Kirk (2001) that time dilation implies that the stripes do not annihilate before reaching the termination shock is incorrect, being derived from the same pitfall that underlies introductory physics Special Relativity paradox problems—events in different frames are not simultaneous. In order to illustrate, and presumably strengthen, their argument, Lyubarsky & Kirk (2001) assume that, in the wind frame, the annihilation wave travels at light speed, the fastest possible destruction of the magnetic field. Suppose the annihilation wave initiates at a stripe’s neutral sheet, and travels in the negative radial direction at light speed. Consider a stationary observer at rest with respect to the wind. Since the annihilation wave travels at the speed of light, the Einstein Law of Velocity Addition says that all observers must measure the wave to travel at light speed. The next stripe’s neutral sheet, a distance $2\pi c/\Omega$ upstream, travels toward the stationary observer at essentially light speed. Thus, the oncoming neutral sheet and the annihilation wave essentially meet at the middle of the stripe ($\pi c/\Omega$), and the magnetic field annihilates in a time $\pi/\Omega$ as measured by the stationary observer. This result is readily confirmed from the exact Lorentz transformation of the initial (annihilation wave starts as the neutral sheet passes the stationary observer) and final (annihilation wave meets oncoming neutral sheet as seen by the stationary observer) events. The analyzes of Coroniti (1990) and Michel (1994), which show that the opposite magnetic polarity stripes annihilate well-within the wind zone, may be flawed for other reasons (many assumptions were made in those calculations), but not for violating Special Relativity.

Recent simulation results by Cerutti & Philippov (2017) confirm their basic conclusion that the magnetic field in the stripes will dissipate before the wind reaches the termination shock.

In conclusion, in CP and this paper, we have presented plausibility arguments that the Crab Pulsar produces a relativistic wind with comparable electromagnetic and ion kinetic energy fluxes. The far asymptotic physical parameters of the Crab Pulsar’s wind depend on the extent to which the striped magnetic field annihilates, and on the extent to which the energetically dominant ions and the pair plasma from the electron polar region mix and combine to form a coherent and steady wind flow.

If energetic ions do dominate the particle outflow from the Crab Pulsar, the interaction of the wind with the Crab Nebula will likely be considerably different than if the wind were purely positronic. As noted long ago by Gallant & Arons (1994), the Larmor radius of an energetic ion in the Nebula’s downstream magnetic field is comparable to the radial distance from the Crab Pulsar to the termination shock ($\approx 3 \times 10^7$ cm) and to the distance between the wisps; thus, the interior of the collisionless ion shock is visible on the plane of the sky. The energetic ion Larmor radius is also comparable to the width of the $\pm 10^7$ opening angle of the X-ray torus (Aschenback & Brinkmann 1975; Weisskopf et al. 2000); thus, the termination shock, which probably occurs partly within a magnetic neutral sheet, will not be one-dimensional, but will likely have a quite complex three-dimensional structure. Within the Crab Nebula, the ballistic motion of the energetic ions is likely to excite both short and long wavelength plasma instabilities. The resulting turbulence may foster the creation of the localized reconnecting neutral sheets that are thought by many to be responsible for the Crab Nebula’s gamma-ray flares (Abdo et al. 2011; Tavani et al. 2011; Cerutti et al. 2014; Lyutikov et al. 2018). The Crab Pulsar

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and Nebula still have a few “Rosetta Stone” hieroglyphics to be translated.

The possibility that the Crab Pulsar might emit a significant energy flux of heavy ions originated in discussions with J. Arons at the ITP in Santa Barbara in 1982. I am pleased to acknowledge discussions with C. F. Kennel, and the opportunity that he provided to revisit the Crab Nebula on the occasion of his 75th birthday celebration Charlie Fest 2014. I would like to thank the anonymous referee for comprehensive comments and suggestions that significantly improved the paper. I also gratefully acknowledge the support of this research by the taxpayers of California through the generosity of the Deans of Physical Sciences J. Rudnick and M. Garcia-Garibay.

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