Generalized uncertainty principle and burning stars

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(Dated: June 29, 2022)

Gamow’s theory of the implications of quantum tunneling on the star burning has two cornerstones including quantum mechanics and equipartition theorem. It has vastly been proposed that both of these foundations are affected by the existence of a non-zero minimum for length which usually appears in quantum gravity scenarios and leads to the Generalized Uncertainty principle (GUP). Mathematically, in the framework of quantum mechanics, the effects of GUP are considered as perturbation terms. Here, generalizing the de Broglie wavelength relation in the presence of minimal length, GUP correction to the Gamow’s temperature is calculated and in parallel, an upper bound for the GUP parameter is estimated.

Introduction

As the first step of star burning, their constituents should overcome the Coulomb barrier to participate in the Nuclear fusion (NF), the backbone of star burning. It means that when the primary gas ingredients have mass $m$ and velocity $v$, then using the equipartition theory, one gets

$$\frac{1}{2}mv^2 = \frac{3}{2}k_BT \geq U_c(r_0),$$

where $k_B$ denotes the Boltzmann constant, leading to

$$T \geq \frac{2Z_iZ_j\epsilon^2}{3kr_0} \simeq 1 \times 10^{10}\frac{Z_iZ_j}{r_0},$$

for temperature required to overcome the Coulomb barrier. Here, $U_c(r_0) = \frac{2Z_iZ_j\epsilon^2}{3k}$ denotes the maximum of the Coulomb potential between the $i$-th and $j$-th particles. Therefore, NF happens whenever the temperature of primary gas is comparable to Eq. (2) which clearly emphasizes that in association with the heavier nuclei, NF happens in higher temperatures. On the other hand, for the temperature of a cloud with mass $M$ and radius $R$, we have

$$T \approx 4 \times 10^6\left(\frac{M}{M_\odot}\right)\left(\frac{R_\odot}{R}\right),$$

where $M_\odot$ and $R_\odot$ are the Sun mass and radius, respectively. Clearly, $T$ and $T$ are far from each other meaning that NF cannot cause the star burning. Therefore, we need a process that decreases Eq. (1) to the values comparable to Eq. (2). Thanks to the quantum tunnelling, overcoming the Coulomb barrier becomes possible which finally lets star burn. Indeed, if the distance between the particles becomes of the order of their de Broglie wavelength ($r_0 \simeq \frac{\hbar}{p = \lambda_Q}$), then the quantum tunnelling happens and simple calculations lead to

$$T \geq \frac{2Z_iZ_j\epsilon^2}{3kr_0} \simeq 9 \times 10^6\frac{Z_i^2Z_j^2}{\lambda_Q},$$

instead of Eq. (2) for the temperature required to launch star burning. $\lambda$ has also been obtained by solving $\frac{p^2}{2m} = U_c(r_0)\bigg|_{r_0 = \lambda_Q}$ which gives

$$\lambda_Q = 2mZ_iZ_j\epsilon^2.$$

This achievement means that the quantum tunneling provides a platform for NF in stars. As an example, for Hydrogen atoms, one can see that the quantum tunneling leads to $T \simeq 9 \times 10^6$ (comparable to (3)) as Gamow’s temperature in which NF is underway. Based on the above argument, it is expect that any change in $p$ affects $\lambda$ and thus the results.

It is also useful to mention here that the quantum tunneling theory allows the above process since the tunneling probability is not zero. Indeed, quantum tunneling is also the backbone of Gamow’s theory on $\alpha$ decay process. Relying on the inversion of Gamow formula in $\alpha$ decay theory, that gives the transmission coefficient, a method has also been proposed for studying the inverse problem of Hawking radiation.

The backbone of quantum mechanics is the Heisenberg uncertainty principle (HUP) written as

$$\Delta x \Delta p \geq \frac{\hbar}{2},$$

where $x$ and $p$ are ordinary canonical coordinates satisfying $[x_i, p_j] = i\hbar\delta_{ij}$, and modified in the quantum scenarios of gravity as

$$\langle \Delta X \rangle \langle \Delta P \rangle \geq \frac{\hbar}{2} \left(1 + \frac{\beta p^2}{\hbar^2}\right).$$

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called GUP where $l_p$ denotes the Planck length, and $\beta_0$ is the GUP parameter. Here, $X$ and $P$ denote the generalized coordinates, and we work in a framework in which $X_i = x_i$, and up to the first order of $\beta$, we have

$$P_i = p_i(1 + \frac{\beta_0 l_p^2}{\hbar})^2 \quad \text{and} \quad [X_i, P_j] = i\hbar(1 + \frac{\beta_0 l_p^2}{\hbar})P_j \delta_{ij} \quad [9].$$

Moreover, GUP implies that there is a non-zero minimum for length as $\langle \Delta X \rangle_{\text{min}} = \sqrt{\beta_0 l_p}$. Indeed, the existence of non-zero mean length also emerges even when the gravitational regime is Newtonian [8], a common restriction. Studies on quantum gravity scenarios [9]. More studies on quantum gravity can be traced in Refs. [10–13]. There are various attempts to estimate the maximum possible upper bound on $\beta_0$ [14–31], and among them, it seems that the maximum estimation for the upper bound is of the order of $10^{78}$ [28]. The implications of GUP on the star evolution [32, 33], and the thermodynamics of various gases [2, 34] have also attracted more attention.

Indeed, the existence of minimal length and gravity leads to the emergence of GUP [9] and it affects thermodynamics [1, 9, 33–37] and quantum mechanics [4, 5], as $P$ can be expanded as a function of $p$. The letter deals with the problem of the effects of GUP on star burning launched by quantum tunnelling. Loosely speaking, we are going to find the effects of minimal length on $T$.

GUP corrections to the tunneling temperature

Up to the first order of $\beta_0$, for thermal energy per particle with temperature $T$, we have [4]

$$\frac{\langle p^2 \rangle}{2m} = \langle K \rangle = \frac{3}{2} K_B T - 3 \beta_0 l_p^2 m K_B^2 T^2. \quad (8)$$

Mathematically, one should find the corresponding de Broglie wavelength by solving equation

$$\frac{p^2}{2m} = U_c(r_0) \bigg|_{r_0=\lambda}. \quad (9)$$

Inserting the result into

$$\frac{3}{2} K_B T - 3 \beta_0 l_p^2 m K_B^2 T^2 \geq U_c(r_0) \bigg|_{r_0=\lambda}, \quad (10)$$

one can finally find the GUP corrected version of Eq. [4].

In order to proceed further and in the presence of the quantum features of gravity, let us introduce the generalized de Broglie wavelength as

$$\lambda_{\text{GUP}} = \frac{\hbar}{P}. \quad (11)$$

It is obvious that as $\beta_0 \to 0$, one obtains $P \to p$ and thus $\lambda_{\text{GUP}} \to \frac{\hbar}{p}$ which is the quantum mechanical result. Indeed, up to the first order of $\beta$, we have

$$\lambda_{\text{GUP}} = \lambda_Q(1 - \frac{\beta_0 l_p^2}{\hbar^2}).$$

Now, inserting $\lambda_{\text{GUP}}$ into Eq. [9], and then combining the results with Eq. [10], we can find

$$T_{\text{GUP}}^+ = \frac{\hbar^2}{4 \beta_0 K_B l_p^2 m} \left(1 \pm \sqrt{1 - 8 \beta_0 l_p^2 m K_B T / \hbar^2} \right). \quad (12)$$

in which Eq. [11] has been used for simplification. In order to have an estimation of the order of $l_p^2 m K_B T / \hbar^2$, let us consider the Hydrogen atom for which $m = \frac{1}{2}$ (here, $m$ is the reduced mass of the Hydrogen nucleus as the primary gas constituents [1]). Now, since $l_p \propto 10^{-35}$, $K_B \propto 10^{-32}$, $\hbar \propto 10^{-34}$, and $T \propto 10^6$, one easily finds $l_p^2 m K_B T / \hbar^2 \propto 10^{27}$. Moreover, because of the effects of GUP in the quantum mechanical regimes are poor [4], a reasonable basic assumption could be that $\beta_0 l_p^2 m K_B T / \hbar^2 \ll 1$. Indeed, if $\beta_0 \ll 10^{27}$, then we always have $\beta_0 l_p^2 m K_B T / \hbar^2 \ll 1$ meaning that $10^{27}$ is an upper bound for $\beta_0$ which is well comparable to previous works [14–31, 34]. Therefore, we are allowed to expand the results.

Expanding the above solutions, and bearing in mind that the true solution should cover $T$ at $\beta = 0$, one can easily find that $T_{\text{GUP}}$ is the proper solution leading to

$$T_{\text{GUP}}^- = T \{1 + 2 \beta_0 (l_p^2 m K_B T / \hbar^2) \}. \quad (13)$$

up to the first order of $\beta_0$. Hence, since it seems that $\beta_0$ is positive [14–31, 34], one can conclude that $T < T_{\text{GUP}}$.

Conclusion

Motivated by the GUP proposal and the vital role of HUP in quantum mechanics and thus, quantum tunneling letting star burn, we studied the effects of GUP on the Gamow’s temperature. In order to continue, GUP modification to the de Broglie wavelength was addressed which finally helped us find GUP correction to the Gamow’s temperature and also estimate an upper bound for $\beta_0 (10^{27})$ which agrees well with previous works [14–31, 34].

Finally and based on the obtained results, it may be expected that GUP also affects the transmission coefficients (Gamow’s formula) [2, 3, 9] meaning that the method of Ref. [2] will also be affected. This is an interesting topic for future study since Hawking radiation is a fascinating issue in black hole physics [85].

Acknowledgments

The authors would like to appreciate the anonymous referees for their valuable comments.
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