Discrete Spectra of Charged Black Holes

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Abstract

Bekenstein proposed that the spectrum of horizon area of quantized black holes must be discrete and uniformly spaced. We examine this proposal in the context of spherically symmetric charged black holes in a general class of gravity theories. By imposing suitable boundary conditions on the reduced phase space of the theory to incorporate the thermodynamic properties of these black holes and then performing a simplifying canonical transformation, we are able to quantize the system exactly. The resulting spectra of horizon area, as well as that of charge are indeed discrete. Within this quantization scheme, near-extremal black holes (of any mass) turn out to be highly quantum objects, whereas extremal black holes do not appear in the spectrum, a result that is consistent with the postulated third law of black hole thermodynamics.

PACS Nos: 04.60.-m, 04.70.-s, 04.70.Dy
1 Introduction

Black holes serve as interesting theoretical laboratories for testing the validity and predictive power of theories of quantum gravity. The robustness of thermodynamic laws associated with black holes, along with their inherent structural simplicity require that any reasonable theory ought to predict some of their generic features. Notable among them are Bekenstein-Hawking entropy associated with horizon area of black holes, and Hawking radiation from those horizons [1]. Together, they ensure that a generalized second of thermodynamics is valid in the presence of black holes. This law was proposed by Bekenstein in the early seventies [2].

Along with these spectacular observations, there also arose the question as to whether the spectra of observables related to black holes were continuous or discrete. Again, by a remarkable set of thought experiments, it was inferred by Bekenstein and collaborators that provided a black hole is far away from extremality, its horizon area can be regarded as an adiabatic invariant. Now, it is well known from quantum theory, that adiabatic invariants are always quantized, and their spectra are equally spaced [3]. Naturally, they proposed that horizon area spectrum is discrete and of the form [4]:

\[ a_n = na_0 , \]  

where \( a_0 \) is a fundamental quantum of area. Recently, Bekenstein and collaborators did an algebraic analysis, and showed that under certain plausible assumptions, the area spectrum was indeed discrete and equally spaced [5]. This feature was confirmed by several other analyses as well, which leads one to believe that spectra of this kind should be features of all black holes. In fact, discrete spectra such as above was found by many authors using diverse approaches [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

In this article, we try to address these questions from a slightly different perspective. In particular, we consider spherically symmetric charged black holes in a generic theory of gravity. The reduced (physical) phase space is just four dimensional. We Euclideanize and impose periodic boundary conditions that reflect the thermodynamic nature of the black holes. By making a canonical transformation, we are able to quantize the resulting theory exactly and obtain a spectrum similar to that proposed by Bekenstein [1] for uncharged black holes. Moreover, our spectrum predicts a remnant
ground state, so that any physical process such as Hawking radiation must stop when this minimum value of horizon area is reached.

The article is arranged as follows: in the next section, we elaborate on the exact quantization procedure adopted by us (following an analysis developed in [11]), and the spectrum obtained thereof. In section (3), we present a rigorous derivation of a slightly modified version of Bekenstein’s hypothesis that the horizon area is an adiabatic invariant. Then, in section (4), we go one step further and show why our spectrum ought to agree with Bekenstein’s one, at least in the uncharged case, by formulating an exact correspondence between the operators in his algebra and the fundamental gravitational degrees of freedom in our analysis. Finally, in section (5) we examine some consequences of our spectrum and conclude by stating some open problems. For further details and intermediate steps, we refer the reader to the original papers [17, 18].

2 Reduced Phase Space Quantization

We start with a generic theory of gravity which describes the dynamics of charged black holes (e.g. it could be Einstein-Maxwell theory, or those that arise in low energy string theory). Since we are primarily interested in the generic and robust features of these black holes, we rid ourselves of unnecessary complications, and consider only those solutions which are spherically symmetric. It is by now well known that under the above assumptions, all such gravity actions can be dimensionally reduced to a simple particle mechanics action in 1-dimension of the form [19, 20, 21]:

\[ I_{\text{red}} = \int dt \left( P_M \dot{M} + P_Q \dot{Q} - H(M, Q) \right), \] (2)

where \( M \) and \( Q \) are the mass and charge of the black hole respectively, and \( P_M \) and \( P_Q \) are the conjugate momenta. The boundary conditions used were those of [22]. The above action automatically ensures that \( M \) and \( Q \) are constants of motion, as required by the generalized Birkhoff theorem. It can be shown that the momentum \( P_M \) has the interpretation of asymptotic Schwarzschild time difference between the left and right wedges of a Kruskal diagram [23, 24, 25]. The reduced phase space of spherically symmetric solutions of generic gravity-electromagnetism systems is thus four dimensional,
spanned by the coordinates \((M, Q, P_M, P_Q)\). Furthermore, we restrict the mass parameter to be non-negative. Note that this describes not only black hole geometries, but also other objects such as spherical stars. Since we are interested in the quantum mechanics of black holes, we restrict the phase space to that of the latter by making use of the following fact: \(P_M\), the conjugate to the mass variable, is effectively the asymptotic “Schwarzschild” time, and is periodic in the Euclideanized formulation of black hole thermodynamics (with period equal to the inverse Hawking temperature \(T_H\) times \(\hbar\)). We therefore go to the Euclidean sector of the theory and impose the additional restriction that \(P_M\) is periodic with the same period. That is:

\[
P_M \sim P_M + \frac{\hbar}{T_H(M, Q)} .
\]  

(3)

Although this may seem ad-hoc at this point, we will see that this one simple (and plausible) assumption helps in deriving satisfactory spectra for both area and charge of the black hole. Also, similar assumptions regarding periodicity (or associating a fundamental time scale with black holes) were made using somewhat complex arguments in the past [9, 10, 14]. Here, we simply proceed with this assumption, and note that this restricts the subspace \((M, P_M)\) into a wedge like region, bounded by the \(M\) axis and the locus of points \(P_M = \hbar/T(M, Q)\) with varying \(M\) (e.g. for a Schwarzschild black hole in four dimensions, it is the straight line \(P_M = 8\pi G_4 M, G_4\) being four dimensional Newton’s constant). The appearance of a wedge may seem a little disturbing, but it was shown in [11, 17] that this wedge can be removed by making the following canonical transformation on the full phase-space:

\[
X = \sqrt{\frac{\hbar(S_{BH}(M, Q) - S_0(Q))}{\pi}} \cos(2\pi P_M T_H(M, Q)/\hbar) ,
\]

(4)

\[
\Pi_X = \sqrt{\frac{\hbar(S_{BH}(M, Q) - S_0(Q))}{\pi}} \sin(2\pi P_M T_H(M, Q)/\hbar) ,
\]

(5)

\[
Q = Q ,
\]

(6)

\[
\Pi_Q = P_Q + \Phi P_M + S'_0(Q) P_M T_H ,
\]

(7)

where \(S_{BH}(M, Q)\) is the Bekenstein-Hawking entropy of the black hole and \(S_0(Q)\) is the value of \(S\) attained at extremality as the mass of the black hole approaches its charge. For example, for Reissner-Nordström black
holes in $d$ spacetime dimensions,

$$S_0(Q) = K_d Q^{(d-2)/(d-3)} / \hbar,$$

(8)

where

$$K_d = \frac{1}{4} \left( \frac{A_{d-2}}{G_d} \right)^{(d-4)/(d-3)} \frac{8 \pi}{(d-2)(d-3)} (d-2)^{(d-2)/(d-3)},$$

(9)

($A_{d-2} = 2 \pi^{(d-1)/2} / \Gamma((d-1)/2)$ is the area of the unit $d-2$ sphere). It is interesting to note that in all cases except $d = 4$, the entropy bound depends explicitly on the gravitational constant $G_d$. $S_0(Q)$ appears in the transformation in order to guarantee that the square-root remains real for all values of the parameters $M$ and $Q$ that correspond to physical black holes (as opposed to naked singularities). Squaring and adding (8) and (9), we get:

$$S_{BH} - S_0(Q) = 2 \pi \frac{\hbar}{\bar{\hbar}} \left( \frac{X^2}{2} + \frac{\Pi_X^2}{2} \right).$$

(10)

The right hand side is immediately recognizable as the Hamiltonian of a linear harmonic oscillator on the $(X, \Pi_X)$ subspace. Quantization is straightforward (with usual identifications $\hat{X} \rightarrow X$, $\hat{\Pi}_X \rightarrow -i \partial / \partial X$), yielding:

$$S_{BH} = 2 \pi \left( n + \frac{1}{2} \right) + S_0(Q), \quad n = 0, 1, 2, \ldots$$

(11)

Before proceeding further, we note that the above spectrum automatically satisfies the extremality bound $S_{BH} \geq S_0(Q)$. In fact, with our choice of factor ordering, it is a strict inequality. Although one might argue that this classical bound may be modified (or even violated) for microscopic black holes, for large black holes, it should evidently hold. Our spectrum ensures that this is indeed the case. Also, note that the (11) implies that (near)-extremal black holes are highly quantum mechanical objects, irrespective of their mass, since they correspond to low values of the quantum number $n$. Although it is somewhat counter-intuitive to think of large, near-extremal black black holes as quantum mechanical, this view is consistent with the thermodynamic interpretation of black holes, since (near)-extremal black holes are associated with extremely small temperatures, signifying transition to quantum regimes. The quantum nature of black holes near extremality, and the breakdown of macroscopic laws in this regime were also found earlier [26, 27].
Another potentially important feature of the spectrum (11) is that, with our choice of factor ordering, extremal black holes \( S_{BH} = S_0(Q) \) are not in the quantum spectrum. This suggests that it may not be possible for non-extremal black holes to decay, or even get arbitrarily close, to extremality. This intriguing possibility has been discussed recently by Medved in [28] who used duality arguments to conclude that back reaction effects prevent Reissner-Nordstrom type black holes in any dimensions from reaching the extremal state.

To complete the analysis of the spectrum, we use the following result from [19]:

\[
\delta P_Q = -\Phi P_M + \delta \lambda ,
\]

where \( \Phi \) is the electrostatic potential on the boundary under consideration, and variation refers to small change in boundary conditions, \( \lambda \) being the gauge parameter at the boundary. This in turn implies that for compact \( U(1) \) gauge group, \( \chi \equiv e\lambda/\hbar = e(P_Q + \Phi P_M) \) is periodic with period \( 2\pi \) (\( e = \) electronic charge). Also, we saw earlier from thermodynamic arguments that \( \alpha \equiv 2\pi P_MT_H(M,Q) \) has period \( 2\pi \). In terms of these ‘angular’ coordinates, the momentum \( \Pi_Q \) in (7) can be written as:

\[
\Pi_Q = \frac{\hbar}{e} \chi + \frac{\hbar}{2\pi} S'_0(Q) \alpha .
\]

Thus, the following identification must hold in the \((Q, \Pi_Q)\) subspace:

\[
(Q, \Pi_Q) \sim \left(Q, \Pi_Q + 2\pi n_1 \frac{\hbar}{e} + n_2 \hbar S'_0(Q) \right) ,
\]

for any two integers \( n_1, n_2 \). Now, wavefunctions of charge eigenstates are of the form:

\[
\psi_Q(\Pi_Q) = \exp \left(iQ\Pi_Q/\hbar \right) ,
\]

which is single valued under the identification (12), provided there exists another integer \( n_3 \) such that:

\[
n_1 \frac{Q}{e} + n_2 \frac{Q}{2\pi} S'_0(Q) = n_3 .
\]

Now, it can be easily shown that the above conditions is satisfied if and only if the following two quantization conditions hold:

\[
\frac{Q}{e} = m
\]
\[
\frac{Q}{2\pi} S'_0(Q) = p, \tag{14}
\]
where \(m\) and \(p\) are any two integers. While the first condition is the familiar charge quantization condition, the second is a new constraint on the \(U(1)\) charge. For example, for Reissner-Nordström black holes, Eq. (8) implies:

\[
\frac{K(d)}{2\pi} \left( \frac{d-2}{d-3} \right) \frac{Q^{(d-2)/(d-3)}}{\hbar} = p. \tag{15}
\]

Together (11) and (15) imply that the horizon area spectrum of the Reissner-Nordström black hole is given by:

\[
S_{BH} = 2\pi \left[ n + \left( \frac{d-3}{d-2} \right) p \right] + \pi. \tag{16}
\]

Using the Bekenstein-Hawking entropy formula

\[
S_{BH} = \frac{A_{BH}}{4 \ell_{Pl}^{d-2}}, \tag{17}
\]
(where \(\ell_{Pl}\) is the Planck length in \(d\)-dimensions), (16) gives the following spectrum of its horizon area

\[
A_{BH} = 8\pi \left[ n + \left( \frac{d-3}{d-2} \right) p \right] \ell_{Pl}^{d-2} + 4\pi \ell_{Pl}^{d-2}. \tag{18}
\]

This is our main result. The quantum number \(p\) determines the charge of the quantum black hole, while \(n\) determines the excitation of the black hole above extremality. As mentioned earlier, although classically the extremality bound can be reached, our analysis predicts the remarkable feature that this classical bound is never saturated due to small vacuum fluctuations of the horizon. Also, note that the ground state of the spectrum is at:

\[
A_{BH}(n=0=p) = 4\pi \ell_{Pl}^{d-2}, \tag{19}
\]

implying that there is a ‘zero-point area’ of Planckian dimensions. This also implies that if for example Hawking evaporation radiates away the energy (and area) of a black hole, then it must stop at the above value. It is tempting to speculate that this Planck sized remnant will retain information that fell into the black hole earlier, thus avoiding the information loss ‘paradox’. Such remnants have been anticipated in many early works in quantum gravity and astrophysics, and there remains a lively debate about their existence in general [29].

6
3 Adiabatic Invariants

Having derived the spectrum (18), we return to Bekenstein’s original reasoning about discrete area spectrum from adiabatic invariants. From a class of novel thought experiments he argued that horizon areas of black holes with charge must be adiabatic invariants. However, here a very similar result can be derived from first principles: consider Eq.(10). Since the right hand side describes a harmonic oscillator, the periodic orbits in phase space naturally give rise to the following adiabatic invariant:

$$J = \oint \Pi_X dX = \frac{A - 4GhS_0(Q)}{4G/\pi}.$$  \hspace{1cm} (20)

Thus for $S_{BH} \gg S_0$ (i.e. far from extremality), the horizon area is indeed an adiabatic invariant. However, close to extremality, the above relation suggests that it is the area above extremality which is an adiabatic invariant. We interpret this as a slight refinement over Bekenstein’s original hypothesis. The advantage of relation (20) is that on the one hand it is consistent with the discrete spectra (18), and on the other hand, it ensures that the extremality bound $S_{BH} \geq S_0$ is always obeyed.

4 Relation to Bekenstein’s Analysis

Now, we examine the relation of our spectrum to that derived by Bekenstein from an algebraic point of view in [5]. The issue was examined in [18] for uncharged black holes, and we review the results here. The extension to charged black holes will be left to a future publication. In [5], Bekenstein and collaborators proposed the existence of a set of linear operators $\{\hat{A}, \hat{R}_{nsn}\}$, where the first operator corresponds to the horizon area observable, and the second creates a single black hole state from vacuum with area $a_n$, in an internal quantum state $s_n$. It is assumed that $s_n \in \{0, 1, \ldots, e^{a_n} - 1\}$, to account for the internal degeneracy associated with the Bekenstein-Hawking entropy. Symmetry, linearity and closure imply that the algebra between these fundamental operators must be of the form:

$$[\hat{A}, \hat{R}_{nsn}] = a_n \hat{R}_{nsn},$$  \hspace{1cm} (21)

$$[\hat{A}, \hat{R}_{nsn}^\dagger] = -a_n \hat{R}_{nsn}^\dagger.$$  \hspace{1cm} (22)
\[
\left[ \hat{A}, \left[ \hat{R}^\dagger_{msm}, \hat{R}_{nsn} \right] \right] = (a_n - a_m) \left[ \hat{R}^\dagger_{msm}, \hat{R}_{nsn} \right] \quad \text{iff} \quad a_n > a_m , \quad (23)
\]
\[
\left[ \hat{R}_{nsn}, \hat{R}_{msm} \right] = \epsilon_{nm}^k \hat{R}_{ksk} \quad (\epsilon_{nm}^k \neq 0, \quad \text{iff} \quad a_n + a_m = a_k) . \quad (24)
\]

Now, it was shown in [5] that the spectrum of the above algebra involves both addition and subtraction of area levels, which is possible if and only if the area levels are equally spaced; i.e.,

\[ a_n = na_0 + \bar{a} \quad n = 0, 1, 2, \cdots \quad (25) \]

where \( \bar{a} \) is a constant which can take any arbitrary value. In [3], \( \bar{a} \) was set to zero so that the ground state area. However, the algebra Eqs.(21-24) does not in any way impose such a constraint, and in fact remains unchanged even if there is a non-zero \( \bar{a} \). For example, for Reissner-Nordstrom black holes, \( a_0 \) and \( \bar{a} \) were chosen to be [18]:

\[ a_0 = 2\bar{a} = 8\pi\ell_{pl}^{d-2} . \quad (26) \]

With this identification, the above spectrum becomes identical to the uncharged version of (18). Although the operators in Eqs.(21-24) have so far been kept abstract, the picture can be completed by their explicit construction:

\[
\hat{R}_{nsn} = (P^\dagger)^n \hat{g}_{sn} , \quad (27)
\]

\[
\hat{A} = (P^\dagger P + 1/2)\bar{a} , \quad (28)
\]

\[
\hat{P}^\dagger = \frac{1}{\sqrt{2}} \left[ \hat{X} - i\hat{\Pi}_X \right] , \quad (29)
\]

which automatically satisfy the algebra:

\[
[\hat{P}, \hat{P}^\dagger] = 1 , \quad (30)
\]

\[
[\hat{P}, \hat{g}_{sm}] = [\hat{P}^\dagger, \hat{g}_{sm}] = 0 , \quad (31)
\]

\[
[\hat{g}_{sm}, \hat{g}_{sn}] = \epsilon_{mn}^k \hat{g}_{sk} \quad \text{where} \quad \epsilon_{mn}^k \neq 0 \quad \text{iff} \quad s_k = s_m + s_n . \quad (32)
\]

Thus we can see that the operators used by Bekenstein to derive equally spaced spectra for black hole can be constructed out of fundamental gravitational degrees of freedom, at least in the context of spherically symmetric uncharged black holes. This makes our analysis and results perfectly consistent with those of Bekenstein.
5 Summary and Conclusions

Finally, let us consider the implications of our results in a physical process in which the black hole emits a photon by making a quantum jump from one level to the next lower level. Assuming that the black hole decays by emitting just one photon with the lowest allowed frequency $\omega_0$ (for simplicity we assume uncharged particle emission and four dimensions), its initial and final masses are $M + \hbar \omega_0$ and $M$ respectively, and using (18) the following relation holds:

$$A_{BH}(M + \hbar \omega_0, Q) - A_{BH}(M, Q) = A_{BH}(n + 1) - A_{BH}(n) = 4\pi \ell_{Pl}^2 .$$

Using $A_{BH} = 4\pi r_+^2$ and $r_\pm = G_4 M \pm \sqrt{(G_4 M)^2 - G_4 Q^2}$, we get:

$$\omega_0 = \frac{(r_+ - r_-)\pi}{A}. \quad (33)$$

In the $Q \to 0$ limit, the above frequency agrees with that found in [5] up to factors of order unity. However, it differs significantly from predictions of loop quantum gravity [30].

To summarize, in this article we have shown how the spectra of black hole observables postulated by Bekenstein can be derived within an explicit, rigorous (given one basic assumption) quantization scheme. Our derivation pertains to the spherically symmetric charged black hole sector of generic theories of gravity. Once non-spherically symmetric modes are introduced, one naturally expects some modifications to the spectrum. In particular, the quantum numbers that we derived are analogous to the principle quantum numbers of the hydrogen atom. Introducing additional modes will bring in more quantum numbers, such as angular momentum, and many more states which, to a first approximation, are highly degenerate. An important open question is to what extent higher order corrections break this degeneracy and cause the spaces between the discrete spectrum values to be “filled in”, potentially restoring the continuous radiation spectrum of Hawking’s original work.

Another intriguing point to note is that quantization conditions (13) and (15) imply that the fundamental charge $e$ is constrained by the following relation:

$$e^{(d-2)/(d-3)} = \left[ \frac{2\pi \hbar}{K(d)} \left( \frac{d - 2}{d - 3} \right) \right] \frac{p}{m^{(d-2)/(d-3)}} , \quad (34)$$
which is the $d$-dimensional fine-structure constant. The above relation can be interpreted in one of the following ways, depending on whether one considers the black hole or the electronic charge as fundamental physical entities. According to the first point of view, even if a single black hole is present in the universe, it would imply that the electronic charge would have to satisfy the condition (34) for some integral values of $p$ and $m$. This is reminiscent of Dirac’s quantization condition, according to which, presence of a single magnetic monopole in the universe of strength $g$ would require all electric charges to be quantized in units of $2\pi\hbar/g$ [31]. This is also in the spirit of Coleman’s ‘Big-Fix Mechanism’ in which he argued that wormholes can fix the constants of nature [32]. Indeed for $d = 4$, condition (34) translates to
\[
\frac{e^2}{\hbar} = \frac{p}{m^2},
\]
which says that the fine structure constant in our universe should somehow be approximated by the above form.

Alternatively, one can take the viewpoint that electronic charges are fundamental, so that charged black holes must be created such that their charges satisfy the conditions (13) and (14). Whichever interpretation one chooses to embrace, we have shown that quantum gravity in general, and black holes in particular, may play a very important role in determining the fundamental constants of nature. In any case, it is certainly clear that Professor Bekenstein’s contributions to the field of black hole dynamics and to our understanding of the universe in general can hardly be over-emphasized.

Acknowledgments
S.D. and G.K. would like to thank Viqar Husain for helpful comments and encouragement. S.D. thanks P. Ramadevi and U. A. Yajnik for useful correspondence, and collaboration on which paper [18] was based. G.K. thanks Valeri Frolov and the gravity group at the Theoretical Physics Institute, University of Alberta for useful discussions. We also acknowledge the partial support of the Natural Sciences and Engineering Research Council of Canada. This work was also supported by the Russian Foundation for Basic Research under the grant No 02-01-00930.
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