Evidence for Direct CP Violation in $B^\pm \to \eta h^\pm$ and Observation of $B^0 \to \eta K^0$

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We report measurements of the branching fractions and CP asymmetries for $B^\pm \to \eta h^\pm$ ($h = K$ or $\pi$) and the observation of the decay $B^0 \to \eta K^0$ from the final data sample of $772 \times 10^6$ $B\overline{B}$ pairs collected with the Belle detector at the KEKB asymmetric-energy $e^+ e^-$ collider. The measured branching fractions are $\mathcal{B}(B^\pm \to \eta K^\pm) = (2.12 \pm 0.23 \pm 0.11) \times 10^{-6}$, $\mathcal{B}(B^{+} \to \eta \pi^{+}) = (4.07 \pm 0.26 \pm 0.21) \times 10^{-6}$ and $\mathcal{B}(B^{0} \to \eta K^{0}) = (1.27^{+0.29}_{-0.23} \pm 0.08) \times 10^{-6}$, where the last decay is observed for the first time with a significance of 5.4 standard deviations ($\sigma$). We also find evidence for CP violation in the charged $B$ modes, $A_{CP}(B^\pm \to \eta K^\pm) = -0.38 \pm 0.11 \pm 0.01$ and $A_{CP}(B^{+} \to \eta \pi^{+}) = -0.19 \pm 0.06 \pm 0.01$ with significances of 3.8$\sigma$ and 3.0$\sigma$, respectively. For all measurements, the first and second uncertainties are statistical and systematic, respectively.

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Charge-Parity (CP) violation plays an important role in any explanation of the observed dominance of matter over antimatter in our Universe. Current experimental knowledge about CP violation is limited. Charmless hadronic $B$ decays constitute sensitive probes for CP violation in the standard model (SM) as well as beyond, and can help to further elucidate this unsolved question. In the SM, the decays $B^\pm \to \eta(0) K^\pm$ and $B^0 \to \eta(0) K^0$ are expected to primarily proceed through $b \to s$ penguin processes and a $b \to u$ tree transition as shown in Fig. 1. The large $B \to \eta' K$ and small $B \to \eta K$ branching fractions can be explained by $\eta - \eta'$ mixing and constructive interference between the amplitudes of the two penguin processes.

The branching fraction of $B^0 \to \eta K^0$ is expected to be lower than that of $B^\pm \to \eta K^\pm$, because the tree diagram in the $B^0 \to \eta K^0$ decay is color suppressed. The destructive combination of penguin amplitudes may interfere with the tree amplitude in $B \to \eta K$, resulting in a large direct CP asymmetry ($A_{CP}$), defined as

$$A_{CP} = \frac{\Gamma[B^- (B^0) \to \eta h^{-0}] - \Gamma[B^+ (B^0) \to \eta h^{+0}]}{\Gamma[B^-(B^0) \to \eta h^{-0}] + \Gamma[B^+(B^0) \to \eta h^{+0}]} \, ,$$

(1)

where $\Gamma(B \to \eta h)$ is the partial width obtained for the $B \to \eta h$ decay, and $h$ denotes $K$ or $\pi$. Similarly, direct CP violation could be sizeable for $B^\pm \to h \pi^{\pm}$ owing to the interference between $b \to d$ penguin and $b \to u$ tree diagrams. Several theoretical calculations with different mechanisms suggest a large $A_{CP}$ for both $B \to \eta K$ and $B \to \eta \pi$, although the sign could be either positive or negative. Previous Belle and BaBar measurements indicate a large negative $A_{CP}$ in $B^\pm \to \eta K^\pm$, but more data are needed to be statistically sensitive to a non-zero $A_{CP}$ in $B^0 \to \eta K^0$.

In this Letter, we report the first observation of $B^0 \to \eta K^0$, and evidence for direct CP asymmetries in $B^\pm \to \eta K^\pm$ and $B^{\pm} \to \eta \pi^{\pm}$ using the final Belle data set. The data sample corresponds to $(772 \pm 11) \times 10^6$ $B\overline{B}$ pairs collected with the Belle detector at the KEKB e$^+ e^-$ asymmetric-energy (3.5 GeV on 8.0 GeV) collider operating at the $\Upsilon(4S)$ resonance. The production rates of $B^{+} B^{-}$ and $B^{0} \overline{B}^{0}$ pairs are assumed to be equal at the $\Upsilon(4S)$ resonance.

The Belle detector [14] is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector, a 50-layer central drift chamber (CDC), an array of aerop.
gel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprising CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside the coil is instrumented to detect $K_S^0$ mesons and to identify muons.

The event selection and $B$ candidate reconstruction methods are similar to those described in Ref. [4]. We select $\eta$ and $\pi^0$ candidates through the decay chains $\eta \rightarrow \gamma \gamma$ ($\eta\gamma\gamma$), $\eta \rightarrow \pi^+\pi^-\pi^0$ ($\eta\pi\pi\pi$) and $\pi^0 \rightarrow \gamma \gamma$. We require the two photons from the $\eta$ and $\pi^0$ decays to have energies ($E_{\gamma i}$, $i = 1, 2$) greater than 50 MeV. Candidate $\pi^0$ mesons are selected from pairs of photons with invariant masses between 115 MeV/c$^2$ and 152 MeV/c$^2$. In the $\eta\gamma\gamma$ reconstruction, the photon energy asymmetry $|E_{\gamma 1} - E_{\gamma 2}|/(E_{\gamma 1} + E_{\gamma 2})$, is required to be less than 0.9 to reduce the large combinatorial background from low-energy photons. Photons in $\eta\gamma\gamma$ reconstruction are not allowed to pair with any other photon having $E_{\gamma} > 100$ MeV, to form a $\pi^0$ candidate. We require the invariant mass of the $\eta\gamma\gamma$ and $\eta\pi\pi\pi$ candidates to be in the intervals (501, 573) MeV/c$^2$ and (538.5, 556.5) MeV/c$^2$, respectively. In order to improve the $\eta$ and $\pi^0$ energy resolution, a mass-constrained kinematic fit is performed after the candidate selection.

Charged tracks that are not used to form $K_S^0$ candidates (see below) are required to have a distance of closest approach with respect to the interaction point (IP) of less than 3.0 cm along the electron beam direction ($z$) and less than 0.3 cm in the transverse plane. Charged kaons and pions are identified using $dE/dx$ information from the CDC, Cherenkov light yields in the ACC, and time-of-flight information from the TOF. This information is combined to form a likelihood ratio, $R_{K/\pi} = L_K/(L_K + L_\pi)$, where $L_K$ ($L_\pi$) is the likelihood of the track being a kaon (pion). Charged tracks with $R_{K/\pi} > 0.6$ ($< 0.4$) are treated as kaons (pions) for $B^\pm \rightarrow \eta K^\pm (B^\pm \rightarrow \eta \pi^\pm)$ reconstruction. A less stringent requirement, $R_{K/\pi} < 0.6$, is used for charged pions in the $\eta\pi\pi$ selection. The efficiencies for the $R_{K/\pi}$ requirement are 84% for kaons, 89% for pions, and 94% for pions in $\eta\pi\pi$ reconstruction. Furthermore, for $B^\pm \rightarrow \eta h^\pm$ and $B^0 \rightarrow \eta K^0$ we reject charged tracks consistent with either the electron or muon hypothesis.

Candidate $K^0_S$ mesons are reconstructed in $K_S^0 \rightarrow \pi^+\pi^-\pi^0$ decays. The $K_S^0$ candidates are required to have an invariant mass lying between 488 MeV/c$^2$ and 508 MeV/c$^2$. The charged tracks for each $K_S^0$ candidate are required to have a distance-of-closest approach with respect to the IP of larger than 0.02 cm in the transverse plane. The angle between the $K_S^0$ momentum and the direction from the IP to the $K_S^0$ decay vertex must be within 0.03 rad. The distance between the two daughter tracks at their point of closest approach in the transverse plane is required to be less than 2.40 cm, and the flight length of the $K_S^0$ is required to be larger than 0.22 cm.

Candidate $B$ mesons are identified using the modified beam-energy-constrained mass $M_{bc} = \sqrt{(E_{\text{beam}}^* - E_B^*)^2 - m_B^2}$ [13], and the energy difference $\Delta E = E_B^* - E_{\text{beam}}^*$, where $E_{\text{beam}}^*$ is the beam energy, and $E_B^*$ and $E_{\text{beam}}^*$ are the energy and modified momentum, respectively, of the $B$ candidate in the $Y(4S)$ rest frame. The energy $E_B^*$ is calculated as $E_B^* = E_{\eta}^* + E_h^*$. The momentum $\vec{p}_B^*$ is calculated according to

$$\vec{p}_B^* = \vec{p}_{\eta}^* + \vec{p}_{\pi}^* \times \sqrt{(E_{\text{beam}}^* - E_B^*)^2 - m_{\eta_B}^2},$$

(2)

where $m_\eta$ is the nominal $\eta$ mass [16]. Since charged tracks are generally measured with a better precision than photons, the $\eta$ decays to neutral particles have worse momentum resolution than primary charged tracks from $B$ decays. The $\vec{p}_B^*$ resolution is improved using Eq. (2) because $\vec{p}_\eta^*$ and $E_{\text{beam}}^*$ are determined more precisely than $\vec{p}_\eta^*$. Events with $M_{bc}$ greater than 5.2 GeV/c$^2$ and $|\Delta E| < 0.3$ GeV are retained for further analysis.

The dominant background arises from $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) continuum events. We use event topology variables to distinguish spherically distributed $B\bar{B}$ events from the jet-like continuum background. First we combine a set of modified Fox-Wolfram moments [17] into a Fisher discriminant. We then compute a likelihood from the product of probability density functions (PDFs) that describe the Fisher discriminant, $\cos \theta_B^*,$ and $\Delta z$ distributions. Here, $\theta_B^*$ is the angle between the $B$ flight direction and the beam direction in the $Y(4S)$ rest frame, and $\Delta z$ is the decay flight-length difference, along the $z$ axis, between vertices of the signal $B$ and the accompanying $\bar{B}$. A likelihood ratio, $R = L_s/(L_s + L_{\eta\eta})$, is formed from signal ($L_s$) and background ($L_{\eta\eta}$) likelihoods, which are obtained from GEANT-based Monte Carlo (MC) simulations. Signal MC events are generated with EVTGEN [13], which invokes the PHOTOS [20] package to take final state radiation into account. We require $R > 0.2$ to suppress continuum background in all modes and then translate $R$ to $R'$, defined as

$$R' = \ln \left( \frac{R - R_{\min}}{R_{\max} - R} \right).$$

(3)

In this expression $R_{\min}$ ($R_{\max}$) is equal to 0.2 (1.0). This translation is convenient, as the $R'$ distributions for signal and backgrounds can be described by a simple sum of Gaussian functions.

Signal yields are extracted by performing an unbinned extended three-dimensional maximum likelihood fit. The likelihood for each $B^\pm$ mode is defined as

$$L = e^{-\sum_i N_i} \prod_i \left( \sum_j N_j^i P_j^i \right),$$

$$P_j^i = \frac{1}{2} \left[ 1 - q^i ACP_j \right] P_j \left( M_{bc}, \Delta E^i, R'^i \right),$$

(4)
is parameterized with an ARGUS function, $M_{\alpha}$ a second-order polynomial, while the $\eta_{\tau}$ distributions in control samples [22].

The continuum background in $\Delta E$ is described by a second-order polynomial, while the $M_{bc}$ distribution is parameterized with an ARGUS function, $f(x) = x\sqrt{1 - x^2} \exp\left[-\xi(1 - x^2)\right]$, where $x$ is $M_{bc}/E_{beam}$ and $\xi$ is a free parameter in the fit [23]. The $R'$ PDF is a double Gaussian function. The background PDFs in both $M_{bc}$ and $\Delta E$ for charmless $B$ decays are modeled with smoothed two-dimensional histograms obtained from a large MC sample, and the $R'$ PDF is a single Gaussian.

We perform a simultaneous fit to $B^{\pm} \to \eta K^{\pm}$ and $B^{0} \to \eta \pi^{\pm}$ candidate events, since these two decay modes can feed into each other. In the likelihood fits, $N_j$ and $A_{CP,j}$ are allowed to vary for the continuum and charmless $B$ backgrounds. Shape parameters of the continuum background PDFs are also floated. The cross-feed background in $\eta K^{\pm}$ ($\eta \pi^{\pm}$) and signal in $\eta \pi^{\pm}$ ($\eta K^{\pm}$) share the same fitting parameters in both $A_{CP}$ and branching fraction. Figure 2 shows the $M_{bc}$, $\Delta E$ and $R'$ projections of the fit to the $B^{0} \to \eta K^0$ sample. The $M_{bc}$ and $\Delta E$ projections for the $B^{+} \to \eta h^{+}$ and $B^{-} \to \eta h^{-}$ samples are shown separately in Fig. 3.

The branching fraction for each mode is calculated by dividing the efficiency-corrected signal yield by the number of $B\bar{B}$ pairs. The dominant systematic errors on the branching fraction come from MC modeling of the $\eta, \pi^0,$ and $K^0$ selection efficiency; these errors are 4.0%, 4.0%, and 1.6%, respectively.
The systematic error due to $R(K/\pi)$ requirement is 0.9% for kaons and 0.8% for pions. It is estimated from the $D^{+} \to D^{0} \pi^{+}(D^{0} \to K^{-} \pi^{+})$ sample. The systematic error due to the charged-track reconstruction efficiency is estimated to be 0.35% per track, which is determined from a study of the $D^{+} \to D^{\pi^{+}}(D^{0} \to \pi^{+}\pi^{-}K_{S}^{0})$ decay. Any difference in the efficiency when the $R$ criterion is applied to data or MC is investigated using the $B^{+} \to D^{0} \pi^{+}(D^{0} \to \pi^{+}\pi^{-}K_{S}^{0})$ sample; the results of this study imply a 0.6% systematic uncertainty. The fitting systematic errors due to the signal PDF modeling are estimated from changes of the fit parameters while varying the calibration factors by one standard deviation. The systematic error due to charmless $B$ background PDF modeling is calculated from the difference observed between the signal yield when the charmless yield is floated in the fit and that when the yield is fixed to the MC expectation. The systematic error due to the uncertainty in the total number of $B\overline{B}$ pairs is 1.4%, and the error due to limited signal MC statistics used to evaluate the efficiency is 0.55%. The systematic errors on $A_{CP}$ arise from detector bias, uncertainties on the detector bias and PDF modeling. The possible detector bias due to the tracking acceptance and $R(K/\pi)$ selection for $A_{CP}(B^{\pm} \to \eta\pi^{\pm})$ is evaluated using the fitted $A_{CP}$ value of the continuum background $^{24}$ $^{25}$. The detector bias in $A_{CP}(B^{\pm} \to \eta K^{\pm})$ is evaluated using the $D^{+} \to \phi\pi^{+}(\phi \to K^{0}K^{-})$ and $D^{0} \to K^{0}\pi^{+}$ samples $^{24}$ $^{25}$. There is a contribution to the $A_{CP}$ systematic uncertainty from the modeling of the signal PDFs. The total systematic errors for $A_{CP}$ are in the range $(8.2-14.2) \times 10^{-3}$.

The statistical significance is evaluated as $\sqrt{-2\ln(L_{0}/L_{\text{max}})}$, where $L_{0}$ is the likelihood value when either the signal yield or $A_{CP}$ is fixed to zero, and $L_{\text{max}}$ is the nominal likelihood value. The total significance ($\Sigma$) including PDF modeling systematic uncertainty is calculated after smearing the likelihood distribution with the appropriate PDF modeling systematic error. In Table 1 we list the fitted signal yields, charge asymmetries, reconstruction efficiencies, and branching fractions. The combined result for the two $\eta$ decay modes is obtained from the combined likelihood function.

In summary, using the final Belle data sample containing $772 \times 10^6 B\overline{B}$ pairs and a three-dimensional fit that maximizes the efficiency, we provide new measurements based on signal yields 2.5 times larger than those reported in our previous publications $^{[9]}$. We find evidence for $CP$ asymmetries in $B^{\pm} \to \eta K^{\pm}$ and $B^{\pm} \to \eta\pi^{\pm}$: $A_{CP}(B^{\pm} \to \eta K^{\pm}) = -0.38 \pm 0.11 \pm 0.01$ and $A_{CP}(B^{\pm} \to \eta\pi^{\pm}) = -0.19 \pm 0.06 \pm 0.01$. The significance of $A_{CP}(\eta K^{+})$ [$A_{CP}(\eta\pi^{+})$] is 3.8$\sigma$ [3.0$\sigma$]. Evidence for $A_{CP}(B^{\pm} \to \eta\pi^{\pm})$ is seen for the first time. We also observe the decay $B^{0} \to \eta K^{0}$ for the first time with a significance of 5.4$\sigma$ and a branching fraction $B(B^{0} \to \eta K^{0}) = (1.27^{+0.33}_{-0.28} \pm 0.08) \times 10^{-6}$. In addition, we report the following new measurements of the branching fractions: $B(B^{\pm} \to \eta K^{\pm}) = (2.12 \pm 0.23 \pm 0.11) \times 10^{-6}$ and $B(B^{\pm} \to \eta\pi^{\pm}) = (4.07 \pm 0.26 \pm 0.21) \times 10^{-6}$. All our branching fraction and $A_{CP}$ measurements supersede the results in Ref.$^{[3]}$.

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