Designing learning trajectory for teaching sequence and series using RME approach to improve students’ problem solving abilities

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Abstract. This study aimed at designing learning trajectory (LT) for teaching Sequence and Series using Realistic Mathematics Education (RME) approach to improve the students’ problem solving ability. The LT designed in this research covered real problems that helped students follow the math learning process to build their knowledge in solving mathematical problems. The LT was developed through a design research that consisted of a cyclic process of preparing the experiment, conducting the experiment, and retrospective analysis. The research’s subject was 34 grade nine students at a junior high school in Nias Selatan, Indonesia. The data were collected through observation, interviews, checklist, videotaping, and analyzing the students’ works. The results showed that the LT for teaching Sequence and Series using RME approach reached the criteria of validity and practicality. The LT also could help students build their own knowledge through problem solving activities to discover sequence and series concepts.

1. Introduction
Problem solving ability was highlighted in many mathematical curricula and has recently become one of the most studied in the field of mathematics education. The National Council of Mathematics Teachers has consistently advocated for problem solving as part of daily math instruction, arguing that solving problems was an important thing to do in learning mathematics [1,2]. Furthermore, Soedjadi stated that mathematical problem solving ability is an ability within the students themselves to use mathematical activities to solve problems in mathematics or other sciences even in everyday life [3]. Beigie also stated that through problem solving, students could learn to improve their understanding of mathematical concepts by working through carefully selected issues which use the application of mathematics in real life problems [4]. The development of mathematical problem solving ability can equip students to think logically, analytically, systematically, critically, and creatively [5]. Therefore, problem solving ability is an important ability that needs to be provided for students to help them learn math as well as solve real life problem.

The development of student problem solving skills is not an easy task. When solving math problems, most students are not able to understand the problem, plan the solution, and apply the solution to the given problem [6]. Other studies also revealed that students poor problem solving ability is caused by their lack of experience in solving nonroutine problems [7]. The same problem was also encountered by the researcher in that students have difficulties in solving mathematical problems due to mathematics teaching that does not connect students' informal knowledge from their lives with their formal knowledge. Mayer defines problem solving as some process that requires the problem solver to find a connection between experience (schemes) that he or she had with the problem he or she is facing and being able to do something to solve the problem [8]. Meanwhile, Gagne defines problem solving as an activity to synthesize between knowledge, rules, concepts,
schemes, or experiences they had and the condition they are facing to find a solution [8]. Based on the description, in order to connect students' thinking between real life problems and their formal knowledge in mathematics, a learning trajectory is required to bridge the informal and formal knowledge.

Learning trajectory is a series of activities and tasks that support the development of students' understanding of a particular instructional purpose [9]. Simon included the empirically supported descriptions of how students' thinking evolves over time. Based on research synthesis, clinical interviews, pedagogical experiments and large-scale evaluation data, learning trajectory was empirically defined as descriptions that help determine how students' informal ideas evolve through mathematical understanding [10]. Learning trajectory in learning is guiding students' instructional decisions [11] and improving their ability to use their thinking [12]. In Mathematics learning, students are expected to be the subject of learning. Therefore, teachers should be able to apply the learning trajectory that requires students' involvement and adapt how they think.

The learning trajectory has attracted the attention of some researchers [13-16]. In these studies, the researchers developed the LTs for various mathematics topics. The results of the researches showed that that the LTs could help the students to reinvent mathematical concepts in meaningful ways, so that they can build their understanding of the topics that they learned.

This learning trajectory is based on realistic mathematics education approach because this approach creates student-centered learning as well as directs students to engage in contextual problem solving. The main principle in RME approach is that mathematics is viewed as a human activity and mathematical learning means doing mathematics [17]. This means that in learning mathematics, student involvement is expected as well as directed to solve mathematical problems related to real life. Furthermore, Freudenthal emphasized that in mathematics learning students should be allowed and supported to create their own ideas and use their own strategies. In other words, they must learn mathematics in their own way [18].

The use of the learning trajectory is expected to improve students' problem solving abilities on sequence and series topics. RME learning that emphasizes on skills process of doing mathematics will lead students to do and solve their own problems by taking advantage of informal knowledge from their life. This is in accordance with a learning trajectory that provides predictions and anticipation of students thinking that will help students solve mathematical problems through mathematization process based on RME approach. Through the mathematization process, the informal knowledge of the students will be linked with the formal knowledge to be studied. This will train students to solve problems and have a positive impact on students' mathematical problem solving abilities.

2. Method

This research used design research approach proposed by Gravemeijer and Cobb [19]. We used this approach because design research aims to better understand the interrelatedness between teaching and learning in order to improve teaching [20]. Design research in this study consisted of a cyclic process of preparing the experiment, conducting the experiment, and retrospective analysis. Gravemeijer and Cobb illustrated the cyclic process as can be seen in Figure 1.

![Figure 1. A cyclic thought process and instruction experiment.](image)

In preparing for the experiment, we determined the end point of the instructions. The goals of our sequence and series lessons were for the students to reinvent the concepts of numerical patterns, arithmetic sequences and series, and geometry sequences and series. Considering that in studying
sequence and series materials it takes the students’ experience to explore the informal way of thinking, we then used the South Nias culture as the starting points of the lesson. After we set the end and the starting points, we designed the HLT that consisted of five main activities and nine sub-activities of solving contextual problems that would facilitate students to do horizontal and vertical mathematizations as well as stimulate students’ thinking and reasoning. Besides, we also formulated the predictions of students’ thinking and solutions, and their anticipations.

In the experiment phase, we tried out the HLT in two cycles. The first try out was conducted in small group that involved six grade nine students at junior high schools in Nias Selatan, Indonesia. After retrospective analysis and re-design processes, the HLT was tried out to 34 grade nine students at the same school. The retrospective analysis involved the research, a teacher, and an observer. Beside focusing our attention to develop the HLT, we also observed and analyzed the impact of the HLT on the development of students’ confidence in using their strategies when solving the contextual problems and development of students’ reasoning during try out. The research data were collected through observation, interviews, checklist, videotaping, and analyzing the students’ works.

3. Results and discussion
The HLT for teaching the sequence and series topics was validated by three mathematics education experts in Indonesia during preparation for the experiment phase. The result showed that the HLT met the validity criteria [21] with the following characteristics: the activities of solving contextual problems in the HLT had potential to facilitate the students to reinvent the sequence and series concept; the activities were well ordered; the HLT suit the fundamental principles and characteristics of RME; and the components in the HLT were designed and consistent between one and another.

The HLT also satisfied the practicality criteria [22], in which it worked as intended during the tryout. The students understood the contextual problems and the conducted ‘doing math’ activities without major obstacle. The probing questions that were prepared as the anticipations of students’ thinking and solutions also helped the students to achieve the goals of the activities. Besides, the time provided for doing the activities of solving the contextual problem was well planned.

The application of the learning trajectory by using the RME approach to the sequence and series learning creates interactive and student-centered learning. Students who are the subject of learning should be able to build their own knowledge based on experience. This is in line with the principle of RME expressed by Gravemeijer and Freudenthal, that the basic principles of RME are the guided reinvention and progressive mathematizing, didactical phenomenology, and self developed models [23, 24]. So in this learning trajectory, the mathematical problems are made as student activity and stimulate students to find mathematical concepts that are solved in their way based on their cognitive level and experience.

The following example is a contextual problem that can stimulate students’ understanding in solving mathematical problems contained in the learning trajectory.

Mr. Fa’a has three children named Nia, Ani, and Adi. The three children will join a singing competition to celebrate Christmas School on January 4, 2018. If that day was November 15, 2017, then there were another 49 days of opportunity to prepare for the clothes that will be worn later. Then Mr. Fa’a gave each of his three children one thousand rupiah coins a day and asked them to keep it. If on the first day Nia obtained 3 coins, Ani 2 coins, and Adi a coin. Then the next day, Nia get 3 coins, while for Ani and Adi each get 2 pieces of coins.

a. Write the pattern of the number of coins received Nia, Ani, and Adi starting on the first day until the 5th day in a sequence of numbers!

b. On the 20th day, what is the number of coins received by Nia, Ani, and Adi?

c. What is the number of coins received by Nia, Ani, and Adi on the 40th day?

d. Can you determine how many coins Nia, Ani, and Adi received for the nth day?

Contextual problems provided greatly helped students’ understanding in solving this problem. This was evident from the student's answers, in which they were able to find the concept of numbers pattern. This solution began in an informal way in which the student determined the number of coins received on a given day by the multiplication operation. Then with the help of tiered questions
students were assisted and directed to find the formal knowledge of the nth term formula on the numbers pattern. One example of student answers can be seen in Figure 2.

4. The number of coins the three children received on the fifth day, namely:
1) Nia = 3 × 1, 3 × 2, 3 × 3, 3 × 4, 3 × 5 = 3, 6, 9, 12, 15;
2) Ani = 2 × 1, 2 × 2, 2 × 3, 2 × 4, 2 × 5 = 2, 4, 6, 8, 10;
3) Adi = 1, 2 × 2 - 1, 2 × 3 - 1, 2 × 4 - 1, 2 × 5 - 1 = 1, 3, 5, 7, 9.

On the 20th day, the number of coins received by three children, namely:
1) Nia = 3 × 20 = 60 coins;
2) 2 × 20 = 40 coins;
3) Adi = 2 × 20 - 1 = 39 coins.

Then, on the 40th day, the number of coins received by the three children, namely:
1) Nia = 3 × 40 = 120 coins;
2) 2 × 40 = 80 coins;
3) Adi = 2 × 40 - 1 = 79 coins.

Furthermore, the number of coins received by the three children on the nth day, namely:
1) Nia = 3n;
2) Ani = 2n;
3) Adi = 2n - 1.

Figure 2. Students answers to find the pattern of numbers.

Implementation of the learning trajectory using RME gives a positive response from students because mathematical problems are connected with everyday life problems, making it easier for students to understand the problem. This suggests that the various situations that students experience in everyday life can be utilized in building an understanding of facts, concepts, and principles of mathematics. The imaginary or real-life situation that students gain from experience makes learning mathematics a useful and meaningful activity that emphasizes reasoning rather than mathematical formulas [24]. The results of research by Zulkardi and Ilma states that context is the first step in learning mathematics. In the reinvention approach, problem contexts play a crucial role [25]. Well-chosen contextual problems provide an opportunity for students to solve problems with an informal solution strategy [26].

Students as learning subjects certainly have different experiences and cognitive levels, thus when given mathematical problems, various solution and strategies will emerge. Through the use of the learning trajectory with the RME approach these different ways of thinking in generalizing the knowledge to be built are accommodated. The same was confirmed by Soedjadi that in general the development of cognitive abilities of children starts with concrete things then gradually leads to the abstract. For every child, the journey from real to abstract can be different. Some are fast and some are slow. The fast ones may not require many stages, but for those who are not fast, it is possible that they go through many stages. Thus for every child different learning trajectory may be required [27]. This means that students are able to find their own strategies to be used in solving contextual
problems based on their experience and cognitive level, which met one of the RME characteristics called ‘students’ free production’ [28,29]. This is evident from the students answer to several problems in each activity led to some strategies. Finding the right solution is related to the problem solving ability of students in planning and determining the solution of the problem. Through the anticipation questions that teachers have provided in the learning trajectory, differences in the way students think in determining the right solution to the given contextual problems are bridged.

This learning situation highly contributes to mathematical problem solving ability. Learning trajectory that uses the RME approach are oriented to real life problems. This makes it easier for students to understand the given problems. Furthermore, in the learning process students are expected to be able to find the concept of sequence and series through the mathematization process, starting from horizontal to vertical mathematization process. This means that students are not allowed to use mathematical formulas directly, but they are directed to find the formula itself through the given contextual problem. The students’ involvement in the mathematization process is a way to practice problem solving skills. The results of the analysis of pretest showed that the average grade of students' initial ability in solving mathematical problems was 48.41. Whereas the result of posttest data analysis which is the average grade of student's mathematical problem solving ability after applying the learning trajectory based on RME reached 74.85.

5. Conclusion

The learning trajectory developed for sequence and series topic meets the criteria of validity, practicality, and effectivity. The result of this design development is beneficial in achieving the learning objectives of sequence and series topics. A learning trajectory that uses a realistic mathematics education approach creates an interactive and enjoyable learning environment. The RME approach is oriented to everyday problems, allowing students to understand the problems and determine the solution of the problem in their own way. Furthermore, this learning activity involved students in the mathematization process from horizontal to vertical mathematization. Through the mathematization process, students' mathematical problem solving abilities are exercised.

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