Cloaking a metal object from an electromagnetic pulse: A comparison between various cloaking techniques

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Abstract

Electromagnetic cloaks are devices that can be used to reduce the total scattering cross section of various objects. An ideal cloak removes all scattering from an object and thus makes this object “invisible” to the electromagnetic fields that impinge on the object. However, ideal cloaking appears to be possible only at a single frequency. To study cloaking from an electromagnetic pulse we consider propagation of a pulse inside a waveguide with a cloaked metal object inside. There are several ways to achieve cloaking and in this paper we study three such methods, namely, the coordinate-transformation cloak, the transmission-line cloak, and the metal-plate cloak. In the case of the two last cloaks, pulse propagation is studied using experimental data whereas the coordinate-transformation cloak is studied with numerical simulations. The results show that, at least in the studied cases where the cloaked object’s diameter is smaller than the wavelength, the cloaks based on transmission-line meshes and metal plates have wider bandwidth than the coordinate-transformation cloak.
I. INTRODUCTION

The possibility of electromagnetic cloaking, or, in other words, the reduction of an object’s total scattering cross section, has intrigued researchers for a long time. Already in the 1960s and 1970s a few publications considered this possibility.\(^1,2\) But it was not until a few years ago when the topic became widely studied among the electromagnetics and physics communities after the publication of papers \([3–6]\). Several methods can be used to achieve electromagnetic cloaking, all of them having their own benefits and drawbacks as can be concluded from recently published review papers on this topic.\(^7,8\) As has been previously concluded,\(^8\) some cloaking methods suffer from inherently strong dispersion that will unavoidably not only limit the cloaking bandwidth but also distort a pulse when it travels through the cloak. In this paper we make a comparative study of cloaking of a metal object from an electromagnetic pulse.

We study three cloaking methods: The coordinate-transformation (CT) cloak,\(^5,6\) the transmission-line (TL) cloak,\(^9–12\) and the metal-plate (MP) cloak.\(^13,14\) Due to the moderate dispersion in the TL and MP cloaks, we expect that the cloaking bandwidth is wider and pulse distortion is mitigated in these cloaks, as compared to the CT cloak. The cloaked object is limited in shape when using the TL cloak (a transmission-line network must fit inside the cloaked object) whereas the MP cloak and the CT cloak can be used to “hide” bulk objects, such as solid metal cylinders in this case.

For the study of pulse propagation in case of the TL and MP cloaks we use transmission data measured in a waveguide with a cloak and a cloaked object inside.\(^10,14\) In case of the CT cloak, we rely on numerical simulations of an idealized cloak having a radially varying permeability\(^6\) \(\mu\) and inserted inside a similar waveguide as the other cloaks. To have physically sound values for \(\mu\) as a function of the frequency, we assume that \(\mu\) has the Lorentzian type dispersion characteristics. Pulse propagation through CT cloaks that are placed in free space and are impinged on by plane waves has been studied analytically and numerically in several papers.\(^15–18\) Since in this study the cloaks are placed inside a waveguide with input and output ports, the distortion of the pulse travelling through the cloaks is very easy to illustrate and compare to the case of the empty waveguide.
II. STUDIED CLOAKING DEVICES

A. Transmission-line cloak

The transmission-line cloak consists of layered two-dimensional networks of parallel-strip transmission lines. The cloak’s outer diameter is approximately 71 mm while the cloaked object consists of 21 metal rods having the diameter 2 mm and the array period 5 mm. The largest outer dimension of the array in the $xy$-plane is 24.5 mm. It is obvious that due to the periodicity of the network an electrically large solid object cannot be cloaked with this method.

Here we use the measured data obtained with a rectangular waveguide having a passband around 3 GHz and the TL cloak enclosing the metal rod array placed inside the waveguide. The cloak is made by etching thin sheets composed of an alloy of bronze and beryllium. See Fig. 1a for an illustration of the measurement setup. The width and height of the waveguide are 86.36 mm and 36.8 mm, respectively and the distance between ports 1 and 2 is 389 mm.

Although the cloaked object in this case is a two-dimensional array of metal rods, it should be noted that with the same cloak also a more complicated three-dimensional object can be cloaked.

B. Metal-plate cloak

The metal-plate cloak consists of layered cylindrical metal plates that have a circular hole cut inside them (the volume which is cloaked). Due to the fact that the height of these waveguide plates decreases radially as moving from the outer surface of the cloak towards the inside, the electric field which is polarized orthogonally to the metal plates travels inside these waveguides around the cloaked region. At the inner surface of the cloak the fields cannot couple into the cloaked region since the fields are concentrated in a very small volume due to the small height of the waveguides. Note that this cloak operates for $z$-polarized electric fields in contrast to a similar cloak geometry which is based on a different design concept.

The inner and outer diameters of the MP cloak are 32 mm and 70 mm, respectively, whereas
the diameter of the cloaked cylinder is 30 mm. The plates of the cloak are made of copper and the cloaked cylinder is made of steel. The measurement data for the MP cloak is obtained with the same waveguide as for the TL cloak. See Fig. 1b for an illustration of the measurement setup.

C. Coordinate-transformation cloak

The coordinate-transformation cloak is formed of an anisotropic material layer, the material parameters of which are designed with the well-known cloak design equations that were used to design the first experimental demonstration of a cloak of this type. To obtain the needed values for \( \mu \) we design the cloak to have a radially dependent component of the permeability \( (\mu_\rho) \) having the optimal values at the design frequency of 3 GHz. See Fig. 1c for an illustration of the simulated structure. Suitable material parameters at the design frequency of 3 GHz are found to be

\[
es_z = \left( \frac{b}{b-a} \right)^2 - j0.002, \tag{1}
\]

\[
\mu_\varphi = 1, \tag{2}
\]

\[
\mu_\rho(\rho) = \left( \frac{\rho-a}{\rho} \right)^2 - j0.006, \tag{3}
\]

where \( \rho \) is the distance from the center of the cloaked object and \( a \) and \( b \) are the inner and outer radius of the cloak, respectively. The losses are modelled according to the experimental setup.

In a realization of such a cloak device, the parameters \( \epsilon_z \) and \( \mu_\varphi \) are constant, but the radial component of the permeability \( \mu_\rho \) must be dispersive (otherwise such values of \( \mu_\rho \) cannot be realized with a passive medium). Here we assume that \( \mu_\rho \) has the Lorentzian dispersion:

\[
\mu_\rho(\rho, f) = 1 - \frac{f_p^2}{f^2 - f_0^2 - j\Gamma f}, \tag{4}
\]
where \( f_p \) is the effective plasma frequency, \( f_0 \) the resonance frequency, and \( f_d \) the damping frequency.

This is a dispersion model typical for an artificial permeability realized with split-ring-resonators. To enable good operation of the cloak and mitigate the effect of dispersion, we position the resonance frequency \( f_0 \) at a much lower frequency than the operation frequency of the cloak. Note that this may not be possible in practical designs due to too large inclusion dimensions.

To satisfy the cloaking condition at the operating frequency of \( f_c = 3 \) GHz, we demand according to (3) that

\[
\mu_\rho(\rho, f_c) = \mu'_{\rho,f_c} - j\mu''_{\rho,f_c} = \left(1 - \frac{a}{\rho}\right)^2 - j0.006, \tag{5}
\]

from which it follows

\[
f_d = \frac{\mu''_{\rho,f_c}}{(1 - \mu'_{\rho,f_c})f_c} (f_c^2 - f_0^2), \tag{6}
\]

\[
f_p^2 = \frac{\mu''_{\rho,f_c}}{f_c f_d} \left[(f_c^2 - f_0^2)^2 + f_c^2 f_d^2\right]. \tag{7}
\]

To have a fair comparison with the other cloaks, here we use the same inner and outer dimensions as for the MP cloak, i.e., \( a = 16 \) mm (inner radius of the cloak) and \( b = 35 \) mm (outer radius of the cloak). The diameter of the cloaked perfectly conducting cylinder is also the same as for the MP cloak, i.e., 30 mm. With \( f_c = 3 \) GHz and \( f_0 = 1 \) GHz and using (3)–(7) we can find the values for the function \( \mu_\rho(\rho, f) \) and assign these values to be continuous in the simulation model of Fig. 1c. See Fig. 2 for the values of the real and imaginary parts of \( \mu_\rho \) as a function of the frequency for three different values of \( \rho \) inside the cloak. According to (1) \( \varepsilon_z \approx 3.39 - j0.002. \)
III. NUMERICAL VERIFICATION OF CLOAKING WITH THE STUDIED DEVICES

Before studying pulse propagation with the waveguide and cloak systems shown in Fig. 1, let us compare the cloaking capabilities of the studied devices using frequency domain simulations where the cloaks are placed in free space. We take the cloaks (and cloaked objects) described in Section II and model them numerically in free space. The cloaks are modelled as being infinitely periodic along the vertical ($z$) direction. By illuminating the cloaks with plane waves having the electric field parallel to the $z$-axis we can compute the total scattering cross sections of the cloaked and uncloaked objects.$^{10}$

The results for the TL cloak and the MP cloak have been already reported$^{10,14}$ and here we compare these with the results for the CT cloak. From Fig. 3 it is obvious that the CT cloak has a much narrower bandwidth as compared to the two other cloaks. This is an expected consequence of the dispersive nature of the permeability in the CT cloak, even though the resonance frequency of the permeability was chosen to be far away from the design frequency of 3 GHz. The minimum of the normalized total scattering cross section (SCS) occurs at 3.04 GHz for the CT cloak.

IV. MEASURED AND SIMULATED TRANSMISSION DATA

The experiments and the transmission measurements for the empty waveguide and the TL and MP cloaks shown in Fig. 1a,b have been already reported$^{10,11,14}$ demonstrating good transmission properties around 3 GHz. Here we compare these experimental results with the numerical results obtained for the CT cloak of the same dimension. The measured transmission magnitude and phase for the TL cloak and the MP cloak are presented in Fig. 4 and the simulated transmission magnitude and phase for the CT cloak are presented in Fig. 5. The simulations of the waveguide with the CT cloak inside are conducted with COMSOL Multiphysics.$^{21}$

The transmission magnitude for the CT cloak has several maxima. The maximum at 3 GHz relates to the designed optimal cloaking frequency, and the other maxima result from resonances inside the cloak. From the simulated field distributions (e.g., animating the time-harmonic
electric field as a function of the phase) this is obvious since at 3 GHz the electromagnetic wave that hits the cloak travels inside the cloak, whereas at the frequencies corresponding to the other maxima, the field oscillates inside the cloak. While the transmitted amplitude is close to 0 dB at the frequencies corresponding to these resonant maxima, the phase of the transmitted wave strongly deviates from the empty-waveguide case as shown in Fig. 5b.

The transmission phase difference between the cloaked and empty cases for various cloaks can be compared. For the TL cloak the phase lags as compared to the empty waveguide and this lag is growing with increasing frequency. This is expected since the wavenumber of waves travelling inside the TL cloak is not equal to the free-space wavenumber. \(^8,9\) In contrast, for the MP cloak at around 3 GHz the transmission phases for cloaked and empty cases are almost identical. Below and above this frequency the phase in the MP cloak has a small lead and lag, respectively.

For the CT cloak we would expect to have identical phase shift for cloaked and empty cases at the design frequency of 3 GHz. However, the matching of the transmission phase occurs at approximately 3.1 GHz. This can be a result of the simplification of the cloak’s material parameters\(^6\) or the influence of losses.

When comparing Figs. 4b and 5b it should noted that the electrical lengths from port 1 to port 2 are different in the two figures. This is because the measured results include the feed probes whereas the simulated results do not, see Fig. 1.

V. PULSE DISTORTION

Next we study pulse propagation inside the waveguides enclosing the different cloaks. This can be investigated analytically by using the frequency-domain transmission data. First we choose the center frequency of a Gaussian pulse \(f_{c,G}\) that we want to transmit through the waveguide and apply a Gaussian filter to create a time-domain signal \(y(t)\)

\[
y(t) = \sin(2\pi f_{c,G} t) e^{-\frac{t^2}{2\sigma^2}},
\]

(8)
where \( \sigma \) is the standard deviation and the center of the pulse corresponds to time \( t = 0 \). For \( f_{c,G} = 3 \text{ GHz} \) and \( \sigma = 7.07 \cdot 10^{-10} \) the pulse looks in the frequency and time domains as shown in Fig. 6.

We apply the Fourier transform to the time-domain pulse and multiply each frequency component by the complex transmission coefficients presented in Section IV. Then by applying the inverse Fourier transform we find the time-domain signal at the output of the waveguide section (at port 2). Fig. 7 presents these time-domain signals for all studied cloaks together with the signals at the output of an empty waveguide when using the input pulse of Fig. 6.

From Fig. 7 we can conclude that for the TL cloak and the MP cloak the pulse shape reasonably well corresponds to the pulse at the output port of an empty waveguide. For the CT cloak, however, the pulse is clearly distorted and there is a considerable “tail” because the energy inside the cloak travels slower than in free space (the slower the closer to the inner part of the cloak\(^8,16\)).

From the time-domain results in Fig. 7 it is difficult to define a numerical measure on how well or bad the cloaks work. Therefore, we calculate the correlation coefficients between the pulse which goes through the empty waveguide and the pulses which go through the cloaks. See Fig. 8 for the results as a function of the inverse of the standard deviation while \( f_{c,G} = 3 \text{ GHz} \) is kept constant. Small values of \( 1/\sigma \) correspond to pulses which are wide in the time-domain and narrow in the frequency domain, whereas large values of \( 1/\sigma \) correspond to pulses which are narrow in the time-domain and wide in the frequency domain.

All the cloaks work well for pulses which are narrow in the frequency domain. As the pulse bandwidth increases, the CT cloak’s operation deteriorates much faster than the operation of the other cloaks. This is expected since the CT cloak has a narrow bandwidth (see Fig. 3) and stronger dispersion as compared to the other cloaks.

Another interesting issue is to study how the choice of the center frequency of the pulse \( (f_{c,G}) \) affects the results. Here we use a constant standard deviation of \( \sigma = 7.07 \cdot 10^{-10} \) and plot the correlation coefficients as functions of \( f_{c,G} \), see Fig. 9. We see that the TL cloak works better for pulses for which the center frequency is significantly lower than the design frequency.
of 3 GHz while the MP cloak works best when the center frequency is approximately 2.9 GHz. This can be explained by Fig. 4b which shows that the optimal matching of the transmission phases of the empty waveguide and the TL cloak occurs at around 2.6 GHz while for the MP cloak the optimal frequency is approximately 2.9 GHz.

The CT cloak appears to work better for pulses with $f_{c,G}$ at frequencies slightly higher than the design frequency of 3 GHz. Again, this can be partly explained by the transmission phase shown in Fig. 5b according to which the optimal matching of the transmission phases for the empty waveguide and the CT cloak occurs at 3.1 GHz. Also the blueshift effect\textsuperscript{15} may cause the cloak to work better for pulses above the design frequency.

VI. CONCLUSIONS

Three different cloak designs have been studied and their operation compared with each other: The transmission-line cloak, the metal-plate cloak, and the coordinate-transformation cloak. The cloaking capabilities of these devices are first studied with numerical frequency-domain simulations of the cloaks in free space with plane waves illuminating the cloaks. This type of comparison enables the study of the reduction of the total scattering cross sections, but it does not give information about distortion of time-domain pulses.

To compare the cloaks’ operation for time-domain signals we use measured and simulated data for the transmission coefficients of rectangular waveguides with the cloaks (and cloaked objects) inside. With this transmission data we can analytically study how time-domain signals travel through the waveguides. It is shown that the transmission-line cloak and the metal-plate cloak work better than the coordinate-transformation cloak for pulses which are narrow in the time-domain. For pulses which are wide in time-domain (or, in other words, which are narrow in frequency domain) all the cloaks offer reasonably good performance.
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FIG. 1: A rectangular waveguide with a cloak and a cloaked object inside. (a) TL cloak – the cloaked object is an array of metal rods. (b) MP cloak – the cloaked object is a metal cylinder. (c) CT cloak – the cloaked object is a metal cylinder. The transmission data of structures in (a) and (b) are obtained experimentally whereas the structure in (c) is simulated (the simulation model is two-dimensional, i.e., the structure is assumed to be infinite along the $z$-axis).
FIG. 2: Frequency dependence of $\mu_\rho$. Solid lines: real part ($\mu'_\rho$), dashed lines: imaginary part ($\mu''_\rho$).

FIG. 3: Simulated total SCS of cloaked objects, normalized to the total SCS of uncloaked objects. The results for the TL cloak$^{10}$ and the MP cloak$^{14}$ are obtained with Ansoft HFSS$^{20}$ whereas the results for the CT cloak are obtained with COMSOL Multiphysics.$^{21}$
FIG. 4: (a) Magnitude and (b) phase of the transmission coefficient. Measured results.\textsuperscript{10,11,14}

FIG. 5: (a) Magnitude and (b) phase of the transmission coefficient. Simulated results. For the empty waveguide the magnitude is 0 dB in this frequency range (above cut-off waveguide with infinite height).
FIG. 6: Magnitude of the pulse at the input port (port 1) of the waveguide. (a) Frequency domain. (b) Time domain.
FIG. 7: Pulse shape in time domain at the output port of the waveguide. $f_{c,G} = 3$ GHz and $\sigma = 7.07 \cdot 10^{-10}$. Solid lines: empty waveguide, dashed lines: waveguide with a cloak inside. (a) TL cloak (measured transmission data used). (b) MP cloak (measured transmission data used). (c) CT cloak (simulated transmission data used).
FIG. 8: Correlation coefficients of the signals transmitted through cloaks and the signal through the empty waveguide. $f_{c,G} = 3$ GHz.

FIG. 9: Correlation coefficients of the signals transmitted through cloaks and the signal through the empty waveguide. $\sigma = 7.07 \cdot 10^{-10}$. 