QCD sum rule study of the masses of light tetraquark scalar mesons

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Abstract

We study the low-lying scalar mesons of light u, d, s flavors in the QCD sum rule. Having all possible combinations of tetraquark currents in the local form, QCD sum rule analysis has been carefully performed. We found that using the appropriate tetraquark currents, the masses of $\sigma$, $\kappa$, $f_0$ and $a_0$ mesons appear in the region of 0.6–1 GeV with the expected ordering. The results are compared with that of the conventional $\bar{q}q$ currents, where the masses are considerably larger.

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The light scalar mesons have been subject to intensive discussions for many years [1]. The expected members are $\sigma(600)$, $\kappa(800)$, $f_0(980)$ and $a_0(980)$ of flavor SU(3) nonet. The existence of $\sigma(600)$ which is denoted as $f_0(600)$ in the Particle Data Group has been confirmed also by a model independent theoretical analysis [2].

Yet, their nature is not fully understood [3,4]. Because they have the same spin and parity as the vacuum, $J^P = 0^+$, they may couple to many different modes. In the conventional quark model, they are $^3P_0$ state of $\bar{q}q$. Their masses are, however, expected to be larger than 1 GeV due to the $p$-wave orbital excitation. Furthermore, the mass ordering in a naive quark mass counting of $m_u \sim m_d < m_s$ implies $m_{\sigma} \sim m_{\kappa} < m_{a_0} < m_{f_0}$. In chiral models, they are regarded as chiral partners of the Nambu–Goldstone bosons ($\pi$, $K$, $\eta$, $\eta'$) [5]. Due to the collective nature, their masses are expected to be lower than those of the quark model. In chiral perturbation theories they are also described as resonances of meson–meson scattering, whose quark content is dominated by $(\bar{q}q)^2$ [6]. Recent discussions on the scalar mesons are then largely motivated by its tetraquark components.

Tetraquark structure of the scalar mesons was proposed long ago by Jaffe with an assumption of strong diquark correlations [7]. Due to the strong attraction in the scalar diquark channel, their masses are expected to be around 0.6–1 GeV with the ordering of $m_{\sigma} < m_{\kappa} < m_{f_0}, a_0$, consistent with the experimental observation [3,4,8]. The form of $\bar{q}q$–$\bar{q}q$ was also proposed [9]. Recent lattice study also showed an indication of the tetraquark for $f_0$ [10].

If such tetraquarks survive, they may be added to members of exotic multiquark states. The subject of the multi-quarks is important, providing a new opportunity to study colored dynamics which has not been reached by conventional hadrons [11]. The problem is also related to the origin of hadron mass as we will briefly discuss here.

In this Letter, we would like to report the results of a systematic study of the masses of the tetraquark scalar mesons in the QCD sum rule. In the QCD sum rule one extracts hadron properties from two-point correlation functions computed by the operator product expansion (OPE) of QCD. The non-perturbative effects are then incorporated by vacuum condensates of QCD operators. By comparing the theoretical correlation functions to...
phenomenological ones, one can determine physical quantities such as masses and coupling constants [12,13].

QCD sum rule analyses become subtle for hadrons containing more quarks such as tetraquarks and pentaquarks. Due to high dimensionality of the interpolating field, one needs to calculate many terms in OPE with high dimension. At the same time, it becomes difficult to find a good Borel window with keeping the convergence of OPE. Another point we would like to address here is the proper choice of hadronic currents (interpolating fields). When the OPE has to be in any way truncated up to certain terms, unless the current is suitably chosen, the resulting OPE and the sum rule may not work. In general, there are several independent currents for a given hadron state. The optimal current can be searched by making their linear combinations, as has been tested recently for the exotic tetraquark state [14].

Let us construct tetraquark currents of \( J^{PC} = 0^{++} \), by establishing the number of independent currents. Following the method in our previous work [14], we adopt the diquark construction, where the diquark and antidiquark have the same color, spin and orbital symmetries. Therefore, they must have the same flavor symmetry, which is either symmetric (\( 6\bar{q}q \)) or antisymmetric (\( 3\bar{q}q \)). Then we assume the ideal mixing in which only isospin symmetry is respected and the currents are classified by the number of strange quarks. Hence, denoting light u, d quarks by \( q \), \( \sigma \) currents are constructed as \( q\bar{q}\bar{q}\bar{q} \), \( \kappa \) currents by \( q\bar{s}\bar{q}\bar{s} \) and \( f_0 \) and \( a_0 \) currents by \( q\bar{s}\bar{q}\bar{s} \).

Using the antisymmetric combination for diquark flavor structure, we arrive at the following five independent currents

\[
S_3^\sigma = (u_T^a C \gamma_5 d_b) (\bar{u}_a \gamma_5 \bar{C} \bar{d}^T_b - \bar{u}_a \bar{C} \gamma_5 C d^T_b),
\]

\[
V_3^\sigma = (u_T^a C \gamma_1 \gamma_5 d_b) (\bar{u}_a \gamma_0 \gamma^\mu \gamma_5 \bar{C} \bar{d}^T_b - \bar{u}_a \bar{C} \gamma_0 \gamma^\mu C d^T_b),
\]

\[
T_3^\sigma = (u_T^a C \gamma_3 d_b) (\bar{u}_a \gamma_0 \gamma^\mu \gamma_5 \bar{C} \bar{d}^T_b + \bar{u}_a \bar{C} \gamma_0 \gamma^\mu C d^T_b),
\]

\[
A_2^\sigma = (u_T^a C \gamma_2 d_b) (\bar{u}_a \gamma_1 \gamma^\mu \gamma_5 \bar{C} \bar{d}^T_b + \bar{u}_a \bar{C} \gamma_1 \gamma^\mu C d^T_b),
\]

\[
P_3^\sigma = (u_T^a C \gamma_1 d_b) (\bar{u}_a \gamma_3 \gamma^\mu \gamma_5 \bar{C} \bar{d}^T_b - \bar{u}_a \bar{C} \gamma_3 \gamma^\mu C d^T_b),
\]

where the sum over repeated indices (\( \mu, \nu, \ldots \) for Dirac, and \( a, b, \ldots \) for color indices) is taken. Either plus or minus sign in the second parentheses ensures that the diquarks form the antisymmetric combination in the flavor space. The currents \( S, V, T, A \) and \( P \) are constructed by scalar, vector, tensor, axial-vector, pseudoscalar diquark and antidiquark fields, respectively. The subscripts 3 and 6 denote the color states of the diquarks (antidiquarks) which are combined into the color representation \( 3_c \) and \( 6_c \) (\( 3_c \) or \( 6_c \)), respectively. The currents for other members are formed by the following replacements in (1), \( \kappa: (u\bar{d})(\bar{u}\bar{d}) \to (u\bar{d})(\bar{u}\bar{d}), f_0: (u\bar{d})(\bar{u}\bar{d}) \to (u\bar{s})(\bar{u}\bar{s}) + (u \leftrightarrow d), a_0: (u\bar{d})(\bar{u}\bar{d}) \to (u\bar{s})(\bar{u}\bar{s}) - (u \leftrightarrow d) \). More details will be discussed in a separate publication [15].

Using the tetraquark current \( q \) which is one of the currents of (1) or their linear combination, we have computed the correlation function

\[
\Pi_{OPE}(q^2) \equiv i \int d^4 \epsilon \epsilon^{q\epsilon} (0) [T(q)(q\epsilon) (0)(0)],
\]

in the OPE up to dimension eight, keeping the current quark masses \( m_u, m_d \) and \( m_s \) finite. As the primary requirement, the spectral densities \( \rho_{OPE} \) must be positive definite. If truncation of OPE is not good, it happens that they become negative sometimes. Using the dispersion relation, it is equivalently written as

\[
\rho_{OPE}(s) \equiv \Im \Pi_{OPE}(s),
\]

where \( \rho_{OPE} = \Im \Pi_{OPE}/\pi \). This is then equated to the integral over the physical (phenomenological) spectral density \( \rho_{phen}(s) \).

The phenomenological spectral density is parameterized as a sum of one pole and continuum contributions. Assuming that the continuum part is approximated by the one of OPE (duality) [13],

\[
\rho_{phen}(s) = f^2 \delta(s - M^2) + \theta(s - s_0) \frac{1}{\pi} \Im \Pi_{OPE}(s),
\]

where \( M \) and \( f \) are the mass and coupling constant of the physical state under investigation. In order to extract physical quantities efficiently by suppressing the continuum contribution, the Borel transformation is performed. Finally we arrive at the sum rule equation

\[
f^2 \exp(-M^2 / M_B^2) = \int_0^{s_0} ds \rho_{OPE}(s) e^{-s/M_B^2},
\]

which determines the mass and the coupling constant. The mass \( M \) is a function of the two parameters \( s_0 \) and \( M_B \). They must be chosen to satisfy (1) rapid convergence of OPE, (2) sufficient amount of pole contribution and (3) weak dependence on \( s_0 \) and \( M_B \). These are important in order to draw reliable conclusions [16].

The use of the \( \delta \)-function in (4) for the scalar mesons might be subject to criticisms, firstly because the observed scalar mesons have wide decay width. The inclusion of a finite width in the QCD sum rule instead of using the \( \delta \)-function is one option to take care of the effect of the decay width. We have performed such analysis in the form of Gaussian and verified that still it is possible to reproduce experimental values of masses [15].

Secondly, the tetraquark currents are expected to couple strongly to two meson states. We argue that such two meson contributions can be computed separately from the short distance method of OPE. By applying the soft-pion theorem [17], the coupling to two meson states can be expressed by a double commutator with the axial charge \( Q_5, (0|\eta|\pi^a \pi^b) \sim (0|[Q_5, [Q_5, \eta]])|0) \). Since one commutator yields the factor \( \bar{q}q \), we find \( \langle \bar{q}q \rangle^4 \) altogether in the two-point correlation function. The dimension of this term is as high as twelve, which is beyond the present study where we compute up to dimension eight. Similarly, as shown in Ref. [18] which also uses the method of QCD sum rule, the coupling of \( s \to pp \) (\( s \) stands for a scalar tetraquark and \( p \) pseudoscalar meson) is of higher order as proportional to \( \langle \bar{q}q \rangle^2 \), consistently implying that the decay width of the \( s \to pp \) is of order \( \langle \bar{q}q \rangle^4 \).
As we will see shortly, the fact that the present QCD sum rule with the OPE up to dimension eight will yield a stable solution indicates that there is a significant component in the scalar meson state which couples to the tetraquark current without going through two mesons. In this case, we expect that the narrow resonance approximation is reliable. The large decay width will then be explained through the coupling to two meson states in the form of high dimension terms of OPE. In fact, we have performed a QCD sum rule analysis by using a peak of finite width, and found that the result does not change much.

We have performed the sum rule analysis using all currents and their various linear combinations. We have found that the results for single currents are not reliable, except for the tensor current $T_0^\rho$, due to either violation of positivity or insufficient convergence of OPE. In fact, we have found good sum rule by a linear combination of $A_0^\rho$ and $V_3^\rho$, where the best choice of the mixing angle turns out to be $\cot \theta = 1/\sqrt{2}$. For $\kappa$, $f_0$ and $a_0$, we have also found that similar linear combinations give better sum rules.

The calculation of OPE is tedious but straightforward. The results up to dimension eight are

\[
\rho^\sigma(s) = \frac{s^4}{11520\pi^6} + \frac{(6\sqrt{2} + 7)(g^2GG)s^2}{9216\pi^6} + (m_u + m_d)\langle \bar{q}q \rangle \left( \frac{s^2}{36\pi^4} + \frac{(6\sqrt{2} + 1)(g^2GG)}{1152\pi^4} \right) + O(m_q^2),
\]

\[
\rho^f(s) = \frac{s^4}{11520\pi^6} - \frac{m_s^2}{572\pi^6} s^3 + \frac{6\sqrt{2} + 7}{9216\pi^6} \left( g^2GG + \frac{m_s}{72\pi^4} \langle \bar{s}s \rangle \right) s^2 + \frac{6\sqrt{2} + 7}{3072\pi^6} m_s^2 g^2GG + \frac{m_s}{128\pi^4} \langle \bar{s}s \rangle \left( \frac{6\sqrt{2} + 7}{48\pi^4} \right) + \frac{\langle \bar{q}q \rangle}{48\pi^4} \left( \frac{6\sqrt{2} + 7}{2304\pi^4} m_s g^2GG \right) + O(m_q^2),
\]

\[
\rho^{f_0}(s) = \frac{s^4}{11520\pi^6} - \frac{m_s^2}{288\pi^6} s^3 + \frac{6\sqrt{2} + 7}{9216\pi^6} \left( g^2GG + \frac{m_s}{36\pi^4} \langle \bar{s}s \rangle \right) s^2 + \frac{6\sqrt{2} + 7}{1536\pi^6} m_s^2 g^2GG - \frac{m_s^2}{6\pi^4} \langle \bar{s}s \rangle s + \frac{m_s^2 g^2GG}{92\pi^4} + \frac{4m_s^2}{9\pi^4} \langle \bar{q}q \rangle^2 + \frac{4m_s^2}{9\pi^4} \langle \bar{s}s \rangle + \frac{6\sqrt{2} + 7}{1152\pi^4} m_s g^2GG \langle \bar{s}s \rangle.
\]

The OPE for $a_0$ takes the same form as for $f_0$. For $\sigma$, terms containing $u,d$ quark masses $m_q$ are small. For instance, the term of $m_q\langle \bar{q}q \rangle$ of dimension four is about ten times smaller than the other term of $g^2GG$. For $\kappa$, $a_0$ and $f_0$, the terms containing strange quark mass are important but those containing $u$ and $d$ quark masses are negligibly small.

We use the following values of condensates [19-21]:

\[
\langle \bar{q}q \rangle = -(0.240 \text{ GeV})^3, \langle \bar{s}s \rangle = -(0.8 \pm 0.1) \times (0.240 \text{ GeV})^3, \langle g^2GG \rangle = (0.48 \pm 0.14) \text{ GeV}^4, m_u = 5.3 \text{ MeV}, m_d = 9.4 \text{ MeV}, m_s(1 \text{ GeV}) = 125 \pm 20 \text{ MeV}, \langle g_s\bar{q}q \rangle = -M_0^2 \langle \bar{q}q \rangle, M_0^2 = (0.8 \pm 0.2) \text{ GeV}^2.
\]

Now let us discuss the feasibility of our QCD sum rule.

The Borel transformed correlation functions are written as power series of the Borel mass $M_B$. Since the Borel transformation suppresses the contributions from $s > M_B^2$, smaller values are preferred to suppress the continuum contributions. However, for smaller $M_B$ convergence of the OPE becomes worse. Therefore, we should find an optimal value of $M_B$. We have found that $M_B \sim 0.4 \text{ GeV}$ for $\sigma$, $0.5 \text{ GeV}$ for $\kappa$ and $0.8 \text{ GeV}$ for $f_0$ and $a_0$, where the pole contributions reach around 50% for all cases, while the convergence is still sufficiently fast [15]. As $M_B$ is increased, the pole contributions decrease, but the resulting tetraquark masses are stable as shown in Fig. 1.

We have also searched the region where the tetraquark mass varies significantly less than the change in $\sqrt{s_0}$. We have found such regions $0.5 < s_0 (\text{GeV}^2) < 1.5$ for $\sigma$, $1 < s_0 < 2$ for $\kappa$, and $1.5 < s_0 < 2.5$ for $f_0$ and $a_0$. In Fig. 2, we show $s_0$ dependence of the masses in these regions. As we see, the mass is stable in a rather wide region of $s_0$. The Borel mass dependence of Fig. 1 is shown for the minimum values of $s_0$.  

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*Fig. 1. Masses of the $\sigma$ (short-dashed), $\kappa$ (solid), $f_0$ and $a_0$ (long-dashed) mesons calculated by the tetraquark currents as functions of the Borel mass $M_B$, with $s_0$ (GeV$^2$) as shown in figures.*
After careful test of the sum rule for a wide range of parameter values of $M_B$ and $s_0$, we have found reliable sum rules, with which we find the masses $m_\sigma = (0.6 \pm 0.1) \text{ GeV}$, $m_\kappa = (0.8 \pm 0.1) \text{ GeV}$, $m_{f_0(500)} = (1 \pm 0.1) \text{ GeV}$. It is interesting to observe that the masses appear roughly in the order of the number of strange quarks with roughly equal splitting.

It would be interesting to observe from Eqs. (6)–(8) that the mass of the $\sigma$ is dominated by the gluon condensate, while other condensates with $m_t$ also play a significant role for other masses. In fact in the SU(3) limit, where all quark masses and condensates take the same values, the three equations become identical. In particular, in the chiral limit where all quark masses vanish, the masses of the scalar mesons are dictated only by the gluon condensate. This property was also observed in a recent publication [22].

Now for comparison, we have also performed the QCD sum rule analysis using the $\bar{q}q$ current within the present framework, although such works have been done before [23–26]. We have computed the OPE up to dimension six in this case, and have verified the previous results. Namely, the masses of the $\bar{q}q$ mesons are considerably heavier than the masses of the tetraquark mesons.

In conclusion we have found that the QCD sum rule analysis with tetraquark currents implies the masses of scalar mesons in the region of 600–1000 MeV with the ordering, $m_\sigma < m_\kappa < m_{f_0(500)}$, while the conventional $\bar{q}q$ currents are considerably heavier. Our conclusion has become rather robust, after we have tested all possible independent tetraquark currents and with their linear combinations.

Our observation supports a tetraquark structure for low-lying scalar mesons. Somewhat non-trivial is that a large part of the mass is due to the gluon condensate rather than chiral condensate. This observation is interesting in relation to the question of the origin of the mass generation of hadrons [27]. To test the validity of the tetraquark structure, it is also important to study decay properties, which is often sensitive to the structure of wave functions. Such a tetraquark structure will open an alternative path toward the understanding exotic multiquark dynamics which one does not experience in the conventional hadrons.

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