Preliminary study of a bridge abutment settlement considering spatial variability of soil properties

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Abstract. The paper focuses on two-dimensional (2D) analyses of a bridge abutment based on a spatially variable soil. The analysis was carried out to evaluate the settlement of the bridgehead. The numerical model consists of a reinforced concrete support structure, embankment and natural soil. Moreover, frictional contact between the soil and the structure is assumed. As it is well known, the natural soils are spatially highly variable materials, and their mechanical and physical parameters in geotechnical designing, such as a bridge abutment, must be considered variable. Thus, the presented analyses will be devoted to determining the settlement of the structure under study, in probabilistic terms due to soil variability. To this end, Sequential Gaussian Simulation was used to describe the spatial variability of soil parameters. This approach uses geostatistical methods to produce different realizations of soil parameters. A data set of several Cone Penetration Tests (CPT) was used. Simulations were used to calculate the histogram of a bridgehead settlement and search the worst case. Abaqus program and some ad hoc scripts written in Python language were used to perform the analyses. The analysis reflects several stages of abutment’s construction. This allows for a comprehensive analysis of a bridge abutment.

1. Introduction
Contemporary bridge structures are putting high demands on their designers. It also concerns their foundations, and in particular their abutments. Consequently, the analysis of the abutment reliability is an important element of the bridge safety assessment. There are only few papers in literature dedicated to the reliability of bridge abutments. The paper by Biernatowski and Puła [1], in which the authors treated the issue in the form of system reliability, can be considered a pioneering work in this field. The components of this system were the five types of abutment failures, namely: horizontal displacement of the abutment, rotation around the lower edge of the base, rotation along a cylinder surface, landslide along the most dangerous slip surface, and exceedance of bearing capacity of subsoil due to soil displacement from under the base of the abutment. The traditional Monte Carlo method was used to evaluate failure probabilities in this article [1]. It is clear that a bridge abutment can be analyzed using some methods typical for retaining structures. As examples of works using probabilistic methods to analyze the safety of retaining walls, the papers by Fenton and others [6], Griffiths and others [7], as well as Zevgolis and Bourdeau [10] can be mentioned. The first two works used a random finite element method (RFEM) to assess reliability (Fenton and Griffiths [5]). In [10], however, approaches analogous to that in [1] were used, i.e., individual types of possible failures were analyzed, using the Monte Carlo method to determine the similarities. However, the consideration of
various failure cases, although it has often been used so far in design assumptions, is a considerable simplification, owing to the coupling between individual types of failure. Therefore, the authors undertook a comprehensive study of the issue, which includes the following elements: numerical modeling of abutment displacements, subjected to various loads at various stages of the construction process; taking into account of the random nature of spatial variability of soil properties; including of elastic soil models; probabilistic estimation of Young’s modulus by simulation methods, and adjustment to reliability-based design.

The current paper presents the initial phase in which the construction of the geostatistical model \cite{2, 3, 4} is based on the results of CPT testing. Next, a numerical model was developed and exemplary calculations of abutment settlement were performed, leading to the assessment of probability of subsidence exceeding the allowable level.

2. Materials and methods

2.1. Methodology

Soil in its natural state is one of the most variable materials. Assessment of spatial variability of soil properties crucial in a safety evaluation of any foundation and consequently of the entire structure. As soil parameters vary, any sampling will be partial and then it will be necessary to estimate soil parameters in the unsampled locations. For this purpose, within the current work, geostatistical simulation was used. Figure 1 shows the basic steps for creating a set of maps of soil parameters.

![Diagram](image)

**Figure 1.** Steps of creating different realization maps of soil parameters.

2.1.1. **Step 1 – Exploratory Data Analysis.** The data set contains nine Cone Penetration Tests (CPT). CPT tests were made in southwestern Poland. CPT tests consist of the cone resistance $q_c$ and of sleeve friction $f_s$ data each measure every two centimetres of depth. The spacing between CPT test points in horizontal direction is 5.8 m. Figure 2 and Figure 3 show the base maps of $q_c$ and $f_s$. 
Figure 2. The base map of cone resistance $q_c$ data.

Figure 3. The base map of sleeve friction $f_s$ data.

Figure 4 and Figure 5 present the histograms of $q_c$ and $f_s$ data. The data distributions of the sample values are strongly skewed, showing clear deviations from normal distribution.

Figure 4. Histogram of the data $q_c$.

Figure 5. Histogram of the data $f_s$. 
2.1.2. Step 2 – Gaussian transformation. From this point forward, the further procedures will be presented only for the data $f_s$, being the same for $q_c$. As the histogram of $f_s$ is skewed, transforming the raw distribution into a normal one is required if a parametric approach is preferred. To this end, we used Gaussian anamorphosis [3]. Anamorphosis is a mathematical function that transforms the standardised Gaussian variable into a variable with any type of distribution. Figure 6 shows the graph of the anamorphosis function fitted to the experimental data. The fitted model is a truncated expansion of Hermit polynomials.

![Graph of the experimental anamorphosis with the fitted model of the sleeve friction data.](image)

Figure 6. Graph of the experimental anamorphosis with the fitted model of the sleeve friction data.

In the present study both $f_s$ and $q_c$ data have been transformed by using a finite expansion of 100 Hermite polynomials. The histogram of Gaussian transformed data of $f_s$ reproduces the Gaussian probability density function quite well.

![Histogram of the normal transformed $f_s$ data.](image)

Figure 7. Histogram of the normal transformed $f_s$ data.
Step 3 – Calculation of the experimental variogram and fitting of the variogram model. The next step is calculation of the experimental variogram. Variogram is a measure of spatial variance of observations. The directional experimental variogram was calculated in two directions - horizontal and vertical. Figure 8 shows the experimental variogram with the fitted model for the normal transformed \( f_s \) data. The theoretical model included a nugget effect, a spherical model with range equal to 0.70 m and a K-Bessel model with scale equal to 2.20 m and parameter equal to 13.00, for the horizontal and vertical direction, respectively. It is worth underlining that “The choice of variogram model is very important because each type yields quite different values for the nugget variance and range, both of which are critical parameters for interpolation” [2]. For this reason, model was tested with cross-validation.

![Experimental variogram with the fitted model](image)

**Figure 8.** Experimental directional variogram with the fitted model for the normal \( f_s \) data.

2.1.3. Step 4 – Sequential Gaussian Simulation. The simulation realizations were generated using Sequential Gaussian Simulation [3, 4], which involves, after fitting a variogram model, to define a regularly spaced grid covering the region of interest and for each realization to draw a random path through the grid, such that each node is visited only once. Figure 9 shows one of the realizations of \( f_s \) as an example. Three hundred simulations were performed. The interpolation grid was defined in 2 dimensions. In each direction the mesh has a dimension 0.50 m. The map is 15.0 m deep and 52.0 m wide.

![Simulation realization of \( f_s \)](image)

**Figure 9.** One map of the realizations of the \( f_s \) parameter.
2.2. Transformation of CPT data into Young’s modulus

Young’s modulus of soil has the greatest impact on the bridge abutment settlement and therefore this parameter was calculated soil attributes. Figure 10 shows the flowchart of CPT data transformation into Young’s modulus. Figure 10 a and b present one of the realizations of \( q_c \) and \( f_s \). The procedure for creating these maps was shown in the previous paragraph. Next step was the calculation of the so-called Soil Behavior Type (SBTn) Index \( I_c \) proposed by Robertson and Wride [9]:

\[
I_c = ((3.47 - \log Q_{tn})^2 + (\log F + 1.22)^2)^{1/2}
\]  \hspace{1cm} (1)

where:
- \( Q_{tn} \) – normalized cone penetration resistance (dimensionless),
- \( F \) – Normalized friction ratio, in %.

Normalized cone penetration and normalized friction ratio are computed according to the following equations:

\[
Q_{tn} = \left( \frac{q_t - \sigma_{v0}}{\sigma_{atm}} \right) \left( \frac{\sigma_{atm}}{\sigma_{v0}} \right)^n
\]  \hspace{1cm} (2)

\[
F = \frac{f_s}{q_t - \sigma_{v0}} \times 100\%
\]  \hspace{1cm} (3)

where:
- \( q_t \) – total tip resistance,
- \( f_s \) – sleeve friction,
- \( \sigma_{v0} \) – total vertical stresses,
- \( \sigma'_{v0} \) – effective vertical stresses,
- \( \sigma_{atm} \) – atmospheric pressure (100 kPa),
- \( n \) – stress exponent.

The following table shows the correspondence between \( I_c \) values and soil type (SBTn) classes.

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
Soil classification (SBTn) & Zone number & SBT index values \\
\hline
Organic soils: peats & 2 & I_c>3.60 \\
Clays: silty clay to clay & 3 & 2.95<I_c<3.60 \\
Silt Mixtures: clayey silt to silty clay & 4 & 2.60<I_c<2.95 \\
Sand Mixtures: silty sand to sandy silt & 5 & 2.05<I_c<2.60 \\
Sands: clean sand to silty sand & 6 & 1.31<I_c<2.05 \\
Gravelly sand to dense sand & 7 & I_c<1.31 \\
\hline
\end{tabular}
\end{center}

Figure 10 (c) shows a derived map of soil classes (SBTn) based on the maps of \( q_c \) and \( f_s \) and the equation (1).

Finally, Young’s modulus is computed according to the following equations:

\[
E = \alpha_E (q_t - \sigma_{v0})
\]  \hspace{1cm} (4)

where:

\[
\alpha_E = 0.015 \left( 10^{0.55I_c+1.68} \right)
\]  \hspace{1cm} (5)

In Figure 10 (d) a derived map of Young’s modulus from one realization of \( q_c \) and \( f_s \) is shown.
Figure 10. Flowchart of CPT data transformation into one realization of Young’s modulus.

Clearly at the varying of the realizations of qc and fs the 300 derived realizations of E₀ were obtained according to the calculation flowchart previously described Figure 10.

2.3. Numerical model

As it has already been mentioned earlier, the present study is the initial part of a larger project which is currently under development. In this work the authors limit themselves to settlement analysis of an abutment imbedded into an elastic and variable subsoil. The numerical model of the bridge abutment, presented in Figure 11a, consists of reinforced concrete support structure, embankment and natural soil. Furthermore, frictional contact was modeled between the bridgehead and soil. The dimensions of the elements are 0.5 m x 0.5 m. The material of the elements is elastic.

| Table 2. The basic parameters of the numerical model |
|-----------------------------------------------------|
| Parameter                                          | Value     |
| Young’s modulus of embankment                       | 80.0 MPa  |
| Young’s modulus of bridgehead                       | 30.0 GPa  |
| Poisson’s ratio                                     | 0.20      |
| Mass density of embankment and soil                 | 1850 kg/m³|
| Mass density of bridgehead                          | 2500 kg/m³|
| Abutment height                                     | 10.0 m    |
| Bridgehead foundation width                         | 6.0 m     |
The grid for the numerical model is the same used in stochastic simulations. As a result of this, it was possible to assign Young’s modulus from the stochastic simulations to the numerical model, which is shown in Figure 11.

Further construction stages were included in the analysis of the abutment. This allows for a comprehensive analysis of a bridge abutment. Figure 12 shows the selected construction stages of the bridge abutment. In the last stage a load was applied. Backfill load is equal to 25 kPa and bridge seat load to 400 kN/m. The computations were done in the system Abaqus.

Figure 11. Numerical model output with one of Young’s module realizations.

Figure 12. Selected construction stages of the bridge abutment using one of $E_0$ realizations.
3. Results

Figure 13 presents the result of a single simulation, which shows the vertical displacement of the bridge abutment. In this paper, the displacement was checked at the point (A) where the bridge is connected to the abutment.

![Figure 13. Example of vertical displacement results in meters. At point A displacement was checked.](image)

From the outcomes of the numerical model 300 realizations of U2 (vertical displacement) were provided. Figure 14 presents the iterative process of calculating U2 at point A.

![Figure 14. The iterative process of calculating U2 at point A.](image)

This allowed to create a histogram of the vertical displacements of the bridgehead at each node of the grid, as it is presented in Figure 15 the vertical displacement at point A. The mean value is equal to 0.021 m and standard deviation is equal to 0.001 m. Moreover, the minimum and maximum values are equal to 0.023 m and 0.020 m, respectively. This means that the displacements are concentrated in
a relatively narrow range. For a bridge designer, the minimum value is the most important information in this case. The histogram is the answer to what is the impact of soil parameters variability on vertical displacements at point A.

![Histogram of the vertical displacement at point A.](image)

**Figure 15.** Histogram of the vertical displacement at point A.

4. Conclusions
This paper has shown how to combine geostatistical stochastic simulation with numerical modelling using finite element methods. Thus, it was possible to assess the influence of soil variability on abutment displacement. Information about the settlement of a bridgehead is very important when designing this type of structure. It gives basic information for serviceability limit state analysis. During the development of the project the authors intend to incorporate more advanced models of subsoil, e.g., a small strain hardening soil model. Then the analysis will be extended to bearing capacity evaluations. An efficient evaluation of failure probabilities, for both ultimate as well as serviceability limit states, allows to use the system under consideration in the process of reliability-based design of bridge abutment.

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