Transverse momentum distribution in hadrons.
What can we learn from QCD?

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Abstract

We discuss some QCD constraints on light-cone $\pi$ meson wave function $\psi(\vec{k}^2_\perp, x)$. The analysis is based on such general methods as dispersion relations, duality and PCAC. We calculate the asymptotical behavior of the wave function ($wf$) at the end-point region ($x \rightarrow 1$ and $\vec{k}^2_\perp \rightarrow \infty$) by analysing the corresponding large $n$–th moments in transverse $\langle \vec{k}^{2n}_\perp \rangle \sim n!$ and longitudinal $\langle (2x-1)^n \rangle \sim 1/n^2$ directions. This information fixes the asymptotic behavior of $wf$ at large $\vec{k}^2_\perp$ (which is turned out to be Gaussian commonly used in the phenomenological analyses).

We discuss one particular application of the obtained results. We calculate the nonleading “soft” contribution to the pion form factor at intermediate momentum transfer. We argue, that due to the specific properties of $\psi(\vec{k}^2_\perp, x)$, the corresponding contribution can temporarily simulate the leading twist behavior in the extent region of $Q^2$: $3\text{GeV}^2 \leq Q^2 \leq 40\text{GeV}^2$, where $Q^2 F(Q^2) \sim \text{const}$. Such a mechanism, if it is correct, would be an explanation of the phenomenological success of the dimensional counting rules at available, very modest energies for many different processes. We discuss some inclusive amplitudes (like Drell Yan and Deep Inelastic) where intrinsic $\pi$ meson structure might be essential. The relation to the valence quark model is also discussed.

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1. Introduction

The main goal of the present paper is the study of the hadronic wave functions with a minimal number of constituents within QCD. To be more specific, we are mainly interested in the $\vec{k}_\perp^2$– behavior of the light cone $wf$ in the transverse direction.

The motivation for this interest is the following. As is known, at asymptotically high energies the parametrically leading contributions to hard exclusive processes can be expressed in terms of the so-called distribution amplitudes $\phi(x)$, which itself can be expressed as an integral $\int \psi(\vec{k}_\perp^2, x) d^2k$ with nonperturbative wave function $\psi(\vec{k}_\perp^2, x)$, see review [1]. Distribution amplitudes $\phi(x)$ depend only on longitudinal variables $x_i$ and not on transverse $\vec{k}_\perp^2$ ones. The same is true for inclusive reactions where structure functions depend on $x$, but not on $\vec{k}_\perp^2$. Thus, any dependence on $\vec{k}_\perp^2$ gives some power corrections to the leading terms. Naively one may expect that these corrections should be small enough already in the few $GeV^2$ region. However, as we argue later, this expectation does not seem work well in intermediate region. Thus, one can say that we study the pre-asymptotic behavior of the exclusive amplitudes.

We shall find that $\psi(\vec{k}_\perp^2, x)$ possesses the quite unusual properties, which lead to the broadening $wf$ in the transverse directions. In terms of observable amplitudes it means that the characteristic scale is not $\sim 1GeV^2$ (as naively one could expect), but $10GeV^2$. What is more important, this scale is not universal, but varies from process to process.

The analysis of some inclusive reactions shows the same result– very often the experimental data can not be explained within the standard scale-invariant description. An explicit introduction of some hadronic (phenomenological) dimensional parameters into the structure functions is often required. Due to the fact that the structure functions can be expressed in terms of the same wave functions as $\sim \int |\psi(\vec{k}_\perp^2, x)|^2 d^2k$, we believe that the similar conclusion (on importance of the pre-asymptotic behavior in the intermediate region) takes place for the inclusive amplitudes as well.

As the simplest application of our $wf$ we consider the $\pi$ meson form factor. Before to explain the qualitative results obtained in this paper, we would like to review a few important steps which have been taken in the investigation of the exclusive amplitudes. We hope that this short historical introduction will help to formulate the problem we want to address in the present analysis.

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3 The formal reason for that can be seen from the following arguments. At large energies the quark and antiquark are produced at small distances $z \sim 1/Q \to 0$, where $Q$ is typical large momentum transfer. Thus one can neglect the $z^2$ dependence everywhere and one should concentrate on the one variable $zQ \sim 1$ which is order of one. One can convince oneself that the standard Bjorken variable $x$ is nothing but Fourier conjugated to $zQ$.  

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In early seventies the famous \textit{dimensional counting rules} were proposed \cite{2}. The predictions of these rules agree well with the experimental data, such as the pion and nucleon form factors, large angle elastic scattering cross sections and so on. This agreement served as a stimulus for further theoretical investigations. The modern approach to exclusive processes was started in the late seventies and early eighties \cite{3}. We refer to the review papers \cite{1},\cite{4},\cite{5} for details.

The main idea of the approach \cite{3} is the separation of the large and small distance physics. At small distances we can use the standard perturbative expansion due to the asymptotic freedom and smallness of the coupling constant. All nontrivial, large distance physics is hidden into the nonperturbative \textit{wf} in this approach. It can not be found by perturbative technique, but rather should be extracted from elsewhere. The most powerful analytical nonperturbative method for such problems is the QCD sum rules \cite{6}, \cite{7}.

The first application of QCD sum rules to the analysis of nonperturbative \textit{wf} was considered more than decade ago \cite{8}. Since then this subject is a very controversial issue \cite{9}-\cite{22} and we are not going to comment these quite opposite points in the present note.

At the same time, the applicability of the approach \cite{3} at experimentally accessible momentum transfers was questioned \cite{14},\cite{13}. In these papers it was demonstrated, that the perturbative, asymptotically leading contribution, is much smaller than the nonleading (”soft”) contributions. Similar conclusion, supporting this result, came from the different side, from the QCD sum rules, \cite{20},\cite{21}, where the direct calculation of the form factor has been presented at $Q^2 \leq 3 GeV^2$. This method, has been extended later for the larger $Q^2 \leq 10 GeV^2$ \cite{14},\cite{22} with the same qualitative result: the soft contribution is more important in this intermediate region than the leading one.

Now we are ready to formulate the question, which we want to address in the present paper.

- If the asymptotically leading contribution can not provide the experimentally observable absolute values, than \textit{how can one explain the very good agreement between the experimental data and dimensional counting rules} \cite{4}, which are supposed to be valid only in the region where the leading terms dominate?

It is clear, that the possible explanation can not be related to the specific amplitude, but instead, it should be connected, somehow, to the nonperturbative wave functions of the light hadrons ($\pi, \rho, p...$) which enter the formulae for exclusive processes. The analysis of the $\pi$ meson form factor, presented below supports this idea.

\footnote{These rules unambiguously predict the dependence of amplitudes on dimensional parameters. In particular, $Q^2 F_\pi(Q^2) \approx \text{constant}.$, $Q^3 F_p(Q^2) \approx \text{constant}.$, $s^2 \frac{d^2}{dt^2}(\gamma p \to \pi^+ n) = f(t/s)$. The experimental data are in a good agreement with these predictions in the large region of $s, Q^2$ at very modest energy and momentum transfer.}
To anticipate the events we would like to formulate here the result of this analysis. The very unusual properties (which will be derived from QCD and not from quark model) of the transverse momentum distribution of the non-perturbative $\pi$ meson wave function lead to the temporarily simulation of the dimensional counting rules by soft mechanism for the $F_\pi(Q^2)$ at the extent range of intermediate momentum transfer: $3 GeV^2 \leq Q^2 \leq 40 GeV^2$. In this region the soft contribution to $F_\pi(Q^2)Q^2$ does not fall-off, as naively one could expect, and we estimate it as $F_\pi(Q^2)Q^2 \simeq 0.3 / 0.4 GeV^2$. The leading twist contribution, after Sudakov suppression, gives, according to [15], [16], [22] a little bit less ($\leq 0.2 GeV^2$).

- Therefore, our answer on the formulated question is the following. The nonperturbative wave functions possess (along with the standard small scale $\vec{k}_{\perp}^2 \sim (330 MeV)^2$) the new, larger scale ($\sim 1 GeV^2$). Precisely this new scale defines the regime where the asymptotical formulae start to work.

We believe that the same features of the $w f$ may affect the analysis of inclusive amplitudes as well where some pre-asymptotic effects might be essential.

The paper is organized as follows. In the next section we define the nonperturbative $w f$ through its moments. We focus on the properties of the two particle leading twist $w f$ and its quark longitudinal and transverse distributions. We recall our previous analysis regarding the nonperturbative $w f$ and formulate the main constraints which have been obtained from QCD analysis. In section 3 we model the $w f$ which satisfies these constraints.

Section 4 is devoted to the calculation of the soft contribution to pion form factor based on the model $w fs$ obtained in the previous section. Let us stress from the very beginning: we are not pretending to have made a reliable calculation of the form factor here. We discuss some very general properties of the amplitudes which are related to the specific features of the $\psi(\vec{k}_{\perp}^2, x)$. We illustrate how these features change the behavior of the form factor in intermediate region of $Q^2$. Section 5 is our conclusion and outlook.

2. Constraints on the nonperturbative wave function $\psi(\vec{k}_{\perp}^2, x)$.

First of all let us review some essential definitions and results about nonperturbative $w f$. We define the pion axial wave function in the following gauge-invariant way:

$$if_\pi q_\mu \phi_A(zq, z^2) = \langle 0| \bar{d}(z) \gamma_\mu \gamma_5 e^{ig \int_{-z}^{z} A_\mu dz_\mu} u(-z)|\pi(q)\rangle = \sum_n \frac{z^n}{n!} \langle 0| \bar{d}(0) \gamma_\mu \gamma_5 (iz_\nu \vec{D}_\nu)^n u(0)|\pi(q)\rangle,$$

where $\vec{D}_\nu \equiv \vec{D}_\nu - \vec{D}_\nu$ and $i\vec{D}_\mu = i\partial_\mu + gA_\mu^a \lambda^a$ is the covariant derivative. From its definition is clear that the set of different $\pi$ meson matrix elements defines the nonperturbative wave function.
The most important part (at asymptotically high \(q^2\)) is the one related to the longitudinal distribution. In this case \(z^2 \simeq 0\) the \(wf\) depends only on one \(zq\)-variable. The corresponding Fourier transformed wave function will be denoted as \(\phi(\xi)\) and its \(n\)-th moment is given by the following local matrix element:

\[
\langle 0| \bar{d}_\gamma \gamma_5 (i \bar{D}_\mu z_\mu)^n u|\pi(q)\rangle = i f_\pi q_\nu (zq)^n \langle \xi^n \rangle = i f_\pi q_\nu (zq)^n \int_{-1}^{1} d\xi \xi^n \phi(\xi) \tag{2}
\]

\(- q^2 \to \infty, \quad zq \sim 1, \quad \xi = x_1 - x_2, \quad x_1 + x_2 = 1, \quad z^2 = 0.\)

Therefore, if we knew all matrix elements (2) (which are well-defined) we could restore the whole distribution amplitude \(\phi(\xi)\). The QCD sum rules approach allows one to find the magnitudes only the few first moments [8]. As is known, this information is not enough to reconstruct the \(wf\); the parametric behavior at \(\xi \to \pm 1\) is the crucial issue in this reconstruction.

To extract the corresponding information, we use the following duality argument. Instead of consideration of the pion \(wf\) itself, we study the following correlation function with pion quantum numbers:

\[
i \int dxe^{iqx} \langle 0| TJ_n^\parallel(x), J_0(0)|0\rangle = (zq)^{n+2} I_n(q^2), \quad J_n^\parallel = \bar{d}_\gamma \gamma_5 (i \bar{D}_\mu z_\mu)^n u \tag{3}
\]

and calculate its asymptotic behavior at large \(q^2\). The result can be presented in the form of the dispersion integral, whose spectral density is determined by the pure perturbative one-loop diagram:

\[
\frac{1}{\pi} \int_0^\infty ds \frac{Im I_n^{pert}(s)}{s - q^2} = \frac{3}{4\pi(n + 1)(n + 3)}. \tag{4}
\]

We assume that the \(\pi\) meson gives a nonzero contribution to the dispersion integral for arbitrary \(n\) and, in particular, for \(n \to \infty\). Formally, it can be written in the following way

\[
\frac{1}{\pi} \int_0^{S_\pi^\parallel} ds Im I_n^{pert}(s) = \frac{1}{\pi} \int_0^\infty ds Im I_n^{pert}(s), \tag{5}
\]

Our assumption means that there are no special cancelations and \(\pi\) meson contribution to the dispersion integral is not zero, i.e. \(S_\pi^\parallel(\parallel) \neq 0\), where we specified the notation for the longitudinal distribution. In this case at \(q^2 \to \infty\) our assumption (5) leads to the following relation:

\[
f_\pi^2 \langle \xi^n \rangle (n \to \infty) \to \frac{3 S_\pi^{\infty}(\parallel)}{4\pi^2 n^2}. \tag{6}
\]

It unambiguously implies the following behavior at the end-point region [4]:

\[
\langle \xi^n \rangle = \int_{-1}^{1} d\xi \xi^n \phi(\xi) \sim 1/n^2, \quad \phi(\xi \to \pm 1) \to (1 - \xi^2). \tag{7}
\]
Few comments are in order. We consider the nonperturbative correlation function \(3\). Thus the behavior \(6\) should be fulfilled for any nonperturbative wf. The perturbative as well as nonperturbative corrections will change the duality interval \(S_n^\pi(\parallel)\) in the formula \(3\) in comparision with perturbative one-loop calculation. However, \(1/n^2\)- behavior remains unaffected.

Thus, our first constraint looks as follows:

\[ \bullet 1 \quad \phi(\xi \to \pm 1) \to (1 - \xi^2). \]

We want to emphasize that we did not use any numerical approximation in this derivation. Therefore, the constraint \(\bullet 1\) has very general origin and it should be considered as a direct consequence of QCD. Only dispersion relations, duality and very plausible assumption formulated above have been used in the derivation \(\bullet 1\).

Now we want to repeat these arguments for the analysis of the transverse distribution. To do so, let us define the mean values of the transverse quark distribution by the following matrix elements:

\[ \langle 0 | \bar{d} \gamma_\nu \gamma_5 (i \overset{\rightarrow}{D}_\mu \ t_\mu)^{2n} u|\pi(q)\rangle = i f_\pi q_\nu (-t^2)^n (2n - 1)!! \begin{pmatrix} \langle \vec{k}_{\bot}^{2n} \rangle \end{pmatrix}. \]  \(8\)

where \(\overset{\rightarrow}{D}_\nu\) is the covariant derivative, acting on the one quark and transverse vector \(t_\mu = (0, \vec{t}, 0)\) is perpendicular to the hadron momentum \(q_\mu = (q_0, 0_\bot, q_z)\). The factor \(\frac{(2n-1)!!}{(2n)!!}\) is introduced to \(8\) to take into account the integration over \(\phi\) angle in the transverse plane: \(\int d\phi \cos(\phi)^{2n} / \int d\phi = (2n - 1)!!/(2n)!!\).

We interpret the \(\langle \vec{k}_{\bot}^{2n} \rangle\) in this equation as a mean value of the quark perpendicular momentum. Of course it is different from the naive, gauge dependent definition like \(\langle 0 | \bar{d} \gamma_\nu \gamma_5 \partial_\perp^2 u|\pi(q)\rangle\), because the physical transverse gluon is participant of this definition. However, the expression \(8\) is the only possible way to definition the \(k_\perp^2\) in the gauge theory like QCD. We believe that such definition is the useful generalization of the transverse momentum conception for the interactive quark system.

To find the behavior \(\langle \vec{k}_{\bot}^{2n} \rangle\) at large \(n\) we can repeat our previous duality arguments with the following result \(\footnote{Here and in what follows we ignore any mild (nonfactorial) \(n\)-dependence.}\):

\[ f_\pi^2 \langle \vec{k}_{\bot}^{2n} \rangle (2n - 1)!! \begin{pmatrix} \langle \vec{k}_{\bot}^{2n} \rangle \end{pmatrix} \sim n! \Rightarrow f_\pi^2 \langle \vec{k}_{\bot}^{2n} \rangle \sim n!. \] \(9\)

This behavior has been obtained in ref.\[24\] by analysing the perturbative series of the specific correlation function at large order. The dispersion relations and duality arguments translate this information into the formula \(3\). Let us repeat again: any nonperturbative wave function should respect eq.\(4\) in spite of the fact that apparently we calculate only the perturbative part (see comment after formula \(4\)).
The nice feature of (9) is its finiteness for arbitrary $n$. It means that the higher moments
\[
\langle \vec{k}_\perp^{2n} \rangle = \int d\vec{k}_\perp^2 d\xi \vec{k}_\perp^{2n} \psi(\vec{k}_\perp, \xi)
\]
do exist. In this formula we introduced the nonperturbative $wf$ $\psi(\vec{k}_\perp, \xi)$, normalized to one. Its moments are determined by the local matrix elements (8). The relations to Brodsky and Lepage notations $\Psi_{BL}(x_1, \vec{k}_\perp)$ \[1\] and to longitudinal distribution amplitude $\phi(\xi)$ introduced earlier, look as follow:

\[
\Psi_{BL}(x_1, \vec{k}_\perp) = \frac{f_\pi}{16\pi^2} \sqrt{6} \psi(\xi, \vec{k}_\perp), \int d\vec{k}_\perp^2 \psi(\vec{k}_\perp, \xi) = \phi(\xi), \int_{-1}^{1} d\xi \phi(\xi) = 1 \tag{10}
\]
where $f_\pi = 133 MeV$. The existence of the arbitrary high moments $\langle \vec{k}_\perp^{2n} \rangle$ means that the nonperturbative $wf$, defined above, falls off at large transverse momentum $\vec{k}_\perp$ faster than any power function. The relation (9) fixes the asymptotic behavior of $wf$ at large $\vec{k}_\perp$. Thus, we arrive to the following constraint:

- $2 \langle \vec{k}_\perp^{2n} \rangle = \int d\vec{k}_\perp^2 d\xi \vec{k}_\perp^{2n} \psi(\vec{k}_\perp, \xi) \sim n! \quad n \to \infty$.

We can repeat our duality arguments again for an arbitrary number of transverse derivatives and large ($n \to \infty$) number of longitudinal derivatives with the following result \[23\]:

- $3 \int d\vec{k}_\perp^2 \vec{k}_\perp^{2k} \psi(\vec{k}_\perp, \xi \to \pm 1) \sim (1 - \xi^2)^{k+1}$.

For the $k = 0$ we reproduce our previous formula for the $\phi$ function: $\phi(\xi \to \pm 1) = \int d\vec{k}_\perp^2 \psi(\vec{k}_\perp, \xi \to \pm 1) \sim (1 - \xi^2)$. The constraint (●3) is extremely important and implies that the $\vec{k}_\perp^2$ dependence of the $\psi(\vec{k}_\perp, \xi)$ comes exclusively in the combination $\vec{k}_\perp^2/(1 - \xi^2)$ at $\xi \to \pm 1$. The byproduct of this constraint can be formulated as follows. The standard assumption on factorizability of the $\psi(\vec{k}_\perp, \xi) = \psi(\vec{k}_\perp^2)\phi(\xi)$ does contradict to the very general properties of the theory. Thus, the asymptotic behavior of the $wf$ turns out to be Gaussian one with the very specific argument:

\[
\psi(\vec{k}_\perp \to \infty, \xi) \sim \exp(-\frac{\vec{k}_\perp^2}{1 - \xi^2}), \tag{11}
\]

We would like to pause here in order to make the following conjecture. The Gaussian $wf$ (reconstructed above from the QCD analysis) not accidentally coincides with the harmonic oscillator $wf$ from constituent quark model. To make this conjecture more clear, let us recall few results from the constituent quark model.

It is well known \[25\] that the equal-time wave functions

\[
\psi_{CM}(q^2) \sim \exp(-q^2) \tag{12}
\]
of the harmonic oscillator in the rest frame give a very reasonable description of static meson properties. Together with Brodsky-Huang-Lepage prescription \[26\], \[27\] connecting the equal-time and the light-cone wave functions
of two constituents (with mass $m \sim 300\text{MeV}$) by identification

$$q^2 \leftrightarrow \frac{k^2_1 + m^2}{4x(1-x)} - m^2, \quad \psi_{CM}(q^2) \leftrightarrow \psi_{LC}(\frac{k^2_1 + m^2}{4x(1-x)} - m^2),$$

one can reproduce the Gaussian behavior (11) found from QCD. It means, first of all, that our identification of the moments (8) defined in QCD with the ones defined in quark model, is the reasonable conjecture.

However, there is a difference. In quark model we do have a parameter which describes the mass of constituent $m \simeq 300\text{MeV}$. We have nothing like that in QCD. This difference has very important consequences which will be discussed later.

We would like to put a few more constraints on the list. But before to do so, we have to emphasize the difference between constraints ($\bullet 1 - \bullet 3$) discussed above and the ones which follow. The first three constraints have very general origin. No numerical approximations have been made in the derivation of the corresponding formulae. The only what have been used are dispersion relations, duality and very plausible assumption formulated above. The main idea of the derivation of all these constraints is one and the same: we calculate some correlation function in QCD using the asymptotic freedom. The dispersion relations and duality arguments transform these properties into the constraints on hadronic matrix elements.

The constraints we are going to discuss now have absolutely different status. They are based on the QCD sum rules with their inevitable numerical assumptions about higher excited states in QCD. Thus, they must be treated as an approximate ones. The well-known constraint of such a kind is the second moment of the distribution amplitude in the longitudinal direction ($\bullet 4$):

$$\langle \xi^2 \rangle \equiv \int d\xi \phi(\xi)\xi^2 \simeq 0.4,$$

(The asymptotic $wf$ corresponds to $\langle \xi^2 \rangle = 0.2$). Such a result was the reason to suggest the “two-hump” shape $wf$ which meets the above requirement. The number cited as the constraint ($\bullet 4$) has been seriously criticized in refs. \cite{9}-\cite{12}. The point for criticism was exactly the assumption about the role of the excited states in the sum rules. We can not answer on this criticism within standard QCD sum rules approach. Thus, in what follows we shall discuss both possibilities: the narrow (asymptotic) $wf$ and the wider one (with larger $\langle \xi^2 \rangle > 0.2$).

The next “numerical” constraint is the second moment of the $wf$ in the transverse direction defined by equation (8) and calculated for the first time

$$\psi_{nucleon}(k^2_{\perp i} \to \infty, x_i) \sim \exp(- \sum \frac{k^2_{\perp i}}{x_i})$$

6The same method can be applied for the analysis of the asymptotical behavior of the nucleon $wf$ which in obvious notations takes the form:

$$\psi_{nucleon}(k^2_{\perp i} \to \infty, x_i) \sim \exp(- \sum \frac{k^2_{\perp i}}{x_i})$$
in \[4\] and independently (with quite different technique) in \[28\]. Both results are in a full agreement to each other:

\[\langle \vec{k}_{\perp}^2 \rangle = \frac{5}{36} \langle \bar{q}q \sigma_{\mu\nu} G_{\mu\nu} \bar{q}q \rangle \sim \frac{5m_0^2}{36} \approx 0.1 GeV^2, \quad m_0^2 \approx 0.8 GeV^2.\]

Essentially, the constraint (\(\bullet 5\)) defines the general scale of all nonperturbative phenomena for the pion. It is not accidentally coincides with 300 MeV which is the typical magnitude in the hadronic physics.

All numerical values obtained within QCD sum rules approach correspond to the normalization point \(\sim 1 GeV^2\). To model the \(w_f\) we need to know their renormalization properties. The anomalous dimensions of the longitudinal operators are well known, see e.g.\[1\], \[4\]. For our particular case we can write it in the following way:

\[\left(\langle \xi^2 \rangle - \frac{1}{5}\right)_{\mu_1} = \left(\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_2)}\right)^{50/9b} \cdot \left(\langle \xi^2 \rangle - \frac{1}{5}\right)_{\mu_2}, \quad b = \frac{11}{3} N_c - \frac{2}{3} n_f = 9 \quad (13)\]

The anomalous dimensions of the higher twist operators (which are related to transverse moments) are less familiar. It has been calculated for the operator describing the mean value of the transverse momentum \(\vec{k}_{\perp}^2\) \[18\]:

\[\langle k_{\perp}^2 \rangle_{\mu_1} = \left(\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_2)}\right)^{32/9b} \cdot \langle k_{\perp}^2 \rangle_{\mu_2}. \quad (14)\]

To study the fine properties of the transverse distribution it is desired to know the next moment. The problem can be reduced to the analysis of the mixed vacuum condensates of dimension seven \[23\]:

\[\langle k_{\perp 4}^4 \rangle = \frac{1}{8} \left\{ -3 \frac{\langle \bar{q}g^2 \sigma_{\mu\nu} G_{\mu\nu} \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle}{4 \langle \bar{q}q \rangle} + \frac{13 \langle g^2 G_{\mu\nu} G_{\mu\nu} q \rangle}{9 \langle \bar{q}q \rangle} \right\}, \quad (15)\]

We analyzed the magnitudes for these vacuum condensates with the following result: the standard factorization hypothesis does not work in this case. The factor of nonfactorizability \(K \simeq 3.0 \div 3.5\) \[23\], \[29\]. The eq.\((15)\) defines the new numerical constraint on the transverse distribution. We prefer to express this constraint not in terms of the absolute values, but rather, in terms of the dimensionless parameter \(R\) which is defined in the following way:

\[R = \frac{\langle k_{\perp 4}^4 \rangle}{\langle k_{\perp 2}^2 \rangle^2} \simeq 3K \cdot \frac{\langle g^2 G_{\mu\nu} G_{\mu\nu} \rangle}{m_0^2} \simeq 5 \div 7, \quad m_0^2 \approx 0.8 GeV^2,\]

where we use the standard values for parameter \(m_0^2\) and gluon condensate \[7\]. We would like to emphasize that the fluctuations of the transverse momentum are large enough. The quantitative characteristic of these fluctuations is parameter \(R \gg 1\). In terms of wave function this property means a very unhomogeneous distribution in transverse direction.

As we already mentioned, to model \(w_f\) we need to know the renormalization properties of the higher twist operators. Unfortunately, we do not know them (except for the lowest one, \(14\)). However, one can argue, that in the large \(N_c\) limit, the main contribution to anomalous dimensions can be found
from the formulae like (15), where matrix elements are expressed in terms of vacuum condensates. In the same $N_c \to \infty$ limit the renormalization properties of these condensates (but not their absolute values) can be estimated (with the same accuracy) by applying the factorization procedure. Thus, one could expect that this prescription gives a reasonable numerical accuracy ($\sim 1/N_c$) for the renormalization properties of higher twist operators. However, the exact calculations are highly desired and welcome [30].

We would like to check this prescription for the operator with known dimension (14). The anomalous dimensions for the chiral condensate and mixed condensate are known:

\[
\langle \bar{q}q \rangle_{\mu_1} = (\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_2)})^{\frac{4}{9}} \cdot \langle \bar{q}q \rangle_{\mu_2}
\]

\[
\langle \bar{q}i g \sigma_{\mu \nu} G_{\mu \nu}^a \frac{\lambda^a}{2} q \rangle_{\mu_1} = (\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_2)})^{\frac{2}{9}} \cdot \langle \bar{q}i g \sigma_{\mu \nu} G_{\mu \nu}^a \frac{\lambda^a}{2} q \rangle_{\mu_2}
\]

Thus, our relation (●5) gives the following prescription for the evolution formula under the renormalization group transformation:

\[
\langle \vec{k}^2_{\perp} \rangle_{\mu_1} \sim \frac{\langle \bar{q}i g \sigma_{\mu \nu} G_{\mu \nu}^a \frac{\lambda^a}{2} q \rangle}{\langle \bar{q}q \rangle} \sim (\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_2)})^{\frac{4}{9}} \cdot \langle \vec{k}^2_{\perp} \rangle_{\mu_1},
\]

instead of exact formula (14). We consider this good numerical agreement as a justification for the analogous estimation for different operators with unknown anomalous dimensions. In particular, we expect that the matrix element $\langle \vec{k}^4_{\perp} \rangle_{\mu}$ (13) is not changed strongly under the renormalization group transformations because after the factorization this operator reduces to the gluon condensate which is renormalization invariant.

At the same time our dimensionless parameter $R$ is changed strongly and we estimate it as follows:

\[
R_{\mu_1} = \frac{\langle \vec{k}^4_{\perp} \rangle}{\langle \vec{k}^2_{\perp} \rangle^2} \sim \frac{\langle g^2 G_{\mu \nu} G_{\mu \nu} \rangle}{\langle \bar{q}q \rangle^2} \sim (\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_2)})^{\frac{4}{9}} \cdot R_{\mu_2}
\]

Let us summarize the results of the section. The constraints (●1 – ●3) have very general origin and should be fulfilled in any phenomenological description based on QCD. The numerical constraints (●4 – ●6) have much less generality because they have been obtained from QCD sum rules with inevitable for this method approximations. All numerical results obtained from QCD sum rules are normalized at $\mu^2 \sim 1GeV^2$. At the same time, the model w.f we are going to construct should be normalized at the lowest possible point which is about $\mu^2 \sim 0.25GeV^2$. We shall use the anomalous dimensions shown above in order to evaluate all numerical constraints at the lowest normalization point.
3. The model wave function.

Let us start our discussion from the analysis of the \( w_f \) motivated by constituent quark model [19], [25]- [27] (CQM). Such a function is known to give a reasonable description of static hadron properties. The Brodsky-Huang-Lepage prescription [27] leads to the following form for the pion \( w_f \):

\[
\psi(\vec{k}^2_\perp, x)_{CQM} = A \exp\left(-\frac{\vec{k}^2_\perp + m^2}{8\beta^2 x(1-x)}\right),
\]

We call this function as the constituent quark model \( w_f \). It satisfies two constraints (\( \bullet 2, \bullet 3 \)), but not (\( \bullet 1 \)) because of the nonzero magnitude for the constituent mass \( m \). We take the standard set for QCD parameters:

\[
\alpha_s(1GeV) = 0.34, \quad \Lambda_{QCD} = 200MeV.
\]

The lowest possible normalization point \( \mu_0 \) is defined as the place where \( \alpha_s(\mu_0) \approx 0.7 \). This corresponds to the QCD sum rules analysis [3], where “almost” renormalization invariant combination \( \alpha_s\langle \bar{q}q \rangle^2 \) comes into the game and it is numerically well known. At the same time the chiral condensate \( \langle \bar{q}q \rangle \) at the lowest possible normalization point is known from PCAC.

We made the standard choice for the parameter \( m \approx 330MeV \) in accordance with its physical meaning. The parameter \( \beta \) is determined from the numerical constraint (\( \bullet 4 \)) for the mean value \( \langle \vec{k}^2_\perp \rangle \). Parameter \( A \) is determined by the normalization eq. (10). As we mentioned earlier, we have to renormalize all moments to the lowest possible point to model \( w_f \). In our case the evaluation of \( \langle \vec{k}^2_\perp \rangle \) is defined by eq. (14). With our set of parameters (20) and (\( \bullet 5 \)) we have the following mean value for \( \vec{k}^2_\perp \) at lowest normalization point:

\[
\langle \vec{k}^2_\perp \rangle_{\mu_0} = 0.14 GeV^2,
\]

which will be used through this paper. This number corresponds to \( \beta \approx 0.3 GeV \) which is within reasonable parametric region. To make function wider in the longitudinal direction (constraint \( \bullet 4 \)) one can insert to formula

---

7Here we neglect all terms in QCM related to spin part of constituents. In particular, we do not consider Melosh transformation and other ingredients of the light cone\( \leftrightarrow \)equal time connection. It does not effect qualitative results presented in the next section.

8Do not confuse our parameter \( \langle \vec{k}^2_\perp \rangle \) which is well defined in terms of QCD matrix element (8) with the one which is defined in terms of CQM as follows:

\[
\langle \vec{k}^2_\perp \rangle_{\bar{q}q} = \frac{\int d^2\vec{k}^2_\perp dx |\psi(\vec{k}^2_\perp, x)_{CQM}|^2 \vec{k}^2_\perp}{\int d^2\vec{k}^2_\perp dx |\psi(\vec{k}^2_\perp, x)_{CQM}|^2}
\]

and should be extracted from somewhere else. Numerically they are not very different, but we prefer to use one and the same procedure to specify parameters for all wave functions.
with additional parameter $g(\mu)$. With this new parameter $g(\mu)$ one can adjust $\langle \xi^2 \rangle$ as appropriate. For the asymptotic distribution amplitude parameter $g = 0$.

We are ready now to discuss the model $w_f$ in QCD. Before to design $\psi_{QCD}(k^2_\perp, x)$, let us explain what do we mean by that. We define the nonperturbative wave function $\psi(k^2_\perp, x, \mu)_{QCD}$ through its moments which can be expressed in terms of the nonperturbative matrix elements $[1]$. As is known, all nonperturbative matrix elements are defined in such a way that all gluon's and quark's virtualities smaller than some parameter $\mu$ (point of normalization) are hidden in the definition of the "nonperturbative matrix elements". All virtualities larger than that should be take into account perturbatively. In particular, all perturbative tails like $1/k^2_\perp$ should be subtracted from the definition of the nonperturbative $w_f$. The same procedure should be applied for the calculation of nonperturbative vacuum condensate $\langle G_{\mu\nu}^2 \rangle$, where the perturbative part related to free gluon propagator $1/k^2$ should be subtracted.

With these general remarks in mind we propose the following form for the nonperturbative wave function $\psi(k^2_\perp, x, \mu_0)_{QCD}$ at the lowest normalization point:

$$\psi(k^2_\perp, x, \mu_0)_{QCD} = A \exp\left(-\frac{k^2_\perp}{8\beta^2x(1-x)}\right) \cdot \left\{1 + g(\mu_0)[(2x - 1)^2 - \frac{1}{5}]\right\}.$$ (23)

In comparison with the constituent quark model the "only" difference is the absence on the mass term $\sim m$ in the exponent. As we shall see in the next section it does make a big difference. This function satisfies all fundamental constraints $([1] - [3])$. The dimensional parameters can be determined from the numerical relations $([4] - [5])$ in the same way as before. Parameter $\beta$ for this parametrization is found to be $\beta \simeq 0.3$ GeV (it corresponds to $R = 2.2$ and $\langle k^2_\perp \rangle = 0.14 GeV^2$)

We would like to emphasize that the nonzero mass in $\psi(k^2_\perp, x)_{CQM}$ was unavoidable part of the wave function. We do not see any room for such term in QCD, because its presence would mean the following behavior of the large moments in longitudinal direction:

$$\langle \xi^n \rangle = \int_{-1}^{1} d\xi \xi^n \phi(\xi) \sim \int_{-1}^{1} d\xi \xi^n \exp\left(-\frac{1}{1-\xi^2}\right) \sim \exp(-\sqrt{n}), \quad n \to \infty.$$ (24)

It is in contradiction to $1/n^2$ behavior $([7], [1])$. Let us stress that such a behavior is the result of the calculation of the correlation function $([3])$. We do not see any possibilities to change this behavior from $1/n^2$ to $\exp(-\sqrt{n})$. If such a behavior were occurred, it would mean that the strong cancellation
between one-loop diagram (4) and higher loop corrections $\sim \alpha_s^k$ takes place. Such a cancellation looks even less probable, if one takes into account that the aforementioned cancellation must take place for each given number $n$ at large $n$. We do not believe that it might happen in QCD.

Up to now we did not discuss the influence of our last “numerical” constraint (6) denoted by $R$. Large number for this parameter means a noticeable fluctuations of the momentum in transverse direction. To satisfy this constraint we need to spread out the $wf$ to make it wider. It can be done in arbitrary way. In particular, one may try to put one more hump apart from the main Gaussian term described by eq. (23). The only requirement is: it has to fall off fast enough at large $k^2_\perp$.

With these remarks in mind we suggest the following QCD motivated $wf$ (we call it as $\psi(k^2_\perp, x, \mu_0)_{QCD+}$) which can be adjusted to satisfy all six requirements mentioned in the previous section:

$$\psi(k^2_\perp, x, \mu_0)_{QCD+} = A\left\{ e^{-\frac{\vec{k}^2_\perp}{8\beta^2 x(1-x)}} + c \cdot e^{-\left(\frac{k^2_\perp}{8\beta^2 x(1-x)} - l\right)^2} \right\} \cdot \{1 + g(\mu)[(2x - 1)^2 - \frac{1}{5}]\}.$$  \hspace{1cm} (25)

The physical meaning of the parameters $c, l$ is clear. Parameter $c$ determines the magnitude of the second hump and parameter $l$ describes its distance from the main term. As we shall see in order to match parameters $l, c$ with the calculated ratio $R$ we need to have the magnitude of the second hump about $1/10$. The maximum of the second hump is located in $k^2_\perp \sim 0.8 GeV^2$ region. To be more specific, we renormalized the parameter $R$ found from QCD to the lowest normalization point according to formula (18). It is found to be $$R(\mu_0) \simeq 3 \div 4$$ \hspace{1cm} (26)

We display this function with parameters $\beta = 0.15$ GeV, $c = 0.15, l = 30$ (which correspond to $R \simeq 4, \langle k^2_\perp \rangle = 0.14 GeV^2$) on Fig.1 at the central point $x = 1/2$.

Let us summarize. We constructed three wave functions. The first one, $\psi_{CQM}$ is motivated by quark model with its specific mass parameters. Two other models are motivated by QCD consideration. Despite of this difference, all these models have Gaussian behavior at large $k^2_\perp$. However, in the case of $\psi_{CQM}$ this behavior is related to the nonrelativistic oscillator model, while for QCD motivated models this behavior is provided by constraints discussed in the previous section.

Contrary to the CQM, the QCD motivated wave functions do not contain the mass parameter $m \simeq 300 MeV$ which is an essential ingredient of any quark model. Such a term is absolutely forbidden from the QCD point of view.
The difference between $\psi_{QCD}$ and $\psi_{QCD+}$ is not fundamental, but rather quantitative. Nevertheless, we believe that it is worth to mention some new effects (noticeable fluctuations of transverse momentum) which the function $\psi_{QCD+}$ brings. The broadening in the transverse direction is the main difference between functions $\psi_{QCD}$ and $\psi_{QCD+}$. In the next section we discuss the contribution to the pion form factor caused by these three wave functions. We shall see the qualitative difference in behavior on $Q^2$, which is our main point.

4. Pion Form Factor.

The starting point is the famous Drell-Yan formula \[31\] (for modern, QCD- motivated employing of this formula, see \[1\]), where the $F_\pi(Q^2)$ is expressed in terms of full wave functions:

$$F_\pi(Q^2) = \int \frac{dx d^2k_\perp}{16\pi^4} \Psi_{BL}(x, k_\perp + (1-x)q_\perp)\Psi_{BL}(x, k_\perp),$$

(27)

where $q^2 = -q_\perp^2 = -Q^2$ is the momentum transfer. In this formula, the $\Psi_{BL}(x, k_\perp)$ is the full wave function; the perturbative tail of $\Psi_{BL}(x, k_\perp)$ behaves as $\alpha_s/\vec{k}_\perp^2$ for large $\vec{k}_\perp^2$ and should be taken into account explicitly in the calculations. This gives the one-gluon-exchange (asymptotically leading) formula for the form factor in terms of distribution amplitude $\phi(x)$ \[3\].

Below is the QCD motivated interpretation of this formula. Let us remind, that the formula (27) takes into account only the valence Fock states. The formula would be exact if all Fock states were taken into account. Besides that, $\vec{k}_\perp^2$ in this formula was originally thought to be the usual (not covariant) perpendicular momentum of the constituents, and not the mean value $\langle \vec{k}_\perp^2 \rangle$ defined in QCD, as a gauge invariant object. However we make the assumption that it is one and the same variable. The physics behind of it can be explained in the following way.

In the formula (27) we effectively take into account some gluons (not all of them), which inevitably are participants of our definition of $wf$. These gluons mainly carry the transverse momentum (which anyhow, does not exceed QCD scale of order $\sim 1GeV$) and/or small amount of the longitudinal momentum. The contributions of the gluons carrying the finite longitudinal momentum fraction are neglected in (27). This is the main assumption. It can be justified by the direct calculation \[1\] of quark-antiquark-gluon (with finite momentum fraction) contribution to $\pi$ meson form factor at large $Q^2$ within the standard technique of the operator product expansion. By technical reasons the corresponding calculation has not been completed, however it was found that the characteristic scale which enters into the game is of order $1GeV^2$. Thus, it is very unlikely that these contributions can be important at $Q^2 \gg 1GeV^2$. The second calculation, which confirms this point, comes from the light cone QCD sum rules \[22\]. This is almost model independent calcu-
lation demonstrates that the quark-antiquark-gluon (with finite momentum fraction) contribution does not exceed 20% at available $Q^2$.

Thus, we expect, that by taking into account the only "soft" gluon contribution (hidden in the definition of $\vec{k}^2_\perp$), we catch the main effect. Again, there is no proof for that within QCD, and the only argumentation which can be delivered now in favor of it, is based on the intuitive picture of quark model, where current quark and soft gluons form a constituent quark with original quantum numbers. No evidence of the gluon playing the role of a valent participant with a finite amount of momentum, is found.

From the viewpoint of the operator product expansion, the assumption formulated above, corresponds to the summing up a subset of higher-dimension power corrections. This subset actually is formed from the infinite number of soft gluons and unambiguously singled out by the definition of nonperturbative $wf$.

In the following, we preserve the notation $\Psi_{BL}(x, \vec{k}_\perp)$ for the nonperturbative, soft part only. It should not confuse the reader.

The formula (27) is written in terms of Brodsky and Lepage notations \[\] ; the relation to our wave function $\psi(\vec{k}_\perp, x)$ is given by formula (11).

With these general remarks in mind, we would like to present the results of calculation, based on three wave functions discussed in the previous section.

The first calculation, based on $\psi_{CQM}(\vec{k}^2_\perp, x)$ is the standard one. The analogous calculations with oscillator-like $wf$ have been done many times with many additional improvements, see i.e. [19]. One can fit the dimensional parameters in such a way that the description at low $Q^2$ will be perfect. However, our goal is different and we are interested in the behavior at large enough $Q^2 \gg 1 GeV^2$. We expect that described here approach makes sense only at high enough $Q^2$. We display the corresponding behavior as a curve 1 on Fig.2.

The main feature of this behavior – it gives very reasonable magnitude for the intermediate region about few $GeV^2$ and it starts to fall off right after that. We expect that any reasonable, well localized, based on quark model wave function with the scale $\sim \langle \vec{k}^2_\perp \rangle \sim m^2$ leads to the similar behavior. Let me stress: we are not pretending to have made a reliable calculation of the form factor here; we displayed this contribution only for the illustrative purposes.

Currently, much more interesting for us is the calculation, based on QCD motivated models. We display the corresponding contribution to the $Q^2 F_x(\vec{k}^2_\perp)$ for the $\psi_{QCD}(x, \vec{k}^2_\perp)$ on Fig. 2 as curve 2. The qualitative difference between this curve and the previous one (curve 1), is much slower fall off at large $Q^2$ for the model $wf \psi_{QCD}$. The qualitative reason for that is the absence of the mass term in $\psi_{QCD}$, see discussion after the formula (24). Precisely this term was responsible for the very steep behavior in the previous calculation with quark model wave function.

The declining of the form factor getting even slower if one takes into
account the property of the broadening of \( \psi \) in transverse direction. The corresponding contribution based on \( \psi_{QCD+} \) is displayed on Fig.2 as curve 3. We notice an additional slowing down of the declining of the magnitude \( Q^2 F(Q^2) \) in the intermediate region of \( Q^2 \). The explanation of this effect is the following. When \( Q^2 \) is getting bigger and bigger, the contribution coming from the overlap of two humps (\( \psi_{QCD+} \) at \( x = 1/2 \) is displayed on Fig.1) starts to grow. These humps are well separated (in order to satisfy constraint (\( \bullet \)6)) from each other by the value of order 1\( GeV^2 \). Therefore, we expect that this contribution starts to grow at high enough \( Q^2 \gg 4 GeV^2 \). As the result, the curve 3 looks more horizontal than the previous one. It is getting lower because the general scale has been changed when we passed from \( \psi_{QCD} \) to \( \psi_{QCD+} \). Let us remind that we keep \( \langle \vec{k}^2_\perp \rangle \) fixed (\( \bullet \5 \)) in all cases. It leads to some changes of dimensional parameters because the small hump in the \( \psi_{QCD+} \) gives a noticeable contribution to \( \langle \vec{k}^2_\perp \rangle \) in spite of the fact that it comes with very small relative weight (coefficient \( c \sim \frac{1}{10} \)).

Our last qualitative remark is some note that the results strongly depend on parameter \( \langle \xi^2 \rangle \). We displayed on Fig.3 the same three curves as on Fig.2 with the only difference in coefficient \( g(\mu) \) (\( \bullet \22 \), \( \bullet \23 \), \( \bullet \25 \)) We set \( g(\mu_0) = 2 \) which corresponds to \( \langle \xi^2 \rangle = 0.3 \). It makes a \( \psi \) wider in longitudinal direction. The soft contribution getting bigger when a wider (in longitudinal direction) wave function is used. The same effect was observed in the recent calculation \( \bullet \22 \), where a quite different method has been used.

The contribution under consideration is subject to Sudakov corrections. An estimate of these corrections reveals that they are small enough in this intermediate \( Q^2 \) region. Besides that, we will be on the safe side if we say that the hard (leading twist) contribution to \( Q^2 F(Q^2) \simeq 0.2 GeV^2 \) \( \bullet \13, \bullet \16, \bullet \22 \). It should be added to the soft terms displayed on Fig.2, Fig.3.

The precise fitting of the pion form factor was not among the goals of this paper. Rather, we wanted to demonstrate how the qualitative properties of a nonperturbative \( \psi \), derived from the QCD analysis might significantly change its behavior.

5. Summary and Outlook.

The main goal of the present paper was the analysis of the nonperturbative \( \psi \) from QCD point of view. We found qualitatively different results in comparison with the wave functions motivated by quark model. We believe that this difference is responsible for the qualitative explanation of dimensional counting rules which work well even at very modest energies.

The standard point of view for the phenomenological success of the dimensional counting rules is the predigest that the leading twist contribution plays the main role in most cases. We suggest here some different explanation for this phenomenological success. Our explanation of the slow falling off of the soft contribution with energy is due to the specific properties of nonperturbative \( \psi \). In particular, we found a new scale \( \sim 1 GeV^2 \) in the
problem, in addition to the standard low energy parameter \( \langle \vec{k}_\perp^2 \rangle \simeq 0.1\text{GeV}^2 \).

Our next remark can be formulated as follows. Exclusive, as well as inclusive amplitudes can be expressed in terms of the \textit{one and the same} particular hadron \( wf \). Therefore, if our explanation (related to specific form of \( \psi(\vec{k}_\perp^2, x) \)) of a temporary simulation of the leading twist behavior is considered as a reasonable one, then:

1. in the analysis of inclusive amplitudes one may expect the same effect (it is our conjecture);
2. one may try to implement the intrinsic transverse momentum dependence into the inclusive calculations.

In particular, one may try to use the following prescription for the \( \pi \) meson distribution function (and analogously for nucleon) at \( x \Rightarrow 1 \):

\[
G_{q/\pi}(x, Q^2) \Rightarrow \left\{ \int \frac{d^2 k_\perp}{x(1-x)} \exp\left(-\frac{\vec{k}_\perp^2}{x(1-x)}\right) \right\} G_{q/\pi}(x, Q^2)
\]

(28)

The analogous formula (without \( x \) dependence in the exponent) has been suggested many years ago [32], see also [33]. Our remark is that this \( x \) dependence is essential point and should be introduced to the formula to satisfy our constraints. In terms of [32], [33] it corresponds to the non-universality of their “constant” \( \langle \vec{k}_\perp^2 \rangle \) which now will depend on \( x \).

To support this conjecture, we would like to mention few inclusive processes where the intrinsic transverse distribution might be essential. First of all, it is Drell-Yan amplitude \( \pi + N \rightarrow \mu^- \mu^+ + X \) which is parametrized as follows (for references and recent development see [34]):

\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \sim 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \nu 2 \sin^2 \theta \cos 2\phi.
\]

(29)

Here \( \theta, \phi \) are angles defined in the muon pair rest frame and \( \lambda, \mu, \nu \) are coefficients. In the naive parton model the coefficient are \( \lambda = 1, \mu = \nu = 0 \). Experimental results do not support this naive prediction. Recently, some improvements have been made [34], but some problems are still remain.

In particular, the Lam-Tung sum rule [35], \( 1 - \lambda - 2\nu = 0 \) is violated by experimental data and the improved model [34] still can not explain the behavior \( 1 - \lambda - 2\nu \) as a function of \( Q^2_\perp \) (\( Q^2_\perp \) is the transverse momentum of the lepton pair).

Due to the fact that \( Q^2 \) is not large enough in this experiment one may expect that the intrinsic distribution (28) might be essential.

Analogously, we would expect that the \( \pi \) meson structure (28) may effect the analysis of azimuthal asymmetries in semi-exclusive amplitudes like \( l + p \rightarrow l' + h + X \). For references and recent development see [36], where intrinsic \( \vec{k}_\perp^2 \) has been introduced in the standard way without \( x \) dependence in the exponent.

One may find many examples like that where the standard parton picture does not work well. We would like to mention here the recent analysis [37]
of the direct photon production ($\pi + p \rightarrow \gamma + X$), with the result that perturbative QCD can not explain the data. Some nonperturbative broadening factor in transverse direction should be implemented. One may hope that formula (28) may improve the agreement with experiment.

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FIGURE CAPTIONS

**Fig.1** QCD Wavefunction (unnormalized) $\psi(\vec{k}_\perp, x, \mu_0)_{QCD+}$ versus transverse momentum $k_\perp$.

**Fig.2** Pion Form Factor $Q^2 \cdot F_\pi(Q^2)$ versus $Q^2$. Line 1 corresponds to the $\psi(\vec{k}_\perp^2, x)_{CQM}$, with parameters, $R \simeq 2.0$ (26), $\langle \vec{k}_\perp^2 \rangle = 0.14 \text{ GeV}^2$ (8), line 2 corresponds to the $\psi(\vec{k}_\perp^2, x, \mu_0)_{QCD}$ with parameters, $R \simeq 2.2$, $\langle \vec{k}_\perp^2 \rangle = 0.14 \text{ GeV}^2$, $\langle \xi^2 \rangle = 0.2$, and line 3 shows the result for two humped (in transverse direction) wavefunction $\psi(\vec{k}_\perp^2, x, \mu_0)_{QCD+}$, with parameters $R \simeq 4.0$, $\langle \vec{k}_\perp^2 \rangle = 0.14 \text{ GeV}^2$, $\langle \xi^2 \rangle = 0.2$.

**Fig.2** The same Pion Form Factor for $\langle \xi^2 \rangle = 0.3$.

FIGURES
Fig. 2

Fig. 3