Dark energy model with spinor matter and its quintom scenario

Yi-Fu Cai\(^1\) and Jing Wang\(^{1,2}\)

\(^1\) Institute of High Energy Physics, Chinese Academy of Sciences, PO Box 918-4, Beijing 100049, People’s Republic of China
\(^2\) Department of Physics Science, Hebei Normal University, People’s Republic of China

E-mail: caiyf@ihep.ac.cn

Received 25 February 2008, in final form 26 May 2008
Published 5 August 2008
Online at stacks.iop.org/CQG/25/165014

Abstract
A class of dynamical dark energy models, dubbed spinor quintom, can be constructed by a spinor field \(\psi\) with a nontraditional potential. We find that, if choosing suitable potential, this model is able to allow the equation-of-state to cross the cosmological constant boundary without introducing any ghost fields. In a further investigation, we show that this model is able to mimic a perfect fluid of Chaplygin gas with \(p = -c/\rho\) during the evolution, and also realizes the quintom scenario with its equation-of-state across \(-1\).

PACS number: 98.80.Cq

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The recent data from type Ia supernovae and cosmic microwave background (CMB) radiation and so on [1–3] have provided strong evidence for a spatially flat and accelerated expanding universe at the present time. In the context of Friedmann–Robertson–Walker (FRW) cosmology, this acceleration is attributed to the domination of a component with negative pressure, called dark energy. So far, the nature of dark energy remains a mystery. Theoretically, the simplest candidate for such a component is a small positive cosmological constant, but it suffers the difficulties associated with the fine tuning and the coincidence problems. So many physicists are attracted by the idea of dynamical dark energy models, such as quintessence [4], phantom [5], k-essence [6], quintom [7] and so on (see [8, 9] for a review). Usually, dark energy models are constructed by scalar fields which are able to accommodate a rich variety of behaviors phenomenologically. However, there is another possibility that the acceleration of the universe is driven by a classical homogeneous spinor field. Some earlier studies on applications of spinor fields in cosmology are given in [10–12]. In recent years there are many works on studying spinor fields as gravitational sources in cosmology, for example: see [13, 14].
for inflation and cyclic universe driven by spinor fields; see [15] for spinor matter in Bianchi type I spacetime; see [16, 17] for a dark energy model with spinor matter and so on.

Although the recent fits to the data in the combination of the 3-year WMAP [18], the recently released 182 SNIa gold sample [19] and also other cosmological observational data remarkably show the consistency of the cosmological constant. It is worth noting that a class of dynamical models with the equation-of-state (EoS) across \(-1\) quintom is mildly favored [20, 21]. In the literature there have been a lot of theoretical studies of quintom-like models. For example, motivated from string theory, the authors of [22] realized a quintom scenario by considering the non-perturbative effects of a generalized DBI action. Moreover, a no–go theorem has been proven to constrain the model building of quintom [23], and according to this no–go theorem there are models which involve higher derivative terms for a single scalar field [24], models with vector field [25], making use of an extended theory of gravity [26], non-local string field theory [27] and others (see, e.g., [28–30]). The similar work applied in scalar–tensor theory has also been studied in [31].

Usually, a quintom model involves a ghost field with its kinetic term to be negative, which leads to quantum instability. In this paper we study the dark energy model with spinor matter. Interestingly, we find that this type of model can realize the quintom scenario with its EoS across the cosmological constant boundary \(w = -1\) without introducing a ghost field. Instead, the derivative of its potential with respect to the scalar bilinear \(\bar{\psi} \psi\), which is defined as the mass term, becomes negative when the spinor field lies in the phantom-like phase. If this model can realize its EoS across \(-1\) more than one time, the total EoS of the universe can satisfy \(w \geq -1\) during the whole evolution which is required by the null energy condition (NEC) [32], and we expect to treat this process as a phase transition merely existing for a short while. Moreover, due to a perfect mathematical property of the spinor field, it is possible to combine the quintom scenario and the picture of Chaplygin gas with the EoS evolving from 0 to \(-1\) smoothly in spinor quintom. In the literature, a dark energy model of Chaplygin gas has been proposed to describe a transition from a universe filled with dust-like matter to an accelerated expanding stage, and hence it has been argued that the coincidence problem of dark energy may be alleviated in this model [33].

Interestingly, in our model we are able to evade the drawbacks of considering the phantom field, of which the kinetic energy is negative and so is unstable in quantum level. We note that [34] has investigated the quantum stability of a phantom phase of cosmic acceleration and shown that a super-acceleration phase can be obtained by quantum effects. Our model is different from that one since the super-acceleration is realized by the background contribution. However, it is interesting that both the two approaches are stable in quantum level since the first order of perturbation theory can be defined.

This paper is organized as follows. In section 2, we simply review the basic algebra of a spinor field in FRW universe which is minimally coupled to Einstein’s gravity. In section 3, we present the condition for the spinor field to realize a quintom scenario and give some detailed examples. In section 4, we provide a unified model of spinor quintom and a perfect fluid of Chaplygin gas by taking certain potentials. By solving the model numerically, we will study the evolution of its EoS and fraction of energy density. Section 5 is the conclusion and discussions of our paper.

2. Algebra of a spinor field

To begin with, we simply review the dynamics of a spinor field which is minimally coupled to Einstein’s gravity (see [35–37] for detailed introduction). Following the general covariance
principle, a connection between the metric $g_{\mu\nu}$ and the vierbein is given by

$$g_{\mu\nu}e^a_\mu e^b_\nu = \eta_{ab},$$  

(1)

where $e^a_\mu$ denotes the vierbein, $g_{\mu\nu}$ is the spacetime metric and $\eta_{ab}$ is the Minkowski metric with $\eta_{ab} = \text{diag}(1, -1, -1, -1)$. Note that the Latin indices represent the local inertial frame and the Greek indices represent the spacetime frame.

We choose the Dirac–Pauli representation as

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix},$$  

(2)

where $\sigma_i$ is Pauli matrices. One can see that the $4 \times 4$ $\gamma^a$s satisfy the Clifford algebra $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$. The $\gamma^a$ and $e^a_\mu$ provide the definition of a new set of Gamma matrices

$$\Gamma^\mu = e^a_\mu \gamma^a,$$  

(3)

which satisfy the algebra $[\Gamma^\mu, \Gamma^\nu] = 2g^{\mu\nu}$. The generators of the spinor representation of the Lorentz group can be written as $\Sigma^{ab} = \frac{1}{4}[\gamma^a, \gamma^b]$. So the covariant derivatives are given by

$$D_\mu \psi = (\partial_\mu + \Omega_\mu)\psi,$$  

$$D_\mu \bar{\psi} = \bar{\psi} \gamma^a \partial_\mu e^a_b,$$  

(4, 5)

where the Dirac adjoint $\bar{\psi}$ is defined as $\psi^* \gamma^0$. The $4 \times 4$ matrix $\Omega_\mu = \frac{1}{2}\omega_{\mu ab} \Sigma^{ab}$ is the spin connection, where $\omega_{\mu ab} = e^a_c \nabla_\mu e^b_b$ are Ricci spin coefficients.

By the aid of the above algebra we can write the following Dirac action in a curved spacetime background [14, 16, 38]:

$$S_\psi = \int d^4x \left[ \frac{i}{2} (\bar{\psi} \Gamma^\mu D_\mu \psi - D_\mu \bar{\psi} \Gamma^\mu \psi) - V(\bar{\psi}\psi) \right].$$  

(6)

Here, $e$ is the determinant of the vierbein $e^a_\mu$ and $V$ stands for the potential of the spinor field $\psi$ and its adjoint $\bar{\psi}$. Due to the requirement of covariance, the potential $V$ only depends on the scalar bilinear $\bar{\psi}\psi$ and ‘pseudo-scalar’ term $\bar{\psi} \gamma^5 \psi$. For simplicity we drop the latter term and assume that there is $V = V(\bar{\psi}\psi)$.

Varying the action with respect to the vierbein $e^a_\mu$, we obtain the energy–momentum tensor

$$T_{\mu\nu} = \frac{\delta S_\psi}{\delta e^a_\mu} e^a_\mu$$  

$$= \frac{i}{4} [\bar{\psi} \Gamma_\nu D_\mu \psi + \bar{\psi} \Gamma_\mu D_\nu \psi - D_\mu \bar{\psi} \Gamma_\nu \psi - D_\nu \bar{\psi} \Gamma_\mu \psi] - g_{\mu\nu} L_\psi.$$  

(7)

On the other hand, varying the action with respect to the field $\psi$, $\bar{\psi}$ respectively yields the equation of motion of the spinor,

$$i\Gamma^\mu D_\mu \psi - \frac{\partial V}{\partial \psi} = 0, \quad iD_\mu \bar{\psi} \Gamma^\mu + \frac{\partial V}{\partial \bar{\psi}} = 0.$$

(8)

Note that we use units $8\pi G = \hbar = c = 1$ and all parameters are normalized by $M_p = 1/\sqrt{8\pi G}$ in the paper.

3. A universe driven by spinor quintom

3.1. Dynamics of a spinor field

In this paper we deal with the homogeneous and isotropic FRW metric,

$$ds^2 = dt^2 - a^2(t) \, dx^2,$$  

(9)
where $a$ stands for the scale factor and we choose today’s scale factor $a_0 = 1$. Correspondingly, the vierbein are given by
\[ e_\mu^0 = \delta_\mu^0, \quad e_i^\mu = \frac{1}{a} \delta_i^\mu. \] (10)

Assuming the spinor field is space-independent, the equation of motion reads
\[ \dot{\psi} + \frac{3}{2} H \psi + i \gamma_0 \nabla V' \psi = 0, \] (11)
\[ \dot{\bar{\psi}} + \frac{3}{2} H \bar{\psi} - i \gamma_0 \nabla V' \bar{\psi} = 0, \] (12)

where a dot denotes a time derivative $\frac{d}{dt}$ and a prime denotes a derivative with respect to $\bar{\psi} \psi$, and $H$ is the Hubble parameter. Taking a further derivative, we can obtain
\[ \ddot{\psi} \bar{\psi} = N a^3, \] (13)

where $N$ is a positive time-independent constant and we define it as today’s value of $\bar{\psi} \psi$.

From the expression of the energy–momentum tensor in equation (7), we get the energy density and the pressure of the spinor field:
\[ \rho_\psi = T_{00} = V, \] (14)
\[ p_\psi = -T_{i}^{i} = V' \bar{\psi} \dot{\psi} - V, \] (15)

where equations (11) and (12) have been applied. The EoS of the spinor field, defined as the ratio of its pressure to energy density, is given by
\[ w_\psi \equiv \frac{p_\psi}{\rho_\psi} = -1 + \frac{V' \bar{\psi} \dot{\psi}}{V}. \] (16)

Simply taking the potential to be power-law-like $V = V_0 \left( \frac{\bar{\psi} \psi}{N} \right)^\alpha$ with $\alpha$ as a nonzero constant, we obtain a constant EoS:
\[ w_\psi = -1 + \alpha. \] (17)

In this case, the spinor matter behaves like a linear-barotropic-like perfect fluid. For example: if $\alpha = \frac{4}{3}$, we can get $\rho_\psi \sim a^{-4}$ and $w_\psi = \frac{1}{4}$, which is the same as radiation; if $\alpha = 1$, then $\rho_\psi \sim a^{-3}$ and $w_\psi = 0$, this component behaves like normal matter.

Furthermore, the spinor matter is able to realize the acceleration of the universe if $\alpha < \frac{2}{3}$. So it provides us with a potential motivation to construct a dynamical dark energy model with the spinor matter. Moreover, as introduced at the beginning of section 1, there is evidence in the recent observations to mildly support a quintom scenario with the EoS of dark energy across $-1$. In the following, we emphasize our investigation on constructing quintom dark energy model with the spinor field, which is called spinor quintom.

3.2. Evolutions of spinor quintom

To keep the energy density positive, one may see that there is $w_\psi > -1$ when $V' > 0$ and $w_\psi < -1$ when $V' < 0$ from equation (16). The former corresponds to a quintessence-like phase and the latter stands for a phantom-like phase. Therefore it requires the derivative of the potential $V'$ to change its sign if one expects a quintom picture. In terms of the variations of $V'$, it shows that there exist three categories of evolutions in spinor quintom:
(1) there is \( V' > 0 \rightarrow V' < 0 \), which gives a quintom-A scenario by describing the universe evolving from quintessence-like phase with \( w_\psi > -1 \) to phantom-like phase with \( w_\psi < -1 \);

(2) there is \( V' < 0 \rightarrow V' > 0 \), which gives a quintom-B scenario for which the EoS is arranged to change from below \(-1\) to above \(-1\);

(3) \( V' \) changes its sign for more than one time, then one can obtain a new quintom scenario with its EoS crossing \(-1\) many times, dubbed quintom-C scenario.

In the following, we will take different potentials of spinor quintom to provide the three kinds of evolutions mentioned above\(^3\).

To begin with, we shall investigate case (i) and provide a quintom-A model. We use the form of potential \( V = V_0[(\bar{\psi}\psi - b)^2 + c] \), where \( V_0, b, c \) are undefined parameters. Then we get \( V' = 2V_0(\bar{\psi}\psi - b) \) and the EoS:

\[
   w_\psi = \frac{(\bar{\psi}\psi)^2 - b^2 - c}{(\bar{\psi}\psi)^2 - 2b\bar{\psi}\psi + b^2 + c}.
\]

(18)

According to equation (13), one finds that \( \bar{\psi}\psi \) is decreasing along with an increasing scale factor \( a \) during the expansion of the universe. From the formula of \( V' \), we deduce that at the beginning of the evolution the scale factor \( a \) is very small, so \( \bar{\psi}\psi \) becomes very large and ensures \( V' > 0 \) at the beginning. Then \( \bar{\psi}\psi \) decreases along with the expanding of \( a \). At the moment of \( \bar{\psi}\psi = b \), one can see that \( V' = 0 \) which results in the EoS \( w_\psi = -1 \). After that \( V' \) becomes less than 0, the universe enters a phantom-like phase. Finally the universe approaches a de Sitter spacetime. This behavior is also obtained by the numerical calculation and is shown in figure 1. From this figure, one can read that the EoS \( w_\psi \) starts the evolution from 1, then mildly increases to a maximum and then begins to decrease. When \( \bar{\psi}\psi = b \), it reaches the point \( w_\psi = -1 \) and crosses \(-1\) from above to below smoothly. After that, the EoS sequentially decreases to a minimal value then increases and eventually approaches the cosmological constant boundary.

In case (ii) we choose the potential as \( V = V_0[-(\bar{\psi}\psi - b)\bar{\psi}\psi + c] \). Then one can obtain \( V' = 2V_0(-2\bar{\psi}\psi + b) \) and the EoS

\[
   w_\psi = \frac{-(-\bar{\psi}\psi)^2 - c}{(-\bar{\psi}\psi)^2 + b\bar{\psi}\psi + c}.
\]

(19)

Initially \( V' \) is negative due to the large values of \( \bar{\psi}\psi \). Then it increases to 0 when \( \bar{\psi}\psi = \frac{b}{2} \) whereinafter changes its sign and becomes larger than 0, in correspondence with the case (ii), dubbed quintom-B model. From figure 2, we can see that the EoS evolves from below \(-1\) to above \(-1\) then finally approaches \(-1\).

In the third case we explore a quintom scenario which gives the EoS across \(-1\) for two times in virtue of the potential

\[
   V = V_0[(\bar{\psi}\psi - b)^2\bar{\psi}\psi + c].
\]

(20)

From the expression of the potential, one has

\[
   V' = V_0(\bar{\psi}\psi - b)(3\bar{\psi}\psi - b),
\]

(21)

\(^3\) Note that we choose the potentials phenomenologically without any constraints from quantum-field theory or other consensus. From the phenomenological viewpoint, this is okay if we treat the background classically while deal with the perturbations in quantum level, just as what is done in inflation theory.
Figure 1. Plot of the evolution of the EoS in case (i) as a function of time. In the numerical calculation we take the potential of the spinor field as $V = V_0((\bar{\psi}\psi - b)^2 + c)$, where we choose $V_0 = 1.0909 \times 10^{-117}$, $b = 0.05$ and $c = 10^{-3}$ for the model parameters. For the initial condition we take $N = 0.051$.

Figure 2. Plot of the evolution of the EoS in case (ii) as a function of time. In the numerical calculation we take the potential of the spinor field as $V = V_0(\bar{\psi}\psi - b(\bar{\psi}\psi + c)$, where we choose $V_0 = 1.0909 \times 10^{-117}$, $b = 0.05$ and $c = 10^{-3}$ for the model parameters. For the initial condition we take $N = 0.051$.

and the EoS

$$w_\psi = \frac{2(\bar{\psi}\psi)^3 - 2b(\bar{\psi}\psi)^2 - c}{(\bar{\psi}\psi)^3 - 2b(\bar{\psi}\psi)^2 + b^2(\bar{\psi}\psi) + c}. \quad (22)$$

Evidently the equation $V' = 0$ has two solutions which are $\bar{\psi}\psi = b$ and $\bar{\psi}\psi = b^3$, thus $V'$ changes its sign two times. From the expression of the EoS, we find that $w_\psi > -1$ in the
beginning. When the value of $\bar{\psi}\psi$ equals $b$, it crosses $-1$ for the first time. After the first crossing, it enters the phantom-like state and continuously descends until it passes through its minimum, then ascends to $\bar{\psi}\psi = b^3/3$ and then experiences the second crossing, and eventually moves up to the quintessence-like phase. This is shown in figure 3. One can see that the big rip can be avoided in this case.

Moreover, taking the component of normal matter with the energy density $\rho_m \propto 1/a^3$ into consideration and the EoS $w_m = 0$, we can see that the EoS of the universe $w_u = w_\psi \rho_\psi/\rho_u$ satisfies the relation $w_u \geq -1$ in this case. As is argued in [32], NEC might be satisfied in the models though $w_\psi < -1$ only stays for a short period during the evolution of the universe.

One important issue of a dark energy model is the analysis of its perturbations. To study this issue, we might be able to learn to what degree the system is stable in both quantum and classical levels. Usually systems with $w < -1$ show some nasty instabilities; for example, a phantom universe suffers a big rip singularity. Although this singularity can be avoided in most quintom models which usually enter a de Sitter expansion in the final epoch, all the scalar models of quintom by now suffer a quantum instability since there are negative kinetic modes from ghost fields. Here we would like to show the perturbation theory of spinor quintom crudely. Since we do not introduce any ghost fields in our model, the crossing of phantom divide is achieved by the spinor field itself and does not perform any particular instabilities.

In order to simplify the derivative, we would like to redefine the spinor as $\psi_N \equiv a^7\psi$. Then perturbing the spinor field, one gives the perturbation equation as follows:

$$
\frac{d^2}{d\tau^2} \delta \psi_N - \nabla^2 \delta \psi_N + a^2[V'2 + i\gamma^0(HV' - 3HV''\bar{\psi}\psi)]\delta \psi_N
= -2a^2V'V''\delta(\bar{\psi}\psi)\psi_N - i\gamma^\mu\partial_\mu[aV''\delta(\bar{\psi}\psi)]\psi_N,
$$

(23)

where $\tau$ is the conformal time defined by $d\tau \equiv dt/a$. Since the right-hand side of the equation decays proportional to $a^{-3}$ or even faster, we can neglect those terms during the late-time evolution of the universe for simplicity.
From the above perturbation equation, we can read that the sound speed is equal to 1 which eliminates the instability of the system in short wavelength. Moreover, when the EoS $w$ crosses $-1$, we have $V' = 0$ at that moment and the eigen function of the solution to equation (23) in momentum space is a Hankel function with an index $\frac{1}{2}$. Therefore, the perturbations of the spinor field oscillate inside the Hubble radius. This is an interesting result, because in this way we might be able to establish the quantum theory of the spinor perturbations, just as what is done in inflation theory.

Note that the above derivative does not mean that spinor quintom is able to avoid any instabilities. We still do not study the effects of the right-hand side of equation (23) which may destroy the system under some certain occasions. Another possible instability may be from the quantum effect that our model is unable to be renormalized. The more detailed calculation will be investigated in the future works.

4. A unified model of quintom and Chaplygin gas

In the above analysis, we have learned that a spinor field with a power-law-like potential behaves like a perfect fluid with a constant EoS. However, it is still obscure to establish a concrete model to explain how a universe dominated by matter evolves to the current stage that is dominated by dark energy. In recent years, another interesting perfect fluid with an exotic EoS $p = -c/\rho$ has been applied into cosmology [33] in the aim of unifying a matter-dominated phase where $\rho \propto 1/a^3$ and a de Sitter phase where $p = -\rho$ which describes the transition from a universe filled with dust-like matter to an exponentially expanding universe. This so-called Chaplygin gas [33] and its generalization [39] has been intensively studied in a literature. Some possibilities for this model motivated by field theory are investigated in [40]. A model of Chaplygin gas can be viewed as an effective fluid associated with D-branes [41, 42], and can also be obtained from the Born–Infeld action [43, 44]. The combination of quintom and Chaplygin gas has been realized by the interacting Chaplygin gas model [45] as well as in the framework of Randall–Sundrum braneworld [46].

The Chaplygin gas model has been thoroughly investigated for its impact on the cosmic expansion history. A considerable range of models was found to be consistent with SN Ia data [47], the CMBR [48], the gamma-ray bursts [49], the x-ray gas mass fraction of clusters [50], the large scale structure [51] and so on.

Here, we propose a new model constructed by spinor quintom which combines the property of a Chaplygin gas. The generic expression of the potential is given by

$$V = \sqrt[1+\beta]{f(\bar{\psi}\psi)} + c, \quad (24)$$

where $f(\bar{\psi}\psi)$ is an arbitrary function of $\bar{\psi}\psi$. Altering the form of $f(\bar{\psi}\psi)$, one can realize both the Chaplygin gas and quintom scenario in a spinor field.

First, let us see how this model recovers a picture of generalized Chaplygin gas. We take $f(\bar{\psi}\psi) = V_0(\bar{\psi}\psi)^{1+\beta}$, and then the potential is given by

$$V = \sqrt[1+\beta]{V_0(\bar{\psi}\psi)^{1+\beta} + c}. \quad (25)$$

Due to this, we obtain its energy density and pressure

$$\rho_{\psi} = \sqrt[1+\beta]{V_0(\bar{\psi}\psi)^{1+\beta} + c}, \quad (26)$$

$$p_{\psi} = -c[V_0(\bar{\psi}\psi)^{1+\beta} + c]^\frac{\beta}{1+\beta}. \quad (27)$$
Now it behaves like a generalized Chaplygin fluid which satisfies the exotic relation $p_\psi = -\frac{c}{\rho_\psi}$.

To be more explicit, we take $\beta = 1$, then get the expressions of energy density and pressure to be

$$\rho_\psi = \sqrt{\frac{N^2}{a^6} + c}, \quad p_\psi = -\frac{c}{\rho_\psi}.$$  \hspace{1cm} (28)

In this case a perfect fluid of Chaplygin gas is given by a spinor field. Based on the above analysis, we may conclude that this simple and elegant model is able to mimic different behaviors of a perfect fluid so that it accommodates a large variety of evolutions phenomenologically.

In succession, we will use this model to realize a combination of a Chaplygin gas and a quintom-A model which is mildly favored by observations. Choosing $f(\bar{\psi}\psi)$ to be $f(\bar{\psi}\psi) = V_0(\bar{\psi}\psi - b)^2$, we get the potential

$$V = \sqrt{V_0(\bar{\psi}\psi - b)^2 + c},$$  \hspace{1cm} (29)

where $V_0, b, c$ are undetermined parameters. So we obtain the derivative of the potential

$$V' = \frac{V_0(\bar{\psi}\psi - b)}{\sqrt{V_0(\bar{\psi}\psi - b)^2 + c}}.$$  \hspace{1cm} (30)

and the EoS

$$w_\psi = -1 + \frac{V_0\bar{\psi}\psi(\bar{\psi}\psi - b)}{V_0(\bar{\psi}\psi - b)^2 + c},$$  \hspace{1cm} (31)

respectively, and the crossing over $-1$ takes place when $\bar{\psi}\psi = b$.

During the expansion $\bar{\psi}\psi$ is decreasing following equation (13). From the formula of the EoS (31), we deduce that at the beginning of the evolution the scale factor $a \to 0$ so $\bar{\psi}\psi \to \infty$. To neglect the terms of lower order of $\bar{\psi}\psi$ and the constants the EoS at early times evolves from 0 which describes the era of matter dominated. Along with the evolution, $w_\psi$ increases from 0 to the maximum and then starts to decrease. When $\bar{\psi}\psi = b$ the EoS arrives at the cosmological constant boundary $w_\psi = -1$ and then crosses it. Due to the existence of the $c$ term, the EoS approaches the cosmological constant boundary eventually. In this case the universe finally becomes a de Sitter spacetime.

Considering a universe filled with normal matter and such a spinor quintom matter, we take the numerical calculation and show the evolution of the EoS in figure 4. Moreover, we display the evolvement of fraction densities of the normal matter $\Omega_m \equiv \rho_m/(\rho_m + \rho_\psi)$ and spinor quintom $\Omega_\psi \equiv \rho_\psi/(\rho_m + \rho_\psi)$ in figure 5. It is evident that there is an exact ratio of these two components from the beginning of evolution, in relief of fine-tuning problems and accounting for the coincidence problem. Then they evolve to be equal to each other in late times. Along with the expansion of $a$, the dark energy density overtakes the matter energy density driving the universe into an accelerating expansion at present and eventually dominates the universe completely, which describes an asymptotic de Sitter spacetime.

5. Conclusion and discussions

The current cosmological observations indicate the possibility that the acceleration of the universe is driven by dark energy with EoS across $-1$, which will challenge the theoretical model building of the dark energy if confirmed further in the future. In this paper we have
Figure 4. Plot of the EoS of the unified model in equation (29) as a function of time. In the numerical calculation we take $V_0 = 3.0909 \times 10^{-239}$, $b = 0.05$ and $c = 9 \times 10^{-241}$. For the initial conditions we take $N = 0.051$.

Figure 5. Plot of the fraction of energy density of dark energy (the violet solid line) and normal matter (the red dash line) as a function of time.

studied various quintom scenarios in virtue of a spinor field and proposed a unified model of spinor quintom and a generalized Chaplygin gas. As shown in the present work, this model can give rise to the EoS crossing the cosmological constant boundary during the evolution by varying the sign of the term $V'$. Compared with other models with $w$ across $-1$ in the literature, so far the present one is also economical in the sense that it merely involves a single spinor field.
Acknowledgments

It is a pleasure to thank Yun-Song Piao, Taotao Qiu, Jun-Qing Xia, Shiping Yang, Xin Zhang and Xinmin Zhang for helpful discussions. CYF acknowledges two anonymous referees for their valuable suggestions. CYF also thanks Haiying Xia and Yi Wang for checking spelling typos. This work is supported in part by the National Natural Science Foundation of China under grant nos. 90303004, 10533010, 10675136 and 10775180 and by the Chinese Academy of Science under grant no. KJCX3-SYW-N2.

References

[1] Perlmutter S et al (Supernova Cosmology Project Collaboration) 1997 Astrophys. J. 483 565 (Preprint astro-ph/9608192)
[2] Riess A G et al (Supernova Search Team Collaboration) 1998 Astron. J. 116 1009 (Preprint astro-ph/9805201)
[3] Seljak U et al (SDSS Collaboration) 2005 Phys. Rev. D 71 103515 (Preprint astro-ph/0407372)
[4] Ratra B and Peebles P J E 1988 Phys. Rev. D 37 3406
[5] Caldwell R R 2002 Phys. Lett. B 545 23 (Preprint astro-ph/0008168)
[6] Arnowitt-Picon C, Mukhanov V F and Steinhardt P J 2000 Phys. Rev. Lett. 85 4438 (Preprint astro-ph/9904034)

Chiba T, Okabe T and Yamaguchi M 2000 Phys. Rev. D 62 023511 (Preprint astro-ph/9912463)
[7] Feng B, Wang X L and Zhang X M 2005 Phys. Lett. B 607 35 (Preprint astro-ph/0404224)
[8] Copeland E J, Saini M and Tsujikawa S 2006 Int. J. Mod. Phys. D 15 1753 (Preprint hep-th/0603057)
[9] Padmanabhan T 2007 Preprint arXiv:0705.2553
[10] Taub A 1937 Phys. Rev. 51 512
[11] Brill D R and Wheeler J A 1957 Rev. Mod. Phys. 29 465
[12] Parker L 1971 Phys. Rev. D 3 346
[13] Parker L 1971 Phys. Rev. D 3 2546 (erratum)
[14] Obukhov Y N 1993 Phys. Lett. A 182 214 (Preprint gr-qc/0008015)
[15] Armendariz-Picon C and Greene P B 2003 Gen. Rel. Grav. 35 1637 (Preprint hep-th/0311129)
[16] Saha B 2001 Mod. Phys. Lett. A 16 1287 (Preprint gr-qc/0009002)
[17] Saha B 2001 Phys. Rev. D 64 123501 (Preprint gr-qc/0107013)
[18] Saha B and Boyadzhiev T 2004 Phys. Rev. D 69 124010 (Preprint gr-qc/0311045)
[19] Ribas M O, Devecchi F P and Kremer G M 2005 Phys. Rev. D 72 123502 (Preprint gr-qc/0511099)
[20] Ribas M O, Devecchi F P and Kremer G M 2007 Preprint arXiv:0710.5155
[21] Chimento L P, Devecchi F P, Forte M and Kremer G M 2007 Preprint arXiv:0707.4455
[22] Cataldo M and Chimento L P 2007 Preprint arXiv:0710.4306
[23] Spergel D N et al (WMAP Collaboration) 2007 Astrophys. J. Suppl. 170 377 (Preprint astro-ph/0603447)
[24] Riess A G et al 2006 Preprint astro-ph/0611572
[25] Zhao G B, Xia J Q, Li H, Tao C, Virey J M, Zhu Z H and Zhang X 2007 Phys. Lett. B 648 8 (Preprint astro-ph/0612728)
[26] Wang Y and Mukherjee P 2006 Astrophys. J. 650 1 (Preprint astro-ph/0604051)
[27] Cai Y F, Li M Z, Lu J X, Piao Y S, Qu T T and Zhang X M 2007 Phys. Lett. B 651 1 (Preprint hep-th/0701016)
[28] Xia J Q, Cai Y F, Qu T T, Zhao G B and Zhang X 2007 Preprint astro-ph/0703202
[29] Li M Z, Feng B and Zhang X M 2005 J. Cosmol. Astropart. Phys. JCAP12(2005)002 (Preprint hep-th/0503268)
[30] Armendariz-Picon C 2004 J. Cosmol. Astropart. Phys. JCAP07(2004)007 (Preprint astro-ph/0405267)
[31] Wei H and Cai R G 2006 Phys. Rev. D 73 083002 (Preprint astro-ph/0603052)
[32] Cai R G, Zhang H S and Wang A 2005 Commun. Theor. Phys. 44 948 (Preprint hep-th/0505186)
[33] Chimento L P, Lukzov R, Maarens R and Quiros I 2006 J. Cosmol. Astropart. Phys. JCAP09(2006)004 (Preprint astro-ph/0605450)
[34] Acef’eva I Y, Koshelev A S and Vernov S Y 2005 Phys. Rev. D 72 064017 (Preprint astro-ph/0507067)
[35] Vernov S Y 2006 Preprint astro-ph/0612487
[36] Koshelev A S 2007 J. High Energy Phys. JHEP04(2007)029 (Preprint hep-th/0701013)
[37] Cai Y F, Li H, Piao Y S and Zhang X M 2007 Phys. Lett. B 646 141 (Preprint gr-qc/0609039)
[38] Cai Y F, Qu T, Piao Y S, Li M and Zhang X 2007 J. High Energy Phys. JHEP10(2007)071 (Preprint arXiv:0704.1090)
