A Fusion Method for Satellite Multi-sensor Based on Improved Support Matrix

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Abstract. In this paper, a satellite multi-sensor data fusion method based on improved support matrix is proposed. The cross-correlation function is used to describe the support degree between multiple sensors, and the measured signals are weighted and fused optimally from the SNR perspective. Compared with the method of distance measurement, this method does not need to set threshold and the calculation process is simpler. Simulation results show that this method can objectively reflect the reliability of each sensor, effectively eliminate redundancy and error information, and select the optimal sensor combination.

1. Introduction
In the process of measuring and controlling the satellite in orbit, the measurement data are uncertain due to the inevitable influence of external environment noise, sensor fault and transmission error, etc. Therefore, multiple sensors are usually used to measure the same target at the same time, and then certain criteria and algorithms are used for data fusion processing to obtain a more accurate conclusion than the measurement of a single sensor.

In the fusion algorithm based on support matrix, distance is often used as support degree. Based on the basic principle of data statistics, a conditional probability distance is defined [1]. A multi-sensor consistent data fusion method based on the nearest statistical distance is proposed [2]. The definition of fusion degree between different sensors in the sense of statistical distance is proposed, and a method of relation matrix that can better avoid subjective factors is given [4]. A data fusion method based on eigenvalues and eigenvectors of support matrix based on confidence distance is proposed [5]. An adaptive weighted fusion algorithm based on confidence distance measure is proposed [6]. In the above methods, a threshold value needs to be set to obtain the effectiveness of the support matrix for the sensor. However, the selection of threshold is based on experience and the calculation process is very complex.

To sum up, this paper proposes a data fusion method for satellite multi-sensor based on improved support matrix. This method describes the degree of support between multiple sensors by means of cross-correlation function and performs optimal weighted fusion of measured signals from the perspective of SNR. It does not need to set a threshold, and the calculation process is simpler.

2. Definition of support matrix
Suppose there are \( n \) independent sensors to measure the same physical quantity, and the measured values follow normal distribution. \( x_i \) and \( x_j \) are the data measured by sensors \( i \) and \( j \) respectively.
Due to the existence of error and interference, there is a certain deviation between the two data. Set \( r_{ij} \) to reflect the deviation of these two data, and call it support degree, where \( r_{ij} \in [0,1] \). The higher \( r_{ij} \) is, the better the compatibility of the two data is, and the lower \( r_{ij} \) is, the worse the compatibility is. Then a support matrix of order \( n \times n \) as shown in Eq. (1) can be obtained.

\[
R_{ij} = \begin{bmatrix}
    r_{11} & r_{12} & \cdots & r_{1n} \\
    r_{21} & r_{22} & \cdots & r_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{n1} & r_{n2} & \cdots & r_{nn}
\end{bmatrix}
\]  

(1)

3. An improved support matrix

In order to reflect more characteristics of measured data and avoid the uncertainty caused by time-domain peak, an improved support matrix based on cross-correlation function is proposed in this paper. It adopts a support function selection method different from the distance measure: cross-correlation function maximum. The cross-correlation function describes the degree of correlation between two sets of random signals at any two different times, which reflects the degree of similarity between them at different times, including peak value, period and waveform, etc. When the cross-correlation function reaches the maximum value, that is, the moment when two groups of data are closest to each other, it is more convincing to use the maximum value to represent the support function than the single peak correlation.

The cross-correlation function is shown as Eq. (2).

\[
d(\tau) = \frac{1}{T} \int_{0}^{T} [f(\tau)g(\tau + T)]d\tau
\]  

(2)

Eq. (2) is discretized, as shown in the following equation.

\[
d(n) = \frac{1}{N} \sum_{m=0}^{m=N-1} [x(m)y(m+n)]
\]  

(3)

Set the confidence measure as follows.

\[
d_{ij} = \max\left(\frac{1}{N} \sum_{m=0}^{m=N-1} [x_i(m)y_j(m+n)]\right)
\]  

(4)

If the maximum value of cross-correlation functions of \( x_i \) and \( x_j \) is \( c_i \) and \( c_j \), then the definition of support degree is defined as the following Eq. (5).

\[
r_{ij} = \exp\left(\frac{d_{ij}}{c} - 1\right)
\]  

(5)

where \( c = \frac{c_i}{c_i + c_j} \times c_i + \frac{c_j}{c_i + c_j} \times c_j \).

If and only if \( i = j \), \( d_{ij} = c \). At this point, the cross-correlation support degree of \( x_i \) is defined as \( r_{ij} = 1 \), indicating that the two are exactly the same. When \( i \neq j \), \( d_{ij} < c \), and \( 0 < r_{ij} < 1 \). At this point, the higher \( r_{ij} \) means the higher correlation, which means the higher support degree, and the lower it is on the contrary.
In order to reflect the comprehensive support degree of $x_i$ by other sensors’ observed values, a comprehensive support function is defined as the following Eq. (6).

$$s_i = \sum_{j=1}^{j=n} r_{ij}, \quad i = 1, 2, \cdots, m$$

(6)

An average support function is defined as the following Eq. (7).

$$p_i = \frac{1}{m-1} \sum_{j=1}^{j=n, j \neq i} r_{ij}, \quad i = 1, 2, \cdots, m$$

(7)

where $p_i < 1$, $p_i$ directly reflects the average support degree of $x_i$ by other sensors’ observed values. The higher $p_i$ is, the higher the support degree will be, and vice versa.

4. Optimal weight fusion algorithm

Optimal weighted data fusion is to determine the optimal weighted coefficient of each group of measured data according to certain criteria and methods. Then the optimal weighted coefficient is multiplied and accumulated to obtain the optimal fusion data. Optimal weighted data fusion can be expressed as follows.

$$\hat{x} = \sum_{i}^{n} h_i x_i$$

(8)

where $h_i$ is the weighting coefficient of each group of data, $\sum_{i}^{n} h_i = 1$ and $h_i > 0$.

Its core is to find an optimal weighting coefficient. At present, the commonly used methods include the noise variance method based on each group of data and the SNR method. Because it is difficult to obtain the noise variance of data in practical application, this paper takes the perspective of signal-to-noise ratio.

Data fusion algorithm can be considered as a multi-group data filtering process. In order to achieve the optimal filtering, the signal-to-noise ratio of the signal is required to reach the highest, that is, the ratio of signal energy $W_s$ to noise energy $W_n$ is the highest. The energy can be represented by the cross-correlation function of the signal.

Set the observed $m$ groups of data are as follows.

$$x = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1l} \\ x_{21} & x_{22} & \cdots & x_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{ml} \end{bmatrix}$$

(9)

The optimal weighting coefficient corresponding to each group is defined as follows.

$$h = (h_1, h_2, \cdots, h_m)$$

(10)

Then the signal after fusion is as the following Eq. (11).

$$s = hx = (h_1, h_2, \cdots, h_m) \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1l} \\ x_{21} & x_{22} & \cdots & x_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{ml} \end{bmatrix}$$

(11)

The cross-correlation function of the signal energy is as follows.
\[ W_s = s^T = h x(h x)^T = h x x^T h^T = h R h^T \]  

where \( R \) is the cross-correlation matrix of the observed signals with non-negative symmetry.

Before the experiment, the effective signal is zero. At this point, the measured signals of each group can be considered as noise signals, denoted as \( V \), then the noise signal after fusion is as the following Eq. (13).

\[
\begin{bmatrix}
v_{11} & v_{12} & \cdots & v_{1l} \\
v_{21} & v_{22} & \cdots & v_{2l} \\
\vdots & \vdots & \ddots & \vdots \\
v_{m1} & v_{m2} & \cdots & v_{ml}
\end{bmatrix}
\]

\[ n = h V = h \]

Then the noise energy is as follows.

\[ W_n = n n^T = h n(h n)^T = h n n^T h^T = h N h^T \]

where \( N \) is the cross-correlation matrix of the noise signals with non-negative symmetry.

The signal and noise energy obtained here are not based on the same length, so divide the energy of signal and noise by their respective lengths to get the energy density \( w_s' \) and \( w_n' \). The SNR is obtained from the energy density, as shown in Eq. (15).

\[
\lambda = \frac{W_s'}{W_n'} = \frac{h R h^T}{h N h^T} \rightarrow \max
\]

Form \( \frac{\partial \lambda}{\partial h} = 0 \), the following equations are established.

\[
\begin{cases}
\frac{\partial \lambda}{\partial h_1} = 0 \\
\frac{\partial \lambda}{\partial h_2} = 0 \\
\vdots \\
\frac{\partial \lambda}{\partial h_m} = 0
\end{cases}
\]

5. Simulation analysis

In the simulation experiment, observation data of four temperature sensors of an in-orbit satellite attitude control system are selected, as shown in Figure 1. The method of this paper is applied to the data fusion of these four groups of data.
By calculating the support matrix and calculating the average support, the data with low support are excluded.

The support matrix is obtained as follows:

\[
\begin{bmatrix}
1 & 0.663 & 0.766 & 0.772 \\
0.663 & 1 & 0.703 & 0.668 \\
0.766 & 0.703 & 1 & 0.781 \\
0.772 & 0.668 & 0.781 & 1
\end{bmatrix}
\]

The average support degree can be obtained from Equation (6).

\[
p = \begin{bmatrix}
0.734 \\
0.678 \\
0.750 \\
0.740
\end{bmatrix}
\]

It can be concluded from the above formula that the data support degree of Sensor 2 is significantly lower than that of the other 3 sensors. Therefore, the second group of data is discarded, and only the first, third and fourth groups of data are used for data fusion. According to Equation (16), the optimal weighted fusion coefficient can be obtained as follows. Subsequently, the fusion coefficient can be used to fuse the 3 groups of selected data.

\[
h_1 = 0.32, \quad h_3 = 0.38, \quad h_4 = 0.30
\]
In order to verify the existence of anomalies in sensor 2's data, spectrum analysis is performed on these four sets of original data, and the analysis results are shown in Figure 2.

![Figure 2. Spectral analysis results of four sets of original observation data](image)

As shown in Figure 2, it can be found that the data spectrum measured by Sensor 2 is indeed abnormal. This proves that the proposed method can effectively eliminate redundancy and error information and thus select the optimal sensor combination.

6. Conclusions

In view of the deficiency of the support matrix based on distance measure, an improved support matrix based on the maximum value method of cross-correlation function is proposed in this paper. It describes the degree of support between multiple sensors by means of cross-correlation function. On this basis, a data fusion method for satellite multi-sensor based on improved support matrix is proposed. This method performs the optimal weighted fusion of the measured signals from the SNR perspective, does not need to set the threshold, and the calculation process is simpler. The simulation experiment is carried out with the in-orbit measurement data of a satellite. The results show that this method can objectively reflect the reliability of each sensor, effectively eliminate redundancy and error information, and select the optimal sensor combination.

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