Increment Dynamic Analysis for One-Story Asymmetric Plan Systems under Hard Soil Site

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Abstract. Considering the multi-dimensional feature of earthquake motions and the torsion response can aggravate destroying of structure, the one-storey asymmetric plan system with three degree of freedom is established. The yield rule of this asymmetric system is determined by two-dimensional yield-surface plasticity function. Bi-directional Earthquake motion records for hard soil site are selected as the excitations of this asymmetric system. The increment dynamic analysis for this system is conducted, and increment dynamic analysis capacity curves are given in which peak ground acceleration is used as earthquake intensity parameter, the ductility factor and the maximum rotation angle are used as performance parameters, respectively. The effect of eccentricity, frequency ratio on responses of the one-storey asymmetric plan system, are analyzed based on the two types of capacity curves.

1. Introduction
The concept of increment dynamic analysis (IDA) was proposed by Bertero [1] in 1977. The method was adopted by the American criteria FEMA350 and FEMA351 as a major analysis method of structure anti-collapse capacity in 2000. In 2002, Vamvatsikos and Cornell proposed a specific application procedure of IDA and applied the procedure in the anti-seismic research of concrete-frame structures and steel-frame structures. Recently, many researchers have applied the IDA method to the analysis, design, and evaluation of structure anti-collapse and anti-seismic performances [2-5].

At present, the studies on IDA and its application in structural seismic analysis have achieved a certain number of results [6, 7], but at least two problems still exist that limit their application in engineering. (1) The IDA method requires a certain number of ground motions as structural excitation, and the intensity amplitude must be adjusted at each level and the dynamic time history analysis must be repeated for each ground motion excitation. Therefore, the IDA calculation for the original structure model is very large and takes a long time, which is not convenient for engineering applications.(2) The corresponding research findings of IDA almost belong to symmetric structures subjected to single direction earthquake motions. However, the torsion response can aggravate destroying of the asymmetry-plan structures [8, 9], and the torsion coupling response will induce the structural space effects that can’t be solved in two dimensional analysis.

For the reasons, the one-story asymmetric system is established, which can be used as an equivalent system of a asymmetric structure. The IDA is conducted based on selected earthquake motion records of hard soil site, and the influence factors are analyzed on the IDA results.
2. One-story asymmetric plan system

2.1. Equation of motion

The system considered is an idealized one-story asymmetric plan system. The resisting plan in the \( x \) and \( y \)-direction are symmetrically located about the centre of mass (CM) but have different stiffness, causing the eccentricities \( e_y \) and \( e_x \) between the centre of stiffness (CS) and the CM. The motion equation of the system is expressed as:

\[
\begin{bmatrix}
mgx & 0 & 0 \\
0 & mg & 0 \\
0 & 0 & mr^2
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_x(t) \\
\ddot{u}_y(t) \\
\ddot{u}_\theta(t)
\end{bmatrix}
+ \frac{2\xi}{\omega_1 + \omega_2}
\begin{bmatrix}
K_x + \omega_1 \omega_2 m & 0 & -e_y K_x \\
0 & K_y + \omega_1 \omega_2 m & e_y K_y \\
-e_x k_x & e_x k_y & K_\theta + \omega_1 \omega_2 mr^2
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_x(t) \\
\ddot{u}_y(t) \\
\ddot{u}_\theta(t)
\end{bmatrix}
+ \begin{bmatrix}
f(u_x, u_y, t) \\
f(u_y, u_y, t) \\
f(u_\theta, u_\theta, t)
\end{bmatrix}
= \begin{bmatrix}
m \ddot{u}_x(t) \\
m \ddot{u}_y(t) \\
0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_x(t) \\
\ddot{u}_y(t) \\
\ddot{u}_\theta(t)
\end{bmatrix}
\]  

(1)

in which \( \omega_1 \) and \( \omega_2 \) are the 1-th and 2-th vibration frequencies of the system, respectively; \( \xi \) is damping ratio; \( u_x(t), u_y(t) \) and \( u_\theta(t) \) are responses of displacement along \( x \), \( y \) and \( \theta \) - directions, respectively; \( u_\theta(t), u_g(t) \) are displacement of ground motions along \( x \) and \( y \)-direction, respectively; \( K_x \) and \( K_y \) are the elastic lateral stiffness in the \( x \) and \( y \)-direction, and \( K_\theta \) is the torsional stiffness; \( r \) is the radius of gyration of the deck about the CM. Defining \( u_\phi = r u_\theta \), the Eq.1 can be deduced as:

\[
\begin{bmatrix}
\ddot{u}_x \\
\ddot{u}_y \\
\ddot{u}_\theta
\end{bmatrix}
+ \frac{2\xi}{\omega_1 + \omega_2}
\begin{bmatrix}
\omega_x^2 + \omega_1 \omega_2 & 0 & -e_y \omega_x^2 \\
0 & \omega_y^2 + \omega_1 \omega_2 & e_y \omega_y^2 \\
-e_x \omega_x^2 & e_x \omega_y^2 & \omega_\theta^2 + \omega_1 \omega_2
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_x \\
\ddot{u}_y \\
\ddot{u}_\theta
\end{bmatrix}
+ \begin{bmatrix}
f(u_x, u_\theta, t) \\
f(u_y, u_\theta, t) \\
f(u_\theta, u_\theta, t) / r
\end{bmatrix}
= \begin{bmatrix}
m \ddot{u}_x(t) \\
m \ddot{u}_y(t) \\
0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_x(t) \\
\ddot{u}_y(t) \\
\ddot{u}_\theta(t)
\end{bmatrix}
\]  

(2)

in which \( e_x/r \) and \( e_y/r \) are regarded as the normalized stiffness eccentricities in \( x \) and \( y \)-direction, respectively. The relative torsional to lateral stiffness is defined by the ratio \( \Omega_\theta = \omega_\theta \omega_1 \), in which \( \omega_\theta = (K_\theta/m)^{0.5} \) and \( \omega_1 = (K_x/mr^2)^{0.5} \) correspond to the natural frequencies of an associated symmetric \((e_x = e_y)\) elastic system with the same mass and stiffness, where \( K_\theta = K_x - e_x^2 K_\theta \) and \( K_\theta \) is the torsional stiffness of the structure about the CS, and then \( \omega_\theta^2 = \Omega_\theta^2 \omega_1^2 + (e_x/r)^2 \omega_y^2 + (e_y/r)^2 \omega_x^2 \).

2.2. Restoring force property

Yield rule of the one-storey asymmetric plan system is determined by two-dimensional yield-surface plasticity function which can be expressed as:

\[
F = \left( \frac{f(u_x, u_\theta, t)}{f_x, y} \right)^2 + \left( \frac{f(u_y, u_\theta, t)}{f_y, y} \right)^2
\]  

(3)

In which \( f_x, s \) and \( f_y, s \) are yield forces of equivalent SDOF systems for \( x \) and \( y \)-directions. The elastic perfectly-plastic response property is presumed as restoring force property of the system in which the rules of loading and unloading are given as follows:

\[
F < 1.0 \quad \text{elastic stage}
\]  

(4a)
When the function $F < 1.0$, representing linear elastic stage, the tangent stiffness matrix $K_t$ of the system is expressed as:

$$K_t = K_e$$

Where $K_e$ is elastic stiffness matrix of the system. When the function $F = 1.0$, representing coupling of responses for two orthogonal components, the tangent stiffness matrix $K_t$ of the system is expressed as:

$$K_t = K_e - \frac{K_e \cdot \frac{\partial F}{\partial \dot{f}} \left( \frac{\partial F}{\partial \dot{f}} \right)^T \cdot K_e}{\left( \frac{\partial F}{\partial \dot{f}} \right)^T \cdot K_e \cdot \frac{\partial F}{\partial \dot{f}}}$$

Where $\frac{\partial F}{\partial \dot{f}}$ is the partial derivative vector defined as the instantaneous rate of change of a function $F$ with respect to the vector $f = \{f(u_x, u_y, t), f(u_y, u_{\theta}, t)\}^T$.

3. **IDA principle**

The IDA procedure is an extension of Pushover analysis, which is also a recently developed dynamic parameter analysis method used in structural seismic performance evaluation. The general process of the IDA procedure is as follows: the acceleration of a ground movement is multiplied by a series of proportional factors so as to continually adjust the intensity measure (IM); thereafter, a set of ground motion records with increasing monotone intensity are generated. Then, a non-linear dynamic analysis of the structure is conducted based on each ground motion record, thereby obtaining the corresponding structural damage measure (DM). Both the IM and its corresponding DM are plotted in a coordinate system with IM as the y-axis and DM as the x-axis, to generate a DM-IM format IDA capacity curve. According to the IDA capacity curve, the seismic demand of the structure and the structural performance characteristics accompanied by the gradually increasing seismic motion can be visually determined, thus the seismic performance of a structure can be evaluated.

4. **Selected earthquake motion records**

In order to analyze the regularities of IDA capacity curves for the one-storey asymmetric system under hard soil site, 8 pair (16) earthquake motion records of hard soil site ($I_0, l_1$) are selected and listed in Table 1.
Table 1. The information of earthquake motion records

| Earthquake Station | Fault distance | Components |
|--------------------|----------------|------------|
| Loma Prieta (89/10/18,Ms7.1) 1652 Anderson Dam 20km | AND270 | AND360 |
| Northridge (94/1/17,Ms6.7) 24157 LA Baldwin Hills 31km | BLD090 | BLD360 |
| Northridge (94/1/17,Ms6.7) 14403 LA 116th St School 42km | 116090,116360 | |
| Landers (92/6/28,Ms7.4) 23 Coolwater 21km | CLW-LN | CLW-TR |
| Imperial Valley (79/10/15,Ms6.9) 6604 Cerro Prieto 27km | H-CPE147 | H-CPE237 |
| Loma Prieta (89/10/18,Ms7.1) 57504 Coyote Lake Dam 22km | CLD195 | CLD285 |
| ChiChi (99/9/20,Ms7.6) Tcu045 24km | TCU045-N | TCU045-W |
| ChiChi (99/9/20,Ms7.6) Tcu047 33km | TCU047-N | TCU047-W |

5. IDA of one-story asymmetric plan system

IDA of one-story asymmetric plan system is conducted based on selected earthquake motions. The system has three degrees of freedom: two mutually perpendicular horizontal components (x and y), and a rotational component. Peak ground acceleration (PGA) was used as the earthquake intensity parameter. IDA capacity curves were constructed using the ductility factor $\mu$ of the x component and the maximum rotation angle $\theta$ of the rotational component as structure performance parameters.

Figures 1 and 2 illustrate $\mu$-PGA format and $\theta$-PGA format IDA capacity curves, respectively, for the hard soil site.

The analysis of the parameters through IDA capacity curves indicates the follows:

1. The eccentricity $e/r$, frequency ratio $\Omega$, and horizontal bidirectional period ratio had little effect on the translational motion of the mass center of one-story asymmetric plan systems. The $\mu$-PGA format IDA capacity curves based on different parameters are basically the same.

2. The eccentricity $e/r$ and frequency ratio $\Omega$ significantly affected the rotational responses of the systems. A systematic analysis using IDA can be conducted.

A comparison between Figure 1 and 2 indicates that the two types of IDA capacity curves are different in shapes. With the increase of earthquake intensity, the increase in the structural deformation of the $\theta$-PGA format IDA capacity curves (Figure 2) tended to reduce; whereas, the $\mu$-PGA format IDA capacity curves (Figure 1) showed the opposite trend.

![Figure 1. $\mu$-PGA format IDA capacity curves](image1)

![Figure 2. $\theta$-PGA format IDA capacity curves](image2)
Figure 3 shows the effect of the frequency ratio $\Omega_\theta$ on IDA capacity curves in $\theta$-PGA format. As shown in Figure 3, with the increase in $\Omega_\theta$, for the same PGA, the maximum rotation angle gradually increased, and the amount of increase was greater for a greater $\Omega_\theta$. In general, the rotation frequency is no more than the translational motion frequency, so the maximum $\Omega_\theta$ was set to 1 in this study. Figure 4 shows the effect of eccentricity $e/r$ on IDA capacity curves in $\theta$-PGA format. As shown in Figure 4, with the increase in the eccentricity, for the same PGA, the maximum rotation angle gradually increased.

**Figure 3.** The influence of $\Omega_\theta$ on IDA curves

**Figure 4.** The influence of $e/r$ on IDA curves
6. Conclusion
The one-story asymmetric plan system is established with three degree of freedom, and the yield rule of the systems is determined by two-dimensional yield-surface plasticity function. Earthquake motion records for hard soil site are selected as the excitations of the asymmetric plan system. IDA are conducted, and the capacity curves are established in which PGA is used as earthquake intensity parameter, and and are used as performance parameters, respectively. The effect of eccentricity and frequency ratio on the two kinds of IDA capacity curves are analyzed. The analysis results as follows:

(1) The eccentricity , frequency ratio , and horizontal bidirectional period ratio have little effect on the translational motion of mass center of one-story asymmetric plan system under earthquake excitation.

(2) The eccentricity and frequency ratio significantly affect the rotation reaction of one-story asymmetric plan system. With the decrease of , for the same PGA, the maximum rotation angle gradually increases, and the amount of increase was greater for a greater . With the increase in eccentricity, for the same PGA, the maximum rotation angle gradually increases.

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