Reheating neutron stars with the annihilation of self-interacting dark matter

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Abstract

Compact stellar objects such as neutron stars (NS) are ideal places for capturing dark matter (DM) particles. We study the effect of self-interacting DM (SIDM) captured by the nearby NS that can reheat NS to an appreciated surface temperature. Recently, DM self-interaction was considered as an negligible effect due to its small geometric cross section in NS. However, we will demonstrate when DM-nucleon cross section $\sigma_{\chi n}$ is much smaller than the current direct search limits, DM self-interaction will dominate the capture process. As a result of small $\sigma_{\chi n}$, DM will not thermalize with NS and its decoupled temperature $T_{\chi}^{\text{dec}}$ is as high as a certain temperature of NS in the early evolution stage. In particular, a higher $T_{\chi}^{\text{dec}}$ will induce a larger DM thermal radius. It increases DM self-capture rate and leads to the stronger DM annihilation rate. The energy injection to NS will be more thus reheat the star. Such effect results from DM self-interaction but it behaves as DM having a relatively large $\sigma_{\chi n}$. The NS temperatures are produced from the interplays between DM-nucleon and DM-DM interactions. In certain parameter region, there are two possible solutions that will generate the same NS temperature. We will also show that the reheating NS surface temperature by SIDM cannot be arbitrary high. It saturates at hundreds of Kelvins depending on the DM mass. The corresponding blackbody peak wavelength is potentially detectable in the future telescopes.

Keywords: infrared telescopes, neutron star temperature, self-interacting dark matter

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I. INTRODUCTION

Dark matter (DM) composes one-fourth of the Universe, however, its essence is still elusive. Many terrestrial detectors are built to reveal the particle nature of DM either from measuring the coupling strength between DM and the Standard Model (SM) particles [1–7] or the indirect signal from DM annihilation in the space [8–13]. But the definitive evidence is yet to come.

A compact stellar object such as neutron star (NS) is a perfect place to capture DM particles even when the DM-nucleon cross section $\sigma_{\chi n}$ is way smaller than the current direct search limits. Investigations on DM in compact stellar objects are studied recently in Refs. [14–23]. Due to the strong gravitational field, DM evaporation mass for NS is less than 10 keV [19]. Therefore, NS is sensitive to a broad spectrum of DM mass from 10 keV to PeV, sometimes it can be even extended to higher mass region. Unlike the Sun, it loses its sensitivity to DM when $m_\chi \lesssim 5$ GeV as a consequence of evaporation [24]. In the later discussion, we will focus on the Weakly Interacting Massive Particle (WIMP) scenario with mass from MeV to 10 TeV.

An old NS has the age greater than billions of years, its temperature might drop out a few Kelvins after processing several cooling mechanism by emitting photons and neutrinos [25, 26]. However, if the residing DM particles in NS can annihilate to SM particles, it could inject energy and heat up the host star [14, 15]. In addition, recent literature also suggests that the halo DM particles constantly bombard NS would deposit their kinetic energy to the star. This is called dark kinetic heating [27]. These two contributions might prevent NS from inevitable cooling and maintain its surface temperature $T_{\mathrm{surface}}$ at [28]

$$T_{\mathrm{surface}} = 2480 \, \text{K} \left( \frac{\sigma_{\chi n} \sigma_{\text{eff}}}{\sigma_c} \right)^{1/4}$$

(1)

where $\sigma_{\chi n}^{\text{eff}}$ is the effective DM-nucleon cross section and $\sigma_c$ the critical DM-nucleon cross section. Their mathematical expressions will be given in the later sections. In principle, NS surface temperature $T_{\mathrm{surface}}$ is different from its interior temperature $T$ but correlated. The relation connecting $T$ and $T_{\mathrm{surface}}$ will be given later.

Besides the DM-nucleon interaction, inconsistencies in the small-scale structure between the observations and the $N$-body simulations [29–38] imply the existence of self-interacting DM (SIDM) [39–45]. The constraint given in Ref. [46, 47] could mitigate such discrepancies
as well as the diversity problem of the galactic rotation curves [48, 49]. It brings us to

\[ 3 \text{ cm}^2 \text{g}^{-1} \leq \sigma_{\chi \chi}/m_{\chi} \leq 6 \text{ cm}^2 \text{g}^{-1} \]  

(2)

where \( \sigma_{\chi \chi} \) is the DM self-interaction cross section. The resulting effect of DM self-capture was considered insignificantly in NS due to it saturates at geometric limit quickly. Furthermore, its impact is unable to compete with the NS capture itself when \( \sigma_{\chi n} \gtrsim 10^{-50} \text{ cm}^2 \). However, current direct searches have put more stringent limits on \( \sigma_{\chi n} \) to test. If it is small enough, DM self-capture will eventually take over. In this region, DM self-interaction will re-enhance the captured DM particles and such effect is in the same way as having a relatively large \( \sigma_{\chi n} \). It also increases the DM annihilation rate so does the energy injection. Our study shows the reheating mechanism is induced by the thermal effect. DM cannot thermalize with NS if \( \sigma_{\chi n} \) is too weak. In such case, DM temperature \( T_{\chi} \) will decouple from NS interior temperature \( T \) in the early stage of evolution. This decoupled temperature \( T_{\chi}^{\text{dec}} \) is in general higher than the current NS interior temperature. A higher \( T_{\chi}^{\text{dec}} \) indicates larger thermal area and has better chance to capture other halo DM particles. Therefore, NS will experience a reheating process when \( \sigma_{\chi n} \) is small enough. Such process is possible to maintain \( T_{\text{surface}} \) up to hundreds of Kelvins.

An old, isolated NS nearby the Solar System emits infrared is a very good candidate to pin down such effect coming from DM self-interaction. The corresponding blackbody peak wavelength is potentially detectable in the future telescopes, e.g. the James Webb Space Telescope (JWST) [50], the Thirty Meter Telescope (TMT) [51] and the European Extremely Large Telescope (E-ELT). In the following context, we consider a nearby NS with age \( t_{\text{NS}} \approx 2 \times 10^9 \) years, mass \( M = 1.44M_{\odot} \) where \( M_{\odot} \approx 1.9 \times 10^{33} \text{g} \) is the Solar mass and radius \( R = 10.6 \text{ km} \). It also has the halo density \( \rho_0 = 0.3 \text{ GeV cm}^{-3} \), the DM velocity dispersion \( \bar{v} = 270 \text{ km s}^{-1} \) and the NS velocity relative to the Galactic Center (GC) \( v_N = 220 \text{ km s}^{-1} \). For discussion convenience, we will use natural unit \( c = \hbar = k_B = 1 \) and \( G = M_{\text{Pl}}^{-2} \) in this paper.

This paper is structured as follows: In Sec. II and Sec. III, we briefly review the formalism of DM captured by NS and the cooling and heating mechanism respectively. In Sec. IV, numerical results are presented as well as the discussion on the reheating effect. We will summarize our work in Sec. V.
II. DM CAPTURED BY NEUTRON STAR

A. DM evolution equation

When the halo DM particles scatter with NS and lose significant amount of energies, they will be gravitationally bounded in the star. The DM number evolution can be characterized by the differential equation

\[ \frac{dN_{\chi}}{dt} = C_c + C_s N_{\chi} - C_a N_{\chi}^2 \]  

where \( N_{\chi} \) is the DM number, \( C_c \) the capture rate due to neutrons in NS, \( C_s \) the DM self-capture rate and \( C_a \) the DM annihilation rate. A general solution to \( N_{\chi} \) is given by

\[ N_{\chi}(t) = \frac{C_c \tanh(t/\tau)}{\tau - 1 - \frac{C_s \tanh(t/\tau)}{2}} \]  

where \( \tau = 1/\sqrt{C_c C_a + C_s^2/4} \) is the equilibrium timescale. In the case of \( t \gg \tau \), \( dN_{\chi}/dt = 0 \). It reaches DM number equilibrium in NS. Hence we have

\[ N_{\chi}(t \gg \tau) \equiv N_{\chi,eq} = \sqrt{\frac{C_c}{C_a}} \left( \sqrt{\frac{R}{4}} + \sqrt{\frac{R}{4} + 1} \right) \]  

where

\[ R \equiv \frac{C_s^2}{C_c C_a} \begin{cases} \gg 1, & C_s \text{-dominant} \\ \ll 1, & C_c \text{-dominant} \end{cases} \]  

Thus, \( R \) signifies how crucial that the DM self-capture is in the DM evolution in NS.

Additionally, we can obtain two solutions to \( N_{\chi} \) when \( dN_{\chi}/dt = 0 \) by examining Eq. (3),

\[ N_{\chi,eq}^{R<1} = \sqrt{\frac{C_c}{C_a}} \quad \text{and} \quad N_{\chi,eq}^{R>1} = \frac{C_s}{C_a}. \]  

That means, either a dominant \( C_c \) or a dominant \( C_s \), can accumulate the same amount of DM particles in NS. In the later discussion, we will show that the similar degeneracy effect will cause the same NS surface temperatures.

B. Rates of DM capture and annihilation

The capture rate due to DM scattering with target neutrons in NS is given by [19]

\[ C_c = \sqrt{\frac{6}{\pi}} \frac{\rho_0}{m_{\chi}} \frac{v_{esc}(r)}{\bar{v}^2} \frac{\bar{v}}{1 - 2GM/R} \xi N_n \sigma_{\chi n}^{\text{eff}} \left( 1 - \frac{1 - e^{-B^2}}{B^2} \right) \]  

where
where $\rho_0$ is the DM density, $\bar{v}$ the DM velocity dispersion, $N_n = M/m_n$ the total number of target neutrons in NS, and $M$ and $R$ are the mass and radius of NS respectively. The suppression factor $\xi = \delta p/p_F$ is due to the neutron degeneracy effect. The momentum transfer in each scattering is $\delta p \simeq \sqrt{2m_r v_{\text{esc}}}$ where $m_r = m_\chi m_n/(m_\chi + m_n)$ the reduced mass and $v_{\text{esc}} \simeq 1.8 \times 10^5 \text{ km s}^{-1}$. Since the DM-nucleon cross section cannot exceed the geometric limit that is given by $N_n \sigma_c = \pi R^2$ where $\sigma_c \simeq 2 \times 10^{-45} \text{ cm}^2$ is the DM-nucleon critical cross section in NS. Thus, $\sigma_{\chi n}^{\text{eff}} \equiv \min(\sigma_{\chi n}, \sigma_c)$. The last factor $B^2 \equiv (3/2)(v_{\text{esc}}/\bar{v})\beta_-$ where $\beta_- = 4m_\chi m_n/(m_\chi - m_n)^2$. Unless $m_\chi \gtrsim 10 \text{ TeV}$, the term in the parentheses is roughly unity.

Another way of capture is due to the halo DM particle scatters with the trapped DM particle. This is DM self-capture and is given by [20]

$$ C_s = \sqrt{\frac{3}{2}} \frac{\rho_0}{m_\chi} v_{\text{esc}}(R) \frac{v_{\text{esc}}(R)}{\bar{v}} \langle \hat{\phi}_\chi \rangle \frac{\operatorname{erf}(\eta)}{\eta} \frac{1}{1 - 2GM/R} \sigma_{\chi \chi}^{\text{eff}}, \quad (9) $$

where $v_{\text{esc}}(R)$ is the escape velocity at the surface of NS. For a rather conservative calculations, we take $\langle \hat{\phi}_\chi \rangle = 1$ [20]. The quantity $\eta = \sqrt{3/2}(v_N/\bar{v})$ where $v_N$ is the NS velocity relative to G.C.

In addition, DM self-capture suffers from the geometric limit as well. The critical cross section for DM self-interaction is defined by $N_\chi \sigma_{\chi \chi}^c = \pi r_{\text{th}}^2$ where [19]

$$ r_{\text{th}} = \sqrt{\frac{9T}{4\pi G \rho_n m_\chi}} \approx 24 \text{ cm } \left( \frac{T_\chi}{10^5 \text{ K}} \cdot \frac{100 \text{ GeV}}{m_\chi} \right)^{1/2}, \quad (10) $$

is the thermal radius of DM and $T_\chi$ is the DM temperature. If DM has higher $T_\chi$, it results in larger thermal area $\pi r_{\text{th}}^2$. Hence we have $\sigma_{\chi \chi}^{\text{eff}} = \min(\sigma_{\chi \chi}, \sigma_{\chi \chi}^c)$. When the geometric limit for DM self-capture attains, we have $C_s N_\chi \propto \sigma_{\chi \chi}^c N_\chi = \pi r_{\text{th}}^2$. The effect of $C_s$ saturates at this point and depends only on the size of $r_{\text{th}}$ regardless of the value of $\sigma_{\chi \chi}$.

In the end, having more and more DM particles accumulated in NS, the chance of DM annihilation becomes appreciated. Thus,

$$ C_a \approx \frac{\langle \sigma v \rangle}{4\pi R^3/3}, \quad (11) $$

and the total annihilation rate is

$$ \Gamma_A = \frac{1}{2} C_a N_\chi^2(t). \quad (12) $$

In the later discussion, we will use thermal relic annihilation cross section $\langle \sigma v \rangle = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ unless otherwise specified.
III. NEUTRON STAR COOLING AND ENERGY INJECTION DUE TO DM ANNHI- LATION

After the birth of NS, it undergoes the cooling mechanism due to neutrino and photon emissions [25, 26]. Nonetheless, if the residing DM particles in NS can annihilate, they will inject extra energies to heat the host star up. The NS interior temperature \( T \) can be described by the following differential equation

\[
\frac{dT}{dt} = -\epsilon_\nu - \epsilon_\gamma + \epsilon_\chi \tag{13}
\]

where \( \epsilon_{\nu,\gamma,\chi} \) are the emissivities due to neutrino, photon emissions, and DM respectively. They are given by [14, 25]

\[
\epsilon_\nu \approx 1.81 \times 10^{-27} \text{ GeV}^4 \text{ yr}^{-1} \left( \frac{n}{n_0} \right)^{2/3} \left( \frac{T}{10^7 \text{ K}} \right)^8 \tag{14}
\]

where \( n \approx 4 \times 10^{38} \text{ cm}^{-3} \) is the NS baryon number density and \( n_0 \approx 0.17 \text{ fm}^{-3} \) the baryon density for the nuclear matter [14]. In this calculation, it is \( n/n_0 \approx 2.3 \). The rate of heat loss due to photon emission is given by \( L_\gamma = 4\pi R^2 \sigma_{\text{SB}} T_{\text{surface}}^4 \) where \( \sigma_{\text{SB}} \) is the Stefan-Boltzmann constant. We can further connect the NS surface temperature \( T_{\text{surface}} \) to the NS interior temperature \( T \) by the relation [26, 52, 53]

\[
T_{\text{surface}} = 0.87 \times 10^6 \text{ K} \left( \frac{g_s}{10^{14} \text{ cm s}^{-2}} \right)^{1/4} \left( \frac{T}{10^8 \text{ K}} \right)^{0.55} \tag{15}
\]

where \( g_s = GM/R^2 = 1.85 \times 10^{14} \text{ cm s}^{-2} \) is the surface gravity. Hence we have the effective photon emissivity [14]

\[
\epsilon_\gamma = \frac{L_\gamma}{4\pi R^3/3} \approx 2.71 \times 10^{-17} \text{ GeV}^4 \text{ yr}^{-1} \left( \frac{T}{10^8 \text{ K}} \right)^{2.2}. \tag{16}
\]

On the other hand, NS heating comes from the contributions of DM annihilation and dark kinetic heating. They are given by

\[
E_\chi = 2m_\chi \Gamma_A = m_\chi C_a N_\chi^2 f_\chi \tag{17}
\]

for annihilation and

\[
K_\chi = C_e E_s \tag{18}
\]

for dark kinetic heating. The factor \( f_\chi \) characterizes the energy absorption efficiency which runs from 0 to 1 and \( E_s = m_\chi (\gamma - 1) \) the DM kinetic energy deposited in NS. It is given by
\( \gamma \simeq 1.35 \) \cite{28}. Therefore, we have
\[
\epsilon_\chi = \frac{\mathcal{E}_\chi + \mathcal{K}_\chi}{4\pi R^3/3}
\tag{19}
\]
for DM.

The last quantity \( c_V \) is the heat capacity of NS. It is expressed as \cite{14}
\[
c_V = \frac{T}{3} \sum_i p_{F,i} \sqrt{m_i^2 + p_{F,i}^2}
\tag{20}
\]
where index \( i \) runs over \( n, p, e \) and the corresponding Fermi momenta are
\[
p_{F,n} = 0.34 \text{ GeV} \left( \frac{n}{n_0} \right)^{1/3},
\]
\[
p_{F,p} = p_{F,e} = 0.06 \text{ GeV} \left( \frac{n}{n_0} \right)^{2/3}.
\]

For calculation convenience, we have expressed all these quantities in terms of natural unit.

IV. SIDM IMPLICATION FOR NEUTRON STAR TEMPERATURE AND DM-NUCLEON CROSS SECTION

Before presenting the numerical results, we briefly introduce the setups of our calculations. In the presence of DM self-interaction, it might come to a time stamp \( t_s \) that DM self-interaction cross section reaches its geometric limit \( \sigma_{\chi\chi}^c = \pi r_{th}^2/N_\chi \). If our numerical procedure detects that the input \( \sigma_{\chi\chi} \) is larger than \( \sigma_{\chi\chi}^c \), it will automatically return \( \sigma_{\chi\chi}^c \) to avoid overestimating the effect of DM self-interaction. Since \( N_\chi \) still changes after \( t > t_s \) unless reaching \( N_{\chi,eq} \), \( \sigma_{\chi\chi}^c \) will adjust itself accordingly as well.

Moreover, \( \sigma_{\chi\chi}^c \) depends on the DM temperature \( T_\chi \) due to its dependence on \( r_{th} \). To thermalize with NS, it costs a timescale \cite{19}
\[
t_{th} \approx \frac{m_\chi^2 m_n p_F}{6\sqrt{2} n_n \sigma_{\chi n} m_n^2 T_\chi}
\tag{21}
\]
where \( n_n \) is the neutron number density of the NS.\footnote{To determine \( t_{th} \) one needs to solve the DM energy loss rate: \( \frac{dE}{dt} = -\xi n_n \sigma_{\chi n} v \delta E \). The detail discussion is beyond the scope of this work. It can be found in Refs. \cite{19, 20, 54} and references therein.} Rigorous discussions on the topic of thermal exchange can be found in Refs. \cite{19, 20, 54}.

During NS cooling, when the NS interior temperature \( T \) drops too low, DM might not have enough time to thermalize with \( T \) within \( t_{NS} \). Hence, in each time stamp \( t \) of our
FIG. 1. The evolution of NS interior temperature $T$ (solid) and DM temperature $T_\chi$ (dashed). DM masses $m_\chi$ are marked with different colors. When $\sigma_{\chi n} = 10^{-58} \text{ cm}^2$, DM carries $m_\chi = 1 \text{ TeV}$ can no longer thermalize with NS since the beginning.

In addition, if $\sigma_{\chi n}$ is too small to have DM particles became thermal equilibrium with NS at the beginning $t = t_0$, we let $T_\chi(t) = T_0$ where $T_0$ is the initial temperature at $t = t_0$ for both NS and DM that we put manually as the initial condition for numerical calculation.

**A. Numerical results**

Our numerical results for the evolutions of NS interior temperature $T$ and the DM temperature $T_\chi$ are shown in Fig. 1. The initial condition is $T_0 = 10^9 \text{ K}$ at $t_0 = 100$ years. The temperature evolution is insensitive to the initial condition. After the first few decades, the effect of $T_0$ becomes negligible. This agrees with Ref. [14]. In the case of $\sigma_{\chi n} = 10^{-55} \text{ cm}^2$, roughly before the first one million years, DM kept in thermal equilibrium with NS. However, when the NS temperature dropped out $T \lesssim 10^8 \text{ K}$, the 1 TeV DM mass cannot thermalize with the NS within $t_{NS}$. Thus, $T_\chi$ decoupled from $T$ and lives on its own. For lighter DM masses, we constantly check if $t_{\text{th}} < t_{NS}$ holds. It returns $T_\chi(t) = T(t)$ for true. On the contrary, if our program finds $t_{\text{th}} > t_{NS}$ at time stamp $t_{\text{dec}}$, we have $T_\chi$ decoupled from the evolution of $T$. After this moment, $T_\chi$ will always freeze at $T_{\chi \text{ dec}} = T(t_{\text{dec}})$ the decoupled temperature.\footnote{Assuming no dark radiation is emitted afterward, thus DM will not suffer from cooling mechanism.}
DM surface temperature $T_{\text{surface}}$ versus $\sigma_{\chi n}$ at $t = t_{\text{NS}}$. DM masses $m_\chi$ are marked with different colors. The plateaus associate with the maximum $T_{\text{surface}}^{\text{max}}$ to each $m_\chi$. We have taken $f_\chi = 1$.

DM particle, it would be easier to thermalize with NS due to more violent thermal motion $v \propto \sqrt{T_\chi/m_\chi}$. Exception happens when $m_\chi \lesssim 1 \text{ GeV}$, its $\sigma_{\chi n}$ is subject to a suppression $\xi = \delta p/p_F$ due to neutron degeneracy effect. If DM can maintain at higher temperature, its thermal radius $r_{\text{th}}$ will be larger. Although the DM self-interaction cross section already saturated at $\sigma_{\chi\chi}^{c}$ in the very early period, with larger $r_{\text{th}}$, $\sigma_{\chi\chi}^{c} \propto \pi r_{\text{th}}^2$ results in larger impact to the DM self-capture rate. The situation is more interesting for $\sigma_{\chi n} = 10^{-58}\text{ cm}^2$. As a consequence of smaller $\sigma_{\chi n}$, all $T_\chi$ decoupled from $T$ far earlier and hotter than those with larger $\sigma_{\chi n}$.

Before continuing the discussion, let’s take a look of Eq. (3). Since DM self-capture usually saturates at this stage with $T_\chi = T_{\chi \text{dec}}$, essentially we have $C_s N_\chi \propto \frac{\sigma_{\chi\chi}}{m_\chi} N_\chi = \pi r_{\text{th}}^2/m_\chi$. Therefore, $dN_\chi/dt = 0$ generally implies

$$\frac{N_{\chi, \text{eq}}^{R \gg 1}}{N_{\chi, \text{eq}}} \propto \sqrt{T_{\chi \text{dec}}/C_a} \frac{1}{m_\chi}$$

(22)

hence

$$\mathcal{E}_\chi \propto m_\chi \Gamma_A \propto \frac{T_{\chi \text{dec}}}{m_\chi}$$

(23)

$^3$ If DM self-capture does not saturate, the scaling is $N_{\chi, \text{eq}}^{R \gg 1} \propto \sigma_{\chi\chi}/(C_a m_\chi)$. Unlike the saturation case, $\mathcal{E}_\chi \propto \sigma_{\chi\chi}^2/(C_a m_\chi)$. Thus, smaller $\langle \sigma v \rangle$ will induce a stronger energy injection.
where the energy injection does not concern with $\langle \sigma v \rangle$. When $dT/dt = 0$, we can simply equal Eq. (16) and Eq. (19). By incorporating Eq. (15), the scaling relation for $T_{\text{surface}}$ in the $C_s$ dominant region is expressed as

$$T_{\text{surface}} = 130 \text{K} \left[ \frac{100 \text{ GeV} \min(T_{\chi}^{\text{dec}}, T_0)}{m_\chi T_0} f_\chi \right]^{1/4} \tag{24}$$

where $T_{\chi}^{\text{dec}}$ is estimated as

$$T_{\chi}^{\text{dec}} \simeq 1.08 \times 10^5 \text{K} \left( \frac{m_\chi}{100 \text{ GeV}} \right)^2 \left( \frac{10^{-55} \text{ cm}^2}{\sigma_{\chi n}} \right) \tag{25}$$

for $m_\chi > 1 \text{ GeV}$ and

$$T_{\chi}^{\text{dec}} \simeq 77 \text{K} \left( \frac{0.1 \text{ GeV}}{m_\chi} \right) \left( \frac{10^{-55} \text{ cm}^2}{\sigma_{\chi n}} \right) \tag{26}$$

for $m_\chi \lesssim 1 \text{ GeV}$. The minimum function in Eq. (24) indicates that $T_{\chi}^{\text{dec}}$ cannot exceed the initial temperature $T_0$. It means that the reheating process is not unlimited. When $\sigma_{\chi n}$ is too weak to make $t_{\text{th}} < t_{\text{NS}}$ at $t = t_0$ even having $T_\chi = T_0$, the corresponding $T_{\chi}^{\text{dec}}$ will always freeze at $T_0$ right after its capture as indicating previously. This corresponds to the largest thermal area $\pi r_{\text{th}}^2$, so does the maximum DM self-capture and energy injection. Therefore, it induces the highest $T_{\text{surface}}^{\max}$ by SIDM. Our scaling relation is valid for $T_0 = 10^9 \text{K}$ which should cover most of the situation.

In Fig. 2, $T_{\text{surface}}$ versus $\sigma_{\chi n}$ is shown with different $m_\chi$. The temperature is calculated at $t = t_{\text{NS}}$. For $\sigma_{\chi n} > 10^{-55} \text{cm}^2$, it is in $C_c$ dominant region. Our numerical calculation agrees with Eq. (1) very well (gray dashed lines). However, for smaller $\sigma_{\chi n} \lesssim 10^{-55} \text{cm}^2$, the self-interaction starts taking over. In this region, $T_{\chi}^{\text{dec}} \propto 1/\sigma_{\chi n}$ thus $T_{\text{surface}} \propto \sigma_{\chi n}^{-1/4}$. Until $T_{\chi}^{\text{dec}} = T_0$, the plateaus to each $m_\chi$ in Fig. 2 represent the associated maximum NS surface temperature $T_{\text{surface}}^{\max}$.

As a remark, in this paper we set the initial NS temperature $T_0 = 10^9 \text{K}$. However, the exact value of $T_0$ is uncertain and highly depends on the nuclear structure of NS in the very early stage of evolution. Although such effect does not directly affect NS cooling in the later stage, it determines $T_{\text{surface}}^{\max}$. If $T_0$ is higher, then in general DM would have a larger $T_{\chi}^{\text{dec}}$ when $\sigma_{\chi n}$ is too small to thermalize with NS since the beginning $t = t_0$. Thus, NS will have a higher $T_{\text{surface}}^{\max}$ than the result we present here. To serve as an illustrative purpose, our setup is rather conservative.

In Fig. 3, a broad scan of $T_{\text{surface}}$ over $m_\chi - \sigma_{\chi n}$ plane are displayed. It is obvious that NS is warmer in the presence of SIDM than without SIDM. Higher temperature region
FIG. 3. A broad scan of $T_{\text{surface}}$ over $m_{\chi} - \sigma_{\chi n}$ plane. The region circled by the blue dashed line is where DM cannot thermalize with NS within $t_{\text{NS}}$ since the beginning of capture. It has $T_{\text{surface}}^{\text{max}}$ to each $m_{\chi}$. Without SIDM, issue on thermalization is irrelevant.

is essentially induced by the larger DM self-capture as discussed earlier. Warmer region for $m_{\chi} \lesssim 1$ GeV is explained by the suppression on $\sigma_{\chi n}$ due to neutron degeneracy effect. Likewise, the thermalization would be more difficult because the thermal motion is less vital for $m_{\chi} > 1$ GeV. Either way leads to a hotter $T_{\chi}^{\text{dec}}$, thus enhances the DM self-capture and $T_{\text{surface}}$ consequently.

**B. Minimum NS temperature and maximum reheating effect**

Here we present our final result in Fig. 4. In this figure, the purple thick line corresponds to the minimum NS surface temperature $T_{\text{surface}}^{\text{min}}$ in the presence of SIDM. It associates to the valleys in Fig. 2 to each $m_{\chi}$. On top of this line, the $\sigma_{\chi n}$ is strong enough to make the capture dominated by $C_c$ effect. Thus, $T_{\text{surface}}$ scales as Eq. (1) very well. Below this line, $C_s$ effect dominates the capture. The NS temperature experiences a reheating due to the increasing capture rate by SIDM.

It is easily noticed that there would be two lines showing the same $T_{\text{surface}}$. For instance, green lines indicate $T_{\text{surface}} = 100$ K. Though one is due to $C_c$ (above the purple line) and the other is due to $C_s$ (below the purple line), they are in general indistinguishable by
FIG. 4. The purple thick line corresponds to the coldest $T_{\text{surface}}^{\text{min}}$ in the presence of SIDM. Below this bound, NS can be reheated due to the increasing DM self-capture efficiency. This reheating region acts like it has a relatively larger $\sigma_{\chi n}$. However, the reheating process saturates at $T_{\text{surface}}^{\text{max}}$ when $\sigma_{\chi n}$ is too small to make $t_{\text{th}} < t_{\text{NS}}$ since $t = t_0$. Thus, $r_{\text{th}}$ is in its maximum so does the DM self-capture. The $T_{\text{surface}}^{\text{max}}$ to each $m_\chi$ is displayed in color gradient. The limits on $\sigma_{\chi n}$ from different direct searches such as DARWIN [3], LUX [4] and XENON1T [7] are shown in the plot as well.

measuring NS surface temperature solely. The color gradient indicates the maximum NS surface temperature $T_{\text{surface}}^{\text{max}}$ to each DM mass. DM with $m_\chi \approx 60\,\text{MeV}$ has the highest $T_{\text{surface}}^{\text{max}} \approx 700\,\text{K}$.

V. SUMMARY

In this work, we found that when DM self-capture dominates the evolution of DM in NS, $T_{\text{surface}}$ will increase by this effect. $T_{\text{surface}}$ will be in the same way as DM has a relatively large $\sigma_{\chi n}$. In addition, there exists a minimum NS temperature $T_{\text{surface}}^{\text{min}} \sim \mathcal{O}(10\,\text{K})$ before $C_s$ dominates the capture process. However, DM self-interaction cross section reaches its geometric limit $\sigma_{\chi \chi}^G$ very quickly and becomes irrelevant to $\sigma_{\chi \chi}$. The value of $\sigma_{\chi \chi}$ determines how fast the DM self-capture attains the geometric limit but not the ultimate fate of DM in NS. Thus, to constrain $\sigma_{\chi \chi}$ from NS temperature may not be possible. Nonetheless, if NS temperature smaller than $T_{\text{surface}}^{\text{min}}$ is observed, it could be inferred that either non-existence of DM self-interaction, inefficient energy absorption $f_\chi$ or even no DM annihilation. On
the other hand, $T_{\text{surface}}$ has a maximum value $T_{\text{surface}}^{\text{max}}$ when DM cannot thermalize with NS since the beginning $t = t_0$. Such features due to SIDM presented in this paper are natural consequences when we consider $T_{\chi}$ does have its own pace in the evolution. Additionally, a scaling relation for $T_{\text{surface}}$ in the $C_s$ dominant region is shown as well.

In a certain parameter space, both $C_c$ and $C_s$ will generate $T_{\text{surface}}$ which are essentially degenerate. The corresponding blackbody peak wavelength is infrared and could be detected by JWST, TMT and E-ELT in the future. If $T_{\text{surface}}$ predicted by such parameter space is observed, its interpretation should be made carefully.

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