Electromagnetic Chirps from Neutron Star–Black Hole Mergers

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Abstract

We calculate the electromagnetic signal of a gamma-ray flare coming from the surface of a neutron star shortly before merger with a black hole companion. Using a new version of the Monte Carlo radiation transport code Pandurata that incorporates dynamic spacetimes, we integrate photon geodesics from the neutron star surface until they reach a distant observer or are captured by the black hole. The gamma-ray light curve is modulated by a number of relativistic effects, including Doppler beaming and gravitational lensing. Because the photons originate from the inspiraling neutron star, the light curve closely resembles the corresponding gravitational waveform: a chirp signal characterized by a steadily increasing frequency and amplitude. We propose to search for these electromagnetic chirps using matched filtering algorithms similar to those used in LIGO data analysis.

Key words: black hole physics – gamma-ray burst: general – gravitational waves

1. Introduction

Tremendous excitement was generated by the recent LIGO discoveries of gravitational waves (GWs) from merging binary black holes (Abbott et al. 2016a, 2016b, 2017b, 2017c). This was only surpassed by the even greater excitement surrounding the simultaneous detection of GWs and gamma-rays from the neutron star merger GW170817/GRB 170817A (Abbott et al. 2017d, 2017f). With the vastly improved sky localization due to the addition of Virgo to the LIGO network, a host galaxy was identified, and numerous multi-wavelength observations were obtained (Abbott et al. 2017a), placing the event in the proper astrophysical context. The galaxy’s redshift could also be used in conjunction with the luminosity distance—measured independently with the GW signal—to place new and unbiased constraints on cosmological parameters, given a sufficiently large sample of events (Nissanke et al. 2010; Abbott et al. 2017e).

Another important advantage of detecting an EM counterpart is that it could give us the ability to combine with sub-threshold GW triggers and improve the statistical significance of marginal signals, potentially lowering the false alarm rate (Blackburn et al. 2015; Clark et al. 2015). It is also conceivable that several weak coincident EM and GW signals could reveal a population of sources which are individually too weak to be detectable by blind GW or EM searches.

As conclusively demonstrated by GW170817, binary neutron star (NS/NS) mergers produce short gamma-ray bursts (SGRB), long predicted theoretically (Eichler et al. 1989; Kochanek & Piran 1993). Similar processes are expected from neutron-star–black-hole (NS/BH) mergers. While GRB 170817A resembled most previous SGRBs in terms of photon energy and flux, the unusually small distance of 40 Mpc implies an extremely under-luminous burst, likely due to the system being off-axis to the line of sight (Abbott et al. 2017f; Goldstein et al. 2017a). For future similar events that occur closer to the limit of LIGO/Virgo sensitivity, the GRB may not be significant, or even detectable at all if the system is sufficiently off-axis. However, for a sub-sample of SGRBs, a precursor gamma-ray flare can be seen roughly 1–10 s before the peak of the GRB (Troja et al. 2010). For a nominal NS/BH binary with masses 1.4 M\textsubscript{\odot} and 10 M\textsubscript{\odot}, this would correspond to hundreds of orbits before merger, with binary separations of 20–30 gravitational radii.

In this paper, we explore the possibility that these precursor flares come from some emission process on the surface of the neutron star as it spirals toward the companion black hole. If so, the resulting gamma-rays will experience the extreme gravitational forces that govern this highly relativistic and dynamical system. These effects will be imprinted on the gamma-ray signal that ultimately reaches a distant observer. We have identified two primary features in these light curves: special relativistic Doppler beaming, and magnification due to gravitational lensing. As we will show below, the detailed properties of the light curve provide information about the binary separation, inclination angle, and orbital period, giving important information complementary to the GW signal.

The most robust feature of this precursor EM light curve is that it is should be locked in phase with the GW signal, also chirping through increasing frequency and amplitude as the system approaches merger. Thus we refer to this particular class of counterparts as “electromagnetic chirps.” We fully expect that data analysis tools similar to those used in GW searches may be fruitful in discovering and interpreting such signals in otherwise noisy data from gamma-ray observatories such as the Gamma-ray Burst Monitor (GBM) on Fermi (T. Dal Canton et al. 2018, in preparation). We remain intentionally agnostic about the specific physical mechanism that produces the gamma-ray flash on the NS surface, but one promising model is that of resonant shattering flares that occur as the binary orbital frequency sweeps through the eigenfrequencies of the NS normal modes during the moments leading up to merger (Tsang et al. 2012; Tsang 2013). When the quadrupolar crust–core interface mode is resonantly excited by tidal interaction, a huge amount of energy can be deposited into the crust, and ultimately released as gamma-rays.
2. Light Curves in Binary Spacetime

We calculate the light curves and spectra from the NS/BH system with the Monte Carlo radiation transport code Pandurata (Schnittman & Krolik 2013). To date, Pandurata has only been applied to problems with stationary Kerr spacetimes. In order to use it for the highly dynamic spacetime of a merging compact binary system, significant modifications were required. First and foremost, we needed to move from the Hamiltonian formalism described in Schnittman & Krolik (2013), appropriate for a system with multiple integrals of motion, to a more generalized Lagrangian approach to solving the geodesic equation for an arbitrary metric and connection, better suited for the binary spacetime.

At the same time, we desired a spacetime formulation that would be computationally efficient for integrating millions of geodesic trajectories. Thus, instead of using full numerical relativity data (typically only available for tens of binary orbits), we instead opted for a relatively simple analytic description of the binary spacetime based on a post-Newtonian (PN) approximation to the orbit (Kelly et al. 2007).

The binary is completely described by two non-spinning point masses with \( m_1 + m_2 = M \) and \( m_1 \geq m_2 \), moving on a circular orbit with binary separation \( D \equiv (GM/c^2)x \) (unless explicitly stated otherwise, we hereby assume units with \( G = c = 1 \)). The angular velocity is given by the 2PN expression

\[
\Omega = \left[ \frac{64}{(1 + 2x)^6} + \frac{4}{x^4} + \left( -\frac{5}{8\eta} + \frac{\eta^2}{256} \right) \right]^{1/2} M^{-1},
\]

with \( \eta \equiv m_1 m_2 / M^2 \).

The orbit is assumed to be instantaneously circular, but we do evolve the separation according to the 2.5PN leading quadrupole radiation reaction terms derived in Peters (1964):

\[
\frac{dx}{dt} = -\frac{64}{5} \frac{1}{x^3}.
\]

Note that these expressions are only used here for simplicity and proof-of-principle. In practice, when searching for counterparts to LIGO triggers, we plan to use the same state-of-the-art formalism of the binary evolution used in the GW analysis (Taracchini et al. 2014; Bohé et al. 2017).

The metric is expressed by the standard 3+1 lapse/shift formalism:

\[
g_{\mu\nu} = \left( \begin{array}{cc}
-\alpha^2 + \beta^2 & \beta_j \\
\beta_i & \gamma_{ij}
\end{array} \right).
\]

Following Campanelli et al. (2006), we use \( \alpha = 2/(1 + \psi^4) \), \( \beta_j = 0 \), and \( \gamma_{ij} = \delta_{ij} \psi^4 \). The conformal factor \( \psi \) is given by

\[
\psi = 1 + \frac{m_1}{2r_1} + \frac{m_2}{2r_2},
\]

with \( r_1 \) and \( r_2 \) being the simple Cartesian distances between the spatial coordinate and the primary/secondary masses. For the Christoffel components \( \Gamma^\rho_{\mu\nu} \), we take the spatial and temporal metric derivatives analytically based on the trajectory as given by Equation (1).

To generate light curves, Pandurata employs a Monte Carlo ray-tracing scheme that is based on shooting a large number of photon packets from the emission region to potential observers at infinity (Schnittman & Krolik 2013). For the NS/BH problem investigated here, we use a simple thermal, optically thick emission model. Each photon is launched from the surface of the NS with radius 10 km, isotropic in the local frame of the star and with a limb-darkening factor appropriate for an optically thick emitter. This gives the photon’s initial position \( x \) and four-velocity \( u \). The photon is then propagated forward along the affine parameter \( \lambda \) according to the standard geodesic formula

\[
\frac{d^2x^\rho}{d\lambda^2} = -\Gamma^\rho_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}.
\]

When a photon packet reaches a simulated detector surface at some large radius, its direction specifies the appropriate pixel in the image plane, exactly like a pinhole camera. Since each photon is also tagged with a time stamp, a movie can be built up over time. Generally, each observer is located at a specific sky position \((\theta, \phi)\), so the vast majority of the Monte Carlo photons never contribute to the image. For circular orbits, we are able to take advantage of the periodic motion of the binary to efficiently map the spatial coordinate to the time coordinate, so a single movie is produced by combining images from observers at all azimuthal positions. Because we are also interested in generating light curves for multiple latitudinal inclination angles, every photon that escapes the system ultimately contributes to some observer’s light curve, restoring a remarkable level of efficiency for the Monte Carlo approach.

In Figure 1, we show four snapshots of the NS/BH system as seen by an observer oriented edge-on to the orbital plane. The binary masses are \( m_1 = 10 M_\odot \) and \( m_2 = 1.4 M_\odot \), and the binary separation is \( D = 10M \). The two dominant effects here are gravitational lensing and relativistic beaming. From weak lensing theory, whenever there is co-alignment of the source, lens, and observer, an Einstein ring is formed, with high magnification. Lensing by a black hole produces an infinite number of concentric Einstein rings, each one at smaller radius and magnification.

The primary Einstein ring can be seen in panel (a), when the NS is on the far side of the BH relative to the observer (a slight misalignment of the system at this snapshot produces the asymmetric ring). The NS then progresses around its orbit to (b), when it reaches the point of maximum blueshift and beaming toward the observer. Also seen in (b) is a demagnified direct image as well as the faint secondary Einstein ring, only appearing a quarter phase later due to the time delay of photons orbiting around the BH before reaching the observer. In panel (c) another Einstein ring is seen, produced when the system is aligned in the order BH–NS–observer, and the photons from the nearer NS are deflected 180° by the BH on the far side of the orbit before returning to the observer. By this time, the NS has already moved roughly a quarter phase out of the way, and thus does not appear co-linear with the BH. Finally, in panel (d) the NS is at the point of maximum redshift, noticeably fainter than in panel (b).

In Figure 2 we show the bolometric light curve corresponding to the snapshots in Figure 1. At this small binary separation of \( 10M \), the orbital velocity is roughly one-quarter of the speed of light, so the dominant feature in the light curve is the Doppler beaming as the NS moves toward and away from the observer. The lensing magnification contributes a strong peak at (a), and a much smaller peak visible at (c). In the bottom panel of
Figure 2 we show the light curve from the same system at binary separation $D = 40M$. The orbital velocity, and thus the relativistic beaming effect, is much smaller, while the lensing still produces a large magnification. The smaller orbital velocity also implies the light-crossing time is shorter relative to the orbital period, so the four phases corresponding to those shown in Figure 1 are more evenly spaced in time.

Figure 3 shows the dependence of the light curve features on observer inclination. Not surprisingly, the sharp lensing feature is only visible at high inclination angles with $i \gtrsim 80^\circ$. Yet even at $i = 30^\circ$, the relativistic beaming is responsible for a significant modulation in the light curve. This is because, at a given frequency, the observed flux scales like the redshift cubed: $I = (v_{\text{obs}}/v_{\text{em}})^3$. The bolometric flux modulation is greater by another factor of the redshift. One simple way to understand this is when the emitter has a blackbody temperature $T_{\text{em}}$, it is observed as a blackbody with temperature $T_{\text{obs}} = T_{\text{em}} (v_{\text{obs}}/v_{\text{em}})$, and of course the total flux scales like $T^4$. For even the moderate inclination of $i = 30^\circ$, the ratio of peak-to-valley flux is $I_{\text{max}}/I_{\text{min}} \approx 3$, while for edge-on systems the modulation is an order of magnitude greater.

For a thermal emission spectrum, the magnitude of the Doppler modulation also depends on the energy band at which it is observed. At energies below the thermal peak, the slope of the spectrum is increasing like $L \sim \nu^2$, so the net effect of beaming is relatively modest: the observer sees a blue(red)-shifted portion of an inherently fainter(brighter) part of the spectrum. The opposite is true at energies above the thermal peak, where the relativistic beaming combines with the inherent shape of the spectrum to enhance the effect of Doppler modulation. These effects are shown in Figure 4, where we have divided the observed spectrum into three broad bands, one covering the thermal peak, and one each above and below. For an inclination angle of $60^\circ$, the bolometric peak-to-valley amplitude is $\sim 10$, while the high-energy flux is actually modulated by nearly a factor of 100. This will have important implications for designing a search strategy for these systems.

On the other hand, the gravitational lensing is achromatic, and generally strongest when the source is moving transverse to the observer, so the lensing modulation will have roughly the same effect on light curves in different energy bands.

3. Electromagnetic Chirps

As shown in the previous section, the gamma-ray light curve from an NS/BH binary is modulated at the orbital frequency with an amplitude that is a function of the line-of-sight velocity. As the orbit shrinks due to gravitational radiation losses, the frequency and amplitude of the gamma-ray light curve increase. In other words, the EM light curve chirps, just like the GW signal.

An NS/BH system with masses $1.4 M_\odot$ and $10 M_\odot$ will complete roughly 700 orbits during the final minute before merger, entirely within the LIGO band. A portion of the light curve corresponding to this period is shown in Figure 5. To
achieve sufficiently high resolution with Pandurata over such a long time, we need only calculate a handful of circular light curves, each at a different orbital separation, and then interpolate between them according to the inspiral evolution as governed by Equation 2). The evolution is cut off at a separation \( D = 10M \), shortly before merger. In practice, the light curve shown in Figure 5 should really be thought of as a modulation window to be multiplied by the inherent flare luminosity, which may only last a few seconds or less, depending on the emissivity model.

At early times, when the orbital velocity is somewhat smaller, the lensing peak of the light curve is larger than that due to the Doppler beaming but, at late times, the beaming dominates the modulation (see Figure 2). As with traditional radial velocity observations of spectroscopic binaries, the Doppler shift modulation will provide a degenerate measurement of the BH mass and the binary inclination. For nearly edge-on systems where the lensing peak is observable, the degeneracy can be broken and an accurate BH mass can be determined, much like in the case of transiting exoplanet systems. With a coincident GW detection, the gravitational waveform will provide a different degenerate measure of the binary masses and inclination. For extremely precise light curve measurements, one could even imagine resolving the width of the lensing peak, in turn giving information about the NS radius and thus the equation of state.

There is also the very real possibility that the source could be an NS/NS binary, with either or both neutron stars initiating the gamma-ray flare(s). In this case, the light curve would be simply repeated with half the period, yielding two beaming and two lensing peaks per orbit. Or if the flares occur at different times (potentially due to different NS masses or magnetic field configurations), the light curve would consist of two isolated flares, modulated by the same chirping signature, but offset in phase by 180°.

4. Observational Potential

To this point, it has not been necessary to specify the physical mechanism that causes the gamma-ray flare on the NS surface. Anything that leads to emission closely tied to the neutron star will give qualitatively the same EM chirp as the light curves computed in the previous sections. One promising model is that of resonant shattering of the NS crust (Tsang et al. 2012; Tsang 2013). In this scenario, the tidal deformation of the NS, modulated by the orbital motion, resonantly excites the crust–core interface eigenmode, leading to the crust shattering and releasing \( \sim 10^{46} \text{ erg} \) on an extremely short timescale. Depending on the NS equation of state, this resonant shattering flare takes place somewhere from 1 to 10 s before...
merger, consistent with SGRB precursors seen with Swift (Troja et al. 2010). Another possible model that similarly deposits a large amount of energy into the NS surface shortly before merger is that of the unipolar inductor, where the accelerating magnetized NS essentially forms an electric generator with the black hole or neutron star companion (Hansen & Lyutikov 2001; McWilliams & Levin 2011; Mingarelli et al. 2015; D’Orazio et al. 2016).

We propose to implement efficient searches for these characteristic chirp signals in the data from existing time-domain observatories such as *Fermi* in conjunction with any LIGO triggers. By using data analysis techniques similar to the matched filtering employed by the LIGO collaboration, we hope to be able to extract long but weak signals from a noisy background. Moreover, if the source parameters are partially known from a coincident GW signal (in particular the chirp mass and the merger time) then the parameters of the EM chirp are strongly constrained, restricting the volume of parameter space that must be explored and further reducing the false alarm background of the search, as well as its computational cost.

Methods for performing such a search in *Fermi/GBM* data are currently being investigated and will be described in a companion paper (Dal Canton et al. 2017). The flux levels required for detection can be conservatively estimated from current, unmodeled burst searches. We estimate that any template-based search will be more sensitive by a factor of a few to 10. The GBM upper limits for EM counterparts to GW170817, LVT151012, GW151226, and GW170104 were in the range of \((5–9) \times 10^{-7} \text{ erg s}^{-1} \text{ cm}^{-2}\), over an energy band of 10-1000 keV (Goldstein et al. 2017b; Racusin et al. 2017). This corresponds to a luminosity of nearly \(10^{39} \text{ erg s}^{-1}\) at a distance of 100 Mpc.

For an NS radius of 10 km and thermal emission, this would require a surface temperature of roughly 500 keV. However, an important caveat in considering the effects of the EM chirp, in the context of these models, is that high luminosities emitted thermally from a small region, like the neutron star surface, will be optically thick to pair production \((\gamma + \gamma \rightarrow e^+ + e^-)\), resulting in a pair-photon fireball that only becomes optically thin at the surface of a much larger photosphere (Goodman 1986).

For photons with energies \(E_\gamma \approx 500\text{ keV}\) we can estimate the optical depth to pair production (see, e.g., Nakar et al. (2005)):

\[
\tau_{\gamma\gamma} \gtrsim \frac{L_\gamma \sigma_T}{4\pi c E_\gamma R_{\text{NS}}} \approx 10^{10} \frac{L_\gamma/(10^{46} \text{ erg s}^{-1})}{R_{\text{NS}}/(10 \text{ km})},
\]

where \(L_\gamma\) is the luminosity of photons near \(E_\gamma\) and \(\sigma_T\) is the Thompson cross-section. Clearly, if all the energy of these flares is deposited into photons above the pair-production threshold, a pair-photon fireball is quickly formed, which will expand relativistically until the pair production freezes out (Goodman 1986), resulting in a large photospheric radius when compared to the orbital and gravitational scales. Additionally, the photospheric surface is accelerated to large Lorentz factor \(\Gamma\), likely washing out any beaming effects from the orbital dynamics.

For blackbody emission, luminosity greater than \(L \gtrsim 10^{42} \text{ erg s}^{-1}\), when confined to \(R_{\text{NS}} \approx 10 \text{ km}\) \((T \gtrsim 20 \text{ keV})\), will have a sufficiently high-energy tail to be optically thick to pair production. Thus, there exists an unfortunate trade-off for potential EM chirp sources. Those brighter than \(L \gtrsim 10^{42} \text{ erg s}^{-1}\) (which is only detectable by GBM out to a few Mpc), will likely be optically thick to pair production and result in a pair-photon fireball, potentially masking the EM chirp signature. Meanwhile, those with luminosities below this threshold will be more difficult to detect at extra-galactic distances. However, non-thermal emission lacking significant contribution above the pair-production threshold may avoid this limitation.

Resonant shattering flares, with luminosities of \(\sim 10^{47–49} \text{ erg s}^{-1}\), consistent with the observed SGRB precursors, while probably detectable with GBM, are likely pair-photon fireballs. On the other hand, emission from a black hole or neutron star companion crossing the NS magnetic field lines in the unipolar inductor model is significantly less energetic, with maximum luminosity scaling as \(L \sim 10^{40} \text{ erg s}^{-1} (B/10^{12} \text{ G})^2 (D/10^6 \text{ cm})^{-2}\), where \(B\) is the NS magnetic field and \(D\) the binary separation (Hansen & Lyutikov 2001).

Despite these potential complications, we are confident that the basic EM chirp remains a robust and potentially very powerful prediction for a counterpart to the GW signal from merging NS binaries. While GW170817 has led to improved rate estimates for NS/NS mergers, the expected rates for BH/NS binaries still cover over three orders of magnitude.
GBM observing an efficiency of 60%, a nominal isotropic luminosity of $10^{48}$ erg s$^{-1}$, and assuming sensitivity five times better than the unmodeled burst search, gives a 90% confidence range for the GBM event rate of $\sim$0.8–41 year$^{-1}$.

In the near future, we encourage any wide-field survey with sufficiently high time resolution to look for these characteristic chirps preceding GW triggers. Serendipitous detection with a wide-field X-ray telescope like Swift would provide sensitivity limits one to two orders of magnitude better than GBM, but over a much smaller area of the sky. In the more distant future, we look forward to using low-frequency GW observatories such as LISA to identify precursor signals that would in turn give an early warning for the time and sky location for ground-based events, triggering more sensitive targeted EM observations (Sesana 2016).

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