Longitudinal electric conductivity and dielectric permeability in quantum plasma with variable frequency of collisions in Mermin’ approach

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Formulas for longitudinal electric conductivity and dielectric permeability in the quantum non-degenerate collisional plasma with the frequency of collisions depending on momentum in Mermin’ approach are received. The kinetic equation in momentum space in relaxation approximation is used. It is shown that when Planck’s constant tends to zero, the deduced formula passes to the corresponding formula for classical plasma. It is shown also that when frequency of collisions of particles of plasma tends to zero (plasma passes to collisionless one), the deduced formula passes to the known Lindhard’ formula received for collisionless plasmas. It is shown, that when frequency of collisions is a constant, the deduced formula for dielectric permeability passes in known Mermin’ formula.

Key words: Lindhard, Mermin, quantum collisional plasma, conductance, Schrödinger–Boltzmann and Vlasov–Boltzmann equations, density matrix, commutator, non-degenerate plasma.

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Introduction

In the known work of Mermin \cite{Mermin1978} on the basis of the analysis of nonequilibrium density matrix in \(\tau\)-approximation has been obtained expression for longitudinal dielectric permeability of quantum collisional plasmas for case of constant collision frequency of plasmas particles.

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Earlier in the work of Klimontovich and Silin [2] and after that in the work of Lindhard [3] has been obtained expression for longitudinal and transverse dielectric permeability of quantum collisionless plasmas. By Kliewer and Fuchs [4] it has been shown, that direct generalisation of formulas of Lindhard on the case of collisional plasmas is incorrectly. This lack for the longitudinal dielectric permeability has been eliminated in the work of Mermin [1]. Next in the work [5] has been given attempt to deduce Mermin’s formula.

For collisional plasmas correct formulas longitudinal and transverse electric conductivity and dielectric permeability are received accordingly in works [7] and [8]. In these works Wigner—Vlasov—Boltzmann kinetic equation in relaxation approximation in coordinate space is used. In work [9] the formula for transverse electric conductivity has been deduced for quantum collisional plasmas with use of the kinetic equation by Mermin’ approach (in momentum space).

In the present article formulas for longitudinal electric conductivity and dielectric permeability in the quantum non-degenerate collisional plasma with the frequency of collisions depending on a momentum by Mermin’ approach are received. The kinetic equation in momentum space in relaxation approximation is used. It is shown, that when Planck’s constant tends to zero, the deduced formula passes to the corresponding formula for classical plasma. It is shown also, that when frequency of collisions of particles of plasma tends to zero (plasma passes to collisionless one), the deduced formula passes to the known Lindhard’ formula received for collisionless plasmas. It is shown, that when frequency of collisions is a constant, the deduced formula for dielectric permeability passes in known Mermin’ formula.

Now considerable interest to research of properties of quantum plasma proceeds [10]–[27].

1. Kinetic Schrödinger—Boltzmann equation for density matrix

Let the vector potential of an electromagnetic field is harmonious, i.e.
changes as \( \varphi(\mathbf{r}, t) = \varphi(\mathbf{r}) \exp(-i\omega t) \). Relation between scalar potential and intensity of the electric field it is given by expression \( \mathbf{E}(\mathbf{q}) = -\nabla \varphi(\mathbf{q}) \). The equilibrium matrix of density looks like

\[
\tilde{\rho} = \frac{1}{\exp \left( \frac{H - \mu}{k_B T} \right) + 1}, \quad \mu = \mu_0 + \delta \mu.
\]

Here \( T \) is the temperature, \( k_B \) is the Boltzmann constant, \( \mu_0 \) is the chemical potential of plasma in an equilibrium condition, \( \delta \mu \) is the correction to the chemical potential, caused presence of variable electric field, \( H \) is the Hamiltonian.

Hamiltonian looks like here \( H = H_0 + H_1 \), where \( H_0 = p^2 / 2m \), \( H_1 = e \varphi \). Here \( m, e \) are mass and charge of electron, \( \mathbf{p} = \hbar \mathbf{k} \) is the electron momentum.

Let’s designate an equilibrium matrix of density in absence of an external field through \( \tilde{\rho}_0 \):

\[
\tilde{\rho}_0 = \frac{1}{\exp \left( \frac{H_0 - \mu_0}{k_B T} \right) + 1}.
\]

Density matrix it is possible to present an equilibrium matrix of density in the form

\[ \tilde{\rho} = \tilde{\rho}_0 + \tilde{\rho}_1. \]

Here \( \tilde{\rho}_1 \) is the correction to the equilibrium matrix of density, caused by presence of an electromagnetic field.

In linear approximation we receive

\[ [H, \tilde{\rho}] = [H_0, \tilde{\rho}_1] + [H_1, \tilde{\rho}_0], \]

and

\[ [H, \tilde{\rho}] = 0. \]

Here \( [H, \tilde{\rho}] = H \tilde{\rho} - \tilde{\rho}H \) is the commutator.

Let’s notice, that the vector \( |\mathbf{k}\rangle \) is the eigen vector of operators \( H \) and \( \mathbf{p} \). Thus

\[
H|\mathbf{k}\rangle = \varepsilon_{\mathbf{k}}|\mathbf{k}\rangle, \quad \langle \mathbf{k}|H = \varepsilon_{\mathbf{k}}\langle \mathbf{k}|, \quad \mathbf{p}|\mathbf{k}\rangle = \hbar \mathbf{k}|\mathbf{k}\rangle, \quad \langle \mathbf{k}|\mathbf{p} = \hbar \mathbf{k}\langle \mathbf{k}|. 
\]
Let’s notice, that for the operator \( L \) the relationship is carried out
\[
\langle k_1 | L | k_2 \rangle = \frac{1}{(2\pi)^3} \int \exp(-ik_1 r) L \exp(i k_2 r) \, dr.
\]

By means of this relation it is received
\[
\langle k_1 | [H_0, \tilde{\rho}_1] | k_2 \rangle = - \langle k_1 | [H_1, \tilde{\rho}_0] | k_2 \rangle.
\]

Let’s write down this equality in the explicit form
\[
\langle k_1 | H_0 \tilde{\rho}_1 | k_2 \rangle - \langle k_1 | \tilde{\rho}_1 H_0 | k_2 \rangle = - \langle k_1 | H_1 \tilde{\rho}_0 | k_2 \rangle + \langle k_1 | \tilde{\rho}_0 H_1 | k_2 \rangle.
\]

From here we receive, that
\[
(\mathcal{E}_{k_1} - \mathcal{E}_{k_2}) \langle k_1 | \tilde{\rho}_1 | k_2 \rangle = (f_{k_1} - f_{k_2}) \langle k_1 | H_1 | k_2 \rangle =
\]
\[
e (f_{k_1} - f_{k_2}) \langle k_1 | \varphi | k_2 \rangle.
\]

Here
\[
\mathcal{E}_k = \frac{\hbar^2 k^2}{2m}, \quad f_k = \frac{1}{\exp \frac{\mathcal{E}_k - \mu_0}{k_B T} + 1}.
\]

Considering, that
\[
\langle k_1 | \varphi | k_2 \rangle = \frac{1}{(2\pi)^3} \int \exp(-i(k_1 - k_2) r) \varphi(r) \, dr = \varphi(k_1 - k_2),
\]
we receive
\[
(\mathcal{E}_{k_1} - \mathcal{E}_{k_2}) \tilde{\rho}_1(k_1, k_2) = e (f_{k_1} - f_{k_2}) \varphi(k_1 - k_2).
\]

The kinetic equation for the density matrix in \( \tau \)–approximation with constant frequency of collisions looks like
\[
i \hbar \frac{\partial \rho}{\partial t} = [H, \rho] + \frac{i \hbar}{\tau} (\tilde{\rho} - \rho),
\]
or
\[
\frac{\partial \rho}{\partial t} = - \frac{i}{\hbar} [H, \rho] + \nu (\tilde{\rho} - \rho). \tag{1.1}
\]

Here \( \tau \) is the average time of free electrons path, \( \nu = 1/\tau \) is the frequency of collisions.
Generally frequency of collisions $\nu$ should depend from electron momentum $p$ (or a wave vector $k$): $\nu = \nu(k)$.

Considering the requirement Hermitian character the equation (1.1) on the density matrix it is necessary to rewrite in the form

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H, \rho] + \frac{\nu(k)}{2} (\tilde{\rho} - \rho) + (\tilde{\rho} - \rho) \frac{\nu(k)}{2}. \quad (1.2)$$

In linear approximation the density matrix we will search in the form

$$\rho = \tilde{\rho}_0 + \rho_1.$$

Then in linear approach the equation (1.2) looks like

$$i\hbar \frac{\partial \rho_1}{\partial t} = H_0 \rho_1 - \rho_1 H_0 + H_1 \rho_0 - \rho_0 H_1 +$$

$$+ i\hbar \frac{\nu(k)}{2} (\tilde{\rho}_1 - \rho_1) + i\hbar (\tilde{\rho}_1 - \rho_1) \frac{\nu(k)}{2}. \quad (1.3)$$

Let’s consider, that $\rho_1 \sim \exp(-i\omega t)$. From here for $\rho_1$ we receive the relation

$$\hbar \omega \langle k_1 | \rho_1 | k_2 \rangle = (E_{k_1} - E_{k_2}) \langle k_1 | \rho_1 | k_2 \rangle - e(f_{k_1} - f_{k_2}) \varphi(k_1 - k_2) +$$

$$+ i\hbar \nu(k_1) \langle k_1 | \tilde{\rho}_1 - \rho_1 | k_2 \rangle + \langle k_1 | \tilde{\rho}_1 - \rho_1 | k_2 \rangle \frac{i\hbar \nu(k_2)}{2},$$

or, having designated

$$\tilde{\nu}(k_1, k_2) = \frac{\nu(k_1) + \nu(k_2)}{2},$$

Let’s rewrite the previous equation in the form

$$\hbar \omega \langle k_1 | \rho_1 | k_2 \rangle = (E_{k_1} - E_{k_2}) \langle k_1 | \rho_1 | k_2 \rangle - e(f_{k_1} - f_{k_2}) \varphi(k_1 - k_2) +$$

$$+ i\hbar \tilde{\nu}(k_1, k_2) \langle k_1 | \tilde{\rho}_1 - \rho_1 | k_2 \rangle. \quad (1.4)$$

From equation (1.4) we receive

$$\langle k_1 | \rho_1 | k_2 \rangle = \rho_1(k_1, k_2) = -e \frac{f_{k_1} - f_{k_2}}{E_{k_2} - E_{k_1} + \hbar (\omega + i\tilde{\nu}(k_1, k_2))} \varphi(k_1 - k_2) +$$

$$+ \frac{i\hbar \tilde{\nu}(k_1, \tilde{\rho}_1 | k_2)}{E_{k_2} - E_{k_1} + \hbar (\omega + i\tilde{\nu}(k_1, k_2))}. \quad (1.5)$$
The relation (1.5) represents the solution of linear Schrödinger—Boltzmann equation, expressed through perturbation to equilibrium matrix of density \( \tilde{\rho}_1(k_1 - k_2) = \langle k_1|\tilde{\rho}_1|k_2 \rangle \). Let’s find this perturbation.

Let’s take advantage of an obvious relation

\[
[H - \mu, \tilde{\rho}] = 0.
\]

In linear approximation from here it is received

\[
[H_0 - \mu_0, \tilde{\rho}_1] + [H_1 - \delta \mu, \tilde{\rho}_0] = 0.
\]

Transforming the first commutator, from here we receive:

\[
[H_0, \tilde{\rho}_1] = [\delta \mu - H_1, \tilde{\rho}_0],
\]

or

\[
[H_0, \tilde{\rho}_1] = [\delta \mu - \epsilon \varphi, \tilde{\rho}_0].
\]

Let’s designate now

\[
\delta \mu_\ast = \delta \mu - \epsilon \varphi.
\]

Then the previous equality write down in the form

\[
[H_0, \tilde{\rho}_1] = [\delta \mu_\ast, \tilde{\rho}_0].
\]

From here we receive that

\[
(\mathcal{E}_{k_1} - \mathcal{E}_{k_2})\langle k_1|\tilde{\rho}_1|k_2 \rangle = -( f_{k_1} - f_{k_2})\delta \mu_\ast(k_1 - k_2),
\]

from which

\[
\langle k_1|\tilde{\rho}_1|k_2 \rangle = -\frac{f_{k_1} - f_{k_2}}{\mathcal{E}_{k_1} - \mathcal{E}_{k_2}}\langle k_1|\delta \mu_\ast|k_2 \rangle,
\]

(1.6)

or, that all the same,

\[
\tilde{\rho}_1(k_1, k_2) = -\frac{f_{k_1} - f_{k_2}}{\mathcal{E}_{k_1} - \mathcal{E}_{k_2}}\delta \mu_\ast(k_1 - k_2).
\]

We have received perturbation to the equilibrium matrix of the density, expressed through perturbation of chemical potential. The last we will find from the preservation law of numerical density.
We put next
\[ k_1 = k + \frac{k}{2}, \quad q_2 = k - \frac{q}{2} \]
and rewrite in this terms equations (1.4), (1.5) и (1.6). We receive following equalities
\[ \hbar \omega \langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \rangle = \left( \varepsilon_{k+q/2} - \varepsilon_{k-q/2} \right) \langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \rangle - \]
\[ -e \varphi(q) (f_{k+q/2} - f_{k-q/2}) + i \hbar \omega \langle k + \frac{q}{2} | \tilde{\rho}_1 | k - \frac{q}{2} \rangle, \quad (1.4') \]
\[ \langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \rangle \equiv \rho_1(q) = -\frac{e \varphi(q) (f_{k+q/2} - f_{k-q/2})}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2} + \hbar \omega + i \hbar \omega} + \]
\[ + \frac{i \hbar \omega \langle k + \frac{q}{2} | \tilde{\rho}_1 | k - \frac{q}{2} \rangle}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2} + \hbar \omega + i \hbar \omega}, \quad (1.5') \]
and
\[ \langle k + \frac{q}{2} | \tilde{\rho}_1 | k - \frac{q}{2} \rangle = -\frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k+q/2} - \varepsilon_{k-q/2}} \langle k + \frac{q}{2} | \delta \mu_* | k - \frac{q}{2} \rangle. \quad (1.6') \]

In this equalities (1.4')–(1.6') the designation
\[ \bar{\nu} = \bar{\nu}(k, q) = \frac{\nu \left( k + \frac{q}{2} \right) + \nu \left( k - \frac{q}{2} \right)}{2} \]
is used.

2. Perturbation of chemical potential

The quantity \( \delta \mu \) (or \( \delta \mu_* \)) is responsible for the local preservation of number of particles (electrons). This local law preservation looks like [1]
\[ \omega \delta n(q, \omega, \bar{\nu}) = q \delta \tilde{j}(q, \omega, \bar{\nu}). \quad (2.1) \]
In equation (2.1) $\delta n(q, \omega, \bar{\nu})$, $\delta j(q, \omega, \bar{\nu}) = j(q, \omega, \bar{\nu})$ are change of concentration and stream density of electrons under action electric field, and

$$
\delta n(q, \omega, \bar{\nu}) = \int \frac{dk}{4\pi^3} \langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \rangle,
$$

$$
\delta j(q, \omega, \bar{\nu}) = \int \frac{\hbar dk}{4\pi^3 m} \langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \rangle.
$$

From equation on the matrix of density follows that

$$
\int \frac{\hbar \omega dk}{4\pi^3} \langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \rangle =
$$

$$
= \int \frac{dk}{4\pi^3} (\mathcal{E}_{k+q/2} - \mathcal{E}_{k-q/2}) \langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \rangle +
$$

$$
- e\varphi(q) \int \frac{dk}{4\pi^3} (f_{k+q/2} - f_{k-q/2}) =
$$

$$
= i\hbar \int \frac{dk}{4\pi^3} \bar{\nu}(k, q) \langle k + \frac{q}{2} | \rho_1 - \rho_1 | k - \frac{q}{2} \rangle.
$$

In this equalities the designation

$$
\bar{\nu} = \bar{\nu}(k, q) = \frac{\nu \left( k + \frac{q}{2} \right) + \nu \left( k - \frac{q}{2} \right)}{2},
$$

is used and conditions

$$
\mathcal{E}_{k+q/2} - \mathcal{E}_{k-q/2} = \frac{\hbar^2}{m} k q,
$$

(2.2)

and

$$
\int \frac{dk}{4\pi^3} (f_{k-q/2} - f_{k+q/2}) = 0
$$

are used.

Therefore last equality can be rewritten in the form

$$
\hbar \int \frac{dk}{4\pi^3} \left( \omega \langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \rangle - \frac{\hbar k q}{m} \langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \rangle \right) =
$$

$$
= i\hbar \int \frac{dk}{4\pi^3} \bar{\nu}(k, q) \langle k + \frac{q}{2} | \rho_1 - \rho_1 | k - \frac{q}{2} \rangle.
$$

Expression according to previous is equal to zero in the left part of this relation, i.e.

$$
\hbar \omega \delta n(q, \omega, \bar{\nu}) - \hbar q \delta j(q, \omega, \bar{\nu}) = 0.
$$
The last is true owing to the law of local preservation of number particles. From here follows, that

\[
\int \frac{d\mathbf{k}}{4\pi^3} \tilde{\nu}(\mathbf{k}, \mathbf{q}) \left\langle \mathbf{k} + \frac{\mathbf{q}}{2}, \tilde{\rho}_1 - \rho_1 \right| \mathbf{k} - \frac{\mathbf{q}}{2} \right\rangle = 0.
\]

This equality is equivalent to the following

\[
\int \frac{d\mathbf{k}}{4\pi^3} \tilde{\nu}(\mathbf{k}, \mathbf{q}) \left\langle \mathbf{k} + \frac{\mathbf{q}}{2}, \tilde{\rho}_1 \right| \mathbf{k} - \frac{\mathbf{q}}{2} \right\rangle = \int \frac{d\mathbf{k}}{4\pi^3} \tilde{\nu}(\mathbf{k}, \mathbf{q}) \left\langle \mathbf{k} + \frac{\mathbf{q}}{2}, \rho_1 \right| \mathbf{k} - \frac{\mathbf{q}}{2} \right\rangle.
\]

Considering earlier received expression (1.4) for \( \langle \mathbf{k}_1 | \tilde{\rho}_1 | \mathbf{k}_2 \rangle \), we have

\[
\delta \mu_\ast (\mathbf{q}, \tilde{\nu}) \int \frac{d\mathbf{k}}{4\pi^3} \tilde{\nu}(\mathbf{k}, \mathbf{q}) \frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2}} = \int \frac{d\mathbf{k}}{4\pi^3} \tilde{\nu}(\mathbf{k}, \mathbf{q}) \left\langle \mathbf{k} + \frac{\mathbf{q}}{2}, \rho_1 \right| \mathbf{k} - \frac{\mathbf{q}}{2} \right\rangle.
\]

Thus for perturbation quantity \( \delta \mu_\ast (\mathbf{q}) \) it is received

\[
\delta \mu_\ast (\mathbf{q}, \omega, \tilde{\nu}) = \frac{1}{B_\nu(q, 0)} \int \frac{d\mathbf{k}}{4\pi^3} \tilde{\nu}(\mathbf{k}, \mathbf{q}) \left\langle \mathbf{k} + \frac{\mathbf{q}}{2}, \rho_1 \right| \mathbf{k} - \frac{\mathbf{q}}{2} \right\rangle.
\]

Here the following designation is accepted

\[
B_\nu(q, 0) = \int \frac{d\mathbf{k}}{4\pi^3} \tilde{\nu}(\mathbf{k}, \mathbf{q}) \frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2}}.
\]

From equation (1.3) we receive

\[
\left[ \varepsilon_{\mathbf{k}_2} - \varepsilon_{\mathbf{k}_1} + \hbar (\omega + i\tilde{\nu}(\mathbf{k}_1, \mathbf{k}_2)) \right] \langle \mathbf{k}_1 | \rho_1 | \mathbf{k}_2 \rangle = -e(f_{k_1} - f_{k_2})\varphi(\mathbf{k}_1 - \mathbf{k}_2) + i\hbar \tilde{\nu}(\mathbf{k}_1, \mathbf{k}_2) \langle \mathbf{k}_1 | \tilde{\rho}_1 | \mathbf{k}_2 \rangle.
\]

Last component in this equality we will replace according to (1.4). We receive, that

\[
\left[ \varepsilon_{\mathbf{k}_2} - \varepsilon_{\mathbf{k}_1} + \hbar (\omega + i\tilde{\nu}(\mathbf{k}_1, \mathbf{k}_2)) \right] \langle \mathbf{k}_1 | \rho_1 | \mathbf{k}_2 \rangle =
\]

\[
- e(f_{k_1} - f_{k_2})\varphi(\mathbf{k}_1 - \mathbf{k}_2) -
\]

\[
- i\hbar \tilde{\nu}(\mathbf{k}_1, \mathbf{k}_2) \frac{f_{k_1} - f_{k_2}}{\varepsilon_{k_1} - \varepsilon_{k_2}} \delta \mu_\ast (\mathbf{k}_1 - \mathbf{k}_2, \omega, \tilde{\nu}).
\]
From this equation we obtain expression for \( \langle k_1 | \rho_1 | k_2 \rangle \):
\[
\langle k_1 | \rho_1 | k_2 \rangle = -\frac{e(f_{k_1} - f_{k_2}) \varphi(k_1 - k_2)}{\mathcal{E}_{k_2} - \mathcal{E}_{k_1} + \hbar(\omega + i\nu(k_1, k_2))} +
\]
\[
+i\hbar \nu(k_1, k_2) \frac{f_{k_1} - f_{k_2}}{\mathcal{E}_{k_2} - \mathcal{E}_{k_1}} \frac{\delta\mu_+(k_1 - k_2, \omega, \nu)}{\mathcal{E}_{k_2} - \mathcal{E}_{k_1} + \hbar(\omega + i\nu(k_1, k_2))},
\]
(2.4)
or, after decomposition on partial fractions,
\[
\langle k_1 | \rho_1 | k_2 \rangle = -\frac{e(f_{k_1} - f_{k_2}) \varphi(k_1 - k_2)}{\mathcal{E}_{k_2} - \mathcal{E}_{k_1} + \hbar(\omega + i\nu(k_1, k_2))} +
\]
\[
+i\hbar \nu(k_1, k_2) \frac{f_{k_1} - f_{k_2}}{\mathcal{E}_{k_2} - \mathcal{E}_{k_1}} \frac{\delta\mu_+(k_1 - k_2, \omega, \nu)}{\mathcal{E}_{k_2} - \mathcal{E}_{k_1} + \hbar(\omega + i\nu(k_1, k_2))},
\]
(2.4')
Passing to variables \( k \) and \( q \), from here we receive
\[
\langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \rangle = -\frac{e(f_{k+q/2} - f_{k-q/2}) \varphi(q)}{\mathcal{E}_{k-q/2} - \mathcal{E}_{k+q/2} + \hbar(\omega + i\nu(k, q))} +
\]
\[
+i\hbar \nu(k, q) \frac{f_{k+q/2} - f_{k-q/2}}{\mathcal{E}_{k-q/2} - \mathcal{E}_{k+q/2}} \frac{\delta\mu_+(q, \omega, \nu)}{\mathcal{E}_{k-q/2} - \mathcal{E}_{k+q/2} + \hbar(\omega + i\nu(k, q))} -
\]
\[
-i\hbar \nu(k, q) \frac{f_{k+q/2} - f_{k-q/2}}{\mathcal{E}_{k-q/2} - \mathcal{E}_{k+q/2}} \frac{\delta\mu_+(q, \omega, \nu)}{\mathcal{E}_{k-q/2} - \mathcal{E}_{k+q/2} + \hbar(\omega + i\nu(k, q))},
\]
(2.4'')
Let’s substitute expression (2.4'') in the formula for perturbation of chemical potential (2.3). On this way for perturbation it is received the following expression
\[
\delta\mu_+(q, \omega, \nu) = -e\varphi(q)\alpha_{\omega, \nu}(q).
\]
(2.5)
Here
\[
\alpha_{\omega, \nu}(q) = \frac{B_{\nu}(q, \omega + i\nu)}{B_{\nu}(q, 0) - iB_{\omega, \nu}(q, 0) + iB_{\omega, \nu}(q, \omega + i\nu)},
\]
\[ B_\nu(q, \omega + i\nu) = \int \frac{dk}{4\pi^3} \tilde{\nu}(k, q) \frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2} + \hbar(\omega + i\nu(k, q))}, \]

\[ B_\nu(q, 0) = \int \frac{dk}{4\pi^3} \tilde{\nu}(k, q) \frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2}} \]

\[ B_{\omega, \nu}(q, 0) = \int \frac{dk}{4\pi^3} \frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k+q/2} - \varepsilon_{k-q/2}} \cdot \frac{\tilde{\nu}(k, q)}{\omega + i\nu(k, q)} \]

\[ B_{\omega, \nu}(q, \omega + i\nu) = \int \frac{dk}{4\pi^3} \frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k+q/2} - \varepsilon_{k-q/2} + \hbar(\omega + i\nu(k, q))} \cdot \frac{\tilde{\nu}(k, q)}{\omega + i\nu(k, q)}. \]

Let’s write out a special case of the formula (2.5), when frequency of collisions of particles of plasma it is constant: \( \tilde{\nu}(k, q) \equiv \nu \equiv \text{const} \). In this case perturbation of chemical potential in Mermin’s designations it is equal

\[ \delta \mu_*(q, \omega, \nu) = -e\varphi(q)\alpha(\omega, \nu). \quad (2.6) \]

Here

\[ \alpha(\omega, \nu) = \frac{(\omega + i\nu)B(q, \omega + i\nu)}{\omega B(q, 0) + i\nu B(q, \omega + i\nu)}. \]

\[ B(q, 0) = \int \frac{dk}{4\pi^3} \frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2}}. \]

\[ B(q, \omega + i\nu) = \int \frac{dk}{4\pi^3} \frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k+q/2} - \varepsilon_{k-q/2} + \hbar(\omega + i\nu)}. \]

3. Electric conductivity and dielectric permeability

Let’s substitute (2.5) in (2.4") and in the received expression we will result similar members. It is as a result received the following expression

\[ \left\langle \mathbf{k} + \frac{q}{2} \right| \rho_1 \left| \mathbf{k} - \frac{q}{2} \right\rangle = \]
Here we will remind, that

$$\alpha_{\omega, \nu}(q) = \frac{B_{\nu}(q, \omega + i\nu) - i B_{\omega, \nu}(q, 0) + i B_{\omega, \nu}(q, \omega + i\nu)}{B_{\nu}(q, 0) - i B_{\omega, \nu}(q, 0) + i B_{\omega, \nu}(q, \omega + i\nu)}.$$  

Density of current $j_e(q, \omega, \nu)$ is calculated through $\langle k_1 | \rho_1 | k_2 \rangle$

$$j_e(q, \omega, \nu) = e \overline{j}(q, \omega, \nu) = \frac{e \hbar}{m} \int \frac{k d k}{4\pi^3} \left\langle k + \frac{q}{2} | \rho_1 | k - \frac{q}{2} \right\rangle. \quad (3.2)$$

Thus intensity of electric field is connected with potential of this field by relation $E(q, \omega) = -i\mathbf{q} \varphi(q, \omega)$, because $\varphi(q, \omega) = e^{i(q r - \omega t)}$.

From here the field potential is equal

$$\varphi(q, \omega) = \frac{i \mathbf{q} E(q, \omega)}{q^2}.$$  

Hence, expression for current density $j_e(q, \omega, \nu)$ by means of relation (3.2) it is possible to rewrite in the form

$$j_e(q, \omega, \nu) = -i \frac{e^2 \hbar}{m q^2} E(q, \omega) \int \frac{q k d k}{4\pi^3} R(k, q, \omega, \nu).$$

Here according to (3.1)

$$R(k, q, \omega, \nu) =$$

$$= \frac{f_{k+q/2} - f_{k-q/2}}{E_{k-q/2} - E_{k+q/2} + \hbar(\omega + i\nu(k, q))} \left(1 - \frac{i\nu(k, q) \cdot \alpha_{\omega, \nu}(q)}{\omega + i\nu(k, q)}\right) +$$

$$+ \frac{f_{k+q/2} - f_{k-q/2}}{E_{k-q/2} - E_{k+q/2}} \cdot \frac{i\nu(k, q) \cdot \alpha_{\omega, \nu}(q)}{\omega + i\nu(k, q)}.$$  

Considering connection of density of the current with intensity of the field, we receive expression for electric conductivity in quantum collisional plasma

$$
\sigma_l(q, \omega, \bar{\nu}) = -ie^2 \hbar \int \frac{q k d k}{4\pi^3} R(k, q, \omega, \bar{\nu}). \quad (3.2)
$$

By means of (3.2) we will write expression for dielectric permeability

$$
\varepsilon_l(q, \omega, \bar{\nu}) = 1 + \frac{4\pi e^2 \hbar}{m \omega q^2} \int \frac{k q d k}{4\pi^3} R(k, q, \omega, \bar{\nu}). \quad (3.3)
$$

Scalar product $k q$ we will express from relation (2.2)

$$
k q = \frac{m}{\hbar^2} \left( E_k + \frac{q}{2} - E_k - \frac{q}{2} \right)
$$

By means of this expression we will copy relations (3.2) and (3.3) in the following form

$$
\sigma_l(q, \omega, \bar{\nu}) = -ie^2 \hbar q^2 \int \frac{d k}{4\pi^3} K(k, q, \omega, \bar{\nu}) \quad (3.4)
$$

and

$$
\varepsilon_l(q, \omega, \bar{\nu}) = 1 + \frac{4\pi e^2 \hbar}{m \omega q^2} \int \frac{d k}{4\pi^3} K(k, q, \omega, \bar{\nu}). \quad (3.5)
$$

Here

$$
K(k, q, \omega, \bar{\nu}) = R(k, q, \omega, \bar{\nu}) \left( \varepsilon_{k+q/2} - \varepsilon_{k-q/2} \right).
$$

Calculating this quantity in an explicit form, we receive

$$
K(k, q, \omega, \bar{\nu}) = -(f_{k+q/2} - f_{k-q/2}) + 
\frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2}} + \hbar(\omega + i\bar{\nu}(k, q)) \left[ \omega + i\bar{\nu}(k, q)(1 - \alpha_{\omega,\bar{\nu}}(q)) \right].
$$

Let’s substitute this expression in equality (3.4). We receive, that

$$
\sigma_l(q, \omega, \bar{\nu}) = -i \frac{e^2}{q^2} \int \frac{d k}{4\pi^3} \frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2}} + \hbar(\omega + i\bar{\nu}(k, q)) \times
\times [\omega + i\bar{\nu}(k, q)(1 - \alpha_{\omega,\bar{\nu}}(q))],
$$
or, that all the same,
\[
\sigma_l(q, \omega, \bar{\nu}) = -\frac{ie^2}{q^2} \left[ \omega B(q, \omega + i\bar{\nu}) + iB_{\bar{\nu}}(q, \omega + i\bar{\nu})(1 - \alpha_{\omega, \bar{\nu}}(q)) \right]. \tag{3.6}
\]

Here
\[
B(q, \omega + i\bar{\nu}) = \int \frac{d\mathbf{k}}{4\pi^3} \frac{f_{k+q/2} - f_{k-q/2}}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2} + \hbar(\omega + i\bar{\nu}(k, q))}.
\]

On the basis of (3.6) we will write out expression for dielectric function
\[
\varepsilon_l(q, \omega, \bar{\nu}) = 1 + \frac{4\pi e^2}{\omega q^2} \times \left[ \omega B(q, \omega + i\bar{\nu}) + iB_{\bar{\nu}}(q, \omega + i\bar{\nu})(1 - \alpha_{\omega, \bar{\nu}}(q)) \right]. \tag{3.7}
\]

From the formula (3.7) it is visible that at \(\omega = 0\) we receive
\[
\varepsilon_l(q, 0, \nu) = 1 + \frac{4\pi e^2}{q^2} B(q, 0).
\]

Thus, at \(\omega = 0\) dielectric function does not depend from frequency of particles collisions of plasma.

At \(\nu = 0\) from (3.7) we receive
\[
\varepsilon_l(q, \omega, 0) = 1 + \frac{4\pi e^2}{q^2} B(q, \omega).
\]

Thus, at \(\nu = 0\) dielectric function will be transformed to the known formula received by Klimontovich and Silin in 1952 and after that by Lindhard in 1954.

We find explicit form of expressions (3.6) and (3.7). For quantity \(1 - \alpha_{\omega, \bar{\nu}}(q)\) we have
\[
1 - \alpha_{\omega, \bar{\nu}}(q) = \frac{[B_{\bar{\nu}}(q, 0) - B_{\bar{\nu}}(q, \omega + i\bar{\nu})] - i[B_{\omega, \bar{\nu}}(q, 0) - B_{\omega, \bar{\nu}}(q, \omega + i\bar{\nu})]}{B_{\bar{\nu}}(q, 0) - iB_{\omega, \bar{\nu}}(q, 0) + iB_{\omega, \bar{\nu}}(q, \omega + i\bar{\nu})}.
\]

We will notice that
\[
\frac{1}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2}} - \frac{1}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2} + \hbar(\omega + i\bar{\nu}(k, q))} = \frac{\hbar(\omega + i\bar{\nu}(k, q))}{(\varepsilon_{k-q/2} - \varepsilon_{k+q/2})(\varepsilon_{k-q/2} - \varepsilon_{k+q/2} + \hbar(\omega + i\bar{\nu}(k, q)))}.
\]
Hence, the first difference from numerator of the previous expression is equal

\[
B_{\nu}(q,0) - B_{\nu}(q,\omega + i\nu) = \\
\hbar \int \frac{dk}{4\pi^3} \frac{\tilde{v}(k,q)(\omega + i\tilde{v}(k,q))(f_{k+q/2} - f_{k-q/2})}{(E_{k-q/2} - E_{k+q/2})(E_{k-q/2} - E_{k+q/2} + \hbar(\omega + i\tilde{v}(k,q)))}.
\]

The second difference from numerator of the previous expression is equal

\[
B_{\omega,\nu}(q,0) - B_{\omega,\nu}(q,\omega + i\tilde{v}) = \\
\hbar \int \frac{dk}{4\pi^3} \frac{\tilde{v}^2(k,q)(f_{k+q/2} - f_{k-q/2})}{(E_{k-q/2} - E_{k+q/2})(E_{k-q/2} - E_{k+q/2} + \hbar(\omega + i\tilde{v}(k,q)))}.
\]

Hence, all numerator is equal

\[
[B_{\nu}(q,0) - B_{\nu}(q,\omega + i\nu)] - i[B_{\omega,\nu}(q,0) - B_{\omega,\nu}(q,\omega + i\tilde{v})] = \\
\hbar \omega \int \frac{dk}{4\pi^3} \frac{\tilde{v}(k,q)(f_{k+q/2} - f_{k-q/2})}{(E_{k-q/2} - E_{k+q/2})(E_{k-q/2} - E_{k+q/2} + \hbar(\omega + i\tilde{v}(k,q)))} = \\
= \omega \int \frac{dk}{4\pi^3} \frac{\tilde{v}(k,q)}{\omega + i\tilde{v}(k,q)} \frac{f_{k+q/2} - f_{k-q/2}}{E_{k-q/2} - E_{k+q/2} + \hbar(\omega + i\tilde{v}(k,q))} = \\
= \omega[b_{\omega,\nu}(k,q,0) - b_{\omega,\nu}(q,\omega + i\tilde{v})],
\]

where

\[
b_{\omega,\nu}(q,0) = \int \frac{dk}{4\pi^3} \frac{\tilde{v}(k,q)}{\omega + i\tilde{v}(k,q)} \cdot \frac{f_{k+q/2} - f_{k-q/2}}{E_{k-q/2} - E_{k+q/2} + \hbar(\omega + i\tilde{v}(k,q))},
\]

\[
b_{\omega,\nu}(q,\omega + i\nu) = \int \frac{dk}{4\pi^3} \frac{\tilde{v}(k,q)}{\omega + i\tilde{v}(k,q)} \cdot \frac{f_{k+q/2} - f_{k-q/2}}{E_{k-q/2} - E_{k+q/2} + \hbar(\omega + i\tilde{v}(k,q))}.
\]

Now we find the denominator

\[
B_{\nu}(q,0) - iB_{\omega,\nu}(q,0) + iB_{\omega,\nu}(q,\omega + i\tilde{v}).
\]
We notice that
\[ B_\nu(q, 0) - B_{\omega, \nu}(q, 0) = \omega \int \frac{dk}{4\pi^2} \frac{f_{k+q/2} - f_{k-q/2}}{E_{k-q/2} - E_{k+q/2}} \cdot \frac{\tilde{v}(k, q)}{\omega + i\tilde{v}(k, q)} = \omega b_{\omega, \nu}(q, 0). \]

Hence, the denominator is equal
\[ B_\nu(q, 0) - iB_{\omega, \nu}(q, 0) + iB_{\omega, \nu}(q, \omega + i\tilde{v}) = \omega b_{\omega, \nu}(q, 0) + iB_{\omega, \nu}(q, \omega + i\tilde{v}). \]

Thus, we have found, that
\[ 1 - \alpha_{\omega, \nu}(q) = \omega \frac{b_{\omega, \nu}(q, 0) - b_{\omega, \nu}(q, \omega + i\tilde{v})}{\omega b_{\omega, \nu}(q, 0) + iB_{\omega, \nu}(q, \omega + i\tilde{v})}. \tag{3.8} \]

According to (3.8) electric conductivity and dielectric permeability in quantum collisional plasma are accordingly equal to
\[
\sigma_l(q, \omega, \tilde{v}) = -i\frac{e^2}{q^2} \omega \left[ B(q, \omega + i\tilde{v}) + iB_\nu(q, \omega + i\tilde{v}) \frac{b_{\omega, \nu}(q, 0) - b_{\omega, \nu}(q, \omega + i\tilde{v})}{\omega b_{\omega, \nu}(q, 0) + iB_{\omega, \nu}(q, \omega + i\tilde{v})} \right]. \tag{3.9}
\]
and
\[
\epsilon_l(q, \omega, \tilde{v}) = 1 + \frac{4\pi e^2}{q^2} \left[ B(q, i\tilde{v}) + iB_\nu(q, i\tilde{v}) \frac{b_{0, \nu}(q, 0) - b_{0, \nu}(q, i\tilde{v})}{iB_{0, \nu}(q, i\tilde{v})} \right]. \tag{3.10}
\]

We put \( \omega = 0 \) in (3.10). Then
\[
\epsilon_l(q, 0, \tilde{v}) = 1 + \frac{4\pi e^2}{q^2} \left[ B(q, i\tilde{v}) + B_\nu(q, i\tilde{v}) \frac{B(q, 0) - B(q, i\tilde{v})}{B_\nu(q, i\tilde{v})} \right] = 1 + \frac{4\pi e^2}{q^2} B(q, 0) \equiv \epsilon_l(q).
\]
4. Non-degenerate plasmas with constant frequency of collisions and comparison with Mermin’ formula

Let’s consider now the case of constant frequency of electrons collisions

\[ \nu(k) = \nu = \text{const}. \]

Then \( \nu(k_1, k_2) = \nu(k, q) = \nu = \text{const}. \) As it was specified above, the quantity \( \alpha_{\omega, \nu}(q) \) for constant frequency of collisions becomes the following form

\[ \alpha_{\omega, \nu}(q) = \frac{(\omega + i\nu)B(q, \omega + i\nu)}{\omega B(q, 0) + i\nu B(q, \omega + i\nu)}. \]

Besides,

\[ B(q, \nu, 0, \omega + i\nu) = \nu B(q, \omega + i\nu), \]

\[ 1 - \alpha_{\omega, \nu}(q) = \omega \frac{B(q, 0) - B(q, \omega + i\nu)}{\omega B(q, 0) + i\nu B(q, \omega + i\nu)}. \]

Hence, according to (3.6) and (3.7) (or (3.9) and (3.10)) it is received accordingly expressions for electric conductivity and dielectric function for quantum collisional plasmas

\[ \sigma_l(q, \omega, \nu) = -\frac{ie^2}{q^2} \frac{\omega (\omega + i\nu)B(q, \omega + i\nu)B(q, 0)}{\omega B(q, 0) + i\nu B(q, \omega + i\nu)}. \quad (4.1) \]

\[ \epsilon_l(q, \omega, \nu) = 1 + 4\pi e^2 \frac{\omega (\omega + i\nu)B(q, \omega + i\nu)B(q, 0)}{q^2 \omega B(q, 0) + i\nu B(q, \omega + i\nu)}. \quad (4.2) \]

Let’s compare the formula (4.2) to corresponding Mermin’s formula (8) of its work [1].

Let’s write out Mermin’s formula (8) for dielectric function [1]

\[ \epsilon_l^{\text{Mermin}}(q, \omega, \nu) = 1 + \frac{(1 + i/\omega\tau)(\epsilon^0(q, \omega + i/\tau) - 1)}{1 + (i/\omega\tau)(\epsilon^0(q, \omega + i/\tau) - 1)/(\epsilon^0(q, 0) - 1)}. \quad (4.3) \]
In expression (4.3) $\varepsilon^0(q, 0)$ is Lindhard’s dielectric function for collisionless plasmas,

$$\varepsilon^0(q, \omega) = 1 + \frac{4\pi e^2}{q^2} B(q, \omega),$$

$$B(q, \omega) = \int \frac{dp}{4\pi^3} \frac{f_{p+q/2} - f_{p-q/2}}{\varepsilon_{p-q/2} - \varepsilon_{p+q/2} + \omega},$$

$$B(q, 0) = \int \frac{dp}{4\pi^3} \frac{f_{p+q/2} - f_{p-q/2}}{\varepsilon_{p-q/2} - \varepsilon_{p+q/2}}.$$

Let’s transform the formula (4.3), noticing, that $1 + i/\omega \tau = (\omega + i\nu)/\omega$, to the form

$$\varepsilon_{\text{Mermin}}^\text{l}(q, \omega, \nu) = 1 + \frac{(\omega + i\nu)[\varepsilon^0(q, \omega + i\nu) - 1][\varepsilon^0(q, 0) - 1]}{\omega[\varepsilon^0(q, 0) - 1] + i\nu[\varepsilon^0(q, \omega + i\nu) - 1]}.$$  (4.4)

Let’s copy Mermin’s formula (4.4) in terms of integrals $B(q, \omega)$ and $B(q, 0)$

$$\varepsilon_{\text{Mermin}}^\text{l}(q, \omega, \nu) = 1 + \frac{4\pi e^2 (\omega + i\nu) B(q, \omega + i\nu) B(q, 0)}{q^2} \frac{\omega B(q, 0) + i\nu B(q, \omega + i\nu)}{\omega B(q, 0) + i\nu B(q, \omega + i\nu)}. $$  (4.5)

The formula (4.5) in accuracy coincides with the formula (4.2).

5. Solution of kinetic equation of Vlasov—Boltzmann

In following two paragraphs we will deduce expression for electric conductivity and dielectric permeability of classical non-degenerate collisional plasmas with any degree non-degeneracy (for any temperature).

We take kinetic Vlasov—Boltzmann equation for collisional plasmas with any temperature

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + e \mathbf{E}(\mathbf{r}, t) \frac{\partial f}{\partial \mathbf{p}} = \nu[f_{eq} - f(\mathbf{r}, \mathbf{v}, t)]. $$  (5.1)
Here $f_{eq}(r, v)$ is the local equilibrium distribution function of Fermi–Dirac (local Fermian)

$$f_{eq} = \frac{1}{1 + \exp \left( \frac{mv^2}{2k_B T} - \frac{\mu(r)}{k_B T} \right)}, \quad (5.2)$$

$k_B$ is the Boltzmann constant, $T$ is the plasmas temperature, $\nu$ is the frequency of electron collisions with plasmas particles, $p = mv$ is the electron momentum, $e$ is the electron charge, $\mu(r)$ is the chemical plasmas potential.

Let’s present the chemical potential in linear approximation as

$$\mu(r) = \mu + \delta \mu(r), \quad \mu = \text{const}.$$  

Let’s spend linearization of the equations (5.1) concerning the absolute Fermian

$$f_0(v, \mu) = \frac{1}{1 + \exp \left( \frac{mv^2}{2k_B T} - \frac{\mu}{k_B T} \right)},$$

or

$$f_0(P, \alpha) = \frac{1}{1 + e^{P^2 - \alpha}}.$$  

Here $P$ is the dimensionless momentum (velocity), $\alpha$ is the dimensionless (reduced) chemical potential,

$$P = \sqrt{\beta v}, \quad \alpha = \frac{\mu}{k_B T}.$$  

For this purpose we will present distribution function of electrons in the form, linear on $\delta \mu(r)$ concerning absolute Fermian

$$f \equiv f(r, v, t) = f_0(v, \mu) + \frac{\partial f_0}{\partial \alpha} e^{i(kr - \omega)} \psi(v). \quad (5.3)$$

Here

$$\frac{\partial f_0}{\partial \alpha} = g(P, \alpha) = \frac{e^{P^2 - \alpha}}{(1 + e^{P^2 - \alpha})^2}.$$  

Making linearization of (5.2) on $\delta \alpha$, we receive, that

$$f_{eq}(r, v) = f_0(v, \mu) + \frac{\partial f_0}{\partial \alpha} \delta \alpha, \quad \delta \alpha = \frac{\delta \mu}{k_B T}. \quad (5.4)$$
Besides, in linear approach a member with the self-consistent field it is equal
\[ E(r, t) \frac{\partial f}{\partial p} = E(r, t) \frac{\partial f_0}{\partial p} = e^{i(kr - \omega t)} \frac{\partial f_0}{\partial \alpha} \frac{v_x}{k_B T}. \] (5.5)

Substituting (5.3) – (5.5) in the equation (5.1), we receive the equation concerning function \( \psi \) from which we find, that
\[ \psi(v) = \frac{\delta \alpha e^{-i(kr - \omega t)} + e^{\tau v_x}}{1 - i\omega T + ik\tau v_x}. \] (5.6)

Let’s find change of quantity of chemical potential of plasma particles \( \delta \mu(r) \) from the law of preservation of number of particles
\[ \int f(r, v, t) d\Omega = \int f_{eq}(r, v, t) d\Omega, \] (5.7)

where
\[ d\Omega = \frac{(2s + 1)d^3p}{(2\pi \hbar)^3}, \]
s is the spin of plasmas particles (electrons), \( s = 1/2 \).

The equation (5.7) will be transformed to the form
\[ e^{i(kr - \omega t)} \int \frac{\partial f_0}{\partial \alpha} \psi(v) d\Omega = \delta \alpha \int \frac{\partial f_0}{\partial \alpha} d\Omega, \] (5.8)

or
\[ e^{i(kr - \omega t)} \int \frac{\partial f_0}{\partial \alpha} \psi(v) d^3v = \delta \alpha \int \frac{\partial f_0}{\partial \alpha} d^3v, \] (5.8')

From equation (5.8) we receive
\[ \delta \alpha = \frac{e^{i(kr - \omega t)}}{b_0} \int \frac{\partial f_0}{\partial \alpha} \psi(v) d^3v. \] (5.9)

Here
\[ b_0 = \int \frac{\partial f_0}{\partial \alpha} d^3v = 4\pi g_2(\alpha), \quad g_2(\alpha) = \int_0^\infty g(P, \alpha)P^2 dP. \]

It is easy to see, that
\[ b_0 = 2\pi f_0(\alpha), \quad f_0(\alpha) = \int_0^\infty \frac{dP}{1 + e^{P^2 - \alpha}} = \int_0^\infty f_0(P, \alpha) dP. \]
Now we will substitute (5.6) in (5.9). We will have

\[ e^{-i(kr-\omega t)}\delta\alpha = \frac{1}{b_0} \int \frac{\partial f_0}{\partial \alpha} \cdot \frac{\delta\alpha e^{-i(kr-\omega t)}}{1-i\omega \tau + i\kappa \tau v_x} d^3v. \] (5.10)

From equation (5.10) we obtain

\[ e^{-i(kr-\omega t)}\delta\alpha = \frac{e\tau}{k_BT} \frac{B_1}{b_0 - B_0} = \frac{e\tau}{k_BT} \frac{B_1/b_0}{1 - B_0/b_0}. \] (5.11)

Here

\[ B_0 = \int \frac{\partial f_0}{\partial \alpha} \frac{d^3v}{1-i\omega \tau + i\kappa \tau v_x}, \]

\[ B_1 = \int \frac{\partial f_0}{\partial \alpha} \frac{v_x d^3v}{1-i\omega \tau + i\kappa \tau v_x}. \]

Thus, function \( \psi \) is constructed

\[ \psi(v) = \frac{B_1}{k_BT} + v_x \frac{1}{b_0 - B_0 + i\omega \tau + i\kappa \tau v_x}. \] (5.12)

6. Electric conductivity and dielectric permeability

From definition of density of a current follows, that

\[ j = \sigma_l e^{i(kr-\omega t)} = e \int v f d\Omega = e \int v_x e^{i(kr-\omega t)} \frac{\partial f_0}{\partial \alpha} \psi(v) d\Omega. \] (6.1)

From this for electri conductivity we receive

\[ \sigma_l = e \int v_x \frac{\partial f_0}{\partial \alpha} \psi(v) d\Omega. \] (6.2)

In more details

\[ \sigma_l = \frac{e^2\tau}{k_BT} \left[ \int \frac{v_x}{1-i\omega \tau + i\kappa \tau v_x} \frac{d\Omega}{b_0 - B_0} + \int \frac{v_x^2}{1-i\omega \tau + i\kappa \tau v_x} \frac{d\Omega}{2\pi \hbar} \right], \] (6.3)

or

\[ \sigma_l = \frac{e^2\tau 2m^3}{k_BT(2\pi \hbar)^3} \left[ \frac{B_1^2}{b_0 - B_0} + B_2 \right], \] (6.4)
where
\[ B_2 = \int \frac{v_x^2 \partial f_0}{1 - i\omega + ik\tau v_x} \, d^3v. \]

We notice that
\[ B_1 = \frac{b_0}{ik\tau} - \frac{1 - i\omega}{ik\tau} B_0. \]
\[ B_2 = -\frac{1 - i\omega}{ik\tau} B_1. \]

Taking into account this relation expression (2.4) will be copied in the form
\[ \sigma_l = \frac{e^2 \tau 2m^3}{k_B T(2\pi \hbar)^3} B_1 \left[ \frac{B_1}{b_0 - B_0} - \frac{1 - i\omega}{ik\tau} \right], \quad (6.5) \]

Now it is necessary to find expression in the square bracket from (6.5)
\[ \frac{B_1}{b_0 - B_1} - \frac{1 - i\omega}{ik\tau} = \frac{1}{1 - \frac{B_0}{b_0}} \left[ \frac{B_1}{b_0} - \frac{1 - i\omega}{ik\tau} \left( 1 - \frac{B_0}{b_0} \right) \right] = \]
\[ = \frac{1}{1 - \frac{B_0}{b_0}} \cdot \frac{\omega}{k}. \]

Hence, (6.5) looks like
\[ \sigma_l = \frac{e^2 \tau 2m^3}{k_B T(2\pi \hbar)^3} \cdot \frac{B_1}{1 - \frac{B_0}{b_0}} \cdot \frac{\omega}{k}. \quad (6.6) \]

Let's replace here \( B_1 \) according to previous and we will rewrite (2.6) in equivalent kind
\[ \sigma_l = \frac{e^2 \tau 2m^3}{k_B T(2\pi \hbar)^3} \cdot \frac{b_0}{ik\tau} - \frac{1 - i\omega}{ik\tau} B_0 \cdot \frac{\omega}{k}. \quad (6.7) \]

We notice that
\[ \frac{b_0}{ik\tau} - \frac{1 - i\omega}{ik\tau} B_0 = \frac{1}{ik\tau} \left[ \int g(v,\alpha) d^3v - \frac{1 - i\omega}{ik\tau} \int \frac{g(v,\alpha) d^3v}{v_x - \frac{\omega + i\nu}{k}} \right], \]
\[
1 - \frac{B_0}{b_0} = 1 + i \frac{1}{2\pi f_0(\alpha) k \tau v_T} \int \frac{g(P) d^3P}{P_x - z'/k'}
\]

where
\[
P = \frac{v}{v_T}, \quad v_T = \frac{1}{\sqrt{\beta}}, \quad z' = \omega + i\nu v_T.
\]

By means of last relation expression of the electric conductivity (6.7) assumes the following form
\[
\sigma_l = -ie^2 \tau 2m^3 \omega 2\pi v_T^3 f_0(\alpha) k_B T (2\pi \hbar)^3 k^2 \tau \times
\]
\[
1 + \frac{z'}{2\pi f_0(\alpha) k} \int \frac{g(P, \alpha) d^3P}{P_x - z'/k} \times
\]
\[
1 + \frac{i\nu}{2\pi f_0(\alpha) v_T k} \int \frac{g(P, \alpha) d^3P}{P_x - z'/k}.
\]

(6.8)

It is easy to calculate, that numerical density (concentration) of non-degenerate plasmas it is equal
\[
N = \frac{f_2(\alpha)}{\pi^2} k_T^3, \quad k_T = \frac{mv_T}{\hbar}, \quad f_2(\alpha) = \int_0^\infty \frac{P^2 dP}{1 + e^{P^2 - \alpha}}.
\]

\(k_T\) is the thermal wave number, \(v_T\) is the thermal velocity of electrons.

Expression (6.8) we will transform to the following form
\[
\frac{\sigma_l}{\sigma_0} = -i \nu \omega f_0(\alpha) \frac{z'}{2\pi f_0(\alpha) k} \int \frac{g(P, \alpha) d^3P}{P_x - z'/k} \times
\]
\[
1 + \frac{i\nu}{2\pi f_0(\alpha) v_T k} \int \frac{g(P, \alpha) d^3P}{P_x - z'/k}.
\]

(6.9)

In considering of \(\sigma_0 = \frac{\omega_p^2}{4\pi} \tau\), where \(\omega_p^2 = \frac{4\pi e^2 N}{m}\), \(\omega_p\) is the plasma (Langmuir) frequency, on basis (6.9) we receive the following expression for dielectric function
\[
\varepsilon_l = 1 + \frac{\omega_p^2 f_0(\alpha)}{k^2 v_T^3 f_2(\alpha)} \frac{z'}{2\pi f_0(\alpha) k} \int \frac{g(P, \alpha) d^3P}{P_x - z'/k} \times
\]
\[
1 + \frac{i\nu}{2\pi f_0(\alpha) v_T k} \int \frac{g(P, \alpha) d^3P}{P_x - z'/k}.
\]

(6.10)
We rewrite the formula (6.10) in the following form

\[ \varepsilon_l = 1 + \frac{x_p^2}{q^2} \cdot \frac{f_0(\alpha)}{f_2(\alpha)} \cdot \frac{1 + (z/q)b_0(z/q)}{1 + (iy/q)b_0(z/q)}, \]

where

\[ z = x + iy = \frac{\omega + i\nu}{kTv_T}, \quad b_0(z/q) = \frac{1}{2f_0(\alpha)} \int_{-\infty}^{\infty} \frac{f_0(\mu, \alpha)d\mu}{\mu - z/q}, \quad q = \frac{k}{k_T}. \]

7. Connection of characteristics of quantum and classical plasma

Let's show, that the basic characteristics of plasma, such, as electric conductivity and dielectric permeability of quantum collisional non-degenerate plasmas, pass in limit, when wave number \( k \) (or Planck’s constant) tends to zero, in corresponding characteristics non-degenerate classical collisional plasmas.

The proof we will spend for electric conductivity. We take expression (4.1) for electric conductivity

\[ \sigma_l(k, \omega, \nu) = -i\frac{e^2}{q^2} \frac{\omega(\omega + i\nu)B(k, \omega + i\nu)B(k, 0)}{\omega B(k, 0) + i\nu B(k, \omega + i\nu)}. \quad (7.1) \]

Here

\[ B(k, \omega + i\nu) = \int \frac{dk'}{4\pi^3} \frac{f_{k'+k/2} - f_{k'-k/2}}{\varepsilon_{k'-k/2} - \varepsilon_{k'+k/2} + \hbar(\omega + i\nu)}, \]

\[ B(k, 0) = \int \frac{dk'}{4\pi^3} \frac{f_{k'+k/2} - f_{k'-k/2}}{\varepsilon_{k'-k/2} - \varepsilon_{k'+k/2}}, \]

\[ f_{k'\pm k/2} = \frac{1}{1 + \exp \left( \frac{\varepsilon_{k'\pm k/2} - \mu}{k_BT} \right)}, \quad \varepsilon_{k'\pm k/2} = \frac{\hbar^2}{2m}(k' \pm \frac{k}{2})^2, \]

\[ \varepsilon_{k'+k/2} - \varepsilon_{k'-k/2} = \frac{\hbar^2}{m}k'k = \frac{\hbar^2}{m}k_xk. \]

We linearize functions \( f_{k'\pm k/2} \) on a wave vector. We receive, that

\[ f_{k'\pm k/2} = f_0(k', \alpha) \pm g(k', \alpha)\frac{\hbar^2k'k}{2mk_BT}, \]
where
\[ f_0(k', \alpha) = \frac{1}{1 + \exp \left( \frac{\hbar^2 k'^2}{2m k_B T} - \alpha \right)}, \]
\[ g(k', \alpha) = \frac{\exp \left( \frac{\hbar^2 k'^2}{2m k_B T} - \alpha \right)}{\left[ 1 + \exp \left( \frac{\hbar^2 k'^2}{2m k_B T} - \alpha \right) \right]^2}. \]

Therefore,
\[ f_{k' + k/2} - f_{k' - k/2} = -g(k', \alpha) \frac{\hbar^2 k'_x k}{m k_B T}. \]

By means of these relations we will present integrals \( B(k, \omega + i\nu) \) in the following form
\[
B(k, \omega + i\nu) = \frac{1}{4\pi^3 k_B T} \int g(k', \alpha) k'_x d^3 k' \frac{k'_x}{k - (\omega + i\nu)m} \frac{m v_T}{\hbar k}. \tag{7.2}
\]

We take the dimensionless variable of integration
\[ P = \frac{\hbar k'}{\sqrt{2mk_B T}} = \frac{\hbar k'}{mv_T} = \frac{k'}{k_T}, \quad k_T = \frac{mv_T}{\hbar}. \]

Then
\[
B(q, \omega + i\nu) = \frac{k_T^3}{4\pi^3 k_B T} \int g(P, \alpha) P_x d^3 P \frac{P_x - z/q}{P_{x - z/q}}, \tag{7.3}
\]
where
\[ g(P, \alpha) = \frac{e^{P^2 - \alpha}}{\left( 1 + e^{P^2 - \alpha} \right)^2}, \quad z = \frac{\omega + i\nu}{v_T k_T} = x + iy, \quad q = \frac{k}{k_T}. \]

It is easy to see, that expression (7.3) is equal
\[
B(q, \omega + i\nu) = \frac{k_T^3 f_0(\alpha)}{2\pi^2 k_B T} + \frac{k_T^3 z}{4\pi^3 k_B T q} \int g(P, \alpha) d^3 P \frac{P_x - z/q}{P_{x - z/q}},
\]
or, that all the same,
\[
B(q, \omega + i\nu) = \frac{N f_0(\alpha)}{2k_B T f_2(\alpha)} \left[ 1 + \frac{z}{2\pi f_0(\alpha) q} \int g(P, \alpha) d^3 P \frac{P_x - z/q}{P_{x - z/q}} \right]. \tag{7.4}
\]
From (7.4) it is clear, that

\[ B(q, 0) = \frac{N f_0(\alpha)}{2k_B T f_2(\alpha)}. \] (7.5)

Substituting (7.4) and (7.5) in (7.1), we receive expression of the longitudinal electric conductivity in quantum collisional non-degenerate plasma in limit, when \( k \to 0 \) (or \( \hbar \to 0 \))

\[
\sigma_l = \frac{\omega \nu f_0(\alpha)}{q^2 v_T^2 k_T^2 f_2(\alpha)} \cdot \left[ 1 + \frac{z}{2\pi f_0(\alpha) q} \int \frac{g(P, \alpha) d^3 P}{P_x - z/q} \right] \cdot \frac{1 + \frac{y}{2\pi f_0(\alpha) q} \int \frac{g(P, \alpha) d^3 P}{P_x - z/q}}{1 + \frac{z}{2\pi f_0(\alpha) q} \int \frac{g(P, \alpha) d^3 P}{P_x - z/q}}. \] (7.6)

We are convinced now, that formulas (6.9) and (7.6) coincide.

8. Conclusion

In the present work formulas for longitudinal electric conductivity and dielectric permeability in quantum collisional non-degenerate plasma with any degree of non-degeneracy are deduced. The general case, when frequency of electron collisions with plasma particles depends on their momentum is considered. For this purpose the kinetic equation concerning a matrix of density with integral of collisions in relaxation form in space of momentum is used.

It is shown, that when Planck’s constant tends to zero, the deduced formulas passes in corresponding formulas for classical plasma. It is shown also, that when frequency of collisions of particles of plasma tends to zero (plasma passes in collisionless one), the deduced formula passes in the known Linhard’s formula received for collisionless plasmas.

It is shown, that when frequency of collisions is a constant, the deduced formula for dielectric permeability passes in known Mermin’s formula.
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