Are there braneworld models that have complete localization properties?

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Abstract. A braneworld model is a model of \((1+3+N)\)-dimensional space-time, called the bulk, with \(N\) extra dimensions where standard model particles are assumed to be localized on a \((1+3)\)-dimensional “surface”, the so-called brane, embedded in the bulk. Gravitational fields, on the other hand, may propagate in the full space, the bulk. Some of the models could explain the weakness of the gravitational force relative to the other fundamental forces of nature. Among various models, the Randall-Sundrum (RS) model, an \(N=1\) braneworld model, has attracted much attention. Unfortunately, the assumption that standard model particles are trapped on the RS brane is not fully fulfilled. Accordingly, one of us with other colleagues proposed a modified RS model. The new model is conformally flat space-time and has much better localization properties. However, it is still not a perfect braneworld model since matter fields in some configurations are not localizable. This paper will elaborate localization properties of the model and discussed possible further modification to have a perfect model in terms of localization properties.

1. Introduction
Along time ago, a space-time model with an extra dimension, the so-called Kaluza-Klein model, was introduced. The extra dimension, with a restriction imposed on it, plays the role of unifying electromagnetic and gravitational forces [1]. This idea then led to models of space-time with more higher dimensions to unify all four fundamental forces [1]. Later, about two-three decades ago, other models of space-time with some extra dimensions were proposed to address some issues such as cosmological constants and hierarchy problems [2-8]. In these models, called the braneworld models, there is also a restriction, i.e. that matter fields are assumed to confine or to localize on a brane, a \((1+3)\)-dimensional spacetime imbedded in a higher, \((1+3+N)\)-dimensional bulk, where \(N\) is the number of the extra dimensions [2-4,9-12]. The gravity, on the other hand, could propagate in the bulk. In fact, this confinement assumption is not fully fulfilled by most introduced models, including the Randall-Sundrum (RS) model, a model with one extra dimension. For the case of fundamental fields that only interact with gravity in a minimal coupling way, this RS model could only localize massless scalar particles [9-12]. This fact then led the authors in [9-11] proposed a modified RS model, say the MRS model. They exhibited that, as far as field confinement is concerned, the MRS model is superior to the original RS model: not only scalar fields, vector fields are also localizable. Later, ref. [12] showed that by revising the definition of adjoint of spinor fields in five-dimension, namely from \(\bar{\Psi} = \Psi^+ \gamma^0\) to \(\bar{\Psi} = \Psi^+ \Gamma^0\) where

...
\[ \Gamma^A = e^B_A y^B \] being the Dirac matrices in a flat space and \( e^B_A \) being the funfbein of the MRS metric, spinor fields are also localised on the brane in the MRS model. Ref. [12] also discussed localization properties of the MRS model for the case of fundamental fields that interact with gauge fields. We will summarize the result of ref. [9,12,13] and discuss further modification of the RS model since there are some configurations where the fields within the configurations are not localised on the MRS brane.

2. The model

The MRS model proposed by [9] corresponds to the following metric

\[ ds^2 = e^{-2k|\rho|}(\eta_{\mu\nu}dx^\mu dx^\nu - dr^2) \]  

(1)

where \( r \) is the extra dimensional coordinate, \( k \) is the warp parameter, and \( \eta_{\mu\nu} = diag(1, -1, -1, -1) \) is the metric of the Minkowski space-time. It turns out that the metric (1) is conformally flat. It differs from the RS metric model

\[ ds^2 = e^{-2k|\gamma|\eta_{\mu\nu}dx^\mu dx^\nu - dy^2} \]  

(2)

in the extra dimension part. Here, \( y \) represents the extra coordinate in the RS model. One may define a new coordinate, say the \( z \) coordinate, as the extra coordinate for the RS model, replacing the \( y \) coordinate through \( dy^2 = e^{-2k|\gamma|}dz^2 \). It turns out that the RS model in the \( z \) coordinate is conformally flat just like the MRS model (1) but with non-exponential warp factor. The metrics (1) and (2) can be related through coordinate transformations: \( e^{-k|\rho|} = 1 - k|y| \), \( dx^\mu = e^{-k|\rho|}dx^\mu \) [9]. In the later, \( X^\mu \) and \( x^\mu \), respectively, represent the \( (1+3) \)-dimensional coordinates in the MRS and RS models. The transformations are singular at \( |y| = 1/k \) showing that both metrics are distinct. The transformations do not fulfil an exact differential, \( dx^\mu = A(x^\mu, y)dx^\mu + B(x^\mu, y)dy \), \( \partial_x A = \partial_x B \), since \( B = 0 \) while \( \partial_y A \neq 0 \) [9]. In addition, one may fine tune the cosmological constant in the RS model to keep vanishing the extra dimension components of the energy-momentum tensor. On the other hand, such a fine tune is impossible for the MRS model [9].

3. Field Localizations

A matter field is called to be localized on a brane of a braneworld model if all integrals over the extra dimensions in the action of the field have finite values. For the case of five-dimensional braneworld models such as the RS and MRS models, writing the extra coordinate as \( x^5 \) and a matter field as \( \Phi(x^\mu, x^5) = \varphi(x^\mu)\chi(x^5) \) the corresponding action has a general form of

\[ S = \sum_i \int d^5f_i(x^5) \int d^4x g_i(x^\mu). \]

(3)

\[ \Sigma_i f_i g_i \] is the Lagrangian of the matter field, the function \( f_i(x^5) \) depends on the metric, \( \chi(x^5) \) and its derivative while \( g_i(x^\mu) \) depends on \( \varphi(x^\mu) \) and its derivative. The summation runs from \( i = 1 \) to a number depending on the complexities of the Lagrangian and the field. For a non-interacting scalar field in the MRS model, as an example, \( i = 1,2 \) with \( f_1(x^5) = e^{-4k|\chi^5|}\chi^{*}\chi \) and \( f_2(x^5) = e^{-4k|\varphi^5|}\varphi^{*}\varphi \) while \( g_1(x^\mu) = \eta^{\mu\nu}\partial_\mu\varphi^{*}\partial_\nu\varphi \) and \( g_2(x^\mu) = \varphi^{*}\varphi \). Thus, the matter field is localized on the brane if \( \int d^5f_i(x^5) \) finite for all \( i \)’s. To evaluate the integrals of \( \int d^5f_i(x^5) \) one must first derive the function \( \chi(x^5) \), which then specifies the functions \( f_i(x^5) \), from the field equation of \( \Phi(x^\mu, x^5) \) (which is \( \partial_M(\sqrt{g}g^{MN}\partial_N\Phi) = 0 \) for a massless scalar field \( \Phi(x^\mu, x^5) \)). The localization properties of the MRS model, summarized from [9,12], are shown in the following tables. We mean non-interacting field in Table 1 is a field that does not interact with other fields except with gravity through immersion of the

\[ \int e^{-2k|\rho|}(\eta_{\mu\nu}dx^\mu dx^\nu - dr^2) \]
field in a curved space-time. Interacting fields in Table 2 are matter fields (scalar, vector, and spinor) that interact with a gauge (vector) field, in addition to interacting with gravity by immersion of the configuration fields in a curved space-time.

**Table 1.** Localizability of non-interacting fields [9,12]

| Field configurations     | Mass modes | Decreasing wrap factor ($k > 0$) | Increasing wrap factor ($k < 0$) | In comparison with the RS model |
|--------------------------|------------|----------------------------------|----------------------------------|---------------------------------|
| Scalar fields            | massless   | localized                         | not-localized                    | same improvement                |
|                          | massive    | localized                         | not-localized                    |                                 |
| Vector fields            | massless   | localized                         | not-localized                    | improvement                     |
|                          | massive    | not-localized                     | not-localized                    | same                            |
| Spinor fields            | massless   | not-localized                     | localized                        | improvement                     |
|                          | massive    | not-localized                     | localized                        |                                 |

**Table 2.** Localizability of interacting fields [12,13]

| Field configurations     | Mass modes | Decreasing wrap factor ($k > 0$) | Increasing wrap factor ($k < 0$) | In comparison with the RS model |
|--------------------------|------------|----------------------------------|----------------------------------|---------------------------------|
| Scalar-vector fields     | massless   | not-localized                    | not-localized                    | same                            |
|                          | massive    | localized                         | not-localized                    | improvement                     |
| Vector-vector fields     | massless   | localized                         | not-localized                    | same                            |
|                          | massive    | not-localized                     | not-localized                    | improvement                     |
| Spinor-vector fields     | massless   | not-localized                     | not-localized                    | same                            |
|                          | massive    | localized                         | not-localized                    | improvement                     |

Notes: All vector (gauge) fields are massless. All configurations are not localized for the massive gauge field case. The mass mode column corresponds to masses of the non-gauge field. The vector-vector field configuration is chosen to be the Yang-Mills one.

**4. Concluding Remarks**

Table 1 and Table 2 show that the MRS model has much better localization properties than the RS model. However, it still does not describe a perfect model since not all possible systems are localizable. For examples, non-interacting (except with gravity) massive vector fields and massless scalar fields that interact with a gauge field could not be localized. In addition, other configurations such as matter fields interacting non-minimally with gravity should also be analysed. The analysis is still underway. Note that, the non-minimal case is possibly more complicated since the Lagrangian of the system considered contains an extra term that depends on a Ricci scalar (or Ricci tensor depending of the types of the non-minimal coupling), in addition to the metric and the field. The Ricci scalars for the RS and MRS models have the forms, respectively [9,12]

\[
R_{RS}(y) = -20k^2 + 16k\delta(y),
\]

\[
R_{MRS}(r) = -12k^2 e^{2kr} + 16k\delta(r)e^{2kr}.
\]  

The non-minimal term with such a form of the Ricci scalar complicates the field equation and thus more likely leads to difficulties in obtaining the function of the extra dimensional part of the field. Obtaining this function is necessary because otherwise we could not analyse the field localization. Accomplishing the analysis of non-minimal coupling cases gives a complete conclusion on superiority, in terms of localization properties, of the MRS model. However, as we already know, even though we neglect non-minimal coupling cases, the MRS model is not a perfect model, meaning that we should modify further the model. Our experience shows that the warp factor \(\exp(-kr)\) plays important role in making the
integrals $\int dx^5 f_i(x^5)$ in some field configurations are finite but in some other configurations it is not able to make the integrals of finite values. One possible and worth trying modification is multiplying the Minkowski metric in eq. (1) by an exponential function like the warp factor but with a negative value of its exponent at any points in the extra dimension and for both decreasing and increasing warp factors. We are investigating this possibility.

5. References

[1] Overduin, J.M. and Wesson, P.S., 1997 *Phys. Rep.* 283, 303
[2] Rubakov, V.A. and Shaposhnikov, M.E., 1983 *Phys. Lett.* B 125, 136
[3] Visser, M., 1985 *Phys. Lett.* B 159, 22
[4] Squires, E.J., 1986 *Phys. Lett.* B 167, 286
[5] Barnaveli, A. and Kancheli, O., 1990 *Sov. J. Nucl. Phys.* 51, 901
[6] Barnaveli, A. and Kancheli, O., 1990 *Sov. J. Nucl. Phys.* 52, 920
[7] Gogberashvili, M., 1999 *Mod. Phys. Lett.* A 14, 2025
[8] Gogberashvili, M., 2000 *Europhys. Lett.* 49, 396
[9] Jones, P., Munoz, G., Singleton, D., and Triyanta, 2013 *Phys. Rev.* D 88, 025048; arXiv:hep-th/1307.3599
[10] Jones, P., Singleton, D., Munoz, G., and Triyanta, 2013 arXiv:hep-th/1309.4790
[11] Triyanta, Singleton, D., Jones, P., and Munoz, G. 2014 *AIP Proc. Conf.* 1617, 96
[12] Wulandari, D., Triyanta, Kosasih, J.S., Singleton, D., and Jones, P., 2017 *Int. J. Mod. Phys.* A 32, 1750191; arXiv:hep-th/1711.08472
[13] Wulandari, D., Triyanta, Kosasih, J.S., and Singleton, D. 2015 *AIP Proc. Conf.* 1677, 040003