Next-to-leading order thermal photon production in a weakly-coupled plasma

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Abstract
We summarize the recent determination to next-to-leading order of the thermal photon rate at weak coupling. We emphasize how it can be expressed in terms of gauge-invariant condensates on the light cone, which are amenable to novel sum rules and Euclidean techniques. For the phenomenologically interesting value of $\alpha_s = 0.3$, the NLO correction represents a 20% increase and has a functional form similar to the LO result.

Keywords: Photons, Hard Probes, Quark-Gluon Plasma, High order calculations, Euclidean methods

1. Introduction
Photons have long been considered a key hard probe of the medium produced in heavy-ion collisions. Experimentally, there are now detailed data on real photon production at RHIC [1–3] and the LHC [4–8]. Photons arising from meson decays following hadronization are subtracted from the data experimentally; theoretically one then needs to deal with several sources: “prompt” photons, produced in the scattering of partons in the colliding nuclei, jet photons, arising from the interactions and fragmentations of jets, thermal photons, produced by interactions of the (nearly) thermal constituents of the plasma and hadron gas photons, produced in later stages.

In this contribution we will concentrate on thermal photons. Specifically, we will consider a weakly-coupled, infinitely extended, static and equilibrated medium and, after briefly summarizing the leading-order perturbative result [9, 10], we will illustrate the NLO, i.e. relative $O(g)$, calculation presented in full detail in [17]. Its motivations are both phenomenological, i.e. an improved knowledge of the rate and, importantly, a first estimate of its associated theory uncertainty, and theoretical. Perturbation theory at finite temperature is known to suffer from slow convergence, at least for thermodynamical quantities; with the present calculation we aimed at exploring the largely uncharted territory of dynamical, time-dependent quantities beyond leading order. A posteriori, the extensive factorizations we found and the possibility of greatly simplifying the calculation through Euclidean and sum rule technologies make this calculation a pattern for future ones of related dynamical quantities, such as jet energy loss and transport coefficients.

1Partial LO results [11] for non-equilibrated, anisotropic media have been presented at this conference in [12][13]. Results for dileptons, in and out of equilibrium, have been presented in [14][15] and [16] respectively.
2. Overview of the calculation

The photon production rate is given at leading order in $\alpha$ by

$$ \frac{(2\pi)^3 d\Gamma}{d^4k} = \frac{1}{2k} \int d^4X e^{-ik\cdot X} \left< j^\mu(0) j^\nu(\infty) \right>, $$

which relates it to the backward Wightman correlator of the electromagnetic current $J^\mu = \sum_{q=uds} e_q \bar{q} \gamma^\mu q$. $K = (k, k) = (k, 0, 0, k)$ is the lightlike momentum of the photon which we choose to be oriented along the $z$ axis. We furthermore assume $k \geq T$, which is the validity region of the LO and NLO calculations. We will work perturbatively in the strong coupling $g$, meaning that we treat the scale $gT$ (the soft scale) as parametrically smaller than the scale $T$ (the hard scale).

At the lowest order in perturbation theory Eq. (1) vanishes, as the two bare Wightman propagators in the simple quark loop it generates cannot be simultaneously put on shell. In other words, on-shell quarks cannot radiate a physical photon. It is then necessary to kick at least one of the quarks out of the loop it generates cannot be simultaneously put on shell. In other words, on-shell quarks cannot radiate a physical photon. If $K$ is the photon momentum, let us call $P$ and $K - P$ the fermion momenta at a current insertion in Eq. (1). Then, as Fig. 1 schematically shows, there are three relevant ways at LO to satisfy momentum conservation.

![Figure 1](image.png)

Figure 1. Left: momentum assignments at one of the two current insertions. Right: the hard (blue), soft (red) and collinear (green) scalings are schematically represented.

In the **hard** region one of the two is far off-shell, $P^2 \sim T^2$. In the **soft** region $K - P$ carries basically the entire photon momentum and $P^2 \sim g^2 T^2$ is soft (all components of order $gT$). Finally, in the **collinear** region, the three vectors are almost collinear and $p^2 \sim g^2 T^2$. We find it convenient to choose $k$ to point along $z$ and define light-cone coordinates $p^+ = (p^0 + p_1)/2$, $p^- = p^0 - p_1$. The three scalings are then conveniently summarized in a $(p^+, p_\perp)$ plane, as shown on the left pane of Fig. [2]. As the figure suggests, the hard and soft regions are logarithmically sensitive to each other and a regulator ($\mu^2_{LO}$) is necessary in intermediate stages of the calculation. In more detail, the hard region is dealt with by evaluating the simple two-loop diagrams in Fig. [3] which, when cut, give rise to the simple $qg \rightarrow q\gamma$ and $q\bar{q} \rightarrow q\gamma$ processes, folded over the thermal distributions of initial and final states. The well known logarithmic IR divergences of these processes are cured by Hard Thermal Loop (HTL) resummation [13] in the soft sector, giving rise to a finite result for their sum [19][20], which corresponds to the elongated diagonal shape in the left pane of Fig. [2]

Collinear processes, as first pointed out in [21][22], also contribute at leading order. The analysis and calculation of [9][10] showed how a soft, small-momentum transfer scattering with the medium constituents can broaden the transverse momentum of the emitting quarks just enough for photon formation to be possible. The long, $O(1/g^2 T)$ formation time allows an arbitrary number of such scattering to contribute at leading order and interfere, in what is called the Landau-Pomeranchuk-Migdal (LPM) effect. Such an example is shown in Fig. [4].

The next-to-leading order ($O(\alpha g^3)$) comes from $O(g)$ corrections to this picture that are of two kinds: loop corrections and mistreated regions. The latter are $O(g)$ slices of the phase space of the LO calculation where intermediate states become soft without causing IR divergences. Integrating over these regions with unresummed matrix elements introduces an $O(g)$ error in the LO calculation, which now needs to be addressed properly. This is done by identifying all such regions, extracting the limiting behavior and subtracting it. We will not concentrate further on the details of this procedure.

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2 Here and throughout this contribution capital letters stand for four-vectors, lowercase italic letters for the modulus of the spatial three-vectors and the metric signature is $(-+++)$, so that $p^2 = p^0^2 - p^2_{\perp}$. 
Loop corrections arise instead from the addition of extra soft gluons, which are penalized by a factor of $g$ only due to their Bose enhancement $T/p^0 \sim T/(gT)$. The soft and collinear regions are both sensitive to these corrections, whereas they are by construction not possible in the hard region. Finally, the slice of phase space corresponding to $T$ due to their Bose enhancement are quarks.

Figure 2. Regions contributing to the leading- (left) and next-to-leading order (right) calculations in the $(p^+, p_\perp)$ plane.

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3. Summary of the calculation

At leading order, the photon rate from the collinear sector reads \[ \frac{d\Gamma_{\text{coll}}}{d^2 k_{\text{coll}}} = \frac{\sigma_{\text{EM}} N_c}{2k(2\pi)^3} \int_{-\infty}^{\infty} dp^+ \left[ \frac{(p^+)^2 + (p^+ + k)^2}{(p^+)^2(p^+ + k)^2} \right] n(k + p^+) [1 - n(p^+)] \lim_{x_\perp \to 0} \text{Im}(2\nabla_{x_\perp} \cdot f(x_\perp)) \] which shows a nice factorization into a DGLAP-like splitting kernel (in square brackets), statistical (Fermi–Dirac) functions for the emitters and finally their transverse size at the emission point. The latter is determined through this Schrödinger-like inhomogeneous differential equation \[ \frac{d\psi}{dx_\perp^2} = \frac{2k}{2p^+(k + p^+)} (m_{\text{soft}}^2 - \nabla_{x_\perp}^2) f(x_\perp) + C(x_\perp) f(x_\perp). \]

This equation has two inputs, the thermal asymptotic mass $m_{\text{as}}$ of the quarks and the scattering kernel $C(x_\perp)$. Since, intuitively, one has that if $p^+ \gg p_\perp \gg p^-$, then in coordinate space $x^+ \gg x_\perp \gg x^-$ and these quantities can be defined through operators supported on the light cone along $x^+$ or on the $x^- = 0$ light front. For the former one has \[ m_{\text{as}}^2 = g^2 C_f \left[ \frac{1}{d_3} \frac{1}{v_\perp F_{\mu \nu} F_{\mu \nu}} + \frac{1}{2 d_3} \frac{\not\psi}{v \cdot D} \not\psi \right] \left| \frac{1}{2 \sqrt{3}} \right| g^2 C_f \left[ \frac{T^2}{6} + \frac{T^2}{12} \right] = \frac{g^2 C_f T^2}{4}. \]
where \( \nu = (1, \nu) \) is a null vector. For the latter instead one needs to consider a Wilson loop in the \((x^\perp, x_\perp)\) plane [26, 27], i.e.
\[
C(x_\perp) = \lim_{x^\perp \to \infty} (x^\perp)^{-1} \log \left\{ \frac{1}{N_c} \text{Tr} \left\{ U(0, x_\perp; x^\perp, x_\perp) U(0, 0; 0, x_\perp) U(x^\perp, 0; 0, 0) U(x_\perp, x_\perp; x^\perp, 0) \right\} \right\} = \text{const},
\]
and based on the analytical properties, dictated by causality, of retarded and advanced amplitudes, is that the thermal \( n \)-point function of \( n \) fields on a spacelike plane is given by the Euclidean Matsubara correlator with an imaginary, discrete component added to the spatial components of the momenta. In the soft sector the lightlike limit can be reached smoothly and furthermore the zero-mode is relevant at LO and NLO: this implies that, for instance, the propagators become the standard, three-dimensional ones of dimensionally-reduced EQCD [29]. Indeed, the leading-order result in Eq. (6) is easily understood as the difference of the massless transverse and massive longitudinal propagators at vanishing \( q_\perp \). With this tremendous simplification the NLO computations of \( C(x_\perp) \) [26] and \( m_\perp^2 \) [25] become much simpler. Their results can be plugged in Eq. (5) and treated as perturbations to obtain the \( O(\gamma) \) correction to \( f(x_\perp) \).

The observation at the base of this Euclideanization is that retarded correlators (e.g. a propagator) are analytical functions in the upper half-plane in any timelike and lightlike variable. This is of extreme importance for the soft sector too. There one has at leading order
\[
\frac{dI^\perp}{dk} = \frac{e^2}{2k(2\pi)^2} \sum_q q^2 d_s n(k) \int \frac{d^4 P}{(2\pi)^4} \text{Tr} \left\{ \gamma_5 S_R(P) - S_A(P) \right\} 2\pi \delta(p^+),
\]
which comes from the first diagram on the left in Fig. 5. At fixed \( p^- \) the HTL-resummed retarded (advanced) soft quark propagator \( S_R(S_A) \) is then guaranteed to be analytical in the upper (lower) half-plane: we can then deform the contour away from the real axis to arcs at constant \( |p^+| \), as shown in Fig. 6. \( p^+ \) is then effectively large, though complex, rendering the integrand considerably simpler. The same can be done when evaluating the four NLO diagrams in Fig. 5 with their intricate HTL-resummed propagators and vertices. After subtracting the counterterm from the mistreated collinear limit we find that the NLO contribution arises only from the \( O(\gamma) \) \( \delta m_\perp^2 \) correction to \( m_\perp^2 \), i.e.
\[
\begin{align*}
\text{L.O.} \quad & \frac{2m_\perp^2}{p^+(p_1^2 + m_\perp^2)} \quad \text{N.L.O.} \quad \frac{2m_\perp^2 + \delta m_\perp^2}{p^+(p_1^2 + m_\perp^2 + \delta m_\perp^2)} \\
\end{align*}
\]
This simplification is also at the base of the recent lattice measurements of \( C(x_\perp) \) and of the related jet-quenching parameter \( \hat{q} \) [20, 22].

\[\text{L.O.} \rightarrow \text{N.L.O.} \]
\[
\begin{align*}
\text{L.O.} \quad & \frac{2m_\perp^2}{p^+(p_1^2 + m_\perp^2)} \quad \text{N.L.O.} \quad \frac{2m_\perp^2 + \delta m_\perp^2}{p^+(p_1^2 + m_\perp^2 + \delta m_\perp^2)} \\
\end{align*}
\]
\[\text{N.L.O.} \rightarrow \text{N.L.O.} \]

\[\text{N.L.O.} \quad \frac{2m_\perp^2 + \delta m_\perp^2}{p^+(p_1^2 + m_\perp^2 + \delta m_\perp^2)} \quad \text{N.L.O.} \quad \frac{2m_\perp^2 + \delta m_\perp^2}{p^+(p_1^2 + m_\perp^2 + \delta m_\perp^2)}
\]
where $\delta$ is defined through this light-cone operator

$$\delta = k p_\perp / (p^+ (p^+ + k))$$

and $\hat{q}(\delta E)$ is a modified version of $\hat{q}$ that keeps track of the evolving $p^-$ component of the momentum as well. It can be defined through this light-cone operator

$$\hat{q}(\delta E) = S \int_{-\infty}^{\infty} dx^+ e^{i x^+ \delta E} \frac{1}{d_\perp} (v_0 F^0_\perp (x^+,0,0) U_\rho(x^+,0,0;0,0,0) \rho_0 F_{\perp 0}(0)),$$

where $v_0 = K/k$. This operator can also be evaluated using Euclidean techniques and reverts to standard $\hat{q}$ for $\delta E \to 0$.

4. Results and conclusions

In Fig. 7 we plot the rate (in units of the leading-log coefficient) and the ratio of LO+NLO correction over LO rate for $N_c = N_f = 3$ and $\alpha_s = 0.3$. Both plots show how in the phenomenologically relevant region the NLO correction represents an $O(20\%)$ increase which comes about from a cancellation between a much larger positive correction from the collinear sector and a negative one from the semi-collinear and soft regions. This cancellation appears largely accidental to us, confirmed by the fact that at larger momenta the negative contribution overcomes the positive one. Hence, we believe that the band formed by these two curves can be taken as an uncertainty estimate of the calculation. We refer to [17] for plots for smaller values of $\alpha_s$, which show a similar trend, as well as for accurate fits of the results.

In conclusion, we have shown how the thermal photon rate can be computed at NLO in $g$ and how causality and analyticity play a big role: in many cases the computationally difficult physics of soft gluons and quarks can be factored into operators supported on light fronts, which can be of two kinds. Either they are the correlators of the much simpler 3D theory, as for $C(x_\perp)$, $\hat{m}^2_{\perp}$, and $\hat{q}(\delta E)$, or they can be mapped on arcs in the complex plane away from the real axis, as for the soft quark contribution. An extension of these techniques to jet propagation and energy loss is underway [33]: there we find that the longitudinal momentum diffusion coefficient can also be mapped on these arcs.

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Figure 7. Left: the function, \( C(k/T) \equiv d\Gamma/d(\text{d}k)(\text{d}\rho_{NIM}(k)/T^3)(3\pi)^{-1} \), parameterizing the photon emission rate for \( N_c = N_f = 3 \) and \( \alpha_s = 0.3 \).

The full next-to-leading order function \((C_{LO+NLO})\) is a sum of the leading-order result \((C_{LO})\), a collinear correction \((\delta C_{coll})\), and a soft + semi-collinear correction \((\delta C_{soft+sc})\). The dashed curve labeled \( C_{LO} + \delta C_{coll} \) shows the result when only the collinear correction is included, with the analogous notation for the \( C_{LO} + \delta C_{soft+sc} \) curve. The difference between the dashed curves provides a uncertainty estimate for the NLO calculation.

Right: the ratio between the LO+NLO rate over the LO rate for the same parameters.

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