Solitons and Jacobi Elliptic Function Solutions to the Complex Ginzburg–Landau Equation

Kamyar Hosseini*1, Mohammad Mirzazadeh2, M. S. Osman3,4, Maysaa Al Qurashi5 and Dumitru Baleanu6,7

1Department of Mathematics, Rasht Branch, Islamic Azad University, Rasht, Iran, 2Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, Rudsat-Vajargah, Iran, 3Department of Mathematics, Faculty of Science, Cairo University, Giza, Egypt, 4Department of Mathematics, Faculty of Applied Science, Umm Alqura University, Makkaah, Saudi Arabia, 5Department of Mathematics, King Saud University, Riyadh, Saudi Arabia, 6Department of Mathematics, Faculty of Arts and Sciences, Cankaya University, Ankara, Turkey, 7Institute of Space Sciences, Magurele-Bucharest, Romania

The complex Ginzburg–Landau (CGL) equation which describes the soliton propagation in the presence of the detuning factor is firstly considered; then its solitons as well as Jacobi elliptic function solutions are obtained systematically using a modified Jacobi elliptic expansion method. In special cases, several dark and bright soliton solutions to the CGL equation are retrieved when the modulus of ellipticity approaches unity. The results presented in the current work can help to complete previous studies on the complex Ginzburg–Landau equation.

Keywords: complex Ginzburg–Landau equation, detuning factor, modified Jacobi elliptic expansion method, solitons, Jacobi elliptic function solutions

INTRODUCTION

One of the very hot topics in many of today’s research is the search for exact solutions of non-linear partial differential (NLPD) equations. As it is known, exact solutions of NLPD equations play a significant role in a wide variety of applied sciences and provide meaningful information about physical phenomena modeled by non-linear partial differential equations. Fortunately, with the development of symbolic computation packages, a variety of systematic methods [1–34] (Hosseini et al., under review) have been proposed to obtain exact solutions of NLPD equations. Many of these methods are based on considering the solution as a finite series in terms of the solutions of well-known ordinary differential equations such as Bernoulli, Riccati, and Jacobi equations. Among these methods, the modified Jacobi elliptic expansion (MJJEE) method [21–27] has achieved a great deal of attention. For instance, El-Sheikh et al. [25] extracted elliptic function solutions of non-linear Boussinesq-like equations using the MJEE method. Hosseini et al. [26] applied the MJEE method to find optical solitons of the Fokas–Lenells equation. Hosseini et al. [27] also obtained exact solutions of a (3+1)-dimensional resonant non-linear Schrödinger equation through the MJEE method. Such results encouraged the authors of the present article to employ the modified Jacobi elliptic expansion method to derive complex wave structures of the following complex Ginzburg–Landau equation [28–34] (Hosseini et al., under review)

\[
\begin{align*}
\imath \frac{\partial u(x,t)}{\partial t} + \alpha_1 \frac{\partial^2 u(x,t)}{\partial x^2} + \alpha_2 |u(x,t)|^2 u(x,t) & - \frac{\alpha_3}{|u(x,t)|^2} \frac{\partial^2 |u(x,t)|^2}{\partial x^2} \\
- \left( \frac{\partial |u(x,t)|^2}{\partial x} \right)^2 - \alpha_4 u(x,t) & = 0,
\end{align*}
\]  

(1)
which describes the soliton propagation in the presence of the detuning factor. In the CGL Equation (1), \( \alpha_1 \) and \( \alpha_2 \) are the coefficients of the group velocity dispersion and the Kerr law non-linearity while \( \alpha_3 \) and \( \alpha_4 \) denote the coefficients of the perturbation effects, especially \( \alpha_4 \) comes from the detuning effect. Recently, the main subject of a lot of studies has been focused on the complex Ginzburg–Landau equation and its exact solutions. Osman et al. [30] reported several complex wave structures to the CGL equation using a modified auxiliary equation method. In another paper, Abdou et al. [31] obtained dark-singular combo solitons of the CGL equation through the extended Jacobi elliptic expansion method. Solitons of the CGL equation were derived by Arshed in [32] with the help of the \( \exp(-\varphi(\epsilon)) \)-expansion method. Rezazadeh [33] listed a series of soliton solutions to the CGL equation using a new extended direct algebraic method, and finally, Arnous et al. [34] employed the modified simple equation method to generate optical solitons of the CGL equation. This paper is arranged as follows: In Section the modified Jacobi elliptic expansion method and its fundamental, the modified Jacobi elliptic expansion method and its fundamental is reviewed. In Section complex Ginzburg–Landau equation and its exact solutions, exact solutions of the CGL equation are given by applying the MJEE method. Section conclusion is devoted to summarizing the conclusions.

THE MODIFIED JACOBI ELLIPTIC EXPANSION METHOD AND ITS FUNDAMENTAL

To explain the basic ideas of the modified Jacobi elliptic expansion method, we consider the following NLPD equation

\[ P (u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \ldots) = 0, \quad P \text{ is a function, } u \text{ is an unknown} \quad (2) \]

and suppose that it can be converted to an ODE as follows

\[ O \left( U, U', U'', \ldots \right) = 0, \quad \frac{d}{d\xi}, \quad (3) \]

through considering the transformation \( u(x, t) = U(\xi) e^{i(\kappa x + \omega t + \theta)} \) which \( \xi = x - vt \).

Now, assume that the non-trivial solution of Equation (3) can be expressed as

\[ U(\xi) = c_0 + \sum_{i=1}^{N} \left( \frac{J(\xi)}{1 + J^2(\xi)} \right)^{i-1} \left( c_1 \frac{J(\xi)}{1 + J^2(\xi)} + d_1 \frac{1 - J^2(\xi)}{1 + J^2(\xi)} \right), \]

\[ c_N \text{ or } d_N \neq 0, \quad (4) \]

where \( c_0, c_1, d_1 \) (1 \( \leq i \leq N \)) are constants to be found later, \( N \) is the balance number, and \( J(\xi) \) is a known function satisfying the following Jacobi elliptic equation

\[ (J'(\xi))^2 = D + EJ^2(\xi) + FJ^4(\xi). \quad (5) \]

The Jacobi elliptic Equation (5) admits the following exact solutions (See Table 1):

| No. | \( D \) | \( E \) | \( F \) | \( J(\xi) \) |
|-----|--------|--------|--------|---------|
| 1   | 1      | \(-m^2 + 1\) | \( m^2 \) | \( sn(\xi) \) |
| 2   | \(1 - m^2\) | \(2m^2 - 1\) | \(-m^2\) | \( cn(\xi) \) |
| 3   | \( m^2 \) | \(-m^2 + 1\) | 1 | \( ns(\xi) \) |
| 4   | \(-m^2\) | \(2m^2 - 1\) | \(1 - m^2\) | \( nc(\xi) \) |

By substituting the solution (4) along with the Jacobi elliptic Equation (5) into Equation (3) and performing a series of calculations, we arrive at a set of non-linear algebraic equations whose solution yields exact solutions of the NLPD Equation (2).

The Jacobi elliptic functions include some interesting properties that are reviewed below [35]:

- \( sn^2(\xi) + cn^2(\xi) = 1 \).
- \( sn(\xi) = sn(\xi, m) \to \tanh(\xi) \) when \( m \to 1 \).
- \( ns(\xi) = (sn(\xi, m))^{-1} \to \coth(\xi) \) when \( m \to 1 \).

COMPLEX GINZBURG–LANDAU EQUATION AND ITS EXACT SOLUTIONS

To gain exact solutions of the CGL equation, we first apply a complex transformation in the form below

\[ u(x, t) = U(\xi) e^{i(\kappa x + \omega t + \theta)}, \quad \xi = x - vt \]

where \( v \) is the velocity of the soliton whereas \( \kappa, \omega \), and \( \theta \) are the wave number, the frequency, and the phase constant, respectively. Such a complex transformation results in

\[ (-\alpha_1 + 4\alpha_3) \frac{d^2U(\xi)}{d\xi^2} + (\kappa^2\alpha_1 + \omega + \alpha_4) U(\xi) - \alpha_2 U^3(\xi) = 0 \quad (6) \]

where the velocity of the soliton is

\[ v = -2\kappa\alpha_1. \]

Now, based on the Jacobi elliptic expansion method and the balance number \( N = 1 \), the solution of Equation (6) is considered as follows

\[ U(\xi) = c_0 + c_1 \frac{f(\xi)}{1 + f^2(\xi)} + c_2 \frac{1 - f^2(\xi)}{1 + f^2(\xi)}, \quad c_2 = d_1, \]

where \( c_0, c_1, \) and \( c_2 \) are constants to be found later. By inserting the above solution along with the Jacobi elliptic Equation (5) into Equation (6) and performing a series of calculations, we will get a set of non-linear algebraic equations as

\[ -k^2\alpha_1 c_0 - k^2\alpha_1 c_1 + 3\alpha_2 c_0^2 c_2 + 3\alpha_2 c_0 c_2^2 + 2\alpha_2 c_2^3 - 4D\alpha_1 c_0 + 16D\alpha_1 c_2 - \omega c_0 - \omega c_2 - \alpha_2 c_0 - \alpha_2 c_2 = 0, \]

\[ -k^2\alpha_1 c_1 + 3\alpha_2 c_0^2 c_1 + 6\alpha_2 c_0 c_1 c_2 + 3\alpha_2 c_1 c_2^2 - 6D\alpha_1 c_1 + 24D\alpha_3 c_1 + E\alpha_1 c_1 - 4E\alpha_3 c_1 - \omega c_0 - \alpha_2 c_0 = 0, \]

\[ -3k^2\alpha_1 c_0 - k^2\alpha_1 c_2 + 3\alpha_2 c_0^3 + 3\alpha_2 c_0^2 c_2 + 3\alpha_2 c_2^2 \]

\[ \text{TABLE 1 | Jacobi elliptic function solutions of Equation (5).} \]

| No. | \( D \) | \( E \) | \( F \) | \( J(\xi) \) |
|-----|--------|--------|--------|---------|
| 1   | 1      | \(-m^2 + 1\) | \( m^2 \) | \( sn(\xi) \) |
| 2   | \(1 - m^2\) | \(2m^2 - 1\) | \(-m^2\) | \( cn(\xi) \) |
| 3   | \( m^2 \) | \(-m^2 + 1\) | 1 | \( ns(\xi) \) |
| 4   | \(-m^2\) | \(2m^2 - 1\) | \(1 - m^2\) | \( nc(\xi) \) |
Now, the following exact solutions to the CGL equation are derived.

\[ u_{1,2}(x,t) = \pm \frac{-8\alpha_1 + 32\alpha_3}{\alpha_2} \pm \text{FP}(x + 2\kappa \alpha_1 t) \]

\[ \times e^{i(-kx + (-k^2\alpha_1 + 8\kappa \alpha_3 - \alpha_4)\theta + t)} \]

\[ u_{10,9}(x,t) = \pm \frac{-\alpha_1 + 4\alpha_3}{2\alpha_2} \pm \text{FP}(x + 2\kappa \alpha_1 t, \frac{1}{2}) \]

\[ \times e^{i(-kx + (-k^2\alpha_1 + \frac{5}{2}\alpha_1 - 10\kappa \alpha_3 - \alpha_4)\theta + t)} \]

where \( \text{FP} \) denotes the Jacobi elliptic function and \( \kappa, \alpha_1, \alpha_3 \) are parameters.

The 3-dimensional graphs of \(|u_1(x,t)|\) and \(|u_3(x,t)|\) which indicate dark and bright soliton solutions have been shown in Figure 1 for a series of the involved parameters.

Now, the following Jacobi elliptic function solutions to the CGL equation are derived.

\[ u_5,6(x,t) = \frac{\alpha_1 - 4\alpha_3}{2\alpha_2} \pm \text{FP}(x + 2\kappa \alpha_1 t) \]

\[ \times e^{i(-kx + (-k^2\alpha_1 + 2\alpha_3 - \frac{1}{2}\alpha_4)\theta + t)} \]

\[ u_7,8(x,t) = \frac{\alpha_1 - 4\alpha_3}{2\alpha_2} \pm \text{FP}(x + 2\kappa \alpha_1 t) \]

\[ \times e^{i(-kx + (-k^2\alpha_1 + 2\alpha_3 - \frac{1}{2}\alpha_4)\theta + t)} \]
Case 3. When $D = m^2, E = -(m^2 + 1)$, and $F = 1$, we obtain

- $c_0 = 0, c_1 = \pm \sqrt{-\frac{32\alpha_1 - 128 \alpha_3}{\alpha_2}}, c_2 = 0, \omega = -k^2 \alpha_1 - 8\alpha_1 + 32\alpha_3 - \alpha_4, m = 1$,
- $c_0 = 0, c_1 = 0, c_2 = \pm \sqrt{-\frac{8\alpha_1 - 32 \alpha_3}{\alpha_2}}, \omega = -k^2 \alpha_1 + 4\alpha_1 - 16\alpha_3 - \alpha_4, m = 1$,
- $c_0 = 0, c_1 = \pm \sqrt{-\frac{8\alpha_1 - 32 \alpha_3}{\alpha_2}}, c_2 = \pm \sqrt{-\frac{2\alpha_1 + 8\alpha_3}{\alpha_2}}, \omega = k^2 \alpha_1 - 2\alpha_1 + 8\alpha_3 - \alpha_4, m = 1$

Now, the following exact solutions to the CGL equation are retrieved

$$u_{11,12}(x,t) = \pm \sqrt{-\frac{32\alpha_1 - 128 \alpha_3}{\alpha_2}} \frac{\coth (x + 2\kappa \alpha_1 t)}{1 + \coth^2 (x + 2\kappa \alpha_1 t)} \times e^{(-x x + (-x^2 \alpha_1 - 8\alpha_1 + 32\alpha_3 - \alpha_4) t + \phi)}$$

$$u_{13,14}(x,t) = \pm \sqrt{-\frac{8\alpha_1 + 32 \alpha_3}{\alpha_2}} \frac{1 - \coth^2 (x + 2\kappa \alpha_1 t)}{1 + \coth^2 (x + 2\kappa \alpha_1 t)} \times e^{(-x x + (-x^2 \alpha_1 + 4\alpha_1 - 16\alpha_3 - \alpha_4) t + \phi)}$$

$$u_{15,16}(x,t) = \pm \sqrt{-\frac{8\alpha_1 - 32 \alpha_3}{\alpha_2}} \frac{\coth (x + 2\kappa \alpha_1 t)}{1 + \coth^2 (x + 2\kappa \alpha_1 t)} \times e^{(-x x + (-x^2 \alpha_1 - 2\alpha_1 + 8\alpha_3 - \alpha_4) t + \phi)}$$

$$u_{17,18}(x,t) = \pm \sqrt{-\frac{8\alpha_1 - 32 \alpha_3}{\alpha_2}} \frac{\coth (x + 2\kappa \alpha_1 t)}{1 + \coth^2 (x + 2\kappa \alpha_1 t)} \times e^{(-x x + (-x^2 \alpha_1 + 4\alpha_1 - 16\alpha_3 - \alpha_4) t + \phi)}$$

The 3-dimensional graphs of $|u_{11}(x,t)|$ and $|u_{13}(x,t)|$ which denote dark and bright soliton solutions have been presented in Figure 2 for a series of the involved parameters.

Case 4. When $D = -m^2, E = 2m^2 - 1$, and $F = 1 - m^2$, we acquire

$$c_0 = \pm \sqrt{-\frac{\alpha_1 + 4\alpha_3}{2\alpha_2}}, c_1 = 0, c_2 = \mp \sqrt{-\frac{\alpha_1 - 4\alpha_3}{\alpha_2}} \sqrt{\frac{2\alpha_2}{32\alpha_3}}$$

$$\omega = -k^2 \alpha_1 + \frac{5}{2} \alpha_1 - 10\alpha_3 - \alpha_4, m = \frac{1}{2}$$

Now, the following Jacobi elliptic function solutions to the CGL equation are derived.

$$u_{19,20}(x,t) = \pm \sqrt{-\frac{-\alpha_1 + 4\alpha_3}{2\alpha_2}} \frac{\alpha_1 - 4\alpha_3}{\alpha_2 \sqrt{\frac{2\alpha_2}{32\alpha_3}}} \frac{1 - \text{nc}^2 (x + 2\kappa \alpha_1 t, \frac{1}{2})}{1 + \text{nc}^2 (x + 2\kappa \alpha_1 t, \frac{1}{2})} \times e^{(-x x + (-x^2 \alpha_1 + 4\alpha_1 - 16\alpha_3 - \alpha_4) t + \phi)}$$

**CONCLUSION**

The complex Ginzburg–Landau equation as a famous NLFP equation was studied successfully in the current paper. A modified version of the Jacobi elliptic expansion method was formally adopted to carry out this goal. A wide variety of exact solutions to the CGL equation in the presence of the detuning factor were derived. Particularly, several dark and bright soliton solutions were derived when the modulus of ellipticity approaches unity. Although, the exact solutions given in the present paper provide useful information about the complex Ginzburg–Landau equation, seeking other exact solutions of the CGL equation can be considered as an important task in our future works.
DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author/s.

AUTHOR CONTRIBUTIONS

KH and DB: conceptualization, project administration, and Supervision. KH: data curation and visualization. KH and MM: formal analysis, investigation, and writing—original draft. MA and DB: funding acquisition. KH, MM, MO, and MA: methodology, software, and validation. KH, MM, MO, MA, and DB: resources. KH, MM, and MO: writing—review editing. All persons who meet authorship criteria are listed as authors, and all authors certify that they have participated sufficiently in the work to take public responsibility for the content, including participation in the concept, design, analysis, writing, or revision of the manuscript.

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**Conflict of Interest:** The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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