Entropy from the foam

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(March 28, 2022)

Abstract

A simple model of spacetime foam, made by \( N \) wormholes in a semiclassical approximation, is taken under examination. We show that the Bekenstein-Hawking entropy is here “quantized” in agreement with the heuristic calculation of Bekenstein.

I. INTRODUCTION

In the early seventies J. Bekenstein \[1\] and S. Hawking \[2\], basing themselves upon considerations originated from quantum field theory, changed drastically the concept of a black hole. Beginning with the simple observation that the area of the horizon of the Schwarzschild black hole is a quadratic function of the mass \( M \), they considered an infinitesimal increment (in natural units \( G = \hbar = c = 1 \))

\[ dM = \kappa dA_{\text{hor}} \quad \kappa = \frac{1}{4M}. \]
This formally resembles the First Law of thermodynamics $dU = TdS$. This analogy of the black hole mechanics with thermodynamics appears reinforced by Hawking’s theorem on black hole mechanics asserting that the horizon area of an isolated black hole never decreases in any transformation. Thus, if two black holes of area $A_1$ and $A_2$ fuse to form a black hole of area $A_{1+2}$, then the Theorem asserts that $A_{1+2} \geq A_1 + A_2$. On the basis of these observation and results, Bekenstein made the proposal that a black hole does have an entropy proportional to the area of its horizon

$$S_{bh} = \text{const} \times A_{hor}. \quad (1)$$

In particular, in natural units one finds that the proportionality constant is set to $1/4G = 1/4l_p^2$, so that the entropy becomes

$$S = \frac{A}{4G} = \frac{A}{4l_p^2}. \quad (2)$$

For a Schwarzschild black hole, for example, one finds the value

$$S = \frac{4\pi (2MG)^2}{4G} = 4\pi M^2 G = 4\pi M^2 l_p^2. \quad (3)$$

In conventional units instead, we find for a generic horizon area that

$$S_{bh} = \frac{1}{8\pi G\hbar} \ln 2k_Bc^3 A_{hor}. \quad (4)$$

1The best way to see this similarity is with a Kerr-Newman black hole, where the incremental formula is

$$dM \equiv \kappa dA_{hor} + \Phi dQ + \vec{\Omega} \cdot d\vec{L}$$

and the analogy with the First Law

$$dU = TdS + PdV$$

is complete.
This formula is identical (except for the factor of \( \ln 2 \) which one may think of as a choice of units of entropy) to the one proposed by Hawking \cite{2} based on consistency with the rate of black hole radiation. The appearance of \( \hbar \) is “... a reflection of the fact that the entropy is ... a count of states of the system” \cite{1}. Following Bekenstein’s proposal on the quantization of the area for nonextremal black holes we have

\[
a_n = \alpha l_p^2 (n + \eta) \quad \eta > -1 \quad n = 1, 2, \ldots
\]

Many attempts to recover the area spectrum have been done, see Refs. \cite{4,5} for a review. Note that the appearance of a discrete spectrum is not so trivial. Indeed there are other theories, based on spherically symmetric metrics in a mini-superspace approach, whose mass spectrum is continuous. Recently a model made by \( N \) coherent wormholes, based on Wheeler’s ideas of a foamy spacetime \cite{8}, has been considered \cite{12}. In that paper, hereafter referred as I, we have computed the energy density of gravitational fluctuations reproducing the same behavior conjectured by Wheeler during the sixties on dimensional grounds. In this paper we wish to apply the ideas of I to a generic area. The result is a quantization process whose quanta can be identified with wormholes of Planckian size. Implications on the black hole entropy are taken under consideration. The rest of the paper is structured as follows, in section II we briefly recall the results reported in Ref. \cite{12}, in section III we compute the area spectrum. We summarize and conclude in section IV.

II. SPACETIME FOAM: THE MODEL

In the one-wormhole approximation we have used an eternal black hole, to describe a complete manifold \( \mathcal{M} \), composed of two wedges \( \mathcal{M}_+ \) and \( \mathcal{M}_- \) located in the right and left sectors of a Kruskal diagram. The spatial slices \( \Sigma \) represent Einstein-Rosen bridges with wormhole topology \( S^2 \times R^1 \). Also the hypersurface \( \Sigma \) is divided in two parts \( \Sigma_+ \) and \( \Sigma_- \) by a bifurcation two-surface \( S_0 \). We begin with the line element

\[
d s^2 = -N^2 (r) \, dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right)
\]  \hspace{1cm} (5)
and we consider the physical Hamiltonian defined on $\Sigma$

$$H_p = H - H_0 = \frac{1}{16\pi l_p^2} \int_\Sigma d^3x \left( N\mathcal{H} + N_i\mathcal{H}^i \right)$$

$$+ \frac{2}{l_p^2} \int_{S_+} d^2x \sqrt{\sigma} \left( k - k^0 \right) - \frac{2}{l_p^2} \int_{S_-} d^2x \sqrt{\sigma} \left( k - k^0 \right),$$

(6)

where $l_p^2 = G$. The volume term contains two constraints

$$\begin{cases}
\mathcal{H} = G_{ijkl} \pi^{ij} \pi^{kl} \left( \frac{l_p^2}{\sqrt{g}} \right) - \left( \frac{\sqrt{\sigma}}{l_p^2} \right) R^{(3)} = 0, \\
\mathcal{H}^i = -2\pi^{ij} = 0
\end{cases}$$

(7)

both satisfied by the Schwarzschild and Flat metric respectively. The supermetric is $G_{ijkl} = \frac{1}{2} \left( g_{ik}g_{jl} + g_{il}g_{jk} - g_{ij}g_{kl} \right)$ and $R^{(3)}$ denotes the scalar curvature of the surface $\Sigma$. By using the expression of the trace

$$k = -\frac{1}{\sqrt{h}} \left( \sqrt{hn^\mu} \right)_{,\mu},$$

(8)

with the normal to the boundaries defined continuously along $\Sigma$ as $n^\mu = (h^{yy})^\frac{1}{2} \delta^\mu_y$. The value of $k$ depends on the function $r, y$, where we have assumed that the function $r, y$ is positive for $S_+$ and negative for $S_-$. We obtain at either boundary that

$$k = -\frac{2r, y}{r}. \quad (9)$$

The trace associated with the subtraction term is taken to be $k^0 = -2/r$ for $B_+$ and $k^0 = 2/r$ for $B_-$. Then the quasilocal energy with subtraction terms included is

$$E_{\text{quasilocal}} = l_p^2 (E_+ - E_-) = l_p^2 \left[ \left( r \left[ 1 - |r, y | \right] \right)_{y=y_+} - \left( r \left[ 1 - |r, y | \right] \right)_{y=y_-} \right].$$

(10)

Note that the total quasilocal energy is zero for boundary conditions symmetric with respect to the bifurcation surface $S_0$ and this is the necessary condition to obtain instability with respect to the flat space. In this sector satisfy the constraint equations (7). Here we consider perturbations at $\Sigma$ of the type

$$g_{ij} = \bar{g}_{ij} + h_{ij},$$

(11)
where $\bar{g}_{ij}$ is the spatial part of the Schwarzschild and Flat background in a WKB approximation. In this framework we have computed the quantity

$$\Delta E (M) = \frac{\langle \Psi | H^{\text{Schw.}} - H^\text{Flat} | \Psi \rangle}{\langle \Psi | \Psi \rangle} + \frac{\langle \Psi | H^\text{quasilocal} | \Psi \rangle}{\langle \Psi | \Psi \rangle},$$

(12)

by means of a variational approach, where the WKB functionals are substituted with trial wave functionals. This quantity is the natural extension to the volume term of the subtraction procedure for boundary terms and it is interpreted as the Casimir energy related to vacuum fluctuations. By restricting our attention to the graviton sector of the Hamiltonian approximated to second order, hereafter referred as $H_{|2}$, we define

$$E_{|2} = \frac{\langle \Psi^\perp | H_{|2} | \Psi^\perp \rangle}{\langle \Psi^\perp | \Psi^\perp \rangle},$$

where

$$\Psi^\perp = \Psi \left[ h^\perp_{ij} \right] = \mathcal{N} \exp \left\{ -\frac{1}{4l_p^2} \left[ \left\langle (g - \bar{g}) K^{-1} (g - \bar{g}) \right\rangle^\perp_{x,y} \right] \right\}.$$  

After having functionally integrated $H_{|2}$, we get

$$H_{|2} = \frac{1}{4l_p^2} \int_\Sigma d^3x \sqrt{g} G^{ijkl} \left[ K^{-1\perp} (x,x)_{ijkl} + (\triangle_2)^a_j K^\perp (x,x)_{iakl} \right]$$

(13)

The propagator $K^\perp (x,x)_{iakl}$ comes from a functional integration and it can be represented as

$$K^\perp (\vec{x}, \vec{y})_{iakl} := \sum_N h^\perp_{ia} (\vec{x}) h^\perp_{kl} (\vec{y}) \frac{2\lambda_N (p)}{2\lambda_N (p)},$$

(14)

where $h^\perp_{ia} (\vec{x})$ are the eigenfunctions of

$$(\triangle_2)^a_j := -\triangle \delta^a_j + 2R^a_j.$$  

(15)

This is the Lichnerowicz operator projected on $\Sigma$ acting on traceless transverse quantum fluctuations and $\lambda_N (p)$ are infinite variational parameters. $\triangle$ is the curved Laplacian (Laplace-Beltrami operator) on a Schwarzschild background and $R^a_j$ is the mixed Ricci tensor whose components are:
\[ R^a_j = \text{diag} \left\{ \frac{-2MG}{r^3}, \frac{MG}{r^3}, \frac{MG}{r^3} \right\} . \]

The minimization with respect to \( \lambda \) and the introduction of a high energy cutoff \( \Lambda \) give to the Eq. \[(12) \] the following form

\[ \Delta E (M) \sim -\frac{V}{32\pi^2} \left( \frac{3MG}{r_0^3} \right)^2 \ln \left( \frac{r_0^3\Lambda^2}{3MG} \right) , \]

where \( V \) is the volume of the system and \( r_0 \) is related to the minimum radius compatible with the wormhole throat. We know that the classical minimum is achieved when \( r_0 = 2MG \). However, it is likely that quantum processes come into play at short distances, where the wormhole throat is defined, introducing a quantum radius \( r_0 > 2MG \). We now compute the minimum of \( \Delta E (M) \), after having rescaled the variable \( M \) to a scale variable \( x = \frac{3MG}{r_0^3\Lambda^2} \). Thus

\[ \Delta E (M) \rightarrow \Delta E (x, \Lambda) = \frac{V}{32\pi^2} \Lambda^4 x^2 \ln x \]

We obtain two values for \( x \): \( x_1 = 0 \), i.e. flat space and \( x_2 = e^{-\frac{1}{2}} \). At the minimum

\[ \Delta E (x_2) = -\frac{V}{64\pi^2} \frac{\Lambda^4}{e} . \]

Nevertheless, there exists another part of the spectrum which has to be considered: the discrete spectrum containing one mode. This gives the energy an imaginary contribution, namely we are discovered an unstable mode \[11,13]. Let us briefly recall, how this appears.

The eigenvalue equation

\[ (\Delta_2) h_{ij} = \alpha h_{ij} \]

can be studied with the Regge-Wheeler method. The perturbations can be divided in odd and even components. The appearance of the unstable mode is governed by the gravitational field component \( h_{11}^{\text{even}} \). Explicitly

\[ -E^2 H (r) \]

\[ = - \left( 1 - \frac{2MG}{r} \right) \frac{d^2 H (r)}{dr^2} + \left( \frac{2r - 3MG}{r^2} \right) \frac{dH (r)}{dr} - \frac{4MG}{r^3} H (r) , \]
where
\[ h_{11}^{\text{even}} (r, \vartheta, \phi) = \left[ H (r) \left( 1 - \frac{2m}{r} \right)^{-1} \right] Y_{00} (\vartheta, \phi) \] (21)
and \( E^2 > 0 \). Eq. (20) can be transformed into
\[ \mu = \frac{\int_0^\bar{y} dy \left[ \left( \frac{dh(y)}{dy} \right)^2 - \frac{3}{2p(y)} h(y) \right]}{\int_0^\bar{y} dy h^2 (y)}, \] (22)
where \( \mu \) is the eigenvalue, \( y \) is the proper distance from the throat in dimensionless form. If we choose \( h (\lambda, y) = \exp (-\lambda y) \) as a trial function we numerically obtain \( \mu = -0.701626 \). In terms of the energy square we have
\[ E^2 = -0.17541 / (MG)^2 \] (23)
to be compared with the value \( E^2 = -0.19 / (MG)^2 \) of Ref. [11]. Nevertheless, when we compute the eigenvalue as a function of the distance \( y \), we discover that in the limit \( \bar{y} \to 0 \),
\[ \mu \equiv \mu (\lambda) = \lambda^2 - \frac{3}{2} + \frac{9}{8} \left[ \bar{y}^2 + \frac{\bar{y}}{2\lambda} \right]. \] (24)
Its minimum is at \( \bar{\lambda} = \left( \frac{9}{32} \bar{y} \right)^{\frac{1}{3}} \) and
\[ \mu (\bar{\lambda}) = 1.2878 \bar{y}^4 + \frac{9}{8} \bar{y}^2 - \frac{3}{2}. \] (25)
It is evident that there exists a critical radius where \( \mu \) turns from negative to positive. This critical value is located at \( \rho_c = 1.1134 \) to be compared with the value \( \rho_c = 1.445 \) obtained by B. Allen in [17]. What is the relation with the large number of wormholes? As mentioned in I, when the number of wormholes grows, to keep the coherency assumption valid, the space available for every single wormhole has to be reduced to avoid overlapping of the wave functions. If we fix the initial boundary at \( R_\pm \), then in presence of \( N_w \) wormholes, it will be reduced to \( R_\pm / N_w \). This means that boundary conditions are not fixed at infinity, but at a certain finite radius and the ADM mass term is substituted by the quasilocal energy expression under the condition of having symmetry with respect to each bifurcation surface.
The effect on the unstable mode is clear: as $N_w$ grows, the boundary radius reduces more and more until it will reach the critical value $\rho_c$ below which no negative mode will appear corresponding to a critical wormholes number $N_{w_c}$. To this purpose, suppose to consider $N_w$ wormholes and assume that there exists a covering of $\Sigma$ such that $\Sigma = \bigcup_{i=1}^{N_w} \Sigma_i$, with $\Sigma_i \cap \Sigma_j = \emptyset$ when $i \neq j$. Each $\Sigma_i$ has the topology $S^2 \times \mathbb{R}^1$ with boundaries $\partial \Sigma_i^\pm$ with respect to each bifurcation surface. On each surface $\Sigma_i$, quasilocal energy gives

$$E_{i \text{ quasilocal}} = \frac{2}{l_p^2} \int_{S_i^+} d^2x \sqrt{\sigma} (k - k^0) - \frac{2}{l_p^2} \int_{S_i^-} d^2x \sqrt{\sigma} (k - k^0).$$

(26)

Thus if we apply the same procedure of the single case on each wormhole, we obtain

$$E_{i \text{ quasilocal}} = l_p^2 (E_i^+ - E_i^-) = l_p^2 (r [1 - |r| y])_{y=y_i^+} - l_p^2 (r [1 - |r| y])_{y=y_i^-}.$$ 

(27)

Note that the total quasilocal energy is zero for boundary conditions symmetric with respect to each bifurcation surface $S_{0,i}$. We are interested in a large number of wormholes, each of them contributing with a term of the type (22). If the wormholes number is $N_w$, we obtain (semiclassically, i.e., without self-interactions)$^3$

$$H_{tot}^{N_w} = H^1 + H^2 + \ldots + H^{N_w}.$$ 

(28)

Thus the total energy for the collection is

$$E_{tot}^{1/2} = N_w H_{1/2}.$$

The same happens for the trial wave functional which is the product of $N_w$ t.w.f.. Thus

$$\Psi_{tot}^\perp = \Psi_1^\perp \otimes \Psi_2^\perp \otimes \ldots \otimes \Psi_{N_w}^\perp = \mathcal{N} \exp N_w \left\{ -\frac{1}{4l_p^2} \left[ (g - \bar{g}) K^{-1} (g - \bar{g}) \right]_{x,y} \right\}$$

$^2$Note that at this approximation level, we are in the same situation of a large collection of $N$ harmonic oscillators whose hamiltonian is

$$H = \frac{1}{2} \sum_{n \neq 0}^{\infty} \left[ \pi_n^2 + n^2 \omega_n^2 \phi_n^2 \right].$$
\[ \Delta E_{Nw}(x, \Lambda) \sim N_w^2 \frac{V}{32\pi^2} \Lambda^4 x^2 \ln x, \]  

where we have defined the usual scale variable \( x = 3MG/\left(r_0^2 \Lambda^2\right)\). Then at one loop the cooperative effects of wormholes behave as one macroscopic single field multiplied by \( N_w^2 \), but without the unstable mode. At the minimum, \( \bar{x} = e^{-\frac{1}{2}} \)

\[ \Delta E(\bar{x}) = -N_w^2 \frac{V}{64\pi^2} \frac{\Lambda^4}{\bar{x}}. \]  

III. AREA SPECTRUM AND ENTROPY

A very important application of the model presented in the previous section is the area quantization. The area is measured by the quantity

\[ A(S_0) = \int_{S_0} d^2x \sqrt{\sigma}. \]  

\( \sigma \) is the two-dimensional determinant coming from the induced metric \( \sigma_{ab} \) on the boundary \( S_0 \). We would like to evaluate the mean value of the area

\[ A(S_0) = \frac{\langle \Psi_F | \hat{A} | \Psi_F \rangle}{\langle \Psi_F | \Psi_F \rangle} = \frac{\langle \Psi_F | \int_{S_0} d^2x \sqrt{\sigma} \rangle | \Psi_F \rangle}{\langle \Psi_F | \Psi_F \rangle}, \]  

computed on

\[ |\Psi_F\rangle = \Psi_1^\perp \otimes \Psi_2^\perp \otimes \ldots \ldots \otimes \Psi_{N_w}^\perp. \]  

If we use the fact that \( \sigma_{ab} = \bar{\sigma}_{ab} + \delta\sigma_{ab} \), where \( \bar{\sigma}_{ab} \) is such that \( \int_{S_0} d^2x \sqrt{\sigma} = 4\pi\bar{r}^2 \) and \( \bar{r} \) is the radius of \( S_0 \), we obtain to the lowest level\(^3\) in the expansion of \( \sigma_{ab} \) that

\(^3\)For lowest level in the expansion of \( \sigma_{ab} \), we mean
\[
A(S_0) = \frac{\langle \Psi_F | \hat{A} | \Psi_F \rangle}{\langle \Psi_F | \Psi_F \rangle} = 4\pi \bar{r}^2. \quad (35)
\]

Suppose to consider the mean value of the area \( A \) computed on a given \textit{macroscopic} fixed radius \( R \). On the basis of our foam model, we obtain \( A = \bigcup_{i=1}^{N} A_i \), with \( A_i \cap A_j = \emptyset \) when \( i \neq j \). Thus

\[
A = 4\pi R^2 = \sum_{i=1}^{N} A_i = \sum_{i=1}^{N} 4\pi \bar{r}_i^2. \quad (36)
\]

When \( \bar{r}_i \to l_p \), \( A_i \to A_{l_p} \) and

\[
A = NA_{l_p} = N4\pi l_p^2. \quad (37)
\]

Thus the \textit{macroscopic} area is represented by \( N \) \textit{microscopic} areas of the Planckian size: in this sense we will claim that the area is quantized. One immediately observes that \( N \) is such that \( N \geq N_{w_c} \), where \( N_{w_c} \) is the critical wormholes number above which we have the stability of our foam model. At this point we can apply the same reasoning of Refs. [18,19,21] to arrive at the Bekenstein-Hawking relation between entropy and area

\[
S = \frac{A}{4l_p^2}. \quad (38)
\]

Note that there is a \( \ln 2 \) numerical factor missing to complete the equality, between this model and the models described in Refs. [18,19,21]. This is principally due to the fact that there is no a degeneracy factor related to the statistics. Here the wormholes are the same. Of course the introduction of a degeneracy factor does not alter the main result and the

\[
\sqrt{\sigma_{ab}} = \exp Tr \ln \sqrt{\sigma_{ab}} = \exp Tr \frac{1}{2} \ln (\bar{\sigma}_{ab} + \delta \sigma_{ab})
\]

\[
= \exp Tr \frac{1}{2} \left[ \ln \bar{\sigma}_{ab} \left( 1 + \frac{\delta \sigma_{ab}}{\bar{\sigma}_{ab}} \right) \right] = \exp Tr \frac{1}{2} \left[ \ln \bar{\sigma}_{ab} + \ln \left( 1 + \frac{\delta \sigma_{ab}}{\bar{\sigma}_{ab}} \right) \right] \simeq \sqrt{\sigma_{ab}} + o(\delta \sigma_{ab}) \quad (34)
\]
only effect will be that of dividing the spacetime covering into equivalence classes with a representative for each class. In our case, we deduce that the entropy is

\[ S = N\pi. \]  \hfill (39)

Moreover this seems also to agree with the conclusions of Bekenstein (4 and Refs. therein), apart the degeneracy factor that in our model seems to be related to the odd or even permutation of any single wormhole wavefunction\[4\]. We can use Eq. (37) to compute the entropy also for other geometries, for example, the de Sitter geometry. Since we know that for this metric the Bekenstein-Hawking relation (38) still holds, we write

\[ S = \frac{3\pi}{l_p^2 \Lambda} = \frac{A}{4l_p^2} = \frac{N4\pi l_p^2}{4l_p^2} = N\pi, \] \hfill (40)

that is\[5\]

\[ \frac{3}{l_p^2 N} = \Lambda. \] \hfill (41)

Thus the cosmological constant \( \Lambda \) is “quantized” in terms of \( l_p \). Note that when the wormholes number \( N \) is quite “large”, \( \Lambda \to 0 \). We could try to see what is the rate of change between an early universe value of the cosmological constant and the value that we observe. In inflationary models of the early universe is assumed to have undergone an early phase with a large effective \( \Lambda \sim (10^{10} - 10^{11} GeV)^2 \) for GUT era inflation, or \( \Lambda \sim (10^{16} - 10^{18} GeV)^2 \) for

\[4\textbf{Remark.} \text{ Entropy is “quantized” as a consequence of (37) and not as a direct application of the definition } S = - \sum p_n \ln p_n. \]

\[5\text{A relation relating } \Lambda \text{ and } G, \text{ via an integer } N \text{ appeared also in Ref. [20]. Nevertheless in Ref. [20], } N \text{ represents the number of scalar fields and the bound from above and below } \] \[ |2G\Lambda/3 - 2| \geq \sqrt{3} \]

comes into play, instead of the equality (11).
Planck era inflation. A subsequent phase transition would then produce a region of space-time with $\Lambda \leq (10^{-42}GeV)^2$, i.e. the space in which we now live. For GUT era inflation, we have (we are looking only at the order of magnitude)

$$10^{20} - 10^{22}GeV^2 = \frac{1}{N}10^{38}GeV^2 \rightarrow N = 10^{16} - 10^{18},$$ \hspace{1cm} (42)

while for Planck era inflation we have

$$10^{32} - 10^{36}GeV^2 = \frac{1}{N}10^{38}GeV^2 \rightarrow N = 10^6 - 10^2,$$ \hspace{1cm} (43)

to be compared with the value of $(10^{-42}GeV)^2$ which gives a wormholes number of the order of

$$10^{-84}GeV^2 = \frac{1}{N}10^{38}GeV^2 \rightarrow N = 10^{122}.$$ \hspace{1cm} (44)

This very huge number is obtained by averaging the area on a cosmological scale by means of Planck scale wormholes. Thus it seems quite reasonable that with the growing of the cosmological radius, we obtain a growing wormholes number covering the horizon area.

IV. CONCLUSIONS

In this paper we have applied the model presented in I to the entropy computation assuming the validity of the Bekenstein-Hawking relation. In this picture the area is “quantized” in the sense that spacetime can be filled by a given integer number of disjoint non-interacting wormholes. This result is in agreement (apart a numerical factor) with the quantized area proposed heuristically by Bekenstein and also with the loop quantum gravity predictions of Refs. [18,19], apart the degeneracy factor missing, principally due to the fact that we have an “ideal Boltzmann gas” of wormholes. A similar result appeared in Ref. [21]. Nevertheless, in Ref. [21], spacetime was assumed ab initio built up of cells of Planckian size, while it seems that, in order to have stability, spacetime needs to be covered by $N$ wormholes of the Planckian size. Nevertheless, even if in this letter also the meaning of the cosmological constant seems to be related to a pure gravitational effect enforcing the idea that $\Lambda$ is not
fundamental but is the effect of quantum fluctuations of the pure gravitational field, giving therefore strong indications of a foamy spacetime, we are not in presence of a model which can cure the well known problems of the still absent theory of quantum gravity. The main reason resides in a cutoff dependent model. Even if one can argue that a Planck length cutoff is quite natural, it is not clear how to compute such a value from first principles, even if it is likely that a relation between this formulation of quantum gravity based on a large number of coherent wormholes and the well accepted string theory could exist.

V. ACKNOWLEDGMENTS

I wish to thank R. Brout, M. Cavaglià, C. Kiefer, D. Hochberg, G. Immirzi, S. Liberati, P. Spindel and M. Visser for useful comments and discussions.
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