We study a class of intersecting D-brane models in which fermions localized at different intersections interact via exchange of bulk fields. In some cases these interactions lead to dynamical symmetry breaking and generate a mass for the fermions. We analyze the conditions under which this happens as one varies the dimensions of the branes and of the intersections.
1. Introduction

The system of $N_c$ D4-branes wrapped around a circle with anti-periodic boundary conditions for the fermions provides an interesting example of gauge-gravity duality [1]. In a certain region of the parameter space of the brane configuration, corresponding to small four-dimensional ’t Hooft coupling, the low energy theory on the D-branes is (3 + 1)-dimensional $SU(N_c)$ Yang-Mills (YM) theory without matter. Unfortunately, in that limit the theory on the branes is hard to solve, even in the large $N_c$ limit.

For large ’t Hooft coupling (and large $N_c$), the dynamics reduces to supergravity in the near-horizon geometry of the $D4$-branes and can be analyzed in some detail. The theory exhibits confinement and has a spectrum of glueballs that can be calculated using supergravity. In this regime the theory on the branes is not pure YM, since the adjoint scalars and fermions living on the $D4$-branes are not decoupled, and their dynamics is not well described by the effective Lagrangian obtained by dimensional reduction of $N = 1$ SYM in $9 + 1$ dimensions. The dynamics can be described in terms of the $(2,0)$ superconformal field theory in $5 + 1$ dimensions compactified on a two-torus with twisted boundary conditions around one of the cycles, but this description is not easy to use.

While the theory with a good supergravity description is not YM, the two are related by a continuous deformation, and one may hope that they are in the same phase. The reason for this is that the $(2,0)$ theory is believed to be a standard QFT; compactifying it on a torus leads to an RG flow, which is expected to be smooth. Changing the parameters associated with the compactification probes different parts of this flow. Since physical properties change smoothly along RG trajectories, it is natural to expect that the supergravity and YM regimes are in the same phase. This means that some qualitative and perhaps even quantitative features of YM theory can be addressed in supergravity. This is of course a general theme in gauge-gravity duality.

A natural way to add dynamical fermions in the fundamental representation of the gauge group to the setup of [1] was proposed in [2] (see e.g. [3-6], for other work on incorporating dynamical quarks into gauge-gravity duality). It involves adding to the $N_c$ color $D4$-branes $N_f$ flavor $D8$ and $\bar{D8}$-branes, which intersect the color branes along an $\mathbb{R}^{3,1}$. The flavor branes and anti-branes are separated by a distance $L$ in the remaining (compact) direction along the $D4$-branes. Left-handed quarks live at the intersection of the $D4$ and $D8$-branes, while right-handed quarks live at the $D4 - \bar{D8}$ intersection. A nice feature of this brane construction is that quarks of different chiralities are physically
separated in the extra dimensions, and the chiral $U(N_f)_L \times U(N_f)_R$ flavor symmetry is manifest.

QCD with massless quarks is obtained in a certain region of the parameter space of the brane configuration where the dynamics is again difficult to analyze. In a different region of parameter space one can analyze the theory by studying the DBI action for the $D8$-branes in the near-horizon geometry of the $D4$-branes. As in the case without quarks, in this regime the theory is not QCD but it is expected to be in the same phase. In particular, it exhibits confinement and chiral symmetry breaking. The latter has a nice geometric realization.

It was pointed out in [7] that the brane construction of [2] has the interesting property that it decouples the scales of confinement and chiral symmetry breaking. By varying the parameters of the brane configuration one can make the energy scale of chiral symmetry breaking arbitrarily higher than that of confinement. In fact, sending the size of the circle that the $D4$-branes wrap to infinity one arrives at a theory which breaks chiral symmetry but does not confine. This theory can again be studied at weak coupling using field theoretic techniques and at strong coupling using supergravity.

The intersecting brane construction of [2,7] is a special case of a much more general class of constructions, obtained by varying the dimensions of the color and flavor branes and that of the intersection. Some other examples were considered recently in [9,10]. The purpose of this paper is to present a more uniform treatment of these and other intersecting brane systems, using field theory and supergravity, as appropriate, to analyze them.

The general setup we will consider is the following. We start with $N_c$ color $Dq$-branes, which generalize the color $D4$-branes of [2,7]. We will take these branes to be non-compact in all directions since we are mainly interested in chiral symmetry breaking. The low-energy theory on the $Dq$-branes is $(q + 1)$-dimensional SYM with sixteen supercharges. For $q < 3$ it is strongly coupled in the infrared and weakly coupled in the UV, while for $q > 3$ it has the opposite behavior. For $q = 3$ this theory is $N = 4$ SYM in $3 + 1$ dimensions, which is conformally invariant. The 't Hooft coupling of the theory on the color branes, $\lambda_{q+1}$, has units of $(\text{length})^{q-3}$.

For $q \leq 4$ the low energy theory of the color branes is a local QFT, which can be decoupled from gravity by taking a certain scaling limit. For $q = 5$ it is a non-local theory known as Little String Theory (LST). This theory has a Hagedorn density of states; the

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1 A similar phenomenon was observed in a different context in [8].
Hagedorn temperature sets the scale of non-locality (see e.g. [11,12] for discussions). For \( q = 6 \), the theory on the branes cannot be decoupled from bulk gravity [13].

In addition to the color branes we have \( N_f \) flavor \( Dp \) and \( Dp \) branes, which are analogs of the flavor \( D8 \)-branes and anti-branes in the construction of [2,7]. The color and flavor branes intersect along an \((r + 1)\)-dimensional spacetime, which we will denote by \( Dq \cap Dp = I_r \). The flavor branes and anti-branes are separated by a distance \( L \) in a direction along the color branes but transverse to the intersection \( I_r \). In the cases of interest, the light degrees of freedom localized at the intersections are fermions, which interact via exchange of fields living on the color branes.

One can think of the flavor branes, and in particular the dynamics of the fermions, as probing the theory of the color branes at the scale \( L \). By changing \( L \) we probe the color theory at different points along its RG trajectory. For \( q \leq 4 \) we expect the dependence on \( L \) to be smooth. For \( q = 5 \) we expect it to be smooth for distances much larger than the scale of non-locality of the underlying LST. For \( q = 6 \) there is no decoupling limit and one has to take into account gravitational effects.

We will study the dynamics of these configurations in the limit \( N_c \to \infty, g_s \to 0 \) with \( g_s N_c \) and \( N_f \) held fixed. This dynamics depends non-trivially on the values of \( p, q \) and \( r \), and on the dimensionless parameter

\[
\lambda_{q+1}^{(eff)}(L) = \lambda_{q+1} L^{3-q},
\]

which can be thought of as the effective coupling of the color degrees of freedom at distance scale \( L \), or energy \( E \sim 1/L \). We will explore the dependence of the low-energy behavior on these parameters, focusing on the question of dynamical symmetry breaking at weak and strong coupling.

We have organized the paper as follows. In section 2 we present a general analysis of the intersecting brane systems we will consider. We review the spectrum of fields localized at a particular intersection, and describe the leading interactions at weak coupling between fields at different intersections. There are two qualitatively different cases that need to be considered, depending on whether or not this interaction has finite range.

The supergravity description of the above intersecting brane systems is obtained by replacing the color branes by their near-horizon geometry and studying the DBI action of the flavor branes in this background. Holography suggests that in some cases (depending on the dimension of the color branes) supergravity provides a useful description of the dynamics at strong coupling.
In the following sections we illustrate the general considerations of section 2 with a number of specific models. In section 3 we discuss configurations with color $D4$-branes. These include the models of [2,7,10] as well as a model with color and flavor $D4$-branes and a (1+1)-dimensional intersection, which we discuss in some detail. In this model there are left and right-moving fermions with the same quantum numbers under $U(N_c) \times U(N_f)$ at each intersection, and one can give them a mass by separating the color and flavor branes in the two dimensions transverse to both. For zero mass we show that the model still exhibits the dynamical breaking of a certain chiral symmetry, both at weak coupling where it is described by a generalization of the Gross-Neveu model, and at strong coupling where it is described by probe brane dynamics in gravity. For non-zero mass this symmetry is explicitly broken.

In section 4 we discuss models with color $Dq$-branes for $q > 4$. These include a IIB model with $D5$-branes intersecting in $1+1$ dimensions. The weakly coupled theory is the Gross-Neveu model, with a slightly different UV cutoff than that of [10]. As the coupling increases (i.e. as $L$ decreases), one probes shorter and shorter distance physics in the LST of the fivebranes. As mentioned above, this theory is non-local. One expects to encounter non-field theoretic behavior when $L$ reaches the scale of non-locality. Indeed, we find that the supergravity analysis gives in this case a continuous set of solutions which exists for some critical value $L = L^*$. These solutions appear to describe the interactions of the fermions localized at the intersection with the continuum of closed string modes propagating in the fivebrane throat. We study a number of additional models with color fivebranes, and find that all of them have similar solutions, with slightly different values of $L^*$.

We also discuss models with color $D6$-branes. At weak coupling they can be studied using field theoretic means. At finite coupling it is not clear that they makes sense due to the absence of a good UV completion (which does not involve gravity). The supergravity analysis predicts the existence of an unstable state with broken symmetry at weak coupling, and is inapplicable for strong coupling.

In section 5 we analyze several models with color $D2$ and $D3$-branes, and find qualitatively similar behavior to the color $D4$-brane case. We conclude in section 6 with a discussion.
2. General results

The brane configurations that we will consider consist of two intersections, $D_q \cap D_p = I_r$ and $D_q \cap \overline{D_p} = I_{r'}$, separated by a distance $L$ in a direction transverse to the $D_p$ and $\overline{D_p}$-branes and along the $D_q$-branes. In this section we develop some tools for analyzing these systems. We start by reviewing some standard facts about such intersections, following [14]. We then go on to a discussion of systems with two intersections in a regime where the coupling between them is weak and the dynamics can be studied using field theoretic techniques. In the last subsection we describe these systems in supergravity, and discuss the implications of holography for them.

2.1. Classification

Consider an intersection of $N_c$ $D_q$-branes and $N_f$ $D_p$-branes\footnote{All the branes here and below are BPS (although the full brane configurations we will study break all supersymmetry). We will not discuss intersecting brane systems that contain non-BPS branes, since the latter have tachyon instabilities that are not of interest here.} along an $\mathbb{R}^{r,1}$ (which we refer to as $D_q \cap D_p = I_r$). We would like to determine the spectrum of massless states living at the intersection, and in particular its chirality with respect to the $U(N_c) \times U(N_f)$ symmetry on the branes.

Imagine that all the spatial directions transverse to the intersection are compactified on circles, so we can apply T-duality in these directions. The spectrum of massless states in $r+1$ dimensions is invariant under these operations. Hence, we can map all intersections to a small class of basic ones, and analyze those. For example, we can T-dualize the $D_p$-branes to $D_9$-branes. The $D_q$-branes turn in the process to $Dr'$-branes, with $r' \geq r$, and the intersection becomes $Dr' \cap D9 = I_{r'}$.

There are four possibilities, $r' = 1, 3, 5, 7$. For $r' = 7$, the spectrum of strings stretched between the color and flavor branes (or $7-9$ strings) contains a NS sector tachyon localized at the intersection. We will not discuss this case further here. For $r' = 5$, the intersection preserves eight supercharges, and the spectrum of $5-9$ strings contains a massless hypermultiplet in the representation $(N_c, \overline{N_f})$ of the gauge group. For $r' = 3$ the massless spectrum contains a Weyl fermion in the same representation, coming from the Ramond sector of $3-9$ strings. All the states in the NS sector are massive. Finally, for $r' = 1$ the system again preserves eight supercharges, which have a particular chirality in the
1 + 1 dimensions along the intersection. The massless spectrum consists of chiral (Weyl) fermions, with the opposite chirality to that of the supercharges.

Note that in all the above cases, the massless spectrum at the intersection coming from \( r' - 9 \) strings is chiral. In cases where \( r' = r \), this means that the original intersection, \( D_q \cap D_p = I_r \), also has a chiral spectrum. When \( r' > r \), the spectrum at the original intersection \( I_r \) is obtained by dimensionless reduction from \( r' + 1 \) to \( r + 1 \) dimensions. As is well known, dimensionally reducing chiral fermions gives non-chiral ones, so the resulting \((r + 1)\)-dimensional spectrum is non-chiral.

A closely related fact is that a transverse intersection, i.e. one with \( q + p - r = 9 \) or equivalently \( r' = r \), has the property that there are no directions of space transverse to both kinds of branes, so they always intersect. This is the geometric counterpart of the fact that one cannot give a gauge invariant mass to chiral fermions.

On the other hand, intersections with \( r' > r \) are not transverse, so the color and flavor branes can be separated in directions transverse to both. Doing so gives a mass to the fermions at the intersection which correspond to strings stretching from one brane to the other. The fact that it is possible to give a gauge invariant mass to the fermions implies that the latter are not chiral.

It should be noted that the discussion above addressed the question of chirality with respect to the \( U(N_c) \times U(N_f) \) symmetry on the branes. Even for non-transverse intersections, the fermions may be chiral with respect to geometric symmetries from the normal bundle to the intersection. The dynamical breaking of such geometric chiral symmetries has been studied in the context of AdS/CFT, see e.g. [15-18].

### 2.2. Weak coupling

In the previous subsection we discussed the spectrum of states localized at a given intersection of the form \( D_q \cap D_p = I_r \). Our main interest in this paper is in the systems with two such intersections described above. The important new feature of such systems is the interaction between modes localized at the two intersections.

The leading interaction between the two intersections is due to exchange of a single color gluon (and, for non-transverse intersections, scalars as well). This gives rise to a quartic interaction proportional to the gauge coupling of the color \( D_q \)-branes,

\[
g_{q+1}^2 = (2\pi)^{q-2} g_s^{q} \ell_s^{q-3} \quad (2.1)
\]
where $\ell_s = \sqrt{\alpha'}$ is the string length and $g_s$ is the string coupling. The \((q+1)\)-dimensional 't Hooft coupling $\lambda_{q+1}$ can be defined in terms of $g_{q+1}$ as

$$\lambda_{q+1} = \frac{g_{q+1}^2 N_c}{(2\pi)^{q-2}}.$$  \hfill (2.2)

The quartic interaction due to single gluon exchange is proportional to

$$\frac{\lambda_{q+1}}{N_c} \int d^{r+1}x d^{r+1}y G_{q+1}(x-y, L)[\psi_L^\dagger(x) \cdot \psi_R(y)][\psi_R^\dagger(y) \cdot \psi_L(x)]$$  \hfill (2.3)

where $\psi_L$ and $\psi_R$ are fermion fields localized at the two intersections, respectively, and

$$G_{q+1}(x, L) = (x^2 + L^2)^{-\frac{1}{2}(q-1)}$$  \hfill (2.4)

is proportional to the \((q+1)\)-dimensional massless propagator over a distance $L$ in the directions along which the flavor branes are separated and distance $x$ along the intersection.\footnote{We have written the interaction in Euclidean spacetime, which is convenient for studying the vacuum structure.}

Each term in squared brackets in (2.3) is a singlet of global $U(N_c)$, and we suppress the flavor labels. The expression (2.3) is schematic. For any given intersection one can write it more precisely, as was done for some cases in \cite{7,10} and will be done for some others below.

The non-local interaction (2.3) is non-singular in the UV. One can think of $L$ as a UV cutoff. The long distance behavior of the theory depends in an important way on whether this interaction has finite range or not. For $q - r > 2$, the integral

$$\int d^{r+1}x G_{q+1}(x, L)$$  \hfill (2.5)

converges, and one can think of $G_{q+1}$ as an \((r+1)\)-dimensional $\delta$-function smeared over a distance of order $L$. Thus, at distances much larger than $L$ one can replace (2.3) by the local interaction

$$\frac{1}{N_c} \times \frac{\lambda_{q+1}}{L^{q-3}} \times L^{r-1} \int d^{r+1}x [\psi_L^\dagger(x) \cdot \psi_R(x)][\psi_R^\dagger(x) \cdot \psi_L(x)].$$  \hfill (2.6)

Each of the factors in front of the integral in (2.6) has a simple interpretation. The first is necessary to get a smooth large $N_c$ limit; the second is the effective coupling $\lambda_{q+1}^{(\text{eff})}(L)$ \hfill (1.1). The third is a power of the UV cutoff, that is needed to account for the scaling
dimension of the local quartic coupling. For example, for \( r = 3 \) (i.e. (3 + 1)-dimensional intersection) the four-Fermi coupling has dimension six, which means that one needs a factor of \( L^2 \) to reach the dimension required of a Lagrangian. For \( r = 1 \) the third factor in (2.6) is absent, in agreement with the fact that the four-Fermi coupling is in this case marginal (more precisely marginally relevant).

The theory with a local interaction (2.6) is solvable in the large \( N_c \) limit. For (1 + 1)-dimensional intersections, such models exhibit dynamical symmetry breaking for arbitrarily weak coupling (an example is the Gross-Neveu model [19], which appears in the example studied in [10], and some other brane configurations that will be mentioned below). For higher dimension (\( r > 1 \)) they typically do not break chiral symmetry at weak coupling. An example is the original NJL model [20], which as we will see appears in string theory as a low-energy model corresponding to a certain brane configuration.

For \( q - r \leq 2 \) the integral (2.5) diverges and the range of the quartic interaction (2.3) is infinite. This makes the analysis above more subtle and we will leave it to future work.

2.3. Supergravity analysis

The discussion of the previous subsection is valid when the effective coupling (1.1) is small. For large \( \lambda_{q+1}^{(\text{eff})}(L) \) the interactions between color and flavor degrees of freedom are strong and one needs to use other tools to analyze them.

The problem without the flavor branes was studied in [13], where a qualitatively different behavior was found for \( Dq \)-branes with \( q \leq 4 \), and for those with \( q = 5,6 \). In the former case the theory on the branes can be decoupled from gravity. As one changes the effective coupling (1.1), the useful description changes from field theory, to ten-dimensional gravity, and sometimes to eleven-dimensional gravity. The important fact for our purposes is that there is a wide range of values of the effective coupling in which the field theoretic description is strongly coupled and one has to use type II supergravity to study the dynamics.

For \( D5 \)-branes the situation is more complex. The low-energy field theory degrees of freedom do not decouple from a continuum of states that live in the throat of the fivebranes (see e.g. [21,11,12]). Gravity in the near-horizon geometry of the fivebranes includes these states. For \( D6 \)-branes, the low-energy theory on the branes cannot be decoupled from gravity at all.

In this subsection we will analyze what happens when one adds to the system the flavor branes and anti-branes discussed above. We will replace the color branes by their
near-horizon geometry and will study the flavor branes and anti-branes using their Dirac-Born-Infeld (DBI) action. We will see that the results are compatible with the above picture, and in particular exhibit qualitatively different behavior for color branes with $q \leq 4$, $q = 5$ and $q = 6$.

We will take the color $D_q$-branes to span the directions $(0, 1, \cdots, q)$, while the flavor $D_p$ and $\overline{D}_p$-branes are stretched in $(0, 1, \cdots, r, q+1, q+2, \cdots, q+p-r)$. The two intersections lie along the $(r+1)$-dimensional space with coordinates $(0, 1, \cdots r)$ and are separated by a distance $L$ in the $x^q$ direction.

The near-horizon geometry of the $D_q$-branes is described by the metric and dilaton

$$
\begin{align*}
    ds^2 &= \left( \frac{U}{R_{q+1}} \right)^{(7-q)/2} dx^2 - \left( \frac{R_{q+1}}{U} \right)^{(7-q)/2} (dU^2 + U^2 d\Omega_{8-q}^2), \\
    e^\Phi &= g_s \left( \frac{R_{q+1}}{U} \right)^{(7-q)(3-q)/4},
\end{align*}
$$

where

$$
R_{q+1}^{7-q} = (2\sqrt{\pi})^{5-q}\Gamma \left( \frac{7-q}{2} \right) g_s N_c = 2^{7-2q}(\sqrt{\pi})^{9-3q}\Gamma \left( \frac{7-q}{2} \right) g_{q+1}^2 N_c.
$$

There is also a RR flux through the $(8-q)$-sphere \((2.7)\).

In the supergravity approximation, the dynamics of the flavor $D_p$-branes in the background \((2.7)\) is described by a DBI action, whose form (suppressing the gauge field on the branes) is given by

$$
S_{Dp} = -T_p \int d^{p+1}x e^{-\Phi} \sqrt{-\det g_{Dp}}
$$

where $g_{Dp}$ is the induced metric on the $Dp$-brane. There are also Chern-Simons couplings in the full action which are important for the analysis of anomalies, but play no role in what follows.

As in the examples studied in \([7,10]\), in solving for the shape of the flavor branes we have to allow for the possibility that the parallel separated $Dp$ and $\overline{D}_p$-brane configuration that we specified at weak coupling is deformed due to the effects of interactions with the color branes.

The brane configuration should still approach a brane and anti-brane at a distance $|\delta x^q| = L$ as $U \to \infty$, and preserve the same symmetry as the intersecting brane system at weak coupling. This implies that the $Dp$-brane wraps $\mathbb{R}^{r,1}$, a spherical subspace $S^{p-r-1}$ of the $(8-q)$-sphere transverse to the color branes, and a curve $U(x^q)$ in the $(U, x^q)$ plane,
which approaches $U \to \infty$ as $x^q = \pm L/2$. The induced metric is then given in terms of $U' = dU/dx^q$ by

$$ds_p^2 = \left( \frac{U}{R_{q+1}} \right)^{\frac{(7-q)/2}{2}} \left[ \eta_{\mu\nu} dx^\mu dx^\nu \right] - \left( \frac{U}{R_{q+1}} \right)^{\frac{(7-q)/2}{2}} \left[ 1 + \left( \frac{R_{q+1}}{U} \right)^{7-q} (U')^2 \right] (dx^q)^2$$

$$- \left( \frac{R_{q+1}}{U} \right)^{\frac{(7-q)/2}{2}} U^2 d\Omega_{p-r-1}^2.$$  \hspace{1cm} (2.10)

Using (2.10) and (2.7) in (2.9) leads to the action

$$S_{Dp} = - C(p, q, r) \int dx^q U^\frac{p}{2} \sqrt{1 + \left( \frac{U}{R_{q+1}} \right)^{2\beta} (U')^2}$$  \hspace{1cm} (2.11)

where we have defined

$$C(p, q, r) = \frac{T_p}{g_s} \text{Vol}(\mathbb{R}^{r,1}) \text{Vol}(S^{p-r-1}) R_{q+1}^{\frac{1}{2}(q-7)(2r-p-q+6)},$$

$$\alpha = (2r - q - p + 6) \frac{7-q}{2} + 2(p - r - 1),$$

$$\beta = \frac{q-7}{2}.$$  \hspace{1cm} (2.12)

For the special case of a transverse intersection these expressions can be simplified by using the relation $p + q - r = 9$.

Since the Lagrangian (2.11) does not depend explicitly on $x^q$, there is a first integral given by

$$U^\frac{p}{2} \sqrt{1 + \left( \frac{U}{R_{q+1}} \right)^{2\beta} (U')^2} = U_0^\frac{p}{2}$$  \hspace{1cm} (2.13)

where $U_0$ is the value of $U$ where $U' = 0$. Solving (2.13) for $U'$ and integrating gives

$$x^q(U) = \pm \frac{1}{R_{q+1}} \int_{U_0}^U \frac{u^\beta du}{\sqrt{(u/U_0)^\alpha - 1}}.$$  \hspace{1cm} (2.14)

The integral can be evaluated in terms of complete and incomplete Beta functions,

$$x^q(U) = \pm \frac{U_0}{\alpha} \left( \frac{U_0}{R_{q+1}} \right)^\beta \left[ B \left( -\frac{\beta+1}{\alpha} + \frac{1}{2}, \frac{1}{2} \right) - B \left( \frac{U_0}{U}; -\frac{\beta+1}{\alpha} + \frac{1}{2}, \frac{1}{2} \right) \right].$$  \hspace{1cm} (2.15)

The boundary conditions $x^q(U \to \infty) \to \pm L/2$ imply that

$$L = 2|x^q(\infty)| = \frac{2U_0}{\alpha} \left( \frac{U_0}{R_{q+1}} \right)^\beta B \left( -\frac{\beta+1}{\alpha} + \frac{1}{2}, \frac{1}{2} \right).$$  \hspace{1cm} (2.16)
Using (2.2), (2.8), (2.12) and dropping constants this can be rewritten as

\[ L^2 \sim U_0^{q-5} \lambda_{q+1}. \] (2.17)

This is the holographic energy – distance relation of [22], with the field theory energy scale \( E \sim 1/L \). The solution (2.15) describes a curved, connected \( Dp \)-brane, which looks like \( Dp \) and \( \overline{Dp} \)-branes connected by a wormhole whose width is determined by \( U_0 \) (2.16).

The case of color fivebranes (\( q = 5 \)) is special: the \( U_0 \) dependence in (2.16) cancels and one finds

\[ L = \frac{2\pi R_6}{p - 1}. \] (2.18)

Thus, in this case a solution exists only for a particular \( L \) of order \( R_6 \) and any width \( U_0 \). The scale (2.18) is of order the non-locality scale of the LST on the fivebranes. Thus, it is natural to suspect that it is associated with interactions between the fermions at the intersections and high energy, non-field theoretic, excitations in the fivebrane theory.

For a given value of \( L \), the equations of motion of the DBI action (2.11) have two solutions. One corresponds to \( U_0 = 0 \) in (2.13) and describes a disconnected \( Dp \) and \( \overline{Dp} \)-brane pair, running along the \( U \) axis at \( x^q = \pm L/2 \). The other is the curved, connected solution (2.15). To determine the ground state of the system, one needs to compare their energies.

The energy difference between the two solutions, \( \Delta E \equiv E_{\text{straight}} - E_{\text{curved}} \) is

\[ \Delta E = \frac{C(p, q, r)}{R_{q+1}^{\beta}} \left( \int_0^\infty du u^{\alpha/2+\beta} - \int_{U_0}^\infty du \frac{u^{\alpha/2+\beta}}{\sqrt{1 - (U_0/u)\alpha}} \right). \] (2.19)

Each integral in (2.19) is separately divergent at large \( u \). The divergence can be regulated as in [1] by regrouping terms, or equivalently by writing the integrals in terms of Beta functions which are then defined by analytic continuation. This procedure should be equivalent to the holographic renormalization reviewed in [23]. One finds

\[ \Delta E = -\frac{1}{\alpha} \frac{C(p, q, r) U_0^{\alpha/2+\beta+1}}{R_{q+1}^3} B \left( -\frac{1}{2} - \frac{\beta + 1}{\alpha}, \frac{1}{2} \right). \] (2.20)

One can check that the sign of \( \Delta E \) depends only on \( q \). For \( q \leq 4 \), \( \Delta E > 0 \), so that the curved solution has lower energy and is the ground state of the system. For \( q = 5 \),
In order to trust the above supergravity analysis the curvature of the metric must be small at the values of $U$ which govern the dynamics, that is at $U \sim U_0$. It was shown in [13] that this is the case provided that the effective coupling (1.1) at the energy $U_0$ is large,

$$\lambda_{q+1} U_0^{q-3} \gg 1 \quad (2.21)$$

For $q \leq 4$, eliminating $U_0$ using (2.17) we find that the validity of supergravity requires

$$\lambda_{q+1} \gg L^{q-3} \quad (2.22)$$

or, equivalently, large effective coupling at the scale $1/L$. We see that for supergravity to be valid, the effective coupling should be large both at the scale $U_0$, and at the scale $1/L$. These two scales are dynamically important; the former sets the dynamically generated mass of the fermions at the intersection, while the latter governs the mass of the low-lying mesons, which can be studied by expanding the DBI action around the background solution (2.15).

The relation between the two scales (2.17) can be rewritten as

$$U_0 \sim \frac{1}{L} \left[ \lambda^{(\text{eff})}_{q+1} (L) \right]^{-\frac{1}{q-3}} \sim \frac{1}{L} \sqrt{\lambda^{(\text{eff})}_{q+1} (U_0)} \quad (2.23)$$

For large effective coupling (2.21), (2.22), $U_0$ is a higher energy scale than $1/L$. Depending on the value of $q$, one of the conditions (2.21), (2.22) can be more restrictive. For $q = 2$ the effective coupling (1.1) decreases as the energy increases. Thus, the condition that the coupling be large at energy $U_0$ (2.21) is more restrictive than that at $1/L$ (2.22). For $q = 3$, the coupling does not run and (2.21) and (2.22) are equivalent. For $q = 4$ the coupling increases with energy so (2.22) is the more restrictive condition.

The cases $q = 5$ and $q = 6$ have to be treated separately. For $q = 5$, we see from (2.18) that the curved solution exists only for a particular $L$ of order $R_6$. Moreover, since the energy difference (2.20) vanishes for this case, we have a continuum of solutions with the same energy, labeled by the width of the throat connecting the flavor $Dp$-branes, $U_0$.\footnote{For the $D6 \cap D2 = I_1$ intersection, plugging into (2.12) one finds $\alpha = 0$, so the present discussion does not apply to this case. The statements in the text apply to all the other intersections with color $D6$-branes.}
As mentioned above, these solutions are associated with the non-locality of the theory of the fivebranes.

For $q = 6$, combining (2.17) and (2.21) leads to

$$\lambda_7 \ll L^3.$$  \hspace{1cm} (2.24)

In this case, validity of the supergravity analysis requires that the effective coupling at the scale $1/L$ be small, while that at the scale $U_0$ should still be large (2.21). Note that the two requirements are consistent since $U_0 \gg 1/L$ and the coupling on the $D6$-branes increases with energy.

At first sight it is surprising that the supergravity approximation should be valid at weak coupling, where we already have a good description of the dynamics in terms of a low-energy field theory. This is a manifestation of the non-decoupling of the $D6$-branes from bulk gravity. The curved brane solution we found in this case describes the interactions of the fermions with gravity, and it is not surprising that it is unstable. The interactions between the fermions and the field theoretic degrees of freedom living on the color branes take place at much smaller values of $U$, and cannot be described using supergravity (see [22] for related comments).

To summarize, we are led to a natural generalization of the picture in [13] to the system with probe D-branes. For $q \leq 4$, the supergravity analysis describes the strong coupling behavior of the intersecting brane system. For $q = 5, 6$ it instead describes the interactions of the fields associated with the intersection with non-field theoretic degrees of freedom, the LST modes living in the throat of the fivebranes for $q = 5$, and gravity modes for $q = 6$.

3. $D4 - D4$ system

Our main purpose in the rest of this paper is to illustrate the general considerations of section 2 in some examples. We have arranged the discussion by the dimension of the color branes. In this section we focus on color $D4$-branes; in the next two we discuss systems with higher and lower-dimensional color branes respectively.

Two models with color $D4$-branes were discussed in detail in [2,7,10]. In both the intersections were transverse. In this section we will study one additional example, with a
non-transverse intersection. The color and flavor branes are in this case both $D4$-branes, and are oriented as follows:

$$
0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9
$$

$$
D_{4c} : x \ x \ x \ x \ x
$$

$$
D_{4f}, \ \overline{D_{4f}} : x \ x \ x \ x \ x
$$

The color $D4$-branes are stretched in $(01234)$ and located at the origin in $(56789)$. The flavor $D4$ and $\overline{D4}$-branes are stretched in $(01567)$ and separated by a distance $L$ in $(234)$. We will take the separation to be in the $x^4$ direction and study the dynamics in the directions common to the different branes, $(01)$.

The subgroup of the Lorentz group preserved by this brane configuration is

$$
SO(1,1)_{01} \times SO(2)_{23} \times SO(3)_{567} \times SO(2)_{89}
$$

Further global symmetry arises from the gauge symmetry on the flavor $D4$-branes,

$$
U(N_f)_{D4} \times U(N_f)_{\overline{D4}}.
$$

Comparing to the discussion of subsection 2.1, we see that since the number of DN directions for strings stretched between the color and flavor branes is equal to six, and this number is invariant under T-duality, this system is T-dual to $D3 - D9$. Therefore $r' = 3$, while the dimension of the intersection is $r = 1$. Indeed, the intersection is not transverse as the color and flavor branes can be separated in the directions $(89)$, which are orthogonal to both. This deformation gives mass to the fermions at each intersection and breaks the $SO(2)_{89}$ symmetry (3.2).

The low-energy degrees of freedom in this case are open strings stretched between color branes, which give rise to $(4+1)$-dimensional SYM theory with sixteen supercharges, and strings stretched between color and flavor branes, which give spacetime fermions. To see how these fermions transform under the global symmetries, one can proceed as follows. If all the branes in (3.1) were extended in the $(89)$ directions instead of being localized in them, the $SO(1,1)_{01} \times SO(2)_{89}$ symmetry in (3.2) would have been extended to $SO(1,3)_{0189}$. The spectrum at each intersection would then be the same as in [2,7], i.e. a left-handed Weyl fermion, $q_L$, at one intersection, and a right-handed one, $q_R$, at the other.
The configuration (3.1) can be obtained by compactifying \((x^8, x^9)\) on a torus, applying T-duality in both directions and decompactifying back, in the process breaking the \(SO(1, 3)\) symmetry back to \(SO(1, 1) \times SO(2)\). The massless states at each intersection are invariant under this operation, so all we have to do is decompose the left and right-handed \(SO(1, 3)\) spinors \(q_L, q_R\) under \(SO(1, 1)_{01} \times SO(2)_{89}\):

\[
q_L = \begin{pmatrix} \chi_{L+} \\ \chi_{R-} \end{pmatrix}; \quad q_R = \begin{pmatrix} \psi_{R+} \\ \psi_{L-} \end{pmatrix}.
\]

\((3.4)\)

\(\chi\) and \(\psi\) denote fermions localized at the two intersections. The subscripts \((L, R)\) and \((+, -)\) on the right hand sides keep track of chirality in \((01)\) and \((89)\), respectively. For example, \(\chi_{L+}\) in \((3.4)\) is a complex, left-moving (one component) spinor field in 1 + 1 dimensions, with charge +1/2 (i.e. half that of a vector) under \(SO(2)_{89}\). Its adjoint, \(\chi^\ast_{L+}\), is a left-moving fermion with the opposite \(SO(2)_{89}\) charge. Note that unlike the \((3 + 1)\)-dimensional system discussed in [2,7] and the \((1 + 1)\)-dimensional one of [10], here there are left and right-handed fermions at each intersection. Thus, the two \(U(N_f)\) factors in \((3.3)\) no longer act purely on left and right-handed fermions.

In addition to their Lorentz charges, the fermions \((3.4)\) transform in the fundamental \((N_c)\) representation of the color gauge group \(U(N_c)\). Under the global symmetry \((3.3)\), the fermions \(\chi_{L+}, \chi_{R-}\) transform as \((\bar{N}_f, 1)\), while \(\psi_{R+}, \psi_{L-}\) transform as \((1, \bar{N}_f)\).

Much of the discussion of the \(D4−D6\) system in [10] carries through with little change. In particular, for \(L \gg \lambda\) the color degrees of freedom are weakly coupled, and the dynamics of the fermions is described by a Lagrangian of the form

\[
S_{\text{eff}} = i \int d^2x \left( q_L^{\dagger} \sigma^\mu \partial_\mu q_L + q_R^{\dagger} \sigma^\mu \partial_\mu q_R \right)
+ \frac{g_5^2}{4\pi^2} \int d^2x d^2y G_5(x - y, L) \left( q_L^{\dagger}(x) \cdot q_R(y) \right) \left( q_R^{\dagger}(y) \cdot q_L(x) \right)
\]

\((3.5)\)

which is obtained by integrating out the color gauge field in the single gluon approximation. \(G_5(x, L)\) is the five-dimensional massless propagator, \((2.4)\).

The \(SU(N_c)\) singlet fermion bilinear that enters the four Fermi interaction \((3.5)\) can be expressed in terms of two-dimensional fermions as follows:

\[
q_L^{\dagger}(x) \cdot q_R(y) = \chi^\ast_{L+}(x) \cdot \psi_{R+}(y) + \chi^\ast_{R-}(x) \cdot \psi_{L-}(y) .
\]

\((3.6)\)

The resulting four-Fermi interaction is not equivalent to a Thirring model for \(U(N_c)\). This is a direct consequence of the fact that the color gluons that are exchanged by the
fermions at the two intersections and give rise to (3.5) include vectors and scalars under the (1 + 1)-dimensional Lorentz group associated with the intersection.

Nevertheless, the theory can be solved at large \( N_c \) using standard methods, as in [10]. The fermion bilinear (3.6) develops dynamically a non-zero vacuum expectation value. This breaks
\[
U(N_f)_{D4} \times U(N_f)_{\overline{D4}} \to U(N_f)_{\text{diag}}.
\] (3.7)

Despite appearances, this symmetry breaking is chiral. Indeed, defining \( Q_1 \) and \( Q_2 \) to be the \( U(1) \) generators in \( U(N_f)_{D4} \) and \( U(N_f)_{\overline{D4}} \), respectively, and \( R \) to be the generator of \( SO(2)_{89} \), the combination
\[
Q_5 = Q_1 - Q_2 + 2R
\] (3.8)
acts chirally on the fermions. The left-moving fermions \( \chi_{L+}, \psi_{L-} \) have charge +2 and −2 respectively, while the right-moving fermions are neutral. The symmetry (3.8) is preserved by the action (3.5), and (if it is a symmetry of the vacuum) prevents the generation of a mass for the quarks (3.4). The quark bilinear (3.6) has charge −2 under it. Thus, if it develops an expectation value, the symmetry is broken and a quark mass can be generated.

A very similar analysis to that of [10] shows that the expectation value (3.6) takes again the form
\[
\langle q_L^\dagger(x) \cdot q_R(0) \rangle = N_c m_f \int_{|k|<\Lambda} \frac{d^2k}{(2\pi)^2} \frac{e^{ik \cdot x}}{k^2 + m_f^2}
\] (3.9)
with the dynamically generated mass \( m_f \) given by
\[
m_f \simeq \Lambda e^{-\frac{\Lambda}{m_f}}.
\] (3.10)

\( \Lambda \) is the UV cutoff of the theory, \( \Lambda \simeq 1/L \). In this case, one can also analyze the system in the presence of mass terms in the Lagrangian which explicitly break the chiral symmetry (3.8),
\[
\delta \mathcal{L}_{\text{eff}} = m_1 \chi_{L+}^* \chi_{R-} + m_2 \psi_{R+}^* \psi_{L-} + \text{c.c.}
\] (3.11)
corresponding to separating the color and flavor branes in the (89) plane. This leads to a straightforward generalization of the analysis in [10].

As discussed in section 2, at strong coupling the dynamics of the fermions and color degrees of freedom can be described by studying the DBI action of the flavor \( D4 \)-branes in the near-horizon geometry of the color branes, (2.11).

5 Note that \( Q_5 \) is a linear combination of a symmetry that is broken by (3.7), \( Q_1 - Q_2 \), and one that is preserved, \( R \).
The U-shaped solution in which the flavor branes and anti-branes are connected by a wormhole is given by \((2.15)\),

\[
x^4(U) = \pm \frac{1}{4^3} \frac{R^{3/2}_{5}}{U^{1/2}_{0}} \left[ B \left( \frac{5}{8}, \frac{1}{2} \right) - B \left( \frac{U^4_{0}}{U^4}, \frac{5}{8}, \frac{1}{2} \right) \right].
\]  \hspace{1cm} (3.12)

The energy difference between the straight brane and anti-brane configuration, and the U-shaped one \((3.12)\) is given by \((2.20)\),

\[
\Delta E = -\frac{1}{4} C(4, 4, 1) U^4_{0} B \left( -\frac{3}{8}, \frac{1}{2} \right) \approx 0.225 C(4, 4, 1) U^4_{0}.
\]  \hspace{1cm} (3.13)

Thus, the vacuum of the theory breaks chiral symmetry both for weak coupling and for strong coupling, in agreement with the general analysis of section 2. One can also analyze the system for finite temperature, as was done for the \(D4 - D8\) case in \([24,25]\) and for the \(D4 - D6\) case in \([10]\).

Overall, we conclude that the \(D4 - D4\) system behaves in a very similar way to the \(D4 - D6\) one analyzed in \([10]\). At weak coupling it reduces to a GN-type model which can be analyzed using field theoretic techniques, and at strong coupling it can be studied using the DBI action for the flavor branes in the near-horizon geometry of the color ones. One advantage of this system is that one can turn on current masses to the fermions and study the dynamics as a function of these masses. Another advantage is that the \(D4 - D4\) brane configuration is simple to lift to M-theory, where the color \(D4\)-brane background goes over to \(AdS_7 \times S^4\), and the flavor \(D4\)-branes and anti-branes become \(M5\)-branes in this background. These and other issues deserve further study.

4. Higher-dimensional color branes

In section 2 we saw that when the color branes are higher than four-dimensional, the supergravity analysis exhibits some qualitative differences from the case when their dimension is four or less. In this section we will examine some examples with color \(D5\) and \(D6\)-branes to study these phenomena in more detail.
4.1. Color fivebranes

In this subsection we discuss a few intersecting brane systems in which the color branes are (5 + 1)-dimensional. They can be further subdivided by the type of flavor branes, the dimension of the intersection and the range of the non-local four-Fermi interaction at weak coupling. In table 1 we list the four systems that will be discussed below.

| flavor branes | dimension of intersection | range of interaction |
|---------------|---------------------------|----------------------|
| $D5$          | $1 + 1$                   | $L$                  |
| $D3$          | $1 + 1$                   | $L$                  |
| $D5$          | $3 + 1$                   | $\infty$            |
| $D7$          | $3 + 1$                   | $\infty$            |

Table 1: Different systems with color $D5$-branes that are discussed in this section.

Our first example is obtained by T-dualizing the $D4 - D6$ system discussed in [10]. It contains color $D5$-branes stretched in (012345), transversally intersecting flavor $D5$ and $\overline{D5}$-branes stretched in (016789). A single intersection of this sort was studied in [26,27]. The main new phenomenon here is the attractive interactions between the fermions at the two intersections.

As before, we can try to analyze this system using QFT techniques at weak coupling, and supergravity at strong coupling. The weak coupling analysis is quite analogous to that of [10]. The fermions at the two intersections, $q_L$, $q_R$, are chiral. Their dynamics is governed by the effective action (3.5), with the coupling $g_5$ replaced by $g_6$ (2.1) and the Green function $G_5(x,L)$ replaced by $G_6(x,L)$, (2.4). It is integrable,

$$\int d^2x G_6(x,L) = \frac{\pi}{2L^2} .$$

Thus, at distances much larger than $L$ the system reduces to the GN model with action

$$S_{gn} = \int d^2x \left[ i q_L^+ \sigma^\mu \partial_\mu q_L + i q_R^+ \sigma^\mu \partial_\mu q_R + \frac{\lambda_{gn}}{N_c} \left( q_L^+(x) \cdot q_R(x) \right) \left( q_R^+(x) \cdot q_L(x) \right) \right]$$

and coupling

$$\lambda_{gn} = \frac{\pi \lambda_6}{2L^2} .$$
In particular, it dynamically breaks chiral symmetry and generates a fermion condensate (3.3), which leads to the fermion mass

\[ m_f \simeq \frac{1}{L} e^{-2\pi/\lambda g_{\text{y}}} . \] (4.4)

As before, this analysis is reliable for \( \lambda_6 \ll L^2 \) and breaks down when this condition is violated. The DBI analysis does give a solution in which the flavor \( D5 \) and \( \overline{D5} \)-branes are connected by a wormhole whose shape is given by (2.15),

\[ x^5(U) = \pm \frac{1}{4} R_6 \left[ B \left( \frac{1}{2}, \frac{3}{2} \right) - B \left( \frac{U_0^4}{U^4}; \frac{1}{2}, \frac{1}{2} \right) \right] . \] (4.5)

However, the asymptotic separation of the \( D5 \) and \( \overline{D5} \)-branes (2.18) is fixed,

\[ L = \frac{1}{2} \pi R_6 . \] (4.6)

For this value of \( L \) there are solutions with arbitrary width \( U_0 \), whose energy is independent of \( U_0 \). This is different from the situation in systems with color \( D4 \)-branes, where there is a solution for generic \( L \), and the width of the wormhole \( U_0 \) is a function of \( L \), growing when \( L \) decreases.

What does this mean for the dynamics of the fermions living at the two intersections? For \( L \gg R_6 \) the field theoretic GN analysis is valid. Chiral symmetry is dynamically broken, and the fermions get a mass (4.4). As \( L \) decreases, the coupling (4.3) grows, and the dynamically generated mass (4.4) does as well. The system probes higher and higher energies in the fivebrane theory. As \( L \) approaches the value (2.18) the dynamics becomes dominated by high energy LST states.

The resulting physics is not field theoretic and its analysis is beyond the scope of this paper. One expects non-smooth behavior at the non-locality scale (4.6), and it is not clear whether the system exists for smaller \( L \). The problem is analogous to the analysis of fivebrane thermodynamics, with the inverse temperature \( \beta \) being the analog of \( L \) and the energy density on the fivebranes an analog of the fermion mass \( m_f \). The curved solution (4.3) is an analog of the Euclidean continuation of the non-extremal fivebrane solution. The latter has the property that the circumference of Euclidean time at infinity, \( \beta \), is independent of the energy density, just like in (4.3) the asymptotic separation between the two arms of the U-shape, \( L \), is independent of the fermion mass (or \( U_0 \)).

In the case of fivebrane thermodynamics it is believed that the Euclidean black hole solution is not continuously connected to the low temperature thermodynamics (see [12]...
for a discussion). It would be interesting to understand whether in our case the solution (4.5) is continuously related to the large \( L \) regime, and what happens for \( L \) smaller than (4.6).

We next move on to the brane system on the second line in table 1, which is obtained by T-duality from the one discussed in section 3. It contains color \( D5 \)-branes stretched in (012345) and flavor \( D3 \) and \( \overline{D3} \)-branes stretched in (0167).

At weak coupling this system reduces to a GN-type model of the sort discussed in section 3. In particular, it exhibits dynamical symmetry breaking. At strong coupling one needs to analyze the DBI action (2.11), which leads to the brane profile

\[
x^5(U) = \pm \frac{1}{2} R_6 \left[ B \left( \frac{1}{2}, \frac{1}{2} \right) - B \left( \frac{U_0^2}{U^2}, \frac{1}{2}, \frac{1}{2} \right) \right].
\]

This looks very similar to the \( D5-D5 \) solution (4.5). As there, the asymptotic separation (2.18) is fixed, \( L = \pi R_6 \). Here too we expect chiral symmetry breaking for sufficiently small values of the coupling \( \lambda_6/L^2 \), and non-smooth behavior when the coupling approaches the critical value (2.18).

The last two lines in Table 1 correspond to systems with (3 + 1)-dimensional intersections. In one, the flavor branes are \( D5 \) and \( \overline{D5} \)-branes stretched in (012367). In the other, they are \( D7 \) and \( \overline{D7} \)-branes stretched in (01236789).

In the model with flavor \( D5 \)-branes, each intersection preserves eight supercharges. Hence the fermions living at a given intersection belong to a hypermultiplet. SUSY is completely broken in the full system and exchange of fields living on the color branes leads to an attractive interaction between the hypermultiplets localized at the two intersections.

At weak coupling, this attractive interaction has a structure similar to (3.5), however unlike the system analyzed in [10] and those discussed earlier in this paper, the Green function \( G_6(x) \) is not integrable in this case:

\[
\int d^4 x G_6(x) = \int \frac{d^4 x}{(x^2 + L^2)^2}
\]

is logarithmically divergent, so the attractive interaction has infinite range. Such systems are in general more subtle than those with a short-range interaction. We hope to return to their study in a separate publication.

In the supergravity approximation, the solution of the equations of motion of the DBI action (2.11) is again given by (4.5), and we find that there is a curved brane solution in the supergravity regime for \( L \) given by (4.6). The interpretation is the same as there.
The situation is similar for the last brane configuration in table 1, which has flavor $D_7$-branes and a $(3 + 1)$-dimensional intersection with the color $D_5$-branes. This model is T-dual to the $D_4 \cap D_8 = I_3$ system studied in [7]. Thus the spectrum is the same as there: a left-handed fermion $q_L$ in the $(N_c, N_f, 1)$ of $U(N_c) \times U(N_f)_L \times U(N_f)_R$ at the $D_5 - D_7$ intersection, and a right-handed fermion $q_R$ in the $(N_c, 1, N_f)$ at the $D_5 - D_7$ one. The leading single gluon exchange interaction between the left and right-handed fermions takes a form similar to (3.5) (or, more precisely, eq. (3.5) in [7]). As in the previous example, this leads to a long-range interaction.

In the supergravity approximation, the solution to the DBI equations of motion is

$$x^5(U) = \pm \frac{1}{6} R_6 \left[ B \left( \frac{1}{2}, \frac{1}{2} \right) - B \left( \frac{U_0^6}{U_6^3}, \frac{1}{2}, \frac{1}{2} \right) \right]. \quad (4.9)$$

It again has a fixed value of $L$ (2.18) at which the dynamics becomes dominated by highly excited LST states.

Before leaving the case of color fivebranes we would like to point out that the curved brane solutions we found for this case are closely related\footnote{In fact for the $D_5 \cap D_3 = I_1$ system they are S-dual.} to the hairpin brane of [28,29] which plays a role in analyzing the dynamics of D-branes propagating in the vicinity of NS5-branes [30,31]. They can be thought of as continuations to Euclidean space of accelerating brane solutions. This also makes it plausible that, as mentioned above, they owe their existence to the interactions of the fields at the intersections with the continuum of modes living in the fivebrane throat, \textit{i.e.} to Little String Theory dynamics [11,12].

4.2. Color sixbranes

As mentioned above, due to lack of decoupling, it is not clear whether intersecting brane systems involving color $D_6$-branes make sense beyond the field theory approximation (\textit{i.e.} for finite $L$). Nevertheless, in this subsection we will discuss two examples of such systems using the tools outlined in section 2.

The first system is obtained by reversing the roles of the $D_4$ and $D_6$-branes in the configuration studied in [10]. Thus, we have $N_c$ color $D_6$-branes stretched in (0156789) and $N_f$ flavor $D_4$ and $\overline{D_4}$-branes stretched in (01234) and separated by a distance $L$ in (56789). At weak coupling the low-energy dynamics is again governed by a GN model which breaks chiral symmetry and generates a mass for the fermions. The GN coupling
\( \lambda_{gn} \) (4.2) is proportional to \( \lambda_7/L^3 \) as can be verified by integrating the \((6 + 1)\)-dimensional propagator (2.4).

From our experience with the \( D5 \)-brane case, we expect the supergravity analysis to be more subtle. The DBI action leads in this case to the solution

\[
x^6(U) = \pm \frac{1}{3} \sqrt{R_7 U_0} \left[ B \left( \frac{1}{3}, \frac{1}{2} \right) - B \left( \frac{U_0^3}{U^3}, \frac{1}{3}, \frac{1}{2} \right) \right].
\]  
(4.10)

The asymptotic separation between the flavor branes and anti-branes is

\[
L = \frac{2}{3} \sqrt{R_7 U_0} B \left( \frac{1}{3}, \frac{1}{2} \right).
\]  
(4.11)

Thus, unlike the fivebrane case, here there is a solution in which the flavor branes are connected by a wormhole for generic \( L \).

As discussed in section 2, the supergravity analysis is valid at large \( L \), (2.24). In that region the dynamics of the fermions is described by the GN model. The curved brane solution (4.10) is not a consequence of that dynamics. Instead, its existence is due to gravitational interactions of the fermions in the vicinity of the \( D6 \)-branes. This dynamics should be unimportant at low energies. Indeed, the solution (4.11) is unstable. The energy difference (2.20) is given by

\[
\Delta E \sim -\frac{1}{3} U_0^2 B \left( \frac{2}{3}, \frac{1}{2} \right) \approx -0.351 U_0^2.
\]  
(4.12)

One can show that the curved brane configuration is unstable to perturbations of the form \( U(x^6) \rightarrow U(x^6) + \delta U \) with \( \delta U \) independent of \( x^6 \). Thus, in the supergravity approximation the vacuum corresponds to straight branes. As mentioned above, the GN analysis implies that the flavor branes do curve towards each other and connect, but this happens at a much smaller value of \( U \) than (4.11) and is well outside the regime of validity of supergravity.

We do not have a good description of chiral symmetry breaking in the strongly coupled regime \( \lambda \gg L^3 \). As mentioned above, it is likely that this is because the system does not exist in that regime, or more generally for any finite \( \lambda/L^3 \).

Our second example consists of \( D6 \) and \( \overline{D6} \)-branes with a \((3 + 1)\)-dimensional intersection (\( i.e. D6 \cap \overline{D6} = I_3 \)). The color branes can be taken to lie in \((0123456)\), while the flavor branes and anti-branes are stretched in \((0123789)\) and separated by a distance \( L \) in \( x^6 \). This configuration is T-dual to the \( D4 - D8 - \overline{D8} \) one studied in [2,7]. The spectrum contains left-handed fermions at one intersection, and right-handed ones at the other.
At weak coupling these fermions interact via the four-Fermi interaction given in eq. (3.5) in [7]. However, here this interaction is local, since

\[
\int d^4 x G_7(x, L) = \int \frac{d^4 x}{(x^2 + L^2)^{\frac{5}{2}}}
\]

is finite. Therefore, the model reduces at long distances \( x \gg L \) to a local NJL model [20], with coupling \( \lambda_7/L \) which has dimension length squared. This model does not break chiral symmetry at arbitrarily weak coupling. Hence the same should be true for the intersecting D-brane system in the limit \( L^3 \gg \lambda_7 \).

The supergravity analysis leads to results that are qualitatively similar to the previous case. The DBI action (2.11) leads to a solution corresponding to flavor branes and anti-branes connected by a wormhole

\[
x^6(U) = \pm \frac{1}{4} \sqrt{R_7 U_0} \left[ B(\frac{3}{8}, \frac{1}{2}) - B(\frac{U_0^4}{U^4}; \frac{3}{8}, \frac{1}{2}) \right].
\]

This solution is again valid for large \( L \), (2.24), and is unstable,

\[
\Delta E \sim -\frac{1}{4} U_0^\frac{7}{2} B(-\frac{5}{8}, \frac{1}{2}) \approx -0.193 U_0^\frac{5}{2}.
\]

5. Lower-dimensional color branes

In this section we will discuss intersecting brane configurations with \( D2 \) and \( D3 \) color branes. The low-energy theories on these branes are renormalizable gauge theories which can be decoupled from gravity. For \( D3 \)-branes this theory is \( N = 4 \) SYM. It is conformal, and its effective coupling (1.1) is independent of the separation \( L \). For \( D2 \)-branes the theory is weakly coupled in the UV and strongly coupled in the IR. Thus, the effective coupling (1.1) grows as \( L \) increases.

Due to the low dimension of the color branes, all our examples involve \((1 + 1)\)-dimensional intersections. Thus, the flavor branes are codimension one or two defects in the gauge theory. The single gluon exchange interaction (2.3) relevant for the weak coupling analysis is a long range one, since the integral (2.3) diverges. We will not discuss the weakly coupled theory in detail here.
5.1. Color threebranes

Our first example contains $N_c$ color $D3$-branes stretched in $(0123)$ and $N_f$ flavor $D7$ and $\overline{D7}$-branes stretched in $(01456789)$ separated by the distance $L$ in $x^3$. This configuration is T-dual to the $D4−D6$ one considered in [10]. Therefore, it has the same massless spectrum of fermions at the intersections – a left-handed fermion $q_L$ transforming in the $(N_c, N_f, 1)$ of $U(N_c) \times U(N_f)_L \times U(N_f)_R$ at the $D3−D7$ intersection and a right-handed fermion $q_R$ in the $(N_c, 1, N_f)$ at the $D3−\overline{D7}$ intersection.

At weak coupling the leading interaction of the fermions is a long range GN coupling due to single gluon exchange. At strong coupling, the $D3$-branes are replaced by their near-horizon geometry, $AdS_5 \times S^5$, and the $D7$-branes are described by the DBI action (2.11). The solution (2.15) takes in this case the form

$$x^3(U) = \pm \frac{1}{6} \frac{R_4^2}{U_0} \left[ B\left(\frac{2}{3}, \frac{1}{2}\right) - B\left(\frac{U_0^6}{U^6}; \frac{2}{3}, \frac{1}{2}\right) \right].$$  \hspace{1cm} (5.1)

The distance between the $D7$ and $\overline{D7}$-branes is

$$L = \frac{R_4^2}{3U_0} B\left(\frac{2}{3}, \frac{1}{2}\right).$$  \hspace{1cm} (5.2)

One can check using (2.20) that this solution has lower energy than the one in which the flavor branes are stretched in $U$ at fixed values of $x^3$. Thus, at strong coupling ($\lambda_4 \gg 1$) the chiral symmetry is dynamically broken, and the fermions get a mass of order $\sqrt{\lambda_4}/L$.

Replacing the flavor branes by $D5$-branes extended in $(014567)$ leads to a T-dual of the $D4−D4$ system studied in section 3. Therefore, the spectrum is the same as there. It consists of fermions $\chi_{L+}, \chi_{R−}$ transforming in $(N_c, N_f, 1)$ and $\psi_{R+}, \psi_{L−}$ transforming in $(N_c, 1, N_f)$ of $U(N_c) \times U(N_f)_D5 \times U(N_f)_{\overline{D5}}$. At weak coupling there are long range interactions between the fermions, the leading of which is given by (3.5), with $G_5$ replaced by $G_4$. At strong coupling, the DBI action of the fivebranes is proportional to that of the $D7$-branes discussed above. Thus, the solution is given again by (5.1), (5.2) and chiral symmetry is broken at strong coupling.

5.2. Color twobranes

A system with color $D2$-branes and flavor $D8$ and $\overline{D8}$-branes was considered recently in [9]. The $D2$-branes can be taken to lie in the directions $(012)$, while the $D8$-branes span the directions $(013456789)$. This system is T-dual to the $D3−D7$ one described above, so
the massless spectrum at the intersection is the same as there. At weak coupling (which in this case means small $L$) it reduces to a non-local GN model, while at strong coupling (large $L$) it breaks chiral symmetry, as follows from the DBI analysis of section 2.3.

Replacing the $D8$-branes by $D6$-branes stretched in the directions (0134567) one finds a $D2 - D6$ system T-dual to the $D4 - D4$ one of section 3. At strong coupling, the DBI analysis of section 2 leads to the solution

$$x^2(U) = \frac{1}{8} \frac{R_3^2}{U_0^2} \left[ B\left(\frac{11}{16}, \frac{1}{2}\right) - B\left(\frac{U_0^8}{U_3^8}, \frac{11}{16}, \frac{1}{2}\right) \right].$$

This solution breaks chiral symmetry and has lower energy than the symmetry preserving one, as in all other cases with $Dq$ color branes with $q \leq 4$.

6. Conclusions

In this paper we presented an analysis of dynamical symmetry breaking in a class of intersecting $D$-brane systems which generalize those investigated earlier in [2,7,9,10]. At weak coupling these models are usefully classified by the range of the effective interaction between fermions localized at the intersections. The models with short range interactions are straightforward to analyze. When the intersection is (1 + 1)-dimensional, they exhibit symmetry breaking of the type studied by Gross and Neveu [19]. In 3 + 1 dimensions they reduce to variants of the Nambu-Jona-Lasinio model [20] and do not lead to symmetry breaking at weak coupling.

We also described the dynamics of these systems in the approximation where we replace the color branes by their near-horizon geometry and study the Dirac-Born-Infeld action of the flavor branes in this geometry. As expected from [13,22], the results of this analysis depend on the dimensionality of the color branes. For $Dq$ color branes with $q \leq 4$ it provides a holographic description of the corresponding field theory at strong coupling. For all such systems the ground state exhibits dynamical symmetry breaking.

For $q > 4$, the DBI analysis does not describe the strong coupling behavior of the field theory at the intersection, but rather the interaction of modes associated with the intersection with other, non-field theoretic degrees of freedom. For $q = 5$ these are Little String Theory modes that propagate in the throat of the fivebranes. Their interactions with the modes at the intersections lead to the existence of flavor brane configurations in the near-horizon geometry, all of the same energy, which are labeled by the width of
the throat, $U_0$. The asymptotic separation of the branes has a particular value, (2.18), for these solutions. We interpreted this value as the non-locality scale of LST for these probes. For $q = 6$ the modes in question are gravity modes of the full theory. Their interaction with the fermions at the intersection leads to the existence of unstable configurations of the flavor branes which exhibit dynamical symmetry breaking.

The main gap in our discussion is the weak-coupling analysis of intersecting brane systems with long-range fermion interactions. These systems are subtle, but also potentially important as they arise in embeddings of QCD in string theory in the limit where the scale of chiral symmetry breaking is much larger than that of confinement \[7\]. We hope to return to them elsewhere.

One of the main motivations for this work was to see whether intersecting brane configurations of the sort described in \[2,7,9,10\] and in this paper, are always in the same phase as far as dynamical symmetry breaking is concerned (in the limit where the color branes are non-compact, so there is no confinement). In all cases where we were able to analyze the dynamics for both weak and strong coupling we found that there was no phase transition between the weak and strong coupling regimes for $Dq$ color branes with $q \leq 4$. It is natural to conjecture that this is always the case. A better understanding of weakly couple systems with long range interactions would allow us to test this conjecture in a wider class of models.

For $q = 5$ one does not expect such smoothness to extend beyond the non-locality scale of the underlying LST, and indeed we found signs of this in the existence of curved branes solutions for a particular value of $L$, (2.18). It would be interesting to understand the physics of these solutions in LST better. For $q = 6$ it is not clear that the brane configurations in question exist for finite $L$, so the question of smoothness in $L$ does not arise.

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