Chiral Dynamics of scalar mesons: radiative $\phi$ decay and $\sigma$ in the medium through $\pi^0\pi^0$ nuclear photoproduction

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Abstract

In order to assess the relevance of chiral dynamics in the scalar sector we address two recent problems: radiative decay of the $\phi$, for which there are quite recent data from Frascati, and the modification of the $\sigma$ properties in the nuclear medium seen through the $\pi^0\pi^0$ photoproduction in nuclei.

1 Introduction

The radiative decays of the $\phi$ into $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ have been the subject of intense study [1, 2, 3, 4, 5, 6, 7]. One of the main reasons for this is the hope that one can get much information about the nature of the $f_0(980)$ and $a_0(980)$ resonances. The nature of the scalar meson resonances has generated a large debate [8], with new ideas brought by the claim that these resonances are dynamically generated from multiple scattering with the ordinary chiral Lagrangians [9,10,11].

These two reactions involving the decay of the $\phi$ are special. Indeed, the $\phi$ does not decay into two pions because of isospin symmetry. But we can bypass this by allowing the $\phi$ to decay into two charged kaons (with a photon attached to one of them) and the two kaons scatter giving rise to the two pions (or $\pi^0\eta$). The loop which appears diagrammatically is proved to be finite using arguments of gauge invariance [12,13,5]. The radiative $\phi$ decay through this mechanism was studied in [5] and the results of lowest order chiral perturbation theory ($\chi PT$) were used to account for the $K^+K^- \rightarrow \pi^0\pi^0$ transition. Since the chiral perturbation theory $K^+K^- \rightarrow \pi^0\pi^0$ amplitude does not account for the $f_0(980)$, the excitation of this resonance has to be taken in addition, something that has been done more recently using a linear Sigma-model in [14].
The work of [7] leads to the excitation of the $f_0(980)$ in the $\pi^0\pi^0$ production, or the $a_0(980)$ in $\pi^0\eta$ production in a natural way, since the use of unitarized chiral perturbation theory ($U\chi PT$), as in [9], generates automatically those resonances in the meson meson scattering amplitudes and one does not have to introduce them by hand.

The experimental situation has also experienced an impressive progress recently. To the already statistically significant experiments at Novosibirsk [15, 16, 17] one has added the new, statistically richer, experiments at Frascati [18,19] which allow one to test models beyond just the qualitative level. In this sense although the predictions of the work of [7], using $U\chi PT$ with no free parameters, provided a good agreement with the experimental data of [15,16,17], thus settling the dominant mechanism as that coming from chiral kaon loops from the $\phi \to K^+K^-$ decay, the new and more precise data leave room for finer details which we evaluate in this paper.

In addition to the mechanisms discussed before we have sequential $V \to VP \to PP\gamma$ process. This mechanism is known to provide the $\omega \to \pi^0\pi^0\gamma$ radiative width with accuracy $[20]$ and has been further extended to study $\rho \to \pi^0\pi^0\gamma$ and other radiative decays in $[21,22]$.

Another novelty of the present work is the consideration of sequential mechanisms involving the exchange of an intermediate axial-vector meson ($J^{PC} = 1^{++}$ or $1^{+-}$), both producing directly the final meson pair or through the intermediate production of kaons which undergo collisions and produce these mesons.

All the mechanisms considered here contribute moderately, but appreciably, to the $\phi$ radiative width. The inclusion of all these mechanisms leads to results compatible with the experimental data of Frascati, particularly if theoretical uncertainties are considered, which is something also done in the present work.

The good agreement with experiment is reached in spite of having in our approach a width for the $f_0(980)$ very small, of the order of 30 MeV, in apparent contradiction with the "visual" $f_0(980)$ width in the experiment, which looks much larger. The reason for this has been recently discussed in $[23,24]$ and stems from the fact that, due to gauge invariance, the amplitude for the process contains as a factor the momentum of the photon, which grows fast as we move down to smaller invariant masses from the mass of the $f_0(980)$ where the photon momentum is very small. This distorts the shape of the resonance, making it appear wider. Our approach, which respects gauge invariance, introduces automatically this factor in the amplitudes.

We shall see that there is some discrepancy of the theoretical results with the data at small invariant masses. We shall discuss this feature, realizing that the results resemble very much the raw data, before the analysis is done to subtract the contribution of $\omega\pi^0$ and to correct for the experimental acceptance. Furthermore, some of the assumptions made in the analysis of $[18]$ might be questionable.
Figure 1: Loop diagrams included in the chiral loop contributions. The intermediate states in the loops are $K^+K^-$. 

2 The $\phi \rightarrow \pi^0\pi^0\gamma$ decay

2.1 Kaon loops from $\phi \rightarrow K^+K^-$ decay

The mechanism for radiative decay using the tensor formulation for the vector mesons have been discussed in [25,7] and we briefly summarize it here. The diagrams considered are depicted in Fig. 1, where the loops contain $K^+K^-$. The vertices needed for the diagrams are obtained from the chiral Lagrangian for vector meson resonances of ref. [26], assuming ideal mixing between the $\phi$ and $\omega$ mesons.

Using arguments of gauge invariance it was proved in [12,13] that the loop functions are convergent and in [27] that the meson meson amplitude factorizes outside the loop integral with on shell value. This information is of much value, since it allows to factorize the on shell meson meson amplitude outside the loop integral. The amplitude for the process is given by

$$ t = -\sqrt{2} \frac{e}{f^2} \epsilon(\phi) \cdot \epsilon(\gamma) \left[ M_\phi G_V \tilde{G}(M_I) + q \left( \frac{F_V}{2} - G_V \right) G(M_I) \right] t_{K^+K^-,\pi^0\pi^0} $$

where $f = 92.4$ MeV and $\tilde{G}$ is the convergent loop function of [12,13]. On the other hand, $G(M_I)$ is the ordinary loop function of two meson propagators which appears in the study of the meson meson interaction in [9] and which is regularized there with a cut off of the order of 1 GeV. In Eq. (1) the $t_{K^+K^-,\pi^0\pi^0} = \frac{1}{\sqrt{3}} t_{K^+K^-,\pi\pi}$ is the transition amplitude with the iterated loops implicit in the coupled channels Bethe Salpeter equation (BS) obtained in [9]. The parameters $F_V, G_V$, for the vector mesons are obtained from their decay into $e^+e^-$, $\mu^+\mu^-$ or two mesons. We take for the calculations $F_V = 156\pm5$ MeV and $G_V = 55\pm5$ MeV (see [28] for a discussion on these values).

2.2 Sequential vector meson exchange mechanisms

Following the lines of [21,22] in the study of $\rho$ and $\omega$ radiative decays and the more recent of [29,15] in the $\phi$ decay, we also include these mechanisms here. They are depicted in
Figure 2: Diagrams for the tree level VMD mechanism.

Fig. 2, where we explicitly assume that the $\phi \to \rho^0 \pi^0$ proceeds via the $\phi - \omega$ mixing.

In order to evaluate these diagrams we use the same Lagrangians as in [5, 21]. In addition we must use the Lagrangians producing the $\phi - \omega$ mixing. We use the formalism of [30]

$$L_{\phi \omega} = \Theta_{\phi \omega} \phi_{\mu} \omega^{\mu}$$

which means that the diagrams of Fig. 2 can be evaluated assuming the decay of the $\omega$ (with mass $M_\omega$) multiplying the amplitude by $\tilde{\epsilon}$ (the measure of the $\phi - \omega$ mixing) given by

$$\tilde{\epsilon} = \Theta_{\phi \omega} / (M_\phi^2 - M_\omega^2)$$

Values of $\Theta_{\phi \omega}$ of the order of $20000 - 29000$ MeV$^2$ are quoted in [31] which are compatible with $\tilde{\epsilon} = 0.059 \pm 0.004$ used in [32] which is the value used here.

The amplitude for the $\phi(q^*) \to \pi_1^0(p_1) \pi_2^0(p_2) \gamma(q)$ decay corresponding to the diagrams of Fig. 2 is given by

$$t = -\mathcal{C} \tilde{\epsilon} \frac{2\sqrt{2}\,egf^2G^2}{3\,M_\omega^2} \left[ \frac{P^2 \{a\} + \{b(P)\}}{M_\rho^2 - P^2 - iM_\rho \Gamma_\rho(P^2)} + \frac{P'^2 \{a\} + \{b(P')\}}{M_\rho^2 - P'^2 - iM_\rho \Gamma_\rho(P'^2)} \right]$$

where $P = p_2 + q$, $P' = p_1 + q$ and $\{a\}, \{b(P)\}$ given in [21].

At this point it is worth mentioning that the theoretical expression for the $V \to P \gamma$ decay widths $\Gamma_{V \to P \gamma} = \frac{4}{3} \alpha C_i \left( \frac{G_\pi^2}{M_\rho M_\omega} \right)^2 k^3$, with $C_i$ SU(3) coefficients given in Table I of [35], gives slightly different results to the experimental values from [36]. For this reason we can follow a similar procedure to that used for the $\eta \to \pi^0 \gamma \gamma$ decay in [35] where the $C_i$ coefficients were normalized so that the theoretical $V \to P \gamma$ decay widths agree with experiment. In the $\phi \to \pi^0 \pi^0 \gamma$ reaction this procedure results in including in Eq. (3) a normalizing factor $\mathcal{C} = 0.869 \pm 0.014$, obtained considering the $V \to P \gamma$ reactions shown in Table I of [34].

### 2.3 Pion final state interaction in the sequential vector meson mechanism

Since the $\pi \pi$ interaction is strong in the region of invariant masses relevant in the present reaction we next consider the final state interaction of the pions in the sequential vector meson mechanism.

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1Note that in [33] a different sign for $\tilde{\epsilon}$ is claimed. This is actually a misprint and the results of that paper are calculated with $\tilde{\epsilon} > 0$ [24].
We must take into account the loop function of Fig. 3a, but on the same footing we must also consider those of Fig. 3b and 3c, where charged pions are produced and allowed to interact to produce the $\pi^0\pi^0$ final state. The thick dot in Fig. 3 means that one is considering the full $\pi\pi \rightarrow \pi\pi$ t-matrix, involving the loop resummation of the BS equation of ref. [9] and not just the lowest order $\pi\pi \rightarrow \pi\pi$ amplitude.

In order to evaluate those diagrams we must calculate the loop function with a $\rho$ and two pion propagators. First let us note that due to isospin symmetry the $\omega\rho^0\pi^0$ coupling is the same as the $\omega\rho^+\pi^-$ or $\omega\rho^-\pi^+$. Next, given the structure of the terms in Eqs. (3) we must evaluate the loop integrals

$$i \int \frac{d^4 P}{(2\pi)^4} \frac{1}{P^2 - M_V^2 + i\epsilon} \frac{1}{(q^* - P)^2 - m_1^2 + i\epsilon} \frac{1}{(q - P)^2 - m_2^2 + i\epsilon}$$

for which we evaluate first the $P^0$ integral analytically and then the three momentum integral numerically with a cut off of 1 GeV.

### 2.4 Kaon loops in the sequential vector meson mechanisms

Next we consider the diagrams analogous to those in Fig. 3 but with kaons and $K^*$ in the intermediate states.

Note that the $\phi VP$ vertices are now not OZI forbidden. They come from the Lagrangian of refs. [5, 21], and all the four $\phi KK^*$ vertices have the same strength.

### 2.5 Sequential axial vector meson mechanisms

Since the mass of the $\phi$ is around 250 MeV higher than the $\rho$ mass, and we are considering sequential vector meson mechanisms with $\rho$ or $K^*$ exchange, we should pay attention to the analogous mechanisms involving vector mesons with a similar mass difference with the $\phi$ on the upper side and these are the axial and vector mesons with $J^{PC} = 1^{++}$ or $1^{+-}$ (see Table 1). Therefore, the $b_1$ or $a_1$ axial vector mesons and the $K_{1B}, K_{1A}$ strange axial vector mesons will play the role of the $\rho$ or the $K^*$ in former diagrams.

Because of the $C$ parity of the states, the Lagrangians for the axial-vector–vector–pseudoscalar couplings have the structure of $< B\{V, P\} >$ for the $b_1$ octet and $< A\{V, P\} >$ for the octet of the $a_1$ [37], where the $<>$ means $SU(3)$ trace. In the last expressions $V$
and $P$ are the usual vector and pseudoscalar $SU(3)$ matrices respectively and $B$ and $A$ are axial vector $SU(3)$ matrices given in [25].

In addition one has to consider an approximate 50% mixture of the $K_{1B}$ and $K_{1A}$ states to give the physical $K_1(1270)$ and $K_1(1400)$ states [37,38,39].

We have modified the original Lagrangian of [37] to treat the vector fields in the tensor formalism of [26]. This formalism has the advantage that without basically changing the rates of the $A \to VP$ decays, one deduces the coupling of the $a_1$ to $\pi\gamma$ using vector meson dominance through $a_1 \to \pi\rho \to \pi\gamma$, with an amplitude which is gauge invariant and which is in agreement with the chiral structure of [26] for the $a_1 \to P\gamma$ couplings and with the experiment. Details are given elsewhere in [40].

We hence use the Lagrangians [40]

\[
\mathcal{L}_{BV_P} = \bar{D} B_{\mu
u} \{V^{\mu\nu},P\} > \\
\mathcal{L}_{AV_P} = i \bar{F} A_{\mu\nu} [V^{\mu\nu},P] >
\]

where the $i$ factor in front of the $\bar{F}$ is needed in order $\mathcal{L}_{AV_P}$ to be hermitian.

In Eq. (5) the fields $W_{\mu\nu} \equiv A_{\mu\nu}, B_{\mu\nu}$ are normalized such that

\[
<0|W_{\mu\nu}|W;\epsilon> = \frac{i}{M_W} [P_{\mu} \epsilon_{\nu}(W) - P_{\nu} \epsilon_{\mu}(W)]
\]

The physical $K_1(1270)$ and $K_1(1400)$, with a mixture around 45 degrees \footnote{It is worth mentioning that in [38] two more possible solutions for the mixing angle around 30 and 60 degrees were found. This uncertainty will be taken into account in the evaluation of the error band in our final results.} found in [37,38,39,40], can be expressed, in terms of the $I = 1/2$ members of the $1^{++}(1^{+-})$ octets, $K_{1B}(K_{1A})$, as

\[
K_1(1270) = \frac{1}{\sqrt{2}} (K_{1B} - iK_{1A}) \\
K_1(1400) = \frac{1}{\sqrt{2}} (K_{1B} + iK_{1A})
\]

With the values for $\bar{D} = -1000 \pm 120$ MeV and $\bar{F} = 1550 \pm 150$ MeV, very similar to those found in [37,38,39], we are able to describe all the $A \to VP$ decays plus the radiative decays of the $a_1 \to \pi\gamma$ [40].

Once again the $\phi$ sequential decay at tree level through $b_1$ exchange is OZI violating and found to be negligible.

\begin{table}[h]
\begin{center}
\begin{tabular}{|c|c|c|c|}
\hline
$J^{PC}$ & $I = 1$ & $I = 0$ & $I = 1/2$ \\
\hline
$1^{+-}$ & $h_1(1235)$ & $h_1(1170), h_1(1380)$ & $K_{1B}$ \\
$1^{++}$ & $a_1(1260)$ & $f_1(1285), f_1(1420)$ & $K_{1A}$ \\
\hline
\end{tabular}
\end{center}
\caption{Octets of axial-vector mesons.}
\end{table}
Table 2: Parameters which uncertainties are relevant in the error analysis. The $f_\pi$ and $\Lambda$ are the $f_\pi$ constant and cutoff of the momentum integral respectively in the loops involved in the unitarized meson-meson rescattering.

| Parameter        | Value         |
|------------------|---------------|
| $C$              | $0.869 \pm 0.014$ |
| $\bar{t}$        | $0.059 \pm 0.004$ |
| $G_V$ (MeV)      | $55 \pm 5$    |
| $F_V$ (MeV)      | $156 \pm 5$   |
| $f_\pi$ (MeV)    | $92.4 \pm 3\%$|
| $\Lambda$ (MeV) | $1000 \pm 50$ |
| $\bar{D}$ (MeV) | $-1000 \pm 120$|
| $\bar{F}$ (MeV) | $1550 \pm 150$|

2.6 Kaon loops from sequential axial vector meson mechanisms

The relevant mechanisms involving axial-vectors are those in which $K$, $\bar{K}$ are created and through scattering lead to the final $\pi^0\pi^0$ state, having also one of the $K_1$ resonances as intermediate state. These are not OZI forbidden and have a nonnegligible contribution.

Since we are using the tensor formulation for the vector mesons this forces us to use the tensor coupling of the photon to the vector mesons obtained from the $F_V$ term of the chiral Lagrangian of ref. [26]:

$$L_{V\gamma} = -\frac{e}{2} \lambda_V V^0_{\mu\nu} (\partial^\mu A^\nu - \partial^\nu A^\mu)$$

with $\lambda_V = 1, \frac{1}{3}, -\frac{\sqrt{2}}{3}$ for $V = \rho, \omega, \phi$ respectively.

3 Results for the $\phi \rightarrow \pi^0\pi^0\gamma$ decay

In Fig. 4 we show the results of the different contributions. We should say that the loops of the sequential vector meson mechanisms involving kaons are relatively important but there is a strong cancellation between the mechanisms with a $\phi$ and an $\omega$ attached to the photon.

Up to now, all the curves shown in the figures have been calculated using the central values of the parameters without considering the uncertainties in their values. In Fig. 5 we show the final result but including an evaluation of the error band due to the uncertainties in the parameters of the model. This error band has been calculated implementing a Monte Carlo gaussian sampling of the parameters within their experimental errors. The parameters of the model which uncertainties are relevant in the error analysis are shown in Table 2.

The errors in $f_\pi$ and $\Lambda$ assumed in the calculations have been chosen such that the quality of the fit to the $\pi\pi$ phase shifts along the lines of [24] is still acceptable within experimental errors.

The parameter with the larger contribution to the error band turns out to be the $G_V$ since the largest contribution, chiral kaon loops form $\phi \rightarrow K^+K^-$ decay, is roughly
Figure 4: Different contributions to the two pion invariant mass distributions of the $\phi \rightarrow \pi^0\pi^0\gamma$ decay: Dashed line: chiral loops of Fig. 1. Dashed-dotted line: chiral loops of Fig. 1 + sequential VMD and its final state interaction. Solid line: idem plus the contribution of the mechanisms involving axial-vector mesons, (full model).

Figure 5: Final results for the $\pi^0\pi^0$ invariant mass distribution for the $\phi \rightarrow \pi^0\pi^0\gamma$ decay with the theoretical error band. Experimental data from [18].
proportional to $G_V$ (up to the term with $q(F_V - G_V)$ in Eq. (11) which would be zero within some vector meson dominance hypotheses \cite{26} and is small with our set of parameters).

The total width and branching ratio obtained in the present work are

$$BR(\phi \to \pi^0\pi^0\gamma) = (1.2 \pm 0.3) \times 10^{-4}$$

(9)

to be compared with the experimental values

$$BR^{exp}(\phi \to \pi^0\pi^0\gamma) = (1.22 \pm 0.10 \pm 0.06) \times 10^{-4} \ [15], (0.92 \pm 0.08 \pm 0.06) \times 10^{-4}, \ [16],$$

$$1.09 \pm 0.03 \pm 0.05) \times 10^{-4} \ [18].$$

In Fig. 6 we can see that our results, considering the error band, fairly agrees with the experimental data except in the region around 500 MeV. The reason of this discrepancy will be further discussed.

4 Results for the $\phi \to \pi^0\eta\gamma$ decay

After the discussion of the former points the consideration of the $\phi \to \pi^0\eta\gamma$ decay requires only minimal technical details which one can see in ref. \cite{28}. We show in Fig. 6 the results for the different mechanisms in this reaction.

Again, in Fig. 7 we have plotted the full model performing the theoretical error analysis\textsuperscript{3}.

\textsuperscript{3}We have also checked that the use of a mixing angle for the strange members of the axial nonets of around 30 or 60 degrees \cite{28,40} turns out in decreasing the lower limit of the error band in around 5\% and 10\% for the $\phi \to \pi^0\pi^0\gamma$ and $\phi \to \pi^0\eta\gamma$ decays respectively.
We can see that when these uncertainties are considered we obtain a theoretical band in acceptable agreement with the experimental data.

The total width and branching ratio obtained are

\[ BR(\phi \rightarrow \pi^0\eta\gamma) = (0.59 \pm 0.19) \times 10^{-4} \]  \hspace{1cm} (10) 

to be compared with the experimental values \( BR^{\text{exp}}(\phi \rightarrow \pi^0\eta\gamma) = (0.88 \pm 0.14 \pm 0.09) \times 10^{-4} \) \[17\], \( (0.90 \pm 0.24 \pm 0.10) \times 10^{-4} \) \[18\], \( (0.85 \pm 0.05 \pm 0.06) \times 10^{-4} \) \[19\].

5 Further discussion of the \( \phi \rightarrow \pi^0\pi^0\gamma \) results.

We would like to comment on the strength that we obtain around 500 MeV in the \( \pi^0\pi^0 \) invariant mass distribution in the \( \phi \rightarrow \pi^0\pi^0\gamma \) decay which appears in contradiction with the experimental analysis. As we saw it comes from accumulation of the novel mechanisms which we have discussed in our paper. Such mechanisms are not considered in other theoretical papers which find a good agreement with the data, after fitting some parameters to the data. Our philosophy has been different and we have not fitted any parameter to the \( \phi \) radiative decay data but simple have considered the different mechanisms that can sizeable contribute to the process. An acceptable agreement with the data is found in the region of the \( f_0(980) \), which is the most important issue concerning this reaction. This is not trivial a priori in view of the very small width of the dynamically generated \( f_0(980) \) (around 30 MeV) that one obtains in the model of \[9\] that we use here and the large ’visual’ width of the \( f_0(980) \) peak in the present experiment. Part of the reason for the agreement comes from the \( q \) factor (photon momentum) in the amplitude, as requirement by gauge invariance and emphasize in \[23,24\], which gives more weight to the amplitude as we move down in the \( \pi\pi \) invariant mass from the upper limit (where \( q = 0 \)). However, as seen in
our results, the inclusion of the new mechanisms and their interference with the dominant one, particularly the contribution of the axial-vector meson exchange mechanisms, also contributes to the widening of the distribution around the $f_0(980)$ peak.

Although the agreement with data at low masses is not very good, we must point out two sources of uncertainty in the experimental spectrum. First, the results in the low and intermediate mass region largely depend on the background subtraction dominated by the non-resonant $\omega\pi^0$ process. The size of this process is difficult to obtain because it has a strong background itself, mostly from the $\phi \to f_0\gamma$ process, as it is discussed in [41]. There, its magnitude has been obtained in a model dependent way assuming some a priori spectrum for the $\phi \to f_0\gamma$ process [41]. In fact, before the subtraction, the raw data resemble much more our calculated spectrum, (see fig. 4 from [18]), and we could think of a slightly smaller $\omega\pi^0$ background.

Additionally, there is some uncertainty in the way the data are corrected to account for the experimental efficiency. This is done in [18] by dividing the observed spectrum by the effect of applying the experimental efficiency on some theoretical distribution. This unfolding procedure depends on the theoretical model used, which we think at low $\pi^0\pi^0$ masses is at least incomplete. In fact, with the unfolding method used, the zero value of the spectrum obtained with the theoretical model of [18] implies unavoidably a zero value for the corrected experimental results. A reanalysis to the light of the present discussion would be most welcome.

6 $\sigma$ meson in a nuclear medium through two pion photoproduction

In the last years there has been an intense theoretical and experimental debate about the nature of the $\sigma$ meson, mostly centered on the discussion about its interpretation as an ordinary $q\bar{q}$ meson or a $\pi\pi$ resonance. The advent of $\chi PT$ showed up that the $\pi\pi$ interaction in s-wave in the isoscalar sector is strong enough to generate a resonance through multiple scattering of the pions. This seems to be the case, and even in models starting with a seed of $q\bar{q}$ states, the incorporation of the $\pi\pi$ channels in a unitary approach leads to a large dressing by a pion cloud which makes negligible the effects of the original $q\bar{q}$ seed. This idea has been made more quantitative through the introduction of the unitary extensions of $\chi PT$ ($U\chi PT$). Even more challenging is the modification of the properties of the $\sigma$ meson at finite nuclear density. Since present theoretical calculations agree on a sizeable modification in the nuclear medium of the $\pi\pi$ scattering in the $\sigma$ region, our purpose here is to find out its possible experimental signature in a very suited process like the $(\gamma, \pi^0\pi^0)$ reaction in nuclei. (This contribution is a summary of the more extended work [42]). This reaction is much better suited than the $(\pi, \pi\pi)$ one to investigate the modification of the $\pi\pi$ in nuclear matter because the photons are not distorted by the nucleus and the reaction can test higher densities.
6.1 Model

For the model of the elementary \((\gamma, \pi\pi)\) reaction we follow [43] which considers the coupling of the photons to mesons, nucleons, and the resonances \(\Delta(1232), N^*(1440), N^*(1520)\) and \(\Delta(1700)\). This model relies upon tree level diagrams. Final state interaction of the \(\pi N\) system is accounted for by means of the explicit use of resonances with their widths. However, since we do not include explicitly the \(\sigma\) resonance, the final state interaction of the two pions has to be implemented to generate it.

The \(\gamma N \rightarrow N\pi^0\pi^0\) amplitude can be decomposed in a part which has in the final state the combination of pions in isospin \(I=0\) and another part where the pions are in \(I=2\).

\[
|\pi^0_1\pi^0_2| = \frac{1}{3} |\pi^0_1\pi^0_2 + \pi^+_1\pi^-_2 + \pi^-_1\pi^+_2| \quad \text{I=0 part} \quad \text{and} \quad -\frac{1}{3} |\pi^0_1\pi^0_2 + \pi^+_1\pi^-_2 + \pi^-_1\pi^+_2| + |\pi^0_1\pi^0_2| \quad \text{I=2 part} \quad (11)
\]

The renormalization of the \(I = 0 \ (\gamma, \pi\pi)\) amplitude is done by factorizing the on shell tree level \(\gamma N \rightarrow \pi\pi N\) and \(\pi\pi \rightarrow \pi\pi\) amplitudes in the loop functions.

\[
T^{I=0}_{(\gamma, \pi^0\pi^0)} \rightarrow T^{I=0}_{(\gamma, \pi^0\pi^0)} \left(1 + G_{\pi\pi} t^{I=0}_{\pi\pi} (M_t)\right) \quad (12)
\]

where \(G_{\pi\pi}\) is the loop function of the two pion propagators, which appears in the Bethe Salpeter equation, and \(t^{I=0}_{\pi\pi}\) is the \(\pi\pi\) scattering matrix in isospin \(I=0\), taken from [44].

In Fig. 8 we show a diagrammatical representation of the the two pion production including their final state interaction.

The multiple scattering of the two final pions can be accounted for by means of the Bethe Salpeter equation,

\[
t = V + V G_{\pi\pi} t \quad (13)
\]

where \(V\) is given by the lowest order chiral amplitude for \(\pi\pi \rightarrow \pi\pi\) in \(I = 0\) and \(G_{\pi\pi}\), the loop function of the two pion propagators can be regularized by means of a cut off or with dimensional regularization. In both approaches it has been shown that \(V\) factorizes with its on shell value in the Bethe-Salpeter equation. Hence, in the Bethe-Salpeter equation the integral involving \(V t\) and the product of the two pion propagators affects only these latter two, since \(V\) and \(t\) factorize outside the integral, thus leading to Eq. (13) where \(V G_{\pi\pi} t\) is the algebraic product of \(V\), the loop function of the two propagators, \(G_{\pi\pi}\), and the \(t\) matrix.
When we renormalize the $I=0$ amplitude in nuclei to account for the pion final state interaction, we change $G$ and $t_{I=0}^{\pi\pi}$ by their corresponding results in nuclear matter \[44\] evaluated at the local density. In the model of [44], the $\pi\pi$ rescattering in nuclear matter was done renormalizing the pion propagators in the medium and introducing vertex corrections for consistency.

In the model for $(\gamma, 2\pi)$ of [43] there are indeed contact terms as implied before, as well as other terms involving intermediate nucleon states or resonances. In this latter case the loop function involves three propagators but the intermediate baryon is far off shell and the factorization of Eq. (12) still holds. There is, however, an exception in the $\Delta$ Kroll Ruderman term, since as we increase the photon energy we get closer to the $\Delta$ pole. For this reason this term has been dealt separately making the explicit calculation of the loop with one $\Delta$ and two pion propagators.

The cross section for the process in nuclei is calculated using many body techniques. From the imaginary part of the photon selfenergy diagram with a particle-hole excitation and two pion lines as intermediate states, the cross section can be expressed as

$$
\sigma = \frac{\pi}{k} \int \frac{d^3\tilde{r}}{(2\pi)^3} \int \frac{d^3\tilde{p}}{(2\pi)^3} \int \frac{d^3\tilde{q}_1}{(2\pi)^3} \int \frac{d^3\tilde{q}_2}{(2\pi)^3} \frac{F_1(\tilde{r}, \tilde{q}_1)F_2(\tilde{r}, \tilde{q}_2)}{2\omega(\tilde{q}_1)2\omega(\tilde{q}_2)} \left| T \right|^2 n(\tilde{p})[1 - n(\tilde{k} + \tilde{p} - \tilde{q}_1 - \tilde{q}_2)]
$$

where $F_i(\tilde{r}, \tilde{q}_i)$ take into account the distortion of the final pions in their way out through the nucleus and are given by

$$
F_i(\tilde{r}, \tilde{q}_i) = \text{exp} \left[ \int_{\tilde{r}_i}^{\infty} dl_i \frac{1}{q_i} Im \Pi(\tilde{r}_i) \right]
$$

where $\Pi$ is the pion selfenergy, taken from a model based on an extrapolation for low energy pions of a pion-nucleus optical potential developed for pionic atoms using many body techniques. The imaginary part of the potential is split into a part that accounts for the probability of quasielastic collisions and another one which accounts for the pion absorption. With this approximation the pions which undergo absorption are removed from the flux but we do not remove those which undergo quasielastic collisions since they do not change in average the shape or the strength of the $\pi\pi$ invariant mass distribution.

### 6.2 Results

In the figure we can see the results for the two pion invariant mass distributions in the $(\gamma, \pi^0\pi^0)$ reaction on $^1H$, $^{12}C$ and $^{208}Pb$. The difference between the solid and dashed
curves is the use of the in medium $\pi\pi$ scattering and $G$ function instead of the free ones, which we take from [44]. As one can see in the figure, there is an appreciable shift of strength to the low invariant mass region due to the in medium $\pi\pi$ interaction. This shift is remarkably similar to the one found in the preliminary measurements of [45].

These results show a clear signature of the modified $\pi\pi$ interaction in the medium. The fact that the photons are not distorted has certainly an advantage over the pion induced reactions and allows one to see inner parts of the nucleus.

Although we have been discussing the $\pi\pi$ interaction in the nuclear medium it is clear that we can relate it to the modification of the $\sigma$ in the medium. We have mentioned that the reason for the shift of strength to lower invariant masses in the mass distribution is due to the accumulated strength in the scalar isoscalar $\pi\pi$ amplitude in the medium. Yet, this strength is mostly governed by the presence of the $\sigma$ pole and there have been works suggesting that the sigma should move to smaller masses and widths when embedded in the nucleus. The present results represent an evidence that the pole position of the $\sigma$ to smaller energies as the nuclear density increases, a phenomenon which would come to strengthen once more the nature of the $\sigma$ meson as dynamically generated by the multiple scattering of the pions through the underlying chiral dynamics.

7 Conclusions

We have demonstrated with two examples the relevance of chiral dynamics in processes involving scalar mesons. The main point to stress is that, since the scalar mesons are generated dynamically in the scheme that we follow, we do not face any unknown coupling and the theory is predictive for any process involving the production of scalar mesons. The agreement with the data obtained here and in many other processes [46] gives strong support to the picture of scalar mesons being dynamically generated by the multiple scattering of the pseudoscalar mesons under the interaction provided by the lowest order chiral Lagrangian.

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Figure 9: Two pion invariant mass distribution for \( \pi^0\pi^0 \) photoproduction in \(^{12}\text{C}\) and \(^{208}\text{Pb}\). Continuous lines: using the in medium final \( \pi\pi \) interaction. Dashed lines: using the final \( \pi\pi \) interaction at zero density. Exp. data from [45].
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