The Beta Type I Generalized Half Logistic Distribution: Properties and Application

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Authors' contributions

This work was carried out in collaboration among all authors. Author POA designed the study, performed the statistical analysis and wrote the first draft of the manuscript. Authors ECN and MAI manage the literature and reviewed the work. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJPAS/2020/v6i230156

Abstract

In a view to obtain a new distribution that is more flexible than the type I generalized half logistic distribution, we used the beta-G generator and the type I generalized half logistic distribution. Some properties of the new distribution including the cumulative distribution function, survival function, hazard function were studied. Estimation of parameters were done using the maximum likelihood estimation method. Application of the derived distribution to lifetime data was illustrated by applying to remission times of bladder cancer patient data and survival times of guinea pigs.

Keywords: Beta-G generator; type I generalized half logistic distribution; guinea pigs; bladder cancer.

2010 Mathematics Subject Classification: 53C25, 83C05, 57N16.

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1 Introduction

Many researchers have generalized both logistic and half-logistic distribution and apply it to different areas of human endeavors. The half logistic distribution was proposed by Balakrishnan [1] and then used to model life-time data. The probability density function (pdf) of the half-logistic distribution is

\[ f(x) = \frac{2e^{\frac{x}{1+e^{x}}}}{(1 + e^{\frac{x}{1+e^{x}}})^2}, \quad 0 \leq x < \infty \]  

(1.1)

with the cumulative distribution function obtained as

\[ F(x) = \frac{e^{\frac{x}{1+e^{x}}}-1}{(1 + e^{\frac{x}{1+e^{x}}})}, \quad 0 \leq x < \infty. \]  

(1.2)

In a bid to generalize 1.1 Olapade[2] obtained a generalized form of the half logistic distribution by including a shape parameter as in 1.3

\[ g(x) = \frac{b^2 e^{\frac{x}{b}}}{\delta (1 + e^{\frac{x}{b}})^{b+1}} \]  

(1.3)

with cumulative distribution function as

\[ G(x) = 1 - \frac{b^b}{(1 + e^{\frac{x}{b}})^b} \]  

(1.4)

The type I generalized distribution has been further generalized, Olapade [3], Bello [4], Femi and Olapade [5] and Femi and Olapade [6]. Introducing the type I generalized half logistic distribution to survival analysis, Awodutire et al [7] studied survival function, hazard function and estimated the parameters under censored observation. Awodutire et.al [8] used the derived survival model to assess the survival times of breast cancer patients in Nigeria using a data collected at a teaching hospital.

One of the appeals of the type I generalized half logistic distribution in the context of reliability theory is that it has non-decreasing hazard rate for all parameter values, a property shared by relatively few distributions which have support on the positive real line. This is a limitation to the type I generalized half logistic distribution as cases may arise for hazard rate that has other rate apart from the monotonically increasing hazard rate. Beta distribution has widespread application as statistical model to handle various kinds of issues in real life experiences. A lot of the finite range distributions encountered in practice can be transformed into the standard beta distribution. The Beta distribution is given as

\[ f(x) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1 - x)^{b-1}dx, \quad 0 < x < 1, \quad a, b > 0 \]  

(1.5)

where \( \Gamma(.) \) is the beta function, \( a, b \) are the shape parameters. Eugene et.al [9] generated a generalized equation for the beta distribution and applied to the normal distribution. This generator has been extensively used in the generation of generalized class distributions. These class distributions came to prominence after the work by Jones [10]. Consider starting from the cumulative distribution function \( G(x) \) of a random variable, Eugene et al. [11] defined a class of generalized distributions from it given by

\[ F(x) = \frac{1}{B(a, b)} \int_0^{G(x)} x^{a-1}(1 - x)^{b-1}dx \]  

(1.6)

where \( a > 0 \) and \( b > 0 \) are two additional parameters whose role is to introduce skewness and to vary tail weight and

\[ B(a, b) = \int_0^1 x^{a-1}(1 - x)^{b-1}dx \]  

(1.7)
Fig. 1. Graphical representation of probability density function of type I generalized half logistic distribution

Fig. 2. Hazard function of different parameter values of type I generalized half logistic distribution

The cumulative distribution function $G(x)$ could be quite arbitrary and $F$ is named the beta $G$ distribution.
Equation 1.5 can also be written as

$$F(x) = I_{G(x)}(a, b)$$

where $I_{G(x)}$ is the incomplete beta function ratio. This further gives the expression as

$$F(x) = \frac{B(G(x); a, b)}{B(a, b)}$$

From equation 1.6, we get the probability distribution function as

$$f(x) = \frac{G(x)^{α-1}[1 - G(x)]^{β-1}g(x)}{B(a, b)}$$

Jose and Manoharan (2016) used the half logistic distribution with Beta-G generator to obtain a distribution called the beta half logistic distribution. The properties of their derived distribution were studied and thereafter applied to remission survival times of bladder cancer patients.

### 2 The Beta Type I Generalized Half Logistic Distribution

In this section, the Beta type I generalized half logistic distribution is derived. Using the p.d.f and c.d.f of the type I generalized half logistic distribution as derived in equations 1.3 and 1.4 in equation 1.10, we have

$$f(x) = \frac{1}{B(a, b)} \left(1 - \frac{2}{1 + e^{\frac{-x}{\delta}}}ight)^{q} \left(\frac{2}{1 + e^{\frac{-x}{\delta}}}ight)^{q(β-1)} e^{\frac{-x}{\delta}}$$

which gives

$$f(x) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \left(1 - \frac{2}{1 + e^{\frac{-x}{\delta}}}ight)^{q} \left(\frac{2}{1 + e^{\frac{-x}{\delta}}}ight)^{q(β-1)} e^{\frac{-x}{\delta}}$$

with cumulative distribution as

$$F(x) = \frac{1}{B(a, b)} \int_{0}^{1} \left(\frac{2}{1 + e^{\frac{-x}{\delta}}}ight)^{q} x^{α-1}(1 - x)^{β-1} dx$$

for $α > 0$, $β > 0$, $ξ > 0$, $δ > 0$ and $q > 0$. Equation 2.3 therefore results to

$$F(x) = I_{G(x)}(a, b) = I_{k}(a, b) = \frac{B(k; a, b)}{B(a, b)}$$

where the $G(x) = k = 1 - \left(\frac{2}{1 + e^{\frac{-x}{\delta}}}\right)^{q}$ and $B(k; a, b)$ is the incomplete beta function.

If we set $h = 1 - G(x)$, then, from equation 2.4, we have the survival function as

$$s(x) = \frac{1}{B(a, b)} \int_{0}^{h} x^{α-1}(1 - x)^{β-1} dx = \frac{B(h; a, b)}{B(a, b)}$$

where

$$h = \left(\frac{2}{1 + e^{\frac{-x}{\delta}}}\right)^{q}$$

and the hazard rate function as

$$h(x) = \frac{\left(1 - \left(\frac{2}{1 + e^{\frac{-x}{\delta}}}\right)^{q}\right)^{(α-1)} q^{2β} e^{\frac{-x}{\delta}}}{B(h; a, b)(1 + e^{\frac{-x}{\delta}})^{q+1}}$$
Fig. 3. Graph showing the curves of Probability Distribution function of Beta type I generalized half logistic distribution.
Fig. 4. Graph showing the curves of Survival function of Beta type I generalized half logistic distribution.

Fig. 5. Graph showing the curves of Survival function of Beta type I generalized half logistic distribution.
Fig. 6. Graph showing the curves of Hazard function of Beta type I generalized half logistic distribution.

Fig. 7. Graph of hazard function of type I generalized half logistic distribution.
3 Derivations for the Cumulative Distribution Function of The Beta Type I Generalized Half Logistic Distribution

In this section, two simple expressions for the c.d.f. of the distribution, depending whether the parameter \( b > 0 \) is real non-integer or integer which will aid in further analysis were derived.

According to Wolfram statistics, If \( jw < 1 \), therefore, For case \( b > 0 \) is real non-integer,

\[
(1 - w)^{b - 1} = \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma(b)}{\Gamma(b - i)!} w^i \quad (3.1)
\]

Using equation 3.1 in 2.3, the cumulative distribution function of the Beta type I generalized half logistic distribution becomes

\[
F(x) = \frac{1}{B(a, b)} \int_0^1 \frac{1}{1 + e^{-x}} x^{a - 1} \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma(b)}{\Gamma(b - i)!} x^i dx
\]

\[
F(x) = \frac{\Gamma(b)}{B(a, b)} \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma(b)}{\Gamma(b - i)!} \int_0^1 \frac{1}{1 + e^{-x}} x^{a - 1} x^i dx
\]

\[
F(x) = \frac{\Gamma(b)}{B(a, b)} \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma(b)}{\Gamma(b - i)!} \int_0^1 \frac{1}{1 + e^{-x}} x^{a + i - 1} dx
\]

\[
F(x) = \frac{\Gamma(a + b)}{\Gamma(a)} \sum_{i=0}^{\infty} \frac{(-1)^i (1 - \frac{2}{1 + e^{-x}})^{a + i}}{\Gamma(b - i)! \Gamma(a + i)}
\]

For case \( b > 0 \) is real integer,

\[
F(x) = \frac{\Gamma(a + b)}{\Gamma(a)} \sum_{i=0}^{b-1} \frac{(-1)^i (1 - \frac{2}{1 + e^{-x}})^{a + i}}{\Gamma(b - i)! \Gamma(a + i)}
\]

The same property holds for 3.2 and 3.3 only that the sum in equation 3.2 is infinite while that of 3.3 is finite.

Furthermore, from the Wolfram Functions Site, it can be seen that for integer \( a \),

\[
I_k(a, b) = 1 - \frac{(1 - t)^b}{\Gamma(b)} \sum_{i=0}^{a-1} \frac{\Gamma(b + i)}{i!} t^i
\]

and for integer \( b \),

\[
I_k(a, b) = \frac{t^a}{\Gamma(a)} \sum_{i=0}^{b-1} \frac{\Gamma(a + i)}{i!} (1 - t)^i
\]

Therefore, for \( a > 0 \) integer

\[
F(x) = I_k(a, b) = 1 - \left( \frac{\frac{2}{1 + e^{-x}}}{\Gamma(b)} \right)^{\frac{b}{a}} \sum_{i=0}^{a-1} \frac{\Gamma(b + i)}{i!} \left( 1 - \left( \frac{2}{1 + e^{-x}} \right) \right)^i
\]

and for \( b > 0 \) integer,

\[
F(x) = I_k(a, b) = \left( \frac{1 - \frac{2}{1 + e^{-x}}}{\Gamma(a)} \right)^{\frac{a}{b}} \sum_{i=0}^{b-1} \frac{\Gamma(a + j)}{j!} \left( 1 - \left( \frac{2}{1 + e^{-x}} \right) \right)^i
\]
It is of importance to note that $f(x)$ is intractable since there is no simple analytic expression for the $g(x)$ and $G(x)$. However, these problem can be solved with the aid of mathematical softwares.

4 Asymptotic Behavior of the Beta Type I Generalized Half Logistic Distribution

Examining the asymptotic properties of the distribution by considering the behavior of the $\lim_{x \to \infty} f(x)$ and $\lim_{x \to 0} f(x)$ as follows:

For, simplification, standardizing the Beta Type I generalized half logistic distribution, we have

$$f(x) = \frac{1}{B(a,b)} \left( 1 - \left( \frac{2}{1 + e^x} \right) q \right)^{(a-1)} \left( \frac{2}{1 + e^x} \right)^{q(b-1)} \frac{2^q e^x}{(1 + e^x)^{q+1}}$$ \hspace{1cm} (4.1)

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{1}{B(a,b)} \left( 1 - \left( \frac{2}{1 + e^x} \right) q \right)^{(a-1)} \left( \frac{2}{1 + e^x} \right)^{q(b-1)} \frac{2^q e^x}{(1 + e^x)^{q+1}}$$

$$\lim_{x \to 0} f(x) = \frac{1}{B(a,b)} \left( 1 - \left( \frac{2}{1 + 0} \right) q \right)^{(a-1)} \left( \frac{2}{1 + 0} \right)^{q(b-1)} \frac{0}{(1 + 0)^{q+1}}$$

$$\lim_{x \to \infty} f(x) = 0$$

$$\lim_{x \to 0} f(x) = 1$$

Therefore, since $\lim_{x \to \infty} f(x)$ and $\lim_{x \to 0} f(x)$, the beta type I generalized half logistic distribution has at least a mode.

5 Sub-Model of the Beta Type I Generalized Half Logistic Distributions

1. When $\xi = 0, q = 1$, then the beta type I generalized half logistic distribution reduces to

$$f(x) = \frac{1}{B(a,b)} \frac{2^q e^x (e^x - 1)^{a-1}}{\delta(1 + e^x)^{a+\delta}}$$ \hspace{1cm} (5.1)

This is the distribution of Joby and Manohara[12]

2. when $a = 1, \xi = 0$, then

$$f(x) = \frac{2^q e^\xi}{\delta(1 + e^\xi)^{b+1}}$$ \hspace{1cm} (5.2)

which is the generalized half logistic distribution defined by Arora[13]
3. when \( a = 1, b = 1, \xi = 0 \)

\[
f(x) = \frac{b^2 e^{\frac{x}{\delta}}}{(1 + e^{\frac{x}{\delta}})^{b+1}}
\]

that is the type I generalized half logistic distribution of Olapade[2]

4. when \( b = 1 \),

\[
f(x) = \frac{q^2 e^{\frac{x}{q}}}{(1 + e^{\frac{x}{q}})^{q+1}}
\]

which is the Lehmann type II generalized half logistic as defined by Awodutire et al.[12](In press)

5. when \( b = 1, \xi = 0, q = 1 \), then

\[
f(x) = ae^{\frac{x}{a}}(1 + e^{\frac{x}{a}})^{-a-1}
\]

which is the pdf of the exponentiated half logistic distribution

6. If \( X \) is a beta random variable with parameters \( a \) and \( b \), then

\[
Q = \ln \left( \frac{2}{2 + \sum_{i=0}^{n} e^{x_i}} - 1 \right)
\]

follows the beta type I generalized half logistic distribution with parameters \( \xi, \delta, q, a, b \).

That is,

\[
Q \sim BTIGHL(\xi, \delta, q, a, b).
\]

Point 6 was useful when simulating data that follow the beta type I generalized half logistic distribution.

7. If \( X \) is distributed as \( BTIGHL(0,1,1,a,b) \), then

\[
L = \ln \left( \frac{2}{1 + e^{x_i}} - 1 \right)
\]

follows the log beta distribution.

### 6. Estimation of Beta Type I Generalized Half Logistic Distribution under Complete Data

This section deals with the maximum likelihood estimators of the unknown parameters for the beta type I generalized half logistic distribution on the basis of complete samples. Let \( X_1, X_2, ..., X_n \) be a set of random variables, assumed to follow the beta type I generalized half logistic distribution, using equation 12; we have the likelihood function as

\[
L(x, a, b, q, \xi, \delta) = \prod_{i=0}^{n} f(x, a, b, q, \xi, \delta)
\]

\[
L(x, a, b, q, \xi, \delta) = \prod_{i=0}^{n} \left( \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \right) \left( 1 - \left( \frac{2}{1 + e^{\frac{x_i}{\delta}}} \right) \right)^n \left( 1 + e^{\frac{x_i}{\delta}} \right)^{(a-1)} \left( 1 + e^{\frac{x_i}{\delta}} \right)^{q(b-1)}
\]

\[
\ln L(x, a, b, q, \xi, \delta) = l = n \ln \left( \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \right) + n(a - 1) \ln \left( 1 - \left( \frac{2}{1 + e^{\frac{x_i}{\delta}}} \right) \right)
\]

\[
\ln L(x, a, b, q, \xi, \delta) = l = n \ln \left( \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \right) + n(a - 1) \ln \left( 1 - \left( \frac{2}{1 + e^{\frac{x_i}{\delta}}} \right) \right)
\]
+ n \ln q + nb \ln 2 + \sum_{i=1}^{n} x_i - \xi - (qb + 1) \sum_{i=1}^{n} 1 + e^{\frac{x_i - \xi}{\delta}} \tag{6.2}
\]

taking the derivatives with respect to the parameters and equating to zero to find the maximum likelihood estimates, we have
\[
\frac{\partial l}{\partial(\cdot)} = 0
\]
\[
\frac{\partial l}{\partial a} = -n \Gamma'(a + b) - n \Gamma(a) + n \ln \sum_{i=1}^{n} \left(1 - \left(\frac{2}{1 + e^{\frac{x_i - \xi}{\delta}}}\right)^q\right) \tag{6.3}
\]
\[
\frac{\partial l}{\partial b} = -n \frac{\Gamma'(a + b)}{\Gamma(a + b)} - n \frac{\Gamma'(b)}{\Gamma(b)} - q \sum_{i=1}^{n} 1 + e^{\frac{x_i - \xi}{\delta}} + nb \ln 2 \tag{6.4}
\]
\[
\frac{\partial l}{\partial q} = q^n 2^q (a - 1) \sum_{i=1}^{n} \left(\frac{1 - e^{\frac{x_i - \xi}{\delta}}}{1 + e^{\frac{x_i - \xi}{\delta}}}\right)^q - \sum_{i=1}^{n} x_i - \xi + (qb + 1) \sum_{i=1}^{n} e^{\frac{x_i - \xi}{\delta}} \tag{6.5}
\]
\[
\frac{\partial l}{\partial \xi} = -\frac{2q(a - 1)}{\delta} \sum_{i=1}^{n} \left(1 - \frac{2}{1 + e^{\frac{x_i - \xi}{\delta}}}\right)^{q-1} \frac{x_i - \xi}{1 + (q - 1) \frac{x_i - \xi}{\delta} + nq - 2} - \frac{n}{\delta} + \frac{nb + 1}{\delta} \frac{x_i - \xi}{\delta} \tag{6.6}
\]
\[
\frac{\partial l}{\partial \delta} = - \sum_{i=1}^{n} \left(1 - \frac{2}{1 + e^{\frac{x_i - \xi}{\delta}}}\right)^{q-1} \frac{x_i - \xi}{\delta} - \sum_{i=1}^{n} \left(1 - \frac{2}{1 + e^{\frac{x_i - \xi}{\delta}}}\right)^{q-1} \frac{x_i - \xi}{\delta} \tag{6.7}
\]
The equations 6.3, 6.4, 6.5, 6.6 and 6.7 are not simple analytical expressions. The maximum likelihood estimates of each parameter is obtained numerically from the non-linear equations using computer programs.

For interval estimation and test of hypothesis on the parameters ($\xi, \delta, q, a, b$), we obtain a 5x5 unit information matrix
\[
M = \begin{bmatrix}
M_{a,a} & M_{a,b} & M_{a,q} & M_{a,\xi} & M_{a,\delta} \\
M_{b,a} & M_{b,b} & M_{b,q} & M_{b,\xi} & M_{b,\delta} \\
M_{q,a} & M_{q,b} & M_{q,q} & M_{q,\xi} & M_{q,\delta} \\
M_{\xi,a} & M_{\xi,b} & M_{\xi,q} & M_{\xi,\xi} & M_{\xi,\delta} \\
M_{\delta,a} & M_{\delta,b} & M_{\delta,q} & M_{\delta,\xi} & M_{\delta,\delta}
\end{bmatrix}
\]

where the corresponding elements are given by taking the second derivatives with respect to the parameters

Under conditions that are fulfilled for parameters, the asymptotic distribution of $\sqrt{n} (\hat{\theta} - \theta)$ is $N_d(0, M(\hat{\theta})^{-1})$ distribution of $\theta$ can be used to construct approximate confidence intervals and confidence regions for the parameters and for the hazard and survival functions. The asymptotic normality is also useful for testing goodness of fit of the beta type I generalized half logistic distribution and for comparing this distribution with some of its special sub-models using one of these two well known asymptotically equivalent test statistics- namely, the likelihood ratio statistic and Wald statistic. An asymptotic confidence interval with significance level $\alpha$ for each parameter $\theta_i$ is given by
\[
ACI(\theta_i, 100(1 - \alpha)) = \hat{\theta} - z_{\alpha/2} \sqrt{M^{\hat{\theta},\theta}_i, \theta} + z_{\alpha/2} \sqrt{M^{\hat{\theta},\theta}_i}
\]
where $M^{\hat{\theta},\theta}_i$ is the $i^{th}$ diagonal element of $K_n(\hat{\theta})^{-1}$ for $i = 1, 2, 3, 4, 5$ and $z_{\alpha/2}$ is the quantile of the standard normal distribution.
6.1 Application to real data

In this section, two data sets were analyzed to reveal the better performance of the beta type I generalized half logistic distribution compared to some existing sub-models (distributions). The distributions are Jose and Manoharan (2016), Olapade[2], Balakrishnan[1].

The maximum likelihood estimates and their corresponding standard error of the model parameters are obtained. For models comparison, criteria like AIC, CAIC and BIC are used. However, the best distribution corresponds to the smaller values of AIC, BIC, CAIC. The data sets used in this section are remission data of bladder cancer patients and survival times of Guinea Pigs.

6.2 Application to Remission time data of Bladder cancer patient

Bladder cancer is a disease in which abnormal cells grow and replicate without control in the bladder. The most common type of bladder cancer is known as transitional cell carcinoma. Using the real data set revealing the remission times (in months) of 128 bladder cancer patients as reported by Lee and Wang (2003). Remission times (X): 0.08, 26.31, 5.32, 2.62, 9.02, 2.09, 13.29, 3.48, 2.75, 4.87, 7.39, 7.32, 6.94, 1.26, 8.66, 1.35, 7.87, 13.11, 23.63, 34.26, 0.20, 2.23, 14.76, 3.52, 0.81, 4.98, 10.06, 0.90, 5.17, 6.97, 16.62, 0.40, 4.26, 5.32, 2.26, 7.62, 3.57, 17.36, 5.06, 5.71, 2.02, 7.09, 5.41, 9.22, 13.80, 25.74, 10.34, 2.07, 11.79, 2.83, 0.50, 14.77, 3.82, 2.46, 3.64, 8.26, 32.15, 5.09, 2.64, 7.26, 14.83, 3.88, 9.47, 6.25, 14.24, 5.85, 25.82, 0.51, 7.93, 2.54, 5.49, 1.46, 3.70, 7.28, 17.14, 9.74, 7.66, 1.40, 3.25, 4.51, 8.53, 3.31, 8.37, 20.28, 22.69, 2.02, 2.69, 4.18, 6.93, 5.34, 12.07, 7.59, 10.66, 15.96, 36.66, 1.05, 8.65, 4.40, 21.73, 79.05, 10.75, 43.01, 1.19, 12.02, 6.54, 2.69, 4.23, 18.10, 5.41, 7.63, 17.12, 12.63, 12.03, 6.76, 46.12, 3.36, 4.33, 11.25, 2.87, 5.62, 11.64, 3.02, 4.34, 4.50, 11.98, 19.13, 1.76, 3.36.

From table 6.2, it was shown that the Beta type I generalized half logistic distribution has the lowest values in terms of AIC, BIC and CAIC and therefore concluded to give the best fit when compared to other distributions.

Fig. 8. Graph of Remission Time of Bladder Cancer Patients
Table 1. Table displaying results of analysis of Remission Time of Bladder Cancer Patients

| Model | Parameter | Estimate | L     | AIC      | BIC      | CAIC     |
|-------|-----------|----------|-------|----------|----------|----------|
| BTIGHL| $\xi$     | 0.414    | 411.93| 833.86   | 832.34   | 824.96   |
|       | $\delta$  | 0.358    |       |          |          |          |
|       | $q$       | 2.810    |       |          |          |          |
|       | $\alpha$  | 1.084    |       |          |          |          |
|       | $b$       | 1.498    |       |          |          |          |
| BHL   | $\alpha$  | 0.00581  | 414.19| 834.38   | 842.94   | 834.2    |
|       | $\alpha$  | 1.1110   |       |          |          |          |
|       | $b$       | 39.3700  |       |          |          |          |
| THLBx | $\alpha$  | 0.0016   | 415.48| 836.96   | 845.52   | 837.15   |
|       | $\theta$  | 0.4373   |       |          |          |          |
|       | $\lambda$ | 81.6600  |       |          |          |          |
| GHL   | $\alpha$  | 0.1440   | 416.64| 837.27   | 842.9755 | 837.37   |
|       | $\alpha$  | 0.9527   |       |          |          |          |

6.3 Application to survival time of Guinea Pig

In further assessing the performance of the distribution, we applied it to a data that represents the survival time (in days) of 72 Guinea Pigs. These pigs were infected with virulent tubercle bacilli. We obtained this data set from the work of Usman, Haq and Talib (2017). The data are as follows: 1.96, 1.36, 1.97, 0.1, 1.63, 2.02, 2.13, 0.33, 1.71, 1.39, 1.44, 1.63, 0.44, 1.68, 0.56, 1.16, 1.76, 0.59, 1.72, 1.60, 3.61, 2.31, 0.72, 1.13, 1.46, 1.15, 0.74, 1.53, 0.77, 1.59, 0.92, 0.93, 0.96, 2.54, 1.24, 1.83, 2.22, 1.00, 1.20, 1.95, 1.00, 1.21, 1.02, 1.12, 3.27, 5.55, 2.93, 1.22, 2.51, 1.05, 1.22, 4.02, 2.15, 2.30, 1.07, 2.54, 2.45, 0.7, 2.78, 0.08, 2.53, 2.40, 1.08, 4.32, 3.42, 2.16, 1.3, 1.08, 3.47, 1.34, 1.09, 4.58.

Table 2. Table displaying results of analysis of Survival Times of Guinea Pigs

| Model | Parameter | Estimate | L     | AIC      | BIC      | CAIC     |
|-------|-----------|----------|-------|----------|----------|----------|
| BTIGHL| $\xi$     | 3.454    | 148.58| 306.34   | 306.97   | 304.64   |
|       | $\delta$  | 1.253    |       |          |          |          |
|       | $q$       | 0.845    |       |          |          |          |
|       | $\alpha$  | 0.582    |       |          |          |          |
|       | $b$       | 0.274    |       |          |          |          |
| ThHL  | $\alpha$  | 1.091    | 151.321| 308.642  | 308.995  | 306.357  |
|       | $\theta$  | 3.9829   |       |          |          |          |
|       | $\lambda$ | 0.9361   |       |          |          |          |
| HL    | $\alpha$  | 1.2152   | 157.341| 314.682  | 315.035  | 316.539  |
|       | $\alpha$  | 1.1261   |       |          |          |          |

Analyzing the data, From table 6.3, it was shown that the Beta type I generalized half logistic distribution has the lowest values in terms of AIC, BIC and CAIC and therefore concluded to give the best fit when compared to other distributions.
Conclusions

The paper studies some general properties of a new distribution called beta type I generalized half logistic distribution. The distribution is a generalization of the type I generalized half logistic distribution, and includes the beta half logistic, exponentiated half logistic, Lehmann type II generalized half logistic and the half logistic distributions as sub models. Applications of the models to real-life data have been cited and shown to give considerable good fits.

Competing Interests

Authors have declared that no competing interests exist.

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