Quasinormal modes for Weyl neutrino field in R-N black holes

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March 24, 2022

Abstract

We employ WKB approximation up to the third order to determine the low-lying quasinormal modes for Weyl neutrino field in R-N black holes, which are the most relevant to the evolution of the field around a black hole in the intermediate stage. It is showed that the quasinormal mode frequencies for Weyl neutrino field in R-N black holes are different from those in Schwarzschild black holes owning to the charge-induced additional gravitation, and the variations of the quasinormal mode frequencies for Weyl neutrino field are similar to those for integral spin fields in R-N black holes.

1 Introduction

It is well known that quasinormal modes (QNM), which dominate in the intermediate stage of field evolution around a black hole, play a two-fold role in classical black hole physics: One is that the existence of QNM ensures the stability of black holes against small perturbations, because the small perturbations will damp following QNM. The other is that a black hole can be identified with its QNM; for they are "footprints" of black holes, since their frequencies only depend on the fundamental parameters of black holes such as mass, charge and angular momentum[1,2,3].
The latter is quite significant from the viewpoint of astrophysical observation because we could determine the fundamental parameters of black holes by detecting their QNM. Recently QNM have also shed light on the quantized area of black holes in quantum gravity such as loop quantum gravity [4, 5, 6, 7, 8, 9, 10], which leads to a renewed interest in QNM of black holes.

There has been much work done on calculation of QNM for different fields around various black holes [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]; however, as far as Weyl neutrino field in charged black holes, namely R-N black holes, is concerned, no work has been done. Nevertheless, with the development of the technology in neutrino detection, the investigation of this case will acquire a practical application in the astrophysical observation: Neutrino fluxes from a star take a more and more important position in our investigating the interior information of the star because neutrino only takes part in the weak interaction except the universal gravitation [25, 26]; especially when a sufficiently massive charged star collapse to a black hole [27], the neutrino fluxes can provide us with the information about the fundamental parameters of the formed black hole due to their QNM behaviors. In addition, this work is of interest itself: In spite of the absence of charge-charge electromagnetic interaction between neutrino and a R-N black hole, the charge of the R-N black hole will influence the evolution of neutrino by way of the charge-induced gravitation, which will cause the QNM frequencies of Weyl neutrino field to deviate from those in Schwarzschild black holes [15]. The present paper serves to work on QNM for Weyl neutrino field in R-N black holes.

In next section, we consider Weyl neutrino field equation in R-N black holes in terms of N-P formalism and its reduction into a pair of one-dimensional equations with supersymmetric partnership. In Section 3, we calculate the low-lying QNM frequencies for Weyl neutrino field using WKB approximation. The conclusion and discussion are presented in Section 4.

2 Weyl neutrino field equation in R-N black holes

Weyl neutrino field equation in curved spacetime [28]

\[ \nabla_{A'} A^A \psi^A = 0 \]  \hspace{1cm} (1)

can be written as [1]

\[ (D + \rho - \varepsilon)\psi^1 + (\delta + \alpha - \pi)\psi^2 = 0 \]  \hspace{1cm} (2)

\[ (\delta + \tau - \beta)\psi^1 + (\Delta + \gamma - \mu)\psi^2 = 0 \]  \hspace{1cm} (3)

With the usual form of the R-N black holes with parameters \( M \) and \( Q \)

\[ ds^2 = \frac{\Delta}{r^2} dt^2 - \frac{r^2}{\Delta} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \]  \hspace{1cm} (4)
where $\Delta = r^2 - 2Mr + Q^2$, if construct the null tetrad-frame in N-P formalism as follows*

$$l^\mu = (l^r, l^\theta, l^\varphi) = \frac{1}{\Delta}(r^2, \Delta, 0, 0)$$

$$n^\mu = (n^r, n^\theta, n^\varphi) = \frac{1}{2r^2}(r^2, -\Delta, 0, 0)$$

$$m^\mu = (m^r, m^\theta, m^\varphi) = \frac{1}{\sqrt{2r}}(0, 0, 1, -i \csc \theta)$$

then non-zero spin coefficients satisfy

$$\rho = \frac{1}{r}, \alpha = -\beta = \cot \theta \frac{\Delta}{2r^3}, \gamma - \mu = \frac{M - r}{2r^2}$$

Therefore, if let

$$\psi_1 = \frac{1}{r} R_-(r) S_- (\theta) e^{-i \omega t} e^{im \varphi}, \psi_2 = \frac{1}{\sqrt{\Delta}} R_+(r) S_+ (\theta) e^{-i \omega t} e^{im \varphi},$$

substitute (5) to (3), and use

$$\frac{d}{dr_*} = \frac{\Delta^*}{\Delta},$$

we have

$$(-i \omega + \frac{d}{dr_*}) R_-(r) = \lambda_1 \frac{\sqrt{\Delta}}{r^2} R_+(r), \frac{1}{\sqrt{2}} (m \csc \theta + \frac{1}{2} \cot \theta + \frac{d}{d\theta}) S_+(\theta) = -\lambda_1 S_-(\theta)$$

$$\quad (i \omega + \frac{d}{dr_*}) R_+(r) = \lambda_2 \frac{\sqrt{\Delta}}{r^2} R_-(r), \sqrt{2} (m \csc \theta - \frac{1}{2} \cot \theta - \frac{d}{d\theta}) S_-(\theta) = -\lambda_2 S_+(\theta)$$

where $\lambda_1, \lambda_2$ are separate constants. Without loss of generalization and for the sake of convenience, we let $\lambda_1 = \lambda_2 = \lambda$. The explicit expression of $\lambda$ can be obtained by combining the angular equations in (9) and (10)

$$\left[ \frac{1}{\sin \theta} \frac{d}{d\theta} (\sin \theta \frac{d}{d\theta}) - \left( \frac{m^2 + \frac{1}{4} + m \cos \theta}{\sin^2 \theta} \right) \right] S_+(\theta) = -\left( \lambda^2 - \frac{1}{4} \right) S_+(\theta)$$

$$\left[ \frac{1}{\sin \theta} \frac{d}{d\theta} (\sin \theta \frac{d}{d\theta}) - \left( \frac{m^2 + \frac{1}{4} - m \cos \theta}{\sin^2 \theta} \right) \right] S_-(\theta) = -\left( \lambda^2 - \frac{1}{4} \right) S_-(\theta)$$

The equations (11) and (12) have as their solutions the spin-weighted spherical harmonics

$$S_{\pm}(\theta) = \pm \frac{1}{2} Y_{l}^{m}(\theta), \lambda^2 = (l + \frac{1}{2})^2$$

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*In our notation, the directional derivatives are the complex conjugate of those in [1]; and the spin coefficients are the minus of those in [1].

$^{\dagger}r_*$ is the tortoise coordinate, defined in [1].
where \( l = \frac{2k-1}{2} \) with \( k \) positive integers, and \( -l \leq m \leq l \). Without loss of generalization, we take \( \lambda = k \) in the following discussion.

Now setting \( Z_\pm(r) = R_+(r) \pm R_-(r) \), and combining the radial equations in (9) and (10), we have
\[
\left( \frac{d}{dr_*} - \lambda \frac{\sqrt{\Delta}}{r^2} \right) Z_+(r) = -i\omega Z_-(r) \tag{14}
\]
\[
\left( \frac{d}{dr_*} + \lambda \frac{\sqrt{\Delta}}{r^2} \right) Z_-(r) = -i\omega Z_+(r) \tag{15}
\]
Finally we readily obtain a pair of one-dimensional equations
\[
\left( \frac{d^2}{dr_*^2} + \omega^2 \right) Z_\pm = V Z_\pm \tag{16}
\]
where
\[
V_\pm = \lambda^2 \frac{\Delta}{r^4} \pm \lambda \frac{d}{dr_*} \left( \frac{\sqrt{\Delta}}{r^2} \right) \tag{17}
\]
When \( Q = 0 \), it is easy to show that the above effective potentials \( V_\pm \) reduce to those for Schwarzschild black holes in [15].

3 QNM for Weyl neutrino field

In this section, we shall only concentrate on calculating the low-lying QNM for Weyl neutrino field using WKB approximation, since the low-lying modes dominate in the evolution of Weyl neutrino field in the intermediate stage.

Note that the equations with respect to \( Z_+ \) and \( Z_- \) in (10) possess the same spectra of QNM, because \( Z_+ \) and \( Z_- \) are related as supersymmetric partnership through (14) and (15); therefore to calculate the spectra of QNM, we need only focus on the equation with \( Z_+ \) in (16)
\[
\left( \frac{d^2}{dr_*^2} + \omega^2 \right) Z = V Z \tag{18}
\]
where
\[
V = \lambda^2 \frac{\Delta}{r^4} + \lambda \frac{d}{dr_*} \left( \frac{\sqrt{\Delta}}{r^2} \right) \tag{19}
\]
here we have written \( V, Z \) as \( V_+, Z_+ \). Furthermore, according to (14) and (15), we find that the QNM frequencies with \( \text{Re}(\omega) < 0 \) are related to those with \( \text{Re}(\omega) > 0 \) by a reflection via the
imaginary axis in the complex plane. Thus we need only calculate the QNM frequencies with \( \text{Re}(\omega) > 0 \).

As is shown in Fig. 1 and Fig. 2, the above effective potential \( V \to 0 \) with \( r_* \to -\infty (r \to r_+) \) or \( r_* \to \infty (r \to \infty) \); in addition, this effective potential \( V \) rises to its maximum at \( r_* (r_{\text{max}}) \). Hence the QNM can be calculated by WKB approach, with the boundary condition of purely "outgoing" waves. The formula for the spectra of QNM in WKB approximation, carried to the third order beyond the eikonal approximation, is given by

\[
\omega^2 = [V_0 + (-2V_0'')^{1/2} \Lambda] - i(n + \frac{1}{2})[(1 + \Omega)]
\]

where

\[
\Lambda = \frac{1}{(-2V'')^{1/2}} \left\{ \frac{1}{8} \left( \frac{V_0^{(4)}}{V_0'} \right) \left( \frac{1}{4} + \alpha^2 \right) - \frac{1}{288} \left( \frac{V_0'''}{V_0''} \right)^2 (7 + 60\alpha^2) \right\}
\]

\[
\Omega = \frac{1}{(-2V'')^{1/2}} \left\{ \frac{5}{6912} \left( \frac{V_0''}{V_0'} \right)^4 (77 + 188\alpha^2) - \frac{1}{384} \left( \frac{V_0'''^2 V_0^{(4)}}{V_0'^3} \right) (51 + 100\alpha^2) \right.
\]

\[
+ \frac{1}{2304} \left( \frac{V_0^{(4)}}{V_0''} \right)^2 (67 + 68\alpha^2) + \frac{1}{288} \left( \frac{V_0'' \cdot V_0^{(5)}}{V_0''^2} \right) (19 + 28\alpha^2) \]

\[
- \frac{1}{288} \left( \frac{V_0^{(6)}}{V_0''} \right) (5 + 4\alpha^2) \right\}
\]

here

\[
\alpha = n + \frac{1}{2}, \quad n = \begin{cases} 0, 1, 2, \cdots, \text{Re}(\omega) > 0 \\ -1, -2, -3, \cdots, \text{Re}(\omega) < 0 \end{cases}
\]

\[
V_0^{(n)} = \left. \frac{d^n V}{dr_*^n} \right|_{r_*=r_{\text{max}}}
\]

According to this analytic approximation formula, we find that the real parts of the frequencies \( \text{Re}(\omega) \) are mainly determined by the maximum of the effective potential \( V \); the more the maximum of the effective potential is, the more the real parts of the frequencies are; the imaginary parts \( \text{Im}(\omega) \) mainly by the mode number \( n \); with the mode number \( n \) on the increase, the imaginary parts decrease fastly. This last point indicates that the QNM with higher mode number \( n \) will

\[^{\dagger}\text{Here it is worth pointing that the QNM spectra for anti-neutrino field in any black hole is the same as that for neutrino field because their solutions are related by the complex conjugation.}^{28}\]
Figure 1: Variation of the effective potential $V$ with $Q$ which increases from right to left (0.0, 0.1, ..., 0.9) in the case of $M = 1, \lambda = 1$.

decay faster than the low-lying modes. Hence, the low-lying QNM are the most relevant to the description of the evolution of fields around black holes. This approximation formula has been used extensively in various cases and is good enough for our purpose: comparing with numerical results, this approximation has been found to be accurate up to around 1% for both the real and the imaginary parts of the frequencies for low-lying modes with $n < \lambda$ \cite{15}.

To proceed, it is convenient to take the mass of the R-N black hole considered as a unit of mass in geometrized units. Later plugging the effective potential $V$ in (19) into the approximation formula above, we obtain the complex QNM frequencies for Weyl neutrino field in R-N black holes. The values for $0 \leq n < \lambda \leq 5$ are listed in Table 1, Table 2, and plotted in Fig. 3, Fig. 4 correspondingly.

Hereby we find the main results as follow.

1. QNM for Weyl neutrino field in Charged black holes show their deviations from Schwarzschild
Figure 2: Variation of the effective potential $V$ with $Q$ which increases from right to left (0.90, 0.93, 0.96, 0.99) in the case of $M = 1, \lambda = 1$. 
| M=1       | Q=0.0     | Q=0.3     | Q=0.6     |
|-----------|-----------|-----------|-----------|
| $\lambda = 1$, $n = 0$ | 0.1765 − 0.1001$i | 0.1799 − 0.1005$i | 0.1919 − 0.1012$i |
| $\lambda = 2$, $n = 0$ | 0.3786 − 0.0965$i | 0.3847 − 0.0970$i | 0.4059 − 0.0981$i |
| $\lambda = 2$, $n = 1$ | 0.3536 − 0.0987$i | 0.3602 − 0.2999$i | 0.3834 − 0.3023$i |
| $\lambda = 3$, $n = 0$ | 0.5737 − 0.0963$i | 0.5827 − 0.0968$i | 0.6140 − 0.0979$i |
| $\lambda = 3$, $n = 1$ | 0.5562 − 0.2930$i | 0.5655 − 0.2943$i | 0.5982 − 0.2973$i |
| $\lambda = 3$, $n = 2$ | 0.5273 − 0.4972$i | 0.5372 − 0.4992$i | 0.5719 − 0.5037$i |
| $\lambda = 4$, $n = 0$ | 0.7672 − 0.0963$i | 0.7792 − 0.0967$i | 0.8208 − 0.0979$i |
| $\lambda = 4$, $n = 1$ | 0.7540 − 0.2910$i | 0.7662 − 0.2924$i | 0.8088 − 0.2956$i |
| $\lambda = 4$, $n = 2$ | 0.7304 − 0.4909$i | 0.7431 − 0.4930$i | 0.7874 − 0.4980$i |
| $\lambda = 4$, $n = 3$ | 0.6999 − 0.6957$i | 0.7131 − 0.6986$i | 0.7595 − 0.7051$i |
| $\lambda = 5$, $n = 0$ | 0.9602 − 0.0963$i | 0.9752 − 0.0967$i | 1.0272 − 0.0979$i |
| $\lambda = 5$, $n = 1$ | 0.9496 − 0.2902$i | 0.9647 − 0.2916$i | 1.0175 − 0.2949$i |
| $\lambda = 5$, $n = 2$ | 0.9300 − 0.4876$i | 0.9455 − 0.4899$i | 0.9996 − 0.4951$i |
| $\lambda = 5$, $n = 3$ | 0.9036 − 0.6892$i | 0.9196 − 0.6923$i | 0.9755 − 0.6992$i |
| $\lambda = 5$, $n = 4$ | 0.8721 − 0.8944$i | 0.8886 − 0.8982$i | 0.9468 − 0.9066$i |

| M=1       | Q=0.7     | Q=0.8     | Q=0.9     |
|-----------|-----------|-----------|-----------|
| $\lambda = 1$, $n = 0$ | 0.1990 − 0.1013$i | 0.2083 − 0.1006$i | 0.2207 − 0.0979$i |
| $\lambda = 2$, $n = 0$ | 0.4182 − 0.0984$i | 0.4348 − 0.0981$i | 0.4581 − 0.0963$i |
| $\lambda = 2$, $n = 1$ | 0.3969 − 0.3025$i | 0.4150 − 0.3009$i | 0.4396 − 0.2944$i |
| $\lambda = 3$, $n = 0$ | 0.6322 − 0.0982$i | 0.6568 − 0.0980$i | 0.6915 − 0.0963$i |
| $\lambda = 3$, $n = 1$ | 0.6172 − 0.2979$i | 0.6428 − 0.2969$i | 0.6787 − 0.2913$i |
| $\lambda = 3$, $n = 2$ | 0.5922 − 0.5042$i | 0.6194 − 0.5019$i | 0.6566 − 0.4914$i |
| $\lambda = 4$, $n = 0$ | 0.8450 − 0.0981$i | 0.8776 − 0.0980$i | 0.9239 − 0.0963$i |
| $\lambda = 4$, $n = 1$ | 0.8336 − 0.2963$i | 0.8670 − 0.2955$i | 0.9142 − 0.2902$i |
| $\lambda = 4$, $n = 2$ | 0.8132 − 0.4988$i | 0.8480 − 0.4971$i | 0.8966 − 0.4875$i |
| $\lambda = 4$, $n = 3$ | 0.7866 − 0.7059$i | 0.8232 − 0.7029$i | 0.8731 − 0.6884$i |
| $\lambda = 5$, $n = 0$ | 1.0573 − 0.0981$i | 1.0981 − 0.0979$i | 1.1559 − 0.0963$i |
| $\lambda = 5$, $n = 1$ | 1.0481 − 0.2956$i | 1.0896 − 0.2949$i | 1.1481 − 0.2897$i |
| $\lambda = 5$, $n = 2$ | 1.0311 − 0.4961$i | 1.0738 − 0.4946$i | 1.1336 − 0.4854$i |
| $\lambda = 5$, $n = 3$ | 1.0082 − 0.7003$i | 1.0523 − 0.6977$i | 1.1135 − 0.6841$i |
| $\lambda = 5$, $n = 4$ | 0.9808 − 0.9078$i | 1.0266 − 0.9040$i | 1.0892 − 0.8855$i |

Table 1: QNM frequencies for Weyl neutrino field in R-N black holes.
Figure 3: QNM frequencies for Weyl neutrino field in R-N black holes.
Figure 4: QNM frequencies for Weyl neutrino field in near extremal R-N black holes.
Table 2: QNM frequencies for Weyl neutrino field in near extremal R-N black holes.

| M=1 | Q=0.90 | Q=0.93 | Q=0.96 | Q=0.99 |
|-----|--------|--------|--------|--------|
| $\lambda = 1, n = 0$ | 0.2207 $- 0.0979i$ | 0.2250 $- 0.0962i$ | 0.2294 $- 0.0936i$ | 0.2336 $- 0.0902i$ |
| $\lambda = 2, n = 0$ | 0.4581 $- 0.0963i$ | 0.4670 $- 0.0950i$ | 0.4773 $- 0.0930i$ | 0.4891 $- 0.0898i$ |
| $\lambda = 2, n = 1$ | 0.4396 $- 0.2944i$ | 0.4485 $- 0.2901i$ | 0.4579 $- 0.2836i$ | 0.4668 $- 0.2740i$ |
| $\lambda = 3, n = 0$ | 0.6915 $- 0.0963i$ | 0.7050 $- 0.0951i$ | 0.7206 $- 0.0931i$ | 0.7390 $- 0.0899i$ |
| $\lambda = 3, n = 1$ | 0.6787 $- 0.2913i$ | 0.6923 $- 0.2873i$ | 0.7076 $- 0.2812i$ | 0.7242 $- 0.2715i$ |
| $\lambda = 3, n = 2$ | 0.6566 $- 0.4914i$ | 0.6701 $- 0.4844i$ | 0.6841 $- 0.4737i$ | 0.6971 $- 0.4578i$ |
| $\lambda = 4, n = 0$ | 0.9239 $- 0.0963i$ | 0.9419 $- 0.0951i$ | 0.9628 $- 0.0931i$ | 0.9876 $- 0.0899i$ |
| $\lambda = 4, n = 1$ | 0.9142 $- 0.2902i$ | 0.9324 $- 0.2864i$ | 0.9531 $- 0.2805i$ | 0.9767 $- 0.2708i$ |
| $\lambda = 4, n = 2$ | 0.8966 $- 0.4875i$ | 0.9148 $- 0.4808i$ | 0.9349 $- 0.4705i$ | 0.9558 $- 0.4543i$ |
| $\lambda = 4, n = 3$ | 0.8731 $- 0.6884i$ | 0.8910 $- 0.6786i$ | 0.9097 $- 0.6638i$ | 0.9265 $- 0.6414i$ |
| $\lambda = 5, n = 0$ | 1.1559 $- 0.0963i$ | 1.1784 $- 0.0951i$ | 1.2046 $- 0.0932i$ | 1.2358 $- 0.0899i$ |
| $\lambda = 5, n = 1$ | 1.1481 $- 0.2897i$ | 1.1708 $- 0.2860i$ | 1.1969 $- 0.2802i$ | 1.2271 $- 0.2705i$ |
| $\lambda = 5, n = 2$ | 1.1336 $- 0.4854i$ | 1.1564 $- 0.4790i$ | 1.1821 $- 0.4689i$ | 1.2102 $- 0.4527i$ |
| $\lambda = 5, n = 3$ | 1.1135 $- 0.6841i$ | 1.1363 $- 0.6746i$ | 1.1611 $- 0.6601i$ | 1.1860 $- 0.6375i$ |
| $\lambda = 5, n = 4$ | 1.0892 $- 0.8855i$ | 1.1117 $- 0.8729i$ | 1.1349 $- 0.8538i$ | 1.1555 $- 0.8251i$ |

For $\lambda$ and $n$ fixed, with the charge $Q$ on the increase, the real parts $\text{Re}(\omega)$ of the frequencies increase monotonously, which agrees with Fig. 1 and Fig. 2. When the charge $Q$ increases, the maximum of the effective potential $V$ also increases. In comparison with the variation of the real parts of the frequencies, their imaginary parts $\text{Im}(\omega)$ seem to change less with $Q$: decrease firstly, later fall to a minimum at the vicinity of $Q = 0.7$, and increase finally. This result shows that the charge-induced effective gravitational potential has an influence on the QNM for Weyl neutrino field, although neutrino does not take part in the charge-charge electromagnetic interaction.

II. In addition, as for a definite $Q$, the real parts $\text{Re}(\omega)$ of the frequencies increase with $\lambda$ for a fixed $n$, and the $\text{Im}(\omega)$ almost remain constant; on the other hand, the $\text{Re}(\omega)$ decrease slowly as the mode number $n$ increases for the same $\lambda$, at the same time, the imaginary parts $\text{Im}(\omega)$ decrease with $n$ fastly, which means the low-lying QNM dominate in the intermediate evolution stage of the field.

III. Despite its spin half, the above variations of QNM frequencies for Weyl neutrino field are the same as those for integral spin fields such as scalar, electromagnetic and gravitational fields in R-N black holes.\[11, 13\]

\(^{3}\)This result is physically reasonable, because the larger $\lambda$ is, the larger the corresponding effective centrifugal potential is.
4 Conclusion and discussion

We have calculated the low-lying QNM frequencies for Weyl neutrino field in R-N black holes using WKB approximation up to the third order. The result shows that the charge-induced effective gravitation makes QNM for Weyl neutrino field in Charged black holes differ from Schwarzchild black holes, and the variations of QNM frequencies for Weyl neutrino field are similar to those for integral spin fields in R-N black holes.

We conclude with two interesting problems worthy of further investigation: One is the calculation of the highly damped asymptotic QNM for Weyl neutrino field in R-N black holes, which is outside our consideration but the most relevant to the quantum charged black holes. The other is the search of low-lying QNM behaviors for charged fermion field in R-N black holes as charged scalar field\[^{[B]}\], where the QNM will be influenced by not only the charge-induced gravitation but also the charge-charge electromagnetic force. One expected result is that the real parts of the QNM for the fermion field with the same charge as that of a R-N black hole will be larger than the neutral fermion field, due to the repulsive electromagnetic interaction, which leads to the rise of the maximum of the effective potential.

Acknowledgement

It is our pleasure to acknowledge Prof. S. Pei for the discussion of neutrino astrophysics. H. Zhang would like to thank Prof. H. T. Cho for many helpful arguments on this work. In addition, this work is supported in part by NSFC(grant 10205002).

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