BIG BANG NUCLEOSYNTHESIS AND THE CONSISTENCY BETWEEN THEORY AND THE OBSERVATIONS OF D, $^3$He, $^4$He, AND $^7$Li

Keith A. Olive

School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA

Abstract

The current status of big bang nucleosynthesis is reviewed. Particular attention is given to the degree at which the theory is consistent with the observation of the light element abundances.

The observational information at hand on the abundances of the four light element isotopes: D, $^3$He, $^4$He, and $^7$Li can be characterized as being either reasonably certain (for $^4$He and $^7$Li); getting certain (for D); or uncertain (for $^3$He). In the cases of $^4$He and $^7$Li, there is a growing wealth of data from extragalactic HII regions¹ on $^4$He and from the surfaces of old Population II halo stars² on $^7$Li. Indeed, we are rapidly approaching the point where our uncertainty in the abundances of these isotopes is dominated by systematic rather than statistical uncertainties. For D, though we have good solar and interstellar medium (ISM) data³, the connection to a primordial abundance through galactic chemical evolution introduces substantial uncertainties. However, recent observations of D in quasar absorption systems are beginning to to yield a more coherent picture for what may be the true primordial abundance of D. In the uncertain category, I would place $^3$He. As for D, we have data in the ISM and there are also $^3$He abundances in planetary nebulae, however in this case, not only our we hampered by our lack of knowledge concerning galactic chemical evolution, but also by our uncertainties in the production of $^3$He in low mass stars.

Overall, there is certainly a broad agreement between the predicted abundances of the light element isotopes from big bang nucleosynthesis⁴ and the abundances inferred from observations which span nearly ten orders of magnitude. Indeed, the standard model theory really only contains one free parameter, namely, the baryon to photon ratio, $\eta$. Agreement between theory
and the observation of all of the four isotopes occurs when $\eta_{10} = 10^{10}\eta$ is in the range, 2.8 – 4. However, at closer level, the above range on $\eta$ is slightly high with regard to what is preferred by $^4$He, and may be too low for D and $^3$He when chemical evolution is combined with the observations. In fact, the lower bound to $\eta$ comes directly from an upper bound to the combined abundance of D and $^3$He and depends on the degree to which $^3$He survives in stellar evolution. Recent data on D and $^3$He may be yielding an inconsistent picture. As argued below, I believe that the most likely source of the problem is our treatment of $^3$He due to the considerable uncertainties in chemical and stellar evolution concerning this isotope. Furthermore, I will show that $^4$He and $^7$Li (abundances that we know best) are already sufficient in constraining the theory and that the abundance of D/H predicted from $^4$He and $^7$Li, appears to be consistent with the recent measurements of D/H in quasar absorption systems, though this requires that some of our beliefs concerning $^3$He need rethinking.

Before moving to the comparison of theory and observation, it will be useful to first briefly review the observational status of the four isotopes considered. More detail on each can be found in refs. 1–3).

There is now a good collection of abundance information on the $^4$He mass fraction, $Y$, O/H, and N/H in over 50 extragalactic HII regions$^{5–7)}$. The observation of the heavy elements is important as the helium mass fraction observed in these HII regions has been augmented by some stellar processing, the degree to which is given by the oxygen and nitrogen abundances. In an extensive study based on the data in refs. 5) and 6), it was found$^{8)}$ that the data is well represented by a linear correlation for $Y$ vs. O/H and $Y$ vs. N/H. It is then expected that the primordial abundance of $^4$He can be determined from the intercept of that relation. The overall result of that analysis indicated a primordial mass fraction, $Y_p = 0.232 \pm 0.003$. In ref. 9), the stability of this fit was verified by a Monte-Carlo analysis showing that the fits were not overly sensitive to any particular HII region. In addition, the data from ref. 7) was also included, yielding a $^4$He mass fraction$^9)$

$$Y_p = 0.234 \pm 0.003 \pm 0.005$$

The second uncertainty is an estimate of the systematic uncertainty in the abundance determination. Though the systematic uncertainty may be somewhat larger$^{10)}$, its precise value will be superfluous to the discussion below.

As I have said above, I also believe that the $^7$Li abundance is reasonably well known. In old, hot, population-II stars, $^7$Li is found to have a very nearly uniform abundance. For stars with a surface temperature $T > 5500$ K and a metallicity less than about 1/20th solar (so that effects such as stellar convection may not be important), the abundances show little or no dispersion beyond that which is consistent with the errors of individual measurements. Indeed, as detailed by Spite$^2)$, much of the work concerning $^7$Li has to do with the presence or absence of dispersion and whether or not there is in fact some tiny slope to a $[\text{Li}] = \log^7\text{Li}/\text{H} + 12$ vs. T or $[\text{Li}]$ vs. $[\text{Fe}/\text{H}]$ relationship. There is $^7$Li data from nearly 100 halo stars, from a variety of sources. I will use the value given in ref. 11) as the best estimate for the mean $^7$Li abundance and its statistical uncertainty in halo stars

$$\frac{\text{Li}}{\text{H}} = (1.6 \pm 0.1^{+0.4+1.6}_{-0.3-0.5}) \times 10^{-10}$$

The first error is statistical, and the second is a systematic uncertainty that covers the range of abundances derived by various methods. The third set of errors in Eq. (2) accounts for the
possibility that as much as half of the primordial $^7$Li has been destroyed in stars, and that as much as 30% of the observed $^7$Li may have been produced in cosmic ray collisions rather than in the Big Bang. Observations of $^6$Li, Be, and B help constrain the degree to which these effects play a role\textsuperscript{12}. For $^7$Li, the uncertainties are clearly dominated by systematic effects.

Turning to D/H, we have three basic types of abundance information: 1) ISM data; 2) solar system information; and perhaps 3) a primordial abundance from quasar absorption systems. The best measurement for ISM D/H is\textsuperscript{13}

$$(D/H)_{\text{ISM}} = 1.60 \pm 0.09^{+0.05}_{-0.10} \times 10^{-5}$$

However, it is becoming apparent that this value may not be universal (or galactic as the case may be) and that there may be some real dispersion of D/H in the ISM\textsuperscript{3}. The solar abundance of D/H is inferred from two distinct measurements of $^3$He. The solar wind measurements of $^3$He as well as the low temperature components of step-wise heating measurements of $^3$He in meteorites yield the presolar $(D + ^3\text{He})/H$ ratio as D was efficiently burned to $^3$He in the Sun’s pre-main-sequence phase. These measurements indicate that\textsuperscript{14,15}

$$\left( \frac{D + ^3\text{He}}{H} \right)_{\odot} = (4.1 \pm 0.6 \pm 1.4) \times 10^{-5}$$

The high temperature components in meteorites are believed to yield the true solar $^3$He/H ratio of\textsuperscript{14,15}

$$\left( \frac{^3\text{He}}{H} \right)_{\odot} = (1.5 \pm 0.2 \pm 0.3) \times 10^{-5}$$

The difference between these two abundances reveals the presolar D/H ratio, giving,

$$(D/H)_{\odot} \approx (2.6 \pm 0.6 \pm 1.4) \times 10^{-5}$$

Finally, there are the recent measurements of D/H in quasar absorption systems. The first of these measurements\textsuperscript{16} indicated a rather high D/H ratio, $D/H \approx 1.9 - 2.5 \times 10^{-4}$. However, a reported measurement\textsuperscript{17} of D/H in a second system seemed to show a very different abundance, $D/H \approx 1 - 2 \times 10^{-5}$. Most recently, a new observation\textsuperscript{18} of the high D/H absorber was made resolving it into two components. The weighted average of these two components indicates that $D/H = (1.9 \pm 0.4) \times 10^{-4}$ in these systems, again calling for a high primordial D/H. It is probably premature to use this value as the primordial D/H abundance in an analysis of big bang nucleosynthesis, but it is certainly encouraging that future observations may soon yield a firm value for D/H. It is however important to note that there does seem to be a trend that over the history of the Galaxy, the D/H ratio is decreasing, something we expect from galactic chemical evolution. Of course the total amount of deuterium astration that has occurred is still uncertain, and model dependent.

There are also several types of $^3$He measurements. As noted above, meteoritic extractions yield a presolar value for $^3$He/H as given in Eq. \textsuperscript{3}. In addition, there are several ISM measurements of $^3$He in galactic HII regions\textsuperscript{19} which also show a wide dispersion

$$\left( \frac{^3\text{He}}{H} \right)_{\text{ISM}} \simeq 1 - 5 \times 10^{-5}$$

Finally there are observations of $^3$He in planetary nebulae\textsuperscript{20} which show a very high $^3$He abundance of $^3$He/H $\sim 10^{-3}$. 
Each of the light element isotopes can be made consistent with theory for a specific range in \( \eta \). Overall consistency of course requires that the range in \( \eta \) agree among all four light elements. \(^3\)He (together with D) has stood out in its importance for BBN, because it provided a (relatively large) lower limit for the baryon-to-photon ratio, \( \eta_{10} > 2.8 \). This limit for a long time was seen to be essential because it provided the only means for bounding \( \eta \) from below and in effect allows one to set an upper limit on the number of neutrino flavors, \( N_\nu \), as well as other constraints on particle physics properties. That is, the upper bound to \( N_\nu \) is strongly dependent on the lower bound to \( \eta \). This is easy to see: for lower \( \eta \), the \(^4\)He abundance drops, allowing for a larger \( N_\nu \), which would raise the \(^4\)He abundance. However, for \( \eta < 4 \times 10^{-11} \), corresponding to \( \Omega h^2 \sim 0.001 - 0.002 \) which is not too different from galactic mass densities, there is no bound whatsoever on \( N_\nu \). Of course, with the improved data on \(^7\)Li, we do have lower bounds on \( \eta \) which exceed \( 10^{-10} \).

In ref. 21, it was argued that since stars (even massive stars) do not destroy \(^3\)He in its entirety, we can obtain a bound on \( \eta \) from an upper bound to the solar D and \(^3\)He abundances. One can in fact limit \(^3\)He which survives as \(^3\)He, for \( g_3 > 0.25 \) as suggested by stellar models, and using the solar data on D/H and \(^3\)He/H, one finds \( \eta_{10} > 2.8 \). This argument has been improved recently, ultimately leading to a stronger limit \( \eta_{10} > 3.8 \) and a best estimate \( \eta_{10} = 6.6 \pm 1.4 \). The problem with this bound, is that it seems to indicate an inconsistency most notably in the high \(^4\)He mass fraction predicted at the large value of \( \eta \). It has been speculated that the cause may be underestimated systematic uncertainties in the \(^4\)He abundance, a problem with chemical evolution, or even a tau-neutrino mass thereby lowering \( N_\nu \) from 3 to 2. Indeed, at the large value of \( \eta \), if \( N_\nu \) is allowed to be adjusted, a value around 2 is needed to match the \(^4\)He mass fraction of 0.234. Even at \( \eta \) around \( \eta_{10} = 3 \), the preferred value for \( N_\nu \), would be well below 3.8.

The limit \( \eta_{10} > 2.8(3.8) \) derived using (8) is really a one shot approximation. Namely, it is assumed that material passes through a star no more than once. (Although even the stochastic approach used in ref. 28) could only lower the bound from 3.8 to about 3.5 when assuming as always that \( g_3 > 0.25 \).) To determine whether or not the solar (and present) values of D/H and \(^3\)He/H can be matched it is necessary to consider models of galactic chemical evolution. In the absence of stellar \(^3\)He production, particularly by low mass stars, it was shown that there are indeed suitable choices for a star formation rate, and an initial mass function, to: 1) match the D/H evolution from a primordial value \( (D/H)p = 7.5 \times 10^{-5} \), corresponding to \( \eta_{10} = 3 \), through the solar and ISM abundances; while 2) at the same time keeping the \(^3\)He/H evolution relatively flat so as not to overproduce \(^3\)He at the solar and present epochs. This was achieved for \( g_3 < 0.3 \). Even for \( g_3 \sim 0.7 \), the present \(^3\)He/H could be matched, though the solar value was found to be a factor of 2 too high. For \( (D/H)p \sim 2 \times 10^{-4} \), corresponding to \( \eta_{10} \sim 1.7 \), though models could be found which destroy D sufficiently, overproduction of \(^3\)He occurred unless \( g_3 \) was tuned down to about 0.1.

In the context of models of galactic chemical evolution, there is however, little justification a priori, for neglecting the production of \(^3\)He in low mass stars. Indeed, stellar models predict that considerable amounts of \(^3\)He are produced in stars between 1 and 3 M\(_\odot\). For \( M < 8M\odot \),
Iben and Truran\textsuperscript{31}) calculate

\begin{equation}
\left(\frac{3\text{He}}{H}\right)_f = 1.8 \times 10^{-4} \left(\frac{M_\odot}{M}\right)^2 + 0.7 \left[\frac{(D + 3\text{He})}{H}\right]_i \tag{9}
\end{equation}

so that at \(\eta_0 = 3\), and \(((D + 3\text{He})/H)_i = 9 \times 10^{-5}\), \(g_3(1M_\odot) = 2.7\). It should be emphasized that this prediction is in fact consistent with the observation of high 3\text{He}/H in planetary nebulae\textsuperscript{20}. Generally, implementation of the 3\text{He} yield in Eq. (9) in chemical evolution models, leads to an overproduction of 3\text{He}/H particularly at the solar epoch\textsuperscript{32,33}. In Figure 1, the evolution of D/H and 3\text{He}/H is shown as a function of time from refs. 14,32). The solid curves show the evolution in a simple model of galactic chemical evolution with a star formation rate proportional to the gas density and a power law IMF (see ref. 32) for details). The model was chosen to fit the observed deuterium abundances. However, as one can plainly see, 3\text{He} is grossly overproduced (the deuterium data is represented by squares and 3\text{He} by circles). Depending on the particular model chosen, it may be possible to come close to at least the upper end of the range of the 3\text{He}/H observed in galactic HII regions\textsuperscript{19}, although, the solar value is missed by many standard deviations.

The overproduction of 3\text{He} relative to the solar meteoritic value seems to be a generic feature of chemical evolution models when 3\text{He} production in low mass stars is included. In ref. 14), a more extreme model of galactic chemical evolution was tested. There, it was assumed that the initial mass function was time dependent in such a way so as to favor massive stars early on (during the first two Gyr of the galaxy). Massive stars are preferential from the point of view of destroying 3\text{He}. However, massive stars are also proficient producers of heavy elements and in order to keep the metallicity of the disk down to acceptable levels, supernovae driven outflow was also included. The degree of outflow was limited roughly by the observed metallicity in the intergalactic gas in clusters of galaxies. One further assumption was necessary; we allowed the massive stars to lose their 3\text{He} depleted hydrogen envelopes prior to...
explosion. Thus only the heavier elements were expelled from the galaxy. With all of these (semi-defensible) assumptions, $^3$He was still overproduced at the solar epoch as shown by the dashed curve in Figure 1. Though there certainly is an improvement in the evolution of $^3$He, without reducing the yields of low mass stars, it is hard to envision much further reduction in the solar $^3$He predicted by these models. The only conclusion that we can make at this point is that there is clearly something wrong with our understanding of $^3$He in terms of either chemical evolution, stellar evolution or perhaps even the observational data.

Given the magnitude of the problems concerning $^3$He, it would seem unwise to make any strong conclusion regarding big bang nucleosynthesis which is based on $^3$He. Perhaps as well some caution is deserved with regard to the recent D/H measurements, although if the present trend continues and is verified in several different quasar absorption systems, then D/H will certainly become our best measure for the baryon-to-photon ratio. Given the current situation however, it makes sense to take a step back and perform an analysis of big bang nucleosynthesis in terms of the element isotopes that are best understood, namely, $^4$He and $^7$Li.

Monte Carlo techniques are proving to be a useful form of analysis regarding big bang nucleosynthesis. In ref. 36), we performed just such an analysis using only $^4$He and $^7$Li. It should be noted that in principle, two elements should be sufficient for constraining the one parameter ($\eta$) theory of BBN. We begin by establishing likelihood functions for the theory and observations. For example for $^4$He, the theoretical likelihood function takes the form

$$L_{BBN}(Y, Y_{BBN}) = e^{-(Y-Y_{BBN}(\eta))^2/2\sigma_1^2}$$

where $Y_{BBN}(\eta)$ is the central value for the $^4$He mass fraction produced in the big bang as predicted by the theory at a given value of $\eta$, and $\sigma_1$ is the uncertainty in that value derived from the Monte Carlo calculations and is a measure of the theoretical uncertainty in the big bang calculation. Similarly one can write down an expression for the observational likelihood function. In this case we have two sources of errors as discussed above, a statistical uncertainty, $\sigma_2$ and a systematic uncertainty, $\sigma_{sys}$. Here, I will assume that the systematic error is described by a top hat distribution. The convolution of the top hat distribution and the Gaussian (to describe the statistical errors in the observations) results in the difference of two error functions

$$L_O(Y, Y_O) = \text{erf} \left( \frac{Y - Y_O + \sigma_{sys}}{\sqrt{2}\sigma_2} \right) - \text{erf} \left( \frac{Y - Y_O - \sigma_{sys}}{\sqrt{2}\sigma_2} \right)$$

where in this case, $Y_O$ is the observed (or observationally determined) value for the $^4$He mass fraction. (Had I used a Gaussian to describe the systematic uncertainty, the convolution of two Gaussians leads to a Gaussian, and the likelihood function would have taken a form similar to that in (10)).

A total likelihood function for each value of $\eta_{10}$ is derived by convolving the theoretical and observational distributions, which for $^4$He is given by

$$L_{total}^{^4\text{He}}(\eta) = \int dY L_{BBN}(Y, Y_{BBN}(\eta)) L_O(Y, Y_O)$$

An analogous calculation is performed for $^7$Li. The resulting likelihood functions from the observed abundances given in Eqs. (1) and (2) is shown (unnormalized) in Figure 2. As one can see there is very good agreement between $^4$He and $^7$Li in vicinity of $\eta_{10} \approx 1.8$.

The combined likelihood, for fitting both elements simultaneously, is given by the product of the two functions in Figure 2, and is shown in figure 3. From Figure 3 it is clear that $^4$He...
Figure 2: Likelihood distribution for each of $^4$He and $^7$Li, shown as a function of $\eta$. The one-peak structure of the $^4$He curve corresponds to its monotonic increase with $\eta$, while the two-peaks for $^7$Li arise from its passing through a minimum.

Figure 3: Combined likelihood for simultaneously fitting $^4$He and $^7$Li, as a function of $\eta$. 

He
η
10
D
H
3
He
H
Figure 4: D/H and $^{3}$He/H as a function of $\eta_{10}$ from BBN along with the one $\sigma$ uncertainty from Monte Carlo calculations. Also shown are the values (demarcated by rectangles) of D/H and $^{3}$He/H consistent with 68% (dashed) and 95% CL (dotted) likelihood values for $\eta_{10}$.

overlaps the lower (in $\eta$) $^{7}$Li peak, and so one expects that there will be concordance, in an allowed range of $\eta$ given by the overlap region. This is what one finds in figure 3, which does show concordance, and gives an allowed (95% CL) range of $1.4 < \eta_{10} < 3.8$. Note that the likelihood functions shown in Figures 2 and 3 are not normalized to unity. An $\eta$ dependent normalization has however been included. Any further normalization would have no effect on the predicted range for $\eta$.

Thus, we can conclude that the abundances of $^{4}$He and $^{7}$Li are consistent, and select an $\eta_{10}$ range which overlaps with (at the 95% CL) the longstanding favorite range around $\eta_{10} = 3$. Furthermore, by finding concordance using only $^{4}$He and $^{7}$Li, we deduce that if there is problem with BBN, it must arise from D and $^{3}$He and is thus tied to chemical evolution or the stellar evolution of $^{3}$He. The most model-independent conclusion is that standard BBN with $N_{\nu} = 3$ is not in jeopardy, but there may be problems with our detailed understanding of D and particularly $^{3}$He chemical evolution. It is interesting to note that the central (and strongly) peaked value of $\eta_{10}$ determined from the combined $^{4}$He and $^{7}$Li likelihoods is at $\eta_{10} = 1.8$. The corresponding value of D/H is $1.8 \times 10^{-4}$ very close to the value of D/H in quasar absorbers.

Since D and $^{3}$He are monotonic functions of $\eta$, a prediction for $\eta$, based on $^{4}$He and $^{7}$Li, can be turned into a prediction for D and $^{3}$He. In Figure 4, the abundances of D and $^{3}$He as a function of $\eta_{10}$ are shown along with the one $\sigma$ uncertainty in the calculations from the Monte Carlo results. The 68% (dashed) and 95% CL (dotted) ranges for D and $^{3}$He as given by our likelihood analysis above are shown by a set of rectangles. The corresponding 95% CL ranges are D/H = $(5.5 - 27) \times 10^{-5}$ and and $^{3}$He/H = $(1.4 - 2.7) \times 10^{-5}$.

In summary, I would assert that one can only conclude that the present data on the abundances of the light element isotopes is consistent with the standard model of big bang nucleosynthesis. Using the the isotopes with the best data, $^{4}$He and $^{7}$Li, it is possible to constrain the theory, and obtain a best value for the baryon-to-photon ratio of $\eta_{10} = 1.8$ with
a 95% CL range of 1.4 to 3.8. This is a rather low value and corresponds to a baryon density $0.005 \leq \Omega_B h^2 \leq 0.014$, and would suggest that much of the galactic dark matter is non-baryonic\(^{37}\). These predictions are in addition consistent with recent observations of D/H in quasar absorption systems. Difficulty remains however, in matching the solar \(^3\)He abundance, suggesting a problem with our current understanding of galactic chemical evolution or the stellar evolution of low mass stars as they pertain to \(^3\)He.

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