THE RAZOR’S EDGE OF COLLAPSE: THE TRANSITION POINT FROM LOGNORMAL TO POWER-LAW DISTRIBUTIONS IN MOLECULAR CLOUDS

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ABSTRACT

We derive an analytic expression for the transitional column density value ($\eta_t$) between the lognormal and power-law form of the probability distribution function (PDF) in star-forming molecular clouds. Our expression for $\eta_t$ depends on the mean column density, the variance of the lognormal portion of the PDF, and the slope of the power-law portion of the PDF. We show that $\eta_t$ can be related to physical quantities such as the sonic Mach number of the flow and the power-law index for a self-gravitating isothermal sphere. This implies that the transition point between the lognormal and power-law density/column density PDF represents the critical density where turbulent and thermal pressure balance, the so-called “post-shock density.” We test our analytic prediction for the transition column density using dust PDF observations reported in the literature, as well as numerical MHD simulations of self-gravitating supersonic turbulence with the Enzo code. We find excellent agreement between the analytic $\eta_t$ and the measured values from the numerical simulations and observations (to within 1.2 $A_V$). We discuss the utility of our expression for determining the properties of the PDF from unresolved low-density material in dust observations, for estimating the post-shock density, and for determining the H1–H2 transition in clouds.

Key words: dust, extinction – galaxies: star formation – magnetohydrodynamics (MHD)

1. INTRODUCTION

Star formation in galaxies occurs in dense molecular environments and is governed by the complex interaction of gravity, magnetic fields, turbulence, and radiation pressure (McKee & Ostriker 2007; Elmegreen 2011). Despite decades of study, the fundamental conditions behind the transition of diffuse atomic gas to cold molecular gas are still relatively unconstrained (Sternberg 1988; Krumholz et al. 2009; McKee & Krumholz 2010; Bialy et al. 2015). The initial conditions imprinted on the diffuse and molecular gas on parsec scales (i.e., the level of turbulence, the cloud density, the structure of the magnetic field) may determine the key properties of the initial mass function (IMF) and the star formation rates in galaxies (Hennebelle & Chabrier 2011). Therefore the properties of diffuse and molecular gas in and around star-forming clouds must be quantified in order to construct a theory of star formation that predicts the IMF.

The density and column density probability distribution functions (PDFs) have been used extensively for understanding the properties of galactic gas dynamics, from the diffuse ionized medium to dense star-forming clouds. The application of the PDF in molecular clouds has included density tracers such as CO (Lee et al. 2012; Burkhart et al. 2013b) and dust (Kainulainen et al. 2009; Froebrich & Rowles 2010; Schneider et al. 2013, 2015; Lombardi et al. 2015). Tracing the PDF using dust emission and absorption provides the largest dynamic range of densities, in contrast to molecular line tracers such as CO, which suffer from depletion and opacity effects (Goodman et al. 2009; Burkhart et al. 2013a, 2013b).

Simulations of self-gravitating MHD turbulence have successfully reproduced the shape and properties of the observational PDFs (Burkhart et al. 2009; Federrath & Klessen 2012, 2013; Collins et al. 2012; Burkhart et al. 2015), suggesting that the gas PDF stems from a combination of turbulence (which induces a lognormal PDF shape in density) and self-gravity (which is characterized by a power-law PDF in density). In more detail, observed and simulated PDFs of Giant Molecular Cloud (GMC) environments, which include supersonic turbulence and self-gravity, suggest that the highest column density regime of the PDF (i.e., above column densities of 1 $A_V$) has a power-law distribution (Burkhart et al. 2009, 2015; Collins et al. 2011, 2012; Federrath & Klessen 2012, 2013; Schneider et al. 2013; Lombardi et al. 2015; Myers 2015), while the lower column density material in the PDF is dominated by turbulent diffuse gas and takes on a lognormal form (Vazquez-Semadeni 1994; Padoan et al. 1997; Federrath et al. 2010; Burkhart & Lazarian 2012; Myers 2015).

The implications for the shape of the gas density PDF in ISM clouds are profoundly linked to the kinematics, star formation rates, and the chemistry of the gas (Federrath & Klessen 2012). Kinematically, the PDF width of the lognormal density distribution can be related to the sonic Mach number of the gas in an isothermal cloud (Federrath & Klessen 2012; Padoan et al. 2014). More recently, the H1 PDF in and around GMCs has been proposed as a tracer of the H1–H2 transition (Burkhart et al. 2015; Imara & Burkhart 2016), as well as a more accurate tracer of the low-density lognormal shape, as opposed to dust emission/absorption data, which have difficulty tracing the lognormal form (Schneider et al. 2013; Lombardi et al. 2015). Burkhart et al. (2015) and Imara & Burkhart (2016) have shown that the lognormal portion of the column density PDF in a sample of Milky Way GMCs is mostly comprised of atomic H1 gas, while the power-law tail is built up by molecular hydrogen (H2). These studies suggest that the transition point in the column density PDF between the lognormal and power-law portions of the column density PDF...
traces important physical processes, such as the H1–H2 transition and the density regime where self-gravity becomes dynamically important.

In this work we derive an analytic formula for the transitional column density from the lognormal portion of the PDF to the power-law form (denoted \( \eta_t \)). We organize the paper as follows. In Section 2 we derive an expression for the transitional column density for a piecewise lognormal and power-law PDF distribution based on the assumption that the PDF is continuous and differentiable. We further demonstrate that \( \eta_t \) is related to the physical parameters such as the sonic Mach number of the gas (i.e., kinematics), the post-shock density, and the power-law index for a self-gravitating isothermal sphere. In Section 3 we compare our analytic expression for the transitional column density to numerical simulations of self-gravitating MHD turbulence run using the Enzo code. In Section 4 we compare our analytic expression for the transitional column density to observations using data from the literature. In Section 5 we discuss our results, followed by our conclusions in Section 6.

2. THE TRANSITION FROM LOGNORMAL TO POWER-LAW TAIL IN THE PDF OF A TURBULENT SELF-GRAVITATING MEDIUM

The lognormal PDF of the gas column density is defined as

\[
p_\eta(\eta) = \frac{1}{\sqrt{2\pi} \sigma_\eta} \exp \left( -\frac{(\eta - \eta_0)^2}{2\sigma_\eta^2} \right),
\]

with \( \eta \) being the logarithm of the normalized column density:

\[
\eta = \ln(\Sigma/\Sigma_0).
\]

The PDF is a normal distribution in \( \eta \), meaning that it is a lognormal distribution in \( \Sigma \). The quantities \( \Sigma_0 \) and \( \eta_0 \) denote, respectively, the mean column density and mean logarithmic column density, the latter of which can be related to the standard deviation \( \sigma_\eta \) by:

\[
\eta_0 = -\frac{1}{2} \sigma_\eta^2.
\]

The lognormal form of the PDF of column density describes the behavior of diffuse H1 and ionized gas (Berkhuijsen & Fletcher 2008; Hill et al. 2008; Burkhart et al. 2010), as well as some star-forming molecular clouds that are not actively star-forming, e.g., see Kainulainen & Tan (2013) and Schneider et al. (2013).

The PDF of the highest column density regime of self-gravitating turbulent clouds has a power-law distribution, as demonstrated in numerical simulations (Collins et al. 2012; Federrath & Klessen 2012, 2013; Burkhart et al. 2015) and observations (Kainulainen et al. 2009; Froebrich & Rowles 2010; Schneider et al. 2013, 2015; Burkhart et al. 2015; Lombardi et al. 2015).

Based on the aforementioned numerical and observational studies, hereafter we consider a piecewise form for the PDF of column density (similar to the assumption of Collins et al. (2012) for the 3D density) that has a lognormal distribution below a transitional column density value, denoted \( \eta_t = \ln(\Sigma_t/\Sigma_0) \), where \( \Sigma_t \) is the transitional column density value. At column densities greater than \( \eta_t \), the PDF is a power-law. The general piecewise function (see also Myers 2015) can be written as

\[
p_\eta(\eta) = \begin{cases} 
N \frac{1}{2\pi \sigma_\eta} \exp \left( -\frac{(\eta - \eta_0)^2}{2\sigma_\eta^2} \right), & \eta < \eta_t \\
N p_\eta \exp[-\alpha \eta], & \eta > \eta_t.
\end{cases}
\]

The mean and width of the lognormal portion are related by \( \eta_0 = -\frac{1}{2} \sigma_\eta^2 \). The normalization \( N \) is determined by the normalization criterion, \( \int_{-\infty}^{\infty} p_\eta(\eta) d\eta = 1 \), and is

\[
N = \left( \frac{\eta_t / \alpha - \eta_0}{1 + \text{erf} \left( \frac{2\eta_t + \sigma_\eta^2}{2\sqrt{2}\sigma_\eta} \right)} \right)^{-1}.
\]

Equation (4), with no further constraints, is completely general. If we assume that \( p_\eta(\eta) \) is continuous and differentiable, we can formulate an analytic estimate for both the transition point, \( \eta_t \), and the normalization of the power law, \( p_0 \). By setting the two parts of Equation (4) as equal at \( \eta_t \) and setting their derivatives as equal, we find

\[
\eta_t = \frac{1}{2} (2|\alpha| - 1) \sigma_\eta^2,
\]

and the amplitude of the power law, \( p_0 \), is then

\[
p_0 = e^{\frac{(\alpha - 1)\sigma_\eta^2}{2\sqrt{2}\pi}}.
\]

The transition column density value between the lognormal and power-law PDFs therefore depends on the slope of the power-law tail (\( \alpha \)) and the standard deviation of the lognormal (\( \sigma_\eta \)) and the mean column density. We note that our expression is similar to that of the transition point derived in Myers (2015), who derived relations between column density and projected radius for column density PDFs that are pure lognormal, pure power law, and the combination of the two as given in Equation (4).

We note that the solution to the transition point should be applicable (and take the same form) for both density (see Collins et al. 2012, Equation (29)) and column density distributions, since both density and column density share the same lognormal4 + power-law form of the PDF. In the following subsection we provide a physical interpretation for \( \eta_t \).

2.1. Physical Interpretation of \( \eta_t \)

The transitional column density \( \eta_t \) is not necessarily a criterion for a critical star formation density, which most likely is farther out in the power-law tail (Federrath & Klessen 2012; Padoan et al. 2014). Rather, \( \eta_t \) represents a transitional point between the dominance of supersonic turbulence in the cloud gas dynamics, which builds the lognormal distribution, to densities where gravity plays an increasingly important role in shaping the distribution.

Given the analytic solution for the transition point of the PDF between the lognormal and power-law tail, we are now in a position to relate the properties of the transition point to the physics of the gas in a GMC. The width of the lognormal PDF

\(^4\) The lognormal (Gaussian) form for column density is applicable under the condition that the central limit theory can be applied, namely when the size of the emitting region is larger than the decorrelation scale of turbulence (Vazquez-Semadeni & Garcia 2001).
\( (\sigma) \) depends on the properties of the turbulence in the GMC, with the primary dependence being on the sonic Mach number. For column density maps, Burkhart & Lazarian (2012) relate the sonic Mach number to PDF width as

\[
\sigma^2 = A \ln[1 + b^2 M^2].
\]

\( A = 0.11 \) is a scaling constant from density to column density. The forcing parameter \( b \) varies from \( b \approx 1/3 \) for purely solenoidal (divergence-free) forcing, to \( b = 1 \) for purely compressive (curl-free) forcing of MHD turbulence (Federrath et al. 2008, 2010; Konstandin et al. 2012; Molina et al. 2012; Federrath & Banerjee 2015; Nolan et al. 2015).

Equation (8) was shown to depend very weakly on the magnetic field (Burkhart & Lazarian 2012). For the 3D density PDF in super-Alfvénic turbulence, Molina et al. (2012) formulated the dependency on the plasma \( \beta_0 \), i.e., the ratio of the gas pressure to magnetic pressure, as

\[
\sigma_{\ln p/p_0}^2 = \ln[1 + b^2 M^2 \beta_0/(\beta_0 + 1)].
\]

The transition density or column density can be further expressed in terms of the sonic Mach number by combining Equations (6) and (8) to find

\[
\eta_t = \frac{1}{2} (2|\alpha| - 1)(A \ln[1 + b^2 M^2]).
\]

The transition density or column density can be further expressed in terms of the post-shock density, \( \rho_{ps} = \rho_0 M_t^2 \), which is the density at which the turbulent energy density is equal to the thermal pressure:

\[
P_{\text{therm}} = \rho_{ps} \sigma_t^2 = \rho_0 \nu^2.
\]

Manipulating this relation we find that \( M_t^2 = \rho_{ps}/\rho_0 \), meaning that Equation (10) becomes

\[
\eta_t = (|\alpha| - 1/2)A \ln \left[ 1 + b^2 \frac{\rho_{ps}}{\rho_0} \right].
\]

In the limit of strong collapse, \( |\alpha| \) tends toward 1.5 (see Figure 2), so the \(|\alpha| - 1/2\) term is of order unity.

Therefore,

\[
\ln(\Sigma_t/\Sigma_0) \approx A \ln \left[ 1 + b^2 \frac{\rho_{ps}}{\rho_0} \right],
\]

so

\[
\Sigma_t/\Sigma_0 \approx \left( 1 + b^2 \frac{\rho_{ps}}{\rho_0} \right)^A.
\]

In the case of a 3D density field (relevant for simulations) we can express the transition density in the same form as the column density (i.e., using Equation (6)) the transition density can be expressed (using Equation (9)) as:

\[
\rho_t/\rho_0 \approx \left( 1 + b^2 \frac{\rho_{ps}}{\rho_0} \beta_0/\beta_0 + 1 \right),
\]

as the exponent \( A \) in Equation (14) accounts for line-of-sight (LOS) effects and radiative transfer in column density (see Burkhart et al. 2013a). Equation (15) can also be extended to account for non-isothermal gas. We note that our expression in Equation (15) is applicable both for an isothermal and non-isothermal equation of state, as Federrath & Banerjee (2015) and Nolan et al. (2015) have extended the relation between the width of the density PDF and the Mach number to include the dependence on polytropic and adiabatic exponents.

The slope of the power-law tail, \( \alpha \), does not have a clear relation to other physical quantities. It depends on the collapse state of the gas and the magnetic pressure (Ballesteros-Paredes et al. 2011; Kritsuk et al. 2011; Collins et al. 2012; Federrath & Klessen 2013; Burkhart et al. 2015).\(^6\)

In the case where the tail is produced only due to gravitational collapse, and if we assume spherical symmetry, the PDF slope of the power-law tail is related to the exponent \( \gamma \) of the radial density profile \( \rho \sim r^{-\gamma} \) (e.g., Shu 1977). Girichidis et al. (2014) showed analytically that column density power-law-tail slopes of \( \alpha = -2.1 \) correspond to the \( \gamma = 2 \) prediction for a collapsing isothermal sphere since \( \alpha = -2/(\gamma - 1) \). We note that \( \alpha = -2.1 \) is a special case and simulations show that collapsing turbulent clouds can have a range of \( \alpha \), which we show in Figure 2 and which also are discussed in other numerical works (Kritsuk et al. 2011; Collins et al. 2012; Federrath & Klessen 2013; Burkhart et al. 2015).

3. NUMERICAL SIMULATIONS

3.1. Numerical Parameters and Methods

We are now in a position to test the analytic relation for the transitional column density given in Equation (6). For this purpose we use simulation data generated by solving the ideal MHD equations including self-gravity using the Adaptive Mesh Refinement (AMR) code Enzo (Bryan et al. 2014) and extended to MHD in Collins et al. (2010). These simulations use a root grid of 128\(^3\) with four levels of refinement, yielding an effective resolution of 2048\(^3\). The Virial parameter \( \alpha_{\text{vir}} \), sonic Mach number \( M_s \), and mean ratio of thermal to magnetic pressure \( \beta_0 \) are chosen here to be:

\[
\alpha_{\text{vir}} = 1, \quad M_s = 9, \quad \beta_0 = 0.2, 2.0, 20.0,
\]

which scale to physical clouds with freefall time \( t_{ff} \), box size \( L_0 \), rms velocity \( v_{rms} \), total mass \( M \), and mean magnetic field \( B_0 \) of:

\[
t_{ff} = 1.1 \text{ Myr}, \quad L_0 = 4.6 \text{ pc}, \quad v_{rms} = 1.8 \text{ km s}^{-1}, \quad M = 5900 \text{ M}_\odot, \quad B_0 = (13, 4.4, 1.3) \mu \text{G}.
\]

These simulations started with initially uniform density and magnetic field, and were driven with a purely solenoidal pattern until a steady state was reached. Then gravity was turned on, at \( t = 0 \) in the present simulations. The turbulent boxes are the same initial conditions as the simulations of Collins et al. (2012), though they are down-sampled to the lower root grid \( 512^3 \) in the present work, \( 128^3 \) in the previous.

\(^6\) We also note that the column density power-law slope \( \alpha \) is related to, but not the same as, the power-law of the 3D density field. The relation between these quantities is also derived in Girichidis et al. (2014) in their Equation (43).
work.) Also, the simulations of Collins et al. (2012) continued driving during the collapse, while the present simulations did not.

These simulations have a post-shock density of $\rho_{\text{ps}}/\rho_0 = M_f^2 = 81$. The density may also be scaled physically using $\rho_0 = 1000 \text{ cm}^{-3}$, yielding a post-shock density of $\rho_{\text{ps}} = 8.1 \times 10^4 \text{ cm}^{-3}$ or a column density of $\Sigma_{\text{ps}} = 6.7 \times 10^{23} \text{ cm}^{-2}$ given a cloud size of 4.6 pc. Typical observational values for the post-shock density range from $300 \text{ cm}^{-3}$ to greater than $4 \times 10^4 \text{ cm}^{-3}$ (Li et al. 2015).

### 3.2. Column Density PDFs

We fit the column density PDF using Equation (4) (i.e., a piecewise lognormal and power-law function) to obtain the transition point value and test the validity of the analytic model outlined in the previous section. We only enforce continuity and leave all variables as free parameters. Determining the location of the transition point presupposes that the piecewise function exists in the correct form of the PDF and the data itself are observed to be continuous. We note that in other works (e.g., Collins et al. 2012; Schneider et al. 2015) the lognormal width, $\sigma$, and power-law slope, $\alpha$, were fit independently to the PDF. We compare the fitted value of the transition point determined from Equation (4), given independent measurements of the power-law slope and lognormal width of the PDF to the value of the transition point determined from fitting Equation (5). In Figure 1 we show an example of our fitting method to a column density PDF from the Enzo simulation at $t = 0.6 t_f$ and for $\beta_0 = 2.0$. The transition point $\eta_t$ is indicated by a green dot, the lognormal portion is a black line, the power-law is a red line, and the actual data are in blue.

### 3.3. Numerical versus Analytic Transitional Column Density

The slope of the power-law tail is expected to flatten with increasing time (the absolute value decreases) and with increasing $\beta_0$ as shown in Figure 2 and in Collins et al. (2012) and Federrath & Klessen (2013). The value of the power-law tail slope is roughly independent of the chosen line of sight (i.e., the relative orientation to the mean magnetic field). The value predicted for the lognormal width in these simulations, given Equation (8), is approximately 0.5, using the variables $M_f = 9$, $b = 1/3$ and $A = 0.11$ as described in Section 2.1. This is consistent with the fit results found here. Since all simulations have the same Mach number, the degeneracy between the normalization $A$ and the compressibility factor $b$ cannot be disentangled.

Given the fitted width of the lognormal and the slope of the power-law tail, we compare the predicted value of the transitional column density $\eta_t$ from Equation (6) to the measured value of the transitional column density, denoted $\eta_t$, found through direct fitting. We present these results in Figure 3. We find good agreement between the predictions of the analytic fit (solid lines) proposed in Section 2 and the simulation results via direct fitting (dashed lines). Given that the lognormal width is roughly constant with time, while the power-law tail slope decreases with time, we observe a slight decrease in the transition point with time, as expected. We also note the overall closer correspondence between the fit and

![Figure 1](image1.png)

**Figure 1.** An example PDF shown at $0.6 t_f$ for $\beta_0 = 2.0$, with the line of sight along the $y$-axis. The fitted transition point $\eta_t$ is indicated by the green dot, the lognormal fit is the black line, the power-law is the red line, and the actual data are the blue line.

![Figure 2](image2.png)

**Figure 2.** Plot of the power-law tail slope, $|\alpha|$ (y-axis), vs. time, $t$ (x-axis), from fitting the column density PDF for the range $0.3 t_f$ to $0.7 t_f$ where the power-law tail is well-developed.

![Figure 3](image3.png)

**Figure 3.** Plot of transition point $\eta_t$ (y-axis) vs. time (x-axis) for each magnetic field strength as predicted by Equation (6) (solid lines) and those found through direct fitting (dashed lines), with the same colors as in Figure 2.
analytic results with lower $\beta$ simulations. The fit and analytic results converge at the later time steps ($t = 0.7t_{\text{ff}}$), where the application of Equation (4) is most appropriate.

4. OBSERVATIONAL COMPARISON

In this section we test our analytic prediction for the transitional column density against observations. In particular, Schneider et al. (2015, hereafter S15) published values of the mean column density $\Sigma_0$, transitional column density ($\Sigma_t$), the width of the lognormal ($\sigma_\alpha$), and the slope of the power-law tail ($\alpha$) for four GMCs with different star formation histories, corrected for foreground and background dust contamination. This provides an observational test for comparing the predicted values of $\Sigma_t$ to the measured value, based on the measured values of $\Sigma_0$, $\alpha$, and $\sigma_\alpha$ and the application of Equation (6). We list the LOS foreground/background corrected parameters as reported in S15 and the analytic predicted value for $\Sigma_t$ in Table 1. We also list the same parameters as measured from the three Enzo simulations at $t = 0.6t_{\text{ff}}$. We compare both simulations and observations in Figure 4.

The values of $\Sigma_{t,\text{fit}}$ from S15 and $\Sigma_{t,\text{Equation (6)}}$ agree in the range of $\Delta A_V = 0.6$–1.7, with an average difference of 1.2$A_V$. The predicted values from Equation (6) are consistently smaller than the reported fitted values in the observations of S15. We discuss the possible reasons for this in the next section.

5. DISCUSSION

5.1. The $\text{H\textsc{i}}$–$\text{H}_2$ Transition and Self-gravity

Recent studies have suggested that the PDF of molecular line tracers and dust tracers is of a power-law form (Lombardi et al. 2015; Schneider et al. 2015), while the neutral diffuse $\text{H\textsc{i}}$ builds-up most of the lognormal portion of the PDF (Burkhart et al. 2015; Imara & Burkhart 2016). In light of these recent studies, the $\text{H\textsc{i}}$ lognormal PDF and $\text{H}_2$ power-law tail PDF may be effectively distinguished by the transition point between the two distributions. The truncation of the $\text{H\textsc{i}}$ lognormal roughly corresponds to the $\text{H\textsc{i}}$–$\text{H}_2$ transition column density in Galactic star-forming clouds (Burkhart et al. 2015; Imara & Burkhart 2016), which suggests that measuring the transitional column density in such clouds could provide constraints on the $\text{H\textsc{i}}$–$\text{H}_2$ transition. The transitional column density is approximately $\Sigma_t = 1 - 5 \times 10^{21}$ cm$^{-2}$ (i.e., $\approx 8$–38 $M\odot$ pc$^{-2}$), which is in the range of the typically quoted $\text{H\textsc{i}}$–$\text{H}_2$ transition value from theoretical works, e.g., $10 M\odot$ pc$^{-2}$ (Sternberg 1988; McKee & Krumholz 2010; Sternberg et al. 2014; Bialy et al. 2015).

Table 1

| Cloud      | $\Sigma_0$ $(A_V)^a$ | $\alpha$ | $\sigma_\alpha$ | $\Sigma_{t,\text{fit}}(A_V)$ | $\Sigma_{t,\text{Equation (6)}}(A_V)$ | Reference |
|------------|----------------------|----------|-----------------|-----------------------------|----------------------------------------|-----------|
| NGC3603    | 3.4                  | -1.31    | 0.52            | 4.9                         | 4.3                                    | S15       |
| Carina     | 3.0                  | -2.66    | 0.38            | 5.5                         | 4.1                                    | S15       |
| Maddalena  | 2.3                  | -3.69    | 0.32            | 4.9                         | 3.2                                    | S15       |
| Auriga     | 1.6                  | -2.54    | 0.45            | 3.5                         | 2.4                                    | S15       |
| $\beta = 0.2$ | 3.4                | $\approx -3.6$ | $\approx 0.50$ | 6.4                         | 6.9                                    | this work |
| $\beta = 2$  | 3.4                  | $\approx -2.9$ | $\approx 0.54$ | 6.2                         | 5.9                                    | this work |
| $\beta = 20$ | 3.4                  | $\approx -2.6$ | 0.53            | 5.7                         | 6.2                                    | this work |

Notes. Simulated clouds are taken from snapshot $t = 0.5t_{\text{ff}}$.

$^a$ Assuming $N(\text{H}_2) = A_V \times 0.94 \times 10^{21}$ cm$^{-2}$/mag$^{-1}$.

Figure 4. Plot of observed transition point (through fitting) vs analytic transition point (Equation (6)) for the three values of plasma $\beta$, along (colored points: blue = $\beta = 20$, green = $\beta = 2$, red = $\beta = 0.2$) with observationally attained transition points (black circles) from nearby molecular clouds from S15.

5.2. Observational Properties of the Low Column Density PDF via $\eta_t$

Several authors (Lombardi et al. 2015; Schneider et al. 2015) have recently noted that dust emission and extinction are problematic probes of the low column density material in molecular clouds. This is because the observed PDF of dust can suffer several biases, including resolution, noise, boundary effects, and line-of-sight contamination. Lombardi et al. (2015) pointed out that while the lognormal portion of the PDF cannot be securely traced by dust, the characteristic break in the power-law regime at low values of extinction/column density (i.e., $\eta_t$) is still unaffected by observational biases.

These studies suggest that $\eta_t$ is a robust observational quantity, even though the properties of the lognormal PDF, such as the width of the lognormal, are not possible to accurately observe in dust tracers. Therefore, using our analytic expression for $\eta_t$, it is possible to estimate the lognormal width of the distribution by measuring the power-law tail slope and value of $\eta_t$. The shape of the low-density portion of the PDF provides an important constraint on the initial conditions of star-forming clouds (i.e., the strength of...
turbulence and comparison to numerical studies) and therefore it is important to quantify this observationally.

Incidentally, the difficulty of constraining the width of the PDF may be the reason that our predicted value for $\eta_t$ differs by about 1.2 $A_v$ from the Herschel observations reported in Table 1 (Schneider et al. 2015), since our prediction depends on the width of the PDF. Since the measured values of $\alpha$ (slope of the power-law) and $\eta_t$ should be robust to observational effects, these two quantities could be used to measure the lognormal width $\sigma_{1/2}$ rather than fitting $\sigma_0$ directly from observations.

6. CONCLUSIONS

The transition point between the turbulence-dominated (lognormal) portion of the PDF and the denser, self-gravitating (power-law) portion of the PDF is an important component of the star formation process. We then test the validity of our expression using both observations from the literature and numerical simulations.

We find that:

1. The expression for $\eta_t$ depends on the mean column density, width of the lognormal portion of the PDF (i.e., the sonic Mach number and driving parameter) and the slope of the power-law portion of the PDF (i.e., power-law index for a self-gravitating isothermal sphere).

2. In the limit of strong collapse, $\eta_t$ represents the post-shock density given by the balance of turbulent and thermal pressure.

3. The values predicted by the analytic expression for $\eta_t$ agree, to within 1.2 $A_v$, with measurements from Herschel dust observations and Enzo AMR simulations.

4. The analytic expression reported in Equation (6) will be useful for determining the properties of the PDF from unresolved low-density material in observations and for estimating the H I–H$_2$ transition in clouds.

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REFERENCES

Ballesteros-Paredes, J., Vázquez-Semadeni, E., Gazol, A., et al. 2011, MNRAS, 416, 1436

Burkhart, B., Stalpes, C., & Collins, D. C. 2015, ApJ, 806, 226

Nolan, C. A., Federrath, C., & Klessen, R. S. 2012, MNRAS, 423, 2680

Myers, P. 2015, ApJ, 806, 226

Berkhuijsen, E. M., & Fletcher, A. 2008, MNRAS, 390, L19

Bialy, S., Sternberg, A., Lee, M.-Y., Le Petit, F., & Roueff, E. 2015, ApJ, 809, 122

Bryan, G. L., Norman, M. L., O’Shea, B. W., et al. 2014, ApJS, 211, 19

Burkhart, B., Collins, D. C., & Lazarian, A. 2015, ApJ, 808, 48

Burkhart, B., Falcke-Gonalves, D., Kowal, G., & Lazarian, A. 2009, ApJ, 693, 250

Burkhart, B., & Lazarian, A. 2012, ApJL, 755, L19

Collins, D. C., Kritsuk, A. G., Padoan, P., et al. 2012, ApJ, 750, 13

Collins, D. C., Padoan, P., Norman, M. L., & Xu, H. 2011, ApJ, 731, 59

Collins, D. C., Xu, H., Norman, M. L., Li, H., & Li, S. 2010, ApJS, 186, 308

Elmegreen, B. G. 2011, ApJ, 731, 61

Federrath, C., & Banerjee, S. 2015, MNRAS, 448, 3297

Federrath, C., & Klessen, R. S. 2012, ApJ, 761, 156

Federrath, C., & Klessen, R. S. 2013, ApJ, 763, 51

Federrath, C., Klessen, R. S., & Schmidt, W. 2008, ApJL, 688, L79

Federrath, C., Roman-Duval, J., Klessen, R. S., Schmidt, W., & Mac Low, M.-M. 2010, A&A, 512, A81

Froebrich, D., & Rowles, J. 2010, MNRAS, 406, 1350

Girichidis, P., Konstadinl, L., Whitworth, A. P., & Klessen, R. S. 2014, ApJ, 781, 91

Goodman, A. A., Rosolowsky, E. W., Borkin, M. A., et al. 2009, Natur, 457, 63

Hennebelle, P., & Chabrier, G. 2011, ApJL, 743, L29

Hill, A. S., Benjamin, R. A., Kowal, G., et al. 2008, ApJ, 686, 363

Imara, N., & Burkhart, B. 2016, ApJ, 829, 102

Kainulainen, J., Beuther, H., Henning, T., & Plume, R. 2009, A&A, 508, L35

Kainulainen, J., & Tan, J. 2013, A&A, 549, 53

Konstadinl, L., Girichidis, P., Federrath, C., & Klessen, R. S. 2012, ApJ, 761, 149

Kritsuk, A. G., Norman, M. L., & Wagner, R. 2011, ApJL, 727, L20

Krumholz, M. R., & McKee, C. F. 2005, ApJ, 630, 250

Krumholz, M. R., McKee, C. F., & Tumlinson, J. 2009, ApJ, 693, 216

Lee, M.-Y., Stanimirović, S., Douglas, K. A., et al. 2012, ApJ, 748, 75

Li, P. S., McKee, C. F., & Klein, R. I. 2015, MNRAS, 452, 2500

Lombardi, M., Alves, J., & Lada, C. J. 2015, A&A, 576, L1

McKee, C. F., & Krumholz, M. R. 2010, ApJ, 709, 308

McKee, C. F., & Ostriker, E. C. 2007, ARA&A, 45, 565

Molina, F. Z., Glover, S. C. O., Federrath, C., & Klessen, R. S. 2012, MNRAS, 423, 2680

Myers, P. 2015, ApJ, 806, 226

Nolan, C. A., Federrath, C., & Sutherland, R. S. 2015, MNRAS, 451, 1380

Padoan, P., Federrath, C., Chabrier, G., et al. 2014, in Protostars and Planets VI, ed. H. Beuther et al. (Tucson, AZ: Univ. Arizona Press), 77

Padoan, P., Jones, B. J. T., & Nordlund, A. P. 1997, ApJ, 474, 730

Padoan, P., & Nordlund, A. 2011, ApJ, 730, 40

Price, D. J., Federrath, C., & Brunt, C. 2011, ApJ, 727, 21

Schneider, N., Andrê, P., Könyves, V., et al. 2013, ApJL, 766, L17

Schneider, N., Ossenkopf, V., Csengeri, T., et al. 2015, A&A, 575, A79 (S15)

Shu, F. H. 1977, ApJ, 214, 488

Sternberg, A. 1988, ApJ, 332, 400

Sterberg, A., Le Petit, F., Ros, E., & Le Bourlot, J. 2014, ApJS, 790, 108

Vazquez-Semadeni, E. 1994, ApJ, 423, 681

Vazquez-Semadeni, E., & Garcia, E. 2001, ApJ, 557, 727