Longitudinal top polarization as a probe of a possible origin of forward-backward asymmetry of the top quark at the Tevatron

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If the forward-backward (FB) asymmetry of top quark ($A_{FB}$) observed at the Tevatron deviates from the SM prediction, there must be $P$-violating interactions in $qq \rightarrow t\bar{t}$. This new interaction will necessarily affect the top spin polarization. In this letter, we perform a model independent analysis on the longitudinal (anti)top polarization ($P_L$ and $P_{\bar{L}}$) using an effective lagrangian with dim-6 four-quark operators relevant for $qq \rightarrow t\bar{t}$, and show that the $P$-odd observable corresponding to the polarization difference ($P_L - P_{\bar{L}}$) gives important informations on the chiral structures of new physics that might be relevant to the $A_{FB}$.

Top physics has entered a new era after its first discovery, due to the high luminosity achieved at the Tevatron and the launch of the Large Hadron Collider (LHC). Most recent results on the top mass and the $t\bar{t}$ production cross section (CDF and D0 Collaborations combined analysis) are: $m_t = (171.3 \pm 1.3)$ GeV and $\sigma_{t\bar{t}} = (7.50 \pm 0.48)$ pb, respectively [1]. Being the heaviest particle observed so far with its mass being near the electroweak breaking (EWSB) scale, the top sector might provide a new window to the EWSB mechanism. Precise determination of top quark properties is essential to address this issue, such as the top compositeness.

The forward-backward asymmetry $A_{FB}$ of the top quark is one of the interesting observables related to top quark. Within the SM, this asymmetry vanishes at leading order in QCD because of $C$ symmetry. At next-to-leading order [$O(\alpha_s^2)$], a nonzero $A_{FB}$ can develop from the interference between the Born amplitude and two-gluon intermediate state, as well as the gluon bremsstrahlung and gluon-(anti)quark scattering into $t\bar{t}$, with the prediction $A_{FB} \sim 0.078$ [2]. The measured asymmetry has been off the SM prediction by $2\sigma$ for the last few years, albeit a large experimental uncertainties. The most recent measurement in the $t\bar{t}$ rest frame is [3]

$$A_{FB} \equiv \frac{N_t(\cos \theta \geq 0) - N_{\bar{t}}(\cos \theta \geq 0)}{N_t(\cos \theta \geq 0) + N_{\bar{t}}(\cos \theta \geq 0)}$$

(1)

with $\theta$ being the polar angle of the top quark with respect to the incoming proton in the $t\bar{t}$ rest frame. The newest number is somewhat lower than the previous one [1], $A_{FB} = 0.24 \pm 0.13 \pm 0.04$, which had stimulated a lot of activities on possible new physics scenarios [4 [23].

Since the central value of the $A_{FB}$ is getting closer to the SM prediction, any new physics effects might be smaller than had been thought previously. Also there is no clear signal for such a new resonance [1]. Therefore, it would be reasonable to assume a new physics scale relevant to $A_{FB}$ is large enough so that production of a new particle is beyond the reach of the Tevatron [14], which makes a key difference between our work and other literatures. Then it is adequate to integrate out the heavy fields, and we can adopt a model independent effective lagrangian approach in order to study new physics effects on $\sigma_{t\bar{t}}$ and $A_{FB}$. If new physics scale is high enough, then their effects on the $t\bar{t}$ production at the Tevatron can be described by dim-6 effective lagrangian. Since the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry has been well established for the light quark system, we assume that $SU(2)_L \times U(1)_Y$ symmetry is linearly realized on the light quark system. And we impose the custodial symmetry $SU(2)_R$ for the light quark sector. Under these assumptions, the dimension-6 operators relevant to the $t\bar{t}$ production at the Tevatron are

$$\mathcal{L}_6 = \frac{g^2}{N^2} \sum_{A,B} \left[ C_{Aq}^B (\bar{q}A\gamma_\mu q_A)(\bar{t}_B\gamma^\mu t_B) + C_{Aq}^{AB} (\bar{q}A T^a \gamma_\mu q_A)(\bar{t}_B T^a \gamma^\mu t_B) \right]$$

(3)

where $T^a = \lambda^a/2$, $\{A,B\} = \{L,R\}$, and $L,R \equiv (1 \mp \gamma_5)/2$ with $q = (u,d)^T, (s,c)^T$ [33]. Our choice of dim-6 operators is basically the same as Ref. [26], except that we use the chiral basis for $t$ and $\bar{t}$. This operator set could be used, for example, to study $t\bar{t}$ production at the Tevatron in case of the composite top scenarios [27].

Before we move to the main subject of this paper, we would like to make a comment on other dim-6 operators that involves $t$ and $\bar{t}$. In principle, there are many more operators that involve $t$, $\bar{t}$ and gluon field strength tensor $G^a_{\mu\nu}$, which have been studied recently in Refs [28] and...
Many of them are however generated at one-loop level, unlike the operators we are considering here and in Ref. [14]. Therefore their effects would be further suppressed by a loop factor $1/(4\pi)^2$ and a power of strong coupling constant $g_s$, relative to the operators we study. Our choice of operators should be enough for the purpose of $t\bar{t}$ production at the Tevatron.

Using the above effective lagrangian, we can calculate the cross section up to $O(1/\Lambda^2)$, keeping only the interference term between the standard model and new physics contributions. The squared amplitude summed (averaged) over the final (initial) spins and colors is given by

$$|\mathcal{M}|_0^2 \approx \frac{4 g_t^4}{9 \sin^2 \theta} \left( \frac{1}{2} \left( C_1 + C_2 \right) \frac{\hat{s}^2}{2\Lambda^2} + \frac{s^2}{2} \right) \left( 1 + \hat{s} \left( C_1 + C_2 \right) \left[ 1 + \hat{s} \left( C_1 + C_2 \right) + \hat{s} \left( C_2 - C_1 \right) \right] \right) \right) \right.$$  \hspace{1cm} (4)

In our previous study [14], we performed a model independent study of $\sigma_{t\bar{t}}$ and $A_{FB}$ considering the interference effects of the SM amplitude and the new physics amplitudes from dim-6 operators, the leading order operators in the effective lagrangian. Here we update the previous results in the light of the new measurement of $A_{FB}$, see Fig. 1. The main results of Ref. [14] can be summarized as follows in terms of two effective couplings $C_1$ and $C_2$:

- $\Delta \sigma_{t\bar{t}} \equiv \sigma_{t\bar{t}}^{SM} \propto (C_1 + C_2)$, whereas $\Delta A_{FB} \equiv A_{FB} - A_{FB}^{SM} \propto (C_1 - C_2)$, i.e., the new physics contributions to the total cross section and $A_{FB}$ are orthogonal. Therefore the new physics can change $A_{FB}$ considerably without affecting $\sigma_{t\bar{t}}$ too much, as long as $C_1 + C_2 \approx 0$.

- In order to have nonzero new physics contribution to $A_{FB}$, we need $C_1 - C_2 \neq 0$. If parity were conserved in the light quark sector in dim-6 operators, one would have $C_{8q}^{LL} = C_{8q}^{RR}$, and $C_{8q}^{LR} = C_{8q}^{RL}$. If parity were conserved in the top quark sector, one would have $C_{tq}^{LL} = C_{tq}^{LR}$ and $C_{tq}^{RR} = C_{tq}^{RL}$. In either case, we end up with the vanishing condition: $(C_1 - C_2) = 0$. Therefore, in order to nonzero new physics contribution from dim-6 operators, one has to break parity $P$ both in the light quark and the top quark sectors. This might be observable in (or constrained by) parity violating effects in nucleon nuclear scattering, for example.

- The usual spin-spin correlation $C$ is strongly correlated with the top quark pair production cross section $\sigma_{t\bar{t}}$, and not with the $A_{FB}$. On the other hand, the newly defined FB spin-spin correlation $C_{FB}$ is strongly correlated with the $A_{FB}$, and thus can be another important check of any anomaly in $A_{FB}$. If there is any deviation in $A_{FB}$, should there be some deviation in $C_{FB}$ too.

- Since $\sigma_{t\bar{t}}$ and $A_{FB}$ depend only on two combinations $C_1$ and $C_2$, we can not know exactly the chiral structure of new physics from these two observables alone. We need another physical observables which are sensitive to independent combinations of coupling constants in dim-6 operators.

It is the purpose of this letter to present new observables which show different dependence on $C_{8q}^{LL}$ from $\sigma_{t\bar{t}}$ and $A_{FB}$. What we propose is the longitudinal polarization of top quark, $P_L \equiv \langle S_t \cdot \vec{n}_t \rangle$, where $\vec{n}_t$ is any unit vector defining the spin quantization axis of the top quark, and similarly for the antitop: $P_{\bar{L}} \equiv \langle \bar{S}_{\bar{t}} \times \vec{n}_{\bar{t}} \rangle$. If we choose $\vec{n}_t(i) = \vec{P}(i)/|\vec{P}(i)|$ with $\vec{P}(i)$ being the momentum vector of $t$ ($\bar{t}$), $P_L (\bar{L})$ becomes the usual helicity of (anti)top quark. Any observables corresponding to the

![FIG. 1: The region in $(C_1, C_2)$ plane that is consistent with the Tevatron data at the 1-$\sigma$ level: $\sigma_{t\bar{t}} = (7.50 \pm 0.48)$ pb and $A_{FB} = (0.158 \pm 0.072 \pm 0.017)$. Also shown are the boundaries of the regions where our effective lagrangian description is valid. For details, we refer to Ref. [14].](image.png)
longitudinal-polarization combinations \((P_L \pm \tilde{P}_L)\) vanish in QCD because of parity \((P)\) conservation. On the other hand, if there is new physics that affects \(A_{FB}\), parity is necessarily broken. Therefore one can expect nonzero \(P\)-violating polarization observables in general, which is the main point of the present work.

2. Now let us study the polarizations of \(t\) and \(\bar{t}\) at the Tevatron using the helicity amplitude method. In particular, we consider the polarization coefficients involving the longitudinal polarizations of \(t\) and \(\bar{t}\) which vanish in QCD due to its \(P\) conservation.

In the center-of-mass frame of the \(t\bar{t}\) pair, the helicity amplitudes for the process \(q(\lambda)\bar{q}(\bar{\lambda}) \to t(\sigma)\bar{t}(\bar{\sigma})\) induced by the dimension-6 operators, Eq. (3), and as well as the SM interactions are given by

\[
\mathcal{M}(\sigma, \bar{\sigma}; \lambda, \bar{\lambda}) = \frac{g^2}{s} \left[ \delta_{ij} \delta_{kl} \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{\text{sing}} + T^a_{ij} T^b_{kl} \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{\text{oct}} \right] 
\]

(6)

where we denote the helicities of the incoming quarks by \(\lambda\) and \(\bar{\lambda}\) and those of the outgoing top quarks by \(\sigma\) and \(\bar{\sigma}\), respectively, with \(\lambda, \bar{\lambda} = \pm\) and \(-\) standing for right- and left-handed particles. The singlet amplitude \(\langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{\text{sing}}\) is irrelevant in our case where we keep only the interference term between the SM and the new physics contributions. The octet amplitude \(\langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{\text{oct}}\) can be written as

\[
\langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{\text{oct}} = \sum_{A,B = L,R} \left( 1 + \frac{\hat{s}}{\Lambda^2} C^{AB} \right) \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{AB} \]

(7)

where the first and the second terms count for the contributions from the SM QCD and the dim-6 operators, respectively. The reduced amplitudes \(\langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{AB}\) are explicitly given by

\[
\langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{AB} = -\frac{m_{t}\sqrt{s}}{2} (1 + A\lambda) s_{\bar{\theta}} \delta_{\lambda,-\bar{\lambda}} \delta_{\sigma,\bar{\sigma}} \left[ \frac{\hat{s}}{4} (1 + A\lambda) (1 + \delta_{3} B\sigma) c_{\theta} - (A + \lambda) \delta_{3} B\sigma \right] \delta_{\lambda,-\bar{\lambda}} \delta_{\sigma,\bar{\sigma}} .
\]

(8)

The top-polarization weighted squared matrix elements can be computed from the helicity amplitudes by a suitable rotation \([30]\) from the helicity basis to a general spin basis:

\[
|\mathcal{M}|^2 = \frac{2 g^4}{s^2} \sum_{\lambda, \bar{\lambda}} \left\{ \text{Tr} \left[ \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle_{\text{oct}} \rho^T \langle \sigma, \bar{\sigma}; \lambda, \bar{\lambda} \rangle^\dagger_{\text{oct}} \rho \right] \right\} 
\]

(9)

where \(\rho\) and \(\tilde{\rho}\) are \(2 \times 2\) polarization density matrices for the top and anti-top, respectively:

\[
\rho = \frac{1}{2} \begin{pmatrix} 1 + P_L & P_T e^{-i\alpha} \\ P_T e^{i\alpha} & 1 - P_L \end{pmatrix},
\]

\[
\tilde{\rho} = \frac{1}{2} \begin{pmatrix} 1 + \bar{P}_L & -P_T e^{-i\alpha} \\ -P_T e^{i\alpha} & 1 - \bar{P}_L \end{pmatrix}.
\]

(10)

Here, \(P_L\) and \(\bar{P}_L\) are the longitudinal polarizations of \(t\) and \(\bar{t}\), respectively, while \(P_T\) and \(\bar{P}_T\) the degrees of transverse polarization with \(\alpha\) and \(\bar{\alpha}\) being the azimuthal angles with respect to the \(t\bar{t}\) production plane.

Neglecting the transverse polarizations, an expansion of the trace in Eq. (9) leads to

\[
|\mathcal{M}|^2 = \frac{g^4}{s^2} \left\{ D_0 + D_1(P_L + \bar{P}_L) 
+ D_2(P_L - \bar{P}_L) + D_3 P_L \bar{P}_L \right\}.
\]

(11)

The polarization coefficients \(D_i (i = 0 - 3)\) are defined in terms of the octet helicity amplitudes by

\[
D_0 = \frac{2}{9} \frac{1}{4} \sum_{\lambda, \bar{\lambda}} \left( |\langle + + ; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 + \langle \langle - - ; \lambda \bar{\lambda} \rangle_{\text{oct}} |^2 \right.
\]

\[
D_1 = \frac{2}{9} \frac{1}{4} \sum_{\lambda, \bar{\lambda}} \left( |\langle + + ; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 - \langle \langle - - ; \lambda \bar{\lambda} \rangle_{\text{oct}} |^2 \right),
\]

\[
D_2 = \frac{2}{9} \frac{1}{4} \sum_{\lambda, \bar{\lambda}} \left( |\langle + - ; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 - \langle \langle - + ; \lambda \bar{\lambda} \rangle_{\text{oct}} |^2 \right),
\]

\[
D_3 = \frac{2}{9} \frac{1}{4} \sum_{\lambda, \bar{\lambda}} \left( |\langle + + ; \lambda \bar{\lambda} \rangle_{\text{oct}}|^2 - \langle \langle - - ; \lambda \bar{\lambda} \rangle_{\text{oct}} |^2 \right) .
\]

(12)

The unpolarized coefficient \(D_0\) gives the squared amplitude summed (averaged) over the final (initial) spins and colors and one may obtain the same expression as Eq. (4) by keeping the terms up to \(O(1/\Lambda^2)\) in \(D_0\). So, the unpolarized coefficient \(D_0\) leads to the total cross section \(\sigma_{t\bar{t}}\) and the forward-backward asymmetry \(A_{FB}\). On the other hand, the coefficient \(D_3\) gives the spin-spin correlations \(C\) and \(C_{FB}\) considered and suggested before.

Note that the other two coefficients \(D_1\) and \(D_2\) are \(P\) violating. Furthermore, the coefficient \(D_1\) is odd under both the CP and CPT transformations \([32]\). In our effective lagrangian approach, new heavy particles are integrated out, and there is no new strong CP-even phase, and so \(D_1\) is zero. However, it could be nonzero when the heavy particle is explicitly included, and we keep the finite decay width of the heavy particle together with possible CP-violating phases in its couplings to light and top quarks. This issue will be discussed in full in the future publication \([31]\).

The other \(P\)-violating coefficient \(D_2\) could be observable at the Tevatron, revealing genuine features of new...
physics responsible for $A_{FB}$. Explicitly, we have obtained

$$D_2 \approx \frac{s}{9 \Lambda^2} \left[ (C'_1 + C'_2) \tilde{\beta}_1 (1 + \tilde{\beta}_3^2) + (C'_1 - C'_2) (5 - 3 \tilde{\beta}_1^2) \tilde{c}_q \right]$$

with

$$C'_1 \equiv C'^{RR}_{sq} - C'^{LL}_{sq}, \quad C'_2 \equiv C'^{LR}_{sq} - C'^{RL}_{sq}.$$ (13)

Therefore $D_2$ will provide additional information on the chiral structure of new physics in $q \bar{q} \rightarrow t \bar{t}$. When we integrate over the polar angle $\hat{\theta}$, only the first term involving

$$(C'_1 + C'_2) = C'^{RR}_{sq} - C'^{LL}_{sq} + C'^{LR}_{sq} - C'^{RL}_{sq}$$

survives. On the other hand, if we separate the forward and the backward top samples and take the difference, the orthogonal combination in the second term survives:

$$(C'_1 - C'_2) = C'^{RR}_{sq} - C'^{LL}_{sq} - C'^{LR}_{sq} + C'^{RL}_{sq}.$$ (14)

For definiteness, we consider the two new observables:

$$D \equiv \frac{(t_{RL}) - \sigma(t_{LR})}{\sigma(t_{RL}) + \sigma(t_{LR})}$$

$$D_{FB} \equiv D(\cos \hat{\theta} \geq 0) - D(\cos \hat{\theta} \leq 0)$$ (15)

which involve the sum and difference of the coefficients $C'_1$ and $C'_2$, respectively. In Fig. 2 we show the $P$-violating spin correlations $D$ and $D_{FB}$ in the $(C'_1, C'_2)$ plane. We observe that $|D|$ and $|D_{FB}|$ could be as large as 0.1 in the region $|C_{1,2}'| (1 \text{ TeV}/\Lambda^2) \lesssim 1$ which can be observed with an event sample of about 100 000 pairs after event selection cuts. Note that there are no experimental constraints on the $D$ and $D_{FB}$ observables yet, but they can be measured with a statistical precision of $\sim 5\%$ using the full anticipated Tevatron data set of 10 fb$^{-1}$ [92].

In principle, the polarization coefficients could be measured by studying the angular distributions of the top-quark decay products. The top and anti-top quarks decay into two $b$ quarks and two $W$ bosons. When both of the $W$ bosons decay leptonically, in the helicity basis, the amplitude squared can be written as

$$|M|^2 = \frac{g^4}{s^2} \left\{ D_0 + D_1 (\cos \theta_+ \ast + \cos \theta_-) + D_2 (\cos \theta_+ - \cos \theta_-) + D_3 \cos \theta_+ \ast \cos \theta_- \right\}$$ (16)

where $\theta_+$ ($\theta_-$) is the angle between the charged lepton $l^+$ ($l^-$) in the top (anti-top) rest frame and the direction of the top (anti-top) in the $t \bar{t}$ rest frame. The $M_{T2}$ variable could be useful to reconstruct the $t \bar{t}$ rest frame even with the two missing neutrinos, which deserves a further study in the future.

3. Now we study specific new physics that could generate the relevant dim-6 operators with corresponding Wilson coefficients. It is impossible to exhaust all the possibilities, and we consider the following interactions of new physics that could generate the models. This interaction lagrangian encompasses many models beyond the SM, and make a good starting point to study the underlying mechanism for the effective lagrangian discussed earlier. If the spin-1 particle has a variable $1$ in the

\begin{align}
\mathcal{L}_{int} &= g_s \sum_{A} V_{8A} \left[ \tilde{q}_A (\tilde{\eta}_A \gamma_\mu T^a q_A) + g_{8A} (\tilde{t}_A \gamma_\mu T^a t_A) \right] \\
&+ g_s \sum_{A} \left[ \tilde{V}_1 A^A \tilde{W}_{8A} \tilde{q}_A (\tilde{t}_A \gamma_\mu T^a q_A) + \tilde{V}_1 A^A \tilde{q}_A (\tilde{t}_A \gamma_\mu T^a q_A) \right] + h.c. \\
&+ g_s \sum_{A} \left[ \tilde{S}_{1A} \tilde{W}_{8A} \tilde{q}_A (\tilde{t}_A T^a q_A) + \tilde{S}_{8A} \tilde{t}_A (\tilde{t}_A T^a q_A) \right] + h.c.,
\end{align}

where $q$ denotes light quarks (either $u$ or $d$ depending on the models). This interaction lagrangian encompasses many models beyond the SM, and make a good starting point to study the underlying mechanism for the effective lagrangian discussed earlier. If the spin-1 particle has both the FC and FV interactions, we may set $V_{8A}$ and $V_{8A}^*$. After integrating out the heavy vector and scalar fields, we obtain the Wilson coefficients as follows:

$$\frac{C_{RR}^{RR}}{\Lambda^2} = -\frac{g_s^2 g_{8A}^R}{m_{8A}^R} - \frac{2 |\eta_{V_{8A}}|^2}{m_{V_{8A}}^2} + \frac{1}{N_C} \frac{|\eta_{S_{8A}}|^2}{m_{S_{8A}}^2}$$

$$\frac{C_{LL}^{LL}}{\Lambda^2} = -\frac{g_s^2 g_{8A}^L}{m_{8A}^L} - \frac{2 |\eta_{V_{8A}}|^2}{m_{V_{8A}}^2} + \frac{1}{N_C} \frac{|\eta_{S_{8A}}|^2}{m_{S_{8A}}^2}$$

$$\frac{C_{LR}^{LR}}{\Lambda^2} = -\frac{g_s^2 g_{8A}^R}{m_{8A}^R} - \frac{2 |\eta_{V_{8A}}|^2}{m_{V_{8A}}^2} + \frac{1}{N_C} \frac{|\eta_{S_{8A}}|^2}{m_{S_{8A}}^2}$$

$$\frac{C_{RL}^{RL}}{\Lambda^2} = -\frac{g_s^2 g_{8A}^L}{m_{8A}^L} - \frac{2 |\eta_{V_{8A}}|^2}{m_{V_{8A}}^2} + \frac{1}{N_C} \frac{|\eta_{S_{8A}}|^2}{m_{S_{8A}}^2}$$

where $m_{V_{8A},SL}$ and $m_{S_{8A},SL}$ denote the masses of vectors $V_{8A,SL}$ and scalars $S_{8A,SL}$, respectively.
with \( i = 1, 8 \). Note that the contributions to the coefficients \( C_{R}^{iR} \) and \( C_{R}^{iL} \) from the FC color-octet vectors may not be vanishing in the coexistence of \( V_{R} \) and \( V_{L} \) and in this case we take \( m_{V_{R}} = m_{V_{L}} = m_{V_{s}} \).

Another interesting possibility is minimal flavor violating interactions of color-triplet \( S_{\alpha}^{3} \) with mass \( m_{S_{\alpha}} \) and color-sextet scalars \( S_{ij}^{\alpha\beta} \) with mass \( m_{S_{ij}} \) with the SM quarks [33]. Here \( \alpha, \beta, \gamma \) and \( i,j,k \) are color and flavor indices, respectively. For example, if we consider the following interactions (Model V and VI in Ref. [33]),

\[
\mathcal{L} = g_{s} \frac{\bar{\psi}_{a} \gamma^{5} \epsilon^{ijk} \bar{u}_{iaR} u_{jaR} S^{\gamma}_{ij} + \bar{u}_{iaR} g_{s} \eta_{i} u_{iaL} S_{ij}^{\alpha\beta} + h.c.]}{2}
\]

the \( u \)-channel exchange of new scalars can contribute to \( u\bar{u} \rightarrow t\bar{t} \), resulting in

\[
C_{RR}^{iR} \left( \frac{m_{S_{ij}}}{\Lambda^{2}} \right)^{2} = \frac{|\eta_{i}|^{2}}{m_{S_{ij}}} + \frac{2|\eta_{i}|^{2}}{m_{S_{ij}}},
\]

Since these new scalars couple only to the right-handed up-type quarks, constraints on the couplings \( \eta_{i} \) and \( \eta_{s} \) from flavor physics are rather weak, and one can accommodate the observed \( A_{FB} \) easily.

In Table II we show the new particle exchanges under consideration and the signs of the couplings induced by them. Note that the particle exchanges with \((C_{1} - C_{2}) > 0 \) are preferred by the positive \( A_{FB} \) at the 1-\( \sigma \) level.

Let us first consider the FV cases. Among the FV interactions with vector or scalar bosons, \( \tilde{V}_{R,SL} , \tilde{S}_{1R,1L} \), and \( S_{13}^{\alpha\beta} \) can give the correct sign for \((C_{1} - C_{2}) \) [14]. But one can not discriminate one model from another only with the \( A_{FB} \) measurement. From Table II we observe that each of the four cases with \( \tilde{V}_{R} , \tilde{V}_{L} , \tilde{S}_{1R} \), and \( \tilde{S}_{1L} \) gives a different sign combination of \( C_{1}^{+} + C_{2}^{+} \) and \( C_{1}^{+} - C_{2}^{+} \). Therefore, a simple sign measurement of \( C_{1}^{+} \) at the Tevatron. However, the flavor-conserving case is considered taking \( g_{s}^{R} g_{s}^{L} (1 \text{ TeV/m}_{V_{s}})^{2} = +1 \).

Unlike the FV cases, the FC color-octet vectors can always accommodate the positive sign of \((C_{1} - C_{2}) \). For the case of \( V_{R} \) (V\( _{L} \)), the couplings \( g_{R}^{R} (g_{L}^{L}) \) and \( g_{R}^{L} (g_{L}^{R}) \) must have different signs to accommodate the positive \( A_{FB} \). In Fig. 3 we also show the predictions of the model with \( V_{R} \) or \( V_{L} \) vector for \( D \) and \( D_{FB} \).

Up to now, we have only one type of couplings by assuming that only one resonance contributes to the \( t\bar{t} \) production at the Tevatron. However, the flavor-conserving color-octet \( V_{R} \) and \( V_{L} \) vectors can coexist in general, and then the situation could be more complicated. In such a general case, all the four couplings \( C_{RR}^{R} \), \( C_{CL}^{L} , C_{DR}^{R} \), and \( C_{RR}^{RR} \) could be nonzero, in contrast to the
Table I: New particle exchanges and the signs of induced couplings $C^{AB}$ ($A, B = R, L$), $C_1 - C_2, C_1' + C_2'$, and $C_1' - C_2'$.

| Resonance | $C^{RR}$ | $C^{LL}$ | $C^{LR}$ | $C^{RL}$ | $C_1 - C_2$ | $C_1' + C_2'$ | $C_1' - C_2'$ | $A_{FB}$ |
|-----------|---------|---------|--------|--------|-----------|-----------|-----------|--------|
| $\tilde{V}_{1R}$ | $-$ | 0 | 0 | 0 | $-$ | $-$ | $-$ | $\times$ |
| $\tilde{V}_{1L}$ | 0 | $-$ | 0 | 0 | $-$ | $+$ | $+$ | $\times$ |
| $\tilde{V}_{sR}$ | $+$ | 0 | 0 | 0 | $+$ | $+$ | $+$ | $\checkmark$ |
| $\tilde{V}_{sL}$ | 0 | $+$ | 0 | 0 | $-$ | $-$ | $-$ | $\checkmark$ |
| $\tilde{S}_{1R}$ | 0 | 0 | 0 | $-$ | $+$ | $+$ | $-$ | $\checkmark$ |
| $\tilde{S}_{1L}$ | 0 | 0 | $-$ | 0 | $+$ | $-$ | $+$ | $\checkmark$ |
| $\tilde{S}_{sR}$ | 0 | 0 | 0 | $+$ | $-$ | $-$ | $+$ | $\times$ |
| $\tilde{S}_{sL}$ | 0 | 0 | $+$ | 0 | $-$ | $-$ | $-$ | $\times$ |
| $S^d_2$ | $-$ | 0 | 0 | 0 | $-$ | $-$ | $-$ | $-$ | $\checkmark$ |
| $S^{d\alpha}_2$ | $+$ | 0 | 0 | 0 | $+$ | $+$ | $+$ | $\checkmark$ |
| $\tilde{V}_{sR}$ | $\pm$ | 0 | 0 | 0 | $\pm$ | $\pm$ | $\pm$ | $\sqrt{(+)}$ or $\times(-)$ |
| $\tilde{V}_{sL}$ | 0 | $\pm$ | 0 | 0 | $\mp$ | $\mp$ | $\mp$ | $\sqrt{(+)}$ or $\times(-)$ |
| $\tilde{V}_{sR}, \tilde{V}_{sL}$ | indef. | indef. | indef. | indef. | indef. | indef. | indef. | indef. |

\begin{align*}
(C_1 + C_2)/\Lambda^2 &= -g_{8q^R}g_{8q}^L(r_q + 1)(r_t + 1)/m_{V_8}^2 \\
(C_1 - C_2)/\Lambda^2 &= -g_{8q^R}g_{8q}^L(r_q - 1)(r_t - 1)/m_{V_8}^2
\end{align*}

with $r_q \equiv g_{8q}^R/g_{8q}^L$ and $r_t \equiv g_{8t}^R/g_{8t}^L$. Any deviation of $r_q$ ($r_t$) from 1 characterizes $P$ violation in the light (top) quark sector. In Fig. 4, we show the 1-σ region in $(r_3, r_q)$ plane taking $g_{8q}^Lg_{8t}^R (1\text{ TeV}/m_{V_8})^2 = +1$. We observe the consistent region lies along the line $r_t = -1$ ($r_q = -1$) with $1 < r_q < 3$ ($1 < r_t < 3$). When $g_{8q}^Lg_{8t}^R (1\text{ TeV}/m_{V_8})^2 = -1$, one may have obtain similar results, except that the green region consistent with $A_{FB}$ would be reflected with respect to the $r_q = 1$ line. In Fig. 5, we show the predictions of the general model with $V_{sR}$ and $V_{sL}$ vectors is considered.

Previous one-coupling cases. In this case, the sum and differences of the couplings can be reparametrized as

\begin{align*}
(C_1 + C_2)/\Lambda^2 &= -g_{8q^R}g_{8q}^L(r_q + 1)(r_t + 1)/m_{V_8}^2 \\
(C_1 - C_2)/\Lambda^2 &= -g_{8q^R}g_{8q}^L(r_q - 1)(r_t - 1)/m_{V_8}^2
\end{align*}

\begin{equation}
\Delta \sigma_{ii} \Delta A_{FB} \propto D D_{FB}.
\end{equation}

Let us note that $\Delta \sigma_{ii} \propto (C_1 + C_2)$, $\Delta A_{FB} \propto (C_1 + C_2)$, $D \propto (C_1' + C_2')$, and $D_{FB} \propto (C_1' - C_2')$. Furthermore, we observe

\begin{equation}
g_{8q^R}g_{8q}^L \left( \frac{\Lambda}{m_{V_8}} \right)^2 = \frac{[(C_1' + C_2') - (C_1 + C_2)][(C_1' + C_2') - (C_1 - C_2)]}{4(C_1' + C_2')}.
\end{equation}
where, for the last term, the relation \((C_1 + C_2)(C_1 - C_2) = (C'_1 + C'_2)(C'_1 - C'_2)\) is used. This explains the linear dependence of \(D\) and \(D_{\text{FB}}\) on \(g_{St}^L g_{St}^L (\Lambda/m_t)^8\) with some finite range coming from the current 1-σ experimental errors on \(\sigma_t\) and \(A_{\text{FB}}\), as shown in Fig. 5. We see that one of \(|D|\) and \(|D_{\text{FB}}|\) could be as large as \(~1\) when the other one is very small, while both of them could be \(~0.1\) simultaneously.

6. In this letter, we extended the model independent study of \(tt\) productions at the Tevatron using dimension-6 contact interactions relevant to \(q\bar{q} \rightarrow tt\), mainly concentrating on the longitudinal (anti)top polarization of \(P_t\) and \(P_{\bar{t}}\) in the helicity frame. As emphasized in Ref. [14], new physics affecting the Tevatron \(A_{\text{FB}}\) necessarily breaks parity unlike QCD. Then the \(P\)-odd top-quark longitudinal polarization observables can be nonzero, in sharp contrast to the case of pure QCD. Therefore, nonvanishing longitudinal polarization observables will be another important aspect of \(P\)-violating new physics relevant to \(q\bar{q} \rightarrow tt\). Most importantly, the longitudinal polarization of (anti)top quark can give another important clue for the chiral structure of new physics, which is completely independent of \(\sigma_t\) or \(A_{\text{FB}}\).

Using the conditions for the couplings of four-quark operators that could generate the FB asymmetry observed at the Tevatron (with the updated data on \(A_{\text{FB}}\)) [14], we studied the possible ranges of longitudinal (anti)top polarization, and their correlations with \(\sigma_t\) and \(A_{\text{FB}}\). Then we considered the \(s\)-, \(t\)-, and \(u\)-channel exchanges of spin-0 and spin-1 particles whose color quantum number is either singlet, octet, triplet or sextet. Our results in Table I encode the predictions for the \(P\)-odd observables corresponding to the polarization difference \((P_t - P_{\bar{t}})\) in various new physics scenarios in a compact and an effective way, when those new particles are too heavy to be produced at the Tevatron but still affect \(A_{\text{FB}}\). If these new particles could be produced directly at the Tevatron or at the LHC, we cannot use the effective lagrangian any more. We have to study specific models case by case including the new particles explicitly, and anticipate rich phenomenology at colliders as well as at low energy. Detailed study of these issues lies beyond the scope of this letter, and will be discussed in the future publications [31].

**Note Added:** While we were finishing this paper, we received three preprints [3-5] which also consider the observables related with the (anti)top polarization. In our work, we note that parity violation is crucial for new physics to make nonzero contributions to \(A_{\text{FB}}\), and the longitudinal polarization of (anti)top quark can give another important clue for the chiral structure of new physics.

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[37] Although we assume the \( SU(2)_L \times SU(2)_R \) chiral symmetry for light quarks, all the explicit models do not satisfy this condition. In that case, one can interpret \( q = u, d, s, c, b \).

[38] Throughout this work, unless explicitly written, we are taking \( C_{AB}^{ij} = C_{AB}^{\alpha \beta} = C_{AB}^{u \bar{u}} \) assuming the \( SU(2)_L \times SU(2)_R \) chiral symmetry. Under this assumption, the down-quark contribution to \( \sigma_{\ell \bar{\ell}} \) and \( A_{FB} \) is suppressed relative to the up-quark one by a factor more than \( \sim 6 \) at the Tevatron.

[39] The \( \tilde{T} \) transformation reverses the signs of the spins and the three-momenta of the asymptotic states, without interchanging initial and final states, and the matrix element gets complex conjugated.

[40] \( C_{1q}^{BR} \) is also induced by color-triplet and sextet scalars, but is not shown, since it is irrelevant here.