Critical Behavior of the $q = 3, 4$-Potts model on Quasiperiodic Decagonal Lattices

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Abstract

In this study, we have performed Monte Carlo simulations of the $q = 3, 4$-Potts model on quasiperiodic decagonal lattices in order to study criticality presents in these systems. Using the single histogram technique together with the finite-size scaling analysis, we estimated the infinite lattice critical temperatures and the leading critical exponents for both $q = 3$ and $q = 4$ states. Our estimates for the critical exponents on quasiperiodic decagonal lattices are in good agreement with the exact ones on 2D periodic lattices, supporting the idea that not only $q = 3$ as well as $q = 4$ Potts model on quasiperiodic lattices belongs to the same universality class as those on 2D periodic lattices.

Keywords: Quasiperiodic decagonal lattices, $q$-Potts model, Critical exponents, Monte Carlo simulation

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1. Introduction

Electron diffraction patterns exhibiting octagonal, decagonal, dodecagonal and icosahedral point symmetry are found in a variety of alloys. The most well known is the icosahedral phase in Al-Mn alloys, which is obtained when these materials are cooled so rapidly that their constituent atoms do not have adequate time to form a crystal lattice. These structures are known as quasi-crystals [1, 2]. In principle, quasi-crystals are characterized as atomic structures that present long-range quasi-periodic translational and long-range orientational order. They can exhibit rotational symmetries otherwise forbidden to crystals. In the last decades, quasi-crystals have attracted significant research attention, mostly because of their stronger magnetic properties and greater elasticity at higher temperatures, when compared to traditional crystals.

One of the most intriguing questions about quasi-crystals is to determine whether its intrinsic complicated structure can change the universality class from its counterpart periodic structure. To this end, Potts model [3] offers a simple and feasible way to study quasi-crystals from this perspective, as it contains both first- and second-order phase transitions. However, given the lack of periodicity of the quasi-periodic lattices, only numerical approaches can be performed. Previous Monte Carlo studies on the ferromagnetic Potts model of quasi-periodic lattices [4–9] have revealed that both the systems belong to the same universality class, despite the fact that the critical temperature of the quasi-periodic lattices is higher than square lattices. However, given the great variety of existing quasi-periodic lattices, this question has not been solved completely. Consequently, this necessitates extensive computational research on the issue of accurately estimating static critical exponents on these lattices. To the best of our knowledge, studies concerning the $q = 4$ Potts model on quasiperiodic lattices have been rarely reported in the literature.

In this study, we have investigated the critical behavior of the ferromagnetic $q = 3, 4$-Potts model on quasi-periodic decagonal lattices (QDL) to accurately estimate the infinite QDL critical temperature and critical exponents for each case. The quasi-periodic lattices analyzed in this study were generated using the strip projection method [10–12] with spins placed in the vertices of the rhombi that make up the QDL (Fig. 1). Periodic boundary conditions were applied on these lattices in...
order to avoid the boundary effects caused by the finite size.

The contents of the article are organized as follows. The section 2 briefly describes the strip projection method adopted for generating the QDL and the periodic boundary conditions used in the simulations. Details of the model and Monte Carlo simulations have been described in section 3. In section 4, we give a rapid description of the FSS relations used in the study. In section 5, we have presented the results for both $q = 3$ and $q = 4$ Potts model and compared them with previous results on quasi-periodic lattices. Finally, in section 6, we make the conclusions.

2. Strip projection method and periodic boundary conditions

The strip projection method used in this study is a powerful technique for constructing periodic and non-periodic lattices. The methodology can be summarized as follows. Firstly, starting from a regular lattice $\mathbb{Z}^n$ whose unit cell, $\phi$, is spanned by the $n$ vectors $[\vec{d}_1, \ldots, \vec{d}_n]$, we can resolve $\mathbb{R}^n$ into two mutually orthogonal subspaces, namely, $\vec{e}_\phi$ and $\vec{e}_\perp$, of dimensions $p$ and $n - p$, respectively, i.e., $\mathbb{R}^n = \vec{e}_\phi \oplus \vec{e}_\perp$. Then, we define a ‘strip’ $s \in \mathbb{R}^n$ as a set of all the points whose positions are given by adding any vector in $\vec{e}_\phi$ to any vector in $\phi$, i.e., $s = \vec{e}_\phi \oplus \phi$. The required lattice, $L^\parallel$, is the projection in $\vec{e}_\phi$ of all points in $\mathbb{Z}^n$ that are included in the strip, i.e., $L^\parallel = \pi^\parallel(\mathbb{Z}^n \cap s)$. The requirement that any point $\vec{x} \in \mathbb{Z}^n$ lies in the strip is equivalent to the condition that the projection of $\vec{x}$ in $\vec{e}_\phi$ lies within the projection of $\phi$ in $\vec{e}_\perp$, i.e.,

$$\vec{x} \in s \Leftrightarrow \vec{x}_\phi \in \phi_\perp,$$

where $\vec{x}_\phi = \pi^\phi(\vec{x})$ and $\phi_\perp = \pi^\phi(\phi)$. Accordingly, the lattice can be defined as follows:

$$L^\parallel = \{\vec{x}_\phi | \vec{x} \in \mathbb{Z}^n, \vec{x}_\phi \in \phi_\perp\}.$$

(2)

One of the ways to describe the projections of the points $\vec{x} \in \mathbb{Z}^n$ given by $\vec{x} = \sum_{i=1}^n u_i \vec{d}_i$ (where the $u_i$’s are integers), onto $\vec{e}_\phi$ and $\vec{e}_\perp$ is to choose an orthogonal basis $\{\vec{b}_1, \ldots, \vec{b}_p\}$ in $\vec{e}_\phi$ and an orthogonal basis $\{\vec{b}_{p+1}, \ldots, \vec{b}_n\}$ in $\vec{e}_\perp$. Put together, they form a new basis $\{\vec{b}_1, \ldots, \vec{b}_n\}$ of $\mathbb{R}^n$. Assuming $b_1 = a_1$, the relationship between the two basis can be given by a rigid rotational operation. By defining a rotation matrix $\rho$, it is possible to determine the projection matrices using the following equations

$$\pi_{ij}^\parallel = \sum_{k=1}^p \frac{\rho_{ik}\rho_{jk}}{\sigma_k}, \quad \pi_{ij}^\perp = \sum_{k=p+1}^n \frac{\rho_{ik}\rho_{jk}}{\sigma_k},$$

(3)

where $\sigma_k = \sum_{j=1}^n \rho_{kj}^2$. The rotation matrix $\rho$ can be split into an $n \times p$ submatrix $\rho^\parallel$ and an $(n - p) \times (n - p)$ submatrix $\rho^\perp$:

$$\rho = \begin{pmatrix} \rho^\parallel \\ \rho^\perp \end{pmatrix}.$$

(4)

For generating the decagonal quasi-periodic lattice, the points of the finite region of a five-dimensional hypercubic lattice ($\mathbb{Z}^5$) are projected onto a two-dimensional subspace ($\vec{e}_\phi$), only if these points are projected inside an rhombic icosahedron, which in this case is the strip. The resulting quasi-periodic lattice is obtained through the standard rotation matrix:

$$\rho = \frac{1}{\sqrt{10}} \begin{pmatrix} 2 & 1/\tau & -\tau & -\tau & 1/\tau \\ 0 & \tau \lambda & -\lambda & -\tau \lambda & 2 \\ -\tau & 1/\tau & 1/\tau & -\tau & 0 \\ 0 & 1 & -\tau & \tau & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix},$$

(5)

where $\tau = (1 + \sqrt{5})$ and $\lambda = \sqrt{(3 - \tau)}$. The decagonal quasiperiodic lattice consists of two types of building blocks, usually represented by a fat rhombus with an acute angle of $2\pi/5$ and a thin rhombus with an acute angle of $\pi/5$, arranged according to specific matching rules. However, the quasi-periodic lattices are not suitable for Monte Carlo simulation with periodic boundary conditions. A more suitable approach is to produce a periodic approximant of these lattices. This can be realized by just replacing the golden number $\tau$ in the sub-matrix $\rho^\perp$ of Eq. (5) by a rational number $F_i/F_{i-1}$.
we have updated our lattices through the Wolff algorithm [13]. For a fixed temperature, we define a Monte Carlo step (MCS) per spin by accumulating the flip times of all spins and then dividing by the total spin number. As usual the Hamiltonian of the $q$ -states ferromagnetic Potts model ($J > 0$) can be written as

$$H = -J \sum_{i<j} \delta(\sigma_i, \sigma_j),$$

where $\delta$ is the Kronecker delta function, and the sum runs over all nearest neighbors of $\sigma_i$. We also define the order parameter $m$ as

$$m = \frac{1}{q-1}(N_{\text{max}}qL^{-2} - 1),$$

where $N_{\text{max}}$ is the maximum number of spins in the same state, $L^2 = N$ is total number of spins. Once the critical region has been established, we have applied the single histogram method [14, 15] along with FSS analysis to obtain highly accurate estimates for the critical temperature and critical exponents. System sizes up to $N = 65391$ were used in these simulations with $1.5 \times 10^6$ MCS per spin performed at a single temperature $T_0$ where $5 \times 10^3$ configurations were discarded for

| $\nu$ | $\alpha/\nu$ | $\beta/\nu$ | $\gamma/\nu$ |
|-------|-------------|-------------|-------------|
| $T_0 = 1.033$ | $0.853 \pm 0.011$ | $0.439 \pm 0.004$ | $0.136 \pm 0.003$ | $1.741 \pm 0.017$ |
| $T_0 = 1.035$ | $0.832 \pm 0.012$ | $0.448 \pm 0.004$ | $0.130 \pm 0.003$ | $1.762 \pm 0.015$ |

Figure 2: Log-log plot of the size dependence of the maximum values of the thermodynamic derivatives $g(L) = dU/dK$ (filled black circle), $\phi_1$ (red triangle) and $\phi_2$ (blue diamond) for the $q = 3$ on the QDL. The simulation temperature was performed at $k_B T_0/J = 1.033$.

Figure 3: Log-log plot of the size dependence of the maximum values of the thermodynamic derivatives $g(L) = dU/dK$ (black circle), $\phi_1$ (red triangle) and $\phi_2$ (blue diamond) for the $q = 4$ on the QDL. The simulation was performed at $k_B T_0/J = 0.943$.

3. Model and Monte Carlo Simulation

In order to study the critical behavior on the QDL, we have updated our lattices through the Wolff algorithm [13]. For a fixed temperature, we define a Monte Carlo step (MCS) per spin by accumulating the flip times of all spins and then dividing by the total spin number. As usual the Hamiltonian of the $q$ -states ferromagnetic Potts model ($J > 0$) can be written as

$$H = -J \sum_{i<j} \delta(\sigma_i, \sigma_j),$$

where $\delta$ is the Kronecker delta function, and the sum runs over all nearest neighbors of $\sigma_i$. We also define the order parameter $m$ as

$$m = \frac{1}{q-1}(N_{\text{max}}qL^{-2} - 1),$$

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thermalization. For each system size considered, we averaged on 100 independent realizations in order to have rough estimates of the statistical errors. We calculated static thermodynamics quantities such as the specific heat, magnetic susceptibility, logarithmic derivatives of the order parameter, and Binder’s fourth-order cumulants \[16, 17\] in the critical region. From the analysis of the location of the maximum values of these quantities and their magnitudes one can estimate the infinite QDL critical temperature and critical exponents, respectively. For the \(q = 3\), we performed simulations at the temperatures \(k_B T_0/J = 1.033\) and \(k_B T_0/J = 1.035\) and we used reweighted probability distribution from \(k_B T/J = 1.028\) to \(k_B T/J = 1.060\) for \(N = 1131, 1785, 2617, 3551, 4659\) and from \(k_B T/J = 1.028\) to \(k_B T/J = 1.048\) for \(N = 5919, 7285, 10445, 14271, 21111, 23543, 29117\) and 65391. For the \(q = 4\), we performed simulations at the temperatures \(k_B T_0/J = 0.940\) and \(k_B T_0/J = 0.943\) and we used reweighted probability distribution from \(k_B T/J = 0.9300\) to \(k_B T/J = 0.960\) for the first five smaller lattices and from \(k_B T/J = 0.930\) to \(k_B T/J = 0.948\) for the remainders. From the fluctuations of \(E\) measurements, we can obtain the specific heat

\[
C(T) = \frac{K^2}{N} (\langle E^2 \rangle - \langle E \rangle^2),
\]

(8)

Similarly, from the fluctuations of \(m\), we can obtain the magnetic susceptibility

\[
\chi(T) = KN(\langle m^2 \rangle - \langle m \rangle^2),
\]

(9)

and the fourth-order magnetization cumulant

\[
U(T) = 1 - \frac{\langle m^4 \rangle}{3 \langle m^2 \rangle^2}.
\]

(10)

We can also obtain the logarithmic derivative of \(n\)-power of \(m\), i.e.,

\[
\phi_n = \frac{\partial}{\partial K} \ln \langle m^n \rangle
\]

\[
= \frac{\langle m^n \rangle |E \rangle}{\langle m^n \rangle} - \langle E \rangle .
\]

(11)

Table 2: Estimates of the ratios of leading critical exponents for \(q = 4\) Potts model on QDL.

| \(\nu\) | \(\alpha/\nu\) | \(\beta/\nu\) | \(\gamma/\nu\) |
|---|---|---|---|
| \(T_0 = 0.940\) | 0.645 ± 0.015 | 1.025 ± 0.011 | 0.118 ± 0.001 | 1.752 ± 0.010 |
| \(T_0 = 0.943\) | 0.634 ± 0.010 | 1.125 ± 0.011 | 0.113 ± 0.002 | 1.795 ± 0.020 |

Figure 4: Size dependence of the effective critical temperatures \(T_c(L)\) estimated from the thermodynamics derivatives \(\phi_1\) and \(\phi_2\). The curves are straight line fits to Eq. [16] with \(\nu = 0.853\). The simulation temperature is the same as in Fig. [4].

Figure 5: Log-log plot of the magnetization \(m\) (measured at the temperature with maximum value of \(dm/dK\)) versus linear size \(L = \sqrt{N}\) for the \(q = 3\) on the QDL.
4. Finite-size scaling relations

4.1. $q = 3$ Potts Model

According to the finite-size scaling theory [18, 19], the free energy of a system of linear dimensional $L$ is described by the scaling ansatz

$$f(t, h) = L^{-d} \tilde{f}(t L^{1/v}, h L^{(\gamma + \beta)/\nu})$$

(12)

where $t = (T - T_c)/T_c$ ($T_c$ is the infinite QDL critical temperature) and $h$ is the magnetic field. The leading critical exponents $\alpha$, $\beta$, $\gamma$ and $\nu$ define the universality class of the system. Considering zero-field regime, the derivatives of the Eq. (12) yield important scaling equations, i.e.,

$$m = L^{-\gamma/v} \tilde{m}(x),$$

(13)

$$C = L^{\beta/v} \tilde{C}(x),$$

(14)

$$\chi = L^{\gamma/v} \tilde{\chi}(x),$$

(15)

where $\tilde{m}$, $\tilde{C}$ and $\tilde{\chi}$ are scaling functions, and $x = t L^{1/v}$ is the temperature scaling variable. In addition, the critical temperature scales as

$$T(L) = T_c + a L^{-1/v},$$

(16)

where $a$ is a constant and $T_L$ is the effective transition temperature for the QDL of linear size $L$. This effective temperature can be obtained by the location of the maximum of any of the above quantities: $\phi_x$, $\phi_y$, $dU/dK$, $C$ and $\chi$. One can also obtain independent estimate of $\nu$ through the slope of the size dependence of the maxima of $\phi_x$ and $dU/dK$.

4.2. $q = 4$ Potts Model

Due to the presence in two-dimensional $q = 4$ Potts model of a marginal operator [20, 21], which is absent in any other two-dimensional Potts model, the leading power-law scaling behavior of this model is modified by multiplicative logarithms. So the Eqs. (12-16) must be modified to allow for these logarithmic corrections. The free energy scaling relation is suitably modified [22, 23] by

$$f(t, h) = L^{-d} \tilde{f}(t L^{1/v}(\ln L)^{y}, h L^{(\gamma + \beta)/\nu}(\ln L)^{\gamma}),$$

(17)

where $y_t = \frac{\hat{\beta}}{\hat{\beta} + \hat{\gamma}}$ and $y_h = \frac{\hat{\gamma} + \hat{\beta}}{2\hat{\gamma}}$. Moreover, considering zero-field regime, the derivatives of the Eq. (17) yield suitable scaling equations for 4-state Potts model:

$$m = L^{-\gamma/v} \tilde{m}(x),$$

(18)

$$C = L^{\beta/v} \tilde{C}(x),$$

(19)

$$\chi = L^{\gamma/v} (\ln L)^{\gamma} \tilde{\chi}(x),$$

(20)

where $\tilde{m}$, $\tilde{C}$ and $\tilde{\chi}$ are scaling functions, and $x = t L^{1/v}$ is the temperature scaling variable. Correspondingly, the critical temperature scales as

$$T(L) = T_c + a L^{-1/v} (\ln L)^{\gamma}$$

(21)

In above equations, the leading critical exponents [24] for the $q = 4$ Potts model on 2D periodic lattices periodic are given by

$$\alpha = \frac{2}{3}, \quad \beta = \frac{1}{12}, \quad \gamma = \frac{7}{6}, \quad \nu = \frac{2}{3}$$

(22)
and the logarithmic-correction exponents \( [27, 25, 26] \) are given by
\[
\hat{\alpha} = -1, \quad \hat{\beta} = -\frac{1}{3}, \quad \hat{\gamma} = \frac{3}{4}, \quad \hat{\nu} = \frac{1}{2}, \quad (23)
\]

5. Results

5.1. \( q = 3 \) Potts Model

From the slope of the linear fit of the size dependence of the maximum of the quantities \( g(L) \equiv \phi_1, \phi_2 \) and \( dU/dK \), we obtain three estimates for \( 1/\nu \). Fig. 2 shows the log-log plot of these quantities, which were obtained by reweighting the data simulated at \( T_0 = 1.033 \), for each system size. We obtained \( \nu = 0.765 \pm 0.013 \) for \( dU/dK \), \( \nu = 0.894 \pm 0.022 \) for \( \phi_1 \) and \( \nu = 0.899 \pm 0.021 \) for \( \phi_2 \). By combining these results, we get \( \nu = 0.853 \pm 0.011 \). Similar analysis was performed for \( k_B T_0/J = 1.035 \), which yielded \( \nu = 0.736 \pm 0.019 \) for \( dU/dK \), \( \nu = 0.876 \pm 0.022 \) for \( \phi_1 \) and \( \nu = 0.883 \pm 0.023 \) for \( \phi_2 \). Combining these results, \( \nu = 0.832 \pm 0.012 \). These estimates are in good agreement with the exact result on the 2D periodic lattice \( (\nu = 5/6) \) and in reasonable agreement with estimates on other quasiperiodic systems \([4, 8, 9]\). After obtaining an estimate for \( \nu \), the infinite QDL critical temperature is estimated by plotting the size dependence of the location of the peaks of \( \phi_1 \) and \( \phi_2 \). In Fig. 3 is shown the finite-size scaling of the effective transition temperatures at \( k_B T_0/J = 1.033 \). We obtained \( T_c = 1.038(7) \) for both \( \phi_1 \) and \( \phi_2 \). Simulating at the temperature \( k_B T_0/J = 1.033 \) with the corresponding \( \nu \) estimated at this temperature, we obtained \( T_c = 1.038(8) \) for both \( \phi_1 \) and \( \phi_2 \). These values of \( T_c \) are higher than the exact value on the 2D periodic lattice \( [27] \) \( k_B T_c/J = 1/\ln(1 + \sqrt{3}) \approx 0.995 \).

Using the Eqs. \([13, 15]\) for the size dependence of the maximum values of \( m, C \) and \( \chi \), we can estimate \( \beta/\nu, \alpha/\nu \) and \( \gamma/\nu \), respectively. In Fig. 5 is shown the log-log plot of \( m \) (measured at the temperature with maximum value of \( dm/dK \) versus linear size of the system \( L \). The slopes of the linear fit of the data obtained by simulating at \( T_0 = 1.035 \) and \( T_0 = 1.033 \) were \( \beta/\nu = 0.130 \pm 0.003 \) and \( \beta/\nu = 0.136 \pm 0.003 \), respectively. While in Fig. 6 is shown the log-log plot of the maximum values of \( C \) versus linear size of the system. Particularly, in this plot we have inserted correction-to-scaling terms \([28]\) in order to improve the fit quality of the data by scaling as
\[
C = aL^{\alpha/\nu}(1 + bL^{-\omega}), \quad (24)
\]
where the proper correlation amplitudes \( a = 1.2 \) and \( b = 2.0 \) and the nonuniversal correction-to-scaling exponents \( \omega = 1.0 \) were chosen in order to minimize the \( \chi^2 \) of the fit. The slopes of the linear fit of the data obtained by simulating at \( T_0 = 1.035 \) and \( T_0 = 1.033 \) were \( \alpha/\nu = 0.448 \pm 0.004 \) and \( \alpha/\nu = 0.439 \pm 0.004 \), respectively. Similarly, Fig. 7 shows a log-log plot of the maximum values of \( \chi \) versus \( L \). The estimated values for \( \gamma/\nu \) at \( T_0 = 1.035 \) and \( T_0 = 1.033 \) were \( \gamma/\nu = 1.762 \pm 0.015 \) and \( \gamma/\nu = 1.741 \pm 0.017 \), respectively. In these figures, the error bars are only statistical and were estimated over 100 different runs for each data point. The estimated ratios of critical exponents along with the average estimate of \( \nu \) at each simulated temperature \( T_0 \) are summarized in Table 1. From Table 1
multiplying the values of the ratios of exponents at each simulated temperature \( T_0 \) by its respective value of \( \nu \), we obtained \( \alpha = 0.373 \pm 0.006 \), \( \beta = 0.108 \pm 0.003 \) and \( \gamma = 1.463 \pm 0.022 \) at \( T_0 = 1.035 \), and \( \alpha = 0.374 \pm 0.006 \), \( \beta = 0.111 \pm 0.003 \) and \( \gamma = 1.50 \pm 0.02 \) at \( T_0 = 1.033 \). On the 2D periodic lattice, the exact values for \( \nu = 3 \) Potts model of the critical exponents are \( \nu = 5/6 \approx 0.833 \), \( \beta = 1/9 \approx 0.111 \), \( \alpha = 1/3 \approx 0.333 \) and \( \gamma = 13/9 \approx 1.444 \).

5.2. \( q = 4 \) Potts Model

In Fig. 10 is shown the log-log plot of the size dependence of the maximum of the quantities \( g(L) = \phi_1, \phi_2 \) and \( dU/dK \). From the slope of the linear fit of these quantities, we obtained \( \nu = 0.592 \pm 0.020 \) for \( dU/dK \), \( \nu = 0.648 \pm 0.015 \) for \( \phi_1 \) and \( \nu = 0.663 \pm 0.011 \) for \( \phi_2 \). Combining these results, we find \( \nu = 0.634 \pm 0.010 \). Similarly at \( k_BT_0/J = 0.940 \), we obtained \( \nu = 0.610 \pm 0.032 \) for \( dU/dK \), \( \nu = 0.663 \pm 0.023 \) for \( \phi_1 \) and \( \nu = 0.662 \pm 0.022 \) for \( \phi_2 \). Combining these results, we find \( \nu = 0.645 \pm 0.015 \). As one can see, these average estimates for \( \nu \) are in reasonable agreement with the exact result on 2D periodic lattices, and especially for \( \phi_1 \) and \( \phi_2 \), we have a very good convergence to this exact value.

The finite-size scaling of the effective transition temperatures is shown in Fig. 8. From the location of the peaks of the quantities \( \phi_1 \) and \( \phi_2 \) and using Eq. (21), we obtained \( T_c = 0.943(4) \). Simulating at the temperature \( k_BT_0/J = 1.043 \) with \( \nu \) estimated at this temperature, we obtained \( T_c = 0.944(9) \). Following the case \( q = 3 \), these values are also higher than the exact value on 2D periodic lattices \( k_BT_c/J = 1/ln(1 + \sqrt{4}) \approx 0.910 \).

Using the Eqs. (18-20) and taking the exact exponents from the Eqs. (22) and (23) in the logarithmic-correction terms, we can estimate \( \beta/\nu, \alpha/\nu \) and \( \gamma/\nu \). In Fig. 10 is shown the log-log plot of the maximum values of \( C \) versus linear size \( L = \sqrt{N} \) for the \( q = 4 \) Potts model on quasiperiodic decagonal lattice. The slopes of the linear fit of the data obtained by simulating at \( T_0 = 0.943 \) and at \( T_0 = 0.940 \) were \( \beta/\nu = 0.113 \pm 0.002 \) and \( \beta/\nu = 0.118 \pm 0.001 \), respectively. In Fig. 11 is shown a log-log plot of the maximum values of \( \chi \) versus \( L = \sqrt{N} \) for the \( q = 4 \) Potts model on the two-dimensional square lattice are \( \nu = 2/3 \approx 0.667 \), \( \beta = 1/12 \approx 0.083 \), \( \alpha = 2/3 \approx 0.667 \) and \( \gamma = 7/6 \approx 1.167 \).

6. Conclusions

In this study, we performed MC simulations of the \( q = 3, 4 \)-Potts model on quasiperiodic decagonal lat-
tices in order to estimate the infinite critical temperature and leading critical exponents for both $q = 3$ and $q = 4$ states. We found that for both $q = 3$ and $q = 4$ states, the infinite critical temperature is higher than that of the square lattice, which results from the different geometric structure between the two models. For $q = 3$ Potts model, the leading critical exponents $\nu$, $\beta$ and $\gamma$ are, within the error precision, in good agreement with the corresponding values of 2D periodic lattices whereas for $q = 4$ Potts model, all the critical exponents are found very close to the exact ones on 2D periodic lattices, being therefore a strong indication that not only $q = 3$ as well as $q = 4$ Potts model on quasiperiodic lattices belongs to the same universality class as those on 2D periodic lattices.

7. Acknowledgements

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