Helicity in atmosphere as a prognostic criterion

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Abstract. It is shown theoretically that the tendency of the height of the isobaric surface of 1000 hPa is proportional to the integral helicity of the geostrophic flow. This result is confirmed by calculations.

1. Introduction

The concept of helicity of the atmospheric velocity field is widely used as a diagnostic characteristic of intense vortices \cite{1}. In some cases, helicity is also a predictor of cyclogenesis for tropical and Mediterranean cyclones \cite{1}, polar mesocyclones \cite{2}, the start of monsoon circulation \cite{3} and others. Previously, the authors demonstrated the prognostic properties of integral helicity using some specific examples \cite{4,5,6}. It is shown that the 1000 hPa constant pressure surface height trend is proportional to integral helicity of a geostrophic flow. This result is confirmed by calculations.

Keywords: helicity, prognostic criterion, 1000 hPa constant pressure surface height trend.

2. The prognostic meaning of integral helicity.

The integral helicity of the velocity field for atmospheric motions is considered in the form

\[ h = \int_{0}^{H_{\text{a}}} s \, dz, \quad s = \vec{V} \cdot \text{rot} \vec{V}, \]

where \( z \) is the height, \( H_{\text{a}} \) is the height of the upper boundary of the atmosphere, \( h \) is the integral helicity, \( \vec{V} \) – is the wind speed vector.

We consider the integrand in the reduced isobaric coordinate system \((x, y, \zeta = p/P, P = 1000 \text{ gPa})\) in an exploded view:

\[ s = u \left( \frac{\partial \tau}{\partial y} - \frac{\partial v}{\partial \zeta} \right) + v \left( \frac{\partial u}{\partial \zeta} - \frac{\partial \tau}{\partial x} \right) + \tau \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right), \]

where \( u, v, \tau \) – components of wind speed. Regrouping the members, we get

\[ s = \left( -u \frac{\partial v}{\partial \zeta} + v \frac{\partial u}{\partial \zeta} \right) + \left( u \frac{\partial \tau}{\partial y} - v \frac{\partial \tau}{\partial x} \right) + \tau \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \]
First expression \((-u \frac{\partial v}{\partial \zeta} + v \frac{\partial u}{\partial \zeta}\)) describes the effect of vertical wind shear in horizontal, second \((u \frac{\partial \tau}{\partial y} - v \frac{\partial \tau}{\partial x})\) — horizontal heterogeneity of vertical currents, third \((\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})\) — dependence on the vertical component of the vortex of velocity and vertical velocity. In the quasi-geostrophic approximation, the second and third expressions in parentheses of the right-hand side vanish, and the first expression describes the geostrophic advection of temperature. Then (2) takes the form

\[
S = \frac{g^2}{l^2} \left( \frac{\partial H}{\partial y} \frac{\partial}{\partial x} \left( \frac{\partial H}{\partial \zeta} \right) - \frac{\partial H}{\partial x} \frac{\partial}{\partial y} \left( \frac{\partial H}{\partial \zeta} \right) \right) = \frac{g^2}{l^2} \left( \frac{\partial H}{\partial \zeta} , H \right) = - \frac{R}{l^2} \zeta A_T, \tag{3}
\]

where \(g\) is the acceleration of gravity, \(l\) is the Coriolis parameter, \(H\) is the geopotential height, \(R\) is the specific gas constant of dry air, and \(A_T = \frac{g}{l}(T,H), \) is the Jacobian.

We use the heat influx equation, the temperature in the standard way through the height of isobaric surface, and the components of wind velocity from geostrophic relations:

\[
-\zeta^2 \frac{\partial}{\partial \zeta} \frac{\partial H}{\partial t} - \frac{R}{g} \zeta A_T = \frac{c^2}{Rg} \tau, \tag{4}
\]

c\(^2\) — static stability parameter.

We get function \(A_T\) with the help of (3) and substitute it into equation (4) in a standard way,

\[
-\zeta^2 \frac{\partial}{\partial \zeta} \frac{\partial H}{\partial t} + \frac{l}{g} \zeta^2 = - \frac{c^2}{Rg} \tau.
\]

Integrating this expression, previously multiplied by \(\zeta^{-2}\), over \(\zeta\) in the range from \(\zeta \to 0\) to \(\zeta = 1\) with boundary conditions

\[
\tau = 0 \quad \text{as} \quad \zeta \to 0 \quad \text{and when} \quad \zeta = 1, \tag{5}
\]

assuming \(c^2 = const\), using the mean value theorem we get that at a certain average level \(\zeta_{av}\), where \(\tau\) reaches the extremum.

\[
\left( \frac{\partial H}{\partial t} \right)_{\zeta=\zeta_{av}} = \frac{l}{g} \int_{0}^{\zeta_{av}} s d\zeta = \frac{l}{g} h_g.
\]

Here \(h_g\) is the integral helicity of the geostrophic flow.

Trend

\[
q = \partial H / \partial t \tag{6}
\]
as \(\zeta \to 0\) is a bounded function due to the upper boundary condition in (5). Then we finally get

\[
q_{\zeta=1} = - \frac{l}{g} h_g \tag{7}
\]

Thus, the tendency of the height of the isobaric surface 1000 hPa is proportional to the integral helicity of the geostrophic flow. This circumstance allows us to theoretically interpret and clearly demonstrate the prognostic value of the integral helicity of the atmospheric motion velocity field.

3. Computational experiments and analysis of relationship between \(h\) and \(H_{1000}\).

For 5 large regions (Atlantic, European Russia, Siberia, the Far East, the Arctic zone of the Russian Federation), calculations were performed with a lead time of 6 and 12 hours of integrated helicity of the wind field \((h)\) and compared with an isobaric surface height of 1000 hPa \((H_{1000})\) over 196 observation periods based on reanalysis data.
The data for 4 observation periods (00, 06, 12, 18 UTC) of the following ECMWF reanalyses were used as initial data:
- over the territory covering the European part of Russia, Siberia and Eastern Europe: May 28, 2017 - June 30, 2018 (136 observation periods),
- over the territory covering the Atlantic and eastern North America: 08/28/2011 - 09/11/2011 (60 periods of observations - tropical storm “Katia”).

Table 1. Calculation Areas.

| Area                                      | borders                |
|-------------------------------------------|------------------------|
| (A) European territory of Russia          | 40 N - 65 N, 25 E - 60 E |
| (B) Siberia                               | 40 N - 65 N, 60 E - 120 E |
| (C) Far East                              | 40 N - 65 N, 120 E - 140 E |
| (D) Arctic                                | 65 N - 80 N, 25 E - 180 E |
| (E) Atlantic and eastern North America    | 15 N - 60 N, 10 W - 90 W |

For terms t+6 and t+12 (hour) are calculated:

\[
\hat{H}_{1000}(t + 6) \approx H_{1000}(t) - \delta t \cdot l \cdot g^{-1} \cdot h(t),
\]

\[
\hat{H}_{1000}(t + 12) \approx H_{1000}(t) - 2\delta t \cdot l \cdot g^{-1} \cdot h(t)
\]

where \(\delta t = 6\) hours, \(l\) is the Coriolis parameter, \(g\) is the acceleration of free fall.

Estimated statistical relationship \(\text{cor}[\hat{H}_{1000}(t+6), H_{1000}(t+6)]\) and \(\text{cor}[\hat{H}_{1000}(t+12), H_{1000}(t+12)]\)

The calculation results are shown in the table 2.

Table 2. Correlation coefficients between calculated (prognostic) (\(\hat{H}\)) and actual (\(H\)) surface heights of 1000 hPa when calculated for 6 and 12 hours.

| Area                                      | \(\text{cor}[\hat{H}_{1000}(t+6), H_{1000}(t+6)]\) | \(\text{cor}[\hat{H}_{1000}(t+12), H_{1000}(t+12)]\) |
|-------------------------------------------|----------------------------------------------------|--------------------------------------------------------|
| (A) European territory of Russia          | 0.940                                              | 0.808                                                  |
| (B) Siberia                               | 0.891                                              | 0.706                                                  |
| (C) Far East                              | 0.878                                              | 0.669                                                  |
| (D) Arctic                                | 0.933                                              | 0.787                                                  |
| (E) Atlantic and eastern North America    | 0.953                                              | 0.881                                                  |

The table 2 shows that all regions are characterized by a high correlation coefficient. The highest values of the correlation coefficient between the calculated and observed values were found over the Atlantic, the European territory of Russia, and the Arctic. Over Siberia and the Far East, the correlation coefficient is slightly lower, apparently due to the weaker geostrophicity of the ongoing processes.

Thus, by calculation experiments it was confirmed that the height of the isobaric surface of 1000 hPa, calculated using expression (7), strongly correlates with the observational data at a lead time of 6-12 hours. This circumstance, taking into account (6), confirms the prognostic value of the integral helicity with respect to the isobaric surface of 1000 hPa.

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“Fundamentals of Breakthrough Technologies in the Interests of National Security”, “Searching Fundamental Scientific Research for the Development of the Russian Arctic”.

References
[1] Kurgansky M V 2017 Izv. Atmos. Ocean. Phys. 53(2), 127–41
[2] Vazaeva N V, Chkhetiani O G, Maksimenkov L O, Kurgansky M V 2017 Integral Characteristics of Polar Lows https://elibrary.ru/item.asp?id=30690481 Accessed on 2019-12-12
[3] Makosko A A, Rubinstein K G 2014 Study of Asian Monsoon Helicity Based on Reanalysis Data and Results of Digital Simulation of Atmospheric Circulation with Account of Nonuniformity of Gravity Dokl. Earth Sci. 459 237-42
[4] Makosko A A, Maksimenkov L O 2018 On the prognostic value of one criterion for assessing the helicity of the velocity field of atmospheric movements Turbulence, dynamics of the atmosphere and climate. -- M. Fizmatkniga. 145-149. ISBN - 978-5-89155-312-5.
[5] Makosko A A, Maksimenkov L O 2019 To the prognostic meaning for the one of criteria for helicity estimation in atmosphere Turbulence, Atmosphere and Climate Dynamics / IOP Conference Series: Earth and Environmental Sciences, 231 012033, pp. 1-7. doi: 10.1088 / 17551315/231/1/012033.
[6] Makosko A A , Maksimenkov L O 2019 A new prognostic indicator of adverse and dangerous weather phenomena is the gradient of the integral helicity of the atmospheric motion velocity field Problems of Risk Analysis, 16( 2), 50-57. doi.org/10.32686/1812-52202019-16-2-50-57.