Optimal Portfolio of Distinct Frequency-Response Services in Low-Inertia Systems

Luis Badesa, Student Member, IEEE, Fei Teng, Member, IEEE, and Goran Strbac, Member, IEEE

Abstract—A reduced level of system inertia due to renewable integration increases the need for cost-effective provision of ancillary services, such as Frequency Response (FR). In this paper, we propose a closed-form solution to the differential equation describing frequency dynamics, which allows to obtain frequency-security algebraic constraints to be implemented in optimisation routines. This is done while considering any finite number of FR services with distinguished characteristics, such as different delivery times and activation delays. The problem defined by these frequency-security constraints can be formulated as a Mixed-Integer Second-Order Cone Program (MISOCP), which can be efficiently handled by off-the-shelf conic optimisation solvers. This paper also takes into account the uncertainty in inertia contribution from the demand side by formulating the frequency-security constraints as chance constraints, for which an exact convex reformulation is provided. Finally, case studies highlighting the effectiveness of this frequency-secured formulation are presented.

Index Terms—Power system dynamics, inertia, frequency response, uncertainty, convex optimisation.

NOMENCLATURE

Indices and Sets

- $i, j, n$: All-purpose indices.
- $k, K$: Index, Set of FR services fully delivered by nadir.
- $l, L$: Index, Set of FR services ramping up by the nadir.
- $s, S$: Index, Set of all FR services.
- $|S|$: Cardinality of set $S$.

Constants

- $\alpha$: Probability of meeting the nadir constraint.
- $\Delta f_{\text{max}}$: Maximum admissible frequency deviation (Hz).
- $\eta$: Probability of meeting the RoCoF constraint.
- $\sigma$: Standard deviation of inertia from demand (MW·s).
- $f_0$: Nominal frequency of the power grid (Hz).
- $H_p$: Forecast for inertia from demand (MW·s).
- $P_{L_{\text{max}}}$: Upper bound for $P_L$ (MW).
- $\text{RoCoF}_{\text{max}}$: Maximum admissible RoCoF (Hz/s).
- $T_s$: Delivery time of FR service $s$ (s).
- $T_{\text{del},s}$: Delay in provision of FR service $s$ (s).

Decision Variables

- $H$: System inertia from thermal generators (MW·s).
- $P_L$: Largest power infeed (MW).
- $R_s$: Maximum FR provision from service $s$ (MW).

Functions and Operators

- $\Phi(\cdot)$: Standard normal cumulative distribution function.
- $\Phi(t)$: Time-evolution of aggregated system FR (MW).
- $P(\cdot)$: Probability operator.

Random Variables

- $H_D$: System inertia from the demand side (MW·s).

I. INTRODUCTION

INCREASING penetration of renewable energy in power grids introduces many challenges, such as uncertainty and variability in generation. Furthermore, most renewable sources, including wind and photovoltaic, do not contribute to system inertia due to being decoupled from the grid by power electronic converters. While system inertia and Frequency Response (FR) were services widely available in grids dominated by thermal generation as by-products of energy production, the increasing scarcity of these services in low-carbon system increments the costs associated to their provision [1]. These frequency services are necessary to contain the frequency drop after a power outage, in order to avoid the tripping of RoCoF relays and/or the activation of Under-Frequency Load Shedding.

In this context, several works have studied optimal strategies to provide inertia and FR [2]–[12]. Authors in [2] studied the design of an ancillary service market for FR, based on solving a constrained optimisation problem that guarantees frequency security. The frequency-security constraints were obtained heuristically from dynamic simulations of the system. A similar approach was used in [3], [4], while the other works focus on deducing the frequency-security region by analytically solving the differential equation describing post-fault frequency dynamics.

To tackle the declining system inertia, FR services with faster delivery have been introduced by system operators [13]. However, a fundamental question yet to be answered is how to optimise the portfolio of FR services from providers with diverse characteristics under different system conditions. References [5]–[9] aggregate the response from all FR providers uniformly, therefore only allowing to consider a single FR service. References [10], [11] co-optimise the two FR services defined in the UK up to date, namely Enhanced Frequency Response (EFR) delivered by one second after the outage, and Primary Frequency Response (PFR) delivered ten seconds after the outage. The only work that integrates different dynamics from generic FR providers is [12], by considering an affine conservative approximation of the frequency dynamics until reaching the nadir. Furthermore, the impact of the activation delays in FR has been shown to affect the system frequency stability [14], which has not been considered in any of the above works.
In the available literature, system inertia has been assumed either fixed in the optimisation of FR [3], [5] or fully controllable by the Unit Commitment (UC) [7], [11]. In fact, a considerable amount of inertia contribution is available from demand, which is not controllable and can only be forecasted: assuming a central authority clearing a pool market using a frequency-secured UC, the inertia from demand can only be forecasted with certain accuracy. Given the risk aversion of system operators, it is necessary to explicitly model such uncertainty, in addition to the uncertainty associated with renewable generation. Furthermore, optimally scheduling the largest online unit has been demonstrated to provide both economic and emission savings [15], [16], which also needs to be co-optimised along with other frequency services.

Given this background, this paper develops an efficient frequency-constrained optimisation framework that recognises and appropriately values the different dynamics of FR services. The contributions are three-fold:

1) This paper proposes closed-form conditions to optimise, for the first time, any finite number of FR services with diverse dynamics while considering any combination of activation delays. The proposed frequency constraints are formulated as Mixed-Integer Second-Order Cone Program (MISOCP), which can be efficiently solved by taking advantage of the recent development of conic optimisation software.

2) The uncertainty associated with the inertia contribution from the demand side is explicitly modeled in the optimisation problem through chance constraints. A convex-reformulation of the chance constraints allows maintaining the problem as an MISOCP.

3) The frequency-constrained Stochastic Unit Commitment (SUC) model is applied to several case studies, which highlight the benefits of a frequency-security framework allowing to co-optimise a diverse portfolio of services.

The rest of this paper is organised as follows: Section II provides the deduction of frequency-security constraints. The proposed analytical model for guaranteeing frequency security is validated through dynamic simulations in Section III. Section IV includes the results from several relevant case studies, while Section V gives the conclusion and proposes future lines of work.

II. CLOSED-FORM CONDITIONS FOR SECURE POST-FAULT FREQUENCY DYNAMICS

In this section we deduce the closed-form conditions for frequency security, that allow to map the sub-second dynamics of transient frequency to any desired time-scale, such as the typical minutes to hour resolution of a UC. Frequency security is respected if sufficient inertia and FR services are available at the time of the power outage to contain and recover the system frequency.

The conditions for a secure post-fault frequency evolution can be deduced from solving the swing equation [17]:

\[ \frac{2(H + H_D)}{f_0} \frac{d\Delta f(t)}{dt} = FR(t) - P_L \]  (1)

Eq. (1) assumes the loss of the largest power infed, therefore representing the N-1 requirement. Load damping has been neglected, as the damping level will be significantly reduced in future power systems that are increasingly dominated by power electronics [5].

The system inertia is aggregated from the inertia provided by all devices, including thermal generators and certain loads such as induction motors, and it includes two components: \( H \) is the controllable term in the UC optimisation, as it is a decision variable which depends on the generators that are scheduled to be online; on the other hand, \( H_D \) is the inertia contribution from demand, which is assumed here to be non-controllable but can be forecasted for time-periods in the near future. The largest possible power outage \( P_L \) can be considered as a decision variable, which would involve part-loading a large generating unit or interconnector [11], [16].

Function \( FR(t) \) in (1) represents the frequency control for Frequency Response, a power injection from several system devices following the outage. We model this function to consider \( |S| \) different FR services, in which each FR service is the slowest.

\[
FR(t) = \begin{cases} \sum_{s \in S} R_s \cdot \frac{T_s}{t} & \text{if } t \leq T_1 \\ R_1 + \sum_{s=2}^{\mid S \mid} R_s \cdot \frac{T_s}{t} & \text{if } T_1 < t \leq T_2 \\ \ldots & \text{if } T_{\mid S \mid - 1} < t \leq T_{\mid S \mid} \\ \sum_{s=1}^{\mid S \mid - 1} R_s + \frac{R_s |S|}{T_{\mid S \mid}} & \text{if } T_{\mid S \mid} < t \leq T_{\mid S \mid + 1} \\ \sum_{s \in S} R_s & \text{if } t > T_{\mid S \mid} \end{cases} \]  (2.1-2.2)

The delivery time \( T_s \) of a service \( s \) is the time by which full FR capacity for the service is delivered. Piecewise function (2) models the delivery of FR from each provider \( s \) as ramping up during the interval \( t \in (0, T_s] \), and constant for \( t > T_s \). This approach, proposed in [5] for a single FR service, is guaranteed to conservatively approximate any controller proportional to frequency deviation, as we further demonstrate in Section III. Note that FR services are ordered from service 1 up to service \( |S| \) in increasing delivery time, i.e. service \( FR_1 \) is the fastest and service \( FR_{|S|} \) is the slowest.

A. Deducing Frequency-Security Constraints

By solving (1), the three constraints that guarantee a secure post-fault frequency evolution can be obtained. The RoCoF constraint is deduced following National Grid’s standard [18], which established that the level of system inertia must be sufficient to limit the highest instantaneous RoCoF at \( t = 0 \):

\[ |RoCoF(t = 0)| = \frac{P_L \cdot f_0}{2(H + H_D)} \leq RoCoF_{\text{max}} \]  (3)

For frequency to stabilise eventually after the fault, the amount of FR available must be at least equal to the power outage. In other words, the steady-state constraint is obtained...
by setting RoCoF to zero in (1) and considering that every FR service has been fully delivered:

$$\sum_{i \in S} R_i \geq P_L \tag{4}$$

Since FR(t) defined in (2) is a piecewise function, the nadir constraint depends on the time-interval when the nadir occurs. For the nadir to take place in a given time-interval $t \in [T_{n−1}, T_n)$, the two following conditions must be met:

$$\left(\sum_{i=1}^{n-1} R_i + \sum_{j=n}^{\lceil|S|\rceil} R_j \frac{T_{n−1}}{T_j} \leq P_L\right) \text{ and } \left(\sum_{i=1}^{n} R_i + \sum_{j=n+1}^{\lceil|S|\rceil} R_j \frac{T_n}{T_j} > P_L\right) \tag{5}$$

Condition (5) states that the power injected from FR becomes greater than the power loss $P_L$ not before $T_{n−1}$ and no later than $T_n$. Before $T_{n−1}$, only the fastest $n−1$ FR services have been fully delivered, while the rest are still ramping up; after $T_n$, the $n$th service has been fully delivered as well.

The solution of (1) for $t \in [T_{n−1}, T_n)$ is:

$$\Delta_f(t) = \frac{f_0}{2(H+H_D)} \left[\sum_{j=n}^{\lceil|S|\rceil} R_j \frac{T_{n−1}}{2T_j} + \sum_{i=1}^{n} R_i \left(t-T_i\right)\right] - P_L \cdot t \tag{6}$$

The time within $t \in [T_{n−1}, T_n)$ at which nadir is exactly reached is given by setting RoCoF to zero in (1) for that given time interval:

$$t_{nadir} = \frac{P_L - \sum_{i=1}^{n-1} R_i}{\sum_{j=n}^{\lceil|S|\rceil} R_j / T_j} \tag{7}$$

By substituting (7) into (6), the condition for respecting the nadir requirement can be deduced:

$$|\Delta_f(t_{nadir})| = |\Delta_f(t = t_{nadir})| \leq \Delta f_{max} \tag{8}$$

Finally, expanding the expression in (8) and enforcing the conditions for $t_{nadir}$ to occur during time-interval $t \in [T_{n−1}, T_n)$, the nadir constraint is obtained as:

$$\begin{cases} \sum_{i=1}^{n-1} R_i + \sum_{j=n}^{\lceil|S|\rceil} R_j \frac{T_{n−1}}{T_j} \leq P_L & (10.1) \\
\sum_{i=1}^{n} R_i + \sum_{j=n+1}^{\lceil|S|\rceil} R_j \frac{T_n}{T_j} > P_L & (10.2)
\end{cases}$$

then enforce:

$$\left(\frac{H+H_D}{f_0} - \sum_{i=1}^{n-1} \frac{R_i T_i}{4 \Delta f_{max}} - \sum_{j=n}^{\lceil|S|\rceil} R_j \frac{T_{n−1}}{T_j}\right) \geq \frac{\left(P_L - \sum_{i=1}^{n-1} R_i\right)^2}{4 \Delta f_{max}} = x_1 \tag{9}$$

As one must consider the possibility of nadir occurring at any time $t \in [0, T_{\lceil|S|\rceil})$ (note that the nadir must occur before $T_{\lceil|S|\rceil}$, as otherwise the steady-state constraint (4) would not hold), $\lceil|S|\rceil$ different nadir constraints must be defined, corresponding to each time-interval $[T_{s−1}, T_s)$ \forall $s \in S$. Only one constraint will be enforced, which is the constraint for which the if-statement in (9) is met. Note that conditional statements in optimisation can be implemented using a big-M formulation with auxiliary binary decision variables [19]. Constraints (9) are nonlinear but are in fact rotated Second-Order Cones (SOCs), therefore convex constraints as $x_1$ and $x_2$ in (9) are nonnegative. SOC Programming generalises

Linear Programming, and recently developed interior-point methods allow to efficiently solve these types of conic optimisation problems to global optimality [20]. Furthermore, SOC Programs are the highest class of conic problems whose mixed-integer counterpart can be solved to global optimality using commercial optimisation packages. Since the nadir constraints in (9) introduce binary variables for implementing the conditional statements, the resulting optimisation problem is an MISOCOP.

In conclusion, constraints (3), (4) and (9) guarantee frequency security in a power system, while considering the dynamics of any finite number $|S|$ of different FR providers.

B. Considering Activation-Delays in Certain FR Services

In Section II-A, every FR service is considered in (2) to start ramping up at the very moment of the power outage. Those FR services would therefore react to any deviation from nominal frequency in the grid, not necessarily caused by the loss of a large power infed. Here we generalise the model to account for some FR services which start providing FR some time after the power outage. In conclusion, constraints (3), (4) and (9) guarantee frequency security in a power system, while considering the dynamics of any finite number $|S|$ of different FR providers.

The FR service defined in (10) is activated $T_{del,i}$ seconds after the fault, a delay which can be driven by either the frequency deadband of a droop control or the communication delay of an activation signal sent to the FR provider. For the following deductions in this section, $FR(t)$ as defined in (2) may now include FR services with an activation delay as the one defined in (10). An example considering four FR services is included in Fig. 1, where services $FR_1$ and $FR_2$ start ramping up at the very moment of the fault (the fault is assumed to happen at $t = 0$) while services $FR_3$ and $FR_4$ have an activation delay.

The RoCoF and steady-state constraints constraints, (3) and (4), remain unchanged while the nadir constraints must be updated if some FR services have an activation delay. Following the same procedure as in Section II-A, the swing equation is solved for the different time-intervals, yielding the following nadir constraint:

$$\sum_{i=1}^{n} R_i \tag{10}$$
\[
\left( \frac{H + H_D}{f_0} - \sum_{k \in K} \frac{R_k(T_k + 2T_{del,k})}{4\Delta f_{max}} + \sum_{l \in L} \frac{R_l T_{del,l}^2}{T_l} \right) \sum_{l \in L} \frac{R_l}{T_l} \geq (P_L - \sum_{k \in K} R_k + \sum_{l \in L} R_l T_{del,l}/T_l)^2 = y_2^2
\]

Note that if every FR service starts ramping up exactly when the fault occurs, i.e. \( T_{del,k} = T_{del,l} = 0 \ \forall k \in K, \forall l \in L \), (11) reduces to (9). Therefore, (11) generalises (9) allowing to consider any combination of activation delays for FR services. The problem defined is still an MISOCP, since (11) is a rotated SOC. In a similar fashion as in (9), as many nadir constraints as intervals defined by the piecewise FR(t) must be included, along with the corresponding conditional statements for nadir to occur in that interval. For the example in Fig. 1, each time-interval is delimited by a tick in the x-axis.

C. Uncertainty in Inertia Contribution from Demand

In this section we propose chance constraints that allow to take into account the inertia contribution from demand subject to forecasting errors. The inertia from demand, \( H_D \), is considered as a random variable for which a forecast is available, along with a distribution on the forecasting error. In order to account for this uncertainty in the frequency-security conditions deduced in Sections II-A and II-B, we modify the RoCoF and nadir constraints to become chance constraints, i.e. constraints that must be met above a pre-defined probability. Here we provide an exact convex reformulation of the nonconvex chance constraints, to allow the system operator to limit the risk of violating each frequency constraint. The error in inertia forecasting is assumed to follow a Gaussian distribution (but any log-concave probability density function still makes the following deductions valid):

\[
H_D \sim N(H, \sigma^2)
\]  

The chance constraint for meeting the RoCoF requirement is given by the following nonconvex constraint, based on (3):

\[
P\left( H_D \geq \frac{P_L \cdot f_0}{2 \cdot \text{RoCoF}_{\text{max}}} - H \right) \geq \eta
\]  

Note that \( H \) is a decision variable since it is the inertia contribution from the generators scheduled to be online in the UC, and therefore \( H \) is not subject to uncertainty.

Since the constraint inside the probability operator in (13) is linear, and making use of the log-concave property of the normal distribution for \( H_D \) [21], the nonconvex chance constraint (13) is equivalent to:

\[
\Phi\left( \frac{-\frac{P_L \cdot f_0}{2 \cdot \text{RoCoF}_{\text{max}}} + H + H}{\sigma} \right) \geq \eta
\]  

Therefore, the exact linear reformulation of the RoCoF chance constraint (13) is:

\[
H + H - \Phi^{-1}(\eta) \sigma \geq \frac{P_L \cdot f_0}{2 \cdot \text{RoCoF}_{\text{max}}}
\]  

The chance constraint for the nadir requirement, using the notation in (11), is:

\[
P\left( \left( \frac{H + H_D}{f_0} + y_1 \right) y_2 \geq y_3^2 \right) = P\left[ g(H_D) \leq 0 \right] \geq \alpha
\]

Where function \( g(H_D) \) is given by:

\[
g(H_D) = - \left( \frac{H + H_D}{f_0} + y_1 \right) y_2 + y_3^2
\]

Function \( g(H_D) \) is linear with respect to the random variable \( H_D \), therefore it also follows a normal distribution [22]:

\[
g(H_D) \sim N\left( \frac{-\frac{H + H}{f_0} + y_1}{\sigma}, \frac{\sigma^2 y_3^2}{f_0} \right)
\]

Again making use of the log-concave property of the normal distribution, (16) becomes:

\[
\Phi\left( \frac{0 - \mu}{\sigma} \right) \geq \alpha
\]

Expanding and rearranging (19):

\[
\left( \frac{H + \frac{1}{H} \Phi^{-1}(\alpha) \sigma + y_1}{f_0} \right) y_2 \geq y_3^2
\]

Constraint (20) is a rotated SOC, therefore it provides a convex reformulation of the chance constraint (16), using the notation for the linear expressions \( y_1, y_2 \) and \( y_3 \) from (11). As in (11), any combination of distinct FR services with or without activation delays can be considered.

III. VALIDATION OF THE FREQUENCY-SECURITY CONSTRAINTS

The frequency-security constraints obtained in Section II are purely mathematical deductions from the swing equation (1), and hence guaranteed to provide the security region entailing no approximation. However, the assumptions for function FR(t) in (2) are conservative, as considering detailed frequency controls in the swing equation would impede to solve it algebraically, and therefore no closed-form frequency-security conditions could be obtained. In this section we demonstrate that the assumptions for FR(t) do indeed underestimate an actual frequency droop control, a demonstration based on comparing FR(t) with an actual dynamic simulation to which we feed the solution of a frequency-constrained optimisation.

For the validation of the dynamic model for post-fault frequency, a generic case including four FR services has been considered: two FR services with no activation delay, FR1 and FR2, with delivery times \( T_1 = 3s \) and \( T_2 = 10s \); an FR service, FR3, with \( T_{del,3} = 0.5s \) and \( T_3 = 5s \); and another FR service, FR4, with \( T_{del,4} = 1s \) and \( T_4 = 8s \). An operating point exactly meeting the nadir constraint (11) was fed into a dynamic simulation in MATLAB/Simulink, for which the dynamics of FR providers are modelled as in Fig. 2. The operating condition exactly meeting the nadir is \( P_L = 1.8GW \), \( H = 180GWs \), \( R_1 = 0.2GW \), \( R_2 = 0.98GW \), \( R_3 = 0.5GW \), \( R_4 = 0.6GW \). A damping term of 0.15GW/Hz was added to the dynamic simulation, in order to analyse the impact of neglecting such support in the frequency constraints.
Fig. 2. Block diagram for the simulation of the system frequency dynamics.

Fig. 3. Post-fault frequency deviation from the dynamic simulation.

Fig. 4. Time-evolution of FR obtained from the dynamic simulation considering the four different providers: FR$_1$ and FR$_2$ do not have an activation delay (black and red lines), while FR$_3$ and FR$_4$ have a delay (green and purple lines). The dashed lines represent the FR profile for each provider assumed in (2).

The results of the dynamic simulation are shown in Fig. 3 and Fig. 4. The nadir is of 0.72Hz, indeed above the $\Delta f_{max} = 0.8$Hz requirement. Although the nadir constraint was binding for this system condition, the 0.08Hz conservativeness in the simulation is due to neglecting the damping support and the linear-ramp assumption for FR delivery in (2).

Although the validation of the frequency constraints has been performed for a simple droop control for frequency dynamics as in Fig. 2, the linear-ramp assumption for FR delivery in (2) can conservatively approximate a generic FR control, as demonstrated by [5]. In other words, the dashed lines in Fig. 4 can be tuned to underestimate other frequency controllers, by simply defining $T_{del,i}$ and $T_s$ appropriately. Therefore, as long as all generic FR services $i$ can deliver $R_i$ MW by the delivery time $T_i$, the proposed nadir frequency-security constraints can guarantee the compliance with the dynamic frequency requirements.

The characteristics of thermal plants in GB’s 2030 system are given in Table I. A 10GW pump-storage unit is included, with 2.6GW rating and 75% round efficiency, corresponding to the Dinorwig plant in Great Britain (GB). Battery Energy Storage Systems (BESS) with 90% efficiency and a 5h tank are also present, with a capacity of 200MW.

Simulations spanning one year of operation were run, with frequency-security requirements of $Rocof_{max} = 0.5$Hz/s and $\Delta f_{max} = 0.8$Hz, while $P_{max}^{L} = 1.8$GW. The quantiles for the SUC were set to 0.005, 0.1, 0.3, 0.5, 0.7, 0.9 and 0.995.

A. Importance of Defining and Co-Optimising New FR Services

Creating new FR services involves a tradeoff between improved market efficiency and increased market complexity. A fundamental question that needs to be answered is “How many and which new FR services should be defined?” While previous models [10], [11] only allow to co-optimise up to two

| Number of Units | Nuclear | CCGT | OCGT |
|-----------------|--------|------|------|
| Rated Power (MW) | 1800 100 30 |
| Min Stable Generation (MW) | 1400 250 50 |
| No-Load Cost (£/h) | 0 4500 3000 |
| Marginal Cost (£/MWh) | 10 47 200 |
| Startup Cost (£) | N/A 10000 0 |
| Startup Time (h) | N/A 4 0 |
| Min Up Time (h) | N/A 4 0 |
| Min Down Time (h) | N/A 1 0 |
| Inertia Constant (s) | 5 4 4 |
| Max FR deliverable (MW) | 0 50 20 |

IV. CASE STUDIES

In order to highlight the importance of co-optimising the provision of distinct FR services, as well as to understand the value of different services under diverse system conditions, several case studies were carried out in a Stochastic Unit Commitment model [23], with the frequency-security constraints deduced in Section II implemented.
distinct FR services, here we show the benefits of optimising additional FR services by applying the proposed frequency-secured formulation, which can efficiently co-optimise any number of FR services. This section therefore focuses on quantifying the value in defining new FR services using the GB 2030 system, while the results give insight on the benefits that new services would bring to any different system.

In this section, some Combined Cycle Gas Turbines (CCGTs) are assumed to provide FR in less than 10s, corresponding to a “fast PFR” service that some gas plants could provide [24]. The generation mix in Table I is considered, and from the total number of CCGTs, 70% are assumed to provide PFR in 10s, 20% have the capability of providing FR in 7s and the 10% remaining have the capability of providing FR in 5s. Four different cases for FR services are defined:

- **Base case:** only EFR and PFR are defined by the system operator, as is current practice in the UK. Therefore, all CCGTs are considered to provide PFR even if some of them can actually achieve faster FR dynamics. EFR is provided by the BESS.
- **Case 1:** a new FR service is defined, FR\(_2\), delivered in 7s (i.e. \(T_2 = 7s\)), therefore the CCGTs with the capability of providing FR in 5s and 7s can provide this FR\(_2\).
- **Case 2:** a new FR service FR\(_2\) is defined with \(T_2 = 5s\), therefore the CCGTs with the capability of providing FR in 5s can provide this FR\(_2\), while the CCGTs with the capability of providing FR in 7s will provide PFR.
- **Case 3:** two new FR services are defined, FR\(_2\) with \(T_2 = 5s\) and FR\(_3\) with \(T_3 = 7s\). This allows to fully extract the value of the different dynamics in FR-provision available.

For each of these cases, two different wind-capacity levels are considered: 40GW and 60GW. The nuclear units are assumed to be fully loaded, therefore \(P_L = P_L^{\text{max}} = 1.8GW\).

The results are presented in Fig. 5, showing the benefits of the three cases referred to the base case. It is interesting to note that Case 2 shows higher benefits than Case 1: by having only 10% of the CCGTs providing FR in 5s in Case 2 (all other CCGTs are assumed to provide PFR in 10s), higher savings can be achieved than in Case 1, where 30% of the CCGTs provide FR in 7s (20% with the capability of providing FR in 7s plus 10% with the capability of 5s). The results clearly demonstrate the complex task of defining new services, which will be a tradeoff between the FR speed of delivery and the amount of provision. Finally, Case 3 shows the benefits from taking full advantage of fast-FR from CCGTs, by defining two new FR services with delivery times of 5s and 7s. For all cases, the savings increase with wind penetration, as higher wind capacity implies lower inertia available from thermal generators, and therefore recognising fast FR services becomes more valuable.

Note that the benefits of co-optimising fast FR services are not only in terms of cost, but also in reduced wind curtailment, as shown in Fig. 5: by defining the new FR services, a lower number of thermal generators need to stay online simply for providing inertia and FR, and therefore more wind power can be accommodated. The wind-curtailment reduction in Fig. 5 is again referred to the base case, for which wind curtailment was of 31.86TWh/year for the 40GW-wind-capacity scenario (which means a 26.36% of wind energy curtailed) and of 79.97TWh/year for the 60GW-wind-capacity (44.10% wind energy curtailed).

As an example of the computational efficiency of the proposed formulation, the simulation for Cases 1 and 2 took roughly 2h50min to run. Note that this simulation represents a whole year of operation of the GB 2030 system, for the computationally intensive Stochastic-UC problem. The SUC optimisations were solved with FICO Xpress 8.0 in a 3.5GHz Intel Xeon CPU with twelve cores and 64GB of RAM, and the duality gap for the MISOCPs was set to 0.5%. Case 3, in which four FR services are defined and therefore an additional binary decision variable is needed for the conditional statements in the nadir constraints (9), took 5h10min to run. Therefore, although defining more FR services increases the computation time of the system scheduling, it is still manageable for the burdensome SUC problem.

### B. Impact of the Availability of Frequency Services on the Benefits from Defining New Services

The previous section has demonstrated that the value of new FR services will depend on the speed of delivery of these services, but another factor must be taken into account: the availability of frequency services, i.e. the number of units/providers that can deliver each FR service. In order to analyse such impact, we consider that one new FR service has been created, and study the following variations: 1) the new FR\(_2\) service is provided by 5%, 10%, 20%, 30% and 40% of the total CCGTs; and 2) same as the previous case, but nuclear units are allowed to part-load 400MW from their rated power (note that nuclear part-loading is co-optimised along with every other frequency service). Furthermore, wind penetrations of 40GW and 60GW are considered, as well as...
C. Impact of Activation-Delays in FR Provision

This section analyses the impact on system operating cost due to activation delays for the different FR services, i.e. the time after the power outage when the FR services are activated and start ramping up. The base case here assumes 30% of the CCGTs providing FR2 with T2 = 5s. Two different wind capacities are studied: 20GW and 60GW. Then, three cases of FR delays are considered: 1) none of the FR services have a delay, therefore they all react to any deviation from nominal frequency; 2) all three FR services have a 0.3s delay; and 3) EFR and PFR have no delay, but FR2 has a 0.3s delay. While the impact of FR delays on the frequency nadir has been studied through dynamic simulations in [14], no previous work has studied their impact on the system operating cost.

The results are presented in Fig. 7, which shows the annual cost of frequency services for each case, that is, the cost related to providing inertia and the different types of FR. This “cost of frequency services” is calculated by taking the cost from the solution of the frequency-secured SUC minus the cost of an SUC with no frequency constraints. By comparing the second case “All Delays” to the first one “No Delays”, it is demonstrated that delays in the delivery of FR services can significantly reduce their value to the system. However, a 0s delay would mean reacting to any deviation from nominal frequency, which would likely have associated wear-and-tear for the device providing FR. This is particularly true for battery storage providing EFR, since the lifetime of the device can be greatly impacted from the frequent charge-discharge switching that would be caused by reacting to any frequency deviation. Finally, the third case “Only FR2 delay” shows that by eliminating the delay in the provision of EFR and PFR, the system costs can be reduced by more than £150m/year for a 60GW-wind scenario. The economic impact of time-delays for FR provision shown in Fig. 7 is very significant for the 60GW-wind scenario, but this impact is limited for the 20GW-wind scenario.

Finally, Fig. 8 presents a sensitivity analysis for the activation-delay in FR2, for the case “Only FR2 delay”, demonstrating that a reduction of just 0.1s in T_{del,2} can have a significant impact in annual system costs: reducing the delay from 0.3s to 0.2s would bring £111m/year savings, for the 60GW-wind scenario.

D. Role of Forecast for Inertia Contribution from Demand

In this section we study the system operating cost under different forecasting scenarios for the inertia contribution from the demand side. The base case here considers 30% of the CCGTs providing FR2 with T2 = 7s and a 40GW wind...
capacity. The system operator is assumed to require a 99% probability for the RoCoF and nadir constraints (15) and (20) to be fulfilled, i.e. $\alpha = \eta = 0.99$.

From this base-case, in which no inertia from demand is taken into account, we consider 4 further cases: 2% and 10% of the total demand at each time-step in the SUC is assumed to provide inertia, with a forecast error of 10% and 35% considered for each case (i.e. $\sigma = 0.1H_0$ or $\sigma = 0.35H_0$). The inertia constant for demand is assumed to be 5s in all cases. The results in Fig. 9 demonstrate that considering the inertia contribution from demand can bring non-negligible economic savings to the system, more so if a higher percentage of the demand is contributing to inertia. Furthermore, the results highlight the importance of achieving an accurate forecast for the inertia contribution from demand: by reducing the forecast error from 35% to 10%, £190m/year additional savings can be obtained, for a case of 10% of demand contributing to inertia.

Furthermore, we analyse in Fig. 10 the detailed operation of the system by considering two cases, each corresponding to a 50-hour period, but showing significantly different levels of net-demand (demand minus wind): a high net-demand period in January, and a low net-demand period in July. The results in Fig. 10, which consider that 2% of the demand provides inertia and the forecast error is 10%, demonstrate how valuable inertia from demand becomes during low net-demand periods: the savings in the second period (average hourly savings of £9.8k) are significantly higher than in the first period (average hourly savings of £0.36k), although there is less inertia available from the demand side in the former. The results clearly demonstrate that market arrangements need to be in place not only to incentivise the demand side to provide inertia, but also to incentivise these inertia-providing loads to consume during periods when low net-demand is expected.

V. CONCLUSION

This paper proposes frequency-security constraints that allow to consider any finite number of FR providers, as well as any combination of activation delays for the different FR services. Uncertainty in system inertia from the demand side is modelled using chance constraints, for which a convex reformulation is provided. The resulting MISOPC formulation allows to efficiently schedule FR considering the different dynamics of providers, co-optimising FR along with inertia and a reduced largest loss. Case studies of frequency-secured Stochastic Unit Commitment have demonstrated the importance of new FR services for achieving cost-efficiency in power systems with significant renewable penetration.

Regarding future work, the reliability of FR provision should be considered, since we have assumed here that all FR providers can deliver the agreed amount of FR. Inertia from the demand side has been shown to provide important savings, and therefore new methods to accurately forecast this inertia should be developed. Finally, the different regional frequencies in a power grid’s buses should be modelled, as this work has considered the uniform-frequency model driven by the Centre of Inertia.

REFERENCES

[1] F. Teng et al., “Provision of ancillary services in future low-carbon UK electricity system,” in 2017 IEEE PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe), Conference Proceedings.

[2] E. Ela et al., “Market designs for the primary frequency response ancillary service. Part I: Motivation and design,” IEEE Transactions on Power Systems, vol. 29, no. 1, pp. 421–431, 2014.

[3] W. Li et al., “Design of a new primary frequency control market for hosting frequency response reserve offers from both generators and loads,” IEEE Transactions on Smart Grid, 2018.

[4] C. Cardozo et al., “Cutting plane approaches for frequency constrained economic dispatch problems,” Electric Power Systems Research, 2018.

[5] I. Chávez et al., “Governor rate-constrained OPF for primary frequency control adequacy,” IEEE Transactions on Power Systems, 2014.

[6] G. Zhang, “New ancillary service market design to improve MW-frequency performance: reserve adequacy and resource flexibility,” PhD thesis, Iowa State University, United States, 2015.

[7] F. Teng et al., “Stochastic scheduling with inertia-dependent fast frequency response requirements,” IEEE Transactions on Power Systems, vol. 31, no. 2, pp. 1557–1566, 2016.

[8] H. Ahmadi et al., “Security-constrained unit commitment with linearized system frequency limit constraints,” IEEE Transactions on Power Systems, vol. 29, no. 4, pp. 1536–1545, 2014.

[9] G. Zhang et al., “Market scheduling and pricing for primary and secondary frequency reserve,” IEEE Transactions on Power Systems, 2018.

[10] V. Trovato et al., “Unit commitment with inertia-dependent and multi-speed allocation of frequency response services,” IEEE Transactions on Power Systems, vol. 34, no. 2, pp. 1537–1548, 2019.

[11] L. Badesa et al., “Simultaneous scheduling of multiple frequency services in stochastic unit commitment,” IEEE Transactions on Power Systems, 2019.

[12] L. E. Sokoler et al., “Contingency-constrained unit commitment in meshed isolated power systems,” IEEE Transactions on Power Systems, vol. 31, no. 5, pp. 3516–3526, 2016.

[13] “System needs and product strategy,” National Grid, Report, 2017.

[14] Q. Hong et al., “Fast frequency response for effective frequency control in power systems with low inertia,” The Journal of Engineering, vol. 2019, no. 16, pp. 1696–1702, 2019.

[15] R. Doherty et al., “Frequency control in competitive electricity market dispatch,” IEEE Transactions on Power Systems, vol. 20, no. 3, pp. 1588–1596, 2005.

[16] L. Badesa et al., “Optimal scheduling of frequency services considering a variable largest-power-infeed-loss,” in 2018 IEEE PES General Meeting, Conference Proceedings.

[17] P. Kundur, Power System Stability and Control, 1st ed. McGraw-Hill Education, 1994.

[18] “System operability framework,” National Grid, Report, 2016.

[19] R. Sioshansi et al., Optimization in Engineering. Springer International Publishing, 2017.

[20] S. Boyd et al., Convex Optimization. Cambridge University Press, 2004.

[21] A. Nemirovski et al., “Convex approximations of chance constrained programs,” SIAM Journal on Optimization, 2007.

[22] A. Papoulis et al., Probability, Random Variables and Stochastic Processes, 4th ed. McGraw-Hill Education, 2002.

[23] A. Nurt et al., “Efficient stochastic scheduling for simulation of wind-integrated power systems,” IEEE Transactions on Power Systems, vol. 27, no. 1, pp. 322–334, 2012.

[24] “Enhanced frequency control capability project,” National Grid, Report, 2017.