The Linearity of the Wesenheit function for the Large Magellanic Cloud Cepheids

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ABSTRACT

There is strong evidence that the period-luminosity (PL) relation for the Large Magellanic Cloud (LMC) Cepheids shows a break at a period around 10 days. Since the LMC PL relation is extensively used in distance scale studies, the non-linearity of the LMC PL relation may affect the results based on this LMC calibrated relation. In this paper we show that this problem can be remedied by using the Wesenheit function in obtaining Cepheid distances. This is because the Wesenheit function is linear although recent data suggests that the PL and the period-colour (PC) relations that make up the Wesenheit function are not. We test the linearity of the Wesenheit function and find strong evidence that the LMC Wesenheit function is indeed linear. This is because the non-linearity of the PL and PC relations cancel out when the Wesenheit function is constructed. We discuss this result in the context of distance scale applications. We also compare the distance moduli obtained from $\mu_0 = \mu_V - R(\mu_V - \mu_I)$ (equivalent to Wesenheit functions) constructed with the linear and the broken LMC PL relations, and find that the typical difference in distance moduli is $\sim \pm 0.03$ mag. Hence, the broken LMC PL relation does not seriously affect current distance scale applications. We also discuss the random error calculated with equation $\mu_0 = \mu_V - R(\mu_V - \mu_I)$, and show that there is a correlation term that exists from the calculation of the random error. The calculated random error will be larger if this correlation term is ignored.

Key words: Cepheids – Distance Scale

1 INTRODUCTION

The Cepheid period-luminosity (PL) relation plays a major role in distance scale studies, which can ultimately be used to determine the Hubble Constant. The calibrating PL relation currently used is based mainly on the Large Magellanic Cloud (LMC) Cepheids, as applied by the $H_0$ Key Project team (Freedman et al. 2001) as well as in other studies (e.g., Saha et al. 2001; Kanbur et al. 2002). The Cepheid PL relation has long been considered to be a linear function of log($P$) within the range of log($P$) $\sim$ 0.3 to log($P$) $\sim$ 2.0, where $P$ is the pulsation period in days.

However, the non-linearity of the LMC PL relation has been proposed by Tammann et al. (2002) and Kanbur & Ngeow (2004), i.e. the LMC data are more consistent with two PL relations and a discontinuity at a period around 10 days. This is illustrated in the lower panels of Figure 1 for the extinction corrected $V$-band LMC PL relation. The existence of two LMC PL relations is further supported by the results from a rigorous statistical test (the $F$-test), as presented in Kanbur & Ngeow (2001) and Kanbur & Ngeow (2004), which shows that the $V$- and $I$-band LMC PL relations are better described by two PL relations. Since the work of Tammann et al. (2002) and Kanbur & Ngeow (2004) are based on the OGLE (Optical Gravitational Lensing Experiment) LMC Cepheids (Udalski et al. 1999b), which are truncated at log($P$) $\sim$ 1.5, Sandage et al. (2004) and Kanbur et al. (2005) used additional data that are available from the literature, especially those with log($P$) $>$ 1.5, to further support the existence of two PL relations in the LMC. These studies of the non-linear LMC PL relations are focused on the optical bands, as they are mainly based on the OGLE data, we hence discuss the non-linear LMC PL relation in the optical bands in this paper. The non-linear $V$-band LMC PL relation as seen from the OGLE data has been verified with the MACHO $V$-band data (Ngeow et al. 2005). In addition, Kanbur & Ngeow (2004) also extended the study of the non-linear LMC PL relations to the $JHK$-bands with the 2MASS data. In Tables 1 & 2 we collect the slopes and the zero-points (ZP) for the long (log[$P$] $>$ 1.0) and short period optical PL rela-
Figure 1. Comparison of the Wesenheit function with the extinction corrected $V$-band PL relation for the LMC Cepheids. On the top panels, we plot the Wesenheit function for the LMC data, which clearly show that the Wesenheit function is linear. We fit the data with a single line regression, using the least-squares method, as indicated on the figures. On the bottom panels, we show the extinction corrected $V$-band PL relations, and separate the data into the short (open circles) and long period (filled circles) Cepheids. The fitted PL relations for long, short and all period Cepheids are drawn as dashed, solid and dotted lines, respectively. We extend the short period PL relation to the longer period ranges in order to compare with the long period PL relations. The data in the left and right panels are taken from Kanbur & Ngeow (2004) and Kanbur et al. (2005), respectively.

The existence of two LMC PL relations suggests that if the Cepheid distance to a galaxy is derived using the Wesenheit function (or the equivalent $\mu_0$ in equation [4]), for example in the $H_0$ Key Project (Freedman et al. 2001, and references therein) or in the Araucaria Project (Gieren et al. 2004; Pietrzyński et al. 2004), then the results might not be affected (see Section 2.3 for details). This is because the Wesenheit function is linear, even though the PL and PC relations that make up the Wesenheit function are not, as shown in Section 2.2. Given that the LMC PL relation is not linear, and the Wesenheit function has been applied in many papers, we would like to examine the effect of broken LMC PL and PC relations on the application of the Wesenheit function to the distance scale. Again, this will be examined in the optical bands because the $V$- and $I$-band LMC PL relations have been frequently applied in the literature (as in, e.g., Freedman et al. 2001).

In addition to the non-linearity of the LMC PL relation, there are some recent studies also suggesting that the Cepheid PL relation is not universal, i.e., the Galactic PL relation is steeper than the LMC counterparts (Tammann et al. 2003; Fouqué et al. 2003; Kanbur et al. 2003; Ngeow & Kanbur 2004; Storm et al. 2004). It is possible that the Wesenheit function may or may not depend on metallicity: Some preliminary studies for both viewpoints can be found in the literature (see, e.g., Moffett & Barnes 1986; Caputo et al. 2000; Baraffe & Allard 2001; Pietrzyński et al. 2004; Storm et al. 2004). However the detailed study of the metallicity dependency of the Wesenheit function is beyond the scope of this paper. This paper only studies the linearity of the Wesenheit Function for the LMC Cepheids.

1 The $I$-band PL relations from Kanbur & Ngeow (2004) are fitted with slightly different definition of the $I$-band mean magnitudes. Hence we refit the $I$-band PL relations with the conventional and reddening corrected $I$-band mean magnitudes. The results from the $F$-test remain unchanged.

Note that these LMC PL relations have been corrected for reddening. The reason that the LMC PL relation is non-linear is because the period-colour (PC) relation for LMC Cepheids is also non-linear across the 10 days period (Tammann et al. 2002; Kanbur & Ngeow 2004; Sandage et al. 2004; Kanbur et al. 2003). The detailed investigation of the physics behind the broken LMC PL and PC relation is beyond the scope of this paper, but it is of great interest for the studies of stellar pulsation and evolution.

The existence of two LMC PL relations suggests that in future distance scale studies, the appropriate LMC PL relation may need to be applied to the long and short period Cepheids, respectively (see, e.g., Kanbur et al. 2003). However, all of the previous applications of the LMC PL relation were based on the linear version (in a sense, it is an approximation of two PL relations). Hence an immediate question that arises is: how does the existence of two LMC PL relations affect previous studies? (A similar question was also asked by Feast 2003.) In this paper we show that if the Cepheid distance to a galaxy is derived using the Wesenheit function (or the equivalent $\mu_0$ in equation [4]), for example in the $H_0$ Key Project (Freedman et al. 2001, and references therein) or in the Araucaria Project (Gieren et al. 2004; Pietrzyński et al. 2004), then the results might not be affected (see Section 2.3 for details). This is because the Wesenheit function is linear, even though the PL and PC relations that make up the Wesenheit function are not, as shown in Section 2.2. Given that the LMC PL relation is not linear, and the Wesenheit function has been applied in many papers, we would like to examine the effect of broken LMC PL and PC relations on the application of the Wesenheit function to the distance scale. Again, this will be examined in the optical bands because the $V$- and $I$-band LMC PL relations have been frequently applied in the literature (as in, e.g., Freedman et al. 2001).
Table 1. Slopes for the LMC PL relations\(^a\). Long and short periods are referred to \(P > 10\) and \(P < 10\) days, respectively.

| Band | Period-Range | TR02  | STR04 | KN04  | KNB05 |
|------|--------------|-------|-------|-------|-------|
| \(V\) | All          | \(-2.760 \pm 0.031\)\(^b\) | \(-2.702 \pm 0.028\) | \(-2.746 \pm 0.043\) | \(-2.736 \pm 0.036\) |
| \(V\) | Short    | \(-2.86 \pm 0.05\) | \(-2.963 \pm 0.056\) | \(-2.948 \pm 0.065\) | \(-2.937 \pm 0.069\) |
| \(V\) | Long     | \(-2.48 \pm 0.17\) | \(-2.567 \pm 0.102\) | \(-2.350 \pm 0.252\) | \(-2.598 \pm 0.161\) |
| \(I\) | All      | \(-2.962 \pm 0.021\)\(^b\) | \(-2.949 \pm 0.020\) | \(-2.965 \pm 0.028\) | \(-2.965 \pm 0.024\) |
| \(I\) | Short    | \(-3.03 \pm 0.03\) | \(-3.099 \pm 0.038\) | \(-3.096 \pm 0.043\) | \(-3.090 \pm 0.041\) |
| \(I\) | Long     | \(-2.82 \pm 0.13\) | \(-2.822 \pm 0.084\) | \(-2.737 \pm 0.179\) | \(-2.918 \pm 0.112\) |
| \(B\) | All      | \(-2.42 \pm 0.08\) | \(-2.683 \pm 0.077\) | \(-2.46 \pm 0.153\) | \(\cdots\) |
| \(B\) | Short    | \(-1.89 \pm 0.62\)\(^c\) | \(-2.151 \pm 0.134\) | \(\cdots\) | \(\cdots\) |

\(^a\) The references are: TR02 = Tammann & Reindl (2002); STR04 = Sandage et al. (2004); KN04 = Kanbur & Ngcoud (2003); KNB05 = Kanbur et al. (2003).

\(^b\) Since Tammann & Reindl (2002) does not give the results from the fit to all Cepheids, we adopted the slopes from Udalski et al. (1999a) because the same dataset is used in both papers.

\(^c\) The number of long period Cepheids in \(B\)-band is \(\sim 13\), hence the error is larger than the others.

Table 2. Same as Table 1 but for the zero-points of the LMC PL relations, by assuming \(\mu_{LMC} = 18.50\)mag.

| Band | Period-Range | TR02  | STR04 | KN04  | KNB05 |
|------|--------------|-------|-------|-------|-------|
| \(V\) | All          | \(-1.458 \pm 0.021\)\(^a\) | \(-1.451 \pm 0.022\) | \(-1.401 \pm 0.030\) | \(-1.412 \pm 0.025\) |
| \(V\) | Short    | \(-1.40 \pm 0.03\) | \(-1.295 \pm 0.036\) | \(-1.284 \pm 0.041\) | \(-1.295 \pm 0.038\) |
| \(V\) | Long     | \(-1.75 \pm 0.20\) | \(-1.594 \pm 0.135\) | \(-1.795 \pm 0.298\) | \(-1.523 \pm 0.198\) |
| \(I\) | All      | \(-1.942 \pm 0.014\)\(^a\) | \(-1.896 \pm 0.015\) | \(-1.889 \pm 0.019\) | \(-1.890 \pm 0.017\) |
| \(I\) | Short    | \(-1.90 \pm 0.02\) | \(-1.806 \pm 0.024\) | \(-1.813 \pm 0.027\) | \(-1.817 \pm 0.026\) |
| \(I\) | Long     | \(-2.09 \pm 0.15\) | \(-2.044 \pm 0.111\) | \(-2.109 \pm 0.212\) | \(-1.909 \pm 0.138\) |
| \(B\) | All      | \(\cdots\) | \(-1.160 \pm 0.029\) | \(\cdots\) | \(\cdots\) |
| \(B\) | Short    | \(-1.18 \pm 0.05\) | \(-0.955 \pm 0.049\) | \(\cdots\) | \(\cdots\) |
| \(B\) | Long     | \(-1.65 \pm 0.74\)\(^b\) | \(-1.364 \pm 0.177\) | \(\cdots\) | \(\cdots\) |

\(^a\) Since Tammann & Reindl (2002) does not give the results from the fit to all Cepheids, we adopted the zero-points from Udalski et al. (1999a) because the same dataset is used in both papers.

\(^b\) The number of long period Cepheids in \(B\)-band is \(\sim 13\), hence the error is larger than the others.

2 THE WESENHEIT FUNCTION AND ITS APPLICATION IN DISTANCE SCALE

2.1 Definition

The Wesenheit function (e.g., see Freedman 1988; Freedman et al. 1991; Greuneweld 2003; Madore 1976; 1992; Madore & Freedman 1991; Moffett & Barnet 1986; Tanvir 1997; Udalski et al. 1999a) is defined as \(W = 1 - R \times \text{colour} \), where \(R\) is the ratio of total-to-selective absorption that has to be adopted. Note that the definition of \(W\) depends on the adopted \(R\) which may be different in different environments. A few variations of \(W\) used in the literature with different combinations of magnitudes and colours are:

\[
W^V = V - R(B - V), \quad R = A_V/E(B - V), \tag{1}
\]
\[
W^V = V - R(V - I), \quad R = A_V/E(V - I), \tag{2}
\]
\[
W^I = I - R(I - 1), \quad R = R_I, \tag{3}
\]
where \(B, V \text{ and } I\) denote the (intensity) mean magnitudes. Similar definitions of \(W\) in near infrared bands can be found, for example, in Persson et al. (2004). The other definition of \(W\), as given by van den Bergh (1972), is different than the one given in equation (1). The van den Bergh version of \(W\) replaces \(R\) by the slope of the constant-period line in the colour-magnitude diagram (CMD). Madore & Freedman (1991) (also in Moffett & Barnet 1986) have pointed out some problems with the van den Bergh version and the advantage of using \(R\) in the definition of \(W\). The biggest advantage of using \(W\) is that it is reddening-free (see, for example, Madore & Freedman 1991), i.e. \(W = V - R(B - V) = V_0 - R(B - V)_0 \equiv W_0\), where \(V_0\) and \((B - V)_0\) denote the intrinsic visual magnitude and colour, as the effect of interstellar extinction on the observed magnitude and colour cancel out for a star (not only for Cepheids). Another advantage of using \(W\) is that the scatter in the \(W\)-log(\(P\)) plot is reduced (Madore 1982; Madore & Freedman 1991; Böhm-Vitense 1995; Tanvir 1997; 1999; Udalski et al. 1999a), as compared to the scatter in the \(V\)- or \(I\)-band PL relations (see Figure I). The remaining scatter is due to the combination of photometric errors and the finite width of the instability strip (for example, see Brodie & Madora 1980). Furthermore, an equivalent definition of \(W\) with the combination of absolute magnitude and colour can be formulated, as \(W_M = M_V - R(M_V - M_I)\), then the distance modulus can be obtained, i.e. \(\mu_W = W - W_M\). It is straightforward to show that the following equation:

\[
\mu_0 = \mu_W - R(\mu_W - \mu_I) \tag{4}
\]
is equivalent to \( \mu_W \), where equation (4) is frequently applied in determining the extra-galactic Cepheid distances (see, for example, Allen & Shanks 2004; Freedman et al. 2001; Ngeow & Kanbur 2004; Freedman et al. 2001; Kanbur et al. 2003; Saha et al. 2001; Tanvir et al. 1999). Therefore, using equation (4) to obtain the distance modulus is equivalent to obtaining the distance modulus by fitting the \( W - \log(P) \) plane with the empirical \( W_M - \log(P) \) relation.

### 2.2 Testing the Linearity of the Wesenheit Function

Both the Wesenheit function and equation (4) can be written as a combination of \( V \) and \( I \)-band PL relations, and they are adopted from the LMC PL relations to derive distances. However, as we mentioned in the Introduction, the LMC PL relations are not linear, hence the applicability of the Wesenheit function and equation (4) is immediately in question. In this sub-section we would like to test the linearity of the Wesenheit function as follows.

The PL relation in bandpass \( \lambda \) can be written as: \( M_\lambda = \alpha_\lambda^X + \beta_\lambda^X \log(P) \). The superscript \( X \) denotes the adopted period range, which is either for short \( (S, \log|P| < 1.0) \), long \( (L, \log|P| > 1.0) \) or all \( (A, \text{short+long}) \) periods. Then the \( V-(V-I) \) Wesenheit function becomes (similar expressions can be derived for other magnitude-colour combinations):

\[
W_X^V = (1 - R)\alpha_X^V + R\alpha_X^I + [(1 - R)\beta_X^V + R\beta_X^I] \log(P),
\]

The linearity of \( W \) demands that:

\[
(1 - R)\beta_X^V + R\beta_X^I = (1 - R)\beta_Y^V + R\beta_Y^I \quad \text{(for slope)}, \quad (6)
\]

\[
(1 - R)\alpha_X^S + R\alpha_X^L = (1 - R)\alpha_Y^S + R\alpha_Y^L \quad \text{(for ZP)}. \quad (7)
\]

By using the slopes in Table 1 we can calculate the values of the left-hand side and right-hand side in equation (6), as well as the slope for \( W^A \) by using the unbroken PL relation with equation (5). The results are summarized in Table 3 with different magnitude-colour combinations. The adopted values for \( R \) in these combinations are: \( R = \frac{\alpha_Y^V}{\alpha_Y^I} \) \( \text{(for slope)} \), and \( R = \alpha_Y^S/\alpha_Y^L \) \( \text{(for ZP)} \). The errors in Table 3 are estimated with the standard formula for propagation of errors, i.e. \( \sigma^2 = (1 - R)^2\sigma_{\beta_Y}^2 + R^2\sigma_{\alpha_Y}^2 \). The same is done for the ZP in Table 4.

It can be seen immediately from Table 3 & 4 that the short period Wesenheit function is consistent with the long period Wesenheit function, as demanded by equation (6) \& (7). Therefore, the Wesenheit function can be regarded as a linear function of \( \log(P) \). Furthermore, the short and long period Wesenheit functions are also consistent with the Wesenheit function obtained from using all Cepheids in the LMC or the linear, unbroken PL relation. Note that the value of \( \sim -3.3 \) for the slope of the Wesenheit function \( \beta_W \) with \( I-(V-I) \) combination also agrees with the values given in Udalski et al. (1999a), \( \beta_W = -3.277 \pm 0.014 \) or in Udalski et al. (2000a), \( \beta_W = -3.300 \pm 0.011 \). The linearity of the Wesenheit function can be immediately seen from the top panels of Figure 1. The residuals of the \( W \) from the fitted regressions in Figure 1 are also plotted as functions of period in Figure 2. If the Wesenheit function is non-linear and can be broken into long and short period Wesenheit functions, as in the LMC PL or PC relations, then the residual plots are expected to show a trend for the long period Cepheids (as in the figure 4 from Kanbur & Ngeow 2004 for the PC relation). However, there is no obvious trend of the residuals seen from Figure 2. This further supports the linearity of the Wesenheit function.

A better and more sophisticated test of the linearity is using the \( F \)-test (Weisberg 1980; Kanbur & Ngeow 2004). The null hypothesis in our \( F \)-test is that a single linear regression is sufficient, while the alternate hypothesis is that two linear regressions are needed to describe the data. These regressions can be obtained with the standard least squares regression method. The setup and the formalism for the \( F \)-test is given in Kanbur & Ngeow (2004). By calculating the

| Dataset | Short | Long | All |
|---------|-------|------|-----|
| V-(V-I) Combination, \( R = 2.45 \). | TR02: \(-3.276 \pm 0.103\) | \(-3.313 \pm 0.403\) | \(-3.255 \pm 0.068\) |
| STR04: \(-3.296 \pm 0.124\) | \(-3.192 \pm 0.253\) | \(-3.307 \pm 0.064\) |
| KNB05: \(-3.312 \pm 0.133\) | \(-3.382 \pm 0.360\) | \(-3.297 \pm 0.079\) |

| I-(V-I) Combination, \( R = 1.55 \). | TR02: \(-3.293 \pm 0.109\) | \(-3.347 \pm 0.423\) | \(-3.275 \pm 0.072\) |
| STR04: \(-3.310 \pm 0.130\) | \(-3.217 \pm 0.266\) | \(-3.332 \pm 0.067\) |
| KNB05: \(-3.327 \pm 0.140\) | \(-3.414 \pm 0.379\) | \(-3.320 \pm 0.083\) |

| V-(B-V) Combination, \( R = 3.24 \). | TR02: \(-4.286 \pm 0.335\) | \(-4.392 \pm 2.134\) | \cdots |
| STR04: \(-3.870 \pm 0.344\) | \(-3.915 \pm 0.613\) | \(-3.875 \pm 0.169\) |
Table 4. Same as Table 3 but for the zero-points.

| Dataset | Short | Long | All |
|---------|-------|------|-----|
|         | $V-(V-I)$ Combination, $R = 2.45$. |      |     |
| TR02    | $-2.625 \pm 0.066$ | $-2.583 \pm 0.468$ | $-2.644 \pm 0.046$ |
| STR04   | $-2.547 \pm 0.079$ | $-2.696 \pm 0.335$ | $-2.541 \pm 0.049$ |
| KNO4    | $-2.580 \pm 0.089$ | $-2.564 \pm 0.676$ | $-2.597 \pm 0.064$ |
| KNB05   | $-2.574 \pm 0.084$ | $-2.469 \pm 0.444$ | $-2.583 \pm 0.055$ |
|         | $I-(V-I)$ Combination, $R = 1.55$. |      |     |
| TR02    | $-2.675 \pm 0.069$ | $-2.617 \pm 0.492$ | $-2.692 \pm 0.048$ |
| STR04   | $-2.598 \pm 0.083$ | $-2.741 \pm 0.352$ | $-2.586 \pm 0.051$ |
| KNO4    | $-2.633 \pm 0.094$ | $-2.596 \pm 0.711$ | $-2.645 \pm 0.067$ |
| KNB05   | $-2.626 \pm 0.089$ | $-2.307 \pm 0.467$ | $-2.631 \pm 0.058$ |
|         | $V-(B-V)$ Combination, $R = 3.30$. |      |     |
| TR02    | $-2.113 \pm 0.206$ | $-2.074 \pm 2.543$ | $\cdots$ |
| STR04   | $-2.397 \pm 0.220$ | $-2.339 \pm 0.810$ | $2.394 \pm 0.132$ |

For an ensemble of Cepheids in a target galaxy, with $N$ of them being used in determining the distance, then the distance modulus for the $i^{th}$ Cepheid in bandpass $\lambda$ (usually $V$ and $I$) is $\mu_{0,i} = m_{0,i} - \beta_{\lambda} \log(P_{i}) - \alpha_{\lambda}$. The distance modulus in bandpass $\lambda$ for these Cepheids can be obtained by taking the unweighted mean to individual Cepheids, i.e., $\mu_{\lambda} = \frac{1}{N} \sum_{i=1}^{N} \mu_{0,i}$. The reddening-free distance modulus for the $i^{th}$ Cepheid can be calculated with equation (4), and it is straightforward to show that:

$$\overline{\mu_{0}} = \frac{1}{N} \sum_{i=1}^{N} \mu_{0,i}$$

$$= \overline{\mu_{V}} - R(\overline{\mu_{V}} - \overline{\mu_{I}}),$$

where $\mu_{0,i} = \mu_{0,V} - R(\mu_{0,I} - \mu_{0,V})$. This procedure (see, e.g., Freedman et al. 2001; Leonard et al. 2003; Kanbur et al. 2003; Thim et al. 2003, 2004), i.e., calculating the distance modulus for individual Cepheids and taking the unweighted average to be the final distance modulus to that target galaxy, is equivalent to fitting the $V$- and $I$-band PL relations to the data and obtaining the $\mu_{V}$ and $\mu_{I}$ that apply to equation (4). If the PL relation is linear, then the single, unbroken PL relation (in both $V$- and $F$-band) can be adopted to fit to all Cepheids to obtain the distance modulus ($\mu_{0}$). However, if the PL relation is non-linear, as in the case of LMC Cepheids, then the long period PL relation should be applied to the long period Cepheids, and similarly for the short period Cepheids. For example, attempts to use the broken LMC PL relations to calibrate the extra-galactic Cepheid distances can be found in Kanbur et al. (2003); Leonard et al. (2003); Thim et al. (2003, 2004).

As mentioned before, $\mu_{0}$ from equation (4) is equivalent to $\mu_{V} = W - W_{M}$, i.e., $\mu_{V} = \mu_{W}$. Since the Wesenheit function (both $W$ and $W_{M}$) for LMC Cepheids is linear (from corollary A or equation [8]), then the $\mu_{0}$ obtained from using either the broken LMC PL relation (for long and short period Cepheids respectively) or the linear LMC PL relation (for all Cepheids) is expected to agree well with each other. In other words, the difference in the distance modulus ($\Delta \mu_{0}$) when using either the linear or the broken PL relation should be small. For example, the difference in distance modulus ($\Delta \mu_{0} = \mu_{0,V} - \mu_{0,I}$) when using either the linear or the broken long period LMC PL relation can be quantitatively estimated if the mean log-period ($\log(P_{L})$) of the long period Cepheids in the target galaxy is known. For the distance modulus obtained with equation (4) & (9), $\Delta \mu_{0}$ is expressed as Kanbur et al. (2003):

$$\Delta \mu_{0} = -\Delta \beta_{W} \log(P_{L}) - \Delta \alpha_{W}$$

where $\Delta \beta_{W}$ and $\Delta \alpha_{W}$ are the differences in slopes and ZPs.
for the linear and the broken long period Wesenheit function. These coefficients can be calculated using the slopes and ZFs for the V-(V - I) Wesenheit function as given in Table 3. If \( \log(P) \) is estimated from equation (11) is around \( \pm 0.03 \) mag, corresponding to \( \pm 1.3 \) mag in distance (in Mpc). Furthermore, for \( 1.0 < \log(P) \) \( < 2.0 \), equation (11) suggests that the maximum \( \Delta \mu_0 \) when using the linear or the broken long period PL relation from Table 4 & 2 is \( \pm 0.07 \) mag. A similar estimation can be done for the short period Cepheids (with \( 0.0 < \log(P) < 1.0 \)) with maximum \( \Delta \mu_0 \) of \( \pm 0.02 \) mag. Even though the value of \( \Delta \mu_0 \) is small, it has a systematic effect on the distance scale because of the different PL relations used.

We give two examples to illustrate the above discussion further:

(i) Kanbur et al. (2003) calculated the distance moduli to 25 HST observed galaxies with the linear and the broken long period LMC PL relations (as \( \sim 98 \) % of the Cepheids detected in these galaxies have period longer that 10 days). By comparing the distance moduli derived from the linear PL relation and the broken long period PL relation, the unweighted average of \( \Delta \mu_0 \) in these 25 galaxies is \( -0.021 \pm 0.002 \) mag., in the sense that the linear PL relation systematically produces slightly smaller distance moduli than the broken long period PL relation. Since the mean log-period for most of these galaxies is \( \sim 1.4 \) (Kanbur et al. 2003), then the difference of \( -0.02 \) mag. is expected and agrees well with the above discussion.

(ii) To compare the distance moduli obtained from using the linear and the broken PL relations, we use the Cepheids in IC 4182 (with the data from Gibson et al. 2000) as an example, because this galaxy contains roughly equal number of short \( (N = 13) \) and long \( (N = 15) \) period Cepheids. The distance modulus from using the linear, unbroken PL relation with all 28 Cepheids \( (\mu_0) \) are compared with the mean distance moduli \( (\overline{\mu}_0) \), from equation (9) obtained from applying the broken long and short period PL relation to the individual long and short period Cepheids, respectively. The PL relations used, include the linear and the broken PL relations, are taken from Table 4 & 2 and the results are presented in Table 3. From this table, \( \mu_0 \) shows a good agreement to \( \overline{\mu}_0 \), with a difference of \( \sim 0.01 \) mag. or smaller.

In short, when using the LMC PL relations to derive the Cepheid distances, we conclude that:

**Corollary B:** The distance moduli are approximately the same when using either the linear PL relation (to all Cepheids) or the means with broken PL relation (to long and short period Cepheids respectively), i.e.,

\[
\mu_0^A \sim \overline{\mu}_0.
\] (12)

Both of the approaches will give similar and consistent distance moduli. Recall that the linear PL relation is the approximation of the broken PL relation, and the accuracy of

\[\text{Table 5. The distance modulus (}\mu_0\text{) to IC 4182.}\]

| LMC PL relation | \( \mu_0^A \)       | \( \overline{\mu}_0 \)   |
|-----------------|---------------------|--------------------------|
| TR02            | 28.267 ± 0.053      | 28.277 ± 0.053           |
| SRT04           | 28.222 ± 0.053      | 28.217 ± 0.053           |
| KN04            | 28.250 ± 0.053      | 28.247 ± 0.053           |
| KNN05           | 28.253 ± 0.053      | 28.257 ± 0.054           |

using the linear PL relation is roughly \( \pm 0.03 \) mag. Therefore, researchers have the freedom to use either the linear LMC PL relation as an approximation or using the correct broken LMC PL relation in deriving the Cepheid distances.

2.3.1 A Note on the Random Errors for \( \mu_0 \) from Equation (4)

By definition, the random error for \( \overline{\mu} = \frac{1}{N} \sum_{i=1}^{N} \mu_i^i \) due to purely statistical fluctuations, is:

\[
\sigma^2 = \frac{1}{N(N-1)} \sum_{i=1}^{N} (\mu_i - \overline{\mu})^2.
\] (13)

This equation holds for the \( \mu_{01} \) and \( \mu_{02} \) (from equation [9]). We can expand the expression for \( \sigma^2 \) by substituting the expression for \( \mu_0^i \) and \( \overline{\mu}_0 \) to equation (13), then:

\[
\sigma^2_0 = (1-R^2) \sigma^2_v + R^2 \sigma^2_I - \text{CORR}_{V,I},
\] (14)

where \( \sigma^2_v \) and \( \sigma^2_I \) are given by equation (13), and \( \text{CORR}_{V,I} = \frac{2(1-R)}{N(N-1)} \sum_{i=1}^{N} (\mu_i - \overline{\mu}_0)(\mu_i - \overline{\mu}_0) \), a term for the correlated residuals from both bands. Tanvir (1997; Freedman et al. 2001), note that the use of equation (13) to estimate the random errors for the Cepheid distance modulus has been practiced in the literature (e.g., see Freedman et al. 2001; Kanbur et al. 2003).

On the other hand, if we apply the standard equation for the propagation of errors (POE) to equation (4) or (10), by ignoring the correlation term, then we have \( \sigma^2_0(\text{POE}) = (1-R)^2 \sigma^2_v + R^2 \sigma^2_I \). By comparing this expression to equation (14), we obtain: \( \sigma^2_0(\text{POE}) = \sigma^2_0 + \text{CORR}_{V,I} \). One can immediately see that \( \sigma^2_0(\text{POE}) \) is greater than \( \sigma^2_0 \) since the term \( \text{CORR}_{V,I} \) is mostly likely to be positive. This is because (a) \( R > 1 \) from the extinction curve; and (b) \( \sum (\mu_i - \overline{\mu})(\mu_i - \overline{\mu}) \) is mostly likely to be positive. The second condition is due to the existence of the period-luminosity-colour relation: if the V-band magnitude for a Cepheid is above (or below) the ridge-line of the V-band PL relation, then the corresponding I-band magnitude will also be above (or below) the ridge-line of the I-band PL relation. The result is that if \( (\mu_i - \overline{\mu}) \) is positive (or negative), then \( (\mu_i - \overline{\mu}) \) is also positive (or negative), and hence the product of these two is positive. Further, if the extinction and/or the correlated errors (from measurement) make \( (\mu_i - \overline{\mu}) \) to be positive/negative, then \( (\mu_i - \overline{\mu}) \) is also going to be positive/negative, and again the product will be positive. Errors estimated from error propagation with equation (10), i.e. \( \sigma^2_0(\text{POE}) \), will ignore the \( \text{CORR}_{V,I} \) term and hence resulted a larger random error than the random error estimated from equation (13). Note that all the errors

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2 Note that the median values of \( \sim 28.25 \) mag. for \( \mu_0^A \) and \( \overline{\mu}_0 \) in Table 3 agree well to the distance modulus obtained from the TRGB (tip of red giant branch) method as given in Sakai et al. (2004): \( \mu_{TRGB} = 28.25 \pm 0.06 \) mag.
discussed here, the $\sigma$, are random errors only, which do not include the systematic errors such as the errors arise from the calibrations, the width of the instability strip, and others (see, e.g., the discussion by Saha et al [2000]).

3 CONCLUSION

Due to the recent discovery of the non-linearity for the LMC PL and PC relations, Feast [2003] has asked the following critical question:

"...is there a significant slope difference between short and long ($\sim 10$days) Cepheids that would seriously affect the calibration and use of PL [and PC] relation?"

From this study, we showed that this problem can be remedied with the application of the Wesenheit function in distance scale studies. This is because the Wesenheit function for the LMC Cepheids is linear (corollary A), as shown in Section 2.2, although the LMC PL and PC relations are not. Therefore, the Cepheid distances obtained with the Wesenheit function or the equivalent $\mu_0$ would not be affected with the recent finding of the broken PL and PC relations. We also found that the typical difference in distance modulus from using the linear or the broken PL relations is about $\pm 0.03$mag. Hence, researchers can choose to apply either the linear or the broken LMC PL relations to obtain the Cepheid distances, without worrying that these two approaches will give inconsistent results, as both approaches are equally applicable in deriving the Cepheid distance (corollary B). Therefore, the broken PL relation found in the LMC Cepheids will not seriously affect the previous applications of the linear LMC PL relation in distance scale studies because the effect is minimal. The question raised by Feast [2003] is essentially answered.

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