A new method for measuring complex relative permittivity of dielectric material by reflection method

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Abstract. In this paper, a new lossless free space method for measuring the complex relative permittivity of dielectric material by reflection method is presented. When this method is used to measure the complex relative permittivity of dielectric material, there is no multi-value problem. The measurement system used in this method consists of a horn lens antenna and a vector network analyzer (VNA). In this paper, HFSS microwave simulation software is used to simulate the complex relative permittivity measurement method proposed in this paper. The simulation results show that this method can accurately measure the complex relative permittivity of dielectric material.

1. Introduction
Accurately and conveniently measuring the complex relative permittivity of dielectric material is of great significance in the field of science and engineering. After a long time of development, people have put forward a variety of methods to measure the complex relative permittivity of dielectric material, such as resonance method [1, 2], waveguide method [3, 4], coaxial probe method [5, 6], free-space method [7, 8] and so on. Each of the complex relative permittivity methods has its advantages and disadvantages. Among many methods to measure the complex relative permittivity of dielectric material, the free-space method is the most simple and lossless one. The most widely used free-space method is the NRW method [9, 10], but the NRW method usually produces multiple value problems due to phase ambiguity. A new free-space reflection method for measuring the complex relative permittivity of dielectric material is presented in this paper. In this method, The complex relative permittivity of dielectric material is determined by using the reflection coefficient of the dielectric material when it is not connected to the metal plate and the reflection coefficient of the dielectric material when it is connected to the metal plate. There is no multi-value problem caused by phase ambiguity in this method.

2. Measurement system and theoretical analysis

2.1. Overall measuring system
The measurement system used in this paper is shown in Figure 1. In Figure 1, the horn antenna fixed on the test bench radiates electromagnetic wave under the excitation of TE 10 mode, and the electromagnetic wave radiated by it are converted into uniform plane wave (TEM wave) after passing through the lens.
Figure 1(a) draws the measurement system when the back end of the dielectric material is not connected with the metal plate. Figure 1(b) draws the measurement system when the back end of dielectric material is connected to the metal plate. Our aim is to derive the complex relative permittivity of the dielectric material from the reflection coefficient \( \Gamma_{in0} \) and the reflection coefficient \( \Gamma_{in} \).

![Figure 1](image1.png)

**Figure 1.** Measurement system of complex relative permittivity of dielectric material

The measurement system of complex relative permittivity of dielectric material as shown in Figure 1(a) and Figure 1(b) can be simplified to the equivalent network model as shown in Figure 2(a) and Figure 2(b) respectively. Both Figure 2(a) and Figure 2(b) of the equivalent network model are composed of two two-port networks cascaded with one one-port network. Planes 0, 1 and 2 in Figure 2 correspond to reference planes 0, 1 and 2 in Figure 1 respectively.

![Figure 2](image2.png)

**Figure 2.** Equivalent network model of complex relative permittivity measurement system

### 2.2. Measuring principle

Let us assume in Figure 1 that the thickness of the dielectric material is \( d \), and the frequency of the uniform plane wave entering the dielectric material is \( f \) and the unit is \( \text{GHz} \), then its angular frequency...
is \( \omega = 2 \pi f \). We assume that the dielectric material is nonmagnetic and its complex relative permittivity is 
\( \varepsilon_r = \varepsilon_r' - j \varepsilon_r'' \).

We assume that the Fresnel reflection coefficient and transmission coefficient of the air-dielectric material interface are \( \Gamma_1 \) and \( \tau_1 \) respectively. The Fresnel reflection coefficient and transmission coefficient of the dielectric material-air interface are \( \Gamma_2 \) and \( \tau_2 \) respectively. The Fresnel reflection coefficient and transmission coefficient of the dielectric material-metal plate interface are \( \Gamma_3 \) and \( \tau_3 \) respectively. The propagation constant of dielectric material is \( \gamma_m \). The network parameters of two-port network \( S_b \), the reflection coefficient \( \Gamma_L' \) and the reflection coefficient \( \Gamma_L'' \) are shown in Formula (1), (2) and (3) respectively.

\[
\begin{bmatrix}
    S_{b11} & S_{b12} \\
    S_{b21} & S_{b22}
\end{bmatrix} =
\begin{bmatrix}
    \Gamma_1 & \tau_1 e^{2\gamma_m d_u} \\
    \tau_2 e^{2\gamma_m d_u} & \Gamma_2 e^{2\gamma_m d_u}
\end{bmatrix}
\]

\( \Gamma_L' = \frac{\sqrt{\varepsilon_r} - \sqrt{\varepsilon_{air}}}{\sqrt{\varepsilon_r} + \sqrt{\varepsilon_{air}}} = \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \)  
\( \Gamma_L'' = \Gamma_3 = -1 \)

The reflection coefficient \( \Gamma_{in}' \) in Figure 1(a) can be obtained from Equation (4):

\[
\Gamma_{in}' = S_{b11} + \frac{S_{b12} S_{b21} \Gamma_L'}{1 - S_{b22}} \Gamma_1' = \Gamma_1' + \frac{\tau_1 \tau_2 \Gamma_2 e^{2\gamma_m d_u}}{1 - \Gamma_2' e^{2\gamma_m d_u}} = \frac{\Gamma_1' - \Gamma_1 e^{-2\gamma_m d_u}}{1 - \Gamma_1' e^{2\gamma_m d_u}}
\]

\( \Gamma_{in}'' = S_{b11} + \frac{S_{b12} S_{b21} \Gamma_L''}{1 - S_{b22}} \Gamma_1'' = \Gamma_1'' - \frac{\tau_1 \tau_2 \Gamma_2 e^{-2\gamma_m d_u}}{1 + \Gamma_2' e^{-2\gamma_m d_u}} = \frac{\Gamma_1'' - \Gamma_1 e^{-2\gamma_m d_u}}{1 + \Gamma_1'' e^{2\gamma_m d_u}}
\]

We take \( e^{-2\gamma_m d_u} \) and combine the equations (4) and (5) together to obtain the system of equations (6):

\[
\begin{cases}
    \Gamma_{in}' = \frac{\Gamma_1' - \Gamma_1 T}{1 - \Gamma_1' T} \\
    \Gamma_{in}'' = \frac{\Gamma_1'' - T}{1 - \Gamma_1'' T}
\end{cases}
\]

By eliminating the variable \( T \), we can obtain the cubic equation of one variable (7). By solving the cubic equation (7) with one variable, we can obtain the Fresnel reflection coefficient \( \Gamma_1' \):

\( (\Gamma_1 - 1)[\Gamma_1^2 + (b + 1)\Gamma_1 + 1] = 0 \)

Where, the value of \( b \) can be obtained from Equation (8):

\[
b = -\frac{\Gamma_{in}' \Gamma_{in}'' + \Gamma_{in}'' + 1}{\Gamma_{in}'}
\]

The cubic equation of one variable (7) has three solutions \( \Gamma_1^{(1)} \), \( \Gamma_1^{(2)} \) and \( \Gamma_1^{(3)} \). The values of these three solutions are shown in Equation (9):
Where, $\Delta$ is the discriminant of the root of the quadratic equation with one variable, and its value is $\Delta = \sqrt{(b+1)^2 - 4}$.

The complex relative permittivity $\varepsilon_{rm}$ of the dielectric material can be obtained from the Fresnel reflection coefficient $\Gamma_1$ of the air-dielectric material interface through Equation (10):

$$\varepsilon_{rm} = \frac{1 - \Gamma_1}{1 + \Gamma_1}$$

Because the Fresnel reflection coefficient at the air-dielectric material interface obtained from the cubic equation of one variable (7) has three solutions $\Gamma_1^{(1)}$, $\Gamma_1^{(2)}$, $\Gamma_1^{(3)}$, correspondingly, three complex relative permittivity $\varepsilon_{rm}^{(1)}$, $\varepsilon_{rm}^{(2)}$ and $\varepsilon_{rm}^{(3)}$ of the dielectric material can be inverted according to Equation (10). The values of these three solutions are shown in Equation (11):

$$\begin{align*}
\varepsilon_{rm}^{(1)} &= \left(\frac{1 - \Gamma_1^{(1)}}{1 + \Gamma_1^{(3)}}\right)^2 = 0 \\
\varepsilon_{rm}^{(2)} &= \left(\frac{1 - \Gamma_1^{(2)}}{1 + \Gamma_1^{(2)}}\right)^2 = \frac{2 + (b+1) - \Delta}{2 - (b+1) + \Delta} \\
\varepsilon_{rm}^{(3)} &= \left(\frac{1 - \Gamma_1^{(3)}}{1 + \Gamma_1^{(3)}}\right)^2 = \frac{2 + (b+1) + \Delta}{2 - (b+1) - \Delta}
\end{align*}$$

Since the complex relative permittivity $\varepsilon_{rm}$ of the dielectric material is not 0, $\varepsilon_{rm}^{(1)}$ is directly excluded. Now let's prove $\varepsilon_{rm}^{(2)} = \varepsilon_{rm}^{(3)}$. The proof process is as shown in Equation (12):

$$\begin{align*}
\frac{\varepsilon_{rm}^{(2)}}{\varepsilon_{rm}^{(3)}} &= \frac{2 + (b+1) - \Delta}{2 - (b+1) + \Delta} \times \frac{2 - (b+1) - \Delta}{2 + (b+1) + \Delta} \\
&= (-1)^2 = 1
\end{align*}$$

To sum up, the complex relative permittivity $\varepsilon_{rm}$ of the dielectric material can be obtained by Equation (13):

$$\varepsilon_{rm} = \varepsilon_{rm}^{(2)} = \varepsilon_{rm}^{(3)} = \frac{2 + (b+1) + \sqrt{(b+1)^2 - 4}}{2 - (b+1) - \sqrt{(b+1)^2 - 4}}$$

Where, the value of $b$ can be obtained from Equation (8).

2.3. System calibration

In order to obtain the reflection coefficients $\Gamma_{in}'$ and $\Gamma_{in}''$ respectively from the reflection coefficients $\Gamma_{in}'$ and $\Gamma_{in}''$, we need to de-embedding the two-port network $S_a$. In this paper, the calibration method in Reference [11] is used to de-embedding the two-port network $S_a$. 

\[\begin{align*}
\Gamma_1^{(1)} &= 1 \\
\Gamma_1^{(2)} &= \frac{-(b+1) + \Delta}{2} \\
\Gamma_1^{(3)} &= \frac{-(b+1) - \Delta}{2}
\end{align*}\]
3. Simulation of measurement method

In order to verify the correctness of measuring complex relative permittivity by free-space reflection method proposed in this paper. In this chapter, the microwave simulation software HFSS is used to simulate and model the measurement system proposed in this paper. The computer used in the simulation has 128GB of memory and a running speed of 2.40GHz. In this paper, HFSS software was used to simulate the calibration process of reference [11], and the two-port network $S_21$ was de-embedded. The simulation of the measurement process of the complex relative permittivity of dielectric material corresponding to Figure 1 is shown in Figure 3. Figure 3 shows the electric field diagram of the measuring system at 6GHz. In Figure 3, the complex relative permittivity of the dielectric material to be tested was set as $3 - 0.5j$, the thickness was set as 10mm, and the distance between the dielectric lens and the material to be tested was 600mm. The thickness of the metal plate in Figure 3(b) is 2mm.

![Figure 3](image)

(a) No metal plate  
(b) Immediately following metal plate

Figure 3. Simulation of measurement process of complex relative permittivity of dielectric materials

According to the principle analysis in Chapter 2, the complex relative permittivity of the material to be measured can be calculated by using the reflection coefficient obtained in Figure 3. Figure 4 shows the calculated value of the complex relative permittivity of the dielectric material in the frequency range 5.5GHz -- 6.5GHz. We compare the calculated complex relative permittivity with the preset permittivity in Figure 4. In Figure 4, the calculated value of the complex relative permittivity fluctuates around the preset value, and the fluctuation error is caused by the simulation error.

![Figure 4](image)

Figure 4. Simulation results of complex relative permittivity
4. Conclusions
In this paper, a new free-space reflection method for measuring the complex relative permittivity of dielectric material is presented, and the principle and measuring system of this method are explained in detail. In this paper, HFSS software is used to simulate the proposed measurement system. The simulation results show that the proposed method can accurately measure the complex relative permittivity of dielectric material.

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