Application of Morgenstern-Price Method in Reversion Analysis of the Loess Strength Parameters

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Abstract. It is difficult to obtain reliable shear strength parameters for the stability analysis and evaluation of high loess slopes. Hence, this paper determines 17 slopes with a height of 40-120 m based on the measured loess slopes in Jingbian-Ansai subregion of northern Shaanxi, China. The model is established using the finite element method, and the comprehensive shear strength parameters of different slope heights of the high slope in the study area are obtained by inversion using the Morgenstern-Price method. The results show that when the height of the slope increases, the cohesion $c$ in the soil increases, and the internal friction angle $\phi$ decreases. According to the calculation results, a comprehensive inversion curve of strength parameters under the slope height of 40 to 120 m is drawn, which can directly evaluate the stability of the loess high slope, which improves the work efficiency and also provides reliable data support in the high side slope geometry design.

1. Introduction
With the country’s construction of the loess plateau has become increasingly active, these high slopes pose a great threat to the safety of people's production and life, and the stability of high loess slopes has gradually been given more attention. Much research shows that the relationship among the position of the sliding surface of the slope, the shear strength parameters and the stability coefficient are key to the evaluation analysis. Methods for obtaining strength parameters through inversion include strength reduction method that gradually reduces the strength parameters of sliding surfaces\textsuperscript{[1]}, numerical simulation methods (FLAC3D), genetic algorithms\textsuperscript{[2]}, etc. The above studies have conducted much research on medium and low loess slopes with slopes below 30 m, but the research on the stability of high loess slopes above 30 m has not reached a unified understanding\textsuperscript{[3]} and lacks an accurate method for analysis.

In addition, most studies take a single slope or landslide as the object, and there is little systematic analysis and research on the variation law of the strength parameters of regional high loess slopes\textsuperscript{[4]}. Hence, this paper analyzes the limit state slopes with obvious signs of damage in the natural high loess slopes in Jingbian-Ansai subregion of northern Shaanxi and the correlation between the slope height and the width of the slopes. We model slopes of different heights and apply the Morgenstern-Price method to inverse their corresponding comprehensive strength parameters. According to the regression equation curve of the slope height and slope width and the comprehensive strength parameters that correspond to the slope height, the relationship curves between the cohesion of different slope heights and the internal friction angle are drawn to calculate the stability factor of the known slope shape and
the design of the proposed slope, also can analyze the safety factor and stability of the known high slope and design the slope shape of the slope to be constructed.

2. High-Slope Measurement and Slope Height-Slope Width Fitting

2.1. Measurement and Statistical Analysis of Limit-state Slopes
In this paper, the regionally distributed loess slopes are mostly distributed along the rivers and ditches of the Xing Zi River, Luo River, and Wuding River (see figure 1). These slopes are located in the Loess Plateau and its surrounding areas; the underlying bedrock has a large burial depth. All slopes in the statistical analysis adopt Wu Weijiang's classification criteria for pure loess landslides[5]. According to incomplete statistics, landslides in the loess layer account for more than one-third of the total number of landslides in the Loess Plateau. Therefore, the types of landslides in the loess layer selected in this paper are typical.

![Figure 1. Slope measurement and sampling points of the loess limit state in the study area.](image)

The premise of the strength parameter inversion is that the selected slopes is in the limit equilibrium state. For a given slope, one can determine whether it is in the limit equilibrium state through field surveys. In response to this problem, this article uses 4 criteria by [6] proposed that a clear concept of the limit-state slope and combined with the natural history analysis method, considering the regional geological background of the study area, this study investigated the signs, scale and morphology of high-slope deformation, sampled different types of slopes to determine their severity, and analyzed and controlled their deformation and damage. In the end, 17 slopes in the limit state were determined in study area, and the slope height was greater than 40 m and less than 130 m (as shown in star of figure 1).

2.2. Research on the Slope Height and Slope Width Fitting of High Loess Slopes
A correlation analysis was performed on the slope height and slope width of 17 limit-state high slopes in the study area. The exponential model of equation (1) was used to fit them through repeated trial calculations, and all degree-of-correlation coefficients are above 0.89. This fitting result provides a basis for arbitrarily setting the slope height when the slope model is established in the later period. Knowing the slope height, the corresponding slope width can be calculated using equation (1) to determine the slope shape.

\[ L = 0.79291H^{1.10242} \]

where \( H \) is slope height, which is measured from the foot of the slope to the top of the slope; \( L \) is slope width, which is the distance between the horizontal projection of the slope shoulder and the slope foot.

3. Inversion Model and Result Analysis
According to the basic principle of [7] inverse strength parameter algorithm, two or more cross-section equations can be simultaneously solved under similar conditions. In the same area, when the slope...
heights are not greatly different, the water content, slip state, and mechanical parameters are similar. Statistical analysis methods are more commonly used in the study of landslide kinematics models like [8] and [9]. Therefore, the cross-section height difference of the joint is controlled at 5 m, and a set of inversion values of shear strength parameters are taken at a height interval of 10 m to represent the comprehensive strength parameters in this range of slope height. With the set slope height, the corresponding slope width is obtained by the slope width-slope height fitting equation (1). The internal friction angle $\phi$ and gravity $\gamma$ are set. The value of $\phi$ is 5-35°, and the value gradient is 5°. The values of gravity $\gamma$ within the range of 20m from the top of the slope is 16.1 kN/m$^3$, the value from 20 to 70m from the top of the slope is 18.0 kN/m$^3$, and the distance from the top of the slope is more than 70m to take the value of 21.0 KN/m$^3$.

The slope model is constructed using the Slope/w module of the finite element software GeoStudio, and the Morgenstern-Price method is used to invert the comprehensive cohesion $c$. Figure 2 shows a slope model with a height of 80 m in the Jianbian-Ansai subregion. The slope models of each area are successively established, the three $c$-$\phi$ relationship curves in each 10-m-slope-height section of each area are intersected at one point, and the coordinates of this intersection point are the comprehensive strength parameters of the slope in the limit state within the range of 10 m, as shown in figure 3. Thus, slope heights of 40 m, 45 m, 50 m, 55 m, 60 m, 65 m, 70 m, 75 m, 80 m, 85 m, 90 m, 95 m, 100 m, 105 m, 110 m, 115 m and 120 m are obtained within the 40 ~ 120 m slope heights of the Jianbian-Ansai subregion inversion of 17 shear strength parameters. Then, the comprehensive strength parameters in the 10-m range of similar slope heights in each subregion are obtained, as shown in table 1.

| Elevation/m | Distance/m |
|-------------|------------|
| 90          | 70         |
| 50          | 30         |
| 30          | 10         |

Figure 2. Using slope/w to establish the strength parameters of 80 m slope model in the Jingbian-Ansai subregion.

Figure 3. Parameters inversion results of slope height of 80~90 m in the Jingbian-Ansai subregion.
4. Inversion Results

According to the above inversion process, the shear strength parameter values of 8 slope height ranges of the Jingbian-Ansai subregion are obtained (see table 1). The inversion values of shear strength parameters of different slope heights are shown in figure 4.

**Table 1.** Inversion of comprehensive strength parameters for different slope heights in Jingbian-Ansai subregion.

| Slope height/m | Jingbian-Ansai subregion | Cohesion c/kPa |
|----------------|--------------------------|----------------|
| 40–50          | 34.92                    | 4.53           |
| 50–60          | 35.08                    | 8.25           |
| 60–70          | 24.63                    | 37.25          |
| 70–80          | 21.12                    | 61.6           |
| 80–90          | 20.2                     | 70.24          |
| 90–100         | 17.48                    | 91.54          |
| 100–110        | 15.01                    | 114.8          |
| 110–120        | 14.98                    | 118.6          |

**Figure 4.** Inversion of strength parameters of different slope heights in the Jingbian-Ansai subregion. The curve in figure 4 show that when the slope height increases, the increase in cohesive force becomes larger, and the decrease in internal friction angle becomes smaller. However, the older the stratum, the lower the $\phi$ value, which is different from the current understanding. There are two reasons for the author's analysis: (1) The stress state of the conventional test is inconsistent with the actual potential sliding surface stress. In conventional tests, 400 kPa is used as the maximum confining pressure, and the stress on the potential sliding surface of high slopes may be much higher than this value. (2) From low stress to high stress, the Mohr-Coulomb envelope does not change nonlinearly, and its slope gradually slows down. Therefore, the results of routine tests overestimate the internal friction angle under high stress (>400 kPa).

When the slope height is lower, the fluctuation of the internal friction angle slightly affects the friction force (anti-sliding force) and hardly affects the stability change. When the slope height is higher, the fluctuation of internal friction angle has a greater impact on friction (anti-sliding force), i.e., the effect of the internal friction angle on the slope stability becomes more obvious when the slope height increases. From the analysis of the principle of limit equilibrium calculation, the potential slip surface of a slope with a small slope height is shallow and steep, the normal stress of the slip surface is small, the anti-sliding effect of the internal friction angle is small, and the stability is mainly a function of
cohesion. The potential slip surface of a slope with a large slope height is deep and gentle, the positive stress of the slip surface is large, and the effect of cohesion on the entire potential slip surface is no longer as obvious as that of a small slope height. At this time, the internal friction angle mainly controls the stability of the slope.

5. Conclusions

1. The high loess slopes in the Jingbian-Ansai subregion increase with the increase in slope height, the cohesion of the loess increases, and the internal friction angle decreases. In contrast, when the slope height increases, the contribution of the internal friction angle to the shear strength becomes more prominent, and the increase in cohesion also increases. When the slope height is in the range of 40 ~ 80 m, the cohesion will increase by 2 ~ 24 kPa for every 10-m increase in slope height; when the slope is in the range of 10 ~ 25m, the cohesion will increase by 25 ~ 45 kPa for every 10-m increase in slope height.

2. Based on the slope height and strength parameter curve obtained by the inversion method, the strength parameters of the high slope can be directly obtained according to the slope height. This method can be used for the initial judgment of the stability coefficient of known slopes and to design the slope shape and strength parameters.

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