UNBOUND GEODESICS FROM THE ERGOSPHERE AND THE MESSIER 87 JET PROFILE

J. Gariel1, G. Marcilhacy1, and N. O. Santos1,2

1 LERMA-UPMC, Université Pierre et Marie Curie, Observatoire de Paris, CNRS, UMR 8112, 3 rue Galilée, Ivry sur Seine F-94200, France; jerome.gariel@upmc.fr, gmarcilhacy@hotmail.com, nilton.santos@upmc.fr
2 School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, UK

Received 2013 May 1; accepted 2013 July 12; published 2013 August 21

ABSTRACT

Assuming that the spin $a$ of the black hole presumably located at the core of the active galactic nucleus Messier 87 takes the value which maximizes the ergospheric volume of the Kerr spacetime, we find the results compatible with the recent observations obtained by high-resolution interferometry on the origin of the jet, which would be located inside the innermost stable circular orbit diameter. Moreover, we find that a flow of unbound geodesics issued from the ergoregion is able to frame the best fits at large scales recently obtained for describing the observed profile of the relativistic jet launched from this central engine.

Key words: black hole physics – galaxies: jets – gravitation

Online-only material: color figures

1. THE JET MODEL

The arrival of high-resolution observations, of the order of microarcseconds, will soon allow us to observe the origin of extragalactic jets, especially the less distant ones like Messier 87 (M87). These data should tip the balance in favor of one of two main kinds of models. The first model type situates the origin of the jet close to the interior of the accretion disk, near the innermost stable circular orbit (ISCO; Blandford & Payne 1982) and gives the leading role to the magnetic field of the disk (Blandford & Znajek 1977, hereafter BZ). The second model type considers that the jet originates inside the ergosphere of the Kerr black hole (BH) situated at the center of the galaxy, for instance as proposed in Williams (1995, 2004), Gariel et al. (2010), and Pacheco et al. (2012), and grants only a secondary role to the magnetic field. The ergoregion, located in the gap (Garofalo 2013) between the retrograde ISCO and the event horizon, plays a key role.

The second-type model (Gariel et al. 2010; Pacheco et al. 2012) is based on the unbound particle geodesics leaving the Kerr BH ergosphere. These particles are produced by a Penrose process (Penrose 1969) inside the ergosphere and, while ejected, follow asymptotic geodesics parallel to the axis of symmetry $z$ at a radial distance

$$\rho = \rho_1 = a \left[ 1 + \frac{Q}{a^2(E^2 - 1)} \right]^{1/2},$$

(1)

where $Q$ is the Carter constant, $a$ is the BH spin, and $E$ is the particle energy. There is a limited number of discrete values for $\rho_1$ for which the energy $E$ of the particles tends to infinity, while near the neighborhood of each of these values, the energy diminishes steeply. This permits the modeling of a thin jet that is highly energetic and collimated.

2. MAXIMAL ERGOSPHERIC VOLUME

Assuming that the Penrose process is the source of production of particles building the jet, it is reasonable to suppose that its frequency increases in proportion to the ergospheric volume, which is the only region where it can take place. Hence, the larger the ergospheric volume is, the larger the amount of particles produced to take part of the jet is, and the more powerful the jet becomes.

The trace of the ergosphere in the plane $(\rho, z)$ is a closed curve, with the equation

$$z^2 = \left[ 1 - a^2 \left(1 - \frac{\rho}{a}\right) \right] \left(1 - \frac{\rho}{a}\right),$$

(2)

depending only on the parameter $a$, the Kerr BH angular momentum per unit mass, and where we normalized its mass $M = 1$. The surface $S$ inside this curve can be calculated from

$$S = \int_{\rho}^{\rho_1} \left[ 1 - a^2 \left(1 - \frac{\rho}{a}\right) \right] \left(1 - \frac{\rho}{a}\right)^{1/2} d\rho,$$

(3)

which, after integration, produces

$$S = \frac{1}{8a^2} \left[ \frac{\pi}{2} - \arcsin(1 - 2a^2) \right] - \frac{1}{4a} (1 - 2a^2)(1 - a^2)^{1/2}.$$

(4)

We can plot the function $S(a)$, Figure 1, for $a \in [0, 1]$ and obtain the value of $a$ where $S$ is maximal:

$$a = 0.877004.$$

(5)

The corresponding trace of the ergosphere, Equation (2), for $a$ given by Equation (5), generating the maximal ergospheric volume, is plotted in Figure 2.

Although the model does not take into account the back reaction and therefore does not require the usual limit $a = 0.998$ excluding the extreme Kerr BH (Thorne 1974), it remains compatible with the existence of such a limit. Maximizing the overall Penrose process at the source of the jet is achieved for the lower value of $a$ (Equation (5)).

3. JET PRODUCED BY M87

The first observation of an extragalactic jet dates back to the year 1918 and concerned the very long and powerful jet produced by M87. Currently, the M87 nucleus distance is estimated at $16.7 \pm 0.6$ Mpc (Blakeslee et al. 2009) and the mass of its central BH is estimated to be $M = (6.2 \pm 0.4) \times 10^9 M_{\odot}$.
(Gebhardt et al. 2011). Its relativistic jet extends for hundreds of kiloparsecs before ending in large radio lobes.

In the framework of the model discussed in Gariel et al. (2010) and Pacheco et al. (2012) briefly recalled in Section 1, it is natural, according to the results of Section 2, to expect that the most energetic jets produced by a Kerr BH of mass $M$ are obtained for $a$ given by Equation (5). Indeed, as we said at the beginning of Section 2, the larger the ergoregion is, the larger the number of massive objects and therefore, the Penrose production will be more frequent. Additionally, the larger the admissible ergoregion is (for the i.c. permitting unbound geodesics), the more numerous the ejections to infinity of products of this process will be. Here, the values of $\rho_1$ for high energies $E$ will prove to be greater than $a$ (cf. Equation (11) below). Then it follows that there is no characteristic hyperbola limiting the admissible ergoregion (see Equation (39) of Pacheco et al. 2012), and consequently, the whole ergoregion is admissible. So, here, there is a maximal possible flux of unbound geodesics with maximal energies.

It is natural to assume too that the jet produced by M87 fits into this model. This value for $a$ is consistent with the few evaluations of the M87 BH spin obtained to date, like $a > 0.65$ in Wang et al. (2008), and more recently $a > 0.8$ in Li et al. (2009).

Hence, we look for particles moving along unbound geodesics with high energy. We take $E = 10^6$, produced in the ergoregion, which means, in two dimensions, produced between the ergosphere and the event horizon situated on the $z$ axis. These particles move asymptotically parallel to the $z$ axis with $\rho \to \rho_1$ when $z \to \infty$. In fact, there are other geodesics, near these perfectly collimated ones, that asymptotically behave as $\rho \to \infty$ when $z \to \infty$; however, they are not very divergent, i.e., they satisfy $\rho \ll z$ for a long distance along the axis.

Now we need to determine the parameter $\rho_1$. In order to do this, we consider the recently obtained high-resolution data (Doeleman et al. 2012).

4. JET NEAR THE CORE OF M87

The data presented in Doeleman et al. (2012) open a new era of resolutions never before achieved, which are of the order of tens of microarcseconds. Hopefully soon even better resolution of microarcseconds can be expected. With these observations, the models considering jets produced by the accretion disk near the ISCO (Blandford & Payne 1982) and models with jets leaving the ergosphere (Williams 1995, 2004; Gariel et al. 2010; Pacheco et al. 2012) will be distinguished. Among these models, the one that is based only on gravity to explain the genesis of jets considers that the Kerr spacetime has the essential features to explain their formation, acceleration, and collimation (Gariel et al. 2010; Pacheco et al. 2012). Other fields, excluding the gravitational, like the magnetic field, are no more than a complement to their formation.

By using the Very Long Baseline Interferometry technique and choosing a frequency of the order of 229 GHz (or wave length 1.3 mm), allowing for a good transparency up to the core of the galaxy with an unprecedented resolution of the order of $40 \pm 1.8 \mu$ as corresponding, for a distance $16.7 \pm 0.6$ Mpc, to a length of the order of $5.5 r_S$, Doeleman et al. (2012) evaluated the maximal diameter of the emerging jet at

$$r_{\text{jetApp}} = 5.5 \pm 0.4 r_S,$$

where $r_S = 2M$ is the Schwarzschild BH radius.

Then, by assuming the validity of the BZ model (Blandford & Znajek 1977), they also assume that the jet starts at the ISCO of the accretion disk and set

$$r_{\text{jetApp}} = r_{\text{ISCOApp}}.$$  

This assumption implies too that the origin of this observation is on the plane of ISCO, i.e., the equatorial plane $z = 0$. The apparent radius of ISCO, plotted in Doeleman et al. (2012) as a function of the spin $a$ (see Figure 3 in Doeleman et al. 2012), is the intrinsic radius corrected by the effect of gravitational lensing caused by its proximity to the core endowed with a Kerr BH. In this figure, the horizontal line representing $r_{\text{jetApp}}$
intersects the prograde radius of ISCO, which evaluates the spin of the BH as $a \approx 0.625 \pm 0.25$ and shows that only the prograde part is viable. We observe that the value of $a$ thus obtained is inside the limit of Wang et al. (2008), but outside the limit of Li et al. (2009).

Next, we interpret the same result in light of the model given by Gariel et al. (2010) and Pacheco et al. (2012). Instead of considering the “apparent” distances, we deduct the lensing and consider the “intrinsic” distances

$$r_{\text{jetInt}} \approx r_{\text{jetApp}} - 1.3,$$

(8)

by observing that the curve $r_{\text{ISCOInt}}(a)$, Figure 3, roughly translated, is about 1.3 below the curve $r_{\text{ISCOApp}}(a)$ given in Figure 3 in Doeleman et al. (2012).

Writing these quantities in terms of Weyl coordinates, given by Equation (1) in Pacheco et al. (2012), we obtain

$$\rho_{\text{jetInt}} = [(r_{\text{jetInt}} - 1)^2 - (1 - a^2)]^{1/2}
= [(r_{\text{jetApp}} - 2.3)^2 - (1 - a^2)]^{1/2}.$$  

(9)

Now we can plot the curves $\rho_{\text{jetInt}}(a)$ for values in the range $r_{\text{jetApp}} \in [5.1, 5.9]$, which are given in Figure 4. In particular, from the central curve, corresponding to the apparent radius $r_{\text{jetApp}} = 5.5$, we deduce for the maximal volume of the ergoregion (5)

$$\rho_{\text{jetInt}} = 3.1637.$$  

(10)

We abandon the hypothesis (7) assumed by Doeleman et al. (2012) and we replace it by

$$\rho_{\text{jetInt}} = \rho_1.$$  

(11)

This allows us to obtain the unbound $\rho_1$-asymptotic geodesic (see the lower curve, asymptotically parallel to the $z$ axis, in Figure 5), with parameters $a = 0.87705$, $E = 10^6$, and $\rho_1 = 3.1637$ after numerical integration of Equation (21) given by Gariel et al. (2010) and initial conditions (i.c.)

$$\rho_i = 0.56524, \ z_i = -0.25,$$  

(12)

which are well inside the ergosphere (see Figure 2).
structure far from the core. To answer this question, we compare the fittings suggested by Asada & Nakamura (2012) to the geodesics studied in the model (Pacheco et al. 2012).

Asada & Nakamura (2012) observe an important changing of the jet slope after a certain altitude of the order $z \approx 10^3$, near the Bondi radius, indicating a divergent tendency. From these observations, they established the following best fitting laws. For $z \in [10^2, 5 \times 10^5]$, 

$$z = 0.2 \left( \frac{\rho}{0.8} \right)^{b_1} \text{ or } \rho = 0.8 \left( \frac{z}{0.2} \right)^{1/b_1}, \quad (13)$$

and for $z \in [10^5, 2 \times 10^7]$, 

$$z = 30 \left( \frac{\rho}{0.2} \right)^{b_2} \text{ or } \rho = 0.2 \left( \frac{z}{30} \right)^{1/b_2}, \quad (14)$$

where $b_1 = 1.73$ is called the parabolic type and $b_2 = 0.96$ is called the conical type. We plot these curves in Figure 5 (see the two straight lines framed by the two geodesics). We seek to compare them with geodesics of the model (Gariel et al. 2010; Pacheco et al. 2012).

To maintain continuity with the previous situation, near the core, studied in Section 4, we keep the parameters $E = 10^6$ and $\rho_1 = 3.1637$ to look for unbound geodesics, not asymptotic to $\rho_1$, by numerical integration of Equation (21) given in Gariel et al. (2010), for different i.c. inside the ergosphere. We plot in Figure 5 “external” and “internal” unbound geodesics, starting from the ergosphere, obtained for the i.c. $\{\rho_i = 0.7, z_i = -0.4\}$ and $\{\rho_i = 0.85, z_i = -0.15\}$, respectively, which frame the two precedent best fits. The initial parts of these geodesics can also be viewed in Figure 2, where they frame a set of various geodesics (not represented in Figure 5, for the sake of clarity) obtained for intermediate i.c., all inside the ergosphere. The “internal” geodesic starts above, very close to, the geodesic defined in Equation (12), and later diverges. The flow of geodesics located between the external one and the internal one diverge also, forming an outer shell, distinct from the perfectly collimated geodesic (12). There is evidence that the jets produced by blazars are composed of an external envelope, called a sheath, and an interior thin beam (Girotetti et al. 2004; De Villiers et al. 2005; Xie et al. 2012). These features appear naturally in our purely gravitational model.

We can see in Figure 5 that the “conical” fit (13) is in good agreement with the “internal” geodesic (and those close to it), which extends even to lower altitudes, up to $z \sim 10^3$ where the fit is not consistent with the observations. In contrast, the “parabolic” fit (13) does not seem able to be equated with any single geodesic of the flow. The flow intersects the fit. That should be interpreted by noting that the successive points emitting the observed radiations belong to the outer portion of the jet. Thus, there is an erosion of the width of the jet during its upward motion, mainly due to its friction with the interstellar medium (ISM). This friction is greater outside, where the density of the ISM is greater, because the centrifugal force due to the BH rotation drags it outside. Without this erosion effect, the external geodesic would remain in the external part of the jet which would remain parallel to the conical fit from $z \sim 10^3$ (or even before), as can be seen in Figure 5.

The internal geodesic presented in Figure 5 is the last one before the asymptotic geodesic to the axis $\rho_1$ (parallel to the $z$ axis). It is the limit before an abrupt changeover to the $\rho_1$ geodesic obtained by fine-tuning of the i.c.. This suggests a bifurcation, but only a qualitative analysis could highlight its possible existence.

The external geodesic, presented in Figure 5, is not the last one. In fact, there are a few more external geodesics, starting from the (left) extremity of the ergoregion (see Figure 2). Again, their asymptotical behavior remains identical (parallel to the same direction), but they no longer cut the “parabolic” fit because the outermost layers are quickly “eroded.” We have chosen to present the first external geodesic that touches the parabolic fit, at the observed nearest boundary (about $z \sim 100$) of the large scale jet.

The collimation is not very strong for the “sheath,” as can be seen in Figure 5: $\rho/z \approx 10^{-1}$, for large $z$. The existence of asymptotical conical geodesics is well known (e.g., see after Equation (45) of Gariel et al. 2010). However, their slopes are theoretically undetermined.

6. DISCUSSION

The simple “geodetical” model proposed here, reflects only the rough profile of the M87 jet, and cannot give a detailed picture of the complex jet phenomenon, with its observed radiations, shocks, blobs, lobes, etc. The complete structure essentially depends on the traversed medium, which is mostly unknown.

More generally, there is presently no known exact solution (with a good asymptotic behavior) to axisymmetric stationary solutions of the general relativity equations with a second member (i.e., in a medium), and all of the further acclimatizations of the Kerr vacuum solution for any medium at best will be only corrective. Hence, all models are faced with these extrinsic difficulties. The BZ model does not escape this rule. However, it can be admitted that there is a predominant phenomenon for launching, accelerating, and collimating the jet. The main differences between the models will come from this fundamental phenomenon. In the BZ model, it is the presence of a magnetic field, even if it is necessary that a process “à la Penrose” remains. In the present model, it is shown that because of these essential features, a magnetic field is not indispensable, essentially because a similar role can be played by the so-called gravito-magnetic field.

However, another important point for adequately modeling the jet phenomenon is to predict the type of ejected particles, especially if they are charged or neutral. For charged particles, the magnetic field can no longer be ignored. Another recent alternative to the BZ effect was also proposed as a model of jets, namely, the oscillating string loop model where the confinement is related to tension of the string loops (see Jacobson & Sotiriou 2009). According to this model, the transmutation effect converting the internal string-loop energy into energy of its translational motion can explain acceleration of the string loops up to ultra-relativistic velocities (see Stuchlí & Kolôs 2012). The string loops can be modeled by magnetic force lines in plasma or by thin isolated flux tubes of plasma (see Christenssen & Hindmarsh 1999; Semenov et al. 2004; Spruit 1981; Cremaschini & Stuchlik 2013). Such a model could also explain the in situ re-accelerations, necessary to give an account of the observed obstrinate continuity of the jet, in spite of the numerous crossed obstacles which tend to disperse or stop it.

Another interesting question concerns the prospective link between the BSW effect (Bañados et al. 2009) and the theoretical existence of infinite energies for unbound geodesics asymptotically parallel to the $z$ axis starting from the ergoregion (Gariel et al. 2010; Pacheco et al. 2012). Though the existence of such large energies produced by the BSW effect does not seem debatable, the possibility of observing them at large distances was
questioned (Bejger et al. 2012; McWilliams 2013). It would be interesting to examine more accurately whether these demonstrations are valid only in the restrictive conditions under which they were made, or if they can have a wider validity. The differences in approach are large and we reserve this question for future investigation. We simply note that in the framework of the superspinning Kerr geometry, ultrarelativistic particles can be observed at the infinity (Patil & Joshi 2011; Stuchlík & Schee 2013).

REFERENCES

Asada, K., & Nakamura, M. 2012, ApJL, 745, L28
Bañados, M., Silk, J., & West, S. 2009, PhRvL, 103, 111102
Bejger, M., Piran, T., Abramowicz, M., & Håkanson, F. 2012, PhRvL, 109, 121101
Blakeslee, J., Jordán, A., Mei, S., et al. 2009, ApJ, 694, 556
Blandford, R. D., & Payne, D. G. 1982, MNRAS, 199, 433
Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 883
Christensson, M., & Hindmarsh, M. 1999, PhRvD, 60, 063001
Cremaschini, C., & Stuchlík, Z. 2013, PhRvE, 87, 043113
De Villiers, J. P., Hawley, J. F., Krolik, H. J., & Hirose, S. 2005, ApJ, 620, 878
Deeoleman, S., Fish, V. L., Schenck, D. E., et al. 2012, Sci, 338, 355
Gariel, J., MacCallum, M. A. H., Marcilhacy, G., & Santos, N. O. 2010, A&A, 515, A15
Garofalo, D. 2013, AdAst, 2013, 213105
Gebhardt, K., Adams, J., Richstone, D., et al. 2011, ApJ, 729, 119
Giroletti, M., Giovani, G., Feretti, L., et al. 2004, ApJ, 600, 127
Jacobson, T., & Sotiriou, T. P. 2009, PhRvD, 79, 065029
Junor, W., Biretta, J. A., & Livio, M. 1999, Natu, 401, 891
Li, Y. R., Yuan, Y. F., Wang, J. M., Wang, J. C., & Zhang, S. 2009, ApJ, 699, 513
McWilliams, S. T. 2013, PhRvL, 110, 011102
Pacheco, J. A. de F., Gariel, J., Marcilhacy, G., & Santos, N. O. 2012, ApJ, 759, 125
Patil, M., & Joshi, P. S. 2011, PhRvD, 84, 104001
Penrose, R. 1969, Rev. Nuovo Cimento (Numero Speciale), 1, 52
Semenov, V., Dyadechkin, S., & Punsly, B. 2004, Sci, 305, 978
Spruit, H. C. 1981, A&A, 102, 29
Stuchlík, Z., & Koliós, M. 2012, JCAP, 10, 008
Stuchlík, Z., & Schee, J. 2013, CQGra, 30, 075012
Thorne, K. S. 1974, ApJ, 191, 507
Williams, R. K. 1995, PhRvD, 51, 5387
Williams, R. K. 2004, ApJ, 611, 952
Wang, J. M., Li, Y. R., Wang, J. C., & Zhang, S. 2008, ApJL, 676, L109
Xie, W., Le, W.-H., Zou, Y.-C., et al. 2012, RAA, 12, 817