Structure of magnetic domain wall in cylindrical microwire

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Abstract—The magnetization in the domain walls (DWs) formed inside the inner core of the amorphous ferromagnetic microwire is studied within a simple analytical model proposed. The influence of the ordering in the internal-stress created outer shell of the wire is included via an effective Dzyaloshinskii-Moriya-like anisotropy and resulting DW textures have been classified. The model is applicable to periodically constricted nanowires (whose core-shell magnetic structure follows from the classification). The model is applicable to periodically constricted nanowires of [9]. We include the influence of the ordering in the internal-stress created outer shell. The ferromagnetic microwire is studied within a simple analytical model proposed. (e.g. and slow cooling stages [5], [6], and its distribution can be established to affect the remagnetization process in the manufacturing process that contains the rapid solidification into micro-domains because of the magnetostatics.

In the present paper, the Landau-Lifshitz-Gilbert (LLG) equation in 3D is analytically solved with application to DWs in the inner core of a soft-ferromagnetic wire that is surrounded by a patterned outer shell. The ferromagnetic domains in the core are magnetized along the wire axis due to the dipole-induced shape anisotropy of the easy-axis type. In the glass-coated amorphous or nanocrystalline ferromagnetic microwires, the patterned magnetic ordering of the outer shell is due to an internal stress and it is dependent on the sign of the magnetoelastic constant [4]. The stress is induced during the manufacturing process that contains the rapid solidification and slow cooling stages [5], [6], and its distribution can be then modified via removal of the glass coating or via a thermal treatment [7], [8]. The patterned magnetic structure of the outer shell can originate from a modulation of the wire shape, thus, the magnetostatic-field modulation as well. (e.g. in periodically constricted nanowires of [9]). We include the effect of the outer-shell patterning on the inner core via a Dzyaloshinskii-Moriya-like contribution to the LLG equation. This type of the effective anisotropy matches the expectation of being dependent of the magnetization derivative over the wire-axis coordinate (a consequence of the shell patterning) with the property of the non-invariance with respect to the space inversion. The non-invariance results from of the chirality of the ordering inside the outer shell.

Upon finding a series of stationary DW solutions to the LLG equation, we carry out a verification of the results with micromagnetic simulations for periodically constricted nanowires and discuss the consequences of the magnetization distribution established on the DW mobility and collisions.

Index Terms—Magnetic domain walls, nanowires.

I. INTRODUCTION

The knowledge of the structure of the domain walls (DWs) in magnetic microwires for sensor applications is of crucial importance with regard to establishing the course of the magnetization process. However, for the amorphous microwires, the strong competition between the exchange and magnetostrictive interactions makes the DW simulations complex [1]. Moreover, systems to consider are huge compared to the magnetostatic exchange length, thus, the dipole interactions are not efficiently included into the simulations, whereas, they are established to affect the remagnetization process in the amorphous glass-coated microwires [2], [3]. This motivates analytical modeling of the DW structure and dynamics.

In the present paper, the Landau-Lifshitz-Gilbert (LLG) equation in 3D is analytically solved with application to DWs in the inner core of a soft-ferromagnetic wire that is surrounded by a patterned outer shell. The ferromagnetic domains in the core are magnetized along the wire axis due to the dipole-induced shape anisotropy of the easy-axis type. In the glass-coated amorphous or nanocrystalline ferromagnetic microwires, the patterned magnetic ordering of the outer shell is due to an internal stress and it is dependent on the sign of the magnetoelastic constant [4]. The stress is induced during the manufacturing process that contains the rapid solidification and slow cooling stages [5], [6], and its distribution can be then modified via removal of the glass coating or via a thermal treatment [7], [8]. The patterned magnetic structure of the outer shell can originate from a modulation of the wire shape, thus, the magnetostatic-field modulation as well. (e.g. in periodically constricted nanowires of [9]). We include the effect of the outer-shell patterning on the inner core via a Dzyaloshinskii-Moriya-like contribution to the LLG equation. This type of the effective anisotropy matches the expectation of being dependent of the magnetization derivative over the wire-axis coordinate (a consequence of the shell patterning) with the property of the non-invariance with respect to the space inversion. The non-invariance results from of the chirality of the ordering inside the outer shell.

Upon finding a series of stationary DW solutions to the LLG equation, we carry out a verification of the results with micromagnetic simulations for periodically constricted nanowires and discuss the consequences of the magnetization distribution established on the DW mobility and collisions.

II. MODEL

The LLG equation in 3D is considered to be of the form

$$-\frac{\partial \mathbf{m}}{\partial t} = J \frac{\mathbf{m} \times \Delta \mathbf{m}}{M} + \frac{\beta}{M} (\mathbf{m} \cdot \hat{i}) \mathbf{m} \times \hat{i} + \gamma \mathbf{m} \times \mathbf{H}$$

$$-\frac{d}{M} \mathbf{m} \times \left( \hat{i} \times \frac{\partial \mathbf{m}}{\partial x} \right) - \frac{\alpha}{M} \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}. \quad (1)$$

Here, \( \hat{i} \equiv (1, 0, 0) \), (the wire is directed along the x axis), \( M = |\mathbf{m}| \), \( \gamma \) denotes the gyromagnetic factor, \( J \) denotes the exchange constant, \( \beta \) and \( d \) determine the strength of the easy-axis anisotropy (of the magnetostatic origin, \( \beta \leq \gamma \mu_0 M^2 \)) and a DM-like anisotropy, respectively, \( \mathbf{H} = (H_x, 0, 0) \) represents the external (longitudinal) magnetic field. The DW solutions satisfy the condition \( \lim_{|x| \to \infty} \mathbf{m} = \pm (M, 0, 0) \).

In the periodically constricted wire, the DM-like anisotropy is a consequence of the presence of the perpendicular to the wire-axis surfaces of the system (rings) which are distributed along the wire. The surface magnetic charges on these rings induce a modulation of the outer-shell ordering, thus, modifying the radial distribution of the anisotropy in the inner core. In the amorphous microwire, the origin of the outer-shell ordering is different while the result of the mutual core-shell interaction is a similar helical ordering of the overall system [9], [10]. The outer-shell magnetization of the glass-coated amorphous microwire is circumferential in the case of the negative-magnetostriction materials while radial in the case of the positive magnetostriction. Additionally, the shell is divided into micro-domains because of the magnetostatics.

Looking for the solution to (1), following [11], we apply the transform

$$m_+ = \frac{2M}{f^* / g + g^* / f}, \quad m_x = M \frac{f^* / g + g^* / f}{f^* / g + g^* / f}, \quad (2)$$
where \( m_+ \equiv m_0 \pm im_z \), and we find tri-linear equations of motion for the complex functions \( g(x,y,z,t), f(x,y,z,t) \) (secondary dynamical variables)

\[
- f_i D_i f^* \cdot g = f [\alpha D_i + J(D_i^2 + D_i^2 + D_i^2)] \\
+ idD_x \cdot g + Jg^* (D_i^2 + D_i^2 + D_i^2) g \cdot g \\
- (\gamma H_x + \beta) |f|^2 g,
\]

\[
- g_i D_i f^* \cdot g = g_i [\alpha D_i - J(D_i^2 + D_i^2 + D_i^2)] \\
- idD_x f^* \cdot g - Jf(D_i^2 + D_i^2 + D_i^2) f^* \cdot f^* \\
+ (\gamma H_x + \beta) |g|^2 f^* ,
\]

where \( D_t, D_x, D_y, D_z \) denote Hirota operators of differentiation \( D_i b(x,y,z,t)c(x,y,z,t) \equiv (\partial/\partial x - \partial/\partial z) b(x,y,z,t)c(x,y,z,t) \left|_{x=x',y=y',z=z',t=t'} \right. \).

### III. DOMAIN-WALL STRUCTURE

For the case \( H_x = 0 \), the stationary single-DW solution to (3) has been found in the form

\[
f_0 = 1, \quad g = \alpha e^{kx+qy+pz},
\]

where

\[
k^2 + q^2 + p^2 - idk = \beta J
\]

and \( \text{Re} k \neq 0 \). I denote \( k = k' + ik'' \), \( q = q' + iq'' \), \( p = p' + ip'' \), where \( k', q', p' \) take real values, rewriting (5) with

\[
k^2 + q^2 + p^2 - k'^2 - q''^2 - p''^2 = \beta J \frac{dk''}{dJ}.
\]

Assuming the DW to be centered at \( (x,y,z) = 0 \), then \( u = e^{i\phi} \), the relevant magnetization profile [the single-DW solution to (1)] is written explicitly with

\[
m_+(x,y,z) = Me^{i(\phi + k' x + q' y + p' z)} \text{sech}[k' x + q' y + p' z],
\]

\[
m_-(x,y,z) = -M \text{tanh}[k' x + q' y + p' z].
\]

For \( q = p = 0 \), the twisted transverse DW solution is obtained unless

\[
k'' = \frac{d}{2J},
\]

\[
k'^2 = \frac{\beta}{J} - \frac{d^2}{4J^2}
\]

while, for \( q' = p' = 0 \) and \( |k'| \ll |q''| = |p''|, |k'| \), the vortex DW of Fig. 1a is found unless the condition (8) is matched with

\[
k'^2 - 2q'^2 = \frac{\beta}{J} - \frac{d^2}{4J^2}.
\]

The solution of the type of the twisted transverse DW is expected not to be relevant to thick wires since it corresponds to a constant magnetization of the wire cross-sections.

Without loose of generality, we assume \( p = 0 \), thus, choosing a DW orientation in the YZ plane. Including an additional condition of minimizing the surface magnetostatic energy (via taking the normal to the nanowire surface to be a hard direction for the magnetization), the number of free parameters can be further reduced. Namely, on the contour of the intersection of the DW-plane \( (k' x + q' y = 0) \) with the surface of the inner core of the wire \( \sqrt{y^2 + z^2} = R \), the magnetization should be aligned in the circumferential direction wherever possible, in particular, at the central \( (x = 0) \) and boundary \( (x = \pm q R/k') \) cross-sections of the wire in the DW-plane region. Therefore, \( q'' = 0, |\phi| = \pi/2 \), \( |p''| R = \pi/2 \). Finally, from (6), one finds (8) to be satisfied, and

\[
k'^2 + q'^2 = \beta J - d^2/4J^2 + (\pi/2R)^2.
\]

An additional “freedom” on the solution is due to independence of (11) of the first derivatives of \( \text{im} \) over \( y \) and \( z \) coordinates, which allows the perpendicular magnetization to take the form

\[
m_+(x,y,z) = Me^{i(\phi + k'' x + p'' y)} \text{sech}[k' x + q' y]
\]

instead of (7). The relevant DW texture shown in Fig. 1b corresponds to a “planar” DW of (12). A similar DW has been found with the micromagnetic simulations for the constricted
nanowires \[9\] , albeit deformed at its ends with a small flixture of the DW plane, as predicted in \[13\].

Using a different from (4) ansatz
\[
f = 1, \quad g \equiv e^{k_x x + i\phi + \eta \arctan(z/y)} \left( \frac{\sqrt{y^2 + z^2}}{R} \right),
\]
we find it to satisfy \[3\] under the conditions \[\eta \leq 0\] and \(\eta > 0\).
we expect \(R = \text{const}\), and we expect \(R = \text{close to } R\). The relevant magnetization
field is plotted in Figs. 1c, 1d, where the DW is seen to be asymmetric relative to the YZ plane. The present structure is
singular at \((x, y, z) \rightarrow (\infty, 0, 0)\) (the DW is infinitely long on the
central line of the wire) and corresponds to a "tubular-
conical" DW observed in \[14\]. We expect the ordering in the
DW to resemble the ordering in the outer shell. Hence,
the solution with \(\eta = 1\) is expected to be valid to the glass-
coated negative-magnetostriction wires, (in particular, Co-rich
microwires), whose shell is circumferentially magnetized.

The solutions with higher values of \(|\eta|\) are expected to describe the
DWs in positive-magnetostriction materials, (in particular, Fe-rich
microwires), while \(\eta\) is expected to be equal to the number of the radially-magnetized microdomains in a single
segment of the outer shell. However, at some compositions,
the outer shell of the negative-magnetostriction microwire can
be of a complex vortex-containing texture, and DWs of \(|\eta| > 1\)
be preferable for such cases as well \[13\]. Obviously, the
conical DW cannot be stable solution to the LLG equation.
The elongated vertex of the DW has to shrink in order to
reduce the exchange energy.

In order to verify the analytical predictions on the DW structure, we have performed micromagnetic simulations of the DWs in constricted Py nanowires (the saturation magnetization \(M = 8.6 \cdot 10^3 \text{A/m}\)). Unlike in \[2\], we have studied infinitely-long wires of the same outer diameter \(D = 150\text{nm}\) varying the diameter of the constriction only, thus, controlling the strength of the helical anisotropy with a single parameter. The discretization size was 5nm, (we utilize the OOMMF package), while the simulations have been initialized with a stepwise magnetization distribution. The outer-shell of the wire is seen to form a bamboo-like domain structure whose period decreases with decreasing the constriction diameter. The stable DWs are found to be of the planar type (Fig. 2a) or of the vortex type (Figs. 2b, 2c) for the constriction diameter bigger or smaller than about 0.8D, respectively, in correspondence to our model that predicts the planar DWs to be formed provided \(\beta > d^2/J\) because of \[11\] (a weak modulation of the surface layer of the wire), while the vortex-
DW formation to allow the opposite relation. Similar vortex
DWs have been found in the circular-cross-section nanowires of the magnetocrystalline easy-plane anisotropy in \[16\].

The relaxation to the vortex DW structure as well as to the planar
DW structure follows the formation of a meta-stable conical
DW (Fig. 2d). We mention that the planar and meta-stable conical DWs have been previously claimed to realize in glassy-
ferromagnetic wires as a result of the competition between
the magnetostatics and complex exchange interactions as well \[13\], \[17\].

With regard to the amorphous microwires, a simple esti-
mation of \(d\) is based on the disturbing of the strain-induced internal magnetic field in the wire \(H \sim 3|\lambda/\sigma|/\mu_0 M\), where \(\lambda\) denotes the magnetoeelastic constant. Here the stress \(\sigma = \sigma_1 + \sigma_\alpha\) consists of the internal-stress contribution that is a compli-
cated function of the parameters of the wire composition and
fabrication conditions (the initial temperatures of the wire and
of the glass cover, an extraction stress) \[6\], and of the applied-
stress contribution. The first differential correction to this
stress due to the ordering inhomogeneity (a magnetostatically-
induced micro-domain structure) is expected to be of the order
of \(|\sigma|/M|\nabla \cdot \mathbf{n}|/M\), where \(l_{ms} = (2A_{xx}/\mu_0 M^2)^{1/2}\) denotes the
magnetostatic exchange length, \((A_{xx} = J M/2\gamma))\), which
gives the estimate \(d \sim 3/2|\lambda/\sigma|l_{ms}\) \(\equiv \gamma K_{ms} l_{ms}\).

IV. IMPLICATIONS FOR DOMAIN-WALL DYNAMICS

Let us denote any stationary single-DW solution to (3)
by \(f_0(x, y, z)\), \(g_0(x, y, z)\). The inclusion of \(H_x \neq 0\) leads to
the dynamical solutions \(f(x, y, z, t) = f_0(x, y, z) = 1, g(x, y, z, t) = e^{i\omega t} g_0(x, y, x), \) where \(\omega = \omega' + i\omega'' = -\gamma H_x/(1 + i\alpha)\), which describes the driven motion of the
DW with the velocity \(v = \omega''/k' \equiv SH_x\) accompanied by
the magnetization rotation about the wire axis with the frequency
\(\omega'\). Here \(S \approx \gamma/|k'|\) denotes the DW mobility. However, the
Walker breakdown is observed in the positive-magnetostriction amorphous and nanocrystalline microwires \[18\]. In the regime of the viscous motion of the DW \(\omega' = 0\), taking the l.h.s.
of \(1\) and \(3\) equal to zero, one finds \(S \approx \gamma \alpha/|k'|\). Since
we consider the magnetization of the outer shell of the wire

Fig. 2. The magnetization in the longitudinal \((z = 0)\) and in the transverse 
\((x = \text{const})\); (a), (b) cross-sections of the constricted Py nanowire in the DW 
area obtained with the micromagnetic simulations. The outer diameter of the wire 
is \(D = 150\text{nm}\), the constriction diameter is: \(0.9D\) (a), \(0.6D\) (b), \(0.5D\) 
(c), \(0.7D\) (d). Stable DWs of the planar type (a), vortex type (b), (c), and
meta-stable (conical-like) DW (d) are shown indicating the sign and value of 
the \(z\) (\(x\)) component of the magnetization at the top (bottom) pictures with 
the colors (red to blue).
to be stationary in the YZ plane, easy directions in the cross-sections of the wire, thus, the viscosity of the inner-core DW occurs in the case of the discrete rotational symmetry of the shell [19]. (for the conical DW of the parameter $|\eta| > 1$).

The measurements of the DW mobility for the glass-coated microwires with dependence on the applied stress has been performed in [20]. In the cited paper, a qualitative analysis for the viscous regime of the DW motion has been performed taking $S = \gamma_{me}/\alpha$, where the width of the DW has been assumed to be $l_{me} \approx (2A_{ex}/K_{me})^{1/2}$. For sufficiently high stress applied, when $k^2 \sim d^2/4J^2 \gg \beta/J$, (the vortex-DW regime), the above assumption corresponds to taking $d \approx K_{me}l_{me}$ instead of the previous-section estimate $d \approx K_{me}l_{ms}$, and it is a consequence of neglecting the dipole-induced anisotropy while assuming the anisotropy in the inner core of the wire to be of the purely magnetostatic origin. In the consequence, the cited authors find $S \propto 1/\sqrt{\sigma}$, (in disagreement with experimental data), while we evaluate $S = \gamma l_{me}^2/l_{ms} \propto 1/\sigma$. However, a dipole-field effect on the DW mobility has been reported for the Fe-rich microwires of large-enough cross sections [2], and for Co-rich microwires and nanowires even [3]. A simple comparison of typical magnetoelastic and magnetostatic exchange lengths of the amorphous microwires ($K_{me} \sim 10^{4} J/m^3$ for Fe-Si-B, $K_{me} \sim 10^{5} J/m^3$ for Co-Si-B, and $\mu_0 M^2 \sim 10^{3} J/m^3$) shows $l_{ms}^2/l_{me}^2 \leq 1/10$ [1, 21]. Since the smallest characteristic exchange length is expected to determine the thickness of the outer-shell DWs, following the previous section, we claim the parameter $d$, thus, the DW mobility to be dependent on the magnetostatic exchange length $l_{ms}$.

Despite the (Hiroti) method that we apply is capable to treat the soliton collisions, the description of the DW collision is complex because of the necessity of including the dissipation. The result of the collision can be predicted simply, however, on the basis of a rule verified for ferromagnetic chains and stripes. When the magnetization in the closing up areas of the colliding DWs is parallel (antiparallel), the walls attract (repulse), thus, they are expected to annihilate each other (to form a 2r-DW) [22]. The circular (helical) ordering in the outer shell enforces the same direction of the magnetization curling in both the colliding DWs, (which manifests in the same sign of $k^2$ and of the parameter $d$ of the model), therefore, we expect the collision to result in the annihilation of the DWs observed in [23].

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