Fate of unoriented bosonic string after tachyon condensation

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Abstract

By a toroidal compactification with a vortex like configuration of tachyon fields, the unoriented bosonic string in 26 dimensions becomes equivalent to 10 dimensional string theory with gauge group $SO(32) \times G$ where $G$ has rank 16. The reduction from $SO(8192)$ to $SO(32)$ is induced by the action of non-abelian Wilson lines which twists the tachyon. The additional enhanced gauge symmetry $G$ appears from closed string sector by the vertex operator construction. We also examine the consistency of the tachyon condensation.
1 Introduction

Some years ago, it was found [1] [2] [3] that the unoriented bosonic string theory with gauge group \( SO(2^{13}) = SO(8192) \) meets tadpole cancellation condition. Although it is interesting to observe that a consistent string theory exists with such a simple field content, it is far from realistic because of the existence of tachyon and the large gauge group.

Recent development of string theory show that a paradigm that every tachyon free, modular invariant string theories (type IIA/B, type I, and heterotic string) should be regarded as different limits of moduli of one unifying model, say M theory. At this moment, however, it is still difficult to understand the status of the modular invariant but tachyonic theories such as bosonic, type O, OA/B, and some heterotic models. The key issue is how to make physical interpretation of tachyons.

Starting from the study of stable non-BPS sates [4]-[9], Sen began to clarify the meaning of tachyon. In parallel D-brane - anti-D-brane system, tachyon plays the rôle of the Higgs fields in the system with spontaneous symmetry breaking. By tachyon condensation with topologically nontrivial winding, D-(\(p-2\))-brane appears at the core of vorticity after “pair annihilation” of D- and anti D-branes. A geometrical basis of such a mechanism was explored by Witten [10] in the language of K-theory.

The purpose of this letter is to analyze a possibility that such a mechanism may be applied to bosonic \( SO(8192) \) theory. The idea is very simple. Originally, the theory has \(2^{13} = 8192 \) 25-branes. After the tachyon condensation, the number of branes should be reduced by halves and dimensions of branes will be decreased by two. If we repeat this step 8 times, we get reduction of \(2^{13} \) branes to \(2^5 = 32 \) branes. If we choose the momentum lattice of toroidal compactification, an extra gauge symmetry \(G\) of rank 16 may appear from the closed string sector. If we take \(G = SO(32)\) the system has the same gauge group as that of the type 0 theory [12] and toroidal compactifications of oriented bosonic closed string. In this way, one may include \(SO(8192)\) theory into a family where a reasonable treatment of remaining tachyon is being developed recently from AdS/CFT correspondence [13].

In [11] Sen has created a method to treat tachyon condensation as a deformation of boundary conformal field theory (BCFT). By following him, we will use three steps to explicitly describe tachyon condensation.

1. Find a insertion of Wilson line which will make tachyon mode anti-
periodic in the compactified directions. It creates a kink in that direction.

2. Find a radius where tachyon vertex has dimension one where it becomes marginal operator.

3. Realize tachyon condensation as the deformation of BCFT.

Sen [11] has developed BCFT of tachyon condensation where the dimension of the brane will be reduced by one by merging two D-branes. Straightforward repetition of his method will reduce the number of branes by a factor of $2^{16}$ while reducing the dimension of the branes by 16. Since we have only $2^{13}$ branes, it should be modified for our purpose. Namely, we need to study the process of pair annihilation of two D-$(p + 2)$ branes to create one D-$p$-brane just as in the superstring cases [7][10][8].

2 Generalized kink configuration and nonabelian Wilson lines

For simplicity, we start from the reduction of two D-$(p + 2)$-branes. Chan-Paton (CP) group is $U(2)$ and we have four CP sectors labeled by $I$ and $\sigma_i$ ($i = 1, 2, 3$). We write two directions where reduction occurs as $x^1, x^2$. At the core of the vorticity, tachyon should behave as

$$T(x) = x^1 \sigma_1 + x^2 \sigma_2 + O(|x|^2).$$

(1)

Let us assume that $x^1$ and $x^2$ directions are toroidary compactified in a radius $R$ and try to achieve the first step of Sen’s scenario. It will be natural to assume from (1) that tachyon field $T(x)$ is anti-periodic in $x^i$ direction when its CP factor is $\sigma_i$ (resp. $i = 1, 2$). We need to find Wilson lines which generate such a change of boundary condition. We denote $X^i(z)$ the string coordinates along the directions $x^i$. Consider the insertion of Wilson line,

$$\exp \left( i \alpha \frac{\sigma_2}{4} \oint \partial_z X^1(z) dz \right),$$

(2)

\footnote{We used the analogy with the superstring case [10]. In bosonic situation, this does not necessarily comes from topological requirement since both tachyons and gauge bosons has $U(2)$ CP factors. Rather we use it simply as a generalization of the kink configuration.}
at the boundary of the world sheet. Although the states with CP factors $I$ and $\sigma_2$ are unaffected, those with CP factors $\sigma_1$ and $\sigma_3$ get some shift of momentum $p_1 \rightarrow p_1 \pm \frac{R}{2}$, because of the commutation relation,

$$[\sigma_2, \sigma_1 \mp i\sigma_3] = \pm 2(\sigma_1 \mp i\sigma_3). \quad (3)$$

More explicitly, if we write the zero-mode wave function of the open string sector as

$$\Psi(x^1, x^2) = \sum_{i=0}^{3} \psi_i(x^1, x^2) \sigma_i, \quad \sigma_0 = 1, \quad (4)$$

the boundary condition is modified to,

$$\Psi(x^1 + 2\pi R, x^2) = h_1 \psi (x^1, x^2)(h_1^{-1}), \quad h_1 = \exp(i\frac{\pi}{2}R\alpha \sigma_2). \quad (5)$$

Momentum along $x^1$ was originally quantized in the unit $1/R$. Therefore if we choose $\alpha = 1/R$ we obtain desired anti-periodicity for the mode with CP factor $\sigma_1$. The factor in the boundary condition becomes $h_1 = \sigma_2$.

To get correct anti-periodicity along $x^2$ direction, we need to include the second Wilson line,

$$\exp \left(i\frac{\alpha}{4} \sigma_1 \int \partial_z X^2(z) dz \right). \quad (6)$$

This time those modes with CP factor $\sigma_2$ and $\sigma_3$ get similar shift of their momentum.

$$\Psi(x^1, x^2 + 2\pi R) = h_2 \psi (x^1, x^2)(h_2^{-1}), \quad h_2 = \exp(i\frac{\pi}{2}R\alpha \sigma_1). \quad (7)$$

We choose $\alpha = 1/R$ again and the deformation factor becomes $h_2 = \sigma_1$. The deformations factor for each Wilson line become non-commutative $h_1 h_2 \neq h_2 h_1$. However, the action on the wave function becomes commutative and there is no ambiguity in the ordering. This happens because of the specific choice of $\alpha$ we made.

At this point it is worth while to discuss the topological aspects of our gauge configuration. If we convert the Wilson lines to the gauge field, it gives constant gauge fields,

$$A_1(x) = \frac{\alpha}{4} \sigma_2 \quad A_2(x) = \frac{\alpha'}{4} \sigma_1 \quad (8)$$
Unlike the usual situation in the Wilson line, it gives the nonvanishing curvature in the gauge background,

$$F_{12} \sim \sigma_3.$$  

(9)

In this sense, the introduction of the second Wilson line is not achieved by the continuous deformation but should be regarded as the discrete transformation.

In the superstring case $[9][10]$, the topologically distinct sectors are labeled by $\pi_1(U(1)) = \mathbb{Z}$. It gives the D-$(p - 2)$-brane charge at the core of vorticity. On the other hand in the bosonic situation, it seems rather doubtful that we may have topologically distinct sectors since $\pi_1(SU(2)) = 0$. However we have to note that the open string wave function transforms in the adjoint representation of $SU(2)$ and is topologically parameterized by $SO(3)$. Since $\pi_1(SO(3)) = \mathbb{Z}_2$, we have a topologically nontrivial sector. The gauge configuration that we have constructed exactly belongs to this sector. This statement can be verified by observing that the monodromy matrices did not commute with each other $h^1h^2 = -h^2h^1$. It gives the distinct elements in $SU(2)$ but results in the consistent boundary condition for the $\Psi$. Since our configuration is topologically nontrivial, it should be regarded as the stable configuration of the open string theory.

To summarize the situation of the boundary condition for the wave function, if we write $(\pm, \pm)$ as the sign of (anti-)periodicity in $x^1$ and $x^2$ direction respectively, four CP sectors are split into,

$$I : (++) \quad \sigma_1 : (+-) \quad \sigma_2 : (+-) \quad \sigma_3 : (--)$$  

(10)

At this point, if we combine all the modes from each CP factor, the momentum distribution is identical to those of rectangular lattice with radius $2R$ as far as open string sector is concerned. If we take T-duality transformation in $x^1$ and $x^2$ directions, one may alternatively view it as winding mode of open string originating from one D-$p$-brane for radius $1/2R$. In this way, we obtained three equivalent descriptions for the same system.

1. two D-$(p + 2)$-branes with Wilson lines compactified with the radius $R$
2. one D-$(p + 2)$-brane with the radius $2R$
3. one D-$p$-brane with the radius $1/2R$
Such an equivalence, however, may not be straightforwardly interpreted as the usual tachyon condensation scenario if we keep the closed string sector in our mind. Namely, the inclusion of Wilson lines do not change the closed string spectrum at all. Therefore, closed strings still live in the space with the radius $R$ or its dual $1/R$. It does not fit with above brane number reduction scenario. To realize a real reduction, we need to keep the original radius $R$ invariant. Such a mechanism occurs only when we introduce the deformation by tachyon fields. We will discuss this issue in section 4.

There is however a positive lesson from the above observation. We may claim that the original $U(2)$ symmetry is broken to $U(1)$ contrary to the usual situation where unbroken symmetry is $U(1) \times U(1)$. The origin of such a phenomena is that we introduced two Wilson lines in mutually non-commuting CP directions $\sigma_1$ and $\sigma_2$. For the reduction from $O(8192)$ to $O(32)$, this claim is already enough and we do not need full tachyon condensation scenario.

While the gauge symmetry from open string sector is reduced, we may choose the radius of the lattice to realize a new gauge symmetry from closed string sector. By taking $R = 1$, we have two pairs of $SU(2)$ current algebras,

$$J^{[I]}_\pm = e^{\pm iX^{[I]}}, \quad J^{[I]}_3 = \partial X^{[I]}, \quad I = 1, 2. \quad (11)$$

This $SU(2) \times SU(2)$ becomes the gauge symmetry of the space-time by using standard vertex operator construction. \[\]

3 Application to $SO(8192)$ theory and reduction in one step

Let us apply this idea to $SO(8192)$ model. To make the discussion clearer, we compactify one extra dimension, say $X^9$ and apply T-duality transformation along that direction. If we introduced an appropriate Wilson line, those 8192 D-branes will be split into $4096 = 2^{12}$ D-branes located at the same point, an orientifold and mirror images of D-branes. The unbroken symmetry becomes $U(2^{12})$. We may introduce non-abelian Wilson lines to reduce the symmetry to $U(2^{11})$. While repeating above procedure for 8 times, the number of branes become $2^{12-8} = 2^4 = 16$. Together with orientifold plane, the surviving D-
branes define $SO(32)$ theory. The extra gauge symmetry from the closed string sector becomes $SU(2)^{\otimes 16}$ if $R = 1$.

To compare this ten dimensional model with type 0 string theory, it is necessary to deform momentum lattice $SU(2)^{\otimes 16}$.

If we deform the lattice, it is not possible to use step by step method. Rather we need to develop the reduction of space-time in one step. If we write $x^I$ ($I = 1 \cdots 16$) our compact directions and $X^I$ as the string variable associated with it. First let us consider rectangular 16 dimensional lattice again. We prepare an explicit representation for gamma matrices in 16 dimensions (256 dimensional representation) as,

\[
\begin{align*}
\Gamma_1 &= \sigma_1 \otimes 1 \otimes \cdots \otimes 1 \\
\Gamma_3 &= \sigma_3 \otimes \sigma_1 \otimes \cdots \otimes 1 \\
\vdots \\
\Gamma_{15} &= \sigma_3 \otimes \sigma_3 \otimes \cdots \otimes \sigma_1 \\
\Gamma_{16} &= \sigma_3 \otimes \sigma_3 \otimes \cdots \otimes \sigma_2.
\end{align*}
\]

Together with these sets, we also need another representation,

\[
\begin{align*}
\tilde{\Gamma}_1 &= \sigma_2 \otimes \sigma_3 \otimes \cdots \otimes \sigma_3 \\
\tilde{\Gamma}_3 &= 1 \otimes \sigma_2 \otimes \cdots \otimes \sigma_3 \\
\vdots \\
\tilde{\Gamma}_{15} &= 1 \otimes 1 \otimes \cdots \otimes \sigma_2 \\
\tilde{\Gamma}_{16} &= 1 \otimes 1 \otimes \cdots \otimes \sigma_1.
\end{align*}
\]

These matrices are chosen to satisfy,

\[
\begin{align*}
[\Gamma_I, \Gamma_J]_+ &= [\tilde{\Gamma}_I, \tilde{\Gamma}_J]_+ = 2\delta_{IJ}, \\
[\Gamma_I, \tilde{\Gamma}_I]_+ &= 0, \\
[\Gamma_I, \tilde{\Gamma}_J]_- &= 0 \ (I \neq J).
\end{align*}
\]

The inclusion of Wilson line,

\[
\prod_{i=1}^{16} \exp \left( \frac{i}{4R} \tilde{\Gamma}_I \oint \partial_z X^I dz \right),
\]

will modify the periodicity of tachyon in CP factor $\Gamma_I$ to $(+, \cdots, +, - , +, \cdots, +)$ where minus sign occurs only at $\tilde{I}$ th position. In other word, we may have condensation of tachyon in the following form at $x^I = 0$,

\[
T(x) = \sum_{I=1}^{16} x^I \Gamma_I + O(|x|^2).
\]
Inclusion of Wilson line (15) will invoke $2^{16}$ sectors of CP factor split into $2^{16}$ points in the half rectangular lattice.

It is now straightforward to generalize the argument to the skew lattice case. To get gauge bosons from the closed string sector, we need to restrict it to a self-dual lattice. We denote $\vec{E}_I$ is the basis of lattice and $\vec{F}_J$ as the dual basis $\vec{F}_I \cdot \vec{E}_J = \delta_{IJ}$. The tachyon mode at the origin should behave as,

$$T(x) = \sum_I \Gamma_I \sin(\vec{E}_I \cdot \vec{x}) \sim \sum_I \Gamma_I (\vec{E}_I \cdot \vec{x}) \quad (x \sim 0).$$  \hfill (17)

Wilson line that invokes such a tachyon mode is

$$\prod_{I=1}^{16} \exp \left( \frac{i}{2} \tilde{\Gamma}_I \oint \vec{F}_I \cdot \partial_z \vec{x} dz \right).$$  \hfill (18)

To obtain $SO(32)$, we take the original lattice as the $SO(32)$ lattice.

## 4 Tachyon condensation scenario

We come back to the consideration of the reduction by two dimensions. So far, there was no restriction on $R$ from the open string sector. To examine the scenario of tachyon condensation, we need to impose the tachyon vertex to have dimension one.

Tachyon fields in twisted direction have mode expansion,

$$T(x) = \sum_{m \in \mathbb{Z}} T_{m+1/2} e^{i(m+1/2)x}.$$  \hfill (19)

The mass squared of $T_{\pm 1/2}$ mode is enhanced to,

$$m^2 = \frac{1}{4R^2} - 1.$$  \hfill (20)

At $R = 1/2$ they become massless and can be used to deform the boundary conformal field theory (BCFT) [11].

The remaining analysis of tachyon condensation is almost parallel to [11] and we will follow it step by step. In this radius, the anti-periodic mode $e^{\pm i2(X/2)}$ defines $SU(2)$ current algebra. The vertex operator for the tachyon condensation (1) is exactly this non-diagonal $SU(2)$ current and we need to make nontrivial change of variable. We introduce the translation operators,

$$h_I : X^I \to X^I + \pi, \quad (I = 1, 2).$$  \hfill (21)
The eigenvalue of each CP sector under $h_I$ is given in (10). Together with $h$, we introduce another $Z_2$ operator $g$ as,

$$g_I : X^I \rightarrow -X^I.$$  \hfill (22)

From massless tachyon mode, we can construct $SU(2)_L^{\otimes 2} \times SU(2)_R^{\otimes 2}$ current algebra,

$$e^{2iX_L^I} = \partial Y^I_L + i\partial Z^I_L$$
$$e^{2iY_L^I} = \partial X^I_L - i\partial Z^I_L$$
$$e^{2iZ_L^I} = \partial X^I_L + i\partial Y^I_L,$$  \hfill (23)

for the left movers and the similar definition for the right movers. We define $h$ and $g$ operator similarly for $Y$ and $Z$. There are some relations between them,

$$h_X = g_Y = g_Z, \quad g_X = h_Y = h_Z g_Z.$$  \hfill (24)

The tachyon condensation is triggered by the inclusion of non-Abelian Wilson line again,

$$\exp \left( i \alpha \frac{\pi}{2} \int dz \partial Y_1^I(z) \sigma_1 \right) \exp \left( i \alpha \frac{\pi}{2} \int dz \partial Y_2^I(z) \sigma_2 \right).$$  \hfill (25)

This time, since we originally have both $\pm 1$ eigenvalues for the operator $h_Y^I$, it is more natural to follow the change of the boundary condition in the translations, $X^I \rightarrow X^I + 4\pi R$. Wilson lines shifts $h_Y^I$ eigenvalues by $\exp(\pm 2\pi i \alpha)$. Since we use the double period, shift of $\alpha$ by 1 will not change the spectrum at all. At $\alpha = 1/2$ they induces deformation factors $\sigma_{1,2}$ again on the wave function. Returning back to shift in $2\pi R$, the eigenvalues of $h_Y$ becomes $\pm i$. We summarize the eigenvalues for $g_Y$ and $h_Y$ in the following table.

|        | $h_Y^I = g_X^I$ | $g_Y^I = h_X^I$ | $h_Y^2 = g_Y^2$ | $g_Y^2 = h_X^2$ |
|--------|----------------|----------------|----------------|----------------|
| $I$    | $\pm 1$        | $+$            | $\pm 1$        | $+$            |
| $\sigma_1$ | $\pm 1$    | $-$            | $\pm i$        | $+$            |
| $\sigma_2$ | $\pm i$     | $+$            | $\pm 1$        | $-$            |
| $\sigma_3$ | $\pm i$     | $-$            | $\pm i$        | $-$            |

Having four values $\pm 1, \pm i$ for $h_Y$ indicates that the momentum are actually quantized in different radius $R = 2$. Of course in making such an
identification we need to combine the contributions from all the sectors. Taking T-duality, we obtain a theory with one D-p brane with compactification radius 1/2.

Since we have come back to the original lattice, the resultant theory may consistently couple to the closed string sector. Of course, since we made non-trivial change of variables for the open string sector, we need to carry out similar task for closed string sector which seems to be quite non-trivial.

More important problem is that if we study the table more carefully on $g_Y$ eigenvalues, we immediately notice that we missed actually 3/4 of the states! For example, for $h_Y$ eigenvalue $(\pm, \pm)$, $g_Y$ eigenvalues are fixed as $(+, +)$ and eigenfunction should be parity even for $Y$ variables. Of course, there are no such restriction for the toroidal compactification in $R = 1/2$.

This phenomena may look like an artifact of the complicated way of introducing non-commutative Wilson lines. However it can be alternatively explained by a simple argument. Originally we have four degeneracy for each momentum because of four CP factors. If we want to arrive at the change of the radius $R = 1/2 \rightarrow 2$ in two directions, the number of momentum mode will be multiplied by $4^2 = 16$. We can not fill all of those states but only 1/4 of them.

It is obviously necessary to clarify why such a missing states could come out. A simple calculation reveals that there seems to be also a similar problem in a superstring scenario.

5 Discussions

Although we can not use the usual scenario of the tachyon condensation, we have shown that $SO(8192)$ theory compactified to ten dimensions may have gauge symmetry $SO(32) \times SO(32)$. Unfortunately, the field content of this theory is rather different from those theories. Especially at the bottom of the spectrum, we still have 528 (symmetric representation of $SO(32)$) tachyons. Since the number of the tachyon is essential to determine the vacuum structure, it is hard to believe that these theories will have common fate.

We are tempted to believe that the toroidal compactification to make tachyon massless is indispensable for the stability of the system. In analogy with standard Higgs mechanism, this may be called as “spontaneous compactification” of the space-time. Although we discussed mainly the com-
pactification to ten dimensions, we believe that unoriented bosonic string will unavoidably experience further compactification until it has only one tachyon from open string mode. This means that we will have a compactification of 24 dimensions to leave only two uncompactified space-time. At this stage, CP factor is just $O(2) = U(1)$ but enhanced gauge symmetry may have rank 24. The final fate of $O(8192)$ theory should be clarified by string theory living in two dimensions. We have one real tachyon in the open string sector. If one may deform it in kink configuration again, we would get a quantum mechanics! However, the last tachyon is neutral in all CP factors, we can not find appropriate Wilson line to deform it. The correct treatment is still mysterious for us.

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