Mathematical model of the firing process in Ladrillera Ocaña, Colombia

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Abstract. The process of heat transfer by conduction is relevant in different context of engineering. Quantify temperature is very important in energy efficiency of firing process. The aim of this work is to establish a mathematical model to determine the firing process at the burner door of the Hoffman kiln of Ladrillera Ocaña, Colombia. For the data acquisition, a virtual instrument was designed for the temperature data collection system. The analytical solution proposed for the model presents a high degree of difficulty. As an alternative, two solution schemes are proposed using numerical methods that are very close to the theoretical model.

1. Introduction
Historically Fourier initiates the study on heat by analyzing the heat transfer phenomenon in steady state; he publishes his works on this matter in 1807 and 1812 and the research around the fourier's methods are an active discipline [1]. The mathematical beauty of fourier's results and the impact they continue to have on engineering applications is demonstrated by the growing research in areas such as combustion, electrical phenomena, fluid mechanics, as shown in the references [2,3].

The study of phenomena related to heat transfer in firing process has had a growing increase in the municipality of Ocaña, Colombia; this interest is directly related to the seminal work that some researchers are carrying out at “Universidad Francisco de Paula Santander Seccional Ocaña”, Colombia. A rigorous study of this situation can be found in the works [4-7].

The main objective of this work is to study the one-dimensional heat transfer phenomenon in the kiln of hoffman type in “Ladrillera Ocaña”, Colombia. For this purpose, we rely on the temperature data recorded following the methodology in [5].

The article's content is organized as follows: the first part establishes the mathematical model related to experimental data and proposes an analytical solution which has a calculation complexity. For this reason, the second part proposes an alternative solution for the mathematical model by means of two numerical schemes. The article ends with a discussion on the numerical results and the relationship with the theoretical model.

2. Mathematical model
The data acquisition system for the kiln in “Ladrillera Ocaña” was designed, following the technique of recollecting data in [5] to record the temperatures of the firing process obtained from
16 type K thermocouples, for which two acquisition cards were used in which 8 thermocouples were installed in each card. In the first acquisition card or block, the internal temperatures in the furnace were recorded; in the second acquisition card, external temperatures were recorded, and then stored through the LabView software in the acquisition report and generate the temperature profiles of the positions of the chamber chosen as reference for the measurements. The data acquisition was performed in the 27nd chamber of the kiln, the production of H 10x40 brick in this was 30,000 u.

The method to establish the mathematical model consisted in modeling the temperatures of the external and internal walls by means of step functions. The Table 1 shows the experimental temperature of the kiln by the LabView software and the step functions \( f_{1}(t) \) and \( f_{2}(t) \).

**Table 1.** The temperatures of the external and internal walls of the kiln modelling by the step functions \( f_{1}(t) \) and \( f_{2}(t) \).

| Time (Hours) | Temperature (°C): external walls \( f_{1}(t) \) | Temperature (°C): internal walls \( f_{2}(t) \) |
|-------------|---------------------------------|---------------------------------|
| 1           | 19.85                           | 30.25                           |
| 2           | 19.97                           | 32.20                           |
| 3           | 19.68                           | 33.64                           |
| 4           | 19.48                           | 36.09                           |
| 5           | 19.65                           | 39.75                           |
| 6           | 20.01                           | 49.75                           |
| 7           | 19.48                           | 57.90                           |
| 8           | 20.02                           | 81.91                           |
| 9           | 21.42                           | 134.41                          |
| 10          | 24.33                           | 343.20                          |
| 11          | 35.89                           | 668.17                          |
| 12          | 48.47                           | 594.34                          |
| 13          | 70.16                           | 747.75                          |
| 14          | 133.44                          | 712.23                          |
| 15          | 144.67                          | 691.35                          |

The mathematical model supporting the experimental data is the following differential equation with boundary values and initial conditions partial differential equation (PDE) 1 it is expressed by the Equation (1), Equation (2) and Equation (3). Functions \( f_{1}(t) \) and \( f_{2}(t) \) represent the step functions generated from the Table 1.

\[
T_{x} = kT_{xx}, \quad 0 < x < l, t > 0 \tag{1}
\]

\[
T(0, t) = f_{1}(t), \quad T(l, t) = f_{2}(t), \quad t > 0 \tag{2}
\]

\[
T(x, 0) = f(x), \quad 0 < x < l. \tag{3}
\]

To solve the PDE1 model following [8], the first step is to transform the boundary conditions (2) into homogeneous conditions, for this purpose a new function in the Equations (4) and Equation (5) is defined.

\[
h(x, t) = f_{1}(t) + \frac{x}{l} \left( f_{2}(t) - f_{1}(t) \right) \tag{4}
\]

Such that \( \bar{T}(x, t) = T(x, t) - h(x, t) \) \tag{5}

Our objective is to observe what type of model the \( \bar{T} \) function verifies. To this goal, we follow the Equations (6), Equation (7), Equation (8) and Equation (9). We notice that,

\[
h_{xx}(x, t) = 0, \quad h_{t}(x, t) = f_{1}''(t) + \frac{x}{l} (f_{2}''(t) - f_{1}''(t)) \tag{6}
\]

We observe that,
\[ \tilde{T}_t - k \tilde{T}_{xx} = -h_t(x, t) \]  

(7)

and

\[ \tilde{T}(0, t) = \tilde{T}(0, t) - h(0, t) = 0, \tilde{T}(l, t) = \tilde{T}(l, t) - h(l, t) = 0 \]  

(8)

\[ \tilde{T}(0, x) = f(x) - h(x, 0) = f(x) - (f_1(0) + \frac{x}{\bar{T}}(f_2(0) - f_1(0))) \equiv \tilde{T}_0(x) \]  

(9)

For the above, the \( \tilde{T} \) function verifies the Equation (10), Equation (11) and Equation (12):

\[ \tilde{T}_t = k \tilde{T}_{xx} - h_t(x, t), \quad 0 < x < l, t > 0 \]  

(10)

\[ \tilde{T}(0, t) = 0, \quad \tilde{T}(l, t) = 0, \quad t > 0 \]  

(11)

\[ \tilde{T}(x, 0) = \tilde{T}_0(x), \quad 0 < x < l. \]  

(12)

First of all, to solve the PDE2 model, we will solve the homogeneous EDP3 model with Equation (13), Equation (14) and Equation (15):

\[ \tilde{T}_t = k \tilde{T}_{xx}, \quad 0 < x < l \]  

(13)

\[ \tilde{T}(x, 0) = 0, \quad \tilde{T}(l, 0) = 0, \quad t \geq 0 \]  

(14)

\[ \tilde{T}(x, 0) = \tilde{T}_0(x), \quad 0 < x < l. \]  

(15)

By applying the separation of variable technique, the solution is the function Equation (16):

\[ T(x, t) = \tilde{T}(x, t) + h(x, t) = \sum_{n=1}^{\infty} \tilde{T}_n(t) \sin \frac{n\pi x}{l} + h(x, t) \]  

(16)

The complexity of the \( T(x, t) \) function indicates the use of another strategy to calculate the solution of the PDEP1 model.

3. Finite difference method for solving the partial differential equation 1 model

3.1. An explicit scheme

The approach to solving the PDE1 model by the explicit method taking as a theoretical foundation [9,10] consists in substituting the unknown function \( T(r, t) \) by the approximations of the partial derivatives that are generated from the corresponding Taylor series:

\[ T_t(r_j, t_n) = \frac{T(r_{j+1}, t_n) - T(r_j, t_n)}{\Delta t} \]  

(17)

\[ T_{xx}(r_j, t_n) = \frac{T(r_{j+1}, t_n) - 2T(r_{j}, t_n) + T(r_{j-1}, t_n)}{(\Delta x)^2} \]  

(18)

Equation (17) and Equation (18) generate a discretized version of the PDE1 model; by substituting these equations, the corresponding recurrence relations are generated in the Equation (19), Equation (20) and Equation (21):

\[ T_t^{k+1} = s(T_t^k + T_{xx}^k) + (1 - s)T_t^k \]  

(19)

\[ T_j^{k+1} = s(T_{j+1}^k + T_{j-1}^k) + (1 - s)T_j^k \]  

(20)
\[ T_n^{k+1} = s(T_{n+1}^{k} + T_{n-1}^{k}) + (1 - s)T_n^{k}, \quad (21) \]

where \( s = \frac{k \Delta t}{(\Delta x)^2} \). For the case \( n = 3 \), the recurrence relations are reflected in the following system Equation (22).

\[
\begin{pmatrix}
1 - 2s & s & 0 \\
-1 & 1 - 2s & s \\
0 & -1 & 1 - 2s
\end{pmatrix}
\begin{pmatrix}
T_1^{k+1} \\
T_2^{k+1} \\
T_3^{k+1}
\end{pmatrix}
= \begin{pmatrix}
sf(t_0) \\
0 \\
sg(t_4)
\end{pmatrix}
\]

\[ (22) \]

3.2. An implicit scheme

The approach to solving the PDE1 model by the implicit method by [11] consists in substituting the unknown function \( T(r, t) \) by the approximations of the partial derivatives that are generated from the corresponding Taylor series.

\[
T_1(t_1, t_n) = \frac{T(r(t_1), t_n) - T(r(t_1), t_n)}{\Delta t} \quad (23)
\]

\[
T_{xx} = \frac{T(x_{j-1}, t_n + 1) + 2T(x_j, t_n + 1) + T(x_{j+1}, t_n)}{(\Delta x)^2} \quad (24)
\]

Equation (23) and Equation (24) generate a discretized version of the PDE1 model, replacing these equations generates the corresponding recurrence relations in the Equation (25), Equation (26) and Equation (27).

\[
T_1^k = -s(T_2^{k+1} + T_1^{k+1}) + (1 + s)T_1^{k+1} \quad (25)
\]

\[
T_2^k = -s(T_3^{k+1} + T_2^{k+1}) + (1 + s)T_2^{k+1} \quad (26)
\]

\[
T_3^k = -s(T_1^{k+1} + T_3^{k+1}) + (1 + s)T_3^{k+1}. \quad (27)
\]

where \( s = \frac{k \Delta t}{(\Delta x)^2} \). For the case \( n = 3 \), the recurrence relations are shown in the following system Equations (28).

\[
\begin{pmatrix}
1 + 2s & -s & 0 \\
-s & 1 + 2s & -s \\
0 & -s & 1 + 2s
\end{pmatrix}
\begin{pmatrix}
T_1^{k+1} \\
T_2^{k+1} \\
T_3^{k+1}
\end{pmatrix}
= \begin{pmatrix}
sf(t_0) \\
0 \\
sg(t_4)
\end{pmatrix}
\]

\[ (28) \]

4. Results

In order to validate the explicit and implicit schemes, we design a code following [12] that implement the numerical methods in the matrix Equation (22) and Equation (28). The parameters used in the code are represented in Table 2.

The temperature variations \( T(x, t) \) for the kiln at different times for the process of heat conduction in one dimension are shown in the Figure 1 and Figure 2. The Figure 1 describe de implementation of the numerical Equation (22) in the case of the explicit method for heat conduction in the kiln. The Figure 2 describe de implementation of the numerical Equation (28) in the case of the implicit method for heat transfer in one dimension. Those modelling approximations are consistent with the experimental results of the work [13].

The Figure 3 and Figure 4 show the results of comparing the linear function \( f(x) \) with the results generated by the explicit method. For the implicit method de graphics are very similar.
Table 2. The parameters used in the design of the code for the explicit and implicit methods.

| Parameters                      | Data                      |
|---------------------------------|---------------------------|
| Kiln wall width                 | 1 meter                   |
| Number of points in the \(x\) direction | 2000                      |
| Step in the \(x\) direction     | 1/50                      |
| Step in the \(t\) direction     | 43200/2000                |
| Thermal diffusion constant      | 0.000004                  |
| Convergence parameter           | 0.2160                    |
| inner wall temperature          | 25 degrees                |

Figure 1. Temperature in the kiln with the explicit method.

Figure 2. Temperature within the explicit method.

Figure 3. Temperature in the iteration 2000 with the explicit method against temperature in steady state conduction.

Figure 4. Error for the temperature for the explicit method in the last iteration.

5. Conclusion

From the behavior of the temperature curves (for the explicit and implicit method) in different instants, we can conclude that \( \lim_{t \to B} T(x,t) = f(x) \), where \( f(x) = (30.25 - 19.85)x + 19.85 \).

The numbers 19.85 and 30.25 are the temperatures of the external and internal walls of the kiln. The behavior of the firing process in the kiln is consistent with the theoretical results when the heat transfer process is one-dimensional while using numerical solutions. However, the figure 4 shows that the error is an increased function. The next step of this investigation is to include the process of heat convection and radiation in the firing process to describe a more realistic behavior.
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