Local hydrodynamics and operator stability of Keplerian accretion disks

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²WARNING!!!! This is an old (2000) manuscript, posted here for archival purposes. It is reproduced verbatim save for this footnote and the postscript which follows the paper. Current address of author: 736 Spokane Ave, Albany, CA 94706 (no current affiliation)
ABSTRACT

We discuss non-self-gravitating hydrodynamic disks in the thin disk limit. These systems are stable according to the Rayleigh criterion, and yet there is some evidence that the dissipative and transport processes in these disks are hydrodynamic in nature at least some of the time. We draw on recent work on the hydrodynamics of laboratory shear flows. Such flows are often experimentally unstable even in the absence of a linear instability. The transition to turbulence in these systems, as well as the large linear transient amplification of initial disturbances, may depend upon the non-self-adjoint nature of the differential operator that describes the dynamics of perturbations to the background state. We find that small initial perturbations can produce large growth in accretion disks in the shearing sheet approximation with shearing box boundary conditions, despite the absence of any linear instability. Furthermore, the differential operator that propagates initial conditions forward in time is asymptotically close (as a function of Reynolds number) to possessing growing eigenmodes. The similarity to the dynamics of laboratory shear flows is suggestive that accretion disks might be hydrodynamically unstable despite the lack of any known instability mechanism.

Subject headings: accretion, accretion disks — hydrodynamics — instabilities
1. Introduction

In this paper we discuss the hydrodynamic stability of thin, non-self-gravitating, Keplerian accretion disks. The study of potential hydrodynamic and magnetohydrodynamic (MHD) instabilities in Keplerian shear flow is important because they may explain transport within accretion disks. (For a review of accretion disk physics, see Frank, King, & Raine (12). A more recent review of accretion disk stability and transport may be found in Balbus & Hawley (3).)

The effective kinematic viscosity $\nu$ of accretion disks is traditionally parametrized by $\alpha$ in the phenomenological relation

$$\nu = \alpha c_s H,$$

where $c_s$ is the sound speed and $H$ is the local disk scale height (26). Inferred values of $\alpha$ for dwarf nova (DN) accretion disks, which are relatively well-studied and for which $\alpha$ is perhaps the most observationally constrained, lie in the range of $\alpha \approx 0.1 - 0.001$. (See Warner (30) for an excellent review of DN disks.) These values for $\alpha$ are many orders of magnitude (e.g., 10–12) larger than back-of-the-envelope estimates for the molecular viscosity. Other disk systems, such as the accretion disks in active galactic nuclei (AGN) and the circumstellar disks in young stellar objects (YSO), are also thought to have effective viscosities that are anomalously large in the same sense (21; 17).

It was originally hoped that the large Reynolds numbers in Keplerian disks would be a source of hydrodynamic turbulence (29). However, Keplerian disks satisfy the Rayleigh stability criterion for inviscid axisymmetric perturbations (5),

$$\frac{d}{dr}(r^2 \Omega)^2 > 0.$$  \hspace{1cm} (2)

It should be noted that this is a necessary, but not sufficient, criterion for stability of the more general case of non-axisymmetric perturbations to a viscous flow. Nonetheless, there
is no well-established instability mechanism in Keplerian hydrodynamic shear flow.

The canonical route to turbulence in disks is the Balbus-Hawley magnetic shearing instability (2; 18). However, there may be some disks that are not sufficiently ionized for the Balbus-Hawley instability to be effective. Ion-neutral collisions in circumstellar disks associated with YSOs may dissipate the magnetic field too quickly for the instability to take hold (27). Gammie & Menou (13) simulated the dynamics of the prototypical DN system SS Cyg, using a code developed by Hameury et al. (16); based on their simulations, they conclude that the Balbus-Hawley instability may not be operating when the disk is in quiescence. In their disk simulations, the magnetic Reynolds number drops below $10^4$, and numerical MHD simulations (19) indicate that the Balbus-Hawley mechanism requires a magnetic Reynolds number $Re_B$ based upon scale height to be larger than approximately $10^4$.

We show that the shearing-sheet approximation with shearing-box boundary conditions shares properties in common with such laboratory flows as Couette flow, Poiseuille flow, Hagen-Poiseuille flow, and Taylor-Couette flow. (For an introduction to these flows, see Drazin & Reid (8).) These shear-dominated laboratory flows exhibit transitions to turbulence even in the absence of a linear instability (28; 24). Hagen-Poiseuille flow (pipe flow), for example, does not possess a linear instability, but is seen to become turbulent in the laboratory at a range of Reynolds numbers, as observed by Reynolds (23).

One property of these flows (10; 28; 14; 22; 4; 15) is that initial perturbations may undergo large transient growth in the absence of any growing eigenmode. Furthermore, the eigenvalue spectrum of these operators is very sensitive to perturbations to the operators, so that the operators are in many cases asymptotically close (as a function of Reynolds numbers) to operators that possess growing eigenmodes, even if the operators themselves do not have growing modes.
2. Local hydrodynamics and shearing box

The shearing-sheet approximation replaces the \((r, \theta)\) coordinates in the disk with a local Cartesian plane \((x, y)\) (see e.g. page 439 of Ryu & Goodman (25) and references therein). We neglect the \(z\) dimension and assume incompressibility. We have then

\[
\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \rho^{-1} \nabla p + 2\Omega \mathbf{\hat{z}} \times \mathbf{v} - 4A \Omega x \mathbf{\hat{x}} = \nu \nabla^2 \mathbf{v},
\]

where \(A\) is Oort’s first galactic constant,

\[
A \equiv -\frac{1}{2} r \partial_r \Omega \bigg|_{r_0} = \frac{3}{4} \Omega_0.
\]

The background flow is \(-2Ax\mathbf{\hat{y}}\). Perturbations \(\mathbf{u}\) to this flow may be described in terms of a streamfunction \(\psi\), so that \(\mathbf{v} = -2Ax\mathbf{\hat{y}} + \mathbf{u}\), and \((u_x, u_y) = (\partial_y \psi, -\partial_x \psi)\). We let

\[
\psi(x, y, t) = \phi(x, t)e^{i\beta y},
\]

and we non-dimensionalize by measuring time in units of \(1/\Omega\) and distance in units of some unspecified \(L\), which we may take to represent, say, the disk scale height. The dimensionful viscosity \(\nu\) is the inverse Reynolds number \(\nu \to R^{-1}\) with the length \(L\) as the defining lengthscale. In the case that \(L\) is the disk scale height, then \(R\) may typically lie in the range \(10^{10}-10^{12}\) approximately, as discussed above.

Taking the curl of (3) and neglecting \(\partial_z u_z\) in the spirit of a 2D analysis, we have then that

\[
\partial_t \phi = \left[\partial_x^2 - \beta^2 \right]^{-1} i\beta \left[(i\beta R)^{-1}(\partial_x^2 - \beta^2) - 2Ax(\partial_x^2 - \beta^2)\right] \phi,
\]

which is a form of the Orr-Sommerfeld equation for viscous shear flow (Reddy et al. 1993). (Note that in Keplerian disks, the nondimensionalized \(A\) is simply \(A = 3/4\).) If we further separate variables, this equation is seen not to be formally self-adjoint in \(x\) in the \(L_2\) norm for \(\phi\). With the addition of rigid boundaries in \(x\), this equation governs 2D perturbations
to Couette flow; at high Reynolds number, its eigenmodes are highly collinear, and its eigenvalues are extremely sensitive to perturbations to the equation.

We seek solutions to equation (6) in the shearing-box system. This is the shearing sheet with the addition of periodic boundary conditions in $y$ and sheared periodic boundary conditions in $x$ (3). Specifically, we consider a square domain of size $L$, and the boundary condition in $x$ is

$$
\psi(0, y, t) = \psi(L, y - 2Alt, t).
$$

Equation (6) is not separable in the shearing box system. However, a complete set of solutions exist; these are the Kelvin modes (20; 11). These modes are Fourier modes in $x$ and $y$, where

$$
\psi(x, y, t) = C_{k(0), \beta}(t)e^{ik(t)x + i\beta y},
$$

with $k(t) = k(0) + 2A\beta t$, and

$$
C_{k(0), \beta}(t) = C_{k(0), \beta}(0) \frac{k(0)^2 + \beta^2}{k(t)^2 + \beta^2} \exp \left[ -R^{-1} \int_0^t (k(\tau)^2 + \beta^2) \, d\tau \right].
$$

To satisfy boundary conditions, we have $\beta = m2\pi$ and $k(0) = n2\pi$. At times $t = j\Delta t$, where $j$ is an integer and $\Delta t = (2Am)^{-1}$, the streamfunction may be written

$$
\psi(x, y, t) = c_{n,m}^{[j]} e^{2\pi i(nx + my)}.
$$

We restrict our attention to the invariant subspace $m = 1$, since these are the least dissipated modes with the potential for growth, and we henceforth drop the subscript $m$.

The streamfunction at timestep $j$ is then completely specified in terms of the state vector $c^{[j]} = (\ldots, c_{-1}^{[j]}, c_0^{[j]}, c_1^{[j]}, \ldots)$. The solution to equation (6) may now be written as a difference equation in $t$,

$$
c^{[j+1]} = \mathcal{L}c^{[j]},
$$

where $\mathcal{L}$ is a matrix operator. It is zero everywhere except on the subdiagonal, reflecting the fact that the $i^{th}$ element of the state vector $c^{[j]}$ is taken from the $(i - 1)^{th}$ element of
(i.e. \( c \) on the previous timestep), adjusted in amplitude according to equation (9).

The matrix \( \mathcal{L} \) provides a convenient representation of the propagator for the initial value problem:

\[
c^{[j]} = \mathcal{L}^{j} c^{[0]}
\]  

(12)

The operator \( \mathcal{L} \) is formally generated by the Orr-Sommerfeld operator \( \mathcal{L}_{OS} \) (which is the right-hand side of equation 6), in the subspace of discrete Kelvin modes described above:

\[
\mathcal{L} = e^{\Delta t \mathcal{L}_{OS}}.
\]  

(13)

The Laplacian in the operator \( \mathcal{L}_{OS} \) is rendered well-defined by the boundary conditions.

3. Transient growth and operator perturbations

In order to discuss growth, we first need a norm to measure the strength of perturbations. We use the \( L_2 \) norm of the streamfunction \( \psi \). In this norm, the magnitude of a perturbation \( c \) is

\[
\|c\| = \int \int \psi^* \psi \, dx \, dy = \sum_{i=-\infty}^{\infty} c_i^* c_i.
\]  

(14)

Given this vector norm, the usual operator norm of a matrix \( \mathcal{A} \) is defined using the vector norm:

\[
\|\mathcal{A}\| = \max_{\|x\|=1} \|\mathcal{A}x\|,
\]  

(15)

\(^{E1}\) and it is equal to the maximum singular value of \( \mathcal{A} \). The norm \( \|\mathcal{L}^{j}\| \) measures the maximum possible growth of perturbations in \( j \) timesteps.

In figure 1, we plot (triangles) the operator norm \( \|\mathcal{L}^{j}\| \) versus \( j \), for a Reynolds number of \( 10^6 \). The “hump” is a typical characteristic behavior for the norm for a non-normal

\(^{E1}\)NOTE TO EDITOR: The output for equation 15 is not quite what I would like: The “max” needs to be directly over the “\( \|x\| = 1 \)”, but I don’t know how to do that.
matrix. We show in figure 2 (triangles) the log of the amount of growth that can be obtained as a function of Reynolds number, which we denote by $G(R)$, where $G(R) = \max \|\mathcal{L}^j\|$ and the maximum is taken over $j$. As $\mathcal{L}$ is represented by a matrix that is zero everywhere except on the subdiagonal, powers of it are easy to compute. The maximum growth scales as

$$G(R) \sim R^{2/3},$$

where the maximum is taken over the values of $j$. This result is also easily obtained analytically from equation (9) in the limit of large $R$. Specifically, for large $R$, we have

$$G(R) \approx (0.092)R^{2/3}$$

The line drawn through the triangles in figure 2 is the analytic approximation above.

This type of transient growth has been remarked upon previously in the literature; it is merely a consequence of the kinematics of the Kelvin modes as described by equation (9). According to the Rayleigh hypothesis, instability occurs whenever there exists an unstable mode. The unbounded exponential growth of an unstable mode is of course unphysical. Large transient growth in the linear limit, as obtained here, simply implies that the strength of the initial perturbation required in order to reach the nonlinear regime is potentially very small, but not infinitesimal. Nonlinear effects may then positively or negatively feedback upon the original perturbation, but we do not discuss these effects here.

This transient growth has implications for the spectra of perturbations to the operator $\mathcal{L}$. We construct matrices $\mathcal{M}$ such that each element of $\mathcal{M}$ is randomly drawn from the unit disk in the complex plane. We then construct the perturbed operator $\tilde{\mathcal{L}}$,

$$\tilde{\mathcal{L}} = \mathcal{L} + \epsilon \mathcal{M}.$$  

The matrix $\mathcal{M}$ couples the Kelvin modes, allowing the possibility for feedback to the transient growth observed in $\|\mathcal{L}^n\|$. Note that we do not rescale $\mathcal{M}$ by its operator norm,
∥M∥ ≈ \sqrt{2 \dim(M)}.

Although in principle \(M\) need not have any eigenvalues with magnitude larger than one, in practice it always will. Given this, for each \(M\) there exits a positive \(\epsilon\) such that the operator \(\tilde{L}\) possesses an eigenvalue \(\lambda\) with \(|\lambda| > 1\), so that the operator \(\tilde{L}\) possesses a growing eigenmode. Such an eigenmode corresponds to a growing Bloch mode,

\[
\phi(x, t) = \xi(x, t)e^{i\omega t},
\]

(19)

where \(\xi(x, t)\) is periodic in \(t\) with period \(\Delta t\). In figure 1 (open squares), we show the growth of the operator norm \(\| (\mathcal{L} + \epsilon M)^n \|\) at a Reynolds number of \(10^6\) and with \(\epsilon = 5 \times 10^{-4}\). The behavior of this perturbed operator diverges exponentially from that of the unperturbed operator in the operator norm, at large times. This reflects the presence of a growing eigenmode. Interestingly, the minimum values of \(\epsilon\) (denoted \(\epsilon_{\text{crit}}\)) required to produce a growing eigenmode for the operator \(\tilde{L}\) do not depend greatly upon the matrix \(M\), for our choice of probability measure for \(M\). Furthermore, the values \(\epsilon_{\text{crit}}\) become asymptotically small as the Reynolds number is increased.

In figure 2 (circles) we plot the log of \(1/\epsilon_{\text{crit}}\) versus the log of the Reynolds number. We perform this calculation for forty different random matrices for each Reynolds number. (This is a more computationally intensive operation than the determination of the maximum operator norm \(\max \| \mathcal{L}^j \|\). We have made extensive use of the LAPACK routine package (1), running on the beowulf cluster parallel virtual machine in the University of Texas Astronomy Department.) The points are the median values of \(1/\epsilon_{\text{crit}}\), and the error bars are the first and third quartile. The relationship is a power law with an index of 0.88 \(\pm\) 0.01. Deviations from this power law at the low end reflect the effects of the discretization of time into timesteps \(\Delta t\). Deviations from the power law at high Reynolds number are due to computational limits on the necessarily finite size of the matrix representations for \(\mathcal{L}\) and \(\mathcal{M}\), as we have confirmed by varying these dimensions. The corresponding points for larger
or smaller matrices fall on top of the points shown here, up until the point at which they turn off of the power law relationship. This point is determined by the value for \( j = j_{\text{peak}} \) at which the operator norm \( \|L^j\| \) reaches its maximum, as matrices of order smaller than \( j \) will not faithfully represent the growth of \( \|L^j\| \). In figure 2 we also show the dependence of \( j_{\text{peak}} \) upon Reynolds number (squares); it is seen to depend upon \( R \) in the form \( n \sim R^{1/3} \), and the line is the theoretical result for large Reynolds number. Given maximum growth \( G(R) \) of the operator norm for the unperturbed operator, we need only perturb the operator by a matrix that is zero everywhere except at the matrix element that couples Kelvin modes at the end of their growth period to the modes that are just beginning their growth period, i.e. \( j_{\text{peak}} \) timesteps earlier. This element need only have an amplitude of \( \epsilon > 1/G(R) \), to create a feedback loop. Roughly, the number of matrix elements that will effectively couple grown modes to the earlier maximally amplified modes may be expected to scale as \( j_{\text{peak}}^2 \); when added in quadrature, the effective feedback then would scale as \( j_{\text{peak}} G(R) \), which grows as \( R^{1.0} \). However, each of these matrix elements do not have the same potential for producing feedback to the Kelvin modes, so that the actual scaling of \( G(R) \) with \( R \) is somewhat less steep.

One physical interpretation of the operator perturbation \( \epsilon M \) is that it potentially represents the differences between the idealized operator \( L \) and the true operator that governs the system. Compressibility, tides, magnetic fields, and so forth, will affect the operator \( L \). While we cannot say that any of these effects will actually result in a new operator \( \hat{L} \) that possesses growing modes, we can say that, at least in the probability measure of random matrices \( \mathcal{M} \) that we have discussed, most perturbed operators of the form \( \hat{L} = L + \epsilon M \) have growing eigenmodes for asymptotically small \( \epsilon \) at high Reynolds number. This result reflects properties of \( L \) itself. Had \( L \) been a normal operator, this result would not obtain, and the spectrum of \( L \) would be very stable to operator perturbations of the form described above.
4. Conclusions and Discussion

The shearing sheet system with shearing box boundary conditions appears, as hypothesized, to share many properties in common with such flows as Couette flow and other shear-dominated flows. The operator $\mathcal{L}$ discussed above is clearly not normal. It does not even possess eigenmodes. However, as we have shown, $\mathcal{L}$ is capable of large transient growth in the operator norm, and it is within $\epsilon$ of possessing growing eigenmodes in the sense described above, where $\epsilon \sim R^{-1}$. In particular, for the range of Reynolds numbers expected in disks as discussed above, $\epsilon$ is on the order of $10^{-8.5}$.

This suggests that Keplerian shear flow may share the same phenomenology as the above-mentioned classic laboratory flows, including turbulence in the nominally stable regions of parameter space. We hypothesize that non-magnetized accretion disks may therefore be turbulent in the absence of a linear instability, as suggested previously by Dubrulle (9) and Richard & Zahn (24). This could explain the presence of transport in DN in quiescence, as well as transport in other cool disks such as those associated with young stars. We suggest that simulations of 2D vertically-symmetric hydrodynamics in the shearing box system should exhibit large transient growth of initial perturbations and operator sensitivity as described above.

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REFERENCES

Anderson, E., et al 1999, LAPACK User’s Guide (3d ed.; Philadelphia: Society for Industrial and Applied Mathematics)

Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214

Balbus, S. A. & Hawley, J. F. 1998, Rev. Mod. Phys., 70, 1

Brosa, U. & Grossmann, S. 1999, Eur. Phys. J. B., 9, 343

Chandrasekhar, S. 1961, Hydrodynamic and Hydromagnetic Stability (New York: Dover)

Coles, D. 1965, J. Fluid Mech., 21, 385

Davies, S. J. & White, C. M. 1928, Proc. R. Soc. London Ser. A. 119, 92

Drazin, P. G., & Reid, W. H. 1981, Hydrodynamic stability (Cambridge: University Press)

Dubrulle, B. 1992, A&A, 266, 592

Farrell, B. F. 1988, Phys. Fluids, 31, 2093

Farrell, B. F., & Ioannou, P. J. 1993, J. Atmos. Sci., 50, 200

Frank, J., King, A. R., & Raine, D. J. 1992, Accretion Power in Astrophysics, Vol. 21 of Cambridge Astrophysics Series (2d ed.; Cambridge: University Press)

Gammie, C. F., & Menou., K. 1998, ApJ, 492, L75

Gebhardt, T., & Grossmann, S. 1993, Z. Phys. B., 90, 475

Grossmann, S. 2000, Rev. Mod. Phys., 72, 603

Hameury, J., Menou, K., Dubus, G., Lasota, J., & Hure, J. 1998, MNRAS, 298, 1048
Hartmann, L. 1998, Accretion Processes in Star Formation, Cambridge Astrophysics Series (Cambridge: University Press)

Hawley, J. F. & Balbus, S. A. 1991, ApJ, 376, 223

Hawley, J. F., Gammie, C. F., & Balbus, S. A. 1996, ApJ, 464, 690

Kelvin, Lord (W. Thompson) 1887, Phil. Mag., Ser. 5, 24, 188

Krolik, J. H. 1999, Active Galactic Nuclei, Princeton Series in Astrophysics (Princeton: Princeton University Press)

Reddy, S. C., Schmid, P. J., & Henningson, D. S. 1993, SIAM J. Appl. Math., 53, 15

Reynolds, O. 1883, Phil. Trans. Roy. Soc., 174, 935

Richard, D. & Zahn, J. 1999, A&A, 347, 734

Ryu, D. & Goodman, J. 1992, ApJ, 388, 438

Shakura, N. I. & Sunyaev, R. A. 1973, A&A, 24, 337

Stone, J. M., Gammie, C. F., Balbus, S. A., & Hawley, J. F. 2000 in Protostars and Planets IV, ed. V. Mannings, A. P. Boss, & S. S. Russel (Tucson: The Universtiy of Arizona Press), 589

Trefethen, L. N., Trefethen, A. E., Reddy, S. C., & Driscoll, T. A. 1992, Science, 261, 578

von Weizsäcker, C. F. 1948, Z. Phys., 124, 614

Warner, B. 1995, Cataclysmic Variable Stars, Vol. 28 of Cambridge Astrophysics Series (Cambridge: University Press)
Fig. 1.— $\|L^j\|$ vs. $j$ for $R = 10^6$. Triangles are the unperturbed operator, squares are for the operator perturbed by a random matrix with matrix elements taken from the unit disk and scaled by $\epsilon = 5 \times 10^{-4}$. Resolution is 201 modes.

Fig. 2.— Log-log plot of $\epsilon_{\text{crit}}^{-1}$ (closed and open circles), $G(R)$ (triangles), and $j_{\text{peak}}$ (squares) versus Reynolds number $R$. Dashed line and dotted line are theoretical curves for large $R$ limit for $G(R)$ and $j_{\text{peak}}$ respectively, and solid line is power law fit to the central part of curve for $\epsilon_{\text{crit}}^{-1}$. Deviation from power law behavior at high $R$ is due to the limitations of numerical resolution, as discussed in text. Closed and open circles are results from runs with 401 and 801 modes respectively.
Fig. 1.

\[ R = 10^6, \quad \epsilon = 5 \times 10^{-4} \]

\[ \|L\| \quad \|L + \epsilon M\| \]

\[ j \text{ (timestep)} \]

Fig. 2.

\[ \log_{10}(\epsilon) \quad \log_{10}(R) \]

\[ \epsilon_{\text{crit}}^{-1} \quad (401 \text{ modes}) \]

\[ \epsilon_{\text{crit}}^{-1} \quad (801 \text{ modes}) \]

\[ \epsilon_{\text{crit}}^{-1} \quad G(t) \]

\[ J_{\text{peak}} \]
5. POSTSCRIPT

The document appearing on the previous pages was submitted to ApJL on 19 Oct 2000 and appears here now without alteration. This paper was not accepted for publication. The primary justifications for this decision were that (1) the streamfunction norm is not a physical norm, (2) the measure of perturbations in the form of random matrices as described does not have an obvious physical basis, (3) it was suggested that the 2D perturbations presented here are essentially uninteresting and that the perturbations should be performed in 3D. Note, for example, that the perturbations that experience the greatest transient amplification in planar Couette flow consist of shear-aligned vortices. These are inherently three-dimensional structures, Squire’s theorem not being relevant. However, 3D streamwise vortices in Keplerian shear flow are quite different from similar perturbations to plane Couette flow, because of the presence of an epicyclic frequency, so to an extent this is where the interesting physics is.

It was also pointed out that the bootstrapping described here “cannot be regarded as a proof of nonlinear instability.” On that the referee and I are wholly agreed. If I implied in my paper that I thought differently, it was unintentional.

In any case, the first objection listed above is quite valid, but on the other hand this problem is easy enough to fix, and switching to a more physical norm such as the energy norm has little material effect on the results.

The second objection is also valid. The problem is that, as the referee put it, “a random perturbation of a system of equations embodying a conservation law will allow an artificial growth of the previously conserved quantity.” True. (Although, strictly speaking, here the unperturbed system of equations is dissipative.) This defect might be remedied by restating the perturbations in terms of, say, noise in the form of random forcing in the system. Putting the perturbations on more physical grounds would have made the paper
much better, and probably would not have required too much work in the end.

The third objection is quite obviously more substantial in the obstacles it presented to me. When I submitted this paper, I had already accepted employment elsewhere working in a completely different subject area. I felt I could fix the other problems raised by the referee, but a 3D analysis would take substantially more work; without question I did not have the time to do such an analysis in place of the 2D results presented here.

I was at the time completely naive about the publication process. This was the first paper I had ever submitted, as is evident by my lack of familiarity with the idiom. Although I was very upset by the report, I simply deferred to the referee on this matter. I reluctantly abandoned the paper. Perhaps he was right, I thought, and only a 3D analysis would be worthy of publication in the peer-reviewed literature.

I sincerely wish to thank the referee for the time and effort he took for his quite thorough and penetrating analysis. In reading the report over now, I see that he was trying to make helpful and encouraging suggestions, but all I heard at the time was a flat “no.” Understanding the editorial process, and reading between the lines of referee reports, it appears, are acquired arts. On the referee’s third objection above, however, I must now respectfully disagree. With perhaps a fair bit of polish, following the remaining suggestions of the referee, this modest work might have been a helpful addition to the literature at the time. I believe now, as I did in October of 2000, that even a 2D analysis of this system was interesting and worthy of appearing in the peer-reviewed literature.

Apparently I am not alone. In the intervening years, other researchers have performed similar, although admittedly more insightful and extensive, analyses of transient amplification in this 2D system. I mention a few noteworthy papers below, some of which were accepted for publication by the referees and editors of other journals. My list of relevant papers is certainly not exhaustive and I apologize to anyone I have left out. I
have been quite happy to see these papers come out, because I believe in the value of this type of analysis. It has been illuminating to see the different approaches that people have taken to this problem. I anticipate that this line of work will ultimately lead to a greater understanding of transport in cool disks.

There are many stories behind my paper, each with its own amusing ironies. These in turn mostly serve now as humorous little lessons to me that I will largely keep to myself. As I have no intention whatsoever of resuming the work presented here, I have decided simply to post it for archival purposes. A more extended discussion appears in the second chapter of my dissertation. So, if anybody wonders why I did not publish my dissertation, here at least is part of the answer. :)

My graduate research advisors, J. Scalo and J. C. Wheeler, deserve my ample thanks for their support. My work certainly did not benefit either of their research programs; I had conceived of the project long before I ended up under their wings. In retrospect, it was clearly a mistake on my part to continue with a project I had begun earlier in my career, but “it seemed like a good idea at the time,” as they say. As I worked autonomously on this project, any mistakes or other gaffes in this paper are entirely my own.

Thanks go to Matt Umurhan for pointing out to me several of the references listed below, and for some fun and enlightening conversations on related topics. I also thank the reader for enduring my effusive commentary. I felt I couldn’t post this paper without an explanation.

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REFERENCES

Ashfordi, N., Mukhopadhyay, B., & Narayan, R. 2005, ApJ, 629, 373, arXiv:astro-ph/0412194

Chagelishvili, G. D., Zahn, J.-P., Tevzadae, A. G., & Lominadze, J. G. 2003, A&A, 402, 401, arXiv:astro-ph/0302258

Mukhopadhyay, B., Ashfordi, N., & Narayan, R. 2005, Proceedings of the 22nd Texas Symposium on Relativistic Astrophysics, Dec 12-17, 2004, arXiv:astro-ph/0501468

Umurhan, O. M., & Regev, O. 2004, A&A, 427, 855, arXiv:astro-ph/0404020

Yecko, P. 2004, A&A, 425, 385.

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