Analysis of Leptogenesis in Supersymmetric Triplet Seesaw Model

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We analyze leptogenesis in a supersymmetric triplet seesaw scenario that explains the observed neutrino masses, adopting a phenomenological approach where the decay branching ratios of the triplets and the amount of CP-violation in its different decay channels are assumed as free parameters. We find that the solutions of the relevant Boltzmann equations lead to a rich phenomenology, in particular much more complex compared to the non-supersymmetric case, mainly due to the presence of an additional Higgs doublet. Several unexpected and counter-intuitive behaviors emerge from our analysis: the amount of CP violation in one of the decay channels can prove to be irrelevant to the final lepton asymmetry, leading to successful leptogenesis even in scenarios with a vanishing CP violation in the leptonic sector; gauge annihilations can be the dominant effect in the determination of the evolution of the triplet density up to very high values of its mass, leading anyway to a sizeable final lepton asymmetry, which is also a growing function of the wash-out parameter \( K \equiv \frac{\Gamma_d}{H} \), defined as usual as the ratio between the triplet decay amplitude \( \Gamma_d \) and the Hubble constant \( H \); on the other hand, cancellations in the Boltzmann equations may lead to a vanishing lepton asymmetry if in one of the decay channels both the branching ratio and the amount of CP violation are suppressed, but not vanishing. The present analysis suggests that in the supersymmetric triplet see-saw model successful leptogenesis can be attained in a wide range of scenarios, provided that an asymmetry in the decaying triplets can act as a lepton–number reservoir.

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I. INTRODUCTION

Leptogenesis is a very appealing explanation of the baryon asymmetry of the Universe, the more so since it can also incorporate the origin of neutrino masses and mixings. In triplet see-saw models, a cosmological lepton asymmetry is produced by the decay of a heavy Higgs triplet, in presence of the Sakharov condition of i) lepton–number violation; ii) CP violation; iii) out–of–equilibrium decay. The same Higgs triplet is also responsible of the dimension–five operator that induces neutrino masses through the see-saw mechanism in the vacuum expectation values. The lepton asymmetry is then transformed to a baryon asymmetry through the mechanism of sphaleron conversion.

This model has already been discussed in the literature with multi Higgs triplets or with additional right-handed neutrinos. As a general feature, it presents some interesting differences compared to the usual see-saw mechanism where the decaying particles producing leptogenesis are assumed to be right–handed neutrinos. In particular, the triplet see-saw is more predictive, since Yukawa matrices can be fixed through neutrino masses, and no unknown Majorana mass matrices are present. An important feature of the triplet leptogenesis is that it suffers from a strong wash–out effect due to gauge–triplet annihilations, which cannot be neglected, particularly, for low mass of the triplet. Even with such a strong annihilation effect, triplets can develop an appropriate lepton asymmetry during their thermal evolution, and this, in the non-supersymmetric version of the model, has been shown to lead to the possibility of a high efficiency for leptogenesis production even for low triplet masses. This is an intriguing aspect that enables us to circumvent the gravitino problem requiring the upper–bound on the reheating temperature of the Universe, \( T_{RH} \lesssim 10^8 \) GeV in supergravity theories.

In this paper, we want to extend the discussion of the properties of supersymmetric triplet see–saw leptogenesis to a general phenomenological scenario, with a minimal set of theoretical assumptions, and to compare it to the non–supersymmetric version of the model. In our study, we will cover the triplet mass down to the TeV range, for which the model has a prospect of being tested in future colliders.

The plan of the paper is as follows. In Section III we introduce the Higgs triplet model and the corresponding Boltzmann equations. In Section III we describe our minimal assumptions and then discuss the produced lepton
asymmetry in the non–supersymmetric version of the model in Section VI and in the supersymmetric one in Section VII. We devote Section VI to our conclusions.

II. THE MODEL AND BOLTZMANN EQUATIONS

In the supersymmetric form of the Higgs triplet model [1], one needs to introduce vector-like pairs of $\Delta = (\Delta^+, \Delta^0, \Delta^0)$ and $\Delta^c = (\Delta^c-, \Delta^c-, \Delta^c0)$ with hypercharge $Y = 1$ and $-1$, allowing for the renormalizable superpotential as follows:

$$W = \lambda_L LL \Delta + \lambda_1 H_1 H_1 \Delta + \lambda_2 H_2 H_2 \Delta^c + M \Delta \Delta^c,$$

(1)

where $\lambda_LL \Delta$ contains the neutrino mass term, $\lambda_L \nu \nu \Delta$. In the supersymmetric limit, the Higgs triplet vacuum expectation value $\langle \Delta^0 \rangle = \lambda_2 (H_2^0)^2 / M$ gives the neutrino mass

$$m_\nu = 2\lambda_L \lambda_2 \frac{v^2}{M},$$

(2)

with $v^2 \equiv \langle H_2^0 \rangle$. Working in the supersymmetric limit, we use the same notations for the bosonic and fermionic degrees of the superfields. A heavy particle $X$, which can be any component of $\Delta, \Delta^c$, decays to the leptonic final states, $LL$, as well as the Higgs final states, $H_1 H_1$ and $H_2 H_2$, the out-of-equilibrium decay of which will lead to a lepton asymmetry of the universe. The corresponding decay rate is $\Gamma \equiv \frac{16}{\lambda_2^2 \lambda_1^2 \lambda_3^2} \frac{M^2}{16 \pi v^2 H(M)} \approx \frac{16}{\sqrt{B_L B_2}} \left( \frac{|m_\nu|}{0.05 \text{ eV}} \right)$.

where $H(M) = 1.66 \sqrt{G_N M^2 / m_{Pl}}$ is the Hubble parameter at the temperature $T = M$, and $B_L, B_2$ are the branching ratios of the triplet decays to $LL$ and $H_2 H_2$, respectively. For the relativistic degrees of freedom in thermal equilibrium $g_*$, we will use the Supersymmetric Standard Model value: $g_* = 228.75$. The parameter $K$ takes the minimum value of $K_{\text{min}} = 32$ for $B_L = B_2 = 1/2$ and gets larger for $B_L$ or $B_2 \ll 1$. For our discussion, we will fix $m_\nu = 0.05 \text{ eV}$, which corresponds to the atmospheric neutrino mass scale.

The resulting lepton asymmetry of the universe is determined by the interplay of the three asymmetries developed in the decay channels $X \rightarrow f_i$ where $f_i = LL, H_1 H_1, H_2 H_2$ for $i = L, 1, 2$, respectively. Their cosmological evolutions crucially depend on the corresponding $K$-values $K_i$ and the CP asymmetries $\epsilon_i$ which are defined by

$$K_i \equiv K B_i \quad \text{and} \quad \epsilon_i \equiv \frac{\Gamma(X \rightarrow f_i) - \Gamma(X \rightarrow \bar{f}_i)}{\Gamma_X}.$$

(4)

The above CP asymmetries follow the relation: $\epsilon_L + \epsilon_1 + \epsilon_2 \equiv 0$. Note here that the model contains non-trivial CP asymmetries $\epsilon_i$ which can be generated after integrating out additional triplets or right-handed neutrinos [3, 4] or from CP phases in the supersymmetry breaking sector [3, 11, 12].

Before discussing how to generate leptogenesis in this model, let us introduce for comparison its non–supersymmetric version. The simplest realization of the triplet model corresponds to the following Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + |D_\mu \Delta|^2 - M^2 |\Delta|^2 + (\lambda_L LL \Delta + M \lambda_H H H \Delta^* + \text{h.c.}),$$

(5)

with the hypercharge assignments: $Y_L = -1/2, Y_H = 1/2$ and $Y_\Delta = 1$. The neutrino mass term is still given by Eq. (2), with the substitutions $v_2 \rightarrow v$ and $\lambda_2 \rightarrow \lambda$, and $v \equiv \langle H^0 \rangle$. For the relativistic degrees of freedom in
number densities, \( n \), (distributions are 2-out to be essential in determining the final lepton asymmetry. Moreover, as a consequence of unitarity, the relation charged under the Standard Model gauge group and thus have a nontrivial gauge annihilation effect which turns annihilation effect, and are accounted for by the functions \( \gamma \).\n
In order to discuss how to generate a lepton asymmetry in the supersymmetric triplet seesaw model let us first consider the general case of a charged particle \( X \) (\( \bar{X} \)) decaying to a final state \( j \) (\( \bar{j} \)) and generating tiny CP asymmetric number densities, \( n_X - n_{\bar{X}} \) and \( n_j - n_{\bar{j}} \). The relevant Boltzmann equations in the approximation of Maxwell–Boltzmann distributions are

\[
\frac{dY_X}{dz} = -zK \left[ \gamma_D(Y_X - Y_{eq}^X) + \gamma_A \frac{Y_{eq}^X}{Y_X} \right]
\]

\[
\frac{dY_z}{dz} = -zK\gamma_D \left[ Y_z - \sum_k 2B_k \frac{Y_{eq}^{X_k}}{Y_k} Y_k \right]
\]

\[
\frac{dY_j}{dz} = 2zK\gamma_D \left[ \epsilon_j(Y_X - Y_{eq}^{X_j}) + B_j(Y_z - 2Y_{eq}^{X_j} Y_j) \right],
\]

where \( Y \)'s are the number densities in unit of the entropy density \( s \) as defined by \( Y_X \equiv n_X/s \approx n_{\bar{X}}/s, \ Y_z \equiv (n_X - n_{\bar{X}})/s, \ Y_j \equiv (n_j - n_{\bar{j}})/s, \) and \( z = M/T \). The quantities \( \epsilon_i \) are defined in Eq. (4).

The evolution of the \( X \) abundance is determined by the decay and inverse decay processes, as well as by the annihilation effect, and are accounted for by the functions \( \gamma_D \) and \( \gamma_A \), respectively. Note that the triplets are charged under the Standard Model gauge group and thus have a nontrivial gauge annihilation effect which turns out to be essential in determining the final lepton asymmetry. Moreover, as a consequence of unitarity, the relation \( 2Y_x + \sum_j Y_j \equiv 0 \) holds, so that one can drop out the equation for \( Y_z \), taking the replacement:

\[
Y_x = -\frac{1}{2} \sum_j Y_j,
\]

in the last of Eqs. (7). In the supersymmetric version of the model, the heavy particle \( X \) can be either of the six charged particles; \( X = \Delta^{\pm}, \Delta^\pm \) or \( \Delta^{0,0} \) for each triplets (\( \Delta, \Delta^c \)). Each of them follows the first Boltzmann equation in Eq. (7) where \( \gamma_D \) and \( \gamma_A \) are given by

\[
\gamma_D = \frac{K_1(z)}{K_2(z)} \quad \gamma_A = \frac{\alpha_s^2 M}{\pi KH(M)} \int_1^\infty dt \frac{K_1(zt)}{K_2(z)} t^2 \beta(t) \sigma(t),
\]

with

\[
\sigma(t) = (14 + 11t_w^2)(3 + \beta^2) + (4 + 4t_w^2 + t_w^4) \left[ 16 + 4(-3 - \beta^2 + \frac{\beta^4 + 3}{2\beta} \ln \frac{1 + \beta}{1 - \beta} \right]
\]

\[
+ 4 \left[ -3 + \left( 4 - \beta^2 + \frac{\beta^2 - 1}{\beta} \right) \ln \frac{1 + \beta}{1 - \beta} \right],
\]

where \( t_w = \tan(\theta_W) \) with \( \theta_W \) the Weinberg angle, and \( \beta(t) = \sqrt{1 - t^2} \). The function \( \gamma_D \) is the ratio of the modified Bessel functions of the first and second kind which as usual takes into account the decay and inverse decay effects in the Maxwell–Boltzmann limit. The function \( \gamma_A \) accounts for the annihilation cross-section of a triplet component \( X \) summing all the annihilation processes; \( X \bar{X} \rightarrow \) Standard Model gauge bosons/gauginos and fermions/sfermions where \( X' \) is some triplet component or its fermionic partner.
The corresponding expression for $\sigma(t)$ in the non-supersymmetric version of the model, accounting for the annihilations of the triplets to the Standard Model gauge bosons and fermions is given by:

$$\sigma(t) = \left( 25 + \frac{41}{2}t_w^4 \right) \frac{\beta^2}{3} + (4 + 4t_w^2 + t_w^4) \left[ 4 + 4(1 - \beta^2 + \frac{\beta^4 - 1}{2\beta} \ln \frac{1 + \beta}{1 - \beta}) \right] + 4 \left[ -1 + \left( 2 - \frac{5}{3}\beta^2 + \frac{(\beta^2 - 1)^2}{\beta} \ln \frac{1 + \beta}{1 - \beta} \right) \right].$$

(12)

The rôle played by annihilation and decay in the determination of the triplet density $Y_X$ can be understood in the following way. When the branching ratios $B_i$ of the different decay channels are all of the same order, inverse decays freeze out at a temperature $z_f$ determined by

$$K z_f^{5/2} e^{-z_f} = 1.$$

(13)

At that temperature the thermal averages of the annihilation and decay rates can be compared by considering the following ratio:

$$\frac{<\Gamma_A>}{<\Gamma_D>} (z_f) \simeq 2 \frac{\alpha_X^2}{\alpha_X} z_f^{-3/2} e^{-z_f},$$

(14)

where $\alpha_X = K H(M)/M$. If the quantity in Eq. (14) is bigger (smaller) than 1 the triplet freeze–out is determined by annihilation (inverse–decay). Thus, in this case, the annihilation effect becomes dominant for

$$M \lesssim 10^{15} z_f e^{-2z_f} \text{ GeV},$$

and so it can be neglected when $M \gtrsim 10^8$ GeV for $K = 32$ and $M \gtrsim 1$ TeV for $K \approx 4300$.

However, if one has $K_i \equiv B_i K << 1$ in one channel, with the same quantity bigger than 1 in the other channels, the condition of Eq. (13) must be modified by using $K_i$ instead of $K$. This leads to a smaller $z_f$ and shifts to higher masses the transition between dominance of annihilation and inverse decay in the determination of the triplet density.

### III. SET-UP FOR THE DISCUSSION

In the following, we will discuss the phenomenology of thermal leptogenesis within the more generic version of the framework introduced in the previous section, i.e. by considering the branching ratios $B_i$ and the CP asymmetries $\epsilon_i$ as free parameters, with the additional constraints $\sum B_i = 1$, $\sum \epsilon_i = 0$, and $|\epsilon_i| \leq 2B_i$ (this last condition ensures that all physical amplitudes are positive, and simply states that the amount of CP violation cannot exceed 100% in each channel). This choice implies 2 free parameters in the non–supersymmetric version of the model and 4 in the supersymmetric one, besides the triplet mass parameter $M$. In order to show our results, we choose to discuss, for every particular choice of the parameters, the amount of CP violation which is needed to provide successful leptogenesis, which we define by the value $\bar{\epsilon}$ that the the biggest of the $|\epsilon_i|$’s must have in order to provide $Y_L = 10^{-10}$ for the final lepton asymmetry (this amount of $Y_L$ leads to the correct baryon asymmetry compatible to observations once reprocessed by sphaleron interactions). Since sphaleron conversion is suppressed at temperatures below Electro-Weak symmetry breaking, in our calculation we stop the evolution of $Y_L$ below $T = m_Z$, with $m_Z$ the Z–boson mass.

The quantity $\bar{\epsilon}$ is inversely proportional to the usual efficiency factor $\eta$, defined by the relation:

$$Y_L = \epsilon_L \eta(Y_X + Y_{\bar{X}})|_{T > M},$$

(15)
FIG. 1: Contour plots of the amount of CP violation \( \bar{\epsilon} \) needed to provide the observed baryon asymmetry in the Universe (as defined in Section III), in the non–supersymmetric version of the triplet see-saw model described in Section IV as a function of the triplet decay branching ratio to leptons \( B_L \), and of the triplet mass \( M \). The marked lower corners are excluded by the conditions \(|\epsilon_i| \leq 2B_i, i = L, H\), while in the upper corners one of the Yukawa couplings becomes bigger than 1 due to Eq. 4.

where \( \eta \) is determined by the amount of wash-out effect by inverse decay and by the suppression of the number density \( Y_X \) by gauge annihilations. One gets \( \eta = 1 \) in the limit where the triplets decay strongly out-of-equilibrium when \( T >> M \).

By fixing \( \bar{\epsilon} \) we reduce by one the number of free parameters. Moreover, since an overall minus sign for the \( \epsilon_i \)'s implies a change of sign in the final value of \( Y_L \), we discuss \( Y_L \) and \( \bar{\epsilon} \) in their absolute values. In this way only the ratios among the \( \epsilon_i \)'s are relevant as input parameters, and we can define the \( \epsilon_i \)'s in such a way that \( \max(|\epsilon_i|) = 1 \).

IV. THE NON-SUPERSYMMETRIC VERSION OF THE MODEL

We start by discussing the non–supersymmetric version of the model. In this case the triplets have two two decay channels, \( X \rightarrow LL, HH \), and the process of triplet annihilations is governed by Eq. 12. In our convention CP violations are fixed, \( \epsilon_L = -\epsilon_H = 1 \) so there are only 2 free parameters, the triplet mass \( M \) and one of the two branching ratios, for example \( B_L \), with \( B_L + B_H = 1 \). The result of our calculation is shown in Figure 1 which is consistent with the result of Ref. 8. In our figure, the curves at constant values of \( \bar{\epsilon} \) are plotted in the \( B_L-M \) plane (in order to blow–up the regions where \( B_L << B_H \) or \( B_L >> B_H \), the two complementary quantities \( B_L \) and \( 1-B_L \) are plotted in logarithmic scale in the range 0,0.5). The marked regions in the lower corners are excluded by the conditions \(|\epsilon_i| \leq 2B_i, i = L, H\), while those in the upper corners are excluded because one of the Yukawa couplings is non–perturbative due to Eq. 2. The figure is symmetric under the exchange \( B_L \rightarrow 1-B_L = B_H \), a feature that can be easily explained by the fact that the parameter \( K \) remains the same 3 and because the identity 8 implies \(|Y_L| = |Y_H|\) at late times (when all the triplets have decayed away).

The change of behavior of the various curves between \( M \approx 10^8 \) and \( M \approx 10^{10} \) GeV signals a transition in the evolution of the triplet number density between decays/inverse–decays and annihilations. When the freeze–out temperature of the triplets is determined by decays/inverse–decays, \( \bar{\epsilon} \) is only a function of the branching ratios (note that the parameter \( K \) does not depend on \( M \), see Eq. 4), so curves are parallel to the vertical axis. On the other hand, when it is the annihilation process that determines the triplet density freeze-out, this strongly suppresses the efficiency \( \eta \) at low values of \( M \) so that higher values of \( \bar{\epsilon} \) are needed in order to obtain successful leptogenesis.

Another important feature that can be seen in the figure is given by the fact that the highest efficiencies (lower values for \( \bar{\epsilon} \)) are reached whenever \( B_L << B_H \) or \( B_H << B_L \). As already discussed in Ref. 9, this is due to the fact...
FIG. 2: Amount $\bar{\epsilon}$ of CP violation needed to provide the baryon asymmetry observed in the Universe, as a function of the triplet mass $M$. Solid lines are calculated in the non-supersymmetric triplet see-saw model of Section IV while dashed ones in the supersymmetric model discussed in Section V, where $B_l = \epsilon_1 = 0$. Each curve corresponds to a different choice of the parameter $B_L = 0.5, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}$ and $10^{-8}$, from top to bottom. The range of each curve is limited by the unitarity and perturbativity constraints shown in Fig. 1 as explained in the text.

that in these cases one of the two decay channels has $K_i << 1$ (even if, due to Eq. 3, $K \gtrsim 32$), and so is “slow” compared to the Hubble expansion, while the other is “fast”. As a consequence of this, the “slow” channel can decay out of equilibrium with efficiency close to 1 and develop a sizeable asymmetry $Y_{\text{slow}}$, while, at the same time, a corresponding asymmetry with opposite sign $Y_x$ is left over in the triplet density (since in this process $Y_x + Y_{\text{slow}}$ is approximately conserved). The quantity $Y_x$ is eventually converted into an asymmetry $Y_{\text{fast}}$ in the fast decay channel (with $|Y_{\text{slow}}| = |Y_{\text{fast}}|$ at later times due to Eq. 8), when the triplets get out of kinetic equilibrium and decay. So, the reason why $Y_{\text{fast}}$ is not erased by the sizeable wash-out effect is clear: due to wash-out, triplets and anti-triplets decay practically with the same rate, but more final particles are produced than final antiparticles because there are more triplets than antitriplets available for decay in the first place. In this way an asymmetry in the triplet density can be stored and eventually converted to a lepton asymmetry, acting in practice as a lepton–number reservoir. This very simple physical picture can be significantly modified if more than two decay channels are present, as will be illustrated in the following sections for the supersymmetric version of the model.

The dependence of the quantity $\bar{\epsilon}$ as a function of the triplet mass $M$ is discussed in Figure 2 where the solid lines correspond to the non-supersymmetric model discussed in this section, while the dashed ones show a supersymmetric modification of the model discussed in the next section. Several features of this figure are worth noticing, as they outline quite general properties of triplet thermal leptogenesis:

- As expected, $\bar{\epsilon}$ is proportional to $B_L$, and the higher values (lower efficiencies) are obtained when $B_l = B_H = 1/2$.
- The lowest value of $\bar{\epsilon}$ is reached at $\bar{\epsilon} \simeq 10^{-8}$. This value corresponds to the limit of out-of-equilibrium decay,
and can be easily obtained from Eq. (15) by setting the efficiency $\eta = 1$. In fact, since $Y_X|_{T>M} \simeq 1/g_*$, where $g_* \simeq 10^2$ is the number of degrees of freedom in the Early Universe, from $Y_L \simeq 10^{-10} \simeq \tilde{\epsilon} 10^{-2}$ one gets $\tilde{\epsilon} \simeq 10^{-8}$. This minimal value for $\tilde{\epsilon}$ is obtained on general grounds that do not depend on the microphysics, so it is not expected to change in the modifications of the model that will be discussed in the next Sections.

- The available range for $M$ at fixed $\tilde{\epsilon}$ is bounded from below by the unitarity constraint, and from above by the perturbativity limit. As shown in Figure 1, the two bounds converge at low $B_L$ or $1 - B_L$ corresponding to small values of $\tilde{\epsilon}$, and eventually meet (outside the bounds of the figure) for $B_L = 1 - B_L \simeq 10^{-8}$. That is why the range of $M$ gets smaller for low values of $\tilde{\epsilon}$, and eventually a particular value of $M \simeq 10^{-10}$ is singled out for which the efficiency reaches its maximum value.

- Two different regimes for $M$ are clearly distinguishable. In particular, the strong loss of efficiency at lower values of $M$ is due to the effect of annihilations in the determination of the triplet freeze–out temperature. This temperature is significantly lowered, with a consequent suppression of the final lepton asymmetry, compared to the case where decays/inverse–decays dominate, which corresponds to the regime of higher values for $M$.

- One realizes that $K$ increases but $K_L = K \ast B_L$ decreases from top to bottom. When the annihilation dominates for lower $M$, the Boltzmann equations show that the quantity $Y_X - Y_{eq}^X$ is determined independently of $K$ and thus the final asymmetry $Y_L$ increases with $K$. As mentioned at the end of Section II, the figure also shows that the dominance of inverse decay starts at larger $M$ for smaller $K_L$.

V. THE SUPERSYMMETRIC VERSION OF THE MODEL

In the supersymmetric version of the model the particle content is enlarged, both because of the additional super-symmetric degrees of freedom (striplets, sleptons and Higgsinos), and from the fact that one more Higgs(+Higgsino) doublet is included. Actually, this latter aspect will turn out to be more relevant than the former for our discussion. In fact, barring possible Susy–breaking effects which are expected to be suppressed for values of $M$ above the Supersymmetry –breaking scale, triplet decay amplitudes to particles belonging to the same supermultiplets are the same, and can be factored out in the Boltzmann equations for asymmetries. This implies that including supersymmetric partners in Eqs. (7) is as simple as multiplying by 2 all the relevant degrees of freedom, and the branching ratios $B_L$, $B_1$ and $B_2$ and CP asymmetries $\epsilon_L$, $\epsilon_1$ and $\epsilon_2$ will refer to a sum over all the members of each supermultiplet. As far as the triplet density $Y_X$ is concerned, the supersymmetric version of the annihilation cross section given in Eq. (11) must be used, where annihilations to supersymmetric particles, as well as coannihilations of the triplets with their fermionic superpartners are taken into account (in our equations we assume that triplets and striplets are degenerate in mass ). This implies a low–temperature annihilation cross–section about a factor 8 bigger compared to the non–supersymmetric case, and a corresponding loss of efficiency at low masses, where annihilation drives triplet freeze–out.

A. Standard Model-like case without $X \rightarrow H_1 H_1$

As long as only the Higgs supermultiplet $H_2$ is included in the model (i.e., in the case with $B_1 = \epsilon_1 = 0$), the resulting phenomenology is not expected to change qualitatively compared to the non-supersymmetric case: from the practical point of view, the supersymmetric case with only $H_2$ corresponds just to the non-supersymmetric one where the degrees of freedom are multiplied by two and the annihilation cross section is about a factor of 8 bigger (changing the degrees of freedom implies also a slight modification of the $K$ parameter of about $\sqrt{2}$ at fixed branching ratios). In order to show this point, in Figure 2 we show with dashed lines the result of a calculation analogous to that
FIG. 3: Same as in Fig. 2. Thin dashed lines are calculated in the supersymmetric model discussed in Section V where \( B_1 = \epsilon_1 = 0 \), while solid and thick–dashed lines show the same quantity for \( B_L = B_2 \approx 1/2 \), \( \epsilon_1 = 1 \) and, from top to bottom, \( B_1 = 10^{-2} \), \( B_1 = 10^{-3} \), \( B_1 \leq 10^{-4} \). For solid curves \( \epsilon_L = 0 \), while for thick-dashed ones \( \epsilon_L = -1 \).

shown by solid ones, where the supersymmetric version of the model with \( B_1=\epsilon_1=0 \) is used. As can be seen, the two models are qualitatively quite similar, the supersymmetric scenario implying worse efficiencies compared to the non–supersymmetric one over the whole range of \( M \). This may be explained by the fact that in supersymmetry both the annihilation cross section (which lowers the efficiency at low \( M \)) and the \( K \) parameter (which reduces it at high \( M \)) are bigger compared to the non-supersymmetric case.

B. Role of the third channel: \( X \to H_1H_1 \)

When non–vanishing \( B_1 \) and \( \epsilon_1 \) are considered, the number of free parameters becomes 4 (two branching ratios and two asymmetries) plus the triplet mass \( M \). In this case, a qualitatively different phenomenology arises compared to the previous cases.

A first remarkable difference is due to the fact that \( B_1 \) is not constrained by Eq. 2 and can be taken arbitrarily small even for high values of \( M \). This implies that out-of-equilibrium decay and very low values of \( \bar{\epsilon} \) are expected to be reached without encountering the upper bound on \( M \) observed in the curves of Fig. 2 due to the non perturbativity of \( \lambda_L \) or \( \lambda_2 \). This is shown in Figure 3 where \( \bar{\epsilon} \) is plotted as a function of \( M \) for \( B_L=B_2=1/2 \), and for very low values of \( B_1 \).

The other important feature of the model is that, as in all scenarios with more than two decay channels, now a hierarchy in the CP violation parameters is possible. This implies that, for instance, in some particular channel CP violation can be suppressed compared to the other two, or even absent. An example of this is again shown in Fig. 3 where the values \( \epsilon_1=1 \) and \( \epsilon_L = 0, -1 \) are assumed for each value of \( B_1 \). As can be seen from Fig. 3 even
the case $\epsilon_L = 0$ can provide leptogenesis with a good efficiency. This fact at first sight might seem quite amusing, since one could wonder how a CP-conserving decay of the triplets to leptons may lead to any lepton asymmetry at all. However the answer to this question is contained in the same mechanism explained in Section IV where the asymmetry in the triplet density produced by out-of-equilibrium decay in a slow decay channel could be fully converted to an asymmetry in the fast one even in presence of a very strong wash–out effect. As already pointed out previously, in the fast channel CP violation produced by triplet decays is negligible even in the case where $\epsilon_i \neq 0$, the final asymmetry being produced only by the fact that the number of decaying triplets is different from the number of decaying antitriplets. So, having $\epsilon_i = 0$ or strongly suppressed in this fast channel doesn’t make any difference.

The existence of a hierarchy in the $\epsilon_i$ parameters can however strongly affect the physical mechanism described above whenever CP violation is suppressed in a slow channel, as can be naively expected since this is the channel that drives leptogenesis. As a matter of fact, if the slow decay channel has a small $\epsilon_i$ it cannot develop a sizeable asymmetry $Y_{\text{slow}}$, while, at the same time, the corresponding asymmetry with opposite sign $Y_x$ left over in the triplet density (due to the approximate conservation of $Y_x + Y_{\text{slow}}$) is also suppressed, leading so to a suppression of the asymmetry also in the other, fast channel.

C. The possibility of cancellations

In the fast channel another important fact may arise: the two mechanisms of asymmetry production (i.e. direct CP violation in the decay and asymmetry in the density of the triplets) may give rise to effects of the same order of magnitude, and, if the sign of the $\epsilon_i$ parameters is the same in the fast and in the slow channels, even cancel out, leading so to a vanishing final asymmetry. In this scenario, which implies a numerical cancellation in the Boltzmann equations, the more populated between $X$ and $\bar{X}$ decays with the lower rate to the corresponding final state $L$ or $\bar{L}$, in such a way that no final asymmetry is produced.

In order to show this effect, in Fig. 4 the parameter $\bar{\epsilon}$ is plotted as a function of $M$ for the same choice of parameters as for the solid lines of Fig. 3 but assuming $\epsilon_L=1$ and $\epsilon_1 = 0.1$ (solid lines) and $\epsilon_1 = -0.1$ (dashed lines). As can be seen in the figure, now peaks arise for $\epsilon_1 = 0.1$ signaling a vanishing efficiency $\eta$, while are not present for $\epsilon_1 = -0.1$. As explained before, this happens because the $\epsilon_i$ parameter corresponding to the slowest decay channel is suppressed compared to the other ones, and the cancellation mechanism described in the previous paragraph may set in when $\epsilon_1$ and $\epsilon_L$ have the same sign.

D. Lepton asymmetry with vanishing $\epsilon_2$

As discussed previously, when its CP-violating term is exactly vanishing, the slow channel no longer drives leptogenesis. If its branching ratio is also much smaller than the other two, the slow channel may be neglected altogether. So, for instance, when $\epsilon_1 = 0$, the case $B_1 << B_{L,2}$, is equivalent to taking $B_1 = 0$ (so, the upper dashed curve in Fig. 2 where $B_1 = 0$ and $B_L = B_2 = 1/2$, is equivalent to the case where $B_1$ is small but non–vanishing). Taking $\epsilon_{L,2}=0$ and very small values for the corresponding $B_{L,2}$ is still equivalent to neglecting the corresponding channel, although, due to Eqs. (2)4, a higher value for the $K$ parameter, and a possible perturbativity constraint at high $M$ are induced.

In order to show this, in Fig. 3 we show with solid lines the case $\epsilon_2=0$, $B_2 << B_L = B_1 \approx 1/2$. As expected, in this scenario the efficiency is very poor, since the slow channel, having vanishing $\epsilon_1$, cannot drive leptogenesis through out-of-equilibrium decay. Moreover, all curves show an upper bound on $M$ due to the perturbativity constraint, that shifts to lower $M$ for smaller $B_2$. As already mentioned, for these curves, due to Eq. (3), the parameter $K$ is very high. Moreover, as expected, $\bar{\epsilon}$ scales with $K \propto \sqrt{B_2}$ (the efficiency is expected to scale as $1/(zL\bar{K})$ for $K >> 1$ and if the inverse–decay effect is important (14)). It is worth noticing now that the relative weight of the two competing
FIG. 4: Same as in Fig. 3 solid lines, but with $\epsilon_L = 1$ and $\epsilon_1 = 0.1$ (solid lines) and $\epsilon_1 = -0.1$ (dashed lines). The presence of peaks in $\bar{\epsilon}$ signals a vanishing efficiency $\eta$.

effects of annihilation and decay/inverse-decay in the determination of the triplet freeze-out temperature depends on $K$, since the rate of latter effect grows with $K$ while the former does not. So, the net consequence of a big $K$ is to suppress the annihilation effect, which, in turn, is instrumental in lowering the efficiency $\eta$ at low values of $M$. As a consequence of this, a larger $K$ reduces the values of $M$ where annihilation starts to dominate, as is observable in Fig. 4 where the change of behavior of all the solid curves at low $M$ is shifted to the left. At lower $M$, when annihilation dominates in the determination of the (s)triplet density, the efficiency grows with $K$.

If, on the other hand, in this same scenario a hierarchy between $B_L$ and $B_1$ is assumed, the presence of another slow channel whose corresponding CP-violation parameter $\epsilon$ is not suppressed can in principle increase the efficiency, allowing, eventually, to reach the values of $\bar{\epsilon}$ typical of early out-of-equilibrium decay. This effect, however, is only possible in the case $B_1 < B_L$; the alternative case $B_L < B_1$ has a much bigger value of $K$, which implies a better efficiency at low mass but a worse one at higher $M$. This is shown in Fig. 4 where the dashed curves, which have $\epsilon_2=0$, $B_2=10^{-4}$, and $B_1=10^{-1}$, $10^{-2}$, $10^{-3}$, $10^{-4}$, $10^{-5}$, $10^{-6}$, from top to bottom, can reach values as low as $\bar{\epsilon} \simeq 10^{-8}$ when $B_1$ is sufficiently small. On the other hand, the dotted curves in the same figure, with $\epsilon_2=0$, $B_2=10^{-4}$, and $B_L=10^{-1}$, $10^{-2}$, $10^{-3}$, $10^{-4}$, from top to bottom have a worse efficiency, as expected. Note, in this last case, the inverse proportionality between $\bar{\epsilon}$ and $K$: this is due to the fact that the relevant parameter is $K_L \propto \sqrt{B_L/B_2}$, so that when $K$ increases $K_L$ gets smaller.
Fig. 5: Same as in Fig. 2, but with $\epsilon_2=0$. From top to bottom, the solid lines correspond to the case when $B_1 = B_L = 1/2$ and $B_2 = 10^{-4}, 10^{-3}, 10^{-2}$; the dashed lines to $B_2 = 10^{-4}$ and $B_1 = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}$; the dotted lines to $B_2 = 10^{-4}$ and $B_L = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$.

E. Lepton asymmetry with vanishing $\epsilon_L$

As pointed out in the previous paragraph, the case $\epsilon_L=0, B_L << B_{1,2}$ is expected to give a very low efficiency for leptogenesis. An example of this case is shown in Fig. 6 where the upper dotted line shows the case: $\epsilon_L = 0, B_L = 10^{-6}, B_2 = 10^{-2}$. Eventually, taking even smaller values of $B_L$ is equivalent to dropping $L$ from the Boltzmann equation. On the other hand, having $\epsilon_L=0, B_1 << B_{L,2}$ leads to high values of the efficiency, as already discussed in Figure 3. Apart from a higher value of $K$, which implies a better efficiency at small masses but a worst one overall, the case $\epsilon_L=0, B_2 << B_{L,1}$ is analogous, since now it is the $H_2$ channel that drives leptogenesis, decaying out-of-equilibrium with a non-suppressed $\epsilon_2$. An example of this scenario is given by the dashed line in Fig. 6 where $\epsilon_L=0, B_2 = 10^{-6}, B_L = 10^{-2}$.

On the other hand, a qualitatively different situation is given by: $\epsilon_L=0, B_{1,2} << B_L$. In this case there are two slow decay channels, and the corresponding $\epsilon_{1,2}$ are not suppressed. However, since $\epsilon_1=-\epsilon_2$, and, due to Eq. 8, at late times $Y_L = -Y_1 - Y_2$, a cancellation between $Y_1$ and $Y_2$ may occur if both quantities reach their out-of-equilibrium value, leading to vanishing $Y_L$. This implies that, in order to reach a good efficiency, some hierarchy between $B_1$ and $B_2$ is needed, in order to have only one slow channel. This is again shown in Fig. 6 by the two solid lines, that correspond to: $\epsilon_L=0, B_1 = 10^{-2}, B_2 = 10^{-6}$ (left) and $\epsilon_L=0, B_1 = 10^{-6}, B_2 = 10^{-2}$ (right). In this case both curves reach a good efficiency, with a less stringent perturbativity limit for the latter due to a smaller value of $K$. In both cases, since $K_{\text{slow}} = B_{\text{slow}}K << 1$, inverse decays in the slow channel freeze out very early, when annihilation still dominates in the determination of the triplet density. As a consequence of this the efficiency is a growing function of $K$, and this explains why the solid curve on the left ($K \simeq 16000$) is below the one on the right ($K \simeq 160$). Besides,
FIG. 6: Same as in Fig. 2, but with $\epsilon_L=0$. The upper dotted line corresponds to $B_L = 10^{-6}$ and $B_2 = 10^{-2}$. For the two solid lines $B_1 = 10^{-2}$, $B_2 = 10^{-6}$ (left) and $B_2 = 10^{-2}$, $B_1 = 10^{-6}$ (right). This last curve corresponds also to the case $B_L = 10^{-2}$, $B_1 = 10^{-6}$. The dashed curve has $B_L = 10^{-2}$ and $B_2 = 10^{-6}$.

in the latter case the curve would remain the same by the exchange ($B_L \leftrightarrow B_2$), i.e.: $B_1 = 10^{-6}$, $B_L = 10^{-2}$. This is due to the fact that the slow channel would remain the same, as well as the corresponding $\epsilon_i$, while also $K$ would be unchanged.

VI. CONCLUSIONS

In this paper we have analyzed the phenomenology of leptogenesis in the supersymmetric triplet see–saw mechanism. Taking the branching ratios $B_i$ of the decay rate of the triplets as free parameters, as well as the CP–violation parameters $\epsilon_i$, with the additional constraints $\sum_i B_i = 1$ and $\sum_i \epsilon_i = 0$, we have calculated the amount $\epsilon$ of CP violation which is needed to provide successful leptogenesis. In the most favourable case of early out-of-equilibrium decay of the triplets to leptons, this number is of order $10^{-8}$. However, it is well known that in this scenario inverse decays and annihilations of triplets (this last effect for lower values of $M$) contribute in general to erase the asymmetry, basically keeping the triplets in thermal equilibrium until late times, when $T < M$ and their number density is exponentially suppressed. An exception to this, within the framework of the non–supersymmetric version of the model, is known to be the case when one branching ratio is much smaller than the other, in such a way that one $K_i = B_i K << 1$ even if $K >> 1$. We have referred to this kind of decay channel as to a slow one, opposed to the fast ones having $K_i >> 1$. In this case it is sufficient that just one slow channel produces a sizeable asymmetry, since a corresponding asymmetry is also developed in the triplet density, which is eventually converted into an asymmetry of the fast channel when the triplets decay. In the supersymmetric version of the Model, this mechanism is still at work. However, mainly because of the interplay of three decay channels instead of two, a richer phenomenology arises:
• the Yukawa coupling $\lambda_i$ of the additional Higgs doublet can be made arbitrarily small at high triplet masses, allowing a good efficiency also for $M \gtrsim 10^{12}$ GeV. In presence of only one Higgs doublet this is not possible due to the perturbativity bound on $\lambda_L$ implied by Eq. (2).

A hierarchy among the CP–violation parameters $\epsilon_i$’s is allowed. Defining them in such a way that $\max(|\epsilon_i|) = 1$, and using the notation $\epsilon_{\text{slow}}$ and $\epsilon_{\text{fast}}$ for the CP–violating parameters in a slow ($K_i << 1$) and fast ($K_i >> 1$) channel, respectively, this enriches the phenomenology, because different combinations are possible:

- $\epsilon_{\text{slow}} = 1$: The efficiency of leptogenesis reaches its maximal value. Inverse decays in the slow channel freeze out early, and annihilations turn out to dominate over inverse decays in the determination of the triplet density up to quite high values of the triplet mass $M$. In these cases the final asymmetry is a growing function of the $K$ parameter. Moreover, the final asymmetry is insensitive to the actual value of $\epsilon_{\text{fast}}$. An apparently surprising example of this situation is when $\epsilon_L = \epsilon_{\text{fast}} = 0$, since in this case even a vanishing $\epsilon_L$ can lead to efficient leptogenesis. An exception to this case is given by the particular situation with $\epsilon_{\text{slow}} = 1$ in two channels, namely when $\epsilon_L << 1$ and the decay channels to $H_1$ and to $H_2$ are both slow with $\epsilon_1 = -\epsilon_2 = 1$. In fact, if $B_1$ and $B_2$ are comparable, a cancellation takes place between the asymmetries in the two channels, leading to a vanishing lepton asymmetry, $Y_L \approx -Y_{H_1} - Y_{H_2} \approx 0$.

- $\epsilon_{\text{slow}} < 1$ and one slow channel: The final lepton asymmetry is suppressed, as it would be naively expected, since $\epsilon_i$ is small in the only available slow channel which is supposed to drive leptogenesis through out-of-equilibrium decays. In this case there are two fast channels with $\epsilon_{\text{fast}} = \pm 1$, and in the channel where $\epsilon_{\text{fast}}$ has the same sign as $\epsilon_{\text{slow}}$ this may lead to a cancellation in the Boltzmann equations, implying a vanishing final asymmetry. Moreover, inverse decays freeze out late in this case ($z_f \sim \ln K >> 1$), and decay is typically dominant over annihilation in the determination of the triplet density, except for very light values of $M$. As a consequence of this the efficiency scales as $1/(z_f K)$ whenever $K >> 1$.

- $\epsilon_{\text{slow}} < 1$ and two slow channels: Since only one $\epsilon_i$ can be small, the other slow channel with unsuppressed $\epsilon_i$ may drive leptogenesis with a good efficiency. In this case, in practice the decay channel with $\epsilon_{\text{slow}} < 1$ drops out from the Boltzmann equation, and a system with just two decay channels is recovered. However, if $\epsilon_{\text{slow}} = \epsilon_{L,2}$, the phenomenology is different compared to the non–supersymmetric case, because the $K$ parameter is much bigger, reducing the efficiency at high masses and improving it at lower ones. Moreover, the unitarity limit is more constraining at high $M$ compared to the non-supersymmetric case.

In conclusion, the present analysis suggests that in the supersymmetric Triplet Seesaw Model successful leptogenesis can be attained in a wide range of scenarios, some of which appear to be non trivial or even counter–intuitive, provided that an asymmetry in the decaying triplets can develop at early times and be eventually converted into a lepton asymmetry, acting in practice as a lepton–number reservoir.

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