Interacting Holographic Dark Energy in Bianchi Type-V Universe with Variable Deceleration Parameter

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Abstract
The present study deals with spatially homogeneous and anisotropic Bianchi Type-V universe filled with interacting dark matter and holographic dark energy. The exact solutions of Einstein’s equations are obtained by using the variable deceleration parameter in the form \( a(t) = \left( \sinh(at) \right)^\frac{1}{2} \) (Chawla et al. [1]). The physical properties of the model are obtained and discussed in details.

Subject Areas
Modern Physics, Theoretical Physics

Keywords
Bianchi Type-V Universe, Holographic Dark Energy, Interacting Dark Fluids, Variable Deceleration Parameter

1. Introduction
The recent remarkable cosmological observations from high red shift Ia supernovae (SNIa) (Perlmutter et al. [2] [3], Riess et al. [4] [5], Astier et al. [6], Spergel et al. [7], Davis et al. [8]) indicate that our universe is accelerating and confirmed later by cross-checks from cosmic microwave background radiation (Bennett et al. [9], Spergel et al. [10]) and large scale structure (Verde et al. [11], Hawkins et al. [12], Abazajian et al. [13] [14] [15], Tegmark et al. [16]) suggests that the universe is spatially flat and dominated by exotic component with large
negative pressure. This component is usually referred to as dark energy (Weinberg et al. [17], Carroll et al. [18], Peebles et al. [19], Padmanabhan et al. [20]). Astronomical observations indicate that the universe consists of approximately 2/3 dark energy and 1/3 dark matter. The nature of dark energy and dark matter is unknown and many radically different models have been proposed. In order to explain anomalous cosmological observations in cosmic microwave background (CMB) at largest angles, Koivisto et al. [21] have suggested cosmological model with anisotropic and viscous dark energy.

Among all dark energy models, a holographic dark energy (HDE) models have received the remarkable attention (Cohen et al. [22], Horava et al. [23], Thomas et al. [24], Li et al. [25]). According to holographic principle, the number of degrees of freedom in a bounded system should be finite and related to the area of its boundary (Hooft et al. [26]) and with the help of this principle, a field theoretical relation between a short distance (ultraviolet) cut off and a long distance (infrared) cut off was established (Cohen et al. [22]) which ensures that the energy in a box of size $L$ which has a cosmological length scale, does not exceed the energy of black hole of the same size. Different dark energy models are due to different types of these cut off.

Bianchi models have been studied by several authors to achieve a better understanding of the observed small amount of anisotropy in the universe. The simple Bianchi family containing flat FRW universe as a special case is the type-I space-times. The Bianchi type-V universe is a generalization of the open universe in FRW cosmology. Hence, its study as dark energy models with non-zero curvature (Coles et al. [27]) in higher dimension is important. Holographic dark energy models have been tested and constrained by various astronomical observations (Zhang et al. [28], Enqvist et al. [29], Shen et al. [30], Chang et al. [31]). The special class models are the models in which holographic dark energy is allowed to interact with dark matter (Carvalho et al. [32], Huang et al. [33], Gong et al. [34] [35], Pavon et al. [36], Wang et al. [37], Perivolaropoulos et al. [38], Nojiri et al. [39], Guberina et al. [40] [41], Guo et al. [42] [43] [44], Hu et al. [45], Li et al. [46], Setare et al. [47] [48], Sadjadi et al. [49], Banerjee et al. [50], Zimdahl et al. [51]). Sarkar et al. [52] [53] [54] have studied non interacting holographic dark energy with linearly varying deceleration parameter in Bianchi type-I and Bianchi type-V universe and also interacting holographic dark energy in Bianchi type-II.

Spatially homogeneous and anisotropic cosmological models play a significant role in the description of large behavior of the universe, and many authors have been widely studied such models in the search of a relativistic picture of the early universe. Anisotropic Bianchi type-I, Bianchi type-II and Bianchi type-V dark energy models have been extensively studied by (Adhav K. S. [55] [56], Pradhan A. et al. [57]). Kumar and Yadav [58] have constructed some Bianchi type-V cosmological models of accelerating universe with dark energy in general relativity by assuming constant deceleration parameter in order to solve Einstein’s field equations. The role of dark energy with variable equation of state parameter
is studied in details within the evolution of Bianchi type-V universe and con-
jointly discovered that dark energy dominates the universe at the present epoch.
Pradhan and Amirhashchi [59] have constructed an accelerating dark energy
model and explored some new exact solutions of Einstein’s field equations in a
spatially and anisotropic Bianchi type-V space-time with minimally interaction
of perfect fluid and dark energy components. Adhav et al. [60] explored aniso-
tropic and homogeneous Bianchi type-I universe field with interacting dark
matter and holographic dark energy. Som and Sil [61] discussed the general
approach of interacting holographic dark energy model.
Motivated by these investigations, we have constructed spatially homogeneous
and anisotropic Bianchi type-V universe field with interacting dark matter and
holographic dark energy. In this paper, we obtained the exact solutions of Ein-
stein’s field equations by using variable deceleration parameter in the form
\[ a(t) = \left( \sinh(at) \right)^{\frac{1}{\beta}}. \]

2. Metric and Field Equations

The Bianchi Type-V metric can be written as
\[ dx^2 = dt^2 - a_1^2(t)dx^2 - a_2^2(t)e^{-2\beta t}dy^2 - a_3^2(t)e^{-2\beta t}dz^2, \]  
where \( a_1(t), \ a_2(t), \ a_3(t) \) are cosmic scale factors and \( \beta \neq 0 \) is an ar-
bbitrary constant.

The Einstein’s field equations in natural limit \((8\pi G = 1 \text{ and } c = 1)\) are,
\[ R_{ij} - \frac{1}{2} g_{ij} R = - \left( {^m T}_{ij} + {^\Lambda T}_{ij} \right), \]  
where \( {^m T}_{ij} = \rho_m u_i u_j \) and \( {^\Lambda T}_{ij} = \left( \rho_\Lambda + p_\Lambda \right) u_i u_j + g_{ij} \rho_\Lambda \)  
are energy momentum tensor for dark matter (pressureless, i.e. \( \omega_m = 0 \)) and
holographic dark energy respectively. Here the quantity \( \rho_m \) is the energy den-
sity of dark matter and \( \rho_\Lambda, \ p_\Lambda \) are energy density and pressure of holograp-
hic dark energy respectively.

In co-moving system, the Einstein field Equations (2.2) for the metric (2.1),
using Equations (2.3) can be written as
\[ \frac{\dot{a}_1 a_2}{a_1 a_2} + \frac{\dot{a}_2 a_1}{a_2 a_1} = \frac{3\beta^2}{a_1^3}, \]  
where \( {^m T}_{ij} = \rho_m u_i u_j \) and \( {^\Lambda T}_{ij} = \left( \rho_\Lambda + p_\Lambda \right) u_i u_j + g_{ij} \rho_\Lambda \)  
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hic dark energy respectively.
\[
\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} = 2 \frac{\dot{a}_1}{a_1}, \tag{2.8}
\]

where an overhead dot (\(\dot{}\)) represents derivative with respect to time \(t\).

On integrating the Equation (2.8), we obtain
\[
\dot{a}_i^2 = \lambda a_i a_j \tag{2.9}
\]

where \(\lambda\) is an integration constant.

On taking \(\lambda = 1\), without loss of generality, the volume scale factor \(V\) and average scale factor \(a\) is given by
\[
V = a^3 = a_1 a_2 a_3 \tag{2.10}
\]

Subtracting Equation (2.5) from Equation (2.6), Equation (2.6) from Equation (2.7), Equation (2.5) from Equation (2.7) and using Equation (2.10), we get
\[
\begin{align*}
\frac{d}{dt} \left( \frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} \right) + \left( \frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3} \right) \frac{\dot{V}}{V} &= 0 \tag{2.11a} \\
\frac{d}{dt} \left( \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} \right) + \left( \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_1}{a_1} \right) \frac{\dot{V}}{V} &= 0 \tag{2.11b} \\
\frac{d}{dt} \left( \frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3} \right) + \left( \frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} \right) \frac{\dot{V}}{V} &= 0 \tag{2.11c}
\end{align*}
\]

On integrating Equations (2.11a)-(2.11c) and using Equations (2.9) and (2.10), the scale factors \(a_1(t)\), \(a_2(t)\) and \(a_3(t)\) can be written as,
\[
\begin{align*}
a_1(t) &= V^{\frac{1}{3}} \tag{2.12a} \\
a_2(t) &= D V^{\frac{1}{3}} \exp \left( X \int \frac{dt}{V} \right), \tag{2.12b} \\
a_3(t) &= D^{-1} V^{\frac{1}{3}} \exp \left( -X \int \frac{dt}{V} \right) \tag{2.12c}
\end{align*}
\]

where \(X\) and \(D\) are constants of integration.

The holographic dark energy density is given by,
\[
\rho_\Lambda = 3 \left( \alpha_1 H^2 + \beta_1 \dot{H} \right) \tag{2.13}
\]

i.e. \(\rho_\Lambda = 3 \left( \alpha_1 H^2 + \beta_1 \dot{H} \right) \) with \(M_p^2 = 8\pi G = 1\) (Granda et al. [62]), where \(\alpha_1\) and \(\beta_1\) are constants.

For the universe, where dark energy and dark matter are interacting with each other, the total energy density \(\rho = (\rho_m + \rho_\Lambda)\) satisfies the equation of continuity as,
\[
\dot{\rho}_m + \dot{\rho}_\Lambda + 3H (\rho_m + \rho_\Lambda + p_\Lambda) = 0 \tag{2.14}
\]

It is assumed that the dark matter component is interacting with the dark energy through an interacting term \(Q\), the continuity equation of matter and dark energy can be obtained as,
\[
\dot{\rho}_m + \left( \frac{\dot{V}}{V} \right) \rho_m = Q \tag{2.15}
\]
where $\omega_{\lambda} = \frac{P_{\lambda}}{\rho_{\lambda}}$ is the equation of state parameter for the holographic dark energy and $Q > 0$ measures the strength of interaction. A vanishing $Q$ implies that the dark matter and dark energy are separately conserved. In view of continuity equations, the interaction between dark energy and dark matter must be a function of the energy density multiplied by a quantity, with units of inverse of time, which can be chosen as the Hubble parameter $H$. It’s a freedom to choose the form of energy density which can be any combination of dark energy and dark matter. Thus interaction between dark energy and dark matter could be expressed phenomenologically in the form as (Guo et al. [43] [44], Amendola et al. [63]),

$$Q = 3b^2 H \rho_m = b^2 \frac{\dot{V}}{V} \rho_m$$

(2.17)

where $b^2$ is the coupling constant.

Cai and Wang [64] have taken the same relation for interacting dark matter and phantom dark energy in order to avoid the coincidence problem.

Using Equations (2.15) and (2.17), we get the energy density of dark matter as,

$$\rho_m = \rho_0 b^2 H^{(b^2 - 1)}$$

(2.18)

where $\rho_0 > 0$ is a real constant of integration.

Using Equations (2.17) and (2.18), we get the interacting term $Q$ as,

$$Q = 3 \rho_0 b^2 H V^{(b^2 - 1)}$$

(2.19)

3. Cosmological Solution for Variable Deceleration Parameter

We consider the deceleration parameter to be a variable

$$q = -\frac{\ddot{a}}{a^2} = -\frac{\ddot{a}}{a H^2} = b(t)$$

variable. (3.1)

and following Pradhan et al. [65] and Chawla et al. [1], we assume the law of variation of scale factor as increasing function of time

$$a(t) = \left[\sinh(\alpha t)\right]^{1/2}$$

(3.2)

Using (3.2) in Equations (2.12a)-(2.12c), we obtain the expressions for scale factors as,

$$a_1(t) = V^{1/3} = a = \left(\sinh(\alpha t)\right)^{1/n}$$

(3.3)

$$a_2(t) = D(\sinh(\alpha t))^{1/n} \exp\left\{X \int \frac{dt}{(\sinh(\alpha t))^{1/n}}\right\}$$

(3.4)
\[ a_s(t) = D^{-1} \left( \sinh(\alpha t) \right)^{1/n} \exp \left( -X \int \frac{dx}{(\sinh(\alpha t))^{1/n}} \right), \]  

(3.5)

where \(X\) and \(D\) are constants of integration.

Using Equations (2.10) and (3.2) in Equations (2.17) and (2.18), we obtained,

\[ Q = \frac{3}{n} \rho_0 b^2 \alpha \coth(\alpha t) \left( \sinh(\alpha t) \right)^{\frac{3}{n}(\nu - 1)}, \]  

(3.6)

\[ \rho_m = \rho_0 \left( \sinh(\alpha t) \right)^{\frac{3}{n}(\nu - 1)}. \]  

(3.7)

Using Equations (3.2)-(3.5) and (3.7) in Equation (2.4) we obtained the density of holographic dark energy as,

\[ \rho_\Lambda = \frac{3\alpha^2}{n^2} \coth^2(\alpha t) - \left( X^2 + \beta^2 \right) \left[ \frac{1}{\sinh(\alpha t)^{1/n}} \right]^2 - \rho_0 \left( \sinh(\alpha t) \right)^{\frac{3}{n}(\nu - 1)} \]  

(3.8)

Using Equation (3.3)-(3.5) in Equation (2.7), we obtained the pressure as,

\[ p_\Lambda = \frac{\alpha^2}{n^2} \left[ (2n - 3) \coth^2(\alpha t) - 2n \right] - \frac{2\alpha X \coth(\alpha t)}{n \sinh(\alpha t)^{1/n}} + \frac{\beta^2 - X^2}{\sinh(\alpha t)^{2/n}}. \]  

(3.9)

The EOS parameter \( \omega_\Lambda \) of holographic dark energy is given by,

\[ \omega_\Lambda = \frac{\alpha^2 \left[ (2n - 3) \coth^2(\alpha t) - 2n \right] \left( \sinh(\alpha t) \right)^{\frac{2}{n}} - 2n \alpha X \coth(\alpha t) \left( \sinh(\alpha t) \right)^{\frac{1}{n}} + n^2 \left( \beta^2 - X^2 \right)}{3\alpha^2 \coth^2(\alpha t) \left( \sinh(\alpha t) \right)^{\frac{2}{n}} - n^2 \rho_0 \left( \sinh(\alpha t) \right)^{\frac{3}{n}(\nu - 1)} - n^2 \left( \beta^2 + X^2 \right)} \]  

(3.10)

The physical parameters such as spatial volume \( V \), Hubble parameter \( H \), expansion scalar \( \theta \) and the time varying deceleration parameter \( q \) are obtained as,

\[ V = a^3 = \left( \sinh(\alpha t) \right)^{3/n} \]  

(3.11)

\[ H = \frac{\alpha}{n} \coth(\alpha t) \]  

(3.12)

\[ \theta = 3H = \frac{3\alpha}{n} \coth(\alpha t) \]  

(3.13)

\[ q = n \left( 1 - \tanh^2(\alpha t) \right) - 1. \]  

(3.14)

The shear scalar \( \sigma \) and mean isotropy parameter \( \Delta \) are given by,

\[ \sigma^2 = \frac{1}{6} \theta^2 = \frac{3\alpha^2}{2n^2} \coth^2(\alpha t), \]  

(3.15)

\[ \Delta = \frac{1}{3} \sum_{i=1}^{3} \left( \frac{H_i - H}{H} \right)^2 = \frac{2n^2 X^2 \left( \sinh(\alpha t) \right)^{2/n}}{3\alpha^2 \coth^2(\alpha t)}. \]  

(3.16)

### 4. Conclusions

In this paper, we have presented spatially homogeneous and anisotropic Bianchi Type-V universe field with interacting dark matter and holographic dark energy.
With the consideration of variable deceleration parameter, we obtained the solutions of Einstein’s field equations.

It is found that the universe approaches to isotropy for large cosmic time as shown by different observational data and dark energy is responsible for expansion of universe. The concluding remarks of the model are as follows.

1) The sign of $q$ represents that the universe is decelerating or accelerating i.e. a positive sign of $q$ represents accelerating universe and negative sign of $q$ represents decelerating universe.

In our model $q > 0$ for $t \to 0$ and $q \leq -1$ for $t \to \infty$ i.e. the model represents the decelerating to accelerating phase and the values of deceleration parameter lie in the phase $-1 \geq q > 0$.

2) From the Equation (3.11), we can say that the spatial volume $V$ is finite at $t = 0$ and expands as $t$ increases and becomes infinite for $t = \infty$.

3) From Equation (3.16), we can conclude that for the large cosmic time (i.e. $t \to \infty$), the anisotropy parameter $\Delta \to 0$. Therefore, for the large cosmic time, the anisotropy of the universe damped out and the universe approaches to an isotropy universe.

4) From the Equation (3.12), it is observed that the directional Hubble parameter diverges for $t = 0$ and converges for $t = \infty$.

5) We observe that $p_{\Lambda}$ (i.e. pressure of dark energy) tends to a negative value for large cosmic time which shows that the universe is accelerating (SNeIa).

6) From the Equation (3.10), the EOS parameter $\omega_{\Lambda} < -1$ for large cosmic time. In this case, the holographic dark energy looks like phantom energy, (Abazajian et al. [15], Ade et al. [66], Riess et al. [4] [67], Zimdahl [51]). Our result is consistent with SNeIa.

7) For $\beta = 0$ in Equation (2.1), the investigated model approaches to Mete, et al. [68].

**Conflicts of Interest**

The authors declare no conflicts of interest regarding the publication of this paper.

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