Erratum: Multivariate Stein factors for a class of strongly log-concave distributions*

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Abstract
The strong continuity argument in [6] did not identify an appropriate Banach space. We do so here. A corrected version has been uploaded to arxiv.org/abs/1512.07392.

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Lem. 3.1 of [6] should read as follows.

Lemma 3.1 (Overdamped Langevin properties). If \( \log p \in C^2(\mathbb{R}^d) \) is strongly concave, then the overdamped Langevin diffusion \((Z_{t,x})_{t \geq 0}\) with infinitesimal generator (1.1) and \( Z_{0,x} = x \) is well-defined for all times \( t \in [0, \infty) \), has stationary distribution \( P \), and satisfies strong continuity on \( L = \{ f \in C^0(\mathbb{R}^d) : \frac{|f(x)|}{1 + \|x\|^2} \to 0 \text{ as } \|x\|^2 \to \infty \} \) with norm \( \|f\|_L \triangleq \sup_{x \in \mathbb{R}^d} \frac{|f(x)|}{1 + \|x\|^2} \), that is, \( \|E[f(Z_t, \cdot)] - f\|_L \to 0 \text{ as } t \to 0^+ \) for all \( f \in L \).

Proof. Consider the Lyapunov function \( V(x) = \|x\|^2 + 1 \). The strong log-concavity of \( p \), the Cauchy-Schwarz inequality, and the arithmetic-geometric mean inequality imply that

\[
(AV)(x) = \langle x, \nabla \log p(x) \rangle + d = \langle x, \nabla \log p(x) - \nabla \log p(0) \rangle + \langle x, \nabla \log p(0) \rangle + d \\
\leq -k\|x\|^2 + \|x\|\|\nabla \log p(0)\| + d \leq \left( \frac{1}{2} - k \right)\|x\|^2 + \frac{1}{2}\|\nabla \log p(0)\|^2 + d \leq k'V(x)
\]

for some constants \( k, k' \in \mathbb{R} \). Since \( \log p \) is locally Lipschitz, [5, Thm. 3.5] implies that the diffusion \((Z_{t,x})_{t \geq 0}\) is well-defined, and [7, Thm. 2.1] guarantees that \( P \) is a stationary distribution. The argument of [4, Prop. 15] with [5, Thm. 3.5] substituted for [5, Thm. 3.4] and [3, Sec. 5, Cor. 1.2] now yields strong continuity. \( \square \)

In addition the final component of the proof of Thm. 1.1 of [6] should read as follows.

Solving the Stein equation Finally, we show that \( u_h \) solves the Stein equation (1.2). Introduce the notation \((P_t h)(x) \triangleq E[h(Z_{t,x})] \). Since \((P_t)_{t \geq 0}\) is strongly continuous on the Banach space \( L \) of Lemma 3.1 and \( h \in L \), the generator \( A \), defined in (1.1), satisfies

\[
h - P_t h = A \int_0^t E_P[h(Z)] - P_s h \, ds \quad \text{for all } \quad t \geq 0
\]
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by [2, Prop. 1.5]. The left-hand side limits in $L$ to $h - \mathbb{E}_P[h(Z)]$ as $t \to \infty$, as

$$
|h(x) - \mathbb{E}_P[h(Z)] - (h(x) - (P_t h)(x))| = \left| \int_{\mathbb{R}^d} \mathbb{E}[h(Z_{t,y})] - \mathbb{E}[h(Z_{t,x})] \ p(y)dy \right|
$$

$$
\leq M_1(h) \int_{\mathbb{R}^d} \mathbb{E}[\|Z_{t,y} - Z_{t,x}\|_2] \ p(y)dy \leq M_1(h) \mathbb{E}_P[\|Z - x\|_2] e^{-kt/2}
$$

for each $x \in \mathbb{R}^d$ and $t \geq 0$. Here we have used the stationarity of $P$, the Lipschitz relation (3.1), the first-order coupling inequality (3.7) of Lemma 3.3, and the integrability of $Z$ [1, Lem. 1] in turn. Meanwhile, the right-hand side limits to $A u_h$, since $A$ is closed [2, Cor. 1.6]. Therefore, $u_h$ solves the Stein equation (1.2).

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