Recent studies about CP violation in the Minimal Supersymmetric Standard Model (MSSM) with complex parameters are reviewed. In order to unambiguously identify the CP-violating phenomena it is necessary to study CP-odd or T-odd observables. In chargino and neutralino production and decay at the International Linear Collider (ILC) triple product asymmetries and asymmetries defined via transverse beam polarization have been analyzed. It has been found that these asymmetries can be measured at the ILC in a large region of the MSSM parameter space and are thus an important tool to establish CP violation in supersymmetry.

1 Introduction

In the Lagrangian of the Minimal Supersymmetric Standard Model (MSSM) many parameters can be complex which can give rise to new CP-violating phenomena [2] and may help to explain the baryon-antibaryon asymmetry of the universe [3]. After the elimination of unphysical phases two complex parameters remain in the neutralino and chargino sector, the U(1) gaugino mass parameter \( M_1 \) and the higgsino mass parameter \( \mu \), whereas the SU(2) gaugino mass parameter \( M_2 \) and the ratio \( \tan \beta \) of the Higgs vacuum expectation values can be chosen real and positive. In addition the SU(3) gaugino (gluino) mass parameter \( M_3 \) and the trilinear scalar couplings \( A_f \) in the sfermion sector can be complex.

The new CP-violating phases are constrained by the experimental bounds on electric dipole moments (EDMs) of electron, neutron and Hg atom. However, these constraints are highly model-dependent. In constrained MSSM scenarios only small values of the phases are allowed, especially the phase of \( \mu \) is strongly limited. In more general supersymmetric (SUSY) models larger phases may be possible due to cancellations between different SUSY contributions to the EDMs or in SUSY models with heavy sfermions in the first two generations [4]. For instance, it has been pointed out recently that for large \( A_f \), phases \( \phi_\mu \sim O(1) \) can be compatible with the EDM constraints [5]. Furthermore, the restrictions on the phases may also disappear if lepton flavor violating terms in the MSSM Lagrangian are included [6, 7]. In conclusion, large phases of SUSY parameters cannot be ruled out by present EDM experiments.

The precise determination of the underlying SUSY parameters including the phases is an important task of the International Linear Collider (ILC) [8]. The parameters \( M_1, M_2, \mu \) and \( \tan \beta \) of the neutralino and chargino sector are expected to be determined with very high precision which can be further enhanced by combining LHC and ILC analyses [9]. The impact of the SUSY CP phases on the MSSM Higgs sector is summarized in [10]. While CP-even observables like production cross sections and decay branching ratios may strongly depend on the new phases, CP-odd observables are necessary to unambiguously determine the phases and establish CP violation [11]. Concerning CP-even observables especially the decays of SUSY particles and Higgs bosons are a sensitive probe of the SUSY phases [12]. CP-odd observables can be constructed in form of rate asymmetries or with the help of triple products, transverse beam polarization or the polarization of final state
particles, for recent studies see e.g. [13]. In this contribution studies about CP-odd triple product asymmetries and asymmetries defined via transverse beam polarization in chargino and neutralino production and decay at the ILC are reviewed, focusing especially on their measurability.

2 Triple product asymmetries

T-odd triple product correlations between momenta and spins of the involved particles allow the definition of CP-odd asymmetries already at tree level [14]. For chargino and neutralino production and subsequent two-body decays CP-odd and T-odd asymmetries based on triple products and their measurability have been thoroughly studied in [15]. Decays involving W and Z bosons and those into sfermions and fermions have been analyzed and it has been found that especially in the latter case large asymmetries up to 30% are possible.

Here, I will focus on two studies about chargino and neutralino production and subsequent three-body decays [16] [17], $e^+e^- \to \tilde{\chi}^\pm_i + \tilde{\chi}^0_j + f f'$. Including full spin correlations between production and decay products of the form $i \epsilon_{\mu\nu\rho\sigma} p_i^\mu p_j^\nu p_k^\rho p_l^\sigma$ (where the $p_i^\mu$ denote the momenta of the involved particles) appear in the amplitude squared in terms, which depend on the spin of the decaying chargino or neutralino [18]. Together with the complex parameters entering the couplings these terms can give real contributions to suitable observables at tree-level. Triple products $T_1 = \vec{p}_{e^-} \cdot (\vec{p}_{f} \times \vec{p}_{f'}(\bar{c}))$ of the initial electron momentum $\vec{p}_{e^-}$ and the two final fermion momenta $\vec{p}_{f}$ and $\vec{p}_{f'}(\bar{c})$ or $T_2 = \vec{p}_{e^-} \cdot (\vec{p}_{\tilde{\chi}^i_j} \times \vec{p}_{\tilde{\chi}^j})$ of the initial electron momentum $\vec{p}_{e^-}$, the momentum of the decaying neutralino or chargino $\vec{p}_{\tilde{\chi}^i_j}$ and one final fermion momentum $\vec{p}_{\tilde{\chi}^j}$ allow the definition of T-odd asymmetries

$$A_T = \frac{\sigma(T_1 > 0) - \sigma(T_1 < 0)}{\sigma(T_1 > 0) + \sigma(T_1 < 0)} = \frac{\int \text{sign}(T_1)|T|^2dL_{\text{Lips}}}{\int |T|^2dL_{\text{Lips}}},$$

where $\int |T|^2dL_{\text{Lips}}$ is proportional to the cross section $\sigma$ of the combined production and decay process. $A_T$ is odd under naive time-reversal operation and hence CP-odd, if higher order final-state interactions and finite-widths effects can be neglected. In the case of chargino production and decay where the asymmetry $A_T$ for the charge-conjugated process is accessible a genuine CP asymmetry

$$A_{CP} = \frac{A_T - \bar{A}_T}{2}$$

can be defined.

The statistical significance $S$ to which above asymmetries can be determined to be non-zero can be estimated in the following way: The absolute error of $A_T$ is given by $\Delta A_T = S \sqrt{A_T^2 + \frac{1}{\sigma L_{\text{int}}}}$, where $S$ denotes the number of standard deviations, $\sigma$ the cross section of the respective process and $L_{\text{int}}$ the integrated luminosity [19]. For $A_T \lesssim 10\%$ it is $\Delta A_T = S \sqrt{A_T^2 + \frac{1}{\sigma L_{\text{int}}}}$ in good approximation and requiring $A_T > \Delta A_T$ for $A_T$ to be measurable one obtains

$$S = \sqrt{A_T^2 \sigma L_{\text{int}}}$$

respectively, assuming that the statistical errors of $A_T$ and $\bar{A}_T$ are independent of each other. $S$ can be used as an estimation of the measurability of the asymmetries. However, in order to determine the final accuracy in the experiment also initial state radiation, beamstrahlung, backgrounds and detector effects have to be included. For neutralino production and decay
this has been analyzed in [20] and it has been found that asymmetries \( \mathcal{O}(10\%) \) are detectable after few years of running of the ILC.

In Figure 1, \( A_{CP} \) and \( S \) are shown for chargino production \( e^+ e^- \rightarrow \tilde{\chi}_j^+ \tilde{\chi}_1^0 \), \( j = 1, 2 \) and subsequent decay \( \tilde{\chi}_1^0 \rightarrow \tilde{\chi}_1^0 \tilde{s} \tilde{c} \) using the triple product \( T_1 = \vec{p}_{e^-} \cdot (\vec{p}_\tilde{c} \times \vec{p}_\tilde{s}) \) [10]. Note that the statistical significance \( S \) is larger than 5 in large regions of the parameter space. However, in order to measure \( A_{CP} \) it is necessary to discriminate the two outgoing quark jets, i.e. to tag the \( c \) jet. The respective \( c \) tagging efficiency will decrease the final significance by about a factor 0.5 but nevertheless large regions of the parameter space can be covered. If instead the production plane is reconstructed by analyzing the decays of the \( \tilde{\chi}_2^- \) in \( e^+ e^- \rightarrow \tilde{\chi}_2^- \tilde{\chi}_1^+ \) also the leptonic decays \( \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 \ell^+ \nu \) can be used to define \( A_{CP} \) using \( T_2 = \vec{p}_{\ell^+} \cdot (\vec{p}_{\tilde{\chi}_1^0} \times \vec{p}_{e^+}) \).
In this case, however, the cross sections are rather small, hence $S$ is always smaller than about 5 despite potentially large asymmetries $A_{\text{CP}}$. 

For neutralino production $e^+ e^- \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_2^0$, $i = 1, \ldots, 4$, with subsequent leptonic three-body decay $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_j^0 \ell^+ \ell^-$, $\ell = e, \mu$, the triple product $T_1 = \vec{p}_{e^-} \cdot (\vec{p}_{e^+} \times \vec{p}_{\ell^-})$ can be used to define the T-odd asymmetry $A_T$, which is directly measurable without reconstruction of the momentum of the decaying neutralino or further final-state analyses. It has been found in [22] that $A_T \gtrsim 10\%$ in large regions of the parameter space for $e^+ e^- \rightarrow \tilde{\chi}_j^0 \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_j^0 + \tilde{\chi}_3^0 \ell^+ \ell^-$, $j = 1, 3$, yielding significances $S$ larger than 5.

3 Asymmetries using transverse beam polarization

The use of transverse beam polarization offers further possibilities to define CP-sensitive observables. As all terms in the squared amplitude $|T|^2$ of respective processes which are sensitive to transverse beam polarization depend on the product of the degrees of transverse beam polarization of both beams the CP-sensitive observables are only accessible if both beams of the ILC can be polarized [21]. The respective terms in $|T|^2$ contain products of the form $i\epsilon_{\mu
u\rho\sigma}t^\mu p^\nu p_\rho^\sigma$, or $i\epsilon_{\mu
u\rho\sigma}t^\mu \ell^\nu p_\rho e^\sigma$, where $t^\mu_{\ell}$ is the 4-vector of the transverse beam polarization of the positron and electron beam, respectively, and the $p^\sigma$ denote the momenta of the involved particles. This in turn allows the definition of CP-odd asymmetries in suitable production and decay processes. In [24] such asymmetries and their measurability have been analyzed for selection production at an $e^- e^-$ collider. In [24] CP-odd asymmetries using transverse beam polarization have been studied for neutralino production and subsequent two-body decays and their measurability has been compared with CP asymmetries accessible with unpolarized or longitudinally polarized beams.

In chargino production $e^+ e^- \rightarrow \tilde{\chi}_j^+ \tilde{\chi}_j^-$ all CP-odd terms in $|T|^2$ vanish because of CPT invariance and the fact that charginos are Dirac particles [26]. Due to the Majorana nature of the neutralinos the respective terms are allowed in neutralino production $e^+ e^- \rightarrow \tilde{\chi}_j^0 \tilde{\chi}_j^0$ and CP-odd asymmetries can be defined by analyzing the azimuthal distributions of the neutralinos [22]:

$$A_{\text{CP}} = \left[ \int_0^{\pi/2} - \int_{\pi/2}^{\pi} \right] A_{\text{CP}}(\theta) \, d\theta,$$

$$A_{\text{CP}}(\theta) = \frac{1}{\sigma} \left[ \int_{\phi+\eta}^{\phi+\eta+\pi/2} - \int_{\phi+\eta+\pi/2}^{\phi+\eta+2\pi} + \int_{\phi+\eta+\pi}^{\phi+\eta+3\pi/2} - \int_{\phi+\eta+3\pi/2}^{\phi+\eta+2\pi} \right] \frac{d^2\sigma}{d\phi d\theta} \, d\phi,$$

where $\phi$ denotes the azimuthal angle of the scattering plane and $\eta$ the orientation of the transverse polarizations. The statistical significance is given by $S = \sqrt{A_{\text{CP}}^2 \sigma L_{\text{int}}}$ or vice versa the necessary integrated luminosity to reach a certain significance by $L_{\text{int}} = S^2/(\sigma A_{\text{CP}}^2)$. 

In Figure 2 $A_{\text{CP}}$ and $L_{\text{int}}$ necessary to reach $S = 5$ are shown for $e^+ e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$, where it can be seen that also the CP-odd asymmetry defined via transverse beam polarization can be measured in large regions of the SUSY parameter space at the ILC [22]. Similarly, the respective asymmetries for $e^+ e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_3^0$ are well measurable in large regions of the parameter space. However, in order to measure $A_{\text{CP}}$ the production plane has to be reconstructed. This is not necessary if the subsequent decays of the neutralinos are included. It has been found in [22] that respective asymmetries including two-body decays of the neutralinos are also measurable in large regions of the SUSY parameter space.

*For a detailed definition see e.g. [22].
Figure 2: Contour lines of (a) the CP-odd asymmetry $A_{\text{CP}}$, Eq. (4), defined with help of transverse beam polarization and (b) the necessary integrated luminosity to reach a significance $S = 5$ for $|M_1/M_2 = 5/3\tan^2\theta_W$, $\phi_{M_1} = 0.5\pi$, $\phi_{\mu} = 0$, $\tan\beta = 5$, $m_{\tilde{\chi}_1^0} = 400$ GeV, $m_{\tilde{\chi}_R} = 150$ GeV, $\sqrt{s} = 500$ GeV and degrees of transverse beam polarizations of (a) $(P_{T,e^-}, P_{T,e^+}) = (100\%, 100\%)$ and (b) $(P_{T,e^-}, P_{T,e^+}) = (80\%, 60\%)$. From [22].

4 Conclusions

Recent studies analyzing CP-odd or T-odd triple product asymmetries or asymmetries defined via transverse beam polarization in chargino and neutralino production and decay have been reviewed. It has been found that these asymmetries are measurable in large regions of the SUSY parameter space and are thus an important tool to search for CP violation in SUSY and to unambiguously determine the SUSY phases.

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