Evaluation of the large deformation grade cloud model of surrounding rock based on combination weighting method

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Abstract. On the basis of the analytic hierarchy process (AHP) and entropy weight method, combined with the statistical results of large deformation under the complex geological conditions of Galigongshan tunnel, the grade cloud model evaluation method of the large deformation of surrounding rock is proposed. Its influencing factors are considered to select the strength stress ratio, maximum principal stress, elastic modulus, and uniaxial compressive strength as evaluation indexes. The weight of each index in the evaluation of large deformation is determined through AHP and the entropy weight method. Meanwhile, the combined weights of each influencing factor index are obtained by the optimal combination weighting method of maximizing the sum of squares of total deviation. Aiming at the uncertainty and randomness of the large deformation evaluation of tunnel surrounding rock, the numerical characteristics of a normal cloud model are used to calculate the determination of the evaluation index, and the evaluation results of the large deformation grade of surrounding rock are obtained. In this study, this method is applied to the large deformation evaluation of the surrounding rock of Galigongshan tunnel engineering. Compared with the results of the fuzzy hierarchical comprehensive evaluation method, the model evaluation method established here is more feasible. It combines subjective and objective weights to obtain a reasonable weight in the practical engineering application of significance, as large deformation provides a reliable reference for the prediction and prevention of disasters.

Keywords: Large deformation of surrounding rock, AHP, Entropy weight method, Optimal combined weighting, Cloud model

1. Introduction
The large deformation of surrounding rock is an unrecoverable and unrestrained deformation under complex geological conditions, such as high in-situ stress [1-3], lithologic conditions, and groundwater, which is a common geological disaster with great harm in underground engineering. This kind of large deformation can cause a series of engineering hazards, but the academic circle has not formed a set of perfect response measures to the large deformation disaster. Most cases are based on the construction experience of relevant surrounding rock large deformation engineering. The classification evaluation of the large deformation of surrounding rock is the basis of the prediction and prevention of large deformation disasters. Combining the whole process of rock stress and deformation curves, Aydan took tangential strain as the evaluation index and divided the extrusion degree of rock mass into the relative deformation [4-6]. On the basis of rock mass constitutive, Hoek connected the strength and stress ratio of rock with the relative deformation of tunnel and established the Hoek soft rock large deformation classification table [7]. On the basis of the engineering example of a large
deformation tunnel in surrounding rock, the classification tunnel is studied by taking stress ratio, initial in-situ stress, and relative deformation as indexes [8]. Following the on-site large deformation monitoring results of the Muzhailing tunnel project on Lanzhou–Chongqing railway, Wang Yonggang classified the large deformation into three grades according to convergence displacement, relative displacement, rock mass compression factor, original in-situ stress, lateral pressure coefficient, deformation modulus, and extrusion deformation characteristics [9]. On the basis of Hoek's classification standard of surrounding rock compression degree, Li Guoliang combined the characteristics of soft rock tunnel and proposed that the large compression deformation can be divided into three grades [10]. Considering the large deformation factor indicators, the present study combines the large deformation prediction criteria of research scholars, according to the principles of representative, independent, and easy access, strength stress ratio, maximum principal stress, elastic modulus, and uniaxial compressive strength as large deformation grading evaluation indexes.

At present, an increasing number of scholars use fuzzy theory [11], artificial neural network [12], extenics theory [13, 14], analytic hierarchy process (AHP) [15], support vector theory [16], and gray theory [17] to establish the influences of the large deformation of surrounding rock and classification methods. These traditional evaluation methods have some deficiencies. AHP is too subjective, and the entropy weight method for sensitivity index difference degree is large. Thus, the proposed combination method to the introduction of the cloud model of large deformation of surrounding rock grade evaluation overcomes the limitations of the traditional evaluation of large deformation of surrounding rock and the entropy weight method. It can also solve the problem of index difference degree of sensitivity. On the basis of the optimal weight method, subjective and objective weights are combined to avoid the problem of unreasonable weight distribution of influencing factors and indicators. The prediction results of this model can objectively reflect the real situation of the large deformation of surrounding rock. A reliable reference for the early warning and prevention of large deformation can also be provided.

2. Establishment of a normal cloud model
Cloud model [18] is a mathematical model that deals with qualitative concepts and quantitative descriptions proposed by Li Deyi et al. Let \( X = \{ x \} \) be the quantitative set, \( A \) be the fuzzy set of \( X \), and the certainty \( \mu(x) \in [0, 1] \) of any element to \( A \) be a random number. According to the concept of cloud model, the large deformation grade index is used to calculate the cloud digital features of a certain grade standard:

\[
\mu(x) = e^{-\frac{(x-c)^2}{\sigma^2}}
\]

\[
E_x = \frac{c_{max} + c_{min}}{2}; E_x = \frac{c_{max} - c_{min}}{6}; H_x = k
\]

where \( c_{max} \) and \( c_{min} \) are the maximum and minimum boundary values of the corresponding grading standards, respectively, and \( k \) is a constant, reflecting the degree of dispersion of cloud droplets that is uniformly set as 0.01 in this paper. For the single boundary variable, the boundary parameters can be determined according to the value range of the variable and then calculated.

3. Determination of the optimal combination weighting of factor index
AHP is a hierarchical weight decision analysis method proposed by American operations researcher Saaty. AHP can be used to solve multi-index decision problems with complex hierarchical structures. This method is flexible and simple but has some subjectivity. According to the scale method, as shown in Table 1, the value of each element in the comparison judgment matrix is determined.
Table 1. Two factor indexes assigned using the proportional scale method

| Scale meaning     | Equally important | A little important | Obviously important | Highly important | Median of the above adjacent comparisons |
|-------------------|-------------------|--------------------|---------------------|-----------------|---------------------------------------|
| 1                 | 3                 | 3                  | 5                   | 7               | 8                                     |
| 2                 | 4, 6, 8           |                    |                     |                 |                                        |

The relative weight of judgment matrix is \( \omega_{ij} \), and the maximum eigenvalue \( \lambda_{max} \) is calculated on the basis of the weight column vector. The consistency of the judgment matrix must carry out consistency text on \( \lambda_{max} \), calculate consistency index, and obtain consistency ratio \( C_R \).

\[
\omega_i = \left( \prod_{j=1}^{n} A_{ij} \right)^{1/n} / \sum_{j=1}^{n} \left( \prod_{i=1}^{n} A_{ij} \right)^{1/n} \quad \lambda_{max} = \frac{\sum_{i=1}^{n} \left( A_{ij} \omega_j / \omega_i \right)}{n} \]

\[
C_R = \frac{\lambda_{max} - n}{n - 1} \quad C_R = \frac{\lambda_{max}}{R_i}.
\]

In the formula, \( R_i \) is the average consistency index, and Table 2 presents the values of \( R_i \) in matrices of different orders. When \( C_R < 0.10 \), it can be determined that the matrix has satisfactory consistency; otherwise, readjusting the relationship among the various factors of the judgment matrix is necessary until the consistency test is met.

Table 2. Average random consistency index \( R_i \) value

| n  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| R  | 0  | 0.5| 0.8| 1.1| 1.2| 1.3| 1.4| 1.4| 1.5| 1.5| 1.5| 1.5| 1.5| 1.5|
| \( R_i \) | 2 | 9 | 2 | 5 | 6 | 9 | 2 | 4 | 6 | 8 | 9 |

The entropy weight method can objectively calculate the weight of each factor index. It can also avoid the situation where the influencing factor index is ignored by taking advantage of its weak sensitivity to the index difference degree. The original data matrix of evaluation objects and evaluation indexes is established. Original data matrix \( X \) is standardized, and the large deformation becomes more intense with a larger factor index. The dimensionless normalization is then performed. Subsequently, the entropy weight \( E_j \) of the evaluation index is calculated for the standardized matrix.

\[
E_j = -\sum_{i=1}^{m} \frac{y_{ij}}{\sum_{i=1}^{m} y_{ij}} \ln \left( 1 - \frac{y_{ij}}{\sum_{i=1}^{m} y_{ij}} \right).
\]

The method of optimal combination weighting is to combine subjective and objective weightings by using the maximum sum of squares of total deviation to obtain the combined weight of the evaluation object. AHP obtains subjective weight vector \( W_1 \), whereas objective weight vector \( W_2 \) is obtained using the entropy weight method. Thus, the combined weight vector is \( W = \theta_1 W_1 + \theta_2 W_2 \) (\( \theta_1, \theta_2 \) is the combination coefficient).

\[
J(W) = \sum_{i=1}^{n} V_i(W) = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{m} \left( y_{ij} - y_{ik} \right) \left( y_{ij} - y_{jk} \right) w_{ij} w_{ik}.
\]

Let matrix \( Y_i \);
Then, objective function \( J(W_c) \) can be expressed as \( J(W_c) = W_c^T Y W_c \) and optimized as follows: \( \max F(\Theta) = \Theta^T W_c^T Y W_c \Theta (\Theta^T \Theta = 1, \Theta \geq 0) \); \( \max F(\Theta) \) is the maximum eigenvalue, and the optimal solution is the unit eigenvector of the maximum eigenvector.

4. Evaluation of the large deformation grade cloud model

Many factors affect large deformation, including not only the physical and mechanical properties of rock but also stress and groundwater. In combination with the large deformation prediction criteria of researchers, strength stress ratio, elastic modulus, maximum principal stress, and uniaxial compressive strength are selected as the evaluation indexes. Large deformation is divided into four grades, and the relationship between the large deformation grade and the four evaluation factor indexes is established. The corresponding indexes and grading relations are shown in Table 3 below.

### Table 3. Relationship between large deformation grade and factor index

| Grade | \( R_c/\sigma_0 \) | I     | II    | III   | IV    |
|-------|-----------------|-------|-------|-------|-------|
| I     | >               | 0.25–0.5 | 0.15–0.25 | <  |
| II    | >               | 1500–2000 | 1000–1500 | <  |
| III   | < 20            | 20–30 | 30–45 | >45   |
| IV    | > 30            | 15–30 | 5–15 | <5    |

In this study, the data related to the typical large deformation of the Gaoligongshan tunnel project are summarized, as presented in Table 4. To use the entropy weight method for evaluating the objective weight of factor index, standardizing the original engineering data is necessary. Among them, the smaller \( R_c/\sigma_0 \), \( E \), and \( R_c \) are, the stronger the grade of large deformation is, and they belong to the larger and better type index. The larger the \( \sigma_{\text{max}} \) is, the weaker the grade of the large deformation is, and it belongs to the smaller and better type index. The data after the standardization are shown in Table 4.

### Table 4. Raw engineering and normalized data

| Serial number | \( R_c/\sigma_0 \) | \( E \) | \( \sigma_{\text{max}} \) | \( R_c \) | \( R_c/\sigma_0 \) | \( E \) | \( \sigma_{\text{max}} \) | \( R_c \) | Grade |
|---------------|-----------------|-------|-----------------|-------|-----------------|-------|-----------------|-------|-------|
| 1             | 0.29            | 1700  | 17              | 5.0   | 0.0             | 0     | 0               | 0     | II    |
| 2             | 0.03            | 1300  | 30              | 1.0   | 0.963           | 0.66  | 1               | 9.0   | IV    |
| 3             | 0.02            | 1100  | 28              | 0.5   | 1               | 1     | 0.84            | 6     | IV    |
| 4             | 0.02            | 1100  | 27              | 0.5   | 1               | 1     | 0.76            | 9     | IV    |
| 5             | 0.02            | 1100  | 23              | 0.5   | 1               | 1     | 0.46            | 2     | IV    |
| 6             | 0.25            | 1700  | 20              | 5.0   | 0.148           | 0     | 0.23            | 1     | II    |
The subjective weight vector of the evaluation index is determined on the basis of AHP. The relative weight of each index is calculated, and the consistency of matrix is checked.

\[
Y_1 = \begin{bmatrix}
1 & 2 & 3 & 5 \\
1/2 & 1 & 2 & 4 \\
1/3 & 1/2 & 1 & 3 \\
1/5 & 1/4 & 1/3 & 1
\end{bmatrix}
\]

\[
\omega_{c_1} = 0.4724; \omega_{c_2} = 0.2854; \omega_{c_3} = 0.1697; \omega_{c_4} = 0.0725
\]

\[
\lambda_{max} = 4.0511; CI = 0.0170; CR = 0.0191
\]

According to \( CR < 0.1 \), the judgment matrix meets the consistency requirements and normalizes, and the subjective weight vector can be obtained as follows:

\[
W_1 = (0.4724, 0.2854, 0.1697, 0.0725)
\]

The objective weight vector of the evaluation index is determined using the entropy weight method. Combined with the normalized large deformation data in Table 4, the normalized matrix is obtained. The entropy value \( E \) of each evaluation index is calculated, and then the weight \( W_2 \) of the evaluation index is calculated according to the entropy value:

\[
E = (0.1164, 0.1608, 0.1254, 0.1548); W_2 = (0.2088, 0.2885, 0.2250, 0.2777)
\]

According to the maximum sum of squares of total deviation, matrix \( Y_1 \) is shown below. The subjective and objective weight vectors are obtained using AHP and the entropy weight method, respectively. Then, \( W = (W_1, W_2) \), and the combined weight \( W_c^{**) \) is obtained.

\[
Y_1 = \begin{bmatrix}
80.676 & 43.614 & 47.088 & 14.555 \\
43.614 & 76.816 & 7.700 & -5.033 \\
47.088 & 7.700 & 87.400 & 7.697 \\
14.555 & -5.033 & 7.697 & 46.857
\end{bmatrix}
\]

\[
W^T Y_1 W = \begin{bmatrix}
48.061 & 36.770 \\
36.765 & 30.471
\end{bmatrix}; W_c^{**} = \begin{bmatrix}
0.2813, 0.2876, 0.2098, 0.2212
\end{bmatrix}^T
\]

In this study, the large deformation data of the Gaoligongshan tunnel are predicted and analyzed. The tunnel is a control project of the Dali–Ruili railway and is approximately 34.5 km. It is characterized by high geothermal, high in-situ stress, and high seismic intensity, including the active neotectonic movement and shallow slope reconstruction. Large deformation occurs due to extremely soft rock strata, expansive soft rock strata or fault zones. By means of field geological investigation, in-situ stress test and inversion, indoor rock physics and mechanics engineering analogy, and other methods, the corresponding values of the original indexes needed for the grade evaluation of the large deformation section of the tunnel are determined, as presented in Table 4. The mathematical characteristics of different grades of large deformation are also calculated, as shown in Table 5. The cloud model distribution is illustrated in Figure 1.
Table 5. Numerical characteristics of each influencing factor index

| Index | $R_c/\sigma_0$ | $E/\text{Mpa}$ | $\sigma_{\text{max}}/\text{Mpa}$ | $R_c/\text{Mpa}$ |
|-------|----------------|----------------|-------------------------------|-----------------|
|       | $E_x$ | $E_n$ | $H_e$ | $E_x$ | $E_n$ | $H_e$ | $E_x$ | $E_n$ | $H_e$ | $E_x$ | $E_n$ | $H_e$ |
| I     | 0.75  | 0.083 | 0.01 | 2500 | 166.7 | 0.01 | 10  | 3.33 | 0.01 | 40  | 3.33 | 0.01 |
| II    | 0.38  | 0.042 | 0.01 | 1750 | 83.33 | 0.01 | 25  | 1.67 | 0.01 | 22.5| 2.5 | 0.01 |
| III   | 0.2   | 0.017 | 0.01 | 1250 | 83.33 | 0.01 | 37.5| 2.5  | 0.01 | 10  | 1.67 | 0.01 |
| IV    | 0.08  | 0.025 | 0.01 | 500  | 166.7 | 0.01 | 50  | 1.67 | 0.01 | 2.5 | 0.83 | 0.01 |

Figure 1. Cloud model distribution of the determination degree of each influencing factor index

According to the comparative analysis of the calculation results in Table 6 and Figure 1, the results of the evaluation model established in this study are basically consistent with the actual situation, indicating that the evaluation method of the large deformation grade cloud model based on combined weight method is feasible. However, the results are different from those of medium and strong large deformation. Our findings are inclined to medium–large deformation, but the possibility of strong large deformation is also high. The results of the fuzzy comprehensive evaluation are inclined to strong large deformation. Thus, this study adopts the combination method to obtain reasonable combination weights and introduces the digital characteristics of the cloud model. The research also claims that large deformation levels of uncertainty have certain advantages. It compares the combination method with the traditional evaluation method that can reflect the actual large deformation. A rapid and effective evaluation of large deformation prediction is conducted in this study, which has an important application in the practical engineering significance.

Table 6. Comparison of prediction grade results of the large deformation of the Gaoligongshan tunnel

| Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------|---|---|---|---|---|---|---|---|---|----|----|----|
| U(I)   | 0.02 | 0 | 0 | 0 | 0 | 0.00 | 0 | 0 | 0 | 0.00 | 0.00 | 0.02 |
| U(II)  | 0.27 | 0.00 | 0.04 | 0.04 | 0.10 | 0.24 | 0.45 | 0.41 | 0.45 | 0.01 | 0.01 | 0 |
| U(III) | 0.00 | 0.24 | 0.05 | 0.24 | 0.00 | 0.28 | 0.23 | 0.28 | 0.24 | 0.24 | 0.05 |
| U(IV)  | 0.00 | 0.09 | 0.26 | 0.26 | 0.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.21 | 0.27 |
| R-1    | II | III | IV | IV | IV | II | II | II | III-I | III-I | IV |
| R-2    | II | IV | IV | IV | IV | II | II | II | IV | IV | IV |

Table note: R-1 represents the evaluation result of this study; R-2 represents the evaluation result of Reference 8.
5. Conclusion
On the basis of AHP and entropy weight method, combined with normal cloud model theory, the evaluation method of large deformation grade is constructed and verified in the Gaoligongshan tunnel project. Conclusions are as follows:
1. The optimal combination weighting method is used to combine subjective and objective weights. The combined weights obtained are more reasonable than those obtained using the traditional evaluation method. The results of fuzzy comprehensive evaluation using only the subjective weight are close to the actual large deformation grade.
2. According to the large deformation data of the Gaoligongshan tunnel project, the cloud model evaluation method based on the combined weighting method is feasible, and the results are only slightly different from those of the actual large deformation, which are closer to the actual engineering results than those obtained using the fuzzy comprehensive evaluation method.
3. The evaluation method of large deformation grade established in this study can predict the field situation quickly and effectively, reflect the actual large deformation grade correctly, and provide a reference for the prediction and prevention of large deformation disasters.

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