THREE-DIMENSIONAL HYDRODYNAMICAL SIMULATIONS OF A PROTON INGESTION EPISODE IN A LOW-METALLICITY ASYMPTOTIC GIANT BRANCH STAR

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ABSTRACT

We use the three-dimensional (3D) stellar structure code djehuty to model the ingestion of protons into the intershell convection zone of a 1 M☉ asymptotic giant branch star of metallicity Z = 10⁻⁴. We have run two simulations: a low-resolution one of around 300,000 zones and a high-resolution one consisting of 2,000,000 zones. Both simulations have been evolved for about 4 hr of stellar time. We observe the existence of fast, downward flowing plumes that are able to transport hydrogen into close proximity to the helium-burning shell before burning takes place. The intershell in the 3D model is richer in protons than the 1D model by several orders of magnitude and so we find no evidence of the convective zone splitting into two, though this could be as a result of insufficient spatial resolution or because the models have not been evolved for long enough. We suggest that the 1D mixing length theory and particularly the use of a diffusion algorithm for mixing do not give an accurate picture of these events. An advective mixing scheme may give a better representation of the transport processes seen in the 3D models.

Key words: convection – hydrodynamics – stars: AGB and post-AGB – stars: carbon – stars: evolution – stars: Population II

Online-only material: animations, color figures

1. INTRODUCTION

The final stage of the life of a low-mass star (one in the mass range 1–6 M☉) is known as the asymptotic giant branch (AGB) phase. During this part of its life, the star consists of an inert core of carbon and oxygen, surrounded by two burning shells with the interior one burning helium and the outer one burning hydrogen. This double shell burning configuration is unstable and there are periodic episodes of runaway helium burning. These are referred to as thermal pulses (see the reviews by Iben & Renzini 1983; Herwig 2005 for more details). During these thermal pulses, helium burning drives a convective region between these helium- and hydrogen-burning shells. The advance of this intershell convection region is checked by the increase in entropy across the hydrogen-burning shell.

As one moves to stars of lower metallicity, the entropy barrier provided by the hydrogen-burning shell is weakened. If the metallicity is low enough, the intershell convection zone can penetrate up into the hydrogen-rich regions (Cassisi et al. 1996; Fujimoto et al. 2000). Protons are then drawn deep into the star where they burn vigorously. This is a proton ingestion episode (PIE). The occurrence of these PIEs in one-dimensional (1D) stellar evolution codes has recently been studied by Campbell & Lattanzio (2008), Lau et al. (2009), Cristallo et al. (2009), Iwamoto (2009), and Suda & Fujimoto (2010). In these codes, if a sufficient quantity of protons are ingested, the energy released when these burn can lead to the intershell convection zone splitting into two parts. The lower part remains driven by helium burning, while the upper part is now sustained by hydrogen burning. While all authors seem to agree that some splitting of the convection zone takes place, there is little agreement on the mass and metallicity regimes where this can occur.

One must take the results of 1D evolution codes with a degree of scepticism: they are dependent on our treatment of convection. Convection, as implemented in the form of mixing length theory (MLT; Böhm-Vitense 1958), has long been the Achilles’ heel of stellar evolution computations and there are several reasons to be wary of it here. First, convection is inherently a 3D process. Are we missing some important property of the way protons are ingested by reducing it to one dimension? Second, mixing is commonly treated as a diffusive process in stellar evolution codes, yet convection is advective in nature. Finally, MLT does not take account of the effect of nuclear burning in downflowing material. Energy released due to nuclear burning will increase the buoyancy of the downward moving element, reducing its speed (see Herwig 2001 for further details). For these reasons, it is fruitful to look at the issue of proton ingestion using hydrodynamical modeling.

Hydrodynamical modeling has advanced to the point where we can study convective processes in some detail. This has been done for a wide variety of situations. Herwig et al. (2006) investigated convection in thermal pulses in two dimensions. Mocák et al. (2009) simulated the core helium flash in low-mass stars in two and three dimensions. Carbon- and oxygen-shell burning in the late stages of the evolution of massive stars have been tackled by Meakin & Arnett (2006, 2007). Meakin (2006) and Arnett & Meakin (2011) have also modeled, in two dimensions, a 23 M☉ star with simultaneously active C-, O-, Ne-, and Si-burning shells. They contrast this 2D model
with 1D stellar evolution calculations, highlighting the fact that convection is advective in nature, rather than diffusive, and that in the case of vigorous convection and burning the diffusion approximation shows large shortcomings.

Hydrogen ingestion at the core helium flash has been modeled using the hydrodynamic code HERAKLES by Mocák et al. (2010). Their simulation was based upon a 1D stellar evolution model for a 0.85 $M_\odot$ star of zero metallicity taken from Campbell & Lattanzio (2008). As an input model, they took the stellar structure at a time when the convective region had already split into two. They then modeled a wedge of this star in both two and three dimensions. They found that convection was not sustained for very long in both the zones. However, there was some uncertainty as to whether this was a numerical artifact brought about by insufficient resolution or by problems with the initial stabilized input model. Further HERAKLES simulations of proton ingestion at the core helium flash (albeit at solar metallicity) by Mocák et al. (2011) suggest that substantial hydrogen ingestion can indeed create two distinct convective regions, with the upper one being driven by hydrogen burning. These authors note that the existing entropy barrier is permeable to chemical transport when multidimensional flows are considered and thus hydrogen can end up in the helium-driven convective region.

Proton ingestion during a very late thermal pulse\(^5\) was modeled using 3D hydrodynamic simulations by Herwig et al. (2011). These simulations dealt only with the gas dynamics and no nuclear burning was included. Their computations were performed on Cartesian grids of 576\(^3\) and 384\(^3\) zones and consisted of three polytropic layers which reflect the stellar structure. An artificial luminosity was added in a shell to represent the driving of the convection zone by nuclear burning. They evolve their simulation for around seven convective turnover times, where the turnover time is estimated to be around 3000 s. Their simulations suggest that large upwelling convective cells dominate the convective flow patterns and that ingestion of hydrogen takes place in downflows that form when these large cells meet each other. It is suggested that Kelvin–Helmholtz instabilities are the likely main mechanism for the entrainment of protons at the convective–radiative boundary.

For this study, we use the DJEHUTY code, which has been developed at Lawrence Livermore National Laboratory and which operates in a parallel environment, to model stars in three dimensions. The major advantage of DJEHUTY is that it allows us to model an entire sphere covering the region of interest (rather than just a wedge of the star) and it also includes a comprehensive suite of nuclear burning reactions. The code is described in extensive detail in Bazán et al. (2003), Dearborn et al. (2005), and Dearborn et al. (2006). Here we shall simply recap the salient details.

The mesh that DJEHUTY uses to compute a star consists of seven distinct sections, with each section being made up of hexahedral cells of various shapes. At the center, the mesh has a cube consisting of $N \times N \times N$ cells. To each of the faces of this cube are attached “arm” segments consisting of $N \times N \times L$ cells. One of the $N \times N$ faces of each segment is attached point by point to the central cube, while the outermost $N \times N$ face is mapped to lie on a spherical surface. Thus, as one proceeds outward through the arm segment, the cells morph from cuboidal structure to wedge shapes so that the mesh transitions from planar to spherical geometry. The number of cells over which this transition takes place can be set by the user. The $N \times L$ faces of adjacent arm segments are also joined in a point by point fashion.

The code has a nuclear network consisting of 21 species: $^1\text{H}$, $^3\text{He}$, $^4\text{He}$, $^12\text{C}$, $^{13}\text{C}$, $^{13}\text{N}$, $^{14}\text{N}$, $^{15}\text{N}$, $^{15}\text{O}$, $^{16}\text{O}$, $^{17}\text{O}$, $^{18}\text{O}$, $^{17}\text{F}$, $^{18}\text{F}$, $^{19}\text{F}$, $^{20}\text{Ne}$, $^{22}\text{Ne}$, $^{24}\text{Mg}$, $^{25}\text{Si}$, $^{32}\text{S}$, and $^{56}\text{Ni}$. The network allows DJEHUTY to be employed on a variety of astrophysical problems including the core helium flash in low-mass stars (Dearborn et al. 2006), extra mixing on the giant branch (Eggleton et al. 2006), and the explosion of white dwarfs and supernovae (Dearborn et al. 2005; Mathews et al. 2005). For the purposes of this work, the network contains all the species we require.

In this paper, we apply DJEHUTY to the problem of proton ingestion during thermal pulses in low-mass AGB stars. In Section 2, we describe how we set up the simulations. Section 3 describes the results of these simulations, which we then compare to the results of 1D stellar evolution calculations in Section 4.

2. THE MODELS

To obtain a starting model to use as an input for the 3D runs, we use the 1D stellar evolution code STARS, originally developed by Eggleton (1971) and updated by many authors (e.g., Pols et al. 1995; Stancliffe & Eldridge 2009). We evolve a 1 $M_\odot$ star of metallicity $Z = 10^{-4}$ from the pre-main sequence to the AGB. We use 999 mesh points, a mixing length parameter of $\alpha = 2.0$ and no convective overshooting is employed. On the AGB, we utilized the AGB-specific modifications outlined in Stancliffe et al. (2004). Mass loss is included using the Reimers’ prescription (Reimers 1975) with $q = 0.4$ up to the TP-AGB, and the prescription of Vassiliadis & Wood (1993) is used for the duration of the TP-AGB.

The model is evolved until just after the peak of the second thermal pulse. As the helium luminosity declines from a peak value of $\log_{10} L_{\text{He}} / L_\odot = 6.63$, the intershell convection zone begins to move outward in mass, ingesting protons up to an abundance of around $X_H \approx 10^{-5}$. It is at this point that we extract a model to use as the input for the 3D runs. The temperature, density, and abundance profiles of the 1D model are shown in Figure 1. Based on the convective velocity as derived from MLT, the turnover time for the intershell is approximately 1 hr.

This 1D model is then mapped into the 3D grid of DJEHUTY. We model the interior of the star from the center out to just below the convective envelope, at a radius of $4 \times 10^{11}$ cm. We have run two separate simulations from the same initial 1D model, the details of which are listed in Table 1. In our low-resolution run, we set the central cube of the grid to be 20 zones wide. In the arms of the grid, we transition from cubic to spherical symmetry over 50 zones, and there are 120 radial zones in each arm segment. This gives a total of about 300,000 zones. In the high-resolution run, we set the central cube to be 40 zones wide, the transition region to be 80 zones wide and the spherical region contains 200 radial zones. This gives a total of about $2 \times 10^6$ zones. In both simulations, we have verified that the transition of the mesh to spherical symmetry is complete before the helium-burning shell is reached: the whole of our region of interest is simulated in a spherically symmetric grid. A slice through the input model showing the structure of the mesh in the low-resolution run is shown in Figure 2.

We note that the 1D code does not possess the same nuclear network as used in the 3D simulations. In the 1D code, only the energetically important species are included, namely, $^1\text{H}$, $^3\text{He}$,
Table 1
Details of the Two Simulations

| Model            | Central cube | Transition region | Radial arm of Zones | Total No. of Zones | Length of Run (hr) |
|------------------|--------------|-------------------|---------------------|--------------------|-------------------|
| Low resolution   | 20           | 50                | 120                 | 2.96 × 10^5        | 4.039             |
| High resolution  | 40           | 80                | 200                 | 1.98 × 10^6        | 4.664             |

Figure 1. Upper panel: density and temperature profiles as a function of radius for our 1D model. Lower panel: abundance profiles of hydrogen and helium-4 as a function of radius for the same model. In both panels, the gray shaded region denotes the extent of the convective zone.

4He, 12C, 14N, 16O, and 20Ne. The abundances of these species are used by DJEHUTY and the abundances of all the other species in the network are set to zero.

3. RESULTS
3.1. The Low-resolution Run

We have simulated around 4 hr of star time with our low-resolution run. This simulation was done using 31 processors and a total of 6.8 CPU years of run time. The total energy generation rate as a function of time is shown in Figure 3. Because the input 1D model has no information regarding the velocity field for convective motions, we must wait for the convection to fully develop and the model to settle down. This process takes around one convective turnover time, which is just under 1 hr of stellar time in the simulation. During this time, the luminosity is dominated by helium burning, as convective motions have not reached out to the areas of the star that contain hydrogen. At this time the energy generated within the star is...
react vigorously, forming a pocket or bubble of $^{13}\text{N}$ that is about $10^6 \ L_\odot$, which is in reasonable agreement with the value from the 1D code. At about 1 hr, the convective motions reach the hydrogen-rich regions below the H-burning shell, and the energy output begins to rise. This builds steadily, reaching a peak luminosity of around $10^{11} \ L_\odot$. This increase in luminosity comes from the occurrence of proton capture reactions. At the start of the simulation, the total mass of hydrogen in the model is $0.01423 \ M_\odot$, and by the end of the run this has fallen to $0.01405 \ M_\odot$. At the same time, the mass of $^{12}\text{C}$ has fallen from $0.28271 \ M_\odot$ to $0.28175 \ M_\odot$ (most of the $^{12}\text{C}$ is locked in the degenerate core and does not take part in any nuclear reactions).

The upper panels of Figure 4 show the density and temperature structure of a 2D slice through the XZ plane of the simulation at a time of around 2 hr. Note that they are both very smooth and spherically symmetric. It is the convective motions and their transport of hydrogen that are key to the simulations. Once convective motions have spread throughout the intershell, we find that protons are typically drawn down in narrow, fast flowing plumes. An example is shown in Figure 4, which is taken from about 2 hr into the simulation. In the lower left-hand panel, we show the hydrogen abundance in the intershell region. In the lower right-hand panel, we show both the abundance of $^{13}\text{N}$ and the velocity field of the model. $^{13}\text{N}$ is a good indicator for where nuclear burning takes place as it is the direct product of the reaction $^{12}\text{C}(p, \gamma)^{13}\text{N}$. Its short half-life (of the order of 10 minutes) means we only see it in active burning regions. When we study the nucleosynthesis in detail.

3.2. The High-resolution Run

The low-resolution run has around $3 \times 10^5$ zones and was run on 31 processors. Across the intershell we have around 20 zones in the radial direction and around 80 zones in the azimuthal directions (recall that the central cube had 20 zones per side and this sets the angular resolution). We ran a second simulation with closer to $2 \times 10^6$ zones. This improves the radial and angular resolution by a factor of about two, and we now have about 40 zones radially across the intershell. The increase in the number of zones comes at the cost of having to increase the number of processors (144 were used here) and the increased resolution reduces the (Courant-limited) time step.

Two modifications to the code were made prior to the high-resolution run. First, the code was made to output the energy generation from various burning reaction groups (e.g., the $pp$-chains, the CNO cycle, the triple-$\alpha$ reaction, etc.). Second, the nuclear reaction network was updated to include neutron-producing reactions and neutrons were added as a species. Specifically, we added the reactions $^{13}\text{C}(\alpha, n)^{16}\text{O}$, $^{17}\text{O}(\alpha, \alpha)^{20}\text{Ne}$, and $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$. The rates used were taken from Caughlan & Fowler (1988). At present, neutrons do not participate in any further reactions (i.e., they are not absorbed by any of the species). We plan to update this in future work when we study the nucleosynthesis in detail.

Figure 5 shows the luminosity from CNO reactions, triple-$\alpha$ burning, and neutron-generating reactions (the dominant neutron source is the $^{13}\text{C}(\alpha, n)^{16}\text{O}$ reaction). After roughly one convective turnover time, the CNO-burning luminosity becomes the dominant source of energy (as is found in 1D simulations which have substantial ingestion of protons, such as those of Cristallo et al. 2009 for thermal pulses and Campbell et al. 2010 for the core helium flash). After about 2.5 hr, energy generation by neutron-producing reactions exceeds energy production via the triple-$\alpha$ reaction. However, the neutron-producing reactions remain of little energetic importance, with energy generation from CNO burning being around two orders of magnitude greater.

A comparison between the total luminosity from both the low- and high-resolution runs is shown in Figure 3. The high-resolution model has a greater luminosity than the low-resolution model in the early, settling down phase (up to about 0.7 hr) by about 50%. In the high-resolution run, the total luminosity increases dramatically from 0.7 hr, reaching $10^{7} \ L_\odot$ by about 1 hr. In contrast, the low-resolution simulation shows some low-level fluctuations in the luminosity after the settling down period and a sharp increase in luminosity does not occur until after 1.25 hr.

The sudden increase in the total luminosity in the high-resolution model is a result of the ingestion of protons into the intershell convection zone. This can be seen when one looks at the contributions to the total luminosity, as shown in Figure 5. CNO cycle reactions become the dominant energy source just prior to 0.8 hr—a clear signature that proton ingestion has begun. Before this point, the triple-alpha reaction was the

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This is not the same as the total mass of H in the star: recall that our 3D model does not include any of the convective envelope.
Figure 4. Slices along the XZ plane of the low-resolution simulation at a time of about 2 hr. In all panels the hydrogen-burning shell is denoted by a solid black line and the helium-burning shell is denoted by a dashed black line. White regions are simply parts of the simulation that have not been included in the plot for the sake of clarity. Top left panel: color map showing the density. Top right panel: color map showing the temperature. Bottom left panel: color map showing the abundance by mass fraction of hydrogen. Bottom right panel: color map showing the abundance by mass fraction of $^{13}$N, with the arrows denoting the velocity of the fluid flow.

dominant energy source. Note that we have substantial activation of the $^{13}$C($\alpha$, n)$^{16}$O reaction in the model as the luminosity from neutron-producing reactions has reached over $10^4$ $L_\odot$. The $^{13}$C is generated when $^{12}$C captures a proton to form the unstable nucleus $^{13}$N which then $\beta$-decays to form $^{13}$C.

With regards to the turbulent convection, we see similar structures in the low- and high-resolution runs. Figure 6 shows slices through the XZ plane of the high-resolution run. We again see plume-like ingestion of hydrogen (see the upper panel of Figure 6). However, in the high-resolution runs we see more individual plumes, possibly as a result of the convective motions being better resolved. The high-resolution simulation shows greater structure in its velocity field, with more individual convective cells being present. The high-resolution simulation also has lower convective velocities.

Figure 7 shows slices through both the low- and high-resolution runs at a time of 1.5 hr. The first thing to note is that the low-resolution run is slightly more compact than the high-resolution run. The $X_{\text{H}} = 0.1$ contour falls just within $3 \times 10^9$ cm from the center, whereas in the high-resolution run the same contour lies just outside this radius (compare the upper and lower panels of Figure 7). This contour also seems to be more spatially separated from the $X_{\text{H}} = 0.01$ contour in the low-resolution model. This is because the low-resolution run mixes down more material from the hydrogen-rich regions, on account of it having a thicker boundary layer and more entrainment because of its lower resolution (see the discussion at the end of this section). Contours of a given hydrogen abundance therefore appear at smaller radii.

The peak $^{13}$N abundances seem comparable in both runs at this point. However, the low-resolution run has more localized pockets of high $^{13}$N abundance while in the high-resolution simulation, the $^{13}$N seems more widely spread. The low-resolution simulation seems to have a much greater degree of
proton ingestion at this point. The $X_H = 10^{-7}$ contour (the red line in Figure 7) has reached almost to the He-burning shell in the lower half of this simulation and there is also significant distortion of the $X_H = 10^{-6}$ contour (denoted in green). In the high-resolution simulation there is less overall distortion of the hydrogen contours, though there are significant downward plumes in evidence (notably on the top- and bottom-left of the plot). In addition, there seem to be differences in the flows in the two simulations. The low-resolution simulation displays significantly faster flows (note the large, whitish arrows in the lower half of the plot). There seems to be significant flow around most of the intershell, with two large eddies accounting for the flow in the lower half of the image. In contrast, the flow in the high-resolution simulation seems to be much slower and split into more, smaller cells.

The upper left panel of Figure 8 shows the radial velocity for a slice through our simulation. In the lower-right quadrant, there is a strong downward plume, with material that is rich in hydrogen flowing at speeds of over 100 km s$^{-1}$. This rapid flow means that the protons do not have time to burn in flight, and in fact strong nuclear burning only occurs once the protons have been transported to close to the helium-burning shell (see the upper-right panel of Figure 8). Following Herwig et al. (2011), we may define a burning timescale for the reaction $^{12}\text{C}(p, \gamma)^{13}\text{N}$ as

$$\tau_{\text{burn}} = \frac{12}{X_{^{12}\text{C}} \rho N_A \langle \sigma v \rangle},$$

where $X_{^{12}\text{C}}$ is the abundance by mass fraction of $^{12}\text{C}$, $\rho$ is the density, $\langle \sigma v \rangle$ is the cross section for the reaction, and $N_A$ is Avagadro’s number. This burning timescale is plotted in the lower panel of Figure 8. Burning will only take place when the timescale for the transport of protons becomes comparable to the burning timescale. With a downflow velocity of around 100 km s$^{-1}$, the entire width of the intershell (around $2 \times 10^4$ km) would be traversed in just 200 s. Burning timescales this short are only reached close to the helium-burning shell, and so we only obtain strong hydrogen burning in such regions as can be seen in the upper-right panel of Figure 8. Perhaps surprisingly, this strong burning does not have a substantial effect on the temperature structure of the model. In the lower-right panel of Figure 8, we plot the temperature contrast of the model. This is the deviation in the local temperature, $T$, from the average temperature over a spherical shell with the same radius, $\langle T \rangle$, measured relative to that average, i.e., $(T - \langle T \rangle)/\langle T \rangle$. Within the intershell, the temperature deviates by no more than around 1% from the spherical average for that radius.

One may question whether we have enough resolution in the high-resolution run. The two runs seem quite dissimilar in behavior. What would happen if we increased the resolution further? We are working on a higher resolution run at present, but it is unlikely to provide a large enough change in resolution. In the 1D evolution code, we use around 600 mesh points to cover...
the intershell region (Stancliffe et al. 2004), whereas there are only about 40 cells in the radial direction between the helium- and hydrogen-burning shells. An order-of-magnitude increase in this resolution, although desirable, is not possible at present.

The less than optimal resolution of our simulations means that we should place two caveats on our results. At the interface between the convective intershell and the hydrogen-rich regions, we are likely to be overestimating the degree of entrainment. Low resolution will lead to a broader entrainment layer with greater viscosity and consequently to more entrainment. That this is so can be seen in the difference between our low- and high-resolution simulations. The low-resolution simulation ingests (entains) more hydrogen, reaching high burning luminosities as a consequence. Until we have properly resolved the boundary layer, we cannot quantify the mass entrainment rate and so we refrain from attempting to do so.

In addition, the simulation may not represent truly turbulent convective motions in the intershell region. Because DJEHTY does not include sub-grid scale physics, the viscosity present in the simulations is numerical and is based upon the grid size used. If the grid size is not sufficiently small, we will not be able to resolve eddies on the smallest scales and our convective flows will not be fully turbulent. Consequently, we may underestimate the degree of mixing that takes place. One must therefore caution that the results of this study may not be borne out by higher resolution studies. It is well known that as one moves from the laminar to the turbulent flow regime the nature of the flow patterns can change dramatically (see, e.g., Miesch et al. 2000). It is possible that at higher resolution, we would not retain such fast-flowing, coherent, downward plumes because of additional turbulence.

We can make a crude estimate of the Reynolds number (Re) of our simulations. Davidson (2004) gives an expression for this quantity in terms of the number of zones (N) in the computational domain and the ratio of the size of the largest eddy (l) to the physical size of the region being simulated (L). His Equation (7.3) states

\[
Re \approx \left( \frac{l}{L} \right)^{\frac{4}{3}} N^{\frac{4}{9}}. \tag{2}
\]

For both our simulations, the size of the largest eddies is comparable to the size of the intershell region so that \(l/L\) is of the order of unity. Consequently, our estimate of the Reynolds number scales (slowly) with the number of zones in the intershell region. For the low-resolution simulation, the intershell has about 20 zones in the radial direction and each of the six arm segments has a cross section of 20 \(\times\) 20 zones. We therefore have roughly 48,000 zones in this region which yields \(Re \approx 120\). For the high-resolution simulation, we have about 45 zones radially across the intershell and each arm has a cross section of 40 \(\times\) 40 zones, yielding a total of 432,000 zones. This gives \(Re \approx 320\). These numbers suggest that our simulations are turbulent, but only marginally so. To be certain we are in the fully turbulent regime, we would need at least an order of magnitude more zones in our intershell.

3.3. Comparison to the Models of Herwig et al.

Our simulations display qualitative agreement with those of Herwig et al. (2011), despite their simulations not including burning of the ingested hydrogen. We also find that hydrogen ingestion takes place where upwelling cells meet and that the ingested material is carried down in narrow, fast flowing plumes. Our radial velocities reach mean values of 30–40 km s\(^{-1}\) (whereas MLT predicts velocities of 1–2 km s\(^{-1}\) for our 1D input model), compared to 12 km s\(^{-1}\) in the Herwig et al. simulations. There are likely two contributions to this difference. First, we have greater energy generation in our model, leading to stronger convective motions. Their model was driven by a total luminosity of \(4 \times 10^7 L_{\odot}\) (whereas MLT predicts velocities of 1–2 km s\(^{-1}\) for our 1D input model), compared to 12 km s\(^{-1}\) in the Herwig et al. simulations. There are likely two contributions to this difference. First, we have greater energy generation in our model, leading to stronger convective motions. Their model was driven by a total luminosity of \(4 \times 10^7 L_{\odot}\), whereas we have a total luminosity reaching up to as much as \(10^8 L_{\odot}\) in the high-resolution case, owing to the release of energy from hydrogen burning which is not included in the Herwig et al. simulations. Second, the Herwig et al. simulations have much higher resolution than ours and are more likely to be truly turbulent.

Figure 7. 2D slice through the model for the low-resolution (upper panel) and high-resolution (lower panel) runs, showing the \(^{13}\)N abundance (color map), contours of \(^1\)H and velocity (gray arrows). The plots are as coincident in time as the output allows (the difference is about 10 s).
4. COMPARISON TO THE 1D MODELS

Perhaps the most striking difference between the 1D and 3D simulations is in the hydrogen-burning luminosity ($L_H$). At the time step that was used as the input model for the 3D simulations, $L_H \approx 5.5 \times 10^2 L_\odot$. The proton ingestion is relatively minor at this point, as can be seen in Figure 1. The intershell convective region extends from around $1.2 \times 10^9$ cm to around $3 \times 10^9$ cm. The H abundance is never greater than $10^{-5}$ in this region and it drops below $10^{-12}$ around $2 \times 10^9$ cm. There is no significant proton abundance below this. This line marks a fairly good approximation of the lower envelope of the hydrogen abundance seen in the 3D simulation. However, the 3D simulation\footnote{Unless otherwise specified, we are referring to the high-resolution simulation.} shows significant variation in the H-abundance in the convective region, reaching as high as $X_H \approx 10^{-4}$ throughout, even down to the He-burning shell. This accounts for the significantly higher H-luminosity found in this simulation.

If we take a mass-weighted average over radial shells in the 3D model and also separate the cells according to their radial motion (i.e., in to up- and downflows), then we obtain the profiles shown in Figure 9. The H abundance in the downflow is clearly greater than that in the upward flow, perhaps by as much as one order of magnitude. It is also clear that the average abundance of H in the convection zone of the 3D model is far higher than in the 1D model and this is the reason for the much higher hydrogen-burning luminosity. We note that close to the upper edge of the convective zone, the hydrogen profile in the 3D model has a similar shape to the 1D model which uses diffusive mixing, i.e., it looks as though the mixing here is more diffusive in nature.
Herwig et al. (2011) also found this to be the case in their simulations. However, as we move away from the boundary, the abundance profiles in the 3D model are noticeably flatter, before finally falling off sharply as the burning shell is approached. We believe that this profile would be better represented by an advective mixing scheme, rather than the diffusive scheme employed by the 1D evolution code. This conclusion underlines the point made by Arnett & Meakin (2011) that convection is employed by the 1D evolution code. This conclusion underlines an advective mixing scheme, rather than the diffusive scheme implemented is currently in progress but is only in its embryonic stages. Here we shall simply speculate on some possible consequences. If the current generation of 1D evolution codes is underestimating the amount of energy generation from hydrogen burning during proton ingestion events, then the depth to which the convective regions to the He-burning shell is so rapid. This rapid transport behavior is not currently accounted for by 1D evolution codes, which use a diffusive approximation for their mixing of chemical species. We believe that the 1D codes could be substantially improved by using an advective, rather than a diffusive, mixing scheme for these episodes. Despite the extremely large hydrogen-burning luminosities found in our 3D simulations, we find no evidence for the convective region splitting into two zones. However, we caution that this may be because the simulations do not have sufficient spatial resolution to resolve the initial splitting or because they have not been evolved for long enough. However, it is also possible that no splitting will occur in these models because the energy injection from proton burning occurs at the base of the convective region, rather than part-way through it. It should also be reiterated that higher resolution simulations are desirable to ensure that the boundary layer at the top of the intershell, and the entrainment of material in this layer, is properly resolved. Higher resolution simulations would also be expected to show more truly turbulent flow than those presented herein.

In Figure 10 we plot the mixing length velocity for the 1D model, along with the mass-weighted average velocities in the up- and downstreams. We note that the 1D MLT velocity is over an order of magnitude slower than the convective motions in the 3D simulation, a result in agreement with the simulations of Herwig et al. (2011). We also note that there is substantial asymmetry between the up- and downflows, particularly toward the base of the convective region. Below a radius of $2 \times 10^9$ cm, the downflow velocity can exceed the upflow velocity by around 7 km s$^{-1}$. Above this point, the two flows have comparable velocity and the upflow is faster than the downflow above a radius of $2.2 \times 10^9$ cm. Note also that this is for the average upward and downward flows. The velocity of fluid elements in these flows can be significantly in excess of these velocities. Peak velocities of over 100 km s$^{-1}$ regularly occur in both the up and downstream flows.

These simulations suggest that there are (at least) two faults with our 1D model. First, the treatment of mixing by a diffusive process is not ideal. Close to the boundary of the convective zone it may be a good approximation, but the farther from the convective boundaries we go, the worse it gets. In the mid-regions of the intershell the fluid flow is advective and the transport of chemical species is rapid: there is not enough time for protons to burn in flight. A two-stream advective scheme may present a better approximation to the actual physical process. In addition, the use of velocities as calculated by MLT may be significantly underestimating the speed of convective motions.

5. CONCLUSIONS

We have modeled the ingestion of protons into the intershell convection zone in a low-metallicity AGB star in a 3D hydrodynamics code. We find substantially higher hydrogen-burning luminosities in our 3D model than in the input 1D evolutionary model. Rapid transport of hydrogen in downflowing plumes leads to vigorous hydrogen burning taking place in close proximity to the He-burning shell. We see no evidence for in-flight proton burning because the transport of protons from H-rich regions to the He-burning shell is so rapid. This rapid transport behavior is not currently accounted for by 1D evolution codes, which use a diffusive approximation for their mixing of chemical species. We believe that the 1D codes could be substantially improved by using an advective, rather than a diffusive, mixing scheme for these episodes. Despite the extremely large hydrogen-burning luminosities found in our 3D simulations, we find no evidence for the convective region splitting into two zones. However, we caution that this may be because the simulations do not have sufficient spatial resolution to resolve the initial splitting or because they have not been evolved for long enough. However, it is also possible that no splitting will occur in these models because the energy injection from proton burning occurs at the base of the convective region, rather than part-way through it. It should also be reiterated that higher resolution simulations are desirable to ensure that the boundary layer at the top of the intershell, and the entrainment of material in this layer, is properly resolved. Higher resolution simulations would also be expected to show more truly turbulent flow than those presented herein.

The ramifications of these simulations for stellar evolution remain to be determined and await the integration of these results into 1D evolution codes, which remain the only way we can investigate stellar behavior over long timescales. This implementation is currently in progress but is only in its embryonic stages. Here we shall simply speculate on some possible consequences. If the current generation of 1D evolution codes is underestimating the amount of energy generation from hydrogen burning during proton ingestion events, then the depth of third dredge-up following one of these episodes is also likely underestimated. Carbon-rich material could be dredged to the surface earlier than current models predict. This could also mean that lower-mass stars, which may not have strong
enough pulses to trigger third dredge-up, could in fact become carbon-rich. This could help to ameliorate the discrepancy between the observed carbon-rich to carbon-normal metal-poor star fraction and the predictions of population synthesis which require there to be more carbon-rich stars than theory currently predicts (see Izzard et al. 2009 for further details of this problem).

There will also be consequences for the nucleosynthesis of these events. The plentiful supply of protons to the $^{12}\text{C}$ rich intershell will generate a substantial quantity of $^{13}\text{C}$. When this undergoes alpha-capture, a substantial quantity of neutrons will be liberated and this may give rise to an $s$-process (see Campbell et al. 2010 for the case of proton ingestion during the core helium flash). In a 1D stellar model in which the splitting of the convective region was delayed (based upon their hydrodynamical simulations) Herwig et al. (2011) reported neutron densities of $10^{15}$ cm$^{-3}$ and significant production of the light $s$-process elements (Rb, Sr, and Zr). This situation was for a higher metallicity object undergoing a very late thermal pulse. Would similar nucleosynthesis take place in a low-metallicity case like the one presented here? This is something that we will look at in future work.

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REFERENCES

Arnett, W. D., & Meakin, C. 2011, ApJ, 733, 78

Barzán, G., Dearborn, D. S. P., Dossa, D. D., et al. 2003, in ASP Conf. Ser. 293, 3D Stellar Evolution, ed. R. M. C. S. Turcotte & S. C. Keller (San Francisco, CA: ASP), 1

Böhmm-Vitense, E. 1958, Z. Astrophys., 46, 108

Campbell, S. W., & Lattanzio, J. C. 2008, A&A, 490, 769

Campbell, S. W., Lugaro, M., & Karakas, A. I. 2010, A&A, 522, L6

Cassisi, S., Castellani, V., & Tomambe, A. 1996, ApJ, 459, 298

Caughlan, G. R., & Fowler, W. A. 1988, At. Data Nucl. Data Tables, 40, 283

Cristallo, S., Piersanti, L., Straniero, O., et al. 2009, PASA, 26, 139

Davidson, P. A. 2004, Turbulence: An Introduction for Scientists and Engineers (Oxford: Oxford Univ. Press)

Dearborn, D. S. P., Lattanzio, J. C., & Eggleton, P. P. 2006, ApJ, 639, 405

Dearborn, D. S. P., Wilson, J. R., & Mathews, G. J. 2005, ApJ, 630, 309

Eggleton, P. P. 1971, MNRAS, 151, 351

Eggleton, P. P., Dearborn, D. S. P., & Lattanzio, J. C. 2006, Science, 314, 1580

Fujimoto, M. Y., Iben, I., Jr., & Hollowell, D. 1990, ApJ, 349, 580

Fujimoto, M. Y., Ikeda, Y., & Iben, I. Jr. 2000, ApJ, 529, L25

Herwig, F. 2001, ApJ, 554, L71

Herwig, F. 2005, ARA&A, 43, 435

Herwig, F., Freytag, B., Hueckstaedt, R. M., & Timmes, F. X. 2006, ApJ, 642, 1057

Herwig, F., Pignatari, M., Woodward, P. R., et al. 2011, ApJ, 727, 89

Hollowell, D., Iben, I., Jr., & Fujimoto, M. Y. 1990, ApJ, 351, 245

Iben, I., Jr., & Renzini, A. 1983, ARA&A, 21, 271

Iwamoto, N. 2009, PASA, 26, 145

Izzard, R. G., Glebbeek, E., Stancliffe, R. J., & Pols, O. R. 2009, A&A, 508, 1359

Lau, H. B. H., Stancliffe, R. J., & Tout, C. A. 2009, MNRAS, 396, 1046

Mathews, G. J., Wilson, J. R., & Dearborn, D. S. P. 2005, Nucl. Phys. A, 758, 467

Meakin, C. A. 2006, PhD thesis, The Univ. Arizona, AZ

Meakin, C. A., & Arnett, D. 2006, ApJ, 637, L53

Meakin, C. A., & Arnett, D. 2007, ApJ, 667, 448

Miesch, M. S., Elliott, J. R., Toomre, J., et al. 2000, ApJ, 532, 593

Mocák, M., Campbell, S. W., Müller, E., & Kifonidis, K. 2010, A&A, 520, A114

Mocák, M., Müller, E., Weiss, A., & Kifonidis, K. 2009, A&A, 501, 659

Mocák, M., Siess, L., & Muller, E. 2011, A&A, 533, A53

Pols, O. R., Tout, C. A., Eggleton, P. P., & Han, Z. 1995, MNRAS, 274, 964

Reimers, D. 1975, Mem. Soc. R. Sci. Liege, 8, 369

Stancliffe, R. J., & Eldridge, J. J. 2009, MNRAS, 396, 1699

Stancliffe, R. J., Tout, C. A., & Pols, O. R. 2004, MNRAS, 352, 984

Suda, T., & Fujimoto, M. Y. 2010, MNRAS, 405, 177

Vassiliadis, E., & Wood, P. R. 1993, ApJ, 413, 641