PLANE TARY MIGRATION AND ECCENTRICITY AND INCLINATION RESONANCES IN EXTRASOLAR PLANETARY SYSTEMS

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Received 2009 February 8; accepted 2009 July 22; published 2009 August 24

ABSTRACT

The differential migration of two planets due to planet–disk interaction can result in capture into the 2:1 eccentricity-type mean-motion resonances. Both the sequence of 2:1 eccentricity resonances that the system is driven through by continued migration and the possibility of a subsequent capture into the 4:2 inclination resonances are sensitive to the migration rate within the range expected for type II migration due to planet–disk interaction. If the migration rate is fast, the resonant pair can evolve into a family of 2:1 eccentricity resonances different from those found by Lee. This new family has outer orbital eccentricity \( e_2 \gtrsim 0.4-0.5 \), asymmetric librations of both eccentricity resonance variables, and orbits that intersect if they are exactly coplanar. Although this family exists for an inner-to-outer planet mass ratio \( m_1/m_2 \gtrsim 0.2 \), it is possible to evolve into this family by fast migration only for \( m_1/m_2 \gtrsim 2 \). Thommes and Lissauer have found that a capture into the 4:2 inclination resonances is possible only for \( m_1/m_2 \gtrsim 2 \). We show that this capture is also possible for \( m_1/m_2 \gtrsim 2 \) if the migration rate is slightly slower than that adopted by Thommes and Lissauer. There is significant theoretical uncertainty in both the sign and the magnitude of the net effect of planet–disk interaction on the orbital eccentricity of a planet. If the eccentricity is damped on a timescale comparable to or shorter than the migration timescale, \( e_2 \) may not be able to reach the values needed to enter either the new 2:1 eccentricity resonances or the 4:2 inclination resonances. Thus, if future observations of extrasolar planetary systems were to reveal certain combinations of mass ratio and resonant configuration, they would place a constraint on the strength of eccentricity damping during migration, as well as on the rate of the migration itself.

Key words: celestial mechanics – planetary systems – planets and satellites: general

1. INTRODUCTION

Extrasolar planet searches have to date yielded about 33 systems with multiple planets, and at least eight of these systems have a pair of planets known or suspected to be in mean-motion resonances. It is well established that the outer two planets in the GJ 876 system are deep in 2:1 resonances, with the retrograde periapse precessions induced by the 2:1 resonances having been observed for more than one full period (Marcy et al. 2001; Laughlin & Chambers 2001; Rivera & Lissauer 2001; Lee & Peale 2002; Laughlin et al. 2005; Rivera et al. 2005). In the GJ 876 system, both of the lowest order, eccentricity-type mean-motion resonance variables

\[
\theta_1 = \lambda_1 - 2\lambda_2 + \omega_1,
\]

\[
\theta_2 = \lambda_1 - 2\lambda_2 + \omega_2,
\]

and hence the secular apsidal resonance variable

\[
\theta_{\text{SAR}} = \omega_1 - \omega_2 = \theta_1 - \theta_2,
\]

librate about \( 0^\circ \), which mean that the periapses are nearly aligned and that conjunctions of the planets occur when both planets are near periapse. In the above equations, \( \lambda_1 \) and \( \lambda_2 \) are the true longitudes of the inner and outer planets, respectively, and \( \omega_1 \) and \( \omega_2 \) are the mean longitudes of periapse. There are three other systems with planets in 2:1 resonances: HD 82943 (Mayor et al. 2004; Ferraz-Mello et al. 2005; Lee et al. 2006; Beaugé et al. 2008), HD 128311 (Vogt et al. 2005), and HD 73526 (Tinney et al. 2006; Sándor et al. 2007), although it should be noted that a pair of planets in 1:1 resonance is a plausible alternative for at least HD 82943 and HD 128311 (Goździewski & Konacki 2006). In addition, the HD 45364 (Correia et al. 2009), 55 Cancri (Marcy et al. 2002; McAruthur et al. 2004), HD 60532 (Desort et al. 2008; Laskar & Correia 2009), and HD 202206 (Correia et al. 2005) systems have planets that are in 3:2, 3:1, 3:1, and 5:1 resonances, respectively. There are uncertainties in converting this data into the fraction of multiple-planet systems with mean-motion resonances. Some of the suspected resonant pairs may not be confirmed eventually (see, e.g., Fischer et al. 2008 for 55 Cancri). On the other hand, the number of resonant pairs that remain undetected could be quite large, because the radial velocity variation due to two planets in resonance (in particular, 2:1) could be indistinguishable from that due to a single planet for certain planetary mass ratio and orbital eccentricities, given the precision levels of the existing radial velocity surveys (Anglada-Escude et al. 2008; Giuppone et al. 2009). Nevertheless, the existing data indicate that \( \sim 20\% \) of multiple-planet systems have mean-motion resonances.

Mean-motion resonances can be easily established during planet formation by the convergent migration of planets due to interactions with the circumstellar gas disk. Two giant planets that are massive enough to open gaps in the disk individually can clear out the disk material between them rather quickly, and the outer planet is forced to migrate inward by the disk material outside its orbit (and the inner planet outward if there is any disk material left inside its orbit; Bryden et al. 2000; Kley 2000). Both hydrodynamic and three-body simulations (with imposed migration for the latter) have shown that the convergence of the orbits naturally leads to capture into mean-motion resonances (Bryden et al. 2000; Kley 2000; Snellgrove et al. 2001; Lee & Peale 2002; Nelson & Papaloizou 2002; Papaloizou 2003; Thommes & Lissauer 2003; Kley et al. 2004, 2005; Lee 2004).
The ubiquity of mean-motion resonances in extrasolar planetary systems and the ease of capture into such resonances by convergent migration have prompted investigations into the variety of stable mean-motion resonance configurations (Lee & Peale 2002, 2003; Beaugé et al. 2003, 2006; Ferraz-Mello et al. 2003; Hadjidemetriou & Psychoyos 2003; Ji et al. 2003; Thommes & Lissauer 2003; Lee 2004; Voyatzis & Hadjidemetriou 2005, 2006; Marzari et al. 2006; Michtchenko et al. 2006). The 2:1 mean-motion commensurability has received most attention, because it is the most common one observed and includes the best case, GJ 876. With the exception of Thommes & Lissauer (2003), all of the works just cited have focused on systems with two planets on coplanar orbits. For small orbital eccentricities, antisymmetric configurations with $\theta_1$ librating about $0^\circ$ and $\theta_2$ about $180^\circ$ (as in the case of the Jovian satellites Io and Europa) are the only stable 2:1 resonance configuration with both $\theta_1$ and $\theta_2$ librating. For moderate to large eccentricities, the Io–Europa configuration is not stable, but there is a wide variety of other stable 2:1 resonance configurations, including symmetric configurations with both $\theta_1$ and $\theta_2$ librating about $0^\circ$ (as in the GJ 876 system), asymmetric configurations with $\theta_1$ and $\theta_2$ librating about angles other than $0^\circ$ and $180^\circ$ (some with intersecting orbits), and antisymmetric configurations with $\theta_1 \approx 180^\circ$ and $\theta_2 \approx 0^\circ$ (and intersecting orbits). Lee (2004) has shown that the sequence of 2:1 resonance configurations that a system with initially coplanar and nearly circular orbits is driven through by continued migration depends mainly on the planetary mass ratio $m_1/m_2$, if the migration rate is sufficiently slow. However, there are stable 2:1 resonance configurations (e.g., those with $\theta_1 \approx 180^\circ$ and $\theta_2 \approx 0^\circ$) that cannot be reached by the convergent migration of planets with constant masses and initially coplanar and nearly circular orbits. If real systems with these configurations are ever found, their origin would require a change in the planetary mass ratio $m_1/m_2$ during migration, multiple-planet scattering in crowded planetary systems, or a migration scenario involving inclination resonances (Lee 2004).

Thommes & Lissauer (2003) have studied the convergent migration of planets with noncoplanar orbits and found that, subsequent to the capture into the 2:1 eccentricity resonances, a capture into the 4:2 inclination resonances (which are the lowest order inclination resonances at the 2:1 commensurability) is possible if $m_1/m_2 \gtrsim 2$. The 4:2 inclination-type mean-motion resonance variables are

$$\phi_{11} = 2\lambda_1 - 4\lambda_2 + 2\Omega_1, \quad (4)$$

$$\phi_{22} = 2\lambda_1 - 4\lambda_2 + 2\Omega_2, \quad (5)$$

where $\Omega_j$ are the longitudes of the ascending node. The simultaneous librations of $\phi_{11}$ and $\phi_{22}$ mean that the mixed resonance variable

$$\phi_{12} = 2\lambda_1 - 4\lambda_2 + \Omega_1 + \Omega_2 = (\phi_{11} + \phi_{22})/2 \quad (6)$$

also librates. As the system enters the inclination resonances, the mutual inclination of the orbits can grow rapidly to tens of degrees. In some cases, the system eventually evolves out of the inclination resonances, and the eccentricity resonances switch to the $\theta_1 \approx 180^\circ$ and $\theta_2 \approx 0^\circ$ configurations mentioned above. Thommes & Lissauer (2003) showed an example with $m_1/m_2 = 3$, which does not have capture into the inclination resonances and remains nearly coplanar throughout its evolution, but we notice that the evolution of the eccentricities and $\theta_j$ is different from that found by Lee (2004) when the outer orbital eccentricity $e_2 \gtrsim 0.45$. These configurations with $e_2 \gtrsim 0.45$ also do not correspond to any of the other eccentricity resonance configurations found by Lee (2004). As we shall see in Section 3, they belong to a new family of 2:1 eccentricity resonances that can be reached by migration if the migration rate is faster than that adopted by Lee (2004) and $m_1/m_2 \gtrsim 2$.

Gap-opening planets undergo type II migration on the disk viscous timescale, whose inverse is

$$\left| \frac{d}{a} \right| \approx \frac{3\nu}{2a^2} = 9.4 \times 10^{-5} \left( \frac{a}{4 \times 10^{-3}} \right) \left( \frac{H/a}{0.05} \right)^2 p^{-1}, \quad (7)$$

(Ward 1997), where $a$ is the semimajor axis of the planet’s orbit about a star of mass $m_0, a \equiv da/dt, \nu = aH^2\Omega$ is the kinematic viscosity, $a$ is the Shakura–Sunyaev viscosity parameter, $H$ is the scale height of the disk, and $P = 2\pi\sqrt{a}/2 \approx 2\pi a^{3/2}/(\Omega m_0)^{1/2}$ is the orbital period. The uncertainties and radial variations in $a$ and $H/a$ mean that the migration rate can be at least a factor of a few faster or slower than $10^{-4}/P$. Although both Thommes & Lissauer (2003) and Lee (2004) performed three-body simulations with imposed inward migration on the outer planet only, it is difficult to determine from the calculations in these papers that the different results at $e_2 \gtrsim 0.45$ for $m_1/m_2 = 3$ are due to different migration rates, because they imposed migration in different ways. Thommes & Lissauer (2003) adopted $H/a_2 \propto a_2^{-1/4}$, so that $a_2$ is independent of $a_2$ (with $a_2 = -10^{-5}$ AU yr$^{-1}$ for most calculations), and they imposed the migration in such a way that the migration does not slow down after the capture of an inner planet into resonance. Lee (2004) adopted constant $H/a_2$ and performed calculations with $a_2/a_2 = -10^{-6}/P_2$ and $-10^{-4}/P_2$, imposed in such a way that the migration slows down by a factor $\beta/(\beta + m_1/m_2)$, where $\beta = a_1/a_2 \approx 2^{-2/3}$ after the capture of an inner planet into 2:1 resonance. In this paper, we examine systematically the effects of different migration rates (within the range expected for type II migration) using three-body integrations with migration imposed in the same way.

We also examine systematically the effects of different eccentricity damping rates during migration. Significant eccentricity damping can prevent the eccentricities from reaching high enough values for capture into the new 2:1 eccentricity resonances or the 4:2 inclination resonances. There is significant uncertainty in both the sign and the magnitude of the net effect of planet–disk interaction on the orbital eccentricity of the planet because of sensitivity to the distribution of disk material near the locations of the Lindblad and corotation resonances (Goldreich & Sari 2003; Ogilvie & Lubow 2003). However, hydrodynamic simulations of two planets orbiting inside an outer disk have shown eccentricity damping of the outer planet, with $K = [e_2/e_1]/[a_2/a_2] \sim 1$ (Kley et al. 2004, 2005). Thommes & Lissauer (2003) and Lee (2004) have reported a small number of simulations with eccentricity damping for noncoplanar and coplanar systems, respectively. In particular, Thommes & Lissauer (2003) have found that the critical value of $K$ for capture into the 4:2 inclination resonances is between 2 and 5 for $m_1/m_2 = 1$.

In Section 2, we describe the numerical methods and initial conditions. In Section 3, we consider coplanar orbits and show that a resonant pair can evolve into a new family of 2:1 eccentricity resonances if the migration rate is faster than that
adopted by Lee (2004) and \(m_1/m_2 \gtrsim 2\), although the new family
exists for \(m_1/m_2 \gtrsim 0.2\). In Sections 4 and 5, we consider
noncoplanar orbits. We show that inclination excitation and
capture into the 4:2 inclination resonances are possible for
\(m_1/m_2 \gtrsim 2\) (as well as \(m_1/m_2 \lesssim 2\)), if the migration rate
is slower than that adopted by Thommes & Lissauer (2003),
and that the maximum value of \(K = e_2/e_1|\dot{a}_2/\dot{a}_2|\) for capture
into the 4:2 inclination resonances is of the order of unity. Our
conclusions are summarized and discussed in Section 6.

2. NUMERICAL METHODS AND INITIAL CONDITIONS

We consider systems consisting of a central star of mass
\(m_0\), an inner planet of mass \(m_1\), and an outer planet of mass
\(m_2\), with \(m_1/m_2\) between 0.1 and 10. Unless stated otherwise,
\((m_1 + m_2)/m_0 = 10^{-3}\). For the migration calculations starting
without eccentricity damping, we consider nonresonant orbits, the planets are initially on circular
orbits, with the ratio of the orbital semimajor axes \(\beta = a_1/a_2 =
1/2\) (far from the 2:1 mean-motion commensurability, where
\(\beta \approx 2^{-2/3}\)), and the outer planet is forced to migrate inward.
The calculations presented in Section 3 are for configurations
with exactly coplanar orbits, while those presented in Sections 4
and 5 are for configurations with initial mutual orbital inclination
\(i_{\text{ini}} = 0\degree\), where the initial invariable plane is used as the \(z\)
= 0 reference plane. Thommes & Lissauer (2003) have found
that the entry into the inclination resonances is not strongly
influenced by the initial value of \(i_{\text{ini}}\), as long as it is \(< 1\degree\).

The three-body integrations with imposed migration are
performed using the code described in Lee (2004), which is a
modified version of the symplectic integrator SyMBA (Duncan
et al. 1998). The outer planet is forced to migrate inward with
a migration rate of the form \(\dot{a}_2/a_2 \propto P_2^{-1}\). The migration slows
down by a factor \(\beta/(\beta + m_1/m_2)\), where \(\beta \approx 2^{-2/3}\), after the
capture of an inner planet into 2:1 resonance (see the paragraph
with Equation (7)). The input and output are in Jacobi orbital
elements, and we apply the forced migration to the Jacobi
\(e_2\) for the calculations with eccentricity damping. To characterize the new family of
2:1 eccentricity resonances, there are also calculations in
Section 3 with a change in \(m_1/m_2\) (and no migration). The
modified SyMBA code used for these calculations is also
described in Lee (2004).

3. A NEW FAMILY OF 2:1 ECCENTRICITY RESONANCES

We begin with migration calculations of coplanar orbits
without eccentricity damping. We consider \(m_1/m_2 = 0.1,
0.3, 0.9, 1.0, 1.5, 2.3, 2.65, 3, 5, \) and 10 (same as in Lee
2004) and \(\dot{a}_2/a_2 = -0.5, -1, -2, -4, \) and \(-8 \times 10^{-4}/P_2\);
Figures 1 and 2 show the evolution of the semimajor axes \(a_1, a_2\),eccentricities \(e_1, e_2\), and eccentricity-type resonance variables \(\theta_1\),
for the calculations with \((m_1 + m_2)/m_0 = 10^{-3}, m_1/m_2 = 3, \) and
\(\dot{a}_2/a_2 = -0.5 \times 10^{-4}/P_2\) and \(-2 \times 10^{-4}/P_2\), respectively.
For \(\dot{a}_2/a_2 = -0.5 \times 10^{-4}/P_2\) (Figure 1), the sequence of
2:1 resonance configurations after resonance capture—from
\((\theta_1, \theta_2) \approx (0\degree, 180\degree)\) at small eccentricities to asymmetric
librations of both \(\theta_1\) and \(\theta_2\) at moderate to large eccentricities—is
equivalent to that found by Lee (2004) for \(|\dot{a}_2/\dot{a}_2| \lesssim 10^{-4}/P_2\),
but with larger libration amplitudes for faster migration rate.
The system eventually becomes unstable at \(t/P_{2,0} \approx 3.96 \times 10^4\),
where \(P_{2,0}\) is the initial outer orbital period. For the faster
migration rate of \(\dot{a}_2/a_2 = -2 \times 10^{-4}/P_2\) (Figure 2),
the sequence of resonance configurations is identical to that shown
in Figure 1 (but with larger libration amplitudes) when \(e_2 \lesssim 0.45\)
(and \(t/P_{2,0} \lesssim 3000\)). However, the system enters a new
family of 2:1 resonance configurations when \(e_2 \gtrsim 0.45\). The
differences between the configurations in Figures 1 and 2 at

![Figure 1](image1)

![Figure 2](image2)
$e_2 \geq 0.45$ are most obvious in the plots of $e_1$ and $\theta_1$. We confirm that the configurations in Figure 2 with $e_2 \geq 0.45$ are stable resonance configurations by integrating the configurations at $t/P_{2.0} = 7000$ and $10^4$ forward with migration turned off and finding stable libration of $\theta_1$, with the centers and amplitudes of libration nearly identical to those just before the migration is turned off.\(^5\)

Migration calculations with different $m_1/m_2$ and $\dot{\alpha}_2/\dot{\alpha}_2$ show that a system can enter the new family of 2:1 eccentricity resonances by fast migration if $m_1/m_2 \geq 2$. For $(m_1+m_2)/m_0 = 10^{-3}$, the transition occurs between $\dot{\alpha}_2/\dot{\alpha}_2 = -1 \times 10^{-4}/P_2$ and $-2 \times 10^{-4}/P_2$ for $m_1/m_2 = 2.65$ and 3, and between $\dot{\alpha}_2/\dot{\alpha}_2 = -2 \times 10^{-4}/P_2$ and $-4 \times 10^{-4}/P_2$ for $m_1/m_2 = 2.3, 5,$ and 10. However, if $\dot{\alpha}_2/\dot{\alpha}_2$ is as fast as $-8 \times 10^{-4}/P_2$, the libration amplitudes are sufficiently large that the system becomes unstable soon after entering the new family. Calculations with twice the total planetary mass [($(m_1+m_2)/m_0 = 2 \times 10^{-3}$)] show that the critical migration rate for entry into the new family is roughly proportional to $(m_1+m_2)/m_0$.

To find the small-libration-amplitude (or near exact resonance) counterpart for this new family and to determine the range of $m_1/m_2$ for which this family exists, we take the large-libration-amplitude configuration at $t/P_{2.0} = 7000$ in Figure 2 and adjust the orbital parameters to obtain a small-libration-amplitude configuration with $m_1/m_2 = 3, e_1 = 0.158, e_2 = 0.702$, $\theta_1 = 1^\circ$, and $\theta_2 = 98^\circ$. This small-libration-amplitude configuration is used as the starting point for two calculations in which $m_1/m_2$ is increased or decreased slowly to find a sequence of configurations with different $m_1/m_2$. The results are shown in Figure 3, with the calculations with $d \ln(m_1/m_2)/dt = -10^{-6}/P_{2.0}$ and $10^{-6}/P_{2.0}$ along the positive and negative time axes, respectively. The inner eccentricity $e_1$ increases (and outer eccentricity $e_2$ decreases) with decreasing $m_1/m_2$, and the system becomes unstable when $m_1/m_2$ is decreased to about 0.2. The resonance configuration for a given $m_1/m_2$ from Figure 3 is then used as the starting point for slow inward ($\dot{\alpha}_2/\dot{\alpha}_2 = -10^{-6}/P_2$) and outward ($\dot{\alpha}_2/\dot{\alpha}_2 = 10^{-6}/P_2$) migration calculations to search for other resonance configurations with the same $m_1/m_2$. (The slow rate of change in $m_1/m_2$ or $\dot{\alpha}_2$ in these calculations ensures that the libration amplitudes and offsets remain small.) Figure 4(a) shows the loci in the $e_1$-$e_2$ plane of the stable resonance configurations from the migration calculations with $m_1/m_2 = 0.3, 1, 3,$ and 10. The initial conditions (dashed lines in Figure 4(a)) are indicated by the triangles in Figure 4(a), and the results from inward (outward) migration extend above (below) the triangles. In addition to the calculations shown in Figure 3, we perform several calculations in which different configurations along the locus shown in Figure 4(a) for $m_1/m_2 = 0.3$ are used as the starting point and $m_1/m_2$ is decreased. In all cases, the system becomes unstable when $m_1/m_2$ is decreased to about 0.2. Thus the new family of 2:1 eccentricity resonances does not appear to exist for $m_1/m_2 \lesssim 0.2$.

The new family of 2:1 eccentricity resonances has $e_2 \geq 0.4$–0.5, asymmetric librations, and intersecting orbits, and it is distinct from any of the families found by Lee (2004). In Figures 4(b)–(d), we compare the new family (labeled IV) with the families (labeled I–III) found by Lee (2004) for $m_1/m_2 = 3, 0.3$, and 10. Comparison of the new family (labeled IV) with the families (labeled I–III) found by Lee (2004) for $m_1/m_2 = (b) 3, (c) 1,$ and (d) 0.3, respectively. It is possible to enter the new family by fast migration only for $m_1/m_2 \gtrsim 2$ because the combination that sequences I and IV come close to each other and that some configurations along sequence I have asymmetric librations and intersecting orbits occur only for $m_1/m_2 \gtrsim 2$ (see the text for details).

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\(^5\) Forced migration causes offsets in the libration centers of the resonance variables (Lee 2004; Murray-Clay & Chiang 2005). Both the offsets and the libration amplitudes increase with the migration rate. The offsets are typically much smaller than the libration amplitudes for the asymmetric configurations but could be noticeable for, e.g., the $(\theta_1, \theta_2) \approx (0^\circ, 180^\circ)$ configuration that the system is first captured into (compare Figures 1 and 2).
1, and 0.3, respectively. Sequence I is the sequence reached by slow migration of planets with constant masses and initially nearly circular orbits; sequence II was found by a combination of calculations in which $m_1/m_2$ is changed and slow migration calculations, and sequence III consists of configurations with $(\theta_1, \theta_2) \approx (180^\circ, 0^\circ)$. For $m_1/m_2 = 3$ (Figure 4(b)), sequences I and IV come close to each other. In addition, like sequence IV, the configurations in sequence I with $e_2 \gtrsim 0.34$ have intersecting orbits, as well as asymmetric librations (Lee 2004). Thus it is possible to jump from sequence I to sequence IV if the libration amplitudes are large due to fast migration, as shown in Figure 2. For $m_1/m_2 = 1$ (Figure 4(c)), sequences I and IV do not come close to each other. For $m_1/m_2 = 0.3$ (Figure 4(d)), although sequences I and IV come relatively close to each other at large $e_1$, the configurations in sequence I at large $e_1$ have $(\theta_1, \theta_2) \approx (0^\circ, 0^\circ)$ and nonintersecting orbits, and it is not possible to jump to sequence IV with asymmetric librations and intersecting orbits. Because the combination that sequences I and IV come close to each other and that some configurations along sequence I have asymmetric librations and intersecting orbits occurs only for $m_1/m_2 \gtrsim 2$, we can understand why it is possible to enter the new family by fast migration only for $m_1/m_2 \gtrsim 2$, even though the new family exists for $m_1/m_2 \gtrsim 0.2$.

The existence of the new family of 2:1 eccentricity resonances was noted by Lee & Thommes (2004). Voyatzis & Hadjidemetriou (2005) have also discovered this family in their search for both stable and unstable asymmetric periodic orbits at the 2:1 resonance for three cases with $m_1/m_2 = 0.54, 1$, and $1.86$. The stable periodic orbits are resonance configurations with zero libration amplitudes. In the range of $m_1/m_2$ studied by Voyatzis & Hadjidemetriou (2005; i.e., $m_1/m_2 \lesssim 2.75$, Lee 2004), sequence II has loci similar to those shown in Figures 4(c) and (d), and Voyatzis & Hadjidemetriou (2005) found that sequences II and IV (the new family) are connected to each other by a sequence of unstable periodic orbits.

4. THE 4:2 INCLINATION RESONANCES

We consider next migration calculations of noncoplanar orbits without eccentricity damping. We perform calculations with initial mutual orbital inclination $i_{\text{mu}} = 0.01$ and migration rate $a_2/a_3 = -0.125, -0.25, \ldots, -4 \times 10^{-4}/P_2$. Figures 5–7 show the results for $m_1/m_2 = 3$, $a_2/a_3 = -2, -0.125$, and $-0.5 \times 10^{-4}/P_2$, respectively. In each figure, we plot the evolution of the inclinations $i_j$ and the inclination-type resonance variables $\phi_{ij}$, as well as $a_j, e_j$, and $\theta_j$. The evolution of $\phi_{i1}$ is nearly identical to that of $\phi_{i2}$, which means that $\Omega_1 - \Omega_2 = (\phi_{i1} - \phi_{i2})/2$ (which is $180^\circ$ initially due to our choice of the initial invariable plane as the $z = 0$ reference plane) is close to $180^\circ$ throughout and the ascending nodes are nearly anti-aligned.

The fast migration calculation shown in Figure 5 is similar to that in Figure 2, but with noncoplanar orbits. The $e_j$ and $\theta_j$ evolve as in the planar case shown in Figure 2 and enter the new family of eccentricity resonances when $e_2 \gtrsim 0.45$. There is no capture into the inclination resonances or excitation of the inclinations. Note, however, that the circulation of the inclination resonance variables $\phi_{ij}$ changes from prograde to retrograde at about the same time as the entry into the new family of eccentricity resonances. The evolution in this figure is that found by Thommes & Lissauer (2003) in their simulation with $m_1/m_2 = 3$. As in the planar case, the noncoplanar calculations with $m_1/m_2 = 3$ and $a_2/a_3$ slower than $-2 \times 10^{-4}/P_2$ do not enter the new family of eccentricity resonances. However, unlike the planar case, the evolution for $a_2/a_3$ slower than $-0.5 \times 10^{-4}/P_2$ is qualitatively different from that for $a_2/a_3 = -0.5$ and $-1 \times 10^{-4}/P_2$. Figure 6 shows a calculation with a slow migration rate ($a_2/a_3 = -0.125 \times 10^{-4}/P_2$). The system is initially captured into the 2:1 eccentricity resonances only, and the initial evolution after capture is similar to that in the planar calculation with slow migration shown in Figure 1.
Figure 6. Same as Figure 5, but for the slow migration rate of $\dot{a}_2/a_2 = -0.125 \times 10^{-4} / P_2$. The system is initially captured into the 2:1 eccentricity resonances only. There is a phase from $t/P_2 \approx 4.7 \times 10^4$ to $5.3 \times 10^4$ with $\phi_{jj}$ changing slowly and $i_j$ increasing rapidly before $\phi_{jj}$ are captured into libration. The system eventually evolves out of the inclination resonances at $t/P_2 \approx 1.0 \times 10^5$, and the eccentricity resonances switch to the $\theta_1 \approx 180^\circ$ and $\theta_2 \approx 0^\circ$ configuration.

Figure 7. Same as Figure 5, but for the intermediate migration rate of $\dot{a}_2/a_2 = -0.5 \times 10^{-4} / P_2$. The rapid inclination excitation phase occurs from $t/P_2 \approx 1.2 \times 10^4$ to $2.4 \times 10^4$. Then $\phi_{jj}$ and $\theta_j$ alternate between libration and circulation for about $6000 P_2$, before $\phi_{jj}$ change to circulation only and the eccentricity resonances to the $\theta_1 \approx 180^\circ$ and $\theta_2 \approx 0^\circ$ configuration.

But starting at $t/P_{2,0} \approx 4.7 \times 10^4$ (when $e_2 \approx 0.45$), the inclination resonance variables $\phi_{jj}$ change very slowly for about $6000 P_{2,0}$, and the inclinations increase rapidly. It is likely that this slow change of $\phi_{jj}$ is associated with the proximity to the separatrix of the inclination resonances, since the circulation/libration period is infinite on the separatrix. We can understand qualitatively the almost exponential growth in the inclinations by noting that the lowest order inclination resonance terms at the 2:1 commensurability in the disturbing potential $\Phi$ are second order and proportional to $i_1 \cos \phi_{11}$, $i_1 \cos \phi_{12}$, and $i_2 \cos \phi_{22}$. Thus the lowest order terms for $d \phi_{jj} / dt \propto i_j^{-1} \partial \Phi / \partial \Omega_j$ are proportional to $i_j \sin \phi_{jj}$ and $i_k \sin \phi_{12}$ (where $k = 2$ for $j = 1$ and vice versa), which can result in exponential growth if $\phi_{jj}$ and $\phi_{12}$ are not equal to $0^\circ$ or $180^\circ$ and change very slowly. At $t/P_{2,0} \approx 5.3 \times 10^4$, both $\phi_{11}$ and $\phi_{22}$ are captured into resonance and librate about $110^\circ$, and the inclinations increase slowly due to the continued migration forcing the system deeper into inclination resonances. We note that the simultaneous librations of $\theta_j$ and $\phi_{jj}$ affect the values of $\theta_j$ and $e_j$ during this phase (compare Figures 1 and 6). As in the case of the eccentricity
resonances, asymmetric libration of $\phi_{jj}$ about an angle other than $0^\circ$ or $180^\circ$ is possible when the inclinations are not small and $d\phi_{ij}/dt$ is not dominated by the lowest order terms in the disturbing potential.\footnote{In the limit of the circular, planar, restricted, three-body problem, one can identify the terms that give rise to asymmetric libration for the $n:1$ (not just $2:1$) exterior resonance as coming from the indirect part of the disturbing potential, and there is a qualitative physical explanation based on the indirect acceleration imparted on the test particle over a synodic period (Pan & Sari 2004; Murray-Clay & Chiang 2005). This type of analysis has not been generalized to either the planar two-planet problem with two resonance variables $\theta_1$ and $\theta_2$ or the inclination resonances.} We confirm that the configuration at, e.g., $t/P_{2,0} \approx 8.0 \times 10^5$ in Figure 6 is indeed in stable inclination resonances by taking that configuration as the starting point for a three-body integration without forced migration and finding stable libration of $\phi_{ij}$ about $110^\circ$ and no secular change in $i_j$ throughout that integration. In Figure 6, the system eventually evolves out of the inclination resonances at $t/P_{2,0} \approx 1.0 \times 10^5$, and the eccentricity resonances switch to the $\theta_1 \approx 180^\circ$ and $\theta_2 \approx 0^\circ$ configuration. As mentioned in Section 1, Thommes & Lissauer (2003) have seen similar switching to the $\phi_{jj}$ and larger fraction of the total evolution time with increasing $\theta_j$.

In contrast to the overall evolution on the migration timescale, the time spent in the phase with $\phi_{ij}$ changing slowly and $i_j$ increasing rapidly is nearly independent of the migration rate for slow migration. Thus this phase takes up a larger and larger fraction of the total evolution time with increasing migration rate, and there is no longer a phase with $\phi_{ij}$ clearly in resonance if the migration rate $d\phi_{ij}/dt$ is as fast as $-0.5$ and $-1 \times 10^{-4}/P$. Even faster migration rate would result in entry into the new family of eccentricity resonances and no inclination excitation, as discussed above. For $d\phi_{ij}/dt = -0.5 \times 10^{-4}/P_{2,0}$ (Figure 7), the rapid inclination excitation phase occurs from $t/P_{2,0} \approx 1.2 \times 10^4$ to $2.4 \times 10^4$. Then $\phi_{ij}$, as well as $\theta_1$, alternate between libration and circulation for about $6000P_{2,0}$, before $\phi_{ij}$ change to circulation only and the eccentricity resonances to the $\theta_1 \approx 180^\circ$ and $\theta_2 \approx 0^\circ$ configuration, with $\theta_1$ nearly circulating but spending most of its time around $180^\circ$. The oscillations of the inclination resonance variables $\phi_{ij}$ between $t/P_{2,0} \approx 1.2 \times 10^4$ to $2.4 \times 10^4$ in Figure 7 might lead one to think that $\phi_{ij}$ are in resonance and librating about equilibrium values and that the rapid increase in the inclinations is due to continued migration forcing the system deeper into inclination resonances. However, this would be inconsistent with our earlier observation that the duration of this phase is nearly independent of the migration rate for slow migration. To show that this rapid inclination excitation is in fact not due to migration forcing, we take the configuration at $t/P_{2,0} = 2.0 \times 10^4$ in Figure 7 as the starting point for a three-body integration without forced migration. The results are shown in Figure 8. As we can see, the inclinations continue to increase rapidly for about $4000P_{2,0}$ even without forced migration. Furthermore, the evolution of all the plotted variables for the first $10^4P_{2,0}$ in Figure 8 without migration is similar to that between $t/P_{2,0} = 2.0 \times 10^4$ and $3.0 \times 10^4$ in Figure 7 with migration. Figure 8 also shows us what would happen if the migration stops due to, e.g., disk dispersal when the system is in the phase with rapid inclination excitation. The inclinations would continue to increase for a while, and the inclination resonance variables would eventually end up in large-amplitude libration (alternating with circulation to varying degree).

Figure 9 summarizes the results for $(m_1 + m_2)/m_0 = 10^{-3}$ and different $m_1/m_2$ and $\tilde{a}_2/a_2$. For $m_1/m_2 \gtrsim 2$ and fast migration (the region labeled E in Figure 9), the eccentricity resonances enter the new family, and there is no capture into inclination resonances or excitation of the inclinations. For slow migration (the region below the solid line in Figure 9), the inclination resonance variables $\phi_{ij}$ are captured into libration after a phase with $\phi_{ij}$ changing slowly and $i_j$ increasing rapidly. The inclination resonance configuration is symmetric with $\phi_{jj}$ librating about $180^\circ$ in the region labeled S with $m_1/m_2 \lesssim 2.5$ (see Thommes & Lissauer 2003 for an example with $m_1/m_2 = 1$), and it is asymmetric with $\phi_{jj}$ librating about an angle other than $0^\circ$ or $180^\circ$ in the region labeled A with $m_1/m_2 \gtrsim 2.5$ (e.g., Figure 6). For intermediate migration rate (and also fast migration rate if $m_1/m_2 \lesssim 2$), we typically see...
the rapid inclination excitation phase, but not a phase with $\phi_{jj}$ clearly in resonance (e.g., Figure 7). The inclination excitation can be partial, with the mutual inclination reaching a maximum of $\approx 1^\circ$ or less, if $m_1/m_2$ is large (in particular $m_1/m_2 = 10$).

In the phase with simultaneous librations of the eccentricity and inclination resonance variables, we can see from the definitions of $\theta_j$ (Equations (1) and (2)) and $\phi_{jj}$ (Equations (4) and (5)) that the arguments of periapse $\omega_j = \sigma_j - \Omega_j = \theta_j - \phi_{jj}/2$ also librate. For $m_1/m_2 \lesssim 2.5$, with symmetric libration of $\phi_{jj}$, $\omega_j$ librate about $\pm 90^\circ$ (i.e., the periapse is on average $90^\circ$ ahead of or behind the ascending node), while for $m_1/m_2 \gtrsim 2.5$ with asymmetric libration of $\phi_{jj}$, the libration of $\omega_j$ is also asymmetric.

5. EFFECTS OF ECCENTRICITY DAMPING

As we mentioned in Section 1, sufficient eccentricity damping can prevent the eccentricities from reaching high enough values for inclination excitation and/or capture into the inclination resonances. In order to study the effects of eccentricity damping, we repeat the noncoplanar calculations in Section 4 with the ratio of eccentricity damping to migration of the outer planet, $K = |\dot{e}_2/e_2|/|\dot{a}_2/a_2|$, ranging from 0.25 to 8.

We consider first the calculations with slow migration ($\dot{a}_2/a_2$ below the solid line in Figure 9). Figure 10 shows the evolution of the mutual inclination $i_{\mu\mu}$ for $m_1/m_2 = 0.3, 1.5$, and 5.0, $\dot{a}_2/a_2 = -0.125 \times 10^{-4}/P_2$, and different $K$. As $K$ increases from zero, the system enters the rapid inclination excitation phase and the subsequent capture into the 4:2 inclination resonances later, because the eccentricities grow slower and slower. However, when $K$ exceeds a critical value, the eccentricities never reach high enough values for inclination excitation and capture into inclination resonances. The critical value of $K$ is $\approx 1.4$ for $m_1/m_2 \lesssim 0.3$, $\approx 2.8$ for $m_1/m_2 \approx 0.9-1.5$, and $\approx 0.7$ for $m_1/m_2 \gtrsim 2.65$ (Figure 11).

For faster migration rate, the effects of eccentricity damping on the evolution of the system can be more complicated. For example, the calculation shown in Figure 12 is similar to that in Figure 7 ($m_1/m_2 = 3$ and $\dot{a}_2/a_2 = -0.5 \times 10^{-4}/P_2$) but with $K = 0.25$. In this case, the eccentricity damping results in clear libration of the inclination resonance variables $\phi_{jj}$ after the rapid inclination excitation phase. Nevertheless, the critical value of $K$ as a function of $m_1/m_2$ shown in Figure 11 also summarizes the results for migration rate up to $\dot{a}_2/a_2 = -2 \times 10^{-4}/P_2$ (the maximum migration rate studied), the critical value of $K$ is modified at
large $m_1/m_2$, with none of the calculations with $m_1/m_2 \geq 5$ showing inclination excitation.

6. CONCLUSIONS

We have investigated the effects of different migration rates on the capture into and evolution in eccentricity and inclination resonances at the 2:1 mean-motion commensurability by the convergent migration of two planets. We focused on systems with orbits that are initially slightly inclined with respect to each other. The system is first captured into the sequence I of 2:1 eccentricity resonances found by Lee (2004), the same as in the case of exactly coplanar orbits. If the migration rate is fast and $m_1/m_2 \gtrsim 2$, the subsequent evolution is also identical to the coplanar case, with the eccentricity resonances entering a new family (sequence IV), and there is no inclination excitation or capture into inclination resonances. The new family of 2:1 eccentricity resonances (with $e_2 \gtrsim 0.4$–0.5, asymmetric librations, and orbits that intersect if they are exactly coplanar) exists for $m_1/m_2 \gtrsim 0.2$, but it is possible to evolve into this family by fast migration only for $m_1/m_2 \gtrsim 2$. If the migration rate is slow, the system subsequently enters a phase with the 4:2 inclination resonance variables $\phi_{jj}$ changing slowly and the inclinations increasing rapidly, before it is captured into 4:2 inclination resonances. The inclination resonance configuration is symmetric, with $\phi_{11} \approx \phi_{22} \approx 180^\circ$, if $m_1/m_2 \lesssim 2.5$ and asymmetric if $m_1/m_2 \gtrsim 2.5$. For intermediate migration rate (and fast migration rate if $m_1/m_2 \lesssim 2$), there is typically a rapid inclination excitation phase, but not a phase with $\phi_{jj}$ clearly in resonance. We have also studied the effects of different eccentricity damping rates during migration and found that the maximum value of $K = |\dot{e}_2/e_2|/|\dot{a}_2/a_2|$ for inclination excitation (which may or may not be followed by a phase with $\phi_{jj}$ clearly in resonance if the migration rate is not slow) ranges from $\approx 0.7$ for $m_1/m_2 \gtrsim 2.65$ to $\approx 2.8$ for $m_1/m_2 \sim 1$. Since the evolution is sensitive to the rates of migration and eccentricity damping within the ranges expected for type II migration due to planet–disk interaction, the discovery of extrasolar planetary systems with certain combinations of mass ratio and 2:1 resonance geometry would place a constraint on the strength of eccentricity damping during migration, as well as on the rate of migration itself.

There are several effects of disk–planet interaction that were neglected in our analysis and may require further investigations. We have focused on inward migration and eccentricity damping of the outer planet, because previous hydrodynamic simulations (e.g., Kley 2000; Kley et al. 2004) have shown that the disk inside the inner planet’s orbit, not just the disk material between the planets, should be cleared rapidly. However, Crida et al. (2008) have recently shown that a better numerical treatment of the inner disk may result in a slower depletion of the inner disk and that the eccentricity damping from the inner disk could be important in explaining the observed eccentricities of resonant pairs such as that in the GJ 876 system. On the other hand, when the nearby disk mass is comparable to the planet mass (i.e., in older, partially depleted disks), planets will undergo type II migration at significantly less than the disk’s viscous advection speed (Syrer & Clarke 1995), so that the inner disk “outruns” the planet and eventually leaves an inner hole, no matter how small the inner boundary radius. The simulations of Thommes et al. (2008) suggest the majority of planets form late enough in their parent disk’s lifetime that such holes are ubiquitous.

Planet–disk interaction can also affect the orbital inclination of a planet. The net effect of inclination damping by secular interactions and excitation by interactions at mean-motion resonances depends on the disk parameters, but any net damping should be on a timescale comparable to or longer than the migration timescale (e.g., Lubow & Ogilvie 2001). This is likely too slow to affect the rapid inclination excitation phase, but may result in equilibrium inclinations if the system is subsequently captured into inclination resonances and the inclinations are excited slowly by continued migration.

We have also neglected the secular apsidal and nodal precessions induced by the disk, which could change the sequence of resonance capture by changing and splitting the locations of the various resonances at the same mean-motion commensu-
retrograde apsidal precession is proportional to \(1/n\). Nances are first order, which means that the resonance-induced nodal precession is roughly constant for small \(e_j\). On the other hand, disk-induced nodal precession could have a larger effect, because the 4:2 inclination resonances are second order and the resonance-induced nodal precession is much larger in magnitude than the disk-induced prograde precession for small \(e_j\). This can be explained by the fact that the 2:1 eccentricity resonances are roughly constant for small \(e_j\), and much larger in magnitude than the 4:2 inclination resonances. If the system is captured into the inclination resonances, the capture requires \(\varpi_{jj} \approx -\dot{\Omega}_j \approx 0\). A better understanding of the capture into the inclination resonances is needed. Since the time spent in the phase with \(\varphi_j\) changing slowly and \(i_j\) increasing rapidly is nearly independent of the migration rate for slow migration (see Section 4), in the limit of very slow migration, there is an almost instantaneous jump in the inclinations at the time of the inclination resonance capture, if we measure time in units of the migration timescale. This is not what one would expect if the capture into the inclination resonances can be modeled by the appearance of additional equilibrium points and separatrices in the Hamiltonian theory of a single second-order resonance (see, e.g., Murray & Dermott 1999). The Hamiltonian approach is based on the assumption that each resonance is encountered individually, which is clearly not the case in our problem. In particular, the system is already in eccentricity resonances when it enters the inclination resonances. If the system is captured into just an inclination resonance, the capture requires \(\dot{\Omega}_j \approx -\dot{\lambda}_1 + 2\dot{\lambda}_2\). On the other hand, if the system is already in eccentricity resonances so that \(\varphi_j \approx -\dot{\lambda}_1 + 2\dot{\lambda}_2\), then the capture into the inclination resonances requires \(\dot{\Omega}_j \approx 0\).

It is a pleasure to thank Stan Peale for informative discussions. This research was supported in part by NASA grant NNG06GF42G (M.H.L.) and a grant from NSERC Canada (E.W.T.).

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