Quantum Field Symbolic Analog Computation: Relativity Model

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Abstract

It is natural to consider a quantum system in the continuum limit of space-time configuration. Incorporating also, Einstein’s special relativity, leads to the quantum theory of fields. Non-relativistic quantum mechanics and classical mechanics are special cases. By studying vacuum expectation values (Wightman functions $W(n; z)$ where $z$ denotes the set of $n$ complex variables) of products of quantum field operators in a separable Hilbert space, one is led to computation of holomorphy domains for these functions over the space of several complex variables, $\mathbb{C}^n$. Quantum fields were reconstructed from these functions by Wightman. Computer automation has been accomplished as deterministic exact analog computation (computation over cells in the continuum of $\mathbb{C}^n$) for obtaining primitive extended tube domains of holomorphy. This is done in a one dimensional space plus one dimensional time model. By considering boundary related semi-algebraic sets, some analytic extensions of these domains are obtained by non-deterministic methods. The novel methods of computation raise interesting issues of computability and complexity. Moreover, the computation is independent of any particular form of Lagrangian or dynamics, and is uniform in $n$, qualifying for a universal quantum machine over $\mathbb{C}^\infty$.

1 Introduction

Recently, there has been considerable interest in what is called quantum computation [1]. The efforts in this regard are to seek improved ways of performing computations or building new types of computing machines. It is hoped that not only faster computation will be achieved [1-3], but that better understanding of computational complexity will come about [4-6].

Quantum computers might appear to be discrete systems, such as a finite collection of spins or qubits [4]. But this is not the only possibility. Instead of thinking of a computer based on a physical system which is understood in
terms of non-relativistic quantum mechanics with spin added on $\frac{1}{2}$, one can also consider a system based on relativistic quantum mechanics [7].

In another direction, the classical discrete digital computer [8] has been generalized to include the possibility of computing over the continuum [9]. This is because the latter way of computing over the continuum is more appropriate to the way we do analysis, physics, and engineering problems. So this is a computing model which is based on classical mechanics. But classical mechanics could also be extended to include relativity, getting relativistic mechanics [10].

Because, in studying atomic phenomena, classical mechanics has been replaced by quantum mechanics, we could also think of more general models of computing [7, 11] based on adding relativity to quantum theory to get relativistic quantum field theory.

It is natural to consider our physical or quantum systems in the continuum limit. In fact Isaac Newton [12], when studying gravitation, found it natural to consider a continuous distribution of matter to model the earth’s gravitational action at external points. From continuum quantum mechanics, by combining relativity, we have quantum field theory.

2 Generalized Quantum Computation

The continuum limit (in space-time configuration) of quantum mechanics together with the incorporation of Einstein’s special relativity leads naturally to the relativistic quantum theory of fields. By taking the limit as the velocity of light $c \to \infty$ we expect to get non-relativistic quantum mechanics. The limit as Planck’s constant $h \to 0$ gives classical mechanics [13]. Quantum field theory includes all of quantum mechanics, classical mechanics, and much more [2]. Included also will be unitary transformations and superposition of amplitudes, which are regarded as prerequisites for quantum computation.

It has been possible to generalize quantum computation to relativistic quantum field computation in a certain model [14]. We expect other forms of quantum computation, namely, those based on non-relativistic quantum mechanics, topological quantum field theories [15] and classical mechanics [16, 17] to be related to this generalization. The relationship should shed light, not only on computing possibilities [18, 19], but also on quantum field theory itself.

3 Quantum Fields

Non-relativistic quantum mechanics is not complete, because radiative corrections have to be made to it, using field theory. In dealing with a system corresponding to an infinite number of degrees of freedom, it is well known historically that formulations of quantum field theory like perturbation theory lead to

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1 Spin used to be added in an ad hoc fashion to non-relativistic quantum mechanics until Dirac produced his relativistic equation for the electron [1].
2 Examples are, discrete anti-unitary symmetries, CPT invariance and the spin statistics connection [20].
infinities resulting in the need for renormalization. Nevertheless, quantum electrodynamics has turned out to be, “the most accurate theory known to man”³. Dirac, Schwinger and Feynman are some of the principal contributors to quantum electrodynamics ⁴ and hence to quantum field theory ⁵. Relativistic covariance is of paramount importance in correctly performing the renormalization process.

It is useful here to work within the Wightman formulation ⁶, ⁷, ⁸ of quantum field theory ⁵. We are dealing with fields in the Heisenberg picture, without using perturbation theory, nor any particular time frame related Hamiltonians. The theory is in terms of analytic functions (Wightman functions) of several complex variables. These functions arise from their boundary values which are vacuum expectation values of the form

$$W_m(x_1, x_2, \ldots x_m) = \langle \Omega, \phi_1(x_1)\phi_2(x_2)\ldots \phi_m(x_m)\Omega \rangle$$

of products of $m$ quantum field operators in a separable Hilbert space. The field operators transform according to appropriate unitary spin representations of the Poincaré (inhomogenous $SL(2, \mathbb{C})$) group. Quantum fields are uniquely reconstructed from these analytic functions by Wightman.

Let the ($m$-point) Wightman functions be denoted by $W(n; z)$ where $z$ denotes the set of $n$ complex variables. Here, $n = sm$ where $s \geq 2$ is the space-time dimension; space-time will consist of 1-time and $(s - 1)$-space dimensions ⁶.

Because these analytic functions are fundamental to the theory, one is led to computation of holomorphy domains for these functions over the space of several complex variables, $\mathbb{C}^n$ ⁹. The mass spectrum is assumed to be reasonable, in the sense that momentum vectors $p^\mu$ lie in the closed forward light cone, with time component $p^0 > 0$ except for the unique vacuum state having $p = 0$.

## 4 Computer Automation

When $s = 2$, i.e. in 1-dimensional space and 1-dimensional time, a system of light-cone coordinates is appropriate ²⁸. In this 2-dimensional space-time, computer automation has been accomplished as deterministic exact analog computation ¹⁴ (computation over “cells” in the continuum of $\mathbb{C}^n$) to obtain primitive extended tube domains of holomorphy for $W(n; z)$. By a series of abstractions the computation is done with essentially reversible logic, programming in the Prolog language, and simulating on a Turing machine.

Just as the classical computer, Turing machine, computes over $\mathbb{Z}$ or equivalently over $\mathbb{Z}_2$, we now have what can be called a complex Turing machine, in fact a, severally complex Turing machine.

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³This statement is attributed to Feynman.
⁴The spectacular history of this is related in [21].
⁵The fruitfulness and utility of this formulation, from a current perspective, is available in [22].
⁶Because it is not known, at present, how to physically understand concepts like closed time-like loops in more than one time dimension, there will be only one dimension in time [20].
The primitive extended tube domains are bounded by analytic hypersurfaces, namely Riemann cuts denoted by $C_{ij}$ and other hypersurfaces denoted by $S_{ij,kl}$ and $F_{ij,kl}$. These domains are in the form of semi-algebraic sets. Since the computation is symbolic, it is also exact, which is important in handling analytic functions.

Because of Lorentz invariance properties of the physics involved, the domains have a structure referred to as complex Lorentz projective spaces\(^7\). Related to this invariance are certain continuum cells over which the computation occurs. Thus this computation is also like analog computation which would otherwise be regarded as impossible to do exactly.

5 Analytic Extensions

In relativistic quantum field theory it is possible to implement the physical requirement of microcausality. There exists quantum microcausality (field operators commute or anticommute) at totally space-like points.

Together with the requirement of permutation invariance of the domains, the edge-of-the-wedge theorem provides enlargements of the original primitive domains of analyticity to analyticity into unions of permuted primitive domains.

Mapping these union domains creates some Boolean satisfiability problems. In fact, the novel methods of computation raise interesting issues of computability and complexity.

6 Non-deterministic Holomorphic Extensions

By the nature of analytic domains in more than one complex variable, it is in general possible to further extend these domains towards the maximal domains called envelopes of holomorphy. By considering boundary related semi-algebraic sets, there are non-deterministic computations of holomorphic extensions of domains. After the guessing step, the verification is by deterministic processes mentioned above.

Built-in permutation invariance has considerable power, just as $n!$ rapidly dominates over $2^n$ for large $n$.

7 Uniformity of Computation

Uniformity in the direction of universal computation has been discussed in \([16]\), in different contexts, including numerical analysis. We do have certain types of uniformity here.

First we note that the computation is independent of any particular form of Lagrangian or dynamics, and is uniform in $n$, qualifying for a universal quantum machine over $\mathbb{C}^\infty$. The latter space is the infinite discrete union $\bigcup_{n=1}^\infty \mathbb{C}^n$.

\(^7\)This is different from Euclidean complex projective spaces, well known in mathematics. The difference is captured in the Hall-Wightman theorem\([20]\).
7.1 Function Order Uniformity

When the program runs for \( s = 2 \), dynamic memory allocation is used through the operating system. Because \( n \) can be input as a variable, only part of the whole memory management cost is outside the program. The program itself is independent of \( n = sn \) and therefore is uniform in \( n \), which is unbounded above. We can call this function order uniformity in \( n_\infty \).

7.2 Space-time Uniformity

In addition, there is uniformity in the dimension \( s \geq 2 \) of space-time, in the following manner. Given a dimension \( s \geq 2 \) of space-time, looking at the semi-algebraic sets defining the primitive extended tube domains of holomorphy (hypersurface boundaries), and at function orders, there are three different classes of orders. These classes comprise, a) lower order W functions, b) intermediate order W functions, and c) high order W functions [30]. Extended tube domains for all high order W functions have the same complicacy. For a) we have \( m \leq s + 1 \), and for c), \( m > s(s - 1)/2 + 2 \). The remaining cases lie in class b). For example, there is no class b) for \( s = 2 \), the most complicated primitive domain being for the 3-point function. If \( s = 3 \), then \( m = 5 \) is the only case in class b). When \( s = 4 \), we have in class b), the cases, \( m = 6, 7 \) and 8.

Since \( s \geq 2 \) is unbounded above, we can call this space-time dimension uniformity in \( s_\infty \).

7.3 co-NHolo Uniformity

The holomorphy envelopes \( H[D_m] \) for different orders \( m \) of Wightman functions are related [29] in the following way.

For \( 0 < r < m \), and relative to \( H[D_m] \),

\[
H[D_m] \subset \bigcap_{\sigma \in P_m} \{ H[\sigma D_{m-r}] \times (\sigma \mathbb{C}^{sr}) \},
\]

where \( \sigma \) denotes permutations in \( P_m \), the permutation group in the \( m \) points of the \( m \)-point W function. In the case of Schlicht domains (analogous to single sheeted Riemann surfaces in \( \mathbb{C} \)), the \( \subset \) sign means set theoretic inclusion.

For example, in \( s = 2 \), the 4-point function cannot be continued beyond the 2-point function Riemann cuts nor the (permuted) 3-point function Källén-Wightman domains of holomorphy.

This is a statement regarding analyticity that does not exist, and thus refers to the complements of the domains of holomorphy; hence the use of the prefix co-. Because computations of analytic extensions of domains are non-deterministic (hence the notation \( N \)), we can say that we have co-NHolo uniformity.
8 Conclusion

We started with relativity and continuum quantum field theory. Arbitrary numbers of particles (optionally with spins) can be created or annihilated. Relying on a fruitful set of models, we have related what appeared to be different models of quantum and classical computation based on non-relativistic quantum mechanics and classical mechanics. Exact deterministic and non-deterministic computation over continuous domains appear naturally. Furthermore there is uniformity in computation over, unbounded above, or arbitrarily high order $n$ of $W(n; z)$ and arbitrarily high dimension $s$ of space-time. In the present context this can be called $co$-$NHolo$ uniformity in $n_\infty$ and $s_\infty$. The novel methods of computation raise interesting issues of computability and complexity, and possibly could shed more light on quantum field theory itself.

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