A Size of $\sim 10 \text{Mpc}$ for the Ionized Bubbles at the End of Cosmic Reionization

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The first galaxies to appear in the universe at redshifts $z \gtrsim 20$ created ionized bubbles in the intergalactic medium (IGM) of neutral hydrogen (H I) left over from the Big-Bang. It is thought that the ionized bubbles grew with time, surrounded clusters of dwarf galaxies\textsuperscript{1,2} and eventually overlapped quickly throughout the universe over a narrow redshift interval near $z \sim 6$. This event signaled the end of the reionization epoch when the universe was a billion years old. Measuring the hitherto unknown size distribution of the bubbles at their final overlap phase is a focus of forthcoming observational programs aimed at highly redshifted 21cm emission from atomic hydrogen. Here we show that the combined constraints of cosmic variance and causality imply an observed bubble size at the end of the overlap epoch of $\sim 10$ physical Mpc, and a scatter in the observed redshift of overlap along different lines-of-sight of $\sim 0.15$. This scatter is consistent with observational constraints from recent spectroscopic data on the farthest known quasars. Our novel result implies that future radio experiments should be tuned to a characteristic angular scale of $\sim 0.5^\circ$ and have a minimum frequency band-width of $\sim 8 \text{ MHz}$ for an optimal detection of 21cm flux fluctuations near the end of reionization.
During the reionization epoch, the characteristic bubble size (defined here as the spherically averaged mean radius of the H II regions that contain most of the ionized volume\(^2\)) increased with time as smaller bubbles combined until their overlap completed and the diffuse IGM was reionized. However, the largest size of isolated bubbles (fully surrounded by H I boundaries) that can be observed is finite, because of the combined phenomena of cosmic variance and causality. Figure 1 presents a schematic illustration of the geometry. There is a surface on the sky corresponding to the time along different lines-of-sight when the diffuse (uncollapsed) IGM was *most recently neutral*. We refer to it as the Surface of Bubble Overlap (SBO). There are two competing sources for fluctuations in the SBO, each of which is dependent on the characteristic size, \(R_{\text{SBO}}\), of the ionized regions just before the final overlap. First, the finite speed of light implies that photons observed from different points along the curved boundary of an H II region must have been emitted at different times during the history of the universe. Second, bubbles on a co-moving scale \(R\) achieve reionization over a spread of redshifts due to cosmic variance in the initial conditions of the density field smoothed on that scale. The characteristic scale of H II bubbles grows with time, leading to a decline in the spread of their formation redshifts\(^1\) as the cosmic variance is averaged over an increasing spatial volume. However, the light-travel time across a bubble rises concurrently. Suppose a signal photon which encodes the presence of neutral gas (e.g. a 21cm line photon), is emitted from the far edge of the ionizing bubble. If the adjacent region along the line-of-sight has not become ionized by the time this photon reaches the near side of the bubble, then the photon will encounter diffuse neutral gas. Other photons emitted at this lower redshift will therefore also encode the presence of diffuse neutral gas, implying that the first photon was emitted prior to overlap, and not from the SBO. Hence the largest observable scale of H II regions when their overlap completes, corresponds to the first epoch at which the light crossing time becomes larger than the spread in formation times of ionized regions. Only then will the signal photon leaving the far side of the HII
region have the lowest redshift of any signal photon along that line-of-sight.

The observed spectra of all quasars beyond $z \sim 6.1$ each show a Gunn-Peterson trough\textsuperscript{3,4}, a blank spectral region at wavelengths shorter than Ly$\alpha$ at the quasar redshift, indicating the presence of H I in the diffuse IGM. The detection of Gunn-Peterson troughs indicates a rapid change\textsuperscript{5,6,7} in the neutral content of the IGM at $z \sim 6$, and hence a rapid change in the intensity of the background ionizing flux. This rapid change implies that overlap, and hence the reionization epoch, concluded near $z \sim 6$. The most promising observational probe\textsuperscript{8,9} of the reionization epoch is redshifted 21cm emission from intergalactic H I. Future observations using low frequency radio arrays (e.g. the Low Frequency Array, LOFAR) will allow a direct determination of the topology and duration of the phase of bubble overlap. Here we determine the expected angular scale and redshift width of the 21cm fluctuations at the SBO theoretically, and show that our determination is consistent with current observational constraints.

We start by quantifying the constraints of causality and cosmic variance. First suppose we have an H II region with a physical radius $R/(1 + \langle z \rangle)$. The light crossing time of this radius is

$$\langle \Delta z^2 \rangle^{1/2} = \left| \frac{dz}{dt} \right|_{\langle z \rangle} \frac{R}{c(1 + \langle z \rangle)};$$

where at the high-redshifts of interest $(dz/dt) = -(H_0\sqrt{\Omega_m})(1 + z)^{5/2}$. Here, $c$ is the speed of light, $H_0$ is the present-day Hubble constant, $\Omega_m$ is the present day matter density parameter, and $\langle z \rangle$ is the mean redshift of the SBO. Note that when discussing this crossing time, we are referring to photons used to probe the ionized bubble (e.g. at 21cm), rather than photons involved in the dynamics of the bubble evolution.

Second, overlap would have occurred at different times in different regions of the IGM due to the cosmic scatter in the process of structure formation within finite spatial volumes\textsuperscript{1}. Reionization should be completed within a region of co-moving radius $R$ when
the fraction of mass incorporated into collapsed objects in this region attains a certain critical value, corresponding to a threshold number of ionizing photons emitted per baryon. The ionization state of a region is governed by the enclosed ionizing luminosity, by its over-density, and by dense pockets of neutral gas that are self shielding to ionizing radiation. There is an offset $\delta z$ between the redshift when a region of mean over-density $\bar{\delta}_R$ achieves this critical collapsed fraction, and the redshift $\bar{z}$ when the universe achieves the same collapsed fraction on average. This offset may be computed from the expression for the collapsed fraction

$$F_{\text{col}}(M_{\text{min}}) = \text{erfc} \left( \frac{\delta_c - \bar{\delta}_R}{\sqrt{2[\sigma^2_R \sigma^2_{R\min} - \sigma^2_R]}} \right),$$

yielding

$$\frac{\delta z}{(1 + \bar{z})} = \frac{\bar{\delta}_R}{\delta_c(\bar{z})} - \left[ 1 - \sqrt{1 - \frac{\sigma^2_R}{\sigma^2_{R\min}}} \right],$$

(2)

where $\delta_c(\bar{z}) \propto (1 + \bar{z})$ is the collapse threshold for an over-density at a redshift $\bar{z}$; $\sigma_R$ and $\sigma_{R\min}$ are the variances in the power-spectrum linearly extrapolated to $z = 0$ on co-moving scales corresponding to the region of interest and to the minimum galaxy mass $M_{\text{min}}$, respectively. The offset in the ionization redshift of a region depends on its linear over-density, $\bar{\delta}_R$. As a result, the distribution of offsets, and therefore the scatter in the SBO may be obtained directly from the power spectrum of primordial inhomogeneities. As can be seen from equation (2), larger regions have a smaller scatter due to their smaller cosmic variance.

Note that equation (2) is independent of the critical value of the collapsed fraction required for reionization. Moreover, our numerical constraints are very weakly dependent on the minimum galaxy mass, which we choose to have a virial temperature of $10^4$K corresponding to the cooling threshold of primordial atomic gas. The growth of an H II bubble around a cluster of sources requires that the mean-free-path of ionizing photons be of order the bubble radius or larger. Since ionizing photons can be absorbed by dense pockets of neutral gas inside the H II region, the necessary increase in the mean-free-path
with time implies that the critical collapsed fraction required to ionize a region of size $R$ increases as well. This larger collapsed fraction affects the redshift at which the region becomes ionized, but not the scatter in redshifts from place to place which is the focus of this Letter. Our results are therefore independent of assumptions about unknown quantities such as the star formation efficiency and the escape fraction of ionizing photons from galaxies, as well as unknown processes of feedback in galaxies and clumping of the IGM.

Figure 2 displays our two fundamental constraints. The causality constraint (Eq. 1) is shown as the blue line, giving a longer crossing time for a larger bubble size. This contrasts with the constraint of cosmic variance (Eq. 2), indicated by the red line, which shows how the scatter in formation times decreases with increasing bubble size. The scatter in the SBO redshift and the corresponding fluctuation scale of the SBO are given by the intersection of these curves. We find that the thickness of the SBO is $\langle \Delta z^2 \rangle^{1/2} \sim 0.13$, and that the bubbles which form the SBO have a characteristic co-moving size of $\sim 60$Mpc (equivalent to 8.6 physical Mpc). At $z \sim 6$ this size corresponds to angular scales of $\theta_{\text{SBO}} \sim 0.4$ degrees on the sky. We have also examined a second model for the formation of H II regions, where ionization is reached when the collapsed fraction divided by the density contrast exceeds a critical value. This model allows for the possibility that the increased recombination rate offsets reionization in overdense regions. The increased range of formation times in this case leads to a slightly larger value for the scale of overlapping H II regions, and we find $\langle \Delta z^2 \rangle^{1/2} \sim 0.2$, $R_{\text{SBO}} \sim 90$Mpc and $\theta_{\text{SBO}} \sim 0.6$ degrees.

A scatter of $\sim 0.15$ in the SBO is somewhat larger than the value extracted from existing numerical simulations$^{11,12}$. The difference is most likely due to the limited size of the simulated volumes; while the simulations appropriately describe the reionization process within limited regions of the universe, they are not sufficiently large to describe the global properties of the overlap phase$^1$. The scales over which cosmological radiative transfer has
been simulated are smaller than the characteristic extent of the SBO, which we find to be $R_{SBO} \sim 70$ co-moving Mpc.

We can constrain the scatter in the SBO redshift observationally using the spectra of the highest redshift quasars. Since only a trace amount of neutral hydrogen is needed to absorb Ly$\alpha$ photons, the time where the IGM becomes Ly$\alpha$ transparent need not coincide with bubble overlap. Following overlap the IGM was exposed to ionizing sources in all directions and the ionizing intensity rose rapidly. After some time the ionizing background flux was sufficiently high that the H I fraction fell to a level at which the IGM allowed transmission of resonant Ly$\alpha$ photons. This is shown schematically in Figure 1. The lower wavelength limit of the Gunn-Peterson trough corresponds to the Ly$\alpha$ wavelength at the redshift when the IGM started to allow transmission of Ly$\alpha$ photons along that particular line-of-sight. In addition to the SBO we therefore also define the Surface of Ly$\alpha$ Transmission (hereafter SLT) as the redshift along different lines-of-sight when the diffuse IGM became transparent to Ly$\alpha$ photons.

The scatter in the SLT redshift is an observable which we would like to compare with the scatter in the SBO redshift. The variance of the density field on large scales results in the biased clustering of sources$^1$. H II regions grow in size around these clusters of sources. In order for the ionizing photons produced by a cluster to advance the walls of the ionized bubble around it, the mean-free-path of these photons must be of order the bubble size or larger. After bubble overlap, the ionizing intensity at any point grows until the ionizing photons have time to travel across the scale of the new mean-free-path, which represents the horizon out to which ionizing sources are visible. Since the mean-free-path is larger than $R_{SBO}$, the ionizing intensity at the SLT averages the cosmic scatter over a larger volume than at the SBO. This constraint implies that the cosmic variance in the SLT redshift must be smaller than the scatter in the SBO redshift. However, it is possible that opacity from
small-scale structure contributes additional scatter to the SLT redshift.

If cosmic variance dominates the observed scatter in the SLT redshift, then based on the spectra of the three $z > 6.1$ quasars\textsuperscript{4,7} we would expect the scatter in the SBO redshift to satisfy $\langle \Delta z^2 \rangle^{1/2}_{\text{obs}} \lesssim 0.05$. In addition, analysis of the proximity effect for the size of the H II regions around the two highest redshift quasars\textsuperscript{13,14} implies a neutral fraction that is of order unity (i.e. pre-overlap) at $z \sim 6.2 - 6.3$, while the transmission of Ly$\alpha$ photons at $z \lesssim 6$ implies that overlap must have completed by that time. This restricts the scatter in the SBO to be $\langle \Delta z^2 \rangle^{1/2}_{\text{obs}} \lesssim 0.25$. The constraints on values for the scatter in the SBO redshift are shaded gray in Figure 2. It is reassuring that the theoretical prediction for the SBO scatter of $\langle \Delta z^2 \rangle^{1/2}_{\text{obs}} \sim 0.15$, with a characteristic scale of $\sim 70$ co-moving Mpc, is bounded by these constraints.

The presence of a significantly neutral IGM just beyond the redshift of overlap\textsuperscript{13,14} is encouraging for upcoming 21cm studies of the reionization epoch as it results in emission near an observed frequency of 200 MHz where the signal is most readily detectable. Future observations of redshifted 21cm line emission at $6 \lesssim z \lesssim 6.5$ with instruments such as the Low Frequency Array (LOFAR), will be able to map the three-dimensional distribution of HI at the end of reionization. The intergalactic H II regions will imprint a 'knee' in the power-spectrum of the 21cm anisotropies on a characteristic angular scale corresponding to a typical isolated H II region\textsuperscript{8}. Our results suggest that this characteristic angular scale is large at the end of reionization, $\theta_{\text{SBO}} \sim 0.5$ degrees, motivating the construction of compact low frequency arrays. An SBO thickness of $\langle \Delta z^2 \rangle^{1/2}_{\text{obs}} \sim 0.15$ suggests a minimum frequency band-width of $\sim 8$ MHz for experiments aiming to detect anisotropies in 21cm emission just prior to overlap. These results will help guide the design of the next generation of low-frequency radio observatories in the search for 21cm emission at the end of the reionization epoch.
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The distances to the observed Surface of Bubble Overlap (SBO) and Surface of Ly$\alpha$ Transmission (SLT) fluctuate on the sky. The SBO corresponds to the first region of diffuse neutral IGM observed along a random line-of-sight. It fluctuates across a shell with a minimum width dictated by the condition that the light crossing time across the characteristic radius $R_{SBO}$ of ionized bubbles equals the cosmic scatter in their formation times. Thus, *causality* and *cosmic variance* determine the characteristic scale of bubbles at the completion of bubble overlap. After some time delay the IGM becomes transparent to Ly$\alpha$ photons, resulting in a second surface, the SLT. The upper panel illustrates how the lines-of-sight towards two quasars (Q1 in red and Q2 in blue) intersect the SLT with a redshift difference $\delta z$. The resulting variation in the observed spectrum of the two quasars is shown in the lower panel. Observationally, the ensemble of redshifts down to which the Gunn-Peterson troughs are seen in the spectra of $z > 6.1$ quasars is drawn from the probability distribution $dP/dz_{SLT}$ for the redshift at which the IGM started to allow Ly$\alpha$ transmission along random lines-of-sight. The observed values of $z_{SLT}$ show a small scatter in the SLT redshift around an average value of $\langle z_{SLT} \rangle \approx 5.95$. Some regions of the IGM may have also become transparent to Ly$\alpha$ photons prior to overlap, resulting in windows of transmission inside the Gunn-Peterson trough (one such region may have been seen in SDSS J1148+5251). In the existing examples, the portions of the universe probed by the lower end of the Gunn-Peterson trough are located several hundred co-moving Mpc away from the background quasar, and are therefore not correlated with the quasar host galaxy. The distribution $dP/dz_{SLT}$ is also independent of the redshift distribution of the quasars. Moreover, lines-of-sight to these quasars are not causally connected at $z \sim 6$ and may be considered independent.
theoretical prediction

allowed observationally
Fig. 2.— Constraints on the scatter in the SBO redshift and the characteristic size of isolated bubbles at the final overlap stage, $R_{SBO}$ (see Fig. 1). The characteristic size of H II regions grows with time. The SBO is observed for the bubble scale at which the light crossing time (blue line) first becomes smaller than the cosmic scatter in bubble formation times (red line). At $z \sim 6$, the implied scale $R_{SBO} \sim 60$ co-moving Mpc (or $\sim 8.6$ physical Mpc), corresponds to a characteristic angular radius of $\theta_{SLT} \sim 0.4$ degrees on the sky. After bubble overlap, the ionizing intensity grows to a level at which the IGM becomes transparent to Ly$\alpha$ photons. The collapsed fraction required for Ly$\alpha$ transmission within a region of a certain size will be larger than required for its ionization. However, the scatter in equation (2) is not sensitive to the collapsed fraction, and so may be used for both the SBO and SLT. The scatter in the SLT is smaller than the cosmic scatter in the structure formation time on the scale of the mean-free-path for ionizing photons. This mean-free-path must be longer than $R_{SBO} \sim 60$Mpc, an inference which is supported by analysis of the Ly$\alpha$ forest at $z \sim 4$ where the mean-free-path is estimated to be $\sim 120$ co-moving Mpc at the Lyman limit (and longer at higher frequencies). If it is dominated by cosmic variance, then the scatter in the SLT redshift provides a lower limit to the SBO scatter. The three known quasars at $z > 6.1$ have Ly$\alpha$ transmission redshifts of $z_{SLT} = 5.9, 5.95$ and 5.98, implying that the scatter in the SBO must be $\gtrsim 0.05$ (this scatter may become better known from follow-up spectroscopy of Gamma Ray Burst afterglows at $z > 6$ that might be discovered by the SWIFT satellite). The observed scatter in the SLT redshift is somewhat smaller than the predicted SBO scatter, confirming the expectation that cosmic variance is smaller at the SLT. The scatter in the SBO redshift must also be $\lesssim 0.25$ because the lines-of-sight to the two highest redshift quasars have a redshift of Ly$\alpha$ transparency at $z \sim 6$, but a neutral fraction that is known from the proximity effect to be substantial at $z \gtrsim 6.2 - 6.3$. The excluded regions of scatter for the SBO are shown in gray. Throughout this Letter, we adopt the latest values for the cosmological parameters as inferred from the Wilkinson Microwave Anisotropy Probe data.