Electromagnetically Induced Transparency In Rydberg Atomic Medium

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Abstract. Due to possessing big principal quantum number, Rydberg atom has some unique properties, for example: its radiative lifetime is long, dipole moment is large, and interaction between atoms is strong and so on. These properties make one pay attention to Rydberg atoms. In this paper we investigate the effects of Rydberg dipole-dipole interactions on electromagnetically induced transparency (EIT) schemes and group velocity in three-level systems of ladder type, which provides theoretical foundation for exploring the linear and nonlinear characteristics of light in a Rydberg electromagnetically-induced-transparency medium.

1. Introduction
Rydberg atoms have unique properties that other neutral atoms don’t have, especially dipole blockade effect [1], which have important research values and application prospects in ultracold plasma physics [2] and quantum information science. When a Rydberg atom excites to the Rydberg state, the strong dipole-dipole interactions will cause the energy shift of excited states of the neighboring Rydberg atoms, which bring adjacent atoms out of two-photon resonance. Other atom is not in Rydberg state because there is blockade effects in system. The dipole blockade effect is characteristic property of Rydberg atom, which provides a new method of manipulation of quantum systems [3]. It has been shown that the dipole blockade effect makes the optical response changed in Rydberg atomic medium. The strong correlation between Rydberg atomic ensemble has attracted intensive studies recently [4], and photonic phase gates and many-atom entanglement have been realized due to Rydberg interaction [5]. On the other hand, nonlinear and quantum optics in Rydberg atomic ensembles has been studied intensively.

In the last decade, some significant achievements of Rydberg atomic properties were achieved in theoretical and experimental studies [6]. Some novel quantum devices have been achieved due to dipole blockade, for example one can use it to make a light source with single photon and design single-photon subtractors and light switch with single photon [7]. There has a prediction on the form For a Wigner crystal of individual photon, Refs. [8] propose a scheme for realize it. Besides, there have some researches on dynamic propagation of light fields in Rydberg media [9]. Mohapatra and his co-workers obtain some significant results when they consider electromagnetically induced transparency of Rydberg atom with blockade effect [10]. Some experiments studied the influence of dipole interactions on CPT and EIT [11] and the strong correlation between Rydberg atoms has been observed [12]. Rydberg blockade effect is a topic of concern in recent years. Along with the technology
of quantum coherent manipulation, more novel and more interesting phenomena will be occurred, which is a motive for us to do this research.

In this paper, we study the linear and nonlinear response of probe light when three-level Rydberg atomic system is in the case of electromagnetically induced transparency. Based on the theory about interaction between light and atom, we build the equations for evolution of our system. According to the method of series of expansion, we obtain the expressions of density matrix elements and calculate the first-order linear expression and third-order nonlinear expression. Then we draw the figures of absorption and dispersion under the appropriate parameters. Finally, we compare the images under the condition of different interactions between atoms and discuss the influences of strong blockade effects on spectra of EIT and group velocity. We show that the linear and nonlinear response experience some changes in three-level Rydberg medium.

2. Theoretical model and numerical analysis

We now investigate an atomic system composed of N Rydberg atoms with a ladder type three-level structure. State \(|1\rangle\) is its ground state, State \(|2\rangle\) is used to stand for the intermediate level, and Rydberg state is \(|3\rangle\). A probe beam with angular frequency \(\omega_p\) and a Rabi frequency \(\Omega_p\) drive the transition between states \(|1\rangle\) and \(|2\rangle\). In addition, we use a strong control beam with a Rabi frequency \(\Omega_c\) and angular frequency \(\omega_c\) to couple with the states \(|2\rangle\) and \(|3\rangle\). Considering the dipole-dipole interaction, we should describe the system by the Hamiltonian

\[
H = \sum_{i=1}^{N}[-(\Delta_2 + \Delta_3)\sigma_3^i - \Delta_2\sigma_{22}^i + \Omega_c(\sigma_{23}^i + \sigma_{32}^i) + \Omega_p^*\sigma_{12}^i + \Omega_p\sigma_{21}^i + \sum_{j=1}^{N}k_{ij}\sigma_{33}^i\sigma_{33}^j],
\]

where \(\Delta_2 = \omega_p - \omega_{12}\) is a detuning between the probe field and atom and \(\Delta_3 = \omega_c - \omega_{23}\) is the coupling detuning, and \(\sigma_{ab}^i = |\alpha_i\rangle\langle\beta_i|\) \((\alpha, \beta = 1, 2, 3)\). The density matrix equations can be written as

\[
\frac{d\rho_{12}}{dt} = -i[\Omega_c\rho_{13} + (1 - \rho_{33} - \rho_{22})\Omega_p]\] - \Gamma_{12}\rho_{12},
\]

\[
\frac{d\rho_{13}}{dt} = -i[-\Omega_p\rho_{23} + \Omega_c\rho_{12} + V_{13}] - \Gamma_{13}\rho_{13},
\]

\[
\frac{d\rho_{23}}{dt} = -i[\Omega_c\rho_{13} - (\rho_{33} - \rho_{22})\Omega_c + V_{23}] - \Gamma_{23}\rho_{23},
\]

\[
\frac{d\rho_{22}}{dt} = -i[-\Omega_p\rho_{12} + \Omega_p\rho_{21} + (\rho_{23} - \rho_{32})\Omega_c] - \gamma_{22}\rho_{22},
\]

\[
\frac{d\rho_{33}}{dt} = -i\Omega_c[(\rho_{32} - \rho_{23})] - \gamma_{33}\rho_{33},
\]

where \(\Gamma_{12} = y_{12} - i\Delta_2\), \(\Gamma_{13} = y_{13} - i\Delta_3\) and \(\Gamma_{23} = y_{23} - i\Delta_3\). We know the Rydberg state \(|3\rangle\) has a long radiative lifetime, so the decay \(y_{13} \rightarrow 0\). For simplicity, the parameters are set as \(y_{23} = y_{12} = \gamma \cdot V_{ab}^i\) are the dipole-dipole interactions between Rydberg atoms.

The above equations can be constrained by \(\rho_{33} + \rho_{22} + \rho_{11} = 1\). Here we discuss the effect of weak probe under the condition of strong coupling mechanism, that is, the intensity of coupling field is much stronger than that of probe light. In this case, we assume the initial state of system is in the state \(|1\rangle\), so the zeroth-order solution will be \(\rho_{11}^{(0)} = 1\), \(\rho_{22}^{(0)} = 0\) and \(\rho_{33}^{(0)} = 0\). In order to calculate the linear and nonlinear expressions, we need to take the steady-state solution of the above equations. The density matrix elements are described as \(\rho_{mn} = \rho_{mn}^{(0)} + \rho_{mn}^{(1)} + \rho_{mn}^{(2)} + \rho_{mn}^{(3)} + \ldots\). In addition, weak probe field can be expressed as \(\Omega_p \rightarrow \Delta \Omega_p\), where \(\Delta\) is a smallness parameter. Substituting expressions \(\rho_{mn}\) and \(\Omega_p\) into the Eqs. (2), we can obtain density matrix elements. Here we need to discuss response of linear and nonlinear effects, therefore, we need third order expression about the matrix element \(\rho_{12}^{(3)}\):

\[
\rho_{12}^{(3)} = \frac{\Omega_p \rho_{22}^{(0)} + i\Omega_c \rho_{13}^{(0)} - i\Omega_c \rho_{12}^{(0)}}{\alpha_{12}^2 + \Gamma_{12}^2},
\]

where we neglect the expressions of two-order density matrix elements. According to the Eqs. (3) and (4), we can numerically analyze the absorption and dispersion of the probe light.
Fig. 1. The linear response for ladder-type system ($\gamma = 2 , \Omega_\gamma = \gamma , \Omega_\rho = 0.01$) changing with the detuning between probe light and atomic transition when setting different values of $V_{13}$, $V_{13} = 0$ in (a), $V_{13} = 1$ in (b) and $V_{13} = 5$ in (c). The lines in (d) stand for dispersion in $V_{13} = 1$ and $V_{13} = 5$, respectively.

According to expressions (3) and (4), we show the linear response in Fig.1 and nonlinear response in Fig.2, respectively. From Fig.1 we can see that when $V_{13} = 0$ in (a), the EIT lines show the features of universal atoms. The resonant part shows destructive coherence, which is caused by dark resonance. However, the figure changes when $V_{13}$ has specific number. There has a negative absorption in figure (b) and the absorption is weaker than that of no interactions in figure (a), which means that the transmission increases and light amplification is realized when considering the interaction. Next we compare figure (b) and figure (c), which are the linear response in $V_{13} = 1$ and $V_{13} = 5$, respectively. It is shown that the peak values of absorption and dispersion increase along with the increase of the interaction, meaning that the linear absorption and dispersion are enhanced. On the other hand, the group velocity also varies due to the interactions. We plot linear dispersion given by the real part of $\rho^{(1)}_{12}$ in (d) in $V_{13} = 1$ and $V_{13} = 5$. It is shown that the slope of the line gets more steeper when the interaction increases. It means that when the dipole-dipole interaction between atoms increases, the group velocity decreases.

It is noted that the variation of the nonlinear response (Fig.2) is very similar to that of the linear response. The Kerr nonlinearity is related to both $V_{13}$ and $V_{23}$. For the first two images, when $V_{13}$ and $V_{23}$ exists in figure (b), a negative absorption appears, meaning the nonlinear absorption gets weaker. The interactions in figure (c) are stronger than that of figure (b), showing that the absorption and dispersion increase as the interactions increase. In figure (d), we can see that when the interactions between atoms increase, the slope of the dispersion becomes bigger, leading to the decrease of the group velocity.
Fig. 2. The nonlinear response for ladder-type system \( (\gamma = 2, \gamma_{23} = \gamma_{33} = 2\gamma, \Omega = \gamma, \Omega_\rho = 0.01) \) changing with the detuning between probe light and atomic transition when setting different values of \( V_{13} \) and \( V_{23} \). \( V_{13} = 0, V_{23} = 0 \) in (a); \( V_{13} = 1, V_{23} = 1 \) in (b); \( V_{13} = 5, V_{23} = 5 \) in (c). The lines in (d) stand for dispersion in \( V_{13} = 1, V_{23} = 1 \) and \( V_{13} = 5, V_{23} = 5 \), respectively.

3. Conclusion
In our work, we theoretically investigate the electromagnetically-induced transparency in Rydberg atomic system with a ladder three-level structure, and our studies include the linear and nonlinear responses. We show that the strong Rydberg blockade effect between Rydberg atoms has a great influence on spectra of electromagnetically induced transparency. The group velocity decreases along with the increase of the interaction. As a result of it, the light transportation is affected. The conclusion provides a theoretical approach to the research of EIT features in three-level Rydberg atomic ensemble.

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