Large Eddy Simulations of a turbulent periodic channel with conjugate heat transfer at low Prandtl number

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Abstract. Thermal striping is one of the possible initiator of pipe rupture. In this framework, thermal fluctuations in a heated periodic channel have been calculated using Large Eddy Simulation (LES). The fluid Prandtl number is set to 0.01 and the friction Reynolds number to 395. The Werner and Wengle Wall Function is used with the Navier-Stokes equations to reduce the computational cost. Satisfactory results can be noticed on the temperature fluctuations for low Prandtl number fluids. Several boundary conditions are considered, namely isothermal, isoflux, and conjugate heat transfer. The impacts of the wall properties on the temperature statistics for conjugated heat transfer boundary conditions are deeply analysed.

1. Introduction

Thermal striping is a phenomenon that needs to be quantified and prevented. It is one of the possible initiator of pipe rupture. This phenomenon is all the more critical for nuclear power plants as it can lead to accidental situations, like in Civaux [1], Phenix [2] or Tsuruga [3]. In these reactors, the mixing between hot and cold fluids (water for Civaux and Tsuruga, sodium for Phenix) in a T-junction induces thermal fatigue in the pipe. The risk of thermal striping may increase in the next generation of power plants because of the use of higher coolant temperatures. Thus, to improve the nuclear power plants safety as request by the GEN IV forum, the fundamental cause of thermal striping must be better understood and critical configurations should be pointed out by experiments [4, 5] or numerical studies. Numerical simulations of the mixing Tee configuration have been performed using several methods as URANS (Unsteady Reynolds Averaged Navier-Stokes) [6, 7], LES (Large Eddy Simulation) [8, 9], or DNS (Direct Numerical Simulation) [10, 11].

Fluid dynamics and heat transfer in the classical periodic channel have been widely considered to improve the understanding of physical phenomena involved in thermal turbulent flows. Moser et al. [12] first use DNS to study the turbulent flow in an isothermal channel up to $Re_\tau = 590$ (see nomenclature at the end for detailed definitions). Tiselj et al. [13] calculated the heat transfer with three different boundary conditions: isothermal, isoflux and conjugate heat transfer. The temperature fluctuations at the fluid-wall interface are shown to be very sensitive to the boundary condition. Hoyas and Jiménez [14, 15] carried out DNS of high Reynolds turbulent channels, up to $Re_\tau = 2003$.

In this framework, to get a better understanding of low Prandtl fluid flows, thermal fluctuations in a heated periodic channel have been calculated. The Prandtl number is set to 0.01, which tallies with liquid metals. LES is used for its suitability to accurately simulate...
turbulent flows with an acceptable computational cost [16]. A wall function for Navier-Stokes equations is used to emphasise this asset as it allows the use of a coarser computational grid. In low Prandtl fluids, the characteristic scales of thermal fluctuations are much bigger than the dynamic ones. Then, even without any wall function for the thermal equation, the thermal field is well resolved. Isothermal and iso-flux boundary conditions are considered along with conjugate heat transfer boundary conditions at a $Re$ number of 395.

The numerical methods are presented in section 2. The computational domain, dimensionless governing equations and the numerical schemes are detailed in this section. First results are shown in section 3. The accuracy of the numerical methods is illustrated, both on dynamic and thermal profiles. Then, in section 4, a parametric study is performed on the governing parameters of the wall conduction equation.

2. Numerical methods
2.1. Computational domain
The computational domain is a streamwise and spanwise periodic channel (see Figure 1). A volume heat source is applied inside the fluid and three cases are considered for the thermal boundary conditions. The first two cases correspond to isothermal or iso-flux thermal boundary conditions. The third boundary condition consists in simulating the wall domain, and completely solving the thermal equation inside the wall. In this configuration, hereafter denoted conjugate heat transfer, the external sides of the upper and lower walls are set adiabatic. A constant volume heat sink is applied inside the walls with a heat sink value set to ensure an instantaneous conservation of the total enthalpy inside the computational domain.

In the present work, we use two types of grids, a coarse grid and a fine grid. In addition to the number of cells (and consequently to the size of cells), the main difference between the two configurations is the use of a wall function for the Navier-Stokes equations with the coarse grid. Regarding the coarse grid, the meshing along wall-to-wall axis ($y$ direction) is regular, with a spacing of $\Delta y^+ = \Delta y/l_r = \Delta y^+Re_r = 20$. This meshing leads to a first grid point located at $y^+ = 10$ which is classical when a wall function is considered. For the fine grid, a growing meshing starting with $y^+ = 1$ near the wall up to $y^+ = 27$ at channel center is used. The details of the two different grids used in this work are given in Table 1. The suffix "WF" indicates the use of a wall function.

For result visualisations, the reduced temperature $\theta^+ = T^+ - T_w^+$ is considered. This operation leads to $\theta_1^+ = 0$ and $\frac{\partial \theta^+}{\partial y^+} |_{y^+=f} = Re_r Pr$. 

Figure 1. Computational domain.
2.2. Governing equations

This configuration is governed by six dimensionless equations as pointed out in [17]. The chosen set of wall parameters is \((K, G, d^+)\):

\[
\begin{align*}
\nabla^+ \cdot \mathbf{u}^+ &= 0 \\
\frac{\partial \mathbf{u}^+}{\partial t^+} &= -\mathbf{u}^+ \cdot \nabla^+ \mathbf{u}^+ + \frac{1}{Re_\tau} \nabla^+2 \mathbf{u}^+ - \nabla^+ P^+ + \mathbf{F}^+ \\
\frac{\partial T^+}{\partial t^+} &= -\mathbf{u}^+ \cdot \nabla^+(T^+) + \frac{1}{Re_\tau Pr} \nabla^+2 T^+ + \frac{\mathbf{u}_i^+}{u_b^+} \\
T_{f,i}^+ &= T_{w,i}^+ \\
\frac{\partial T_I^+}{\partial y^+_i} &= \frac{1}{K \sqrt{G}} \frac{\partial T_{w,i}^+}{\partial y^+_i} \\
\frac{\partial T^+}{\partial t^+} &= \frac{1}{GRe_\tau Pr} \nabla^+2 T^+ - \frac{K}{d^+ \sqrt{G}}
\end{align*}
\]

These equations represent the modelling of mass (1), of momentum (2), of enthalpy inside the fluid (3), the boundary condition for the temperature at fluid-wall interfaces (4), the boundary condition for heat flux at fluid-wall interfaces (5) and the conservation of enthalpy inside the wall (6).

The volumic force \(\mathbf{F}\) is calculated at each iteration to keep a constant mass flow rate over the time, and so a constant \(Re_\tau\). The value of the mass flow rate is calculated according to the \(Re_\tau\) value, thanks to Dean correlation [18]: \(Re_\tau = 0.175 Re_b^{7/8}\). \(Re_b\) substitutes for Reynolds number based on bulk velocity. \(\mathbf{u}_i^+\) represents the dimensionless fluid volume heat source, and \(\frac{K}{d^+ \sqrt{G}}\) the wall volume heat sink. We assume here that all the energy generated inside the fluid is absorbed inside the wall. Equations (4) to (6) can be replaced by \(T_{i}^+ = T_{i \text{imp}}^+\) in case of the isothermal boundary condition and by \(\frac{\partial T}{\partial y^+_i} = \Phi_{\text{imp}}\) in case of the isoflux boundary condition.

2.3. Numerical procedures

The numerical simulations presented hereafter have been performed with the code ISIS [19]. Finite volume numerical schemes with staggered grid are used for the velocity field. For time derivations, a 2\(^{nd}\) order Crank-Nicholson scheme is used. A MUSCL scheme is adopted for the thermal convective transfers. Inside the wall, wall-normal thermal exchanges are implicitied while cross diffusion transfers are explicited. The eddy viscosity is obtained using the WALE model while the eddy diffusivity is obtained using a dynamic method [20, 21]. A constant subgrid scale Prandtl number model was rejected here because the constant value is barely known at such a low Prandtl number. For non-square cells, the Scotti correction [22] is applied for the filter cutoff length \(\Delta\).

This work aims at to show the suitability of using a dynamic wall law for numerical simulations of low Prandtl number fluid flows. As explained in [23], since thermal scales are much bigger than dynamic ones, it can be very attractive to use a wall function for the Navier-Stokes equations.
equations. The computational cost is reduced and the simulation accuracy remains satisfactory. In this work, the Werner and Wengle wall function \[24\] is considered with the coarse grid. The second grid is refined so as to be able to set a no slip condition.

Statistical moments are computed over the same physical times. They are computed over time and homogeneous space dimensions (over plans parallel to the wall surface). A first period is used to fully develop turbulence, then a second period to compute the statistical moments. The development time corresponds to 70 channel rounds and the statistics computational time to 164 channel rounds at bulk velocity. In terms of \( t_\tau = \frac{h}{u_\tau} \), these times correspond to 11.9\( t_\tau \) and 27.7\( t_\tau \) respectively.

3. Results
In this section, the velocity and temperature profiles obtained with a coarse and a fine grids are compared with the results of the DNS of \[25\]. These authors performed DNS of a periodic channel at \( Re_\tau = 395 \) and \( Re_\tau = 590 \), for the three kinds of boundary conditions.

First, have a look on velocity profiles on Figure 2. This figure shows dimensionless mean velocity (Figure 2a), and dimensionless velocity fluctuations (Figure 2b). It can be noticed that the results of both the coarse and the fine grids fit well with the DNS profiles, whether it is about mean or fluctuations profiles.

![Figure 2](image)

**Figure 2.** Mean (a) and root mean square (b) of dimensionless longitudinal velocity \( u^+ \)

Figure 3 shows mean dimensionless temperature inside the wall (a) (only for the conjugate heat transfer case) and inside the fluid (b). The three different boundary conditions are analysed: isoflux, isothermal, and conjugate heat transfer boundary conditions. Type of boundary condition has no impact on mean profiles inside the wall. The small discrepancies near \( y^* = 395 \) may be ascribed to the mesh grid. After dimensionless operation, this difference can be noted near the channel center. The Use of a conjugate heat transfer boundary condition remove this difference with the DNS. Figure 4 shows dimensionless temperature fluctuations in the same parts of the domain as Figure 3. At the wall interface, the maximum fluctuation level is get with the isoflux boundary condition and the minimum with the isothermal boundary condition. Regarding conjugate heat transfer, the fluctuation level at fluid-wall interface is in between that evaluated with the isothermal and the isoflux boundary conditions.

On Figures 2, 3 and 4, we can observe that using the coarse grid with a wall function for the Navier-Stokes equations does not alter much the results. This observation remains true for mean and fluctuating temperatures, inside the solid and the fluid domains. The use of a wall function seems a good compromise between result accuracy and computational cost. The simulation time
Figure 3. Mean of dimensionless temperature $\theta^+$ in the wall for conjugate heat transfer case (a) and in the fluid (b).

Figure 4. Root mean square of the dimensionless temperature $\theta^+$ in the wall for the conjugate heat transfer case (a) and in the fluid (b).

is divided by a factor of three while the maximum difference does not exceed 10% (it can be noticed at the channel center for the mean temperature). Consequently, this strategy is pursued hereafter to carry out a parametric study on the physical properties of the wall in the conjugated heat transfer boundary condition case.

4. Parametric study
In this section, the mean and fluctuation profiles of temperature for several wall parameters at $Re_\tau = 395$ are discussed. Chosen parameters are detailed in Table 2. The coarse grid and the wall function for Navier-Stokes equation are used. Conjugate heat transfer is set as boundary condition. Several simulations have been performed with a number of cells inside the wall set between 20 and 80 but no differences were observed. Then, this number has been set to 20 in the following. As it is shown in equation (6), three wall parameters are significant: The diffusivity ratio $K$, the diffusivity ratio $G$ and the dimensionless wall thickness $d^*$ (or $d^\ast$). For this parametric study, seven cases have been considered. The case 0 is used as reference (which corresponds to the conjugate heat transfer of the previous section). Impacts of parameters $K$, $G$, $d^*$ are studied in cases (1,2), (3,4) and (5,6) respectively.
| Case ID | $K$  | $G$  | $d^*$ |
|---------|------|------|------|
| 0       | 1    | 1    | 300  |
| 1       | 0.1  | 1    | 300  |
| 2       | 10   | 1    | 300  |
| 3       | 1    | 0.1  | 300  |
| 4       | 1    | 10   | 300  |
| 5       | 1    | 1    | 100  |
| 6       | 1    | 1    | 590  |

Table 2. Parameters series for parametric study

Figure 5. Mean of dimensionless temperature $\theta^+$ in the wall for conjugate heat transfer case (a) and in the fluid (b)

Figure 6. Root Mean Square of dimensionless temperature $\theta^+$ in the wall for conjugate heat transfer case (a) and in the fluid (b)

Any of the three parameters mentioned above has an impact on the mean temperature profile inside the fluid, as it can be seen on Figure 5. However, the parameter $K$ has the greatest influence on the mean temperature profile inside the wall. In fact, $K$ represents the thermal inertia of the wall with regard to the thermal inertia of the fluid. The thermal energy produced inside the fluid has to be evacuated inside the wall. If the wall material has a weak thermal
inertia with regard to the fluid one (high $K$ value), the wall temperature at external interface is very low. The opposite is observed with a low $K$, which corresponds to a high wall thermal inertia. This property has a direct impact on the temperature fluctuations profiles (see Figure 6). If the thermal inertia of wall is weak, the temperature fluctuations generated inside the fluid can easily penetrate inside the wall. Consequently, for a high $K$ value, the temperature fluctuations level at wall interface, and so through the wall, is higher than for low $K$ value corresponding to a high thermal inertia of the wall.

Here, impact of parameter $G$ is analysed while keeping a constant value of $K$. According to equation (5), $G$ influences temperature mean value inside the wall (see Figure 5). Three phenomena induced by $G$ explain the temperature fluctuation profiles. The first one is located near the fluid-wall interface. It corresponds to the feedback from wall temperature to fluid temperature. A high thermal diffusivity in the wall (small $G$) smooths the temperature profile inside the fluid and reduces the fluctuations near the interface. The two other phenomena appear everywhere in the wall, but have opposite effects. First, a high $\kappa_w$ (small $G$) allows a better transmission of temperature fluctuations through the wall depth. The result is a higher temperature fluctuation level for small $G$. However, there is a third phenomenon. A high $\kappa_w$ smooths temperature fluctuations by cross diffusion on wall-parallel planes. The result is a smaller temperature fluctuation level for small $G$. The first of these phenomena is visible on Figure 6 near the wall. Then, because temperature fluctuations decrease faster (in $y^*$ direction) for high values of $G$, we can conclude that the second phenomenon detailed above is predominant with regard to the third one.

The wall thickness influences mean and fluctuating profiles for several reasons. As the total heat sink through the wall does not change with the wall thickness, for a thin wall, the volumic sink is higher than for a thick one. Consequently, for a thin wall, the temperature gradient is higher, and so $T_\tau$. Since dimensionless temperature is obtained by dividing temperature by $T_\tau$, for a thin wall, we obtain a smaller dimensionless mean temperature in absolute value, and a bigger one in algebraic value (with the dimensionless operation, temperature is non-positive inside the wall). This result can be observed on Figure 5. Moreover, when the wall is too thin, it hasn’t enough thermal inertia to absorb the thermal fluctuations.

5. Conclusion
In this paper, we have shown that, for simulations of low Prandtl number fluid flows, the use of a wall function can reduce the computational cost without modifying the mean and fluctuation temperature profiles. This conclusion has been obtained by comparing the results from fine and coarse grids with DNS results. Besides, a parametric study about the three governing parameters of wall conduction has been performed using the Werner and Wengle wall function. The effusivity ratio $K$ and the diffusivity ratio $G$ impact both mean and fluctuating temperature profiles inside the wall, and the fluctuating temperature profile inside the fluid. Finally, the dimensionless wall thickness $d^+$ impacts both mean and fluctuating temperature profiles inside the wall, but has no influence on the fluid temperature statistics, as long as the wall is thick enough.

This simulation strategy, using a wall function for Navier-Stokes equation with a coarse grid to reduce the computation and keeping acceptable results, will be enforced to more complex flow configurations in future works, starting with the mixing of two liquid metal flows in a Tee junction.

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Nomenclature

**Materials properties**
- $\nu$ = kinematic viscosity
- $\lambda$ = thermal conductivity
- $\rho$ = density
- $c_p$ = specific heat at constant pressure
- $\kappa$ = thermal diffusivity ($\lambda/(\rho c_p)$)
- $d$ = wall thickness
- $l_f$ = fluid property
- $l_w$ = wall property
- $l_{i,f}$ = fluid-wall interface property

**Variables**
- $u$ = velocity vector $< u, v, w >$
- $T$ = temperature
- $\theta$ = $T - T_w$
- $t$ = time
- $q_w$ = wall-to-fluid heat flux
- $\tau$ = Reynolds stress tensor

**Characteristic variables**
- $u_\tau = \sqrt{\tau/\rho}$
- $u_b = \int_0^{2h} \int_0^{L_z} u/(2hL_z)$
- $l_\tau = \nu/\nu_\tau$
- $t_\tau = h/\nu_\tau$
- $T_\tau = \left. \kappa_{f}/\kappa \frac{\partial T}{\partial y} \right|_{y=0} = \frac{q_w}{\rho_f c_p/\kappa_w}$
- $P_\tau = \rho u_\tau^2$

**Dimensionless variables**
- $()^+ = \text{normalized by } u_\tau, P_\tau, T_\tau, t_\tau \text{ or } h$
- $()^* = \text{normalized by } l_\tau$
- $Re_\tau = \text{friction Reynolds number } (u_\tau h/\nu)$
- $Pr = \text{Prandtl number}$
- $K = \text{effusivity ratio } (\sqrt{\lambda_f \rho_f c_p}/\sqrt{\lambda_w \rho_w c_p})$
- $G = \text{diffusivity ratio } (\kappa_f/\kappa_w)$

**Operators**
- $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$
- $\nabla^+ = (\partial/\partial x^+, \partial/\partial y^+, \partial/\partial z^+)$

References

[1] Gourdin C, Chapuliot S, Magnaud J P, Mermou F and Monavon A 2007 *Trans. SMiRT 19* (Toronto, Canada) p G05/4

[2] Chellapandi P, Velusamy K, Biswas A, Rama R, Bhoje S B, Vaidyanathan G and Chetal S C 2002 *Validation of fast reactor thermomechanical and thermohydraulic codes*, vol tecdoc1318 (IAEA) chap Thermal hydraulics and thermohydraulic analysis of thermal striping in a mixing Tee-junction of secondary sodium circuit of PHENIX reactor, pp 63–88

[3] Hoshino T, Aoki T and Kutomi Y 2000 *Proceedings of ICONE 8* vol 8

[4] Igarashi M, Tanaka M, Kawashima S and Kamide H 2002 *Proceedings of ICONE 10* vol 3 (Arlington, U.S.A.) pp 383–390

[5] Walker C, Simiano M, Zboray R and Prasser H M 2009 *Nucl. Eng. Design* 239 116–126

[6] Frank T, Lifante C, Prasser H M and Menter F 2010 *Nucl. Eng. Design* 240 2313–2328

[7] Merzari E, Khakim A, Ninokata H and Baglietto E 2009 *Nucl. Eng. Design* 240 1191–1204

[8] Kuhn S, Braillard O, Niceno B and Prasser H M 2010 *Nucl. Eng. Design* 240 1548–1557

[9] Aulery F, Toutant A, Brillant G, Monod R and Bataille F 2012 *Applied Thermal Engineering* 37 38–43

[10] Simonneau J P, Noe H and Menant B 1997 *5th International Conference on Nuclear Engineering* (Nice, France) pp ICONES–2145

[11] Fukushima N, Fukagata K, Kasagi N, Noguchi H and Tanimoto K 2003 *The 6th ASME-JSME Thermal Engineering Joint Conference* (Hawaii) pp TED–AJ03–582

[12] Moser R D, Kim J and Mansour N N 1999 *Phys. Fluids* 11 943–945

[13] Tiselj I, Bergant R, Markvo B, Bajšić I and Hetsroni G 2001 *J. Heat Trans. (ASME)* 123 849–857

[14] Hoyas S and Jiménez J 2006 *Phys. Fluids* 18 011702

[15] Hoyas S and Jiménez J 2008 *Physics of Fluids* 20 101511

[16] Brillant G, Husson S and Bataille F 2008 *Int. J. Heat Fluid Flow* 29 1670–1678

[17] Monod R, Brillant G, Toutant A and Bataille F 2012 *Int. Comm. Heat Mass Transfer in review process –*

[18] Dean R B 1978 *J. Fluids Eng. (ASME)* 100 377–377

[19] Suard S, Lapuerta C, Babik F and Rigollet L 2011 *Nucl. Eng. Design* 241 3645–3657

[20] Brillant G, Husson S and Bataille F 2006 *J. Appl. Mech., Trans ASME* 73 360–367

[21] Germano M, Piomelli U, Moin P and Cabot W H 1991 *Phys. Fluids A* 3 1760–1765

[22] Scotti A and Piomelli U 2001 *Phys. Fluids* 13 1367–1384

[23] Grötzbach G 2011 *Nucl. Eng. Design* 241 4379–4390

[24] Werner H and Wengle H 1993 *Selected papers from the 8th Symposium on Turbulent Shear Flows* pp 155–168

[25] Tiselj I and Cizelj L 2012 *Nucl. Eng. Design in review process*