Magnification of subwavelength field distributions using a tapered array of metallic wires with planar interfaces and an embedded dielectric phase compensator

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\textbf{Abstract.} We report the magnification of subwavelength field distributions using a tapered array of metallic wires with planar front and back interfaces through numerical simulations and experiments. It is demonstrated that subwavelength images with a resolution of one-fifteenth of a wavelength can be transferred to a distance of three wavelengths with a threefold magnification. We also propose embedding a dielectric phase compensator in the tapered array to compensate the phase differences introduced by the different lengths of wires and significantly improve the operational bandwidth of the image transmission and magnification device.

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1. Introduction

Conventional imaging systems are restricted by the so-called diffraction limit: any source details below the half-wavelength at the frequency of operation cannot be resolved at the image plane. However, it has been proposed in [1] that a planar lens formed by the left-handed material (LHM) [2] can be used to image source information with a spatial resolution below the diffraction limit. A lens formed by LHM with $\epsilon = \mu = -1$ is termed the ‘perfect lens’ and the principle of its operation is based on the negative refraction of propagating waves and the amplification of evanescent field components [1]. It is well known that evanescent waves carry subwavelength source details and decay exponentially in positive-index materials. The uniqueness of LHM is that evanescent waves show growing behaviour when propagating inside the medium. Hence, the subwavelength details that are lost in the region between the source and a perfect lens can be restored to create a perfect image [1]. The amplification of evanescent waves in LHM is due to the resonant excitation of surface plasmons at the interfaces. However, such an effect is sensitive to losses in LHM and thus limits the maximum thickness of the LHM slab [3]. Furthermore, the mismatch of LHM with its surrounding medium also limits the imaging capability of LHM lenses [4].

Recently, it has been suggested that one use an alternative way to transfer subwavelength source details to an image plane at a significant distance. The principle of operation, named ‘canalization’ [5], is based on the fact that for certain types of devices, evanescent wave components can be transformed into propagating waves, and therefore the source field can be delivered to its back interface with little or no deterioration. In contrast to the case of LHM, such devices are less sensitive to losses. Recent work on subwavelength imaging using anisotropic materials (based on the canalization principle) includes [6]–[8]. Moreover, the work presented in [9] shows that on introducing nonlinearity one is able to suppress the diffraction, and this is useful for future subwavelength imaging devices even when the dispersion curve is not flat.

One typical example of structures operating on the canalization principle is the wire medium, i.e. a structure formed by an array of parallel conducting wires [10]. The thickness of the wire medium needs to be equal to an integer multiple of a half-wavelength at the operating frequency (due to Fabry–Perot resonance) in order to avoid reflections between the source and the structure. Two sets of experiments have been carried out at microwave frequencies to demonstrate the canalization of the TE-polarized wave (transverse electric field with respect to the slab interface; the capacitively loaded wires are aligned in parallel with the slab interface) [11] and the TM-polarized wave (transverse magnetic field; the unloaded wires are aligned perpendicular to the slab interface) [10]. It is also shown numerically that
subwavelength details can be transferred through the wire medium at frequencies up to terahertz (THz) and infrared range [12]–[15]. The limitations of subwavelength imaging using wire medium slabs have been analytically studied in [16], and the experimental validations of these analytical findings are available in [17].

Recently, there has been growing interest in the development of structures that are capable of magnifying subwavelength field distributions in the visible frequency range [18]–[21]. These structures allow source details to be transferred a certain distance while maintaining the same patterns with a linearly magnified or enlarged scale. The structures that can provide such functionalities in the visible frequency range can be constructed using either a stack of alternating layers of dielectric–plasmonic materials arranged uniaxially in Cartesian or cylindrical coordinates [6], or the conventional [12] and plasmonic wire medium [22]. In microwave frequencies, the simultaneous enhancement and magnification of evanescent field patterns have been demonstrated experimentally with the help of double cylindrical polariton-resonant structures by Alitalo et al [23], and numerically using a tapered array of metallic wires simulated using a commercial full-wave electromagnetic solver FEKO [24]. In [24], all the wires of the tapered array have the same length (tuned to the Fabry–Perot resonance to avoid reflections from the structure). Hence both front and back interfaces of the wire medium are spherical, which is inconvenient in practice since most of the scanning surfaces are planar. In this paper, we report the numerical simulations and experimental results of a tapered array of metallic wires with planar front and back interfaces. We also propose embedding a dielectric phase compensator in the tapered array to compensate the phase differences introduced by different lengths of wires and significantly improve the operational bandwidth of the device.

2. Numerical simulations of the tapered array of wires with and without the phase compensator

A schematic diagram of the simulated tapered array of wires is shown in figure 1. The wires are modelled as perfect electric conductors (PECs) and the number of wires is 21 × 21. The radius of the wires is 1 mm and the length of the longest wires (four corner ones) is 1000 mm. The length of the shortest wire in the centre is 959.17 mm. The periods of the wires at the front and back interfaces are 10 and 30 mm, respectively, which allows a threefold magnification of the source distribution. A crown-shaped near-field source is used in the simulations and is placed at a distance of 3 mm from the front interface of the tapered array (the source plane; see figure 1). A finite-integral technique (FIT)-based commercial electromagnetic solver CST Microwave Studio™ is used to simulate the structure.

The proposed tapered array of wires operates on the canalization principle; therefore, it is capable of magnifying field patterns comprising TM-polarized incident waves (both propagating and evanescent) with any transverse components of the wave vector. The canalization principle also requires the length of the wire medium to be integer multiples of a half-wavelength (λ/2) at the frequency of operation, in order to satisfy the Fabry–Perot resonant condition and avoid destructive reflections from the structure. However, for the case of the tapered array of wires with planar interfaces (see figure 1), since the wires have unequal lengths, the optimum operating frequency cannot be determined directly. Therefore, we plot the distributions of the z-component of electric field at the front interface (source plane) and the back interface (image plane) of the tapered array at different frequencies around 900 MHz (at this frequency, the
average length of the wires is approximately $3\lambda$). Figure 2 shows field distributions at four chosen frequencies: 880, 910, 940 and 1000 MHz.

As analysed in [17], the wire medium excites strong surface waves below the Fabry–Perot resonant frequency and introduces reflections above it. The Fabry–Perot resonant frequencies for wires in the central region are considerably higher than the frequencies for wires near the edge area, due to different lengths of the wires. Therefore, in figure 2, at low frequencies ($< 940$ MHz), the lengths of wires near the edge of the tapered array satisfy the Fabry–Perot resonant condition and have good subwavelength imaging resolution, but strong surface waves occur in the central region. These surface waves propagate along the surface of the tapered array and significantly disturb the entire source distribution. Despite this effect, the image distributions near the edge still show good correspondence with the source, as shown by the field distributions at 910 MHz in figure 2. At even lower frequencies (below 880 MHz, not shown), surface waves completely distort the source distribution and cause the image distribution to be significantly different. On the other hand, at frequencies above 940 MHz, the image distribution has good subwavelength resolution ($< \lambda/5$), but strong reflections occur in the edge area, as can be seen from the field distributions at 1000 MHz in figure 2. The above discussion allows one to conclude that the operating bandwidth of the tapered array of wires (without surface waves or reflections) is extremely narrow, or the optimum operating frequency does not exist. Note that the ‘pixel’-like maxima in all the simulated field distributions (as well as the measured ones, shown in a later part of the paper) in the image plane are due to the fact that the field is recorded at a very small distance ($\leq 1$ mm) from the back interface of the tapered array.

For proper operations of the canalization principle, it is required that all wires of the wire medium have the same electrical length, which can be different from the actual physical length of the wires. Therefore, we may use a dielectric phase compensator to compensate the phase differences introduced by different physical lengths of the wires, in order to still allow the
Figure 2. Simulated distributions of the $z$-component of electric field at the front interface (source plane) and back interface (image plane) of the tapered array of wires at frequencies of (a) 880, (b) 910, (c) 940 and (d) 1000 MHz (refer to the coordinate system in figure 1). The fields are normalized to their maximum values at each frequency.

tapered array of wires to be used for imaging and magnification of a large area. The schematic diagram of the tapered array with a dielectric phase compensator is shown in figure 3. The front surface of the phase compensator is planar and aligned with the middle plane of the tapered array, as illustrated in figure 3(b). The thickness of the phase compensator is maximum in the centre and gradually reduces towards the edges and corners (the thickness is zero at the four corners). For each individual wire, the thickness can be calculated using the phase matching conditions:

$$k_1 l_1 = k_1 (l_2 - h) + k_2 h,$$

$$k_2 = \sqrt{\varepsilon_r} k_1,$$

where $k_1$ and $k_2$ are the wave numbers in free space and the dielectric phase compensator, respectively, $l_1$ is the length of the longest wire in the tapered array, $l_2$ is the length of the wire for which the thickness is calculated, and $h$ and $\varepsilon_r$ are the thickness and dielectric constant of the phase compensator, respectively. After simple manipulations, the thickness of the phase compensator for each wire can be calculated as

$$h = \frac{l_1 - l_2}{\sqrt{\varepsilon_r} - 1}.$$
Figure 3. A tapered array of metallic wires with planar front and back interfaces and a dielectric phase compensator: (a) perspective view and (b) side view. A crown-shaped near-field source is used in simulations of image magnification.

In our case, \( l_1 = 1000 \text{ mm} \) and the dielectric constant is chosen as \( \varepsilon_r = 4 \) (lossless). After calculating the thicknesses, the resulting shape of the phase compensator can be well approximated by a part of a sphere with radius \( r = 571.43 \text{ mm} \). It is interesting to notice that in equation (2), the thickness of the phase compensator is independent of frequency. Hence the phase compensator is capable of compensating phase differences for the tapered array of wires at all wavelength scales. Furthermore, it is worth mentioning that at low frequencies, the phase differences introduced by different lengths of wires are less significant and the tapered array of wires serves as a good subwavelength image magnification device.

The phase-compensated tapered array of wires is also simulated using CST Microwave Studio™, and the distributions of the z-component of electric field at the source and image planes at the four chosen frequencies of 840, 880, 920 and 960 MHz are plotted in figure 4. The images contain subwavelength distributions with a resolution of about \( \lambda/5 \), which indicates that the source distributions with a resolution of \( \lambda/15 \) can be transferred and magnified. The introduced dielectric phase compensator shifts the operating frequencies of the tapered array of wires lower (about 30–50 MHz), as can be identified from the fact that the field pattern at 940 MHz in figure 2 is similar to the ones at 880 and 920 MHz in figure 4. Nevertheless, the patterns above 880 MHz in figure 4 do not contain either surface waves or reflections introduced by the tapered array, and vary slowly with the increase of frequencies, in contrast to figure 2. This demonstrates that the operational bandwidth of the tapered array of wires is significantly improved. When the frequency is further increased (above 960 MHz, not shown), reflections from the tapered array start to appear and disturb the source and image distributions; similarly, at lower frequencies (below 850 MHz), surface waves are excited, which also distort the source and image distributions.

So far we have shown that introducing a dielectric phase compensator can improve the magnification capability of a tapered array of wires with planar front and back interfaces through numerical simulations. In the following, the above numerical simulation results are validated by experiments.
3. Experimental validations

In the experiments, we have fabricated a tapered array of wires using long copper wires. The dimensions of the tapered array follow those shown in figure 1. A crown-shaped loop antenna printed on a 2-mm-thick Duroid substrate with relative permittivity $\varepsilon_r = 2.33$ is used as the near-field source. Detailed dimensions of the crown-shaped source can be found in figure 5. The supporting frames at both the front and back ends of the tapered array are made of foam, whose permittivity is close to that of free space. The dielectric phase compensator is made of acetal, which has a dielectric constant $\varepsilon_r = 3.7–3.9$ and very low dielectric loss in the microwave frequency range.

The fact that one of the interfaces is planar simplifies the fabrication to cutting a three-dimensional (3D) surface only on one side of the acetal block. The compensator has been manufactured using a 3D milling machine C-TEK KM-80D with a CNC controller CNT 830. For the given frequency range (about 1 GHz) where the wavelength is about 30 cm, the tolerance can be chosen within the range of 1–2 mm. It significantly decreases the cutting time, taking into account the large dimensions of the compensator ($60 \times 60$ cm$^2$). The fabrication process contains two major operations: drilling 441 holes for wires to pass through, and

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Figure 5. The fabricated tapered array of metallic wires with planar front and back interfaces and a dielectric phase compensator. A crown-shaped near-field source is used in the measurement of image magnification. The phase compensator is made of acetal and its largest thickness is 4 mm (in the centre).

Figure 5. The fabricated tapered array of metallic wires with planar front and back interfaces and a dielectric phase compensator. A crown-shaped near-field source is used in the measurement of image magnification. The phase compensator is made of acetal and its largest thickness is 4 mm (in the centre).

An automatic mechanical near-field scanning device is used in the measurement. A 1-mm-long monopole made from the central core of a coaxial cable with 2 mm diameter is used as the probe to detect the near field of the tapered array of wires. The probe is oriented perpendicular to the interfaces of the device to capture the \( z \)-component of the electric field (refer to the coordinate system in figure 5). The scanned area is \( 19 \times 19 \, \text{cm}^2 \) with 96 steps in each direction at the tapered array’s source plane (the vertical area 2 mm away from the front side of the crown-shaped source), and \( 60 \times 60 \, \text{cm}^2 \) with 101 steps at the image plane (the vertical area 1 mm away from the back interface of the tapered array). All the measurements are performed in an anechoic chamber to avoid interferences, as illustrated in figure 5.

The measurement is performed for the tapered array of wires with the embedded dielectric phase compensator to demonstrate its bandwidth enhancement capability. A series of frequencies are considered in the measurement, and the results at four frequencies of 860, 890, 900 and 940 MHz are chosen and presented in figure 6. For convenience of illustration, here we use the same coordinate system as in the simulation setup, as shown in figure 5.

From figure 6, one could identify similar results to the simulated ones shown in figure 4: at low frequencies (specifically, below 890 MHz), the lengths of the wires near the edge area of the tapered array satisfy the Fabry–Perot resonant condition; however, in the central region, strong surface waves are excited and propagate along the surface of the tapered array. Both source and image distributions are disturbed by the surface waves. The results at 860 MHz in
Figure 6. Measured distributions of the perpendicular electric field component (with respect to the interfaces of the tapered array of wires) at the front interface (source plane) and back interface (image plane) of the tapered array with an embedded dielectric phase compensator at frequencies of (a) 860, (b) 890, (c) 900 and (d) 940 MHz (refer to the coordinate system in figure 5). The fields are normalized to their maximum values at each frequency.

Figure 6 show an example of such a case. At frequencies above 890 MHz, the field distributions in the source plane vary slowly with frequency and are magnified with little distortion. The measurement results in figure 6 show good correspondence with the simulated ones, and both results demonstrate that the tapered array of wires with an embedded phase compensator has a much larger operational bandwidth, compared with the tapered array of wires without the phase compensator.

4. Conclusion

In conclusion, we have demonstrated the image magnification capability of a tapered array of metallic wires through numerical simulations and experiments. The results show that subwavelength images with a resolution of $\lambda/15$ can be transferred a distance of three wavelengths with threefold magnification. However, due to the different lengths of wires in the conventional tapered array, the Fabry–Perot resonant condition cannot be simultaneously met by the entire device, which leads to its very narrow bandwidth of operation or the optimum frequency of operation cannot be found. Therefore, we propose embedding a dielectric phase
compensator within the tapered array of wires to compensate the phase differences introduced by different lengths of wires in free space. Both numerical simulation and experimental results confirm that the operational bandwidth of the phase compensator loaded tapered array of wires is significantly improved. Furthermore, since dielectric phase compensators are extremely flexible in the design of their shapes and thicknesses, one may construct a tapered array of wires for scanning arbitrary shaped object planes.

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