Magnetization plateaus in the Ising limit of the multiple-spin exchange model on plaquette chain.

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We consider the Ising spin system, which stems out from the corresponding Multiple-spin exchange (MSE) Hamiltonian, on the special one–dimensional lattice, diamond-plaquette chain. Using the technique of transfer-matrix we obtain the exact expression for system free energy with the aid of which we obtain the magnetization function. Analyzing magnetization curves for varies values of temperature and coupling constants we found the magnetization plateaux at $1/3$ and $2/3$ of the full moment. The corresponding microscopic spin configurations are unknown by virtue of high frustration.

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The Heisenberg model is widely recognized as a lattice model for magnetism of materials. However, this model by no means is universal, because it based on several assumptions. One of these assumptions consists in the pair character of exchange interactions. This means that only the exchange processes of no more than two particles (nearest-neighbour or next-nearest-neighbour and so on) are taken into consideration:

$$H_{\text{Heis}} = 2J \sum_{(i,j)} P_{ij}. \quad (1)$$

Here $P_{ij}$ are the pair exchange operators, which implement the transposition of two spin states in $i$-th and $j$-th sites of the lattice:

$$P_{ij}|\xi_i\rangle \otimes |\xi_j\rangle = |\xi_j\rangle \otimes |\xi_i\rangle. \quad (2)$$

For the SU(2) spins and $s = 1/2$ the expression for $P_{ij}$ is

$$P_{ij} = \frac{1}{2} (1 + \sigma_i \cdot \sigma_j), \quad (3)$$

$\sigma_i$ are the Pauli matrices. The generalization of this picture is known since 60-s and called the multiple-spin exchange (MSE) model. This model describes the system of almost localized fermions via the concept of many particle permutation. The general form of MSE Hamiltonian is

$$H_{\text{ex}} = - \sum_{n,\alpha} J_{n\alpha} (-1)^p P_n. \quad (4)$$

Here $P_n$ denotes the $n$–particle cyclic permutation operator, $J_n$ is corresponding exchange energy and $p$ is the parity of the permutation, which indicates how much pair transpositions contains in the given cyclic permutation. Many peculiar magnetic and thermodynamical properties of $^3$He adsorbed on graphite surface can be understood only within the framework of MSE model. Recently the significant role of MSE interaction was revealed in low-dimensional cuprate compounds, what initiated the interest toward the two-leg spin ladders with four-spin cyclic interaction. The MSE model by itself exhibits rich phase structure even at classical level. Another interesting feature of MSE model is the possibility of complex magnetic behavior, including such a phenomena as magnetization plateaux, which was established in the MSE model on triangular lattice. The study of the magnetization plateaux is one of the main directions of nowadays investigations on macroscopic non–trivial quantum effects in condensed matter physics, which have a number of fundamental and applied importance. Despite the purely quantum origin of this effect it was shown recently that, magnetization plateaux can appear in the Ising spin systems also, exhibiting in some cases fully qualitative correspondence with its Heisenberg counterpart. The latter fact is very important because it can serve for more profound understanding of magnetization plateaux physics, can provide it with new methods and can initiate a search of novel magnetic materials with huge axial anisotropy.

I. THE MODEL

We consider a one-dimensional lattice consists of corner shared so–called diamond plaquettes (See Fig. 1).
The diamond plaquette is a square plaquette of 4 spins with nearest neighbor interaction and additional bound, connecting two opposite spins. If we consider the Hamiltonian of MSE model and restrict ourself with the two-, three-, and four–spin exchanges, which is the simplest case, we arrive at the following Hamiltonian:

\[
H = J \sum_{\langle i,j \rangle} \mathbf{\sigma}_i \cdot \mathbf{\sigma}_j + K \sum_p h_p - \mu H \sum_i \mathbf{\sigma}_i^z,
\]

where \( J = J_3 - J_2 / 2 \) and \( K = -J_4 / 4 \). This model have been successfully applying for describing the properties of solid \(^3\)He films. In our case we omit the off–diagonal part of the Hamiltonian which is equivalent to the formal replacement of all spin operators by simple Ising variables which take values ±1. Putting the system, describing by the Hamiltonian onto the lattice, depicted in Figure 1, we get the following model:

\[
-\beta \mathcal{H} = \sum_i \alpha_1 s_i s_{i+1} + \alpha_2 (s_i + s_{i+1}) (t_i + \tau_i) + \alpha_3 (t_i \tau_i + s_i s_{i+1} t_i \tau_i) + h (s_i + t_i + \tau_i),
\]

where

\[
\alpha_1 = \beta (J_3 - J_2 / 2 - J_4 / 4), \quad \alpha_2 = \beta (J_3 / 2 - J_2 / 2 - J_4 / 4), \quad \alpha_3 = -\beta J_4 / 4.
\]

Here the Ising spins placed in the corners of diamond plaquette chain denoted by \( t_i \) and \( \tau_i \) and spins placed in the middle line by \( s_i \). The system allows the exact calculation of the partition function, which can be represented as a trace of the N-th power of the corresponding transfer-matrix

\[
Z = \sum_{\{s,t,\tau\}} e^{-\beta \mathcal{H}} = \text{Sp} T^N.
\]

The transfer-matrix is

\[
T = \begin{pmatrix}
2e^{\alpha_1 + h} (e^{2\alpha_3 \cosh(4\alpha_2 + 2h)} + e^{2\alpha_3}) , & 2e^{-\alpha_1} (\cosh(2h) + 1) \\
2e^{-\alpha_1} (\cosh(2h) + 1) , & 2e^{\alpha_1 - h} (e^{2\alpha_3 \cosh(4\alpha_2 - 2h)} + e^{2\alpha_3})
\end{pmatrix}.
\]
The maximal eigenvalue is

\[
\lambda = A \cosh(h) + B \cosh(3h) + \sqrt{\sum_{k=0}^{3} C_k \cosh(2kh)},
\]  

(10)

where the coefficients are

\[
A = e^{\alpha_1 - 4\alpha_2 + 2\alpha_3} + 2e^{\alpha_1 - 2\alpha_3},
\]

(11)

\[
B = e^{\alpha_1 + 4\alpha_2 + 2\alpha_3},
\]

(12)

\[
C_0 = e^{2\alpha_1} \left( 6e^{-4\alpha_1} + 2e^{-4\alpha_2} - 2e^{-4\alpha_3} - e^{4\alpha_3} \cosh(8\alpha_2) \right),
\]

(13)

\[
C_1 = e^{2\alpha_1 + 4\alpha_3} \left( 1 + \frac{1}{2} e^{-8\alpha_2} + 2e^{-8\alpha_3} \right) - 4e^{2\alpha_1} \cosh(4\alpha_2) + 8e^{-2\alpha_1},
\]

(14)

\[
C_2 = e^{2\alpha_2} \left( 2e^{-4\alpha_1} + 2e^{4\alpha_2} - e^{4\alpha_3} \right),
\]

(15)

\[
C_3 = e^{2\alpha_1 + 8\alpha_2 + 4\alpha_3}.
\]

(16)

Having the partition function we can obtain the free energy of the system per one spin in the thermodynamical limit:

\[
f = -\frac{1}{\beta} \lim_{N \to \infty} \frac{\log \lambda^N}{3N} = -\frac{1}{3\beta} \log \lambda.
\]

(17)

Then, using the conventional thermodynamical relations \( m = -\left( \frac{\partial f}{\partial H} \right) \), we obtain the magnetization per spin as an explicit function

\[
m = \frac{A \sinh(h) + 3B \sinh(3h) + \sqrt{\sum_{k=1}^{3} C_k \sinh(2kh)}}{3 \left( A \cosh(h) + B \cosh(3h) + \sqrt{\sum_{k=0}^{3} C_k \cosh(2kh)} \right)}.
\]

(18)

Having this function we can draw the plots of the magnetization processes for all finite temperatures and arbitrary values of coupling constants. First of all, due to its geometry the system is highly frustrated in case of antiferromagnetic effective coupling constants. This means that even at \( T = 0 \) the ground state of the system is disordered, because the arrangement of spins on lattice precludes satisfying all interaction simultaneously. However, it is possible to choose such MSE coupling constants at which the frustration will be partially or entirely removed in the Ising limit. Among the variety of magnetization curves obtained for different sets of \( J_2, J_3 \) and \( J_4 \) the most remarkable is the one with two magnetization plateaux at \( m = 1/3 \) and \( m = 2/3 \) in the units of full moment(Fig. 2). At that region of coupling constants the system is highly frustrated and it is not so easy to determine the microscopic spins configurations corresponding to these plateaux. Apparently they are some complex periodic structures with spatial period at least 3 for 1/3-plateau and 6 for 2/3-plateau. The investiga-
FIG. 2: The magnetization curve for Ising plaquette chain at $T/J_3 = 0.17$, $J_2/J_3 = 1.5$ and $J_4/J_3 = 1.7$.

tion of the entire MSE model on diamond plaquette chain may change the picture obtained by us drastically. On the one hand, the appearance of other plateaux is possible, on the other hand, the plateaux pertinent the Ising system might not survive in quantum case. The typical example is the simple $S = 1/2$ Heisenberg chain which is gapless, whereas the corresponding Ising system exhibits the plateau at $m = 0$. Let us mention also that apparently among the overdoped $RCuO_2 + x$ (R=Y, La, etc.) compounds [12] the ones are possible whose magnetic lattice is analogous to that considered here.

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