The Effect of Lensing on the Large-Scale CMB Anisotropy

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ABSTRACT

We first compare the CMB lensing model of Seljak (1996) with the empirical model of Lieu & Mittaz (2005) to determine if the latter approach implies a larger effect on the CMB power-spectrum. We find that the empirical model gives significantly higher results for the magnification dispersion, $\sigma$, at small scales ($\theta < 30'$) than that of Seljak (1996), assuming standard cosmological parameters. However, when the empirical foreground model of Lieu & Mittaz is modelled via correlation functions and used in the Seljak formalism, the agreement is considerably improved at small scales. Thus we conclude that the main difference between these results may be in the different assumed foreground mass distributions. We then show that a foreground mass clustering with $\xi(r) \propto r^{-3}$ gives an rms lensing magnification which is approximately constant with angle, $\theta$. In Seljak’s formalism, we show this can lead to a smoothing of the CMB power-spectrum which is proportional to $e^{-\sigma^2 l^2/2}$ and which may be able to move the first CMB acoustic peak to smaller $l$, if the mass clustering amplitude is high enough. Evidence for a high amplitude of mass clustering comes from the QSO magnification results of Myers et al (2003, 2005) who suggest that foreground galaxy groups may also be more massive than expected, implying that $\Omega_m \approx 1$ and that there is strong galaxy anti-bias, $b \approx 0.2$. We combine the above results to demonstrate the potential effect of lensing on the CMB power spectrum by showing that an inflationary model with neither CDM nor a cosmological constant and that predicts a primordial first peak at $l = 330$ might then fit at least the first acoustic peak of the WMAP data. This model may be regarded as somewhat contrived since the fit also requires high redshift reionisation at the upper limit of what is allowed by the WMAP polarisation results. But given the finely-tuned nature of the standard $\Lambda$CDM model, the contrivance may be small in comparison and certainly the effect of lensing and other foregrounds may still have a considerable influence on the cosmological interpretation of the CMB.

Key words: gravitational lensing, cosmic microwave background, dark matter

1 INTRODUCTION

The standard $\Lambda$CDM model gives an excellent fit to the CMB power spectrum from WMAP and other experiments (Hinshaw et al. 2003, 2006). The reduced chi-squared values are impressive for a model with 7 basic parameters and fitting over 40 approximately independent data points. The model also appears to give an excellent fit to independent datasets such as the 2dFGRS galaxy power spectrum (Cole et al. 2005).

Impressive though these fits are, the standard model still appears to have a variety of more fundamental problems (see Shanks, 2005 and references therein). For example, the CDM particle still has its own associated problems. The limits from the Cryogenic Dark Matter Search (Akerib et al, 2004) and other experimental constraints when combined with the WMAP result for the CDM particle density, now leave only a small amount of ‘generic’ parameter space allowed by some of the basic supersymmetric models for the neutralino (e.g. Ellis et al 2003, Barenboim & Lykken 2006); this is known as the ‘neutralino overdensity’ problem. There is also what is known as the ‘gravitino problem’. If the neutralino is the lightest supersymmetric particle and the gravitino is the next-to-lightest then if the neutralino is stable as required for a CDM candidate then the gravitino may be also massive and unstable; if its lifetime is longer than 0.01 secs then the decay of these particles into other particles such as baryons could upset the Big Bang prediction of the light element abundances (e.g. Cyburt et al 2003). Thus the usual agreement of light element abundances with the baryon density may not be taken for granted if the neutralino is the CDM particle. In some sense, the invoking of...
the CDM particle could appear to have over-specified the model in terms of solving the baryon nucleosynthesis problem.

The cosmological constant or dark energy presents further fundamental problems. Indeed, string theory prefers a negative $\Lambda$ rather than the observed positive $\Lambda$ (e.g. Witten, 2001). But it is the small size of the cosmological constant that undoubtedly presents the biggest difficulty for anyone assessing how successful the standard model is in explaining the current data. In the standard model, just after inflation the ratio of the energy density in radiation to the energy density in the vacuum was one part in $10^{52}$. This means that the standard model ends up potentially having more fine-tuning than implied in the original ‘flatness’ problem before it was solved by inflation (Guth 1981). It is this problem that presents the difficulty in assessing the success of the standard model where the low reduced chi-squared, from fits to the CMB power-spectrum has to be balanced against the huge degree of freedom allowed by the invocation of a very small cosmological constant.

Here we shall take the view that the fundamental complications of the present standard model suggest that it may still be worthwhile to explore if other models may explain the data. This may imply the introduction of further model parameters which may locally increase the model’s complication, but if they then allow the dropping of the hypothesis of the cosmological constant and even the CDM particle, the model’s complication will be globally decreased.

CMB foregrounds will form our main escape route from the tight constraints set by the CMB power-spectrum. One example will be the evidence for the effects of high redshift ($z \approx 10$) reionisation from the WMAP polarisation results (Kogut et al 2003, Page et al 2006). This already decreases the height of the acoustic peaks by $\approx 20\%$ relative to the multipoles at larger scales even if the reionisation is homogeneous. But galaxy clusters in the CMB foreground at lower redshifts provide several other ways of contaminating the CMB signal. The hot gas in the richer clusters scatter the CMB photons via the SZ effect and there are claims from the cross-correlation analysis of WMAP data with Abell clusters that the SZ decrements may extend up to 0.5 degree scales (Myers et al 2004). They can also gravitationally affect the CMB photons primarily via the Sachs-Wolfe effect and also by gravitationally lensing the CMB photons. Although the effects of foreground lensing are generally claimed to be small, we shall start by considering the claims by Lieu & Mittaz (2005) that the effects of lensing on the CMB power spectra may be bigger than previously estimated. Lieu & Mittaz analysed lensing by foreground groups and clusters on the CMB foreground as the magnification in the empty voids and the magnification in the clusters separately and show that in terms of the average magnification $\langle \eta \rangle$ they cancel, so that the same ‘light conservation’ result applies in this inhomogeneous case as in the homogeneous case treated e.g. by Weinberg (1976). They show that the same result applies when strong lensing is taken into account. Here we follow Lieu & Mittaz and define $\eta = (\theta' - \theta)/\theta$ as the magnification ‘contrast’; the effect of the scattering is to increase the average angular size $\theta$ of the source by this fractional amount.

The basic relations are for the expected value of $\eta$ in the case where the source is far behind the lens from equation (18) of Lieu & Mittaz:

$$\langle \eta \rangle = \frac{3}{2} \Omega_{\text{groups}} H_0^2 \int_0^{\pi f} dx [1 + z(x)] \frac{(x_s - x)x}{x_s},$$

and equation (31) of Lieu & Mittaz gives for $\delta \eta$

$$\langle \delta \eta \rangle^2 = \frac{8\pi^3}{3} n_0 x_f^2 \left(1 - \frac{3x_f}{2x_s} + \frac{3x_s^2}{5x_f^2}\right) \sum p_{r,i} \sigma_{\text{eff}}^2 f(R_j, b_{\text{min}}).$$

We shall argue that a low $H_0$, $\Omega_{\text{baryon}} = 1$ model may give an excellent fit to the CMB TT power spectrum with no need to invoke either a CDM particle or dark energy.

Hence, in Section 2 we review and modify the CMB lensing results of Lieu & Mittaz. In Section 3 we introduce the CMBFAST lensing formalism and compare with the results of Lieu & Mittaz. In Section 4 we use these results together with cosmological parameters determined from the QSO lensing results of Myers et al (2003, 2005) to predict a significantly increased effect of lensing on the CMB. In Section 5 we then consider the baryonic cosmological model that has an intrinsic CMB first peak at $l \approx 330$. We discuss how homogeneous reionisation at high redshift and then the effect of lensing can significantly improve agreement with the observed first peak at $l \approx 220$. In Section 6 we discuss our conclusions.

2 LENSING MODEL OF LIEU AND MITTAZ (2005)

We shall first consider the lensing model of Lieu & Mittaz (2005). These authors have suggested that the effects of lensing by foreground groups and clusters on the CMB can be much larger than previously calculated. This area has been controversial in the past with some authors claiming large effects of lensing on the CMB (Fukushige et al., 1994, Ellis et al., 1998) while others find much smaller effects (e.g. Bartelmann & Schneider, 2001). The lensing effects claimed by Lieu & Mittaz are significantly larger than those claimed by Bartelmann & Schneider and somewhat larger than claimed by Seljak. This may be because Lieu & Mittaz use strong-lensing formulae appropriate for clusters modelled as isothermal spheres or NFW profiles rather than the weak-lensing approximations used by Seljak and Bartelmann & Schneider, although all the above authors assume the weak-lensing Born approximation in terms of considering only small deviations from the original photon path. Another reason may be that they use empirically determined models for the foreground structure which may produce larger effects than standard ΛCDM models.

Lieu & Mittaz treat the demagnification in the empty voids and the magnification in the clusters separately and show that in terms of the average magnification $\langle \eta \rangle$ they cancel, so that the same ‘light conservation’ result applies in this inhomogeneous case as in the homogeneous case treated e.g. by Weinberg (1976). They show that the same result applies when strong lensing is taken into account. Here we follow Lieu & Mittaz and define $\eta = (\theta' - \theta)/\theta$ as the magnification ‘contrast’; the effect of the scattering is to increase the average angular size $\theta$ of the source by this fractional amount.

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where
\[ f(R_j, b_{\text{min}}) = \ln \left( \frac{R_j}{b_{\text{min}}} \right) - \frac{8}{\pi^2} \]
and \( p_{ij} \) is the probability of finding a group with velocity dispersion \( \sigma_i \) and radius \( R_j \).

In one case, they use a space density of groups which corresponds to that observed in the group sample of Ramella et al. (1999, 2002) and assuming the other group parameters such as the velocity dispersion, \( \sigma \). This produces \( \langle \eta \rangle \approx 0.099 \) and \( \delta \eta \approx 0.093 \), assuming the groups continue with the same comoving space density out to \( z_f = 1 \) in a \( \Omega_m = 0.27 \) model. Lieu & Mittaz note that this assumption may not be justifiable for groups because of the effects of dynamical evolution. For their Abell cluster model they find \( \langle \eta \rangle \approx 0.099 \) and \( \delta \eta = 0.064 \). Lieu & Mittaz argue that their assumption of no dynamical evolution of the cluster density may have more empirical support in the case of clusters than groups. One point to note is that Lieu & Mittaz use the line-of-sight rms velocity dispersion and the full 3-D rms velocity dispersion should be \( \sqrt{3} \) bigger. Although \( \langle \eta \rangle \) would remain unchanged, \( \delta \eta \) would then increase by a factor of 3, implying \( \delta \eta = 0.28 \) for the Ramella et al groups and \( \delta \eta = 0.19 \) for the Abell clusters.

These values for \( \delta \eta \) apply in the coherent regime of scattering where a CMB patch has a mean angular size less than \( \alpha \), the size of a group at an average redshift. At larger scales, in the incoherent regime of scattering, the effective \( \delta \eta \) reduces according to \( \delta \eta \propto 1/\theta \) (see equations 33, 34 of Lieu & Mittaz). For the groups and clusters considered by Lieu & Mittaz, the division between these regimes occurs at \( \approx 10 \text{arcmin} \). The form of \( \delta \eta \) \( = \sigma(\theta)/\theta \) in the terminology of Seljak, 1996 with \( \theta \) is given in Fig. 1 for the modified group and cluster models of Lieu & Mittaz. We shall compare these to the standard CMBFAST estimates of the magnification function in the next section.

Lieu & Mittaz note that in cases where \( \delta \eta \) is large compared to \( \langle \eta \rangle \) a skewed distribution for \( \eta \) may be implied due to negative density fluctuations cutting off at \( \rho = 0 \) (Dyer-Roeder, 1972, empty beam). Although this may also apply to the models above and elsewhere in this paper, here we shall simply assume a symmetric magnification distribution which is applied using equation A7 of Seljak (1996).

### 3 LENSING MODEL OF SELJAK (1996)

We next discuss the more usual lensing formalism of Seljak (1996) as implemented in CMBFAST. This is based on inferring the fractional magnification as \( \sigma(\theta)/\theta \) \( = \delta \eta \) in the terminology of Lieu & Mittaz) from the mass power spectrum using the variation of Limber’s formula given in equations (7) of Seljak (1996). Using the CMBFAST source code, \( \sigma(\theta)/\theta \) can be generated. The \( \Lambda \)CDM model with \( \Omega_m = 0.27, \Omega_l = 0.73 \) (as assumed by Lieu & Mittaz) with the non-linear extension of Peacock & Dodds (1994) gives the result shown in Fig. 1. It can be seen that the \( \sigma(\theta)/\theta \) drops like \( 1/\theta \) at large scales as expected in the incoherent regime while flattening at smaller scales. At \( \theta > 100 \text{arcmin} \),

\[ C(\theta) = (2\pi)^{-1} \int_0^\infty dl e^{-l^2/2} \tilde{C}(l)J_0(l) \]

where \( C_1 \) is the CMB angular power spectrum and \( \tilde{C}(\theta) \) is the lensed angular correlation function. This equation has the appearance of a Hankel transform but the dependence of \( \sigma \) on \( \theta \) means that this is only true under the assumption that the \( \theta \) dependence is slow.

When equation A6 is used with the standard model, only small effects are seen on the CMB (see Seljak, Fig. 2). Indeed, we found that any \( \sigma(\theta)/\theta \) that flattens at scales
smaller than 1 degree even with amplitudes as high as $\sigma(\theta)/\theta = 0.45$, generally gave small lensing effects on the first acoustic peak. Since the lensing dispersion is close to that of Lieu & Mittaz, similar results are found under their assumptions.

A case of interest is the one where we start by assuming that $\sigma(\theta)$ is a constant. This means that $\sigma(\theta)/\theta \propto 1/\theta$ i.e. the usual large-scale behaviour extends to the smallest scales. It also means that the combination $e^{-\sigma^2(\theta)C_2/C_1}$ is the lensed power spectrum. By adjusting the size of $\sigma$ it is then possible to smooth small scale peaks away and even to partially smooth away and hence move the first peak. For the moment, we simply assume $\sigma = 0.005$ rad. in the case shown in Fig. 1 for $\theta > 7'$ . The question is whether this amplitude and form of $\sigma(\theta)/\theta$ can be justified physically. Equation 7 of Seljak(1996) implies that $\sigma = $constant requires the power spectrum of the density, $P(k)$, ($\propto P_g \propto k^4$ where $P_g$ is the power spectrum of the potential) to have the form $P(k) \sim k^0$ in the range of interest which implies that $\xi(r) \sim r^{-3}$, since Fourier transforming the density power spectrum, $P(k) \propto k^n$, to the correlation function implies $\xi(r) \sim r^{-(3+n)}$ asymptotically for $-3 < n < 0$. Thus it appears that the constant $\sigma$ model will require a correlation function for the mass which is steeper than the observed galaxy correlation function, $\xi(r) \sim r^{-1.8}$. As can be seen, the amplitude of $\sigma$ is also much higher than for the standard model. But we shall see next that such an amplitude may be suggested by considerations of QSO lensing and that the $\sigma = $constant form shown in Fig. 1 then can be used to ‘move’ the first acoustic peak.

We now further draw the comparison with the results of Lieu & Mittaz by expressing their model in power-spectrum or correlation function terms. These authors assume a random distribution of Abell clusters each with a singular isothermal sphere for its mass distribution. According to Peebles (1974) a model with randomly placed monolithic clusters with density profile $\rho(r) \propto r^{-4}$ implies a correlation function with $\xi(r) \propto r^{-(2n-3)}$, $\epsilon = 2$ as assumed by Lieu & Mittaz gives $\gamma = 1.0$ whereas $\epsilon = 2.4$ gives $\gamma = 1.8$. Fourier transforming the correlation function to the power spectrum (as above) we find that isothermal spheres of large radius lead to a power spectrum of slope $k^{-2}$ which can be assumed to apply in the range $0.01 < k < 10$ h Mpc$^{-1}$.

When we take the value for the local galaxy correlation function scale length of $r_0 = 6h^{-1}$Mpc we find that, assuming $\epsilon = 2.4$ for the Abell cluster model, $\sigma(\theta)$ is approximately the same at large scales as the linear theory $\Lambda$ model (see Fig. 1). Thus, as expected, in the Seljak formalism an approximately unbiased mass correlation function with a -1.8 power-law gives approximately the same result for $\sigma/\theta$ as the $\Lambda$CDM model with non-linear extension.

If we now assume a model in the spirit of Lieu & Mittaz where the mass is in galaxies and all the galaxies are in randomly distributed Abell clusters, this will give a mass correlation function that extends as a -1.8 power-law to $>10h^{-1}$Mpc with a correlation length of $r_0 = 10.6h^{-1}$Mpc (from equation 23 of Peebles, 1974). This then produces a $\sigma/\theta$ result which would be 1.7x higher than that for $r_0 = 6h^{-1}$Mpc shown in Fig. 1. This model gives a generally flatter result with $\theta$ than the LM cluster (or group) model, agreeing with the LM cluster model at 1 arcmin, lying a few times lower at 10 arcmin and a few times higher at 100 arcmin. The cut in the cluster mass profile at radius $\approx 2h^{-1}$Mpc assumed by Lieu & Mittaz makes the comparison less valid at larger scales. An $\approx 2.5 \times$ bigger correlation function scale length would be required to reach the constant $\sigma$ model even at large scales ie $r_0 = 27h^{-1}$Mpc. Also, as we have seen the model implied by the -1.8 power law for $\xi$ is significantly flatter than the constant $\sigma$ model discussed above and the SIS model with $\epsilon = 2$ and $\gamma = 1$ as actually assumed by Lieu & Mittaz would give an even flatter slope.

We conclude that Lieu & Mittaz and Seljak appear to obtain reasonably similar results for $\sigma/\theta$ when similar input models are assumed. Although the Lieu & Mittaz group/cluster models show more power at small scales than the $\Lambda$CDM model, when we assume mass correlation functions which should be reasonably appropriate for the Lieu & Mittaz cluster model in the Seljak formalism, improved agreement is seen at small scales. Therefore in terms of the discussion (e.g. Lewis & Challinor 2006) of how the results of Lieu & Mittaz relate to earlier work, there may be no serious disagreement, with their assumed higher amplitude of foreground mass clustering probably being a bigger factor in explaining any difference than their partial inclusion of strong lensing in their lensing formalism. But in the wider context of affecting the first peak position, such differences between the models are academic because none of the $\sigma(\theta)/\theta$ models which flatten at small scales produces the simple smoothing of the CMB first peak of the constant $\sigma$ model; models with more small-scale power and a larger mass clustering amplitude are required. We next look to see if there is any evidence that the size of the magnification dispersion may be currently underestimated.

4 QSO MAGNIFICATION RESULTS

4.1 Group masses from QSO lensing applied to results of Lieu & Mittaz.

We now consider the possible implications if galaxy group masses were as big as those found in the QSO magnification results of Myers et al (2003, 2005). Myers et al (2003) found that the QSO-group cross-correlation function fitted SIS mass profiles with $\sigma = 1125$km$s^{-1}$. This velocity dispersion is comparable to the values found for rich clusters, although the sky density of these groups is $\approx 1$deg$^{-2}$ compared to $\approx 0.1$deg$^{-2}$ for the Abell clusters. Myers et al (2005) also found that 2QZ QSO lensing by individual galaxies again produced more anti-correlation than expected from the standard model. Here they analysed the results using the galaxy power spectrum after Gaztanaga (2003) and found that in the standard cosmology an anti-bias with $b \approx 0.1$ on $\approx 1h^{-1}$Mpc scales would be required to explain the strength of anti-correlation seen. Guimaraes, Myers & Shanks (2005) showed that these results were not due to selection effects by showing, for example, that the same cluster finding algorithms in the Hubble volume N-body simulation produced much smaller lensing effects.

Now we discuss below other observations which disagree with the results of Myers et al (2003, 2005). Nevertheless, here we consider the possibility of whether galaxy groups may be dynamically young and the virialised assumption therefore unjustified. In this case the group lensing masses
may be more correct, implying that the group masses are bigger than expected and comparable to the mass of Abell clusters. If the full factor of ten increase observed by Myers et al for QSO lensing over what is expected for the concordance model is translated directly into the results of Section 2 for CMB lensing, then $≈ 3\times$ bigger values of $\delta \eta$ will be found, resulting in $\delta \eta \approx 0.6$ for both groups and clusters in the above model.

However, Myers et al pointed out that their space density of high mass groups implies a Universe with a mass density more consistent with that of an Einstein-de Sitter model, rather than the $\Lambda$CDM model. The space density of groups reported by Myers et al is $n_0 = 3\pm 1\times 10^{-4}h^3\text{Mpc}^{-3}$, close to the value of Ramella et al as used by Lieu & Mittaz, $(n_0 = 4.4\times 10^{-4}h^3\text{Mpc}^{-3})$. Taking a radius of $2h^{-1}\text{Mpc}$ with $b_{\text{min}} = 7h^{-1}\text{kpc}$ and substituting these in the SIS result for $\delta \eta$, we find $\delta \eta = 0.15$. Here we have assumed that the group and cluster distribution is now unevolved out to $z_r = 0.5$; our cluster and group densities are essentially determined within this $z$ range, reducing uncertainties due to evolution. We note that Lieu & Mittaz find an average value of the group velocity dispersion of $270\text{km/s}$ whereas the weighted sum in their eqn (32) corresponds to an average value of $460\text{km/s}$. Clearly the more massive groups, in the tail of the distribution, dominate the deflection effect. If the same effect applies for the groups of Myers et al, the rms dispersion would rise to $\delta \eta = 0.45$ in the coherent regime, as shown by the dotted line marked ‘LM groups (Myers)’ in Fig. 1.

This is the extra dispersion in the CMB anisotropy caused by coherent scattering. Following Lieu & Mittaz we calculate the scale above which the scattering of light by the galaxy groups becomes incoherent rather than coherent. In the EdS model the angular diameter distance at the average group distance of $z = 0.25$ is $d_A = 500h^{-1}\text{Mpc}$. Therefore our assumed group diameter of $4h^{-1}\text{Mpc}$ subtends an angle of $\approx 0.05\text{deg}$ at this redshift. At smaller $\theta$, $\delta \eta$ is constant, and at larger angles $\delta \eta \propto 1/\theta$ (see Fig. 1). Without the effect of the tail, the results are similar to the $\Omega_m = 1$ results of Seljak (1996) although the $\delta \eta$ extends to larger scales before starting to decrease. With the tail, the model gives considerably higher values for $\delta \eta$ (see Fig. 1, dotted line) than found by Seljak for the standard $\Lambda$CDM model (long-dashed line).

4.2 Galaxy masses from QSO lensing applied to results of Seljak (1996).

We next apply the galaxy lensing results of Myers et al (2005) to the results of Seljak (1996). For the standard $\Lambda$ cosmology, Myers et al found values of the bias $b \approx 0.05-0.15$ at sub-Mpc scales, depending on the methodology. In the EdS cosmology, they found values in the range $b \approx 0.15-0.3$. Little evidence of scale dependence was seen in the data, so these results may be assumed to also apply at larger scales. Scaling $\sigma/\theta$ from the unbiased results of CMBFAST in the $\Lambda$ and EdS cosmologies according to the above derived anti-bias, the results for $\sigma/\theta$ clearly rise by $1/b$ i.e. by $\approx 10\times$ in the $\Lambda$ case and $\approx 5\times$ in the EdS case. Clearly these results are only approximate. In a future paper we shall relate the QSO galaxy lensing result of Myers et al (2005) to the CMB result more directly via equations (A14) of Myers et al (2005) and (7) of Seljak (1996).

5 APPLICATION TO THE MODEL OF SHANKS (1985)

An example of a model which initially has the first acoustic peak at $l = 330$ rather than $l = 220$ is the Einstein-de Sitter, baryon dominated, low $H_0$ model of Shanks (1985); Shanks et al. (1991); Shanks (2005). The $C_l$ from this model also produces too high a peak height, so to obtain a fit to the data we need to assume that the optical depth to reionisation is $\tau = 0.4$ which is at the high end of what has been suggested by the large-scale WMAP polarisation results (Kogut et al 2003, Page et al 2006). In fact as shown by CMBFAST, as the $\tau$ goes up, the polarisation power spectra quickly saturate and it becomes more difficult to reject higher values.
of $\tau$. The resulting model $C_\ell$ is compared to the WMAP data in Fig. 2.

We then use the $\sigma = \text{constant}$ model from Fig. 1. There we have seen that this model has a high amplitude but one which may be motivated by the QSO magnification results of Myers et al (2003, 2005). The exponentially sharp increase in smoothing as $f$ increases that then results from equation A7 of Seljak is the sort of behaviour which might be required to smooth the baryon model into the WMAP data. The result of applying this lensing smoothing function is shown as the solid line in Fig. 2 and it can be seen to give an improved fit to the first acoustic peak. (Note that we are assuming that $I^2C_{gl,2}(\theta) \ll 1$ in equations A5 and A6 of Seljak in applying his equations A6 and A7 at this $\sigma =$ constant case where $\sigma(\theta)/\theta \approx 1$ at $\theta < 20'$. ) The question of whether the rise in $\sigma$ towards the smallest scales is self-consistent with the baryon model also arises. Certainly the linear theory baryon model does not produce such a rise, indeed showing less small scale power than the standard model. However, the non-linear power spectrum of the baryon model after top-down collapse is not easy to calculate and this leaves open the possibility that the baryon mass power spectrum may still be as steep as required to produce the $\sigma =$ constant model. As we have seen, a mass correlation function as steep as $r^{-3}$ approximates the behaviour needed, compared to the observed galaxy correlation function’s $r^{-1.8}$ behaviour. However, galaxy correlation functions as steep as $r^{-2.2}$ are seen in samples of early-type galaxies on $r < 1h^{-1}$Mpc scales (Ze- havi et al 2005, Ross et al 2006). The amplitude would have to be of order $r_0 \approx 25h^{-1}$Mpc implying strong anti-bias of the type suggested by the QSO magnification results.

Note that we have only postulated the $\sigma = \text{constant}$ model in Fig. 1 and not derived it from equation (7) of Seljak (1996). Therefore, although the model has $\sigma(\theta)/\theta > 1$ for $\theta < 20$ arcmin, we have not violated the condition that equation (7) only applies in the case $\sigma(\theta)/\theta < 1$ because of its assumption of the Born approximation. The main assumptions we therefore have made in deriving the lensed $C_\ell$ in Fig. 2 is that $I^2C_{gl,2}(\theta) \ll 1$ in the application of Seljak’s equation A7 as noted above. Nevertheless, the lensed result in Fig. 2 should still be regarded as only an approximation.

It has also not been possible to fit the second peak. To address this issue, we could consider using the $\delta$ effect caused by the known presence of hot gas in the rich Abell clusters to re-create the second peak after its era- sure by the lensing. Myers at al (2004) have statistically detected $\delta$ decrements around Abell clusters in WMAP data out to radii of 30 arcmin. These decrements simply add power to the power spectrum on cluster-sized scales. Assuming a random distribution of circular ‘clusters’, with a sky density of 3deg$^{-2}$, radius 12 arcmin and with ‘decrement’ $\Delta T/T \approx -3 \times 10^{-3}$, we can produce a quite significant ‘second peak’ (see Fig. 1). Although the match to the second peak is only approximate, this simple model with just a ‘top-hat’ $\delta$ profile at least serves to indicate the principle of this approach. The observed spectral index of the WMAP CMB fluctuations which effectively constrain the $\delta$ contribution to the first acoustic peak (Huffenberger & Seljak 2004) are much less effective constraints against a significant $\delta$ contribution to the second peak.

Further work is needed to check if the CMB EE and TE polarisation power spectra from WMAP can be reproduced by the above model (Kogut et al 2003, Page et al 2006). For example, it is known that lensing will produce B modes as well as E modes in the polarisation signal (Zaldarriaga & Seljak 1998) and it will be interesting to see if this provides a constraint on the current approach.

For the moment, we conclude that using high-redshift reionisation to reduce the first two acoustic peaks, then lensing to move the first peak to larger scales and finally $\delta$ to recreate the second peak, it appears possible to achieve an approximate fit to the WMAP CMB fluctuation power spectrum. Although this approach may be criticised as fine-tuning CMB foreground effects to achieve a fit to the data, it may also be viewed as being much less fine-tuned than the approach taken via exotic particles and dark energy, as employed to construct the standard $\Lambda$CDM model.

6 CONCLUSIONS

We first compared models for the effect of foreground galaxy clusters and groups on the CMB power-spectrum. We have shown that the lensing effects considered by Lieu & Mittaz may have bigger effects than they suggested, if the full 3-D velocity dispersion is used rather than the 1-D dispersions assumed by these authors. This change produces $\approx 3 \times$ bigger results for $\sigma(\theta)/\theta$ than they suggested, which will increase the effect on the CMB peaks.

We find that the empirical model of Lieu & Mittaz then gives significantly higher results for the magnification dispersion at small scales ($\theta < 30'$) than those of Seljak (1996) assuming standard cosmological parameters. We have also shown that under the same foreground assumptions, now modelled in mass correlation function terms, the lensing formalism of Lieu & Mittaz produces approximately the same results as the that of Seljak (1996). But both formalisms in the case of the standard $\Lambda$CDM foreground or in the empirical cases considered by Lieu & Mittaz tend to produce $\sigma(\theta)$ relations which are too flat towards small $\theta$ and have too small amplitudes to modify significantly the first peak of the CMB power spectrum.

In conjunction with a model that assumed lensing dispersion $\sigma(\theta) = \text{constant}$ we then used the QSO magnification results of Myers et al (2003, 2005) to show that, if galaxy groups have bigger than expected masses, there could be significant effects of gravitational lensing on even the first peak of the CMB. Although the results of Myers et al remain controversial (Scranton et al. 2005, Mournichas & Shanks 2007), it is clear that at least in principle, lensing can affect the interpretation of the CMB power spectrum. But to have a significant effect on the first peak, the mass clustering power spectrum would then need to have been significantly underestimated in previous studies with anti-bias, $b \approx 0.2$, being required.

There are many pieces of evidence which favour the standard model bias value of $b \approx 1$. For example, redshift space distortion analyses imply $\beta = \Omega_m h / b \approx 0.5 \pm 0.1$ (e.g. Ratcliff et al 1996, Hawkins et al 2003.) But such analyses tend to assume that the galaxies trace the mass with linear bias and in the case where groups contain as much mass as clusters this assumption does not apply. Even here we note that the infall velocity patterns around the rich environments of passive galaxies in 2dFGRS are very
similar to those around the poorer environments of active galaxies (Madgwick et al. 2003), suggestive of the evidence from the QSOs that groups dominated by spirals may have as much associated mass as the richer environments of the early-types.

But still, the most direct previous constraints on the mass power spectrum come from estimates of cosmic shear. The QSO lensing results are again in clear contradiction with these since they are mainly consistent with $b = 1$. For example, Seljak (1996, Fig. 1) suggests that the cosmic shear upper limits on $\sigma(\theta)/\theta$ are close to the SCDM predictions and well below the constant $\sigma$ model in Fig. 1. Yet even in the case of cosmic shear there are possible systematic effects which may cause underestimates of the mass density and overestimates of $b$. For example, Hirata & Seljak (2004) have suggested that intrinsic galaxy alignments may prove difficult to disentangle from the effects of gravity on the shapes of galaxies. They suggest that even with the availability of perfect redshift information to remove physically associated pairs of galaxies, the effect of lensing may still be corrupted by physical alignment if the foreground tidal field dynamically elongates the foreground galaxy while elongating the background galaxy via its lensing effect. This could cause the cosmic shear to be significantly underestimated in present samples and explain the discrepancy with the QSO lensing result (see also Mandelbaum et al., 2006).

Although the agreement in form between the galaxy and QSO power spectra and WMAP are impressive, there is the possibility that the galaxies do not trace the mass on the largest scales and may be affected by scale-dependent bias. The high masses for groups suggested by the QSO lensing result (see also Mandelbaum et al., 2006).

The QSO lensing results are again in clear contradiction with these since they are mainly consistent with $\sigma(\theta)/\theta$.

We have taken the view here that the crucial observation is the high fraction of baryons in Abell clusters such as Coma. This ‘baryon catastrophe’ already forces the CDM model to include $\Lambda$ to allow a model with a lower $\Omega_m$ to be compatible with zero spatial curvature and inflation (White et al 1993). If the X-ray gas is to explain the missing mass in Coma, routes must be found to reconcile WMAP with the adiabatic $\Omega_{\text{baryon}} = 1$ prediction and we have shown that cosmic CMB foregrounds may form such a route. If the model can be shown to satisfy other constraints such as the WMAP polarisation power spectra, the result could be a model which is less finely-tuned than the standard model and is based on known physics, without recourse to undetected neutralinos or postulated dark energy.

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