Error localization of finite element updating model based on element strain energy

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Abstract. An error localization indicator based on modal strain energy changes is proposed and used for selecting design parameters to be updated in model updating process. Taking an aero-engine combustor casing structure as an example, the ‘supermodel’ of combustor casing was established and validated with the test data and the reduced model (also called the design model) was built with the simplification of modelling. By comparing the modal strain energy changes between ‘supermodel’ and design model of combustor casing, the error locations of the reduced combustor casing modelling was highlighted by the error localization indicator. Then, the updating parameters of the design model were selected as the areas with significant variations of modal strain energy changes based on the error localization indicator. Defining the updating object function with the minimum of natural frequency errors between the FE model prediction and the modal test data, model updating of the design combustor casing model based sensitivity analysis method was carried out using the experimental modal data. After model updating, the maximum frequency error of the first ten modes was decreased from 27.1% to 1.2%, compared with the test data. The result shows the effectiveness of the proposed method and certain significance in parameter selection for model updating.

1. Introduction
With the development of computer technology, the finite element model has been widely used in the prediction of the dynamic characteristics of aero engine components and the whole engine structure. Finite element model can be considered as discretization of the actual structure. Due to some assumptions and inevitably engineering simplifications, some errors may exist in the finite element modelling. Generally speaking, the errors of the finite element modelling can be mainly classified into three aspects [1, 2]: the discrete error, the parameter error and the boundary condition error. Model updating with the experimental or reference data is widely used to eliminate or reduce the errors in the model in order that the dynamic prediction of the updated finite element model can accurately demonstrate the real structure’s dynamic behaviours.

At present, the most widely used model updating method in engineering application is based on the sensitivity analysis. Friswell and Mottershead reviewed the updating methods based on sensitivity analysis of modal parameters, including the parameter estimation and weighting parameter selection problem, and a lynx helicopter frame model is updated [1, 3, and 4]. Chen and Zang et al. used modal test data to update the intermediate casing of a complex aircraft engine, and the finite element prediction results are in good agreement with the experimental test [2, 5]. Zang [6] conducted a comprehensive assessment of model updating methods and pointed out that the iterative sensitivity methods based on the modal characteristics and frequency response function have the greatest potential in engineering
applications. In fact, model updating is an inverse dynamic problem, the improper updating parameter selection will lead to fail in updating or parameters lose physical meanings. Therefore, how to locate the error in finite element model and select the correct updating parameter is the key step to solve the problem. Ladevêze [7] discussed the effect of discrete of finite element model on the dynamics characteristics from the mathematical view. Mottershead studied the element size influence on model updating [8], and pointed out that the updating model should be verified to meet the updating requirements. Lallement [9] and Xiang Jinwu [10] proposed the balance of dynamic equation to locate the source of the error, but the error sources were located on the model nodes or dofs which lack of physical meanings and do not suit for engineering. Linderholt and Friswell both proposed forward the selection method and the parameter selection method of the optimal subspace [11, 12], which are used to identify the model error sources and the parameter selection in damage detection. Kim [13, 14] put forward the method of combining residual vector and multi objective optimization to guide the parameter selection in model updating. These methods can help parameter selecting in model updating, but the problem of error location still exists in model updating.

In recent years, methods based on the modal strain energy have been widely used to localize the structural damage in structural health monitoring. Link used the modal strain energy of the substructure to indicate the error in the model [15]. Shi [16, 17] and Law [18] detected the damage of beam and building structures by using the method of element modal strain energy. Hu took the plate, cylinder and composite laminates as examples to demonstrate the damage detection method based on modal strain energy [19-21]. All results showed the advantages of modal strain energy in parameter selection and error location. Because of the limited modal testing points and test noises, it is still difficult to use test data to calculate the modal strain energy and locate errors for complex structures. Zang etc. used the data from the supermodel instead of the test data to locate the errors in the finite element model, based on the element modal strain energy. The method was successfully applied to a complex intermediate casing structure [22, 23]. This paper proposes an error location method based on supermodel and the variation of modal strain energy. Taking a complex combustor casing structure as an example, the supermodel of the casing and the simplified design model were established. Error indicator is built up, based on variations of modal strain energy between the supermodel and the simplified design model. Then, the design parameters of the FE model were selected for model updating according to the error indicator’s result. The updating result demonstrates the effectiveness of the proposed method.

2. METHODOLOGY OF ERROR LOCATION AND MODEL UPDATING

2.1. Error Indicator based on modal strain energy

A finite element design model always contains some errors in stiffness or mass in general. The mass error can be eliminated through the mass check in the modeling process, but mostly, the stiffness error needs to be updated in order to get the correct stiffness property of the finite element model. Generally, the dynamic equation of undamped system is described as below

$$ (K - \lambda_i M)\varphi_i = 0 $$

(1)

Where $K, M$ is the global stiffness matrix and mass matrix of the system, $\lambda_i, \varphi_i$ is the $i^{th}$ eigenvalue and mass normalized eigenvector respectively. Because of the structural simplification and discretization of elements, the stiffness error would be introduced into the finite element design model. The dynamic equation of the design model system can be described as follows

$$ (K + \Delta K - \lambda_i^d M)\varphi_i^d = 0 $$

$$ \Delta K = \sum \Delta k $$

(2)

Where $\Delta K$ represents the stiffness error of the design model, $\lambda_i^d, \varphi_i^d$ is the eigenvalue and mass normalized eigenvector of the design model. The modal strain energy of each element in the model was defined as
\[ MSE_{i,j} = \frac{1}{2} \phi_i^T K \phi_i \]
\[ MSE'_{i,j} = \frac{1}{2} \phi_i^{d,r} K_i^{d,r} \phi_i^{d,r} \]

Premultiplication \( \phi_i^T \) of equation(1) and \( \phi_i^{d,r} \) of equation(3), we can get

(a) \[ \lambda_i = \sum \phi_i^T K \phi_i = 2 \sum MSE_{i,j} \]

(b) \[ \lambda_i^{d,r} = \sum \phi_i^{d,r} K_i^{d,r} \phi_i^{d,r} = 2 \sum MSE'_{i,j} \]

If equation(4b) minuses equation (4a), we can have

\[ \Delta \lambda_i = \lambda_i^{d,r} - \lambda_i = \sum (MSE'_{i,j} - MSE_{i,j}) \]

It can be seen that the variation of model strain energy reflects the eigenvalue changes, the error indicator can be therefore defined as follows

\[ \chi_j = |MSE'_{i,j} - MSE_{i,j}| \]

If the \( j^{th} \) element of the model has no error in stiffness, then the indicator \( \chi_j \) can be expressed as

\[ \chi_j = \frac{1}{2} [\phi_i^{d,r} K \phi_i - \phi_i^{d,r} K \phi_i] \]

When the \( j^{th} \) element or substructure has errors in stiffness, the indicator \( \chi_j \) can be expressed as

\[ \chi_j = \frac{1}{2} [\phi_i^{d,r} K \phi_i - \phi_i^{d,r} \Delta K \phi_i - \phi_i^{d,r} K \phi_i] \]

Then, the normalized error indicator is defined as

\[ \chi'_{N,j} = \frac{\chi_j}{\Delta \lambda_i} \]

The normalized error indicator can conveniently flag the stiffness errors in the model. It should be noticed that the indicator is a kind of tool to locate the potential stiffness errors in the model but it is difficult to assess how much the stiffness errors exist exactly.

2.2. Model updating: Inverse Eigen-sensitivity Method

The sensitivity-based model updating method is widely used in engineering applications. Generally, the objective function to be minimized is the weighted sum of squares of the error in the modal parameters, given by, at the \( j^{th} \) iteration by

\[ J(\delta \theta) = z^T W_j (\delta z - S \delta \theta)^T W_j (\delta z - S \delta \theta) \]

\( \delta \theta \) represents the updating parameter perturbation,

\[ \delta \theta = 0 - \theta_j \]

\( \delta z \) is the residual error between the test modal parameters and the predicted modal parameters,

\[ \delta z = z - z_j \]

\( S \) is the sensitivity matrix. The elements of the natural frequencies’ sensitivity with respect to the parameter in the matrix are obtained by

\[ \frac{\partial \omega_i}{\partial p_r} = \frac{1}{2 \pi f_i} \phi_i \frac{\partial (K - \lambda \phi_i M)}{\partial p_r} \phi_i \]

Where \( p_r \) is the \( r \text{th} \) updating parameter at the \( j^{th} \) iteration, and \( z_j \) is the predicted modal parameter at the \( j^{th} \) iteration. The \( i \text{th} \) natural frequency (in Hz) is given by \( f_i \) and the corresponding eigenvalue and mass normalized eigenvector are \( \lambda_i \) and \( \Phi_i \), respectively. In the iterative updating process, \( \theta \) represents the \((j + 1)^{th}\) estimation parameters after the iteration.
\( \mathbf{W}_e \) is a weighting matrix that is used to reflect the importance of each element of the residual vector, and should be a symmetric and positive definite matrix. Equation (10) is solved by minimizing \( J \) with respect to \( \delta \theta \), and leads to the following estimate of the updating parameters after the iteration,

\[
\delta \theta = \left[ \mathbf{S}_i^T \mathbf{W}_e \mathbf{S}_i \right]^{-1} \mathbf{S}_i^T \mathbf{W}_e \delta \mathbf{z}
\]

Hence the \((j+1)\)th updating parameter vector is given by

\[
\theta_{j+1} = \theta_j + \left[ \mathbf{S}_i^T \mathbf{W}_e \mathbf{S}_i \right]^{-1} \mathbf{S}_i^T \mathbf{W}_e \delta \mathbf{z}
\]

The updating parameter \( \theta \) is constrained to lie in a limited range, given by

\[
\theta_{lb} < \theta < \theta_{ub}
\]

In this paper, we select the natural frequency errors of the paired modes between the FE model and the test model for the updating residuals.

3. Finite element analysis and modal test configuration of a combustor casing

3.1. Modal testing of a combustor casing

A modal test was performed using traditional hammer excitation and acceleration transducers. The casing was fastened with four rubber bands at the four proportional spacing bolt holes. The vibration signals were measured by the PCB transducers, and recorded by the four-channel DP data acquisition unit. The ICATS modal test and analysis tool is used to obtain the natural frequencies, damping ratios, and mode shapes, with the frequency response functions (FRFs) generated by the DP software. Because of the approximate symmetric character of the casing structure, two transducers were located with an angel to identify the repeated modes. All modes within 1000Hz were considered, especially those within 500Hz. Figure 1 shows the experimental configuration of the Combustor Casing and the test grid of the measurement points. Figure 2 shows the tested natural frequencies and mode shapes of the first ten measured modes. Clearly, the modes pairs (1/2), (3/4), (5/6), (7/8), and (9/10) are the repeated modes.

![Figure 1](image1.png)  
Figure 1. Experimental setup for modal testing of the Combustor Casing

| Mode | Frequency |
|------|-----------|
| 1st  | 62.80Hz   |
| 2nd  | 63.97Hz   |
| 3rd  | 169.32Hz  |
| 4th  | 170.14Hz  |
| 5th  | 292.52Hz  |
3.2. Supermodel analysis and design model analysis of the combustor casing

As only 70 nodes were used to build the test model in modal testing, it is too difficult to establish the error indicator of the stiffness parameter with the test mode shapes due to the limitation of test nodes. So the supermodel that is the refined and detailed high fidelity model with 3D solid elements was built and its analytical mode shapes were taken to replace the test mode shapes based on the assumption that the dynamic characteristic of supermodel can approximately represent that of real structure. The geometric structure of the complex combustor casing was shown in Figure 3(a). There are 72 bolt holes located on the flange, fuel pipe holes in the middle of the casing and 90 stator vanes in the inlet of the casing. In order to keep the detailed features of the casing, the combustor casing supermodel was meshed with 10 nodes tetra element and the global element length is 3mm. The totally supermodel has 1,190,000 elements and 11,000,000 dofs. The supermodel of combustor casing was shown in Figure 3(b).

The design model was built in two steps. First, all the small features were removed and the casing was made smooth. Then, the design model of the casing was meshed with 10 nodes tetra element and a proper element size of 35mm, shown in Figure 3(c). The dofs of design model were around 330,000, almost decreased to 97% in comparing with the supermodel. The mode shapes of design model are shown in Figure 4. Modes 1 and 2 are the repeated modes of the first nodal line vibration mode of the casing outlet. Modes 3 and 4 is the repeated modes of the second nodal line vibration mode, and Modes 5 and 6 and Modes 9/10 , the third and the fourth nodal line vibration modes respectively. Modes 7 and 8 are the repeated modes of the first nodal line vibration mode of the inner area of the casing.

Figure 2. First ten measured modes of Combustor Casing

Figure 3. models of Combustor Casing: (a) Geometric structure, (b) Supermodel, (c) Design model

Figure 4. First ten modes of the design model of the combustor casing
Correlation analysis of the design model and the supermodel with the test data was shown in Table 1. The absolutely maximum frequency error of paired mode between supermodel and experimental data is 3.8% in Mode 5, the frequency errors are relatively large in Modes 6, 9 and 10 but still less than 3.8%. The MAC values of paired modes are larger than 0.77 and the highest value is 0.92 in Modes 5 and 6. Because of the symmetric characteristic of the casing, the mode shapes of paired modes will have similar shape but with a rotation angle and that may cause the decrease of the MAC value. The correlation results of supermodel and experimental data also indicates that the supermodel’s dynamic characteristic can represent the true dynamics property of the real combustor casing. Therefore, the mode shapes from the supermodel instead of the test mode shapes can be used for the error localization of the design model. The frequency errors of the design model, compared with the test, are relative larger than those from the supermodel. The frequency errors of Modes 7 and 8 are up to 27.1%. Even for Modes 1 and 2, the natural frequency errors were 7.2% and 5.2% respectively. The MAC values of paired modes between the design model and the test data are still larger than 0.6. The results show that the design model’s simplification and modeling is proper and the model can be updated.

| Mode | Exp data Freq/Hz | Supermodel Freq/Hz | Error/% | MAC | Design model Freq/Hz | Error/% | MAC |
|------|------------------|--------------------|---------|-----|----------------------|---------|-----|
| 1    | 62.8             | 62.0               | -1.3    | 0.77| 67.3                 | 7.2     | 0.78|
| 2    | 64.0             | 63.2               | -1.2    | 0.77| 67.3                 | 5.2     | 0.78|
| 3    | 169.3            | 165.8              | -2.1    | 0.85| 171.3                | 1.2     | 0.88|
| 4    | 170.1            | 166.4              | -2.2    | 0.88| 171.3                | 0.7     | 0.88|
| 5    | 292.5            | 281.4              | -3.8    | 0.92| 298.7                | 2.1     | 0.75|
| 6    | 299.7            | 288.6              | -3.7    | 0.92| 299.0                | -0.2    | 0.74|
| 7    | 312.1            | 312.7              | 0.2     | 0.80| 396.6                | 27.1    | 0.68|
| 8    | 313.2            | 313.6              | 0.1     | 0.80| 356.9                | 26.4    | 0.70|
| 9    | 423.7            | 408.0              | -3.7    | 0.91| 436.2                | 2.9     | 0.60|
| 10   | 427.9            | 412.3              | -3.6    | 0.91| 435.6                | 1.8     | 0.60|

### 3.3. Error location of the design model based on modal strain energy

As the number of design model elements are quite large and value of each element error indicator can be easily affected by the element matches between the supermodel and the design model. In order to overcome this problem, we divided the design model into several parts. The principles of parts division mainly follow two aspects. First, the same simplification areas are divided as a part, such as the flange where the bolt holes were simplified. Second, each part should be the symmetrical or approximate symmetrical, since the modal experimental results show the true dynamic characteristic of the casing is symmetrical. The design model, shown in Figure 5, was divided into 10 parts, include the flange parts (parts 2 and 7) and fuel tube hole part (part 5) and so on.

![Figure 5. Design model of Combustor Casing with division](image-url)
Correspondingly, the supermodel was also divided into 10 parts by the same way. The mass of each part between both models keeps equal by adjusting the density, the elastic module of each part of the design model is selected as the parameters to localize the errors in the model with the proposed method. By comparing the modal strain energy of design model and that of supermodel, the normalized error indicator was shown in Figure 6. The normalized error indicators of the first and second modes show that the parameters $E_2, E_1, E_4, E_8, E_{10}$ have relative larger errors than other parameters. The errors of the parameters $E_1, E_2, E_3, E_4, E_6$ are larger than others from the results of the $3^{rd}$ mode to the $6^{th}$ mode. From the results of the $7^{th}$ mode and the $8^{th}$ mode, we can see that the parameter $E_4$ dominated the error and the parameter $E_{10}$ had a limited influence on these modes. Both parameters caused the natural frequency errors of these modes over 26.4%. As shown in Figure 5, the part four is the inner area of the casing which the mode has a relative large vibration in this region. This can be explained that the vibration mode shape reflects the modal strain energy distribution, so modal strain energy of the $7^{th}$ and $8^{th}$ modes almost concentrated in this area. The $9^{th}$ mode and $10^{th}$ mode show that the parameter $E_1, E_3, E_4, E_6$ have some error. When we select the updating parameter based on the error indicator, we focus on the mode which has a relative large frequency error, especially mode 1, 2, 7, and 8. So, we selected parameters $E_1, E_3, E_4, E_6, E_{10}$ as the updating parameters.

![Figure 6](image-url)
3.4. Model updating and result analysis of combustor casing
Taking the $E_2, E_3, E_4, E_8, E_{10}$ as updating parameters, the minimization of frequency errors of paired modes between design model and test data were selected as the objective function. After 5 iterations, the objective function and updating parameter tend to be converged. The changes of updating parameter and the frequency errors in the convergence process are shown in Figure 7. After updating, parameters $E_2, E_3, E_4$ increased over 20% because the simplification of the chamber made the stiffness loss. Parameters $E_8, E_{10}$ decreased because the big fuel tube holes were filled and the mesh size of stator vanes was large that increased the stiffness of the structure.

![Figure 7. Convergence of Updating parameter(a) and frequency error(b)](image)

The correlations between design model and test data of combustor casing before and after updating are shown in Table 2. The frequency errors of the 7th and 8th modes reduced from over 26.4% to less than 0.3% respectively, frequency errors of the 1st and 2nd modes also decreased to less than 1.2%. The maximum frequency error of all modes is less than 1.2%. The MAC values of the 7th and 8th modes increased to over 0.77, modes 9 and 10 to 0.72. The MAC values of modes 5 and 6 have small decreases while others keep almost unchanged. Results demonstrated capability of the proposed method to localize the errors in the model and this approach can provide some guidance for the parameter selection in model updating.

| Mode No. | Frequency Error/% | MAC |
|----------|------------------|-----|
|          | Initial | Updated | Initial | Updated |
| 1        | 7.2     | 1.2     | 0.78    | 0.79 |
| 2        | 5.2     | -0.6    | 0.78    | 0.78 |
| 3        | 1.2     | 0.2     | 0.88    | 0.80 |
| 4        | 0.7     | 0.2     | 0.88    | 0.85 |
| 5        | 2.1     | 1.1     | 0.75    | 0.62 |
| 6        | -0.2    | -1.2    | 0.74    | 0.62 |
| 7        | 27.1    | 0.3     | 0.68    | 0.77 |
| 8        | 26.4    | -0.3    | 0.70    | 0.79 |
| 9        | 2.9     | 0.6     | 0.60    | 0.72 |
| 10       | 1.8     | -0.6    | 0.60    | 0.72 |

4. Conclusions
An error location method based on modal strain energy is proposed and applied to selecting the parameters for model updating of a complex combustor casing structure. Modal test of the combustor casing was performed by the traditional method to get natural frequencies and mode shapes and also further used to validate the supermodel. Because of the limitation in test, the mode shapes from valid supermodel are implied to replace the test data for localizing errors in the simplified design model of
the combustor casing structure. The simplified design model was divided into several parts reasonably and the errors of parameters in model was successfully located using the error indicator based on modal strain energy. Sensitivity-based model updating method using the selected parameters from the error location method was performed. The updated results show that the proposed error location method can help to select the updating parameter in model updating procedure for engineering applications.

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Acknowledgments

The support of the National Natural Science Foundation of China (Project No. 51175244, 11372128), AVIC Commercial Aircraft Engine co., LTD and the Collaborative Innovation Centre of Advanced Aero-Engine are gratefully acknowledged. This study has also been made with the aid of the Foundation of Graduate Innovation Centre in NUAA and the Fundamental Research Funds for the Central Universities. The project number is KFJJ20150201.