Progressive trade credit policy in a supply chain with and without stock-out for supplier’s lead time under inflationary and fuzzy environment

Chaman Singh* and S.R. Singh

Department of Mathematics, Acharya Narendra Dev College (University of Delhi), New Delhi – 19, India; Department of Mathematics, D.N. College, Meerut, Uttar Pradesh, India

(Received 1 September 2014; accepted 9 February 2015)

In this paper, we have developed a progressive trade credit model with and without stock-out for retailer under supplier’s promised lead time incorporating the effect of inflation and time value of money. Supplier is not always in a position to fulfill retailer’s request at any time, and thus forced to adopt new policies to deal with the situation. To deal with the situation, the supplier does not hold the inventory indeed, but produces the items whenever they are demanded and promises a certain lead time. Therefore, the retailer has the choices to have the shortage or not during the promised lead time period. This paper investigates different situations where the supplier promises the lead time and permissible delay in payments under the circumstances of retailer’s choice to have the shortages or not. Finally, numerical examples along with sensitivity analysis are presented to illustrate the proposed model.

Keywords: fuzzy environment; inflation; lead time; trade credit

1. Introduction

In a supply chain, the supplier prefers that the retailer should place the orders well in advance of his requirement. However, the retailer prefers the supplier to fulfil orders immediately without facing any backlog at the supplier’s site. Hence, the supply chain faces an enticement problem in which both the supplier and the retailer want the other party to bear the consequences. This forces the supplier to rethink about his strategies, and to adopt the new policies. Thus, instead of instant replenishment, the supplier promises a lead time and starts the production after getting the order, and guarantees shipment of each order on time at the end of the promised lead time. The supplier’s promised lead time reduces the retailer’s risk from uncertain supply, but extends the retailer’s horizon beyond his standard replenishment time and provides the opportunity to retailer to rethink about his policies, and he may decide whether to have a shortage or not.

Liao and Shyu (1991) assumed that lead time can be reduced through crashing cost. Treville, Shapiro, and Hameri (2004) examined the role of lead time reduction in order to improve demand chain performance. Hsu, Wee, and Teng (2007) developed an inventory model with expiration date and uncertain lead time. Leng and Parlar (2009) discussed lead time reduction in a two-level supply chain with non-cooperative equilibria versus coordination under profit-sharing contract. Singh and Singh (2010a) developed a supply chain model with stochastic lead time and fuzzy ramp-type demand under imprecise partially backlogging rate for expiring items. Glock (2012) studied methods of reducing lead time and their impact on the safety stock and the expected total costs of a \((Q, s)\) continuous review inventory control system, and proposed that lead time may be reduced by crashing set-up and transportation time, by increasing the production rate, or by reducing the lot size.

Advance orders decrease the supplier’s risk from uncertain demand and excess inventory holding cost and deterioration cost. A cost–benefit analysis of this interaction, and the resulting inventory costs, determines who pays for the promised lead-time agreement. Thus, to ensure the retailer’s agreement to the promised lead time policy, the supplier may also offer the retailer a delay period, known as trade credit period. It makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier.

Hwang and Shinn (1997) examined an economic order quantity model under conditional permissible delay in payments. Chang and Dye (2001) developed an inventory model with permissible delay in payments and partial backlogging for deteriorating items. Soni and Shah (2008) derived optimal ordering policy for stock-dependent demand under progressive payment scheme.

*Corresponding author. Email: chamansingh07@gmail.com

© 2015 The Author(s). Published by Taylor & Francis. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/Licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.
Chung and Huang (2009) discussed an ordering policy with allowable shortage and permissible delay in payment. Hu and Liu (2010) determined an optimal replenishment policy for the economic production quantity model with permissible delay in payment and with shortages allowed. Khanra, Ghosh, and Chaudhuri (2011) proposed an economic ordering quantity model for deteriorating items with time-dependent quadratic demand under the condition of permissible delay in payment. Maity and Abadi (2012) discussed the joint control of inventory and its pricing for non-instantaneously deteriorating items under the condition of permissible delay in payment, with shortages allowed and partially backlogged. Uthayakumar and Rameswari (2013) explored a supply chain model with variable lead time under credit policy. Jana, Maity, and Roy (2013) developed a three-layer supply chain-integrated production-inventory model with permissible delay in payments in uncertain environments. Yang and Tseng (2014) developed a three-echelon inventory model with permissible delay in payments under controllable lead time and backorder consideration.

Today, the study of the supply chain model in a fuzzy environment is gaining phenomenal importance around the globe. Fuzziness grants authenticity to the model in the sense that it allows vagueness in the whole set-up which brings it closer to reality. Zadeh (1965) first introduced the fuzzy set theory. Maity and Maiti (2007) developed a two-storage inventory model with lot size-dependent lead time in fuzzy environment. Singh and Singh (2010b) developed a two-echelon supply chain model with imperfect production under imprecise and inflationary environment. Yadav, Singh, and Kumari (2012) developed a two-warehouse inventory model with stock-dependent demand using genetic algorithm in fuzzy environment. Singh and Singh (2013) explored a model from the perspective of vendor and buyers for deteriorating items with shortages under inflationary and imprecise environment.

In the existing literature, most of the researchers have discussed supply chain models either considering the lead time problem or permissible delay in payments separately. Very few who considered both these issues in a single problem did not consider the imprecise nature of parameters.

In this paper, we investigate different situations where the supplier promises a lead time and permissible delay in payments under the circumstances of retailer’s choice to have the shortages or not, incorporating the effect of inflation and time value of money, considering that all the cost parameters as well as selling price of retailer, supplier’s production rate, initial demand rate and interest earned and the interest charged rate involved in the model are imprecise in nature. Finally, numerical examples along with sensitivity analysis are presented to demonstrate the developed model and the solution procedure.

2. Assumptions and notations
The following assumptions and notations have been adopted to discuss the proposed model:

(1) Demand rate is a exponentially decreasing function of time, that is, \( D(t) = ae^{-bt} \) where \( a > 0 \), \( 0 < b < 1 \).
(2) Deterioration starts as soon as items are produced.
(3) Lead time of the supplier is a decision variable.
(4) Inflation and time value of money are considered.

The following notations have been used in our study:

| Symbol | Description |
|--------|-------------|
| \( \theta \) | The deterioration rate of the inventory, where \( \theta \) is a constant |
| \( P \) | The supplier’s production rate |
| \( R \) | Constant representing the difference between the discount rate and inflation rate |
| \( L \) | Lead time of the supplier |
| \( \delta \) | Constant partial backlogging rate for the retailer |
| \( M \) | The first credit offer for the retailer to settle the account |
| \( N \) | The second credit offer for the retailer to settle the account |
| \( T \) | Replenishment cycle length |
| \( B \) | Backlogged inventory |
| \( \overline{TP}_t(M) \) | Revenue generated by the retailer through sales during the time interval \([0, M]\) |
| \( \overline{TP}_t(N - M) \) | Revenue generated by the retailer through sales during the time interval \([M, N]\) |
| \( I_s(t) \) | Inventory level of the supplier at any time \( t \) |
| \( I_r(t) \) | Inventory level of the retailer at any time \( t \) |

3. Development of model
In order to study the model in fuzzy environment, the following parameters have been used which are fuzzy in nature and represented by the triangular fuzzy numbers.

3.1. Supplier’s inventory model
The supplier receives an order from the retailer of \( Q \) units of quantities; once the supplier gets the order, he starts the production and takes \( L \) units of time to produce the required \( Q \) units of items and completes the order at time \( t = L \). The supplier’s inventory is depicted in Figure 1 and governed by Equation (1).

\[
\dot{I}_s(t) = P - \theta I_s(t), \quad 0 \leq t \leq L \tag{1}
\]

with condition \( I_s(0) = 0 \) solution of Equation (1) is:

\[
I_s(t) = \frac{P}{\theta}(1 - e^{-\theta t}) \tag{2}
\]
\[ \delta_t = (s_{t1}, s_{t2}, s_{t3}) \] _fuzzy selling price for the retailer \$/unit_

\[ \tilde{p}_t = (p_{t1}, p_{t2}, p_{t3}) \] _fuzzy purchasing cost for the retailer \$/unit_

\[ \tilde{s}_s = (s_{s1}, s_{s2}, s_{s3}) \] _fuzzy selling price for the supplier \$/unit_

\[ \tilde{c}_s = (c_{s1}, c_{s2}, c_{s3}) \] _fuzzy production cost for the supplier \$/unit_

\[ \tilde{c}_{1s} = (c_{1s1}, c_{1s2}, c_{1s3}) \] _fuzzy holding cost for the retailer \$/unit_

\[ \tilde{c}_{2s} = (c_{2s1}, c_{2s2}, c_{2s3}) \] _fuzzy deterioration cost for the supplier \$/unit_

\[ \tilde{c}_{3s} = (c_{3s1}, c_{3s2}, c_{3s3}) \] _fuzzy ordering cost for the retailer \$/unit_

\[ \tilde{c}_{4s} = (c_{4s1}, c_{4s2}, c_{4s3}) \] _fuzzy shortage cost per unit backordered for the retailer \$/unit_

\[ \tilde{c}_{5s} = (c_{5s1}, c_{5s2}, c_{5s3}) \] _fuzzy lost sale cost for the retailer \$/unit_

\[ \tilde{c}_{1t} = (c_{1t1}, c_{1t2}, c_{1t3}) \] _fuzzy holding cost for the retailer \$/unit_

\[ \tilde{c}_{2t} = (c_{2t1}, c_{2t2}, c_{2t3}) \] _fuzzy deterioration cost for the retailer \$/unit_

\[ \tilde{c}_{3t} = (c_{3t1}, c_{3t2}, c_{3t3}) \] _fuzzy ordering cost for the supplier \$/unit_

\[ \tilde{c}_{4t} = (c_{4t1}, c_{4t2}, c_{4t3}) \] _fuzzy shortage cost per unit backordered for the supplier \$/unit_

\[ \tilde{c}_{5t} = (c_{5t1}, c_{5t2}, c_{5t3}) \] _fuzzy lost sale cost for the supplier \$/unit_

\[ \tilde{I}_c = (I_{c11}, I_{c12}, I_{c13}) \] _fuzzy interest earned per unit time by the retailer_

\[ \tilde{I}_{c1} = (I_{c11}, I_{c12}, I_{c13}) \] _fuzzy interest charged per unit by the retailer when the retailer pays during \([M, N]\)\_

\[ \tilde{I}_{c2} = (I_{c21}, I_{c22}, I_{c23}) \] _fuzzy interest charged per \$ in account by the supplier when the retailer pays during \([N, T]\)\_

\[ \tilde{P}_s = \tilde{c}_s \int_0^L \tilde{P}_e \tilde{e}^{\tilde{r}_t} dt = \frac{\tilde{c}_s}{\tilde{r}} (1 - e^{-\tilde{r}_t L}). \] (6)

The present worth of the set-up cost of the supplier in fuzzy sense is:

\[ \tilde{SEC_s} = \tilde{c}_{3s}. \] (7)

The present worth of the sales revenue of the supplier: Since the supplier offers a credit period to settle the account, therefore, instead of instant payment, the retailer takes his time to settle the account at time \(t = M\) and thus the present worth of the supplier’s sales revenue in fuzzy sense is:

\[ \tilde{SR}_s = \tilde{s}_s Q e^{-\tilde{r} M}. \] (8)

The present worth of the total profit of the supplier in fuzzy sense is:

\[ \tilde{TP}_s = \tilde{SR}_s - \tilde{PC}_s - \tilde{HC}_s - \tilde{DC}_s - \tilde{SEC}_s. \] (9)

### 3.2. Retailer’s inventory model with stock-out in crisp environment

Initially, the retailer places an order at time \(t = 0\). Due to the supplier’s lead time, the order reaches the retailer at time \(t = L\); during this time, the retailer backlogs the shortages that partially arise due to the absence of stock. After the arrival of stock, the retailer first clears the backlog. Then, the inventory is depleted due to the combined effect of demand and deterioration and reaches the zero level at time \(t = T\). The retailer’s inventory is represented in Figure 2.

Differential equations governing the retailer’s model are as follows:

\[ I_{c1}(t) = -\delta a e^{-\delta t}, \quad 0 \leq t \leq L, \] (10)

\[ I_{c2}(t) = -\theta I_{c2}(t) - a e^{-\delta t}, \quad L \leq t \leq T. \] (11)
Solving Equation (10) with initial condition $I_{11}(0) = 0$, one can have
\[ I_{11}(t) = \frac{-\delta a}{b}(1 - e^{-bt}), \quad 0 \leq t \leq L. \] (12)

The total backlogged stock at time $t = L$ is:
\[ B = -I_{11}(L) = \frac{\delta a}{b}(1 - e^{-\beta L}). \] (13)

The stock left at time $t = L$, after clearing the backlogged items is:
\[ q = Q - B = Q - \frac{\delta a}{b}(1 - e^{-\beta L}). \] (14)

Solving Equation (11) with initial condition $I_{22}(L) = q$, one can have
\[ I_{22}(t) = \left\{ q + \frac{ae^{-\beta t}}{(\theta - b)} \right\} e^{-(\theta - L) - \frac{ae^{-\beta t}}{(\theta - b)}}, \quad L \leq t \leq T. \] (15)

Using the boundary condition $I_{22}(T) = 0$, one has
\[ q = \frac{a}{(\theta - b)}(e^{-\beta t}e^{(\theta - b)T} - e^{-\beta L}). \] (16)

Using Equations (14) and (16), the total ordered quantity is:
\[ Q = \frac{a}{(\theta - b)}(e^{-\beta t}e^{(\theta - b)T} - e^{-\beta L}) + \frac{\delta a}{b}(1 - e^{-\beta L}). \] (17)

Using Equations (3) and (17), one can have
\[ T = \frac{1}{(\theta - b)} \ln \left[ \left\{ \frac{(\theta - b)e^{bL}}{a} \right\} \frac{ae^{bL}}{(\theta - b)} + \frac{P}{\theta} (1 - e^{-\theta L}) \right] - \frac{\delta a}{b} (1 - e^{-\beta L}). \] (18)

The present worth of the holding cost of the retailer: The retailer possesses the inventory during the time interval $[L, T]$. Hence, the retailer’s holding cost in fuzzy sense is:
\[ \tilde{H}C_r = \tilde{c}_{1}\left[ \left\{ q + \frac{ae^{-\beta L}}{(\theta - b)} \right\} e^{\theta L} \frac{e^{-(\theta + \gamma)L} - e^{-(\theta + \gamma)T}}{\theta + r} \right. \]
\[ \left. - \frac{a}{(\theta - b)}(e^{-(b+r)\theta L} - e^{-(b+r)\theta T}) \right]. \] (19)

The present worth of the deterioration cost of the retailer: The retailer’s deteriorating cost in fuzzy sense is:
\[ \tilde{D}C_r = \tilde{c}_{2}\left[ \left\{ q + \frac{ae^{-\beta L}}{(\theta - b)} \right\} \frac{e^{\theta L}}{\theta + r} e^{-(\theta + \gamma)L} - e^{-(\theta + \gamma)T} \right. \]
\[ \left. - \frac{a\theta}{(\theta - b)(b + r)}(e^{-(b+r)\theta L} - e^{-(b+r)\theta T}) \right]. \] (20)

The order is done at the beginning of each cycle; therefore, the present worth of the ordering cost of the retailer in fuzzy sense is
\[ \tilde{O}C_r = \tilde{c}_{3}. \] (21)

The present worth of the shortage cost of the retailer due to supplier’s lead time: Shortages are partially backlogged by the retailer during the starting time of the cycle. Hence, the present worth of the shortage cost of the retailer in fuzzy sense is:
\[ \tilde{S}C_r = \tilde{c}_{4}\delta a \left[ \frac{1}{r} (1 - e^{-\gamma r}) - \frac{1}{b + r} (1 - e^{-(b+r)L}) \right]. \] (22)

The shortages incurred in the initial phase of the cycle are partially lost. Hence, the retailer has to bear an extra expense of the lost sale cost. Thus, the present worth of the lost sale cost of the retailer due to the supplier’s lead time in fuzzy sense is
\[ \tilde{L}C_r = \tilde{c}_{5}(1 - \delta) a \frac{1}{b + r} (1 - e^{-(b+r)L}). \] (23)

Since the retailer has the privilege to settle the account at time $t = M$, therefore, the present worth of the purchasing cost of the retailer in fuzzy sense is:
\[ \tilde{P}C_r = \tilde{p}rQe^{-rM}. \] (24)
[\[L, T\]], therefore, the present worth of the retailer’s sales revenue in fuzzy sense is:

\[
\tilde{\text{SR}}_e = \tilde{s}_t \left[ B e^{-rL} + \int_L^T a e^{-bL} e^{-rT} dt \right]
\]

\[
= \tilde{s}_t \left[ B e^{-rL} + \frac{a}{(b + r)} \left( (1 + (b + r)L) e^{-(b+r)L} - (e^{-(b+r)}e^{rL}) \right) \right].
\]  

(25)

3.2.1. Case 1 \(T \leq M\) (see Figure 3)

The supplier offers a credit period of time \(M\), at which the retailer can settle the account at time \(t = M\) and the supplier charges nothing for it. The retailer sells the back-log amount at time \(t = L\) and deposits the revenue in an interest-bearing account. The retailer sells the remaining inventory units during the time interval \([L, T]\) and deposits the revenue in the same interest-bearing account. Therefore, the present worth of the interest earned in fuzzy sense is:

\[
\tilde{\text{IE}}_1 = \tilde{s}_t \tilde{I}_e \left[ \frac{Be^{-rL}}{r} (e^{-rL} - e^{-rM}) \right.
\]

\[
+ \frac{a}{(b + r)} \left( (1 + (b + r)L)e^{-(b+r)L} - (1 + (b + r)T)e^{-(b+r)T} \right)
\]

\[
+ \frac{a}{br} (e^{-bL} - e^{-bT})(e^{-rT} - e^{-rM}) \right].
\]  

(26)

In this case, the retailer sells all the units during the time interval \([0, T]\) and pays for \(Q\) units in full to supplier at time \(M \geq T\).

Therefore, the present worth of the interest charges is zero, that is,

\[
\tilde{\text{IC}}_1 = 0.
\]  

(27)

The present worth of the total profit of the retailer in fuzzy sense is:

\[
\tilde{\text{TP}}_1 = \tilde{\text{SR}}_e + \tilde{\text{IE}}_1 - \tilde{\text{HC}}_t - \tilde{\text{DC}}_t - \tilde{\text{SC}}_t - \tilde{\text{TC}}_t - \tilde{\text{PC}}_t - \tilde{\text{OC}}_t - \tilde{\text{IC}}_1.
\]  

(28)

The present worth of the total profit of the supply chain in fuzzy sense is:

\[
\tilde{\text{TP}}_1 = \tilde{\text{TP}}_s + \tilde{\text{TP}}_{1t}.
\]  

(29)

Using Equation (18), one can see that \(\tilde{\text{TP}}_1\) and \(\tilde{\text{TP}}_{1t}\) are the function of \(L\) only.

3.2.2. Case 2 \(M < T \leq N\) (see Figure 4)

In this case, the retailer sells units and deposits the revenue into an interest-earning account at an interest rate \(I_e\) per unit per year and earns interest on the sold items during the time interval \([0, M]\). Therefore, the present worth of the interest earned by the retailer during \([0, M]\) in fuzzy sense is:

\[
\tilde{\text{IE}}_2 = \tilde{s}_t \tilde{I}_e \left[ \frac{Be^{-rL}}{r} (e^{-rL} - e^{-rM}) \right.
\]

\[
+ \frac{a}{(b + r)} \left( (1 + (b + r)L)e^{-(b+r)L} - (1 + (b + r)M)e^{-(b+r)M} \right). \]  

(30)

The retailer has to pay for \(Q\) units ordered at time \(t = 0\) at the rate of \(P\), per unit at time \(t = M\) to the supplier. Through sales of the product, the retailer generates revenue in the time interval \([0, M]\). Thus, the present worth of the total revenue generated by the retailer during the time interval \([0, M]\) in fuzzy sense is:

\[
\tilde{\text{TP}}_{1t}(M) = \tilde{s}_t \tilde{I}_e \left[ B e^{-rL} + \int_L^M a e^{-bL} e^{-rT} dt \right]
\]

\[
= \tilde{s}_t \left[ B e^{-rL} + \frac{a}{(b + r)} \left( (1 + (b + r)L)e^{-(b+r)L} \right. \right.
\]

\[
- \left. (1 + (b + r)M)e^{-(b+r)M} \right]. \]  

(31)

Thus, the present worth of the total money of the retailer at time \((t = M)\) in fuzzy sense is

\[
\tilde{\text{TP}}_{1t}(M) + \tilde{\text{IE}}_2.
\]  

(32)

Based on the difference in the total amount of money in the account and the purchasing cost \(\tilde{p}_e Q e^{-rM}\) at time \((t = M)\), there arise two sub-cases:

(i) \(\tilde{\text{TP}}_{1t}(M) + \tilde{\text{IE}}_2 \geq \tilde{p}_e Q e^{-rM}\).

(ii) \(\tilde{\text{TP}}_{1t}(M) + \tilde{\text{IE}}_2 < \tilde{p}_e Q e^{-rM}\).

3.2.2.1. Sub-case 2.1 \(\tilde{\text{TP}}_{1t}(M) + \tilde{\text{IE}}_2 \geq \tilde{p}_e Q e^{-rM}\). In this sub-case, the retailer has enough money in his account to settle the account at time \(t = M\). Therefore, the present worth of interest charges in fuzzy sense is:

\[
\tilde{\text{IC}}_{21} = 0.
\]  

(33)

Since the retailer settles the account at time \(t = M\), he deposits the excess amount in an interest-bearing account and sells the remaining inventory up to time \(t = T\) and earns interest on both the remaining amount and the sold

---

**Figure 5.** Retailer’s inventory level when \(M < N \leq T\).
items in the time interval $[M, T]$. Therefore, the present worth of the interest earned in fuzzy sense is:

$$\hat{I}_{E_{2.1}} = \hat{I}_{E_2} + \left[ \hat{T}P_r(M) + \hat{I}_{E_2} - \hat{p}_rQe^{-rM} \right] \frac{\hat{I}_c}{r} \left( e^{-rM} - e^{-rT} \right)$$

$$+ \frac{\hat{s}_r \hat{I}_c a}{(b + r)^2} \left\{ (1 + (b + r)M)e^{-(b+r)M} \right.$$

$$- (1 + (b + r)T)e^{-(b+r)T} \right\}. \quad (34)$$

Hence, the present worth of the total profit of the retailer in fuzzy sense is:

$$\hat{T}P_{r2.1} = \hat{S}R + \hat{I}_{E_{2.1}} - \hat{H}C_t - \hat{D}C_t - \hat{S}C_t - \hat{L}C_t - \hat{P}C_t - \hat{O}C_t - \hat{I}C_{2.1}. \quad (35)$$

The present worth of the total profit of the supply chain in fuzzy sense is:

$$\hat{T}P_{2.1} = \hat{T}P_s + \hat{T}P_{r2.1}. \quad (36)$$

Using Equation (18), one can see that $\hat{T}P_{2.1}$ and $\hat{T}P_{r2.1}$ are the function of $L$ only.

3.2.2.2. Sub-case 2.2 $\hat{T}P_r(M) + \hat{I}_{E_2} < \hat{p}_rQe^{-rM}$. In this subcase, the retailer does not have enough money in his account to settle the account at time $t = M$. In this case, the supplier permits another credit period to the retailer to settle the unpaid balance and charges interest on the unpaid amount at the rate of $\hat{I}_c$ per unit per year. Thus, the retailer will have to pay interest on the unpaid balance in fuzzy sense:

$$\hat{U}_1 = \hat{p}_rQe^{-rM} - \left( \hat{T}P_r(M) + \hat{I}_{E_2} \right)$$

at time $t = M$, at the rate of $\hat{I}_c$ per unit per year to the supplier. Since $T < N$, therefore, the retailer sells all the units up to time $t = T$ and settles the account at time $t = T$. Therefore, the present worth of the interest charged in fuzzy sense is:

$$\hat{I}C_{2.2} = \left[ \hat{p}_rQe^{-rM} - \left( \hat{T}P_r(M) + \hat{I}_{E_2} \right) \right] \frac{\hat{I}_c}{r} \left( e^{-rM} - e^{-rT} \right). \quad (38)$$

The retailer wants to settle the account at time $t = M$, but he does not have enough money in his account to settle the account at time $t = M$. Thus, he pays interest on the unpaid balance at time $M$ at the rate of $\hat{I}_c$ to the supplier. Now, he sells the remaining inventory up to time $t = T$ and earns interest on sold items in the time interval $[M, T]$. Therefore, the present worth of the interest earned by the retailer in fuzzy sense is:

$$\hat{I}_{E_{2.2}} = \hat{I}_{E_2} + \frac{\hat{s}_r \hat{I}_c a}{(b + r)^2} \left\{ (1 + (b + r)M)e^{-(b+r)M} \right.$$

$$- (1 + (b + r)T)e^{-(b+r)T} \right\}. \quad (39)$$

Hence, the present worth of the total profit of the retailer in fuzzy sense is:

$$\hat{T}P_{r2.2} = \hat{S}R + \hat{I}_{E_{2.2}} - \hat{H}C_t - \hat{D}C_t - \hat{S}C_t - \hat{L}C_t - \hat{P}C_t - \hat{O}C_t - \hat{I}C_{2.2}. \quad (40)$$

The present worth of the total profit of the supply chain in fuzzy sense is:

$$\hat{T}P_{2.2} = \hat{T}P_s + \hat{T}P_{r2.2}. \quad (41)$$

Using Equation (18), one can see that $\hat{T}P_{2.2}$ and $\hat{T}P_{r2.2}$ are the functions of $L$ only.

3.2.3. Case 3 $M < N \leq T$ (see Figure 5)

The present worth of the interest earned by the retailer on the sold items in time interval $[M, N]$ in fuzzy sense is:

$$\hat{I}_{E_3} = \hat{s}_r \int_{M}^{N} e^{-(b+r)t} \, dt$$

$$= \frac{\hat{s}_r a \hat{I}_c}{(b + r)^2} \left\{ (1 + (b + r)M)e^{-(b+r)M} \right.$$

$$- (1 + (b + r)N)e^{-(b+r)N} \right\}. \quad (42)$$

The present worth of the interest charged on remaining amount in time interval $[M, N]$ in fuzzy sense is:

$$\hat{I}C_3 = \left[ \hat{p}_rQe^{-rM} - \left( \hat{T}P_r(M) + \hat{I}_{E_2} \right) \right] \frac{\hat{I}_c}{r} \left( e^{-rM} - e^{-rN} \right). \quad (43)$$

The present worth of the revenue generated by the retailer through sales during the time interval $[M, N]$ in fuzzy sense is:

$$\hat{T}P_r(N - M) = \hat{s}_r \int_{M}^{N} e^{-bt} \, dt$$

$$= \frac{\hat{s}_r a}{(b + r)} \left\{ e^{-(b+r)M} - e^{-(b+r)N} \right\}. \quad (44)$$

Based on the total amount of money in the account at time $t = M$, that is, $\hat{T}P_r(M) + \hat{I}_{E_2}$, the total amount of money in the account at time $t = N$, that is, $\hat{T}P_r(N - M) + \hat{I}C_3$, and the present worth of the purchasing cost $\hat{p}_rQe^{-rM}$, there are three possible sub-cases:

(i) $\hat{T}P_r(M) + \hat{I}_{E_2} \geq \hat{p}_rQe^{-rM}$,

(ii) $\hat{T}P_r(M) + \hat{I}_{E_2} < \hat{p}_rQe^{-rM} \& \hat{T}P_r(N - M) + \hat{I}C_3 \geq [\hat{p}_rQe^{-rM} - (\hat{T}P_r(M) + \hat{I}_{E_2})]e^{-rM} + \hat{I}C_3$,

(iii) $\hat{T}P_r(M) + \hat{I}_{E_2} < \hat{p}_rQe^{-rM} \& \hat{T}P_r(N - M) + \hat{I}C_3 < [\hat{p}_rQe^{-rM} - (\hat{T}P_r(M) + \hat{I}_{E_2})]e^{-rM} + \hat{I}C_3$.

3.2.3.1. Sub-case 3.1 $\hat{T}P_r(M) + \hat{I}_{E_2} \geq \hat{p}_rQe^{-rM}$. In this sub-case, the retailer has enough money in his account to settle the account at time $t = M$. Therefore, this sub-case
is similar to the sub-case 2.1. Thus, we have
\[
\hat{T}\bar{P}_{3.1}(L) = \hat{T}\bar{P}_{2.1}(L).
\] (45)

3.2.3.2. Sub-case 3.2.
\[
\hat{T}\bar{P}_{r}(M) + \hat{IE}_2 < \tilde{\alpha}Qe^{-rM} \& \hat{T}\bar{P}_{r}(N - M) + \hat{IE}_3
\]
\[
\geq [\tilde{\alpha}Qe^{-rM} - (\hat{T}\bar{P}_{r}(M) + \hat{IE}_2)]e^{-rN} + \hat{IC}_3.
\]

In this sub-case, the retailer has enough money to settle the account at time \( t = N \). Therefore, the present worth of the interest charges in fuzzy sense is:
\[
\hat{IC}_{3.2} = \hat{IC}_3
\]
\[
= \left[ \tilde{\alpha}Qe^{-rM} - (\hat{T}\bar{P}_r(M) + \hat{IE}_2) \right] \frac{\hat{IC}_1}{r} (e^{-rM} - e^{-rN}).
\] (46)

Let
\[
\hat{U}_1 = [\hat{T}\bar{P}_r(N - M) + \hat{IE}_3] - \left[ \tilde{\alpha}Qe^{-rM} - (\hat{T}\bar{P}_r(M) + \hat{IE}_2) \right] e^{-rN} + \hat{IC}_3,
\]
and the retailer deposits the excess amount in an interest-bearing account, sells the remaining inventory during the time interval \([N, T]\) and deposits the generated revenue in the same interest-bearing account and earns interest on both the excess amount and the revenue generated. Therefore, the present worth of interest earned in fuzzy sense is:
\[
\hat{IE}_{3.2} = \hat{IE}_2 + \hat{IE}_3 + \frac{\hat{U}_1\hat{I}_1}{r} (e^{-rN} - e^{-rT})
\]
\[
+ \frac{\hat{S}_a\hat{a}_\varepsilon}{(b + r)^2} \left\{ (1 + (b + r)N)e^{-(b+r)N} - (1 + (b + r)T)e^{-(b+r)T} \right\}.
\] (47)

Hence, the present worth of the total profit of the retailer in fuzzy sense is:
\[
\hat{T}\bar{P}_{3.2} = \hat{SR}_r + \hat{IE}_{3.2} - \hat{rH}_\varepsilon - \hat{DC}_r - \hat{SC}_r - \hat{LCC}_r - \hat{fPC}_r
\]
\[
- \hat{OC}_r - \hat{IC}_{3.2}.
\] (48)

The present worth of the total profit of the supply chain in fuzzy sense is:
\[
\hat{T}\bar{P}_{3.2} = \hat{T}\bar{P}_s + \hat{T}\bar{P}_{3.2}.
\] (49)

Using Equation (18), one can see that \( \hat{T}\bar{P}_{3.2} \) and \( \hat{T}\bar{P}_{3.2} \) are the functions of \( L \) only.

3.2.3.3. Sub-case 3.3.
\[
\hat{T}\bar{P}_r(M) + \hat{IE}_2 < \tilde{\alpha}Qe^{-rM} \& \hat{T}\bar{P}_r(N - M) + \hat{IE}_3
\]
\[
< [\tilde{\alpha}Qe^{-rM} - (\hat{T}\bar{P}_r(M) + \hat{IE}_2)]e^{-rN} + \hat{IC}_3.
\]

In this sub-case, the retailer does not have enough money in his account to settle the account at time \( t = N \). He pays \( \hat{T}\bar{P}_r(M) + \hat{IE}_2 \) at time \( t = M \) and \( \hat{T}\bar{P}_r(N - M) + \hat{IE}_3 \) at \( t = N \). Thus, the retailer has to pay interest on the unpaid balance \( \hat{U}_1 = \tilde{\alpha}Qe^{-rM} - (\hat{T}\bar{P}_r(M) + \hat{IE}_2) \) at the interest rate \( \hat{IC}_1 \) during the time interval \([M, N]\) and the unpaid balance
\[
\hat{U}_2 = [\tilde{\alpha}Qe^{-rM} - (\hat{T}\bar{P}_r(M) + \hat{IE}_2)]e^{-rN} + \hat{IC}_3 - (\hat{T}\bar{P}_r(N - M) + \hat{IE}_3)]
\]
\[\text{at the interest rate } \hat{IC}_2 \text{ at time } (t = N). \text{ Therefore, the present worth of the interest charges in fuzzy sense is:}\]
\[
\hat{IC}_{3.3} = \hat{IC}_3 + \frac{\hat{U}_2\hat{I}_2}{r} (e^{-rN} - e^{-rT}).
\] (50)

Now, he sells the remaining inventory up to the time \( t = T \) and earns interest on sold items in time interval \([N, T]\). Therefore, the present worth of the interest earned in fuzzy sense is:
\[
\hat{IE}_{3.3} = \hat{IE}_2 + \hat{IE}_3 + \frac{\hat{S}_a\hat{a}_\varepsilon}{(b + r)^2} \left\{ (1 + (b + r)N)e^{-(b+r)N} - (1 + (b + r)T)e^{-(b+r)T} \right\}.
\] (51)

The present worth of the total profit of the retailer in fuzzy sense is:
\[
\hat{T}\bar{P}_{3.3} = \hat{SR}_r + \hat{IE}_{3.3} - \hat{rH}_\varepsilon - \hat{DC}_r - \hat{SC}_r - \hat{LCC}_r - \hat{fPC}_r
\]
\[
- \hat{OC}_r - \hat{IC}_{3.3}.
\] (52)

The present worth of the total profit of the supply chain in fuzzy sense is:
\[
\hat{T}\bar{P}_{3.3} = \hat{T}\bar{P}_s + \hat{T}\bar{P}_{3.3}.
\] (53)

Using Equation (18), one can see that \( \hat{T}\bar{P}_{3.3} \) and \( \hat{T}\bar{P}_{3.2} \) are the functions of \( L \) only.

Representing the total profit in fuzzy sense by triangular fuzzy number, we have
\[
\hat{T}\bar{P}_j = (TP_{j1}, TP_{j2}, TP_{j3}), \text{ for } j = 1, 2, 1, 2, 2, 2, 3, 1, 3, 2, 3, 3 \quad (54)
\]
and the membership function of total profit is given by
\[
\mu_{\hat{T}\bar{P}}(x) = \begin{cases} 
\frac{x - TP_{j1}}{TP_{j2} - TP_{j1}}, & TP_{j1} \leq x \leq TP_{j2}, \\
\frac{TP_{j3} - x}{TP_{j3} - TP_{j2}}, & TP_{j2} \leq x \leq TP_{j3}, \\
0, & \text{otherwise.}
\end{cases}
\] (55)

The \( \alpha \)-cut, \( TP_j(\alpha) \) of \( \hat{T}\bar{P}_j \) consists of points \( x \) such that \( \hat{T}\bar{P}_j(\alpha) = \{x : \mu_{\hat{T}\bar{P}}(x) \geq \alpha \} \). Since the total profit is a triangular fuzzy number, so \( \alpha \)-cut, of \( \hat{T}\bar{P}_j \) is \( TP_j(\alpha) = [TP_{j1}(\alpha), TP_{j2}(\alpha), TP_{j3}(\alpha)], \alpha \in [0, 1], \) where \( TP_{j1}(\alpha) = TP_{j1} + \alpha(\hat{TP}_{j2} - TP_{j1}) \) and \( TP_{j3}(\alpha) = TP_{j3} - \alpha(\hat{TP}_{j3} - \hat{TP}_{j2}). \) In order to find the optimal value of decision variables, defuzzification of profit expression is performed by signed
distance. Using this method, the equivalent crisp profit expression is:

\[
F(\tilde{T}_j) \equiv d(\tilde{T}_j, \tilde{0}) = \frac{TP_{j1} + 2TP_{j2} + TP_{j3}}{4},
\]

for \( j = 1, 2.1, 2.2, 3.1, 3.2 \) and 3.3. (56)

Now, our problem is to find the optimal value of \( L \) which maximizes the function \( F(\tilde{T}_j) \) for \( j = 1, 2.1, 2.2, 3.1, 3.2 \) and 3.3. We have the following optimization problem:

Maximize \( F(\tilde{T}_j) \) for \( j = 1, 2.1, 2.2, 3.1, 3.2 \) and 3.3. (57)

In order to reach the optimal policy, we will use the algorithm presented in Figure 6.

4. Computational algorithm to calculate optimal policy (see Figure 6)

5. Numerical example

To illustrate the model, following data are considered which is fuzzy in nature and represented by the non-zero triangular fuzzy numbers.

\[ \tilde{s}_r = (20, 22, 26), \tilde{p}_r = (9, 10, 12), \tilde{s}_s = (9, 10, 12), \tilde{c}_s = (1.9, 2, 2.2), \tilde{c}_{1r} = (0.40, 0.50, 0.55), \theta = 0.07, \]

Figure 6. Algorithm to find the optimal policies.
Table 1. Optimal results for integrated profit policy for fuzzy model.

| $L$  | $T$    | $Q$    | $TP_t$ | $TP_s$ | $TP$   |
|------|--------|--------|--------|--------|--------|
| 4.1357 | 14.3046 | 714.59 | 8493.37 | 2861.83 | 11355.20 |

$\tilde{c}_{2r} = (0.5, 0.60, 0.65), \tilde{c}_{4r} = (0.90, 1.00, 1.20),
\tilde{c}_{5r} = (1.90, 2.00, 2.15), \tilde{c}_{1s} = (0.40, 0.50, 0.55),
\tilde{c}_{2s} = (0.5, 0.60, 0.65), \tilde{c}_{3r} = (105, 120, 130),
\tilde{c}_{3s} = (70, 80, 85), \tilde{I}_e = (0.11, 0.12, 0.14), M = 7, N = 9,
\tilde{I}_{c1} = (0.12, 0.14, 0.15), \tilde{I}_{c2} = (0.16, 0.17, 0.18), P = 200,
a = 50, b = 0.02, r = 0.05, \delta = 0.6.$

Following the algorithm chart presented in Figure 6, optimal results are obtained using the software Mathematica 7.0 and are presented in Table 1. The concavity of total profit function is shown in Figures 7–9.

When all the cost parameters are fuzzy in nature, it is found that the supplier gets an order of $Q = 714.59$ units at time $t = 0$, the supplier takes a lead time $L = 4.1357$ to produce the items, thus fulfilling the order at time $t = 4.1357$. As a result, the retailer is forced to have the shortage for the period $t = 4.1357$; the retailer partially backlogs the demand during the shortage period and fulfills when the inventory reaches him. Thereafter, the retailer’s inventory is depleted due to the combined effect of demand and deterioration and reaches the zero level at time $t = 14.3046$. The retailer’s present worth of the total profit during this cycle is $8493.37$, the supplier’s present worth of the total profit is $2861.83$ and the present worth of the total supply chain profit is $11355.20$.

5.1. Particular case: If all the parameters in the model are crisp in nature:

To illustrate the model when all the parameters are crisp in nature, we have considered the following data in appropriate units as follows:

$s_r = 22, p_r = 10, s_s = 10, c_s = 2, c_{1t} = 0.5, c_{2r} = 0.6,$
$c_{3r} = 120, c_{4r} = 1, c_{5r} = 2, c_{1s} = 0.5, c_{2s} = 0.6, c_{3s} = 80,$
$\delta = 0.6, I_e = 0.12, I_{c1} = 0.14, I_{c2} = 0.17, a = 50,$
$b = 0.02, r = 0.05, \theta = 0.07, M = 7, N = 9, P = 200.$

Table 2. Optimal results for integrated profit policy for crisp model.

| $L$  | $T$    | $Q$    | $TP_t$ | $TP_s$ | $TP$   |
|------|--------|--------|--------|--------|--------|
| 4.1397 | 14.1716 | 718.78 | 8253.28 | 2750.99 | 11004.27 |
Following the algorithm presented in Figure 6, optimal results are obtained using the software Mathematica 7.0 and are presented in Table 2.

It is found that the supplier gets an order of $Q = 718.78$ units, at time $t = 0$, takes a lead time $L = 4.1397$ to produce the items, thus fulfilling the order at time $t = 4.1397$. Therefore, the retailer is forced to have a shortage for the period $t = 4.1397$; the retailer partially backlogs the demand during the shortage period and fulfils the backlogged demand when the inventory reaches him. Therefore, the retailer’s inventory is depleted due to the combined effect of demand and deterioration and reaches the zero level at time $t = 14.1716$. The present worth of the retailer’s total profit during this cycle is $8253.28$, the supplier’s present worth of the total profit is $2750.99$ and the present worth of the total supply chain profit is $11,004.27$.

6. Sensitivity analysis

In order to study the effect of various parameters on the optimal policy, a sensitivity analysis was done and the obtained results are presented in Tables 3–8.

6.1. Observations from sensitivity analysis

1) From Table 3, it is seen that if the first credit offer $M$ is 4 units and the second credit offer increases from 7 to 9 units, the retailer’s profit decreases, whereas the supplier’s profit and the total profit of the supply chain increase and the optimal ordering quantities also increase. If the first credit offer $M$ is 5 units, the retailer’s total profit is maximum when the second credit offer is $N = 8$ units. The supplier’s profit and the total profit of the supply chain increase and the optimal ordering quantities also increase. If the first credit offer $M$ is 6 units and the second credit offer increases from 7 to 9 units, the retailer’s total profit and the supplier’s total profit increase and the total profit of the supply chain increases and the optimal ordering quantities also increase. It may be understood as the credit period increases as the retailer purchases more items and as a result, the retailer’s total profit, the supplier’s total profit and the total profit of the supply chain all increase.

2) From Table 4, it is observed that as the production rate $P$ increases, the retailer’s profit, the supplier’s profit and the total profit of the supply chain all increase and the optimal ordering quantities also increase. It may be understood as the production rate increases, the supplier takes less time to produce the required items. Therefore, the retailer orders more items and as a result, the retailer’s profit, the supplier’s profit and the total profit of the supply chain all increase.

3) From Table 5, it is clear that as the demand parameter increases from 30 to 70, the retailer’s profit, the supplier’s profit and the total profit of the supply chain all increase and the optimal ordering quantities also increase. It is a well-known fact that as the demand rate increases, total sales increase and as a result, the retailer orders more items and therefore, the retailer’s profit, the supplier’s profit and the total profit of the supply chain all increase.

4) From Table 6, we see that as the backlogging rate $\delta$ increases from 0.4 to 0.8, the retailer’s profit, the supplier’s profit and the total profit of the supply chain all increase and the optimal ordering quantities also increase. It is well understood that as the backlogging rate increases, the retailer purchases more items and as a result, the retailer’s profit, the supplier’s profit and the total profit of the supply chain all increase.

5) From Table 7, it is observed that as the deterioration rate $\theta$ increases from 0.05 to 0.09, the retailer’s profit, the supplier’s total profit and the total profit of the supply chain all decrease and the optimal ordering quantities also decrease. We all know very well that deterioration has a negative effect on the profits as the deterioration rate increases, total profit decreases. Thus, the results from Table 7 match the reality.

6) From Table 8, one can see that as the inflation rate $r$ increases from 0.06 to 0.09, the retailer’s profit, the supplier’s total profit and the total profit of the supply chain all decrease and the optimal ordering quantities also decrease. It is a well-known fact that inflation has a negative effect on the total profit. Thus, the results from Table 8 match the reality.

7. Special cases

7.1. Retailer’s inventory model without stock-out in crisp environment

In order to avoid the stock-out, the retailer places the order well in time knowing that the supplier will promise lead time, so that replenishment quantities reach the retailer at the time when his inventory reaches the zero level. The retailer’s inventory can be represented in Figure 10 and governed by the differential equation (58) as follows:

$$I_t'(t) = -\theta I_t(t) - ae^{-bt}, \quad 0 \leq t < T. \quad (58)$$

By solving Equation (58) with initial condition $I_t(0) = Q$, one can have

$$I_t(t) = \left\{Q + \frac{a}{(\theta - b)}\right\} e^{-\theta t} - \frac{ae^{-bt}}{(\theta - b)}, \quad 0 \leq t \leq T. \quad (59)$$
Using the boundary condition $I_r(T) = 0$, one can have
\[
T = \frac{1}{(\theta - b)} \ln \left[ \frac{(\theta - b)Q + a}{a} \right].
\] (60)

The present worth of the holding cost of the retailer in fuzzy sense is:
\[
\tilde{\text{HC}}_r = \tilde{c}_{1r} \left[ (Q + \frac{a}{(\theta - b)}) \frac{1 - e^{-(\theta + r)T}}{(\theta + r)} - \frac{a}{(\theta - b)(b + r)} (1 - e^{-(b + r)T}) \right].
\] (61)

The present worth of the deterioration cost of the retailer in fuzzy sense is:
\[
\tilde{\text{DC}}_r = \tilde{c}_{2r} \left[ \frac{Q + \frac{a}{(\theta - b)}}{\theta} \frac{\theta}{(\theta + r)}(1 - e^{-(\theta + r)T}) \right.
\]
\[
- \frac{a\theta}{(\theta - b)(b + r)} (1 - e^{-(b + r)T}) \].
\] (62)

The present worth of the ordering cost of the retailer: Since the order is done $L$ units before the time when the inventory is actually needed. Therefore, the present worth
of the ordering cost of the retailer in fuzzy sense is:

$$\tilde{OC}_t = \tilde{\varepsilon}_3 e^{-r(t-L)} = \tilde{\varepsilon}_3 e^{Lt}. \quad (63)$$

The present worth of the purchasing cost of the retailer: Since the retailer has the privilege to settle the account at time \(t = M\), therefore, the present worth of the purchasing cost of the retailer in fuzzy sense is:

$$\tilde{PC}_t = \tilde{p}_t Q e^{-rM}. \quad (64)$$

The present worth of the sales revenue of the retailer: The retailer sells the inventory during the time interval \([0, T]\); therefore, the present worth of the sales revenue in fuzzy sense is:

$$\tilde{SR}_t = \tilde{s}_t \int_0^T a e^{-br} e^{-r} dt = \frac{\tilde{s}_t a}{b+r} (1 - e^{-(b+r)T}). \quad (65)$$

7.1.1. Case 1 \(T \leq M\)

The supplier offers a credit period of time \(M\), at which the retailer can settle the account at time \(t = M\) and the supplier charges nothing for it. The retailer sells the inventory during \([0, T]\) and deposits the revenue in an interest-bearing account to earn interest. Therefore, the present worth of the interest earned in fuzzy sense is:

$$\tilde{IE}_1 = \tilde{s}_t \tilde{I}_e \left[ -\frac{a}{(b+r)^2} \left( 1 - (1 + (b+r)T)e^{-(b+r)T} \right) + \frac{a}{br} \left( 1 - e^{-bT} \right) \left( e^{-rT} - e^{-rM} \right) \right]. \quad (66)$$

In this case, the retailer sells all the units during \([0, T]\) and pays for \(Q\) units in full to the supplier at time \(M \geq T\). Therefore, the present worth of interest charges in fuzzy sense is zero, that is,

$$\tilde{IC}_1 = 0. \quad (67)$$

The present worth of total profit of the retailer in fuzzy sense is:

$$\tilde{TP}_1 = \tilde{SR}_t + \tilde{IE}_1 - \tilde{HC}_t - \tilde{DC}_t - \tilde{PC}_t - \tilde{OC}_t - \tilde{IC}_1. \quad (68)$$

The present worth of the total profit of the supply chain in fuzzy sense is:

$$\tilde{TP} = \tilde{TP}_1 + \tilde{TP}_r. \quad (69)$$

7.1.2. Case 2 \(M < T \leq N\)

In this case, the retailer sells units and deposits the revenue into an interest-earning account at an interest rate \(\tilde{I}_e\) per unit per year during \([0, M]\) and earns interest on the sold items during \([0, M]\). Therefore, the present worth of the interest earned by the retailer during \([0, M]\) in fuzzy sense is:

$$\tilde{IE}_2 = \frac{\tilde{s}_t \tilde{I}_e a}{(b+r)^2} \left( 1 - (1 + (b+r)M)e^{-(b+r)M} \right). \quad (70)$$

Through sales of the product, the retailer generates revenue in the time interval \([0, M]\). Thus, the present worth of the total revenue generated by the retailer during \([0, M]\) in fuzzy sense is:

$$\tilde{TP}_r(M) = \tilde{s}_t \int_0^M a e^{-br} e^{-r} dt = \frac{\tilde{s}_t a}{(b+r)} \left( 1 - e^{-(b+r)M} \right). \quad (71)$$

Thus, the present worth of the total amount of the retailer at time \((t = M)\) is \(\tilde{TP}_r(M) + \tilde{IE}_2\). Based on the difference in total amount of money in the account and the purchasing cost \(\tilde{p}_t Q e^{-rM}\) at time \((t = M)\), there arise the following two sub-cases:

(i) \(\tilde{TP}_r(M) + \tilde{IE}_2 \geq \tilde{p}_t Q e^{-rM}\).

(ii) \(\tilde{TP}_r(M) + \tilde{IE}_2 < \tilde{p}_t Q e^{-rM}\).

7.1.2.1. Sub-case 2.1 \(\tilde{TP}_r(M) + \tilde{IE}_2 \geq \tilde{p}_t Q e^{-rM}\). In this sub-case, the retailer has enough money in his account to settle the account at time \((t = M)\). Therefore, the present worth of interest charges in fuzzy sense is:

$$\tilde{IC}_{2.1} = 0. \quad (72)$$

Since the retailer settles the account at time \((t = M)\), now he deposits the excess amount in an interest-bearing

| Parameter | values | \(L\) | \(T\) | \(Q\) | \(TPr\) | \(TPs\) | TP |
|-----------|--------|------|------|------|-------|-------|-----|
| \(R\)     | 0.06   | 3.9622 | 13.7726 | 697.18 | 7578.88 | 2403.77 | 9982.65 |
|           | 0.07   | 3.8518 | 13.3665 | 675.24 | 6987.15 | 2088.41 | 9075.56 |
|           | 0.08   | 3.7080 | 12.9573 | 653.16 | 6467.06 | 1803.06 | 8270.12 |
|           | 0.09   | 3.5660 | 12.5485 | 631.16 | 6008.83 | 1545.78 | 7554.61 |
account and sells the remaining inventory up to the time \( (t = T) \) and earns interest on both the remaining amount and the sold items in time interval \([M, T]\). Therefore, the present worth of total interest earned in fuzzy sense is:

\[
\tilde{I}_{E2.1} = \tilde{I}_E + [\tilde{T}\tilde{P}_r(M) + \tilde{I}_E - \tilde{p}_1Qe^{-rM}] \frac{\tilde{I}_e}{r}(e^{-rM} - e^{-rT}) + \frac{\tilde{s}_tI_a}{(b + r)^2} \{ (1 + (b + r)M)e^{-(b+r)M} - (1 + (b + r)T)e^{-(b+r)T} \}.
\]

(73)

The present worth of the total profit of the retailer in fuzzy sense is:

\[
\tilde{T}\tilde{P}_{2.1} = \tilde{S}\tilde{R}_r + \tilde{I}_{E2.1} = \tilde{H}\tilde{C}_r - \tilde{D}\tilde{C}_r - \tilde{P}\tilde{C}_r - \tilde{Q}\tilde{C}_r - \tilde{I}\tilde{C}_{2.1}.
\]

(74)

7.1.2.2. Sub-case 2.2 \( \tilde{T}\tilde{P}_r(M) + \tilde{I}_E < \tilde{p}_1Qe^{-rM} \). In this sub-case, the retailer does not have enough money in his account to settle the account at time \( (t = M) \). Therefore, the present worth of the total profit of the retailer in fuzzy sense is:

\[
\tilde{I}_{C2.2} = [\tilde{p}_1Qe^{-rM} - (\tilde{T}\tilde{P}_r(M) + \tilde{I}_E)] \frac{\tilde{I}_e}{r}(e^{-rM} - e^{-rT}).
\]

(76)

The retailer wants to settle the account at time \( t = M \), but does not have enough money in his account. Thus, he pays interest on the unpaid balance at time \( M \) at the rate of \( \tilde{I}_{c1} \) to the supplier. Now he sells the remaining inventory up to time \( (t = T) \) and earns interest on the sold items in time interval \([M, T]\). Therefore, the interest earned in fuzzy sense is:

\[
\tilde{I}_{E2.2} = \tilde{I}_E + \frac{\tilde{s}_tI_a}{(b + r)^2} \{ (1 + (b + r)M)e^{-(b+r)M} - (1 + (b + r)T)e^{-(b+r)T} \}.
\]

(77)

Hence, the present worth of total profit of the retailer in fuzzy sense is:

\[
\tilde{T}\tilde{P}_{2.2} = \tilde{S}\tilde{R}_r + \tilde{I}_{E2.2} - \tilde{H}\tilde{C}_r - \tilde{D}\tilde{C}_r - \tilde{P}\tilde{C}_r - \tilde{Q}\tilde{C}_r - \tilde{I}\tilde{C}_{2.2}.
\]

(78)

7.1.3. Case 3 \( M < N \leq T \)

The present worth of the interest earned by the retailer in fuzzy sense is:

\[
\tilde{I}_E = \tilde{s}_t\int_M^N e^{-(b+r)\theta} d\theta = \frac{\tilde{s}_tI_a}{(b + r)^2} \{ (1 + (b + r)M)e^{-(b+r)M} - (1 + (b + r)N)e^{-(b+r)N} \}.
\]

(80)

The present worth of interest charged on unpaid amount in time interval \([M, N]\) in fuzzy sense is:

\[
\tilde{I}_C = [\tilde{p}_1Qe^{-rM} - (\tilde{T}\tilde{P}_r(M) + \tilde{I}_E)] \frac{\tilde{I}_e}{r}(e^{-rM} - e^{-rN}).
\]

(81)

Based on the total amount of money in the account at time \( t = M \), that is, \( \tilde{T}\tilde{P}_r(M) + \tilde{I}_E \), the total amount of money in the account at time \( t = N \), that is, \( \tilde{T}\tilde{P}_r(N - M) + \tilde{I}_E \), and the present worth of the purchasing cost \( \tilde{p}_1Qe^{-rM} \), there arise three sub-cases:

(i) \( \tilde{T}\tilde{P}_r(M) + \tilde{I}_E \geq \tilde{p}_1Qe^{-rM} \).

(ii) \( \tilde{T}\tilde{P}_r(M) + \tilde{I}_E < \tilde{p}_1Qe^{-rM} \) and \( \tilde{T}\tilde{P}_r(N - M) + \tilde{I}_E \geq \tilde{p}_1Qe^{-rM} \).

(iii) \( \tilde{T}\tilde{P}_r(M) + \tilde{I}_E < \tilde{p}_1Qe^{-rM} \) and \( \tilde{T}\tilde{P}_r(N - M) + \tilde{I}_E < \tilde{p}_1Qe^{-rM} \).

7.1.3.1. Sub-case 3.1 \( \tilde{T}\tilde{P}_r(M) + \tilde{I}_E \geq \tilde{p}_1Qe^{-rM} \). In this sub-case, the retailer has enough money in his account to settle the account at time \( t = M \). Therefore, this sub-case is similar to the sub-case 2.1; thus, we have

\[
\tilde{T}\tilde{P}_{3.1} = \tilde{T}\tilde{P}_{2.1}
\]

(83)

7.1.3.2. Sub-case 3.2

\[
\tilde{T}\tilde{P}_r(M) + \tilde{I}_E < \tilde{p}_1Qe^{-rM} \) and \( \tilde{T}\tilde{P}_r(N - M) + \tilde{I}_E \geq \tilde{p}_1Qe^{-rM} \).

In this sub-case, the retailer has enough money to settle the account at time \( t = N \). Therefore, the present worth of
the interest charge in fuzzy sense is:

\[ \tilde{IC}_{3.2} = \tilde{IC}_3 \]

\[ = \tilde{p}_t Q e^{-rM} - \left[ \tilde{TP}_t(M) + \tilde{IE}_2 \right] \frac{\tilde{I}_e}{r} (e^{-rM} - e^{-rT}) \].

(84)

Let

\[ \tilde{U}_1 = \left[ \tilde{TP}_t(N - M) + \tilde{IE}_3 \right] - \left[ \tilde{p}_t Q e^{-rM} - \left( \tilde{TP}_t(M) + \tilde{IE}_2 \right) \right] e^{-rN} + \tilde{IC}_3 \].

(85)

Now the retailer deposits the excess amount in the interest-bearing account, sells the remaining inventory during the time interval \([N, T]\) and deposits the generated revenue in the same interest-bearing account and earns interest on both the excess amount and the revenue generated. Therefore, the present worth of interest earned in fuzzy sense is:

\[ \tilde{IE}_{3.2} = \tilde{IE}_2 + \tilde{IE}_3 + \frac{\tilde{U}_1 \tilde{I}_e}{r} (e^{-rN} - e^{-rT}) \]

\[ + \frac{\tilde{I}_e a \tilde{I}_e}{(b + r)^2} \left[ (1 + (b + r)N)e^{-(b+r)N} \right. \]

\[ \left. - (1 + (b + r)T)e^{-(b+r)T} \right]. \]

(86)

Thus, the present worth of the total profit of the retailer in fuzzy sense is:

\[ \tilde{TP}_{3.2} = \tilde{SR}_e + \tilde{IE}_{3.2} - \tilde{HC}_t - \tilde{DC}_t - \tilde{PC}_t - \tilde{OC}_t - \tilde{IC}_{3.2}. \]

(87)

The present worth of total profit of the supply chain in fuzzy sense is:

\[ \tilde{TP}_{3.2} = \tilde{TP}_e + \tilde{TP}_{3.2}. \]

(88)

7.1.3.3. Sub-case 3.3. \( \tilde{TP}_t(M) + \tilde{IE}_2 < \tilde{p}_t Q e^{-rM} \) & \( \tilde{TP}_t(N - M) + \tilde{IE}_3 < \left[ \tilde{p}_t Q e^{-rM} - (\tilde{TP}_t(M) + \tilde{IE}_2) \right] e^{-rN} + \tilde{IC}_3 \)

Since the retailer does not have enough money in his account to settle the account at time \((t = N)\), he pays \( \tilde{TP}_t(M) + \tilde{IE}_2 \) at time \((t = M)\) and \( \tilde{TP}_t(N - M) + \tilde{IE}_3 \) at \((t = N)\). Thus, the retailer has to pay interest on the unpaid balance \( \tilde{U}_1 = \tilde{p}_t Q e^{-rM} - (\tilde{TP}_t(M) + \tilde{IE}_2) \) with the interest rate \( IC_1 \) during the time interval \([M, N]\) and unpaid balance \( \tilde{U}_2 = \left[ \tilde{p}_t Q e^{-rM} - (\tilde{TP}_t(M) + \tilde{IE}_2) \right] e^{-rN} + \tilde{IC}_3 - (\tilde{TP}_t(N - M) + \tilde{IE}_3) \) with interest rate \( IC_2 \) at time \((t = N)\). Therefore, the present worth of interest charges in fuzzy sense is:

\[ \tilde{IC}_{3.3} = \tilde{IC}_3 + \frac{\tilde{U}_2 \tilde{I}_e}{r} (e^{-rN} - e^{-rT}). \]

(89)

The retailer wants to settle the account at time \( t = N \), but he does not have enough money in his account. Thus, he pays interest on the unpaid balance at time \( t = N \) at

| Table 9. Optimal results for integrated profit policy without shortages. |
|----------------|---|---|---|---|---|
| \( L \) | \( T \) | \( Q \) | \( TP_t \) | \( TP_e \) | \( TP \) |
| 5.2262 | 12.5763 | 875.38 | 6866.30 | 3139.05 | 10005.35 |

the rate of \( IC_2 \) to the supplier. Now, he sells the remaining inventory up to time \( t = T \) and earns interest on sold items in time interval \([N, T]\). Therefore, the present worth of the interest earned in fuzzy sense is:

\[ \tilde{IE}_{3.3} = \tilde{IE}_2 + \tilde{IE}_3 + \frac{\tilde{S}_a \tilde{I}_e}{(b + r)^2} \left[ (1 + (b + r)N)e^{-(b+r)N} \right. \]

\[ - (1 + (b + r)T)e^{-(b+r)T} \right]. \]

(90)

Hence, the present worth of the total profit of the retailer in fuzzy sense is:

\[ \tilde{TP}_{3.3} = \tilde{TP}_e + \tilde{TP}_{3.3}. \]

(91)

The present worth of total profit of the supply chain in fuzzy sense is:

\[ \tilde{TP}_{3.3} = \tilde{TP}_e + \tilde{TP}_{3.3}. \]

(92)

Using Equations (3) and (60), one can see that \( \tilde{TP}_t(L) \) for \( j = 1, 2, 1, 2, 2, 3, 1, 2, 2, 3, 3 \) is a function of \( L \) only. Thus, our problem is to find the optimal policy which maximizes the function \( \tilde{TP}_t(L) \) for \( j = 1, 2, 1, 2, 2, 3, 1, 2, 2, 3, 3 \). Therefore, the optimization problem can be stated as follows:

Maximize : \( \tilde{TP}_t(L) \) for \( j = 1, 2, 1, 2, 2, 3, 1, 2, 2, 3, 3 \).

(93)

In order to reach the optimal policy, we will use the algorithm presented in Figure 6.

7.2. Optimal policies for special cases

7.2.1. If supplier promised the lead time, but the retailer does not want to have shortages

Using the same data as in Example 1, optimal results are calculated numerically with the help of software, Mathematica 7.0 and presented in Table 9.

7.2.2. If the retailer is the decision-maker in the supply chain with retailer's stock-out model

In this case, the retailer is the decision-maker in the supply chain and decides the optimal policy and the results are presented numerically in Table 10.

7.2.3. If the supplier promises the lead time, but does not offer any credit period

In this case, the supplier promises lead time, but does not offer any credit period to retailer to settle the account; thus,
the retailer has to pay in full for purchased items at the time of purchasing. Optimal results are calculated numerically and are presented in Table 11.

7.3. Observations from special cases

(1) From Table 9, it is observed that the retailer decides not to have the shortages and orders for the replenishment well in time. It is found that the total supply chain profit decreases by 9.07% as well as the retailer suffers a loss of 16.8%. This happens because the profits on demand backlogged during the shortage period dominate the loss due to shortage cost and the lost sale cost. Hence, the retailer will be willing to have the shortage during the promised lead time.

(2) From Table 10, it is observed that if the integrated profit maximization policy is compared with the individual profit maximization policy decided by the retailer, there is a loss of 0.66% in the present worth of total supply chain profit. Thus, the integrated profit maximization policy is more profitable as compared to the individual profit maximization policy.

(3) From Table 11, it is clear that, if the supplier promises the lead time but does not offer any credit period to the retailer, the retailer’s total profit decreases by 95.26%. As a result, the total supply chain profit also decreases. Thus, the credit offer policy is more profitable as compared to policy when the supplier does not offer any credit period and also good for the longevity of the supply chain.

8. Summary and concluding remarks

In this paper, we studied progressive trade credit model with and without stock-out under supplier’s promised lead time, incorporating the effect of inflation and time value of money under imprecise environment. Every supplier wants to minimize his costs like holding cost and the deterioration cost, as well as the supplier is not always in a position to fulfil the retailer’s request at any time, thus forced to adopt new policies to deal with the situation. To deal with the situation, the supplier does not hold the inventory indeed, but produces the items whenever they are demanded and promises a certain lead time. The existence of lead time brings the study in close proximity to reality. Therefore, the retailer has choices to have the shortage or not for the lead time period. Fuzzy model deals with all the fluctuations of the cost parameters, selling prices, demand rate as well as the production rate and gives better insights to the decision-makers to deal with market fluctuations and be prepared for the market ups and downs. Special cases also study how retailer’s shortage affects the optimal policies and the total profit of the supply chain. On the other hand, if the retailer is the decision-maker in the supply chain, then the total profit of the supply chain also decreases. Thus, an integrated policy results in profit making, as compared with total profits obtained from the independent decisions made by the retailer. It is also observed that if the supplier promises the lead time but does not offer any credit period to the retailer, the retailer’s profit decreases up to great extent, as well as total profit of the supply chain decreases unexpectedly. Results from the sensitivity also support the fact that the model proposed in this paper is practically applicable to the realistic situations and matches the reality. Thus, this study is particularly useful for the supply chain systems where the supplier and the retailer form a strategic alliance with mutually beneficial objectives to deal with realistic market-oriented situations.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Chang, H. J., & Dye, C. Y. (2001). An inventory model for deteriorating items with partial backlogging and permissible delay in payments. International Journal of Systems Science, 32(3), 345–352.

Chung, K. J., & Huang, C. K. (2009). An ordering policy with allowable shortage and permissible delay in payments. Applied Mathematical Modelling, 33(5), 2518–2525.

Glock, C. H. (2012). Lead time reduction strategies in a single-vendor-single-buyer integrated inventory model with lot size-dependent lead times and stochastic demand. International Journal of Production Economics, 136(1), 37–44.

Hsu, P. H., Wee, H. M., & Teng, H. M. (2007). Optimal ordering decision for deteriorating items with expiration date and uncertain lead time. Computers & Industrial Engineering, 52(4), 448–458.

Hu, F., & Liu, D. (2010). Optimal replenishment policy for the EPQ model with permissible delay in payments and allowable shortages. Applied Mathematical Modelling, 34(10), 3108–3117.

Hwang, H., & Shin, S. W. (1997). Retailer’s pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. Computers and Operations Research, 24(6), 539–547.
Jana, D. K., Maity, K., & Roy, T. K. (2013). A three-layer supply chain integrated production-inventory model under permissible delay in payments in uncertain environments. *Journal of Uncertainty Analysis and Applications, 1*(1), 1–17.

Khanra, S., Ghosh, S. K., & Chaudhuri, K. S. (2011). An EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment. *Applied Mathematics and Computation, 218*(1), 1–9.

Leng, M., & Parlar, M. (2009). Lead-time reduction in a two-level supply chain: Non-cooperative equilibria vs. coordination with a profit-sharing contract. *International Journal of Production Economics, 118*(2), 521–544.

Liao, C. J., & Shyu, C. H. (1991). An analytical determination of lead time with normal demand. *International Journal of Operations and Productions Management, 11*, 72–78.

Maihami, R., & Abadi, I. N. K. (2012). Joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging. *Mathematical and Computer Modelling, 55*(5), 1722–1733.

Maity, M. K., & Maiti, M. (2007). Two-storage inventory model with lot-size dependent fuzzy lead-time under possibility constraints via genetic algorithm. *European Journal of Operational Research, 179*, 352–371.

Singh, C., & Singh, S. R. (2010a). Supply chain model with stochastic lead time under imprecise partially backlogging and fuzzy ramp-type demand for expiring items. *International Journal of Operational Research, 8*(4), 511–522.

Singh, C., & Singh, S. R. (2010b). Two-echelon supply chain model with imperfect production for weibull distribution deteriorating items under imprecise and inflationary environment. *International Journal of Operations Research and Optimization, 1*(1), 9–25.

Singh, C., & Singh, S. R. (2013). Vendor-buyers relationship model for deteriorating items with shortages, fuzzy trapezoidal costs and inflation. *Yugoslav Journal of Operations Research, 23*(1), 73–85.

Soni, H., & Shah, N. H. (2008). Optimal ordering policy for stock-dependent demand under progressive payment scheme. *European Journal of Operational Research, 184*, 91–100.

Treville, S. D., Shapiro, R. D., & Hameri, A. P. (2004). From supply chain to demand chain: the role of lead time reduction in improving demand chain performance. *Journal of Operations Management, 21*(6), 613–627.

Uthayakumar, R., & Rameswari, M. (2013). Supply chain model with variable lead time under credit policy. *The International Journal of Advanced Manufacturing Technology, 64*, 389–397.

Yadav, D., Singh, S. R., & Kumari, R. (2012). Inventory model of deteriorating items with two-warehouse and stock-dependent demand using genetic algorithm in fuzzy environment. *Yugoslav Journal of Operations Research, 22*(1), 51–78.

Yang, M. F., & Tseng, W. C. (2014). Three-echelon inventory model with permissible delay in payments under controllable lead time and backorder consideration. *Mathematical Problems in Engineering, 2014*, Article ID 809149, 16 pages.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*, 338–353.