ON CONTINUITY OF CORRESPONDENCES OF PROBABILITY MEASURES IN THE CATEGORY OF TYCHONOV SPACES

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Abstract. We consider the correspondence assigning to every Radon measure on two Tychonoff coordinate spaces the set of probability measures with these marginals. It is proved that this correspondence is continuous.

1. Introduction

In this paper we continue to investigate the relationship between joint probability distribution on product spaces and their marginal distributions. This problem is actual because of its application in economic and game theories. In particular, we consider the correspondence assigning to every probability measure on two coordinate spaces the set of probability measures with these marginals. Game theory regards a probability measure, for instants, as a mixed strategy of players and a continuity of the correspondence is a sufficient condition of existence of Nash equilibrium of the static game in mixed strategies.

Similar problems were considered by Bergin (1999) for metrizable coordinate spaces and Zarichnyi (2003) for the compact case. The natural extension of these results is to take under consideration spaces of probability measures in Tych. There are some extensions of functor $P$ onto this category proposed by Chigogidze (1984) and Fedorchuk (1991). In this paper we consider the correspondence on the space of Radon measures and as a corollary we derive the result for construction $P_\beta$.

The paper is organized as follows. Section 2 provides necessary definitions and preliminary results which we need for the proof of the main theorem. The continuity of correspondences is established in Section 3.

2. Definitions and preliminaries

For the Tychonoff space $X$ let us consider the space

$$P_\beta(X) = \{\mu \in P(\beta X) : \text{supp}(\mu) \subset X \subset \beta X\},$$

where $\beta X$ is the Stone-Cech compactification of $X$ and $\text{supp}(\mu)$ is the support of measure $\mu$. This construction extends functor $P: \text{Comp} \to \text{Comp}$ to the functor $P_\beta: \text{Tych} \to \text{Tych}$ and was proposed by Chigogidze (1984) (see also Teleiko and Zarichnyi (1999)).

In order to consider two other extensions let us recall some definitions. The set of all Borel measures on Tychonoff space $X$ is denoted by $M(X)$. The weak-* topology on $M(X)$

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Lemma 2.2. 

i) For every \( \lambda \in \alpha \) measure \( M \) and this implies that \( \psi \phi \)ments.

Consider the sequence of functions \( \| \Lambda(\psi f) \| \leq 1 \) for every net \( \{ \tau \} \) exists dense \( (C \)dense functionals. Varadarajan (1961) proved that the measure \( \mu \) is Radon \( (\tau\)-smooth) if and only if there exists dense \( (\tau\)-smooth) bounded linear functional \( \Lambda \) such that for every \( f \in C_b(X) \) we have \( \Lambda(f) = \int f d \mu \) and \( \| \Lambda \| = \| \mu \| \).

Similarly, bounded linear functional \( \Lambda \) is called dense if \( \Lambda(f) \rightarrow 0 \) for any net \( \{ f_\alpha \} \subset C_b(X) \) such that \( \| f_\alpha \| \leq 1 \) for every \( \alpha \) and \( f_\alpha \rightarrow 0 \) uniformly on compact subsets of \( X \).

Recall, that bounded linear functional \( \Lambda \) is called \( \tau \)-smooth if \( f_\alpha \downarrow 0 \) implies that \( \Lambda(f_\alpha) \rightarrow 0 \) for every net \( \{ f_\alpha \} \subset C_b(X) \).

Further, we will assume these notions equivalent and use both techniques without comments.

Denote by \( u_X : C(\beta X) \rightarrow C(\beta X) \) the map defined by the formula \( u_X(\varphi)(\mu) = \mu(\varphi) \).

The map is a linear operator with \( \| u_X \| = 1 \).

For every \( \varphi \in C_b(X) \) we can restrict the function \( u_X(\varphi) \) on \( \hat{P}(X) \).

Define the map \( \psi_X : P^2(\beta X) \rightarrow P(\beta X) \) by the formula \( \psi_X(M)(\varphi) = M(u_X(\varphi)) \).

Lemma 2.1. \( \psi_X(\hat{P}^2(X)) \subseteq \hat{P}(X) \).

Proof. Consider the sequence of functions \( \{ \varphi_\alpha \}_{\alpha \in \Gamma} \) such that \( \| \varphi_\alpha \| \leq 1 \) for every \( \alpha \in \Gamma \) and \( \varphi_\alpha \rightarrow 0 \) uniformly on compact subsets of \( X \). It is easy to show that \( \| u_X(\varphi_\alpha) \| \leq 1 \) for every \( \alpha \in \Gamma \) and \( u_X(\varphi_\alpha) \rightarrow 0 \) uniformly on compact subsets of \( \hat{P}(X) \). Therefore for every radon measure \( M \in \hat{P}^2(X) \) we have \( \psi_X(M)(\varphi_\alpha) = M(u_X(\varphi_\alpha)) \rightarrow 0 \) and this implies that \( \psi_X(M) \in \hat{P}(X) \). \( \square \)

Lemma 2.2. i) For every \( \mu \in \hat{M}(X) \) and \( \nu \in \hat{M}(Y) \) such that \( \| \mu \| = \| \nu \| \) there exists \( \lambda \in M(X \times Y) \) such that \( \hat{M} \text{pr}_1(\lambda) = \mu \) and \( \hat{M} \text{pr}_2(\lambda) = \nu \).

ii) The statement is true if we substitute \( \hat{M} \) with \( M_\beta \).
Proof. i). Let us first assume that $\mu \in \hat{P}(X)$ and $\nu \in \hat{P}(Y)$. Denote by $i_x : Y \to X \times Y$ an embedding defined for every $x \in X$ by the formula $i_x(y) = (x, y)$. Define $f_\nu : X \to \hat{P}(X \times Y)$ in the following manner: $f_\nu(x) = \hat{P}i_x(\nu)$ for every $x \in X$. Let us check whether the measure
\[ \lambda = \psi_{X \times Y}(\hat{P}f_\nu(\mu)) \in \hat{P}(X \times Y) \]
satisfies the conditions of the lemma. For every $f \in C_b(X)$
\[ \hat{P}pr_1(\lambda)(f) = \psi_{X \times Y}(\hat{P}f_\nu(\mu))(f \circ pr_1) = \hat{P}f_\nu(\mu)(u_{X \times Y}(f \circ pr_1)) = \mu(u_{X \times Y}(f \circ pr_1) \circ f_\nu). \]
Since the function
\[ (u_{X \times Y}(f \circ pr_1) \circ f_\nu)(x) = u_{X \times Y}(f \circ pr_1)(f_\nu) = u_{X \times Y}(f \circ pr_1)(\hat{P}i_x(\nu)) = \hat{P}i_x(\nu)(f \circ pr_1) = \nu(f \circ pr_1 \circ i_x) = \nu(f(x)) = f(x) \]
we have that $u_{X \times Y}(f \circ pr_1) \circ f_\nu \equiv f$ and $\hat{P}pr_1(\lambda) = \mu$.
Analogically for every $g \in C_b(Y)$
\[ \hat{P}pr_2(\lambda)(g) = \psi_{X \times Y}(\hat{P}f_\nu(\mu))(f \circ pr_2) = \mu(u_{X \times Y}(g \circ pr_2) \circ f_\nu). \]
Since the function
\[ (u_{X \times Y}(g \circ pr_2) \circ f_\nu)(x) = \nu(g \circ pr_2 \circ i_x) = \nu(g) \]
this implies that $\hat{P}pr_2(\lambda)(g) = \mu(\nu(g)) = \nu(g)$.

According to Lemma 2.1 $\lambda \in \hat{P}(X \times Y)$. We denote this measure by $\lambda = \mu \otimes \nu$ and call the tensor product of the measures $\mu$ and $\nu$.

Now, if $c = \|\mu\| = \|\nu\| \neq 1$ then it is easy to check that the measure $c(\frac{\mu}{c} \otimes \frac{\nu}{c})$ satisfies the conditions of the lemma.

ii). Let us denote by $K_\mu = \text{supp}(\mu)$ and $K_\nu = \text{supp}(\nu)$. By the definition of the functor $P_\beta$ the sets $K_\mu$ and $K_\nu$ are compacta. This implies that measures $\frac{\mu}{c} \in P(K_\mu)$ and $\frac{\nu}{c} \in P(K_\nu)$. The statement ii) of the lemma follows from the bicommutativity of the functor $P$. \qed

Lemma 2.3. Let $\lambda \in \hat{P}(X \times Y)$. Then for every open set $V \subset X \times Y$ and $\varepsilon > 0$ there exist elements of the base $\tilde{V}_1, ..., \tilde{V}_k$ such that $\tilde{V}_i \subset V$ for every $i = 1, ..., k$ and
\[ \lambda(V) - \lambda(\bigcup_{i=1}^{k} \tilde{V}_i) < \varepsilon. \]

Proof. The sets $\tilde{V}_1, ..., \tilde{V}_k$ are elements of the base means that there are open sets $\tilde{V}_i' \subset X$ and $\tilde{V}_i'' \subset Y$ such that $\tilde{V}_i = \tilde{V}_i' \times \tilde{V}_i''$ for every $i = 1, ..., k$. Since $\lambda$ is a Radon measure there exists a compactum $K \subset V$ such that $\lambda(V \setminus K) < \varepsilon$. Consider a covering by elements of the base $\{\tilde{V}_i\}_{i \in \Gamma}$ of the compactum $K$ such that $\tilde{V}_i \subset V$ for every $i \in \Gamma$. We can find a finite number of elements of the covering $\tilde{V}_1, ..., \tilde{V}_k$ such that $K \subset \bigcup_{i=1}^{k} \tilde{V}_i \subset V$. This condition proves the lemma. \qed

Lemma 2.4. Let $\lambda \in \hat{P}(X \times Y)$, $\mu = \hat{P}pr_1(\lambda)$, $\nu = \hat{P}pr_2(\lambda)$ and $\varepsilon > 0$. Then for open sets $V, V' \subset X$ such that $\mu(V \setminus V') < \varepsilon$ it is satisfied that $\lambda(V \setminus V') \times W < \varepsilon$ for every open set $W \subset Y$. Analogically, for open sets $W, W' \subset Y$ such that $\nu(W \setminus W') < \varepsilon$ it is satisfied that $\lambda(V \times (W \setminus W')) < \varepsilon$ for every open set $V \subset Y$. \qed
Proof. Since $\mu = \hat{P}\text{pr}_1(\lambda)$ this implies that $\mu(V) = \lambda(\text{pr}_1^{-1}(V)) = \lambda(V \times Y)$. Then for every open set $W \subset Y$ we have that

$$\lambda((V \setminus V') \times W) < \lambda((V \setminus V') \times Y) = \mu(V \setminus V') < \varepsilon.$$ 

Another statement of the lemma can be proved in the same manner. \hfill $\square$

**Lemma 2.5.** $\lambda \in \hat{P}(X \times Y)$, $\mu = \hat{P}\text{pr}_1(\lambda)$, $\nu = \hat{P}\text{pr}_2(\lambda)$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$. Then for open sets $V, V' \subset X$ such that $\mu(V \setminus V') < \varepsilon_1$ and for open sets $W, W' \subset Y$ such that $\nu(W \setminus W') < \varepsilon_2$ it is satisfied that $\lambda((V \times W) \setminus (V' \times W')) < \varepsilon_1 + \varepsilon_2$.

**Proof.** It is easy to check that

$$\lambda((V \times W) \setminus (V' \times W')) < \lambda(((V \setminus V') \times W) \cup (V \times (W \setminus W'))) \leq \lambda((V \setminus V') \times W) + \lambda(V \times (W \setminus W')) < \varepsilon_1 + \varepsilon_2. \hfill \square$$

**Lemma 2.6.** Let $\lambda \in \hat{P}(X \times Y)$ and let $V_1, \ldots, V_n$ be open sets in $X \times Y$ and $\varepsilon_0 > 0$. Then for every $\varepsilon > 0$ there exist pairwise disjoint elements of base $W_1, \ldots, W_m \subset X \times Y$ and a number $\delta > 0$ such that

(i) $\lambda(V_i) - \sum_{\{j: W_j \subset V_i\}} \lambda(W_j) < \varepsilon$ for every $i = 1, \ldots, n$,

(ii) $O(\lambda, W_1, \ldots, W_m, \delta) \subset O(\lambda, V_1, \ldots, V_n, \varepsilon_0)$,

(iii) $(\text{pr}_1(W_{j'}) \cap \text{pr}_1(W_{j''}^r) = \emptyset) \lor (\text{pr}_1(W_{j'}) = \text{pr}_1(W_{j''}^r))$ for $l = 1, 2$ and $j', j'' = 1, \ldots, m$.

**Proof.** According to Lemma 2.2 of Bogachev (1999) we can assume that $V_1, \ldots, V_n$ are pairwise disjoint. By Lemma 2.2 we can find elements of base $\tilde{V}_{ij} \subset V_i$, $j = 1, \ldots, k_i$ such that

$$\lambda(V_i) - \lambda(\bigcup_{j=1}^{k_i} \tilde{V}_{ij}) < \frac{\varepsilon}{2}. \hfill \square$$

Since every set $\tilde{V}_{ij}$ is an element of the base it can be represented as $\tilde{V}_{ij} = \tilde{V}_{ij}' \times \tilde{V}_{ij}''$ for some open sets $\tilde{V}_{ij} \subset X$ and $\tilde{V}_{ij}'' \subset Y$. Following Lemma 2.2 of Bogachev (1999) there exist pairwise disjoint open sets $W_1, \ldots, W_m' \subset X$ and $W_1'', \ldots, W_m'' \subset Y$ such that

$$\mu(\tilde{V}_{ij}') - \sum_{\{s: W_s' \subset \tilde{V}_{ij}'\}} \mu(W_s') < \frac{\varepsilon}{4k}$$

and

$$\nu(\tilde{V}_{ij}'') - \sum_{\{t: W_t'' \subset \tilde{V}_{ij}''\}} \nu(W_t'') < \frac{\varepsilon}{4k}$$

for every $i = 1, \ldots, n$ with $k = \max\{k_1, \ldots, k_n\}$. Consider all pairwise products of the type $W_q' \times W_s''$ for $q = 1, \ldots, m'$, $s = 1, \ldots, m''$ which is in at least one of the set $\tilde{V}_{ij}$ for $j = 1, \ldots, k_i$ and $i = 1, \ldots, n$. Let us denote these products as $W_1, \ldots, W_m$. Clearly, they are pairwise disjoint and $I_j = \{i: W_j \subset V_i\}$ is a singleton. In addition, Lemma 2.5 implies that for every
\[ j = 1, \ldots, k_i \]

\[
\lambda(\tilde{V}_{ij}) - \sum_{\{t: W_t \subset V_{ij}\}} \lambda(W_t) = \lambda(\tilde{V}_{ij}^f \times \tilde{V}_{ij}^m \setminus ((\bigcup_{q: W_q \subset V_{ij}^f} W_q^f) \times (\bigcup_{s: W_s \subset V_{ij}^m} W_s^m)))
\]

\[
< \frac{\varepsilon}{4k} + \frac{\varepsilon}{4k} < \frac{\varepsilon}{2k}.
\]

Let us show that for every \( i = 1, \ldots, n \) we have \( \lambda(\bigcup_{j=1}^{k_i} \tilde{V}_{ij}) - \sum_{q=1}^{m} \lambda(W_q) < \varepsilon \). For \( j = 1, \ldots, k_i \) the inequality

\[
\lambda(\tilde{V}_{ij} \setminus (\bigcup_{q=1}^{m} W_q)) \leq \lambda(\tilde{V}_{ij} \setminus (\bigcup_{q: W_q \subset V_{ij}} W_q)) < \frac{\varepsilon}{2k},
\]

is satisfied. Using this fact we can see that

\[
\lambda(\bigcup_{j=1}^{k_i} \tilde{V}_{ij} \setminus (\bigcup_{q=1}^{m} W_q)) \leq \sum_{j=1}^{k_i} \lambda(\tilde{V}_{ij} \setminus (\bigcup_{q=1}^{m} W_q)) < k_i \frac{\varepsilon}{2k} < \frac{\varepsilon}{2}.
\]

The condition (i) is easily implied from the last equation.

Indeed, for every \( i = 1, \ldots, n \)

\[
\lambda(V_i) - \sum_{q=1}^{m} \lambda(W_q) = \lambda(V_i) - \lambda(\bigcup_{j=1}^{k_i} \tilde{V}_{ij}) + \lambda(\bigcup_{j=1}^{k_i} \tilde{V}_{ij}) - \sum_{q=1}^{m} \lambda(W_q) < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.
\]

In order to prove (ii) it is necessary to find \( \delta \) such that for \( i = 1, \ldots, n \)

\[ \lambda'(V_i) - \lambda(V_i) > -\varepsilon_0 \]

for every \( \lambda' \in O(\lambda, W_1, \ldots, W_m, \delta) \). The condition (i) implies that for \( \delta < \frac{\varepsilon_0}{2m} \) and \( \varepsilon < \frac{m}{2} \) we have

\[
\lambda'(V_i) - \lambda(V_i) > \sum_{\{q: W_q \in V_i\}} \lambda'(W_q) - \sum_{\{q: W_q \in V_i\}} \lambda(W_q) - \varepsilon
\]

\[
> \sum_{\{q: W_q \in V_i\}} (\lambda'(W_q) - \lambda(W_q)) - \varepsilon > -m\delta - \varepsilon > -\varepsilon_0.
\]

The statement (iii) is obvious by the algorithm.

The Lemma is proved. \( \square \)

Further we need the notion of the restriction of the measure. We endow the set

\[
\{\psi \in C_b(X, [0, 1]), \psi|_A \equiv 1\}
\]

with the reverse pointwise order relation. For every \( \mu \in M(X) \) and Borel subset \( A \subset X \) define \( \mu|_A(\varphi) = \lim\{\mu(\varphi \cdot \psi: \psi \in C_b(X, [0, 1])), \psi|_A \equiv 1\} \) for every \( \varphi \in C_b(X) \) (see Teleiko and Zarichnyi (1999)). It is easy to show that for every Borel set \( B \subset X \) we have that \( \mu|_A(B) = \mu(A \cap B) \).

Let us define the map \( \hat{\chi}: \hat{P}(X \times Y) \to \hat{P}(X) \times \hat{P}(Y) \) by the formula

\[
\hat{\chi}(\lambda) = (\hat{P}_{pr_1}(\lambda), \hat{P}_{pr_2}(\lambda))
\]

for every \( \lambda \in \hat{P}(X \times Y) \).
Theorem 3.1. The map $\hat{\chi}$ is open.

Proof. Let $\lambda^0 \in \hat{P}(X \times Y)$ and let $O(\lambda^0, V_1, ..., V_n, \varepsilon)$ be its neighborhood in the weak-* topology for open sets $V_i \subset X \times Y$ and $\varepsilon > 0$. According to Lemma 2.6 we assume without loss of generality that the sets $V_i$ are pairwise disjoint and there are exist open sets $W'_i \subset X$ and $W''_i \subset Y$ such that $V_i = W'_i \times W''_i$ and $(W'_i \cap W'_j = \emptyset) \lor (W''_i = W''_j)$ and $(W''_i \cap W''_j = \emptyset) \lor (W''_i = W''_j)$ for $i, i_1, i_2 = 1, ..., n$. Let $\mu^0 = \hat{P}\rho_1(\lambda^0)$ and $\nu^0 = \hat{P}\rho_2(\lambda^0)$. Denote by $m' = \text{Card}\{W'_i, i = 1, ..., n\}$ and $m'' = \text{Card}\{W''_i, i = 1, ..., n\}$ and let $\delta < \varepsilon$. We adopt the notation $W_{qs} = W'_q \times W''_s$ for every $q \in \{1, ..., m'\}$ and $s \in \{1, ..., m''\}$.

In order to prove the theorem it is sufficient to show that every pair of measures

$$(\mu, \nu) \in O(\mu^{0}, W'_1, ..., W'_m, \delta) \times O(\nu^{0}, W''_1, ..., W''_m, \delta)$$

has a preimage in $O(\lambda^0, V_1, ..., V_n, \varepsilon)$.

For every $(q, s) \in \{1, ..., m'\} \times \{1, ..., m''\}$ we define

$$\alpha'_{qs} = \frac{\lambda^0(W_{qs})\mu(W'_q)}{\mu^0(W'_q)}$$

and analogically

$$\alpha''_{qs} = \frac{\lambda^0(W_{qs})\nu(W''_s)}{\nu^0(W''_s)}.$$

In addition we set $\alpha_{qs} = \min(\alpha'_{qs}, \alpha''_{qs})$, $\beta'_{qs} = \alpha'_{qs} - \alpha_{qs}$ and $\beta''_{qs} = \alpha''_{qs} - \alpha_{qs}$.

Let $\mu_q = \mu|_{W'_q}$ and $\nu_s = \nu|_{W''_s}$ be the restrictions of the measures $\mu$ and $\nu$ on the sets $W'_q$ and $W''_s$ respectively for every $q \in \{1, ..., m'\}$ and $s \in \{1, ..., m''\}$. Denote

$$\lambda_{qs} = \alpha_{qs}(\frac{\mu_q}{\mu(W'_q)} \otimes \frac{\nu_s}{\mu(W''_s)}).$$

if $\lambda^0(W'_q \times W''_s) \neq 0$ and $\lambda_{qs} = 0$ otherwise. It is clear that

$$\lambda_{qs}(X \times Y) = \lambda_{qs}(W'_q \times W''_s) = \alpha_{qs}.$$

We define the measures

$$\tilde{\lambda} = \sum_{q=1}^{m'} \sum_{s=1}^{m''} \lambda_{qs} \in \hat{M}(X \times Y), \tilde{\mu} = \mu - \hat{M}\rho_1(\tilde{\lambda}) \in \hat{M}(X) \text{ and } \tilde{\nu} = \nu - \hat{M}\rho_2(\tilde{\lambda}) \in \hat{M}(Y).$$

Since for every $q \in \{1, ..., m'\}$ we have

$$\tilde{\mu}(W'_q) = \mu(W'_q) - \sum_{s=1}^{m''} \alpha_{qs} \geq \mu(W'_q) - \sum_{s=1}^{m''} \alpha'_{qs} = \frac{\mu(W'_q)\lambda^0(W'_q \times (Y \setminus (\bigcup_{s=1}^{m''} W''_s))))}{\mu^0(W'_q)} \geq 0,$$

this implies that the measures $\tilde{\mu}$ and $\tilde{\nu}$ are non-negative. Moreover, it is clear that $||\tilde{\mu}|| = ||\tilde{\nu}|| \geq 0$. Therefore by Lemma 2.2 there exists non-negative measure $\tilde{\lambda}$ such that $\hat{M}\rho_1(\tilde{\lambda}) = \hat{P}(X \times Y)$.
\( \tilde{\mu} \) and \( \tilde{\mathcal{M}} \mathbf{Pr}_2(\tilde{\lambda}) = \tilde{\nu} \). Let us set \( \lambda = \tilde{\lambda} + \tilde{\lambda} \). Obviously, \( \lambda \in \tilde{\mathcal{P}}(X \times Y) \). Besides, for every \( q \in \{1, \ldots, m'\} \) and \( s \in \{1, \ldots, m''\} \) we have
\[
\lambda(W_{qs}) - \lambda^0(W_{qs}) = \tilde{\lambda}(W_{qs}) - \lambda^0(W_{qs}) = \alpha_{qs} - \lambda^0(W_{qs}) = \lambda^0(W_{qs})(\min\{\mu(W_{qs}'), \nu(W_{qs}'')\} - 1).
\]
The condition \((\mu, \nu) \in O(\mu^0, W_1', \ldots, W_m', \delta) \times O(\nu^0, W_1'', \ldots W_m'', \delta)\) implies that
\[
\min\{\mu(W_{qs}'), \nu(W_{qs}'')\} \alpha_{qs} \geq \min\{\mu^0(W_{qs}), \nu^0(W_{qs}''\} - 1 > -\delta \min\{\mu^0(W_{qs}), \nu^0(W_{qs}'')\}.
\]
and therefore
\[
\lambda(W_{qs}) - \lambda^0(W_{qs}) > -\delta \lambda^0(W_{qs}) > -\delta > -\varepsilon.
\]
Thus, \( \lambda \in O(\lambda^0, W_{11}, \ldots, W_{m'm''}, \varepsilon) \subset O(\lambda^0, V_1, \ldots, V_n, \varepsilon) \). The theorem is proved. \( \square \)

Analogically, we define the map \( \chi_\beta = \tilde{\chi}_{P_\beta}(X \times Y) \).

**Corollary 3.2.** The map \( \chi_\beta \) is open.

The proof of the corollary is directly implied from Theorem 3.1 and Lemma 2.2 ii).

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