MEAN SQUARE CORDIAL LABELING ON STAR RELATED GRAPHS

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Abstract. A Mean Square Cordial Labeling (MSCL) of a Graph G(V,E) with p vertices and q edges is a bijection from V to {0, 1} such that each edge uv is assigned the label \( \left\lceil \frac{(f(u)^2 + f(v)^2)}{2} \right\rceil \) where \( \left\lceil x \right\rceil \) (ceil( x)) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. In this paper it is analyzed that some star related graphs like Bistar graph B(k,k), Subdivision of a star graph S(Sk), Coconut tree CT(k,k), Merge graph Cn*Skn, Graph PmΘ Kn, and Banana tree BT(k, k) admits mean square cordial labeling.

I. INTRODUCTION

Graph theory plays an important role for automatic graph generation in computer science technology applications such as database design, software engineering, circuit designing, networks and data mining. Due to the involvement of the researchers for the past 60 years, over 200 graphs labeling techniques [1] have been discussed in thousands of research papers. For basics terms and notations we follow Harary [2]. Cordial labeling was introduced by Cahit [3] and Ponraj et al[4] were initiated the mean cordial labeling of a graph. Mean square cordial labeling introduced by A.Nellai murugan et al and they have discussed it for some special graphs[5].Moreover they have discussed the mean square cordial labeling for some tree and cycle related graphs[6,7]. Dhanalakshmi et al have discussed mean square cordial labeling related to some cyclic and acyclic graphs and its rough approximations [8,9].In this paper it is analyzed that some star related graphs like Bistar graph B(k,k), Subdivision of a star graph S(Sk), Coconut tree CT(k,k), Merge graph Cn*Skn, Graph PmΘ Kn, and Banana tree BT(k, k) admits mean square cordial labeling.

II. PRELIMINARIES

Definition 1: Let G = (V,E) be a graph with p vertices and q edges. “A Mean Square Cordial labeling of a Graph G(V,E) with p vertices and q edges is a bijection from V to {0, 1} such that each edge uv is assigned the label \( \left\lceil \frac{(f(u)^2 + f(v)^2)}{2} \right\rceil \) where \( \left\lceil x \right\rceil \) (ceil( x)) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1 .

Definition 2: Bistar graph Bn,n is obtained by joining the apex(centre) vertices of two Kn,1 by an edge.

Definition 3: Banana tree as defined is graph obtained by connecting one leaf of each of n copies of an K-star graph with single root vertex that is distinct from all the stars.

Definition 4: The graph PnΘ Kn,n is obtained by replacing each vertex of a path graph by a central vertex of a star graph

Definition 5: Merge graph Cn*Kn,n is obtained by attaching an apex vertex of a star graph Kn,n with any jth vertex of a cycle Cn.
Definition 6: Let $G$ be a graph. The subdivision graph $S(G)$ is obtained from $G$ by subdividing each edge of $G$ with a vertex.

Definition 7: Coconut tree $CT(m,n)$ is the graph obtained from the path $P_m$ by adding a star graph $S_n$ at the end vertex of a path $P_m$.

III. MAIN RESULTS

Theorem 1: Bistar graph $B(k, k)$ admits mean square cordial labeling, $\forall k \geq 2$

Proof: Let $V(B(k, k)) = \{(u_0, v_0, u_i, v_i): i \text{ varies from } 1 \text{ to } k\}$ and $E(B(k, k)) =\{(u_0u_i): i \text{ varies from } 1 \text{ to } k\} \cup \{(v_0v_i): i \text{ varies from } 1 \text{ to } k\} \cup \{(u_0v_0)\}$ where $u_0$ and $v_0$ are the apex vertices and $u_i$ and $v_i$ be the pendent vertices corresponding to the apex vertices $u_0$ and $v_0$ respectively.

Here $|V| = 2k + 2$ and $|E| = 2k + 1$

Define $f$ maps from $V(G)$ to $\{0, 1\}$

\[
\begin{align*}
    f(u_0) &= 0 \\
    f(u_i) &= 0, \ i \text{ varies from } 1 \text{ to } k \\
    f(v_0) &= 1, \ i \text{ varies from } 1 \text{ to } k \\
    f(v_i) &= 1, \ i \text{ varies from } 1 \text{ to } k
\end{align*}
\]

Hence the labeling of edges are

\[
\begin{align*}
    f(u_0u_i) &= 0, \ i \text{ varies from } 1 \text{ to } k \\
    f(v_0v_i) &= 1, \ i \text{ varies from } 1 \text{ to } k \\
    f(u_0v_0) &= 1, \ i \text{ varies from } 1 \text{ to } k
\end{align*}
\]

Table: 1 vertex and edge cardinality for MSCL of a Bistar graph $B(k, k)$

| $|V_f(t)|$ | $0$ | $1$ |
|---|---|---|
| $|V_f(t)|$ | $k + 1$ | $k + 1$ |
| $|E_f(t)|$ | $k$ | $k + 1$ |

It is very clear that above labeling pattern satisfies the condition of MSCL. Hence the theorem is proved.

Illustration 1: MSCL of a Bistar graph $B(4,4)$ is shown in figure 4.2.2.
Theorem 2: Subdivision of a star graph $S(S_k)$ admits mean square cordial labeling , $\forall k \geq 2$

Proof: Let $V(S(S_k))= \{(u_0, u_i, v_i): i \text{ varies from 1 to } k\}$ and $E(S(S_k))=\{(u_0u_i): i \text{ varies from 1 to } k\} \cup \{(u_iv_i): i \text{ varies from 1 to } k\}$ where $u_0$, $u_i$, and $v_i$ are the apex vertices, $v_i$ be the pendent vertices and $u_i$ be the subdivision vertex of $u_0v_i$ respectively.

Here $|V| = 2k + 1 \text{ and } |E| = 2k$

Define $f$ maps from $V(G)$ to $\{0, 1\}$

$f(u_0) = 0$

$f(u_i) = 0, i \text{ varies from 1 to } k$

$f(v_i) = 1, i \text{ varies from 1 to } k$

Hence the labeling of edges are

$f(u_0u_i) = 0, i \text{ varies from 1 to } k$

$f(u_iv_i) = 1, i \text{ varies from 1 to } k$

Table: 2 vertex and edge cardinality for MSCL of a Subdivision of a star graph $S(S_k)$

| $v_f(i)$ | $k+1$ | $k$ |
|----------|-------|-----|
| $e_f(i)$ | $k$   | $k$ |

It is very clear that above labeling pattern satisfies the condition of MSCL. Hence the theorem is proved.

Illustration 2: MSCL of a Subdivision of a star graph $S(S_k)$ is shown in figure 4.2.3

![Figure 2]

Theorem 3: Coconut tree CT $(k,k)$ admits mean square cordial labeling , $\forall k \geq 2$

Proof: Let $V(CT(k,k))= \{(u_i, v_i): i \text{ varies from 1 to } k\}$ and $E(CT(k,k))=\{(u_iu_{i+1}): i \text{ varies from 1 to } k\} \cup \{(u_iv_i): i \text{ varies from 1 to } k\}$ where $u_i$ and $v_i$ are the vertices on a path and the pendent vertices of a star graph respectively.

Here $|V| = 2k \text{ and } |E| = 2k - 1$

Define $f$ maps from $V(G)$ to $\{0, 1\}$

$f(u_i) = 0, i \text{ varies from 1 to } k$

$f(v_i) = 1, i \text{ varies from 1 to } k$

Hence the labeling of edges are

$f(u_iu_{i+1}) = 0, i \text{ varies from 1 to } k-1$

$f(u_iv_i) = 1, i \text{ varies from 1 to } k$
Table: 3 vertex and edge cardinality for MSCL of a coconut tree CT(k,k)

| T   | 0 | 1 |
|-----|---|---|
| \(v_f(t)\) | \(k\) | \(k\) |
| \(e_f(t)\) | \(k-1\) | \(k\) |

It is very clear that above labeling pattern satisfies the condition of MSCL. Hence the theorem is proved.

Illustration 3: MSCL of a Coconut tree CT(4,4) is shown in figure 4.2.4

Figure(3)

Theorem 4: The merge graph \(C_k \ast S_k\) admits mean square cordial labeling, \(\forall k \geq 3\)

**Proof:** Let \(V(C_k \ast S_k) = \{u_i, v_i : i \text{ varies from } 1 \text{ to } k\}\) and \(E(G) = \{(u_i u_{i+1} : i \text{ varies from } 1 \text{ to } k-1) \cup (u_1 u_k) \cup (u_i v_i : i \text{ varies from } 1 \text{ to } k)\}\) where \(u_i\) and \(v_i\) are the vertices on a cycle and pendent vertices of a star graph respectively.

Here \(|V| = 2k\) and \(|E| = 2k\).

Here \(f\) maps \(V(C_k \ast S_k)\) to \(\{0, 1\}\).

- \(f(u_i) = 0, i \text{ varies from } 1 \text{ to } k\)
- \(f(v_i) = 1, i \text{ varies from } 1 \text{ to } k\)

Hence the labeling of edges are

- \(f((u_i u_{i+1}) = 0, i \text{ varies from } 1 \text{ to } k - 1\)
- \(f(u_1 u_k) = 0\)
- \(f(u_i v_i) = 1, i \text{ varies from } 1 \text{ to } k\)

Table: 4 vertex and edge cardinality for MSCL of a merge graph \(C_k \ast S_k\)

| T   | 0 | 1 |
|-----|---|---|
| \(v_f(t)\) | \(k\) | \(k\) |
| \(e_f(t)\) | \(k\) | \(k\) |
It is very clear that the above labeling pattern satisfies the condition of MSCL. Hence the theorem is proved.

*Illustration 4:* MSCL of a merge graph C_{6}*S_{6} is shown in figure 4.2.5

**Theorem 5:** The graph P_{m}\Theta K_{1,n} admits mean square cordial labeling , \( \forall m \geq 1, n \geq 2 \)

**Proof:** Let \( V(P_{m}\Theta K_{1,n}) = \{ v_{i}, u_{j}^{i} : i \text{ varies from } 1 \text{ to } m; j \text{ varies from } 1 \text{ to } n \} \) and \( E(G) = \{ [(v_{i}, v_{i+1}^{i}) : i \text{ varies from } 1 \text{ to } m-1] \cup [(v_{i}, u_{j}^{i}) : i \text{ varies from } 1 \text{ to } m, j \text{ varies from } 1 \text{ to } n ] \} \) where \( v_{i} \) and \( u_{j}^{i} \) are the vertices of path and pendent vertices of a star graph attached to the vertices of the path graph respectively.

Here \( |V| = m + nm \) and \( |E| = m + nm - 1 \)

Here \( f \) maps \( V(P_{m}\Theta K_{1,n}) \) to \{0,1\}.

Case (i): \( m \) and \( n \) are even

\[
 f(v_{i}) = 0, i \text{ varies from } 1 \text{ to } \frac{m}{2} \\
 1, i \text{ varies from } \frac{m+2}{2} \text{ to } m \\
 f(u_{j}^{i}) = 0, i \text{ varies from } 1 \text{ to } \frac{m}{2}, j \text{ varies from } 1 \text{ to } n \\
 1, i \text{ varies from } \frac{m}{2} + 1 \text{ to } m, j \text{ varies from } 1 \text{ to } n \\
\]

Hence the labeling of edges are

\[
 f(v_{i}v_{i+1}^{i}) = 0, i \text{ varies from } 1 \text{ to } \frac{m}{2} - 1 \\
 1, i \text{ varies from } \frac{m}{2} \text{ to } m - 1 \\
\]
\[ f(v_i u'_j) = 0, i \text{ varies from } 1 \text{ to } \frac{m}{2}, j \text{ varies from } 1 \text{ to } n \]
\[ 1, i \text{ varies from } \frac{m+1}{2} \text{ to } m, j \text{ varies from } 1 \text{ to } n \]

Table: 5.1 vertex and edge cardinality for MSCL of a \( P_m \oplus K_{1,n} \), m and n are even

| T   | \( v_f(t) \) | \( e_f(t) \) |
|-----|--------------|--------------|
| 0   | \( \frac{m+nm}{2} \) | \( \frac{m+nm}{2} \) |
| 1   | \( \frac{m+nm}{2} \) | \( \frac{m+nm}{2} \) |

Case (ii) : m is odd and n is even
\[ f(v_i) = 0, i \text{ varies from } 1 \text{ to } \frac{m+1}{2} \]
\[ 1, i \text{ varies from } \frac{m+3}{2} \text{ to } m \]

Subcase (i) : If \( i = \frac{m+1}{2} \)
\[ f(u'_j) = 0, j \text{ varies from } 1 \text{ to } n/2 \]
\[ 1, j \text{ varies from } (n/2)+1 \text{ to } n \]

Subcase (ii) : If \( i \neq \frac{m+1}{2} \)
\[ f(u'_j) = 0, i \text{ varies from } 1 \text{ to } \frac{m-1}{2}, j \text{ varies from } 1 \text{ to } n \]
\[ 1, i \text{ varies from } \frac{m+3}{2} \text{ to } m, j \text{ varies from } 1 \text{ to } n \]

Hence the labeling of edges are
\[ f(v_i v_{i+1}) = 0, i \text{ varies from } 1 \text{ to } \frac{m-1}{2} \]
\[ 1, i \text{ varies from } \frac{m+1}{2} \text{ to } m-1 \]

Subcase (i) : If \( i = \frac{m+1}{2} \)
\[ f(v_i, u_j) = 0, \] j varies from 1 to \( n/2 \)

1, j varies from \((n/2)+1\) to \( n \)

Subcase (ii): If \( i \neq \frac{m+1}{2} \)

\[ f(v_i, u_j) = 0, i \text{ varies from } 1 \text{ to } \frac{m-1}{2}, \] j varies from 1 to \( n \)

1, i varies from \( \frac{m+3}{2} \) to \( m \), j varies from 1 to \( n \)

\[ f(v_i, u_j) = 0, i \text{ varies from } 1 \text{ to } \frac{m-1}{2}, \] j varies from 1 to \( n \)

1, i varies from \( \frac{m+3}{2} \) to \( m \), j varies from 1 to \( n \)

Table 5.2 vertex and edge cardinality for MSCL of a \( P_m \Theta K_{1,n} \), m is odd and n is even

| \( T \) | \( v_f(t) \) | \( e_f(t) \) |
|------|-------------|-------------|
| 0    | \( \frac{m+nm+1}{2} \) | \( \frac{m+nm-1}{2} \) |
| 1    | \( \frac{m+nm+1}{2} \) | \( \frac{m+nm-1}{2} \) |

Case (iii): m is odd and n is odd

\[ f(v_i) = 0, i \text{ varies from } 1 \text{ to } \frac{m+1}{2} \]

1, i varies from \( \frac{m+3}{2} \) to \( m \)

Subcase (i): If \( i = \frac{m+1}{2} \)

\[ f(u_j) = 0, j \text{ varies from } 1 \text{ to } (n-1)/2 \]

1, j varies from \((n+1)/2\) to \( n \)

Subcase (ii): If \( i \neq \frac{m+1}{2} \)

\[ f(u_j) = 0, i \text{ varies from } 1 \text{ to } \frac{m-1}{2}, j \text{ varies from } 1 \text{ to } n \]

1, i varies from \( \frac{m+3}{2} \) to \( m \), j varies from 1 to \( n \)

Hence the labeling of edges are
\[ f(v_{i+1}) = 0, i \text{ varies from } 1 \text{ to } \frac{m-1}{2} \]

\[ 1, i \text{ varies from } \frac{m+1}{2} \text{ to } m-1 \]

**Subcase (i):** If \( i = \frac{m+1}{2} \)

\[ f(v_i u^j_i) = 0, j \text{ varies from } 1 \text{ to } (n-1)/2 \]

\[ 1, j \text{ varies from } (n+1)/2 \text{ to } n \]

**Subcase (ii):** If \( i \neq \frac{m+1}{2} \)

\[ f(v_i u^j_i) = 0, i \text{ varies from } 1 \text{ to } \frac{m-1}{2}, j \text{ varies from } 1 \text{ to } n \]

\[ 1, i \text{ varies from } \frac{m+3}{2} \text{ to } m, j \text{ varies from } 1 \text{ to } n \]

Table: 5.3 vertex and edge cardinality for MSCL of a \( P_m \Theta K_{1,n} \), \( m \) is odd and \( n \) is odd

| \( T \) | \( 0 \) | \( 1 \) |
|---|---|---|
| \( |v_f(t)| \) | \( \frac{m+nm}{2} \) | \( \frac{m+nm}{2} \) |
| \( |e_f(t)| \) | \( \frac{m+nm}{2} - 1 \) | \( \frac{m+nm}{2} \) |

Case (iv): \( m \) is even and \( n \) is odd

\[ f(v_i) = 0, i \text{ varies from } 1 \text{ to } m \]

\[ f(u^j_i) = 0, i \text{ varies from } 1 \text{ to } m, j \equiv 0 \mod 2 \]

\[ 1, i \text{ varies from } 1 \text{ to } m, j \equiv 1 \mod 2 \]

Hence the labeling of edges are

\[ f(v_i v^j_{i+1}) = 0, i \text{ varies from } 1 \text{ to } m-1 \]

\[ f(v_i u^j_i) = 0, i \text{ varies from } 1 \text{ to } m, j \equiv 0 \mod 2 \]

\[ 1, i \text{ varies from } 1 \text{ to } m, j \equiv 1 \mod 2 \]

Table: 5.4 vertex and edge cardinality for MSCL of a \( P_m \Theta K_{1,n} \), \( m \) is even and \( n \) is odd
It is very clear that the above labeling pattern satisfies the condition of MSCL. Hence the theorem is proved.

Illustration 5.1: MSCL of a graph $P_4 \Theta K_{1,4}$ is shown in figure 4.2.6.1

![Figure 5.1](image1)

Illustration 5.2: MSCL of a graph $P_3 \Theta K_{1,4}$ is shown in figure 4.2.6.2

![Figure 5.2](image2)
Illustration 5.3: MSCL of a graph $P_5 \Theta K_{1,3}$ is shown in figure 4.2.6.3

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.3.png}
\caption{Figure(5.3)}
\end{figure}

Theorem 6: The Banana graph $BT(m,n)$, where $m=n$ admits mean square cordial labeling , $\forall m \geq 2, n \geq 2$

Proof: Let $V(BT(m,n)) = \{u_0, u_i, u_j, u_j', j\}$ varies from 1 to m; $i$ varies from 1 to n} and $E(G) = \{[(u_0u_j') : j$ varies from 1 to m] $\cup [(u_iu_j') : i$ varies from 1 to m, j varies from 1 to n} $\}$ where $u_0, u_i, u_j'$ and $u_j'$ are the root vertex, central vertex of a star graph, one leaf of a star graph attached to the single root vertex and the remaining pendent vertices of a star graph respectively.

Here $|V| = m + nm + 1$ and $|E| = m + nm$

Here f maps $V(BT(m,n))$ to $\{0,1\}$.

Case (i): $m$ and $n$ are even

\begin{align*}
f(u_0) &= 0 \\
f(u_j) &= 0, j$ varies from 1 to $\frac{m}{2}$ \\
&\quad 1, i$ varies from $\frac{m+2}{2}$ to $m$ \\
f(u_j') &= 0, i$ varies from 1 to $n$, j$ varies from 1 to $\frac{m}{2}$ \\
&\quad 1, i$ varies from 1 to $n$, j$ varies from $\frac{m}{2} + 1$ to $n$
\end{align*}

Hence the labeling of edges are

\begin{align*}
f(u_0u_j') &= 0, j$ varies from 1 to $\frac{m}{2}$ \\
&\quad 1, i = 1, j$ varies from $\frac{m}{2} + 1$ to $n$
\end{align*}
\[ f(u, u') = 0, i \text{ var ies from } 1 \text{ to } n, j \text{ var ies from } 1 \text{ to } \frac{m}{2} \]

\[ 1, i \text{ var ies from } 1 \text{ to } n, j \text{ var ies from } \frac{m}{2} + 1 \text{ to } n \]

Table: 6.1 vertex and edge cardinality for MSCL of a Banana tree BT(k,k) , k is even

| T   | 0                                | 1                                |
|-----|----------------------------------|----------------------------------|
| \( |v_f(t)|\) | \(\frac{m+mn+2}{2}\) | \(\frac{m+mn}{2}\) |
| \( |e_f(t)|\) | \(\frac{m+mn}{2}\) | \(\frac{m+mn}{2}\) |

Case (ii): m and n are odd

\( f(u_0) = 0 \)

Subcase (i): \( j = m/2 \)

\[ f(u_i) = 0, i \text{ var ies from } 1 \text{ to } \frac{n-1}{2} \]

\[ 1, i \text{ var ies from } n+1 \text{ to } n \]

\( f(u_j) = 0 \)

Hence the labeling of edges are

\[ f(u_0u_i') = 0 \]

\[ f(u_ju_j') = 0, i \text{ var ies from } 1 \text{ to } \frac{n-1}{2} \]

\[ 1, i \text{ var ies from } \frac{n+1}{2} \text{ to } n \]

Subcase (ii): \( j \neq m/2 \)

\( f(u_0) = 0 \)

\( f(u_j) = 0, j \text{ var ies from } 1 \text{ to } \frac{m-1}{2} \]

\[ 1, j \text{ var ies from } \frac{m+3}{2} \text{ to } m \]
\[ f(u'_i) = 0, j \text{ varies from } 1 \text{ to } \frac{m-1}{2}, i \text{ varies from } 1 \text{ to } n \]

\[ 1, j \text{ varies from } \frac{m+3}{2} \text{ to } m, i \text{ varies from } 1 \text{ to } n \]

Hence the labeling of edges are

\[ f(u_u) = 0, j \text{ varies from } 1 \text{ to } \frac{m-1}{2} \]

\[ 1, j \text{ varies from } \frac{m+3}{2} \text{ to } m \]

\[ f(u_u') = 0, j \text{ varies from } 1 \text{ to } \frac{m-1}{2}, i \text{ varies from } 1 \text{ to } n \]

\[ 1, j \text{ varies from } \frac{m+3}{2} \text{ to } m, i \text{ varies from } 1 \text{ to } n \]

Table: 6.2 vertex and edge cardinality for MSCL of a Banana tree BT(k,k), k is odd

| \( V_j(i) \) | 0 \( \frac{m+nm+2}{2} \) | 1 \( \frac{m+nm}{2} \) |
|---|---|---|
| \( E_j(i) \) | \( \frac{m+nm}{2} \) | \( \frac{m+nm}{2} \) |

It is very clear that the above labeling pattern satisfies the condition of MSCL. Hence the theorem is proved.

Illustration 6.1: MSCL of a Banana tree BT(5,5) is shown in figure 4.2.7.2
**IV. CONCLUSION**

Mean square cordial labeling is widely used in social networks such as Facebook, Research gate, Orkut, Linked in etc., In this paper it is analyzed that some star related graphs like Bistar graph $B(k,k)$, Subdivision of a star graph $S(S_k)$, Coconut tree $CT(k,k)$, Merge graph $C_k*S_k$, Graph $P_mΘ K_{1,n}$ and Banana tree $BT(k,k)$ admits mean square cordial labeling.

**V. FUTURE SCOPE**

Graph operations like union, intersection, corona of two graphs etc., can also be discussed for mean square cordial labeling in future.

**ACKNOWLEDGEMENTS**

We offer our sincere thanks to the referee for the valuable suggestion for revision of the paper.

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