Low-energy direct muon transfer from H to Ne$^{10+}$, S$^{16+}$, and Ar$^{18+}$ using two-state close-coupling approximation to Faddeev-Hahn-type equation

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Abstract

We perform three-body calculation of direct muon-transfer rates from thermalized muonic hydrogen isotopes to bare nuclei Ne$^{10+}$, S$^{16+}$, and Ar$^{18+}$ employing integro-differential Faddeev-Hahn-type equations in configuration space with two-state close-coupling approximation scheme. All Coulomb potentials including the strong final-state Coulomb repulsion are treated exactly. A long-range polarization potential is included in the elastic channel to take into account the high polarizability of the muonic hydrogen. The transfer rates so calculated are in good agreement with recent experiments. We find that the muon is captured predominantly in the $n = 6, 9, \text{ and } 10$ states of muonic Ne$^{10+}$, S$^{16+}$, and Ar$^{18+}$, respectively.

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I. INTRODUCTION

A theoretical description of three-body charge-transfer reactions in physics is both challenging and interesting and is a subject of active research \[1-8\], as the study of this fundamental problem provides a testing ground of the underlying physical model. This is even more so when the mass of the exchanged charged particle is not negligible compared to the other two and when there is a strong final-state Coulomb interaction. The first feature invalidates the commonly-used simplifying Born-Oppenheimer-type approximation scheme \[2\] and calls for a detailed three-body description. The second feature demands a proper dynamical treatment of the final-state Coulomb interaction.

The direct muon (\(\mu\)) transfer rates at low energies from the muonic-hydrogen isotopes \(^1\text{H}_\mu (p\mu)\) and \(^2\text{H}_\mu (d\mu)\) to bare nuclei \(X^{Z+}\) of large charge \(Z\):

\[
(\text{H}_\mu)_{1s} + X^{Z+} \rightarrow (X_\mu)^{(Z-1)+} + \text{H}^+
\]  

have been the subject of many experimental investigations \[9-17\], where \(\text{H}\) stands for the hydrogen isotope proton (\(p\)) or deuteron (\(d\)) and \(X^{Z+}\) stands for the target nuclei. Investigations of the low-energy muon-transfer reactions in collisions of muonic hydrogen \(\text{H}_\mu\) with nuclei \(X^{Z+}\) are of importance for the muon-catalyzed fusion cycle \[18\]. Although, there have been many experiments of muon transfer with nuclei of large charge \(Z\), the theoretical investigations are limited to nuclei \(X^{Z+}\) with \(Z = 1\) \[7\] and \(Z = 2\) \[8\]. The theoretical description of these reactions at low energies has both the complicating features mentioned above and becomes extremely complicated as the charge \(Z\) of the nuclei increases. This is due to the strong electromagnetic field of the nuclei simultaneously responsible for a large initial-state polarization and a strong final-state Coulomb interaction. Also, the large mass of the transferred muon compared to the electron leads to additional difficulties compared to the electron-transfer reaction where a Born-Oppenheimer-type approximation is efficient \[2\]. In addition, a large number of open channels in the muon-transfer reactions even at zero energy complicates the theoretical treatment. It is difficult to incorporate these effects properly in a dynamical three-body calculation. This is why there are no published work to date on a three-body dynamical calculation of these muon-transfer rates for nuclei with charge \(Z > 3\). These aspects demand a careful three-body quantum mechanical treatment with exact inclusion of the final-state Coulomb interaction.

The recent theoretical activities in charge transfer are centered around problems with much weaker polarization and Coulomb interactions compared to those encountered in muon transfer involving bare nuclei like argon or sulphur. In electron-hydrogen-atom \[1\], deuteron-hydrogen-atom \[2\], positron-hydrogen-atom \[4\], electron-positronium-atom \[5\], hydrogen-positronium-atom \[6\] and \(H-\mu-H\) \[7\] systems the final-state Coulomb interactions are zero compared to \(17e^2/\rho\) in the case of \(p-\mu-\text{Ar}^{18+}\) considered here, where \(e\) is the electronic charge and \(\rho\) is the radial separation in the final state. In proton-deuteron \[8\] and \(H-\mu-\text{He}\) \[8\] systems
there is a small final-state Coulomb repulsion, e.g., $e^2/\rho$. The initial-state polarization is also much smaller in those studies [1–8]. The large charge of the bare nuclei complicates substantially the analysis of the present muon transfer problems.

Recently, we presented a theoretical formalism for the study of such muon-transfer reactions using two-component integro-differential Faddeev-Hahn-type equations [19] in configuration space with close-coupling approximation scheme [20], and applied it to the study of muon transfer from muonic hydrogen isotopes to He$^{2+}$ and Li$^{3+}$ [21], and C$^{6+}$ and O$^{8+}$ [22]. The transfer rates $\lambda_Z$ calculated there are in good agreement with experiments, whenever available. The formalism of Ref. [22] seems to be appropriate for the study of the problem of muon transfer from muonic hydrogen isotopes to nuclei with large charge.

Among such transfer reactions, muon transfers from muonic hydrogen isotopes to Ne$^{10+}$, S$^{16+}$, and Ar$^{18+}$ have been the center of active interest to different experimental groups. There have been several experiments for muon transfer from $p\mu$ to argon which yielded transfer rates centered around the following five different values: \((16.3 \pm 0.9) \times 10^{10}\text{s}^{-1}\) [9], \((12.0 \pm 1.9) \times 10^{10}\text{s}^{-1}\) [10], \((14.6 \pm 1.4) \times 10^{10}\text{s}^{-1}\) [11], \((35 \pm 6) \times 10^{10}\text{s}^{-1}\) [12] and \((98 \pm 15) \times 10^{10}\text{s}^{-1}\) [13]. For muon transfer from $d\mu$ to argon the experimental transfer rates are \((8.6 \pm 0.4) \times 10^{10}\text{s}^{-1}\) [9] and \(9.4 \times 10^{10}\text{s}^{-1}\) [14]. The experimental situation in the case of argon is quite controversial with widely different values for the transfer rates, specially in the case of $p\mu$. Despite this intense experimental activity in these muon-transfer reactions there are no quantum dynamical calculations valid at low energies for these transfer rates. In view of the above interest and controversy, in this paper we undertake the challenging three-body study of direct muon transfer from $p\mu$ and $d\mu$ to argon using the formulation of Ref. [22]. To test our approach, in addition, we apply it to study muon transfer from hydrogen isotopes to neon and sulphur where experimental results are available. In the case of Sulphur the experimental transfer rate from $p\mu$ is $8.9 \times 10^{10}\text{s}^{-1}$, whereas that from $d\mu$ is $11 \times 10^{10}\text{s}^{-1}$ [15]. In the case of neon the experimental transfer rate from $d\mu$ is $10.1 \times 10^{10}\text{s}^{-1}$ [16].

Although there is a very large number of open channels in these problems, for a given nuclei the muon is transferred predominantly to a few (muonic) atomic labels of the heavy nuclei $X^{Z+}$ [23–25]. For example, it was first noted by Gershtein [25] and reconfirmed later [22] that the muon is captured mostly in the $n = 4$ states of C$^{6+}$, and $n = 5$ states of O$^{8+}$ [17,22]. Also these transfers take place mostly to the final muonic-atomic states with low angular momenta and transfer rates are negligible for muonic atomic states with angular momenta $l > 2$. Semiclassical description of these muon transfer reactions has been very useful in explaining many qualitative and quantitative features [24,26]. Using a semiclassical model based on potential curves of the two-centered $p\mu X^{Z+}$ system, Holzwarth et al. [23] demonstrated that muon transfer to F$^{9+}$ takes place essentially to the $n = 6$ level of the $(F\mu)^{8+}$ system. They also showed that transfer to the nearby levels of the $(F\mu)^{8+}$ atom is negligible compared to the $n = 6$ level. From similar consideration of semiclassical barrier
penetration, Haff et al. [24] derived the following formula for the state \( n \) of the \((X_\mu)(Z-1)^+\) atom to which the muon from \( H_\mu \) is predominantly transferred in reaction (1):

\[
n = I_n \left[ \frac{Z(1 + 2Z^{1/2})}{1 + 2/Z^{1/2}} \right]^{1/2},
\]

(2)

where \( I_n \) denotes the integral part. For \( C^{6+}, O^{8+} \) and \( F^{9+} \) this formula leads to \( n = 4, 5, \) and \( 6, \) respectively, in agreement with calculation [23,25]. The formula (2) is expected to work even better for heavier nuclei \( \text{Ne}^{10+}, \text{S}^{16+} \) and \( \text{Ar}^{18+} \) where the channelization to a final specific state \( n \) should be more perfect with muon transfer to nearby \( n \) states heavily suppressed.

The correct dynamical formulation should include all open transfer channels and we included them in a previous study on muon transfer with light nuclear targets [21]. However, it is quite impossible now to treat even this reduced number of open transfer channels in a quantum calculation with heavier targets due to convergence difficulties in the presence of the large final-state Coulomb interaction mentioned above. Hence, in the present treatment we use a two-channel model to calculate transfer to a single final state, where we include the elastic and one transfer channel. Different sets of equations are used for the different final states. Eventually, the total transfer rate is calculated by summing the different contributions. After calculation we find that in both cases \((p\mu \) and \( d\mu)\) the muon is captured predominantly in the \( n = 6 \) state of \( \text{Ne}^{10+}, \) \( n = 9 \) state of \( \text{S}^{16+}, \) and \( n = 10 \) state of \( \text{Ar}^{18+} \) in complete agreement with formula (2) and in this pioneering theoretical study we present results for these cases. The transfer is highly suppressed to other values of \( n \) and higher \( l \) states.

In the Faddeev-Hahn-type equation [19], the wave function is broken up into components with proper asymptotic behavior in different physical channels. Consequently, these wave-function components are much smoother functions of configuration-space variables than the Schrödinger wave function which is the sum of all these components. Hence, this approach simplifies the solution procedure and the correct asymptotic behavior of the solution in different channels can be incorporated easily. In addition, these equations allow us to introduce explicitly a polarization potential in the initial channel. All Coulomb potentials including the strong final-state Coulomb repulsion are treated exactly (without approximation or parameters) in this formalism. The effect of strong polarization of the muonic hydrogen by the bare nuclei is accounted for by a polarization potential with a cut off parameter. By a proper inclusion of the polarization potential in the intermediate region, the present transfer rates are found to be essentially independent of this parameter. The correct inclusion of the final-state Coulomb interaction has the advantage of building in the proper asymptotic behavior of the wave function in a low-order close-coupling type approximation [27,28]. Hence as in Ref. [22] we make a two-state close-coupling approximation to the Faddeev-Hahn-type equation in the present study and find that a numerical solution using the present scheme leads to very encouraging agreement with recent experimental transfer rates.
The theoretical framework for the present study is based on the formalism developed in Refs. [20–22] which was used for the study of muon transfer from muonic hydrogen atoms to lighter charged nuclei H+, He2+, Li3+, C6+, and O8+. In the dynamical equations in Refs. [21,22] the final-state Coulomb interaction in the transfer channel is treated exactly without approximation. In addition, as in Ref. [22], here we explicitly include a polarization potential in the elastic channel. The presence of the strong Coulomb interaction and the associated large polarization make the present calculational scheme far more complicated numerically compared to those of Refs. [21,22]. In a coupled-channel approach for atomic processes, the coupling to infinite number of p-wave states is responsible for generating the polarization potential [29,30]. As it is impossible to include all such states in a numerical scheme, the commonly accepted procedure is to replace these coupling terms by the polarization potential as in Ref. [31].

In Sec. II we present a brief account of the theoretical formulation. In Sec. III we report the present numerical results and finally, in Sec. IV we present some concluding remarks.

II. THEORETICAL FORMULATION

A detailed account of the theory has already appeared [21,22]. For the sake of completeness we report here a summary of the theoretical development. Here we describe the dynamical equations we use based on the close-coupling approximation to Faddeev-Hahn-type two-component equations [21]. We use units \( e = \hbar = m_\mu = 1 \), where \( m_\mu(e) \) is the muonic mass (charge), and denote the heavy nucleus \( X^{2+} \) by 1, the hydrogen isotope(s) by 2 and muon by 3. Below the three-body breakup threshold, following two-cluster asymptotic configurations are possible in the system 123: \((23)−1\) and \((13)−2\). These two configurations correspond to two distinct physical channels, denoted by 1 and 2, respectively. These configurations are determined by the Jacobi coordinates \( (\vec{r}_{j3}, \vec{\rho}_k) \):

- \( \vec{r}_{13} = \vec{r}_3 - \vec{r}_1 \), \( \vec{\rho}_2 = (\vec{r}_3 + m_1 \vec{r}_1)/(1 + m_1) - \vec{r}_2 \),
- \( \vec{r}_{23} = \vec{r}_3 - \vec{r}_2 \), \( \vec{\rho}_1 = (\vec{r}_3 + m_2 \vec{r}_2)/(1 + m_2) - \vec{r}_1 \),

where \( i = 1, 2, 3 \) are coordinates and masses of the particle \( i \), respectively.

Let us introduce the total three-body wave function as a sum of two components

\[
\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \Psi_1(\vec{r}_{23}, \vec{\rho}_1) + \Psi_2(\vec{r}_{13}, \vec{\rho}_2)
\]  

where \( \Psi_1(\vec{r}_{23}, \vec{\rho}_1) \) is quadratically integrable over the variable \( \vec{r}_{23} \), and \( \Psi_2(\vec{r}_{13}, \vec{\rho}_2) \) over \( \vec{r}_{13} \). The components \( \Psi_1 \) and \( \Psi_2 \) carry the asymptotic boundary condition for channels 1 and 2, respectively. The second component is responsible for pure Coulomb interaction in the final state. These components satisfy the following set of two coupled equations

\[
[E - (H_0 + V_{23}(\vec{r}_{23})) - U_{\text{pol}}(\vec{\rho}_1)]\Psi_1(\vec{r}_{23}, \vec{\rho}_1) = [(V_{23}(\vec{r}_{23}) + V_{12}(\vec{r}_{12})) - U_C(\vec{\rho}_2)]\Psi_2(\vec{r}_{13}, \vec{\rho}_2) \tag{4}
\]

\[
[E - (H_0 + V_{13}(\vec{r}_{13})) - U_C(\vec{\rho}_2)]\Psi_2(\vec{r}_{13}, \vec{\rho}_2) = [(V_{13}(\vec{r}_{13}) + V_{12}(\vec{r}_{12})) - U_{\text{pol}}(\vec{\rho}_1)]\Psi_1(\vec{r}_{23}, \vec{\rho}_1) \tag{5}
\]
where \( E \) is the center-of-mass energy, \( H_0 \) the total kinetic energy operator, \( V_{ij}(\vec{r}_{ij}) \) the pair potential \((i \neq j = 1, 2, 3)\), \( U_C \) the final-state Coulomb interaction given by

\[
U_C(\vec{p}_2) = \frac{(Z - 1)Z'}{\rho_2}
\]

with \( Z \) the charge of the heavy nuclei and \( Z' (= 1) \) the charge of the hydrogen isotope. Here \( U_{pol} \) is the polarization potential given by

\[
U_{pol}(\vec{p}_1) = -\frac{9Z^2}{4\rho_1^4} \quad \text{for} \quad \rho_1 > \Lambda
\]

and zero otherwise. The value of the cut-off parameter \( \Lambda \) has to be chosen appropriately. By adding equations (4) and (5) we find that they are equivalent to the Schrödinger equation.

Distortion potentials are useful in phenomenological models of scattering in atomic physics [31]. Although, unnecessary in a complete solution of the Schrödinger equation, they facilitate the numerical effort in a simplified model. We have included the proper polarization potential in the initial channel. Although, a polarization potential exists in the final rearrangement channel, the most important interaction in this channel is the Coulomb repulsion which has been exactly included in our description.

Because of the strong final-state Coulomb repulsion, it is very difficult to solve the multichannel model equations based on Eqs. (4) and (5). Hence, for solving (4) and (5) we expand the wave function components in terms of bound states in initial and final channels, and project these equations on these bound states. The expansion of the wave function is given by

\[
\Psi_1(\vec{r}_{23}, \vec{p}_1) \approx \frac{f^{(1)}_{1s}(\rho_1)}{\rho_1} R^{(Z')}_{1s,\mu_1}(|\vec{r}_{23}|)/4\pi
\]

\[
\Psi_2(\vec{r}_{13}, \vec{p}_2) \approx \frac{f^{(2)}_{nlL}(\rho_2)}{\rho_2} R^{(Z)}_{nl,\mu_2}(|\vec{r}_{13}|) \{ Y_L(\hat{r}_1) \otimes Y_l(\hat{r}_1) \}_{00}
\]

where \( nlL \) are quantum numbers of the three-body final-state, \( \mu_1 = m_3m_2/(m_3 + m_2) \), \( \mu_2 = m_3m_1/(m_3 + m_1) \), \( Y_l\)'s the spherical harmonics, \( R^{(Z')}_{nl,\mu_1}(|\vec{r}|) \) the radial part of the hydrogen-like bound-state wave function for reduced mass \( \mu_1 \) and charge \( Z \), \( f^{(1)}_{1s}(\rho_1) \) and \( f^{(2)}_{nlL}(\rho_2) \) the unknown expansion coefficients. This prescription is similar to that adopted in the close-coupling approximation. After a proper angular momentum projection, the set of two-coupled integro-differential equations for the unknown expansion functions can be written as

\[
\left[ (k_1^{(1)})^2 + \frac{\partial^2}{\partial \rho_1^2} - 2M_1 U_{pol}(\vec{p}_1) \right] f^{(1)}_{1s}(\rho_1) = g_1 \sqrt{(2L + 1)} \int_0^\infty d\rho_2 f^{(2)}_{nlL}(\rho_2)
\]

\[
\times \int_0^\pi d\omega \sin \omega R^{(Z')}_{1s,\mu_1}(|\vec{r}_{23}|) \left( -\frac{Z'}{|\vec{r}_{23}|} + \frac{Z}{|\vec{r}_{12}|} - U_C(\vec{p}_2) \right) R^{(Z)}_{nl,\mu_2}(|\vec{r}_{13}|)
\]

\[
\times \rho_1 \rho_2 C^{000}_{L000} Y_{lm}(\nu_2, \pi)/\sqrt{4\pi}
\]

\[
(10)
\]
\[ \left[ (k_{n}^{(2)})^{2} + \frac{\partial^{2}}{\partial \rho_{2}^{2}} - \frac{\mathcal{L}(\mathcal{L} + 1)}{\rho_{2}^{2}} - 2M_{2}U_{\text{C}}(\tilde{\rho}_{2}) \right] f_{nlC}^{(2)}(\rho_{2}) = g_{2}\sqrt{(2\mathcal{L} + 1)} \]
\[ \times \int_{0}^{\infty} d\rho_{1} f_{1s}^{(1)}(\rho_{1}) \int_{0}^{\pi} d\omega \sin \omega R_{nl,\rho_{2}}^{(Z)}(\mid\vec{r}_{13}\mid) \left( -\frac{Z}{\mid\vec{r}_{13}\mid} + \frac{Z}{\mid\vec{r}_{12}\mid} - U_{\text{pol}}(\rho_{1}) \right) \]
\[ \times R_{1s,\mu_{1}}^{(Z')}(\mid\vec{r}_{23}\mid)\rho_{2}\mathcal{C}_{00_{0}}^{(0)}Y_{lm}(\nu_{1}, \pi)/\sqrt{4\pi}. \] (11)

Here \( k_{1}^{(1)} = \sqrt{2M_{1}(E - E_{1s}^{(2)})}, k_{n}^{(2)} = \sqrt{2M_{2}(E - E_{n}^{(1)})} \) with \( M_{1}^{-1} = m_{1}^{-1} + (1 + m_{2})^{-1} \) and \( M_{2}^{-1} = m_{2}^{-1} + (1 + m_{1})^{-1} \), \( E_{n}^{(j)} \) is the binding energy of pair \( (j3) \) and \( g_{j} = 4\pi M_{j}/\gamma^{3} \) \( (j = 1, 2, \gamma = 1 - m_{1}m_{2}/((1 + m_{1})(1 + m_{2})) \). \( C_{lm}^{T_{0_{0}0}} \) the Clebsch-Gordon coefficient, \( L \) the total angular momentum, \( \omega \) the angle between the Jacobi coordinates \( \rho_{1} \) and \( \rho_{2} \), \( \nu_{1} \) the angle between \( \vec{r}_{23} \) and \( \vec{r}_{13} \) and \( \nu_{2} \) the angle between \( \vec{r}_{23} \) and \( \vec{r}_{13} \). To find unique solution to \( (10) \) and \( (11) \), appropriate boundary conditions are to be considered. We impose the usual condition of regularity at the origin \( f_{1s}^{(1)}(0) = 0 \) and \( f_{nlC}^{(2)}(0) = 0 \). Also for the present scattering problem with \( 1 + (23) \) as the initial state, in the asymptotic region, two solutions to \( (10) \) and \( (11) \) satisfy the following boundary conditions

\[ f_{1s}^{(1)}(\rho_{1}) \rho_{1} \sim \sin(k_{1}^{(1)} \rho_{1}) + K_{11}^{nl} \cos(k_{1}^{(1)} \rho_{1}) \] (12)
\[ f_{nlC}^{(2)}(\rho_{2}) \rho_{2} \sim \sqrt{v_{1}/v_{2}} K_{12}^{nl} \cos(k_{2}^{(2)} \rho_{2} - \eta/2k_{2}^{(2)} \ln 2k_{2}^{(2)} \rho_{2} - \pi\mathcal{L}/2) \] (13)

where \( K_{ij}^{nl} \) are appropriate coefficients. For scattering with \( 2 + (13) \) as the initial state, we have the following conditions

\[ f_{1s}^{(1)}(\rho_{1}) \rho_{1} \sim \sqrt{v_{1}/v_{2}} K_{21}^{nl} \cos(k_{1}^{(1)} \rho_{1}) \] (14)
\[ f_{nlC}^{(2)}(\rho_{2}) \rho_{2} \sim \sin(k_{2}^{(2)} \rho_{2} - \eta/2k_{2}^{(2)} \ln 2k_{2}^{(2)} \rho_{2} - \pi\mathcal{L}/2) \]
\[ + K_{22}^{nl} \cos(k_{2}^{(2)} \rho_{2} - \eta/2k_{2}^{(2)} \ln 2k_{2}^{(2)} \rho_{2} - \pi\mathcal{L}/2) \] (15)

where \( v_{i} \) \( (i = 1, 2) \) is the velocity in channel \( i \). The Coulomb parameter in the second transfer channel is \( \eta = 2M_{2}(Z - 1)/k_{n}^{(2)} \) \( [29] \). The coefficients \( K_{ij}^{nl} \) are obtained from the numerical solution of the Faddeev-Hahn-type equations. The cross sections are given by

\[ \sigma_{1s \rightarrow nl}^{tr} = \frac{4\pi(2L + 1)}{k^{(1)2}} \frac{(K_{12}^{nl})^{2}}{(D - 1)^{2} + (K_{11}^{nl} + K_{22}^{nl})^{2}} \] (16)

where \( D = K_{11}^{nl} K_{22}^{nl} - K_{12}^{nl} K_{21}^{nl} \). When \( k^{(1)} \rightarrow 0 \), \( \sigma_{1s \rightarrow nl}^{tr} \sim 1/k_{1}^{(1)} \). The transfer rates are defined by

\[ \lambda_{1s \rightarrow nl}^{tr} = \sigma_{1s \rightarrow nl}^{tr} \nu_{1} N_{0} \] (17)

where \( \nu_{1} \) is the relative velocity of the incident fragments and \( N_{0} \) the liquid-hydrogen density chosen here as \( 4.25 \times 10^{22} \text{ cm}^{-3} \). We note that \( \lambda_{1s}^{tr}(k^{(1)} \rightarrow 0) \sim \text{const} \). In our model the total muon transfer rate is

\[ \lambda_{\text{tot}}^{tr} = \sum_{nl} \lambda_{1s \rightarrow nl}^{tr}. \] (18)
III. NUMERICAL RESULTS

We employ muonic atomic unit: distances are measured in units of $a_\mu$, where $a_\mu$ is the radius of muonic hydrogen atom. The integro-differential equations are solved by discretizing them into a linear system of equations as in Refs. [21,22]. As we are concerned with the low-energy limit, only the total angular momentum $L = 0$ is taken into account. Even at zero incident energy, the transfer channels are open and their wave functions are rapidly oscillating Coulomb waves. A large number of discretization points is needed for achieving convergence.

First Eqs. (10) and (11) are solved setting the polarization potential to zero. The numerical convergence for the system of equations including the elastic and a transfer channel (at a time) is obtained after moderate effort. Finally, the total transfer cross section is calculated by adding the results of different two-channel contributions. In this case we need up to 700 discretization points per channel adequately distributed between 0 and $70a_\mu$. This relatively large number of points is necessary to properly deal with the large final-state Coulomb interaction which could be as large as $17e^2/\rho$ in the case of argon. Some 20 to 30 discretization points per channel would be enough for uncharged fragments in the initial and final state.

Next the calculations are repeated in the presence of polarization potential. It is more difficult to obtain convergence with the polarization potential (7) which is taken to be zero at small distances below the cut off $\Lambda$. In this case to get numerical convergence we have to integrate to very large distances — up to $300a_\mu$. We need up to 2000 discretization points per channel to obtain convergence. More points are employed near the origin and less at large distances. For example, near the origin we take up to 60 equally spaced points per unit length interval $a_\mu$; in the intermediate region ($\rho = 15 - 30a_\mu$) we take up to 8 equally spaced points per unit length interval, and in the asymptotic region ($\rho = 30 - 300a_\mu$) we take up to 6 equally spaced points per unit length interval.

The short-range potential of the present problem extends to about $R = 20a_\mu$. It is customary to take the cut off $\Lambda$ of the polarization potential much larger than the range $R$ of the short-range potential. For $\Lambda < R$, the polarization potential becomes much larger than the short-range potential and for $\Lambda = 0$ it diverges. The results for the cross section and the transfer rates can be very sensitive to the value of the cut off for $\Lambda < R$. However, the polarization potential should be effective in the intermediate region: $\infty > \Lambda > R$. We find that for $\infty > \Lambda > R$, the results for the transfer rates are very weakly dependent on $\Lambda$. We considered the polarization potential in this asymptotic region $\rho_1 > \Lambda \simeq 120a_\mu$. For a variation of $\Lambda$ in this region from $120a_\mu$ to about $160a_\mu$, we find the transfer cross sections to be reasonably constant and the reported transfer rates of this study are the averages of these cross sections. If we increase $\Lambda$ beyond $160a_\mu$ the effect of the polarization potential gradually decreases and transfer rates gradually tend towards the rates calculated without
the polarization potential.

We present partial muon-transfer rates $\lambda_{nl}^{\text{tr}}$ and total transfer rates $\lambda_{\text{tot}}^{\text{tr}}$. We calculate the low-energy muon-transfer rates from $(p \mu)_1s$ and $(d \mu)_1s$ to Ne$^{10+}$, S$^{16+}$, and Ar$^{18+}$. We find in our calculation that in all cases the transfer takes place predominantly to the angular momentum states $l = 0, 1$ of the $n = 6$ orbital of (Ne$\mu$)$^{9+}$, $n = 9$ orbital of (S$\mu$)$^{15+}$, and $n = 10$ orbital of (Ar$\mu$)$^{17+}$. The transition to other states of (X$\mu$)$^{(Z-1)+}$ is negligibly small. Hence in this work we only present muon-transfer rates for the $l = 0, 1$ states of the above orbitals of neon, sulphur, and argon. The low-energy partial rates $\lambda_{1s \rightarrow nl}^{\text{tr}} / 10^{10}$ s$^{-1}$ and total rates $\lambda_{\text{tot}}^{\text{tr}} / 10^{10}$ s$^{-1}$ with and without the polarization potential are presented in Tables I to V together with the results of experimental works. In all cases the partial transfer rates without the polarization potential saturates to a reasonably constant value for $E < 0.01$ eV. A similar behavior is also observed in the presence of the polarization potential. In all cases presented, the rate of transition to the s state of the muonic atomic orbital is larger than that to the p state both in the presence and absence of polarization potential. For example, in the case of Ne$^{10+}$, the muon-transfer rate to the 6s state of (Ne$\mu$)$^{9+}$ is larger than that to the 6p state, and so on.

First we consider the results in Table I for muon transfer from muonic protium $(p \mu)_1s$ to Ar$^{18+}$. In this case we find that a constant value for the transfer rate is achieved for $E \leq 0.04$ eV and we present results up to this energy. The present total transfer rate of $(12.9 \pm 0.4) \times 10^{10}$ s$^{-1}$ is in good agreement with experiments of Refs. [9][11]. However, it disagrees strongly with experiments of Refs. [12][13]. In the case of muon transfer from $(d \mu)_1s$ to Ar$^{18+}$, we find in Table II that the total transfer rate has decreased in this case compared to that in Table I, in agreement with the observed experimental trend [9]. For the rates reported in Table II a constant value is obtained for $E \leq 0.04$ eV. The present total transfer rate of $(5.3 \pm 0.4) \times 10^{10}$ s$^{-1}$ for $(d \mu)_1s$ to argon is in reasonable agreement with the experimental rates of Refs. [14][15].

In the cases of muon-transfer rates from hydrogen isotopes to muonic sulphur and neon reported below in Tables III, IV, and V, a constant value for the rate was not obtained till $E = 0.04$ eV, as in the case of argon above. Hence in these cases we report the results of our calculation for energies 0.01 and 0.04 eV with the experimental energy lying in between. In Table III the results for muon transfer from muonic protium $(p \mu)_1s$ to S$^{16+}$ are shown. The present total transfer rate of $(12.0 \pm 0.4) \times 10^{10}$ s$^{-1}$ is in reasonable agreement with the experimental rate $8.9 \times 10^{10}$ s$^{-1}$ of Ref. [16]. In the case of muon transfer from muonic deuterium $(d \mu)_1s$ to S$^{16+}$, we find from Table IV that the total transfer rate of $(12.7 \pm 0.4) \times 10^{10}$ s$^{-1}$ is also in good agreement with the experimental rate $11.0 \times 10^{10}$ s$^{-1}$ of Ref. [16]. The transfer rate has increased in this case compared to that in Table III for muon transfer from protium in agreement with experimental observation [16]. Finally, In Table V we exhibit muon transfer from muonic deuterium $(d \mu)_1s$ to Ne$^{10+}$. The present total transfer rate of $(8.4 \pm 0.2) \times 10^{10}$ s$^{-1}$ at 0.01 eV is in good agreement with experimental rate $10.1 \times 10^{10}$ s$^{-1}$.
From Tables I to V we find that in most cases the transition rates calculated in the present model without the polarization potential is already in reasonable agreement with experiment. This suggests that the present model based on the close-coupling approximation to coupled Faddeev-Hahn-equations is very suitable for muon transfer reactions with neon, sulphur, and argon. A similar conclusion can be made from our previous study on muon transfer reactions with hydrogen [20], helium, lithium [21], carbon, and oxygen [22]. Although, the effect of polarization in these cases is expected to be large on the observables of the elastic-channel observables at low energies, this effect is not found to be so pronounced on the muon-transfer rates calculated in this work.

IV. CONCLUSION

We have calculated muon-transfer rates from muonic protium and deuterium to bare nuclei neon, sulphur, and argon employing a full three-body quantum-mechanical description of rearrangement scattering by solving the Faddeev-Hahn-type equations [19] using close-coupling approximation. The exact inclusion of the final-state Coulomb interaction in the rearrangement channel (without approximation) guarantees the correct asymptotic form of the wave function in this channel. We also included a polarization potential at large distances in the initial channel beyond a cut off distance $\Lambda$. In these problems the short-range part of the interaction extends to about 20 muonic atomic units ($25a_\mu$). By varying $\Lambda$ in the intermediate range between $120a_\mu$ to $160a_\mu$ we find that the transfer rates are almost independent of the cut off $\Lambda$. This makes the reported transfer rates of this work quasi-independent of cut off. It is shown that in the present approach, the application of a close-coupling-type ansatz leads to satisfactory results for direct muon-transfer reactions from muonic hydrogen to bare nuclei neon, sulphur, and argon. The present rates are in good agreement with experiments [11–16]. Our calculation also reproduces the observed experimental trend of transfer rates to sulphur and argon from protium and deuterium [9]. The transfer rate increases with the decrease of mass of the hydrogen isotope in the case of argon; the reverse is true for sulphur.

It is interesting to note from the works of Refs. [20–22] and the present investigation that a low-order approximation to the Faddeev-Hahn-type equations as considered here produces very good muon-transfer rates from hydrogen isotopes to bare nuclei at low energies. In previous studies it was noted that low-order approximation to these equations also produces very accurate scattering observables at low energies for electron-hydrogen [32] and positron-hydrogen systems [27] in agreement with the variational results. It is well-known that similar low-order approximation to the Schrödinger equation leads usually to poor results at low energies [1,4,31]. Hence low-order approximation to the Faddeev-Hahn-type equations is a very attractive alternative for studying low-energy Coulomb charge-transfer reactions in...
general.

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TABLES

Table I. Low energy partial $\lambda_{1s \rightarrow nl}^{tr}/10^{10}$ s$^{-1}$ and total $\lambda_{tot}^{tr}/10^{10}$ s$^{-1}$ muon transfer rates reduced to liquid-hydrogen density $N_0 = 4.25 \times 10^{22}$ cm$^{-3}$ from muonic protium $(p\mu)_{1s}$ to hydrogen-like excited state of muonic argon $(Ar_{\mu})_{n=10}^{17+}$ together with some experimental results $\lambda^{tr}/10^{10}$ s$^{-1}$.

| Energy (eV) | $(nl)$ | $U_{pol}(\rho_1) = 0$ | With polarization | Experiment |
|------------|--------|-----------------------|-------------------|------------|
|            |        | $\lambda_{1s \rightarrow nl}^{tr}$ | $\lambda_{tot}^{tr}$ | $\lambda_{1s \rightarrow nl}^{tr}$ | $\lambda_{tot}^{tr}$ | $\lambda_{H_\mu+Ar}^{tr}$ |
| 0.04       | 10s    | 5.0 ± 0.2             | 8.1 ± 0.2         | 16.3 [9]   | 12.0 [10] | 14.6 [11] |
|            | 10p    | 3.9 ± 0.2             | 8.9 ± 0.4         | 4.8 ± 0.2  | 12.9 ± 0.4 | 35 [12]   | 98 [13]   |
| 0.06       | 10s    | 5.0 ± 0.2             | 7.9 ± 0.2         |            |           |           |
|            | 10p    | 3.9 ± 0.1             | 8.9 ± 0.3         | 4.6 ± 0.1  | 12.5 ± 0.3 |           |
| 0.1        | 10s    | 4.8 ± 0.1             | 6.8 ± 0.2         |            |           |           |
|            | 10p    | 3.8 ± 0.1             | 8.6 ± 0.3         | 4.1 ± 0.1  | 10.9 ± 0.3 |           |

Table II. Same as in Table I from muonic deuterium $(d\mu)_{1s}$ to muonic argon $(Ar_{\mu})_{n=10}^{17+}$

| Energy (eV) | $(nl)$ | $U_{pol}(\rho_1) = 0$ | With polarization | Experiment |
|------------|--------|-----------------------|-------------------|------------|
|            |        | $\lambda_{1s \rightarrow nl}^{tr}$ | $\lambda_{tot}^{tr}$ | $\lambda_{1s \rightarrow nl}^{tr}$ | $\lambda_{tot}^{tr}$ | $\lambda_{H_\mu+Ar}^{tr}$ |
| 0.04       | 10s    | 1.3 ± 0.1             | 3.4 ± 0.2         |            |           |           |
|            | 10p    | 0.9 ± 0.1             | 2.2 ± 0.2         | 1.9 ± 0.2  | 5.3 ± 0.4  | 8.6 [9]   | 9.4 [14] |
| 0.06       | 10s    | 1.2 ± 0.1             | 3.2 ± 0.2         |            |           |           |
|            | 10p    | 0.9 ± 0.1             | 2.1 ± 0.2         | 1.8 ± 0.2  | 5.0 ± 0.4  |           |
| 0.1        | 10s    | 1.1 ± 0.1             | 2.8 ± 0.1         |            |           |           |
|            | 10p    | 0.8 ± 0.1             | 1.9 ± 0.2         | 1.4 ± 0.1  | 3.2 ± 0.2  |           |

Table III. Same as in Table I from muonic protium $(p\mu)_{1s}$ to muonic sulphur $(S_\mu)_{n=9}^{15+}$

| Energy (eV) | $(nl)$ | $U_{pol}(\rho_1) = 0$ | With polarization | Experiment |
|------------|--------|-----------------------|-------------------|------------|
|            |        | $\lambda_{1s \rightarrow nl}^{tr}$ | $\lambda_{tot}^{tr}$ | $\lambda_{1s \rightarrow nl}^{tr}$ | $\lambda_{tot}^{tr}$ | $\lambda_{H_\mu+S}^{tr}$ |
| 0.01       | 9s     | 6.5                   | 8.2 ± 0.2         |            |           |           |
|            | 9p     | 3.1                   | 9.6               | 3.8 ± 0.2  | 12.0 ± 0.4 | 8.9 [16] |
| 0.04       | 9s     | 6.9                   | 8.4 ± 0.2         |            |           |           |
|            | 9p     | 3.4                   | 10.3              | 4.1 ± 0.2  | 12.5 ± 0.4 |           |
Table IV. Same as in Table I from muonic deuterium \((d\mu)_{1s}\) to muonic sulphur \((S_{\mu})_{n=9}^{15+}\)

| Energy (eV) | \((nl)\) | \(U_{\text{pol}}(\rho_1) = 0\) | With polarization | Experiment |
|------------|--------|-------------------------------|-------------------|------------|
|            | \(\lambda_{1s\rightarrow nl}^{\text{tr}}\) | \(\lambda_{\text{tr}}^{\text{tot}}\) | \(\lambda_{1s\rightarrow nl}^{\text{tr}}\) | \(\lambda_{\text{tr}}^{\text{tot}}\) | \(\lambda_{H_{\mu}+S}^{\text{tr}}\) |
| 0.01       | 9s     | 6.8                           | 7.9 ± 0.2         |            |
|            | 9p     | 4.0                           | 10.8              | 4.8 ± 0.2  | 12.7 ± 0.4 | 11.0      |
| 0.04       | 9s     | 8.7                           | 9.7 ± 0.2         |            |
|            | 9p     | 4.4                           | 13.1              | 4.9 ± 0.2  | 14.6 ± 0.4 |

Table V. Same as in Table I from muonic deuterium \((d\mu)_{1s}\) to muonic neon \((Ne_{\mu})_{n=6}^{9+}\)

| Energy (eV) | \((nl)\) | \(U_{\text{pol}}(\rho_1) = 0\) | With polarization | Experiment |
|------------|--------|-------------------------------|-------------------|------------|
|            | \(\lambda_{1s\rightarrow nl}^{\text{tr}}\) | \(\lambda_{\text{tr}}^{\text{tot}}\) | \(\lambda_{1s\rightarrow nl}^{\text{tr}}\) | \(\lambda_{\text{tr}}^{\text{tot}}\) | \(\lambda_{H_{\mu}+S}^{\text{tr}}\) |
| 0.01       | 6s     | 6.9                           | 8.2 ± 0.2         |            |
|            | 6p     | < 0.1                         | 7.0               | < 0.2      | 8.4 ± 0.2  | 10.1      |
| 0.04       | 6s     | 4.7                           | 6.2 ± 0.2         |            |
|            | 6p     | < 0.05                        | 4.75              | < 0.1      | 6.3 ± 0.2  |