Gravitational Lensing and Gravitomagnetic Time Delay

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We derive the delay in travel time of photons due to the spin of a body both inside a rotating shell and outside a rotating body. We then show that this time delay by the spin of an astrophysical object might be detected in different images of the same source by gravitational lensing; it might be relevant in the determination of the Hubble constant using accurate measurements of the time delay between the images of some gravitational lens systems. The measurement of the spin-time-delay might also provide a further observable to estimate the dark matter content in galaxies, clusters, or super-clusters of galaxies.

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One of the basic confirmations of Einstein general relativity is the measurement of the deflection of path of electromagnetic waves propagating near a mass due to the spacetime curvature generated by a central body \[1, 2\]. To-day this effect is reported to agree with its general relativistic value with accuracy of about 1 part in \[10^4\], by using VLBI, Very Long Baseline radio Interferometry to observe the gravitational deflection of radio waves from quasars and radio galaxies produced by the Sun \[3\]. The VLBI measurement constrains the parameter \(\omega\) of scalar-tensor theories to be larger than about 3500 \[4\]. Depending from the relative position, distance and alignment of source, observer and deflecting mass, several images of the same source may be observed. This phenomenon is the well known gravitational lensing. Several examples of gravitational lensing have been discovered; a well known case is the Gravitational Lens G2237 + 0305, or Einstein Cross, where the light bending produces four images of the same quasar as observed from Earth \[5, 6\].

The other well known general relativistic effect due to the spacetime curvature generated by a central body is the delay in the travel time of electromagnetic waves propagating near the central mass, or Shapiro time delay \[7, 8, 9\]; this effect is to-day tested with accuracy of about \(10^{-3}\) by observing the delay of radio waves propagating near the Sun with active reflection using transponders on the Viking spacecraft orbiting Mars or on its surface.

All these phenomena are due to a static mass and can be derived using the static Schwarzschild metric. If an axially symmetric body has a steady rotation around its symmetry axis an external solution is the stationary Kerr metric. In the weak-field and slow-motion limit, the Kerr metric is, in Boyer-Lindquist coordinates, the Lense-Thirring metric \[2, 11\]. Einstein’s general theory of relativity \[2\] predicts that when a clock co-rotates arbitrarily slow around a spinning body and returns to its starting point, it finds itself advanced relative to a clock kept there at ”rest” (in respect to ”distant stars”), and a counter-rotating clock finds itself retarded relative to the clock at rest \[2, 11\]. Indeed, synchronization of clocks all around a closed path near a spinning body is not possible, and light co-rotating around a spinning body would take less time to return to a fixed point than light rotating in the opposite direction. Similarly, the orbital period of a particle co-rotating around a spinning body would be longer than the orbital period of a particle counter-rotating on the same orbit. Furthermore, an orbiting particle around a spinning body will have its orbital plane ”dragged” around the spinning body in the same sense as the rotation of the body, and small gyroscopes that determine the axes of a local, freely falling, inertial frame, where ”locally” the gravitational field is ”unobservable,” will rotate in respect to ”distant stars” because of the rotation of the body. This phenomenon—called ”dragging of inertial frames” or, ”frame dragging,” as Einstein named it—is also known as Lense-Thirring effect. The Lense-Thirring effect has been observed in the orbits of the LAGEOS satellites \[12\]. In general relativity, all these phenomena are the result of the rotation of the central mass.

However, Einstein’s gravitational theory predicts peculiar phenomena also inside a rotating shell. In a well known paper of 1918, Thirring published a solution of Einstein’s field equation representing the metric inside a rotating shell to first order in \(\frac{\omega}{c}\), mass over radius of the shell, and to first order in \(\omega\), angular velocity of the shell \[13\]. In 1966 Brill and Cohen derived the metric inside a shell with arbitrary mass, this solution is a lowest order series expansion in the angular velocity of the spherical shell on the Schwarzschild background of any mass, valid both inside and outside the shell \[14\]. An extension of the Brill-Cohen results to higher orders on \(\omega\) was then
published in 1985 by Pfister and Braun [14]. However, the exact solution representing the spacetime geometry inside a shell with arbitrary mass and rotating with arbitrary angular velocity is still unknown. In the following we consider only the weak-field and slow-motion metric derived by Thirring.

The gravitomagnetic potential, i.e. the non-diagonal part of the metric in standard isotropic PPN coordinates [1], $h_{0i}$, at the post-Newtonian order, outside a stationary rotating body with angular momentum $J$, is [2]:

$$h^{\text{ext}}(X) \equiv -\frac{2 J \times X}{|X|^3},$$

where $X$ is the position vector. If $J = (0, 0, J)$, in spherical coordinates: $h_{0\phi}^{\text{ext}} \equiv \frac{-2}{r} \sin^2 \theta$. Inside a thin shell of total mass $M$ and radius $R$, with stationary rotation around a Z-axis with small angular velocity $\omega$, we have [2]:

$$h^{\text{int}}(X) = -\frac{4M}{3R} \omega \times X = \left(\frac{4M}{3R} \omega Y, -\frac{4M}{3R} \omega X, 0\right)$$

Let us derive time delay and deflection of electromagnetic waves due to spin and quadrupole moment of the central body; we keep only post-Newtonian terms in the following derivation. Deflection by spin was analyzed in [14, 15], and time delay by spin is considered in [14, 16]. Strong gravitational lensing due to a Schwarzschild black hole is treated in [17]. In the following, the quasi-Cartesian coordinate system $(x, y, z)$ is a system of isotropic coordinates with the z-axis directed towards the observer on Earth, while $(X, Y, Z)$ is the standard PPN system of isotropic coordinates attached to the deflecting body. To relate the coordinates $(x, y, z)$ and $(X, Y, Z)$ we use the standard Euler’s angles ($\phi, \beta, \gamma$). If the deflecting body is symmetric with respect to Z, the shape of the body is invariant for rotations of $\phi$. We can thus choose $\phi = 0$. Since we analyze the behaviour of photons, moving with speed $c \equiv 1$, we neglect the $U^2$ terms in the post Newtonian metric, where $U$ is the Newtonian potential. In the new coordinate system $(x, y, z)$ we then have:

$$ds^2 = (-1 + 2U) dt^2 + (1 + 2U) \delta_{ij} dx^i dx^j$$

$$+ \frac{4J}{r^3} (y \cos \beta - z \cos \gamma \sin \beta) dt dx$$

$$- \frac{4J}{r^3} (x \cos \beta - z \sin \gamma \sin \beta) dt dy$$

$$+ \frac{4J}{r^3} (x \cos \gamma \sin \beta - y \sin \gamma \sin \beta) dt dz$$

in $U$, for simplicity, we have only included mass and quadrupole moment of the deflecting body.

We have chosen the quasi-Cartesian coordinates so that the emitting and the deflecting bodies have the same $x$ and $y$ coordinates, (see fig.1), but a different $z$ coordinate: source, lens and observer are aligned. We have chosen this simple configuration since, in this paper, we are only interested to analyze the time delay due to spin. However, there is an additional time delay, called geometrical time delay [20], due to the different geometrical path followed by different rays. Depending on the geometry of the system, this additional term may be very large and may be the main source of time delay. However, if we compare the time delay of photons that follow the same geometrical path we can neglect the geometrical time delay, as in the case of two light rays with the same impact parameter but on different sides of the deflecting object. For a small deflection angle, the contribution to the travel time delay from the different path length traveled, due to the small deflection, is of the order of $\Delta T \approx 2 \cos \gamma$. Depending on the geometrical configuration considered, this delay may need to be included in the total time delay. In a following paper we shall analyze higher order time delay and compare it with the gravitomagnetic time delay. Here, for simplicity, we neglect any geometrical time delay. Thus, from the condition of null arc length, $ds^2 = 0$, solving with respect to $dt$, we have at the lowest order in $U$ and $g_{0z}$: $dt \approx g_{0z} dz = (1 + 2U) dz$. Thus, by integrating $dt$ from $z_1$ to $z_2$ corresponding, respectively, to the position of source and observer, if $z_1 \approx z_2 \equiv \pm b$ (see fig.1) we get:

$$\Delta T = 2 b + 4M \ln \left(\frac{b}{b}\right) + \frac{4 J \cos(\alpha + \gamma) \sin \beta}{b^2}$$

$$2 M R^2 J_2 \cos 2(\alpha + \gamma) \sin \beta^2$$

In this expression the first term is the time that a radio pulse takes to travel from the source to Earth in the absence of a central mass: $M = 0$; the second term is the Shapiro time delay; the third one is the gravitomagnetic
time delay and the last term is the additional time delay due to the quadrupole moment, $J_2$, of the deflecting body.

Together with the positive spin-time-delay of a counter-rotating photon relative to a co-rotating photon, there is a negative deflection of the path of the counter-rotating photon, and a positive deflection of the co-rotating photon, due to the spin of the lens. This negative deflection gives rise to a negative time delay, due to the decrease in the path travelled, and to a positive time delay due to the increase in the standard Shapiro delay by the mass of the lens due to the decrease of the distance from the central lens. These two additional contributions, negative geometrical delay and positive Shapiro delay, are equal and opposite and cancel out so that the only remaining effect is the positive spin-time-delay of the counter-rotating photon and the negative spin-time-delay of the co-rotating photon.

To write the deflection of electromagnetic waves due to spin and quadrupole moment of the deflecting body we use the geodesic equation in the weak field approximation. When source, deflecting body and observer are aligned, after some calculations we then get [11]:

$$\delta = -\frac{4M}{b} - \frac{4J_1 M R^2 \sin^2 \beta}{b^3}$$

where the first term is the standard deflection by a spherical object of mass $M$, the second term is the deflection by the angular momentum, $J_1$, and the third one the additional deflection due to the quadrupole moment, $J_2$, of the central body.

Let us now study the possibility of measuring the spin-time-delay and determining the angular momentum $J$ of the central deflecting body by measuring total time delay. We consider a simple case in which the source is behind the lens and there are three light rays with the same impact parameter $b$, propagating along one axis. We assume to be able to measure, or determine, the following quantities: total time delay between the three rays, $\Delta T_{12}$ and $\Delta T_{13}$; deflection angles $\delta_1$, $\delta_2$ and $\delta_3$; equatorial radius $R$ of the deflecting body and distances of source and lens from the observer. In this way we are able to determine the angle $\alpha$ for each light beam and the impact parameter $b$ (see fig. [1]), and we can write a system in which the only unknown quantities are: angular momentum, $J$, quadrupole moment, $J_2$, mass, $M$, and Euler’s angle $\beta$ and $\gamma$. Solving this system we can, in principle, determine the time delay due to the angular moment $J$ and the other unknown quantities. Detailed calculations are given in [11]. Of course, for other configurations in which the source is not exactly aligned with lens and observer, the difference in path traveled and the corresponding difference in Shapiro time delay can be the main source of relative time delay; one would then need to model and remove these delays between the different images on the basis of the observed geometry of the system. Nevertheless, in special cases, for example if we observe four images of the source and if the angle $\alpha$ of each deflected ray differs by about $\pi$, such as in the Einstein Cross, we can directly eliminate the time delay due to the quadrupole moment and thus determine the spin-time-delay.

Let us now calculate the time delay due to the spin of some astrophysical sources. For the sun ($M_\odot = 1.477$ km, $R_\odot = 6.96 \cdot 10^8$ km and $J_{2\odot} \approx 1.7 \cdot 10^{-7}$ \textbf{[12]}) by considering two light rays with impact parameters $b \approx R_\odot$ and $-b$, and, for simplicity, with $\gamma = 0$ and $\beta = \frac{\pi}{2}$, the gravitomagnetic and quadrupole-moment time delays, according to [11], are: $\Delta T_{12}^J = \frac{4M \omega^2}{b^2} = 1.54 \cdot 10^{-11}$ sec, and $\Delta T_{12}^{J_2} = 4 \frac{J_2 M R^2}{b^3} = 3.35 \cdot 10^{-12}$ sec. The time delay due to the Sun spin could then, in principle, be measured using an interferometer at a distance of about $8 \cdot 10^{10}$ km, by detecting by gravitational lensing photons emitted by a laser on the side of the Sun opposite the detector, and travelling on opposite sides of the Sun. To derive the time delay due to the lensing galaxy of the Einstein cross \textbf{[13]} we assume a simple model for rotation and shape of the central object. Details about this model can be found in [21]. The angular separation between the four images is about $0.9\arcsec$, corresponding to a radius of closest approach of about $R \approx 650 h_{75}^{-1}$ pc, and the mass inside a shell with this radius $R$ is $\sim 1.4 \cdot 10^{10} h_{75}^{-1} M_\odot$ \textbf{[8]}. Let us assume: $J_2 \approx 0.1$ and $J \approx 10^{23} km^2 h_{75}^{-2}$, we then have $\Delta T_{12}^J = \frac{4M \omega^2}{b^2} \approx 4 \min$, and $\Delta T_{12}^{J_2} = 4 \frac{J_2 M R^2}{b^3} \approx 8 \text{ hr}$. Thus, at least in principle, one could measure the time delay due to the spin of the lensing galaxy by removing the larger quadrupole-moment time delay by the previously described method; of course, as in the case of the Sun, one should be able to accurately enough model and remove all the other delays, due to other physical effects, from the observed time delays between the images. As a third example we consider the relative time delay of photons due to the spin of a typical cluster of galaxies; the precise calculations are shown in a following paper. We consider a cluster of galaxies of mass $M_C \approx 10^{14} M_\odot$, radius $R_C \approx 5$ Mpc and angular velocity $\omega_C \approx 10^{-8} \text{ km s}^{-1}$; depending on the geometry of the system and on the path followed by the photons, we then find relative time delays ranging from a few minutes to several days \textbf{[11]}.

Let us now analyze the time delay in the travel time of photons propagating inside a rotating shell with $\omega = (0, 0, \omega)$. Inside the shell it is not possible to synchronize clocks all around a closed path. Indeed, if we consider a clock co-rotating very slowly along a circular path with radius $r$, when back to its starting point it is advanced with respect to a clock kept there at rest (in respect to distant stars). The difference between the time read by the co-rotating clock and the clock at rest is equal to:

$$\delta T = -\int \frac{g_{00} - g_{0\phi}}{\sqrt{-g}} dx = \frac{8M}{3R} \pi \omega r^2$$

For a shell with finite thickness we just integrate \textbf{[14]} from the smaller radius to the larger one.

Let us now consider a co-rotating photon traveling with an impact parameter $r$ on the equatorial plane of a
galaxy (see fig. 4). The time delay due to the rotation of the external mass for every infinitesimal shell with mass $dm = 4\pi \rho R^2 dR'$ and radius $R' \geq |r|$, is [11]:

$$\Delta t_{dm} = \int_{|r|}^{R'} \int_{r^2 - r'^2}^{R^2 - r'^2} (h_{0x}) dx = \frac{8 dm}{3} \frac{\sqrt{R'^2 - r'^2}}{\rho R'}$$

By integrating the second term in this expression from $|r|$ to the external shell radius $R$, we have:

$$\Delta T = \frac{32\pi}{3} \omega T \int_{|r|}^{R} \rho R' \sqrt{R'^2 - r'^2} dR'$$

This is the time delay due to the spin of the whole rotating mass of the external shell. From this formula we can easily calculate the relative time delay between two photons traveling on the equatorial plane of a rotating shell, with impact parameters $r_1$ and $r_2$.

Finally, let us calculate the time delay corresponding to some astrophysical configurations. In the case of the "Einstein cross" [7], we assume, to get an order of magnitude of the effect, that the lensing galaxy has an external radius $R \simeq 5$ kpc; after some calculations based on the model given in ref. [21], the relative time delay of two photons traveling at a distance of $r_1 \simeq 650$ pc and $r_2 \simeq -650$ pc from the center, using [11] in the case $r_1 \simeq -r_2$, is: $\Delta T \simeq 30$ min. If the lensing galaxy is inside a rotating cluster, or super-cluster, to get an order of magnitude of the time delay, due to the spin of the mass rotating around the deflecting galaxy, we use typical super-cluster parameters: total mass $M = 10^{15} M_\odot$, radius $R = 70$ Mpc angular velocity $\omega = 2 \cdot 10^{-18}$ s$^{-1}$ [22]. If the galaxy is in the center of the cluster and light rays have impact parameters $r_1 \simeq 15$ kpc and $r_2 \simeq -15$ kpc (of the order of the Milky Way radius), the time delay, applying formula [7] in the case $r_1 \simeq -r_2$ and $\rho = \text{const}$, is: $\Delta t \simeq 1$ day.

If the lensing galaxy is not in the center of the cluster but at a distance $r = aR$ from the center, with $0 \leq a \leq 1$ and $R$ radius of the cluster, by integrating [7] between $r = aR$ and $R$, when $r_1 \simeq r_2$ we have: $\Delta t = \frac{32\pi}{3} \omega |r_1 - r_2| \rho (1 - a^2)^{1/2} (1 - 4a^2) R^3$. Thus, if the lensing galaxy is at a distance of 10 Mpc from the center of the cluster, the relative time delay due to the spin of the external rotating mass between two photons with $(r_1 - r_2) \simeq 30$ kpc, is: $\Delta T \simeq 0.9$ day.

Promising candidates to observe time delay due to spin are systems of the type of the gravitational lens B0218+357 [23], where the separation angle between the images is so small that also the delay between the images is very small. In B0218+357 the separation angle between the two images is 335 milliarcsec and the time delay is about 10.5 days. In such systems, the time delay due to the spin of the external mass, or of the central object, might be comparable in size to the total delay. In addition, in the system B0218+357 is observed an Einstein ring the diameter of which is the same as the separation of the images. In such configurations, the Einstein ring can provide strong constraints on the mass distribution in the lens; this, in turn, can be used in order to separate the time delays due to a mass distribution non-symmetrical with respect to the images. The accurate measurement of the delay between the images of some gravitational lens systems is used as a method to provide estimates of the Hubble constant, the time delay due to spin might then be relevant in the corresponding modeling of the delay in the images. The measurement of time delay is possible, in the case of B0218+357, because the source is a strongly variable radio object, thus one can determine the delay in the variations of the images. In this system it is possible to observe clear variations in total flux density, percentage polarization and polarization position angle at two frequencies. For B0218+357 the measured delay is 10.5 ± 0.4 day [23]. Therefore, since the present measurement uncertainty in the lensing time delay is of the order of 0.5 day [23], the gravitomagnetic time delay might already be observable.

In conclusion, we have derived and studied the "spin-time-delay" in the travel time of photons propagating near a rotating body, or inside a rotating shell due to the angular momentum. We found that there may be an appreciable time delay due to the spin of the body, or shell, thus spin-time-delay must be taken into account in the modeling of relative time delays of the images of a source observed at a far point by gravitational lensing. This effect is due to the propagation of the photons in opposite directions with respect to the direction of the spin of the body, or shell. If other time delays can be accurately enough modeled and removed from the observations, the larger relative delay due to the quadrupole moment of the lensing body can be removed, for some configurations of the images, by using special combinations of the observables; thus, one could directly measure the spin-time-delay due to the gravitomagnetic field of the lensing body. In order to estimate the relevance of the spin-time-delay in some real astrophysical configurations, we have considered some possible astrophysical cases. We analyzed the relative time delay in the gravitational lensing images caused by a typical rotating galaxy, or cluster of galaxies. We then analyzed the relative spin-time-delay when the path of photons is inside a galaxy, a cluster, or super-cluster of galaxies rotating around the deflecting body; this effect should be large enough to be detected from Earth. The measurement of the spin-time-delay, due to the angular momentum of the external massive rotating shell, might be a further observable for the determination of the total mass-energy of the external body, i.e. of the dark matter of galaxies, clusters and super-clusters of galaxies. Indeed, by measuring the spin-time-delay one can determine the total angular momentum of the rotating body and thus, by estimating the contribution of the visible part, one can determine its dark-matter content. The estimates presented in this paper are preliminary because we need to analyze the spin-time-delay in the case
of some particular, known, gravitational-lensing images; furthermore, we need to estimate the size and the possibility of modeling other sources of time delay in these known systems. Nevertheless, depending on the geometry of the astrophysical system considered, the relative spin-time-delay can be a quite large effect.

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