Polarization Asymmetry Zero in Heavy Quark Photoproduction and Leptoproduction

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Abstract

We demonstrate two novel features of the sea-quark contributions to the polarized structure functions and photoproduction cross sections, a zero sum rule and a zero crossing point of the polarization asymmetry, which can be traced directly to the dynamics of the perturbative tree-graph gluon-splitting contributions. In particular, we show that the Born contribution of massive quarks arising from photon-gluon fusion gives zero contribution to the logarithmic integral over the polarization asymmetry $\int \frac{d\nu}{\nu} \Delta \sigma(\nu, Q^2)$ for any photon virtuality. The vanishing of this integral in the Bjorken scaling limit then implies a zero gluon-splitting Born contribution to the Gourdin-Ellis-Jaffe sum rule for polarized structure functions from massive sea quarks. The vanishing of the polarization asymmetry at or near the canonical position predicted by perturbative QCD provides an important tool for verifying the dominance of the photon-gluon fusion contribution to charm photoproduction and for validating the effectiveness of this process as a measure of the gluon polarization $\Delta G(x, Q^2)$ in the nucleon. The displacement of the asymmetry zero from its canonical position is sensitive to the virtuality of the gluon in the photon-gluon fusion subprocess, and it can provide a measure of intrinsic and higher-order sea quark contributions.

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Deep-inelastic polarized lepton-polarized nucleon scattering provides a unique testing ground for quantum chromodynamics, challenging our theoretical understanding of hadron structure[1]. The Bjorken sum rule[2] for the non-singlet polarized nucleon structure functions gives a remarkable connection of deep inelastic scattering to the weak axial coupling $g_A$. The prediction that the QCD radiative corrections to the Bjorken sum rule are the inverse of the radiative corrections to the $e^+e^-\rightarrow$ hadrons cross section at their commensurate momentum transfer and energy scales provides a fundamental test of perturbative QCD[3]. There are also many important non-perturbative QCD aspects of the polarized structure functions such as the contributions to nucleon spin from valence and sea quarks[4], the role of polarized gluons arising through the axial coupling anomaly, the influence of quark orbital angular momentum, and the apparently large and negative contribution from the strange sea. It is even possible that the $s$ and $\bar{s}$ momentum and helicity distributions are distinctly different in the nucleon[5].

In this paper we will demonstrate two unexpected features of the sea-quark contributions to photoproduction and leptoproduction cross sections: a zero sum rule and a zero crossing point of the longitudinal polarization asymmetry, which can be traced directly to the dynamics of the perturbative tree-graph gluon-splitting contributions. The vanishing of the polarization asymmetry at or near the canonical position predicted by perturbative QCD provides an important tool for verifying the dominance of the photon-gluon fusion contribution to charm photoproduction for real or virtual photons and for validating the effectiveness of this process as a measure of the gluon polarization $\Delta G(x, Q^2)$ in the nucleon. The displacement of the asymmetry zero from its canonical position is sensitive to the virtuality of the gluon in the photon-gluon fusion subprocess, and it can provide a measure of intrinsic and higher-order sea quark contributions to the polarized structure functions.
It is well known from the work of Burkert and Ioffe\[6\] that the polarized deep-inelastic structure function \(g_1(x, Q^2)\) is an analytic extension into the Bjorken scaling region of the polarized photo-absorption cross section \(\Delta \sigma(\nu, Q^2)\), the same cross section difference that appears for real photons in the Drell-Hearn-Gerasimov (DHG) sum rule\[7\]. The sum rule we discuss here follows from a set of superconvergence relations for the DHG integral\[8\] and provides new insights into the role of polarized gluons and the gluon anomaly contribution from light sea quarks to the Gourdin-Ellis-Jaffe (GEJ) singlet sum rule\[9\]. In particular, we shall show that the Born contribution of massive quarks arising from gluon splitting gives zero contribution to the logarithmic integral over the polarization asymmetry \(\int \frac{d\nu}{\nu} \Delta \sigma(\nu, Q^2)\) for any photon virtuality. The vanishing of this integral then implies in the Bjorken scaling limit a zero gluon-splitting Born contribution to the Gourdin-Ellis-Jaffe sum rule \(\int_0^1 dx g_1^p(x, Q^2)\) from massive sea quarks, in agreement with the triangle graph calculation of Carlitz, Collins, and Mueller\[10\]. It also gives new insights into the factorization scheme-dependence of the interpretation of the gluon anomaly contribution for massless sea quarks emphasized by Bodwin and Qiu\[11\].

The diagrammatic analysis of polarized deep-inelastic scattering starts with the virtual photon-proton polarized asymmetry \(A^{\gamma*N}\). According to the factorization theorem, this can be separated into the convolution of hard and soft pieces:

\[
A^{\gamma* N}(x, Q^2) = A^{\gamma* g}_h \left(x, \frac{Q^2}{\mu^2}\right) \otimes \Delta G(x, \mu^2) + A^{\gamma* q}_h \left(x, \frac{Q^2}{\mu^2}\right) \otimes \Delta q(x, \mu^2),
\]

where \(\mu^2\) is the factorization scale, \(\Delta G\) and \(\Delta q\) are the polarized gluon and quark distributions respectively, and \(A^{\gamma* g}_h\) (\(A^{\gamma* q}_h\)) is the hard part of the polarized photon-gluon (quark) asymmetry. To be more precise, the asymmetries \(A\) are the differences of imaginary parts of forward current-current matrix elements when the photon has the same or opposite helicities to that of the target\[12\].

Although the polarization asymmetry \(A^{\gamma* N}\) is a physical quantity, the identifi-
cation of the individual contributions given in Eq. (1) have an intrinsic ambiguity for light mass sea quarks related to the precise prescription by which the hard parts are defined. Two general regularization schemes have been proposed in the literature: the gauge-invariant scheme, which breaks chiral symmetry, and the chiral-invariant scheme, which breaks gauge invariance. In the gauge-invariant scheme, hard gluons do not contribute to the first moment of $g_1^p(x)[11]$. Since the anomaly corresponds to the quantum breaking of the chiral symmetry which allows for the creation of pairs of definite chirality even in the zero mass limit, a net sea-quark polarization is expected[14]. In this case the anomaly is then identified with the sea quark helicity density inside a gluon.

The chiral-invariant scheme is implemented by imposing a sharp perpendicular quark momentum $k_\perp$ cutoff to separate hard and soft gluon momenta in the photon-gluon asymmetry diagram calculation (see Fig. 1). In this case, the effect of the anomaly from light sea quarks is shifted from the helicity-dependent quark distribution to the hard gluon asymmetry. Then the integral of this hard part is the anomaly, independent of the infrared regulator that has been used to regulate collinear divergences, and the first moment of $g_1^p(x, Q^2)$ contains an anomalous gluon contribution $-(\alpha_s/2\pi)\Delta G$.

In this paper we shall utilize a “light-cone factorization scheme” which is similar to the chiral-invariant scheme, but has a physical, frame-independent parton model interpretation: the quark distributions $q(x, \mu_{\text{fact}})$ are defined from the absolute square of the light-cone Fock state wavefunctions integrated up to invariant mass $\mathcal{M} < \mu_{\text{fact}}[15]$. This can also be interpreted as a cutoff in parton virtuality. For example, a gluon with light-cone momentum fraction $x = k^+/P^+$ in a Fock state of the proton with invariant mass $\mathcal{M}$ has Feynman virtuality $k^2 = x(M_p^2 - \mathcal{M}^2)$. Unlike the sharp perpendicular quark momentum $k_\perp$ cutoff, this definition of the parton distributions
is invariant under Lorentz boosts. The light-cone factorization scheme is however gauge-dependent since the light-cone Fock wavefunctions describe particles defined in the physical $A^+ = 0$ light-cone gauge.

![Photon-gluon fusion diagrams](image)

**Figure 1:** Photon-gluon fusion diagrams.

Let us examine in more detail polarized inclusive $c\bar{c}$ pair production in deep inelastic scattering. The dominant contribution to the partonic subprocess at any photon virtuality is expected to derive from the photon-gluon fusion diagrams $\gamma^* g \rightarrow c\bar{c}$ shown in Fig. 1. The polarized gluon distribution then contributes to the helicity-dependent charm structure functions at order $\alpha_s$, in accordance with the first term of Eq. (1). Charm photoproduction is the basis of proposed experiments at SLAC[16] and HERMES[17] because of its sensitivity to the polarized gluon distribution.

Let us consider the logarithmic integral of the polarized virtual photoabsorption cross section:

$$
\int_0^{x_{\text{max}}} A^{\gamma* g}(x) \, dx \propto \int_{s_{\text{th}}}^{\infty} \frac{ds}{s + Q^2} [\Delta \sigma(s, Q^2)],
$$

(2)
where $\Delta \sigma(s) = \sigma_p - \sigma_A$ is the difference of cross sections for parallel and antiparallel photon-gluon helicities. Here $x_{\text{max}} = Q^2/(Q^2-k^2+4m^2)$, $s_{\text{th}} = 4m^2$, $s = (q+k)^2$, and $q^2 = -Q^2(\leq 0)$ and $k^2(\leq 0)$ are the virtualities of the photon and gluon respectively.

In fact, the Born contribution to the integral is zero when the gluon is on its mass shell $k^2 = 0$, but for any photon virtuality $q^2$. To prove this, consider the process $\gamma a \rightarrow bc$, where $a, b$, and $c$ are arbitrary fields of a renormalizable theory (as long as $a$ carries nonzero spin). Then one can show from the absence of an anomalous moment of particle $a$ at lowest order, that the logarithmic integral of the helicity-dependent part of the photoabsorption cross section must vanish in Born approximation, i.e.

$$\int_{\nu_{\text{th}}}^\infty \frac{d\nu}{\nu} \Delta \sigma_{\text{Born}}(\nu) = 0,$$

where $\nu$ is the photon laboratory energy. This is a remarkable consequence of the Drell-Hearn-Gerasimov sum rule, reflecting the canonical couplings of the fundamental particles in gauge theory. One of its most interesting applications is a novel method for measuring the $W$ magnetic and quadrupole moments to high precision in electron-photon collisions.

In our application of this classical sum rule to polarized deep inelastic scattering, the virtual photon plays the role of the target, and the gluon plays the role of the on-shell photon. Since the photon is spacelike, we change variables from $\nu$ to $s = (p+q)^2$, and analytically continue to negative $q^2$. We then obtain:

$$\int_{s_{\text{th}}}^\infty \frac{ds}{s+Q^2} \Delta \sigma_{\text{Born}}(s, Q^2) = 0,$$

which is exactly the Born heavy quark integral of equation (2). Thus we have shown that the Born heavy-quark integral vanishes not only in the scaling limit, but for any $Q^2 = -q^2$.

The DHG integral of the virtual photon-gluon fusion contribution for massless gluons ($k^2 = 0$) can be calculated explicitly, and in agreement with Eq. (4), the
Born contribution can be shown to vanish for any value of the $Q^2$ and quark mass $m$. The natural physics variable is the velocity $\beta$ of the quark in the CM. If we define $y = 1 - \beta^2 = 4m^2/s$ and $a = Q^2/4m^2$, the DHG integral reads:

$$\int_0^1 \frac{dy}{(1 + ay)^3} \{(1 - ay)L + (3 - ay)\beta\} = 0,$$

(5)

where $L = \log((1 - \beta)/(1 + \beta))$.

The vanishing of the DGH integral also implies that there must be a value of $s = s_0$ and $\beta = \beta_0$ where $\Delta\sigma(s_0) = 0$. In fact, the the polarization asymmetry will reverse its sign for any value of the ratio $a = Q^2/4m^2$. In terms of the Bjorken variable, $x = Q^2/(s + Q^2) = ay/(1 + ay)$, the zero occurs in the range $0 < x < 1/2$ when

$$x_0 = \frac{3\beta_0 + L_0}{4\beta_0 + 2L_0}.$$

(6)

Figure 2 shows the crossing point as a function of $Q^2/4m^2$ for zero gluon virtuality $k^2 = 0$.

The vanishing of the polarization asymmetry at or near the canonical position predicted by perturbative QCD can provide an important tool for verifying the dominance of the photon-gluon fusion contribution to charm photoproduction and for validating the effectiveness of this process as a measure of the gluon polarization $\Delta G(x, Q^2)$ in the nucleon.

Since the gluon entering the fusion process in sea-quark leptoproduction is a constituent of the target hadron, it is always spacelike $k^2 < 0$, and the sum rule is not exactly zero. The virtuality of the gluon can in fact be determined from the measurement of the lepton and heavy pair kinematics. However, a zero crossing point still occurs even for $k^2$ not zero. It can be calculated from the complete virtual photon-
virtual gluon polarization asymmetry: [19, 20]

\[
\mathcal{A}(x, Q^2, K^2) = -\frac{\alpha_s}{2\pi} \sqrt{1 - \frac{4m^2}{x^2}} \left\{ (2x - 1) \left( 1 - \frac{2xK^2}{Q^2} \right) \right.
\times \left[ 1 - \sqrt{1 - \frac{4m^2}{x^2}} \frac{1}{\sqrt{1 - \frac{4x^2K^2}{Q^2}}} \ln \frac{1 + \sqrt{1 - \frac{4m^2}{x^2}} \sqrt{1 - \frac{4x^2K^2}{Q^2}}}{1 - \sqrt{1 - \frac{4m^2}{x^2}} \sqrt{1 - \frac{4x^2K^2}{Q^2}}} \right] + 
\left. \left( x - 1 + \frac{xK^2}{Q^2} \right) \frac{2(1 - \frac{4x^2K^2}{Q^2} - \frac{K^2}{m^2}x(2x-1)(1-\frac{2xK^2}{Q^2})}{(1 - \frac{4x^2K^2}{Q^2} - \frac{K^2}{m^2}x(x-1 + \frac{2xK^2}{Q^2}))} \right\},
\]

where \( K^2 = -k^2 \), and \( s = \frac{Q^2(1-x) - K^2x}{x} \) is the invariant mass squared of the photon-gluon system. The result for the crossing point is shown in Fig. 3 as a function of \( K^2/Q^2 \), for different \( Q^2/4m^2 \) values. The crossing point becomes insensitive to the gluon virtuality for small values of \( Q^2/4m^2 \).

The asymmetry zero and the underlying physics of the fusion process for virtual gluons should be experimentally accessible. Consider the process \( \gamma^* q \rightarrow \bar{c}q' \). Clearly the virtuality of the exchanged gluon can be large since the quark emitting it can recoil at large momentum transfer. This is the regime where the two heavy quarks
and the emitting quark all appear as final state jets at large transverse momentum.

The inclusive deep inelastic cross section is given by an integration $\alpha_s(k^2)dk^2/k^2$ over the gluon virtuality corresponding to the available span of transverse momentum of the emitting quark. The region of small virtuality dominates, with a logarithmic tail extending to the kinematic limits. If this three-jet final state is identified in an experiment, then one would find zero contribution to the GEJ sum rule for events in which the $q'$ has small transverse momentum compared to the mass of the heavy quark $c$. The anomaly contribution $-(\alpha_s/2\pi)\Delta G$ in the scaling region $Q^2 >> 4m_c^2$ thus derives from events where the gluon is off-shell with $-k^2 \gtrsim m_c^2$, i.e., where the quark $q'$ recoils with transverse momentum of order $m_c$ or larger. There is no anomalous contribution for $-k^2 \ll m_c^2$. The physics of the asymmetry zero thus reflects the basic spin dynamics of the $\gamma^* g \to c\bar{c}$ process, and it is connected to basic principles underlying the gluon anomaly.

The asymmetry zero should also be a measurable effect in the case of the polarized photon structure function. Consider $\gamma^*\gamma \to Q\bar{Q}$. The asymmetry zero in the polarized
photon-photon cross section has the same expression as given before in terms of $Q^2$ and the heavy quark pair mass. The target photon can be polarized using a back-scattered laser beam, and the virtual photon polarization tracks with that of the scattered electron.

Since our analysis applies only to the extrinsic contribution arising from photon-gluon fusion, the displacement of the asymmetry zero from its canonical position can provide a test for the presence of non-perturbative contributions to the charm photon- and electroproduction cross sections. For example, the multiply-connected (intrinsic) contributions[21] to the charm structure function may give a non-zero contribution to the first moment of $g_1^p(x)$ and displace the crossing point predicted by the photon-gluon fusion contributions. Although the intrinsic charm probability in the nucleon is of order 0.6%[22], the intrinsic contribution can dominate the charm structure function at large momentum fraction $x_{BJ} > 0.2$ or near threshold. The suppression of the intrinsic bottom quarks is larger. On the other hand, there could be substantial intrinsic effects in the case of strange quarks[19, 5].

We have thus demonstrated two unexpected features of the sea-quark contributions to the polarized structure functions, a zero sum rule and a zero crossing point, which can be traced directly to the dynamics of the perturbative tree-graph gluon-splitting contributions. In particular, we have shown that the Born contribution of massive quarks arising from the photon-gluon fusion subprocess gives zero contribution to the logarithmic integral over the polarization asymmetry $\int \frac{d\nu}{\nu} \Delta\sigma(\nu, Q^2)$ for any photon virtuality. The vanishing of this integral in the Bjorken scaling limit then implies a zero gluon-splitting Born contribution to the Gourdin-Ellis-Jaffe sum rule for $\int_0^1 dx g_1^p(x, Q^2)$ from massive sea quarks as long as the gluon virtuality can be neglected. If the gluon virtuality $\langle k^2 \rangle$ is small compared to the quark mass, the corrections will be of order $\langle k^2 \rangle/m^2$. The displacement of the asymmetry zero from
its canonical position in photon energy provides a measure of intrinsic and higher order sea quark contributions, as well as the virtuality of the gluon in the photon-gluon fusion subprocess.

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