Locating strongly coupled color superconductivity using universality and data for trapped ultracold atoms

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Abstract

Cold fermionic atoms are known to enter the universal strongly coupled regime as their scattering length $a$ gets large compared to the inter-particle distances. Recent experimental data provide important critical parameters of such system. We argue that quarks may enter the same regime due to marginal binding of diquarks, and if so one can use its universality in order to deduce such properties as the slope of the critical line of color superconductivity, $dT_c/d\mu$. We further discuss limitations on the critical temperature itself and conclude that it is limited by $T_c < 70 \text{ MeV}$.

1. Color superconductivity (CS) in dense quark matter in general appears due to attractive interaction in certain scalar diquark channels. More specifically, three mechanisms of such attraction were discussed in literature are: (i) the electric Coulomb interaction (see e.g. [1]) (ii) instanton-induced ’t Hooft interaction [2,3] and the magnetic interaction [4].

Unlike electrons in ordinary superconductors (for which the driving attraction is a complicated exchange of collective excitations like phonons), quarks naturally may have color charges leading to their mutual electric attraction. However this straightforward mechanism of CS is significantly weakened by the electric screening, at the “Debye mass” scale $M_D \sim g\mu$. It thus leads to rather weak pairing, with the gaps $\Delta \sim 1 \text{ MeV}$ [1].
The second mechanism of pairing is due to the *instanton-induced* forces between light quarks, also known as the ’t Hooft interaction. It is related to fermionic zero mode of instantons and thus it is very flavor-dependent. In particularly, it only exists for two quark of different flavors (such as $ud$, $us$ and $ds$). Therefore the simplest setting in which it was considered is the 2-flavor (no $s$-quark) problem [2,3], for 3-flavor case see [5]. The instanton mechanism have re-activated the field, because it have demonstrated for the first time that large (and thermodynamically significant) pairing may actually appear in cold quark matter. Since forces induced by small-size instantons are not affected by screening that much, they can be much stronger than (i), leading to gaps about 2 orders of magnitude larger $\Delta \sim 100 \text{MeV}$ [2,3].

However such gaps are rather uncertain because they are so large that they may fall outside of applicability range of the usual BCS-like mean field theory. Unfortunately, an analytic theory of strong pairing is still in its infancy. The main idea of this letter is to get around this “calculational” problem by using the universality arguments and an appropriate atomic data.

Magnetic mechanism of pairing (iii) has been pointed out by Son [4]: it dominates at very high densities ($\mu > 10 \text{GeV}$) when two others get strongly screened. It leads to gaps $\Delta \sim \mu \exp\left(-\frac{3\pi^2}{\sqrt{2}g(\mu)}\right)$, large in absolute magnitude but small relatively to $\mu$.

Different colors and flavors of quarks lead to to multiple possible condensates, and indeed color superconductivity may exist in many different phases depending on $T, \mu$ and quark masses. In particular, they may have chiral and translational symmetries being either preserved or broken. We will not go into this vast subject: the reader for definiteness may think about a single $ud$ scalar condensate, or the so called 2SC phase. Its symmetries are similar to the electroweak part of the Standard Model, with the fundamental color representation of the $ud$ condensate, breaking the color group and making 5 gluons massive and leaving 3 (of the unbroken SU(2)) massless.

The estimates for the *maximal pairing* possible are obviously very interesting for applications. One of them is a possibility that a superconducting quark matter is present at the central region of compact “neutron” stars. Another is a possibility that either a region of the CS phase on the phase

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1. Additional argument [2] why one should believe such large gaps: the *same* interaction but in $\bar{q}q$ channel is responsible for chiral symmetry breaking, producing the gap (the constituent quark mass) as large as 350-400 MeV. Furthermore, in the *two-color* QCD, the so called Pauli-Gursey symmetry relates these two condensates directly.
diagram, or at least a region where diquarks are bound can be reached via heavy ion collisions. In both cases the issue of the gap magnitude (or critical temperature) is crucial for making those possibilities real.

2. Let us now introduce the main idea of this letter. The interaction (scattering) of two quarks is maximally enhanced (to its unitarity limit) if there is a marginal state in the diquark channel, a bound state with near-zero binding or a virtual state at small positive energy. In atomic systems such situations are generically called a “Feshbach resonance”, tuned by external magnetic field. Transition between these two possibilities is reflected in specific behavior of the condensation, known as BEC-to-BCS transition. In the middle, with resonance at exactly zero binding, the interaction is at its maximum, limited by unitarity for the relevant partial wave. (It is s-wave with $l=0$ in all cases considered.)

In quark-gluon plasma the existence of marginally bound states of quarks and gluons and their possible role in liquid-like behavior at RHIC has been pointed out in [6]. For any hadronic states one can argue that they dissolves at large $T$ or $\mu$, and thus existence of some lines of zero binding on the phase diagram are unavoidable. The issue was so far studied only for small $\mu$ and high $T$ relevant for RHIC heavy ion program: and indeed there are theoretical, lattice and experimental evidences that e.g. charmonium ground state does not melt till about $T \sim 400$ MeV [7, 8]. One can also follow discussion of such lines for glueball and light quark mesons in [7], charmed mesons in [9], baryons and multi-gluon chains in [10].

In this letter we explore consequences of the idea that there is a diquark marginal binding line, more or less trailing the phase boundary line on the phase diagram. If this is the case (which cannot be proven at this time), it must cross the CS critical line, separating the CS region into a BCS-like and BEC-like. (For general information on BEC-BCS transition and its possible presence in quark matter see e.g. a lecture [11].)

In Fig.1(a) we show schematic phase diagram of QCD, in coordinates baryonic chemical potential $\mu$ (per quark)- temperature $T$. Lines of zero binding$^2$ of a $qq$ pair [6] starts at $\mu = 0$ at the temperature $T_{qq}$ very close to the critical line $|T_{qq} - T_c| \ll T_c$: the reason for that is that effective color attraction in $qq$ state is only 1/2 of that in mesons such as charmonium. The

$^2$Similar “curves of marginal stability” (CMS) of certain states are known in various settings: e.g. they have an important role in confinement of supersymmetric gauge theories [12].
lines associated with (color triplet) diquark $qq_3$ are the lower (blue) dashed and dash-dotted lines: below the former diquark binding is below zero and above the later it does not exist at all, even as a Cooper pair.

3. How can one relate atomic and quark systems, if at all? The way it can be done is due to the so called “universality” of the system. As the scattering length gets large $a \to \infty$, it cannot enter the answers any more. As a result, there remain so few parameters on which the answer may depend³ on that those can be absorbed by selection of the appropriate units.

Atomic 50-50 mixture of two “spin” states is characterized by the density $n$ and the atomic mass $m$. Quantum mechanics adds $\hbar$ to the list of possible quantities, and so one has only 3 input parameters, which can be readily absorbed by selecting proper units of length, time and mass. Thus the pressure (or mean energy) at infinite and zero $a$ can only be related by some universal numerical constant $p_\infty/p_0 = (1 + \beta)$. We do not know how to get its value from any theory (other than quantum Monte-Carlo simulations or other brute force methods), but it has been measured experimentally (e.g. by the very size of the trapped system). The same should hold for transport properties: e.g. viscosity of such universal gas can only be $\eta = \hbar n \alpha_\eta$ where $\alpha_\eta$ is some universal coefficient [14]. Similarly, the critical temperature must be simply proportional to the Fermi energy

$$T_c = \alpha_T E_F$$

with the universal constant $\alpha_T$.

Experimental progress in the field of strongly coupled trapped fermionic atoms is quite spectacular; unfortunately this author is certainly unqualified to go into its discussion. Let me just mention one paper, which killed remaining doubts about superfluidity: a discovery by the MIT group of the quantized vortices, neatly organized into the usual lattice [13]. We will however need only the information about the value of the transition temperature. Duke group lead by J.E.Thomas (Kinast et al) have for some time studied collective vibrational modes of the trapped system. Their frequencies in strong coupling regime are well predicted by hydrodynamics and universal equation of state, with little variation with temperature. Their dampings however

³The next parameter of scattering amplitude, the effective range, is about 3 orders of magnitude smaller than interparticle distance for trapped atoms, and thus completely irrelevant. Although similar parameter for quarks are not that small, we will assume it does not affect the universal results too much.
show significant $T$-dependence: in fact Kinast et al. [16] have found two distinct transitions in its behavior. The lowest break in damping is interpreted as the phase transition to superfluidity, it corresponds to

$$\alpha_{T_c} = \frac{T_c}{E_F} = .35$$ (2)

where $E_F$ stands for the Fermi energy of the ideal Fermi gas at the center of the trap. In another (earlier) set of experiments [15] there has also been found a change in the specific heat, at $\alpha_{T_c} = .27$. In spite of some numerical difference between these two values, the Duke group indicates that both are related to the same phenomenon$^4$. At another temperature

$$\alpha_2 = \frac{T_2}{T_F} \approx 0.7 - 0.8$$ (3)

the behavior of the damping visibly changes again. Kinast et al interpret it as a transition to a regime where not only there is no condensate of atomic pairs, but even the pairs themselves are melted out.

4. There are of course important differences between quarks and atoms. First, quarks have not only spin but also flavor and color, so there are $3 \times N_f$ more Fermi surfaces. However, in the first approximation one may focus only at one pair of them (say u-d quarks with red-blue colors) which are actually paired. Second, atoms are non-relativistic while quarks are in general relativistic and in matter may have some complicated dispersion laws. Since the gaps are large, one cannot use a standard argument that close to Fermi surface only Fermi velocity is important. Nevertheless, we will assume that quark quasiparticles have dispersion laws which can be approximated by a simple quadratic form

$$E(p) = M_1 + \frac{p^2}{2M_2} + ...$$ (4)

where dots are for $O(p^4)$ terms we ignore. In matter $M_1$ and $M_2$ need not be the same. It then implies that e.g. the relation between the critical $T$ and chemical potential should read

$$T_c = \alpha_{T_c}(\mu/3 - M_1)$$ (5)

$^4$They do not provide any error bars on the value of $T$, as the absolute value of the temperature is obtained in rather indirect calibration procedure. The reader may take the spread of about 20% as an estimate of uncertainties involved.
and similarly for the second point. (The factor 3 appears because baryon number is counted per baryon, not quark.)

The $M_2$ can be used to set the units as explained above, and we will not actually need its value. The $M_1$ is needed, but since it makes a simple shift of the chemical potential, it can be eliminated by differenciation. Thus we get a prediction of the slope of the critical line

$$d\alpha_T/c/3 \approx 0.1$$

The intersection of CMS for diquarks with the SC critical line (the point S (strong) in our phase diagram Fig.1(a)) should thus be at this line, at which the boundary of superconducting phase crosses with the zero energy of the bound state. The second critical point associated with disappearance of pairs (identified with the (blue) dash-dotted line and point D in Fig.1(a)) should thus be at the line with the slope $\alpha_2$.

In order to plot the line on the phase diagram one needs the value of the $M_1$, which unfortunately is not known. To set the upper bound one may simply take $M_1 = 0$ and draw two straight lines pointing to the origin, see Fig.1(b). As $M_1$ grows, the lines slide to the right, as is shown by another line with a (randomly chosen) value $M_1 = 100 \text{MeV}$.

Finally, we turn to “realistic” phase diagram, with numerical values extracted from experiment. We know for sure that matter is released at the so called chemical freezeout lines indicated by points in Fig.1(b). These points are the ends of adiabatic cooling paths. Unfortunately we do not know how high above them those curves start at a given collision energies. However it is general expected that this line more or less traces the critical line, being few MeV below it, into the hadronic phase. One may then conclude that the upper limit on $T_c$ of CS is about 70 MeV (intersection of the upper solid lines with the freezeout line). The disappearance of pairing (dashed lines) is thus expected below $T_2 = 150 \text{MeV}$.

Finally, comparing these results with theoretical expectations and experimental capabilities, we conclude that (i) if there is a strongly coupled CS, its critical temperature should definitely be below $T_c < 70 \text{MeV}$. This maximal value is amusingly close to what was obtained from the instanton-based calculations $[2, 3]$; (ii) it is unlikely that any heavy ion collisions can reach the CS domain, even at the maximal coupling. A penetration into the region $T = 100 - 150 \text{MeV}, \mu = 500 - 600 \text{MeV}$ in which non-condensed bound diquarks may exist, is however quite likely, both in the low-energy RHIC runs and in future GSI facility FAIR; (iii) if that happens, one may think of some
further uses of universality, e.g. about relating the transport properties in both systems. One may in particularly ask whether the universal viscosity extracted from vibrations of trapped atoms (like that in [14], but at appropriate $T$) can or cannot describe the hydrodynamics of the corresponding heavy ion collisions.

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Figure 1: (a) Schematic phase diagram for QCD, in the plane baryon chemical potential - temperature. M (multifragmentation) point is the endpoint of nuclear gas-liquid transition. E is a similar endpoint separating the first order transition to the right from a crossover to the left of it. (Black) solid lines show phase boundaries, dashed lines are curves of marginal stability of indicated states. Two dash-dotted straight lines are related with bounds from atomic experiments we discuss in the text, they intersect with unbinding of diquark Cooper pairs (D) and most strongly coupled point (S), which is at the maximum of the transition line ans is also a divider between BCS-like and BEC-like color superconductor. (b) Compilation of experimental data on chemical freezeout parameters from different experiments according to [17]. The squares and circles are for fits at mid-rapidity and all particles, respectively. Two solid lines are the phase transition lines with the quark effective mass $M_1 = 0$ and $100\ MeV$, two dashed lines show pair unbinding lines for the same masses.