A model for inter-laminar shear stress in laminated composites

W Zaki, V Nguyen and R Umer
Khalifa University, 127788 Abu Dhabi, UAE
wael.zaki@kustar.ac.ae

Abstract. The paper presents an analytical model for the estimation of shear stress at the interface of adhesively bonded layers in laminated composites. For this purpose, a new shear stress function is proposed that accounts for the influence of shear lag in laminates assembled using adhesive layers of different types and thicknesses. In addition to the estimation of interfacial stress, the function can be used to determine the difference in normal strains and axial displacements in adjoining layers.

1. Introduction
Composite laminates are used in a multitude of engineering applications because of the ability to achieve superior and directionally prevalent mechanical performance per unit weight compared to bulk materials [1]. In this regard, a key factor governing the overall mechanical performance of laminates is the efficiency of load transfer at the interface of adjoining layers, which is assumed to be perfect by conventional laminate theories. In reality, the transfer efficiency is affected by the development of interlaminar shear stress in the interface between these layers [2]. This shear stress is accounted for in this paper by means of a newly developed stress function. The parameters of the function are determined by fitting against numerical simulation results using finite element analysis. Experimental validation is then carried out for the case of a 3-layer adhesively bonded aluminium laminate.

2. Materials and methods
The model for interlaminar shear proposed in this work is derived based on the shear lag model (SLM) [3,4], which describes stress distribution and mechanical properties of the interface [5,6]. The model is developed for a laminate consisting of \( n \) metallic sheets, as shown in figure 1, each of width \( a \), thickness \( t \), and length \( 2L \). The metal sheets are bonded by means of adhesive layers of matching width and length, of thickness \( s \).

![Figure 1. Adhesively bonded laminate.](image-url)
The variation of the average normal stress $\sigma_{a,m}$ in the longitudinal direction $x$ of layer $m$ of the adhesive, where $m = 1$ to $n$, is related to the shear flow by means of the equation

$$\frac{d\sigma_{a,m}}{dx} = \frac{\tau_k - \tau_{k-1}}{t} = \frac{\tau_{m-1} - \tau_m}{t}$$

where $k = m - 1$ is the interface number.

The shear stress is proposed to be related to the difference in average normal strain $\epsilon_a$ in layers $m - 1$ and $m$ as follows:

$$\tau_k = f(E_a) \cdot (\epsilon_{a,m-1} - \epsilon_{a,m})$$

where $f(E_a)$ is a function of the Young modulus $E_a$ of the adhesive, which is assumed here to have the following expression:

$$f(E_a) = A \cdot e^{B \cdot E_a} + C \cdot e^{D \cdot E_a}$$

in which $A, B, C, D$ are scalar parameters that depend on the properties of the laminate. This expression of $f(E_a)$ is based on observations made in earlier work [2].

Once the parameters of the stress function $f(E_a)$ are determined, the distribution of shear stress in interface $k$ can be found from the difference in normal strain $(\epsilon_{a,m-1} - \epsilon_{a,m})$. Moreover, if one side of the laminate is fixed, the average axial displacement of the tip in each layer $m$ is obtained by the integral

$$u_{a,m} = \int_0^L \epsilon_{a,m} dx$$

### 2.1. Characterization of the parameters of the stress function

The general procedure for determining the parameters of the function $f(E_a)$ consists in fitting the analytical expression of the function to results of numerical simulation of the laminates. These results give access to numerical estimates of $f(E_a)$. As an example, finite element analysis of a 3-layer aluminium laminate with the parameters listed in Table 1 was carried out for adhesives with Young’s moduli ranging from 0.9 to 5 GPa.

| Parameter | Value |
|-----------|-------|
| $L$       | 25 mm |
| $t$       | 1 mm  |
| $s$       | 0.05 mm |

In each case, the average tip displacement of the laminate was calculated and fitted to its expression in terms of $f(E_a)$, resulting in the values of $A, B, C, D$ listed in Table 2.

| Parameter | Value          |
|-----------|----------------|
| $A$       | 21.878 GPa     |
| $B$       | 0.004969 GPa$^{-1}$ |
| $C$       | -18.43 GPa     |
| $D$       | -1.779 GPa$^{-1}$ |

A plot of the fitted function versus numerical simulation results is shown in figure 2.
In this case, very good accuracy is achieved with a maximum relative error not exceeding 2% in magnitude (figure 3).

2.2. Experimental validation
Experimental validation was carried out for the case of 3-layer AA 1100 H-14 aluminum laminates, bonded using the Cytec PRISM EP2400 thermosetting epoxy resin. The aluminum layers consisted of plates of thickness $t = 0.9$ mm. The surfaces of the plates were abraded to uniform roughness resulting in a uniform mat appearance, then cleaned using alcohol and water in preparation for the application of the adhesive layer. The adhesive was then deposited uniformly on the surfaces to be joined, and the layers were clamped to avoid side-ways sliding. The resin was cured at 178 C for two hours then cooled and maintained at room temperature for 24 hours. The resulting Young modulus for the cured resin was $E_a = 3.4$ GPa according to manufacturer’s specifications. The laminate was finally loaded in an Instron 5969 UTM equipped with a 50 kN load cell and the tip displacement measured using extensometers of gauge lengths 25, 50 and 75 mm.
As shown in Table 3, the experimental measurements are predicted to good accuracy by the analytical model, using the fitted stress function reported earlier.
3. Conclusion
A model for shear stress at the interface of adhesively bonded laminates was proposed based on SLM. The model introduces a novel analytical stress function to relate the shear stress that develops in an interface to the difference in axial strain, and therefore the elongation, in adjoining layers. The function is assumed to present exponential dependence on the Young modulus of the adhesive and features 4 scalar parameters that can be determined by fitting to numerical simulation results. Experimental validation of the proposed model was successfully carried out for the case of a 3-layer aluminum laminate bonded using a thermosetting resin. The work contributes to improving our understanding of the properties and behavior of adhesive interfaces and presents strong potential for application by simplifying the design and analysis of adhesively bonded composite laminates.

4. References
[1] Marimuthu R, Sundaresan M.K and Rao G.V 2003 J. Inst. Eng. 32–38.
[2] Viet N.V, Wang Q and Kuo W.S 2012 Compos: Part B 43 332.
[3] Jiang G and Peters K 2008 Int. J. Solids. Struct. 45 4049.
[4] Nairn J.A 2005 Adv. Compos Lett 06.
[5] Gao X.L and Li K 2005 Int. J. Solids. Struct 42 1649–67.
[6] Her S.C, Liu S.J 2014 Eng. Comput. 353–364.

Acknowledgements
Dr. Wael Zaki would like to acknowledge the financial support of Khalifa University through KUIRF research grant no. 210114.