Interaction and Identification of the Doubly Heavy Di-Hadronic Molecules

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Abstract

We study the interesting problem of interaction and identification of hadronic molecules seems to be deuteron- like structure. In particular we proposed binding mechanism mostly as One Boson Exchange potential plus screen Yukawa like potential in relative S-wave state and revisited Weinberg’s model-independent approach to distinguish the molecule from confined (elementary) state. We predicts some dimesonic, dibaryonic and meson-baryon molecular states involving heavy quarks (c, b or η, δ) namely $D D^*$, $D_0 D^*$, $D^{*\pm} D_1^0$, $\Sigma_c \Sigma_c$, $\Sigma_c \Sigma_c$, $\Sigma_c \Sigma_c$, $\Sigma_b \Sigma_b$, $\Sigma_b \Sigma_b$ with their possible quantum numbers.
Since a few decades, tremendous efforts had been made theoretically as well as experimentally for searches of the hadronic molecules. Apart from Deuteron, we still waiting for such strong molecular candidates which also expected in the fundamental theory (QCD). In the last few years there have been several experimental discoveries of manifestly narrow exotic resonances X(3872) \[1\], Z(4430)\(^+\) \[2\], Y(4260) \[3\], Z\(_b\)(10610)/(10650) \[4\], P\(_c\)(4450) \[5\] and many more. All these narrow resonance lie very close to the two hadron threshold, seems to be deuteron-like systems. The scanning of the internal structure of these resonances and looking towards molecular interpretation, one needs to know two prerequisite (i) interaction between hadrons (ii) identification as a composite state from bare fundamental state.

As a hadronic molecule, deuteron has been well-established and studied state \[6-9\]. It is known for the bound state of proton and neutron. Thus, deuteron being healthy model for the study and predictions of hadronic molecules. In general for such bound state, genuinely, a simple question arises- How and why two hadrons make a bound state? According to definition of molecule, a sufficient attractive potential strength needed to have a bound state. These strength will parameterized by an effective coupling constant. Indeed, in the relation to the first question, ‘how’- various realistic potentials like full-Bonn (CD-Bonn), Nijmegen-Group of, Paris-Group of potential etc. had been developed (see Ref.\[8\] for review). The exchange of particles had basis for all these realistic potential and to be known as a One Boson Exchange potential, exploring the initial idea of Yukawa’s pion exchange for explaining the nuclear force. In OBE, the long, mid and short distance interaction have been introduced through exchange of meson and range of the force depends on mass of the exchange mesons. To give precise answer for second question that why two color neutral hadron make bound system being difficult. Because, it requires very elegant knowledge of fundamental interaction at very short distance where the OBE has limitation to explain such interaction. We propose the residual dipole like interaction between two color neutral hadrons. The uneven color charge field distribution between two hadrons tend to residual dipole effect. The screening of color charge field tends to vacuum polarization and creates quark anti quark pairs. Let us note some points (i) the strength of this screen type interaction must carried the asymptotic behavior as per QCD (ii) the strength of the interaction increases tends two hadronic states to bare confinement state (iii) the attraction and repulsion depending on respective aliment of the residual dipoles in spin-isospin space. Thus the coupling with hadrons to the created quark antiquark pair becomes spin-isospin dependent.
Thus the condition for existence for the molecular system are: (i) the kinetic energy of the system must be less i.e. the hadrons should be heavy enough to get bound state (ii) the two hadron should carry lowest orbital angular momentum \((l=0)\) (iii) molecular state should be loosely bound (i.e. below threshold) and narrow. We emphasize that the S-wave OBE plus screen type potential could apply, in principle, for binding mechanism between two hadrons. As we mentioned, the second prerequisite or challenge is identification of the molecular state form the bare elementary state.

In the Sixties, Weinberg \[10\] suggested in a sophisticated way that the deuteron were a composite particle. In his novel work, he try to show an elegant model-independent way to identified whether a particle is a bare elementary state or a composite state. The conclusion was based on a generalization of Levinson’s theorem which gives the formulas for scattering length \(a_s\) and effective range \(r_e\) in terms of \(Z\), where \(Z\) is the ”field renormalization” constant \[10\],

\[
a_s = \left[2(1 - Z)/(2 - Z)\right] R + \mathcal{O}(1/\beta)
\]

\[
r_e = \left[-Z/(1 - Z)\right] R + \mathcal{O}(1/\beta)
\]

Where \(R \equiv \sqrt{2\mu\epsilon}\), \(\epsilon\) is the binding energy and \(\mu\) is the reduced mass of the composite system. The \(\mathcal{O}(1/\beta)\) is the range of the force and could be calculated if one know the information of the interaction well. In order to determine the particle as a bare elementary or in composite state he argued that the renormalization constant \(Z\) takes the value \(0 \leq Z \leq 1\). If \(Z=0\) then the particle is in a pure composite state while for \(Z=1\) it become purely elementary. This argument had been discussed by other authors \[11, 12\] previously and followed by Weinberg \[10\]. For the case \(Z=0\) (the deuteron as a composite particle) the Eq(1) becomes \(a_s = R\) and \(r_e = \mathcal{O}(1/\beta)\) which is in agreement with the experimental vales : \(a_s = +5.41\) fm, \(r_e = +1.75\) fm. In addition he were made remarked for such investigation as \[10\]; (i) the composite particle must couple to a two-particle channel with threshold not too much above the composite particle mass (ii) composite particle must be in a S-wave \((l=0)\) (iii) composite particle must be stable. In the case of deuteron all these three prerequisite are all most satisfied. But deuteron consist lightest baryons preserves stability. Usually in other cases one could get (i) and (ii) satisfied. V. Baru et. al. \[13\] have also applied this formalism to study the state \(a_0(980)\) and \(f_0(980)\) in the light meson sector.

In the present study, our aim is to draw attention on some interesting possibilities for molec-
ular (deuteron-like) structure with proposed interactions and identification with observables. Thus we have approximate interaction potential as discussed and revisited Weinberg’s approach in order to determine the composite state from bare state. Let us describe the interaction potential in terms of the well defined One Boson Exchange (OBE) potential and phenomenological attractive screen Yukawa-like potential. The light mesons under consideration for the OBE Potential are as follows: Pseudoscalar meson \((ps) = \pi, \eta\), Scalar meson \((s) = \sigma, \delta\) and Vector meson \((v) = \omega, \rho\). The OBE potential is just the sum of the all one meson exchange, namely

\[
V_{OBE} = V_{ps} + V_s + V_v
\]

where the individual one meson exchange interaction potential expressed as (in s-wave)

\[
V_{ps} = \frac{1}{12} \left[ \frac{g_{\pi qq}^2}{4\pi} \left( \frac{m_\pi}{m} \right)^2 e^{-m_\pi r_{ij}} \frac{r_{ij}}{r_{ij}} \left( \tau_i \cdot \tau_j \right) + \frac{g_{\eta qq}^2}{4\pi} \left( \frac{m_\eta}{m} \right)^2 e^{-m_\eta r_{ij}} \frac{r_{ij}}{r_{ij}} \left( \sigma_i \cdot \sigma_j \right) \right] \tag{3}
\]

\[
V_s = -\frac{g_{\sigma qq}^2}{4\pi} m_\sigma \left[ 1 - \frac{1}{4} \left( \frac{m_\sigma}{m} \right)^2 \right] e^{-m_\sigma r_{ij}} r_{ij} + \frac{g_{\delta qq}^2}{4\pi} m_\delta \left[ 1 - \frac{1}{4} \left( \frac{m_\delta}{m} \right)^2 \right] e^{-m_\delta r_{ij}} r_{ij} \left( \tau_i \cdot \tau_j \right) \tag{4}
\]

\[
V_v = \frac{g_{\omega qq}^2}{4\pi} \left( \frac{e^{-m_\omega r_{ij}}}{r_{ij}} \right) + \frac{1}{6} \frac{g_{\rho qq}^2}{4\pi} \left( \frac{1}{m^2} \right) \left( \tau_i \cdot \tau_j \right) \left( \sigma_i \cdot \sigma_j \right) \left( \frac{e^{-m_\rho r_{ij}}}{r_{ij}} \right) \tag{5}
\]

and the additional screen Yukawa-like potential introduced as

\[
V_Y = -\frac{\alpha_s}{r_{ij}} e^{-\frac{\alpha_s r_{ij}^2}{2}} \tag{6}
\]

here, \(\alpha_s\) is the residual running coupling constant and \(c\) taken as screen fitting parameter. Hence, the net interaction potential given as

\[
V_{hh} = V_{OBE} + V_Y \tag{7}
\]

It is interesting to analyze the individual contribution of meson exchange in OBE. The overall contribution form OBE is very less. For which spin-isospin dependent interaction in the pseudoscalar meson exchange\((\pi, \eta)\) i.e. one pion exchange gives shallow attraction
TABLE I. The threshold of hadronic molecules consist of doubly heavy quark (c,b or \(\bar{c},\bar{b}\)). The binding energy, mass and root mean square radius with possible S-wave quantum numbers are presented and compared with possible exotic states. All hadron masses are taken from PDG [15].

| Candidate | Molecular Interpretation | S-wave Threshold | Exp. mass [MeV] | Expected mass [MeV] | This Work mass [MeV] | rms \(\sqrt{r^2}\) [fm] |
|-----------|-------------------------|------------------|----------------|---------------------|---------------------|-----------------|
| Deuteron  | \(p - n\)               | 0(1+)            | 1877.84        | 1875.6              | 2.224               | 1.72            |
| X(3872)   | \(D - \bar{D}^*\)       | 0(1+)            | 3871.84        | 3871.68 ± 0.17      | 0.160               | 11.64           |
| Y(4260)   | \(D_0 - \bar{D}^*\)     | 0(1−)            | 4328.28        | 4259 ± 8            | 69.28               | 11.70           |
| Z(4430)\(^+\) | \(D^{++} - \bar{D}^{*+}\) | 0(0−)            | 4431.58        | 4430                | 1.580               | 12.20           |
| \(P_c(4450)\) | \(\Sigma_c - \bar{D}^*\) | \(\frac{1}{2}(\frac{3}{2}−)\) | 4460.72        | 4449.8 ± 1.7        | 10.92               | 47.96           |
| \(\Sigma_c - \bar{D}^*\) | \(\frac{1}{2}(\frac{3}{2}−)\) | 4525.78        | 51.69           | 4474.0              | 0.79               |
| \(\Sigma_b - \bar{D}^*\) | \(\frac{1}{2}(\frac{5}{2}−)\) | 7819.98        | 49.34           | 7770.6              | 0.90               |
| \(\Sigma_b - \bar{D}^*\) | \(\frac{1}{2}(\frac{5}{2}−)\) | 7839.08        | 50.52           | 7788.5              | 0.85               |
| \(\Sigma_c - \Sigma_c\) | 0(1+)                      | 4907.48        | 14.66           | 4892.8              | 0.95               |
| \(\Sigma_c - \Sigma_c\) | 0(0+)                      | 12.86           | 4894.6          | 1.04                |
| \(\Sigma_b - \Sigma_b\) | 0(1+) | 11626 | 9.41 | 11616.5 | 1.01 |
| \(\Sigma_b - \Sigma_b\) | 0(0+) | 9.11 | 11616.8 | 1.03 |

while the \(\eta\) exchange gives almost negligible contribution to the long range tail. The central interaction term of the phenomenological scalar particle \(\sigma\) gives very strong attraction in the mid-range while \(\delta\) contributes opposite to the \(\sigma\). Whereas central \(\omega\) exchange contributes very strong repulsion at short distance while spin-isospin dependent interaction of the \(\rho\) contributes very fed attraction coherent with \(\pi\). Here we need to marked two points on the overall contribution (attraction/repulsion) of the OBE potential that its contribution strongly related to the coupling constant of the each meson exchange and on the spin-isospin channels.

The reasonable estimates of the coupling constant are given in the most of the realistic potentials [6–8] developed to reproduced NN-phase data and explain the deuteron properties. These estimates reproduced the empirical data reasonably well. Indeed, it is sensible to take
the quark-meson coupling in the present interaction scheme. The quark-meson coupling with respect to nucleon-meson coupling constant could be derive by Goldberger-Treiman relation. To fit empirical data one could take suitable estimate of it and could control the effective strength of the interaction. We have scaled the strength of all the coupling constant in a ratio with respect to pion-quark coupling which extracted by using Goldberger-Treiman relation.

$$g_{\pi qq} = \frac{3m_q}{5m_N^2} g_{\pi NN}$$  \hspace{1cm} (8)

By extracting the ratio followed by coupling constants in OBE with respect to $\pi NN$ listed in then using the same ratio to get meson-quark coupling constants with respect to $\pi qq$ coupling constant such that we could manage the overall strength of the OBE in a viable way.

For a quick inspection, we have confined our calculations for some interesting states (see Table-1). We preserves the generalize detailed spectroscopy of the di-hadronic systems in
the subsequent study. We have solve non-perturbative non-relativistic Schrdinger equation with above discussed interaction potential in the variational approach by using hydrogen-like trial wave function. The finite size effects due to extended structure of the mesons in OBE potential are included in the line of Eq.(F.9) of Ref.

By scaling the coupling strength of OBE potential and fitting $c=0.065$ GeV for which we could get approximate binding energy of the deuteron consistence with empirical value. Thus, we fixed these parameters for all further calculation. The calculated results leads to following the most likely di-hadronic molecular candidates such as $D\bar{D}^*(DD^*), D^*\bar{D}, B\bar{B}^*, B^*\bar{B}, D_0\bar{D}, D^*\bar{D}, \Sigma_c\bar{D}, \Sigma_c\bar{B}, \Sigma_c\Sigma_c, \Sigma_b\Sigma_b, \Sigma_b\Sigma_b$ etc.. The binding energy of the respective channels and thresholds are shown in the Table-1. These channels are compared with those states for which they are expected to be a bound state hadronic molecule. The hadronic bound states have also predicted in Ref. [16] where interaction potential were assumed as propositional to the spin-isospin.

The extremely famous state $X(3872)$ have been extensively studied as $D\bar{D}^*(DD^*)$ molecule. This state is just below the $D\bar{D}^*$ threshold and its very small binding energy and narrow width made this state spacial. The binding energy of the $X(3872)$ is very small form its natural energy scale (could be estimated by $m^2_\pi/2\mu$, $\mu$ is the reduced mass of two hadrons [17]) which is about 10 MeV if it consist of $D\bar{D}^*$. Deuteron is only another known case whose binding energy is very small (2.22 MeV) to the natural scale which is about 20 MeV. Eric Braaten in [17] explain the Low-energy Universality for $X(3872)$ very near to the threshold and large scattering length $a_s$. The Low-energy Universality defined as : the low energy few-body observables for non-relativistic particles with short-range interactions and a large scattering length have universal features that are insensitive to the details of the mechanism that generates the large scattering length. Such that if $a_s > 0$ it predicts shallow two body bound state. A shallow s-wave bound state leads to scattering length large compared to the natural length scale $1/m_\pi \simeq 1.41$ fm (in the present study $1/\beta \simeq 1.89$ fm) and it implies that as scattering length increases the probability for molecular interpretation increases [17].

The natural scale for the binding energy increases by ten time in the present study with proposed interaction. One can analyze from Table-1 that the expected binding energy for deuteron, $X(3872)$ and $Z(4430)$ are very small to the natural scale while the calculated one are overestimated from expected for all compared states, except deuteron. Thus, it requires the fine tunning of potential parameters to get consistent expected binding energy. Indeed,
these results extracting positive scattering length $a_s$ (see Table-2). The scattering could not be exactly calculable because it desire the accurate information of the interaction which leads very fine tuning of the potential parameters. As mentioned by Weinberg \[10\] that for any elementary bare-state $a_s$ would be less than $R$, and $r_e$ would be large and negative. With the expected value of binding energy for the deuteron, the scattering length and effective range are in agreement with experimental values ($a_s=5.41$ fm, $r_e=1.75$ fm) for Z=0.1 whereas effective range gains negative value from Z=0.4 and tend to larger as $Z \to 1$ which indicates that the state is composite. The same analogy could apply to the other states. The X(3872) with large positive value of $a_s$ but negative $r_e$ from Z=0.2 which strongly required $Z<0.2$ for dominance of the molecular structure and same happens with state $Z(4430)^+$ for $Z<0.4$. Guo-Zhan Meng in \[18\] studied low energy scattering of $D^*$ and $D_1$ meson using quenched lattice QCD and reported scattering length $a_s=2.52$ fm and effective range $r_e=0.7$ fm in $J^P = 0^-$ channel. The small and positive effective range leads to molecular interpretation of the state $Z(4430)^+$ composed of $D^* \bar{D}_1$. Whereas, with the expected binding energy tends $a_s=5.2$ fm and $r_e=1.9$ fm for $Z=0$ (pure molecule) while $r_e$ becomes negative and large for $Z>0.4$. Moreover, the Y(4260) for which the calculated binding energy is underestimated. While, with the expected binding energy and with proposed interaction the effective range drive negative value from Z=0.8 with large and positive $a_s$. If this state has driven a molecular composition then $a_s$ and $r_e$ must be such that $Z<0.8$, similar for the state $P_c(4450)$ for which the effective range gets negative value from Z=0.6.

From these analysis, apart from deuteron, with deep binding Y(4260) and $P_c(4450)$ highlighted as strong candidate for hadronic molecule while X(3872) and $Z(4430)^+$ seems to be extended structure. Still, to determine whether these states have molecular or confined structure, one needs reasonable estimates of scattering length and effective range either form experimental observables or from lattice QCD, once its known then we could strongly conclude and identify the substructure of the state. In conclusion, we strongly feel that there must be dipole like interaction between two color neutral states such that color screening tends vacuum polarization. This type interaction could explore the certain decays of the particles. Still, the proposed interaction is in a primary stage and need to investigate elaborately.

Our aim of the study to explore the interaction between two hadrons and their identification as a molecule. With our proposed interaction, we have found di-hadronic bound state in
attractive spin-isospin channels. In [5] the another state was reported as \( P_c(4380) \) with \( P_c(4450) \). \( P_c(4380) \) is not predicted with our approach. In [16] also noted that if \( P_c(4380) \) is \( \Sigma_c - D^{*} \) bound state it requires deep binding about 80 MeV, moreover, it needs spin splitting about 70 MeV which is very much unlikely with heavy flavour hadrons. This two states \( P_c(4380) \) and \( P_c(4450) \) have opposite parities is also an interesting issue.

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