Analysis of multi-representation ability to solve algebra problem

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Abstract. The phenomenon that occurs in everyday life, one of the mathematical topic that was difficult to understand by students is algebraic. In algebra, there are many symbols that make students difficult to understand and understand the concept of algebraic material. This research aims to describe the ability of multi-representation in solving algebraic problems based on students’ mathematical skills (high, moderate and low). Furthermore, the type of research used was descriptive research. The subject of research in this study is class VIII students SMP Negeri 13 Singkawang, odd semester of the school year 2017/2018 Overall the research results show the multi-representation that the student presented to the verbal representation of the average percentage of 54.46%, The average visual representation of the percentage was 59.09% while for symbol representation was obtained an average percentage of 48.92%. In this case, a highly capable subject presents a solution through various representations (verbal, visual and symbol) For medium-capable subjects, it was able to give an answer with symbolic and verbal representation and lacking in visual representation presentation. As for low-capability subjects are less able to present solutions in visual, verbal and symbolic representations.

1. Introduction
Algebra is an essential branch of mathematics and is considered a difficult and abstract lesson for students. The students should be able to understand the problems in using mathematical models and to determine the problems contained in the problem [1,2]. Algebra is one of the materials in mathematics learning that requires multi-representation in solving the problem and requires the development of verbal multi-representation ability, images, diagrams and mathematical expressions. To solve algebraic problems in math learning, students need to have mathematical reasoning [3,4]. Thinking algebra requires symbols and mathematical tools to analyze various conditions by presenting mathematical information related to words, diagrams, tables, graphs and mathematical equations that use mathematical findings such as calculating unknown values, proving and determining relationships between functions [5,6]. The online dynamic module facilitated students development of geometric thinking and integration of multiple modes of thinking, noting, "the geometric representations in the presence of algebraic and arithmetic modes appear to help learners begin to consider the different state aspects of a concept" [7]. In this case, multi-representation in mathematics is representing the process of mathematics in many different ways, including verbal, drawing, diagrams, graphs and mathematical expressions [8]. The ability of representation is the ability to apply various representations in explaining mathematical concepts and mathematical problems. In this case, there are three main functions of multi-representation, including complementary cognitive processes, helping to limit the likelihood of other...
interdisciplinary errors, and establishing a deeper understanding of the concept [9]. Multi-representations are highly relevant and necessary in establishing and developing students' learning understanding of the scientific concepts in depth [10].

In addition to acting as an understanding tool, multi-representations are also related to students' ability and readiness. In the higher stages, students' ability and readiness to learn mathematics have been at a certain level. Students no longer need a concrete model offering, but can also be other mathematical representations such as symbols, tables, graphs, verbal descriptions with their modellers and mathematical expressions in interpreting mathematical problems [6,11]. Students who use more than one representation, when solving a problem achieve a higher score compared to students who do not use it. Furthermore, in multi-representation capability is also influenced by the student's mathematical ability factor. Math skills students have an important role in learning mathematics because there is a connection between one material and another [11,12]. So in the event of rapid or slow students in mastering the material is influenced by the level of mathematics skills of students. Therefore, it is necessary to identify early identification of how multi-representation skills students are reviewing from the ability of mathematics. Based on the results of a survey of junior high school students using Google forms, showing that the main factors that cause students to experience difficulties in learning algebra are: lack of understanding of concepts and procedures, most students tend to memorize formulas without accompanying understanding and meaning of the formulas given, and lack of ability to solve problems where students tend to be oriented. In mathematics lessons, there are many problems solving mathematics in various ways from a variety of representations that serve to activate the understanding of the concept of students. The use of multi-representation helps students develop and enhance students' understanding of concepts [13]. Effective multi-representation learning for students as well as students using multi-representations tends to be correct and precise in resolving and generating answers [14]. The learners have various alternative representations in the pouring of ideas related to mathematical materials tested [15].

Based on the background above researchers want to find out more about analysis of multi-representation capabilities in resolving algebraic problems, in hopes that the results of this research can be based on a foothold in development and improvement of multi-representation ability in solving mathematical problems

2. Method
The type of research used was a qualitative descriptive. In research, the descriptive data obtained can be derived from the manuscript interviews, field notes, photographs, videotape, personal documents, notes or memos, and other official documents [16]. The research venue was held at first junior high School 13 Singkawang class VIII on the odd semester of the academic year 2017/2018. The research subject was determined by purposive sampling. The subject of this study was the VIII-B students of the state Junior high School 13 Singkawang with the number of 31 students. The selected subject amounted to 3 students consisting of 1 student of high mathematics, one student of moderate mathematics and 1 student of low mathematics. Group determination based on the recapitulation of written test results conducted at the time of research. The study uses instruments consisting of essays in essay form, interview and observation guidelines. Before conducting research instruments used tested the validity of the first two validators of members who are competent in the field of mathematics education Faculty of Mathematics STKIP Singkawang has competence in the field of qualitative research. For data analysis techniques consist of data reduction, data presentation and withdrawal of conclusions. Furthermore, the validity of data. Validity tests in qualitative research include credibility test, transferability, dependability and confirmability [17].

3. Result and Discussion
Results of initial test analysis of multi-representation ability of 31 students state Junior high School 13 Singkawang presented in Table 1.
Tabel 1. Recapitulation Frequency distribution score initial test multi-representation ability

| No | Score     | Frequency | Percentage (%) | Category  |
|----|-----------|-----------|----------------|-----------|
| 1  | X ≥ 67    | 8         | 25.80          | High      |
| 2  | 33 ≤ X ≤ 67 | 13        | 41.94          | Moderate  |
| 3  | X ≤ 33    | 10        | 32.26          | Low       |
| Sum|           | 31        | 100%           |           |

The results of data analysis in Table 1, indicating that the ability to multi-representation in resolving algebraic problems, students of the state Junior high School 13 Singkawang, the majority belong to the category of medium. Furthermore this initial test data was used to select one person's subject from each category. Based on the analysis on the test of the overall multi-representation ability, It was shown in Figure 1. Recapitulation of multi-representation ability of the students overall.

![Multirepresentation Ability](image)

Figure 1. The average percentage of students' multi-representation capabilities overall

Data analysis results obtained the most prominent ability is a visual representation of 59.09%, followed by a verbal representation of 54.46% while the ability to less multi-representation is a symbol representation of 48.92%. It will be described by the student analysis results in solving the multi-representation capability of algebraic material based on the following multi-representation capability indicators.

3.1. Analysis of mathematical skills in high, moderate and low categories on representations with visual indicators: Problem number 1
At the dining table there are five spoons, three plates and four cups. My little brother, took a spoon and one plate. How many cutlery at the table? Illustrate the above problem in the form of images! Students' answers based on subject ability categories (high, medium and low) include.

3.1.1. High subject mathematical ability

![Resolution with Images](image)

Therefore, the rest of the tableware at the dining table is four spoons, two plates and four glasses.

Figure 2. Answer from subject AY
Based on Figure 2, AY was capable of translating solving problems from verbal representation into the form of visual representation correctly. In this case the subject is able to perform a verbal representation to the visuals appropriately. Based on the subject interview writes the answer in the image form (visual representation). It can be said that the subject can do the completion by representing the solution that it works in the form of verbal and visual representation (can change the verbal representation into visual representation). The subject explains that, the subject is familiar with the questions presented and can transform into a visual representation form. The subject works in order to think step-by-step and potential enough to use visual representation offerings.

3.1.2. Moderate subject mathematical ability

From the results of the ND subject answer (Figure 3.), it appears that the subject can draw. But the answer given is not exactly this because the subject is still mistaken and confused in illustrating the verbal form into the visual representation (the image). Based on the results of the interview the subject is asked to re-examine the answering process from the results of answers done to see a verbal representation in resolving the problem into the form of visual representation. Seen the subject trying to pour his thinking in visual form in solving the problem. In understanding the problem, the subject takes time to read over and over. The subject expresses the feeling of lack of confidence with the image he made. The subject also states less understanding and is somewhat erroneous in working on the problems presented in the form of text (words) into the form of images (visuals).

3.1.3. Low subject mathematical ability

The BS subject does not answer correctly. This is because the subject's visual representation has not been used optimally to resolve the issue. In addition the subject was still confused in changing the problem given to the correct image. This is due to the low conceptual understanding of the subject and erroneously presenting the image with the concept of algebraic thus causing the answer given was not corrected. Based on the results of interviews obtained by the subject can draw but the results of the examination of answers through visual representation results are made inappropriate, the subject states that confused how to work. The problem can be understood by the subject. But the subject does not write important information and find it difficult to connect and discover the relationship between the representation so that it is constrained in the problem-solving process.
3.2. Analysis of mathematical skills in high, moderate and low categories on representations with verbal indicators: Problem number 2

In problem 2, the subject is instructed to change from image representation to verbal representation. The problem is presented in Figure 5.

Figure 5. Problem number 2

3.2.1. High subject mathematical ability

The AY subject translates a visual representation into verbal form, using a language that was structured, systematic and understandable. AY writes “Anita buys nine candies and four lollipops. Anita then gives three candies to her friend. What’s the rest of candies and lollipop Anita?” as a verbal representation of Figure 5. This means that the subject was able to understand and recognize other forms of representation in providing solutions to the problems presented. Based on the interviews done, the subject has already believed in working on the right outcome, evidenced that the subject can work to restore all the solutions correctly. Then the subject rechecks the settlement it works with explaining in words. Visible subjects can choose the right strategy to solve the problem. In addition, the subject is able to understand the problem well. The subject mentions the known things and mentions what is asked appropriately. Can write a step (way) through appropriate procedures to solve the problem.

3.2.2. Moderate subject mathematical ability

From the answers written by the ND subject can be seen to manifest a visual representation into the form of verbal representation, but the answers were written only a part of the true and precise mathematically. ND writes “I have 9 candies and lollipops. 3 Candy and gave my younger sisters three candies and Lollipops.” as a verbal representation of Figure 5. Based on the results of interviews conducted by researchers to the subject is seen when the subject checks the answer again, the subject of difficulty in writing words well and systematically. The subject says less careful and less thorough in changing the visual representation into the form of verbal representation.

3.2.3. Low subject mathematical ability

BS subject was able to make interpretation of mathematical phenomena from visual representation into the verbal form, but still erroneously. BS writes “Fadli bought 9 candies and Lollipop then Kayla asked for 3 candies. How much is the rest of the candies and Lollipop Fadli after given to Kayla =?” as a verbal representation of Figure 5. BS using symbolic markings in the form (=), this is because the subject was less understanding and familiar form of representation as a solution for completion. Based on the results of visible interviews the subject is able to plan and present data using a verbal representation that explains in terms of words (able to translate the data in planning the completion of only verbal or words). In examining the completion of the problem, the subject can write important information in a different form of representation. Only the subject has not been able to find the relationship between the related representations, of the settlement result.

3.3. Analysis of mathematical skills in high, moderate and low categories on representations with symbolic indicators: Problem Number 3

Mr. Tanto, a successful rice merchant in Singkawang City. Once Mr. Tanto got orders from Kuala market and Roban market simultaneously. Kuala market ordered three sacks of rice, while Roban market
ordered five sacks of rice. Rice is now available in Pak Tanto's warehouse as many as four sacks of rice. What is the rest of Pak Tanto rice if only fulfill the Kuala market order?

3.3.1. High subject mathematical ability

The AY subject was able to create mathematical models appropriately and present the concept of algebra by translating the question of a verbal representation into the form of visual representation and symbolic representation (see Figure 6). Students were also able to present solutions with the right procedures. Based on the results of the interviews, researchers see that the subject is convinced of the answer he works. When asked to re-check the solution written. The subject can resolve the problem using the appropriate symbolic representation. The subject can explain the question of verbal words into symbolic representation, with proper concepts and procedures.

3.3.2. Moderate subject mathematical ability

Based on Figure 7, the ND subject in providing solution completion does not use equations or models of mathematics and improper results. The ND subject has not understood the working concepts and procedures of the given problem. In the process of interviews, the subject is asked to re-check the answers to the questions he works. High anxiety seen when conducting an answer check. It also appears that the subject made mistakes in doing calculations. The subject states that in working on the matter, the subject is less thorough and not careful in reading.
3.3.3. Low subject mathematical ability

The BS subject cannot present concepts or relationships between concepts contained in the matter (see Figure 8). In addition the subject problem solving capabilities are low. It can be seen the subject of BS experiencing confusion and error in creating mathematical models and determining the final solution of the problems presented. Based on the results of the interview conducted by the researcher, from the results of the interview the subject does not conduct a review of the completion and presentation of the data that he has done. When working on the subject was directly answered, but rarely wrote the known and asked, and erroneously in understanding the representation of the symbol well that had an error in the calculation. In interviews conducted against a low subject, there is also an error checking the count so that it is wrong in writing the final result. It is demonstrated from the inaccuracy of the subject, when asked to return to work on the matter.

The description of the analysis results in detail the ability of multi-representation of students, which is seen from all research subjects, whether the subjects are highly capable, moderate and low in changing the phenomenon of mathematical multi-representation ability given on the indicator of visual representation, verbal and symbolic, there is a difference in presenting the form of representation. In this case only a highly capable subject, which can or is able to transform mathematical phenomena into various representations (visual representation, verbal representation and symbolic representation). While for moderate and low ability to change mathematical phenomenon into verbal representation.

Subjects with high ability in question number 1, 2 and 3 in transforming the mathematical phenomenon into visual, verbal and symbolic forms, in this case, are able to interpret the mathematical phenomenon of verbal representation form into a form of visual representation, then it can change the form of visual representation of the form of verbal representation, otherwise it can also change the form of verbal representation into symbolic representation form. This means that high-ability subjects can alter the mathematical phenomenon of a diverse representation, giving a description of the mathematical problem solving can be easily understood.

Subjects with moderate ability to question numbers 1.2 and 3 in transforming mathematical phenomena into the form of visual representation, verbal representation and symbolic representation. In this case the capable subject is being able to make interpretation of the mathematical phenomenon a symbolic representation into the form of verbal representation. This means the subject with the ability to be able to understand and recognize other forms of representation in providing solutions in resolving the mathematical problems given, even though it is not good enough. Medium-capable subjects have also not been able to make interpretations of transforming mathematical phenomena from verbal representations into visual representations and poorly in converting verbal representations into symbolic representations. This is because the subject is erroneous and confused in illustrating the verbal form into the visual representation form, while changing the verbal representation to the symbolic representation of the subject with the moderate ability to not understand the concepts and procedures that are not precise in resolving the mathematical problems given [6].

Subjects with low ability of number 1, 2 and 3 in transforming mathematical phenomena into the form of visual representation, verbal representation and symbolic representation. In this case the subject with low ability has not been able to make mathematical interpretation of the form of verbal representation into the form of visual representation this is due to erroneous, confused and low
conceptual understanding. In addition, low ability subjects have not been able to change the form of verbal representation into the symbolic representation of this because the lack of understanding and lack of proper representation as a solution to the mathematical problems given. But low-capability subjects are very good at making interpretations of transforming mathematical phenomena from visual form to verbal forms.

Further analysis results of multi-representation capabilities in visual indicators, verbal and symbolic overall is 1) erroneous and confused in illustrating verbal form into visual representation, the visual representation of the students has not been used optimally to solve the problem, 3) Low conceptual understanding of students and erroneously in presenting images with the concept of algebraic, 4) lack understanding and know the proper form of representation, 5) have not understood the correct working, and 6) Low student problem solving skills.

The research was in line with the research results [18], showing that the mathematics representation ability of high-ability subjects in solving every geometry problems always presents an appropriate visual representation, mathematical reform expressions inappropriate symbols, and using verbal representation at each stage of problem-solving; The middleability subjects present visual representation but less representative, using mathematical expressions and verbal representations; and low-ability subjects did not present the right visual representation, use a little mathematical expression and tend only to use verbal representations and cannot answer all problems.

4. Conclusion
This study shows that the multi-representation that the student presented to the verbal representation of the average percentage of 54.46%, the average visual representation of the percentage of 59.09% while for symbol representation was obtained an average percentage of 48.92%. In this case: 1) highly capable subjects are able to present solutions through various representations (verbal, visual and symbol), able to use structured, systematic and understandable language; 2) medium-capable subjects, able to present a solution with verbal representation as well as lacking in presentation of visual representation and symbolic representation this is due to not understand the concept and procedures of the work of the problems given; 3) low-capacity subjects, less able to present solutions in visual representation this was due to low conceptual understanding, in addition, it does not understand the symbolic representation was due to low problem-solving skills, as well as low-capacity subjects are less able to present verbal representation because of lack of understanding and know the exact form of representation.

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