A microscopic approach to understand Brownian dynamics in viscoelastic fluid

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We present an entirely microscopic formulation of viscoelasticity of a fluid starting from the microscopic Stokes-Oldroyd B Model assuming instantaneous hydrodynamic friction, and show that linearization leads to a form for the frequency dependent viscosity that can be directly applied to the Langevin equation. Interestingly, the calculated expression of viscosity can be directly mapped to the Jeffreys’ model which is essentially macroscopic in nature with the bulk viscoelasticity of the fluid being characterized by a complex elastic modulus $G(\omega)$. Further, we demonstrate that the concerned Green’s function is same as that in an incompressible, low Reynolds’s number Newtonian fluid with the simple incorporation of frequency dependence in the viscosity term. We proceed to evaluate the trajectory of a free Brownian particle in a viscoelastic environment using our formalism, and calculate parameters such as the power spectral density, the autocorrelation function and the mean-square displacement, which we then extend to the particle confined in a harmonic potential in the fluid.

I. INTRODUCTION

The study of complex fluids is of paramount importance primarily since biological entities which sustain life exist in such fluidic environments - whose complexity is typically manifested by a ‘viscoelastic’ nature [1–5], wherein the fluid exhibits both viscous and elastic properties [6]. Understandably, there exists a strong interest in the scientific community to understand the exact nature of viscoelastic fluids and to measure their rheological properties. Most models that attempt to analyze visco-elasticity are essentially “macroscopic” in nature, and attempt to understand the fluid in terms of its bulk viscous and elastic properties, and their response to external stresses. The foremost and simplest among the models developed is the Maxwell model [7, 8], which has been improved further into the generalized Maxwell or Jeffrey’s model [9], and can provide the relation between stress and shear-rate in linear viscoelastic fluids, which can then be used to calculate frequency dependent viscoelastic parameters of the medium. These measurements facilitate “macrorheological” measurements of fluid parameters using commercial rheometers that require several ml of sample volume, and typically determine the rheological properties at low frequencies [10]. Thus, the rheological properties of viscoelastic fluids are expressed in terms of a frequency dependent complex elastic modulus $G(\omega)$, whose real part represents storage and imaginary part represents loss. Finally, $G(\omega)$ gives rise to a complex frequency dependent viscosity $\eta(\omega)$ using $\eta(\omega) = \frac{G(\omega)}{-i\omega}$ - the real and imaginary components of which have similar associations as those of $G(\omega)$.

In contrast, a ‘microscopic’ approach towards comprehending the viscoelastic properties of a complex fluid would attempt to link the basic constituents of the fluid [11], often conceptualized as polymer chains of different length scales embedded and moving inside a Newtonian viscous fluid - and the macroscopic rheological properties of the fluid. Such a link would enable the synthesis of complex fluids much more simpler, since although the microscopic features of a polymer chain are often well known, the exact effect they would have in determining the macroscopic rheological properties of the fluid is very difficult to predict. Microscopic modeling would thus be an excellent predictive tool, which remains not very accessible due to the associated high computational cost involved with the introduction of additional microscopic variables. The rheological effects of the polymer chains can, however, be studied on a microscopic scale, by determining the the trajectory of a Brownian particle embedded in the fluid, which is manifested in the mean squared displacement (MSD) of the particle. The viscous and elastic properties may then be illustrated by considering the particle to be attached to a dashpot and a spring, respectively. In this approach, the Maxwell model provides a comprehensive description of the viscoelastic effects of the fluid on the particle trajectory assuming instantaneous hydrodynamic friction. A so-called ‘Maxwell’ fluid is also characterized by a characteristic time-scale $\tau_m$, which marks a transition from a high frequency elastic regime to a low-frequency purely viscous regime. The effects of the solvent’s dissipation is incorporated in the generalized Maxwell’s or Jeffrey’s model which interprets this as a background viscosity $\eta_\infty$, and also redefines the frequency regimes where purely viscous (very low and very high frequencies) and elastic (intermediate frequencies) behaviour is displayed by the fluid. Then, the drag force on the particle is evaluated by solving the Navier Stokes equations with the no-slip boundary condition and integrating over the stress tensor on...
the particle surface. This treatment enables “microrheology”, which is growing very rapidly due to its capability to explore small-scale intrinsic properties of complex fluids over a wide frequency band \[12, 13, 19\]. Microrheology - as the name suggests - employs Brownian probes embedded in the medium and measures their position autocorrelations induced by thermal fluctuations (passive microrheology \[12, 13, 19\]), or response to external forces (active microrheology \[20, 22\]), in order to determine the viscoelastic properties of the fluid. Now, an interesting exercise would be to consider viscoelasticity at the level of the polymer chains themselves, as well as incorporate solvent properties such as incompressibility into a model that would express the effects of viscoelasticity into a simple expression of frequency dependent viscosity, and compare it with the existing “macroscopic” models, as well as determine the trajectory of Brownian probes, both free and in confinement enabled by optical traps - which would thus cast the problem in a microrheological context.

In this paper, we perform this exercise rather thoroughly. We start with the Stokes-Oldroyd-B model \[23, 24\] - an existing microscopic model of viscoelasticity - and linearize it to understand its behavior to small external perturbations. We then formulate a hydrodynamic Green’s function - which, finally yields a frequency dependent complex viscosity. Interestingly, though we linearize the quasi-linear Stokes-Oldroyd-B equation, the expression for the complex viscosity differs from that obtained from the stress-strain relation of Maxwell model. However, we observe a qualitative similarity in terms of the viscosity contributions of the solvent and the polymer chains in the total frequency dependent expression of viscosity for the fluid. Furthermore, we determine the thermal trajectory of a Brownian particle - both free and confined in a harmonic oscillator potential as is the case in optical tweezers - in a low-Reynold’s number, incompressible, viscoelastic fluid. It is clear that the statistical properties of the phenomena are connected to the rheological properties of the fluid and thus facilitate accurate microrheology.

II. THEORY

The Stokes-Oldroyd-B equations - having a microscopic origin - for an incompressible, low Reynolds number fluid are given by

\[ \nabla \cdot \mathbf{u} = 0 \]  
\[ -\nabla p + \mu_s \Delta \mathbf{u} = -\nabla \cdot \mathbf{S} - \mathbf{f} \]  
\[ \frac{\partial \mathbf{S}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{S} - (\mathbf{S} \nabla \mathbf{u} + (\nabla \mathbf{u})^T \mathbf{S}) + \frac{\mathbf{S}}{\lambda} = \frac{\mu_p}{\lambda} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \]

where, \( \mathbf{u}, p, \mathbf{S}, \mathbf{f} \) are the fluid velocity, pressure, polymer contribution to the stress tensor, and external force, respectively. \( \mu_s \) is the zero-frequency solvent viscosity, \( \mu_p \) is the zero-frequency polymer viscosity and \( \lambda \) is the polymer relaxation time scale. \( \mu_p = \mu_0 - \mu_s \) where \( \mu_0 \) is the viscosity of the solution at \( \omega = 0 \). We linearize above equations by considering \( \mathbf{f} = \epsilon \mathbf{g}, \mathbf{u} = \epsilon \mathbf{u}', \mathbf{p} = \epsilon \mathbf{p}' \) and \( \mathbf{S} = \epsilon \mathbf{T} \) where \( \epsilon \ll 1 \). Now \[2\] and \[3\] become

\[-\nabla \mathbf{p}' + \mu_s \Delta \mathbf{u}' + \mathbf{g} = -\nabla \cdot \mathbf{T} \]  
\[ \frac{\partial \mathbf{T}}{\partial t} - \frac{\mu_p}{\lambda} (\nabla \mathbf{u}' + (\nabla \mathbf{u}')^T) + \frac{\mathbf{T}}{\lambda} = 0 \]

Clearly, \[5\] relates stress tensor with shear-rate tensor as it is in Maxwell model of viscoelastic fluid. Equations \[1\], \[4\] and \[5\] can be written in component form as

\[ \partial_t u_i' = 0 \]  
\[ -\partial_i p' + \mu_s (\partial_i^2 u_j' + \partial_j^2 u_i' + \partial_k^2 u_i') + g_i = -\partial_j \mathbf{T}_{ij} \]  
\[ \frac{\partial \mathbf{T}_{ij}}{\partial t} - \frac{\mu_p}{\lambda} (\partial_i u_j' + \partial_j u_i') + \frac{\mathbf{T}_{ij}}{\lambda} = 0 \]

where Einstein summation has been used. Now, after performing a Fourier transform on \[6\], \[7\] and \[8\], we obtain

\[ k_i \tilde{u}_i' = 0 \]  
\[ -ik_i \tilde{p}' - \mu_s k^2 \tilde{u}_i' + \tilde{g}_i = -ik_j \tilde{T}_{ij} \]  
\[ -i\omega \tilde{T}_{ij} - \frac{\mu_p}{\lambda} (ik_i \tilde{u}_j' + ik_j \tilde{u}_i') + \frac{\tilde{T}_{ij}}{\lambda} = 0 \]

where the rules of four dimensional Fourier transforms were used. Multiplying \( k_j \) with \[11\] we obtain,

\[ -i\omega k_j \tilde{T}_{ij} - \frac{\mu_p}{\lambda} (ik_i k_j \tilde{u}_j' + ik_j k_i \tilde{u}_i') + \frac{k_j \tilde{T}_{ij}}{\lambda} = 0 \]

Now, using \[9\] we finally obtain,

\[ k_j \tilde{T}_{ij} = \frac{i \mu_p}{\lambda} \frac{k^2}{(-i\omega + \frac{1}{\lambda})} \tilde{u}_i' \]

so that, after plugging \[12\] into \[10\] we have,

\[ -ik_i \tilde{p}' - \mu_s k^2 \tilde{u}_i' + \tilde{g}_i = \left( \frac{\mu_p}{\lambda} \frac{k^2}{(-i\omega + \frac{1}{\lambda})} \right) \tilde{u}_i' \]  
\[ \nabla p'(r, \omega) + \left[ \mu_s + \frac{\mu_p}{(-i\omega \lambda + 1)} \right] \Delta \mathbf{u}'(r, \omega) + \mathbf{g}(r, \omega) = 0 \]
We can also perform an inverse Fourier transform to get
\[ \nabla \cdot \mathbf{u}(r, \omega) = 0 \] (15)

(14) and (15) are similar to the stokes equation in low Reynolds’s number, incompressible Newtonian fluid which are given below.
\[ -\nabla P + \mu \Delta \mathbf{v} + \mathbf{F} = 0 \] (16)
\[ \nabla \cdot \mathbf{v} = 0 \] (17)

\( P, \mathbf{v}, \mathbf{F} \) are pressure, velocity, external force respectively, while \( \mu \) is the fluid viscosity. The above two equations can be written in \((r, \omega)\) space where they appear as
\[ -\nabla P(r, \omega) + \mu \Delta \mathbf{v}(r, \omega) + \mathbf{F}(r, \omega) = 0 \] (18)
\[ \nabla \cdot \mathbf{v}(r, \omega) = 0 \] (19)

Hence, we can compare (14) and (18), and can conclude that
\[ \mu(\omega) = \left[ \mu_s + \frac{\mu_p}{-i\omega\lambda + 1} \right] \] (20)

is the expression of frequency dependent viscosity in an incompressible, low Reynold’s number viscoelastic fluid. Thus, the frequency dependent dynamic complex modulus is \( G^*(\omega) = -i\omega \mu(\omega) = G'(\omega) - iG''(\omega) \). The real part \( G'(\omega) \) represents the storage modulus and \( G''(\omega) \) refers to the loss modulus of the fluid. The result can be compared to the expression obtained from the Maxwell model [9], where the frequency dependent viscosity is given by
\[ \mu_M(\omega) = \frac{\eta_0}{1 - i\omega\tau_M} \]
where \( \eta_0 \) is the zero-frequency viscosity, and \( \tau_M \) is the Maxwell time described earlier. This is modified to the generalized Maxwell model or Jeffrey’s model which gives
\[ \mu_M(\omega) = \eta_\infty + \frac{\eta_0}{1 - i\omega\tau_M} \]
where \( \eta_\infty \) is the viscosity of the solution at \( \omega \to \infty \). It is clear that the inclusion of \( \eta_\infty \) is performed in order to ensure that a physically meaningful result is obtained at very large values of \( \omega \), where the behaviour of the fluid is essentially viscous. However, \( \eta_\infty \) is defined as the background viscosity that arises due to the constituent mesoscopic particles of the viscoelastic material being immersed in a solvent [9]. This can directly be mapped to the solvent viscosity \( \mu_s \) that we define in our approach. Thus, it is interesting to note that while the Maxwell model and Jeffrey’s model both originate macroscopically, and do not directly include the incompressibility (Eq.11) of the fluid and the momentum balance condition (Eq.22), our microscopic model which considers both these conditions, give rise to very similar results. Note that \( \lambda \) in our model is the ratio of viscosity of the solution to its elasticity, so that it has a dimension of time, and can again be interpreted to be of similar significance to the Maxwell model, since when the medium becomes purely viscous then \( \lambda \to \infty \) and from (20), \( \mu(\omega) = \mu_s \). This is identical to the behaviour of \( \tau_M \) in the Maxwell model(s). \( G^*(\omega) \) and \( \mu(\omega) \) are plotted in Fig.1. It is clear from the figure that the fluid becomes purely viscous as \( \omega \to \infty \) as the real part of \( \mu \) approaches \( \mu_s \) and the imaginary part tends to zero.

A. Green’s function calculation:

Taking inner product of equation (13) with \( ik \) and using equation (9) we obtain,
\[ k^2 \tilde{p}_i + ik \tilde{g}_i = 0 \]

In three dimensions, the above equation can be written as
\[ k^2 \tilde{p}'(\mathbf{k}, \omega) = -ik \cdot \tilde{g} \]
\[ \tilde{p}'(\mathbf{k}, \omega) = -ik \cdot \tilde{g} \]

Using Eq.21 into the three dimensional form of Eq.13, we get
\[ k \left( k \cdot \tilde{g} \frac{k}{k^2} \right) + \mu(\omega) k^2 \tilde{u}' = \tilde{g} \]
\[ \tilde{u}'(\mathbf{k}, \omega) = \frac{1}{\mu(\omega) k^2} \left[ \tilde{g} - k \left( \frac{k \cdot \tilde{g}}{k^2} \right) \right] \] (22)
So,
\[ p'(\mathbf{r}, \omega) = \frac{1}{2(2\pi)^3} \int_{R^3} dk \left( -ik \cdot \tilde{g} \right) \exp(i \mathbf{k} \cdot \mathbf{r}) \] (23)
\[ u'(\mathbf{r}, \omega) = \frac{1}{2(2\pi)^3 \mu(\omega)} \int_{R^3} dk \frac{1}{k^2} \left[ \tilde{g} - k \left( \frac{k \cdot \tilde{g}}{k^2} \right) \right] \exp(i \mathbf{k} \cdot \mathbf{r}) \] (24)
If \( \mathbf{g} \) is a constant force then Eq.23 gets solved to
\[ p'(\mathbf{r}, \omega) = -\mathbf{g} \cdot \nabla \left( \frac{1}{4\pi r^3} \right) = \frac{\mathbf{r} \cdot \mathbf{g}}{4\pi r^3} \] (25)
where we used
\[ \nabla \left( \frac{1}{4\pi r} \right) = \frac{i}{(2\pi)^3} \int_{R^3} dk \frac{k}{k^2} \exp(i \mathbf{k} \cdot \mathbf{r}) \]
and Eq.24 becomes
\[ u'(\mathbf{r}, \omega) = \frac{\mathbf{g}}{4\pi \mu(\omega) r} - \frac{\mathbf{g}}{\mu(\omega)} \cdot \nabla \nabla \left( \frac{r}{8\pi} \right) \] (26)
Figure 1: The real and imaginary part of the dynamic complex modulus \( G' \) and \( G'' \) are plotted against the angular frequency \( \omega \) in (a), while the frequency dependency of \( \mu(\omega) \) is shown in (b). \( \mu_p \) and \( \mu_s \) are chosen to be equal so that the polymer concentration in the solution is relatively low so that the viscoelasticity of the fluid is manifested clearly, but is not large enough for the fluid to display non-linear characteristics. The dotted gray line represents \( 1/\lambda \) frequency. Clear, this is the transition frequency of the fluid where viscous nature starts to dominate over the elastic nature.

where we make use of

\[
\nabla \nabla \left( \frac{F}{8\pi} \right) = \frac{1}{(2\pi)^3} \int_{R^3} dk \, \frac{kk}{k^4} \exp (i k \cdot r)
\]

and \( kk \cdot g = (g \cdot k)k \). Since, \( \nabla r = \frac{r}{r} \) and the tensor product \( \nabla r = I \), we have

\[
u'(r, \omega) = \frac{1}{8\pi \mu(\omega) r} \left( \frac{r r}{r^2} \right) \cdot g \quad (27)
\]

Now, in general, the velocity field at a point can be expressed as

\[
u'(r, \omega) = \int \mathcal{G}(r - r', \omega) \cdot g(r', \omega) dr'
\]

where

\[
\mathcal{G}(r) = \frac{1}{8\pi \mu(\omega)r} \left( \frac{rr}{r^2} \right) \quad (28)
\]

is the hydrodynamic Green’s function or Oseen tensor in a viscoelastic fluid. \( g(r', \omega) \) is a force acting only in a single point \( r' \) on the fluid. Under no-slip boundary condition, the above Green’s function can be used to calculate the hydrodynamic drag force on a spherical particle [25]. Since, \( \mu(\omega) \) is only a function of frequency and has no spatial dependency, the translational drag force will be

\[
F = 6\pi \mu(\omega) a_0 \left[ \left( 1 + \frac{a_0^2}{6} \nabla^2 \right) u_1 - (U - u^\infty) \right]
\]

where \( a_0 \) is the radius of the particle, \( u_1 \) is the velocity field caused by other means evaluated at the sphere center, \( U \) is the velocity of the particle and \( u^\infty \) is the velocity of the uniform background flow evaluated at the center of the sphere.

B. Brownian motion in a harmonic oscillator potential:

The generalized Langevin equation describing the one dimensional trajectory of a Brownian spherical particle of mass \( m \) in a harmonic oscillator potential in this fluid is given by

\[
m \ddot{x}(t) = - \int_{-\infty}^{t} \gamma(t - t') \ddot{x}(t')dt' - kx(t) + \xi(t) \quad (29)
\]

where \( \gamma(t) \) is the time dependent friction coefficient, \( k \) is the trap stiffness and \( \xi(t) \) is the correlated thermal noise with correlation \( \langle \xi(t) \xi(t') \rangle = 2k_B T \gamma(t - t') \). Since, the fluid has low Reynold’s number and the mass of the colloid is very small, momentum of the particle relaxes in a negligible time scale so that the effect of inertia is negligible, and the above equation in frequency domain can be written as:

\[
0 = i \omega \gamma(\omega)x(\omega) - kx(\omega) + \xi(\omega) \quad (30)
\]

\[
x(\omega) = \frac{\xi(\omega)}{(-i \omega \gamma(\omega) + k)} \quad (31)
\]

Here, \( \gamma(\omega) = 6\pi \mu(\omega)a_0 \). So, the power spectral density (PSD) is given by,

\[
\langle x(\omega)x^\ast(\omega) \rangle = \frac{2k_B T}{\gamma_0} \left[ \frac{\left( \frac{k_B T}{\gamma_0} - \omega^2 \right)}{\omega^2} + \omega^2 \left( \frac{1}{\gamma_0} - \frac{1}{\gamma_0} \right) \right] \quad (32)
\]

where, \( \gamma_0 = 6\pi \mu_a a_0 \) and we used \( \langle \xi(\omega)\xi^*(\omega) \rangle = 2k_B T \times Re[\gamma(\omega)] \). The position autocorrelation function (ACF) is the inverse Fourier transform of the PSD, and is given by
The above expressions for free Brownian particles in such fluid can be obtained making \( k \to 0 \). Then clearly, the above expressions of PSD, ACF and MSD become

\[
\langle x(\tau) x(0) \rangle = \frac{1}{\nu} \left[ \frac{a - \frac{b}{4}(c - \nu)^2}{\left( \frac{c - \nu}{2} \right)^2 + \frac{a}{\nu} (c + \omega_0)} \exp \left( - \frac{c - \nu}{2} \right) \right] \tag{33}
\]

\[
\langle \Delta x^2(\tau) \rangle = 2 \left[ \langle x^2(0) \rangle - \langle x(\tau) x(0) \rangle \right]
\]

\[
= \frac{2}{\nu} \left[ \frac{a - \frac{b}{4}(c - \nu)^2}{\left( \frac{c - \nu}{2} \right)^2 + \frac{a}{\nu} (c + \omega_0)} \left( 1 - \exp \left( - \frac{c - \nu}{2} \right) \right) \right]
\]

\[
+ \frac{\frac{b}{4}(c + \nu)^2 - a}{\left( \frac{c + \nu}{2} \right)^2 + \frac{a}{\nu} (c + \omega_0)} \left( 1 - \exp \left( - \frac{c + \nu}{2} \right) \right) \right] \tag{34}
\]

\[
\langle x(\omega) x^*(\omega) \rangle = \frac{2k_B T}{\gamma_0} \frac{1}{\omega^2 + \left( \frac{k}{\gamma_0} \right)^2} \tag{35}
\]

\[
\langle x(\tau) x(0) \rangle = \frac{k_B T}{k} \exp \left( - \frac{k}{\gamma_0} \tau \right) \tag{36}
\]

\[
\langle \Delta x^2(\tau) \rangle = \frac{2k_B T}{k} \left( 1 - \exp \left( - \frac{k}{\gamma_0} \tau \right) \right) \tag{37}
\]

which are the expressions in a viscous medium in an optical tweezers of stiffness \( k \). The power spectral density, auto correlation function and the mean-square displacement functions are plotted for different \( \lambda \) in Fig.2.

1. Free Brownian particle:

The above expressions for free Brownian particles in such fluid can be obtained making \( k \to 0 \). Then clearly, the above expressions of PSD, ACF and MSD become
III. CONCLUSIONS

In conclusion, we determine the frequency-dependent viscosity for a complex fluid from a microscopic origin, where we take into account the boundary conditions of fluid mechanics including incompressibility and low Reynolds number as is relevant for microrheology applications. Thus, we start from the Stokes-Oldroyd-B model, which we linearize assuming small external perturbations. A comprehensive solution yields an expression for frequency dependent viscosity $\eta(\omega)$, which we compare to that obtained using the generalized Maxwell (Jeffrey’s) model. We observe that akin to the parameters $\eta_0$ and $\eta_\infty$ which represent the zero and infinite frequency values of the solvent in Jeffrey’s model, our model yields the parameters $\mu_p$ and $\mu_s$, which are the polymer chain and solvent zero-frequency viscosities, respectively.

Interestingly, our model couples the polymer relaxation time scale $\lambda$ to $\mu_p$, which is finally responsible for the frequency dependence of the viscosity of the complex fluid. In contrast, the time-scale $\tau_M$ appearing in the Jeffrey’s model - which marks the transition from elastic to viscous behaviour - is coupled with the elastic modulus $G_\infty$ of the material. This difference arises from the fact that the Maxwell model does not consider the contributions of the polymer chains and the solvent separately - it being based on Maxwell’s initial assumption of a viscoelastic material being made up of a viscous element (represented by $\eta_0$) having an elastic component ($G_\infty$). The contribution of the polymer chains is somewhat ad hoc, with a so-called ‘background’ viscosity $\eta_\infty$ being added to the frequency dependent term for the final expression of $\eta(\omega)$. On the other hand, our treatise makes no such assumption of this form, and instead starts with the well-known equations for a “Boger” elastic fluid, where we have polymer particles being immersed in a solvent [27]. Yet, the final expression we obtain is similar to that obtained using the Jeffrey’s model which justifies the intuition involved in the latter. After having obtained the expression for $\eta(\omega)$, we proceed to use it in solving the Langevin equations for Brownian motion for a free particle, and a particle confined in a harmonic potential well as is the case in optical tweezers. We determine the various parameters for particle dynamics including the MSD, and autocorrelations of the position fluctuations in time and frequency domain. Most importantly, we observe that the MSD reduces to the well-known expression for a viscous fluid in the diffusive limit when we have $\mu_p \rightarrow 0$ and $\lambda \rightarrow \infty$. This acts as an important consistency check and gives us confidence in the correctness of our theoretical treatment. Our expression for $\eta(\omega)$ will prove to be very useful in microrheology and measurements of viscoelasticity of complex fluids, which we are presently performing in our laboratory. We would also like to point out that the entire theoretical treatment has been performed in the limit of instantaneous hydrodynamic friction, where we have neglected vorticity diffusion in the fluid which will lead to retardation effects. This is what we intend to report in the immediate future.
where we will also attempt to solve the entire non-linear Stokes-Oldroyd-B equations, that should therefore provide a general form for $\eta(\omega)$.

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