Transverse current response of graphene at finite temperature: plasmons and absorption

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Abstract

We calculate the linear transverse current–current response function for graphene at finite temperature and chemical potential. Within the random phase approximation, we then discuss general aspects of transverse plasmons beyond the local response such as their dependence on temperature and on the surrounding dielectric media. We find, for example, maximal confinement of this mode for a homogeneous dielectric medium with refractive index $n \simeq 40$. Confinement can be further enhanced by placing the graphene sheet inside an optical cavity, but there exists a critical width below which no transverse mode can be sustained. For zero doping and finite temperature, there are no well-defined transverse plasmonic excitations in contrast to the longitudinal channel. We also discuss the absorption of electromagnetic radiation in single and double layer systems for s and p polarizations and point out that the theoretical limit of 50% is reached for s-polarized light with an incident angle of $\theta \approx 89^\circ$.

Keywords: graphene, current response, random phase approximation, transverse plasmons, absorption, multilayer, confinement

(Some figures may appear in colour only in the online journal)

1. Introduction

Over the last few years, plasmonics has become a celebrated field to which deep and intense research is being targeted. The term plasmon alludes to a collective harmonic movement of electrons [1] entailed to equally oscillating electromagnetic fields, and was born in the context of studying losses of fast electrons when moving through metallic foils [2, 3]. This concept also emerges as the existence of electromagnetic modes at the interface between a metal and a dielectric [4, 5].

Its popularity lies in both the theoretical and practical interest that its understanding offers. On the one hand, plasmons manifest many-body effects between electrons associated with the long range Coulomb interaction [6, 7]; on the other hand, their striking features, among which their high confinement to the two-dimensional sheet plays a main role, suggest various applications in cutting-edge electronics and optoelectronics [8]. This motivates the desire to extend this exploration to systems of present physical interest which offer a theoretical challenge and promise much regarding engineering or applied physics.

Graphene is one of the most promising candidates [9–13]. Its two-dimensional honeycomb lattice structure leads to an effective low-energy theory which is governed by a spin-independent massless Dirac Hamiltonian. The physical implications underlying this fact have been comprehensively studied and spread to a wide extent of different topics [14]. Throughout this paper, we are mainly interested in those related to its current–current correlation function. In other words, we will continuously allude to the interaction of electrons in graphene with electromagnetic radiation. Some main results covering this aspect have been outlined in [15]. A constant value for the real part of the conductivity above twice the Fermi energy, i.e., an absorption value equal to 2.3% dependent only on universal constants for any high enough frequency, deserves to be highlighted [16, 17]. At the same
time, several issues still demand further research and suggest that this topic is far from being exhausted. For example, the real part of the conductivity shows a plateau between the intra- and inter-band energy regions [18] whose numerical value cannot yet be accounted for theoretically [19].

The current response function is an indispensable tool for characterizing plasmons. Its tensorial character allows its decomposition in two channels: the longitudinal and transverse channel depending on whether the vector potential \( \mathbf{A}(\mathbf{q}, \omega) \) is parallel or perpendicular to the wavevector \( \mathbf{q} \). Plasmons arising from each channel correspond to transverse magnetic or transverse electric modes (TM, TE) [6]. Those linked to the longitudinal channel have outshone the others because they are associated with the density response and thus charge accumulation, for transverse plasmons they move forward and backward along the direction determined by \( \mathbf{q} \). Whereas for longitudinal plasmons they move parallel or perpendicular to \( \mathbf{q} \). Therefore, \( \chi_{ij}(\mathbf{q}, \omega) \) will provide the result of the longitudinal (transverse) component of the response function when \( \mathbf{q} = q \hat{x} \) (\( \mathbf{q} = q \hat{y} \)).

Due to their transverse nature, the dispersion relation is closely pinned to the light cone and they are confined to energies between 1.667 and twice the Fermi energy (for larger energies they are strongly damped) [32]. Here, we want to discuss this transverse light–matter coupling in more detail, such as the influence exerted by finite temperature, the possibility of inducing transverse plasmons in the undoped graphene sheet, the circumstances that demand that the local approximation for the response function be discarded, the changes induced by modifying the dielectric surroundings, the highest spatial confinement within reach or their behavior when sheets are embedded between an optical cavity.

The paper is organized as follows. In section 2, we calculate the current–current response function at finite temperature and doping for both the longitudinal and transverse channels as the fundamental tool for discussing light–matter interaction within linear response. We then analyze the questions mentioned above: (i) effects related to finite temperature; (ii) effects related to various dielectric media and the necessity of going beyond the local response; (iii) effects related to placing graphene inside a vertical cavity. We complement this discussion by calculating the transmissivity of electromagnetic radiation for s and p polarizations for a single and double layer structure, and close with a summary and outlook.

2. Response functions at finite temperature and doping

The (bare) response function \( \chi_{ij}(\mathbf{q}, \omega) \) allows us to determine the value of currents arising in a medium due to the presence of a total (external) vector potential \( \mathbf{A}(\mathbf{q}, \omega) \) (\( \mathbf{A}^{\text{ext}}(\mathbf{q}, \omega) \)). That is, as linear response theory states for a homogeneous system, \( \mathbf{j}(\mathbf{q}, \omega) = \mathbf{j}_{ij}(\mathbf{q}, \omega) \mathbf{A}(\mathbf{q}, \omega) \) and \( \mathbf{j}(\mathbf{q}, \omega) = \mathbf{j}^{(0)}_{ij}(\mathbf{q}, \omega) \mathbf{A}^{\text{ext}}(\mathbf{q}, \omega) \), where [37]

\[
\mathbf{j}_{ij}(\mathbf{q}, \omega) = \left( \chi_{ij}(\mathbf{q}, \omega), \chi_{ij}(\mathbf{q}, \omega), \chi_{ij}(\mathbf{q}, \omega) \right)
\]

(1)

(analogous expression is satisfied for the bare response, with the superscript (0) in every \( \chi \) letter, that is omitted above for the sake of simplicity) and

\[
\chi_{ij}^{(0)}(\mathbf{q}, \omega) \equiv -\frac{i}{\hbar} \lim_{\epsilon \to 0^+} \int_0^\infty dt \langle \mathcal{A}_z(t), \hat{B}_{-\mathbf{q}}(0) \rangle e^{i\mathbf{q} \cdot \mathbf{x}} e^{-\epsilon t}.
\]

(2)

\( S \) being the total area of the system. From now on, every property of the full response \( \chi_{ij}(\mathbf{q}, \omega) \) will be shared with the bare one unless the contrary is specified. \( \hat{\chi}_{ij}(\mathbf{q}, \omega) \) is a tensor of the same dimension of the space under study (2 in case of graphene). We can choose a reference frame with one axis parallel to any given \( \mathbf{q} \) in such a way that \( \hat{\chi}_{ij}(\mathbf{q}, \omega) \) will become diagonal [37]. In that case, we can decouple the response in two channels: the longitudinal and the transverse one, alluding respectively to \( \mathbf{A}(\mathbf{q}, \omega) \) being parallel or perpendicular to \( \mathbf{q} \). Therefore, \( \chi_{ij}(\mathbf{q}, \omega) \) will provide the result of the longitudinal (transverse) component of the response function when \( \mathbf{q} = q \hat{x} \) (\( \mathbf{q} = q \hat{y} \)).

Differing the isospin–isospin response function only in a multiplicative factor with respect to the current–current one, from now on, we will deal with \( \chi_{\sigma, \sigma}(\mathbf{q}, \omega) \). The Lehmann representation yields

\[
\chi_{\sigma, \sigma}^{(0)}(\mathbf{q}, \omega) = \frac{1}{8} \lim_{\epsilon \to 0^+} \sum_k \sum_{\lambda, \lambda'} \frac{N^{(0)}_{k, \lambda} - N^{(0)}_{k+\mathbf{q}, \lambda'}}{\hbar \omega + \epsilon_{k, \lambda} - \epsilon_{k+\mathbf{q}, \lambda'} + i\epsilon} 
\]

\[
\times |\langle \chi_\lambda (\tilde{k}) | \sigma | \chi_\lambda (\tilde{k} + \mathbf{q}) \rangle |^2.
\]

(5)

In this expression, we will introduce the wavefunctions

\[
\chi_\lambda (\tilde{k}) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ \lambda \phi_i \end{array} \right)
\]

(6)
and the noninteracting Fermi statistics, \( n^{(0)}_{\kappa\lambda} \). \( \lambda = +1 (-1) \) corresponds to the upper (lower) band and \( \phi_{k} \) is the angle between \( k \) and \( \hat{k} \).

It is also interesting to note that, by virtue of the continuity equation, the density–density and the longitudinal isospin–isospin response functions are related [38]

\[
\chi_{\sigma\rho}(q, \omega) = \frac{\omega q f}{\omega^2} [(\sigma^+ \rho - \rho \sigma^+)] + \frac{\omega q^2}{\omega^2} \chi_{\sigma\rho}(q \hat{x}, \omega).
\]  

(7)

On the other hand, the transverse channel is completely independent of the density response.

On the basis of all previous considerations, we can write the isospin–isospin bare response functions as follows, with \( f \) being independent of the density response.

\[
\text{Re} \left( \chi_{\sigma\rho}^{(0)}(q, \omega, T, \mu) \right) = -\frac{E_{\text{max}}}{4\pi \hbar^2 v_F^2} + \frac{g}{4\pi \hbar v_F^2} \times \sum_{\alpha=\pm} \left[ \frac{\omega^2 2\hbar k T \log[1 + \exp(\frac{\omega q}{2\hbar k T})]}{\hbar^2 v_F^2} + \Theta(\omega - \nu q f)(\omega, \nu q f) \times \left[ G_{\alpha}^{(0)}(q, \omega, T, \mu) - G_{-\alpha}^{(0)}(q, \omega, T, \mu) \right] + \Theta(\nu q f - \omega f)(\omega, \nu q f) \times \left[ -\frac{\pi}{2} \Theta(-\alpha) + H^{(0)}_{\alpha}(q, \omega, T, \mu) \right] \right];
\]

\[
\text{Im} \left( \chi_{\sigma\rho}^{(0)}(q, \omega, T, \mu) \right) = \frac{g}{4\pi \hbar v_F^2} \sum_{\alpha=\pm} \left[ \Theta(\nu q f - \omega f)(\omega, \nu q f) \times \left[ G_{\alpha}^{(0)}(q, \omega, T, \mu) - G_{-\alpha}^{(0)}(q, \omega, T, \mu) \right] + \Theta(\nu q f - \omega f)(\omega, \nu q f) \times \left[ -\frac{\pi}{2} \Theta(-\alpha) + H^{(0)}_{\alpha}(q, \omega, T, \mu) \right] \right];
\]

\[
\text{with}
\]

\[
f_\beta(\omega, \nu q f)(\omega) = \frac{g}{2} \left[ 1 - \frac{\nu q f^2}{\omega^2} \right]^{-\beta/2};
\]

\[
G_{\pm}^{(0)}(q, \omega, T, \mu) = \int_{-\infty}^{\infty} \frac{u[1 - \frac{u}{\nu q f}]^{\beta/2}}{\exp[\frac{\nu q f u}{\omega} - 2\mu u] + 1} du;
\]

\[
H^{(0)}_{\alpha}(q, \omega, T, \mu) = \int_{-\infty}^{\infty} \frac{|u|^{1/2}[1 - \frac{u}{\nu q f}]^{\beta/2}}{\exp[\frac{\nu q f u + 2\mu u}{\omega}] + 1} du.
\]

\( E_{\text{max}} \) is an ultraviolet cutoff which is canceled by the diamagnetic contribution, as required by gauge invariance [38]. We include the spin and valley degeneracies as \( g = g_s g_v \). Given the electronic density \( n \), the chemical potential is determined by

\[
\int_{-\infty}^{\infty} d\nu(\nu)(n_{\nu}(\nu) - \Theta(-\nu)) = n,
\]

(13)

where the density of states is \( \nu(\nu) = g|\nu|/(2\pi \hbar v_F^2) \).

These are the main analytical results of this work. They generalize others as the ones given by [38, 39], whose validity is restricted to \( T = 0 \), or those of [24], where only the longitudinal channel (or, equivalently, the density–density response) is taken into account.

### 3. Transverse response and plasmons

So far, we count on equations (8) and (9) for the bare current–current response. Those formulas deal with the case of independent electrons, that is, they cannot manifest the existence of plasmons. However, by means of the random phase approximation (RPA), we can easily obtain a current–current response function which takes into account the interaction between electrons. Reference [25] assures that without neglecting retardation, we can write

\[
\chi_{\nu_{\beta j}}^{\text{RPA}}(q, \omega) = \frac{\chi_{\nu_{\beta j}}^{(0)}(q, \omega)}{1 - d_{\nu_{\beta j}}^0};
\]

\[
\text{for both the longitudinal (} q = \hat{q} \text{) and transverse (} q = q \hat{y} \text{) channels knowing the respective propagators } d_{\nu_{\beta j}}^0 \text{ and } d_{\nu_{\beta j}}^1:\n\]

\[
d_{\nu_{\beta j}}^0 = \frac{q^2 - (\nu_\omega/c)^2}{2 \nu_\omega}; \quad d_{\nu_{\beta j}}^1 = -\frac{\mu^0}{2 q^2}.
\]

(14)

In turn,

\[
q' = \sqrt{q^2 - (\nu_\omega/c)^2}.
\]

(16)

The singularities of equation (14) in the longitudinal (transverse) channel will tell when longitudinal (transverse) plasmons arise. Both correspond to collective current oscillations of the electrons, but the character of the oscillations is quite different due to the parallel (perpendicular) direction of their movement with respect to \( q \) and consequently their (in)dependence on density fluctuations.

Regarding transverse plasmons, the analysis of the denominator of equation (14) was carried out by [32], demonstrating the existence of TE modes in graphene. However, its conclusions are restricted to zero temperature, suspended graphene in vacuum and the local \( q = 0 \) approximation of the conductivity.

Here, we investigate what happens if those simplifications are discarded. Our standpoint will be able to yield the answer to this and many other questions, as outlined in the introduction. In the following sections, it is our aim to offer a better understanding of TE modes in graphene by means of the result for the current–current response functions presented in equations (8) and (9).
Figure 1. Solid lines: dispersion relation for transverse plasmons at $T = 0$ (setting $\text{Im}(\chi_{p,h}^{(0)}(q,\omega)) = 0$). Color: value of the loss function—equation (17)—for a given temperature. Left: $T = 0$; right: $T = 0.02T_F$ (roughly 27 K for $E_F = 0.12$ eV).

Figure 2. Same as in figure 1. Left: $T = 0.04T_F$ (roughly 53 K for $E_F = 0.12$ eV); right: $T = 0.1T_F$ (roughly 134 K for $E_F = 0.12$ eV). Whereas in the first case the dispersion relation structure is still recognizable, in the last one it is completely lost.

3.1. Transverse plasmons and temperature

A suitable tool for inquiring about the dispersion relation of plasmons is the loss function, which can be defined as

$$S(q, \omega) = -\text{Im}(\chi_{p,h}^{\text{RPA}}(\vec{q}, \omega)).$$  \hspace{1cm} (17)

Aside from reproducing the singularities of equation (14) at $T = 0$, it gives information about how they broaden due to damping arising from the bare response (damping coming, in our case, from electron–hole excitations) when $T > 0$ or whenever $\text{Im}(\chi_{p,h}^{(0)}(\vec{q}, \omega)) \neq 0$.

It is already known that density plasmons hold (although, of course, damped) for room temperature [40, 41] or higher [25] ($T \sim T_F$) in the sense that the loss function is not excessively smeared around the singularity present at $T = 0$.

However, transverse plasmons behave in a different manner regarding this aspect. To demonstrate so, let us focus on figures 1 and 2. There, we represent the dispersion relation of transverse plasmons at $T = 0$ with a solid line\(^3\), whereas the colors correspond to the values of the loss function at the specified temperature. It is convenient to plot the difference of frequency with respect to the light cone $cq - \omega$ in units of $E_F/\hbar$ against the wavevector $q$ in units of $v_F k_F/c$.

We can see that at very low temperatures, the loss function still suggests the structure of the dispersion relation of transverse plasmons: they are so far well defined. However, raising $T$ to $0.1T_F$ implies that no trace of them is preserved. This is due to the vicinity of the plasmon dispersion to the region of interband transition and also due to the small spectral weight of the transverse plason suppressed by $1/c^2$ with $c \approx c_F = 300$. The effect of frequency shifting as a consequence of the influence of $T$ is also worth mentioning [25]. Not only does temperature determine the extent to which longitudinal plasmons are damped, but it also displaces the dispersion relation towards higher energies.

\(^3\) $\text{Im}(\chi_{p,h}^{(0)}(\vec{q}, \omega))$ is set equal to 0 for $\omega > E_F/\hbar(2 - q/k_F)$, i.e. outside the Pauli blocking zone, where the loss function is really smeared around the solid line plotted because of electron–hole excitations.
From figures 1 and 2, an analogous result can be inferred for the transverse channel. Nevertheless, here we observe a red instead of a blue shift; the transverse plasmon thus becomes more localized but finally fades out.

### 3.2. Plasmons at zero doping

Another interesting consideration involving temperature is its ability to induce plasmonic excitations at zero doping. When $E_F$ and $T$ equal 0, neither electric nor magnetic modes can be present in a graphene layer. However, finite temperature involves thermally activated electron–hole excitations, allowing longitudinal (slightly damped) plasmons to appear [40]. In fact, the case of $E_F = 0$ and $T > 0$ ($T$ being sufficiently low) can be shown to be equivalent to doped graphene at $T = 0$ with the Fermi energy $E_F \equiv 2 \ln 2 k_BT$ [41]. In other words, the role of temperature is equivalent to inducing a nonzero value of doping.

Thus basically two forces compete in the context of this mechanism to make plasmons appear: the excitation of carriers being favorable to their emergence and the increase of damping claiming to make them vanish. Whereas in the longitudinal case there are well-defined oscillations, we find that for transverse plasmons they are completely washed out. We can understand this by comparing the energy scale set by the temperature $T$ with the energy scale given by $E_F$:

$$k_BT/E_F = 1/(2 \ln 2) \simeq 0.7.$$  \hfill (18)

Since already for $T = 0.1T_F$ there is no clear maximum in the loss function (see figure 2), the scale set by equation (18) seems too high to induce transverse plasmons at zero doping.

Figure 3 confirms this intuition: the loss function for $T = 1$ K is completely diluted and does not reproduce the plasmon dispersion relation respective to $T = 0$ and nonzero doping given by $E_F$. Lowering the temperature even more does not involve any change, which can be expected since equation (18) is a scale-invariant universal result. Thus, we have reached a remarkable conclusion concerning a difference between transverse and longitudinal plasmons: only the latter can be found at zero doping.

### 4. Influence of dielectric media

It is well known how longitudinal plasmons change their dispersion relation when graphene sheets (e.g. monolayer and double layer systems) are embedded between different dielectrics [25]. However, transverse plasmons do not behave similarly even in the simplest case, that is, a single sheet lying on a substrate. They exhibit an extreme sensibility to a slight difference in the refractive index of the two surrounding media to the extent of vanishing for $|n_2 - n_1| \sim 10^{-7}$ at room temperature [42]. This is due to the fact that the dispersion relation of transverse plasmons is extremely pinned to the light cone, such that when two different light cones exist and are sufficiently separated, they rapidly vanish.

Thus to focus on transverse plasmons and inquire about how the dispersion relation can substantially change due to their dielectric surroundings (permittivity and permeability), we will keep the vicinity of the graphene layer homogeneous with the same refractive index $n$. Within this constraint, we can analyze the consequences of modifying $n$.

#### 4.1. Influence of the refractive index

Increasing the refractive index $n$ decreases the speed of light and as a consequence the light cone is shifted to greater values of $q$ for a fixed energy $h\omega$. Since the plasmon relation must lie in the evanescent region where $\omega < cq/n$, the current–current response $\chi^{(2)}(q, \omega)$ involved in the present situation must be evaluated at larger $q$. As we mentioned before, previous discussions of the dispersion relation as the one in [32] would not be valid for high enough values of $n$, since they are only based on the local response ($q = 0$). The relevance of this dependence is shown in figure 4, where the real part of the transverse current response is shown as a function of the energy for $q/k_F \sim 0–0.6$. The remarkable differences between curves imply significant changes in $\omega(q)$. Thus equations (8) and (9) will provide us with the ability to characterize transverse plasmons for high values of the refractive index.

Some results are shown in figure 5, where we focus on undamped plasmons at $T = 0$. They can be compared with the solid black line of figures 1–3, respective to $\epsilon\mu = 1$. It is interesting to note that, as we increase $n$ starting from $n = 1$, the highest value of the curves moves further and further away from the light cone (left-hand side of figure 5). But if we keep increasing $n$, it turns back (right-hand side of figure 5). Therefore, there is a maximal separation of the dispersion relation from the light cone which determines the fastest decay possible of the fields away from the graphene sheet [25]. The confinement of transverse plasmons can thus be increased by...
Figure 4. \( \operatorname{Re}(\chi^{(0)}_{j,h}(q\hat{y},\omega)) \) at \( T = 0 \) for different values of \( q/k_F : 5 \times 10^{-3} \) (blue), 0.15 (red), 0.25 (green), 0.37 (magenta), 0.5 (solid black) and 0.6 (dashed).

Figure 5. Dispersion relation of transverse plasmons for different refractive index \( n \) of the homogeneous dielectric medium embedding graphene. We only show the undamped region at \( T = 0 \). Curves are normalized to \( q_l = n_0\omega/c \) with \( \omega_l \) the lowest frequency for which transverse plasmons appear.

Figure 6. Same as figure 1, but for \( T = 0.02T_F \) and \( n = 20.3 \). The loss function indicates the presence of damped transverse plasmons.

a factor of \( 10^2 \) by embedding the layer within a dielectric with \( n \simeq 40 \).

Regarding now the influence of temperature, there are no substantial differences with respect to the case \( \epsilon\mu = 1 \) (section 4). Figure 6 represents the loss function, which allows to see how at low temperatures the structure of the dispersion relation of plasmons is maintained. Once again, for \( T > 0.1T_F \), it is completely diluted. Following the reasoning stated in equation (18), neither will it be possible to have transverse plasmons for \( E_F = 0 \) induced by a finite \( T_F \).

4.2. Influence of an optical cavity

Density plasmons are an excellent way of enclosing radiation in small regions. These dimensions just depend on the separation of the dispersion relation from the light cone: the decay length can be written as \( \lambda = 2\pi/q' \), with \( q' = \sqrt{q^2 - (n_0\omega/c)^2} \) [25]. For energies of the order of \( E_F/\hbar \), we can neglect retardation effects \( (q' \simeq q) \), yielding \( \lambda \sim 2\pi/k_F \) and decay lengths of the order of 10 nm.

However, once again the situation turns inside out when considering the transverse channel. The proximity to the light cone implies that \( q' \) (even its maximum value, which can be estimated from figure 5) is now much smaller than that of its longitudinal counterpart. For suspended graphene, for example, we find a minimum value for undamped plasmons of \( \lambda \sim 10^{-4} \) m. It is our aim to address the question of the extent to which dodging this limitation is possible and whether confinement can be achieved by relying on a multilayer system.

Let us thus consider a single sheet of graphene embedded between four dielectrics (figure 7). For the sake of simplicity, we will set \( \mu_1 = \mu_4 \rightarrow 0 \) and \( \mu_2 = \mu_3 = 1 \), but keep the velocity of light constant and equal within the whole sample. This makes media 1 and 4 impenetrable by s-polarized electromagnetic waves and one might expect to force a faster decay of the potential vector \( \vec{A} \) in the \( z \)-direction.

To analyze this problem, we make the ansatz

\[
\vec{A}_j(\vec{r},t) = M_j\hat{y}\exp[-q'z + i(qx-\omega t)] + N_j\hat{y}\exp[q'z + i(qx-\omega t)]
\]

(19)

for every medium \( j \), setting \( M_1 = N_1 = M_4 = N_4 = 0 \), and apply Maxwell’s equations in every interface. The transverse
current—current response of graphene arises when attending to the frontier between dielectrics 2 and 3 (vacuum in our case), as well as RPA turns up when writing the surface current as [37] $\mathbf{j}(q, \omega) = \mathbf{j}^{(0)}(q, \omega) / \omega \Lambda(q, \omega)$ (notice the presence of $\mathbf{j}^{(0)}$ instead of $\mathbf{j}$). The resulting system of linear equations for $[M_j, N_j]$ with $j = \{2, 3\}$ will have a nontrivial solution only when its determinant is zero ($d > 0$):

$$2q' + \mu_0 \chi_{j,0}^{(0)}(q^{\dagger}, \omega) \tan(dq') = 0. \quad (20)$$

In that case,

$$N_2 = M_3 = -e^2 dq' M_2; \quad N_3 = M_2; \quad (21)$$

where $M_2$ is related to the field amplitude.

Let us now define the scale for the decay length. Denoting $q'_0$ as the retarded wavevector which is most separated from the light cone and thus related to the maxima in figure 5, we can define $\lambda = 2\pi/q'_0$ as the length scale within which the transverse plasmon will be confined. Solving equation (20) numerically, we find a solution only for layer separation $d > 0.15\lambda$. Thus, transverse plasmons in suspended graphene can be maximally confined to length scales of the order of $10^5$ m, still much larger than the previously commented associated to their longitudinal counterparts. Relaxing the boundary conditions to other values does not significantly change these conclusions.

For short enough distances, it can be seen that the decay in the perpendicular direction to the frontiers differs quite a lot from being exponential, rather becoming practically linear. The plot for several values of $d$ appears in figure 8, closing our discussion about transverse plasmons regarding the spatial decay of the electromagnetic fields attached to them.

To sum it up, this section has mainly remarked on the disparity between longitudinal and transverse plasmons as for confinement in the outskirts of the sheet, the latter being significantly more spread in space. This emerges from the nature underlying their dispersion relation (namely its proximity to the light cone) and can be hardly eluded even with the aid of setups like the one with impenetrable dielectrics shown in figure 7.

5. Absorption in single and double layer systems

In the previous sections, we have highlighted some aspects concerning the evanescent spectrum. Here, we will also cover some others related to the propagating modes, i.e., the absorption of s- and p-polarized light by a single and double layer graphene structure.

\[ \text{Figure 7. Schematic view of the cavity with graphene suspended in air. Media 1 and 4 are semi-infinite dielectrics/superconductors.} \]

\[ \text{Figure 8. Modulus of confined vector field of transverse plasmons for graphene in a cavity. Blue curves correspond to } d/\lambda \text{ equal to 0.17, 0.25, 0.35, 0.5 and 0.6 (starting from the inner curve). For } d \leq 0.15\lambda, \text{ no confined modes exist. Red curve: graphene in free space (media 1 and 4 absent). The length scale is given by } \lambda = 2.65 \times 10^{-4} \text{ m.} \]

5.1. Single layer structures

The amount of energy transmitted, reflected and absorbed is encoded in the Fresnel coefficients which emerge from the application of Maxwell’s equations in the frontier between two dielectrics when a layer of graphene is placed separating them [43, 44]. For s-polarized light, the Fresnel coefficients for the parallel (conserved) component are given by [25]

$$t_s = \frac{2\mu_2 q'_1}{\mu_2 q'_1 + \mu_1 q'_2 + \mu_1 \mu_2 \mu_0 \chi_{j,0}^{(0)}(q^{\dagger}, \omega)}, \quad (22)$$

$$r_s = \frac{\mu_2 q'_1 - \mu_1 q'_2 - \mu_1 \mu_2 \mu_0 \chi_{j,0}^{(0)}(q^{\dagger}, \omega)}{\mu_2 q'_1 + \mu_1 q'_2 + \mu_1 \mu_2 \mu_0 \chi_{j,0}^{(0)}(q^{\dagger}, \omega)}. \quad (23)$$

The absorption is then:

$$A_s = 1 - |t_s|^2 - |r_s|^2 \frac{\mu_1 q'_2}{\mu_2 q'_1}. \quad (24)$$

Equivalent equations are obtained for p-polarized light by replacing $\mu_i \rightarrow q'_i, q'_i \rightarrow \epsilon_i$ and $\chi_{j,0}^{(0)}(q^{\dagger}, \omega) \rightarrow \chi_{j,0}^{(0)}(q^{\dagger}, \omega).$

Their application to the simplest case, i.e., suspended graphene, reveals that the maximum absorption (2.3%) for p-polarization corresponds to normal incidence, whereas when dealing with s-polarization, it is reached when

$$\left( \frac{q_c^*}{\omega} \right)^2 = 1 - \left( \frac{\pi \alpha}{2} \right)^2; \quad \alpha = \frac{e^2}{4\pi \varepsilon_0 \hbar c}. \quad (25)$$

The angle of incidence is given by $\sin \theta = qc/\omega \Rightarrow \theta \simeq 89^\circ$, and the absorption is exactly 50% which resembles the theoretically highest absorption by a single interface [45].

Also for graphene on a substrate, differences between both polarizations are important close to and beyond the light cone. Figure 9 shows the results for a dielectric with $\mu = 1$

\[ \text{Note that these expressions slightly differ from standard textbook notation.} \]
Figure 9. Absorption for graphene on a substrate with $\mu = 1$ and $n = 1.5$ for p- and s-polarized light at $T = 300$ K. Incidence occurs from the substrate such that there is a critical angle for total reflection.

Figure 10. Transmission of incident light through a double layer graphene structure with dielectric media $\epsilon_1 = \epsilon_3 = 5$, $\epsilon_2 = 1$ and $\mu = 1$. Shown is the difference $T_g - T_0$, with $T_g$ ($T_0$) the transmission with (without) graphene. The distance separating the graphene layers is $d = 14.8$ $\mu$m. The light cones respective to media 1 and 2 are denoted by the solid and dotted black lines, respectively.

and $n = 1.5$. It can be realized that in the zone where total reflection should take place in the absence of graphene, some absorption ($\approx 3\%$) is found for p-polarization, but more than twice as much for s-polarization.

5.2. Double layer structures

Other setups with more layers may also yield interesting results. In [25], two sheets of graphene separating three dielectrics with permittivities $\epsilon_1$, $\epsilon_2$ and $\epsilon_1$ ($\epsilon_1 > \epsilon_2$) give rise to perfect transmission between the two light cones in the regime of strong light–matter coupling (for energies of the order of the fine-structure constant times the Fermi energy). The results can be interpreted as surface plasmon mediated extraordinary transmission similar to the one through sub-wavelength apertures [46]. This analysis was carried out for p-polarization and here we want to extend it to s-polarization. Results are shown in figure 10, where no perfect transmission shows up $^5$.

Our interpretation is the aforementioned fragility of transverse plasmons, which vanish for large layer separation in this staging unless the refractive indices $n_1$ and $n_2$ are sufficiently close to each other. As a consequence, there is no means of tunneling s-polarized light through transverse plasmons coupling between layers.

6. Conclusions

In summary, we have calculated the linear current–current response function for graphene at finite temperature and chemical potential. This analytical result enables the characterization and study of plasmons from a quite general standpoint.

This paper was focused on the transverse channel, whose collective oscillations remained not as well studied as the ones of the longitudinal channel. As for them, we have analyzed the strong influence that temperature exerts when compared to their counterparts, in the sense that they vanish much earlier when $T$ increases. Red shifting of the dispersion relation or the infeasibility of inducing their existence at zero doping due to a finite temperature also make for clear differences.

Moreover, the influence of the dielectric surroundings on transverse plasmons has been targeted. The main consequences are encapsulated in the evolution of the dispersion relation as a result of modifying the refractive index in which graphene is embedded. We have described these curves for a wide range of values of the refractive index which made it necessary to go beyond local response and showed that maximal confinement is obtained for $n \approx 40$. This confinement can be enhanced by placing graphene inside a cavity consisting of a perfect diamagnet, i.e., $\mu = 0$. It reaches a maximum for a certain distance below which no transverse plasmons can be sustained which can be used for a sensor.

In section 5, we commented on some aspects of the absorption of electromagnetic radiation by a single and double layer system due to the presence of graphene, where the polarization of light as well as the incident angle give rise to fluctuations up to $\approx 50\%$.

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$^5$ The colored region inside the light cone for $q \to 0$ is due to high reflectivity of graphene at low frequencies.
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