Non-equilibrium Thermodynamics of a Resistance Element

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This paper describes the thermal characterization of a coaxial calculable resistor, part of traceability chain of the capacitance unit to the quantum Hall effect, in construction at Inmetro. The realization of the capacitance unit (farad) is related to the QHE by a calculable coaxial resistor, and three low-uncertainty coaxial bridges: the two-terminal and the four-terminal bridges, already in operation at Inmetro; and the quadrature bridge, in final stage of construction.

Keywords: Measurement, measurement standards, precision measurements, calculable resistor

I. INTRODUCTION

One of the mid-term projects of the Laboratory of Metrology of Electrical Standards at Inmetro (Lampe) is to establish the traceability of the capacitance unit all the way to the quantum Hall effect (QHE) [1]. The traceability chain from the farad to the quantum Hall resistance (QHR) begins with the AC-DC transference of the resistance unit through a calculable resistor. The next steps consist in comparisons in high-precision coaxial in-phase and quadrature bridges [2–4].

Volatility in the value of this calculable resistor can be caused by the degradation of the resistive element surface on welds, which brings as an immediate effect the increase of the contact resistance at junction points and very likely a reduction of the repetitivity of the measurements. Furthermore, if the element finds itself under mechanical strain, there can be a slow migration of macroscopic physical properties of the resistor, which may cause long-term drift [5].

We propose the introduction of a electrically inert support for the resistive element – usually of difficult handling due to its reduced dimensions – in order to allow its better settlement to the internal terminals in soldering, as well as a finer control of its nominal value. To this end, a resistor prototype was built, and some of its physical properties were studied in this work.

In the sections that follow, some brief comments are made on general thermal properties of metals. Details on the resistor element and its support construction are presented in Sec. II. Still in that section, we present a physical characterization of the resistive element, which can be useful for future reference, and some conclusions are left to Sec. III.

II. CONSTRUCTION

In the design adopted by Lampe, the resistor is made of a nickel, chrome, aluminum and silicon alloy, named isaohm. As a rule, the resistor construction must be guided by the symmetries of the electromagnetic fields induced by the resistive element. The trivial symmetry, which gives rise to the most simple construction, is the cylindrical one [3, 6]. The coaxial resistor made at Inmetro follows the 4-shutter, 2-plane design described in Refs. [7, 8]. The resistor element consists of a isaohm wire of ϕ0.014 mm, approximately 11 cm long. A few physical properties of isaohm are presented in Ref. [9].

A sheet of epoxy laminate of width 1.60 mm and plane dimensions 1.030×16.700 cm was cut in order to host two copper islands 11.080 cm apart. The two ends of the isaohm wire that make the resistive element are then locked to the islands, together with ordinary ϕ1 mm copper wires to allow resistance measurements in the 4-terminal scheme.

The resistor so constructed has nominal value 1080.02 Ω, and final linear resistivity about 8.5% above the result of measurements performed before the wire soldering process. This set makes the test prototype for the calculable resistor to be used in the derivation of the AC ohm from QHR measurements [10].

A. Thermal Inertia and Characteristic Time

The resistive element described in Sec. II has thermal inertia and capacity which depend on equilibrium thermodynamic properties, such as specific heats, and bulk and surface transport properties, such as the electrical and thermal conductivities, and heat transference rates. If the frequency f of the function applied to the resistor (voltage or current) is such that f−1 << τ (where τ is the thermal stabilization time of the resistor), the thermal oscillations induced by thermoelectric effects [11] become irrelevant in practice.

This makes the high thermal integration time a desirable property of resistors employed in electrical standardization, at the same time that τ becomes an important construction parameter. A rough estimate of the order

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of magnitude of \( \tau \) can be realized through a lumped parameter model for the heat conduction in the resistive element analogous to the RC electrical circuit (see Fig. 1). According to this model \[^{[12]}\]

\[
\tau \sim \ell c_V \rho/h,
\]

where \( \ell \) is the resistive element length, \( c_V \) is the specific heat of the isohm alloy, \( \rho \) its mass density and \( h \) the convection heat transfer rate through its surface. To isohm, \( c_V \approx 0.46 \, J \cdot K^{-1} \cdot g^{-1} \) at \( 20^\circ C \); \( \rho \approx 8 \cdot Kg \cdot m^{-3} \) \[^{[9]}\]; and \( h \sim 0, 10 \, J \cdot m^{-2} \cdot K^{-1} \cdot s^{-1} \) \[^{[12]}\], and it follows that \( \tau \sim 400 \, s \). Under these conditions, thermal oscillations are integrated for frequencies above \( \tau^{-1} \sim 2 \cdot 10^{-3} \, Hz \), while capacitance bridge measurements are usually taken at \( 10^4 \, rad/s \) (approximately \( 1.6 \, kHz \)) \[^{[13]}\].

### B. Thermal Characteristics

An active temperature control setup allowed that measurements were performed under periodic fluctuations of temperature. The system consists basically of a periodic signal generator that commands the thermostat of a chamber, in which the test resistor is contained. Four-terminal resistance measurements are then made through a 8-digit multimeter (Fig. 2).

This kind of measurement conditions, taken together with the lumped parameter model adopted for heat conduction in the resistive element \[^{[12]}\], enabled us to estimate, even if only roughly (because of the low resolution of the environmental control thermostats), the characteristic time for thermal stabilization \( \tau \), defined in Sec. II A. Still according to the model, we expect that

\[
\tau^{-1} = \omega \tan^{-1} (\theta),
\]

where \( \theta \) is the phase difference between the temperature and resistance measurements made on the resistor; and \( \omega \) is the oscillation frequency, given in \( rad \cdot s^{-1} \).

Figure 3 shows how does normalized electrical resistance \( R \) (superior curve) and multimeter internal temperature \( T \) as functions of time \( t \). From phase difference measurements on the curves showed in this picture, we obtain \( \tau \approx 420 \, s \), which is in excellent agreement with the previous estimate made in Sec. II A.

and swings in phase with the external laboratory temperature, as we verified from direct measurements. The mean external and multimeter internal temperatures are \( 23.0 \, ^{\circ}C \) and \( 34.7 \, ^{\circ}C \), respectively.

From the phase difference measurements between the curves showed in this figure, we obtain \( \omega \approx 2.51 \cdot 10^{-3} \, rad \cdot s^{-1} \) and \( \theta \approx 1.4 \, rad \), which gives \( \tau \approx 420 \, s \). This estimate, made entirely from resistance and temperature time evolution measurements on the test prototype, is in excellent agreement with the estimate made on Sec. II A, entirely based on simple dimensional analysis.
III. CONCLUSIONS

At the early stages of the construction of a prototype calculable resistor, we have run measurement tests under conditions of periodic varying temperature. The resistor behavior when subjected to such perturbation allowed the estimation of the characteristic time $\tau$ for thermal stabilization from the time dispersion of resistance measurements.

However, due to the cylindrical construction symmetry of the resistor, $\tau$ can also be independently estimated from thermodynamic (equilibrium) and unsteady-state transport properties, as the heat transference rate or the electrical and thermal conductivities, using a simple lumped parameter model, such as the ones described in Ref. [12], for instance.

Both estimates of $\tau$ were compared, and good agreement was found. This suggests the application of such models in the study and characterization of calculable resistors such as Lampe’s, with only but very few restrictions, if any whatsoever. Among other things, the estimation of $\tau$ can enable us to get a good grip of the stabilization times of calibration systems, or aid us to correctly quantify thermal properties of standard impedances in non-steady-state regimes, usually of difficult characterization.

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