Consistent QFT description of non-standard neutrino interactions

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Neutrino oscillations are precision probes of new physics beyond the Standard Model. Apart from neutrino masses and mixings, they are also sensitive to possible deviations of low-energy interactions between quarks and leptons from the Standard Model predictions. In this paper we develop a systematic description of such non-standard interactions (NSI) in oscillation experiments within the quantum field theory framework. We calculate the event rate and oscillation probability in the presence of general NSI, starting from the effective field theory (EFT) in which new physics modifies the flavor or Lorentz structure of charged-current interactions between leptons and quarks. We also provide the matching between the EFT Wilson coefficients and the widely used simplified quantum-mechanical approach, where new physics is encoded in a set of production and detection NSI parameters. Finally, we discuss the consistency conditions for the standard NSI approach to correctly reproduce the quantum field theory result.

Introduction. Precision measurements at low energies are sensitive probes of fundamental interactions, which complement collider searches. Neutrino oscillation experiments [1] are a specific class thereof where one observes a characteristic oscillatory dependence of the neutrino detection rate as a function of the neutrino energy and the distance between the neutrino source and detector. The large body of oscillation data so far has established the existence of at least three distinct neutrino states with different masses [2,3], which is consistent with the predictions of the Standard Model (SM) supplemented with dimension-5 terms leading to Majorana masses for the SM neutrinos [4]. Within this paradigm, the neutrino mass squared differences and the angles of the PMNS mixing matrix have been measured with good accuracy. This opens the door to also probe and constrain new physics (NP), by which we mean NSI between neutrinos and matter that arise from physics beyond the SM (BSM) [5-21]. To this end, however, one needs a map between fundamental parameters of BSM models and observables in oscillation experiments. In this paper we construct such a map for the EFT of the SM degrees of freedom, in which NP modifies the charged-current interactions between neutrinos, charged leptons, and quarks. We also discuss the consistency conditions for the widely used simplified approach, where NP is parametrized by a set of NSI production and detection parameters, to correctly reproduce the quantum field theory (QFT) result.

QFT description. Oscillation probability can be rigorously derived in the framework of quantum field theory. Various derivations are available in the literature in the absence of NSI (see e.g. [25,27]). Below we give an expression valid for completely general interactions between neutrinos and matter. Consider neutrinos produced in the process $S \rightarrow X_{\alpha} \nu$ (e.g. beta decay of a nucleus in a reactor, or pion decay), where $X_{\alpha}$ is one or more body final states containing one of the charged leptons $\ell_{\alpha} = (e, \mu, \tau)$. The neutrinos are detected via the process $\nu T \rightarrow \bar{Y}_{\beta}$ (e.g. inverse beta decay), where $Y_{\beta}$ contains a charged lepton $\ell_{\beta}$. The production and detection can be described by QFT amplitudes $M_{\alpha k}^p \equiv M(S \rightarrow X_{\alpha} \nu_k)$ and $M_{\beta \ell}^D \equiv M(\nu_T \rightarrow Y_{\beta})$, where the index $k$ labels neutrino mass eigenstates. The information about fundamental parameters, is encoded in $M_{\alpha k}^p$ and $M_{\beta \ell}^D$, which should be then connected to observables. For the source (S) and target (T) states separated by a macroscopic distance $L$, the observable is the differential rate of detected events $R_{\alpha \beta} \equiv dN_{\alpha \beta}/dt dE_{\nu}$ given by

$$R_{\alpha \beta} = \frac{\kappa}{E_{\nu}} \sum_{k,l} e^{-i \frac{m_{\nu_k}^2 \cdot \delta}{2E_{\nu}}} \int d\Pi_P d\Pi_D M_{\alpha k}^P M_{\beta \ell}^D \int d\Pi_D d\Pi_{\beta k} M_{\beta k}^D M_{\beta \ell}^D$$

where $\kappa = N_S N_T / (32\pi L^2 m_{S} m_{T})$. A compact derivation of this formula is presented in Appendix A, where we also enumerate its limitations. Above, $\Delta m_{\nu_k}^2 \equiv m_{k}^2 - m_{l}^2$ is the mass squared difference between the neutrino eigenstates. The phase space elements $d\Pi_P d\Pi_D$ for the production and detection processes are defined in the standard way: $d\Pi = \frac{d^3 k_{E_{\nu}}}{(2\pi)^3 2E_{\nu}} \cdot \frac{d^3 k_{E_{\nu}}}{(2\pi)^3 2E_{\nu}} (2\pi)^4 g^4(\mathcal{P} - \sum k_i)$, where $\mathcal{P}$ is the total 4-momentum of the initial state and $k_i$ are the 4-momenta of the final states. For the production $d\Pi_P$ includes the neutrino phase space $\frac{d^3 k_{E_{\nu}}}{(2\pi)^3 2E_{\nu}}$ and we define $d\Pi_{P} \nu$ via $d\Pi_{P} \nu \equiv d\Pi_{P} dE_{\nu}$. The $\int$ sign in Eq. (1) involves integration and sum/average over all unobserved degrees of freedom, such as angular variables and spins. Finally, complex conjugated amplitudes are denoted with a bar, $N_{S,T}$ are the number of source and target particles, $m_{S,T}$ are their masses, and $E_{\nu}$ is the neutrino energy, which is an observable in typical experiments.

The rate in Eq. (1) involves integration and sum/average over all unobserved degrees of freedom, such as angular variables and spins. Finally, complex conjugated amplitudes are denoted with a bar, $N_{S,T}$ are the number of source and target particles, $m_{S,T}$ are their masses, and $E_{\nu}$ is the neutrino energy, which is an observable in typical experiments.
trino differential flux $\Phi_\alpha = \frac{N_\alpha N_T}{4 \pi L^2} \frac{d\Phi^F}{dm_{\nu_\alpha}}$, and the detection cross section at the target. The source decay rate $\Gamma_\alpha^P$ (with an emission of $\ell_\beta$ and summed over neutrino mass eigenstates) and the detection cross section $\sigma_\beta^D$ (with an emission of $\ell_\beta$ and summed over neutrino mass eigenstates) can be calculated by the usual means in QFT. We have

$$\Phi_\alpha \sigma_\beta = \frac{\kappa}{\mathcal{E}_\nu} \int d\Pi^P \sum_{k} |\mathcal{M}_{\alpha k}|^2 \int d\Pi_D \sum_{l} |\mathcal{M}_{\beta l}|^2.$$  \hspace{1cm} (2)

One can define the $\nu_\alpha \rightarrow \nu_\beta$ oscillation probability as the ratio of the rate of detected events in Eq. (1) to the no-oscillation expression in Eq. (2), finding

$$P_{\alpha \beta} = \sum_{k,l} e^{-i \frac{\Delta m_{\alpha \beta}^2}{2E_{\nu}}} \int d\Pi^P \mathcal{M}_{\alpha k}^* \mathcal{M}_{\alpha l}^T \int d\Pi_D \mathcal{M}_{\beta l}^* \mathcal{M}_{\beta k}.$$  \hspace{1cm} (3)

This formula appears in Ref. [28] in a slightly different form without explicit phase space integration. Oscillation probability is an intuitive and widely employed concept, however strictly speaking $P_{\alpha \beta}$ is not an observable. For this reason in the rest of this paper we work with the rate in Eq. (1).

**QM-NSI description.** The machinery of QFT is rarely employed in the neutrino literature. Instead, a simpler quantum mechanical (QM) description of neutrino oscillations is most often used. One defines flavor states as linear combinations of mass eigenstates: $|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle$, where $U$ is the unitary PMNS mixing matrix. In this language, the NSI effects are encoded in the matrix parameters $\epsilon_{\alpha \beta}^{s,d}$, which correspond to non-standard effects in neutrino production and detection, respectively [19][29][30]. They are defined by the mismatch between pure flavor states and the neutrino states produced at the source and detected at the target, namely [2]:

$$|\nu^s_\alpha\rangle = \frac{(1 + e^\epsilon)}{N^s_\alpha} |\nu_\alpha\rangle, \quad |\nu^d_\alpha\rangle = \frac{(1 - e^\epsilon)}{N^d_\alpha} |\nu_\alpha\rangle,$$  \hspace{1cm} (4)

with the normalization $N^s_\alpha = \sqrt{[(1 + e^\epsilon)(1 + e^{\epsilon^*})]_{\alpha \alpha}}$, $N^d_\alpha = \sqrt{[(1 + e^{\epsilon^*})(1 + e^\epsilon)]_{\alpha \alpha}}$. For anti-neutrinos Eq. (1) holds with $|\nu^s_\alpha\rangle \rightarrow |\nu^d_\alpha\rangle^*$. The probability of $|\nu^s_\alpha\rangle$ oscillating into $|\nu^d_\beta\rangle$ is given by $P^\text{QM}_{\alpha \beta} = |\langle \nu^d_\beta | e^{-iH\Delta t} | \nu^s_\alpha \rangle|^2$, where in the absence of the matter effects in the propagation the Hamiltonian is $H_\beta_\alpha = \sum_k U_{\alpha k} m_{\nu_k}^2 U^*_{\beta k}/(2E_\nu)$. In this approach, which we refer to as the QM-NSI formalism, the event rate is then given by [2]

$$R^\text{QM}_{\alpha \beta} = \Phi^\text{SM}_\alpha \sigma^\text{SM}_\beta P^\text{QM}_{\alpha \beta} (N^s_\alpha N^d_\beta)^2$$  \hspace{1cm} (5)

$$= \Phi^\text{SM}_\alpha \sigma^\text{SM}_\beta \sum_{k,l} e^{-i \Delta m_{\alpha \beta}^2} [x^s_{\alpha k} x^s_{\alpha l} x^s_{\beta k} x^s_{\beta l}],$$

where $x^s_{\alpha \beta} \equiv (1 + e^\epsilon^*) U^*$ and $x^d_{\alpha \beta} \equiv (1 + e^\epsilon) T U$. Above, $\Phi^\text{SM}_\alpha$ and $\sigma^\text{SM}_\beta$ are the incident flux and detection cross section calculated in the absence of NSI. The normalization factors $N^s_\alpha N^d_\beta$ cancel in the observable rate and thus one could have omitted them altogether [2]: their only role is to ensure that $P_{\alpha \beta} \leq 1$, that is it can be interpreted as a probability.

Results from oscillation experiments are often presented or recast as constraints on the NSI parameters $\epsilon_{\alpha \beta}^{s,d}$. However, the utility of the latter hinges on whether they can be unambiguously connected to more fundamental parameters of the microscopic theory. Only after such matching the coefficients $\epsilon_{\alpha \beta}^{s,d}$ determined in different experimental settings can be meaningfully compared and combined. In the following we discuss this issue, and illustrate it with physically relevant examples. We will define the conditions under which the NSI parameters can indeed provide an adequate description of NP effects in neutrino oscillation. Conversely, we will show examples where this is not the case.

**Matching QFT and QM-NSI results.** One could try to match the QM-NSI and QFT language starting from the definition in Eq. (1). This however would be problematic, as such concepts as neutrino flavor states or production and detection states are murky in a QFT framework when general neutrino interactions with matter are allowed. Therefore, we will follow a pragmatic approach and match the observable rates predicted by the QFT (Eq. (1)) and QM-NSI frameworks (Eq. (5)). This comparison will allow us to determine the map between the NSI $\epsilon^{s,d}$ and the Lagrangian parameters, or else conclude the map does not exist.

In this paper we focus on NP in charged current interactions between neutrino and matter. The microscopic theory we consider is the EFT of the SM degrees of freedom at the energy scale $\mu \approx 2$ GeV, in which NP modifies the effective 4-fermion interactions between leptons and quarks (extensions to other theories and interactions are straightforward using our approach). At leading order in this EFT the neutrino interactions with matter can be parametrized by the Lagrangian

$$\mathcal{L} = - \frac{2V_{ud}}{u^2} \left\{ (1 + \epsilon \gamma^5)|\bar{\nu}_\alpha\rangle \gamma^\mu P_L |\bar{\ell}_\beta\rangle \bar{\ell}_\alpha \gamma_\mu P_L |\nu_\beta\rangle + \frac{1}{2} |\bar{\nu}_\alpha\rangle \gamma^\mu P_R |\bar{\ell}_\beta\rangle \bar{\ell}_\alpha \gamma^\mu P_R |\nu_\beta\rangle - \frac{1}{2} \frac{1}{2} |\bar{\nu}_\alpha\rangle \gamma^\mu P_L |\bar{\ell}_\beta\rangle \bar{\ell}_\alpha \gamma^\mu P_L |\nu_\beta\rangle + \frac{1}{4} |\bar{\nu}_\alpha\rangle \gamma^\mu P_R \gamma^\mu P_L |\bar{\ell}_\beta\rangle \bar{\ell}_\alpha \gamma^\mu P_L |\nu_\beta\rangle \right\},$$  \hspace{1cm} (6)

where $v \equiv (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV, $V_{ud}$ is a CKM matrix element, $\sigma^{\mu \nu} = i[\gamma^\mu, \gamma^\nu]/2$, and $P_{LR}$ are the usual chirality projectors $(1 \mp \gamma_5)/2$. The quarks $u, d$, and charged leptons $\ell_\alpha$ are in the basis where their kinetic and mass terms are diagonal. For the neutrino fields the kinetic terms are diagonal but the mass terms are not: they are connected to the mass eigenstates by the unitary rotation via the PMNS matrix, $\nu_\alpha = \sum_k U_{\alpha k} \nu_k$. In this EFT the effects of NP are parametrized by the Wilson coefficients $[\mathcal{X}]_{\alpha \beta}$, which encode new interactions between quarks and leptons mediated by BSM particles heavier than $\sim 2$ GeV. For example, non-zero $\epsilon_R$ can arise in left-right symmetric models due to the $W'$
boson coupling to right-handed quarks and mixing with the SM $W$, while non-zero $\epsilon_{S,P,T}$ are generally predicted in leptoquark models. More generally, $\epsilon_X$ can be connected to parameters of the weak-scale EFT, known as the SMEFT \cite{31,32,33}.  

\text{SM interactions}. To warm up, let us first calculate the oscillation probability in the limit of SM interactions, which corresponds to setting all $\epsilon_X = 0$. In this case, which was studied in Ref. \cite{25}, the amplitudes can be decomposed as $M_{\alpha k}^{D(P)} = V_{\alpha k}^{(s)} A_{\alpha k}^{D(P)}$. The functions $A_{\alpha k}^{P,D}$ are independent of the neutrino mass index $k$ up to negligible corrections, whereas they do depend on the charged-lepton flavor index $\alpha$ (which we omit to ease the notation). They also depend on the kinematic and spin variables in the production and detection processes, and they appear in the observables integrated/averaged over by $\int d\Pi_{P,D}$. All in all the rate in Eq. (1) can be written as  
\begin{equation}
R_{\alpha \beta}^{\text{SM}} = \Phi_\alpha^{\text{SM}} \sigma_\beta^{\text{SM}} \sum_{k,l} e^{-\frac{\Delta m_{kl}^2}{2E_{\nu}}} U_{\alpha k}^* U_{\beta l} U_{\beta l}^* U_{\beta l}^*,
\end{equation}
where the SM flux and cross-section are given by  
\begin{equation}
\phi_\alpha^{\text{SM}} = \frac{N_S \int d\Pi_P |A_P|^2}{8\pi \sin^2 \theta_C L^2}, \quad \sigma_\beta^{\text{SM}} = \frac{N_T \int d\Pi_D |A_D|^2}{4E_{\nu} m_T}.
\end{equation}

Exactly the same result is obtained from Eq. (5) in the limit $\epsilon_{\alpha \beta}^{s,d} = 0$.  

\text{V-A interactions}. A less trivial example is when NP enters only via V-A interactions: $[\epsilon_L]_{\alpha \beta} \neq 0$ \cite{5,6,34}. In the V-A case the map between NSI and Lagrangian parameters is well-defined, unambiguous, and simple. The NSI parameters for production and detection are the same up to Hermitian conjugation.  

General case. For general NP interactions in Eq. (6), the production and detection amplitudes can be decomposed as  
\begin{equation}
M_{\alpha k}^{P} = U_{\alpha k}^* A_{\alpha k}^P + \sum_{X} [\epsilon_X U]_{\alpha k}^* A_{X}^P,
\end{equation}
\begin{equation}
M_{\beta k}^{D} = U_{\beta k} A_{\beta k}^D + \sum_{X} [\epsilon_X U]_{\beta k} A_{X}^D.
\end{equation}

The sum above goes over all types of interactions in Eq. (6): $X = L, R, S, P, T$. We stress that $A_{X}^{P,D}$ will typically have completely different dependence on kinematic and spin variables for different $X$. Plugging this decomposition into Eq. (1) we obtain a lengthy expression, which we nevertheless quote in full:

\begin{equation}
R_{\alpha \beta} = \Phi_\alpha^{\text{SM}} \sigma_\beta^{\text{SM}} \sum_{k,l} e^{-\frac{\Delta m_{kl}^2}{2E_{\nu}}} [U_{\alpha k}^* U_{\beta l} + p_{XL}(\epsilon_X U)_{\alpha k} U_{\beta l} + p_{XL} U_{\alpha k}^* (\epsilon_X U)_{\beta l} + p_{XY}(\epsilon_X U)]_{\alpha k} (\epsilon_Y U)_{\beta l}]
\end{equation}

where we define the production and detection coefficients  
\begin{equation}
p_{XY} = \frac{\int d\Pi_P A_P^* A_X^P}{\int d\Pi_P |A_P|^2}, \quad d_{XY} = \frac{\int d\Pi_D A_D^P A_X^D}{\int d\Pi_D |A_D|^2}.
\end{equation}

We show in Appendix \[\text{I}\] the expressions of the above coefficients for different processes. For anti-neutrinos Eq. (12) holds with $U \leftrightarrow U^*$ and $\epsilon_X \leftrightarrow \epsilon_X^\dagger$. The formulas for the neutrino oscillation probability are collected in Appendix \[\text{C}\].

At the linear level in $\epsilon$ the QFT expression in Eq. (12) matches the QM-NSI one in Eq. (5), provided the NSI parameters are expressed by the EFT parameters as  
\begin{equation}
\epsilon_{\alpha \beta}^s = \sum_X p_{XL} [\epsilon_X]_{\alpha \beta}, \quad \epsilon_{\beta \alpha}^d = \sum_X d_{XL} [\epsilon_X]_{\alpha \beta}.
\end{equation}

Therefore the QM-NSI formalism can approximate the correct oscillation probability obtained from the general EFT as long as the deviation from the SM, encoded in the coefficients $[\epsilon_X]_{\alpha \beta}$, is sufficiently small. If non-V-A interactions are involved, the NSI parameters obtained via the matching in Eq. (12) may be a function of the neutrino energy. The production and detection parameters depend on the same $\epsilon_X$ parameters, but they do not satisfy anymore the relation $\epsilon^s = \epsilon^d$ valid for the V-A case.

Beyond the linear approximation the QM-NSI formalism fails in general because no matching can be found to connect with the QFT result. This is one of our main results. The consistency condition for the matching in
The observable in Eq. (1) may depend on NP in two distinct ways. One is direct, e.g. through a dependence of the production and detection amplitudes \( \mathcal{M}^{P,D} \) on the NP parameters \( \epsilon_{X,Y} \) of the Lagrangian in Eq. (4). The other is indirect, due to NP "polluting" the observable used for determination of the SM parameters. This is the case for the CKM parameter \( V_{ud} \) in Eq. (6). If NP is present, \( \beta \) decay experiments determine a combination of \( V_{ud} \) and \( \epsilon_{X,Y} \) parameters, and in this case the value of \( V_{ud} \) cannot be just taken from PDG. This indirect effect is ignored in most of the prior neutrino literature, even though it is of the same order as the direct effects, leading to incorrect results. For instance, indirect and direct effects generated by the coefficient \( |\epsilon_{X}e\beta| \) cancel at all orders, making this coefficient unobservable in solar oscillation experiments.

The rate in Eq. (1) is decomposed into the product of the oscillation probability (Eq. (3)) and the no-oscillation result (Eq. (2)). From the general QFT viewpoint this decomposition may seem artificial, as the rate in Eq. (1) is directly observable in neutrino experiments. Nonetheless, there are advantages of defining the oscillation probability (Eq. (3)) and the no-oscillation result (Eq. (2)).
Neutrino Process & NSI Matching with EFT \\
\(\nu_e\) produced in beta decay & \(\epsilon_{e\beta}^s = [\epsilon_L - \epsilon_R - \frac{m_2}{g_A} \frac{m_2}{g_A} (\epsilon_L^T \epsilon_L)_{e\beta}\) \\
\(\nu_e\) detected in inverse beta decay & \(\epsilon_{\beta e}^d = [\epsilon_L + \frac{1 - 3g_5^2}{1 + 3g_5^2} (R - \frac{m_2}{g_A} (\epsilon_L^T \epsilon_L)_{e\beta} - \frac{3g_5}{1 + 3g_5^2} \epsilon_S)]_{e\beta}\) \\
\(\nu_\mu\) produced in pion decay & \(\epsilon_{\mu\beta}^s = \left[\epsilon_L - \epsilon_R - \frac{m_2^2}{m_\mu (m_\mu + m_\tau)} (\epsilon_L^T \epsilon_L)_{\mu\beta}\right]\)

**TABLE I.** Summary of the matching between NSI parameters and EFT Wilson coefficients. See Appendix for further details and discussion of the validity of the QM-NSI approach in each case. In our conventions the matching is the same for neutrinos and antineutrinos.

We find that there are no such “zero-distance effects” at linear order. Let us note that in the \(\alpha = \beta\) case the rate itself is affected by linear effects in \(|\epsilon_X|_{\alpha\alpha}\), but they come from NP modifying the neutrino flux and detection cross-section in Eq. (2). At quadratic order, zero-distance effects do appear in general. On the other hand, they vanish at all orders in the \(\alpha = \beta\) case with \(V-A\) interactions, i.e. \(P_{\alpha\alpha}^{V-A}(L = 0) = 1\). Our results are relevant for the study of zero-distance effects since they are quadratic and, in the \(\alpha = \beta\) case, necessarily non-\(V-A\).

In conclusion, the main results of this paper are: i) The expression in Eq. (12) for the event rate in neutrino oscillation experiments including nonstandard charged-current interactions described by the EFT Lagrangian in Eq. (6); ii) The matching in Eq. (14), valid at the linear level in NP, between the EFT coefficients that describe the underlying interactions and the QM-NSI parameters; iii) The consistency condition in Eq. (15) for that matching (and the simplified QM-NSI approach itself) to be valid to all orders in NP parameters.

Our results are particularly relevant for analyses of oscillation data when effects of non-\(V-A\) physics (or equivalently \(\epsilon^s \neq \epsilon^d\)) are taken into account [7, 9, 11, 13, 15, 17]. We clarify the microscopic meaning and validity of the long list of existing analyses of oscillation data carried out within the traditional QM-NSI approach. Their discovery potential can now be consistently analyzed and compared, among themselves and together with non-oscillation probes that are sensitive to the same underlying physics.

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Appendix A: Oscillations in QFT

In this appendix we derive the master formula in Eq. (1) describing the number of neutrino events detected at a distance $L$ from the source, taking into account possible neutrino oscillations and nonstandard charged-current interactions. Our approach follows similar steps as Ref. [25]. The two main differences are: 1) we allow for general non-SM charged-current interactions in neutrino production and detection, and 2) we work with time-independent packets for the source and target particles, which greatly simplifies further mathematical transformations. Of course, the source is necessarily unstable, hence the latter assumption will lead to one subtlety in the derivation below.

We consider an experimental setup where neutrinos are produced in a process $A_x \rightarrow X_\alpha \nu$, and detected via the process $\nu B_y \rightarrow Y_\beta$. Here $X_\alpha$ and $Y_\beta$ are $n_x$- and $n_y$-body final states ($n_i \geq 1$). The indices $\alpha$ and $\beta$ indicate that these states contain charged lepton $\ell_\alpha$ and $\ell_\beta$ respectively, but otherwise their precise identity is irrelevant for this discussion. $A_x$ and $B_y$ are both one-body particle states localized in the coordinate space, describing the neutrino source (e.g. a beta-decaying nucleus in a reactor) and target (e.g. a proton in a detector). We will work in the time-independent approximation where the states $A_x$ and $B_y$ are represented by wave-packets of scattering in-states which do not change in time. We parametrize them as

$$|A_x\rangle = \int \frac{d^3p_A}{\sqrt{2E_A(2\pi)^3}} \phi_x(p_A)|\vec{p}_A\rangle_{\text{in}}, \quad |B_y\rangle = \int \frac{d^3p_B}{\sqrt{2E_B(2\pi)^3}} \phi_y(p_B)|\vec{p}_B\rangle_{\text{in}}, \quad (A1)$$

where $E_j = \sqrt{m_j^2 + |p_j|^2}$, for $j = A,B$, $|\vec{p}_j\rangle_{\text{in}}$ are momentum eigenstates normalized as $\langle \vec{q}_j | \vec{p}_j \rangle = (2\pi)^3 2E_j \delta^3(\vec{p}_j - \vec{q}_j)$, and the states are normalized as $\langle A_x | A_x \rangle = |B_y \rangle \langle B_y | = 1$. For simplicity we choose Gaussian wave packets for both states with the same spread $\sigma$ in the position space:

$$\phi_x(p) = (2\sigma \sqrt{\pi})^{3/2} \exp (-|\vec{p}|^2 \sigma^2/2 + i\vec{p} \vec{z}). \quad (A2)$$

The wave packet describes a particle at rest localized near $\vec{z}$ with the uncertainty of order $\sigma$.

The idea is to treat the neutrino production and detection together as a single process [25]:

$$A_x B_y \rightarrow X_\alpha Y_\beta, \quad (A3)$$

rather than consider the neutrino production and detection separately. In this approach, neutrino is merely an intermediate particle in the amplitude. The outgoing states are approximated by pure momentum eigenstates with the eigenvalues $\vec{k}_i, \, i = 1 \ldots n$, where $n = n_x + n_y$ is the number of particles in the final states. We are interested in the transition probability for this process:

$$N_{\alpha \beta} = |\langle X_\alpha Y_\beta | A_x B_y \rangle|^2 = \Pi_i \int \frac{d^3k_i}{(2\pi)^3} |_{\text{out}} \langle k_1 k_2 \ldots k_n | A_x B_y \rangle|^2. \quad (A4)$$

Plugging the wave packets for the initial states, and using $\langle_{\text{out}} \langle k_1 k_2 \ldots k_n | p_A p_B \rangle_{\text{in}} = (2\pi)^4 \delta^4(p_A + p_B - \sum k_i)\mathcal{M}$ we obtain

$$N_{\alpha \beta} = \frac{1}{(2\pi)^8} \int \frac{d^3p_A}{\sqrt{2E_A}} \frac{d^3p_B}{\sqrt{2E_B}} \frac{d^3p'_A}{\sqrt{2E_A}} \frac{d^3p'_B}{\sqrt{2E_B}} \phi_x(p_A)\phi_y(p_B)\phi_x(p'_A)\phi_y(p'_B) \delta^4(p'_A + p'_B - p_A - p_B) d\Pi_n \mathcal{M} \mathcal{M}', \quad (A5)$$

where $d\Pi_n = (2\pi)^4 \delta^4(p_A + p_B - \sum k_i)\Pi_i\int \frac{d^3k_i}{(2\pi)^3 \sqrt{2E_i}}$ is the n-body phase space for the final-state particles, and $\mathcal{M} \equiv \mathcal{M}(p_A p_B \rightarrow k_1 \ldots k_n)$, $\mathcal{M}' \equiv \mathcal{M}(p'_A p'_B \rightarrow k_1 \ldots k_n)$ are the usual amplitudes calculated by Feynman diagrams. Tacitly, $N_{\alpha \beta}$ involves sum/average over all non-observed degrees of freedom, such as polarizations of the initial- and final-state particles.

Up to this point, we followed the classic derivation of the cross section formula, see e.g. Ref. [41]. What distinguishes the case at hand is the particular choice of the initial states $\langle A_x | \, B_y \rangle$ describing two spatially separated particles (rather than head-on beams as in the cross section case). Furthermore, the amplitude for this process is dominated by the kinematic region where $q \equiv p_A - p_X = p_Y - p_B$ is close to the neutrino mass shell, $q^2 \approx m_\nu^2$. In that region, unitarity requires the amplitude to factorize into the production, detection, and propagation parts:

$$\mathcal{M} = \sum_k \mathcal{M}(p_A \rightarrow k_X q_{kq}) \mathcal{M}(q_{0k} p_B \rightarrow k_{Yq}) \equiv \sum_k \frac{\mathcal{M}_{\alpha k} \mathcal{M}^B_{k\beta}}{q^2 - m_k^2 + i\epsilon}, \quad (A6)$$

where the sum goes over all neutrino eigenstates, and the amplitudes in the numerator are evaluated for all particles on-shell, including the neutrinos. We can also factorize the phase space: $\int d\Pi_n = \int \frac{d^3q}{2\pi} d\Pi_P d\Pi_D$, where the first
factor is the $X+$neutrino phase space, next, it is convenient to isolate the phases in the wave packets by rewriting $\phi_\alpha(p) = \phi(p)e^{i\vec{p}\vec{z}}$. Finally, we trade one delta function for a time integral using $\delta(E_A + E_B - E'_A - E'_B) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(E_A + E_B - E'_A - E'_B)t}$. This leads to

$$N_{\alpha\beta} = \frac{1}{(2\pi)^3} \int dt \frac{d^3p_A}{\sqrt{2E_A}} \frac{d^3p_B}{\sqrt{2E_B}} \phi(p_A) \tilde{\phi}(p_B) \delta(p'_A + p'_B - \tilde{p}_A - \tilde{p}_B)$$

where $\tilde{L} = \tilde{y} - \tilde{x}$. The next step is to perform the $q_0^2$ integral, treating it as a contour integral:

$$N_{\alpha\beta} = \frac{1}{(2\pi)^3} \int dt \frac{d^3p_A}{\sqrt{2E_A}} \frac{d^3p_B}{\sqrt{2E_B}} \phi(p_A) \tilde{\phi}(p_B) \delta(p'_A + p'_B - \tilde{p}_A - \tilde{p}_B)e^{i(E_A + E_B - E'_A - E'_B)t}$$

where $\Delta m_{kl}^2 = m_k^2 - m_l^2$. Above, the amplitudes are now evaluated at $q_0 = \sqrt{|q|^2 + m_f^2}$, that is for on-shell neutrinos.

At this point we introduce a number of approximations that are appropriate for the description of broad classes of real-life neutrino experiments:

1. The intermediate neutrinos are relativistic, hence in the production and detection amplitudes we can set $q_0 = |q|$. The dependence on the neutrino masses survives only via the $\Delta m_{kl}^2$ factor in Eq. (A8).

2. The wave packets are localized in an area much larger than the inverse mass of the source and target particles, $\sigma \gg m_{A,B}$, such that $|p_{A,B}| \sim \sigma^{-1} \ll m_{A,B}$. In the subsequent analysis we will only keep terms of $O(\sigma^{-1})$ and ignore those of $O(\sigma^{-2})$. In particular, we can approximate $E_A \approx m_A$ and $E_B \approx m_B$.

3. We ignore the dependence of the amplitudes on $p_j$ or $p'_j$ from which it follows that $\mathcal{M}' = \mathcal{M}$. Given the assumption in the previous point, this present assumption is safe whenever the amplitudes are dominated by a velocity-independent term.

With the above assumptions Eq. (A8) simplifies to

$$N_{\alpha\beta} = \int d\Pi_d d\Pi_d \sum_{kl} \mathcal{M}_{\alpha k}^{P} \mathcal{M}_{\beta k}^{D} \mathcal{M}_{\alpha l}^{P} \mathcal{M}_{\beta l}^{D} \frac{(-i)e^{i(p_{\tilde{A}} - p_{\tilde{B}})\tilde{L}}}{\Delta m_{kl}^2 - 2q_0(p_{\tilde{B}} - p_{\tilde{B}})}$$

Due to our approximations, after replacing $e^{i(E_A + E_B - E'_A - E'_B)t} \approx 1$ nothing in the integrand depends on $t$ and thus the integral is infinite. This singularity could in fact be expected: due to using time-independent wave packets for the source $|A_x\rangle$ we tacitly integrate the rate of the $A_x B_y \rightarrow XY$ process from $t = -\infty$ to $t = +\infty$. In a physical situation, however, $A_x$ is unstable, appears at a finite time $t_0$, and decays after a finite time $t_0 + T$. Outside this window the process $A_x B_y \rightarrow XY$ cannot occur. With this in mind, we drop the integration over $t$, and obtain the following result for the rate, that is the number of events per unit time:

$$\frac{dN_{\alpha\beta}}{dt} = \sum_{kl} \mathcal{M}_{\alpha k}^{P} \mathcal{M}_{\beta k}^{D} \mathcal{M}_{\alpha l}^{P} \mathcal{M}_{\beta l}^{D} \frac{(-i)e^{i(p_{\tilde{A}} - p_{\tilde{B}})\tilde{L}}}{\Delta m_{kl}^2 - 2q_0(p_{\tilde{B}} - p_{\tilde{B}})}$$

Next, it is convenient to change the integration variables as $\tilde{p}_j^\pm = p_j \pm p'_j$. Afterwards we can trivially perform the Gaussian integral over $d^3p_A d^3p_B$ and eliminate the integral over $d^3p_A$ using the $\delta^3$. We also fix the coordinate frame such that $\tilde{L} = (0, 0, L)$, so that the z-axis connects the source and the target. This leads to

$$\frac{dN_{\alpha\beta}}{dt} = \sum_{kl} \mathcal{M}_{\alpha k}^{P} \mathcal{M}_{\beta k}^{D} \mathcal{M}_{\alpha l}^{P} \mathcal{M}_{\beta l}^{D} \frac{(-i)e^{iLp_z}}{\Delta m_{kl}^2 - 2q_0 - i\epsilon}$$
where we simplified the notation $\vec{p} \equiv \vec{p}_\nu = -\vec{p}_\lambda$. In the integration over $p_z$, the principal value is suppressed by the rapidly oscillating $e^{iLp_z \nu}$, and is neglected, which leaves the contribution from the pole at $p_z = (\Delta m^2_{kl} - 2q_ip_i)/2q_z$:

$$
\frac{dN_{\alpha\beta}}{dt} = \frac{1}{(2\pi)^3(2m_A)(2m_B)} \int \frac{d^3p_d d^3p_D}{(2\pi)^3(2m_A)(2m_B)} \left( \frac{1}{8q_z^2} \right) \exp \left( - \frac{(\Delta m^2_{kl} - 2q_ip_i)^2}{8q_z^2} \right)
$$

where $i = x, y$. Note that $q_i$ is the neutrino momentum in the “wrong”, off-axis direction transverse to $\vec{L}$, thus $|q_i| \ll |q_z|$ as long as $L \gg \sigma$. Therefore we can drop $q_i/q_z$ factors everywhere, except when they are multiplied by $L$. Then we can trivially perform the Gaussian integral over $p_x$ and $p_y$, which leads to

$$
\frac{dN_{\alpha\beta}}{dt} = \frac{1}{32\pi m_A m_B^2} \int \frac{d^3p_d d^3p_D}{(2\pi)^3(2m_A)(2m_B)} \left( \frac{1}{8q_z^2} \right) \exp \left( - \frac{L \Delta m^2_{kl}}{2q_z} \right) \mathcal{M}^P_{\alpha k} \mathcal{M}^D_{\beta k} \tilde{\mathcal{M}}^P_{\alpha l} \tilde{\mathcal{M}}^D_{\beta l} \frac{1}{q_z} \exp \left( - \frac{\Delta m^4_{kl} \sigma^2}{8q_z^2} \right) \exp \left( - \frac{(q_x^2 + q_y^2)L^2}{2q_z^2 \sigma^2} \right).
$$

The oscillatory factor $e^{-i2\pi L/L_{osc}}$ appears here for the first time in this derivation, with the oscillation length $L_{osc} = \frac{4\pi q_z}{\Delta m^2_{kl}} \approx \frac{4\pi E_\nu}{\Delta m^2_{kl}}$. In the QFT approach it arises because of interference between distinct neutrino mass eigenstates $k \neq l$ entering the propagators in Eq. [A7], which in turn is possible due to momentum uncertainty described by the initial state wave packets. The oscillations become suppressed by the factor $e^{-\Delta m^4_{kl} \sigma^2/8q_z^2}$ when the packet size becomes comparable to the oscillation length [42]. A condition for oscillations to be possible is

$$
\sigma \lesssim \frac{E_\nu}{\Delta m^2_{kl} \sigma} \approx L_{osc}.
$$

On the other hand, in our approach we do not find exponential suppression of the oscillations proportional to the distance $L$ travelled by the neutrino. The usual argument for this suppression [42, 43], due the decoherence of wave packets corresponding to different neutrino eigenstates traveling at different speeds, does not apply in the static situation considered here.

Due to the last exponential factor in Eq. (A13) the neutrino production angle $\theta \approx \sqrt{q_x^2 + q_y^2}/q_z$ must be such that $L\theta \lesssim \sigma$. This has a simple physical interpretation: the neutrino must hit the target within its position uncertainty described by the wave packet. Neutrinos emitted with $L\theta \gtrsim \sigma$ simply miss the target and do not contribute to the probability of the $A_x B_y \rightarrow XY$ process. With this in mind, on the final transformation we trade $q_z = E_\nu \cos \theta \approx E_\nu$, and $q_x^2 + q_y^2 = E_\nu^2 \sin^2 \theta \approx E_\nu^2 \theta^2$. The production phase space contains the neutrino phase space $d^3p_d = E_\nu dE_\nu d\cos \theta d\phi$. One more assumption we need is that neutrinos are produced isotropically, that is $\mathcal{M}^P_{\alpha k} \mathcal{M}^D_{\beta k} \tilde{\mathcal{M}}^P_{\alpha l} \tilde{\mathcal{M}}^D_{\beta l}$ integrate/summed over unobserved degrees of freedom is independent of the angular variables $\theta, \phi$. This assumption is satisfied in typical neutrino experiments where the source is unpolarized. The integral over $\theta$ can be evaluated order by order in $\sigma^2/L^2$, leading to

$$
\frac{dN_{\alpha\beta}}{dt \, dE_\nu} = \frac{1}{32\pi L^2 m_A m_B} \int d^3p_d d^3p_D \sum_{kl} \exp \left( - \frac{L \Delta m^2_{kl}}{2E_\nu} \right) \mathcal{M}^P_{\alpha k} \mathcal{M}^D_{\beta k} \tilde{\mathcal{M}}^P_{\alpha l} \tilde{\mathcal{M}}^D_{\beta l} \exp \left( - \frac{\Delta m^4_{kl} \sigma^2}{8E_\nu^2} \right),
$$

where $d^3p_D = d^3p_d dE_\nu$. Note the geometric $1/L^2$ factor in front, which is of course expected intuitively. Mathematically, it appears due to integrating over the neutrino production angles in the phase space, where the contribution of off-axis neutrinos is exponentially suppressed and only the small cone $\theta \lesssim \sigma/L$ effectively contributes to the transition rate. The dependence on the size $\sigma$ of the initial wave packets has canceled out, except in the last exponential factor, which can be ignored in the limit $\sigma \Delta m^2_{kl} \ll E_\nu$. In that limit, after multiplying the rate by the number of source and detector particles $N_{S,T}$ we obtain the master formula in Eq. [1].

**Appendix B: NSI matching**

The matching between the NSI parameters $\epsilon^e$ and $\epsilon^d$ to the EFT Wilson Coefficients $\epsilon_X$ depends on the specific processes in which neutrinos are produced or detected, as shown in Eq. (14). The process dependence is encoded in the production and detection coefficients $p(d)_{X \nu}$ defined in Eq. (13). With the production and detection coefficients at hand, we can verify whether the consistency condition in Eq. (15) is satisfied. If it is not, the matching is only valid at the linear order in $\epsilon_X$, whereas at higher orders it fails. In the latter case, the QM-NSI approach does not reproduce the correct dependence on EFT parameters beyond the linear order in $\epsilon_X$. Here we list the production and detection coefficients $p(d)_{X \nu}$ for different processes: 

1. **Production process** $p(d)_{X \nu}$ for neutrino production process $\nu$. 
2. **Detection process** $p(d)_{X \nu}$ for detector particles. 

For a detailed discussion of these processes, refer to the original source.
detection coefficients for nuclear decay, inverse beta decay and pion decay and discuss the validity of the matching
in each case. Neutrino detection through non-elastic processes (quasi-elastic, deep-inelastic or resonances) are more
involved and we leave them for future work.

Reactor electron anti-neutrinos $\bar{\nu}_e$ are produced via beta decays of nuclear fission products. To calculate
the corresponding amplitudes we assume that only the Gamow-Teller type decays are important (see Ref. [32] for
further details). With this assumption the non-zero coefficients are

$$p_{LL} = -p_{RL} = 1, \quad p_{TL} = -p_{TR} = -\frac{g_T}{g_A} \frac{m_e}{f_T(E_\nu)},$$

$$p_{RR} = 1, \quad p_{TT} = \frac{g_T^2}{g_A^2},$$  \hspace{1cm} (B1)

which gives the following matching with NSI parameters

$$\epsilon^s_{e\beta} = \sum_X p_{XL}[\epsilon_X]_{e\beta} = [\epsilon_L]_{e\beta} - [\epsilon_R]_{e\beta} - \frac{g_T}{g_A} \frac{m_e}{f_T(E_\nu)} [\epsilon_T]_{e\beta}.$$  \hspace{1cm} (B2)

Here $g_A = 1.251(33)$ and $g_T = 0.987(55)$ are the axial and tensor nucleon charges [36–38], and $m_e$ is the electron
mass. $f_T(E_\nu)$ is a function that depends on the nuclear decays taking place in the reactor, which was calculated
using certain approximations in Ref. [32]. We see that the relation in Eq. (15) is not satisfied for the tensor case:

$$\frac{m_e^2}{f_T^2(E_\nu)} \neq 1,$$

which implies that reactor anti-neutrino production cannot be described by the QM-NSI formalism in the
presence of tensor interactions. Moreover, the energy dependence at the linear level (entering via $p_{TT}$) is not there
at the quadratic level (because $p_{TT}$ is a constant), which will be missed if we use Eq. (B2) in Eq. (5). For the left- and
right-handed interactions the matching is valid at all orders.

A common detection process of low-energy neutrinos is the inverse beta decay. For this case we find the following
detection coefficients:

$$d_{LL} = 1, \quad d_{RL} = \frac{1 - 3g_A^2}{1 + 3g_A^2}, \quad d_{SL} = d_{SR} = -\frac{gs}{1 + 3g_A^2} \frac{m_e}{E_\nu - \Delta}, \quad d_{TL} = -d_{TR} = \frac{3g_A^2}{1 + 3g_A^2} \frac{m_e}{E_\nu - \Delta},$$

$$d_{RR} = 1, \quad d_{SS} = \frac{g_A^2}{1 + 3g_A^2}, \quad d_{TT} = \frac{3g_T^2}{1 + 3g_T^2},$$  \hspace{1cm} (B3)

where $g_S = 1.02(10)$ [37–38, 41] is the scalar nucleon charge and $\Delta = m_p - m_n \sim 1.3 \text{ MeV}$. The NSI detection
parameters can thus be related to EFT parameters as

$$\epsilon^s_{e\beta} = \sum_X d_{XL}[\epsilon_X]_{e\beta} = [\epsilon_L]_{e\beta} + \frac{1 - 3g_A^2}{1 + 3g_A^2} [\epsilon_R]_{e\beta} - \frac{m_e}{E_\nu - \Delta} \left( \frac{gs}{1 + 3g_A^2} [\epsilon_S]_{e\beta} - \frac{3g_A^2}{1 + 3g_A^2} [\epsilon_T]_{e\beta} \right).$$  \hspace{1cm} (B4)

The consistency condition in Eq. (14) is satisfied only for the $V-A$ case, and fails if other NSI are present.

Finally, we consider muon neutrino production from pion decays at rest. The non-zero production coefficients in
this case are

$$p_{LL} = -p_{RL} = 1, \quad p_{PL} = -p_{PR} = -\frac{m_\mu^2}{m_\mu (m_u + m_d)},$$

$$p_{RR} = 1, \quad p_{PP} = \frac{m_\mu^4}{m_\mu^2 (m_u + m_d)^2}.$$  \hspace{1cm} (B5)

The NSI production parameters can thus be related to EFT parameters as

$$\epsilon^s_{\mu\beta} = \sum_X p_{XL}[\epsilon_X]_{\mu\beta} = [\epsilon_L]_{\mu\beta} - [\epsilon_R]_{\mu\beta} - \frac{m_\mu^2}{m_\mu (m_u + m_d)} [\epsilon_P]_{\mu\beta}.$$  \hspace{1cm} (B6)

Since the consistency condition in Eq. (15) is satisfied for all the interactions involved in the pion decay, the above
matching will be valid to all orders. This holds for neutrino production through any 2-body decay of a spin-zero
particle.
Appendix C: Oscillation probability

In the main body of this paper the basic quantity we worked with was the event rate \( R_{\alpha\beta} \) in Eq. (1), which is an observable in neutrino experiments. In the neutrino literature the oscillation probability in Eq. (3) is often used. It is not strictly speaking an observable in the general context when non-SM interactions are present. Nevertheless, it is a simple quantity bounded between 0 and 1 with an intuitive interpretation. For completeness, in this appendix we explicitly list relevant expressions for the oscillation probability.

In the QM-NSI approach the oscillation probability is given by

\[
P^{\text{QM}}_{\alpha\beta} = (N^a_N^d)^{-2} \sum_{k,l} e^{-i \frac{\Delta m_{kl}^2}{2 E_{\nu}}} |x_s|_{ak} |x_d|_{\beta k} |x_d|_{\beta l}^*,
\]  

where \( x_s \equiv (1 + \epsilon^s) U^* \) and \( x_d \equiv (1 + \epsilon^d) U \), and the normalization factors are

\[
(N^a_N^d)^2 = [(1 + \epsilon^s)(1 + \epsilon^s)^\dagger]_{\alpha\alpha} [(1 + \epsilon^d)(1 + \epsilon^d)^\dagger]_{\beta\beta}.
\]

In the QFT approach the oscillation probability depends on the parameters of the EFT Lagrangian in Eq. (6) as

\[
P^{\text{QFT}}_{\alpha\beta} = N_{\alpha\beta}^{-1} \sum_{k,l} e^{-i \frac{\Delta m_{kl}^2}{2 E_{\nu}}} \left[ U^*_{ak} U_{\alpha l} + \sum_X p_{XL}(\epsilon_X U)^*_{ak} U_{\alpha l} + \sum_{X,Y} p_{XY}(\epsilon_X U)^*_{ak}(\epsilon_Y U)_{\alpha l} \right] \times \left[ U_{\beta k} U^*_{\beta l} + \sum_X d_{XL}(\epsilon_X U)_{\beta k} U^*_{\beta l} + \sum_{X,Y} d_{XY}(\epsilon_X U)_{\beta k}(\epsilon_Y U)_{\beta l} \right].
\]

where the coefficients \( p(d)_{XY} \) are defined in Eq. (13) and the normalization factor is

\[
N_{\alpha\beta} = \left[ 1 + \sum_X p_{XL} \epsilon_X^s + \sum_X p^*_{XL} \epsilon_X^s + \sum_{X,Y} p_{XY} \epsilon_Y^s \epsilon_X^s \right]_{\alpha\alpha} \left[ 1 + \sum_X d_{XL} \epsilon_X + \sum_X d^*_{XL} \epsilon_X^s + \sum_{X,Y} d_{XY} \epsilon_Y^s \epsilon_X^s \right]_{\beta\beta}.
\]

The QM-NSI and QFT probabilities can be matched as in Eq. (14) only when the conditions \( p_{XY} = p_{XL} p_{LY} \) and \( d_{XY} = d_{XL} d_{LY} \) are satisfied for each \( X, Y \) for which \( \epsilon_X, \epsilon_Y \) are non-zero. In the case of \( V-A \) interactions we have \( \epsilon_{LL} = 1 \) and the consistency conditions are automatically satisfied. The SM limit corresponds to \( \epsilon_X = 0 \) in the EFT, or \( \epsilon^s = \epsilon^d = 0 \) in the QM-NSI approach, in which case we recover the familiar expression

\[
P^{\text{SM}}_{\alpha\beta} = \sum_{k,l} e^{-i \frac{\Delta m_{kl}^2}{2 E_{\nu}}} U^*_{ak} U_{\alpha l} U_{\beta k} U^*_{\beta l}.
\]
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