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Jump-induced mixed-mode oscillations through piecewise-affine maps. (English)

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Summary: Mixed-mode oscillations (MMOs) are complex oscillatory patterns in which large-amplitude oscillations (LAOs) of relaxation type alternate with small-amplitude oscillations (SAOs). MMOs are found in singularly perturbed systems of ordinary differential equations of slow-fast type, and are typically related to the presence of so-called folded singularities and the corresponding canard trajectories in such systems. Here, we introduce a canonical family of three-dimensional slow-fast systems that exhibit MMOs which are induced by relaxation-type dynamics, and which are hence based on a “jump mechanism”, rather than on a more standard canard mechanism. In particular, we establish a correspondence between that family and a class of associated one-dimensional piecewise affine maps (PAMs) which exhibit MMOs with the same signature. Finally, we give a preliminary classification of admissible mixed-mode signatures, verifying results of B. Rajpathak et al. [Chaos 22, No. 3, 033126, 9 p. (2012; Zbl 1319.37032)] in the process, and we illustrate our findings with numerical examples.

MSC:

34E15 Singular perturbations for ordinary differential equations
34C26 Relaxation oscillations for ordinary differential equations
34E17 Canard solutions to ordinary differential equations

Keywords:

mixed-mode oscillation; relaxation oscillation; piecewise affine map; slow-fast system; geometric singular perturbation theory

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