Observable $r$, Gravitino Dark Matter, and Non-thermal Leptogenesis in No-Scale Supergravity

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Abstract

We analyse the shifted hybrid inflation in a no-scale supersymmetric $SU(5)$ GUT model which naturally circumvents the monopole problem. The no-scale framework is derivable as the effective field theory of the supersymmetric (SUSY) compactifications of string theory, and yields a flat potential with no anti-de Sitter vacua, resolving the $\eta$ problem. The model predicts a scalar spectral tilt $n_s$ compatible with the most recent measurements by the Planck satellite, while also accommodating large values of the tensor-to-scalar ratio $r$ ($\sim 0.0015$), potentially measurable by the near-future experiments. Moreover, the proton decay lifetime in the presence of the dimension-5 operators is found to lie above the current limit imposed by the Super-Kamiokande experiment. A realistic scenario of reheating and non-thermal leptogenesis is employed, wherein the reheating temperature $T_r$ lies in the $(2 \times 10^6 \lesssim T_r \lesssim 2 \times 10^9)$ GeV range, and at the same time realizing gravitino as a viable dark matter (DM) candidate.

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1 Introduction

Inflation provides a successful phenomenological framework for addressing cosmological puzzles such as the size, the age, the homogeneity and the (approximate) geometrical flatness of the Universe [1]. It also explains large-scale structures, the smallness of the primordial density perturbations measured in the cosmic microwave background (CMB), as well as the small tilt \( n_s = 0.965 \) in it and the inconsistency of this CMB with the Gaussian white-noise spectrum [2]. The success of inflation has motivated several attempts to relate it to the Standard Model (SM) of particle physics, and at the same time to a candidate quantum theory of everything, including gravity, such as string theory. The characteristic energy scale of inflation is presumably intermediate between SM and quantum gravity, and inflationary models may thus provide a welcome bridge between the two of them.

A connection between inflation and a viable quantum theory of gravity at some very high scale and the SM at the electroweak (EW) scale makes models based on no-scale supergravity (SUGRA) a particularly attractive choice [3, 4, 5]. No-scale SUGRA appears generically in models with ultraviolet completion using string theory compactifications [6]. It is known to be the general form of the 4-dimensional effective field theory derivable from string theory that embodies low-energy supersymmetry. Moreover, no-scale SUGRA is an attractive framework for constructing models of inflation [7, 8] because it naturally yields a flat potential with no anti-de Sitter ‘holes’, resolving the so-called \( \eta \) problem. No-scale inflation also comfortably accommodates values of the \( r \) and \( n_s \) perfectly compatible with the most recent measurements of the Planck satellite, and potentially very similar to the values predicted by the original Starobinsky model [9]. For more detailed studies in the context of Starobinsky model, see Refs. [10, 11, 12, 13, 14, 15, 16].

Hybrid inflation is one of the most promising models of inflation that can be naturally realized within the context of SUGRA theories [19, 20, 21, 22, 23, 24, 25]. In SUSY hybrid inflation, the scalar potential along the inflationary track is completely flat at the tree level. The inclusion of radiative corrections to this potential provides the necessary slope required to drive the inflaton towards the SM vacuum. In such a scenario [19], the CMB temperature anisotropy \( \delta T/T \) is of the order \( (M/m_P) \), where \( M \) is the breaking scale of the parent gauge group \( G \) of the Universe at the start of inflation, and \( m_P = 2.4 \times 10^{18} \) GeV is the reduced Planck mass. In order for the self-consistency of the inflationary scenario to be preserved, \( M \) turns out to be comparable to the scale of grand unification,
M_{GUT} \sim 10^{16} \text{ GeV}, hinting that \( G \) may be the GUT gauge group.

In the standard hybrid model of inflation, \( G \) breaks spontaneously to its subgroup \( H \) at the end of the inflation \([19, 26]\), which leads to topological defects such as copious production of magnetic monopoles by the Kibble mechanism \([27]\). These magnetic monopoles dominate the energy budget of the Universe, contradicting the cosmological observations. In the shifted \([28]\) or smooth \([30, 31, 32]\) variants of the hybrid inflation, \( G \) is instead broken during inflation, and in this way the disastrous monopoles are inflated away.

In this article, we study shifted hybrid inflation formulated in the framework of no-scale SUGRA. In our model, the \( SU(5) \) gauge group is spontaneously broken down to the \( G_{SM} \) by the vacuum expectation value (VEV) of the \( \Phi_{24} \) adjoint Higgs superfield. By generating a suitable shifted inflationary track wherein the \( SU(5) \) is broken during inflation, the monopole density can be significantly diluted. The predictions of our model are consistent with the Planck’s latest bounds \([2]\) on \( n_s \) and \( r \). Moreover, a wide range of the \( T_r \) is obtained which naturally avoids the gravitino problem. A model of non-thermal leptogenesis via right-handed neutrinos is studied in order to explain the observed baryon asymmetry of the Universe (BAU). As compared to the \( SU(5) \) model with shifted hybrid inflation studied outside the no-scale framework in \([28]\), relatively large values of \( r \) (\( \sim 0.0015 \)) are obtained here, potentially measurable by the future experiments.

The layout of the paper is as follows. Sec. 2 presents the basic description of the model including the field content, the superpotential and the global minima of the potential. The inflationary trajectories and the dimension-5 proton decay are discussed in Sec. 3. The inflationary setup and the theoretical details of reheating with non-thermal leptogenesis are provided in Sec. 4 and Sec. 5, respectively. The numerical analysis of the prospects of observing primordial gravity waves, and of leptogenesis and gravitino cosmology, is presented in Sec. 6. Finally, Sec. 7 summarizes our findings.

2 The Supersymmetric \( SU(5) \) Model with \( U(1)_R \) Symmetry

In the model we consider, the matter fields of the minimal supersymmetric standard model (MSSM) reside in the following representations of the \( SU(5) \) supermultiplets.

\[
\begin{align*}
F_i & \equiv 10_i = Q_i (3, 2, 1/6) + u^c_i (3, 1, -2/3) + e^c (1, 1, 1), \\
F_i & \equiv 5_i = d^c_i (3, 1, 1/3) + \ell_i (1, 2, -1/2), \\
\nu_i & \equiv 1_i = \nu^c_i (1, 1, 0),
\end{align*}
\]

(2.1)

where \( i \) is the generation index (\( i = 1, 2, 3 \)) and \( \nu^c_i \) are right-handed neutrino superfields. The Higgs sector constitutes of a pair of 5-plet superfields \( \{ H \equiv H_5, \overline{H} \equiv \overline{H}_5 \} \) that contain the colour Higgs triplets and the two doublets of the MSSM, a gauge-singlet
superfield $S$, and a 24-plet superfield $\Phi$ that belongs to the adjoint representation. This superfield $\Phi$ is responsible for breaking the $SU(5)$ gauge symmetry down to the SM gauge group by acquiring a non-zero VEV in the hypercharge direction. These Higgs superfields are decomposed under the $G_{SM}$ as

\[
H = H_T(3, 1, -1/3) + H_u(1, 2, 1/2),
\]
\[
\overline{H} = \overline{H}_T(3, 1, 1/3) + H_d(1, 2, -1/2),
\]
\[
\Phi = \Phi_{24}(1, 1, 0) + W_H(1, 3, 0) + G_H(8, 1, 0)
\]
\[
+ X_H(3, 2, -5/6) + \overline{X}_H(3, 2, 5/6).
\]

Following [28], the $U(1)_R$-charge assignments of the various superfields are

\[
R\left(S, \Phi, H, \overline{H}, F_i, \overline{F}_i, \nu_i\right) = \left(1, 0, 2, 3, 3, 1, 10, 10, 1\right).
\]

The $U(1)_R$-symmetric superpotential of the $SU(5)$ group containing the above superfields is written as

\[
W = \kappa S \left(\mu^2 - Tr(\Phi^2) - \beta \frac{Tr(\Phi^3)}{\kappa M_*}\right) + \gamma H \Phi H + \delta \overline{H} H
\]
\[
+ y_{ij}^{(u)} F_i F_j H + y_{ij}^{(d,e)} F_i \overline{F}_j H + y_{ij}^{(\nu)} \nu_i \overline{\nu}_j H + m_{ij} \nu_i \nu_j + \lambda_{ij} \frac{Tr(\Phi^2) \nu_i \nu_j}{M_*},
\]

where $\kappa$, $\gamma$, and $\lambda_{ij}$ are dimensionless couplings, while $\delta$ is a parameter with unit mass-dimension and $M_*$ is a high cut-off scale (compactification scale in a string model or the Planck scale) $M_{GUT} \lesssim M_* \lesssim m_P$. The Yukawa couplings $y_{ij}^{(u)}$, $y_{ij}^{(d,e)}$, and $y_{ij}^{(\nu)}$ in the second line above generate the quark and lepton masses after the EW symmetry-breaking, whereas the last two terms generate the right-handed neutrino masses, and are thus relevant for leptogenesis in the post inflationary era.

### 2.1 The Global SUSY Minima

The first line of the superpotential in Eq. (2.4) contains the terms responsible for the shifted hybrid inflation, and can be rewritten in component form as

\[
V_F \supset \sum_i \left|\frac{\partial W}{\partial z_i}\right|^2 = \sum_i \left|\kappa S \phi_i + \frac{3 \beta}{4 M_*} d_{ijk} \phi_i \phi_j \phi_k - \gamma T^i_{ab} \phi^a H_b + \delta H_a H_b\right|^2
\]
\[
+ \kappa^2 \left|\mu^2 - \frac{1}{2} \sum_i \phi_i^2 - \frac{\beta}{4 M_*} d_{ijk} \phi_i \phi_j \phi_k\right|^2 + \sum_b \left|\gamma T^i_{ab} \phi^a H_b\right|^2 + \left|\delta H_b\right|^2,
\]

Following the adjoint basis $\Phi = \phi_i T^i$ with $Tr(T_i T_j) = \delta_{ij}/2$ and $d_{ijk} = 2 \text{Tr}(T_i \{T_j, T_k\})$. Here the indices $i$, $j$, and $k$ run from 1 to 24, whereas $a$ and $b$ run from 1 to 5. The global $F$-term potential is obtained from $W$ as

\[
V_F \supset \sum_i \left|\frac{\partial W}{\partial z_i}\right|^2 = \sum_i \left|\kappa S \phi_i + \frac{3 \beta}{4 M_*} d_{ijk} \phi_i \phi_j \phi_k - \gamma T^i_{ab} \phi^a H_b\right|^2 + \sum_b \left|\gamma T^i_{ab} \phi^a H_b\right|^2
\]
\[
+ \kappa^2 \left|\mu^2 - \frac{1}{2} \sum_i \phi_i^2 - \frac{\beta}{4 M_*} d_{ijk} \phi_i \phi_j \phi_k\right|^2 + \sum_b \left|\gamma T^i_{ab} \phi^a H_b\right|^2 + \left|\delta H_b\right|^2,
\]

where $\kappa$, $\gamma$, and $\lambda_{ij}$ are dimensionless couplings, while $\delta$ is a parameter with unit mass-dimension and $M_*$ is a high cut-off scale (compactification scale in a string model or the Planck scale) $M_{GUT} \lesssim M_* \lesssim m_P$. The Yukawa couplings $y_{ij}^{(u)}$, $y_{ij}^{(d,e)}$, and $y_{ij}^{(\nu)}$ in the second line above generate the quark and lepton masses after the EW symmetry-breaking, whereas the last two terms generate the right-handed neutrino masses, and are thus relevant for leptogenesis in the post inflationary era.
where $z_i$ are the scalar components of the Higgs superfields. The global SUSY minimum of the above potential lies at the following VEVs of the fields

$$\langle S \rangle = \langle H_a \rangle = \langle \overline{H}_a \rangle = 0,$$

(2.7)

while $\langle \phi_i \rangle$ satisfy the condition

$$\sum_{i=1}^{24} \langle \phi_i \rangle^2 + \frac{\beta}{2\kappa M_*} d_{ijk} \langle \phi_i \rangle \langle \phi_j \rangle \langle \phi_k \rangle = 2\mu^2.$$

(2.8)

The VEV matrix $\langle \Phi_i \rangle = \langle \phi_i \rangle T^i$ can be aligned in the hypercharge $(i = 24)$ direction using the $SU(5)$ transformation

$$\langle \Phi_{24} \rangle = \frac{\langle \phi_{24} \rangle}{\sqrt{15}} (1, 1, 1, -3/2, -3/2),$$

(2.9)

such that $\langle \phi_i \rangle = 0, \forall i \neq 24$ and $\langle \phi_{24} \rangle \equiv \upsilon/\sqrt{2}$, where $d_{242424} = -1/\sqrt{15}$ and $\upsilon$ satisfies

$$4\mu^2 = \upsilon^2 - \frac{\beta}{2\sqrt{30}\kappa M_*} \upsilon^3.$$

(2.10)

The $D$-term contribution to the potential,

$$V_D = \frac{g_5^2}{2} \sum_i \left( f^{ijk} \phi_j \phi_k + T_i \left( |H_a|^2 - |\overline{H}_a|^2 \right) \right)^2,$$

(2.11)

also vanishes for $\phi = \phi^*$ and $|\overline{H}_a| = |H_a|$.

## 3 Inflationary Trajectories

The scalar potential in Eq. (2.6) can be rewritten in terms of the dimensionless variables

$$y = \frac{\phi_{24}/\mu}{\sqrt{2}}, \quad w = \frac{S/\mu}{\sqrt{2}},$$

(3.1)

as

$$\tilde{V}(w, y) = \frac{V(w, y)}{\kappa^2 \mu^4} = \left( 1 - y^2 + \alpha y^3 \right)^2 + 2w^2 y^2 \left( 1 - \frac{3\alpha y}{2} \right)^2,$$

(3.2)

with $\alpha = \beta\mu/\sqrt{30}\kappa M_*$. This dimensionless potential exhibits the following three extrema

$$y_1 = 0,$$

(3.3)

$$y_2 = \frac{2}{3\alpha},$$

(3.4)

and

$$y_3 = \frac{1}{3\alpha} + \frac{1}{3\sqrt{2}\alpha} \left( \sqrt{2 - 27\alpha^2} + \sqrt{(2 - 27\alpha^2)^2 + 4(9\alpha^2w^2 - 1)^3} \right. \right.$$

$$\left. \left. - \sqrt[3]{-2 + 27\alpha^2 + (2 - 27\alpha^2)^2 + 4(9\alpha^2w^2 - 1)^3} \right) \right.$$

(3.5)
Figure 1: Normalized scalar potential \( \bar{V}(w, y) = V(w, y)/\kappa^2\mu^4 \) as a function of dimensionless variables \( w \) and \( y \), for different values of parameter \( \alpha \). The standard hybrid inflation potential is reproduced for \( \alpha = 0 \) in panel (a). For \( \alpha \neq 0 \), an additional shifted trajectory appears which lies higher than the standard trajectory for \( \alpha < \sqrt{2/27} \) and lower than the standard trajectory for \( \alpha > \sqrt{2/27} \). These shifted hybrid inflation potentials are shown in panels (b) and (c) for \( \alpha = 0.25 \) and \( \alpha = 0.3 \), respectively.

for any constant value of \( w \). The dimensionless potential \( \bar{V}(w, y) \) is displayed in Fig. 1 for different values of \( \alpha \). The first extremum \( y_1 \) with \( \alpha = 0 \) corresponds to the standard hybrid inflation for which \( \{y = 0, w > 1\} \) is the only inflationary trajectory that evolves at \( w = 0 \) into the global SUSY minimum at \( y = \pm 1 \) (panel (a)). For \( \alpha \neq 0 \), a shifted trajectory appears at \( y = y_2 \), in addition to the standard trajectory at \( y = y_1 = 0 \), which is a local maximum (minimum) for \( w < \sqrt{4/27\alpha^2 - 1} \) (\( w > \sqrt{4/27\alpha^2 - 1} \)). For \( \alpha < \sqrt{2/27} \approx 0.27 \), this shifted trajectory lies higher than the standard trajectory (panel (b)). In order to have suitable initial conditions for realizing inflation along the shifted track, we assume \( \alpha > \sqrt{2/27} \), for which the shifted trajectory lies lower than the standard trajectory (panel (c)). Moreover, to ensure that the shifted inflationary trajectory at \( y_2 \) can be realized before \( w \) reaches zero, we require \( \alpha < \sqrt{4/27} \approx 0.38 \). Thus, for \( 0.27 < \alpha < 0.38 \), while the
Figure 2: The proton lifetime for the decay $p \rightarrow K^+ \bar{\nu}$ as a function of SUSY breaking scale $M_{\text{SUSY}}$ for different values of $\tan \beta$. The curves are drawn for $SU(5)$ symmetry breaking scale $M_{\alpha}$ fixed at $2 \times 10^{16} \text{ GeV}$ with $\alpha = 0.3$. The black and gray dashed lines represent the Super-Kamiokande ($\tau_p = 5.9 \times 10^{33} \text{ years}$) bounds and future Hyper-Kamiokande expected bounds ($\tau_p = 3.2 \times 10^{34} \text{ years}$) on proton lifetime, respectively.

inflationary dynamics along the shifted track remain the same as for the standard track, the $SU(5)$ gauge symmetry is broken during inflation, hence alleviating the magnetic monopole problem. As the inflaton slowly rolls down the inflationary valley and enters the waterfall regime at $w = \sqrt{4/27\alpha^2 - 1}$, its fast rolling ends the inflation, and the system starts oscillating about the vacuum at $w = 0$ and $y = y_3$.

### 3.1 Dimension-5 Proton Decay

Upon breaking of the $SU(5)$ symmetry, the last two terms in the $W$ in Eq. (2.5) can be rewritten as,

$$W_H \supset \left( \delta - \frac{3\gamma \phi_{24}^0}{2\sqrt{30}} \right) H_u H_d + \left( \delta + \frac{\gamma \phi_{24}^0}{\sqrt{30}} \right) H_T H_T \equiv \mu_H H_u H_d + M_{H_T} H_T H_T,$$

(3.6)

where $\mu_H$ is identified with the usual MSSM $\mu$-parameter, which is taken to be of the order of TeV scale, and $M_{H_T}$ is the color-triplet mass parameter given by

$$M_{H_T} \simeq \frac{10\gamma M_{\alpha}}{3\sqrt{30} \left( \frac{4}{\sqrt{7}} - \alpha^2 \right)}.$$

(3.7)
The above mass-splitting between the Higgs doublet and triplet can be addressed by fine tuning of the parameters $\tilde{\delta}$ and $\tilde{\gamma}$, such that

$$\tilde{\delta} \simeq \frac{3\tilde{\gamma}\phi_2^0}{2\sqrt{30}}.$$  

However, the fermionic components of $H_T$, the color-triplet Higgsinos, contribute to the proton decay via a dimension-5 operator, which typically dominates the gauge boson mediated dimension-6 operators.

The proton lifetime for the decay $p \to K^+\bar{\nu}$ can be approximated by the following formula [35],

$$\tau_p \simeq 4 \times 10^{31} \times \sin^4 2\beta \left( \frac{M_{\text{SUSY}}}{1 \text{ TeV}} \right)^2 \left( \frac{M_{H_T}}{10^{16} \text{ GeV}} \right)^2 \text{yrs},$$  (3.8)

The proton lifetime $\tau_p$ is shown in Fig. 2 as a function of $M_{\text{SUSY}}$ for different values of $\tan \beta$, using Eq. (2.10), (3.7) and (3.8). The curves are drawn for the SU(5)-breaking scale $M_\alpha$ fixed at $2 \times 10^{16}$ GeV, with $\alpha = 0.3$. It can be seen that the proton lifetime is consistent with the experimental bound, $\tau_p > 5.9 \times 10^{33}$ years, from Super-Kamiokande [36], for $M_{\text{SUSY}} \gtrsim 12.5$ TeV and can be observed by Hyper-Kamiokande [37]. Our model thus remains safe from proton decay even while adequately addressing the doublet-triplet splitting problem.

4 No-Scale Shifted Hybrid Inflation

The Kähler potential with a no-scale structure, after including contributions from the relevant fields in the model, takes the following form

$$K = -3m_p^2 \log \Delta,$$  (4.1)

with

$$\Delta = T + T^* - \frac{\text{Tr}(\Phi^2) + S\bar{S} + \bar{H}H + \nu^c\nu^c}{3m_p^2} + \gamma \frac{(S\bar{S})^2}{3m_p^4} + \zeta \frac{\text{Tr}(\Phi^4)}{3m_p^6} + \sigma \frac{(S\bar{S})^3}{3m_p^8},$$  (4.2)

where $\gamma$, $\zeta$ and $\sigma$ are dimensionless couplings, and $T$ and $T^*$ are Kähler complex moduli fields given as $T = (u + iv)$, so that $T + T^* = 2u$ with $u = 1/2$. The $F$-term SUGRA scalar potential is given by

$$V_F = e^{K/m_P^2} \left[ (K_{ij})^{-1} (D_{zi}W) (D_{zj}W)^* - \frac{3|W|^2}{m_P^2} \right],$$  (4.3)

where we have defined

$$D_{zi}W \equiv \frac{\partial W}{\partial z_i} + \frac{\partial K}{\partial z_i} \frac{W}{m_P^2}, \quad K_{ij} \equiv \frac{\partial^2 K}{\partial z_i \partial z_j^*},$$  (4.4)
and $D_z^i W^* = (D_z W)^*$. 

Since SUSY is temporarily broken along the inflationary trajectory, the radiative corrections to the above $V_F$ along with the soft SUSY-breaking potential $V_{\text{soft}}$ can lift its flatness, while also providing the necessary slope for driving inflation. For a detailed discussion on the mass spectrum of the model, see Ref. [28]. The effective contribution of the one-loop radiative corrections can be calculated using the Coleman-Weinberg formula as

$$V_{1\text{-loop}} = \kappa^2 M_\alpha^4 \left( \frac{\kappa^2}{16\pi^2} [F(M_\alpha^2; x^2) + 11 \times 25 F(5M_\alpha^2; 5x^2)] \right),$$

where, for a non-canonically normalized field $x \equiv |S|/M_\alpha$,

$$F(M_\alpha^2, x^2) = \frac{1}{4} \left( x^4 + 1 \right) \ln \left( \frac{x^4 - 1}{x^4} \right) + 2x^2 \ln \left( \frac{x^2 + 1}{x^2 - 1} \right) + 2 \ln \left( \frac{\kappa^2 M_\alpha^2 x^2}{Q^2} \right) - 3,$$

with $M_\alpha^2 = \mu^2 \left( \frac{4}{27\alpha^2} - 1 \right)$, and $Q$ being the renormalization scale.

As for the breaking of SUSY, in this study we consider the scenario wherein it is communicated gravitationally from the hidden sector to the observable sector. Following [39], the soft SUSY-breaking potential thus reads

$$V_{\text{soft}} \simeq a m_{3/2} \kappa M_\alpha^3 x + M_S^2 M_\alpha^2 x^2 + \frac{8 M_\phi^2 M_\alpha^2}{9\alpha^2 (4/27\alpha^2 - 1)},$$

with

$$a = 2 |A - 2| \cos (\arg S + \arg |A - 2|).$$

Here $m_{3/2}$ is the gravitino mass, $A$ is the complex coefficient of the trilinear soft SUSY-breaking terms, $a$ and $M_S$ are the coefficients of the soft linear and mass terms for $S$, respectively, while $M_\phi$ is the soft mass parameter for the $\phi$ field. The complete effective scalar potential during inflation is then given as

$$V(x) \simeq V_F + V_{1\text{-loop}} + V_{\text{soft}}$$

$$\simeq \kappa^2 M_\alpha^4 \left[ \left( 1 + 2 \left( 2\gamma - \frac{1}{3} \right) x^2 - \frac{1}{(1 - 27\alpha^2/4)} \right) \left( \frac{M_\alpha}{m_P} \right)^2 
+ \left( \frac{16 (1 - 2\zeta)}{3 (4 - 27\alpha^2)^2} + \frac{8x^2 (1 - 12\gamma)}{3 (4 - 27\alpha^2)} - x^4 \left( 16\gamma^2 + \frac{14}{3} \gamma + \frac{9\sigma}{2} - \frac{5}{9} \right) \right) \left( \frac{M_\alpha}{m_P} \right)^4
+ \frac{\kappa^2}{16\pi^2} \left( F(M_\alpha^2, x^2) + 11 \times 25 F(5M_\alpha^2, 5x^2) \right) \right]
+ a m_{3/2} \kappa M_\alpha^3 x + M_S^2 M_\alpha^2 x^2 + \frac{8 M_\phi^2 M_\alpha^2}{9\alpha^2 (4/27\alpha^2 - 1)},$$

where the $F$-term scalar potential along the shifted trajectory in the $D$-flat direction has been obtained from Eq. (4.3). Finally, the action of our model is given by

$$\mathcal{A} = \int dx^4 \sqrt{-g} \left[ \frac{m_p^2}{2} \mathcal{R} - K_j^i \partial_{[x^i} \partial^j x^j} - V(x) \right],$$

(4.9)
where $\mathcal{R}$ is the Ricci scalar. Introducing a canonically normalized field $z$ satisfying

$$
\left(\frac{dz}{dx}\right)^2 = \frac{\partial^2 K}{\partial S^\dagger \partial S} \quad \text{with} \quad S^\dagger = S = x M_\alpha \quad \text{and} \quad \phi_{24} = \frac{\sqrt{2} M_\alpha y_3}{(4/27 \alpha^2 - 1)},
$$

requires modification of the slow-roll parameters, as shown later in section 6.

## 5 Reheating with Non-thermal Leptogenesis

A complete inflationary scenario should be followed by a successful reheating that satisfies the constraint, $T_r < 10^9$ GeV, from gravitino cosmology, and generates the observed BAU. At the end of the inflationary epoch, the system falls towards the SUSY vacuum and undergoes damped oscillations about it. The inflaton (oscillating system) consists of two complex scalar fields $S$ and $\theta = (\delta \phi + \delta \bar{\phi})/\sqrt{2}$. The canonical normalized inflaton field can be defined as

$$
\delta \tilde{\phi} = \langle J_0 \rangle \delta \phi,
$$

with

$$
\delta \phi = \phi - \langle \phi_{24} \rangle y_3, \quad \text{and} \quad J_0 \equiv \left. \frac{dz}{dx} \right|_{\text{Minimum}} = \left(1 - \frac{\langle \phi_{24} \rangle^2 y_3^2}{6 m_P^2} + \frac{\zeta \langle \phi_{24} \rangle y_3^2}{12 m_P^4} \right)^{-1/2},
$$

where

$$
\langle \phi_{24} \rangle y_3 \equiv \frac{\sqrt{2} M_\alpha y_3}{(4/27 \alpha^2 - 1)}.
$$

The decay of the inflaton field, with its mass given by

$$
\tilde{m}_{\text{inf}}^2 = \frac{d^2 V(\phi)}{dz^2} = \frac{1}{J_0} \left( \frac{d^2 V(\phi)}{d\phi^2} \right) = \frac{m_{\text{inf}}^2}{J_0} = \frac{2 \kappa^2 M_\alpha y_3^2 (1 - 3 \alpha y_3/2)^2}{J_0 (4/27 \alpha^2 - 1)},
$$

gives rise to the radiation in the Universe. The inflaton decay into the higginos and the right-handed neutrinos is induced by the superpotential terms,

$$
W \supset \left( \gamma \bar{H} \Phi H + \lambda_{ij} \text{Tr}(\Phi^2) \nu_i \nu_j \right) M^*.
$$

The Lagrangian terms relevant for inflaton decay into neutrinos are\textsuperscript{40, 41},

$$
\mathcal{L}_{\text{inf} \to \nu_i \nu^c_j} = -\frac{1}{2} e^{K/2 m_P^2} \left( K \phi W_{\phi, \nu_i^c \nu_j^c} + W_{\phi, \nu_i \nu_j} - 2 \Gamma_{\phi, \nu_i \nu_j} W_{\phi, \nu_i \nu_j} \right) \phi \nu_i \nu^c_j + \text{h.c.}
$$

$$
\rightarrow -g_{\nu_i \nu_j} \left( \delta \tilde{\phi} \nu_i \nu^c_j + \text{h.c.} \right) + \ldots,
$$

where $W_{\phi, \nu_i \nu_j}$ is the second derivative of $W$ with respect to $\nu^c$, and $g_{\nu_i \nu_j}$ is the effective inflaton-neutrino coupling, defined as

$$
g_{\nu_i \nu_j} = \frac{\sqrt{2} \lambda_{ij} M_\alpha y_3 J_0^4}{M_* (4/27 \alpha^2 - 1)^{1/2}}.
$$
The decay width in the flavor-diagonal basis is thus given by [42]

\[
\Gamma_{\text{inf} \rightarrow \nu^c_i \nu^c_i} = \frac{g^2_{\nu_i}}{64\pi} m_{\text{inf}}^2 \left( 1 - \frac{4M_{\nu_i}^2}{m_{\text{inf}}^2} \right)^{3/2},
\]

\[
= \frac{g^8_0}{16\pi} m_{\text{inf}}^2 \left( \frac{M_{\nu_i}}{\langle \phi_{24} \rangle y_{3i}} \right)^2 \left( 1 - \frac{4M_{\nu_i}^2}{m_{\text{inf}}^2} \right)^{3/2}, \tag{5.8}
\]

where \( M_{\nu_i} \) represent the eigenvalues of the soft neutrino mass matrix. Note that this term violates the lepton number by two units, \( \Delta L = 2 \). In addition, the Dirac mass terms for the neutrinos are obtained from \( W \supset y^{(\nu)}_{ij} \nu^c_i \bar{f}_j H \rightarrow m_{\nu_D_{ij}} \nu^c_i \nu^c_j \) upon EW symmetry-breaking. The small neutrino masses, consistent with the results from the neutrino oscillation experiments, are obtained by integrating out the heavy right-handed neutrinos, so that

\[
m_{\nu_D_{\alpha\beta}} = -\sum_i y^{(\nu)}_{ia} y^{(\nu)}_{ib} \frac{v_u}{M_{\nu_i}}. \tag{5.9}
\]

The Dirac mass matrix above can be diagonalised by a unitary matrix \( U_{ai} \) as \( m_{\nu_D_{\alpha\beta}} = U_{ai} U_{bj} m_{\nu_D} \), with \( m_{\nu_D} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \).

The Lagrangian relevant for the inflaton decay to \( H \) and \( \bar{H} \) is

\[
\mathcal{L}_{\text{inf} \rightarrow H\bar{H}} = -\frac{1}{2} e^{K/2m_{\text{inf}}^2} \left( K_{ab} W_{HH} + W_{\phi HH} - 2\Gamma_{\phi HH} W_{HH} \right)^* m_{\text{inf}} \phi H\bar{H} + \text{h.c.}
\]

\[
\rightarrow -g_H m_{\text{inf}} \left( \delta\phi H\bar{H} + h.c. \right) + ..., \tag{5.10}
\]

with the effective coupling

\[
g_H = \frac{\gamma}{2} J_3^0. \tag{5.11}
\]

This leads to the partial decay width [42],

\[
\Gamma_{\text{inf} \rightarrow H\bar{H}} = \frac{g^4_H}{8\pi} m_{\text{inf}}. \tag{5.12}
\]

The reheating temperature of the Universe depends on the combined decay width \( \Gamma = \Gamma_{\text{inf} \rightarrow \nu^c_i \nu^c_i} + \Gamma_{\text{inf} \rightarrow H\bar{H}} \) as

\[
T_r = \left( \frac{72}{5\pi^2 g^*} \right)^{1/4} \sqrt{\Gamma m_P}. \tag{5.13}
\]

Assuming a standard thermal history, the number of e-folds, \( N_0 \), can be written in terms of the \( T_r \) as [43]

\[
N_0 = 54 + \frac{1}{3} \ln \left( \frac{T_r}{10^9 \text{ GeV}} \right) + \frac{2}{3} \ln \left( \frac{V(x)^{1/4}}{10^{15} \text{ GeV}} \right). \tag{5.14}
\]

The ratio of the lepton number density to the entropy density in the limit \( T_r < M_{\nu_i} \leq m_{\text{inf}}/2 \leq M_{\nu_2,3} \) is defined as

\[
\frac{n_L}{s} \sim \frac{3}{2} \frac{\Gamma_{\text{inf} \rightarrow \nu^c_i \nu^c_i}}{\Gamma} \frac{T_r}{m_{\text{inf}}} \epsilon_{cp}, \tag{5.15}
\]

\[ \]
where $\epsilon_{cp}$ is the CP-asymmetry factor, which is generated from the out-of-equilibrium decay of the lightest right-handed neutrino. For a normal hierarchical pattern of the light neutrino masses, this factor becomes [44]

$$
\epsilon_{cp} = \frac{3}{8\pi} \frac{M_{\nu_3} m_{\nu_3} \delta_{\text{eff}}}{v_u^2},
$$

(5.16)

where $m_{\nu_3}$ is the mass of the heaviest light neutrino, $v_u$ is the VEV of the Higgs doublet $H_u$, and $\delta_{\text{eff}}$ is the CP-violating phase.

The lepton asymmetry from experimental observations is [45],

$$
| n_L/s | \approx (2.67 - 3.02) \times 10^{-10}.
$$

(5.17)

In the numerical estimates discussed below, we take $m_{\nu_3} = 0.05$ eV, $|\delta_{\text{eff}}| \leq 1$, $v_u = 174$ GeV, while assuming large $\tan \beta$. A successful baryogenesis is usually generated through the sphaleron process [46, 47], where an initial lepton asymmetry, given by

$$
n_L/s \lesssim 3 \times 10^{-10} \left( \frac{\Gamma_{\text{inf} \rightarrow \nu \nu}}{\Gamma} \right) \left( \frac{m_{\nu_3}}{10^6 \text{ GeV}} \right) \left( \frac{M_{\nu_3}}{0.05 \text{ eV}} \right) \delta_{\text{eff}},
$$

(5.18)

is partially converted into baryon asymmetry as $n_B/s = -0.35 n_L/s$.

### 6 Numerical analysis

#### 6.1 Inflationary Predictions

The inflationary slow-roll parameters can be expressed as

$$
\epsilon = \frac{1}{4} \left( \frac{m_P}{M_\alpha} \right)^2 \left( \frac{V''(x)}{V(x) z'(x)} \right)^2, \quad \eta = \frac{1}{2} \left( \frac{m_P}{M_\alpha} \right)^2 \left( \frac{V'''(x)}{V(x) z'(x) V''(x)/V'(x)} - \frac{V''(x) z''(x)}{V(x)(z'(x))^3} \right),
$$

$$
\text{and} \quad s^2 = \frac{1}{4} \left( \frac{m_P}{M_\alpha} \right)^4 \left( \frac{V'(x)}{V(x) z'(x)} \right) \left( \frac{V'''(x) V''(x)}{V(x)(z'(x))^3} - \frac{3 V''(x) z''(x)}{V(x)(z'(x))^2} \right),
$$

(6.1)

where a prime denotes a derivative with respect to $x$. In terms of these parameters, we obtain

$$
r \simeq 16 \epsilon, \quad n_s \simeq 1 + 2 \eta - 6 \epsilon, \quad \text{and} \quad \frac{dn_s}{d \ln k} \simeq 16 \epsilon \eta - 24 \epsilon^2 + 2 s^2,
$$

(6.2)

where the last quantity gives the running of $n_s$. The number of $e$-folds is given by

$$
N_0 = 2 \left( \frac{M_\alpha}{m_P} \right)^2 \int_{x_*}^{x_0} \left( \frac{V(x) z'(x)^2}{V'(x)} \right) dx,
$$

(6.3)
where $x_0$ is the field value at the pivot scale and $x_e$ is the field value at the end of inflation (i.e., when $\epsilon = 1$). Finally, the amplitude of curvature perturbation $\Delta R$ is obtained as

$$\Delta^2 R = \frac{V(x)}{24\pi^2 \epsilon(x)}. \quad (6.4)$$

The results of our numerical calculations are displayed in Figs. 3 and 4, which show the ranges of $\kappa$, $S_0$, and $T_r$ in the $\tilde{\gamma} - \sigma$ plane. The color map in Fig. 3 corresponds to $r$ in the top two panels and to the coupling $\gamma$ in the bottom panel. The light-shaded region in all the panels implies that the inflaton predominantly decays into the neutrinos, whereas the dark-shaded region represents a Higgsino-dominant decay. In obtaining these results, we have used up to second-order approximation on the slow-roll parameters, and have set $\zeta = 0$, $\alpha = 0.3$, and $x_e = 1$. Moreover, we have fixed the $SU(5)$ gauge symmetry-breaking scale $M_\alpha$ to $M_{\text{GUT}} = 2 \times 10^{16}$ GeV and $n_s$ to the central value (0.9655) of the bound from Planck’s data. Finally, the soft masses have been fixed at 12.5 TeV, in order to avoid the dimension-5 proton decay. We restrict ourselves to the parameter region with the largest possible values of $r$ observable by the near-future experiments highlighted below.

In our analysis the soft SUSY contributions to the inflationary trajectory are suppressed, while the radiative and SUGRA corrections, parametrised by $\gamma$ and $\sigma$, play the dominant role. To keep the SUGRA expansion under control we impose $S_0 \leq m_P$. We further require $\sigma \gtrsim -1$ and $\{2 \times 10^6 \lesssim T_r \lesssim 2 \times 10^9\}$ GeV. These constraints appear in Figs. 3 and 4 as the boundaries of the allowed region in the $\tilde{\gamma} - \sigma$ plane.

### 6.2 Observable Primordial Gravitational Waves

The tensor-to-scalar ratio $r$ is the canonical measure of primordial gravitational waves and the next-generation experiments are gearing up to measure it. One of the highlights of PRISM [48] is to detect $r$ as low as $5 \times 10^{-4}$, and a major goal of LiteBIRD [49] is to attain a measurement of $r$ within an uncertainty of $\delta r = 0.001$. Furthermore, the CORE [50] experiment is forecast to have sensitivity to $r$ as low as $10^{-3}$, and PIXIE [51] aims to measure $r < 10^{-3}$ at the $5\sigma$ level. Other future missions include CMB-S4 [52], which has the goal of detecting $r \gtrsim 0.003$ at greater than $5\sigma$ or, in the absence of a detection, reaching an upper limit of $r < 0.001$ at the 95% confidence level, and PICO [53], which aims to detect $r = 5 \times 10^{-4}$ at $5\sigma$.

The explicit dependence of $r$ on $\kappa$ and the $SU(5)$ symmetry-breaking scale $M_\alpha$ is given by the following approximate relation obtained by using the normalization constraint on $\Delta R$

$$r \simeq \left( \frac{2\kappa^2}{3\pi^2 \Delta^2 R} \right) \left( \frac{M_\alpha}{m_P} \right)^4. \quad (6.5)$$
that the above equation gives indeed the case. For fixed the dark shaded region corresponds to Higgsino dominant channel. One can see that, for fixed the light shaded region corresponds to the neutrino dominant channel whereas, displays the range of tensor to scalar ratio Figure 3: Variation of These approximate values are very close to the actual values obtained in our numerical ratio are given by

\[ n_s \simeq 1 + \left( \frac{m_P}{M_\alpha} \right)^2 \left[ - \frac{\frac{z'(x_0)}{z(x_0)^3}}{4} \left( \frac{M_\alpha}{m_P} \right)^2 x_0^2 \left( 2\gamma - \frac{1}{3} \right) + \frac{4}{3} \frac{14 \gamma + 16\gamma^2 - \frac{5}{9} + \frac{9}{2} \sigma}{16\pi^2} F'(5x_0) \right] + \frac{1}{z(x_0)^2} \left( 4 \frac{M_\alpha}{m_P} \right)^2 \left( 2\gamma - \frac{1}{3} \right) + 12 \left( \frac{M_\alpha}{m_P} \right)^4 x_0^2 \left( \frac{14 \gamma + 16\gamma^2 - \frac{5}{9} + \frac{9}{2} \sigma}{3} \right) + \frac{275\kappa^2}{16\pi^2} F'(5x_0) \right] \right]^{-1}, \quad (6.6)\]
whereas the dark shaded region corresponds to Higgsino dominant channel.

The light shaded region corresponds to the neutrino dominant channel whereas the dark shaded region corresponds to Higgsino dominant channel.

Solving these two equations simultaneously, we obtain

\[ r \approx 3 \left( \frac{m_P}{m_{\alpha}} \right)^2 \frac{1}{z(x_0)^2} \left[ 4 \left( \frac{m_{\alpha}}{m_P} \right)^4 x_0^2 \left( 2 \gamma - \frac{1}{3} \right) + \frac{275\kappa^2}{16\pi^2} F'(5x_0) \right] \]

with

\[ z(x_0) \approx \sqrt{1 + \left( \frac{m_{\alpha}}{m_P} \right)^2 x_0^2}. \]

Solving these two equations simultaneously, we obtain \( \gamma \sim 0.17027, \sigma \sim -0.15726 \) for \( S_0 \approx m_P, n_s \approx 0.9655, r \approx 0.0013 \). Similarly, for \( S_0 \approx 0.027m_P, n_s \approx 0.9655, r \approx 7.5 \times 10^{-7} \) we obtain \( \gamma \sim 0.1683, \sigma \sim -1 \). Both these estimates are in good agreement with our numerical results displayed in Figs. 3 and 4. Thus, for couplings \( 0.1678 \lesssim \gamma \lesssim 0.1707 \) and \( -1 \lesssim \sigma \lesssim -0.1525 \), we obtain \( n_s \) compatible with the Planck constraints and \( r \) in the \( 7.5 \times 10^{-7} - 1.5 \times 10^{-3} \) range. At the same time, given our chosen values of \( M_{\alpha} \) and \( M_{\text{SUSY}} \) that yield these results, the proton lifetime lies above the lower bound from the Super-K experiment and should be testable by the future Hyper-K experiment.

Fig. 4 is of particular importance in connection with the non-thermal leptogenesis and shows the variation of \( T_r \) in the \( \tilde{\gamma} - \sigma \) plane. The color map displays the range of right handed neutrino mass \( 4.0 \times 10^{13} \lesssim M_{\nu_1} \lesssim 1.6 \times 10^{15} \) GeV in the left panel and inflaton mass \( 8.3 \times 10^{13} \lesssim \tilde{m}_{\text{inf}} \lesssim 3.3 \times 10^{15} \) GeV in the right panel. Imposing the kinematic condition,

\[ \frac{\tilde{m}_{\text{inf}}}{M_{\nu_1}} \geq 2, \]  

and using \( n_L/s = 3 \times 10^{-10} \), Eq. (5.18) can be written as,

\[ \frac{\Gamma_{\text{inf} \to \nu^\ell\nu^\ell}}{\Gamma_{\text{inf} \to HH}} = \left( \frac{T_r}{2 \times 10^6 \text{ GeV}} \right), \] 

\[ \text{(6.9)} \]
which implies $T_r > 4 \times 10^6$ GeV for the Higgsino-dominant decay of the inflaton, and $T_r < 4 \times 10^6$ GeV for the neutrino-dominant channel.

6.3 BBN Constraints on Reheating Temperature and Gravitino Cosmology

Another important constraint on $T_r$ comes from gravitino cosmology, as it depends on the SUSY-breaking mechanism and the gravitino mass. As noted in [18, 51, 55, 56], one may consider the case of

- a) a stable gravitino as the LSP;
- b) an unstable long-lived ($\tilde{\tau} \gtrsim 1$ sec) gravitino with $m_{3/2} < 25$ TeV;
- c) an unstable short-lived ($\tilde{\tau} < 1$ sec) gravitino with $m_{3/2} > 25$ TeV.

In models based on SUGRA, the relic abundance of a stable gravitino LSP is given [57, 58, 59] by

$$\Omega_{3/2} h^2 = 0.08 \left( \frac{T_r}{10^{10} \text{ GeV}} \right) \left( \frac{m_{3/2}}{1 \text{ TeV}} \right) \left( 1 + \frac{m_{\tilde{g}}^2}{3m_{3/2}^2} \right),$$  \hspace{0.5cm} (6.10)$$

where $m_{\tilde{g}}$ is the gluino mass, $h$ is the present Hubble parameter in units of 100 km sec$^{-1}$ Mpc$^{-1}$, and $\Omega_{3/2} = \rho_{3/2}/\rho_c$.$^6$ A stable LSP gravitino requires $m_{\tilde{g}} > m_{3/2}$, while

6Taking into account only the dominant QCD contributions to the gravitino production rate. In principle there are extra contributions descending from the EW sector, as mentioned in [58] and recently revised in [59].

$^6$ $\rho_{3/2}$ and $\rho_c$ are the gravitino energy density and the critical energy density of the present-day Universe, respectively.
current LHC bounds on the gluino mass are around 2.3 TeV \cite{60}. It follows from Eq. (6.10) that the overclosure limit, $\Omega_{3/2} h^2 < 1$, puts a severe upper bound on $T_r$, depending on $m_{3/2}$. Fig. 5 shows the gluino mass as a function of $T_r$ for $m_{3/2} = (10, 100)$ TeV, with the condition that $\Omega_{3/2} h^2$ does not exceed the observed DM relic abundance, $\Omega h^2 \leq 0.126$ \cite{2}, of the Universe. We see that $(m_\tilde{g} > m_{3/2})$ is satisfied for the entire range of the plotted parameter space, implying that the gravitino is consistently realized as the LSP in the model, and acts as a viable DM candidate.

When the gravitino is the next-to-LSP instead, the role of the LSP (and hence the DM) can be played by the lightest neutralino, $\tilde{\chi}_1^0$, which has two origins: thermal and non-thermal relic. Its thermal production consists of the standard freeze-out mechanism of the weakly interacting massive particles (WIMPs), whereas the non-thermal production proceeds via the decay of the gravitino, itself produced during the reheating process \cite{61, 62}. However, since the density of the thermal relic is strongly model-dependent, we do not take into account its effect in calculating the density parameter here.

An unstable gravitino can be long-lived or short-lived \cite{63, 64}. For $m_{3/2} < 25$ TeV, a gravitino lifetime of $\tilde{\tau} \gtrsim 1$ sec \cite{65, 66} can be sufficiently long to result in the cosmological gravitino problem \cite{63}. The fast decay of gravitino may affect the abundances of the light nuclei, thereby ruining the success of the big-bang nucleosynthesis (BBN) theory. To avoid this problem, one has to take into account the BBN bounds on $T_r$, which are conditioned on $m_{3/2}$ in gravity mediated SUSY-breaking as

$$T_r \lesssim 2 \times 10^9 \text{ GeV} \quad \text{for} \quad m_{3/2} \gtrsim 10 \text{ TeV}. \quad (6.11)$$

A long-lived gravitino scenario is therefore consistent with the BBN bounds \cite{61, 62} for the entire ($12.5 \leq m_{3/2} \leq 25$) TeV range, given the $T_r$ in our model.

For a short-lived gravitino, the above $T_r$ bounds from BBN do not apply, and it can decay into the $\tilde{\chi}_1^0$ LSP, with the resultant abundance given by

$$\Omega_{\tilde{\chi}_1^0} h^2 \simeq 2.8 \times 10^{11} \times Y_{3/2} \left( \frac{m_{\tilde{\chi}_1^0}}{1 \text{ TeV}} \right), \quad (6.12)$$

where the gravitino yield is defined as

$$Y_{3/2} \simeq 2.3 \times 10^{-12} \left( \frac{T_r}{10^{10} \text{ GeV}} \right). \quad (6.13)$$

Requiring $\Omega_{\tilde{\chi}_1^0} h^2 \lesssim 0.126$ leads to the relation

$$m_{\tilde{\chi}_1^0} \gtrsim 19.6 \left( \frac{10^{11} \text{ GeV}}{T_r} \right). \quad (6.14)$$

The prediction of $\{2 \times 10^6 \lesssim T_r \lesssim 2 \times 10^9\}$ GeV by our model with gravity-mediated SUSY-breaking easily satisfies the $m_{\tilde{\chi}_1^0} \geq 18$ GeV \cite{67} limit, thus making the short-lived gravitino a viable scenario also.
7 Summary

To summarize, we have investigated various cosmological implications of a generic model based on the $SU(5)$ gauge symmetry formulated in the framework of no-scale supergravity, highlighting the issues of dimension-5 proton decay, inflation, gravitino, as well as baryogenesis via non-thermal leptogenesis. The breaking of $SU(5)$ gauge symmetry suffers from the magnetic monopole problem. Employing the shifted extension of hybrid inflation the monopole density is diluted and remain within the observable limit. The model yields large values of the tensor-to-scalar ratio ($r \sim 0.0015$) that are potentially measurable by future experiments and favors values of the scalar tilt $n_s$ that are consistent with current constraints. Moreover, the model avoids rapid proton decay via dimension-5 operators and also provides for baryogenesis via non-thermal leptogenesis, with low reheating temperature ($2 \times 10^6 \lesssim T_r \lesssim 2 \times 10^9$) GeV consistent with gravitino cosmology.

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