Schema building profiles among elementary school students in solving problems related to operations of addition to fractions on the basis of mathematic abilities

S Gembong1,*, S T Suwarsono2 and Prabowo1
1Universitas Negeri Surabaya, Ketintang, Gedung K.9, Surabaya 60231, Jawa Timur, Indonesia
2Universitas Sanata Dharma, Mrican, Jogyakarta 55002, Indonesia
*Corresponding author: gembongretno2@gmail.com

Abstract. Schema in the current study refers to a set of action, process, object and other schemas already possessed to build an individual’s ways of thinking to solve a given problem. The current study aims to investigate the schemas built among elementary school students in solving problems related to operations of addition to fractions. The analyses of the schema building were done qualitatively on the basis of the analytical framework of the APOS theory (Action, Process, Object, and Schema). Findings show that the schemas built on students of high and middle ability indicate the following. In the action stage, students were able to add two fractions by way of drawing a picture or procedural way. In the Stage of process, they could add two and three fractions. In the stage of object, they could explain the steps of adding two fractions and change a fraction into addition of fractions. In the last stage, schema, they could add fractions by relating them to another schema they have possessed i.e. the least common multiple. Those of high and middle mathematic abilities showed that their schema building in solving problems related to operations odd addition to fractions worked in line with the framework of the APOS theory. Those of low mathematic ability, however, showed that their schema on each stage did not work properly.

1. Introduction
To understand the concept of mathematics begins by manipulating mental construction [1]. Mental reconstruction here means the creation of action which is interiorized into process, then encapsulated to become object. Object can be de-encapsulated into process. Action, process, object and other schemas are organized into a schema abbreviated as APOS.

In line with the viewpoint, for someone to comprehend a mathematic concept he or she needs to begin by manipulating mental construction through some actions. The actions then are internalized or reflected and contemplated to become process which finally crystalized to become object. Object will be de-encapsulated back into process. Action, process and object will be managed into schema to be used in problem solving. Illustrate mental structure and schema mechanism to construct mathematic knowledge as follows [1-2].
Figure 1. Mental structure and schema mechanism to construct mathematic knowledge

The characteristics of the stage of action, process, object and schema are described in Figure 1 as follows [3]. In the action stage, what someone does is simply applying formulas or directly using formulas given, using existing algorithms, copying examples, and detailed steps to do transformation. Activities in action are procedural ones. In the process stage, an individual can: do transformation without external guidance, reflect transformation steps without doing it in reality, and retrieve transformation steps without doing it in reality. In this step, comprehension is still simply procedural rather than conceptual one. In the object stage an individual can: do actions on objects, decapsulate an object back to where it comes or divide a schema already thematized into components, and identify characteristics of a concept. In the schema stage, an individual can: relate action, process, and object of a concept to those of another concept, interconnect objects and processes in various ways, comprehend realtions among action, process, object and other characteristics already comprehended, and comprehend rules/formulas required.

Some research that has been done is generally only seen in the student's difficulties to solve the problem of fractions. No studies have focused on student schemas [4-5]. Based on these considerations, in this study focused on the formation of student schemas in solving the problem of fractions, especially on the problem of addition operations of fractions.

1.1. Genetic decomposition

In the following it is briefly described analysis of genetic decomposition as the operationalization of APOS theory in the concept of addition of fractions.

1.1.1. Action. Performance in the action stage are procedural activities [6]. For an example, “How much is \( \frac{1}{2} + \frac{1}{4} \). Action of a student toward the problem can be by equating the denominators of the two numbers. For instance, \( \frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} \), then \( \frac{2}{4} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4} \). The student does activities of adding fractions actively by equating the denominators in such a way that it can be stated the sum of \( \frac{1}{2} + \frac{1}{4} \) is \( \frac{3}{4} \). Another action can be performed is by drawing a sketch as Figure 2.

Figure 2. Sketch of \( \frac{1}{2} + \frac{1}{4} \)

1.1.2. Interiorizing: from action to process. If action is done repeatedly and reflection is done on it, such action has been interiorized into a process [6-7]. For instance, “How much is the sum of \( \frac{1}{2} + \frac{1}{4} \)?
in interiorizing the sum, a student does not do an action, he or she, instead, does it in imagination and can explain the process to find the sum, even though by equating the denominators. He or she can imagine to find the sum of $\frac{1}{2} + \frac{1}{4}$ by equating the denominators.

1.1.3. Encapsulating: from process to object. Encapsulating the process to find the sum of a fraction is determined when students can show that the sum of the number has certain characteristics [7-8]. For example, “How much is $\frac{1}{2} + \frac{1}{4}$?” Those who have encapsulated the sum of the fractions as object can explain that it can be solved by equating the denominators because of the characteristics that the two numbers have different denominators. If, however, each number has the same denominator, they can explain that the sum of the fraction is the same as the sum of each numerator divided by the denominators.

1.1.4. Thematizing: from schema to object. Thematizing is the construction which relates separated action, process, and object for an object in such a way that creates a schema. Thematizing addition of fractions as a schema involves certain relationship between addition to integers and concept of the sameness of fractions. A student is categorized into able to thematize addition to fractions of an object if he or she successfully shows the certain relationship between addition of integers and sameness concept of fractions [9]. For an example, “How much is $\frac{1}{2} + \frac{1}{4}$?” Students who have thematized the addition to fractions are able to explain that the addition is a process of identifying the sum of the two with different denominators, and able to explain the relationship between the integers added by relating it to the sameness of a fraction.

As has been described, schema building profile in the present study refers to construction of action, process, object, and other schemas done by an individual in the information to solve problems related to operations of addition to fractions. Action which is done by a subject is viewed from the viewpoint of the subject activities in adding or modifying the schemas already possessed. To reveal the action, it is investigated the efforts made by the subject to comprehend a concept related to problems of fractions. Such action can be manifested in the subject’s behaviors such as using concrete objects, pictures, or prior knowledge. If he or she can reflect the action to solve a problem, the action becomes a process. Then, it is investigated the process as whether or not it has become a cognitive object and what other schemas are involved by the subject to solve problems related to addition of fractions.

2. Methods
2.1. The kind of research and research subject
The present study is qualitative. The researcher explored the schema construction of elementary school students in solving problems related to operations of addition to fractions on the bases of mathematic abilities. To uncover the existing schema construction, students were given problems related to operations of fractions. Based on student worksheets, an in depth interview was carried out. The interview was meant to reveal student schema constructions in solving problems related to operations of fractions.

The subjects of the present study were the fifth graders of the elementary school Garon, Kawedanan District, Magetan Regency in the schooling year of 2016-2017. Three students representing high, middle, and low mathematic abilities.

2.2. Research instruments
The main instrument of the current study was the researcher himself. The researcher, therefore, was actively involved in the field following the subject activities. Data were gathered through interviews. The interviews were initiated by distributing student worksheets containing operations of addition to
fractions. Each worksheet contained 2 equally problems, i.e (1) there is a cake after a party was over. In the following day Budi and Tono ate the cake. Budi ate a half of the cake, while Tono ate a third of the cake. How much cake did Budi and Tono eat?, (2) Mr Ali has a rectangular rice field. It will be given to his two children. The first child gets a quarter while the second one gets a third. How large is the rice field given by Mr Ali to his two children?. Both problems were given interchangely and an indepth interview followed. This is to assure valid data.

2.3. Data analyses
Based on the characteristics of APOS theory as previously explained, Table 1 presents the indicators of the schema construction in solving problems related to operations of addition to fractions.

| Schema Components | Indicators |
|-------------------|------------|
| 1. Action         | 1.1. Able to add two fractions by drawing. |
|                   | 1.2. Able to add two fractions procedurally. |
| 2. Process        | 1.1. Able to add two fractions |
|                   | 1.2. Able to add three fractions |
| 3. Object         | 3.1. Able to explain steps of adding fractions |
|                   | 3.2. Able to change fractions into addition to fractions |
| 4. Schema         | 4.1. Able to add fractions by relating to other schemas or concepts already possessed for instance the least common multiple or the greatest common divisor |
|                   | 4.2. Able to convert sketches of a fraction into sketches of addition to fractions |
|                   | 4.3. Able to explain the steps to add two fractions and explain satisfactorily the reasons behind a given action |

Indicators on Table 3.1 were used by the researcher as the basis to analyse data qualitatively.

3. Results and discussion
This section provides excerpts of the interviews and answer sheets of Subject S1 (high mathematic abilities), Subject S2 (middle mathematic abilities), and Subject S3 (low mathematic abilities). The three Subjects were selected from the fifth graders of elementary school. The following are tiny part of the complex research results.

3.1. Action stage (A)
In adding two fractions by drawing (A1), Subjects S1 and S2 drew sketches one by one. Regarding the results of the addition, Subjects S1and S2 did not apply different marks on the numbers being added. They did action A1 well. Subject S3 could not draw satisfactorily [9-10].

In adding fractions procedurally (A2), Subjects S1and S2 equated the denominators. They were able to add correctly. By so doing, they could do A2 satisfactorily. Subject S3 encountered difficulties to do addition to fractions procedurally [5,9].

3.2. Process stage (P)
In adding two fractions (P1), Subjects S1 and S2 equated the denominators by identifying the least common multiple of each denominator. They could do it (P1) perfectly. Subject S3 could do it (P1) if the two denominators added are the same. They find it difficult if the denominators are different [5,9].

In adding three fractions (P2), Subjects S1 and S2 equated the denominators by identifying the least common multiple of each denominator. They could do it (P2) perfectly. Subject S3, in contrast, found it difficult to add three fractions (P2).
3.3. Object stage (O)
In explaining the steps to add fractions (O₁), Subjects S1 and S2 were able to explain in details the steps to add fractions. Subject S3, however, were difficult to explain them.

To change fractions into addition to fractions (O₂), Subject S1 were able to do by changing the denominator or without changing the denominator. Subject S2 changed only the numerators into other numbers whose sum remained the same and did not change the denominators. Subjects S1 and S2 were able to do O₂ satisfactorily. Subject S3, on the contrary, could not.

3.4. Schema stage (S)
In adding fractions by relating to another schema or concept they already possessed for instance. the least common multiple or the greatest common divisor (S₁), Subjects S1 and S2: (1) if the denominator is the same, they directly adding them, (2), if the denominators of the fractions are not the same, they equated the denominators by identifying the multiples of each denominator. Then, they replaced each fraction new denominators. Then, they added them. Hence, they involved another schema i.e. identifying each denominator’s multiples. Subject S1 and S2 were able to relate schemas they already possessed. In schema S₁, Subject S1 and S2’s schemas were already affirmed [10]. In this stage, Subject S3 encountered difficulties [5,9].

![Figure 3](image-url)

**Figure 3.** Schema construction subject S

- **A**: Action
- **P**: Process
- **O**: Object
- **S**: Schema

**T**: Adding two fractions by way of drawing

**T₁**: Adding two fractions by applying formulas (equating the denominator)

**S**: Adding fractions by relating them to another schema they already possessed i.e. the least common multiple.

**O**: Explaining the steps to add fractions

**P**: Adding two fractions based on examples usually done

**P₁**: Adding three fractions

**O₁**: Changing fractions into addition to fractions

**O₂**: Interrelated

**P**: Interrelated if needed
In drawing a sketch for a fraction to become a sketch of addition to fractions (S2), Subject S1 and S2 changed first the number into sum of fractions. Next, they created the sketch of the addition operation. This means they in schema (S1) were able to relate new schema to object. Subject S3 in this stage encountered problems.

To explain what to do when while adding two fractions and explain the right reasons behind a given action on the schema (S1), Subjects S1 and S2: (1) if the denominator is the same, they directly added them, (2) if the denominators of the two fractions are not the same, they quoted them by identifying the multiples of each denominator. This means that S1 and S2 were able to relate action, process, object and another needed schema that is the least common multiple. In this stage, Subject S3 encountered difficulties.

Based on the data analysis according to the analytical framework of APOS above [1,3], the schema construction of Subject S is illustrated in Figure 3.

4. Conclusion
Referring to the APOS theory as explained in the results and discussion sections, it is concluded that the schema construction among elementary school students in solving problems related to operations to addition of fractions is as follows.

Students of high mathematic ability (subject S1) and middle (subject S2) can perform well both action and process. In the stage of action both subjects were able to add two fractions by way of drawing and procedures. In the stage of process both subjects were able to add two and three fractions. In the stage of object both subjects were able to explain the steps of adding two fractions and to change a fraction into addition to fractions. Finally, in the stage of schema, they were able to add fractions by relating them to another schema they already possessed i.e. the least common multiple. In this stage they could explain what to do to add two fractions but their reasons for the actions were less proper.

Subjects S1 and S2 showed complete schemas when they solved problems related to addition to fractions. Those of low mathematic ability (S3) showed incomplete schemas. They were found difficulties in each stage of action, process, and object.

Based on the conclusion, the researcher suggests that teachers pay attention to students of low mathematic abilities in the lessons of addition to fractions.

References
[1] Arnon I, Cottrill J, Dubinsky E, Oktac A, Fuentes S R, Trigueros M and Weller K 2014 APOS Theory Framework for Research and Curriculum Development in Undergraduate Mathematics Education (New York: Springer-Verlag New York)
[2] Martinez I G and Parragues M 2017 J. Math. Behav. 46 128
[3] Mulyono M 2012 Kreano, Jurnal Matematika Kreatif-Inovatif 3(1) 49-58
[4] Ni Y and Zhou Y-D 2010 J. Educ. Psychologist 40 27
[5] Sankaran S, Sampath H and Sivaswamy J Proceedings of the 17th International Conference on Computers in Education (Hong Kong: Asia-Pacific Society for Computers in Education) p 341
[6] Jitendra A 2007 J. Educ. Psychol. 99 115
[7] Planell R M and Delgado A C 2016 J. Math. Behav. 43 111
[8] DeVries D and Armon I 2004 Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education (Bergen: International Group for the Psychology of Mathematics Education) p 55
[9] Lazic B, Abramovich, Mirela M, and Romano D A 2017 Int. Electronic J. Math. Educ. 8 749
[10] Wijayanti K 2016 J. Phys.: Conf. Ser. 824 1