Dark energy and global rotation of the Universe

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We discuss the problem of universe acceleration driven by global rotation. The redshift-magnitude relation is calculated and discussed in the context of SN Ia observation data. It is shown that the dynamics of considered problem is equivalent to the Friedmann model with additional non-interacting fluid with negative pressure. We demonstrate that the universe acceleration increase is due to the presence of global rotation effects, although the cosmological constant is still required to explain the SN Ia data. We discuss some observational constraints coming from SN Ia imposed on the behaviour of the homogeneous Newtonian universe in which matter rotates relative local gyroscopes. In the Newtonian theory \( \Omega_{r,0} \) can be identified with \( \Omega_{\omega,0} \) (only dust fluid is admissible) and rotation can exist with \( \Omega_{r,0} \leq 0.033 \) at 1\( \sigma \) level.

We are also beyond the model and postulate the existence of additional matter which scales like radiation matter and then analyse how that model fits the SN Ia data. In this case the limits on rotation coming from BBN and CMB anisotropies are also obtained. If we assume that the current estimates are \( \Omega_{m,0} \sim 0.3, \Omega_{\omega,0} \sim 10^{-5} \), then the SN Ia data show that \( \Omega_{r,0} \geq -0.01 \) (or \( \omega_0 < 2.6 \cdot 10^{-17} \text{ rad/s} \)). The statistical analysis gives us that the interval for any matter scaling like radiation is \( \Omega_{r,0} \in (-0.01, 0.04) \).

I. INTRODUCTION

We consider a homogeneous universe which is more general than the FRW model, in which matter additionally rotates relative to local gyroscopes. The motion of the fluid in such a universe is described by the scalar expansion \( \theta \), the rotation tensor \( \omega_{ab} \), and the shear tensor \( \sigma_{ab} \). The homogeneous rotation of fluid as a whole is usually called the global rotation of the universe. Applying these concepts we must remember that CMB strongly restricts (indirectly from observations) the value of angular velocity and rotation can exist with \( \Omega_{r,0} \leq 0.033 \) at 1\( \sigma \) level. These observations show that \( \theta_0 = 3H_0, \omega_0 \lesssim \theta_0/3, \sigma_0 \lesssim \theta_0/4 \).

The propagation equation for \( \theta \), known as the Raychaudhuri equation, for the perfect fluid with energy-momentum tensor \( T_{ab} = (\rho + p)u_a u_b + pg_{ab} \) (where \( \rho \) and \( p \) are the energy density and pressure, respectively), is

\[
\dot{\theta} - \dot{u}_a \frac{\partial \theta}{\partial u^a} + \frac{1}{3} \theta^2 + 2(\sigma^2 - \omega^2) + \frac{1}{2} (\rho + 3p) - \Lambda = 0
\]

(1)

where \( \dot{u}_a \equiv u_a{}^b \dot{b}^b \) is the acceleration vector; we shall use a dot to denote the rate of change of any quantity as measured by an observer moving with 4-velocity \( u^a \); and \( \omega^2 = \omega_{ab}\omega^{ab}/2, \sigma^2 = \sigma_{ab}\sigma^{ab}/2 \) are the scalars of rotation and shear, respectively; and \( \Lambda \) is the cosmological constant.

If we define a representative length \( l \) along a particle world line by

\[
\frac{i}{l} = \frac{1}{3} \Theta
\]

(2)

then \( l \) represents the volume behaviour of the fluid completely. For example from \( l \) one can define the Hubble function \( H \) and the deceleration parameter \( q \) by

\[
H \equiv \frac{1}{l} \dot{l} \quad q \equiv -\frac{l}{1} H.
\]

(3)
Using definition (2) and (3), equation (1) can be rewritten in the form

\[ \dddot{l} = 2(\omega^2 - \sigma^2) + \dot{\omega}^a_a - \frac{1}{2}(\rho + 3p) + \Lambda. \]  

(4)

This shows how the acceleration of the universe (the curvature of curve \( l(t) \)) is directly determined at each point of spacetime. Let us note that \( \Lambda \) acts as a constant repulsive force whereas rotation as a variable repulsive force.

When \( \omega^2, \sigma^2, \) and \( \dot{\omega}^a_a \) are given as a function of \( l \) we can integrate equation (4). To simplify matter we take \( \dot{u}_a = 0 \) (because the acceleration vector represents the effects of non-gravitational forces it vanishes when a particle moves along a geodesic, which would necessarily follow in the case of dust).

It has been shown that spatially homogeneous, rotating, and expanding universes with the perfect fluid have the non-vanishing shear \([6, 7]\). This is quite contrary to the case of the homogeneous Newtonian cosmology where many such solutions are known. These homogeneous shear-free solutions are independent of the pressure which may be set equal to zero or a constant. This difference in the two theories seems to be both surprising and interesting since Ellis’ theorem has a purely local character, and it is completely independent of the strength of the gravitational field \([6, 7, 8]\).

Let us consider solutions with \( \sigma = 0 = \dot{u} \). In this case \( \omega^2 = \omega_0^2/l^4 \) where \( \dot{\omega} = 0 \). Then we can integrate the Raychaudhuri equation using the conservation equation

\[ \dot{\rho} + \Theta(\rho + p) = 0. \]  

(5)

The occurrence of term \( p \) in the factor \( (\rho + p) \) is a special relativistic effect \([6, 7]\).

In the considered case of \( \omega \Theta \neq 0 \) we obtain the generalised Friedmann equation

\[ 3\dot{l}^2 - \frac{\omega_0^2}{l^2} - \frac{\mu l^3}{l} - \Lambda l^2 = -3k \]  

(6a)

\[ \dot{k} = 0 \]  

(6b)

where \( \mu = \text{const}, p = 0 \) and \( l(t) = a(t) \) is the scale factor. From the mathematical point of view equation (6) is a first integral of system (4).

Equation (6) can be treated as basic equations in a Newtonian homogeneous cosmology. Solutions of this equation represent shear-free Newtonian cosmologies which are in general both expanding \( (i \neq 0) \) and rotating \( (\omega_0 \neq 0) \). Equation (6) is called the Heckmann-Schücking equation \([9]\).

If we consider models with \( \omega = 0 = \dot{u}, \sigma \Theta \neq 0 \), and the Ricci tensor \( ^3R_{ab} \) is isotropic then we obtain \( \sigma^2 = \Sigma^2/l^6 \) where \( \Sigma = 0 \). We can then integrate the Raychaudhuri equation to obtain the generalised Friedmann equation

\[ 3\dot{l}^2 - \frac{\Sigma^2}{l^4} - \frac{\mu l^3}{l} - \Lambda l^2 = -3k \]  

(7a)

\[ \dot{k} = 0 \]  

(7b)

where \( l = (a_1 a_2 a_3)\!^{1/3} \) is an average scale factor.

Therefore, it seems reasonable to assume that \( \sigma \) is sufficiently small compared with \( \omega \) since the shear falls off more rapidly than the rotation \([2, 3, 4]\).

In the case of dust \( \sigma^2 \propto a^{-6} \) whereas \( \omega^2 \propto a^{-4} \). The conservation of angular momentum gives \( \omega \rho a^5 = \text{const} \) \([6, 7]\).

From equation (7) we see that the effect of anisotropy is like in the FRW model with stiff matter. In our further analysis of observational effects we consider equation (6) as a simplest model in which the effect of global rotation can be investigated. However, we also consider the presence of additional non-interacting radiation matter which can be treated as a simple extension beyond the Newtonian model.

### II. EFFECT OF GLOBAL ROTATION ON ACCELERATION OF THE UNIVERSE

The supernovae observations indicate that the Universe’s expansion has started to accelerate during recent cosmological times, and CMB observations suggest that the Universe is dominated by a dark energy component, with negative pressure, driving the acceleration \([10, 11]\). While the most obvious candidate for such a component is the vacuum energy a plausible alternative is the dynamical vacuum energy or quintessence. However, these models usually face fine-tuning problems, because there is a question of explaining why the vacuum energy dominates the Universe only recently \([12]\).
To study the effect of the global rotation on the acceleration of the universe we formally introduce rotation to the model by definition of

$$\rho_{\text{eff}} = \rho_m + \rho_\omega = \rho_{m0}a^{-3} + \rho_{\omega0}a^{-4} + \Lambda$$  \hspace{1cm} (8a)

$$p_{\text{eff}} = \frac{1}{3}\rho_\omega - \Lambda$$  \hspace{1cm} (8b)

where $\rho_{\omega0} = -2\omega_0^2 < 0$ and $\rho_\omega = \frac{1}{3}\rho_m$ (like for radiation matter).

Therefore, in the case of dust filled universe, the dynamical effect of global rotation is equivalent to an additional non-interacting fluid with negative pressure.

In order to take into account the effects of rotation we introduce

$$\Omega_\omega = \frac{\rho_\omega}{3H_0^2} = \frac{2\omega_0^2}{3H_0^2} \left(\frac{a}{a_0}\right)^{-4}$$  \hspace{1cm} (9a)

$$\Omega_m = \frac{\rho_m}{3H_0^2} = \frac{\rho_{m0}}{3H_0^2} \left(\frac{a}{a_0}\right)^{-3}$$  \hspace{1cm} (9b)

For our purpose it is also useful to rewrite the dynamical equations to a new form using dimensionless quantities

$$x = \frac{a}{a_0}, \quad T = |H_0|t$$

with $H = \dot{a}/a$, $\rho_{cr,0} \equiv 3H_0^2$ and the subscript 0 means that a quantity with this subscript is evaluated today (at time $t_0$). Additionally we define $\Omega_{k,0} = -3k/6H_0^2$ and $\Omega_{\Lambda,0} = \Lambda/3H_0^2$.

The basic dynamical equations are then rewritten as

$$\dot{x}^2 = \frac{1}{2}\Omega_{k,0} + \sum_i \Omega_{i,0}x^{1-3\gamma_i}$$  \hspace{1cm} (10a)

$$\ddot{x} = -\frac{1}{2}\sum_i \Omega_{i,0}(1 + 3\gamma_i)x^{-2-3\gamma_i}$$  \hspace{1cm} (10b)

where $i = (m, \omega, \Lambda)$. The above equations can be represented as the two-dimensional dynamical system

$$\dot{x} = y$$  \hspace{1cm} (11a)

$$\dot{y} = -\frac{1}{2}\sum_i \Omega_{i,0}(1 + 3\gamma_i)x^{-2-3\gamma_i}$$  \hspace{1cm} (11b)

or by the Hamiltonian dynamical system with the Hamiltonian given in the form

$$\mathcal{H} = \frac{1}{2}\dot{x}^2 + V(x) \equiv 0$$  \hspace{1cm} (12)

and with the potential

$$V(x) = -\frac{1}{2}\Omega_{k,0} - \sum_i \Omega_{i,0}x^{1-3\gamma_i}.$$  \hspace{1cm} (12)

The system should be considered on the zero energy level.

The form of (12) can be useful in particle-like description for the simplest model with global rotation, whereas form (11) is helpful in the analysis of dynamics on a phase plane $(x,y)$.

The system under consideration can be identified after taking

$$w_1 = 1/3 \quad \text{(effect of rotation or radiation)}$$

$$w_2 = 0 \quad \text{(effect of dust matter)}$$

$$w_3 = -1 \quad \text{(effect of } \Lambda, \rho = \Lambda)$$

As an example of application of these equations consider the case of $\Omega_{m,0}, \Omega_{\omega,0}, \Omega_{\Lambda,0} \neq 0$. Then our Universe accelerates provided that the potential $V$ is a decreasing function of its argument

$$-\frac{dV}{dx} = -\Omega_{\omega,0}x^{-3} - \frac{1}{2}\Omega_{m,0}x^{-2} + \Omega_{\Lambda,0}x > 0,$$  \hspace{1cm} (13)
i.e., if $\Omega_{m,0} = 0$, the universe always accelerates for every $x$.

For $\Omega_{\Lambda,0} = 0$ the Universe accelerates provided that

$$x < \frac{\Omega_{\omega,0}}{\Omega_{m,0}}$$

where $\Omega_{\omega,0} < 0$ and $\sum_i \Omega_{i,0} + \Omega_{k,0} = 1$ ($i = m, \omega, \Lambda$).

Our Universe accelerates at present provided that

$$-2 - \Omega_{m,0} + 4\Omega_{\Lambda,0} + 2\Omega_{k,0} > 0$$

or $\Omega_{\Lambda,0} > 0.425$ where we assume $\Omega_{k,0} = 0$, $\Omega_{m,0} = 0.3$. We can see that rotation lowers the value of cosmological constant needed to explain the SN Ia data.

### III. MAGNITUDE-REDSHIFT RELATION IN THE MODEL

The important test to verify whether rotation may represent “dark energy” (which can be called true dark radiation because causes the acceleration of the Universe) is to compare rotation effects with the supernovae type Ia data. The answer is that global rotation may be seriously taken as a candidate to describe only part of dark energy and the cosmological constant is still required.

It is well known that cosmic distance measures, like the luminosity distance, depend sensitively on the spatial geometry (curvature) and dynamics. Therefore, luminosity depends on the present densities of the different components of matter content and their equations of state. For this reason, the magnitude-redshift relation for distant objects is proposed as a potential test for cosmological models and play an important role in determining cosmological parameters.

Let us consider an observer located at $r = 0$ at the moment $t = t_0$ who receives light emitted at $t = t_1$ from a source of absolute luminosity $L$ located at the radial distance $r_1$. Of course the cosmological redshift $z$ of the source is related with $t_1$ and $t_0$ by the relation $1 + z = a(t_0)/a(t_1)$. If the apparent luminosity of the source measured by the observer is $l$, the luminosity distance $d_L$ of the source, defined by

$$l = \frac{L}{4\pi d_L^2} \quad (14)$$

is

$$d_L = (1 + z)a_0 r_1. \quad (15)$$

For historical reasons, the observed and absolute luminosities are defined in terms of K-corrected observed and absolute magnitudes $m$ and $M$, respectively ($l = 10^{-2m/5} \cdot 2.52 \cdot 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-2}$, $L = 10^{-2M/5} \cdot 3.02 \cdot 10^{35} \text{ erg s}^{-2}$) [13]. When written in terms of $m$ and $M$, equation (14) yields

$$m(z, M, \Omega_{m,0}, \Omega_{\Lambda,0}) = M + 5 \log_{10}[D_L(z, \Omega_{m,0}, \Omega_{\Lambda,0})] \quad (16)$$

where

$$M = M - 5 \log_{10} H_0 + 25 \quad (17)$$

and

$$D_L(z, \Omega_{m,0}, \Omega_{\Lambda,0}) \equiv H_0 d_L(z, \Omega_{m,0}, \Omega_{\Lambda,0}, H_0)$$

is the dimensionless luminosity distance in Mpc.

By using expression for the FRW spacetime metric we obtain coordinate distance $r_1$, appearing in [15]

$$\psi(r_1) = \int_{a_0/(1+z)}^{a_0} \frac{da}{a\dot{a}} = -\int_0^0 dr \frac{dr}{\sqrt{1-kr^2}} \quad (18)$$
with
\[
\psi(r_1) = \sin^{-1} r_1 \quad \text{for} \quad k = +1
\]
\[
\psi(r_1) = r_1 \quad \text{for} \quad k = 0
\]
\[
\psi(r_1) = \sinh^{-1} r_1 \quad \text{for} \quad k = -1.
\]
By using Hamiltonian constraint \([12]\) for the model with dust matter, cosmological constant, curvature, and global rotation we obtain
\[
\psi(r_1) = \frac{1}{a_0 \dot{H}_0} \int_0^z \Omega_{k,0}(1 + z')^2 + \Omega_{m,0}(1 + z')^3 + \Omega_{\omega,0}(1 + z')^4 + \Omega_{\Lambda,0} \right)^{-1/2} dz'.
\]
We obtain finally
\[
D_L((z, \Omega_{m,0}, \Omega_{\Lambda,0}, \Omega_{\omega,0}) = \frac{(1 + z)}{\sqrt{K}} \left( \sqrt{K} \int_0^z [(1 - \Omega_{m,0} - \Omega_{\omega,0} - \Omega_{\Lambda,0})(1 + z')^2
\]
\[
+ \Omega_{m,0}(1 + z')^3 + \Omega_{\omega,0}(1 + z')^4 + \Omega_{\Lambda,0} \right)^{-1/2} dz').
\]
where
\[
\xi(x) = \sin x \quad \text{with} \quad K = -\Omega_{k,0} \quad \text{when} \quad \Omega_{k,0} < 0
\]
\[
\xi(x) = x \quad \text{with} \quad K = 1 \quad \text{when} \quad \Omega_{k,0} = 0
\]
\[
\xi(x) = \sinh x \quad \text{with} \quad K = \Omega_{k,0} \quad \text{when} \quad \Omega_{k,0} > 0
\]
and
\[
\Omega_{k,0} = \frac{k}{a_0}.
\]
Thus for given \(M, \Omega_{m,0}, \Omega_{\Lambda,0}, \Omega_{k,0}, \Omega_{\omega,0}\), equations \([16]\) and \([21]\) give the predicted value of \(m(z)\) at a given \(z\).

IV. MAGNITUDE-REDSHIFT RELATION IN THE MODEL — RESULTS

We decided to test our model using the Perlmutter sample \([14]\). To avoid any possible selection effect we choose the full sample without excluding any supernova from that sample. It means that our basic sample is Perlmutter sample A. We test our model using the likelihood method \([11]\).

Firstly, we should estimate the value of \(M\) (equation \([17]\)) from the full sample of 60 supernovae taking \(\Omega_{\omega,0} = 0\) (the pure Perlmutter & Riess model). We obtain value of \(M = -3.39\) (we also assume that the present value of the Hubble constant is \(H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}\) what is in very good agreement with the result of Efstathiou et al. \([14]\) and Vishwakarma \([15]\) (Vishwakarma obtains \(M = 24.03\) for \(c = 1\), i.e., \(M = -3.365\)). Also the value of \(\chi^2\) obtain for Perlmutter’s flat model is 96.5 what is in very good agreement with Perlmutter’s results (see Table 3 in \([11]\)). (Some marginal differences are probably because in our analysis we include both errors in measurements of magnitude and radial distances).

We consider the pure Newtonian model with \(\Omega_{\omega,0} < 0\) and assume that \(\Omega_{m,0} \sim 0.3\) \([16, 17]\). Using the minimalization procedure, described below, with aforementioned assumptions we obtain the density distribution for \(\Omega_{\omega,0}\). The results are presented on Fig. \([1]\) Here we find that the limit for \(\Omega_{\omega,0} > -0.033\) on the 1\(\sigma\) level, while \(\Omega_{\omega,0} > -0.065\) on the 2\(\sigma\) level.

The analysis of the pure Newtonian model is presented on Fig. \([2]\) with the magnitude-redshift relation for real data (marked with asterisks) and for predicted values by models. The top line is the pure Perlmutter flat model with \(\Omega_{m,0} = 0.28, \Omega_{\Lambda,0} = 0.72\). The bottom line is the pure flat model with the cosmological constant \(\Omega_{\Lambda,0} = 0\). Between these models there are located our models with \(\Omega_{\omega,0} = -0.01\) best-fitted model (lower curve) and best-fitted flat model (upper curve). The latter model curve overlaps the Perlmutter model curve. One could observe that the difference between our lower best-fitted model and the Einstein-de Sitter model with \(\Omega_{\Lambda,0} = 0\) is the largest for \(z\) between

\[z \approx 2.23 - 3.23\]
FIG. 1: The density distribution for $\Omega_{\omega,0}$ in the Newtonian model. We obtain the limit $\Omega_{\omega,0} > -0.03$ on the $1\sigma$ level, while $\Omega_{\omega,0} > -0.06$ on the $1\sigma$ level.

0.6 and 0.7 and significantly decreases for higher redshifts. There are significant differences between predictions of these models and Perlmutter’s one where differences to the pure flat model increase for higher redshifts. It gives us possibility to discriminate between the Perlmutter model and our model when data from supernovae more distant than $z \sim 1$ could be available. It is very important because for present data our model is only marginally better than the Perlmutter model.

We can also admit that the total matter content scales like radiation. It means that the contribution coming from $\Omega_{\omega,0}$ is included in $\Omega_{r,0}$. Therefore, in the more detailed analysis we assumed that $\Omega_{k,0} \in [-1, 1]$, $\Omega_{m,0} \in [0, 1]$. From the formal point of view then we obtain the best fit ($\chi^2 = 94.7$) for $\Omega_{k,0} = -1.0$, $\Omega_{m,0} = 0.54$, $\Omega_{\omega,0} = 0.15$, $\Omega_{\Lambda,0} = 1.31$, which is completely unrealistic. However, we should note that we obtain, in fact, a three-dimensional ellipsoid of possible models depending on $\Omega_{m,0}$, $\Omega_{\omega,0}$, $\Omega_{\Lambda,0}$. It is more complicated than in the case of Perlmutter’s analysis when he obtains only two-dimensional ellipsoid (depends only on $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$). But, knowing the best-fit values has no enough scientific relevance, if not also confidence levels for parameter intervals are presented. On the Fig. 4 we show the levels of constant $\chi^2$ on the plane ($\Omega_{\Lambda,0}, \Omega_{m,0}$) minimalized over the rest of parameters. The figure shows the preferred value of $\Omega_{\Lambda,0}, \Omega_{m,0}$. The minimalization procedure confirms the chosen value of $M = -3.39$, because it is a best-fitted value for the flat models. Now, we would like to obtain confidence contours in the $\Omega_{\Lambda,0}, \Omega_{m,0}$ plane.

Since from the formal point of view we have no a priori constraints on cosmological parameters we assume here that $\Omega_{k,0}$ and $\Omega_{m,0}$ are of any value. The result of our analysis are presented on the Fig. 5. This figure shows the confidence levels of 2-dimensional distribution of $(\Omega_{m,0}, \Omega_{\Lambda,0})$. It is analogous to the confidence level figure obtained by Perlmutter.

Another considered case is the flat model ($\Omega_{k,0} = 0$) where we obtain “corridors” of possible models (we presented confidence contours in $\Omega_{\Lambda}, \Omega_{m}$ plane Fig. 6). The formal best-fitted flat model is $\Omega_{m,0} = 0.12$, $\Omega_{\omega,0} = 0.12$, $\Omega_{\Lambda,0} = 0.76$, and...
\[ \chi^2 = 95.7. \] In probably a more realistic case we obtain for flat model \( \Omega_m,0 = 0.28, \Omega_\omega,0 = 0.02, \) i.e., \( \Omega_\Lambda,0 = 0.73. \) For that model \( \chi^2 = 95.9. \) For the flat model with low rotation \( \Omega_m,0 = 0.33, \Omega_\omega,0 = -0.01, \) i.e., \( \Omega_\Lambda,0 = 0.68, \chi^2 = 96.0. \) The value of \( \chi^2 \) is practically the same in all three cases. It clearly shows that statistical analysis is not sufficient for discrimination between statistically available models. To choose the physically plausible model we need additional information which can be obtained for example from extragalactic astronomy investigations (especially estimations for \( \Omega_m,0 \) and \( \Omega_\Lambda,0 \) are useful).

It is interesting to observe how presence of non-zero (but rather small) \( \Omega_\Lambda,0 \) with realistic rotation changes our results. For example for \( \Omega_\Lambda,0 = 0.1, \Omega_m,0 = 0.28, \Omega_\omega,0 = -0.01, \) i.e. \( \Omega_\Lambda,0 = 0.63 \chi^2 = 96.2, \) while for \( \Omega_\Lambda,0 = -0.1, \Omega_m,0 = 0.38, \Omega_\omega,0 = -0.01, \) i.e. \( \Omega_\Lambda,0 = 0.63, \chi^2 = 95.8. \) It shows interesting possibility if separately we could find value for rotation \( \Omega_\omega,0 \) and matter \( \Omega_m,0, \) than we could test the value of \( \Omega_\Lambda,0 \) more precisely than with the models without rotation.

From Ref. [14, 17] we obtain that value \( \Omega_m,0 \) should be not far from 0.3. With this assumption we could find from Fig. 4 that \( \Omega_\omega,0 \) should satisfy \( \Omega_\omega,0 > -0.01 \) which gives critical angular velocity \( \omega_0 = 2.6 \cdot 10^{-19} \text{ rad/s}, \) is in a good agreement with other limits, however it should be pointed that our limit is weaker, Li [12] suggested \( \omega_0 = 6 \cdot 10^{-21} \text{ rad/s}. \) In terms of density parameter that limit requires \( \Omega_\omega,0 > -5.3 \cdot 10^{-6} \) whereas to obtain Ciufolini and Wheeler’s limit \( 7 \) is required \( \Omega_\omega,0 > -1.4 \cdot 10^{-4}. \)

One should note that we give our analysis without excluding any supernovae from Perlmutter’s data. However, from formal point of view, when we analyse full Perlmutter’s sample A, all analysed models should be rejected even on the confidence level 0.99. One of the reasons could be the fact that assumed errors of measurements are too low. Nevertheless, another solution is usually suggested. We can exclude 2 supernovae as outliers and 2 as likely reddened ones from the sample of 42 high redshift supernovae and eventually 2 outliers from the sample of 18 low redshift supernovae (Perlmutter’s sample B and C, respectively). We decided to use full Perlmutter’s sample A as our basic sample because rejecting any supernovae from the sample could be the source of not fully controlled selection effect.

On the other side such procedure also could be useful. It is the reason that we decided to check our analysis using Perlmutter’s samples B and C. It does not significantly change our result, but increases quality of the fit. The formal best-fit for sample B (56 supernovae) is \( (\chi^2 = 57.5) \) what gives \( \Omega_\Lambda,0 = -0.3, \Omega_m,0 = 0.2, \Omega_\omega,0 = 0.17, \) i.e., \( \Omega_\Lambda,0 = 0.93. \) For the flat model we obtain \( (\chi^2 = 57.6) \) \( \Omega_m,0 = 0.03, \Omega_\omega,0 = 0.19, \) i.e., \( \Omega_\Lambda,0 = 0.78, \) while for “realistic” model \( (\Omega_m,0 = 0.28, \Omega_\omega,0 = 0.03) \) \( \Omega_\Lambda,0 = 0.69 \chi^2 = 57.7. \) For the flat model with small rotation \( \Omega_\omega,0 = -0.01, \Omega_m,0 = 0.34, \) i.e., \( \Omega_\Lambda,0 = 0.67, \chi^2 = 57.8. \)
FIG. 3: Levels of constant $\chi^2$ on the plane ($\Omega_{m,0}, \Omega_{\Lambda,0}$) minimalized over the rest of parameters. The figure shows the preferred value of $\Omega_{\Lambda,0}, \Omega_{m,0}$.

The formal best-fit for sample C (54 supernovae) ($\chi^2 = 53.6$) gives $\Omega_{k,0} = -0.1$, $\Omega_{m,0} = 0.11$, $\Omega_{\omega,0} = 0.18$, i.e., $\Omega_{\Lambda,0} = 0.81$, while for flat model $\Omega_{m,0} = 0.05$, $\Omega_{\omega,0} = 0.19$, i.e., $\Omega_{\Lambda,0} = 0.86$, $\chi^2 = 53.6$, while for “realistic” model $\Omega_{m,0} = 0.24$, $\Omega_{\omega,0} = 0.07$, i.e., $\Omega_{\Lambda,0} = 0.84$, $\chi^2 = 53.6$. For the flat model with small rotation $\Omega_{\omega,0} = -0.01$, $\Omega_{m,0} = 0.36$, i.e., $\Omega_{\Lambda,0} = 0.65$, $\chi^2 = 53.7$.

It again confirms our conclusion that on the base of pure statistical analysis we could only select “corridor” of possible models. However, if we assume that the Universe is flat $\Omega_{k,0} = 0$, we obtain estimations for $\Omega_{m,0}, \Omega_{\omega,0}$ what seems to be realistic.

One should note that we also could separately estimate the value of $M$ for sample B and C. We obtain $M = -3.42$ what is again in very good agreement with result of Efstathiou et al. [14] (what for the “combined” sample obtain the value of $M = -3.45$). However, if we use that value in our analysis it does not change significantly our results (value of $\chi^2$ does not change more then 1 what is marginal effect for $\chi^2$ distribution for 53 or 55 degrees of freedom.

We also analyse the influence of rotation for the age of the Universe. The results are presented on Fig. 6. If we assumed that $\Omega_{m,0} = 0.3$ and $H_0 = 65$ km/s/Mpc then small rotation $\Omega_{\omega,0} = -0.01$ increases the age of the Universe from $14.57 \cdot 10^{10}$ yr to $15.17 \cdot 10^{10}$ yr.
Finally, let us study the angular diameter test for our universe. The angular diameter of a galaxy is defined by

\[ \theta = \frac{d(z + 1)^2}{d_L}, \]  

where \( d \) is a linear size of the galaxy. In a pure flat dust model universe \( \theta \) has the minimum value for \( z_{\text{min}} = 5/4 \). It is particularly interesting to notice that for flat models with \( \Omega_{\Lambda,0} \neq 0 \) the dark radiation can increase the minimum value of \( \theta \) toward the largest \( z_{\text{min}} \) and smaller \( \Theta_{\text{min}} \), while the ordinary radiation lowers this value.

We present influence of rotation for the angular diameter \( \Theta(z) \) as a function of redshift \( z \). For the flat model with \( \Omega_{m,0} = 0.3 \) as shown in Fig. 4 the rotation causes the minimum to move right (higher \( z \)) and the minimum value of \( \Theta(z) \) decreases. However, because there are small differences between predicted \( \Theta(z) \) in all considered cases then verifying the observational test could be difficult.
FIG. 5: Confidence levels on the plane \((\Omega_{m,0}, \Omega_{\omega,0})\) minimalized over the rest of parameters for the flat model. The figure shows the ellipsoid of the preferred value of \(\Omega_{m,0}, \Omega_{\omega,0}\). The results prefer the positive value of \(\Omega_{\omega,0}\), while the negative values are allowed (i.e., rotation can exist).

FIG. 6: The angular diameter \(\Theta\) for the flat model with rotation for \(\Omega_{m,0} = 0.3\) and \(\Omega_{r,0} = 0.1, 0, -0.02\) (top, middle, bottom). The minima for these cases are 1.364, 1.605, 1.707, respectively. The rotation causes the minimum to move right (towards to higher \(z\)) and the minimum value of \(\Theta\) decreases.
V. CONCLUSIONS

We discuss the problem of universe acceleration driven by global rotation. We demonstrate that the universe acceleration increase due to the presence of global rotation effects, although the cosmological constant is still required to explain SN Ia data. In this cosmology, the Friedmann equation is modified by appearance of extra term which diminishes with cosmic scale factor as \(-a^{-4}\). Our model suggests limit for rotation \(\Omega_{\omega,0} > -0.033\) (at one \(\sigma\) level) if considered in the Newtonian model, however in the extended model with additional matter which scales like radiation (not necessary \(\Omega_{r,0} < 0\)) we obtain the more safely limit for rotation \(\Omega_{\omega,0} > -0.01\) (at one \(\sigma\) level).

Our limit is weaker than that which can be obtained from BBN (\(\Omega_{\omega,0} = -1.23\Omega_{\gamma,0}\)) and CMB (\(\Omega_{\omega,0} = -0.41\Omega_{\gamma,0}\)) where present value of \(\Omega_{\gamma,0}\) is estimated as \(\Omega_{\gamma,0} = 2.48h^{-2} \cdot 10^{-5}\) [18].

We showed that, although the observational constraint from SN Ia allows only a small contribution from ‘dark radiation’ (however, when in the pure Newtonian model \(\Omega_{\omega,0} < 0\) a much wider range of negative values of \(\Omega_{\omega,0}\) are allowed. We can find the strict analogy between the considered analysis of the observational constraints on the global rotation in the model and the search for observational constraints on dark radiation in brane cosmology. The corresponding term in brane cosmology scales just like radiation with a constant \(\rho_0\) or both positive and negative \(\rho_{\omega,0}\) \(\rho_{\omega,0}\) are possible mathematically. Dark radiation should strongly affect both the Big-Bang nucleosynthesis (BBN) and the cosmic microwave background (CMB). Ichiki et al. [19] used such observations to constrain both the magnitude and the sign of dark radiation in the case when term \(\rho^2\) coming from the brane is negligible (it rapidly decays as \(a^{-8}\) in the early radiation dominated universe). Therefore, the presence of the term is insignificant during the later nucleosynthesis. In such an approximation we recover the considered model in which dark radiation mimics radiation or rotation. Let us note negative contribution coming from the global rotation presence can reconcile the tension between the observed \(^4\)He and \(D\) abundance [14]. The application of these results gives also the possible
constraints on global rotation term from BBN and from the power spectrum of CMB anisotropies.

We obtain the limit for $\Omega_{\omega,0}$ from BBN as $-7.21 \cdot 10^{-5}$, while the limit from CMB is $-2.41 \cdot 10^{-5}$. The present extragalactic data suggest $\omega_0 = 6 \cdot 10^{-21} \text{rad/s}$. This gives the strongest limit for $\Omega_{\omega,0} > -5.3 \cdot 10^{-6}$. Therefore, we can conclude that the present observational data of SN Ia give the weaker limit for rotation then obtained by other methods. However, let us note that the obtained limitations are constructed in independent manner.

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