Mapping Hawking temperature in the spinning constant curvature black hole spaces into Unruh temperature

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We established the equivalence between the local Hawking temperature measured by the time-like Killing observer located at some positions $r$ with finite distances from the outer horizon $r_+$ in the 5-dimensional spinning black hole space with both negative and positive constant curvature, and the Unruh temperature measured by the Rindler observer with constant acceleration in the 6-dimensional flat space by employing the globally embedding approach.

Keywords: Black hole; Hawking temperature; Unruh temperature; Globally embedding approach.

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1. Introduction

In the past decade it was shown that, for both Hawking\cite{123} and Unruh\cite{413} effects, temperature and entropy emerge from information loss associated with real and accelerated-observer horizons, respectively. It was firstly demonstrated in\cite{405} that an observer with a constant acceleration $a$ in de Sitter space will detect a temperature given by $\sqrt{a^2 + 1/l^2}/2\pi$, where $l$ is the de Sitter radius. This result was soon generalized by Deser and Levin\cite{678} that the local Hawking temperatures $\sqrt{a^2 + 1/l^2}/2\pi$ measured by accelerated detectors in (anti-)de Sitter (AdS/dS) geometries can be
obtained from their corresponding Rindler observers with constant accelerations by using the Global Embedding Approach (GEA). For more examples on the equivalence, such as Banados-Teitelboim-Zanelli, Schwarzschild, Schwarzschild-AdS (dS), and Reissner-Nordström solutions into higher dimensional Minkowskian spaces etc., see \[9\] \[10\] \[11\] \[12\] and references therein.

On the other hand, the so-called BTZ (Banados-Teitelboim-Zanelli) black hole solutions \[13\] \[14\] have played the important role in understanding microscopic degrees of freedom of black hole. The BTZ black hole, which is constructed by identifying points along the orbit of a Killing vector in a three dimensional anti-de Sitter space, is an exact solution of Einstein field equations with a negative cosmological constant in three dimensions. This kind of black hole has a topology of $M_2 \times S^1$, where $M_2$ denotes a conformal Minkowski space in two dimensions. Following the same logic, one can construct analogues of the BTZ solution, the so-called constant curvature (CC) black holes in higher $n \geq 4$ dimensional AdS spaces \[15\] \[16\] \[17\]. Because $n$-dimensional anti-de Sitter space has the topology $R_{n-1} \times S^1$, the CC black holes have topology of $M_{n-1} \times S^1$, which is quite different from the known topology of $M_2 \times S^{n-2}$ for the usual black holes in $n$ dimensions. In addition, the exterior region of these CC black holes is time-dependent and thus, there is no global timelike Killing vector \[15\]. Because of this feature, it is difficult to discuss Hawking radiation and thermodynamics associated with these black holes. For example, see \[18\] \[19\] \[20\] and references therein.

Comparing with the anti-de Sitter case, the $n$-dimensional de Sitter space has the different topology $R_1 \times S^{n-1}$. Similar to the negative constant curvature case, a positive constant curvature spacetime was constructed in \[21\] by identifying points along a rotation Killing vector in a de Sitter space. Such solutions turn out to be counterparts of the three-dimensional Schwarzschild de Sitter solution in higher dimensions, and have an associated cosmological horizon. There is a parameter in the solution, which can be explained as the size of cosmological horizon.

The equivalence between the lower dimensional Hawking temperature and higher dimensional Unruh temperature has been well established for the general spinning BTZ black hole in \[9\] \[26\] and the spinless negative/positive CC black hole in \[22\]. It should be noted that the Hawking temperature obtained by Cai and Myung \[22\] is the same as that obtained from semi-classical tunneling method \[23\]. Recently, a reduced approach to the study of Hawking/Unruh effects including their unification was proposed by authors of \[25\]. The primary goal of this work is to generalize the equivalence for the spinless negative/positive CC black hole established in \[22\] to the spinning one. The rest of this paper is organized as follows. In section II, we briefly review the construction of the spinless negative/positive CC black hole from the (anti)-de Sitter space by identifying the points along a boost direction. In section III and IV, we firstly construct the 5-dimensional spinning black hole with negative and positive constant curvature, respectively, and then establish the equivalence between the Hawking temperature in 5-dimension and the Unruh temperature in 6-dimension. Finally, we conclude in section V.
2. review on the spinless constant curvature black hole

In this section, we begin with a brief review the construction of the spinless CC black hole from the (anti-)de Sitter space by identifying the points along one boost direction. The $n$-dimensional (anti-)de Sitter space is defined as (the universal covering) of the hypersurface embedded into a $(n+1)$-dimensional Minkowskian space, satisfying
\[-x_0^2 + x_1^2 + \cdots + x_{n-1}^2 \mp x_n^2 = \mp l^2, \tag{1}\]

here and in the following paragraph of this section, we denote the upper and lower sign for anti-de Sitter and de Sitter space, respectively. The (anti-)de Sitter space admits the boost along Killing vector $\xi = (r+/l)(x_{n-1}\partial_n \pm x_n\partial_{n-1})$ with norm $\xi^2 = (r_+^2/l^2)(\mp x_{n-1}^2 + x_n^2)$.

In what follows, we will firstly discuss the anti-de Sitter case, in which the $(n+1)$-dimensional flat space has two timelike coordinates
\[ds^2 = -dx_0^2 + dx_1^2 + \cdots + dx_{n-1}^2 - dx_n^2. \tag{2}\]

To go further in the discussion, let us introduce the local Kruskal coordinates $(y_\alpha, \phi)$ on anti-de Sitter space (in the region $\xi^2 > 0$),
\[x_\alpha = \frac{2y_\alpha}{1 - y^2}, \quad \alpha = 0, \cdots, n - 2, \tag{3}\]
\[x_{n-1} = \frac{lr}{r_+} \sinh \left( \frac{r_+\phi}{l} \right), \tag{4}\]
\[x_n = \frac{lr}{r_+} \cosh \left( \frac{r_+\phi}{l} \right) \tag{5}\]

where $r$ and $y^2$ are defined as
\[r = r_+ \frac{1 + y^2}{1 - y^2}, \tag{6}\]
and $y^2 = \eta_{\alpha\beta}y^\alpha y^\beta$ [$\eta_{\alpha\beta} = \text{diag}(-1, 1, \cdots, 1)$], respectively. The coordinate ranges are $-\infty < \phi < \infty$ and $-\infty < y^2 < 1$ with the restriction $-1 < y^2 < 1$. By using Kruskal coordinate, the induced metric on the anti-de Sitter space reads
\[ds^2 = \frac{l^2(r + r_+)^2}{r_+^2} dy^\alpha dy^\beta \eta_{\alpha\beta} + r^2 d\phi^2. \tag{7}\]

Without losing any information, we shall restrict the discussion to the five dimensional case. The negative CC black hole can be easily obtained by introducing the local “spherical” coordinates $(t, r, \theta, \chi)$
\[y_0 = f \cos \theta \sinh(r_+ t/l), \quad y_2 = f \sin \theta \sin \chi, \tag{8}\]
\[y_1 = f \cos \theta \cosh(r_+ t/l), \quad y_3 = f \sin \theta \cos \chi, \tag{9}\]

with $f(r) = ((r - r_+)/(r + r_+))^{1/2}$. (Note that these coordinates, with ranges $0 < \theta < \pi/2$, $0 \leq \chi < 2\pi$, $-\infty < t < \infty$, and $r_+ < r < \infty$, do not cover the whole
Using these new coordinates, the metric (7) can be casted into the Schwazschild form
\[ ds^2 = l^2 N^2 d\Omega_3^2 + N^{-2} dr^2 + r^2 d\phi^2 , \]
with \( N^2(r) = (r^2 - r_+^2)/l^2 \) and
\[ d\Omega_3^2 = -\cos^2 \theta dt^2 + \frac{l^2}{r_+^2}(d\theta^2 + \sin^2 \theta d\chi^2) . \]

In this coordinate frame, the Killing vector, generated a boost, becomes into \( \xi = \partial_\phi \) with norm \( \xi^2 = r^2 \), and the spinless negative CC black hole can be obtained by identifying the points along the Killing vector \( \xi \)
\[ \phi \sim \phi + 2n\pi , \quad n \in \mathbb{Z} . \]
The horizon for the negative CC black hole in these coordinates is located at \( r = r_+ \), the point where \( N^2 \) vanishes.

The positive CC black hole can be constructed analogously, by identifying the points along the same Killing vector \( \xi = \partial_\phi \) with norm \( \xi^2 = r^2 \). In details, we firstly embed the \( n \)-dimensional de Sitter space into a \( (n+1) \)-dimensional Minkowskian space
\[ ds^2 = -dx_0^2 + dx_1^2 + \cdots + dx_{n-1}^2 + dx_n^2 . \]
Furthermore, we define the Kruskal coordinates \( (y_\alpha, \phi) \) on the \( n \)-dimensional de Sitter space in the region \( 0 \leq \xi^2 \leq r_+^2 \),
\[ x_\alpha = \frac{2l y_\alpha}{1 - y_\alpha^2} , \quad \alpha = 0, \cdots , n - 2 ; \]
\[ x_{n-1} = \frac{lr}{r_+} \sin \left( \frac{r_+ \phi}{l} \right) ; \]
\[ x_n = \frac{lr}{r_+} \cos \left( \frac{r_+ \phi}{l} \right) ; \]
with
\[ r = r_+ \frac{1 - y^2}{1 + y^2} , \quad y^2 = \eta_{\alpha\beta} y^\alpha y^\beta , \]
\[ \eta_{\alpha\beta} = \text{diag}(-1, 1, \cdots , 1) . \]
Here the coordinate range is \( -\infty < y_\alpha < +\infty \), and \( -\infty < \phi < +\infty \) with the restriction \(-1 < y^2 < 1\) in order to keep \( r \) positive. Under the Kruskal coordinate frame, the induced metric takes the same form with (7). We can also introduce Schwarzschild coordinates to describe the solution. Using local spherical coordinates \( (t, r, \theta, \xi) \) defined as (5) with \( f(r) = [(r_+ - r)/(r + r_+)]^{1/2} \), and the coordinate ranges are \( 0 < \theta < \pi/2 \), \( 0 \leq \chi < 2\pi \), \( -\infty < t < +\infty \), and \( 0 < r < r_+ \), we find that the solution can be expressed as
\[ ds^2 = l^2 N^2 d\Omega_3^2 + N^{-2} dr^2 + r^2 d\phi^2 , \]
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with $N^2(r) = (r_+^2 - r_-^2)/l^2$ and

$$d\Omega_2^2 = -\cos^2 \theta dt^2 + \frac{l^2}{r_+^2}(d\theta^2 + \sin^2 \theta d\chi^2).$$

(19)

Like what happens for the negative constant curvature case, the spinless positive CC black hole can be constructed by identifying the points along the Killing vector $\xi = \partial_\phi$ with norm $\xi^2 = r^2$

$$\phi \sim \phi + 2n\pi, \quad n \in \mathbb{Z}.$$  

(20)

In these coordinates $r = r_+$ is the cosmological horizon. The only difference is that $N^2 = (r^2 - r_+^2)/l^2$ in the negative CC black hole space there is replaced by $N^2 = (r_+^2 - r_-^2)/l^2$ in the positive one.

3. spinning negative constant curvature black hole

In this section, we firstly construct the 5-dimensional spinning negative CC black hole by virtue of the GEA, then we calculate both the local Hawking temperature observed by a time-like Killing observer in the 5-dimensional hypersurface and the corresponding Unruh temperature observed by a Rindler observer in the 6-dimensional flat space.

By using the “spherical” coordinate $(t, \rho, \theta, \phi, \chi)$

$$x_0 = l \sinh \rho \cos \theta \sinh \left(\frac{r_+}{l} t - \frac{r_-}{l} \phi\right),$$

$$x_1 = l \sinh \rho \cos \theta \cosh \left(\frac{r_+}{l} t - \frac{r_-}{l} \phi\right),$$

$$x_2 = l \sinh \rho \sin \theta \sin \chi,$$

$$x_3 = l \sinh \rho \sin \theta \cos \chi,$$

$$x_4 = l \cosh \rho \sin \left(\frac{r_+}{l} \phi - \frac{r_-}{l} t\right),$$

$$x_5 = l \cosh \rho \cos \left(\frac{r_+}{l} \phi - \frac{r_-}{l} t\right),$$

(21)

with $r_+$ and $r_- (r_+ > r_-)$ two arbitrary real constants with dimensions of length, the 5-dimensional spinning negative CC black hole can be obtained

$$ds^2 = \cos^2 \theta \left[-N^2 l^2 dt^2 + r^2 (d\phi + N^\phi dt)^2\right]$$

$$+ N^{-2} dr^2 + l^2 \frac{r^2 - r_-^2}{r_+^2 - r_-^2} (d\theta^2 + \sin^2 \theta d\chi^2)$$

$$+ l^2 \frac{r^2 - r_-^2}{r_+^2 - r_-^2} \sin^2 \theta \left(\frac{r_+}{l} d\phi - \frac{r_-}{l} dt\right)^2,$$

(22)

by identifying

$$\phi \sim \phi + 2n\pi, \quad n \in \mathbb{Z}.$$  

(23)
where
\[ N^2 = \left( r^2 - r_+^2 \right) \left( r^2 - r_-^2 \right) \frac{l^2 r^2}{l^2 r_+^2} \],
(24)
\[ N^\phi = -\frac{r_+ r_-}{r^2} \],
(25)
and
\[ r^2 = r_+^2 \cosh^2 \rho - r_-^2 \sinh^2 \rho \].
(26)
In these coordinates, \(-\infty < t < \infty\), \(0 < \theta < \pi/2\), \(0 \leq \chi < 2\pi\) and \(r_+ < r < \infty\).

In the 5-dimensional spinning space, the local observer is chosen as the one whose trajectory follows the time-like Killing vector \(\zeta = \partial_t + \frac{r_-}{r_+} \partial \phi\), namely the “time-like Killing observer” \((O_K)\)
\[ \phi = \frac{r_-}{r_+} t, \quad r, \theta, \chi = \text{const.} \],
(27)
then the proper velocity of \((O_K)\) is
\[ u^\mu = (t, r, \theta, \phi, \chi) = \begin{pmatrix} \frac{r_+}{\cos \theta \sqrt{(r^2 - r_+^2)(r^2 - r_-^2)}} & 0, 0, \frac{r_-}{\cos \theta \sqrt{(r^2 - r_+^2)(r^2 - r_-^2)}} \end{pmatrix} \],
(28)
and the proper acceleration for \((O_K)\) reads
\[ a_5^\mu = \begin{pmatrix} 0, \frac{r^2 - r_+^2}{l^2 r}, -\frac{(r_+^2 - r_-^2) \tan \theta}{l^2 (r^2 - r_+^2)}, 0, 0 \end{pmatrix} \].
(29)
Thus the \(a_5\) equals
\[ a_5 = \frac{\sqrt{(r^2 - r_+^2) + \tan^2 \theta (r_+^2 - r_-^2)}}{l \sqrt{r^2 - r_+^2}} \].
(30)
The Killing vector tangent to the worldline of the time-like observer \((O_K)\) is
\[ \zeta = \partial_t + \frac{r_-}{r_+} \partial \phi, \]
consequently, the Hawking temperature which enters the thermodynamical relations can be obtained through the surface gravity [\(\kappa \equiv -\frac{1}{2} (\nabla^\mu \xi^\nu) (\nabla_\mu \xi_\nu)\)]
\[ 2\pi T_{HK} = \kappa = \frac{r_+^2 - r_-^2}{r_+ l} \].
(31)
This result is consistent with the one which is obtained by embedding the spinning CC black hole into a Chern-Simons supergravity theory \([16]\). However, as pointed out in \([22]\), the method used in \([16]\) has two drawbacks, one is that the result cannot be degenerated to the spinless case, the other is that it cannot be generalized to other dimensions. Along the integral curves generated by the Killing vector \(\zeta\) the line element becomes
\[ ds^2 = \cos^2 \theta \left[ -N^2 (\frac{r_+}{l})^2 dt^2 + r^2 (\frac{r_-}{r_+} dt + N^\phi dt)^2 \right] + \cdots , \]
\[ = -\cos^2 \theta \left( \frac{r_+^2 - r_-^2}{r_+} \frac{r^2 - r_-^2}{r_+^2} \right) dt^2 + \cdots . \]
(32)
Armed with the above results, we can define the Tolman temperature which is the local Hawking temperature observed by the time-like Killing observer \((O_K)\) located at \(r\)

\[
2\pi T = \frac{\kappa}{\sqrt{-g_{00}}} = \frac{\sqrt{r_+^2 - r^2}}{l \cos \theta \sqrt{r^2 - r_+^2}},
\]

where \(\hat{g}_{00}\) is the Tolman redshift factor which can be read off from (32).

On the other hand, the worldline of the time-like Killing observer in the 5-dimensional spinning black hole space coincides with the trajectory of Rindler observer with the constant acceleration

\[
a_0^2 = x_1^2 - x_0^2 = l^2 \frac{r_+^2 - r^2}{r_+^2 - r_-^2} \cos^2 \theta,
\]

thus, the corresponding Unruh temperature reads

\[
2\pi T_{DU} = a_0 = \frac{\sqrt{r_+^2 - r_-^2}}{l \cos \theta \sqrt{r^2 - r_+^2}} = \sqrt{\frac{1}{l^2} + a_0^2}.
\]

From (33) and (35) we can easily see that the local Hawking temperature measured by a time-like Killing observer in the 5-dimensional space is nothing but the Unruh temperature observed by a Rindler observer in the 6-dimensional flat space.

### 4. spinning positive constant curvature black hole

Analogously, the 5-dimensional spinning positive CC black hole can also be constructed by using the “spherical” coordinate \((t, \rho, \theta, \phi, \chi)\)

\[
x_0 = l \sin \rho \cos \theta \sinh \left(\frac{r_+ - r}{l} + \phi \right),
\]

\[
x_1 = l \sin \rho \cos \theta \cosh \left(\frac{r_+ - r}{l} - \phi \right),
\]

\[
x_2 = l \sin \rho \sin \theta \sin \chi,
\]

\[
x_3 = l \sin \rho \sin \theta \cos \chi,
\]

\[
x_4 = l \cos \rho \sin \left(\frac{r_+ - r}{l} \phi + \frac{r_- - r}{l} \right),
\]

\[
x_5 = l \cos \rho \cos \left(\frac{r_+ - r}{l} \phi + \frac{r_- - r}{l} \right).
\]

Plugging (36) into the 6-dimensional Minkowskian metric and identifying \(\phi \sim \phi + 2n\pi\) with \(n \in \mathbb{Z}\), one can derive the induced metric which describes a 5-dimensional spinning black hole with positive constant curvature

\[
d^2 = \cos^2 \theta \left[ -N^2 l^2 dt^2 + r^2 (d\phi + N^\phi dt)^2 \right] + N^{-2} dr^2
\]

\[
+ l^2 \frac{r_+^2 - r^2}{r_+^2 + r_-^2} \left( d\theta^2 + \sin^2 \theta d\chi^2 \right) + l^2 \frac{r_+^2 + r_-^2}{r_+^2 + r_-^2} \sin^2 \theta \left( \frac{r_+}{l} d\phi + \frac{r_-}{l} dt \right)^2,
\]
where
\[ N^2 = \frac{(r^2_- - r^2)(r^2 + r^2_+)}{l^2 r^2}, \quad (38) \]
\[ N^\phi = \frac{r^2_+}{r^2}, \quad (0 < r_- < r_+), \quad (39) \]
and
\[ r^2 = r^2_+ \cos^2 \rho - r^2_\phi \sin^2 \rho, \quad 0 < r < r_+. \quad (40) \]
The ranges of other “spherical” coordinates \((t, \theta, \phi, \chi)\) are the same as the case with negative constant curvature. In these coordinates, the cosmological horizon locates at \(r_+\) and the ranges \(0 < r < r_+\) represents the interior of the horizon \(^{24}\).

The 5D CC black hole is constructed by indentifying \(\phi \sim \phi + 2n\pi, \quad n \in \mathbb{Z}\) \(^{(41)}\),

The time-like Killing observer \((\mathcal{O}_K)\) is chosen as
\[ \phi = -\frac{r_-}{r_+} t, \quad r, \theta, \chi = \text{const.}, \quad (42) \]
then the corresponding proper velocity is
\[ u^\mu = (t, r, \theta, \phi, \chi) = \begin{pmatrix} \frac{r_+}{\cos \theta \sqrt{(r^2_+ - r^2)(r^2_+ + r^2_\phi)}}, 0, 0, \frac{-r_-}{\cos \theta \sqrt{(r^2_+ - r^2)(r^2_+ + r^2_\phi)}}, 0 \end{pmatrix}. \quad (43) \]
The proper acceleration for observer \((\mathcal{O}_K)\) can be derived by differentiating the proper velocity \(^{(43)}\) with respect to the proper time
\[ a^\mu_5 = \begin{pmatrix} 0, -\frac{r^2 + r^2_\phi}{l^2 r}, -\frac{(r^2_+ + r^2_\phi) \tan \theta}{l^2 (r^2_+ - r^2)}, 0, 0 \end{pmatrix}, \quad (44) \]
thus the \(a_5\) equals
\[ a_5 = \sqrt{\frac{(r^2_+ + r^2_\phi) + \tan^2 \theta (r^2_+ + r^2_\phi)}{l \sqrt{r^2_+ - r^2}}}. \quad (45) \]
The Killing vector tangent to the worldline of the time-like observer \((\mathcal{O}_K)\) reads \(\zeta = \partial_t - r_-/r_+ \partial_\phi\), so the Hawking temperature reads
\[ 2\pi T_{\text{HK}} = \kappa = \frac{r^2_+ + r^2_\phi}{r_+ l}. \quad (46) \]
This result is consistent with those obtained in \(^{24}\). Along the integral curves generated by the Killing vector \(\zeta\), the line element becomes
\[ ds^2 = \cos^2 \theta \left[ -N^2 l^2 dt^2 + r^2 \left( \frac{r_-}{r_+} dt + N^\phi dt \right)^2 \right] + \cdots, \]
\[ = -\cos^2 \theta \frac{(r^2_+ - r^2)(r^2_+ + r^2_\phi)}{r^2_+} dt^2 + \cdots. \quad (47) \]
The local Hawking temperature measured by the time-like Killing observer ($O_K$) located at $r$ equals the Hawking temperature in (46) rescaled by a redshift factor

$$2\pi T = \frac{\kappa}{\sqrt{-g_{00}}} = \frac{\sqrt{r_+^2 + r_-^2}}{l \cos \theta \sqrt{r_+^2 - r^2}},$$

where the redshift factor are derived in (47).

The corresponding Unruh acceleration and temperature can also be obtained by using the same logic in the negative case

$$a_6^{-2} = x_1^2 - x_0^2 = l^2 \frac{r_+^2 - r^2}{r_+^2 + r_-^2} \cos^2 \theta,$$

and

$$2\pi T_{DU} = a_6 = \frac{\sqrt{r_+^2 + r_-^2}}{l \cos \theta \sqrt{r_+^2 - r^2}} = \sqrt{\frac{1}{l^2} + a_5^2}.$$  

As we expect that the temperatures (48) and (50) coincide exactly.

5. Conclusion

In this work, we established the equivalence between the local Hawking temperature measured by the time-like Killing observer located at some positions $r$ with finite distances from the outer horizon $r_+$ in the 5-dimensional spinning CC black hole space, and the Unruh temperature measured by the Rindler observer with constant acceleration in the 6-dimensional flat space by employing the globally embedding approach. For the spinning black hole with negative constant curvature, the local Hawking or Unruh temperature equals to $\sqrt{r_+^2 - r^2}/2\pi l \cos \theta \sqrt{r_+^2 - r^2}$, while for the positive case, the temperature reads $\sqrt{r_+^2 + r_-^2}/2\pi l \cos \theta \sqrt{r_+^2 - r^2}$, where $l$ is the radius of (anti-)de Sitter spaces and $\theta$ is the spherical coordinate of time-like Killing observer. Our results can recover the previous one in both the general spinning BTZ black hole [17] when one sets $\theta = 0$ and the spinless black hole with negative/positive constant curvature [22] when one sets both $\theta = 0$ and $r_- = 0$.

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