About Possibility for Examination of Gravity Theories Using the Precise Measurement of Particle Lifetime

Kh.M. Beshtoev

Joint Institute for Nuclear Research, Joliot Curie 6, 141980 Dubna, Moscow region, Russia

Abstract

An approach for examination of gravitational theories using precision measurements of particle lifetime is proposed. The expressions describing dependence of particle lifetime on gravitational potential in Einstein’s and Newton’s gravity theories are obtained. In the case of Newton’s gravity there is a dependence of the particle velocity direction from the direction of matter location, which creates the gravitational potential. If the external gravitational field is spherical symmetric then there would be no possibility to distinguish these types of gravity. It is found that the deposit of gravitational potential of the Universe (in the case of uniformly distribution of matter in the Universe) in particle lifetime is approximately one percent. On the basis of the available experimental data it is found that deposit of asymmetric gravitational field is \( \phi \simeq 2 \times 10^{-4} \), i.e., if the experimental precision of particle lifetime measurements will be several units of \( 10^{-4} \), then we could see this effect.

In reality, the lifetime of elementary particles can be defined by effective masses of these particles in the external gravitational field. The expressions for effective masses of particles in the external gravitational field for two gravity type theories are obtained. These masses can be used at computation of the decay probability (or lifetime) of particles by standard methods. It is shown that in this case it is also possible to distinguish these two types of gravity theories.

I. Introduction

Probably, brain of scientists has already settled the idea that gravitational interactions are connected with space curvature and our space
is a curved one [1]. Nevertheless, this problem demands a subsequent investigation.

a). Gravitational Red Shift.

In the work [2] it was shown, that radiation spectrum (or energy levels) of atoms (or nuclei) in the gravitational field has a red shift since the effective masses of radiant electrons (or nucleons) change in this field. This red shift is equal to the red shift of the radiation spectrum in the gravitational field measured in existing experiments [3, 4]. The same shift must arise when the photon (or $\gamma$ quantum) is passing through the gravitational field if it participates in gravitational interactions. Then, on experiments one must detect double red shift. Absence of the double shift in the experiments means that photons (or $\gamma$ quanta) are passing through the gravitational field without interactions (see also Ref. [5]).

b). Advance of Perihelion of Planets.

It is necessary to note that in the work [6, 7] expression for the advance of the perihelion of Mercury was obtained in the case of flat space, which is mathematically equivalent to the Einstein’s expression.

c). Photon Deflection in Gravitational Field.

It is well known, that only massive bodies and particles participate in the Newton’s theory of gravitation (i.e. body and particle having the rest mass). Since the photons have no rest mass, the usage of the ”mass” $m_{ph}$ obtained in the formula

$$m_{ph} = \frac{E_{ph}}{c^2} = \frac{h\nu}{c^2},$$  

(1)

is a hypothesis to be checked of. The check has shown (see above or [2] ) that there are no photons (or $\gamma$ - quanta) red shift when they pass through the gravitational field. It is obvious, that since they have no rest masses (or a gravitational charge), they cannot participate in the gravitational interactions. Then, the deflection of photons in gravitational fields must not exist either. The question arises: How could the deflection of photons appear in the gravitational field, if they do not participate in these interactions? It is clear that this question calls for an answer. Let’s note, that the given question has been discussed in work [8] (see also references in [8]), where it was shown that from the
available experimental data it is not impossible to come to a conclusion that photons are deflected in the gravitational field.

Probably, in order to clarify these problems it is necessary to perform some experiments. I would like to indicate yet one more experimental possibility for examination of the idea of connecting gravitational interactions with space curvature (the Einstein’s theory of gravity).

II. About Possibility for Examination of Gravitational Theories by Using Precise Lifetimes of Relativistic Particles

If particle’s lifetime depends on gravity field (potential) in the decay points, then we have a chance to examine how its lifetime change in dependence of gravitational potential and velocity of the particle. Further on, we will obtain an expression for particle lifetime depending on gravitation potential and its velocity in the framework of Einstein’s and Newton’s theory gravity. It is necessary to stress that in this case we use the gravitational potential in the point of particle location and it includes the masses \( M \) of all objects, which create this potential, i.e., in this approach will be used potential but not potential gradient (see [3, 4]).

II. 1. The Proper \( \tau \) and World \( x^0 \) Times

In the General relativistic theory [1], it is supposed that influence of gravitation is reduced to appearance of a space-time curvature described by the curvature tensor \( g_{\alpha\beta}; \alpha, \beta = 0, 1, 2, 3 \), and the source of gravitational field is the energy-momentum tensor \( T_{\alpha\beta} \). In case of a weak gravitational field \( \varphi/c^2 \ll 1 \) from this curved space, we may come to flat (quasi-flat) space, where gravitational field contribution is determined by the potential \( \varphi(x) \) [1]. Naturally, the \( \varphi \) is equivalent to the Newton gravitational potential. Particularly simple expressions are obtained in the case of stationary (time independent) gravitational field. In this case the nondiagonal terms of curvature tensor are equal to zero, and

\[
g_{oo} = 1 + 2\varphi/c^2. \quad (2)
\]
We will consider the case of stationary field since it presents interest to us. Then connection between proper time $\tau$ (i.e., time when we take into account the gravity) and the world time $x_o/c$ (the time without the gravity) is determined by the following expression:

$$x_o = \frac{\tau}{\sqrt{g_{oo}}} c = inv,$$  \hspace{1cm} (3)

or

$$\tau = \frac{1}{c} \sqrt{g_{oo}} x_o.$$ \hspace{1cm} (4)

In the case of weak gravitational fields from (4), we come to the following expression:

$$\tau = \frac{x_o}{c} (1 + \frac{\varphi}{c^2}).$$ \hspace{1cm} (5)

For the photon frequency $\omega$ in the external gravitational field $\varphi$ we have the following expression

$$\omega = \omega_o (1 - \frac{\varphi}{c^2}),$$ \hspace{1cm} (6)

where $\omega_o$ is photon frequency in the absence of gravitational field.

We will work in the framework of the method which is used in [1] (L.D. Landau, E.M. Lifshitz) where $\varphi > 0$ and in the case of necessity it needs to change on $\Delta \varphi$ which may be either positive or negative.

Since in the flat space the time $x_o$ duration is identical to time $x'_o$ duration in the proper reference system of a physical object, then the relativistic transformation between them has the standard form

$$x'_o = \gamma x_o,$$ \hspace{1cm} (7)

and the general formula for the proper time of relativistic object in gravitational field takes the form

$$x'_o = \gamma (1 + \frac{\varphi}{c^2}) x_o.$$ \hspace{1cm} (8)

Then for

$$\tau' = \frac{x'_o}{c}, \hspace{1cm} \tau_o = \frac{x_o}{c},$$ \hspace{1cm} (9)

we have

$$\tau' = \gamma (1 + \frac{\varphi}{c^2}) \tau_o.$$ \hspace{1cm} (10)
The expression (10) describes duration of time for identical physical processes of moving and a resting objects in the given reference system in presence of gravity.

We will consider $\tau'$ as a lifetime (or decay time) of the particle with velocity $v$ in the stationary external gravitational field $\varphi$.

The photon frequency transformation, emitted by relativistic object in gravitational field, is determined in a similar way (however see Ref. [6, 7]).

In the Newton’s gravity case the analogous expression for time duration has the following form:

$$\tau' = \gamma \left(1 + \frac{\varphi}{c^2} \frac{1}{\sqrt{1 + \sin^2 \theta (\gamma^2 - 1)}}\right) \frac{x_0}{c},$$

(11)

where angle $\theta$ is angle between direction of the particle velocity $\vec{v}$ and unity vector $\vec{n}$ to the center of gravitational system.

From Exp. (10) we can see that in Einstein’s case the fraction

$$\frac{\tau'}{\gamma} = (1 + \frac{\varphi}{c^2}) \frac{x_0}{c} = \text{inv},$$

(12)

is invariant. But in the Newton’s case the value $\frac{\tau'}{\gamma}$

$$\frac{\tau'}{\gamma} = (1 + \frac{\varphi}{c^2} \frac{1}{\sqrt{1 + \sin^2 \theta (\gamma^2 - 1)}}) \frac{x_0}{c},$$

(13)

is not invariant and depends on $\gamma$ and velocity direction.

Now we have to discuss a question related to the required precision for measurement of particles lifetime. For this purpose we must know the estimation of value $\frac{\varphi}{c^2}$ created by Universe matter in the experimental point (i.e. at the Earth surface).

Let us fulfill estimation of $\frac{\varphi}{c^2}$ of our Universe. For estimation of average value of $\varphi$ we can use the following expression:

$$d\varphi = G \frac{dM}{R} = G 4\pi \rho \frac{R^2 dR}{R} = 4\pi \rho RdR,$$

(14)

where $G$ is gravitational constant, $\rho$ is average matter density of the Universe. Then

$$\left| \frac{\varphi}{c^2} \right| = 4\pi G \rho \int_0^{R_o} RdR = 2\pi G \rho R_o^2,$$

(15)
where $\rho \sim 3.0 \cdot 10^{-31} \text{g/cm}^3$ is the Universe matter density, $R_o$ is the Universe radius and $R_o \sim 10^{10}$ years [9] (we presume that $c$ the light velocity equal to one). And then the average value of $\frac{\mathcal{F}}{c^2}$ is

$$| \frac{\mathcal{F}}{c^2} | \simeq 1.25 \cdot 10^{-2}.$$  \hspace{1cm} (16)

Obviously, in the modern experiments we can principally reach one percent precision and to measure the effects connected with the gravitational field of the Universe matter. However, in previous considerations we presumed that the matter is distributed symmetrically in the Universe, and then we cannot distinguish the Einstein’s and Newton’s gravity (see Exp. (11), (13)). In order to distinguish, which of these gravity theories is realized in Nature, the matter distribution in the Universe must be asymmetrical. In any case there is an asymmetry since the Earth is not placed in the center of the Universe. At present, there are found such asymmetries [9] and estimation of the gravitational potential of this one gives that it is order of (if we take into account the dark matter, i.e. neutrino masses and et cetera)

$$\frac{\mathcal{F}}{c^2} \simeq 2 \cdot 10^{-4}.$$  \hspace{1cm} (17)

From the Exp. (17), we can see that the experimental precision of decay time measurements must be several units of $10^{-4}$ in order to see this effect.

II.2. Lifetime of Elementary Particles in a Stationary Gravitational Field

In reality, the lifetime of elementary particles can be defined by effective masses of these particles in external gravitational field. In this case we must use particle effective masses, computing their lifetimes by the standard methods.

Expression for energy of a physical object in Einstein’s theory (when the object is moving along the world line) has the following form [1]:

$$E_o = m_o c^2 g_{oo} \frac{dx^o}{ds} = m_o c^2 \frac{dx^o}{\sqrt{g_{oo}(dx^o)^2 - dl^2}}.$$ \hspace{1cm} (18)
Introducing the object velocity $v$ (i.e., observer time)

$$v = \frac{dl}{d\tau} = \frac{cdl}{\sqrt{g_{oo}dx^o}},$$

(19)

into the reference system of the observer, we obtain expression of energy of the object in the given system

$$E_o = \frac{m_oc^2 \sqrt{g_{oo}}}{\sqrt{1 - \frac{v^2}{c^2}}}.$$  

(10)

At $\frac{\varphi}{c^2} \ll 1$

$$\sqrt{g_{oo}} \approx 1 + \frac{\varphi}{c^2},$$

(21)

and we get

$$E_o \approx \frac{mc^2(1 + \frac{\varphi}{c^2})}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \Delta E = \frac{m_o\varphi}{\sqrt{1 - \frac{v^2}{c^2}}}.$$  

(22)

From Exp. (22) we can come to a conclusion that effective mass of an object $M'$ (or a particle) in external stationary gravitational field is

$$M' = M(1 + \frac{\varphi}{c^2}).$$

(23)

As we have already stressed above, in reality the lifetime of elementary particles can be defined by effective masses of these particles in the external gravitational field. In this case, we should use particle effective masses while computing their lifetimes. Naturally, this effect can be observed in experiment.

For example, lifetime $\tau(\pi)$ of $\pi^{\pm}$ mesons at their lepton decays ($\pi \rightarrow l + \bar{\nu}_l$) is described by the following expression:

$$\tau(\pi) = \frac{1}{\Gamma(\pi)},$$

(24)

where

$$\Gamma(\pi) = \frac{G_F^2 f^2 \pi^2 \cos^2 \theta m_l^2 m_{\pi}}{8\pi} (1 - \frac{m_l^2}{m_{\pi}^2})^2,$$

(25)

and $f_{\pi}$ is the pion decay constant, $G_F$ is Fermi constant, $\theta$ is mixing angle, $m_l$ is lepton mass, $m_{\pi}$ is pion mass.
The lifetime of relativistic pion is defined by usage of standard relativistic transformations. From Exp. (22), (23) it is well seen that in the case of the Einstein’s gravity, there must not be any dependency of the lifetime of elementary particle on the external gravitational potential, i.e., there must not be a visible effect.

In the case of Newton’s gravity, in contrast to the case of Einstein’s gravity, there must be dependence of external gravitational potential as well as direction and value of particle velocity, determined by Exp. (13).

III. Conclusion

This work proposes an approach for examination of gravitational theories taking use of the precision lifetime measurements of elementary particles. We obtained expression for dependence of particle lifetimes of external gravitational potential in the case of Einstein’s and Newton’s gravity. In the case of Newton’s gravity, there is a dependence of velocity direction from the direction of matter location, which creates a gravitational potential. If the external gravitational field is spherical symmetric then there is no possibility to distinguish between these types of gravity. It is found that the deposit of gravitational potential of the Universe (in the case of uniformly distribution of matter in the Universe) in particle lifetime is about one percent. On basis of the available experimental data it is found that deposit of asymmetric gravitational field is \( \phi_c \simeq 2 \cdot 10^{-4} \), i.e., if the experimental precision of particle lifetime measurements is several units of \( 10^{-4} \), then we could see this effect and it would be the confirmation of Newton’s theory gravity (see Introduction of this work).

In reality, the lifetime of elementary particles can be defined by effective masses of these particles in the external gravitational field. The expressions for effective masses of particles in the external gravitational field for two gravity type theories are obtained. These masses can be used at computation of the decay probability (or lifetime) of particles by standard methods. It is shown that in this case it is also possible to distinguish these two types of gravity theories.

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