On the matrix elements of $\Delta B = 0$ operators in the heavy meson decay widths

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Abstract

We determine the chiral corrections to the matrix elements of the $\Delta B = 0$ four-quark operators which are relevant to the studies of the ratios of lifetimes of heavy-light mesons as well as to the power corrections to the inclusive semileptonic heavy-to-light decays. The chiral logarithmic corrections computed here can be combined with the forthcoming estimates of the corresponding matrix elements on the lattice to provide the reliable physics result of the well known bag-parameters $B_{1,2}$ and $\varepsilon_{1,2}$.

1. Phenomenological introduction: The matrix elements of $\Delta B = 0$ operators enter several phenomenological studies of which the most important ones are the analyses of the spectra of inclusive semileptonic decays of heavy mesons and the lifetime ratios of heavy-light mesons.

1.1. Power correction to $\Gamma(B \to X_u e\nu)$: Controlling the power corrections in the spectra of inclusive semileptonic heavy to light decays has been—and still is—an important obstacle when aiming at the reliable extraction of the corresponding CKM parameters [1]. This is particularly important in the case of $|V_{ub}|$. In ref. [2] it has been shown that the $1/m_b^3$-corrections involve the matrix elements of dimension-6 four-quark operators of the flavour structure $\Delta B = 0$. More specifically

$$
\Gamma(B \to X_u e\nu)_{1/m_b^3} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2 \times \frac{-16\pi^2}{m_b^3} \frac{1}{2m_B} \langle B|O_{V-A} - O_{S-P}|B \rangle, \quad (1)
$$
where the matrix elements are conveniently expressed in terms of bag parameters, $B_{1,2}$, as

$$\langle B|O_{\nu-A}^\mu|B\rangle \equiv \langle B|\bar{b}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma_\mu(1-\gamma_5)b|B\rangle = f_B^2 m_B^2 B_1,$$

$$\langle B|O_{\nu-A}^\mu|B\rangle \equiv \langle B|\bar{b}(1-\gamma_5)u(1-\gamma_5)b|B\rangle = f_B^2 m_B^2 B_2,$$

with $f_B$ being the $B$-meson decay constant. Therefore what is actually needed in eq. (1) is the difference of bag parameters, $B_1 - B_2$. The early estimates of $B_{1,2}$ in the framework of QCD sum rules in the static heavy quark limit were reported in ref. [4], and recently extended to the full QCD case [5]. They were also computed on the lattice. In ref. [6] the authors found that in the static heavy quark limit $B_1 - B_2$ is zero, while the lattice study with the propagating heavy quark indicated that $B_1 - B_2$ can be quite different from zero [7]. We are now in the era of ever better unquenched lattice QCD studies and a new lattice computation of the bag parameters of the $\Delta B = 0$ operators is clearly desired. In recent years it became evident that the control over the chiral extrapolation is essential in order to reduce the systematic uncertainties in the results of the lattice QCD studies. In this paper we provide the chiral corrections associated with the bag parameters computed in the static heavy quark limit.

1.2. $B_1 - B_2$ from $D$’s: Before we turn to the question of chiral corrections, let us see if we can get some information about the size of $B_1 - B_2$ from the available information on $D$-decays. The expression for the inclusive semileptonic decay width up to and including the terms $\propto 1/m_c^3$, and neglecting the small contribution $\propto |V_{cd}|$ can be written as

$$\Gamma(D \to X\nu) = \frac{G_F^2 m_c^5}{192\pi^3}|V_{cs}|^2 \eta(z) \left[ \frac{1}{2m_D} \left( I_0(z)\langle D|\bar{c}c|D\rangle - \frac{I_1(z)}{m_c^2}\langle D|\bar{c}g_s\sigma G c|D\rangle \right) - \frac{16\pi^2}{2m_c} f_D^2 m_D (B_1 - B_2) \right],$$

where the phase space factor

$$I_0(z) = (1 - z^2)(1 - 8z + z^2) - 12z^2 \log z, \quad I_1(z) = (1 - z)^4,$$

and $z = m_s^2/m_c^2$, while the numerical parameterisation of the $\alpha_s$-correction to the partonic decay width $\eta(z)$ reads [8],

$$\eta(z) \approx 1 - \frac{2\alpha_s}{3\pi} \left( \pi^2 - \frac{31}{4} \right) (1 - z^{1/2})^2 + \frac{3}{2}.$$

The equation of motion allows us to write

$$\bar{c}c = \bar{c}\gamma c + \frac{1}{2m_c^2} \left( \bar{c}(iD_\perp)^2 c + \bar{c}\frac{g_s}{2}\sigma G c \right) + O(1/m_c^3),$$

which in the standard notation

$$\mu_\pi^2 = -\frac{1}{2m_D}\langle D|\bar{c}(iD_\perp)^2 c|D\rangle, \quad \mu_G^2 = \frac{1}{2m_D}\langle D|\bar{c}\frac{g_s}{2}\sigma B c|D\rangle,$$

1Also standard is the notation in terms of $\lambda_{1,2}$, the parameters measuring the kinetic and chromomagnetic energy of the heavy quark inside a heavy-light system. The relation to $\mu_G^2, \mu_\pi^2$ is: $\lambda_1 = -\mu_\pi^2$, and $\lambda_2 = \mu_G^2/3$. 

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can be written as
\[ \frac{1}{2m_D} \langle D|\bar{c}c|D \rangle = 1 - \frac{\mu_\mu^2 - \mu_b^2}{2m_c^2}. \] (8)

Finally eq. (3) becomes
\[ \Gamma(D \to Xe\nu) = \frac{G_F^2m_c^5}{192\pi^3} |V_{cs}|^2 I_0(z)\eta(z) \left\{ 1 + \frac{1}{2m_c^2} \left[ \mu_\pi^2 \left( 1 - 4I_1(z)/I_0(z) \right) \mu_G^2 \right] \right\} - \frac{G_F^2m_c^2}{12\pi} |V_{cs}|^2 \eta(z)f_D^2m_D (B_1 - B_2). \] (9)

Clearly the heavy quark expansion applied to the decay of charmed mesons is expected to converge much slower than in the case of B-mesons. It is however interesting to use the available information on charmed modes to bound the $B_1 - B_2$ value. Concerning the parameters appearing in the first line of eq. (9) we can use $\mu_G^2 = (3/4)[m_D^2 - m_D^2] = 0.41$ GeV$^2$, while the value of $\mu_\pi^2$ is still somewhat vague. Recent experimental fits to the moments of the semileptonic $b \to c$ decay spectrum [9, 10, 11] quote $m_c \approx 1.1(1)$ GeV and $\mu_\pi^2 \approx 0.5(1)$ GeV$^2$ in the so-called kinetic scheme [12] and at $\mu = 1$ GeV. When converted to the \(\overline{\text{MS}}\) scheme, the charm quark mass is $m_c(m_c) = 1.2(1)$ GeV, consistent with the estimates based on the lattice QCD simulations $m_c(m_c) = 1.32(3)$ GeV, and $1.30(3)$ GeV [13], as well as with the recent QCD sum rule study $m_c(m_c) = 1.29(1)$ GeV [14].

If, in addition, we take $f_D = 208(4)$ MeV [15], $\tau_{D^\pm} = 1.040(7)$ ps, $\tau_{D^0} = 0.410(15)$ ps [16], and the recently measured semileptonic branching fractions [17]:

\[ B(D^+ \to Xe\nu) = (16.13 \pm 0.20 \pm 0.33)\%, \quad B(D^0 \to Xe\nu) = (6.46 \pm 0.17 \pm 0.13)\%. \] (10)

then we get that the terms including $1/m_c^2$-corrections saturate the experimental value for the branching ratio to about 85%. If we now assume the $1/m_c^2$ term fully saturates the rate, we get $B_2(m_c) - B_1(m_c) \approx 0.04$. When evolving those bag parameters to the $m_b$-scale [3, 19], the difference is further increased by about 30%, leading us to $B_2(m_b) - B_1(m_b) \approx 0.05$.

After plugging that number in eq. (1), and by taking $f_B = 0.2$ GeV, we get
\[ B(B \to Xu\nu)_{1/m^3} < 0.04 \times |V_{ub}|^2, \] (11)

which is (comfortably) a very small number. Of course this exercise is only a speculation, while for the reliable estimate of $B(B \to Xu\nu)_{1/m^3}$ a direct non-perturbative method should be employed to compute the matrix elements [2].

1.3. B-meson lifetimes: When studying the B-meson lifetimes, due to the nonleptonic decay modes two more operators enter the game
\[ \langle B|T_\nu^{\mu A}|B \rangle \equiv \langle B_q|b \overline{b} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma_\mu (1 - \gamma_5) b|B_q \rangle = f_B^2m_B^2 \varepsilon_1, \]
\[ \langle B|T_\nu^{\mu A}|B \rangle \equiv \langle B_q|b \overline{b} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma_\mu (1 - \gamma_5) b|B_q \rangle = f_B^2m_B^2 \varepsilon_2 , \] (12)

\[ \text{Very recently the other charm factory (BES) presented similar results but with the final muon instead of electron [15]. Their values are fully consistent with those given in eq. (10), measured by CLEOc, but the error bars are an order of magnitude larger.} \]
where $\lambda^A$ are the Gell-Mann matrices. The bag parameters in eqs. \[12\] were first introduced in this form in ref. [3] in which the authors studied the lifetime difference of hadrons containing one valence $b$-quark. Voloshin, however, realised in ref. [2] that the light flavour content of the operators can be different from the light valence quark of the $B$-meson. Those are the (in)famous “eye-contractions” which are extremely difficult to study non-perturbatively. In addition, an extra penguin operator contribution was singled out in ref. [19]. The contribution of the non-valence and the penguin operators are expected to be negligible in the case of the meson lifetime ratios due to the light flavour symmetry of the spectator quark. That argument however does not apply to the power correction to the semileptonic decays (3). In terms of bag parameters and neglecting the eye-contractions as well as the penguin operator contributions, but including the NLO QCD-corrections to the Wilson coefficients computed in refs. [19] [20], the master formulas for the lifetime ratios of $B$-mesons read

$$\frac{\tau(B^{\pm})}{\tau(B_d)} = 1 + 0.07(2) \times B_1 + 0.011(3) \times B_2 - 0.7(2) \times \varepsilon_1 + 0.18(5) \times \varepsilon_2,$$

$$\frac{\tau(B_s)}{\tau(B_d)} = 1 + 0.007(2) \times [B_1^s - B_1] - 0.009(2) \times [B_2^s - 0.9B_2]
+ 0.15(4) \times [\varepsilon_1^s - 1.1\varepsilon_1] - 0.18(5) \times [\varepsilon_2^s - 0.9\varepsilon_1],$$

(13)

where the superscript “$s$” has been used to distinguish the bag parameters for the case of valence strange quark. In this case it is even more important to have a good handle on the $\varepsilon_{1,2}$ parameters whose impact is enhanced by the size of the Wilson coefficients (numerical values of which are displayed above).

2. Chiral corrections: One of the main problems in relating the bag parameters $B_{1,2}$ and $\varepsilon_{1,2}$ computed on the lattice to the physical bag parameters is the necessity to perform the chiral extrapolation of matrix elements computed with the light quark masses directly accessible on the lattice ($1 > m_q/m_s \gtrsim 1/4$) down to the physical limit ($m_q/m_s \approx 1/25$).\(^3\)

The expressions derived in chiral perturbation theory provide an important guidance in that respect. The chiral corrections to the matrix elements of the whole basis of four-quark $\Delta B = 2$ operators were recently computed in refs. [21] [22]. We showed in ref. [22] that the validity of the formulas derived in heavy meson chiral perturbation theory (HMChPT) may be questionable for the quarks not lighter than about the third of the strange quark mass, because of the nearness of the scalar heavy-light mesons (or more precisely, of the heavy-light $(1/2)^+$-doublet). In other words, unless one wants to deal with a very large number of low energy constants, the adequate HMChPT expressions are only those with $N_f = 2$ light quark flavours (i.e. with the pion loops only). In this paper we do not return to that issue. Instead we focus on the chiral corrections to the bag parameters of the $\Delta B = 0$ operators introduced above.

2.1. Framework: As in ref. [22], in the present paper we work in the static heavy quark limit and use the HMChPT lagrangian already described in detail in ref. [22]. We chose a

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\(^3\)Here and in the following $m_q \equiv m_u = m_d$. 
basis of operators

\[
O_{V-A} = \bar{b}\gamma_\mu(1 - \gamma_5)qL\bar{q}L\gamma^\mu(1 - \gamma_5)b,
\]
\[
T_{V-A} = \bar{b}\gamma_\mu(1 - \gamma_5)t^AqL\bar{q}Lt^A\gamma^\mu(1 - \gamma_5)b,
\]
\[
O_{S-P} = \bar{b}(1 - \gamma_5)qL\bar{q}L(1 + \gamma_5)b,
\]
\[
T_{S-P} = \bar{b}(1 - \gamma_5)t^AqL\bar{q}Lt^A(1 + \gamma_5)b.
\]

The heavy quark spin \((S)\) and the chiral symmetry \((U_L, U_R)\) transformations act on the heavy and light quark respectively like, \(b \rightarrow Sb\) (i.e., \(\gamma_0 b = b\)), and \(q_{L,R} \rightarrow U_{L,R}q_{L,R}\). Since the colour structure (short distance) does not influence the chiral logarithms (long distance) \cite{23}, we need to consider only two of the above operators which we choose to be \(O_{V-A}\) and \(O_{S-P}\). In HMChPT we need the bosonised forms of these operators, which are built up from the heavy-light \((\frac{1}{2})\)-doublet fields, \(H_q(v) = \frac{1 + \gamma_5}{2} [P^a_{\mu}(v)\gamma^\mu - P^a_5(v)\gamma_5]q\), and the pseudo-Goldstone fields, \(\Sigma = \exp(2i\phi/f)\), where \(\phi\) is the usual matrix of pseudo-Goldstone bosons. Under the heavy quark and chiral symmetry the field \(H_q(v)\) transforms as \(H_q \rightarrow SH_qU^\dagger_{q'}\), while \(\Sigma\) transforms as \(\Sigma \rightarrow U_L\Sigma U^\dagger_{R'}\). The standard procedure then consists in introducing \(\xi = \sqrt{\Sigma} = \exp(i\phi/f)\), which transforms as \(\xi \rightarrow U_L\xi U^\dagger_L = U_L^\dagger\xi U_{R'}\). By a simple chiral and heavy quark spurion analysis of the operators \(O_{V-A}, O_{S-P}\) we then obtain their most general bosonized form within HMChPT, namely

\[
O_{V-A}^q = \sum_X \tau_1X \text{Tr}[(\xi H)q\gamma_\mu(1 - \gamma_5)X] \text{Tr}[X\gamma^\mu(1 - \gamma_5)(H\xi^\dagger)q] + \sum_{X,q'} \delta_1X \text{Tr}[H_q\gamma_\mu(1 - \gamma_5)X] \text{Tr}[X\gamma^\mu(1 - \gamma_5)H_{q'}] + \text{c.t.},
\]
\[
O_{S-P}^q = \sum_X \tau_2X \text{Tr}[(\xi H)q(1 - \gamma_5)X] \text{Tr}[X(1 + \gamma_5)(H\xi^\dagger)q] + \sum_{X,q'} \delta_2X \text{Tr}[H_{q'}(1 - \gamma_5)X] \text{Tr}[X(1 + \gamma_5)H_{q}] + \text{c.t.},
\]

where “c.t.” stands for counterterms, and \(X \in \{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}\). The contraction of Lorentz indices and HQET parity conservation requires the same \(X\) to appear in both traces in the products. Any insertion of \(\phi\) can be absorbed via equation of motion, \(\#H(v) = H(v)\), while any nonfactorizable contribution with a single trace over Dirac matrices can be reduced to the form written in (14) by using the \(4 \times 4\) matrix identity

\[
4 \text{Tr}(AB) = \text{Tr}(A)\text{Tr}(B) + \text{Tr}(\gamma_5A)\text{Tr}(\gamma_5B) + \text{Tr}(A\gamma_\mu)\text{Tr}(\gamma^\mu B) + \text{Tr}(A\gamma_\mu\gamma_5)\text{Tr}(\gamma_5\gamma^\mu B) + \frac{1}{2} \text{Tr}(A\sigma_{\mu\nu})\text{Tr}(\sigma^{\mu\nu} B).
\]
flavours. Once saturated by the external heavy-light meson states of the light flavour \( q \) only the eye-contraction with \( q = q' \) will contribute. However at one loop in HMChPT these contractions will not produce any chiral logarithmic correction to the matrix elements of the above operators. Their effect can show up at two or more loops though.

After calculating the above traces, and by retaining the pseudo-Goldstone fields \( \phi \) up to quadratic order, we obtain:

\[
O^q_{V-A} = 4\hat{\tau}_1(P_{\mu}^\dagger P_{\mu}^q + P_{\nu}^\dagger P_{\nu}^q) \left[ \delta q_q \left( 1 + \frac{\hat{\delta}_1}{\hat{\tau}_1} + \frac{i}{\hat{\tau}_1 f} (\phi q_q - \phi q'_q) \right) \right.
\]
\[
+ \frac{1}{2 f^2} \left( 2\phi q_q \phi q'_q - \delta q_q (\phi,\phi) q_q - (\phi,\phi) q_q \delta q'_q \right) + \ldots ,
\]
\[
O^q_{S-P} = 4(\hat{\tau}_2^* P_{\mu}^\dagger P_{\mu}^q + \hat{\tau}_2 P_{\mu}^\dagger P_{\mu}^q) \left[ \delta q_q \left( 1 + \frac{\hat{\delta}_2^*}{\hat{\tau}_2} + \frac{i}{\hat{\tau}_2 f} (\phi q_q - \phi q'_q) \right) \right.
\]
\[
+ \frac{1}{2 f^2} \left( 2\phi q_q \phi q'_q - \delta q_q (\phi,\phi) q_q - (\phi,\phi) q_q \delta q'_q \right) + \ldots ,
\]

where, for simplicity, we do not display the counterterms and we used

\[
\hat{\tau}_1^{(s)} \equiv \tau_1 + \tau_1 \gamma_5 - 4(\tau_1 \gamma_\mu + \tau_1 \gamma_\nu \gamma_5) - 12\tau_1 \gamma_\mu \gamma_\nu ,
\]
\[
\hat{\tau}_2 \equiv \tau_2 - \tau_2 \gamma_5 - \tau_2 \gamma_\nu + \tau_2 \gamma_\mu - \tau_2 \gamma_5 - \tau_2 \gamma_\mu + \tau_2 \gamma_\nu - \tau_2 \gamma_5 ,
\]
\[
\hat{\tau}_2^* \equiv \tau_2 \gamma_\nu - \tau_2 \gamma_\mu + \tau_2 \gamma_\nu - \tau_2 \gamma_\mu .
\]

Similarly \( \hat{\delta}_1, \hat{\delta}_2^* \) stand for the combinations of \( \delta_{1,2}^{(s)} \) couplings appearing in eq. (14).

### 2.2. Chiral loop corrections

The computation of the chiral loop corrections to our operators is by now standard. It involves 6 diagrams which are shown in fig. [1]. Four graphs are factorizable and two are not. Of factorizable diagrams we have the self energy contributions \((a)\) and \((b)\) in fig. [1] which give rise to the wave function renormalisation corrections \( (\delta Z_q) \) which can be found in e.g. [22], and two tadpole graphs which represent the loop corrections to the weak currents composing the four-quark \( \Delta B = 0 \) operator \((c)\) and \((d)\) in fig. [1]. Finally there are two nonfactorizable graphs: tadpole \((e)\), and “sunset” \((f)\). An alerted reader may notice the absence of the mixed terms, i.e. the ones involving an exchange of a pseudo-Goldstone boson between the weak operator and the HMChPT interaction Lagrangian. Those contributions drop out due to heavy vector meson transversality \((v \cdot \varepsilon_{F*} = 0)\). Compared to the situation we encountered in the computation of the chiral loop corrections to the matrix elements of \( \Delta B = 2 \) operators [22], in the present situation the sum of diagrams \((c), (d)\) and \((e)\) vanishes.

The resulting expressions read

\[
f_q^2 B_1^q = (f_q^2 B_1)^{\text{Tree}} \left\{ 1 + \delta Z_q - \frac{1}{2 f^2} \left( 2g_2^2 t^i i^\dagger + X_1(t^i t^i + t^i t^i - 2t^i t^i) \right) \right\} q q I_0(m_i) + \text{c.t.} ,
\]
\[
f_q^2 B_2^q = (f_q^2 B_2)^{\text{Tree}} \left\{ 1 + \delta Z_q - \frac{1}{2 f^2} \left( 2Y_2 g_2^2 t^i i^\dagger + X_2(t^i t^i + t^i t^i - 2t^i t^i) \right) \right\} q q I_0(m_i) + \text{c.t.} ,
\]

(18)
Figure 1: The graphs giving the non-zero contribution to the NLO chiral corrections to the matrix elements of the $\Delta B = 0$ operators discussed in this paper. The double lines correspond to the heavy-light mesons, and the dashed ones to the pseudo-Goldstone bosons. $\Delta B = 0$ operators are denoted by “$\otimes\otimes$”, while the strong vertices coming from the HMChPT lagrangian are denoted by the full dots, “•”. 
where $I_0(m_i) = (m_i/4\pi)^2 \log(m_i^2/\mu^2)$, $Y_2 = (B_2/B_2)^\text{Tree}$, $X_i = (\hat{\tau}_i/B_i)^\text{Tree} \approx (1 - \delta_i/\hat{\tau}_i)$, and $t^i$ are the $SU(N)$ generator matrices. Summation over “$i$” in the above expressions is understood.

3. Results: On the basis of the expressions derived in the previous section we now discuss our results. We will first give the explicit formulas for the chiral corrections that might be particularly useful to the lattice practitioners. We will then make some important assumptions which will allow us to infer a few phenomenological implications.

3.1. Message relevant to the lattice QCD studies: In our previous paper we showed that due to the nearness of the $(\frac{1}{2})^+$-doublet of the heavy-light mesons, only the pion loop contributions are a safe prediction of this (HMChPT) approach which then can be used to guide the chiral extrapolations of the heavy-light meson quantities computed on the lattice. This was shown to be the case for the decay constants, Standard Model and SUSY bag parameters parameterising the matrix elements of the $\Delta B = 2$ operators [22], pionic couplings $g$, $\tilde{g}$ and $h$ [25], as well as for the Isgur-Wise functions [24]. The same holds true in this case. Therefore the relevant expressions to be used in the lattice extrapolations in the light quark mass are those derived in $SU(2)$-theory and they read:

$$f^2_{qB_1} = \alpha B_1^\text{Tree} \left[ 1 - \frac{9g^2}{(4\pi f)^2} m^2_\pi \log \frac{m^2_\pi}{\mu^2} + o_1(\mu)m^2_\pi \right],$$

$$f^2_{qB_2} = \alpha B_2^\text{Tree} \left[ 1 - \frac{9g^2(1 + Y_2)}{2(4\pi f)^2} m^2_\pi \log \frac{m^2_\pi}{\mu^2} + o_2(\mu)m^2_\pi \right],$$

(19)

or by recalling that

$$f_q = \alpha \left[ 1 - \frac{1 + 3g^2}{(4\pi f)^2} \frac{3}{4} m^2_\pi \log \frac{m^2_\pi}{\mu^2} + c_f(\mu)m^2_\pi \right],$$

(20)

for the bag parameters we have

$$B_1 = B_1^\text{Tree} \left[ 1 + \frac{1 - 3g^2}{(4\pi f)^2} \frac{3}{2} m^2_\pi \log \frac{m^2_\pi}{\mu^2} + b_1(\mu)m^2_\pi \right],$$

$$B_2 = B_2^\text{Tree} \left[ 1 + \frac{1 - 3g^2Y_2}{(4\pi f)^2} \frac{3}{2} m^2_\pi \log \frac{m^2_\pi}{\mu^2} + b_2(\mu)m^2_\pi \right],$$

(21)

where $g^2$ can be computed separately on the lattice as in ref. [26], and the parameters of the fit are $B_{1,2}^\text{Tree}$ and the counterterms $b_{1,2}(\mu)$. It is worth emphasizing that the $\mu$-dependence in the chiral logarithms cancels against the one in the low energy constants. The situation with the chiral corrections to the matrix element $O_{S-P}$ is similar to what we discussed in ref. [22] where for the non-Standard Model $\Delta B = 2$ operators the new low energy constant “$Y$” appeared. Its value is likely to be very close to unity as it represents the following ratio

$$Y_2 = \frac{\langle B^*|O_{S-P}|B^* \rangle}{\langle B|O_{S-P}|B \rangle},$$

(22)
and it can be relatively easily evaluated on the lattice. 4 Finally, let us stress once again that thanks to the identity

\frac{1}{2} \lambda_{ab}^A \lambda_{cd}^A = \delta_{ad} \delta_{bc} - \frac{1}{3} \delta_{ab} \delta_{cd}, \quad (23)

the chiral logarithms to the bag parameters \( \varepsilon_{1,2} \) are of the same as those in \( B_{1,2} \) parameters but their low energy constants are of course different. To be fully explicit:

\[ \varepsilon_1 = \varepsilon_1^{\text{Tree}} \left[ 1 + \frac{1}{2} \lambda^{2} \frac{3}{2} \frac{m_{\pi}^{2}}{\mu^{2}} \log \frac{m_{\pi}^{2}}{\mu^{2}} + \frac{b'(\mu)m_{\pi}^{2}}{\mu^{2}} \right], \]

\[ \varepsilon_2 = \varepsilon_2^{\text{Tree}} \left[ 1 + \frac{1}{2} \lambda^{2} \frac{3}{2} \frac{m_{\pi}^{2}}{\mu^{2}} \log \frac{m_{\pi}^{2}}{\mu^{2}} + \frac{b'(\mu)m_{\pi}^{2}}{\mu^{2}} \right], \quad (24) \]

with

\[ Y''_2 = \frac{\langle B^*|T_{S-P}|B^* \rangle}{\langle B|T_{S-P}|B \rangle}. \quad (25) \]

3.2. Back to phenomenology: In the early phenomenological applications the formulas derived in HMChPT were used to estimate the size of the hadronic quantities by using the theory with \( N_f = 3 \) light flavour and by neglecting the counterterms (or, at best, estimating them by means of some quark model). Nowadays we also know that the \( \frac{1}{2} \)\(^+\) states should be included if one is to use HMChPT with \( N_f = 3 \). In what follows, the \( \frac{1}{2} \)\(^+\) contributions will be neglected too, which is an extra assumption. To get the difference \( B_2 - B_1 \) we will proceed along these lines and impose \( B_{1,2}^{\text{Tree}} = 1 \), like in the vacuum saturation approximation, and neglect the counterterms, to obtain

\[ B_2 - B_1 = \frac{3g^2(1-Y_2)}{(4\pi f)^2} \left( \frac{3}{2} m_{\pi}^{2} \log \frac{m_{\pi}^{2}}{\mu^{2}} + m_{K}^{2} \log \frac{m_{K}^{2}}{\mu^{2}} + \frac{1}{6} m_{\eta}^{2} \log \frac{m_{\eta}^{2}}{\mu^{2}} \right), \quad (26) \]

which for \( g^2 \approx 0.3, f = 120 \text{ MeV}, \) and \( \mu = 1 \text{ GeV} \) gives

\[ B_2 - B_1 = 0.21 \left( 1 - Y_2 \right). \quad (27) \]

This is as far as one can get at this stage, since there is no information available concerning the size of \( Y_2 \). We reiterate that it can be computed on the lattice as indicated in eq. (22).

Note in passing that if we use \( B_2 - B_1 = 0.05 \) as inferred in introduction from the \( D \)-decays, we would obtain \( Y_2 \approx 0.8 \). Similarly, for the bag parameters entering eq. (13) we have

\[ B_1^q = 1 + \frac{1}{2} \lambda_{ab}^A \lambda_{cd}^A \left( \frac{3}{2} m_{\pi}^{2} \log m_{\pi}^{2} + m_{K}^{2} \log m_{K}^{2} + \frac{1}{6} m_{\eta}^{2} \log m_{\eta}^{2} \right) = 0.98, \]

\[ B_1^s = 1 + \frac{1}{2} \lambda_{ab}^A \lambda_{cd}^A \left( 2m_{K}^{2} \log m_{K}^{2} + \frac{2}{3} m_{\eta}^{2} \log m_{\eta}^{2} \right) = 0.96, \quad (28) \]

while for

\[ B_2^q = 0.77 + 0.21 Y_2, \quad B_2^s = 0.59 + 0.37 Y_2. \quad (29) \]

\(^4\)Notice that the complexity related to the matching of the operator computed on the lattice to its counterpart renormalised in the continuum renormalisation scheme completely cancels in that ratio.
which, together with the assumption that $\varepsilon_{1,2}^{\text{Tree}} = \varepsilon_{1,2}^{\text{VSA}} = 0$ brings eq. (13) to

\[
\frac{\tau(B^\pm)}{\tau(B_d)} = 1.077 + 0.002 \ Y_2 \quad (1.076 \pm 0.008)^{\text{exp}},
\]

\[
\frac{\tau(B_s)}{\tau(B_d)} = 1.001 - 0.002 \ Y_2 \quad (0.950 \pm 0.019)^{\text{exp}}.
\]

where in the parentheses we also give the experimental values \[^{[16]}\]. We see that in spite of the assumptions the current experimental information does not allow to constrain appreciably the value of the coupling $Y_2$.

4. **Summary**: In this paper we presented the result of our calculation of the chiral corrections to the matrix elements of four-quark $\Delta B = 0$ operators that are relevant to the phenomenology of the lifetime ratios of the heavy-light mesons and to the inclusive semileptonic decay spectra \[^{[27]}\]. The calculation of the chiral corrections can be combined with the lattice calculations of the $\Delta B = 0$ matrix elements to either extrapolate the lattice data towards the physical light quark masses, and/or to fix the counterterm coefficients $b_{1,2}(\mu)$ in eq. (21), the couplings $Y_2$ \[^{[22]}\] and $Y_2'$ \[^{[25]}\] and the tree level bag parameters (in terms of chiral expansion).

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