Collective energy absorption of ultracold plasmas through electronic edge-modes

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New Journal of Physics 14 (2012) 053039 (14pp)
Received 16 January 2012
Published 25 May 2012
Online at http://www.njp.org/
doi:10.1088/1367-2630/14/5/053039

Abstract. We investigate the collective dynamics of electrons in ultracold neutral plasmas driven by an oscillating radio-frequency field. We point out the importance of a sharp density drop at the plasma boundary that arises due to unavoidable charge imbalances, and show that this plasma edge provides the major mechanism for energy absorption from the external field. Using a cold fluid theory, we derive the corresponding absorption frequency and validate our findings by microscopic molecular dynamics simulations. The proposed edge-mode is shown to provide a consistent explanation for the observed absorption spectra measured in different experiments. Understanding the response of the electronic plasma component to weak external driving is essential since it grants experimental access to the time-evolving density and temperature of ultracold plasmas.

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1. Introduction

In recent years, ultracold plasmas, produced by photoionization of laser-cooled atoms [1–4] or molecules [5], have opened up new avenues for studying various plasma physics phenomena, ranging from collective behavior [6–13], plasma expansion [14–19] and strong correlations [20–29] through to low-energy atomic processes [30–37]. Experimentally, these studies rely on the ability to probe the time-evolving state of the plasma that is characterized, e.g., by the density and temperature of the different plasma constituents.

Optical techniques such as absorption imaging [21, 38] or fluorescence measurements [18, 26, 39] have allowed us to probe directly the distribution of the positions and velocities of the ions in the course of the plasma expansion into vacuum. Tracking the dynamics of the electrons has turned out to be more subtle due to their lack of an internal structure. One possible approach is to apply a weak radio-frequency (rf) field and record the yield of electrons that escape due to resonant rf heating. Provided the relation between the plasma parameters and the resonance frequency is known, this technique allows us to deduce the density [14, 40] and temperature [41] of the electronic plasma component. In an early experiment [14], such a relation has been deduced by assuming local resonant absorption at positions $\mathbf{r}$ where the electron density $n_e(\mathbf{r})$ is such that the local plasma frequency

$$\omega_p(\mathbf{r}) = \sqrt{\frac{4\pi e^2 n_e(\mathbf{r})}{m_e}}$$

(1)

matches the frequency $\omega$ of the applied rf field ($m_e$ is the mass of the electrons). Within this model one finds that the average energy absorption is peaked around the average plasma frequency $\bar{\omega}_p = \sqrt{4\pi e^2 \bar{n}}$, which is determined solely by the average density $\bar{n} = \langle n^2 \rangle / \langle n \rangle$. The observed density evolution extracted from this simple relation was shown to be in excellent agreement with theoretical calculations [15, 17] for a wide range of initial plasma parameters. Later, a different calculation suggested [6] that the plasma exhibits a continuous excitation spectrum and absorbs energy through a damped quasi-mode, which results in a broad absorption resonance around a frequency $\sim 0.37\bar{\omega}_p$. Apparently, this relation underestimates the densities measured in [14]. However, it was found to give a good description of subsequent rf-absorption measurements [40]. These contradictory theoretical and experimental results suggest that neither the picture of local energy absorption nor the quasi-mode permits a consistent interpretation of existing experiments, which is needed for a reliable probe of ultracold plasmas.

Here, we resolve this seeming discrepancy and reveal the existence of an additional mode that provides a unifying consistent description of all previous experiments. This eigenmode arises from one property which has not been taken into account so far, namely the fact that the experimentally realized plasmas—despite being quasi-neutral in their interior—exhibit an overall charge imbalance

$$\delta = (N_i - N_e)/N_i$$

(2)

between the number $N_i$ of ions and the number $N_e$ of electrons. This imbalance results from a prompt loss of electrons upon the creation of the plasma [1, 42] and, more importantly, from electrons that continue to leave at later times during the plasma expansion due to the presence of small electric fields [43]. Since the plasma always neutralizes in its interior region,
this charge imbalance creates a rather sharp outer edge of the electron density, as opposed to the smoothly decreasing ion density \([17, 18]\). We will demonstrate that this results in a strong energy absorption at the plasma edge and produces a sharp absorption resonance that depends strongly on the charge imbalance \(\delta\). It is shown that this edge-mode provides a consistent explanation for the energy absorption reported in all previously reported experiments \([14, 40]\).

The paper is organized as follows. In section 2, we review the cold fluid theory for the rf-driven electron plasma. This is used to determine the plasma edge from which we calculate the electronic eigenmodes. We also determine the corresponding spectrum of energy absorption by directly calculating the power absorbed by the plasma electrons as a function of the frequency of the applied rf field. These results are confirmed by detailed comparisons with large-scale molecular dynamics (MD) simulations in section 3. All three approaches give results in excellent agreement with each other, providing an exhaustive characterization of the identified eigenmode and its role in energy absorption. Finally, in section 4, we analyze previous rf-absorption measurements, showing excellent agreement with our calculations.

2. Cold fluid theory

Due to the large mass ratio between the plasma ions and the much lighter electrons, their respective dynamics takes place on vastly disparate timescales. In particular, the time on which the electrons respond to the rapidly oscillating rf field is much shorter than the timescale of the ion motion driven by the slow expansion of the plasma \([14–17]\). For the purposes of this work, we can, hence, assume a stationary ionic density distribution \(n_i\). Typically, an ultracold plasma is produced by photo-ionizing a spherically Gaussian cloud of trapped atoms and, consequently, also exhibits a Gaussian density distribution

\[
n_i = n_0 \exp \left( -\frac{r^2}{2\sigma^2} \right).
\]

The magneto-optical trapping potential used to confine the atoms has no considerable effect on the produced ions so that the plasma expands freely into the surrounding vacuum \([2]\). As confirmed by independent optical measurements \([18]\), the plasma expands self-similarly \([44]\), preserving its Gaussian shape. We can, hence, consider a Gaussian ion density of the form (3) with a time-dependent width \(\sigma(t)\) that produces an attractive electrostatic confinement potential for the rapidly moving electrons.

We investigate their dynamics using a hydrodynamical cold fluid description where one assumes a vanishing electron temperature \(T_e\). In practice, however, the electron temperature needs to be chosen sufficiently high (typically \(\gtrsim 100\) K) in order to avoid recombination of electrons into Rydberg states \([33, 36]\), which would lead to a rapid loss of the plasma constituents and the heating of the electrons \([14, 31]\). Nevertheless, even for such temperatures the electronic Debye length

\[
\lambda_D = \sqrt{\frac{k_B T_e}{4\pi e^2 n_e}}
\]

is still much smaller than the size \(\sigma\) of the plasma, so that the cold fluid assumption provides a good description of the dominant mechanism of energy absorption. A cold fluid analysis of the eigenmodes of finite plasmas \([45]\) has been successfully applied, e.g., to confined non-neutral ion plasmas \([46–49]\) and more recently to spherically confined complex plasmas \([50, 51]\).
The cold fluid equations describing the evolution of the electronic density \( n_e(r, t) \) and their hydrodynamic velocity \( u_e(r, t) \) read

\[
\frac{\partial}{\partial t} n_e + \nabla \cdot (n_e u_e) = 0, \tag{5}
\]

\[
m_e \left[ \frac{\partial}{\partial t} u_e + (u_e \cdot \nabla) u_e \right] = -e n_e (E + E_{rf} \text{e}^{i \omega t}) - m_e v n_e u_e. \tag{6}
\]

The first equation is the continuity equation. The second one describes the velocity field dynamics determined by the electrostatic meanfield \( E \) of the plasma charges, the external rf field \( E_{rf} \text{e}^{i \omega t} \), oscillating at a frequency \( \omega \), and collisional dissipation, modeled by a phenomenological damping term proportional to the damping rate \( v \). The plasma meanfield is obtained from the Poisson equation

\[
\nabla E = 4 \pi e (n_i - n_e) , \tag{7}
\]

which couples equations (5) and (6).

First consider their stationary steady-state solution, \( n_e^{(0)} = 0 \) and \( u_e^{(0)} = 0 \), determined by \( \dot{n}_e^{(0)} = 0, \dot{u}_e^{(0)} = 0 \). This gives \( u_e^{(0)} = E^{(0)} = 0 \). Consequently, the Poisson equation (7) requires that \( n_e = n_i \), since the electrons tend to screen the positive ionic charges and therefore neutralize the plasma. However, in the presence of a finite positive charge imbalance (see equation (2)), the electrons can only neutralize the interior of the plasma up to a critical radius \( R_0 \) beyond which the electron density drops to zero. Assuming an abrupt density drop and using equation (2), the critical radius is readily obtained from the charge imbalance \( \delta \) and the electron and ion density distributions

\[
\delta = 1 - \int_0^{R_0} 4 \pi r^2 n_i dr
\]

\[
= 1 + \frac{2}{\pi} R_0 \frac{R_0^2}{\sigma} \exp \left( -\frac{R_0^2}{2 \sigma^2} \right) + \text{erf} \left( \frac{R_0}{\sqrt{2} \sigma} \right) , \tag{8}
\]

where we have substituted the Gaussian ion distribution to obtain the expression in the second line. In figure 1(a), we show stationary electron density distributions obtained from MD simulations, which will be described in more detail in section 3. The MD simulations indeed yield a rather sharp drop around the critical radius \( R_0 \) given by equation (8), as indicated by the vertical dotted line. Due to the small but finite electron temperature the electron component still has a finite screening length given by equation (4). As a result, we find an exponential broadening of the plasma edge on the scale of the local Debye length

\[
\lambda_D(R_0) = \left( \frac{k_B T_e}{4 \pi e^2 n_i(R_0)} \right)^{\frac{1}{2}} . \tag{9}
\]

We can account for this effect by using a simple function

\[
n_e^{(0)}(r) = n_i f(r) = \exp \left( -\frac{r^2}{2 \sigma^2} \right) \begin{cases} 1, & \text{if } r \leq R_\lambda, \\ \cosh \left( \frac{R_\lambda - r}{\lambda_D} \right)^{-1}, & \text{if } r > R_\lambda, \end{cases} \tag{10}
\]

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which smoothly interpolates between the inner and outer regions of the plasma and yields an exponential drop on the length scale $\lambda$, beyond a radius $R_\lambda$. Knowing the charge imbalance $\delta$, the latter can be obtained self-consistently from equation (10) and

$$\delta = 1 - \int_0^\infty 4\pi r^2 f(r)dr.$$  

Identifying the broadening parameter $\lambda$ with the local Debye length $\lambda_D(R_0)$, these two equations yield sufficiently accurate agreement with our MD results and can thus be used for a simple description of the electron distribution in ultracold neutral plasma without having to resort to elaborate numerical simulations.

### 2.1. Eigenmodes

Having determined the equilibrium state of the electrons, we can now investigate their collective response to the applied rf field. Instead of directly solving the respective nonlinear equations (5) we apply linear response theory and expand their solution around the steady state

$$n_e = n_e^{(0)} + n_e^{(1)} e^{i\omega t},$$

$$u_e = u_e^{(1)} e^{i\omega t},$$

$$E = E^{(1)} e^{i\omega t},$$

in terms of small perturbations $\delta n_e$, $\delta u_e$, and $\delta E$ induced by the rf-driving field. Substituting this ansatz into equation (5) and introducing the electrostatic perturbation potential $\phi$, defined by $\nabla \phi = -E^{(1)}$, gives to leading order in the perturbation

$$\left(\frac{\omega_{pe}^2}{\omega^2 + i\omega v} - f(r)\right) \nabla^2 \phi(r) - [\nabla f(r)] \cdot [\nabla \phi(r)] = -E_{rf}(r) \nabla f(r).$$  

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**Figure 1.** (a) Electron density for a spherical Gaussian-shaped plasma with a charge imbalance $\delta = 0.2$ obtained from an MD simulations of 40 000 electrons with three different screening length. Panel (b) shows the same distributions obtained self-consistently from equations (10) and (11).
where \( \omega_{p0} = \omega_p(0) \) is the peak plasma frequency. To obtain the eigenmodes of the system we can set \( E_{\alpha f} = 0 \) and \( \nu = 0 \). In this case equation (13) reduces to
\[
\nabla(\epsilon \nabla \phi) = 0, 
\]
which is simply the Laplace equation for a medium with a dielectric function
\[
\epsilon(r) = 1 - \frac{\omega_{p0}^2}{\omega^2} f(r) = 1 - \frac{\omega_p(r)^2}{\omega^2}.
\]
To determine the eigenfrequencies of equation (14), we consider a sharp drop in the density at the plasma edge, i.e. set \( \lambda = 0 \) in equation (10). In order to exploit the radial symmetry of the equilibrium electron density distribution, \( \sim f(r) \), we expand the perturbation potential
\[
\phi(r) = \phi(r, \theta, \varphi) = \sum \phi_\ell(r) P_\ell(\cos \theta)
\]
in terms of the Legendre polynomials \( P_\ell(\cos \theta) \). The radial potential \( \phi_\ell(r) \) represents a perturbation of the well-defined angular momentum \( \ell \) and, according to equation (14), has to fulfill the boundary condition
\[
\epsilon(r) \frac{d \ln \phi_\ell^+}{dr} \bigg|_{r=R_0} = - \frac{d \ln \phi_\ell^-}{dr} \bigg|_{r=R_0} 
\]
at the interface \( R_0 \), where \( \phi_\ell^+ \) and \( \phi_\ell^- \) denote the potentials for \( r < R_0 \) and \( r > R_0 \), respectively. From equation (17) one can obtain a formal expression for the eigenmode frequency,
\[
\omega = \frac{\omega_p(R_0)}{\sqrt{1 - \left(\frac{d \ln \phi_\ell^+}{dr} \bigg|_{r=R_0}\right)^{-1} \frac{d \ln \phi_\ell^-}{dr} \bigg|_{r=R_0}}},
\]
which can be evaluated by solving equation (14) for the potentials \( \phi_\ell \) under the matching condition equation (17).

It is instructive to first consider the simplest situation of a homogeneous plasma density \( f(r) = \Theta(R - r) \). In this case, \( \epsilon(r) = \text{const} \) inside and outside the plasma, such that equation (14) can be readily solved. Requiring that \( \phi_\ell \) must be finite at \( r = 0 \) and decrease as \( r \to \infty \), one obtains \( \phi_\ell^+ \propto r^\ell \) and \( \phi_\ell^- \propto r^{-\ell-1} \). Substituting these solutions into our expression (18) for the eigenmode frequency gives \( \omega/\omega_p = [\ell/(2\ell + 1)]^{1/2} \). These are the familiar multipole excitation modes for a homogeneous plasma sphere, where, in particular, the dipole mode (\( \ell = 1 \)) has the well-known resonance frequency \( \omega = \omega_p/\sqrt{3} \), corresponding to surface-plasmon excitations [52].

For a truncated Gaussian density \( n_e(r) = n_i f(r) \) of the electrons, we solve equation (14) numerically by propagating the logarithmic derivative of \( \phi_\ell^- \) outwards, starting from \( r = 0 \), where \( \phi_\ell^- \propto r^\ell \). At the boundary \( R_0 \), the propagated solution has to match the free solution \( \phi_\ell^- \propto r^{-\ell-1} \) outside the plasma, which fixes the eigenmode frequency \( \omega \). In the experiments [14, 40], a linearly polarized driving field \( E_{\alpha f} = E_{\alpha f} e_z \), and we therefore focus in this work on this situation. Consequently, the only relevant mode is the dipole mode with \( \ell = 1 \).

Its frequency as a function of the charge imbalance \( \delta \) is shown in figure 2. The dependence on \( \delta \) originates from the cutoff radius \( R_0 \), which is related to the charge imbalance via equation (8) as discussed above. As one can see, the modes form a smooth function between
two simple expressions corresponding to the limits of a neutral system and a highly non-neutral plasma. The latter case, $R_0 \ll \sigma$, implies a nearly homogeneous plasma within a small sphere of radius $R_0 \rightarrow 0$, such that the eigenmode frequency approaches that of a homogeneous plasma sphere ($\omega_p(0)/\sqrt{3}$), as indicated by the horizontal dashed line in figure 2. In the opposite limit, the resonance frequency approaches the plasma frequency $\omega_p(R_0)$ at the boundary. As will be described below, this indicates that energy is predominantly absorbed at the plasma edge.

2.2. Energy absorption spectrum

In order to demonstrate this point, we now explicitly calculate the energy absorption spectrum by solving equation (13) in the presence of the external driving field $E_{rf}(e^{i\omega t})$ and collisional damping, i.e. $\nu > 0$. The radial density of the power absorbed by the electron plasma is given by

$$p_\delta(r, \omega) = \frac{1}{4} \int_0^\pi \text{Re}(F \cdot j^*) \sin \theta \, d\theta,$$

where $j = en_e u$ is the electronic current density and $F = E_{rf} + E$ is the total electric field acting on the electrons. Since $E = E^{(1)}e^{i\omega t}$ and $u = u^{(1)}e^{i\omega t}$ the current density is $j = e n^{(0)} e^{i\omega t} u^{(1)}$. Using equation (5) the perturbed velocity field can be expressed in terms of the electric field as

$$u^{(1)} = -\frac{\nu + i\omega}{v^2 + \omega^2 m_e} \left( E_{rf} + E^{(1)} \right),$$

such that the absorbed power density

$$p_\delta(r, \omega) = \left(16\pi \right)^{-1} \frac{\omega_p(r)^2 \nu}{\omega^2 + \nu^2} \int_0^\pi |E_{rf} + E^{(1)}|^2 \sin \theta \, d\theta$$

Figure 2. Dipole mode frequency of an ultracold plasma as a function of its charge imbalance $\delta$ equation (2), obtained from solving the eigenvalue problem equation (14) (solid line). The dashed line corresponds to the limit of a homogeneous spherical plasma and the dashed-dotted line indicates the local plasma frequency (1) at the plasma boundary $R_0$ given by equation (8).
can be calculated only from the perturbed plasma electric field $E^{(1)} = -\nabla \phi$. The latter is obtained by solving equation (13) in a similar way to that described above. Starting from the plasma center, where $\phi_1 \propto r^\ell (\ell = 1)$, the logarithmic derivative of $\phi_1$ is propagated outwards well beyond the plasma edge. To propagate across the plasma edge, we use the softened density distribution (10) that smoothly interpolates between the inner and outer plasma regions. Due to the presence of the rf-field term in equation (13) this does not yield an additional boundary condition at $R_0$. However, the potential $\phi_1(r)$ has to be matched to the free solution $\phi_r \propto r^{-2}$ for $r \gg R_0$, which uniquely determines $\phi(r)$ and yields the absorbed power density according to equation (21).

In figure 3, we show the resulting spatially resolved absorption spectrum, $4\pi r^2 p_\delta(r, \omega)$, for a neutral plasma ($\delta = 0$) and for a plasma with a finite charge imbalance $\delta = 0.2$. As can be seen, local energy absorption prevails in both cases. Integrating $p_\delta(r, \omega)$ over $r$ gives the energy absorption spectrum

$$P_\delta(\omega) = 4\pi \int_0^{R_0} r^2 p_\delta(r, \omega),$$

which is shown in the left part of figures 3(a) and (b), respectively. For $\delta = 0$ (figure 3(a)) this resembles the absorption spectrum obtained in [6], which peaks at the quasi-mode frequency $\omega \approx 0.22\omega_p(0)$. Moreover, the spatially resolved absorption spectrum for $\delta = 0$ of figure 3(a) provides a simple unifying picture for the two models put forward in [14] and [6]. One first observes that the power density $p_0(r, \omega)$ is strongly peaked around the local plasma frequency, i.e. at radii $r_p(\omega)$ for which $\omega = \omega_p(r)$. This corresponds to the picture of local resonant energy absorption used in [14]. Assuming that the absorption strength is independent of the radial position, spatial integration (equation (22)) yields an absorption resonance at $\omega \approx \bar{\omega}_p = 0.6\omega_p(0)$ as found in [14]. While the absorption is indeed local, the present, more detailed mode analysis reveals that the absorption has an additional radial dependence, due to

\[\text{Figure 3.} \text{ Double differential power density } p_\delta(r, \omega) \text{ (see equation (21)) for plasma with (a) } \delta = 0 \text{ and (b) } \delta = 0.2. \text{ The absorbed power is strongly peaked around local plasma frequency equation (1), as indicated by the black dotted line. The red line in the left part of each panel shows the energy absorption spectrum obtained by spatial integration, as given in equation (22).}\]
the spatially varying density gradient. Including this variation and integrating over the local absorption around \( r = r_p \) yields the quasi-mode resonance at \( \omega \approx 0.37\tilde{\omega}_p = 0.22\omega_p(0) \) found in [6].

Along this line, one can immediately understand the finite-\( \delta \) modification of the local (or quasi-mode) absorption, which can be estimated from the neutral-plasma absorption spectrum \( p_0(r, \omega) \) via the truncated integration

\[
P^{\text{qm}}_\delta(\omega) \approx 4\pi \int_0^{R(\delta)} r^2 p_0(r, \omega).
\]  

(23)

As shown in figure 4, this also yields a broad absorption, truncated at low frequencies due to the lack of small densities that would facilitate resonant absorption at small frequencies.

However, at finite \( \delta \) there appears an additional sharp maximum extending well above the broad quasi-mode background (left part of figure 3(b)). This additional resonance originates from strong energy absorption at the plasma edge where local neutrality is maximally violated. In this region, the electrons experience a much larger net electric field, which, consequently, results in a much stronger energy absorption.

The calculated absorption spectrum \( P_\delta(\omega) \) is shown in figure 4 for different charge imbalances \( \delta \). In order to draw a connection to the eigenmode analysis of the previous section, we have chosen a very small broadening parameter \( \lambda/\sigma = 10^{-4} \). This value is sufficiently small to not affect the resulting absorption spectrum and therefore resembles the sudden density drop assumed in the derivation of equation (18) for the eigenfrequency. As demonstrated in figure 4, the edge-mode absorption peaks appear exactly at the eigenmode frequencies derived above. Hence, energy is predominantly absorbed around the plasma edge at the eigenfrequency.
equation (18) shown in figure 2, and greatly exceeds the quasi-mode contribution [6] for typical charge imbalances that occur in ultracold plasmas.

3. Numerical simulations

To further substantiate these findings, we have also performed MD simulations of large electron plasmas. As before, the density distribution of the ion is given by a stationary Gaussian, which provides a spherically symmetric electrostatic potential well

\[ U(r) = -N_i e^2 \frac{r}{r \text{erf} \left( r \sqrt{2}\sigma^2 \right)} \]  

(24)

that confines the electrons. The dynamics of the electrons is described by propagating the classical equations of motion for each of the \( N_e \) electrons that evolve under the influence of the confinement potential (24) and the mutual electron–electron interaction. One simulation proceeds as follows. First, we equilibrate the electron plasma to a prescribed temperature \( T_e \). This yields the electron density distributions shown in figure 1(a). Subsequently, we apply a weak oscillating field of amplitude \( E_{rf} \) and frequency \( \omega \). During this phase we record the kinetic energy of the electrons, which increases linearly in time due to the action of the rf field. The corresponding slope yields directly the energy absorption rate.

The calculation of the absorption spectrum for different charge imbalances requires many such simulation runs. This is made feasible by using the so-called fast-multipole method (FMM) [53, 54] to calculate the electron–electron interaction, which yields a speedup of order \( O(N_e) \) compared to a direct force calculation.

In order to relate our simulations to the predictions of the cold fluid theory, we choose comparably low temperatures that correspond to the Coulomb coupling parameters \( \Gamma = \frac{e^2}{(ak_B T_e)} \) \( a = (4\pi n_e/3)^{-1} \) is the Wigner–Seitz radius) in the range \( \Gamma = 0.2 \ldots 20 \). This places the simulated plasmas well into the strongly correlated fluid regime. However, as shown in previous work on non-neutral plasmas, the resulting correlations even in the very strongly coupled regime have only a minor effect on the eigenmode frequencies of the plasma [48], justifying the neglect of particle correlations in the cold fluid theory described above.

The simulated absorption spectra for different values of the charge imbalance are included in figure 4. The MD results show a sharp peak on top of the broad quasi-mode, in quantitative agreement with the cold fluid description (see equations (21) and (22)) and the eigenmode analysis (see equation (18)).

A closer inspection of figure 4 reveals that the MD results for the edge-mode resonance are slightly shifted towards larger frequencies for small charge imbalances. This is due to the fact that we have used a very small broadening parameter \( \lambda/\sigma = 10^{-4} \) in the cold fluid calculation of the absorption spectrum. The MD simulation yields a larger edge broadening on a spatial scale \( \lambda_D(R_0) \) (see figure 1). This spatial broadening decreases the net electric field at the plasma edge and therefore weakens the edge-mode resonance and, in addition, leads to a small shift of the resonance frequency. This is demonstrated in figure 5, where we show the absorption spectrum obtained from the cold fluid equations for two different widths \( \lambda \) of the plasma edge. One can account for these finite temperature effects within the cold fluid description by identifying \( \lambda \) with the Debye screening length \( \lambda_D(R_0) \) (see equation (9)) at the plasma edge. Accurate measurements of the resonance frequency that can resolve such corrections in conjunction with such calculations would therefore yield a probe of the electron temperature.

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4. Comparison with experiments

In the experiments \cite{14, 40}, energy absorption has been measured by recording an enhanced evaporation of resonantly rf-heated electrons, which were accelerated towards a detector by a small electric extraction field $E_{\text{extr}}$. Since the resonance frequency depends on the plasma density and the density decreases as the plasma expands into vacuum, the evaporation peaks appear at different times with varying frequencies of the applied rf-field. Provided the plasma expansion dynamics is known, this allows us to extract the frequency of the absorption resonance as a function of the plasma density.

As mentioned above, the plasma expands self-similarly. The width of the Gaussian density distribution increases according to \cite{44}

$$\sigma(t) = \sqrt{\sigma_0 + v^2 t^2},$$

where the expansion velocity $v^2 = k_B T_e(0)/M$ is given by the initial electron temperature $T_e(0)$ and the mass $M$ of the ions \cite{2, 15}. The density, consequently, decreases by a factor $\sigma(0)^3/\sigma(t)^3 = (1 + v^2 t^2/\sigma(0)^2)^{-3/2}$.

However, since the extraction field $E_{\text{extr}}$ constantly extracts electrons during plasma expansion, the charge imbalance also changes dynamically and therefore affects the time evolution of the absorption resonance. The influence of the extraction field on the electron evaporation dynamics was investigated recently in \cite{43}, where it was shown experimentally and theoretically that the electron loss $N_i - N_e$ is a universal function of a single parameter $\alpha = E_{\text{extr}} \sigma(t)^2/(4\pi N_i)$. Using the universal curve found in \cite{43} one can determine $\delta(t) = N_{\text{esc}}(t)/N_i$ by optimizing a single quantity, the extraction field $E_{\text{extr}}$. Combined with the ion density dynamics from the self-similar description of the plasma expansion, this allows us to extract the absorption resonance frequency as a function of the charge imbalance $\delta$ from the electron evaporation yield measured in \cite{14, 40}.

The result of this analysis is shown in figure 6. Both experiments consistently fall on the same universal curve, which coincides with the edge-mode for sufficiently large $\delta$. For small
charge imbalances \( \delta \to 0 \) the experimental results seem to approach the quasi-mode limit. This behavior may be attributed to the increasing effects of the edge broadening with decreasing charge imbalance. With decreasing charge imbalance the plasma edge \( R_0 \) moves outwards towards lower densities such that the local Debye length \( \lambda_D(R_0) \) decreases. Consequently, for a given temperature and density of the electron the broadening becomes stronger with decreasing \( \delta \). As shown above, this shifts the edge-mode resonance towards larger frequencies, as observed in figure 6. Moreover, the edge-mode resonance weakens with increasing \( \lambda_D(R_0) \). Consequently, one expects the broad quasi-mode to dominate the resonant energy absorption, which is consistent with the behavior shown in figure 6. A more recent experimental work [55], which reports on a more detailed study of rf-driven energy absorption with particular emphasis on the predictions of this work, firmly establishes the importance of the edge-mode for energy absorption in rf-driven ultracold plasmas and demonstrates excellent agreement with the derived resonance for a wider range of charge imbalances.

5. Summary

In conclusion, we have presented an exhaustive theoretical analysis of energy absorption by ultracold plasmas driven by an external electric rf field. We have pointed out the importance of unavoidable charge imbalances between electrons and ions that lead to a truncated electron distribution, a generic but so far scarcely discussed feature of realistic ultracold plasmas. The resulting plasma edge gives rise to an additional absorption resonance, termed edge-mode, that for the first time provides a consistent explanation for all previous rf heating measurements [14, 40, 55]. The calculations show that edge-mode absorption takes place at the plasma boundary, which supports resonant electron escape as observed experimentally, in contrast to continuous quasi-mode absorption that takes place predominantly inside the plasma. Our analysis yields a simple universal curve \( \omega/\omega_p \) that depends on a single parameter, the charge
imbalance $\delta$. This greatly simplifies the experimental analysis of the plasma response, paving the way for a precise probe of ultracold electron plasmas by rf-heating measurements.

The notion of the truncated electron distribution could also explain additional resonances found experimentally [40]. Future work extending the present calculations to finite temperatures may help us to establish their suggested relation to the Tonks–Dattner resonances.

Acknowledgment

We thank Dan Dubin, Kevin Twedt and Steve Rolston for useful discussions.

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