Magnetic moment and electric dipole moment of the \(\tau\)-lepton

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Abstract. Limits on the anomalous magnetic moment and the electric dipole moment of the \(\tau\) lepton are calculated through the reaction \(e^+ e^- \rightarrow \tau^+ \tau^- \gamma\) at the \(Z_1\)-pole and in the framework of a left-right symmetric model. The results are based on the recent data reported by the L3 Collaboration at CERN LEP. Due to the stringent limit of the model mixing angle \(\phi\), the effect of this angle on the dipole moments is quite small.

1. Introduction

The anomalous moments for the electron and muon have been measured with very high precision [1] compared to those of tau for which there are only upper limits [2-5].

In general, a photon may couple to a tau through its electric charge, magnetic dipole moment or electric dipole moment. This coupling may be parametrised using a matrix element in which the usual \(\gamma\mu\) is replaced by a more general Lorentz-invariant form

\[
\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2m_\tau}\sigma^{\mu\nu}q_\nu + F_3(q^2)\sigma^{\mu\nu}\gamma^5q_\nu,
\]

where \(m_\tau\) is the mass of the \(\tau\) lepton and \(q = p' - p\) is the momentum transfer. The \(q^2\)-dependent form-factors, \(F_i(q^2)\), have familiar interpretations for \(q^2 = 0\): \(F_1(0) \equiv Q_\tau\) is the electric charge; \(F_2(0) \equiv a_\tau = (g - 2)/2\) is the anomalous magnetic moment; and \(F_3 \equiv d_\tau/Q_\tau\), where \(d_\tau\) is the electric dipole moment.

Our aim in this paper is to analyze the reaction \(e^+ e^- \rightarrow \tau^+ \tau^- \gamma\). We use recent data collected with the L3 detector at CERN LEP [5] in the \(Z_1\) boson resonance. The analysis is carried out in the context of a left-right symmetric model [6-8] and we attribute a magnetic moment and an electric dipole moment to the tau lepton. Processes measured in the resonance serve to set limits on the tau magnetic moment and electric dipole moment. We take advantage of this fact to set limits for \(a_\tau\) and \(d_\tau\) for different values of the mixing angle \(\phi\) [9-11], which is consistent with other constraints previously reported [3-5].
Table 1. Limits on the $a_\tau$ magnetic moment and $d_\tau$ electric dipole moment of the $\tau$-lepton for different values of the mixing angle $\phi$ in the $Z_1$ resonance, i.e., $s = M_{Z_1}^2$.

| $\phi$ | $a_\tau$ | $d_\tau$ (10^{-16} \text{ ecm}) |
|--------|----------|-----------------------------|
| -0.009 | 0.06     | 3.27                        |
| -0.005 | 0.059    | 3.25                        |
| 0       | 0.058    | 3.22                        |
| 0.004   | 0.057    | 3.21                        |

2. The Total Cross Section

We calculate the total cross section of the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ using the Breit-Wigner resonance form [12,13]:

$$\sigma(e^+e^- \rightarrow \tau^+\tau^-\gamma) = \frac{4\pi(2J + 1)\Gamma_{e^+e^-} - \Gamma_{\tau^+\tau^-\gamma}}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2},$$

(1)

where $\Gamma_{e^+e^-}$ is the decay rate of $Z_1$ to the channel $Z_1 \rightarrow e^+e^-$ and $\Gamma_{\tau^+\tau^-\gamma}$ is the decay rate of $Z_1$ to the channel $Z_1 \rightarrow \tau^+\tau^-\gamma$.

The total cross section is given by

$$\sigma(e^+e^- \rightarrow \tau^+\tau^-\gamma) = \int \frac{d^2}{48\pi} \left[ \frac{e^2 a_{\tau}^2}{4m_{\tau}^2} + d_{\tau}^2 \right] \frac{\left[ \frac{1}{2}(a^2 + b^2) - 4a^2x_W + 8a^2 x_W^2 \right]}{x_W(1 - x_W)^2} \left[ \frac{\left[ \frac{1}{2}(a^2 + b^2) - 4a^2x_W + 8a^2 x_W^2 \right] (s - 2\sqrt{s} E_\gamma) + \frac{1}{2} b^2 E_\gamma^2 \sin^2 \theta_\gamma}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2} \right] E_\gamma dE_\gamma d\cos \theta_\gamma,$$

(2)

where $x_W \equiv \sin^2 \theta_W$.

After evaluating the limit when the mixing angle is $\phi = 0$, the expression for $a$ and $b$ is reduced to $a = b = 1$ and Eq. (2) is reduced to the expression (4) given in Ref. [4].

3. Results

As was discussed in Refs. [5], $N \approx \sigma(\phi, a_\tau, d_\tau) L$. Using the Poisson statistic [5,14], we require that $N \approx \sigma(\phi, a_\tau, d_\tau) L$ be less than 1559, with $L = 100 \text{ pb}^{-1}$, according to the data reported by the L3 Collaboration Ref. [5]. Taking this into consideration, we put a bound for the tau lepton magnetic moment and the electric dipole moment as a function of the $\phi$ mixing parameter. We show the value of this bound for values of the $\phi$ parameter in Table 1.

4. Conclusions

We have determined a limit on the magnetic moment and the electric dipole moment of the tau lepton in the framework of a left-right symmetric model as a function of the mixing angle $\phi$, as shown in Table 1.

In summary, we conclude that the estimated limit for the tau lepton magnetic moment are almost independent of the experimental allowed values of the $\phi$ parameter of the model. In the limit $\phi = 0$, our bound takes the value previously reported in the literature [5].
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