Current constraints on interacting holographic dark energy

Qiang Wu\textsuperscript{1} § Yungui Gong\textsuperscript{1} ‡ Anzhong Wang\textsuperscript{1} † and J. S. Alcaniz\textsuperscript{2}∗
\textsuperscript{1}CASPER, Physics Department, Baylor University, Waco, TX 76798, USA and
\textsuperscript{2}Observatório Nacional, 20921-400, Rio de Janeiro - RJ, Brasil

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Although there is mounting observational evidence that the cosmic expansion is undergoing a late-time acceleration, the physical mechanism behind such a phenomenon is yet unknown. In this paper, we investigate a holographic dark energy (HDE) model with interaction between the components of the dark sector in the light of current cosmological observations. We use both the new gold sample of 182 type Ia supernovae (SNe Ia) and the 192 SNe Ia ESSENCE data, the baryon acoustic oscillation measurement from the Sloan Digital Sky Survey and the shift parameter from the three-year Wilkinson Microwave Anisotropy Probe data. In agreement with previous results, we show that these observations suggest a very weak coupling or even a noninteracting HDE. The phantom crossing behavior in the context of these scenarios is also briefly discussed.

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I. INTRODUCTION

The current idea of a negative-pressure dominated universe seems to be inevitable in light of the impressive convergence of the recent observational results (see, e.g., \textsuperscript{[1, 2, 3, 4, 5]}). This in turn has led cosmologists to hypothesize on the possible existence of an exotic dark component that not only would explain these experimental data but also would reconcile them with the inflationary flatness prediction ($\Omega_{\text{total}}^\text{flat} = 1$). This extra component, or rather, its gravitational effects is thought of as the first observational piece of evidence for new physics beyond the domain of the standard model of particle physics and constitutes a link between cosmological observations and a fundamental theory of nature (for some dark energy components of the dark sector in the light of current cosmological observations. We use both the new gold sample of 182 type Ia supernovae (SNe Ia) and the 192 SNe Ia ESSENCE data, the baryon acoustic oscillation measurement from the Sloan Digital Sky Survey and the shift parameter from the three-year Wilkinson Microwave Anisotropy Probe data. In agreement with previous results, we show that these observations suggest a very weak coupling or even a noninteracting HDE. The phantom crossing behavior in the context of these scenarios is also briefly discussed.

On the other hand, based on the effective local quantum field theories, the authors of Ref. \textsuperscript{8} proposed a relationship between the ultraviolet (UV) and the infrared (IR) cutoffs due to the limit set by the formation of a black hole (BH). The UV-IR relationship in turn gives an upper bound on the zero point energy density $\rho_{\text{UV}} \leq M^2_p L^{-2}$, which means that the maximum entropy is of the order of $S^{3/4}_{BH}$. This zero point energy density has the same order of magnitude as the dark energy density \textsuperscript{5}, and is widely referred to as the holographic dark energy (HDE) \textsuperscript{10} (see also \textsuperscript{11}). However, the HDE model based on the Hubble scale as the IR cutoff seems not to be able of leading to an accelerating universe \textsuperscript{9}. A solution to this matter was subsequently given in Ref. \textsuperscript{10} that discussed the possibilities of the particle and the event horizons as the IR cutoff, and found that only the event horizon identified as the IR cutoff gave a viable HDE model \textsuperscript{10}. The HDE model based on the event horizon as the IR cutoff was found to be consistent with the observational data \textsuperscript{12}.

A subsequent development concerning the idea of a holographic dark energy is the possibility of considering interaction between this latter component and the dark matter in the context of a holographic dark energy model with the event horizon as the IR cutoff. As an interesting result, it was shown that the interacting HDE model realized the phantom crossing behavior \textsuperscript{13}, which is also obtained in the context of non-minimally coupled scalar fields (see, e.g., \textsuperscript{14} and references therein). Other recent discussions on interacting HDE models can be found in \textsuperscript{15, 16, 17}.

In this paper, we test the viability of the interacting HDE model discussed in Ref. \textsuperscript{13} by using the new 182 gold supernovae Ia (SNe Ia) data \textsuperscript{2}, the 192 ESSENCE SNe Ia data \textsuperscript{3}, the baryon acoustic oscillation (BAO) measurement from the Sloan Digital Sky Survey (SDSS) \textsuperscript{4}, and the shift parameter determined from the three-year Wilkinson Microwave Anisotropy Probe (WMAP3) data \textsuperscript{5}. We organized this paper as follows. In Sec. II, we review the basic equations of the interacting HDE model considered in this paper. The choices of the IR cutoff are also discussed in this Sec. II. We fit the model based on the event horizon as the IR cutoff to the observational data above mentioned in Sec III. We end this paper by summarizing our main conclusions in Sec. IV.

II. INTERACTING HDE MODEL

We consider a spatially flat Friedmann-Robertson-Walker universe with dark matter, HDE and radiation. Due to the interaction between the two dark components, the balance equations between them can be written as

\begin{equation}
\dot{\rho}_m + 3H\rho_m = \Gamma ,
\end{equation}

\begin{equation}
\dot{\rho}_D + 3H(1 + \omega_D)\rho_D = -\Gamma
\end{equation}

\textsuperscript{8}Electronic address: qiang.wu@baylor.edu
\textsuperscript{9}Electronic address: yungui.gong@baylor.edu
\textsuperscript{10}Electronic address: anzhong.wang@baylor.edu
\textsuperscript{11}Electronic address: alcaniz@on.br
where the HDE density is
\[ \rho_D = 3c^2 M_p^2 L^{-2}. \]  
(3)

In the above equations, \( L \) is the IR cutoff, \( M_p = 1/\sqrt{8\pi G} \) is the reduced Planck mass, \( \omega_D \) is the equation of state of the HDE, \( \Gamma = 9b^2 M_p^2 H^3 \) is a particular interacting term with the coupling constant \( b \), and the subscript 0 means the current value of the variable. The HDE, dark matter and radiation density parameters are defined, respectively, as \( \Omega_D = \rho_D/(3H^2M_p^2) \), \( \Omega_m = (\rho_m)/(3H^2M_p^2) \), and \( \Omega_r = \rho_r/(3H^2M_p^2) \), and \( \Omega = \rho_c/(3H^2M_p^2) \) for the transition from deceleration to acceleration. As suggested by Li [10], one can choose the future event horizon or particle horizon as the IR cutoff. For the future event horizon,
\[ L(t) = a(t) \int_t^\infty \frac{dt'}{a(t')} = a \int_a^\infty \frac{da'}{H'a'^2}, \]  
(4)
wheras for the particle horizon,
\[ L(t) = a(t) \int_0^t \frac{dt'}{a(t')} = a \int_0^a \frac{da'}{H'a'^2}. \]  
(5)

Substituting Eqs. (1) and (4) into Eq. (3) and taking the derivative with respect to \( x = \ln a \), we obtain
\[ \rho_D' \equiv \frac{d\rho_D}{dx} = -6M_p^2 H^2 \Omega_D \pm \frac{6M_p^2}{c} H^2 \Omega_D^{3/2}, \]  
(6)
where the upper (lower) sign is for the event (particle) horizon. Since \( \rho_D' \equiv d\rho_D/dt = \rho_D'H \), Eq. (2) can be written as
\[ \rho_D' + 3(1 + \omega_D)\rho_D = -9M_p^2 b^2 H^2. \]  
(7)

Combining Eqs. (6) and (7), we obtain the equation of state of this interacting holographic dark energy, i.e.,
\[ \omega_D = -\frac{1}{3} \pm \frac{2}{3} \frac{\sqrt{\Omega_D}}{c} - \frac{b^2}{\Omega_D}. \]  
(8)

When the interaction is absent, \( b^2 = 0 \), it is clear from the above expression that we cannot choose the particle horizon as the IR cutoff. In [17], it was argued that the effective equation of state of the HDE in the interacting case should be
\[ \omega_D^{\text{eff}} = \omega_D + \frac{b^2}{\Omega_D} = -\frac{1}{3} + \frac{2}{3} \frac{\sqrt{\Omega_D}}{c}. \]  
(9)

Based on this effective equation of state, it was concluded that there was no phantom crossover even for an interacting HDE model. In fact, by combining the Friedmann equation with Eqs. (1) and (2), we obtain the acceleration equation
\[ \dot{H} = -4\pi G (\rho + p). \]  
(10)

For a flat universe, the physical consequence of the phantom dark energy is a super-acceleration when the dark energy dominates. Note that it is \( \omega_D \), not \( \omega_D^{\text{eff}} \), that appears in the acceleration equation (10). Therefore, the effective equation of state seems not to show the true physical meaning of the equation of state of the HDE, and \( \omega_D \) should be used instead.

Substituting Eq. (8) into Eq. (11) and applying the definition of \( \Omega_D \), we have
\[ \frac{H'}{H} = -\frac{\Omega_D}{2\Omega_D} \pm \sqrt{\frac{\Omega_D}{c}}. \]  
(11)
On the other hand, substituting $\dot{H} = H'H$ and $p_D = \omega_D \rho_D$ into Eq. (10), we obtain
\[
\frac{H'}{H} = \frac{1}{2} \Omega_D \pm \Omega_D^{3/2} \frac{c}{3} + \frac{3}{2} b^2 - \frac{3}{2} \Omega_\gamma.
\] (12)

Now, combining Eqs. (11) and (12), we find the differential equation for $\Omega_D$, i.e.,
\[
\frac{\Omega_D'}{\Omega_D} = 1 - \frac{\Omega_D \pm \Omega_D^{3/2} c}{(1 - \Omega_D) - 3b^2 + \Omega_\gamma},
\] (13)
a result that is consistent with Eq. (5) of Ref. [18] when the radiation term $\Omega_\gamma$ is neglected.

If we choose the particle horizon as the IR cutoff, the current acceleration requires that $\omega_{D0} < -1/3 - (\Omega_{m0} + 2c_{\gamma0})/3\Omega_{D0}$. From Eq. (13), we also obtain a lower bound on $b^2$,
\[
b^2 > \frac{\Omega_{m0}}{3} + \frac{2}{3} \Omega_{D0}^{3/2} c + \frac{2}{3} \Omega_{\gamma0}.
\] (14)

The past deceleration and the transition from deceleration to acceleration requires that $\Omega_{D0} > 0$, so Eq. (13) gives the upper bound on $b^2$,
\[
b^2 \leq \frac{\Omega_{m0}}{3} \left(1 - 2 \frac{\Omega_{D0}}{c}\right) + \frac{1}{3} \Omega_{\gamma0}.
\] (15)

By comparing Eqs. (14) and (15), we see that the upper bound is lower than the lower bound, so that the inequalities are not satisfied. The model based on the particle horizon as the IR cutoff is not, therefore, a viable dark energy model. In what follows, we consider only the HDE based on the event horizon. As discussed in [18], the interaction $\Gamma$ cannot be too strong and the parameters $b^2$ and $c$ are not totally free; they need to satisfy some constraints. Following [18], we take $0 \leq b^2 \leq 0.2$ and $\sqrt{\Omega_D} < c < 1.255$.

### III. OBSERVATIONAL CONSTRAINTS

#### A. The data

There are three parameters $\Omega_{m0}$, $c$ and $b^2$ in the interacting HDE model since $\Omega_{D0} = 1 - \Omega_{m0} - \Omega_{\gamma0}$ and $\Omega_{\gamma0} \sim 10^{-5}$. In order to place limits on them and test the viability of the model, we apply both the 182 gold SNe Ia [2] and the 192 ESSENCE SNe Ia data [3] to fit these parameters by minimizing
\[
\chi^2 = \sum_i \left[\frac{\mu_{obs}(z_i) - \mu(z_i)}{\sigma_i}\right]^2,
\] (16)
where the extinction-corrected distance modulus $\mu(z) = 5 \log_{10}(d_L(z)/\text{Mpc}) + 25$, $\sigma_i$ is the total uncertainty in the $\mu_{obs}$ observations, and the luminosity distance is given by
\[
d_L = (1 + z) \int_0^z \frac{dz'}{H(z')} = \left[\frac{c(1 + z)^2}{H(z)\sqrt{\Omega_D(z)}} - (1 + z)\frac{c}{\sqrt{\Omega_{D0}H_0}}\right],
\] (17)
where $z = a_0/a - 1$. In all the subsequent analyses, we have marginalized the Hubble parameter $H_0$.

In addition to the SNe Ia data, we also use the BAO measurement from the SDSS data [4, 5]
\[
A = \frac{\sqrt{\Omega_{m0}}}{E(0.35)^{1/3}} \left[1 - \frac{0.35}{0.35} \frac{dz}{E(z)}\right]^{2/3}
\] (18)
and the CMB shift parameter measured from WMAP3
data \[5, 20\]

$$R = \sqrt{\Omega_{m0}} \int_{0}^{z_{fs}} \frac{dz}{E(z)} = 1.70 \pm 0.03,$$

(19)

where the dimensionless function \(E(z) = H(z)/H_0\) and \(z_{fs} = 1089 \pm 1\). In order to obtain the distance, we need to find out the evolution of \(\Omega_D(z)\) and \(H(z)\), so we need to solve Eqs. (12) and (13) numerically. Since the derivatives in Eqs. (12) and (13) are with respect to \(x = \ln a\), we need to rewrite them with respect to \(z\). We find

$$\frac{dH}{dz} = -\frac{H}{1 + z} \left( \frac{1}{2} \Omega_D + \frac{3}{2} c^{3/2} + \frac{3}{2} b^2 - \frac{3}{2} \Omega_\gamma \right),$$

(20)

and

$$\frac{d\Omega_D}{dz} = \frac{\Omega_D}{1 + z} \left[ (1 - \Omega_D)(1 + 2\sqrt{\Omega_D}) - 3b^2 + \Omega_\gamma \right].$$

(21)

By solving numerically the above equations, we then obtain the evolutions of \(\Omega_D\) and \(H\) as a function of the redshift.

B. Results

In Figs. (1) and (2) we show the results of our statistical analyses. Figure (1a) shows the \(c - b^2\) plane for the joint analysis involving the 192 ESSENCE SNe Ia data \[3\] and the other cosmological observables discussed above. For this analysis the best fit values are \(\Omega_{m0} = 0.27^{+0.04}_{-0.03}\), \(c = 0.85^{+0.18}_{-0.02}\), and \(b^2 = 0.002^{+0.01}_{-0.002}\) (at 68.3% c.l.) with \(\chi^2_{\text{min}} = 196.16\). If we fix \(c = 1\), we find \(\Omega_{m0} = 0.26^{+0.04}_{-0.03}\) and \(b^2 = 0.005^{+0.008}_{-0.005}\) (at 68.3% c.l.) with \(\chi^2 = 198.96\). The plane \(\Omega_{m0} - c\) is shown in Panel (1b). Figure (2a) shows the same parametric space \(c - b^2\) when the 192 ESSENCE data is replaced by the new 182 gold sample \[2\]. In this case, we find \(\Omega_{m0} = 0.29 \pm 0.04\), \(b^2 = 0^{+0.01}_{-0.04}\), \(c = 0.88^{+0.07}_{-0.07}\) (at 68.3% c.l.) with \(\chi^2 = 158.66\). The plane \(\Omega_{m0} - c\) for this latter combination of data is also shown in Panel (2b). We note that from both combinations the value of \(b^2\) is very close to 0, which suggests a very weak coupling or a noninteracting HDE. Such a result is also in agreement with the limits found in Ref. \[13\]. By fixing \(b^2 = 0\), we also fit the HDE model without interaction to the observational data discussed above. These results are summarized in Table I. For the sake of comparison, we also list the best-fit results for a flat \(\Lambda\)CDM model.

Finally, by fixing the value of the matter density parameter at \(\Omega_{m0} = 0.27\) we show, in Fig. (3a), the evolutions of \(\Omega_D\) and \(\omega_D\) with the scale factor \(a\). The behavior of \(H/\dot{H}\) is also shown in Fig. (3b). The curves displayed in these Panels are complementary in the sense that from them, we see that while \(\omega_D\) crosses the cosmological constant barrier to the phantom region, \(\dot{H}\) increases from negative to positive values. The distinctive future superacceleration, which is an evidence of a phantom behavior, is apparent from Panel (3b).

IV. CONCLUSIONS

We have analyzed the observational viability of an interacting HDE model characterized by the parameters
From both analyses (involving the 192 SNe Ia ESSENCE and BAO and shift parameter from CMB data for the intervals of the parameters $\Omega_{m0}$, $c$ and $b^2$ displayed in Table I. From both analyses (involving the 192 SNe Ia ESSENCE data and the new 182 Gold sample) we have also found that a very weak coupling or even a noninteracting HDE model is favored ($b^2 \simeq 0$). By using the fitting results for the parameters, we have shown that the expansion rate in the interacting HDE model can be super-accelerated, i.e. $\dot{H} > 0$, which is the key physical consequence of the so-called phantom behavior [19]. This phantom regime is obtained after a transition from $\omega_D > -1$ to $\omega_D < -1$ (phantom crossing behavior), which is a feature well studied in the context of some non-minimally coupled scalar fields [14].

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### Table I: The best-fit results for the HDE parameters

| Model | Results | Gold | Gold+\(A+\mathcal{R}\) | ESSENCE | ESSENCE+\(A+\mathcal{R}\) |
|-------|---------|------|----------------|---------|----------------|
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