Parity-violating DIS and the flavour dependence of the EMC effect

I. C. Cloét, 1 W. Bentz, 2 and A. W. Thomas 1

1 CSSM and ARC Centre of Excellence for Particle Physics at the Terascale, School of Chemistry and Physics, University of Adelaide, Adelaide SA 5005, Australia
2 Department of Physics, School of Science, Tokai University, Hiratsuka-shi, Kanagawa 259-1292, Japan

Isospin-dependent nuclear forces play a fundamental role in nuclear structure. In relativistic models of nuclear structure constructed at the quark level these isovector nuclear forces affect the u and d quarks differently, leading to non-trivial flavour dependent modifications of the nuclear parton distributions. We explore the effect of isospin dependent forces for parity-violating deep inelastic scattering on nuclear targets and demonstrate that the cross-sections for nuclei with \( N \neq Z \) are sensitive to the flavour dependence of the EMC effect. Indeed, for nuclei like lead and gold we find that these flavour dependent effects are large.

Understanding the mechanisms responsible for the change in the per-nucleon deep inelastic scattering (DIS) cross-section between the deuteron and heavier nuclei remains one of the most important challenges confronting the nuclear physics community. In the valence quark region this effect is characterized by a quenching of the nuclear structure functions relative to those of the free nucleon and is known as the EMC effect [1]. This discovery has led to a tremendous amount of experimental and theoretical investigation [2,4]. However, after the passage of more than 25 years there remains no broad consensus regarding the underlying mechanism responsible for the EMC effect.

Early attempts to explain the EMC effect focused on detailed nuclear structure investigations [5] and the possibility of an enhancement in the pionic component of the nucleon in-medium [6,7]. The former studies were unable to describe the data and the latter explanation appears to be ruled out by Drell-Yan measurements of the anti-quark distributions in nuclei [8]. Other ideas included the possibility of exotic, six-quark bags in the nucleus [9] or traditional short-range correlations [10]. It has also been argued that the EMC effect is a result of changes in the internal structure of the bound nucleons brought about by the strong nuclear fields inside the nucleus [11]. Many of these approaches can explain the qualitative features of the EMC effect but the underlying physics mechanisms differ substantially.

To make further progress in our understanding of the mechanism responsible for the EMC effect, it has become clear that we require new experiments that reveal genuinely novel features of this effect. In this Letter we propose an important step in this direction, namely the exploitation of parity-violating DIS (PVDIS), which follows from the interference between photon and \( Z^0 \) exchange. When used in conjunction with the familiar electromagnetic DIS data, it becomes possible to obtain explicit information about the quark flavour dependence of the nuclear parton distribution functions (PDFs). This will allow the predictions of any model of the EMC effect to confront new experimental information and hence provide important insights into this longstanding puzzle.

The parity violating effect of the interference between photon and \( Z^0 \) exchange, in the DIS of longitudinally polarized electrons on an unpolarized target, leads to the non-zero asymmetry:

\[
A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}, \tag{1}
\]

where \( \sigma_L \) and \( \sigma_R \) denote the double differential cross-sections for DIS of right- and left-handed polarized electrons, respectively. In the Bjorken limit \( A_{PV} \) can be expressed as [12]

\[
A_{PV} = \frac{G_F Q^2}{4\sqrt{2} \pi \alpha_{em}} \left[a_2(x_A) + 1 - (1 - y)^2 \right]^{1/2} a_3(x_A), \tag{2}
\]

where \( x_A \) is the Bjorken scaling variable of the nucleus multiplied by \( A \), \( G_F \) is the Fermi coupling constant and \( y = \nu / E \) is the energy transfer divided by the incident electron energy. The \( a_2 \) term in Eq. (2) originates from the product of the electron weak axial current and the quark weak vector current and has the form

\[
a_2(x_A) = -2 g_A^u F_{2A}^u(x_A) + 2 g_A^d F_{2A}^d(x_A) = 2 \sum_q e_q^u g_V^u q_A^+(x_A) + \sum_q e_q^d g_V^d q_A^+(x_A). \tag{3}
\]

The plus-type quark distributions are defined by \( q_A^+ (x_A) = q_A(x_A) + \bar{q}_A(x_A) \), \( e_q \) is the quark charge, \( g_A^e = -1/2 \) [13] and the quark weak vector couplings are [13]

\[
g_V^u = \frac{1}{2} \left( - \frac{4}{3} \sin^2 \theta_W - \frac{1}{3} \cos^2 \theta_W \right), \quad g_V^d = \frac{1}{2} \left( \frac{2}{3} \sin^2 \theta_W \right). \tag{4}
\]

where \( \theta_W \) is the weak mixing angle. The parity violating \( F_2 \) structure function of the target arising from \( \gamma Z \) interference is labelled as \( F_{2A}^Z(x_A) \), while \( F_{2A}^\gamma(x_A) \) is the familiar electromagnetic structure function of traditional DIS. The parton model expressions for these structure functions are [14]

\[
F_{2A}^Z = 2 x_A \sum_q e_q g_V^u q_A^+, \quad F_{2A}^\gamma = x_A \sum_q e_q^2 q_A^+. \tag{5}
\]

The \( a_3 \) term in Eq. (3) is given by

\[
a_3(x_A) = -2 g_V^e F_{2A}^Z(x_A) + 4 g_V^e \sum_q e_q^2 g_A^u q_A(x_A) = -4 g_V^e \sum_q e_q^2 q_A^+(x_A). \tag{6}
\]
where \( g_V^q = -\frac{1}{2} + 2 \sin^2 \theta_W \), \( g_\lambda^q = -g_V^{d_\lambda} = \frac{1}{2} \) \(^{13}\) and \( q_A(x) = g_A(x) - q_A(x) \). This term is suppressed in the parity-violating asymmetry, \( A_{PV} \), because of its \( y \)-dependent prefactor and the fact that \( g_V^q \ll g_\lambda^q \). Therefore, the \( a_2 \) term will not be considered any further in this Letter.

The parity violating structure function, \( F^Z_{2A} \), has a different flavour structure from that of \( F_{2A} \) and, as a consequence, \( a_2(x) \) is sensitive to flavour dependent effects. To illustrate this we expand \( a_2 \) about the \( u_A \simeq d_A \) limit, by ignoring heavy quark flavours we obtain

\[
a_2(x) \simeq \frac{9}{5} - 4 \sin^2 \theta_W - \frac{12}{25} u_A^+(x) - d_A^+(x),
\]

\( a_2(x) \simeq \frac{9}{5} - 4 \sin^2 \theta_W \).

A measurement of \( a_2(x) \) will therefore provide information about the flavour dependence of the nuclear quark distributions and when coupled with existing measurements of \( F_{2A} \) a reliable extraction of the flavour dependent quark distributions becomes possible. Alternatively, if the isovector correction term in Eq. \( 2 \) is known, then the parity violating asymmetry provides an independent method with which to determine the weak mixing angle. For example, if we ignore heavy quark flavours, quark mass differences \( ^{14}\) \( ^{16}\) and electroweak corrections, the \( u \)- and \( d \)-quark distributions of an isoscalar target will be identical, and in this limit Eq. \( 7 \) becomes

\[
a_2(x) = \frac{9}{5} - 4 \sin^2 \theta_W.
\]

This result is analogous to the Paschos-Wolfenstein ratio \(^{17}\) \(^{18}\) in neutrino DIS, which motivated the NuTeV collaboration measurement of \( \sin^2 \theta_W \) \(^{19}\) \(^{20}\). An important advantage of \( a_2(x) \) as a measure of the weak mixing angle is that in the valence quark region strange quark effects are almost absent, which eliminates the largest uncertainty in the NuTeV measurement of \( \sin^2 \theta_W \) \(^{20}\). Also the isovector correction term in Eq. \( 7 \) does not depend on \( \sin^2 \theta_W \) and thus a measurement of \( a_2(x) \) at each value of \( x_A \) constitutes a separate determination of \( \sin^2 \theta_W \). More importantly however, in the context of this work, is that \( a_2 \) is sensitive to flavour dependent nuclear effects that influence the quark distributions of nuclei. Indeed, because of this sensitivity, a measurement of \( a_2 \) on a target with \( N > Z \) would provide an excellent opportunity to test the importance of the isovector EMC effect \(^{18}\) \(^{20}\) for the interpretation of the anomalous NuTeV result for \( \sin^2 \theta_W \).

To determine the nuclear quark distributions and investigate their isospin dependence we use the Nambu–Jona-Lasinio (NJL) model \(^{21}\) \(^{22}\), which is a QCD motivated low energy chiral effective theory characterized by a 4-fermion contact interaction between the quarks. The NJL model has a long history of success in describing mesons as \( q\bar{q} \) bound states \(^{23}\) \(^{24}\) and more recently as a self-consistent model for free and in-medium baryons \(^{25}\) \(^{28}\).

The NJL interaction Lagrangian can be decomposed into various \( q\bar{q} \) and \( qq \) interaction channels via Fierz transformations \(^{29}\), where relevant details to this discussion are given in Ref. \(^{28}\). The scalar \( q\bar{q} \) interaction dynamically generates a constituent quark mass via the gap equation and gives rise to an isoscalar-scalar mean field in-medium. The vector \( q\bar{q} \) interaction terms are used to generate the isoscalar-vector, \( \omega_0 \), and isovector-vector, \( \rho_0 \), mean-fields in-medium. The \( qq \) interaction terms give the diquark \( t \)-matrices with poles corresponding to the scalar and axial-vector diquark masses. The nucleon vertex function and mass are obtained by solving the homogeneous Faddeev equation for a quark and a diquark, where the static approximation is used to truncate the quark exchange kernel \(^{26}\). To regulate the NJL model we choose the proper-time scheme, which eliminates unphysical thresholds for nucleon decay into quarks, and hence simulates an important aspect of QCD, namely quark confinement \(^{20}\) \(^{32}\).

To self-consistently determine the strength of the mean scalar and vector fields, an equation of state for nuclear matter is derived from the NJL Lagrangian using hadronization techniques \(^{30}\). In a mean-field approximation the result for the energy density is \(^{30}\)

\[
\mathcal{E} = \mathcal{E}_V - \frac{\rho_{0}^{2}}{4G_{\omega}} - \frac{\rho_{0}^{3}}{4G_{\rho}} + \mathcal{E}_p + \mathcal{E}_n,
\]

where \( G_{\omega} \) and \( G_{\rho} \) are the \( qq \) couplings in the isoscalar-vector and isovector-vector channels respectively. The vacuum energy, \( \mathcal{E}_V \), has the familiar Mexican hat shape and the energies of the protons and neutrons moving through the mean scalar and vector fields are labelled by \( \mathcal{E}_p \) and \( \mathcal{E}_n \) respectively. Minimizing the effective potential with respect to each vector field gives the following relations:

\[
\omega_0 = 6G_{\omega}(\rho_p + \rho_n) \quad \rho_0 = 2G_{\rho}(\rho_p - \rho_n),
\]

where \( \rho_p \) is the proton and \( \rho_n \) the neutron density. The vector field experienced by each quark flavour is given by \( V_u = \omega_0 + \rho_0 \) and \( V_d = \omega_0 - \rho_0 \).

The parameters of the model are determined by reproducing standard hadronic properties, such as masses and decay constants, as well as the empirical saturation energy and density of symmetric nuclear matter. The empirical symmetry energy of nuclear matter, namely \( a_4 = 32 \text{ MeV} \), is used to constrain \( G_{\rho} \), giving \( G_{\rho} = 14.2 \text{ GeV}^{-2} \). A discussion of the model parameters can be found in Ref. \(^{28}\).

Using medium modified quark distributions – calculated following the techniques of Refs. \(^{25}\) \(^{27}\) – we determine \( a_2(x) \) for symmetric and asymmetric nuclear matter. In each case the total baryon density, \( \rho_B = \rho_p + \rho_n \), is kept fixed and only the proton-neutron ratio is varied. In Figs. \(^{1}\) we present our results for \( a_2(x) \) in nuclear matter with a proton-neutron ratio equal to that of iron (top) and lead (bottom). The full result, which includes the effects from Fermi motion and the scalar and vector mean-fields, is represented by the solid line. The dot-dashed line is the naive expectation where the nuclear quark distributions are obtained from the free proton and neutron PDFs without modification. The dotted line is the result for isoscalar nuclear matter, which is given by Eq. \(^{3}\) and maybe a reasonable approximation to nuclei such as \(^{12}\)C and \(^{40}\)Ca. When evaluating \( g_V^q \) we
The EMC effect can be defined for both electromagnetic and parity violating DIS via the ratio

$$ R^i = \frac{F^i_{2A}}{F^i_{2A,\text{naive}}} = \frac{F^i_{2A}}{Z F^i_{2p} + N F^i_{2n}}, $$

where $i \in \{\gamma, \gamma Z\}$. The corresponding proton and neutron structure functions are respectively labelled by $F^i_{2p}$ and $F^i_{2n}$.

The leading correction to $a_2$ is isovector, as illustrated in Eq. (7). As a consequence, the difference between the naive and full results of Figs. [1] is primarily caused by the non-zero $\rho^0$ mean-field. This is precisely the same effect which eliminates 1 to 1.5σ [18, 20] of the NuTeV discrepancy with respect to the Standard Model in their measurement of $\sin^2 \theta_W$. Thus, quite apart from the intrinsic importance of understanding the dynamics of quarks within nuclei, the observation of these large flavour dependent nuclear effects illustrated in Figs. [1] would be direct evidence that the isovector EMC effect plays an important role in interpreting the NuTeV data. It would also indicate the importance of flavour dependent effects in our understanding of the EMC effect in nuclei like lead and gold, a point we will return to shortly.

The $a_2$ function is potentially sensitive to charge symmetry violation (CSV) effects as well, which are a consequence of the light quark mass differences and electroweak corrections [14, 16]. Including only the CSV correction, Eq. (7)

$$ a_2(x) \simeq \frac{9}{5} - 4 \sin^2 \theta_W - \frac{6}{25} \frac{\delta u^+(x) - \delta d^+(x)}{u^+_p(x) + d^+_p(x)}, $$

becomes

$$ a_2(x) \simeq \frac{9}{5} - 4 \sin^2 \theta_W - \frac{6}{25} \frac{\delta u^+(x) - \delta d^+(x)}{u^+_p(x) + d^+_p(x)}, $$

where $\delta u^+ \equiv u^+_p - d^+_n$ and $\delta d^+ \equiv d^+_p - u^+_n$. These effects are largely independent of the in-medium effects already discussed [20] and using the MRST parametrizations of Ref. [33], with their central value of $\kappa = -0.2$, we find this correction to be negligible on the scale of Figs. [1]. However, if these CSV effects turn out to be larger than expected, they can be constrained via measurements on isospin symmetric nuclei, where the isovector EMC corrections are much smaller.

![Figure 1](image1.png)

**Figure 1.** Asymmetric nuclear matter results for $a_2(x_A)$ obtained by using the $Z/N$ ratio of iron (top) and lead (bottom). In each figure the dotted line is the isoscalar result, the dot-dashed line the naive expectation where no medium effects have been included and the solid line is the full result.

![Figure 2](image2.png)

**Figure 2.** The solid line in each figure is our full result for the EMC effect in electromagnetic DIS on nuclear matter, with the $Z/N$ ratio chosen to be that of iron (top) and lead (bottom). The dash-dotted line illustrates the EMC ratio for PVDIS on the same target. The dotted and dashed lines show the EMC effect in the $u$ and $d$ quark sectors, respectively. The data in each figure is for isoscalar nuclear matter and is taken from Ref. [34].
while $F_{2A}$ is the structure function of the target. The naive structure function $F_{2A,n_{\text{naive}}}$ has no medium effects whatsoever, and therefore, in this limit $R^i$ would be unity. Expressing the EMC effect in terms of the quark distributions we find the parton model expressions

$$R^i \simeq \frac{4 u_A^i + d_A^i}{4 u_f^i + d_f^i}, \quad R^{\gamma Z} \simeq \frac{1.16 u_A^\gamma + d_A^\gamma}{1.16 u_f^\gamma + d_f^\gamma},$$

(12)

where $q_f$ are the quark distributions of the target if it were composed of free nucleons. For an isoscalar target we have $R^i = R^{\gamma Z}$ (modulo electroweak, quark mass and heavy quark flavour effects). However, for nuclei with $N \neq Z$ these two EMC effects need not be equal. The solid line in Figs. 2 illustrates our EMC effect results for $F_{2A}$ in nuclear matter, with $Z/N$ ratios equal to that of iron (top) and lead (bottom), while the corresponding EMC effect in $F_{2A}^{\gamma Z}$ is represented by the dot-dashed line. We find that as the proton-neutron ratio is decreased, the EMC effect in $F_{2A}$ increases, whereas the EMC effect in $F_{2A}^{\gamma Z}$ is slightly reduced. Consequently, for $N > Z$ nuclei we find that $R^i < R^{\gamma Z}$ on the domain $x_A \gtrsim 0.2$, which is the domain over which our valence quark model can be considered reliable.

The fact that $u_A/u_f < d_A/d_f$ and as a consequence $R^i < R^{\gamma Z}$ in nuclei with a neutron excess is a direct consequence of the isovector mean field and is a largely model independent result. In Ref. [18] it was demonstrated that the isovector mean field leads to a small shift in quark momentum from the $u$ to the $d$ quarks, and hence, the in-medium depletion of $u_A$ is stronger than that of $d_A$ in the valence quark region. Because $u_A$ is multiplied by a factor four in the ratio $R^i$, the depletion is more pronounced for this ratio than for $R^{\gamma Z}$, where the $d$-quark quickly dominates as $Z/N$ becomes less than one. The results presented in Figs. 2 demonstrate that the flavour dependence of the EMC effect is potentially large in nuclei like lead and gold.

We have shown that an accurate comparison of the electromagnetic and parity violating DIS cross sections have the potential to pin down the flavour dependence of the EMC effect. The most direct determination of this flavour dependence (c.f. Figs. 2) would involve charged current reactions on heavy nuclei at an electron-ion collider [35] or with certain Drell-Yan reactions [36][37]. However, such experiments will not be possible for ten to twenty years. On the other hand, accurate measurements of PVDIS on heavy nuclei should be possible at Jefferson Lab after the 12 GeV upgrade [38] and would therefore provide a timely, critical test of an important class of models which aim to describe the modification of the nuclear structure functions. These experiments would complement alternative methods to access the quark substructure of nuclei, for example, the measurement of the EMC effect for spin structure functions [25][28], and as a corollary, would also offer a unique insight into the description of nuclear structure at the quark level. Finally, they would constitute a direct test of the isovector EMC effect correction to the NuTeV measurement of $\sin^2 \theta_W$.

ACKNOWLEDGEMENTS

The work is supported by the ARC Centre of Excellence in Particle Physics at the Terascale and an Australian Laureate Fellowship FL0992247 (AWT).