On the Relationship between Leptogenesis and Low-energy Observables

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Abstract. We present some recent results on the connection between leptogenesis and low energy observables in the framework of specific flavour structures for the fundamental leptonic mass matrices with zero textures.

1. Introduction
The origin of fermion masses and mixing remains a puzzle. The experimental discovery of neutrino masses and mixing occurred recently and has rendered this puzzle even more interesting. Mixing in the leptonic sector differs significantly from mixing in the quark sector, furthermore it is not yet known whether or not there is CP violation in the leptonic sector.

There have been many attempts at solving the flavour puzzle either through symmetries or the assumption of special textures for the fermionic masses, such as for instance texture zeros in the leptonic Yukawa couplings [1–6], which is the main focus of this talk, or universal strenght of Yukawa couplings [7]. The underlying idea for special textures is also based on possible symmetries. In all cases the aim is to reduce the number of free parameters leading to definite predictions allowing to test the model.

All experimental data on flavour physics and CP violation in the quark sector seem to be in agreement with the Standard Model (SM) and its KM mechanism with a few hints for potential deviations. In particular the consistency of $|V_{ub}|/|V_{cb}|$ and $\sin 2\beta$ is not very good at present [8]. Recent measurements on $D^0 - \bar{D}^0$ mixing [9–11] are only consistent with the SM if long distance contributions are dominant. There are also hints related to $b \to s$ penguin transitions [12, 13]. The recent experimental flavour-tagged determination of mixing induced CP violation in $B_s \to J/\Psi \Phi$ decays [14], [15] also hints at potential deviations from the SM predictions which might be accounted for within the framework of models with $Q = 2/3$ vector like quarks [16] which lead to deviations of unitarity of the CKM matrix as well as Z mediated flavour changing neutral currents.

There are strong expectations for the discovery of physics beyond the SM at the LHC [12, 17, 18], with strong motivations. In fact the SM leaves many open questions. In particular it is not yet known whether, or not, there is a Higgs scalar boson or if there are more than one. It is by now established that the strength of CP violation is not sufficient to generate the observed baryon asymmetry of the Universe (BAU). Almost all extensions of the SM, including supersymmetry have new sources of CP violation. Many new physics scenarios have flavour
changing neutral currents, (FCNC). Furthermore the SM does not provide a good candidate for dark matter. In addition the discovery of neutrino masses and leptonic mixing which is one of the most important recent developments in Particle Physics is a clear indication of physics beyond the SM. In fact it is well known that in the Standard Model (SM), neutrinos are strictly massless, due to the absence of right-handed neutrinos, together with exact \( B - L \) conservation. One of the simplest ways of accommodating neutrino masses is through the introduction of at least two right-handed neutrinos, leading to the seesaw mechanism [19–23]. Apart from offering an elegant explanation for the smallness of neutrino masses, the seesaw mechanism also provides a framework to create the observed baryon asymmetry of the Universe (BAU). This is achieved through leptogenesis, a mechanism first suggested in Ref. [24], where out-of-equilibrium decays of heavy right-handed neutrinos create a lepton asymmetry which in turn is converted into a baryon asymmetry, through sphaleron interactions [25,26]. In the context of the seesaw mechanism it is natural to ask whether there is a connection between leptogenesis and low energy observables. Furthermore, such a connection would be necessary in order to test leptogenesis, assuming the heavy neutrinos masses lie outside the reach of future experiments.

2. Low-energy observables

Let us assume that lepton number is violated at a high energy scale, leading to the generation at low energies of an effective left-handed Majorana neutrino mass matrix. In the mass eigenstate basis, the charged weak current can be written:

\[
\mathcal{L}_W = -\frac{g}{\sqrt{2}} \bar{l}_L \gamma_\mu U_{jk} \nu_{kL} + h.c.,
\]  

(1)

where \( U \) denotes the leptonic mixing matrix at low energies, usually named the Pontecorvo, Maki, Nakagawa and Sakata (PMNS) matrix [27–29]. Although in the seesaw framework \( U \) is not exactly unitary, in the standard seesaw type I framework, \( U \) is unitary to a high degree of accuracy. Once \( 3 \times 3 \) unitarity is assumed, \( U \) is characterized by six parameters which are usually taken as three mixing angles and three CP violating phases. There are nine low energy observables, namely the three light neutrino masses and the six parameters characterizing \( U \) [30]. The question is then whether it is possible to relate leptogenesis to the low energy observables.

Several authors have discussed phenomenological implications from sizeable deviations from unitarity of the matrix \( U \), [31, 32] which can naturally occur, for instance, for “light heavy neutrinos”, i.e., heavy neutrinos with masses of order one TeV or even lower. Such light heavy neutrinos could have implications for physics at the LHC [33]. This is not the framework we are considering here.

3. Leptogenesis and Low-energy Observables

In the minimal seesaw framework, the SM is extended only through the inclusion of right-handed components for the neutrinos which are singlets of \( SU(2) \times U(1) \). Frequently, one right-handed neutrino component per generation is introduced. This will be the case in what follows. In fact, neutrino masses can be generated without requiring the number of right-handed and left-handed neutrinos to be equal. Present observations are consistent with the introduction of two right-handed components only. In this case one of the three light neutrinos would be massless. There are also scenarios where more than three right-handed neutrinos are included [34].

With one right-handed neutrino component per generation the number of fermionic degrees of freedom for neutrinos equals those of all other fermions in the theory. However neutrinos are the only known fermions which have zero electrical charge and this allows one to write Majorana mass terms for the singlet fermion fields. After spontaneous symmetry breakdown (SSB) the
leptonic mass term is of the form:

\[
\mathcal{L}_m = -\left[\bar{\nu}_L^0 m_D^0 \nu_R^0 + \frac{1}{2} \nu_R^{0T} C M_R \nu_R^0 + \bar{\nu}_L^0 m_L^0 \nu_R^0 \right] + h.c. = \\
-\frac{1}{2} n_{L}^T C \mathcal{M}^* n_{L} + \bar{\nu}_L^0 m_{L}^0 \nu_R^0 + h.c.
\]

(2)

with the \(6 \times 6\) matrix \(\mathcal{M}\) given by:

\[
\mathcal{M} = \left( \begin{array}{cc} 0 & m_D^T \\ m_D & M_R \end{array} \right)
\]

(3)

the upperscript 0 in the neutrino (\(\nu\)) and charged lepton fields (\(l\)) is used to indicate that we are still in a weak basis (WB), i.e., the gauge currents are still diagonal. The charged current is given by:

\[
\mathcal{L}_W = -\frac{g}{\sqrt{2}} W^\mu_\mu \bar{\nu}_L^0 \gamma^\mu \nu_R^0 + h.c.
\]

(4)

Since the Majorana mass term is gauge invariant there are no constraints on the scale of \(M_R\). The (standard) seesaw limit consists of taking this scale to be much larger than the scale of the Dirac mass matrices \(m_D\) and \(m_{\ell}\). The Dirac mass matrices are generated from Yukawa couplings after SSB and are therefore at most of the electroweak scale. As a result the spectrum of the neutrino masses splits into two sets, one consisting of very heavy neutrinos with masses of the order of that of the matrix \(M_R\) and the other set with masses obtained, to a very good approximation, from the diagonalisation of an effective Majorana mass matrix given by:

\[
m_{\text{eff}} = -m_D^0 \frac{1}{M_R} m_D^T
\]

(5)

This expression shows that the light neutrino masses are strongly suppressed with respect to the electroweak scale. There is no loss of generality in choosing a WB where \(m_{\ell}\) is real diagonal and positive. The diagonalization of \(\mathcal{M}\) is performed via the unitary transformation:

\[
V^T \mathcal{M}^* V = D
\]

(6)

where \(D = \text{diag}(m_1, m_2, m_3, M_1, M_2, M_3)\), with \(m_i\) and \(M_i\) denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write \(V\) and \(D\) in the following block form:

\[
V = \left( \begin{array}{cc} K & G \\ S & T \end{array} \right) ; \\
D = \left( \begin{array}{cc} d & 0 \\ 0 & D \end{array} \right).
\]

(7)

The neutrino weak-eigenstates are then related to the mass eigenstates by:

\[
\nu_{iL}^0 = V_{i\alpha} \nu_{\alpha L} = (K, G) \left( \begin{array}{cc} \nu_{iL} \\ N_{iL} \end{array} \right) \left( \begin{array}{cc} \cdots \alpha = 1, 2, 3 \\ i = 1, 2, 3 \end{array} \right)
\]

(8)

and the leptonic charged current interactions are given by:

\[
\mathcal{L}_W = -\frac{g}{\sqrt{2}} \left( \bar{\nu}_{iL} \gamma^\mu K_{ij} \nu_{jL} + \bar{\nu}_{iL} \gamma^\mu G_{ij} N_{jL} \right) W^\mu + h.c.
\]

(9)

with \(K\) and \(G\) being the charged current couplings of charged leptons to the light neutrinos \(\nu_{j}\) and to the heavy neutrinos \(N_{j}\), respectively.
In the seesaw limit the matrix $K$ coincides to an excellent approximation with the unitary matrix $U$, the PMNS matrix, that diagonalises $m_{\text{eff}}$ of Eq. (5):
\[ -U^\dagger m_D^1 M_R m_D^T U^* = d \] (10)
and the matrix $G$ verifies the exact relation:
\[ G = m_D T^* D^{-1} \] (11)
and is therefore very suppressed.

The lepton number asymmetry generated through CP violating decays of the $j$-th heavy Majorana neutrino into the different leptonic families has been computed [35], [36], [37], [38], [39] in the single flavour approximation, and shown to be proportional to:
\[ A^j \propto \sum_{k \neq j} \text{Im}(m_D^\dagger m_D)_{jk} (m_D^\dagger m_D)_{jk} \] (12)
in the weak basis (WB) where $M_R$ is diagonal. In the seesaw framework the matrix $m_D$, given in the WB where $M_R$ is real and diagonal, can be written [40]:
\[ m_D = iU \sqrt{dR} \sqrt{D} \] (13)
where $R$ is a general complex orthogonal matrix. This equation is derived from Eq. (10). The matrix $R$ is relevant for leptogenesis since:
\[ m_D^\dagger m_D = -\sqrt{D}R^\dagger dR\sqrt{D}. \] (14)
Eq. (14) shows that in general unflavoured leptogenesis is independent of the presence of CP violation at low energies [41,42]. Unflavoured leptogenesis requires the matrix $R$ to be complex. Notice that in the WB where the charged lepton mass matrix and $M_R$ are real and diagonal, all CP violating phases appear in $m_D$.

The general seesaw framework contains a large number of free parameters. The introduction of zero textures and/or the reduction of the number of right-handed neutrinos to two, allows to reduce the number of parameters. In this work only zero textures imposed in the fundamental leptonic mass matrices are considered and, in particular, zero textures of the Dirac neutrino mass matrix, $m_D$ in the WB where $M_R$ and $m_l$ are real and diagonal. Zero textures of the low energy effective neutrino mass matrix are also very interesting phenomenologically [43]. The physical meaning of the zero textures that appear in most of the leptonic mass ansätze was analysed in a recent work [44] where it is shown that some leptonic zero texture ansätze can be obtained from WB transformations and therefore have no physical meaning.

In general, zero textures reduce the number of CP violating phases, as a result some sets of zero textures imply the vanishing of certain CP-odd WB invariants [4]. This is an important fact since clearly zero textures are not WB invariant, therefore in a different WB the zeros may not be present making it difficult to recognise the ansatz. Furthermore, it was also shown [4] that starting from arbitrary leptonic mass matrices, the vanishing of certain CP-odd WB invariants, together with the assumption of no conspiracy among the parameters of the Dirac and Majorana mass terms, one is automatically lead to given sets of zero textures in a particular WB. The general procedure to obtain relevant CP-odd WB invariants was outlined for the first time in Ref. [45]

From Eq. (13) it is clear that one zero in $(m_D)^{ij}$ corresponds to having an orthogonality relation between the $i$th row of the matrix $U_\nu \sqrt{d}$ and the $j$th column of the matrix $R$:
\[ (m_D)^{ij} = 0 : \quad (U_\nu)^{ik} \sqrt{d_{kl}} R_{lj} = 0 \] (15)
Ibarra and Ross [3] showed that, in the seesaw case with only two right-handed neutrinos, a single zero texture, has the special feature of fixing the matrix $R$, up to a reflection, without imposing any further restriction on light neutrino masses and mixing, whilst two zeros in $m_D$ also imply restrictions on $U$ and $\sqrt{d_i}$.

Four zero textures in the context of seesaw with three righthanded neutrinos are studied in detail in Ref. [5]. It is shown that four is the maximum number of zeros in textures compatible with the observed leptonic mixing and with the additional requirement that none of the neutrino masses vanishes. It is also shown that such textures lead to important constraints both at low and high energies, and allow for a tight connection between leptogenesis and low energy parameters. It is possible in all cases to completely specify the matrix $R$ in terms of light neutrino masses and the PMNS matrix. These relations are explicitly given in Ref. [5].

Leptogenesis in the single flavour approximation relies on the assumption that washout effects are not sensitive to the different flavours of charged leptons into which the heavy neutrino decays. It was pointed out that flavour matters in leptogenesis whenever the mass of the lightest heavy neutrino is lower than $10^{12}$ GeV [46–51].

The separate lepton $i$ family asymmetry generated from the decay of the $k$th heavy Majorana neutrino depends on the combination [48] $\text{Im}(m_D^+ m_D)_{k,k'}(m_D^+ m_D)_{k,i}(m_D^+ m_D)_{ik}$. It is clear from these expressions that in the case of flavoured leptogenesis there are additional sources of CP violation since the PMNS matrix does not cancel out. This gives rise to the new possibility of having viable leptogenesis even in the case of $R$ being a real matrix. For some of the early examples of models with viable leptogenesis with $R$ real see [52–55]. In Ref. [56] it is argued that the possible effects of unmeasurable seesaw parameters render it difficult to directly relate flavoured leptogenesis to low energy CP violation.

Examples of CP-odd WB invariant conditions relevant for leptogenesis in the single flavour approximation are [42]:

\[ I_1 \equiv \text{ImTr}[m_D^+ m_D M_R^T M_R M_R^* M_D^* M_D M_R] = 0 \] (16)
\[ I_2 \equiv \text{ImTr}[m_D^+ m_D (M_R^T M_R)^2 M_R^* M_D^* M_D M_R M_R] = 0 \] (17)
\[ I_3 \equiv \text{ImTr}[m_D^+ m_D (M_R^T M_R)^2 M_R^T M_D^* M_D M_R M_R M_R] = 0 \] (18)

In the case of flavoured leptogenesis there is still the possibility of generating the required CP asymmetry even for $m_D^+ m_D$ real. In this case additional CP-odd WB invariant conditions are required since those written above cease to be necessary and sufficient. A simple choice are the WB invariants $\bar{I}_i (i = 1, 2, 3)$, [42] obtained from $I_i$, through the substitution of $m_D^+ m_D$ by $m_D^+ h_i m_D$ where $h_i = m_D m_i^T$. For example one has:

\[ \bar{I}_1 = \text{ImTr}(m_D^+ h_i m_D M_R^T M_R M_R^* m_D^T h_i m_D M_R) \] (19)

and similarly for $\bar{I}_2, \bar{I}_3$. As it was the case for $I_i$, CP invariance requires that $\bar{I}_i = 0$. The latter CP-odd WB invariant conditions are sensitive to the additional phases appearing in flavoured leptogenesis.

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References

1. G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim, I. Masina, M. N. Rebelo and C. A. Savoy, Phys. Rev. D 67 (2003) 073025 [arXiv:hep-ph/0211001].
2. V. Barger, D. A.Dicus, H. J. He and T. j. Li, Phys. Lett. B 583 (2004) 173 [arXiv:hep-ph/0310278].
3. A. Ibarra and G. G. Ross, Phys. Lett. B 591 (2004) 285 [arXiv:hep-ph/0312138].
4. G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Phys. Lett. B 633 (2006) 345 [arXiv:hep-ph/0510412].
5. G. C. Branco, D. Emmanuel-Costa, M. N. Rebelo and P. Roy, Phys. Rev. D 77 (2008) 053011 [arXiv:0712.0774 [hep-ph]].
6. S. Choubey, W. Rodejohann and P. Roy, Nucl. Phys. B 808 (2009) 272 [arXiv:0807.4289 [hep-ph]].
7. G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Nucl. Phys. B 686 (2004) 188 [arXiv:hep-ph/0311348].
8. C. Amsler et al. [Particle Data Group], Phys. Lett. B 667 (2008) 1.
9. Babar Collaboration, B. Aubert et al., Phys. Rev. Lett. 98, 211802 (2007), hep-ex/0703020.
10. Belle Collaboration, M. Staric et al., Phys. Rev. Lett. 98, 211803 (2007), hep-ex/0703036.
11. Belle Collaboration, K. Abe et al., Phys. Rev. Lett. 99, 131803 (2007), arXiv:0704.1000.
12. M. Artuso et al., Eur. Phys. J. C 57 (2008) 309 [arXiv:0801.1833 [hep-ph]].
13. R. Fleischer, arXiv:0802.2882.
14. D0 Collaboration, V. M. Abazov et al., arXiv:0802.2255.
15. CDF Collaboration, T. Aaltonen et al., Phys. Rev. Lett. 100, 161802 (2008), arXiv:0712.2397.
16. F. J. Botella, G. C. Branco and M. Nebot, arXiv:0805.3995 [hep-ph].
17. F. del Aguila et al., Eur. Phys. J. C 57 (2008) 183 [arXiv:0801.1800 [hep-ph]].
18. M. Raidal et al., Eur. Phys. J. C 57 (2008) 13 [arXiv:0801.1826 [hep-ph]].
19. P. Minkowski, Phys. Lett. B 67 (1977) 421.
20. T. Yanagida, in Proc. of the Workshop on Unified Theory and Baryon Number in the Universe, KEK, March 1979.
21. S. L. Glashow, in “Quarks and Leptons”, Cargèse, ed. M. Lévy et al., Plenum, New York, 1980, p. 707.
22. M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, Stony Brook, Sept 1979.
23. R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
24. M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.
25. F. R. Klinkhamer and N. S. Manton, Phys. Rev. D 30 (1984) 2212.
26. V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155 (1985) 36.
27. B. Pontecorvo, Sov. Phys. JETP 7 (1958) 172 [Zh. Eksp. Teor. Fiz. 34 (1957) 247].
28. Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.
29. B. Pontecorvo, Sov. Phys. JETP 26 (1963) 984 [Zh. Eksp. Teor. Fiz. 53 (1967) 1717].
30. R. N. Mohapatra et al., Rept. Prog. Phys. 70 (2007) 1757 [arXiv:hep-ph/0502133].
31. See for instance: S. Antusch, M. Blennow, E. Fernandez-Martinez and J. Lopez-Pavon, arXiv:0903.3986 [hep-ph] and references therein.
32. See also: W. Rodejohann, arXiv:0903.4590 [hep-ph] and references therein.
33. F. del Aguila and J. A. Aguilar-Saavedra, Nucl. Phys. B 513 (2009) 22 [arXiv:0808.2468 [hep-ph]].
34. J. R. Ellis and O. Lebedev, Phys. Lett. B 653 (2007) 411 [arXiv:0707.3419 [hep-ph]].
35. J. Liu and G. Segre, Phys. Rev. D 48 (1993) 4609 [arXiv:hep-ph/9304241];
36. M. Flanz, E. A. Paschos, U. Sarkar, Phys. Lett. B 345, 248 (1995) [Erratum, ibid. B 382, 447 (1996)];
37. L. Covi, E. Roulet, F. Vissani, Phys. Lett. B 384, 169 (1996);
38. A. Pilaftsis, Phys. Rev. D 56 (1997) 5431 [arXiv:hep-ph/9707245];
39. W. Buchmuller and M. Plumacher, Phys. Lett. B 431 (1998) 354 [arXiv:hep-ph/9710460];
40. J. A. Casas and A. Ibarra, Nucl. Phys. B 618 (2001) 171 [arXiv:hep-ph/0103065].
41. M. N. Rebelo, Phys. Rev. D 67 (2003) 013008 [arXiv:hep-ph/0207236].
42. G. C. Branco, T. Morozumi, B. M. Nobre and M. N. Rebelo, Nucl. Phys. B 617 (2001) 475 [arXiv:hep-ph/0107164].
43. P. H. Frampton, D. L. Glashow and D. Marfatia, Phys. Lett. B 536, 79 (2002) [arXiv:hep-ph/0201008].
44. G. C. Branco, D. Emmanuel-Costa, R. G. Felipe and H. Serodio, arXiv:0711.1613 [hep-ph].
45. J. Bernabéu, G. C. Branco and M. Gronau, Phys. Lett. B 169, 243 (1986).
46. R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B 575 (2000) 61 [arXiv:hep-ph/9911315].
47. T. Endoh, T. Morozumi and Z. h. Xiong, Prog. Theor. Phys. 111 (2004) 123 [arXiv:hep-ph/0308276].
48. T. Fujihara, S. Kaneko, S. K. Kang, D. Kimura, T. Morozumi and M. Tanimoto, Phys. Rev. D 72 (2005) 016006 [arXiv:hep-ph/0505076].
49. A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, JCAP 0604 (2006) 004 [arXiv:hep-ph/0601083].
[50] E. Nardi, Y. Nir, E. Roulet and J. Racker, JHEP 0601 (2006) 164 [arXiv:hep-ph/0601084].
[51] A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, JHEP 0609 (2006) 010 [arXiv:hep-ph/0605281].
[52] G. C. Branco, A. J. Buras, S. Jager, S. Uhlig and A. Weiler, JHEP 0709 (2007) 004 [arXiv:hep-ph/0609067].
[53] S. Pascoli, S. T. Petcov and A. Riotto, Phys. Rev. D 75 (2007) 083511 [arXiv:hep-ph/0609125].
[54] G. C. Branco, R. Gonzalez Felipe and F. R. Joaquim, Phys. Lett. B 645 (2007) 432 [arXiv:hep-ph/0609297].
[55] S. Uhlig, JHEP 0711 (2007) 066 [arXiv:hep-ph/0612262].
[56] S. Davidson, J. Garayoa, F. Palorini and N. Rius, Phys. Rev. Lett. 99 (2007) 161801 [arXiv:0705.1503 [hep-ph]].