Comparison between GFDM and VOFDM

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This document provides a comparison of the transmission techniques used in Generalized Frequency Division Multiplexing (GFDM) and Vector-OFDM (VOFDM). Within the document both systems are coarsely described and common and distinct properties are highlighted.

1 Mathematical Description of GFDM and VOFDM

In the following we describe the signal processing necessary for creating a transmit signal. For simplicity we only consider the pure modulation, without addition of CP or more advanced waveform engineering techniques.

1.1 V-OFDM

VOFDM initially proposed in 2001 by X.G. Xia [12] is a multicarrier technique that is able to deal with spectral nulls and better exploits frequency diversity than conventional OFDM. Further studies on VOFDM have been carried out in e.g [1 4 5]. The following mathematical description of the VOFDM transmitter is adapted from the derivation in [5]. However, in the present description we have focused on the formulation of the transmitter using a single matrix representation, which eases understanding of the system.

Consider a data vector \( \vec{d} \) of length \( N \),

\[
\vec{d} = [d_0, d_1, d_2, \ldots, d_{N-1}].
\] (1)

Now, this data vector is reshaped into a matrix \( D \) of \( M \) rows and \( L \) columns such that \( N = LM \) and

\[
D = \begin{pmatrix}
  d_0 & d_M & d_{2M} & \ldots & d_{(L-1)M} \\
  d_1 & d_{M+1} & d_{2M+1} & \ldots & d_{(L-1)M+1} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  d_{M-1} & d_{2M-1} & d_{3M-1} & \ldots & d_{LM-1}
\end{pmatrix}.
\] (2)

The columns of \( D \) denote the vector blocks (VB) of length \( M \) and \( L \) VBs form a VOFDM frame. Then, at the transmitter an IFFT along the rows of \( D \) (i.e. of the VBs) is performed and the resulting time-domain signal \( \vec{x} \) is given by stacking the transformed VBs on top of each other. This operation can be written by

\[
\vec{x} = \text{vec}(DF_L^H V \vec{d}),
\] (3)

where \( \text{vec} \) performs the vectorization operation (i.e. stacking the columns of the argument on top of each other) and \( F_L \) denotes the \( L \)-point DFT matrix. Using properties of the Kronecker product \( \otimes \), we obtain the final description of the linear V-OFDM modulation

\[
\vec{x} = (F_L^H \otimes I_M) \vec{d},
\] (4)

and \( V \) denotes the VOFDM modulation matrix.
1.2 GFDM

GFDM was initially proposed in 2009 by G. Fettweis et al. [2] as a flexible multicarrier technique for future wireless systems. Further research on GFDM has been published in e.g. [10, 6, 11] and others towards 5G systems. GFDM transmission can be described with several notations, such as time-domain description as in [9], frequency-domain description as in [8] or a low-complexity description in the time domain as in [3]. In the following mathematical description we derive the GFDM modulation matrix based on the description in [3]. There, the transmit data $d_m$ for each subsymbol is repeated over the full block and then multiplied by the pulse shaping filter which is shifted to the corresponding subsymbol position. Consider a GFDM data block $D$

$$D = \begin{pmatrix} d_0,0 & d_0,1 & \ldots & d_0,M-1 \\ d_1,0 & d_1,1 & \ldots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ d_{K-1},0 & d_{K-1},1 & \ldots & d_{K-1,M-1} \end{pmatrix},$$ (5)

where $K$ denotes the number of subcarriers and $M$ denotes the number of subsymbols (SSs). An $K$-point IFFT is performed along the columns of $D$, i.e. separately for each SS. Then each SS is repeated $M$ times. Finally each SS is multiplied element-wise by a circularly rotated version of the pulse shaping filter and all subsymbols are summed up to generate the GFDM transmit signal.

$$\bar{x} = \sum_{m=0}^{M-1} \text{diag}(C_{mK}\vec{g})R_MF_K^Hd_m$$ (6)

$$= \left[ \text{diag}(C_0\vec{g}) \text{ diag}(C_K\vec{g}) \ldots \text{ diag}(C_{(M-1)K}\vec{g}) \right] \text{vec}(D),$$ (7)

where $R_M = I_K \otimes 1_M$ performs a $M$-fold repetition of its argument, $C_l$ performs a circular shift of $l$ elements of its argument and diag($\cdot$) returns a diagonal matrix with its argument on the diagonal. $A$ is the resulting GFDM modulation matrix, that is also derived in [9] and also used in [7].

2 Interpretation and Comparison of Both Systems

V-OFDM modulation defines data in the frequency domain in the rows of $D$. The data is transferred to time domain by IFFT operation on the rows of $D$. Finally, reading the resulting matrix column-wise, the corresponding time-domain signals for each row in $D$ are transmitted in an interleaved way. This can be understood as if each time-domain signal is upsampled by factor $L$ and transmitted with a delay equal to its row number, resembling a time domain multiplexing (TDM) system.

The upsampling operation in time corresponds to a spectrum repetition. Moreover, transmission with a time offset corresponds to a phase rotation in the frequency domain. Hence, the V-OFDM modulation can be illustrated as in Fig. 1. For each row in $D$, the spectrum is repeated $M$ times and afterwards depending on the row index the phase is rotated. Eventually, all spectra are summed up to yield the complete transmit signal in the frequency domain. From this description we see that each data symbol in $D$ is spread onto the entire transmit spectrum, yielding frequency diversity in frequency selective channels. On the other hand, no spectral agility is available, as always the full bandwidth is occupied, regardless of symbol allocation.

In contrast, the modulation of GFDM can also be understood as explicitly separating the signals in frequency domain in sub-bands. In GFDM the upsampled data on each subcarrier is circularly convolved with the pulse shaping filter that is shifted to the appropriate subcarrier frequency. The cyclic time convolution turns to element-wise multiplication in the frequency domain. Furthermore, upsampling in the time domain again corresponds to repetition in the frequency domain as in the VOFDM system. This technique is illustrated in Fig. 2.

Hence, similar to both GFDM and VOFDM is a repetition of the transmitted spectrum of one data
1) Spectrum  2) Repeat  3) Phase rotation

Figure 1: Illustration of VOFDM modulation in the frequency domain, for $M = 3$.

1) Spectrum  2) Repeat  3) Filtering

Figure 2: Illustration of GFDM modulation in the frequency domain, for $K = 3$

The severe difference between both systems is the applied window for element-wise multiplication in the frequency domain, corresponding to convolution in time domain.

In GFDM, the signal is multiplied with a spectrally localized window, that corresponds to a smooth function in the time domain (i.e. the pulse shaping filter). In contrast, VOFDM utilizes a rotating exponential that corresponds to a Dirac function in the time domain. Accordingly, in GFDM both subsymbols and subcarriers are localized in time and frequency domains, compared to VOFDM where all data spreads over the entire spectrum and VB are interleaved in the time domain. From the medium access point of view, VOFDM is more like CDMA using different spreading codes. GFDM is more like FDMA.

Accordingly we can conclude that GFDM and VOFDM have similarities in their spectral structure. However, the most significant difference is the used frequency domain window. The localized window of partially GFDM wastes frequency diversity compared to VOFDM. On the other hand, it allows to implement spectrally agile systems that leave certain subcarriers empty for other systems. A comparison summary is given in Tab. 1.

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3Ignoring the domain where the data is defined. For VOFDM its defined in the frequency domain, whereas in GFDM data on one subcarrier is defined in time domain.
Table 1: Comparison of GFDM and VOFDM.

| Aspect                        | GFDM                                      | VOFDM                                      |
|-------------------------------|-------------------------------------------|--------------------------------------------|
| Frequency domain structure    | Spectrum repetition for each data matrix row | rectangular matrix row                     |
| Frequency domain window       | localized (sparse) filter                  | multiplication with complex                |
| Filter change row to row      | circular shift by one subcarrier           | exponential of increasing frequency        |
| Localization                  | localized in time and frequency            | interleaved in time and frequency          |

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