Simple criterium for CP conservation in 2HDM

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Abstract. We find a set of necessary and sufficient conditions for the CP conservation in the most general 2HDM in terms of observable quantities. This set contains two simple and relatively easily testable conditions instead of the more complex conditions usually discussed.

1. Introduction

The detailed version of this paper was prepared in collaboration with M. Krawczyk [1]. Maria passed away on May 24, 2017. In this report I omit some details, which look me interesting only for participants of Higgs studies.

CP violation is one of the important yet not well understood aspect of the fundamental physics. The modern LHC data [2],[3] allow us to conclude that the observed particle $h(125)$ is a Higgs boson with spin-CP parity $0^{++}$ but only under the assumption that this particle has a definite parity. Generally in many models $h(125)$ does not possess a definite parity. In this case the data give no information about $h(125)$ parity [4].

In the Standard Model the CP violation is described by the CKM matrix, but its origin remains unclear. The extension of SM with two Higgs doublets, called the Two Higgs Doublet Model (2HDM), has been introduced in 1974 with the main aim to provide an extra source of CP violation [5]. Later, many variants of 2HDM were considered with different features, providing various physical realizations. Each of these variants is described by Lagrangian with many parameters. The choice of the set of these parameters, describing the same physical reality is ambiguous. In non-minimal models such as 2HDM the current situation, with the properties of the observed Higgs boson resembling those of the SM Higgs (SM-like scenario [7] or alignment limit [8]), can be described by different non-equivalent sets of parameters.

We consider in the report two types of problems which appear in the study of CP violation in these models\textsuperscript{5}F.

(A) Let us have a variant of 2HDM, constructed as a model for the description of some set of phenomena.

\textit{How to know whether the CP symmetry is violated or not in this model without detailed calculations of each particular effect?}

(B) Consider 2HDM as an approximation to the description of the Nature. How do we know whether the CP violation is described by this approximation, or whether the observed CP violation is an effect of yet another, weaker interaction.

\textit{How to check the presence of CP violation in the experiments with particles that appear in 2HDM?}
2. 2HDM

The 2HDM describes a system of two spinless isospinor fields $\phi_1$, $\phi_2$ with hypercharge $Y = 1$. The most general form of the 2HDM potential is

$$V = \frac{\lambda_1}{2} (\phi_1^\dagger \phi_1)^2 + \frac{\lambda_2}{2} (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2^\dagger) (\phi_2^\dagger \phi_1) + \left[ \frac{\lambda_5}{2} (\phi_1^\dagger \phi_2^\dagger)^2 + \lambda_6 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_7 (\phi_2^\dagger \phi_2) (\phi_1^\dagger \phi_1) \right] + \text{h.c.}$$  \hspace{1cm} (1)

The potential parameters are restricted by the requirement that the potential is positive at large quasiclassical values of $\phi$ (positivity constraints). We assume also that these parameters are not too big so that one can use estimates based on the lowest non-trivial approximation of the perturbation theory.

After Electroweak Symmetry Breaking the 2HDM contains 3 neutral Higgs bosons $h_a \equiv h_{1,2,3}$ (in general with indefinite CP parity) and the charged Higgs bosons $H^{\pm}$, with the masses $M_a$ and $M_{\pm}$, respectively (the numbering of $h_a$ is independent of the masses $M_a$).

2.1. Reparametrization freedom

2HDM describes system of two fields with identical quantum numbers. Therefore, its description in terms of original fields $\phi_i$ or in terms of their linear superpositions $\phi'_i$ with corresponding transformation of the parameters of the Lagrangian are equivalent; this statement verbalizes the reparameterization (RPa) freedom of the model. We refer to these different choices as different RPa bases. The RPa group consists of RPa transformations $\hat{F}$ of the form:

$$\left( \begin{array}{c} \phi_1' \\ \phi_2' \end{array} \right) = \hat{F}_{\text{gen}}(\theta, \tau, \rho) \left( \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right), \quad \hat{F}_{\text{gen}} = e^{i\rho_0} \begin{pmatrix} \cos \theta e^{i\rho/2} & \sin \theta e^{i(\tau+\rho/2)} \\ -\sin \theta e^{-i(\tau+\rho/2)} & \cos \theta e^{-i\rho/2} \end{pmatrix}. \quad (2)$$

A subgroup of the RPa group — the rephasing group RPh — describes a freedom of adjusting the relative phase of the fields $\phi_i$. The parameter $\rho_0$ describes an overall phase transformation of the fields and can be ignored since it does not affect the parameters of the potential. The parameter $\rho$ describes the RPh symmetry transformation of system.

The $U(1)_{EM}$ symmetry preserving ground state of this system is given by a global minimum of the potential and reads

$$\langle \phi_1 \rangle = \left( \begin{array}{c} 0 \\ v_1/\sqrt{2} \end{array} \right), \quad \langle \phi_2 \rangle = \left( \begin{array}{c} 0 \\ v_2 e^{i\xi/\sqrt{2}} \end{array} \right), \quad (3)$$

with the standard parameterization $v_1 = v \cos \beta$, $v_2 = v \sin \beta$.

2.2. The Higgs basis

We use below the RPa basis with $v_2 = 0$ (the Higgs, or Georgi, basis [9]), in which the 2HDM potential can be written in the form [10] with the change of lowercase letters of (1) to capital:

$$V_{HB} = \frac{\Lambda_1}{2} (\Phi_1^\dagger \Phi_1 - v^2) + \frac{\Lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \Lambda_3 (\Phi_1^\dagger \Phi_1 - v^2) (\Phi_2^\dagger \Phi_2) + \Lambda_4 (\Phi_1^\dagger \Phi_2^\dagger) (\Phi_2^\dagger \Phi_1) + \left[ \frac{\Lambda_5}{2} (\Phi_1^\dagger \Phi_2^\dagger)^2 + \Lambda_6 (\Phi_1^\dagger \Phi_1 - v^2) (\Phi_2^\dagger \Phi_2) + \Lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right] + M_{\pm}^2 (\Phi_2^\dagger \Phi_2). \quad (4)$$

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3. Relative couplings

In the discussion below we use the relative values of couplings for each neutral Higgs boson \( h_a \):

\[
\chi_a^P = \frac{g_a^P}{g_{\text{SM}}} , \quad \chi_a^\pm = \frac{g(H^+H^-h_a)}{2M_Z^2/v} , \quad \chi_a^{H^+W^-} = \frac{g(H^+W^-h_a)}{M_W/v} ; \quad (a = 1, 2, 3) .
\] (5)

The quantities \( \chi_a^P \) (where \( P = V (W, Z) \), \( q = t, b, ..., \ell = \tau, ... \)) are the ratios of the couplings of \( h_a \) with the fundamental particles \( P \) to the corresponding couplings for the would-be SM Higgs boson with mass \( M_a \). The other couplings describe the interactions of \( h_a \) with the charged Higgs boson \( H^\pm \). Couplings \( \chi_a^V \) and \( \chi_a^\mp \) are real due to Hermiticity of Lagrangian, they are directly measurable. Couplings \( \chi_a^q, \chi_a^\ell \) and \( \chi_a^{H^+W^-} \) are generally complex. Both the real and imaginary parts of the Yukawa couplings \( \chi_a^q \) and \( \chi_a^\ell \) can be measured in principle using distributions of Higgs bosons decay products in \( h_a \to \bar{q}q, h_a \to \ell\ell \). Besides, we have [10]

\[
\chi_a^Z = \frac{g(Zh_a h_0)}{M_Z/v} = -\varepsilon_{abc} \chi_a^V .
\] (6)

There are useful sum rules among these couplings, namely

\[
(a) \sum_a (\chi_a^V)^2 = 1 , \quad (b) (\chi_a^V)^2 + |\chi_a^{H^+W^-}|^2 = 1 .
\] (7)

The absolute value of the coupling \( \chi_a^{H^+W^-} \) is fixed by the sum rule (7 b) and is well measurable. The unitarity of the rotation matrix describing transition from components of fields \( \phi_i \) to the physical Higgs fields \( h_a \) allows to obtain the following relations for couplings \( \chi_a^{H^+W^-} \) (the factor \( e^{i\rho} \) represents the rephasing freedom in the Higgs basis):

\[
\chi_1^{H^+W^-} = \chi_1^{H^-W^+*} = -e^{i\rho} \frac{\chi_1^V \chi_2^V - i\chi_3^V}{\sqrt{1 - (\chi_2^V)^2}} ,
\]

\[
\chi_2^{H^+W^-} = \chi_2^{H^-W^+*} = e^{i\rho} \frac{\chi_2^V}{\sqrt{1 - (\chi_1^V)^2}} ,
\]

\[
\chi_3^{H^+W^-} = \chi_3^{H^-W^+*} = -e^{i\rho} \frac{\chi_2^V \chi_3^V + i\chi_1^V}{\sqrt{1 - (\chi_2^V)^2}} .
\] (8)

Note, that here we discuss couplings which appear in the Lagrangian. Radiative corrections (RC) change these couplings; however in most cases these corrections are small and therefore the corresponding observables differ weakly from those presented in the Lagrangian. In this sense we treat the latter ones as being measurable. Therefore the identities (6)-(8) are valid with accuracy to RC.

When describing interactions of the Higgs bosons with the gauge bosons \( V = W, Z \), one should distinguish interactions of different Lorentz structure: the vectorial \( h_a V_\mu V^\mu \), tensor \( h_a V_\mu V_\nu \epsilon^{\mu\nu\alpha\beta} V_{\alpha\beta} \) and axial-tensor \( h_a V_\mu V_{\nu\alpha} \bar{V}^{\nu\beta} V^{\alpha\mu} \) interactions, with the coupling constants \( g_a^V \), \( g_a^{VT} \) and \( g_a^{VT} \), respectively (here \( V_\mu = \partial_\mu V_\nu - \partial_\nu V_\mu \)). These interactions can be separated in the experiment from each other by the study of angular correlations in the decays like \( h_a \to ZZ \to e^+e^-\mu^+\mu^- \) [11]. In the 2HDM tensor and axial-tensor interactions appear only due to RC (mainly from t-loops). In most cases their contributions to the observed decays rates of \( h_a \) are hardly observable. The axial-tensor interaction can imitate CP violation even if it is absent in the model.
4. A minimal complete set of observables in 2HDM

In Ref. [10] a minimal complete set of directly measurable quantities (observables) defining the 2HDM was found \((a = 1, 2, 3)\), namely:

\[
v.e.v. \text{ of Higgs field } v = 246 \text{ GeV}, \quad \text{masses of Higgs bosons } M_a, M_{\pm},\n\]

2 (out of 3) couplings \(\chi_a^V\), 3 couplings \(\chi_a^\pm\), quartic coupling \(g(H^+H^-H^+H^-)\). (9)

The parameters of the potential in the Higgs basis are expressed through these observables and free parameter \(\rho\) (it appears in \(\Lambda_{5,6,7}\) via couplings \(\chi_a^{H^+H^-}\) given in eq. (8)):

\[
\begin{align*}
A_1 &= \sum_a (\chi_a^V)^2 M_a^2/v^2; \quad A_5 = \sum_a (\chi_a^{H^+H^-})^2 M_a^2/v^2; \quad A_4 = (\sum_a M_a^2 - M_{\pm}^2)/v^2 - A_1; \\
A_3 &= 2 (M_{\pm}^2/v^2) \sum_a \chi_a^V \chi_a^\pm; \quad A_6 = \sum_a (\chi_a^V \chi_a^{H^+H^-}) M_a^2/v^2; \\
A_7 &= 2 (M_{\pm}^2/v^2) \sum_a \chi_a^{H^+H^-} \chi_a^\pm; \quad A_2 = 2g(H^+H^-H^+H^-).
\end{align*}
\] (10)

The parameters of the potential (1) with the values of \(\tan \beta\) and \(\xi\) defined in (3) are obtained from parameters (10) with the aid of transformation (2) in the following form:

\[
\left(\frac{\phi_1}{\phi_2}\right) = \tilde{F}_{HB} \left(\frac{\Phi_1}{\Phi_2}\right), \quad \tilde{F}_{HB} = \tilde{F}_{gen}(\theta = -\beta, \tau = \xi, -\rho).
\] (11)

5. Conditions for a CP conservation

In the Higgs models like 2HDM neutral scalar particles coincide with their antiparticles. In such models one can discuss the P-parity violation, but not C-parity. When in addition we consider fermions, the P-parity violation is transformed to the CP violation (see e.g. [6]).

5.1. Basic facts

The CP symmetry is conserved in the 2HDM and similar models containing spinless bosons if

- Each observable physical neutral spinless boson has definite P-parity \((12a)\)

- There are no P-violating interactions between these bosons. \((12b)\)

In the 2HDM we denote neutral spinless P-even bosons as \(h_1, h_2\) (they are called often as \(h\) and \(H\)) and P-odd boson as \(h_3\) (it is called often as \(A\)). The condition \((12b)\) means the absence of the interactions \((i, j, k = 1, 2)\)

\[
h_i h_j h_3, \quad h_3 h_3 h_3, \quad h_i h_j h_k h_3, \quad h_i h_3 h_3 h_3.
\] (13)

It is well known that in the 2HDM the CP conservation holds if

There exists the RPa basis in which:

(a) all parameters of potential are real; (b) relative phase (3) \(\xi = 0\). \((14)\)

It is worth mentioning that the condition \((14a)\) forbids the explicit CP violation while both conditions \((14a)\) and \((14b)\) together forbid the spontaneous CP violation.

The statement \((12)\) only describes CP conservation, but does not provide a criterion for CP conservation in the considered model. The description \((14)\) is RPa basis-dependent. Below we discuss both \(A\) and \(B\) set-ups of our problem, presented in the Introduction.
5.2. Method of the CP-odd basis-independent invariants
Many authors found the basis independent criterium for CP violation or conservation in terms of parameters of the Higgs potential; this corresponds to problem (A) stated in the Introduction. For this goal they constructed the RPb basis-invariant CP-odd combinations of parameters of the potential and as a condition for CP conservation they demanded vanishing of all these invariants. For 2HDM three such invariants $\text{Im} J_{1,2,3}$ were found in refs. [12, 13].

To solve the corresponding problem (B) the invariants [12, 13] for 2HDM were expressed via measurable quantities in refs. [14]. Note that in this approach there are four CP-odd invariants but one should check vanishing only of two of them (see e.g. [6]). Since the choice of these two invariants is not fixed from beginning, the presented set contains three conditions, instead of necessary two. In our opinion these equations are too complicated and their experimental verification, discussed in [12]-[15], requires very complex procedure.

5.3. A direct criterium for CP conservation
In this approach we start with a description of CP conservation (12) and use only observables. In the CP conserving 2HDM, all $h_a$ should have definite P-parity. In particular, one of them is P-odd, while two others are P-even (12a). Therefore, the necessary condition for a CP conservation is an existence of one neutral Higgs boson (we denote it $h_3$), which doesn’t couple to the CP-even states $VV$ and $H^+ H^-$. There exists an neutral Higgs boson $h_3$ for which
\[
\begin{align*}
\text{vectorial coupling } g_3^V &\equiv g(h_3VV) = 0 \\
g_3^\pm &\equiv g(h_3H^+H^-) = 0.
\end{align*}
\]

Now, one has to check condition (12b). In order to do this, we substitute Eqs. (15), (8) into (10), choosing $\rho = 0$. The choice $\rho = 0$ together with constraint $\chi_3^V = 0$ in (8) makes both $\chi_3^{H^+W^-}$ real while $\chi_3^{H^+W^-}$ is imaginary. Next, we insert these $\chi_3^{H^+W^-}$ into (10). The parameter $\Lambda_5$ contains real negative quantity $(\chi_3^{H^+W^-})^2$ while in $\Lambda_6$ and $\Lambda_7$ this imaginary $\chi_3^{H^+W^-}$ is multiplied by $\chi_3^V$ or $\chi_3^\pm$, which are zeros (15). Hence, with this choice all parameters of potential $\Lambda_a$ in the Higgs basis are real. Therefore, in view of the statement (14), the CP-symmetry of model is not violated. In particular, the CP violating interactions (13) do not appear. Therefore the conditions (15) are necessary and sufficient for establishing CP conservation in the boson sector of 2HDM (problem B).

The Eqs. (10), after substitution of Eqs. (15), (11), can be treated as a solution of problem (A).

It is useful to note that in view of identity (6), the first condition (15) can be written also in the form $g(Zh_1h_2) = 0$. In some cases, checking this form of condition may be more convenient. (Such form of criterium for CP non-violation was discussed in Ref. [18].)

6. Possibilities for a verification
The verification of the CP conservation requires observation of all scalars of the model. In the SM-like scenario realized in Nature this looks difficult (see e.g. [17]). Moreover, one should check if some measurable quantities are equal to zero. In any case, these measurements cannot be performed with a high accuracy. Nevertheless, one can not hope for a high accuracy in testing CP conservation in the 2HDM.

Earlier we proposed conditions for CP conservation, based only on condition (12a), without checking up of condition (12b), in the form $\prod\limits_a \chi_a^V = 0$, $\prod\limits_a \chi_a^\pm = 0$ [16, 17].
If in future the experiments show, within the definite experimental uncertainties, an agreement with criteria (15), the important problem arises for the further studies, namely whether one can expect that the more accurate measurements will show violation of CP in our 2HDM, i.e. violation of criteria (15), or the observed CP violation is in fact an effect beyond approximation given by 2HDM, and some new weaker interactions should be implemented in the description of Nature. (This situation is similar to that in atomic physics. Atomic interaction (QED) conserves P-parity. Parity non-conservation appears at the smallest distances due to next level weak interaction. It is observed as a small effect in rare atomic transitions.)

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