One- and Two-Neutron Halos in Effective Field Theory

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Work done in collaboration with
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Outline

- Neutron Halos
  - Overview, motivation
  - Some experimental results

- EFT For One-Neutron Halos
  - Analysis of experimental data on Carbon-19

- EFT For Two-Neutron Halos
  - Implications of a measurement of the Carbon-22 matter radius
  - Coulomb dissociation of Carbon-22
Signature

- Low separation energy for one or more neutrons, core tightly bound
- Large cross section for transfer and break-up reactions, large matter radius
- Enhanced charge radius and dipole moment

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Alan Stonebraker for APS Physics
Motivation

- Nuclear reactions of astrophysical significance
- Nuclear structure away from the line of stability
- “Universality” – connection to other systems with large scattering length (nucleons, cold atoms near Fesbach resonance...)

Signature

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- Large cross section for transfer and break-up reactions, large matter radius
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Matter radii of nuclei deduced by Glauber model calculations from reaction cross section data.
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Momentum distribution of $^{18}\text{C}$ from neutron removal of $^{19}\text{C}$.
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Momentum distribution of $^{18}$C from neutron removal of $^{19}$C.

$^{19}$C break-up on Pb. Curves are calculated using Woods-Saxon wavefunction at $S_n = 0.53$ MeV.
• Direct reaction. Eikonal or semiclassical approximation.

• Perturbation theory to first order. Higher orders small.  
  Typel and Baur (2001, 2008)

• Virtual photons $\rightarrow$ real photons 
  \[ \sigma = \sum_{\pi L} \int \frac{d\omega}{\omega} N_{\pi L}(\omega) \sigma_{\gamma \pi L}(\omega) \]

• Dipole excitation, e.g. higher multipoles smaller by factor of $10^5$ for $^{11}\text{Li}$.  
  Bertulani (2009)

\[ \frac{d\sigma}{dE} = \frac{16\pi^3}{9} \alpha N_{E_1}(B + E) \frac{dB(E_1)}{e^2 dE} \]
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Halo EFT

Bertulani, Hammer and van Kolck (2002)
Bedaque, Hammer and van Kolck (2003)

- Degrees of freedom: halo neutron and the core.

- Symmetries: invariance under Galilean transformation, translation, rotation...

- Exploit separation of scales: $\sqrt{mB} \sim M_\text{lo} << M_\text{hi} \sim R^{-1}$.

- Systematic expansion in $M_\text{lo}/M_\text{hi}$.

- Short distance physics (at scale $M_\text{hi}$ and beyond) of the core unresolved, but its impact on low energy observables taken care of by renormalization.
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$^{19}$C: $J^\pi = 1/2^+$, $B = 0.58$ MeV

$^{18}$C: $R = 2.7$ fm$^\dagger$, $J^\pi = 0^+$, $E^* = 1.62$ MeV

$M_{lo}/M_{hi} \sim 0.5$

NNDC, BNL
$^\dagger$Simple estimate based 1.2 $A^{1/3}$ law

1/3
\[ \mathcal{L} = \mathcal{N}^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) \mathcal{N} + c^\dagger \left( i\partial_0 + \frac{\nabla^2}{2M} \right) c \\
+ d^\dagger \left[ \eta \left( i\partial_0 + \frac{\nabla^2}{2(M+m)} \right) - \Delta \right] d - g \left[ d^\dagger \mathcal{N} c + c^\dagger \mathcal{N}^\dagger d \right] \]

Kaplan, Savage and Wise (1998); Gegelia (1998); van Kolck (1998); Birse, McGovern and Richardson (1998)
\[ \mathcal{L} = N^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) N + c^\dagger \left( i\partial_0 + \frac{\nabla^2}{2M} \right) c \\
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\[ T = -\frac{2\pi}{\mu} \frac{1}{-\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik} \]
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\[ T = -\frac{2\pi}{\mu} \frac{1}{-\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik} \]

\[ a = \left( \frac{2\pi}{\mu g^2} \Delta + \kappa \right)^{-1} \]

\[ r_0 = -\eta \frac{2\pi}{\mu^2 g^2} \]
\[ \mathcal{L} = N^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) N + c^\dagger \left( i\partial_0 + \frac{\nabla^2}{2M} \right) c \\
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Assume naturalness: shape parameter, \( P \), enters at N3LO. Stay at N2LO.
| Term | Order |
|------|-------|
| $\left(\frac{M_{lo}}{M_{hi}}\right)^{-1}$ | LO |
| $\left(\frac{M_{lo}}{M_{hi}}\right)^{2}$ | N3LO |
| $\left(\frac{M_{lo}}{M_{hi}}\right)^{4}$ | N5LO |

*cf. Beane and Savage (2001); Hammer and Phillips (2011); Rupak and Higa (2011); Rupak, Fernando and Vaghani (2012) for similar analysis and calculations with other nuclei*
Extracting Effective Range Parameters

\[
\frac{dB(E1)}{e^2 dE} = \frac{12}{\pi^2} \frac{\mu^3}{M^2} Z^2 \frac{\gamma_0}{1 - r_0 \gamma_0} \frac{p^3}{(\gamma_0^2 + p^2)^4},
\]

cf. Bertulani and Baur (1988) for LO result

\[
\frac{1}{a} + \frac{1}{2} r_0 \gamma_0^2 - \gamma_0 = 0; \quad ANC = \sqrt{\frac{2 \gamma_0}{1 - r_0 \gamma_0}}
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Extracting Effective Range Parameters

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\]

Data: Nakamura et al, RIKEN (1999, 2003); Calculation: Acharya and Phillips (2013)

\[a = (7.75 \pm 0.35(\text{stat.}) \pm 0.3(\text{EFT})) \text{ fm} ; \quad r_0 = (2.6^{+0.6}_{-0.9}(\text{stat.}) \pm 0.1(\text{EFT})) \text{ fm}\]

\[B = (575 \pm 55(\text{stat.}) \pm 20(\text{EFT})) \text{ keV}\]
Prediction: Momentum Distribution

- Width sensitive to $B$; ANC only affects height.
- Data with normalization unavailable for high $Z$ target. Nuclear break-up background too strong for low $Z$ ones.
- Uncertainty in absolute energy scale $\rightarrow$ also fit position $\rightarrow$ width is the only prediction.

Data: Bazin et al, NSCL (1998); Calculation: Acharya and Phillips (2013)
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At LO, dressed two-body propagators are renormalized by using two-body scattering lengths as input.

Three-body contact interaction enters at LO.

Bedaque, Hammer and van Kolck (1998)
\[ \Psi(\vec{p}, \vec{q}) = + 2 \times \]

\[ \mathcal{F}(k^2) = \int d^3p \int d^3q \, \Psi(\vec{p}, \vec{q}) \, \Psi(\vec{p} - \vec{k}, \vec{q}) = 1 - \frac{1}{6} k^2 \langle r^2 \rangle + \ldots \]

Canham and Hammer (2008)
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Canham and Hammer (2008)
The Point Core Limit

\[ mB \langle r_0^2 \rangle \equiv f \left( \frac{E_{nn}}{B}, \frac{E_{nc}}{B}; A \right); \quad B = S_{2n} \]
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Observation of a Large Reaction Cross Section in the Drip-Line Nucleus $^{22}$C

K. Tanaka,$^1$ T. Yamaguchi,$^2$ T. Suzuki,$^2$ T. Ohtsubo,$^3$ M. Fukuda,$^4$ D. Nishimura,$^4$ M. Takechi,$^{4,1}$ K. Ogata,$^5$ A. Ozawa,$^6$
### Observation of a Large Reaction Cross Section in the Drip-Line Nucleus $^{22}\text{C}$

K. Tanaka, T. Yamaguchi, T. Suzuki, T. Ohtsubo, M. Fukuda, D. Nishimura, M. Takechi, K. Ogata, A. Ozawa

|                     | $^{20}\text{C}$ | $^{21}\text{C}$ | $^{22}\text{C}$ |
|---------------------|-----------------|-----------------|-----------------|
| Spin and Parity     | $0^+$           | $1/2^+$         | $0^+$           |
| Binding/Virtual Energy | $S_n = 2.9 \text{ MeV}$ | $E_{nc} = \text{?}$ | $S_{2n} = 0.42(94) \text{ MeV}$ |
|                     | NNDC, BNL (2013) |                 | Horiuchi and Suzuki (2006) |
| RMS matter radius   | $2.97(5) \text{ fm}$ | $-$             | $5.4(9) \text{ fm}$ |
|                     | Ozawa et al (2001) |                 | Tanaka et al, RIKEN (2010) |
Spin and Parity

| 20C bound | 21C unbound | 22C bound |
|-----------|------------|----------|
| 0^+       | 1/2^+      | 0^+      |

Binding/Virtual Energy

|          | 20C | 21C | 22C |
|----------|-----|-----|-----|
| S_n = 2.9 MeV | NNDC, BNL (2013) | E_{nc} = ? | S_2n = 0.42(94) MeV | Horiuchi and Suzuki (2006) |
| S_2n = -0.14(46) MeV |  |  | Gaudefroy et al (2012) |
| RMS matter radius | 2.97(5) fm | - | 5.4(9) fm |
| Ozawa et al (2001) | | | Tanaka et al, RIKEN (2010) |

- $\sqrt{(mS_{2n}[^{22}\text{C}])} \sim M_{lo}$, $\sqrt{(mS_{n}[^{20}\text{C}])}$, $(\sqrt{<r^2[^{20}\text{C}]>})^{-1} \sim M_{hi}$
- $E_{nc}$ unknown $\rightarrow$ treat as free parameter; $\sqrt{(mE_{nc})}$ as $M_{lo}$.
- $B = S_{2n}[^{22}\text{C}]$ not well constrained by experiments $\rightarrow$ Treat as free parameter.
• 1-σ experimental error bar $\rightarrow B < 100$ keV

• Excited Efimov states not possible unless $E_{\text{nc}} < 1$ keV.

• $|a_{\text{nc}}| < 2.8$ fm. Mosby et al (2013)

cf. Hagen, Hagen, Platter and Hammer for study of Efimov states in Ca isotopic chain using Halo EFT, coupled cluster theory and interactions from Chiral EFT.
Coulomb dissociation of Carbon-22

cf. Ershov et al (2012) for a non-EFT calculation (hyperspherical harmonic model)
Hagen, Platter and Hammer (2013) for charge form factor calculation

\[
\hat{d} = 2eZrY_1^0(\hat{r})/(A + 2)
\]

\[
\mathcal{M}_{PWIA} = \langle pq; lm\lambda\mu | \hat{d} | \Psi_{in} \rangle
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Final State Interactions

- $|l - \lambda| \leq 1 \leq l + \lambda$ in the final state.

- $l = 1$ suppressed. But $l = 0, \lambda = 1$ enters at LO.

- Final state wavefunction has to be constructed with all S-wave two-body interactions included.
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\[
|\Phi\rangle \rightarrow |\Phi\rangle + \hat{G}_0 \sum_{i=n,c} \hat{t}_i (|\Phi\rangle + |F_i\rangle),
\]

\[
|F_i\rangle = \hat{G}_0 \sum_{j \neq i} \hat{t}_j (|\Phi\rangle + |F_j\rangle)
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$$|F_i\rangle = \hat{G}_0 \sum_{j \neq i} \hat{t}_j (|\Phi\rangle + |F_j\rangle)$$
We applied Halo EFT to study Coulomb dissociation of $^{19}$C and determined the $S_n$ and the ANC of the $^{18}$C – $n$ system with high accuracy. $S_n$ agrees with momentum distribution data; ANC remains to be tested.

1-σ experimental error on the matter radius of $^{22}$C puts an upper bound of about 100 keV on its $S_{2n}$.

Absence of low lying virtual states in $^{21}$C rules out Efimov states in $^{22}$C.

Forthcoming data on Coulomb dissociation of $^{22}$C is expected to provide better estimates of the $^{22}$C two-neutron separation energy and the $^{21}$C virtual state energy.
Backups
\[ H' = \int d^3 r_1 d^3 r_2 \frac{\rho_1^{ch}(r_1 - R_1)\rho_2^{ch}(r_2 - R_2)}{|r_1 - r_2|} = \frac{Z_1 Z_2 e^2}{R(t)} \]

\[
\frac{d\sigma_C}{dE_\gamma} (E_\gamma) = \frac{1}{E_\gamma} \sum_{\pi L} N_{\pi L} (E_\gamma) \sigma_{\gamma \pi L} (E_\gamma)
\]

\[
N_{E1}(\omega, R) = 2 \frac{Z_1^2 \alpha^2}{\pi \beta^2} \left( \xi K_0(\xi) K_1(\xi) - \frac{\beta^2}{2} \xi^2 \left( (K_1(\xi))^2 - (K_0(\xi))^2 \right) \right)
\]

\[
\frac{d\sigma}{dQ d^3 p/(2\pi)^3} = 24\pi^2 \frac{Z_1^2 \alpha^2}{\gamma^2 \beta^2} \omega^2 Z_{eff}^{(1)} \langle r \rangle_{01}^2
\]

\[
\sum_{M_1 M_2} i^{M_1 - M_2} \chi_{M_1}(Q) \chi_{M_2}^*(Q) G_{E1M_1} (1/\beta) G_{E1M_2}^*(1/\beta) Y_1^{M_1}(\hat{p}) Y_1^{M_2}*(\hat{p}),
\]

\[
B(E1) = \frac{3}{4\pi} \left( \frac{Ze}{A} \right)^2 \langle r_1^2 + r_2^2 + 2r_1 \cdot r_2 \rangle = \frac{3}{\pi} \left( \frac{Ze}{A} \right)^2 \langle r_{c,2n}^2 \rangle,
\]
The kernel of the Faddeev equations involves integrals of the form,
\[
\frac{1}{2} \int_{-1}^{1} dx \, P_n(x) \frac{1}{E - \frac{q^2}{a} - \frac{q'^2}{b} - qq'\frac{x}{c} + i\epsilon} = \frac{c}{qq'} \, Q_n \left( \frac{c}{qq'} \left[ E - \frac{q^2}{a} - \frac{q'^2}{b} \right] + i\epsilon \right) \\
= (-1)^{n+1} \frac{c}{qq'} \, Q_n \left( \frac{c}{qq'} \left[ -E + \frac{q^2}{a} + \frac{q'^2}{b} \right] - i\epsilon \right),
\]
where \( q \) is the external variable and \( q't \) is the integration variable.

For \( n = 0 \),
\[
Q_0(x \pm i\epsilon) = \begin{cases} 
\frac{1}{2} \log \frac{|x + 1|}{|x - 1|}, & |x| > 1 \\
\frac{1}{2} \log \frac{|x + 1|}{|x - 1|} \pm \frac{i\pi}{2}, & |x| < 1,
\end{cases}
\]
and for \( n = 1 \),
\[
Q_1(z) = \frac{1}{2} \int_{-1}^{1} dx \, \frac{x}{z - x} = -1 + \frac{z}{2} \int_{-1}^{1} dx \, \frac{1}{z - x} = -1 + zQ_0(z)
\]
\[
\Rightarrow Q_1(x \pm i\epsilon) = \begin{cases} 
-1 + \frac{x}{2} \log \frac{|x + 1|}{|x - 1|}, & |x| > 1 \\
-1 + \frac{x}{2} \log \frac{|x + 1|}{|x - 1|} \pm \frac{i\pi}{2} x, & |x| < 1.
\end{cases}
\]
\[ F_n(q;0010) = \sqrt{\pi} \int_{-1}^{1} d(\hat{q}, \hat{q}') P_1(\hat{q}, \hat{q}') G_0^n(\pi_{1}(q, K_n \hat{q}'), q; E) t_n(E; K_n) Y_1^0(\hat{K}_n) \]

\[ + \int_{0}^{\infty} \frac{dq' q'^2}{2\pi^2} \frac{1}{2} \int_{-1}^{1} d(\hat{q}, \hat{q}') P_1(\hat{q}, \hat{q}') G_0^n(\pi_{1}(q, q'), q; E) t_n(E; q') F_n(q'; 0010) \]

\[ + \sqrt{\pi} \int_{-1}^{1} d(\hat{q}, \hat{q}') P_1(\hat{q}, \hat{q}') G_0^n(\pi_{0}(q, K \hat{q}'), q; E) t_c(E; K) Y_1^0(\hat{K}) \]

\[ + \int_{0}^{\infty} \frac{dq' q'^2}{2\pi^2} \frac{1}{2} \int_{-1}^{1} d(\hat{q}, \hat{q}') P_1(\hat{q}, \hat{q}') G_0^n(\pi_{0}(q, q'), q; E) t_c(E; q') F_c(q'; 0010), \]

and

\[ F_c(q;0010) = 2\sqrt{\pi} \int_{-1}^{1} d(\hat{q}, \hat{q}') P_1(\hat{q}, \hat{q}') G_0^n(\pi_{2}(q, K_n \hat{q}'), q; E) t_n(E, K_n) Y_1^0(\hat{K}_n) \]

\[ + \int_{0}^{\infty} \frac{dq' q'^2}{2\pi^2} \int_{-1}^{1} d(\hat{q}, \hat{q}') P_1(\hat{q}, \hat{q}') G_0^n(\pi_{2}(q, q'), q; E) t_n(E; q') F_n(q'; 0010). \]