Rotating Super Black Hole as Spinning Particle

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We give a review of the works devoted to the treatment of the Kerr super black hole solution as a spinning particle. The real, complex and stringy structures of the Kerr and super-Kerr geometries are discussed, as well as the recent results on the regular matter source for the Kerr spinning particle.

It is shown that the source has to represent a rotating bag-like bubble having (A)dS interior and a smooth domain wall boundary. The given by Morris supersymmetric generalization of the U(I) x U'(I) field model ( which was used by Witten to describe cosmic superconducting strings ) is considered, and it is shown that this model can be adapted for description of superconducting bags having a long range external electromagnetic field and another gauge field confined inside the bag.

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1 Introduction

The Kerr rotating black hole solution displays some remarkable features indicating a relation to the structure of the spinning elementary particles. In particular, in the 1969 Carter [1] observed, that if three parameters of the Kerr-Newman metric are adopted to be \( (\hbar=c=1) \), \( e^2 \approx \frac{1}{137} \), \( m \approx 10^{-22} \), \( a \approx 10^{22} \), \( ma = 1/2 \), then one obtains a model for the four parameters of the electron: charge, mass, spin and magnetic moment, and the gyromagnetic ratio is automatically the same as that of the Dirac electron. Investigations along this line [2, 3, 4, 5, 7] allowed to find out stringy structures in the real and complex Kerr geometry and to put forward a conjecture on the baglike structure of the source of the Kerr-Newman solution. The earlier investigations [2, 15, 5] showed that this source represents a rigid rotator (a relativistic disk) built of an exotic matter with superconducting properties. Since 1992 black holes have paid attention of string theory. In 1992 the Kerr solution was generalized by Sen to low energy string theory [8], and it was shown [19] that near the Kerr singular ring the Kerr-Sen solution acquires a metric similar to the field around a heterotic string. The point of view has appeared that black holes can be treated as elementary particles [9]. On the other hand, a description of a spinning particle based only on the bosonic fields cannot be complete, and involving fermionic degrees of freedom is required. Therefore, the spinning particle must be based on a super-Kerr-Newman black hole solution [20] representing a natural combination of the Kerr spinning particle and superparticle model. Angular momentum \( L \) of spinning particles is very high \( |a| = L/m \geq m \), and the horizons of the Kerr metric disappear. There appears a naked ring-like singularity which has to be regularized being replaced by a smooth matter source. In this review we consider a source representing a rotating superconducting bag with a smooth domain wall boundary described by a supersymmetric version of the \( U(I) \times U'(I) \) field model [26]. In fact, this model of the Kerr-Newman source represents a generalization of the Witten superconducting string model [18] for the superconducting baglike sources [7].
2 Complex source of Kerr geometry and its stringy interpretation

The Kerr-Newman solution can be represented in the Kerr-Schild form

\[ g_{\mu\nu} = \eta_{\mu\nu} + 2he^3_{\mu}e^3_{\nu}, \] (1)

where \( \eta_{\mu\nu} \) is metric of an auxiliary Minkowski space \( \eta_{\mu\nu} = diag(-1, 1, 1, 1) \), and \( h \) is a scalar function. Vector field \( e^3 \) is null, \( e^3_{\mu}e^{3\mu} = 0 \), and tangent to PNC (principal null congruence) of the Kerr geometry. The Kerr PNC is twisting i.e. corresponding to a vortex of a null radiation. One of the main peculiarities of the Kerr geometry is singular ring representing a branch line of the Kerr space on the ‘positive’ \( (r > 0) \) and ‘negative’ \( (r < 0) \) sheets which are divided by the disk \( r = 0 \) spanned by this ring. The Kerr singular ring is exhibited as a pole of the function \( h(r, \theta) = \frac{rmr^2}{2 + a^2 \cos^2 \theta} \), where \( r \) and \( \theta \) are the oblate spheroidal coordinates. The Kerr PNC is in-going on the ‘negative’ sheet of space, it crosses the disk \( r = 0 \) and turns into out-going one on the ‘positive’ sheet. The simplest solution possessing the Kerr singular ring was obtained by Appel in 1887 (!) \[10\]. It can be considered as a Newton or a Coulomb analogue to the Kerr solution. On the real space-time the singular ring arises in the Coulomb solution \( f = e/\tilde{r} \), where \( \tilde{r} = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} \), when the point-like source is shifted to a complex point of space \( (x_0, y_0, z_0) \rightarrow (0, 0, ia) \). Radial distance \( \tilde{r} \) is complex in this case and can be expressed in the oblate spheroidal coordinates \( r \) and \( \theta \) as \( \tilde{r} = r + ia \cos \theta \). The source of Kerr-Newman solution, like the Appel solution, can be considered from complex point of view as a ”particle” propagating along a complex world-line \[11, 14\] parametrized by complex time.

The objects described by the complex world-lines occupy an intermediate position between particle and string. Like the string they form the two-dimensional surfaces or the world-sheets in the space-time. It was shown that the complex Kerr source may be considered as a complex hyperbolic string which requires an orbifold-like structure of the world-sheet. In many respects this source is similar to the ‘mysterious’ \( N = 2 \) string of superstring

\[ ^2 \text{Besides, the Kerr PNC is geodesic and shear free, it represents a bundle of twistors and can be described by the Kerr theorem [12-14].} \]
theory shedding a light on the puzzle of its physical interpretation. As we have already mentioned, there is one more stringy structure in the Kerr geometry connected with the Kerr singular ring. In fact both these stringy structures are different exhibitions of some membrane-like source. This source has a complex interpretation alongside with a real image in the form of a rotating bubble which will be discussed further.

The Kerr PNC may be obtained from the complex source by a retarded-time construction. The rays of PNC are the tracks of null planes of the complex light cones emanated from the complex world line \( x_0^{\mu}(\tau) \). The complex light cone with the vertex at some point \( x_0 \) of the complex world line \( x_0^\mu(\tau) \): \((x_\mu - x_{0\mu})(x^\mu - x_{0}^\mu) = 0\), can be split into two families of null planes: "left" planes \( x_L = x_0^0(\tau) + \alpha e^1 + \beta e^3 \) spanned by null vectors \( e^1 \) and \( e^3 \), and "right" planes \( x_R = x_0^0(\tau) + \alpha e^2 + \beta e^3 \), spanned by null vectors \( e^2 \) and \( e^3 \). The Kerr PNC arises as the real slice of the family of the "left" null planes of the complex light cones which vertices lie on the straight complex world line \( x_0(\tau) \).

Only the cones lying on the strip \(|\text{Im}\tau| \leq |a|\) have a real slice. Therefore, the ends of the resulting complex string are open. To satisfy the complex boundary conditions, an orbifold-like structure of the worldsheet must be introduced \([11, 14]\), which is closely connected with the above mentioned Kerr’s twosheetedness.

### 3 Super-Kerr-Newman geometry

A supergeneralization of the Kerr-Newman solution can be obtained as a natural combination of the Kerr spinning particle and superparticle \([20]\). In fact, the complex structure of the Kerr geometry suggests the way of its supergeneralization.

Note, that any exact solution of the Einstein gravity is indeed a trivial solution of supergravity field equations. The supergauge freedom allows one to turn any gravity solution into a form containing spin-3/2 field \( \psi_\mu \) satisfying the supergravity field equations. However, since this spin-3/2 field can be gauged away by the reverse transformation, such supersolutions have

\[^3\text{The Kerr’s tetrad null vector-forms are: } e^1 = d\zeta - Y dv, \quad e^2 = d\bar{\zeta} - \bar{Y} dv, \quad e^3 = du + \bar{Y} d\zeta + Y d\bar{\zeta} - YY dv, \quad e^4 = dv + he^3, \text{ where the Cartesian null coordinates } u, v, \zeta, \bar{\zeta} \text{ are used.}\]
to be considered as *trivial*. The hint how to avoid this triviality problem follows from the complex structure of the Kerr geometry. In fact, from the complex point of view the Schwarzschild and Kerr geometries are equivalent and connected by a *trivial* complex shift.

The *non-trivial* twisting structure of the Kerr geometry arises as a result of the complex *shift of the real slice* concerning the center of the solution [13, 11]. Similarly, it is possible to turn a *trivial* super black hole solution into a *non-trivial*. The *trivial supershift* can be represented as a replacement of the complex world line by a superworldline \( X^\mu_0(\tau) = x^\mu_0(\tau) - i\theta\sigma^\mu\bar{\zeta} + i\zeta\sigma^\mu\theta \), parametrized by Grassmann coordinates \( \zeta, \bar{\zeta} \), or as a corresponding coordinate replacement in the Kerr solution

\[
x^\mu = x^\mu + i\theta\sigma^\mu\bar{\zeta} - i\zeta\sigma^\mu\theta; \quad \theta' = \theta + \zeta, \quad \bar{\theta}' = \bar{\theta} + \bar{\zeta},
\]

Assuming that coordinates \( x^\mu \) before the supershift were the usual c-number coordinates one sees that coordinates acquire nilpotent Grassmann contributions after supertranslations. Therefore, there appears a natural splitting of the space-time coordinates on the c-number ‘body’-part and a nilpotent part - the so called ‘soul’. The ‘body’ subspace of superspace, or B-slice, is a submanifold where the nilpotent part is equal to zero, and it is a natural analogue to the real slice of the complex case.

Reproducing the real slice procedure of the Kerr geometry in superspace one has to use the replacements:

- complex world line \( \to \) superworldline,
- complex light cone \( \to \) superlightcone,
- real slice \( \to \) body slice.

Performing the body-slice procedure to superlightcone constraints

\[
s^2 = [x^\mu - X^\mu_0(\tau)][x^\mu - X^\mu_0(\tau)] = 0,
\]

one selects the body and nilpotent parts of this equation and obtains three equations. The first one is the discussed above real slice condition of the complex Kerr geometry claiming that complex light cones can reach the real slice. The nilpotent part of (2) yields two B-slice conditions

\[
[x^\mu - x^\mu_0(\tau)](\theta\sigma_\mu\bar{\zeta} - \zeta\sigma_\mu\bar{\theta}) = 0; \quad (\theta\sigma\bar{\zeta} - \zeta\sigma\bar{\theta})^2 = 0.
\]
These equations can be resolved by representing the complex light cone equation via the commuting two-component spinors $\Psi$ and $\tilde{\Psi}$:

$$x_\mu = x_{0\mu} + \Psi \sigma_\mu \tilde{\Psi}.$$ 

"Right" (or "left") null planes of the complex light cone can be obtained keeping $\Psi$ constant and varying $\tilde{\Psi}$ (or keeping $\tilde{\Psi}$ constant and varying $\Psi$.) As a result we obtain the equations $\bar{\Psi} \bar{\theta} = 0$, $\bar{\Psi} \bar{\zeta} = 0$, which in turn are conditions of proportionality of the commuting spinors $\bar{\Psi}(x)$ determining the PNC of the Kerr geometry and anticommuting spinors $\bar{\theta}$ and $\bar{\zeta}$, these conditions providing the left null superplanes of the supercones to reach B-slice. It also leads to $\bar{\theta} \bar{\theta} = \bar{\zeta} \bar{\zeta} = 0$, and equation (5) is satisfied automatically.

Thus, as a consequence of the B-slice and superlightcone constraints we obtain a non-linear submanifold of superspace $\theta = \theta(x)$, $\bar{\theta} = \bar{\theta}(x)$. The original four-dimensional supersymmetry is broken, and the initial supergauge freedom which allowed to turn the super geometry into trivial one is lost. Nevertheless, there is a residual supersymmetry based on free Grassmann parameters $\theta^1$, $\bar{\theta}^1$.

The above B-slice constraints yield in fact the non-linear realization of broken supersymmetry introduced by Volkov and Akulov [23, 24] and considered in N=1 supergravity by Deser and Zumino [21]. It is assumed that this construction is similar to the Higgs mechanism of the usual gauge theories and $\zeta^\alpha(x)$, $\bar{\zeta}^{\dot{\alpha}}(x)$ represent Goldstone fermion which can be eaten by appropriate local supertransformation $\epsilon(x)$ with a corresponding redefinition of the tetrad and spin-3/2 field. Complex character of supertranslations in the Kerr case demands to use in this scheme the N=2 supergravity [22]. We omit here details referring to [24] and mention only that in the resulting exact solution the torsion and Grassmann contributions to tetrad cancel, and metric takes the exact Kerr-Newman form. However there are the extra wave fermionic fields on the bosonic Kerr-Newman background propagating along the Kerr PNC and concentrating near the Kerr singularity (traveling waves). Solution contains also an extra axial singularity which is coupled topologically with singular ring threading it.
4 Baglike source of the Kerr-Newman solution

The above consideration of super-Kerr-Newman solution is based on the massless fields providing description of the rotating super-black-hole. It could be the end of story since the source of a rotating black hole is hidden behind the horizons.

However, the value of angular momentum for spinning particles is very high regarding the mass parameter and the horizons disappear uncovering the Kerr singular ring. To get a regularized solution the massless fields of the black hole solution have to get a mass in the core region forming a matter source removing the Kerr singularity and twosheetedness of the Kerr space.

Obtaining a regularized Kerr source represents an old problem. In the first disk-like model given by Israel \[2\] a truncation of the negative sheet was used. As a result there appeared a source distribution on the surface of the disk \( r = 0 \). Analyzing the resulting stress-energy tensor Hamity showed \[15\] that this disk has to be in a rigid relativistic rotation and built of an exotic matter having zero energy density and negative pressure. In the development of this model given by López \[5\] the truncation is placed at the coordinate surface \( r = r_e = \frac{c^2}{2m} \) (where \( h = 0 \)), and the region \( r < r_e \) is replaced by Minkowski space. As a result the source takes the form of the highly oblate and infinitely thin elliptic shell of the Compton radius \( a = \frac{1}{2m} \) and of the thickness of the classical Dirac electron radius \( r_e \). For small angular momentum the source takes the form of the Dirac electron model, a charged sphere of the classical size \( r_e \). The fields out of the shell have the exact Kerr-Newman form. Interior of the shell is flat. The shell is charged and rotating, and built of a superconducting matter. In corotating space one sees that matter has a negative pressure and zero energy density.

The López source represents a bubble with an infinitely thin domain wall boundary. In the paper \[7\] an attempt was undertaken to get the source of the Kerr-Newman solution with a smooth matter distribution. \[4\] Retaining the metric in the Kerr-Schild form \( (1) \) and the form (and main properties) of the Kerr PNC, it was assumed that function \( h(r, \theta) \) takes a more general form \( h = \)

\[4\] This problem is actual for black hole physics, too. See for example \[25\] and references therein.

\[5\] First attempts in this direction were undertaken in the papers \[27\], \[29\].
where the function $f(r)$ is continuous and takes the usual Kerr-Newman form $f_{KN}(r) = mr - e^2/2$ in the external region. In the same time, in a neighborhood of the Kerr disk $r \leq r_0$ (the core region) including the Kerr singularity, the function $f(r)$ has to satisfy some conditions of regularity to provide finiteness of the metric and the stress-energy tensor of source, which is determined by the Einstein equations for this metric.

It was shown that this regularity is achieved for the function $f(r) \sim r^n$ with $n \geq 4$. In the case $n = 4$, $f(r) = f_0(r) = \alpha r^4$, (in the nonrotational case $a = 0$) space-time has a constant curvature in the core and generated by a homogenous matter distribution with energy density $\rho = \frac{1}{8\pi}6\alpha$. Therefore, assuming that matter in the core has a homogenous distribution one can estimate the boundary of the core region $r_0$ as a point of intersection of $f_0(r)$ and $f_{KN}(r)$. Regularity of the stress-tensor demands continuity of the function $f(r)$ up to first derivative, therefore, the resulting smooth function $f(r)$ must be interpolating between functions $f_0(r)$ and $f_{KN}(r)$ near the boundary of the core $r \approx r_0$.

Let us now mention that general metric (1) can be expressed via orthonormal tetrad as follows

$$g_{\mu\nu} = m_\mu m_\nu + n_\mu n_\nu + l_\mu l_\nu - u_\mu u_\nu,$$

and the corresponding stress-energy tensor of the source (following from the Einstein equations) may be represented in the form

$$T_{\mu\nu}^{(af)} = (8\pi)^{-1}[(D + 2G)g_{\mu\nu} - (D + 4G)(l_\mu l_\nu - u_\mu u_\nu)],$$

where $u_\mu$ is the unit time-like four-vector, $l_\mu$ is the unit vector in radial direction, and $n_\mu, m_\mu$ are two more space-like vectors. Here

$$D = -f''/(r^2 + a^2\cos^2 \theta),$$

$$G = (f' r - f)/(r^2 + a^2\cos^2 \theta)^2,$$

and the Boyer-Lindquist coordinates $t, r, \theta, \phi$ are used. The expressions (4), (8), (9) show that the source represents a smooth distribution of rotating confocal ellipsoidal layers $r = \text{const}$. Like to the results for singular (infinitely thin) shell-like source [15, 1], the stress-energy tensor can be diagonalized in a comoving coordinate system showing that the source represents a relativistic rotating disk. However, in
this case, the disk is separated into ellipsoidal layers each of which rotates rigidly with its own angular velocity \( \omega(r) = \frac{a}{(a^2 + r^2)} \). In the comoving coordinate system the tensor \( T_{\mu\nu} \) takes the form

\[
T_{\mu\nu} = \frac{1}{8\pi} \begin{pmatrix}
2G & 0 & 0 & 0 \\
0 & -2G & 0 & 0 \\
0 & 0 & 2G + D & 0 \\
0 & 0 & 0 & 2G + D
\end{pmatrix},
\]

(10)

that corresponds to energy density \( \rho = \frac{1}{8\pi} 2G \), radial pressure \( p_{\text{rad}} = -\frac{1}{8\pi} 2G \), and tangential pressure \( p_{\text{tan}} = \frac{1}{8\pi} (D + 2G) \).

Setting \( a = 0 \) for the non-rotating case, we obtain \( \Sigma = r^2 \), the surfaces \( r = \text{const.} \) are spheres and we have spherical symmetry for all the above relations. The region described by \( f(r) = f_0(r) \) is the region of constant value of the scalar curvature invariant \( R = 2D = -2f''_0(r^2) = -24\alpha \), and of a constant value of energy density. If we assume that the region of a constant curvature is closely extended to the boundary of source \( r_0 \) which is determined as a root of the equation

\[
f_0(r_0) = f_{KN}(r_0),
\]

(11)

then, smoothness of the \( f(r) \) in a small neighborhood of \( r_0 \), say \( |r - r_0| < \delta \), implies a smooth interpolation for the derivative of the function \( f(r) \) between \( f'_0(r)|_{r=r_0-\delta} \) and \( f'_{KN}(r)|_{r=r_0+\delta} \). Such a smooth interpolation on a small distance \( \delta \) shall lead to a shock-like increase of the second derivative \( f''(r) \) by \( r \approx r_0 \).

In charged case for \( \alpha \leq 0 \) (AdS internal geometry of core) there exists only one positive root \( r_0 \), and second derivative of the smooth function \( f''(r) \) is positive near this point. Therefore, there appears an extra tangential stress near \( r_0 \) caused by the term \( D = -f''(r)/(r^2 + a^2 \cos^2 \theta)|_{r=r_0} \) in the expression (11). It can be interpreted as the appearance of an effective shell (or a domain wall) confining the charged ball-like source with a geometry of a constant curvature inside the ball. The case \( \alpha = 0 \) represents the bubble with a flat interior which has in the limit \( \delta \to 0 \) an infinitely thin shell. It corresponds to the López model.

\[\text{Similar interpretation was also given in [27], however, the expression for stress-energy does not contain very important D term there.}\]
The point \( r_e = \frac{e^2}{2m} \) corresponding to a “classical size” of electron is a peculiar point as a root of the equation \( f_{KN}(r) = 0 \). It should be noted that by \( \alpha = 0 \) the equation (11) yields the root \( r_0 = r_e \). For \( \alpha < 0 \) position of the roots is \( r_0 < r_e \), and for \( \alpha > 0 \) one obtains \( r_0 > r_e \). Therefore, there appear four parameters characterizing the function \( f(r) \) and the corresponding bag-like core: parameter \( \alpha \) characterizing cosmological constant inside the bag \( \Lambda_{in} = 6\alpha \), two peculiar points \( r_0 \) and \( r_e \) characterizing the size of the bag and parameter \( \delta \) characterizing the smoothness of the function \( f(r) \) or the thickness of the domain wall at the boundary of core.

The internal geometry of the ball is de Sitter one for \( \alpha > 0 \), anti de Sitter one for \( \alpha < 0 \) and flat one for \( \alpha = 0 \).

Let us consider peculiarities of the rotating Kerr source. In this case the surfaces \( r = \text{const.} \) are ellipsoids described by the equation \( \frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} = 1 \). Energy density inside the core will be constant only in the equatorial plane \( \cos \theta = 0 \). Therefore, the Kerr singularity is regularized and the curvature is constant in string-like region \( r < r_0 \) and \( \theta = \pi/2 \) near the former Kerr singular ring. The ratio \( \frac{\text{stress}|_{\theta=0}}{\text{stress}|_{\theta=\pi/2}} < (r_e/a)^4 = e^8 < 10^{-8} \) shows a strong increase of the stress near the string-like boundary of the disk.

### 4.1 Field model for the bag: From superconducting strings to superconducting bags

The known models of the bags and cosmic bubbles with smooth domain wall boundaries are based on the Higgs scalar field \( \phi \) with a Lagrange density of the form \( L = -\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{\lambda^2}{8} (\phi^2 - \eta^2)^2 \) leading to the kink planar solution (the wall is placed in \( xy \)-plane at \( z = 0 \))

\[
\phi(z) = \eta \tanh(z/\delta),
\]

where \( \delta = 2\frac{\eta}{\lambda} \) is the wall thickness. The kink solution describes two topologically distinct vacua \( <\phi> = \pm \eta \) separated by the domain wall.

The stress–energy tensor of the domain wall is

\[
T^\nu_\mu = \frac{\lambda^2 \eta^4}{4} \cosh^{-4}(z/\delta) \text{diag}(1, 1, 1, 0),
\]

indicating a surface stress within the plane of the wall which is equal to the energy density. When applied to the spherical bags or cosmic bubbles [30, 31],

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the thin wall approximation is usually assumed $\delta \ll r_0$, and a spherical domain wall separates a false vacuum inside the ball $(r < r_0) < \phi >_{in} = -\eta$ from a true outer vacuum $< \phi >_{out} = \eta$.

In the gauge string models, the Abelian Higgs field provides confinement of the magnetic vortex lines in superconductor. Similarly, in the models of superconducting bags, the gauge Yang-Mills or quark fields are confined in a bubble (or cavity) in superconducting QCD-vacuum.

A direct application of the Higgs model for modelling superconducting properties of the Kerr source is impossible since the Kerr source has to contain the external long range Kerr-Newman electromagnetic field, while in the models of strings and bags the situation is quite opposite: vacuum is superconducting in external region and electromagnetic field acquires a mass there from Higgs field turning into a short range field. An exclusion represents the $U(I) \times \tilde{U}(I)$ cosmic string model given by Vilenkin-Shellard and Witten [17, 18] which represents a doubling of the usual Abelian Higgs model. The model contains two sectors, say $A$ and $B$, with two Higgs fields $\phi_A$ and $\phi_B$, and two gauge fields $A_\mu$ and $B_\mu$ yielding two sorts of superconductivity $A$ and $B$. It can be adapted to the bag-like source in such a manner that the gauge field $A_\mu$ of the $A$ sector has to describe a long-range electromagnetic field in outer region of the bag while the chiral scalar field of this sector $\phi_A$ has to form a superconducting core inside the bag which must be unpenetrable for $A_\mu$ field.

The sector $B$ of the model has to describe the opposite situation. The chiral field $\phi_B$ must lead to a $B$-superconductivity in outer region confining the gauge field $B_\mu$ inside the bag.

The corresponding Lagrangian of the Witten $U(I) \times \tilde{U}(I)$ field model is given by [18]

$$L = -(D^\mu \phi_A)(\overline{D_\mu \phi_A}) - (\overline{D^\mu \phi_B})(\overline{D_\mu \phi_B}) - \frac{1}{4} F_{A\mu\nu} F^{A\mu\nu} - \frac{1}{4} F_{B\mu\nu} F^{B\mu\nu} - V,$$  \hspace{1cm} (14)

where $F_{A\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $F_{B\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ are field stress tensors, and the potential has the form

$$V = \lambda (\phi_B \phi_B - \eta^2)^2 + f (\phi_B \phi_B - \eta^2) \tilde{\phi}_A \phi_A + m^2 \tilde{\phi}_A \phi_A + \mu (\tilde{\phi}_A \phi_A)^2.$$  \hspace{1cm} (15)

Two Abelian gauge fields $A_\mu$ and $B_\mu$ interact separately with two complex scalar fields $\phi_B$ and $\phi_A$ so that the covariant derivative $D_\mu \phi_A = (\partial + i e A_\mu) \phi_A$
is associated with $A$ sector, and covariant derivative $\tilde{D}_\mu \phi_B = (\partial + igB_\mu)\phi_B$ is associated with $B$ sector. The model fully retains the properties of the usual bag models which are described by $B$ sector providing confinement of $B_\mu$ gauge field inside bag, and it acquires the long range electromagnetic field $A_\mu$ in the outer-to-the-bag region described by sector $A$. The $A$ and $B$ sectors are almost independent interacting only through the potential term for scalar fields. This interaction has to provide synchronized phase transitions from superconducting $B$-phase inside the bag to superconducting $A$-phase in the outer region. The synchronization of this transition occurs explicitly in a supersymmetric version of this model given by Morris [26].

### 4.2 Supersymmetric Morris model

In Morris model, the main part of Lagrangian of the bosonic sector is similar to the Witten field model. However, model has to contain an extra scalar field $Z$ providing synchronization of the phase transitions in $A$ and $B$ sectors.\

The effective Lagrangian of the Morris model has the form

$$L = -2(D^\mu \phi)(D_\mu \phi) - 2(\tilde{D}^\mu \sigma)(\tilde{D}_\mu \sigma) - \partial^\mu Z \partial_\mu \bar{Z} \right. \\
- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} F_B^{\mu\nu} F_{B\mu\nu} - V(\sigma, \phi, Z),$$ (16)

where the potential $V$ is determined through the superpotential $W$ as

$$V = \sum_{i=1}^{5} |W_i|^2 = 2|\partial W/\partial \phi|^2 + 2|\partial W/\partial \sigma|^2 + |\partial W/\partial Z|^2. \right.$$ (17)

The following superpotential, yielding the gauge invariance and renormalizability of the model, was suggested:

$$W = \lambda Z(\sigma \bar{\sigma} - \eta^2) + (cZ + m)\phi \bar{\phi}, \right.$$ (18)

where the parameters $\lambda$, $c$, $m$, and $\eta$ are real positive quantities.

The resulting scalar potential $V$ is then given by

$$V = \lambda^2(\bar{\sigma} \sigma - \eta^2)^2 + 2\lambda c(\bar{\sigma} \sigma - \eta^2)\phi \bar{\phi} + c^2(\bar{\phi} \phi)^2 + \right. \\
2\lambda^2 \bar{Z} Z \sigma \bar{\sigma} + (c\bar{Z} + m)(cZ + m)\phi \bar{\phi}. \right.$$ (19)

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7In fact the Morris model contains five complex chiral fields $\phi_i = \{Z, \phi_-, \phi_+, \sigma_, \sigma_+\}$. However, the following identification of the fields is assumed $\phi = \phi_+$; $\tilde{\phi} = \phi_-$ and $\sigma = \sigma_+; \bar{\sigma} = \sigma_-$. In previous notations $\phi \sim \phi_A$ and $\sigma \sim \phi_B$.

8Superpotential is holomorphic function of $\{Z, \phi, \bar{\phi}, \sigma, \bar{\sigma}\}$. 12
4.2.1 Supersymmetric vacua

From (17) one sees that the supersymmetric vacuum states, corresponding to the lowest value of the potential, are determined by the conditions

\[ F_\sigma = -\partial \bar{W} / \partial \bar{\sigma} = 0; \]  
\[ F_\phi = -\partial \bar{W} / \partial \bar{\phi} = 0; \]  
\[ F_Z = -\partial \bar{W} / \partial \bar{Z} = 0, \]

and yield \( V = 0 \). These equations lead to two supersymmetric vacuum states:

I) \( Z = 0; \quad \phi = 0; \quad |\sigma| = \eta; \quad W = 0; \)

and

II) \( Z = -m/c; \quad \sigma = 0; \quad |\phi| = \eta \sqrt{\lambda/c}; \quad W = \lambda m \eta^2 / c. \)

We shall take the state I for external region of the bag, and the state II as a state inside the bag.

The treatment of the gauge field \( A_\mu \) and \( B_\mu \) in \( B \) is similar in many respects because of the symmetry between \( A \) and \( B \) sectors allowing one to consider the state \( \Sigma = \eta \) in outer region as superconducting one in respect to the gauge field \( B_\mu \). Field \( B_\mu \) acquires the mass \( m_B = g\eta \) in outer region, and the \( \tilde{U}(I) \) gauge symmetry is broken, which provides confinement of the \( B_\mu \) field inside the bag. The bag can also be filled by quantum excitations of fermionic, or non Abelian fields. The interior space of the Kerr bag is regularized in this model since the Kerr singularity and twofoldedness are suppressed by function \( f = f_0(r) \). However, a strong increase of the fields near the former Kerr singularity can be retained leading to the appearance of traveling waves along the boundary of the disk.

4.3 Supersymmetric bubble based on the Morris field model

It is shown in [7] that in the planar thin wall approximation, and by neglecting the gauge fields there is a supersymmetric BPS-saturated domain wall solution interpolating between supersymmetric vacua I) and II). This
domain wall displays the usual structure of stress-energy tensor with a tangential stress. The non-zero components of the stress-energy tensor take the form
\[
T_{00} = -T_{xx} = -T_{yy} = \frac{1}{2} \left[ \delta_{ij} (\Phi^i_z)(\Phi^j_z) + V \right]; \quad (25)
\]
\[
T_{zz} = \frac{1}{2} \left[ \delta_{ij} (\Phi^i_z)(\Phi^j_z) - V \right], \quad (26)
\]
where \( \Phi_i = \{ Z, \phi_-, \phi_+, \sigma_-, \sigma_+ \} \). One can estimate the mass and energy of a bubble formed by such a domain wall in global supersymmetry setting vacuum I) as external one and vacuum II) as an internal vacuum. Using the Tolman relation
\[
M = \int dx^3 \sqrt{-g} (-T_{00} + T_{11} + T_{22} + T_{33}),
\]
replacing coordinate \( z \) on radial coordinate \( r \), and integrating over sphere one obtains
\[
M_{\text{bubble}} = -4\pi \int V(r) r^2 dr = -4\pi \int (\Phi^i_r)^2 r^2 dr. \quad (27)
\]
The resulting effective mass is negative, which is caused by gravitational contribution of the tangential stress. The repulsive gravitational field was obtained in many singular and smooth models of domain walls [35, 28, 33, 34]. One should note, that similar gravitational contribution to the mass caused by interior of the bag will be \( M_{\text{gr.int}} = \int Dr^2 dr = -\frac{2}{3} \Lambda r_0^3 \). It depends on the sign of curvature inside the bag and will be negative in de Sitter case and positive in AdS one.

The total energy of a uncharged bubble forming from the supersymmetric BPS saturated domain wall is
\[
E_{0\text{bubble}} = E_{\text{wall}} = 4\pi \int_0^\infty \rho r^2 dr \approx 4\pi r_0^2 \epsilon_{\text{min}}, \quad (28)
\]
where \( r_0 \) is radius of the bubble, and
\[
\epsilon_{\text{min}} = W(0) - W(\infty) = \lambda m^2/c.
\]
Corresponding total mass following from the Tolman relation will be negative
\[
M_{0\text{bubble}} = -E_{\text{wall}} \approx -4\pi r_0^2 \epsilon_{\text{min}}. \quad (29)
\]
It is the known fact showing that the uncharged bubbles are unstable and form the time-dependent states [33, 34].
For charged bubbles there are extra positive terms: contribution caused by the energy and mass of the external electromagnetic field

\[ E_{e.m.} = M_{e.m.} = \frac{e^2}{2r_0}, \quad (30) \]

and contribution to mass caused by gravitational field of the external electromagnetic field (determined by Tolman relation for the external e.m. field)

\[ M_{\text{grav.e.m.}} = E_{e.m.} = \frac{e^2}{2r_0}. \quad (31) \]

As a result the total energy for charged bubble is

\[ E_{\text{tot.bubble}} = E_{\text{wall}} + E_{e.m.} = 4\pi r_0^2 \epsilon_{\text{min}} + \frac{e^2}{2r_0}, \quad (32) \]

and the total mass will be

\[ M_{\text{tot.bubble}} = M_0\text{bubble} + M_{e.m.} + M_{\text{grav.e.m.}} = -E_{\text{wall}} + 2E_{e.m.} = -4\pi r_0^2 \epsilon_{\text{min}} + \frac{e^2}{r_0}. \quad (33) \]

Minimum of the total energy is achieved by

\[ r_0 = \left( \frac{e^2}{16\pi \epsilon_{\text{min}}} \right)^{1/3}, \quad (34) \]

which yields the following expressions for total mass and energy of the stationary state

\[ M_{\text{tot}}^* = E_{\text{tot}}^* = \frac{3e^2}{4r_0}. \quad (35) \]

One sees that the resulting total mass of charged bubble is positive, however, due to negative contribution of \( M_0\text{bubble} \) it can be lower than BPS energy bound of the domain wall forming this bubble. This remarkable property of the bubble models (‘ultra-extreme’ states for the Type I domain walls in [33]) allows one to overcome BPS bound [36] and opens the way to get the ratio \( m^2 \ll e^2 \) which is necessary for particle-like models.
4.4 Baglike source in supergravity

In supergravity the scalar potential has a more complicated form \[24, 33, 34, 32\]
\[ V_{sg} = e^{k^2 K}(K^{ij}D_i W D_j WW - 3k^2 W W), \quad (36) \]
where $K$ is Kähler potential $K^{ij} = \frac{\partial^2 K}{\partial \Phi_i \partial \Phi_j}$, and $k^2 = 8\pi G_N$, $G_N$ is the Newton constant. In the small $kW$ limit, this expression turns into potential of global susy. In this approximation, the above treatment of the charged domain wall bubble will be valid in supergravity. The preserving supersymmetry vacuum state has to satisfy the condition $D_i W \equiv W_i + k^2 K_i W = 0$. This condition is satisfied for the internal vacuum state II) only in the limit $k^2 \to 0$ since $W = \lambda m^2/c$ inside the bag, and $D_i W \approx k^2 K_i W$ there. In the order $k^2$ the vacuum state II) does not preserve supersymmetry. There appears also an extra contribution to stress-energy tensor having the leading term
\[ T_{\mu\nu} = 3(k^2 / 8\pi)e^{k^2 K}|W|^2 g_{\mu\nu}, \quad (37) \]
and yielding the negative cosmological constant $\Lambda = -3k^4 e^{k^2 K}|W|^2$ and to anti-de Sitter space-time for the bag interior. General expression for cosmological constant inside the bag has the form
\[ \Lambda = k^4 e^{k^2 K} \sum_i \{k^2 |K_i W|^2 - 3|W|^2\}. \quad (38) \]
It yields AdS vacuum if $k^2 |K_i W|^2 - 3|W|^2 < 0$.

In the same time the vacuum state I) in external region has $W = 0$ and $\Lambda = 0$, and it preserves supersymmetry for strong chiral fields.

5 Conclusion

A regularized source of the Kerr-Newman solution is considered having the structure of a rotating bag with AdS interior and a smooth domain wall boundary.

We show that the Witten superconducting string model can be generalized and adapted forming a charged supersymmetric superconducting bag with AdS interior and with a long range external gauge field which is necessary for description of charged black holes.

Since 1989 a successive accumulation of evidences is observed relating the structure of Kerr geometry with physics of elementary particles.
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