Surface quality and cylindricity of ultrasonic elliptical vibration–assisted centerless grinding of micro-rod YAG single crystals

Yuxiu Hu1,2 · Chen Li1,2 · Xin Wang2 · Yanquan Geng1 · Guijian Xiao3 · Feihu Zhang1,2

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Abstract
Micro-rod YAG single crystals are the preferred laser crystals for laser gain mediums with a high power. However, brittle fracture and crack damages are easy to occur in the grinding process of micro-rod brittle crystals due to their large length-to-diameter ratio, high brittleness, and high hardness. In this work, the modal, frequency, and harmonic response of the transducer under ultrasonic elliptical vibration are analyzed by using finite element simulation. Then, the mechanical structure of ultrasonic elliptical vibration system was designed and optimized based on the ultrasonic elliptical vibration theory and finite element simulation. To verify the reliability of the transducer, ultrasonic vibration experiments were carried out to measure the resonance frequency, amplitude, and impedance characteristics of the transducer. The vibration synthesis experiments under different phase differences and voltages were performed to verify the rationality of the structural design of the ultrasonic elliptical vibration system. An experimental platform of ultrasonic elliptical vibration–assisted centerless grinding (UEVCG) was developed, and UEVCG tests of micro-rod YAG crystals were performed. The influences of the voltage, phase difference, and pallet inclined angle on surface roughness, peak-to-valley value, and cylindricity of the micro-rod YAG crystals were systematically analyzed. The ultrasonic elliptical vibration parameters were optimized based on the range analysis results of the orthogonal test. The results indicated that ultrasonic elliptical vibration effectively improved the surface quality and cylindricity of the micro-rod YAG crystals compared with traditional grinding. This work will not only enhance the understanding of the ultrasonic elliptical vibration principle, but also provide a technical support for precision and high-efficiency machining of micro-rod brittle materials.

Keywords Ultrasonic elliptical vibration–assisted centerless grinding · Cylindricity · Surface quality · YAG single crystal

1 Introduction
Micro-rod YAG single crystals are the preferred laser crystals for laser gain mediums with a high power due to their high gain, low threshold, and stable physical and chemical properties [1–3]. In addition to developing advanced growth technology of laser crystals with low defects, hard and brittle crystal elements must achieve their satisfactory surface integrity through precision machining technologies, such as grinding and polishing [4–6]. At present, the plastic damage mechanism of the laser crystals was revealed by the nanoindentation and nanoscratch tests combined with the cross-sectional TEM observation [7–9], which was dominated by polycrystalline nanocrystalline, dislocations, stacking faults, and lattice distortions. In addition, the ultra-smooth surface of planar laser crystal elements can be achieved by polishing and ductile grinding technologies [10–12]. However, micro-rod brittle crystals have large length-to-diameter ratio, high brittleness, and high hardness; therefore, brittle fracture and crack damages are easy to occur in the grinding process [13–15], which seriously affect the service accuracy and life of the solid-state lasers. At present, centerless grinding is the most effective machining technology for micro-rod work materials [16–20], which can avoid the concentrated stress caused by the workpiece clamping and improve the
roundness and surface quality of the workpiece. Barrenetxea et al. [16] performed centerless grinding tests of rod workpieces based on an active damping system, and the results demonstrated that centerless grinding technology effectively suppressed the chatter of the machine tool and improved the accuracy and productivity of the workpiece. Xu and Wu [18, 19] performed centerless grinding tests of rod stainless steels on a surface grinder, and a precision rod workpiece with a roundness of 0.9 μm was achieved by using the optimized process parameters. Hashimoto et al. [20] reviewed the development history of the centerless grinding technology, and they pointed that centerless grinding had a broad application prospect in the industrial production and high-precision manufacturing of micro-rod elements.

To improve the surface integrity of the hard and brittle components, many efforts have been made to reduce the brittle damages generated in the grinding process of hard and brittle materials, such as combining ultrasonic vibration into the traditional grinding [21–25], using intelligent algorithms to optimize grinding parameters [26, 27], and developing innovative grinding fluid [28, 29]. Many scholars demonstrated that ultrasonic vibration effectively improved the ground surface quality, decreased the subsurface damage, and decreased the wheel wear [21–25]. Li et al. [21] performed ultrasonic vibration–assisted grinding tests of deep holes of zirconia ceramics, and the results indicated that compared with traditional grinding, ultrasonic vibration–assisted grinding decreased the edge-chipping size and improved the surface integrity of the holes. Wang et al. [22] analyzed the power spectrum density during ultrasonic vibration–assisted grinding of sapphire single crystals, and they found that power spectrum density could well characterize the wheel wear and ultrasonic vibration effectively decreased the brittle fractures and wheel wear. Li et al. [24] studied the damage mechanism and force modeling during the ultrasonic vibration–assisted grinding of SiC ceramics, and the results demonstrated that compared with traditional grinding, ultrasonic vibration–assisted grinding decreased the subsurface damage depth, surface roughness value, and grinding force. Kumar et al. [25] summarized the advances in the machining of brittle optical materials, and they believed that ultrasonic vibration could reduce the brittle damages and improve the surface quality during the machining of brittle optical materials. Moriwaki and Shamoto [30] found that a high-frequency voltage with a phase difference excited two transducers to generate elliptical vibration, and proposed ultrasonic elliptical vibration–assisted machining technology based on the ultrasonic vibration technology. Compared with the traditional grinding technology, the ultrasonic elliptical vibration–assisted centerless grinding (UEVCG) technology can obtain a larger range of elliptical vibration and a larger amplitude amplification, which has been widely used in the high-efficiency and precision machining of micro-rod difficult-to-machine materials [31–37]. Wu et al. [31, 32] fabricated the micro-scale cylindrical components of tungsten carbide using the UEVCG technology, and a cylindrical component of 60 μm in diameter and 15 mm in length was achieved, which demonstrated that UEVCG had significant advantages in fabricating micro-rod elements with a large aspect ratio. Xu and Wu [33, 34] conducted UEVCG tests of K-grade cemented carbide to fabricated micro-rod components, and a micro-rod component with an aspect ratio of 310:1 and a diameter of 42 μm was successfully achieved, which indicated that UEVCG was an effective method to manufacture micro-rod components with a large aspect ratio. Fan et al. [35, 36] performed UEVCG tests of rod carbon steels, and the results demonstrated that the UEVCG technology effectively improved the cylindricity and surface quality of the work material. Nevertheless, there is hardly report on ultrasonic vibration–assisted centerless grinding of rod laser crystals, which hinders the precision and industrial production of rod laser crystals.

In this work, the modal, frequency, and harmonic response of the transducer under the ultrasonic elliptical vibration were analyzed by using finite element simulation. Then, the mechanical structure of ultrasonic elliptical vibration system was designed and optimized based on the ultrasonic elliptical vibration theory and finite element simulation. To verify the reliability of the transducer, ultrasonic vibration experiments were carried out to measure the resonance frequency, amplitude, and impedance characteristics of the transducer. The vibration synthesis experiments under different phase differences and voltages were performed to verify the rationality of the structural design of the ultrasonic elliptical vibration system. An experimental platform of ultrasonic elliptical vibration–assisted centerless grinding was developed, and UEVCG tests of micro-rod YAG single crystals were performed. The influences of voltage, phase difference, and pallet inclined angle on surface roughness, peak-to-valley (PV) value, and cylindricity of the micro-rod YAG single crystals were systematically analyzed, based on which the processing parameters were optimized. The results will not only enhance the understanding of the ultrasonic elliptical vibration principle, but also provide a technical support for precision and high-efficiency machining of micro-rod brittle solids.

2 Design of ultrasonic elliptical vibration device

2.1 Structural design of ultrasonic transducer

The structure of the ultrasonic transducer consists of front cover, back cover, piezoelectric ceramics, and fastening bolt. The function of the front cover was to transmit the energy
generated by the piezoelectric ceramics and impedance conversion, which was made of duralumin. The back cover was made of 45# steel, which was used to decrease the energy dissipation. PZT-8 piezoelectric ceramics were used in the design of the ultrasonic transducer, which had low mechanical loss, high piezoelectric constant, and high electromechanical conversion coefficient. The detailed parameters of covers and PZT-8 piezoelectric ceramics were given in Tables 1 and 2, respectively.

The longitudinal propagation speed of the sound wave in PZT-8 piezoelectric ceramics is 3560 m/s, so the lengths of the ultrasonic transducer, front cover, back cover, and piezoelectric ceramics are calculated as 92.1 mm, 46 mm, 14 mm, and 20 mm, respectively. The number of piezoelectric ceramic sheets is generally set as an even number; therefore, four piezoelectric ceramic sheets with a thickness of 5 mm are used in the ultrasonic transducer. As shown in Fig. 1, the thickness of the flange ring is set as 1 mm to reduce the influence of the flange ring on the vibration of the ultrasonic transducer.

2.2 Structure simulation and vibration analysis of ultrasonic transducer

The 3D model of the ultrasonic transducer was established in SolidWorks, and then, the mode analysis was performed in the workbench. The materials of the front cover, back cover, and piezoelectric ceramics were duralumin, 45# steel, and PZT-8 piezoelectric ceramics, respectively. In this model, the number of the mesh and node are 97,596 and 197,952, respectively. The mesh quality is 0.86, which indicates that the quality of the mesh division is appropriate.

The elastic, piezoelectric constant, and dielectric constant matrices of PZT-8 piezoelectric ceramics are given in Eqs. (1)–(3).

Table 1 Physical properties of the front and back covers

| Physical properties | Duralumin | 45# steel |
|---------------------|-----------|-----------|
| Density (kg/m³)     | 2700      | 7800      |
| Elastic modulus (GPa)| 66        | 210       |
| Poisson’s ratio     | 0.3       | 0.3       |
| Longitudinal wave velocity (m/s) | 5068 | 5170 |
| Acoustic impedance (Mrayl)| 13.9 | 41.6 |
| Wave number         | 37.19     | 36.84     |

Table 2 Physical properties of PZT-8 piezoelectric ceramics

| Physical properties | PZT-8 |
|---------------------|-------|
| Density (kg/m³)     | 7600  |
| Piezoelectric constant | 26.8 \times 10^8 |
| Dielectric impermeability | 1.78 \times 10^8 |
| Acoustic impedance (Mrayl) | 29.4 |
| C_{33}^E             | 11.5 \times 10^{10} |
| C_{33}^D             | 15.9 \times 10^{10} |

\[ [s] = \begin{bmatrix} s_{33} & s_{13} & 0 & 0 & 0 \\ s_{13} & s_{11} & s_{12} & 0 & 0 \\ s_{13} & s_{11} & s_{12} & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & s_{46} & 0 \\ 0 & 0 & 0 & 0 & s_{44} \end{bmatrix} = \begin{bmatrix} 14.9 & 8.11 & 8.11 & 0 & 0 \\ 8.11 & 14.9 & 8.11 & 0 & 0 \\ 8.11 & 8.11 & 13.2 & 0 & 0 \\ 0 & 0 & 0 & 3.4 & 0 \\ 0 & 0 & 0 & 0 & 3.13 \\ 0 & 0 & 0 & 0 & 3.13 \end{bmatrix} \]

\[ [d] = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & 0 & 0 \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -4.1 \\ 0 & 0 & 14.0 \\ 0 & 0 & 0 \\ 0 & 10.3 & 0 \\ 10.3 & 0 & 0 \end{bmatrix} \]

\[ [\varepsilon] = \epsilon_0 \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} 900 & 0 & 0 \\ 0 & 900 & 0 \\ 0 & 0 & 600 \end{bmatrix} \]

where \( \epsilon_0 \) is the permittivity of vacuum, which is equal to \( 8.8542 \times 10^{-12} \) F/m.

The transducer will output the maximum displacement and efficiency when the voltage frequency is equal to the resonance frequency. The flange ring is set as a fixed constraint. The results of the modal analysis of the ultrasonic transducer are shown in Fig. 2, which indicates that only longitudinal vibration with a frequency of 29,968 Hz occurs in the results of the 10th mode of the ultrasonic transducer. The vibration frequency of the simulated results is close to the design index of 28 kHz, and the error is approximately 7.03%.
Harmonic response analysis is usually used to determine the steady-state response of a linear structure subjected to a sinusoidal load [38, 39]. The relationship between the amplitude and frequency under different voltages was analyzed in this work. The range of the analysis frequency was chosen as 29.5–30.5 kHz according to the modal analysis results. The voltage of the piezoelectric ceramic was selected as 100 V. The simulated results of the amplitude and phase are shown in Fig. 3.

As shown in Fig. 4, the displacement reaches to a maximum value when the frequency is 29,968 Hz. The output displacement is the same as the phase of the input voltage, and the maximum elongation length of the transducer is 6 μm.

2.4 Vibration analysis of ultrasonic transducer

The ultrasonic transducer is shown in Fig. 5a, where the piezoelectric ceramic is clamped between the front and back covers by the fastening bolt. The flange ring is between the front cover and piezoelectric ceramics. The fastening bolt is used to connect with other components. As shown in Fig. 5b, an impedance analyzer (Keysight, E4990A) is used to analyze the impedance characteristics to obtain the resonant frequency. The impedance curves of the two ultrasonic transducers are shown in Fig. 5c and d, respectively. The resonant frequencies are 28,145 Hz and 27,810 Hz, respectively. The anti-resonant frequencies are 29,830 Hz and 29,617 Hz, respectively. When the ultrasonic transducer is in the resonant state, the internal impedance is the smallest, and the phase is equal to 0. The equivalent impedance of the two ultrasonic transducers in resonant state is 105.2 Ω and 86.4 Ω, respectively. When the ultrasonic transducer is in anti-resonant state, the internal impedance is the largest, and the phase is equal to π/2. The equivalent impedance of the two ultrasonic transducers in anti-resonant state is 6777.1 Ω and 3650.8 Ω, respectively.

The vibration amplitude under different driving voltages can be determined by measuring the vibration displacement.
The displacement amplitude for the end face of the front cover is measured by a laser displacement sensor (Keyence, LK-H020) whose repetition accuracy and sampling period are 0.2 μm and 2.5 μs, respectively. The vibration measurement system is shown in Fig. 6, and the voltage parameters and measurement results are given in Table 3.

When the input voltage is 200 peak-to-peak voltage (V_{p-p}), the vibration displacement curve and its partial enlarged view are shown in Fig. 7a and b, respectively. The peak, valley, and amplitude values are 6.4 μm, −4 μm, and 10.4 μm, respectively. Peak and valley values are not symmetrical about the coordinate axis due to the drift in the acquisition process. The peak, valley, and amplitude values of the two transducers under different voltages are shown in Fig. 7c and d, which indicate that the vibration amplitude is approximately proportional to the voltage.
Structural design and analysis of ultrasonic elliptical vibration transducer

Suppose that a particle vibrates in two mutually perpendicular directions at the same frequency and with a certain phase difference, and the displacement equation of this particle in the $X$ and $Y$ directions is shown in Eq. (4).

$$
\begin{align*}
X &= A_1 \sin (\omega t + \varphi_0) \\
Y &= A_2 \sin (\omega t + \varphi_0 + \varphi)
\end{align*}
$$

The coordinate equation can be obtained by eliminating the parameter $t$, as shown in Eq. (5).

$$
\frac{X^2}{A_1^2} + \frac{Y^2}{A_2^2} - 2 \frac{XY}{A_1 A_2} \cos \varphi = \sin^2 \varphi
$$

The inclination angle of the major axis of the ellipse is shown in Eq. (6).

$$
\theta = \frac{1}{2} \arctan \frac{2A_1 A_2 \cos \varphi}{A_1^2 - A_2^2}
$$

The major axis and minor axis of the ellipse can be calculated by Eq. (7).

$$
\begin{align*}
a^1 &= \frac{2 \sin^2 \varphi}{\left( \frac{1}{A_1^2} + \frac{1}{A_2^2} \right) + \sqrt{\left( \frac{1}{A_1^2} - \frac{1}{A_2^2} \right)^2 + 4 \cos^2 \varphi}} \\
b^2 &= \frac{2 \sin^2 \varphi}{\left( \frac{1}{A_1^2} + \frac{1}{A_2^2} \right) - \sqrt{\left( \frac{1}{A_1^2} - \frac{1}{A_2^2} \right)^2 + 4 \cos^2 \varphi}}
\end{align*}
$$

When the vibration frequencies of two simple harmonic vibrations perpendicular to each other are the same, a stable
elliptical trajectory can be synthesized. In addition, the synthesized elliptical trajectory can be adjusted by changing the amplitude ratio and phase difference. For ultrasonic vibration transducer, the elliptical trajectory can be adjusted by changing the voltage and phase difference of the piezoelectric ceramics.

The diagrammatic sketch of the overall structure of two transducers is shown in Fig. 8a, and the vibration joint is connected with two transducers. Figure 8b shows the modal analysis result of the transducer when the vibration frequency is 27,911 Hz. The voltage amplitude and frequency between the piezoelectric ceramic sheets are 100 V and 27,911 Hz, respectively. The initial phases of the transverse and longitudinal transducers are 0° and 45°, respectively. The amplitudes and displacements of two transducers are shown in Fig. 8c–f, which indicate that the elliptical vibration is well realized.

Ultrasonic elliptical vibration transducer is shown in Fig. 9a, which shows that the elliptical vibration is generated at the end face of the vibrating joint. It is driven by two high-frequency alternating voltages, as shown in Fig. 9b. A digital signal generator (AFG1022) generates sinusoidal voltage with a phase difference, and a two-channel power amplifier (ATA-2042) amplifies the voltage signal and connects it to two transducers. Then, elliptical vibration is generated on the end face of the vibration joint.

The displacement trajectory can be obtained by the end-face displacement of the transverse and longitudinal transducers during the vibration process. When the voltage is 160 V\text{p-p}, the synthetic trajectories under different phase differences are shown in Fig. 10. Because the resonant frequencies of the two transducers are different and the longitudinal wave attenuates during the vibration process, therefore, there is a small error in the synthesized elliptical trajectory. When the phase difference is 15°, the trajectory vibrates at a major axis of the ellipse, whose major axis and minor axis are 14.6 μm and 5.4 μm, respectively. With the increase of the phase difference, the major axis becomes shorter and the minor axis becomes longer. When the phase difference is 90°, the synthesized trajectory is approximately a circle.

When the phase difference is 45°, the synthetic trajectories under different voltages are shown in Fig. 11. Both major and short axes increase as the voltage increases; therefore, the area of the ellipse becomes larger. When the voltage is low, the shape of the ellipse is irregular. With the increase of voltage, the shape of the ellipse becomes regular and the vibration amplitude of the transducer increases. Therefore, high voltage should be selected in the grinding test.

3 Experiment

The schematic diagram of ultrasonic elliptical vibration–assisted centerless grinding of micro-rod YAG single crystals is shown in Fig. 12a, where \( n_w, V_f, R_w, \) and \( \varphi \) are the wheel rotational speed, feed speed, workpiece radius, and pallet inclined angle, respectively. The ultrasonic elliptical
Fig. 8  a The structure diagram of ultrasonic elliptical vibration transducer. b The vibration mode of transducer. c The vibration amplitude of harmonic response of transverse transducer. d The vibration amplitude of harmonic response of longitudinal transducer. e The unidirectional displacement of transducer. f The synthetic trajectory of transducer.

Fig. 9  a Ultrasonic elliptical vibration transducer and b drive system
The vibration transducer can produce high-frequency vibration at the end face, which drives the rotation of the workpiece. The position of the transducer and plate is fixed. The grinding wheel moves downward in the radial direction during the grinding process. As shown in Fig. 12b, ultrasonic elliptical vibration–assisted centerless grinding tests of microrod YAG single crystals were performed on an ultrasonic elliptical centerless grinding device. The guide wheel was replaced by the elliptical vibration device, so the rotation of the workpiece changes periodically. The grinding platform was built on a precision grinder with a feed resolution of 1 µm, and worktable is an electromagnetic chuck with a movement range of 300 mm × 500 mm. The lifting range and rotation speed of the spindle is 300 mm and 2800 r/min, respectively. A sliding table with a groove is placed under the plate, whose lifting range is 10 mm. The inclined plane and vibration device are fixed by the electromagnetic chuck. The ultrasonic elliptical vibration device is driven by the signal generator when the voltage is amplified by the power amplifier.
Metal-bonded diamond wheel was used in grinding experiment, whose diameter, width, and abrasive size were 200 mm, 15 mm, and 40 μm, respectively. The positional relationship between the work material and UEVCG experimental device is shown in Fig. 12c. The YAG workpiece before grinding is shown in Fig. 12d, whose diameter, length, and surface roughness were 1.2 mm, 12 mm, and 0.91 μm in Ra, respectively. The micro-rod YAG crystals were ground to 1.0 mm by ultrasonic elliptical vibration–assisted centerless grinding. The grinding depth is 1 μm. The influences of the abrasive size, grinding speed and feed speed, and grinding depth on the grinding process of laser crystals have been reported in other papers [11–13]. Therefore, this paper focused on the influences of the voltage, phase difference, and pallet inclined angle. As shown in Table 4, the orthogonal test was designed to analyze the ultrasonic vibration parameters on the grinding quality. Before the grinding tests, the wheels were shaped and dressed by electrolytic dressing method. Copper electrode, discharge voltage of 100 V, pulse width of 20 μs, and pulse gap of 50 μs were used during the truing and dressing process. After the grinding, the surface roughness of the work material was measured by a profilometer (Form Talysurf PGI, UK), and the PV value and cylindricity were measured by a laser confocal microscope (OLS3000, Japan).

### Table 4 Orthogonal test design of UEVCG of micro-rod YAG crystals

| Factors                       | Parameter 1 | Parameter 2 | Parameter 3 | Parameter 4 |
|-------------------------------|-------------|-------------|-------------|-------------|
| A Voltage (V<sub>pp</sub>)     | 40          | 80          | 120         | 160         |
| B Phase different (°)         | 30          | 45          | 60          | 75          |
| C Pallet inclined angle (°)   | 30          | 45          | 60          | 75          |

4 Results and discussions

The optical image of the surface of the ground micro-rod YAG crystal is shown in Fig. 13a. The measured size and number of the scanning points are 640 μm × 640 μm and 1024 × 1024, respectively. The 3D morphology of the ground surface and its cross-sectional profile are shown in Fig. 13b and c, respectively. According to the points of the cross-sectional profile, the radius and coordinates of the circle center were fitted by using the least squares method, as shown in Fig. 13d. It can be found that the radius and coordinates are approximately 446.3 μm, 333.8 μm, and −382.3 μm, respectively. For each workpiece, 20 groups of fitted radius and coordinates can be obtained. The minimum value of the circumscribed circle diameter containing 20 circle centers was taken as the straightness error. The roundness of each fitted circle can be expressed by the radius difference between
two concentric circles containing the cross-sectional profile. The maximum value of the 20 roundness values was chosen as the roundness error. The error of the cylindricity can be calculated by the errors of the maximum straightness and half of the maximum roundness, as shown in Eq. (8). The experimental and range analysis results of the orthogonal tests are shown in Tables 5 and 6, respectively.

$$f_{\text{Cylindricity}} = f_{\text{Straightness(max)}} + \frac{1}{2} f_{\text{Roundness(max) }}$$ \hspace{1cm} (8)

Table 6 shows that the voltage has the greatest influence on the surface roughness and cylindricity, and the phase difference has the smallest influence on the surface and cylindricity. The phase difference and voltage have the greatest and smallest influences on the PV value, respectively. The

| Group | Voltage (V) | Phase difference (°) | Pallet inclined angle (°) | Roughness, Ra (μm) | PV value (μm) | Cylindricity (μm) |
|-------|-------------|----------------------|---------------------------|---------------------|---------------|-------------------|
| 1     | 40          | 30                   | 30                        | 0.78                | 5.66          | 5.23              |
| 2     | 40          | 45                   | 45                        | 0.94                | 3.73          | 4.30              |
| 3     | 40          | 60                   | 60                        | 1.06                | 4.80          | 3.52              |
| 4     | 40          | 75                   | 75                        | 0.91                | 4.44          | 4.05              |
| 5     | 80          | 30                   | 45                        | 0.56                | 4.20          | 4.42              |
| 6     | 80          | 45                   | 60                        | 1.04                | 3.87          | 3.76              |
| 7     | 80          | 60                   | 75                        | 0.68                | 3.45          | 4.31              |
| 8     | 80          | 75                   | 30                        | 1.08                | 3.96          | 5.15              |
| 9     | 120         | 30                   | 60                        | 0.95                | 4.88          | 5.54              |
| 10    | 120         | 45                   | 75                        | 0.98                | 4.60          | 6.04              |
| 11    | 120         | 60                   | 30                        | 1.30                | 5.62          | 5.80              |
| 12    | 120         | 75                   | 45                        | 0.68                | 3.44          | 3.10              |
| 13    | 160         | 30                   | 75                        | 1.20                | 5.08          | 2.58              |
| 14    | 160         | 45                   | 30                        | 0.71                | 2.03          | 2.90              |
| 15    | 160         | 60                   | 45                        | 1.16                | 5.50          | 3.08              |
| 16    | 160         | 75                   | 60                        | 1.40                | 4.28          | 3.34              |
optimized grinding parameters are given in Table 7 based on the range analysis results.

Under the same grinding parameters, the surface roughness and 3D surface morphology of micro-rod YAG crystals measured in traditional centerless grinding and ultrasonic elliptical vibration-assisted centerless grinding are shown in Fig. 14a and b, respectively. The surface roughness and PV value measured in traditional centerless grinding were 0.91 μm and 9.29 μm, respectively, and they were 0.56 μm and 5.48 μm measured in UEVCG. The results indicated that ultrasonic elliptical vibration improved the surface quality of the micro-rod YAG crystals compared with traditional grinding. In addition, the 3D surface morphologies also showed that the cylindricity in UEVCG was better than that in traditional grinding. This is because ultrasonic elliptical vibration increases the cutting arc length of the single abrasive and enlarges the interaction area between the abrasives and workpiece [40–42], which results in the decrease of maximum undeformed chip thickness, grinding force, and wheel wear and in the increase of the brittle-to-ductile transition depth [43–45]. In addition, there is no regulating wheel in UEVCG technology of this work, which can avoid the

| Table 6 Range analysis results of the orthogonal test | Roughness, Ra (μm) | PV value (μm) | Cylindricity (μm) |
|---------------------------------------------------|-------------------|---------------|-------------------|
|                                                   | A     | B     | C     | A     | B     | C     | A     | B     | C     |
| \( \bar{K}_1 \)                                 | 0.92  | 0.92  | 0.97  | 4.66  | 5.96  | 4.32  | 4.20  | 3.60  | 3.44  |
| \( \bar{K}_2 \)                                 | 0.83  | 0.86  | 0.83  | 4.87  | 3.56  | 5.22  | 3.30  | 3.22  | 2.80  |
| \( \bar{K}_3 \)                                 | 0.98  | 1.05  | 1.11  | 4.64  | 4.84  | 4.46  | 3.85  | 4.05  | 3.65  |
| \( \bar{K}_4 \)                                 | 1.12  | 1.02  | 0.94  | 4.22  | 4.03  | 4.39  | 2.98  | 3.82  | 3.96  |
| \( R_i \)                                       | 0.29  | 0.16  | 0.28  | 0.65  | 2.40  | 0.90  | 1.22  | 0.83  | 1.16  |

| Table 7 The optimized grinding parameters of UEVCG of micro-rod YAG crystals |
|-------------------------------------------------|-----------------|
| A voltage (V<sub>p-p</sub>) | B phase difference (°) | C pallet inclined angle (°) |
| Roughness | 80 | 45 | 45 |
| PV value  | 160 | 45 | 30 |
| Cylindricity | 160 | 45 | 45 |

Fig. 14 Surface roughness and 3D surface morphology measured in a traditional centerless grinding and b ultrasonic elliptical vibration-assisted centerless grinding.

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roundness errors caused by the regulating wheel [31–34]. Therefore, compared with the traditional grinding, UEVCG effectively reduced the surface roughness and cylindricity error of the ground workpiece.

5 Conclusions

- The modal, frequency, and harmonic response of the transducer under ultrasonic elliptical vibration are analyzed by using finite element simulation. Then, the mechanical structure of ultrasonic elliptical vibration system was designed and optimized based on the ultrasonic elliptical vibration theory and finite element simulation.
- To verify the reliability of the transducer, ultrasonic vibration experiments were carried out to measure the resonance frequency, amplitude, and impedance characteristics of the transducer. The vibration synthesis experiments under different phase differences and different voltages were performed to verify the rationality of the structural design of the ultrasonic elliptical vibration system. The results indicated that the elliptical vibration trajectory was well synthesized by adjusting the phase difference and voltage.
- An experimental platform of ultrasonic elliptical vibration–assisted centerless grinding was developed, and UEVCG tests of micro-rod YAG crystals were performed. The influences of voltage, phase difference, and pallet inclined angle on surface roughness, PV value, and cylindricity of the micro-rod YAG crystals were systematically analyzed, based on which the processing parameters were optimized. The surface roughness and PV value measured in traditional centerless grinding were 0.91 μm and 9.29 μm, respectively, and they were 0.56 μm and 5.48 μm measured in UEVCG. The results indicated that ultrasonic elliptical vibration improved the surface quality and cylindricity of the micro-rod YAG crystals compared with traditional grinding.

Author contribution Chen Li contributed in the ideal and paper writing, Xin Wang contributed in the grinding experiment and paper writing, and Yuxiu Hu, Feihu Zhang, Yanquan Geng, and Guijian Xiao contributed in the grinding experiment and paper writing.

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Declarations

Ethical approval and consent to participate Not applicable.

Consent for publication We would like to submit the manuscript entitled “Surface quality and cylindricity of ultrasonic elliptical vibration-assisted centerless grinding of micro-rod YAG single crystals,” for your consideration for publication in the International Journal of Advanced Manufacturing Technology. The manuscript is approved by all authors for publication. On behalf of the co-authors, we declare that the work described was an original research that has not been published previously, and not under consideration for publication elsewhere, in whole or in part.

Conflict of interests The authors declare no competing interests.

References

1. Li C, Zhang Q, Zhang Y, Zhang F, Wang X, Dong G (2020) Nanoindentation and nanoscratch tests of YAG single crystals: an investigation into mechanical properties, surface formation characteristic, and theoretical model of edge-breaking size. Ceram Int 46(3):3382–3393
2. Zhang Z, Jin Z, Mu Q, Yang H, Han X (2021) Optimization of CMP solution for yttrium aluminum garnet crystal. Diamond Abras Eng 41:82–88
3. Li C, Hu Y, Huang S, Meng B, Piao Y, Zhang F (2022) Theoretical model of warping deformation during self-rotating grinding of YAG wafers. Ceram Int 48(4):4637–4648
4. Zhang Y, Wang Q, Li C, Piao Y, Hou N, Hu K (2022) Characterization of surface and subsurface defects induced by abrasive machining of optical crystals using grazing incidence X-ray diffraction and molecular dynamics. J Adv Res 36:51–61
5. Li C, Hu Y, Zhang F, Geng Y, Meng B (2023) Molecular dynamics simulation of laser assisted grinding of GaN crystals. Int J Mech Sci. https://doi.org/10.1016/j.ijmecsci.2022.107856
6. Zhang T, Jiang F, Huang H, Lu J, Wu Y, Jiang Z, Xu X (2021) Towards understanding the brittle–ductile transition in the extreme manufacturing. Int J Extreme Manuf 3(2):022001
7. Li C, Piao Y, Zhang F, Zhang Y, Hu Y, Wang Y (2023) Understand anisotropy dependence of damage evolution and material removal during nanoscratch of MgF2 single crystals. Int J Extreme Manuf. https://doi.org/10.1088/2631-7990/ac9eed
8. Li C, Zhang F, Meng B, Rao X, Zhou Y (2017) Research of material removal and deformation mechanism for single crystal GGG (Gd3Ga5O12) based on varied-depth nanoscratch testing. Mater Des 125:180–188
9. Li C, Piao Y, Meng B, Zhang Y, Li L, Zhang F (2022) Anisotropy dependence of material removal and deformation mechanisms during nanoscratch of gallium nitride single crystals on (0001) plane. Appl Surf Sci 578:152028
10. Zhang Z, Jin Z, Guo J, Han X, Mu Q, Zhu X (2019) A novel chemical mechanical polishing slurry for yttrium aluminum garnet crystal. Appl Surf Sci 496:143601
11. Li C, Wu Y, Li X, Zhang F, Huang H (2020) Deformation characteristics and surface generation modelling of crack-free grinding of GGG single crystals. J Mater Process Technol 279(5):11677
12. Li C, Piao Y, Meng B, Hu Y, Li L, Zhang F (2022) Phase transition and plastic deformation mechanisms induced by self-rotating grinding of GaN single crystals. Int J Mach Tools Manuf 172:103827

13. Li C, Li X, Wu Y, Zhang F, Huang H (2019) Deformation mechanism and force modelling of the grinding of YAG single crystals. Int J Mach Tools Manuf 143:23–37

14. Zhang D, Shao Z, Geng D, Jiang X, Liu Y, Zhou Z, Li S (2021) Feasibility study of wave-motion milling of carbon fiber reinforced plastic holes. Int J Extreme Manuf 3(1):010401

15. Zhu D, Peng X, Xu X, Yang Z, Li W, Yan S, Ding H (2020) Robotic grinding of complex components: a step towards efficient and intelligent machining – challenges, solutions, and applications. Robot Comput Integr Manuf 65:101908

16. Barrenetxea D, Mancisidor I, Beudaert X, Munno J (2018) Increased productivity in centerless grinding using inertial active dampers. CIRP Ann 67(1):337–340

17. Garitaonandia I, Fernandez M, Albizuri J, Hernández J, Barrenetxea D (2010) A new perspective on the stability study of centerless grinding process. Int J Mach Tools Manuf 50(2):165–173

18. Xu W, Wu Y (2012) Simulation investigation of through-feed centerless grinding process performed on a surface grinder. J Mater Process Technol 212(4):927–935

19. Xu W, Wu Y (2011) A new in-feed centerless grinding technique using a surface grinder. J Mater Process Technol 211(1):141–149

20. Hashimoto F, Gallego I, Oliveira J, Barrenetxea D, Takahashi M, Sakakibara K, Sakakibara K, Stålhe B, Staad G, Ogawa K (2012) Advances in centerless grinding technology. CIRP Ann 61(2):747–770

21. Li S, Ma C, Sun J (2021) R. Zhao B, Chang B, W. Westphal S, M. M. Moro A. K. Zhang D, Shao Z, Geng D, Jiang X, Liu Y (2022) Phase transition in centerless grinding technology. CIRP Ann 61(2):747–770

22. Wang Q, Liang Z, Bai S, Wu Y, Jia S (2021) Power spectrum density characterization of grinding wheel surface in ultrasonic vibration spiral grinding. Diamond Abras Eng 41:58–64

23. Kang R, Song X, Dong Z, Pan Y, Zhang V, Bao Y (2021) Study on surface integrity of tungsten alloy processed by ultrasonic elliptical vibration cutting. Surface Technol 50(11):321–328. https://doi.org/10.16490/j.cnki.issn.1001-3660.2021.11.034

24. Li C, Zhang F, Meng B, Liu L, Rao X (2017) Material removal mechanism and grinding force modelling of ultrasonic vibration assisted grinding for SiC ceramics. Ceram Int 43(3):2981–2993

25. Kumar S, Tong Z, Jiang X (2022) Advances in the design and manufacturing of novel freeform optics. Int J Extreme Manuf 4(3):032004

26. Lv R, Liu H, Wang Z, Zhu D (2022) WPMAVM: weighted plus-minus allowance variance minimization algorithm for solving matching distortion. Robot Comput Integr Manuf 76:102320

27. Lv Y, Peng Z, Qu C, Zhu D (2020) An adaptive trajectory planning algorithm for robotic belt grinding of blade leading and trailing edges based on material removal profile model. Robot Comput Integr Manuf 66:101987

28. Huang S, Wu H, Jiang Z, Huang H (2021) Water-based nanosuspensions: formulation, tribological property, lubrication mechanism, and applications. J Manuf Process 71:625–644

29. Li C, Li X, Huang S, Li L, Zhang F (2021) Ultra-precision grinding of Gd2Ga5O12 crystals with graphene oxide coolant: material deformation mechanism and performance evaluation. J Manuf Process 61:417–427

30. Moriwaki T, Shamoto E (1995) Ultrasonic elliptical vibration cutting. CIRP Ann 44(1):31–34

31. Wu Y, Fan Y, Kato M (2006) A feasibility study of microscale fabrication by ultrasonic-shoe centerless grinding. Precis Eng 30(2):201–210

32. Wu Y, Fan Y, Kato M, Kuriyagawa T, Suyoki K, Tachibana T (2004) Development of an ultrasonic elliptical-vibration shoe centerless grinding technique. J Mater Process Technol 155:1780–1787

33. Xu W, Wu Y (2018) A novel approach to fabricate high aspect ratio micro-rod using ultrasonic vibration-assisted centreless grinding. Int J Mach Sci 141:21–30

34. Xu W, Wu Y (2019) Piezoelectric actuator for machining on macro-to-micro cylindrical components by a precision rotary motion control. Mech Syst Signal Process 114:439–447

35. Fan Y, Wu Y, Kato M, Tachibana T, Suyoki K, Kuriyagawa T (2004) Design of an ultrasonic elliptical-vibration shoe and its performance in ultrasonic elliptical-vibration-shoe centerless grinding. JSME Int J Ser C Mech Syst Mach Elem Manuf 47(1):43–51

36. Fan Y, Tang K, Yin S, Zhu J, Wu Y (2010) New ultrasonic elliptical vibration centerless grinding technique. Nanotechnol Precis Eng 8(6):484–490

37. Cui Q, Cheng K, Chen S, Ding H (2017) An innovative investigation on the workpiece kinematics and its roundness generation in through-feed centreless grinding. Proc Inst Mech Eng Part B: J Eng Manuf 231(7):1131–1143

38. Peng Y, Jiang T, Guo Y, Wang Z, Wu Y (2012) Effect mechanism of elliptic vibration Assistance on the cutting of brittle materials. Adv Mater Res 472:499–504

39. Zhang Y, Wu T, Li C, Wang Y, Geng Y, Dong G (2022) Numerical simulations of grinding force and surface morphology during precision grinding of leucite glass ceramics. Int J Mach Sci 231:107562

40. Zhao B, Chang B, Wang X, Bie W (2019) System design and experimental research on ultrasonic assisted elliptical vibration grinding and nano-ZrO2 ceramics. Ceram Int 45(18):24865–24877

41. Wang Y, Liang Z, Zhao W, Wang X, Wang H (2020) Effect of ultrasonic elliptical vibration assistance on the surface layer defect of M-plane sapphire in microcutting. Mater Des 192:108755

42. Peng Y, Liang Z, Wu Y, Guo Y, Wang C (2012) Characteristics of chip generation by vertical elliptic ultrasonic vibration-assisted grinding of brittle materials. Int J Adv Manuf Technol 62(5):563–568

43. Liu X, Yu D, Chen D, Yang S, Wen Y, Xiao Y (2021) Self-tuned ultrasonic elliptical vibration cutting for high-efficient machining of micro-optics arrays on brittle materials. Precis Eng 72:370–381

44. Han L, Zhang J, Chen J, Zhang J, Liu H, Yan Y, Sun T (2020) Influence of vibration parameters on ultrasonic elliptical vibration cutting of reaction-bonded silicon carbide. Int J Adv Manuf Technol 108(1):427–437

45. Li L, Xu J, Ji M, Yin Y, Chen M (2022) On crack suppression mechanisms of ultrasonic elliptical vibration cutting of 3Y-TZP ceramics. Ceram Int 48(19):28308–28326

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