Strength evaluation of the relocatable building made of the polymer composite material

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Abstract. The article deals with the problem of improving the efficiency of the frame relocatable building structures by perfecting structural forms, introducing sound and lightweight materials, in particular, polymer composite materials (PCM). For this purpose, we have developed an engineering calculation algorithm using CAD / CAE systems and considered various options for deposition of PCM layers. The analytical formulas that determine the Eulerian instability for PCMs are compared with the values in ANSYS.

1. Introduction

A primary structural component of relocatable buildings and structures is a high-strength frame made of metal or wooden profiles. It is possible to increase further the efficiency of frame building structures through the improvement of structural forms and the use of strong and lightweight materials; currently, it is one of the urgent tasks of capital construction. The replacement of metal or wood profiles with multilayer polymer composite materials (PCM) is of particular interest for the construction industry. The use of this type of materials is especially relevant for construction of relocatable buildings in hard-to-reach places, as it is in oil and gas industry, geological exploration, where weight and frequent assembly and disassembly are of prime importance [1]. Such structures can include relocatable buildings.

However, despite good perspectives, PCMs have not found wide application in load bearing elements of building constructions. This is hampered by the complicated methods for calculating structures that do not find practical use. In this regard, it is relevant to create an engineering calculation that allows evaluating the behavior of a structure and its individual elements, as well as to develop computer methods for modelling and calculating building structures taking into account the PCM nonlinearity.

The paper considers the construction of a model of PCM-based relocatable building frame using modern CAD / CAE systems.

An analogue of this construction is the Mongolian yurt, with its frame made of wood. Lattice folding walls made of battens are pivotally fixed to each other, the yurt is covered with felt, over which a tarp is stretched. The roof is a dome formed of poles fixed in the center (figure 1(a)).

During the operation of this design, there is a loss of stiffness of the wooden frame and the stability of the elements, a decrease in strength during assembly-disassembly (2-3 times a year), the appearance of cracks at the joints of the rails and wear of the elements on the lower parts of the supports (figure...
1(b)). To increase the strength and reduce the weight of the frame, wooden elements were replaced with PCM (figure 2).

Figure 1. (a) traditional Mongolian tent construction “yurt”; (b) wear of wooden elements.

Figure 2. Relocatable building construction.

The improved construction contains a transparent dome (position 1 in figure 2), and roof (position 2), which has an umbrella type construction. The roof (position 2) frame consists of radially arranged rods (position 3), connected at one end to a center pillar (position 4), at the other end to a lathed wall (position 5) mounted in a circle and assembled from intersecting strips (position 6), which are fixed with belts (position 7). The center pillar (position 4) has a form of a hollow metal pipe (position 8). A patent for a utility model was obtained for the developed frame construction made from PCM [2].

2. Research methodology

To study the stress-strain state of the PCM structure, we used numerical methods of engineering analysis and application software packages Mathcad, SolidWorks, ANSYS, taking into account the characteristics of material and the actual loading conditions. A three-dimensional model of a tent-type construction using SolidWorks computer-aided design system (CAD) is developed. We carried out a series of numerical experiments taking into account the properties of the material, the direction of the fibers in each layer. The calculation procedure is presented in figure 3.

At the stage of specifying the preliminary model, we determined the form of the future structure and its elements related to the technology for manufacturing structural elements from PCM, operating conditions, research and development work. At the second stage, we calculated the elastic characteristics of PCM taking into account the filler and matrix material, the direction and size of fibers using Mathcad system. Next, we created a geometric model of the structure using the graphical interface of ANSYS program or CAD systems for complex objects (figure 4(a)). We carried out a preliminary check of the developed CAD-model of the construction, so that the structural elements made of PCM were of the “shell” type. At the last stage, we made a calculation of the construction.

Load summary was drawn up in accordance with the requirements of the Russian Set of Rules 20.13330.2016 [3] and presented in table 1. When calculating wind directions, snow loads, dead load of
the structure were taken into account, but temperature climatic effects (small quantity of the 2nd degree) were disregarded.

**Figure 3.** An algorithm for calculating the strength of structural elements made of PCM.

**Figure 4.** A strip made of the composite material: (a) 3D model of the relocatable building frame; (b) a finite element model; (c) equivalent von Mises stresses (Pa).

**Table 1.** Load calculation.

| Area load per 1 m² (kPa) | Snow loads 1 | Snow loads 2 | Snow loads 3 | Wind loads 1 |
|--------------------------|--------------|--------------|--------------|--------------|
|                          | 3.5          | 5.34         | 4            | –0.545       |
As an example, we have selected a section of the frame – a lathed wall of the relocatable structure, which is under the maximum load. The frame of the structure is presented as a package of layers, with the properties of individual layers (monolayer is made of fiberglass) specified (figure 5). Fiberglass has the following characteristics:

- $E = 4.15 \times 10^4$ MPa,
- $E = 4.21 \times 10^4$ MPa,
- $G = 1.25 \times 10^3$ MPa,
- $\mu = 0.28$.

The monolayer thickness is $b = 0.2$ mm. The scheme of reinforcement is $[0/90/\pm45]$. We have determined the stiffness characteristics of the composite package by the known stiffness characteristics of the monolayer [4]:

\[
E_{11} = E_1 c^4 + E_2 s^4 + G_{12} s^2 + \mu_{21} E_1 2 s^2 c^2; \\
E_{22} = E_1 s^4 + E_2 c^4 + G_{12} s^2 + \mu_{21} E_1 2 s^2 c^2; \\
E_{12} = (E_1 + E_2) s^2 c^2 - G_{12} s^2 + \mu_{21} E_1 (c^2 + s^2); \\
E_{33} = (E_1 + E_2) s^2 c^2 + G_{12} s^2 - \mu_{21} E_1 2 s^2 c^2; \\
E_{13} = E_1 s^3 c^3 - E_2 s^3 c - G_{12} s c^2 + \mu_{21} E_1 (s^3 c - sc^3); \\
E_{23} = E_1 s^3 c^3 - E_2 s^3 c + G_{12} s^2 c^2 + \mu_{21} E_1 (sc^3 - s^3 c),
\]

where $c = \cos \phi; c_2 = \cos 2\phi; s = \sin \phi; s_2 = \sin 2\phi; \bar{E}_1 = E_1 / (1 - \mu_{12}\mu_{21}); \bar{E}_2 = E_2 / (1 - \mu_{12}\mu_{21}); E_1\mu_{21} = E_2\mu_{12}$.

Here $E_1, E_2, G_{12}, \mu_{12}$ are the stiffness characteristics of the monolayer. The approximate stiffness characteristics of the multi-layered CM with a sufficient accuracy for design calculations are determined by the formulas:

\[
E_x = A_{11} - A_{12}^2 / A_{22}; \\
E_y = A_{22} - A_{12}^2 / A_{11}; \\
G_{xy} = A_{33}; \\
\mu_{xy} = A_{11} / A_{22}; \\
\mu_{yx} = \mu_{xy} = E_y / E_x = A_{22} / A_{11},
\]

where $A_{ij} = \frac{1}{H} \sum_{k=1}^{n} h_k E_{ij}^k; i, j = 1, 2, 3; H$ – thickness of laminated package; $h_k$ – thickness; $k$ – layer; $n$ – total number of layers in a package. The results of filling out these formulas in Mathcad system are presented in table 2.

**Table 2.** The stiffness characteristics of the multi-layered CM.

| $E_x$ (Pa) | $E_y$ (Pa) | $G_{xy}$ (Pa) | $\mu_{xy}$ | $\mu_{yx}$ |
|-----------|-----------|---------------|------------|------------|
| $3.924 \times 10^4$ | $3.248 \times 10^4$ | $7.118 \times 10^3$ | 0.248 | 0.205 |
Further, we calculated rods in ANSYS Mechanical system. To calculate the fiberglass rod, SOLID-46 multilayer element is used. This is a multilayer version of an 8-node three-dimensional solid element with three degrees of freedom for each node ($U_x, U_y, U_z$). The element is designed to simulate multilayer shells or multilayer solids [5]. The numerical solution of the nonlinear problem was carried out by the stepwise method. This method is based on the non-linear static calculation with a gradual increase in the load to the level at which the model becomes unstable. This allows to include such material features in the model as initial form deviations, plastic properties, gaps, etc.

The results of calculating the strain-stress state of the frame are presented in figure 4(c). The analysis of the numerical experiment shows that the upper part of lathed wall carries the highest load. Peak axially displacements in $U_y$-direction are observed in the setting of [0°/60°/120°] reinforcement scheme and are 0.5 mm modulo. The least $U_y$ axial displacement takes place, when reinforcement scheme is [0°/±45°], and makes 0.467 mm modulo. The maximum stress on the supports of strips is 195.66 MPa. When reinforcing [0°/±45°], the structural frame can withstand maximum stresses of 82.989 MPa. The calculation results are presented in table 3.

| Results of numerical calculations | $[0°/90°/±45°]$ | $[0°/60°/120°]$ | $[0°/±45°]$ |
|----------------------------------|----------------|----------------|-------------|
| Normal displacements $U_x$ (mm)  | 0.253          | 0.251          | 0.252       |
| Normal displacements $U_y$ (mm)  | −0.475         | −0.499         | −0.467      |
| Normal displacements (mm)        | 0.097          | 0.097          | 0.097       |
| Normal stresses $\sigma_x$ (MPa) | −127.760       | −125.690       | −123.660    |
| Normal stresses $\sigma_y$ (MPa) | −85.400        | −85.927        | −82.989     |
| Normal stresses $\sigma_z$ (MPa) | −193.930       | 190.330        | −195.660    |

The loss of stability is dangerous for thin-walled rods made of PCM, [6]; for their study we considered a linearized equation of quartic derivative in the following form:

$$\nu^{IV} + k^2 \nu^{IV} = 0,$$

where $k^2 = P / E_x I_{\text{min}}$; $D = \frac{E_x h^3}{12(1-\mu_{xy}\mu_{yx})}$; $I_{\text{min}}$ - axial moment of inertia; $E_x$ - elastic modulus; $h$ - rod thickness; $\mu_{xy}, \mu_{yx}$ - Poisson's ratio in the corresponding axis direction.

The solution to equation (1) is following:

$$\nu = A_1 \sin(kx) + A_2 \cos(kx) + A_3 x + A_4,$$

where $A_1, A_2, A_3, A_4$ are constants. Four boundary conditions are specified for this rod depending on the fixation of ends: 1) $\nu(0) = 0$; 2) $\nu''(0) = 0$; 3) $\nu(l) = 0$; 4) $\nu''(l) = 0$. Substituting the boundary conditions into the general solution (2), we obtain a system of four homogeneous linear equations (3) in the $A_i$ unknowns:

$$\begin{align*}
A_1 \sin(0) + A_2 \cos(0) + A_3 \cdot 0 + A_4 &= 0; \\
-A_1 k^2 \sin(0) - A_2 k^2 \cos(0) &= 0; \\
-A_1 \sin(kl) + A_2 \cos(kl) + A_3 l + A_4 &= 0; \\
-A_1 k^2 \sin(kl) - A_2 k^2 \cos(kl) &= 0.
\end{align*}$$
If the determinant of this system (3) is equal to zero, we can find the eigenvalues of the $P_n$ problem and the corresponding eigenfunctions. The smallest eigenvalue gives the critical load value, and the corresponding eigenfunction describes the form of the curved rod axis at loss of stability:

$$v_n(x) = A_1 \cdot \sin \left( \frac{\pi n}{l} \cdot x \right).$$

We have obtained the value of Euler's critical load for a rod at the loss of stability:

$$\frac{P \cdot 12 \left(1 - \mu_{xy} \cdot \mu_{yx}\right)}{E_i h_i^3} = \left( \frac{\pi n}{l} \right)^2,$$

then for $n = 1$:

$$P_{cr} = \frac{\pi^2 E_i h_i^3}{12 \cdot l^2 \left(1 - \mu_{xy} \cdot \mu_{yx}\right)}.$$

The compared critical load values depending on the length of rods, which have been obtained analytically and in ANSYS, result in allowable convergence for engineering calculations (figure 6).

![Figure 6. Euler's critical load for [0°; ±45°] reinforcement scheme and the critical load obtained in ANSYS.](image)

Using this diagram, we can determine the permissible length of relocatable building rods for a given type of laying.

3. Conclusion
Numerical calculations confirmed our preliminary analytical calculations for the application of various materials and reinforcement schemes. For fiberglass quasi-isotropic reinforcement scheme is the best in terms of strength and stiffness of the product received. The reinforcement scheme [0°; ±45°] provides sufficient stiffness of the structure, maximum displacements of the frame strips does not exceed 0.467 mm, so that it corresponds to the strength of wooden structures. In addition, in the improved construction made of PCM 42 strips are used instead of 63 wooden, as a result its weight decreases by 2.5 times.
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