sAVSS: Scalable Asynchronous Verifiable Secret Sharing in BFT Protocols

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Abstract—This paper introduces a new way to incorporate verifiable secret sharing (VSS) schemes into Byzantine Fault Tolerance (BFT) protocols. This technique extends the threshold guarantee of classical Byzantine Fault Tolerant algorithms to include privacy as well. This provides applications with a powerful primitive: a threshold trusted third party, which simplifies many difficult problems such as a fair exchange.

In order to incorporate VSS into BFT, we introduced sAVSS, a framework that transforms any VSS scheme into an asynchronous VSS scheme with constant overhead. By incorporating Kate et al.’s scheme [1] into our framework, we obtain an asynchronous VSS that has constant overhead on each replica—the first of its kind.

We show that a key-value store built using BFT replication and sAVSS supports writing secret-shared values with about a 30% – 50% throughput overhead with less than 35 millisecond request latencies.

I. INTRODUCTION

Combining the power of Byzantine Fault Tolerant (BFT) replication with secret sharing, one can build a decentralized service that acts over private values in a coordinated manner by consensus decrees. This powerful combination can be leveraged in various ways to build an automated, decentralized threshold trusted third party (T3P). For example, it may be used to build a decentralized T3P escrow. Crucially, escrowed secrets will be opened by consensus decree, not necessarily requiring client interaction. One can easily derive a fair-exchange from such an escrow service: One party confidentially stores one value, another party confidentially stores a second value; a consensus decree opens both. Another example use-case is a decentralized T3P certification authority (CA). The CA employs some policy that automates certification decisions. The CA utilizes threshold signing to certify documents, and again, if and when to certify is decided automatically by consensus. Using polynomial secret sharing, multiple values entrusted to a decentralized T3P may be aggregated without client involvement. Simple additive aggregations are trivial to implement. Arbitrary multi-party computation is possible (e.g., [2], [3]), though more costly.

In all of these use-cases, the enabling core is a mechanism called Verifiable Secret Sharing (VSS) [4] used for populating a decentralized service with secret values. Our use of VSS weaves it into BFT replication in order to automate the handling of secret values. For example, in a private key-value store we designed, a client request to store an entry is broken into two parts, public and private. The public part works as a normal BFT replication request. However, replicas delay their participation in the ordering protocol until they obtain a verifiable share of the private component of the store request. The private part of a client protocol utilizes VSS for the client to directly share the private entry.

Partly due to the need to weave VSS into a BFT replication engine, the setting of interest to us is asynchronous. Relying on synchrony requires making conservative assumptions about the network delay, whereas asynchronous protocols move at the speed of the network. Even worse, incorporating a synchronous VSS scheme into a BFT replication engine would require the replication engine to make a synchrony assumption even if one was not required for the BFT protocol to work.

The best known Asynchronous VSS (AVSS) solutions require a client (dealer) to incur quadratic communication and message complexities in order to store a single value [1]. This requires each replica to process and store a linear number of bits, which means that the performance overhead due to the addition of secret sharing increases linearly with the number of replicas. When AVSS and BFT replication solutions were originally developed, most BFT solutions were aimed for systems of four ($f = 1$) or seven ($f = 2$) replicas. However, today, BFT replication is being revisited at scale in blockchain systems of hundreds or thousands of replicas. Incurring such a large degradation in service performance for privacy may be prohibitively expensive.

To scale out AVSS, this paper introduces a new verifiable secret sharing framework called sAVSS. sAVSS is a framework that, given a VSS scheme, constructs an AVSS scheme whose performance is only a constant factor away from the original VSS scheme. We instantiate sAVSS in two ways: (1) using Kate, et al.’s secret sharing scheme [1], which gives us an AVSS that has constant time share verification and share recovery and (2) using Pedersen’s secret sharing scheme [5] which, while only providing linear time share verification and recovery, has cheaper cryptographic operations. Our framework is based on one key concept: the recovery polynomial. The recovery polynomial is a single polynomial that encodes
recovery information for \( f \) shares. Thus, by only sharing a small, constant number of additional polynomials, the client can enable all \( 3f + 1 \) shares to be recovered.

We intertwine sAVSS in a BFT replication system and build a full private key-value store solution. Our key-value store performs well in practice, with only a 30\% to 50\% throughput overhead over a nonprivate key-value store with request latencies less than 35 milliseconds.

This paper contributes a new framework for constructing asynchronous verifiable secret sharing schemes through the use of recovery polynomials, sAVSS. We then instantiate sAVSS using two verifiable secret sharing schemes and benchmark the overhead that our new framework adds. Finally, we incorporate our two instantiations into PBFT \([6]\) and evaluate a private, Byzantine Fault Tolerant key value store.

II. TECHNICAL OVERVIEW

In this section, we provide a high-level, informal overview of the core technique we develop to solve the asynchronous VSS problem. A precise description and details are given in the following sections.

A. The Asynchronous VSS Problem

In the asynchronous VSS problem, a dealer shares to a group of \( n \) participants a polynomial \( s \). The API for sharing is denoted \( \text{vssShare} \). If the sharing completes anywhere, then eventually every non-faulty participant completes the sharing.

The basic method for secret sharing (API: \( \text{vssShare} \)) is to provide participant \( i \) a point \((x_i, s(x_i))\) on the polynomial \( s \). The method fulfills two key properties, hidden and binding:

- Loosely speaking, hiding means that for a polynomial \( s \) of degree \( f \), any \( k = f + 1 \) shares suffice to reconstruct it via interpolation (API: \( \text{vssReconstruct} \)), and that no combination of \( f \) or less reveal any information about it.
- Binding means that every participant receives, in addition to its private share, a global commitment \( c \) to the polynomial \( s \) that binds the share it receives as a verifiable valid share of \( s \) (API: \( \text{vssVerify} \)).

In asynchronous settings, a dealer can wait for at most \( n - f \) participants to acknowledge receiving a valid share, before it inevitably may walk away. Note that it is possible for the dealer to walk away before all of the honest replicas have a valid share. The asynchronous VSS problem requires that if the dealer (or any participant) completes the share protocol, then every correct participant can eventually reconstruct its share using a distributed protocol with \( f + 1 \) correct participants: Participants contribute recovery information (API: \( \text{vssRecoverContrib} \)), which is validated by the recipient (API: \( \text{vssRecoverVerify} \)) and then combined to reconstruct the missing share (API: \( \text{vssRecover} \)).

AVSS in Byzantine Fault Tolerance

There are a few design goals to meet when using AVSS for state machine replication. For example, it is acceptable for a Byzantine client to lose the hiding guarantee. However, every sharing must always be binding otherwise the replicated state machine can be in an inconsistent state.

Additionally, there are many different Byzantine Fault Tolerance (BFT) algorithms in the literature that have been optimized to perform under certain circumstances. For example, some BFT algorithms \([7]\), \([8]\) have often incorporated a linear “fast-path” suitable for cases where there are few failures. In particular, this search for more optimized performance in the common case is something that we forsee continuing in the BFT literature.

Thus, it is important for a secret sharing scheme to have minimal overheads in the common case. In particular, a verifiable secret sharing scheme used in BFT must meet the requirement that \( \text{vssShare} \) only incurs \( O(1) \) overhead for the replicas. This ensures that the same techniques will be reusable for more scalable BFT protocols that work with larger clusters.

B. Existing Solutions

The seminal work by Shamir \([9]\) introduced the idea of employing polynomial interpolation, a technique that was used before for error correction codes, to share a secret with unconditional security. A line of work emanated from this result and addressed many additional features, such as share verifiability, asynchrony, and proactive share refresh.

Share verifiability tackles the problem of a malicious dealer that equivocates and maliciously shares values that are inconsistent. There are many such schemes with different properties, from classical works such as Feldman \([10]\) and Pedersen \([5]\)’s schemes to newer works such as Kate et al. \([1]\) and SCAPE \([11]\). sAVSS can take any of these works as input and construct a verifiable secret sharing scheme that also handles asynchrony.

Original solutions for asynchronous VSS in the information-theoretic setting were introduced in the context of Byzantine agreement and secure MPC \([12]\). They incur communication complexity of \( O(n^6 \log n) \) and message complexity \( O(n^5) \).

AVSS. The first practical asynchronous VSS solution in the computational setting was introduced by Cachin et al. \([13]\). We will refer to it by the name AVSS. To cope with asynchrony, AVSS uses a bivariate secret polynomial \( \hat{s}(\cdot, \cdot) \). Share \( i \) consists of two univariate polynomials, \( \hat{s}(i, \cdot) \), \( \hat{s}(\cdot, i) \), and so the dealer sends \( O(n) \) information to each participant. A missing \( i^{th} \) share can be reconstructed from \( f + 1 \) evaluations of \( \hat{s}(i, \cdot) \) and \( f + 1 \) evaluations of \( \hat{s}(\cdot, i) \), incurring linear communication overhead per recovery, for an overall recovery complexity of \( O(n^2) \) messages and \( O(n^3) \) bits.

Additionally, participants need to verify that all shares are bound to the same polynomial. AVSS makes use of Pedersen polynomial commitments \([5]\) for all polynomials \( \hat{s}(i, \cdot) \), \( \hat{s}(\cdot, i) \), \( i = 1..n \). This commitment scheme leverages the hardness of discrete log in a multiplicative group of order \( q \) with generator \( g \). A commitment \( c(v) \) to a value \( v \in \mathbb{Z}_q \) is a value \( g^v h^r \), where \( h \) is another element of the group and \( r \) is a secret drawn at random from \( \mathbb{Z}_q \). A Pedersen commitment to a polynomial \( s(\cdot) \in \mathbb{Z}_q[x] \) consists of a set of commitments to \( n \) values, i.e., \( c(s(\cdot)) = \{ (x_i, c(s(x_i))) \}_{i=1}^n \). Given any pair \((x, s(x))\), it is possible to verify that this point is on \( s(\cdot) \) using the commitment’s homomorphic properties, i.e., that
for any $v_1, v_2 \in \mathbb{Z}_q$, $c(v_1)c(v_2)$ is a valid commitment to $v_1 + v_2 \mod q$.

AVSS weaves into the sharing protocol the dissemination of commitments [14] while incurring message complexity $O(n^2)$ and communication complexity $O(n^3)$.

eAVSS-SC. Kate et al. [1] introduces a polynomial commitment that has constant size. This commitment scheme leverages the hardness of the $q$-Strong Diffie-Hellman assumption in some group with order $q$ where $q$ is a generator. In Kate et al., a commitment $c(s(\cdot))$ is defined as $c(s(\cdot)) = g^{s(\cdot)}$, where $\tau$ is unknown to all participants and $s(\tau)$ is the polynomial $s$ evaluated at $\tau$. To commit to a particular evaluation (or share) $s(i)$, the dealer also produces a witness, $g^{\frac{s(i) - \hat{s}(i)}{r}}$. Given any triple of share, witness and commitment, it is possible to verify the share is indeed the evaluation of the polynomial at that point using a bilinear map. The technique was employed by Backes et al. [15] to construct an asynchronous VSS scheme called eAVSS-SC that incurs both message and communication complexities $O(n^2)$.

In eAVSS-SC, a dealer chooses, in addition to the secret polynomial $s$, another $n$ polynomials $\hat{s}_i$, $i = 1..n$. $\hat{s}_i$ encodes share $i$ of $s$ for recovery purposes. Each of $s$, $\hat{s}_i$, has a constant-size polynomial commitment due to the scheme by Kate et al. [1]. The commitments are constructed such that a commitment of $\hat{s}_i$ validates it as a share of $s$. Using the homomorphism of the commitments, eAVSS-SC weaves into the sharing protocol the dissemination of commitments while incurring both message complexity and bit complexity $O(n^2)$.

The need for a new scheme. All of the above schemes fail to satisfy our two requirements above. In particular, the dealer computes $O(n^2)$ commitment values and sends $O(n^2)$ bits. Hence, to date, all asynchronous VSS solutions require a dealer, who wants to store a single secret to a system, to incur $O(n^2)$ communication complexities, which means that a replica must process $O(n)$ bits per request. This can be quite prohibitive for moderate $n$ values, e.g., $n = 1,000$, and infeasible for $n = 50,000$.

Fundamentally, prior asynchronous VSS schemes allow share recovery by having the dealer enumerate all pairwise responses between replicas during recovery. In other words, if replica $i$ is helping replica $i$ recover, the dealer has shared with $i$ the response to send to $i$. However, such an approach must require quadratic bandwidth on the dealer. To get around this difficulty, we make use of a distributed pseudorandom function (DPRF), allowing these recovery responses to be generated dynamically using information shared in the setup phase.

C. sAVSS

Our solution, named sAVSS (for ‘scalable AVSS’), is the first in which the replica work is constant per sharing in the common case.

Our approach is to use proactive secret sharing [16] in order to construct recovery polynomials to help a replica recover a share. Informally, suppose that a replica $i$ has share $s(i)$, which is simply a point on the polynomial $s$. In order to recover this share, the other replicas construct a Recovery Polynomial (RP). A Recovery Polynomial $s_i$ is random at every point except for $i$, where $s_i(i) = 0$. Thus, if the recovering replica $i$ receives shares of the sum of the original polynomial and the masking polynomial, $s_i(\cdot) + s(\cdot)$, it can recover its own share without obtaining any information about any other share.

However, adapting this technique to the asynchronous setting is nontrivial. First, each RP must be well defined for each $i$ and having every honest replica agree on the same polynomial in the presence of adversarial nodes is very expensive. We resolve this by having our dealer construct the recovery polynomials for each replica. Since we are already checking that the dealer is sharing the secret consistently, doing this for the recovery polynomials is easy. Additionally, a dishonest dealer does not need any privacy guarantees, which means that we only need to check for malicious behavior that hurts the consistency of the replica’s shares.

While this approach solves the need to agree on a recovery polynomial, the dealer still needs to share $n$ recovery polynomials which will incur a quadratic cost. To fix this problem, we first make the observation that there is no particular need for the constraint that $s_i(i) = 0$ as long as replica $i$ knows what the value of $s_i(i)$ is. Our scheme uses a distributed pseudorandom function in order to communicate the value of that point efficiently. Now that the value of $s_i(i)$ can be any random point, we can actually encode multiple replica’s recovery polynomials as one recovery polynomial. To ensure that all points of $s_i$ are random, we encode $f$ points into one recovery polynomial. Thus, the dealer only needs to construct four recovery polynomials for the entire cluster.

We now present a high level description of our protocol. Let a Recovery Polynomial $s_j$ encode $f$ secret values. The dealer partitions the secret shares of $s$ into $\ell = \lceil n/f \rceil$ groups, and uses $\ell$ RPs $s_j$, $j = 1..\ell$, to encode the corresponding groups. Every one of original $n$ shares of $s$ is encoded in one of the RPs. The dealer shares both $s$ and the $s_j$’s among the $n$ participants, and participants use the $s_j$’s for share recovery.

More specifically, an RP $s_j$ is a random polynomial of degree $f$ that has $f$ pre-defined points. For $(j-1)f \leq i <jf$, the recovery polynomial $s_j$ is constructed so that $s_j(i) = y_i$, where $y_i = F(i)$ for a DPRF $\mathcal{F}$ with reconstruction threshold $f$. (In our actual construction, $y_i = \mathcal{F}(r, i)$ for a random value $r$, to ensure that $s_j$ is distinct each time. But we elide $r$ for our discussion here.) The dealer shares each $s_j$ among the $n$ participants, as well as $s$.

To recover its share, participant $i$ probes other participants, to which each participant responds with its share of $s + s_j$ for $j = \lceil i/f \rceil$. Each participant $i$ can construct its response from its shares of $s$ and $s_j$. In addition, participant $i$ also responds with their shares of $\mathcal{F}(i)$, i.e., of the secret value $s_j(i)$. (In API terms, $\text{vssRecoverContribution}$ returns a share of $s + s_j$ and a share of $\mathcal{F}(i)$.) Participant $i$ then reconstructs $s + s_j$ in full and computes $(s + s_j)(i) - \mathcal{F}(i) = s(i)$ (API: $\text{vssRecover*}$). To verify a recovery share (\text{vssRecoverVerify*}), first participant $i$ validates each share of $s + s_j$ that it receives against the commitment $c(s + s_j)$, which it computes from $c(s)$ and $c(s_j)$. Then it validates the recovery result against the commitment
c(s). A technicality to note here is that in the Kate et al. commitment scheme, each share must be accompanied by a witness used for commitment validation. Participants need to recover witnesses for validation. The witness for participant i could be encoded into the RPs, at the expense of sharing polynomials over much larger fields. Luckily, this is not necessary; witnesses can be reconstructed leveraging homomorphism over the witnesses of the participants that participate in the recovery protocol (details in Section V-B2).

If validation fails, then participant i can prove to the other participants that the dealer is bad. In that case, different from AVSS and eAVSS-SC, participants expose the dealer’s secret.

The complexities incurred by different participants at different steps of the sAVSS protocol instantiated with Kate et al. [1] are as follows. A dealer provides each of n participants shares and constant-size commitments on ℓ+1 polynomials. The total communication complexity is O(tn), or simply O(n) in the usual case where ℓ is a (small) constant. Upon each recovery request, a participant sends a constant amount of information to the requestor, for a total O(t) communication for t requests. Finally, each participant requiring share recovery obtains shares from other participants incurring O(n) communication, for a total O(tn) communication for t requests.

III. SHARE RECOVERY IN VERIFIABLE SECRET SHARING

In this section, we detail our VSS protocol and its security. We begin with the definitions of distributed pseudorandom functions and verifiable secret sharing in Section III-A and Section III-B, respectively. We will then detail our goals (Section III-C), further assumptions on which our scheme builds (Section III-D), our construction (Section III-E) and its security (Section III-F).

Note that our proofs are applicable to any schemes that satisfy our descriptions below. Our particular instantiations are described in Section V. To highlight the generality of our descriptions, we instantiate our secret sharing scheme described in Section III-B in two ways [1], [5], of which one gives us the desired asymptotic complexity while the other uses more inexpensive cryptographic operations.

A. Distributed Pseudorandom Functions

A distributed pseudorandom function (DPRF) is a pseudorandom function that requires the cooperation of k replicas out of n total replicas to evaluate [17]. A DPRF \( \mathcal{F} \) provides the following interfaces, where \( |n| = \{1, \ldots, n\} \):

- \( \text{dprfInit} \) is a randomized procedure that returns a set of pairs \( \{(dpk_i, dsk_i)\}_{i \in [n]} \leftarrow \text{dprfInit}(1^n, k, n, D, R) \). Each dpk_i is public key, and each dsk_i is its corresponding private key.
- \( \text{dprfContribution} \) is a randomized procedure that returns a contribution d \( \leftarrow \text{dprfContribution}(dsk_i, x) \) if \( x \in D \) and failure (⊥) otherwise.
- \( \text{dprfVerify} \) is a deterministic procedure that returns a boolean value. We require that \( \text{dprfVerify}(dpk_i, x, d) \) returns true if d is output from \( \text{dprfContribution}(dsk_i, x) \) with nonzero probability, for the private key dsk_i corresponding to dpk_i.

- \( \text{dprfEval} \) is a deterministic procedure that returns a value \( y \leftarrow \text{dprfEval}(x, \{d_i\}_{i \in I}) \), where \( y \in R \), if \( x \in D \), \( |I| \geq k \) and for all \( i \in I \), \( \text{dprfVerify}(dpk_i, x, d_i) \) returns true. Otherwise, \( \text{dprfEval}(x, \{d_i\}_{i \in I}) \) returns ⊥.

Security for a distributed pseudorandom function is defined as follows. An adversary \( \mathcal{A} \) is provided inputs \( \{(dpk_i)_{i \in [n]}, k, n, D, R\} \leftarrow \text{dprfInit}(1^n, k, n, D, R) \). In addition, \( \mathcal{A} \) is given oracle access to \( n + 1 \) oracles. The first \( n \) oracles, denoted \( \langle O_{F,i} \rangle_{i \in [n]} \), each supports two types of queries. \( \mathcal{A} \) can invoke \( O_{F,i, \text{contribution}}(x) \), which returns \( \text{dprfContribution}(dsk_i, x) \), or it can invoke \( O_{F,i, \text{commitment}} \), which returns \( dsk_i \). The last oracle provided to \( \mathcal{A} \) is denoted \( O_{F} : D \rightarrow R \) and is instantiated as one of two oracles, either \( O_{F}^{\text{real}} \) or \( O_{F}^{\text{rand}} \). Oracle \( O_{F}^{\text{real}} \), on input \( x \), selects a subset \( I \subseteq [n] \) at random of size \( |I| = k \), invokes \( d_i \leftarrow O_{F,i, \text{contribution}}(x) \) for each \( i \in I \), and returns \( \text{dprfEval}(x, \{d_i\}_{i \in I}) \). Oracle \( O_{F}^{\text{rand}} \) is instantiated as a function chosen uniformly at random from the set of all functions from D to R. For any \( x \in D \), let \( I_x \) be the oracle indices such that for each \( i \in I_x \), \( \mathcal{A} \) invokes \( O_{F,i, \text{contribution}}(x) \). Then, \( \mathcal{A} \) is legitimate if \( |I_x| < k \) for every x for which \( \mathcal{A} \) invokes \( O_{F}(x) \). Finally, \( \mathcal{A} \) outputs a bit. We say that the distributed pseudorandom function is secure if for all legitimate adversaries \( \mathcal{A} \) that run in time polynomial in \( \kappa \),

\[
P \left( \mathcal{A}^{(O_{F,i}^{\text{real}})}_{F, i \in [n]}, O_{F}^{\text{real}} \left( \langle dfpk_i \rangle_{i \in [n]}, k, n, D, R \right) = 1 \right) - P \left( \mathcal{A}^{(O_{F,i}^{\text{rand}})}_{F, i \in [n]}, O_{F}^{\text{rand}} \left( \langle dfpk_i \rangle_{i \in [n]}, k, n, D, R \right) = 1 \right) \]

is negligible in \( \kappa \).

B. Verifiable Secret Sharing

Verifiable Secret Sharing (VSS) is a way to share a secret so that it requires a coalition of k replicas out of n total replicas in order to reconstruct the secret. A VSS scheme provides the following interfaces:

- \( \text{vssInit} \) is a randomized procedure that returns \( \langle q, \{vpk_i, vsk_i\}_{i \in [n]} \rangle \leftarrow \text{vssInit}(1^n, k, n) \). Here, q is a prime of length \( \kappa \) bits. Each vpk_i is a public key, and each vsk_i is its corresponding private key.
- \( \text{vssShare} \) is a randomized procedure that produces \( \langle c, \{u_i\}_{i \in [n]} \rangle \leftarrow \text{vssShare}(s, q, \{vpk_i\}_{i \in [n]}) \). Here, \( s \in Z_q[x] \) is a degree k–1 polynomial, and q and \( \{vpk_i\}_{i \in [n]} \) are as output by vssInit. The value c is a commitment, and each \( u_i \) is a share.
- \( \text{vssVerify} \) is a deterministic procedure that returns a boolean. We require that \( \text{vssVerify}(vpk_i, c, u_i) \) return true if \( \langle c, u_i \rangle \) (i.e., with arbitrary \( \{u_j\}_{j \neq i} \)) is output from vssShare(s, q, \{vpk_j\}_{i \in [n]}) with nonzero probability.
- \( \text{vssReconstruct} \) is a deterministic procedure that returns a value \( s \leftarrow \text{vssReconstruct}(c, \{vpk_i, u_i\}_{i \in [n]}) \) where \( s \in Z_q[x] \) of degree k–1, if \( |I| \geq k \) and for all
\( i \in I \), \( \text{vssVerify}(\text{vpk}_i, c, u_i) \) returns true. Otherwise, \( \text{vssReconstruct}(c, \{\{\text{vpk}_i, u_i\}\}_{i \in I}) \) returns \( \perp \).

The security of a VSS scheme lies in its hiding and binding properties.

1. **Hiding:** A hiding adversary \( A_V \) is provided inputs \( q \) and \( \{\text{vpk}_i\}_{i \in \mathbb{N}} \), where \( (q, \{\{\text{vpk}_i, \text{vsk}_i\}\}_{i \in \mathbb{N}}) \leftarrow \text{vssInit}(\kappa, k, n) \), and access to \( n+1 \) oracles. The first \( n \) oracles are denoted \( (O_{V,i})_{i \in \mathbb{N}} \); each \( O_{V,i} \) is initialized with \( \text{vsk}_i \) and can be invoked as described below. The last oracle provided to \( A_V \) can invoke \( A \) with two inputs \( s_0, s_1 \in \mathbb{Z}_q \). When invoked, \( O_{V,i}^s \) generates a random \( s \in \mathbb{Z}_q \) of degree \( k-1 \) such that \( s(0) = s_b \) and performs \( (c, \{u_i\}_{i \in \mathbb{N}}) \leftarrow \text{vssShare}(s, q, \{\text{vpk}_i\}_{i \in \mathbb{N}}) \), providing \( c \) to \( A_V \) and \( \{u_i\}_{i \in \mathbb{N}} \) to \( O_{V,i} \). The oracles \( (O_{V,i})_{i \in \mathbb{N}} \) can be invoked by \( A_V \) as follows. \( A_V \) can invoke \( O_{V,i}.\text{contrib}(c) \), which returns the share \( u_i \) provided to \( O_{V,i} \) with commitment \( c \) by \( O_{V,\hat{i}}^c \). \( A_V \) can also invoke \( O_{V,i}.\text{commit}(c) \), which returns \( \text{vsk}_i \), and all \( \{c, u_i\} \) pairs received from \( O_{V,i}^c \). For any \( c \), let \( I_c \) be the oracle indices such that for each \( i \in I_c \), \( A_V \) invokes \( O_{V,i}.\text{commit}(c) \). Then, \( A_V \) is legitimate if \( |I_c| < k \) for every \( c \). Finally, \( A_V \) outputs a bit. We say that the VSS \( V \) is hiding if for all legitimate adversaries \( A_V \) that run in time polynomial in \( \kappa \),

\[
\Pr \left( \left( A_{V(i)}^{O_{V,i}}, i \right) \left( q, \{\text{vpk}_i\}_{i \in \mathbb{N}} \right) = 1 \right) - \Pr \left( \left( A_{V(i)}^{O_{V,i}}, i \right) \left( q, \{\text{vpk}_i\}_{i \in \mathbb{N}} \right) = 1 \right)
\]

is negligible in \( \kappa \).

2. **Binding:** A binding adversary \( A_V \) is provided inputs \( q, \{\{\text{vpk}_i, \text{vsk}_i\}\}_{i \in \mathbb{N}} \leftarrow \text{vssInit}(\kappa, k, n) \). \( A_V \) outputs \( \{u_i\}_{i \in I} \) and \( \{\hat{u}_i\}_{i \in \hat{I}} \). We say that VSS \( V \) is binding if for all binding adversaries \( A_V \) that run in time polynomial in \( \kappa \),

\[
\Pr \left( \text{vssReconstruct}(c, \{\{\text{vpk}_i, u_i\}\}_{i \in I}) = s \quad \land \quad \text{vssReconstruct}(c, \{\{\text{vpk}_i, \hat{u}_i\}\}_{i \in \hat{I}}) = \hat{s} \quad \land \quad s \neq \perp \land \hat{s} \neq \perp \land s \neq \hat{s} \right)
\]

is negligible in \( \kappa \), where the probability is taken with respect to random choices made in \text{vssInit} and by \( A_V \).

**C. Goals**

Given such a VSS scheme \( V \) and a DPRF \( \mathcal{F} \), our goal is to construct a new VSS scheme \( V^* \) that provides both a \text{vssInit}, \text{vssShare}, \text{vssVerify}, and \text{vssReconstruct} algorithms (denoted \text{vssInit}*, \text{vssShare}*, \text{vssVerify}*, and \text{vssReconstruct}*) for \( V^* \), respectively as defined in Section III-A as well as three more algorithms, denoted \text{vssRecoverContrib}*, \text{vssRecoverVerify}*, and \text{vssRecover}*. We allow the \text{vssShare} algorithm to accept additional arguments (a set of private keys for a DPRF) and to return an additional value \( r \) that is provided as input to all procedures except for \text{vssInit}*. The algorithms \text{vssRecoverContrib}*, \text{vssRecoverVerify}*, and \text{vssRecover} together permit a replica to recover its share from other replicas, and behave as follows:

- \text{vssRecoverContrib} is a randomized procedure that returns \( v^*_i \leftarrow \text{vssRecoverContrib}^*(c^*, r, \text{vpk}_i^*, u_i^*, i) \) where \( v^*_i \) is a recovery share with properties described below.
- \text{vssRecoverVerify} is a deterministic procedure that returns a boolean. \text{vssRecoverVerify}^*(c^*, r, v^*_i, \text{vpk}_i^*, i) must return true if \( v^*_i \) is output from \text{vssRecoverContrib}^*(c^*, r, \text{vpk}_i^*, u_i^*, i) \) if \( |I| \geq k \), \text{vssRecoverVerify}^*(c^*, r, v^*_i, \text{vpk}_i^*, i) \) returns true for all \( i \in I \), and \text{vssRecover}^*(\text{vpk}_i^*, c^*, r, u_i^*) \) returns true. Otherwise, \text{vssRecover}^*(c^*, r, \{\{\text{vpk}_i^*, v_i^*\}\}_{i \in I}, i, \text{vpk}_i^*) \) returns \( \perp \).

Due to the additional interfaces above, we change the definition of hiding security as follows. Each oracle \( O_{V*,i} \) additionally supports a query \( O_{V,i}.\text{recover}(c^*, i) \) that returns \( v^*_i \leftarrow \text{vssRecoverContrib}^*(c^*, r, \text{vpk}_i^*, u_i^*, i) \). For any \( c^* \), let \( I_{c^*} \) be the oracle indices such that for each \( i \in I_{c^*} \), \( A_V \) invokes \( O_{V,i}.\text{commit}(c^*) \), or \( \{O_{V,i}.\text{recover}(c^*, i)\}_{i \in I} \) where \( |I| \geq k \). Then, \( A_V \) is legitimate if \( |I_{c^*}| < k \) for every \( c^* \).

**D. Assumptions on Underlying VSS**

Our construction combines an existing VSS scheme with a DPRF for which, if \( (q, \{\{\text{vpk}_i, \text{vsk}_i\}\}_{i \in \mathbb{N}}) \leftarrow \text{vssInit}(\kappa, k, n) \), then \( R = \mathbb{Z}_q \) and each share \( u_i \) output from \text{vssShare} is in \( \mathbb{Z}_q \). In addition, we require that the VSS offer additional procedures, as follows.

- There is a procedure \text{vssMakeSecret} that creates \( s \leftarrow \text{vssMakeSecret}(q, \{(x_i, y_i)\}_{i \in I}) \) where \( s \in \mathbb{Z}_q \) is of degree \( |I| \), and so that if \( \langle c, \{u_i\}_{i \in \mathbb{N}} \rangle \leftarrow \text{vssShare}(s, q, \{\text{vpk}_i\}_{i \in \mathbb{N}}) \) then \( u_i = y_i \) for any \( i \in I \).
- There is a procedure \text{vssCombineCommitments} such that if \( \text{vssReconstruct}(c, \{\{\text{vpk}_i, u_i\}\}_{i \in I}) = s \) then \( \text{vssReconstruct}(\hat{c}, \{\text{vpk}_i, \hat{u}_i\}_{i \in \hat{I}}) = \hat{s} \) where \( s, \hat{s} \neq \perp \), and if \( \hat{c} \leftarrow \text{vssCombineCommitments}(c, \hat{c}) \) then \( \text{vssReconstruct}(\hat{c}, \{\text{vpk}_i, (u_i + \hat{u}_i)\}_{i \in I}) = s + \hat{s} \).

An example of such a scheme is that due to Kate et al. [1].

**E. VSS Scheme with Recovery**

Below we describe the procedures that make up the VSS scheme \( V^* \). The algorithms are expressed in terms of constants \( n \) (the number of replicas), \( k \) (the reconstruction threshold), and \( \ell = \lceil n/(k - 1) \rceil \). Each share \( u_i^* \) and commitment \( c^* \) is a zero-indexed vector of \( \ell + 1 \) elements. We denote the \( j \)-th element of each by \( u_{i,j}^* \) and \( c_{i,j}^* \), respectively, for \( 0 \leq j \leq \ell \). Line numbers below refer to Figure 1.
vssInit* initializes the underlying VSS \( \mathcal{V} \) in line 2 as well as a DPRF \( \mathcal{F} \) in line 3. The public key \( vpk_i^* \) for replica \( i \) consists of its public key \( vpk_i \) for \( \mathcal{V} \) and its public key \( dpk_i \) for \( \mathcal{F} \) (line 7) and similarly for the private key \( vsk_i^* \) (line 6).

vssShare* is modified to take in all of the private keys \( \{dsk_i\}_{i \in [n]} \) for the DPRF \( \mathcal{F} \), as well as the other arguments included in its definition in Section III-B. (For this reason, our construction requires each node to have a distinct set of parameters for its sharings, i.e., produced by its own call to vssInit*.) This enables the dealer to evaluate \( \mathcal{F} \) itself, which it does on \( (r,i) \) for each \( i \in [n] \) (lines 10-12), where \( r \) is a new, random \( \kappa \)-bit nonce (line 9). The resulting values \( \{y_i\}_{i \in [n]} \) are divided into \( t \) groups of size \( k - 1 \), each group being used to construct a set of \( k - 1 \) points 

\[
\text{Points}_t \leftarrow \{(i, y_i) \mid (j - 1)(k - 1) < i \leq j(k - 1)\}
\]

on which vssMakeSecret is invoked (line 14). The resulting \( s_j \in \mathbb{Z}_q[x] \) is then shared using \( \mathcal{V} \) (line 16). Recall that by the definition of vssMakeSecret, each \( u^*_j[i] \) thus produced satisfies \( u^*_j[j] = y_i \). Of course, the input secret \( s \) is also shared (line 17). The results of these sharings are grouped according to replica index \( i \) and returned as \( u^*_i \) for each \( i \in [n] \), along with all of the shared commitments to \( c^* \) and the nonce \( r \) (line 18).

vssVerify* and vssReconstruct* operate in the natural way. vssVerify* verifies the commitment \( c^*[0] \) and share \( u^*_i[0] \) (line 21) in the sharing of \( s \), as well as verifying the commitment \( c^*[j] \) and share \( u^*_j[j] \) (line 23) in the sharing of \( s_j \). In addition, it verifies (intuitively) that \( u^*_j[j] = y_i \) (line 25). The latter two verifications are skipped if \( u^*_j[1] = \bot \) (line 23), which occurs if the share \( u^*_i \) was recovered (see below). In this case, \( u^*_j[1] = \bot \) for all \( j \in [t] \) (or should be, and so any \( j \in [t] \) for which \( u^*_j[1] \neq \bot \) is just ignored). vssReconstruct* simply uses vssVerify* to verify each share \( u^*_i \) provided as input (line 33) and then submits \( c^*[0] \) and the inputs \( \{(vpk_i, u^*_i[0])\}_{i \in [t]} \) to vssReconstruct to reconstruct \( s \) (line 35).

vssRecoverContribution \((c^*, r, vsk^*_i, u^*_i, i)\) is invoked at replica \( i \) to construct its contribution to enable replica \( i \) to reconstruct its share \( u^*_i \). vssRecoverContribution returns \( u^*_i[0] \) blinded by \( u^*_j[1] \) (line 41) where \( j \leftarrow \lceil i / (k - 1) \rceil \). Then, so that replica \( i \) can recover its share of the original secret, replica \( i \) also returns its share of the DPRF scheme \( \mathcal{F} \) evaluated at \( (r, i) \) (line 39).

vssRecoverVerify \((c^*, r, v_i^*, vpk^*_i, i)\) is executed by replica \( i \) to verify that replica \( i \) performed vssRecoverContribution* correctly. The output of vssRecoverContribution* contributed by replica \( i \) is passed into vssRecoverVerify* as \( v_i^* \) and is parsed into its constituent components in line 44. First, the DPRF contribution \( d_i \) is checked on line 36 to ensure that it corresponds to a correct evaluation of the DPRF scheme \( \mathcal{F} \) at the point \((r, i)\). vssRecoverVerify* then combines the commitments (line 38) and uses vssVerify (line 39) to check that the blinded share \( u \) was created correctly. If both checks pass, then vssRecoverVerify* returns true.

vssRecover* \((c^*, r, \{\langle vpk_i^*, v_i^* \rangle\}_{i \in [t]}, \hat{i}, vpk^*_i)\) is executed at replica \( \hat{i} \) to recover its share \( u^*_i \). In particular, \( u^*_i[0] \) will be a share of the original polynomial for \( \hat{i} \). vssRecover* first invokes vssRecoverVerify* to make sure that the share sent by each replica \( i \in I \) is correct (line 55). vssRecover* then leverages vssReconstruct (line 60) to reconstruct a polynomial \( s \in \mathbb{Z}_q[x] \) that is the sum of the polynomial originally shared in vssShare* that resulted in commitment \( c^*[0] \) and the \( j \)-th masking polynomial \( s_j \) that resulted in commitment \( c^*[j] \), where \( j = \lceil i / (k - 1) \rceil \). vssRecover* then evaluates \( s(i) \) and subtracts \( s_j(i) = \text{dprfEval}(\langle r, \hat{i} \rangle, \{d_i\}_{i \in I}) \) (lines 60-62) to obtain \( u^*_i[0] \).

**F. Security**

Below, we prove that our modified VSS scheme still satisfies the security properties guaranteed by the underlying VSS protocol.

**a) Hiding:** Suppose that the underlying DPRF scheme \( \mathcal{F} \) and VSS scheme \( \mathcal{V} \) are secure, and let \( \mathcal{A}_V \) be a hiding adversary for \( \mathcal{V}^* \). We claim that if \( \mathcal{A}_V \) is legitimate, then for any commitment \( c^* \), the set of indices \( I \) for which \( \mathcal{A}_V \) obtains the shares \( \{u^*_i[0]\}_{i \in I} \) produced in line 17 (i.e., in its invocation of \( \mathcal{O}_{\mathcal{V}^*} \), that returned \( c^* \)) satisfies \( |I| < k \). To see why, note that \( \mathcal{A}_V \) can obtain \( u^*_i[0] \) for any \( i \) in one of three ways:


(i) by invoking \( \mathcal{O}_{\mathcal{V}^*}.\text{commit} \); (ii) by invoking \( \mathcal{O}_{\mathcal{V}^*}.\text{contribute} \); or (iii) by invoking \( \mathcal{O}_{\mathcal{V}^*}.\text{recover} \) at each \( i \in I \) where \( |I| \geq k \), in which case \( \mathcal{A}_V \) can recover \( u^*_i[0] \) using the vssRecover* routine (line 62). Critically, invoking \( \mathcal{O}_{\mathcal{V}^*}.\text{recover} \) at each \( i \in I \) where \( |I| < k \) yields no useful information about \( u^*_i[0] \), since when \( |I| < k \), the value \( y_i \) (line 61) and so \( u^*_i[0] = s(i) - y_i \) (line 62) cannot be predicted nonnegligibly better than guessing it at random, due to the security of the DPRF scheme \( \mathcal{F} \) (i.e., (1)). Because \( \mathcal{A}_V \) is legitimate, it thus obtains \( u^*_i[0] \) for only fewer than \( k \) values of \( i \), and so by the security of \( \mathcal{V} \), its success (in the sense of (2)) is negligible in \( \kappa \).

**b) Binding:** A binding adversary \( \mathcal{A}_V \) is provided inputs \( \langle q, \{\langle vpk_i^*, vpk^*_i \rangle\}_{i \in [n]} \rangle \leftarrow \text{vssInit}^*(\kappa, k, n) \), and succeeds if it outputs \( c^*, \{u^*_i[0]\}_{i \in I} \) and \( \{\hat{u}^*_i\}_{i \in I} \) for which

\[
\begin{align*}
\text{vssReconstruct}^*(c^*, r, \{\langle vpk_i^*, u^*_i \rangle\}_{i \in I}) &= s \\
\land \quad \text{vssReconstruct}^*(c^*, r, \{\langle vpk_i^*, \hat{u}^*_i \rangle\}_{i \in I}) &= \hat{s} \\
\land \quad s &\neq \bot \land \hat{s} \neq \bot \land s \neq \hat{s}
\end{align*}
\]

Let \( s \) and \( \hat{s} \) be values satisfying this condition. Then,

\[
\begin{align*}
s &= \text{vssReconstruct}(c^*[0],\{\langle vpk_i, u^*_i[0]\rangle\}_{i \in I}) \\
\hat{s} &= \text{vssReconstruct}(c^*[0],\{\langle vpk_i, \hat{u}^*_i[0]\rangle\}_{i \in I})
\end{align*}
\]

where \( \langle q, \{\langle vpk_i, vsk_i \rangle\}_{i \in [n]} \rangle \leftarrow \text{vssInit}(\kappa, k, n) \) (see lines 2 and 16). That is, breaking binding for \( \mathcal{V}^* \) implies breaking binding for \( \mathcal{V} \), and so if \( \mathcal{V} \) ensures the binding property, then so does \( \mathcal{V}^* \).

**IV. A PRIVATE, BYZANTINE FAULT-TOLERANT KEY-VALUE STORE**

In this section we describe how to incorporate sAVSS into PBFT [6]. PBFT works with arbitrary applications, but for simplicity, we will use a simple application that captures how
secret values are handled in our PBFT extension: a private, replicated key/value service that tolerates Byzantine faults of replicas. The service provides two APIs, replicated key/value service that tolerates Byzantine faults and a private, signed by its sender so that its origin is known, subject to a private, signed by its sender so that its origin is known, subject to a private, signed by its sender so that its origin is known, subject to.

We represent the state of the replicated service as a key-value store. Every client in the system is allowed to view all keys in the store. However, the service maintains a (potentially dynamic) access control policy that specifies for every client the values it is allowed to open. Under these assumptions, we provide the standard guarantees provided by a Byzantine fault tolerant protocol:

- **Linearizability** [20]. If a client sends a request to the replicated service, then the service’s response is consistent with an execution where the client’s request was executed instantaneously at some point between when the request was sent and the response was received.
- **Liveness.** If the network is synchronous, then every client request will get a response.

In addition to these standard properties, our design offers the following privacy property:

- **Privacy.** A value written to a key by a correct client where the access-control policy prohibits access by any faulty client, remains hidden from $f$ Byzantine servers.

A. Setup and Log

In addition to setting up authenticated communication channels among all parties, in a setup phase, $vssInit^*$ is called for.
every client in the system and is part of the public/private key infrastructure. The client takes the role of the dealer in vssInit∗ while each replica takes the role of a participant. In particular, each client knows the secret key for all replicas corresponding to its invocation of vssInit∗.

Every replica stores a full copy of the K-V store. For each key there are two value entries, a public value (keyed K-pub) and a private value (keyed K-priv). A replica maintains a bounded log of pending commands, waiting to be committed. The size of the log cannot grow beyond a certain system-wide parameter W. Once an entry in the log becomes committed, it is applied to the K-V store. No replica or leader starts handling slot j + W in the sequence of commands before it learns that 2f + 1 replicas have committed all commands up to j. When a replica learns that all commands up to j have been committed by 2f + 1 replicas, it evicts them from the log.

B. Views

Our solution employs a classical framework [6], [21] that revolves around an explicit ranking among proposals via view numbers. Replicas all start with an initial view, and progress from one view to the next. They accept requests and respond to messages only in their current view.

In each view there is a single designated leader. In a view, zero or more decisions may be reached. This strategy separates safety from liveness: It maintains safety even if the system exhibits arbitrary communication delays and again up to f Byzantine failures; it provides progress during periods of synchrony.

If a sufficient number of replicas suspect that the leader is faulty, then a view change occurs and a new leader is elected. The mechanism to trigger moving to a higher view is of no significance for safety, but it is crucial for liveness.

C. Common Mode Protocol

A client put is split into two parts, public and private. The public part is concerned with setting sequence ordering of requests. The private part stores a private value.

More specifically, in a put(K, V) request, the client privately shares V by via vssShare∗, and sends the corresponding shares to every replica in the system. The public part of put(K, V) consists of a client sending a put(K, cv) request to the current leader. cv is a global commitment to the polynomial s that binds the shares of each replica as a verifiable valid share of s.

The leader waits until its local log has length < W. It then extends its local log with the put request, and sends a pre-prepare (ordering-request) containing its log tail. We discuss below the protocol for a leader to pick an initial log when starting a new view.

A replica accepts a pre-prepare from the leader of the current view if it has valid format, if it extends any previous pre-prepare from this leader, if its log has fewer than W pending entries, and if the replica received a valid share corresponding to c. If the leader pre-prepare message has a valid format, but the replica did not receive the corresponding share for it, it starts a timer for share-recovery (see more below).

Upon accepting a pre-prepare, a replica extends its local log to include the new request and broadcasts a prepare message to all replicas that includes the new log tail. Replicas wait to collect a commit-certificate, a set of 2f + 1 prepare responses for the current log tail. Then the replica broadcasts a commit message carrying the commit-certificate to the other replicas. A decision is reached in a view on a new log tail when 2f + 1 distinct replica have sent a commit message for it.

When a replica learns that a put(K, V) request has been committed to the log, it inserts to its local key-value store two entries, a global entry ((K||public), cv) containing the global commitment to V, and a private entry ((K||private), share) containing the replica’s private share. The replica then responds to the client with an put acknowledgement message containing K and c. A client waits to receive 2f + 1 put responses to complete the request. Figure 2a depicts the put io path, and Figure 2b the put io path when shares are missed.

The client get(K) protocol consists of sending the get request to the current leader. The pre-prepare, prepare and commit phases of the ordering protocol are carried as above, without the need to wait for shares. At the final stage, when a replica executes the get requests, it returns its share to the client in a response. If the replica is missing its share, it initiates the share-recovery protocol. The client waits to receive f + 1 valid get responses. It uses vssVerify∗ to verify each response, and vssReconstruct∗ to reconstruct the secret value from the responses.

D. Share-Recovery Protocol

There are several circumstances in the protocol when a replica discovers it is missing its private share of a re-
quest and needs to recover it (see above). To initiate share-
recovery, a replica broadcasts a recovery request. Other repli-
cas respond to a share-recovery request with the output of vssRecoverContribution. After receiving a response, the original replica uses vssRecoverVerify* to check the response. If the response is valid, it is stored and if it is invalid, it is dropped. When it receives $f+1$ valid responses, the replica uses vssRecover* to recover its missing secret share.

E. Common mode performance

The common mode protocol incurs the following perfor-
mance costs. The client interaction with the BFT replicated
service is linear, since it needs to populate all replicas with
shares. Additionally, the client collects $f+1$ responses from
servers.

The communication among the replicas to achieve an order-
ing decision is quadratic. There are several practical variants
of BFT replication that achieve linear communication during
periods of synchrony and when a leader is non-faulty (e.g., [8].
[22], [23]) These improvements are left outside the scope of
this paper, however, our linear AVSS protocol is designed
so it can be incorporated within them without increasing the
asymptotic complexity of the common mode.

In terms of latency, the sharing protocol is non-interactive
and single-round, hence it can be performed concurrently
with the leader broadcast. Recovery incurs extra latency since
each replica must ask at least $f+1$ correct replicas for
their contribution. In the original BFT protocol, recovering a
missing request only requires asking 1 correct replica for the
request data. In both cases, the recovery protocol is interactive
and single-round, so there are no asymptotic increases in
latency. However, in practice, there will be a difference in
latency between the two scenarios.

Remaining Items. For completeness, we describe the view
change and state transfer protocols in Appendix B. A proof of
linearizability, liveness and privacy appears in Appendix A.

V. INSTANTIATION AND IMPLEMENTATION

We first see how to instantiate our distributed pseudorandom
function using the techniques presented in Naor, et al. [17].
We then cover two ways to instantiate our verifiable scheme,
using Pedersen secret sharing [5] and one from Kate, et
al. [1]. Finally, we go over implementation details and the
programming API that we expose for application developers.

A. Distributed Pseudorandom Function Instantiation

Our distributed pseudorandom function $\mathcal{F}$ consists of four
algorithms: dprfInit, dprfContribution, dprfVerify, and dprfEval.
Our implementation defines them as follows:

- $\text{dprfInit}(1^\kappa, k, n, D, \mathbb{Z}_q)$, first chooses a generator $h$ of
  $\mathbb{G}$ of order $q$. A $k$ out of $n$ secret sharing of a private
  value $\alpha \in \mathbb{Z}_q^*$ is produced using Shamir secret sharing
  [9], of which the shares are $\{\alpha_i\}_{i \in [n]}$. $\text{dprfContribution}_i$ is set
to $\langle h^\alpha, \{ h^{\alpha_i} \}_{i \in [n]} \rangle$ for all $\alpha_i \in \mathbb{Z}_{q^n}$.
  $\text{dprfContribution}_i$ is set to $\beta_i$ for all $\beta_i \in \mathbb{Z}_{q^n}$.
  $\text{dprfEval}(x)$ first computes $f_i(x) = H(x)^{\alpha_i}$
  where $H: \{0,1\}^* \to \mathbb{G}$ is a hash function that is
  modeled as a random oracle. Here, $\alpha_i$ is obtained from the
  $\text{dsk}_\alpha$. Let $r$ be a randomly generated element of $\mathbb{Z}_q^*$. Then,
  we let $c_i \leftarrow H'(H(x), h, f_i(x), h^{\alpha_i}, H(x)^\alpha, h^\alpha)$,
  where $H': \{0,1\}^* \to \mathbb{Z}_q^*$ is a hash function modeled as a
  random oracle. We set $z_i \leftarrow \alpha_i + r \mod q$. $\text{dprfContribution}_i$
  then outputs $(f_i(x), z_i, \alpha_i)$.

- $\text{dprfVerify}(dpk_{i, x}, d)$ first extracts $f_i(x)$, $\alpha_i$, and $z_i$
  from $d$. Then, $h$ and $h^{\alpha_i}$ are extracted from
  $\text{dsk}_\alpha$. Finally, $\text{dprfVerify}$ returns true if $c_i =
  H'(H(x), h, f_i(x), h^{\alpha_i}, H(x)^\alpha, f_i(x)^\alpha, h^\alpha)$. $\text{dprfVerify}_i$
  first verifies each $d_i$ using $\text{dprfVerify}$. If any of the verifications returns false,
  then $\text{dprfEval}$ returns $\bot$. Otherwise, we extract $f_i(x)$ values from each $d_i$. Since the exponents of $f_i(x)$ were
  shared in the exponent using Shamir secret sharing, $\text{dprfEval}$ uses Lagrange interpolation in the exponent to
  get the value of $\mathcal{F}$ at $x$, hashes it into an element of $\mathbb{Z}_q^*$
  and outputs that value.

B. Verifiable Secret Sharing Instantiations

For a verifiable secret sharing scheme, we require the
functions vssInit, vssShare, vssVerify, and vssReconstruct. To
be used in our construction, we require a few additional func-
tions, namely: vssMakeSecret and vssCombineCommitments.
To support addition, we simply require replicas to add the
respective shares, by definition of vssCombineCommitments.
We now define all of these functions for two secret sharing
schemes.

1) Pedersen Secret Sharing: We describe how to fit the
secret sharing scheme from Pedersen [5] into our framework.

- $\text{vssInit}(1^\kappa, k, n)$ first chooses a safe prime $q = 2q’ + 1$
  at least $\kappa$ bits in length, for $q’$ a prime. Also, we let $g$
  and $h$ be two distinct generators of the quadratic residues
  $QR(\mathbb{Z}_q^*)$ of $\mathbb{Z}_q^*$ such that $\log_g(h)$ is unknown. Then, $\text{vpk}_i$
  is set to $(g, h)$ for all $i$. $\text{vsk}_i$ is set to $\bot$ for all $i$. The return value of $\text{vssInit}$ is
  $(q, \{\text{vpk}_i, \text{vsk}_i\}_{i \in [n]}^n)$.

- $\text{vssShare}(s, q, \{\text{vpk}_i\}_{i \in [n]}^n)$ first extracts the public key
  and gets $g$ and $h$ as defined in $\text{vssInit}$. Set $s_j$ to be the coefficient of the $x^j$ term in $s$ and $\beta_i(x)$ is the evaluation
  of $s$ at point $i$. Pick $t_i \in \mathbb{Z}_q[x]$ to be a random polynomial of
  degree $k-1$. Similarly, we let $t_j$ to be the coefficient of $x^j$ term in $t$ and $\beta_i(t)$ be the evaluation of $t$ at point $i$. Now,
  we set $u_i$ to be $\langle s, \beta_i(t) \rangle$. Here, all linear operations on
  $u_i$ values are just done element-wise. Set $c$ to be
  $\{g^s, h^\beta_i\}_{i \in [n]}$. Then, $\text{vssShare}$ returns $\{c, \{u_i\}_{i \in [n]}\}$.

- $\text{vssVerify}(\text{vpk}_i, c, u_i)$ first extracts $\{g^s, h^\beta_i\}_{i \in [n]}$ from $c$. Then, $s(i), \beta_i(t)$ is extracted from $u_i$. We return true if
  $g^{s(i)}h^{\beta_i(t(i))} = \prod_{j=0}^{k-1}(g^{s_j}h^{\beta_j(t(i))})^i$ and false otherwise.

- $\text{vssReconstruct}(c, \{\text{vpk}_i, u_i\}_{i \in I}^n)$ first calls $\text{vssVerify}(\text{vpk}_i, c, u_i)$ for all $i \in I$. If all of $\text{vssVerify}$ calls return true, then we continue. Otherwise, $\text{vssReconstruct}$
  returns $\bot$. Then, we extract $s(i), \beta_i(t)$ from each $u_i$. Finally, we simply do a Lagrange interpolation in order to
  identify the unique degree $k-1$ polynomial in $\mathbb{Z}_q[x]$.
that goes through the points \((i, s(i))\) for all \(i \in I\) and return that value.

- **vssMakeSecret**\((q, \{(x_i, y_i)\}_{i \in I})\) does a Lagrange interpolation in order to identify the unique degree \(k - 1\) polynomial in \(Z_q[x]\) that goes through \((x_i, y_i)\) and returns that as \(s\).

- **vssCombineCommitments**\((c, \hat{c})\) extracts \(\{g^{s(i)} h^{t(i)}\}_{j \in [k]}\) from \(c\) and \(\{g^{\hat{s}(j)} h^{\hat{t}(j)}\}_{j \in [k]}\) from \(\hat{c}\). Then, it returns \(\hat{c} = \{(g^{s(i)}) (g^{\hat{s}(j)})\}_{j \in [k]}\).

2) **Kate et al. Secret Sharing:** We describe how to fit the secret sharing scheme from Kate et al. [1] into our framework. Note that this secret sharing scheme also has a witness, which proves that a particular share is consistent with the polynomial commitment. Witnesses are additively homomorphic as well and can be manipulated the same way as the shares can. In particular, we can perform polynomial interpolation in order to take a set of \(f\) witnesses and obtain the witness for any other share. Additionally, we only need to send a witness when we transmit the corresponding share. Thus, witnesses only increase the communication overhead by a constant factor. In the description below, we assume that we have the witness corresponding to each share.

- **vssInit**\((k, n)\) first chooses a safe prime \(q\) at least \(\kappa\) bits in length. Then, we initialize two groups of order \(q\): \(G\) and \(G_t\) such that there exists a bilinear map \(e: G \times G \rightarrow G_t\). We then generate a \(\tau \in Z_q\) and pick a generator \(g \in G\). Set \(vpk_i\) to be \(\langle G, G_t, e, g, \{g^j\}_{j \in [f]} \rangle\) and \(vsk_i\) to be \(\bot\) for all \(i\). Then, we delete \(\tau\). Finally, **vssInit** returns \(\{q, \{\{vpk_i, vsk_i\}\}_{i \in [n]}\}\).

- **vssShare**\((s, q, \{vpk_i\}_{i \in [n]}\) first extracts the public key and gets \(g\) and \(\{g^j\}_{j \in [f]}\). Let \(s_j\) be the coefficient of the \(x^j\) term in \(s\) and \(s(i)\) be the evaluation of \(s\) at point \(i\). We now compute \(g^{s(i)}\) by computing \(\prod_{j=0}^{f-1} (g^j)^{s_j}\) and assign it to \(c\). Now, using polynomial division, we can compute the coefficients of \(\frac{(x^j - s(i))}{x - i}\) which will allow us to compute \(g^{\frac{d(x) - s(i)}{x - i}}\). Finally, **vssShare** returns \(\{s, q, \{vpk_i\}_{i \in [n]}\}\).

- **vssVerify**\((vpk_i, c, u_i, g, g^q, q)\) first extracts \(g^{s(i)}\) from \(c\), \(s(i)\) from \(u_i\), and \(e, g, g^q\) from \(vpk_i\). We also have access to the value \(g^{\frac{d(x) - s(i)}{x - i}}\) since the witness for the share is transmitted along with the share. Then, **vssVerify** returns true if \(e(g^{s(i)} g^q)\) equals \(e(g^{\frac{d(x) - s(i)}{x - i}}, g^q)) e(g, g)^{s(i)}\) and false otherwise.

- **vssReconstruct**\((c, \{vpk_i, u_i\}_{i \in I}\) first calls **vssVerify**\((vpk_i, c, u_i\) for all \(i \in I\). If all of **vssVerify** calls return true, then we continue. Otherwise, **vssReconstruct** returns \(\bot\). Then, similarly to the Pedersen scheme, we extract \(s(i)\) from each \(u_i\) and do Lagrange interpolation to identify the original polynomial and return that value.

- **vssMakeSecret**\((q, \{(x_i, y_i)\}_{i \in I}\) works the same way as it does in the Pedersen scheme above.

- **vssCombineCommitments**\((c, \hat{c})\) first extracts \(g^{s(i)}\) from \(c\) and \(g^{\hat{s}(j)}\) from \(\hat{c}\). We then set \(\hat{c}\) to \((g^{s(i)})(g^{\hat{s}(j)})\) and return that value.

**C. Implementation**

We implement a secret shared BFT engine by layering PBFT [6] with our secret sharing scheme. Our implementation consists of 4700 lines of Python and 4800 lines of C. We optimize our design for multicore environments, with one network thread running on a core which never blocks. Additionally, we use one thread for every other core in order to do all cryptographic operations that are required by PBFT and our secret sharing scheme. We use elliptic curve signatures with the secp256k1 library for all signature checking operations and the Relic library [24] for all other cryptographic operations related to our scheme. We also make a few optimizations for the Kate and Pedersen secret sharing schemes in order to make them faster.

a) **Kate et al.:** Kate et al.’s secret sharing scheme lends itself for extensive caching during setup time. Once the powers \(g^{s(i)}\) are known for all \(j\), we construct precomputation tables for each coefficient so that all exponentiations during runtime leverage these tables for efficiency. In the sharing step, we first use the well-known Horner’s method to optimize the share evaluation. However, we also note that each intermediate value obtained in Horner’s method when evaluating \(s(i)\) is also the coefficient of the quotient polynomial \(\frac{d(x) - s(i)}{x - i}\) which means that we can do the necessary division required for free before using our precomputation tables to evaluate the quotient at \(\tau\). In the share verification step, we note that every verification requires the value of \(e(g^q, g)\) so we can precompute that as well to save a bilinear map operation. Also in the share verification phase, we note that the division of \(g^q\) only has \(n\) possible values which means that we can precompute all of these values as well. Finally, when doing Lagrange interpolation, we know that the indices range from \(0\) to \(n - 1\) and in the denominator, we need to compute the produce of differences of these indices. Thus, to avoid taking inverses, we simply take inverses of all \(n\) values of the differences which means that during runtime, we only have to do multiplications.

b) **Pedersen:** Pedersen’s secret sharing scheme does not lend itself to as much caching since most of the values are unknown beforehand. However, we do generate precomputation tables for both \(g\) and \(h\) during setup and compute the inverses to make Lagrange interpolation easier.

**VI. Evaluation**

Our evaluation seeks to answer two basic questions. First, we investigate the costs of each API call in our secret sharing scheme. Then, we look at how expensive it is to incorporate our secret sharing scheme into a Byzantine Fault Tolerant key value store. We instantiate sAVSS using the DPRF in Naor et al. [17] and two different VSS schemes: Pedersen’s VSS scheme [5] and Kate et al.’s VSS scheme [1]. We call the Pedersen instantiation Ped-sA VSS and the Kate et al. instantiation KZG-sA VSS. Note that the latter scheme has constant overhead on the replicas per sharing, while Ped sAVSS has linear overhead on the replicas but with cheaper cryptographic operations. We instantiate the BFT algorithm using PBFT [6].
and build a Byzantine Fault Tolerant key value store with secret shared state.

Our implementation uses the Relic [24] cryptographic library and, for our elliptic-curve algorithms, the BN_P254 curve.

A. Microbenchmarks

For our microbenchmarks, we evaluate each function in our full asynchronous verifiable secret sharing scheme. We vary the number of replicas from 4 to 211 and measure the latency and throughput of each operation. We use EC2 c5.xlarge instances in order to run our microbenchmarks, which have 4 virtual CPUs per instance.

The module that implements our secret sharing scheme optimizes for throughput, while compromising slightly on latency. Each API call runs on a single core; the task is run to completion and the result is returned in the order that the tasks were enqueued. This maximizes for throughput due to the lack of cross core communication, but at the expense of request latency as many of the underlying cryptographic operations can leverage multicore environments to execute faster.

Each microbenchmark ran for at least 60 seconds and collected at least 30 samples. Before computing the final statistic, we ignored any requests that were completed in the first 10 seconds and the last 10 seconds of the run. We report the aggregate throughput during the run and the mean and standard deviation of the latency of each request completed in our run.

1) vssShare* Microbenchmark: Figure 3a shows that Ped-sAVSS can sustain more sharings per second than KZG-sAVSS for all cluster sizes. At \( n = 4 \), Ped-sAVSS does 1.2 times more sharings per second than KZG-sAVSS. However, this difference increases as the cluster size increases, with Ped-sAVSS sustaining 58 times more sharings per second than KZG-sAVSS at \( n = 199 \).

This performance difference between Ped-sAVSS and KZG-sAVSS is due to the underlying VSS scheme. KZG-sAVSS computes witnesses for each share, which involves evaluating a polynomial in the elliptic curve group. Additionally, we see that the throughput decrease is quadratic since evaluating each share (or witness) takes \( O(n) \) CPU time and there are \( n \) shares so vssShare* takes \( O(n^2) \) time for both KZG-sAVSS and Ped-sAVSS. For the above reasons, we also see that KZG-sAVSS has a higher latency than Ped-sAVSS as we see in Figure 3b. This discrepancy also increases as the size of the cluster increases.

Figure 4 shows the size of the share and associated metadata that is sent to each replica after the client computes a share, which is equal to the disk space that the replica needs to store a secret shared value. Since Ped-sAVSS has a linear overhead per sharing to each replica, we see the bandwidth and storage footprint increasing linearly with the cluster size. Ped-sAVSS requires about 1KB of storage at \( n = 4 \) and increases to 23KB at \( n = 211 \). Meanwhile, KZG-sAVSS only requires each replica to store 860 bytes of information irrespective of the cluster size. Note that, in both instances, we are storing a single 254 bit integer.

2) vssVerify* Microbenchmark: Figure 5 shows that the throughput and latency of verifying a share, which is done by the replicas upon receiving a share. We see that at \( n = 4 \), Ped-sAVSS has 12 times higher throughput and 38 times lower latency. At \( n = 211 \), Ped-sAVSS only outperforms KZG-sAVSS by a factor of 5.4 on throughput and is only 5.4 times faster. We also see another trend: KZG-sAVSS’s latency and throughput stays constant at 117 operations per second with a 350 millisecond mean latency irrespective of the cluster size. Meanwhile, Ped-sAVSS’s throughput decreases and latency increases as the number of replicas in the cluster increases. We see that KZG-sAVSS asymptotically is better than Ped-sAVSS, but Ped-sAVSS’s cheaper cryptographic operations still allows it to outperform KZG-sAVSS.

3) vssReconstruct* Microbenchmark: vssReconstruct* has almost identical performance between KZG-sAVSS and Ped-sAVSS, as we see in Figure 6a. vssReconstruct* does not include the time taken to run vssVerify* since share verification happens when the message itself is verified. Figure 6b shows that vssReconstruct* can occur at high throughput, with 45000 operations per second with 4 replicas. However, vssReconstruct*’s throughput drops off quadratically as the cluster size increases, only being able to do 1100 operations per second with \( n = 211 \) replicas. Figure 6c shows a similar
performance story, with the latency increasing quadratically as the cluster size increases though even at $n = 211$ the latency is fairly low at 46 milliseconds.

The reason we see the quadratic behavior in $\text{vssReconstruct}^*$ is that $\text{vssReconstruct}^*$ does a quadratic number of modular multiplications in a 254 bit prime field. All multiplicative inverses are precomputed during setup, which makes the runtime of $\text{vssReconstruct}^*$ very fast even though there is a quadratic dropoff.

4) $\text{vssRecoverContrib}^*$ Microbenchmark: Figure 7 shows that $\text{vssRecoverContrib}^*$ throughput and latency is independent of the cluster size for both KZG-sAVSS and Ped-sAVSS. Additionally, Ped-sAVSS’s $\text{vssRecoverContrib}^*$ has exactly half the throughput (430 ms vs 860 ms) and twice the latency (118 ms vs 59 ms) of KZG-sAVSS. The constant CPU cost is due to $\text{vssRecoverContrib}^*$ only computing a share of a DPRF point evaluation and its associated verification proof. The reason that Ped-sAVSS’s $\text{vssRecoverContrib}^*$ operation is exactly half as performant than KZG-sAVSS is that Ped-sAVSS has two polynomial shares that must be recovered while KZG-sAVSS has one.

5) $\text{vssRecoverVerify}^*$ Microbenchmark: Figure 8 shows that KZG-sAVSS’s $\text{vssRecoverVerify}^*$ operation has higher throughput and lower latency than Ped-sAVSS. KZG-sAVSS has a 1.18 times higher throughput at a cluster size of $n = 4$, with the performance gap increasing to 13 times at larger cluster sizes of $n = 211$. The difference in latency is also similar, with KZG-sAVSS’s $\text{vssRecoverVerify}^*$ taking 130 milliseconds for all cluster sizes while Ped-sAVSS starts at 150 milliseconds at $n = 4$ and increases to 1.7 seconds at $n = 211$.

This performance difference occurs since $\text{vssRecoverVerify}^*$ must combine commitments and witnesses from the contributions received from $\text{vssRecoverContrib}^*$. KZG-sAVSS performs this computation using a constant number of elliptic curve multiplications whereas Ped-sAVSS computes this using a linear number of elliptic curve multiplications. Thus, as the cluster size increases, Ped-sAVSS’s performance also degrades accordingly.

6) $\text{vssRecover}^*$ Microbenchmark: Similar to $\text{vssReconstruct}^*$, share verification via $\text{vssRecoverVerify}^*$ happens in our implementation upon receiving each share from a replica, and the costs of these $\text{vssRecoverVerify}^*$ operations are not included in the $\text{vssReconstruct}^*$ results shown in Figure 9. $\text{vssRecover}^*$ thus incurs costs primarily due to interpolation (like $\text{vssReconstruct}^*$), evaluation of the DPRF and interpolation of any witnesses. Therefore, asymptotically, we see in Figure 9a and Figure 9b that $\text{vssRecover}^*$ behaves similarly to $\text{vssReconstruct}^*$ but with an order of magnitude lower throughput and an order of magnitude higher latency.

B. Incorporating sAVSS into PBFT

We incorporate sAVSS into a PBFT implementation in order to implement a threshold trusted third party (T3P). We instantiate our T3P using KZG-sAVSS and Ped-sAVSS, which we will refer to as KZG-T3P and Ped-T3P. We also implement and evaluate a key-value store on top of KZG-T3P, Ped-T3P, and PBFT.

To generate load in our evaluation, a client sends PUT requests asynchronously to the primary. The client pregenerates the requests to send to the cluster and loops through them once they are finished. For our throughput experiments, the clients asynchronously send enough requests at a time to saturate the system without oversaturating it. For our latency benchmarks,
V. Related Work

This paper makes two primary contributions: an asynchronous verifiable secret sharing scheme sAVSS that has linear dealer cost and a threshold trusted third party (T3P) built by combining sAVSS with a Byzantine Fault Tolerant state machine. We discuss the related works in both areas below.

A. Proactive secret sharing

A treatment of the prior work in verifiable secret sharing and asynchronous verifiable secret sharing is given in Section II-B. Unlike those prior works though, another way to approach share recovery is through proactive secret sharing. Using proactive secret sharing for share recovery would require sending a random polynomial that has nothing in common with the original shared polynomial except for the share that the recovering replica is interested in.

Prior work in proactive secret sharing [16] is difficult to apply directly to the problem of share recovery, however. These works [16], [25] assume a synchronous broadcast channel that delivers to all replicas instantaneously, which greatly simplifies the problem of agreeing on a random recovery polynomial. PVSS [26] does not make any such assumption and can be used in sAVSS, but it suffers from an exponential setup cost in the number of faults it tolerates, making it unusable for tolerating more than a few faults. MPSS [27] uses a Byzantine agreement protocol in order to explicitly agree on the random recovery polynomial, which would add a few additional rounds to sAVSS if used in share reconstruction. Although closest in “spirit” to proactive recovery schemes, sAVSS addresses only the share phase. It is left for future work to see whether proactive share recovery can be expedited with techniques borrowing from sAVSS.

B. Privacy in BFT

Methods to store data across n storage nodes in a way that ensures the privacy, integrity, and availability of the data despite up to k of these nodes being compromised is a theme that has been revisited numerous times in the last 30 years (e.g., [28–32]). The proposals in this vein of research often do not defend against the misbehavior of the data writers. In particular, a data writer might deploy data to the storage nodes in a way that makes data recovery impossible or ambiguous, in the sense that the data reconstructed depends on which correct nodes cooperate to do so. Protecting against corrupt data writers is one of the primary goals of verifiable secret sharing and its derivatives, for which we’ve surveyed the most directly related works in Section II.

With the rise of blockchains supporting smart contracts, there has been a resurgence of activity in finding ways to add privacy guarantees to Byzantine fault-tolerant algorithms, and indeed this is one motivation behind our work. Another class of approaches to this problem uses zero knowledge proofs [33], [34] for privacy. These approaches provide a very strong guarantee where it is impossible for anyone (other than the data owner) to recover the sensitive data, but where anyone can validate that the data satisfies some prespecified properties. However, such systems only work for a limited set of applications, rather than general purpose state machines that we target here. Additionally, these systems do not have any control over the data itself; i.e., the sensitive data must be managed by the owner, which is not suitable for a large class of applications.
CALYPSO \cite{35} resolves this through the use of a publicly verifiable secret sharing scheme, but they require two BFT clusters—one for access control and one for secret management. Thus, their protocol requires more replicas to operate. Additionally, CALYPSO requires the access-control policy to be specified ahead of time by the client, whereas sAVSS can easily allow dynamic access-control policies.

VIII. CONCLUSION

This paper introduces a new method for creating asynchronous verifiable secret sharing (AVSS) schemes for use in Byzantine Fault Tolerance (BFT) protocols called sAVSS. We apply this method to two VSS schemes and incorporate the resulting AVSS schemes into PBFT. We then implement a Byzantine Fault Tolerant key value store and evaluate the effectiveness of our scheme.

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\end{itemize}
In this appendix, we show why our composition of sAVSS along with PBFT is secure. We do this by first observing under what conditions the share recovery protocol will terminate. To show linearizability and liveness, we map every execution of our modified PBFT algorithm to the original PBFT algorithm. Thus, since the original PBFT algorithm satisfies linearizability and liveness, so does our modified algorithm. Then, we show privacy separately.

### a) Share Recovery Protocol Termination:
We claim that the share recovery protocol will always terminate if \( f + 1 \) replicas have successfully completed the sharing and the network eventually delivers all messages. To see why, recall that a replica that is missing its share needs the output of \( \text{vssRecoverContribution}^* \) from \( f + 1 \) replicas. If \( f + 1 \) replicas are honest, then they will faithfully call \( \text{vssRecoverContribution}^* \) and send the output to a replica that is missing its share. The missing share can then be recovered by using \( \text{vssRecover}^* \) to terminate the share recovery protocol.

### b) Normal Case Protocol:
In the normal case protocol, we only have changed how the client constructs requests. If a client is honest, then we can simply ignore the secrets being shared in the request and have the client send regular requests in the original run of the PBFT algorithm. The requests are consistent due to the binding property of our verifiable secret sharing (VSS) scheme. If a client is dishonest and sends an invalid share to the replica, then in the original PBFT protocol, the request from another replica in the system. In the modified protocol, this is done using the share recovery protocol so we simply wait to deliver the request messages in the original run until the share recovery protocol terminates. Note that if the share recovery protocol never terminates in the modified protocol, that means that less than \( f + 1 \) honest replicas have the request. This means that strictly less than \( 2f + 1 \) total replicas have the request, making it impossible for this request to be prepared. Therefore, if the modified normal case protocol never terminates, then neither does the original protocol. Thus, we see that liveness is unchanged from the original PBFT protocol.

Additionally, through the binding property of our underlying VSS scheme, we know that if a request has been committed, all secret values must be consistently shared. Thus, we see that the linearizability property also follows from linearizability in the original protocol along with binding.

### c) Checkpoint Protocol:
The checkpoint protocol is identical to a case where the state of the replicated service contains only the commitments of the secret values instead of the secret values themselves. Thus, by the binding property of the underlying VSS scheme, we have a one to one mapping from a run of the checkpoint protocol for our modified PBFT algorithm and the original PBFT algorithm. Therefore, if the original PBFT’s checkpoint algorithm provides liveness and linearizability, then so does our modified algorithm.
d) **State Transfer Protocol:** The state transfer protocol can be mapped back similarly to the normal case protocol. A replica receiving the value of a key using the share recovery protocol in our modified PBFT would have been receiving the plaintext value of the key in the original PBFT algorithm. We simply delay the plaintext value of the key until the share recovery protocol completes in our modified PBFT protocol. Additionally, in the state transfer protocol, we know that the share recovery protocol will complete since at least $2f + 1$ replicas have the state at the last checkpoint. This means that at least $f + 1$ honest replicas have the state, which is sufficient to guarantee termination during periods of synchrony.

e) **Privacy:** Our modified PBFT protocol achieves privacy through the hiding property of the VSS protocol. The hiding property says that a legitimate adversary (i.e. one that has at most $f$ shares of the secret) cannot do nonnegligibly better than guessing the secret at random. Thus, privacy is satisfied unless an adversary gets at least $f + 1$ shares of a value. However, this means that some correct replica has shared the secret with the adversary which contradicts our threshold assumption. Thus, we see that our modified PBFT protocol preserves linearizability and liveness while also guaranteeing privacy.

**APPENDIX B**

**PBFT State Transfer and View Change**

In this section, we describe the PBFT state transfer and view change protocols.

**A. View Change Protocol**

The view change protocol changes the leader. The core mechanism for transferring safe values across views is for a new leader to collect a set $P$ of view-change messages from a quorum of $2f + 1$ replicas. Each replica sends a view-change message containing the replica's local state: Its local request-log, and the commit-certificate with the highest view number it responded to with a commit message, if any.

The leader processes the set $P$ as follows.  
1) Initially, it sets a leader-log $G$ to an empty log.  
2) If any view-change message contains a valid commit-certificate, then it selects the one with the highest view number and copies its log to $G$. Share recovery is triggered for any requests in $G$ that the leader is missing its private share.

The leader sends a new-view message to all replicas. The message includes the new view number, the set $P$ of view-change messages the leader collected as a leader-proof for the new view, and the leader-log $G$. A replica accepts a new-view message if it is valid, and adopts the leader log. It may need to roll back speculatively executed requests, and process new ones. As usual, processing may entail triggering share-recovery for any requests where the replica is missing its private share.

**B. State Transfer Protocol**

We present a modified version of the PBFT state transfer protocol that is simpler and more suited when TCP is used for the underlying network protocol. When a replica has fallen behind, it sends a state transfer request along with its current sequence number to at least $f + 1$ replicas. Some replica will respond with the most recent valid checkpoint messages and the messages from the normal case protocol that were missed by the slow replica. In addition, the response will contain only the values of the keys that have changed since the sequence number known to the slow replica as well as the full requests that came after the last checkpoint.