Design optimization and analysis of a vaned diffuser based on the one-dimensional impeller-diffuser throat area model

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Abstract. The paper presents a design optimization method for a type of centrifugal compressor’s vaned diffuser with using a one-dimensional (1D) mathematical model to determine the best throat area ratio of the impeller to the diffuser. The research aims to verify of the 1D throat area mathematical model and to study the influence of vaned diffuser structure on the impeller-diffuser matching and the stage performance. The results of CFD analysis and experiments have proved that 1D throat area model can predict the throat area ratio of the impeller to the vaned diffuser accurately for the same choking mass flow at the required rotating speed. After optimization, the compressor performances and the matching between impeller and vane diffuser are considerably improved as well. The results and conclusions indicate that quick geometry modification of vaned diffusers can achieve excellent performance optimization and provide the theoretical basis for the selection of variable vaned diffusers.

Keywords: Vaned Diffuser; 1D Throat Area Model; Geometry Modification; Optimization

1. Nomenclature

| Symbol | Description                                      | Subscript/Superscript | Unit      |
|--------|--------------------------------------------------|-----------------------|-----------|
| $A^\star$ | Throat area ($m^2$)                              |                       |           |
| $D$    | Diameter ($m$)                                   |                       |           |
| $\Delta h$ | Enthalpy rise ($J/kg$)                         |                       |           |
| $m$    | Mass Flow Rate ($kg/s$)                         |                       |           |
| $M_{u_2}$ | Tip speed Mach number (-)                      |                       |           |
| $n$    | Polytropic exponent (-), $n_z = 1.5$            |                       |           |
| $T$    | Temperature ($K$)                                | 1                     | Inlet of the impeller |
| $u$    | Flow speed ($m/s$)                               | 2                     | Outlet of the impeller |
| $V$    | Volume flow rate ($m^3/s$)                      | $d$                   | Diffuser   |
| $\gamma$ | Isentropic exponent (-), $\gamma = 1.4$        | i                     | Impeller   |
| $\eta$ | Polytropic efficiency (Total-Total) (-)         | $t$                   | Total or Blade Tip |
| $\lambda$ | Work input coefficient, $\lambda = \Delta h_i/u_{i,2} \approx 0.8$ |                       |           |
| $\pi$  | Pressure ratio (Total-Static) (-)               |                       |           |
| $\rho$ | Density ($kg/m^3$)                              |                       |           |
| $\phi$ | Flow coefficient (-)                            |                       |           |
| $\chi$ | Matching coefficient (-)                        |                       |           |

Subscripts & Superscripts
2. Introduction
Centrifugal compressor stages of turbochargers for widespread applications are required to combine high efficiency along with wide flow ranges of both mass flow and enough pressure ratio, which is dramatically influenced by the matching of the centrifugal impeller and the diffuser. Extensive profound and in-depth studies and structure optimizations has been done for single components, most of which are related to the impeller as the impeller has a significant effect on the performance of the compressor stage. However, other components including the diffuser also have a great influence on the stage performance. Special attention paid to the diffuser has not been as much as that paid to the impeller in past studies of the component matching, as the matching is one of the most complex aspects of compressor design. Transonic flow and rotor-stator interaction in the space between the impeller and the diffuser is highly unsteady and complex with strong spanwise and circumferential distortions. Klassen [1] found that better matching of impeller and the diffuser improve peak efficiency of the stage by testing a centrifugal compressor with a backswep blade impeller and a vaned and vaneless diffuser. Tamaki [2] investigated how different diffuser throat areas influence the surge flow based on experiment results of a centrifugal compressor with 11 different vaned diffusers. Some Researchers believe that the component mismatching is a far more crucial factor than structure details of diffuser or impeller vanes to cause the poor performance of high-pressure ratio compressors. [3, 4] On the other hand, engineers have persistently developed empirical 1D model analysis for the preliminary design in past decades. Galvas [5] proposed a Single-Zone model for calculating the off-design performance of centrifugal compressors with channel diffusers. Japikse [6] developed a two-Zone model with assuming a ‘jet-wake’ structure in the impeller passage. Rossetti [7] provided an optimum design procedure for an aerodynamic radial diffuser. Li [8] developed a method of one-dimensional calculations with optimized combination of loss models and redesigned a high-pressure centrifugal compressor. Similar Equations based on a similar 1D gas dynamic formulation were proposed in several publications as well. [2, 9-10]

This paper presents an efficient optimization method for the vaned diffuser with using a one-dimensional (1D) mathematical model to determine the best throat area ratio of the impeller to the diffuser. Firstly, the original compressor cannot meet expected matching requirements [11]. The purpose of the optimization is to broaden the operation range of the low-speed zone and improve the expected matching characteristics at operating points. The 1D model method is applied to define the best throat area ratio of the impeller to the diffuser, so that both components have the same choking mass flow. Then three optimization plans of geometric parameters of the vaned diffuser are proposed for quick design. The study expects to verify that the 1D impeller-diffuser throat area model is a feasible preliminary design method, which increases the accuracy of forecasting the maximum flow capacity of compressors under given design operating conditions. After optimization by the best plan, based on CFD and experiment results of the optimized compressor stage according to the required operation condition points, the matching characteristics of the diffuser with the impeller and the stage performance of the compressor has been improved. The optimized centrifugal compressor can meet the performance requirement of design points at a large scale of rotating speeds.

3. One-Dimensional Model on Throat Area Ratio
For a compressor with a vaned diffuser, both the impeller and the vaned diffuser respectively have a throat area. The mass flow of choking condition depends on the throat area and local flow conditions. It can be either the diffuser or the impeller to decide the choke mass flow of the compressor stage. It is vital for matching to ensure that both components can operate at high Mach number near peak efficiency and have the same throat choke margin at the same choking mass flow. Therefore, if the impeller and the vaned diffuser match well, they should have the same choke mass flow.

Dixon and Hall [8] provided two equations to predict $\phi_\text{i}$ and $\phi_\text{d}$ as the maximum flow coefficient of the impeller and diffuser. In the equations, $n$ is given the value of 1.5 as the polytropic exponent based on the impeller total-total efficiency $\eta$. $\gamma$ is given the value of 1.4 as the isentropic exponent.
is the work coefficient based on enthalpy rise $\Delta h$ and the flow speed $u_2$ at the outlet of the impeller. All of the parameters are explained in the table at the beginning of the paper.

$$\phi_{ii}' = \frac{1}{M_{u_2}^2 D_2^2} \left[ 1 + \left( \frac{\gamma - 1}{2} \right) \left( \frac{D_1}{D_2} \right)^2 M_{u_2}^2 \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\phi_{i}' = \frac{1}{M_{u_2}^2 D_2^2} \left[ 1 + \left( \frac{\gamma - 1}{2} \right) \lambda M_{u_2}^2 \right]^{\frac{\gamma+1}{2}}$$

Rodgers [9] and Tamaki [10] proposed the matching coefficient $\chi$, so that the maximum flow and widest range can occur at design speed when the diffuser and impeller choke at the same time:

$$\phi_{ii}' = \chi \phi_{ii}' \quad \chi \approx 1$$

$$A_{d}' = \frac{A_i'}{1 + (\gamma - 1) \lambda M_{u_2}^2} \left[ 1 + \left( \frac{\gamma - 1}{2} \right) \left( \frac{D_1}{D_2} \right)^2 M_{u_2}^2 \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Based on Eq. (1), (2) and (3), a method to judge whether the impeller and the vaned diffuser match can be proposed as Eq. (4). For a given rotating speed, the ratio of the impeller and the diffuser throat areas if both choke at the same mass flow can be obtained by Eq. (4). From the results of the throat area model, a variation of throat area ratio for a variation of tip-speed Mach number can be determined, as the Mach number is proportional to the rotating speed if the impeller structure is defined. In the next chapter, design optimization plans of the vaned diffuser are established based on the 1D model computation results to optimize matching and performance of the compressor.

4. CFD Simulation and Quick Optimization Plans

4.1. The Geometry and Performance Requirement of the Compressor

| Table 1. Computation Settings |
|-----------------------------|
| Fluid Type | Flow Model | Turbulence Model | Rotor-Stator |
| Real Gas | Turbulent Navier-Stokes | S-A | Non-Reflecting |

The process of geometry modification, mesh generation, numerical computation and post-processing in the paper is completed by NUMECA software. Table 1 shows the key settings of numerical computation. The data of mass flow, pressure ratio and efficiency in the paper are all normalized.

Geometries of the impeller and the original stage are shown in Figure 1. The impeller has 8 main blades and 8 splitter blades, and the vaned diffuser has 15 wedge vanes. The throat area of each passage in the impeller and the diffuser is respectively 1948.02 mm² and 440.70 mm². The vane height and vane angle of the diffuser is respectively 14mm and 67.13°.
For the convenience of preliminary design and comparison, the CFD simulation and analysis of models in the 3rd chapter only involve the impeller and the vaned diffuser, not including the volute. The Grid independence study has been checked as Table 2 shows. Table 3 shows performance requirements of design operation points from the matched diesel engine.

![Geometries of the Impeller](a)  ![Geometries of the Stage](b)

**Figure 1.** Geometries of the Impeller (a) and the Stage (b)

| Grids/millions | Normalized Mass Flow | Normalized Pressure Ratio | Normalized Efficiency |
|----------------|----------------------|--------------------------|-----------------------|
| 0.8            | 1                    | 1                        | 1                     |
| 1.6            | 0.999                | 1.001                    | 1.001                 |
| 2.4            | 1.002                | 1.003                    | 1.003                 |

**Table 2.** Grid Independence Study

| Point | Normalized Mass Flow | Normalized Efficiency | Pressure Ratio | Surge Margin/% |
|-------|----------------------|-----------------------|----------------|----------------|
| A     | 0.994                | 0.951                 | 0.847          | 15             |
| B     | 0.885                | 0.963                 | 0.715          | 15             |
| C     | 0.752                | 0.957                 | 0.602          | 20             |

**Table 3.** Operating Design Points of the Compressor

4.2. *CFD Results of the Original Compressor*

![Compressor Map](a)  ![Efficiency Curves](b)

**Figure 2.** The Compressor Map (a) and Efficiency Curves (b) (Original)
Figure 2 shows the compressor map and efficiency curves of the original selected compressor. Wide high efficiency range can be observed. However, one can see that the required operation points locate far away from the high efficiency region of the compressor, leading to poor matching between the compressor and the engine. To solve this problem, either a new compressor with high mass flow rate should be selected or a new design on the impeller/diffuser should be performed. Since the flow capacity of the compressor depends on the throat of the impeller and vane diffuser, a re-matching between the impeller and diffuser offers an efficiency way of compressor re-design.

4.3. Quick Design Optimization Plans of the Vaned Diffuser

According to the physical theory of the 1D throat area model, small operation range in low-speed area means the vaned diffuser’s throat is too small to pass required flow, which will cause choking in the diffuser area. Therefore, the throat area of the diffuser must be expanded to achieve larger flow capacity, as the potential of the impeller has been suppressed. The maximum choking mass flow of both components at the expected design speed line should be as equal as possible. The expected goal of optimization is to match three design points and achieve shifts in mass flow, efficiency and operation ranges to meet required performance of compressors at low, medium and high speeds.

As the distribution of three design points on the map in Figure 2, the medium rotating speed is approximately from 30000 to 32000 rpm. To achieve a good distribution of efficiency circles and even operating ranges of different speed lines, the target rotating speeds for the input of 1D throat area model are set to be 30000 rpm and 32000 rpm and two expected throat areas can be obtained. The total diffuser throat area is the product of the throat area of each vane passage (EVP) and vane blades.

The easiest method to realize the change of diffuser throat area is by modify the vane angle. Thus, details of optimization design plans for the vaned diffuser are determined in Table 4. Plan 1 and Plan 2 are designed by changing vane angles. To study the influence of vane solidity, Plan 3 has the same vane angle as Plan 2 just with two more vanes to 17 vanes, which leads to a reduce of the throat area.

| Plan   | Vane Angle(°) | Throat Area (EVP)(mm²) | Vanes | Throat Area (Total)(mm²) | Expected Design Speed of Throat Area Ratio (rpm) |
|--------|---------------|------------------------|-------|-------------------------|-----------------------------------------------|
| Original | 67.13         | 333.50                 | 15    | 5003                    | 34000                                         |
| Plan 1  | 63.00         | 405.12                 | 15    | 6077                    | 30000                                         |
| Plan 2  | 65.12         | 368.40                 | 15    | 5526                    | 32000                                         |
| Plan 3  | 65.12         | 306.79                 | 17    | 5215                    | ≈33000                                        |

5. Optimization Results Analysis

5.1. Comparison of Plans

Four compressor maps of Plan Original, 1, 2, 3 obtained by CFD simulation are shown in Figure 3.

Plan 1 (Figure 3(b)): For the largest diffuser throat, peak efficiency of the compressor is pretty much higher because of less loss. However, with the increase of the rotational speed, the peak efficiency of speed lines decreases. The high-efficiency circles deviate to the conditions of low and medium speed. The relative surge margin is narrower at high-speed lines. Considering that, the total pressure of the compressor is close to the surge condition when the total pressure of the compressor begins to decrease, at 35000 rpm the compressor is on the verge of surge as it just leave the low efficiency and choking zone. Therefore, Plan 1 cannot meet the matching requirement at high speed.

Plan 2 (Figure 3(c)): Plan 2 has a moderate throat area, which is between the Original and Plan 1. The performance of Plan 2 compressor has been improved as follows: (1) From Figure 3 it can be seen that the operation ranges of all speed lines obviously increased and are very balanced. The surge margins of speed lines from 29000 rpm to 32000 rpm are all above 20% and at other speed lines the surge margin
is still above 15%. (2) Three design points are located in the optimum operation zone and inside the 1.00 normalized efficiency circle. (3) The efficiency of Plan 2 compressor has improved from the Original from 1% to 2%, and the flow capacity of Plan 2 has increased 5%-10%.

Plan 3 (Figure 3(d)) has a smaller throat area than Plan 2 with the similar problem of the Original that the operation range is not wide enough for the low-speed area. As the number of vanes rises, the map moves left for about 0.02 normalized mass flow compared to Plan 2, which means mass flow reduces.

![Compressor Maps of Plans](image)

Figure 3. Compressor Maps of Plans

Figure 4 (a) shows the peak efficiency of Plan 2 is improved at all speed lines comparing to the original compressor. Both the surge margins and the operating ranges of all speed lines meet the requirement in Figure 4 (b). Eventually, Plan 2 becomes the final design plan of the optimized compressor, as Plan 2 can achieve the best balance of performance. At the same time, the surge margin of the compressor at all speed lines, especially the low-speed lines, can be significantly broadened. Both the efficiency and mass flow of the Plan 2 compressor improved considerably by almost 2%. 

![Figure 4](image)
Figure 4. Comparison of Efficiency (a) and Pressure Ratio (b) of Plan Original with Plan 2

5.2. Flow Field Analysis of Plan 2

(a) Entropy - 32000rpm Peak Eff. 
(b) Entropy - 35000rpm Peak Eff.

(c) Static Pressure - 32000rpm Peak 
(d) Static Pressure - 35000rpm Peak Eff.
Figure 5. Meridional Average Flow Field of two peak efficiency conditions

Figure 5 shows meridional average flow field distributions of entropy, static pressure and total pressure of 32000 rpm and 35000 rpm peak efficiency points.

It can be concluded in entropy distribution figures that most of flow loss in the compressor is concentrated in the tip region of the impeller and the diffuser, which is caused by tip clearance leakage loss and shock-induced boundary layer separation. The tip loss is gradually accumulated along the flow direction to the diffuser outlet. The strength of tip shock wave and gap leakage flow increase with the rotational speed, leading to the increase of tip loss, which is completely mixed with the mainstream at the diffuser outlet.

The static pressure lifting in the compressor maintains a good gradient near the peak efficiency point. The variety of gradient is uniform in the radial flow section and pressure isolines are almost vertical to that in the meridional flow passage.

At different rotating speeds, the total pressure of the flow is increasing, which increases slower in the impeller and faster in the diffuser. Flow in the diffuser has a good diffusion state.

Figure 6. Blade-to-Blade Relative Mach Number Flow Field of the 32000rpm choking condition
Figure 6 is blade-to-blade relative Mach number flow field of the choking condition at 32,000rpm speed line, which is the design rotating speed and is expected that the impeller and the diffuser choke at the same time by the 1D throat model. From 95% blade height of the impeller (Figure 6(a)), it is shown that relative Mach number is above the sound speed (the black isoline) in the entire flow field of the impeller inlet, which means choke in the impeller. From 50% vane height of the diffuser (Figure 6(b)), relative Mach number is also above the sound speed (the black isoline) in the entire flow field of the diffuser, which means choke in the diffuser as well. Therefore, the impeller and the diffuser have been designed to choke at the same time with the same mass flow by the 1D throat model. The accuracy of 1D throat model is verified.

5.3. Experimental Validation of the Optimization

Performance maps of the compressor stage obtained by CFD computation compared to experiment data of the test for a compressor prototype are given with three design points in Figure 7. CFD results of the optimized compressor are in good agreement with the experimental results. Design points A, B and C are all in high-efficiency circle (above 0.98). The final plan can achieve good matching and better performance with broadened operation ranges of the compressor at all speed lines.

6. Conclusions

The paper presents an optimization design for a type of centrifugal compressor based on the method of one-dimensional impeller-diffuser throat area mathematical model. Conclusions of the study could be drawn as follows:

The increase of diffuser throat area can effectively increase the choking mass flow in low-speed and middle-speed areas, but it has little effect on the choking mass flow in high-speed areas. The choking condition of the compressor is more likely to occur in the diffuser at low and middle speed lines, while the choking condition of the compressor is more likely to occur in the impeller inlet at high-speed lines.

The vaned diffuser is optimized and redesigned based on the 1D optimization results and the matching of vaned diffusers. After optimization, the entire stage efficiency has been increased by approximately 2%. The 1D impeller-diffuser throat area mathematical model has been verified to predict the throat area that make the impeller and the vaned diffuser choke at the same mass flow. The final plan can achieve good matching and better performance with broadened operation ranges of the compressor at all speed lines.

Quick modification of the vane installation angle of the vaned diffuser has been realized in the paper. The new designed compressor can achieve better matching and the optimization of the stage...
performance, which can be directly shown in the compressor map. The 1D impeller-diffuser throat area model provide the theoretical basis for the research of variable vaneed diffusers, which can modify vane angles under different operating conditions.

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