On determining characteristic length scales in pressure gradient turbulent boundary layers

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Abstract. In the present work we analyze three methods used to determine the edge of pressure gradient turbulent boundary layers: two based on composite profiles, the one by Chauhan et al. (Fluid Dyn. Res. 41:021401, 2009) and the one by Nickels (J. Fluid Mech. 521:217–239, 2004), and the other one based on the condition of vanishing mean velocity gradient. Additionally, a new method is introduced based on the diagnostic plot concept by Alfredsson et al. (Phys. Fluids 23:041702, 2011). The boundary layer developing over the suction side of a NACA4412 wing profile, extracted from a direct numerical simulation at $Re_c = 400,000$, is used as the test case. We find that all the methods produce robust results with mild or moderate pressure gradients, but stronger pressure gradients (with $\beta$ larger than around 7) lead to inconsistent results in all the techniques except the diagnostic plot. This method also has the advantage of providing an objective way of defining the point where the mean streamwise velocity is 99% of the edge velocity, and shows consistent results in a wide range of pressure gradient conditions, as well as flow histories. Therefore, the technique based on the diagnostic plot is a robust method to determine the boundary layer thickness (equivalent to $\delta_{99}$) and edge velocity in pressure gradient turbulent boundary layers.

1. Introduction & Motivation
Contrary to internal flows, where the outer length scale $\delta$ is exactly defined through the geometry, the counterpart for semi-confined flows such as boundary layers “can hardly be exactly defined” [1]. The problem is commonly circumvented by using integral quantities such as the displacement thickness $\delta^*$, e.g. through usage of the Rotta-Clauser outer scale $\Delta = U_\infty \delta^*$. In the present paper + denotes normalization with either the friction velocity $u_\tau = \sqrt{\tau_w/\rho}$ (where $\tau_w$ is the mean wall shear stress and $\rho$ is the fluid density) or the viscous length $\ell^* = \nu/u_\tau$ (with $\nu$ being the fluid kinematic viscosity). Other common work-arounds include employing analytical expressions for the wake, i.e., the law of the wake, from which $\delta$ and other boundary layer parameters are extracted through fits of the data. For a detailed discussion on the employment of the law of the wake or composite velocity profile descriptions see the seminal work by Coles [3, 4] and more recent work by Nickels [5] and Nagib et al. [6]. The aforementioned investigations emphasize that the use of similarity laws (or ad hoc empirical relations) for the determination of boundary-layer parameters is crucial when dealing with experimental profiles, due to lack of measurements

Note that Rotta himself attributes the introduction of this length scale to himself, cf. “the present writer proposed the introduction of the dimensionless wall distance $yu_\tau/(\delta^* U_\infty)$ instead of $y/\delta^*$ [2].
close to the wall and/or sparse data around the boundary-layer edge. The problem is, however, also acute with numerical data even for the “simple” case of zero pressure gradient (ZPG) flat plate turbulent boundary layers (TBLs) as discussed in Schlatter and Örlü [8]. For less “simple” TBL configurations, such as those developing on curved surfaces and/or with (strong) pressure gradients, the problem becomes even more apparent, and commonly used wake descriptions are unable to accurately describe the data. One reason for the fallacy is attributed to the velocity gradient that is not necessarily zero for \( y > \delta \), i.e., \( U(y > \delta) \neq \) constant as can be observed in Figure 1, which shows TBL profiles along the suction side of a wing [9, 10]. Due to the relatively low Reynolds numbers \( Re \) and the strong pressure gradient conditions (where similarity laws usually fail) the edge of the boundary layer, its velocity \( U_e \) (which is identical to the freestream velocity \( U_\infty \) in the case of ZPG TBLs), the boundary layer thickness \( \delta \), and the integral parameters (for which \( \delta \) is needed to truncate the integrals), are all ill-defined. Another reason for the fallacy comes from the fact that composite profiles often assume that the classical or a modified logarithmic law applies, as well as a two-layer structure of the boundary layer exists. It is not clear whether these laws and layer structures are valid in the case of large-defect TBLs, in particular non-equilibrium ones, although as will be discussed in §4 available composite profiles are able to represent the data well except close to separation.

More sophisticated indicators can of course be employed such as the vorticity (enstrophy) or intermittency (see e.g. the discussion in Ref. [11]), but these are often not at hand (in case of most experiments) or are simply not documented. Other approaches, such as the irrotational velocity defect method, are problematic with simulations due to the fact there is not a region of irrotational flow, and with experiments where the irrotational region may be affected by blockage effects induced by the wind tunnel walls. The purpose of the present paper is to briefly review some of the commonly used methods for TBLs with pressure gradients and surface curvature and introduce a method that is based on the diagnostic plot concept by Alfredsson and co-workers [12, 13, 14] which — as will be shown here — is believed to provide a robust method for the determination of the edge velocity and boundary layer thickness, in particular, when it comes to low \( Re \) and/or strong pressure gradients. Moreover, this method only requires knowledge of the mean and fluctuating streamwise velocity profiles, which also in the case of experiments are accurately measured close to the freestream.

The present article is organized as follows: in §2 a description of the analyzed datasets is provided; in §3 the techniques considered for the analysis are described in detail; in §4 we present the results and discussion; and in §5 we summarize the conclusions of the present work.

2. Employed data sets

The main test case considered in the present study was the flow around a NACA4412 wing section at a moderate Reynolds number of 400,000 (based on chord length and freestream velocity), which is shown in Figure 1. This database was obtained through direct numerical simulation (DNS), using the spectral element code Nek5000 [15], and a full description of the setup is given by Vinuesa et al. [9]. This flow is a good test case to assess the merits and deficiencies of the various methods, due to the fact that the boundary layers developing over the wing are subjected to the effect of wall curvature, as well as pressure gradients. More precisely, the boundary layer developing over the suction side of the wing experiences conditions from very weak adverse pressure gradients (APG), up to very strong APGs and incipient separation close to the trailing edge. The TBL on the pressure side experiences a mild favorable pressure gradient (FPG) throughout the whole wing. The maximum Reynolds number in this database was \( Re_\theta \simeq 2,800 \)

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2 This is mainly caused by the habit to measure in the wall-normal direction with a logarithmic spacing, see e.g. the relevant comment by George [7] with regard to some well-known data sets: “... experiments are lacking sufficient detail in the outer 50% of the boundary layer to determine the crucial outer length scale parameters (\( \delta_{99} \) or even \( \delta_{95} \)) to within even a few percent.”
Figure 1. Turbulent boundary-layer profiles in inner-units at the three highlighted positions along the suction side of a wing. Data extracted from a DNS of the flow around a NACA4412 wing profile, at a Reynolds number based on freestream velocity and chord length of 400,000 [9, 10]. The linear and logarithmic profiles (with $\kappa = 0.41$ and $B = 5.2$) are shown to ease comparison and emphasize their fallacy for strong pressure gradient conditions (and low $Re$). The insets show the respective velocity gradients around the boundary-layer edge.

where $\theta$ is the momentum thickness. The analysis of the boundary layers developing over the wing was supplemented with the assessment of wind tunnel measurements at higher Reynolds numbers, namely the measurements at the National Diagnostic Facility (NDF) from the Illinois Institute of Technology (IIT) by Nagib et al. [16]. In this database, simple APG and FPG configurations were considered in the Reynolds number range $10,000 < Re_\theta < 40,000$.

3. Techniques

3.1. Common techniques

Since the introduction of the law of the wake [3], various alterations and improvements for the original description of the velocity-deficit have been proposed. A “well-known and useful property of the defect law is that it avoids the awkward problem of defining the thickness $\delta$ for a boundary layer” [3]. Composite velocity profiles, i.e., linear combinations of law of the wall and law of the wake descriptions, are widely used and are useful for similarity laws. Two recent ones, by Nickels [5] and by Chauhan et al. [17], will be employed here. Note that the first one was developed in particular for pressure gradient TBLs, and the second one has been successfully applied to several canonical wall-bounded turbulent flows. In particular, they will also serve as a validation of the diagnostic plot concept for the flow cases in which the composite profiles are known to work well.

The main advantage of composite profiles is the fact that they can help to alleviate certain problems in experimental data, namely when near-wall measurements are not available or are
inaccurate, or when measurements are sparse across the boundary layer. These issues may lead to problems when evaluating boundary-layer parameters [18], and recent studies by Rodríguez-López et al. [19] and Vinuesa and Nagib [20] have shown the potential of composite profiles in developing correcting schemes for high-Re boundary layer measurements. However, the use of composite profiles at very low Re or under strong APGs requires pre-processing steps, which complicates their straightforward application in such cases. Nevertheless, and as will be discussed in §4, composite profiles provide consistent results when used to determine the boundary layer edge in boundary layers subjected to mild pressure gradients and at moderate Reynolds numbers.

Another technique investigated in the present study is the condition of a vanishing mean velocity gradient, \(dU/dy \approx 0\), which has been shown to be effective in defining the boundary layer edge in ZPG and mild pressure gradient boundary layers (even in consecutive sequences of APG and FPG [21]). It is important to note that this technique cannot be used in flows with strong streamline curvature due to the fact that the streamwise velocity necessarily varies in the wall-normal direction within the irrotational flow region. Moreover, in §4 we show that this method also exhibits problems with strong pressure gradients due to the fact that the streamwise velocity is not constant and equal to \(U_{\infty}\) beyond the boundary layer edge as shown in Figure 1, and therefore \(dU/dy\) does not reach 0. This leads to the introduction of arbitrary thresholds to define \(\delta\), which is also not desirable when analyzing wide ranges of pressure gradient conditions.

3.2. The ‘diagnostic plot’ concept
An alternative method to determine the boundary layer edge is in the following introduced and based on the diagnostic-plot concept [13], in which the streamwise velocity root mean square (rms) \(u'\) value is plotted against the mean velocity \(U\) normalised by the boundary-layer edge velocity \(U_e\) rather than the wall distance. Although the diagnostic plot was originally proposed as a means for testing the adequacy of experimental data [12], it soon showed promising results to scale ZPG TBL data (but also pipe and channel flows) covering a wide \(Re\) range throughout the logarithmic and wake layers [14]. This is made apparent in Figure 2 (left), where ZPG TBL data from DNS [8] and experiments [22] covering a range of \(670 \leq Re_\theta \leq 4,300\) and \(2,500 \leq Re_\theta \leq 19,000\), respectively, are brought to scale \(Re\)-independent and linearly dependent on \(U/U_e\). As apparent, the \(Re\)-independent region extends with increasing Reynolds number. The success of the diagnostic scaling could also be extended to turbulent boundary layers on rough walls when taking the roughness function into the scaling [23], and with pressure gradients\(^3\) when accounting for the shape factor [26], as well as to free-shear flows such as mixing layers, jets and wakes [24].

A useful feature of the diagnostic plot is that it is constructed such that the turbulence intensity diminishes in the free stream, i.e. \(u'/U \rightarrow 0\) for \(y/\delta \rightarrow \infty\). For the purpose of the present paper, the velocity ratio of \(U/U_e = 0.99\) is of particular interest, since it relates to the wall-normal location \(\delta_{99}\) commonly employed as the boundary-layer thickness \(\delta\). The straightforward idea is therefore to use the diagnostic plot concept, which is free of differentiation or fitting operations and employs merely the turbulence intensity, to determine the boundary-layer thickness, by simply finding the local turbulence intensity that corresponds to \(U/U_e = 0.99\). This value is around 2% when scaled as in Figure 2 (right).

4. Results and Discussion
The scaling of the boundary layer developing over the suction side of the wing is analyzed in Figure 3, which reveals interesting similarities between low-\(Re\) data subjected to a strong APG and high-\(Re\) ZPG data. In this figure we also show that an appropriate scaling for both the mean velocity defect \(U_e - U\) and the streamwise variance \(u'^2\) is the one proposed

\(^3\) Castro [24] had made similar observations when considering equilibrium APG data near separation [25].
Figure 2. (Left) ZPG TBL data from DNS [8] and experiments [22] covering a range of $670 \leq Re_\theta \leq 4,300$ (black) and $2,500 \leq Re_\theta \leq 18,700$ (blue). Arrow points in direction of increasing $Re$. Red line depicts a straight line of form $u'/U = a - b(U/U_e)$, with $a = 0.286$ and $b = 0.255$ [14]. Lowest three $Re$ profiles from DNS (i.e. $Re_\theta < 1500$) are shown as dashed lines. Zoom into the wake region is shown in the inset. (Right) Same as (left), but with the shape factor $H_{12}$ taken into account in order to compare with the diagnostic plots in case of pressure gradient effects [26].

by Zagarola and Smits [27], denoted here as ‘ZS’, in which $U_{ZS} = U_e \delta^*/\delta$ is considered as the velocity scale, and the boundary-layer thickness $\delta$ is taken as the length scale. This scaling was also shown to be adequate for APG boundary layers by Maciel et al. [28]. The velocity profiles shown in Figure 3 are expressed in the direction tangential to the wing surface at each location, and the $y$ coordinate is measured in the direction normal to the wall. Profiles are shown between $x/c = 0.15$ and 0.98, which according to Vinuesa et al. [9] corresponds to a Reynolds number range $500 < Re_\theta < 2,800$. The pressure gradient can be quantified in terms of the so-called Rotta–Clauser pressure-gradient parameter $\beta = -\Delta/u_t dU_e/dx_t$ (where $dU_e/dx_t$ is the derivative of the tangential edge velocity with respect to the tangential direction $x_t$), which is in the range $0 < \beta < 85$ in this particular dataset. Figure 3 (left) shows that the outer region of the mean velocity defect profiles tend to be regrouped as higher Reynolds numbers and larger APGs are reached, and in Figure 3 (center) it is possible to observe how the scaled outer region of the streamwise variance shows a decreasing trend with $Re$ and $\beta$. Interestingly, Figure 3 (right) also shows the collapse of the diagnostic plot in the outer region at higher $Re$ and larger APG, when appropriately scaled with the shape factor $H_{12} = \delta^*/\theta$, reminiscent of the behavior at high Reynolds numbers in ZPG boundary layers. The purpose of the present study is to develop a robust method to determine the boundary layer edge, or more precisely, a practical representation of the boundary layer thickness such as $\delta_{99}$. Note that the edge velocity $U_e$ (and for that matter $H_{12}$) is not known a priori since that requires knowing $\delta_{99}$, but if one uses as a first approximation $U_e \approx \max(U)$, then the values of $u'/U_e \sqrt{H_{12}}$ at $U_e/U_e = 0.99$ are around 0.02, see inset in Figure 2 (right). This motivated the use of an iterative method, where the diagnostic plot is initially computed for all the profiles using $U_e \approx \max(U)$, then $\delta_{99}$ is defined as the location where $u'/U_e \sqrt{H_{12}} = 0.02$. Note that this location corresponds to $U_e/U_e = 0.99$, therefore a new $U_e$ can be defined for the next iterative step. It should be noted that the value of 0.02 was deduced from ZPG TBL data in order not to be biased by the ambiguities related to $\delta_{99}$, $U_e$ and $H_{12}$. This leads to the fact that, as apparent from the intersection highlighted in the inset from Figure 3, the profiles for the pressure gradient cases.
Figure 3. (Left) Mean velocity defect and (center) streamwise variance scaled with $U_ZS$ and $\delta$; (right) diagnostic plot scaled with the shape factor $H_{12}$. Profiles extracted from the suction side of the wing, in the range $0.15 < x/c < 0.98$, with red indicating data closer to the leading edge and blue profiles towards the trailing edge of the wing. The arrow also shows increasing downstream location, which is associated with higher Reynolds number but also larger $\beta$. Note the similarities between the high-$Re$ TBL data shown in Figure 2 and the high-$\beta$ cases presented here.

are forced through the point $u' / (U \sqrt{H_{12}}) = 0.02$ and $U/U_e = 0.99$. This scheme converged to the final $H_{12}$ value after one iteration in most of the profiles, and after few iterative steps in the profiles near separation. Therefore, it is a robust criterion to determine the boundary layer thickness $\delta_{99}$. It is also important to note that $\delta_{99}$ is an arbitrary representation of the boundary layer thickness and it is problematic when $Re \to \infty$, but it is widely used in the literature for practical purposes, and that is why it is also adopted in the present study.

The results shown for the wing at $Re_\theta$ up to 2,800 are extended to higher Reynolds numbers up to $Re_\theta = 40,000$ in Figure 4, where the experimental data by Nagib et al. [16] is shown. These measurements include APG, FPG and strong FPG measurements approximately in the range $-0.5 < \beta < 0.5$, and as it can be seen in Figure 4 (left) and (center) the Zagarola–Smits scaling is also adequate for these higher $Re$ cases since the outer region of the mean velocity defect and the streamwise variance profiles tend to be regrouped with increasing Reynolds number. In addition to this, Figure 4 (right) also shows very good collapse of the diagnostic plot scaled with $H_{12}$, Note that although the slope may change slightly depending on the pressure gradient (as also observed by Castro [24]) and the history effects of the flow, the value 0.02 is robust and accurate enough for the purposes of the present study. This is a very reassuring test for the use of the diagnostic plot concept to determine the boundary-layer edge in pressure-gradient boundary layers, due to the wide range of pressure gradient and Reynolds-number conditions, but also the very different history effects exhibited by the datasets under consideration.

The various methods are then used to determine the boundary-layer thickness of the profiles developing over the suction side of the wing in the range $0.15 < x/c < 0.98$, and the results are shown in Figure 5. For the two composite profile methods, i.e., the one by Nickels [5] and the one by Chauhan et al. [17], previous processing of the data with the diagnostic plot (or any other pre-treatment) is necessary in order to provide initial estimations for the boundary layer thickness. Note that in both cases we performed curve fits without prescribing any variable, and the boundary layer thickness $\delta$ was one of the fitting parameters. It is also important to
Figure 4. (Left) Mean velocity defect and (center) streamwise variance scaled with $U_{ZS}$ and $\delta$; (right) diagnostic plot scaled with the shape factor $H_{12}$. Experimental results by Nagib et al. [16] in several pressure gradient configurations, where blue corresponds to APG, red to FPG and green to strong FPG cases.

note that, as can be observed in Figure 5 (left), in the composite profile by Chauhan et al. the fitting parameter $\delta$ corresponds to $\delta_{100}$, whereas in the case of Nickels it is $\delta_{95}$. With adequate initial estimations of the boundary-layer thickness, both composite profile approaches provide consistent results up to around $x/c \approx 0.85$, after which the APG is strong (with $\beta$ values larger than 7), and especially the composite law of Chauhan et al. [17] leads to scattered values of $\delta$. Let us recall that the composite law of Nickels [5] was developed for pressure gradient boundary layers, whereas the one by Chauhan et al. [17] inherits a number of parameters in the wake function specifically developed for ZPG boundary layers. The method based on the mean velocity gradient $dU/dy$, with an arbitrary threshold of $10^{-3}$, leads to results similar to the diagnostic plot up to $x/c \approx 0.8$, which means that the boundary layer thickness it provides is approximately $\delta_{99}$ in this range. Beyond this point the gradient does not decrease to $10^{-3}$, and therefore the position of the minimum of $dU/dy$ is taken as the boundary-layer edge. This lead to values of $\delta$ different from the ones calculated from the diagnostic plot after $x/c = 0.8$, which also supports the need to avoid arbitrary thresholds in order to obtain robust results.

Finally, the results from the method based on the diagnostic plot are also shown in Figure 5 (left). As shown in Figure 3 (right), the point where $u'/U_{\sqrt{H_{12}}}$ is approximately 0.02 is found for each profile, which corresponds to the location where $U/U_e \approx 0.99$, and therefore the boundary layer thickness obtained with this approach corresponds to $\delta_{99}$. Figure 5 (left) shows that the results are robust throughout the whole suction side of the wing, and since this method is based on an objective way of determining the point where $U/U_e \approx 0.99$, we believe that it is the best method to determine the boundary layer thickness in pressure gradient TBLs. To further show the consistency of the results, in Figure 5 (right) we show the values of $\delta_{99}$ obtained from the two composite approaches, and compare them with the diagnostic plot and the velocity gradient. The four methods exhibit excellent agreement up to $x/c \approx 0.8$, which is the region where the APG is mild or moderate, and beyond this point the diagnostic plot and the composite from Nickels [5] (provided an initial $\delta$ is used, which is here obtained through the diagnostic plot) are still in very good agreement until almost the trailing edge.
5. Conclusions

The present study is focused on the determination of the edge of pressure-gradient turbulent boundary layers, i.e. \( U_e \) and \( \delta \), which can be quite problematic due to the fact that in such boundary layers the mean velocity is not necessary constant beyond the layer edge. The merits and shortcomings of the following four methods are analyzed: (i) curve fit to the composite profile by Chauhan et al. [17], developed for ZPG TBLs; (ii) fit to the composite profile by Nickels [5], which was developed for pressure gradient TBLs; (iii) find the point where the mean velocity gradient \( dU/dy \) is approximately zero, with a threshold of \( 10^{-3} \) in inner units (in case that the gradient does not go below the threshold, then the position of its minimum is considered as the edge); (iv) use of the diagnostic plot to find \( U/U_e \simeq 0.99 \), which is given by the value \( u/(U\sqrt{H/12}) \simeq 0.02 \). Note that methods (i) and (ii) require an initial estimation for \( \delta \), which is taken as the value obtained from method (iv). Without such a pre-treatment the composite profiles would not result in such smooth curves as presented in Figure 5 towards the stronger pressure gradient cases.

All the methods are tested on turbulent boundary-layer profiles developing over the suction side of a NACA4412 wing profile, extracted from a recent DNS at Reynolds number 400,000 [9, 10]. While the four methods provide robust results when the pressure gradient is mild or moderate, stronger pressure gradients \( (\beta > 7) \) lead to inconsistent results in all the techniques except the diagnostic plot one. It is important to note that although the composite profile approaches produce acceptable results, they require initial estimations of \( \delta \) which have to be obtained by other means (in this case, they were obtained with the diagnostic plot method). Moreover, in the strong pressure gradient cases the composite profiles do not fit the velocity profiles, although the values of \( \delta \) they provide are acceptable. This is due to the particular formulation of the wake function, which is not general enough to represent the wake encountered in such pressure gradient TBLs. The diagnostic plot method has the additional advantage of determining the point where \( U \) is 99% of the edge velocity in an objective way, and shows
consistent results over a wide range of pressure gradient conditions, as well as flow histories. The robustness of the diagnostic plot was also checked with the pressure gradient TBL experiments by Nagib et al. [16]. In addition to this, it is based on data that is usually available in most experimental studies, namely mean and streamwise turbulence intensity profiles. Therefore, the method based on the diagnostic plot described in the present study is a robust technique to determine the boundary layer thickness in pressure gradient TBLs.

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