Linear Distillation Learning

Abstract

Deep Linear Networks do not have expressive power but they are mathematically tractable. In our work, we found an architecture in which they are expressive. This paper presents a Linear Distillation Learning (LDL) a simple remedy to improve the performance of linear networks through distillation. In deep learning models, distillation often allows the smaller/shallow network to mimic the larger models in a much more accurate way, while a network of the same size trained on the one-hot targets can’t achieve comparable results to the cumbersome model. In our method we train students to distill teacher separately for each class in dataset. The most striking result to emerge from the data is that neural networks without activation functions can achieve high classification score on a small amount of data on MNIST and Omniglot datasets. Due to tractability, linear networks can be used to explain some phenomena observed experimentally in deep non-linear networks. The suggested approach could become a simple and practical instrument while further studies in the field of linear networks and distillation are yet to be undertaken.

1 Introduction

Deep learning has shown to be a reliable approach of approximating very complex dependencies by means of learning highly parametrized models. One of the challenges is the intractability of deep neural architectures. Often we cannot answer the questions, when a model will perform well and why. One possible way to approach this problem is studying simple architectures with similar dynamics to understand more complex architectures. Another way is using interpretable architectures in cases where is important to understand data dependencies.

Since the first neural networks already used threshold gates, linear networks became uninteresting in practise. On the contrary, from the research perspective, the study of linear networks leads to new insights in deep learning methods performance and provides theoretical models for generalization dynamics. Linear networks allow studying the structure of error function landscapes and apply this knowledge in nonlinear cases. Our research seeks to create a mathematically tractable, scalable and simple model, which can help us to bridge the gap between the efficacy and lucidity of a deep learning model.

We propose a novel framework we name Linear Distillation Learning, which allows achieving expressive results in case of limited data. The approach is based on using a linear function for each class in dataset, which is trained to simulate output of teacher linear network for each class separately. When the model is trained, we can apply classification by novelty detection for each class. Our framework can be presented as distilling randomized prior functions for data. Since our prior functions are linear, in couple with bootstrap methods it provides a Bayes posterior. We tested our architectures on tasks with different amounts of data on MNIST and Omniglot datasets.

Preprint. Under review.
Despite linear networks are very weak to represent complex nonlinear transformations, the approach we propose allows linear network to be trained due to the transformations carried out on the data by random linear network are linear too.

The remaining paper will be structured in the following way. In Sections 2, we discuss related works in linear network analysis and distillation learning application. In Section 3, 4, 5 we provide motivation why we use distillation as a method for linear networks training and gradually describe the components of our architecture. Furthermore, we hope that this split promotes the understanding of the final method. In Section 6, we present description and results of empirical evaluations. Section 7 concludes the paper and outline the further research.

2 Related Work

2.1 Deep Linear Networks

Deep linear networks (DLN) are deep feedforward networks with no nonlinearities. Output of each layer is multiplication of its weights to its input and the output of such network can be computed by matrix multiplication of all weight matrices:

$$\hat{y}^i = W_l W_{l-1} \cdots W_1 x^i$$  \hspace{1cm} (1)

where $\{(x^i, y^i)\}_{j=1}^n$ are data points and $\{W_i\}_{i=1}^l$ are weigh matrices of all $l$ layers. The total loss for such network is the sum:

$$L(W_1, \ldots, W_l) = \frac{1}{N} \sum_{i=1}^{N} \ell (\hat{y}^i, y^i)$$  \hspace{1cm} (2)

where $\ell (\hat{y}^i, y^i)$ is object loss.

Increasing depth of DLN does not increase its expressive power. Linear networks are trivial from representational perspective. However, they are in a focus of intensive research because, being simple and tractable, they share many of non-linear model proprieties. The study of linear networks loss dynamics is a source of novel research questions, helpful insights and brand new ways of looking at certain aspects of deep learning.

DLN may be useful in understanding deep learning phenomena such as regularization [24], initialization and momentum [25]. Understanding of DLN also leads to new insights in neural networks generalization properties and unsupervised learning algorithms [21].

One of the main DLNs’ advantages is that they exhibit highly nonlinear learning dynamics with increasing depth. The loss function of DLN is non-convex and behave similarly to loss functions of deep nonlinear networks: it may demonstrate nonlinear hyperbolic dynamics, may have plateaus and sudden performance transitions at learning. This is why they can be used to understand the loss surfaces of deep neural networks (DNN) [9][10].

2.2 Knowledge Distillation

Knowledge distillation (KD) is a method of transferring “knowledge” from one machine learning model called teacher to another one called student. The idea behind KD is that a teacher network is a high-capacity model with desired high performance and a student network is a more lightweight model [1][19][27]. A student cannot match the teacher, but the distillation process brings the student closer to the predictive power of the teacher. Distillation idea was brought to the neural network community by Hinton et. al. [7].

In distillation learning, knowledge is transferred by training a student model, using a soft target distribution for comparison with the output layer. This distribution is produced by a cumbersome model with a high temperature in its output softmax

$$S_i = \frac{\exp (z_i/T)}{\sum_j \exp (z_j/T)}$$  \hspace{1cm} (3)

with $z_i$ are logits and $T$ is temperature. Another scenario of knowledge distillation training is transferring knowledge from an ensemble of highly regularized models to a smaller model [7].
Distillation can also be applied for adversarial permutation [15], born-again neural networks [6] and Global Additive Explanations [26]. Furthermore, Sau and Balasubramanian [20] proposed to add random perturbations into soft labels for simulating learning from multiple teachers.

Surprisingly, a distillation method often allows smaller student network to be trained to mimic the larger and deeper models very accurately, while the student trained on the one-hot hard targets cannot achieve the same results. The clear reason for this awaits to be discovered.

2.3 Random Network Distillation

Random Network Distillation (RND) method was first proposed by Burda et. al [3]. The idea was to use distillation for creating a function of curiosity in a reinforcement learning agent. RND was used for exploration in environments with sparse rewards. In this setting, distillation allows the agent to determine whether the states were visited or not, and therefore use curiosity for the exploitation.

Let $O$ be a set of observable states. Predictor is a network $\hat{f} : O \rightarrow \mathbb{R}^k$ that is trained to predict behavior of target $f : O \rightarrow \mathbb{R}^k$ during interaction with the environment using MSE $\|\hat{f}(x; \theta) - f(x)\|^2$ for updating parameters $\theta_f$. If the difference between the predictions of the random network and the predictor at some environment state $S$ is large, the agent receives a higher curiosity reward. In fact, this can be considered as a model of novelty detection [18], the training process of which is performed via distillation of a random network. An important claim is that $\hat{f}$ can simulate the behavior of $f$ if their expressive powers are identical [3].

In our research, we are studying if the most inexpressive networks can be used for novelty detection. We show empirically that a linear network is also capable of novelty detection if the requirement of identical power is met.

3 Linear Network Distillation

Consider a classification problem with object set $X = \mathbb{R}^d$ and label set $Y = \{1, \ldots, C\}$. We are given a labelled dataset $D = \{x_i^{'}, y_i^{'}, y_i\}_{i=1}^N$, where $x_i^{'}, y_i^{'}, y_i \in \mathbb{R}^k$. The classification is performed by distance-based learning with some classifier $A : \mathbb{R}^k \rightarrow Y$ that works with object representation is some $k$-dimensional space. Consider also some target function $Q_\phi : X \rightarrow \mathbb{R}^k$, which maps objects to this vector space, which is DLN or just a matrix.

3.1 Class-Dependent Linear Distillation

Our idea is to create linear predictor $P_{\theta_c} : X \rightarrow \mathbb{R}^k$ for each class $c$ that would simulate behavior of the target on this class. Each predictor is trained to map objects $x_i^c$ of class $c$ into representation $Q_\phi(x_i^c)$.

Due to functions $\{P_{\theta_c}\}$ and $Q_\phi$ are linear, we can denote them in matrix form. For example, $Q_\phi$ is can be considered as a matrix multiplication $W_1W_2 \cdots W_1$. During training, labels $y_i^c \in \{0, C\}$ are used to activate one of the predictors $\{P_{\theta_c}\}_{c=1}^C$. The process of training uses MSE:

$$\mathcal{L}(P_{\theta_c}) = \frac{1}{N} \sum_{i=1}^N \|P_{\theta_c}x_i^c - W_1W_2 \cdots W_1x_i^c\|^2$$  \hspace{1cm} (4)

At the model evaluation step, we make prediction using the distance between $P_{\theta_c}(x_i^c)$ and $Q_\phi(x_i^c)$ for each class $c$: resulting label is chosen as $\arg\min_c \|P_{\theta_c}(x) - Q_\phi(x)\|^2$.

It is important to note, that despite each of $\{P_{\theta_c}\}$ is linear, composition of their results cannot be expressed with a linear function. Nevertheless, on each step of learning process, both teacher and student are linear.

We replaced the classification problem with the problem of approximating a linear function with linear functions for different classes, and call this method One to Many Distillation (O2MD).
3.2 Bayesian interpretation of Class-Dependent Distillation

Following the analogies in RND, our distillation framework can be presented as randomized prior functions for data $D = \{x^i, y^i\}_{i=1}^n$. Osband et al. [12] show in the paper that bootstrap approaches [23] and randomized prior functions provide a Bayes posterior in the linear case and in comparison to Exact Bayes provides much cheap computing.

In this setting, we investigate a distribution over functions $G_\theta = P_\theta + Q_\phi$, where parameters $\theta$ are specified by minimizing the expected prediction error with regularization $R(\theta)$ [12]. In our formulation, we have specific distribution $G_{\theta_c}$ for each class $c$:

$$\theta_c = \arg \min_{\theta_c} \mathbb{E}_{(x^i, y^i) \sim D} \left( (P_{\theta_c} + Q_\phi) \left( x^i_c \right) - y^i \right)^2 + R(\theta_c). \quad (5)$$

Parameters $\phi$ are drawn from prior $q(\phi)$ over the parameters of mapping $Q_\phi$ and after updating on the evidence we can extract from the posterior. In our case, if we set $y^i$ to 0 for every class, according to RND each distillation error is a quantification of uncertainty in predicting the constant zero function

$$\theta_c = \arg \min_{\theta_c} \mathbb{E}_{(x^i, y^i) \sim D} \left\| P_{\theta_c} \left( x^i_c \right) + Q_\phi \left( x^i_c \right) \right\|^2. \quad (6)$$

By default, we interpret our model as an unbiased ensemble with shared parameters, but in practice we can actually consider the model as ensemble of target-predictor networks for each class. In this setting, predictions and target in each ensemble are taken as the sum of target and predictor functions.

During bootstrapping with zero target, ensemble without priors has almost zero predictive uncertainty as $x$ becomes large and negative [13] which leads to to arbitrarily poor decisions [14].

4 Pretraining Linear Target Network

In previous Section, we have presented approach for learning linear predictors with linear target. In this Section, we present two approaches for selecting this target.

First, we will formulate desirable properties of target. Due to the predictors simulate target on corresponding classes, when we compare outputs, we can more clearly distinguish one of the predictor if dissimilarity with other classes is much larger. For example, if the target output for class 1 is very different from the output for other classes, the trained predictor $P_{\theta_1}$ for this class will be closer to the target than other ones. It may be possible if the target outputs for each class are far from each other.

4.1 Random teachers for target

One of the way to choose $Q_\phi$ that map classes in distinct regions is directly train it to do this. For example, by updating our target $\phi$ parameters using distillation from some teacher.

In our case, we noticed an interesting feature of distillation: teacher does not have to be able to generalize data. It would be sufficient if teacher function just map the train data classes in different regions of its output space. In contrast to the standard paradigm of distillation, our teacher is a set of random initialized linear functions $\{P_{c_i}\}_{c=1}^C$ for each class $c$ that map data in different spaces by default. At step $i$ we choose a linear network from $\{P_{c_i}\}_{c=1}^C$ corresponding to label $y^i$ and use its transformation as a target for our $Q_\phi$. Learning/optimization is performed by minimizing loss:

$$L(Q_\phi) = \frac{1}{N} \sum_{i=1}^N \| W_1 W_{i-1} \cdots W_1 x^i_c - P_{c_i} x^i_c \|^2. \quad (7)$$

We call this method Many to One Random Distillation (M2ORD). The prediction accuracy of the model trained in this way is sub-optimal. It seems to be difficult to learn data distribution from the teachers’ outputs because the transformation the teachers apply to the data is not linear.
4.2 Orthonormal teachers for target

The idea of orthogonalization is to make \( \{ P_\zeta c \}_{c=1}^C \) functions orthogonal to each other to maintain distinct mapping for each class. We used the Gram–Schmidt process \(^8\) to make teacher functions orthogonal. This method is used in linear algebra for orthonormalizing a set of vectors. We create \( \{ P_\zeta c \}_{c=1}^C \) for each class with size of \((n, d)\), then join them in one matrix \( A \) size of \((C \cdot n, d)\), where \( d > C \cdot n \).

We use singular vector decomposition (SVD) \(^4\) to obtain an orthogonal matrix. SVD of a matrix \( A \) is the factorization of \( A \) into the product of three matrices: \( A = UDV^T \), where \( U \) is orthogonal. After obtaining results of SVD, we take \( U \), and split it into \( C \) matrices \( \{ P_\zeta c \}_{c=1}^C \). Each such matrix \( P_\zeta c \) consists of strings in \( U \) corresponding to the objects of class \( c \). As a result, all rows of all teacher matrices are linearly independent and project points into subspaces that are orthogonal.

\[
\|P_{\zeta 1}\| = \|P_{\zeta i}\| = \cdots = \|P_{\zeta c}\| = 1 \quad P_{\zeta 1} \perp P_{\zeta i} \perp \cdots \perp P_{\zeta c}
\]

We call this method **Many to One Orthonormal Distillation** (M2OOND).

5 Bidirectional Distillation

In the previous two Section we presented ideas of O2MD for learning linear predictor functions using distillation and M2O*D for learning linear target function using distillation. In this Section, we combine this two ideas in a method we call **Bidirectional Distillation**.

After initialization, our predictors \( \{ P_\theta c \}_{c=1}^C \) are equal to \( \{ P_\zeta c \}_{c=1}^C \), so at the first step we pretrain our function \( Q_\phi \) by \( \{ P_\theta c \}_{c=1}^C \) in M2O*D and then distill this knowledge back into \( \{ P_\theta c \}_{c=1}^C \) by O2MD training.
The learning procedure is presented in Figure 1. During Bidirectional Distillation, we alternate O2MD and M2OD in different proportions, we consistently train them a certain number of iterations, allowing to be updated several times in each epoch.

6 Experiments

In this Section, we describe the results of experiments, comparing One-To-Many and Bidirectional variants of a LDL model against a deep fully connected neural network, logistic regression and Naive linear model. In Naive settings, our predictors trained without target. Each predictor training like an autoencoder, but without any dimensionality reduction, during prediction we just measure distance between each predictor output and sample $x_i$.

All of our experiments are formulated within few-shot problem framework. Each model was provided with a set of $m$ labelled samples from $C$ classes. In a few-shot terminology, $C$ is traditionally called a way and $m$ is called a shot. The task is $C$-class classification. Unlike traditional few-shot learning models, our approach does not imply knowledge transfer between episodes, being thus more similar to small-sample learning [22]. To avoid bias between small number of samples and reported results, each model is trained for 100 independent trials and average accuracy is reported. We ran experiments on two datasets with images of low resolutions, namely MNIST and Omniglot. Our experiments are conducted on a machine with two separate NVIDIA GeForce GTX 1080 Ti, 4 core Intel i5-6600K processors and 47.1 GB RAM memory. We use PyTorch 1.1 with CUDA 10 support for implementation of all models presented in the study. All datasets were uploaded into memory preliminary for faster computation.

6.1 Comparison of the target network pretraining methods

In Figure 2, the difference of data transformations between random and orthogonal teacher is displayed. During the training, we noticed that orthogonal teachers M2OOND do not give an advantage to M2ORD, since the transformation that teacher function sets is not linear. As a result, the training of the target function using orthogonal teacher does not differ from the training with random. This method can be used in a nonlinear case where target function can approximate such subtle teachers transformations.

![First and Second Principal Components colored by digit](image)

(a) Random linear networks

![First and Second Principal Components colored by digit](image)

(b) Orthogonal linear networks

Figure 2: Output of teacher by PCA on MNIST dataset

6.2 Experiments and results on MNIST

For experiments on the MNIST dataset, a small number of samples from each of the digits were given to the model, thus the way was always set to 10. After a model was trained on the given samples, accuracy was calculated on the whole official test part of the MNIST dataset for the easier comparison with well-known approaches. Models were trained on the shots of sizes 1, 10, 50, 200, 300. For testing models, we performed no data preprocessing and image augmentation. As a result, a single sample was a flattened representation of the $28 \times 28$ grayscale image. We also chose learning rates
between $1e-3$, $1e-4$ and $5e-5$ values. We noticed that large numbers of epochs were not sufficient for the linear setup and chose the total number epochs to be not greater than 10 for all training shots.

Linear distillation approaches were compared with logistic regression and fully connected network. Fully connected network was chosen between configurations of one and two hidden layers of sizes 64, 256 or 1024. Most promising results were reported for each of the shot value. Results comparing the One-To-Many and Bidirectional Distillation models for different number of shots are shown in Table 1. In our results it is no surprise that accuracy increased with the size of the training dataset, but it slowed down when the size of the dataset reached the size of about thousand samples.

| Samples per class | Naive | LogReg | MLP | O2MD | BidirDistill |
|-------------------|-------|--------|-----|------|--------------|
| 1                 | 0.127 | 0.316  | 0.448 | 0.426 | 0.436 |
| 10                | 0.777 | 0.679  | 0.749 | 0.801 | 0.800 |
| 50                | 0.903 | 0.839  | 0.881 | 0.912 | 0.917 |
| 100               | 0.898 | 0.870  | 0.926 | 0.934 | 0.917 |
| 200               | 0.942 | 0.892  | 0.929 | 0.953 | 0.953 |

An important property of our architecture that is learn fairly quickly, on a small training epochs. The convergence of each student to the teacher is presented in Figure 3, where students are $\{P_{θ_c}\}_{c=1}^C$, and the teacher is our $Q_φ$ network.

![Figure 3: Test loss curve for Bidirectional Distillation model. The model is trained for 10 epochs on MNIST dataset with 5 samples per class](image)

Training a Bidirectional Distillation model and a O2MD version have advantages over the classic deep fully connected network on a small amount of data. Bidirectional training allows much more faster converging to an almost best capabilities of the model after the first epochs, because the target pre-trained on predictors simplifies training. An example of the training process comparing MLP with two hidden layers and the distillation models is depicted in Figure 4. The y axis shows accuracy on the full test part of the dataset measured after each epoch/sample.

### 6.3 Experiments and results on Omniglot

The Omniglot dataset consists of 1623 characters from 50 different alphabets, where each of the characters was drawn by 20 different people. Like authors of Matching Networks [28], we augmented existing classes with rotations by multiples of 90 degrees and used 1200 characters from training and the remaining character classes for evaluation. We resized images to the size of $28 \times 28$ pixels and obtained the very same model settings as for the previous dataset.

We experimented with different learning rates ($1e-3$, $1e-4$, $1e-5$) and optimizer types (sgd, adam, adadelta) and reported results from most promising configurations. We tested our models on the way $N$ of sizes 3, 5 and 10 with shots $m$ of size 1, 3, 5, and 10. After training on the given $N \cdot m$
Figure 4: Accuracy curve for the MLP, One-tp-Many and Bidirectional Distillation models. The models are trained for 10 epochs on MNIST dataset with 1, 5 and 10 samples per class.

...samples, we measured the accuracy on the \( N \) unseen samples from these \( N \) classes (one sample per class). For the One-To-Many and Bidirectional setups, we provided results for 784 and 2000 networks’ output dimensions. To provide unbiased results, we averaged results over 100 independent runs. Results comparing studied models for different number of shots are shown in Table 2.

| WAY | SHOT | O2MD OUTPUT DIMENSION | O2MD OUTPUT DIMENSION |
|-----|------|------------------------|------------------------|
|     |      | 784                    | 2000                   |
| 3   | 1    | 0.563                  | 0.593                  |
|     | 3    | 0.683                  | 0.720                  |
|     | 5    | 0.800                  | 0.803                  |
|     | 10   | 0.797                  | 0.830                  |
| 5   | 1    | 0.428                  | 0.454                  |
|     | 3    | 0.614                  | 0.626                  |
|     | 5    | 0.666                  | 0.674                  |
|     | 10   | 0.806                  | 0.778                  |
| 10  | 1    | 0.305                  | 0.342                  |
|     | 3    | 0.480                  | 0.536                  |
|     | 5    | 0.572                  | 0.585                  |
|     | 10   | 0.685                  | 0.689                  |

Our model shows acceptable results mainly on a small number of ways. Our model is sensitive to hyperparameters settings, but nevertheless, is able to learn classification with proper configuration.

7 Conclusion and Future Work

In this paper we present an architect based on the several methods of random function distillation using linear networks. The first method is a training of a single network to simulate the behaviour of another on a particular class. The second method is a training of a single network to predict the behavior of many other networks for each class in dataset. Our architecture can be considered through the Bayesian lens too. The motivation for our work was to create an architecture which consists of linear functions capable of classifying on a small dataset. We tested our model on several datasets and showed results comparable to the results of nonlinear models on small amounts of data.

For the Omniglot dataset, we tested our architecture in the few-shot learning paradigm. Our model does not have the key concept of the paradigm to preserve knowledge in the learning process between sub-datasets (episodes). This is an important issue for future research, so the further studies will be focused on using our method in the few-shot tasks on a higher level. There is abundant room for further progress in using distillation as a method of learning, and explore open questions in this area. The development of linear networks is substantial area for further research. In further work, we will explore open questions around linear networks, to fully reveal their potential and capabilities.
8 Acknowledgments

We would like to thank Artem Zholus for productive discussions.

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