Higgs boson: a composite dilaton and its mass dependence on the constituent mass anomalous dimension

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The Higgs boson can be a composite dilaton as discussed by Eichten and Lane\textsuperscript{[5,6]}, following this idea we perform a Bethe-Salpeter equation (BSE) evaluation of scalar boson masses in order to verify how these masses can be smaller than the composition scale. The calculation is developed with an ansatz for the constituent self-energy dependent on its mass anomalous dimension (γ), and we obtain a relation showing how the scalar mass decreases as γ is increased. We also discuss how fermionic corrections to the BSE shall decrease the scalar mass, which is one of the arguments in favor of the composite dilaton case.

I. INTRODUCTION

The discovery of the Higgs boson at the LHC\textsuperscript{[1,2]} completed the Standard Model (SM), where a scalar boson sector is present as proposed long ago\textsuperscript{[3]}. In many extensions of this model this scalar sector is even larger than the SM one, although experimental signals of new particles belonging to this sector are still missing. Furthermore, there are theoretical shortcomings about this scalar sector\textsuperscript{[4,5]}

The absence of signals of a large scalar boson sector in the experimental data, as well as a possible explanation of a light Higgs boson has been beautifully discussed recently in Ref.\textsuperscript{[6,7]}. In that reference it is argued that the Higgs boson is a Gildener-Weinberg dilaton\textsuperscript{[8]}, but a composite dilaton\textsuperscript{[9]}. Composite scalar bosons appear in the context of Technicolor theories (TC)\textsuperscript{[6,11]}, which usually have a composition scale of order of Λ\textsubscript{H} ≥ 1TeV. However, a long standing problem is why the Higgs boson mass (m\textsubscript{H}) is smaller than this scale, and why there are not signals of many other possible composite states of this scheme.

As remarked in Ref.\textsuperscript{[6,7]} a composite Gildener-Weinberg dilaton should result from a composite effective potential that, at leading order, is proportional to

\[ V(\Phi)_{lo} \propto \Phi^4, \]  

(1)

where Φ is a composite effective field. The effective potential at larger order will have contribution from different particles, including fermions, gauge bosons and scalars. The Higgs boson mass appears at next order of this potential involving all masses present in the theory, and leading naturally to a light mass depending on cancellations between these masses, particularly influenced by the corrections of fermionic masses as clearly explained in Ref.\textsuperscript{[7]} (see, for instance, Eq.(2) of that reference).

A light composite scalar boson is also generated when the strong interaction theory (or TC) has a large mass anomalous dimension (γ) as proposed by Holdom\textsuperscript{[12]}, and discussed by many others\textsuperscript{[12,23]}. In this case the self-energy of the new fermions (or technifermions) responsible for the composite states is characterized by a large mass anomalous dimension, resulting in mass diagrams whose calculation do not scale with the naive dimensions. This self-energy at large momenta is proportional to

\[ \Sigma_C(p^2) \propto \frac{\mu_C^3}{p^2}(p^2/\mu_C^2)^{\gamma/2}. \]  

(2)

where \( \mu_C \) is the typical composition or dynamical mass scale and γ the mass anomalous dimension. It is now known that according to the mass anomalous dimension, this self-energy may vary asymptotically from a 1/p\textsuperscript{2} behavior, up to a very slowly decreasing logarithmic behavior with the momentum\textsuperscript{[26,27]}. It is usually assumed that large γ values appear in gauge theories with higher fermionic representations or with a large number of fermions. However, Eq.(2) can be quite modified just coupling two strongly interacting theories\textsuperscript{[27,28]}

The idea of a composite Gildener-Weinberg dilaton is quite natural. In a strong interaction gauge theory the effective potential for composite operators\textsuperscript{[29]} leads naturally to a potential free from a Φ\textsuperscript{2} term, which is the main point in the calculation of the Gildener-Weinberg potential. The absence of a Φ\textsuperscript{2} term in the composite effective potential was explicitly verified in an Abelian model\textsuperscript{[30]}, as well as in a non-Abelian gauge theory\textsuperscript{[31,32]}. The absence of a Φ\textsuperscript{2} term in the composite effective potential is a consequence of assuming that the fermionic propagator used to compute the effective potential obey the Schwinger-Dyson equation of the constituent fermions. Although this fact is not usually discussed in the literature, we stress that it is quite relevant to complement the arguments of Refs.\textsuperscript{[6,7]}

It must be noticed that the effects of different mass anomalous dimensions and presence of massive fermions that lower the scalar mass relation are all intertwined.
They modify the mass relation between scalar boson and other particles. A full (and lengthy) calculation of an effective potential for composite operators can be performed within a specific constituent model. However, the large order terms of the effective potential are quite dependent on the constituent mass anomalous dimension, and do not provide simple mass relations as appear in Refs. [6, 7]. Therefore, it is interesting to obtain a simple estimate of the scalar composite mass as a function of $\gamma$, as well as a determination of how much fermionic contributions can decrease this scalar mass.

In this work we calculate the scalar composite mass with the help of Bethe-Salpeter equations (BSE), obtaining a relation between the scalar mass and $\gamma$. We present a fit of such relation expecting that it could be tested by other methods. This calculation is shown in Section II. In Section III we perform a simple order of magnitude estimate of fermionic loop effects that can lower the scalar mass value. Our results are discussed in the case of the QCD scalar boson (the sigma meson) and in the composite Higgs case. Section IV contains our conclusions.

II. THE SCALAR MASS: BSE AND MASS ANOMALOUS DIMENSION

The scalar boson mass can be calculated using BSE once we specify the propagators and vertices of the strong interaction that binds such scalar boson. For the scalar case the Bethe-Salpeter equation has the form

$$\chi(p, q) = -i \int \frac{d^4k}{(2\pi)^4} S(g + \alpha p) K_{\mu\nu}(p, k, q) S(q - \beta p),$$

$$K_{\mu\nu}(p, k, q) = \gamma_{\mu}\chi(p, k)\gamma_{\nu} G_{\mu\nu}(k - q),$$

(3)

where $\alpha + \beta = 1$ ($\alpha$ and $\beta$ characterize the fraction of momentum carried by the constituents), although the result is not dependent on these quantities. As a first approximation we shall choose $\alpha = \beta = 1/2$. $G_{\mu\nu}$ is the gauge boson propagator in the Landau gauge. There are different models for this quantity which will be discussed ahead.

The fermion propagator is given by

$$S^{-1}(q) = gA(q^2) - B(q^2),$$

(4)

From now on we shall assume $A(q^2) = 1$ and $B(q^2) = \Sigma(q^2)$. Usually in this type of calculation the self-energy $\Sigma(q^2)$ is obtained from the numerical solution of the Schwinger-Dyson equation (SDE) for the fermionic propagator. In our case this self-energy is going to be obtained by one ansatz that is a function of the mass anomalous dimension $\gamma$.

We recall that $\Sigma(q^2)$ at large momenta in QCD (or any asymptotically free non-Abelian gauge theory) is given by

$$\Sigma(q^2) \propto \frac{\langle \bar{\psi}\psi \rangle^2}{q^2} \left( \frac{q^2 + m^2}{\mu^2} \right)^{-\kappa/2},$$

(5)

assuming for the fermionic condensate $\langle \bar{\psi}\psi \rangle \approx \mu^3$ where $\mu$ is the dynamically generated mass.

The ansatz that we assume for the self-energy has the form

$$\Sigma(q^2) = \frac{\mu^3}{q^2 + \mu^2} \left( \frac{q^2 + \mu^2}{\mu^2} \right)^{\kappa},$$

(6)

where

$$\kappa = \gamma/2.$$  

(7)

Eq. (6) behaves in the infrared region as a mass $\mu$ and decays at large momenta as prescribed in Eq. (5), mapping the SDE in the full Euclidean space, and will allow us to obtain BSE solutions for the scalar mass as a function of $\gamma$. Note that $\kappa$ is just a parameter that when $\kappa = 0$ the self-energy behaves asymptotically as $1/p^2$, and when $\kappa \to 1$ Eq. (6) behaves like

$$\Sigma(q^2) \approx \mu \left[ 1 + \delta_1 \ln \left( \frac{(q^2 + \mu^2)}{\mu^2} \right) \right]^{-\delta_2},$$

(8)

where $\delta_1$ and $\delta_2$ are obtained from $\gamma$ when expanded as a function of the running coupling $g^2(q^2)$.

The BSE solution appear as an eigenvalue problem for $p^2 = M^2$, where $M$ is the bound state mass. The variables are $p, q, k$. $k$ is integrated and we remain with a equation in $q$ that will have a solution for $p^2 = M^2$. In the Eq. (6) $\chi$ can be projected into four coupled homogeneous integral equations, that implies for projection of the scalar component

$$\chi(p, q) = \chi_{S0} + \rho \chi_{S1} + q \chi_{S2} + [p, q] \chi_{S3},$$

(9)

which are functions of $p^2$, $q^2$ and $p.q$.

It is possible to expand $\chi(p, q)$ in terms of Tscheby- shev polynomials, and these equations can be truncated at a given order determined by the relative size of the next-order functions. In accordance with Ref. 33, a satisfactory solution can be obtained by keeping only some terms, like $\chi_{S(0)}$, $\chi_{S(1)}$, $\chi_{S(2)}$.

To set up the problem we follow closely the work of Ref. 34, where the BSE was solved for the scalar boson constituents with masses $m_a$ and $m_b$, and for simplicity we assume

$$m_a = m_b = m = \Sigma(x + \frac{1}{4}p^2),$$

(10)

where it was assumed $\alpha = \beta$ (each constituent carries half of the momentum), and $x = q^2/\Lambda^2$, where $\Lambda = \Lambda_{QCD}$ (or $\Lambda = \Lambda_H$, the TC mass scale) the characteristic mass scale of the bounding force.

The different components of Eq. (9), $\chi_{S(i)}$ for $i = 0, .., 3$, are given by

$$\chi_{S(i)} = 2\left[ (x - \frac{1}{4}p^2 - m^2)J_i \right] S_{ij} + \Delta \chi_{S(i)},$$

(11)

with

$$I_{S0} = \frac{2}{3\pi} \int dyy\chi_{S0} K_1,$$

(12)
where \( y = k^2/\Lambda^2 \) and
\[
K_1(x, y) = \frac{3}{16\pi^2} \int d\theta \sin^2 \theta G(x, y, \cos \theta),
\] (13)
\[
J_1 = \frac{2}{\pi} \int_0^\pi d\theta \frac{\sin^2 \theta}{D(p^2, q^2, pq \cos \theta)}
\] (14)
and in the Eq. \[14\]
\[
D(p^2, q^2, pq \cos \theta) = ((q + \frac{1}{2} p)^2 + m^2)((q - \frac{1}{2} p)^2 + m^2).
\] (15)

In the above equation we can expand \((q + \frac{1}{2} p)^2 + m^2\) and \((q - \frac{1}{2} p)^2 + m^2\) in Taylor series. Just keeping the first-order derivative terms for \( m \), we have that the function \( J_1 \) can be written in the form
\[
J_1 = \frac{2}{c_1 + c_2 + c_3} \left[ \frac{c_2}{D_1} + \frac{c_3}{D_2} + d_1 \left( \frac{c_1}{D_1} - \frac{c_3}{D_2} \right) \right]
\] (16)
where, in our approximation, we have
\[
c_1 = c_3 = x + \frac{1}{4} p^2 + m^2
\]
\[
c_2 = c_4 = 1 + 2 m m'
\]
and \( m' \) is the derivative of \( m \) with respect to the momentum. In addition, as a consequence of \( \alpha = \beta \) we obtain
\[
d_1 = 0
\]
\[
D_1 = D_2 = c_1 + \sqrt{c_1^2 - p^2 x c_2}.
\]

In Eq. \[11\], the term \( \Delta \chi_{S0}^{(0)} \) represent corrections to the leading-order results of \( \chi_{S0}^{(0)} \), that correspond to \( \chi_{S1}^{(0,1)} \), \( \chi_{S2}^{(0,1)} \) and \( \chi_{S3}^{(0,1)} \). With the approximations considered here we obtain
\[
\Delta \chi_{S0}^{(0)} = -\frac{4}{3\pi} m J_1 \int dyy \chi_{S2}^{(0)} \sqrt{xy(3K_6 - 2\sqrt{xy} K_3)} - \frac{2}{3\pi^2} (J_3 - J_1) \int dyy \chi_{S3}^{(0)} \left\{ 2 \sqrt{xy} K_6 - \frac{8}{3} xy K_3 \right\} - \frac{4}{\pi} \left\{ (-x + \frac{1}{4} p^2 + m^2) J_2 \right\} \int dyy \chi_{S0}^{(1)} \sqrt{\frac{2}{x} K_6} - \frac{2}{\pi} \left\{ (-x + \frac{1}{4} p^2 + m^2) J_1 \right\} \int dyy \chi_{S0}^{(2)} \left( \frac{4}{3} K_7 - K_1 \right).
\] (17)

In the Eq. \[17\], the lowest order terms \( \chi_{S(1-3)}^{(0)} \) are given by
\[
\chi_{S1}^{(0)} = 0
\] (18)
\[
\chi_{S2}^{(0)} = \frac{4}{\pi} m J_1 I_{S0} + \frac{2}{3\pi} \left( -x + \frac{1}{4} p^2 + m^2 \right) \int dyy \chi_{S2}^{(0)} \left( \frac{3}{8} \sqrt{\frac{2}{x} K_6} - 2y K_3 \right),
\] (19)
and
\[
\chi_{S3}^{(0)} = \frac{3}{2} J_1 I_{S0}.
\] (20)

While the higher order terms by
\[
\chi_{S0}^{(1)} = -6 \left( -x + \frac{1}{4} p^2 + m^2 \right) \frac{J_2}{x p^2} I_{S0},
\] (21)
and
\[
\chi_{S0}^{(2)} = -3 \left( \frac{1}{x p^2} \right) \left( -x + \frac{1}{4} p^2 + m^2 \right) \left( 4 J_3 - J_1 \right) I_{S0}.
\] (22)

Since we are dealing with a scalar boson case with equal mass constituents the equations of Ref. \[34\] are also simplified and we have:
\[
J_2 = 0
\] (23)
\[
J_3 = \frac{1}{D_1^2}
\] (24)
where in the Eqs. \[19\] \[19\]
\[
K_3(x, y) = \frac{3}{16\pi^2} \int d\theta \frac{\sin^4 \theta}{x + y - 2 \sqrt{xy} \cos \theta} G(x, y, \cos \theta),
\] (25)
\[
K_6(x, y) = \frac{3}{16\pi^2} \int d\theta \sin^2 \theta \cos \theta G(x, y, \cos \theta),
\] (26)
\[
K_7(x, y) = \frac{3}{16\pi^2} \int d\theta \sin^4 \theta G(x, y, \cos \theta).
\] (27)

These equations can be solved making use of suitable expressions for the main Green functions (i.e. propagators and vertices). The gauge boson propagator is given by
\[
G(k^2) = \frac{16\pi}{3} \left[ \frac{\pi d}{k^2 \ln(\pi k^2 + x)} \right] + G_{IR}(k^2),
\] (28)
where \( G(x, y, \cos \theta) = \Lambda^2 G(k - q)^2 \), and \( G_{IR}(k^2) \) is an assumed form of the interaction at infrared momenta. In Ref. \[34\] this contribution was chosen to be of the form
\[
G_{IR}(k^2) = \frac{16\pi}{3} \alpha k^2 e^{-\frac{k^2}{\lambda^2}},
\] (29)
and in the gaussian ansatz for the $G_{IR}(k^2)$, $\omega \in [0.4,0.6]$GeV$^2$. According to Ref. [34] the parameters used in the QCD case are given by

$$a = (0.387 \text{GeV})^{-4}, \quad \omega = (0.510 \text{GeV})$$

$$d = 12/(33 - 2n_f), \quad n_f = 5$$

$$\mu \approx \Lambda_{QCD} = 0.228 \text{GeV}, \quad x_0 = 10.$$  \hspace{1cm} (30)

More recent expressions for these Green functions were formulated in Refs. [35-39], where the two free parameters in Eq. (29), $\omega$ and $a = D$, are parameterized by $\langle \sigma \rangle^3 = D\omega = \text{const}$, and the fitted values of $\langle \sigma \rangle$ depend on the form that is assumed for the dressed-gluon quark vertex. Considering the recent results reported in Ref. [27], it is possible to verify that these choices do not modify substantially the numerical results.

To find the eigenvalues $p^2 = M^2$, we consider an interactive process

$$\Delta(p^2) = \chi(p^2,0) - \chi(p^2,q^2),$$  \hspace{1cm} (31)

where the function $\chi(0,0) = \chi_{\text{free}}(0)$ is fixed to some arbitrarily chosen value. For a given value of $p^2$, not an eigenvalue, $\Delta(p^2)$ is not zero, and we therefore obtain a system of inhomogeneous integral equations which are solved by iteration. The eigenvalue where $p^2 = M^2$ can then be computed by finding $p^2$ such that we obtain $\Delta(p^2) = 0$.

Assuming Eqs. (4-29) in Fig.1 we present the results obtained for $\Delta(p^2)$, given by Eq. (31), considering the anzats Eq. [9] for $m$. In this figure we normalize ours results for $M_S$ in terms of

$$M_S = 2 \Lambda_{QCD},$$

associated to a negligible $\gamma$.

The choice of this normalization is based on the result described in Ref. [10], where Delbourgo and Scadron verified analytically with the help of the homogeneous Bethe-Salpeter equation (BSE), that the sigma meson mass is given by $m_\sigma = 2 \mu_{\text{dyn}}$. In this calculation it is assumed that the dynamically generated quark mass behaves (for large $p^2$) as $m_{\text{dyn}} \sim \mu_{\text{QCD}}^2 q^2$, which corresponds to the case where $\kappa = 0$ in the ansatz proposed in Eq. [9]. In this way, based upon this normalization, we can follow the behavior of how $M_S$ resulting from Eq. [31] is influenced by $\gamma$.

In Fig.(1a) the black line corresponds exactly to the case where $\kappa = 0$, while the blue line to $\kappa = 0.2$. In Fig. (1b) we consider the cases where $\kappa = 0.3$ (red line) and $\kappa = 0.4$ (green line), the Fig.(1c) is a composition of the previous results, where we indicate for each curve the value assumed for $\kappa$. Note that in Fig1.(a-b) in the upper right corner, we describe $M_S$ found in each case observing that for a given $z = p^2/\Lambda^2_{QCD}$ we have

$$M_S = \sqrt{2} \Lambda_{QCD},$$  \hspace{1cm} (32)

Finally, in Fig.(1d), we present the behavior of $M_S(\gamma)$ obtained for $\kappa$ in the range $\in [0,0.95]$. We obtained a very simple fit to the data with $R^2 = 0.977$ which corresponds to

$$A(\gamma) = \frac{M_S(\gamma)}{\Lambda_{QCD}} = \frac{2.15}{1 + \gamma/2}^{5.34}.$$  \hspace{1cm} (33)

It is clear that Fig.(1d) may be slightly dependent on the propagators and vertex that we assumed here. However, we expect that the behavior of this curve can be tested by other methods, and more importantly it shows how the scalar composite mass should behave as we vary the constituent mass anomalous dimension.

We can now focus on the Higgs boson case. In Fig.2 we extend the results obtained for QCD in the case of a $SU(3)$ TC model. As a first approach, since TC is based upon an analogy with the dynamics of QCD, we can use the equations obtained for QCD to determine, by appropriate rescaling, the behavior of $M_S^{TC}(\gamma)$. Hence, we can estimate $\omega_{TC}, a_{TC}$ from the QCD analogue using the following scaling relation

$$(\omega, a)_{TC} = \sqrt{\frac{N_{TC}}{3}} \frac{\Lambda_{TC}}{\Lambda_{QCD}} (\omega, a)_{QCD},$$

where $\Lambda_H = \Lambda_{TC} = 1$TeV. In this case the results for $M_S(\gamma) = M_S^{TC}(\gamma)$ follow from the normalization $M_S = 2 \Lambda_{TC}$; and we include in Fig.(2d) the dot-dashed line in red, which corresponds to the observed Higgs boson mass for the purpose of comparison with the $M_S(\gamma)$ behavior. In the region where $\kappa \sim 0.8$, we recover the result obtained for the extreme walking behavior [41], where for a $SU(3)$ TC model in that reference, assuming
for $\Sigma(p^2)_{TC}$ the behavior given by Eq. (31), we obtained $M_H \sim O(110)$ GeV.

In Fig. (2d), we present the behavior for $M_S^{TC}(\gamma)$ obtained in the range $\kappa \in [0, 0.95]$. The fit obtained with $R^2 = 0.988$ corresponds to

$$A(\gamma) = \frac{M_S^{TC}(\gamma)}{\Lambda_{TC}} = \frac{2.1}{(1 + \gamma/2)^{5.12}}.$$  

(34)

This result shows how the composite Higgs boson mass may vary with the constituent mass anomalous dimension. However, as we will discuss in the next section, it is not only the value of $\gamma$ that modif\ies these estimates. Note that Eq. (34) do not differ appreciably from Eq. (33) due to the fact that we have chosen the TC gauge theory as a QCD rescaled version.

### III. SCALAR MASSES: THE EFFECT OF FERMIONS

It is interesting to see that the problem of understanding a possible light (composite) Higgs boson also happens in the case of the sigma meson. The sigma meson is the QCD scalar composite now known as $f_0(500)$. A standard BSE calculation gives $m_\sigma = 670$ MeV [42], which is larger than its experimental value. The detailed work of Ref. [42] deals with possible contributions that may lower the estimate of this mass. A composite $J = 0$ state may have many contributions to its mass. In the case of the sigma meson it is not even clear how much of its composition is due to different quarks, even more the amount of its mass that is due to gluons, although it is already a puzzle the fact that a simple BSE mass estimate gives a result larger than the experimental value.

If we go back to the linear sigma model at constituent level we know that it couples to fermions as

$$\mathcal{L}_\sigma = \lambda \bar{\Psi} \Psi \phi,$$

(35)

where $\phi$ stands for $\sigma$. This coupling imply that the $\sigma$ mass obtains contributions from the BSE diagram shown in Fig. (3a), that comes with a negative sign due to the effect of a fermion loop. Eq. (35) also describe the Higgs Yukawa coupling to fermions. In particular, when we consider a composite Higgs boson the coupling to fermions is more sophisticated, we may even have fermionic contributions to the BSE like the one shown in Fig. (3b), involving the exchange of extended TC gauge bosons (ETC) [11]. However, as the gauge bosons of Fig. (3b) are very heavy, the vertex in that figure can be reduced to an effective vertex as shown in Fig. (3c) and the final BSE mass contribution can be reduced to the one of Fig. (3a).

The fermionic contribution to the BSE would be given by

$$\Pi(p^2) = m^2_{f(t)} Tr \int_0^\Lambda d^4q \chi(q^2) \mathcal{F}(q,p,f(t)) \chi(q^2),$$

where the vertex of the BSE (due to a fermion $f$ or to the top quark $t$) reads

$$m_{f(t)} \propto \chi(p^2),$$

(37)

where we stress the effect of the large top quark mass in the calculation of the Higgs boson mass. A full calculation of the BSE including fermionic corrections is a lengthy work and is under study; it may affect the sigma as well as the Higgs boson mass estimate. In the case of the Higgs boson we can resort to a simple estimate of the loop of Fig. (3a), i.e. a correction of $M_\phi^2$ represented by $\delta M_\phi^2$, is given by

$$i\delta M_\phi^2 = \frac{\lambda^2 N_f}{(2\pi)^4} \int d^4q \frac{\Sigma^2(q^2)}{(q^2 - \Sigma^2(q^2))^2},$$

(38)

where $N_f$ is the number of fermions ($f$) in the loop, and the biggest effective coupling $\lambda$ (when $f=$ top quark) is given by

$$\lambda = m_\ell(\gamma)/v,$$  

(39)

and $v$ is the standard model vacuum expectation value ($v = 2 M_W/g_\theta$).

Eq. (38) is enough to verify the order of magnitude that we shall obtain when solving the complete system of BSE for the scalar boson. Considering the anzats described by Eq. (1), in euclidean space we obtain the following expression for $\delta M_\phi^2(\gamma)$

$$\delta M_\phi^2(\gamma) = \frac{\lambda^2 N_f}{4\pi^2} f(\gamma) \Lambda_{TC}^2$$

(40)
where

\[ f(\gamma) = \frac{1-\gamma}{3-\gamma} \quad (41)\]

The behavior of Eq.(41) with \( \gamma \) is an artifact of our approximation, since the \( \gamma \) running with momentum was not considered, and its value has to be bounded so that the scalar boson wave function is quadratically integrable [43, 44].

The contribution due to the fermion loop indicated in Fig.(3a), particularly in the extreme walking behavior or massive top case for the self-energy (i.e. \( \gamma \to 2(\text{or } \kappa \to 1) \)), tends to decrease \( M_2^2 \) according to Eq.(40), and in this case this contribution lowers the estimate of the composite scalar boson mass.

In Fig.(3c), the vertex \( \lambda \) can be approximately represented by

\[ \lambda \approx \frac{g_w}{\pi} N_F \lambda_{ETC} \frac{\Lambda_{ETC}}{M_W} \left( \frac{\Lambda_{ETC}^2}{\Lambda_{ETC}^2} \right)^{1-\kappa}, \quad (42)\]

where \( N_F \) is the number of technifermions that couple to the fermions (f) in the loop, and we assumed the existence of an ETC gauge theory with coupling \( \alpha_{ETC} \). The effective charge, \( \lambda_{ETC} = C_{ETC} \alpha_{ETC} \), involve the ETC coupling and the appropriate ETC Casimir operator eigenvalues \( C_{ETC} \). The top quark makes the most significant contribution in the loop described in Fig.(3), as discussed in Refs. [45, 46] in the extreme walking behavior, its mass can approximately be expressed by \( m_t(\gamma) \approx N_F \lambda_{ETC} \Lambda_{ETC} \), so we can write the vertex \( \lambda \) as

\[ \lambda \approx \frac{g_w}{\pi} m_t(\gamma) \left( \frac{\Lambda_{ETC}^2}{\Lambda_{ETC}^2} \right)^{1-\kappa}. \quad (43)\]

In the limit when \( \kappa \to 0 \), we have

\[ \delta M_2^2(0) \approx 0, \]

while in the limit when \( \kappa \to 1(\text{or } \gamma \to 2) \), we recover the effective coupling of the top described in Eq.(39)

\[ \lambda \propto \frac{g_w}{\pi} m_t(\gamma) \approx \frac{m_t(\gamma)}{v} \]

and we obtain

\[ \delta M_2^2(2) \approx -\frac{3\lambda^2}{4\pi^2} \Lambda_{ETC}^2 \]

\[ \approx -\frac{3}{\pi^4} \left( \frac{m_t(2)}{v} \right)^2 \Lambda_{ETC}^2. \quad (44)\]

At this point we should highlight that in the parameterization of the estimate presented by Eq.(44), \( m_t(2) \) is model dependent. In Fig.(3), the number of technifermions \( F (N_Q: \text{techniquarks or } N_L: \text{technileptons}) \) that generate the \( m_t(2) \) mass depends on ETC interactions. However, we can consider as an illustrative example the model described in Ref.[46], where in Fig.(3) of that reference, we present the diagrams that contribute to \( m_t(2) \) in this work. As a result of these contributions \( m_t(2) \) was estimated to be on the order of \( \sim 100 \text{GeV}, \) what leads to

\[ |\delta M_2(2)| \approx 70 \text{GeV}. \quad (45)\]

As we have seen, the determination of \( m_t(2) \) is model dependent, however, assuming that it is possible to elaborate a more realistic ETC model, where in principle \( m_t(2) \) can be of the same order of the observed top quark mass, the positivity condition of \( M_2^2 > 0 \), which is given by the smallest BSE solution \( (M_S(2)) \) minus the fermionic correction described by Eq.(44), leads to the following intriguing theoretical limit

\[ \frac{M_S(2)}{\Lambda_{ETC}} > \frac{\sqrt{3} m_t}{\pi^2 v}. \quad (46)\]

If we assume \( \Lambda_{ETC} \approx 1 \text{TeV} \) and use the known \( m_t \) and \( v \) values the bound of Eq.(46) is exactly of the order of the known Higgs boson mass.

Note that these are very rough estimates originated by the existence of radiative corrections due to TC and ETC as appear in Fig.(3). The effect of fermion loops inevitably decreases the scalar bound state mass. A full BSE calculation should also involve the dependence of all Green’s functions on the mass anomalous dimensions and fermion masses of the scalar boson constituents. Actually, the result of this section can be seen only as a correction to the BSE result of the previous section, which

FIG. 3. In Fig.(3b) we indicate the BSE diagram that should introduce fermionic corrections to \( M_2^2 \) in the case of a composite Higgs boson. As the ETC gauge bosons depicted in this diagram are very heavy, the vertex appearing in Fig.(3b) can be reduced to the one of Fig.(3c). Therefore, Fig.(3a) is the result of Fig.(3b) when we assume the effective vertex, and it reduces the scalar boson mass.
is dependent on the ansatz proposed in Eq.\((\text{[63]}\)) whereas a complete calculation should rely on a self-energy obtained from the Schwinger-Dyson equation, which is beyond the scope of this work. It is also opportune to recall that even the sigma meson mass calculation may have corrections of similar type (generated by electroweak bosons exchange), that can lower the BSE evaluation of its mass.

**IV. CONCLUSIONS**

We commented in the introduction that the Higgs boson may indeed be a composite dilaton as claimed in Refs. [8, 9]. We draw attention to results in the literature that agree with such claim, stressing that the absence of a \(\Phi^2\) composite term is what happens in the effective potential for composite operators calculations, when the self-energy of the constituents used to compute the effective potential is an exact solution of the Schwinger-Dyson equation. This fact can be observed in the works of Refs. [30–32], and do lead to an effective Schwinger-Dyson equation. This fact can be observed in the works of Refs. [30–32], and do lead to an effective potential without a \(\Phi^2\) term.

In Section II we computed the BSE in the case of a scalar boson with constituents of same mass. The calculation was performed with the help of an ansatz for the constituent self-energy dependent on the mass anomalous dimension. The exact calculation should be performed for one specific theory using SDE solutions. However, our result indicates how the scalar masses, no matter we are talking about the sigma meson or the Higgs boson, can vary with the mass anomalous dimension as shown in Eqs.\((\text{[63]}\) and \([\text{[64]}]\). We hope that this behavior can be tested by other methods.

In Section III we call attention to the fact that a full BSE calculation should include diagrams like the one of Fig.\((\text{[3]}\). The effect of such diagrams is to lower the scalar boson mass. As a simple estimate of this effect we have computed the fermionic contribution of the radiative corrections induced by Fig.\((\text{3a})\). Of course, our calculation is very rough but it shows that this effect cannot be neglected. The bound of Eq.\((\text{[10]}\) is an example of the balance between the different contributions to scalar masses.

Our results are consistent with the ideas of Refs. [6, 7], showing that the Higgs boson may surge as a composite dilaton, and its mass can be smaller than the composition scale, as long as we have large anomalous dimensions and the effect of fermions, like the top quark, included into the calculation. Actually, we may have a delicate balance between the mass anomalous dimension of the fermionic constituents and the contribution of fermions that contribute negatively to the Higgs boson mass. Therefore, we believe that there is a systematic path to perform a realistic Higgs boson mass calculation using BSE and DSE, assuming a given TC gauge group, and varying data such as the number of fermions and others until obtaining the experimental value of the Higgs boson mass.

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