Optimization of the robot motion law by the criterion of minimizing maximum instantaneous power and electric motor size

A Volkov, A Kornilova, O Matsko, A Mosalova

Peter the Great St.Petersburg Polytechnic University, Russia, 195251, St.Petersburg, Politechnicheskaya str., 29

volkov-and-I@yandex.ru, anfly@list.ru, onmatsko@gmail.com, mosalova_anna@mail.ru

Abstract. Maximum instantaneous power consumption of robot drives determines requirements for power supply systems and dimensions of drives for robots and different process machines. The tasks of minimizing peaks in power consumption and drive sizes are acquired when creating autonomous robots operating under restrictions for installed capacity and weight-size parameters of the equipment. For a large number of such robots there are no process restrictions for types of applied motion laws and their numerical characteristics: maximum speeds and accelerations. The motion law type and especially its parameters are traditionally determined by the designer preferences without any justification. These are, for example, restrictions for maximum accelerations or speeds. The first is usually associated with the need to ensure strength and accuracy of the robot, the second - with safety of service personnel in the robot working area. Using the example of one of the most common motion laws the relation of instantaneous power and energy consumption on the payload are studied. Results of this research can be used as a basis for robot drives design in view of accepted criteria when optimal parameters of the motion law are calculated for each drive operation cycle depending on anticipated or actual load.

1. Introduction

One of the main requirements for equipment designed for operation in the Arctic zone and especially for autonomous operation is the minimum power consumption and the minimum value of instantaneous electrical power consumed. Various papers are devoted to energy saving in robotics [1, 2]. This issue is usually solved when designing new equipment by optimizing design parameters or during operation - by developing appropriate algorithms and motion laws for the drives of existing robots and other process machines [3, 4, 5]. In addition, balancing vertically moving robot masses can make a significant contribution to energy saving [6]. Energy consumption can also be reduced by applying rational parameters of the motion law and this may be accompanied by different variations in the maximum instantaneous power consumption. A typical motion cycle of the robot with any degree of freedom consists of three sections of the trajectory: acceleration, uniform movement and deceleration. For development of the motion law the initial data are often dimensionless travel \( \rho \) and time. They can be achieved with any known motion law of the machine output link and with different values of relative acceleration and deceleration time. In this case, there is a maximum and a minimum value of instantaneous power: the first corresponds to the highest value of the power consumed, and
the second corresponds to the maximum value of recuperation power or dissipation power [1]. In case of
cumulative deceleration the maximum negative instantaneous power corresponds to the maximum
power generated by the electric drive motor. When decelerating with energy dissipation it can be
dissipated by means of a brake resistor when the motor is operating in generator mode, as in the first
case, or by means of a friction brake.

For drives with inertial load there is an optimal speed tachogram which has the form of an isosceles
trapezoid [6] providing a minimum of instantaneous power. It is important for the designer of such
drives to have answers to the following questions. Firstly, how optimization by the criterion of
instantaneous power minimization affects the energy costs of the motion cycle. Secondly, how the
load value affects the values of the optimal parameters of the motion law. Thirdly, what is the
efficiency of optimization.

2. General motion equations of the drive

In this paper the acceleration and deceleration times are assumed to be equal and the law of speed
variation is symmetrical with respect to the middle of the motion period $T$ which does not violate the
commonality of views and significantly simplifies mathematical calculations and graphics. For
rectangular law of acceleration variation (uniform acceleration motion, uniform motion and uniformly
decelerated motion) relation between travel $\rho(\alpha, \zeta)$, speed $\dot{\rho}(\alpha, \zeta)$, acceleration $\ddot{\rho}(\alpha, \zeta)$ and time $\alpha$ to
have the following form:

$$
\rho(\alpha, \zeta) = \begin{cases} 
\frac{a^2}{2\zeta(1-\zeta)}, & 0 \leq \alpha \leq \zeta \\
\frac{\zeta - 2a}{\zeta(1-\zeta)}, & \zeta < \alpha \leq 1 - \zeta \\
\frac{(1-\alpha)^2}{2\zeta(1-\zeta)} + 1, & 1 - \zeta < \alpha \leq 1 
\end{cases}
$$

$$
\dot{\rho}(\alpha, \zeta) = \begin{cases} 
\frac{1}{\zeta(1-\zeta)}, & 0 \leq \alpha \leq \zeta \\
\frac{1}{1-\zeta}, & \zeta < \alpha \leq 1 - \zeta \\
\frac{1}{\zeta(1-\zeta)}, & 1 - \zeta < \alpha \leq 1 
\end{cases}
$$

$$
\ddot{\rho}(\alpha, \zeta) = \begin{cases} 
\frac{1}{\zeta(1-\zeta)}, & 0 \leq \alpha \leq \zeta \\
0, & \zeta < \alpha \leq 1 - \zeta \\
\frac{1}{\zeta(1-\zeta)}, & 1 - \zeta < \alpha \leq 1 
\end{cases}
$$

where $\rho(\alpha, \zeta) = s/s_0$, $s$ is the displacement, $s_0$ is the travel, $\alpha = t/T$, $t$ is the current time, $\zeta = \tau/T$ is the
relative acceleration time, $\tau$ is the acceleration time, $\dot{\rho}(\alpha, \zeta) = vT/s_0$, $v$ is the speed, $\ddot{\rho}(\alpha, \zeta) = aT^2/s_0$, $a$
is the acceleration. Diagrams of dimensionless acceleration, speed and travel versus time are shown in
Fig.1. Similarly, equations for other motion laws, such as sine, cosine, triangular, etc., can be written.

3. Relationship between instantaneous drive power and motion law parameters

In general, the expression for dimensionless instantaneous power $\eta(\alpha, \zeta)$ has the following form:

$$
\eta(\alpha, \zeta, f) = \frac{NT^3}{ms_0^2} = \dot{\rho}(\alpha, \zeta)(\ddot{\rho}(\alpha, \zeta) + f)
$$

where $N$ is the instantaneous power, $m$ is the reduced mass, $f = FT^2/ms_0$ is the reduced force and $F$
is the resistance force. Expression (4) is written for the case when the drive overcomes gravity. The
Fig.2 shows diagrams of the instantaneous power versus time for different values of the drive relative
acceleration time for rectangular law of acceleration variation and inertial load.
Fig. 2 confirms the assumption made earlier that there is an optimal acceleration time that provides a minimum value of the maximum instantaneous power. Fig. 3 shows the relation of instantaneous power on time for inertial load and three values of dimensionless force and the optimal acceleration time for each of them equal to 0.33 s, 0.25 s and 0.21 s, respectively. The diagrams show that when the resistance force increases from 0 to 10 the maximal dimensionless instantaneous power increases proportionally from 6.75 to 20.28. Given that the maximum instantaneous power for the considered motion law is required at the end of the acceleration section an expression to calculate it can be written as:

$$
\eta_m(\zeta, f) = \frac{1}{1 - \zeta} \left[ \frac{1}{\zeta(1 - \zeta)} + f \right] 
$$

(5)

Fig. 4 shows relations of the maximum instantaneous power versus acceleration time for different resistance forces (0, 5 and 10). The graphs indicate that as the load increases the optimal acceleration time decreases and the optimization efficiency increases.
\[ \zeta_m(f) = \frac{2fu(f) + f(3^{1/2}i - 1) - fu(f)^2(3^{1/2}i + 1) + 9(3^{1/2})i - 9}{6fu(f)} \]  

where \[ u(f) = \left[ 3 \left( -\frac{3f^2 + 27f + 81}{f^3} \right) + 1 \right]^{1/3} \]

The dependence of the optimal acceleration time on load is shown in Fig. 5. It follows from the graph that when the load increases from zero to infinity, the optimal acceleration time decreases from 1/3 to 0, and it depends solely on the load.

The following formula is proposed to evaluate the relation between the acceleration time optimization efficiency and the load value:

\[ \delta(f) = \frac{\eta_m(\zeta_m(f); f) - \eta_m(0.5; f)}{\eta_m(0.5; f)} \times 100\% \]  

(7)

Here, the percentage increase in the minimum value of the maximum instantaneous power is determined for each load when changing from the optimal acceleration time to the relative acceleration time \(\zeta = 0.5\), which corresponds to the triangular law of speed variation: motion is carried out without a steady speed section. The diagram of optimization efficiency versus load is presented in Fig. 6. The diagram shows that when the load increases from zero to infinity, the efficiency of optimizing the acceleration time increases from 18.5 to 100%.

4. Relationship between the energy consumed by the drive for one operation cycle and the acceleration time

The equation to determine dimensionless operation of the drive in a single cycle is the following:

\[ A(\zeta, f) = \begin{cases} \int_{0}^{1-\zeta} \eta(\alpha, \zeta, f) \, d\alpha, & f \leq \frac{1}{\zeta(1-\zeta)} \\ \int_{0}^{1-\zeta} \eta(\alpha, \zeta, f) \, d\alpha, & f > \frac{1}{\zeta(1-\zeta)} \end{cases} \]  

(8)

The first expression in curly brackets corresponds to the case when deceleration is performed using a generator mode of an electric motor with energy dissipation in the brake resistor or a mechanical brake. The second expression corresponds to the case when resistance forces are sufficient to ensure the specified braking rate, and at this stage the drive continues to operate in traction mode. The transition condition from one calculation formula to another is derived from the expression for instantaneous power (4) written for the deceleration start point \(\alpha = 1 - \zeta\):

\[ \rho(1 - \zeta, \zeta) \left[ \tilde{\rho}(1 - \zeta, \zeta) + f \right] = 0 \]  

(9)

By solving (9) with respect to \(f\) it is possible to deduce an expression for resistance force as a function of acceleration time at which motion in the deceleration section will be carried out only under the action of resistance forces, as shown in the middle graph in Fig.3:
\[ f(\zeta) = \frac{1}{\zeta(1 - \zeta)} \]  

(10)

If there are high resistance forces in the deceleration section the drive continues to operate and the integration of the power function should be performed at the interval \([0, 1]\), while at lower forces, energy is dissipated and the engine does not operate in traction mode and integration is performed at the interval \([0, 1 - \zeta]\). The work - acceleration time diagrams for different resistance forces and the resistance force that provides deceleration due to the resistance forces are shown in Fig.7 and Fig. 8.

Analysis of the diagrams presented in Fig.7 showed the following. Firstly, the nature of relationship can change from monotonically increasing to monotonically decreasing as the force increases from zero to infinity. Secondly, when curves move from monotonically increasing to monotonically decreasing in the range of \(2.2 < \zeta < 3.8\) there are minor extremes (minimums).

Analysis of the function (10) and the graph in Fig.5 shows that the relation has an asymptote and therefore the resistance force that provides a stop only by runout equals four dimensionless units. The limit force is determined by the type of motion law, and the absolute value of this force is determined by the formula:

\[ F = \frac{4s_0}{T^2} \]  

(11)

It is important to note here that this force is equal to twice the inertia force that occurs on the uniform acceleration section \(s_0\) during the time \(T\). The value of this force can be called critical. To evaluate optimization efficiency of the motion law parameters a coefficient of optimization efficiency for energy consumption per cycle is introduced:

\[ \delta_1(\zeta, f) = \frac{A(0.5; f) - A(\zeta, f)}{A(\zeta, f)} \times 100\% \]  

(12)

Diagrams of dependence \(\delta_1(\zeta, f)\) on the resistance force for different values of the acceleration time are shown in Fig.9. Fig.10 shows the relation of \(\delta_1(\zeta, f)\) on the acceleration time at different load values.

Analysis of plots in Fig.9 has showed the following. Firstly, when the load increases from zero to critical the efficiency decreases from 300\% to -11\% (in actual systems at \(\zeta < 0.1\) this range will narrow to 224\%...-11\%). A negative part of this range means that energy consumption is no longer decreasing, but increasing. Secondly, when the load increases further \((f > 4)\) the optimization efficiency changes from -11\% to 0 \%. Here, the consumption is more than the optimal value, but it decreases as the acceleration time increases. Thirdly, when loads are greater than the critical value there is a situation where changing the parameter \(\zeta\) does not affect energy consumption.
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Figure 9. Relation of energy consumption optimization efficiency on the load for different acceleration times

Figure 10. Relation of energy consumption optimization efficiency on the acceleration time under various loads

The plots in Fig. 10 can be interpreted as follows. Firstly, for loads less than critical there may be an extreme point of efficiency (see the plot $\delta 1(\zeta, 3)$). Secondly, for loads that differ significantly from the critical load the efficiency of energy optimization is proportional to the acceleration time.

The greatest interest for the designer is the relationship between the maximum instantaneous power consumption and the energy consumption per a single cycle. Formulas (5) and (8) are used to plot these graphs, and the variable parameter is assumed to be the dimensionless acceleration time $\zeta$. In most practical cases the acceleration time is in the range of $0.05 < \zeta < 0.5$. Diagrams of these relationships for different loads are presented in Fig.11.

Figure 11. Relation between the maximum instantaneous power on energy costs for different loads

The analysis of the above diagrams showed the following. Firstly, there is a range of $\zeta$ parameter values in which a significant decrease of the minimum value of the maximum instantaneous power (up to 2.5 times) is accompanied by a slight decrease of energy consumption per cycle (up to 1.2 times). This range starts from $\zeta = 0.05$. Secondly, for a relative time range starting from $\zeta = 0.5$ and loads $f \leq 4$ a slight decrease of the minimum value of the maximum instantaneous power (up to 1.3 times) is accompanied by a slight increase of energy consumption (up to 1.1 times). Thirdly, the "loop" on the graph at $f = 3$ indicates the existence of a pair of minimum values of the maximum instantaneous
power and energy consumption which corresponds to two different values of $\zeta$. Fourthly, at high loads ($f > 4$) there are ranges of $\zeta$ where a significant decrease of the minimum value of the maximum instantaneous power occurs with constant energy consumption.

5. Conclusion
The presented studies showed the following. Firstly, optimization of the motion laws of drives for the rectangular law of acceleration variation allows decreasing the maximum value of instantaneous power consumption by tens of percent depending on the load. Secondly, the proposed approach to the choice of parameters of the motion law allows in some cases to reduce energy consumption along with power. Thirdly, the results obtained can be used for multi-criteria optimization of the drive, where instantaneous power, energy consumption and weight - size parameters of the drive are selected as criteria depending on the power of the electric motor.

The results obtained can be used not only in the design of new machines, but also to optimize the motion laws of the existing equipment drives which in most cases will require minimal costs due to their reprogramming. In addition, optimization of the motion law is of great practical interest for equipment that works with loads changing in each new cycle.

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