Verification of Time of Concentration Equation for Improved Drainage Design

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Abstract
This study focused on verifying the Time of concentration Equation for a more reliable design of drains. Tracer studies were carried out in 1000m long rectangular drains at the University of Nigeria Nsukka. Sodium Chloride which was used as the tracer was introduced at station 1 and samples were collected after 20secs simultaneously at the 9 collection points. Flow parameters including velocity and slope of channel were determined. The collected samples were analyzed in the laboratory for determination of chloride concentration in mg/l. The maximum concentration was 42.11mg/l. Thereafter the time of concentrated was computed by dividing the summation of product of concentration and measured time with the summation of concentration. Through regression analysis, a modified formula for computing time of concentration ($t_c$). Model verification was carried out using experimental data. The results from obtained from the modified Equations were compared respectively with computed results from the commonly used Kirpich formula and measured times of concentration. The results showed no correlation between measured and computed values. There were also marked variations between results from existing and modified formulae. There is therefore need for further verification and validation of the modified Equation. This is necessary in view of the fact that a reliable prediction of time of concentration is key to a cost effective drainage design.
Keywords: Drain, Time of concentration, Runoff, Flow rate, Slope, Watershed

1. Introduction

Drainage is an important factor in determining the performance of roads. Most times, when a road pavement fails, it is due to inadequacies in drainage. It is a regular phenomenon to see road pavements deteriorate yearly after the rainy season leading to avoidable expensive major maintenance works on the roads. This is largely due to poor or absence of drainage works that are supposed to take away runoff from the road. A good drainage philosophy on roads will reduce to minimum the untimely repair and replacement of many miles of prematurely deteriorated road. (Usman, 2001). Surface drainage is the removal of excess water from the surface of the land. This is normally accomplished by shallow ditches, also called open drains (Chow, 1988). An open Channel is a waterway, canal or conduit in which a liquid flows with a free surface (Chanson, 2001).

Hydrologic design is the process of assessing the impact of hydrologic events on a water resource system and choosing values for the key variables of the system so that it will perform adequately (Chow et al 2003). The design of channels or structures to handle storm runoffs involves the determination of peak rate of runoff volume, and the time distribution of runoff rates. Characterization of hydrologic processes of a watershed in the context of drainage design requires estimation of time-response characteristics, which was used in hydrologic models and influence model response to rainfall from real or design storms (Rousel et al., 2005). A major parameter for consideration is the Time of Concentration (t_c) which is the time required for water to flow from the remotest part of the watershed to the outlet (Haan et al, 1994). From hydraulic point of view, the remotest point is defined as the point with the longest travel time to the watershed outlet (UBDM, 2013). It consists of the summation of the individual travel times for consecutive components of the drainage system. The time of concentration has been conventionally tackled as a constant. However, theoretical proof and empirical evidence imply that t_c exhibits significant variability against rainfall making its definition and definition and estimation a hydrological paradox (Ellen et al, 2018). It is a concept used in hydrology to measure the response of a watershed to a rainfall event. The time of concentration is useful in predicting flow rates that would result from hypothetical storms which are based on statistically derived return periods (Monjo, 2016). It is one of the basic watershed characteristics that influence the shape of runoff hydrograph. It is the time required for an entire watershed to contribute to runoff at the point of interest for hydraulic design.

Time of concentration is a function of Length of watershed (L) and velocity for a particular watercourse. Along but steep flow path may actually have a shorter travel time than a short but relatively flat flow path. The designer must identify the flow path along which the longest travel time is likely to occur. For most practical design projects, t_c is made up of multiple travel time components. The t_c affects both the slope and peak of the runoff hydrograph. Inaccurate estimation of Tc could lead to hydraulic failure of drains. Underestimation results in high value of rainfall intensity (i) used in Rational formula to compute peak flow. On the other hand overestimation results in higher values of i being read from intensity- duration –
frequency curves. This leads to designs that are not cost effective. Many urban areas in Nigeria have experienced different levels of flooding incidences arising from different degrees and/or type of drainage failure (Nwaogazie and Duru, 2002).

Many empirical equations have been proposed for quick estimation of time of concentration which is taken as rainfall duration. It is therefore necessary to verify the applicability of these models with field data.

Common Equations for time of concentration $t_c$ (Chin 2000, Chow et al 1988).

(i) **FAA (US Federal Aviation Administration) Equation**

$$t = \frac{G(1.1-c)L^{0.5}}{(100S)^{0.33}}$$

Where $L$ = Longest watercourse length in

$S$ = Slope of watershed

$G$ = A coefficient representing type of ground cover.

The equation is recommended by American Society of Civil Engineers (ASCE) because it uses the widely recognized Rational constant.

(ii) **Kirpich Equation** developed in 1940 reported by Agunwamba (2001)

$$t_c = \frac{0.01947(L)^{0.77}}{S^{0.5}}$$

Where

$t_c$ = time of concentration in mins

$L$ = flow path distances in m

$S$ = surface slope

(iii) **Izzard Formula**

Izzard (1946) developed a formula for computing rainfall intensity:

$$t_c = \frac{526.423bl^{1/3}}{(ci)^{2/3}}$$

where,

$$b = \frac{2.8 \times 10^{-5}i + C_r}{s^{7/3}}$$

$C_r$ = retardance coefficient with values 0.007 for smooth asphalt surface, 0.012 for concrete pavement, 0.017 for tar and gravel pavement and .06 for dense blue grass turf.

The surface slope $s$ is estimated from the contour map. Between two points A and B,

$$Slope(S) = \frac{Elevation at point A - Elevation at point B}{Distance between A and B}$$
(iv) Kerby Formula

For small watersheds where overland flow and overland flow length are important components of overall travel time, the Kerby Equation can be used. The Kerby Equation is (NMDOT, 2018)

\[ T_{ov} = K (L \times N)^{0.467} \times S^{0.235} \]  

Where \( T_{ov} \) = Overland flow time of concentration in minutes

- \( K \) = unit conversion coefficient, in which \( k = 0.828 \)
- \( L \) = the overland flow length, feet
- \( N \) = a dimensionless retardance coefficient
- \( S \) = the dimensionless slope of terrain conveying the overland flow

The first step in designing drains is to determine the desired return period that will withstand the greatest rainfall that has ever occurred. It is often more economical to have a periodic failure than to design for a very intense storm unless human life is endangered.

The frequency of occurrence can be expressed in the Return period which is defined as the average number of years expected between occurrence of a storm of a given duration and intensity. The return period can be determined by using Weibull or Gumbell frequency analysis selected entirely based on the extent of damage anticipated if the flood exceed. The return period is chosen from 2 to 5 years, if damage due to flooding is small, and 20 to 100 years when damage is great. (Agunwamba, 2001).

In view of obvious limitations of empirical formulae with regard to adaptation in environments outside environments where data for calibration of these models were generated, it was necessary to verify the common formulae using field data from a tropical environment. This study therefore focused on the verification of Kirpich Equation using measured data at University of Nigeria, Nsukka.

2. Materials and Methods

The experimental sitewas at the Nsukka campus of University of Nigeria in Enugu state south eastern Nigeria (Fig 1). The area is located on 6°51’24’’N/7°23”45’E.
The topography of the area is hilly which leads to high runoff velocities. During rainfalls, 12 samples were simultaneously collected from marked stations along the flowing drainage and current meter revolutions noted at 100m intervals using propeller NO 1 (225630).

Runoff samples were collected from a drain running from the new National Library to Odim Gate. The parameters of interest included:

i. Length (m)
ii. Slope
iii. Velocity (m/s)
iv. Discharge (m³/s)

The length was measured in m using measuring tape. The slope was determined from the experimental plan by computing the difference in spot height divided by the distance between the locations.

The velocity of flow was measured with the aid of the Current meter. For propeller no1, 

\[ n > 4.67 \]

\[ v = 0.0546 + 0.047 
\]

\[ n < 4.67 \]

\[ v = velocity (m/s) \]

Where \( n = \frac{\text{Revolutions}}{\text{time}} \)

The discharge \( Q \) was computed from the formula
Where \( A \) is cross sectional Area (m\(^2\))

Tracer studies were carried out to determine actual time of flow. Industrial salt (Sodium chloride) served as the tracer in this study. It was introduced at station 1 and samples were collected after 20 secs simultaneously at the 9 collection points. The collected samples were analyzed in the Laboratory for chloride concentration. Silver Nitrate (AgNO\(_3\)) was used as reagent while Potassium Chlorate was used as indicator in the chloride test.

The chloride concentration was calculated from the formula:

\[
C = \frac{(A-B) \times N \times 35450}{V}
\]  

(8)

\( C \) = Chloride concentration (mg/l)

\( V \) = Volume of sample

Graphs of chloride concentration against time of flow were plotted. Thereafter Time of concentration was computed from the formula;

\[
t_c = t_f + t_{a}
\]  

(9)

Where \( t_a = \sum \frac{ct}{\sum C} \text{ (secs)} \)  

(10)

\( t_c \) is measured time of concentration

\( t_f \) is time from origination of flow

\( C \) is concentration (mg/l)

Using the values of length of slope and slope, corresponding values of time of concentration were also computed using Kirpich formula (Equation 2).

3. Results and Analysis

Values of Chloride concentration against time for selected samples collected at locations A1, F1, A2, H3, D3 and F3 are plotted in Figs 2 – 7. The graphs indicate peak concentrations points. The maximum concentration obtained was 42.11mg/l.
Figure 2. Chloride Concentration against time (A1)

Figure 3. Chloride Concentration against time (F1)

Figure 4. Chloride Concentration against time (A2)

Figure 5. Chloride Concentration against time (H3)
Figure 6. Chloride Concentration against time (D3)

Figure 7. Chloride Concentration against time (F3)

Measured and Calculated times of concentrations are presented in Tables 1. The results show marked variations in the values.

Table 1. Measured and Calculated Values of $t_c$

| Location | Measured Time $T_m$ (mins) | Length (m) | Slope | Computed $t_c$ (mins) (Kirpich formula) |
|----------|---------------------------|------------|-------|-----------------------------------------|
| A1       | 2.021                     | 100        | 0.070 | 2.524                                   |
| B1       | 2.097                     | 200        | 0.045 | 5.407                                   |
| C1       | 2.126                     | 300        | 0.050 | 7.009                                   |
| D1       | 2.201                     | 400        | 0.038 | 10.035                                  |
| E1       | 2.218                     | 500        | 0.050 | 10.387                                  |
| F1       | 2.230                     | 600        | 0.042 | 13.042                                  |
| G1       | 2.739                     | 700        | 0.040 | 15.049                                  |
| H1       | 2.764                     | 800        | 0.039 | 16.890                                  |
| I1       | 2.98                      | 900        | 0.039 | 18.494                                  |
| A2       | 1.768                     | 100        | 0.070 | 2.542                                   |
| B2       | 2.08                      | 200        | 0.045 | 5.407                                   |
| C2       | 2.338                     | 300        | 0.050 | 7.009                                   |
| D2       | 2.521                     | 400        | 0.038 | 10.035                                  |
| E2       | 2.688                     | 500        | 0.050 | 10.387                                  |
| F2       | 2.885                     | 600        | 0.042 | 13.042                                  |
| G2       | 3.021                     | 700        | 0.040 | 15.049                                  |
| H2       | 3.109                     | 800        | 0.039 | 16.890                                  |
| I2       | 3.20                      | 900        | 0.039 | 18.494                                  |
| A3       | 1.75                      | 100        | 0.070 | 2.542                                   |
| B3       | 2.083                     | 200        | 0.045 | 5.407                                   |
| C3       | 2.229                     | 300        | 0.050 | 7.009                                   |
| D3       | 2.648                     | 400        | 0.038 | 10.035                                  |
| E3       | 2.750                     | 500        | 0.050 | 10.387                                  |
| F3       | 2.837                     | 600        | 0.042 | 13.042                                  |
| G3       | 3.086                     | 700        | 0.040 | 15.049                                  |
| H3       | 3.167                     | 800        | 0.039 | 16.890                                  |
| I3       | 3.20                      | 900        | 0.039 | 18.494                                  |
From Equation 2

\[ t_c = \frac{aL^b}{S^c} \]  

(11)

\[ \log T = \log a + b \log L - c \log S \]  

(12)

Using Multiple Regression analysis;

\[ \sum y = a_0 + a_1 \sum X_1 + a_2 \sum X_2 \]  

(13)

\[ \sum y = a_0 + a_1 \sum X_1 + a_2 \sum X_2 \]  

(14)

\[ \sum y = a_0 + a_1 \sum X_1 + a_2 \sum X_2 \]  

(15)

Where N is No of Tests = 27 (18 was used for variation purposes)

X₁ is Log of slope

X₂ is Log of length

Y is Log of measured time.

Applying values from Table …, Regression Analysis was carried out based on 18 samples.

The solution gives \( a_0 = 2, 5204, a_1 = 1.7502 \) and \( a_2 = 0.7692 \).

Since \( a_0 = \log a; a = 331.43 \).

Similarly \( b = 1.7502, c = 0.7692 \).

Thus;

\[ t_c = \frac{331.43L^{1.7502}}{S^{0.7692}} \]  

(16)

Equation is a modified \( t_c \) equation using data from University of Nigeria Nsukka. Values of \( t_c \) computed with the modified formula (Equation) for sections A₁ to I₁ are presented in Table 2.

The results show no correlation with measured values.

Table 2. \( T_c \) values modified Equation

| Location | Computed \( T_c \) from Equation… (x10⁵) (mins) |
|----------|-----------------------------------------------|
| A₁       | 1.36                                          |
| B₁       | 3.25                                          |
| C₁       | 7.16                                          |
| D₁       | 9.60                                          |
| E₁       | 17.52                                         |
| F₁       | 21.07                                         |
| G₁       | 26.58                                         |
| H₁       | 32.93                                         |
| I₁       | 40.50                                         |

4. Conclusion and Recommendations

The study has attempted to verify the application of the popular Kirpich Equation for determining the time of concentration using field data. Based on regression analysis, a
modified Equation was formulated. Results indicate no correlation between measured and computed values of $t_c$. There were also marked variations between values obtained using the existing Equation and values obtained using the modified formula.

The result of this study raises a serious question on the application of empirical formulae in environments different from the locations where the Equations were derived. We recommend that the modified $t_c$ Equation be further verified and validated with data from other locations before application in the estimation of runoff. This is in view of the fact that accurate estimation of $t_c$ is key to a reliable design of drains. Overestimation of $t_c$ for a given Return period leads to lower values of rainfall intensity constant ($i$) and consequently reduces the value of peak flow. This will consequently lead to under design of drains.

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