Judgment and correction of lost speed Keyphasor pulse of rotating machinery

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Abstract: The order analysis of vibration signal of rotating machinery depends on the speed Keyphasor signal, and the Keyphasor pulse will be lost in actual measurement. In order to calculate the rotational speed accurately, a method to judge whether the Keyphasor pulse is lost is given, and a method to recover the lost pulse based on cubic spline interpolation is proposed. The simulation based on LabVIEW shows that the method given in this paper can well judge and recover the lost Keyphasor pulse.

1. Introduction
The order analysis method is usually used to extract the characteristic frequency and spectrum analysis of rotating machinery. Whether the order analysis of vibration signal is accurate depends on the measurement and calculation of speed. In the actual measurement system, a circle of black-and-white color band is pasted on the shaft, and the photoelectric sensor is used to pick up the speed Keyphasor pulse signal. In the actual measurement process, if the sampling rate is too low and some external interference factors will lead to pulse loss. The loss of pulse will cause the calculated speed profile to be incorrect, incorrect resampling time in angular domain, and incorrect order analysis results. This paper presents how to judge the pulse loss and how to correct it.

2. Judgment Basis of Loss of Speed Key Phase Pulse
In actual sampling process, the loss of speed key phase pulse will lead to abnormal values of calculated speed, which will lead to incorrect re-sampling time in angular domain. Therefore, it is necessary to judge and restore whether the key phase pulse is missing before calculating the speed profile. During the operation of rotating machinery, there are generally three processes: start-up, smooth operation and stop, start-up is the acceleration stage, smooth operation can be seen as the uniform speed stage and stop is the deceleration stage. Whether the speed is increased or decreased, the speed will not change suddenly. It can be seen as uniform acceleration and deceleration, which is the prerequisite for judging whether the speed Keyphasor pulse is lost[1][4].

Assuming that the actually collected Keyphasor time scale vector is \( T[N] \) and the time scale vector element \( T[i] \) is represented by \( t_i \), whether the Keyphasor pulse is lost can be judged according to the change of time interval \( \Delta t_i (\Delta t_i = t_{i+1} - t_i, i \in (0,1,2,\cdots,N-1)) \) between two adjacent pulses.
2.1 Judgment basis of single pulse loss in acceleration stage

Figure 1 shows the Keyphasor pulse sequence diagram in the speed up stage. In normal state, \( \frac{\Delta t_{i-1}}{\Delta t_i} > 1 \). If a pulse is lost at \( t_i \), \( t_i' = t_{i+1} \) and \( \frac{\Delta t_{i-1}}{\Delta t_{i-2}} = \frac{t_i' - t_{i-1}}{t_{i+1} - t_{i-2}} \). In the acceleration stage, \( t_{i+1} - t_i = t_{i+2} - t_{i+1} \), so \( \frac{t_{i+1} - t_{i-1}}{t_{i+2} - t_{i+1}} = 1 + \frac{\Delta t_{i-1}}{\Delta t_i} > 2 \). Therefore, when a pulse is lost during acceleration, \( \frac{\Delta t_{i-1}}{\Delta t_i} > 2 \).

![Figure 1 Loss of single pulse at acceleration](image1)

2.2 Judgment basis of single pulse loss in deceleration stage

Fig. 2 is the sequence diagram of Keyphasor pulse in deceleration stage. In normal state, \( \frac{\Delta t_{i+1}}{\Delta t_i} > 1 \). If a pulse is lost at \( t_i \), \( t_i' = t_{i+1} \) and \( \frac{\Delta t_{i+1}}{\Delta t_{i-2}} = \frac{t_i' - t_{i+1}}{t_{i+2} - t_{i+1}} \). In the deceleration stage, \( t_{i+1} - t_i < t_{i+2} - t_{i+1} \), so \( \frac{t_{i+1} - t_{i-1}}{t_{i+2} - t_{i+1}} = \frac{\Delta t_{i+1} + \Delta t_{i-1}}{\Delta t_{i-2}} > 2 \). Therefore, when a pulse is lost during acceleration, \( \frac{\Delta t_{i+1}}{\Delta t_{i-2}} > 2 \).

![Fig. 2 loss of single pulse during deceleration](image2)

2.3 Judgment basis of single pulse loss in constant speed stage

In the constant velocity stage, \( \frac{\Delta t_i}{\Delta t_{i-1}} = \frac{t_i' - t_{i-1}}{t_{i+1} - t_i} = \frac{t_{i+1} - t_{i-1}}{t_{i+2} - t_{i+1}} = 2 \). Due to the influence of power fluctuation, load change, vibration and other factors, the ideal uniform rotation of the rotating shaft does not exist, but the rotating speed will fluctuate near a stable value, and the fluctuation is very small, so \( \frac{\Delta t_{i-1}}{\Delta t_i} \approx 2 \).

To sum up, the judgment condition of single pulse loss is: \( \frac{\Delta t_i}{\Delta t_{i-1}} \geq 2 \) or \( \frac{\Delta t_{i-1}}{\Delta t_i} \geq 2 \).

2.4 Judgment basis for continuous multiple pulse loss

In practical measurement, the rare extreme case is the continuous loss of multiple pulses. As shown in Figure 3, the speed is increased. If the number of lost pulses is \( m \), then \( t_i' = t_{i+m} \) and \( \frac{\Delta t_i}{\Delta t_{i-1}} = \frac{t_i' - t_{i-1}}{t_{i+1} - t_i} = \frac{t_{i+m} - t_{i+m+1}}{t_{i+2} - t_{i+1}} = 2 \).

![Fig. 3 multi pulse loss state during speed increase](image3)
\[
\frac{t_{i+m} - t_{i-1}}{t_{i+m+1} - t_{i+m}} = \frac{t_{i+m} - t_{i+m-1} + t_{i+m-2} + t_{i+m-3} - \cdots - t_i + t_{i-1}}{t_{i+m+1} - t_{i+m}}.
\]

In the acceleration stage, \( t_{i+m+1} - t_{i+m} < t_{i+m} - t_{i+m-1} \leq \cdots \leq t_{i+1} - t_i \), therefore, \( \frac{\Delta t_i'}{\Delta t_i} = \frac{t_{i+1} - t_i}{t_{i+1} - t_i} > m + 1 \). Similarly, it can be deduced that in the deceleration stage, \( \frac{\Delta t_i'}{\Delta t_i} = m + 1 \). Therefore, the judgment condition for the loss of m pulses is: \( (\frac{\Delta t_i}{\Delta t_{i-1}} \geq m + 1) \) or \( (\frac{\Delta t_{i-1}}{\Delta t_i} \geq m + 1) \) \( (1) \)

The number of lost pulses should be an integer, so the number of lost pulses is \( m = \text{int} \left( \frac{\Delta t_{i-1}}{\Delta t_i} \right) - 1 \) or \( m = \text{int} \left( \frac{\Delta t_i}{\Delta t_{i-1}} \right) - 1 \)

3. Interpolated recovery of lost pulses

3.1 Algorithm analysis of lost pulse recovery

If equation (1) is satisfied, m pulses are lost and can be recovered by interpolation. The simplest is to use linear interpolation. The algorithm of linear interpolation is: \( t_{i+n} = t_{i-1} + \frac{t_{i+1} - t_i}{m+1} n, n = 1,2,3,\cdots, m \). For linear interpolation, \( \Delta t_{i+k} = t_{i+k+1} - t_{i+k} = \frac{t_{i+1} - t_i}{m+1}, k \in [1, m] \), that is, linear interpolation is only applicable to the uniform stage. For the acceleration stage, \( \Delta t_{i+k} \) should decrease with the increase of \( k \), and for the deceleration stage, \( \Delta t_{i+k} \) should increase with the increase of \( k \). Therefore, linear interpolation is not suitable for the acceleration stage. For the acceleration stage, cubic spline interpolation can accurately recover the lost pulse.

According to the circular motion equation, the key phase time scale of speed is a function of rotation angle, and the function is \( t = f(\theta) \). Let \( \theta_0 = 0, a = \theta_0 < \theta_1 < \cdots < \theta_{N-1} = b \) function \( t = f(\theta) \) is monotonic and satisfies the condition of piecewise cubic spline interpolation in both speed-up stage and speed-down stage. The cubic spline interpolation function set on interval \( \theta \in [\theta_i, \theta_{i+1}] \) is: \( S_i(\theta) = a_i + b_i(\theta - \theta_i) + c_i(\theta - \theta_i)^2 + d_i(\theta - \theta_i)^3 \) for \( i \in [0, N-2] \), and the cubic spline equation is shown in formula (2)\[^3\]:

\[
\begin{pmatrix}
2h_0 & h_0 & 0 & 0 & \cdots & 0 & 0 & m_0 \\
h_0 & 2(h_0 + h_1) & h_1 & 0 & \cdots & 0 & 0 & m_1 \\
0 & h_1 & 2(h_1 + h_2) & h_2 & \cdots & 0 & 0 & m_2 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & h_{N-3} & 2(h_{N-3} + h_{N-2}) & h_{N-2} & \cdots & 0 & m_{N-2} \\
0 & \cdots & 0 & h_{N-2} & 2(h_{N-2} + h_{N-3}) & h_{N-3} & \cdots & m_{N-1} \\
0 & \cdots & 0 & 0 & h_{N-2} & h_{N-3} & \cdots & m_{N-}\[1pt] \\
0 & \cdots & 0 & 0 & 0 & h_{N-2} & h_{N-3} & 0 \\
\end{pmatrix}
\begin{pmatrix}
m_0 \\
m_1 \\
m_2 \\
\vdots \\
m_{N-2} \\
m_{N-1} \\
\end{pmatrix}
= \begin{pmatrix}
f[\theta_0, \theta_0] - f'(a) \\
f[\theta_b, \theta_0] - f[\theta_0, \theta_1] \\
f[\theta_0, \theta_2] - f[\theta_0, \theta_1] \\
f[\theta_3, \theta_0] - f[\theta_2, \theta_1] \\
f[\theta_2, \theta_2] - f[\theta_0, \theta_2] \\
f[\theta_0, \theta_3] - f[\theta_2, \theta_1] \\
\end{pmatrix}
\]

(2)

In the equation, \( m_i = S''_i(\theta_i) \), \( f[\theta_i, \theta_{i+1}] \) is the mean difference, \( f'(a) \) and \( f'(b) \) are boundary conditions, \( h_i = \theta_{i+1} - \theta_i \), and \( h_i = 2\pi \) under normal conditions. If it is judged that m pulses are lost in interval \([t_i, t_{i+1}]\) according to equation (1), then \( h_i = m \times 2\pi \).

The cubic polynomial is fitted according to the Newton interpolation formula, and the boundary conditions \( f'(a) \) and \( f'(b) \) can be obtained:

\[
f'(a) = \frac{-12h_0 + 15h_1 - 9h_2 + 2h_3}{12h_2} \\
f'(b) = \frac{-2h_3 + 4h_2 - 8h_1 + 4h_0}{12h_2}
\]

(3) (4)

Solve the equations (2) and store the results in the quadratic differential array \( m[N] \). According to the Keyphasor time scale array \( T[N] \) and the calculated quadratic differential array \( m[N] \), the coefficients of cubic spline interpolation function \( S_i(\theta) \) are calculated as follows: \( a_i = t_i; b_i = \frac{t_{i+1} - t_i}{h_i} - \frac{h_i}{6} (m[i + 1] + 2m[i]); c_i = \frac{m[i]}{2}; d_i = \frac{m[i+1] - m[i]}{6h_i}, i \in [0, N-2]. \)

3.2 Algorithm flow of interpolation to recover lost pulse

For each rotation speed Keyphasor pulse, the rotation angles corresponding to intervals \([t_i, t_{i+1}]\) and \([t_k, t_{k+1}]\) are \( 2\pi \) in normal state. However, due to the possible loss of pulse, \( h_i \neq h_k \), the Keyphasor pulse vector should be scanned before solving equation (2), the corresponding \( h_i \) of each
interval should be calculated and stored in vector $h[N]$. Combined with vector $T[N]$, boundary conditions $f'(a)$ and $f'(b)$, the results of solving equations (2) are stored in the quadratic differential array $m[N]$, and then the function coefficients are obtained. According to the function coefficients, the cubic spline function $S_i(\theta)$ on interval $[t_i, t_{i+1}]$ can be determined.

According to the previous algorithm analysis, before calculating the speed, it is necessary to scan circularly to judge whether each Keyphasor time scale interval $[t_i, t_{i+1}]$ is missing pulses, and if so, interpolate and recover. The algorithm flow is shown in Figure 4. In Fig. 4, the variable $h$ records the $h_i$ value corresponding to the vector $h[N]$ and the current interval $[t_i, t_{i+1}]$. It is used to calculate the coefficient of cubic spline function $S_i(\theta)$.

![Fig. 4 flow chart of lost pulse judgment and interpolation recovery](image)

4. Simulation verification

Use LabVIEW software to simulate a 1000-6000-1000 rotation photoelectric pulse signal collected at a sampling rate of 10K. After shaping, the pulse is shown in Figure 5. The speed Keyphasor time scale vector $T[N]$ has 292 elements in total. Artificially delete three pulse time scales $t_{10}$, $t_{11}$, and $t_{12}$ in the speed Keyphasor time scale vector, and calculate the speed using the Lagrange 7-point differential interpolation formula$^{[2]}$. The speed curve is shown in Figure 6 (the speed in the figure is angular speed). Based on LabVIEW, the program is written according to the algorithm flow in Figure 4, the lost three pulse time scales are interpolated and recovered, and the recalculated speed curve is shown in Figure 7.

Table 1 shows the comparison before and after Keyphasor pulse interpolation. It can be seen that using cubic spline interpolation, $\Delta t_i$ is decreasing and conforms to the law of increasing speed.
Combined with figure 3 and table 1, it can be seen that 
\[ \frac{\Delta t_{i-1}}{\Delta t_i} = \frac{t_13 - t_9}{t_14 - t_{13}} \approx 4.54, \quad \text{int} \left( \frac{\Delta t_{i-1}}{\Delta t_i} \right) = 3 + 1, \]
m = 3. The results verify the correctness of equation (2) as the judgment basis.

The order analysis system of vibration signal of rotating machinery is developed by the author using LabVIEW. The angular domain signal generated under normal conditions is shown in Fig. 8, and the angular domain signal is shown in Fig. 9 when the pulse is lost. It can be seen that if the Keyphasor pulse is lost, the result of order analysis of rotating machinery vibration will be incorrect\[1\][2].

5. Conclusion
The method given in this paper can be used to judge whether the Keyphasor time scale is lost and the number of keyphasors lost. The lost Keyphasor pulse time scale can be recovered by using the interpolation method given in this paper. The corrected speed Keyphasor time scale vector is used to calculate the speed, which ensures the correctness of the angle domain signal generated in the later stage, and finally ensures the correctness of the final order analysis.

Reference
[1] Liu, Q.H. Design of the Unsteady Vibrating Signals Order Analysis System for the Rotating Machinery. J. Journal of Physics: Conf. Series, 2020, 1601:011001. (doi:10.1088/1742-6596/1601/1/011001)
[2] Liu, Q.H. Measurement and Calculation of Speed Profile for Rotating Machinery. J. Journal of Physics: Conf. Series, 2019, 1314:012118. (doi:10.1088/1742-6596/1314/1/012118)
[3] ZHU, J.M. 2000 Analysis of unsteady vibration signal. J. Journal of Vibration and Shock, PP 86-88. (In Chinese)
[4] J. Tamas. 1995 Introduction to numerical calculation. M. Nanjing University Press, PP 207-229. (In Chinese)