dMFEA-II: An Adaptive Multifactorial Evolutionary Algorithm for Permutation-based Discrete Optimization Problems

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ABSTRACT

The emerging research paradigm coined as multitasking optimization aims to solve multiple optimization tasks concurrently by means of a single search process. For this purpose, the exploitation of complementarities among the tasks to be solved is crucial, which is often achieved via the transfer of genetic material, thereby forging the Transfer Optimization field. In this context, Evolutionary Multitasking addresses this paradigm by resorting to concepts from Evolutionary Computation. Within this specific branch, approaches such as the Multifactorial Evolutionary Algorithm (MFEA) has lately gained a notable momentum when tackling multiple optimization tasks. This work contributes to this trend by proposing the first adaptation of the recently introduced Multifactorial Evolutionary Algorithm II (MFEA-II) to permutation-based discrete optimization environments. For modeling this adaptation, some concepts cannot be directly applied to discrete search spaces, such as parent-centric interactions. In this paper we entirely reformulate such concepts, making them suited to deal with permutation-based search spaces without loosing the inherent benefits of MFEA-II. The performance of the proposed solver has been assessed over 5 different multitasking setups, composed by 8 datasets of the well-known Traveling Salesman Problem (TSP) and Capacitated Vehicle Routing Problems (CVRP). The obtained results and their comparison to those by the discrete version of the MFEA confirm the good performance of the developed dMFEA-II, and concur with the insights drawn in previous studies for continuous optimization.

CCS CONCEPTS

- Theory of computation → Bio-inspired optimization; Random search heuristics; Theory of randomized search heuristics; Mathematics of computing → Evolutionary algorithms;

KEYWORDS

Transfer Optimization, Evolutionary Multitasking, Multifactorial Optimization, Discrete Optimization, Traveling Salesman Problem

1 INTRODUCTION

The main motivation behind the recent Transfer Optimization paradigm is that real-world optimization problems hardly occur in isolation [21]. Thus, the key idea on which this paradigm relies is the exploitation of what has been learned by optimizing one task when facing another problem or task. To tackle this paradigm, three different categories of Transfer Optimization can be distinguished: sequential transfer, multitasking and multiform optimization [15, 21]. Among these three classes, multitasking is arguably the one that has attracted most attention by the current community [18, 41], which is devoted to simultaneously solving different optimization problems or tasks by dynamically analyzing existing synergies and complementarities among them.

Given the context above, this manuscript is focused on Evolutionary Multitasking (EM, [31]), a branch of Transfer Optimization that relies on concepts from Evolutionary Computation for the simultaneous solving of different problems [2, 14]. In the last few years, several EM proposals have been reported in the literature to deal with several discrete, continuous, single-objective and multi-objective optimization problems at the same time [17, 22, 39, 45]. From the algorithmic point of view, in most of the aforementioned studies EM has been materialized by means of the so-called Multifactorial Optimization (MFO) strategy, which hinges on the definition of a unique factor for each individual to influence the search of
We assess the performance of our proposed dMFEA-II by considering there is global consensus in the community that until 2017, the paramount importance of the correlation is crucial in order to positively leverage the transfer of optimization tasks. Several influential contributions in the community known as negative transfer [3]. This negative transfer has been reported by some recent studies as the central pitfall of multitasking, becoming a priority in the formulation of new schemes [6, 47]. Among them, the brand new Multifactorial Evolutionary Algorithm II (MFEA-II, [3]) is an adaptive extension of the aforementioned MFEA, incorporating the capability to dynamically learn how much knowledge should be transferred across tasks.

As evinced by the literature so far, MFEA-II has so far been tested over continuous optimization problems, using experimental environments composed by up to 6 tasks. The lack of applications with alternative problem flavors, and wider experimental setups, comprise the main source of motivation of this research work. Specifically, we elaborate on adapting MFEA-II to permutation-based combinatorial problems, giving rise to the discrete MFEA-II (dMFEA-II). Despite the simple formulation of our research hypothesis, the adaptation beneath dMFEA-II is not straightforward, as the naive version of MFEA-II is comprised by concepts and operators that cannot be directly applied to permutation-based discrete search spaces. An example supporting this statement is the parent-based crossover parameter matrix, crucial for the search procedure of MFEA-II. We assess the performance of our proposed dMFEA-II by considering 8 instances of the well-known Traveling Salesman (TSP, [26]) and Capacitated Vehicle Routing (CVRP, [36]) problems, which are combined to yield 5 multitasking environments with heterogeneous search spaces and varying degrees of phenotypical relationship. Results obtained by dMFEA-II are compared to those of the discrete version of the MFEA, aimed at the confirmation of the same findings drawn from [3] for continuous optimization environments.

The remainder of the article is organized as follows. Section 2 sets the background and related work. Next, Section 3 exposes in detail the main features of the proposed dMFEA-II. The experimentation setup and discussion of the results are given in Section 4. Finally, Section 5 concludes the paper.

2 BACKGROUND AND RELATED WORK

There is global consensus in the community that until 2017, the concept of EM was only formulated within the framework of MFO [8]. In the last years, several approaches have embraced this concept [27, 33, 42, 44], with MFEA at the spearhead [20]. Additional alternatives to MFO have been also proposed in terms of new algorithmic schemes, such as the multitasking multi-swarm optimization in [38], or the coevolutionary multitasking schemes in [7, 34].

Deeper into mathematical details, MFO can be formulated by considering an environment comprising K tasks or problems to be simultaneously solved. This environment is therefore made up by as many search spaces as tasks to be faced. Therefore the objective function for the k-th task $T_k$ is denoted as $f_k : \Omega_k \rightarrow \mathbb{R}$, where $\Omega_k$ is the search space of task $T_k$. Assuming that all tasks should be minimized, the main objective is to find a group of solutions $\{x_1, \ldots, x_K\}$ such that $x_k = \arg \min_{x \in \Omega_k} f_k(x)$. In general, a MFO algorithm operates on a population $\mathcal{P}$ of candidate solutions (individuals), where each $x_\mathcal{P} \in \mathcal{P}$ should belong to a unified search space $\Omega_\mathcal{P}$. Each search space $\Omega_k$ is mapped to $\Omega_\mathcal{P}$ through the use of an encoding/decoding function $\xi_k : \Omega_k \mapsto \Omega_\mathcal{P}$. Consequently, every individual $x_\mathcal{P} \in \mathcal{P}$ should be encoded as $x_{\sigma,k} = \xi_k^{-1}(x_\mathcal{P})$ to represent a task-specific solution $x_{\sigma,k}$ for each of the K tasks. Departing from these definitions, in every MFO solver four different features are associated with each individual $x_\mathcal{P}$ of the population $\mathcal{P}$: the Factorial Cost, Factorial Rank, Scalar Fitness and Skill Factor. These features permit to sort, select and/or discard individuals along the search, as they dictate the contribution of every individual to the population considering that K tasks are optimized [3]:

- **Factorial Cost** $\Psi_p^K$ of an individual $x_\mathcal{P} \in \mathcal{P}$ is given by its fitness value for task $T_k$, so that each solution in the population retains a list $\{\Psi_1, \Psi_2, \ldots, \Psi_K\} \in \mathbb{R}^K$ of factorial costs.
- **Factorial Rank** $r_p^K$ of an individual $x_\mathcal{P}$ in a given task $T_k$ is its relative rank within the population in ascending order of $\Psi_p^K$. Similarly to the factorial cost, each individual can be characterized by a factorial rank list $\{r_1, r_2, \ldots, r_K\} \in \mathbb{N}^K$.
- **Scalar Fitness** $\varphi_p^K$ of an individual $x_\mathcal{P}$ is given by its best factorial rank over all tasks as $\varphi_p = 1/\min_{k \in \{1,\ldots,K\}} r_p^K$. The scalar fitness permits to compare different individuals in MFEA.
- **The Skill Factor** $t_p$ is the task in which $x_\mathcal{P}$ performs best, namely, $t_p = \arg \min_{k \in \{1,\ldots,K\}} r_p^K$. As we will show later, the skill factor plays a crucial role in MFEA by establishing which population members are selected for crossover.

When operating on the population of individuals via evolutionary methods, EM emerges as an effective paradigm for tackling multiple problems simultaneously. This efficiency is due to i) the parallelism granted by having a population of individuals, which eases the concurrent application of evolutionary operators and the dynamic estimation of latent synergies between tasks [32]; and ii) the exchange of genetic material among individuals through crossover methods, allowing all tasks to interact with each other. Among them, the specific MFEA approach is based on bio-cultural schemes of multifactorial inheritance. We depict in Algorithm 1 the pseudo-code of the basic MFEA, which has four key characteristics:

- **Unified search space**: one of the main design challenges when modeling a MFEA is the definition a unified space $\Omega_\mathcal{P}$, which should be able to represent all feasible solutions of the K tasks.
- **Assortative mating**, which is based on the principle that individuals are more inclined to interact with others belonging to the same cultural background. For this reason, genetic operators...
used in MFEA are committed to follow this principle, promoting interactions among solutions with the same skill factor. We again recommend [20] for more details on this procedure.

- **Selective evaluation**: every newly created individual is measured only on one task. This procedure guarantees the computational feasibility of the method. Specifically, each new solution is evaluated in the task corresponding to the skill factor of its parent. When mating two parents, the skill factor of the offspring is selected randomly among those of the parents.

- **Scalar fitness based selection**, which can be conceived as an elitist replacement strategy that uses the scalar fitness (namely, the best relative rank of the individual over all tasks) as the control parameter. In other words, the best \( P \) solutions (considering both newly generated individuals and the current population) in terms of scalar fitness survive for the next generation.

### Algorithm 1: Pseudocode of MFEA

1. Randomly draw a population of \( |P| = P \) individuals \( \{x_p\}_{p=1}^P \) with \( x_p \in \Omega_U \)
2. Evaluate each generated individual for the \( K \) problems
3. Calculate the skill factor \( \tau_p \) of each \( x_p \)
4. while termination criterion not reached
   a. Set \( Q = \emptyset \)
   b. while individuals still to select do
      i. Randomly sample w/out replacement \( x_{p'} , x_{p''} \in P \)
      ii. if \( \tau_{p'} = \tau_{p''} \) then
         - \( [x_A, x_B] = \text{IntrataskCX}(x_{p'}, x_{p''}) \)
         - Set \( \tau_A = \tau_{p'} \)
         - else if \( \text{rand} \leq \text{RMP} \) then
            - \( [x_A, x_B] = \text{InterTaskCX}(x_{p'}, x_{p''}) \)
            - Set \( \tau_A = \text{rand}(\tau_{p'}, \tau_{p''}) \)
            - else
               - Compute \( x_A = \text{mutation}(x_{p'}) \), and set \( \tau_A = \tau_{p'} \)
               - Compute \( x_B = \text{mutation}(x_{p''}) \), and set \( \tau_B = \tau_{p''} \)
               - Evaluate \( x_A \) for task \( \tau_A \), and \( x_B \) for task \( \tau_B \)
               - \( Q = Q \cup \{x_A, x_B\} \)
      end
   end
5. Select the best \( P \) individuals in \( P \cup Q \) as per their \( \varphi_p \)

### Algorithm 2: Inter-task crossover procedure of MFEA-II

1. if \( \tau_{p'} \neq \tau_{p''} \) then
2. if \( \text{rand} \leq \text{RMP}_{\tau_{p'}, \tau_{p''}} \) then
   a. \( [x_A, x_B] = \text{IntertaskParentCentricCX}(x_{p'}, x_{p''}) \)
   b. Update \( x_A = \text{mutation}(x_A) \), and \( x_B = \text{mutation}(x_B) \)
   c. Set \( \tau_A = \text{rand}(\tau_{p'}, \tau_{p''}) \)
   d. \( \tau_B = \text{rand}(\tau_{p'}, \tau_{p''}) \)
   else
      a. Randomly select \( x_{p_1} \in P \) with \( \tau_{p_1} = \tau_{p'} \) and \( p_1 \neq p' \)
      b. \( x_A = \text{IntrataskParentCentricCX}(x_{p'}, x_{p_1}) \)
      c. Update \( x_A = \text{mutation}(x_A) \), and set \( \tau_A = \tau_{p'} \)
      d. Randomly select \( x_{p_2} \in P \) with \( \tau_{p_2} = \tau_{p''} \) and \( p_2 \neq p'' \)
      e. \( x_B = \text{IntertaskParentCentricCX}(x_{p''}, x_{p_2}) \)
      f. Update \( x_B = \text{mutation}(x_B) \), and set \( \tau_B = \tau_{p''} \)

In recent years, a manifold of contributions have been inspired by the algorithmic principles of MFEA. In [48], for example, a discrete adaptation of the canonical MFEA is proposed and applied to one of the problems addressed on this present paper: the CVRP. The research work in [46] also goes in the same direction by introducing the discrete unified encoding, which has thereafter served as a reference when dealing with different discrete problems via MFEA. Furthermore, authors in [17] implemented an improved variant of the MFEA, endowing the meta-heuristic with a dynamic resource allocating strategy. A similar solver is presented in [43] for dealing with multiobjective optimization tasks. Other applications of MFEA can be found in [39] for the composition of semantic web services, and in [30] for evolving deep reinforcement learning models.

Despite this success, MFEA, EM and the wider field of Transfer Optimization are also in the focus of few critical researchers, who question the operation of the methods implemented so far. Mainly, these skeptical voices refer to the difficulty of avoiding negative transfers and reacting to their existence [40]. In fact, it is well accepted that the performance of Transfer Optimization algorithms is directly related to the synergies between the problems involved [8, 31, 49]. For this reason, the community is striving to propose new methods to cope with this situation, favoring positive transfers, and making optimization algorithms adaptive to avoid negative influences among tasks [5, 28]. This is in fact the main purpose of the recently proposed adaptive variant of MFEA, coined as MFEA-II [3]. MFEA-II introduces new algorithmic ingredients that make its search resilient against negative information transfer. The next section describe these ingredients and their adaptation for efficiently solving permutation-based problems.

### 3 PROPOSED dMFEA-II APPROACH

The main novelty introduced by MFEA-II with respect to its predecessor is the introduction of a transfer parameter matrix, which dictates the way in which the inter-tasks relationships are conducted, and whose entries are evolved based on the information generated during the course of the multitasking search. In accordance with the claims in [3], the initial phases of MFEA-II are the same as those in MFEA. With this, the main differential factor is the incorporation of the online RMP learning module, and its foundry within the optimization process. This learning module is in charge of building and managing the dynamic RMP matrix, which dictates the extent of genetic transfer across individuals with different **skill task** (see line 11 in Algorithm 1). Another feature of MFEA-II is the inter-task crossover procedure, activated when individuals with different skill tasks should interact, and fully conducted using parent-centric operators [13]. In other words, lines 11-16 in Algorithm 1 are replaced by those in Algorithm 2. The main contribution of the present paper is specifically the adaptation of these steps in order to deal with discrete optimization environments.

Before proceeding further, we now pause at the main rationale for the need of this adaptation. As mentioned previously, the search
process of MFEA-II hinges on parent-centric operators, such as the Simulated Binary Crossover [10], the Polynomial Mutation [12] or the Gaussian Mutation [23] with small variance. These operators are known to produce individuals close to their parents in the unified search space \( \Omega_U \). All these operators were originally conceived for continuous optimization problems [11, 16], and have no clear correspondence for discrete problems as the permutation-based ones considered in this work.

The second issue when adapting the canonical MFEA-II to combinatorial optimization problems is the mutation mechanism in use. While in MFEA local perturbations are only conducted when \( \tau_p \neq \tau_{p'} \) and \( \text{rand} > \text{RMP} \) (lines 15 and 16 of Algorithm 1), in MFEA-II each generated offspring also undergoes small parent-centric mutation [3]. This new procedure requires a reformulation when solving permutation-based problems, in which operators such as 2-opt, 3-opt, swapping or insertion [24, 25] involve a small change in the individual. This small perturbation, along with the previous crossover, could lead to drastically modified \( x_A \) and \( x_B \) individuals. This potentially intensified change of the produced offspring clashes with the main search behavior of MFEA-II. This same trend also holds for the dynamic RMP matrix, which has also been adapted to the typology of problems addressed in this paper.

We now describe the main modifications conducted over the basic MFEA-II in order to properly face permutation-based problems:

1) First of all, dMFEA-II implements a simple strategy for dynamically adapting the RMP matrix to the search performance. Following the philosophy of the online RMP learning module for continuous scenarios described in [3], we have designed a reliable alternative strategy, simple but effective, to dictate the intensity and frequency of the interactions of tasks of different kind. First, as for the continuous MFEA-II, in our dMFEA-II RMP is not a single parameter but a symmetric \( K \times K \) matrix, with \( K \) denoting the number of optimization tasks. The entries of this matrix are real-valued in the range \([0.0, 1.0]\), so that \( \text{RMP}_{k,k'} \) indicates the probability of conducting an inter-task crossover between tasks \( k \) and \( k' \). All \( \text{RMP}_{k,k'} \) are initially set to a relatively high value (e.g. 0.95) in order to facilitate all task interactions in the initial stages of the search. Furthermore, two additional control parameters are defined: \( \Delta_{\text{dec}} \) and \( \Delta_{\text{inc}} \). These parameters are set to a real value withing the interval \([0.0, 1.0]\), and determine the evolution of each \( \text{RMP}_{k,k'} \) entry in the following manner: each time a new individual is created (e.g. \( x_A \) as per lines 3 to 5 of Algorithm 3), its factorial cost is calculated and compared to the parent \( x_p \) from which its \textit{skill task} \( \tau_A \) has been inherited. In case \( x_A \) obtains a better performance in the \textit{skill task} of its parent, we can ensure that the genetic transfer between tasks \( \tau_p \) and \( \tau_{p'} \) has been positive. Thus, \( \text{RMP}_{\tau_p,\tau_{p'}} = \text{inc} \cdot \Delta_{\text{inc}} \) is incremented using \( \Delta_{\text{inc}} \) parameter control as \( \text{RMP}_{\tau_p,\tau_{p'}} = \min\{0.1, \text{RMP}_{\tau_p,\tau_{p'}} / \Delta_{\text{inc}}\} \). Otherwise, we can categorize the transfer as negative, decrementing the value of \( \text{RMP}_{\tau_p,\tau_{p'}} \) as \( \text{RMP}_{\tau_p,\tau_{p'}} = \max\{0.1, \text{RMP}_{\tau_p,\tau_{p'}}, \Delta_{\text{dec}}\} \). A lower bound of \( \text{RMP}_{\tau_p,\tau_{p'}} \) is set to maintain a minimum knowledge exchange between any two tasks. Lastly, the intra-task crossover conducted in dMFEA-II if \( \text{rand} > \text{RMP}_{\tau_p,\tau_{p'}} \) are also parent-centric, so that the evaluation and comparison of the produced individuals update \( \text{RMP}_{\tau_p,\tau_{p'}} \) and \( \text{RMP}_{\tau_p,\tau_{p'}} \) by following the previous rules.

2) In order to counteract the aforementioned intensification of changes imprinted to the offspring, dMFEA-II introduces a \textit{mutation parameter} \( P_m \in [0, 1] \) to control whether a new individual \( x_A \) or \( x_B \) should undergo mutation.

3) The \textit{dynamic discrete parent-centric operator} for both intra-task and intra-task crossover designed for the dMFEA-II is based on the fulfillment of two different considerations. The first one is its parent-centric nature. In other words, created individuals should not be far away with respect to their parents (a small \textit{leap} in the search space). This first consideration can be realized by just fixing one of the parents as dominant, and limiting the amount of genetic material transferred from the other parent. The second factor is the dynamic nature of the operator. By virtue of this feature, the crossover function adapts its operation to the synergies arisen between optimization tasks over the search. This entails that if the complementarity shown among tasks \( k \) and \( k' \) is high, the amount of genetic material transferred between these tasks should also be high, and vice versa. In this way, since the RMP matrix should dynamically reflect the effectiveness of knowledge sharing between tasks, we use the values in this matrix for materializing the dynamic parent-centric characteristic of the crossover in dMFEA-II.

| Algorithm 3: Crossover strategy of dMFEA-II |
| --- |
| 1 if \( \tau_p \neq \tau_{p'} \) then |
| 2 if \( \text{rand}1 \leq \text{RMP}_{\tau_p,\tau_{p'}} \) then |
| 3 \( |x_A, x_B| = \text{InterTaskParentCentricCX}(x_{p'}, x_{p''}) \) |
| 4 if \( \text{rand}2 < P_m \) then |
| 5 \( x_A = \text{mutation}(x_A); x_B = \text{mutation}(x_B) \) |
| 6 \( \tau_A = \text{rand}(\tau_{p'}, \tau_{p''}) \) and \( \tau_B = \text{rand}(\tau_{p'}, \tau_{p''}) \) |
| 7 Update RMP \( \tau_p, \tau_{p''} \) using \( \Delta_{\text{inc}} \) or \( \Delta_{\text{dec}} \) |
| 8 else |
| 9 Randomly select \( x_{p1} \in \mathcal{P} \) with \( p1 = \tau_p \) and \( p1 \neq p' \) |
| 10 \( x_A = \text{IntrataskParentCentricCX}(x_{p1}, x_{p1}) \) |
| 11 if \( \text{rand}2 \leq P_m \) then |
| 12 \( x_A = \text{mutation}(x_A) \) |
| 13 \( \tau_A = \tau_p \) and update RMP \( \tau_p, \tau_p \) |
| 14 Randomly select \( x_{p2} \in \mathcal{P} \) with \( p2 = \tau_p \) and \( p2 \neq p'' \) |
| 15 \( x_B = \text{IntrataskParentCentricCX}(x_{p2}, x_{p2}) \) |
| 16 if \( \text{rand}2 \leq P_m \) then |
| 17 \( x_B = \text{mutation}(x_B) \) |
| 18 set \( \tau_B = \tau_{p''} \) and update RMP \( \tau_{p''}, \tau_{p''} \). |

In light of the above, Algorithm 3 summarizes the scheme proposed in our dMFEA-II for the inter-task crossover procedure, which replaces lines 11-16 in Algorithm 1 and the whole pseudocode depicted in Algorithm 2. To begin with, a permutation encoding is employed as unified representation \( \Omega_U \) for \( \mathcal{P} \), as also done in other studies [46, 48]. Having said this, if \( k \) problems are to be addressed, and representing the dimensionality of each instance \( T_k \), as \( D_k \in \mathbb{N} \), a solution \( x_p \) is represented as a permutation of the integer set \( \{1, 2, \ldots, D_{\max}\} \), where \( D_{\max} = \max\{k \in [1, \ldots, K]\} D_k \) (maximum dimension among the \( K \) tasks). Hence, if an individual \( x_{p'} \) is going to be measured on a task \( T_k \) whose \( D_k < D_{\max} \) only integers lower than \( D_k \) are considered for producing the solution \( x_k \) of \( T_k \).
Without loss of generality we consider the Order Crossover (OX, [9]) to exemplify how we translated this concept to the specific case study presented in this paper. The main principles of OX is to randomly choose two different cutting points in the problem solution, in order to define the segment of the individual (cutting window) that decides the amount of genetic material transferred from one parent to another. The first change done to adapt OX to this parent-centric feature is to limit the size of the cutting window to a fraction \( W \in [0, 1] \) of the total dimension. This maximum size, along with the value of \( \text{RMP}_{k,k'} \), would set the amount of genetic material transferred from task \( T_k \) to \( T_{k'} \) as \( W \cdot \text{RMP}_{k,k'} \cdot D_k \), where \( D_k \) is the dimensionality of task \( T_k \). Namely, for tasks with a fully positive synergy in terms of knowledge transfer (\( \text{RMP}_{k,k'} = 1.0 \)), the size of the shared material would be equal to \( W \cdot D_k \). Finally, if the amount of elements transferred is so low that it is not possible to ensure a variability between the parent and the generated child, a \( a - opt \) mutation is conducted to ensure that offspring and parents differ. We have coined this modified crossover operator as Dynamic OX (dOX), which will be later used in the experimental part of the study. Depending on the problems under consideration, other crossover functions could be also considered and reformulated to incorporate the dynamic and parent-centric nature of dMFEA-II.

4 EXPERIMENTATION AND RESULTS

In order to shed light over the performance of the proposed dMFEA-II approach, an experimental benchmark has been designed considering both TSP and CVRP instances to be simultaneously optimized. Readers interested on these classical problems are referred to recent surveys such as [4, 35]. In particular, we assess the efficiency of dMFEA-II and its MFEA counterpart over 8 TSP and CVRP instances, which are combined to yield 5 different test scenarios. All TSP instances have been obtained from the TSPLIB repository [37]: berlin52, eil151, st70 and eil176. Sizes of these instances are 52, 51, 70 and 76, respectively. On the other hand, the CVRP instances are part of the Augerat Benchmark [1]: P-n50-k7, P-n50-k8, P-n55-k7 and P-n55-k8. The dimensions of these cases are 50 in the first two datasets and 55 in the remaining two. We have opted for related instances, as e.g. all the CVRP or e1151-e1176; and non-related instances, such as berlin52 and st70 or any TSP instance compared to a CVRP one. In this way, we ensure that when facing the experimentation environments, dMFEA-II deals with both positive and negative sharing of knowledge.

| Parameter          | Value       | Parameter          | Value       |
|--------------------|-------------|--------------------|-------------|
| Population size    | 200         | Population size    | 200         |
| Intra-task CX      | OX [9]      | CX                 | OX          |
| Mutation           | 2-opt [28]  | Mutation           | 2-opt       |
| Initial values of RMP\(_{k,k'}\) | 0.95   | RMP                | 0.9         |
| Parent-centric CX  | Dynamic OX (dOX) | \( P_m \)       | 0.2         |
| \( \Lambda_{inc} / \Lambda_{dec} \) | 0.99 / 0.99 |                     |             |

Table 1: Parameter values for dMFEA-II and MFEA.

Thus, 5 different multitasking environments have been constructed for the tests. Each of these scenarios implies that both dMFEA-II and MFEA should solve all the datasets assigned to the environment simultaneously. The main criterion for generating these particular environments is twofold: i) to exploit the possible genetic synergies of the instances and analyze the reaction of the dMFEA-II to negative interactions, and ii) to reach significant findings over a diverse group of multitasking environments, involving each TSP and CVRP instance in exactly the same number of environments. Four of these environments are composed by 4 different instances, while the last one, namely TE_8, contemplates all the 8 problem instances. TE_4_1 is comprised by the four TSP datasets, while the four CVRP datasets are included in TE_4_2. The rest of multitasking setups comprise both TSP and CVRP instances. First, TE_4_3 is composed by e1151, ber1152, P-n50-k7 and P-n50-k8. Lastly, TE_4_4 consists of st78, e1176, P-n55-k7 and P-n55-k8.

For the sake of reproducibility of this research work, parameters employed for the developed methods are summarized in Table 1. Some of these parameters, such as the population size \( P \), W (cutting window size), \( \Delta_{inc} \) and \( \Delta_{dec} \), have been tuned after an exhaustive search process not shown for lack of space. Other parameter values have been set as per other related works [3, 20, 46]. All individuals in the population \( P \) have been initialized uniformly at random. As termination criterion, each solver finished its execution after \( I = 6 \cdot 10^5 \) objective function evaluations. To deal with CVRP problems, a set of zeros are dynamically inserted in the solution as control integers, with the aim of meeting the capacity constraints. Each multitasking configuration has been run 20 times to account for the statistical significance of performance gaps found along the tests. Lastly, all the experimentation has been conducted on an Intel Xeon E5-2650 v3 computer, with 2.30 GHz and a 32 GB RAM.

4.1 Results and Discussion

Table 2 summarizes the outcomes attained by both dMFEA-II and MFEA for all the five test environments described above. Specifically, the table shows the average and standard deviation (computed over 20 independent runs) of the fitness obtained for each instance and multitasking configuration. Moreover, we provide the known optima for each TSP and CVRP instance. However, it is important to set clear, at this point, that the objective of the designed experimental benchmark is not to reach the optimal solution of the instances under consideration, but rather to use them as a reference of the performance of the designed multitasking approach.

The simulation outputs furnished by the implemented dMFEA-II confirm that this method reaches a better performance than its discrete MFEA counterpart in 22 out of the 24 comparisons that can be established throughout the considered test environments. These findings concur with the conclusions drawn by Bali et al. in [3], namely, that the and learning and adaptation of the parameters driving evolutionary multitasking methods permit to better handle the transfer of negative knowledge, and to leverage even further the existence of synergies among tasks. Going deeper into the results, we observe that dMFEA-II outperforms MFEA in all the eight TSP-VRP instances evolved jointly in TE_8. Furthermore, if we analyze the difference in the results reached in all test environments comprising 4 tasks and in TE_8, dMFEA-II appears to scale better and more resiliently to modifications in the problem instances to
solve. Specifically, the results of MFEA degrade significantly when the size of the test environment increases from four to eight simultaneous tasks. Focusing on berlin52, for example, we see that MFEA obtains an average fitness value of 8130.3 in TE_4_1, 8154.0 in TE_4_2, and a much worse 8222.5 in TE_8. This phenomenon does not occur in dMFEA-II, which maintains its performance in every multitasking environment, even improving it in some cases for TE_8. This is symptomatic of its adaptability, and evinces the superiority of dMFEA-II when compared to the discrete MFEA.

A Wilcoxon Rank-Sum test has been applied over the obtained results to verify the statistical significance of the aforementioned performance gaps. As an example of the analysis conducted in this regard, we comment on the most complex test environment, TE_8. For properly performing this Wilcoxon Rank-Sum test, we have compared the outcomes obtained for all the instances separately, establishing the confidence interval at 90%. In this way, a white circle (○) in Table 2 means that dMFEA-II outperforms MFEA with statistical significance. On the contrary, the gray circle (□) indicates the non-existence of evidences for ensuring the statistical significance of the performance gap. Thus, the Wilcoxon Rank-Sum test confirms that dMFEA-II significantly outperforms MFEA in 6 of 8 datasets embedded in environment TE_8. Specifically, the obtained average z-value is −2.44. Considering that the critical z-value is −1.64, and since −2.44 < −1.64, these results strengthen the significance of the performance differences at 90% confidence level. For this reason, we can finally conclude that dMFEA-II is statistically better than MFEA for the multitasking configurations deployed in this experimental study.

| Method       | TSP instances     | CVRP instances     |
|--------------|-------------------|--------------------|
|              | ber11n52 | s15l1 | s17b   | e15l1 | s17b   | s17b   | s17b   | s17b   | s17b   |
| dMFEA-II     | 8074.8   | 531.1  | 712.2  | 585.3  | -      | -      | -      | -      | -      |
|              | -       | -      | -      | -      | -      | -      | -      | -      | -      |
| MFEA         | 8130.3   | 447.5  | 747.7  | 597.0  | 275.3  | 51.4   | 21.17  | 8.23   | -      |
|              | -       | -      | -      | -      | -      | -      | -      | -      | -      |
| Wilcoxon test| ○       | ○      | •      | -      | -      | -      | -      | -      | -      |

| Method       | TSP instances     | CVRP instances     |
|--------------|-------------------|--------------------|
|              | ber11n52 | s15l1 | s17b   | e15l1 | s17b   | s17b   | s17b   | s17b   | s17b   |
| dMFEA-II     | 8151.8   | 447.6  | 747.6  | 597.0  | 275.3  | 51.4   | 21.17  | 8.23   | -      |
|              | -       | -      | -      | -      | -      | -      | -      | -      | -      |
| MFEA         | 8145.0   | 449.1  | 747.6  | 597.0  | 135.3  | 10.99  | 26.98  | 30.54  | -      |
|              | -       | -      | -      | -      | -      | -      | -      | -      | -      |
| Wilcoxon test| •       | ○      | •      | -      | -      | -      | -      | -      | -      |

| Method       | TSP instances     | CVRP instances     |
|--------------|-------------------|--------------------|
|              | ber11n52 | s15l1 | s17b   | e15l1 | s17b   | s17b   | s17b   | s17b   | s17b   |
| dMFEA-II     | 8154.0   | 449.1  | 747.6  | 597.0  | 135.3  | 10.99  | 26.98  | 30.54  | -      |
|              | -       | -      | -      | -      | -      | -      | -      | -      | -      |
| MFEA         | 8120.8   | 462.3  | 818.1  | 605.1  | 625.3  | 714.4  | 659.2  | 659.2  | -      |
|              | -       | -      | -      | -      | -      | -      | -      | -      | -      |
| Wilcoxon test| -       | -      | -      | -      | -      | -      | -      | -      | -      |

| Method       | TSP instances     | CVRP instances     |
|--------------|-------------------|--------------------|
|              | ber11n52 | s15l1 | s17b   | e15l1 | s17b   | s17b   | s17b   | s17b   | s17b   |
| dMFEA-II     | 8222.5   | 462.3  | 818.1  | 605.1  | 625.3  | 714.4  | 659.2  | 659.2  | -      |
|              | -       | -      | -      | -      | -      | -      | -      | -      | -      |
| MFEA         | 8135.8   | 447.6  | 747.6  | 597.0  | 135.3  | 10.99  | 26.98  | 30.54  | -      |
|              | -       | -      | -      | -      | -      | -      | -      | -      | -      |
| Wilcoxon test| □       | ○      | •      | -      | -      | -      | -      | -      | -      |

Table 2: Results obtained by dMFEA-II and MFEA for all the test environments, and statistical significance of the performance gaps as per the Wilcoxon Rank-Sum test.

5 CONCLUSIONS AND FUTURE WORK

This work has presented dMFEA-II, an adaptation of the Multi-factorial Evolutionary Algorithm II to permutation-based discrete optimization problems. Specifically, we have elaborated on how the novel ingredients that MFEA-II introduces over its predecessor MFEA have been adapted to deal with solutions encoded as permutations, yielding a new algorithmic proposal that blends together 1) a novel dynamic strategy to update the matrix of evolutionary parameters controlling the exchange of knowledge between tasks; and 2) a new dynamic parent-centric crossover operator suited to deal with permutation-based solutions. For showcasing the application of the proposed dMFEA-II, extensive experiments have been performed using eight different TSP and CVRP instances. We have compared the results attained by dMFEA-II with the ones reached by the discrete variant of MFEA introduced by Yuan et al. in [46], over five multitasking setups comprising different combinations of the aforementioned problem instances. Results have been conclusive: dMFEA-II outperforms MFEA, with statistical significance, thereby aligning with the claims in [3] regarding the intrinsic value of adaptivity in Evolutionary Multitasking.

Several research directions are planned for the near future departing from the conclusions drawn from this study. First, we will further analyze the scalability of the introduced dMFEA-II using a larger number of TSP and VRP instances. In addition, a critical step is a deeper analysis of the update dynamics of the RMP matrix developed in our dMFEA-II, in order to better understand its behavior along the search process. Finally, we will extrapolate the developed method to other problems arising from other domains with combinatorial optimization at their core.

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REFERENCES

[1] Ph Augerat, Jose Manuel Belenguer, Enrique Benavent, A Corberán, D Naddef, and G Rinaldi. 1995. Computational results with a branch and cut code for the capacitated vehicle routing problem. Vol. 34. IMAG.
[2] Thomas Bäck, David B Fogel, and Zbigniew Michalewicz. 1997. Handbook of evolutionary computation. CRC Press.
[3] Kavitesh Kumar Bali, Yew-Soon Ong, Abhishek Gupta, and Puay Siew Tan. 2020. Multifactorial Evolutionary Algorithm with Online Transfer Parameter Estimation: MFEA-II. IEEE Transactions on Evolutionary Computation 24(1) (2020), 69–83.
[4] Jose Caceres-Cruz, Pol Arias, Daniel Guimarans, Daniel Riera, and Angel A Juan. 2015. Rich vehicle routing problem: Survey. ACM Computing Surveys (CSUR) 47, 2 (2015), 32.
[5] Yajiao Cai, Deining Feng, Shunkai Fu, and Hui Tian. 2019. Multitasking differential evolution with difference vector sharing mechanism. In 2019 IEEE Symposium Series on Computational Intelligence (SSCI). IEEE, 3039–3046.
[6] Yongliang Chen, Jinghui Zhong, Liang Feng, and Jun Zhang. 2019. An adaptive archive-based evolutionary framework for many-task optimization. IEEE Transactions on Emerging Topics in Computational Intelligence (2019).
[7] Mer-Ying Cheng, Abhishek Gupta, Yew-Soon Ong, and Zhi-Wei Ni. 2017. Coevolutionary multitasking for concurrent global optimization. With case studies in complex engineering design. Engineering Applications of Artificial Intelligence 64 (2017), 13–24.

[8] Bingshui Da, Yew-Soon Ong, Liang Feng, A Kai Qin, Abhishek Gupta, Zexuan Zhu, Chuan-Kang Ting, Ke Tang, and Xin Yao. 2017. Evolutionary multitasking for single-objective continuous optimization: Benchmark problems, performance metric, and baseline results. arXiv preprint arXiv:1706.03470 (2017).

[9] Lawrence Davis. 1985. Jobj shop scheduling with genetic algorithms. In International Conference on Genetic Algorithms and Their Applications, Vol. 140.

[10] Kalyanmoy Deb, Ram Bhusan Agrawal, et al. 1995. Simulated binary crossover for continuous search space. Complex systems 9, 2 (1995), 115–148.

[11] Kalyanmoy Deb, Ashish Anand, and Dhiraj Joshi. 2002. A computationally efficient evolutionary algorithm for real-parameter optimization. Evolutionary computation 10, 4 (2002), 371–395.

[12] Kalyanmoy Deb and Mayank Goyal. 1996. A combined genetic adaptive search (GeneAS) for engineering design. Computer Science and informatics 26 (1996), 30–45.

[13] Kalyanmoy Deb, Dhiraj Joshi, and Ashish Anand. 2002. Real-coded evolutionary algorithms with parent-centric recombination. In Proceedings of the 2002 Congress on Evolutionary Computation. CEC’02 (Cat. No. 02TH8600), Vol. 1, IEEE, 61–66.

[14] Javier Del Ser, Eneko Osaba, Daniel Molina, Xin-She Yang, Sancho Salcedo-Sanz, David Camacho, Swagatam Das, Ponnuthurai N Suganthan, Carlos A Coello, and Francisco Herrera. 2019. Bio-inspired computation: Where we stand and what’s next. Swarm and Evolutionary Computation 48 (2019), 220–250.

[15] Liang Feng, Yew-Soon Ong, Ah-Hwee Tan, and Ivor W Tang. 2015. Memes as building blocks: a case study on evolutionary optimization+ transfer learning for routing problems. Memetic Computing 7, 3 (2015), 159–180.

[16] Carlos Garcia-Martinez, Manuel Lorenzo, Francisco Herrera, Daniel Molina, and Ana M Sánchez. 2008. Global and local real-coded genetic algorithms based on parent-centric crossover operators. European Journal of Operational Research 185, 3 (2008), 1088–1113.

[17] Maoguo Gong, Zedong Tang, Hao Li, and Jun Zhang. 2019. Evolutionary Multitasking with Dynamic Resource Allocating Strategy. IEEE Transactions on Evolutionary Computation 23(5) (2019), 858–869.

[18] Abhishek Gupta and Yew-Soon Ong. 2016. Genetic transfer or population diversification? Deciphering the secret ingredients of evolutionary multitask optimization. In IEEE Symposium Series on Computational Intelligence, 1–7.

[19] Abhishek Gupta, Yew-Soon Ong, B Da, L Feng, and Stephanus Daniel Handoko. 2015. Landscape synergy in evolutionary multitasking. In IEEE Congress on Evolutionary Computation. 3076–3083.

[20] Alphonse Gungor, Zongben Xu, and Baoding Liu. 2016. Multifactorial optimization via explicit multipopulation evolutionary framework. Information Sciences 352 (2015), 1555–1570.

[21] Zhengping Liang, Weiqi Liang, Xiiju Xu, and Zexuan Zhu. 2020. A two stage Adaptive Knowledge Transfer Evolutionary Multi-tasking Based on Population Distribution for Multi/Many-Objective Optimization. arXiv preprint arXiv:2001.00810 (2020).

[22] Shen Lin. 1965. Computer solutions of the traveling salesman problem. Bell System Technical Journal 44, 10 (1965), 2245–2269.

[23] Javier Del Ser, Eneko Osaba, Javier Del Ser, and Francisco Herrera. 2020. Simultaneously Evolving Deep Reinforcement Learning Models using Multifactorial Optimization. arXiv preprint arXiv:2002.12133 (2020).

[24] Yew-Soon Ong. 2016. Towards evolutionary multitasking: a new paradigm in evolutionary computation. In Computational Intelligence, Cyber Security and Computational Models. Springer, 25–26.

[25] Yew-Soon Ong and Abhishek Gupta. 2016. Evolutionary multitasking: a computer science view of cognitive multitasking. Cognitive Computation 8, 2 (2016), 125–142.

[26] Eneko Osaba, Aritz D Martínez, Jesús L Lobo, Javier Del Ser, and Francisco Herrera. 2020. Multifactorial Cellular Genetic Algorithm (MFCGA): Algorithmic Design, Performance Comparison and Genetic Transferability Analysis. arXiv.cs.NE/2003.10768.

[27] Eneko Osaba, Javier Del Ser, Xin-Sheng Yang, Andres Igesias, and Akemi Galvez. 2020. COEBA: A Coevolutionary Bat Algorithm for Discrete Evolutionary Multitasking. arXiv.cs.NE/2003.11628.

[28] Eneko Osaba, Xin-Sheng Yang, and Javier Del Ser. 2020. Is the Vehicle Routing Problem Dead? An Overview Through Bioinspired Perspective and a Prospect of Opportunities. In Nature-Inspired Computation in Navigation and Routing Problems. Springer, 57–84.

[29] G. Reinelt. 1991. TSPLIB: A traveling salesman problem library. ORSA Journal on Computing 3, 4 (1991), 376–384.

[30] Hui Song, AK Qin, Pei-Wei Tsai, and JJ Liang. 2019. Multitasking Multi-Swarm Optimization. In IEEE Congress on Evolutionary Computation. 1937–1944.

[31] Chen Wang, Hui Ma, Gang Chen, and Sven Hartmann. 2019. Evolutionary Multitasking for Semantic Web Service Composition. arXiv preprint arXiv:1902.06370 (2019).

[32] Bingshui Da, Yew-Soon Ong, and Lei Wang. 2019. Rigorous Analysis of Multi-Factorial Evolutionary Algorithm as Multi-Population Evolution Model. International Journal of Computational Intelligence Systems 12, 2 (2019), 1121–1133.

[33] Yew-Soon Ong and Zhe Wang. 2015. Parting ways and reallocating resources in evolutionary multitasking. In IEEE Congress on Evolutionary Computation. 2404–2411.

[34] Heng Xiao, Gen Yokoya, and Toshiharu Hatanaka. 2019. Multifactorial PSO-FA Hybrid Algorithm for Multiple Car Design Benchmark. In IEEE International Conference on Systems, Man and Cybernetics. 1926–1931.

[35] Shuangshuang Yao, Zhihong Ding, Xiangpeng Wang, and Lei Ren. 2020. A Multitask multibjective multifactorial optimization algorithm based on decomposition and dynamic resource allocation strategy. Information Sciences 511 (2020), 18–35.

[36] Gen Yokoya, Heng Xiao, and Toshiharu Hatanaka. 2019. Multifactorial optimization using Artificial Bee Colony and its application to Car Structure Design Optimization. In 2019 IEEE Congress on Evolutionary Computation (CEC). IEEE, 3484–3490.

[37] Yanan Yu, Anmin Zou, Zexuan Zhu, Quzhen Lin, Jian Yin, and Xiaoliang Ma. 2019. Multifactorial Differential Evolution with Opposition-based Learning for Multi-tasking Optimization. In IEEE Congress on Evolutionary Computation. 1989–1995.

[38] Yuan Yuan, Yew-Soon Ong, Abhishek Gupta, Puay Siew Tan, and Hua Xu. 2016. Evolutionary multitasking in permutation-based combinatorial optimization problems: Realization with TSP, QAP, LOP, and JSP. In IEEE Region 10 Conference. 3157–3164.

[39] Jun Zhang, Weiwen Zhou, Xiaolin Chen, Wen Yao, and Lu Cao. 2019. Multi-Source Selective Transfer Framework in Multi-Objective Optimization Problems. IEEE Transactions on Evolutionary Computation (2019).

[40] Lei Zhou, Liang Feng, Jingzheng Zhong, Yew-Soon Ong, Zexuan Zhu, and Edwina Sha. 2016. Evolutionary multitasking in combinatorial search spaces: A case study in capacitated vehicle routing problem. In IEEE Symposium on Computational Intelligence. 1–8.

[41] Lei Zhou, Liang Feng, Jingzheng Zhong, Zexuan Zhu, Bingshui Da, and Zhou Wu. 2018. A study of similarity measure between tasks for multifactorial evolutionary algorithm. In Proceedings of the Genetic and Evolutionary Computation Conference Companion. ACM, 229–230.