Multiple-Channel Resonance Scattering of One-Dimensional Ultracold Spinor Bosons

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So far the interaction of ultracold atoms can only be tuned within one particular scattering channel near a resonance, where the spinor structure of atomic isotopes is destroyed due to the typically large magnetic field. In this Letter, we propose a scheme to realize multiple-channel scattering resonance (MCSR) of ultracold bosons in one-dimension while still keeping their spinor structure. The MCSR refers to a simultaneous scattering resonance among all different scattering channels, including those breaking SU(2) and SO(2) spin rotation symmetries. Essential ingredients for MCSR include the 3D interactions, the confinement potential and a spin-flipping field. Near MCSR a many-body spinor system exhibits exotic spin density distributions and pair correlations, which are significantly different from those near a single-channel resonance.

Introduction. Easy access to strong coupling regime of interacting particles and full liberation of the bosonic and fermionic spin degree of freedom comprise two unique and important features of ultracold atomic gases. Especially, the former allows the exploration of intriguing properties of strongly correlated many-body systems, such as the BCS-BEC crossover, the universal thermodynamics, and the Tonks and super-Tonks continuity of one-dimensional (1D) gas. For the latter, taking advantage of the high (hyperfine) spin structure of atomic isotopes and the SU(2)-invariant interaction at low fields, the atomic spinor system has been shown to exhibit diverse spin textures in the ground state, and interestingly coherent spin-exchange dynamics. Despite all these achievements, little attention has been paid to spinor system with strong interactions. An essential reason is this requires the combination of multiple-chance scattering channels with the spinor structure, which are significantly different from those near a single-channel resonance.

In this Letter, we aim at generating strong coupling of particles, which respectively induces spin flips and tunes the interaction in a single scattering channel. We show that this system exhibits new physics incorporating both features of multiple-spin degree of freedom and strong coupling of particles, which manifests themselves in generating exotic low-energy scattering properties and significant many-body effects, as summarized below:

(A) The low-energy effective scattering will break SU(2) and SO(2) symmetries, i.e., the scattering process will no longer conserve any component of the total spin of incident particles.

(B) By tuning the rf field or magnetic field, all scattering channels will simultaneously go across the resonance, named as multiple-channel scattering resonances (MCSR).

Near MCSR, a many-body system exhibits very different properties from those near a single-channel resonance. First, the spin-flip process is greatly enhanced, even in the presence of a weak rf field. Second, despite of the strong intra-species repulsion, the system exhibits evidently attractive correlations. These properties are experimentally detectable through the measurements of spin densities and two-body correlation functions.

Model. We consider two-species bosons (denoted as $\uparrow, \downarrow$) in 1D geometry subject to tight transverse harmonic traps (with frequency $\omega_{\perp}$) and a rf field (with strength $\Omega$). The Hamiltonian for two such atoms located at $(r_1, r_2)$ is given by $H = \sum_{i=1}^{2} H_i^{(0)} + U$, where

$$H_i^{(0)} = -\frac{\nabla_{r_i}^2}{2m} + \frac{m}{2} \omega^2_{\perp} (x_i^2 + y_i^2) + h_i^{(0)}, \quad (1)$$

$$U = \sum_{M=1,0,-1} U_{MM} \delta(r_1 - r_2) |M\rangle \langle M|. \quad (2)$$

Here $h_i^{(0)} = \frac{\nabla^2_{r_i}}{2m} + \Omega \sigma^z_i$ is the non-interacting Hamiltonian along the 1D tube ($z$), and $\sigma^z$ is Pauli matrix inducing spin-flip. $U$ characterizes scattering in three channels classified by total magnetization $M$, and specifically $|M = 1\rangle = |\uparrow\uparrow\rangle_2$, $|M = 0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)\sqrt{2}$.
\[ |M = -1\rangle = | \downarrow_{12}\rangle; \]  
\[ U \]  
in each \( M \)-channel is associated with a s-wave scattering length \( a_M \), via  
\[ 1/U_{MM} = 4\pi a_M/m - 1/V \sum_k 1/(k^2/m). \]  

To realize above model Hamiltonian, one can use \( F = 1 \) alkali isotopes such as \( ^{87}\text{Rb} \), with \( |m_F = 1(0)\rangle \equiv | \uparrow (\downarrow) \rangle \).

Through a FR at \( B_0 = 1007G \), the scattering length \( a_1 \) can be tuned efficiently (but not \( a_0, a_{-1} \)). At magnetic field \( B \sim B_0 \), the large Zeeman splitting between \( \uparrow \) and \( \downarrow \) spins can be effectively eliminated by applying a rf field and tuning its frequency on resonance with the splitting.

In the frame rotating at the rf frequency, a spinor bosonic system (without Zeeman splitting due to external \( B \)) can be created. Note that the third component of \( F = 1 \) isotopes, \( |m_F = -1\rangle \), can be adiabatically eliminated due to the large quadratic Zeeman shift at \( B \sim B_0 \).

For the reduced 1D system, the Hamiltonian is given by \( h = \sum_{j=1}^2 \hat{h}_j^{(0)} + u \), with
\[ u = \sum_{MN} u_{MN} \delta(z_1 - z_2) |M\rangle \langle N|. \]  

Here \( u_{MN} \) is the effective 1D coupling strength between channel \( M \) and \( N \). In the following we will calculate \( u \) by matching the two-body solutions of \( H \) and \( h \), with the only criterion that they produce the same low-energy scattering property.

Two-body formalism. We study the full scattering wavefunction \( |\Psi\rangle \) according to \( H|\Psi\rangle = E|\Psi\rangle \), with low energy \( E \ll E_{th} + 2\omega_\perp \) where \( E_{th} = \omega_\perp - 2\Omega \) is the threshold energy. We only consider the relative motion here since it is interaction-relevant and can be decoupled from the center-of-mass motion. Given \( U \) in \( (2) \), we assume

\[ \langle \mathbf{r}|U|\Psi\rangle = \delta(\mathbf{r}) \sum_{M = -1,0,1} F_M |M\rangle, \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2 \]  

Further utilizing the Lippman-Schwinger equation

\[ U|\Psi\rangle = T|\Psi^{(0)}\rangle, \]  
where the T-matrix follows \( T = U + UG_0 T \), \( |\Psi^{(0)}\rangle \) is the incident wave function and \( G_0 = (E - H_j^{(0)} - H_0^{(0)} + i0^+)^{-1} \) is the non-interacting Green function, we obtain the scattering amplitudes \( \{F_M\} \) in \( (4) \) via following matrix equation
\[ \sum_N \left[ U^{-1} - \tilde{G}_0 \right]_{MN} F_N = \tilde{\Psi}^{(0)}_M; \]  

with \( \tilde{G}_0 = G_0(\mathbf{r} = \mathbf{0}), \tilde{\Psi}^{(0)}_M = (M|\Psi^{(0)}(\mathbf{r} = \mathbf{0})\rangle \). Here \( U \) and \( G_0 \) are both \( 3 \times 3 \) matrices expanded in the \( \{|M\rangle \langle N|\} \) spin basis. After solving \( \{F_M\} \) from \( (5) \), we obtain the wave function as

\[ \Psi(\mathbf{r}) = \Psi^{(0)}(\mathbf{r}) + \sum_M F_M G_0(\mathbf{r}) |M\rangle. \]  

When \( z \to \infty \), the wave function \( (6) \) is frozen at the lowest transverse mode \( (n_x = n_y = 0) \), i.e.,

\[ \Psi(\mathbf{r}) \to \phi_0(x)\phi_0(y)\psi(z). \]  

Here \( \psi \) describes the effective scattering process along \( z \), which can be equally obtained based on the reduced 1D Hamiltonian \( h \). Similarly we define the 1D scattering amplitudes \( f_M \), and find that Eqs.\((15)\) are still applicable as long as \( \{r, \mathbf{r}, \Psi, \Psi^{(0)}, F_M, U, G_0\} \) are respectively replaced by \( \{z, \mathbf{r}, \psi, \psi^{(0)}, f_M, U, g_0\} \), where
\[ g_0 = (E - E_{th} - h_1^{(0)} - h_2^{(0)} + i0^+)^{-1} \]  
is non-interacting Green function in 1D. Given Eq.\((7)\), we get \( f_M = F_M \phi_0^{(0)}(0) \) and finally relate \( u \) in \( (3) \) to \( U \) in \( (2) \) as

\[ u = |\phi_0(0)|^2 [U^{-1} - \tilde{G}_0^{\sigma z}]^{-1}, \quad \tilde{G}_0^{\sigma z} = \tilde{G}_0 - |\phi_0(0)|^2 g_0. \]  

Here \( \tilde{G}_0^{\sigma z} \) is the Green function constructed by all excited transverse modes \( n_x + n_y > 0 \). As we will see later, its structure is essential to induce the multiple-channel scattering resonances in \( u \).

Multiple-channel scattering resonance. Cooperating with the confinement and 3D interactions, the rf field can result in multi-channel effective scattering in the low-energy 1D space. This is achieved through the virtual scattering processes to higher transverse modes, as schematically shown in Fig.1. Without rf field (Fig.1a), an initial spin state, \( | \uparrow \uparrow \rangle (M = 1) \), at the ground state mode \( (n: n_x = n_y = 0) \), can only be scattered by 3D interaction \( U \) to the same spin state in higher modes \( (n': n_x + n_y > 0) \), and then back to itself in \( n \), which process renormalizes the effective \( u_{11} \). When rf field is switched on, two additional processes can occur (Fig.1b and 1c). The rf field could flip spins in \( n' \) once or twice and finally be scattered to a different spin state \( | \uparrow \downarrow \rangle (M = 0) \) or \( | \downarrow \downarrow \rangle (M = -1) \) in the ground mode \( n \). These processes respectively renormalize \( u_{10} \) and \( u_{11}, -1 \), which are originally absent if without rf field.

More accurately, the physics illustrated above can be reflected in the exact expression of \( \tilde{G}_0^{\sigma z} \) (Eq.\((8)\)), which include both diagonal and off-diagonal elements contributed from all orders of scattering processes involving all excited modes. Consequently, \( u-Matrix \) also have non-zero off-diagonal elements, which break both SU(2) and SO(2) symmetries in the spin-spin scattering process. This exactly demonstrates the multiple-channel scattering as summarized previously by (A) in the introduction.
Moreover, due to the intrinsic entanglement between different scattering processes, Eq.⑧ further predicts an exotic phenomenon in the low-energy scattering, namely the multiple-channel scattering resonances (MCSR), which refers to a simultaneous divergence of effective couplings in different scattering channels (different $\nu$-matrix elements). For threshold scattering ($E = E_{th}$), the MCSR occurs when

$$|U^{-1} - G^\nu_{0}(E = E_{th}, \Omega_{res})| = 0.$$  \hspace{1cm} (9)

Here $\Omega_{res}$ is the strength of rf field required by MCSR. Remarkably, it means that by tuning a single parameter ($\Omega$ or scattering length $a_M$ in an arbitrary $M$-channel), the spinor system can be driven to strongly coupling regime in multiple scattering channels. This demonstrates (B) in the introduction.

In Fig.2, we show the general features of MCSR by numerically solving Eqs.⑧⑨. Fig.2(a) gives $\Omega_{res}$ as a function of one interaction parameter $a_\perp/a_1$ ($a_\perp = \sqrt{2/(m\omega_\perp)}$ is confinement length), while $a_0$ and $a_\perp$ are both fixed and far off resonance. This resembles the actual case of $^{87}$Rb in realistic experiments[23]. At $\Omega = 0$, different scattering channels are decoupled and we recover the result of CIR within the single $M = 1$ channel at $a_\perp/a_1 = 1.4617$. At finite $\Omega$, it is found that the resonance position shifts to BEC side with larger $a_\perp/a_1$. More importantly, the structure of $\nu$ is drastically different from what CIR predicted (dashed curves in Fig.2(b1,b2,c1,c2)). Especially, by tuning $\Omega$ we find non-cyclically different from what CIR predicted (dashed curves in Fig.2(b1,b2,c1,c2)).

FIG. 2: Multiple-channel scattering resonances with tunable $a_1$ and fixed $a_0 = a_{-1} = a_{\perp}/4[23]$. $\Omega$ is scaled by $\omega_\perp$, and $u_{11}$, $u_{10}$ are scaled by $2/(m\omega_\perp)$. (a): Resonance position $\Omega_{res}$ as functions of $a_\perp/a_1$. (b1,b2) or (c1,c2): $u_{11}$, $u_{10}$ as functions of $\Omega$ at fixed $a_\perp/a_1 = 1.6$ or as functions of $a_\perp/a_1$ at fixed $\Omega = 0.1\omega_\perp$, corresponding to the vertical [or horizontal] arrow in (a). For comparison, CIR predictions are shown by dashed lines.

To further understand the mechanism of MCSR, one can follow the traditional way in understanding FR[20] and CIR[17], by constructing a virtual "closed-channel" bound state. Whenever such a bound state at zero energy, the MCSR in the low-energy space will occur. More discussions on how this mechanism works for MCSR and its difference from that for a single-channel resonance will be presented in Supplementary materials[25].

Though MCSR predicts a simultaneous resonance in all scattering channels, not all resonances have visible widths as to be easily detected in realistic experiments[23]. Given that $a_1$ is tunable through FR while $a_0$ and $a_{-1}$ are far off resonance, only those scatterings associated with $M = 1$, namely $u_{11}$, $u_{10}$ and $u_{1-1}$, are more likely to be strongly affected. Further, among three of them $u_{1-1}$ require higher order of spin-flip in virtual scattering processes (see Fig.1(c)), hence in the presence of a weak $\Omega(\ll \omega_\perp)$ its resonance width is expected to be much narrower than that of $u_{11}$ and $u_{10}$. We have verified these expectations by numerical simulations[23]. In the following, we will explore the many-body effect due to strong couplings of $u_{11}$ and $u_{10}$ near MCSR, while the other channels are approximated as non-interacting.

Many-body effects. Given $u_{11}$ and $u_{10}$ from two-body solutions, we write down the many-body Hamiltonian for 1D spinor bosons subject to an additional harmonic trap,

$$H = \sum_\sigma \int dz \Psi_\sigma^\dagger(z) \left( \frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + \frac{1}{2} m \omega_T z^2 \right) \Psi_\sigma(z)$$

$$+ \Omega \int dz \left( \Psi_\uparrow^\dagger(z) \Psi_\downarrow(z) + h.c. \right)$$

$$+ \frac{u_{11}}{2} \int dz \Psi_\uparrow^\dagger(z) \Psi_\uparrow(z) \Psi_\downarrow(z) \Psi_\downarrow(z)$$

$$+ \frac{u_{10}}{\sqrt{2}} \int dz \left( \Psi_\uparrow^\dagger(z) \Psi_\uparrow(z) \Psi_\downarrow(z) \Psi_\downarrow(z) + h.c. \right).$$ \hspace{1cm} (10)

It is clear that the interaction part of $H$ includes various terms breaking SU(2) and SO(2) symmetries, such as $\rho \cdot \rho$, $\sigma_z \cdot \sigma_z$, $\sigma_z \cdot \sigma_z$, $\sigma_z \cdot \sigma_z$, where $\rho$ and $\sigma_z$ are respectively the number density and spin densities.

Based on Hamiltonian (10), we will first use exact diagonalization method to solve the ground state of a four-particle system. The results obtained not only are relevant to the cluster system[27,28], but also serve as a benchmark for a many-body system. We focus on the large repulsion limit of $u_{11}$, where $u_{10}$ can also be tuned large and positive using MCSR. To highlight the significance of $u_{10}$, we compare three different cases: (i) strongly repulsive spin-↑ bosons without rf field ($\Omega = 0$) and $u_{10} = 0$; (ii) $\Omega \neq 0$ but still $u_{10} = 0$; (iii) $\Omega \neq 0$ and $u_{10} \neq 0$. Among them, case (iii) is what we are most interested in and also the general one near MCSR.

In Fig.3 we show the spin density distributions, $\rho_\sigma(z) \equiv \langle \Psi_\sigma^\dagger(z) \Psi_\sigma(z) \rangle$, and the two-body correlation functions, $g_{\sigma\sigma'}(z,z_0) \equiv \langle \Psi_\sigma^\dagger(z) \Psi_{\sigma'}(z_0) \Psi_{\sigma'}(z_0) \Psi_\sigma(z) \rangle$. 

"
channel resonance, the case (iii), with $u$ property (as summarized by (C)). As the characterized by a dip of energy, and they still maintain fermion-like correlations, the residual $\uparrow$ alone will generate a lot more $\downarrow$ with determinate $\uparrow$ well approximated by the absolute value of a Slater $(i)$, the spin-$\uparrow$ and $\downarrow$ parameters, $(a_1,b_1)$ provides a way to avoid strong repulsion between $\uparrow$-spins if $u_{10}$ from zero, the original dip of up-spin correlations at $z = z_0 = 0$ gradually vanishes and turns to a peak, implying the up-spins gradually lose their repulsive nature and turn attractive. Accordingly all spins are attracted to the trap center and produce more pronounced density distribution (than non-interacting case)(Fig.2(a4)). Indeed, assume a many-body system at sufficiently large $u_{10}$, one would expect the spins being polarized along $-\hat{x}$, giving $\sigma_z(z) = -2\rho_\uparrow(z) = -2\rho_\downarrow(z)$. Eventually the total interaction becomes $\frac{1}{2}(u_{11} - 2\sqrt{2}u_{10}) \int dz\rho_\uparrow^2(z)$, which becomes purely attractive if $u_{10} \gg u_c = u_{11}/(2\sqrt{2})$.

Concluding remark. In conclusion, we have demonstrated that a multiple-channel scattering resonance (MCSR) can be achieved for spinor bosons confined in 1D geometry. The two-body and many-body properties revealed in this Letter are expected to be easily probed in current cold atoms experiments.

Finally, we remark that the proposal of MCSR widely applies to other high-spin systems which allow more-than-one collision channels, and other confined geometries such as 2D. Moreover, rf field can be replaced by any field that allows spin-flips, such as a rotating magnetic field generating spin-orbit couplings. Given the wide applicability and special properties of MCSR, we expect this new type of scattering resonance will induce a lot more intriguing many-body physics in the subject of spinor systems, such as the ground state structure and topological defects of a 2D BEC, the pairing superfluidity of spinor systems, such as the ground state structure and topological defects of a 2D BEC, the pairing superfluidity of spinor systems, such as the ground state structure and topological defects of a 2D BEC.

With $z_0 = 0$, for $N = 4$ system in cases (i,ii,iii). For case (i), the spin-$\uparrow$ bosons are fermionalized, with wave function well approximated by the absolute value of a Slater determinant $\psi_\uparrow(z) = \sqrt{\prod_{i=1}^{N-1} |\phi_i(z)|^2}$ (here $\phi_i$ is the eigen-function of 1D harmonic oscillator, with level index $i = 0, \ldots, N - 1$). Consequently the density is given by $\rho_\uparrow(z) = \sum_{i=1}^{N-1} |\phi_i(z)|^2$, and the two-body correlation by $g_{\uparrow\uparrow}(z, z_0) = \sum_{x, y} \phi_x(z)\phi_y(z_0) - \phi_y(z)\phi_x(z_0)|^2$, which all share the same properties of identical fermions (see gray curves in Fig.3(a1,b1)). When turn on a weak rf, it provides a way to avoid strong repulsion between $\uparrow$-spins by flipping them to $\downarrow$, and thus the ground state will be tremendously changed. If $u_{10}$ is absent (case (ii)), rf alone will generate a lot more $\downarrow$ than $\uparrow$ (see Fig.3(a2)). The residual $\uparrow$ are there to take advantage of the rf energy, and they still maintain fermion-like correlations, characterized by a dip of $g_{\uparrow\downarrow}$ at $z = z_0 = 0$ (Fig.3(b2)).

Compared with cases (i,ii) which describe a single-channel resonance, the case (iii), with $u_{10}$ present and describing MCSR, will show a very different ground state property (as summarized by (C)). As the $u_{10}$ term follows the form of $\int dx\rho_\uparrow(z)\sigma_x(z)$, the spin-flip process will be drastically enhanced through $u_{10}$. As shown in Fig.3(a3) and (a4), the spin numbers get more balanced as $u_{10}$ increases. One may thus expect this $u_{10}$ term just simply enhance the effective rf strength, $\Omega_{\text{eff}}$. However, this is not true, as it also generates significant interaction effect and modifies the pair correlations. As seen from Fig.3(b3) and (b4), when increasing $u_{10}$ from zero, the original dip of up-spin correlations at $z = z_0 = 0$ gradually vanishes and turns to a peak, implying the up-spins gradually lose their repulsive nature and turn attractive. Accordingly all spins are attracted to the trap center and produce more pronounced density distribution (than non-interacting case)(Fig.2(a4)). Indeed, assume a many-body system at sufficiently large $u_{10}$, one would expect the spins being polarized along $-\hat{x}$, giving $\sigma_z(z) = -2\rho_\uparrow(z) = -2\rho_\downarrow(z)$. Eventually the total interaction becomes $\frac{1}{2}(u_{11} - 2\sqrt{2}u_{10}) \int dz\rho_\uparrow^2(z)$, which becomes purely attractive if $u_{10} \gg u_c = u_{11}/(2\sqrt{2})$.

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Finally, we remark that the proposal of MCSR widely applies to other high-spin systems which allow more-than-one collision channels, and other confined geometries such as 2D. Moreover, rf field can be replaced by any field that allows spin-flips, such as a rotating magnetic field generating spin-orbit couplings. Given the wide applicability and special properties of MCSR, we expect this new type of scattering resonance will induce a lot more intriguing many-body physics in the subject of spinor systems, such as the ground state structure and topological defects of a 2D BEC, the pairing superfluidity of high-spin fermions, the interplay effect with spin-orbit correlations and so on.

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The background scattering lengths are 

\[ a_1 \approx a_0 \approx a_{-1} = 5.3 \text{nm} \]

\[ a_1 \] can be tuned through a FR at 

\[ B = 1007 \text{G} \]

Transverse confinement frequency is chosen to be

\[ \omega_\perp = (2\pi)400 \text{KHZ} \]

giving confinement length

\[ a_\perp = 24 \text{nm} \]

This gives

\[ a_\perp/a_0 \approx a_\perp/a_{-1} \approx 4 \]

and

\[ a_\perp/a_1 \] is highly tunable. Near FR of

\[ a_1 \]

the Zeeman splitting between

\[ m_F = 1 \text{ and } m_F = 0 \]

is

\[ 0.63 \text{GHZ} \]

while between

\[ m_F = 0 \text{ and } m_F = -1 \]

is

\[ 0.77 \text{GHZ} \]

due to the large quadratic Zeeman shift. Thus

\[ m_F = -1 \]

can be adiabatically eliminated if the rf frequency matches

\[ 0.63 \text{GHZ} \].

To avoid the system being polarized by rf field, we consider

\[ \Omega \ll E_F \]

where \( E_F \) is the Fermi energy of identical fermions with the same density as bosons.

The ratio of

\[ u_{10} \text{ to } u_{11} \]

near MCSR is directly given by the ratio of their individual width, which is tunable through the rf strength and interaction parameters. For a weak rf field, this ratio cannot be too large, so the system will hardly be fully polarized or turn purely attractive.

See a recent review by V. Galitski and I. B. Spielman, Nature 494, 49 (2013).
SUPPLEMENTARY MATERIALS

In this supplementary material we present more details for the derivation of Eq.(8) in the main text, and more discussions on the properties of the multiple-channel scattering resonances.

I. Derivation of $u$-matrix (Eq.(8) in the text)

First, we decouple the non-interacting two-body Hamiltonian as follows,
\[
H_{1}^{(0)} + H_{2}^{(0)} = \left(-\frac{\nabla_{x}^{2} + \nabla_{y}^{2} + \nabla_{z}^{2}}{4m} + m\omega_{\perp}^{2}(X^{2} + Y^{2})\right) + \left(-\frac{\nabla_{x}^{2} + \nabla_{y}^{2} + \nabla_{z}^{2}}{m} + \frac{m}{4}\omega_{\perp}^{2}(x^{2} + y^{2}) + \Omega(\sigma_{x}^{2} + \sigma_{z}^{2})\right)
\]
(11)

Here the first bracket represents the Hamiltonian for center-of-mass motion $R \equiv (X, Y, Z) = \frac{r_{1} + r_{2}}{2}$ with effective mass $2m$; the second bracket includes the relative motion $r \equiv (x, y, z) = r_{1} - r_{2}$ with mass $m/2$, and the spin part according to a transverse magnetic field. Since the interaction $U$ is only relevant to the relative motion and their spins, we only consider the second bracket in Eq.(11) when solving two-body problem in the following.

The eigen-state for non-interacting Hamiltonian (second bracket in Eq.(11)) can be expressed as $|n_{x}, n_{y}, k\rangle|\alpha_{1}\beta_{2}\rangle$, where the first part describes the relative motion with wave function
\[
\langle x, y, z|n_{x}, n_{y}, k\rangle = \phi_{n_{x}}(x)\phi_{n_{y}}(y) e^{ikz} \sqrt{L_{z}},
\]
(12)

and the second part shows the spin configuration with $\alpha, \beta = +$ or $-$, and $|\pm\rangle = (|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}$. The corresponding energy spectrum is
\[
E_{n_{x}, n_{y}, k; \alpha_{1}\beta_{2}} = (n_{x} + n_{y} + 1)\omega_{\perp} + \frac{k^{2}}{m} + (\epsilon_{\alpha} + \epsilon_{\beta}),
\]
(13)

here $\epsilon_{\pm} = \pm\Omega$. The threshold scattering energy is therefore $E_{th} = \omega_{\perp} - 2\Omega$ when $n_{x} = n_{y} = 0$, $k = 0$, $\alpha = \beta = -$.

Making use of the Lippman-Schwinger equation $U|\Psi\rangle = T|\Psi^{(0)}\rangle$, and the T-matrix $T = U + UG_{0}T$, we get
\[
U|\Psi\rangle = U|\Psi^{(0)}\rangle + UGU|\Psi\rangle,
\]
(14)

then combining with Eq(4) in the text, we obtain the matrix equation expanded in $\{|M\rangle|N\rangle\}$ spin space, (see also Eq.(5) in the text)
\[
\begin{bmatrix}
U
\end{bmatrix}^{-1} = \begin{bmatrix}
\tilde{G}_{0} \equiv G_{0}(r = 0)
\end{bmatrix}
\]
(15)

here matrix $(U) = \text{diag}(U_{1}, U_{0}, U_{-1})$; $\begin{bmatrix}\tilde{G}_{0}(r)\end{bmatrix}_{MN} = \langle r, M|G_{0}|0, N\rangle$; $\begin{bmatrix}\tilde{\Psi}^{(0)}_{0}\end{bmatrix}_{M} = \langle M|\Psi^{(0)}(r = 0)\rangle$. With the information of $F_{M}$ from above matrix equation, the wf can be deduced straightforwardly, which is Eq.(6) in the text.

For low-energy scattering $E \ll E_{th} + 2\omega_{\perp}$ and at $z \to \infty$, in the full wave function all excited $n$ (with $n_{x} + n_{y} > 0$) modes decays away except the lowest $n_{0}(n_{x} = n_{y} = 0)$, i.e., the wave function is effectively propagating in 1D as
\[
\Psi(r) \rightarrow \phi_{0}(x)\phi_{0}(y)\left\{\psi^{(0)}(z) + g_{0}(z)(f_{1}|1\rangle + f_{0}|0\rangle + f_{-1}|1\rangle - 1\right\}
\]
(16)

with $f_{M} = F_{M}\phi_{0}^{2}(0)$, and $g_{0}$ is the non-interacting Green function for 1D system (see definition in the text). Alternatively, the wave function along $z$ can be generated effectively through a 1D interaction
\[
\langle z|u|\psi\rangle = \delta(z)(f_{1}|1\rangle + f_{0}|0\rangle + f_{-1}|1\rangle - 1\right),
\]
(17)

together with the Lippman-Schwinger equation
\[
\begin{bmatrix}
\begin{bmatrix}
u
\end{bmatrix}
\end{bmatrix}^{-1} = \begin{bmatrix}
\tilde{g}_{0} \equiv g_{0}(z = 0)
\end{bmatrix}
\]
(18)
Compare Eq. (15) with Eq. (15), and recall the relations that $f_M = F_M \phi_n^2(0)$, $\tilde{\psi}_M^{(0)} = \tilde{\Psi}_M^{(0)}/\phi_0^2(0)$, we obtain
\[
\left( \left( \begin{array}{c} u \\ \phi \end{array} \right) \right)^{-1} = \frac{1}{|\phi(0)|^2} \left( \left( \begin{array}{c} U \\ \phi \end{array} \right) \right)^{-1} \left( \begin{array}{c} g_0 \\ \phi \end{array} \right),
\]
which gives rise to Eq. (8) in the text.

Next we show the detailed procedure how to evaluate $\tilde{G}_0^{\mu\nu}$ and $u$–matrix in Eq. (8). To expand $\tilde{G}_0^{\mu\nu}$, one needs to insert a complete set of eigen-states \{$(n_x, n_y, k|\alpha_1 \beta_2)$\} and sum over all contributions from these energy states. We will see in the following that for a diagonal element of $\tilde{G}_0^{\mu\nu}$, the summation will have ultraviolet divergence, which will be compensated by the same divergence in $U^{-1}$. Eventually each element of $u$ is physically a finite value.

As the ultraviolet divergence in energy space corresponds to the short-range singularity of the wave function ($\sim 1/r$) as inter-particle distance $r \to 0$, in the following we will try to evaluate the Green function in coordinate space and summing over the $1/r$ singularities. Explicitly, take one diagonal element ($M = N = 1$) as example, we have
\[
[G_0^{\mu\nu}(r)]_{11} = \langle r; M = 1|G_0^{\mu\nu}|0, M = 1\rangle = \sum_{n_1, n_2 > 0} \phi_{n_1}(x)\phi_{n_2}(0) f_{n_2}(2\pi)^3 2 \int_{-\infty}^{\infty} dk e^{ikz} \left( \sum_{\alpha, \beta} \langle n_1 + n_2 \omega_{\perp} \rightarrow \frac{k^2}{m} - (\epsilon_{\alpha} + \epsilon_{\beta} + 2\Omega + i\delta) \rangle \right)
\]
with $\Delta E = E - E_{th}$; $\langle M = 1|\alpha_1 \beta_2\rangle = \xi_{\alpha}^{(1)} \xi_{\beta}^{(1)}$ and $\xi_{\alpha}^{(1)} = 1/\sqrt{2}$. Note that in order for the non-zero value of above equation as $r \to 0$, $n_x + n_y$ should be an even integer ($= 2, 4, \ldots$), therefore as long as $\Delta E < 2\omega_{\perp}$ the denominator inside the bracket is always negative. For this low-energy scattering ($\Delta E \ll 2\omega_{\perp}$), Eq. (20) is transformed to
\[
-\sum_{\alpha, \beta} \langle n_1 + n_2 \omega_{\perp} \rightarrow \frac{k^2}{m} - \epsilon_{\alpha} - \epsilon_{\beta} + 2\Omega + i\delta \rangle \right)
\]
and $A(t)$ can be obtained by making use of the propagator of 1D harmonic oscillator,
\[
A(t) = \left( \sum_{n_1 = 0}^{\infty} e^{-n_1 \omega_{\perp} t} \phi_{n_1}(x)\phi_{n_1}(0)\right) \left( \sum_{n_2 = 0}^{\infty} e^{-n_2 \omega_{\perp} t} \phi_{n_2}(y)\phi_{n_2}(0)\right) - \phi_0(x)\phi_0(0) - \phi_0(y)^2
\]
\[
= \frac{1}{\pi a^2} \left( \frac{1}{1 - e^{-2\omega_{\perp} t}} e^{-\frac{x^2 + y^2}{2a^2} \coth(\omega_{\perp} t)} - e^{-\frac{x^2 + y^2}{2a^2}} \right). (a_{\perp} = \sqrt{\frac{2}{m\omega_{\perp}}})
\]

Further using the identity
\[
\int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikx} e^{-\frac{k^2}{4}} = \frac{1}{a_{\perp} \sqrt{2\pi} \tau} e^{-\frac{x^2}{2a^2} \tau}, (\tau = \omega_{\perp} t)
\]
Eq. (21) is further reduced to
\[
-\sum_{\alpha, \beta} \langle n_1 + n_2 \omega_{\perp} \rightarrow \frac{k^2}{m} - \epsilon_{\alpha} + \epsilon_{\beta} + 2\Omega \rangle \right)
\]
It is then noticed this integral has singularity at $\tau \to 0$ and as $r = \sqrt{x^2 + y^2 + z^2} \to 0$, and the singularity is given by
\[
-\frac{m}{(2\pi)^{3/2}a_{\perp}} \int_{0}^{\infty} \frac{d\tau}{2\pi} e^{-\frac{2\tau}{4\pi}} = -\frac{m}{4\pi r}.
\]

Apart from the singularity term, the (physical) constant term in the asymptotic form
\[
\lim_{r \to 0} G_0^{\mu\nu}(r)_{11} = -\frac{m}{4\pi r} + C_{11}
\]
can be extracted as
\[
C_{11} = -\sum_{\alpha, \beta} \langle n_1 + n_2 \omega_{\perp} \rightarrow \frac{k^2}{m} - \epsilon_{\alpha} + \epsilon_{\beta} + 2\Omega \rangle \right)
\]

(27)
Recalling that \( \frac{1}{\nu_{11}} = \frac{m}{4\pi a_e} - \frac{1}{\hbar} \sum_k \frac{1}{(k^2 + m)} = \frac{m}{4\pi a_e} - \text{lim}_{\epsilon \to 0} \frac{m}{4\pi \epsilon} \), and combining with Eq.(26), we get the element

\[
[U^{-1} - \tilde{G}_0^{ex}]_{11} = \frac{m}{4\pi a_e} - C_{11}. \tag{28}
\]

Similarly one can obtain diagonal \( C_{MM} \) for \( M = 0, -1 \), by replacing \( M = 1 \) with \( M = 0, -1 \) in Eq.(27). The one-dimensional integration in term of imaginary time \( \tau \) can be performed straightforwardly by numerics. In the case of \( \Omega = 0 \), we have \( C_{MM} = \frac{m}{4\pi a_e} c_0 \) that are identical for all \( M \), with \( c_0 = 1.46 \) obtained previously for conventional CIR (see Ref.[2] in the manuscript).

The same strategy can be used to calculate off-diagonal elements of matrix \( \left( U^{-1} - \tilde{G}_0^{ex} \right) \), and one can find that the divergence of these elements at short distance will be absent. Explicitly we have \( (M \neq N) \)

\[
C_{MN} = -\sum_{\alpha,\beta} \langle M \alpha_1 \beta_2 | \langle \alpha_1 \beta_2 | N \rangle \rangle \frac{m}{(2\pi)^{b/2} a_e} \int_0^\infty e^{\frac{\Delta E}{\tau}} e^{-\frac{\epsilon_0 + \epsilon_2 + 2\Delta}{\tau}} \left( \frac{1}{1 - e^{-2\tau}} - 1 \right); \tag{29}
\]

\[
\left[ U^{-1} - \tilde{G}_0^{ex} \right]_{MN} = -C_{MN}. \tag{30}
\]

Knowing all the elements of matrix \( \left( U^{-1} - \tilde{G}_0^{ex} \right) \), the \( u^- \)-matrix can be obtained through Eq.(8) in the text.

### II. Properties of Multiple-channel scattering resonances

Similar to that for Feshbach resonances, a physical interpretation for multiple-channel scattering resonances can be obtained through a two-channel model, where an open channel and a closed channel are introduced respectively with projection operators \( P \) and \( Q \). Here the open \( (P) \) channel refers to scattering within the lowest transverse mode \((n_x = n_y = 0)\), while the closed \( (Q) \) channel refers to scattering involving higher transverse modes \((n_x + n_y > 0)\). With projections \( P \) and \( Q \), the two-body schrodinger equation can be divided into two equations,

\[
H_{PP} \Psi_P + H_{PQ} \Psi_Q = E \Psi_P; \quad H_{QP} \Psi_P + H_{QQ} \Psi_Q = E \Psi_Q, \tag{31}
\]

with \( H_{\mu\nu} = H_{\nu\mu} = H_{\mu} \) and \( \Psi_{\mu} = H_{\mu} \Psi \) \((\mu, \nu = P \) or \( Q)\). By solving these equations, one can obtain the effective schrodinger equation for the open-channel state \( \Psi_P \) as \( H_{eff} \Psi_P = E \Psi_P \), with effective Hamiltonian

\[
H_{eff} = H_{PP} + \frac{1}{E - H_{QQ}} H_{QP}. \tag{32}
\]

The second term of above equation incorporates all contributions from the virtual scattering processes involving higher transverse modes \((\text{closed channel})\), which renormalize the effective scattering within the lowest transverse mode \((\text{open channel})\). As the eigen-value of \( H_{QQ} \) can be adjusted by interaction parameters or the strength of rf field, it can be tuned crosses \( E_{th} \) and cause a divergence of \( H_{eff} \) according to Eq.(32). This gives rise to the scattering resonance in the open channel.

We denote the eigen-states of \( H_{QQ} \) as \( \tilde{\Psi}_Q \), and write \( H_{QQ} \tilde{\Psi}_Q = E_Q \tilde{\Psi}_Q \). \( E_Q = E_{th} \) determines the scattering resonances for open channel. In Fig.4 we plot three \( E_Q \) evolving with the rf strength or interaction parameters, and the place when one of these bound states across \( E_{th} \) gives the location of MCSR. Different from the closed-channel bound state in a single-channel- resonance, here in MCSR each bound state is highly entangled in spin space, which is a certain superposition of all \( M \)–states. Whenever such a dressed-spin state across threshold, resonances will simultaneously occur in multiple spin-collision channels, as seen from Fig.2(b1,b2,c1,c2) in the main text.

Using the formula in Eq.(32), one can evaluate the resonance width in different collision channels, which is proportional to \( H_{PQ} H_{PQ} = \langle \Psi_P | H | \tilde{\Psi}_Q \rangle \langle \tilde{\Psi}_Q | H | \Psi_P \rangle \). Explicitly, in our case we write the \( a_1 \)-tuned resonances as

\[
u_{MN} = \frac{W_{MN}}{a_1/a_1 - C}, \quad W_{MN} \propto \langle \Psi_P^M | H | \tilde{\Psi}_Q \rangle \langle \tilde{\Psi}_Q | H | \Psi_P^N \rangle, \tag{33}
\]

where \( C \) the resonance position of \( a_1/a_1 \), \( | \Psi_P^N \rangle \) is the open-channel wave function when projected to \( | N \rangle \) spin state, and \( W_{MN} \) the resonance width of \( \nu_{MN} \).
can be estimated through the perturbation theory, i.e., the vicinity of above 1D resonances the closed channel is mainly composed by $\omega_1$ fixed $\Omega = 0$

From this estimation, one can see that under the condition $\Omega 

FIG. 4: Virtual bound state energies $E_Q$ (shifted by $E_{th}$) as functions of $\Omega$ at fixed $a_+/a_1 = 1.6$ (a), or as functions of $a_+/a_1$ at fixed $\Omega = 0.1\omega_1$ (b). Here (a) and (b) respectively follow the vertical or horizontal arrows in Fig. 2(a) in the main text. $E_Q$, $\Omega$ are all scaled by $\omega_1$.

Given that $a_1$ can be tuned large through FR, while $a_0$ and $a_{-1}$ are far off resonances (small positive values), in the vicinity of above 1D resonances the closed channel is mainly composed by $|M = 1\rangle$ states, and its wave function can be estimated through the perturbation theory, i.e.,

$$\tilde{\Psi}_Q(r) = \tilde{\Psi}_1^{(0)}(r)|M = 1\rangle + \tilde{\Psi}_0^{(0)}(r)\sqrt{2\Omega}\tilde{\Psi}_0^{(0)}\tilde{\Psi}_1^{(0)} |M = 0\rangle + \tilde{\Psi}_{-1}^{(0)}(r)\frac{2\Omega^2(\tilde{\Psi}_0^{(0)}|\tilde{\Psi}_0^{(0)}\tilde{\Psi}_1^{(0)}|M = -1)}{(E_1^{(0)} - E_0^{(0)})(E_1^{(0)} - E_{-1}^{(0)})}|M = -1\rangle, \quad (34)$$

here $\tilde{\Psi}_M^{(0)}(r)$ and $E_M^{(0)}$ are respectively the eigen-function and eigen-energy of $H_{QQ}$ in $M \leftrightarrow M$ scattering channel at $\Omega = 0$. Given above interaction parameters, we have $E_1^{(0)} \gg E_0^{(0)} = E_{-1}^{(0)} \approx -1/(ma_0^2)$ or $\approx -1/(ma_{-1}^2)$. Above perturbation theory is valid when $\Omega/|E_1^{(0)} - E_0^{(0)}| \ll 1$.

Given Eq. (33) and Eq. (34), one can obtain the relative widths of all $\{W_{MN}\}$. For instance, we have the ratios

$$\frac{W_{10}}{W_{11}} \sim \frac{\Omega}{E_1^{(0)} - E_0^{(0)}}, \quad \frac{W_{00}}{W_{11}} \sim \frac{\Omega^2}{(E_1^{(0)} - E_0^{(0)})^2}, \quad \frac{W_{1-1}}{W_{11}} \sim \frac{\Omega^2}{(E_1^{(0)} - E_0^{(0)})(E_1^{(0)} - E_{-1}^{(0)})}, \quad \ldots \quad (35)$$

From this estimation, one can see that under the condition $\Omega/|E_1^{(0)} - E_0^{(0)}| \ll 1$, only $W_{11}$ and $W_{10}$ would have visible widths, while the others (in comparison to $W_{11}$ and $W_{10}$) are too narrow to be observable in reality.

Above analyses from two-channel model have also been verified by our numerical calculations. Fig. 5 shows that all the elements of $u$-matrix simultaneously go to infinity as $a_+/a_1$ across resonance position. However, under the conditions specified before Eq. (34), only the resonances of $u_{11}$ and $u_{10}$ have visible widths. The other components of $u$-matrix are only large enough if extremely close to the resonance position (with very narrow width).

Finally, for the two visible resonances of $u_{11}$ and $u_{10}$, their ratio near the multiple-channel scattering resonances is given by the ratio of their individual resonance width, which depend on both $\Omega$ and $a_{1,0,-1}$ as shown by Eq. (35). This ratio is therefore tunable through the rf strength and interaction parameters. Again, under the conditions specified before Eq. (34) and in view of Eq. (35), the ratio of $u_{10}$ to $u_{11}$ near MCSR cannot be too large, therefore the system will not be polarized along $-\hat{x}$ and will not turn purely attractive. However, as long as the energy scale of $u_{10}$ exceeds $E_F$ (the chemical potential of strongly repulsive spinless bosons, or the Fermi energy of identical fermions with the same density as bosons), the presence of $u_{10}$ will have significant effect to the density distribution and pair correlation of the system, as demonstrated in the main text.
FIG. 5: (a1) Diagonal and (a2) off-diagonal matrix elements of \( u \) as functions of \( a_{\perp}/a_{\parallel} \) at fixed \( \Omega = 0.1\omega_{\perp} \). The other parameters, \( a_0 = a_{\perp} = a_{\parallel}/4 \), are the same as those in Fig.2 in the main text. Insets show magnified plots of \( u \)-matrix elements except for \( u_{11} \) and \( u_{10} \). (b) Resonance widths \( W_{MN} \) defined in Eq.(33). \( u \)-elements and \( W_{MN} \) are both scaled by \( 2/(m\omega_{\perp}) \).