Spin Precession and Avalanches

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In many magnetic materials, spin dynamics at short times are dominated by precessional motion as damping is relatively small. In the limit of no damping and no thermal noise, we show that for a large enough initial instability, an avalanche can transition to an ergodic phase where the state is equivalent to one at finite temperature, often above that for ferromagnetic ordering. This dynamical nucleation phenomenon is analyzed theoretically. For small finite damping the high temperature growth front becomes spread out over a large region. The implications for real materials are discussed.

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Ferromagnetic systems that are subject to slowly changing external magnetic fields very commonly show avalanche-like responses. This leads to hysteresis, as avalanches occur over a very fast timescale resulting in irreversibility and entropy production.

A large amount of experimental and theoretical work has been devoted to understanding aspects of this behavior, such as Barkhausen noise, which demonstrates that there is often a large degree of reproducibility in the mesoscopic dynamics on repeated cycling of the field and also interesting critical properties. With advances in experimental techniques, direct tests of the reproducibility of magnetic memory have been undertaken recently which highlighted the prominent role of sample disorder.

The theoretical treatments to date have largely relied on simplified models such as the Ising model to understand these complex systems. The dynamics of such models have been purely relaxational, the extreme limit of large damping, whereby a spin is flipped if the energy of the system is decreased by doing so, the excess energy being transferred out of spin degrees of freedom, to for example, phonons. These models have had a great deal of success in describing many features of disorder ferromagnets, showing fascinating properties, for instance “Return Point Memory” (RPM). With advances in experimental techniques, direct tests of the reproducibility of magnetic memory have been undertaken recently which highlighted the prominent role of sample disorder.

However real magnets are typically dominated by precessional effects on short enough time scales. The Landau-Lifshitz-Gilbert (LLG) equation, contains a precessional term and a dissipative one

\[ \frac{ds}{dt} = -s \times (B - \gamma s \times B), \]  

where \( s \) is a microscopic magnetic moment, \( B \) is the local effective field, and \( \gamma \) is a damping coefficient. \( \gamma \) measures the relative importance of damping to precession. It ranges from about .01 to 1 in real materials. Therefore in many magnetic materials, there should be an interesting short time regime where it makes sense to regard the damping as a perturbation. In fact, we will show that there is a short time scale in which the magnetic response to the applied field change is strongly influenced by the finite level of damping in real materials. Furthermore, even macroscopic and long-time properties, such as hysteresis loops are influenced, by the level of damping and other materials properties, that we shall explain in more detail below.

Throughout this paper, we only consider the case of zero thermal noise. This is because we will see that effective finite temperature behavior is found even in this case and we want to carefully separate out these two effects.

If we adiabatically lower the external field, taken to be in the \( z \) direction, at some point the system will go unstable and have an avalanche. This will involve the nonlinear and possibly chaotic motion of its spins that interact through ferromagnetic and dipolar interactions.

We will first analyze what happens during the avalanche when we set \( \gamma \) to zero. This will describe the dynamics of the system for short time scales. In this case, the dynamics conserve energy. If the dynamics are sufficiently nonlinear, we might expect the system to be ergodic, which means that its equilibrium behavior is well described by the microcanonical ensemble. In this case, the avalanched state is not static but is one of a system at a finite temperature, and we will see that this temperature can be quite high, even above the ferromagnetic transition temperature. However there are two reasons why ergodicity may break down. The first being if the Hamiltonian has strict rotational symmetry about an axis leading to conservation of angular momentum in that direction.

The second reason why ergodicity may be broken is more subtle and to our knowledge, this is the first time a scenario of this kind has been proposed. After the system initiates an avalanche, energy is transmitted into neighboring spins, some of which will be in the form of spin waves. This will propagate energy away from the avalanche region which will decrease the temperature of the avalanche spins, implying that as time progresses, the avalanche region becomes cooler. So if neighboring spins are not recruited, the avalanche will be extinguished. In this sense, the spin waves act as a damping term even if there is no damping in the LLG equation.
For long times, still assuming $\gamma = 0$, the energy will be distributed in all the degrees of freedom of the entire system, which means in the limit of infinite system size, the temperature of the system will have dropped back down to zero. One is then left with a system that has produced only a sub-system-size avalanche and has got trapped in another local minimum.

Therefore we propose that an avalanche that is initially large enough, will propagate through the whole system causing it to go into a state of statistical mechanical equilibrium, often at a high temperature (for $\gamma = 0$). If the initial avalanche is small, the avalanche will usually die out instead, leading to only a finite number of spins changing the sign of $s_z$.

We now turn to two dimensional numerical experiments to support these claims and study the case of finite damping. Most real experiments on avalanche dynamics have been effectively two dimensional \(^\text{[3]}\). Dipolar forces were not included as they complicate the analysis by adding an additional parameter. Their effects will be the subject of future work.

We consider a Hamiltonian that couples nearest neighbor spins on a two dimensional square lattice and contains an anisotropy term where the orientation of the easy axis is randomized slightly about the $z$-axis and there is disorder in the ferromagnetic coupling.

$$\mathcal{H} = -\sum_{<i,j>} J_{ij} s_i \cdot s_j - \alpha \sum_i (s_i \cdot \hat{n}_i)^2 - B_{ext} \sum_i s_{i,z} \quad (2)$$

where the $J_{ij}$’s are the ferromagnetic coupling constants drawn from a uniform distribution with nonzero positive mean. $\alpha$ is a measure of the anisotropy. We choose the $\hat{n}_i$ to be random but biased towards the $z$-axis (out of plane). See ref. \(^\text{[3]}\) for details. This system is placed in an external field $B_{ext}$.

The system was started at high field and the field was lowered adiabatically by evolving the system at fixed $B_{ext}$ until, to a high accuracy, there was no further change in spin variables. To obtain convergence the damping was made finite, $\gamma = 1$. After this the field was lowered again. An avalanche was defined to occur when the maximum $s_z$ among all the spins changed by a finite amount $\Delta = 1$. At that point, a successive approximation scheme was initiated to find the precise field at which the transition takes place, further evolution can then proceed using the same procedure. When an avalanche of desired size was detected, the system was restarted with the same external field and pre-avalanche configuration but now with a different value of damping and the evolution of the system was recorded.

A common scenario is to find that the whole system will avalanche for sufficiently low damping, but will have a sub-system-size avalanche when the damping is above some critical threshold, that depends on the precise configuration right before the avalanche. This is demonstrated for a $128 \times 128$ spin system in Fig. \(^\text{[4]}\). At $\gamma = 0.9$, the post-avalanche cluster of avalanched spins has an approximate diameter of 8 lattice spacings. However at $\gamma = 0.8$, the whole system avalanches. Fig. \(^\text{[4]}\) shows greyscale images of the motion of the system for $\gamma = 0.8$ (a), and $\gamma = 0.01$ (b), during their avalanches. The intensity is proportional to $|ds/dt|$. In both cases, the system was started in the same configuration right before the avalanche. The motion of the system is confined to the cluster’s surface for $\gamma = 0.8$ but is spread out for $\gamma = 0.01$ in a ring-like structure, in which the magnetic system has an elevated effective temperature. In the case of no damping, many islands in front of the avalanche’s main boundary form and this elevated temperature range encompasses the entire avalanche region. After completion of the system-size avalanche, the entire system continues to move indefinitely in an ergodic phase.

From these observations, one expects the hysteresis loop to change as one varies the damping in these simulations. This has been verified directly.

For $\gamma = 0$, the highly chaotic phase is energy conserving and if there are no other invariants of motion, then the system should be well described by a microcanonical ensemble. For a large system, this ensemble is well known to be equivalent to the canonical ensemble (at finite temperature). We verified by direct simulation of $32 \times 32$ size systems that this was the case by measuring static and dynamic correlation functions, for example, $\langle s_i(0) \cdot s_j(t) \rangle$.

The finite temperature of the avalanche is often higher than the transition temperature. It is high because the pre-avalanche metastable configuration has an energy much higher than the minimum $T = 0$ state. When the system gets out of its trapped static configuration it therefore has a lot of excess energy. By considering the Ising model with low disorder and estimating the critical external field for avalanches to take place, we find that the system can have an energy close to zero, which is consistent with our numerical results of a high post-avalanche temperature \(^\text{[11]}\).

We next turn to the mechanism by which this ergodic region spreads. As mentioned above, the effective temperature of the ergodic phase is quite high, with large amplitude motion over a short time-scale. The pre-avalanche configuration is static and when these two re-
regions are connected together, there will be energy transfer between them. One would expect that over a large scale, Fourier’s law should hold, so that that the temperature in the ergodic region will heat up the metastable region. Given thermal energy, the metastable region now has the opportunity to tunnel into the stable phase.

We will now construct a simple one dimensional model that attempts to capture the above physics. We use a variable \( \phi_i \) to denote if site \( i \) is part of an ergodic region, \( \phi_i = 1 \), or metastable region, \( \phi_i = 0 \). There is a temperature field, which starts off being zero in the metastable region and a non-zero constant \( T_0 \) for the ergodic sites that seed the avalanche. This temperature corresponds to the energy released per spin when it becomes part of the ergodic region. The equation

\[
\frac{\partial T}{\partial t} = D \nabla^2 T + T_0 \frac{\partial \phi_i}{\partial t} - \nu T
\]  

(3)

describes thermal diffusion with diffusion coefficient \( D \), but adds a source term when the region becomes ergodic. In this case, \( \nabla^2 \) is a discretized second derivative, \( \propto T_{i+1} - 2T_i + T_{i-1} \). We have also included for the sake of generality, a last term, \( \nu T \), which is related to the damping in the system. This gives a time scale for the temperature to die out. For example in fig. 1(a), large \( \nu \) corresponds to high temperature only on the surface of the cluster, whereas for low \( \nu \), fig. 1(b), it persists over a fairly thick surface layer. However the case of \( \nu = 0 \) (i.e. no damping) will be the focus of study below.

The probability per unit time that \( \phi_i \) will go from 0 to 1 is \( r(t) \), which we can take for example to be of the Arrhenius form \( r_A = \nu_0 \exp(-1/T_i) \), where \( T_i \) is the (dimensionless) temperature on site \( i \).

Together these define a simple model for the disappearance of the metastable region. We are now in a position to analyze under what circumstances an avalanche propagates and when it dies out. We find that for sufficiently large \( T_0 \) and initial width \( w_0 \), of the ergodic seeded region, the avalanche propagates indefinitely, but dies out if these two quantities are too small.

As the avalanche propagates, the temperature at the surface will be \( T_0 \) implying that the temperature in the interior will be almost constant with the same value. As the temperature diffuses to sites with \( \phi = 0 \) , there is a finite probability of them tunneling to \( \phi = 1 \).

To estimate the boundary as a function \( T_0 \) and width \( w \), we assume that for small \( T \) that \( r(T) \) is a rapidly increasing function of \( T \). If the initial ergodic region is a top hat function at temperature \( T_0 \), then for large \( w \), there is a long time when the temperature field next to the boundary will approach \( T_0/2 \). This time \( \tau \) will scale as \( w^2 \). Therefore the probability \( p \) that a site next to the boundary will change to \( \phi = 1 \) should equal \( r(T_0/2)w^2 \) for \( p < < 1 \). If it does not succeed in tunneling during this time \( \tau \), the avalanche will die out, otherwise it will continue to propagate. As the avalanche spreads, \( w \) increases meaning that \( p \) increases so that it is easier to seed new sites. Therefore we expect the requirement for an avalanche to propagate is that

\[
w_0 = \frac{c}{r^{1/2}(T_0/2)}
\]  

(4)

where \( c \) is a constant. We have checked this numerically for the case of Arrhenius tunneling function \( r = r_A \), as mentioned above. The results are shown in fig. 2. The plus symbols are the 50 percent probabilities of an avalanche propagating indefinitely, below the line, they will die out. The solid line is a best fit for the constant \( c \) in \( w_0 = c \exp(1/T_0) \). Given the simplicity of the estimate, the agreement is excellent.

Local one dimensional models of avalanches with randomness will often not show a transition to propagation because there is a finite probability that at some time it will encounter conditions causing extinction. The difference here is that the temperature field widens with increasing time. So if a site adjacent to the ergodic region fails to tunnel, in the mean tunnel time, the temperature field will take longer to die away with increasing \( w \), and therefore the probability of extinction rapidly goes to zero as the width increases, again for \( \nu = 0 \).

Therefore for a zero temperature magnetic system with no spin damping, we expect that an initial disturbance will likely propagate if its size is above some threshold value, causing a transition between a static configuration and one at some effective finite temperature. Of course, if there is any coupling to degrees of freedom other than spin, so that \( \gamma > 0 \), the motion will die out and the spins will stop moving. In such a case, the avalanched spins will lose their energy and cool down to zero temperature. The system is locally being annealed at a finite rate.

One might expect that the inclusion of more realistic non-relaxational dynamics would not influence critical properties, because the ergodic region has a finite length scale for \( \gamma > 0 \). If \( \gamma \) is small, one would expect these effects to shrink the critical regime. However there is also a possibility that the critical behavior would be altered, and this deserves closer scrutiny. Work on spin systems, with a conserved order parameter (e.g. the Heisenberg

![FIG. 2: (Color Online) The boundary between avalanche propagation and extinction for the one dimensional model discussed in the text. The “+” symbols are the numerical values and the solid line is an analytical fit to the data.](image_url)
model without disorder) has shown that precessional motion is relevant to the dynamic critical behavior in equilibrium. Also, it is interesting to note that the few experiments available on hysteresis criticality do not seem to agree quantitatively with Random Field Ising Model calculations based upon relaxational dynamics.

We now consider the issue of RPM. For the proof of it to be valid, a no passing rule must be satisfied related to earlier work on charge density waves. If there are two configurations with the spins in the first all more negative than in the second, then continued evolution under the same field will not guarantee that the second set of spins will remain more negative, as is required by this no passing rule. This is because the motion of the spins is effectively thermal and fluctuations can flip a spin in the first system above the value of the first system. This is very different than the relaxational dynamics needed to give RPM, where this can never happen. On the other hand, over a large enough scale, it may be unlikely that a coarse grained variable will violate RPM but nevertheless it is possible for this to occur.

For the simplified model described above eqn. finite damping \( \nu > 0 \) can be analyzed. In this case it is quite similar to heat balance models for explosive crystallization, which show many interesting properties. Instead of the temperature field at the front widening indefinitely, it should be of finite width, leading to a finite probability, per unit time, of the avalanche dying out. Thus one expects that in one dimension, propagation will always terminate eventually. In higher dimensions, because the surface area of the front is increasing, we do expect to see infinite sized avalanches.

Their are many interesting further questions to investigate. If the total perpendicular angular momentum is only weakly broken by interactions, then what effect does this have on the dynamics? How does precessional motion effect the size of the critical region of avalanche dynamics? What is the effect of including disorder on the simplified model, eqn. And last, can these considerations be extended to better understand avalanches in granular media?

In conclusion, we have shown that avalanches and hysteretic behavior in spin systems are strongly influenced by precession and the strength of damping, where small damping makes a system more prone to large avalanches. However even with no damping term, coupling to spin waves can lead to the termination of avalanches. For finite but small damping, the growth front becomes spread out over a large region, for which the spins inside can be described, for short times, by an ergodic system at high temperature and, for longer times, slowly anneal to a low temperature state.

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[1] H.Barkhausen, Z. Phys. 20, 401 (1919); P.J. Cote and L.V. Meisel, Phys. Rev. Lett. 67 1334 (1991); J.P. Sethna, K.A. Dahmen and C.R. Myers Nature 410 242(2001); B. Alessandro, C. Beatrice, G. Bertotti, and A. Montorsi, J. Appl. Phys. 68 2901 (1990); ibid. 68 2908 (1990); J.S. Urbach, R.C. Madison, and J.T. Markert, Phys. Rev. Lett. 75 276 (1995); O. Narayan, Phys. Rev. Lett. 77 3855 (1996); S. Zapperi, P. Cizeau, G. Durin, and H.E. Stanley, Phys. Rev. B 58 6353 (1998); G. Durin and S. Zapperi, Phys. Rev. Lett. 84 4705 (2000).

[2] J.P. Sethna, K. Dahmen, S. Kartha, J.A. Krumhansl, B.W. Roberts and J.D. Shore, Phys. Rev. Lett. 70, 3347 (1993).

[3] J.P. Sethna, K.A. Dahmen and O. Perkovic, cond-mat/0406320 (2005).

[4] T. M.S. Pierce, C.R. Buechler, L.B. Sorensen, E.A. Jagla, J.M. Deutsch, T. Mai, O. Narayan, J.J. Turner, S.D. Kevan, K.M. Chesnel, J.B. Kortright, O. Hellwig, E. E. Fullerton, J.E. Davies, K. Liu, and H. Durn. Phys. Rev. Lett. 94, 017202 (2005).

[5] F.H. de Leeuw, R. van den Doel and U. Enz, Rep. Prog. Phys. 43, 689 (1980).

[6] Q. Peng and H.N. Bertram, J. Appl. Phys. 81, 4384 (1997); A. Lyberatos, G. Ju, R.J.M. van de Veerdonk, and D. Weller, J. Appl. Phys. 91, 236 (2002).

[7] S.K. Ma and G.F. Mazenko, Phys. Rev. B 11 4077 (1975).

[8] In practice, disorder and dipole interactions will break this, albeit weakly in some cases.

[9] J.M. Deutsch and T. Mai Phys. Rev. E, 72, 016115 (2005).

[10] If the energy were positive this would correspond to a negative temperature where correlations should become antiferromagnetic. We have searched for this interesting regime but have not found any systems displaying this behavior.

[11] E. M. Purcell and R. V. Pound, Phys. Rev. 81 279 (1951).

[12] There is an additional length scale associated with the lattice constant. Our analysis assumes that initially, the next spin flip comes from the site nearest the boundary. That is not always the case, because reducing the lattice constant sufficiently will also allow other sites to avalanche. In this regime the correct scaling is \( r(T_0/2)w^3 = \text{constant} \).

[13] A. Berger, A. Inomata, J. S. Jiang, J. E. Pearson, and S. D. Bader, Phys. Rev. Lett. 85 4176 (2000); J. Marcos, E. Vives, L. Manosa, M. Acet, E. Duman, M. Morin, V. Novak, and Antoni Planes, Phys. Rev. B 67 224406 (2003).

[14] A.A. Middleton and D.S. Fisher, Phys. Rev. B, 47 3530 (1993).

[15] R.B. Gold, J.F. Gibbons, T.J. Magee, J. Peng, R. Ormond, R. Ormond, V.R. Deline, and C.A. Evans, in Laser and Electron Beam Processing of Materials, edited by C.W. White and P.S. Peercy (Academic, New York, 1980), p. 221.

[16] W. van Saarloos and J.D. Weeks, Phys. Rev. Lett. 51 1046 (1983).

[17] M. Karttunen, N. Provatas, T. Ala-Nissila, and M. Grant, J. Stat. Phys. 90 1401 (1998).