The existence of a quark gluon plasma phase in which quarks and gluons are weakly interacting degrees of freedom at temperatures \( T > \Lambda \) is suggested by the renormalization group flow of the effective QCD coupling constant and the analogy with the plasma phase of QED. Lattice simulations indicate that this phase transition is first order and for pure \( SU(2) \) occurs at \( T^2 \sim 2.1 \times \) string tension \( \Lambda \). Although QCD is asymptotically free, the perturbative analysis of the high-temperature phase is plagued by infrared (IR) divergences \( \delta \). The best one can presently achieve perturbatively at high temperatures is a resummation of the infrared-safe contributions \( \delta \). The situation is somewhat embarrassing, since one naively might hope that an asymptotically free theory allows for an accurate perturbative description of the high temperature phase. The IR-problem encountered in the perturbative high temperature expansion in fact is part of the more general problem of defining a non-abelian SU(2) connection \( \tilde{A}_\mu = (W^1_\mu, W^2_\mu, A_\mu) \) in Euclidean space is uniquely specified by the BRST algebra. A critical continuum model that are defined by the same BRST algebra is perhaps of some interest.

We will see that a certain SL(2,R) symmetry associated with this gauge fixing is spontaneously broken by ghost-antighost condensation at arbitrarily small coupling \( \epsilon \). The corresponding massless Goldstone states form a BRST-quartet and do not contribute to physical quantities such as the free energy. Screening “masses” in a certain sense thus arise naturally in an \( SU(2) \) gauge theory in covariant Abelian gauges. They cure the IR-problem of the perturbative skeleton expansion for the free energy.

The critical continuum action of the lattice model \( \mathcal{L} \) in Euclidean space is uniquely specified by the BRST algebra, the field content and power counting. Decomposing the non-abelian \( SU(2) \) connection \( \tilde{A}_\mu = (W^1_\mu, W^2_\mu, A_\mu) \) in terms of two real vector bosons (or one complex one) and a U(1)-connection \( A_\mu \) (the “photon” of the model), the loop expansion is defined by the Lagrangian,

\[
\mathcal{L} = \mathcal{L}_{\text{inv.}} + \mathcal{L}_{\text{AG}} + \mathcal{L}_{\text{aGF}} .
\]

Here \( \mathcal{L}_{\text{inv.}} \) is the usual \( SU(2) \)-invariant Lagrangian written in terms of the vector bosons and the photon \( \overline{W} \),

\[
\mathcal{L}_{\text{inv.}} = \mathcal{L}_{\text{matter}} + \frac{1}{4} (G_{\mu
u} G^{\mu\nu} + G^a_{\mu
u} G^a_{\mu\nu}) ,
\]

with

\[
G_{\mu
u} = \partial_\mu A_\nu - \partial_\nu A_\mu - g \epsilon^{ab} W^a_\mu W^b_\nu
\]

\[
G^a_{\mu
u} = D^a_\mu W^b_\nu - D^a_\nu W^b_\mu
\]

\[
= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \epsilon^{ab} (A_\mu W^b_\nu - A_\nu W^b_\mu) .
\]

\( \mathcal{L}_{\text{AG}} \) partially gauge-fixes to the maximal Abelian subgroup \( U(1) \) of \( SU(2) \) in a covariant manner,

\[
\mathcal{L}_{\text{AG}} = \frac{F^a F^a}{2\alpha} - \bar{c}^a M^{ab} c^b - g^2 \alpha (\bar{c}^a \epsilon^{ab} c^b)^2 ,
\]

where \( \alpha \) is a parameter.

Latin indices take values in \( \{1, 2\} \) only, Einstein’s summation convention applies and \( \epsilon^{12} = -\epsilon^{21} = 1 \), vanishing otherwise. All results are in the \( \overline{\text{MS}} \) renormalization scheme.

The non-abelian infrared divergences in the perturbative high-temperature phase is a resummation of the infrared-safe contributions \( \delta \). The best one can presently achieve perturbatively at high temperatures \( \delta \) is plagued by infrared (IR) divergences \( \delta \). The corresponding massless Goldstone states form a BRST-quartet and do not contribute to physical quantities such as the free energy. Screening “masses” in a certain sense thus arise naturally in an \( SU(2) \) gauge theory in covariant Abelian gauges. They cure the IR-problem of the perturbative skeleton expansion for the free energy.
with
\[
F^a = D^a_{\mu} W^b_{\mu} = \partial_{\mu} W^b_{\mu} + g A_{\mu} \varepsilon^{ab} W^b_{\mu},
\]
\[
M^{ab} = D^{ac}_{\mu} D^{b}_{\mu} + g^2 \varepsilon^{ac} \varepsilon^{bd} W^a_{\mu} W^d_{\mu}.
\] (5)

Note that \(L_{U(1)} = L_{\text{inv.}} + L_{\text{AC}}\) is invariant under \(U(1)\)-gauge transformations and under an on-shell BRST symmetry \(s\) and anti-BRST symmetry \(\bar{s}\), whose action on the fields is
\[
s A_{\mu} = g \varepsilon^{ab} c^b W^a_{\mu}, \quad s W^a_{\mu} = D^a_{\mu} \bar{c}^b, \quad s c^a = 0, \quad sc^a = F^a / \alpha,
\] (6)

with an obvious extension to include matter fields. Contrary to most other proposals for mass generation \[3\], the BRST algebra Eq. (6) closes on-shell on the set of \(U(1)-\)invariant functionals: on functionals that depend only on \(W, A, c\) and the matter fields, \(s^2\) for instance effects an infinitesimal \(U(1)\)-transformation with the parameter \(\frac{1}{2} \varepsilon^{ab} c^b\). The algebra Eq. (6) thus defines an equivariant cohomology and ensures the perturbative renormalizability and unitarity \[4\] of the model. Note that the physical sector comprises states created by composite operators of \(A, W\) and the matter fields in the equivariant cohomology of \(s\) or \(\bar{s}\). They are BRST closed and \(U(1)\)-invariant.

The corresponding equivariant (anti-) BRST symmetry of the LGT is valid also non-perturbatively and it was shown \[3\] that expectation values of physical observables of the \(U(1)\)-LGT are the same as those of the original \(SU(2)\)-LGT for any \(\alpha > 0\). Note that formally setting \(\alpha = 0\) and solving the constraint \(F^a = 0\) as in \[1\] is not the same as taking the limit \(\alpha \to 0\). The reason is inherently non-perturbative and nicely exhibited by the lattice calculation \[3\]: without the quartic ghost interaction, Gribov copies of a configuration conspire to give vanishing expectation values for all physical observables. No matter how small, the quartic ghost interaction is required to have a normalizable partition function and expectation values of physical observables that are identical with those of the original \(SU(2)\)-LGT.

\(L_{\text{GF}}\) in Eq. (1) has been added “by hand” to fix the remaining \(U(1)\) gauge invariance and define the perturbative series of the continuum model unambiguously. I will assume a conventional covariant gauge-fixing term,
\[
L_{\text{aGF}} = \frac{\left(\partial_{\mu} A_{\mu}\right)^2}{2 \xi}.
\] (7)

However, none of the following conclusions depend on the gauge-fixing of the Abelian subgroup – they in particular do not depend on \(\xi\).

The Lagrangian Eq. (1) also is invariant under a global bosonic \(SL(2, \mathbb{R})\) symmetry generated by
\[
\Pi^+ = \int \bar{c}^a(x) \frac{\delta}{\delta c^a(x)}, \quad \Pi^- = \int c^a(x) \frac{\delta}{\delta \bar{c}^a(x)}.
\] (8)

and the ghost number \(\Pi = [\Pi^+, \Pi^-]\). This \(SL(2, \mathbb{R})\) symmetry is is also realized in the lattice regularized model \[3\] and not anomalous. The conserved currents corresponding to \(\Pi^\pm\) are \(U(1)\)-invariant and BRST, respectively anti-BRST exact,
\[
\Pi^+ = c^a D^a_{\mu} c^b, \quad \Pi^- = \bar{c}^a D^a_{\mu} \bar{c}^b = \bar{s} \varepsilon^a W^a_{\mu}.
\] (9)

I will argue that this global \(SL(2, \mathbb{R})\) symmetry of the model is spontaneously broken to the noncompact abelian subgroup generated by the ghost number \(\Pi\). Because the currents Eqs. (3) are \(\Pi\)-BRST exact, a spontaneously broken \(SL(2, \mathbb{R})\) symmetry is accompanied by a BRST-quartet of massless Goldstone states with ghost numbers \(2, 1, -1\) and \(-2\). They are \(U(1)\)-invariant \(c - c, c - W, \bar{c} - W\) and \(c - \bar{c}\) bound states. It is important to note that BRST quartets do not contribute to physical quantities \[1\] such as the free energy. The spontaneous symmetry breaking in this sense is similar to a (dynamical) Higgs mechanism in the adjoint.

An order parameter for the spontaneous breaking of the \(SL(2, \mathbb{R})\) symmetry is
\[
\langle \varepsilon^{ab} c^b \rangle = \frac{1}{2} \langle \Pi^- (\varepsilon^{ab} c^b) \rangle = -\frac{1}{2} \langle \Pi^+ (\varepsilon^{ab} c^b) \rangle.
\] (10)

To perturbatively investigate the consequences of \(\langle \varepsilon^{ab} c^b \rangle \neq 0\), the quartic ghost interaction in Eq. (1) is linearized using an auxiliary scalar field \(\rho(x)\) of canonical dimension two. Adding the quadratic term
\[
\mathcal{L}_{\text{aux}} = \frac{1}{2g^2} (\rho - g^2 \lambda \varepsilon^{ab} c^b)^2
\] (11)
to the Lagrangian of Eq. (1), the tree level quartic ghost interaction vanishes at \(\lambda^2 = \alpha\) and is then formally of \(O(g^4)\), proportional to the difference \(Z^2 - Z_\alpha\) of the renormalization constants of the two couplings \[1\].

We shall see that the perturbative expansion about a non-trivial solution \(\rho = v \neq 0\) to the gap equation
\[
\frac{v}{g^2} = \sqrt{\alpha} \langle \varepsilon^a(x) \varepsilon^{ab} c^b(x) \rangle \big|_{\rho = v},
\] (12)

is much better behaved in the infrared. Note that Eq. (12) is \(U(1)\)-invariant and therefore does not depend on the \(U(1)\) gauge-fixing Eq. (7). Let us for the moment assume that a unique non-trivial solution to Eq. (12) exists in some gauge \(\alpha\); we return to this conjecture below. The consequences for the IR-behavior of the model are dramatic. Defining the quantum part \(\sigma(x)\) of the auxiliary scalar \(\rho\) by

\[2\] This is analogous to the decoupling of the Goldstone quartets of the weak interaction in \(R_\xi\) gauges \[1\].

\[3\] The discrete symmetry \(c^a \to \bar{c}^a, \bar{c}^a \to -\bar{c}^a, \rho \to -\rho\) relating \(s\) and \(\bar{s}\) also ensures that \(\rho\) only mixes with \(\bar{c}^a \varepsilon^{ab} c^b\).
the momentum representation of the Euclidean ghost propagator at tree level becomes
\[
\langle \phi^a \phi^b \rangle_p = \frac{g^2 \delta^{ab} + \sqrt{\alpha} \varepsilon^{ab}}{p^2 + \alpha v^2}.
\]

Feynman’s parameterization of this propagator allows an evaluation of loop integrals using dimensional regularization that is only slightly more complicated than usual. More importantly, the ghost propagator is regular at Euclidean momenta when \( v \neq 0 \). Its complex conjugate poles at \( p^2 = \pm \sqrt{\alpha} v^2 \) can furthermore not be interpreted as due to asymptotic ghost states \([12]\).

When \( v \neq 0 \), the W-boson is massless only at tree level and (see Fig. 1) acquires the finite mass \( m_W^2 = g^2 \sqrt{\alpha} v^2 / (16 \pi) \) at one loop,
\[
\Gamma_W^a \Gamma_W^b = \frac{g^2 \sqrt{\alpha} v^2}{16 \pi} \delta_{ab} \delta_{\mu
u} \delta^{ab}.
\]

Fig. 1. The finite one-loop contribution to the W mass.

Technically, the one-loop contribution is finite because the integral in Eq. (14) involves only the \( g^0 \)-part of the ghost propagator Eq. (14). Since \( p^2 / (p^4 + \alpha v^2) = \alpha v^2 / (p^2 + \alpha v^2) + 1 / p^2 \), the \( v \)-dependence of the loop integral is IR- and UV-finite. The quadratic UV-divergence of the 1\( / p^2 \) subtraction at \( v = 0 \) is canceled by the other, \( v \)-independent, quadratically divergent one-loop contributions – (in dimensional regularization this scale-invariant integral vanishes by itself). \( m_W^2 \) furthermore is positive due to the overall minus sign of the ghost loop. The sign of \( m_W^2 \) is crucial, for it indicates that the model is stable and (as far as the loop expansion is concerned) does not develop tachyonic poles at Euclidean \( p^2 \) for \( v \neq 0 \). Conceptually, the local mass term proportional to \( \delta_{ab} \delta^{ab} \) is finite due to the BRST symmetry Eq. (1), which excludes a mass counter-term. The latter argument implies that contributions to \( m_W^2 \) are finite to all orders of the loop expansion.

\[
\frac{1}{g^2} + \sigma
\]

Fig. 2. \( \Gamma_{\sigma \sigma}(v, p^2) \) to order \( g^0 \).

If the model is stable at \( v \neq 0 \), the 1PI 2-point function \( \Gamma_{\sigma \sigma}(v, p^2) \) of the scalar must not vanish at Euclidean \( p^2 \) either. To order \( g^0 \), \( \Gamma_{\sigma \sigma}(p^2) \) is given by the 1\( / g^2 \) term that arises from Eq. (11) upon substitution of Eq. (13) and the one-ghost-loop contribution shown in Fig. 2. Since a non-trivial solution to the gap equation Eq. (12) relates \( 1 / g^2 \) to a loop integral of zeroth order in the coupling, we may use Eq. (12) to lowest order to obtain a “tree-level” expression for \( \Gamma_{\sigma \sigma}(v, p^2) \) of order \( g^0 \). Evaluating the loop integrals, one obtains the real, positive and monotonic function
\[
\Gamma_{\sigma \sigma}(x := \frac{\sqrt{\alpha} v^2}{p^2}) = \left\{ -1 + 2 \sqrt{1 - 4 \alpha} \coth(\sqrt{1 - 4 \alpha} x) \right\} \frac{32 \pi^2 \alpha^{-1}}{2} + \{ x \to -x \}.
\]

The anomalous dimension \( \gamma_v \) of the expectation value is simultaneously found to be
\[
\gamma_v = -\frac{d \ln Z_v}{d \ln \mu} = \frac{g^2}{16 \pi^2} (2\alpha - \beta_0) + O(g^4),
\]
where \( \beta_0 \) is the lowest order coefficient of the \( \beta \)-function of this model \( (\beta_0 = (22 - 2n_f) / 3 \) with \( n_f \) quark flavors in the fundamental representation as matter).

Using the relation between \( \mu, g^2 \) and the asymptotic scale parameter \( \Lambda_{\text{MS}} \), we may rewrite Eq. (17) as
\[
\ln \frac{\alpha v^2}{\mu^4} = \frac{16 \pi^2}{g^2} \left( \frac{2}{\beta_0} - \frac{1}{\alpha} \right) + 2 + O(\ln g, g^2).
\]

Apart from an anomalous dimension, the non-trivial solution \( v \) at sufficiently small coupling is thus proportional to the physical scale \( \Lambda_{\text{MS}} \) in the particular gauge \( \alpha = \beta_0 / 2 \). The anomalous dimension \( \gamma_v \) in Eq. (18) is furthermore of order \( g^4 \) at \( \alpha = \beta_0 / 2 \). The terms of order \( \ln g \) in Eq. (18) thus also vanish in this particular gauge and higher order corrections to the asymptotic value of
\( v^2 = \frac{2}{\beta_0} e^2 \Lambda^4_{MS} (1 + O(g^2)) \) 

(20)

and determine the \( O(g^2) \) corrections in Eq. (20) order by order in the loop expansion of the gap equation Eq. (12). At \( \alpha = \beta_0/2 \) the lowest order solution Eq. (20) remains accurate to order \( g^2 \) at any finite order of the loop expansion. This does not imply that other gauges are any less physical, but it does single out \( \alpha = \beta_0/2 \) as the gauge in which a perturbative evaluation of the gap equation Eq. (12) is consistent at sufficiently small values of \( g^2 \). (In QED the hydrogen spectrum to lowest order is \( \propto g^2/\alpha \) at one loop thus lead to a term of order \( g/\alpha \) at one loop is proportional to \( 3g^2/\alpha \).)

Gauge dependent interaction terms proportional to \( g/\alpha \) at one loop thus lead to a term of order \( g^2/\alpha^2 \) in the longitudinal part of the \( W \) self-energy only. The transverse part of the \( W \) self-energy is regular in the limit \( \alpha \to 0 \). Taking \( \alpha \) to vanish thus is rather tricky: Eq. (21) implies that the longitudinal part of the \( W \)-propagator at one loop is proportional to \( 3g^2p^2 \ln(p^2) \) at large momenta and no longer vanishes in this limit. Higher order loop corrections similarly contribute to the longitudinal propagator as \( \alpha \to 0 \). \( \gamma_W \) does not depend on the gauge parameter \( \xi \) at one loop, due to an Abelian Ward identity that also gives the QED-like relation \( Z_A = Z_g^{-2} = Z_\xi \) between the renormalization constants of the photon, of the coupling \( g \) and of the gauge parameter \( \xi \).

The anomalous dimension of the gauge parameter \( \alpha \) at sufficiently small \( g^2 \) is negative for positive values of \( \alpha \) when \( \beta_0 < 6 + 2\sqrt{3} \). With \( \gamma_\alpha < 0 \), the effective gauge parameter tends to decrease at higher renormalization scales \( \mu \) and direct integration of Eq. (21) gives a vanishing \( \alpha \) at a finite value of the coupling \( g^2 \). As already noted above, the loop expansion, however, is valid only if \( g^2 / \mu^2 \ll 1 \) and \( g^2 / \alpha \ll 1 \). But Eq. (21) does show that there is no \textit{finite} UV fixed point for the gauge parameter and that \( \alpha \) effectively vanishes at least as fast as \( g^2 \) as \( \mu \to \infty \) for any gauge at finite \( g^2 \). Eq. (20) nevertheless is the asymptotic solution to Eq. (12) in the sense that it is valid at arbitrary small coupling if one chooses the gauge at that coupling to be \( \alpha(g) = \beta_0/2 \). This is compatible with the asymptotic vanishing of the effective gauge parameter only if higher order corrections lead to an anomalous dimension \( \gamma_\alpha \) that effectively remains of order \( g^2 \) even as \( g, \alpha(g) \to 0 \). Since the gauge sector becomes strongly coupled when \( g^2/\alpha(g) \to 0 \), and the loop expansion does not give the correct behavior of Eq. (18) in this limit, this is at least conceivable.

Let me finally say that the non-trivial solution to the gap equation apparently persists to arbitrarily high temperatures. The (unique) non-trivial solution at \( T = 0 \) is a consequence of the scale anomaly \( \gamma_\alpha \) and the Goldstone quartet of the spontaneously broken \( \text{SL}(2,\mathbb{R}) \) symmetry does not contribute to the free energy. The renormalization point dependence of Eq. (17) and the associated UV-divergence of the loop integral are an indication of this. The character of the solution to Eq. (12) does, however, change dramatically with temperature \( T \). At low temperatures \( v(T) \) deviates only marginally from Eq. (20), whereas at high temperatures \( v(T \sim \infty) \propto T^2/\ln^2(T/\Lambda) \).

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