Simulation of pension plan supplemental cost based on withdrawal rate interpolation and different benefit

Endang Sri Kresnawati and Yulia Resti*
Jurusan Matematika Fakultas MIPA Universitas Sriwijaya, Jl. Raya Palembang-Prabumulih Km.32 Inderalaya 30662, Ogan Ilir, Sumatera Selatan, Indonesia

Email: *yulia_resti@mipa.unsri.ac.id

Abstract. One factor that affects the supplemental cost is the probability that a participant will survive in employment to the retirement age. The probability depends on the mortality rate, the disability rate, the withdrawal rate, and the normal retirement rate. All of the rates are presented in one-year interval except the withdrawal rate. This paper uses the cubic spline interpolation to get the withdrawal rates at one year intervals, whereactuarial calculation method used is Projected Unit Credit (PUC), and pension benefits are calculated using two assumptions, namely the final average and the carrier accrued salary. The interpolation results show that first, the probability of withdrawal at the age of 25 for participants entering the pension program at the age of 20 differs from ages of 21, 22, 23, and 24. Likewise the probability of withdrawal at 30 years of age for participants entering a pension program at the age of 25 is different from ages 26, 27, 28 and 29. The same applies to other ages. The younger the age enters the pension program, the greater the unfunded liabilities that can arise. The unfunded liabilities for the final salary benefit assumption is smaller than the carrier average salary benefit assumption. The unfunded liabilities for entry at a 1-year interval are greater than 5-year intervals.

1. Introduction
The Projected Unit Credit (PUC) method is one of actuarial method in pension funding. Contribution of participants in the PUC method consists of normal cost (NC) and supplementary cost (SC). NC is the difference between accrued liability at the beginning and end of the year, while SC is an amortization of accrual liability that are not funded. SC appears because the present value of benefits is greater than the present value of the normal cost, and can arise either before the establishment of the system or after the system starts. Before establishment of the system, SC can arise due to actuarial cost assumptions, whereas after the system starts, SC can arise due to changes in benefits, changes in actuarial assumptions, or actuarial losses.

The SC is influenced by the interest rate, the benefit, and the probability that a participant will survive in employment to the retirement age. Simulation of SC as well as simulation of actuarial liability (AL) from various perspectives and methods are useful for pension funds in making their investment decisions. Stock is one of the investment instruments chosen by many pension funds. According to [3], significant factor that influence return on investment on stock is Bank Indonesia (BI) interest rates. [2] who uses fixed interest rates and market interest rates to simulate AL for Netherlands Pension Fund show that the values of AL are not different significantly. The different SC can be found using the different benefit assumptions, namely 1) the final salary at one year before retirement, 2) the average salary since entering the pension program until reaching retirement age.
The probability that a participant will survive in employment to the retirement age probability depends on the mortality rate, the disability rate, the withdrawal rate, and the normal retirement rate. All the rates are given in a one-year age unless the withdrawal rate is given at a five-year interval. The presentation of the rate in a one-year age can provide a more detailed calculation in pension funds, especially for simulation of SC. Interpolation can be used to obtain the withdrawal rate in a one-year age. The purpose of this paper is to make a simulation of SC based on the different benefit and the withdrawal rate interpolation in a one-year age. This simulation results can provide an overview of the effects of the entry age - withdrawal rate in the pension program on SC that have not been done by [1] and [2].

2. Methodology

Let $C_n$ be the benefit coefficient of addition costs for payment of unfunded liabilities at $n$, $b_x$ be the amount of benefits received by participants at age $x$, $v^{r-x}$ be the present value of the interest rate $i$ during the participation period $(r - x)$, $r_x P_x^{(T)}$ be the probability that a participant age $x$ will survive in employment to age $r$, and $\bar{a}_r$ be the lifetime annuity since a pensioner aged $r$. The supplemental cost of pension funds for the participant who valuated at age $x$ for retirement at age $r$ for the PUC methods is defined as ([6]),

$$(SC_n)_x = C_n b_x v^{r-x} r_x P_x^{(T)} \bar{a}_r$$

where

$$C_n = \frac{B_x}{B_r - B_x}$$ (2)

$$b_x = \frac{B_x}{r - x}$$ (3)

$$v^{r-x} = (1 + i)^{-(r-x)}$$ (4)

$$r_x P_x^{(T)} = \prod_{t=0}^{r-x} (1 - q_x^{(m)}(t) (1 - q_x^{(d)}(t)) (1 - q_x^{(w)}(t)) (1 - q_x^{(r)}(t))$$ (5)

$$\bar{a}_r = \sum_{t=0}^{\infty} t P_r^{(m)} v^t = \sum_{t=0}^{\infty} \left(1 - t q_r^{(m)} \right) v^t$$ (6)

In this paper, supplemental cost simulations are based on 2 benefit calculation assumptions, namely the final salary at age $(r - 1)$, and the average salary during work $(r - y)$ years (carrier average salary).

Let the final salary at age $(r - 1)$ is denoted as $s_{r-1}$. For proportion of $k$, the benefit based on the final salary is defined as,

$$B_r = k s_{r-1} (r - y)$$ (7)

For the salary at age $x$ is $s_x$, the assumption of salary increase per year is $j$, the salary at age $(x + t)$ is defined as,

$$s_{x+t} = s_x (1 + j)^t$$ (8)

for $t = 1, 2, \cdots, r - x - 1$. 
Let the average salary during active work \((r - y)\) years (the carrier average salary) is \(\bar{S}_{r-y}\) formulated as,

\[
\bar{S}_{r-y} = \frac{1}{r - y} [s_y + \cdots + s_x + s_{x+1} + \cdots + s_{r-1}]
\]  

(9)

So the benefit based on the average salary is defined as,

\[
B_r = \bar{S}_{r-y}(r - y)
\]  

(10)

Let the final average salary be formulated as

\[
\bar{F}_s = \frac{s_{r-1} + s_{r-2} + \cdots + s_{r-n}}{n}
\]  

(11)

So the benefit based on the final average salary is defined as,

\[
B_r = k \bar{F}_s(r - y)
\]  

(12)

Each of \(q_{x+t}^{(w)}\) in (5) is obtained from a corresponding table which is presented at 1-year interval except the table of withdrawal. The table presents the probability of withdrawal for participants entering the pension program at a certain age at 5-year intervals.

Suppose the participant enters the pension plan at the age of \(y_s\) and resigns at the age of \(x_s\) with the probability of withdrawal \(q_{x_s}^{(w)}\), where \(q_{x_s}^{(w)} = q_{x_t}^{(w)}\). In the withdrawal table as presented in Table 1, the value of \(q_{x_s}^{(w)}\) is only available for \(y\) at 5-year intervals, i.e. when \(y_s = 20, 25, 30, \cdots\).

| \(x\) | 20  | 25  | 30  | 35  | 40  | 45  | 50  |
|------|-----|-----|-----|-----|-----|-----|-----|
| 20   | 0.2431 | 0.2245 | 0.2071 | 0.1908 | 0.1757 | 0.1616 | 0.1486 |
| 21   | 0.2071 | 0.2245 | 0.2431 | 0.2245 | 0.2071 | 0.1908 | 0.1757 |
| 22   | 0.1908 | 0.1757 | 0.1908 | 0.1757 | 0.1506 | 0.1365 | 0.1232 |
| 23   | 0.1757 | 0.1506 | 0.1757 | 0.1506 | 0.1506 | 0.1506 | 0.1506 |
| 24   | 0.1616 | 0.1506 | 0.1616 | 0.1506 | 0.1506 | 0.1506 | 0.1506 |
| 25   | 0.1486 | 0.1365 | 0.1486 | 0.1365 | 0.1365 | 0.1365 | 0.1365 |
| 26   | 0.1365 | 0.1232 | 0.1365 | 0.1232 | 0.1232 | 0.1232 | 0.1232 |
| 27   | 0.1232 | 0.1100 | 0.1232 | 0.1100 | 0.1100 | 0.1100 | 0.1100 |

The values of \(q_{x_s}^{(w)}\) for 1-year intervals, i.e. when \(y_s = 21, \cdots, 24, 26, \cdots, 29, \cdots, 49\) can be obtained using the cubic spline interpolation which is formulated as a solution of the following linear equation system,

\[
a_0 + a_1 x + a_2 x^2 + a_3 x^3 = q_{x_s}^{(w)}
\]  

(13)
For $= 1,2,3,4$, Equation (13) can be written as,

\begin{align*}
q_1(x) &= a_0 + a_1 x_1 + a_2 x_1^2 + a_3 x_1^3 = q_1^{('w)} \\
a_0 + a_1 x_2 + a_2 x_2^2 + a_3 x_2^3 = q_2^{('w)} \\
a_0 + a_1 x_3 + a_2 x_3^2 + a_3 x_3^3 = q_3^{('w)} \\
a_0 + a_1 x_4 + a_2 x_4^2 + a_3 x_4^3 = q_4^{('w)}
\end{align*}

Simulation in this paper is based on the retirement age is 56 years, the salary when the initial period of participation is Rp 3,000,000 per month, an interest rate of 7.2587 % (interest rate average of Bank Indonesia in years 2006-2017), a salary growth of 8% and unfunded liabilities are evaluated at the age of 50 years.

3. Result and Discussion

3.1. Interpolation for withdrawal rate $q_{x_s}^{('w)}$

Table 2 presents the cubic spline interpolation results for 1-year interval of $q_{x_s}^{('w)}$. The values are the solutions of (12) for $x = 21, 22, 23, \ldots, 54$ when $y = 21, \ldots, 29, \ldots, 49$. In Table 1, the probability of withdrawal at the age of 25 for participants entering the pension program at the age of 20, is the same as the participant who enters the pension program at the ages of 21, 22, 23, 24 and 25. The interpolation results shown in Table 2 show that the probability of withdrawal at the age of 25 years for participants entering the pension program at the age of 20 is different from the ages of 21, 22, 23, 24, and 25.

| $x$ | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | \ldots | 50 |
|-----|----|----|----|----|----|----|----|----|--------|----|
| 20  | 0.2431 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | \ldots | 0 |
| 21  | 0.2245 | 0.2386 | 0 | 0 | 0 | 0 | 0 | 0 | \ldots | 0 |
| 22  | 0.2071 | 0.2126 | 0.2332 | 0 | 0 | 0 | 0 | 0 | \ldots | 0 |
| 23  | 0.1908 | 0.1798 | 0.2018 | 0.2268 | 0 | 0 | 0 | 0 | \ldots | 0 |
| 24  | 0.1757 | 0.1774 | 0.1723 | 0.1920 | 0.2197 | 0 | 0 | 0 | \ldots | 0 |
| 25  | 0.1616 | 0.2133 | 0.1651 | 0.1650 | 0.1831 | 0.2119 | 0 | 0 | \ldots | 0 |
| 26  | 0.1486 | 0.1488 | 0.2042 | 0.1538 | 0.1577 | 0.1749 | 0.2036 | 0 | \ldots | 0 |
| 27  | 0.1365 | 0.1373 | 0.1369 | 0.1957 | 0.1435 | 0.1506 | 0.1673 | 0.1950 | \ldots | 0 |
| \vdots | \vdots | \vdots | \vdots | \vdots | \vdots | \vdots | \vdots | \vdots | \vdots | \vdots |
| 54  | 0.0354 | 0.0238 | 0.0154 | 0.0086 | 0.0034 | 0.0354 | 0.0294 | 0.0223 | \ldots | 0.0345 |

3.2. Simulation of $(SC)_n$.

Simulation of $(SC)_n$ was evaluated for a participant who entire at the age of 21, 22, 23, \ldots, 49 years and resign at the age of 50 years for different benefit calculation assumptions; the proportion of final salary at age $(r - 1)$ and the average salary during work $(r - y)$ years. The simulation results of $(SC)_n$ for 5-years and 1-year interval of $q_{x_s}^{('w)}$ respectively are presented in Table 3. From Table 3, some information is obtained, namely, first, the younger the age enters the pension program, the greater the unfunded liabilities that can arise. Second, the unfunded liabilities for the final salary benefit.
assumption is smaller than the carrier average salary benefit assumption. Third, the unfunded liabilities for entry at a 1-year interval are greater than 5-year intervals.

Table 3. Simulation of \((SC)_{50}\) for different benefit assumptions in 5 and 1-year interval of \(q^{(w)}_{x}\)

| y   | Final Salary 5-year interval | Final Salary 1-year interval | Carrier Average Salary 5-year interval | Carrier Average Salary 1-year interval |
|-----|-----------------------------|-----------------------------|----------------------------------------|----------------------------------------|
| 21  | 186,843,127                 | 194,802,020                 | 315,286,220                            | 328,716,360                            |
| 22  | 171,338,861                 | 179,413,304                 | 281,865,075                            | 295,148,131                            |
| 23  | 106,497,562                 | 107,993,532                 | 251,665,246                            | 259,727,102                            |
| 24  | 97,479,970                  | 98,249,978                  | 198,501,394                            | 200,989,624                            |
| 26  | 93,210,387                  | 95,591,091                  | 238,970,461                            | 245,074,050                            |
| 27  | 85,035,288                  | 88,393,190                  | 221,268,945                            | 230,006,486                            |
| ... | ...                        | ...                        | ...                                    | ...                                    |
| 49  | 906,130                     | 920,960                     | 1,148,235                              | 1,167,028                              |

4. Conclusion
Application the cubic spline interpolation shows that the probability of withdrawal at the age of 25 for participants entering the pension program at the age of 20, is different from the ages of 21, 22, 23, and 24. Likewise for other ages. The younger the age enters the pension program, the greater the unfunded liabilities that can arise. The unfunded liabilities for the final salary benefit assumption is smaller than the carrier average salary benefit assumption. Thus, the unfunded liabilities for entry at a 1-year interval are greater than 5-year intervals.

References
[1] diBartolomeo and Dan 2012 Asset Allocation and Risk Assessment for Pension Schemes Inclusive of Funding Guarantees, 25th Annual Research Conference, Del Mar, CA, August, Modeling Financial Markets Worldwide.
[2] Johannsson and Stefan 2016 Evaluation of pension fund assets and liabilities: Using variable versus fixed discount rates. Thesis, Master of Science in Engineering Management. Reykjavik University, Iceland.
[3] Prakoso and Boentoro 2013 Analisis Pengaruh BI Rate, Return on Asset, dan Return on Equity terhadap Return Saham PT. Tambang Batubara Bukit Asam (Persero) Tbk. Periode 2007-2011.
[4] Widiyanti M H, Taufik and Isnurhadi H 2008 The Influence of Traditional Accounting and Economic Value Added Approaches on Stock Returns of Banks Listed on Jakarta Stock Exchange. Proceedings of The MFA Conference 2008, Parallel Session II (B).
[5] Pension Fund Statistics 2016 Indonesia Financial Services Authority, www.ojk.go.id
[6] Winklevoss H E 1993 Pension Mathematics with Numerical Illustrations, 2nd Edition. Pension Research Council, Wharton School of the University of Pennsylvania and University of Pennsylvania Press, Philadelphia.