The (2+1)-dimensional Gross-Neveu model with a $U(1)$ chiral symmetry at non-zero temperature

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Abstract

We present results from numerical simulations of the (2+1)-dimensional Gross-Neveu model with a $U(1)$ chiral symmetry and $N_f = 4$ fermion species at non-zero temperature. We provide evidence that there are two different chirally symmetric phases, one critical and one with finite correlation length, separated by a Berezinskii-Kosterlitz-Thouless transition. We have also identified a regime above the critical temperature in which the fermions acquire a screening mass even in the absence of chiral symmetry breaking, analogous to the pseudogap behaviour observed in cuprate superconductors.
1 Introduction

Phase transitions in QCD and model field theories have been studied intensively over the last decade both analytically and numerically. Understanding the phase diagram of QCD is becoming increasingly important in view of recent experimental efforts to create and detect the quark-gluon plasma in the relativistic heavy ion collisions at BNL and CERN.

Since the problem of chiral symmetry breaking and its restoration is intrinsically non-perturbative the number of available techniques is limited and most of our knowledge about the phenomenon comes from lattice simulations. Because of the complexity of QCD with dynamical fermions, studies so far have been done on lattices with modest size and in various cases the results are distorted by finite size and discretization effects. The three-dimensional Gross-Neveu model (GNM$_3$) has been proved to be an interesting and tractable model to study chiral phase transitions both numerically by means of lattice simulations and analytically in the form of large-$N_f$ expansions [1–10].

In this letter we present results of numerical simulations of the $U(N_f)$-$V$-invariant GNM$_3$ with a $U(1)$ chiral symmetry at non-zero temperature. This model is described by the following continuum Lagrangian density (we work in Euclidean space throughout):

$$L = \bar{\Psi}_i(\partial \psi + m_0)\Psi_i - \frac{g^2}{2N_f}[(\bar{\Psi}_i \Psi_i)^2 - (\bar{\Psi}_i \gamma_5 \Psi_i)^2].$$  \hspace{1cm} (1)

We treat $\Psi_i$, $\bar{\Psi}_i$ as four-component Dirac spinors and the index $i$ runs over $N_f$ fermion species. In the chiral limit $m_0 \to 0$, the $U(1)$ invariance is given by

$$\Psi \mapsto e^{i\theta \gamma_5} \Psi ; \quad \bar{\Psi} \mapsto \bar{\Psi}e^{i\theta \gamma_5}.$$  \hspace{1cm} (2)

This Lagrangian is considerably easier to treat, both numerically and analytically, if it is bosonized by introducing auxiliary fields $\sigma$ and $\pi$:

$$L = \bar{\Psi}_i(\partial \psi + m_0 + \sigma + i\gamma_5 \pi)\Psi_i + \frac{N_f}{2g^2}(\sigma^2 + \pi^2).$$  \hspace{1cm} (3)

At tree level, the fields $\sigma$ and $\pi$ have no dynamics; they are truly auxiliary fields. However, they acquire dynamical content by dint of quantum effects arising from integrating out the fermions. The model is renormalizable in the $1/N_f$ expansion unlike in the loop expansion.

Apart from the obvious numerical advantages of working with a relatively simple model in a reduced dimensionality there are several other motivations for studying such a model: (i) at $T=0$ for sufficiently strong coupling it exhibits spontaneous chiral symmetry breaking and the pion field $\pi$ is the associated Goldstone boson; (ii) the spectrum of excitations contains both baryons and mesons, i.e. the elementary fermions and the composite fermion–anti-fermion states; (iii) the model has an interacting continuum limit for a critical value of the coupling $1/g_0^2$, which has a numerical value $\approx 1.0/a$ in the large-$N_f$ limit if a lattice regularisation is employed [2]; (iv) numerical simulations of the model with chemical potential $\mu \neq 0$ show qualitatively correct behaviour, unlike QCD simulations [3, 4].

The phase diagram of GNM$_3$ with various global symmetries at non-zero temperature and density has been studied extensively in [3–10]. More specifically [5, 6], it was shown
that the thermally induced phase transition of the $Z_2$-symmetric model belongs to the twodimensional Ising universality class in accordance with the dimensional reduction scenario [11] which predicts that the long-range behaviour at the chiral phase transition is that of the $(d - 1)$ spin model with the same symmetry, because the IR region of the system is dominated by the zero Matsubara mode of the bosonic field. Outside the non-trivial region the critical properties of the system are described by mean field theory according to the large-$N_f$ prediction. It was also shown that the two-dimensional Ising scaling region is suppressed by a factor $N_f^{-1/2}$ [3].

At leading order in $1/N_f$ the $U(1)$ model has a second order chiral phase transition at $T_c = \frac{m_f}{2\ln 2}$ [4], where $m_f$ is the fermion dynamical mass at zero temperature. The leading order effective potential has the same form as the discrete symmetry case with the replacement $\sigma^2 \to \sigma^2 + \pi^2$. This conclusion is expected to be valid only when $N_f$ is strictly infinite, i.e. when the fluctuations of the bosonic fields are neglected, since otherwise it runs foul of the Coleman-Mermin-Wagner (CMW) theorem [12], which states that in two-dimensional systems the continuous chiral symmetry must be manifest for all $T > 0$. Next-to-leading order calculations demonstrated that the symmetry is restored for arbitrarily small $T < 0$ [7–10]. As argued in [4, 11] the model is expected in accordance with the dimensional reduction scenario to undergo a Berezinskii-Kosterlitz-Thouless (BKT) transition [13] at $T_{BKT}$ which is associated with the unbinding of vortices like in the two-dimensional $XY$ model. In terms of the reduced temperature $t \equiv T - T_{BKT}$ the scaling behaviour of the correlation length, susceptibility and the specific heat is given [13] by

$$\xi(t) \sim e^{at^{-\nu}}, \quad \chi(t) \sim \xi^{2-\eta}, \quad C_v \sim \xi^{\alpha/\nu} + \text{constant},$$

where for $t \to 0^+$, $\nu = 1/2$, $\eta = 1/4$ and $\alpha = -d\nu = -1$. It is easier to visualize this scenario if we use the “modulus-phase” parametrization $\sigma + i\pi \equiv \rho e^{i\theta}$. In two spatial dimensions logarithmically divergent infrared fluctuations do not allow the phase $\theta$ to take a fixed direction and therefore prevent spontaneous symmetry breaking via $\langle \theta \rangle \neq 0$. The critical temperature $T_{BKT}$ is expected to separate two different chirally symmetric phases: a low $T$ phase, which is characterized by power law phase correlations $\langle e^{i\theta(x)} e^{-i\theta(0)} \rangle \sim x^{-\eta(T)}$ at distances $x \gg 1/T$ but no long range order (i.e. a spinwave phase where chiral symmetry is “almost but not quite broken”), and a high $T$ phase which is characterized by exponentially decaying phase correlations with no long range order. In other words for $0 \leq T \leq T_{BKT}$ there is a line of critical points characterised by a continuously varying $0 \leq \eta(T) \leq \frac{1}{4}$. However, since the amplitude $\rho$ is neutral under $U(1)$, the dynamics of the low temperature phase do not preclude the generation of a fermion mass $m_f \propto \rho$ whose value may be comparable with the naive prediction of the large-$N_f$ approach; this was demonstrated in $d = (1 + 1)$ in [4], in which results from an exactly soluble fermionic model [15] are generalised to GNM$_2$. The effect may be understood in terms of the spectral function $\rho(s)$. For orthodox mass generation, the spectral function in the broken chiral symmetry phase is a simple pole $\rho(s) \propto \delta(s - m_f^2)$. By contrast in the BKT scenario [4] the function is modified to a branch cut $\rho(s) \propto (\sqrt{s} - m_f)^{-1}\theta(\sqrt{s} - m_f)$; although chiral symmetry is manifest, the propagating fermion constantly emits and absorbs massless scalars and hence has indefinite chirality.

In [10] it is shown that in GNM$_3$ with $T > 0$, next-to-leading order corrections cause fermion mass generation to occur for $T < T_*$, where $T_* > T_{BKT}$, i.e. it predicts a “pseudogap.”
phase in which the system is non-critical, chiral symmetry is manifest, and yet the energy required to create a spin-$\frac{1}{2}$ excitation does not vanish. Such behaviour is interesting because it is characteristic of the underdoped phase of certain cuprate superconductors [10].

We have attempted to check the validity of (a) the BKT scenario, and (b) mass generation without chiral symmetry breaking, by performing lattice simulations. Our results with $N_f = 4$ show that the symmetry is restored for a large range of non-zero temperatures. The susceptibility of the order parameter diverges at a temperature which we associate with $T_{BKT}$. We also verify the prediction of [10] that the fermions have non-zero mass in a region where the system is not critical, i.e. for $T > T_{BKT}$. Our simulations have not shown evidence for a second phase transition to a massless phase at $T = T_*$, as predicted by large-$N_f$ methods [10], although we cannot rule it out.

2 Numerical Simulations

The model in its bosonised form can be formulated on the lattice using the following action:

$$S_{lat} = \sum_{x,y} \bar{\chi}(x) M_{x,y} \chi(y) + \frac{1}{8} \sum_x \bar{\chi}(x) \chi(x) \sum_{\langle \tilde{x}, x \rangle} \sigma(\tilde{x}) + i \epsilon(x) \sum_{\langle \tilde{x}, x \rangle} \pi(\tilde{x}) + \frac{1}{2g^2} \sum_x [\sigma^2(\tilde{x}) + \pi^2(\tilde{x})],$$

(5)

where $\chi$ and $\bar{\chi}$ are Grassmann-valued staggered fermion fields defined on the lattice sites, the auxiliary field $\sigma$ is defined on the dual lattice sites and the symbol $\langle \tilde{x}, x \rangle$ denotes the set of 8 dual lattice sites $\tilde{x}$ surrounding the direct lattice site $x$ [4]. The fermion kinetic operator $M$ is given by

$$M_{x,y} = \frac{1}{2} \left[ \delta_{y,x+\delta} - \delta_{y,x-\delta} \right] + \frac{1}{2} \sum_{\nu=1,2} \eta_\nu(x) \left[ \delta_{y,x+\nu} - \delta_{y,x-\nu} \right],$$

(6)

where $\eta_\nu(x)$ are the Kawamoto-Smit phases $(-1)^{x_0 + \ldots + x_{\nu-1}}$ and the symbol $\epsilon(x)$ denotes the alternating phase $(-1)^{x_0 + x_1 + \ldots + x_2}$. The simulations were performed by using the standard hybrid Monte Carlo algorithm in which complex bosonic pseudofermion fields $\Phi$ are updated using the action $\Phi^i(M \Phi^{\dagger})^{-1} \Phi$. According to the discussion in [4], simulation of $N$ staggered fermions describes $N_f = 4N$ continuum species; the full symmetry of the lattice model in the continuum limit, however, is $U(N_f/2)_V \otimes U(N_f/2)_V \otimes U(1)$ rather than $U(N_f)_V \otimes U(1)$. At non-zero lattice spacing the symmetry group is smaller: $U(N_f/4)_V \otimes U(N_f/4)_V \otimes U(1)$. In this study we have used $N = 1$.

In order to study the behaviour of the chiral symmetry first at $T = 0$ and then for $T > 0$ we set the fermion bare mass to zero. Without the benefit of this symmetry breaking interaction, the direction of symmetry breaking changes over the course of the run so that $\Sigma \equiv \frac{1}{V} \sum_x \sigma(x)$ and $\Pi \equiv \frac{1}{V} \sum_x \pi(x)$ average to zero over the ensemble. It is in this way that the absence of spontaneous breaking of this continuous symmetry on a finite lattice is enforced. The next best thing to measure is an effective “order parameter” $|\Phi| \equiv \sqrt{\Sigma^2 + \Pi^2}$, which is a projection onto the direction of $\Phi^a \equiv (\Sigma, \Pi)$ separately for each configuration [17]. In the case of chiral symmetry breaking $|\Phi|$ differs from the true order parameter $(\Sigma)$ extrapolated to the chiral limit, because in the absence of a symmetry breaking term it
is impossible to disentagle the fluctuations of the latter from those of the Goldstone modes. Including the effects of the latter will give $|\Phi| > \Sigma_0$, where $\Sigma_0$ denotes the value of $\Sigma$ in the chiral limit. The simulations at $T = 0$ were performed on $24^2 \times 36$ lattices for $\beta \equiv 1/g^2 = 0.45 - 0.775$. In Fig.4 we plot $|\Phi|$ and the fermion mass $m_f$ versus $\beta$. We fitted these two observables to the scaling functions $|\Phi| = a_1(\beta_{c}^{\text{bulk}} - \beta)^{\beta_{\text{mag}}}$ and $m_f = a_2(\beta_{c}^{\text{bulk}} - \beta)^{\nu}$ for $\beta = 0.625 - 0.750$. We extract $\nu = 1.08(5)$, $\beta_{\text{mag}} = 1.1(1)$ which are consistent with the values of the large-$N_f$ prediction $\beta = \nu = 1$ [2]. The values of the associated bulk critical couplings are: $\beta_{c}^{\text{bulk}} = 0.87(3)$ from the order parameter fit and $\beta_{c}^{\text{bulk}} = 0.86(1)$ from the fermion mass. At very strong coupling (small $\beta$) the data deviate slightly from the fitted scaling functions due to discretization effects.

In order to study the effects of non-zero temperature we performed simulations on asymmetric lattices with constant temporal extent $L_t = 4$ and spatial extents $L_s = 30, 50, 100, 150$. We vary $T$ by changing $\beta$. At $\beta_{c}^{\text{bulk}}$ the lattice spacing becomes zero and $T \to \infty$. Simulations of the $Z_2$ model with $L_t = 4$ show a thermally induced phase transition at $\beta_{c}^{Z_2} = 0.565(3)$, which should be compared with the value $\beta_{c} = 0.76$ expected in the $N_f \to \infty$ limit on a lattice with $L_t = 4$. For $L_s = 100$ and 150 we accumulated approximately 30,000 to 45,000 trajectories for the range of couplings $\beta \leq 0.54$ and 10,000 to 20,000 for larger values of $\beta$. The trajectory length was approximately 1.5 and was chosen at random from a Poisson distribution in order to decrease autocorrelation times. For $L_s = 30$ and 150 we chose a trajectory length of approximately 1.0 and accumulated 40,000 to 50,000 trajectories for $\beta \leq 0.54$ and 20,000 to 30,000 for larger values of $\beta$. In Fig.3 we plot $|\Phi|$ versus $\beta$ for the various lattice sizes together with the results from simulations of the $Z_2$ model on lattice sizes $4 \times 50^2$ and $4 \times 100^2$. It is clear that the order parameter of the $Z_2$ model is independent of the lattice size until just before the transition at $\beta = \beta_{c}^{Z_2}$, whereas in the $U(1)$ model $|\Phi|$ has a strong size dependence for a large range of values of $\beta$, i.e. it decreases rapidly as the spatial volume increases in accordance with the expectation that chiral symmetry should be restored for $T > 0$. We believe that the signal for chiral symmetry breaking at very low $T$ is a result of finite size corrections. The finite spatial extent $L_s$ provides a cut-off for the divergent correlation length and according to the BKT scenario the slow decay of the correlation function $\langle e^{i\theta(x)} e^{-i\theta(0)} \rangle$ with exponent $\eta(T) < 0.25$ for $T < T_{BKT}$ ensures a non-zero magnetization even in a system with very large size. In other words on small lattices the spinwave phase is expected to look like a broken phase. A similar effect has been observed in the two-dimensional Gross-Neveu model with a discrete symmetry at non-zero $T$ [3]: For large-$N_f$ the density of kinks (whose condensation is responsible for the restoration of chiral symmetry at $T > 0$) is exponentially suppressed and therefore on lattices with spatial extent smaller than the kink size the system looks as if it is in the broken phase.

In Fig.3 we plot the susceptibility of the order parameter

$$\chi = V(\langle|\Phi|^2\rangle - \langle|\Phi|\rangle^2),$$

measured on lattices with spatial size $L_s = 30, 50, 100, 150$. It is clear that $\chi$ has a peak for $L_s = 30, 50, 100$ at $\beta_{BKT}(L_s) \simeq 0.54, 0.53, 0.515$ respectively. The values of $\chi$ near the $L_s = 150$ peak are very noisy and we don’t plot them in this graph. This is clear evidence of a phase transition, with a critical coupling $\beta_{BKT}$ significantly less than $\beta_{c}^{Z_2}$. The transition occurs at a much lower temperature than the critical temperature of the GNM$_3$ with a $Z_2$
symmetry, because the infrared fluctuations are stronger in the continuous symmetry case.

Another interesting observation is that in the low temperature phase the susceptibility has stronger size dependence than in the high temperature phase and the error bars at low $T$ are much larger than at high $T$ despite the fact that the statistics at low $T$ are much larger than at high $T$. This is evidence that the system is critical in the low $T$ phase in accordance with the BKT scenario.

Another useful quantity to measure is the so-called specific heat $C_v$, which we calculated from the fluctuations of the bosonic action $S_b = \frac{1}{2} \sum_x [\sigma^2(x) + \pi^2(x)]$.

$C_v$ is given by

$$C_v = \frac{\beta^2}{V} \left( \langle S_b^2 \rangle - \langle S_b \rangle^2 \right),$$

and is plotted versus coupling for $L_s = 50, 100, 150$ in Fig. 4. It is clear that (a) $C_v$ has a broad peak at $\beta \simeq 0.50$ and (b) it does not show any divergent behaviour or significant finite size effects which is consistent with the BKT scenario, according to which $\alpha = -2$.

Analogous behaviour has been observed in numerical studies of the $XY$ model [19].

Now we discuss the issue of fermion mass generation. Since the temporal direction of our lattices is short, in order to study the asymptotic behaviour of the fermion propagator we have chosen to measure the spatial fermion correlator $G_s(x)$ along one of the spatial lattice axes, which yields information on spatial correlations in the thermal medium rather than the spectrum of excitations. The quantity analogous to mass is the so-called screening mass $M_s$; for free, massless continuum fermions this quantity is given by the lowest Matsubara mode for fermions, ie. $M_s = M_0^{\text{cont}} = \pi T$. On a finite temporal lattice this becomes $M_0^{\text{lat}} = \sinh^{-1} [\sin(\pi/L_t)]$. For $L_t = 4$ the screening mass in the absence of dynamical mass generation is $M_0^{\text{lat}} \approx 0.658$. If a dynamical mass $M$ is generated, then naively we expect

$$M_s^2 = M_0^2 + M^2.$$

We calculated the screening masses from the exponential decay of the spatial correlation functions using an ansatz for the fit which is motivated by the form of the free propagators [20]:

$$G_s(x) = A(1 - (-1)^x) \sinh(M_s(x - L_s/2)) + B(1 + (-1)^x) \cosh(M_s(x - L_s/2)).$$

The screening masses extracted from the simulations of GNM$_3$ with $U(1)$ symmetry on lattices with varying $L_s$, together with the results of the $Z_2$ model from simulations on a $4 \times 100^2$ lattice, are shown in Fig. 5. It is clear that the fermions have non-zero screening mass $M$ in the sense that $M_s > M_0$, even when the order parameter is very small. The screening mass is independent of $L_s$ for almost all values of $\beta$ – in particular we observe that $M_s^{U(1)}$ is comparable with $M_s^{Z_2}$ for $\beta < \beta_c^{Z_2}$. In the $Z_2$ case we ascribe mass generation to orthodox chiral symmetry breaking. For $U(1)$ the situation is more subtle, as shown in Fig. 5 where we plot the spatial fermion correlator of both models on a $4 \times 100^2$ lattice at $\beta = 0.51$. The $Z_2$ data are non-vanishing on all timeslices, consistent with the standard scenario of mass generation via broken chiral symmetry. By contrast, the vanishing of $G_s(x)^{U(1)}$ for even $x$, corresponding to $B = 0$ in (10), signals a manifest chiral symmetry, which follows because the $U(1)$ symmetry of staggered fermions implies that the only non-vanishing elements of the propagator are $G_{eo}$ and $G_{oe}$. By comparing Fig. 4 with Fig. 5 we infer that the fermions
are massive, i.e. $M > 0$, in the phase where both chiral symmetry is restored and the system is not critical, i.e. the susceptibility is finite and size independent. This implies the existence of a pseudogap phase for $\beta \lesssim \beta_{BKT} \lesssim \beta_{c2}$.

Strictly speaking, in demonstrating that $M_s > M_0$ we have not excluded the possibility that the relation (9) could be replaced by $M_s^2 \simeq A(T)M_0^2 + M^2$ with $A - 1 \gg M^2/M_0^2$, implying a temperature-dependent modification of the speed of light rather than mass generation [21]. The similarity of $M_s^{U(1)}$ and $M_s^{Z_2}$ at low $T$ where $A \approx 1$ suggests this is unlikely to be the dominant effect for $\beta \lesssim \beta_{c2}$; however for $\beta \gtrsim 0.56$ the two models begin to diverge, and the level ordering switches. Since the departure from the free-field result in this regime is greater for the $U(1)$ model (which has twice as many scalar fields) as for the $Z_2$ model, it is conceivable that the effect is due to in-medium modifications of the massless fermion propagator, and could be calculable in a large-$N_f$ framework. To understand the physics of this regime the two effects need to be disentangled via a more detailed study of the dispersion relation for the spatial correlator, i.e. the value of $M_s$ for several distinct Matsubara modes. This will require simulations closer to the continuum limit, i.e. on lattices with $L_t \geq 8$. For this reason we cannot at this stage identify a second phase transition at $\beta = \beta_s$ where mass generation switches off. It is possible that such a transition may be an artifact of the large-$N_f$ approach [10].

3 Summary and Outlook

Our numerical study of the $U(1)$-symmetric GNM$_3$ provided evidence for: (a) existence of two chirally symmetric phases separated by a transition which is possibly a BKT transition in agreement with the dimensional reduction scenario; (b) dynamical mass generation without symmetry breaking in accordance with the prediction of the large-$N_f$ calculation and (c) the existence of a pseudogap phase where the system is not critical (i.e. $T > T_{BKT}$) and the fermions acquire a non-zero mass. Our results are another example of what was shown by Witten [14] in GNM$_2$ with a $U(1)$ symmetry at $T = 0$, i.e. that the large-$N_f$ expansion is a reliable guide to the properties of the model as long as one interprets the results carefully. In [14] it was shown that although large-$N_f$ predicts spontaneous symmetry breaking which is in contradiction with the CMW theorem, the scalar field correlator falls off like $x^{-1/N_f}$, i.e. there is an “almost long range order” together with fermion mass generation which does not break chiral symmetry because the physical fermion is a superposition of positive and negative chirality states and has zero net chirality. The results of [4], where it was shown that for GNM$_3$ with a $Z_2$ chiral symmetry at non-zero $T$ the region (in the space of $m/T$) of the large-$N_f$ behaviour squeezes out the two-dimensional Ising region by a factor $N_f^{-1/2}$, also support the spirit of Witten’s statement.

We are currently extending our work in various directions. We wish

- to increase the statistics near and below the transition in order to perform finite size scaling to extract the exponents $\eta(T)$.
- to repeat the simulations with $L_t = 8$ in order to study the fermion dispersion relation.
• to study the $N_f$-dependence of our various results, which can be compared with analytic predictions [10].

We also plan to study the GNM$_3$ with an $SU(2)_L \otimes SU(2)_R$ chiral symmetry at non-zero temperature. According to the dimensional reduction scenario this model’s critical properties at non-zero $T$ should be the same as the two-dimensional spin model with an $O(4)$ symmetry, i.e. it is expected to have a transition at $T = 0^+$, and to have dynamical mass generation for $T > 0$.

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Figure 1: Order parameter $|\Phi|$ and fermion mass $m_f$ at $T = 0$ vs. coupling $\beta$ on a $24^2 \times 36$ lattice.

Figure 2: Order parameter $|\Phi|$ vs. $\beta$ for $L_t = 4$ for both $U(1)$ and $Z_2$ symmetric models.
Figure 3: Susceptibility $\chi$ vs. $\beta$.

Figure 4: Specific heat $C_v$ vs. $\beta$. 
Figure 5: Fermion screening mass $M_s$ vs. $\beta$. The horizontal line shows the lowest Matsubara mode $M^{\text{lat}}_0$.

Figure 6: Fermion spatial correlators $G_s(x)$ extracted from simulations of $U(1)$- and $Z_2$-symmetric GNM$_3$ on $4 \times 100^2$ lattices at $\beta = 0.51$. 

