Thermodynamical properties of the Undulant Universe

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Abstract

Recent observations show that our universe is accelerating by dark energy, so it is important to investigate the thermodynamical properties of it. The Undulant Universe is a model with equation of state $\omega(a) = -\cos(b \ln a)$ for dark energy. Due to the term, $\ln a$, neither the event horizon nor the particle horizon exists in the Undulant Universe. However, the apparent horizon is a good holographic screen, and can be seen as a boundary of keeping thermodynamical properties. The Universe is in thermal equilibrium inside the apparent horizon, i.e. the Unified First Law and the Generalized Second Law of thermodynamics are confirmed. As a thermodynamical whole, the evolution of the Undulant Universe behaves very well in the current phase. However, considering the unification theory and the real universe, the failure of conversation law indicates that the model of the Undulant Universe needs further consideration.

\textit{Key words:}

cosmology, the Undulant Universe, dark energy, thermodynamics

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1 Introduction

Many cosmological observations (1; 2; 3; 4), such as the type Ia Supernova (SN Ia), Wilkinson Microwave Anisotropy Probe (WMAP), the Sloan Digital Sky Survey (SDSS) etc. support that our Universe is experiencing an accelerated expansion, and dark energy (DE) with negative pressure, contributes about 72% of the matter content to the present Universe. For DE, we have the equation of state (EOS) $P_{\Lambda} = \omega \rho_{\Lambda}$. However, the nature of DE is known so little. In order to get more knowledge of DE, many researchers have discussed it from thermodynamical perspective widely. Such as thermodynamics of DE with constant $\omega$ in the range $-1 < \omega < -1/3$ (5), $\omega = -1$ in the de Sitter spacetime and anti-de Sitter spacetime (6), $\omega < -1$ in the Phantom field (7; 8), and $\omega = \omega_0 + \omega_1 z$ (9) and so on. More discussions on the thermodynamics of DE can be found in (10; 11; 12; 13; 14). In this paper, we also investigate a model from the thermodynamical perspective to find some more interesting properties of DE. G. 't Hooft found that the 3-dimensional world is an image of data. The data can be stored on a 2-dimensional holographic film, and the information on the projection is called hologram (29). Later, Fischler and Susskind applied holography to the standard cosmological context (30), and considered the world as a hologram (31). The entire information about the global 3+1-dimensional spacetime can be stored on particular hypersurfaces (called holographic screens). In other words, the degrees of freedom of a spatial region reside on the surface of the region. The total number of the degrees of freedom does not exceed the Bekenstein-Hawking bound (32), which is a universal entropy bound within a given weakly gravitational volume. The holographic principle is helpful to understand the DE.

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Usually we treat the Universe as a whole, and a global spacetime. So some conclusions on BH can be used to the Universe. Hawking temperature, BH entropy, and BH mass satisfy the First Law of thermodynamics \( dM = TdS \) (21). In terms of the definition of the rationalized area (22), if a charged BH is rotating, solving \( dM \), we get \( dM = (\kappa/8\pi)dA + \Omega_0 dJ + V_0 dQ \). Where \( \kappa, A, \Omega_0, J, V_0, Q \) are the surface gravity, area, dragged velocity, angular momentum, electric potential and charge of a BH respectively. This expression is analogous with the First Law of thermodynamics \( dE = TdS - PdV \). This law suggests a connection between thermodynamics and BH physics in general, and between entropy and BH area in particular. Bekenstein conjectured these analysis at first, and Hawking discovered that the BH can emit particles according to the Planck spectrum. So we get the effective temperature on the horizon of the BH, \( T = \kappa/2\pi \) (23), and the entropy, \( S = A/4 \) (24). Then we can study many gravitational systems in the framework of thermodynamics. Consequently, our Universe can be considered as a thermodynamical system (25; 26; 27; 28), and the thermodynamical properties of BH can be generalized to spacetime enveloped by the cosmological horizon. In other words, the thermodynamical laws should be satisfied in the Universe.

We use the Planck units \( c = G = k_B = \hbar = 1 \), where \( G \) is Newton’s constant, \( \hbar \) is Planck’s constant, \( c \) is the speed of light, and \( k_B \) is Boltzmann’s constant respectively. The Planck units of energy density, mass, temperature, and other quantities are converted to CGS units.

This paper is organized as follows: In Sec. 2, we review the introduction to the Undulant Universe. In Sec. 3, we discuss the cosmological horizons of the Undulant Universe. In Sec. 4 and Sec. 5, we study the Unified First Law of thermodynamics and the Generalized Second Law of thermodynamics on the apparent horizon respectively. In Sec. 6, we present conclusions and discus-
Our homogeneous and isotropic Universe follows the dynamics of an expanding Robertson-Walker (RW) spacetime ($i, j = 1, 2$):

$$ds^2 = g_{ij}dx^i dx^j + r^* d\Omega^2 = dt^2 - a^2(t)(\frac{dr^2}{1 - \kappa r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2).$$ (1)

Where we choose a class of comoving coordinate system $(t, r, \theta, \phi)$ for RW metric, $x^i = (t, r)$ are the arbitrary coordinates spanning the radial 2-spheres $(\theta, \phi) = \text{constant}$, $a(t)$ is the expansion scale factor of the Universe, $r^* = ar$, is defined in the usual way from the proper area of a 2-spheres $x^i = \text{constant}$: $\mathcal{A} = 4\pi r^{*2}$, and $d\Omega$ is the metric of 2-dimensional unit sphere respectively.

The spatial hypersurfaces of the Friedmann Universe have positive, zero and negative curvatures for $\kappa = +1, 0$ and $-1$. And its metric is:

$$g_{ij} = \begin{pmatrix} -1 & 0 \\ 0 & \frac{a^2}{1 - \kappa r^2} \end{pmatrix}.$$ (2)

The evolution of the Universe is governed by the Friedmann equation:

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho - \frac{\kappa}{a^2}.$$ (3)

Where $H$ is the Hubble parameter, the energy density $\rho$ is a sum of different components: $\rho = \sum_i \rho_i$, evolving differently as the Universe expands. To characterize the energy density of each component, we define density parameter:

$$\Omega_i = \frac{\rho_i}{\rho_c}.$$ (4)
where the critical density:

$$\rho_c = \frac{3H^2}{8\pi G}. \quad (5)$$

Observational evidence, extended by detailed observations \(^{(15)}\), presents a flat Universe. The mass-energy of the Universe includes 0.05 ordinary matter and 0.22 nonbaryonic dark matter, and is dominated by DE. The densities in matter and vacuum are of the same order of magnitude. The phase of the Universe transferred from radiation dominated to matter dominated in the past, and recently, the vacuum energy is dominated at $z \sim 1.5$. In other words, if the vacuum energy becomes dominated at any other epoch, the Universe would have evolved completely different history. To solve this cosmic coincidence problem, many workers put forward many ideas \(^{(16; 17)}\), but all of them involve fine tuning problem in some way. \(^{(18)}\)

To avoid these problems, we investigate Undulant Universe characterized by alternating periods of acceleration and deceleration \(^{(19)}\), with an EOS:

$$\omega(a) = -\cos(b \ln a), \quad (6)$$

and the Hubble parameter:

$$H^2(a) = H_0^2(\Omega_{M0}a^{-3} + \Omega_{\Lambda0}a^{-3}\exp[\frac{3}{b}\sin(b \ln a)]). \quad (7)$$

Where $\Omega_{M0}, \, \Omega_{\Lambda0}$ are the density parameter of matter and DE respectively, and the dimensionless parameter $b$ controls the frequency of the accelerating epochs. Note that in the limit of small values of $b$, the EOS above approaches the cosmological constant, $\omega \sim -1$. As discussed in \(^{(20)}\), the best fit values for the parameters $b = 0.06 \pm 0.01$, so we choose $b = 0.05$ in the following discussion. From FIG.1, we can see the evolution of $\omega$ for the Undulant Uni-
verse. To demonstrate the nowadays Universe being not particular clearly, we choose two periods, three periods, and four periods respectively on large range of scales to discuss in the following discussion.

At different epochs, $\omega$ behaves in the same way that oscillates as a cosine function, and the average of $\omega$ over one period is zero. Consequently, the cosmic coincidence problem is solved, and no fine tuning is required. The range of possible destinies for the Universe, even in the near case, is very broad indeed. The Universe needn’t evolve toward the cataclysm of terminal inflation or recollapse necessarily, but might go on with the sedate drift of a big slink.

3 cosmological horizons of the Undulant Universe

In general, global holographic screens are highly non-unique (34). We concentrate on a class of Universes with well-defined holographic screens of finite area. There are many kinds of horizons for the boundary (35), and we will
discuss some of them below.

We concentrate on the quasi-local of the cosmological horizon. Because the DE dominated Universe is asymptotic flatness at infinity, in the asymptotic regions the matter is diluted. Although there can be significant matter in the interior of the spacetime, we can assume that there just DE contributes to the energy on the quasi-local of the cosmological horizon, i.e. $\Omega_0^\Lambda = 1$, $\Omega_0^M = 0$. So the Hubble parameter becomes:

$$H^2(a) = H_0^2a^{-3}\exp\left[\frac{3}{b}\sin(b\ln a)\right].$$

There are two quite different horizon concepts in cosmology, to which cosmologists have devoted their attention at various times. Firstly, for a global observer, the radius of an event horizon (EH) at cosmic time $t$ can be written as:

$$R_E = a(t)\int_t^{t_f}\frac{dt}{a(t)}.$$  \hspace{1cm} (9)

When $\int_t^{t_f} \frac{dt}{a(t)} < \infty$. Where $t_f$ is the final cosmic time, and generally, $t_f \to +\infty$. Consequently, for the Undulant Universe:

$$\int_t^{t_f} \frac{dt}{a(t)} = \int_{a}^{+\infty} \frac{da}{a^2H} = \int_{a}^{+\infty} \frac{da}{H_0a^{0.5}\exp\left[\frac{3}{b}\sin(b\ln a)\right]},$$

Eq.(12) $\to +\infty$, which results from the oscillating term, $\ln(a = \infty) \to +\infty$, so sine term has not a certain value, and the integral is infinity. We can understand it intuitively. Because we can’t ensure the final phase of the Universe, the radius is unknown. Different final phase can evolve a different history, in other words, our recent Universe can evolve into any phase in the future. Secondly, the radius of a particle horizon (PH) at cosmic time $t$ can be written
as:

\[ R_P = a(t) \int_{t_i}^{t} \frac{dt}{a(t)}. \]  

(11)

When \( \int_{t_i}^{t} \frac{dt}{a(t)} < \infty \). Where \( t_i \) is the initial cosmic time, and generally, \( t_i \to -\infty \). For the Undulant Universe:

\[ \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{0}^{a} \frac{da}{a^2H} = \int_{0}^{a} \frac{da}{H_0 a^{0.5} \exp\left[\frac{1.5}{b} \sin(b \ln a)\right]}. \]  

(12)

Eq. (14) \( \to \infty \). Similarly, the integral is unknown because of the uncertainty of \( \ln(a = 0) \). We cannot give a certain PH, i.e. any phase in the past can evolve into our recent Universe. According to the discussion above, we find the Undulant Universe really remove the coincidence problem, and the nowadays phase is not particular any more. Whatever the phase is in the past or in the future, our Universe can be the recent phase.

However, we have to find some other horizons to study the thermodynamical properties of the Undulant Universe. Hawking and Ellis define an apparent horizon (AH), which is the outer boundary of a connected component of a trapped region. This 2-dimensional AH can necessarily be extended to a 3-dimensional, time-evolved AH over some finite range of \( t \) (45). The AH is local in time, and it is easy to locate in the numerical relativity. To simplify the calculation, we define the physical radius as the form of Eq.(3):

\[ r^* = ar, \]  

(13)

the hypersurface can be set as:

\[ f = g^{ij} r^*_i r^*_j. \]  

(14)
For the AH, \( f = 0 \). Then we get the radius of an AH is

\[
r_A^* = a r_A = H_0^{-1} a^{1.5} \exp\left[-\frac{1.5}{b} \sin(b \ln a)\right].
\] (15)

The radius depends on the details of the matter distribution in the Universe. In generic situation, the AH evolves with time and visibility of the outside antitrapped region depends on the time development of the AH. Because there is no EH, the spatial region outside of the horizon at a given time might be observed. The change of the radius varying with time during a Hubble time is:

\[
t_H \frac{d \ln r_A^*}{dt} = 1.5 [\cos(b \ln a) - 1].
\] (16)

We notice that from the FIG.2, the AH displays an oscillating while ascending trend, and there is remarkable ascending trend without periodic variation for any period in \( \omega \). Furthermore, the ascending trends are similar at different epochs. Namely, our Universe is not particular in the cosmological evolution. FIG.3 shows that the AH does not change significantly over one Hubble scale, so the equilibrium thermodynamics still can be applied here.

4 The Unified First Law of thermodynamics on the apparent horizon

As a thermodynamical system, the Unified First Law of thermodynamics \( dE = TdS - PdV \) on the AH of the Undulant Universe should be satisfied \( (36) \). Especially, the work term \( PdV \) should be zero at infinity \( (37) \).

Now we calculate the flux of energy through the proper area of a 2-spheres \( x^0 = t, x^1 = r^* \), where the time \( t \) is allowed to vary slowly, so, \( t \) can be treated
Fig. 2. The radius of an AH in units of $H_0^{-1}$ as function of $a$ at different epochs, corresponding to three different $\omega$ in FIG.1 respectively.

Fig. 3. The change of the radius of an AH $t_H d \ln r_A^* / dt$ varies with time during a Hubble time at different epochs, corresponding to three different $\omega$ in FIG.1 respectively.

as constant. The surface gravity on the AH is

$$\kappa = -f'/2 \big|_{r=r_A^*} = 1/r_A^*,$$

(17)

so the Hawking temperature is

$$T_A = \kappa/2\pi = 1/2\pi r_A^*.$$

(18)
The amount of energy flux (38) crossing the AH within the time interval \( dt \) is

\[
dE_A = 4\pi r_A^* T^{\mu\nu} u_\mu u_\nu dt = 4\pi r_A^* (\rho + P) dt \\
= 1.5H_0^{-1} a^{0.5}[1 - \cos(b \ln a)] \\
\times \exp[-\frac{1.5}{b} \sin(b \ln a)] da.
\]  

(19)

The entropy on the AH is

\[
S_A = A/4 = \pi r_A^2,
\]  

(20)

and its differential form is

\[
dS_A = 2\pi r_A^* dr_A^*.
\]  

(21)

Thus we can obtain

\[
T_A dS_A = dr_A^* \\
= 1.5H_0^{-1} a^{0.5}[1 - \cos(b \ln a)] \\
\times \exp[-\frac{1.5}{b} \sin(b \ln a)] da.
\]  

(22)

Comparing Eq.(21) with Eq.(24), we have proved the result \( dE_A = T_A dS_A \). So the Unified First Law of thermodynamics on the AH is confirmed, and the work that has been done is really zero. From the FIG.4, we find no periodic variation in any periods of \( \omega \). FIG.5 demonstrates some details in the top panel of FIG.4. At different epochs, as shown in FIG.5, the basic feature for the change of the energy is similar, and \( T_A dS_A \) and \(-dE_A\) are symmetrically distributed around the level line. Actually, due to the oscillation of \( \omega \), the variations of energy are some fluctuations in the Standard Universe, which is \( \Lambda \)CDM with \( \Omega_{\Lambda 0} = 1 \) and \( \Omega_{M0} = 0 \).

However, if we add the matter term, or \( k \neq 0 \) to Eq(8), the Unified First Law is broken. Namely, when the Universe is deceleration, or the curvature be-
Fig. 4. $T_A dS_A$ in units of $H_0^{-1}$ (thick red line) and $-dE_A$ in units of $H_0^{-1}$ (dashed green line) as function of $a$ at different epochs, corresponding to three different $\omega$ in FIG.1 respectively. And the level line is the change of energy in the Standard Universe (dotted black line).

comes large generally near the singularity, the Unified First Law needs more investigation. In other words, for the Undulant Universe, the standard calculation for the Unified First Law of thermodynamics is only confirmed in a particular situation. We can speculate that the DE has scalar vector form near the singularity (43). Another way is to find some new calculation.

5 The Generalized Second Law of thermodynamics on the apparent horizon

The preferred screen is the cosmological horizon. The AH is a good cosmological horizon, and the Unified First Law of thermodynamics on the AH is satisfied. So we can continue to discuss the Generalized Second Law of thermodynamics.

For the Generalized Second Law, we now state a version of the cosmic holographic principle based on the cosmological AH: the particle entropy inside the AH can never exceed the AH gravitational entropy, i.e. the entropy inside
Fig. 5. $T_A dS_A$ (thick red line) in units of $H_0^{-1}$ and $-dE_A$ in units of $H_0^{-1}$ (dashed green line) as function of $a$ at different epochs, which demonstrate some details in the top panel of FIG.4. And the level line is the change of energy in the Standard Universe (dotted black line).

the AH plus the AH gravitational entropy never deceases (39). We can obtain the entropy of the Universe inside the AH through the Unified First Law of thermodynamics:

$$TdS_i = dE_i + PdV = Vd\rho + (\rho + P)dV. \tag{23}$$

In terms of Eq.(25), the energy inside the AH is $E_i = 4\pi r_A^3 \rho / 3$, and the volume is $V = 4\pi r_A^3 / 3$. Combining Eq.(6) with Eq.(7), when $\Omega_{\Lambda 0} = 1$, and differentiating $\rho = \rho_{\Lambda}$, we get $d\rho = 3HdH/4\pi$. According to the Zeroth Law of thermodynamics (40), we know that on a stationary surface in the thermodynamical equilibrium, the temperature is constant Hawking temperature. However, the temperature of the viscous matter is higher than the Hawking temperature. At the same time, because the Universe expands and the DE is dominant, the temperature declines rapidly and becomes lower than the Hawking temperature. We might define $T = uT_H$ (41), where $u$ is a real constant, $0 < u \leq 1$. Indeed the parameter $u$ shows the deviation from Hawking temperature. Therefore, we obtain
\[ dS_i = \frac{V \rho + (P + P) dV}{T} = \frac{V \rho + (1 + \omega) \rho dV}{uT_H} \]  
\[ = \frac{1.5 H_0^{-1}}{u} \pi a^2 \left[ 3 \cos(b \ln a) - 1 \right] \left[ \cos(b \ln a) - 1 \right] \]  
\[ \times \exp\left[ -\frac{3}{b} \sin(b \ln a) \right] da. \]  
(24)

When \( u = 1 \), \( dS_i \) is minimum. So it is enough to study how the minimal differential entropy evolves. For the entropy of the AH, we get

\[ dS_A = \frac{d^r_A}{T_A} = 3 H_0^{-1} \pi a^2 [1 - \cos(b \ln a)] \]  
\[ \times \exp\left[ -\frac{3}{b} \sin(b \ln a) \right] da. \]  
(25)

The total differential entropy is \( dS_{\text{total}} = dS_A + dS_i \), and all of the variations of the entropy vary with \( a \). As shown in FIG.6, \( dS_i \leq 0 \) all the time, but \( dS_A \geq 0 \) and \( dS_{\text{total}} \geq 0 \). All of these cases at different epochs are similar. FIG.7 demonstrates some details in the top panel of FIG.6. we find \( dS_i \geq 0 \) at different epochs in the small ranges. \( dS_i \) are small fluctuations in the cosmological evolution.

Therefore, \( dS_{\text{total}} \geq 0 \) all the time, which is the minimal total differential entropy. Correspondingly, the real total differential entropy should not decrease. In a word, the Generalized Second Law of thermodynamics is confirmed on the AH. We know from (43; 44), \( \omega(a) < -1 \) is meaningless physically because of the negative entropy, while \( \omega(a) = -\cos(b \ln a) > -1 \) all the time. The confirmation of the Generalized Second Law for the Undulant Universe is another support to this conclusion.
Fig. 6. $dS_i$ (thick dotted green line), $dS_A$ (thick solid red line), and $dS_i + dS_A = dS_{total}$ (thick dashed blue line) in units of $H_0^{-2}$ as functions of $a$ at different epochs, corresponding to three different $\omega$ in FIG.1 respectively. The varied entropy of the Standard Universe is the level line (thin dotted black line).

Fig. 7. $dS_i$ (thick dotted green line), $dS_A$ (thick solid red line), and $dS_i + dS_A = dS_{total}$ (thick dashed blue line) in units of $H_0^{-2}$ as functions of $a$ within three different small ranges, and at different epochs, which demonstrate some details in the top panel of FIG.6. The varied entropy of the Standard Universe is the level line (thin dotted black line).

6 Conclusion

The properties of DE have been studied since long time ago, and many models have been put forward. Although there is no evidence to give an exact model
consistent with the real world, researchers have been successful in many aspects. In this paper, we study the Undulant Universe on its thermodynamical properties.

We have shown that the Undulant Universe is a model to solve the cosmic coincidence problem without fine tuning, i.e. nowadays phase of the Universe is not particular. Thus we can come from any phase in the past, and go to any phase in the future. We find that neither the EH nor the PH exists in the Undulant Universe, which is consistent with our conclusion above. However, we find another cosmological horizon, the AH, with a definition coming from topology. We choose a special expression for the AH, whose evolution can be seen from FIG.2. There is no periodicity. No matter how the Universe evolves, we can always find a AH, and all of their evolution is similar. So we can consider the Universe as a thermodynamical system inside the AH. And because of no significant change over one Hubble scale, as shown in FIG.3, the equilibrium thermodynamics can be used.

The Four Laws of BH thermodynamics \( \text{(40)} \) are successful. However these conjectures should be proved in the Universe. The thermodynamical First Law \( dM = TdS \) on the AH is expressed as \( dE_A = T_A dS_A \). Because the work term \( PdV \) is zero at infinity, we have ignored it. According to our deduction, the Unified First Law of thermodynamics is satisfied perfectly. As shown in FIG.4, there is no periodic variation in any periods of \( \omega \), and the basic feature for the change of the energy is similar at different epochs. FIG.5 presents the variations of energy in small ranges, and there are some fluctuations comparing with the Standard Universe. The fluctuations are due to the oscillation of \( \omega \).

Holography can be applied to the standard cosmological context, so we consider the world as a hologram. The entire information about the global 3+1-
dimensional spacetime can be stored on particular hypersurfaces. The connection between entropy and information is well known \(^{(42)}\). Accordingly, the total entropy of the Universe increases in the process. We obtain the entropy inside the AH by the Unified First Law of thermodynamics. Because the temperature of the Universe dominated by the DE is not higher than the Hawking temperature, we choose the maximal temperature to get the minimal total differential entropy. The total differential entropy is calculated from 
\[ dS_{total} = dS_A + dS_i. \]

The temperature of the cosmological material will decline as the Universe expands, so \( dS_i \leq 0 \). Its accompanying loss of energy can cause a back reaction on the cosmic dynamics, so that the horizon area increases, and \( dS_A \geq 0 \). We find that the total entropy of the AH does not decrease with time, \( dS_{total} \geq 0 \). In the small ranges, the entropy of the visible Universe decreases at different epochs, \( dS_i \geq 0 \), which locally are small fluctuations in the cosmological evolution. Thus the Generalized Second Law of thermodynamics is satisfied on the AH. As shown in FIG.6 and FIG.7, whenever and wherever the Universe evolves, the variations of the entropy are similar. Although we just discuss the minimal total differential entropy, the Generalized Second Law of thermodynamics remains confirmed in the condition of the real world.

In conclusion, the AH of the Undulant Universe is a good holographic screen, and can be seen as a boundary of keeping thermodynamical properties. The Undulant Universe is in thermal equilibrium inside the AH, and behaves very well in its cosmological evolution. In spite of some undulant departure from the Standard Universe, the Undulant model solves some coincidence problems in the Standard Universe.

In summary, the investigation into the Universe is very intriguing from the thermodynamical perspective, however, there are still some problems to be solved. In fact, the acceleration of our Universe is temporary, and there ever
was and will be a matter dominated phase or singularity. Given this we should consider matter dominated or large curvature case. As we know, the conversation law always works. When adding the matter term and \( k \neq 0 \) to the Undulant Universe, e.g., when the Universe is decelerating or near singularity, the law is broken. This bad result means that, the energy in the Undulant Universe \( \omega(a) = -\cos(b \ln a) \) maybe some other form at these epochs to keep the conversation law confirmed. Another possibility is that the entropy should be redefined. However, the success of our standard deduction encourages the definition in the Undulant Universe. Furthermore, thermodynamical properties demand to be explored more deeply in the future, and there are some more cosmological horizons expected to be studied.

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