The Optimization of Surface Quality and Smoothness Property with Orthogonal Function Algorithm

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Abstract. In free surface fitting, the method of increasing control points to improve surface fitting accuracy is generally utilized. But with the increment of control points, the order of surface continuity decreases. According to the disadvantages of the traditional method of surface reconstruction, the surface fitting algorithm with orthogonal function is proposed in the paper. It avoids irregular points to implement surface fitting, which can ensure the smoothness of the surface. 3x3 order surface and 36 control points are used in surface fitting with the maximum error 0.0770mm, and the surface patches are G2 continuous. Experimental results show that the proposed method realizes the best matching of surface control points, surface curvature and reconstruction precision. The quality of surface reconstruction is greatly improved.

1. Foreword
CAD model reconstruction is to transform the discrete data collected by three-dimensional optical scanning or three-coordinate measurement into continuous surface, and regenerate the geometric model representing the object. Surface reconstruction is the most important link in reverse engineering[1]. Generally, a large number of discrete points are obtained by scanning the measured surface by three-dimensional scanner and other equipment according to the characteristics of the surface. Surface reconstruction is realized by fitting the surface with appropriate algorithm.

In surface reverse engineering, common surface models include Bessel surface, B-spline surface and NURBS surface[2-4]. Triangular Bezier surface has the characteristics of flexible fitting and fast reconstruction. It has been widely used in surface fitting of three-dimensional scattered point set. However, the number of triangular patches constructed by this method is huge, and the generated surface model does not meet the surface reconstruction standard. When data exchange with general three-dimensional software, some information will be lost. The B-spline surface reconstruction method inherits the advantages of triangular Bezier fitting, such as strong adaptability and flexibility, and solves the problems of local surface fitting and standard surface description. Its main drawbacks are that there are some problems such as poor accuracy and multiple geometric shape definitions when describing the constructed surface, so it is difficult to form a definite surface description method[5].Ma W[6] divided the discrete data obtained by optical scanning into regions and interpolates them, and then fits the interpolation points to generate NURBS surface. Because NURBS surface is the standard for describing the shape of industrial products, the fitting method has the characteristics of stable solution, good compatibility and smooth surface. However, the points fitted by this method are obtained by interpolation operation, which results in the deviation between the surface and the point cloud data. Peng Fangyu[7] proposed the least squares method to fit the surface, which is based on curve fitting. In order to reduce the fitting error of the least squares surface, the method of
increasing the number of control points is usually used to make the smoothness of the reconstructed surface worse [8].

In order to overcome the shortcomings of the surface reconstruction algorithm mentioned above, a free-form surface fitting optimization algorithm based on orthogonal function is presented. The method is a theoretical innovation based on the least square method. Its purpose is to make the number of control points of surface fitting match the surface-point cloud deviation and surface smoothness characteristics of the reconstructed surface optimally. The deviation between the surface and the original point cloud is not affected under the requirement of smoothness of free-form surface.

2. Surface Fitting Based on Orthogonal Function

2.1. The Description of the Algorithm
Mathematical representation of surfaces (such as Bezier surfaces and NURBS surfaces) involves two perpendicular directions. The whole surface is a basic function in both directions. The two basis functions are expressed by U and V functions. The parametric equation of a surface contains two parameters, U and V. The corresponding surface is also expressed by parametric curves in U and V directions.

The process of surface optimization using orthogonal function is as follows: Firstly, discrete points are obtained by three-dimensional scanning, point set selection is made according to the principle of minimum distance, and free-form surface is interpolated for partitioned data, and this surface is consistent with the trend of the whole reconstructed surface. Orthogonal function is used as least squares fitting surface, and orthogonal polynomials of scatter sets is defined according to given data and weight.

Orthogonal function surface fitting makes the reconstructed surface consistent with the discrete points, and because the surface does not pass through the discrete data points with irregular positions, it also makes the surface smoother. The essence of the algorithm is to transform the integral operation into summation operation, then to orthogonalize the point set to obtain polynomials, and use polynomials as operational factors to obtain orthogonal function fitting curves. Finally, the fitting curves are converted to NURBS surfaces, which solves the problem of non-convergence caused by using general polynomials to reconstruct surfaces.

2.2. Establishment of Fitting Curve Equation
In the process of NURBS surface reconstruction, discrete data points are fitted by step-by-step approximation method, and the control points of the curves are obtained by multiple iterations. In the point cloud data acquired by three-dimensional scanning, orthogonal polynomial with the point set is used as the basis function group, and \( \phi_j(x) \) is \( j \) degree polynomial, \( \omega(x_i)(i = 0, 1, 2, ..., m) \) is weight factor corresponding to discrete points, and the following conditions are satisfied.

\[
(\phi_j, \phi_k) = \sum_{i=0}^{m} \omega(x_i) \phi_j(x_i) \phi_k(x_i) = \begin{cases} 0 & j \neq k \\ A_k & j = k \\ \end{cases}, \quad k, j = 0, 1, 2, ..., n; \ n \leq m
\]

(1)

Matrix \( A^T \) is the transposing matrix of matrix \( A \), and \( A^T A \) is diagonal matrix, and it satisfies the following condition.

\[
A^T A = \begin{bmatrix}
\phi_0, \phi_0 \\
\phi_1, \phi_1 \\
\vdots \\
\phi_n, \phi_n
\end{bmatrix}
\]

(2)

And the solution of the normal equation \( A^T A \omega = A^T Y \) is as shown below.
Orthogonal polynomial \( \{ \varphi_j(x) \} \) is constructed on discrete point sets \( \{ x_i \} \). The following recursive method is used.

\[
\begin{align*}
\varphi_0(x) &= 1, \\
\varphi_1(x) &= (x - \alpha_1)\varphi_0(x), \quad j = 1, 2, \ldots, n - 1 \\
\varphi_{j+1}(x) &= (x - \alpha_j)\varphi_j(x) - \beta_j \varphi_{j-1}(x)
\end{align*}
\]

Here,

\[
\begin{align*}
\alpha_j &= \frac{\sum_{i=0}^{n} x_i \varphi_j^i(x_i)}{\sum_{i=0}^{n} \varphi_j^i(x_i)}, \quad j = 0, 1, \ldots, n - 1; \\
\beta_j &= \frac{\sum_{i=0}^{n} \varphi_j^i(x_i)}{\sum_{i=0}^{n} \varphi_{j-1}^i(x_i)}, \quad j = 1, 2, \ldots, n - 1;
\end{align*}
\]

Linear combination of orthogonal polynomials \( \{ \varphi_j(x) \} \) is used for least squares curve fitting as long as the coefficient \( a^*_j \) is calculated out according to the formula.

\[
a^*_j = \frac{(f, \varphi_j)}{(\varphi_j, \varphi_j)} = \frac{\sum_{i=0}^{n} \omega(x_i) f(x_i) \varphi_j(x_i)}{\sum_{i=0}^{n} \omega(x_i) \varphi_j^2(x_i)}, \quad j = 0, 1, \ldots, n
\]

Finally, the \( n \) order polynomial fitting curve of the surface can be obtained by accumulating \( a^*_k \varphi_k(x) \) into \( S(x) \) step by step.

\[
y = S(x) = \sum_{j=0}^{n} a^*_j \varphi_j(x) = a^*_0 \varphi_0(x) + a^*_1 \varphi_1(x) + \cdots + a^*_n \varphi_n(x)
\]

After the fitting curve is obtained, the evenly swept curve is used to fit the final surface, and the generation process from the curve to the surface can be realized by professional three-dimensional reverse software. The value of \( n \) can be set beforehand or determined according to error in the process of solving. This is the best method of surface fitting by polynomial function at present. In the surface fitting algorithm with orthogonal function, the point cloud obtained by three-dimensional scanning is the data source for solving fitting parameters, and it is also the basis for the quality optimization of reconstructed surface. In the process of solving, the trend surface reflecting the shape of the object is generated according to the discrete points, and then the corresponding points of the discrete points on the fitting surface are found by calculating the function values. Finally, the smooth surface of the discrete points is reconstructed by setting polynomial equations and solving the equations.

3. The Analysis of Surface Quality

The main factors to evaluate the surface quality are the reconstruction error and the fairness of the surface. In this experiment, the validity of the proposed algorithm is verified by the three-dimensional point cloud data shown in Figure 1. The traditional least squares method and the optimization algorithm based on orthogonal function are used to reconstruct the surface, and the smoothness, curvature continuity and surface fitting accuracy of the surface are analyzed and compared.
3.1. The Analysis of Surface Fairness
Taking the point cloud data in Figure 1 as an example, the least squares algorithm is used to fit the point cloud data. In order to ensure the accuracy of surface fitting, 4 x 4 order and 49 control points are used to fit the surface in the direction of U and V. The results of surface fitting are shown in Figure 2. The simulated external strong illumination ray is used to detect the surface. The projection line distribution is shown in Figure 3. Some areas are distorted and burred, and the surface is not smooth enough. The curvature of curved surface is analyzed by Imageware reverse software[9], as shown in Figure 4. Curvature comb changes sharply, curvature changes abruptly, and the curvature order of curved surface reduces.

By using orthogonal function surface optimization algorithm, high-precision surface reconstruction can be achieved by using only 3x3 order and 36 control points in the U and V directions of the surface. The constructed surface is shown in Figure 5. The surface is detected by reflecting line. The result of detection is shown in Figure 6. The projection lines are evenly distributed and there is no distortion of the curve. The curvature characteristics of the reconstructed surface are analyzed by Imageware reverse software, as shown in Figure 7. The results show that the curvature changes smoothly in the same direction. There is no sudden change in curvature, which meets the requirements of precise surface fairing.

3.2. The Analysis of Reconstruction Error
Reconstructing deviation is surface-point cloud deviation, which refers to the degree of deviation between the three-dimensional surface constructed from discrete point cloud and the point cloud data obtained from three-dimensional measurement. The deviation value reflects the coincidence between the fitting surface and the original data. Under the condition of meeting the requirement of surface fairing feature, the deviation of surface reconstruction should be reduced as far as possible, so as to
ensure that the original design idea of surface fitting can be achieved and the required design accuracy can be satisfied.

The deviation between the reconstructed surface and the original point cloud is analyzed with the proposed algorithm and the least square algorithm. The least square algorithm is used to fit the surface. The minimum error between the surface and point cloud is 0.1402 mm, and the maximum error is 0.2241 mm, and the color difference distribution is shown in Figure 8. With the proposed algorithm, the minimum error is 0.0352 mm, and the maximum error is only 0.0770 mm. The color difference distribution is shown in Figure 9.

| The algorithm            | Maximum deviation(mm) | Average deviation(mm) | Standard deviation(mm) |
|--------------------------|------------------------|------------------------|-------------------------|
| The least square algorithm | 0.2241                 | 0.1822                 | 0.0593                  |
| The proposed algorithm    | 0.0770                 | 0.0552                 | 0.0155                  |
From Table 1, we can see that compared with the traditional least squares surface reconstruction, the average deviation of fitting is reduced to 0.0552 mm, the average deviation between point cloud and surface is reduced by 69.7%, and the accuracy of surface reconstruction is significantly improved by the proposed algorithm.

4. Conclusions

In the paper, various algorithms of three-dimensional surface reconstruction are described, and their advantages and disadvantages are compared. The influence of surface order and control points on surface quality is analyzed when fitting surface by least square method. Aiming at the disadvantage that the least square method can not give consideration to both reconstruction accuracy and surface fairness, an optimization algorithm based on orthogonal function is presented. It fits surface by local interpolation, avoids the ill-conditioned problem of solving equation appearing in traditional polynomial fitting surface, and realizes the compatibility between surface fairness and reconstruction accuracy.

The optimization algorithm based on orthogonal function is used to reconstruct the surface of discrete point cloud. The maximum deviation between the surface and point clouds is 0.0770 mm, and the average deviation is only 0.0552 mm, and the adjacent quadrilateral surfaces satisfy G2 continuity. The example shows that the optimization algorithm based on orthogonal function makes the reconstructed surface have high fitting accuracy and smooth fairness.

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