Grand Unification and B & L Conservation

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A review of baryon and lepton conservation in supersymmetric grand unified theories is given. Proton stability is discussed in the minimal SU(5) supergravity grand unification and in several non-minimal extensions such as the SU(3)³, SO(10) and some string based models. Effects of dark matter on proton stability are also discussed and it is shown that the combined constraints of dark matter and proton stability constrain the sparticle spectrum. It is also shown that proton lifetime limits put severe constraints on the event rates in dark matter detectors. Future prospects for the observation of baryon and lepton number violation are also discussed.

1 Introduction

While string theory purports to unify all the fundamental interactions of physics including gravity, there is currently no string model which is fully viable. Pending the discovery of such a model, it is imperative that one look to the bottom up approach to unification. Grand unification is such an approach wherein one attempts to unify the electro-weak and the strong interactions in a single framework. In this review we will focus on such an approach, specifically, the approach of supersymmetric and supergravity grand unification. Our aim is to discuss the phenomena of baryon and lepton number conservation and their violation in such theories. The grand unification approach is expected to be valid up to the scale $M_G \sim 2 \times 10^{16}$ GeV but it could be valid even up to the string scale of

$$M_{str} \approx 5 \times g_{string} \times 10^{17} \text{ GeV}$$

Beyond that scale one is in the domain of quantum gravity where the full framework of string unification is needed for consistency.

The outline of the paper is as follows: In Sec.2 we discuss the LEP data which lends support to the ideas of supersymmetry and gauge coupling unification. In Sec.3 we give a general discussion of the various sources of baryon number (B) and lepton number (L) violation in SUSY theories. In Sec.4 we
discuss proton decay from B & L violating dimension five operators. In Sec.5 we discuss proton decay within the framework of supergravity unification. In Sec.6 we discuss B & L violation in non-minimal models. These include the $(SU(3))^3$ models, the unflipped and the flipped no-scale models, and SO(10) models. Effects of textures on proton lifetime are discussed in Sec. 7 and effects of dark matter constraints on proton lifetime are discussed in Sec.8. A brief discussion of proton decay in gauge mediated breaking of supersymmetry is given in Sec.9, and a discussion of Planck effects on proton decay is given in Sec.10. Exotic decay modes of the proton are discussed in Sec.11. In Sec.12 we give a brief discussion of the relation between SUSY GUTS and strings. Conclusions and prospects are given in Sec.13.

2 Grand Unification and LEP Data

While the LEP data extrapolated to high scales exhibits unification in the minimal SUSY SU(5), one finds that there could be a small discrepancy in the predicted value of $\alpha_s$ and experiment, i.e., that the predicted value of $\alpha_s$ lies $\sim (1 - 2)\sigma$ higher than the world average. Such deviations can be corrected by Planck scale corrections, which can produce $\sim (1 - 2)\sigma$ effects on $\alpha_s$. In fact, effects of such a size are expected because of the proximity of the grand unification scale to the Planck scale, and so effects of size $O(M_G/M_{\text{Planck}})$, i.e., $O(\text{few\%})$ are quite natural. A source of such corrections in supergravity grand unification is from the gauge kinetic energy function $f_{\alpha\beta}$ where

$$[A\delta_{\alpha\beta} + \frac{c}{2M_{\text{Planck}}}d_{\alpha\beta\gamma}\Sigma^\gamma]F^\alpha_{\mu\nu}F^\beta_{\mu\nu}$$  \hspace{1cm} (2)$$

Analyses show that values of $c \sim O(1)$ give agreement with LEP data. One may also use the LEP data to put a limit on the range of $c$. One finds

$$-1 \leq c \leq 3, \hspace{1cm} LEP \hspace{0.1cm} D A T A$$  \hspace{1cm} (3)$$

It is also possible to get 1-2 $\sigma$ effects on $\alpha_s$ from extensions of SU(5), such as, for example, in the missing doublet model with Peccei-Quinn symmetry.

3 Sources of Baryon Number Violation in SUSY

We discuss now the main sources of B & L violation in supersymmetric theories. These consist of baryon and lepton number violation from (i)lepto-quark exchange, (ii) dimension 4 operators, (iii) dimension dimension 5 operators, and (iv) higher dimensional operators. A summary of the important operators is given in Table 1.
Table 1: Baryon and lepton number violating operators

| Dim | Operator | B & L violation | comment |
|-----|----------|-----------------|---------|
| dim 6 | qqql | $\Delta B = 1$, $\Delta L = 1$ |         |
| dim 9 | qqqqq | $\Delta B = 2$, $\Delta L = 0$ | $n - \bar{n}$ oscillation |
| SUSY | (qqq)$_F$ | $\Delta B = 1$, $\Delta L = 0$ |         |
| dim 5 | (qqql)$_F$ | $\Delta B = 1$, $\Delta L = 1$ | mass scale $\sim 10^{16}$ GeV |
| dim 7 | (qqqqqq)$_F$ | $\Delta B = 2$, $\Delta L = 0$ | $n - \bar{n}$ oscillation |

The experimental possibilities for testing the baryon and lepton number violations are (i) the proton decay decay experiments, (ii) experiments that look for double beta decay, and (iii) proposed experiments for the test of $n - \bar{n}$ oscillation. Of these the first two are the ones that have been most vigorously pursued. In this review we will mostly focus on the implications of B & L violations on proton stability.

3.1 Proton Decay via Lepto-quark Exchange

As mentioned already there are various sources of proton decay in grand unified theories. Most grand unified theories allow for baryon and lepton number violation because quarks and leptons belong to the same multiplets and proton decay in these theories can proceed via lepto-quark exchange. In SU(5) models the dominant mode via lepto-quark exchange is $p \rightarrow e^+ \pi^0$ which gives a lifetime for this mode of

$$\tau(p \rightarrow e^+ \pi^0) \approx \left( \frac{M_V}{3.5 \times 10^{14} \text{GeV}} \right)^4 10^{31 \pm 1} \text{ yr}$$

(4)

For the non-supersymmetric SU(5) the lifetime is too small to be consistent with experiment. For the supersymmetric SU(5) one estimates $\tau(p \rightarrow e^+ \pi^0)$ to be

$$1 \times 10^{35 \pm 1} \text{ yr}$$

(5)

The current experimental limit for this decay mode is

$$\tau(p \rightarrow e^+ \pi^0) > 2.1 \times 10^{33} \text{ yr, (90\%CL)}$$

(6)

It is expected that in the future Super K will reach a sensitivity of

$$\tau(p \rightarrow e^+ \pi^0) > 1 \times 10^{34} \text{ yr, (90\%CL)}$$

(7)

Thus the $e^+ \pi^0$ mode in SUSY SU(5) may be on the edge of detection if Super-K and Icarus reach their maximum sensitivity. However, as mentioned already in supersymmetric theories there are other sources of B & L violation, such as dimension 4 and dimension 5 operators.
3.2 p Decay via Dimension 4 Operators

Supersymmetric theories generically have dimension four operators which violate baryon and lepton number. Thus, for example, in the minimal supersymmetric standard model (MSSM) one has in general dimension four operators in the superpotential of the form

\[ W = \lambda_u Qu^c H_2 + \lambda_d Qd^c H_1 + \lambda_e Le^c H_1 + \mu H_1 H_2 \\
+ (\lambda_B u^c d^c d^c + \lambda'_L Qd^c L + \lambda''_L LLe^c) \]  (8)

Here the terms proportional to $\lambda'_B$, $\lambda'_L$, and $\lambda''_L$ induce B & L violation. Suppression of fast p decay requires

\[ \lambda'_B \lambda'_L < \left( \frac{m_d^2}{10^{16} \text{GeV}} \right)^2 \sim O(10^{-26}) \]  (9)

In the MSSM one eliminates fast p decay via a discrete R symmetry $R = (-1)^{3B+L+2S}$. However, in general R symmetry which is a global symmetry is not preserved by gravitational interactions. Thus for example, worm holes can generate dimension 4 operators and catalyze p decay. To protect against fast p decay of the above type one must promote the global R symmetry to a gauge symmetry, since gauge symmetries are protected against wormhole effects. Even if the local symmetry breaks down leaving behind a residual discrete symmetry, that residual discrete symmetry will be sufficient to protect against fast p decay induced by worm hole effects. R parity is an interesting symmetry in that it appears that it is the only $\mathbb{Z}_2$ symmetry which is free of anomalies with just the MSSM spectrum. Thus there is a possibility that it could arise in an automatic fashion from the spontaneous breakdown of groups that contain a $U(1)_{B-L}$ such as $SU(4)_{C}$ and SO(10). It should be noted, however, that even if a theory is originally free of dangerous dimension four operators, such operators can be induced from higher dimensional operators via spontaneous symmetry breaking. For example, one may have a dimension five operator in SO(10) which contains an SU(5) singlet field $\nu^c$. The following provide examples where spontaneous VEV formation of an SU(5) singlet generates dangerous dimension four operators

\[ \frac{1}{M_P} (u^c d^c d^c \nu^c) \rightarrow (u^c d^c d^c), \quad \frac{1}{M_P} (QL d^c \nu^c) \rightarrow (QL d^c) \]  (10)

4 Dimension 5 B and L Violation and p Decay

In MSSM one can write many dimension 5 operators that violate B and L number such as $QQQL$, $u^c e^c d^c e^c$, $QQQH_1$, $Qu^c e^c H_1$ etc. These operators
contribute to p decay at the loop level. The only operators that arise in the minimal SU(5) GUT model and generate observable p decay are the first two operators in the list above, i.e., $QQQL$, and $u^c d^c e^c$.

B and L violating dimension five operators occur in most supersymmetric theories and string theories and lead to p instability in these models. In SUSY grand unified models proton decay via dimension five operators is governed by the interaction

$$\bar{H}_1 J + \bar{K} H_1 + \bar{H}_i M_{ij} H_j$$

(11)

where $H_1, \bar{H}_1$ are the Higgs triplets, $J$ and $\bar{K}$ are matter currents, and $M_{ij}$ is the Higgs triplet mass matrix. The suppression of p decay in these theories can come about if

$$(M^{-1})_{11} = 0$$

(12)

A suppression of this type can occur by discrete symmetries, by non-standard embeddings, or by the presence of additional Higgs triplets. However, in most SUSY/string models (except for the flipped $SU(5) \times U(1)$ models) one does not have a a natural suppression, and a suppression requires a doublet-triplet splitting in the Higgs multiplets. Many mechanisms for doublet-triplet splittings have been discussed in the literature, such as (i) the sliding singlet mechanism, which works for SU(n) for $n \geq 6$, (ii) the missing partner mechanism, (iii) the mechanism of VEV alignment, (iv) the mechanism where the Higgs doublets are pseudo-Goldstone, and (v) the mechanism with more than one adjoint Higgs.

In the following we will discuss p decay via dimension 5 operators in several SUSY GUTS: SU(5), (SU(3))$^3$, SO(10), the flipped and the unflipped no-scale models, with the most emphasis on the simplest SUSY GUTs, i.e., the minimal SU(5) model. The p decay in the minimal SU(5) model is governed by

$$W_Y = -\frac{1}{8} f_{1ij} \epsilon_{uvwxy} H_1^u M_{ij}^{uv} M_j^{xy} + f_{2ij} \bar{H}_2 u_m M_{ij}^{uv} M_j^{vw}$$

(13)

where $M_{ix}$ and $M_{ix}^{xy}$ ($i=1,2,3$) are the 5 and 10 of SU(5) which contain the three generations of quarks and leptons, and $H_1, H_2$ are the 5,5 of Higgs. After the breakdown of the GUT symmetry there is a splitting of the Higgs multiplets where the Higgs triplets become superheavy and the Higgs doublets remain light by one of the mechanisms listed earlier. One can now integrate on the Higgs triplet field and obtain an effective interaction at low energy given below

$$LLLL = \frac{1}{M} \epsilon_{abc}(P f^u V)_ij (f^d_2)_{kl} (\bar{u}_{LBi} \bar{d}_{Lcj} (\bar{e}_{Lk} (Vu_L)_{al} - \nu_{Lk} d_{Lal}) + ...) + H.c.$$ \hspace{0.5cm} (14)

$$RRRR = -\frac{1}{M} \epsilon_{abc}(V^t f^u)_ij (PV f^d)_{kl} (\bar{e}_{Ri} u_{Raj} \bar{u}_{Rck} \bar{d}_{Rbl} + ...) + H.c.$$ \hspace{0.5cm} (15)
where \( V \) is the CKM matrix and \( f_i \) are the Yukawa couplings which are related to the quark masses by

\[
m_u^i = f_u^i (\sin 2\theta_W / e) M_Z \sin \beta, \quad m_d^i = f_d^i (\sin 2\theta_W / e) M_Z \sin \beta
\]

and \( P_i \) are generational phases

\[
P_i = (e^{i\gamma_i}), \quad \sum_i \gamma_i = 0; \quad i = 1, 2, 3
\]

Both LLLL and RRRR interactions must be taken into account in a full analysis and their relative strength depends on the part of the parameter space where their effects are computed.

The operators of Eq.(14) are dimension five operators which must be dressed via the exchange of gluinos, charginos and neutralinos. The dressings give rise to dimension six operators. These dimension six operators are then used in the computation of proton decay. In the dressings one takes into account the L-R mixings, where for the up squark mass matrix one has

\[
\left( \begin{array}{c}
m_u^2 \\
m_u (A_u m_0 - \mu c t n \beta) \\
(A_u m_0 - \mu c t n \beta)
\end{array} \right)
\]

(17)

The mass diagonal states are given by

\[
\tilde{u}_R = \cos \delta_u \tilde{u}_1 + \sin \delta_u \tilde{u}_2, \quad \tilde{u}_L = -\sin \delta_u \tilde{u}_1 + \cos \delta_u \tilde{u}_2
\]

\[
sin 2\delta_u = -2m_u (A_u m_0 - \mu c t n \beta) / (\tilde{m}_{u1}^2 - \tilde{m}_{u2}^2)
\]

(18)

and similarly for the down squarks and the leptons. In supergravity grand unification the values of the trilinear couplings \( A_u, A_d, A_e \) are all related to the single parameter \( A_0 \). After dressing of the dimension 5 by the gluino, the chargino and the neutralino exchange diagrams one finds baryon and lepton number violating dimension six operators with chiral structures LLLL, LLRR, RRLL and RRRR which enter in the proton decay analysis.

4.1 Effective Lagrangian Approach

The B and L violating interaction one gets from the fundamental SU(5) Lagrangian is in terms of quarks and leptons, while the p decays involve physical mesons and baryons. There are various techniques for bridging the gap, i.e., in going from the fundamental to the phenomenological interactions, such as the Bag model, lattice QCD, and effective Lagrangians. Currently the most efficient of these approaches is that of effective Lagrangians. Here the basic
technique consists in finding the effective interactions in terms of mesons and baryons with the same chiral structures as LLLL, LLRR, etc. that appear in the fundamental Lagrangian. This effort is facilitated by utilizing the transformation properties of these terms under $SU(3)_L \times SU(3)_R$ as shown below:

| Chiral structure | $SU(3)_L \times SU(3)_R$ \ rep |
|-----------------|----------------------------------|
| LLLL            | $(8, 1)$                         |
| LLRR            | $(3^*, 3)$                       |
| RRLL            | $(3, 3^*)$                       |
| RRRR            | $(1, 8)$                         |

In the effective lagrangian approach one finds combinations of mesonic and baryonic fields with the same chiral transformation properties as the dimension six B & L violating quark fields. For this purpose one defines first a pseudo-goldstone mass matrix

\[
\begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\
\pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\
K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta
\end{pmatrix}
\]  

(19)

Similarly, one defines a baryon mass matrix so that

\[
\begin{pmatrix}
\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\
\Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\
\Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda
\end{pmatrix}
\]  

(20)

Defining

\[
\xi = e^{\frac{i}{2}M}
\]  

(21)

one can find combinations of mesons and baryon mass matrices with the following transformations

| meson – baryon structure | $SU(3)_L \times SU(3)_R$ \ rep |
|--------------------------|----------------------------------|
| $\xi B \xi^\dagger$      | $(8, 1)$                         |
| $\xi^\dagger B \xi^\dagger$ | $(3^*, 3)$                       |
| $\xi B \xi$              | $(3, 3^*)$                       |
Using the above technique one can write an effective Lagrangian with the same chiral transformation properties as the fundamental Lagrangian in terms of quarks. Currently the effective lagrangian approach is the most reliable approach to the computation of proton decay amplitudes.

In the minimal SU(5) model the dominant decay modes of the proton involve pseudo-scalar bosons and anti-leptons, i.e.,

$$\bar{\nu}_i K^+, \bar{\nu}_i \pi^+; i = e, \mu, \tau$$

$$e^+ K^0, \mu^+ K^0, e^+ \pi^0, \mu^+ \pi^0, e^+ \eta, \mu^+ \eta$$

The relative strength of these decay modes depends on various factors, such as quark masses, CKM factors, and third generation effects in the loop diagrams etc. denoted by $y_{\text{tK}}^i$ below. The various decay modes and some of the factors that control these decays modes are summarized in Table 1.

| Mode       | quark factors | CKM factors |
|------------|---------------|-------------|
| $\bar{\nu}_e K$ | $m_d m_c$     | $V_{11} V_{21} V_{22}$ |
| $\bar{\nu}_\mu K$ | $m_s m_c$     | $V_{21} V_{22}$ |
| $\bar{\nu}_\tau K$ | $m_b m_c$     | $V_{31} V_{21} V_{22}$ |
| $\bar{\nu}_e \pi_1, \bar{\nu}_e \pi_2$ | $m_d m_c$     | $V_{11} V_{21}$ |
| $\bar{\nu}_\mu \pi_1, \bar{\nu}_\mu \pi_2$ | $m_s m_c$     | $V_{21} V_{21}$ |
| $\bar{\nu}_\tau \pi_1, \bar{\nu}_\tau \pi_2$ | $m_b m_c$     | $V_{31} V_{21}$ |
| $e^+ K$      | $m_d m_u$     | $V_{11} V_{12}$ |
| $\mu^+ K$    | $m_s m_u$     |             |
| $\pi^+ \eta$ | $m_d m_u$     | $V_{11} V_{21}$ |
| $\mu^+ \eta$ | $m_s m_u$     |             |

The order of magnitude estimates can be gotten using

$$m_u V_{11} : m_s V_{21} : m_t V_{31} \approx 1 : 50 : 500$$

Typically the most dominant mode is $\bar{\nu} K$. It is governed by the interaction

$$L_6(N \to \bar{\nu} K) = ((\alpha_2)^2(2M M_{\tilde{W}} \sin 2\beta)^{-1} P_2 m_c m_1^d V^*_{11} V_{21} V_{22})$$

$$(F(\bar{\tilde{c}}, \tilde{d}_i, \tilde{W}) + F(\bar{\tilde{c}}, \tilde{d}_i, \tilde{W}))(1 + y_{iK}^t + (y_{i}^\bar{W} + y_{i}^\bar{Y}) \delta_{i2} + \Delta_{i2}^K) \alpha_i L$$

$$(1 + y_{iK}^t - (y_{i}^\bar{W} y_{i}^\bar{Y}) \delta_{i2} + \Delta_{i2}^K) \beta_i L + (y_{1}(R) \alpha_3^R + y_{2}(R) \beta_3^R) \delta_{i3})$$

(24)
where

\[ \alpha^L_i = \epsilon_{abc}(d_{aL}\gamma^0u_{bL})(s_{cL}\gamma^0\nu_{iL}) \]
\[ \alpha^R_i = \alpha^L_i(d_{L,R}u_{L,R}) \]
\[ \beta_{L,R}^i = \alpha^L_i(d \rightarrow s) \] (25)

In the above \( y^k_{tK} \) is the third generation contribution

\[ y^k_{tK} = \frac{P_2}{P_3} m_4 V_{31}^V \frac{F(\tilde{t}, \tilde{d}_i, \tilde{W}) + F(\tilde{t}, \tilde{e}_i, \tilde{W})}{F(\tilde{e}, \tilde{d}_i, \tilde{W}) + F(\tilde{e}, \tilde{e}_i, \tilde{W})} \] (26)

where \( F \) are dressing loop integrals and \( y_{\tilde{g}} \) are corrections from the gluino exchange, and \( y_{\tilde{Z}} \) are corrections from neutralino exchange. The second and the third generation squark loop contributions can interfere both constructively and destructively. The \( p \) lifetime is enhanced when there is destructive interference. Further, there are situations when the \( \bar{\nu}\pi^+ \) mode can be significantly enhanced so that it becomes comparable to the \( \bar{\nu}K^+ \) mode [17].

4.2 The \( \bar{\nu}K \) Mode

We discuss now the details of the \( \bar{\nu}K \) mode which as already pointed out is most often the most dominant mode in the nucleon decay in the minimal SU(5) model. The decay width of the \( p \rightarrow \bar{\nu}_i K^+ \) mode is given by

\[ \Gamma(p \rightarrow \bar{\nu}_i K^+) = \left( \frac{\beta_p}{M_H} \right)^2 |A|^2 |B_i| C \] (27)

Here the factors \( A \) and \( B_i \) are given by

\[ A = \frac{\alpha^2}{2M_W^2} m_m V_{21}^V A L A S \] (28)
\[ B_i = \frac{1}{\sin 2\beta} \frac{m_m V_{21}^V}{m_m} \left[ P_2 B_{2i} + \frac{m_4 V_{31}^V}{m_m} V_{22} P_3 B_{3i} \right] \] (29)
\[ B_{3i} = F(\tilde{u}_i, \tilde{d}_j, \tilde{W}) + (\tilde{d}_j \rightarrow \tilde{e}_j) \] (30)

\[ F(\tilde{u}_i, \tilde{d}_j, \tilde{W}) = [E \cos\gamma_- \sin\gamma_+ \tilde{f}(\tilde{u}_i, \tilde{d}_j, \tilde{W}_1) + \cos\gamma_+ \sin\gamma_- \tilde{f}(\tilde{u}_i, \tilde{d}_j, \tilde{W}_1)] \]
\[ -1 \frac{\delta_{i3} m_4 \sin 2\delta_{ui}}{2 \sqrt{2} M_W \sin \beta} [E \sin\gamma_- \sin\gamma_+ \tilde{f}(\tilde{u}_{i1}, \tilde{d}_j, \tilde{W}_1) - \cos\gamma_- \cos\gamma_+ \tilde{f}(\tilde{u}_{i1}, \tilde{d}_j, \tilde{W}_2) \]
\[-(\tilde{u}_{i1} \rightarrow \tilde{u}_{i2})] \] (31)
where $\tilde{f}$ is given by

$$\tilde{f}(\tilde{u}_i, \tilde{d}_j, \tilde{W}_k) = \sin^2 \delta_{u_i} \tilde{f}(\tilde{u}_1, \tilde{d}_j, \tilde{W}_k) + \cos^2 \delta_{u_i} \tilde{f}(\tilde{u}_2, \tilde{d}_j, \tilde{W}_k)$$  \hspace{1cm} (32)

and where

$$f(a, b, c) = \frac{m_c}{m_b^2 - m_c^2} \left[ \frac{m_b^2}{m_a^2 - m_b^2} \ln \left( \frac{m_b^2}{m_a^2} \right) - (m_a \rightarrow m_c) \right]$$ \hspace{1cm} (33)

In Eq.(30) $\gamma_\pm = \beta_+ \pm \beta_-$ and

$$\sin 2\beta_\pm = \frac{(\mu \pm \tilde{m}_2)}{[4\nu_\pm^2 + (\mu \pm \tilde{m}_2)^2]^{1/2}}$$

$$\sqrt{2}\nu_\pm = M_W (\sin \beta \pm \cos \beta)$$

$$\sin 2\delta_{u_3} = -\frac{2(A_1 + \mu\csc \beta)m_t}{m_t^2 - m_{\tilde{t}_2}^2}$$

$$E = 1, \sin 2\beta > \mu\tilde{m}_2/M_W^2$$

$$= -1, \sin 2\beta < \mu\tilde{m}_2/M_W^2$$ \hspace{1cm} (34)

$C$ in Eq.(27) is a current algebra factor and is given by

$$C = \frac{m_N}{32\pi f_\pi^2} \left[ (1 + \frac{m_N(D + F)}{m_B}) (1 - \frac{m_N^2}{m_B^2}) \right]^2$$ \hspace{1cm} (35)

where $f_\pi, D, F, \ldots$ etc are the chiral Lagrangian factors with the numerical values: $f_\pi = 139$ MeV, $D=0.76, F=0.48, m_N=938$ MeV, $m_K=495$ MeV, and $m_B=1154$. Finally, in Eq.(27) $\beta_p$ is defined by

$$\beta_p U_L^\gamma = \epsilon_{abc}\epsilon_{\alpha\beta} < 0 |d_{aL}^\alpha u_{bL}^\beta u_{cL}^\gamma| p >$$ \hspace{1cm} (36)

where the lattice gauge analysis gives\cite{10}

$$\beta_p = (5.6 \pm 0.5) \times 10^{-3} GeV^3$$ \hspace{1cm} (37)

### 4.3 Vector Meson Decays Modes of the Proton

In addition to the nucleon decay modes involving pseudo-scalar bosons and anti-leptons, one also has in general decay modes involving vector bosons and anti-leptons. The source of these modes are the same baryon number violating dimension six quark operators that give rise to the decay modes that give rise
to pseudoscalar and ant-lepton modes. The vector decay modes of the proton are

\[ \bar{\nu}_i K^*, \bar{\nu}_i \rho, \bar{\nu}_i \omega; i = e, \mu, \tau \]
\[ e K^*, \mu K^*, e \rho, \mu \rho, e \omega, \mu \omega \]

(38)

However, the vector meson decay modes have generally smaller branching ratios than the corresponding pseudo-scalar decay modes.

5 Supergravity Analysis

We shall work here in the framework of supergravity models where supersymmetry is broken in the hidden sector by a superhiggs phenomenon and the breaking communicated gravitationally to the physical sector. The simplest case corresponds to when the superhiggs coupling are generation blind. Here after breaking of supersymmetry and of the gauge group and after integrating out the superhiggs fields and the heavy fields of the theory, one finds that the supersymmetry breaking potential below the GUT scale is given by

\[ V_{SB} = m_0^2 z_a z^*_a + \left( A_0 W^{(3)} + B_0 W^{(2)} + h.c. \right) \]

(39)

and in addition one has a universal gaugino mass term

\[ L^\lambda_{\text{mass}} = -\frac{1}{2} \tilde{\tilde{\lambda}}^\alpha \lambda^\alpha \]

(40)

In the above, \( W^{(3)} \) is the trilinear part of the superpotential and \( W^{(2)} \) is the bilinear part which under the constaint of R parity invariance is given by \( W^{(2)} = \mu_0 H_1 H_2 \). Thus we find that the supersymmetry breaking sector contains just four parameters.

Supergravity unification possesses the remarkable feature that the electro-weak symmetry breaking can be manufactured by radiative effects. The radiative electro-weak symmetry breaking is governed by the following Higgs potential

\[ V_H = m_1^2(t)|H_1|^2 + m_2^2(t)|H_2|^2 - m_3^2(t)(H_1 H_2 + h.c.) \]
\[ + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + \Delta V_1 \]

(41)

where \( \Delta V_1 \) is the one loop correction to the Higgs potential and the parameters \( m_i^2(t) \) satisfy

\[ m_1^2(0) = m_0^2 + \mu_0^2; i = 1, 2; \quad m_3^2(0) = -B_0 \mu_0 \]

(42)
while the gauge coupling constants satisfy the GUT relation $\alpha_3(0) = \alpha_2(0) = \alpha_G = (5/3)\alpha_Y(0)$. The breaking of the electroweak symmetry is accomplished by a satisfaction of the constraints ($\tan \beta = H_2 / H_1$):

$$
\frac{1}{2} M_Z^2 = (\mu_1^2 - \mu_2^2 \tan^2 \beta) / (\tan^2 \beta - 1)
$$

$$
\sin 2\beta = (2m_3^2) / (\mu_1^2 + \mu_2^2)
$$

$$
\mu_i^2 = m_i^2 + \Sigma_i
$$

(43)

where $\Sigma_i$ is the one loop correction from $\Delta V_1$. On using the radiative breaking constraint the low energy SUSY parameters can be chosen to be

$$
m_0, m_{1/2}, A_0, \tan \beta, \text{sign} \mu
$$

(44)

Supergravity unification exhibits the phenomenon of scaling over most of the parameter space of the model. This arises because over most of the parameter space of the model one has $\mu^2 >> M_Z^2$ which gives

$$
m_{\tilde{W}_1} \sim \frac{1}{3} m_{\tilde{g}} (\mu < 0); \quad m_{\tilde{W}_1} \sim \frac{1}{4} m_{\tilde{g}} (\mu > 0)
$$

$$
2m_{\tilde{Z}_1} \sim m_{\tilde{W}_1} \sim m_{\tilde{Z}_2}; \quad m_{\tilde{Z}_3} \sim m_{\tilde{Z}_4} \sim m_{\tilde{W}_2} >> m_{\tilde{Z}_1}
$$

(45)

In addition one also has $m_{H^0} \sim m_A \sim m_{H^\pm} >> m_h$. Corrections to scaling are $O(M_Z/\mu)$. While these corrections are small over most of the parameter space of the model, they can become significant in the region of small $\mu$.

5.1 Non-universalities of Soft SUSY Breaking Parameters

Universalities of the soft SUSY breaking parameters at the GUT scale are a consequence of the assumption of a generation independent Kahler potential in the supergravity analysis. However, more generally one can have generational dependence and string based analyses indeed show such a dependence. From the phenomenological viewpoint, these generational dependences cannot be arbitrary but are severely constrained by flavor changing neutral currents (FCNC). However, it is possible to introduce significant amounts of non-universalities in the Higgs sector and in the third generation sector without upsetting the FCNC constraints. Thus, for example, Higgs sector non-universalities can be parametrized at the GUT scale so that

$$
m_{H_1}^2 = m_0^2(1 + \delta_1), m_{H_2}^2 = m_0^2(1 + \delta_2)
$$

(46)

where a reasonable range for $\delta_i$ is $|\delta_i| \leq 1$. Similarly one can parametrize the non-universalities in the third generation sector.
5.2 \( p \) Decay in mSUGRA

We give here a numerical analysis of the maximum \( p \) lifetime for the minimal supergravity model. The upper bounds on the proton lifetime for different values of \( m_0 \) as a function of the gluino mass are given in Fig. 1. These upper limits are gotten by integrating over the allowed range of the parameter space, i.e., \( \tan \beta \) and \( A_0 \) for fixed values of \( m_0 \) and \( m_{\tilde{g}} \), consistent with radiative breaking of the electro-weak symmetry. The analysis is carried out under the constraint on the Higgs triplet mass such that \( M_{H_3} \leq 10 M_G \). The theoretical upper bounds may be compared with the current experimental lower bound on the \( p \to \bar{\nu}K^+ \) mode of

\[
\tau(p \to \bar{\nu}K) > 5.5 \times 10^{32} \text{ yr}
\]

and the lower limits that the super Kamiokande (SuperK) hopes to achieve in the future for this mode which is

\[
\tau(p \to \bar{\nu}K) > 2 \times 10^{33} \text{ yr}
\]

The analysis shows that the current lower limit on the \( \bar{\nu}K^+ \) mode does not exhaust the parameter space for the naturalness limit of \( m_0 \leq 1 \) TeV. However, most of the parameter space for this naturalness limit will be exhausted when SuperK reaches its expected sensitivity of \( 2 \times 10^{33} \text{ yr} \).
6 B and L Violation in Non-minimal Models

In this section we will discuss B and L violation in various non-minimal models such as the \((SU(3))^3\) models, unflipped and flipped no-scale models, and SO(10) models.

6.1 \((SU(3))^3\) Models

\((SU(3))^3\) models arise naturally in many Calabi-Yau string model constructions which below the compactification scale have the gauge structure \(E_6 \times N = 1\) supergravity. After Wilson line breaking, the \(E(6)\) can break to \((SU(3))^3\) and one can make contact with low energy physics. The most studied models of this type are the three generation models such as \(CP^3 \times CP^3 / Z_3\), and \(CP^2 \times CP^3 / Z_3 \times Z'_3\). In models of this type the massless sector of the theory falls into \(27 + \bar{27}\) of \(E_6\). The \(27\)-plet decomposes under \(SU(3)_C \times SU(3)_L \times SU(3)_R\) so that (see, e.g, refs. 36, 37, 38),

\[
27 = L(1, 3, 3) + Q(3, \bar{3}, 1) + Q^c(3, 1, 3)
\]

and one has a similar decomposition of \(\bar{27}\) into \(\bar{L} + \bar{Q} + \bar{Q}^c\). The particle content of \(L, Q, Q^c\) is as follows

\[
L = (l, H, H', e^c, \nu^c, N); \quad Q = (q, D); \quad Q^c = (q^c, D^c)
\]

where \(l, q, \ldots\) etc are the \(SU(2)_L\) doublets and \(D\) and \(D^c\) are the \(SU(2)_L\) singlet quarks. The coupling structure of the theory is given by

\[
(27)^3 = \lambda_1 duD + \lambda_2 u^c d^c D^c + \lambda_3 [-HH'N - H\nu^c l + H' e^c l]
+ \lambda_4 [N D^c e^c + D^c u^c D^c + q l D^c + q l u^c - q H u^c - q H' d^c] (51)
\]

There are several sources of proton instability in the these. We list the two dominant sources among these: (a) From B and L violating interactions which contain a \(D\) or a \(D^c\). They lead to nucleon decay from the B and L violating dimension 5 operators after D and \(D^c\) terms are eliminated; (b) \(D - d\) mixing arising from \(N\) and \(\nu\) VEV growth. This interaction provides the other dominant source of \(p\) decay in this theory. The superpotential with the \(D-d\) mixing after \(N\) and \(\nu\) VEV growth has the form

\[
W_{3-D-d}^v = [M_1 DD^c + M_2 Dd^c + M_3 dd^c] (52)
\]

Diagonalization of the mass matrix leads to interactions which lead to proton decay.
Superstring models of Calabi-Yau type with $(SU(3))^3$ gauge group can generate neutrinoless double beta decay ($\beta\beta^0\nu$) after R parity violation. Thus as discussed $\nu^c$ VEV growth violates lepton number and R parity and one can obtain an effective Lagrangian of the type

$$L_{\text{eff}} = g\bar{\chi}_1^0 e_L d_u^a$$

Since $\chi_1^0$ is Majorana, the interaction allows a neutrinoless double beta decay. The predictions give

$$g^2/m_{\chi_1^0} < 10^{-15}\text{GeV}$$

while the current experimental limits correspond to

$$g^2/m_{\chi_1^0} < 10^{-13}\text{GeV}$$

Thus the predicted level of double beta decay in Calabi-Yau models lies below the current level of sensitivity of experiment by about two orders of magnitude.

6.2 $P$ Decay in No Scale Models

An important sub-class of supergravity models are the so-called no scale models. In such models the F term contribution to the scalar potential of the models, i.e.,

$$V = e^{-G[\frac{1}{2}G_{AB}G^{AB} - 3]} + \frac{1}{2}ReF_{\alpha\beta}^{-1}D^\alpha D^\beta$$

vanishes because one in on an Einstein manifold. In a class of no-scale models one has $m_0 = A_0$ and SUSY breaking is driven by the universal gaugino mass term $m_{1/2}$. In this class of models one typically finds $m_{\tilde{q}} < m_{\tilde{g}}$. Unfortunately unflipped no-scale models of the above type violate experimental p lifetime limits in the interesting part of the parameter space since proton decay is governed roughly by the ratio $m_{\tilde{g}}/m_{\tilde{q}}^2$, and the constraint $m_{\tilde{q}} < m_{\tilde{g}}$ tends to destabilize the proton.

6.3 Proton Decay in the Flipped SU(5) Models

In the flipped SU(5) model some particles assigned to the $\bar{5}$-plets ($\bar{5}_i; i = 1, 2, 3$) and the 10-plets ($T_i; i = 1, 2, 3$) are flipped, i.e.,

$$u^c \leftrightarrow d^c, \quad e^c \leftrightarrow \nu^c$$

Thus one has three generations of matter of the type

$$\bar{F}_i = (u^c_i, L_i = (e_i, \nu_{e_i})); \quad T_i = (q_i = (u_i, d_i), d^c_i, \nu^c_i); \quad e^c_i; \quad i = 1 - 3$$
The Higgs sector consists of the usual one pair of $5 + \bar{5}$ of Higgs $h, \bar{h}$, i.e.,

$$h = (H, D), \quad \bar{h} = (\bar{H}, \bar{D})$$

and two pairs of $10, \bar{10}$ of Higgs $H_i, \bar{H}_i (i=1,2)$ so that

$$H_i = (q_{H_i}, d_{cH_i}, \nu_{cH_i}), \quad \bar{H}_i = (q_{\bar{H}_i}, d_{c\bar{H}_i}, \nu_{c\bar{H}_i}); \quad i = 1, 2$$

The effective potential in this theory is

$$W = \lambda_1 TT h + \lambda_2 T \bar{F} h + \lambda_3 \bar{F} l^e h + \lambda_4 H H h + \lambda_5 H \bar{H} \bar{h} + \lambda_6 \bar{F} \bar{H} \phi$$

where $\phi$'s are the singlet fields. Symmetry breaking occurs via VEV formation of $H, \bar{H}$ fields

$$< \nu_{H_i}^c >= M_i, < \nu_{\bar{H}_i}^c >= \bar{M}_i$$

The Higgs doublet mass matrix arises from

$$\mu h \bar{h} \rightarrow \mu H \bar{H}, \quad \delta_2 H \bar{F} h \rightarrow \delta_2 M L \bar{H}$$

and thus has the matrix form (with column labelled by $\bar{H}$ and rows by $H$ and $L$):

$$M_{doub} = \left( \begin{array}{c} \mu \\ \delta_2 M \end{array} \right)$$

The usual scenarios consider the hierarchy: $M_i \sim \bar{M}_i \sim M >> M' >> \mu$. The light Higgs is gotten by setting $\delta_2 = 0$. Regarding the Higgs triplet sector, in addition to the $D$ and $\bar{D}$ one also has color triplets from the $10$ and $\bar{10}$ of Higgs. Thus there is mixings between the $D$ and $\bar{D}$ terms and the color triplet terms from the $10$ and $\bar{10}$ of SU(5). After symmetry breaking mass contributions arise as follows:

$$\mu h \bar{h} \rightarrow \mu D \bar{D}$$

$$\delta_1 T F h \rightarrow \delta_1 M d^c D, \lambda_4 T T h \rightarrow M d^c H$$

$$\lambda_5 H \bar{H} h \rightarrow \lambda_5 M d^c \bar{H}, \lambda_5 M' H \bar{H} \rightarrow M' d^c H$$

Often one sets $\delta_1 = 0$. With these assumptions the mass matrix in the Higgs triplet sector is (columns: labelled by $\bar{D}, d^c_H$; rows: labelled by $D, d^c_H$):

$$M_{trip} = \left( \begin{array}{cc} \mu & \lambda_4 M \\ \lambda_5 M & M' \end{array} \right)$$
Diagonalization gives the Higgs triplets a superheavy mass $O(M)$. The effective dimension five operator arising from $\mu D \bar{D}$ is of size

$$\mu/M^2 << 1/M$$

(67)

Thus in the flipped model the dimension five operators which mediate proton decay would be suppressed by a $\mu/M^2$ rather than the usual $1/M$ factor as in the standard SU(5) model. However, it is possible to upset this suppression in some cases.

6.4 Proton Decay in SO(10) Models

SO(10) models have several interesting features. One of these is that SO(10) allows for the doublet-triplet splitting via VEV alignment. To achieve this one considers a Higgs sector of the type

$$W_{d-t} = 10_1 45_1 10_2 + 10^2 \phi$$

(68)

where $10_1$ is the Higgs that couples with matter and $\phi$ is a singlet. $45_1$ gets a VEV $O(M_G)$ in the B-L direction giving the mass matrices

$$M_{doub} = \begin{pmatrix} 0 & 0 \\ 0 & \phi \end{pmatrix}$$

(69)

$$M_{trip} = \begin{pmatrix} 0 & a_1 \\ -a_1 & \phi \end{pmatrix}$$

(70)

The Higgs doublet is light but the higgs triplets develop an effective mass

$$M_{PD}^{-1} = M_{11}^{-1} = \phi/a_1^2$$

(71)

In proton decay analyses in SO(10) models one has in addition to the matrix element $\beta_p$ defined by Eq.(35) also the matrix element $\alpha_p$ defined by

$$\alpha_p U^*_{\ell L} = \epsilon_{abc} \epsilon_{\alpha \beta} <0| \bar{d}^a_{uL} \bar{u}^\beta_{bL} u^\gamma_{cL} |p>$$

(72)

where $|\alpha_p| = |\beta_p|$ but the phase of $\alpha_p/\beta_p$ remains to be specified. This phase can affect proton decay rates.

SO(10) models generally involve large $\tan \beta$. Because of that there are some features specific to these models. Thus, for example, one finds that the gluino contributions can be very significant and comparable to the contributions from the chargino exchange. Further, it is found that the contributions of the dimension six LLRR operators may be comparable to or even dominate the
Figure 2: Plot in the $\sin^2\theta_W - \alpha_3(M_Z)$ plane for various $M_{PD}$ values. The dashed line is for the case of the lower bound on $M_{PD} = 1.2 \times 10^{16}$ GeV in the minimal SU(5) model. The dot-dashed line corresponds to the lower bound on $M_{PD} = 2.7 \times 10^{17}$ GeV in the minimal SO(10) model. (Taken from ref.[45]).

LLLL contribution in these models. However, there is a potential problem in SO(10) regarding proton stability vs unification of gauge couplings using LEP data. To see the problem one notices that the mass scale necessary to suppress p decay to the current experimental value is

$$(M^{-1})_{11} > \tan\beta(0.57 \times 10^{16}) \text{ GeV}$$

which for $\tan\beta \sim 50$ requires a GUT mass of $2.5 \times 10^{17}$ for suppression of p decay to the current experimental limit. However, a mass scale of this size upsets unification of gauge couplings and one needs large threshold corrections to get agreement with experimental values. Fig.2 illustrates the problem of the high scale in SO(10). One finds from Fig.2 that for the experimental value of $\sin^2\theta_W(M_Z)_{\overline{MS}}$ which is 0.23122 $\pm$ 0.00002 the value of $\alpha_s$ corresponding to $M_{PD} = 2.7 \times 10^{17}$ is many standard deviations away from the current experimental limit of $\alpha_s(M_Z) = 0.118 \pm 0.003$. Thus as stated large threshold corrections are necessary to get consistency. There are a great variety of SO(10) models, and details of the model affect critically the nature of supersymmetric signals. Thus in a class of SO(10) models discussed recently the charged lepton modes of the proton such as $l^+ \pi^0$, $l^+ K^0$ and $l^+ \eta$ ($l = e, \mu$), can become prominent. There are also other approaches in the non-minimal extensions where proton decay is correlated directly with the ansatz on the Yukawa couplings of the dimension five operators.
7 Effects of Textures on P Decay

Gut models give poor predictions for quark-lepton mass ratios. In SU(5) \( m_b/m_{\tau} \) is in good agreement with experiment but \( m_s/m_{\mu} \) and \( m_d/m_e \) are not. One needs textures, i.e.,

\[
W_d = H_1 A^E e^c + H_2 d^c A^D q + H_3 w^c A^U q
\]

where \( A^E, A^D, \& A^U \) are the texture matrices. The simplest choice for these are the the Georgi-Jarlskog choice

\[
A^E = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad A^D = \begin{pmatrix} 0 & F e^{i\phi} & 0 \\ F e^{-i\phi} & E & 0 \\ 0 & 0 & D \end{pmatrix}
\]

\[
A^U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}
\]

where A-F have a hierarchy of the type

\[
A \sim O(1), \quad B, D \sim O(\epsilon) \\
C, E \sim O(\epsilon^2), \quad F \sim O(\epsilon^3), \quad \epsilon \ll 1
\]

A possible origin of the parameter \( \epsilon \) is from the ratio of mass scales, e.g.,

\[
\epsilon = \frac{M_{GUT}}{M_{str}}
\]

In the context of supergravity unified models this ratio can arise from higher dimensional operators. In the energy domain below the string scale after integration over the heavy modes of the string one has an effective theory of the type

\[
W = W_3 + W_4 + W_5 + ...
\]

where \( W_n(n > 3) \) are suppressed by the string (Planck) scale and in general contain the adjoints which develop VEVs \( \sim O(M_{GUT}) \). After VEV formation of the heavy fields

\[
W_n \sim O\left(\frac{M_{GUT}}{M_{string}}\right)^{n-3} \times \text{operators in } W_3
\]

With the above one can generate mass hierarchy with \( \lambda_{yuk} \sim O(1) \). One can also compute textures in the Higgs triplet sector.
\[ W_i = H_1 B^E q + H_2 u^c B^U e^c + \epsilon_{abc}(H_1 d^c B^D u^c + H_2 u^c C^U d^c) \] 

(81)

However, the Planck scale expansion is not unique and consequently the Higgs triplet texture are not unique, and a dynamical principle is needed to achieve uniqueness. One such principle is the assumption of an exotic sector wherein fields in a minimal vector like representation couple to both the hidden sector fields and fields in the physical sector. If the exotic fields gain superheavy masses their elimination will lead to a specific set of Planck scale interactions. Such an assumption indeed leads to unique Higgs triplet textures of the form:

\[ B^E = \begin{pmatrix} 0 & aF & 0 \\ a^*F & \frac{16}{3} E & 0 \\ 0 & 0 & \frac{2}{3} D \end{pmatrix} \] 

(82)

\[ B^D = \begin{pmatrix} 0 & -\frac{8}{27} F & 0 \\ -\frac{8}{27} F & -\frac{4}{3} E & 0 \\ 0 & 0 & -\frac{2}{3} D \end{pmatrix} \] 

(83)

\[ B^U = \begin{pmatrix} 0 & \frac{4}{3} C & 0 \\ \frac{4}{3} C & 0 & -\frac{2}{3} B \\ 0 & -\frac{2}{3} B & A \end{pmatrix} \] 

(84)

\[ C^U = B^U, \ a = (-\frac{19}{27} + e^{i\phi}) \] 

(85)

Inclusion of textures gives a moderate modification of p decay branching ratios. The p lifetime for the $\bar{\nu}K^+$ mode is enhanced by a factor of $\sim (\frac{9}{30} m_\mu)^2$. p decay modes hold important information on GUT physics. Further, the textures affect in a differential way the various decay modes which in turn can be used to test theories of textures.\[\text{[27][31]}\]

8 Effects of Dark Matter on Proton Stability

In this section we discuss the effects of dark matter constraints on proton stability. There is now a great deal of convincing evidence for the existence of dark matter in the universe and most of this dark matter must be non-baryonic. In supersymmetric theories with R parity invariance the least massive supersymmetric particle (LSP) is absolutely stable and hence a candidate for dark matter. A priori there are many possible LSP candidates for such non-baryonic dark matter in supersymmetry such as the gravitino, sneutrino, neutralino, etc. However, detailed analyses show that over most of the parameter space of the SUGRA models, it is the lightest neutralino ($\chi_1$) which is the LSP and thus a
candidate for cold dark matter. The LSP neutralino is an admixture of four
neutral states, i.e.,
\[ \tilde{\chi}_1 = n_1 \tilde{W}_3 + n_2 \tilde{B} + n_3 \tilde{H}_1 + n_4 \tilde{H}_2 \]  
(86)
where \( \tilde{B} \) is the Bino, \( \tilde{W}_3 \) is the neutral Wino, and \( \tilde{H}_1, \tilde{H}_2 \) are the neutral Higgsinos. In the scaling region when \( \mu^2 >> M_Z^2 \) one finds
\[ n_1 \sim -\frac{M_Z^2}{2m_{\chi_1}\mu} \sin 2\theta_W \sin \beta, \]
\[ n_2 \sim 1 - \frac{1}{2} \frac{M_Z^2}{\mu^2} \sin^2 \theta_W, \]
\[ n_3 \sim -\frac{M_Z}{\mu} \sin \theta_W \sin \beta, \]
\[ n_4 \sim \frac{M_Z}{\mu} \sin \theta_W \cos \beta \]  
(87)
The above analysis shows that the LSP is mostly a Bino in the scaling region
which is most of the parameter space of the model. In the region when \( \mu \) is
small scaling breaks down, and one can have a large Higgsino component for
the LSP. However, recent analyses show that this possibility may be close to
being eliminated.\(^5\)

8.1 Cosmological Constraints

The amount of dark matter in the universe puts rather stringent constraints
on supersymmetric models\(^6,7\). These constraints arise from the allowed range
of \( \Omega_{\chi_0}h^2 \) where \( \Omega_{\chi_0} = \rho_{\chi_0}/\rho_c \). Here \( \rho_{\chi_0} \)
is the neutralino matter density, and \( \rho_c \) is the critical matter density
\[ \rho_c = 3H_0^3/8\pi G_N = 1.88h_0^2 \times 10^{-29} \text{gm/cm}^3 \]  
(88)
and \( h_0 \) is the Hubble parameter \( H_0 \) in units of 100 km/s.Mpc. The number
density of \( \chi_1 \)'s obeys the Boltzman equation
\[ \frac{dn}{dt} = -3Hn - <\sigma v>(n^2 - n_0^2) \]  
(89)
where \( <\sigma v> \) is the thermal average of the neutralino annihilation cross-
section and \( v \) is the relative neutralino velocity. At the ”freeze-out” temperature \( T_f \) the \( \chi_1 \) decouple from the background, and integration from \( T_f \) to

\[ \Omega_{\chi_1} h^2 \sim 2.48 \times 10^{-11} \left( \frac{T_f}{T_{\gamma}} \right)^3 \left( \frac{T_{\gamma}}{2.73} \right)^3 \frac{N_f^{1/2}}{J(x_f)} \]  
(90)
where \( J(x_f) = \int_0^{x_f} dx (\sigma v)(x) GeV^{-2} \), \( N_f \) is the number of massless degrees
of freedom at the freezeout, \( x_f = kT_f/m_{\chi_1} \), \( T_{\gamma} \) is the current background
temperature, and \((T_{\tilde{\chi}_1^0} / T_\gamma)^3\) is the reheating factor. In the analysis below we shall assume the relic density constraint of

\[
0.1 \leq \Omega_{\chi_1^0} h^2 \leq 0.1
\]

which encompasses a wide range of cosmological models. The imposition of the dark matter constraint reduces the proton lifetime by a very significant amount for values of the gluino mass greater than about 500 GeV\(^{35}\). The reduction factors as a function of the gluino mass are listed in Table 2 (taken from ref.\(^{35}\)). One finds that the reduction factors lie in the range 10-30. Taking into account corrections due to Yukawa textures, and other uncertainties in the calculations, SuperK should be able to test the region \(m_{\tilde{g}} > 500\) GeV when it achieves a sensitivity of \(1 \times 10^{33}\). These results are exhibited in Fig.3 for the minimal supergravity model.

8.2 Effects of Proton Lifetime Constraint on Event Rates

The imposition of proton lifetime constraint has an important effect on the event rates in dark matter analysis\(^{55}\). One finds that in addition to the drastic reduction in the allowed range of the gluino mass, there is also a very significant reduction in the magnitude of the maximum event rate curves. In Fig.4 we exhibit the maximum event rate as a function of the neutralino mass. We find
Figure 4: Plots of the the maximum and the minimum of event rates for the scattering of neutralinos off germanium target as a function of the neutralino mass. The relic density constraint $0.1 < \Omega h^2 < 0.4$ is imposed. The solid curves are for the case of $m_0 \leq 1$ TeV and without $p$ decay constraint. The dashed curves are for the case with the same relic density constraint, but with $m_0 \leq 5$ TeV, and under the proton lifetime constraint $\tau(p \rightarrow \bar{\nu}K) > 1 \times 10^{32}$ yr. The minimum dashed curve coincides with the lower solid curve. [taken from Ref.[35]].

that the reduction of the maximum event rate curve can be a factor of 10 to 100. This reduction arises from the fact that the proton lifetime constraint eliminates certain parts of the parameter space which give rise to large event rates.

| gluino mass (GeV) | reduction factor when $0.1 < \Omega h^2 < 0.4$ |
|------------------|-----------------------------------------------|
| 500              | 29.5                                          |
| 550              | 23.2                                          |
| 600              | 18.6                                          |
| 650              | 15.3                                          |
| 700              | 12.9                                          |
| 750              | 11.3                                          |
| 800              | 9.8                                           |

9 Proton Stability in Gauge Mediated Breaking of Supersymmetry

In the simplest model of gauge mediated supersymmetry breaking (GMSB), one has that SUSY breaking arises from VEV formation of a chiral superfield $\hat{S}$ which couples to messenger chiral superfield $(\phi_i, \bar{\phi_i})$ in vector like represent-
tations of the MSSM group with a superpotential of the form
\[ W = \sum C_i S \phi_i \bar{\phi}_i \]  
(92)
where in SU(5) one has \((\phi, \bar{\phi}) = (5, \bar{5}), \text{ or } (10, \bar{10})\). The information on the breaking of supersymmetry is communicated by gauge interactions to the physical sector. Masses for the gauginos \((\tilde{M}_a)\) arise at the one loop level via gauge interactions, i.e.,
\[ \tilde{M}_a = \frac{\alpha_a}{4\pi} \Lambda, \quad \Lambda = \frac{< F >}{< S >} \sim 10^2 \text{TeV} \]  
(93)
where \(\sqrt{F}\) is the SUSY breaking scale, \(\Lambda\) is the cutoff and typically \(\sqrt{F} \sim \Lambda\). Masses for the scalars arise at the two loop level, i.e,
\[ \tilde{m}_i^2 = \sum_a 2C_{iF}^a \left( \frac{\alpha_a}{4\pi} \right)^2 \Lambda^2 \]  
(94)
where \(C_{iF}\) are the Casimir co-efficients for the field \(i\).

There are many extensions of this simplest GMSB version including models with the messenger fields in incomplete multiplets. However, in these scenarios it is difficult to generate \(\mu\) and \(B\mu\) terms (where \(B\mu\) is the value of \(B_0\mu_0\) at the electro-weak scale), and one needs non-gauge interactions for their generation. Further, because of the low value of supersymmetry breaking scale \(\sqrt{F}\) in these theories, i.e., \(\sqrt{F} \sim 10^2 \text{ TeV}\), the gravitino is the LSP with a mass \(m_{3/2} < 1 \text{ Kev}\). Thus the gravitino cannot be a CDM in these models.

There are significant constraints that arise in these models in a GUT framework. First it is noted that the unification of the gauge couplings using the LEP data along with the \(b - \tau\) unification puts severe constraints on the model. This type of constraint, however, can be softened by inclusion of the Planck scale corrections as in Ref. However, it is further found that the limits on \(\tan\beta\) from proton stability and the limits on it from radiative breaking of the electro-weak symmetry under the constraint that the bilinear and the trilinear soft couplings vanish at the messenger scale eliminate all GUT models considered in ref except for one or two isolated cases.

10 Planck Effects and Proton Decay
In the R-G analysis of the gauge coupling constants \(\alpha_i\) there is an overlap of GUT threshold effects and of the Planck scale effects. For example, for SU(5) and for \(Q \sim M_G\) one has
\[ \alpha_i^{-1}(Q) = \alpha_G^{-1} + C_{iG} \ln \left( \frac{M_G}{Q} \right) + \frac{eM}{2M_P} \alpha_G^{-1} n_i \]  
(95)
where $M_a$ are the superheavy GUT thresholds, and $n_i = (-3, -1, 2)$ for the SU(2), U(1) and SU(3)$_C$ sectors. It is easily seen that the Planck effects characterized by $n_i$ can be absorbed in the GUT thresholds by a rescaling. Thus for the minimal SU(5) model rescaling gives

$$\alpha_i^{-1}(Q) = \alpha_i^{eff-1} + C_{ia} \ln \left( \frac{M_a^{eff}}{Q} \right)$$  \hspace{1cm} (96)

where

$$M_a^{eff} = M_a e^{k_a C_P} ; k_a = (-\frac{3}{5}(\Sigma), \frac{3}{10}(V), 5(H_3))$$  \hspace{1cm} (97)

and where

$$\alpha_i^{eff-1} = \alpha_i^{-1} - \frac{15}{2\pi} C_P, \quad C_P = \frac{\pi e M}{\alpha_G M_P}$$  \hspace{1cm} (98)

While the RG analysis involves the effective parameters $M_V^{eff}$ and $M_{H_3}^{eff}$, the proton decay lifetime is determined by $M_V$ and by $M_{H_3}$. Thus the RG analysis along with proton lifetime measurements can allow one to measure the size of the Planck correction, i.e., the value of $c$. For the $p \rightarrow \bar{\nu} K^+$ one finds

$$p \rightarrow \bar{\nu} K^+, \quad c = \frac{\alpha_G}{10} \frac{M_P}{\pi M_V} \ln \frac{M_{H_3}^{eff}}{M_V^{eff}}$$  \hspace{1cm} (99)

and for the $p \rightarrow e^+ \pi^0$ mode one finds

$$p \rightarrow e^+ \pi^0, \quad c = \frac{100}{3} \sqrt{2} \frac{\alpha_{3/2}^{3/2} M_P}{\pi M_V} \ln \frac{M_V}{M_{H_3}^{eff}}$$  \hspace{1cm} (100)

11 Exotic $p$ Decay Modes

$p$ decay modes discussed so far are all of the type where a proton decays into an anti-lepton and a meson, i.e.,

$$p \rightarrow e^+ \pi^0$$

$$p \rightarrow \bar{\nu} K^+ (\pi^+)$$

$$p \rightarrow \mu^+ K^0$$  \hspace{1cm} (101)

These decay modes arise in the minimal SU(5) and SO(10) models where the interactions obey the $B - L$ conservation. However, it is possible to include interactions where $B - L$ conservation is violated. Thus, for example, one may consider an interaction of the type

$$5_M \bar{5}_M \bar{1}_0 H$$  \hspace{1cm} (102)
which can generate $\Delta(B - L) = 2$ transitions. Such interactions will allow proton decay modes with a lepton and mesons such as

\[
\begin{align*}
d + d + s &\rightarrow \mu^- \\
n &\rightarrow \mu^- K^+ \\
p &\rightarrow \mu^- \pi^+ K^+, ..
\end{align*}
\]

(103)

Thus p decay modes distinguish among the varieties of GUT interactions and can provide important insights into GUT physics.

12 Connection with String/M Theory

Proton stability is a very strong constraint on string model building. Most string models contain interactions which violate R parity and can generate rapid proton decay. However, even with R parity invariance models with string GUTs will show generically the same typical B and L violating interactions that one has in ordinary GUT models. Recently, there has been an exhaustive study of string theories which perturbatively allow grand unification. One finds that indeed it is possible to find string models with interesting unified gauge groups such as SU(5), SO(10), E(6) etc. However, simultaneous satisfaction of other desirable properties such as $N = 1$ space time supersymmetry, three chiral families, and massless adjoints needed to break the gauge symmetry is not so easy, and one needs to go to models with higher Kac-Moody levels for their satisfaction. Recently, there has been a great deal of work on models of this type and explicit models at Kac-moody level three with the above properties have been constructed. Unfortunately, there are several problems of a phenomenological level still to be overcome before such models can become viable.

String Guts is one of the many ways in which one can reconcile LEP data and the unification of gauge couplings within string theory. Without string GUTS one will have the Standard Model gauge group emerging directly at the string scale, and to reconcile the LEP data on the gauge coupling constants with this high scale, one needs some extra effects, such as string thresholds and extra vector like representations at an intermediate state below the string scale.

Horava and Witten have suggested a new possibility for the unification of the gauge couplings within the framework of string/M theory. It is within the framework of the conjecture that the strongly coupled limit of the $E_8 \times E_8$ heterotic string is M theory on $R_{10} \times S_1/Z_2$ with gravity propagating in the bulk and the gauge fields living on the hyperplanes. If M theory compactifies on an $M_4 \times CY \times S_1/Z_2$ the unification of gauge coupling constants can arise
at the GUT scale with the MSSM spectrum with the appropriate identification of the Calabi-Yau compatification radius with the inverse of the GUT scale. One has in addition unification of gauge coupling constants with gravity arising at this scale because of an extra running of the gravitational coupling due to the opening up of the fifth dimension at a scale which is an order of magnitude below the GUT scale. This picture is very close to the supergravity GUT picture as far as the particle sector of the theory is concerned.

13 Conclusions/Prospects

LEP data appears to support ideas of both grand unification and of supersymmetry. Thus SUSY/SUGRA GUT may be an important way station to the Planck scale where unification of all interactions occurs. In SUSY/SUGRA GUTS one needs R parity invariance to eliminate B and L violating dimension 4 operators which lead to rapid proton decay. B and L violating dimension 5 operators of GUT models allow one to probe via proton decay a majority of the parameter space of the minimal SUGRA model within the naturalness constraint of \( m_0 \leq 1 \text{ TeV} \) and \( m_{\tilde{g}} \leq 1 \text{ TeV} \) if the Super-K and ICARUS experiments can reach the expected sensitivity of \( 2 \times 10^{33} \text{ y} \) and \( 10^{34} \text{ y} \) respectively for the \( \bar{\nu}K^+ \) mode. With the inclusion of dark matter constraints one finds that if the proton decay is not observed the gluino mass must lie below 500 GeV within any reasonable naturalness constraint. The simultaneous p stability and dark matter constraints will be tested in the near future in p decay experiments as well by experiments at the Tevatron, LEP2 and the LHC.

Acknowledgments

This work was supported in part by NSF grant number PHY-96020274 and PHY-9722090.

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