Power Controlled Adaptive Sum-Capacity of Fading MACs with Distributed CSI

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Abstract

We consider the problem of finding optimal, fair and distributed power-rate strategies to achieve the sum capacity of the Gaussian multiple-access block-fading channel. In here, the transmitters have access to only their own fading coefficients, while the receiver has global access to all the fading coefficients. Outage is not permitted in any communication block. The resulting average sum-throughput is also known as ‘power-controlled adaptive sum-capacity’, which appears as an open problem in literature.

This paper presents the power-controlled adaptive sum-capacity of a wide-class of popular MAC models. In particular, we propose a power-rate strategy in the presence of distributed channel state information (CSI), which is throughput optimal when all the users have identical channel statistics. The proposed scheme also has an efficient implementation using successive cancellation and rate-splitting. We propose an upperbound when the channel laws are not identical. Furthermore, the optimal schemes are extended to situations in which each transmitter has additional finite-rate partial CSI on the link quality of others.

I. INTRODUCTION

The multiple access channel (MAC) is a widely studied model in information theory, where many users communicate to a single entity using a shared medium. With its natural applications in wireless communications, the so called fading MAC with additive white Gaussian noise is one of the popular MAC models. In here, the channel from each user to the receiver is modeled by a multiplicative fading channel.

In order to find the rate-tuples at which reliable communication is possible over the fading MAC model, it is important to make assumptions about the amount of channel knowledge available at the transmitters and the receiver. It is natural to assume that the receiver has access to the fading coefficients, by means of pilot symbols. In other words, the receiver has full CSI. On the other hand, the same is not true about the transmitter. We consider a model where each transmitter is fully aware of its own fading coefficient (individual CSI), but that of no other. Towards the latter sections of this paper, we relax this assumption and equip the transmitter with finite-rate partial CSI of other links.

We consider a slow fading model, which is modeled by block fading: the fading coefficients are constant over a block of channel uses, over which the codeword lasts. The transmitters, thus, are not allowed to take advantage of the ergodic nature of the fading process during coding, but may employ adaptive power and rates. This particular situation is motivated by systems involving occasional (opportunistic) access to a shared medium, such as in a cognitive radio or a sensor network with a star topology. Here, multiple users wish to communicate their data to the receiver over the awarded time slot in a fair but distributed fashion. These systems may lack the global user coordination information to do conventional multiplexing strategies like TDM. However, some limited coordination information can be made available or gleaned from the network, for example, the total number of active users participating in a given slot. It is natural to look for within-block coding in these systems and demand that communication in each block to be outage-free, while allowing for adaptively controlling the power and transmission-rates based on the available channel knowledge. Furthermore, the power-adaptation strategy should respect the corresponding average transmit power at each of the users.

There is considerable literature on multiaccess fading channels with instantaneous CSI. The Shannon capacity of a Gaussian MAC with CSI available only at the receiver is evaluated rigorously in [1]. The optimal power control strategies to achieve capacity for the case of complete channel state information at the transmitters (CSIT) are given in [2] and [3]. Coming to partial side information at the transmitters, [4] gives the capacity region of a fading MAC under very general notions of CSI at the transmitters. These notions can be specialized to nearly all practical scenarios including individual transmitter CSI. However, our work differs from [4] due to the block-fading assumption and the requirement of no outage in each block. The ergodic averaging inherently used in evaluating the Shannon capacity region in [4] turns out to be essential because of the absence of complete CSI. Alternate notions of capacity motivated by different practical scenarios have also been investigated: delay-limited capacity for the fading MAC is dealt with in [3] while [5] defines the notions of expected capacity and capacity with outage for information unstable single-user channels. Other related works which consider partial CSI in a fading MAC setup are [6], where non-causal CSI is considered, and its generalization and unification with causal CSI in [7].

The model that we consider in this paper, i.e. block fading MAC with individual CSI, is applicable to a wide-range of situations. The model can effectively capture a MAC with random access, where the availability of a packet for transmission is indicated by binary fading states, available at the respective transmitters [8], [9], [10]. Building on this, [11] generalized
the fading MAC model and enforced an additional requirement that the probability of error approaches zero for every state realization, or block. Clearly, the underlying assumption of a sufficiently large block-length allows the construction of capacity achieving coding strategies, with error-probabilities vanishing in the blocklength. An important utility in this case is the throughput, which is the average data-rate over blocks. A single letter characterization in terms of the block-wise MAC capacity regions is provided in [11] for the DM-MAC and fading AWGN MAC. Averaging the possible communication rates over different state realizations will result in the adaptive capacity region in the presence of distributed CSI. In a Gaussian setting, the users can also adapt their power in addition to the rates [12]. The maximal sum-throughput in this case is named as the power controlled adaptive sum-capacity (see Section 23.5.2 of [12]), which is the maximal empirical average of the sum-rates achieved in each block. In [11], [12], it is also shown that the general power-controlled adaptive capacity region corresponds to a convex problem, and the sum-capacity is numerically determined for two-state fading MACs using convex programming techniques. The evaluation of adaptive sum-capacity for the popular Rayleigh fading is an open problem [11]. We present a solution to this problem in the current paper, which is almost closed form (in terms of a water-filling formula), along with several other interesting results. Recently, other approaches connecting random access and MAC with distributed CSI has been reported [13].

Another related work considers rateless coding in a distributed MAC set-up [14]. Here, a set of users try to convey their respective message, but the slot (block) duration is not fixed. Each user is unaware of the link quality of other users or even the number of users in the system. A communication round ends when all the messages are successfully recovered at the receiver. The receiver employs a feedback beacon signal for synchronizing the rounds of communication. Interestingly, it is demonstrated that the distributed nature of access results in negligible loss of throughput efficiency. As opposed to this, here we consider fixed slot-duration, and there is no feedback signal to the transmitters. Nevertheless, we are still able to construct a rate-splitting scheme, paving the way for low complexity throughput-optimal strategies, albeit in conjunction with an unconventional decoding architecture.

For most parts of the paper we consider identical channel statistics across users. This is a reasonable assumption in many practical systems. The strategies that we propose have a simpler structure when the average powers are also the same across users. We identify this special case by the name identical users. More precisely, throughout the paper, the usage identical users is synonymous with the following two constraints.

- the fading statistics are iid across users.
- each user has the same average power constraint.

While we state the results for arbitrary power constraints at the transmitters, we will first single out the identical users case, for the ease of exposition.

Our main results are summarized as follows:

1) We introduce a fair, simple and distributed policy called the ‘alpha-midpoint’ strategy for the Gaussian multiple-access block-fading channel.
2) The alpha-midpoint strategy achieves the power controlled adaptive sum-capacity when the channel statistics are identical.
3) We propose a low-complexity rate-splitting scheme that allows the alpha-midpoint strategy to be implemented through successive decoding.
4) For identical users, we compute the power-controlled adaptive sum-capacity when each user is additionally given some symmetric finite-rate partial CSI of the other links.
5) For non-identical channel statistics, an upper bound to the power-controlled adaptive sum-capacity is presented.

The organization of the paper is as follows. Section II will introduce the system model and some notations, and also defines the notion of power-controlled adaptive sum-capacity, which is our utility of interest. In Section III we introduce a communication strategy utilizing the available individual CSI, called the midpoint strategy. To implement the communication strategy with low complexity successive cancellation, we propose a rate-splitting strategy in Section IV along with an unconventional decoding architecture. The power-controlled adaptive sum-capacity is evaluated in Section III-B. Section V extends our results to the case where additional partial finite-rate side information on the other links is available at the transmitters. Bounds for non-identical channels are detailed in Section VI. We also describe some interesting connection between our model and the so called L-out-of-K MAC (LOOK channel), this is done in Section VII. Section VIII concludes the paper with a discussion of results and possible extensions.

II. System Model and Definitions

Consider \( L \) users communicating with a single receiver. These users transmit real-valued signals \( X_i \), encountering real-valued fades \( H_i \). If \( Y \) is the value of the received signal at a (discrete) time instant we have

\[
Y = \sum_{i=1}^{L} H_i X_i + Z
\]
where $Z$ is an independent Gaussian noise process. The fading space $\mathcal{H}_i$ of the $i$-th user is the set of values taken by $H_i$, and the joint fading space $\mathcal{H}$ is the set of values taken by the joint fading state $\tilde{H} = (H_1, H_2, \cdots, H_L)$. Similar vector quantities of user-wise parameters, like rate, power, channel state realization, will be denoted similarly, i.e., with an overbar symbol.

We consider a slow-fading model, where each channel coefficient stays constant within a block and varies across blocks in an i.i.d. fashion. While we demand reliable communication within each block, the utility that we consider is the average sum-throughput, or average sum-capacity, where the average is over different fading realizations or blocks. A more precise definition of our utility is given later in this section. In Gaussian channels, rate-expressions usually take a logarithmic form, and in this paper, all logarithms are expressed to the base of 2, that the rates we talk about are expressed as bits.

We assume that the (stationary and ergodic) fading processes $H_i$ are independent, and their distributions are known to all the transmitters and the receiver. In addition, we have individual CSIT, i.e. each transmitter knows its own channel fading coefficient $H_i$ but that of no other. The receiver knows all the channel coefficients. The transmitters have individual average power constraints, i.e.

$$\int_{h} P_i(h) d\Psi_i(h) \leq P_i^{avg}, \quad 1 \leq i \leq L,$$

where $\Psi_i(\cdot)$ is the cumulative channel law (cdf) of user $i$. The users can adapt the rate (and power) according to their own channel conditions. Apart from the notation changes, our model and objectives are similar in spirit to the those presented in [12] (see Section 23.5), [11]. In fact, the terminology power-controlled adaptive sum-capacity is borrowed from these references, which we explicitly compute in the present paper.

The adaptive nature of communication naturally leads to the following notion of a power-rate strategy.

**Definition 1.** A power-rate strategy is a collection of mappings $(P_i, R_i) : \mathcal{H}_i \mapsto \mathbb{R}_+ \times \mathbb{R}_+ ; i = 1, 2, \cdots, L$. Thus, in the fading state $H_i$, the $i$-th user expends power $P_i(H_i)$ and employs a codebook of rate $R_i(H_i)$.

Let $C_{MAC}(\bar{h}, \bar{P}(\bar{h}))$ denote the capacity region of a Gaussian multiple-access channel with fixed channel gains of $\bar{h} = h_1, \cdots, h_L$ and respective power allocations $\bar{P}(\bar{h}) = (P_1(h_1), \cdots, P_L(h_L))$. We know that,

$$C_{MAC}(\bar{h}, \bar{P}(\bar{h})) = \left\{ \bar{R} \in \{\mathbb{R}_+\}^L : \forall S \subseteq \{1, 2, \cdots, L\}, \sum_{i \in S} R_i \leq \frac{1}{2} \log \left(1 + \sum_{i \in S} |h_i|^2 P_i(h_i)\right) \right\}$$

**Definition 2.** We call a power-rate strategy as feasible if it satisfies the average power constraints for each user i.e. $\forall i \in \{1, 2, \cdots, L\}, \quad \mathbb{E}_{H_i} P_i(H_i) \leq P_i^{avg}$.

**Definition 3.** A power-rate strategy is termed as outage-free if it never results in outage i.e.

$$\forall \bar{h} \in \mathcal{H}, (R_1(h_1), \cdots, R_L(h_L)) \in C_{MAC}(\bar{h}, \bar{P}(\bar{h}))$$

Notice that the definition of outage-free implicitly makes use of the fact that the blocklength is sufficiently large for achievable strategies to reach acceptable error-probabilities. While a very large or infinite block-length may be required to drive the error vanishingly small, we save on the notation by the simpler definition of outage-free as given above.

Let $\Theta_{MAC}$ be the collection of all feasible power-rate strategies which are outage-free. Let us now specialize the definitions to the case of identical channel statistics, i.e. the cdf of each user is $\Psi(h)$. For any strategy $\theta \in \Theta_{MAC}$, the throughput is

$$T_\theta = \sum_{i=1}^{L} E[R^\theta_i(H_i)] = \sum_{i=1}^{L} \int_{h} R^\theta_i(h) d\Psi(h)$$

where the superscript $\theta$ is used to identify the feasible power-rate strategy employed, i.e. $R^\theta_i(h)$ is the rate allocated to user $i$ while observing fading coefficient $h$. The corresponding transmit power is denoted as $P^\theta_i(h)$.

**Definition 4.** The power controlled adaptive sum-capacity is the maximum (average) throughput achievable, i.e. $C_{sum}(\Psi) = \max_{\theta \in \Theta_{MAC}} T_\theta$.

One of the main results of the paper is the computation of the power-controlled adaptive sum-capacity for several popular fading models. In the special case of a single user channel ($L = 1$), the adaptive sum-capacity is well known, as it becomes a full CSI model. We denote the single user-capacity with an average power constraint of $P_a$ as $C_1(\Psi, P_a)$, which can be evaluated using a water-filling formula [15],

$$C_1(\Psi, P_a) = \frac{1}{2} \int_{h} \Psi(h) \log(1 + |h|^2 P^w(h))$$

where $P^w(h)$ is the water-filling power.
where
\[ P^*(h) = \left( \frac{1}{\lambda} - \frac{1}{|\hat{h}|^2} \right)^+ \quad \text{and} \quad \int d\Psi(h) P^*(h) = P_a. \] (6)

The single user water-filling formula is considered to be closed form for all practical purposes. Our results for the MAC with distributed CSI also take the form of similar water-filling formulas. Thus, we will re-use the notation \( C_1(\Psi, P_a) \) several times in this paper. Let us now focus on computing \( C_{\text{sum}}(\Psi) \) for \( L > 1 \).

### III. Power Controlled Adaptive Sum-capacity \( C_{\text{sum}}(\Psi) \)

Throughout this section, we consider identical fading statistics across all users of a \( L \)-user MAC with distributed CSI. The main result of this section is to compute the power-controlled adaptive sum-capacity \( C_{\text{sum}}(\Psi) \) for arbitrary \( \Psi(\cdot) \). We first state the main result and then explain its structure and implications, before providing the proof.

**Theorem 5.** Given independent and identically distributed channels according to the c.d.f \( \Psi(h) \),

\[ C_{\text{sum}}(\Psi) = C_1(\Psi, \sum_{i=1}^{L} P_i^{\text{avg}}). \] (7)

Before proving this, it is instructive to observe the structure of the result. The result states that \( C_{\text{sum}}(\Psi) \) is same as the capacity of a single user channel with cdf \( \Psi(\cdot) \) and average power \( \sum_{i=1}^{L} P_i^{\text{avg}} \). It has an element of surprise in the first look, as if there is some degeneracy in the problem statement. While this is not the case, the single user result essentially comes from the fact that communication has to be outage-free in every block. The result is restating that the worst joint distribution of the MAC fading-states is a highly correlated one, in which the same fading value is observed across users.

We propose a distributed strategy which achieves \( C_{\text{sum}}(\Psi) \), termed as alpha mid-point strategy in this paper. In order to clearly present the ideas behind this scheme, we first consider a special case of the model. The proof of Theorem 5 is detailed in Section [III-B]. The special case that we consider now is that of identical users, i.e. users having identical channel statistics and similar average powers.

#### A. Identical Users and Mid-point Strategy

For identical users with individual CSI, TDMA is a natural scheme for communication. Let us first compute the achievable rates for TDMA and then consider possible alternatives.

1) **Plain TDMA:** In plain TDMA the transmitters employ a simple ‘taking turns’ policy. Each block is divided into sub-blocks with only one user transmitting in that sub-block. This requires some extra coordination such as agreeing on an ordering for the users. The channels for the users are now orthogonal and they may water-fill over their own sub-blocks to improve throughput. Thus, we obtain the power-rate strategy corresponding to plain TDMA as:

\[ P_i(h_i) = \left( \frac{1}{\lambda_i} - \frac{1}{|h_i|^2} \right)^+ \]
\[ R_i(h_i) = \frac{1}{2L} \log \left( 1 + L|h_i|^2 P_i(h_i) \right) \]

where \( \lambda_i \) is chosen such that \( \mathbb{E}_{H_i} P_i(H_i) = P_i^{\text{avg}} \). The actual power employed by the user in its sub-block is \( LP_i(H_i) \) and the full transmission rate supported thereby is chosen.

2) **The Midpoint Rate Strategy:** We now present an appealing alternative to TDMA, which also has some advantages over TDMA. This strategy is distributed in nature, and we call it the midpoint strategy. Consider a block in which the fading vector is \( h_1, h_2, \ldots, h_L \) and let the respective powers be \( P_1(h_1), P_2(h_2), \ldots, P_L(h_L) \) (as part of some feasible power strategy). Each user assumes that all others are identical to itself (in terms of fading coefficients and transmit powers) and constructs the symmetric MAC region based on this assumption. It then chooses the maximal equal-rates point for operation. Thus we have

\[ R_i^{\text{mid}}(h_i) = \frac{1}{2L} \log \left( 1 + L|h_i|^2 P_i(h_i) \right) \] (8)

**Lemma 6.** The midpoint rate strategy is outage-free, i.e.

\[ \forall \hat{h} \quad R_i^{\text{mid}} \in C_{\text{MAC}}(\hat{h}, \bar{P}). \]
Proof: The lemma follows directly from the concavity of the logarithm function, i.e. \( \forall S \subset \{1, 2, \cdots, L\} \),

\[
\sum_{i \in S} R_{i, \text{mid}}(h_i) = \frac{1}{2L} \sum_{i \in S} \log (1 + L|h_i|^2 P_i(h_1)) \\
\leq \frac{1}{2 |S|} \sum_{i \in S} \log (1 + |S|h_i^2 P_i(h_i)) \\
\leq \frac{1}{2} \log \left( 1 + \sum_{i \in S} |h_i|^2 P_i(h_i) \right).
\]

As \( P_1, \cdots, P_L \) are arbitrary, the users’ power strategies can now be decoupled. The best power strategy for each user would thus be to water-fill over its own channel and we obtain

\[
P_i(h_i) = \left( \frac{1}{\lambda_i} - \frac{1}{|h_i|^2} \right)^+
\]

where \( \lambda_i \) is chosen such that \( \mathbb{E}_{H_i} P_i(H_i) = P_i^{\text{avg}} \). In addition to its apparent simplicity, it also turns out that the midpoint strategies achieve the power-controlled adaptive sum-capacity for identical users - a special case of Theorem 5.

Notice that the throughput achieved by the midpoint strategy is identical to that achieved by plain TDMA. We compare this in Figure 2 with the optimal opportunistic-TDMA (O-TDMA) possible in the presence of complete CSIT [2]. The figure shows the throughputs of the two schemes for a normalized Rayleigh fading channel, corrupted with AWGN of unit gain. The users are assumed to have identical average powers. The advantage of plain TDMA over the midpoint strategy is its simplicity in decoding, since only \( L \) single-user decoders are needed. However, the price for this is paid in the extra coordination required to set up an ordering for transmission between users. The midpoint strategy avoids this coordination, albeit at the cost of incurring joint decoding. We show in the next section that this cost can be ameliorated through rate-splitting and successive decoding.

B. Unequal Average Powers

We now prove Theorem 5 in two steps. First, we construct an upperbound to \( C_{\text{sum}}(\Psi) \). The second step generalizes the mid-point strategy to construct an achievable scheme which meets the proposed upper-bound.

1) An Upperbound: Let us consider a single link with cdf \( \Psi(h) \), let \( \Theta_s(P) \) be the collection of all single-user power allocation schemes \( \{P_1(h)\} \) such that

\[
\int P_1(h) d\Psi(h) = \tilde{P}.
\]

Let \( P_{\text{sum}} = \sum_{i=1}^{L} P_i^{\text{avg}} \), also recall the definition of throughput in (4).

Lemma 7. The throughput \( T_\theta \) obeys,

\[
T_\theta \leq C_1(\Psi, P_{\text{sum}}), \forall \theta \in \Theta_{\text{MAC}}.
\]
Lemma 8. The alpha-midpoint strategy is outage free.

Proof: For any $S \subseteq \{1, 2, \ldots, L\}$,

$$\sum_{i \in S} R_i^\alpha(h_i) = \sum_{i \in S} \alpha_i \frac{1}{2} \log \left( 1 + |h_i|^2 \frac{P_i(h_i)}{\alpha_i} \right)$$

by concavity of the logarithm. Clearly the chosen rate tuple across users is within $C_{MAC}(\tilde{h}, \tilde{P}(\tilde{h}))$ for every block, ensuring that there is no outage.

We now show the optimality of alpha-midpoint schemes.

Lemma 9.

$$\max_{\theta \in \Theta_{MAC}} T_\theta = C_1(\Psi, P_{sum})$$

Proof: We will specialize our alpha-midpoint strategy to achieve $C_1(\Psi, P_{sum})$. To this end, choose for $1 \leq i \leq L$,

$$\alpha_i = \frac{P_{i\text{avg}}}{\sum_{i=1}^L P_{i\text{avg}}}$$

and $P_i(h) = \alpha_i P^\alpha(h)$.

\[ \text{Fig. 2. The midpoint strategy is only a constant off the full CSI bound.} \]

Proof:

$$T_\theta \overset{\alpha}{\leq} \frac{1}{2} \int_h d\Psi(h) \log \left( 1 + |h|^2 \sum_{i=1}^L P_i^\theta(h) \right) \overset{(11)}{\leq} \max_{\theta \in \Theta_{MAC}} \frac{1}{2} \int_h d\Psi(h) \log \left( 1 + |h|^2 P(h) \right). \quad (12)$$

Here (a) follows from (4), by applying the sum-rate upper bound on a MAC with received signal power $\sum_i |h_i|^2 P_i^\theta(h)$. The second inequality results from relaxing the individual power constraints to a single average sum-power constraint.

It is clear that water-filling of the inverse fading gains is the optimal strategy in a point to point fading channel under an average power constraint. Thus the last expression above is indeed $C_1(\Psi, P_{sum})$.

2) Alpha-midpoint strategy: The alpha-midpoint strategy is a generalization of the midpoint scheme that we introduced earlier. Let $\alpha$ be a vector of non-negative values with $\sum_i \alpha_i = 1$. In alpha-midpoint strategy, the rate chosen by user $i$ while encountering a fading coefficient of $h_i$ is,

$$R_i^\alpha(h_i) = \alpha_i \frac{1}{2} \log \left( 1 + |h_i|^2 \frac{P_i(h_i)}{\alpha_i} \right), \quad (13)$$

where $P_i(h_i)$ is the transmitted power, chosen such that

$$\int P_i(h) d\Psi(h) = P_i^{\text{avg}}.$$
where $P^*(h)$ is given in (6), with $P_a$ replaced by $P_{sum}$. Notice that,

$$
\int P_i(h) d\Psi(h) = \alpha_i \int P^*(h) d\Psi(h) = \alpha_i P_{sum} = P_i^{avg}.
$$

Furthermore, by (13)

\[
\sum_{i=1}^{L} \int R_i^\alpha(h_i) d\Psi(h_i) = \sum_{i=1}^{L} \frac{\alpha_i}{2} \int \log(1 + |h|^2 \frac{P_i(h_i)}{\alpha}) d\Psi(h) = \sum_{i=1}^{L} \frac{\alpha_i}{2} \int \log(1 + |h|^2 P^*(h)) d\Psi(h) = \frac{1}{2} \int \log(1 + |h|^2 P^*(h)) d\Psi(h) \left( \sum_{i=1}^{L} \alpha_i \right) = C_1(\Psi, P_{sum})
\]

This completes the proof of Theorem 5.

### IV. Rate Splitting and Successive Decoding

The alpha mid-point strategies have a fairly simple structure. In this section, we show that these strategies can be implemented by low complexity successive decoding architectures. To this end, we present an asymptotically optimal rate-splitting strategy that mitigates the requirement of joint decoding, replacing the joint decoder with low complexity successive decoding architectures. To this end, we present an asymptotically optimal rate-splitting strategy

The length of each round is determined by a feedback link from the receiver, which announces the next round via a beacon. Our scenario requires that the communication occur within a fixed block or time slot, and there is no assumption of such a feedback link.

We will first construct rate-splitting schemes for identical users. Extensions to arbitrary average powers is done in a separate subsection. We will write the received signal power for user $i$, i.e. $P_i|h_i|^2$ as simply $\gamma_i$, throughout this section. Assume that the users have different (received) powers $\gamma_1, \gamma_2, \cdots, \gamma_L$. For simplicity, we will assume that the additive noise is of unit variance. The values of $\gamma_i$ may change with each block of communication depending on the individual fading conditions. Each user is unaware of the fade values and transmit powers of the rest of the users and, consequently, the interference they may cause.

The encoding and decoding are done as: each user splits itself into $L$ virtual users and splits its power, perhaps unequally, among these users. Each user can be visualized as a ‘stack’ of virtual users. For decoding, we use a successive cancellation based single-user decoder, which decodes one of the virtual users assuming all other virtual users which are not yet decoded are interference. It then chooses the rates $r_i^l$ by considering all the other virtual users in the same and lower layers as interference, i.e.,

$$
r_i^l = \frac{1}{2} \log \left( 1 + \frac{\gamma_i^l}{1 + (L - 1)\gamma_i^l + L \sum_{j=1}^{l-1} \gamma_j^l} \right).
$$

However, in the actual setting, the interference encountered from the other users are substantially different from that accounted for in the denominator of (17). So a layer by layer decoding may fail, as the virtual users are not chosen according to the actual channel conditions. Surprisingly, it turns out that this can be compensated by not strictly adhering to a layer by layer decoding. In particular, the receiver retains the freedom to decode the topmost hitherto undecoded layer of any transmitter, irrespective of the number of layers which were already decoded. It is, in fact, this freedom that allows the transmitters to choose the virtual rates without knowledge of interference from the other users.

**Lemma 10.** Assuming layer-wise rate allocation as per (17), it is always possible to find a virtual user which can be decoded correctly, i.e. with arbitrarily small error probability.
Proof: We prove this by induction. Assume that layers (virtual users) above \( l_k \) have been decoded for the \( k^{th} \) transmitter. Choose:

\[
\kappa = \arg \max_k \sum_{j=1}^{l_k} \gamma_j^k
\]

For user \( \kappa \), the remaining interference for decoding layer \( l_\kappa \):

\[
1 + \sum_{j=1}^{l_\kappa-1} \gamma_j^\kappa + \sum_{k \neq \kappa} \sum_{j=1}^{l_k} \gamma_j^k \leq 1 + \sum_{j=1}^{l_\kappa} \gamma_j^\kappa - \gamma_{l_\kappa}^\kappa
\]

\[
= 1 + \sum_{j=1}^{l_\kappa} \gamma_j^\kappa + (L-1) \sum_{j=1}^{l_\kappa} \gamma_j^\kappa
\]

The inequality follows directly from the choice of \( \kappa \). The RHS is the expected interference for the \( l_\kappa \)th virtual user of transmitter \( \kappa \). Thus, as the actual interference is less than the expected interference, this virtual user can be correctly decoded. In other words, the user with the ‘best’ received SNR can always be chosen for decoding.

Theorem 11. As \( N_v \to \infty \) and \( \forall j,l, \gamma^j_l \to 0 \), the rate achieved by all the users approach their midpoint rate.

Proof: Using (17) we have

\[
R_i = \sum_{j=1}^{N_v} \frac{1}{2} \log \left( 1 + \frac{\gamma_i^j}{1 + (L-1)\gamma_i^j + L \sum_{j=1}^{l_i-1} \gamma_i^j} \right)
\]

Under the given conditions, we can use the same method as in Lemma 1 of [14] to show that:

\[
\lim_{N_v \to \infty} R_i = \frac{1}{2} \int_0^1 \frac{dy}{1+L\gamma_i} = \frac{1}{2L} \log (1 + L \gamma_i)
\]

A. Unequal Average Powers

We will construct two levels of splitting in the presence of unequal average powers. In particular, we first split each user \( k \) into \( N_k \) virtual users, in such a way that each virtual user has an identical average transmit power constraint of \( P_v \), irrespective of the user index \( k \). Thus,

\[
\sum_{i=1}^{N_v} P_v = P_v^{avg}.
\]

Evaluating the maximal average rate for the \( L' = \sum_{k=1}^{L} N_k \) virtual users under the midpoint strategy of (8) will also yield \( C_1(\Psi, P_{sum}) \). To see this, notice that the total-rate obtained by the \( N_k \) layers of user \( k \) is

\[
N_k \frac{1}{L'} \int \frac{1}{2} \log(1 + |h|^2 P^*(h))d\Psi(h),
\]

where \( P^*(h) \) is the single-user water-filling allocation with an average power of \( L' P_v = \sum_{k=1}^{L} P_v^{avg} = P_{sum} \). Notice that \( N_k / L' \) is nothing but the \( \alpha_k \) in (16), proving that the above strategy can achieve the same rates as the alpha-midpoint scheme. Furthermore, since the midpoint rates are achievable by single user decoding techniques [17], alpha midpoint rates can also be achieved by low complexity schemes.

1sometimes the transmit-power of users may not be commensurate, however we can choose a slightly lower power level for some of the users, with negligible loss of performance.
V. Finite-rate CSI on other links

Up to this point, we have assumed only individual CSI. In this section, we wish to study the effect of additional partial information about the other links. To keep things simple, we consider identical users with the specified cdf $\Psi(h)$ and an average power of $P_{\text{avg}}$. Extensions to unequal average power constraints are possible, but not covered here. To start with, we consider 1 bit of additional partial CSI, i.e., each transmitter gets one bit of information from every other link, in addition to its own individual CSI. The individual fading components are assumed to be independently chosen. The additional link-information given to other transmitters is only a function of the fading parameter of this link. Thus, the model captures situations where the extra bit is obtained through transmitter cooperation or cribbing. It is crucial that the receiver has no say on the partial CSI. If the receiver decides the conveyed bit, then the throughput is same as that of the full CSI [2].

The partial CSI contains link quality information: let us assume it to be chosen from the set $\{G, B\}$, where we used $G$ for good and $B$ for bad. A natural separation between $G$ and $B$ is a link gain threshold. In particular, the partial CSI bit $\hat{h}_k$ of transmitter $k$ is

$$\hat{h}_k = \begin{cases} G & \text{if } |h_k| \geq h_T \\ B & \text{otherwise}, \end{cases}$$

for some fixed positive threshold $h_T$. By slight abuse of notation, we will say that link $j$ is in state $G$ (and call it good user), and denote the probability of that event by $\mu(G)$. Using the same token, $1 - \mu(G) = \mu(B)$. Let $C_{PSI}$ be the maximum attainable throughput with 1 bit additional CSI on each of the other links, along with individual CSI.

**Theorem 12.** For $L$ identical users,

$$C_{PSI} = C_1(\Psi^1, L P_{\text{avg}}),$$

where the cdf $\Psi^1(\cdot)$ is such that,

$$d\Psi^1(h) = d\Psi(h) \left( [\mu(B)]^{L-1} \mathbb{1}_{\{h \in B\}} + (1 + \zeta) \mathbb{1}_{\{h \in G\}} \right)$$

and the parameter

$$\zeta = \sum_{m=1}^{L-1} \binom{L-1}{m} [\mu(B)]^m [\mu(G)]^{L-1-m} \frac{m}{L-m}. \quad (20)$$

**Proof:** Recall the definition of $C_1(\cdot, \cdot)$ given in (5)–(6). We explain the proof for $L = 2$, which contains all the essential features. The proof is relegated to appendix A.

It is instructive to compare the advantages of 1 bit of extra CSI, which we demonstrate for a two user identical Rayleigh fading links of unit second moment, see Figure 4. The threshold value $h_T$ for 1-bit CSI was taken as unity.

One immediate question is the sensitivity of the results with respect to the fading threshold. We argue that it is not that crucial and the results are robust. In fact, taking the median of the fading distribution seems to be natural choice for many
models. Numerical results show that even for moderate power-levels the difference from the the best choice of the threshold is barely noticeable.

Extending to multiple bits of CSI is straightforward when the users are identical and the additional CSI bits are generated in a symmetric manner, i.e. $n - 1$ identical threshold values define the $\log_2 n$ bits of partial CSI about each link. For each $0 \leq m \leq n$, all users who experience a link-fading in the threshold bracket $[h_m, h_{m+1})$, $0 \leq m \leq n$ will form group $m$, where $h_0 = 0$ and $h_{n+1} = \infty$. An optimal strategy is to let all the users in group $m$ transmit at their respective midpoint rates, if there are no users in any group above $m$.

VI. NONIDENTICAL CHANNEL STATISTICS

Our second result is a generalization to non-identical channel statistics. In this case, we do not know the optimal schemes, but we provide an upperbound, which seems to be close for several channels of practical interest. W.l.o.g consider non-negative valued fading coefficients (by taking modulus), and let the respective cdf of the individual channels be $\{F_1(\cdot), F_2(\cdot), \cdots, F_L(\cdot)\}$.

We will assume each of them to be right continuous and define the corresponding inverse functions as

$$\forall \gamma \in [0, 1], \quad F_k^{-1}(\gamma) = \min h : F_k(h) \geq \gamma.$$  \hfill (21)

For convenience, we will denote $F_k^{-1}(\cdot)$ by $h_k^2(\cdot)$.

**Lemma 13.** For non-identical channels defined by the c.d.f.s $F_k(\cdot), 1 \leq k \leq L$, the maximal sum-rate $C_{\text{sum}}$ is bounded by

$$C_{\text{sum}} \leq \max_{g_{\text{MAC}}} \int_0^1 \frac{1}{2} \log \left( 1 + \sum_{k=1}^L h_k^2(x) P_k[h_k(x)] \right) dx.$$  \hfill (22)

**Proof:** Imagine that the range of each cdf $F_k, 1 \leq k \leq L$ in $[0, 1]$ is divided into $n$ equal segments. Let the inverse map of the $j^{th}$ segment of cdf $F_k$ be $h_k^2(j/n)$. The lemma states that for each segment $j$, the MAC formed by the corresponding inverse maps of this segment should obey the sum-rate constraint.

Notice that different channel values are coupled in the above bound (through their cdf structure), and we can maximize the power allocation on these coupled fading vectors, thus obtaining a bound to the RHS of (22). By using Lagrange optimization as in [2], we get the following lemma.

**Lemma 14.**

$$C_{\text{sum}} \leq \sum_{k=1}^L \int_0^1 \frac{1}{2} \log(1 + h_k^2(x) P_k(x)) \alpha_k(x) dx$$  \hfill (23)

where

$$P_k(x) = \left( \frac{1}{h_k^2(x)} - \frac{1}{h_k^2(x)} \right)^+ \quad \text{and} \quad \int_0^1 P_k(x) \alpha_k(x) dx = P_k^{\text{avg}}.$$  

In here, $\alpha_k(x)$ are non-negative functions such that $\sum_{k=1}^L \alpha_k(x) = 1, \forall x \in (0, 1).$
Proof: Maximizing (22) over all coupled channel vectors will yield the bound in (23). The detailed proof is available in appendix B.

While the existence of \( \lambda_k \) and the functions \( \alpha_k(x) \) is enough for the proof, numerical algorithms are required to find these, except for special cases. One such case where the algorithm is straightforward is when the channel coefficients are generated by the same law, but scaled by different average gains. In this case, \( \alpha_k(x) = \frac{P_{avg}^k}{\sum P_{avg}^k} \) for all \( x \in (0,1) \) and the water-filling formula can be evaluated using single-user water-filling. For example, consider a 2–user Rayleigh faded MAC, with \( E|h_2|^2 = 2E|h_1|^2 = 2 \), and \( P_{avg}^1 = P_{avg}^2 \). Figure 5 compares our upperbound against the rates resulting from an adaptation of the alpha-midpoint strategy. The lower bound is obtained by considering a symmetric Rayleigh fading channel with \( E|h|^2 = 2 \) and \( \alpha_1 = \frac{1}{3} \) and \( \alpha_2 = \frac{2}{3} \). For this strategy, the sum-power is taken as \( P_{avg}^1 + P_{avg}^2 \).

![Fig. 5. Upper and Lower bounds to adaptive sum-capacity](image)

This not only demonstrates the utility of our upper-bound, but also that the alpha mid-point strategy is a good scheme.

VII. CONNECTIONS WITH THE LOOK CHANNEL

We will show the suitability of our strategies in presence of individual CSI to the so called LOOK MAC setting. The LOOK channel is a multiple access model which models the variability of the active user set. In particular, of the \( K \) available transmitters, at most \( L \) are active in a communication epoch. The communication scheme should accommodate any allowed choice of the active user set. The capacity region of a discrete LOOK MAC model is known, see [18] for history and other details. Specifically, a \( K \)– dimensional rate-vector is termed as achievable, if for every subset \( S \) of indices having cardinality at most \( L \), the rate-coordinates of \( S \) belong to the corresponding \( |S| \)– user capacity region. Notice that in the discrete memoryless case, the capacity region is non-enlarging with \( K \), and it can possibly decrease due to the additional requirements of the incoming users. Furthermore, strategies like time-sharing are not viable here, as there is no proper coordination between the users. In reality, the users may not even know the identity of each other. Computation of the capacity region for even moderate \( K \) seems infeasible due to the curse of dimensionality.

We will now consider the Gaussian LOOK channel and extend it to a block-fading setup. In a block-fading LOOK channel, each communication epoch corresponds to a block. There are \( K \) users in the system, of which \( L \) are chosen uniformly at random. Communication has to be outage-free in every block or epoch. There is individual CSI at the transmitters and we consider identical users with power \( P_{avg} \) and fading cdf \( \Psi(h) \). The sum throughput \( C_{LOOK} \) for this system is given by,

\[
C_{LOOK} = \frac{1}{2} \int \log (1 + Lh^2 P(h)) d\Psi(h),
\]

where

\[
P(h) = \left( \frac{1}{\lambda} - \frac{1}{|h|^2} \right)^+ \text{ and } \int P(h) d\Psi(h) = \frac{K}{L} P_{avg}.
\]

The proof follows in a straightforward manner by each active-user employing its midpoint rate for communicating, as in (8). We do not repeat the steps here. Notice that for any fixed \( L \), the throughput goes unbounded with \( K \), contradictory to what one expect in a discrete memoryless case. This is due to the fact that each active user can scale his/her power to make up for its inactivity period.
We will show the proof for a

**Proof of Theorem 12**

interesting to relax this assumption and compare the performance, in terms of non-identical users or asymmetric CSI. While channels are also straightforward. The current paper focussed on the adaptive sum-capacity, which actually is a step towards situations, we are currently extending our work to take care of asymmetric channel statistics. Further extensions to MIMO states be

As for the third and fourth terms, the information on who has the better channel is readily available to both parties here. Let us consider the first term in the summation of the right hand side. By suitably integrating, it can be written as a single integral,

\[ R_1 + R_2 = \int_{G \times G} \left( R_1(h_1, \hat{h}_2) + R_2(\hat{h}_1, h_2) \right) d\Psi(h_1, h_2) + \int_{B \times B} \left( R_1(h_1, \hat{h}_2) + R_2(\hat{h}_1, h_2) \right) d\Psi(h_1, h_2) \]

\[ + \int_{B \times G} \left( R_1(h_1, \hat{h}_2) + R_2(\hat{h}_1, h_2) \right) d\Psi(h_1, h_2) + \int_{G \times B} \left( R_1(h_1, \hat{h}_2) + R_2(\hat{h}_1, h_2) \right) d\Psi(h_1, h_2). \]  

(25)

Consider the first term in the summation of the right hand side. By suitably integrating, it can be written as a single integral,

\[ \mu(G) \int_G \left( R_1(h, G) + R_2(G, h) \right) d\Psi(h) \leq \frac{\mu(G)}{2} \int_G \log \left( 1 + h^2(P_1(h, G) + P_2(G, h)) \right) d\Psi(h), \]

which is the sum-rate bound of the corresponding MAC. Similarly, for the second term,

\[ \mu(B) \int_B \left( R_1(h, B) + R_2(B, h) \right) d\Psi(h) \leq \frac{\mu(B)}{2} \int_B \log \left( 1 + h^2(P_1(h, B) + P_2(B, h)) \right) d\Psi(h). \]

(27)

As for the third and fourth terms, the information on who has the better channel is readily available to both parties here. Let us now consider only those channel states \((h_1, h_2) \in \{(G \times B) \cup (B \times G)\}\). Let the average power expenditure on these channel states be \(P_{GB}\). Suppose we relax our assumption, and give full CSI to each transmitter whenever one of the links is in state \(G\) and the other in \(B\). Furthermore, let us enforce only a average sum-power constraint of \(P_{GB}\) in these states. In such a system, only the better user transmits with an appropriate power \[2\]. This fact can be utilized along with \(26\) and \(27\) to equivalently write the maximum throughput as

\[ J^* = \max \frac{\mu(B)}{2} \int_B \log \left( 1 + h^2(P_1(h, B) + P_2(B, h)) \right) d\Psi(h) + \frac{\mu(G)}{2} \int_G \log \left( 1 + h^2(P_1(h, G) + P_2(G, h)) \right) d\Psi(h) \]

\[ + \frac{\mu(B)}{2} \int_B \left( P_1(h, B) + P_2(B, h) \right) d\Psi(h) + \frac{\mu(G)}{2} \int_G \left( P_1(h, G) + P_2(G, h) \right) d\Psi(h) \leq 2P_{avg}. \]

where the maximization is over \(P_1(\cdot, \cdot)\) and \(P_2(\cdot, \cdot)\). Furthermore, the original individual power constraint is relaxed to an average sum-power constraint of the form,

\[ \mu(B) \int_B \left( P_1(h, B) + P_2(B, h) \right) d\Psi(h) + \mu(G) \int_G \left( P_1(h, G) + P_2(G, h) \right) d\Psi(h) + \mu(B) \int_B \left( P_1(h, B) + P_2(B, h) \right) d\Psi(h) \leq 2P_{avg}. \]

Notice that our integration now is over just one variable. Let us denote,

\[ P_B(h) = \frac{P_1(h, B) + P_2(B, h)}{2} \quad \text{and} \quad P_G(h) = \frac{P_1(h, G) + P_2(G, h)}{2}. \]
By the concavity of logarithm, the maximization can be bounded in terms of the new variable as

\[
J^* \leq \max \frac{\mu(B)}{2} \int_B \log \left( 1 + h^2 2P_B(h) \right) d\Psi(h) + \frac{\mu(G)}{2} \int_G \log \left( 1 + h^2 2P_G(h) \right) d\Psi(h) + \frac{\mu(B)}{2} \int_G 2\log(1 + h^2 P_B(h))d\Psi(h). \tag{29}
\]

The power constraint, in the new notation, is

\[
\mu(B) \int_B 2P_B(h)d\Psi(h) + \mu(G) \int_G 2P_G(h)d\Psi(h) + \mu(B) \int_G 2P_B(h)d\Psi(h) \leq 2P^{avg}. \tag{30}
\]

Let the RHS of (29) be denoted as \(J^{**}\). Further simplification is possible by treating the variable \(h\) as one which belongs to a single-user channel with appropriate distribution and an average power of \(2P^{avg}\).

**Lemma 16.** For 2 identical-users with individual cdf \(\Psi(\cdot)\), the maximal throughput with partial CSI is \(C_1(\Psi', 2P^{avg})\), where

\[
d\Psi'(h) = \begin{cases} 
  d\Psi(h)\mu(B) & \text{if } h \in B \\
  d\Psi(h)(1 + \mu(B)) & \text{if } h \in G
\end{cases}
\]

**Proof:** First we show that

\[
C_1(\Psi', 2P^{avg}) \geq J^{**}.
\]

For the single user channel \(\Psi'(h)\), consider two power allocation schemes \(\hat{P}\) and \(\tilde{P}\) such that

\[
\hat{P}(h) = \begin{cases} 
  2P_B(h), h \in B \\
  2P_G(h), h \in G
\end{cases}
\]

and

\[
\tilde{P}(h) = \begin{cases} 
  2P_B(h), h \in B \\
  P_B(h), h \in G
\end{cases}
\]

If we use \(\hat{P}\) for a fraction \(\frac{\mu(G)}{1 + \mu(B)}\) of the times over \(\Psi'(h)\), and \(\tilde{P}\) for the remaining fraction, the throughput is

\[
\frac{\mu(B)}{2} \int_B \log(1 + h^2 P_B(h))d\Psi(h) + \frac{1 + \mu(B)}{2} \frac{\mu(G)}{1 + \mu(B)} \int_G \log(1 + h^2 P_G(h))d\Psi(h) + \frac{1 + \mu(B)}{1 + \mu(B)} \int_G 2\log(1 + h^2 P_B(h))d\Psi(h), \tag{34}
\]

which is indeed \(J^{**}\). Notice that an average power constraint of \(2P^{avg}\) is maintained under this allocation. Let us now show that \(C_1(\Psi', 2P^{avg})\) is indeed achievable for our MAC with partial CSI model. Let \(P'(h)\) be the optimal single-user power allocation for the channel \(\Psi'(h)\). Consider the following power allocation in (28).

\[
P_1(h, G) = P_2(h, G) = 0, \forall h \in B; \quad P_1(h, B) = P_2(B, h) = P'(h), \forall h \in G
\]

\[
P_1(h, G) = P_2(h, G) = \frac{P'(h)}{2}, \forall h \in G; \quad P_1(h, B) = P_2(B, h) = \frac{P'(h)}{2}, \forall h \in B
\]

The users will choose the midpoint rates whenever both users are either in \(B\) or in \(G\). In other cases, only the better user is active. Clearly the power constraints are met and the throughput is indeed \(C_1(\Psi', 2P^{avg})\).

For \(L > 2\) users, if there are \(K \geq 1\) links in \(G\), only those links with \(h_k \in G\) will transmit at their respective \(K\)–user mid-point rates. On the other hand, if no links are in \(G\), all \(L\) users transmit at their respective \(L\)–user mid-point rates. The power allocation can be effectively determined by single user water-filling of the cdf \(\Psi'(h)\) given in Theorem 12.

**APPENDIX B**

**Proof of Lemma 16** Notice that we assume arbitrary channel statistics for the links. The following proposition on MAC captures the essential idea behind the result.

**Proposition 17.** For a given fixed \(L\)–user MAC with link gains \(h_1, \ldots, h_L\) and respective average transmit powers \(P_1, \ldots, P_L\), the maximal sum-rate can be achieved by time-sharing.

**Proof:** Let user \(i\) transmit for a fraction of time \(\beta_i\) with power \(\frac{P_i}{\beta_i}\), at its single user capacity. By choosing \(\beta_i = \frac{h_i^2 P_i}{\sum_k h_k^2 P_k}\) we get,

\[
\sum_i R_i = \sum_{i=1}^L \frac{\beta_i}{2} \log(1 + \sum_{k=1}^L h_k^2 P_k), \tag{35}
\]
which is indeed the MAC sum-rate bound.

Let us now relax the maximization in (22). In particular, we replace \( P_k(h(x)) \) by \( P_k(\bar{h}(x)) \), where \( \bar{h}(x) \) is the global fading vector corresponding to the same c.d.f. value \( x \) at each transmitter. Thus our relaxed optimization problem is,

\[
\max \frac{1}{2} \int_0^1 \log(1 + \sum_k h_k^2(x) P_k(\bar{h}(x))) dx,
\]

such that

\[
\int_0^1 P_k(\bar{h}(x)) dx = P_{k,\text{avg}}, \forall k.
\]

By defining Lagrange multipliers, \( \lambda_i, 1 \leq i \leq L \), one for each constraint, we can equivalently maximize the cost \( J \), where

\[
J = \frac{1}{2} \int_0^1 \log(1 + \sum_k h_k^2(x) P_k(\bar{h}(x))) dx - \sum_k \lambda_k \int_0^1 P_k(\bar{h}(x)) dx.
\]

Taking derivative w.r.t to \( P_k(\cdot) \) and applying the boundary conditions

\[
\frac{1}{2} \frac{h_k^2(x)}{1 + \sum_{k=1}^L h_k^2(x) P_k(\bar{h}(x))} - \lambda_k \geq 0, 1 \leq k \leq L,
\]

where the inequality becomes equality for the active user-set (ones which are allocated non-zero power at a given value of \( x \)). Therefore, we can conclude that power is allocated to user \( j \) only if

\[
\frac{h_j^2(x)}{\lambda_j} \geq \frac{h_k^2(x)}{\lambda_k}, \forall i \neq j.
\]

Let \( \zeta(x) \) be the maximum value of \( \frac{h_j^2(x)}{\lambda_j} \) over \( 1 \leq j \leq L \). Each active-user will achieve \( \zeta(x) \). However, Proposition \( 17 \) will suggest that the active users can time share and achieve the sum-rate. The power chosen by an active user is

\[
P_i(h_i(x)) = \max \{0, \frac{1}{\lambda_i} - \frac{1}{h_i^2(x)} \}.
\]

The instantaneous received power from active user \( i \) while in its transmitting time-fraction is \( \frac{h_i^2(x)}{\lambda_i} - 1 \). However, the fraction of time given to each active user is dependent on the channel-laws and average powers. Thus \( \alpha_i(x) \) in \( 23 \) is the time-fraction for the active user \( i \), for a given set of channels determined by the cdf index \( x \). This concludes the proof of Lemma \( 14 \).

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