AN EXECUTIVE MODEL FOR NETWORK-LEVEL PAVEMENT MAINTENANCE AND REHABILITATION PLANNING BASED ON LINEAR INTEGER PROGRAMMING

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ABSTRACT. Although having too many details can complicate the planning process, this study involves the formulating of an executive model having a broad range of parameters aimed at network-level pavement maintenance and rehabilitation planning. Four decomposed indicators are used to evaluate the pavement conditions and eight maintenance and rehabilitation categories are defined using these pavement quality indicators. As such, some restrictions called “technical constraints” are defined to reduce complexity of solving procedure. Using the condition indicators in the form of normalized values and developing technical constraints in a linear integer programming model has improved network level pavement M&R planning. The effectiveness of the developed model was compared by testing it under with-and-without technical constraints conditions over a 3-year planning period in a 10-section road network. It was found that using technical constraints reduced the runtime in resolving the problem by 91%, changed the work plan by 13%, and resulted in a cost increase of 1.2%. Solving runtime reduction issues can be worthwhile in huge networks or long-term planning durations.

1. Introduction. Pavement Management Systems (PMS) help maintain pavement conditions at an acceptable level and provide the highest return on investments. A large portion of the annual budgets of transportation agencies is allocated to pavement maintenance and rehabilitation (M&R). Due to limited budgets, M&R work planning is critical involving work assignments to pavement sections at appropriate times. Setting objectives and developing a model for optimization purposes are important in the planning process. Additionally, including effective factors in the model is essential for efficient decision making.

Single-objective optimization for M&R work planning deals with a function in which one independent variable is optimized. However, its main drawback is its sub-optimal response compared to that obtained from multi-objective planning [15]. Several studies have been conducted on multi-objectives optimization in engineering. Pishvae & Razmi proposed a multi-objective fuzzy mathematical programming model for designing an environmental supply chain under inherent uncertainty of input data for minimizing multiple environmental impacts besides the traditional objective of cost minimization [18]. Another study developed a robust design for

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a multiproduct, multi-echelon, closed-loop logistic network model in an uncertain environment. The model included a general network structure that took into consideration both forward and reverse processes that could be used in various industries, such as electronics, digital equipment, and vehicles [19]. In addition, various pavement management studies have developed and utilized multi-objective optimization models such as [6, 25, 14]. Wu et al. developed an optimization model with two objectives: maximization of the network level of service and minimization of the total M&R cost [26]. Moreover, an optimization model was used within deterministic pavement management systems [27]. The model was aimed at determining the least-cost maintenance and rehabilitation strategy to be implemented in a road network, taking into account the applicable technical and budgetary constraints [3]. Another research developed a bi-objective deterministic optimization model which simultaneously satisfied the objectives of both minimization of total maintenance costs and maximization of performance of the road network [12]. Also, an optimization methodology for county paved roads has been devised that identifies the best mix of preservation projects within budget, maximizing traffic (passengers and trucks traffic) on treated roads, maximizing the weighted average PSI, and minimizing risk [20].

There are also various optimization approaches for developing decision-making models in PMS. The Analytic Hierarchy Process (AHP) is the simplest and most popular method for multi-objective M&R action prioritization [16, 17]. Others are the life-cycle cost analysis for probabilistic model using fuzzy logic [2] and the application of Artificial Neural Network (ANN) for solving multi-objective optimization problems in PMS [6, 4, 5, 11]. Also, an optimization tool based on a hybrid Greedy Randomized Adaptive Search Procedure (GRASP) has been developed. This tool facilitates the design of optimal maintenance programs subject to budgetary and technical restrictions and explore the effects of different budgetary scenarios on overall network conditions [28].

Integer programming employment for budgeting at network level is among the other useful methods for work planning in PMS [21]. Each method has its own advantages and drawbacks but since integer programming is the exact solution to an optimization problem, it is preferable for small networks where optimal solutions can be arrived at fairly quickly. Several studies utilized different types of integer programming and imposed various constraints to the objective function. A pavement management problem is formulated as a constrained integer linear programming model subject to budgetary and improvement-requirement constraints [1]. A comparison of the multi-year multi-constraint optimization methodology and the year-by-year multi-constraint technique using mixed integer programming in the US state of Virginia found the former to be superior [21]. Also, a case study in Indiana state identified the optimal pavement M&R strategy for a given section using a mixed-integer nonlinear programming model [9].

A comprehensive study by by Swei et al. identified decomposed condition indicators as an important study area in PMS that needs further investigation in model development [24]. Although numerous studies have been conducted on model development based on integer programming, most utilized compound condition indicators [23, 7, 13].

In this research, an integer programming-based model is formulated. In the model, in addition to incorporation of 4 separate condition indicators that each corresponds to a specific distress, one compound indicator is taken into account. Having
many decomposed indicators made direct application of specific constraints, namely technical constraints, into the model possible. These constraints specify the condition ranges under which performing the M&R actions is not justified. Generally, there are five M&R categories that most models can choose from, that is, localized safety; localized preventive; global preventive; major above critical pavement quality; and major below critical pavement quality [22]. In this research, eight M&R categories are defined using four pavement quality indicators reflecting thermal distress, structural, skid and roughness conditions, and another that reflects a specific combination of them. Clearly, this level of detail and its application in pavement M&R planning has led to increased model accuracy as well as a higher degree of problem difficulty and complexity. In previous studies, due to the complexity of the problem, especially in large scale issues or long term planning periods, less attention has been paid to increasing the number of details. Therefore, the development of a mixed integer programming model which can be used for a large-scale network or for long-term planning periods while also addressing the issue of a greater number of details is one of the innovations of this study. Moreover, with more details in the M&R action assignment process, it is expected that the network level planning becomes nearly as precise as project level planning. Also three separate objective functions including single and multi-objective ones are developed to compare the M&R cost and the pavement network quality that each function produces. So, M&R action assignments on the sections seek to minimize overall expenditure while satisfying the financial and technical criteria relating to available budgets and pavement quality and performance, respectively, by taking into account the defining constraints.

2. Literature review. In this section, a comparison of the applied methodologies mentioned above is summarized in Table 1. As can be seen, there are few studies that utilize more than one condition indicator. Accordingly, this study attempts to develop a model involving a larger number of condition indicators that allow the M&R planning process to be more accurate and precise.

3. Methodology. In this study, the following steps are taken to develop, run, and evaluate the work plan model at the network level. First, the required input and output parameters for the deterministic model are defined and three separate objective functions formulated using integer programming. Then, the response of each objective function for a case study is computed. Finally, the best objective function is determined based on the minimum overall cost.

3.1. Model development. The outputs of the work plan model suggest when and which treatment(s) should be applied on a section. Accordingly, there are three subscripts which correspond respectively to the M&R actions, the sections, and the time period (year) attributed to the response variable. As listed in Table 2, in addition to them, index \( b \), which is a transformation of pavement condition value to a value between 0 to 1 is used in the developed model to have the rounded sections quality with a precision of 0.1 taken into account. In the other word, the index \( b \) is used to discretize continuous values of variables and parameters. It is assumed that index \( t \) indicates the end of the time period (year).

As shown in Table 3, there are 8 categories of M&R actions with an ID number attributed to them.
### Table 1. Comparison of applied methodologies in investigated studies

| Study | Optimization Method | Level of Study | Formulation | Model Type | Condition Indicator |
|-------|---------------------|----------------|-------------|------------|---------------------|
| (TF Fwa, et al., 1996) | ✓ | ✓ | GA | Det | PSI |
| (TF Fwa, et al., 1998) | ✓ | ✓ | GA | Robust | PCI |
| (Smilowitz & Madanat, 2000) | ✓ | ✓ | LMDP | Condition State | Condition State |
| (TF Fwa, et al., 2000) | ✓ | ✓ | GA | Robust | PCI |
| (Ferreira, et al., 2002) | ✓ | ✓ | GA | Det | PSI, IRI, SN |
| (Chen & Flintsch, 2007) | ✓ | ✓ | LCPA | Fuzzy | PCI |
| (Wu, et al., 2008) | ✓ | ✓ | GP & AHP | Det | Condition State |
| (Aboza & Ashur, 2009) | ✓ | ✓ | CLIP | Det | PCR |
| (Wu & Flintsch, 2009) | ✓ | ✓ | MDP | Prob | Condition State |
| (Meneses & Ferreira, 2010) | ✓ | ✓ | GA | Det | PSI |
| (Mouzami, et al., 2011) | ✓ | ✓ | AHP | Fuzzy | PCI |
| (Irfin, et al., 2012) | ✓ | ✓ | MINLP | Prob | RRI |
| (Guo & Zhang, 2013) | ✓ | ✓ | Knapsack | Det | PCI |
| (McHarg & Madanat, 2013) | ✓ | ✓ | LP | Prob | Condition State |
| (Matthew & Isaac, 2014) | ✓ | ✓ | GA | Det | PCI |
| (Meneses & Ferreira, 2015) | ✓ | ✓ | LCCA | Det | PSI |
| (Sinha & Ksibiati, 2016) | ✓ | ✓ | GRASP | Det | PCI |
| (Swei, et al., 2016) | ✓ | ✓ | MINLP | Det & Prob | PCR |
| Current study | ✓ | ✓ | MILP | Det | \( \hat{q}(q_f, q_t, q_s, q_r), q_o \) |

MuOb - Multi Objective; SiOb - Single Objective; Net - Network; Pro - Project; Det - Deterministic; Prob - Probabilistic

### Table 2. List of indices

| Variable Index | Description |
|----------------|-------------|
| \( t, (t \in \{1,2,\ldots,T\}) \) | Period (year) |
| \( n, (n \in \{1,2,\ldots,N\}) \) | No. of Section |
| \( m, (m \in \{1,2,\ldots,M\}) \) | M&R actions category |
| \( b, (b \in \{1,2,\ldots,B\}) \) | Auxiliary index corresponding to condition |

The model parameters and variables are listed in Tables 4 and 5, respectively. The response \( (x) \) is a binary variable in which 1 indicates implementation of a treatment to a section at the end of a specific year and 0 means otherwise. It can be found in Table 5 that there are four independent indicators of \( q\hat{x}(q_f, q_t, q_s \text{ and } q_r) \) as well as a compound indicator of \( q_o \) to quantify pavement condition with respect to fatigue (structural) condition, thermal distresses, skid resistance, roughness, and a linear combination of them, respectively. Notably, the values of these indicators are normalized to be between 0 and 1.

Generally, there is the Administration Cost (AC) and User Cost (UC) included in work planning optimization models. The AC relates to M&R action expenses paid by the administration while UC comprises the sum of Vehicle Operation Cost (VOC), Delay Cost (DC), and Crash Cost (CC) that are indirect road-user costs [10]. In this research, VOC, which depends on the pavement quality, and DC, which is due to treatment type and the time duration of road closure, are used...
Table 3. List of M&R action categories

| ID No. | Action Category                                      | Policy          |
|--------|------------------------------------------------------|-----------------|
| 1      | Localized safety maintenance                         | Temporary       |
| 2      | Localized preventive maintenance                     | Preventive      |
| 3      | Level 1: Global preventive maintenance with the objective of improving thermal distresses | Preventive      |
| 4      | Level 2: Global preventive maintenance with the objective of improving skid resistance in addition to the level 1 objective | Preventive      |
| 5      | Level 3: Global preventive maintenance with the objective of surface irregularity correction in addition to improving the level 2 objective | Preventive      |
| 6      | Surface rehabilitation                               | Corrective      |
| 7      | Deep rehabilitation                                  | Corrective      |
| 8      | Reconstruction                                       | Corrective      |

in the developed model. Table 6 lists the types of influential costs on objective functions utilized for the cost analysis.

In this research, three separate objective functions are defined. Since the objective functions are cost-based, they should be minimized. The first function \( \text{ob}_1 \) has a similar process to conventional budget allocation in transportation agencies in Iran. The \( \text{ob}_1 \), as formulated in Eq (1), maximizes the condition-based benefit across the network and minimizes the differences between the available and required budget. Therefore, the most possible is spent from the available budget.

\[
\text{ob}_1 = \min \left[ \sum_{n=1}^{N} \sum_{t=1}^{T} v_{a_{n,t}} \times (1 - q_{o_{n,t}}) + \sum_{t=1}^{T} \left( b_{u_{t}} - \sum_{n=1}^{N} \sum_{m=1}^{M} c_{o_{n,m,t}} \times z_{n,m,t} \right) \right] \tag{1}
\]

The second function \( \text{ob}_2 \) minimizes the overall cost (M&R operating cost and user cost). In the \( \text{ob}_2 \), as formulated in Eq (2), the aim is to minimize the sum of AC, VOC, and DC.

\[
\text{ob}_2 = \min \left[ \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{t=1}^{T} \left( c_{o_{n,m,t}} \times z_{n,m,t} + c_{e_{n,m,t}} \times x_{n,m,t} \right) 
+ \sum_{n=1}^{N} \sum_{t=1}^{T} \left( c_{ub_{n,t}} \times c_{uq_{b_{n,t}}} - c_{b_{n,t}} \times \left( \frac{c_{ub_{n,t}} - c_{uc_{n,t}}}{c_{rq_{n}}} \right) \right) 
+ \left( c_{uc_{n,t}} \times c_{uq_{g_{n,t}}} - c_{g_{n,t}} \times c_{rq_{n}} \times c_{uq_{g_{n,t}}} \times \left( \frac{c_{uc_{n,t}} - c_{uq_{g_{n,t}}}}{1 - c_{rq_{n}}} \right) \right) \right] \tag{2}
\]

The third function \( \text{ob}_3 \) is a multi-objective function which minimizes the overall cost and maximizes pavement condition at the end of the analysis period. \( \text{ob}_3 \) expresses the \( \text{ob}_3 \) where, in addition to minimization of all costs, the maximization of pavement network condition at the end of the analysis period is going to be achieved. Since the objective function is to be minimized, the consumed pavement
### Table 4. Model parameters

| Variable | Description | Domain |
|----------|-------------|--------|
| $\mu$    | A large value (close to infinity) | $\mu \rightarrow \infty$ |
| $\epsilon$ | A small value (close to zero) | $\epsilon \rightarrow 0$ |
| $va_{n,t}$ | Pavement financial value of section $n$ at the year $t$ while is in the best condition | $va_{n,t} \in [0, \infty)$ |
| $drop\dot{x}_{n,t}$ | Condition drops from $t = 1$ to the target year of $t$ if no M&R action is performed | $drop\dot{x}_{n,t} \in [0, 1]$ |
| $dropv\dot{x}_{n,m,t,b}$ | Condition drops from the year of performing M&R action $(m)$ on a pavement section $(n)$ with condition index $b$ to the target year of $t$ | $dropv\dot{x}_{n,m,t,b} \in [0, 1]$ |
| $inq\dot{x}_n$ | Initial condition | $inq\dot{x}_n \in [0, 1]$ |
| $im\dot{x}_m$ | Condition improvement | $im\dot{x}_m \in [0, 1]$ |
| $cr\dot{x}_n$ | Lower threshold of condition | $cr\dot{x}_n \in [0, 1]$ |
| $crlo_{n,s}$ | Lower threshold of overall condition | $crlo_{n,s} \in [0, 1]$ |
| $crh\dot{x}_n$ | Upper threshold of condition | $crh\dot{x}_n \in [0, 1]$ |
| $crho_{n}$ | Upper threshold of overall condition | $crho_{n} \in [0, 1]$ |
| $but$ | Allocated budget | $but \in [0, \infty)$ |
| $co_{n,m,t}$ | M&R action cost (operating cost) | $co_{n,m,t} \in [0, \infty)$ |
| $crq_{n}$ | Critical condition in the vehicle operation cost (VOC) vs overall condition curve | $crq_{n} \in [0, 1]$ |
| $cuc_{n,t}$ | VOC at the critical condition | $cuc_{n,t} \in [0, \infty)$ |
| $cub_{n,t}$ | VOC at the worst condition (0) | $cub_{n,t} \in [0, \infty)$ |
| $cug_{n,t}$ | VOC at the best condition (1) | $cug_{n,t} \in [0, \infty)$ |
| $ce_{n,m,t}$ | Delay cost due to performing M&R actions | $ce_{n,m,t} \in [0, \infty)$ |
| $k\dot{x}$ | Coefficient of $\dot{x}$ condition in overall condition equation | $k\dot{x} \in [0, 1]$ |
| $c$ | Deterioration constant in overall condition equation | $c \in [0, 1]$ |

$\dot{x}$ can be replaced by $f, t, s$ or $r$ which respectively related to Fatigue, Thermal distress, Skid or Roughness.

Financial value ($va$) is defined and considered and which should be minimized in order to have maximized pavement condition at the end of the analysis period. In this way, a multi objective function has been changed to a single objective function by a linear combination of weights or a weighted sum method [8]. The ob3 was solved twice, with and without technical constraints, to assess the impact of them on runtime, overall cost, and M&R actions assignment.
Table 5. Variables

| Variable   | Description                                                                 | Domain                      |
|------------|-----------------------------------------------------------------------------|-----------------------------|
| $x_{n,m,t}$ | Binary variable for M&R action $m$, section $n$ and year $t$ in the M&R work planning | $x_{n,m,t} \in \{0,1\}$    |
| $q_{n,t}$  | Condition indicator                                                         | $q_{n,t} \in \{0,1\}$      |
| $w\dot{x}_{n,t,b}$ | Auxiliary variable of condition                                             | $w\dot{x}_{n,t,b} \in [0.5 - 10\epsilon, 1]$ |
| $k_w\dot{x}_{n,t,b}$ | Binary variable determining whether (1) or not (0) auxiliary variable of condition is in index $b$ | $k_w\dot{x}_{n,t,b} \in \{0,1\}$ |
| $cub_{n,t}$ | Binary variable determining whether (1) or not (0) the condition is in the poor zone in the VOC vs overall condition curve | $cub_{n,t} \in \{0,1\}$ |
| $cuq_{n,t}$ | Binary variable determining whether (1) or not (0) the condition is in the good zone in the VOC vs overall condition curve | $cuq_{n,t} \in \{0,1\}$ |
| $cb_{n,t}$  | Auxiliary variable for the effect of poor condition on the VOC              | $cb_{n,t} \in [0,1]$        |
| $cg_{n,t}$  | Auxiliary variable for the effect of good condition on the VOC              | $cg_{n,t} \in [0,1]$        |
| $qo_{n,t}$  | Overall condition indicator                                                | $qo_{n,t} \in [0,1]$        |
| $k_{n,m,t}$ | Intensity of distress variable                                             | $k_{n,m,t} \in [0,1]$       |
| $z_{n,m,t}$ | Auxiliary variable for intensity of distress                               | $z_{n,m,t} \in [0,1]$       |
| $dax_{n,t}$ | Condition drop                                                             | $dax_{n,t} \in [0,1]$       |
| $ddx_{n,t}$ | Auxiliary variable for condition drop                                      | $ddx_{n,t} \in [0,1]$       |
| $d\dot{x}_{n,m,t,t'}$ | Condition drop at $t$ once M&R action was performed at $t$ | $d\dot{x}_{n,m,t,t'} \in [0,1]$ |

$x$ can be replaced by $f, t, s$ or $r$ which respectively related to Fatigue, Thermal distress, Skid or Roughness.

Table 6. Types of influential costs used in this study

| ID | Cost                                                                 |
|----|----------------------------------------------------------------------|
| $ce$ | Delay cost due to performing M&R actions                             |
| $co$ | Operating cost                                                       |
| $cu$ | Vehicle Operation cost                                               |
| $va$ | Consumed pavement financial value compared to the highest pavement value |

\[
o3 = \min \left[ \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{t=1}^{T} (co_{n,m,t} \times z_{n,m,t} + ce_{n,m,t} \times x_{n,m,t}) \\
+ \sum_{n=1}^{N} \sum_{t=1}^{T} \left( (cub_{n,t} \times cuq_{n,t} - cb_{n,t} \times \left( \frac{cub_{n,t} - cuc_{n,t}}{crq_{n}} \right) \right) \\
+ \left( cuc_{n,t} \times cuqg_{n,t} - (cg_{n,t} - crq_{n} \times cuqg_{n,t}) \times \left( \frac{cuc_{n,t} - cuqg_{n,t}}{1 - crq_{n}} \right) - cuqg_{n,t} \right) \\
+ \sum_{n=1}^{N} va_{n,T} \times (1 - qo_{n,T}) \right] \] (3)
Equations (4) to (45) present the restrictions of the model. Equations (4) to (9) indicate the constraints of auxiliary variables of pavement condition. These equations convert the continuous values of the input parameters corresponding to (8) indicate the constraints of auxiliary variables of pavement condition. These equations convert the continuous values of the input parameters corresponding to boundary conditions (between 0 - 1) into discretized values with increments of 0.1.

$$w\dot{x}_{n,t,b} =$$

$$\frac{1}{0.1} \left( \max \left\{ \min \{\max\{0,(inq\dot{x}_n - drop\dot{x}_{n,t})\}\} - \varepsilon - 0.1 \times b + 0.1,0.1 \right\} \right) \forall n,b \quad t = 1 (4)$$

$$\frac{1}{0.1} \left( \max \left\{ \min \{\max\{0,(iq\dot{x}_{n,t-1} - da\dot{x}_n)\}\} - \varepsilon - 0.1 \times b + 0.1,0.1 \right\} \right) \forall n,t,b \quad t \neq 1 (5)$$

$$w\dot{x}_{n,t,1} =$$

$$\frac{1}{0.1} \left( \max \left\{ \min \{\max\{0,(inq\dot{x}_n - drop\dot{x}_{n,t})\}\} - \varepsilon,0.1 \right\} \right) \forall n \quad t = 1 (6)$$

$$w\dot{x}_{n,t,1} =$$

$$\frac{1}{0.1} \left( \max \left\{ \min \{\max\{0,(iq\dot{x}_{n,t-1} - da\dot{x}_n)\}\} - \varepsilon,0.1 \right\} \right) \forall n \quad t \neq 1 (7)$$

$$kw\dot{x}_{n,t,b} \geq (1 - w\dot{x}_{n,t,b}) \forall n,t,b (8)$$

$$kw\dot{x}_{n,t,b} \geq (1 - w\dot{x}_{n,t,b}) \times \mu \forall n,t,b (9)$$

Equations (10) to (13) define the constraints of pavement deterioration rate and yield the deterioration rate for each condition indicator.

$$d\ddot{x}_{n,t'} = (drop\ddot{x}_{n,t'} - drop\ddot{x}_{n,t' - 1}) + \mu \times \left[ \sum_{b \in B} \sum_{m=1}^{M} x_{n,m,t} \right] \forall n,t' (t' < t) (10)$$

$$d\ddot{x}_{n,m,t'} = \left[ \sum_{b \in B} (kw\dot{x}_{n,t,b} \times drop\dot{x}_{n,m,t' - t,b}) \right] + \mu \times (1 - x_{n,m,t}) \forall n,m,t,t' \quad (t' = t + 1) (11)$$

$$d\ddot{x}_{n,m,t'} = \left[ \sum_{b \in B} (kw\dot{x}_{n,t,b} \times drop\dot{x}_{n,m,t' - t,b}) \right] - \sum_{b \in B} (kw\dot{x}_{n,t,b} \times drop\dot{x}_{n,m,(t' - 1,b)}) \forall n,m,t,t' \quad (t' \neq t + 1) (12)$$
\[ \text{d} \hat{x}_{n,t'} = \min_{\forall m,t(t'<t')} \{ d \hat{x}_{n,m,t',s} , \text{d} \hat{x}_{n,t',s} \} \quad \forall n,t'(t' > 1) \quad (13) \]

Subsequently, Equation (14) shows the constraints of the pavement condition at each year and defines to compute the values of the pavement condition indicators at the end of each year.

\[
q \hat{x}_{n,t} = \begin{cases} 
\min \left\{ 1, \left( \max\{0, (inq \hat{x}_{n,t} - drop \hat{x}_{n,t})\} + \sum_{m=1}^{M} (im \hat{x}_{m} \times x_{n,m,t}) \right) \right\}, & \forall n, t \in \{1\} \\
\min \left\{ 1, \left( \max\{0, (q \hat{x}_{n,t-1} - \text{d} \hat{x}_{n,t})\} + \sum_{m=1}^{M} (im \hat{x}_{m} \times x_{n,m,t}) \right) \right\}, & \forall n, t \in \{2, \ldots, T\} 
\end{cases}
\]  

t (14)

Equation 15 is the constraint of the overall condition computation and gives the linear relation between the overall condition \((q_o)\) and condition indicators \((q_f, q_t, q_s, q_r)\).

\[
q_o_{n,t} = kf \times q_f_{n,t} + kt \times q_t_{n,t} + ks \times q_s_{n,t} + kr \times q_r_{n,t} + c \\
\forall n, t \quad (15)
\]

With the aid of Equations (16) to (18), the calculation of M&R costs based on distress intensity level is possible. As such, these equations define the constraints relating to the intensity of distress for localized maintenance application.

\[
k_{n,m,t} = \begin{cases} 
1 - (kf \times inq_f_{n,t} + kt \times inq_t_{n,t} + ks \times inq_s_{n,t} + kr \times inq_r_{n,t} + c); & m \in \{1, 2\}, \forall n, t \in \{1\} \\
1; & m \in \{3, \ldots, 8\}, \forall n, t \in \{1\} 
\end{cases}
\]  

t (16)

\[
k_{n,m,t} = \begin{cases} 
1 - qo_{n,t-1}; & m \in \{1, 2\}, \forall n, t \neq 1 \\
1; & m \in \{3, \ldots, 8\}
\end{cases}
\]  

t (17)

\[
z_{n,m,t} \leq x_{n,m,t} \\
z_{n,m,t} \leq k_{n,m,t} \\
z_{n,m,t} \leq k_{n,m,t} - (1 - x_{n,m,t}) \quad \forall n, m, t \quad (18)
\]

The constraint for budget allocation (Equation (19)) relates to the restriction on the annual budget.

\[
\sum_{m} \sum_{n} (co_{n,m,t} \times z_{n,m,t}) \leq bu_{t} \quad \forall t \quad (19)
\]

Equation (20) is the constraint that determines the applications of localized maintenance or corrective rehabilitation which is done based on cost comparisons. The equation imposes a restriction that recommends a single corrective M&R action.
instead of several localized preventive M&R actions if the cost of the former is less than the latter.

\[ c_{n,1,t} \times z_{n,1,t} \leq c_{n,m,t} \quad \forall n,t \quad m \in \{6,7,8\} \quad (20) \]

Equations (21) to (29) are used for user cost computations. The constraints of auxiliary variables for vehicle operation cost are defined by Equations (21) to (25). The constraints of vehicle operation cost are defined by Equations (26) to (29).

\[ cuqb_{n,t} \leq 1 - (kf \times invf_n + kf \times invt_n + ks \times inqs_n + kr \times inqr_n + c - crq_n) \quad \forall n,t (t = 1) \quad (21) \]

\[ cuqb_{n,t} \leq 1 - (qo_{n,t-1} - crq_n) \quad \forall n,t (t > 1) \quad (22) \]

\[ cuqq_{n,t} < 1 - (crq_n - (kf \times invf_n + kt \times invt_n + ks \times inqs_n + kr \times inqr_n + c)) \quad \forall n,t (t = 1) \quad (23) \]

\[ cuqq_{n,t} < 1 - (crq_n - qo_{n,t-1}) \quad \forall n,t (t > 1) \quad (24) \]

\[ cuqb_{n,t} + cuqq_{n,t} = 1 \quad \forall n,t \quad (25) \]

\[ cb_{n,t,s} = (kf \times invf_n + kt \times invt_n + ks \times inqs_n + kr \times inqr_n + c) \times cuqb_{n,t} \quad \forall n,t (t = 1) \quad (26) \]

\[ cb_{n,t,s} \leq cuqb_{n,t,s} \quad \forall n,t (t > 1) \quad (27) \]

\[ cb_{n,t,s} \leq qo_{n,t-1,s} \quad \forall n,t (t > 1) \quad (27) \]

\[ cb_{n,t,s} \geq qo_{n,t-1,s} - (1 - cuqb_{n,t,s}) \]

\[ cg_{n,t,s} = (kf \times invf_n + kt \times invt_n + ks \times inqs_n + kr \times inqr_n + c) \times cuqq_{n,t} \quad \forall n,t (t = 1) \quad (28) \]

\[ cg_{n,t,s} \leq cuqq_{n,t} \quad \forall n,t (t > 1) \quad (29) \]

\[ cg_{n,t,s} \leq qo_{n,t-1} - (1 - cuqq_{n,t}) \]

Equations (30) and (31) are the constraints for avoiding the application of more than one M&R action from a M&R policy in each year. These restrictions do not allow the application of more than one strategy from Table 3 within a year.

\[ x_{n,1,t} + x_{n,2,t} + x_{n,6,t} + x_{n,7,t} + x_{n,8,t} \leq 1 \quad \forall n,t \quad (30) \]

\[ \sum_{m} x_{n,m,t} \leq 1 \quad \forall n,t \quad m \neq 2 \quad (31) \]

In addition, extra constraints, or technical constraints, are defined according to the existence criteria in Table 7 (Equations (32) to (45)). These constraints determine
the appropriate limits for some threshold values which creates the zone for ignoring M&R action.

\[ crho_n + 1 - qo_{n,t-1} \geq x_{n,8,t} \quad \forall n, t(t > 1) \] (32)

\[ crho_n + 1 - (kf \times inqf_n + kt \times inqt_n + ks \times inqs_n + kr \times inqr_n + c) \geq x_{n,8,t} \quad \forall n, t(t = 1) \] (33)

\[ crlo_n + 1 - qo_{n,t-1} \geq x_{n,1,t} \quad \forall n, t(t > 1) \] (34)

\[ crlo_n + 1 - (kf \times inqf_n + kt \times inqt_n + ks \times inqs_n + kr \times inqr_n + c) \geq x_{n,1,t} \quad \forall n, t(t = 1) \] (35)

\[ qo_{n,t-1} \geq crlo_n \times x_{n,m,t} \quad \forall n, t(t > 1) \quad m \in \{2, 3, 4, 5\} \] (36)

\[ (kf \times inqf_n + kt \times inqt_n + ks \times inqs_n + kr \times inqr_n + c) \geq crlo_n \times x_{n,m,t} \quad \forall n, t(t = 1) \quad m \in \{2, 3, 4, 5\} \] (37)

\[ qf_{n,t-1} \geq crlf_n \times x_{n,m,t} \quad \forall n, t(t > 1) \quad m \in \{2, 3, 4, 5\} \] (38)

\[ inqf_n \geq crlf_n \times x_{n,m,t} \quad \forall n, t(t = 1) \quad m \in \{2, 3, 4, 5\} \] (39)

\[ crht_n + 1 - qt_{n,t-1} \geq x_{n,3,t} \quad \forall n, t(t > 1) \] (40)

\[ crht_n + 1 - inqt_n \geq x_{n,3,t} \quad \forall n, t(t = 1) \] (41)

\[ crhs_n + 1 - qs_{n,t-1} \geq x_{n,4,t} \quad \forall n, t(t > 1) \] (42)

\[ crhs_n + 1 - inqs_n \geq x_{n,4,t} \quad \forall n, t(t = 1) \] (43)

\[ crhr_n + 1 - qr_{n,t-1} \geq x_{n,5,t} \quad \forall n, t(t > 1) \] (44)

\[ crhr_n + 1 - inqr_n \geq x_{n,5,t} \quad \forall n, t(t = 1) \] (45)
Table 7. Limits of condition indicators for ignoring M&R action

| m  | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8          |
|----|------------|------------|------------|------------|------------|------------|------------|------------|
| qo | < crl <    | < crl *    | < crl      | < crl      | -          | -          | * * crh <  | -          |
| qf | -          | < crl      | < crl      | < crl      | < crl      | -          | -          | -          |
| qg | -          | crh <      | -          | -          | -          | -          | -          | -          |
| qs | -          | -          | crh <      | -          | -          | -          | -          | -          |
| qr | -          | -          | -          | crh <      | -          | -          | -          | -          |

* The < crl indicates the lower threshold value for a condition indicator in which performing M&R action over sections with values of less than that is not justified.
** The crh < indicates the upper threshold value for a condition indicator in which performing M&R action over sections with values greater than that is not justified.
- No limitations on performing M&R action.

3.2. **Problem solving procedure.** This study develops an integer programming-based model involving three separate objective functions including single and multi-objective ones. A multi-objective function is changed to a single objective function by a linear combination of weights or the weighted sum method. Gams software is used to solve the linear integer programming problem. The settings of the software are modified to meet the specifications of the solving algorithm of the work planning problem. This particular software is used due to its wide application and accurate problem-solving in linear models although any other software that can solve mathematical models is also suitable for the purpose of this research. The CPLEX algorithm is selected to solve the model since this study deals with Mix Integer Programming (MIP).

4. **Results and discussions.** Since the runtime for problem-solving is directly related to the length of the analysis period and the size of the pavement network, a small pavement network and short analysis period have been selected for evaluating the model and comparing the objective functions. A 3-year data collection on pavement conditions and M&R implementation was carried out covering 10 pavement network sections in Mashhad, Iran.

Table 8 indicates the M&R actions assigned to sections for each year based on section number and M&R action identity according to the numbers defined in Table 2. The reason why ob2 is not assigned any treatments to any section in the third year is because of the assumption on implementation of treatments at the end of each year. Clearly, performing M&R action improves pavement conditions from the end of each year on and has no impact on it within the same year. Also it is assumed that the term relating to VOC in ob2 is affected by pavement conditions at the beginning of the year. Therefore, the application of any action at the third year has no influence on the pavement condition or VOC at the beginning of the third year. Consequently, based on the objective function calculation, the lack of an M&R action assignment in the third year results in an overall cost reduction.

Fig 1 shows each cost type separately for each objective function. According to the lowest va value, ob1 leads to the highest pavement network quality as the aim of this objective function is to maximize quality regardless of expenditure from the available budget. As shown in Table 9, the ob1 suggests spending 99 percent of the available budget because the conventional approach is to utilize as much as possible
of the available funds allocated. Despite the adequacy of financial resources, ob3 offers no treatment for some sections in the third year due to constraints such as not implementing M&R action on sections that are in fairly good condition and/or cost analysis that shows unjustified benefit from performing it.

FIGURE 1. Cost comparisons from different objective functions

Table 8. M&R actions assignment results

| n  | ob1  | ob2  | ob3  |
|----|------|------|------|
|    | t=1  | t=2  | t=3  | t=1  | t=2  | t=3  | t=1  | t=2  | t=3  |
| 1  | Noting | 6     | 2     | Noting | 6     | 2     | Noting | 6     | 2     |
| 2  | 5     | 5     | 6     | Noting | 5     | 6     | Noting | 6     | 2     |
| 3  | 6     | 6     | 2     | 6     | Noting | 6     | 4     | Noting |
| 4  | 6     | 2     | 2     | 6     | 2     | Noting | 6     | 2     | Noting |
| 5  | 6     | 6     | 6     | 6     | Noting | 6     | Noting | Noting |
| 6  | 5     | 2     | 2,5   | 5     | 2     | Noting | 6     | 2     | Noting |
| 7  | 6     | 5     | 6     | 6     | Noting | 6     | 2     | Noting |
| 8  | Noting | 6     | 2,5   | 6     | 2     | Noting | 6     | 2     | Noting |
| 9  | 4     | 2,4   | 6     | 2,4   | 6     | 2     | 4     | 2     | Noting |
| 10 | 2     | 5     | 2     | Noting | Noting | Noting | Noting | Noting | 2     |

Table 9. Percentage of allocation to the available budget in each year.

| t (year) | ob1  | ob2  | ob3  | Budget ($1000) |
|----------|------|------|------|----------------|
| 1        | 99.94| 99.97| 99.97| 2518           |
| 2        | 99.74| 6.85 | 80.68| 2946           |
| 3        | 99.99| 0    | 3.37 | 3450           |
| Total    | 99.89| 30.50| 56.21| 8914           |

Fig 2 shows the overall cost including ce, co, and cu for whole analysis period plus va at the end of the analysis period (third year) for each objective function. The reason for adding the va of the last year is the goal to decrease overall costs while striving for high pavement network conditions at the end of the analysis.
period. As shown in Fig 2, ob3 results in the least overall cost and, as such, can be chosen as the best objective function. In this research, in order to assess the impact of technical constraints on the results of the best objective function (ob3), it was also solved without any technical constraints that addressed by Equations (32) to (45). As presented in Table 10, there are only 4 (out of 30) of the M&R actions in the ob3 without any technical constraints differently from ob3 with technical constraints. Such a small difference between the two approaches (13%) proves the efficiency of the selected and defined constraints. As shown in Table 11, adding technical constraints in ob3 results in a 91% decrease in runtime for solving the problem. However, as depicted in Fig 3, having constraints in ob3 increases overall cost slightly.

As can be seen, although the use of technical constraints increased costs by about $232,000 the amount represents only 1.2% of the total figure. On the other hand, the model solved for a sample case study with a small size network (10 sections) and in a 3-year period. Certainly, in large-scale networks or for long durations, solving a model may be either too time-consuming or not feasible due to over-computational complexity. Therefore, taking into account the time element in solving the problem becomes much more important. Since the increase in cost is negligible, it is well-justified to use technical constraints in solving ob3.

Fig 4 illustrates both overall cost reduction in accordance with ob1 overall cost and runtime in gams for different objective functions. It can be observed that there is a reduction in overall cost from ob1 to ob3, and that ob2 and ob3 with technical constraints require the least time to solve the problem.

5. Conclusion. Initially, most studies on network level pavement M&R planning focused on optimization methods based on the maximization of pavement performance or minimizing administration costs that are subjected to budgetary constraints or minimal performance limitations. Although the dimensions of the networks expanded as the studies developed, the number of M&R treatments used is still constant and limited to about 4 cases in most of the later studies. This research takes into account a larger number of indicators corresponding to a unique distress or a combination of distresses as well as more M&R actions in the assignment process thus allowing the network-level work planning to became closer to project-level one. Overall, the conclusions of this research can be summarized as follows:
Table 10. With and without technical constraints comparison of M&R action assignments

| n  | \(ob3\) t = 1 | \(ob3\) t = 2 | \(ob3\) t = 3 | \(ob3\) Without t = 1 | \(ob3\) Without t = 2 | \(ob3\) Without t = 3 |
|----|---------------|---------------|---------------|-----------------|-----------------|-----------------|
| 1  | Noting       | 6             | 2             | Noting          | 6               | 2               |
| 2  | Noting       | 6             | 2             | Noting          | 6               | 2               |
| 3  | 6            | 4             | Noting        | 6               | 4               | Noting          |
| 4  | 6            | 2             | Noting        | 6               | 2               | Noting          |
| 5  | 6            | Noting        | 6             | Noting          | Noting          | Noting          |
| 6  | 5            | 2             | 2             | 2,5             | 2               | 2               |
| 7  | 6            | 2             | Noting        | 4               | 6               | Noting          |
| 8  | 6            | 2             | Noting        | 6               | 2               | Noting          |
| 9  | 2,4          | 2             | Noting        | 2,4             | 2               | Noting          |
| 10 | Noting       | Noting        | 2             | Noting          | 5               | 2               |

Table 11. With and without technical constraints runtime comparisons

| Objective Function | \(ob1\) | \(ob2\) | \(ob3\) | \(ob3\) Without |
|--------------------|--------|--------|--------|-----------------|
| Solution Time (min)| 46     | 1      | 9      | 108             |

![Graph](image.png)

Figure 3. Overall cost comparison in \(ob3\) with and without technical constraints

- The technical constraints in the model preserve the accuracy of response while significantly reducing solution-time. This advantage is applicable for huge networks or long-term planning durations and helps avoid expending too much time in solving M&R planning issues.
- Taking consumed pavement financial value into account in generating a workplan model leads to lower overall costs in the analysis period.
- Conventional budget allocations aimed at maximizing pavement network quality using as much as possible from available budgets increases the overall cost of work planning compared to the approach with VOC and AC minimization.
The network-level work planning becomes closer to project-level one as more M&R actions can be applied in the assignment process by taking into consideration the various indicators corresponding to a unique distress or a combination of distresses.

This study can be helpful to other researchers in this area since it employs a linear integer programming model. As such, it would be worthwhile for other researchers and scholars to develop this model by using heuristic and Meta heuristic methods like genetic algorithm (GA) and others which also reduce runtime in solving the problem. Also, this study can be considered by other researchers as it takes into account uncertainty in parameters such as budgeting, pavement performance deterioration, traffic estimations, and other similar items.

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