In this paper, an adaptive neural network control method is described to stabilize a continuous stirred tank reactor (CSTR) subject to unknown time-varying delays and full state constraints. The unknown time delay and state constraints problem of the concentration in the reactor seriously affect the input-output ratio and stability of the entire system. Therefore, the design difficulty of this control scheme is how to debar the effect of time delay in CSTR systems. To deal with time-varying delays, Lyapunov–Krasovskii functionals (LKFs) are utilized in the adaptive controller design. The convergence of the tracking error to a small compact set without violating the constraints can be identified by the time-varying logarithm barrier Lyapunov function (LBLF). Finally, the simulation results on CSTR are shown to reveal the validity of the developed control strategy.

1. Introduction

To eliminate the effect of nonlinearities and uncertainties appearing in nonlinear systems, the fuzzy logic systems (FLSs) and neural networks (NNs) [1, 2] are useful tools, which avoid the requirement that nonlinearities must be known or can be linearly parameterized. Therefore, adaptive neural or fuzzy control methods are usually applied for the nonlinear single-input-single-output (SISO) systems in [3–8] and multi-input-multi-output (MIMO) systems in [9–12]. In recent years, more and more scholars have paid attention to the practical field. By employing neural networks or fuzzy logic systems, many stable control strategies for CSTR (a common chemical engineering reactor, usually with multiple reactors connected to each other for material conversion) have been caused widespread concern in [13–19] currently. A robust adaptive control method for the nonlinear CSTR system was proposed to ensure that the entire system remains stable in [20]. A novel controller design method was reported in [21] for a CSTR system. In practical CSTR systems, the concentration or temperature must be considered and controlled within certain limits to ensure the stability and safety of the system [22]. Hence, it is meaningful to consider the constraint problem in CSTRs. It is noteworthy that these above methods neglect the constraint limitations of operating spaces and safety specifications on system states.

Constraints could be seen everywhere in practical systems, for instance, car suspension [23, 24], motor servo systems [25, 26], mobile manipulators [27], and chemical systems [28]; the violation of constraints is a main source of reducing control quality and even causing instability. Logarithm barrier Lyapunov functions were first proposed for constant output constraints in [29] to ensure that system output stays within the prescribed barrier functions. Afterwards, several constraint control methods, such as Logarithm-BLFs, Tangent-BLFs, and Integral-BLFs, were presented for output constraint in [30] and full state constraints in [31–34]. In recent years, novel state-dependent nonlinear-transformation-based state-constrained control schemes have been developed, see, for example, [31, 32]. The new solution that completely removes the feasibility conditions for virtual controllers is first proposed in [31] by constructing a nonlinear state-dependent function. Besides, the authors in [32] propose a unified barrier function upon the constrained states, with which the original constrained
A nonlinear system is transformed into an equivalent "non-constrained" system. From a practical point of view, comparing with constant constraint, the time-varying constraint is more general. Hence, based on time-varying BLFs, some novel adaptive constraint controllers were designed, such as nonlinear strict-feedback systems and robotic manipulator with time-varying output constraint in [35, 36] and active suspension systems and nonlinear input saturated systems with time-varying state constraints in [37]. However, a major obstacle was that these control methods were proposed for delay-free systems.

The time-delay control problem always attracts the attention of the researchers. Some relative strategies were developed for the treatment of several categories of nonlinear time delayed systems. Some new impulsive delay control methods were proposed for nonlinear impulsive delay differential systems in [38–40], and in order to achieve the system stability, the change rates of states and impulses were imposed. Employing appropriate LKFs, the robust adaptive backstepping control strategies in [41–43] were reported for nonlinear strict-delay-feedback systems to debar time delays. Afterwards, based on [41–43], some adaptive NN and fuzzy controllers in [44–47] were further constructed to debar the effect of unknown time-varying delays appearing in several classes of nonlinear systems. In practical application, it must take times to transfer materials from one reactor to the next in the CSTR system, which creates time delay. In [48, 49], two adaptive state feedback controllers were designed for the CSTR system with unknown time delays.

According to the above discussion, this study develops an adaptive NN control strategy for CSTRs with both time-varying delays and full time-varying state constraints. As the best of our knowledge, there are only few results solving the stability problem of such CSTRs in the existing literatures. The primary contributions are as follows:

1. Under the adaptive control framework, the problem of time-varying delays and time-varying full state constraints is considered for CSTRs simultaneously, which is more in line with the needs of engineering systems. By using the time-varying barrier Lyapunov functions, all the states never violate the prescribed time-varying ranges.

2. It is inevitable that there will be time delays in the transfer of substances in the reactor, resulting in the corresponding delay of the reaction time in the next stage. The time delayed functions are decomposed into a number of positive continuous functions based on the separation technique. With the help of the Lyapunov–Krasovskii functionals, these positive continuous functions with delayed states are eliminated.

Finally, the construct integrity of the closed-loop nonlinear system is obtained by Lyapunov theory and the effectiveness of the proposed control scheme is proved by giving the simulation results.

The paper is divided into the following parts. Certain preliminaries are assumed during Section 2. Then, the adaptive NN control and stability analysis which are designed by backstepping technique are recommended during Section 3. Section 4 demonstrates certain experimental results. In Section 5, the conclusion is presented.

2. Preliminaries and Problem Formulation

We discuss the CSTR systems with two reactors A and B, as shown in Figure 1 [45]:

\[
\begin{align*}
\dot{C}_A &= -\eta_A C_A - \frac{H_A(C_A, C_A(t - \tau_A))}{\theta_A} + \frac{(1 - R_B)C_B}{D_A} + \xi_A(t, C_A(t - \tau_A)), \\
\dot{C}_B &= -\eta_B C_B - \frac{H_B(C_B, C_B(t - \tau_B))}{\theta_B} + \frac{R_A C_A(t - \tau_A)}{D_B} + \frac{R_B C_B(t - \tau_B)}{D_B} + \frac{F u(t)}{D_B + \xi_B(t, C_B(t - \tau_B))}
\end{align*}
\]  

where \( R_i \) denotes the recycle flow rate, \( \theta_i \) is the reactor residence time, \( \eta_i \) stands for the reaction constant, \( F \) is the feed rate, \( D_i \) is the reactor volume, \( H_i \) is the nonlinear function representing the complex behavior of the systems, \( \xi_i \) is nonlinear function for describing the system uncertainties and external disturbances, \( C_i \) denotes the reactor \( i \) concentration, and \( \tau_i \) denotes time-varying delay, \( i = A, B \).

The control objective is to develop an adaptive neural controller to ensure that the concentrations \( C_A \) and \( C_B \) of the producing reactors \( A \) and \( B \) never overstep the correspondent barrier functions, and the boundedness of all signals in the CSTR systems are obtained.

Remark 1. Comparing with the CSRT systems with only constant time delays in [44, 45], the considered systems in this paper with unknown time-varying delays are more important and general. In addition, to improve the control performance and system stability, full time-varying state constraints’ problem is also considered in the design process. Hence, the developed control approach is more responsive to actual engineering needs.

When \( \xi_A = 0 \) and \( \xi_B = 0 \), the entire CSTR system achieves dynamic equilibrium; assume \( H_A = C_A + C_A(t - \tau_i) \) and \( H_B = C_B(t) \). We can get that the equilibrium points \( C^*_A \) and \( C^*_B \) satisfy...
\[
\begin{align*}
\left\{ \frac{2}{\theta_A} + \eta_A \right\} C_A^* &= \frac{1 - R_B C_B^*}{D_A} \\
\eta_B C_B^* + \frac{1}{\theta_B} C_B^* &= \frac{R_A C_A^* + R_B C_B^*}{D_B}
\end{align*}
\]

Letting \( x_1 = C_A(t) - C_A^* \) and \( x_2 = C_B(t) - C_B^* \), the constraints of \( x_i, i = 1, 2 \), can be regarded as the constraints of the concentrations \( C_A \) and \( C_B \); we can further have

\[
\begin{align*}
\dot{x}_1 &= -\eta_A x_1 - \frac{x_1(t - \tau_1)}{\theta_A} + x_2 \left( \frac{1 - R_B}{D_A} \right) + \xi_A(t, C_A(t - \tau_1)), \\
\dot{x}_2 &= -\eta_B x_2 - \frac{x_2(t)}{\theta_B} + \frac{R_A x_1(t - \tau_1)}{D_B} - \frac{2C_B^* x_2(t)}{\theta_B} + \frac{R_B x_2(t - \tau_2)}{D_B} + \frac{F(t)}{D_B} + \xi_B(t, C_B(t - \tau_2)).
\end{align*}
\]

Let \( \varphi_1 = (1 - R_B)/D_A \) and \( \varphi_2 = F/D_B \);

\[
\begin{align*}
g_1(x_1(t)) &= -\eta_A x_1 - \frac{x_1}{\theta_A}, \\
g_2(x_2(t)) &= -\eta_B x_2 - \frac{x_2(t)}{\theta_B} - \frac{2C_B^* x_2(t)}{\theta_B},
\end{align*}
\]

\[
\begin{align*}
\bar{\xi}_A(t, x_1(t - \tau_1)) &= \xi_A(t, C_A(t - \tau_1)), \\
\bar{\xi}_B(t, x_2(t - \tau_1)) &= \xi_B(t, C_B(t - \tau_1)),
\end{align*}
\]

\[
\begin{align*}
h_1(x_{1,\tau}(t)) &= \frac{x_1(t - \tau_1)}{\theta_A} + \xi_A(t, x_1(t - \tau_1)), \\
h_2(x_{2,\tau}(t)) &= \frac{R_A x_1(t - \tau_1)}{D_B} + \frac{\bar{\xi}_A(t, x_1(t - \tau_1)) + \frac{R_B x_2(t - \tau_2)}{D_B}}{D_B}.
\end{align*}
\]

For brevity, thus, (3) can be rewritten as

\[
\begin{align*}
\left\{ \frac{2}{\theta_A} + \eta_A \right\} C_A^* &= \frac{1 - R_B C_B^*}{D_A} \\
\eta_B C_B^* + \frac{1}{\theta_B} C_B^* &= \frac{R_A C_A^* + R_B C_B^*}{D_B}
\end{align*}
\]
\[
\begin{aligned}
\dot{x}_1 &= \varphi_1 x_2(t) + g_1(x_1(t)) + h_1(x_{1,r}(t)), \\
\dot{x}_2 &= \varphi_2 u(t) + g_2(x_2(t)) + h_2(x_{2,r}(t)), \\
y(t) &= x_1(t),
\end{aligned}
\]  
(5)

where \(x_i, i = 1, 2\), are the state variables, \(u \in \mathbb{R}\) and \(y(t)\) are control input and output, respectively, \(x_2 = [x_1, x_1^T, g_1(.)\) and \(h_1(.)\) are known and unknown smooth nonlinear functions, respectively, and \(g_1(0) = h_1(0) = 0\), \(\varphi_i(.)\) is the continuous function, and \(\tau_i(.)\) is the unknown time-varying time delay satisfying \(\tau_i(t) \leq \tau_{\text{max}}\) and \(\dot{\tau}_i(t) \leq \bar{\tau} \leq 1\), with \(\tau\) and \(\tau_{\text{max}}\) being two known constants. Here, \(|x_i| \leq k_i(t), i = 1, 2\), with \(k_i(.)\) are known functions.

A continuous function \(f(z)\) can be approximated by the radial basis function neural networks (RBFNN) as

\[
f(z) = W^T I(z),
\]

(6)

where \(W = [W_1, W_2, \ldots, W_d]^T \in \mathbb{R}^k\) denote the adjustable weight vector, \(k\) expresses the number of neuron, and \(f(z) = [I_1(z), I_2(z), \ldots, I_k(z)]^T\) signify the basis function vector. There is a smooth vector function \(f(z) \in \mathbb{R}\) and ideal weights \(W^*\); hence, the smooth function \(f(z)\) can be approximated by the RBFNN as follows:

\[
f(z) = W^* I(z) + \theta(z), \quad \forall z \in \Omega \subseteq \mathbb{R}^q.
\]

(7)

We choose \(W^*\) as follows:

\[
W^* = \arg\min \sup \|f(z) - W^T I(z)\|
\]

(8)

in which the error \(\theta(z)\) fulfills \(|\theta(z)| \leq \epsilon\) within \(\epsilon > 0\). During the thesis, undermentioned Gaussian basis function \(I_i(z)\) will be employed:

\[
I_i(z) = \exp \left[-\frac{(z - i)^T (z - i)}{u_i^2}\right],
\]

(9)

where \(i = [i_1, i_2, \ldots, i_d]^T\) depicts the center of the receptive field and \(u_i\) represents the width of the Gaussian function within, \(i = 1, 2, \ldots, k\).

\textit{Assumption 1} (see [35]). There exist positive constants \(K_i, i = 1, 2\) and \(j = 0, 1, 2\), such that, for \(t > 0\), the functions \(k_i(.)\) and the \(j\)th time derivative satisfy \(|k_i(t)| \leq K_i^{(0)}\) and \(|k_i^{(j)}(t)| \leq K_i^{(j)}\).

\textit{Assumption 2}. The smooth functions \(h_i, i = 1, 2\), satisfy

\[
h_i^2(x_i(t - \tau_i(t))) \leq z_i^2(t - \tau_i(t))a_i^2(\tau_i(t - \tau_i(t)))
\]

(10)

where \(a_i(.)\) is a positive smooth function.

\textit{Assumption 3} (see [32]). There exist a positive function \(Y_0(t) : R_+ \to R_+\) and the constants \(Y_i > 0, i = 1, 2\), so that the desired trajectory \(y_d(t)\) and its time derivatives satisfy \(|y_d(t)| \leq Y_0(t)\) and \(|y_i^{(j)}(t)| \leq Y_i\).

\textbf{Remark 2}. Assumption 1 shows that the function and the \(j\)th time derivative less than or equal to positive constants. Assumptions 1–3 are used to prove the stability of the system; all the signals in the closed-loop system are bounded.

\textbf{Lemma 1} (see [23]). For all \(|\xi| < 1\), such that the following inequality holds:

\[
\log \frac{1}{1 - \xi^2} - \frac{\xi^2}{1 - \xi^2} \leq \frac{\xi^2}{1 - \xi^2}.
\]

(11)

\textbf{3. The Controller Design and Stability Analysis}

An adaptive NN control approach will be investigated in this section. It is used to solve the control problem of time delays and state constraints. The time variable \(t\) is omitted except for delay terms to improve the readability of this paper.

Step 1: define the error \(z_i = x_i\) and a compact set \(\Omega = \{z_i | k_{b_i, i} = 1, 2\}\) with \(k_{b_i, i} = 1, 2\) are known positive constants, and we have

\[
\dot{z}_i = \dot{x}_i = \varphi_i x_2(t) + g_1(x_1(t)) + h_1(x_{1,r}(t)).
\]

(12)

Consider the time-varying BLF candidate:

\[
V_{D1} = \frac{1}{2} \log \frac{k_{b_i}^2}{k_{b_i} - z_i^2} + \frac{1}{2} W_i^T I_1 - \bar{W}_i I_1^T.
\]

(13)

where \(\Gamma_1\) is a positive constant matrix satisfying \(\Gamma_1 = \Gamma_1^T > 0\), \(\bar{W}_i = W_i - W_i^*\) represents the estimate error, and \(\bar{W}_i\) is the estimation weight of the optimal weight \(W_i^*\). We define

\[
\psi_{b_i} = \frac{z_i}{k_{b_i}}\]

(14)

\[
\mu_i = \frac{1}{k_{b_i}^2 - z_i^2}, \quad i = 1, 2.
\]

Combining (13) and (14), one can obtain

\[
\dot{V}_{D1} = \frac{\psi_{b_i}}{k_{b_i}^2 - z_i^2} \dot{z}_i - \frac{k_{b_i} \psi_{b_i}^2}{k_{b_i}^2(1 - \psi_{b_i}^2)} + \bar{W}_i^T \Gamma_1^T \bar{W}_i.
\]

(15)

Based on (12) and (15), we obtain

\[
\dot{V}_{D1} = \frac{\psi_{b_i}}{k_{b_i}^2(1 - \psi_{b_i}^2)} \psi_{b_i} \varphi_i x_2(t) + g_1(x_1(t)) + \bar{W}_i^T \Gamma_1^T \bar{W}_i
\]

\[
+ \frac{\psi_{b_i}}{k_{b_i}^2(1 - \psi_{b_i}^2)} h_1(x_{1,r}(t)) - \frac{k_{b_i} \psi_{b_i}^2}{k_{b_i}^2(1 - \psi_{b_i}^2)}
\]

(16)

Using Young’s inequality, one has
Based on (24) and (25), (23) can be written as

\[ V_{D1} \leq \mu_1 z_1 \left[ G_1(Z_i) + \varphi_1(z_2 + a_1) - \frac{k_b}{\kappa b_i} z_1 \right] - z_1 H_1 \]

\[ + \tilde{W}_1^T \Gamma^{-1}_1 \tilde{W}_1 + \frac{c \exp(-\tau_{\text{max}}) h_i^2(x_{1,r(t)})}{2} \]

By using the NN approximation property, the continuous function \( G_1(Z_i) \) can be approximated as

\[ G_1(Z_i) = W_1^T I_1(Z_i) + \theta_1(Z_i), \]

where \( I_1(Z_i) \in R \) indicates the Gaussian basis function and \( \theta_1(Z_i) \) indicates inherent approximation error of the NNs satisfying \( |\theta_1(Z_i)| \leq \tilde{\theta}_1 \) with \( \tilde{\theta}_1 \) being a positive constant; \( Z_i = x_1 \).

The middle controller and the adaptive law are designed as

\[ \alpha_i = \frac{1}{\varphi_1} \left( -k_1 z_1 - K_i z_1 - \tilde{W}_1^T I_1(Z_i) - \frac{1}{2\gamma_1} \mu_1 z_1^2 - \frac{1}{2} \varphi_1 z_1 \right), \]

\[ \tilde{W}_1 = \Gamma_i \left[ \mu_1 z_1 I_1(Z_i) - \sigma_i \tilde{W}_1 \right], \]

where \( k_1, \gamma_1, \) and \( \sigma_i \) are the positive constants.

Denote \( K_i = \sup \left( (k_i/\kappa b_i)^2 + \varepsilon_i \right) \) with \( \varepsilon_i > 0 \) being a small constant.

According to (20)-(22), (19) becomes

\[ V_{D1} \leq \mu_1 z_1 \left[ -k_1 z_1 - \frac{1}{2} \mu_1 z_1^2 + \varphi_1 z_2 - \frac{1}{2} \varphi_1 z_1 + \theta_1 (Z_i) \right] - z_1 H_1 \]

\[ + \alpha_i \left( -k_1 z_1 - \frac{1}{2} \mu_1 z_1^2 + \varphi_1 z_2 - \frac{1}{2} \varphi_1 z_1 + \theta_1 (Z_i) \right) - \tilde{W}_1^T \tilde{W}_1 \]

By employing Young’s inequality, we can obtain

\[ \mu_1 z_1 \theta_1 (Z_i) \leq \frac{1}{2\gamma_1} \mu_1 z_1^2 + \frac{\gamma_1}{2} \tilde{\theta}_1^2, \]

\[ \mu_1 \varphi_1 z_1 z_2 \leq \mu_1 \varphi_1 \frac{1}{2} \left( z_1^2 + z_2^2 \right). \]

Based on (24) and (25), (23) can be written as

\[ V_{D1} \leq \frac{1}{2} \mu_1 \varphi_1 z_2^2 - k_1 \mu_1 z_1^2 + \frac{\gamma_1}{2} \tilde{\theta}_1^2 \]

\[ - \sigma_i \tilde{W}_1^T \tilde{W}_1 + \frac{c \exp(-\tau_{\text{max}}) h_i^2(x_{1,r(t)})}{2} - z_1 H_1. \]

Step 2 : since \( z_2 = x_2 - a_1 \), its time derivative is given as follows:
\[
\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = \varphi_2 u + g_2(x_2) + h_2(x_{2, \tau(t)}) - \dot{\alpha}_1, \tag{27}
\]

where
\[
\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial x_1} [\varphi_1 x_2 + g_1(x_1) + h_1(x_{1, \tau(t)})] + \frac{\partial \alpha_1}{\partial W_1} \dot{W}_1 + \sum_{i=0} \frac{\partial \alpha_1}{\partial k_{b_i}^2} k_{b_i}^{(i+1)}. \tag{28}
\]

By using Young's inequality, it has
\[
\psi_{b_1} H_2(x_{2, \tau(t)}) \leq \frac{\exp(\tau_{\max})}{2c} \left( \frac{\psi_{b_1}}{k_{b_1} (1 - \psi_{b_2})} \right)^2 + \frac{c \exp(-\tau_{\max}) H_2^2(x_{2, \tau(t)})}{2}, \tag{31}
\]

\[
-\frac{\psi_{b_2}}{k_{b_1} (1 - \psi_{b_2})} \frac{\partial \alpha_1}{\partial x_1} h_1(x_{1, \tau(t)}) \leq \frac{\exp(\tau_{\max})}{2c} \left( \frac{\psi_{b_1}}{k_{b_1} (1 - \psi_{b_2})} \frac{\partial \alpha_1}{\partial x_1} \right)^2 + \frac{c \exp(-\tau_{\max}) h_1^2(x_{1, \tau(t)})}{2}. \tag{32}
\]

The BLF candidate is given as
\[
V_{D2} = V_{D1} + \frac{1}{2} \log \frac{k_{b_2}^2}{k_{b_2} - z_2} + \frac{1}{2} \bar{W}_2^T \Gamma_2^{-1} \bar{W}_2, \tag{29}
\]

where \(\Gamma_2 = \Gamma_2^{-1} > 0\) is a constant matrix and \(\bar{W}_2 = \bar{W}_2 - W_2^*\) denotes the estimate error with \(\bar{W}_2\) being the estimation weight of \(W_2^*\).

Based on (27), differentiating \(V_{D2}\), one obtains
\[
\dot{V}_{D2} = \dot{V}_{D1} + \frac{\psi_{b_1}}{k_{b_1} (1 - \psi_{b_2})} [\varphi_2 u + g_2(x_2)]
\]

\[
+ h_2(x_{2, \tau(t)}) - \frac{\partial \alpha_1}{\partial x_1} \varphi_1 x_2 - \frac{\partial \alpha_1}{\partial x_1} g_1(x_1)
\]

\[
- \frac{\partial \alpha_1}{\partial x_1} h_1(x_{1, \tau(t)}) - \frac{\partial \alpha_1}{\partial W_1} \dot{W}_1 - \frac{1}{2} \sum_{i=0} \frac{\partial \alpha_1}{\partial k_{b_i}^2} k_{b_i}^{(i+1)} - \frac{k_{b_1} \psi_{b_2}}{k_{b_1} (1 - \psi_{b_2})} + \bar{W}_2^T \Gamma_2^{-1} \bar{W}_2. \tag{30}
\]

The unknown function \(G_2(Z_2)\) is concluded as
\[
G_2(Z_2) = g_2(x_2) - \frac{\partial \alpha_1}{\partial x_1} [g_1(x_1) + \varphi_1 x_2]
\]

\[
- \frac{\partial \alpha_1}{\partial W_1} \dot{W}_1 - \sum_{i=0} \frac{\partial \alpha_1}{\partial k_{b_i}^2} k_{b_i}^{(i+1)} + \frac{1}{\mu_2} H_2 + \exp(\tau_{\max}) \frac{\mu_2 \bar{z}_2}{2c} \left[ 1 + \left( \frac{\partial \alpha_1}{\partial x_1} \right)^2 \right], \tag{33}
\]

where \(H_2 = z_2 q_2^2(z_2)/2\) with \(q_2(\cdot)\) being a continuous and smooth function.

By using the NNs, the continuous unknown function \(G_2(Z_2)\) is approximated as
\[
G_2(Z_2) = W_2^T I_2(Z_2) + \vartheta_2(Z_2),
\]
where \(Z_2 = [x_1, x_2, W_1^T]^T\), and \(I_2(Z_2) \in \mathbb{R}\) indicates the Gaussian basis function, and \(\vartheta(Z_2)\) indicates inherent approximation error of the NNs satisfying \(|\vartheta(Z_2)| \leq \vartheta_2\) with \(\vartheta_2\) being a positive constant.

The virtual controller and the adaptive law are given as
\[
u = \frac{1}{\varphi_2} \left( -k_2 z_2 - k_2 z_2 - \dot{W}_2^T I_2(Z_2) - \frac{1}{2} \mu_2 \dot{z}_2^2 - \frac{2}{\mu_2} \dot{z}_2 \right),
\]
(35)
\[
\dot{W}_2 = \Gamma_2 \left[ \mu_2 z_2 I_2(Z_2) - \sigma_2 \dot{W}_2 \right],
\]
(36)
where \(k_2, \varphi_2, \) and \(\sigma_2\) are the positive constants.

By using Young’s inequality, it has
\[
\mu_2 z_2 \vartheta_2(Z_2) \leq \frac{\varphi_2}{2} \dot{z}_2^2 + \frac{1}{4 \varphi_2} \dot{\vartheta}_2^2.
\]
(37)
Along with (31)–(37), we obtain
\[
\dot{V}_{D2} \leq \dot{V}_{D1} - \frac{1}{2} \mu_1 \varphi_1 \dot{z}_2^2 - k_2 \mu_2 \dot{z}_2^2 + \frac{1}{2} \varphi_1 \dot{\vartheta}_1^2 - \sigma_2 \dot{W}_2^T \dot{W}_2 - z_2 H_2 + \sum_{i=1}^{c} \frac{1}{2} \exp(-\tau_{\max}) h_i^2(x_{i,r(t)}).
\]
(38)
In Step 1, we can obtain
\[
\dot{V}_{D1} \leq -k_1 \mu_1 \dot{z}_1^2 + \frac{1}{2} \mu_1 \varphi_1 \dot{z}_2^2 - z_1 H_1 + \frac{1}{2} \varphi_1 \dot{\vartheta}_1^2 - \sigma_1 \dot{W}_1^T \dot{W}_1 + \frac{1}{2} \exp(-\tau_{\max}) h_1^2(x_{1,r(t)}).
\]
(39)
Hence, (38) becomes
\[
\dot{V}_{D2} \leq - \sum_{i=1}^{c} k_1 \mu_1 \dot{z}_1^2 - \sum_{i=1}^{c} z_i H_i + \sum_{i=1}^{c} \frac{1}{2} \varphi_1 \dot{\vartheta}_1^2 - \sigma_1 \dot{W}_1^T \dot{W}_1 + \frac{1}{2} \exp(-\tau_{\max}) \left( h_1^2(x_{1,r(t)}) + \frac{1}{2} h_2^2(x_{2,r(t)}) \right).
\]
(40)
Based on Assumption 1 in [45], the delay term in (40) becomes
\[
h_1^2(x_{1,r(t)}) \leq \bar{q}_1(z_1(t - t_1(t))) + \sum_{j=1}^{l} \sum_{i=1}^{k} \bar{W}_{ij}^*(z_j(t - t_i(t))),
\]
(41)
where \(\bar{q}_1\) and \(\bar{W}_{ij}\) represent positive definite and sufficiently smooth known functions with \(\bar{q}_1(x) = x^2 \bar{q}_1(x)\) and \(\bar{W}_{ij}(x) = x^2 \bar{W}_{ij}(x)\) (with \(\bar{q}_1(x)\) and \(\bar{W}_{ij}(x)\) being continuous and smooth functions).

Noting the term \(\sigma_1 \dot{W}_1^T \dot{W}_1\), the following inequality can be obtained:
\[
- \sigma_1 \dot{W}_1^T \dot{W}_1 = - \sigma_1 \dot{W}_1^T (\dot{W}_1 + W_1^*) \leq - \frac{1}{2} \sigma_1 \|\dot{W}_1^*\|^2 + \frac{1}{2} \sigma_1 \|W_1^*\|^2.
\]
(42)
Thus, (40) can be rewritten as
\[
\dot{V}_{D2} \leq - \sum_{i=1}^{c} k_1 \mu_1 \dot{z}_1^2 - \sum_{i=1}^{c} z_i H_i + \sum_{i=1}^{c} \frac{1}{2} \varphi_1 \dot{\vartheta}_1^2 - \sigma_2 \dot{W}_2^T \dot{W}_2 - \sigma_1 \dot{W}_1^T \dot{W}_1 + \frac{1}{2} \exp(-\tau_{\max}) \left( 2z_1^2(t - t_1(t)) \right) \bar{q}_1(z_1(t - t_1(t))) + \frac{1}{2} \sum_{i=1}^{2} \sigma_1 \|W_i^*\|^2 + \frac{1}{2} \sum_{i=1}^{2} \sigma_1 \|\dot{W}_i\|^2 + z_1^2(t - t_1(t)) \psi_{211}(z_1(t - t_1(t))) - \frac{1}{2} \sum_{i=1}^{2} \sigma_1 \|\dot{W}_i\|^2 + z_1^2(t - t_2(t)) \psi_{212}(z_1(t - t_2(t))).
\]
(43)
Choose the Lyapunov function candidate as
\[
V = V_{D2} + V_L,
\]
(44)
in which, LKF \(V_L\) is considered as
\[
V_L = \frac{1}{2} \sum_{m=1}^{2} \int_{t_m}^{t} (3 - m) \exp(s - t) z_m^2(s) q_m(z_m(s)) ds + \frac{1}{2} \sum_{m=1}^{2} \sum_{i=1}^{m-1} \sum_{j=1}^{m} \exp(-\tau_2(t)) z_j^2(t - t_1(t)) \psi_{mji}(z_j(t - t_1(t))).
\]
(45)
The time derivative of \(V_L\) is given as
\[
\dot{V}_L = \frac{1}{2} \sum_{m=1}^{2} (3 - m) z_m^2 q_m(z_m) + \frac{1}{2} \sum_{m=1}^{2} \sum_{i=1}^{m-1} \sum_{j=1}^{m} \exp(-\tau_m(t)) z_j^2(t - t_m(t)) q_m(z_m(t - t_m(t)))
\]
(46)
Noting \( \tau_1 \leq \tau_{\text{max}} \), according to (43) and (46), the time derivative of \( V \) can be obtained as

\[
\dot{V} \leq - \frac{2}{\gamma_i} k_i \mu_i z_i^2 + \frac{1}{2} \sum_{i=1}^{m} \frac{1}{2} \sum_{i=1}^{m} \sigma_i \| W_i \|^2 - \frac{2}{\gamma_i} \sum_{i=1}^{m} \frac{1}{2} \sum_{i=1}^{m} \sigma_i \| W_i \|^2 - \frac{2}{\gamma_i} \sum_{i=1}^{m} \frac{1}{2} \sum_{i=1}^{m} \sigma_i \| W_i \|^2 - V_L.
\]

where

\[
\Lambda_1 = \frac{c \exp(-\tau_{\text{max}})}{2} \left( 2z_i^2(t - \tau_i(t))q_i(z_i(t - \tau_i(t))) \right)
\]

\[
+ z_i^2(t - \tau_i(t))q_i(z_i(t - \tau_i(t))) - \frac{c \exp(-\tau_{\text{max}})}{2}
\]

\[
\times \frac{2}{\gamma_i} \sum_{m=1}^{m-1} (3 - m) z_m^2 \left( t - \tau_m(t) \right) q_m \left( z_m(t - \tau_m(t)) \right) = 0,
\]

\[
\Lambda_2 = \frac{c \exp(-\tau_{\text{max}})}{2} \left( z_i^2(t - \tau_i(t)) \psi_{211}(z_i(t - \tau_i(t))) \right)
\]

\[
+ z_i^2(t - \tau_i(t)) \psi_{212}(z_i(t - \tau_i(t))) - \frac{c \exp(-\tau_{\text{max}})}{2}
\]

\[
\times \frac{2}{\gamma_i} \sum_{m=1}^{m-1} \sum_{j=1}^{m} \sum_{i=1}^{m} \psi_j(t - \tau_i(t)) \psi_{mj}(z_j(t - \tau_i(t))) = 0,
\]

\[
\Lambda_3 = \frac{1}{2} \sum_{m=1}^{m} (3 - m) z_m^2 \Delta_{am}(z_m) + \frac{1}{2} \sum_{m=1}^{m} \sum_{j=1}^{m} \sum_{i=1}^{m} z_j \psi_{mj}(z_j) - \frac{3}{2} \sum_{i=1}^{m} \Delta_i H_i = 0.
\]

So, \( V \) further becomes

\[
\dot{V} \leq - \frac{2}{\gamma_i} k_i \mu_i z_i^2 + \frac{1}{2} \sum_{i=1}^{m} \sigma_i \| W_i \|^2 - \frac{2}{\gamma_i} \sum_{i=1}^{m} \frac{1}{2} \sum_{i=1}^{m} \sigma_i \| W_i \|^2 - V_L.
\]

With the help of Lemma 1, we have

\[
- \frac{2}{\gamma_i} k_i \mu_i z_i^2 \leq - \frac{2}{\gamma_i} \log \frac{k_i}{k_{\text{bi}}} - z_i.
\]

Then, (49) is further shown as

\[
\dot{V} \leq - \frac{2}{\gamma_i} \log \frac{k_i}{k_{\text{bi}}} - z_i + \frac{1}{2} \sum_{i=1}^{m} \frac{1}{2} \sum_{i=1}^{m} \sigma_i \| W_i \|^2 - \frac{2}{\gamma_i} \sum_{i=1}^{m} \frac{1}{2} \sum_{i=1}^{m} \sigma_i \| W_i \|^2 - V_L.
\]

According to (13), (29), (44), and (45), one has

\[
\dot{V} \leq - \rho V + q,
\]

where \( p = \min\{2k_i, \sigma_i \lambda_{\text{min}}(V_i), 1, i = 1, 2, 3\} \),

\[
q = \frac{1}{2} \sum_{i=1}^{m} \sigma_i \| W_i \|^2 + \frac{1}{2} \sum_{i=1}^{m} \sigma_i \| W_i \|^2.
\]

**Theorem 1.** Consider the CSTR systems (1); if Assumptions 1–3 hold, the proposed virtual controller (21), the actual controller (35), and the adaptation laws (22) and (36) are utilized to ensure the boundedness of closed-loop system.
signals, and all the state systems remain within the corresponding constraint set.

Proof. According to (53), one can obtain
\[
V(t) \leq \left( V(0) - \frac{q}{p} \right) e^{-pt} + \frac{q}{p} V(0) + \frac{q}{p} \tag{55}
\]

From (13), we can get that every term is positive, so, the following term satisfies
\[
\frac{1}{2} \log \left( \frac{1}{1 - \psi_i} \right) \leq V(t) \leq V(0) e^{-pt} + \frac{q}{p} \tag{56}
\]

\[
\frac{1}{2} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i \leq V(0) e^{-pt} + \frac{q}{p} \tag{57}
\]

From (56) and (57), the error signal is bounded:
\[
-\Delta_i \leq z_i(t) \leq \Delta_i, \tag{58}
\]

where \(\Delta_i = k_i(t) (1 - e^{-2(V(0)e^{pt} - 2q/p)^{1/2}})\).

The adaptation laws can be obtained:
\[
\| \tilde{W}_i \| \leq \sqrt{2\lambda_{\max}(\Gamma_i)} (V(0)) e^{-pt} + \frac{q}{p}. \tag{59}
\]

Then, according to Assumptions 1–3, it is known that \(|k_i(t)| \leq Y_1 + K_i^{1b}\) and \(y_i(t) - \Delta_i \leq x_i(t) \leq y_i(t) + \Delta_i\), so we can obtain \(-k_i(t) \leq x_i(t) \leq k_i(t)\). Due to the definition of \(a_1\), meanwhile, the boundedness of \(y_i, k_i, k_i, y_i\), \(x_i\), and \(x_i\), it is easy to obtain \(a_2\) is bounded and \(x_2 = z_2 + a_1\) is bounded. From the definition in (35), it can be obtained that the controller \(u\) is bounded by the same way. Therefore, all the signals in the closed-loop system are bounded. The proof completes. \(\square\)

Remark 3. CSTRs’ system is a complex and nonlinear system. It has time-varying delayed states. This paper adopts the mechanism modeling method; according to the kinetic equation, by conservation of mass and conservation of energy, the nonlinear model equation is derived. The Lyapunov–Krasovskii functional is used to eliminate these continuous functions with the delayed state in this paper so that the CSTRs system with the delayed state is stable.

In the case of cascade reaction between two reaction reactors, there will be time delays in the transfer of substances in the reactor, which has a strong delay characteristic. At the same time, the CSTR system is nonlinear. In the reaction process, the reactant concentration and other parameters are easy to fluctuate with time-varying, and the temperature in the chemical reaction system must be controlled within a certain safety limit since its data are constrained. Therefore, the time delay and state constraints of the CSTR system should be considered when designing the controller.

4. Simulation Example

In this part, the simulation example is shown to demonstrate the effectiveness of the presented method for CSTR systems (1) with both time-varying full state constraints and time-varying delays.

According to the considered CSTR systems, system states \(x_1\) and \(x_2\) are constrained by \(-k_{1i}(t) \leq x_1(t) \leq k_{1i}(t)\) and \(-k_{2i}(t) \leq x_2(t) \leq k_{2i}(t)\) with \(k_{1i}(t) = 0.4e^{-2t} + 0.005\) and \(k_{2i}(t) = 2e^{-2t} + 0.05\). The equilibrium points are \(C_A^* = 14/9\) and \(C_B^* = 7/3\). The system parameters are chosen as \(\eta_A = \eta_B = 0.5, \theta_A = \theta_B = 2, R_A = R_B = 0.5, D_A = D_B = 0.5, F = 0.5, \xi_A(t, C_A(t - \tau_1(t))) = 0.5 \sin(t)x_1(t - \tau_1(t)), \) and \(\xi_B(t, C_B(t - \tau_2(t))) = 0.5 \times \exp(0.01x_2(t - \tau_2(t)))\cos(t) \times x_2^2(t - \tau_2(t))\) with time-varying delays \(\tau_1(t) = 0.25 - 0.2 \sin(0.5t)\cos(2t)\) and \(\tau_2(t) = 0.55 - 0.5 \sin(0.5t)\). The initial conditions are selected as \(x_1(0) = 0.1, x_2(0) = 0, W_1 = W_2 = 0, \) and \(G_1 = 0.4\text{diag}(\text{ones}(1, 30)), \) and \(G_2 = 0.2\text{diag}(\text{ones}(1, 30))\). The design parameters are selected as \(k_1 = 1.1, k_2 = 12, y_1 = 0.5, y_2 = 2, \) and \(e_1 = e_2 = 0.01\).

The simulation results are shown as Figures 2–5 to certify the effectiveness of the presented control method. Figure 2
illustrates the responses of system state $x_1$ and the corresponding constrained functions. In Figure 3, it obtains the trajectories of $x_2$ and its constrained functions. It is worth stressing that system states $x_1$ and $x_2$ always stay within the prescribed compact set in Figures 2 and 3. From Figures 4 and 5, the boundedness of control input and adaptation laws are obtained.

**5. Conclusion**

The time delays and constraints are often occurring in the chemical reactor system, which also are main limitation factors of system performance severely. This paper has studied the tracking problem of the continuous stirred tank reactor containing the time-varying delays and full state constraints simultaneously. By constructing appropriate LKF s and LBLF, the effects of unknown time-varying delays were eliminated and the time-varying full states are never violated. By employing the proposed control method, we can make the conclusion that the tracking error can converge into a small set of zero and ensure all the signals in the system are bounded. The simulation results are proved to the rationality and effectiveness of the scheme. The future research directions should focus on the intergral BLF-based finite-time adaptive control for a CSTR system.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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