Learning calculus with geogebra at college

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Abstract. Calculus was one of important and fundamental subjects for engineering students. However, there was many students had difficulties on this subject. On the other hand, Geogebra, one of mathematics software, was developed to represent mathematical object. Therefore, learning calculus with Geogebra was expected to make students understand the material. The aim of this research was to describe and analyse how learning integral calculus by using Geogebra give impact for students. This study was qualitative research. The data was collected by observation, questionnaire, and task. The result of this research showed that more than 50% students gave positive respond about using Geogebra on calculus learning. Geogebra was useful to help students understanding the material of integral calculus such as area between two curves, surface area, and volume of rotary object. Moreover, learning calculus with Geogebra gave higher motivation for students and made joyful learning. Based on the task, most of students could represent the function that given in problem to the graph and solved the problems exactly.

1. Introduction
Calculus is one of the branches of mathematics. In university, calculus become basic subject that should be understood by students because it is used to another subject which more complex. Calculus is not only thought in mathematics department but also in other department that connected with mathematics such as engineering department. This figure has great impact on all branches of science, especially determining the variables can be controlled and those that cannot. Calculus materials related to limits, derivatives, and integrals are directly related and responsible for understanding models of mathematics which relevant to daily lives [1].

In this study, the activities to construct students’ knowledge-particularly in applied of integral on rotary object-were part of fundamental concepts on Calculus. In particular, Calculus has concept of the rate change (differentiation) and cumulative growth (integration) which are very essential. Moreover, it focuses on proficiency on symbolic methods for differentiation and integration and also the application to solve the various problems [2].

Regarding to the curricula developing in Universitas Muhammadiyah Surakarta, there are two parts of calculus becoming prerequisite subject, namely Differential and Integral Calculus. In the Differential Calculus, students learn about the concept of limits, concept of differential, and how to calculate and apply it. In the Integral Calculus, student studies about the formula and technique to determine the integral from a function and application of integral such as calculated area that bounded by curves, volume, and surface area.

According to the result of final exam in academic year 2016/2017, there was no more than 30% of students who attend calculus classes could get grade B and over. It showed that Calculus was one of the difficult subjects in mathematics. Based on previous research, students tend to have difficulties in solving the calculation of integral, particularly in arithmetic skill, number fact skill, information skill,
and language skill [3]. In this research, students had main difficulties especially on arithmetic skill which was needed critical thinking since they should determine the exact technique to solve calculus problems. Moreover, the majority error of students when solving integral problems is process skill error where the students could not determine the suitable formula and the algebra manipulation [4]. Those researches focused on calculating the integral, while it did not explain the error of students in application of integral. There were some application of Integral that was learnt by students such as area between curves, length of arch, volume of rotary object. Based on observation, when studied application of integral, most of students still confused to imagine the area from the function that given. Usually, students could not determine the area that bounded by the curve, also determined the object that rotated by axis x or ordinate y. If they could not determine the area asking on the task, they could not solve the problems. This study focused on how students construct the knowledge related to represent the object becoming rotating object which were fundamental step before solving calculus problems called procedural understanding.

In this globalization era, technology is developing fast in all aspects of life includes in education. Integration of Information, Communication, and Technology (ICT) in education likes computer-based instruction [5]. ICT integration has a great effectiveness for both teachers and the students [5]. In particular, engineering students benefited from the integration of the mathematical software on both integral calculus conceptual understanding and procedural understanding [6]. In mathematics field, there are many kinds of software that can be used to understand mathematical concept, visualize the object of mathematics, also calculate the data. One of the software that common use in mathematics is Geogebra. Geogebra is free access software that could be downloaded at www.geogebra.com. The use of Geogebra is one of the best choices to teach students about application of integral. Geogebra has many beneficial for making geometry, algebra, and statistics easier to understand. Based on related research, using mathematics software could support on learning Calculus especially on visualization activities [7]. The aim of using Geogebra on this particular material is constructing student’s knowledge how they rotary object could be shaped. Using Geogebra construction is great concept even though it is not as easy as it sounds [8]. Besides, Geogebra related to Algebra, Geometry and Calculus directly and also more than 80% of the students did not suppose to have background knowledge to attend in the activities involving the software [9]. Then how does Geogebra could be implemented on learning Calculus? This aims of this research are describing the implementation of Geogebra on learning Calculus, representing respond students about Geogebra, and identifying students task using Geogebra.

2. Method
This research was qualitative description which explained the implementation of Geogebra on learning Calculus and respond of students while using it. The subject was students of Industrial Engineering Universitas Muhammadiyah Surakarta year academic 2017/2018 which had Integral Calculus class. The data was collected by observation, questioner, and task. The observation was done when implementing Geogebra on Calculus class, the questioner was used to know how students respond about learning Calculus with Geogebra and using it, then the task was used to identify how students could sketch the graph by using Geogebra. The data was analysed by data reduction, presentation, and conclusion. In addition, the data was validated by using two researchers which analyse about it, namely investigator triangulation.

3. Results and Discussions
The results and discussions of this study would be classified according to the researchs’ purposes, namely implementation of Geogebra in Calculus classes, identification of students’ task, and students’ response to Geogebra.

3.1 Implementation of Geogebra in Calculus Instruction
The Integral Calculus class took place at Universitas Muhammadiyah Surakarta which consist of 82 students of Industrial Engineering in two classes. Implementing Geogebra on learning Calculus was still
limited on application of integral material. It was because the aim of using GeoGebra in this research was visualizing the graph and determining the area that required on the problem. The implementation of GeoGebra was done in material about area, length of arch, volume of rotary object, and surface of rotating object. However, this research was focused on material about volume of rotary object as this material was more difficult than others. The implementation of GeoGebra in this research was doing in 150 minutes in length and consisted of four steps, namely introduction, simulation, practice, and task.

### 3.1.1 Introduction

To begin with, students should know the fundamental material of rotary-object volume. Previous knowledge of individual gains weight regarding learning outcome each individuals, even if collaborative knowledge construction has greater influences in this area [10]. Therefore, it was really important for teacher starting the lesson with the prior knowledge of students. In this stage, teacher explained the prerequisite of material which known as area between two graphs. The main aim of this lesson was encouraging students to represent the given function to two-dimensional graph. Students learn this material in conventional way by using their paper and ruler to sketch graph.

Besides, teacher describe GeoGebra, particularly the usage and facilities that served by GeoGebra. GeoGebra is open source application that could be downloaded by everyone at https://www.geogebra.org/. GeoGebra orders so many menus that could be represented mathematical concept and especially for integral calculus material, GeoGebra could be used to sketch the graph both in cartesian and polar coordinate, determine rotary object, and calculate the value of integral. In the introduction stage, students were known about the layout, toolbar, menus, and how to operate GeoGebra.

The most important aspect using GeoGebra for this study is that GeoGebra could help students discover relation between mathematical object and their graphical representation [11].

### 3.1.2 Simulation

After introduce prior knowledge and Geogebra, the next stage was simulation. The aim of simulation was demonstrating how using GeoGebra in volume of rotary object material. At this step, teacher demonstrated three main cases, namely volume of rotary object which rotated by axis x, volume of rotary object which rotated by ordinate y, and volume of rotary object between two curves. To make perspective of the curve become rotary object, it could be used “surface” command. Surface was used to make perspective of an object. The format of this command is \( \text{Surface} \left[<\text{Expression}>, <\text{Expression}>, <\text{Parameter Variable 1}>, <\text{Start Value}>, <\text{End Value}>, <\text{Parameter Variable 2}>, <\text{Start Value}>, <\text{End Value}> \right] \).

The first material about rotary object which rotated by axis x was presented in Figure 1. It was given a function \( f(x) = x^3 - x^2 + 2 \). If the curve was rotated by axis x and bounded by \(-1\) to \(1\), then how it could be sketch in GeoGebra. Those were the stages.

i. The function \( f(x) = x^3 - x^2 + 2 \) should be written in the input bar and it would became a curve that presented in Graphics layout.

ii. Making slider by choosing icon of slider, then made a slider for \( \alpha \) angle from \(0^\circ \) until \(360^\circ\).

iii. For making rotary object such as presented in Figure 1, it should be written \( \text{Surface} \left[ a, f(a) \cos(b), f(a) \sin(b), a, -1, 1, b, 0, \alpha \right] \) in the input bar for running the command.
Figure 1. The Rotary Objects Rotating by Axis $x$

This was very fundamental material on determining rotary object. Students should know how the function could be represented on the graph and also determining the bound from the problems. Rotary object by axis $x$ was the easiest one since they did not needed exchange the function became its invers. Besides, students supposed to know the rotary object were shaped by rotating $360^\circ$, hence they should make slider from $0^\circ$ to $360^\circ$.

The second material which showed at Figure 2 was rotary object that rotated by ordinate $y$. It was given a function $f(x) = x^3 + 1$. If the curves were rotated by ordinate $y$ and bounded from -1 to 2, then how it could be sketch in Geogebra. The steps were mentioned below.

i. Determining the invers of the function $f(x)$. It was $g(x) = \sqrt[3]{x - 1}$.

ii. Sketching the graph by written the function $f(x) = x^3 + 1$ also $g(x) = \sqrt[3]{x - 1}$ in the input bar.

iii. Making slider by choosing icon of slider, then made a slider for $\alpha$ angle from $0^\circ$ until $360^\circ$.

iv. It should be written Surface $[g(a) \cos(b), a, g(a) \sin(b), a, -1, 2, b, 0, \alpha]$ in the toolbar for running the command for making rotary object such as presented in Figure 2.
This second example was more complex than the first one since this case needed skill to find the invers of the function in order to make the graph. The providing function was a function in variable $x$, while the problem ask to make rotary object by rotating on ordinate $y$. Therefore, to make the command surface could run well, students should input the invers function.

The last material was rotary object which bounding from two curves. It was given two functions $f(x) = 2x^2 + x + 1$ and $g(x) = x^3 + 2$. If the area between that curves was rotated by axis $x$, then how it could be sketch in Geogebra. The stages were:

i. Sketching the graph by written the function $f(x) = 2x^2 + x + 1$ also $g(x) = x^3 + 2$ in the input bar.

ii. Determining the intersection point from the two curves, namely point A and point B for the point that located in the intersection area.

iii. Making slider by choosing icon of slider, then made a slider for $\alpha$ angle from $0^\circ$ until $360^\circ$.

iv. It should be written Surface $[a, f(a) \cos(b), f(a) \sin(b), a, x(A), x(B), b, 0, \alpha]$ and also Surface $[a, g(a) \cos(b), g(a) \sin(b), a, x(A), x(B), b, 0, \alpha]$ in the toolbar for running the command for making rotary object such as presented in Figure 3.
This last simulation was the most difficult ones as there were two functions given. For this case, students should understand the example one first. The students have represented each functions to two-dimensional graph, then they should determine two intersection points among two functions. These points would become bounds of the area. The next step was similar with the previous example include making slider; however it had difference on surface command.

These three kinds of examples gave consecutively as the first example was basic, the second was intermediate, and the last was the most complex problem. The complexity of problems were very important and teacher should begin with the easier and progress to more difficult and complicated until the problems solved [8]. Moreover, even this lesson using technology, students could construct their knowledge from the basic material and progress to represent the two-dimensional form, then how they could determine the bounds of area and finally rotated the object based on the problem. It suited with other research which explained that the knowledge construction processes of students occurred in technological context. In particular, supporting technology on the construction processes consisted of (1) explore representative case, (2) transition from one construct to other complex construct, (3) construct unexpected construct, and (4) justify the constructed relations [12].

### 3.1.3 Practice and Task

After the simulation stage, the students practiced to sketch the rotary object by using Geogebra. Based on the observation, most of students enjoyed and interested to use Geogebra. Moreover, they had big enthusiastic to make visualization of the object on Geogebra. Besides, students gave more attention toward the explanation and the material. In fact, students did not have any difficulties on operating Geogebra since they used technology easily in their daily activities. However, for particular software such as Geogebra, they needed to learn in special terms like the formula, the command, the special function, and many others. There were some problems might face by student when using Geogebra at the first time, such as the students’ unfamiliarity with the software, the learning curve that goes with it, and the usual network problems in a computer laboratory [8]. Therefore, teacher should provide to coach students in order to develop students’ skill on using Geogebra.

The last stage was giving task for student to know their understanding on rotary object material using Geogebra. The task that gave for student was drawing a rotary object bounding from -1 to 1 with rotating the curve \( f(x) = x^3 + 2 \) as far as 360° toward ordinate \( y \). The task was done in group in order to make
students solved the problems each other and teach others who did not understand the material yet. Besides, using discussion group for construction problems was also equally essential [8] and students who attended in collaborative learning are more satisfied with their outcome and could get better grade[13]. The task was also collected on Schoology, a learning management system that used to do blended learning, therefore the teacher could be analysing students understanding when using Geogebra. The aim of using Schoology at this lesson was to combine the face-to-face learning with online learning, so students could do their assignment in flexible time and place. There were some benefits using blended learning in terms of effectiveness and content knowledge; blended learning was better than face-to-face learning to make developing students’ learning processes and blended learning provided various multimedia content knowledge for students anytime and anywhere [14].

3.2 Identification of Students’ Task

Based on the task students collected, all of students could use Geogebra in order to draw the rotary object. However, there were some mistakes that found in their answers. After analysing students task, the mistakes could be classified to three types of answers which show how their ability to operate the Geogebra. The first was correct answer, the second was doing mistake on rotating object, and the last was doing mistake on rotating object and bounding the object.

The Figure 4 presented the correct answer from students. There was 53.97% students could solve the problem correctly. From this Figure, it could be analysed that students could sketch the graph \( f(x) = x^3 + 2 \) and the inverse \( g(x) = \sqrt[3]{x - 2} \) correctly, and also make slider with angle from 0° until 360°. Besides, they could use the surface command to represent the rotary object. Moreover, one of essential aspect was determining the bound of object. Therefore, they could make the rotary object appropriate as the instruction.

As students at this group could solve the problems perfectly, it means that they understand all of the process to shape rotary object. their ability to represent the function into two-dimensional object was really great as they could determine the inverse function. Besides, they understood that the bounds of the area were limited between −1 to 1 which gave impact to the formula they used. If they did not pay attention with the bounds, then the rotary object would be shaped in different form. The last, they could construct their idea to build rotary object that rotated toward ordinate \( y \).
Based on the analyses of the student task, there was 14.9% student that made mistake on rotating object. The students’ answers in this group were showed in the Figure 5. It can be seen the students could make sketch from the function $f(x) = x^3 + 2$, then the slider and also could use the surface command with the bound of object too. However, they did mistake on rotating as the instruction ask for rotating toward ordinate $y$, but this object was rotated toward axis $x$.

![Figure 5. The False Answer with Mistakes on Rotating Object](image)

In this case, students had mistake on rotating the curve. In particular, students did well on representing the function into two-dimensional graphs and making slider from $0^\circ$ to $360^\circ$. Surprisingly, they did not determine the inverse function which means that they did not think carefully whether the curve would be rotated toward axis $x$ or ordinate $y$. It was also probably they did not understand how to rotate the object through ordinate $y$. Therefore, they did not get the proper rotary object consider to the problems.

The third type of the students answer was presented in Figure 6. The identification showed that 3.75% students did mistakes with direction of rotating and also determined the bound of the area. The picture shows that students could draw the curve from function $f(x) = x^3 + 2$ and made slider with angel from $0^\circ$ to $360^\circ$. Besides, they could use the surface command. However, they did mistakes on rotating toward axis $x$ since the instruction ask the rotating toward ordinate $y$. In addition, they did not give any appropriate bound at the format on surface command, thus the rotary object was false.
This last category was similar with the second one. The main error was rotating the object toward axis $x$. Additionally, students also did not give bounds on the curve. Thus, the rotary object was not proper as the problem ask. In fact, this category had the most mistakes among others. It indicated that there were a few students did not understand how to construct idea in order to build the rotary object.

3.3 Students’ response to Geogebra

Another point was that how students respond to the use of Geogebra in calculus learning. The students’ response was analysed by using questionnaire which consisted of four main questions. The first was students’ opinion toward learning Calculus by using Geogebra. In this study, there were some notes such as students said that using Geogebra on learning Calculus needed much time and they tended to focus on doing some tasks rather than doing the calculation. In particular, Geogebra was only used at the basic material like interpretation of function to curve and representation two-dimension object to three-dimension object and it still lack for operating or calculating the integral. Even the use of Geogebra still limited on the representation of object into three-dimensional object, many students felt satisfy as they could imagine the real object. From the percentages, there was 68.25% students stated that it was enjoyable.

Some of them said that there was no needed to sketch the object manually as they used Geogebra and it was easier. These positive response concurred with other vast research [15–18].

The second respond about whether easy or no for students using Geogebra by themself. The result was 36.5% students stated that it was easy, 31.75% said that it was easy enough, and the rest was said that it was not easy. Some students stated that it was not easy because they think that they were not familiar on using Geogebra yet. In contrary, many of them feel easy to use Geogebra since it was user friendly and also they could see the tutorial on youtube or website of Geogebra.

The next was about whether using Geogebra made easy to understand the material or no. The outcome was 34.9% students said that Geogebra make easier to understand the material, 33.33% students stated that it was easy enough, and the rest was contending that Geogebra did not make easier to understand the material. Most of students believe that using Geogebra was effective since they just input the function, then the object could be drawn on Geogebra screen. Besides, Geogebra made students easier to imagine the abstract object. The properties and the relationship of the object and image could be observed easily and simultaneously, the incorporation of GeoGebra could accelerate the learning process [16]. However, there was a note that teacher was still needed to explain the material clearly.
since they just sketch the object. Although dynamic software provide positive impact upon learners’ thus becoming potential tools in teaching mathematics, the use of mathematical software need to be further considered in its utility and feasibility [19]. Students in the traditional group pay more prominence to procedural fluency, while students in the computer-based learning group attach more significance to conceptual competencies in terms of instructional objects [20]. The last, students did not have any difficulties on using Géogebra because they believe that they just needed to practice more.

There were some findings of this research. Firstly, Student gave more attention on classroom as they were interested in visualization of object by using Géogebra. This was supported by other research which concluding that the activities on learning that using Géogebra could increase students motivation to participate in subsequent activities [1]. Secondly, more than a half of the number of students in the class gave positive impact about using Géogebra on learning Calculus. According to other research, the use of Géogebra had a positive effect on understanding knowledge of the students [21]. Besides, using Géogebra on Differential and Integral Calculus facilitating students’ understanding about the content, providing space discussion new technologies [22]. Lastly, Géogebra was user-friendly software that could be used and learnt for everyone. It was supported by other research showing that, Géogebra could be alternative of teacher since it contained the interactive task. Géogebra was effective integration on primary mathematics [23].

4. Conclusion
To sum up, learning calculus with Géogebra could make students give more attention on the material, but it still limited on application of integral material. In addition, most of students gave positive respond about using Géogebra on learning Calculus, although some of them still confused when used Géogebra by their self. More than 50% students could sketch the graph by Géogebra correctly.

References
[1] Nobre C N, Rezende M and Meireles G 2016 The use of geogebra software as a calculus teaching and learning tool Inf. Educ. 15 253–67
[2] Tall D 1997 Functions and Calculus (International Handbook of Mathematics Education)
[3] Machromah I U, Eriska M, Purnomo R, Febriyanti K and Ayu H 2017 Arithmetics Skill : Kesulitan Utama Mahasiswa dalam Menyelesaikan Soal Kalkulus Integral Seminar Matematika dan Pendidikan Matematika UNY pp 365–72
[4] Machromah I U, Eriska M, Purnomo R, Surakarta U M and Tengah J 2017 Process skill error: the majority student’s error in problem solving of integral calculus J. Daya Matenatis 5 358–76
[5] Ghavifekr S, Athirah W, Rosdy W and Teaching W A W 2015 Teaching and learning with technology : effectiveness of ict integration in schools Int. J. Res. Educ. Sci. 1 175–91
[6] Salleh T S A and Zakaria E 2012 The Effects of Integrating technology on students’ conceptual and procedural understandings in integral calculus Asian Soc. Sci. 8 8–16
[7] Sari C K, Machromah I U and Safika K N S 2017 Pemanfaatan Teknologi dalam Pembelajaran Kalkulus: Mendukung atau Sia-sia? pp 59–76
[8] Laigo G R, Bhatti A H, P L K and Yohannes H M G 2016 Revisiting geometric construction using geogebra Electron. J. Math. Technol. 10 35–43
[9] Pereira L R, Silva J M da and Jardim D F 2017 Practices for Geometry Teaching Using Geogebra New Perspectives in Science Education
[10] Ertl B and Mandl H 2008 Effects of individual prior knowledge on collaborative knowledge construction and individual learning outcomes Proceedings of the Conference Knowledge Construction in E-learning
[11] Dikovic L 2009 Implementing dynamic mathematics resources with geogebra at the college level Int. J. Emerg. Technol. Learn. 4 51–4
[12] Anabousy A and Tabach M 2018 Using Geogebra To Enhance Students’ Inquiry Activity Proceedings of the 13th International Congress on Mathematical Education
[13] Burke A 2011 Group Work : How to Use Groups Effectively J. Eff. Teach. 11 87–95

10
[14] Irawan V T, Sutadji E and Widiyanti E 2017 Blended learning based on schoology: Effort of improvement learning outcome and practicum chance in vocational high school Cogent Educ. 11 1–10
[15] Shadaan P and Eu L K 2013 Effectiveness of using geogebra on students’ understanding in learning circles Malaysian Online J. Educ. Technol. 1 1–11
[16] Chua G L L, Tengah K A, Shahrill M, Tan A and Leong E 2017 Analysing Students’ Perspectives on Geometry Learning from The Combination of Van Hiele Phase-Based Instructions and Geogebra Proceding of the 3rd International Conference on Education pp 205–13
[17] Arbain N and Shukor N A 2015 The effects of GeoGebra on students achievement Procedia - Soc. Behav. Sci. 172 208–14
[18] Horzum T and Ünlü M 2017 Pre-service mathematics teachers’ views about geogebra and its use acta didact. Napocensia 10 77–90
[19] Tarmizi R A, Fauzi A, Ayub M, Bakar K A and Yunus A S 2010 Effects of technology enhanced teaching on performance and cognitive load in calculus 4 109–20
[20] Sevimli E 2016 Do calculus students demand technology integration into learning environment? case of instructional differences Int. J. Educ. Technol. High. Educ. 13 1–18
[21] Dikovic L 2009 Applications geogebra into teaching some topics of mathematics at the college level ComSIS 6 198–203
[22] Santos G G S, Santos J A dos and Silva M A A e 2013 Use in teaching geogebra differential and integral calculus: an experience with students in higher education J. Brazilian Conf. Geogebra 1 56–67
[23] Žilinskienė I and Demirbilek M 2015 Use of geogebra in primary math education in lithuania: an exploratory study from teachers’ perspective Inf. Educ. 14 127–42