A Binary Neutron Star GRB Model

J. R. Wilson*,†, J. D. Salmonson*, & G. J. Mathews†

*University of California, Lawrence Livermore National Laboratory, Livermore, California 94550
†University of Notre Dame, Department of Physics, Notre Dame, Indiana 46556

Abstract. In this paper we present the preliminary results of a model for the production of gamma-ray bursts (GRBs) through the compressional heating of binary neutron stars near their last stable orbit prior to merger.

Recent numerical studies of the general relativistic (GR) hydrodynamics in three spatial dimensions of close neutron star binaries (NSBs) have uncovered evidence for the compression and heating of the individual neutron stars (NSs) prior to merger [1,2]. This effect will have significant effect on the production of gravitational waves, neutrinos and, ultimately, energetic photons. The study of the production of these photons in close NSBs and, in particular, its correspondence to observed GRBs is the subject of this paper.

The gamma-rays arise as follows. Compressional heating causes the neutron stars to emit neutrino pairs which, in turn, annihilate to produce a hot electron-positron pair plasma. This pair-photon plasma expands rapidly until it becomes optically thin, at which point the photons are released. We show that this process can indeed satisfy three basic requirements of a model for cosmological gamma-ray bursts:
1) sufficient gamma-ray energy release ($> 10^{51}$ ergs) to produce observed fluxes,
2) a time-scale of the primary burst duration consistent with that of a “classical” GRB ($\sim 10$ seconds),
3) peak of photon number spectrum matches that of “classical” GRB ($\sim 300$ keV).

NEUTRON STAR HEATING

The method for solving the general relativistic field equations in three spatial dimensions has been discussed in [1,2]. At each time slice we obtain an exact (to numerical accuracy) instantaneous solution to the GR field equations. The hydrodynamic equations are then evolved for the moving matter against these GR fields. This method ignores gravitational waves, however in [1] it was shown that the effect is very small; $\frac{\dot{J}}{\omega J} \sim 10^{-4}$, where $J$ is the angular momentum and $\omega$ is the angular frequency of the NSB.

The computational evolution calculation of NSBs and their GR fields begins by generating two identical non-spinning neutron stars with an initial mass and an EOS. The stars are allowed to evolve until a stationary orbit is achieved with a
prescribed initial angular momentum. A variety of systems have been studied over a range of star masses, equations of state and initial angular momentums. The key result to report here is that the proper baryonic density of the stars was observed to increase prior to the stars reaching their last stable orbit. This compression and heating can be parameterized in terms of $U^2$, the squared amplitude of the spatial components of the 4-velocity (Figure 1). See [2–4] for discussion of why the compression is not adiabatic.

In [2] it is argued that the gravitational binding energy will be converted into thermal energy. This thermal energy will be radiated via neutrino luminosity. The time scale for the energy emitted up to collapse ($t < 0$) can be estimated from the gravitation wave emission

$$E(t) = \frac{E_{tot}}{[1 - (64/5) m^{5/3} \omega_0^{8/3} t]^1/2}$$

where $\omega_0$ is the final angular orbital velocity.

Some of the $\nu\overline{\nu}$ pairs emanating from the surface of a hot neutron star will annihilate to create $e^+e^-$ pairs. In order to calculate an estimate of the efficiency of this process, the numerical supernova model of Mayle & Wilson [5,2] was used. It was found that the $\nu + \overline{\nu} \rightarrow e^+ + e^-$ reaction is 3% efficient at the end of a standard supernova calculation when the neutron star at the center is at its hottest and most compact. This simulation includes all GR effects, except the neutrinos are assumed to travel in straight lines. In the strong field environment around a NSB the neutrino trajectories will be significantly bent, thus increasing the chances of $\nu\overline{\nu}$ annihilation. New estimates show that this bending will augment $\nu\overline{\nu}$ annihilation by a factor of $\sim 3$. So the efficiency of $\nu + \overline{\nu} \rightarrow e^+ + e^-$ is estimated to be $\sim 10\%$. It is also found that the entropy per baryon of the pair plasma is greater than $10^{10}$ so relatively very few baryons are liberated from the surface of the star.

From Figure 1 we see that a typical star emanates $\sim 3 \times 10^{52}$ ergs of gravitational energy as $\nu\overline{\nu}$ pairs. Thus we have $\sim 6 \times 10^{52}$ ergs in $\nu\overline{\nu}$ pairs from both stars. A 10% efficiency in $\nu + \overline{\nu} \rightarrow e^+ + e^-$ gives us $\sim 6 \times 10^{51}$ ergs of energy in the form of...
a $e^+e^-$ pair plasma-photon gas. The temperature of this gas near the NS is several MeV.

**THE GAMMA RAY BURST**

Having roughly defined the initial parameters of the hot $e^+e^-$ pair wind blowing off of a NS, we wish to follow its evolution and characterize the observable gamma-ray emission. It is important to note that there are no free parameters in this model, barring uncertainties in understanding and correctly calculating the physics; our signature either corresponds to an experimentally observed phenomenon (i.e. GRBs) or it does not.

The expanding $e^+e^-$ pair plasma emanating from a NS is modeled as a spherically symmetric special relativistic fluid by the following hydrodynamic equations:

$$\frac{\partial D}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 DV)$$

$$\frac{\partial E}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 EV) - P \left[ \frac{\partial \gamma}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \gamma V) \right]$$

$$\frac{\partial S}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 SV) - \frac{\partial P}{\partial r}$$

where the radial 4-velocity $U$, Lorentz factor $\gamma$ and coordinate velocity $V$ are defined as

$$U \equiv \frac{S}{D + \Gamma E}, \quad \gamma \equiv \sqrt{1 + U^2}, \quad V \equiv \frac{U}{\gamma}.$$

$D$ and $E$ are the coordinate densities of baryons and thermal energy ($e^+e^-$ and photons) respectively.

The total energy equation, including photons and $e^+e^-$ pairs, is

$$E_{tot} = aT^4 + E_{pairs}(T).$$

To track the $e^+e^-$ pairs we define a pair equation

$$\frac{\partial N_{pairs}}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 N_{pairs} V) + \overline{\sigma v} N_{pairs}(N_{pairs}^0(T) - N_{pairs})/\gamma^2$$

where the coordinate pair number density is $N_{pair}$ and $\overline{\sigma v}$ gives the mean pair annihilation rate. $N_{pairs}^0(T)$ is given by a Fermi integral with a chemical potential of zero.

To model the material blown off the surface of a NS we inject baryon and pair-photon energy densities into the innermost zone at a rate determined by the time derivative of the heating energy given in Equation (1).
The hydrodynamic equations are evolved, allowing the plasma to expand. Once the system becomes transparent to Compton scattering, assuming no further scattering, the calculation is stopped and the photon signal is analyzed. In the results presented here we have set $E_{\text{tot}} = 10^{51}$ ergs. Since the entropy per baryon of the wind is quite high we define the rate of injection of baryons as $D = 10^{-10} \dot{E}$.

Since the photons and $e^+e^-$ pairs appear to decouple at virtually the same time throughout the entire fireball (radius $\sim 10^{12}$ cm), we take this event to be instantaneous and to occur when the cloud becomes optically thin to Compton scattering. We then look at two observables, the time integrated number spectrum $N(\epsilon)$ and the total energy received as a function of time $E(t)$.

To get the spectrum, as mentioned above, we assume that the $e^+e^-$ pairs and photons are in thermodynamic equilibrium when they decouple. Thus the photons in the fluid frame (denoted with a prime) make up a Planck distribution. We calculate the observed number spectrum, per photon energy $\epsilon$, per steradian, of a relativistically expanding spherical shell with radius $R$, thickness $dR$ in cm, velocity $v$ where $c=1$, Lorentz factor $\gamma$ and fluid-frame temperature $T'$ to be

$$N(\epsilon) = 4\pi R^2 dR \frac{\epsilon T'}{v \gamma} \log \left[ \frac{1 - \exp[-\gamma \epsilon(1 + v)/T']} {1 - \exp[-\gamma \epsilon(1 - v)/T']} \right]$$

which has a maximum at $\epsilon_{\text{max}} \cong 1.39\gamma T' \text{ eV}$ for $\gamma \gg 1$. We may then sum this spectrum over all shells of our fireball to get the total spectrum shown in Figure 2 (Top). Since we a priori assume the photons are thermal, our spectrum has a high frequency exponential tail.

A key feature of this spectrum is that its peak agrees with observation. It is interesting to note that the bulk of the photons have a fluid frame temperature of only $\epsilon \sim 5 - 15$ eV, but are Lorentz boosted by $\gamma \sim 10^4 - 10^5$. Thus our spectrum derives from a more relativistic fluid than other models. The photons to be observed at early times have about twice the energy of the later photons.

To acquire the observed light curve $E(t)$ we consider two effects. First is the relative arrival time of the first light from each shell. Second is the shape of the light curve from a single shell [6]. We find that, for our Plank distribution of photons, a relativistically expanding shell of radius $R$ will have a time profile $E(\tau > R/c) \sim (\frac{R}{ct})^4$.

In Figure 2 (Bottom) we see an example of $E(t)$ for NSB of equal star mass. Variation in the ratio of star mass in the NSB effects the relative compression and heating rate of each star, thus allowing a variety of GRB durations. We estimate a range of burst durations from several seconds to a few $10^3$ seconds.

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REFERENCES

1. Wilson, J. R., Mathews, G. J. & Marronetti, P., Phys. Rev. D. 54, 1317 (1996).
2. Mathews, G. J., & Wilson, J. R., Astrophysical J. 482, 929—941, (1997).
FIGURE 2. (Top) Photon number spectrum $N_{37}(\epsilon) \times 10^{37}$ photons/keV/4$\pi$ from $e^+e^-$ pair plasma fireball. (Bottom) Light curve $E_{49}(t) \times 10^{49}$ ergs/second/4$\pi$.

3. Wilson, J. R., & Mathews, G. J., submitted Phys. Rev. D (1997).
4. Mathews, G. J., submitted Phys. Rev. Lett. (1997).
5. Wilson, J. R., & Mayle, R. W., Phys. Rep. 227, 97, (1993).
6. Fenimore, E. E. et al., Astrophysical J. 473, 998, (1996).