Decay of $Z$ into Two Light Higgs Bosons

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Abstract

If the standard electroweak gauge model is extended to include two or more Higgs doublets, there may be a neutral Higgs boson $h$ which is light (with a mass of say 10 GeV) but the $hZZ$ coupling is suppressed so that it has so far escaped experimental detection. However, the effective $hhZZ$ coupling is generally unsuppressed, hence the decay of $Z$ into two light Higgs bosons plus a fermion-antifermion pair may have an observable branching fraction, especially if $h$ decays invisibly as for example in the recently proposed doublet Majoron model.
1 Introduction

In the standard model, the Z boson may decay into the Higgs boson H and a fermion-antifermion pair. From the absence of such events, it has been deduced that $m_H$ is greater than about 60 GeV[1]. On the other hand, in extensions of the standard model with two or more scalar doublets, there may be a neutral Higgs boson $h$ with $m_h < 60$ GeV which has escaped experimental detection so far because the $hZZ$ coupling is suppressed[2]. This situation can be the natural consequence of a particular theoretical model, an example[3] of which will be discussed in this paper.

Our main observation is that the effective $hhZZ$ coupling is generally unsuppressed, hence the decay of Z into two h’s plus a fermion-antifermion pair may have an observable branching fraction. Note that the decay $Z \rightarrow hh$ is strictly forbidden because of angular-momentum conservation and Bose statistics. In Section 2 we formulate our analysis in the context of two Higgs doublets. We obtain the condition for the possible existence of a light $h$ which would not conflict with present data. In Section 3 we identify all the contributions to the effective $hhZZ$ coupling. We show that the gauge-sector contributions alone are probably not large enough for our proposed process to be observable, but the addition of scalar-sector contributions may make it so. In Section 4 we focus on the recently proposed doublet Majoron model[3] where $h$ decays invisibly and discuss a related issue. Finally in Section 5 there are some concluding remarks.

2 Two Higgs Doublets

Consider the following Higgs potential $V$ for two $SU(2) \times U(1)$ scalar doublets $\Phi_{1,2} = (\phi_{1,2}^1, \phi_{1,2}^0)$:

$$V = \mu_1^2 \Phi_1 \Phi_1^* + \mu_2^2 \Phi_2 \Phi_2^* + \mu_{12}^2 (\Phi_1 \Phi_2^* + \Phi_2 \Phi_1^*)$$
\[
\begin{align*}
&+ \frac{1}{2} \lambda_1 (\Phi_1^+ \Phi_1^-)^2 + \frac{1}{2} \lambda_2 (\Phi_2^+ \Phi_2^-)^2 + \lambda_3 (\Phi_1^+ \Phi_1^-)(\Phi_2^+ \Phi_2^-) \\
&+ \lambda_4 (\Phi_1^+ \Phi_2^-)(\Phi_2^+ \Phi_1^-) + \frac{1}{2} \lambda_5 (\Phi_1^+ \Phi_1^-)^2 + \frac{1}{2} \lambda_6^* (\Phi_2^+ \Phi_2^-)^2. \\
\end{align*}
\]

(1)

Assume \( \lambda_5 \) to be real for simplicity. Define \( \tan \beta \equiv v_2/v_1 \) as is customary, where \( v_{1,2} = \langle \phi_{1,2}^0 \rangle \) are the usual two nonzero vacuum expectation values. The charged Higgs boson is then given by

\[ H^\pm = \sin \beta \phi_1^\pm - \cos \beta \phi_2^\pm, \]

(2)

\[ m_{H^\pm}^2 = -\mu_{12}^2 (\tan \beta + \cot \beta) - (\lambda_4 + \lambda_5) (v_1^2 + v_2^2); \]

(3)

the pseudoscalar neutral Higgs boson is given by

\[ A = \sqrt{2} (\sin \beta \text{Im}\phi_1^0 - \cos \beta \text{Im}\phi_2^0), \]

(4)

\[ m_A^2 = -\mu_{12}^2 (\tan \beta + \cot \beta) - 2\lambda_5 (v_1^2 + v_2^2); \]

(5)

and the two scalar neutral Higgs bosons \( \sqrt{2} \text{Re}\phi_{1,2}^0 \) have the following mass-squared matrix:

\[ \mathcal{M}^2 = \begin{bmatrix}
-\mu_{12}^2 \tan \beta + 2\lambda_1 v_1^2 & \mu_{12}^2 + 2(\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2 \\
\mu_{12}^2 + 2(\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2 & -\mu_{12}^2 \cot \beta + 2\lambda_2 v_2^2
\end{bmatrix}. \]

(6)

Let us rotate to the basis of \( H = \sqrt{2} (\cos \beta \text{Re}\phi_1^0 + \sin \beta \text{Re}\phi_2^0) \) which couples singly to the Z, and \( H' = \sqrt{2} (\sin \beta \text{Re}\phi_1^0 - \cos \beta \text{Re}\phi_2^0) \) which does not. Then

\[ \mathcal{M}_{11}^2 = 2[\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + 2(\lambda_3 + \lambda_4 + \lambda_5) \sin^2 \beta \cos^2 \beta](v_1^2 + v_2^2), \]

(7)

\[ \mathcal{M}_{22}^2 = -\mu_{12}^2 (\tan \beta + \cot \beta) + 2(\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4 - 2\lambda_5) \sin^2 \beta \cos^2 \beta(v_1^2 + v_2^2), \]

(8)

\[ \mathcal{M}_{12}^2 = \mathcal{M}_{21}^2 = -\sin 2\beta[\lambda_1 \cos^2 \beta - \lambda_2 \sin^2 \beta - (\lambda_3 + \lambda_4 + \lambda_5) \cos 2\beta](v_1^2 + v_2^2). \]

(9)

Hence \( H' \) may well have escaped experimental detection if its mixing with \( H \) is small. This is clearly the case if \( \sin 2\beta << 1 \). Without loss of generality, let us consider \( v_1 << v_2 \), i.e. \( \sin \beta \approx 1 \) and \( \cos \beta \approx 0 \); and assume \( \mathcal{M}_{22}^2 \) to be small. Then \( h \approx H' \) and

\[ m_h^2 \approx \mathcal{M}_{22}^2 - \mathcal{M}_{21}^2 \mathcal{M}_{12}^2 / \mathcal{M}_{11}^2 \]

\[ \approx -\mu_{12}^2 \tan \beta + 2\lambda_1 v_1^2 \left[ 1 - \frac{(\lambda_3 + \lambda_4 + \lambda_5)^2}{\lambda_1 \lambda_2} \right]. \]

(10)
The second term in the above is naturally small, but the first term is not, unless \( \mu_{12}^2 \) is fine-tuned to be of order \( v_3/v_2 \). This means that unless \( \mu_{12}^2 = 0 \) from a symmetry requirement of the model, it is not likely that a light neutral Higgs boson has so far escaped experimental detection. Furthermore, since the \( hAZ \) coupling is unsuppressed in this limit, the nonobservation of \( Z \rightarrow hA \) at the \( e^+e^- \) collider LEP at CERN means that \( m_h + m_A > M_Z \).

Comparing Eq. (10) with Eq. (5), we see that \( \lambda_5 \neq 0 \) is another necessary condition. By the same token, the nonobservation of \( Z \rightarrow H^+H^- \) requires \( \lambda_4 + \lambda_5 \neq 0 \).

In the minimal supersymmetric standard model (MSSM),

\[
\lambda_1 = \lambda_2 = -\lambda_3 - \lambda_4 = \frac{1}{4}(g_1^2 + g_2^2), \quad \lambda_5 = 0. \tag{11}
\]

Hence \( m_h \approx m_A \approx -\mu_{12}^2 \tan \beta \) for large \( \tan \beta \) and the absence of \( Z \rightarrow hA \) events implies \( m_h > M_Z/2 \). This means that \( Z \) decay into two \( h \)'s would not be possible kinematically and our proposal cannot be realized in the MSSM. On the other hand, a natural model with \( \mu_{12}^2 = 0 \) and \( \lambda_5 \neq 0 \) does exist. It is the recently proposed doublet Majoron model\[3\], details of which will be discussed later in Section 4.

### 3 Effective hhZZ Coupling

We assume \( h \) to be light and \( A \) to be heavy so that \( Z \rightarrow hA \) is kinematically forbidden, whereas \( Z \rightarrow hh \bar{f}f \) is allowed, \( f \) being either a quark or a lepton. The contributing diagrams are given in Fig. 1. (We have assumed that Yukawa couplings of \( f \) to \( h \) are negligible. Actually there is an important exception if the \( b \) quark has a large Yukawa coupling to both \( h \) and \( A \). In that case, the \( Z \rightarrow \bar{b}b \) rate may be enhanced to explain the \( R_b \) excess observed at LEP, but then \( Z \rightarrow \bar{b}b + h(A) \) should become observable\[4\] with a branching fraction of order \( 10^{-4} \).) We can eliminate diagram (c) because it is negligible for large \( \tan \beta \). As for diagrams (b) and (d), they may be suppressed for large values of \( m_H \) and \( m_A \) respectively.
The only model-independent contribution is that of diagram (a). The fundamental $hhZ^\mu Z^\nu$ coupling is always unsuppressed, with Feynman rule given by $ig^2g^{\mu\nu}/2\cos^2\theta_W$, same as that for $HHZ^\mu Z^\nu$. Let the $Z\bar{f}f$ coupling be given by $(g/\cos\theta_W)Z^\mu\bar{f}\gamma_\mu(a + b\gamma_5)f$, where $a = I_{3L}/2 - Q\sin^2\theta_W$ and $b = -I_{3L}/2$, $I_{3L}$ and $Q$ being the weak-isospin projection of $f_L$ and electric charge of $f$ respectively. Integrating out the momenta of the two $h$'s, we obtain the differential decay rate of $Z \to hh\bar{f}f$ as a function of the energies $E_3$ and $E_4$ of the fermion-antifermion pair, and the angle $\theta_{34}$ between them:

$$\frac{d\Gamma}{dE_3dE_4d\cos\theta_{34}} = \frac{g^6(a^2 + b^2)}{3072\pi^5\cos^6\theta_W} \frac{E_3^2 E_4^2 (3 - \cos\theta_{34})\sqrt{1 - \Delta}}{M_Z^2 [M_Z^2 - 2E_3E_4(1 - \cos\theta_{34})]^2},$$

(12)

where

$$\Delta = \frac{4m_h^2}{M_Z(M_Z - 2E_3 - 2E_4) + 2E_3E_4(1 - \cos\theta_{34})}.$$  

(13)

We have assumed in the above that $m_f$ can be neglected. The kinematical constraints are

$$0 < E_3 < \frac{1}{2}M_Z \left[ 1 - \frac{4m_h^2}{M_Z^2} \right], \quad -1 < \cos\theta_{34} < 1,$$

(14)

$$0 < E_4 < \frac{1}{2}M_Z \left[ \frac{M_Z - 2E_3 - 4m_h^2/M_Z}{M_Z - E_3(1 - \cos\theta_{34})} \right].$$

(15)

We integrate the above numerically and sum over all fermion species, i.e. the quarks $u, d, s, c, b$, and the three families of leptons $\nu_i, l_i$. The resulting total branching fraction for $Z \to hh\bar{f}f$ as a function of $m_h$ is shown in Fig. 2. Using the experimental Z width of 2.5 GeV, the branching fraction is thus only about $1.8 \times 10^{-8}$ for $m_h = 10$ GeV. With the accumulation of about $8 \times 10^6$ Z decays at LEP up to the end of 1993, this amounts to only 0.15 event. This means that if only diagram (a) is important, we do not expect this decay to be readily observable. The contribution of diagram (d) is also small, i.e. comparable in magnitude to that of diagram (a), because $m_A$ must be greater than $M_Z - m_h$ and there are no other adjustable parameters.

The contribution of diagram (b) depends on the $hhH$ coupling with Feynman rule given by $-i\sqrt{2}(\lambda_3 + \lambda_4 + \lambda_5)v_2$ for large $\tan\beta$. Now $m_H^2 \simeq 2\lambda_2v_2^2$ in this limit, hence the effective
$hhZZ$ contribution in diagram (b) is given by

$$\frac{ig^2 g^{\mu\nu}}{2 \cos^2 \theta_W} \frac{2(\lambda_3 + \lambda_4 + \lambda_5)v_2^2}{(k_1 + k_2)^2 - 2\lambda_2 v_2^2},$$  \hspace{1cm} (16)$$

where $k_{1,2}$ are the four-momenta of the two $h$’s. An enhancement of the $Z \to hh\bar{f}f$ rate is thus possible if the ratio $(\lambda_3 + \lambda_4 + \lambda_5)/\lambda_2$ is large enough. Further enhancement occurs if $m_H$ is not much greater than its experimental lower bound of about 60 GeV. Assuming that diagram (b) dominates, we show in Fig. 3 the branching fraction $B$ of $Z \to hh\bar{f}f$ for various values of $|\lambda_3 + \lambda_4 + \lambda_5|$, $m_h$, and $m_H$. We note that $|\lambda_3 + \lambda_4 + \lambda_5|$ should not be too large, because $\lambda_1 - (\lambda_3 + \lambda_4 + \lambda_5)^2/\lambda_2$ is constrained by the smallness of $m_h^2$ to be at most of order unity and we want to avoid having to fine-tune $\lambda_1$. We see from Fig. 3 that there is a significant region in parameter space with $B > 10^{-6}$, in which case our proposed process may in fact become observable.

So far we have not specified how the scalar doublets couple to quarks and leptons. If we consider the usual case of $\Phi_1(\Phi_2)$ coupling to down(up)-type quarks and charged(neutral) leptons, then $h$ decays mainly into $b\bar{b}$ if kinematically allowed. The final state of our process would then contain $b\bar{b}\bar{b}\bar{b} +$ another fermion-antifermion pair. The background to this from second-order QCD (quantum chromodynamics) radiative corrections has not been calculated, but we estimate it to be of order $10^{-5}$ to $10^{-6}$. This would make it very difficult for the observation of $Z \to hh\bar{f}f$ unless its branching fraction is much greater than $10^{-6}$. On the other hand, in some theoretical models, $h$ decays invisibly[5]. It may appear at first sight that this would be more difficult to detect experimentally[6]. Actually the opposite is true at LEP because the well-defined missing energy provides a signature with essentially no background and better limits are possible if $h$ decays invisibly than otherwise[4]. In the following section we will discuss the scalar sector of the recently proposed doublet Majoron model which has all the desired properties for the possible observation of $Z \to hh\bar{f}f$, i.e. $\mu_{12}^2 = 0$, $\lambda_5 \neq 0$, and $h$ decays invisibly to two massless Goldstone bosons.
4 Doublet Majoron Model

The scalar sector of the doublet Majoron model[3] consists of three $SU(2) \times U(1)$ doublets with lepton number assignments $L = 0, 1$ and $-1$. The Lagrangian is assumed to conserve $L$, hence terms of the form $\Phi_i^\dagger \Phi_j$ for $i \neq j$ are not allowed. The Higgs potential is also assumed to be symmetric under the interchange of the two scalar doublets with $L = \pm 1$. As all three doublets acquire nonzero vacuum expectation values, $L$ is spontaneously broken, resulting in the appearance of a massless Goldstone boson called the Majoron which is a decay product of $\nu_\tau$ in this model. Because of the interchange symmetry, the Majoron $J$ and its neutral and charged partners are odd under a discrete $Z_2$ symmetry whereas the other scalar particles are even. The Higgs potential consisting of only the latter two scalar doublets is of the form of Eq. (1). In the notation of Ref. 3, it can easily be shown that

$$
\begin{align*}
\lambda_1 &= 2\lambda_0, \\
\lambda_2 &= \lambda + \frac{1}{2}\eta_{12}, \\
\lambda_3 &= \eta + \zeta + \xi, \\
\lambda_4 &= -\xi, \\
\lambda_5 &= -\xi; \\
\mu_{12}^2 &= 0, \\
v_2 &= v_0, \\
v_1 &= \sqrt{2}v_L.
\end{align*}
\quad (17)
\quad (18)
$$

Furthermore $v_1$ is required to be small, say of order 20 GeV, in this model for various cosmological and astrophysical reasons[4]. We have thus all the features necessary for a light $h$ which may have escaped experimental detection. In addition, the dominant decay of $h$ in this model is into two Majorons ($JJ$) which interact very weakly and are thus invisible. The signature of $Z \to hh\bar{f}f$ is then $Z \to \bar{f}f +$ missing energy. This decay mode is free of QCD backgrounds and may be observable if its branching fraction is greater than $10^{-6}$.

Suppose we replace $h$ by $J$ and consider the decay of $Z \to JJ\bar{f}f$. It appears at first sight that this may have an enhanced branching fraction, using the same argument as we have presented. Actually, this is not the case. The reason is that $J$ is a Goldstone boson, so that it can always be parametrized exponentially, i.e. as a phase. Hence it has only derivative couplings in this representation and the analog of diagram (a) does not exist.
addition, the analog of diagram (b) cannot be enhanced; in the limit of \( m_h = m_H \) it is in fact suppressed by a factor \( (k_1 \cdot k_2) / (\lambda_3 + \lambda_4 + \lambda_5) v_2^2 \) relative to that of \( Z \to h\bar{h}f \). In the linear representation where \( J \) is treated on the same footing as the other particles, this suppression manifests itself as a necessary cancellation between diagrams (a) and (b), and we arrive at the same physical amplitude. In other words, the \( hJJ \) and \( HJJ \) couplings are not arbitrary, but are related to the other parameters of the model in a completely determined way.

5 Concluding Remarks

If there are two or more Higgs doublets, a light neutral \( h \) may exist which has so far escaped experimental detection. This is not possible in the minimal supersymmetric standard model, but is natural in any other model where \( \mu_{12}^2 = 0 \) and \( \lambda_5 \neq 0 \), as in the doublet Majoron model\(^3\). The process \( Z \to h\bar{h}f \) is then of interest because it may be observable for a reasonable \( hhH \) coupling if \( m_H \) is not much greater than its current lower bound of 60 GeV. This is especially so if \( h \) decays invisibly, as in the doublet Majoron model.

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References

[1] ALEPH Collaboration, D. Buskulic et al., Phys. Lett. B313, 299 (1993); DELPHI Collaboration, P. Abreu et al., Nucl. Phys. B373, 3 (1992); L3 Collaboration, B. Adeva et al., Phys. Lett. B303, 391 (1993); OPAL Collaboration, M.Z. Akrawy et al., Phys. Lett. B253, 511 (1991).

[2] ALEPH Collaboration, D. Buskulic et al., Phys. Lett. B313, 312 (1993).

[3] H. Kikuchi and E. Ma, Phys. Lett. B335, 444 (1994); Phys. Rev. D 51, R296 (1995).

[4] E. Ma and D. Ng, Univ. of Calif., Riverside Report No. UCRHEP-T144 (April 1995), to appear in Proc. of the International Symposium on Vector Boson Self-Interactions (February 1995).

[5] R.E. Shrock and M. Suzuki, Phys. Lett. 110B, 250 (1982). L.F. Li, Y. Liu, and L. Wolfenstein, Phys. Lett. 159B, 45 (1985); E.D. Carlson and L.B. Hall, Phys. Rev. D 40, 3187 (1989).

[6] A.S. Joshipura and S. D. Rindani, Phys. Rev. Lett. 69, 3269 (1992).
Figure Captions

Fig. 1: The four tree-level diagrams that contribute to $Z \rightarrow hh\bar{f}f$ in extensions of the standard model with two Higgs doublets. Wavy lines and solid lines represent $Z$-bosons and fermion-antifermion pairs, respectively. Dashed lines represent scalar fields and are labeled as $h$ (the light Higgs scalar), $H$ (the heavy Higgs scalar), or $A$ (the Higgs pseudoscalar).

Fig. 2: The total branching fraction $B$ for $Z \rightarrow hh\bar{f}f$ due to the model-independent contribution of Fig.1a alone as a function of the light Higgs scalar mass $m_h$.

Fig. 3: The total branching fraction $B$ for $Z \rightarrow hh\bar{f}f$ due to the heavy Higgs scalar $H$ mediated contribution of Fig.1b alone for various values of $\eta \equiv |\lambda_3 + \lambda_4 + \lambda_5|$ (see Eq. (1)), $m_h$, and $m_H$. The decay width of $H$, due primarily to $H \rightarrow hh$ and $H \rightarrow \bar{b}b$ (where we assume the b-quark Yukawa coupling of the standard model), has been taken into account. (a): $B$ when $m_h = 10$ GeV as a function of $m_H$ for different values of $\eta$. (b): $B$ when $m_h = 10$ GeV as a function of $\eta$ for different values of $m_H$. (c): $B$ when $\eta = 1$ as a function of $m_h$ for different values of $m_H$. 