Metallic nanolayers: a sub-visible wonderland of optical properties [Invited]

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It was predicted long ago that ultra-thin metallic films must exhibit unusual optical properties for radiation frequencies from the radio-frequency domain to the infrared domain. A film would remain highly reflective even when it is orders of magnitude thinner than a skin depth at any frequency. Only when the film is a few nanometers thick (depending on the material but not on the frequency) do its reflectivity and transmittivity become equal, while its absorption peaks at 50%. It has been confirmed experimentally and new directions and applications were proposed. We review the electromagnetic theory of the phenomenon and recent developments in the field and present some new results. © 2018 Optical Society of America

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1. INTRODUCTION

The optics of metals is a prominent part of optical physics and related technologies, in particular, electromagnetic (EM) devices from radio to microwave to infrared wavelengths; we will call it the sub-visible (SV) domain. Of utmost importance to optics is the capability of a polished metal surface to serve as an almost ideal mirror, including the visible domain. This quality was well known to humankind from ladies’ mirrors in ancient Egypt to the legend of Archimedes’ use of soldiers’ shields to focus sunlight into the enemy’s ship sails, to radars, telescopes, and other modern reflectors.

Due to the very high conductivity of metals, $\sigma$, the major properties of metallic mirrors in a dielectric environment are as follows: (1) their reflectivity $R$ is very close to 1 at any frequency $\omega$ within the SV domain, and their absorption, $Q = 1 - R$, is respectively tremendously low; (2) the electrical field at the reflecting boundary nearly vanishes (i.e., the reflected EM wave is almost of the same amplitude but not on the frequency) its do its reflectivity and transmittivity become equal, while its absorption peaks at 50%. It has been confirmed experimentally and new directions and applications were proposed. We review the electromagnetic theory of the phenomenon and recent developments in the field and present some new results. © 2018 Optical Society of America

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that is greatly overlooked in the general literature is that at that thickness one attains impedance matching between the environment and the metallic layer, resulting in maximum absorption. In a free-space environment, the $d_{pk}$-layer’s impedance, $Z_{pk}$, is exactly half of that of a vacuum, $Z_0 = 377\Omega$, $Z_{pk} = Z_0/2$, and it does not depend even on a specific metal (see details in Sections 6 and 7). It would be reasonable to call $d_{pk}$ an impedance-matching thickness.

The effect has by now been verified and explored both theoretically and experimentally. That research included early [7–10] microwave (mm) and recent [11–14] millimeter (mm) wave experiments; applications to the visualization of micro-wave modes using thermoluminescence sensors [15–17], broadband mm wave spectroscopy in resonators at cryogenic temperatures [18–20], and the theoretical and experimental study of EM properties of periodic multilayers of metallic films (photonic crystals) [21,22]; and a proposal to attain 100% EM absorption in a standing wave [6]. However, it would be not an overstatement to note that aside from those studies, this strongly pronounced and physically transparent effect remained little known to the research community in the optics of metals, making it a blind spot in the field (the choice of term “visible” here is not accidental). The theoretical and experimental tools required for its exploration are not overly sophisticated and were available for almost a century, yet even mention of it is lacking not only from classical texts on electrodynamics but also from recent reviews on the subject. The objective of this paper is to make a consistent review of the major features of the optical properties of ultra-thin metallic films (including new results), their underlying physics, and experimental results.

A brief overview of optical properties of semi-infinite metallic layers in the SV domain is found in Section 2. Sections 3 and 4 are on the electrodynamics and optical properties of the layers of finite thickness, and Section 5 is on electrical currents. Section 6 treats the problem in terms of impedance theory, in particular for an arbitrary input/output environment. Section 7 addresses the issue of how the size-dependent conductivity affects the optical properties of the layers, and Section 8 addresses experimental results and consideration for future experiments. Section 9 is on the wave interference at metallic films resulting in 100% absorption (blackbody effect), and Section 10 briefly discusses potential applications and outlook.

## 2. SEMI-INFINITE METALLIC LAYERS

Major EM properties of semi-infinite (or sufficiently thick, $d \gg \delta$) metallic layers, found in any “old goldies” texts, such as, e.g., [23–27], can be represented by a succinct formula for the reflectivity, $R$, and the absorption, $Q$, of the layer for a normal EM-wave incidence [1–3],

$$Q = 2\delta k = \sqrt{2\omega/\pi\sigma}; \quad R = 1 - Q; \quad P = 0,$$

where $k = 2\pi/\lambda = \omega/c$ is a wave number, $\lambda = 2\pi c/\omega$ is a free-space wavelength for a $\omega$-monochromatic wave, $\sigma$ is a dc conductivity of the layer (we use here Gaussian units, see Appendix A, whereby $[\sigma] = s^{-1}$), and

$$\delta = c/\sqrt{2\pi\sigma\omega}$$

is a skin depth. For metals (and other highly conductive materials, with $\sigma \gg \omega/2\pi$ or $\lambda \gg c/\sigma$), one has $\delta \ll \lambda$, so that $Q = 1 - R \ll 1$; this is due to the large and almost purely imaginary dielectric constant of metal, $\epsilon_{in} \propto i\sigma/\omega$ (see Appendix A). As an example, for a silver layer at $\lambda = 1$ m, $\delta \approx 3.5$ $\mu$m (respectively, for $\lambda = 1$ cm, $\delta \approx 0.35$ $\mu$m). Thus, the EM properties of the system, $R$, $Q$, and $\delta$, depend on both frequency of incident light $\omega$ and conductivity of the layer, $\sigma$, as expected. A much less appreciated (yet known, see, e.g., [25]) fact is that the total electrical current near the metallic surface, $J$, induced by the incident wave, remains the same for any metal and wavelength, and its amplitude, $J_{\infty}$, depends only on the incident amplitude $E_{in}$ as

$$J_{\infty} = cE_{in}/2\pi = 2E_{in}/Z_0,$$

where $Z_0 = 4\pi/c$ is the free-space wave impedance (see Appendix A). A simple explanation of that is the following: due to almost full reflection of light at the surface of a semi-infinite layer and formation of standing wave in the free space with a node at the surface, an $E$-field there almost vanishes, while the magnetic field peaks out, reaching amplitude $H(x = 0) = 2E_{in}$, where $x$ is the distance from the surface. Thus, we have a rare situation of an almost purely magnetic wave, although it rapidly decays inside the layer, $H(\infty) = 0$. For a plane wave in a good metal (see details in Section 5 below), due to Ampere’s law, this $H$-field induces local currents $j(x)$, $dH/dx = -j(x)Z_0$, see below, Eq. (30), so that $J_{\infty} = \int_{-\infty}^{\infty} j(x)dx = H(x = 0)/Z_0$, which results in Eq. (3).

It has to be noted though that the ratio $E_{\infty}/j_{\infty} = Z_0/2$ is not the impedance of the metal surface; a respective impedance, $Z_{\infty}$, must relate the current $J_{\infty}$ to the $E$-field amplitude, at the surface, $E_{in}(x = 0) = E_{in}k\delta(1 - i)$ (see Section 3 below) and not to the much larger amplitude of the incident wave, $E_{in}$, so that

$$Z_{\infty} = E(x = 0)/J_{\infty} = Z_0/2k\delta(1 - i); \quad Z_{\infty} \ll Z_0.$$  

(4)

Considering free space as a transmission line for a plane wave, $Z_{\infty} \ll Z_0$ corresponds to its short-circuiting, hence full reflection, as one would expect. Rewriting Eq. (4) as

$$\frac{2}{1 - i} \frac{Z_{\infty}}{Z_0} = \sqrt{\frac{\omega}{2\pi\sigma}} = \sqrt{k\Lambda}, \quad \text{with}$$

$$\Lambda = \frac{c}{2\pi\sigma} = k\delta^2 = \frac{2}{Z_0\sigma} = O(1)\Lambda,$$

(5)

we introduce a new frequency-independent characteristic length scale of a layer, $\Lambda$, which will become one of the major “characters” of the story for very thin layers. It is a characteristic scale at which one presumes that the layer conductivity, $\sigma$, remains the same as the bulk conductivity $\sigma_0$, the reflection would be significantly reduced, and the transition respectively increased. For good metals, this new scale has atomic size (and even less than that) and is not only many orders of magnitude smaller than the wavelength of incident light, but also of the skin depth. In real layers, however, the conductivity, $\sigma$, depends of the layer thickness, $d$, and is greatly reduced as $d \to 0$ (due to the fact the mean free path of electrons, $l$, with $\sigma \propto l$, gets “clipped” by the walls); see Section 7 below. This “clipping” results in the formation of another scale, $\lambda_N$, which will finally determine the depth, $d_{pk}$, at which the layer will universally
reach the point where the reflectivity, \( R \), will be exactly equal to the transmittivity, \( T \), with \( R = T = 0.25 \), and absorption will peak at \( Q = 0.5 \) (see Fig. 1) for the case of a silver layer and normal incidence.

This new scale is of the order of 1–2 nm and is defined as \( \lambda_N = \sqrt{\Lambda_0 \ell_0} \approx 8.2 \times N_e^{1/3} \), where \( \Lambda_0 = \Lambda(d \to \infty) \), \( \ell_0 = \ell(d \to \infty) \) is the bulk mean free path of electrons, and \( N_e \) is the number density of free electrons (for details, see Section 7).

The conductivity \( \sigma \) in the SV domain remains essentially the same as for a dc current \([2,3]\), i.e., the entire phenomenon is of a (quasi)-static nature. This greatly simplifies the theory of the optical properties of metal in particular thin metal films in that domain and makes the entire SV domain so special. However, at the upper energy end of this domain, the interaction of radiation with quasi-free electrons, at least in semi-infinite layers, reaches the point where the skin depth becomes smaller than the mean free path of electrons, \( \ell_0 \), which results in the so-called anomalous skin effect \([28,29,30]\). The interaction then becomes non-local: the electrons driven by the field near the surface run away into a no-field area. For example, in the case of silver, the wavelength at which this happens is in the sub-mm domain, \( \lambda \approx 0.23 \) mm. For shorter wavelength radiation, an absorption is increasing; however, the metal mirrors still remain well-reflecting even in the visible domain. However, for thin layers (see Section 7), the onset of non-locality shifts to shorter wavelengths, since \( \ell_0 \) gets closer to the layer thickness \( d \); a quasi-static model of optical constants of nano-layers holds true to the mid-infrared domain. A review of the electronic properties of various metals and their related optical properties from the classical Drude–Lorentz model to the quantum theory for various metals and frequencies can be found in \([31,32,33,34]\).

For the higher photon energies, as, e.g., in the ultraviolet (UV) spectral domain, the quasi-free electrons can be regarded as an over-dense plasma (see also Appendix A), having now almost real yet negative dielectric constant \( \epsilon \approx 1 - \lambda^2/\lambda_{pl}^2 < 0 \) with the plasma wavelength \( \lambda_{pl} \) found in the 200–300/nm range determined by the density of electrons. Beyond that threshold, metals can be viewed as a plain under-dense plasma, with their dielectric constants approaching that of a free space, \( 0 < \epsilon < 1 \). Further into the X-ray domain, the number of free electrons undergoes large jump-like increases as photon energy increases near the so-called \( K, L, M, \) and \( N \) absorption edges, which are due to resonant photo-ionization of bound electrons from respective shells into the conduction state \([35–37]\). These jumps would affect the optical constants of the metal surface and may be used for various applications, in particular for narrow-line transition radiation by electron beams traversing a multi-layer, super-lattice structure of metallic layers generating almost coherent radiation in the soft \([38]\) and hard X-ray \([39]\) domains. In this paper we limit our consideration only to the quasi-static, i.e., SV spectral domain.

3. FIELDS IN A LAYER OF FINITE THICKNESS

The optical properties of thin metal layers (in both the visible and far-infrared domains) with a great deal of experimental data have been reviewed in \([40–43]\) (relevant research has been done on absorption by small metal particles in the infrared \([44]\)). Some of them clearly pointed to the layers’ ability to sustain high reflectivity at surprisingly small thickness (see, e.g., \([45,46]\)); yet, a general picture of the existence of universal maximum absorption and a corresponding spatial scale related only to the number density of free electrons (Section 7 here) does not seem to have transpired yet. It is also interesting to note that a tremendous amount of work on the physics of superconductivity, especially on high-temperature (high-\( T_c \)) superconductivity, has been done by studying infrared and far-infrared optical properties of thin films of those materials (see, e.g., review \([47]\)), with some of them emphasizing that the spatial scale of strongly absorbing films are below skin depth (see, e.g., \([48]\)).

This and the following sections are to provide a basic understanding of how the ultra-thin metallic films reach the state of maximum absorption, \( Q = 0.5 \), and to show that this behavior is really universal. While looking for the fields and current in a (non-magnetic) layer of finite thickness, for our purposes, we consider only normal incidence, i.e., the waves propagating in the \( x \) axis normal to the layer; see Fig. 1. In this section, we consider only a free space as an environment, where the layer is bounded between \( x = 0 \) and \( x = d \). (The results are extended to an arbitrary environment in Section 6.) All the calculations in this paper are based on the most simplifying assumptions sufficient to bring up and quantitatively describe major features of the phenomenon discussed: the layer has flat surfaces, and the metal in it is homogeneous (i.e., not granulated), so that there is no scattering of light at the layer; the conduction electrons are regarded as a gas of non-interacting particles following the Drude model \([49]\), which may get scattered by ions and the surfaces of the layer. A \( \omega \)-monochromatic wave is incident normally at the layer at the point \( x = 0 \). (For the incidence close to the normal one, the results are expected to be not much different, since the refractive index of metals is large.)

The wave is linearly polarized in the \( y \) axis, both incident fields \((1/2)E(x)e^{-i\omega t} + c.c. \) and \((1/2)H(x)e^{-i\omega t} + c.c. \) have the same amplitude \( E_{in} \), and \( E_x = E_y = E_z = 0 \) everywhere. We designate \( E \equiv E_x \) and \( H \equiv H_y \). In this case, Eqs. (A1) and the respective wave equation, e.g., for \( E \), are written as:

\[
\frac{dE}{dx} = i\kappa H; \quad \frac{dH}{dx} = -i\kappa E; \quad \frac{d^2E}{dx^2} + \kappa^2 E = 0, \quad (7)
\]

![Fig. 1. Theoretically calculated refractivity, \( R \), transmittivity, \( P \), and absorption, \( Q \), of a silver layer versus its thickness \( d \); \( d_{pl} \approx 12.6 \) Å \([5]\). Insert: a normal plane wave incidence.](image-url)
where \( k = \omega/c = 2\pi/\lambda \) and \( \lambda = 2\pic/\omega \) are the free-space wavenumber and wavelength of the wave, respectively. The free-space dielectric constant is \( \varepsilon = 1 \), whereas inside the layer, \( 0 \leq x \leq d \), under a “good metals” condition, \( \varepsilon_m \gg 1 \) or \( \sigma \gg \omega \), \( \varepsilon_m \) can be well approximated as a purely imaginary quantity (see also Appendix A),

\[
\varepsilon_m \approx \frac{4i\sigma}{\omega} = \frac{2i}{k\Lambda} = \frac{2i}{k^2/\delta^2}, \quad \Lambda = \frac{\varepsilon}{2\pi}, \quad (8)
\]

where a scale \( \Lambda \) [5] was introduced in Eq. (6), and \( \delta \) is as defined in Eq. (2). The incident \( E \)-field of amplitude \( E_{in} \), \( H \)-field of amplitude \( H_{in} \), and reflected fields of amplitude \( E_{rif} \) and \( H_{rif} \) at \( x < 0 \) are written, respectively, as

\[
E(x)/E_{in} = H(x)/E_{in} = e^{ixk},
\]

\[
E_{rif}(x)/E_{in} = -H_{rif}(x)/E_{in} = re^{-ixk}.
\]

The transmitted fields behind the layer, i.e., at \( x > d \), will be sought for as

\[
E_{ts}/E_{in} = H_{ts}/E_{in} = pe^{i(x-d)k},
\]

where \( p \) and \( \rho \) are the coefficients or reflection and transmission, respectively, to be found from boundary conditions at the surfaces of the layer. [The solution in Eq. (10) for the transmitted field takes into account the so-called Zemfereff’s condition in infinity \((x \to +\infty)\) by ruling out a back-propagating wave \( e^{-ixk} \) at \( x > d \). Inside the layer, the fields \( E_m \) and \( H_m \) are superpositions of forward \((\pm)\) and backward \((-\pm)\) propagating waves with normalized amplitudes \( a^\pm \) as

\[
E_m(x)/E_{in} = a^+ e^{ik_x a^+ x} + a^- e^{-ik_x a^- x};
\]

\[
H_m(x)/E_{in} = n_m(a^+ e^{ik_x a^+ x} - a^- e^{-ik_x a^- x}),
\]

with

\[
k_m = k\sqrt{\varepsilon_m} = \frac{1 + i}{\delta} = kn_m; \quad nm = \sqrt{\varepsilon_m} = \frac{1 + i}{\delta}.
\]

where \( n_m \) is a (complex) refractive index of metal. The constants \( a^\pm \) are found from boundary conditions at \( x = 0 \) and \( x = d \) for \( E \) and \( H \) (to be continuous at a boundary). Using Eqs. (10) and (11), we have at \( x = 0 \)

\[
a^+ + a^- = 1 + r; \quad n_m(a^+ - a^-) = 1 - r
\]

and at \( x = d \),

\[
a^+ e^{ik_m d} + a^- e^{-ik_m d} = n_m(a^+ e^{ik_m d} - a^- e^{-ik_m d}) = \rho.
\]

From these equations, the solution for \( a^\pm \) is then

\[
a^\pm = \frac{-2(n_m \pm 1)e^{\pm ik_m d}}{(n_m - 1)^2 e^{ik_m d} - (n_m + 1)^2 e^{-ik_m d}}.
\]

Using the fact that \( \varepsilon_m \gg 1 \), hence \( n_m \pm 1 \approx n_m \pm 1/n_m \), we simplify Eq. (14) as

\[
a^\pm \approx \frac{1}{n_m \sinh(2/n_m - ik_m d)},
\]

For very thin layers, \( d \ll \delta \), we have

\[(a^\pm) \approx (A/2)/(A + d) = \text{const}\]

[for \( E(x) \) see Eq. (20) below], i.e., both the counter-propagating waves are of almost the same magnitude. Equation (11) yields now for the fields inside the layer

\[
\frac{E_m(x)}{E_{in}} \approx \frac{2 \cosh[ik_m(x - d) + 1/n_m]}{n_m \sinh(2/n_m - ik_m d)},
\]

\[
\frac{H_m(x)}{E_{in}} \approx \frac{2 \sinh[ik_m(x - d) + 1/n_m]}{\sinh(2/n_m - ik_m d)},
\]

hence,

\[
H_m(x)/E_{in}(x) \approx n_m \tanh[ik_m(x - d) + 1/n_m],
\]

in particular, for a semi-infinite layer, \( d \to \infty \),

\[
H_m(x)/E_{in}(x) = \text{const} = n_m.
\]

In the most interesting case of a thin layer, \( d \ll \delta \), we have

\[
\frac{E_m(x)}{E_{in}} \approx \frac{\Lambda}{\Lambda + d}; \quad \frac{H_m(x)}{E_{in}} \approx \frac{\Lambda + 2(d - x)}{\Lambda + d},
\]

i.e., \( E_m(x)/E_{in} = \text{const} \), so the electrical field \( E_m \) is homogeneous inside the layer.

4. REFLECTIVITY, TRANSMITTIVITY, AND ABSORPTION

From Eq. (13), \( r = (a^+ + a^-) - 1 \); hence, using Eq. (14),

\[
r = -\frac{(\varepsilon_m - 1)(\varepsilon_m + 1)}{(n_m - 1)^2 e^{i\omega_0 d} - (n_m + 1)^2 e^{-i\omega_0 d}}.
\]

If \( d = 0 \), we have \( r = 0 \), as expected. If \( d \to \infty \), the terms \( e^{i\omega_0 d} \) tend to zero, so that

\[
r_{d \to \infty} = -(n_m - 1)/(n_m + 1) \approx 1 - k\delta(1 - i),
\]

and consistent with Eq. (1), we have

\[
R_{\infty} = |r|^2 = 1 - 2k\delta.
\]

For \( \varepsilon_m \gg 1 \), similar to Eq. (15), Eq. (21) is simplified as

\[
r \approx -[\sin[(1 - i)d/\delta∉)][\sin[(1 - i)(d/\delta + k\delta)]].
\]

In the same way, we have the transmission coefficient, \( p \),

\[
p = [(1 - i)k\delta]/[\sin[(1 - i)(k\delta + d/\delta)]].
\]

Both of them can be further simplified for the case of a very thin layer, \( d \ll \delta \) [5],

\[
r \approx -d/(d + \Lambda); \quad p \ll \Lambda/(d + \Lambda).
\]

Translating Eqs. (26) for thin layers into the formulas for reflectivity, \( R = |r|^2 \), transmittivity, \( P = |p|^2 \), and energy losses, \( Q = 1 - (R + P) \), we get

\[
R = (1 + \Lambda/d)^{-2}; \quad P = (1 + d/\Lambda)^{-2};
\]

\[
Q = 2\left(\sqrt{\Lambda/d + \sqrt{d/\Lambda}}\right)^{-2}.
\]

Interestingly enough, Eq. (27) provides us with a relationship that does not explicitly include any parameters of the incident field or the metal,

\[
\sqrt{R} + \sqrt{P} = 1; \quad Q = 2\sqrt{RP}.
\]

Notice that the first of these equations, written as \( p = 1 + r \), is not related to the conservation of full momentum in the system; indeed, that conservation should include the momentum, \( p_m \), transferred to the layer,

\[
p_m = 1 - p - r = 2d/(d + \Lambda),
\]

which originates a radiation pressure on the layer. In the semi-infinite layer case, \( p_m \approx 2 \), i.e., is maximal, as expected, whereas
\( p_m = 0 \) at \( d = 0 \), and finally \( p_m = 1 \) at \( d = \Lambda \). However, Eqs. (27) and (28) uphold the conservation of radiation energy, \( R + P + Q = 1 \).

5. ELECTRICAL CURRENT IN THE LAYER

As long as the solution for electrical and magnetic fields Eqs. (17) and (20) are known, the electrical current \( j(x) \) in the layer is found as

\[
j(x) = -Z_0 \frac{\partial H}{\partial x} = \sigma E. \tag{30}\]

For a thin layer, \( d \gg \delta \), the field \( E \) is almost constant, Eq. (20), so the current \( j(x) \) is also evenly distributed along the depth. The full current, \( j(d) \), in the layer is

\[
j(d) = \int_{x=0}^{x=d} j(x) dx = \frac{H_m(x=0) - H_m(x=d)}{Z_0} \tag{31}\]

or by using the second equation in Eq. (17), we have

\[
\frac{j(d)}{E_{in}} \approx \frac{2}{Z_0} \frac{\sinh(ik_m d/2)}{\sinh(ik_m d/2 - 1/n_m)}. \tag{32}\]

For a very thin layer, \( d \ll \delta \), we have

\[
\frac{j(d)}{E_{in}} = \frac{d}{Z_0 d + \Lambda} = \frac{|r|}{Z_0}. \tag{33}\]

In a semi-infinite layer, \( d \gg \delta \), or \( k_m d \gg 1 \), we have from Eq. (32) [see also Eq. (3)]

\[
j(d \to \infty)/E_{in} = 2/Z_0, \tag{34}\]

as long as \( d > \Lambda \), the total current, is almost the same! Furthermore, the efficient layer resistance per square centimeter (cm) coincides exactly with the half-impedance of a vacuum. Thus, the layer almost always makes the same radiating antenna from \( d \gg \delta \) down to \( d \sim \Lambda \).

6. TRANSMISSION LINE ANALOGY: ARBITRARY ENVIRONMENT

It is instructive and revealing to describe EM wave propagation through a thin metallic layer as a transmission line problem, by using wave impedances of the line and its components. A simple impedance algebra allows for an easy generalization of our results to a system with an arbitrary (i.e., not just free space) environment algebra allows for an easy generalization of our results to a system with an arbitrary (i.e., not just free space) input/output environment, which may include, e.g., dielectric or semiconductor substrate. At the same time if offers a plain “electrical engineering” view of the phenomenon.

Let us start with a free-space environment. Both of the impedances of the “incident” and “output” arms of the line are \( Z_0 \), and in view of quasi-static nature of the problem, a layer can be regarded as a lump circuit (even for a semi-infinite layer), which in the case of \( d \ll \delta \) is simply a resistor. Thus, for all purposes, using Eq. (17) with \( x = 0 \) and Eq. (32), the impedance \( Z_L \) of a layer of a thickness \( d \) is found as

\[
Z_L(d) = E_m(x=0)/j(d), \quad Z_L(d) \approx -\frac{(Z_0/2n_m) \cosh(ik_m d/2 - 1/n_m) \sinh(ik_m d/2)}{\cosh(ik_m d/2 - 1/n_m)} \tag{35}\]

In the limit \( d \to \infty \), we have

\[
Z_\infty/Z_0 = n_m^{-1} = k\delta(1-i)/2. \tag{36}\]

We can also find \( Z_\infty \) directly from the wave solution Eq. (19). Indeed, for a plain “traveling” wave, a wave impedance \( Z \) in electrodynamics is usually defined as

\[
Z = (E/H)_{SL} = Z_0(E/H)_{Gauss}, \tag{37}\]

where subscripts refer to a respective unit system. This is still true even if the wave goes through an absorbing material, if there is no retroreflection inside it. In a semi-infinite metallic layer, the ratio \( E/H \) remains constant, as in Eq. (19), since the wave propagates only away from the interface [see, e.g., Eq. (11) with \( a^* = 0 \) as follows from Eq. (15) with \( d \to \infty \), and thus we have \( Z_\infty/Z_0 = E_m(x)/H_m(x) = 1/n_m \), which coincides with Eq. (36). In the limit \( d \ll \delta \), we have for the layer impedance \( Z_L \)

\[
Z_L/Z_0 \approx 1/ie_m kd = \Lambda/2d. \tag{38}\]

The coefficient of reflection, \( r \), of the layer, if the wave is incident from \( x \to -\infty \), can be evaluated by assuming that the incidence line with \( Z_{in} = Z_0 \) is loaded by the impedance \( (Z_\Sigma)_{0} \) formed by two elements connected in parallel; the layer, with its impedance \( Z_L \), and the output line with its impedance, \( Z_0 \), i.e.,

\[
(Z_\Sigma)_{0} = Z_L^{-1} + Z_0^{-1}, \tag{39}\]

so that a transmission line theory yields

\[
r = \frac{(Z_\Sigma)_{0} - Z_0}{(Z_\Sigma)_{0} + Z_0} = -\frac{Z_0}{Z_0 + 2Z_L}; \quad R = |r|^2. \tag{40}\]

Similarly, the coefficient of transmission \( p \) is evaluated as

\[
p = \frac{2(Z_\Sigma)_{0}}{(Z_\Sigma)_{0} + Z_0} = \frac{2Z_L}{Z_0 + 2Z_L}; \quad P = |p|^2, \tag{41}\]

so that for \( d \ll \delta \), they coincide with the respective result from Eq. (20). Finally, the energy losses are as

\[
Q = 1 - (P + R) = 4Z_0 Z_L/j(Z_0 + 2Z_L)^2, \tag{42}\]

which peaks \((Q_{peak} = 0.5)\) at \( Z_L = Z_0/2 \), as expected.

The transmission line results can be readily generalized to the case whereby semi-infinite dielectric materials sandwiching the metallic layer are different. Assuming that the material of the wave incidence has a refractive index \( n_1 \) and the output one—the index \( n_2 \), so that their respective wave impedances are \( Z_i = Z_0/n_i \) with \( i = 1, 2 \), we define “input/out load impedance” \( Z_\Sigma \) as

\[
Z_\Sigma^{-1} = Z_L^{-1} + Z_2^{-1}, \tag{43}\]

and use it to generalize Eqs. (40) and (41) as

\[
r = \frac{Z_\Sigma - Z_1}{Z_\Sigma + Z_1} = -\frac{Z_1 Z_2 + Z_1 Z_L - Z_2 Z_L}{Z_1 Z_L + Z_1 Z_L + Z_2 Z_L}, \tag{44}\]

\[
= \frac{Z_0 Z_L + n_2 - n_1}{Z_0 Z_L + n_2 + n_1}; \quad R = |r|^2; \tag{45}\]

\[
p = \frac{2Z_\Sigma}{Z_\Sigma + Z_1} = \frac{2Z_2 Z_L}{Z_1 Z_2 + Z_1 Z_L + Z_2 Z_L}, \tag{46}\]

\[
= 2n_1/(Z_0/Z_L + n_2 + n_1); \quad P = (n_2/n_1)|p|^2. \tag{47}\]

We then obtain the energy losses as

\[
Q = 1 - (P + R) = \frac{4Z_1 Z_2 Z_3}{(Z_1 Z_2 + Z_1 Z_L + Z_2 Z_L)^2}, \tag{48}\]

\[
= 4n_1(Z_0/Z_L)/(Z_0/Z_L + n_2 + n_1)^2. \tag{49}\]
If \( n_1 = n_2 = 1 \), the results from Eqs. (43)-(45) coincide with Eqs. (41)-(42), including the case \( d \ll \delta \), when they coincide with Eq. (27). The losses in Eq. (46) then reach their maximum,

\[
Q_{pk} = \frac{n_1}{n_1 + n_2} \quad \text{when} \quad \frac{Z_0}{Z_L} = 2 \frac{d_{pk}}{\Lambda} = n_1 + n_2, \tag{47}
\]

while the reflectivity \( R \) and transmittivity \( P \) are as

\[
R = \frac{n_1^2}{(n_1 + n_2)^2}, \quad P = \frac{n_1 n_2}{(n_1 + n_2)^2}, \quad \frac{R}{P} = \frac{n_2}{n_1}. \tag{48}
\]

Note that if in Eq. (47) \( n_1 \gg n_2 \), the losses \( Q \) greatly increase; this may happen if the dielectric at the entrance is highly optically dense, as in some semiconductors, or if the output medium consists of plasma near critical frequency, when \( 0 < n_2 \ll 1 \). For example, if one uses silicon (Si) or gallium arsenide (GaAs), both of which have refractive index \( n \sim 4 \), as an input medium (\( n_1 \)), and an air as an output one (\( n_2 = 1 \)), one would have a highly absorbing and low reflecting layer \( Q_{pk} = 0.8, R = 0.04, \) and \( P = 0.16 \) at \( d_{pk}/\Lambda = 2.5 \).

Having in mind that, e.g., in free space the maximum absorption happens when the layer impedance matches exactly half of vacuum impedance, regardless of the specific material, it might be perhaps appropriate to call the entire phenomenon an impedance-match absorption.

### 7. SIZE-AFFECTED CONDUCTIVITY AND NEW FUNDAMENTAL THICKNESS SCALE

For ultra-thin films, Eq. (27) suggests a very simple and transparent dependence of optical properties \( R, Q, \) and \( P \) versus film thickness \( d \)—assuming that the conductivity \( \sigma \) and the scale \( \Lambda = c/2\pi \sigma \) from Eq. (8) are constants that do not depend on \( d \). (The resulting scale \( \Lambda \) is around or even less than one angstrom.) But at very small \( d \) this is not true anymore, so that \( \sigma \) versus \( d \) dependence has to be taken into consideration. The major parameter through which the specific conductivity of good metal is affected by the film size, \( d \), is the mean free-path of electrons, \( l(d) \), with the conductivity \( \sigma \) being proportional to \( l(d) \) [50,51],

\[
\frac{\sigma(d)}{\sigma_0} = \frac{l(d)}{l_0} ; \quad \text{with} \quad \sigma_0 = \frac{N_e e^2 l_0}{\sqrt{2mW_F}}, \tag{49}
\]

where \( \sigma_0 \) and \( l_0 \) are, respectively, specific bulk conductivity and bulk mean free path, \( N_e \) is number density of conduction electrons, \( m \) is the electron mass, and \( W_F \) is the Fermi energy of an electron gas at a given temperature; for most applications, the temperatures are less than \( 10^3-10^4 \) K, so \( W_F \) is the same as for absolute zero [52], \( W_F \approx W_0 = (h k_F)^2/2m, \) where \( k_F = (3\pi^2 N_e)^{1/3} \) is the Fermi wave vector, which reduces \( \sigma_0 \) in Eq. (49) to

\[
\sigma_0 = \alpha k_F^2 c l_0 / 3\pi^2 ; \quad \text{with} \quad \alpha = e^2/\hbar c = 1/137, \tag{50}
\]

where \( \alpha \) is the fine structure constant. The experimental and theoretical data for \( l_0 \) can be found in many publications; the latest extensive study, for 20 metals using numerical calculations over the Fermi surface, can be found in [53]. The size effect, i.e., how \( \sigma \) and \( l \) depend on \( d \), represents fundamental interest as well as application challenges for nano-electronics.

Sufficient for our purposes here is a classical Fusch–Sondheimer model [50,51], which assumes a so-called spheric Fermi surface, whereby the major factor affecting \( \sigma \) and \( l \) is electron scattering at the layer surfaces. Within that model, under the most realistic assumption that electrons scatter at the surface in a purely diffuse way, the dimensionless mean-free path of electrons and the conductivity, \( \theta = l(d)/l_0 = \sigma(d)/\sigma_0 \), versus the layer width, \( \xi = d/l_0 \), is as [51]

\[
\theta = 1 - \frac{3}{2\xi} \int_1^\infty (e^{-\xi} + 1 - e^{-\xi^2})dt \tag{51}
\]

or, in terms of exponential integral \( E_1(\xi) = \int_1^\infty e^{-\tau}d\tau \) [54],

\[
\theta = 1 - 3(1 - e^{-\xi^2}/8\xi^3 \approx [(-10 - \xi + \xi^2) e^{-\xi} + \xi(12 - \xi^2)E_1(\xi)]/16. \tag{52}
\]

In the limit of a very thick layer, \( \xi \gg 1 \), the solution of Eq. (51) is \( \theta \approx 1 - 3/8\xi^2 \), whereas in the limit of a very thin layer, \( \xi \ll 1 \), which is of most physical interest, the solution is \( \theta \approx (3\xi/4) \ln(1/\xi) \).

To simplify the analysis of the result of Eq. (52), we find that for all practical purposes, in particular in the above limits, it is interpolated with great precision within an entire range \( 0 < d < \infty \) by

\[
\theta = (4 + \xi^{-2})^{-1} + (3\xi/4) \ln(1 + \xi^{-1}); \tag{53}
\]

see the comparison of Eqs. (52) and (53) in Fig. 2. A layer thickness, \( d_{pk} \), where the absorptions peaks out, \( Q_{pk} = 0.5 \), according to Eq. (27), satisfies the condition \( \Lambda(d_{pk}) = d_{pk} \).

Thus, to solve Eq. (53), we recall that \( l(d)/l_0 = \Lambda_0/\Lambda(d) \), replace \( \Lambda(d) \) by (unknown yet) \( d_{pk} \) as

\[
\frac{\Lambda_0}{d_{pk}} = \frac{1}{4 + (l_0/d_{pk})^2} + \frac{3}{4} \left( \frac{d_{pk}}{d_0} \right) \ln \left( 1 + \frac{l_0}{d_{pk}} \right), \tag{54}
\]

and define a new fundamental spatial scale, \( \lambda_N \), directly related to \( d_{pk} \), since \( d_{pk} = O(\lambda_N) \):

\[
\lambda_N = \sqrt{\Lambda_0 l_0} = \frac{3\sqrt{8\pi}}{\sqrt{\alpha}} N_e^{-1/3} \approx 8.2 \times N_e^{-1/3}. \tag{55}
\]
As opposed to Λ0, it does not depend on the free electron path l0 and therefore on temperature, the same as the Fermi wave vector, kF. (While dpk still depends on l0, this dependence is logarithmically weak at l0/ΛN ≫ 1.) Using this scale, we introduce now dimensionless variables,

\[ \eta = \frac{l_0}{\Lambda_N} = \sqrt{\frac{l_0}{\Lambda_0}} = 0.122 \times N^{2/3}l_0 \quad \text{and} \quad \zeta = \frac{l_0}{d_{pk}}, \quad (56) \]

where \( \eta \) is a “free electron figure of merit” (\( \eta \gg 1 \)) for either good metals, as, e.g., for Ag, or long mean free path of electrons, as, e.g., for Bi), and \( \zeta \) is an inverse position of peak absorption weighted by \( l_0 \). Equation (54) is rewritten then in the form

\[ \eta^2 = 4\zeta^2/[3 \ln(1 + \zeta) + \zeta/(1 + \zeta^2/4)]. \quad (57) \]

It makes it convenient to plot and analyze the figure of merit, \( \eta \), versus dimensionless peak position, \( s_{pk} \equiv d_{pk}/\Lambda_N = \eta/\zeta \). For poor conductors, we have \( \eta, \zeta \ll 1 \) (i.e., \( l_0/\Lambda_N \ll 1 \)), the solution of Eq. (57) is \( \zeta = \eta^2 \), or \( l_0^2/\Lambda_N^2 = l_0^2/d_{pk} \), and since \( \Lambda_N^{-1} = \Lambda_0^{-1} \), we have \( d_{pk} = \Lambda_0 \), as expected when \( \sigma = \sigma_0 \), i.e., the absorption peak position coincides with that predicted by a simple theory within which \( \sigma = \text{const} = \sigma_0 \). In general, to find \( d_{pk}/\Lambda_N \) for a given \( \mu \), we need to inversely solve Eq. (57) for \( \zeta(\eta) \). This is greatly simplified for good metals, whereby \( \eta, \zeta \gg 1 \) and \( s_{pk} = O(1) \). Equation (57) then reduces to

\[ s_{pk} \approx 2\sqrt{3} \ln(\eta/s_{pk}), \quad \text{and a good estimate can be obtained via fast converging iterations, whereby} \quad s_0 = 1 \quad \text{and} \quad s_n = 2\sqrt{3} \ln(\eta/s_{n-1}), \quad \text{by, e.g., using} \quad n = 2 \quad \text{or even} \quad n = 1. \]

Equation (53) allows us to generate plots of reflectivity \( R \), transmittivity \( T \), and energy losses \( Q = 1 - R - T \) versus the thickness \( d \) using Eq. (27) (having in mind now that for any given \( d \) the parameter \( \Lambda = \Lambda(d) \) depends now on \( d \), via \( \Lambda_0/\Lambda(d) \) in Eq. (53)). Figure 3(a) shows those plots for the example of silver film [5]. One can see that the absorption, \( Q \), has a peak, \( Q_{pk} \approx 0.5 \) (and \( R = P = 0.25 \)), at the thickness \( d_{pk} \approx 12.6 \, \text{Å} \), as predicted by Eq. (57) (or simplified calculations for \( s_{pk} \); see the preceding paragraph), based on the known data (see Table 1) that the characteristic length \( \Lambda_0 = c/(2\pi \sigma_0) \) for silver is \( \Lambda_0 \approx 0.84 \, \text{Å} \), and the mean free path of electrons is \( l_0 = 533 \, \text{Å} \). Based on those two numbers, we also estimate the new spatial scale as \( \Lambda_N = \sqrt{\Lambda_0 l_0} \approx 21.2 \, \text{Å} \) and the silver figure of merit as \( \eta = \sqrt{l_0/\Lambda_0} \approx 25.2 \). As one can see from Table 1, good metals (Ag, Cu, Au, and Al) have their conductivity \( \sigma_0 \) of the same order, which is also true for their mean free path of electrons, \( l_0 \approx 200 - 500 \, \text{Å} \), and the scales \( \Lambda_0 \approx 1 \, \text{Å} \) and \( \Lambda_N \approx 20 \, \text{Å} \), resulting in \( d_{pk} \approx 12 \, \text{Å} \). The theoretically calculated reflectivity \( R \), transmittivity \( T \), and absorption \( Q \) versus the thickness \( d \) for a silver layer are shown in Fig. 1.

In view of the results of Section 6, it is important to evaluate how the major characteristics of thin films changed for the input/output environment different from free propagation, e.g., when \( n_1 + n_2 = 2 \), where \( n_1 \) and \( n_2 \) are the refraction coefficients of input and output media, respectively. Having in mind that due to Eq. (47), \( d_{pk} = \Lambda(d_{pk})(n_1 + n_2)/2 \), where \( d_{pk} \) is the thickness that corresponds to peak absorption, \( Q_{pk} \) in

\[ R, T, A \]

\[ d, \, \text{nm} \]

Fig. 3. Experimental and theoretical data [12] for reflectivity, transmittivity, and absorption of aluminum film versus its thickness (in nm).

Eq. (47), the ratio \( \zeta = l_0/d_{pk} \) is determined now by Eq. (57) modified as

\[ \zeta = \eta^2 \ln(2)/2(n_1 + n_2); \quad (58) \]

for good metals, \( \zeta \gg 1 \), it can be further simplified as

\[ \zeta \approx \eta^2 \ln(\zeta)/2(n_1 + n_2); \]

for either \( n_1 + n_2 \), and not on individual indices \( n_i \), while the optical characteristics \( Q_{pk} \) [Eq. (47)] \( R \), and \( P \) [Eq. (48)] depend on \( n_i \)'s separately. At that, if \( n_1 = n_2 = 1 \), we still have \( Q_{pk} = 1/2 \) and \( R = P = 1/4 \), as in free propagation. Similar to Eq. (57), Eq. (58) is readily solved numerically by fast converging iterations, starting with \( \zeta_0 = \eta \). For example, for aluminum at \( n_1 = 1, n_2 = 1.5 \) (air+glass) we have \( d_{pk} = 12.6 \, \text{Å} \), while at \( n_1 = 1, n_2 = 4 \) (air+silicon), \( d_{pk} = 18.7 \, \text{Å} \), with \( Q_{pk} = 0.8 \).

8. EXPERIMENTAL OBSERVATION

One of recent publications on experimental measurements of the effect and observation of the peak absorption in nanometer (nm) films was the work by Andreev and co-workers [12], who studied the optical properties \( R, P, \) and \( Q \) of thin aluminum film using radiation with \( \lambda = 8 \, \text{mm} \). Their results are depicted in Fig. 3 [12], for an Al film deposited on a glass substrate with refractive index \( n = 1.5 \); Fig. 3(a) is for the configuration whereby the wave is incident upon Al film from air, and Fig. 3(b) from the substrate. (In those plots, the reflectivity is denoted by \( R \), i.e., the same as in this paper, whereas \( T \)
denotes transmission, i.e., corresponds to $P$ in this paper, and $A$ denotes absorption, corresponding to $Q$ here.) Theoretical plots were calculated for the film environment consisting of two different materials (air and glass), using formulas similar to Eqs. (44)–(46), and the size-dependent conductivity using equations similar to Eq. (53) in the limit $d \ll l_0$. As one can see, the experiment shows a great qualitative agreement with the theory, which is also true for quantitative agreement at the thicknesses $d$ greater than 20 Å (2 nm). However, the position $d_{pk}$ of the maximum absorption is almost double of that predicted by the theory; the authors’ explanation is that at that thickness (1–2 nm), a thin film undergoes a structural transformation whereby it becomes granulated (which may depend very much on the way the film was prepared [40]) or even breaks up into islands, which results in much faster reduction of averaged conductivity, hence the shift of peak of absorption to a greater thickness. We also note that in strongly granulated films, the “absorption” calculated as $Q = 1 - P - R$ could be very much due to strong scattering [40] and not due to real losses in the film. Having in mind possible future experiments to find out whether the theory based on the assumption of a homogeneous layer is still good around the peak of absorption, one might be interested to use a metal with lower intrinsic bulk conductivity and thus greater $d_{pk}$, and hence a still relatively unperturbed structure with greater number of atomic layers. One can see from Table 1, for the metals with lower bulk conductivity (Ca, Na, W, Mo, Ni, and K) but roughly the same $l_0$, the scale $\Lambda_N$ and peak position $d_{pk}$ predictably increase up to ~45 Å and ~31 Å (for Ni), respectively; this should make it easier to measure all the related effects in a more homogeneous structure.

Semi-metals such as tin, graphite, bismuth, telluride, and their chemical compounds (including most recently developed semi-metal polymers [56]) may be even more promising potential candidates for further explorations of the phenomena considered here, for they could have a much longer mean free path of electrons and may provide an arena of almost ideal homogeneous (i.e., granulation-free) layers that can be much easier to use for cleaner experiments. A good example is bismuth (Bi), which has $\sigma_{0,Bi}^G \approx 0.76 \times 10^{16} \text{s}^{-1}$ [55] (see Table 1). Having a mean free path of electrons about 3 μm at $T \sim 300K$, it would exhibit impedance-math absorption close to 50% at the thickness near 80 nm, which allows us to have it as a free-standing film. At thicknesses below 20–30 nm and low temperatures, Bi becomes a semiconductor [57], which would make it a different and even more interesting game.

### Table 1. Bulk Conductivity $\sigma_0$, Mean Free Path of Electrons, $l_0$, Characteristic Scales $\Lambda_0$ and $\Lambda_N$, Free Electron Figure of Merit $\eta$, and Peak Thickness $d_{pk}$ for Various Metals

| Metal        | $\sigma_0(G)$ $10^{27}s^{-1}$ | $l_0$ Å | $\Lambda_0$ Å | $\Lambda_N$ Å | $\eta$ | $d_{pk}$ Å |
|--------------|------------------------------|---------|----------------|---------------|-------|-------------|
| Silver (Ag)  | 5.67                         | 533     | 0.842          | 21.18         | 25.17 | 12.6        |
| Copper (Cu)  | 5.36                         | 399     | 0.891          | 18.33         | 21.76 | 11.2        |
| Gold (Au)    | 4.07                         | 377     | 1.173          | 20.13         | 17.70 | 13.4        |
| Aluminum (Al)| 3.40                         | 189     | 1.404          | 16.29         | 11.60 | 11.1        |
| Calcium (Ca) | 2.67                         | 354     | 1.788          | 25.16         | 14.07 | 16.5        |
| Sodium (Na)  | 1.89                         | 309     | 2.539          | 28.01         | 11.03 | 19.2        |
| Tungsten (W) | 1.70                         | 155     | 2.808          | 20.86         | 7.43  | 15.5        |
| Molybdenum (Mb)| 1.69                      | 112     | 2.825          | 17.79         | 6.30  | 13.8        |
| Nickel (Ni)  | 1.30                         | 587     | 3.673          | 46.43         | 12.64 | 31.0        |
| Potassium (K)| 1.25                         | 315     | 3.820          | 34.69         | 9.08  | 24.7        |
| Bismuth (Bi) | 0.076                        | 29,490  | 63             | 1360          | 21.6  | 828         |

*The data for $l_0$ for the first 10 metals are from [53]; the data for Bi are from [55].

## 9. FREE-SPACE TERMINATOR AND COHERENT BROADBAND INTERFEROLOGY

In waveguides or transmission lines, the full absorption of incident wave is attained by a terminator whose impedance matches that of the waveguide—but not in free space. However, it was demonstrated in [6] that such a “blackbody” (BB) can be realized by using a thin metal layer of exactly the thickness $d_{pk}$ (which has only half of the impedance of free space) in a Sagnac interferometer. It would then provide 100% absorption (hence zero reflection) for the entire spectrum of incident radiation in one position and almost full transparency in another; such a device might be of great interest to many applications. The effect is due to ideal coherence between incident and transmitted radiation for all the frequencies involved; because of the tremendously low distance of propagation, the phase of the transmitted wave is exactly the same as that of the incident wave, while the reflected wave has an opposite phase, which is true for the entire spectrum.

To realize this effect one needs counter-propagating waves of the same amplitude, running normally to the layer (without the layer they would form a standing wave). Because of the amplitude reflection coefficient, the reflection of a straight-propagating (“+”) incident wave of the unity amplitude at $d = d_{pk}$ will form a back-propagating wave, $E_{+}^{(t)}$, with the amplitude $-0.5$. At the same time, if a back-propagating (“−”) incident wave has exactly the same phase at the film as the “+” incident wave, its transmitted portion, $E_{-}^{(t)}$, will have the same phase, and $p = 0.5$, so that $E_{+}^{(t)} = -E_{-}^{(t)}$, and similarly, $E_{-}^{(t)} = -E_{+}^{(t)}$. Thus, there will be no waves escaping from the film into any direction, and the energy of both waves will be fully absorbed!

In such a case, the layer is to be located in the anti-node (i.e., maximum of electric field) of the original standing wave. Because of the amplitude reflection coefficient, the reflection of a straight-propagating (“+”) incident wave of the unity amplitude at $d = d_{pk}$ will form a back-propagating wave, $E_{+}^{(t)}$, with the amplitude $-0.5$. At the same time, if a back-propagating (“−”) incident wave has exactly the same phase at the film as the “+” incident wave, its transmitted portion, $E_{-}^{(t)}$, will have the same phase, and $p = 0.5$, so that $E_{+}^{(t)} = -E_{-}^{(t)}$, and similarly, $E_{-}^{(t)} = -E_{+}^{(t)}$. Thus, there will be no waves escaping from the film into any direction, and the energy of both waves will be fully absorbed!
where brackets \( \langle \rangle \) stand for time averaging, \( \langle \zeta \rangle = \int_{t_0}^{t_\infty} \zeta(\tau) d\tau / t_\infty \) as \( t_\infty \to \infty \) \((F(\tau) = F(-\tau))\), the respective spectrum is

\[
S(\omega) = \frac{1}{\pi} \int_0^{\infty} F(\tau) \cos(\omega \tau) d\tau.
\]  (59)

If one of the output channels, e.g., channel 2, behind the film (Fig. 4) is blocked, the output signal in channel 1, \( E_{1,\text{out}} \), is formed then by the input 2, \( E_{2,\text{in}} \), that gets through the entire loop without change of sign and attenuated by the factor of 2 and by the input 1, \( E_{1,\text{in}} = E_{2,\text{in}} \), that gets reflected by the thin film with the same attenuation, but with the change of sign, so that

\[
E_{1,\text{out}}(\tau) \propto E_{1,\text{in}}(\tau) - E_{2,\text{in}}(\tau - \tau),
\]  (60)

where \( \tau = t - t_0 \) is a retarded time, with \( t_0 = L/c \) being a full time delay of the light to go around the full ring of the length \( L \). The normalized BB reflection at the fixed delay \( \tau \) is then

\[
R_{1,\text{BB}}^{(BB)}(\tau) = \langle E_{1,\text{out}}^2(\tau) \rangle / \langle E_{2,\text{in}}^2(0) \rangle.
\]

Using Eq. (60) and having in mind that due to Eq. (59), \( F(\tau) = \int_0^{\infty} S(\omega) e^{i \omega \tau} d\omega \) and \( F(0) = 1 = \int_0^{\infty} S(\omega) d\omega \), we have

\[
R_{1,\text{BB}}^{(BB)}(\tau) = \frac{1 - F(\tau)}{2} = 2 \int_0^{\infty} S(\omega) \sin^2 \left( \frac{\omega \tau}{2} \right) d\omega.
\]  (61)

Note that \( R_{1,\text{BB}}^{(BB)}(0) = 0, R_{1,\text{BB}}^{(BB)}(\tau) \propto 0(\tau^2) \) as \( \tau \to 0 \), and for \( F(\infty) = 0 \), we have \( R_{1,\text{BB}}^{(BB)}(\infty) = 1/2 \). A typical example is a Gaussian spectrum, \( S(\omega) \), with an arbitrary bandwidth \( \Delta \omega \) centered around some frequency \( \omega_0 \);

\[
S(\omega) = [\exp(-\omega^2) + \exp(-\omega^2)](2\Delta \omega / \sqrt{\pi})^{-1},
\]  (62)

where \( s_\pm = (\omega_0 \pm \omega) / \Delta \omega \). The total reflectivity is then

\[
R_{1,\text{BB}}^{(BB)}(\tau) = [1 - \cos(\omega_0 \tau)] X / 2,
\]  (63)

with \( X = \exp(-[\tau \Delta \omega/2]^2) \). Figure 5 depicts \( R_1 \) versus \( x \) for various ratios \( \Delta \omega / \omega_0 \). With both of the output channels opened, the full output signal is \( E_{\text{out}}(\tau) \propto E_{\text{in}}(\tau) - (1/2)[E_{\text{in}}(\tau - \tau) + E_{\text{in}}(\tau + \tau)] \), similar to Eq. (61), and the full BB reflection is

\[
R_{\Sigma,\text{BB}}^{(BB)}(\tau) = [3 - 4F(\tau) + F(2\tau)] / 8,
\]  (64)

in the case of the Gaussian spectrum from Eq. (6), we have

\[
R_{\Sigma,\text{BB}}^{(BB)}(\tau) = [3 - 4 \cos(\omega_0 \tau) X + \cos(2\omega_0 \tau) X^2] / 8.
\]  (65)

For monochromatic input, \( \Delta \omega = 0 \), we have \( R_{\Sigma,\text{BB}}^{(BB)} \propto \sin^4(\omega_0 \tau / 2) \). In the case of white-like noise, i.e., there is no distinct central frequency of the signal, \( \Delta \omega \gg \omega_0 \), or simply \( \omega_0 = 0 \), Eqs. (63) and (65) are reduced to

\[
R_1 = (1 - X) / 2; \quad R_{\Sigma} = R_1^2 (3 + 2X + X^2) / 2.
\]  (66)

At \( \tau \Delta \omega / q \ll 1 \), we have \( R_{1,\text{BB}}^{(BB)}(\tau) \approx q^2 / 8 \) while \( R_{\Sigma,\text{BB}}^{(BB)}(\tau) \approx 3q^4 / 64 \), i.e., double channel detection is much more sensitive to the high-frequency details of the spectrum. In general, using both reflectivities, Eqs. (63) and (65), one may substantially enhance the temporal and spectral resolution because of simultaneous auto-correlation at two different delay times, \( \tau \) and \( 2\tau \). Figure 5 depicts \( R_1 \) versus \( \omega_0 \tau \) for various ratios \( \Delta \omega / \omega_0 \).

10. APPLICATIONS AND OUTLOOK

Having a nanometer-thick film absorbing 50% or even 100% of incident power of an extremely broad spectrum may have quite a few promising applications. We will discuss a few of them, yet there is no doubt that there could be others. Besides, one can expect some interesting directions of research related to such films.

So far we discussed a coherent spectroscopy of signals with super-broad, almost white-noise spectrum. To measure such signals, most of the elements of the Sagnac interferometer must be metallic, including all the mirrors, and the semi-transparent mirror should be made also the same way as the blackbody element, i.e., by again using a very thin metallic layer. This is necessary to extinguish any possible resonant or frequency-sensitive effects if the mirrors are made of dielectric layers. This kind of spectroscopy would be appropriate for the sub-visible domain down to the mid-infrared. It is well suited for terahertz technology; other applications may include the detection of high-frequency coherent features that may allow for detecting

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Fig. 4. Ring (Sagnac) interferometer using a blackbody element (thin metallic film) BB. Notations: M, metallic mirrors; D, intensity detector.
an information transmission in "pseudo-white-noise" signal, and potentially, in mw radiation from the space that may be helpful in the detecting extraterrestrial signals, as well as in low-level signal such as primordial thermal radiation. In all these potential applications, the important factor is that in contrast to regular auto-correlation techniques, whereby the auto-correlation signal at small delay times \( \tau \) is finite, the BB interferometry produces a zero output at \( \tau = 0 \), which may greatly increase its sensitivity compared to that of a regular auto-correlation. For certain applications, e.g., for primordial radiation, special care should be taken with blackbody radiation of the BB element (same as the other mirrors) by cooling it down with, e.g., liquid helium.

Further modification and enhancement of the BB interferometry, especially for a narrowband signal, may be attained by employing more than one metallic layer and using ensuing resonances. For example, for the monochromatic radiation with wavelength \( \lambda \), if the spacing between layers with \( Q = 1/2 \) is \( \lambda/2 \), the system is fully transparent, if irradiated from both directions. Inversely, a reflection resonance would exist if the spacing is \( \lambda/4 \). In this case, the amplitude of reflection of each of the counter-propagating waves is \( r = -2/5 \), and the transmission is \( t = 1/5 \). If the couple is positioned strictly at the center of the ring interferometer, the amplitude reflection for each of the waves is \( -1/5 \), and the intensity reflectivity is thus 4%. This is not far from total zero as with a single layer, but the system has substantial selectivity to the frequency. In thin-metal multi-layered structure [21,22] the resonant effect will be enhanced.

Another feasible application might be related to the use of \( Q = 0.5 \) layers for detecting and imaging/visualization of IR on mw radiation by covering a metallic film with a thermoluminescent layer (i.e., whose luminescence strongly depends on temperature) continuing along the line of the original research [15–17] by using more advanced materials (see, e.g., [58]). If such a material is preliminarily irradiated by, e.g., UV radiation and then by infrared, the spots where infrared is stronger will be heated up enough to trigger visible thermoluminescence from such spots. The visible optical image is expected to then have a very high spatial resolution, since the heat transfer along the layer would be negligible due to its extremely small thickness.

Another expected effect is related to the nonlinearity of the layers slightly thinner than the peak absorption, e.g., less than 1–2 nm. At the thicknesses corresponding to the formation of isolated “islands” of metal, which are still close to each other, the local field due to formation of plasmons can become enhanced by orders of magnitude, and due to closely packed islands induce tunnelling transitions and discharges, i.e., strongly nonlinear effects that may result in high harmonics generation.

It would be of great application interest to develop a tool of fast and efficient modulation of optical properties of ultra-thin films, especially in the vicinity of maximal absorption, by using an electro-optical effect to control behavior of free electrons in a film, as, e.g., in [59].

Application-wise, it would be interesting to use inexpensive “artificial” metal-like polymers, i.e., highly conductive doped polycarbonate whose electrical conductivity can be varied over the range of eleven orders of magnitude [60], polyanilin [61], and others (see review [62]). Related to that would be development of controllably produced 2D spatial modulation of the conductivity of the thin film, thus allowing the opportunity to design 2D photonic nano-crystals [63,64] with easily designable patterns.

As we have already discussed earlier, semi-metals present an interesting opportunity to study impedance-matching films, as most of them have a very long mean free path of electrons; Bi would be probably the most promising. Actually, it was the element that was first predicted [65,66] and experimentally observed [67,68] to show quantum-size effect at low temperature. In that effect, when the film thickness becomes comparable with the effective de Broglie wavelength of electrons, it would exhibit oscillations of its properties versus, e.g., its thickness. It would be of fundamental interest to explore the possibility of a time-dependent analogy of this effect in phase-matching Bi films under modulation of the incident radiation.

11. CONCLUSION

In conclusion, we reviewed major features of the frequency-independent reflection of the radiation from ultra-thin metallic layers within the large so-called sub-visible domain from the r.f to mw to mm to mid-infrared, or even infrared. We demonstrated a very universal optical property of such layers: they remain almost ideally reflectant (and almost non-absorbing) at the thicknesses orders of magnitude shorter that the skin layer at any frequency, down to a certain depth scale, typically a few mm, which depends only on the number density of free electrons. Near that scale, the optical parameters undergo a dramatic change whereby the reflectivity becomes equal to the transmittivity (25%), while 50% of the incident energy is absorbed (under certain arrangements, the absorption can go up to 100%). From the general EM point of view, this situation corresponds to a layer’s wave impedance matching exactly half of the impedance of free space. A major role in this scale formation is played by the size-affected conductivity directly related to the mean free electron path being “clipped” by the walls of the film. We also considered an arbitrary environment (a metal film sandwiched between dielectrics with different refractive indexes). We pointed out quite a few feasible applications of the phenomenon and related research direction.

APPENDIX A: MAXWELL EQUATIONS IN GAUSSIAN UNITS

We used here the Maxwell equations in Gaussian units and the Drude model for metal as a gas of quasi-free electrons,

\[
\nabla \times \vec{E} = \frac{1}{c} \frac{\partial \vec{H}}{\partial t}; \quad \nabla \times \vec{H} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi \vec{j}}{c},
\]

(A1)

where \( \vec{E} \) and \( \vec{H} \) are, respectively, electrical and magnetic fields, \( \vec{j} = 0 \) current density, and \( \sigma \) is a dc conductivity of a metallic layer; \( \sigma = 0 \) outside the layer. (The Drude model is a quite adequate model of conductivity for optical properties of metals in the sub-visible domain, while their thermal properties are not considered here.) In Gaussian units, \( \sigma \) is measured by the same unit as frequency, i.e., \( [\sigma] = \text{s}^{-1} \). The SI units for the

\[
\sigma = \frac{2\pi n \varepsilon_0 \omega}{c^2}
\]

where \( n \) is the number density of free carriers, \( \varepsilon_0 \) is the electric constant, \( \omega \) is the angular frequency. The zero of the dc conductivity is put at the skin depth. On the other hand, the results of section 6 are quite universal, and have been used in many applications. In that effect, when the film thickness becomes comparable with the effective de Broglie wavelength of electrons, it would exhibit oscillations of its properties versus, e.g., its thickness. It would be of fundamental interest to explore the possibility of a time-dependent analogy of this effect in phase-matching Bi films under modulation of the incident radiation.
conductivity $\sigma [(\Omega \cdot m)^{-1}]$ (or resistivity $\rho = 1/\sigma$, used often in the literature, can be converted to Gaussian units as

$$\sigma^G / \sigma^{SI} = \rho^{SI} / \rho^G \approx 9 \times 10^9 \Omega \cdot m/s. \quad (A2)$$

For $\omega$-monochromatic radiation, we represent, as usual, any field, $\vec{F}(\vec{r}, t)$, as a product of time-independent amplitude, $\vec{F}(\vec{r})$, and time-dependant exponents $\vec{F}(\vec{r}, t) = (1/2)\vec{F}(\vec{r})e^{-i\omega t} + c.c.$, and rewrite Eq. (A1) as

$$\nabla \times \vec{E} = i k \vec{H}; \quad \nabla \times \vec{H} = -i c(\omega)\vec{k} \vec{E}, \quad (A3)$$

where $k = \omega/c = 2\pi/\lambda$ and $\lambda = 2\pi c/\omega$ are, respectively, the wavenumber and wavelength of the wave in a free space, and $c$ is a dielectric constant; in free space in Gaussian units we have $\epsilon = 1$, and inside the layer, $\epsilon = \epsilon_m = 1 + 4\pi \sigma / \omega$. Under a “good metal” condition, $|\epsilon_m| \gg 1$, or $\sigma \gg \omega$, $\epsilon_m$ can be well approximated by a purely imaginary quantity, Eq. (8), where a scale $\Lambda$ [5] was introduced in Eq. (6), and skin depth $\delta$ is as defined in Eq. (2). Dropping a “vacuum” term “1” in $\epsilon_m$ is equivalent to neglecting the term $(1/\epsilon)(\partial \vec{E} / \partial t)$ in the second equation in Eq. (A1), which, if we use current, $j$, can be also rewritten as a magneto-quasi-static equation:

$$\nabla \times \vec{H} = Z_0 \vec{j}; \quad Z_0 = 4\pi / c, \quad (A4)$$

where $Z_0$ is the wave impedance of a free space in Gaussian units (in SI units $Z_0 = 120\pi\Omega \approx 377\Omega$). [It is worth noting that a plasma model of free electrons, and a related formula for dispersion $\epsilon = 1 - \omega_p^2 / \omega(\omega + i/\tau)$, where $\omega_p = \sqrt{4\pi N_e e^2/m}$ is plasma frequency, while meaningful at higher frequencies, is of little use in the quasi-static case, where the induced dynamics is much slower than the relaxation time $\tau = \ell_0 / v_F$, where $\ell_0$ is a mean free path of electrons, and $v_F = \sqrt{2W_e / m}$ is a Fermi velocity [see Eq. (49)]]. $\omega \tau \ll 1$. However, the above formula for $\epsilon_m$ is still consistent with the plasma formula for $\epsilon$ in the limit $\omega \tau \rightarrow 0$, having in mind the relationship in Eq. (49).

APPENDIX B: SIDE NOTES: METALLIC FILMS, CIRCA 1962–1964

In 1961, NASA launched the first inflated balloon satellite, Echo 1, to be used as a passive reflector of microwave radiation to detect the traces of the balloon in space by monitoring the deceleration of the balloon in time. It was followed by a much larger balloon satellite, Echo 2, launched in 1964. Both of them were made of thin mylar film coated by a very thin aluminum foil to facilitate a mirror-like reflection of the radiation. In 1962, this author, freshly graduated with his MS degree in general physics in 1961 with a focus on “radiophysics,” worked at a government R&D lab near Moscow that was developing inflated balloons for meteorological and reconnaissance purposes for the Russian Air Force. He was asked to look into possible applications of Echo-like satellites; the lab was looking into a way to join the rapidly growing space industry.

However, soon the idea of copying the Echo satellite was abandoned: the rocket-happy “big boys” of the Russian space industry apparently were not much interested. But the author kept playing with the subject, starting with the reflectivity of aluminum foil—there were plenty of aluminum-coated mylar films around, and he did some experimenting with them, trying to get voice-modulated and electrostatic-controlled reflection of large mirrors with a film stretched over a large rigid rim, and a primitive yet efficient telescope: toys, basically. His training called for the use of good theory; as a warm-up exercise, he calculated the reflection and transmission of thin metallic foils. His expectation was that in a microwave domain, a foil should start losing its reflectivity and increase its transparency when its thickness is just around skin depth. But that was not what happened; to his great surprise, the reflectivity kept staying close to 100%, even when the foil thickness got orders of magnitude lower than that. Greatly puzzled, he kept repeating his calculations—with the same result. None of the sources available to him indicated anything like that either. He finally showed his result to the lab bosses, emphasizing that one can now reduce the weight of a potential satellite—not a small feat in those days. He was met with derisive comments about his “elite training.”

He wrote his paper anyway, and it took more than a year to fight reviewers off; it was accepted then by a decision of a willful editor in chief (Prof. B. Z. Katsenelenbaum, who checked out all the calculations by himself (can you find an editor like that these days?), and published in 1964 [5]. The author even got a national award for “best paper by a young scientist,” but it was meaningless for his further research career in Russia anyway, especially considering his increasing involvement in dissident human rights activity. He never returned to the subject again (until 2005, when Boris Zeldovich came up with a new twist on it, and they published a paper [6] on the subject). He got his Ph.D. on a completely different subject (high-order subharmonics in nonlinear parametric oscillators) from which he did his research for his MS degree and published it (as a sole author too) even before the thin-film paper. Closer to the end of the 1960s, he switched to lasers and nonlinear optics, including predictions of self-bending effect, and later on, optical bistability and switching at nonlinear interfaces.

In 1979, carrying two suitcase and an empty wallet, he came as a refugee to the U.S., where he immediately got back to his research on nonlinear optics at MIT, continued later on at Purdue and then Johns Hopkins, in particular on nonlinear interfaces, hysteretic relativistic resonances of a single cyclotron electron, sub-femtosecond pulses, and shock waves in cluster explosions. A whimsical but lucky part of all of that was that it was the Air Force (again) Office of Scientific Research, this time of the U.S., that kept supporting him for 35 years; his steadfastly encouraging and supportive program manager all that time was Dr. Howie Schlossberg, while his diverse and forever shifting research interests strayed far away from those early subjects.

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