Fundamental constants and tests of general relativity - Theoretical and cosmological considerations

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Abstract. The tests of the constancy of the fundamental constants are tests of the local position invariance and thus of the equivalence principle. We summarize the various constraints that have been obtained and then describe the connection between varying constants and extensions of general relativity. To finish, we discuss the link with cosmology, and more particularly with the acceleration of the Universe. We take the opportunity to summarize various possibilities to test general relativity (but also the Copernican principle) on cosmological scales.

Keywords: Fundamental constant, gravitation, cosmology

1. Introduction

Physical theories usually introduce constants, i.e. numbers that are not, and by construction can not be, determined by the theory in which they appear. They are contingent to the theory and can only be experimentally determined and measured.

These numbers have to be assumed constant for two reasons. First, from a theoretical point of view, we have no evolution equation for them (since otherwise they would be fields) and they cannot be expressed in terms of other more fundamental quantities. Second, from an experimental point of view, in the regimes in which the theories in which they appear have been validated, they should be constant at the accuracy of the experiments, to ensure the reproducibility of experiments. This means that testing for the constancy of these parameters is a test of the theories in which they appear and allow to extend the knowledge of their domain of validity.

Indeed, when introducing new, more unified or more fundamental, theories the number of constants may change so that the list of what we call fundamental constants is a time-dependent concept and reflects both our knowledge and ignorance (Weinberg, 1983). Today, gravitation is described by general relativity, and the three other interactions and whole fundamental fields are described by the standard model of particle physics. In such a framework, one has 22 unknown constants [the Newton constant, 6 Yukawa couplings for the quarks and 3 for the leptons, the mass and vacuum expectation value of the Higgs field, 4 parameters for the Cabibbo-Kobayashi-Maskawa matrix, 3 coupling constants, a UV cut-off to which one must add the speed of light and the Planck constant; see e.g. Hogan (2000)].

Since any physical measurement reduces to the comparison of two physical systems, one of them often used to realize a system of units, it only gives access to dimensionless numbers. This implies that only the variation of dimensionless combinations of the fundamental constants can be measured and would actually also correspond to a modification of the physical laws [see e.g. Uzan (2003), Ellis and Uzan (2005)]. Changing the value of some constants while letting all dimensionless numbers unchanged would correspond to a change of units. It follows that from the 22 constants of our reference model, we can pick 3 of them to define a system of units (such as e.g. c, G and h to define the Planck units) so that we are left with 19 unexplained dimensionless parameters, characterizing the mass hierarchy, the relative magnitude of the various interactions etc.
Indeed, this number can change with time. For instance, we know today that neutrinos have to be somewhat massive. This implies that the standard model of particle physics has to be extended and that it will involve at least 7 more parameters (3 Yukawa couplings and 4 CKM parameters). On the other hand, this number can decrease, e.g. if the non-gravitational interactions are unified. In such a case, the coupling constants may be related to a unique coupling constant $U$ and a mass scale of unification $M_U$ through

$$\frac{1}{\alpha_i}(E) = \frac{1}{U} + \frac{b_i}{2} \ln \frac{M_U}{E};$$

where the $b_i$ are numbers which depends on the explicit model of unification. This would also imply that the variations, if any, of various constants will be correlated.

The tests of the constancy of fundamental constants take all their importance in the realm of the tests of the equivalence principle (Will, 1993). This principle, which states the universality of free fall, the local position invariance and the local Lorentz invariance, is at the basis of all metric theories of gravity and implies that all matter fields are universally coupled to a unique metric $g$ which we shall call the physical metric,

$$S_{matter}(g);$$

The dynamics of the gravitational sector is dictated by the Einstein-Hilbert action

$$S_{grav} = \frac{c^2}{16 \pi G} \int g^{\frac{1}{2}} R \; d^4x;$$

General relativity assumes that both metrics coincide, $g = g$. The test of the constancy of constants is a test of the local position invariance hypothesis and thus of the equivalence principle. Let us also emphasize that it is deeply related to the universality of free fall since if any constant $c_i$ is a space-time dependent quantity so will the mass of any test particle so that it will experience an anomalous acceleration

$$a = \frac{\partial}{\partial t} \frac{m}{c^2} \frac{\partial \ln m}{\partial c_i};$$

which is composition dependent [see Uzan (2003) for a review].

In particular, this allows to extend tests of the equivalence, and thus tests of general relativity, on astrophysical scales. Such tests are central in cosmology in which the existence of a dark sector (dark energy and dark matter) is required to explain the observations.

Necessity of theoretical physics in our understanding of fundamental constants and on deriving bounds on their variation is, at least, threefold:

1. it is necessary to understand and to model the physical systems used to set the constraints. In particular one needs to determine the effective parameters that can be observationally constrained to a set of fundamental constants;
2. it is necessary to relate and compare different constraints that are obtained at different space-time positions. This often requires a space-time dynamics and thus to specify a model;
3. it is necessary to relate the variation of different fundamental constants through e.g. unification.

This text summarizes these three aspects by first focusing, in section 2, on the various physical systems that have been used, in section 3, on the theories describing varying constants (focusing on unification and on the link with the universality of free fall). Section 4 summarizes the links with cosmology where our current understanding of the dynamics
Table I. Summary of the systems considered to set constraints on the variation of the fundamental constants. We summarize the observable quantities (see text for details), the primary constants used to interpret the data and the other hypotheses required for this interpretation. [:fine structure constant; : electron-to-proton mass ratio; \( g_i \): gyromagnetic factor; \( E_r \): resonance energy of the samarium-149; \( \tau \): lifetime; \( B_D \): deuterium binding energy; \( Q_{np} \): neutron-proton mass difference; \( q^\prime \): neutron lifetime; \( m_e \): mass of the electron; \( m_N \): mass of the nucleon].

| System             | Observable            | Primary constraints | Other hypothesis         |
|--------------------|-----------------------|---------------------|--------------------------|
| Atomic clock       | \( \ln l \)          | \( g_i \); \( \tau \) |                          |
| Oklo phenomenon    | isotopic ratio        | \( E_r \)           | geophysical model        |
| Meteorite dating   | isotopic ratio        |                     |                          |
| Quasar spectra     | atomic spectra        | \( g_p \); \( \tau \) | cloud properties         |
| 21 cm              | \( T_{21} \)          | \( g_p \); \( \tau \) | cosmological model       |
| CMB                | \( T \)               | \( \tau \)          | cosmological model       |
| BBN                | light element abundances | \( Q_{np} \); \( m_e \); \( m_N \); \( B_D \) | cosmological model       |

of the cosmic expansion calls for the introduction of another constant: the cosmological constant, if not of a new sector of physics, the “dark energy”.

2. Physical systems and constraints

2.1. Physical systems

The various physical systems that have been considered can be classified in many ways.

First, we can classify them according to their look-back time and more precisely their space-time position relative to our actual position. This is summarized on Fig. 1 which represents our past-light cone, the location of the various systems (in terms of their redshift \( z \)) and the typical level at which they constrain the time variation of the fine structure constant. This systems include atomic clocks comparisons \( (z = 0) \), the Oklo phenomenon \( (z \approx 4) \), meteorite dating \( (z \approx 3) \), both having a space-time position along the world line of our system and not on our past-light cone, quasar absorption spectra \( (z = 2\text{ - }4) \), cosmic microwave background (CMB) anisotropy \( (z = 10^8) \) and primordial nucleosynthesis (BBN, \( z = 10^8 \)). Indeed higher redshift systems offer the possibility to set constraints on an larger time scale, but at the prize of usually involving other parameters such as the cosmological parameters. This is particularly the case of the cosmic microwave background and primordial nucleosynthesis, the interpretation of which require a cosmological model [see Uzan (2003) for a review and Uzan and Leclercq (2005) for a non-technical introduction].

The systems can also be classified in terms of the physics that they involve in order to be interpreted (see Table I). For instance, atomic clocks, quasar absorption spectra and the cosmic microwave background require only to use quantum electrodynamics to draw the primary constraints, so that these constraints will only involve the fine structure constant \( g_i \), the ratio between the proton-to-electron mass ratio and the various gyromagnetic factors \( g_i \). On the other hand, the Oklo phenomenon, meteorite dating and nucleosynthesis require nuclear physics and quantum chromodynamics to be interpreted (see below).
Figure 1. Summary of the systems that have been used to probe the constancy of the fundamental constants and their position in a space-time diagram in which the cone represents our past light cone. The shaded areas represent the comoving space probed by different tests with respect to the largest scales probed by primordial nucleosynthesis.

2.2. Setting Constraints

For any system, setting constraints goes through several steps that we sketch here.

First, any system allows us to derive an observational or experimental constraint on an observable quantity \( O(G_k; X) \) which depends on a set of primary physical parameters \( G_k \) and a set of external parameters \( X \), that usually are physical parameters that need to be measured or constrained (e.g. temperature,...). These external parameters are related to our knowledge of the physical system and the lack of their knowledge is usually referred to as systematic uncertainty.

From a physical model of the system, one can deduce the sensitivities of the observables to an independent variation of the primary physical parameters

\[
    \frac{\partial \ln O}{\partial \ln G_k} = \frac{\partial^2 \ln O}{\partial \ln G_k \partial \ln X}
\]

As an example, the ratio between various atomic transitions can be computed from quantum electrodynamics to deduce that the ratio of two hyperfine-structure transition depends only on \( g_I \), and while the comparison of fine-structure and hyperfine-structure transitions depend on \( g_I \) and \( g_f \). For instance (Dzuba et al. (1999); Karshenboim(2005))

\[
    \frac{C_{\text{Rb}}}{C_{\text{Cs}}} / \frac{C_{\text{Rb}}}{C_{\text{K}}}, 0.249; \quad \frac{C_{\text{K}}}{C_{\text{Rb}}} / \frac{C_{\text{K}}}{C_{\text{Cs}}}, 2.83;
\]

The primary parameters are usually not fundamental constants (e.g. the resonance energy of the samarium \( E_r \) for the Oklo phenomenon, the deuterium binding energy \( B_D \) for nucleosynthesis etc.) The second step is thus to relate the primary parameters to (a choice of) fundamental constants \( c_i \). This would give a series of relations (see e.g. Müller et al. (2004))

\[
    \ln G_k = X \ln c_i \ln c_i;
\]
The determination of the parameters $d_{ki}$ requires first to choose the set of constants $c_i$ (do we stop at the masses of the proton and neutron, or do we try to determine the dependencies on the quark masses, or on the Yukawa couplings and Higgs vacuum expectation value, etc.; see e.g. Dent et al. (2008) for various choices) and also requires to deal with nuclear physics and the intricate structure of QCD. In particular, the energy scales of QCD, $\tilde{Q}_{\text{CD}}$, is so dominant that at lowest order all parameters scales as $c_i \tilde{Q}_{\text{CD}}$ so that the variation of the strong interaction would not affect dimensionless parameters and one has to take the effect of the quark masses.

As an example, the Oklo phenomenon allows to draw a constraint on the value of the energy of the resonance. The observable $O$ is a set of isotopic ratios that allow to reconstruct the average cross-sections for the nuclear network that involves the various isotopes of the samarium and gadolinium (this involves assumptions about the geometry of the reactor, its temperature that falls into $X$). It was argued (Damour and Dyson, 1996) on the basis of a model of the samarium nuclei that the energy of the resonance is mainly sensitive to so that the only relevant parameter is $d_{1;}$. The level of the constraint $0.9 \times 10^{-7} < \approx < 1.2 \times 10^{-7}$ that is inferred from the observation is thus related to the sensitivity $d_{1;}$.

2.3. CONSTRAINTS ON THE FINE STRUCTURE CONSTANT

An extended discussion of the different constraints on the time variation of the fine structure constant can be found in Uzan (2003, 2004). We just summarize the state of the art in Fig. 2 which depicts the constraints on $\alpha$ for different redshift bands. In the cases where the constraints involve other constants, we have assumed for simplicity that only $\alpha$ was allowed to vary. This is the case in particular for quasar absorption spectra (see the contribution by P. Petitjean in this volume).

The typical constraints on cosmological timescales (of order of 10 Gyr) is $< 10^{-6}$ which would correspond to a constraint $= 10^{-16} \text{yr}^{-1}$ if one assumes (but there a priori no reason to support it) that the rate of change is constant over time (i.e. $\langle t \rangle$ is a linear function of the cosmic time). For comparison, under the same hypothesis, Oklo would give the constraint $= 5 \times 10^{-17} \text{yr}^{-1}$. But indeed these constraints are complementary since they concern different spacetime positions.

2.4. UNIFICATION AND CORRELATED VARIATIONS

In the context of the unification of the fundamental interactions, it is expected that the variations of the various constants are not independent. By understanding these correlations we can set stronger constraints at the expense of being more model-dependent. We only illustrate this on the example of BBN, along the lines of Coc et al. (2007).

The BBN theory predicts the production of the light elements in the early universe: the abundances synthesized rely on the balance between the expansion of the universe and the weak interaction rates which control the neutron to proton ratio at the onset of BBN (see Peter and Uzan (2005) for a textbook introduction). Basically, the abundance of helium-4 is well approximated by

$$\gamma_p = 2 \left( \frac{(n=p)_f \exp( \frac{t_{\text{nu}}}{T})}{1 + (n=p)_f \exp( \frac{t_{\text{nu}}}{T})} \right)$$

where $(n=p)_f = \exp( \frac{Q_{np} = kT_f}{G_F})$ is the neutron to proton ratio at the freeze-out time determined (roughly) by $G_F^2 (kT_f)^3 = \frac{8}{3} \frac{G N}{m_p} (kT_f)^2$, $N$ being the number of relativistic degrees of freedom; $Q_{np} = m_n \rightarrow m_p$, $t_{\text{nu}}$ is the neutron lifetime, $G_F$ the Fermi constant and $t_{\text{nu}}$ the time after which the photon density becomes low enough for the photo-dissociation of the
Figure 2. Constraints on the variation of the fine structure constant obtained for the various physical systems as a function of redshift or look-back time, assuming the standard CDM model.

deuterium to be negligible. As a conclusion, the predictions of BBN involve a large number of fundamental constants. In particular, $t_D$ depends on the deuterium binding energy and on the photon-to-baryon ratio, $\eta$. Besides, one needs to include the effect of the fine structure constant in the Coulomb barriers (Bergström et al., 1999). For a different analysis of the effect of varying fundamental constants on BBN predictions see e.g. Müller et al. (2004), Landau et al. (2006), Coc et al. (2007), Dent et al. (2007).

It follows that the predictions of the BBN are mainly dependent of the effective parameters $G_k = (G; m_e; \eta; \mu_{DP}; B_D; \cdots)$ while the external parameters are mainly the cosmological parameters $X = (\eta; n; \cdots)$. The dependence of the predictions on the independent variation of each of these parameters can be determined and it was found (Flambaum and Shuryak (2002), Coc et al. (2007); Fig. 3) that the most sensitive parameter is the deuterium binding energy, $B_D$ (see also Dent et al. (2007) for a similar analysis including more parameters). The helium and deuterium data allow to set the constraints

$$7.5 \times 10^{-2} < \frac{B_D}{Q_{np}} < 6.5 \times 10^{-2}; \quad 8.2 \times 10^{-2} < - < 6 \times 10^{-2};$$

$$4 \times 10^{-2} < \frac{Q_{np}}{Q_{np}} < 2.7 \times 10^{-2}$$

on the independent variations of these parameter. More interestingly (and more speculative!) a variation of $B_D$ in the range $(7.5; 4) 10^{-2}$ would be compatible with the helium and deuterium constraint while reconciling the spectroscopically determined lithium-7 abundance (Coc et al., 2007) with its expected value from WMAP.

In a second step, the parameters $G_k$ can be related to a smaller set of fundamental constants, namely the fine structure constant $\alpha$, the Higgs VEV $v$, the Yukawa couplings $h_i$ and the QCD scale $\alpha_{QCD}$ since $Q_{np} = (m_n; m_p; a_{QCD} + (h_d + h_u)v, m_e = h_e v, n = G_Y^2 m_e^3 (Q = m_e) )$ and $G_F = \frac{G_Y}{2\sqrt{2}}$. The deuterium binding energy can be expressed in terms of $h_n$, $\nu$ and $\alpha_{QCD}$ (Flambaum and Shuryak, 2003) using a sigma nuclear model.

\[ \text{Figure 2. Constraints on the variation of the fine structure constant obtained for the various physical systems as a function of redshift or look-back time, assuming the standard CDM model.} \]
or in terms of the pion mass (Epelbaum et al., 2003). Assuming that all Yukawa couplings vary similarly, the set of parameters $G_k$ reduces to the set of constants $f; v; h; g_{QCD}$ (again in units of the Planck mass).

Several relations between these constants can however be found. For instance, in grand-unified models the low-energy expression of $g_{QCD}$,

$$g_{QCD} = \frac{m_e m_B m_t}{3} \exp^{2+7/9} \left( \frac{2}{3} \right)$$

for $m_t > m_e$ yields a relation between $f; v; h; g_{QCD}$ so that one actually has only 3 independent constants. Then, in all models in which the weak scale is determined by dimensional transmutation, changes in the Yukawa coupling $h_t$ will trigger changes in $v$ (Ibanez and Ross, 1982). In such cases, the Higgs VEV can be written as

$$v = M_p \exp \left( \frac{8 \pi^2 c}{h_t^2} \right),$$

where $c$ is a constant of order unity. It follows that we are left with only 2 independent constants. This number can even be reduced to 1 in the case where one assumes that the variation of the constants is trigger by an evolving dilaton (Damour and Polyakov (1994); Campbell and Olive (1995)). At each stage, one reduces the number of constants, and thus the level of the constraints, at the expense of some model dependence. In the latter case, it was shown that BBN can set constraints of the order of $j < 4 \times 10^5$ (Coc et al., 2007).

### 3. Theories with “varying constants”

#### 3.1. Making a constant dynamical

The question of whether the constants of nature may be dynamical goes back to Dirac (1937) who expressed, in his “Large Number hypothesis”, the opinion that very large (or small) dimensionless universal constants cannot be pure mathematical numbers and must not occur in the basic laws of physics. In particular, he stressed that the ratio between the gravitational and electromagnetic forces between a proton and an electron, $G m_e m_p / e^2 = 10^{-40}$, is of the same order as the inverse of the age of the universe in atomic units, $e^2 H_0 = m_e c^2$. He stated that these were not pure numerical coincidences but instead that these big numbers were not pure constants but reflected the state of our universe. This led him to postulate that $G$ varies\(^1\) as the inverse of the cosmic time. Diracs’ hypothesis is indeed not a theory and it was shown later that a varying constant can be included in a Lagrangian formulation as a new dynamical degree of freedom so that one gets both a new dynamical equation of evolution for this degree of freedom and a modification of the other field equations with respect to their form derived under the hypothesis it was constant.

Let us illustrate this on the case of scalar-tensor theories, in which gravity is mediated not only by a massless spin-2 graviton but also by a spin-0 scalar field that couples universally to matter fields (this ensures the universality of free fall). In the Jordan frame, the

\(^1\) Dirac hypothesis can also be achieved by assuming that $e$ varies as $t^{-2}$. Indeed this reflects a choice of units, either atomic or Planck units. There is however a difference: assuming that only $G$ varies violates the strong equivalence principle while assuming a varying $e$ results in a theory violating the Einstein equivalence principle. It does not mean we are detecting the variation of a dimensionful constant but simply that either $e^2 \rightarrow c$ or $G m_e^2 \rightarrow -c$ is varying.
action of the theory takes the form

\[ S = \frac{Z}{16G} \int d^4x \sqrt{g} \left[ \mathcal{L}'(\varphi) + \frac{g}{2} \mathcal{Z}'(\varphi) \mathcal{Z}'' + \frac{g}{2} \mathcal{U}'(\varphi) \right] + S_{\text{matter}}[\varphi, \mathcal{Z}] \]  

(4)

where \( G \) is the bare gravitational constant. This action involves three arbitrary functions \( (F, Z, \text{and } \mathcal{U}) \) but only two are physical since there is still the possibility to redefine the scalar field \( \varphi \). \( F \) needs to be positive to ensure that the graviton carries positive energy. \( S_{\text{matter}} \) is the action of the matter fields that are coupled minimally to the metric \( g \).

In the Jordan frame, the matter is universally coupled to the metric so that the length and time as measured by laboratory apparatus are defined in this frame.

It is useful to define an Einstein frame action through a conformal transformation of the metric \( g = F(\varphi)g \). In the following all quantities labelled by a star (*) will refer to Einstein frame. Defining the field \( \varphi' \) and the two functions \( A(\varphi') \) and \( V(\varphi') \) (see e.g. Esposito-Farèse and Polarski, 2001) by

\[ \frac{d^2}{d\varphi'} = \frac{3}{4} \frac{d \ln F(\varphi)}{d\varphi'} + \frac{1}{2F(\varphi)}; A(\varphi') = F^{-1}(\varphi') \; 2V(\varphi') = U(\varphi')F^{-2}(\varphi') \]

the action (4) reads as

\[ S = \frac{1}{16G} \int d^4x \sqrt{g} \left[ 2g \mathcal{R} + 2g \mathcal{Z} \mathcal{Z}' + 4\mathcal{V} \right] + S_{\text{matter}}[A, g] \]  

(5)

The kinetic terms have been diagonalized so that the spin-2 and spin-0 degrees of freedom of the theory are perturbations of \( g \) and \( \varphi \) respectively.

The action (4) defines an effective gravitational constant \( g_0 = G = F = G A^2 \). This constant does not correspond to the gravitational constant effectively measured in a Cavendish experiment. The Newton constant measured in this experiment is \( G_{\text{cav}} = G A^2 (1 + \frac{2}{\varphi}) \) where the first term, \( G A^2 \), corresponds to the exchange of a graviton while the second term \( G A^2 \left( \frac{2}{\varphi} \right) \) is related to the long range scalar force. The gravitational constant depends on the scalar field and is thus dynamical.

The post-Newtonian parameters can be expressed in terms of the values of \( g \) and \( \varphi \) today as

\[ \frac{\mathcal{P}_{\text{PN}}}{\mathcal{P}_{\text{PN}}} = 1 = \frac{2}{1 + \frac{2}{\varphi}}; \quad \frac{\mathcal{P}_{\text{PN}}}{\mathcal{P}_{\text{PN}}} = 1 = \frac{2}{1 + \frac{2}{\varphi}} \]  

(6)

The Solar system constraints imply \( \varphi \) to be very small, typically \( \frac{2}{\varphi} < 10^{-5} \) while \( \varphi \) can still be large. Binary pulsar observations (Esposito-Farèse, 2005) impose that \( \varphi \) > 4.5 and that \( G_{\text{cav}} < 10^{-12} \text{yr}^{-1} \).

3.2. General Dangers

Given the previous discussion, it seems a priori simple to cook up a theory that will describe a varying fine structure constant by coupling a scalar field to the electromagnetic Faraday tensor as

\[ S = \frac{Z}{16G} \int d^4x \sqrt{g} \left[ R^{(\mathfrak{g})} \right] + \frac{1}{4} B^2 \mathcal{F}^2 + \mathcal{P} \]  

\[ m_{\mathfrak{g}A}(Z) = 98.25 \left( \frac{Z}{A^{1.3}} \right) \text{MeV} \]
This implies that the sensitivity of the mass to a variation of the scalar field is expected to be of the order of

\[ f_{\sigma(\Lambda_Z)} = \theta m_{\sigma(\Lambda_Z)} \left( 10^2 \frac{Z}{\Lambda_{4+3}} \right) \theta(0) : \]  

(8)

It follows that the level of the violation of the universality of free fall is expected to be of the level of \( 10^{-12} \times (\Lambda_1 Z_1 ; \Lambda_2 Z_2) (\theta \ln B)^5 \). Since the factor \( X (\Lambda_1 Z_1 ; \Lambda_2 Z_2) \) typically ranges as \( O \left( 0.1 \times 10 \right) \), we deduce that \( \theta \ln B \) has to be very small for the Solar system constraints to be satisfied. It follows that today the scalar field has to be very close to the minimum of the coupling function \( \ln B \).

Let us mention that such coupling terms naturally appear when compactifying a higher-dimensional theory. As an example, let us recall the compactification of a 5-dimensional Einstein-Hilbert action (Peter and Uzan (2005), chapter 13)

\[ S = \frac{1}{12} \frac{Z_5}{G_5} \]  

Decomposing the 5-dimensional metric \( g_{AB} \) as

\[ g_{AB} = g_{\Lambda M} + \frac{\Lambda_{\Lambda M}}{\Lambda^2} \left( \frac{\Lambda}{M} \right)^2 \]  

where \( M \) is a mass scale, we obtain

\[ S = \frac{1}{16} \frac{Z_4}{G} \]  

(9)

where the 4-dimensional gravitational constant is \( G = 3 \times G_5 = 4 \pi \). The scalar field couples explicitly to the kinetic term of the vector field and cannot be eliminated by a redefinition of the metric: this is the well-known conformal invariance of electromagnetism in four dimensions. Such a term induces a variation of the fine structure constant as well as a violation of the universality of free-fall. Such dependencies of the masses and couplings are generic for higher-dimensional theories and in particular string theory. It is actually one of the definitive predictions for string theory that there exists a dilaton, that couples directly to matter (Taylor and Veneziano, 1988) and whose vacuum expectation value determines the string coupling constants (Witten, 1984).

For example, in type I superstring theory, the 10-dimensional dilaton couples differently to the gravitational and Yang-Mills sectors because the graviton is an excitation of closed strings while the Yang-Mills fields are excitations of open strings. For small values of the volume of the extra-dimensions, a T-duality makes the theory equivalent to a 10-dimensional theory with Yang-Mills fields localized on a D3-brane. When compactified on an orbifold, the gauge fields couple to fields \( M_i \) living only at these orbifold points with couplings \( c_i \) which are not universal. Typically, one gets that \( M_2 = e^2 \left( \frac{2}{V_5 M_5} + c_i M_i \right) \) while \( g_{22}^2 = e^2 \left( \frac{2}{V_6 M_6} + c_i M_i \right) \). Unfortunately, the 4-dimensional effective couplings depend on the version of the string theory, on the compactification scheme and on the dilaton.

### 3.3. Ways Out

While the tree-level predictions of string theory seem to be in contradiction with experimental constraints, many mechanisms can reconcile it with experiment. In particular, it has been claimed that quantum loop corrections to the tree-level action may modify the coupling function in such a way that it has a minimum (Damour and Polyakov, 1994). As explained in the former paragraph, the dilaton needs to be close to the minimum of the
coupling function in order for the theory to be compatible with the universality of free fall. In the case of scalar-tensor theories, it was shown that when the coupling function enjoys such a minimum, the theory is naturally attracted toward general relativity (Damour and Nordtvedt, 1993). The same mechanism will apply if all the coupling functions have the same minimum (see the contribution by T. Damour in this volume for more details). In that particular model the mass of any nuclei will typically be of the form

\[ m_i = Q_{CD} (1 + a_q \frac{m_q}{Q_{CD}} + a_e); \]

where \( a_q \) and \( a_e \) are sensitivities. It follows that composition independent effects (i.e. \( J_{PPN}^{\nu} J_{PPN}^{\nu} J_{PPN}^{\nu} J_{PPN}^{\nu} J_{Q=G} \)) and composition dependent effects (\( J_{Q=G} \)) will be of the same order of magnitude, dictated by the difference of the value of the dilaton today compared to its value at the minimum of the coupling function.

Another possibility is to invoke an environmental dependence, as can be implemented in scalar-tensor theories by the chameleon mechanism (Khoury and Weltman, 2004) which invokes a potential with a minimum that does not coincide with the one of the coupling function.

4. Links with cosmology

The comparison of various constraints requires a full cosmological model. In particular, one cannot assume that the time variation of the constant is linear either with time or redshift (as used to compare the typical magnitudes of the constraints in \( \times 2.3 \)). Besides, cosmology tends to indicate that a new constant, the cosmological constant, has to be included in our description. We briefly address these two points by focusing on the tests that can be performed on cosmological scales and on the status of our cosmological model.

4.1. Cosmological Evolution

The cosmological dynamics is central to apply the least coupling principle. Since the dilaton is attracted towards the minimum of the coupling function during its cosmological evolution, deviations from general relativity are expected to be larger in the early universe. In particular BBN can set bounds on the deviation from general relativity that are stronger than those obtained from Solar system experiments (Damour and Pichon (1999), Coc et al. (2006)).

4.2. About Dark Energy

The construction of any cosmological model relies on 4 main hypotheses: (H1) a theory of gravity; (H2) a description of the matter contained in the Universe and their non-gravitational interactions; (H3) symmetry hypothesis; (H4) an hypothesis on the global structure, i.e. the topology of the Universe.

These hypotheses are not on the same footing since H1 and H2 refer to the physical theories. These two hypotheses are however not sufficient to solve the field equations and we must make an assumption on the symmetries (H3) of the solutions describing our Universe on large scales while H4 is an assumption on some global properties of these cosmological solutions, with same local geometry.

Our reference cosmological model is the CDM model. It assumes that gravity is described by general relativity (H1), that the Universe contains the fields of the standard model of particle physics plus some dark matter and a cosmological constant, the latter
two having no physical explanation at the moment. It also deeply involves the Copernican principle as a symmetry hypothesis (H3), without which the Einstein equations usually can not be solved, and usually assumes that the spatial sections are simply connected (H4). H2 and H3 imply that the description of the standard matter reduces to a mixture of a pressureless and a radiation perfect fluids. This model is compatible with all astronomical data which roughly indicates that $\rho_0 \approx 0.73$, $\rho_m \approx 0.27$, and $\kappa_0 \approx 0$. Cosmology thus roughly imposes that $j_0 \ll H_0^2$, that is $H_0 \approx 10^{26} \text{ m}^{-1} 10^{41} \text{ GeV}^{-1}$. Notice that it is disproportionately large compared to the natural scale fixed by the Planck length

\[ \phi < 10^{-120} M_p \times 10^{-47} \text{ GeV}^4 \]  

at the heart of the cosmological constant problem.

The assumption that the Copernican principle holds, and the fact that it is so central in drawing our conclusion on the acceleration of the expansion, splits the investigation into two avenues. Either we assume that the Copernican principle holds and we have to modify the laws of fundamental physics or we abandon the Copernican principle, hoping to explain dark energy without any new physics but at the expense of living in a particular place in the Universe. In the former case, the models can be classified in terms of 4 universality classes summarized on Fig. 3).

The goal in this section is to summarize some attempts to test both the Copernican principle and general relativity on cosmological scales.

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**Figure 3.** Summary of the different classes of physical dark energy models. As discussed in the text, various tests can be designed to distinguish between them. The classes differ according to the nature of the new degrees of freedom and their couplings. Left column accounts for models where gravitation is described by general relativity while right column models describe a modification of general relativity. In the upper line classes, the new fields dominate the matter content of the universe at low redshift. Upper-left models (class A) consist of models in which a new kind of gravitating matter is introduced. In the upper-right models (class C), a light field induces a long-range force so that gravity is not described by a massless spin-2 graviton only. In this class, Einstein equations are modified and there may be a variation of the fundamental constants. The lower-right models (class D) correspond to models in which there may exist an infinite number of new degrees of freedom, such as in some class of braneworld scenarios. These models predict a modification of the Poisson equation on large scales. In the last class (lower-left, class B), the distance duality relation may be violated. From Uzan (2007).
4.3. BEYOND THE COPERNICAN PRINCIPLE

The Copernican principle implies that the spacetime metric reduces to a single function, the scale factor $a(t)$ that can be Taylor expanded as $a(t) = a_0 + H_0 (t - t_0) + \frac{1}{2} \Theta_0 H_0^2 (t - t_0)^2 + \cdots$. It follows that the conclusions that the cosmic expansion is accelerating ($q_0 < 0$) does not involve any hypothesis about the theory of gravity (other than the one that the spacetime geometry can be described by a metric) or the matter content, as long as this principle holds.

While isotropy around us seems well established observationally, homogeneity is more difficult to test. The possibility that we may be living close to the center of a large underdense region has sparked considerable interest, because such models can successfully match the magnitude-redshift relation of type Ia supernovae without dark energy (see the contribution by S. Sarkar in this volume).

The main difficulty in testing the Copernican principle lies in the fact that most observations are located on our past light-cone. Recently, it was realized that cosmological observations may however provide such a test (Uzan et al., 2008). It exploits the time drift of the redshift that occurs in any expanding spacetime, as first pointed out in the particular case of Robertson-Walker spacetimes for which it takes the form $z = (1 + z)H_0 H(z)$. Such an observation would give informations on the dynamics outside the past light-cone since it compares the redshift of a given source at two times and thus on two infinitely close past light-cones (see Fig. 4-right). It follows that it contains informations about the spacetime structure along the worldlines of the observed sources that must be compatible with the one derived from the data along the past light-cone.

For instance, in a spherically symmetric spacetime, the expression depends on the shear, $(z)$, of the congruence of the wordlines of the comoving observers evaluated along our past light-cone (Uzan et al., 2008),

$$z = (1 + z)H_0 H(z) \frac{1}{3} \left( \frac{\ddot{z}}{\dot{z}} \right) (z):$$

It follows that, when combined with other distance data, it allows to determine the shear on our past light-cone and one can check whether it is compatible with zero, as expected for any Robertson-Walker spacetime.

In a RW universe, one can go further and determine a consistency relation between several observables since $H^{-1}(z) = D^0(z) 1 + \kappa_0 H_0^2 D^2(z) 1^2$, where a prime stands for $\dot{z}$ and $D^2(z) = D_L(z) = (1 + z)$; this relation is independent of the Friedmann equations. It follows that in any Robertson-Walker spacetime the consistency relation,

$$1 + \kappa_0 H_0^2 \frac{D_L(z)}{1 + z} \left[ D_L(z) \right]^2 \frac{d}{dz} \left[ \frac{D_L(z)}{1 + z} \right]^2 = 0;$$

between observables must hold whatever the matter content and the field equations, since it derives from pure kinematical relations that do not rely on the dynamics.

$z(z)$ has a typical amplitude of order $z = 5 \times 10^{10}$ on a time scale of $t = 10$ yr, for a source at redshift $z = 4$. This measurement is challenging, and impossible with present-day facilities. However, it was recently revisited in the context of Extremely Large Telescopes (ELT), arguing they could measure velocity shifts of order $\frac{v}{10 \text{ cm/s}}$ over a 10 years period from the observation of the Lyman- forest. It is one of the science drivers in design of the CODEX spectrograph (Pasquini et al., 2005) for the future European ELT. Indeed, many effects, such as proper motion of the sources, local gravitational potential, or acceleration of the Sun may contribute to the time drift of the redshift. It was shown (Uzan et al., 2008), however, that these contributions can be brought to a 0.1% level so that the cosmological redshift is actually measured.
Figure 4. Left: Most low-redshift data are localized on our past light-cone. In a non-homogeneous spacetime there is no direct relation between the redshift that is observed and the cosmic time, needed to reconstruct the expansion history. Right: The time drift of the redshift allows to extract information about two infinitely close past light-cones. \( z \) depends on the proper motions of the observer and the sources as well as the spacetime geometry.

Let us also stress that another idea was also proposed recently (Goodman, 1995; Caldwell and Stebbins, 2008; Clarkson et al. 2008, see the contribution by R. Caldwell in this volume). This idea is based on the distortion of the Planck spectrum of the cosmic microwave background.

4.4. Testing General Relativity on Astrophysical Scales

Extracting constraints on deviations from GR on cosmological scales is difficult because large scale structures entangle the properties of matter and gravity. On sub-Hubble scales, one can, however, construct tests reproducing those in the Solar system. For instance, light deflection is a test of GR because we can measure independently the deflection angle and the mass of the Sun.

On sub-Hubble scales, relevant for the study of the large-scale structure, the Einstein equations reduce to the Poisson equation

\[ \nabla^2 \phi = 4 \pi G \rho, \quad \rho = \rho_m \delta \Theta, \]

relating the gravitational potential and the matter density contrast.

As first pointed out by Uzan and Bernardeau (2001), this relation can be tested on astrophysical scales, since the gravitational potential and the matter density perturbation can be measured independently from the use of cosmic shear measurements and galaxy catalogs. The test was recently implemented with the CFHTLS-weak lensing data and the SDSS data to conclude that the Poisson equation holds observationally to about 10 Mpc (Doré et al., 2007). The main limitation in the applicability of this test is due to the biasing mechanisms (i.e. the fact that galaxies do not necessarily trace faithfully the matter field) even if it is thought to have no significant scale dependence at such scales.

4.4.1. Toward a post-CDM formalism

The former test of the Poisson equation exploits one rigidity of the field equations on sub-Hubble scales. It can be improved by considering the full set of equations.

Assuming that the metric of spacetime takes the form

\[ ds^2 = (1 + 2 \kappa)dt^2 + (1 - 2 \gamma^2)dx^i dx^j \]

relating the gravitational potential and the matter density contrast.

As first pointed out by Uzan and Bernardeau (2001), this relation can be tested on astrophysical scales, since the gravitational potential and the matter density perturbation can be measured independently from the use of cosmic shear measurements and galaxy catalogs. The test was recently implemented with the CFHTLS-weak lensing data and the SDSS data to conclude that the Poisson equation holds observationally to about 10 Mpc (Doré et al., 2007). The main limitation in the applicability of this test is due to the biasing mechanisms (i.e. the fact that galaxies do not necessarily trace faithfully the matter field) even if it is thought to have no significant scale dependence at such scales.
on sub-Hubble scales, the equation of evolution reduces to the continuity equation \( \frac{0}{m} + \frac{m}{a} = 0 \), where \( \frac{m}{a} \) is the divergence of the velocity perturbation and a prime denotes a derivative with respect to the conformal time, the Euler equation \( \frac{m}{a} + H = 0 \), where \( H \) is the comoving Hubble parameter, the Poisson equation (11) and \( = . \)

These equations imply many relations between the cosmological observables. For instance, decomposing \( m \) as \( D(t)(x) \) where \( \text{encodes the initial conditions} \), the growth rate \( D(t) \) evolves as \( D + 2H D = 0 \). This equation can be rewritten in terms of \( p = \ln a \) as time variable (Peter and Uzan, 2005) and considered not as a second order equation for \( D(t) \) but as a first order equation for \( H^2(a) \)

\[
(H^2)^0 + 2 \frac{3}{a} + \frac{D^0}{D^0} H^2 = 3 \frac{m a H^2 D}{a^2 D^0}
\]

where a prime denotes a derivative with respect to the conformal time, the Euler equation implies that \( m = (m a t o g) m a t \) with \( (m a t o g) = \ln D(a) = \ln a \).

We conclude that the perturbation variables are not independent and the relation between them are inherited from some assumptions on the dark energy. Phenomenologically, we can generalize the sub-Hubble equations to

\[
0 + m = 0 \quad ; \quad 0 + H = + S_{de}; \quad k^2 = 4 G F (k;H) + de; \quad ( ) = de;
\]

(14) (15)

We assume that there is no production of baryonic matter so that the continuity equation is left unchanged. \( S_{de} \) describes the interaction between dark energy and standard matter. \( de \) characterizes the clustering of dark energy, \( F \) accounts for a scale dependence of the gravitational interaction and \( de \) is an effective anisotropic stress. It is clear that the CDM corresponds to \( (F; de; S_{de}) = (1;0;0) \) The expression of \( (F; de; S_{de}) \) for quintessence, scalar-tensor, \( f(R) \) and DGP models and more generally for models of the classes A-D can be found in Uzan (2007).

From an observational point of view, weak lensing survey gives access to \( + \), galaxy maps allow to reconstruct \( q = b m a t \) where \( b \) is the bias, velocity fields give access to \( m \). In a CDM, the correlations between these observables are not independent since, for instance \( h g i = \alpha h^2 m a t l \) and \( h g i = 8 G m a t a h^2 m a t l \). Various ways of combining these observables have been proposed, construction of efficient estimators and forecast for possible future space missions designed to make these tests as well as the possible limitations (arising e.g. from non-linear bias, the effect of massive neutrinos or the dependence on the initial conditions) are now being extensively studied (Zhang et al., 2007; Jain and Zhang, 2007; Amendola et al., 2008; Song and Koyama, 2008).
The study of fundamental constants provides tests of general relativity that can be extended on astrophysical scales. These constraints can be useful in our understanding of the origin of the acceleration of the cosmic expansion. They can be combined with tests of general relativity based on the large scale structure of the universe.

In the coming future, we can hope to obtain new constraints from population III stars ($z \approx 15$) and 21cm absorption ($z \approx 30-100$) and improved constraints from QSO by about a factor 10 (and probably more with ELT).

Interpreting the coupled variations of various constants is also challenging since it usually implies to deal with nuclear physics and QCD. It also offers a window on unification mechanisms and on string theory.

All these aspects make the study of fundamental constants a lively and promising topic.

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