COMPTON SCATTERING ON PION AND PION POLARIZABILITIES\footnote{This work was supported by the Deutsche Forschungsgemeinschaft (SFB 201)}

L.V. FIL’KOV, V.L. KASHEVAROV

Lebedev Physical Institute, Moscow 117924, Russia

The Compton scattering on a charged pion is studied using the dispersion relations. Unknown parameters of the \(\sigma\) meson are found from a fit to the experimental data for \(\gamma\gamma \rightarrow \pi^0\pi^0\) process.

In the present work we investigate the elastic \(\gamma\pi^\pm\) scattering in the energy region up to \(\sqrt{s} \approx 1\) GeV (where \(s\) is the square of the total energy in \(\gamma\pi\) c.m.s.). With this aim we construct dispersion relations (DRs) at fixed square of the momentum transfer \(t\) with one subtraction for the helicity amplitudes \(M_{++}\) and \(M_{-+}\) of the elastic \(\gamma\pi\) scattering.

\[
\text{Re}M_{++}(s, t) = \text{Re}\mathcal{M}_{++}(s = \mu^2, t) + B_{++}
\]

\[
\frac{(s - \mu^2)}{\pi} P \int_4 \left[ \frac{1}{s' - s} \right] \left[ \frac{1}{s' - \mu^2} \right] ds'
\]

\[
B_{++} + \frac{1}{4\mu^2} \left[ \frac{1}{(s' - s)(s' - \mu^2)} - \frac{1}{(s' - u)(s' - \mu^2 + t)} \right] ImM_{++}(s', t)
\]

where \(B_{++}\) is the Born term. The subtraction function \(\text{Re}\mathcal{M}_{++}(s = \mu^2, t)\) is determined with help of the DR at fixed \(s = \mu^2\) with one subtraction where the subtraction constant is expressed through the difference of the pion polarizabilities \((\alpha - \beta)\):

\[
\text{Re}M_{++}(s = \mu^2, t) = \text{Re}M_{++}(s = \mu^2, t) - B_{++}(s = \mu^2, t) = 2\pi\mu(\alpha - \beta) +
\]

\[
\frac{t}{\pi} P \left\{ \int_4 \left[ \frac{1}{s' - s} \right] \left[ \frac{1}{s' - \mu^2} \right] ds' \right\} - \int_4 \left[ \frac{1}{s' - \mu^2} \right] ImM_{++}(s', u = \mu^2) ds'
\]

The DRs for the amplitude \(M_{+-}(s, t)\) have the same expressions (1) and (2) with substitutions: \(ImM_{++} \rightarrow ImM_{+-}\), \(B_{++} \rightarrow B_{+-} = B_{++}/\mu^2\) and \(2\pi\mu(\alpha - \beta) \rightarrow 2\pi/\mu(\alpha + \beta)\). The DRs for the \(\gamma\pi^\pm\) scattering are saturated by the contributions of the \(\rho(770)\), \(b_1(1235)\) and \(a_2(1320)\) mesons in the \(s\) and \(u\) channels and \(\sigma\), \(f_0(980)\) and \(f_2(1270)\) mesons in the \(t\) channel. The parameters of the \(\rho\), \(\omega\), \(\phi\), \(b_1\) and \(a_2\) mesons are given by the Particle Data Group. The masses and decay widths of the \(f_0\) and \(f_2\) mesons are chosen to get the best fit.
Table 1: The $\sigma$ meson parameters determined by the fit

|   | $m_\sigma$(MeV) | $\Gamma_\sigma$(MeV) | $\Gamma_{\sigma\to\gamma\gamma}$(keV) | $\chi^2$ | $(\alpha - \beta)_{\pi^0}$ |
|---|----------------|---------------------|--------------------------------------|---------|-----------------------------|
| a | 524 ± 52       | 1075 ± 390         | 0.52 ± 0.20                          | 0.40    | -2.5                        |
| b | 544 ± 33       | 1170 ± 285         | 0.61 ± 0.13                          | 0.42    | -1.9                        |

The parameters of the $\sigma$ meson are not known. In the present work the parameters of the $\sigma$ meson are found from a fit to the experimental data for the $\gamma\gamma \to \pi^0\pi^0$ process. As the $\gamma\pi$ elastic scattering and the process $\gamma\gamma \to \pi^0\pi^0$ should be described by the common analytical function, we use the same DRs for description of both of these processes. In the case of reaction $\gamma\gamma \to \pi^0\pi^0$ the DRs are saturated by the $\rho$, $\omega$ and $\phi$ mesons in the $s$ and $u$ channels and $\sigma$, $f_0$ and $f_2$ mesons in the $t$ channel. The fitting parameters are the mass, the full width and the width of the decay into $\gamma\gamma$ of the $\sigma$ meson. An additional parameter is the polarizabilities difference of $\pi^0$ meson ($\alpha - \beta$). We consider two variants of the fitting: a) ($\alpha - \beta$) is determined during fit procedure through the dispersion sum rule (DSR) ; b) ($\alpha - \beta$) is fixed from calculation in the framework of chiral perturbation theory ($\chi PT$) equal to -1.9 (in units of $10^{-4}$fm$^3$). The polarizabilities sum ($\alpha + \beta$) is calculated with help of the DSR : $(\alpha + \beta)_{\pi^0} = 0.8$.

The parameters of $\sigma$ meson obtained by the fitting in the energy region from $\sqrt{t} = 270$ MeV up to 825 MeV ($t$ is the square of the total energy in $\gamma\gamma$ c.m.s.) are listed in table 1. As it is evident from the table, the parameters of the $\sigma$, found for the ”a” and ”b” variants are in good agreement. Fig. 1 demonstrates a description of the experimental data of the process $\gamma\gamma \to \pi^0\pi^0$ in the energy region up to $\sqrt{t} \approx 2$ GeV using the found values of the $\sigma$ meson parameters.

If one takes the parameters of $\sigma$ meson obtained for the variant ”b” to evaluate the $(\alpha - \beta)_{\pi^0}$ with help of DSR, we find $(\alpha - \beta)_{\pi^0} = -2.3$. It differs from the value (-1.9) used for fitting in this case. This difference could be a measure of accuracy of calculation of $(\alpha - \beta)_{\pi^0}$ in the framework of this DSR.

An application of the found parameters of the $\sigma$ meson for evaluation of the difference of $\pi^\pm$ meson polarizabilities with help of the DSR gives: $(\alpha - \beta)_{\pi^\pm} = 7.8$ for the variant ”a” and 8.1 for ”b”. These values are bigger than ones predicted in $\chi PT$ ($\sim 5.6$), smaller than values obtained in the frame of DSRs early ($10.9-10.6$) but close to the prediction of the quark confinement model ($7.05$).

Fig.2 shows the results of calculation of $\gamma\pi^\pm$ back scattering cross section with help of the DRs for the fitting variants ”a” (solid line) and ”b” (painted line) description of the experimental data for $\gamma\gamma \to \pi^0\pi^0$ process in the energy region of these resonances. The parameters of the $\sigma$ meson are not known.
line). The dashed line figure corresponds to \((\alpha - \beta)_{\pi^{\pm}} = 0\). It is evident from this figure, that the value of the calculated cross section of \(\gamma\pi^{\pm}\) scattering is practically independent of the variant of fitting.

For the \(\gamma\pi^{\pm}\) forward scattering the cross section is determined by the contribution of the Born term, the sum \((\alpha + \beta)_{\pi^{\pm}}\) and the \(\rho\), \(b_1\) and \(a_2\) mesons. The results of calculation of the forward scattering cross section are given in Fig.3 where the solid curver corresponds to account of all contributions, the dashed curver shows the same contributions but with \((\alpha + \beta)_{\pi^{\pm}} = 0\) and the dotted one is contribution of the Born+\((\alpha + \beta)_{\pi^{\pm}}\). This figure demonstrates big contribution of the sum \((\alpha + \beta)_{\pi^{\pm}}\) in the energy region near 1GeV.

The relative contributions of \((\alpha - \beta)_{\pi^{\pm}}\) and \((\alpha + \beta)_{\pi^{\pm}}\) into the back and forward cross sections, respectively, as function of the energy \(\sqrt{s}\) are shown in Fig.4. As follows from this figure the relative contributions of the \((\alpha - \beta)_{\pi^{\pm}}\) and \((\alpha + \beta)_{\pi^{\pm}}\) grow with energy and exceed 100\% in the region of 1GeV. This result permits to determine the pion polarizabilities with high accuracy from the experimental data for the elastic \(\gamma\pi^{\pm}\) scattering in this energy region.

So, using the DRs we found the mass, full width and decay width into \(\gamma\gamma\) of the \(\sigma\) meson from the fit to the experimental data for the \(\gamma\gamma\rightarrow\pi^0\pi^0\) process. The analysis of the calculation of cross section of elastic \(\gamma\pi^{\pm}\) scattering showed: 1) when extracting the \((\alpha - \beta)\) and \((\alpha + \beta)\) from the pion Compton scattering data it is necessary to take into account corrections to the low energy expression; 2) the data for the elastic \(\gamma\pi^{\pm}\) scattering in the energy region up to \(\sim 1\)GeV (together with the \(\gamma\gamma\rightarrow\pi^0\pi^0\) data) could be used both for determination of the pion polarizability values with high enough precision and for research of the \(\sigma\) meson.

The authors would like to thank J. Ahrens, R. Beck, D. Drechel, A. L’vov and Th. Walcher for helpful discussions.

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Figure 1: Cross section for $\gamma\gamma \rightarrow \pi^0\pi^0$ process in energy region up to 2 GeV

Figure 2: Back scattering cross section for $\gamma\pi^\pm \rightarrow \gamma\pi^\pm$ process

Figure 3: Forward scattering cross section for $\gamma\pi^\pm \rightarrow \gamma\pi^\pm$ process

Figure 4: The relative contributions of $(\alpha - \beta)_\pi^\pm$ (solid line) and $(\alpha + \beta)_\pi^\pm$ (dashed line)