Reaction cross sections of hypernuclei and the shrinkage effect

T. Akaishi and K. Hagino
Department of Physics, Tohoku University, Sendai 980-8578, Japan

We calculate the reaction cross sections for $^6\text{Li}$ and $^7\text{Li}$ on a $^{12}\text{C}$ target at 100 MeV/nucleon using the Glauber theory. To this end, we assume a two-body cluster structure for $^6\text{Li}$ and $^7\text{Li}$, and employ the few-body treatment of the Glauber theory, that is beyond the well known optical limit approximation. We show that the reaction cross section for $^7\text{Li}$ is smaller than that for $^6\text{Li}$ by about 4%, reflecting the shrinkage effect of the $\Lambda$ particle.

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One of the main interests in hypernuclear physics is the change of the nuclear structure due to a few $\Lambda$ particles, which is referred to as the impurity effect [1–10]. A $\Lambda$ particle can enter the interior of a nucleus since it does not receive the Pauli exclusion principle from nucleons, and attracts the surrounding nucleons. Various theoretical analyses and experimental measurements have suggested that this effect appears as a shrinkage of a nucleus, that is, the change of the nuclear size [11,12]. Particularly, a large shrinkage effect is expected for light nuclei which have cluster structures [1–3], since clusters are in general weakly bound in these nuclei. The shrinkage of hypernuclei has been investigated experimentally with the $\gamma$-ray spectroscopy [4,5]. In the experiment of Ref. [4], the electric quadrupole transition probability $B(E2)$ from the excited $5/2^+$ state to the ground state in $^7\text{Li}$ was measured. The observed $B(E2)$ value of $^7\text{Li}$ was smaller than that of $^6\text{Li}$ by about 33%. This corresponds to about 19% shrinkage of the intercluster distance if one assumes the two-body cluster structure with core+deuteron (that is, $\alpha+d$ and $^3\text{He}+d$ for $^6\text{Li}$ and $^7\text{Li}$, respectively) [3].

In this paper, we investigate the shrinkage effect of $\Lambda$ particle using reaction cross sections with the Glauber theory. The reaction cross section is defined as a sum of all cross sections except for the elastic scattering, and it has played an important role in the discussion of a density distribution for neutron-rich nuclei, such as a “halo” structure of exotic nuclei [17–21]. Classically, if a projectile and a target nucleus are assumed to be spheres with a radius of $R_p$ and $R_T$, respectively, the reaction cross section $\sigma_R$ is given as $\sigma_R = \pi(R_p + R_T)^2$. The experimental reaction cross sections have indeed shown that the reaction cross section increases for nuclei which have a halo structure [1], and the reaction cross sections have thus been used as a standard tool to extract the nuclear size of neutron-rich nuclei [18]. It is thus of intriguing to study the reaction cross sections of hypernuclei and discuss their size.

In order to extract the nuclear size and matter distributions, the Glauber theory [22] has often been used [23–26]. Notice that the Glauber-type analyses need only the ground state information. This makes our study complementary to the method with electromagnetic transitions, which involves both the ground and excited states.

We choose $^6\text{Li}$ and $^7\text{Li}$ nuclei as projectiles and $^{12}\text{C}$ as a target. Since these projectile nuclei are known to have a two-body cluster structure [3,27], we adopt the semi-microscopic cluster model [27,28] in order to obtain the ground state wave functions. In this model, the intercluster potentials between the core and deuteron are constructed based on the core+p+n structure. For the $s$-wave state (that is, the ground state), it is given as,

$$V(R) = \int dr \left[ V_{CN}(R + r/2) + V_{CN}(R - r/2) \right] \cdot |\psi_d(r)|^2. \tag{1}$$

where $\psi_d(r)$ is an $s$-state wave function for the relative motion between the proton and the neutron in the deuteron cluster. $V_{CN}$ is the potential between the nucleons and the core nucleus. In our calculations, we employ an exponential form for the deuteron wave function [29]:

$$\psi_d(r) = \sqrt{2\alpha e^{-\alpha r}/\sqrt{4\pi r}} \quad \alpha = 0.2316 \text{fm}^{-1}$$

and a Gaussian-type potential between $\alpha$ particle and nucleon [30]:

$$V_{\alpha N}(r) = -v_0 e^{-\beta r^2} \quad v_0 = 40.45 \text{MeV}, \beta = 0.189 \text{fm}^{-2}$$

For $^7\text{Li}$ nucleus, one also has to add the $\Lambda$-nucleon potential to $V_{CN}$, that is, $V_{CN} = V_{\alpha N} + V_{\Lambda N}$. In order to construct it, we fold a $\Lambda$-nucleon potential $v_{\Lambda N}$ in the free space with the $\Lambda$ particle density in the $\alpha$ cluster,

$$V_{\Lambda N}(r) = \int dr' \rho_{\Lambda}(r')v_{\Lambda N}(r-r') \quad \tag{2}$$

For a Gaussian density distribution, $\rho_{\Lambda}(r) = (\pi\beta^2_{\Lambda})^{-2/3}e^{-r^2/\beta^2_{\Lambda}}$ and a Gaussian $\Lambda$-nucleon potential, $v_{\Lambda N}(r) = -v_0 e^{-r^2/\beta^2}$, this can be calculated analytically as

$$V_{\Lambda N}(r) = -v_0 \left( \frac{b^2_{\Lambda}}{b^2_{\Lambda} + b^2_{\nu}} \right)^{3/2} \exp \left[ -\frac{r^2}{b^2_{\Lambda} + b^2_{\nu}} \right]. \tag{3}$$

We numerically solve the Schrödinger equation with the potential $V(R)$, Eq. (1). The ground state is identified.

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1 Experimentally, the interaction cross sections, defined as a sum of cross sections in which the nucleon number changes, are actually measured instead of the reaction cross sections at intermediate and high energies. At these energies the interaction and the reaction cross sections do not differ much, especially for weakly bound systems, because inelastic scattering cross sections are small.
as the state with the node of 1. In this paper, we use the same width and strength parameters, $b_\Lambda, b_\alpha$, and $\nu_0$ as those in Ref. [6].

Figure 1 shows the intercluster radial wave functions $u(R)$ so obtained, where $u(R)$ is defined with the intercluster wave function $\psi_{\text{rel}}(R)$ as $\psi_{\text{rel}}(R) = \frac{u(R)}{\rho_{\text{rel}}(R)}$. The dashed and solid lines are for $^6\text{Li}$ and $^7\Lambda$Li, respectively. The figure also shows the one-body densities $\rho(r)$ obtained as

$$\rho(r) = \int dR |\psi_{\text{rel}}(R)|^2 \left[ \rho_1(r + a_1 R) + \rho_2(r - a_2 R) \right],$$

where $a_1$ and $a_2$ are defined as $M_2/(M_1 + M_2)$ and $M_1/(M_1 + M_2)$, respectively, $M_1$ and $M_2$ being the mass of the cluster 1 and 2, respectively. We use Gaussian-type densities for all the nuclei obtained with the semi-microscopic cluster model. The reaction cross sections in the projectiles (see also Ref. [31] for an alternative method which is beyond the OLA). The reaction cross sections in the FB are given as

$$\sigma_{\text{FB}}(\Lambda) = 2\pi \int b db \left[ 1 - |\langle \psi_{\text{rel}}|S_1(b_1)S_2(b_2)|\psi_{\text{rel}} \rangle|^2 \right],$$

where $S_i(b_i)$ is the scattering matrix between the cluster $i$ in the projectile and the target nucleon evaluated in the OLA. For $d$ and $\alpha$, $S_i(b)$ is given by

$$S_i(b) = \exp \left[ -\frac{\sigma_{\text{NN}}}{2} \int ds \rho_1^{(z)}(-b + s)\rho_T^{(z)}(s) \right].$$

Since we employ the Gaussian-type densities for all the clusters including the target, it can be obtained analytically as

$$S_i(b) = \exp \left[ -\frac{\sigma_{\text{NN}}}{2} \frac{A_i A_T}{\pi(\gamma_i^2 + \gamma_T^2)} e^{-b^2/(\gamma_i^2 + \gamma_T^2)} \right].$$

For a hypernucleus $^4\Lambda$He Eq. (11) is extended as

$$S_{^4\Lambda\text{He}}(b) = \exp \left[ -\frac{\sigma_{\text{NN}}}{2} \int ds \rho_1^{(z)}(-b + s)\rho_T^{(z)}(s) \right] - \frac{\sigma_{\text{AN}}}{2} \int ds \rho_\alpha^{(z)}(-b + s)\rho_T^{(z)}(s),$$

where $\sigma_{\text{AN}}$ is the total cross section for $\Lambda$-nucleon scattering. We consider the energy region of $E \approx 100$ MeV/nucleon, where experimental cross sections for both $\sigma_{\text{NN}}$ and $\sigma_{\text{AN}}$ are available, although $\sigma_{\text{AN}}$ has not
been determined accurately. We take $\sigma_{NN} = 55.2 \text{ mb}$ and $\sigma_{AN}$ in the range from 10 to 30 mb.

Table I shows the reaction cross sections obtained with the FB and the OLA calculations for $^6\text{Li}$ and $^7\text{Li}$. We notice that the OLA yields always larger reaction cross sections compared to the FB, indicating an importance of the cluster correlation in these nuclei. If one considers only the shrinkage of the nucleon distribution in $^7\text{Li}$, neglecting the contribution of the $\Lambda$ particle to the reaction cross section (that is, setting $\sigma_{AN} = 0$), the reaction cross section for $^7\text{Li}$ decreases by about 5.4 % in the FB and 7.4 % in the OLA compared to the reaction cross section for $^6\text{Li}$. By including the $\Lambda$ particle contribution to the reaction cross section, the reduction is in the range of 4.7-3.6 % in the FB and 6.9-5.9 % in the OLA. Although these values are somewhat smaller than the reduction in the rms radius, these results clearly indicate that the shrinkage effect in hypernuclei can be studied also with the reaction cross section.

In summary, we have investigated reaction cross sections for $^6\text{Li}$ and $^7\text{Li}$ nuclei incident on a $^{12}\text{C}$ target at 100 MeV/nucleon. We have first applied for these projectile nuclei the semi-microscopic cluster model in order to construct the ground state wave functions and the one-body densities. For a reaction theory, we have used the few-body (FB) treatment for the projectiles in the Glauber theory in order to take into account the cluster correlations. We have found that the reaction cross section for $^7\text{Li}$ is always smaller than that for $^6\text{Li}$ by about a few percent. This fact suggests that the shrinkage effect induced by a $\Lambda$ particle in hypernuclei can be studied with reaction cross sections as an alternative to $\gamma$-ray spectroscopy. It would be interesting if a measurement of reaction cross sections could be realized for hypernuclei in some future.

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| $^6\text{Li}$ | FB  | OLA |
|--------------|-----|-----|
| $^7\text{Li}$ | $^{12}\text{C}$ target at 100 MeV/nucleon.
| $^7\text{Li}$ | 825.9 | 880.6 |
| $^7\text{Li}$ | (10 mb) | 786.7 | 819.9 |
| $^7\text{Li}$ | (20 mb) | 791.7 | 824.4 |
| $^7\text{Li}$ | (30 mb) | 796.5 | 828.8 |

Table I: The reaction cross sections $\sigma_R$, in the units of mb, for $^6\text{Li}$ and $^7\text{Li}$ nuclei incident on a $^{12}\text{C}$ target at 100 MeV/nucleon. The values for $\Lambda$-nucleon scattering cross section $\sigma_{AN}$, are given in the parentheses. Both the results based on the few-body (FB) treatment and the optical limit approximation (OLA) for the Glauber theory are shown.
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