On the paradoxical evolution of the number of photons in a new model of interpolating Hamiltonians

C. Valverde\textsuperscript{1,2} and B. B. Baseia\textsuperscript{3,4}

\textsuperscript{1}Campus de Ciências Exatas e Tecnológicas, Universidade Estadual de Goiás, 75001-970 Anápolis, GO, Brazil
\textsuperscript{2}Universidade Paulista, 78.45-090 Goiânia, GO, Brazil
\textsuperscript{3}Instituto de Física, Universidade Federal de Goiás, 74.690-900 Goiânia, GO, Brazil
\textsuperscript{4}Departamento de Física, Universidade Federal da Paraíba, 58.051-970 João Pessoa, PB, Brazil

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We introduce a new Hamiltonian model which interpolates between the Jaynes-Cummings model and other types of such Hamiltonians. It works with two interpolating parameters, rather than one as traditional. Taking advantage of this greater degree of freedom, we can perform continuous interpolation between the various types of these Hamiltonians. As applications we discuss a paradox raised in literature and compare the time evolution of photon statistics obtained in the various interpolating models. The role played by the average excitation in these comparisons is also highlighted.

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\section{Introduction}

The Jaynes-Cummings model (JCM), proposed in 1963 \cite{1}, constitutes an excellent theoretical approach to describe analytically the interaction of a two level atom with a single mode of a quantized radiation field. The field frequency may belong either to the optical domain or to the microwave one. In the first case the researchers use common atoms \cite{2} whereas in second case they use (highly excited) Rydberg atoms \cite{3}. The issue was also extended to other systems, as (i) in nanocircuits operating in microwave domain, either through the substitution of the atom by a Copper-pair box (CPB) and the field by a nanomechanical resonator in nanocavities \cite{4}; (ii) or the CPB inside a chip \cite{5,6}; (iii) substituting the atom by quantum a dot embedded in a photonic-crystal \cite{7}; (iv) using spin in quantum-dot arrays \cite{8}, etc. In spite of its simplicity the JCM gives exact solutions of the Schrödinger equation in many examples that occur in such physical systems.

The JCM has been employed in the study of various fundamental quantum aspects involving the matter-radiation. To give some examples we mention: collapse and revival of the atomic inversion \cite{9}; the Rabi frequency of oscillation for a given atomic transition acted upon by a light field \cite{10}; nonclassical statistical distributions of light fields \cite{11}, antibunching effect \cite{12}; squeezed states \cite{13,14}, and others.

An alternative model that maintains various characteristics of the JCM and offers advantages in certain situations was proposed by Buck-Sukumar in 1981, abbreviated as BSM \cite{15}. It is called intensity-dependent JCM, since it substitutes the JCM interaction $\lambda (\sigma_+ \hat{a} + \sigma_- \hat{a}^\dagger)$ by another interaction that includes the number operator $\hat{n}$, in this way: $\lambda (\sigma_+ \hat{R} + \sigma_- \hat{R}^\dagger)$ with $\hat{R} = \hat{a} \sqrt{\hat{n}}$ and $\hat{R}^\dagger = \sqrt{\hat{n}} \hat{a}^\dagger$. In the previous expressions $\hat{a}$ ($\hat{a}^\dagger$) stands for annihilation (creation) operator, $\sigma_-$ ($\sigma_+$) is lowering (raising) operator, $(\hat{n} = \hat{a}^\dagger \hat{a})$ is the number operator, and $\lambda$ stands for the atom-field coupling. This model also leads to analytical solution of the Schrödinger equation. It has been argued that its physical simulation in laboratory could be implemented via matrices of waveguides \cite{16}; optical analogies of quantum systems realized in waveguide arrays have recently impacted the field of integrated optical structures \cite{17}. In particular, SUSY photonic lattices can be used to provide phase matching conditions between large number of modes allowing the pairing of isospectral crystals \cite{18,21}. In spite of its apparent theoretical nature the BSM has attracted the attention of various researchers in the quantum optical community. \cite{21,34}.

In 1992 P. Shanta, S. Chaturvedi, and V. Srinivasana (SCS-model) proposed an extension of the intensity-dependent JCM \cite{22}. This model interpolates between the JCM and the BSM. In this approach the authors assumed the modified Hamiltonian,

$$H_1 = \omega \hat{N}' + \frac{1}{2} \omega_0 \sigma_z + \lambda (\sigma_+ \sqrt{\hat{N}'} + 1 \hat{a} + \sigma_- \hat{a}^\dagger \sqrt{\hat{N}'} + 1),$$

where $\hat{N}'$ is the number operator and the operators $\hat{a}$, $\hat{a}^\dagger$ are quons operators satisfying the the commutation relation $\hat{a} \hat{a}^\dagger - \hat{a}^\dagger \hat{a} = 1$; $q$ is a c-number restricted to the interval $q \in [1, -1]$. Accordingly, quons would stand for particles intermediate between bosons ($q = 1$) and fermions ($q = -1$). The authors then use specific connections between the operator $\hat{N}'$ and $\hat{a}$ and $\hat{a}^\dagger$ and prove that the SCS model interpolates between the BSM and JCM in the limits $q = 1$ and $q = 0$, respectively, with $q$ playing the role of the interpolating parameter. However, although being a creative approach, here we will not take it forward because we are restricting ourselves to photonic field, not to quons. According to Ref. \cite{29}.
there are other nonlinear models in this context, but it
treats the coupled system only approximately [35].

Another type of intensity-dependent JCM, was proposed
in 2002 by S. Sivakumar [23], named here as
Sivakumar model (SM). This model also interpolates
between various Hamiltonian models, including the JCM,
that provides a continuous and exact interpolation be-
tween. In Sec. II we briefly discuss this class of Hamiltonian,

\[ H = \omega \hat{K} \hat{K} + \frac{1}{2} \omega_0 \hat{\sigma}_z + \lambda (\hat{\sigma}_+ \hat{K} + \hat{K}^\dagger \hat{\sigma}_-), \]  

(2)

where \( \hat{K} = \sqrt{1 + k a^\dagger a} \) and \( \hat{K}^\dagger = a^\dagger \sqrt{1 + k a^\dagger a} \) stand
respectively for annihilation (\( \hat{a} \)) and creation (\( \hat{a}^\dagger \)) operators. The change from \( \hat{a} \) to \( \hat{K} \) aims to get a convenient
deformed algebra for various theoretical applications, as
in group theory, field theory, and others. As established
in [23, 24], for \( k = 0 \) one has the Heisenberg-Weyl alge-
bra generated by \( \{ \hat{a}, \hat{a}^\dagger, \hat{I} \} \) and for \( k = 1 \) one finds the
SU(1,1) algebra. For all values of \( k \) the algebra is closed,

\[ [\hat{K}, \hat{K}^\dagger] = 2\hat{K}_0, \quad [\hat{K}_0, \hat{K}^\dagger] = k \hat{K}^\dagger, \quad [\hat{K}_0, \hat{K}] = -k \hat{K}, \]

(3)

with \( \hat{K}_0 = k \hat{a}^\dagger \hat{a} + \frac{1}{2} \). We note some resemblance be-
tween the Hamiltonian in Eq. (2) and that given by
the BSM for \( k = 1 \). As pointed out by the authors, the BSM
is only reached when the mean photon number of the
field satisfies the condition \( k (\hat{a}^\dagger \hat{a}) >> 1 \), leading the term
\( \sqrt{1 + k \hat{a}^\dagger \hat{a}} \) to an approximate form of BSM \( \sqrt{\hat{n}}, \hat{n} = \hat{a}^\dagger \hat{a} \).

A somewhat ‘similar’ model, also intensity-dependent,
was proposed in 2014 by Rodríguez-Lara [21], named here
as Rodrigues-Lara model (RLM), constituting a general-
ization of BSM since it substitutes the operator \( \hat{R} = \hat{a} \sqrt{\hat{n}} \)
of the BSM by the operator \( \hat{R} = \hat{a} \sqrt{\hat{n} + 2 k} \). The RLM
recovers the BSM in the limit \( k \to 0 \), but it includes the
counter-rotating terms, due to the form of the interaction
Hamiltonian,

\[ H_{int} = \lambda (\sqrt{\hat{n} + 2 k \hat{a}^\dagger \hat{a}} \sqrt{\hat{n} + 2 k}) \hat{\sigma}_z, \]

(4)

where the decomposition \( \hat{\sigma}_z = \hat{\sigma}_+ + \hat{\sigma}_- \) explains the
appearance of counter-rotating terms \( \hat{a}^\dagger \hat{\sigma}_+ \) and \( \hat{a} \hat{\sigma}_- \). As
well known, separately they do not conserve energy. Also,
due to the inclusion of the counter rotating terms, this
model puts a restriction on the average number of pho-
tons.

In this report we present a generalized Hamiltonian
that provides a continuous and exact interpolation be-
tween various Hamiltonian models, including the JCM,
BSM, SM, and RLM. The plan of the paper is as follows.
In Sec. II we briefly discuss this class of Hamiltonian,
showing its interpolating property. In Sec. III we obtain
the solution of the Schrödinger equation in this extended
scenario. In Sec. IV we give some applications, in the Sec.
V we calculate Mandel parameter. The Sec. VI
contains comments and the conclusion.

II. INTENSITY-DEPENDENT COUPLING
MODEL HAMILTONIAN

The Hamiltonian described by the JCM, widely re-
ferred to as the JCM in the rotating wave approximation,
is given in the form,

\[ \hat{H} = \omega \hat{a}^\dagger \hat{a} + \frac{1}{2} \omega_0 \hat{\sigma}_z + \lambda_0 (\hat{\sigma}_+ \hat{a} + \hat{a}^\dagger \hat{\sigma}_-), \]

(5)

where \( \omega \) stands for the field frequency, \( \omega_0 \) is the
atomic frequency, and \( \lambda \) stands for atom-field coupling. Now, our
mentioned class of interpolating Hamiltonians is obtained
substituting \( \hat{H} \) by \( \hat{\mathcal{H}} \), given by

\[ \hat{\mathcal{H}} = \omega \hat{a}^\dagger \hat{a} + \frac{1}{2} \omega_0 \hat{\sigma}_z + \lambda (\hat{\sigma}_+ \hat{R} + \hat{\sigma}_- \hat{R}^\dagger). \]

(6)

where \( \hat{R} = \hat{a} \sqrt{\xi \hat{n} + \delta} \) and \( \hat{R}^\dagger = \sqrt{\xi \hat{n} + \delta \hat{a}^\dagger} \), for \( \xi \geq 0 \) and
\( 0 < \delta \leq 1 \).

Here it is easily seen that the Hamiltonian in Eq. (6)
interpolates between the various interaction models of
Hamiltonians, as follows:

- the Jaynes-Cummings model (JCM) [1] for \( \xi = 0 \)
  and \( \delta = 1 \),
- the Buck-Sukumar model (BSM) [15] for \( \xi = 1 \) and
  \( \delta = 0 \),
- the Sivakumar model (SM) [23] for \( \xi = k \) and \( \delta = 1 \),
- the Rodrígues-Lara model (RLM) [21] for \( \xi = 1 \)
  and \( \delta = 2k \).

Some basic properties involving these atomic and field
operators are,

\[ [\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2 \hat{\sigma}_\pm, \quad [\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z, \]

(7)

\[ [\hat{a}, \hat{a}^\dagger] = 1, \quad [\hat{a}, \hat{n}] = \hat{a}, \quad [\hat{a}^\dagger, \hat{n}] = -\hat{a}^\dagger, \]

(8)

\[ [\hat{R}, \hat{n}] = \hat{R}, \quad [\hat{R}^\dagger, \hat{n}] = -\hat{R}^\dagger, \quad [\hat{R}, \hat{R}^\dagger] = 2 \hat{R}_0 = \delta + \xi + 2k \hat{n}, \]

(9)

with \( \hat{R}_0 = \frac{\delta + \xi}{2} + \xi \hat{n} \); thus we have a closed algebra in this scenario,

\[ [\hat{R}, \hat{R}^\dagger] = 2 \hat{R}_0, \quad [\hat{R}_0, \hat{R}^\dagger] = \xi \hat{R}^\dagger, \quad [\hat{R}_0, \hat{R}] = -\xi \hat{R}. \]

(10)

The Eq. (6) can be rewritten in the form,

\[ \hat{\mathcal{H}} = \hat{\mathcal{H}}_A + \hat{\mathcal{H}}_I, \]

where,

\[ \hat{\mathcal{H}}_A = \omega (\hat{a}^\dagger \hat{a} + \frac{1}{2} \hat{\sigma}_z), \]

(11)

\[ \hat{\mathcal{H}}_I = \frac{1}{2} \Delta \omega \hat{\sigma}_z + \lambda (\hat{\sigma}_+ \hat{R} + \hat{\sigma}_- \hat{R}^\dagger), \]

(12)
with $\Delta \omega = \omega_0 - \omega$. Next we can use the Eqs. (7) and (9) to show that $\hat{H}_A$ and $\hat{H}_I$ are constant of motion, namely,

$$[\hat{H}, \hat{H}_A] = [\hat{H}, \hat{H}_I] = [\hat{H}_A, \hat{H}_I] = 0. \quad (13)$$

All essential dynamic properties contained in a state of the atom-field system described by any of the previous interpolating Hamiltonians, can also be described by the interpolating Hamiltonian proposed here, $\hat{H}_I$, considering that $\hat{H}_A$ contributes only for general phase factors, usually not relevant.

**III. FIELD FLUCTUATIONS**

Let us consider a simple example assuming the system in resonance, $\Delta \omega = 0$

$$\hat{H}_I = \lambda (\hat{\sigma}_+ \hat{R} \hat{\sigma}_- \hat{R}^\dagger). \quad (14)$$

Now, to analyze the time evolution of the coupled atom-field system we solve the time dependent Schrödinger equation using the Hamiltonian in Eq. (14),

$$i \frac{d|\Psi(t)\rangle}{dt} = \hat{H}_I |\Psi(t)\rangle. \quad (15)$$

We can write the formal solution of Eq. (15) as,

$$|\Psi(t)\rangle = \hat{U}(t) |\Psi(0)\rangle, \quad (16)$$

where $\hat{U}(t) = \exp(-i\hat{H}_I t)$, is the (unitary) evolution operator. Next, using the expression

$$e^{-\beta \hat{n}} \equiv \sum_{n=0}^{\infty} \frac{(-\beta)^n}{n!} \hat{n}^n, \quad (17)$$

and decomposing the above sum in their even and odd terms, plus the use of the two following relations

$$\langle \sigma^\dagger \sigma \rangle = \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n}}{(2n)!} \hat{n}^{2n} = \frac{(\hat{R} \hat{R}^\dagger)^2}{4}, \quad (18)$$

**IV. PARADOXICAL EVOLUTION OF AVERAGE NUMBER OF PHOTONS**

The paradox concerned with the time evolution of the average number of photons, discussed by Luis [36], used the JCM. Here we treat this paradox for the various interpolating Hamiltonians mentioned above. This is obtained directly from our Hamiltonian by an appropriate choice of the pair $\xi$ and $\delta$.

The mean number of photons of the field is calculated as,

$$\langle \hat{n} \rangle = Tr(\hat{n} \hat{\rho}) = \sum_{n=0}^{\infty} \langle n | \hat{n} \hat{\rho} | n \rangle, \quad (22)$$

where $\hat{\rho} = |\psi\rangle \langle \psi|$ is the density operator.

In this section we study the dynamic behavior of the average number of photons, $\langle \hat{n}(t) \rangle = \langle \psi(t) | \hat{n} | \psi(t) \rangle$, $\langle \hat{n}(t) \rangle_g = \langle \psi_g | \hat{n} | \psi_g \rangle$ and $\langle \hat{n}(t) \rangle_e = \langle \psi_e | \hat{n} | \psi_e \rangle$, where

$$|\psi_g\rangle = \cos(\lambda t \sqrt{\hat{A}}) |\psi\rangle |g\rangle, \quad (23)$$

$$|\psi_e\rangle = -i \hat{R} \sin(\lambda t \sqrt{\hat{A}}) |\psi\rangle |e\rangle. \quad (24)$$

For small times the following average values are obtained,

$$\langle \hat{n}(t) \rangle_g = \langle \hat{n} \rangle - 2 \lambda^2 t^2 [\xi (\langle \hat{n} \hat{n} \rangle - \langle \hat{n} \rangle \langle \hat{n} \rangle) + \delta \langle \hat{n} \rangle], \quad (25)$$

and

$$\langle \hat{n}(t) \rangle_e = \langle \hat{n} \rangle - 2 \lambda^2 t^2 [\xi (\langle \hat{n} \hat{n} \rangle - \langle \hat{n} \rangle \langle \hat{n} \rangle) + \delta \langle \hat{n} \rangle]. \quad (26)$$
\[
\langle \hat{n}(t) \rangle_c = \frac{\langle \hat{n} \hat{A} \rangle}{\langle \hat{A} \rangle} - 1 - \frac{\lambda^2 t^2}{3\langle \hat{A} \rangle^2} \left[ \langle \hat{n} \hat{A}^2 \rangle - \langle \hat{n} \hat{A} \rangle \langle \hat{A} \rangle \right], \tag{26}
\]

\[
\langle \hat{n}(t) \rangle = \langle \hat{n} \rangle \left( 1 - \lambda^2 t^2 \delta \right) - \lambda^2 t^2 \xi \langle \hat{n}^2 \rangle, \tag{27}
\]

where \( \langle \hat{A} \rangle = \xi \langle \hat{n}^2 \rangle + \delta \langle \hat{n} \rangle, \langle \hat{n} \hat{A} \rangle = \xi \langle \hat{n}^3 \rangle + \delta \langle \hat{n}^2 \rangle, \langle \hat{n} \hat{A}^2 \rangle = \xi^2 \langle \hat{n}^5 \rangle + 2 \xi \delta \langle \hat{n}^4 \rangle + \delta^2 \langle \hat{n}^3 \rangle, \langle \hat{A}^2 \rangle = \xi^2 \langle \hat{n}^4 \rangle + 2 \xi \delta \langle \hat{n}^3 \rangle + \delta^2 \langle \hat{n}^2 \rangle. \]

Regardless of the types of interpolations, i.e., JCM \( \leftrightarrow \) BSM \( \leftrightarrow \) SM \( \leftrightarrow \) RLM, and eventually others obtained by varying the pair \( \xi \) and \( \delta \), the essential features of the paradox discussed in Ref. [36] remain for all these interpolation models. Now, for small times, the following relation is valid, irrespective of the interpolating model.

\[
\langle \hat{n}(t) \rangle_c > \langle \hat{n}(t) \rangle > \langle \hat{n}(t) \rangle_g. \tag{28}
\]

In the plots of Fig. (1) we have assumed the initial field in a coherent state, assuming the average number of photons \( \langle \hat{n} \rangle = 3 \). Here we have used mathematical expressions more general than those in Eqs. [25, 26 and 27], hence the following plots are not restricted to small times. We observe in Fig. (1 (a)) the occurrence of the mentioned paradox, which starts immediately and remains up to \( \lambda t \approx 0.7 \) for the JCM; in Fig. (1 (b)) \( \lambda t \approx 0.3 \) for the BSM; in Fig. (1 (c)) \( \lambda t \approx 0.27 \) for the SM; and in Fig. (1 (d)) \( \lambda t \approx 0.25 \) for the RLM.

Hence, these results show that the paradox raised by A. Luis [36] using the JCM happens no matter what kind of Hamiltonian model used within the class considered here.

V. ESTATÍSTICA SUB-POISSONIANA

A quantized photon field with sub-Poissonian statistics is characterized when the variance is smaller than the average number of photons, namely: \( \langle \Delta \hat{n}^2 \rangle = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 \); the opposite characterizes a super-Poissonian photon field and if \( \langle \Delta \hat{n}^2 \rangle = \langle \hat{n} \rangle \) the photon field exhibits Poissonian statistics, characterizing all coherent states. The Mandel parameter tells us what kind of statistics the field displays \( [37] \); it is given by the relation,

\[
Q = \frac{\langle \Delta \hat{n}^2 \rangle}{\langle \hat{n} \rangle} - 1. \tag{29}
\]

So, when \( Q > 0 \) the field is super-Poissonian; when \( Q < 0 \) it is sub-Poissonian; and Poissonian for \( Q = 0 \).

In Fig. (2), we represent our Hamiltonian in Eq. (12) interpolating between the four Hamiltonians: JCM, BSM, SM and RLM.

Fig. (3) exhibits various plots of the Mandel parameters in these different models of Hamiltonian. The various plots show that, by conveniently adjusting the pair of parameters \( \xi \) and \( \delta \) in the present model Hamiltonian we can interpolate continuously from the JCM to the BSM, the SM, and the RLM. In these interpolations we have observed in which way the Mandel parameter modifies during the time evolutions, as shown in Fig. (6), plots (a), (b), and (c); also, this interpolation occurs in a softly way, from the JCM to BSM. The same happens for the interpolation from the JCM to the SM, shown in Fig. (3), plots (d), (e), and (f); and also from the JCM to the RLM, Fig. (3), plots (g), (h), and (i).

We can note in Fig. (4) that, when we compare the case where the system state has a small average excitation \( \langle \hat{n} \rangle \) with those having larger values of \( \langle \hat{n} \rangle \), the Mandel parameters for different Hamiltonians differ.

![FIG. 1. Evolution of \( \langle \hat{n}(t) \rangle_c \) (solid curve) \( \langle \hat{n}(t) \rangle \) (dashed curve) and \( \langle \hat{n}(t) \rangle_g \) (dotted curve), for an initial coherent state with \( \langle \hat{n} \rangle = 3 \); a) JCM, b) BSM, c) SM and d) RLM.](image1)

![FIG. 2. Evolution of \( Q(\lambda t) \), for an initial coherent state with \( \langle \hat{n} \rangle = 3 \); a) JCM, b) BSM, c) SM and d) RLM.](image2)
sensitively from each other for small values of \(\langle \hat{n} \rangle\), the region where the quantum nature of the system state is more evident. Contrarily, for larger values \(\langle \hat{n} \rangle\) the corresponding plots are very similar. In these examples we are analyzing the Mandel parameter close to BSM, Fig.(4 a) and d)), with other close to SM, Fig.(4 b), and e), and another close to RLM, Fig.(4 c) and f). This shows a great sensitivity of the system to the parameters \(\xi\) and \(\delta\) in the quantum regime of small numbers, as usually expected. In addition, for small values of \(\langle \hat{n} \rangle\) the field state exhibits a greater sub-Poissonian effect.

VI. CONCLUSION

We have proposed a (two parameters) interpolating Hamiltonian. It allows one to extend from (a) the JCM, (b) the BSM, (c) the SM, and (d) the RLM. This new Hamiltonian employs the basic operators \(R = \sqrt{\xi} \hat{n} + \delta\), \(R^\dagger = \sqrt{\xi} \hat{n} + \delta \hat{a}^\dagger\), and \(R_0 = \frac{\hat{a} + \hat{a}^\dagger}{2} + \frac{\xi}{2} \hat{n}\) which form a closed algebra. As mentioned before, it contains all essential dynamic properties contained in a state of the atom-field system described by the previous interpolating Hamiltonians. To give an example we have verified that, essentially, the results found in the paradox discussed by A. Luis [36] in the JCM remains in the scenario of this extended Hamiltonian (see Fig. [1]), no matter the chosen extension, say: from (a) to (b), from (a) to (c), and from (a) to (d). We have also calculated the Mandel parameter to obtain the evolution of the statistical properties of the system state and their time evolution when we pass from our interpolating model to another after appropriate choices of the pair \(\xi, \delta\). In these time evolutions we have highlighted the influence of the average excitation \(\langle \hat{n} \rangle\), when large or small, upon the statistical properties of the system. From what we have learned in quantum optics, concerning the degradation caused by decoherence effects affecting quantum states [35], for practical purposes this result would lead us to give priority to states with smaller excitations, the quantum region of small numbers, where some types of interpolating Hamiltonians have problems [39].

VII. ACKNOWLEDGEMENTS

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