MHD bioconvective flow of a thermally radiative nanoliquid in a stratified medium considering gyrotactic microorganisms

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Abstract.

The impact of gyrotactic microorganisms of a stratified flow of a thermally radiative NL with heat absorption is highlighted. In addition, magneto NL with an inclined magnetic field is included. Suitable transformations are adopted to convert the governing PDEs into nonlinear ODEs. Homotopy analysis method (HAM) is employed to solve these ODEs analytically. The impact of sundry parameters on VP, TP, NPVFP, MMDP, SFC, LNN and LDMM are graphically explained. We compare our results to available results in literature survey.

Keywords: Nanoliquid, Heat generation/absorption, Gyrotacticro microorganisms, Radiation, Stratification.

1. Introduction

Most of the engineering and industrial processes, the HT phenomenon is essential. The ordinary fluids, like, ethylene, oil, water, glycol, toluene are poor HT properties, since they have poor thermal conductivity. Many scientists tried in several ways to raise the thermal conductivity. One of the simplest method is to suspend nano-sized particles, such as gold, titanium, aluminum, copper, iron or their oxides in the ordinary liquids to enhance its thermal properties. These liquids are used in microchips, fuel cells, microelectronics, solid state lightening, bio-medicine, etc. The NL flow over a stretching tube was analyzed by Ahmed et al.[1]. Kasmani et al.[2] found the analytical and numerical solutions of viscous NL flow past a moving wedge. Chemically reactive NL flow over a wedge with suction and heat absorption was analyzed by Kasmani et al.[3]. They found that the HT coefficient enhances with raising the values of chemical reaction parameter. Some useful studies in this directions are ([4]-[6]). Bioconvection is the microscopic convection of liquid which is created by density gradient when swimming of motile microorganisms. It is used in bio-fuel, promising renewable power source, bio-diesel and hydrogen gas. The stability of bioconvection in a porous medium was examined by Kuznetsov and Avramenko[7]. Nguyen-Quang et al.[8] analyzed the stability of gravitactic micro-organisms in a porous medium. The impact of bioconvective NL with gyrotactic microorganisms was
explored by Mutuku and Makinde[9]. Akbar and Khan[10] studied the MHD NL flow with gyrotactic microorganisms.

Stratification acts a vital role in engineering and industrial mechanisms and it arises in temperature gradient, variations of concentration or combination of different liquids with different densities. For example, flows occur in rivers, ground water reservoirs, lakes, seas, etc. Free convective flow of a thermally stratified non-isothermal plate was analyzed by Yang et al.[11]. Cheng[12] investigated the double stratification and HMT analysis of power-law fluid over a porous medium. They proved that the thermal stratification parameter enhances the HT gradient. Numerical solution of chemically reactive Williamson fluid with thermal and solutal stratification was derived by Rehman et al.[13]. They proved that the MT rate suppresses with enhancing the solutal stratification. Khan et al.[14] studied the flow of a Williamson NL over a non-linear SS. They considered the fluid viscosity is depends on temperature and thermal diffusion. They have seen that the LNN is an enhancing function of TSP. The problem of MHD second grade NL flow with stratification was studied by Khan et al.[15].

The above literature analysis shows that the impact of radiative NL flow in a stratified medium with the presence of S/I and TS is not analyzed. Hence our main aim is to fill this gap. Characteristics of heat A/G and magneto-hydro-dynamics along with JH and VD are examined. Gyrotactic microorganisms is also retained. The formulating equations are analytically solved by HAM, see ([16]-[19]).

2. Mathematical Formulation

Let us take the MHD mixed convective flow of a radiative NL past a SS. The horizontal liquid layer was induced by inclined magnetic field and this field is omitted because of less value of magnetic RN. The HMT and MMT rates are investigated with the effects of JH, VD and stratification. The microorganisms are imposed into the NL to stabilize the nanoparticles due to bioconvection. The velocity and gravitactic microorganisms direction are not affected by nanoparticles. The motion of the microorganisms can be divided into random and directional components. Under the above assumptions, the flow problem is given by, see Alsaedi et al.[20]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho_f} \frac{\partial^2 u}{\partial x^2} - \frac{\sigma}{\rho_f} B_0^2 \sin^2 \alpha \frac{\partial u}{\partial y} + v \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho_f} \left[(1 - C_\infty) \rho_f \beta g (T - T_\infty) - (\rho_p - \rho_f) g (C - C_\infty) - (N - N_\infty) g \gamma (\rho_m - \rho_f) \right], \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho c_p} \frac{16 \sigma^* T_\infty}{3 k^* T_{yy}} + \tau \left[ \frac{D_B}{T_\infty} \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right] \tag{3}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}, \tag{4}
\]

\[
u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} + \frac{b W_c}{(C_w - C_0)} \left[ \frac{\partial}{\partial y} \left( N \frac{\partial C}{\partial y} \right) \right] = D_m \frac{\partial^2 N}{\partial y^2} \tag{5}
\]

where \( v \) and \( u \) are the velocity components in \( y \) and \( x \) directions, \( \mu \) is the viscosity of the suspensions of NL and microorganisms, \( \rho_f \) is the density of NL, \( \sigma \) is the electrical conductivity, \( B_0 \) is the magnetic field intensity, \( \alpha \) is the inclination angle of magnetic field, \( C_\infty \) is the ambient concentration of the nanoparticles, \( \beta \) is the volume expansion coefficient, \( g \) is the gravity, \( T_\infty \)
is the ambient temperature of the nanoparticles, \( C \) is the concentration of the nanoparticles, \( N \) is the concentration of microorganisms, \( N_{\infty} \) is the ambient concentration of microorganisms, \( \rho_m \) is the density of the microorganisms particles, \( \sigma^* \) - Stefan Boltzmann constant, \( k^* \) - mean absorption coefficient, \( \tau \) is the ratio of heat capacity of nanoparticles divided by heat capacity of NL, \( D_B \) is the Brownian motion diffusion coefficient, \( D_f \) is the thermophoretic diffusion coefficient, \( c_p \) is the specific heat, \( Q \) is the heat A/G coefficient, \( b \) is the chemotaxis constant, \( W_c \) is the maximum cell swimming speed, \( D_m \) is the microorganisms diffusion coefficient.

The boundary conditions can be expressed as,

\[
\begin{align*}
  u &= U_w = ax, \ v = V_w, \ T = T_w = T_0 + b_1 x, \\
  C &= C_w = C_0 + c_1 x, \ N = N_w = C_0 + d_1 x \ \text{at} \ y = 0, \\
  u &\to 0 \quad \frac{\partial u}{\partial y} \to 0, \ T \to T_\infty = T_0 + b_2 x, \\
  C &\to C_\infty = C_0 + c_2 x, \ N \to N_\infty = N_0 + d_2 x, \ \text{as} \ y \to \infty,
\end{align*}
\]

(6)

where \( U_w \) is the stretching velocity \( a > 0 \) is the stretching ratio and \( b_1, b_2, c_1, c_2, d_1, d_2 \) are positive constants.

Now, the following dimensionless variables are introduced:

\[
\begin{align*}
  \eta &= \sqrt{\frac{a}{\nu}} y, \ u = ax f'(\eta), \ v = \pm\sqrt{a} v f(\eta), \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} , \\
  \phi(\eta) &= \frac{C - C_\infty}{C_w - C_\infty}, \ \zeta(\eta) = \frac{N - N_\infty}{N_w - N_\infty},
\end{align*}
\]

(7)

Substituting Equation (7) in Equations (2)-(5) we have

\[
\begin{align*}
  f''' + f f'' - f'^2 - Ha \sin^2 \alpha f' + Gr \left( \theta - N r \phi - R b \zeta \right) &= 0, \\
  f(0) &= f_w, \ f'(0) = 1, \ \theta(0) = 1 - S_1, \ \phi(0) = 1 - S_2, \ \zeta(0) = 1 - S_3, \\
  f'(\infty) &= 0, \ \theta(\infty) = 0, \ \phi(\infty) = 0, \ \zeta(\infty) = 0
\end{align*}
\]

(8)

the corresponding boundary conditions are

\[
\begin{align*}
  H_a &= \sqrt{\frac{\beta \gamma (1 - C_w) (T_w - T_0)}{\rho_m k_0}} \text{ is the HN}, \ \lambda = \frac{\beta \gamma (1 - C_w) (T_w - T_0)}{\rho_m k_0} \text{ is the the MCP}, \ N_r = \frac{(\rho_w - \rho_f) (C_w - C_0)}{\beta \rho_f (T_w - T_0)} \text{ is the RP}, \\
  R_B &= \frac{\gamma (N_w - N_0) (\rho_m - \rho_f)}{\beta \rho_f (1 - C_w) (T_w - T_0)} \text{ is the BRN}, \ \rho_r = \frac{\mu c_p}{k} \text{ is the PN}, \ R = \frac{4 a^3 T_\infty}{k \nu} \text{ is the RP}, \\
  N_t &= \frac{\tau D_B (T_w - T_0)}{\alpha T_\infty} \text{ is the TMP}, \ N_b = \frac{\tau D_B (C_w - C_0)}{\alpha} \text{ is the BMP}, \ E_c = \frac{U_w^2}{\rho f (T_w - T_0)} \text{ is the EN}, \\
  H_g &= \frac{Q}{a c_p} \text{ is heat generation (}> 0) \text{ and absorption (< 0) parameter}, \ L_e = \frac{\nu}{\mathbf{D}_m} \text{ is the LN}, \\
  L_b &= \frac{\nu}{\mathbf{D}_m} \text{ is the BLN}, \ P_c = \frac{\mathbf{D}_w}{\mathbf{D}_m} \text{ is the BPN}, \ \Omega = \frac{N_w - N_\infty}{N_w - N_{\infty}} \text{ is the MCDP}, \ f_w = \frac{- V}{\sqrt{\nu}} \text{ is the S/I}
\end{align*}
\]

The dimension form of the SFC, LNN, LSN and LDMM are defined as

\[
\begin{align*}
  C_f &= \frac{T_w}{\mathbf{U}_w^2/2}, \ Nu = \frac{x q_w}{k (T_w - T_0)}, \ Sh = \frac{x r_w}{D_B (C_w - C_0)}; \ N_n = \frac{x s_w}{D_m (N_w - N_0)}
\end{align*}
\]
Figure1. The h curves of $f''(0)$, $\theta'(0)$, $\phi'(0)$ and $\zeta'(0)$

where

$$\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} \text{ is the wall shear stress, } q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} \text{ is the surface heat flux,}$$

$$r_w = -D_B \left( \frac{\partial C}{\partial y} \right)_{y=0} \text{ is the nanoparticle mass flux from the surface and}$$

$$s_w = -D_m \left( \frac{\partial N}{\partial y} \right)_{y=0} \text{ is the motile microorganisms flux.}$$

Then the reduced SFC, LNN, LSN and LDMM are given by

$$\frac{1}{2} C_f Re \frac{i}{2} = f''(0); \quad Nu/Re \frac{i}{3} = -\left( 1 + \frac{4}{3} R \right) \theta'(0); \quad Sh/Re \frac{i}{2} = -\phi'(0); \quad Nn/Re \frac{i}{3} = -\zeta'(0).$$

3. HAM Solution

The resultant equations (8) - (11) with boundary conditions (12) are analytically solved using HAM. Let $f_0(\eta) = 1 - e^{-\eta}$, $\theta_0(\eta) = (1 - S_1)e^{-\eta}$, $\phi_0(\eta) = (1 - S_2)e^{-\eta}$ and $\zeta_0(\eta) = (1 - S_3)e^{-\eta}$ are the initial approximations. The linear operator are $L_f = \frac{d^2f}{d\eta^2} - \frac{df}{d\eta}$, $L_\theta = \frac{d^2\theta}{d\eta^2} - \theta$, $L_\phi = \frac{d^2\phi}{d\eta^2} - \phi$ and $L_\zeta = \frac{d^2\zeta}{d\eta^2} - \zeta$ with $L_f \left[ e_1 + e_2 e^\eta + e_3 e^{-\eta} \right] = 0$, $L_\theta \left[ e_4 e^\eta + e_5 e^{-\eta} \right] = 0$, $L_\phi \left[ e_6 e^\eta + e_7 e^{-\eta} \right] = 0$ and $L_\zeta \left[ e_8 e^\eta + e_9 e^{-\eta} \right] = 0$ where $e_j (j = 1 - 9)$ are constants. After applying the $m$th order HAM equations, we get the followings

$$f_m(\eta) = f^*_m(\eta) + e_1 + e_2 e^\eta + e_3 e^{-\eta} \quad \theta_m(\eta) = \theta^*_m(\eta) + e_4 e^\eta + e_5 e^{-\eta} \quad \phi_m(\eta) = \phi^*_m(\eta) + e_6 e^\eta + e_7 e^{-\eta} \quad \zeta_m(\eta) = \zeta^*_m(\eta) + e_8 e^\eta + e_9 e^{-\eta}$$

here the particular solutions are $f^*_m(\eta), \theta^*_m(\eta)$ and $\zeta^*_m(\eta)$.

The HAM solution contains the auxiliary parameters $h_f, h_\theta, h_\phi$ and $h_\zeta$ and these parameters adjust the convergence of the HAM solutions. The $h_f, h_\theta, h_\phi$ and $h_\zeta$ curves are shown in Figure 1. The value of the auxiliary parameters in the whole region of $\eta$ is $h_f = h_\theta = h_\phi = h_\zeta = -1.0$ for better solution. Table 1 represents the different order of $-f''(0)$, $-\phi'(0)$, $-\theta'(0)$ and $-\zeta'(0)$. It is clear that 15th order is sufficient for both velocity, temperature, nanoparticle volume fraction and motile microorganisms density profiles. Table 2 provides the comparison of $-f''(0)$ with different combination of $Ha, \alpha, \lambda, Nr, Rb, S_1, S_2$ and $S_2$ between our results and published results and seen that the our results are in good agreement.
Table 2. Comparative outcomes of $\frac{1}{2}C_fRe^{\frac{1}{2}}$ for different values of $Ha, \alpha, \lambda, Nr, Rb, S_1, S_2$ and $S_2$ with Alsaedi et al. [20].

| $Ha$ | $S_2$ | $S_1$ | $Rb$ | $Nr$ | $\lambda$ | $\alpha$ | $\frac{1}{2}C_fRe^{\frac{1}{2}}$ Present Study | $\frac{1}{2}C_fRe^{\frac{1}{2}}$ Alsaedi et al. [20] |
|------|------|------|------|------|------|------|--------------------------|--------------------------|
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0069                  | −1.0069                  |
| 0.3  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0766                  | −1.0766                  |
| 0.5  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.1424                  | −1.1424                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0069                  | −1.0069                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0766                  | −1.0766                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.1100                  | −1.1100                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0472                  | −1.0472                  |
| 0.3  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0182                  | −1.0182                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −0.9898                  | −0.9898                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0766                  | −1.0766                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0873                  | −1.0873                  |
| 0.5  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0981                  | −1.0981                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0766                  | −1.0766                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0853                  | −1.0853                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0941                  | −1.0941                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0830                  | −1.0830                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0958                  | −1.0958                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.1084                  | −1.1084                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0762                  | −1.0762                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0749                  | −1.0749                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0737                  | −1.0737                  |
| 0.1  | 0.1  | 0.2  | 0.1  | 0.1  | 0.1  | 0.1  | −1.0722                  | −1.0722                  |
Table 3. SKC, LNN, LSN and LDNMM for different values of $fw$, $\alpha$, $M$, $\lambda$, $R$, $Hg$, $S_1$, $S_2$ and $S_2$.

| $fw$ | $\alpha$ | $Ha$ | $\lambda$ | $R$  | $Hg$ | $S_1$ | $S_2$ | $\frac{1}{2}C_fRe^{\frac{1}{2}}$ | $Nu/Re^{\frac{1}{2}}$ | $Sh/Re^{\frac{1}{2}}$ | $Nu/Re^{\frac{1}{2}}$ |
|------|--------|------|--------|-----|-----|------|------|-----------------|-----------------|-----------------|-----------------|
| 0.3  | 0.1    | 0.1  | −0.5   | 0.1 | 0.1 | 0.1  |       | −0.857929       | 0.916787        | 0.469022        | 1.15425         |
| 0.3  | 0.0    | 0.1  | −0.5   | 0.1 | 0.1 | 0.1  |       | −0.939586       | 0.99116          | 0.485898        | 1.28107         |
| 0    | −1.07887 | 1.11515 | 0.516871 | 1.49758 |
| 0.3  |       |       | −0.5   | 0.1 | 0.1 | 0.1  |       | −1.23878        | 1.25453         | 0.557172        | 1.74772         |
| 0.5  |       |       | −0.5   | 0.1 | 0.1 | 0.1  |       | −1.35666        | 1.35066         | 0.590067        | 1.93312         |
| 0.3  | 0.3    | 0.1  | −0.5   | 0.1 | 0.1 | 0.1  |       | −1.16932        | 1.28507         | 0.561696        | 1.76199         |
|      | −1.18293 | 1.27906 | 0.560781 | 1.75915 |
|      | −1.20459 | 1.26952 | 0.559352 | 1.75468 |
|      | −1.23878 | 1.25453 | 0.557172 | 1.74772 |
| 0.3  | 0.1    | 0.1  | −0.5   | 0.1 | 0.1 | 0.1  |       | −1.3285        | 1.30125         | 0.56421         | 1.76969         |
|      | −1.27198 | 1.24005 | 0.555147 | 1.74109 |
|      | −1.39633 | 1.18652 | 0.548433 | 1.71738 |
|      | −1.50988 | 1.13866 | 0.543617 | 1.69748 |
|      | −1.63463 | 1.09905 | 0.548691 | 1.68846 |
|      | −2.53752 | 1.11962 | 0.84959 | 1.90850 |
| 0.3  | 0.3    | 0.1  | −0.5   | 0.1 | 0.1 | 0.1  |       | −1.26692        | 1.24687         | 0.552341        | 1.73869         |
|      | −1.83464 | 1.26902 | 0.566084 | 1.76452 |
|      | −1.12925 | 1.28255 | 0.574233 | 1.77983 |
|      | −1.04961 | 1.30157 | 0.585701 | 1.80043 |
|      | −0.997406 | 1.31415 | 0.595354 | 1.81313 |
| 0.3  | 0.1    | 0.1  | −0.5   | 0.1 | 0.1 | 0.1  |       | −1.24107        | 1.19227         | 0.486629        | 1.6704          |
|      | −1.23878 | 1.25453 | 0.557172 | 1.74772 |
|      | −1.23667 | 1.31217 | 0.61532 | 1.81313 |
|      | −1.23469 | 1.36596 | 0.664253 | 1.86476 |
|      | −1.23110 | 1.46421 | 0.74243 | 1.95008 |
| 0.3  | 0.3    | 0.1  | −0.5   | 0.1 | 0.1 | 0.1  |       | −1.24076        | 1.35080         | 0.485711        | 1.66891         |
|      | −1.23878 | 1.25453 | 0.557172 | 1.74772 |
|      | −1.23464 | 1.08693 | 0.679424 | 1.88206 |
|      | −1.22633 | 0.856104 | 0.842083 | 2.05935 |
|      | −1.20269 | 0.481097 | 1.07385 | 2.29765 |
| 0.3  | 0.1    | 0.1  | −0.5   | 0.1 | 0.1 | 0.1  |       | −1.233336       | 1.34108         | 0.51294         | 1.70384         |
|      | −1.24958 | 1.07993 | 0.64718 | 1.83748 |
|      | −1.26031 | 0.90333 | 0.739246 | 1.92991 |
|      | −1.27630 | 0.63467 | 0.881248 | 2.07364 |
|      | −1.28688 | 0.45303 | 0.978533 | 2.17290 |
| 0.3  | 0.1    | 0.1  | −0.5   | 0.1 | 0.1 | 0.1  |       | −1.23914        | 1.25152         | 0.62464         | 1.81195         |
|      | −1.23878 | 1.25453 | 0.55717 | 1.74772 |
|      | −1.23844 | 1.25754 | 0.48970 | 1.68397 |
|      | −1.23810 | 1.26054 | 0.42222 | 1.62071 |
|      | −1.23744 | 1.26656 | 0.28723 | 1.49568 |
| 0.3  | 0.1    | 0.1  | −0.5   | 0.1 | 0.1 | 0.1  |       | −1.23981        | 1.25421         | 0.55693         | 1.85413         |
|      | −1.23878 | 1.25453 | 0.55717 | 1.74772 |
|      | −1.23776 | 1.25485 | 0.55741 | 1.64134 |
|      | −1.23673 | 1.25517 | 0.55765 | 1.53497 |
|      | −1.23467 | 1.25582 | 0.55814 | 1.32229 |
Figure 2. The VP for various values of $\text{Ha}$ (a), $f_w$ (b) and $\lambda$ (c).

Abbreviations

| Abbreviation | Description |
|--------------|-------------|
| HAM          | homotopy analysis method |
| LNN          | local Nusselt number |
| MMDP         | motile microorganisms density profile |
| SFC          | skin friction coefficient |
| VP           | velocity profile |
| HN           | Hartman number |
| BRP          | buoyancy ratio parameter |
| PN           | Prandtl number |
| RP           | radiation parameter |
| BMP          | Brownian motion parameter |
| LN           | Lewis number |
| BPN          | bioconvection Peclet number |
| S/IP         | suction/injection parameter |
| TSP          | thermal stratification parameter |
| MDSP         | motile density stratification parameter |
| SS           | stretching surface |
| FT           | fluid temperature |
| PDE          | Partial differential equation |
| MT           | mass transfer |
| VD           | viscous dissipation |
| MMT          | Motile microorganism transfer |
| LSH          | Local Sherwood number |
| HAM          | homotopy analysis method |
| LDMM         | local density of motile microorganism |
| MHD          | magnetohydrodynamics |
| NPVF         | nanoparticle volume fraction |
| TP           | temperature profile |
| MCP          | mixed convection parameter |
| BRN          | bioconvection Rayleigh number |
| MDSP         | motile density stratification parameter |
| TMP          | thermophoresis motion parameter |
| EN           | Eckert number |
| BLN          | bioconvection Lewis number |
| MCDP         | microorganisms concentration |
| MSP          | mass stratification parameter |
| NL           | nanoliquid |
| FV           | fluid velocity |
| HT           | heat transfer |
| ODE          | ordinary differential equation |
| HMT          | heat and mass transfer |
| A/G          | Absorption/generation |
| JH           | Joule heating |
| RN           | Reynold’s number |
Figure 3. The TP for various values of $f_w$ (a), $R$ (b), $H_g$ (c), $S_1$ (d), $N_t$ (e) and $N_b$ (f).

Figure 4. The NPVFP for various values of $f_w$ (a) and $S_2$ (b).
4. Results and Discussion

In this section, we presented the graphical and numerical results for VP, TP, NPVF, MMDP, SFC, LNN, LDMM for different values of physical parameters. Table 3 presents the SFC, LNN and LDMM for different values of $fw, \alpha, Ha, \lambda, R, Hg, S_1, S_2$ & $S_3$. It is found that the surface shear stress diminishes with strengthening the values of $fw, \alpha, Ha$ & $S_1$ and growing with upgrading the values of $\lambda, R, Hg, S_2$ & $S_3$. The heat transfer gradient declines with rising the values of $\alpha, Ha, Hg$ & $S_1$ and it growing with escalating the values of $fw, \lambda, R, S_2$ & $S_3$. The local Sherwood number diminishes with raising the values of $fw, \alpha, S_2$ & $S_3$ and the opposite trend was obtained for increasing values of $\alpha, Ha$ & $S_2$. The local density of motile microorganism becomes smaller with developing the values of $\alpha, S_2$ & $S_3$ and it growing for increasing the values of $fw, Ha, \lambda, R, Hg$ & $S_1$.

Figures 2(a-c) portray the VP for different values of $fw, Ha$ and $\lambda$. It is seen from these figures that fluid velocity suppresses with enhancing $Ha$ and $fw$ values. Physically, the magnetic field generates a drag force that opposes the motion of the fluid. This causes to decrease the fluid temperature. The reverse trend is obtained for $\lambda$ values. The impact of $fw, R, Hg, S_1, Nt$ and $Nb$ for temperature profile was shown in Figures 3(a-e). We found that the FT increases with enhancing the values of $R, Hg, Nt$ and $Nb$ and it reduces with raising the values of $fw$ and $S_1$. In general, thermal radiation leads to enrich the energy transport to the fluid and causes to increase the FT. The fluid thermal conductivity increases in the presence of nanoparticles. Physically, thermophoresis parameter increase the fluid thermal conductivity and this causes to increase the FT. Figures 4(a-b) illustrated the effect of $fw$ and $S_2$ on nanoparticle volume fraction profile and found that the nanoparticle volume fraction decreases with increasing $fw$ and $S_2$ values. The variations of $fw$ and $S_3$ on MMDP is displayed in Figures 5(a-b). It is seen that the motile density suppresses with rising the values of $fw$ and $S_3$.

5. Conclusions

In this paper, we analyze the impact of gyrotactic microorganisms of a stratified flow of MHD nanofluid with heat absorption is highlighted. In this analyzes, we found that the fluid velocity suppresses for escalating the values of Hartmann number and suction/injection parameter. The fluid temperature improves when enhancing the values of heat generation/absorption parameter and diminishes for rising the values of thermal stratification parameter. The nanoparticle volume fraction is strengthen when the large values of mass stratification parameter is given. The motile microorganisms density suppresses with enhancing the motile density stratification parameter. The surface shear stress diminishes with strengthening the values S/I P, HT gradient declines with rising the values of $\alpha$, LSN diminishes with raising the values of $R$ and the LDMM becomes smaller with developing the values of $\alpha$. 
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