Matching the Electroweak Penguins $Q_7$ and $Q_8$

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We report on recent advances on the computation of the matrix elements of the electroweak penguins $Q_7$ and $Q_8$ which are relevant for the $\Delta I = 3/2$ contribution to $\epsilon'_K$ in the chiral limit. The matching of scale and scheme dependences between Wilson coefficients and these matrix elements is done analytically at NLO in $\alpha_S$.

Direct CP violation in $K \to \pi\pi$ is usually parameterized with

$$\epsilon'_K \simeq \frac{i}{\sqrt{2}} \frac{\text{Re}(a_{2})}{\text{Re}(a_0)} \left[ \frac{\text{Im}(a_0)}{\text{Re}(a_0)} + \frac{\text{Im}(a_2)}{\text{Re}(a_2)} \right] e^{i\delta_2 - \delta_0}, \quad (1)$$

where $iA_I \equiv a_I \exp(i\delta_I)$ are $K \to \pi\pi$ isospin invariant amplitudes and $\delta_I$ are final state interaction (FSI) phases. The ratio $\text{Re}(a_0)/\text{Re}(a_2) \simeq 22.2$ is an experimentally well known quantity.

In the limit $m_u = m_d$ and $\alpha^2_{QED} = 0$, and neglecting the tiny electroweak corrections to $\text{Re}(a_2)$, one gets

$$\text{Im}(a_2) \cong \frac{3}{5} \frac{F_0^2}{m_K - m_\pi^2} \frac{\text{Im}(e^2 G_E)}{G_{27}} \quad (2)$$

including FSI to all orders in CHPT and up to $O(p^4)$ non-FSI corrections. The coupling $G_{27}$ modulates the 27-plet operator describing $K \to \pi\pi$ at $O(p^2)$ in CHPT. Its value can be obtained from a fit of $K \to \pi\pi$ and $K \to \pi\pi\pi$ to $O(p^4)$ amplitudes.

The coupling $G_E$ appears in CHPT to $O(e^2 p^0)$.

$$\mathcal{L}_{\Delta S = 1} = C F_0^6 e^2 G_E \left( U^\dagger Q U \right)_{23} + O(p^2)(G_{27}, \cdots). \quad (3)$$

The constant $C$ was defined in [1], $F_0$ is the pion decay constant in the chiral limit, $Q$ is a $3 \times 3$ matrix collecting the three light quarks electric charges and $U = \exp(i\sqrt{2} \Phi / F_0)$ with $\Phi$ a $3 \times 3$ matrix collecting the octet pion, kaon, and eta pseudo-scalar boson fields.

In the Standard Model, there are just two operators contributing to $\text{Im}(e^2 G_E)$; namely, the so-called electroweak penguins, $Q_7$ and $Q_8$. In the chiral limit, these operators form a closed system under QCD corrections. Its anomalous dimensions mixing matrix is known to NLO in the NDR and HV schemes.

There has been recently a lot of work devoted to calculate $\text{Im}(e^2 G_E)$, both analytically and using lattice QCD. Here, we report on the work presented in [17] and to which we refer for explicit formulas and further references. For a review of the method we used and the matching procedure see [18]. Using this method, one can write exact results for the coupling $\text{Im}(e^2 G_E)$ in terms of integrals of full two-point functions in the chiral limit. They can be related to spectral functions via dispersion relations and resummation of the effect of all higher dimensional operators in the OPE of the relevant two-point functions.

The coupling $\text{Im}(e^2 G_E)$ can be written as

$$- \frac{3}{5} F_0^6 \text{Im}(e^2 G_E) = -6 \text{Im} C_7(\mu_R) \langle 0 | Q_7 | 0 \rangle (\mu_R) + \text{Im} C_8(\mu_R) \langle 0 | Q_8 | 0 \rangle (\mu_R). \quad (4)$$

In the chiral limit and in the NDR scheme (for the HV scheme expressions see [13]), the matrix
elements above are
\[
\langle 0|Q_7|0 \rangle^{\text{NDR}}_{\chi}(\mu_R) = \frac{3}{32\pi^2} \left( 1 + \frac{\alpha_S}{24 \frac{\pi}{\pi}} \right) A_{LR}(\mu_R) + \frac{1}{48} \frac{\alpha_S}{\pi} B_{SP}(\mu_R) \tag{5}
\]
\[
\langle 0|Q_8|0 \rangle^{\text{NDR}}_{\chi}(\mu_R) = \left( 1 + \frac{23}{12} \frac{\alpha_S}{\pi} \right) B_{SP}(\mu_R) + \frac{3}{32\pi^2} \frac{9}{2} \frac{\alpha_S}{\pi} A_{LR}(\mu_R) \tag{6}
\]
with
\[
A_{LR}(\mu_R) \equiv \int_0^{s_0} dt t^2 \ln \left( \frac{t}{\mu_R^2} \right) \frac{1}{\pi} \text{Im}\Pi_{LR}^T(t) \tag{7}
\]
and
\[
B_{SP}(\mu_R) \equiv 3\langle 0|\bar{q}q|0 \rangle \frac{2}{\pi \alpha_S} \text{Im}\Pi_{SP}^{(0-3)}(\mu_R) + \frac{1}{16\pi^2} \int_0^{\hat{s}_0} dt t \ln \left( \frac{t}{\mu_R^2} \right) \frac{1}{\pi} \text{Im}\Pi_{SS+PP}^{(0-3)}(\mu_R) \tag{8}
\]
\(s_0\) and \(\hat{s}_0\) are the onsets of local QCD duality. These results are exact in the chiral limit. The two-point function \(\Pi_{LR}^T(Q^2)\) is the transverse part of \(\langle 0|V_\nu - A_\nu\rangle^{(3)}(x)\langle V_\nu + A_\nu\rangle^{(3)}(0)|0\rangle\) in momentum space, and the two-point function \(\Pi_{SS+PP}^{(0-3)}(Q^2)\) is the singlet minus triplet combination of \(\langle 0|(S+iP)(S-iP)|0\rangle|0\rangle\) in momentum space, both in the chiral limit. See [17] for their explicit expressions. In this reference, we show how scheme and scale dependences of the Wilson coefficients match analytically the ones of the matrix elements. This was done there explicitly in the NDR and HV schemes at \(O(\alpha_S^2)\).

The first term in [8] comes from a disconnected diagram and is order \(N_c^0\). The second term comes from connected diagrams and is order \(N_c^2\). We also know that \(\Pi_{SS+PP}^{(0-3)}(Q^2)\) has an OPE which starts at \(1/Q^4\) modulated by \(O(\alpha_S^2)\) coefficients times \(\langle 0|Q_7^1|0\rangle\) and \(\langle 0|Q_8^1|0\rangle\). As a consequence, it fulfills exactly a first Weinberg sum rule-like [13] and very approximately a second one [17].

Apart of fulfilling the two Weinberg sum rules-like, notice that the kernel of the sum rule [8] we are interested in has zeroes at \(t=0\) and at \(t = \mu^2\) which we chose to be \(\mu^2 = s_0\). One expects therefore that keeping just the first pole in each channel is a good approximation for estimating the leading OPE behavior as happens in \(\pi^+ - \pi^0\) electromagnetic mass difference.

In [17] we discussed this sum rule using the known pion and the \(\eta_1\) poles and including the first pion prime as a narrow width. The \(\pi'\) contribution can be improved using a Breit-Wigner shape and the results do not change much due to the phenomenologically small coupling of the \(\pi'\). The scalar counterpart is more delicate and more work is needed. But as a first estimate, we used two hadronic models [20,21] that fulfill the QCD short-distance constraints and produce values for \(L_4\), \(L_6\) and \(L_8\) compatible with phenomenology. The scalar form factor in [20] is obtained from data and dispersion relations up to 1 GeV and Breit-Wigner shapes above. The result of using these models agreed with the results of naive narrow widths for the lowest scalar resonances. These were constructed to fulfill the short-distance QCD constraints and also produced reasonable values for \(L_0\) and \(L_8\). Here, we have also tried Breit-Wigner shapes for the scalar mesons instead of narrow widths and again find results in the same ball park. Now, we also have used the scalar form factors obtained in [22] where the lowest scalar triplet and singlet resonances are generated dynamically for energies up to 1 GeV and Breit-Wigner shapes above and we get similar results.

In all the estimates, we got negative corrections to the first term in [8] in the region between \(-10\%\) to \(-30\%\). Though the scalar-pseudoscalar sum rule [8] cannot obviously be used at a quantitative level at present, the results above indicate that is difficult to have corrections larger than \(\pm 30\%\) to the first term in [8].

The chiral limit OPE of \(\Pi_{LR}^T(Q^2)\) starts at \(1/Q^6\) modulated by known coefficients times \(\langle 0|Q_7^1|0\rangle\) and \(\langle 0|Q_8^1|0\rangle\). Using this, one arrives at [8][13][17]
\[
M_2 \equiv \int_0^{s_0} dt t^2 \frac{1}{\pi} \text{Im}\Pi_{LR}^T(t) \simeq \frac{4\pi}{3} \frac{\alpha_S(s_0)}{\pi} \left( 1 + \frac{25}{8} \frac{\alpha_S(s_0)}{\pi} \right) \langle 0|Q_8^1|0 \rangle^{\text{NDR}}(s_0). \tag{9}
\]
This is the relation that we use for the quantitative analysis of \(\langle 0|Q_8^1|0 \rangle\).

This sum rule and the one in [8] can be calcu-
lated using the excellent ALEPH \cite{23} and OPAL \cite{24} tau data. For the details of how we use the tau data we refer to \cite{17}. Only to mention here that we have generated around 100,000 tau data distributions according to their covariance matrix and assigned to each one a single value of $s_0$. This value is the highest one allowed by data where the second Weinberg sum rule (WSR) in (7) has a zero at one power less in the kernel. In addition, the sum differs from the one we are interested in just by importance with respect to the low energy region. We have chosen the second WSR since it differs from the one we are interested in just by one power less in the kernel. In addition, the sum rule in (7) has a zero at $t = \mu^2 = s_0$. We also used the highest value available by the data -which is always between 2 GeV$^2$ and 3 GeV$^2$- since one also expects QCD-Hadron duality to work better there.

Combining the ALEPH and OPAL results, we obtain

$$M_2 = -[1.9 \pm 1.0] \cdot 10^{-3} \text{ GeV}^6$$

(10)

and always $M_3 > 0$ in agreement with the recent analysis in \cite{12}.

If we use $\text{Im} \Pi_{SS+PP}^{(0-3)}(t) = 0$ in (8) and $\langle 0|\overline{q}q|0\rangle_{\chi_{MS}}(2 \text{ GeV}) = -(0.018 \pm 0.004) \text{ GeV}^3$ from \cite{25}, we get

$$M_2 = -[2.0 \pm 0.9] \cdot 10^{-3} \text{ GeV}^6.$$  

(11)

This result is very compatible with the FESR analysis result we got in (10).

For our final result in the NDR scheme, using $\alpha_S(2 \text{ GeV}) = 0.32$ (see more details in \cite{12}), we quote

$$\langle 0|Q_6|0\rangle_{\chi}^{\text{NDR}}(2 \text{ GeV}) =$$

$$1.20 \pm 0.60 \pm 0.15 \cdot 10^{-3} \text{ GeV}^6 =$$

$$1.2 \pm 0.7 \cdot 10^{-3} \text{ GeV}^6.$$  

(12)

Where the first error is purely experimentally and takes into account both ALEPH and OPAL results as well as a possible variation of the local duality onset $s_0$ and the second one is from the unknown $O(\alpha_S^3)$ terms in (4) assuming a geometrical series.

The sum rule in (7) is much better behaved and with smaller error bars due to to the zero at $t = \mu^2 = s_0$. Using again the same strategy we explained above for the analysis of sum rule (4), we get

$$A_{LR}(2 \text{ GeV}) = (4.35 \pm 0.50) \cdot 10^{-3} \text{ GeV}^6$$

(13)

combining ALEPH and OPAL results. See \cite{17} for further details.

Comparison with other recent determinations is made in Table 1. For the results in the cases $B_3^X(2 \text{ GeV}) = B_3^X(2 \text{ GeV}) = 1$ and $\text{Im} \Pi_{SS+PP}^{(0-3)}(t) = 0$ we used $\langle 0|\overline{q}q|0\rangle_{\chi_{MS}}(2 \text{ GeV}) = -(0.018 \pm 0.004) \text{ GeV}^3$ from \cite{25}, which is in agreement with the most recent sum rule determinations of this condensate and of light quark masses -see \cite{24} for instance- and the lattice light quark masses world average in \cite{27}.

Within the present accuracy of $(0|\overline{q}q|0)$, the disconnected contribution to (8) -third line in Table 1- is perfectly compatible with our full result -fourth line in Table - as well as the results from \cite{24,23,21,20,19} , so that we cannot conclude a large deviation from the large $N_c$ result within the present accuracy. Notice that we include in this result -third line in Table 1- $O(\alpha_S)$ corrections that are indeed leading order in $1/N_c$ (see \cite{17} and \cite{17}) which are usually disregarded in the factorization approaches, this makes the chiral limit $B_3^X(2 \text{ GeV})$ parameter larger than one by around 20% to 30%.

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Table 1
Comparison of NDR results for $\mu_R = 2$ GeV and $n_f = 3$ flavors in units of $10^{-3}$ GeV$^2$.

| Method | $-60 \langle 0|Q_7|0\rangle_{\chi}$ | $(0|Q_8|0)_{\chi}$ |
|--------|---------------------------------|-----------------|
| $B_s^+ (2 \text{ GeV}) = B_s^0 (2 \text{ GeV})$ | $3.2 \pm 1.3$ | $1.0 \pm 0.4$ |
| This Work, [17] Im$\Pi^{(0-)}_{SS+PP} = 0$ | $2.4 \pm 0.3$ | $1.2 \pm 0.5$ |
| This Work, [17] Data & Duality FESR | $2.4 \pm 0.3$ | $1.2 \pm 0.7$ |
| Cirigliano et al., [11] Data & Fitted FESR | $2.1 \pm 0.3$ | $1.5 \pm 0.3$ |
| Cirigliano et al., [12] Weighted Data & Fitted FESR | $2.2 \pm 0.3$ | $1.6 \pm 0.4$ |
| Cirigliano et al., [11] Weighted Data | $1.6 \pm 1.0$ | $2.1 \pm 0.6$ |
| Knecht et al., [10] $N_c \to \infty$, MHA | $1.1 \pm 0.3$ | $2.3 \pm 0.7$ |
| Narison, [9] Data & Tau-like FESR | $2.1 \pm 0.6$ | $1.4 \pm 0.4$ |
| Donini et al., [13] Lattice (Wilson) | $1.5 \pm 0.4$ | $0.7 \pm 0.2$ |
| CP-PACS Coll., [14] Lattice (Chiral) | $2.4 \pm 0.3$ (stat.) | $1.0 \pm 0.2$ (stat.) |
| RBC Coll., [15] Lattice (Chiral) | $2.7 \pm 0.3$ (stat.) | $1.1 \pm 0.2$ (stat.) |

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