Abstract

We discuss time-dependent perturbations (induced by matter fields) of a black-hole background in tree-level two-dimensional string theory. We analyse the linearized case and show the possibility of having black-hole solutions with time-dependent horizons. The latter exist only in the presence of time-dependent ‘tachyon’ matter fields, which constitute the only propagating degrees of freedom in two-dimensional string theory. For real tachyon field configurations it is not possible to obtain solutions with horizons shrinking to a point. On the other hand, such a possibility seems to be realized in the case of string black-hole models formulated on higher world-sheet genera. We connect this latter result with black hole evaporation/decay at a quantum level.
Recently much attention has been concentrated on black-hole background configurations of two-dimensional string theory after Witten’s observation [1] that such models can be represented as exactly solvable (finite) Wess-Zumino conformal field theories. It was also suggested [2, 3] that such theories will be consistent with quantum mechanics even during the process of black-hole evaporation [4]. The reason for this is the existence of a particular type of infinite-dimensional quantum hair, the so-called ‘W-hair’, carried by the black holes as a result of an enormous stringy gauge symmetry that mixes the various string levels [1]. Due to these symmetries, the two-dimensional phase space of the matter tachyon fields is preserved under time evolution, provided that one takes into account the discrete massive string states. As a result the modifications [7] to the usual quantum mechanical evolution equations for the density matrix of the matter system interacting with the black hole are viewed as artefacts of the truncation of the string spectrum to the massless modes only [8]. For instance, it has been shown in [9] that the exactly marginal operator that turns on static (massless) tachyon backgrounds in the coset black-hole model of [1], necessarily involves the entire spectrum of massive string modes. This makes the system of the light modes of the string in the presence of black-hole backgrounds ‘open’, thereby leading to apparent modifications of quantum mechanics for the massless string states [8] in a string field theory framework.

As argued in [10] the stringy black holes can ‘evaporate’, but unlike the local field theory case [4] their evaporation is non-thermal, resembling an ordinary decay of a massive string state. There are various decay channels and selection rules that characterize the decay process, which involve higher excited string (discrete) states, in addition to the massless tachyon fields. Their precise form is not yet fully known, although significant advances have been made in this direction [11, 12].

Motivated by these results, we would like to examine in some detail, in the present letter, time-dependent perturbations of static black-hole solutions of two-dimensional (target) space-time string theory. We should remark that to our knowledge time-dependent perturbations of local field theory black holes induced by scalar fields have been studied, so far, explicitly only in the context of five-dimensional Kaluza-Klein theories, where the scalar field is associated with the component of the metric pertaining to the extra-dimension [13]. In a similar context it has also been argued [14] that $O(d, d)$-transformations of static two-dimensional string black-hole backgrounds could absorb the singularity, at the cost of introducing an extra target-space dimension. Our approach here will be different in that we shall examine time-dependent perturbations of two-dimensional string black holes by avoiding the introduction of extra dimensions. We shall show that there are solutions, at least in the linearized approximation, describing time-varying horizons, but only in the

---

1In the case of two-dimensional target space the only propagating string degrees of freedom are the massless ‘tachyon’ fields. The higher-spin string states are non-propagating with definite energies and momenta [6, 8]; nevertheless, these (quasi-topological) modes play an important rôle in the physics [2, 3].
presence of time-dependent configurations for the tachyon fields. However it seems
that for real matter-field configurations it is not possible to obtain solutions with
horizons shrinking down to a point, a possibility that seems to be realized in the case
of complex matter fields. As we shall discuss, this latter result might be connected
with the decay of the black hole induced by higher genera [10]. As is well-known
in string theory, higher genus effects can be effectively represented as additional
renormalization counterterms of the tree-level $\sigma$-model couplings [15]. The effect of
higher genera is to add extra marginal operators which do not exist in the tree-level
$\sigma$-model background theory. Slightly relevant deformations lead to imaginary parts
in the effective target-space action, as a result of the circulation of these modes along
the string loops.

We start our discussion from the low-energy effective action with dilaton, tachyon
and graviton fields for a two-dimensional string theory. For simplicity we restrict
ourselves to one ($\sigma$-model) loop order, which from an effective field theory point of
view corresponds to a large-$k$ Wess-Zumino model, with $k$ the level parameter [1]. In
this truncated theory conformal invariance conditions should always be understood
as approximate solutions where corrections of the order of the Planck mass are
suppressed. The action takes the form

$$I_{\text{eff}} = \frac{1}{2g^2} \int d^2x \sqrt{G} e^\Phi \left\{ R + (\partial \Phi)^2 + (\partial T)^2 - 8T^2 + \Lambda \right\}$$  \hspace{1cm} (1)$$

where $G_{\mu\nu}$ is a (Euclidean signature) metric in a two-dimensional target space-time,
g is the gravitational coupling and $\Lambda$ is a cosmological constant arising from the
non-criticality of the dimension of the string space-time [16]. The usual ambiguities
of the tachyon potential in string theory [17] have been fixed here to quadratic
configurations for the tachyon field. The equations of motion obtained from (1) are,

$$D_\mu D_\nu \Phi = G_{\mu\nu} \{ D^2 \Phi + \frac{1}{2} (\partial \Phi)^2 - \frac{1}{2} (\partial T)^2 + 4T^2 - \frac{1}{2} \Lambda \} + \partial_\mu T \partial_\nu T$$

$$0 = -2D^2 \Phi - (\partial \Phi)^2 + R + \Lambda - 8T^2 + (\partial T)^2$$

$$0 = D^2 T + (\partial_\mu \Phi \partial^\mu T) + 8T.$$  \hspace{1cm} (2)$$

In the absence of matter, $T = 0$, the first of these equations imposes the solution [1]
$\Phi = \ln \cosh^2 Q r$ and the invariant line element acquires the form $ds^2 = dr^2 + tgh^2 Q r dt^2$,
where $Q^2 = \Lambda$. The existence of such a solution is guaranteed by the fact that when
$T = 0$ the vector $\partial_\mu \Phi$ is hypersurface-orthogonal and geodesic, according to a theorem
of general relativity [18]. Changing variables [19], $\ln \cosh^2 Q r = 2Q \rho - \ln \alpha$, leads to
the gauge $\Phi = 2Q \rho - \ln \alpha$ ($\alpha$ is an arbitrary constant), in which the line element
acquires the form

$$ds^2 = \frac{d\rho^2}{1 - \alpha e^{-2Q \rho}} + (1 - \alpha e^{-2Q \rho}) dt^2.$$  \hspace{1cm} (3)$$

where $\alpha$ plays the role of the mass of the black hole [1, 19]. It is interesting to
note that in the limiting case $\alpha \to 0$, one discovers the $Q$-graviton discrete state of
Polyakov [3]. The excitation of this, as well as higher-spin topological string states, might thus be interpreted as a general feature of the last stage of the black-hole evaporation [2], and has important consequences for the existence of the $W$-hair of the black hole, and the consistency with quantum mechanics [3].

To incorporate time dependence in these solutions, in the presence of non-trivial matter, $T \neq 0$, it is convenient to write them in the form, following [20]:

\[
D_\mu D_\nu \Phi - \frac{1}{2} G_{\mu\nu} D^2 \Phi = \partial_\mu T \partial_\nu T - \frac{1}{2} G_{\mu\nu} T (\partial T)^2
\]

\[
R = D^2 \Phi - (\partial T)^2
\]

\[
D^2 \Phi + (\partial \Phi)^2 = \Lambda - 8T^2
\]

\[
0 = D^2 T + (\partial_\mu \Phi \partial^\mu T) + 8T.
\]

(4)

From the five independent equations (4) we may relax one, since we are effectively working with conformal invariance conditions of $\sigma$-models and hence the Curci-Paffuti relation [21] applies

\[
\frac{1}{2} D_\nu \beta^\Phi = (D^\mu + D^\mu \Phi) \beta G_{\mu\nu}^\Phi + \frac{1}{2} (D_\nu T) \beta^T.
\]

(5)

Below we choose to relax the second of the equations (4). Using the hypersurface orthogonality property of the gradient vector $\partial_\nu \Phi$ [18], we are free to choose the gauge

\[
\Phi = Q \rho
\]

\[
ds^2 = \frac{1}{g(\rho, t)} d\rho^2 + A(\rho, t) g(\rho, t) dt^2,
\]

(6)

where $g, A \geq 0$ outside the (outer) horizon, and $A$ is a regular function of $\rho, t$. Using the notation $\partial_t$ for a time derivative and $'$ for a spatial-one, we can write the equations (4) in component form

\[
\frac{\partial_t g}{g} = \frac{2}{Q} \partial_t TT'
\]

\[
\frac{A'}{A} = -\frac{2}{Q} (T')^2 + \frac{2}{Ag^2} \frac{(\partial_t T)^2}{Q}
\]

\[
g' + (g - 1)Q = -\frac{g A'}{2 A} - \frac{8T^2}{Q}
\]

\[
\frac{\partial_t g}{Ag^2} \partial_t T - \frac{g A'}{2 A} T' = gT'' + (g' + gQ) T' + 8T + \frac{1}{Ag} \beta_T T - \frac{1}{2g} \frac{\partial_t A}{A^2} \partial_t T.
\]

(7)

It is readily checked that the above system of four equations with three unknowns is compatible. Following the standard iterative method to solve this non-linear system,
we introduce a small parameter $\epsilon$, which serves as a book-keeping parameter of the order of linearity. We rewrite the system of equations (7) as

$$ \frac{\partial_t g}{g} = -\frac{2\epsilon}{Q} \partial_t TT' $$

$$ g' + (g-1)Q = \frac{g}{Q} (T')^2 - \frac{\epsilon(\partial_t T)^2}{QgA} - \frac{\epsilon 8T^2}{Q} $$

$$ gT'' + (g' + gQ)T' + 8T + \frac{1}{g\sqrt{A}} \partial_t(\frac{\partial T}{\sqrt{A}}) = \frac{\epsilon g}{Q} (T')^3 + \frac{\epsilon}{Qg} (\frac{\partial T}{\sqrt{A}})^2 T' $$

$$ \frac{A'}{A} = -\frac{2\epsilon}{Q} (T')^2 + \frac{2\epsilon}{Qg^2A} (\partial_t T)^2 $$

(8)

and look for solutions for $g$ and $A$ of the form $g = g_0 + \epsilon g_1 + \ldots$, $A = A_0 + \epsilon A_1 + \ldots$. In the static case, the system of equations (8) is satisfied with the choice $g_0 = A_0 = 1$ and $T_0 = (\mu_1 + \mu_2 \rho)e^{-\frac{1}{2}Q\rho}$. This corresponds to the solution of [20], and it is compatible with the result for the configurations of the tachyon field expected on general grounds from Liouville theory [22], where $\mu_1$ might be identified with the Liouville (world-sheet) cosmological constant.

The non-static case can be dealt with iteratively. As an initial step we choose

$$ g_0 = 1, \quad A_0 = f(t)^2 $$

(9)

with $T_0$ satisfying the linear equation

$$ T_0'' + QT'_0 + 8T_0 + \frac{1}{f} \partial_t(\frac{\partial T_0}{f}) = 0. $$

(10)

In this and the following expressions we only consider the time-dependent part of the matter (tachyon) fields, because we are only interested in the effects of time-dependent perturbations on the black-hole background.

To first order in $\epsilon$ the equations for $A_1, g_1$ become:

$$ \partial_t g_1 = -\frac{2}{Q} \partial_t T_0T'_0 $$

$$ g'_1 + g_1Q = \frac{1}{Q} T'_0 - \frac{1}{Q} \frac{(\partial_t T_0)^2}{f^2} - \frac{8}{Q} T_0^2 $$

$$ A'_1 = \frac{2}{Q} (-f^2(T'_0)^2 + (\partial_t T_0)^2). $$

(11)

Making use of the transformation $g_1 = e^{-Q\rho} u_1$ we can write the equations for $g_1$ in the form

$$ \partial_t u_1 = -\frac{2}{Q} e^{Q\rho} \partial_t T_0T'_0 $$

$$ u'_1 = \frac{e^{Q\rho}}{Q} (T_0'^2 - (\frac{\partial T_0}{f})^2 - 8T_0^3). $$

(12)
Setting \( \chi = \int_0^t f(\tau) d\tau \) and \( T_0 = e^{-Q\chi} \hat{T}_0 \), we observe that \( \hat{T}_0 \) is harmonic in \((\rho, \chi)\)-space,

\[
(\partial_{\rho\rho} + \partial_{\chi\chi}) \hat{T}_0 = 0.
\]

The general solution is \( \hat{T}_0 = ReF(z) \), where \( F(z) \) is an arbitrary analytic function of \( z = \rho + i\chi(t) \). It is not difficult to see, then, that the general solution for \( g(\rho, \chi(t)) \) takes the form

\[
g(\rho, \chi) = 1 + e^{-Q\rho} \int_0^\chi d\lambda \left( \frac{Q}{4} (\partial_{\chi} \hat{T}_0)^2 \right)|_{\rho=0, \chi=\lambda} + e^{-Q\rho} \int_0^\rho d\lambda \left( \frac{Q}{2} (\partial_{\rho} \hat{T}_0)^2 \right)|_{\rho=\lambda}.
\]

The general solution for \( A(\rho, t) \) is given by

\[
A(\rho, t) = f(t)^2 \{ 1 + e^{-Q\lambda} \left( (\partial_\chi \hat{T}_0)^2 - (\partial_\rho \hat{T}_0 - \frac{Q}{2} \hat{T}_0)^2 \right)|_{\rho=\lambda} d\lambda \} + h(t).
\]

where \( f(t) \geq 0, h(t) \) are functions (otherwise arbitrary) that guarantee the regularity of \( A(\rho, t) \). Without loss of generality we set \( f(t) = 1 \) for simplicity.

We can now arrive at some general conclusions regarding the solutions. We assume a polynomial form for the tachyon, which is compatible with a weak-field expansion: \( F(z) = \alpha_0 + \alpha_1 z + ... \alpha_n z^n \), with \( \alpha_i, i = 1, 2, ... n \) real. In case one is interested only in the time-dependent parts of the tachyon, as most relevant for the question on the evaporation addressed in this work, one can drop the static part [20] and hence set \( \alpha_0 = \alpha_1 = 0 \).

When \( n = 2m + 1, m = \) positive integer, we observe that

\[
g(\rho, \chi(t)) = 1 + e^{-Q\rho} (c + \alpha_2 \rho \chi^4) = \left( \frac{2m+1}{Q} \kappa - \frac{1}{2} + \left( \frac{2m+1}{Q} \kappa \right)^2 \right) - (2m+1)\kappa \rho \left( \frac{2m+1}{2} \kappa^2 \rho^2 \right) + O(\chi^{4m-2}),
\]

with \( \kappa \equiv \frac{\alpha_{2m+1}}{\alpha_{2m}} \).

The appearance of the negative coefficient of the \( \rho^2 \) term always implies the existence of an event horizon which is slightly expanding with increasing time (see fig. 1a).
The existence of non-vanishing horizons also characterizes the even order \( n = 2m \) polynomial solutions. This becomes clear already from the boundary value \( g(0, \chi(t)) \),

\[
g(0, \chi(t)) = 1 + c - \frac{\epsilon}{2} \alpha_{2m}^2 \chi(t)^{4m} + O[\chi^{4m-2}].
\]

(17)

The difference in this case, as compared with the previous one, is that the horizon increases considerably with increasing time (see fig. 1b). This class of solutions might be considered as corresponding to absorption by the black hole of time-dependent matter; in view of energy conservation of the matter-black-hole system, such a process will increase the mass of the black hole and consequently its horizon. In principle the increase is unbounded. However, from a conformal field theory point of view, the scattering of light particles off a static black-hole background \([23, 11]\) leads to excitations of discrete (higher-spin) states that constitute the internal degrees of freedom of the black hole. The latter then decay, emitting light particles whose number is restricted by appropriate selection rules \([11]\). An important feature is the irreversibility of the decay process that seems to characterize the stringy black holes \([24]\). In view of these results, solutions with horizons expanding to spatial infinity should be considered as not representing an exact conformal field theory. Probably when the rest of the backgrounds, corresponding to the topological modes of the string, are taken into account, the phenomenon of the uncontrollable increase in the horizon disappears.

We expect the above conclusions, concerning the existence of event horizons non-shrinkable to a point, to hold for any solution of (13) for the tachyon field. A simple argument for this is based on the fact that one is mainly interested in the behaviour of the solution at finite time and space intervals, and we know that any analytic function in the region \( 0 \leq \rho \leq L, 0 \leq \chi \leq T \), with \( T >> 0 \), can be resolved in orthogonal polynomials whose highest power coefficient is real.

It should be noticed at this stage that a similar behaviour of non-shrinking horizons characterizes the second-order (in \( \epsilon \)) solutions of the non-linear system \([8]\), as becomes clear from the relevant expressions for the metric field. For the case of \((2m + 1)\)-degree polynomials, the second-order analysis yields for the metric at the origin \( \rho = 0 \)

\[
g_2(0, \chi) = \epsilon^2 \alpha^4 \chi^{8m}(\frac{1}{8} + \frac{1}{4} \lambda + \frac{3}{4} \lambda^2 - \frac{1}{2} \lambda^3 - \frac{3}{2} \lambda^4),
\]

(18)

with \( \lambda \equiv n \kappa / Q \), and \( n \) denotes the degree of the solution.

The situation is slightly different for the even-degree polynomials \( n = 2m \), where the second-order correction in the leading-\( \chi \) behaviour becomes positive:

\[
g_2(\rho, \chi) = 1 + e^{-Q \rho}[c - \frac{\epsilon}{2} \alpha^2 \chi^{4m} + \frac{1}{8} \epsilon^2 \alpha^4 \chi^{8m} e^{-Q \rho} + \ldots].
\]

(19)

However, as becomes clear from fig. 3, this still leads to non-shrinking horizons for the black-hole solution.
The above considerations are indicative of some sort of general behaviour that characterizes tree-level stringy black holes, which appear as stable solutions of the conformal invariance conditions (in the approximation where the Regge slope $\alpha' \to 0$). Higher-genus corrections on the world-sheet, however, can change the situation drastically, as we shall discuss shortly below.

At the moment, some comments are in order concerning the perturbative nature of the polynomial configurations for the matter fields. The solutions (16)–(19) have been obtained for $\epsilon$ small. This, however, does not prevent one from analysing the behaviour at large times, provided that $\epsilon \alpha^2 \chi^{4m} \leq 1$, and thus justifying restriction to the highest relevant powers. It should be remarked at this stage that even in the region $\epsilon \alpha \chi^{4m} > 1$, where perturbative calculations break down, the form of the solutions, at least up to second order in $\epsilon$, remains the same, as can be seen by simple inspection of the relevant equations. However this is only a formal observation, and in order to extrapolate the results for $\chi \to \infty$ safely, one needs non-perturbative information.

The above solutions have been obtained in Euclidean formalism, but can be analytically continued to Minkowski space-times. This does not affect our conclusions, given that the time dependence of the relevant terms appears through powers of $\chi$ that are multiples of 4. It should be noticed, though, that at the level of the Wess-Zumino representation of the two-dimensional (static) black-hole solution the spectrum of the Euclidean black hole is not the same as that of the Minkowskian one [25]. Despite this, there are certain features of the Euclidean formalism, as for example the formal character of selection rules in black-hole decay [11], that can be transcribed to the Minkowski (physical) case by simple analytic continuation. It is in this sense that we apply this method here. From a conformal field theory point of view, we are implicitly working with Minkowski space coset models (and their deformations to include matter fields), and analytically continue to Euclidean signature only at the level of the effective action for computational easiness.

The above considerations have indicated that there exist no time-dependent solutions with horizons shrinking to a point in the case of two-dimensional stringy black holes formulated at a tree-level on the world sheet. In fact, it becomes evident from (16) that it is not possible to obtain a shrinking horizon with a real tachyon field, at least within a perturbative framework. On the other hand, such a possibility may arise in the case of complex tachyon fields. The latter may be considered as an effective description of higher-genus world-sheet effects. Indeed, as shown in [14] as a result of the regularization of modular infinities by analytic continuation, the one-string-loop (torus) analysis shows the appearance of a phase in front of the Einstein-dilaton terms in the effective action (1), but not in front of the matter (tachyon) sector. In order to keep the space-time geometry real, this would lead, at least naïvely, to an effectively complex tachyon field, which would invalidate the
analysis leading to (10). In this way one could obtain black-hole solutions with hori-
zens shrinkable to a point, thereby allowing for a possibility of ‘evaporation’/decay
along the lines of ref. [10]. However a rigorous proof still awaits a consistent genus
expansion of string theory in a closed form. For the two-dimensional (target) space-
time case, this could be achieved in a matrix-model representation of the stringy
black hole. From this point of view, the situation would be described by induced
time-dependent deformations in the light sector of string theory.

The evaporating black-hole solutions in string theory would be characterized by
the coherence-preserving target space $W_\infty$-symmetries (W-hair) which are responsi-
ble for a mixing of the string mass levels [2, 3, 11]. The size of the black-hole could
be macroscopic or microscopic. Microscopic (virtual) black holes have been argued
[4] to appear in fluctuations of the metric field in a quantum gravity framework. In
string field theory, such virtual black holes would lead to the apparent modifications
of the light string mode quantum mechanics as a result of the level-mixing $W_\infty$-
symmetries [8]. Although at present we have not constructed explicitly such exact
string configurations, however, we expect them to exist on the basis of the above
discussion and quite general arguments [10]. We leave these considerations, as well
as attempts to solve exactly the systems of coupled equations discussed above, for
future work.

**Acknowledgements**

We are grateful to A. Petridis for his invaluable help with the computer. One of
us (G.A.D.) thanks the Theory Division of CERN for its hospitality during the last
stage of this work.
References

[1] E. Witten, Phys. Rev. D44 (1991), 314.

[2] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B267 (1991), 465.

[3] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B272 (1991), 261.

[4] S. Hawking, Comm. Math. Phys. 43 (1975), 199; *ibid* 87 (1982), 395.

[5] A. Sengupta and S. Wadia, Int. J. Mod. Phys. A6 (1991), 1961;
D. Gross, I. Klebanov and M.J. Newman, Nucl. Phys. B350 (1991), 621;
D. Gross and I. Klebanov, Nucl. Phys. B352 (1991), 671.

[6] A.M. Polyakov, Mod. Phys. Lett. A6 (1991), 635.

[7] J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, Nucl. Phys. B241 (1984), 381.

[8] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, CERN and Texas A & M Univ. preprint, CERN-TH.6595/92; ACT-17/92; CTP-TAMU-58/92 (1992), Phys. Lett. B in press.

[9] S. Chaudhuri and J. Lykken, Fermi Lab preprint FERMI-PUB-92/169-T, June 1992.

[10] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B276 (1992), 56.

[11] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B284 (1992), 23;
*ibid* 43.

[12] A.M. Polyakov, Princeton Univ. preprint PUPT-1289 (1991).

[13] A. Tomimatsu, Progr. Theor. Phys. 76 (1986), 639.

[14] M. Gasperini, J. Maharana and G. Veneziano, CERN and Torino Univ. preprint CERN-TH.6634/92; DFTT-50/92 (1992).

[15] W. Fischler and L. Susskind, Phys. Lett. B171 (1986), 383; *ibid* B173 (1986), 262.

[16] R.C. Myers, Phys. Lett. B199 (1987), 37;
I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, Phys. Lett. B211 (1988), 393; Nucl. Phys. B328 (1989), 117.

[17] T. Banks, Nucl. Phys. B361 (1991), 166.

[18] H. Stefani, *General Relativity* (Cambridge Univ. Press, 1982).
[19] G. Mandal, A. Sengupta and S. Wadia, Mod. Phys. Lett. A6 (1991), 1685.

[20] S.P. de Alwis and J. Lykken, Phys. Lett. B269 (1991), 264.

[21] G. Curci and G. Paffuti, Nucl. Phys. B286 (1987), 399.

[22] J. Polchinski, Nucl. Phys. B346 (1990), 253.

[23] R. Dijkraaf, H. Verlinde and E. Verlinde, Nucl. Phys. B371 (1992), 269.

[24] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, CERN and Texas A & M Univ. preprint, CERN-TH.6534/92; ACT-12/92; CTP-TAMU-47/92, and CERN-TH.6536/92; ACT-13/92; CTP-TAMU-47/92, to be published in Phys. Lett. B.

[25] J. Distler and P. Nelson, Nucl. Phys. B374 (1992), 123.
Figure Captions

**Fig. 1.** The behaviour of the metric tensor as a function of $\rho$ and $\chi$ to order $\epsilon$ in the case of: (a) odd-degree polynomial solutions, and (b) even-degree polynomial solutions. Clearly one sees the existence of black-hole horizons non-shrinkable to a point.

**Fig. 2.** The behaviour of the metric tensor as a function of $\rho$ and $\chi$ to order $\epsilon^2$ in the case of: (a) odd-degree polynomial solutions, and (b) even-degree polynomial solutions. The existence of horizons non-shrinkable to a point also characterizes the solutions to this order.