Herwig++ 2.7 Release Note

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Abstract
A new release of the Monte Carlo event generator Herwig++ (version 2.7) is now available. This version comes with a number of improvements including: an interface to the Universal FeynRules Output (UFO) format allowing the simulation of a wide range of new-physics models; developments of the Matchbox framework for next-to-leading order (NLO) simulations; better treatment of QCD radiation in heavy particle decays in new-physics models; a new tune of underlying event and colour connection parameters that allows a good simultaneous description of both Tevatron and LHC underlying event data and the double-parton scattering parameter $\sigma_{\text{eff}}$.

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1 Introduction

The last major public version (2.6) of Herwig++ is described in great detail in [1–5]. This release note describes all changes since version 2.6. The manual will be updated to reflect these changes and this release note is only intended to highlight these new features and the other minor changes made since version 2.6.

Please refer to [1] and the present paper if using version 2.7 of the program.

1.1 Availability

The new program version, together with other useful files and information, can be obtained from the following web site:

http://herwig.hepforge.org/

In order to improve our response to user queries, all problems and requests for user support should be reported via the bug tracker on our wiki. Requests for an account to submit tickets and modify the wiki should be sent to herwig@projects.hepforge.org.

Herwig++ is released under the GNU General Public License (GPL) version 2 and the MCnet guidelines for the distribution and usage of event generator software in an academic setting, which are distributed together with the source, and can also be obtained from

http://www.montecarlonet.org/index.php?p=Publications/Guidelines

2 Beyond the Standard Model Simulations

2.1 Universal FeynRules Output

In order to avoid having to explicitly code the Feynman rules for new-physics models, Herwig++ now includes an interface to programs which automatically calculates Feynman rules via the Universal FeynRules Output (UFO) format [6]. This format can be generated by a number of programs, such as FeynRules [7,8] and Sarah [9–11].

The directory containing the UFO output can automatically be converted to generate the code required by Herwig++ using

ufo2herwig NAME_OF_UFO_DIRECTORY

which will automatically generate both code and input files. The code can be compiled using the automatically generated Makefile and events simulated using the automatically generated LHC-FRModel.in input file.

We expect that following this release there will be no further hard-coded new-physics models added to Herwig++ and that future models will be included using the UFO interface.
2.2 Other Changes

We have added the R-parity violating supersymmetric model including the option of both bi- and tri-linear R-parity violating couplings, although not both simultaneously. In addition later releases in the 2.6 series had already included Little Higgs models with and without the conservation of T-parity.

The machinery for the automatic generation of particle decays in new-physics models has been restructured to allow the simulation of higher-multiplicity decays. So far only the 4-body decays of scalar bosons to fermions, which are important for the decays of the Higgs boson, the lightest top squark in supersymmetric models and the lightest tau slepton in R-parity violating models, are included. However, the infrastructure will allow the inclusion of further 4-body and higher-multiplicity decays in the future if required.

3 NLO Development

3.1 Decay Chains

Previous versions of Herwig++ have included next-to-leading (NLO) POSitive Weight Hardest Emission Generator (POWHEG) scheme corrections [12, 13] to a number of processes. In this release additional POWHEG-style corrections are included for the Standard Model decay \( t \to Wb \) and a range of Beyond the Standard Model (BSM) decays. These corrections allow the highest \( p_T \) emission in the parton shower to be generated using the real-emission matrix element. However, we have not implemented the \( \bar{B} \) function that would be required to simulate the decays with full NLO accuracy. The correction has been documented in detail in [14] together with examples illustrating the impact of the correction on a number of decays.

3.1.1 Top Quark Decay

Simulation of the hardest emission in the decay \( t \to Wb \) is implemented, using the POWHEG scheme, in the SMTopPOWHEGDecayer class. The singular regions of the real-emission matrix element are separated using the massive Catani-Seymour dipole formalism in [15]. The implementation was validated in [14] by comparing distributions generated using the POWHEG-style correction with those simulated with the existing Herwig++ hard and soft matrix element corrections. Whilst the two approaches were found to give very similar results, the matrix element corrections consistently produced slightly softer distributions. This difference is due to the soft matrix element correction being applied to multiple hard emissions in the shower, whereas the POWHEG correction applies only to the hardest.

3.1.2 Decays of BSM Particles

The POWHEG-style correction for two-body BSM decays is implemented in the GeneralTwo-BodyDecayer class. The real-emission matrix element is calculated in the decayer class of the leading-order decay using internal helicity-amplitude code. The Catani-Seymour dipole formalism [15, 16] is used to describe the singular behaviour of the real-emission matrix element. In this approach, dipoles describing quasi-collinear radiation from massive vector bosons are not well defined. Therefore, fermion to fermion vector, scalar to scalar vector and tensor to vector vector decays are limited to cases in which final-state coloured vector particles are massless. The vector to fermion fermion and vector to scalar scalar decays do, however, include radiation from massive incoming vector particles. Decays are performed in the rest frame of the decaying
Table 1: Spin combinations for which the POWHEG-style correction has been applied. Corrections to the decays marked * are not included for massive, coloured vector particles.

| Incoming  | Outgoing                |
|-----------|-------------------------|
| Scalar    | Scalar Scalar           |
| Scalar    | Scalar Vector*          |
| Scalar    | Fermion Fermion         |
| Fermion   | Fermion Scalar          |
| Fermion   | Fermion Vector*         |
| Vector    | Scalar Scalar           |
| Vector    | Fermion Fermion         |
| Tensor    | Fermion Fermion         |
| Tensor    | Vector Vector*          |

Table 2: Colour flows for which the POWHEG-style correction has been applied. For tensor particles, corrections are only included for colour flows marked †.

| Incoming | Outgoing |
|----------|----------|
| 0        | 3 3†     |
| 0        | 8 8†     |
| 3        | 3 0      |
| 3        | 3 0      |
| 3        | 3 8      |
| 3        | 3 8      |
| 8        | 3 3      |

particle and therefore the dipole describing the singular behaviour of this particle will only contain a universal soft contribution. This is a well defined, spin-independent function.

Table 1 shows the combinations of incoming and outgoing spins for which the POWHEG-style correction is included. Each spin structure is implemented for the colour flows given in Table 2. However, scenarios with coloured structure have not been considered and, therefore, decays involving incoming tensor particles are limited to colour flows in which the tensor is a colour singlet.

The POWHEG-style correction in the GeneralTwoBodyDecayer class may also be used to generate the hardest final-state emission in $2 \rightarrow 2$ processes that proceed via a colourless $s$-channel resonance. This is done by simulating the decay of the intermediate particle, including the emission from the POWHEG-style correction, and replacing the final-state particles in the hard $2 \rightarrow 2$ process with those generated in the decay. This procedure is performed in the Evolver class and will automatically be attempted for all suitable $2 \rightarrow 2$ processes when POWHEG corrections are switched on.

3.2 Matchbox 2.0/3

Several improvements and enhancements have been developed for the Matchbox NLO framework, of which we introduce version 2.0/3 along with this release.

In particular, the fixed-order part has been extensively tested and pushed to calculating state-of-the-art $2 \rightarrow 4$ processes at hadron colliders. The interfaces to implement new processes have been significantly simplified, and implementations of various scale choices are now available in a much more transparent way. A generic interface to matrix elements provided via the Binoth Les Houches Accord version 2 [17] is now included. Matchbox also provides facilities to implement subtraction algorithms different to the default Catani-Seymour dipole [16] implementation [18].

The matching structures have been rewritten completely to reflect recent developments on the structure of consistent NLO matching and the various ways to assign uncertainties to be

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Footnote:

1 For example, $q\bar{q} \rightarrow G \rightarrow q\bar{q}$, where $G$ is the lightest Randall-Sundrum graviton.
described in detail elsewhere [19]. Currently, matching to the dipole shower is fully supported and a preliminary version of POWHEG-type matching is available as well. Since the matching subtractions are handled in a very generic way, we foresee matching to the default angular ordered shower in one of the forthcoming versions.

A number of processes at $e^+e^-$, $ep$ and $pp$ colliders are included at NLO, amongst them $pp \to V(\pm jet)$, where $V = W, Z$ including the decay to lepton pairs, and $gg \to h$ and $b\bar{b} \to h$ inclusive Higgs production. Electroweak Higgs plus two and three jet production at NLO, as recently presented in [20], is based on the Matchbox framework and will become publicly available soon.

4 Higgs boson pair production

Higgs boson ($H$) pair production in the Standard Model (SM) at hadron colliders is dominated by the gluon fusion initial states [21–31]. At leading order, the process is loop-initiated, and the dominant diagrams contributing towards it are shown in Fig. 1, where the loops can contain either a top quark or a bottom quark.

Figure 1: The Higgs pair production diagrams contributing to the gluon fusion process at LO are shown.

The triangle diagram can only contain initial-state gluons in a spin-0 state, whereas the box contribution can contain both spin-0 and spin-2 configurations. Therefore, there are two Lorentz structures involved in the box diagram matrix element. At LO, we may write, schematically

$$\sigma^{LO}_{HH} = \left| \sum_{q} (\alpha_q C^{(1)\text{tri}}_{q} + \beta_q C^{(1)\text{box}}_{q}) \right|^2 + \left| \sum_{q} \gamma_q C^{(2)\text{box}}_{q} \right|^2,$$

where $C^{(1)\text{tri}}_{q}$ represents the matrix element for the triangle contributions and $C^{(i)\text{box}}_{q}$ represents the matrix element for the two Lorentz structures ($i = 1, 2$) coming from the box contributions [21, 36], for each of the quark flavours $q = \{t, b\}$.

The parameters $\alpha_q$, $\beta_q$ and $\gamma_q$ for quark flavour $q$ are given in terms of the Standard Model Lagrangian parameters by

$$\alpha_q = \lambda y_q,$$

$$\beta_q = \gamma_q = y_q^2,$$

where $q = \{t, b\}$. The dimensionless (and normalised) triple Higgs coupling $\lambda$ is defined by

$$G_{HHH} = \frac{\lambda 3 M_H^2}{v},$$

$^2$Other modes, like $qq \to qqHH, VHH, t\bar{t}HH$ are a factor of 10-30 smaller [32–35].
where \( v \simeq 246 \text{ GeV} \) is the Higgs vacuum expectation value. \( y_q \) is the \( Hq\bar{q} \) coupling (as defined after electroweak symmetry breaking and assumed to be real), normalised to the SM value, defined by

\[
G_{HQ} = y_q \frac{M_q}{v}, \quad (Q = \{B, T\}).
\]  

The implementation of the process in the Herwig++ event generator was performed using the available HPAIR code [37]. The model is present in Contrib/HiggsPair.

The following options currently exist for the Higgs pair production process:

- **Process**: All, \( gg\text{ToTriangleTohh} \), \( gg\text{ToBoxTohh} \), \( gg\text{ToHToh} \), where, respectively, they correspond to: ‘all SM \( gg \to hh \) subprocesses’, ‘only SM \( gg \to hh \) triangle subprocess’, ‘only SM \( gg \to hh \) box subprocess’, ‘all \( gg \to hh \) subprocess, with heavy Higgs’. The heavy Higgs mass should be set in the input file with PDG id 35.

- **SelfCoupling**: set to the value of \( \lambda \) as defined by Eq. 3.

- **hhHCoupling**: set to the value of the (light Higgs-light Higgs-heavy Higgs) coupling.

Validation of the implementation has been performed using an equivalent MadGraph model [38, 39], implemented in a similar (but independent) way using functions taken from HPAIR. We have made comparisons of several distributions and the total cross section between the two implementations. We have also confirmed that the total cross section output obtained by using Herwig++ matches that obtained using HPAIR at leading order for various PDF sets and the scale choice \( \sqrt{s} \).

The final result of Ref. [45] has \( \sigma_{\text{eff}} \approx 15 \text{ mb} \) and gives a good description of the underlying event data from Tevatron’s lowest energy point [46], \( \sqrt{s} = 300 \text{ GeV} \) to the LHC’s highest [47], \( \sqrt{s} = 7 \text{ TeV} \).

Herwig++ version 2.7 is released together with the tune of Ref. [45], UE-EE-5-MRST, by default. Other related tunes can be obtained from the Herwig++ tunes page.

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3HPAIR was modified to use the LHAPDF library [40].
6 Uncertainties in Shower Algorithms

Evaluating uncertainties within parton-shower algorithms is becoming more and more important. Though no fully detailed accord is yet available, we introduce some first measures for scale variations using both parton-shower modules.

6.1 Angular Ordered Shower

Within the angular-ordered shower, a variation of the argument of the strong coupling can be set using

\[ \text{set } /\text{Herwig/Shower/AlphaQCD:RenormalizationScaleFactor } x \]

which will result in evaluating \( \alpha_s(q_{\perp}^2) \) for each shower emission, where \( q_{\perp}^2 \) is the dynamic scale chosen by the shower for each emission. We note that such variations are being discussed as typically overestimating the uncertainty, and improved schemes may be provided in the future.

A variation of the hard veto scale (as relevant to processes with jets at the level of the hard process), can be achieved by using

\[ \text{set } /\text{Herwig/Shower/Evolver:HardScaleFactor } x \]

which will scale the \( p_{\perp} \) veto value for shower emissions by the factor \( x \).

6.2 Dipole Shower

The dipole shower implementation naturally supports variations of all the scales involved, \emph{i.e.} renormalization, factorization and hard shower scales (similar to a resummation scale). The scale factors can be set through the following interfaces

\[ \text{cd } /\text{Herwig/DipoleShower} \]

\[ \text{set DipoleShowerHandler:RenormalizationScaleFactor } x \]

\[ \text{set DipoleShowerHandler:FactorizationScaleFactor } x \]

\[ \text{set DipoleShowerHandler:HardScaleFactor } x \]

The hard scale factor setting is ignored, when running in MC@NLO mode, and the corresponding value is obtained from the matching object in place to ensure consistency. The corresponding value in the matching object can be set by

\[ \text{set } /\text{Herwig/MatrixElements/Matchbox/DipoleMatching:HardScaleFactor } x \]

7 Other Changes

A number of other more minor changes have been made. The following changes have been made to improve the physics simulation:

- A new matrix element for single top production, including both \( s \)- and \( t \)-channel mechanisms, has been implemented in the \textit{MEPP2SingleTop} class.

- The parametrization of the kinematics for emissions in the dipole shower has been made more efficient.
Decays of $W$ and $Z$ bosons now use the POWHEG decayers by default.

The width treatment in BSM decay chains has been greatly improved and is now switched on by default in the `.model` files. To get the old behaviour, use

```
set /Herwig/NewPhysics/NewModel:WhichOffshell Selected
```

The handling of beam remnants has been improved in multiple contexts, leading to a much lower error rate in hadronic collisions. An additional `[VeryHard]` option for the `RemnantOption` switch in the `ClusterFissioner` class has been added to disable any special treatment of beam remnant clusters. The default remains unchanged.

The Higgs boson mass is now set to 125.9 GeV (from PDG 2013 update).

A number of technical changes have been made:

- To help with the coming transition to C++-11, we provide the new `--enable-stdcxx11` configure flag. Please try to test builds with this flag enabled and let us know any problems, but do not use this in production code yet. In future releases, this flag will be on by default.

- Many new Rivet analyses have been included in the `Tests` directory.

- The header structure and compilation of the shower module has been improved. The parameters relating the evolution scale and Sudakov decomposition for radiation generated in the shower have been grouped into one `struct`.

- The boolean flag describing whether or not POWHEG corrections were present in the `HwMEBase` and `HwDecayerBase` classes has been changed to an enumeration to allow better control over the types of emission.

- If any decay modes are selectively disabled, using the `BranchingRatioReweighter` class as post-handler, *i.e.*

  ```
  create Herwig::BranchingRatioReweighter BRreweight
  insert LHCGenerator:EventHandler:PostDecayHandlers 0 BRreweight
  ```

  will cause all reported cross sections to include the branching ratio factor(s) from the previous stages correctly. Care should be taken not to use this option in processes, such as Higgs boson production in association with a $W^\pm$ or $Z^0$ boson, where the cross section already includes the branching ratio factor.

- The search path for repository `read` command is now configurable on the command line with the `-i` and `-I` switches. By default, the installation location is now included in the search path, so that

  ```
  Herwig++ read LEP.in
  ```

  will work in an empty directory. The current working directory will always be searched first. The rarely used `Herwig++ init` command has been made consistent with `read` and `run` and should now be used without the `-i` flag.

- Support for setting quark masses different from the values in the `ParticleData` objects in the `O2AlphaS` class implementing the strong coupling has been enabled.
• The various switches to turn off the compilation of BSM models have been unified into a single `--disable-models` switch. A new flag `--disable-dipole` can be used to turn off the compilation of the Dipole shower and Matchbox modules.

• It is now possible to use anomalous vertices with the MEfftoVH matrix-element class.

• A Matcher object has been added for charged, rather than all, leptons.

• New scale choices have been added in the GeneralHardME class to support testing against Madgraph results.

• A number of changes have been made to the automatic calculation of running width effects in BSM models to improve the limits on the offshellness of the particles, speed and numerical stability

• The consistency checks performed on SLHA and Les Houches event files have been significantly improved to reduce cases of GIGO.

• The option of only including two-body decay modes in the running width calculations to speed up the calculation has been added.

The following bugs have been fixed:

• Various array bounds errors have been corrected.

• Issues with inconsistent C++ and FORTRAN compilers using OS X have been improved.

• Vertex positions involving pseudo-vertices, that Herwig++ inserts for technical reasons, are now set correctly.

• The scale settings for MPI and the parton shower in POWHEG events has been changed to fix reported anomalies in jet rates in POWHEG processes. NLO PDFs are now also set consistently in the example input files.

• The interactive shell no longer quits following an error.

• Possible division by zero errors in BSM branching ratio calculations have been fixed.

• The limits on the momentum fraction allowed in forced splittings in the remnant handling have been fixed.

• The colour rearrangement of beam clusters with each other is no longer allowed.

• Various fixes have been made to ensure that the labelling of remnant clusters is correct after colour rearrangement and additional soft scatters.

• Changes have been made to improve checkpointing in the decayer and matrix-element classes. The regular dump files are now consistent.

• A bug affecting the lifetime and vertex positions in the decay $\pi^0 \rightarrow e^+e^-\gamma$ has been fixed. The virtual photon is no longer included in the event record.

• A bug, introduced in Herwig++ 2.6.0, in the diffraction code which would abort any runs has been fixed.
• A bug in showering from $gg \rightarrow h$ in the dipole shower has been fixed.

• Various minor inconsistencies in the general hard matrix elements and decayers, which did not effect the built-in models, have been fixed in order to support models generated from the UFO format.

• A number of code features which caused warnings with recent compilers have been fixed.

• A number of minor fixes to the SUSY couplings have been made to improve the consistency and handling of running masses effects.

• Various issues with colour flows in BSM models introduced when sextet particles were added have been fixed.

• The generation of QED radiation from massless charged particles coming from Les Houches event files has been forbidden to avoid numerical stability problems.

8 Summary

Herwig++ 2.7 is the ninth version of the Herwig++ program with a complete simulation of hadron-hadron physics and contains a number of important improvements with respect to the previous version. The program has been extensively tested against a large number of observables from LHC, LEP, Tevatron and B factories. All the features needed for realistic studies for hadron-hadron collisions are present. As always, we look forward to feedback and input from users, especially from the Tevatron and LHC experiments.

Our next major milestone is the release of version 3.0, which will be at least as complete as HERWIG in all aspects of LHC and linear-collider simulation.

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