Mainly axion cold dark matter from natural supersymmetry

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By eschewing finetuning from the electroweak and QCD sectors of supersymmetry (natural supersymmetry or SUSY), and by invoking the Kim-Nilles solution to the SUSY µ problem, one is lead to models wherein the dark matter is comprised of a mixture of axions and higgsino-like WIMPs. Over a large range of Peccei-Quinn breaking scale $f_a \sim 10^9 - 10^{12}$ GeV, one then expects about 90-95% axion dark matter. In such a scenario, both axion and WIMP direct detection may be expected.

PACS numbers: 12.60.-i, 95.35.+d, 14.80.Ly, 11.30.Pb

The recent discovery of a Higgs-like boson with mass $m_h \approx 125$ GeV at the CERN LHC is a triumph of modern particle physics [1]. But it brings with it a conundrum: why is the Higgs mass so small? In the Standard Model (SM), one may calculate

$$m_h^2(\text{phys}) = m_h^2(\text{tree}) + \delta m_h^2$$ (1)

where the radiative correction $\delta m_h^2 \sim \frac{3f_t^2}{8\pi^2}A^2 + \cdots$ where $f_t \sim 1$ is the top quark Yukawa coupling and $A$ is a high energy cutoff/regulator which denotes the limit of validity of the effective theory. If the SM is to be valid at energy scales $\Lambda$ far beyond $m_{\text{weak}} \approx 100$ GeV, then an enormous finetuning will be required to maintain $m_h \sim 125$ GeV.

Supersymmetric (SUSY) theories of particle physics provide all-orders cancellations of the quadratic divergences thus stabilizing the Higgs mass. In SUSY, $\delta m_h^2$ is instead logarithmically divergent: naively,

$$\delta m_h^2(\text{SUSY}) \sim -\frac{3f_t^2}{8\pi^2}(m_{Q3}^2 + m_{U3}^2 + A_t^2) \ln (\Lambda^2/m_{\text{SUSY}}^2)$$ (2)

where $m_{Q3}^2, m_{U3}^2$ and $A_t^2$ are soft SUSY breaking terms related to the top- and bottom-squark masses and $m_{\text{SUSY}} \sim \sqrt{m_{t1} m_{b1}}$. To avoid finetuning, third generation squarks and low $\Lambda$ are thus required to maintain $m_h \sim 125$ GeV.

A second possibility for SUSY finetuning which is independent of $A$ occurs purely at the weak scale, where the $Z$ mass is given by

$$\frac{m_z^2}{2} \approx -m_{H_u}^2 - \Sigma_w^0 - \mu^2$$ (3)

which is valid for the ratio of Higgs field vacuum expectation values (vevs) $\tan \beta \equiv v_u/v_d$ over its typical range of $3 - 60$. Here, $m_{H_u}$ is a soft SUSY breaking Higgs mass, $\mu$ is the superpotential Higgs/higgsino mass and $\Sigma_w^0$ collects various radiative corrections (expressions are provided in the Appendix of Ref. [4]). For many models, $m_{H_u}$ is driven radiatively to negative TeV-scale values signaling the breakdown of electroweak symmetry. A large positive value of $\mu^2$ must be imposed (finetuned) to obtain the measured $Z$-mass, $m_Z \approx 91.2$ GeV. To avoid large uncorrelated cancellations in the $Z$ mass, then one expects $|\mu|$ and $|m_{H_u}| \sim m_Z$, or of order $100 - 200$ GeV [4, 5]. This has enormous implications for SUSY phenomenology, as then the lightest SUSY particle, often touted as a WIMP dark matter candidate, is expected to be largely higgsino-like. Higgsino-like WIMPs with mass $m_{H_u} \sim 100 - 200$ GeV develop a thermally-produced dark matter abundance $\Omega_{\text{DM}} h^2 \sim 0.005 - 0.01$, i.e. typically a factor $10 - 15$ below the WMAP/Planck measured value [2].

Supersymmetric models with low finetuning (not too heavy 3rd generation squarks and low $\mu$) are referred to as natural SUSY [2, 3, 4] since they are devoid of large electroweak finetuning.

A third possibility for finetuning occurs in the QCD sector. To implement 't Hooft’s solution to the $U(1)_A$ problem (i.e. why there are three and not four light pions), the term

$$\frac{\bar{\theta}}{32\pi^2} F_{\mu\nu} \tilde{F}_A^{\mu\nu}$$ (4)

should occur in the QCD Lagrangian, where $\bar{\theta} = \theta + \arg(\det(\mathcal{M}), \mathcal{M}$ is the quark mass matrix, $F_{\mu\nu}$ is the gluon field strength and $\tilde{F}_A^{\mu\nu}$ is its dual. Measurements of the neutron electric dipole moment (EDM) require $\bar{\theta} \lesssim 10^{-10}$, so that one might require an enormous cancelation within $\theta$ [7]. Alternatively, the PQWW solution [8] introduces an axion field $a$; the additional axion contributions to Eq. 4 allow for $\bar{\theta}$ to dynamically settle to zero, thus solving the so-called strong $CP$ problem.

In SUSY theories, the axion enters as one element of an axion superfield which necessarily contains also a spin-0 $R$-parity even saxion $s$ and a spin-1/2 $R$-parity-odd axino $\tilde{a}$. Calculations of the saxion and axino masses within the context of supergravity [9] imply $m_s \sim m_{\tilde{a}} \sim m_{3/2}$ where the gravitino mass $m_{3/2}$ is expected to be of order the TeV scale. If the lightest neutralino ($\bar{\nu}$, e.g. the higgsino $\tilde{Z}_1$) is the lightest SUSY particle (LSP) in $R$-parity conserving theories, then one would expect dark matter to be comprised of two particles: the axion as well as the higgsino-like SUSY WIMP. The saxion and axino couplings to matter are suppressed by the PQ breaking...
scale $f_a$ which may range from $f_a \sim 10^9 - 10^{16}$ GeV \[7\]. While the saxion and axino are expected to play little or no role in terrestrial experiments, they can have an enormous impact on dark matter production in the early universe.

The PQ symmetry required to solve the strong CP problem can be implemented in two ways. In the SUSY KSVZ model \[10, 11\], the axion superfield couples to exotic heavy quark/squark superfields $Q$ and $\bar{Q}$ which carry PQ charges. The loop-induced axino-gluino-gluon coupling leads to a thermal axino production rate proportional to the re-heat temperature $T_R$ at the end of inflation. It also allows for axino decays $\tilde{a} \rightarrow g \bar{g}$ or $\tilde{a} \rightarrow \gamma Z_i$ (with $i = 1 - 4$). In SUSY KSVZ, axinos are sufficiently long-lived that they almost always decay after neutralino freeze-out $T_n \sim m_{\tilde{Z}_1}/25$ \[12\]. Axinos, produced at a sufficient rate, may induce neutralino re-annihilation at the axino decay temperature $T_D = \sqrt{\lambda_3 M_P/(\pi^2 \langle T_H^2 \rangle /90)}^{1/4}$ which also augments the neutralino abundance \[12, 13\] (here, $M_P$ is the reduced Planck mass $\sim 2 \times 10^{18}$ GeV). Saxions may be produced thermally \[13\] (again proportional to $T_R/f_a^2$) or via coherent oscillations \[15\] (proportional to $f_a^2$, so important at very large $f_a \sim 10^{14} - 10^{16}$ GeV). They may decay via $s \rightarrow g \bar{g}$, which leads to entropy dilution of all relics present at the time of decay, or to $g \bar{g}$ or other SUSY modes, which can augment the neutralino abundance. Depending on the combinations of PQ charges $q_i$ and PQ vevs $v_i$, saxions should also decay via $s \rightarrow a a$ or $s \rightarrow a a$ for $\xi = \sum_i q_i^a v_i^2/f_a^2 \sim 1$. The first of these leads to production of dark radiation which is stringently limited by WMAP/Planck \[16\] parametrized in terms of the number of additional neutrinos present in the universe: $\Delta N_{\nu f} < 1.6$ at 95% CL. If the saxion or axino decays occur much after $T_{\text{BBN}} \sim 1$ MeV, then light elements produced during Big Bang Nucleosynthesis may become dis-associated leading also to severe constraints \[17\].

Alternatively to SUSY KSVZ, in the SUSY DFSZ model \[18, 21\] the PQ superfield couples directly to the Higgs superfields leading to thermal production rates which are independent of $T_R$ \[20, 21\]. The saxion and axino thermal yields (number density over entropy density) are then given by

$$Y_s^{\text{TP}} \simeq 10^{-7} \zeta_a \left( \frac{\mu}{\text{TeV}} \right)^2 \left( \frac{10^{12} \text{ GeV}}{f_a} \right)^2$$

(6)

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(7)

where the $\zeta_i$ are model-dependent constants of order unity. Saxions can also be produced via coherent oscillations with a yield given by

$$Y_s^{\text{CO}} = 1.9 \times 10^{-6} \left( \frac{\langle T_H \rangle}{m_s} \right)^{\frac{1}{2}} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^2$$

(8)

assuming an initial saxion field amplitude of $s_0 \sim f_a$. Along with these, neutralinos will be produced via thermal freeze-out as usual given by

$$Y_{\tilde{Z}}^{\text{fr}} = \frac{(90/\pi^2 \langle T_H \rangle)^{1/2}}{4(\sigma v) M_P T_R}$$

(9)

where $T_R$ is the freeze-out temperature and $\langle \sigma v \rangle$ is the thermally averaged neutralino annihilation cross section times relative velocity. Axions will also be produced at the QCD phase transition via coherent oscillations \[23\] given by

$$\Omega_a^{\text{std}} h^2 \simeq 0.23 f(\theta_1) \theta_1^2 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6}$$

(10)

where the misalignment angle $0 < \theta_1 < \pi$ and $f(\theta_1) = \ln \left( \frac{1 - \theta_1^2/\pi^2}{\theta_1^2/\pi^2} \right)^{7/6}$. Along with the above processes, neutralinos can be produced via axino decays. In SUSY DFSZ, the dominant modes include: $\tilde{a} \rightarrow Z_i \phi$ (where $\phi = h, H, A$), $Z_i Z$ ($i = 1 - 4$), $W^{\pm} H^{\mp}$ and $W^{\pm} W^{\mp}$ ($j = 1 - 2$). Summing over decay modes and neglecting phase space factors, the axino width is

$$\Gamma_\tilde{a} \simeq \frac{\mu}{4\pi} \left( \frac{v}{v_{PQ}} \right)^2 m_\tilde{a}$$

(11)

where $c_H$ is an order one parameter for the axino (saxion) coupling arising from Eq. \[4\] and $v_{PQ} \equiv \sqrt{\sum_i q_i^a v_i^2} \sim f_a$ in terms of PQ charges and vevs. This tends to greatly exceed the value obtained in SUSY KSVZ.

For illustration, we adopt the Standard Underabundance SUSY benchmark (SUA) scenario from Ref. \[4, 24\] which assumes a higgsino mass scale $\mu \sim 150$ GeV while...
sparticles which do not contribute to naturalness are at high masses $\gg 1$ TeV, safely beyond the LHC reach. Our results are hardly sensitive to the selected benchmark so long as $|\mu| \sim m_Z$ while the remaining sparticles are quite heavy. The axino decay temperature for the SUA point using the complete decay widths is shown in Fig. 1. The spike is due to a resonant axino-neutralino mixing effect when $|m_a - m_{\tilde{Z}_1}| \lesssim \mu v / f_a$ and can be ignored in our analyses. Comparing against the neutralino freeze-out temperature for a 135 GeV higgsino-like LSP ($T_f \sim 5$ GeV), we see that $T_a^\text{dec}$ tends to exceed $T_f$, for $f_a \lesssim 10^{12}$ GeV. In this case, the axino-produced neutralinos will thermalize and their abundance is determined by the usual thermal freeze-out. For higher $f_a \gtrsim 10^{12}$, the axino decay will augment the neutralino abundance, and if the decay-produced neutralinos are sufficiently abundant, they re-annihilate at $T_a^\text{dec}$ with an abundance given by $Y_{\text{ann}} \approx \frac{1}{2} (T_f \rightarrow T_a^\text{dec})$. This, since $T_a^\text{dec} < T_f$, gives an increased yield over the thermal expectation. If the temperature at which the axino density equals the radiation density, $T_a^\text{dec} = 4m_a / 3$, is smaller than the decay temperature, axinos temporarily dominate the energy density of the universe. This hardly happens in SUSY DFSZ as $\mu \gtrsim 10^{-4} f_a$ is required.

Saxions, produced thermally and non-thermally, can decay via $s \rightarrow h h, H H, h H, A A, H^+H^-, Z Z, W^+W^-, ZA, W^\mp H^\mp$, $Z\tilde{Z}, Z\nu$, and $W^\pm\tilde{W}^\mp$ (also to fermions and sfermions). For large $m_s$, the width is dominated by

$$\Gamma(s \rightarrow \text{Higgsinos}) \approx \frac{c_s^2}{32\pi} \left( \frac{\mu}{v_{\text{WQ}}} \right)^2 m_s.$$  \hspace{1cm} (12)

In addition, for $\xi \sim 1$, the decay $s \rightarrow aa$ may be sizable, leading to dark radiation $24$, or $s \rightarrow \tilde{a} \tilde{a}$ may occur as well, further augmenting the LSP abundance. The saxion decay temperature $T_a^\text{dec}$ is shown in Fig. 2. The spike is due to a resonant saxion-Higgs mixing effect when $|m_a^2 - m_h^2| \lesssim B_{\mu v} / f_a$ and can be ignored in our analyses. For $f_a \lesssim 10^{12}$ GeV, saxions tend to decay before the neutralino freeze-out. A comparison of the saxion radiation equality temperature $T_a^\text{eq}$ against the decay temperature $T_a^\text{dec}$ shows that saxions dominate the energy density of the universe only when $f_a \gtrsim 10^{14}$ GeV for which $Y_s^{\text{CO}}$ is large enough.

Let us now examine the contributions of neutralinos and axions to the observed dark matter density expected in the SUSY DFSZ model. Our main result is shown in Fig. 3 assuming $m_a = m_s = 5$ TeV. Starting at $f_a = 10^9$ GeV as required by astrophysical constraints, the neutralino abundance $\Omega_{\tilde{Z}_1} h^2 \approx 0.01$ is given by the standard thermal freeze-out over a large range of $f_a$ extending all the way up to $f_a \sim 10^{12}$ GeV. In this regime, the axion abundance can always be found by adjusting $\theta_i$ such that the summed abundance meets the measured value: $\Omega_{\tilde{Z}_1} h^2 + \Omega_s h^2 = 0.12$. The required value of $\theta_i$ is shown in Fig. 4. For very low $f_a \sim 10^9$ GeV, a large value of $\theta_i \sim \pi$ is required, and $\Omega_s h^2$ is dominated by the anharmonicity term. As $f_a$ increases, the assumed initial axion field value $\theta_i f_a$ increases, so the required misalignment angle $\theta_i$ decreases. Values of $\theta_i \sim 1$ are found around $f_a \sim 2 \times 10^{11}$ GeV for both $\xi = 0$ and 1. In this entire region with $f_a \sim 10^9 - 10^{12}$ GeV, we expect from natural SUSY that the relic higgsino abundance lies at the standard freezeout value, comprising about 5-10% of the total dark matter density, while axions would comprise 90-95% of the abundance. Thus, over the commonly considered range of $f_a$, we expect mainly axion cold dark matter from natural SUSY, along with a non-negligible fraction of higgsino-like WIMPs.

As $f_a$ increases beyond $10^{12}$ GeV, axinos and saxions decay later than the neutralino freeze-out, and neutralino re-annihilation at temperatures $T < T_f$ increases the neutralino abundance as can be seen from the rising curves of $\Omega_{\tilde{Z}_1} h^2$ in Fig. 3 for both $\xi = 0$ and 1. The neutralino abundance rapidly rises as $f_a$ increases excluding basically the region of $f_a \gtrsim 2 \times 10^{12}$ GeV (7 $\times 10^{12}$ GeV) by neutralino dark matter overproduction for $\xi = 0$ (1). If we increase $m_a = m_s$ to 10 (20) TeV, then the upper bound on $f_a$ moves to 3 $\times 10^{12}$ (4 $\times 10^{12}$) GeV for $\xi = 0$. For $\xi = 0$, the $\Omega_{\tilde{Z}_1} h^2$ curve rises steadily with $f_a$ due to increasing production of saxions from coherent oscillations and their dominant decays to

\[\text{FIG. 1: Decay temperature of DFSZ axinos vs. PQ scale $f_a$ for the SUA benchmark.}\]

\[\text{FIG. 2: Decay temperature of DFSZ saxons vs. PQ scale $f_a$ for the SUA benchmark with $m_a = 2$ TeV.}\]
SUSY particles. This leads to subsequent neutralino re-annihilation at decreasing temperatures $T_D$. For $\xi = 1$, the dominant saxion decay mode is $s \rightarrow aa$, and decay-produced neutralinos come mainly from thermal axino production which decreases as $f_a$ increases. One sees that $\Omega_{\tilde{Z}} h^2$ turns over and briefly reaches $\Omega_{\tilde{Z}} h^2 \approx 0.12$ at $f_a \sim 3 \times 10^{13}$ GeV before beginning again a rise due to increasing non-thermal saxion production. It is important to note that for $\xi \sim 1$ and $f_a \gtrsim 10^{14}$ GeV, too much dark radiation is produced ($\Delta N_{\text{eff}} > 1.6$) (not shown here) and thus large $f_a$ is excluded by overproduction of both dark radiation and WIMPs.

Summary: Supersymmetry with not too heavy top squarks, low higgsino mass $\mu \sim 100 - 200$ GeV and PQWW solution to the strong CP problem successfully avoids high finetuning in three sectors of the theory while evading LHC constraints. The SUSY DFSZ model, wherein Higgs superfields carry PQ charge, also provides a solution to the SUSY $\mu$ problem. In such models, over a large range of PQ breaking scale $f_a \sim 10^9 - 10^{12}$ GeV, saxions and axinos typically decay before neutralino freeze-out so that the higgsino portion of dark matter is expected to lie in the 5-10% range while axions would comprise the remainder: 90-95%. The relic higgsinos ought to be detectable at ton scale noble liquid detectors, even with a depleted local abundance, while indirect detection should be more limited since expected rates go as the depleted abundance squared. Prospects are bright for microwave cavity detection of axions since the range of $f_a$ where mainly axion dark matter is expected should be accessible to experimental searches. While corroborative searches for natural SUSY with light higgsinos is limited at the LHC, a definitive higgsino search should be possible at $e^+e^-$ colliders with $\sqrt{s}$ up to 500 − 600 GeV.

Acknowledgements: We thank V. Barger for comments on the manuscript and A. Lessa for collaboration on the early phase of this work. This work was supported in part by the US Department of Energy, Office of High Energy Physics.

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