Notes on the Conway-Kochen Twin Argument

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ABSTRACT

This is a revision of my original posting, in which I raised objections to part of the Conway Kochen arument. I now agree with them that their recent reply answers my original concerns. In the first part of these notes (identical to the original), I give a reformulation of the part of the Conway-Kochen result that closes the contextuality loophole in the original Kochen-Specker (KS) theorem. In the second part (modified in this revision) I review my concerns connected with the finite time needed to make a measurement, and briefly indicate how Conway and Kochen have responded to them.
(1) **Review of the KS argument.** Let \( n \) be a general spin direction, and \( H \) a set of hidden variables, which we postulate to determine the squared spin of a spin-1 particle in all directions. That is,

\[
S_n^2 = F(H, n)
\]  

for all \( n \) with a fixed set of hidden variables \( H \). To reproduce quantum mechanics, KS impose two constraints:

\[
S_n^2 \text{ takes only values } 0, 1 \quad ,
\]  

(2a)

For every orthogonal triple \( x, y, z \), one has \( S_x^2 + S_y^2 + S_z^2 = 2 \) .

(2b)

Clearly, the constraints of Eq. (2a,b) imply that two of \( S_x^2, S_y^2, S_z^2 \) are 1, and one is 0. The KS argument proceeds from constructing a set of directions, referred to below as KS directions, for which Eqs. (1a,b) and (2) are in contradiction, implying that there exists no function \( F(H, n) \) that satisfies the quantum mechanical constraints.

(2) **Contextuality.** Let us introduce a restricted notion of contextuality as follows. Let us define a “relevant parameter” \( r \) to be one for which (i) \( r \) must be specified at the start of the measurement, in addition to \( n \), to uniquely describe a spin-measuring apparatus, and (ii) \( r \) must be given at least 2 distinct values to measure \( S_n^2 \) for all directions in a set of KS directions. We shall assume further that the \( r \) value is not changed in the course of a single measurement. [Example: A Stern-Gerlach apparatus depends on 2 directions, the beam axis (say, \( x \)) and the axis of the inhomogeneous magnetic field (say, \( z \)). To measure \( S_z^2 \), the beam axis can be chosen to be any direction in a plane perpendicular to \( z \). To specify the apparatus uniquely, one has to specify the axis \( x \), so (i) is satisfied. Since no single axis is perpendicular to all of the directions in a KS direction set, (ii) is also satisfied.] Now we can
state the contextuality loophole: if \( S_n^2 = F(H, n, r) \), where the value of \( r \) influences whether
the function \( F \) takes the value 0 or 1, then the KS contradiction is avoided.

(3) **The Conway-Kochen argument to close the loophole.** Let us introduce the following three assumptions, in addition to the constraints of Eq. (2a,b):

**TWIN – reduced symmetrical version:** Consider a spin-0 state of two spin-1 particles in an EPR setup with observer \( A \) measuring one particle, and observer \( B \) measuring the other particle. Then if \( A \) chooses direction \( n \) and measures \( S_n^2 \), \( B \) must get the same answer \( S_n^2 \) on the same direction \( n \), irrespective of the distance \( d_{AB} \) between the two observers.

**FREE:** Each experimenter can choose the relevant parameter \( r \) for his/her apparatus and complete a measurement in time \( dt \). (Here, for simplicity, \( dt \) is taken as the minimum and the maximum time for a new measurement.)

**REL:** Classical information, such as the value of the relevant parameter, propagates with at most a finite signal velocity \( c \).

**The Conway-Kochen argument**

To rule out contextuality as defined above, proceed as follows:

by TWIN \( S_n^2 = F_A(H, n, r_A) = F_B(H, n, r_B) \)

by FREE \( r_A, r_B \) can be set to new values, and a measurement performed, in time \( dt \).

by REL If we take \( d_{AB} >> cdt \), then \( F_A \) cannot depend on \( r_B \) and \( F_B \) cannot depend on \( r_A \), because there is insufficient time for a signal to propagate from \( A \) to \( B \). Hence we conclude that

\[
F_A(H, n, r_A) = F_B(H, n, r_B) = F_A(H, n) = F_B(H, n) \ ,
\]

and we are back to the original KS contradiction.
My concern with respect to finite measurement time. Suppose we try to extend the no-go theorem to rule out a contextual dependence of the form $F(H, n, i_{\text{past}})$, where $i_{\text{past}}$ is all information in the intersection of the past light cones of the observers $A$ and $B$. Since a measurement takes a finite time $dt$, and since setting up the apparatus for different directions may take differing times, this intersection increases (see below) in the course of a measurement, and need not be the same for potential measurements in all directions in a KS set.

To make things concrete, suppose there is a dependence on a signal emanating from a distant extra-galactic source, that arrives at identical times at $A$ and $B$ (this latter is not an essential restriction). Then Eq. (3) is replaced by

$$F_A(H, n, S(t)) = F_B(H, n, S(t)).$$

(4)

This does not imply that $F_A(H, n, S(t)) = F_A(H, n)$, and if the dependence on $S(t)$ influences whether $F$ takes the value 0 or 1, then the KS contradiction may be avoided.

Can this potential problem be evaded by taking the limit $d_{AB} \to \infty$, thereby squeezing the intersection of the past cones of $A$ and $B$ back to the infinite past? The answer is “no” for physically realizable configurations, again because of the finiteness of propagation velocities. In a twin experiment, the particles measured by $A$ and $B$ proceed outwards from a common initial point. If the outward velocities were equal to $c$, the intersection of the past light cones of $A$ and $B$ would remain a constant, equal to the past light cone of the initial point. However, the spin-1 particles used in the KS argument must be massive (zero mass spin-1 particles, such as photons, have two states of helicity $\pm 1$, but no state of helicity 0). Hence the particles measured by $A$ and $B$ must move apart with velocities less than $c$, which
implies that the intersection of their past light cones is a monotone increasing function of time.

(5) **Conway and Kochen’s reply** One can avoid this problem as follows: Let $H'$ be the information contained within the intersection of the past light cones of observers $A$ and $B$ at the latest time potentially involved in a measurement over a KS set of directions. Since the intersection of the past light cones is monotone increasing in time, $H'$ represents all information that is used by observers $A$ and $B$, and in particular, $H'$ contains both the information $H$ and the signal $S(t)$ of Eq. (4). If one replaces $H$ in Eq. (1) by $H'$, one finds, by the original KS argument, that a functional relation

$$S_n^2 = F(H', n)$$

is excluded, which is the result asserted by Conway and Kochen, and with which I now agree. My original concern about this argument was that some measurements in the KS set would be at times before all of the information in $H'$ had arrived, but this is in fact not a problem, since the statement that the result of these measurements is independent of this later information is not in conflict with relativistic causality.