Bright soliton dynamics for seventh degree nonlinear systems with higher-order dispersion

Citation for published version:
Guo, Y, Cheng, Q, Okacha, Y, Ahmed, KA, Wang, Y & Wang, W 2021, 'Bright soliton dynamics for seventh degree nonlinear systems with higher-order dispersion', AIP Advances, vol. 11, no. 8, 085102. https://doi.org/10.1063/5.0054195

Digital Object Identifier (DOI):
10.1063/5.0054195

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Publisher's PDF, also known as Version of record

Published In:
AIP Advances

Publisher Rights Statement:
© 2021 Author(s).

General rights
Copyright for the publications made accessible via Heriot-Watt Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy
Heriot-Watt University has made every reasonable effort to ensure that the content in Heriot-Watt Research Portal complies with UK legislation. If you believe that the public display of this file breaches copyright please contact open.access@hw.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.
Bright soliton dynamics for seventh degree nonlinear systems with higher-order dispersion

Yunsong Guo, Quan Cheng, Yahia Okacha, Karmand Abdulla Ahmed, Ying Wang, and Wei Wang

AFFILIATIONS
1 School of Science, Jiangsu University of Science and Technology, Zhenjiang 212100, China
2 Laboratory of Advanced Optics, Jiangsu University of Science and Technology, Zhenjiang 212100, China
3 Institute of Photonics and Quantum Sciences, School of Engineering and Physical Sciences, Heriot-Watt University Edinburgh, Edinburgh EH14 4AS, United Kingdom

ABSTRACT
In this study, we investigate the typical systems modeled by the (3+1)-dimensional as well as (1+1)-dimensional Schrödinger equations incorporating third-order dispersion effects, higher-order scattering effects, and cubic–fifth–seventh degree nonlinear interactions. We use the F-expansion method and the self-similar method to solve the higher-order Schrödinger equation for one-dimensional and three-dimensional settings, respectively, identifying typical bright soliton solutions under appropriate system settings. The bright soliton features are demonstrated analytically in regions around the soliton peak region. Pictorial bright soliton features are demonstrated for the three-dimensional setting as well as one-dimensional setting. Our work shows the applicability of the theoretical treatment utilized in studying bright soliton dynamics for systems with third-order dispersion and seventh degree nonlinearity.

© 2021 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/). https://doi.org/10.1063/5.0054195

I. INTRODUCTION
Due to the balance of the dispersion effect and scattering effect,1–3 solitons exhibit robust stability in the process of propagation and interaction.4 It is an extremely important phenomenon in modern physics, which has attracted extensive attention in theoretical and experimental research due to its unique properties. The nonlinear Schrödinger equation (NLSE) is an appropriate choice for theoretical study of soliton behavior and plays an important role in nonlinear optics,5 condensate matter physics,6 and plasma physics.7–10 Specifically, when the dispersion effects of the NLSE are considered up to the third order, besides the cubic–fifth degree nonlinearity (in the atomic system) caused by the two-body and three-body effects,11 the nonlinearity up to the seventh degree12–15 by incorporating the multi-body effect should be considered. In addition, the multi-body effect of the quantum system should also be considered, so the higher-order scattering effect should be incorporated.

In this work, we study the (3+1)-dimensional as well as (1+1)-dimensional higher-order Schrödinger equations incorporating the third-order dispersion effect and the cubic–fifth–seventh degree nonlinear interaction,16 and we use the F-expansion17,18 and the self-similar method19–21 to solve the three-dimensional as well as one-dimensional higher-order NLSE under appropriate parametric settings. We first identify the bright soliton solutions by deriving the one-dimensional solution of the bright soliton type, then the three-dimensional bright soliton solution is identified, and we then derive the key characteristics of bright solitons around the location of the soliton peak, demonstrating typical bright soliton features. The obtained analytical results can be used to guide the experimental detection of bright solitons in (3+1)-dimensional as well as (1+1)-dimensional systems with high-order dispersion and seventh degree nonlinear effects.

This work is organized as follows: in Sec. II, the third-order NLSE incorporating cubic–fifth–seventh degree nonlinearity and high-order dispersion terms is analyzed, followed by the
introduction of the F-expansion method. In Sec. III, we use the F-expansion method to solve such a NLSE in the one-dimensional setting, with typical bright soliton analytical features demonstrated and pictorially displayed. Section IV presents the derivation of the bright soliton solution of this category of NLSE in the three-dimensional setting, with typical bright soliton features pictorially demonstrated. Section V presents typical application scenario demonstration of our work and stability analysis of our derived bright soliton solutions. Section VI gives the conclusion remarks.

II. NONLINEAR SCHRÖDINGER EQUATION MODEL AND F-EXPANSION METHOD

A. Nonlinear Schrödinger equation involving cubic-fifth-seventh degree nonlinearity and higher-order dispersion terms

Incorporating higher-order dispersion and cubic–fifth–seventh degree nonlinear interaction effects, the (1 + 1)-dimensional Schrödinger equation is expressed as follows:

\[
\begin{align*}
\frac{i}{\partial t} + a_1 \frac{\partial^2 \psi(x,t)}{\partial x^2} + i a_2 \frac{\partial^3 \psi(x,t)}{\partial x^3} + b_1 |\psi(x,t)|^2 \psi(x,t) \\
+ b_2 |\psi(x,t)|^4 \psi(x,t) + b_3 |\psi(x,t)|^6 \psi(x,t)
\end{align*}
\]

\[
= i \left( \lambda_1 \frac{\partial \psi(x,t)}{\partial x} \right)^2 \psi(x,t) + \lambda_2 \frac{\partial \psi(x,t)}{\partial x} \frac{\partial^2 \psi(x,t)}{\partial x^2} + \lambda_3 \frac{\partial \psi(x,t)}{\partial x} \frac{\partial^3 \psi(x,t)}{\partial x^3} \right). 
\]

(1)

The function \( \psi(x,t) \) denotes the complex valued function of \( x \) and \( t \), where \( x \) and \( t \) denote the space and time coordinates, respectively, in cold atomic systems for example. Additionally, \( t \) denotes the spatial coordinate, and \( x \) denotes time in the optical system. \( a_1 \) and \( a_2 \) denote second and third-order dispersion coefficients, respectively. Furthermore, \( b_1, b_2, \) and \( b_3 \) denote the parameters of the cubic, fifth, and seventh degree nonlinear interactions that are attributed to two-body and three-body effects, respectively, in the atomic system. The parameters \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) denote the coefficients of the leading and higher-order scattering terms. To proceed, we give a general introduction of the F-expansion method to be used.

B. F-expansion method

In order to solve Eq. (1), we use the F-expansion method, which solves the nonlinear partial differential equation taking the following form:

\[ H(u, u_t, u_{xx}, u_{xxx}, \ldots) = 0. \]

Equation (2) is expressed as the polynomial of an unknown function \( u(x,t) \) and its partial derivatives of various orders. The unknown function \( u(x,t) \) is to be expressed by the polynomial of base-function \( F(\xi) \), with \( F(\xi) \) defined as a function of \( \xi = px + qt \) as

\[
\left( \frac{dF(\xi)}{d\xi} \right)^2 = a_0 F^n(\xi) + a_{n-1} F^{n-1}(\xi) + \cdots + a_3 F^3(\xi)
+ a_2 F^2(\xi) + a_1 F(\xi) + a_0, \quad n \geq 4,
\]

(3)

where \( n, n-1, \ldots, a_3, a_2, a_1, \) and \( a_0 \) are constants. Furthermore, if we differentiate both sides of Eq. (3) with respect to \( \xi \), we obtain the transformed equation for \( F(\xi) \) as

\[
2 \frac{d^2 F(\xi)}{d\xi^2} = n a_0 F^{n-1}(\xi) + (n-1) a_{n-1} F^{n-2}(\xi) + \cdots
+ 3 a_3 F^2(\xi) + 2 a_2 F(\xi) + a_1.
\]

(4)

\[
u(x,t) \text{ is expressed as}
\]

\[
u(x,t) = \sum_{i=0}^{m} h_i(t) F^i(\xi), \quad h_m(t) \neq 0,
\]

(5)

where usually \( h_i(t) \) is just a parametric constant (also constant in this work), which is to be determined by the ensuing equation solving steps. By substituting Eq. (5) into Eq. (2) and using Eq. (3) or Eq. (4), we reach the polynomial in terms of \( F(\xi) \) and \( F(\xi) \) times \( dF(\xi)/d\xi \) is set by balancing between the highest-order nonlinear term and highest-order differential term. For example, if in Eq. (2), the highest order for the nonlinear term is \( k (k \geq 2) \), nonlinear term \( u^k \) for example, the highest-order for the differential term is \( r (\frac{d^r}{d\xi^r}) \), for example, we need at least two highest order terms \( F^r \) of order \( s \) in Eq. (2) by substituting Eq. (5) into Eq. (2) and utilizing formula (4) or (5), so that \( h_m \) can be nonzero. The highest order (of \( F^r \)) from the nonlinear term is \( s_1 = km \), the highest order (of \( F^s \)) from the differential term is \( s_2 = r(n/2 - 1) + m \), by letting \( s_1 = s_2 = km = r(n/2 - 1) + m \), we can solve for \( m \). For example, for a traditional cubic nonlinear Schrödinger equation and \( n = 4 \) in definition (3), the solution is \( m = 1 \) [so that ansatz for \( u(x,t) \) is \( u(x,t) = h_1 F(\xi) + h_0 \)]. The resultant analytical form for Eq. (2) is a polynomial of \( F \) on the left-hand side (LHS) and zero on the right-hand side, and the coefficients of the LHS polynomial are expressions of \( h_i(t) \) and \( a_i \).

In order to solve Eq. (2), the coefficients of all terms are set to zero. This generates a series of ordinary differential equations (ODEs, for functions \( h_i(t) \)) or algebraic equations (for parameters \( h_i \)), whose solutions determine the formulation of Eq. (5), so that Eq. (1) is solved accordingly. According to the definitions (4) and (5) of the base function \( F(\xi) \), it is of elliptical function feature in most scenarios with its precise analytical form determined by the coefficient values solved by the ODEs mentioned above.

III. SOLITON SOLUTION FOR (1 + 1)-DIMENSIONAL NLSE WITH SEVENTH DEGREE NONLINEARITY AND HIGHER-ORDER DISPERSION

We consider the traveling wave solution format of Eq. (1) as follows:

\[
\Psi(x,t) = \phi(\xi) e^{A t + B x},
\]

(6)

where \( \xi = px + qt, A, B, p, \) and \( q \) are constants to be determined. \( \phi \) is the modulus of wave function \( \Psi \), that is \( \phi = |\Psi| \). Substituting Eq. (6) into Eq. (1), the expressions of each differential term take the following form:

\[
\Psi_t = \left( i \phi^\prime + B \phi \right) e^{A t + B x},
\]

(7a)

\[
\Psi_{xx} = \left( p^2 \phi^{\prime\prime\prime} + 2i B p \phi^\prime - B^2 \phi \right) e^{A t + B x},
\]

(7b)

\[
\Psi_{xxx} = \left( p^3 \phi^{\prime\prime\prime\prime} + 3i p^2 B \phi^{\prime\prime} - 3p B^2 \phi^\prime - i B^3 \phi \right) e^{A t + B x},
\]

(7c)

\[
\partial \Psi = \left( i \phi^\prime - A \phi \right) e^{A t + B x},
\]

(7d)
where \( \phi' = \frac{d\phi}{dx} \) and \( \phi'' = \frac{d^2\phi}{dx^2} \). Substituting Eq. (7) into Eq. (1), the real part equation of Eq. (1) is

\[
(b + \lambda_1 B)\phi^2 + \lambda_2 B \phi' + \frac{1}{\lambda_3 B} - [a + i B^2 + y B^3] \phi + \frac{1}{a + i B^2 + y B^3} \phi'' = 0.
\]

(8)

The imaginary part of Eq. (1) takes the following form:

\[
3\lambda_1 B \phi^2 \phi' + 5\lambda_2 B \phi^4 \phi' + 7\lambda_3 B \phi^6 \phi' - [q + 2apB + 3pB^2] \phi' + y \phi^3 \phi'' = 0.
\]

(9)

With the appropriate choice of constants \( A \) and \( q \) such that \( A + iaB^2 + yB^3 = 0 \) in Eq. (8), and in Eq. (9), \( q + 2apB + 3pB^2y = 0 \). At the same time, choosing an appropriate value of \( p \),

\[\gamma p = a + 3yB.\]

(10)

Equations (8) and (9) are self-consistent and are unified as the following equation:

\[
\phi'' = \beta_1 \phi^2 + \beta_2 \phi^4 + \beta_3 \phi^6,
\]

(11)

where

\[
\beta_1 = -\frac{b + \lambda_1 B}{a^2} + 3yB^2, \quad \beta_2 = \frac{a^2 + \lambda_2 B}{a^2} + 3yB^2, \quad \beta_3 = \frac{\lambda_3 B}{a^2 + \lambda_2 B}. \]

(12)

Utilizing \( \phi'' = \frac{1}{2} \frac{d\phi^2}{dx} \) and with the following definition:

\[w = \phi^2 = |u|^2.\]

(13)

After integration (we require that the integration constant is zero, since \( \xi \to 0, \varphi \to 0 \), and \( \frac{1}{\xi^2} \to 0 \)), Eq. (11) is transformed to the following form:

\[
\sqrt{\frac{w}{(w - c_1)^2 - c_3^2}} dw = c_5 d\xi,
\]

(14)

where \( c_1 = \frac{1}{2} \frac{\beta_1}{\beta_2^2}, c_2 = \frac{1}{2} \frac{\beta_2}{\beta_2^2}, c_3 = 2\beta_3^2 \), and it can be seen that \( w \) is symmetrical about axis \( \xi = 0 \) in Eq. (14), when \( \xi \) varies from 0 to +\( \infty \), \( w(\xi) \) monotonically increases from the minimum value \( c_1 + c_2 \) to infinity, according to Eq. (13), \( \varphi \) monotonically decreases from the maximum value \( \sqrt{\frac{1}{c_1 + c_2}} \) at \( \xi = 0 \) to the zero value when \( \xi \to \infty \), which are typical features of bright solitons. Such a bright soliton feature is visually shown (in Fig. 1) via numerical evaluation of Eq. (14). According to the traveling wave ansatz [analytical solution (6)] of \( \varphi \), the solution of \( \varphi \) is just a bright soliton solution (with a positive peak of a constant shape and tends to zero as \( \xi \to 0 \), which is the feature of bright soliton).

When \( \xi \to 0, w \to c_1 + c_2, \) near \( \xi = 0, \) ignoring higher-order terms of \( \xi, \) the approximate analytical form of Eq. (14) is

\[
\sqrt{\frac{c_1 + c_2}{(w - c_1)^2 - c_3^2}} dw = c_5 d\xi.
\]

(15)

The approximate analytical solution of Eq. (15), that is, Eq. (14), is

\[\varphi = w^{-\frac{1}{2}} = \sqrt{\frac{1}{c_1 + c_2}} \text{sech} (c_5 \xi).\]

(16)

It is just the classical form of solution of the bright soliton category, which is a traveling wave with stable shape that comes into being by the balance between nonlinear and dispersion effects with the center peak possessing the highest positive amplitude. Figure 1 shows the bright soliton solution plot for \( |\varphi| \) by numerically evaluating Eq. (14) with a typical setting of \( c_5 (i = 1, 2, 3) \). With such a typical bright soliton solution for one-dimensional Eq. (1), we can build its three-dimensional solution based on the self-similar method, which will be studied in Sec. IV.

IV. BRIGHT SOLITON BEHAVIOR OF THREE-DIMENSIONAL SEVENTH DEGREE NONLINEAR SYSTEM WITH HIGHER-ORDER DISPERSION

Incorporating the third-order dispersion and seventh degree nonlinear effects, the three-dimensional nonlinear Schrödinger equation, which is the three-dimensional analog of Eq. (1), takes the following form:
\[
\frac{\partial \psi(r,t)}{\partial t} + i\omega_3 \sum_{i=1}^{3} \frac{\partial^3 \psi(r,t)}{\partial x_i^3} + \omega_3 \sum_{i=1}^{3} \frac{\partial^2 \psi(r,t)}{\partial x_i^2} + g_3 \frac{\partial \psi(r,t)}{\partial (\hat{n} \cdot \vec{r})} \left[ \frac{\partial \psi(r,t)}{\partial \hat{n}} \right] + g_3 \frac{\partial \psi(r,t)}{\partial (\hat{n} \cdot \vec{r})} \left[ \frac{\partial \psi(r,t)}{\partial \hat{n}} \right] = 0,
\]

where \( \hat{n} \) is the velocity direction vector of system flow, \((x_1, x_2, x_3) = (x, y, z) \). To solve Eq. (20), We consider its self-similar ansatz (analytical solution, ...) of the form \( \psi(r,t) = \psi_{3D}(\hat{r}, t) \) as follows:

\[
\psi_{3D}(\hat{r}, t) = \psi_{1D}(v(x,y,z), \tau(t)) e^{i A'(x,y,z)},
\]

where \( \psi_{1D}(v(x,y,z), \tau(t)) = \psi(p(x,y,z) + q(t)) e^{i A(x,y,z)} \) is the analytical solution of one-dimensional analog (1) of Eq. (20), and \( \varphi = |\psi_{1D}| \) is the modulus of \( \psi_{1D} \).

Substituting Eq. (21) into Eq. (20), we obtain

\[
\frac{\partial (\tau(t))}{\partial t} \frac{\partial \psi_{1D}(v,t)}{\partial \tau} + \sum_{i=1}^{3} \left( -3\omega_1 \frac{\partial A'}{\partial x_i} + \omega_2 \right) \left( \frac{\partial \psi_{1D}}{\partial x_i} \right) + \sum_{i=1}^{3} \frac{\partial^2 \psi_{1D}}{\partial x_i^2} + 2\omega_2 \left( \frac{\partial A'}{\partial x_i} \right) \frac{\partial \psi_{1D}}{\partial x_i} + g_3 |\psi_{1D}|^2 \psi_{1D} + g_4 |\psi_{1D}|^4 \psi_{1D} + g_5 |\psi_{1D}|^6 \psi_{1D} + g_6 \sum_{i=1}^{3} \frac{\partial \psi_{1D}}{\partial x_i} \frac{\partial |\psi_{1D}|^2 \psi_{1D}}{\partial \tau} + g_7 \sum_{i=1}^{3} \frac{\partial \psi_{1D}}{\partial x_i} \frac{\partial |\psi_{1D}|^4 \psi_{1D}}{\partial \tau} + g_8 \sum_{i=1}^{3} \frac{\partial \psi_{1D}}{\partial x_i} \frac{\partial |\psi_{1D}|^6 \psi_{1D}}{\partial \tau} = 0.
\]

The self-similar formulation requires that Eq. (22) has the same form as Eq. (1), so the coefficient of the first derivative of \( \psi_{1D} \) with respect to \( v \) in Eq. (22) must be zero, this requires that \( A'(x,y,z) \) and \( v(x,y,z) \) are linear functions of \( x, y, z \), and \( \tau(t) \) is a linear function of \( t \) as follows:

\[
A'(x,y,z) = n_1 x + n_2 y + n_3 z,
\]

\[
v(x,y,z) = k_1 x + k_2 y + k_3 z,
\]

\[
\tau(t) = t.
\]

Substituting Eqs. (24) and (25) into Eq. (21), we obtain the following one-dimensional equation that \( \psi_{1D} \) satisfies:

\[
\frac{\partial \psi_{1D}(v,t)}{\partial t} + i\omega_3 \sum_{i=1}^{3} \frac{\partial^3 \psi_{1D}(v,t)}{\partial x_i^3} + \omega_3 \sum_{i=1}^{3} \frac{\partial^2 \psi_{1D}(v,t)}{\partial x_i^2} + g_3 \frac{\partial \psi_{1D}(v,t)}{\partial (\hat{n} \cdot \vec{r})} \left( \frac{\partial \psi_{1D}(v,t)}{\partial \hat{n}} \right) + g_3 \frac{\partial \psi_{1D}(v,t)}{\partial (\hat{n} \cdot \vec{r})} \left( \frac{\partial \psi_{1D}(v,t)}{\partial \hat{n}} \right) = 0,
\]

where

\[
\omega_1' = \omega(k_1'^2 + k_3'^2 + k_2'^2),
\]

\[
\omega_2' = -3\omega_1 a(n_1 k_2^2 + n_1 k_3^2 + n_1 k_1^2) + \omega_2 (k_1'^2 + k_2'^2 + k_3'^2),
\]

\[
\omega_3' = -3\omega_1 a (n_1 k_3^2 + n_1 k_2^2 + n_1 k_1^2) + 2\omega_2 a (n_1 k_1 + n_1 k_2 + n_1 k_3) = 0.
\]

The degree of freedom for \( n \) is 2, and we can choose the direction of \( \hat{n} \) as \( (1, 0, 0) \); Eq. (27c) becomes

\[
-k_1 a (4a^2 + 3a - 2) = 0.
\]

The solution of the above equation is \( a = 0.425 \). \( \psi_{1D} \) has the form of analytical solution of one-dimensional equation [Eq. (1)], so

\[
\psi_{3D}(\hat{r}, t) = \psi_{1D}(v(x,y,z), \tau(t)) e^{i [(dk_1 + a) x + dk_3 y + dk_2 z + \beta t]},
\]

where \( q, d, and \beta \) are constants determined by the initial conditions of the system. \( \psi_{1D} \) is the form of bright soliton solution. When \( pv + q t \rightarrow 0 \), \( \psi_{1D} \propto \text{sech}^2 \left( \frac{x}{2} \right) \), so \( \psi_{1D} \propto \text{sech} \left[ \left( pk_1 x + pk_2 y + pk_3 z \right) + q t \right] e^{i [(dk_1 + a) x + dk_3 y + dk_2 z + \beta t]} \). The precise waveforms are shown in Fig. 2 by precise numerical evaluation of self-similar ansatz \( \psi_{1D} \). From Eq. (29), we can see that the flow velocity is along the direction of \( \psi_{1D} \), \( \psi_{1D} \propto \text{sech}^2 \left( \frac{x}{2} \right) \), so \( \psi_{1D} \propto \text{sech} \left[ \left( pk_1 x + pk_2 y + pk_3 z \right) + q t \right] e^{i [(dk_1 + a) x + dk_3 y + dk_2 z + \beta t]} \).

V. PRACTICAL APPLICATION OF SOLITON DYNAMICS IN SEVENTH DEGREE NONLINEAR SYSTEM AND STABILITY ANALYSIS

The seventh degree nonlinearity related analysis has important applications in the non-Kerr medium, where the propagation of
optical pulse with ultrashort femtosecond features has the following form:

$$E_i = i\alpha E_{i0} + i\gamma_1 |E_i|^2 E + \alpha_2 (|E_i|^2 E)_t + \alpha_3 E_{i0} + i\gamma_2 E_{i0}$$
$$+ \alpha_4 (|E_i|^4 E)_t + i(\gamma_2 |E_i|^2 E)_t + \alpha_6 (|E_i|^6 E)_t$$
$$+ i\gamma_3 |E_i|^6 E + \alpha_7 (|E_i|^2 E)_i + \alpha_8 (|E_i|^4 E)_i + \alpha_9 (|E_i|^6 E)_t.$$

(30)

This (1 + 1)-dimensional equation [Eq. (30)] (with seventh degree nonlinearity but with only second-order dispersion) is very similar to that shown in Fig. 1 of this work. In addition, the bright soliton profile of the complex value electric field $E$ in Eq. (30) as shown in Fig. 3 in the non-Kerr medium work is very similar to that shown in Fig. 1 of this work.

With regard to the applicability of our theoretical analysis to experimental observation, it is important to perform some stability analysis of the base bright soliton solution $|\psi_{10}\rangle = \varphi = \varphi$ in Eq. (16), with $\varphi$'s amplitude $I(\lambda_1, \lambda_2, \lambda_3) \approx \sqrt{\frac{\varphi}{\varphi_0^0}}$, soliton peak distribution width $\delta \approx \frac{\varphi_0}{\varphi}$. According to the analytical expressions for $\lambda_1$, $\lambda_2$, and $\lambda_3$ following Eq. (14), $\delta$ only depends on the cubic nonlinearity constant $\lambda_1$ via $\lambda_1$ through $\delta$, so it is stable relative to the variation of fifth degree and seventh degree nonlinear strength constants $\lambda_2$ and $\lambda_3$. This means that the soliton is stable (against wave shape spreading) relative to variations of quintic and seventh degree nonlinear strength constants and combining Eq. (12), $\frac{\partial I}{\partial \lambda_1} \approx \frac{\partial I}{\partial \lambda_1} (\lambda_1 - \lambda_1^3/3)$, and $\frac{\partial I}{\partial \lambda_3} \approx \frac{\partial I}{\partial \lambda_3} (\lambda_3 - \lambda_3^3/3)$.

VI. CONCLUSION

In this study, we investigated the (3 + 1)-dimensional as well as (1 + 1)-dimensional higher-order cubic–fifth–seventh degree nonlinear Schrödinger equation (NLSE) with higher-order dispersion effects. We focused on identifying the bright soliton behavior of the system modeled by the higher-order NLSE. We use the F-expansion method and self-similar approach to determine the bright soliton solution. Through numerical evaluation of the inexplicit analytical form of the bright soliton solution, the typical bright soliton characteristics of the three-dimensional solution as well as the one-dimensional solution are identified, and then, the analytical expression of bright soliton solution around the peak region is derived analytically, with the exact characteristics of bright soliton solutions and stability feature illustrated. The theoretical results obtained from the NLSE model with seventh degree nonlinear interaction and the high-order dispersion effect can be used as an experimental guide for detecting the behavior of bright solitons in optical or ultracold atomic systems with seventh degree nonlinearity and higher-order dispersion.

AUTHORS’ CONTRIBUTIONS

Y.G. and Q.C. contributed equally to this work.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation (NSF) of China under Grant No. 11547024 and the Postgraduate Research and Practice Innovation Program of Jiangsu Province under Grant No. KYCX20_3113.

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study. Our study is theoretical and analytical work of the corresponding theoretical models. We do not use previously published data in our work.

REFERENCES

1. M. Shen, H. Ding, Q. Kong, L. Ruan, S. Pang, J. Shi, and Q. Wang, “Self-trapping of two-dimensional dipole solitons in nonlocal media,” Phys. Rev. A 82, 043815 (2010).
2. N. J. Zabusky and M. D. Kruskal, “Interaction of solitons in a collisionless plasma and the recurrence of initial states,” Phys. Rev. Lett. 15, 240 (1965).
3. Y. S. Kivshar and B. Luther-Davies, “Dark optical solitons: Physics and applications,” Phys. Rep. 298, 81 (1998).
4. G. I. Stegeman and M. Segev, “Optical spatial solitons and their interactions: Universality and diversity,” Science 266, 1518 (1999).
5. G. P. Agrawal, Nonlinear Fiber Optics (Academic, San Diego, 2007).
6. D. S. Liu, Z. T. Fu, S. K. Liu, and Z. G. Wang, “Stationary periodic solutions and asymptotic series solutions to nonlinear evolution equations,” Chin. J. Phys. 42(2), 127–134 (2004).
7. S. K. Adhikari, “Coupled Bose–Einstein condensate: Collapse for attractive interaction,” Phys. Rev. A 63, 043611 (2001).
8. S. K. Adhikari and L. Salasnich, “Superfluid Bose–Fermi mixture from weak coupling to unitarity,” Phys. Rev. A 78, 043616 (2008).
9. V. Y. Belashov and S. V. Vladimirov, Solitons in Dispersive Complex Media (Springer, Berlin, 2005).
10. M. Ablowitz and H. Segur, Solitons and the Inverse Scattering Transform (SIAM, Philadelphia, 1981).
11. R. R. He and H. M. Li, “Analytical solitary-wave solutions of the generalized nonautonomous cubic-quintic nonlinear Schrödinger equation with different external potentials,” Phys. Rev. E 83, 066607 (2011).
12. A. S. Reyna and C. B. de Araújo, “Spacial phase modulation due to quintic and septic nonlinearities in metal colloids,” Opt. Express 22, 22456 (2014).
13. A. S. Reyna, K. C. Jorge, and C. B. de Araujo, “Two-dimensional solitons in a quintic-septimal medium,” Phys. Rev. A 90, 063835 (2014).
14. Y.-F. Chen, K. Beckwitt, F. W. Wise, B. G. Aitken, J. S. Sanghera, and I. D. Aggarwal, “Measurement of fifth- and seventh-order nonlinearities of glasses,” J. Opt. Soc. Am. B 23, 347 (2006).
15. J. Jayabalan, A. Singh, R. Chari, S. Khan, H. Srivastava, and S. M. Oak, “Transient absorption and higher-order nonlinearities in silver nanoplatelets,” Appl. Phys. Lett. 94, 181902 (2009).
16. H. Triki, A. Biswas, D. Milović, and M. Belić, “Chirped femtosecond pulses in the higher-order nonlinear Schrödinger equation with non-Kerr nonlinear terms and cubic–quintic–septic nonlinearities,” Opt. Commun. 366, 362–369 (2016).
17. Y. Zhou, M. Wang, and Y. Wang, “Periodic wave solutions to a coupled KdV equations with variable coefficients,” Phys. Lett. A 308, 31 (2003).
18. M. A. Abdou, “The extended F-expansion method and its application for a class of nonlinear evolution equations,” Chaos, Solitons Fractals 31, 95 (2007).
19 Y. Wang and S. Zhou, “Soliton dynamics for trapped Bose–Einstein condensate with higher-order interaction,” AIP Adv. 7, 085006 (2017).

20 Y. Wang, Y. Chen, J. Dai, L. Zhao, W. Wen, and W. Wang, “Soliton evolution and associated sonic horizon formation dynamics in two-dimensional Bose–Einstein condensate with quintic-order nonlinearity,” Chaos 31, 023105 (2021).

21 J. X. Fei and C. L. Zheng, “Exact projective excitations of a generalized $(3 + 1)$-dimensional Gross–Pitaevskii system with varying parameters,” Chin. J. Phys. 51, 200–208 (2013).

22 M. Wang, X. Li, and J. Zhang, “The $(G'/G)$-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics,” Phys. Lett. A 372, 417–423 (2008).