The BigDAWG Polystore System and Architecture

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Abstract—Organizations are often faced with the challenge of providing data management solutions for large, heterogeneous datasets that may have different underlying data and programming models. For example, a medical dataset may have unstructured text, relational data, time series waveforms and imagery. Trying to fit such datasets in a single data management system can have adverse performance and efficiency effects. As a part of the Intel Science and Technology Center on Big Data, we are developing a polystore system designed for such problems. BigDAWG (short for the Big Data Analytics Working Group) is a polystore system designed to work on complex problems that naturally span across different processing or storage engines. BigDAWG provides an architecture that supports diverse database systems working with different data models, support for the competing notions of location transparency and semantic completeness via islands and a middleware that provides a uniform multi-island interface. Initial results from a prototype of the BigDAWG system applied to a medical dataset validate polystore concepts. In this article, we will describe polystore databases, the current BigDAWG architecture and its application on the MIMIC II medical dataset, initial performance results and our future development plans.

I. INTRODUCTION

Enterprises today encounter many types of databases, data, and storage models. Developing analytics and applications that work across these different modalities is often limited by the incompatibility of systems or the difficulty of creating new connectors and translators between each one. For example, consider the MIMIC II dataset [1] which contains deidentified health data collected from thousands of critical care patients in an Intensive Care Unit (ICU). This publicly available dataset (http://mimic.physionet.org/) contains structured data such as demographics and medications; unstructured text such as doctor and nurse reports; and time-series data of physiological signals such as vital signs and electrocardiogram (ECG). Each of these components of the dataset can be efficiently organized into database engines supporting different data models. For example, the structured data in a relational database, the text notes in a key-value or graph database and the time-series data in an array database. Analytics of the future will cross the boundaries of a single data modality, such as correlating information from a doctor’s note against the physiological measurements collected from a particular sensor. Further, the same dataset may be stored in different data engines and leveraged based on the data engine that provides the highest performance response to a particular query.

Such analytics on complex datasets call for the development of a new generation of federated databases that support seamless access to the different data models of database or storage engines. We refer to such a system as a polystore in order to distinguish it from traditional federated databases that largely supported access to multiple engines using the same data model.

As a part of the Intel Science and Technology Center (ISTC) on Big Data, we are developing the BigDAWG, short for Big Data Analytics Working Group, polystore system. The BigDAWG stack is designed to support multiple data models, real-time streaming analytics, visualization interfaces, and multiple databases. The current version of BigDAWG shows significant promise and has been used to develop a series of applications for the MIMIC II dataset. The BigDAWG system supports multiple data stores; provides an abstraction of data and programming models through “islands”; a middleware and API that can be used for query planning, optimization and execution; and support for applications, visualization and clients. Initial results of applying the BigDAWG system to diverse data such as medical imagery or clinical records has shown the value of a polystore system in developing new solutions for complex data management.

The remainder of the article is organized as follows: Section II expands on the concept of a polystore databases and the execution of polystore queries. Section III describes the current BigDAWG architecture and its application to the MIMIC II dataset. Section IV describes performance results on an initial BigDAWG implementation. Finally, we conclude and discuss future work in Section V.

II. POLYSTORE DATABASES

With the increased interest in developing storage and management solutions for disparate data sources coupled with our belief that “one size does not fit all” [3], there is a renewed interest in developing database management systems (DBMSs) that can support multiple query languages and complete functionality of underlying database systems. Prior work on federated databases such as Garlic [4], IBM DB2 [5] and others [6] have demonstrated the ability to provide a single interface to disparate DBMSs. Other related work in parallel databases [7] and computing [8], [9] have demonstrated the high performance that can be achieved by making use

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of replication, partitioning and horizontally scaled hardware. Many of the federated database technologies concentrated on relational data. With the influx of different data sources such as text, imagery, and video, such relational data models may not support high performance ingest and query for these new data modalities. Further, supporting the types of analytics that users wish to perform (for example, a combination of convolution of time series data, gaussian filtering of imagery, topic modeling of text, etc.) is difficult within a single programming or data model.

Consider the simple performance curve of Figure 1 which describes an experiment where we performed two basic operations – counting the number of entries and extracting discrete entries – on a varying number of elements. As shown in the figure, for counting the number of entries, SciDB outperforms PostGRES by nearly an order of magnitude. We see the relative performance reversed in the case of extracting discrete entries.

Many time-series, image or video storage systems are most efficient when using an array data model [10] which provides a natural organization and representation of data. Analytics on these data are often developed using linear algebraic operations such as matrix multiplication. In a simple experiment in which we performed matrix multiplication in PostGRES and SciDB, we observed nearly three orders of magnitude difference in performance time (for a $1000 \times 1000$ dense matrix multiplication, PostGRES takes approximately 166 minutes vs. 5 seconds in SciDB).

These results suggest that analytics in which one wishes to perform a combination of operations (for example, extracting the discrete entries in a dataset and using that result to perform a matrix multiplication operation) may benefit from performing part of the operation in PostGRES (extracting discrete entries) and the remaining part (matrix multiplication) in SciDB.

Extending the concept of federated and parallel databases, we propose a “polystore” database. Polystore databases can harness the relative strengths of underlying DBMSs. Unlike federated or parallel databases, polystore databases are designed to simultaneously work with disparate database engines and programming/data models while supporting complete functionality of underlying DBMSs. In fact, a polystore solution may include federated and/or parallel databases as a part of the overall solution stack. In a polystore solution, different components of an overall dataset can be stored in the engine(s) that will best support high performance ingest, query and analysis. For example, a dataset with structured, text and time-series data may simultaneously leverage relational, key-value and array databases. Incoming queries may leverage one or more of the underlying systems based on the characteristics of the query. For example, performing a linear algebraic operation on time-series data may utilize just an array database; performing a join between time-series data and structured data may leverage array and relational databases respectively.

In order to support such expansive functionality, the BigDAWG polystore system (Figure 2) utilizes a number of features. “Islands” provide users with a number of programming and data model choices; “Shim” operations allow translation of one data model to another; and “Cast” operations allow for the migration of data from one engine or island to another. We go into greater depth of the BigDAWG architecture in Section [III].

III. BigDAWG ARCHITECTURE

The BigDAWG architecture consists of four distinct layers as described in Figure 2: database and storage engines; islands; middleware and API; and applications. In this section, we...
discuss the current status of each of these layers as well as how they are used with the MIMIC II dataset.

A. Database and Storage Engines

A key design feature of BigDAWG is the support of multiple database and storage engines. With the rapid increase in heterogeneous data and proliferation of highly specialized, tuned and hardware accelerated database engines, it is important the BigDAWG support as many data models as possible. Further, many organizations already rely on legacy systems as a part of their overall solution. We believe that analytics of the future will depend on many, distinct data sources that can be efficiently stored and processed only in disparate systems. BigDAWG is designed to address this need by leveraging many vertically-integrated data management systems.

For the MIMIC II dataset, we use the relational databases PostgreSQL and Myria [11] to store clinical data such as demographics and medications. BigDAWG uses the key-value store Apache Accumulo for freeform text data and to perform graph analytics [12]. For the historical waveform time-series data of various physiological signals, we use the array store SciDB [13]. Finally, for streaming time-series data, our application uses the streaming database S-Store [14].

B. Islands

The next layer of the BigDAWG stack is its islands. Islands allow users to trade off between semantic completeness (using the full power of an underlying database engine) and location transparency (the ability to access data without knowledge of the underlying engine). Each island has a data model, a query language or set of operators and one or more database engines for executing them. In the BigDAWG prototype, the user determines the scope of their query by specifying an island within which the query will be executed. Islands are a user-facing abstraction, and they are designed to reduce the challenges associated with incorporating a new database engine.

We currently support a number of islands. For example, the D4M island exposes support for iteration over and efficient casting between the MyriaX, PostgreSQL and SciDB databases. We also support a number of degenerate islands that connect to a single database engine. These degenerate islands provide support for the full semantic power (programming and data model) of a connected database at the expense of location transparency.

C. BigDAWG Layer

The BigDAWG middleware consists of a number of components required to support the multiple islands, programming languages and query types that BigDAWG supports.

1) BigDAWG middleware: The BigDAWG middleware, is responsible for receiving queries, query planning, determining efficient execution strategies, maintaining history of previous queries, and maintaining a record of previous query performance. The architecture of the middleware is shown in Fig. 3. The BigDAWG middleware consists of four modules - planner, monitor, migrator and executor. For a new query, it is first passed to the planner which interacts with the monitor to develop a complete query plan. This is then passed to the executor which leverages the migrator as needed to complete the query.

2) BigDAWG API: The BigDAWG interface provides a simple API to execute polystore queries. The API layer consists of server and client facing components. The server components incorporate the many possible islands which connect to database engines via lightweight connectors referred to as shims. Shims essentially act as an adapter to go from the language of an island to the native language of an underlying database engine. In order to specify how a user is interacting with an island, a user specifies a scope in their query. A scope of a query allow an island to correctly interpret the syntax of the query and allows the Island to select the correct Shim that is needed to execute a part of the query. Thus, a cross-island query may involve multiple Scope operations. For example, let us suppose we have two tables \( A \) and \( B \) in a relational and array database, respectively. Suppose that we want to perform the cross-island query \( \text{ARRAY}(\text{multiply}(\text{RELATIONAL}(\text{select * from A, ...}), B) \) which takes all the data in table \( A \) and multiplies it with all the data in table \( B \). In this case, the inner operation \((\text{RELATIONAL}(\ldots))\) invokes the Relational scope and the outer operation invokes the Array scope. Moving the data between two engines can be done through the Cast operation. The Cast operation sends information about the translation between data models and moves the data as needed. In the example query, this may imply that the results

![Figure 3](image-url)
3) Polystore Queries: Efficient query execution is a key goal of the BigDAWG system. A key challenge is that the data being queried is likely to be distributed among two or more disparate data management systems. In order to support different islands, efficient data movement is also critical. Moreover, efficient execution may also depend on system parameters such as available resources or usage that are prone to change. To illustrate the mechanics of a polystore query, in this section we describe the simplest case where there is no replication, partitioned objects, expensive queries or attempts to move objects for load balancing. Given an incoming query, an execution plan for the query is based on whether the query is in a training or production phase.

The training phase is typically used for execution of queries that are new (either the query is new or the system has changed significantly since the last time a particular query was run) or are believed to have been poorly executed. In the simplest case, the training phase consists of queries that arrive with a “training” tag. In the training phase, we allow the query execution engine to generate a good query plan using any number of available resources. First, the query planner parses the query and assigns the scope of each piece of the query to a particular island. Pieces of the resulting subquery that are local to a particular storage engine are encapsulated into a container and given an identifying signature. For the remaining elements of the query (remainder), which correspond to cross-system predicates, we generate a signature by looking at the structure of the remainder, the objects being referenced and the constants in the query. If the remainder signature has been seen before, a query plan can be extracted. If not, the system decomposes the remainder to determine all possible query plans which are then sent to the monitor.

To execute the query, the monitor feeds the queries to the executor, plus all of the containers which are then passed to the appropriate underlying storage engine. For the cross-engine predicates, the executor decides how to perform each step. The executor runs each query, collects the total running time and other usage statistics and stores the information in the monitor database. This information can then be used to determine the best query plan in the production phase.

In the production phase, when a query is received it is first matched against the various signatures in the monitor database and the optimizer selects the closest one. The BigDAWG optimizer also compares the current usage statistics of the system and compares it against the usage statistics of the system when the training was performed. If there are large differences, the optimizer may select an alternate query plan that more closely resembles the current resources or system usage or recommend that the user rerun the query under the training phase under the current usage. In cases where the signature of the incoming query do not match with existing signatures, the optimizer may suggest the query run in training mode or construct a list of plans as done in the training phase and have the monitor pick one at random. The remaining plans can then be run in the background of the system when it is underutilized. Over time, these plans are then added to the monitor database.

D. Applications and Visualizations

Polystore applications, visualizations and clients may need to interact with disparate database and storage engines. Through the BigDAWG API and middleware, these applications can use a single interface to any number of underlying systems. In order to minimize the impact to existing applications, the “island” interface allows users to develop their applications using the language(s) or data model(s) that most efficiently (or easily) represents the queries or analytics they are developing (or have already developed). For example, an application developed using SQL can leverage the relational island or a scientific application can leverage the array island. In both cases, the applications may talk to the same underlying data engines.

In our current implementation, BigDAWG supports a variety of visualization platforms such as Vega [20] and D3 [21]. Most recently, applying BigDAWG to the MIMIC II dataset allowed for the development of a number of polystore applications:

1) **Browsing**: This screen provides an interface to the full MIMIC II dataset which is stored in different storage engines. This screen utilizes the open source tool ScalaR [22].

2) **“Something interesting”**: This application uses SeeDB [23] to highlight interesting trends and anomalies in the MIMIC II dataset.

3) **“Text Analytics”**: This application performs topic modeling of the unstructured doctor and nurse notes directly in PostGRES.

4) **“Heavy Analytics”**: This application looks for hemodynamically similar patients in a dataset by comparing the signatures of historical ECG waveforms using Myria. We discuss this particular application in detail in Section IV-B.

5) **“Streaming Analytics”**: This application performs analytics on streaming time-series waveforms and can be used for extract-transform-load (ETL) via the data migrator into another database such as SciDB.

IV. **BigDAWG PERFORMANCE**

The current reference implementation of the BigDAWG system satisfies two key performance goals: 1) The polystore architecture of Figure 2 can provide low overhead access to data in disparate engines and 2) Polystore queries can outperform “one size fits all” solutions. In this section, we discuss performance results with respect to these two goals.

A. **BigDAWG overhead**

Providing low overhead access to data is an important element of the BigDAWG system. Low overhead ensures that the BigDAWG middleware and “island” architecture do not
penalize clients or applications for using BigDAWG. Supporting low overhead queries is especially important for applications such as interactive analytics and visualizations [24]. In Figure 4, we show the overhead of executing queries to a single data engine via BigDAWG compared with the time taken for directly querying the database engine through its native interface. As we can observe, for most queries, the overhead incurred by using BigDAWG is a small percentage of the overall query time. There is a minimum overhead incurred which may be a larger percentage for queries of shorter duration.

B. Polystore Analytic: Classifying Hemodynamic Deterioration

To demonstrate the performance advantages offered by a polystore in executing a complex analytical query, we replicated a process described by Saeed & Mark [25]. This process begins by identifying temporally-similar patterns in the physiologic measurements of patient data found in the MIMIC II dataset. These patterns are used as input to a classifier which identifies subsequent patients as being likely (or not) to experience hemodynamic deterioration.

Using the process described by Saeed & Mark, we first compute the Haar-basis transform [26] over the ECG waveforms of training patients. For each patient, we then binned the coefficients over each temporal scale and concatenated the resulting histograms into a single patient vector. We then normalized each patient vector by applying a term frequency-inverse document frequency (TF-IDF) computation. Finally, we classified a test patient by performing a $k$-nearest neighbor computation using these frequency-adjusted vectors.

We first executed this workflow on our polystore prototype under each of the Myria and SciDB degenerate islands. We then performed a multi-island execution designed to capture performance advantages that exist between each of these systems. This execution first computes the Haar-basis transform on SciDB and casts the intermediate coefficients to Myria, where the TF-IDF and $k$-NN computation is performed.

We trained a classifier under each configuration using 256-minute ECG vectors drawn from 600 patients present in the MIMIC II dataset and classified a single test patient. Each execution was performed on a cluster comprised of eight m4.large Amazon EC2 instances (https://aws.amazon.com/).

As illustrated in Figure 5, we found that performance under the hybrid configuration (32 seconds) exceed performance under both the Myria and SciDB islands in isolation (77 and 240 seconds, respectively).

Our results highlight that substantial performance differences exist between systems when executing this complex analytical query. For example, performance of the TF-IDF and $k$-NN computations are substantially faster in Myria than SciDB while the wavelet transform time under SciDB greatly exceeds that of Myria.

For this analytical query, the ability to capture these performance differences under a polystore yields a substantial performance benefit. More generally, our results support the notion that overall query performance may be improved by identifying and leveraging relative strengths of disparate database systems within a polystore, and that this improvement far exceeds the cost of inter-system data casts.

V. CONCLUSION AND FURTHER WORK

Future analytics will require access to disparate database management systems. Previously developed federated and parallel database engines provided a first step towards the solution but were largely limited to working with single data or programming models. The concept of polystore systems extends these concepts to support multiple query languages and disparate DBMSs. We described our architecture for such a polystore system, BigDAWG. A reference version of the BigDAWG architecture has been built and applied to the diverse medical dataset - MIMIC II. Initial performance results validate that a polystore approach to data management can be applied without excessive overhead. Further, initial results on a polystore medical application reinforce the notion that we can achieve greater performance when using multiple storage engines that are optimized for particular operations and data models.

There are many areas of potential improvement of the BigDAWG system. For example, we are interested in developing more complex query planning and execution capabilities, increasing the number of supported islands and engines, and applying BigDAWG to a greater variety of datasets.

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Fig. 5: Polystore analytic applied to medical dataset for 256 minute ECG vectors drawn from 600 patients. The polystore (Myria+SciDB) execution strategy outperforms a “one size” approach of using just Myria or SciDB.

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