Predictions for flow harmonic distributions and flow factorization ratios at RHIC

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Abstract

Data obtained at RHIC can be reproduced by relativistic viscous hydrodynamic simulations by adjusting the viscosity and initial conditions but it is difficult to disentangle these quantities. It is therefore important to find orthogonal observables to constrain the initial conditions separately from the viscosity. New observables have been measured at the LHC and shown to be sensitive to initial conditions and less to medium properties, specifically factorization breaking ratios appears to be promising. Here we consider two models (NeXus and TRENTO) that reproduce these quantities reasonably well at the LHC and then we make predictions for RHIC. While both models lead to similar results for the scaled flow harmonic distributions, they predict very different patterns for the factorization breaking ratios at RHIC.
I. INTRODUCTION

Relativistic heavy ion collisions are being performed at RHIC and LHC to study the Quark Gluon Plasma. It was first discovered as a new state of matter created at high temperature and nearly zero net baryonic density. Now the challenge is to create it at non-zero net baryonic density, explore its phase diagram and unravel its conjectured critical point [1–4]. Experimental facilities around the world are participating in this effort or planning to (BES-RHIC in the USA [5, 6], HADES [7] and FAIR [8–11] in Germany, NICA in Russia [12, 13], J-PARC in Japan).

However, one of the largest uncertainties in the description of the Quark Gluon Plasma is the initial state because it cannot be directly probed by experiments. Rather, experiments only measure the after math of a heavy-ion collisions, following the pre-equilibrium hydrodynamic state, hydrodynamics, and hadronic interactions [14–17]. Thus, one must disentangle different contributions to the final measured flow observables that come from the medium itself vs. the initial state. This is even more challenging at low beam energies where finite $\mu_B$ effects begin to play a role.

Many data can be reproduced by adjusting suitably the fluid viscosities and its initial conditions and it is difficult to disentangle these quantities [14]. However, in recent years it has become clear that some quantities are fairly independent of the fluid transport properties and reflect fluctuations in its initial state [18]. This is the case (as detailed below) of the scaled harmonic flow distributions and flow factorization ratios which have been measured at the LHC. In fact these data provide a rather strong test of initial condition. Therefore, it would be important to have such measurements made at RHIC at least at the highest energy where the equation of state is known from lattice simulations and there are fewer competing effects. Since these measurements have not been made yet, one can test if a model has correct fluctuations by comparing with harmonic flow distributions [19–21] and flow factorization data [22, 23] obtained at LHC energy and then make predictions for RHIC top energy. In this paper, we illustrate this with two initial condition models: NeXus [24] and TRENTO [25].

Here we systematically study the beam energy scaling of the flow distributions and factorization breaking and identify specific centralities and kinematic cuts that are the most promising for distinguishing initial state models. Then we make predictions for top RHIC
II. RELEVANT EXPERIMENTAL OBSERVABLES

A. Flow harmonic distributions

In this work our primary focus will be on the flow harmonics that can be measured at RHIC. In order to calculate them from our theoretical models we expand the particle azimuthal distribution as a Fourier series:

\[
\frac{dN}{d\phi} = \frac{N}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\phi - \Psi_n)] \right],
\]

(1)

with

\[
V_n = v_n e^{i\Psi_n} = \frac{\int d\phi e^{i\phi} \frac{dN}{d\phi}}{\int d\phi \frac{dN}{d\phi}}
\]

(2)

where \(V_n\) is the flow vector and \(\phi\) is the particle transverse momentum azimuthal angle. The magnitude and orientation of the flow vectors, \(v_n\) and \(\Psi_n\), vary from an event to another due to quantum fluctuations in the initial conditions.

Because of the event-by-event fluctuations that occur in the experiment, hydrodynamic models must also incorporate these same fluctuations in order to reproduce experimental results. Thus, one runs a large set of initial conditions (events) through relativistic hydrodynamic simulations, each which produces its own flow harmonic \(v_n\) for that specific event. Then, one can construct a \(v_n\) distribution \(P(v_n)\) from this ensemble of events, which can be directly compared to unfolded distributions from ATLAS [19, 20] and ALICE [21] experimental data.

In fact, these distributions offer a stringent test of initial conditions. Many models fail to reproduce these observables. This is the case [19, 20] of Glauber [26–28] and MC-KLN [29] initial conditions (variations have also been studied [30, 31]). The following models provide satisfactory results: IP-Glasma [32, 33] as shown in [34], EKRT [35] as studied in [36], AMPT [37] and TRENTO [25] as discussed in [38]. Below we show that NeXus also leads to reasonable results.

The scaled harmonic distributions \(P(v_n/\langle v_n \rangle)\) provide information about the amount of fluctuations, and are known to be relatively independent of viscosity [36, 39] (and small scale

energies.
structure [30, 40]), which can be understood as follows. For a given centrality class, where the average value of \( v_n \) is \( \langle v_n \rangle \), the initial conditions drive the magnitude of \( v_n \) compared to \( \langle v_n \rangle \) (so do not cancel in \( v_n/\langle v_n \rangle \)) while the fluid properties are approximately similar for a given event and the average of events (so should cancel in \( v_n/\langle v_n \rangle \)). Thus, the width of the \( P(v_n/\langle v_n \rangle) \) distributions depends on having the right amount of fluctuations in the initial conditions.

A more precise understanding of the relationship between \( P(v_n/\langle v_n \rangle) \) and the initial conditions can be obtained by defining the event eccentricities:

\[
\epsilon_{m,n} e^{i\Phi_{m,n}} = \frac{\int r dr d\phi r^m e^{in\phi} \rho(r, \phi)}{\int r dr d\phi r^m \rho(r, \phi)}
\]  

(3)

where \( r \) and \( \phi \) are the spatial radius and azimuthal angle. When \( m = n \), the simplified notations \( \epsilon_n \) and \( \Phi_n \) are used, which we will consider here. For \( n = 2 \) or 3, it has been shown that in a given centrality window, \( v_n \propto \epsilon_n \) holds not just in average [41, 42] but approximately for each event [36, 39, 43]. One expects therefore \( P(v_n/\langle v_n \rangle) \sim P(\epsilon_n/\langle \epsilon_n \rangle) \) for \( n = 2 \) and 3 and can even been shown for high \( p_T \) and heavy flavor [44–47]. Deviations from this are expected e.g. \( v_2 \) departs from linear growth with \( \epsilon_2 \) for non-central collisions [36, 39, 43, 48], small systems [49], and low beam energies [50]. For \( n > 3 \), the simple relation \( v_n \propto \epsilon_n \) also breaks. For instance, \( v_4 \) is expected to be linear in \( \epsilon_4 \) for central collisions while it has an \( \epsilon_2^2 \) behavior for non-central ones (see e.g. [41, 42]), so the relation \( P(v_4/\langle v_4 \rangle) \sim P(\epsilon_4/\langle \epsilon_4 \rangle) \) might hold but only for central collisions. Scalings such as \( v_n \propto \epsilon_{m,n} \) were studied in [36, 43, 51] and analytical approximations for the eccentricity distributions were proposed in [52–55].

**B. Flow factorization ratios**

Above we considered momentum integrated observables, however more detailed information can be obtained from differential quantities, such as the Pearson correlation between flow vectors in different momentum range known as the flow factorization ratio [56]:

\[
r_n(p_1, p_2) = \frac{V_{n\Delta}(p_1, p_2)}{\sqrt{V_{n\Delta}(p_1, p_1)V_{n\Delta}(p_2, p_2)}}
\]  

(4)
where the $V_n\Delta(p_1, p_2)$ are the Fourier coefficients of the averaged pair correlation $dN_{\text{pairs}}/d^3p_1d^3p_2$. It can be re-written as:

$$r_n(p_1, p_2) = \frac{\langle v_n(p_1)v_n(p_2) \cos n(\psi_n(p_1) - \psi_n(p_2)) \rangle}{\sqrt{\langle v_n^2(p_1) \rangle \langle v_n^2(p_2) \rangle}}.$$  \hspace{1cm} (5)

In hydrodynamics $r_n$ is $\leq 1$ because of the fluctuations in the event-plane angles $\psi_n(p_T)$ and in the harmonic flow magnitudes $v_n(p_T)$ ($\langle v_n(p_1)v_n(p_2) \rangle \neq \sqrt{\langle v_n^2(p_1) \rangle \langle v_n^2(p_2) \rangle}$), and as also happened with scaled flow, factorization breaking is not very sensitive to viscosity [22, 57, 58], but can probe the transverse size of the fluctuations in initial conditions [40, 57].

Data for these ratios were obtained at the LHC by CMS [22] and ALICE [23], providing another strong test of initial condition models. It was observed in [22, 58] that neither MC-Glauber nor MC-KLN was compatible with data in the whole range of $p_T$ intervals and centralities. Though leading to reasonable results for $r_2$, AMPT, TRENTO and IP-Glasma predict a drop in $r_3$ for large $p_T$ differences stronger than in data [33, 38].

III. HEAVY-ION SIMULATIONS

A. Initial conditions

Here we compare two different initial conditions models relevant for both LHC (PbPb 2.76 TeV) and RHIC (AuAu 200 GeV) energies. NeXus [24, 59, 60] is based on a partonic string model inspired on Gribov-Regge theory. Among models that have initial nucleon-size fluctuations, NeXus has the advantage to have been thoroughly tested at top RHIC energy and provides a coherent picture of data [61–68], and has also been compared to some of the LHC data [69, 70].

We will then contrast NeXus with the TRENTO initial condition framework that is a phenomenological model that parameterizes the initial conditions. Then the parameter space is constrained using a Bayesian analysis [25, 71]. We use the central values from their Bayesian analysis such that $p = 0$, $k = 1.6$ and $\sigma = 0.51$. This parameters set has been shown to fit to charged particle yields, $\langle p_T \rangle$, and event-by-event flow fluctuations [18, 72, 73]. Additionally, many other theoretical predictions exist from Trento that remain to be confirmed [74–77]. However, we note that correlations of $\langle p_T \rangle$ and $v_n$ fail to reproduce experimental data [78, 79], it is not clear if this deviations arises from the initial state or hydrodynamic response.
B. Relativistic hydrodynamics

In order to connect initial conditions to the final flow harmonics, each initial condition is fed into a 2+1 or 3+1 relativistic hydrodynamic simulation that is followed by a hadronic phase. Following the hadronic phase then one can reconstruct the particle spectra (in this paper we consider only all charged particles). After the particle spectra is obtained, one can calculate the relevant experimental observables following the description in Sec. II. Each initial condition model requires specific parameters in the hydrodynamic simulation in order to reproduce experimental data. Thus, for simplicity’s sake, we couple each individual initial condition to the respective hydrodynamic model that they are commonly coupled to. In this paper we then consider NeXSPHeRIO (NeXus+SPheRIO) and Trento+v-USPhydro and will describe each respective set-up below. However, we remark that both codes use the same numerical solver know as Smoothed Particle Hydrodynamics, which is a Lagrangian method [80] to solve the equations of motion. There are two crucial differences between SPHeRIO and v-USPhydro, which are the dimensionality and out-of-equilibrium effects: SPHeRIO is a 3+1 ideal hydrodynamic code whereas v-USPhydro is a 2+1 viscous hydrodynamic code.

First we discuss the 3+1 ideal hydrodynamic framework of NeXSPHeRIO [80]. The equation of state has a critical point introduced phenomenologically [81]. A Cooper-Frye freeze out is used with temperatures adjusted to reproduce observables for each centrality window. Tests of this code against known solutions can be found in [80].

In contrast, v-USPhydro [82, 83] can incorporate both shear and bulk viscosity in 2+1 dimensions where longitudinal boost invariance is assumed. For simplicity, only shear viscosity is considered where $\eta/s \sim 0.05$ and assumed constant. TRENTO initial conditions are used. The most state-of-the-art Lattice QCD equation of state [72] at $\mu_B = 0$ coupled to the particle list PDG16+ [84] that includes all *-**** states from the Particle Data group. A constant temperature $T_{FO} = 150$ MeV Cooper-Frye freeze out is employed. Tests of this code against known solutions were presented in [82, 83].

IV. RESULTS

First we check how these models perform for the ATLAS scaled flow harmonic distributions at PbPb 2.76 TeV [20] for $v_2$, $v_3$, and $v_4$. Fig. 1 shows that results coming from
NeXus initial conditions agree with data. It has already been shown that TRENTO can reproduce these results for parameters similar to our own in [38] so we do not repeat those results here. Therefore, both NeXus and TRENTO have the right amount of fluctuations in the initial conditions, since scaled flow distributions approximately follow scaled eccentricity distributions. As expected, the eccentricity and flow harmonics distributions are nearly the same for these centralities (0-5% and 20-25%). The one exception is for $v_4$ in the 20-25% bin, which is unsurprising since $v_4$ experiences both a linear and non-linear response (from mixing with $v_2$). In fact, reproducing the sign change of $v_4$ is an open problem in the field [85].

![Graph of scaled eccentricity and flow harmonics distributions](image)

FIG. 1. Comparison of scaled $\epsilon_n$ and $v_n$ distributions from NeXSPheRIO ($|\eta| < 2.5$) with ATLAS data ($|\eta| < 2.5$) [20] in the range $0.5 \text{ Gev} < p_t$ for Pb+Pb collisions at 2.76 TeV. (As expected the right side tail of the scaled $\epsilon_n$ may be narrower than the $v_n$’s one.)

We now turn to the energy dependence of these distributions: the scaled eccentricity distributions are not expected to depend strongly on the beam energy, so the same should hold for the scaled $v_n$ distributions. Predictions for $P(v_n/\langle v_n \rangle)$ at 5.02 A TeV as well as
data and comparison at PbPb 2.76 TeV were presented for IP-Glasma [33], AMPT and TRENTO [38]. No difference was observed between these two LHC energies in all cases. In figure 2, NeXSPheRIO $v_n$ distributions are shown for LHC and top RHIC energies. They are also fairly similar even though the energies are more different. From Fig. 2 we see that RHIC has slightly larger fluctuations than LHC for both $v_2$ and $v_3$, this is consistent with [50, 72] which showed that $v_2 \{4\} / v_2 \{2\}$ is smaller at RHIC than the LHC (implying that fluctuations are larger at RHIC).

![Comparison of scaled $v_n$ distributions for NeXSPheRIO ($|\eta| < 2.5$) at LHC and RHIC energies for the 20-30% centrality window and 0.5 GeV < $p_T$ in the case of Pb+Pb collisions at 2.76 TeV.](image)

Finally, predictions for scaled $v_n$ distributions in Au+Au collisions at 200 GeV at various centralities are shown in fig. 3 and 4, comparing TRENTO and NeXus. Unsurprisingly, they are relatively equivalent but we do find that there are some subtle differences in their centrality dependence. TRENTO appears to have fewer fluctuations in central collisions compared to NeXus for both $v_2$ and $v_3$. Additionally, for $v_2$ in peripheral collisions it appears to be reversed in that NeXus then has slightly narrower fluctuations than TRENTO.

We now turn to the flow factorization ratios. As mentioned in Sec. II B, compared to CMS data [22], TRENTO provides good results for $r_2$ in all centralities but $r_3$ exhibits a too large drop for all trigger $p_T$’s and centralities [38]. As can be seen in Fig. 5, similar results hold for NeXus. This effect is more pronounced in central collisions, which also happens to
FIG. 3. Predictions for scaled $v_2$ distributions for NeXus and TRENTO initial conditions for Au+Au collisions at 200 GeV.

coincide with the $v_2$ to $v_3$ puzzle [86–92]. Additionally, it also appears that our theoretical calculations demonstrate the largest suppression compared to the data when $p_T^{(a)}$ is large. This may indicate some physical mechanism that is suppressing sources of fluctuations and is not yet included in our simulations plays a role at higher $p_T$. Even for mid-central collisions we find a similar discrepancy for $r_3$, however, the deviation is significantly smaller.

We can address the energy dependence of these ratios. Predictions for $r_n$ at PbPb 5.02 TeV as well as data and comparison at 2.76 TeV were presented for IP-Glasma [33], AMPT and TRENTO [38]. Some difference was observed between these two LHC energies with a tendency towards more factorization breaking at lower energy. A comparison of factorization breaking for LHC and RHIC energies with TRENTO initial conditions is shown in Figs. 6 and 7. Factorization breaking is always larger at RHIC than at LHC, for $r_2$ (less for $r_3$) up to 40% centrality. For less central collisions, it becomes smaller than at LHC. Generally, central collisions at higher $p_T$ appear to have the largest factorization breaking. However, we note
that this is precisely the regime where our predictions struggle to reproduce experimental results so this regime still requires more theoretical effort.

We note also that $r_3$ has a much weaker centrality dependence compared to $r_2$, which is likely due to the fact that $v_2$ also has a geometrical component to it when one varies centrality whereas $v_3$ is entirely fluctuations driven. Because $v_3$ is primarily driven by fluctuations, which exist regardless of the centrality class then it is not surprising that there is not a strong centrality dependence.

Next we compare central NeXus initial conditions between RHIC and the LHC. A larger breaking is seen at RHIC than at LHC for $r_2$ (and not for $r_3$) in Fig. 8, which is very similar to what we found for the TRENTO results in Figs. 6-7. We note, however, that the difference in $r_2$ for higher $p_T$ demonstrates a smaller difference between RHIC and LHC compared to what was seen for the TRENTO initial conditions. Thus, it would be interesting to have factorization breaking measurements from RHIC to further distinguish between initial state models.
FIG. 5. Comparison of $r_n$ from NeXSPheRIO ($|\eta| < 2.5$) with CMS data [22] ($|\eta| < 2.4$) for Pb+Pb collisions at 2.76 TeV.

In order to understand how higher energies imply smaller breaking of factorization, we have selected 600 events in a central window and solved the ideal hydrodynamic equations at RHIC. Then exactly the same events had their energy density scaled by a factor of 5, and the hydrodynamic equations were solved. The results shown in Fig. 9, indicate that the
factorization breaking for $n = 2$ (and less for $n = 3$) is stronger at RHIC and this appears to be directly connected to the overall energy density scale, which leads to a longer lifetime at RHIC vs. the LHC. Thus, one can argue that at high-energies due to the longer lifetime of hydrodynamics, one would expect $r_n \rightarrow 1$.

To see which part of the flow vector was more affected, the following quantities (for
FIG. 7. Comparison of $r_3$ for Trento at LHC and RHIC energies for various centrality windows.

$p_T^{(a)} = 2.5 \text{GeV}$) were computed:

$$r_{vn} = \left\langle v_n(p_T^{(a)})v_n(p_T^{(b)}) \right\rangle / \sqrt{\left\langle v_n^2(p_T^{(a)}) \right\rangle \left\langle v_n^2(p_T^{(b)}) \right\rangle}$$  \hspace{1cm} (6)$$

and

$$r_{\psi n} = \left\langle \cos n(\psi_n(p_T^{(a)}) - \psi_n(p_T^{(b)})) \right\rangle.$$  \hspace{1cm} (7)$$
Eq. (6) is a Pearson coefficient between different $p_T$ bins of the flow harmonics and Eq. 7 demonstrates the decorrelation of event plane angles across $p_T$. Note similar studies have been performed in [45] to study high $p_T$ flow harmonics.

Although these quantities are not independent and their product is not exactly $r_n$, we expect that their individual behaviors can help to understand the physical picture. In Fig. 9, the $r_{v_n}$ curves are seen to be almost energy independent, while the $r_{\psi_n}$ ones are energy dependent. The energy independence of $r_{v_n}$ implies that the magnitude of the $v_n$’s vs. $p_T$ generally scales in the same way regardless of the beam energy. The $r_{\psi_n}$ can be understood, since longer lifetimes (LHC energy) lead to different event planes tending to have the same direction due to pressure gradients in the hydrodynamic expansion. The longer that one runs hydrodynamics, the more the event plane angles are aligned.

Now that we have better understood the scaling of the factorization with beam energy and subtle differences between Trento and NeXus initial conditions, we present the predictions of the factorization breaking ratios at RHIC for TRENTO and NeXus in Fig.s 10-11. We find that for $r_2$ low $p_T$ values are nearly identical, it is only for $p_T^{(a)}$ in bins above $p_T^{(a)} > 2$ GeV that one can distinguish between TRENTO and NeXus. Additionally, we find that high $p_T^{(a)}$ in central collisions is by far the most important region to distinguish between different
FIG. 9. Comparison of factorization breaking for initial conditions at RHIC top energy and the same initial conditions with energy density multiplied by a factor 5. Top row: $r_2$ and $r_3$, middle row: $r_{v2}$ and $r_{v3}$, bottom row: $r_{\psi 2}$ and $r_{\psi 3}$ (definitions in text). $p_T^{(a)} = 2.5\, GeV$.

Comparing our results for both $r_2$ and $r_3$, we find that $r_2$ also is a better candidate for distinguishing initial state models. While subtle differences exist for $r_3$ for TRENTO vs. NeXus in Fig. 11, they would require much more precise experimental data to distinguish between models.
FIG. 10. Predictions for $r_2$ for NeXus and TRENTO models for Au+Au collisions at 200 A GeV.

V. CONCLUSION

Because RHIC also is more sensitive to a variety of medium effects compared to the LHC (i.e. the finite $\mu_B$ equation of state, diffusion, criticality etc), it is that much more important to find medium independent observables that can help to constrain initial state effects. In this paper we concentrate on two quantities that are not very sensitive to viscosity and
compare predictions from two initial condition models, NeXus and TRENTO (with $p = 0$
configuration). We found that the scaled $v_n$ distributions that were used to rule out initial
condition models at the LHC do not provide new constraints at top RHIC energy. Here we
argue that factorization breaking will be sensitive to the choice in the initial state at RHIC
and outline the best centrality and $p_T$ windows to study. Generally, we find that $r_2$ is a

FIG. 11. Predictions for $r_3$ for NeXus and TRENTO models for Au+Au collisions at 200 A GeV.
better candidate for distinguishing initial state models and specifically in central collisions and using higher $p_T$ cuts.

Our main conclusions that we have found from the factorization breaking ratios, $r_n$, is that they seem to be an excellent tool to constrain initial condition models at top RHIC energy: 1) models that lead to fairly similar results at the LHC have very different predictions for RHIC, 2) factorization breaking can be substantially stronger at RHIC (for central collisions). In a previous paper [40], we argued that smearing the initial hot spots (without altering large scale structure) has little effect on a range of variables (integrated $v_n$, scaled $v_n$ distributions, normalized symmetric cumulants, event-plane correlations, $p_T$ distributions and $v_n(p_T)$) but modifies $r_n$. The difference between $r_n$’s for NeXus and TRENTO, that have different scales for their initial inhomogeneities, are a nice illustration of this.

In order to better understand how the factorization breaking observable scales with beam energy, we have calculated two new quantities that correlate either the magnitude of flow harmonics across $p_T$, $r_{vn}$, or the correlation between event-plane angles across $p_T$, $r_{\psi n}$. We found that the correlation between overall magnitudes of flow harmonics is not strongly sensitive to the lifetime of hydrodynamics. In contrast, though, the correlation of event plan angles is more sensitive to the lifetime of the system.

While not done in this initial study, it may be interesting to also calculate the sensitivity of factorization breaking ratios at RHIC for small systems and lower beam energies. We leave this as a potential future work.

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