Computing the Transfer Pricing for a Multidivisional Firm Based on Cooperative Games

Abstract. This paper suggests a novel cooperative game-theoretic approach to solve the transfer pricing problem for a multidivisional firm considering that each division purchases goods from an upstream division in the supply chain. The formulation of the transfer pricing problem considers costs and taxes in a vertically integrated supply chain. We conceptualize the transfer pricing as a multi-objective problem and present a method for finding the strong Nash equilibrium that maximizes the utility of the entire multidivisional firm. The approach consists on determining a scalar $\lambda^*$ and the corresponding strategies $\nu^*(\lambda^*)$ fixing specific bounds on the Pareto front. Bounds correspond to restrictions imposed by each division over the Pareto front that determine the maximum and minimum transfer price legally authorized. For ensuring the existence of a unique strong Nash equilibrium, we employ a penalized regularization method for poly-linear functions. We implement a recurrent procedure for finding the extremal point. For finding the strong Nash equilibrium, we select the Pareto optimal strong strategy $\nu^*(\lambda^*)$ with minimal distance to the utopia point. We show that the regularized functional of the game decreases and converges, proving the existence and uniqueness of strong Nash equilibrium. The usefulness of the method is successfully demonstrated by a numerical example.

Keywords: Transfer pricing cooperative games strong Nash equilibrium multidivisional firm regularization multi-objective.

1. Introduction

1.1. Brief review

The problem of transfer pricing employs optimization goals to set internal prices of goods or services that are sold between subsidiaries or divisions of a firm [8]. Several pricing policies are followed in practice: marginal cost, market price, average cost, etc. and particular solutions (negotiations) adapted to specific situations. In a
decentralized system, each divisional manager identifies independent opportunities other than the opportunity of trading internally with other divisions. Then, each division estimates the performance measures (revenues or profits) of each opportunity and selects the best opportunity. As a result, the transfer pricing system focuses on the maximization of divisional income, leading the divisions to achieve the goals of the firm through sub-optimization. However, sub-optimizations by individual divisions do not result in an optimum for the firm.

The transfer pricing problem needs to solve transfer prices considering that each division independently purchases goods from an upstream division in the supply chain. Transfer prices are determined in such a way that divisional sub-optimizations imply an overall optimum for the firm as whole. Then, transfer pricing problem has to consider the allocation of a global profit between subsidiaries or divisions of a firm. As a result, it is usually employed for tax avoidance by firms that lower profits in subsidiaries or divisions located in high-tax places and increase profits in divisions located in low-tax places. Because it is considered an instrument for allocating the total profit of a multidivisional firm, transfer pricing plays a fundamental role in the decision making for buying or selling subsidiaries or divisions [15].

1.2. Related works

Related works presented in the literature have identified that the transfer pricing problem is inherently a multi-objective problem for optimally and developed transfer pricing methodologies based on mathematical programming and game theory. The seminal paper on transfer pricing was presented by Hirshleifer[15] who considering a system that focus on the maximization of divisional income suggested that the precise transfer price for a good is the marginal cost of the producing division. Horst[16] explored the profit-maximizing strategy for a monopolistic firm selling to two national markets simultaneously. Kassicieh[18] presented a model of transfer pricing that is developed further to include all of the issues that affect the total profits of the multinational corporation through transfer prices and expanded to include economic externalities and interdependencies such as nonlinear cost functions and functional relationships among demand, supply and transfer prices. Luft and Libby[22] examined experienced managers' judgments about the effects of market price and accounting profit information on negotiated transfer prices. Lakhal[19] developed a method using a mathematical programming model and providing companies an opportunity to work proactively with the internal revenue service in a cooperative manner in order to avoid costly audit and litigation. Lakhal et al.[20] proposed a framework and methodology for profit sharing and transfer-pricing between network companies presenting a paradigm that enables maximization of operating profits by the manufacturing-network in its
larger supply chain, suggesting a departure from the model that maximizes profits for the individual company within the sphere of its own supply chain. Shunko and Gavirneni[25] showed that randomness in a supply chain magnifies the impact of transfer prices and analyzed possible reasons behind this behavior and summarized the impact of various supply chain parameters on the magnitude of profit improvement. Rosenthal[24] developed a cooperative game that provides transfer prices for the intermediate products in a vertically integrated supply chain. Leng and Parlarb[21] considered the transfer pricing decision for a multidivisional firm with an upstream division and multiple downstream divisions solving the problem using a cooperative game which computes the Shapley value-based transfer prices for the firm. Huh and Park[17] compared the supply chain profits for transfer pricing suggesting that the firm-wide and divisional profits tend to be higher under the cost-plus method than they are under the resale-price method. Hammami and Frein[14] developed an optimization model for supply chain addressing decisions, costs, and complexity factors integrating transfer pricing to derive a series of insights that may be helpful for companies and governments. Clempner and Poznyak[8] proposed a solution to the transfer pricing problem from the point of view of the Nash bargaining game theory approach.

1.3. Main results

Transfer pricing is related to the pricing of an intermediate product or service within a firm. The products or services are collaborative transferred or relocated between divisions of the firm. As a consequence, transfer pricing refers to the allocation of profits in a supply chain. Encouraged by the significant impact of transfer pricing methods for tax purposes on operational decisions and the corresponding profits of a supply chain we propose a model for a multidivisional firm in a vertically integrated supply chain. We suggest a new game-theoretic approach to solve transfer prices considering that each division purchases goods from an upstream division in the supply chain, and in which transfer prices are determined in such a way that divisional sub-optimizations imply an overall optimum for the firm as whole. The main results of this paper are as follows:

- We propose a method for computing the transfer pricing for a multidivisional firm.
- We suggest a multi-objective approach for representing a cooperative game.
- For computing the strong Nash equilibrium, the method finds the Pareto optimal point with minimal distance to the utopia point.
The Tikhonov’s regularization [3, 2, 13] method is used to guarantee the convergence to a single (strong) equilibrium point.

We propose a method based on a penalized programming approach for computing the regularized strategies of the game.

We develop a programming method to solve the successive single-objective constrained problems that arise from taking the regularized functional of the game.

We implement a recurrent procedure based on the gradient method to solve the regularized problem and finding the Pareto optimal strategies.

We show that in the regularized problem the functional of the game decreases and finally converges, proving the existence and uniqueness of strong Nash equilibrium.

In addition, we present the convergence conditions and compute the estimate rate of convergence of variables $\mu$ and $\delta$ corresponding to the step size parameter of the regularized penalty method.

We provide all the details needed to implement the method in an efficient and numerically stable way.

The usefulness of the method is successfully demonstrated by a numerical example.

1.4. Organization of the paper

The remainder of the paper is organized as follows. The next Section suggests the formulation of the problem for transfer pricing. Section 3 presents the cooperative game model describing the Pareto front, the restrictions over jurisdictions imposed by governments and the strong Nash equilibrium of the problem. Section 4 describes the regularized penalty function optimization method. Section 5 concludes the paper with some remarks and future work. The appendix presents the proofs of the theorems.

2. Formulation of the problem

Transfer pricing refers to the pricing of an intermediate product or service within a firm. This product or service is transferred between divisions of the firm. Thus, transfer pricing is closely related to the allocation of profits in a supply chain.

Problem: The transfer pricing problem consists in fixing transfer prices in such a way that divisional sub-optimizations imply an overall optimum for the firm as whole.

Remark 1. No division, or subset of divisions, can obtain a greater profit from being outside the supply chain.
Computing the Transfer Pricing for a Multidivisional Firm Based on Cooperative Games

An example is as follows. American Petroleum (AM) is the world’s top producer of petroleum and natural gas hydrocarbons. AM has a refinery called Refinery American Petroleum (RAM) in U.S.A. which produces A-oil. Petroleum products are materials derived from crude oil (petroleum) as it is processed in oil refineries. RAM established a corporation CAM as a subsidiary to distribute product A-oil in Europe. Then, AM sells petroleum to RAM, that sells the A-oil to CAM which then sells it to approximately 1000 third-party retailers in Europe. The function performed by CAM are that of purchase of inventories of product A-oil and the sale of these inventories to final retailers. Both, RAM and CAM buy products (petroleum and A-oil) at negotiated transfer prices in order to maximize the global profit of the firm. In calculating the product's price, it must be considered the costs and taxes involving for the sales transactions of petroleum from company AM to company RAM, those involving the sales transactions of A-oil from company RAM to company CAM and finally, the costs and taxes involving the sales to the retailers.

The formulation of the transfer price for the multidivisional firm considers a vertically integrated supply chain. The game consists of \( N \) divisions or players (denoted by \( l = 1, \ldots, N \)) which jointly make their decisions to maximize the global profits of the firm. For solving the problem, the multidivisional firm should compute the optimal strategy under an allocation scheme able to get the global profit maximization. We develop a cooperative game model for computing the transfer pricing for a multidivisional firm based on a multi-objective approach.

Let us consider for \( l = 1, \ldots, N - 1 \), intermediate goods are shipped from level \( l \) to level \( l + 1 \), i.e., downstream along the supply chain. Then, we develop the utility functions for divisions. Division \( l \) makes its market pricing decision \( p_l \) and sells \( q_l \) units of its final products. Following [1], we can consider that \( q_l \) is determined by a linear demand function, i.e., \( q_l = \alpha - \beta p_l \) where \( \alpha, \beta > 0 \). Because of the existence of economies of scale, we consider that the division's unit production cost is dependent on the production quantity. Then, the unit production cost which is incurred by division \( l \) when the division sells \( q_l \) units of intermediate products is represented by \( c_l(q_l) \). As well, we represent the taxes that a division \( l \) has to pay as function depending on the product and the quantity represented by \( r_l(p_l'q_l) \). We do not consider any specific function for the

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1 This example describes the treatment for transfer pricing cost-taxation purposes and the aim is to illustrate the key points to take into consideration.
costs and the taxes, and we use the general form $c^i(q^i)$ and $r^i(p^i, q^i)$ for our analysis. Then, our variables are defined as follows:

- Market prices $p^i$ for one intermediate good (sold from $l$ to $l+1$).
- Quantities $q^i$ of intermediate good/shipped from/to $l+1$ for $l=1, \ldots, N-1$.
- Production costs $c^i(q^i) := \theta^i_c q^i, \theta^i_c \geq 0$ (e.g.: transactional costs, raw materials, components, and their per period inventory costs in dollars) at each level $l = 1, \ldots, N$.
- Taxes $r^i(p^i, q^i) := \theta^i_r p^i q^i, \theta^i_r \geq 0$, at each level $l = 1, \ldots, N$.

We suppose that all the division are located in different marketing areas, therefore they face independent demands $q^i$, costs $c^i(q^i)$ and taxes $r^i(p^i, q^i)$. Then, the division's utility $U^i$ for each level $l = 1, \ldots, N$ is given by

$$U^i(p^i, q^i) = p^i q^i - c^i(q^i) - r^i(p^i, q^i)$$

$$U^i(p^i, p^{i-1}, q^i) = (p^i - p^{i-1}) q^i - c^i(q^i) - r^i(p^i, q^i)$$

for $l = 2, \ldots, N$ where $l = 1$ represents the first division and $l = 2, \ldots, N$ the rest of the divisions on the vertically integrated supply chain.

3. The cooperative game model

3.1. Pareto set

The Pareto set can be defined as [6, 7, 11, 12]

$$\mathcal{P} := \left\{ v^*(\lambda) := \max_{v \in \mathcal{V}} W(v|\lambda), \lambda \in S^N \right\}$$

(1)

Where

$$W(v|\lambda) := \lambda U^1(p^1, q^1) + \sum_{l=2}^N \lambda U^l(p^l, p^{l-1}, q^l)$$

$$v = \left\{ (p, q) : p = (p_1, \ldots, p_N)^T \text{ and } q = (q_1, \ldots, q_N)^T \right\}$$

$$S^N := \left\{ \lambda \in \mathbb{R}^N : \lambda \geq 0, \sum_{l=1}^N \lambda_l = 1 \right\}$$

with the Pareto front given by

$$U(v^*(\lambda)) = \left[ U^1(v^*(\lambda)) U^2(v^*(\lambda)) \ldots U^N(v^*(\lambda)) \right]$$

3.2. Constraints

Multidivisional firms look for coordinate its divisions in order to maximize the global profit of the firm after cost-tax through the transfer pricing decision. For instance, if a given upstream division is located in a lower-cost-tax jurisdiction, then increasing the transfer price results in an increase in the firm’s after-cost-tax profit. To prevent

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multidivisional firms from intentionally transferring their profits from high-cost-tax jurisdictions to low-cost-tax jurisdictions governments establish bounds over the transfer pricing. Bounds correspond to restrictions imposed over each division on the Pareto front that determine the maximum and minimum transfer price legally authorized. The bounds determine specific decision area where the Pareto optimal strategies can be selected.

The bounds over the Pareto front [7,11] are defined as follows

\[ p^l \in \left[p^l_-, p^l_+\right], q^l \in \left[q^l_-, q^l_+\right] \quad P_{adm} = \bigcap_{l=1}^{N} \left[p^l_-, p^l_+\right], Q_{adm} = \bigcap_{l=1}^{N} \left[q^l_-, q^l_+\right] \]

Remark 2. For practical reasons and improving the flexibility of the model we propose to establish bounds on both \( p \) and \( q \).

3.3. Optimal weights selection

Let define the \( \min \) and \( \max \) allowed bounds as follows

\[ F_{l}^+ = \min_{\lambda \in S^N} U^l(\nu^l(\lambda)) \quad F_{l}^- = \max_{\lambda \in S^N} U^l(\nu^l(\lambda)) \quad \text{for } l = 1, \ldots, N \]

Suppose that these bounds are a priori given as

\[ u^l(\nu^l(\lambda)) \in \left[U_{l}^+, U_{l}^-\right] \quad \text{for } l = 1, \ldots, N \]

According to [4,5] define the optimal \( \lambda^* \in S^N \) that corresponds to the strong Nash equilibrium of the problem

\[ \lambda^* = \arg \min_{\lambda \in S^N} \left\{ U^l - U(\nu^l(\lambda)) \right\} \quad U^* = \left(U^{1*}, \ldots, U^{N*}\right) \quad \text{for } \lambda \in S^N \]

\[ U_{l}\text{corresponds with the utopian point} \quad \text{subject to} \]

\[ u^l(\nu^l(\lambda)) \in \left[U_{l}^+, U_{l}^-\right] \quad \text{for } l = 1, \ldots, N \]

Lemma 3. The problem (3) formulated above is feasible iff

1. \([U_{l}^+, U_{l}^-] \subseteq \left[F_{l}^-, F_{l}^+\right]\) \[ \text{for } l = 1, \ldots, N \]
2. \([U_{l}^+, U_{l}^-] \cap \left[U_{2}^-, U_{2}^+\right] \cap \cdots \cap \left[U_{N}^-, U_{N}^+\right] \neq \emptyset \) \[ \text{for } \mathbf{P} \neq \emptyset \]

3.4. Final result

- The optimal individual utility is given by

\[ U_{l}(\nu^l(\lambda)) = U_{l}^*(\nu^l(\lambda)) \]

- The optimal global utility

113

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Remark 4. The proposed model can find the collaborative equilibrium point of both, \( p \) by fixing the value of \( q \) and, \( p \) and \( q \) simultaneously, depending on the case.

4. Regularization method

4.1. Regularized poly-linear programming problem

Consider the following poly-linear programming problem

\[
f(x) = \alpha_0 \sum_{j=1}^{N} c_j x_j + \sum_{j=1}^{N} \sum_{j'=1}^{N} a_{j,j'} x_j x_{j'} + \sum_{j=1}^{N} \sum_{j'=1}^{N} a_{j,j'} x_j x_{j'} + \lambda + \alpha_{N-1} \sum_{j=1}^{N} \sum_{j'=1}^{N} \sum_{j''=1}^{N} c_{j,j',j''} x_j x_{j'} x_{j''} + \alpha_0 \sum_{j=1}^{N} \sum_{j'=1}^{N} a_{j,j'} x_j x_{j'} + \alpha_0 \sum_{j=1}^{N} \sum_{j'=1}^{N} a_{j,j'} x_j x_{j'}
\]

\[
\text{subject to } \quad \begin{cases} x \in \mathbb{R}^N : x \geq 0, V_i x = b_i, V_i x - b_i = 0 \end{cases}
\]

Introducing the “slack” vectors \( u \in \mathbb{R}^{M_1} \) with nonnegative components, that is, \( u_j \geq 0 \) for all \( j = 1, \ldots, M_1 \), the original problem (6) can be rewritten as

\[
\begin{align*}
\min_{x \in X_{adm}, u \geq 0} & \quad f(x) \\
\text{subject to } & \quad X_{adm} = \{ x \in \mathbb{R}^N : x \geq 0, V_i x = b_i, V_i x - b_i = 0 \}
\end{align*}
\]

Define by \( X^* \subseteq X_{adm} \) the set of all solutions of the problem (7).

Following [9] and [10] consider the penalty function

\[
F_{k, \delta}(x, u) = f(x) + k \left[ \frac{1}{2} \| V_i x - b_i \| + \frac{1}{2} \| V_i x - b_i + u \| + \frac{\delta}{2} \left( \| x \| + \| u \| \right) \right]
\]

where the parameters \( k \) and \( \delta \) are positive. Obviously, the unconstraint on \( x \) the optimization problem

\[
\min_{x \in X_{adm}, u \geq 0} F_{k, \delta}(x, u)
\]

has a unique solution since the optimized function (8) is strongly convex [23] if \( \delta > 0 \). Notice also that

\[
\text{arg} \min_{x \in X_{adm}, u \geq 0} F_{k, \delta}(x, u) = \text{arg} \min_{x \in X_{adm}, u \geq 0} F_{k, \delta}(x, u)
\]

where \( \mu = k^{-1} > 0 \) and

DOI: 10.24818/18423264/52.1.18.07
Computing the Transfer Pricing for a Multidivisional Firm Based on Cooperative Games

\begin{equation}
F_{\mu, \delta}(x, u) = \mu f(x) + \frac{1}{2} \|b - b_0\|^2 + \frac{1}{2} \|c - c_0 + u\|^2 + \frac{\delta}{2} \|f + f\|^2
\end{equation}

**Proposition 5.** If the penalty parameter \( \mu \) tends to zero by a particular manner, then we may expect that \( x^*(\mu, \delta) \) and \( u^*(\mu, \delta) \), which are the solutions of the optimization problem

\[
\min_{x, u \in X_{adm}, \delta \geq 0} F_{\mu, \delta}(x, u)
\]

tend to the set \( X^* \) of all solutions of the original optimization problem (7), that is,

\[
\rho_{X^*}^{\mu, \delta}(x^*, u^*) \to 0 \quad \text{as} \quad \mu \to 0
\]

where \( \rho_{X^*}^{\mu, \delta}(x^*, u^*) \) is the Hausdorff distance defined as

\[
\rho_{X^*}^{\mu, \delta}(x^*, u^*) = \min_{x, u \in X^*} \|x - x^*\|
\]

Below we define exactly how the parameters \( \mu \) and \( \delta \) should tend to zero to provide the property (11).

**Theorem 6.** Let us assume that

1. The bounded set \( X^* \) of all solutions of the original optimization problem (7) is not empty and the Slater’s condition holds, that is, there exists a point \( x \in X_{adm} \) such that \( V_b x < b \).

2. The parameters \( \mu \) and \( \delta \) are time-varying, i.e.,

\[
\mu = \mu_n, \quad \delta = \delta_n, \quad n = 0, 1, 2, \ldots
\]

such that

\[
0 < \mu_n \downarrow 0, \quad \frac{\mu_n}{\delta_n} \downarrow 0 \quad \text{where} \quad n \to \infty.
\]

Then

\[
x_n^* := x^*(\mu_n, \delta_n) \to x^{**}, \quad u_n^* := u^*(\mu_n, \delta_n) \to u^{**}
\]

where \( x^{**} \in X^* \) is the solution of the original problem (7) with the minimal weighted norm which is unique, i.e.,

\[
\|x^{**}\| \leq \|x^*\| \quad \text{for all} \quad x^* \in X^*
\]

and

\[
u^{**} = b_1 - V_1 x^{**}.
\]

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Proof. See Appendix A.

We also need the following corollary.

**Corollary 7.** Under the assumptions of the Theorem 6 above there exist positive constants $C_\mu$ and $C_\delta$ such that

$$
|u_n^* - u_m^*| = |u_n^* - u_m^*| \leq C_\mu |p_n - p_m| + C_\delta |\delta_n - \delta_m|
$$

(17)

4.2. The Gradient Method

Consider the following recurrent procedure for finding the extremal point

$$
z^{**} = \begin{pmatrix} x^{**} \\ u^{**} \end{pmatrix}
$$

$$
z_n = \left [ z_{n-1} - \gamma_n \frac{\partial}{\partial z_n} F_{\mu_n, \delta_n} (z_{n-1}) \right ], \quad z := \begin{pmatrix} x \\ u \end{pmatrix}
$$

(18)

where $[z_n] = \begin{cases} z_n & \text{if } z_n \geq 0 \\
0 & \text{if } z_n < 0 \end{cases}$ and

$$
\frac{\partial}{\partial z_n} F_{\mu_n, \delta_n} (z_{n-1}) = \begin{pmatrix} \frac{\partial}{\partial x_n} F_{\mu_n, \delta_n} (x_{n-1}, u_{n-1}) \\
\frac{\partial}{\partial u_n} F_{\mu_n, \delta_n} (x_{n-1}, u_{n-1}) \end{pmatrix} = \begin{pmatrix} \mu_n \frac{\partial}{\partial x_n} f(x_{n-1}) + V_0 f(x_{n-1}) V_0 \delta_n - \delta_{n-1}^* \\
V_0 \delta_n - \delta_{n-1}^* + V_0 [V_0 x_{n-1} - b_0 + u_{n-1}] + V_0 \delta_{n-1} \end{pmatrix}
$$

**Remark 8.** (Convergence of the Gradient Method) If

$$
\sum_{n \geq 0} \gamma_n = \infty, \quad \gamma_n \to 0, \quad \frac{|\mu_n - \mu_{n+1}|}{\gamma_n} \to 0, \quad \frac{|\delta_n - \delta_{n+1}|}{\gamma_n} \to 0
$$

(19)

then

$$
W_n = \left | z_n - z^{**} \right | \to 0
$$

(20)

Let us select the parameters of the algorithm (18) as follows:

$$
\delta_n = \begin{cases} \delta_0 \left [ 1 + \ln (n - n_0) \right ] & \text{if } n \leq n_0 \\
\delta_0 \left [ 1 + (n - n_0) \right ] & \text{if } n > n_0 \end{cases}, \quad \mu_n = \begin{cases} \mu_0 & \text{if } n < n_0 \\
\mu_0 \left [ 1 + \ln (n - n_0) \right ] & \text{if } n \geq n_0 \end{cases}
$$

$$
\gamma_n = \begin{cases} \gamma_0 \left [ 1 + (n - n_0) \right ] & \text{if } n \leq n_0 \\
\gamma_0 & \text{if } n > n_0 \end{cases}, \quad \delta, \mu, \gamma > 0, \delta_0, \mu_0, \gamma_0 > 0
$$

(21)
Computing the Transfer Pricing for a Multidivisional Firm Based on Cooperative Games

To guarantee the convergence of the suggested procedure, by the property \( \frac{\mu_n}{\delta_n} \to 0 \) and by the conditions (19) we should have
\[
\delta \leq \mu, \gamma \geq \delta, \gamma + \delta \leq 1. \quad (22)
\]

Let us prove the following simple result.

**Remark 9.** Suppose that for a nonnegative sequence \( \{u_n\} \) the following recurrent inequality holds
\[
u_n \leq u_{n-1} (1 - \alpha_n) + \beta_n
\]
where numerical sequences \( \{\alpha_n\} \) and \( \{\beta_n\} \) satisfies
\[
\alpha_n \in (0,1], \beta_n \geq 0, \nu_n > 0 \quad \text{for all} \ n = 0, 1, \ldots
\]
\[
\sum_{n=0}^{\infty} \alpha_n = \infty, \sum_{n=0}^{\infty} \beta_n \nu_n < \infty, \quad \lim_{n \to \infty} \frac{\nu_{n+1}}{\alpha_n \nu_n} = \theta < 1
\]

Then
\[
u_n = o(u_{n-1})
\]

5. Numerical example

5.1. A priori data required for the solution of the problem

- \( p_i \in [p_i^{-}, p_i^{+}], q_i \in [q_i^{-}, q_i^{+}] \quad l = 1, \ldots, \mathcal{N}; \quad p_i \in [1000,9000], q_i \in [4100,15000], p^{2} \in [10100,340000], q_{1}^{3} \in [50,300], q_{2}^{3} \in [100,500], q_{3}^{3} \in [150,700] \)
- \( c^{l}(q^{l}) = \theta^{l} q^{l}, \quad \theta^{l} \geq 0, \quad l = 1, \ldots, \mathcal{N}; \quad \theta_{1}^{l} = 0.1, \quad \theta_{2}^{l} = 0.1, \quad \theta_{3}^{l} = 0.1 \).
- \( r^{l}(p^{l} q^{l}) = \theta^{l} p^{l} q^{l}, \quad l = 1, \ldots, \mathcal{N}. \quad \theta_{1}^{l} = 0.1, \quad \theta_{2}^{l} = 0.2, \quad \theta_{3}^{l} = 0.3 \).
- \( p_{0}^{1} = 4500; p_{0}^{2} = 10000; p_{0}^{3} = 180000; q_{0}^{1} = 150; q_{0}^{2} = 300; q_{0}^{3} = 450. \)

5.2. Linear constraints interpretation

Let us represent the problem above
\[
P := \left\{ v(\lambda) = \arg \max_{v \in \mathcal{V}, \lambda \in \mathcal{N}} \max_{q_{0}^{i}} W(v|\lambda), \lambda \in \mathcal{S}^{\mathcal{N}} \right\}
\]

Where

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in the form of a poly-linear function optimization with linear constraints, namely,
\[ W(v | \lambda) \rightarrow \max_v A_{\text{ineq}} v - b_{\text{ineq}}, v \geq 0 \]
where (for \( N = 3 \))
\[
A_{\text{ineq}} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0 \\
-1 & 0 & \cdots & 0 & 0 \\
\end{bmatrix}
\]
\[
b_{\text{ineq}} = \begin{bmatrix}
p_1 \\
p_2 \\
\vdots \\
p_3 \\
p_1 \\
\end{bmatrix}
\]

5.3. Regularized penalty function

Let us introduce the following regularized penalty function:
\[
F_{\mu, \beta}(v, u) = \mu W(v | \lambda) + \frac{1}{2} \| A_{\text{ineq}} v - b_{\text{ineq}} + u \|_2^2 + \frac{\beta}{2} \| v \|_1 + \alpha \| u \|_1
\]
for which we have
\[
\frac{\partial}{\partial v} F_{\mu, \beta}(v, u) = \mu \frac{\partial}{\partial v} W(v | \lambda) + A_{\text{ineq}} \left( A_{\text{ineq}} v - b_{\text{ineq}} + u \right) + \beta v
\]
\[
\frac{\partial}{\partial u} F_{\mu, \beta}(v, u) = A_{\text{ineq}} v - b_{\text{ineq}} + (1 + \beta) u
\]
with
\[
\frac{\partial}{\partial v} W(v | \lambda) = \begin{bmatrix}
\frac{\partial}{\partial v_1} W(v | \lambda) \\
\frac{\partial}{\partial v_2} W(v | \lambda) \\
\vdots \\
\frac{\partial}{\partial v_N} W(v | \lambda)
\end{bmatrix}
\]

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5.4. The recurrent procedure

For maximizing the utility let us consider the following recurrent procedure for finding the extremal point \( z^{**} = \begin{pmatrix} v^{**} \\ u^{**} \end{pmatrix} \):

\[
z_n = \left[ z_{n-1} + \gamma_n \frac{\partial}{\partial z} F_{\mu_n \delta_n} (z_{n-1}) \right], \quad z = \begin{pmatrix} v \\ u \end{pmatrix}
\]

(29)

where \( [z_i] = \begin{cases} z_i & \text{if } z_i \geq 0 \\ 0 & \text{if } z_i < 0 \end{cases} \) and the first-order derivative of \( F_{\mu_n \delta_n} (z_{n-1}) \)

\[
\frac{\partial}{\partial z} F_{\mu_n \delta_n} (z_{n-1}) = \begin{pmatrix}
\lambda_1 (q_1^1 - \theta q_1^1) + \lambda_2 (-q_1^1) \\
\lambda_2 (q_1^2 - \theta q_1^2) + \lambda_3 (-q_1^2) \\
\lambda_3 (q_1^3 - \theta q_1^3) \\
\lambda_3 (p^1 - \theta q^1 p^1) \\
\lambda_3 (p^2 - \theta q^2 p^2) \\
\lambda_3 (p^3 - \theta q^3 p^3) \\
A_{mq} - b_{mq} + (1 + \delta \kappa) u
\end{pmatrix}
\]

Figure 4. Transfer price for a multidivisional firm.

Figure 4 presents the Pareto front of the transfer pricing. Computing the strong Nash equilibrium, we have that \( z^* = \begin{bmatrix} 0.0256, 0.0 \\ 256, 0.9487 \end{bmatrix} \), \( p^* = \begin{bmatrix} 0.0505, 0.1 \end{bmatrix}, q^* = \begin{bmatrix} 497, 1.9645 \end{bmatrix}, U^* = \begin{bmatrix} 837, 7.8584 \end{bmatrix} \times 10^7, W^* = 7.4648 \times 10^7 \).
6. Conclusion and future work

The transfer pricing problem results essential in resource allocation and global firm profit for a multidivisional firm which employs optimization goals to set internal prices of goods that are sold between subsidiaries or divisions of the firm. Usually, each division estimates the revenues or profits of each opportunity and selects the best opportunity. As a result, the transfer prices system focuses on the maximization of divisional income, leading the divisions to achieve the goals of the firm through sub-optimization. However, sub-optimizations by individual divisions do not result in an optimum for the firm. This paper presented a new modeling framework for transfer pricing schemes based on cooperative game theory considering cost and tax policies. The study provided decision makers with a useful tool for supporting the design of global strategies for transfer-pricing, considering different variables of optimization (price and quantity) and establishes a flexible model after cost-taxes profitable configuration. The proposed model integrate financial issues for vertically integrated supply chains. There are relatively few models of global profit maximization reported in the literature for large-scale organizations. Accordingly, we consider that the implementation of transfer pricing models for global profit maximization represents a fundamental opportunity that must be seriously considered by large-scale multidivisional firms.

We proposed a method for computing the transfer pricing for a multidivisional firm based on a multi-objective approach for representing a cooperative game. For computing the strong Nash equilibrium we employed the minimal distance to the utopia point. We suggested the use of the Tikhonov’s regularization to guarantee the convergence to a single (strong) equilibrium point. We proposed a method based on a penalized programming approach for computing the regularized strategies of the game. We developed a programming method to solve the successive single-objective constrained problems that arise from taking the regularized functional of the game. In addition, we implemented a recurrent procedure based on the gradient method to solve the regularized problem and finding the Pareto optimal strategies. We proved that in the regularized problem the functional of the game decreases and finally converges, proving the existence and uniqueness of strong Nash equilibrium. We also presented the convergence conditions and compute the estimate rate of convergence of variables μ and δ corresponding to the step size parameter of the regularized penalty method. We provided all the details needed to implement the method in an efficient and numerically stable way.

In terms of future work, there exist a number of challenges left to address. One interesting technical challenge is that of developing a numerical method for implementing the results presented in this paper. We also are considering to extend the
Computing the Transfer Pricing for a Multidivisional Firm Based on Cooperative Games

We prove that the Hessian matrix $H$ associated with the penalty function $(10)$ is strictly positive definite for any positive $\mu$ and $\delta$, i.e., we prove that for all $x \in \mathbb{R}^N$ and $u \in \mathbb{R}^{V_1}$

$$H = \begin{bmatrix} \frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial x^2} & \frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial u \partial x} \\ \frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial u \partial x} & \frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial u^2} \end{bmatrix} > 0$$

(A.1)

To prove that, by the Schur lemma $[23]$, it is necessary and sufficient to prove that

$$\frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial x^2} > 0, \quad \frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial u^2} > 0$$

$$\frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial u \partial x} > \frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial u \partial x} \left[ \frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial u^2} \right]^{-1} \frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial u \partial x}$$

(A.2)

We have

$$\frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial x^2} = \mu \frac{\partial^2 f(x)}{\partial x^2} + V_0^TV_0 + V_1^TV_1 + \delta I_{N \times N}$$

$$\mu \frac{\partial^2 f(x)}{\partial x^2} + \delta I_{N \times N} > I_{N \times N}$$

$$x^* = \min_{x \in \mathbb{R}^n} \lambda_{\mu \delta} \left( \frac{\partial^2 f(x)}{\partial x^2} \right) > I_{M \times M} > 0$$

By the Schur lemma

$$\frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial x^2} = \mu \frac{\partial^2 f(x)}{\partial x^2} + V_0^TV_0 + V_1^TV_1 + \delta I_{N \times N} > \frac{\partial^2 F_{\mu,\delta}(x,u)}{\partial u^2}$$

$$(I + \delta)^{-1} V_1^T V_1$$

implying, $\mu \frac{\partial^2 f(x)}{\partial x^2} + V_0^TV_0 + \frac{\delta}{1+\delta} V_1^TV_1 + \delta I_{N \times N} > 0$ which holds for any $\delta > 0$ by the condition (13) since

$$\left( \mu + \frac{\delta}{1+\delta} V_1^TV_1 \right) I_{N \times N} = \delta(1+\alpha(1)) I_{N \times N} > 0$$

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So, $H > 0$ which means that the penalty function (10) is strongly convex and, hence, has a unique minimal point defined below as $x^*(\mu, \delta)$ and $u^*(\mu, \delta)$. By the strictly convexity property (30) for any $z = \begin{pmatrix} x \\ u \end{pmatrix}$ and any vector

$$z_n^* := \left( \begin{array}{c} x_n^* = x^*(\mu_n, \delta_n) \\ u_n^* = u^*(\mu_n, \delta_n) \end{array} \right)$$

for the function $F_{\mu, \delta}(x, u) = F_{\mu, \delta}(z)$ we have

$$0 \geq \left( \begin{array}{c} x_n^* - z \\ u_n^* - u \end{array} \right)^T \frac{\partial}{\partial x} F_{\mu_n, \delta_n} \left( x_n^* - x \right) + \frac{\partial}{\partial u} F_{\mu_n, \delta_n} \left( u_n^* - u \right) + \frac{1}{\delta_n} \left( \begin{array}{c} V_0(x_n^* - x) \\ V_1(x_n^* - u) \end{array} \right) + \delta_n \left( \begin{array}{c} x_n^* - x \\ u_n^* - u \end{array} \right)^T x_n^* + \delta_n \left( \begin{array}{c} x_n^* - x + (1 + \delta) u \\ u_n^* - u + (1 + \delta) u_n \end{array} \right)^T u_n^*$$

(A.3)

Selecting in (A.3) $x := x^* \in X^*$ ($x^*$ is one of admissible solutions such that $V_0 x^* = b_0$) and $u := b_1 - V_1 x_n^*$ we obtain

$$0 \geq \mu_n \left( x_n^* - x \right)^T \frac{\partial}{\partial x} f(x_n^*) + \frac{1}{\delta_n} \left( V_0 x_n^* - b_0 \right) + \left( x_n^* - x \right)^T x_n^* + \frac{1}{\delta_n} \left( V_1 x_n^* - b_1 \right) + \frac{1}{\delta_n} \left( u_n^* - u - V_1 x_n^* \right)^T u_n^*$$

(A.4)

Dividing both sides of this inequality by $\delta_n$ we get

$$0 \geq \frac{\mu_n}{\delta_n} \left( x_n^* - x \right)^T \frac{\partial}{\partial x} f(x_n^*) + \frac{1}{\delta_n} \left( V_0 x_n^* - b_0 \right) + \left( x_n^* - x \right)^T x_n^* + \frac{1}{\delta_n} \left( V_1 x_n^* - b_1 \right) + \frac{1}{\delta_n} \left( u_n^* - u - V_1 x_n^* \right)^T u_n^*$$

(A.4)

Notice also that from (A.3), taking $x = x_n^*$ and $u = 0$, it follows

$$0 \geq \left[ \left( V_1 x_n^* - b_1 \right)^T \left( V_1 x_n^* - b_1 \right) \right]$$

implying

$$1 \geq \left[ \frac{\mu_n}{\delta_n} \right] \left[ \left( V_1 x_n^* - b_1 \right)^T \left( V_1 x_n^* - b_1 \right) \right]$$

which means that the sequence $\left\{ \mu_n \right\}$ is bounded. In view of this and taking into account that by the supposition (13) $\frac{\mu_n}{\delta_n} \to 0$, from (A.4) it follows

$$\text{const} = \limsup_{\delta_n \to \infty} \left[ \left( x_n^* - x \right)^T f(x_n^*) + \left( u_n^* - u \right)^T f(u_n^*) \right]$$

(A.5)

From (A.5) we may conclude that

DOI: 10.24818/18423264/52.1.18.07
Computing the Transfer Pricing for a Multidivisional Firm Based on Cooperative Games

\[
\left\| x^*_n - b_0 \right\| + \left\| V_1 x^*_n - x \right\| + (1 + \delta_n) \left\| V_1 x^*_n - b_1 + (1 + \delta_n) b_0 \right\| = O(\delta_n) \tag{A.6}
\]

and

\[
V_0 x^*_n - b_0 = 0, \quad V_1 x^*_n - V_2 x^* = V_1 x^*_n - b_1 + u_n^*, \n\]

where \( x^*_n \in X^* \) is a partial limit of the sequence \( \{x^*_n\} \) which, obviously, may be not unique. The vector \( u_n^* \) is also a partial limit of the sequence \( \{u_n\} \). Denote by \( \hat{x}_n \) the projection of \( x^*_n \) to the set \( X_{adm} \), namely,

\[
\hat{x}_n = \text{Pr}_{X_{adm}}(x^*_n) \tag{A.7}
\]

and show that

\[
\|x^*_n - \hat{x}_n\| \leq C_1 \delta_n, C = \text{const} > 0. \tag{A.8}
\]

From (A.6) we have

\[
\left\| (x^*_n - b_1 + u_n^*) \right\| \leq C_1 \delta_n, C_1 = \text{const} > 0
\]

implying

\[
V_1 x^*_n - b_1 \leq C_1 \delta_n e = u_n^* \leq C_1 \delta_n e, \|
\]

where the vector inequality is treated in component-wise sense. Therefore

\[
\|x^*_n - x\| \leq \max_{V_1 x_n \in X_{adm}} \min_{V_1 x_n \in X_{adm}} \left| x - y \right| = d(\delta_n) \tag{A.9}
\]

Introduce the new variable

\[
x = \frac{x - \nu_n x}{1 - \nu_n}, \quad \nu_n > 0 \quad \text{such that} \quad \nu_n = \frac{C_1 \delta_n}{C_1 \delta_n + \min_{j=1, \ldots, m_1} \left| V_1 x - b_1 \right|} < 1.
\]

For new variable

\[
x = \frac{x - \nu_n x}{1 - \nu_n}
\]

we have

\[
V_1 \tilde{x} - b_1 \leq \frac{C_1 \delta_n}{C_1 \delta_n + \min_{j=1, \ldots, m_1} \left| V_1 x - b_1 \right|} \left( \min_{j=1, \ldots, m_1} \left| V_1 x - b_1 \right| \right) < 0
\]

and therefore

\[
d(\delta_n) \leq \max_{V_1 x_n \in X_{adm}} \left| \frac{x - \nu_n x}{1 - \nu_n} - x \right| = \frac{\nu_n^2}{(1 - \nu_n)} \max_{V_1 x_n \in X_{adm}} \left| x - x \right| \leq C_2 \delta_n, \quad 0 < C_2 < \infty.
\]

In view of that \( x^*_n - \hat{x}_n \leq \sqrt{d(\delta_n)} \leq \sqrt{C_2 \delta_n} \) which proves (A.8). The last step is to prove the inequality

\[
0 \geq (x^*_n - x)^T x^*_n \quad \text{for any} \ x^*_n \leq X^*.
\]

(A.10)

DOI: 10.24818/18423264/52.1.18.07
From (A.4) we get

\[
0 \geq (x^*_n - x^*)^T \frac{\partial}{\partial x} f(x^*_n) + \frac{1}{\mu_n} \left[ (x^*_n - x^*) + \frac{\delta_n}{\mu_n} (x^*_n - x^*) \right] \cdot \left[ (x^*_n - x^*) + \frac{\delta_n}{\mu_n} (x^*_n - x^*) \right]
\]

(11.1)

By the strong convexity property we have (see Corollary 21.4 in [23])

\[
(x - y)^T \frac{\partial}{\partial x} f(x) + \frac{\partial}{\partial y} f(y) \geq 0 \quad \text{for any } x, y \in \mathbb{R}^N \text{ which, in view of the property (A.8), implies}
\]

\[
(x^*_n - x_n)^T \frac{\partial}{\partial x} f(x^*_n) = o\left(\frac{\delta_n}{\mu_n}\right) (x_n - x^*)^T \cdot \frac{\partial}{\partial x} f(x_n) \geq (x_n - x^*)^T \frac{\partial}{\partial x} f(x^*_n) \geq 0
\]

and

\[
(x^*_n - x^*)^T \frac{\partial}{\partial x} f(x^*_n) = (x^*_n - x_n)^T \frac{\partial}{\partial x} f(x_n)
\]

\[
(x_n - x^*)^T \frac{\partial}{\partial x} f(x_n) = o\left(\frac{\delta_n}{\mu_n}\right) (x_n - x^*)^T \left( \frac{\partial}{\partial x} f(x^*_n) - \frac{\partial}{\partial x} f(x_n) \right)
\]

Since any polynomial function is Lipschitz continuous on any bounded compact set, we can conclude that

\[
\left| \frac{\partial}{\partial x} f(x^*_n) - \frac{\partial}{\partial x} f(x_n) \right| \leq \text{Const} \left| x^*_n - x_n \right| = O\left(\frac{\delta_n}{\mu_n}\right)
\]

which gives

\[
(x^*_n - x_n)^T \frac{\partial}{\partial x} f(x^*_n) = O\left(\frac{\delta_n}{\mu_n}\right) \quad \text{which by (11.1) leads to}
\]

\[
0 \geq (x^*_n - x_n)^T \frac{\partial}{\partial x} f(x^*_n) + \frac{\delta_n}{\mu_n} (x^*_n - x^*)^T x^*_n = o(1), 0 \geq \frac{\delta_n}{\mu_n} (x^*_n - x^*)^T x^*_n
\]

(A.12)

Dividing both sides of the inequality (A.12) by \(\frac{\mu_n}{\delta_n}\) and in view (A.8) we finally obtain

\[
0 \geq \frac{\mu_n}{\delta_n} (x^*_n - x^*)^T x^*_n = o(1), 0 \geq \frac{\delta_n}{\mu_n} (x^*_n - x^*)^T x^*_n
\]

(A.13)

This, by (13), for \( n \rightarrow \infty \) leads to (A.10). Finally, for any \( x^* \leq x^* \) it implies

\[
0 \geq (x^*_n - x^*)^T x^*_n = (x^*_n - x^*)^T + (x^*_n - x^*)^T x^* \geq (x^*_n - x^*)^T x^*
\]

This inequality exactly represents the necessary and sufficient condition that the point \( x^* \) is the minimum point of the function \( \| x^*_n \|^2 \) on the set \( X^* \). Obliviously, this point is unique and has a minimal norm among all possible partial limits \( x^*_n \).

Theorem is proven.

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Computing the Transfer Pricing for a Multidivisional Firm Based on Cooperative Games

REFERENCES

[1] T. Baldenius and S. Reichelstein (2006), External and Internal Pricing in Multidivisional Firms. Journal of Accounting Research, A.15(1):1–28;
[2] F. Bauer and M. A. Lukas (2011), Comparing Parameter Choice Methods for Regularization of Ill-Posed Problems. Mathematics and Computers in Simulation, 81(9):1795–18A.12;
[3] W. Cheng, C.-L. Fu, and Z Qian (2007), A Modified Tikhonov Regularization Method for a Spherically Symmetric Three-Dimensional Inverse Heat Conduction Problem. Mathematics and Computers in Simulation, 75(3-4):97–112;
[4] J. B. Clempner and A. S. Poznyak (2016), Analyzing an Optimistic Attitude for the Leader Firm in Duopoly Models: A Strong Stackelberg Equilibrium Based on a Lyapunov Game Theory Approach. Economic Computation and Economic Cybernetics Studies and Research; ASE Publishing; 4(50):41-60;
[5] J. B. Clempner and A. S. Poznyak (2015), Computing the Strong Nash Equilibrium for Markov Chains Games. Applied Mathematics and Computation, 265:911–927;
[6] J. B. Clempner and A. S. Poznyak (2016), Solving the Pareto Front for Nonlinear Multiobjective Markov Chains Using the Minimum Euclidian Distance Optimization Method. Mathematics and Computers in Simulation, 119:1A.13–160;
[7] J. B. Clempner (2016), Necessary and Sufficient Karush-Kuhn-Tucker Conditions for Multiobjective Markov Chains Optimality. Automatica, 71:1A.6–1A.13;
[8] J. B. Clempner and A. S. Poznyak (2017), Negotiating the Transfer Pricing Using the Nash Bargaining Solution. International Journal of Applied Mathematics and Computer Science, 27(4):853-864;
[9] J. B. Clempner and A. S. Poznyak (2018), A Tikhonov Regularized Penalty Function Approach for Solving Polylinear Programming Problems. Journal of Computational and Applied Mathematics,328:267-286;
[10] J. B. Clempner and A. S. Poznyak (2018), A Tikhonov Regularization Parameter Approach for Solving Lagrange Constrained Optimization Problems. Engineering Optimization, to be published;
[11] J. B. Clempner and A. S. Poznyak (2018), Constructing the Pareto Front for Multiobjective Markov Chains Handling a Strong Pareto Policy Approach. Computational and Applied Mathematics, 37, 1, 567-591;

DOI: 10.24818/18423264/52.1.18.07
Julio B. Clempner, Alexander S. Poznyak

[12] J. B. Clempner and A. S. Poznyak (2018), Using Manhattan Distance for Computing the Multiobjective Markov Chains Problem. International Journal of Computer Mathematics, to be published;
[13] J. B. Clempner and A. S. Poznyak (2018), Sparse Mean-Variance Customer Markowitz Portfolio Selection for Markov Chains: A Tikhonov's Regularization Penalty Approach. Optimization and Engineering, to be published;
[14] R. Hammami and Y. Frein (2014), Redesign of Global Supply Chains with Integration of Transfer Pricing: Mathematical Modeling and Managerial Insights. International Journal of Production Economics, 158:267–277;
[15] J. Hirshleifer (1956), On the Economics of Transfer Pricing. Journal of Business, 29:172–184;
[16] T. orsHt (1971), The Theory of the Multinational Firm: Optimal Behavior under Different Tariff and Tax Rates. Journal of Political Economy, 79(5):1059–1072;
[17] W. T. Huh and K. S. Park (2013), Impact of Transfer Pricing Methods for Tax Purposes on Supply Chain Performance under Demand Uncertainty. Naval Research Logistics, 60(4):269–293;
[18] S. K. Kassicieh (1981), International Intra-company Transfer Pricing. Operations research, 29(4):817–828;
[19] S. Lakhal, S. HMida and U. Venkatadri (2005), A Market-driven Transfer Price for Distributed Products Using Mathematical Programming. European Journal of Operational Research, 162(3):690–699;
[20] S. Lakhal (2006), An Operational Profit Sharing and Transfer Pricing Model for Network-Manufacturing Companies. European Journal of Operational Research, 175:5A.14–565;
[21] M. Leng and M. Parlarb (2012), Transfer Pricing in a Multidivisional Firm: A Cooperative Game Analysis. Operations Research Letters, A.11(5):A.74–A.79;
[22] J. Luft and R. Libby (1997), Profit Comparisons, Market Prices and Managers' judgments about Negotiated Transfer Prices. The Accounting Review, 77(2):217–229;
[23] A. S. Poznyak (2008), Advanced Mathematical Tools for Automatic Control Engineers. Deterministic Technique, volume 1. Elsevier, Amsterdam, Oxford;
[24] E. C. Rosenthal (2008), A Game Theoretic Approach to Transfer Pricing in a Vertically Integrated Supply Chain. International Journal of Production Economics, 115:5A.13–5A.23;
[25] M. Shunko and S. Gavirneni (2007), Role of Transfer Prices in Global Supply Chains with Random Demands. Journal of Industrial And Management Optimization, 3(1):99–117.