**UN-INTEGRATED PDFS IN CCFM**

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The un-integrated parton distribution functions (uPDFs) obtained from a CCFM evolution are studied in terms of the intrinsic transverse momentum distribution at low scales. The uPDFs are studied for variations of the renormalization and factorization scales.

1 Introduction

Un-integrated parton distributions are best suited to study the details of the perturbative as well as the non-perturbative QCD evolution. The general form of the integral equation for the parton evolution is:

\[ xA(x, k_t, \bar{q}) = xA_0(x, k_t, \bar{q}) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(\text{order}) \Delta_s(\bar{q}) \tilde{P}(z, q, k_t) xA\left(\frac{x}{z}, k'_t, q\right) \]  

with \( \bar{q} \) being the evolution (factorization) scale. The ordering condition of the evolution is specified by \( \Theta(\text{order}) \) which in the case of CCFM is given by \( \Theta(\bar{q} - zq) \). The first term of the rhs of eq. (1) gives the contribution of non-resolvable branchings between the starting scale \( Q_0 \) and the factorization scale \( \bar{q} \) and is given by:

\[ xA_0(x, k_t, \bar{q}) = xA_0(x, k_t, Q_0) \Delta_s(\bar{q}, Q_0) \]  

where the Sudakov form factor \( \Delta_s(q_2, q_1) \) describes the probability of no radiation between \( Q_0 \) and \( \bar{q} \). The second term of the rhs of eq. (1) describes the details of the QCD evolution, expressed by the convolution of the splitting function \( \tilde{P} \) with the parton density and the Sudakov form factor \( \Delta_s \).

2 The starting distribution \( xA_0(x, k_t, Q_0) \)

The non-resolvable branching contribution is directly related to the initial (starting) distribution \( A_0(x, k_t, Q_0) \), and especially to the intrinsic \( k_t \) distribution. Often a Gaussian type distribution with width \( k_0 \) is assumed and \( A_0 \) can be parameterized as:

\[ xA_0(x, k_t, Q_0) = N x^{p_0} (1 - x)^{p_1} \exp\left(-k_t^2/k_0^2\right) \]  

where \( N \) is a normalization constant, and \( p_0, p_1 \) being parameters to be determined experimentally.

In the following sections the dependence of the choice of \( p_0, p_1 \) and \( k_0 \) will be investigated. The CCFM evolution equation is applied to the starting distribution...
$x A_0(x, k_t, Q_0)$ and then convoluted with the off-shell matrix elements. The simple gluon splitting function is applied \[1,2\],

$$P_{gg}(z, \bar{q}, k_t) = \frac{\bar{\alpha}_s(q^2(1-z)^2)}{1-z} + \frac{\bar{\alpha}_s(k_t^2)}{z} \Delta_{ns}(z, q^2, k_t)$$  \hspace{1cm} (4)

with $q = p_t/(1-z)$ and the non-Sudakov form factor $\Delta_{ns}$ is defined by:

$$\log \Delta_{ns}(z, q^2, k_t) = -\bar{\alpha}_s \int_z^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \cdot \Theta(k_t - q) \Theta(q - z'q)$$  \hspace{1cm} (5)

The collinear cutoff $Q_g = 1.3$ GeV regulates the region of $z \to 1$, applied both to the real emissions as well as in the Sudakov form factor for the virtual corrections, which is equivalent to using the plus prescription of the splitting function $P_+$. Due to the angular ordering in CCFM a kind of random walk in the propagator gluon $k_t$ can be performed, and values of small $k_t$ can be reached. A cut $k_t^{cut}$ is introduced to avoid this region.

The CCFM evolution equations have been solved numerically \[1\] using a Monte Carlo method. In Tab. 1 the fit results for the un-integrated gluon densities are summarized. The input parameters are varied such that after convolution with the off-shell matrix elements the results agree best with the measured structure function $F_2(x, Q^2)$ in the range $x < 5 \cdot 10^{-3}$ and $Q^2 > 4.5$ GeV$^2$ as measured at H1 \[3,4\] and ZEUS \[5,6\]. The starting distribution is given by eq.(3), where the parameter $p_1 = 4$ is kept fixed, but $p_0$ and the width of the Gaussian $k_0$ was adjusted to give the smallest $\chi^2/points$. The results are given in Tab. 1 and labeled as set A0.

Equally good fits can be obtained using different values for the soft cut $k_t^{cut}$ and a different value for the width of the intrinsic $k_t$ distribution set B0. Motivated by the success of the saturation model \[7\], the parameterization of GBW can also serve as a starting distribution:

$$x A_0(x, k_t, Q_0) = \frac{3\sigma_0}{4\pi^2\alpha_s} R_0^2(x) k_t^2 \exp \left(-R_0^2(x) k_t^2\right)$$  \hspace{1cm} (6)

with the parameters given in \[7\]. As seen from Tab. 1 a reasonable description of the data can be achieved set GBW.

The renormalization scale dependence of the final cross section can be estimated by changing the scale used in $\alpha_s$ in the off-shell matrix element. Since here we are using the LO $\alpha_s$ matrix elements, any scale variation will change the cross section. In order to obtain a reasonable result, the uPDF was fitted to describe $F_2$ by varying the scale $\mu_r$. The set A- (set B-) correspond to a scale $\mu_r = 0.5 p_t$ and the set A+ (set B+) correspond to a scale $\mu_r = 2 p_t$ by: $0.5 \cdot \mu_r$ and $2 \cdot \mu_r$. The resulting $\chi^2/points$ is also shown in Tab. 1. However changing the scale $\mu_r$ and $\mu_f$ simultaneously with the same factor results in a much larger $\chi^2$, which is understandable, since the length of the evolution ladder is changed by $\mu_f$.

The CCFM evolution is performed in an angular ordered region of phase space and the factorization scale is related to the maximum allowed angle for any emission. In CCFM the scale $\mu_f$ (coming from the maximum angle) can be written as:

$$\mu_f = \sqrt{s + Q_{\perp}^2}$$  \hspace{1cm} (7)
with \( \hat{s} \) being the invariant mass of the \( q\bar{q} \) subsystem, and \( Q_\perp \) its transverse momentum. In this definition, the scale \( \mu_f \) is related to the quark pair.

In order to investigate the uncertainties coming from the specific choice of the evolution scale given in eq.(7), another definition is applied, relating the factorization scale only to the quark (or anti-quark): \( \mu_f = \frac{p_t}{1-z} \) with \( p_t \) being the transverse momentum of the quark(anti-quark) and \( z = \frac{k_t}{\sqrt{\hat{s}Q_\perp}} \). This definition follows closely the definition of the rescaled transverse momentum \( \bar{q} \) in CCFM. In Tab. 1 is shown, that also with this definition a reasonable description of the data can be achieved, but the parameter \( p_0 \) of the starting distribution is changed significantly.

| set | \( \mu_f \) | \( \mu_x \) | \( p_0 \) | \( k_0 \) (GeV) | \( k_{\text{cut}} \) (GeV) | \( \chi^2/\text{points} \) |
|-----|--------------|------------|------|--------------|--------------|----------------|
| A0  | \( \sqrt{\hat{s}+Q_\perp^2} \cdot \mu_f \) | 0          | 1.33 | 1.33         | 276/248 = 1.1 |
| A0− | \( \sqrt{\hat{s}+Q_\perp^2} \cdot 0.5 \cdot \mu_f \) | -0.01     | 1.33 | 1.33         | 433/248 = 1.75 |
| A0+ | \( \sqrt{\hat{s}+Q_\perp^2} \cdot 2 \cdot \mu_f \) | -0.01     | 1.33 | 1.33         | 316/248 = 1.3  |
| A1  | \( \frac{p_t}{1-z} \) | -0.1      | 0.8  | 0.3          | 275/248 = 1.1  |
| B0  | \( \sqrt{\hat{s}+Q_\perp^2} \cdot \mu_f \) | 0          | 0.8  | 0.25         | 356/248 = 1.4  |
| B0− | \( \sqrt{\hat{s}+Q_\perp^2} \cdot 0.5 \cdot \mu_f \) | 0.01      | 0.8  | 0.25         | 483/248 = 1.95 |
| B0+ | \( \sqrt{\hat{s}+Q_\perp^2} \cdot 2 \cdot \mu_f \) | 0.01      | 0.8  | 0.25         | 297/248 = 1.2  |
| B1  | \( \frac{p_t}{1-z} \) | -0.1      | 0.8  | 0.25         | 702/248 = 2.8  |
| GBW | \( \sqrt{\hat{s}+Q_\perp^2} \cdot 1 \) | -         | -    | 0.25         | 349/248 = 1.4  |

Table 1. The different settings of the CCFM un-integrated gluon densities. The initial distribution is parameterized as 
\( xA_0(x,k_t,Q_0) = N x^{p_0}(1-x)^{p_1} \cdot \exp \left( -k_t^2/k_0^2 \right) \) with \( p_1 = 4 \) and \( Q_g = 1.33 \) GeV. In the last line the starting distribution from the parameterization of GBW [7] is used. In the last column, the \( \chi^2/\text{points} \) to HERA F2 data [3,4,5,6] is given (for \( x < 5 \cdot 10^{-3} \) and \( Q^2 > 4.5 \) GeV²).

3 \( k_t \) dependence of \( xA(x,k_t,\mu_f) \)

In Fig. 1 the uPDFs (as described in Tab. 1) are shown as a function of \( k_t \) for different values of \( x \). Also shown is the contribution of the first term of the rhs of eq. (1). It can be seen, that the influence of the intrinsic \( k_t \) distribution is concentrated at small values of \( k_t \), whereas at values \( k_t \gtrsim 1 \) GeV the perturbative evolution is responsible. The specific choice of the \( k_{\text{cut}}^2 \) results in different distributions at small values of \( k_t \). It is also interesting to observe, that the dip in \( k_t \) visible in Fig. 1 (upper plot) is directly related to \( k_{\text{cut}}^2 \).

It is interesting to observe, that even with very different starting distributions, the uPDFs after perturbative evolution are similar (at larger \( k_t \)). Thus the small \( k_t \) region provides new information on the non-perturbative part of the parton density functions, which can only be investigated by using uPDFs. Especially the question of saturation effects at small \( k_t \) can be directly investigated with uPDFs. The difference of using a Gaussian intrinsic \( k_t \) distribution to a form motivated by
the saturation model of GBW is significant. It is also important to note, that these differences are washed out for inclusive distributions, as all uPDFs are able to describe $F_2$ at a similar level.

$$\eta = 10 \text{ GeV}$$

Figure 1. Comparison of the different sets of un-integrated gluon densities obtained from the CCFM evolution as described in the text.

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References

1. H. Jung, G. Salam, *Eur. Phys. J. C* **19** (2001) 351, hep-ph/0012143.
2. M. Hansson, H. Jung, The status of CCFM unintegrated gluon densities, DIS 2003, St. Petersburg, Russia, [hep-ph/0309009](http://arxiv.org/abs/hep-ph/0309009).
3. H1 Collaboration, S. Aid et al., *Nucl. Phys. B* **470** (1996) 3.
4. H1 Collaboration, C. Adloff et al., *Eur. Phys. J. C* **21** (2001) 33.
5. ZEUS Collaboration; M. Derrick et al., *Z. Phys. C* **72** (1996) 399.
6. ZEUS Collaboration; S. Chekanov et al., *Eur. Phys. J. C* **21** (2001) 443.
7. K. Golec-Biernat, M. Wusthoff, *Phys. Rev. D* **60** (1999) 114023.