Building a model of the process of grinding screws for ball-screw transmission

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Abstract. Studies of domestic and foreign scientists show that the radius of the thread profile has a significant impact on the performance of BS transmissions: load capacity, rigidity, smoothness and durability. Therefore, the accuracy of the thread profile is high requirements. In order to ensure the high quality of transmission functioning, it is very important to obtain a minimum deviation on the entire length of the propeller of parameters such as the moment of idling, the changes of which characterize the smoothness of the work, and the rigidity on which it depends, precision of movement and installation in the required position of the machine's working organs. Therefore, special attention is paid to ensuring the stability of the thread profile radius. For ensuring the stability of the radius of the thread profile, the article developed a mathematical model of the thread grinding process that ensures the preservation of the circle profile. It is shown that the obtained mathematical model requires experimental verification in the conditions of grinding the thread of the rolling screws and, if necessary, its corresponding refinement.

1. Introduction

Studies by national and foreign scientists show that the thread profile radius has a significant impact on the performance of ball-screw transmission (BST): load capacity, rigidity, smooth operation and durability, and therefore high demands are placed on the accuracy of the thread profile (tolerance on the radius of the groove profile in depending on the size is 0.007 ... 0.015 mm). However, even if these requirements are met, changes in the size of the threaded surface within the tolerance field lead to a change in the load capacity of the transmission by 1.6 times, its rigidity by 1.4 times and durability by 2.1 times [1-6, etc.]. To ensure a high quality of transmission operation, it is very important to obtain a minimum deviation over the entire length of the screw of such parameters as the idle time, changes of which characterize smooth operation, and rigidity, on which the accuracy of movement and installation of the working bodies of the machine into the required position depends. Therefore, special attention in the manufacture of screws and nuts is paid to ensure stability of the thread profile radius.

As follows from the technical requirements, for screws with a thread pitch of 10 mm, changes in the radius of the profile should not exceed 5 microns, and for screws with a thread pitch of 5 mm - 2.5 microns. When this condition is met, the calculated dispersion field of the load capacity, rigidity and durability of the transmission does not exceed 20% of their normal values.

As shown in [7-10], grinding wheels intended for machining a threaded groove are dressed with diamond needles and rollers and provide a constant dressing radius within the operation. Therefore, the radius of the profile of the wheels after editing at all junctions is the same and equal to the required
radius of the profile of the threaded groove. In the grinding process due to uneven wear, the profile of the wheel changes, leading to a deviation of the thread profile from the set. To ensure the required accuracy of the thread profile, it is necessary to create conditions that increase the stability of the profile of the wheel. In production, this is achieved by reducing the rate of metal removal, increasing the frequency of straightening the wheel, or by the simultaneous influence of these factors. The first way to improve the stability of the profile of the grinding wheel leads to a decrease in processing performance. Frequent straightening of the wheel due to loss of accuracy of the profile leads to an increase in the time of dressing per cycle to the workpiece and to a decrease in the life of the wheel to complete wear. The latter is explained by the fact that up to 85 ... 95% of the volume of abrasive material is removed during editing [11]. Therefore, improving the stability of the profile of the grinding wheel is an important task of the technological process, providing the required accuracy of the thread profile. When using expensive wheels, the requirement to ensure the stability of their profiles increases.

2. Initial position.
It was shown in [12] that when grinding the threads of screws, the shape of the profile of the grinding wheel remains close to semicircular, and the radius of the profile increases. This occurs as a result of more intense wear on the top of the wheel profile. The wear rate of the abrasive wheel increases with increasing intensity of metal removal [13-15, etc.]. It is possible to increase the stability of the thread profile by controlling the amount of wear on the wheel in various sections of the profile in two ways: by reducing the wear rate at the top of the profile or by increasing the wear rate of the wheel on the side sections of its profile.

When grinding a semicircular profile thread, the removed allowance is equal to \( Z = \sum_{i=1}^{n} t_{0i} \), where \( t_{0i} \) is the cutting depth at the top of the circle profile at the \( i \)-th transition. Therefore, the productivity of the operation is determined by the intensity of metal removal by the apex of the profile of the wheel at each of their transitions, i.e., the cutting depth \( t_{0i} \) and the peripheral speed of the workpiece \( V_i \). In this regard, the first way to improve the performance of the profile wheel will lead to a decrease in processing stability. Therefore, it is advisable to increase the stability of the profile in the second way, increasing the thickness of the removed metal layer in the lateral sections (in this case, the productivity of the process will not only not decrease, but, on the contrary, will increase). This can be achieved if the profile of the threaded groove received for grinding differs from the profile of the wheel so that the thickness of the layer to be removed on the side sections is greater than with the grinding pattern \( r = r_p \) (where \( r_p \) is the radius of the thread profile before machining).

The distribution of the cutting depth along the profile of the wheel in order to ensure the constancy of the thread profile is difficult, because the wear rate of the grinding wheel depends on many factors (physical and mechanical properties of the material being processed, the characteristics of the wheel, the presence and properties of CLTE, processing conditions, etc.). As a result of this, the profile of the threaded groove realizing such a distribution is different from the arc of a circle.

3. Materials and methods of research.
The main initial data for determining the profile of a threaded groove are the radius of the profile of the wheel \( r \) and the depth of cut at the vertex \( t_0 \). The radius of the profile of the circle is equal to the required radius of the profile of the thread after each working stroke. Therefore, as a mathematical model of the process of grinding the thread, ensuring the preservation of the geometric shape and radius of the profile of the circle, we take the dependence of the profile of the threaded groove entering the grinding on the radius of the profile of the circle \( r \) and the cutting depth \( t_0 \) at its top:

\[
Z_p = f(r, t_0),
\]

where \( Z_p \) is a curve describing the profile of a threaded groove.
The profile of the threaded groove can be described using the radius-vector $R_\alpha$. Based on the circuit shown in figure 1, we get:

$$R_\alpha = r - t_\alpha,$$

(2)

where $t_\alpha$ – cutting depth measured normal to the profile of the circle.

**Figure 1.** The scheme for determining the profile of the threaded groove, ensuring the stability of the profile of the wheel.

During the grinding process, the wheel will maintain its profile when its wear $U$ in the radial direction is the same throughout the profile. The wear of the wheel to its profile in this case should be equal to:

$$U_\alpha = U_0 \cos \alpha,$$

(3)

where $U_0$ – wear of the top of the profile of the grinding wheel.

Condition (3) is ensured for a certain cutting depth at each profile point and, therefore, for a certain law of variation of the cutting depth $t_\alpha$ along the profile of the circle. It was established in [13–16] that when grinding flat and cylindrical surfaces of steel billets, the wear of a wheel depends on the processing conditions:

$$U = C_u t^S V^Z.$$

(4)

where $C_u$ – coefficient depending on the properties of the processed material, CLTE and characteristics of the wheel. When grinding the threads of screws of BST, the values of $S$ and $V$ practically do not change along the thread profile. With this in mind, from formulas (3) and (4) we obtain:

$$\frac{U_\alpha}{U_0} = \left(\frac{t_\alpha}{t_0}\right)^x \cos \alpha.$$

From here we find the distribution of the depth of cut along the profile of the wheel

$$t_\alpha = t_0 \left(\cos \alpha\right)^{-1}.$$

Substituting the value of $t_\alpha$ in the formula (2), we obtain an expression describing the profile of the threaded groove received for grinding:

$$R_\alpha = r - t_0 \left(\cos \alpha\right)^{-1}.$$

Transform this formula:

$$R_\alpha = r \left(1 - \frac{t_0}{r} \left(\cos \alpha\right)^{-1}\right).$$

(5)

The expression in parentheses depends on the ratio $t_0/r$ and is constant at $(t_0/r) = \text{const}$. Moreover, the value of $R_\alpha$ is directly proportional to $r$. Those, at $(t_0/r) = \text{const}$, the shape of the profiles of the threaded grooves is the same, and the value of $r$ plays the role of a scale factor. According to [17], one can take $x = 1.6$. Then

$$t_\alpha = t_0 \cos^{0.625} \alpha$$

(6)
As follows from formulas (5) and (7), the profile of the threaded groove is different from the circular arc and has a complex shape. To form a threaded groove, you must apply a wheel with the same profile. Obtaining this profile is a difficult task, the solution of which requires the creation of special complex and expensive devices for straightening the wheel. The production experience and the results of studies on the wear of abrasive wheels [14, 15, 19, etc.] allow us to make the assumption that it is inadvisable to create such a profile due to the complex interaction of physical phenomena that occur during grinding, which quickly violate the profile of the wheel. As a result, the effect of the created complex profile will disappear. In this regard, in order to simplify the design of the control devices and instrumentation for checking the profile, it is necessary to make the assumption that the effect on the approximation processing error of the complex calculation profile is simpler.

To select an approximating profile, we consider the shape of the calculation profile made in accordance with formula (7) and shown in figure 1. The profile is designed for grinding a circle with a profile radius \( r = 3 \text{ mm} \) (the most common lead screw profile radius) with a cutting depth \( t_0 = 0.3 \text{ mm} \) (usually when grinding the thread, the cutting depth does not exceed 0.3 mm) in the range of angles \( \alpha = 0^\circ \) to \( 75^\circ \). With a smaller depth of cut, the deviation of the calculated profile from the profile of the circle is less. From figure 1 it can be seen that the calculated profile is close to the arc of a circle. Therefore, we consider the possibility of approximating the profile described by formula (7) with an arc of a circle of radius \( r_p \).

From figure 1 we get
\[
(1 - \cos \alpha) (1 - \cos \alpha) + 2 \cos \alpha = \frac{2}{\cos \alpha}.
\] (8)

At \( \alpha = 0 \)
\[
r_p = r - t_0 + a.
\] (9)

Thus, to determine the value of \( r_p \), it is necessary to know the value of \( a \). To find \( a \), we substitute in the formula (8) the values of \( R_a \) from expression (7) and \( r_p \) from formula (9). Solving the obtained equation with respect to \( a \), we find
\[
a = K t_0.
\] (10)

\[
1 - 0.5 \frac{t_0}{r} (1 + \cos \alpha)^{0.625} = \frac{1 - \cos \alpha}{1 - (\cos \alpha)^{0.625}} - \frac{t_0}{r} \times \frac{1 - \cos \alpha}{1 - (\cos \alpha)^{0.625}}.
\] (11)

Substituting the value of \( a \) from formula (10) into formula (7) and assuming \( A = 1 - K \), we obtain
\[
r_p = r - t_0 (1 - K) = r - A t_0.
\] (12)

It follows from formula (8) that with a decrease in \( a \), the value of \( r_p \) decreases. This leads to an increase in the depth of cut in the side sections of the profile. With decreasing \( a \), the coefficient \( K \) in formula (10) decreases, and, consequently, the coefficient \( A \) in formula (12) increases. That is, the larger the coefficient \( A \), the greater the cutting depth \( r_p \).

To determine the radius of the profile of a threaded groove approximating the calculated profile (7), it is necessary to know the value of the coefficient \( K \). From formula (11) it can be seen that the coefficient \( K \) depends on the ratio \( t_0/r \) and varies in profile depending on the angle \( \alpha \). Consider the influence of these factors on the value of \( K \). When grinding the threads of screws of BST, the range of change in the ratio is 0...0.1, and the thread profile is limited to an angle of 71°. The values of the coefficient \( K \) calculated by formula (11), depending on the angle \( \alpha \) and the ratio \( t_0/r \), are presented in table 1, and graphical dependencies \( K = f(\alpha, t_0/r) \) are shown in figure 2.
From the calculation results presented in table 1 and figure 2, it can be seen that the value of the coefficient $K$ changes with a change in the angle $\alpha$. These changes do not exceed 12% relative to the average coefficient $K_{\text{mean}} = \frac{\sum K_{i}}{6}$.

### Table 1. The values of the coefficient $K$ depending on the angle $\alpha$ and the ratio $t_0/r$.

| $t_0/r$ | $\alpha$, grad | $K_{\text{mean}}$ | $K_{\text{max}} - K_{\text{min}}$, $\%$ |
|---------|----------------|-------------------|----------------------------------------|
| 0       | 5              | 0.624             | 0.03                                   |
| 0.03    | 15             | 0.628             | 0.05                                   |
| 0.05    | 30             | 0.642             | 0.05                                   |
| 0.10    | 45             | 0.666             | 0.03                                   |
| 0.15    | 60             | 0.704             | 0.05                                   |
| 0.20    | 75             | 0.769             | 0.03                                   |
| 0.25    |                 | 0.672             | 0.03                                   |

The ratio of the coefficient $K$ along the thread profile $K_{\text{max}} - K_{\text{min}}$. Change in the coefficient $K$, not exceeding $\pm 3.4\%$ of the average value, defined as the arithmetic mean of $K_{\text{mean}}$ at $t_0/r = 0$, and $K_{\text{mean}}$ at $t_0/r = 1$: $K = \frac{0.672 + 0.716}{2} \approx 0.7$. Substituting this value in formulas (10) and (12), we obtain

$$a = 0.7t_0, \quad r_p = r - 0.3t_0.$$  \hspace{1cm} (13)  \hspace{1cm} (14)

Thus, as a result of approximation, a simple formula (14) is obtained. In order to determine the possibility of using this formula, we find the errors introduced by approximating the profile of another circle in the distribution of the cutting depth $t_\alpha$ along the profile of the wheel. To do this, we determine the distance $\Delta t_\alpha$ between the indicated curves at various angles $\alpha$ and compare it with the cutting depth $t_\alpha$. The quantity $\Delta t_\alpha$ is equal to

$$\Delta t_\alpha = t_\alpha - t'_\alpha.$$  \hspace{1cm} (15)

where $t_\alpha$ and $t'_\alpha$ – the cutting depth on the side sections of the grinding wheel profile when grinding the threaded groove with the calculated profile (7) and approximated (14), respectively.

The value of $t_\alpha$ is determined by the formula (6). To determine the cutting depth, we find the coordinates of the points of intersection of the line passing at an angle $\alpha$ to the $Y$ axis (figure 1), with the profile of the wheel ($x, y$) and with the profile of the thread ($x_p, y_p$). Then

$$t'_\alpha = \sqrt{(x - x_p)^2 + (y - y_p)^2}.$$  \hspace{1cm} (16)

The equation of the circular arc of the thread profile in the $XOY$ coordinate system (figure 1) has the form:
The equation of a line passing through the origin at an angle \( \alpha \) to the \( Y \) axis is described by the formula

\[ x^2 + (y + a)^2 = r_p^2. \]  \hspace{1cm} (17)

Solving equations (17) and (18) together, we obtain the coordinates:

\[ x_p = \left( r_p^2 - a^2 \sin^2 \alpha - a \cos \alpha \right) \sin \alpha; \quad y_p = \left( r_p^2 - a^2 \sin^2 \alpha - a \cos \alpha \right) \cos \alpha; \]

From figure 1 we find the coordinates of the intersection of line (17) with the profile of the wheel:

\[ x' = r \sin \alpha, \quad y = r \cos \alpha. \]

Substituting the obtained coordinates \( x, y, x_p, y_p \) in formula (16), we obtain

\[ t' = r - \sqrt{r_p^2 - a^2 \sin^2 \alpha - a \cos \alpha}, \]  \hspace{1cm} (19)

Taking into account the values \( t', \) and values of \( t_0 \) from formula (6), expression (15) will have the following form:

\[ \Delta t_a = t_0 \cos^0.625 \alpha - r + \sqrt{r_p^2 - a^2 \sin^2 \alpha - a \cos \alpha}. \]

Substituting in this formula the values of \( a \) and \( r_p \) from formulas (13) and (14), we obtain:

\[ \Delta t_a = t_0 \cos^0.625 \alpha - r + \sqrt{(r - 0.3t_0)^2 - 0.49t_0^2 \sin^2 \alpha - 0.7t_0 \cos \alpha}. \]  \hspace{1cm} (20)

To assess the degree of influence of the approximation on the change in the depth of cut \( t_0 \) using formula (20), we find the relation \( \Delta t_a/t_0 \):

\[ \frac{\Delta t_a}{t_0} = \cos^0.625 \alpha - 0.7 \cos \alpha - \frac{r}{t_0} + \sqrt{\left( \frac{r}{t_0} - 0.3 \right)^2 - 0.49 \sin^2 \alpha}. \]  \hspace{1cm} (21)

The relative error \( \Delta t_a/t_0 \) depends on the ratio \( r/t_0 \) and varies with the angle \( \alpha \). The value of the \( r/t_0 \) ratio when grinding the thread exceeds \( r/t_0 > 10 \). The values of \( \Delta t_a/t_0 \) calculated by formula (21), depending on the \( r/t_0 \) ratio and angle \( \alpha \), are presented in table 2.

| \( r/t_0 \) | \( \alpha \), deg. | 0 | 15 | 30 | 45 | 60 | 75 |
|------------|-----------------|---|---|---|---|---|---|
| 10         | 0               | 0.0007 | 0.0015 | -0.0024 | -0.0025 | -0.0751 |
| 20         | 0               | 0.0016 | 0.0047 | 0.0040 | -0.0109 | -0.0631 |
| 50         | 0               | 0.0021 | 0.0065 | 0.0077 | -0.0053 | -0.0562 |
| 100        | 0               | 0.0022 | 0.0072 | 0.0090 | -0.0035 | -0.0538 |

From table 2 it follows that for the chosen value of the coefficient \( K=0.7 \) as a result of approximation, the change in the cutting depth in most of the profile of the grinding wheel does not exceed 1% and reaches 7.5% only at the extreme sections (at \( r/t_0=10 \)). When grinding threads even with small cutting depths \( t_0 = 0.3 \) mm, these changes do not exceed 0.01 mm, which is a fairly small value.

According to studies [20-24 et al.], dedicated to the performance of abrasive wheels, the properties of wheels of the same marking and even within the same wheel fluctuate over a wide range, which, when grinding the same material with the same processing conditions, leads to a significant spread in the wear rate of the wheel. Therefore, we can assume that changes in the depth of cut in the lateral sections caused by the approximation of the calculated profile by an arc of a circle will not lead to a significant change in the wear rate. So, according to formula (4), the wear of the circle \( U \) is directly proportional to the value of \( t' \). When grinding the threads of the lead screws \( x=1.6 \). Then, a change in the cutting depth by 7.5% will lead to a change in the wear rate of \( \approx 12\% \), which is an insignificant
fraction of the scattering field of the values of $U$ [20, 22]. Therefore, with a sufficient degree of accuracy for practice, the calculated complex profile can be approximated by an arc of a circle.

4. Conclusion

As a result of theoretical research, a method has been developed to increase the stability of the profile of the wheel by increasing the wear of the wheel in the side sections by increasing the cutting depth on them. To implement the method, a mathematical model of the thread grinding process has been developed, which ensures the preservation of the profile of the grinding wheel. This model represents the dependence of the profile of the threaded groove that is received for grinding on the radius of the profile of the grinding wheel and the depth of cut at its top. The profile of the threaded groove, ensuring the stability of the profile of the wheel during wear, is complex. With a small error, this profile can be approximated by an arc of a circle. This allows us to simplify the design of the control devices and instrumentation to check the thread profile and grinding wheel.

The developed dependencies and formulas are obtained using the results of studies of wheel wear during grinding of flat and cylindrical surfaces of steel workpieces. Therefore, the developed mathematical model requires experimental verification under the conditions of grinding the threads of the ball-screw transmission and, if necessary, its corresponding refinement.

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