Yang-Mills fields for Cosets

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ABSTRACT

We consider theories with degenerate kinetic terms such as those that arise at strong coupling in \( N = 2 \) super Yang-Mills theory. We compute the components of generalized \( N = 1, 2 \) supersymmetric sigma model actions in two dimensions. The target space coordinates may be matter and/or Yang-Mills superfield strengths.
1 Introduction

Yang-Mills theory [1] is the essential ingredient in our understanding of all fundamental interactions. Part of its beauty comes from its rigid geometric structure: Once a gauge group is chosen, the basic degrees of freedom of the gauge sector are specified. In this note, we present a generalization of this structure, and find Yang-Mills fields that in some sense are gauge fields for cosets.

It has long been known that in gauge theories with scalar fields, nonminimal couplings may modify the gauge-field kinetic terms. In particular, positive definite kinetic terms were constructed for noncompact groups in [2, 3]. Here we consider another possibility: semi-definite terms which give rise to gauge theories where part of the gauge group is auxiliary. As an example, consider an $SU(2)$ gauge theory with an adjoint representation scalar field $\varphi^i$. Then, using the methods of [3], the most general gauge invariant action with no more than two derivatives is

$$S_0 = \int d^Dx \left[ g_1(|\varphi|) \left( \delta_{ij} - \frac{\varphi^i \varphi^j}{|\varphi|^2} \right) (f_{MN}^i f_{MN}^j + h_1(|\varphi|) \nabla_M \varphi^i \nabla_M \varphi^j) ight. \\
+ g_2(|\varphi|) \left( \frac{\varphi^i \varphi^j}{|\varphi|^2} \right) (f_{MN}^i f_{MN}^j + h_2(|\varphi|) \nabla_M \varphi^i \nabla_M \varphi^j) + V(|\varphi|) \right], \quad (1.1)$$

Minimal coupling implies $g_1 = g_2 = h_1 = h_2 = 1$, and the usual positive definite case arises when these functions are all positive. We want to consider the case when $g_1$ or $g_2$ vanishes; in the first case, only the gauge field for a $U(1)$ subgroup remains dynamical, and the $SU(2)/U(1)$ gauge fields become auxiliary; this is analogous to the discussion of “composite gauge-fields” (see, e.g., [4]). When $g_2$ vanishes, we have a new situation: the $U(1)$ gauge-field is auxiliary, and only the gauge-fields for the coset $SU(2)/U(1)$ remain dynamical.

In [3], a general theory of invariant nonminimal kinetic terms was developed; the main object there was to construct positive definite kinetic terms for noncompact gauge groups. However, exactly the same techniques can be used to construct degenerate kinetic terms such as those described above. Indeed, using the techniques of [3], we can couple gauge fields to nonlinear sigma-models in arbitrary dimensions, and select which gauge fields are dynamical and which are auxiliary. As in the examples above, we can “keep” either the fields of a subgroup, or of a coset. In this sense, we have constructed a gauge theory for cosets. A possible application of this would be to effective actions with fewer gauge fields than local gauge symmetries.

Though in two dimensions an action such as (1.1) is renormalizable, in higher dimensions it is more interesting when considered as the low energy limit of an effective action. Indeed, specific functions $g_1, g_2, h_1, h_2$ were given for the $D = 4, N = 2$ super...
Yang-Mills low energy effective action in [5]. As shown by two of us, though $g_2$ is always positive and hence the $U(1)$ gauge field remains dynamical for all values of $\langle |\varphi| \rangle$, along a certain curve, $g_1$ vanishes, and then actually changes sign; we interpret this as a signal that the coset gauge fields become nondynamical and disappear from the spectrum [6].

Extended super Yang-Mills theories contain physical scalars as superpartners of the gauge-fields; this means that in superspace, there are scalar super field-strengths which can be used to construct nonlinear sigma models. This has been studied for $D = 4$, $N = 2$ theories (the “Kählerian vector multiplet” [7, 3]), where it gives rise to a limit of “special Kähler geometry”. Here we consider the analogous sigma-models in two dimensions (similar constructions can be done in higher dimensions). For $N = 1$, $D = 2$, the super field-strength is a scalar superfield that transforms in the adjoint representation of the gauge group, and can be used to coordinatize any manifold that is invariant under the gauge-symmetry and has dimension less than or equal to the dimension of the group. Similarly, for $N = 2$, $D = 2$, the super field-strength is a complex twisted (covariantly) chiral superfield (again in the adjoint representation) [9], and can be used to coordinatize any Kähler manifold that is invariant under the gauge-symmetry and has dimension less than or equal to twice the dimension of the group. (Generalizations exist for actions that include twisted super Yang-Mills multiplets; these give rise to manifolds with torsion).

In the next section we collect the basics of $N = 1$ Yang-Mills theory in $D = 2$ and give the reduction to $N = 0$ of a general nonlinear action including matter-fields. In Sec. 3, we turn to $N = 2$, recapitulate the basic definitions and representations, and reduce a general action, containing both covariantly chiral and twisted chiral fields, to $N = 1$. We find that it takes on the same form as for the usual nonlinear $\sigma$-model. In both sections, we give $SU(N)$ examples analogous to (1.1).

2 $N = 1$

We first briefly review $N = 1$, $D = 2$ super Yang-Mills theory. The covariant superspace derivatives $\nabla_\pm \equiv D_\pm + i\Gamma_\pm$ satisfy the algebra

$$(\nabla_\pm)^2 = \nabla_{\pm\pm}, \quad \{\nabla_+, \nabla_-\} = F \equiv F^i T_i,$$  \hspace{1cm} (2.1)$$

where $\nabla_{\pm\pm} \equiv \partial_{\pm\pm} + i\Gamma_{\pm\pm}$, $\partial_{++} \equiv \partial$, $\partial_{--} \equiv \bar{\partial}$ are the usual (anti)holomorphic derivatives on the world-sheet and $T_i$ are the Lie algebra generators satisfying $[T_i, T_j] = c_{ij}^k T_k$. The super field-strength $F$ is an unconstrained scalar except that the Bianchi identities

\[ c_{ij}^k T_k. \]
imply:
\[ \nabla_+ \nabla_- F = -[\nabla_{++}, \nabla_{--}] . \] (2.2)

The standard component fields are defined by

\[ \varphi \equiv F|, \quad A \equiv \Gamma_{++}|, \quad \bar{A} \equiv \Gamma_{--}|, \quad \lambda_+ \equiv \nabla_+ F|, \quad \lambda_- \equiv \nabla_- F|, \]

\[ f \equiv \partial \bar{A} - \bar{\partial} A + i[A, \bar{A}] = -\nabla_+ \nabla_- F| . \] (2.3)

We consider a superspace action for the Yang-Mills multiplet for a gauge group \( G \)
\[ S_{N=1} = \frac{1}{2\pi} \int d^2 z \nabla^2 \left[ (g_{ij}(F) + b_{ij}(F)) \nabla_+ F^i \nabla_- F^j \right] , \] (2.4)

where \( i, j \) are group indices, \( g \) is some \( G \)-invariant metric, and \( b \) is a two-form that is invariant modulo exact terms. The minimal case occurs when \( b = 0 \) and \( g \) is the killing metric \( k_{ij} \). Typical nonminimal terms are:

\[ g_{ij}(F) = g_0(F)k_{ij} + g_1(F)F^i F^j + ... \] (2.5)

\[ b_{ij} = b_0(F)F^k c_{ijk} + ... \] (2.6)

We may also consider nonminimal couplings to extra matter multiplets. These are described by scalar superfields \( \phi^\mu \) that coordinatize some manifold which admits an action of the gauge group generated by Killing vector fields \( k^\mu_i(\phi) \). The scalars have component expansions:

\[ X^\mu \equiv \phi^\mu|, \quad \psi^\mu_\pm \equiv \nabla_\pm \phi^\mu|, \quad S^\mu \equiv \frac{1}{2}[\nabla_+, \nabla_-]\phi^\mu| , \] (2.7)

where the gauge covariant derivatives are

\[ \nabla \phi^\mu \equiv D\phi^\mu + i\Gamma^i k^\mu_i . \] (2.8)

The nonminimal kinetic term for the Yang-Mills multiplet may depend on these scalars, and the general formalism of [3] can be applied. Furthermore, the gauge fields couple to the scalar kinetic term as described in [8]; in many cases, this is simply a matter of minimal coupling using the covariant derivatives (2.8) in an action analogous to \( S_{N=1} \) (2.4) with \( \{ F^i \} \rightarrow \{ F^i, \phi^\mu \} \).

Using the definitions of the components (2.3), we find that the action \( S_{N=1} \) (2.4), including matter-fields \( \phi^\mu \), has the component expansion

\[ S_{N=1} = \frac{1}{2\pi} \int d^2 z \left\{ (g_{\alpha\beta}(Y) + b_{\alpha\beta}(Y))(-\nabla_{++} Y^\alpha \nabla_{--} Y^\beta) \right\} . \]
\[ + g_{\alpha\beta} (\psi_+^\alpha D_- \psi_+^\beta + \psi_+^\alpha D_+ \psi_-^\beta) \]

\[ + \frac{1}{2} R_{(-)\alpha\beta\gamma\delta} \psi_+^\delta \psi_+^\gamma \psi_-^\alpha - i \psi_+^\alpha \lambda_-^\delta \tilde{k}_\alpha + i \lambda_+^\delta \tilde{k}_\alpha \psi_-^\alpha \]

\[ - (S_\alpha + \frac{i}{2} b_{\beta\alpha} \varphi^i k_\beta^i - \Gamma_{\alpha\beta\gamma}^\delta g_{\delta\gamma} \psi_+^\beta)^2 \]

\[ - \frac{1}{4} (\varphi^i \tilde{k}_{i\alpha})^2 + i \varphi^i (\tilde{k}_{i\alpha})_{\gamma} \psi_+^\alpha \psi_-^\gamma \}

(2.9)

where the symbolic squaring of expressions within parenthesis is shorthand for the scalar product in the metric \( g_{\alpha\beta} \) and

\[ D_{\pm\pm} \psi_\mp^\beta \equiv \nabla_{\pm\pm} \psi_\mp^\beta + (\nabla_{\pm\pm} Y) \Gamma_{(\pm)\gamma}^{\beta} \psi_\mp^\gamma , \]

\[ \nabla \psi^\beta \equiv \partial \psi^\beta + i \Gamma^i_{\gamma} k_i^\beta \psi^\alpha , \]

\[ \tilde{k}_{i\alpha} \equiv (g_{\alpha\beta} + b_{\alpha\beta}) k_i^\beta . \]

(2.10)

The connections are defined as

\[ \Gamma_{(\pm)\alpha\beta}^\gamma \equiv \Gamma_{(0)\alpha\beta}^\gamma \pm T_{\alpha\beta}^\gamma \]

(2.11)

with \( \Gamma_{(0)\alpha\beta}^\gamma \) the metric (Christoffel) connection and the torsion defined as

\[ T_{\alpha\beta\gamma} \equiv \frac{1}{2} (b_{\alpha\beta,\gamma} + b_{\gamma[a,\beta]}). \]

(2.12)

The curvature tensor formed from \( \Gamma_{(\pm)\alpha\beta}^\gamma \) is denoted by \( R_{(\pm)\alpha\beta\mu\nu} \); denotes the \( \Gamma^-\) covariant derivative w.r.t. \( Y \), and we have used a collective notation for the fields:

\[ Y^\alpha \equiv \left( \varphi^i X^\mu \right) , \quad \psi_\mp^\alpha \equiv \left( \lambda_\pm^\beta \psi_\pm^\beta \right) , \quad S^\alpha \equiv \left( f^i S^\mu \right) , \quad k_i^\alpha \equiv \left( c^i_{jk} k_j^k \right) , \]

(2.13)

where \( c^i_{jk} \) are the structure constants of the Lie algebra under consideration. To separate the \( N = 0 \) Yang Mills field strength \( f^i \) from the auxiliary matter field \( S^\mu \) we rewrite the \( S^\alpha \) term in (2.9) as follows

\[ - \left[ S^\mu + i \frac{1}{2} b^\mu_{\alpha\beta} \varphi^i k_i^\alpha - \Gamma_{\alpha\beta\gamma}^\delta g_{\delta\gamma} \psi_+^\beta + g^{\mu\alpha} g_{\alpha i} \left( f^i + i \frac{1}{2} b_{\beta\alpha} \varphi^j k_j^\beta - \Gamma_{\alpha\beta\gamma}^\delta \psi_+^\gamma \psi_-^\delta \right) \right] \]

\[ - \left( f^i + i \frac{1}{2} b_{\beta\alpha} \varphi^j k_j^\beta - \Gamma_{\alpha\beta\gamma}^\delta \psi_+^\gamma \psi_-^\delta \right) \tilde{g}_{ik} \left( f^k + i \frac{1}{2} b_{\beta\gamma} \varphi^j k_j^\beta - \Gamma_{\alpha\beta\gamma}^\delta \psi_+^\delta \psi_-^\gamma \right) , \]

(2.14)
where we have used the metric $g_{\mu\nu}$ in the first square and $\tilde{g}_{ik} \equiv (g_{ik} - g_{i\mu}g^{\mu\nu}g_{\nu k})$.

We are particularly interested in cases when the metric $\tilde{g}$ is degenerate; then, though the action is invariant under the full gauge group, some of the component gauge and scalar fields are auxiliary. We consider two examples for the gauge group $SU(N)$: Let

$$ g_{ij} = \left( \frac{1}{|F|^2} \right) \left[ g_1 \left( \delta^{ij} - \frac{F^i F^j}{|F|^2} \right) + g_2 \frac{F^i F^j}{|F|^2} \right], \quad (2.15) $$

where $g_1 = 0, g_2 = 1$ or $g_1 = 1, g_2 = 0$. We include no matter fields and put $b_{ij} = 0$. The bosonic part of the lagrangian in (2.9) is then proportional to (1.1) with $h_1 = h_2 = 1$ and $g_1, g_2$ as above. (The proportionality factor is $1/|F|^2$.)

3 $N = 2$

The same phenomenon as described in the previous section occurs for $N = 2, D = 2$ super Yang-Mills. In part because this is how we discovered the phenomenon, and in part because of the curious interplay of these ideas and Kähler geometry, we now give a detailed discussion.

In two dimensions, the $N = 2$ Yang-Mills supermultiplet is described by a gauge-covariantly twisted chiral superfield [9]. It is well known that ordinary chiral superfields can be interpreted as the complex coordinates of a Kähler manifold [10]. In particular, the superspace Lagrangian

$$ K = \ln(\Phi^\mu \bar{\Phi}^\mu), \quad \mu = 1, ..., n + 1, \quad (3.1) $$

gives rise to a $\sigma$-model on the manifold $CP(n)$, with $\Phi^\mu$ the natural homogeneous coordinates (chiral superfields). Because of Kähler-invariance, the component action depends not on all the $\Phi^\mu$, but only on their ratios. If we replace $\Phi^\mu$ by the field strength of a nonabelian gauge multiplet, we find a system that does not depend on all the usual gauge fields, but rather only on certain combinations; nevertheless, we maintain the full gauge invariance of the system.

We begin by reviewing some background material.

3.1 Chiral, twisted chiral, and variant superfields

Extended supersymmetric multiplets are generally described by constrained superfields (see, for example, [11]). In our case ($D = N = 2$), we will work with chiral superfields $\Phi$ that satisfy the constraints

$$ \bar{D}_+ \Phi = 0, \quad D_+ \bar{\Phi} = 0, \quad (3.2) $$
where the spinor derivatives $D, \bar{D}$ as usual satisfy the supersymmetry algebra
\begin{equation}
\{D_+, \bar{D}_+\} = \partial, \quad \{D_-, \bar{D}_-\} = \bar{\partial}.
\end{equation}

The constraints (3.2) have the general solution:
\begin{equation}
\Phi = i\bar{D}_+ \bar{D}_- \Psi, \quad \bar{\Phi} = iD_+ D_- \bar{\Psi}.
\end{equation}

The chiral superfield reduces to a complex unconstrained $N = 1$ scalar superfield.

We will also work with twisted chiral superfields $\chi$ that satisfy twisted constraints \cite{9}
\begin{equation}
\bar{D}_+ \chi = D_- \chi = 0, \quad D_+ \bar{\chi} = \bar{D}_- \bar{\chi} = 0,
\end{equation}
which can be solved by
\begin{equation}
\chi = i\bar{D}_+ D_- \Psi, \quad \bar{\chi} = iD_+ \bar{D}_- \bar{\Psi}.
\end{equation}

Just as for the chiral case, when we project to $N = 1$ superspace, the twisted chiral superfield reduces to a complex unconstrained $N = 1$ scalar superfield.

As explained in \cite{12}, though these solutions are general, they are not unique: there exist variant multiplets satisfying the same constraints. For example, a variant multiplet is found by constraining $\Psi = \bar{\Psi} \equiv V$. The effect of this is to replace a complex auxiliary field by a real field and the divergence(curl) of a vector field. Since, in two dimensions, vector fields are not dynamical, except for possible topological issues, this does not change the physical content of the theory. In particular, for the twisted case, the variant multiplet has a familiar interpretation \cite{9}: It is the superfield strength of a $U(1)$ gauge multiplet.

### 3.2 $D = 2, N = 2$ Yang-Mills theory

In the $N = 2$ case, the gauge covariant derivatives obey the constraints:
\begin{align*}
\nabla_\pm & \equiv \bar{D}_\pm + i\Gamma_\pm, \quad \nabla_\pm \equiv \{\nabla_+, \bar{\nabla}_+\} \quad \nabla_\pm \equiv \partial_\pm + i\Gamma_\pm,
\{\bar{\nabla}_+, \nabla_-\} & = \mathcal{F}, \quad \{\nabla_+, \bar{\nabla}_-\} = \bar{\mathcal{F}}, \quad \nabla^2_\pm = \bar{\nabla}^2_\pm = 0,
\end{align*}

where $\mathcal{F}$ is a complex superfield strength. The Bianchi identities imply that $\mathcal{F}$ is a gauge covariantly twisted chiral superfield (c.f. 3.5):
\begin{equation}
\bar{\nabla}_+ \mathcal{F} = \nabla_- \mathcal{F} = 0.
\end{equation}
A convenient solution to the constraints can be found in a chiral representation, i.e., in a representation where the gauge parameter is a chiral superfield $\Lambda$:

$$
\nabla_\pm = \bar{D}_\pm, \quad \nabla_\pm = e^{-V} D_\pm e^V, \quad (3.9)
$$

where $V$ is an unconstrained prepotential that transforms as

$$
e^{V'} = e^{i\bar{\Lambda}} e^V e^{-i\Lambda}. \quad (3.10)
$$

Note that in chiral representation, since the gauge parameter is chiral, and hence complex, $\nabla \neq (\nabla)^\dagger$, etc.

We can also consider a twisted super Yang-Mills theory with a twisted chiral gauge parameter which couples to covariantly twisted chiral matter and has a covariantly chiral field strength. The twisted theory is constructed by interchanging $\nabla_-$ with $\bar{\nabla}_-$ in (3.7-3.9) above.

The $N = 1$ components of the untwisted multiplet are defined as:

$$F^i \equiv F^i + \bar{F}^i, \quad \phi^i \equiv -i(F^i - \bar{F}^i), \quad (3.11)$$

where $F^i$ is the $N = 1$ Yang-Mills field strength and $\phi^i$ is $N = 1$ matter.

### 3.3 $CP(n)$ $\sigma$-models

Sigma-models in $N = 2$ superspace are described by a super-Lagrangian that is just a function of chiral (and/or twisted chiral) superfields and their complex conjugates: $K(\Phi, \bar{\Phi})$. In the case with only chiral or twisted chiral superfields, the superfields can be interpreted as complex coordinates on a Kähler manifold with Kähler potential $K$ [10]. The metric is just the complex Hessian of $K$, and hence is unchanged if one adds any holomorphic function to it; in superspace, this Kähler-invariance arises because any function of purely chiral superfields is itself chiral, and hence annihilated by the full superspace measure $D^2 \bar{D}^2$.

A particularly simple description exists of the $\sigma$-model on $CP(n)$; the superspace Lagrangian is:

$$K = \ln(\Phi^\mu \bar{\Phi}^\mu), \quad \mu = 1, \ldots, n + 1, \quad (3.12)$$

where $\Phi^\mu$ are homogeneous coordinates. Because of Kähler-invariance, the component action depends not on all the $\Phi^\mu$, but only on their ratios; equivalently, the model is invariant under the gauge transformation

$$\Phi^\mu \rightarrow \Lambda \Phi^\mu, \quad \bar{\Phi}^\mu \rightarrow \bar{\Lambda} \bar{\Phi}^\mu, \quad (3.13)$$

for an arbitrary chiral superfield $\Lambda$. Then one can choose the gauge $\Phi^{n+1} = 1$.

We now consider generalized $\sigma$-models constructed out of $N = 2$ superfields.
3.4 Kähler manifolds and gauge theories

In this section we derive the $N = 1$ super-component action for a general superspace Lagrangian

$$L_{N=2} = K(\Phi^A, \bar{\Phi}^B), \quad (3.14)$$

where $A \in (\mu, i)$, and $\Phi^\mu$ ($\bar{\Phi}^i$) are covariantly (twisted) chiral superfields; $\Phi^\mu$ is thus either a matter field covariant with respect to an untwisted Yang-Mills symmetry or a field strength of a twisted Yang-Mills multiplet whereas $\Phi^i$ is either a matter field covariant with respect to a twisted Yang-Mills symmetry or a field strength of an untwisted Yang-Mills multiplet. In deriving the $N = 1$ action we will make the assumption that $K(\Phi^A, \bar{\Phi}^B)$ is invariant under local gauge transformations. This makes it possible to use covariant superspace derivatives in the superspace measure. In the language of Kähler geometry we are assuming that $K$ is invariant under the isometry we have chosen to gauge. This is not the most general possibility, but is sufficient for our purposes.

To rewrite the invariant Lagrangian (3.14) in $N = 1$ language, we define new covariant derivatives

$$\nabla_1^\pm \equiv \bar{\nabla}_\pm + \nabla_\pm, \quad \nabla_2^\pm \equiv i(\bar{\nabla}_\pm - \nabla_\pm). \quad (3.15)$$

In terms of these derivatives the covariant (twisted) chirality constraints become

$$\nabla_2^\pm \Phi^A = -i\varepsilon_\pm(A)\nabla_1^\pm \Phi^A, \quad \nabla_2^\pm \bar{\Phi}^A = i\varepsilon_\pm(A)\nabla_1^\pm \bar{\Phi}^A, \quad (3.16)$$

where no summation over $A$ is intended and the sign-factor is

$$\varepsilon_\pm(\mu) = 1, \quad \varepsilon_\pm(i) = \pm 1. \quad (3.17)$$

We take the $N = 1$ algebra to be the one spanned by $\nabla^1$ (c.f. (2.1)):

$$\{\nabla_1^\pm, \nabla_1^\mp\} = 2\nabla^\pm, \quad \{\nabla_1^\pm, \nabla_1^\pm\} = F. \quad (3.18)$$

Using (3.15), the action corresponding to (3.14) may be written

$$S = \int d^2z d^2\theta d^2\bar{\theta} K(\Phi^A, \bar{\Phi}^B) = \int d^2z \nabla_+ \nabla_- \nabla_+ \nabla_- K(\Phi^A, \bar{\Phi}^B) |$$

$$= -\frac{1}{4} \int d^2z \nabla_+^1 \nabla_-^1 \nabla_+^2 \nabla_-^2 K(\Phi^A, \bar{\Phi}^B) |, \quad (3.19)$$

where $|$ denotes the projection onto the $\theta_2$-independent part. Letting the $\nabla^2$’s act on $K$ we find, using (3.16),

$$S = -\frac{1}{2} \int d^2z d^2\theta (G_{IJ} + B_{IJ}) \nabla_+ \phi^I \nabla_- \phi^J, \quad (3.20)$$
where $\nabla \equiv \nabla^1$ and $\theta \equiv \theta^1$ is the corresponding Fermi coordinate. The new indices have the range $I \in (A, \bar{A})$ and the target space metric and antisymmetric tensor field are

$$G_{IJ} = \begin{pmatrix}
0 & K_{\mu\nu} & 0 & 0 \\
K_{\bar{\mu}\nu} & 0 & 0 & 0 \\
0 & 0 & 0 & -K_{\bar{i}j} \\
0 & 0 & -K_{\bar{i}j} & 0
\end{pmatrix} \tag{3.21}$$

$$B_{IJ} = \begin{pmatrix}
0 & 0 & 0 & -K_{\mu\bar{j}} \\
0 & 0 & -K_{\bar{\mu}j} & 0 \\
0 & K_{\bar{i}\nu} & 0 & 0 \\
K_{\nu\bar{i}} & 0 & 0 & 0
\end{pmatrix}. \tag{3.22}$$

The $N = 0$ component action is found by substituting (3.20,3.21,3.22) into (2.9); a form of it has been previously given in [13].

Above we have not interpreted the twisted covariantly chiral fields as either matter-fields or Yang-Mills fields. If $\Phi^A$-fields are all matter fields, the $N = 1$ component fields are simply their $\theta_2$-independent parts. If, for some $A$, $\Phi^A = F^i$ (the $N = 2$ field strengths), we also have to display the $N = 1$ components (3.11). For the special case with no $\Phi^\mu$’s and all $\Phi^i = F^i$, and the Kähler potential $K = ln|\Phi^i \bar{\Phi}^i|$ we obtain $B_{IJ} = 0$ and a $G_{IJ}$ that gives

$$S = \int d^2z d^2\theta \left\{ \left( \frac{1}{F^2 + \phi^2} \right) \left[ \left( \delta^i_j - \frac{F^i F^j + \phi^i \phi^j}{(F^2 + \phi^2)} \right) (\nabla^+ F^i \nabla^- F^j + \nabla^+ \phi^i \nabla^- \phi^j) \right.ight.$$  
$$- \frac{F^{[i} \phi^{j]}}{(F^2 + \phi^2)} \nabla^+ F^j \nabla^- \phi^i \left\} \right. \tag{3.23}$$

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