High Precision, Low Excitation Capacitance Measurement Methods from 10 mK- to Room-Temperature

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Capacitance measurement is a useful technique in studying quantum devices, as it directly probes the local particle charging properties, i.e., the system compressibility. Here we report one approach which can measure capacitance from mK to room temperature with excellent accuracy. Our experiments show that such a high-precision technique is able to reveal delicate and essential properties of high-mobility two-dimensional electron systems.

I. INTRODUCTION

Capacitance contains useful information of electronic devices, as it directly probes their electrical charging properties. Recently, the capacitance measurement at cryogenic temperature has attracted significant attention in quantum studies and has revealed a series of quantum phenomena. Although high precision capacitance measurement is widely used in studying classical devices such as field effect transistors and diodes, it is extremely difficult to be performed on quantum devices. Firstly, the quantum phenomena usually emerge in fragile systems so that the excitation should be sufficiently low to preserve the quantum properties of the device. Secondly, many quantum devices can only be studied in cryostats which host the low temperature and high magnetic field environment. The meter-long cables connecting samples and room temperature instruments have ~100 pF capacitance, which is orders-of-magnitude larger than the devices themselves. Thirdly, the total power dissipation at the cryogenic sample stage must be limited to sub-µW in order to maintain the low-temperature environment. This limits the use of active devices.

In order to reduce the crosstalk and signal leaking between cables, many reported works use the cryogenic preamplifier to isolate the input and output signal. Unfortunately, the preamplifiers usually dissipate more than ~10 µW heat at the sample stage, which is sufficiently high to cause a noticeable temperature raise. Sometimes, indirect probes of capacitance such as penetration field, etc., are used which can provide qualitative information about quantum phase transitions.

In this report, we introduce a new approach for high precision capacitance measurement from 10 mK- to room-temperature. We install a passive bridge at the sample stage, and use a voltage-controlled-variable-resistance to in-situ tune its balance. We use radio frequency excitation to increase the output signal, and develop a high sensitivity radio frequency (RF) lock-in technique to analyze the bridge output.

II. METHOD

The passive bridge at the sample stage (Fig. 1(a)) consists of the device under test $C_{DUT}$, the reference capacitor $C_{ref}$, the reference resistor $R_{ref}$ and a voltage-controlled-variable-resistor $R_h$. The excitation voltage $V_{in}$ is differentially coupled to the $V_{in}^+$ & $V_{in}^-$ ports of the bridge by a RF transformer, and the output signal $V_{out}$ is the voltage difference between the midpoints of the capacitor and resistor arms. The bridge dissipates only ~10 nW when $V_{in}$ is 1 mV rms, mostly caused by $R_{ref}$ and $R_h$. We can tune the balance of the bridge with $R_h$, which is the drain-to-source resistance of a high electron mobility transistor (model ATF35143). The bridge reaches its balance point when the balance condition

$$\frac{C_{DUT}}{C_{ref}} = \frac{R_h}{R_{ref}}, \quad (1)$$

is achieved, signaled by the $|V_{out}|$ minimum (see Fig. 1(b)). Two low-frequency current inputs $I_1$ and $I_2$ are injected into the bridge. They flow into the ground through $R_h$ and $R_{ref}$ respectively, and generate a voltage difference $(V_{R_h}^+ - V_{R_h}^-) = I_1 \cdot R_h - I_2 \cdot R_{ref}$. We can in-situ measure $R_h$ and $R_{ref}$ simultaneously with lock-in technique by locking the frequency of $(V_{R_h}^+ - V_{R_h}^-)$ to be the same as $I_1$ and $I_2$. Therefore, as long as $C_{ref}$ is stable, the absolute value of $C_{DUT}$ can be deduced by Eq. [1]. The $|V_{out}|$ vs. $R_h$ curve is “V”-shaped, so we call this procedure as the “V-curve” procedure for brevity.

The output impedance of this bridge is about $1 \frac{2\pi f}{(C_{ref}+C_{DUT})}$, where $f$ is the frequency of $V_{in}$. $C_{DUT}$ and $C_{ref}$ are typically in the order of 0.1 pF, corresponding to output impedance as large as ~1 Ω when $f = 1$ kHz. Therefore, we increase $f$ to ~100 MHz so that the output impedance decreases to about 10 kΩ. We use standard 50 Ω coaxial cable and impedance matched RF preamplifier at room temperature for broadband measurement. When the excitation amplitude is ~1 mV rms, $V_{out}$ is only about ~1 µV rms and 0.1% change in $C_{DUT}$ corresponds to ~1 nV rms variation in the output. In order to analyze such a small signal, we combine the superheterodyne and lock-in techniques (see Fig. 1(c)). The signal source generates three single-frequency signals $V_{in}, V_{LO}$ and $V_{ref}$ with frequencies $f_{in}, f_{LO}$ and $f_d$, respectively, where $f_d = |f_{in} - f_{LO}|$. $V_{in}$ is sent to the bridge as the voltage excitation. The bridge output $V_{out}$ is amplified by a low-noise-amplifier, mixed with $V_{LO}$.
and low-pass-filtered, resulting in an audio frequency signal \( V_d \). We use a digital audio-frequency lock-in unit to measure the amplitude \( |V_d| \) and phase \( \theta_d \) in reference to \( V_{ref} \), where \( |V_d| \propto |V_{out}| \) and \( \theta_d \) is the same as the phase difference between \( V_{out} \) and \( V_{in} \) but differs by a constant. For simplicity, we quote \( |V_d| \) and \( \theta_d \) as the amplitude \( |V_{out}| \) and the (relative) phase \( \theta \) (from \( V_{in} \)) of the bridge output.

III. CALIBRATION

We calibrate our setup at room temperature by measuring fixed-value capacitors whose nominal value \( C_{nom} \) ranges from 0.1 to 10 pF (see Fig. 2(a)). We select different \( C_{ref} \) that is comparable with \( C_{DUT} \). The y-axis is the measured \( C_{DUT} \) using the "V-curve" procedure with \( \approx 1.5 \text{ mV}_{\text{rms}} \) excitation amplitude and \( \approx 110 \text{ MHz} \) frequency. The horizontal and vertical error bars are deduced from the tolerance of \( C_{DUT} \) and \( C_{ref} \), which dominate the inaccuracy at \( < 1 \text{ pF} \). The measured \( C_{DUT} \) matches its nominal value \( C_{nom} \) even for capacitors as small as \( 0.1 \text{ pF} \), evidencing that our setup is capable of measuring the absolute capacitance value\(^1\[9\].

The "V-curve" procedure can measure the absolute value of \( C_{DUT} \) with decent accuracy, however, continuously monitoring \( C_{DUT} \) while sweeping particular physical parameter, such as magnetic field, temperature, etc, is of more interest. We can deduce \( C_{DUT} \) from \( V_{out} \) using the \( V_{out} \) vs. \( R_h \) relation obtained by the "V-curve" procedure. At the vicinity of the balance point, the bridge output \( V_{out} \) is approximated as

\[
V_{out} \propto \left( \frac{C_{DUT}}{C_{ref} + C_{DUT}} - \frac{R_h}{R_{ref} + R_h} \right) \cdot V_{in} + V_0,
\]

(2)

where \( V_0 \) represents the leaking signal. \( V_{out} \) is a vector and can be decomposed into two orthogonal scalar components using its amplitude \( |V_{out}| \) and phase \( \theta \)

\[
\begin{align*}
V_x &= |V_{out}| \cdot \cos(\theta), \\
V_y &= |V_{out}| \cdot \sin(\theta),
\end{align*}
\]

(3)

Note that, in the ideal case when \( V_0 = 0 \), \( V_{out} \) changes its sign, or \( \theta \) changes by \( 180^\circ \) as we tune the bridge through its balance point. We set \( \theta = -90^\circ \) at the balance point so that \( V_x = V_{out} \) is the expected bridge output (Fig. 1(b)). In a careful measurement, leaking signal \( V_0 \) mostly comes from the capacitive coupling between the input and output cables and has a \( 90^\circ \) phase shift from \( V_{in} \). At the vicinity of the balance point, \( V_x \approx V_0 \) and \( V_x \) has a linear dependence on \( \frac{R_h}{R_{ref} + R_h} \), see Fig. 1(b). From the symmetry between the resistors and capacitors in the bridge, we assume a single parameter, the sensitivity \( S \), can describe the dependence of \( V_x \) on both \( R_h \) and \( C_{DUT} \) by

\[
S = \frac{\partial V_x}{\partial \frac{R_h}{R_{ref} + R_h}} = \frac{\partial V_x}{\partial \frac{C_{DUT}}{C_{ref} + C_{DUT}}}. \tag{4}
\]

\( S \) can be obtained by linearly fitting \( V_x \) with \( \frac{R_h}{R_{ref} + R_h} \) while keep \( C_{DUT} \) fixed. Note that \( S \) is inversely proportional to the output impedance \( 1/2\pi f(C_{ref} + C_{DUT}) \) and corresponding corrections might be necessary. Thereafter, we can deduce \( C_{DUT} \).
from $V_{\text{out}}$ by
\[
\frac{C_{\text{DUT}}}{C_{\text{ref}} + C_{\text{DUT}}} = \frac{R_h}{R_{\text{ref}} + R_h} - \frac{V_{\text{in}}}{S}.
\]

Figs. 2(b-d) examine the feasibleness of the "monitoring"-mode. The device-under-test is two back-to-back connected tunable capacitance diodes (Infineon BB837) whose capacitance $C_T$ can be controlled by the diode reverse voltage $V_R$. $V_R$ has a 9.99 V DC component which biases $C_{\text{DUT}} = 0.5C_T$ to about 500 fF, as well as a 20 mVpp, 1000-second-period square-wave AC component which induces a small capacitance variation, see Fig. 2(b). The frequency and amplitude of $V_{\text{in}}$ are 110 MHz and $\sim 1$ mV$_{\text{rms}}$, respectively. We show the measured capacitance using the "monitoring"-mode in Fig. 2(c). The measured $C_{\text{DUT}}$ is about 487.5 fF when $V_R=10$ V, and increases by 1.5 fF when $V_R$ decreases by 20 mV; consistent with the BB837 datasheet. The standard deviation of $C_{\text{DUT}}$ is about 0.12 fF at 0.3 Hz measurement bandwidth. In short, this method can resolve $\lesssim 240$ ppm variation of a $\lesssim 0.5$ pF capacitor within a second. Fig. 2(d) shows $C_{\text{DUT}}$ measured for 72 hours, which drifts by $< 1$ fF in 72 h. The variation of $C_{\text{DUT}}$ is in good agreement with the room temperature fluctuation, likely introduced by the device itself.

IV. MEASUREMENTS AT MK-TEMPERATURE

We install the capacitance setup into an Oxford Triton 400 dilution refrigerator, and study the gate-to-2D capacitance of a high-mobility two-dimensional electron gas (2DEG) sample at mK temperature in Fig. 3(b). The sample consists an AlGaAs/GaAs/AlGaAs quantum well structure grown by molecular beam epitaxy, as illustrated in Fig. 3(a). The mobility is about 2 m$^2$/V·s. The 650-Å-wide GaAs quantum well resides 1665 Å below the surface, bound by AlGaAs spacer-layers on both sides. We grow three Si-$\delta$-doping-layers in the spacer-layers, two in the surface-side one, and one in the substrate-side one. At zero front gate bias, a bilayer-like 2DEG forms inside the quantum well, and another 2DEG forms at the interface between the substrate-side AlGaAs spacer-layer and the GaAs buffer-layer, see Fig. 3(a). We evaporate two concentric gates and measure the gate-to-gate capacitance, i.e. the two gate-to-2D capacitors which are serial-connected by the 2DEG. The inner gate radius is $60 \, \mu$m, and the gap between the two gates is $20 \, \mu$m. The outer-gate is much bigger than the inner-gate, so that $C_{\text{DUT}}$ is approximately the inner-gate-to-2D capacitance. An isolation capacitor is used between the inner-gate and the bridge so that a DC gate bias $V_{\text{FG}}$ can be applied. The frequency and amplitude of $V_{\text{in}}$ is 17 MHz and $\sim 0.4$ mV$_{\text{rms}}$, respectively. We don’t see any increase of the mixing chamber temperature ($\lesssim 17$ mK) when turn on the measurement, evidencing that the heat load at the sample stage is negligible.

Fig. 3(b) shows the $C_{\text{DUT}}$ as a function of $V_{\text{FG}}$ measured at 250 mK. The electron charging falls into four scenarios, corresponding to the four plateaus seen in the Fig. 3(b) data. Depletion: The 2DEG under the inner-gate is completely de-
electron distribution of 2DEG at zero gate voltage. I, II, and III indicate the effective charge position of the topmost 2DEG in the three cases of (b). (b) The measured gate-to-2D capacitance $C_{2D}$. The four $C_{2D}$ plateaus indicate four different working scenarios of the device: Depletion, Cases I, II, and III. The black and red double-arrow lines mark the capacitance value and the voltage range for each plateau. (c) $C_{2D}$ vs. $V_{FG}$ at 1.1 T perpendicular magnetic field. The red lines mark the periods of oscillation of the 250-mK trace.

When subjected into a large perpendicular magnetic field $B$, the electrons of a 2DEG are quenched into discrete Landau levels, leading to density-dependent compressibility. When we tune the 2DEG density by sweeping $V_{FG}$ (see Fig. 3(c)), we observe a $C_{\text{DUT}}$ oscillation where each period corresponds to the occupation of two Landau levels (two spins) in the topmost 2DEG. We extracted the oscillation period $\Delta V$ for the three cases and list them in Table I. The reciprocal ratio of $\Delta V$ is similar to the ratio of $C_{\text{DUT}}$, consistent with our expectation that $C_{\text{DUT}}$ is inversely proportional to $V^2_{\text{FG}}$ as $C_{\text{DUT}} \propto \frac{2\hbar e^2}{\alpha V^2_{\text{FG}}}$. Here, $\hbar$ is the Planck’s constant, $e$ is the electron charge and $B$ is the magnetic field. Note that the capacitance oscillation is strong in Case I and II, but heavily damped in Case III. At lower temperature 20 mK, the oscillation becomes more pronounced in case II, but remain roughly unchanged in case III. This is possibly because the two subbands have spatially separated charge distribution which smooths the compressibility oscillation.

We can also calculate the density of each 2DEG using the relation $n_i = \frac{\epsilon_0 \epsilon_r}{\epsilon} \cdot \frac{\Delta V}{d_i}$, where $\epsilon = 13$ is the relative dielectric constant, $\epsilon_0$ is the permittivity of vacuum and $\Delta V$ is the width of the plateau in the Case-$i$ region, see Fig. 3(b). We find that $n_2/n_1 = 0.39$ agrees with the reciprocal ratio of the distance from the doping layer to the corresponding 2DEG (355 Å : 872Å = 0.41, see Fig. 3(a)).
V. CONCLUSION

We have introduced a high-precision, low-excitation capacitance measurement method for 10 mK- to room-temperature experiments. We are able to measure the absolute capacitance value using the “V-curve” procedure, and monitor the variation of capacitor using $V_{\text{out}}$. With about 1 mVrms excitation voltage, we can resolve 240 ppm variation of a 500 fF capacitor. We measure the gate-to-2D capacitance of a high-mobility two-dimensional electron system at mK-temperature and extract consistent, essential information of the device. The results demonstrate that our capacitance bridge can detect extremely small capacitance fluctuation with mV-excitation at 10 mK-temperature.

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19. In Fig. 2(a), $C_{\text{OUT}}$ slightly deviates from $C_{\text{nom}}$ when $C_{\text{OUT}} \neq C_{\text{ref}}$. This is not surprising because the bridge output is less sensitive to $C_{\text{OUT}}$ in this case, so that the leaking signal causes more deviation.

20. The equivalent voltage noise of the whole setup at the input of the RF preamplifier has a spectrum density of about 8 nV/√Hz.

21. The mixing chamber temperature increases from $\sim 5$ mK to $\sim 17$ mK after we install three semi-rigid coaxial cables. In a separate report, we use extra-thin flexible coaxial wires and the mixing chamber temperature can be kept below 10 mK during the measurement.

22. The isolation capacitor is 22 nF in this measurement, which needs to be much larger than the device capacitance.

23. In this manuscript, all quoted temperatures are mixture chamber plate temperatures of the dilution refrigerator.

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