The CaFe project: Optical Fe II and near-infrared Ca II triplet emission in active galaxies: simulated EWs and the co-dependence of cloud size and metal content

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ABSTRACT

Aims. Modelling the low-ionisation lines (LILs) in active galactic nuclei (AGNs) still faces problems in explaining the observed equivalent widths (EWs) when realistic covering factors are used and the distance of the broad-line region (BLR) from the centre is assumed to be consistent with the reverberation mapping measurements. We re-emphasise this problem and suggest that the BLR ‘sees’ a different continuum from that seen by a distant observer. This change in the continuum reflected in the change in the net bolometric luminosity from the AGN is then able to resolve the above problem.

Methods. We carefully examine the optical Fe II and near-infrared (NIR) Ca II triplet (CaT) emission strengths with respect to Hβ emission using the photoionisation code CLOUDY and a range of physical parameters. Prominent among these parameters are (a) the ionisation parameter (U), (b) the local BLR cloud density (n_HII), (c) the metal content in the BLR cloud, and (d) the cloud column density. Using an incident continuum for I Zw 1—a prototypical Type-1 narrow-line Seyfert galaxy—our basic setup is able to recover the line ratios for the optical Fe II (i.e. R_CaFe) and for the NIR CaT (i.e. R_CaT) in agreement with the observed estimates. Nevertheless, the pairs of (U,n_HII) that reproduce the conforming line ratios do not relate to agreeable line EWs. We therefore propose a way to mitigate this issue. The LIL region of the BLR cloud does not see the same continuum emitted by the accretion disc as that seen by a distant observer; rather it sees a filtered version of the original continuum which brings the radial sizes into agreement with the reverberation mapped estimates for the extension of the BLR. This is achieved by scaling the radial distance of the emitting regions from the central continuum source using the photoionisation method in correspondence with the reverberation mapping estimates for I Zw 1. Taking inspiration from past studies, we suggest that this collimation of the incident continuum can be explained by the anisotropic emission from the accretion disc, which modifies the spectral energy distribution such that the BLR receives a much cooler continuum with a reduced number of line-ionising photons, allowing reconciliation in the modelling with the line EWs.

Results. (1) The assumption of the filtered continuum as the source of BLR irradiation recovers realistic EWs for the LIL species, such as the Hβ, Fe II, and CaT. However, our study finds that to account for the adequate R_CaFe (Fe II/Hβ flux ratio) emission, the BLR needs to be selectively overabundant in iron. On the other hand, the R_CaT (CaT/Hβ flux ratio) emission spans a broader range from solar to super-solar metallicities. In all these models, the BLR cloud density is found to be consistent with our conclusions from prior studies, that is, n_HII ~ 10^13 cm^−3 is required for the sufficient emission of Fe II and CaT. (2) We extend our modelling to test and confirm the co-dependence between metallicity and cloud column density for these two ionic species (Fe II and CaT), further allowing us to constrain the physical parameter space for the emission of these LILs. Adopting the estimates from line ratios that diagnose the metallicity in these gas-rich media—which suggest super-solar values (Z ~ 5–10 Z⊙)—, we arrive at cloud columns that are of the order of 10^{24} cm^{-2}. (3) Finally, we test the effect of inclusion of a micro-turbulent velocity within the BLR cloud and find that the Fe II emission is positively affected. An interesting result obtained here is the reduction in the value of the metallicity by up to a factor of ten for the R_CaFe cases when the microturbulence is invoked, suggesting that microturbulence can act as an apparent metallicity controller for the Fe II. On the contrary, the R_CaT cases are relatively unaffected by the inclusion of microturbulence.

Key words. accretion, accretion disks – radiative transfer – methods: data analysis – galaxies: active – quasars: emission lines – galaxies: abundances

1. Introduction

The broad-line region (BLR) of active galactic nuclei (AGNs) consists of two basic components emitting the high-ionisation lines (HILs) and low-ionisation lines (LILs), and the two regions have different physical conditions and show different dynamical motion (Collin-Souffrin et al. 1988; Marziani et al. 2019a). The LILs come from a denser region closer to the mid-plane of the black hole accretion-disc system; one that is perpendicular to the spin axis of the black hole. LILs play a very important role in two aspects, the first being in black hole mass measurements using the Hβ and Mg II lines (Czerny et al. 2019; Homayouni et al. 2020; Zajaček et al. 2020, 2021; Martínez-Aldama et al. 2020). The typical values for time-lags reported for the LILs (e.g. Hβ (IP: 13.6 eV) and Mg II (IP: 15.04 eV^1)) are found to be longer than those shown for the HILs (e.g. C IV (IP: 64.49 eV) and He II (IP: 54.42 eV)) from reverberation mapping studies (Peterson & Wandel 1999; Horne et al. 2021). Secondly, LILs play an important role in defining the quasar main

¹ The values for the ionisation potential (IP) are taken from NIST Atomic Spectra Database Ionization Energies.
sequence, which additionally involves the optical Fe II emission (Boroson & Green 1992; Sulentic et al. 2000; Shen & Ho 2014; Marziani et al. 2018; Panda et al. 2018, 2019a). Here, the optical Fe II emission refers to the 4434–4684 Å blend bluewards of the Hβ emission line. This definition for the optical Fe II is employed throughout this paper.

Despite their importance and huge efforts to study them, the modelling of LILs is inherently difficult (e.g. Collin-Souffrin et al. 1986; Joly 1987; Korista et al. 1997). In the case of the Fe II lines, this difficulty is connected with the large number of transitions which should be incorporated into the radiative transfer computations (e.g. in CLOUDY, Ferland et al. 2017). In spectral analyses, observational (Boroson & Green 1992; Véron-Cetty et al. 2004; Kovačević et al. 2010; García-Rissmann et al. 2012) templates are frequently used. The difficulty in understanding the Fe II emission has led us in search of other reliable, simpler ionic species such as Ca II and O I (Martínez-Aldama et al. 2015b, and references therein) which would originate from the same part of the BLR and have the potential to play a similar role in quasar main sequence studies. Here, the Ca II emission refers to the Ca II triplet (CaT), that is, the infrared (IR) triplet emitting at λ8498 Å, λ8542 Å, and λ8662 Å. We refer readers to Panda et al. (2020, hereafter P20) for an overview of the issue of CaT emission in AGNs and its relevance to the Fe II emission.

In P20, we compiled an up-to-date catalogue of quasars with spectral measurements of the strengths of the optical Fe II and NIR CaT emission; for example, for the Fe II, this is the measured intensity ratio of the optical Fe II blend between 4434 and 4684 Å to Hβ, denoted \( R_{\text{Fe II}} \). Similarly, for the CaT, this refers to the intensity ratio of the Ca T emission to Hβ, denoted \( R_{\text{CaT}} \). Our findings in P20 reinforced the existing tight correlation (Martínez-Aldama et al. 2015b) between the strengths of the two aforementioned ionic species with a higher significance.

We also performed a suite of CLOUDY\(^2\) photoionisation models to derive the \( R_{\text{Fe II}}-R_{\text{CaT}} \) correlation from a theoretical standpoint with emphasis on the important roles played by the ionisation parameter and the local cloud density. We touched upon the effect of metallicity and cloud column density and showed their marked contribution to this correlation, albeit qualitatively.

While P20 was devoted to justifying the connection between the optical Fe II and NIR CaT, there we used only the line ratios and did not address the basic problem with the LIL-emitting region, which is the inability of our model to reproduce the observed equivalent widths (EWs) of the lines. The main goal of the present paper is to match the modelled data with the observations in terms of EWs and flux ratios of the lines of these two ions, to constrain the relative location of Fe II and CaT, and to determine the metallicity required to optimise their emission strengths. Additionally, we investigate the effect of the cloud column densities (\( N_{\text{HI}} \)) on the net emission strengths of Fe II and CaT, which, for a given local mean density of the BLR cloud, can be used to estimate the size of the BLR cloud. Treatment of the metallicity and cloud column density is done in a heuristic manner and the obtained inferences are gauged against prior observed measurements for I Zw 1.

The paper is organised as follows: in Sect. 2, we describe our photoionisation modelling setup, which is line with our approach in P20. The novelty of this part of the work lies in (i) the appropriate treatment of the issue of the EWs in terms of the covering factor for the line species; and (ii) the systematic treatment of the metallicity and cloud column density, unlike P20, where we assumed only two representative cases for each entity, namely \( Z = 0.2 Z_\odot \) and \( 5 Z_\odot \), at \( N_{\text{HI}} = 10^{24} \) cm\(^{-2} \), and \( N_{\text{HI}} = 10^{24} 3 \) cm\(^{-2} \) and \( 10^{25} \) cm\(^{-2} \) at \( Z = Z_\odot \). In Sect. 3, we analyse the results from the photoionisation models and check for inconsistencies with regards to the line EWs of Hβ, optical Fe II, and CaT such as those noted in P20, and propose a way to bring the results from photoionisation modelling into agreement with the observational estimates. We discuss certain aspects of the results and their implications in Sect. 4. The key findings from this study are then summarised in Sect. 5.

2 \( \text{https://gitlab.mublab.org/cloudy/cloudy/-/wikis/home} \)

\[^3\] According to the standard photoionisation theory, the ratio of the hydrogen ionising photon density to the total hydrogen density is denoted by the dimensionless ionisation parameter \( U \), such that:

\[
U = \frac{Q_{\text{H}}}{4 \pi r^2 n_{\text{H}} c},
\]

where \( Q_{\text{H}} \) is the number of hydrogen ionising photons emitted by the central object, \( r \) is the separation between the centre of the source of ionising radiation and the illuminated face of the line-emitting medium, \( n_{\text{H}} \) is the total hydrogen (or mean cloud) density, and \( c \) is the speed of light. All parameters are given in cgs units. In accordance with P20, we perform a suite of CLOUDY (version 17.02, Ferland et al. 2017) models\(^3\) by varying the mean cloud density, \( 10^{10.5} \leq n_{\text{H}} \leq 10^{13} \) (cm\(^{-3} \)), the ionisation parameter, \( -4.25 \leq \log U \leq -1.5 \), and the metallicity, \( 0.1 Z_\odot \leq Z \leq 10 Z_{\odot} \), at a base cloud column density of \( N_{\text{HI}} = 10^{23} \) cm\(^{-2} \). The cloud column density command allows the user to set the size (d) of the line-emitting medium given by the relation: \( d = \frac{N_{\text{HI}}}{n_{\text{HI}}} \), where \( N_{\text{HI}} \) and \( n_{\text{HI}} \) have their usual meaning. Other cases of cloud column densities are explored in later sections. As in P20, we use a constant shape for the ionising continuum, one that is appropriate for the nearby NLS1, I Zw 1\(^2\).

The bolometric luminosity of I Zw 1 is \( L_{\text{bol}} \sim 4.32 \times 10^{45} \) erg s\(^{-1} \). This is obtained by applying the bolometric correction prescription from Netzer (2019):

\[
k_{\text{bol}} = c \times \left( \frac{L_{5100}}{10^{45}} \right)^d,
\]

where, \( c = 40 \) and \( d = -0.2 \), and \( L_{5100} \) is measured in erg s\(^{-1} \). For I Zw 1, \( L_{5100} \sim 3.48 \times 10^{44} \) erg s\(^{-1} \) (Persson 1988). Huang et al. (2019) performed the first reverberation mapping campaign for this source and obtained a value for the \( L_{5100} = 3.19^{+0.27}_{-0.25} \times 10^{44} \) erg s\(^{-1} \) which agrees well with the previous estimate from Persson (1988) within 1\(\sigma\) uncertainty. As we also use the \( R_{\text{Fe II}} \) and \( R_{\text{CaT}} \) estimates from their study (Persson 1988), we incorporate their \( L_{5100} \) value for our study.

Compared to the range of \( n_{\text{HI}} \) and \( U \) explored in P20, both entities are extended by 1 dex to explore possible solutions in a low-density and low-ionisation regime. As opposed to P20 (where we additionally included dust in the BLR), in this paper we do not impose any limitation on the log \( U - \log n_{\text{HI}} \) space. The model assumes a distribution of cloud densities at various radii from the central illuminating source to mimic the gas distribution around the close vicinity of the AGN. The range of metallicities incorporated here is inspired by the works on the quasar main sequence,
where a distribution of quasars is used ranging from the low-\(R_{\text{FeII}}\) ‘normal’ Seyfert galaxies, which can be modelled with a sub-solar assumption, to the narrow-line Seyfert galaxies (NLS1s), especially the extreme FeII emitters, which are proposed to have super-solar metallicities (Laor et al. 1997a; Negrete et al. 2012; Marziani et al. 2019a; Śniegowska et al. 2021). Also, the range of cloud column density used is in agreement with previous works, mainly in Ferland & Persson (1989), Matsuoka et al. (2007, 2008), and Negrete et al. (2012) and further extension shown in P20. The line fluxes and \(R_{\text{FeII}}\) and \(R_{\text{CaT}}\) estimates are extracted from these simulations.

In the following sections, we analyse the results from the photoionisation models and check for inconsistencies with regards to the line EWs of \(H\beta\), optical FeII, and CaT. We then apply a simple radiation filtering to the incident continuum to mimic the incoming radiation seen by the BLR cloud which scales down the radial distance of the BLR cloud in agreement with the reverberation mapping results. This filtering also brings the EWs and their corresponding covering factors into consistency with the observed data. Next, by imposing an additional constraint on the obtained estimates for \(R_{\text{FeII}}\) and \(R_{\text{CaT}}\) from the photoionisation models to the observations, we are left with a small set of solutions that agree on all three counts mentioned above.

3. Results

3.1. First analysis

The results from our base setup are shown in Fig. 1. The panels in this figure show the log \(U\)–log \(n_H\) parameter space colour-coded as a function of the flux ratios (\(R_{\text{FeII}}\) or \(R_{\text{CaT}}\)). The five panels show the setup as a function of increasing metallicity content considered in the BLR cloud model, that is, \(Z = 0.1Z_\odot, 0.3Z_\odot, Z_\odot, 3Z_\odot\), and 10\(Z_\odot\). These figures are constructed from models that take into account a cloud column density of \(N_H = 10^{24}\) cm\(^{-2}\). We illustrate the effect of other cloud column densities in Sect. 3.5. Note that \(U\) represents the ionisation parameter of the medium, and we later discuss whether or not its value should be estimated directly from the observed continuum.

In Fig. 1, for the lowest metallicity case (log \(Z_0 = -1\), top-left panel), the maximum \(R_{\text{FeII}}\) recovered is \(-0.575\) (for log \(U = -1.75\), log \(n_H = 12.25\)). For log \(Z_0 = -0.5\), this maximum rises to \(-0.906\) (for log \(U = -1.75\), log \(n_H = 12\)). This value of maximum \(R_{\text{FeII}}\) further increases when the metallicity is raised to solar and super-solar values. At solar metallicity, the maximum \(R_{\text{CaT}}\) recovered is \(-0.557\) (for log \(U = -4.25\), log \(n_H = 12.5\)), which is quite close to the estimates for I Zw 1 reported by Persson (1988) and Marinello et al. (2016) of 0.513 \pm 0.130 and 0.564 \pm 0.080, respectively. Requesting higher-than-solar metallicities in the case of \(R_{\text{CaT}}\) recovers values that are yet to be confirmed observationally. Hence, from this base model analysis, we find that we can indeed recover the \(R_{\text{CaT}}\) estimates that are consistent with the observed estimates if we take into account approximately solar metallicity values.

3.2. The problem of the EWs and covering factors in the LIL region

These results are in line with our conclusions obtained in P20, where we showed that our photoionisation models can predict \(R_{\text{FeII}}\) and \(R_{\text{CaT}}\) based on their flux ratios, and the modelled estimates were found to be in line with the measured values from an up-to-date observational sample of 58 sources (see Table 1 in P20). Furthermore, the measured correlation (almost one-to-one) between the two ratios was matched by both the modelled and observed data. We re-affirm this in the previous section with the agreement extended to the radial distance of the BLR in terms of the emitting regions of these two ions.

However, the proper model should reproduce not only the line ratios but also the line intensities, which are reflected in the line EWs. Therefore, we now also measure the line EWs from the grid of models. To do so, we use continua that are possibly close to the lines under consideration. For estimating the EWs for \(H\beta\) and FeII, we use one of the CLOUDY default continuum values, at \(\lambda = 4885.36\) Å. We checked for differences with the habitually considered continuum level, that is at 5100 Å, and found good agreement (they differ by \(-0.2\%)\). On the other hand, for the CaT emission, the triplet is located in the NIR part of the spectrum, and therefore a different continuum level is required to properly estimate the EWs. This was required in previous observational works (see e.g. Martínez-Aldama et al. 2015b; Marinello et al. 2016) because of the additional contamination of the disc continuum by the reprocessed torus contribution. To mitigate this issue, we use another default CLOUDY continuum at \(\lambda = 8329.68\) Å, which is closer to the triplet and overlaps with the continuum windows in the NIR used in prior studies.

To estimate the covering factors for FeII and \(H\beta\) predicted by CLOUDY, we first derive an average EW estimate from observations of a large sample of quasars that have similar physical properties to I Zw 1. We consider a subset of the DR14 Quasar Catalogue (Rakshit et al. 2020) wherein the selected subset contains quasars that have \(FWHM(H\beta) \lesssim 4000\) km s\(^{-1}\), which are also referred to as Population A sources. Population A sources can be understood as the class that includes local NLS1s as well as more massive high accretors which are mostly classified as radio-quiet (Marziani & Sulentic 2014) and have \(FWHM(H\beta) \lesssim 4000\) km s\(^{-1}\). Previous studies found that the Population A sources typically have Lorentzian-like \(H\beta\) profile shapes (Sulentic et al. 2002; Zamfir et al. 2010), in contrast to Population B sources, which have broader \(FWHM(H\beta) \gtrsim 4000\) km s\(^{-1}\), are pre-dominantly ‘jetted’ sources.
Padovani et al. (2017), and have been shown to have Hβ profiles that are a better fit with Gaussian (for sources with still higher full width at half maxima (FWHMs), we observe disc-like double Gaussian profiles in Balmer lines). This subset from Rakshit et al. (2020) contains 48017 sources (about 9% of the total number of sources in the DR14 catalogue). In addition to
the FWHM limit, which limits the sources within the Population A type (Marziani et al. 2018, and references therein), we employ a quality cut on the estimated EWs from the catalogues by limiting the errors associated with the EW(Hβ) measurements within 20%. This reduces the sample to 28,252 sources. The estimated mean and standard deviation values (in Å) for this subset are 48.84 and 52.42 for Fe II and 68.13 and 46.90 for Hβ, respectively. We also estimated the mean and standard deviation for the EWs for the Hα measurements in this subset which gave us 299.05 and 141.67, respectively. Going by the arguments for the typical values for the Balmer decrement in AGNs (here, Hα/Hβ) ≃ 3 (see Fig. 3 in Dong et al. 2008) we recover an average EW for Hβ of ≃100 Å given the EW(Hα). Therefore, for simplicity, we assume a generic value of 40 Å for EW(Fe II) and 100 Å for EW(Hβ) in our study. These generic values are confirmed by the sample of 58 sources in P20 containing the observations from Persson (1988), Martínez-Aldama et al. (2015a, b), and Marinello et al. (2016, 2020).

To compare the observed EWs to the model predictions we require a certain covering fraction (or factor) that scales the modelled EWs. The covering factors associated with the line species are grossly over-predicted: for Hβ, the derived EWs from the models are quite low, requiring covering factors ≥100% for 547 out of the 660 models (these 660 models include all five metallicities cases) to be comparable to the observed values. This implies that most of the model predictions of line intensities are lower by up to two orders of magnitude recovered from these models. In the following section, we propose a way to mitigate this issue and to recover EWs from the models with reasonable covering factors.

### 3.3. A simple proposition

We consider three cases of covering factor for the LIL region that have a typical EW(Fe II) = 40 Å recovered for Population A-type sources: at 30%, 45%, and at a more liberal 60%. These values for the covering factors of the LIL region are representative and agree with the values from the traditional single-cloud and the locally optimised cloud (LOC) models (Baldwin et al. 1995; Korista & Goad 2001). The need for high covering factors is substantiated to explain the strengths of the emission lines in the BLR and the lack of the Lyman continuum absorption suggesting a flattened distribution of the BLR and the distant observer seeing the source at relatively small viewing angles (Gaskell 2009, see also Fig. 7 in the current paper). The results of previous studies have suggested that the covering factors of the BLR and the torus are similarly based on the following reasoning: (A) If the torus had a lower covering factor than the BLR we would see the BLR in absorption against the central continuum source in some objects near the type-2 viewing position. This is never seen. (B) On the other hand, if the BLR had a lower covering factor, some regions of the dusty torus would see direct radiation from the central source. This cannot be the case for much of the torus because it would then be unable to exist as close in as is seen (Gaskell et al. 2007; Gaskell 2009, and references therein). We therefore assume here that the covering factors for the two entities (the LIL region and the torus) are similar and substantiate the assumed covering factors from prior statistical studies on large quasar samples to recover the covering factors for the torus. Previous observational studies estimated the covering factors, for example by using the ratio of IR to optical-UV luminosity (Roseboom et al. 2013) for luminous type 1 (or unobscured) quasars from large surveys in those wavebands. These latter authors estimate a mean value for the covering factor of 0.39+0.23−0.15. On the other hand, Gupta et al. (2016) consider the ratio of the mid-IR luminosity to the bolometric luminosity to estimate the covering factors for a sample of radio-loud and radio-quiet sources. For their radio-quiet sample, these authors estimate a median covering factor of ~0.29. In addition to these estimates, Mor et al. (2009) estimated the covering factor for 1 Zw 1 to be ~63%. This was achieved by fitting the Spitzer/IRS (2–35 µm) spectrum for the source using a clumpy torus model in addition to models for dusty narrow line region clouds and dust, where the latter was modelled using a black-body distribution.

The locations of the solutions that agree with the EWs in addition to the observed flux ratios are shown in Fig. 2 using special symbols. The underlying grid is identical to the respective panels shown already in Fig. 1. We find that the solutions for $R_{\text{FeII}}$ are only plausible now at higher metallicities, of the order of $\sim$10$Z_{\odot}$. This is in line with the observational evidence suggesting super-solar metallicities in excess of $10Z_{\odot}$ (Hamann & Ferland 1992; Shin et al. 2013; Śniegowska et al. 2021). Models with lower metallicity values ($Z \lesssim 3Z_{\odot}$) require covering factors that are above the requested limit ($\geq 60\%$) and henceforth are not considered. On the other hand, for CaT emission, the flux ratios can still be produced from models that are at solar metallicities, although the covering factor required in such cases is higher ($\geq 45\%$). Increasing the metallicity to higher than solar, we have more optimal solutions in terms of low covering factor (see lower panels in Fig. 2). For completeness, we also check for plausible solutions at higher than $10Z_{\odot}$ by considering two additional cases: at $20Z_{\odot}$ and $100Z_{\odot}$. We notice that in $20Z_{\odot}$ models (see Fig. 3), the solutions for $R_{\text{FeII}}$ are pushed to lower ionisation parameters albeit at similar densities. There are limited solutions for the $R_{\text{CaT}}$ case; these suggest radial sizes lower than $R_{\text{FeII}}$ by a factor of two, and smaller than the Hβ reverberation mapping estimate. There are no solutions that are in agreement for any of the three chosen covering factors for the $10Z_{\odot}$ metallicity case. Hence, an increase in the metallicity up to $\sim 20 Z_{\odot}$ works well for $R_{\text{FeII}}$ estimates in the case of I Zw 1-like sources but not for corresponding $R_{\text{CaT}}$ emission. For the $R_{\text{CaT}}$ emission, metallicity values $Z \lesssim 10 Z_{\odot}$ are found to be suitable to explain the EWs and the flux ratios. In summary, the solutions that reproduce agreement on both optimal EWs and the flux ratios are obtained without significant change in the density, log $n_1 \approx 11.75$. However, the new solutions are now nearly 2 dex lower in the ionisation parameter for $R_{\text{FeII}}$, that is, log $U \approx -3.5$ (the maximum value for $R_{\text{FeII}}$ in the left panels of Fig. 1 correspond to log $U \approx 1.75$).

Next, to assess the radial size of these emitting regions, we investigate the coupled distribution between the ionisation parameter and local cloud density. As has been previously explored in Negrete et al. (2012, 2014) and Marziani et al. (2019b) and in P20, we take the product of the ionisation parameter and the local cloud density ($U \cdot n_1$); this entity bears resemblance to ionising flux, and for a given number of ionising photons emitted by the radiating source can be used to estimate the size of the BLR ($R_{\text{BLR}}$):

$$R_{\text{BLR}} \,[\text{cm}] = \sqrt{\frac{Q(H)}{4\pi n_1 u_1 c}} \approx \sqrt{\frac{L_{\text{bol}}}{4\pi n_1 u_1 c}} \approx \frac{2.294 \times 10^{22}}{n_1 u_1}, \quad (3)$$

5 For lower covering factors (e.g., ~10%), we have one solution each for $R_{\text{FeII}}$ and $R_{\text{CaT}}$, i.e. at log $U = -3.5$, log $n_1 = 11.75$ at log $Z[Z_{\odot}] = 1$ (for $R_{\text{FeII}}$), and, log $U = -3.5$, log $n_1 = 11.5$ at log $Z[Z_{\odot}] = 1$ (for $R_{\text{CaT}}$). This pair of solutions is unanimously recovered for all cases of the covering factors considered in this work.
where $R_{\text{BLR}}$ is the distance of the emitting cloud from the ionising source which has a mean local density, $n_H$, and receives an ionising flux that is quantified by the ionisation parameter, $U$. Here, $Q(H)$ is the number of ionising photons, which can be equivalently expressed in terms of the bolometric luminosity of the source per unit energy of a single photon, that is, $h\nu$. Here, we consider the average photon energy, $h\nu = 1$ Rydberg (Wandel et al. 1999; Marziani et al. 2015). The specific value of the bolometric luminosity corresponds to 1 Zw1.

The H$\beta$-based $R_{\text{BLR}}$ for I Zw 1 was estimated to be $1.827 \times 10^{17}$ cm ($\approx 37.2 \pm 4.5$ light-days) obtained from the dedicated reverberation mapping campaign for this source (Huang et al. 2019). The authors estimate the source to be a super-Edington accretor with a dimensionless accretion rate ($M$, Wang et al. 2014) = 203.9$^{+61.0}_{-65.8}$. In order to validate the deviation of the source from the standard $R_{\text{BLR}}-L_{5100}$ relation (Bentz et al. 2013), they estimate the contribution of the host galaxy and subsequently recover an AGN luminosity of $L_{5100} = 3.19 \pm 0.27 \times 10^{44}$ erg s$^{-1}$. This value for the AGN luminosity of the source is well within the estimate from Persson (1988) of $L_{5100} = 3.48 \times 10^{44}$ erg s$^{-1}$ within 1$\sigma$ uncertainty. The position of this source on the $R_{\text{BLR}}-L_{5100}$ diagram (see Fig. 4 in Huang et al. 2019) is almost at the boundary of the quoted scatter in the $R_{\text{BLR}}-L_{5100}$ relation by Bentz et al. (2013), which is at 0.19$\pm$0.02 dex. This is also reflected in the conclusion of Huang et al. (2019), who state that the source follows the empirical $R_{\text{BLR}}-L_{5100}$ relation.

On the other hand, the super-Edington accretors have been found to show shorter lags compared to their low-accreting counterparts, which brings into question the validity of the standard $R_{\text{BLR}}-L_{5100}$ relation for these sources (Du et al. 2016; Yu et al. 2020). Corrections have been proposed to the standard $R_{\text{BLR}}-L_{5100}$ relation, for example, linking to a dependence on accretion rate (Du et al. 2016; Martínez-Aldama et al. 2019) and suggestion of a ‘new’ $R_{\text{BLR}}-L_{5100}$ relation are being put forward, which uses observables such as $R_{\text{FeII}}$ (Du & Wang 2019) and $R_{\text{CaT}}$ (Martínez-Aldama et al. 2021). We test the hypothesis that this new $R_{\text{BLR}}-L_{5100}$ relation including $R_{\text{FeII}}$ is suitable for I Zw 1. We use the two epochs of spectral information containing the $R_{\text{FeII}}$ estimate from Persson (1988) and Marinello et al. (2016), i.e. $1.778 \pm 0.050$ and $2.286 \pm 0.199$, respectively. For the earlier epoch, we set the $L_{5100}$ luminosity to $3.48 \times 10^{44}$ erg s$^{-1}$ and recover the H$\beta$-based $R_{\text{BLR}} \approx 18.68$ light-days for $R_{\text{FeII}} = 1.778$. Next, for the more recent epoch, we set the $L_{5100}$ luminosity to $3.19 \times 10^{44}$ erg s$^{-1}$ as per Huang et al. (2019) and get the $R_{\text{BLR}} \approx 11.93$ light-days for the $R_{\text{FeII}} \approx 2.286$. Both these $R_{\text{BLR}}$ estimates indicate that the lags thus obtained from this new $R_{\text{FeII}}-L_{5100}$ relation are shorter by up to a factor of three than the lag value reported for I Zw 1 by Huang et al. (2019). With this in mind, we consider that the standard $R_{\text{BLR}}-L_{5100}$ applies for this source and proceed accordingly.

The similarity in the location of the emitting region for Fe II and H$\beta$ has been studied previously. The proximity of the ionisation potential for the two ions suggests that they are produced in relatively close-by regions under similar physical conditions (Panda et al. 2018, and references therein). Hu et al. (2015) found that the time delays of H$\beta$ and optical Fe II are mostly similar, although there is scatter in their FWHM correlation, which may suggest that Fe II is emitted from a larger region relative to H$\beta$. Next, we therefore consider the emitting region for Fe II and H$\beta$ to have significant overlap and for simplicity use the H$\beta$ radius obtained from Huang et al. (2019) as a proxy for the Fe II.

Now, with the agreement on the flux ratios for Fe II and CaT, and their EWs in harmony with the observational evidence, we are left with the problem of matching the radial distances from the continuum source. We have a discrepancy between the radius of the emitting region suggested by the reverberation mapping and the one obtained using the photoionisation method. In our case, the value of the radial distance obtained using Eq. (3) for the physical parameters $log \ U = -3.5$ and $log n_H = 11.75$ is $1.720 \times 10^{18}$ cm. We call this radius $R_{\text{BLR}}^{\text{FeII}}$. This value reproduces the EWs for the LILs and flux ratios of the lines in agreement with the observed values. On the other hand, from the H$\beta$ reverberation mapping of I Zw 1, we have the radial distance at $1.827 \times 10^{17}$ cm. We call this radius $R_{\text{BLR}}^{\text{FeII}}$. To match these radii and recover the expected ionisation parameter, the luminosity incident on the BLR cloud responsible for the line reverberation needs to be scaled. We perform this scaling by employing the scaling relation between the radius–luminosity that agrees with...
Fig. 3. Same as Fig. 2 but for $Z \approx 20Z_{\odot}$ for $R_{\text{FeII}}$ (upper panel) and $R_{\text{CaT}}$ (lower panel).

the low-ionisation emitting region, i.e. $R_{\text{BLR}} \propto L^{0.5}$. Hence, we have the following relation:

$$L' = L \times \left( \frac{R_{\text{BLR}}}{R_{\text{BLR}}^{0.5}} \right)^2,$$

where $L$ and $L'$ are the monochromatic luminosities for the photoionisation and the reverberation method, respectively. Thus, in our case, this scaling value is ~0.011 (i.e. only about 1% of the original AGN continuum irradiating the BLR). This simply translates back to the lowering of the ionisation parameter by nearly 2 dex in our photoionisation modelling that reproduces the flux ratios, the EWs within reasonable covering factors, and scales down the Fe\textsc{II} radial distance from the central source in agreement with the reverberation measurements. The implications of this filtering are discussed in Sect. 4.1.

3.4. Salient features of the Fe\textsc{II} and Ca\textsc{II} emission from photoionisation

As a result of the consideration of the proper values of the line, the EW has led us to an entirely different parameter space in the ionisation parameters than in P20 and in Sect. 3.1. We now change our approach to search for the proper parameter space. We perform a three-step refinement to extract the final solutions for the log $U$–log $n_{\text{H}}$ pertaining to the two parameters $R_{\text{FeII}}$ and $R_{\text{CaT}}$. Step 1 is matching the EWs for the H2, Fe\textsc{II}, and Ca\textsc{II} simultaneously within the requested covering factors (30%, 45%, and 60%). Then, the selected solutions are gauged against the radial distance that is within 20% of the value obtained from the $R_{\text{FeII}}$–$L_{5100}$ for the luminosity of $\zeta$ 20 1 ($\sim 3.48 \times 10^{44}$ erg s$^{-1}$). The reported time delay for $\zeta$ 20 1 in Huang et al. (2019) has an associated mean uncertainty of ~13%.

The last step in the refinement is matching the modelled estimates with the observed line flux ratios for both the ions $R_{\text{FeII}}$ and $R_{\text{CaT}}$. This is what gives us the solution marked at the top of the simulation grid in Figs. 2 and 3.

The solutions that fully satisfy the observational constraints are a small subsection of the original grid, as illustrated in Figs. 2 and 3. To better demonstrate why P20 solutions with much higher ionisation parameters are favoured, we replot all solutions in Fig. 4. The grid points from three panels for metallicity at $Z_{\odot}$, $-3Z_{\odot}$, and $10Z_{\odot}$ for both $R_{\text{FeII}}$ and $R_{\text{CaT}}$ are extracted from the log $U$–log $n_{\text{H}}$ space and reported here in terms of the radial distance (as referred to in previous sections, the product of $U$ and $n_{\text{H}}$ for a fixed ionising continuum gives the size of the line emitting region) versus the two flux ratios. The grid points are colour-coded with the corresponding ionisation parameters. First considering the $R_{\text{FeII}}$ cases (left panels in Fig. 4), we can see that the maximum emission in $R_{\text{FeII}}$ is nearly 2 dex larger, suggesting that the radial distance here is approximately ten times greater, which is explored in the previous sections. The vertical and horizontal patches in the plots indicate the $R_{\text{FeII}}$ estimates within 2σ of the observed estimates and the radial sizes converted into $U_{\text{HII}}$ scales. Here, σ is taken as the maximum value of the error quoted from the two reported estimates from Persson (1988) and Marinello et al. (2016). Such a liberal range is considered whilst keeping in mind that the observed and modelled estimates show subtle differences; for example, in the photoionisation modelling with CLOUDY, the code considers 371 levels accounting for ~68 635 transitions for the Fe\textsc{II} atom, which are evaluated only up to ~11.6 eV (Verner et al. 1999). In the analysis of the optical spectrum for $\zeta$ 20 1, there is a need to supplement the fitting procedure with broad Gaussians in addition to the Fe\textsc{II} pseudo-continuum generated from CLOUDY to minimise the residuals (Negrete et al. 2012, Panda & Martínez–Aldama, in prep.). This difference is highlighted by the subtle discrepancies in certain Fe\textsc{II} line transitions belonging to the $^4F$ group (mostly the 37 and 38 multiplets in the 4550 Å and 4580 Å wavebands). Kovačević et al. (2010) mitigate this problem by supplying line intensities found in the observational spectrum of $\zeta$ 20 1 in addition to the Fe\textsc{II} line transitions expected from standard photoionisation involving line recombination and collisional excitation processes. We apply the same approach (values with 2σ uncertainties) while evaluating the $R_{\text{CaT}}$ panels. The overlapping region between the vertical and horizontal patches marks the acceptable region for the solutions to the $R_{\text{FeII}}$. As can be seen from the three left panels, the solutions are in best agreement when the BLR cloud has metallicity $Z = 10Z_{\odot}$. The gradual increase in overall modelled distribution with an increase in metallicity suggests that the BLR clouds indeed require an overabundance of iron. On the other hand, for the $R_{\text{CaT}}$ case, solutions with relatively low ionisation parameters can successfully achieve the required $R_{\text{CaT}}$ estimates. Unlike the $R_{\text{FeII}}$ cases, here $R_{\text{CaT}}$ can be modelled with a wider range of metallicities, $Z_{\odot} \leq Z \leq 10Z_{\odot}$. Although in the higher-than-solar metallicity cases, the solutions that belong to the inter-junction of the appropriate radial distances and observed $R_{\text{CaT}}$ values in the plots show an increasing number of solutions that prefer higher ionisation parameters (log $U$ $\leq -4.0$). These trends reveal that the emitting regions of the two species (Fe\textsc{II} and Ca\textsc{II}) show significant overlap, although the results from this analysis suggest that the BLR cloud needs to be selectively overabundant in iron in order to optimize the Fe\textsc{II} emission. In contrast, sufficient Ca\textsc{II} emission can be produced in a wider range of abundances ranging from solar to super-solar values. This points toward different formation channels for the two species, because iron is
Fig. 4. Non-monotonic behaviour of \( R_{\text{cell}} \) versus \( U_{\text{H}} \) colour-coded with respect to \( X \) (left panels). Corresponding cases for \( R_{\text{CaT}} \) are shown in the right panels. The panels represent the three sets of high-metallicity cases: \( Z = 0 \) (top), 0.5 (middle), and 1 (bottom). The pale-blue vertical strip identifies the \( U_{\text{H}} \) product yielding the reverberation-based \( R_{\text{cell}} \) (black dashed line) within the uncertainties. The pale-orange horizontal strip identifies the observed \( R_{\text{cell}} \) and \( R_{\text{CaT}} \) values (black dashed lines) from Persson (1988) and Marinello et al. (2016) within uncertainties. A column density of \( N_{\text{H}} = 10^{24} \) cm\(^{-2} \) is assumed.

3.5. Co-dependence of metallicity and cloud column density

In P20, we explored, in a rather limited manner, the increasing trend of \( R_{\text{cell}} \) and \( R_{\text{CaT}} \) estimates as a function of increasing column density. We considered two additional cases in terms of column density apart from the base value of \( N_{\text{H}} = 10^{24} \) cm\(^{-2} \), namely \( 10^{25} \) and \( 10^{25.5} \) cm\(^{-2} \), limiting our models within the realms of the optically thin regime, that is, the optical depth \( \tau = \sigma_T \cdot N_H \), \( \tau \sim 1−2 \) for optically thin medium, which implies \( N_H \sim 10^{24.5}−10^{24.5} \) cm\(^{-2} \). Here, \( \sigma_T \) is the Thompson’s scattering cross-section and \( N_H \) is the cloud column density. There is a clear hint that the real scenario perhaps points towards a collective increase in both metallicity and cloud column density. This supports the arguments in favour of using very high metallicities \( Z \gtrsim 5 Z_{\odot} \) to recover the \( R_{\text{cell}} \) estimates for the strong Fe II emitters (Nagao et al. 2006; Negrete et al. 2012; Śniegowska et al. 2021) which has strong implications for the BLR cloud properties, especially their density and radial distributions. In this
section, we test this connection between the two aforementioned parameters in terms of the $R_{\text{FeII}}$ and $R_{\text{CaT}}$ estimates they recover.

From the analyses in the previous sections, the parameter values for ionisation and local cloud density, that is, log $U/n_t$, that best reproduce the $R_{\text{FeII}}$ and $R_{\text{CaT}}$ in agreement with the observed flux ratios, and keeping the BLR cloud within the limits of the $R_{\text{BLR}}$ as estimated from the reverberation mapping and constrained for the EWs (even at covering factor ~10%), are ~−3.5 and ~11.75 (cm$^{-3}$), respectively. We therefore fix these two values in the subsequent modelling and study the effect of the metallicity within $Z_{0} \leq Z \leq 100Z_{\odot}$ with a step size of 0.25 dex (in log-space) and the cloud column density within $10^{20} \leq N_{H} \leq 10^{24}$ with a step size of 0.5 dex (in log-space).

Here, the modelled range for metallicity is extended to higher values than are assumed above to test their relevance in the BLR LILs emission (Leighly 2004; Negrete et al. 2012; Martínez-Aldama et al. 2015b). Also, at the expected radial extensions for the LILs, the clouds are relatively cold and can clump together. In addition, the lowering of the net radiation pressure keeps the cloud relatively extended (Marconi et al. 2008; Netzer 2009). On the other hand, having a larger cloud column allows species like FeII to increase their ionic fraction compared to Hβ and thereby produce enough emission to account for the $R_{\text{FeII}} \geq 1$, as often seen for the high FeII-emitters belonging to the extreme Population A (see Bruhweiler & Verner 2008; Panda et al. 2018, 2019a, and references therein).

In Fig. 5, we demonstrate the strong degeneracy between the two quantities (metallicity and cloud column density) as a function of the recovered values for $R_{\text{FeII}}$ and $R_{\text{CaT}}$. It is clearly seen that the same value of $R_{\text{FeII}}$ or $R_{\text{CaT}}$ can be derived by widely different combinations of column density and metallicity. The main plots are in log–log space so that the large extent of the intensity ratio against the fifth-order stretch of cloud column density can be appreciated. $R_{\text{FeII}}$ and $R_{\text{CaT}}$ estimates have been reported from prior spectroscopic observations for I Zw 1: (a) $R_{\text{FeII}}$ and $R_{\text{CaT}}$ estimates from Persson (1988) are 1.778 ± 0.050 and 0.513 ± 0.130, respectively; (b) $R_{\text{FeII}}$ and $R_{\text{CaT}}$ estimates from Marinello et al. (2016) are 2.320 ± 0.110 and 0.564 ± 0.083, respectively. We use these measurements and overlay them on Fig. 5 with the quoted uncertainties in the measured values. For the $R_{\text{FeII}}$ case, the models that have metallicities $Z \leq 3Z_{\odot}$ cannot account for the expected intensity ratio, not even for the lower limit from Persson (1988), even at the highest column density ($10^{25}$ cm$^{-2}$) considered in the analysis. We start to enter the optimal regime with $Z \sim 5Z_{\odot}$ and higher. The inset plot zooms in on the optimal range of solutions in terms of the $R_{\text{FeII}}$ recovered (note the linear scale used here for the y-axis), and the column density and metallicity values needed to obtain that value. In principle, BLR cloud with the smallest size can reproduce the optimal $R_{\text{FeII}}$ emission, although in this case, the models require extremely high metallicity ($100Z_{\odot}$). Such an inverse behaviour between metallicity and cloud column size is no surprise because these clouds are effectively made of mostly hydrogen and helium, which exist in the front-facing part (or the fully ionised zone) of the cloud, while heavier and more metallic elements tend to occur in deeper parts of the cloud as revealed by the increase in the ion fractions for the latter as a function of the depth within the BLR cloud (see Fig. 4 in Negrete et al. 2012). As we increase the column size, the $R_{\text{FeII}}$ estimate can still be obtained with lower metallicity values. For $R_{\text{CaT}}$, the trend between the $R_{\text{CaT}}$ and cloud column density is rather monotonic in log–log space. Similar to the $R_{\text{FeII}}$, smaller cloud sizes suggest higher metallicity, yet solutions with almost solar values for metallicity are sufficient to recover the required $R_{\text{CaT}}$ emission for cloud column sizes that are similar to the $R_{\text{FeII}}$ case, i.e. $N_{H} \geq 10^{24}$ cm$^{-2}$. Therefore, a degeneracy between these two quantities, that is, metallicity and cloud column density, sustains. We discuss this conundrum between the metallicity and cloud column density in the BLR clouds and highlight ways to break this degeneracy in Sect. 4.
Fig. 6. Effect of microturbulence: The first and third columns are from the original models without any microturbulence for $R_{\text{FeII}}$ and $R_{\text{CaT}}$, respectively. The second and fourth columns are the corresponding cases with microturbulence = 20 km s$^{-1}$. Each column consists of the five cases of metallicity considered in this work. The overlaid symbols have the same meaning as shown previously in Fig. 2.

3.6. Microturbulence: a metallicity controller?

Another important aspect to consider in the optimisation of the Fe II emission is the effect of microturbulence, which has been noted to provide additional excitation (Baldwin et al. 2004; Bruhweiler & Verner 2008). The velocity field around a black hole might be a superposition of different kinematic components, such as Doppler motions, turbulence, shock components, in/outflow components, and rotation. Different velocity components result in different profiles, and the final profile is a convolution of different components (Kollatschny & Zetzl 2013). Furthermore, local turbulence substantially affects the Fe II spectrum in photoionisation models by facilitating continuum and line-line fluorescence. Increasing the microturbulence can increase the Fe II strength and give better agreement between the predicted shape of the Fe II blends and observation (Shields et al. 2010). The effect of microturbulence has been carefully investigated in our previous works (Panda et al. 2018, 2019b), where a systematic rise in $R_{\text{FeII}}$ estimates is obtained by increasing the microturbulence up to 10–20 km s$^{-1}$. After this limit, the $R_{\text{FeII}}$ tends to drop and for 100 km s$^{-1}$ reaches values similar to zero microturbulent velocity. We tested the effect of the microturbulence in the context of this study, in particular whether or not this entity works similarly for boosting the CaT. We considered a microturbulence value of 20 km s$^{-1}$ and re-run our models. The results are summarised in Fig. 6 for the two ions side by side. As expected, for the $R_{\text{FeII}}$ case, the microturbulence leads to comparable $R_{\text{FeII}}$ estimates for lower metallicity. For example, the case with no microturbulence with solar metallicity gives $R_{\text{FeII}}$ values similar to the microturbulence = 20 km s$^{-1}$ at 0.3 $Z_{\odot}$. This effect is seen in other metallicity cases as well. For the preferred solution with $\sim 10 Z_{\odot}$ for the case with no microturbulence, upon invoking this parameter, we achieve the solution with the $\sim 3 Z_{\odot}$ models. On the other hand, for the $R_{\text{CaT}}$ cases, the results are almost similar between the two versions, indicating that the CaT emission is probably less prone to fluorescence effects. We overlay the solutions that agree with the line EWs for the three cases of covering factors similar to Figs. 2 and 3. For a much lower covering factor ($\sim$10%), we find that with the inclusion of microturbulence in the medium, $R_{\text{FeII}}$ estimates closer to those of Marinello et al. (2016) are more probable with an ionisation parameter of log $U$ $\sim$ $\sim$3.5 and density of log $n_{H}$ $\sim$ 11.5, albeit at 10 $Z_{\odot}$. For the same low covering factor, there is a unique solution satisfying for $R_{\text{CaT}}$, i.e. for log $U$ $\sim$ $\sim$3.25 and log $n_{H}$ $\sim$ 11.5 also at 10 $Z_{\odot}$, which shows that the two ions can have significant overlap in their emitting regions. This is another confirmation of the nearly 1:1
correlation obtained in P20 between the $R_{\text{CaT}}$ and $R_{\text{FeII}}$. There are solutions with higher covering factors that agree with the line EWs at metallicities $Z \lesssim 10 Z_\odot$ in the $R_{\text{FeII}}$ cases with ionisation parameter as high as $\log U \gtrsim -2.75$, yet the cloud densities remain nearly unchanged at $\log n_{\text{H}} \gtrsim 11.5$ (cm$^{-3}$). These latter solutions then require larger covering factors (>30%) to account for the optimal emission in Fe II. The last panel of $R_{\text{FeII}}$ cases with microturbulence included (10 $Z_\odot$) show significant overlap with the solutions realised from $Z \approx 20 Z_\odot$ models for $R_{\text{FeII}}$ (see Fig. 3). The effect of microturbulence is only a secondary effect seen from the spectra as this affects mostly the wings of the broad line profiles (Goad et al. 2012); the effect becomes quite difficult to estimate properly as these features become increasingly close to the noise level.

4. Discussions
The problem in reproducing the EWs of LILs has been discussed in the literature. Several explanations are possible: additional mechanical heating is perhaps necessary (e.g. Collin-Souffrin et al. 1986; Joly 1987), or a multiple cloud approach, with part of the radiation scattered and/or re-emitted between different clouds, or the BLR does not see the same continuum as the observer (Korista et al. 1997) due to an intervening medium such as a wind component (Leighly 2004). This wind component is often seen in high-ionisation lines in the UV, such as C IV, of an AGN spectrum typically belonging to the Population A type (Marziani et al. 2018, and references therein) like I Zw 1. In our computations, we recover the proper Fe II ratio and EW for low values of the ionisation parameter which indicates that we do not need extra energy to produce stronger lines but must instead reduce the incident flux; otherwise the medium is over-ionised and line production efficiency drops. This favours the option of the intervening medium. We applied the latter hypothesis to analyse I Zw 1 and its LILs of the BLR. This scenario is also supported by the recent observational findings of Wolf et al. (2020), who found that the Fe II-emitting region is shielded from the central source for a sample of $\sim$2100 Type-I AGNs (see Martínez-Aldama et al. 2015b, for a similar inference on CaT). We illustrate this scenario in Fig. 7 wherein the key assumption is that the broad-band spectral energy distribution (SED) seen by the BLR is different from the one that is perceived by a distant observer. This hypothesis can be perceived as the combined effect of (a) geometrical effects, and (b) radiation filtering due to obscuration. Nevertheless, other effects, such as lensing and limb darkening, which modify the inclination dependence, could additionally supplement in recovering this radiation filtering. The flattened disc geometry results in an inclination-dependent flux ($\cos \theta$, where $\theta$ is the angle between the symmetry axis and the line-of-sight of the observer). On the other hand, the filtering, or rather the collimation of the continuum can be a result of the anisotropy in the radial structure of the disc. The anisotropy originating from the accretion disc has been suggested by previous studies (Wang et al. 2014, and references therein) pointing away from a generally assumed geometrically thin accretion disc especially in the regions at close vicinity of the black hole. Such a geometry inhibits the radiation coming from the inner, hotter region, making it possible for the BLR to receive a continuum that is a fraction of the original ionising flux. This is a valid assumption in the case where the observer is systematically at an offset in the viewing angle to the BLR cloud.

4.1. Analysing the change in the shape of the SED
A primary finding of the present study is that BLR clouds that have sizes of the order of $\sim 10^{12}$ cm need to see about 1% of the SED and have optimised properties in order to reproduce all the observational constraints for the LILs in extreme objects like I Zw 1. Here, we discuss the implications of these findings and their justification based on prior studies.

Wang et al. (2014) studied in detail the solutions for the structure of accretion discs from sub-Eddington accretion rates to extremely high, super-Eddington rates (where the dimensionless accretion rate, $\dot{M}/\dot{M}_{\text{Edd}}$, changes). The appearance of sharper funnels in the innermost region (below $3 R_g$, where $R_g$ is the gravitational radius) as the accretion rate increases significantly modifies the thin-disc geometry that applies under the Shakura & Sunyaev (1973) regime. The authors note that the aspect ratio ($h = H/r$, where $H$ is the height of the disc and $r$ is the distance along the radial direction, both in units of $R_g$ such that $h$ is dimensionless) of the funnel for these slim discs is insensitive to the black hole mass and the shape of a slim disc has three notable features: (1) a funnel that develops in the innermost region $[dH/dr > 0]$; (2) a flattened part $[dH/dr < 0$ and $h \sim 1]$; and (3) a geometrically thin part ($h \sim 10^{-2}$), approaching the Shakura & Sunyaev (1973) regime, in which the funnel disappears. The authors further investigate the effect of the SEDs received by the BLR (or a distant observer) with different inclinations to the disc. In the present study, the BLR clouds are considered to be relatively close to the midplane of the system (see Fig. 7) such that the inclination angles (to the symmetry axis) subtended by these clouds are relatively high, around $75^\circ$--$85^\circ$. On the other hand, the face-on view of NLS1 sources such that the inclination angle subtended to the distant observer is relatively small (Wu & Han 2001; Panda et al. 2019a) is supported by the small widths of the Balmer lines due to the projection effect (Osterbrock & Pogge 1985; Bian & Zhao 2004).

7 Here the angle is estimated using the relation $90^\circ - \tan^{-1}(d/r)$, where $H$ is the peak height attained by the BLR cloud and $r$ is the corresponding radial distance of the BLR cloud from the black hole. This picture of the BLR considers clouds accelerated under the combined influence of radiation pressure and gravity. We refer the readers to Naddaf et al. (2021) for more details.
Recent studies by Rakshit et al. (2017) who used the SDSS DR12 data to construct an NLS1 catalogue also found that, statistically, the NLS1s have smaller viewing angles in comparison to their broad-line counterparts. In Wang et al. (2014), the authors notice the anisotropy of the radiation field clearly with their study and note that: (1) the flux received by the clouds (or the distant observer) dramatically decreases with increasing inclination by a factor of 30 (i.e. going from $\theta = 10^\circ$ to $\theta = 80^\circ$), which is much steeper than what is recovered with the simple cos$\theta$ dependence; and (2) the SEDs are significantly softened by self-shading at higher inclinations, resulting in the lack of photoionising photons required for the emission lines. We highlight the relative change in the bolometric flux content as a function of the source’s inclination with respect to the distant observer in Fig. 8 for two sets of slim disc SEDs (at $M = 100$ and 500, Jian-Min Wang, priv. comm.) at a black hole mass of $10^7 M_\odot$ similar to the estimate from Huang et al. (2019), i.e. $9.30^{+1.35}_{-2.76} \times 10^6 M_\odot$. The selection of the lower value of inclination was based on the Mor et al. (2009) estimate for the viewing angle from torus fitting to Spitzer/IRS 2–35 μm spectra for T2w1 suggesting a value of $\theta \approx 8^\circ$. We chose a SED model with $\theta = 10^\circ$ to mimic the SED seen by a distant observer. On the other hand, for the BLR, we chose a modelled SED with a higher inclination angle ($\theta = 80^\circ$). We estimate the flux ratio between the two cases of inclination ($\theta = 10^\circ$ and $80^\circ$) at 0.1 keV. The choice of the reference energy is according to Wang et al. (2014) that drives the Hβ emission. For the $M = 100$ case, we obtained a flux ratio $=96.67\%$. For a higher accretion rate, for example $M = 500$, we obtain a value ($=99.92\%$) consistent with the filtering factor we obtain from the scaling of the BLR radius. These flux ratios, when estimated at 1 Rydberg give us 62.5% and 90.62% for $M = 100$ and 500, respectively.

The anisotropic emission from the accretion disc has been tested more rigorously in past studies (Runnoe et al. 2013; Xu 2015; Lasota et al. 2016, and references therein), such as in the context of estimating quasar bolometric corrections considering thin accretion discs as the source of the continuum emission including relativistic effects (Nemmen & Brotherton 2010). In such cases, the authors find a slump in the net integrated luminosity for a source over an order of magnitude when the viewing angle increases from $\theta = 10^\circ$ to $\theta = 80^\circ$ (see Fig. 3 in Nemmen & Brotherton 2010). The relative decrease in this luminosity as a function of the increasing viewing angle can be even higher under Newtonian approximation where the bolometric luminosity is related to the integrated luminosity assuming isotropic emission such that:

$$L_{\text{bol}} = \frac{1}{2 \cos \theta} \int_{\nu_0}^{\nu_1} L_{\nu} \, d\nu,$$

where $\nu_0$ and $\nu_1$ are the frequencies bounds for thin disc radiation. On the other hand, strong light-bending effects in quasar microlensing events can cause differential lensing distortion of the X-ray and the optical emission and can significantly change the X-ray-to-optical flux ratios in such sources (Chen et al. 2015).

Narrow-line Seyfert galaxies with high accretion rates are typically shown to have a soft-X-ray excess (Arnaud et al. 1985) in their broadband SED (Jin et al. 2012a,b; Marziani et al. 2018; Ferland et al. 2020). The interstellar medium blocks our view of this spectral region, thus necessitating the use of indirect methods to predict the emission from this part of the radiation field (Kubota & Done 2018, and references therein). This component helps to bridge the absorption gap between the UV downturn and the soft-X-ray upturn (Elvis et al. 1994; Laor et al. 1997b; Richards et al. 2006) and changes the far-UV and soft-X-ray part of the spectrum, affecting the FeII line production (Panda et al. 2019b). Therefore, there is a need to expand the study to construct more realistic SEDs for T2w1 and similar sources which will be undertaken in subsequent work.
4.2. Degeneracies with metallicity and cloud size in the BLR

Additional constraints from high signal-to-noise-ratio rest-frame UV spectra for I Zw 1 can help narrow down the possibilities for the metallicity. There are quite a few metallicity indicators; for example, Al III/He II ≳ 0.40, a fairly unbiased estimator for the metallicity (see Snigewa et al. 2021, for an overview). Another line ratio frequently used is Si IV/He I, which gives ≳ 0.89, suggesting a metallicity ∼ 10 Z⊙, and slightly above solar, respectively. However, another ratio, N II/He II flux ratio gives a value of ∼ 5.78 suggesting Z ∼ 10 Z⊙, although this ratio is quite sensitive to change in ionisation parameter (Wang et al. 2012). Other ratios, such as C IV/He II and Si IV/He II also point towards similarly high metallicities (Z ∼ 10 Z⊙), although they are not so reliable due to issues related to blending with other species which becomes cumbersome unless better quality spectra are available. Therefore, using the Al III/He II flux ratio coupled with the photoionisation-based estimates in this work puts the column density required for R FeII at ∼ 10^22 cm^-2. More recent works suggest a slightly higher value for these line ratios; for example, Al III/He II ≳ 5.55 ± 2.728 if the λ1900 Å blend is fitted with a combination of a blueshifted component that is characteristic of the low-density high-ionisation outflow component, and a broad component that is typical for the high-density low-ionisation part of the BLR (Negrete et al. 2012, Paola Marziani, priv. comm.). Certainly, a higher S/N is needed to properly account for the issues mentioned above. The increased availability of optical–UV and NIR spectroscopic measurements, especially with the advent of the upcoming ground-based 10-metre-class (e.g. Maunakea Spectroscopic Explorer, Marshall et al. 2019) and 40-metre-class (e.g. The European Extremely Large Telescope, Evans et al. 2015) telescopes; and space-based missions such as the James Webb Space Telescope and the Nancy Grace Roman Space Telescope, would certainly be a welcome addition to help break this degeneracy.

On the other hand, for the cloud sizes, Ferland et al. (2009) find that the minimum column density required is ∼ 10^23 cm^-2 for gravity to overpower radiation pressure and allow infall of clouds as found by Hu et al. (2008). Using arguments based on virial determinations of the black hole mass in AGNs by Marconi et al. (2008), Netzer (2009) also concludes that the column densities must substantially exceed ∼ 10^23 cm^-2 to avoid excessive effects of radiation pressure on the orbital velocities of the BLR clouds. Thus, there may be limited freedom to vary the column density to produce the wide range of optical Fe II strength observed which then restricts the parameter space within ∼ 2 dex in column density without accounting for significant electron scattering effects that start to become important at higher optical depths. With such constraints on the column densities and from Fig. 5, we expect metallicities no greater than ∼ 30 Z⊙ but still ≳ 5 Z⊙ in order to efficiently produce the required R FeII values in this case. From the point of view of recent advances in observations, only very recently are we starting to resolve the inner parsec scales in nearby AGNs using interferometric techniques (GRAVITY Collaboration 2018, 2020) but mapping individual BLR clouds is something that remains elusive.

5. Conclusions

In this article, we examine the R FeII and R CaT emission in the broad-line regions of active galaxies. We probe the parameter space in terms of (a) ionisation parameter, (b) the BLR cloud density, (c) the metal content in the BLR cloud, and (d) the size of the BLR cloud in terms of the cloud column density. We incorporate the observational broad-band SED of the prototypical narrow-line Seyfert 1 galaxy, I Zw 1, which serves as the incident continuum that photoionises the BLR cloud. In our previous paper (P20) and the first attempt of this paper, we are successful in reproducing the respective flux ratios (R FeII and R CaT), although it was noticed that the pairs of log U − log n H that correspond to the best estimates for the flux ratios do not recover reasonable line EWs for these low-ionisation lines, including Hβ. We evaluate the EWs for the entire grid of models and optimise the line EWs within reasonable covering factors (between 30% and 60%) and recover the flux ratios obtained from two epochs of prior observations for this source (Persson 1988; Marinello et al. 2016). These new solutions are found to have ionisation parameters (log U ∼ 3.5) that are ∼ 2 dex smaller than previous results, although the local cloud density remains almost unchanged (log n H ∼ 12 cm^-3). This points towards a significant reduction in the flux that is incident on the BLR cloud compared to what has been assumed before. We achieve this reduction in the flux by scaling the radial distance of this LIL-emitting region obtained from our photoionisation modelling to the radial distance obtained from the reverberation mapping estimate for I Zw 1 using the standard radius–luminosity relation (R BLR ∝ L^{1/5}). This suggests that the broad-line region cloud does not ‘see’ the same continuum emitted from the accretion disc as that seen by a distant observer, and that rather it sees a filtered, colder continuum that implies a lowering in the number of line-ionising photons that irradiate the BLR. This in turn suggests smaller radial sizes than predicted earlier by photoionisation modelling. This screening of the accretion disc continuum can be tied to the flux as a function of the cosi, where i is the inclination angle to the symmetry axis. Additionally, there can be a lowering of the photon flux due to anisotropy in the disc structure causing self-shadowing effects as the accretion rate increases in addition to light-bending effects. Our results are applicable to Type-1 Narrow-line Seyfert galaxies with high Fe II emission, i.e. I Zw 1-like sources.

Independently from this aspect, our study still finds that the BLR needs to be selectively overabundant in iron in order to account for the adequate R FeII emission. This is suggested by the requirement of higher-than-solar metallicities (Z ∼ 10 Z⊙) to optimise the emission of optical Fe II. On the other hand, the R CaT emission spans a broader range in metallicity, from solar to super-solar metallicities. In all these models, the BLR cloud density is found to be consistent with our conclusions from prior works, i.e. n H ∼ 10^23 cm^-3 is required for the sufficient emission of Fe II and CaT. We further our modelling to test and confirm the co-dependence between the metallicity and the cloud column density for these two ions and constrain the effective cloud sizes using metallicity constraints from UV line ratios shown to be effective tracers of the metal content in the BLR. Finally, we test the effect of inclusion of a turbulent velocity within the BLR cloud which informs us that the R FeII emission is positively affected by the inclusion of the microturbulence parameter. An interesting result obtained here is that when the microturbulence is invoked, there is a reduction in the value of the metallicity required to obtain optimal R FeII estimates, suggesting that microturbulence can act as a metallicity controller for the Fe II. On
the contrary, the $R_{\text{corr}}$ cases are rather unaffected by changes in microlurbulence.

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