Revisiting $K\pi$ puzzle in the pQCD factorization approach

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Abstract

In this paper, we calculated the branching ratios and direct CP violation of the four $B \to K\pi$ decays with the inclusion of all currently known next-to-leading order (NLO) contributions by employing the perturbative QCD (pQCD) factorization approach. We found that (a) Besides the 10% enhancement from the NLO vertex corrections, the quark-loops and magnetic penguins, the NLO contributions to the form factors can provide an additional ~ 15% enhancement to the branching ratios, and lead to a very good agreement with the data; (b) The NLO pQCD predictions are $A_{\text{dir}}^{\text{CP}}(B^0 \to K^+\pi^-) = (-6.5 \pm 3.1)\%$ and $A_{\text{dir}}^{\text{CP}}(B^+ \to K^+\pi^0) = (2.2 \pm 2.0)\%$, become well consistent with the data due to the inclusion of the NLO contributions.

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I. INTRODUCTION

The four \( B \rightarrow K \pi \) decays play an important role in the precision test of the standard model (SM) and the searching for the new physics beyond the SM \([1]\). The branching ratios of these four decays have been measured with high precision \([1,2]\), but it is still very difficult to interpret the so-called “\( K \pi \)”-puzzle: why the measured direct CP violation \( A_{\text{CP}}(B^0 \rightarrow K^+\pi^-) \) and \( A_{\text{CP}}(B^\pm \rightarrow K^\mp\pi^0) \) are so different? At the quark level, \( B^0 \rightarrow K^+\pi^- \) and \( B^+ \rightarrow K^+\pi^0 \) decay differ only by sub-leading color-suppressed tree and the electroweak penguin. Their CP asymmetry are expected to be similar, but the measured values differ by \( 5\sigma \) \([1,3]\): \( A_{\text{CP}}^{\text{exp}}(B^0 \rightarrow K^+\pi^-) = -0.087 \pm 0.008 \) while \( A_{\text{CP}}^{\text{exp}}(B^+ \rightarrow K^+\pi^0) = 0.037 \pm 0.021 \).

In Ref. \([4]\), the authors studied the “\( K \pi \)” puzzle in the pQCD factorization approach, took the NLO contributions known at 2005 into account, and provided a pQCD interpretation for the large difference between \( A_{\text{CP}}(B^0 \rightarrow K^+\pi^-) \) and \( A_{\text{CP}}(B^\mp \rightarrow K^\mp\pi^0) \). In this paper, we re-calculate these four \( B \rightarrow K \pi \) decays with the inclusion of all currently known NLO contributions in the pQCD approach, especially the newly known NLO corrections to the form factors of \( B \rightarrow (K,\pi) \) transitions \([5]\).

The paper is organized as follows. In Sec.II we calculate the decay amplitudes for the considered decay modes. The numerical results, some discussions and short summary, are presented in Sec.III.

II. DECAY AMPLITUDES IN THE PQCD APPROACH

In the pQCD approach, we treat the \( B \) meson as a heavy-light system, and consider the \( B \) meson at rest for simplicity. By using the light-cone coordinates, the \( B \) meson momentum \( P_B \) and the two final state mesons’ momenta \( P_2 \) and \( P_3 \) (for \( M_2 \) and \( M_3 \), respectively) can be written as

\[
P_B = \frac{M_B}{\sqrt{2}} (1,1,0_T), \quad P_2 = \frac{M_B}{\sqrt{2}} (1-r_3^2,r_2^2,0_T), \quad P_3 = \frac{M_B}{\sqrt{2}} (r_3^2,1-r_2^2,0_T),
\]

where \( r_i^2 = m_i^2/M_B^2 \) are very small for \( m_i = (m_\pi,m_K) \) and will be neglected safely. Putting the light quark momenta in \( B, M_2 \) and \( M_3 \) meson as \( k_1, k_2, \) and \( k_3, \) respectively, we can choose

\[
k_1 = (x_1 P_B^+, 0, k_{1T}), \quad k_2 = (x_2 P_2^+, 0, k_{2T}), \quad k_3 = (0, x_3 P_3^-, k_{3T}).
\]

The decay amplitude after the integration over \( k_{1T}^{-2} \) and \( k_{3T}^+ \) can then be written as

\[
A(B_d \rightarrow M_2 M_3) \sim \int dx_1 dx_2 dx_3 b_1 b_2 b_3 \cdot \text{Tr} \left[ C(t) \Phi_B(x_1,b_1) \Phi_{M_2}(x_2,b_2) \Phi_{M_3}(x_3,b_3) H(x_i,b_i,t) S_i(x_i) e^{-S(t)} \right],
\]

where \( b_i \) is the conjugate space coordinate of \( k_{iT} \). \( C(t) \) is the Wilson coefficient evaluated at scale \( t \), the hard function \( H(k_1,k_2,k_3,t) \) describes the four quark operator and the spectator quark connected by a hard gluon. The wave function \( \Phi_B(k_1) \) and \( \Phi_{M_i} \) describe the hadronization of the quark and anti-quark in the \( B \) meson and \( M_i \) mesons. The
Sudakov factor $S_t(x_i)$ and $e^{-S(t)} = e^{-S_B(t) - S_{M_2}(t) - S_{M_3}(t)}$ can together suppress the soft dynamics effectively \[6\].

For the $B$ meson, we adopt the widely used distribution amplitude $\phi_B$ as in Refs. \[7–9\]

$$
\phi_B(x, b) = N_B x^2 (1 - x)^2 \exp \left[ -\frac{1}{2} \left( \frac{x m_B}{\omega_B} \right)^2 - \frac{\omega_b^2 b^2}{2} \right],
$$

(4)

where the normalization factor $N_B$ depends on the values of the shape parameter $\omega_B$ and the decay constant $f_B$ and defined through the normalization relation $\int_0^1 dx \phi_B(x, b = 0) = f_B/(2\sqrt{6})$. The shape parameter $\omega_b = 0.40 \pm 0.04$ has been fixed \[6\] from the fit to the $B \to \pi$ form factors derived from lattice QCD and from Light-cone sum rule. For the light $\pi$ and $K$ mesons, we adopt the same set of distribution amplitudes $\phi_{A,P,T}^{\pi,K}(x_i)$ as those defined in Ref. \[10\] and being used widely for example in Refs.\[9, 11, 12\].

### A. Leading-order contributions

In the pQCD factorization approach, the leading order contributions to $B \to K \pi$ decays come from the eight Feynman diagrams as shown in Fig.1. Following Ref. \[12\], we here also use the terms $(F_{e}^{LL}, F_{e}^{LR}, F_{e}^{SP})$ and $(M_{e}^{LL}, M_{e}^{LR}, M_{e}^{SP})$ to describe the contributions from the factorizable emission diagrams (Fig.1(a) and 1(b)) and non-factorizable emission diagrams (Fig.1(c) and 1(d)) through the $(V - A)(V - A)$, $(V - A)(V + A)$ and $(S - P)(S + P)$ operators, respectively. In a similar way, we also adopt $(F_{a}^{LL}, F_{a}^{LR}, F_{a}^{SP})$ and $(M_{a}^{LL}, M_{a}^{LR}, M_{a}^{SP})$ to stand for the contributions from the factorizable annihilation diagrams (Fig.1(e) and 1(f)) and non-factorizable annihilation diagrams (Fig.1(g) and 1(h)). From the analytic calculations we obtain all relevant decay amplitudes for the four $B \to K \pi$ decays:

By evaluating the emission diagrams Fig.1(a)-1(d), for example, we find the following
decay amplitudes

\[
F_{e}^{LL} = -F_{e}^{LR} = 16\pi C_{F} M_{B}^{2} \int_{0}^{1} dx_{1} dx_{3} \int_{0}^{\infty} b_{1} db_{1} b_{3} db_{3} \phi_{B}(x_{1}) \times \left\{ \left[ (x_{3} + 1)\phi_{3}^{A}(x_{3}) + r_{3}(1 - 2x_{3}) (\phi_{3}^{P}(x_{3}) + \phi_{3}^{T}(x_{3})) \right] \cdot h_{a}(x_{1}, x_{3}, b_{1}, b_{3}) \ E_{e}(t_{a}) + 2r_{3}\phi_{3}^{P}(x_{3}) \cdot h_{b}(x_{1}, x_{3}, b_{1}, b_{3}) \ E_{e}(t_{b}) \right\},
\]

(5)

\[
F_{e}^{SP} = 32\pi C_{F} M_{B}^{2} \int_{0}^{1} dx_{1} dx_{3} \int_{0}^{\infty} b_{1} db_{1} b_{3} db_{3} \phi_{B}(x_{1}) r_{2} \times \left\{ \left[ r_{3}(2 + x_{3})\phi_{3}^{P}(x_{3}) - r_{3}x_{3}\phi_{3}^{T}(x_{3}) + \phi_{3}^{A}(x_{3}) \right] \cdot h_{a}(x_{1}, x_{3}, b_{1}, b_{3}) \cdot E_{e}(t_{a}) + 2r_{3}\phi_{3}^{P}(x_{3}) \cdot h_{b}(x_{1}, x_{3}, b_{1}, b_{3}) \cdot E_{e}(t_{b}) \right\},
\]

(6)

\[
M_{e}^{LL} = \frac{64}{\sqrt{6}} \pi C_{F} M_{B}^{2} \int_{0}^{1} dx_{1} dx_{2} dx_{3} \int_{0}^{\infty} b_{1} db_{1} b_{2} db_{2} \phi_{B}(x_{1}) \times \left\{ \left[ \bar{x}_{2}\phi_{2}^{A}(x_{2}) - x_{3}r_{3} \left( \phi_{3}^{P}(x_{3}) - \phi_{3}^{T}(x_{3}) \right) \right] \cdot h_{c}(x_{1}, b_{1}, b_{2}) \ E_{e}'(t_{c}) + \left[ \left( -x_{2} - x_{3}\phi_{3}^{A}(x_{3}) + x_{3}r_{2} \left( \phi_{3}^{P}(x_{3}) + \phi_{3}^{T}(x_{3}) \right) \right] \cdot h_{d}(x_{1}, b_{1}, b_{2}) \ E_{e}'(t_{d}) \right\},
\]

(7)

\[
M_{e}^{LR} = \frac{64}{\sqrt{6}} \pi C_{F} M_{B}^{2} \int_{0}^{1} dx_{1} dx_{2} dx_{3} \int_{0}^{\infty} b_{1} db_{1} b_{2} db_{2} \phi_{B}(x_{1}) \times \left\{ \left[ \bar{x}_{2}\phi_{2}^{P}(x_{2}) + \phi_{2}^{T}(x_{2}) \right] \phi_{3}^{A}(x_{3}) + x_{3}r_{3} \left( \phi_{3}^{P}(x_{3}) + \phi_{3}^{T}(x_{3}) \right) \left( \phi_{2}^{P}(x_{2}) - \phi_{2}^{T}(x_{2}) \right) + \bar{x}_{2}r_{3} \left( \phi_{3}^{P}(x_{3}) - \phi_{3}^{T}(x_{3}) \right) \left( \phi_{2}^{P}(x_{2}) + \phi_{2}^{T}(x_{2}) \right) \right] \cdot h_{c}(x_{1}, b_{1}, b_{2}) \ E_{e}'(t_{c}) - \left[ x_{2} \left( \phi_{2}^{P}(x_{2}) - \phi_{2}^{T}(x_{2}) \right) \phi_{3}^{A}(x_{3}) + x_{2}r_{3} \left( \phi_{3}^{P}(x_{3}) + \phi_{3}^{T}(x_{3}) \right) \left( \phi_{2}^{P}(x_{2}) - \phi_{2}^{T}(x_{2}) \right) \right] + x_{3}r_{3} \left( \phi_{3}^{P}(x_{3}) + \phi_{3}^{T}(x_{3}) \right) \left( \phi_{2}^{P}(x_{2}) + \phi_{2}^{T}(x_{2}) \right) \right] \cdot h_{d}(x_{1}, b_{1}, b_{2}) \ E_{e}'(t_{d}) \right\},
\]

(8)

\[
M_{e}^{SP} = \frac{64}{\sqrt{6}} \pi C_{F} M_{B}^{2} \int_{0}^{1} dx_{1} dx_{2} dx_{3} \int_{0}^{\infty} b_{1} db_{1} b_{2} db_{2} \phi_{B}(x_{1}) \times \left\{ \left[ \left( x_{2} - x_{3} - 1 \right)\phi_{3}^{A}(x_{3}) + x_{3}r_{3} \left( \phi_{3}^{P}(x_{3}) + \phi_{3}^{T}(x_{3}) \right) \right] \cdot h_{c}(x_{1}, b_{1}, b_{2}) \ E_{e}'(t_{c}) + \left[ x_{2}\phi_{3}^{A}(x_{3}) + x_{3}r_{3} \left( \phi_{3}^{P}(x_{3}) - \phi_{3}^{T}(x_{3}) \right) \right] \cdot h_{d}(x_{1}, b_{1}, b_{2}) \ E_{e}'(t_{d}) \right\},
\]

(9)

where \( r_{2} = m_{2}/m_{B} \), \( r_{3} = m_{3}/m_{B} \) and \( C_{F} = 4/3 \) is a color factor. The explicit expressions for the convolution functions \( E_{e}(t_{a}) \) and \( E_{e}'(t_{c,d}) \), the hard scales \( t_{a,b,c,d} \), and the hard functions \( h_{a,b,c,d}(x_{i}, b_{l}) \) can be found in Ref. [9]. By evaluating the annihilation diagrams Fig.1(e)-1(h) we can find the corresponding decay amplitudes \( F_{a}^{LL,LR,SP} \) and \( M_{a}^{LL,LR,SP} \), similar with those as given in Eqs.(34-38) in Ref. [13].

Taking into account the contributions from different Feynman diagrams, the total decay
FIG. 2: The typical Feynman diagrams for currently known NLO contributions: the vertex corrections (a-d); the quark-loop (e-f); the chromo-magnetic penguins (g-h); and the NLO contributions to form factors (i-l).

The explicit expressions for $B^0 \rightarrow K^+\pi^-$ and $B^+ \rightarrow K^+\pi^0$ decays can be written explicitly as:

$$A(B^0 \rightarrow K^+\pi^-) = V_{ub}^* V_{us} \left[ f_K a_1 F_{eL}^{LL} + C_1 M_{eL}^{LL} \right]$$

$$-V_{tb}^* V_{ts} \left\{ f_K (a_4 + a_{10}) F_{eL}^{LL} + f_K (a_6 + a_8) F_{eSP}^{SP} + (C_3 + C_9) M_{eL}^{LL} \right.$$ 

$$+ (C_5 + C_7) M_{eR}^{LR} + f_B \left[ \left( a_4 - \frac{a_{10}}{2} \right) F_{a}^{LL} + \left( a_6 - \frac{a_8}{2} \right) F_{a}^{SP} \right]$$

$$+ \left( C_3 - \frac{C_9}{2} \right) M_{a}^{LL} + \left( C_5 - \frac{C_7}{2} \right) M_{a}^{LR} \right\}, \quad (10)$$

$$\sqrt{2} A(B^+ \rightarrow K^+\pi^0) = V_{ub}^* V_{us} \cdot \left\{ [a_1 f_K + a_2 f_{\pi}] F_{eL}^{LL} + (C_1 + C_2) M_{eL}^{LL} + a_1 f_B F_{a}^{LL} + C_1 M_{a}^{LL} \right\}$$

$$-V_{tb}^* V_{ts} \cdot \left\{ (a_4 + a_{10}) (f_K F_{eL}^{LL} + f_B F_{a}^{LL}) + (a_6 + a_8) (f_K F_{eSP}^{SP} + f_B F_{a}^{SP}) \right.$$ 

$$+ (C_3 + C_9) \left( M_{eL}^{LL} + M_{aL}^{LL} \right) + (C_5 + C_7) \left( M_{eR}^{LR} + M_{aR}^{LR} \right)$$

$$+ \left( \frac{3}{2}(-a_7 + a_9) f_{\pi} F_{eL}^{LL} + \frac{3}{2} C_8 M_{eSP}^{SP} + \frac{3}{2} C_{10} M_{aL}^{LL} \right) \right\}, \quad (11)$$

where $a_i$ is the combination of the Wilson coefficients $C_i$ with the definitions: $a_{1,2} = C_{2,1} + \frac{C_{1,2}}{3}$, $a_i = C_i + \frac{C_{i+1}}{3}$ ( $a_i = C_i + \frac{C_{i+1}}{3}$ ) for $i = 3, 5, 7, 9$ ( $i = 4, 6, 8, 10$ ) respectively. The explicit expressions for $B^0 \rightarrow K^0\pi^0$ and $B^+ \rightarrow K^0\pi^+$ decays are similar with those as shown in Eqs. (10, 11).

**B. NLO contributions**

Based on the power counting rule in the pQCD factorization approach [4], the following NLO contributions should be included [4]:
(1) The Wilson coefficients $C_i(M_W)$ at NLO level \cite{14}, the renormalization group evolution matrix $U(t, m, \alpha)$ at NLO level and the strong coupling constant $\alpha_s(t)$ at two-loop level\cite{11}.

(2) The currently known NLO contributions to hard kernel $H^{(1)}(\alpha_s^2)$ include \cite{4, 5, 15}:

(a) The vertex correction (VC) from the Feynman diagrams Fig.2(a)-2(d);
(b) The NLO contributions from the quark-loops (QL) as shown in Fig.2(e)-2(f);
(c) The NLO contributions from the operator $O_{8g}$ as shown in Fig.3(g)-3(h) \cite{15};
(d) The NLO contributions to the form factors as shown in Fig.2(i)-2(l) \cite{5}.

The still missing NLO parts in the pQCD approach are the $O(\alpha_s^2)$ contributions from hard spectator diagrams and annihilation diagrams, as illustrated by Fig.5 in Ref. \cite{13}. According to the general arguments as presented in Ref. \cite{4} and explicit numerical comparisons of the contributions from different sources for $B \to K\pi$ decays as made in Ref. \cite{13} one generally believe that these still missing NLO parts should be very small and can be neglected safely. The major reasons are the following:

1. For the non-factorizable spectator diagrams in Fig.1(c)-1(d), their LO contributions are strongly suppressed by the isospin symmetry and color-suppression with respect to the factorizable emission diagrams Fig.1(a)-1(b). The NLO contributions from Figs.5(a)-5(d) in Ref. \cite{13} are higher order corrections to small LO quantities.

2. For the annihilation spectator diagrams at leading order, i.e. Figs.1(e)-1(h), they are power suppressed and generally much smaller with respect to the contributions from the emission diagrams Fig.1(a)-1(b). The NLO contributions from Figs.5(e)-5(h) in Ref. \cite{13} are also the higher order corrections to the small LO quantities.

3. Taking $B^+ \to K^+\eta$ decay as an example, as shown in Eq.(87) of Ref. \cite{13}, the relative strength of the individual LO contribution $\mathcal{M}^{a+b}$ from the emission diagrams, $\mathcal{M}^{c+d}$ and $\mathcal{M}^{anni}$ from the spectator and the annihilation diagram respectively can be evaluated through the following ratio:

$$|\mathcal{M}^{a+b}|^2 : |\mathcal{M}^{c+d}|^2 : |\mathcal{M}^{anni}|^2 = 3.23 : 0.02 : 0.33. \quad (12)$$

One can see directly from the above ratio that the contribution from emission diagram is indeed dominant, while the contribution from $\mathcal{M}^{c+d}$ ( $\mathcal{M}^{anni}$ ) is less than 1% (10%) of the dominant one.

Based on about reasonable arguments and explicit numerical examinations, one can see that the still missing NLO parts in the pQCD approach are higher order corrections to those small LO quantities, and therefore should be very small and can be neglected safely. For more details of numerical comparisons, one can see Ref. \cite{13}.

The vertex corrections from the Feynman diagrams as shown in Figs. 2(a)-2(d), have been calculated years ago in the QCD factorization approach \cite{16, 17}. Since there is no end-point singularity in the evaluations of Figs.2(a)-2(d), it is unnecessary to employ the $k_T$ factorization theorem here \cite{4}. The NLO vertex corrections will be included by adding a same vertex function $V_i(M)$ to the corresponding Wilson coefficients $a_i(\mu)$ as in Refs. \cite{9, 16, 17}. 

6
For the $b \to s$ transition, the contributions from the various quark loops are given by

$$H_{\text{eff}} = -\sum_{q=u,c,t} \sum_{q'} \frac{G_F}{\sqrt{2}} V_{qb} V_{qs}^* \frac{\alpha_s(\mu)}{2\pi} C^{(q)}(\mu, l^2) \left[ \bar{s}_i \gamma_\rho (1 - \gamma_5) T^a b \right] \left( \bar{q}'_j \gamma^\rho T^a q' \right),$$  

(13)

where $l^2$ is the invariant mass of the gluon, which attaches the quark loops in Figs. 2e and 2f. The expressions of the functions $C^{(q)}(\mu, l^2)$ for $q = (u, c, t)$ can be found easily in Refs. [4, 9].

The magnetic penguin is another kind penguin correction induced by the insertion of the operator $O_{8g}$, as illustrated by Fig. 2(g) and 2(h). The corresponding weak effective Hamiltonian contains the $b \to sg$ transition can be written as

$$H_{\text{mp}}^{\text{eff}} = -\frac{G_F g_s}{\sqrt{2} m_b} V_{tb} V_{ts}^* C_S^{(8g)} \left[ \bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) T_{ij} G^{\mu\nu} b_j \right],$$  

(14)

where $i, j$ are the color indices of quarks, $C_S^{(8g)} = C_{8g} + C_5$ [4] is the effective Wilson coefficient.

For the sake of convenience we denote all current known NLO contributions except for those to the form factors by the term Set-A. For the four $B \to K\pi$ decays, the Set-A NLO contributions will be included in a simple way:

$$A_{\pi K} \to A_{\pi K} + \sum_{q=u,c,t} \xi_q M^{(q)}_{\pi K} + \xi_t M^{(g)}_{\pi K},$$

$$A_{K\pi} \to A_{K\pi} + \sum_{q=u,c,t} \xi_q M^{(q)}_{K\pi} + \xi_t M^{(g)}_{K\pi},$$  

(15)

where $\xi_q = V_{qb} V_{qd}^*$, $\xi'_q = V_{qb} V_{qs}^*$ with $q = (u, c, t)$, while the decay amplitudes $M^{(q)}_{M_i, M_j}$ and $M^{(g)}_{M_i, M_j}$ are of the form:

$$M^{(q)}_{\pi^{-} K^{+}} = -8m_B^4 \frac{G_F^2}{\sqrt{2} N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_3 \phi_B(x_1) \left\{ \left[ (1 + x_3) \phi^A_{\pi}(x_3) \phi^A_K(x_2) ight. ight.$$

$$+ 2r_\pi \phi_K^P(x_2) \phi^A_\pi(x_3) + r_\pi (1 - 2x_3) \phi^A_K(x_2) \left( \phi^P_\pi(x_3) + \phi^T_\pi(x_3) \right)$$

$$+ 2r_\pi \phi_K^P(x_2)(2 + x_3) \phi^P_\pi(x_3) - x_3 \phi^T_\pi(x_3) \right\]$$

$$\cdot \alpha_2^2(t_a) h_e(x_1, x_3, b_1, b_3) \exp[-S_{ab}(t_a)] C^{(q)}(t_a, l^2)$$

$$+ \left[ 2r_\pi \phi^A_K(x_2) \phi^P_\pi(x_3) + 4r_\pi \phi_K^P(x_2) \phi^P_\pi(x_3) \right]$$

$$\cdot \alpha_2^2(t_b) h_e(x_1, x_3, b_3, b_1) \exp[-S_{ab}(t_b)] C^{(q)}(t_b, l^2) \right\},$$

(16)
\[ \mathcal{M}_{\pi^- K^+}^{(q)} = -16m_B^6 \frac{C_F^2}{2\sqrt{N_c}} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 b_3 db_3 \phi_B(x_1) \cdot \left\{ \left[ (1 - x_3) \left[ 2\phi^A_\pi(x_3) + r_\pi(3\phi^P_\pi(x_3) + \phi^T_\pi(x_3)) \right. \right. \right.
\left. + r_\pi x_3(\phi^P_\pi(x_3) - \phi^T_\pi(x_3)) \left. \right] \phi^A_K(x_2) \right.
\left. - r_K x_2 (1 + x_3)(3\phi^P_K(x_2) - \phi^T_K(x_2))\phi^A_\pi(x_3) \right.
\left. - r_\pi r_K (1 - x_3)(3\phi^P_K(x_2) + \phi^T_K(x_2))(\phi^P_\pi(x_3) - \phi^T_\pi(x_3)) \right.
\left. - r_\pi r_K x_2 (1 - 2x_3)(3\phi^P_K(x_2) - \phi^T_K(x_2))(\phi^P_\pi(x_3) + \phi^T_\pi(x_3)) \right\} \cdot \alpha_s^2(t_a) h_q(x_i, b_i) \exp[-S_{cd}(t_a)] C_{8g}^{eff}(t_a) + \left[ 4r_\pi \phi^A_K(x_2)\phi^P_\pi(x_3) + 2r_\pi r_\pi x_2(3\phi^P_K(x_2) - \phi^T_K(x_2))\phi^P_\pi(x_3) \right] \alpha_s^2(t_b) h_q^f(x_i, b_i) \exp[-S_{cd}(t_b)] C_{8g}^{eff}(t_b), \]

where the expressions of the Sudakov factors \( S_{i\delta}(t_i) \) and \( S_{cd}(t_i) \), the functions \( C^{(q)}(t_a, l^2) \) and \( C^{(q)}(t_b, l^2) \), can be found easily in Refs. [13,14].

In Ref. [5], the authors derived the \( k_T \)-dependent NLO hard kernel \( H^{(1)} \) for the \( B \to \pi \) transition form factor. Here we quote their results directly, and extend the expressions to the \( B \to K \) transitions under the assumption of \( SU(3) \) flavor symmetry. At the NLO level, the hard kernel function \( H \) can then be written as

\[ H = H^{(0)}(\alpha_s) + H^{(1)}(\alpha_s^2) = [1 + F(x_1, x_3, \mu, \mu_f, \eta, \zeta_1)] H^{(0)}(\alpha_s), \]

where the expression of the NLO factor \( F(x_1, x_3, \mu, \mu_f, \eta, \zeta_1) \) can be found in Eq. (56) of Ref. [5].

III. NUMERICAL RESULTS AND DISCUSSIONS

In numerical calculations, the following input parameters will be used [11] (all the masses, QCD scale and decay constants are in units of GeV):

\[ \Lambda_{QCD} = 0.25, \quad m_W = 80.40, \quad m_B = 5.28, \quad m_\pi = 0.14, \quad m_K = 0.494; \]
\[ f_\pi = 0.13, \quad f_K = 0.16, \quad \tau_{B^0} = 1.528 \text{ ps}, \quad \tau_{B^+} = 1.643 \text{ ps}. \]

For the CKM matrix elements in the Wolfenstein parametrization, we use \( \lambda = 0.2254, \quad A = 0.817, \quad \bar{\rho} = 1.367^{+0.019}_{-0.018} \) and \( \bar{\eta} = 0.348 \pm 0.013 \) [11]. For the Gegenbauer moments and other relevant input parameters, we use [10]

\[ a_1^\pi = 0, \quad a_1^K = 0.06, \quad a_2^\pi = a_2^K = 0.25 \pm 0.15, \quad a_4^\pi = -0.015, \quad a_4^K = 0, \]
\[ \rho_\pi = m_\pi/m_0^\pi, \quad \rho_K = m_K/m_0^K, \quad \eta_3 = 0.015, \quad \omega_3 = -3.0, \]

with the chiral mass \( m_0^\pi = 1.4 \pm 0.1 \text{ GeV} \), and \( m_0^K = 1.6 \pm 0.1 \text{ GeV} \).
From the decay amplitudes and the input parameters, it is straightforward to calculate the branching ratios and CP violating asymmetries for the four considered $B \to K \pi$ decays \[1, 9\].

In Table I and II, we show the LO and NLO pQCD predictions for the branching ratios and the direct CP violating asymmetries of the considered four $B \to K \pi$ decays. In Table I and II, we list only the central values of the LO pQCD predictions in column two, and the central values and the major theoretical errors simultaneously in column four. The first error arises from the uncertainty of $\omega_B = 0.40 \pm 0.04$ GeV, the second one from the uncertainty of $a_2^{\pi,K} = 0.25 \pm 0.15$, and the third one is induced by the variations of both $m_0^K = 1.6 \pm 0.1$ GeV and $m_0^\pi = 1.4 \pm 0.1$ GeV. The errors induced by the uncertainties of other input parameters are very small and have been neglected. As a comparison, we also show the partial pQCD predictions obtained in this work (labeled by Set-A in column three) and those as given in Ref. \[4\] in the column five, where the same Set-A NLO contributions are included. One can see from those numerical results that:

1. For branching ratios, the central values of pQCD predictions as given in column three in Table I are smaller than those as shown in column five by about thirty percent, such difference are largely induced by the change of the lower cutoff of the hard scale $t$ from $\mu_0 = 0.5$ GeV in Ref. \[4\] to $\mu_0 = 1$ GeV here, because it may be conceptually incorrect to evaluate the Wilson coefficients at scales down to 0.5 GeV \[9\] \[18\]. For direct CP violating asymmetries, as shown in the third and fifth column of Table II, the changes of the pQCD predictions due to the variation of $\mu_0$ are rather small, this is consistent with the general expectation.

2. Analogous to the case for $B \to K \eta^{(')}$ decays as shown explicitly in Table VIII and IX in Ref. \[13\], the NLO contributions to the decay amplitudes from the vertex, the quark-loop and the magnetic penguins are largely canceled from each other, and in turn leaving only a roughly 10% enhancement to the LO pQCD predictions of the branching ratios.

3. As listed in Table I of Ref. \[19\], the NLO contribution to the form factor for $B \to \pi$ ($B \to K$) transition can provide a 18% (15%) enhancement to the corresponding LO result:

$$F_0^{LO}(0)(B \to \pi) = 0.22 \pm 0.04 \longrightarrow F_0^{NLO}(0)(B \to \pi) = 0.26 \pm 0.04,$$

$$F_0^{LO}(0)(B \to K) = 0.27 \pm 0.05 \longrightarrow F_0^{NLO}(0)(B \to K) = 0.31 \pm 0.05. \quad (23)$$

Such enhancement to form factors $F_0^{B \to \pi}(0)$ and $F_0^{B \to K}(0)$ can in turn result in an additional 12% to 18% enhancement to branching ratios relative to the results in the third column with the label ”Set-A”, as illustrated clearly by the numerical results in column four of Table I, and consequently lead to a very good agreement between the NLO pQCD predictions and the measured values within errors.

4. For $A^{dir}_{CP}(B^0 \to K^0 \pi^0)$ and $A^{dir}_{CP}(B^+ \to K^0 \pi^+)$, the pQCD predictions agree well with the data.

5. At the leading order, the pQCD predictions for $A^{dir}_{CP}(B^0 \to K^+ \pi^-)$ and $A^{dir}_{CP}(B^+ \to K^+ \pi^0)$ are indeed similar in both the sign and the magnitude, $-12.6\%$ vs $-8.6\%$, as generally expected. After the inclusion of the NLO contributions, however, they
TABLE I: The LO and NLO pQCD predictions for branching ratios $B r(B \to K \pi)$ (in units of 10$^{-6}$), the previous pQCD predictions in Ref. [4] and the relevant data [1, 2] also be listed in last two columns.

| Decay modes | LO | Set-A | NLO: This Work | pQCD [4] | Data |
|-------------|----|-------|----------------|---------|------|
| $B^0 \to K^0 \pi^0$ | 6.3 | 6.6 | $7.4^{+2.2+1.3+0.9}_{-1.5-1.2-0.9}$ | $9.1^{+5.6}_{-3.3}$ | $9.9 \pm 0.5$ |
| $B^0 \to K^+ \pi^-$ | 14.4 | 15.3 | $17.7^{+5.5+2.6+2.0}_{-3.8-2.4-2.0}$ | $20.9^{+15.6}_{-6.3}$ | $19.6 \pm 0.5$ |
| $B^+ \to K^+ \pi^0$ | 10.1 | 10.6 | $12.5^{+4.0+1.7+1.3}_{-2.8-1.6-1.2}$ | $13.9^{+10.0}_{-5.6}$ | $12.9 \pm 0.5$ |
| $B^+ \to K^0 \pi^+$ | 17.5 | 18.4 | $21.5^{+6.7+3.4+2.8}_{-4.7-3.1-2.3}$ | $24.5^{+13.6}_{-8.1}$ | $23.8 \pm 0.7$ |

TABLE II: The same as in Table I, but for the pQCD predictions for the direct CP violations $A_{CP}^{dir}(B \to K \pi)$ (in units of 10$^{-2}$).

| Decay modes | LO | Set-A | NLO: This Work | pQCD [4] | Data |
|-------------|----|-------|----------------|---------|------|
| $A_{CP}^{dir}(B^0 \to K^0 \pi^0)$ | $-2.2$ | $-7.0$ | $-7.9^{+0.3+0.8+0.4}_{-0.23-0.9-0.5}$ | $-7 \pm 3$ | $0 \pm 13$ |
| $A_{CP}^{dir}(B^+ \to K^0 \pi^+)$ | $-0.75$ | 0.40 | $0.38^{+0.09+0.02+0.03}_{-0.11-0.07-0.05}$ | $0 \pm 0$ | $-1.5 \pm 1.2$ |
| $A_{CP}^{dir}(B^0 \to K^+ \pi^-)$ | $-12.6$ | $-6.4$ | $-6.5^{+2.1}_{-2.0} \pm 2.3 \mp 0.3$ | $-9_{-8}^{+6}$ | $-8.7 \pm 0.8$ |
| $A_{CP}^{dir}(B^+ \to K^+ \pi^0)$ | $-8.6$ | 2.0 | $2.5^{+7.1}_{-1.8} \pm 1.2 \pm 0.1$ | $-1_{-5}^{+3}$ | $3.7 \pm 2.1$ |

become rather different as can be seen from Table II. The NLO pQCD predictions, consequently, become agree well with the data. One can also see that the pQCD predictions for $A_{CP}^{dir}(B^0 \to K^+ \pi^-)$ and $A_{CP}^{dir}(B^+ \to K^+ \pi^0)$ remain basically unchanged when the NLO corrections to the form factors are taken into account.

In summary, we studied the $B \to K \pi$ decays by employing the pQCD factorization approach. We focus on checking the effects of all currently known NLO contributions to the branching ratios and direct CP violations of the considered decay modes, especially the rule of the NLO corrections to the form factors $F_0^{B \to \pi}(q^2)$ and $F_0^{B \to K}(q^2)$. Based on the numerical calculations and the phenomenological analysis, the following points have been observed:

1. Besides the 10% enhancement from the Set-A NLO contributions, the NLO contributions to the form factors can provide an additional $\sim 15\%$ enhancement to the branching ratios, and lead to a very good agreement with the data.

2. With the inclusion of all known NLO contributions, the NLO pQCD predictions are

$$A_{CP}^{dir}(B^0 \to K^+ \pi^-) = (-6.5 \pm 3.1)\%, \quad A_{CP}^{dir}(B^+ \to K^+ \pi^0) = (2.2 \pm 2.0)\%,$$ (24)

where the theoretical errors have been added in quadrature, which agree well with the data.
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