Photoproductions of $K^+\Lambda$ and $K^0\Lambda$ Studied by Using Isobar Model

S Clymton and T Mart
Departemen Fisika, FMIPA, Universitas Indonesia, Depok 16424, Indonesia
E-mail: samsonclymton@gmail.com, terry.mart@sci.ui.ac.id

Abstract. We have investigated the neutral kaon photoproduction off a neutron by using an isobar model based on our previous study. The unknown parameters were extracted by fitting the model prediction to experimental data. We analyzed the two $\Lambda$ channels, i.e., $K^+\Lambda$ and $K^0\Lambda$ channels, simultaneously. We used two different approaches in the fitting process which lead to two different models. We compare the agreement between model prediction and experimental data for both models quantitatively. The result reveals the effect of including the $K^0\Lambda$ data on the model and thus improves our understanding on the kaon photoproduction process.

1. Introduction
Kaon photoproduction off the nucleon has been continuously studied since five decades ago. A number of theoretical models have been proposed to explain the result of experimental data which improves our understanding of the baryon structures. From the six isospin channels that are possible for this process two of them have abundant experimental data and have been thoroughly studied. However, the problem arises as the proton is replaced by the neutron in the target. The short lifetime of a neutron and the difficulty to detect neutral kaon make the experiments a daunting task. Fortunately, in recent years, there is an effort to measure this reaction through deuteron target by CLAS collaboration [1]. They have successfully produced 361 data points of differential cross section with energies up to 2.5 GeV. These data are useful for a better understanding on kaon photoproduction, thus improving our knowledge of baryon structure.

From the theoretical side, models for investigation of the $\Lambda$ final state of kaon photoproduction have been produced in recent years with significant improvements. Our previous study, for instance, has included nucleon resonances with spins up to $9/2$ [2, 3] and has been updated in view of recent experimental data [4]. The latest model yields nice agreement with experimental data and provides the resonance properties that contribute to the $K^+\Lambda$ photoproduction. In this paper, we report on the extension of this model that includes the $K^0\Lambda$ photoproduction. Besides for the sake of completeness the inclusion of this channel provides an important constraint on the resonance properties, especially on those that have strong coupling to the $\Lambda$ final state. Our previous analyses on this channel can be found in Refs. [5, 6].

2. The Models
In this paper we use the same formalism as in our previous study [4]. We extend the model by introducing new parameters to accommodate the differences between photo decay amplitude of
nucleon resonance to proton and neutron. We perform two kind of fittings to create two different models. The first model makes use of a new parameter that is considered as free parameter. The mass, width and coupling constant of all resonances are fixed and their values are taken from our previous study. In the second model all parameters are considered as free parameters. The free parameters are adjusted to fit experimental data of both $K^0\Lambda$ (361 data) and $K^+\Lambda$ (9003 data) photoproduction simultaneously. The parameters are adjusted in the fitting process until the value of $\chi^2/N$ reaches its minimum. We use the common definition of $\chi^2/N$, i.e.,

$$\chi^2/N = \frac{1}{N_{\text{data}}} \sum_{i=1}^{N_{\text{data}}} \left[ \frac{\sigma_i(\text{exp}) - \sigma_i(\text{calc.})}{\Delta \sigma_i(\text{exp})} \right]^2,$$

where $\sigma_i(\text{expt})$ and $\sigma_i(\text{calc.})$ are the $i$-th experimental and calculated observables, respectively, and $\Delta \sigma_i(\text{expt})$ is the corresponding experimental error bar. The fitting process has been performed by using the MINUIT code.

3. The Resonance Position

Understanding the structure of baryon is one of the dreams of the particle physicists. This structure is related to the spectrum of its excited states called the resonances. In the Particle Listing of Particle Data Group there are numerous numbers of resonances with the corresponding properties [8]. However, compared to the quark model predictions the number of resonances is much smaller. These yet unlisted resonances are referred to as the "missing resonances". One of the reason of this phenomenon is due to the limited analysis of the reactions that are strongly coupled to these resonances. Clearly, establishing a reliable model to study these resonances becomes the highest priority in hadronic physics researches.

In our previous study, we have presented a number of important resonance properties by using two different methods, i.e., the Breit-Wigner and the pole position ones. We found that these methods could produce different results if we used different backgrounds in the model [7]. Furthermore, the result could also change due to the addition of new experimental data. We perform a simple analysis by calculating the shift of resonance position. The considered resonance position is determined from the Breit-Wigner parameter and pole position methods. The Breit-Wigner parameter consists of mass and width of resonance. This mass determines the position of resonance in the energy distribution of cross section. The width of this resonance position does not have meaningful information but it is needed to describe the resonance property. The pole position is calculated by using both Breit-Wigner parameters. This value can be obtained by finding the root of the denominator in the amplitude, i.e.,

$$s_p - m_{BW}^2 + im_{BW} \Gamma(s_p) = 0 \ .$$

with $s_p = W_p^2$ indicates the pole position and $\Gamma$ is the energy-dependent width that is proportional to $\Gamma_{BW}$. The shift values of this resonance position are defined as

$$\Delta m_{BW} = \frac{|m_{BW}(1) - m_{BW}(2)|}{m_{BW}(1)},$$

$$\Delta \Gamma_{BW} = \frac{|\Gamma_{BW}(1) - \Gamma_{BW}(2)|}{\Gamma_{BW}(1)},$$

$$\Delta W_p = \frac{|W_p(1) - W_p(2)|}{|W_p(1)|} \ .$$

where the notations (1) and (2) refer to the position according to the first and second model, respectively. These values will be used in the following discussion.
Table 1. Comparison between the $\chi^2/N$ obtained in Model 1 and Model 2 for $K^+\Lambda$ channel, $K^0\Lambda$ channel, and both channels.

| Model      | $\chi^2/N(K^+\Lambda)$ | $\chi^2/N(K^0\Lambda)$ | $\chi^2/N$ |
|------------|--------------------------|--------------------------|------------|
| Model 1    | 1.49                     | 3.33                     | 1.56       |
| Model 2    | 1.50                     | 0.96                     | 1.48       |

4. Results and discussion

The two proposed models provide the main discussion of this study. We call the first model as "Model 1" and the second model as "Model 2". The result of the fitting process for both models are presented in Table 1. Clearly, compared to Model 1 a better agreement with experimental data is exhibited by Model 2. However, Model 2 can not reproduce $K^+\Lambda$ data better than Model 1. This is expected, because in Model 2 we let all parameters as free parameter, and as a consequence the fit adjusted these parameters to reproduce the data. Nevertheless, these slight changes produce a significant agreement with $K^0\Lambda$ data. This is probably due to the adjustment of the resonances position and the strength in the model that are compatible for data in both channels. It is a proof that the $K^0\Lambda$ data become a stringent constraint to the resonances properties.

From Table 2, we analyze how the $K^0\Lambda$ data change the resonance properties, especially the resonance positions. We have two kind of positions, the Breit-Wigner and the pole ones. The mass shift of the resonances are below 1%, except for the $N(1875)$ state. However, it is different in the case of the resonance width. The latter changes significantly, with the largest one is found in the $N(1875)$ state. From our previous study [7], we have found a similar phenomenon; the $N(1875)$ is the most unstable state under the change of the background. Therefore, we believe that this resonance requires special investigation in the future study because of its unstable nature. Furthermore, whether this resonance is really important in this reaction is also important to prove.

The Breit-Wigner position is determined through the mass but not the width. However, we cannot disregard the drastic changes of the widths as shown in Table 2. Therefore, we investigate the shift of its pole position that originates from both the Breit-Wigner parameters.

Table 2. The shift value of nucleon resonance positions ($\Delta M_{BW}$, $\Delta \Gamma_{BW}$ and $\Delta W_p$) due to the effect of including $K^0\Lambda$ data (Model 2) from our previous study (Model 1). Values shown below are in percentage (%).

| Resonance | $J^P$ | $\Delta M_{BW}$ | $\Delta \Gamma_{BW}$ | $\Delta W_p$ |
|-----------|-------|-----------------|----------------------|-------------|
| N(1440)   | 1/2+  | 0.0             | 0.0                  | 0.0         |
| N(1520)   | 3/2-  | 0.0             | 0.0                  | 0.0         |
| N(1535)   | 1/2-  | 0.0             | 0.0                  | 0.0         |
| N(1650)   | 1/2-  | 0.0             | 0.0                  | 0.0         |
| N(1675)   | 5/2-  | 0.0             | 0.0                  | 0.0         |
| N(1680)   | 5/2+  | 0.0             | 0.0                  | 0.0         |
| N(1700)   | 3/2-  | 0.2             | 6.2                  | 0.2         |
| N(1710)   | 1/2+  | 0.0             | 0.0                  | 0.0         |
| N(1720)   | 3/2+  | 0.0             | 3.2                  | 0.2         |
| N(1860)   | 5/2+  | 0.0             | 0.0                  | 0.0         |
| N(1875)   | 3/2-  | 1.6             | 43.8                 | 2.1         |

| Resonance | $J^P$ | $\Delta M_{BW}$ | $\Delta \Gamma_{BW}$ | $\Delta W_p$ |
|-----------|-------|-----------------|----------------------|-------------|
| N(1880)   | 1/2+  | 0.0             | 0.0                  | 0.0         |
| N(1895)   | 1/2-  | 0.0             | 1.5                  | 0.1         |
| N(1900)   | 3/2+  | 0.0             | 3.8                  | 0.2         |
| N(1990)   | 7/2+  | 0.0             | 15.4                 | 0.7         |
| N(2000)   | 5/2+  | 0.0             | 0.0                  | 0.0         |
| N(2060)   | 5/2-  | 0.0             | 0.0                  | 0.0         |
| N(2120)   | 3/2-  | 0.0             | 0.0                  | 0.0         |
| N(2190)   | 7/2-  | 0.1             | 0.0                  | 0.1         |
| N(2220)   | 9/2+  | 0.0             | 24.0                 | 1.1         |
| N(2250)   | 9/2-  | 0.0             | 0.0                  | 0.0         |
Figure 1. Comparison of the total cross section calculated in the Model 1 (slashed line), Model 2 (solid line) and experimental data. Data are obtained from the CLAS [1] collaborations.

Figure 2. Same as in Fig. 1, but for the angular (top panels) and energy (bottom panels) distributions of differential cross section.

The change in this value is the same as the change in the mass value, it is under 1%, except for the two resonances, i.e., the $N(1875)$ and $N(2220)$ states. It is interesting to see the behavior of the $N(2220)$ resonance and the $N(2250)$ one. The reason is because the two resonances are complementary to each other, as was found in our previous study [7] as well as in the present work. From this result, we are tempted to say that we just need one $9/2$ resonance for this mass region. Nevertheless, a more careful and thorough study is strongly needed to rule out one of these resonances, because both resonances have four-star rating in the particle data listing of PDG [8].

The total and differential cross sections obtained from Model 1, Model 2 and experimental data are compared in Figs. 1 and 2. Figure 1 exhibits that a single and broad peak is existing in the total cross section. This is different from the $K^+\Lambda$ total cross section that shows two
peaks. As shown in Fig. 1, Model 1 reproduces the experimental data poorly and it is almost similar as in the case of $K^+\Lambda$, it shows two peaks but the second one is much smaller in order to fit the $K^0\Lambda$ data. This result is expected because we did not change the background and all resonance properties, except the nucleon resonance couplings. If we considered all parameters as free parameters then the result is shown by the solid lines (Model 2). This model seems to be more convergent in the high energy region and can explain experimental data much better than Model 1. Probably, this is due to the adjusted background that helps to suppress the divergent behavior in the high energy region which is less flexible in Model 1. Near the peak, the resonance strength and position in Model 2 are distributed in order to relieve the underestimated cross section near 1.8 GeV. The result is a much better cross section near the peak. However, the total cross section magnitude near 1.8 GeV is still far from perfect. Presumably, this is due to the lack of resonances whose mass is around 1.8 GeV. On the contrary, numerous resonance states are existing near 1.7 GeV and 1.9 GeV. This could also be an indication of the existence of missing resonances in this reaction.

The differential cross section tells us the information on the angular behavior of cross section that is absent in the total cross section. In Fig. 2 we can clearly see that Model 1 overpredicts the cross section at certain regions. At low energies, Model 1 overestimates the cross section near $\cos\theta = 0$. This is presumably due to the strong Born terms contribution in that region, while in Model 2 the contribution is suppressed. The same behavior is also found in the forward and backward regions at high energies. The large cross section of Model 1 in this region is presumably suppressed in Model 2 due to the destructive interference with hyperon resonances, since the contribution is large in the backward region. On the other hand, in the high energy region, it is slightly suppressed due to the small contribution of hyperon resonances. Furthermore, it also causes some changes in the resonance position of Model 2.

The energy distribution of differential cross section exhibits the overestimated cross section of Model 1 at $W \approx 1.7$ GeV. It also provides us with the information on how the models are different in distributing the resonance contribution. In general, we found that both models have similar pattern of resonances but differ in their strength. The one peak phenomenon shown by Model 2 that we see in the total cross section comes actually from two peaks that have the same strength. This is clearly seen at $\cos\theta = 0.45$. The shift in the second peak is also shown in Fig. 2. Probably this is the effect of the $N(1875)$ state that changes its position in Model 2.

5. Summary and conclusion
We have proposed two different models that make use of different free parameters to simultaneously reproduce the $K^0\Lambda$ and $K^+\Lambda$ data. We found that Model 2 yields the best agreement with experimental data. We also found a number of resonances that shift their positions after the inclusion of the $K^0\Lambda$ data. Moreover, the strength of each resonance is also changing. Thus, the $K^0\Lambda$ data yield a tough constraint for phenomenological models that try to explain the resonances whose contribution to the $K^+\Lambda$ channel is important.

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