Numerical investigation of flow past a circular cylinder controlled by electromagnetic force

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Abstract. The flow past a circular cylinder controlled by Electromagnetic force is calculated by a finite volume method in order to investigate the control effect of the electromagnetic force on the three-dimensional vortical structures and the dynamics characteristics. The Reynolds number considered are from 200 to 1000, during which the wake transition occurs and the three-dimensional flow structures develop. The electromagnetic force (i.e. Lorentz force) is utilized to suppress the boundary layer separation and modify the vortical structures of the flow past the cylinder in low-conducting electrolyte. It is found that the spanwise wavelength of mode A and mode B instability are about 3.3 and 0.83 times cylinder diameter, respectively. The streamwise vortex becomes finer as the Reynolds number increase. The three dimensional flow structures during the period of vortex shedding may be suppressed effective by the Lorentz force, leading to drag reduction and vibration suppression.

1. Introduction
It is well known that vortex shedding occurs in the wake of a circular cylinder when the Reynolds number \( Re \) exceeds a critical value (\( Re \approx 49, \) \( Re \) based on the diameter of cylinder). The alternate shedding of vortices leads to fluctuation of drag and lift forces and may cause structure vibrations and acoustic noises, which in some cases can trigger damage. Therefore, control of the vortex shedding of a bluff body is of great importance from a practical engineering viewpoint.

The control methods developed in the past 20 years can be categorized into three groups: passive, active open-loop and active closed-loop controls [1]. In the case of low-conductivity electrolyte (such as seawater) the Lorentz force can be utilized to modify the boundary layer and control the vortex shedding, which is an active control strategy.

Few results have been reported on the three-dimensional wake of the cylinder controlled by Lorentz force. The objective of the present study is to numerically investigate the vortical structures and dynamic characteristics of the cylinder wake controlled by the Lorentz force.

2. Governing equations and numerical scheme

2.1 Governing equations

The incompressible Navier-Stokes equations with Lorentz force in dimensionless form read
\[ \nabla \cdot \mathbf{U} = 0 \]  \hspace{1cm} (1)

\[ \frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{U} + \mathbf{N} \]  \hspace{1cm} (2)

where \( \mathbf{U} \) and \( p \) are the velocity and pressure of the fluid, and \( \mathbf{F} \) refers to the dimensionless Lorentz force. The Reynolds number \( Re \) is based on the diameter of cylinder \( D \). The interaction parameter

\[ N = \frac{j_0 B_0 D}{\rho u_{\infty}^2} \]

is the ratio of Lorentz force to the inertial force of the fluid \((N=2 \text{ used in this article})\), where \( u_{\infty} \) is the free-stream velocity, \( j_0 \) and \( B_0 \) are the current density and magnetic field at the surface of the electrode and magnetic, respectively. The surface of the cylinder is flush mounted with alternating electrodes and magnets along the streamwise, therefore, the Lorentz force acts tangential to the surface of the cylinder along the streamwise. The distribution of the streamwise Lorentz force can be described as \([2, 3]\):

\[ |F| = e^{-a(x^2+y^2)^{3/2}} \]  \hspace{1cm} (3)

where \( r \) is the radius of the cylinder, \( a \) is a constant showing the electromagnetic penetration into the flow which is mainly defined by the electrode spacing. It can be found that the Lorentz force decays exponentially in radial direction and is linear in azimuthal direction.

### 2.2 Spatial and temporal discretization

As shown in figure 1, the computational domain is a \( 40 D \times 10 D \times 10 D \) cuboid. Periodic boundary condition (b. c.) is applied on the spanwise domain boundary and inflow b. c. is applied on the side domain boundary. The cylinder surface is non-slip b. c and the outflow is pressure b. c.. The domain is spatially discretized using cubic finite volumes organise hierarchically as an octree [4-6]. The fundamental feature of tree type discretization is that each cell can be divided into eight sub-cells. As the cross-section mesh shown in figure 1, the mesh is adapted dynamically by vorticity criteria to follow the vortical evolution in the wake in order to save the computation time.

**Figure 1.** Computational details of flow past a circular cylinder.

The fractional-step projection method is used to implement temporal discretization \([4, 5]\). The advection term is discretized using the Bell-Colella-Glaz second-order upwind scheme. The advection
term on the solid surface is treated using cell-merging technique. The diffusion term is discretized using an implicit Crank-Nicolson scheme. Second-order convergence in space and time is achieved.

3. Flow past a circular cylinder

Figure 2 shows the instantaneous vortical structures by \( \lambda \) definition[7]. At \( Re=200 \), the primary Karman vortex becomes undulate and bifurcates to a three-dimensional flow (mode A). At \( Re=250 \), the primary vortex become more twisty and the streamwise vortex is clearly observed. At \( Re=300 \), finer-scale streamwise vortex pairs (mode B) are formed in the near wake. The spanwise wavelength of mode A and mode B is about 3.3 \( D \) and 0.83 \( D \), respectively, which agrees well with [8]. The nonuniformity of the streamwise vortex may be caused by the interaction of the spanwise vortex and the streamwise vortex after shedding from the cylinder. At \( Re=1000 \), more streamwise vortex are generated and interact with the spanwise vortex, leading to an increasing disorder in the far wake. The scale of the streamwise vortex becomes finer as Reynolds number increase.

![Figure 2. Vortical structures: (a)\( Re=200 \); (b)\( Re=250 \); (c)\( Re=300 \); (d)\( Re=1000 \).](image)

The drag coefficient and lift coefficient are defined as

\[
C_D = \frac{F_x}{\frac{1}{2} \rho u_c^2 S} \quad \text{and} \quad C_L = \frac{F_y}{\frac{1}{2} \rho u_c^2 S}
\]

Where \( F_x \) and \( F_y \) are the drag and lift force, respectively. \( S \) is the projected area of the cylinder perpendicular to the direction of the free-stream, \( S=D \times H \), where \( H \) is the spanwise length of the cylinder. \( C_D \) denotes the time-averaged drag coefficient and \( C_L \) is the amplitude of lift coefficient. As figure 3 shows, the drag and lift force coefficients decrease as \( Re \) increase.
4. Control effect of Lorentz force
The boundary layer separation and vortical structures may be suppressed by the Lorentz force. The momentum is directly transported into the boundary layer by the Lorentz force without associated mass flux, making the Lorentz force an appealing approach for flow control.

Figure 3. Variations of the drag and lift coefficients.

Figure 4. Vortical structures controlled by Lorentz force (Re=300).

Figure 5. Drag and lift force coefficients (Re=300).
Figure 4 shows the vortical structures of flow past the cylinder at Reynolds number of 300 controlled by Lorentz force. The flow is from left to right and the Lorentz force is applied at nondimensional time $t$ of 100. From the evolving vortical structures, it can be found that the shedding of spanwise primary vortex and the accompanying streamwise vortex are suppressed effectively by the Lorentz force. As time goes on, the vortex shed before the Lorentz force is applied is transported downstream out of the domain.

As figure 5 shows, the force coefficients become quasi-periodic at Reynolds number of 300 for the flow past a circular cylinder. The mean drag coefficient is about 1.396 without control and reduced to 0.67 controlled by the Lorentz force, leading to 28% of the drag reduction. The oscillation of the lift force is completely eliminated.

5. Discussion
Computations have been carried out for three-dimensional flow past a circular cylinder controlled by Lorentz force for Reynolds number varying from 200 to 1000. The spanwise wavelength of mode A and mode B are found to be about 3.3 and 0.83 times cylinder diameter, respectively. The boundary layer separation and vortex shedding is suppressed for the flow controlled by Lorentz force, leading to drag reduction and vibration suppression.

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