The mathematical model of characteristics of the convective unstable atmosphere taking into account microphysical processes in clouds

I E Kuznetsov\textsuperscript{1}, O V Strashko\textsuperscript{1}, V V Dorofeev\textsuperscript{1} and D V Gotsev\textsuperscript{1}

\textsuperscript{1}Federal State Public Military Higher Education Institution Military Educational and Scientific Center of the Air Force N.E. Zhukovsky and Yu.A. Gagarin Air Force Academy (Voronezh), Ministry of Defense of the Russian Federation, Starykh Bolshevikov 54A, Voronezh, Russia, 394064

e-mail: vaiumet@mail.ru, strashko57@yandex.ru, dor1950@yandex.ru, ilja74@mail.ru

Abstract. The new approach to obtaining diagnostic and prognostic information on characteristics of the atmosphere in the conditions of its convective instability based on mathematical model operation of the processes proceeding in conditions a cloud - and sludge formations and influencing dynamics of variability of meteorological fields is offered. Its application ensures increase in effectiveness and safety of functioning of organizational and technical systems.

1. Introduction
It is known that dangerous meteorological phenomena occurs in the environment, which is extremely variable under different conditions. The energy store lability in a boundary layer is one of the important factors influencing mode fluctuations of a convectively unstable atmosphere. As laboratory research shows [1] the interaction of convective and turbulent flow in this layer causes change in range of meteorological magnitude. Turbulence supports the development of the convection (i.e. it induces the growth of temperature gradient in hard layers, and causes spontaneous convection). However while the turbulence grows there is enforcement in exchange with the outer air, weak correlation between vertical velocity and temperature fluctuations, that suppresses heat convection. On the one hand those factors effect greatly on the formation of the meteorological fields, but on the other hand they are difficult to predict.

Nowadays a wide range of mathematical models [2] is used to calculate those factors. Capabilities of mathematical models have been broadening nowadays. That is due to the development of the science of atmosphere and the advance of numerical mathematics as well [3]. However, processes related to the realization of energy store lability in a boundary level and influencing the dynamics of meteorological fields change are not taken into account by existing models to the full extend. There are two classical methods to obtain information about energy store of convectively unstable atmosphere: a parcel method and a slice method. Convection models, on which the methods are based, allow to get the characteristics of the atmosphere: atmospheric instability energy magnitude, vertical distribution of rising currents velocity, to determine the altitude of upper and lower border of a convective cloud. It should be noted that each method can be applied to calculate convection parameters in a period of its maximum development during different convection processes. So, while the slice method is used to assess the convection parameters during the air mass processes, the parcel method permits descriptions
of adiabatic processes in central part of heavy cumulonimbus clouds with no involvement of outer air.

2. Description of the method

Being calculated on the base of adiabatic convection, convectively unstable atmosphere parameters are coherent with their extreme values available from the experience. However they are hardly suitable for forecasting current atmosphere characteristics. Therefore the article provides mathematical model of the atmosphere, based on nonadiabatic approach of convection description. The approach to the fullest degree takes into account thermal, electrical and dynamical condition of outer air involved in the cloud formation.

The purpose of the study is to increase the effectiveness of convective unstable characteristics forecast by developing and applying mathematical model taking into account interconnection between microphysical, electrical and thermodynamic processes.

The proposed composite model is constructed by functional integration of a number of particular models: hydrodynamic process model, microphysics processes model and electrical model processes.

Hydrodynamic block model consists of differential nonlinear equation of motion, describing moist convection, buoyant force with implication coefficient.

Microphysical block describes the process of interaction of particles in the field of force of gravity, carrying them over by air currents, as well as coagulation of cloud particles influenced by an electrical field of a cloud. Electrical block calculates electrical field influence upon the effectiveness of charged drops collision. The degree of given relation is determined by the size of the particles, sign and value of their charges. Besides the drop polarity effect, electrical field causes change in the speed of movement of charged particles, if their size is small enough. The less the relative speed of movement, the stronger the electrical force impetus. That is why electrical fields influence strongly current processes [4]. Interrelations and interdependence of given equations enable to estimate energy store of cloudy atmosphere.

In a formalized aspect the model can be represented by simultaneous equation of movement in terms of the Exner function. The equation is linearized in thermodynamic variables and describes advective and turbulent transport of substances, the force of buoyancy, friction and atmospheric pressure gradients [5]:

\[
\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} = -\bar{\theta}_{bcgr} \cdot \nabla \pi' + \nabla g \left( \frac{\theta'}{\theta} - s' \right) + \left[ \vec{V} \times \vec{f} \right] + \frac{1}{\rho} \Delta \vec{V}
\]  

(1)
equation of continuity for deep convection calculating hydrostatic air compressibility [6]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \sigma w, \quad \sigma = -\frac{d}{dz} [\ln \rho_{bda} (z)] \cong 10^{-4} \text{km}^{-1}, \quad \rho_{bda} (z) = \rho_0 \left( \frac{T_0}{T_0 - \gamma_z} \right)^{(\frac{\gamma_z}{2} - 1)}
\]  

(2)

thermodynamic equations calculating generated heat of water aggregate transition (3-4) [7].

\[
\frac{\partial \theta}{\partial t} + (\vec{V} \cdot \nabla) \theta = \frac{1}{\rho C_p} \left[ L_c M_c + L_s \sum_s M_s + L_f M_f \right] + \Delta' \theta, \quad \theta = T \left( \frac{1000}{P} \right)^{0.288}
\]  

(3)

\[
\frac{\partial s}{\partial t} + (\vec{V} \cdot \nabla) s = -M_c - M_s + \Delta' s
\]  

(4)
microphysics equations (5) [8]:

\[
\frac{dm}{dt} = 4\pi \rho_{air}^2 \frac{dr}{dt}, \quad \frac{dr_w}{d\tau} = D \rho_w \frac{1}{r_w} \frac{M}{P} E_w (f - 1), \quad \frac{dr_i}{d\tau} = D \frac{\rho_{air}}{\rho_i} \frac{1}{r_i} \frac{M}{P} E_w (f - \frac{E_i}{E_w})
\]  

(5)
equation, calculating electrical processes in the air [9]:
\[ E(r^+, r) = \frac{1}{r^+} \left[ \frac{45 \cdot (\varepsilon - 1) \cdot q^2}{16 \rho g \cdot (\varepsilon + 2) \cdot (r^+/r - 1)} \right]^{\frac{2}{3}} \]  

where \( q = K \cdot r^2 \), \( \left( \vec{V} \cdot \nabla \right) = u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \), \( \Delta' = \frac{\partial^2}{\partial x^2} K \frac{\partial}{\partial x} + \frac{\partial^2}{\partial y^2} K \frac{\partial}{\partial y} + \frac{\partial^2}{\partial z^2} K \frac{\partial}{\partial z} \), \( \pi = C_p \left( \frac{K}{p_0} \right) \), \( p_0 \) - arbitrary value of background pressure (for ease of computation \( p_0 = 1000 \) hPa), \( \xi = \frac{R_c}{C_p} \), \( (R_c \text{ and } C_p) \) - (specific gas constant and heating capacity under uniform pressure of dry air), \( \overrightarrow{n_x^2} \) - ort of axis z, \( \overrightarrow{V} = \{u, v, w\} \) - velocity vector, \( u(\vec{r}), v(\vec{r}), w(\vec{r}) \) - components of velocity vector of air currents in a cloud; \( [\vec{V} \times \vec{T}] \) - characteristics of earth angular velocity; \( \theta(\vec{r}) \) - potential temperature; \( \pi(\vec{r}) = C_p \theta(p(x, y, z) / 1000)^R/C_p \) - dimensionless pressure; \( \bar{\theta} \) - average potential temperature; \( s(\vec{r}) \) - specific air humidity; \( Q_s(\vec{r}) \) - the total ratio of the mixture of liquid and solid phases in the cloud; \( \sigma(z) \) - a parameter that takes into account the change in the air density with the altitude; \( P(\vec{r}) \) and \( T(\vec{r}) \) - pressure and temperature accordingly; \( L_e, L_s, L_f \) - specific condensation heat, sublimation and freezing accordingly; \( \pi'(\vec{r}), \theta'(\vec{r}), s'(\vec{r}) \) - departure of dimensionless pressure, potential temperature and specific humidity from their background values in the outer air \( \pi_{bcgr}(\vec{r}), \theta_{bcgr}(\vec{r}) \) and \( s_{bcgr}(\vec{r}) \); \( M_c, M_s \) - changes in the specific humidity due to the steam diffusion on droplets and crystals; \( M_f \) - the mass of the dropping-liquid water freezing per unit time in the unit of air volume; \( K(\vec{r}) \) - turbulent diffusion ratio, \( r \) - the radius of cloud particles; \( r^+ \) - the initial radius of cloud particles; \( q \) - is the charge of the aerosol particles, \( E(r^+, r) \) - coagulation ratio, \( g, \varepsilon \) - physical constants, \( \rho_{da}(z) \) - acoustic perturbation of the background density, \( \rho_0 \) - air density at the Earth’s surface, \( T_0 \) - air temperature at the surface of the Earth, \( \gamma_z \) - vertical temperature gradient, \( g \) - acceleration of gravity.

**3. Calculating**

In this connection initial and boundary assumptions look like:

\[
\begin{align*}
\theta(x, y, z, 0) &= \theta_{t0}(x, y, z), \\
\theta(0, y, z, t) &= \theta_{x0}(y, z, t), \\
\theta(x, 0, z, t) &= \theta_{y0}(x, z, t), \\
\theta(x, y, 0, t) &= \theta_{z0}(x, y, t), \\
s(x, 0, z, t) &= s_{x0}(y, z, t), \\
s(0, y, z, t) &= s_{x0}(y, z, t), \\
s(x, 0, z, t) &= s_{y0}(x, z, t), \\
s(x, y, 0, t) &= s_{z0}(x, y, t), \\
\varphi(0, y, z, t) &= \varphi_{x0}(y, z, t), \\
\varphi(x, 0, z, t) &= \varphi_{y0}(x, z, t), \\
\varphi(x, y, 0, t) &= \varphi_{z0}(x, y, t), \\
\varphi(L_x, y, z, t) &= \varphi_{xL}(y, z, t), \\
\varphi(x, L_y, z, t) &= \varphi_{yL}(x, z, t), \\
\varphi(x, y, L_z, t) &= \varphi_{zL}(x, y, t),
\end{align*}
\]

where \( \varphi = \{u, v, w\} \).

Consequently the atmospheric conditions will be described as a sum of atmospheric processes, formalized in equations (1) – (6), corresponding a priory information will permit to get prognostic properties of atmospheric parameters.

The Marchuk splitting method [10] and the finite-difference scheme for solution of differential equation were used when implementing mathematical model of convectively unstable atmosphere. Differential equation system was solved in three phases in each time sample. During the first phase in a time area \( (t; t + \frac{\Delta t}{5}) \) the following equations were solved:
The interplay between humidity and temperature on ice and water particles formed in the atmosphere is taken into account.

That is described by the following equations:

\[
\frac{\partial u}{\partial t} + C_x^u \left( u - \frac{\partial K}{\partial x} \right) \frac{\partial u}{\partial x} + C_y^u \left( v - \frac{\partial K}{\partial y} \right) \frac{\partial u}{\partial y} + C_z^u \left( w - \frac{\partial K}{\partial z} \right) \frac{\partial u}{\partial z} = 0,
\]

\[
\frac{\partial v}{\partial t} + C_x^v \left( u - \frac{\partial K}{\partial x} \right) \frac{\partial v}{\partial x} + C_y^v \left( v - \frac{\partial K}{\partial y} \right) \frac{\partial v}{\partial y} + C_z^v \left( w - \frac{\partial K}{\partial z} \right) \frac{\partial v}{\partial z} = 0,
\]

\[
\frac{\partial w}{\partial t} + C_x^w \left( u - \frac{\partial K}{\partial x} \right) \frac{\partial w}{\partial x} + C_y^w \left( v - \frac{\partial K}{\partial y} \right) \frac{\partial w}{\partial y} + C_z^w \left( w - \frac{\partial K}{\partial z} \right) \frac{\partial w}{\partial z} = 0,
\]

\[
\frac{\partial T}{\partial t} + C_T \left( u - \frac{\partial K}{\partial x} \right) \frac{\partial T}{\partial x} + C_T \left( v - \frac{\partial K}{\partial y} \right) \frac{\partial T}{\partial y} + C_T \left( w - \frac{\partial K}{\partial z} \right) \frac{\partial T}{\partial z} = 0,
\]

\[
\frac{\partial s}{\partial t} + C_s \left( u - \frac{\partial K}{\partial x} \right) \frac{\partial s}{\partial x} + C_s \left( v - \frac{\partial K}{\partial y} \right) \frac{\partial s}{\partial y} + C_s \left( w - \frac{\partial K}{\partial z} \right) \frac{\partial s}{\partial z} = 0,
\]

\[
\frac{\partial f_1}{\partial t} + C_f \left( u - \frac{\partial K}{\partial x} \right) \frac{\partial f_1}{\partial x} + C_f \left( v - \frac{\partial K}{\partial y} \right) \frac{\partial f_1}{\partial y} + C_f \left( w - \frac{\partial K}{\partial z} \right) \frac{\partial f_1}{\partial z} = 0,
\]

\[
\frac{\partial f_2}{\partial t} + C_f \left( u - \frac{\partial K}{\partial x} \right) \frac{\partial f_2}{\partial x} + C_f \left( v - \frac{\partial K}{\partial y} \right) \frac{\partial f_2}{\partial y} + C_f \left( w - \frac{\partial K}{\partial z} \right) \frac{\partial f_2}{\partial z} = 0,
\]

where \( C_{x,y,z}^{u,v,w} f_1, f_2 \) - selected empirical constants taking into account dimensions of calculated characteristics under mass transfer.

During the second phase in a temporary area \( (t + \frac{2}{5} \Delta t; t + \frac{3}{5} \Delta t) \) interplay between humidity and temperature on ice and water particles formed in the atmosphere is taken into account.

That is described by the following equations:

\[
\phi_{i,j,k}^{t+2/5} = h_t \left[ C_x^\phi \left( K_{i+1,j,k}^t - K_{i,j,k}^t \right) \phi_{i+1,j,k}^{t} - \phi_{i,j,k}^{t} \right] h_x +
C_y^\phi \left( K_{i+1,j,k}^t - K_{i,j,k}^t \right) \phi_{i+1,j,k}^{t} - \phi_{i,j,k}^{t} \] h_y +
C_z^\phi \left( K_{i,j+1,k}^t - K_{i,j,k}^t \right) \phi_{i,j+1,k}^{t} - \phi_{i,j,k}^{t} \] h_z \right]
\]

\[
\frac{\partial r_w}{\partial t} = D \frac{r_{air}}{r_{ow}} \mu E_w (f - 1) \frac{\partial r_i}{\partial t} = D \frac{r_{air}}{r_{ow}} \mu E_w (f - \frac{E_i}{E_w}) , f = \frac{sp}{0.622 E_w} ,
\]

\[
\frac{\partial M_k}{\partial t} = 4 \pi r_w^2 \frac{\partial r_w}{\partial t} = 4 \pi r_i^2 \frac{\partial r_i}{\partial t} ,
\]

\[
\frac{\partial s_k}{\partial t} = \frac{\partial M_k}{\partial t} \frac{R_c T}{pV_c + 0.608 R_c \frac{\partial M_c}{\partial t}} , \frac{\partial s_c}{\partial t} = \frac{\partial M_c}{\partial t} ,
\]

\[
\frac{\partial T}{\partial t} = -L_k \frac{\partial s_k}{\partial t} - L_c \frac{\partial s_c}{\partial t} , - \frac{\partial s_k}{\partial t} - \frac{\partial s_c}{\partial t} ,
\]

\[
\frac{\partial f_1}{\partial t} = -C_f \frac{\partial M_k}{\partial t} , \frac{\partial f_2}{\partial t} = -C_f \frac{\partial M_c}{\partial t} ,
\]

where \( C_{f_1, f_2} \) - scaling factor for recounts of ice and water particles rate of change of mass in probability \( f_1, f_2 \); \( V_c \) - elementary infinitesimal volume of space, where specific humidity \( s \) is recounted.
During the third phase \((t + \frac{3}{5} \Delta t; t + \Delta t)\) a temporary area air velocity is recounted:

\[
\frac{\partial u}{\partial t} = l_v, \quad \frac{\partial v}{\partial t} = l_u, \quad \frac{\partial w}{\partial z} = g \left( \frac{T - T_{bcgr}}{T_{bcgr}} + 0.61 (s - s_{bcgr}) - Q_s \right)
\]

(15)

According to the calculations values of water content \(W\) and ice are calculated:

\[
W = \int_0^\infty m_{f1} dm, \quad I = \int_0^\infty m_{f2} dm
\]

(16)

To solve the discussed above equations we have used the finite-difference scheme which simplifies algorithm of characteristics calculating:

\[
\frac{\partial \phi}{\partial t} = \frac{\phi_{x,y,z}^{t+\Delta t} - \phi_{x,y,z}^t}{\Delta t}, \quad \frac{\partial \phi}{\partial x} = \frac{\phi_{x+\Delta x,y,z}^t - \phi_{x,y,z}^t}{\Delta x}, \quad \frac{\partial \phi}{\partial y} = \frac{\phi_{x,y,z+\Delta y}^{t+\Delta t} - \phi_{x,y,z}^t}{\Delta y},
\]

(17)

\[
\frac{\partial^2 \phi}{\partial x^2} = \frac{\phi_{x+2\Delta x,y,z}^t - 2\phi_{x+\Delta x,y,z}^t + \phi_{x,y,z}^t}{\Delta x^2}, \quad \frac{\partial^2 \phi}{\partial y^2} = \frac{\phi_{x,y,z+2\Delta y}^{t+\Delta t} - 2\phi_{x,y,z+\Delta y}^t + \phi_{x,y,z}^t}{\Delta y^2},
\]

(18)

\[
\frac{\partial^2 \phi}{\partial x \partial y} = \frac{\phi_{x+\Delta x,y+\Delta y,z}^t - \phi_{x,y+\Delta y,z}^t - \phi_{x+\Delta x,y,z}^t + \phi_{x,y,z}^t}{\Delta x \Delta y}, \quad \frac{\partial^2 \phi}{\partial x \partial z} = \frac{\phi_{x+\Delta x,y,z+\Delta z}^t - \phi_{x,y,z+\Delta z}^t - \phi_{x+\Delta x,y,z}^t + \phi_{x,y,z}^t}{\Delta x \Delta z},
\]

(19)

\[
\frac{\partial F}{\partial m} = \frac{F_{x,y,z}^{t,m+\Delta m} - F_{x,y,z}^{t,m}}{\Delta m},
\]

(20)

where \(\phi = \{u, v, w, s, T, f_1, f_2\}, \quad F = \{f_1, f_2\}\). Based upon described schemes we have arrived at the following solution for the first stage of splitting:

\[
u_i^{t+2/5} = 2h_t \left[ C_x \left( \frac{K_{i+1,j,k}^t - K_{i,j,k}^t}{h_x} - u_{i,j,k}^t \right) \frac{\phi_{i+1,j,k}^t - \phi_{i,j,k}^t}{h_x} + C_y \left( \frac{K_{i,j+1,k}^t - K_{i,j,k}^t}{h_y} - u_{i,j,k}^t \right) \frac{\phi_{i,j+1,k}^t - \phi_{i,j,k}^t}{h_y} + C_z \left( \frac{K_{i,j,k+1}^t - K_{i,j,k}^t}{h_z} - u_{i,j,k}^t \right) \frac{\phi_{i,j,k+1}^t - \phi_{i,j,k}^t}{h_z} \right] + \phi_{i,j,k}^t
\]

(21)

where \(\phi = \{u, v, w, s, T, f_1, f_2\}\).

There is the following finite-difference scheme for the second stage of splitting

\[
r_{w_{i,j,k}}^{t+1} = h_t D \frac{r_{ow}^t}{r_{w_{i,j,k}}^t} \frac{\mu E_w}{pM} (f_{i,j,k}^t - 1) + r_{w_{i,j,k}}^t
\]

(22)
The algorithm of calculating water and ice content is represented by the formulas:

\[ r_{t+1}^{i,j,k} = \left(1 - \alpha \right) r_{t}^{i,j,k} + \alpha \left( \frac{E^t_{i,j,k}}{E_w} \right) r_{w_{i,j,k}}^{t} + \alpha \left( \frac{E^t_{i,j,k}}{E_w} \right) r_{l_{i,j,k}}^{t} \]

\[ f_{i,j,k}^{t} = \frac{s_{i,j,k}^{t}}{0.622 E_w} \]

\[ M_{k_{i,j,k}}^{t+1} = 4 h_t \pi \left( r_{w_{i,j,k}}^{t} \right)^2 \frac{r_{w_{i,j,k}}^{t+1} - r_{w_{i,j,k}}^{t}}{h_t} + M_{k_{i,j,k}}^{t} \]

\[ M_{c_{i,j,k}}^{t+1} = 4 h_t \pi \left( r_{i,j,k}^{t} \right)^2 \frac{r_{i,j,k}^{t+1} - r_{i,j,k}^{t}}{h_t} + M_{c_{i,j,k}}^{t} \]

\[ s_{k_{i,j,k}}^{t+1} = \frac{3 h_t}{5} \frac{M_{k_{i,j,k}}^{t+1} - M_{k_{i,j,k}}^{t}}{h_t} + R_c T_{i,j,k}^{t+1} + \frac{pV_{el} - 0.608 R_c}{M_{k_{i,j,k}}^{t+1} - M_{k_{i,j,k}}^{t}} + s_{k_{i,j,k}}^{t+2/5} \]

\[ s_{c_{i,j,k}}^{t+1} = \frac{3 h_t}{5} \frac{M_{c_{i,j,k}}^{t+1} - M_{c_{i,j,k}}^{t}}{h_t} + R_c T_{i,j,k}^{t+1} + \frac{pV_{el} - 0.608 R_c}{M_{k_{i,j,k}}^{t+1} - M_{k_{i,j,k}}^{t}} + s_{c_{i,j,k}}^{t+2/5} \]

\[ T_{i,j,k}^{t+2/5} = \left( \frac{L_e s_{k_{i,j,k}}^{t+1} - s_{k_{i,j,k}}^{t}}{C_p} + \frac{L_c s_{c_{i,j,k}}^{t+1} - s_{c_{i,j,k}}^{t}}{C_p} \right) \frac{3 h_t}{5} + s_{i,j,k}^{t+2/5} \]

\[ s_{i,j,k}^{t+1} = \left( \frac{s_{k_{i,j,k}}^{t+1} - s_{k_{i,j,k}}^{t}}{h_t} \right) - \left( \frac{s_{c_{i,j,k}}^{t+1} - s_{c_{i,j,k}}^{t}}{h_t} \right) \frac{3 h_t}{5} + s_{i,j,k}^{t+2/5} \]

\[ f_{l_{i,j,k}}^{t+1/2} = - \frac{3 h_t}{5} C_{f_1} \frac{M_{k_{i,j,k}}^{t+1} - M_{k_{i,j,k}}^{t}}{h_t} + f_{l_{i,j,k}}^{t+1/2,5} \]

\[ f_{l_{i,j,k}}^{t+1/2,5} = - \frac{3 h_t}{5} C_{f_2} \frac{M_{c_{i,j,k}}^{t+1} - M_{c_{i,j,k}}^{t}}{h_t} + f_{l_{i,j,k}}^{t+2/5} \]

There is the following finite-difference scheme for the third stage of splitting:

\[ u_{i,j,k}^{t+1} = \frac{3 h_t}{5} v_{i,j,k}^{t+2/5} + u_{i,j,k}^{t+2/5} \]

\[ v_{i,j,k}^{t+1} = \frac{3 h_t}{5} u_{i,j,k}^{t+2/5} + v_{i,j,k}^{t+2/5} \]

\[ w_{i,j,k}^{t+1} = \frac{3 h_t}{5} g \left( T_{i,j,k}^{t} - T_{beg}^{i,j,k} \right) + 0.61 \left( s_{i,j,k}^{t} - s_{beg}^{i,j,k} \right) - Q_s + w_{i,j,k}^{t+2/5} \]

The algorithm of calculating water and ice content is represented by the formulas:

\[ W_{i,j,k} = \sum_{i=0}^{\infty} i m h_{m} f_{i,j,k}^{t+1} h_{m} \]

\[ I_{i,j,k} = \sum_{i=0}^{\infty} i m h_{m} f_{i,j,k}^{t+1} h_{m} \]

Complexity of evaluation by formulas available has made us use special software, such as Matrix Laboratory environment, which relieved us from a number of computational problems, reduced the calculation error, allowed to visualize the resulting data.
To assess the working efficiency of the model, velocity vector projection of the airflow in Cartesian system was calculated. Here the initial conditions were varying in the following way:

\[
\begin{align*}
    u_1(\vec{r}) &= 10 \text{ (ms}^{-1}) , v_1(\vec{r}) = 0 \text{ (ms}^{-1}) , w_1(\vec{r}) = 0 \text{ (ms}^{-1}) , \\
    u_2(\vec{r}) &= 0 \text{ (ms}^{-1}) , v_2(\vec{r}) = 10 \text{ (ms}^{-1}) , w_2(\vec{r}) = 0 \text{ (ms}^{-1}) , \\
    u_3(\vec{r}) &= 10 \text{ (ms}^{-1}) , v_3(\vec{r}) = 0 \text{ (ms}^{-1}) , w_3(\vec{r}) = 0 \text{ (ms}^{-1}).
\end{align*}
\]

The obtained field of vertical fluctuations of the airflow is illustrated on figure 1 and figure 2 under still-air conditions.

We can conclude from the analysis of the simulation data that the most considerable fluctuations of airflow velocity occur under the genesis of atmospheric convection in the lower quarter of the cloud. Those areas form in a mesoscale way and have a great impact on meteo quantity in a close to a cloud space. Positive fluctuations occur in the central part of a cloud. In the upper part of a cloud positive quantity fluctuations of airflow velocity take less space than in its lower parts. The obtained parameters of airflows closely match experimental observations, carried out in FGBU "TZAO" under conditions of convective unstable atmosphere [11].

**Figure 1.** Velocity field of airflow at the moment of convection genesis.

**Figure 2.** Velocity field of airflow in 15 minutes after the convection genesis.

**Figure 3.** Velocity field of airflows at the altitude of 1, 2, 3 km under different atmosphere lability energy value (E) if E= 0-1 kJ/kg.

**Figure 4.** Velocity field of airflows at the altitude of 1, 2, 3 km under different atmosphere lability energy value (E) if E= 0-1 kJ/kg.
Further research was carried out in view of different atmosphere lability energy. The obtained results are illustrated on figure 3 and figure 4.

**Figure 5.** Velocity field of an airflow at the moment of cloud genesis under still-air conditions.

**Figure 6.** Velocity field of an airflow at the moment of cloud genesis under disturbance of air environment.

4. Conclusion

The results presented in the work are the following.

1. To obtain diagnostic and prognostic information about atmosphere conditions at the point of time \( t \) and the eventual points of time, it is necessary to know only initial and boundary conditions of the investigated field given at the point of time \( t \) and presented in equations (7). It should be noted that if we use meteorological conditions data as initial and boundary conditions, received via remote sensing instruments, we can increase the efficiency of the obtained results.

2. The conducted numerical model experiment allowed us to obtain effective space and time steps (on \( x \) and \( y \) axis - 500m, vertical spacing – 250 m, time step – 3 sec), under which a satisfactory reproducibility of calculations with full-scale measurement data of the convectively unstable atmosphere was achieved.

3. Model analysis revealed that when the convective clouds form, cyclonic motion of air about a cloud axis occurs. With the increase of energy of instability of the atmosphere the speed of cyclonic circulation increases. However the surrounding atmosphere layers are involved in the motion of air either.

4. The assessment of effectiveness of the discussed model was carried out by means of comparing the model results with conventional ways of obtaining information about convectively unstable atmosphere conditions (radio location and radiosonde) on the territory of the Krasnodar region during summer. Having carried out the investigation we can conclude that the represented model efficiently calculates atmosphere characteristics under convective instability. Average measurements of speed and wind direction for instance differ from the measured ones by 10-15%.

Thus, the developed multitemporal model allows us to obtain diagnostic and prognostic information about atmosphere conditions under convective instability with high space-time resolution and quality, meeting the requirements of the consumers of hydro meteorological information.

5. References

[1] Kornienko E 1982 *Trudy’UkrNII* 187 3-25 (in Russian)

[2] Semenov M, Nesterov V, Kuznetsov I, Solovyov A and Meleshenko P 2016 *Matec Web of Conference* 83 7004
[3] Matveev L T 2000 *Fizika atmosfery*’ (Sankt-Peterburg: Gidrometeoizdat) p. 779 (in Russian)
[4] Nguyen Hang T T, Meleshenko P A, Semenov M E, Kuznetsov I E and Gorlov V A 2016 Klinskikh fresh look at Lorenz-like system Progress *Electromagnetic Research Symp.* (Shanghai) p 2255-2259 DOI: 10.1109/PIERS.2016.7734922
[5] Potashnik E L and Kuznetsov A D 2010 *Modelirovanie oblakov* (Sankt-Peterburg: Rossijskij gosudarstvennij gidrometeorologicheskij universitet) p 444 (in Russian)
[6] Shapovalov VA, Prodan KA and Mashukov I 2011 *Doklady Vserossijskoj konferencii po fizike oblakov i aktivnym vozdeystviym na gidrometeorologicheskie processy*’ (Nal’chik) p 100 (in Russian)
[7] Kogan E L, Mazin I P, Sergeev B N and Hvorost’yanov V I 1984 *Chislennoe modelirovanie oblakov* (Moscow: Gidrometeoizdat) p 186 (in Russian)
[8] Rodzhers R R 1979 *Kratkij kurs fiziki oblakov* (Leningrad: Gidrometeoizdat) p 231 (in Russian)
[9] Kuznetsov I E, Semenov M E, Kanishcheva O I and Meleshenko P A 2016 On the interaction of electromagnetic waves with charged aerosol particles in atmosphere Progress in Electromagnetic Research Symp. (Shanghai) pp 3542-45 DOI: 10.1109/PIERS.2016.7735357
[10] Marchuk G I 1977 *Metody’ vy’chislitel’noj matematiki* (Moscow: Nauka) p 352 (in Russian)
[11] Sitnikov N M, Borisov Y A, Akmulin D V, Chekulaev I I, Efremov D I, Sitnikova V I, Ulanovsky A E and Popovicheva O B 2014 *International conf. COSPAR* (Moscow) 1 12-20