Possible violation of the optical theorem in LHC experiments

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Abstract

The optical theorem (OT), allowing the determination of the total cross section for a hadron–hadron scattering from the imaginary part of the forward elastic scattering amplitude, is believed to be an unavoidable consequence of the conservation of probability and of the unitary S matrix. This is a fundamental theorem which contains an imaginary part of the forward elastic scattering amplitude that is not directly measurable. The impossibility of scattering phenomena without the elastic channel is considered to be a part of the quantum magic. However, if one takes seriously the idea that the hadrons are extended particles, one may define a unitary S matrix such that one cannot prove the OT. Moreover, data violating the OT do exist, but they are not conclusive due to the uncertainties related to the extrapolation of the differential elastic cross-section to the forward direction. These results were published several years ago, but they were forgotten. In this paper we will recall these results in an understandable way, and we will give the additional arguments why the OT can be violated in high energy strong interaction scattering and why it should be tested and not simply used as a tool in LHC experiments.

Keywords: violation of the optical theorem, unitary S matrix, high energy hadron scattering, Large Hadron Collider data, total and elastic cross sections

1. Introduction

Using classical mechanics (CM), Rutherford described with success a scattering of alpha particles on a thin foil of gold as a scattering of point-like charged particles by point-like positively charged particles in the target. This description gave a clear indication that each atom contains a positively charged and heavy nucleus attracting negatively charged point-like electrons. There are no point-like physical objects in Nature, but the point-like approximation (PLA)—used often in CM with success to describe, for example, the motion of planets around the Sun—seemed to be justified, because diameters of nuclei were of the order of $10^{-15}$ m and diameters of atoms of the order of $10^{-10}$ m.

In quantum mechanics (QM), elastic scattering phenomena are described in a center of mass frame (CMS) as a scattering of some complex valued probability wave on a scattering center. For Coulomb elastic scattering, one obtains the same formula as the formula found by Rutherford.

In QM, for a scattering by short range potentials, one obtains a startling relation called the optical theorem (OT) between the imaginary part of the elastic forward scattering amplitude and the total elastic cross section.

The proof of the OT can be generalized to cover various non-elastic scattering phenomena in atomic and molecular physics [1–3]. The OT is also proven in the relativistic S matrix theory [4, 5]. The conservation of probability implies the unitarity of the S matrix. Since the OT is proven using explicitly the unitarity equation, it is considered to be a fundamental law and is used as an important tool in various theoretical models and in the analysis of the experimental data. In particular, it is used as a constraint in the maximum likelihood fits to elastic differential cross-section data.

Many years ago while trying to understand why, using the S matrix, one could not have high energy scattering without an elastic channel, we succeeded in proving [6–9] that it is possible to construct a unitary S matrix without the OT. Simply stated, instead of defining a scattering operator $T$ using the decomposition $S = I + iT$, we defined a unitary scattering operator $\tilde{S}$ by a formula $I \oplus \tilde{S}$, where $\tilde{S}$ was acting on only two particle initial state vectors with impact parameters smaller than the effective range of strong interactions.
Our definition was consistent with an intuitive picture of colliding extended hadrons which, in order to interact strongly, must be close to each other.

The decomposition $S = I + i T$ is analogous to wave phenomena. However, following Bohr, we can say that we are dealing in hadron-hadron scattering instead, with particle-like phenomena in which particle beams are prepared using the laws of relativistic CM and classical electrodynamics. These laws fail to describe what happens during a strong interaction scattering, and we must use some other theoretical models in order to make predictions about the type of particles produced and about the branching ratios of different reaction channels.

It is obvious that hadrons cannot be treated as point-like particles when we are studying a deep inelastic electron–hadron scattering or a strong hadron–hadron scattering. One can expect that a completely new physics is needed to explain these phenomena. This is why, in order to explain multiple hadron production observed in ultra-high energy cosmic rays (UHECR) data, several quite successful statistical and thermodynamical models of extended hadrons were proposed in the past by Fermi [10], Hagedorn [11], Frautschi [12], and several other researchers.

An extensive review of these models was given by Feinberg [13]. Various improved variants of these models continue to be used [14]. Other extended hadron models explaining the confinement of quarks in hadrons have been constructed: the so-called MIT bag model [15], the chiral bag model [16], and their modifications.

In spite of the fact that the use of PLA leads to infinite self-energies in classical electrodynamics and to infinities in quantum field theory (QFT) requiring renormalization, PLA is still maintained for parton–parton (pp) interactions in the so-called standard model (SM), which is successfully used in elementary particle physics.

In the SM the confinement of quarks is taken for granted, and one uses the parton phenomenology together with the renormalizable quantum chromodynamics (QCD) theory [17–19]. A hadron is described as composed of a number of point-like constituents called partons: colored quarks and gluons. These point-like ‘free constituents’ from two colliding hadrons interact instantaneously and incoherently, producing in general several quark–antiquark pairs and gluons which recombine in the process of hadronization to form the final particles. The parton interactions are described by QCD and experimentally determined, generalized parton distribution (GPD) functions [17], which are available worldwide.

The comparison of the SM with experimental data is a difficult task requiring many free parameters, various phenomenological inputs, and a Monte Carlo simulation of events [18]. Moreover, perturbative calculations of parton–parton interactions break down for small momentum transfers essential for the study of the elastic scattering [19].

Therefore to study elastic cross sections, single and multichannel eikonal models and/or Regge phenomenology are used with mixed success [20–22]. In all these models and in the data analysis, the OT is used as an important constraint. All recent reported values for the total cross sections [23–25] have been obtained using the OT.

One can only conclude that physicists using the SM, both theoreticians and experimentalists, are unaware of the fact that the violation of the OT can be made consistent with the unitary $S$ matrix. For this reason we review in some detail and in a different way our forgotten results [6–9].

After proving that the OT could be violated, we inspected various elastic scattering data and found the confirmation of our doubts [26]. However, the only conclusion we could arrive at was that a reliable direct test of the OT is impossible, since for the same set of data one could get very good fits to the differential elastic cross sections: one drastically violating the OT and another consistent with it [8, 26].

Therefore we decided to search for other implications of our model, in which initial two-particle states were mixed quantum states with respect to some distance sensitive quantum number, such as the impact parameter. Pure and mixed statistical ensembles have different properties [27]. Any sub-ensemble of a pure ensemble has the same properties as the initial ensemble. For a mixed ensemble one can find sub-ensembles with different properties. To be able to detect such differences, one should search for some fine structure in the experimental data [27] using non-parametric statistical compatibility tests which we called purity tests [28–31].

However, if one reanalyzes Tevatron, LHC and UHECR data without using the OT, one probably will find more indications that the OT may be violated. There are several problems with the description of the elastic pp and other scattering data [19, 21, 22], which we will discuss in a subsequent, more technical paper. If a model using some theoretical assumption (such as the OT) and several free parameters compares reasonably well with the experimental data [21], it does not prove that the particular theoretical assumption used is correct [6, 8, 27].

One can expect that with growing total collision energies presently available at LHC, inelastic scattering channels will be favored progressively, and the elastic scattering due to strong interactions will gradually be suppressed, violating various bounds deduced using the OT.

If hadrons are really extended particles, they must collide to interact strongly; then all allowed inelastic channels are open. If they are far enough apart and miss each other, there is no strong interaction. The elastic channel still could be open due to the Coulomb scattering for larger values of impact parameters but not due to strong interactions [6–9].

If confirmed by the experimental data, this idea would be a major discovery showing the importance of a constant reconsideration of quantum foundations and the need for a critical analysis of well-established models and theorems. Such reconsideration allows elucidating quantum paradoxes and finding limitations and generalizations of the present formalism. It can also lead to new experimental discoveries and new applications.

This QTAP conference is a part of a series of Växjö conferences on the foundations of quantum mechanics, gathering physicists, mathematicians, and philosophers to present many important ideas and results [35–37].

Since many participants in this conference are not experts in the domain of particle physics, we will keep our
explanations simple and will start with a pedagogical intro-
duction to the notion of a scattering cross section in CM and
QM. We will also recall the proofs of the OT in QM and in
the relativistic S matrix theory. The plan of this paper is the
following:

1. Scattering cross sections in CM and in QM.
2. Optical theorem in CM.
3. Optical theorem in relativistic S=matrix approach.
4. Unitary S matrix without the optical theorem.
5. Data violating the optical theorem.
6. Conclusions.

2. Total and differential cross-sections

The total cross section \( \sigma \) represents the effective interaction
area of one beam and one target particle perpendicular to the
beam of incoming particles. It depends in general on particles
involved, the energy of the beam, etc. The immobile target can
be replaced by another beam as in the intersecting storage ring
(ISR) or in LHC experiments. For elastic scattering of two
hard spheres with radiuses \( r \) and \( R \):

\[ \sigma = \pi (R + r)^2 \]

If the interaction area \( \sigma \) is very small, and if \( 1 \text{ cm}^2 \) of a
thin target contains \( N \) target particles, then the probability \( p \)
that one beam particle will be scattered by the target can be
estimated as \( p = N \sigma / \text{cm}^2 \). Since \( p = (I_0 - I_m)/I_0 \) where \( I_0 \) is a
flux of incoming particles and \( I_m \) is a flux of non-scattered
particles, therefore

\[ \sigma = \frac{N}{N_0} \]

(1)

where \( N_0 = (I_0 - I_m) \times 1 \text{ cm}^2 \) is a number of beam particles
scattered by \( 1 \text{ cm}^2 \) of the target per unit of time. Total cross sections
for hadron-hadron scattering are very small; therefore they
are measured in barns (b), mb, \( \mu b \), etc. where \( 1 \text{ b} = 10^{-28} \text{ m}^2 \).

For long-range interactions such as those described by a
Coulomb potential, total cross sections are infinite, and only
the so-called differential cross section \( d\sigma = \sigma(\theta, \phi) \) has a physical
meaning, which is the area such that a beam particle passing
by this area is scattered into a solid angle \( d\Omega \): \( \sin \theta d\theta d\phi \):

\[ d\sigma(\theta, \phi) = \frac{N_d(\Omega)}{N_0} \]

(2)

where \( N_0 \) is the number of scattered beam particles in \( d\Omega \)
per unit time. Integrating \( d\sigma(\theta, \phi) \) over all angles, one obtains
the total cross section \( \sigma \) if it is finite.

In CM a scattering of one beam and one target particle is
described in the CMS as a scattering of some fictitious point-
lke particle with reduced mass \( m \) on some immobile scat-
tering center described by a potential \( U(r) \). If the interaction
has a cylindrical symmetry, then \( d\Omega = 2\pi r \sin \theta \; dr \; d\theta \), and one can
find that \( \sigma(\theta) = \sigma(\theta) = 2\pi b \; db \) where \( b = b \theta \) is the impact parameter of a beam particle. Using this functional relation
for \( U(r) = \alpha/r \), Rutherford obtained his famous formula:

\[ d\sigma = \left( \frac{\alpha}{m v^2} \right)^2 \frac{d\Omega}{\sin^2 \left( \frac{\theta}{2} \right)} \]

(3)

and explained with success the scattering of a beam of
alpha particles on a foil of gold.

3. Scattering in CM and the optical theorem

To describe the elastic scattering in QM, one solves a reduced
relative motion wave equation in the CMS [1–3]:

\[ \frac{\hbar^2}{2\mu} \Delta u - Vu = Eu \]

(4)

with the asymptotic boundary condition for \( r \to \infty \):

\[ u \approx A \left( e^{i\theta} + f(\theta) \frac{\Delta r}{r} \right) \]

(5)

The differential cross section is calculated using the ratio
of the outgoing and incoming probability density currents
according to the formula:

\[ \sigma(\theta, \phi) = \sigma(\theta) = \frac{J_r^2}{J_i} = \left| f(\theta) \right|^2 \]

(6)

where \( J = Re (\bar{\Psi}_n \frac{n}{\mu_1} \Psi \Psi) \).

Using the partial wave expansion of \( u \), one obtains:

\[ f(\theta) = (k)^{-1} \sum_{l=0}^{\infty} (2l + 1) \sin \delta e^{i\phi} P(\cos \theta) \]

(7)

and the total cross section:

\[ \sigma = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l + 1) \sin^2 \delta_l \]

(8)

By comparing (7) and (8), one obtains immediately the
OT for the scattering of probability waves by short-range
potentials:

\[ \sigma = \frac{4\pi}{k^2} \text{Im} f(0) \]

(9)

As we mentioned in the introduction, the OT is also
proven for the scattering with various non-elastic channels
opened. In a high energy relativistic domain, to describe the
scattering of elementary particles one must abandon methods
of nonrelativistic QM and use a relativistic S matrix [3–5].

4. Optical theorem in relativistic S matrix theory

When two hadrons collide, various outcomes are possible and
are called channels of reaction: \( 1 + 2 \to 1 + 2 \),
\( 1 + 2 \to 1 + 3 + \ldots + N \) or \( 1 + 2 \to 3 + 4 + \ldots + N \). It is clearly
more complicated than phenomena described by the scattering
of probability waves on some potential.
In QFT and in the relativistic $S$ matrix approach, initial two-particle states are represented by vectors $|i\rangle$ in a Fock space, and final states are represented by vectors $|f\rangle$. The probability $P_g$ for obtaining a particular final state $|f\rangle$ from the initial state $|i\rangle$ is:

$$P_g = \left| \langle f|S|i \rangle \right|^2$$  \hspace{1cm} (10)

where $S$, called $S$ matrix, is a unitary operator.

The OT is obtained by using a unitarity condition $S$ $S' = S^* S = I$ and a decomposition:

$$S = I + iT$$  \hspace{1cm} (11)

where $T$ is called a scattering operator.

From the unitarity condition $S' S = I$ using (11), one obtains $(I+iT)^*(I+iT) = I$, and subsequently

$$-i < a \left| (T - T^*) \right| a > = < a \left| T^* T \right| a >$$  \hspace{1cm} (12)

where $|a\rangle$ is some initial two-particle state vector.

Finally, by placing a spectral decomposition of the identity operator between $T^*$ and $T$ we obtain:

$$2 \text{Im} \langle a|T|a \rangle = \sum \langle a|T^*|\rho \rangle \langle b|T|a \rangle$$

$$= \sum \langle \rho|T|a \rangle^2$$  \hspace{1cm} (13)

By interpreting $f_{ab} = \langle b|T|a \rangle$ as a probability amplitude for obtaining a final state $|b\rangle$ after the interaction of the particles in the state $|a\rangle$, one recovers the OT:

$$\text{Im} f_{ab} = C(s) \sigma$$  \hspace{1cm} (14)

where $C(s)$ is some function of CMS energy of initial particles and $f_{ab}$ is a forward elastic scattering amplitude. For a binary reaction $1 + 2 = 3 + 4$, the amplitude $f_{ab}$ depends on two kinematical relativistic variables $s = (p_1 + p_2)^2 = (p_3 + p_4)^2$ and $t = (p_3 - p_1)^2$ where $p_i$ are four-momenta of the particles. For the elastic scattering in the CMS, one finds $t = -2p^2(1 - \cos \theta)$ where $p$ is the modulus of the linear momentum of the first particle and $\theta$ is its scattering angle. The forward elastic scattering $t$ and $\theta$ are equal to 0.

### 5. Unitary $S$ matrix without the optical theorem

Various particle beams prepared in accelerators are manipulated using classical relativistic mechanics and electrodynamics and projected on some targets. To describe the motion of free hadrons, their internal degrees of freedom such as a quark structure are not important, and a point-like approximation is completely justified when they are far apart.

We may therefore describe hadron–hadron scattering as a two-step process:

1. Using the information about prepared beam profiles and properties of the target, we find the probabilities that two hadrons collide with a particular impact parameter $b$.

2. For $b$ smaller than the effective range of strong interactions, we find probabilities for observing any particular final state $f$. These probabilities depend on $f$, on quantum numbers $\mu$ describing all relevant external and internal degrees of freedom of the initial colliding pairs, and of course on a detailed model for strong interactions.

Let us construct a mathematical model based on a unitary $S$ matrix having these properties [6–9]. First of all we split the quantum numbers $\mu$ into two sets such that there is a strong interaction only if $\mu \in A$ and $|\mu\rangle \in H_2$. This splitting implies the splitting of the Hilbert space of all states into a direct sum of three Hilbert spaces:

$$H = H_1 \oplus H_2 \oplus H_3$$  \hspace{1cm} (15)

where, $H_1 = \{ |\mu\rangle, \mu \notin A \}$, $H_2 = \{ |\mu\rangle, \mu \in A \}$ and $H_3$ contains all other possible final states. Since pairs of particles are prepared with different impact parameters, the initial state for the scattering is described by a density operator:

$$\rho_i = \sum \rho(\mu)|\mu\rangle \langle \mu|$$  \hspace{1cm} (16)

The sum in (15) and in equations which follow can be replaced by an integral over values of the impact parameter. According to our model, a unitary $S$ matrix can be written now in a form:

$$S = I \oplus \tilde{S}$$  \hspace{1cm} (17)

where $\tilde{S}$ is a unitary scattering operator acting from a subspace $H_2$ into a subspace of all possible final states. In our model, (17) replaces (11), and one cannot prove the OT.

Using (17) and the final density matrix, $\rho_f = \rho_i S$ one finds the probability $P_g = Tr \left( |f\rangle \langle f| \rho_f \right)$ for finding a final state $f$:

$$P_g = \sum_{\mu \in A} \rho(\mu) \left| \langle f|\mu \rangle \right|^2 + \sum_{\mu \notin A} \rho(\mu) \left| \langle f|\mu \rangle \right|^2$$  \hspace{1cm} (18)

For final states produced after strong interactions took place, $\langle f|\mu \rangle$ vanish, and only the second term in (18) corresponds to the scattering, due to strong interactions:

$$P_g = \sum_{\mu \notin A} \rho(\mu) \left| \langle f|\mu \rangle \right|^2$$  \hspace{1cm} (19)

### 6. Data violating the optical theorem

It is difficult to test the OT in a reliable way since it requires the knowledge of a non-measurable imaginary part of forward elastic scattering probability amplitude.

However, using the OT, one may prove [19, 26, 33] that a forward differential elastic cross section due to strong interactions for any spin state described by a density matrix $\rho$
has to satisfy the following inequality:

$$\left| \frac{d\sigma}{dt} \right|_{t=0} \geq \frac{(\sigma_{tot})^2}{16\pi \hbar^2}$$  \hspace{1cm} (20)$$

In the most general form, the equation (20) does not contain \( \hbar \) [19], because one is using the units in which \( c = 1 \) and \( \hbar = 1 \). One also usually omits the absolute value sign on the left hand side, replacing in fact a negative \( dt \) by \( -dt = 2p^2 \sin \theta \, d\theta \), which is a slightly misleading notation. To proof (20) it is sufficient to observe that \( d\Omega = 2 \pi \sin \theta \, d\theta \), \( -dt = (p \, d\tau) \, d\Omega \), \( p = h \, k \), \( |f|^2 \geq (\text{Im} \, f)^2 \) and use the optical theorem (9).

The direct test of (20) consists of estimating the value of the elastic differential cross section in the forward direction (for \( t = 0 \)) and comparing it with the data for total cross sections.

The main difficulty is the extrapolation to the region in which we do not have any data points. The fits to the data are done using some reasonable formulas containing free parameters on which the experimentalists reached a consensus. However, using these reasonable formulas and the same set of data one can show [8, 26, 32–34] that the OT is consistent with the data or that it is dramatically violated.

Our model also has other serious implications for any theoretical description of hadron-hadron scattering [6, 8, 9]. We are assuming that initial two hadron states are mixed quantum states with respect to the impact parameter. The impact parameters cannot be controlled during the preparation of the beams and the targets; therefore, their statistical distribution may depend on the geometry of the beams and other factors [28].

In order to find such effects, one may use the purity tests we discussed in several papers [28–31]. One may find more details in our recent preprint1. These details, together with other topics, will be published in a more technical paper addressed to elementary particle physicists.

7. Conclusions

We proved that one can describe short range hadron-hadron interactions by a unitary S matrix without being able to prove the OT. Therefore the OT is not a fundamental law of nature, and it could be violated in high energy hadron-hadron scattering.

The violation of the OT would be confirmed if elastic scattering cross sections at LHC and beyond were more suppressed than was allowed by the OT constraint. This constraint has been called the unitarity constraint, which is misleading since one can have a unitary S matrix without the OT [6, 8, 26].

In LHC we have a huge amount of inelastic scattering data with an average number of tracks observed larger than 25. The main objective of the LHC experiment is a search for the Higgs particle and study of the production and the properties of charmed and other heavy flavor hadrons. In some sense the main objective is a search for a New Physics requiring the modification of the SM [18].

Pursuing this goal, one should not overlook another possibility for a New Physics—namely, Strong Interactions without the Optical Theorem, which, if confirmed, would be a major discovery.

To be able to make this discovery, one has to reanalyze Tevatron, LHC, and UHECR data without using the OT constraint.

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1 Our older papers can be easily uploaded using http://w4.uqo.ca/kupcma01/homepage.htm.
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