Environment-mediated charging process of quantum batteries

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We study the charging process of open quantum batteries mediated by a common dissipative environment in two different scenarios. In the first case, we consider a two-qubit system as a quantum charger-battery model. Where the battery has the capability to properly charge under non-Markovian dynamics in a strong coupling regime, without any external power and any direct interaction with the charger, i.e., a wireless battery charging happens. In fact, the environment plays a major role in the charging of the battery, while this does not happen in the weak coupling regime. In the second scenario, we show the effect of individual and collective spontaneous emission rates on the charging process of quantum batteries by considering a two-qubit system in the presence of Markovian dynamics such that each one can be charged through an external field. Contrary to previous claims for individual environments, our results demonstrate that the battery can be satisfactorily charged in non-Markovian and Markovian dynamics. We also present a robust battery by taking into account subradiant states and an intermediate regime. Moreover, we propose an experimental setup to explore the ergotropy in the first scenario.

I. INTRODUCTION

Exploiting the non-classical effects in order to energy storage is a fundamental issue. A quantum battery (QB) is a two-level system used to temporarily store energy from an external field and transfer it to a consumer [1–8]. On the other word, QBs are considered as a collection of N independent subsystems, in which they employ a temporary charging field for extraction or storage. Operationally, an optimal QB needs to have two important factors: First, the ability to store maximum energy with the least amount of time or the maximum average charging power. We define the maximum energy extracted from a system as charge or ergotropy. In fact, the ergotropy is the maximum extractable work under a unitary cyclic process [9, 10]. Second, the capability to fully discharge energy in the time required or the skill of extracting useful work. Therefore, designing protocols to accomplish these two objectives is particularly significant. Recently, one of the most important research purposes is to investigate the positive and negative effects of quantum concepts on the performance of QBs.

In a very new view, the QBs have been considered as open systems, where the battery, the charger, or both are in contact with an external environment [12–15]. This is logical because real quantum systems are related to their environment and this is what happens in practice. In reality, the unavoidable interaction between the system and environment leads to the decoherence of the system, where quantum properties of the system may be destroyed due to the interaction. Therefore, one cannot ignore the dissipative effects of the environment on the stable charging process of open QBS. However, this is not a new issue and so far many protocols have been developed to stabilize the charging status of close QBS [16–18], but the role of common reservoirs with non-Markovian dynamics has not been yet explored. This leads us to a question: may an environment act as a charger? where transfers energy to a QB.

As mentioned above we investigate the environmental effects on a charging process. However, ergotropy is used to determine the maximum energy that can be extracted from a QB during a unitary process. But we here describe an approach that the energy is firstly transferred from a charger to a battery under the decoherence effects and we obtain the instantaneous battery state. Then, we study the instantaneous ergotropy after the charging process, i.e., the battery is disconnected from the charger and is coupled to a consumption hub.

Therefore, in this paper, we analyze the role of environmental effects on QB efficiency by considering two scenarios. In the first scenario we explore a Charger-battery model in a common reservoir under non-Markovian evolution. Our results indicate that in the strong coupling regime, it can be possible that the battery appropriately charged. In contrast, in the weak coupling regime, it may not be charged, so we need an external power to charge the battery. The advantage of this model is that in the former, an external field is not required to charge the battery, and the charging process occurs only by the environment.

The second scenario is a two-qubit system in the common environment under the Born-Markov approximation, such that each qubit can be charged through an external driving field. Therefore, we investigate this model in two different approaches: with and without the charger-mediated charging process. In the former, two qubits are regarded as a QB in the ground state that we show the battery cannot charge by means of the environment then the driving fields have to be applied. In the latter, the charger-battery model is taken where we illustrate the battery can be properly charged in Markovian dynamics against what is claimed in Ref. [13], the situation in which qubits interact with independent reservoirs. We also discuss the model presented in Ref. [14] with a different perspective. Where we are comparing the destructive and constructive effects of individual and collective spontaneous emission rates in the charging process of the batteries as well as the coupling strengths by defining different regimes.

In addition, we figure out a way to stabilize a N-cell QB fully charged against the environment by means of assuming subradiant initial states and establishing the intermediate
regime. In the following, we bring up an optical experimental setup to examine the amount of extractable work in the first scenario.

The rest of the paper is organized as follows. We investigate the open QB in a common environment under two different scenarios. We present the first model: non-Markovian dynamics in Sec. II and the second model: Markovian dynamics in Sec. III. The optical setup is discussed in Sec. IV. The conclusion is summarized in Sec. V.

II. FIRST SCENARIO: NON-MARKOVIAN DYNAMICS

We describe the quantum charger-battery model as a two-qubit system coupled to a common bosonic reservoir [23, 24]. The qubits are simulated as two-level systems with excited state |e⟩ and ground state |g⟩ where we assume two qubits have the same transition frequency ω0 = ωB = ω0. The total Hamiltonian of the system in the rotating-wave approximation (RWA) described as follows [23, 24] (with ħ = 1)

\[ H = \sum_{j=C,B} \omega_0 \sigma_j^+ \sigma_j^- + \sum_k \omega_k a_k^\dagger a_k + \sum_k (g_{12} \sigma_C^+ + g_{22} \sigma_B^+) a_k + \text{H.c.}, \]

where the first and second term denotes the free Hamiltonian of the two-qubit system and the reservoir, respectively. \( \sigma_j^+ \) and \( \sigma_j^- \) are the Pauli raising and lowering operators for the jth qubit, respectively, \( a_k^\dagger \) (\( a_k \)) is the annihilation (creation) operator of the \( k \)th mode of the field with \( \omega_k \). The second line in the Eq. (1) describes the interaction of the system with the reservoir where \( \mu_{ij} \), \( \mu_{i} \) is the coupling constant between the charger/battery and the \( k \)th mode of the field, in which \( \mu_i \) is a dimensionless real parameter. The relative interaction strength is defined as \( c_i = \mu_i/\mu_T \) and the collective coupling constant as \( \mu_T = (\mu_1^2 + \mu_2^2) \).

Notice that there is no direct coupling between the charger and QB in Eq. (1), accordingly, we study wireless charging of the QB where the environment plays a mediated role in the charging process.

Presume the initial state of the whole system as follows

\[ |\Psi(0)\rangle = |\psi_0\rangle_C|g_B\rangle + |\psi_0\rangle_C|e_B\rangle \otimes |0\rangle_E, \]

where \( |0\rangle_E \) is the vacuum state and \( \psi_0, (i = 1, 2) \) are the probability amplitudes. Now, in the lack of Born-Markov approximation and by using the Lorentzian spectral density for the environment, the density operator of the charger-battery system provided that non-Markovian evolution can be written as [23, 24]

\[ \rho_{CB}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ |v_1(t)|^2 & v_1(t)v_2^*(t) & 0 & 0 \\ v_1^*(t)v_2(t) & |v_2(t)|^2 & 0 & 0 \\ 0 & 0 & 1 - |v_1|^2 - |v_2|^2 \end{pmatrix}, \]

where

\[ v_1(t) = [c_1^2 + c_1^2 \kappa(t)]v_{01} - c_1c_2 [1 - \kappa(t)]v_{02}, \]

\[ v_2(t) = -c_1c_2 [1 - \kappa(t)]v_{01} + [c_1^2 + c_2^2 \kappa(t)]v_{02}, \]

with

\[ \kappa(t) = e^{-\lambda t/2} \{ \cosh \left( \frac{\chi t}{2} \right) + \frac{\chi}{\lambda} \sinh \left( \frac{\chi t}{2} \right) \}, \]

in which \( \chi = \sqrt{\lambda^2 - 4R^2} \). In the above equation, \( \lambda \) is the width of the Lorentzian spectrum and \( R = \xi/\mu_T \) where \( \xi \) is the vacuum Rabi frequency. One can define the parameter \( R = R/\lambda \) to distinguish the strong coupling regime (\( R \gg 1 \)) from the weak one (\( R \ll 1 \) [23, 24]).

To characterize the maximal amount of energy that can be extracted from a QB, the ergotropy is introduced as [9, 10]

\[ W = Tr(\rho_B H_B) - Tr(\sigma_{p_B} H_B). \]

in which, \( H_B \) and \( \rho_B \) are the Hamiltonian and the state of the battery. \( \sigma_{p_B} \) is called the passive state of \( \rho_B \) that the extractable work from it is zero under cyclic unitary processes [11].

Then, the ergotropy of the QB can be computed from the reduced density matrix of the QB, thus according to Eq. (3), the density matrix is

\[ \rho_B(t) = |v_2(t)|^2 |e_B\rangle \langle e_B| + [1 - |v_2(t)|^2] |g_B\rangle \langle g_B|. \]

Hence, the ergotropy can be obtained as

\[ W = \hbar \omega_0 (2|v_2(t)|^2 - 1) \Theta(|v_2(t)|^2 - \frac{1}{2}), \]

in which \( \Theta(x - x_0) \) is Heaviside function. Where, the maximum ergotropy is \( W_{\text{max}} = \hbar \omega_0 \).

At this point, let’s us consider an initial entangled state as

\[ |\Phi(0)\rangle = \alpha_- |\varphi_-\rangle + \alpha_+ |\varphi_+\rangle, \]

where \( \alpha_{\pm} = \langle \varphi_{\pm} | \alpha_0 \rangle \) and \( |\varphi_+\rangle = c_1 |e_C\rangle |g_B\rangle + c_2 |g_C\rangle |e_B\rangle \). Also, \( |\varphi_-\rangle \) is the subradiant state of the Hamiltonian (1) that his state does not decay in time, it is a decoherence-free state, and takes the following form [23, 24]

\[ |\varphi_-\rangle = c_2 |e_C\rangle |g_B\rangle - c_1 |g_C\rangle |e_B\rangle. \]

According to Eq. (4), the amplitudes \( v_1(t) \) and \( v_2(t) \) can be written as [23, 24]

\[ v_1(t) = c_2\alpha_- + c_1 \kappa(t) \alpha_+, \]

\[ v_2(t) = -c_1\alpha_+ + c_2 \kappa(t) \alpha_+ \]

Thus, the amount of the ergotropy depends on the specific initial state \( |\varphi_{\pm}\rangle \) and on the value of the coefficient \( \kappa(t) \).

In the following, we study the behavior of ergotropy for different initial states in both the weak and the strong coupling regimes in terms of \( \lambda t \). First, we consider the initial charger-battery state as \( |\Phi(0)\rangle = |e_C\rangle |g_B\rangle \) with \( c_1 = 1/\sqrt{2} \) where the battery is empty and the charger has maximal energy. Second,
we choose Eq. (10) that an amount of entanglement exists between the charger and the battery. The ergotropy time evolution of the former and the latter are plotted in Fig. 1 and Fig. 2, respectively.

In Fig. 1, dashed red line and solid blue line indicate \( R = 30 \) and \( R = 0.3 \), respectively. As can be seen the ergotropy is nearly \( 0.9\tilde{W}_{\text{max}} \) at \( \lambda t \approx 0.1 \) and zero for \( \lambda t > 1.5 \) for the strong coupling regime whereas it is always zero for the weak one.

Figs. 2 illustrates the time evolution of ergotropy for \( R = 30 \). Here, the solid blue line, dot-dot-dashed black line, dotted red line, dashed green line and dot-dashed magenta line represents \( (\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{3}}{\sqrt{5}}), (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}), (\frac{\sqrt{3}}{\sqrt{7}}, 0.92), (\frac{\sqrt{3}}{\sqrt{5}}, 0.5), (\frac{1}{\sqrt{2}}, 0.2) \) for the pair of parameters \((c_1, \alpha_-)\), respectively.

As can be observed the battery is almost fully charged \( W \approx 0.95\tilde{W}_{\text{max}} \) at \( \lambda t = 0.1 \) in the strong coupling regime. The inset of Fig. 2 shows the long-time behavior of the ergotropy and one can see that it tends to 0.125\( \tilde{W}_{\text{max}} \) for long time limit. This happens when there is an amount of entanglement in the initial state whereas the ergotropy is zero for long times in Fig. 1.

We have also investigated the ergotropy for \( R = 0.3 \) that where we realize the ergotropy reaches approximately the amount of 0.125\( \tilde{W}_{\text{max}} \), for \( c_1 = \alpha_- = \frac{\sqrt{3}}{\sqrt{5}} \), however, it is zero for other initial states. In addition, the ergotropy is always zero in the weak coupling regime with \( R = 0.1 \), hence the battery does not charge in a such situation.

Clearly, these observations underline that in the absence of any direct interaction between the charger and the battery, the battery can be properly charged by the environment. Also, we understand the weak coupling regime is not an eligible candidate for optimal battery charging. In contrast, the strong coupling regime can be used in a desirable way in the charging process of a battery.

### III. SECOND SCENARIO: MARKOVIAN DYNAMICS

In this section, by employing the Born-Markov approximation, rotating-wave approximation, and applying the external laser fields, the master equation of the two-qubit system can be expressed as [25]

\[
\frac{\partial \rho(t)}{\partial t} = -i[H_s, \rho(t)] - \frac{1}{2} \sum_{i,j=1}^{2} \Gamma_{ij}(\rho(t)\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+\rho(t) - 2\sigma_j^+\rho(t)\sigma_i^-),
\]

where

\[
H_s = \sum_{i=1}^{2} (\omega_0)i\sigma_i^0 + \sum_{i \neq j=1}^{2} \Omega_{ij}\sigma_i^+\sigma_j^- + H_L.
\]

denotes the Hamiltonian of the two qubits where \( \Omega_{ij} \) represents the environment induced coherent (dipole-dipole) interaction between the qubits and \( H_L \) indicates the coupling between the qubits and external field with the Rabi frequency \( \Omega_L \) and the angular frequency \( \omega_L \). Given by

\[
H_L = \sum_{i=1}^{2} [\Omega_L i\sigma_i^0 e^{i\omega(t)} + H.c.].
\]

In Eq. (12), the parameters \( \Gamma_{ij} \) being spontaneous emission rates where \( \Gamma_i = \Gamma_{ij} \) is the individual spontaneous emission rate of the \( i \)-th atom, and \( \Gamma_{ij} = \Gamma_{ji}, i \neq j \) are collective spontaneous emission rates due to the coupling between the qubits through the environment. Notice that the collective interactions between the qubits leads to the modified dissipative decay rates and the coherent coupling \( \Omega_{ij} \). It has been demonstrated that both the collective parameters \( \Gamma_{ij} \) and \( \Omega_{ij} \) are dependent on the interatomic separation. As an example,
for large separations i.e., \( r_{12} \gg \lambda \) (with the resonant wavelength \( \lambda \)), the parameters are \( \Gamma_{ij} = \Omega_{ij} \approx 0 \) [25]. Now, suppose \( \Omega_{12} = \Omega_{21} = \Omega, \Gamma_i = \Gamma, \) and \( \Gamma_{ij} = \gamma \) for \( i \neq j \), respectively.

To obtain the dynamics of state \( \rho \) at a generic time instant \( t \), we solve Eq. 12 numerically. To this end we write the \( \rho \) in the matrix form as following

\[
\rho(t) = \begin{pmatrix}
\rho_{11}(t) & \rho_{12}(t) & \rho_{13}(t) & \rho_{14}(t) \\
\rho_{21}(t) & \rho_{22}(t) & \rho_{23}(t) & \rho_{24}(t) \\
\rho_{31}(t) & \rho_{32}(t) & \rho_{33}(t) & \rho_{34}(t) \\
\rho_{41}(t) & \rho_{42}(t) & \rho_{43}(t) & \rho_{44}(t)
\end{pmatrix}
\]

In the following, one can inquire two models: (i) a charger-battery protocol, single-cell \( QB \), for charging process without any external coherent field as the first scenario in the previous section, (ii) two-qubit system is regarded as a \( QB \), two-cell \( QB \), where each of the qubits is charged by laser.

According to Eqs. (6) and (15), the analytical expression of the ergotropy in the single-cell model can be evaluated as [12]

\[
\mathcal{W}(t) = \frac{\omega_0}{2} \left( \sqrt{4[r_{12}(t) + r_{34}(t)]^2 + (2[r_{11}(t) + r_{33}(t)] - 1)^2} + 2[r_{11}(t) + r_{33}(t)] - 1 \right)
\]

and for the two-cell case as

\[
\mathcal{W}(t) = \omega_0(-2\eta_1(t) - \eta_2(t) - \eta_3(t) + 2r_{11}(t) + r_{22}(t) + r_{33}(t)),
\]

where \( \eta_i \)'s are the eigenvalues of \( \rho \) such that \( \eta_1 \leq \eta_{i+1} \).

### A. Single-cell quantum battery

Here, we consider the first qubit as the charger and the second one as the battery in the absence of a external coherent field i.e., \( l_1 = l_2 = 0 \). As the previous section, the maximum ergotropy is \( \mathcal{W}_{\text{max}} = h\omega \).

### B. Two-cell quantum battery

Let’s consider the two-qubit system as a \( QB \), where the lasers are turned on such that \( l_1 = l_2 = l \) and \( \omega_0 = \omega_L = \omega \).

In Figs. 5 and 6 the time dependence of the battery ergotropy \( \mathcal{W}/\mathcal{W}_{\text{max}} \) as a function of \( \hbar t \) is plotted for different regimes. Here, the highest value of ergotropy is \( \mathcal{W}_{\text{max}} = 2\hbar\omega \), hence we have normalized it to the unit. The initial state of two cells battery is regarded as \( |\varphi\rangle_B = |gg\rangle \) in Fig. 5 while it is considered as \( |\varphi\rangle_B = |gg\rangle \) with \( c_1 = 1/\sqrt{2} \) in Fig. 6.

By comparing the spontaneous emission rate \( \Gamma \) with the coupling strength \( l \) and the dipole-dipole interaction parameter \( \Omega \), one can also find four different regimes. Where \( \Omega \gg \Gamma \),...
demonstrates that collective emission decay rate does not play an effective role in ergotropy dynamics. In addition, Fig. 5 demonstrates that the ratio \( \Gamma/l \) is more significant than \( \Gamma/\Omega \) because by reducing the former the ergotropy tends to the unit. This implies that the driving external fields play a substantial role in this scenario.

Figure 6 displays the time evolution of the ergotropy for the battery initial state as \( |\varphi_1\rangle \) and \( c_1 = 1/\sqrt{2} \). Here, the parameters are the same as in Fig. 5. As can be seen unlike the previous case, the dot-dot-dashed-dashed darker orange line is coincided with solid blue line and the ergotropy does not change over time. Indeed, it remains constant for intermediate regime, i.e., \( W = 0.5W_{\text{max}} \) for all times. This indicates the energy value of two cells battery equals to the energy of a single-cell fully charged battery, i.e., \( W = 0.5W_{\text{max}} = \hbar \omega \). Moreover, our results imply that by considering \( 2N \) qubits such that every two qubits in a common reservoir takes into account as a single cell battery Fig. 8, then we have \( W = N(\hbar \omega) \). Notice that this amount of energy is equal to the amount of \( N \) single-cell \( QB \) that are completely charged.

We emphasize that we find a stable battery with the amount of energy \( N \hbar \omega \) that keeps its energy and is not affected by the destructive effects of the environment by taking into account \( 2N \) qubits that every two-qubit state is subradiant state under intermediate regime.

Indeed, it is worth mentioning that this strategy for the robust battery can be applied for the first scenario (non-Markovian dynamics) by regarding two qubits as \( QB \) with initial subradiant state \( |\varphi_\cdot\rangle \). We stress that this stable battery may also be examined in a lab, note the considerations in the next section.

In addition, we observe the ergotropy decays for other regimes. Therefore, we realize that the subradiant state \( |\varphi_\cdot\rangle \) is not always a decoherence-free state in Born-Markov approximation. Also, in \( l \gg \Gamma \) regime the battery charge decays to zero monotonically, while the battery discharges very fast in the \( l \ll \Gamma \) regime. Therefore, the \( QB \) is more stable in the presence of a strong external driving field compared to the weak one. Similar to the above, in this case the ratio \( \Gamma/l \) is
more efficient factor in the stability of quantum batteries. As it increases, the battery discharge time becomes longer.

IV. EXPERIMENTAL STUDY OF WIRELESS-LIKE QUANTUM BATTERY

In order to measure the ergotropy experimentally, we introduce an optical setup that has been suggested in [26, 27]. We define a two-qubit system based on degrees of freedom of a single photon. The horizontal and the vertical polarization are regarded as the ground and the excited state of the first qubit as well as the $HG_{10}$ and the $HG_{10}$ mode are considered as the ground and the excited state of the second qubit. We show the experimental setup in Fig. 8. For more information and the details of the experiment, we refer to [26, 27]. If we prepare the total initial state as $|\Phi(0)\rangle = |e\rangle_1 |g\rangle_2 \otimes |0\rangle_3$, then the ergotropy can be measured as

$$\mathcal{W} = \hbar \omega_0 (2 \langle P_{ge} \rangle - 1) \Theta (\langle P_{ge} \rangle - \frac{1}{2}),$$

(18)

in which $\langle P_{ge} \rangle = (I_5 + I_6)/I_T$ is the population of $|ge\rangle$ where $I_i$ is the intensity of the corresponding output $O_i$ ($i = 1, 2, ..., 8$), and $I_T = \sum_{i=1}^{8} I_i$.

Therefore, by measuring the intensity of the outputs one can investigate the ergotropy behavior in the wireless-like charging process of $QB$.

V. CONCLUSION

In summary, we have investigated the dynamics of ergotropy for quantum batteries in common dissipative bosonic environments. To this purpose, we have considered the time evolution of a two-qubit system mediated by a common environment with two approaches: non-Markovian dynamics and Markovian dynamics. In the first approach we have shown that the charger-mediated charging process, where the battery can be fully charged when we are in a strong coupling regime and also provided an optical experimental setup to evaluate the amount of extractable work in this scenario. Furthermore, we have studied the second approach with and without the charger-mediated charging process. Our results demonstrate that in the latter despite the collective and individual decay rates a strong external power can play an essential role in the optimal charging process of open quantum batteries while in the former the battery is properly charged by the environment with the lack of lasers. Also, our models lighting the way to have stable and robust quantum batteries in the future. Moreover, we have found that in some scenarios, initial quantum correlations between the charger and the battery or between the battery components may not be a useful resource for further extractable work.

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[1] R. Alicki and M. Fannes, Phys. Rev. E 87, 042123 (2013).
[2] K. V. Hovhannisyan, M. Perarnau-Llobet, M. Huber, and A. Acín, Phys. Rev. Lett. 111, 240401 (2013).
[3] F. C. Binder, S. Vinjanampathy, K. Modi, and J. Goold, New J. Phys. 17, 075015 (2015).
[4] F. Campaioli, F. A. Pollock, F. C. Binder, L. Céleri, J. Goold, S. Vinjanampathy, and K. Modi, Phys. Rev. Lett. 118, 150601 (2017).
[5] D. Ferraro, M. Campisi, G. M. Andolina, V. Pellegrini, and M. Polini, Phys. Rev. Lett. 120, 117702 (2018).
[6] T. P. Le, J. Levinsen, K. Modi, M. M. Parish, and F. A. Pollock, Phys. Rev. A 97, 022106 (2018).
[7] D. Rossini, G. M. Andolina, and M. Polini, Phys. Rev. B 100, 115142 (2019).
[8] G. M. Andolina, D. Farina, A. Mari, V. Pellegrini, V. Giovannetti, and M. Polini, Phys. Rev. B 98, 205423 (2018).
[9] A. E. Allahverdyan, R. Balian, T. M. Nieuwenhuizen, Europhys. Lett. 67, 565 (2004).
[10] G. Francica, J. Goold, F. Plastina, M. Paternostro, npj Quantum Inf. 3, 12 (2017).
[11] K. V. Hovhannisyan, M. Perarnau-Llobet, M. Huber, and A. Acín, Phys. Rev. Lett. 111, 240401 (2013).
[12] D. Farina, G. M. Andolina, A. Mari, M. Polini, and V. Giovannetti, Phys. Rev. B 99, 035421 (2019).
[13] F. H. Kamin, F. T. Tabesh, S. Salimi, F. Kheirandish, A. C. Santos, arXiv:1910.07751 [quant-ph].
[14] J. Q. Quach, W. J. Munro, arXiv:2002.10044 [quant-ph].
[15] S. Zakavati, F. T. Tabesh, S. Salimi, arXiv:2003.09814 [quant-ph].
[16] A. C. Santos, B. Çakmak, S. Campbell, N. T. Zinner, Phys. Rev. E 100, 032107 (2019).
[17] A. C. Santos, A. Saguia, M. S. Sarandy, arXiv:1912.03675v1 [quant-ph].
[18] F. Pirmoradian and K. Mølmer, Phys. Rev. A 100, 043833 (2019).
[19] J. Oppenheim, M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. 89, 180402 (2002).
[20] J. Goold, M. Huber, A. Riera, L. del Rio, and P. Skrzypczyk, J. Phys. A 49, 143001 (2016).
[21] M. N. Bera, A. Riera, M. Lewenstein, and A. Winter, Nat. 8, 2180 (2017).
[22] G. Manzano, F. Plastina, and R. Zambrini, Phys. Rev. Lett. 121, 120602 (2018).
[23] S. Maniscalco, F. Francica, R. L. Zaffino, N. Lo Gullo, and F. Plastina, Phys. Rev. Lett. 100, 090503 (2008).
[24] F. Francica, S. Maniscalco, J. Piilo, F. Plastina, and K.-A. Suominen, Phys. Rev. A 79, 032310 (2009).
[25] Z. Ficek, R. Tanaś, Physics Reports, 372, 369–443, (2002).
[26] M. H. M. Passos, W. F. Balthazar, A. Z. Khoury, M. Hor-Meyll, L. Davidovich, and J. A. O. Huguenin, Phys. Rev. A 97, 022321 (2018).
[27] M. Hor-Meyll, A. Auyuanet, C. V. S. Borges, A. Aragão, J. A. O. Huguenin, A. Z. Khoury, and L. Davidovich, Phys. Rev. A 80, 042327 (2009).