Unstable behaviour of RPT when testing turbine characteristics in the laboratory

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Abstract. A reversible pump turbine is a machine that can operate in three modes of operation i.e. in pumping mode, in turbine mode and in phase compensating mode (idle speed). Reversible pump turbines have an increasing importance for regulation purposes for obtaining power balance in electric power systems. Especially in grids dominated by thermal energy, reversible pump turbines improve the overall power regulating ability. Increased use of renewables (wind-, wave- and tidal power plants) will utterly demand better regulation ability of the traditional water power systems, enhancing the use of reversible pump turbines. A reversible pump turbine is known for having incredible steep speed – flow characteristics. As the speed increases the flow decreases more than that of a Francis turbines with the same specific speed. The steep characteristics might cause severe stability problems in turbine mode of operation. Stability in idle speed is a necessity for phasing in the generator to the electric grid. In the design process of a power plant, system dynamic simulations must be performed in order to check the system stability. The turbine characteristics will have to be modelled with certain accuracy even before one knows the exact turbine design and have measured characteristics. A representation of the RPT characteristics for system dynamic simulation purposes is suggested and compared with measured characteristics. The model shows good agreement with RPT characteristics measured in The Waterpower Laboratory. Because of the S-shaped characteristics, there is a stability issue involved when measuring these characteristics. Without special measures, it is impossible to achieve stable conditions in certain operational points. The paper discusses the mechanism when using a throttle to achieve system stability, even if the turbine characteristics imply instability.

1. Introduction
In all reaction turbines, the speed of rotation will have an influence on the flow. In general, a low specific speed Francis turbine will have steeper characteristics than a high specific speed Francis. This is mainly due to the acting centripetal forces. Dependent of the ratio between inlet and outlet diameter, the pumping effect will be different in different machines. For a low specific speed Francis, the inlet diameter is much larger than the outlet diameter, hence the centripetal forces works against the driving pressure. This results in a throttling effect, which decreases the flow when the speed of rotation increases. If the outlet diameter is the larger one, the effect will be opposite, the flow increases when the speed of rotation increase, see Figure 1.
A RPT is a compromise between a pump and a turbine; in fact the geometry is more similar to a centrifugal pump than that of a Francis turbine. In order to have a stable pump, the pump outlet angle must be tilted backwards. The Figure 1, right figure below the principal difference. Both the difference in diameters and the smaller inlet angle makes the RPT characteristics steeper than that of a Francis turbine with the same specific speed.

A simulation model is developed that takes into account the pumping behaviour of Francis turbines as the speed of rotation increases. The model includes both the effect of the increased difference between the inlet and outlet diameter, and the effect of the small inlet angle (i.e. the outlet angle in pumping mode of operation) [3].

Figure 2 shows simulated $N_{ED}$-$Q_{ED}$ diagram for a RPT compared with measurement on a model turbine in the Waterpower laboratory at NTNU. The similarity is so good, that we have confidence that the numerical model of the RPT can be used for further analysis.

![Figure 1: The effect of centripetal force on a low and high specific speed Francis turbine. The figure to the right shows the principal difference between a Francis turbine and a RPT](image)

![Figure 2: Simulation of RPT characteristics compared with model measured characteristics (left). Flow-Head characteristics at one of rotation (right)](image)
2. Flow-Head characteristics
In prototype operation, the speed of rotation is of course the synchronous speed. Because of variation in the head, a more relevant representation is the Flow – Head characteristics

The definitions of $N_{ED}$ and $Q_{ED}$ are:

$$N_{ED} = \frac{nD}{\sqrt{gH}} \quad \text{and} \quad Q_{ED} = \frac{Q}{D^2\sqrt{gH}} \quad (1)$$

The characteristics may be transformed, representing a given speed of rotation, one characteristic for each guide vane opening. The same S-shape of the $N_{ED}$-$Q_{ED}$ characteristics can be observed also in the Flow-Head characteristics. The characteristics get a negative slope as the head decreases, see Figure 3.

![Figure 3: Measured $Q_{ed}$-$N_{ed}$ characteristics (left) transformed to a Flow-Head characteristics for constant speed of rotation, different guide vane opening (right)](image)

3. Stability criteria
The turbine characteristic is often characterized as stable or unstable. In reality a characteristic by its own can be neither stable nor unstable, it has to be seen in relation to the water power system as a whole. [1]

A water power system with no surge shaft can by represented by the differential equation:

$$\frac{L}{gA} \frac{dQ}{dt} = H_a - H_f - H_t \quad (2)$$

The head loss is: $H_f = K_f Q^2$. The turbine head, $H_t$, consists of two terms, one term is similar to the valve equation (as for a Pelton turbine) the other term is a function of the speed of rotation. It is important to note that the different $K$’s in these equations are not constants, but transients in interaction with the system.

$H_t$ can be represented by:

$$H_t = (K_r + K_n)Q^2 \propto K_r Q^2 \quad \text{where} \quad K_n = f(n) \quad (3)$$

With a perturbation $q$ on the steady state flow $Q_0$ inserted in equation 2:
\[ \frac{L}{gA} \frac{d(Q_0 + q)}{dt} = H_0 - K_f (Q_0 + q)^2 - K_i (Q_0 + q)^2 \]  \hspace{1cm} (4)

Subtracting \( \frac{L}{gA} \frac{dQ_0}{dt} = H_0 - K_f Q_0^2 - K_i Q_0^2 \)

and neglecting second order terms gives the differential equation for the perturbation:

\[ \frac{dq}{dt} = - \frac{Ag}{L} (2K_f Q_0 + 2K_i Q_0)q \]  \hspace{1cm} (5)

Separation of the variables:

\[ \frac{dq}{q} = - \frac{Ag}{L} (2K_f + 2K_i)Q_0 dt \]  \hspace{1cm} (6)

The solution is:

\[ q = e^{\frac{Ag}{L} (-2K_f - 2K_i)Q_0} \]  \hspace{1cm} (7)

This is unstable when: \( K_f + K_i < 0 \) \hspace{1cm} (8)

With a negative slope for the turbine in the QH-diagram we have the necessary conditions for instability in the system. In the laboratory, the head loss is small, i.e. \( K_f \approx 0 \), so the instability basically occurs when the turbine characteristic is negative. The Figure 4 below, shows the characteristic of one guide vane opening. The system is stable at high head. Reducing the head, the system becomes unstable.

**Figure 4:** Stable and unstable operational point as the system head changes shown in Flow-Head diagram
The negative slope of the turbine characteristics as shown in the Figure 4, means that $K_t$ is negative. Since the stability criterion for the system is $K_f + K_t > 0$ it follows that the additional head loss in the system will have to be $K_f > -K_t$ to have stable conditions in the system as a whole.

The stability problem during RPT tests occurs when approaching the runaway curve in the $N_{ED}$-$Q_{ED}$ diagram, which is the point where the turbine Flow-Head characteristic becomes negative, approaching the criteria for instability in eq.8.

Using the definition of $N_{ED}$ and $Q_{ED}$, the Q-H characteristics for different speed of rotation can also be determined. The characteristics are shown in Figure 5.

With a given reservoir head indicated by the dotted line, the unstable operation point will be reached by increasing speed of rotation and there will be problems to measure the characteristics further on.

4. Stabilization by means of a throttle valve

In the laboratory, there is mounted a throttle valve in front of the model RPT. The principal system is as shown in Figure 6.
At the Waterpower Laboratory at NTNU, we have the possibility to perform tests in an open loop system, which is beneficial when addressing stability issues.

By adding the loss characteristic of the throttle to the turbine characteristic, one can see in the Figure 7 that even if the turbine characteristics are unstable, the turbine + throttle characteristic become stable. In the figure, the Turbine head and the head of the turbine + throttle, i.e. the System head, are shown. In the equations for $N_{ED}$ and $Q_{ED}$, it is the turbine head that is comes in.

![Figure 7: Illustration of how the operational point of the system becomes stable even if the turbine characteristics imply instability](image)

5. Simulations
The test arrangement at The Waterpower Laboratory has been modeled with the RPT running in open loop, with a free surface reservoir as head water and free surface tail water. At the up-stream side of the turbine, there is a pressure tank acting as an air cushion. The turbine model mentioned in the introduction is used for the turbine characteristics. The stability criterion, ch. 3, is developed as static criteria, so in order to visualize oscillations, the air cushion is necessary to introduce a compliancy in the system.

In addition to the turbine model mentioned, the simulation model includes the equation of motion for the upper pipe, the continuity and the equation of stat for the air cushion compliance and water hammer equations for the pressure pipe.
Starting with normalized head 1 with a speed of rotation which brings the operational point near instability, and then giving a small perturbation of head, the Figures 8 and 9 show the transient performance as the perturbation is increased.

Utterly increasing the head perturbation, the turbine will take off and end up with negative flow, as shown in Figure 9. The performance will stabilize at negative flow.

**Figure 8:** Behaviour as the RPT reaches the unstable operational point

**Figure 9:** With higher perturbation in head, the turbine will stabilize at negative flow, more or less following the turbine

Utterly increasing the head perturbation, the turbine will take off and end up with negative flow, as shown in Figure 9. The performance will stabilize at negative flow.

6. Conclusion
Measuring the S-curves of a RPT always gives challenges. Using a throttle valve to stabilize the system is well known by the turbine laboratories. [2]. The intention of this paper is to explain the reason why, even if the RPT characteristic are unstable, the system becomes stable. How the real behavior of the turbine is, when instability occurs, is shown by the simulations.

When performing laboratory tests and this happening occurs, the observation is that the RPT goes from positive flow to negative flow in a relative short time. We have, up till now, only preliminary
measurements of head, flow and speed of rotation to verify the simulated transient behavior. The main problem is to measure the transient flow. With good results, we have previously used the Gibson method for transient flow measurements, and in near future, we intend to use this method to do measurements in the transient period, verifying or falsifying the simulations.

The normal assumption regarding this transient behavior is that the operation point jumps from positive to negative flow as illustrated in Figure 10.

![Figure 10](image_url)

*Figure 10:* Simulated (left) and measured (right) performance, two wicket gate openings in $N_{ed}$-$Q_{ed}$ diagram.

However, we have reasons to believe that the performance follows the characteristics more like that the simulation shows and the preliminary measurements indicate. *Natura non facit saltus* [Leibnitz]

| Symbol list |
|-------------|
| Symbol | Quantity | Unit |
| H | Head | m |
| Q | Flow | m$^3$/s |
| D | Turbine outlet diameter | m |
| $N_{ED}$ | Dimension less rotational speed | - |
| $Q_{ED}$ | Dimension less flow | - |
| $K_t,K_n,K_r$ | Loss factor, turbine | |
| $K_f$ | Loss factor, system | |
| g | Gravitational constant | m/s$^2$ |
References
[1] Staubli T et al 2010 Starting pump-turbines with unstable characteristics Conf.: Hydro 2010
[2] Dörfler P K et al 1998 Stable Operation Achieved on a Single-Stage Reversible Pump-Turbine Showing Instability at No Load Proc. XIX IAHR Symposium on Hydraulic Machinery and Cavitation, Singapore
[3] Nielsen T K and Olimstad G Dynamic 2010 Behaviour of Reversible Pump-Turbines in Turbine Mode of Operation The 13th Internat. Symp. on Transport Phenomena and Dynamics of Rotating Machinery, Honolulu, Hawaii