New Class of Algebraic Fuzzy Systems Using Cubic Soft Sets with their Applications

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Abstract:
In this paper, we stratify the connotation of cubic soft sets to \( \rho - \) algebras, and introduce the new class of cubic soft algebras like \( \mathcal{R}(\mathcal{SACSe}) \) and \( \mathcal{R}(\mathcal{SACS}) \). We show that the \( R - \) union of two cubic soft \( \rho - \) sualgebras might not be cubic soft \( \rho - \) sualgebra. Furthermore, we show the sufficient condition to satisfy that the \( R - \) union of two cubic soft \( \rho - \) sualgebras is cubic soft \( \rho - \) sualgebra. Moreover, some of their basic characteristics are given.

Keywords: soft sets, cubic sets, fuzzy sets, \( \rho - \) algebra, \( \rho - \) subalgebra.

1. Introduction

An addition of the connotation of a fuzzy set (FS) is shown by Zadeh [1] to consider the interval-valued fuzzy sets (IVFSs). Next, the general ideas of algebraic fuzzy system (AFS) are brightened up by inserting the notion of a fuzzy subsets. Jun et al. [2] studied some operations such as P/R-union, P/R-intersection on cubic sets. They display several related distinctive to find the solutions for intricate problems in engineering, economics, and environment. Sometimes it is not easy to use traditional methods to get good results, because of different uncertainties model for those problems. Therefore, we need to deal with non-classical mathematical tools, so some non-classical sets are studied and their applications are given like fuzzy sets, intuitionistic fuzzy sets, cubic sets, permutation sets, vague sets, soft sets and rough sets see([3]-[15]).

In 2017, the connotations of \( \rho - \) algebra/subalgebra/ideal, \( \overline{\rho} - \) ideal and permutation topological \( \rho - \) algebra were first fixed by Mahmood and Abud Alradha [16], they applied the connotation of soft sets to consider the soft \( \rho - \) algebra and soft edge \( \rho - \) algebra [17]. In 2020 [18], Mahmood and Hameed introduced some concepts of fuzzy algebras like fuzzy \( \rho - \) subalgebra (\( F_{\rho} - \mathcal{S} \)), fuzzy \( \rho - \) ideal...
The mathematical idea of soft sets is a fresh notion was studied by D. Molodtsov [19]. This theory is applied in many directions such as fuzzy sets theory, algebra, Riemann integration, topologies and so on, see ([20]-[28]). The connotation of soft BCK / BCL – algebras is fixed by Jun and others see [29].

In this work, new connotation of cubic soft algebras like \((A,SAC_S)\) and \((A,SACS)\) are shown. We explained that the \(R\)-union of two cubic soft \(\rho\) – saulgebras might not be cubic soft \(\rho\) – saulgebra. Moreover, the sufficient condition is given to satisfy that the \(R\)-union of two cubic soft \(\rho\) – saulgebras is cubic soft \(\rho\) – saulgebra. Also, some of their basic characteristics are given.

2. Preliminary

We will include in this section some definitions that are necessary for our work.

**Definition 2.1**: ([16]) A \(\rho\) – algebra \((\rho - A, (\Gamma, \rho, \ell))\) is a constant \(\ell\) in \(\Gamma\) with binary operation \(\circ\) such that:

i) \(\omega \circ \omega = \ell\),

ii) \(\ell \circ \omega = \ell\),

iii) \(\omega \circ \varnothing = \ell = \varnothing \circ \omega\) imply that \(\varnothing = \omega\),

iv) \(\omega \circ \varnothing = \varnothing \circ \omega = \ell, \forall \varnothing \neq \omega \in \Gamma - \{\ell\}\).

A non-empty subset \(I\) of \((\rho - A)\) \(\Gamma\) is called \(\rho\) – subalgebra \((\rho - SA)\) of \(\Gamma\) if \(\varnothing \circ \omega \in I\) whenever \(\varnothing, \omega \in I\).

**Definition 2.2**: ([1]) Let \(\Gamma\) be non-empty set. A fuzzy set (FS) \(\zeta\) of the set \(\Gamma\) is a mapping \(\zeta : \Gamma \rightarrow [0,1]\). The set of all (FSs) in \(\Gamma\) is referred as \(I^\Gamma\). Define a relation \(\leq\) on \(I^\Gamma\) as follows:

\(\varphi \leq \eta, \forall \varphi, \eta \in I^\Gamma\) \(\iff (\varphi(\omega) \leq \eta(\omega), \forall \omega \in \Gamma)\) (1)

The operations \((\vee)\) and \((\wedge)\) are defined on \(I^\Gamma\) by:

\((\varphi \vee \eta)(\omega) = \max\{\varphi(\omega), \eta(\omega)\}, \quad (2)\)

\((\varphi \wedge \eta)(\omega) = \min\{\varphi(\omega), \eta(\omega)\}, \forall \omega \in \Gamma\)

For any \(\varphi \in I^\Gamma\) it’s complement is referred by \(\varphi^c\) and it is defined by

\(\varphi^c(\omega) = 1 - \varphi(\omega), \forall \omega \in \Gamma\) (3)

Also, if \(\{\varphi_i \mid i \in \Psi\}\) is a family of (FSs), where \(\Psi\) is an index set. Then we can define \((\vee)\) and \((\wedge)\) by:

\((\vee_{i \in \Omega} \varphi)(\omega) = \sup\{\varphi_i(\omega) \mid i \in \Psi\}, \quad (4)\)
\((\bigwedge_{i \in I} \varphi_i)(\omega) = \inf\{\varphi_i(\omega) \mid i \in I\}, \forall \omega \in \Gamma\)

**Definition 2.3** ([30])

A closed subinterval \(\hat{I} = [\hat{0}^-, \hat{0}^+]\) of \(I = [0,1]\) is called an interval number (IN), where \(0 \leq \hat{0}^- \leq \hat{0}^+ \leq 1\). The set of all interval numbers (INs) is refereed as \([I]\).

Now, we show the definitions of two operations on \([I]\) which are refined minimum and refined maximum (briefly, \(r\) min and \(r\) max). We also show the definitions of the symbols \(\geq^\prime\), \(\leq^\prime\) and \(=^\prime\) on \([I]\) as follows:

\[
\begin{align*}
    r \min\{\hat{0}, \hat{\nu}\} &= [\min\{\nu^-, \nu^\prime\}, \min\{\nu^+, \nu^\prime\}], \\
    r \max\{\hat{0}, \hat{\nu}\} &= [\max\{\nu^-, \nu^\prime\}, \max\{\nu^+, \nu^\prime\}], \tag{5}
\end{align*}
\]

\(\hat{0} \geq^\prime \hat{\nu} \iff \nu^- \geq \nu^\prime\) and \(\nu^+ \geq \nu^\prime\),

Also, we say \(\hat{0} >^\prime \hat{\nu}\) (resp., \(\hat{0} <^\prime \hat{\nu}\)), that means \(\hat{0} \geq^\prime \hat{\nu}\) and \(\hat{0} \neq \hat{\nu}\) (resp., \(\hat{0} \leq^\prime \hat{\nu}\) and \(\hat{0} \neq \hat{\nu}\)). If \(\{\hat{\nu}_\alpha \in [I] \mid \alpha \in \Psi\}\) is a family of (INs). We define

\[
\begin{align*}
    r \min_{r \text{ inf} \Psi} \hat{\nu}_\alpha &= [\inf_{r \text{ inf} \Psi} \nu^\prime_\alpha, \inf_{r \text{ inf} \Psi} \nu^\prime_\alpha], \\
    r \max_{r \text{ sup} \Psi} \hat{\nu}_\alpha &= [\sup_{r \text{ sup} \Psi} \nu^\prime_\alpha, \sup_{r \text{ sup} \Psi} \nu^\prime_\alpha]. \tag{6}
\end{align*}
\]

The complement of any \(\hat{\nu} \in [I]\) is symbolized by \(\hat{\nu}^\prime\) and defined as:

\[
\hat{\nu}^\prime = [1 - \nu^+, 1 - \nu^-]. \tag{7}
\]

If \(\Gamma \neq \Phi\), we say \(\hat{\varphi} : \Gamma \to [I]\) is an interval-valued fuzzy set (IVFS) in \(\Gamma\). The set of all interval-valued fuzzy sets (IVFSs) in \(\Gamma\) is symbolized by \([I]^{\Gamma}\). Also, for each \(\hat{\varphi} \in [I]^{\Gamma}\) and \(\omega \in \Gamma\), we say \(\hat{\varphi}(\omega) = [\hat{\varphi}_+^\prime(\omega), \hat{\varphi}_-^\prime(\omega)]\) is the degree of membership of \(\omega\) to \(\Gamma\), where \(\hat{\varphi}_-^\prime : \Gamma \to I\) and \(\hat{\varphi}_+^\prime : \Gamma \to I\) are (FSs) in \(\Gamma\) and they are called a lower fuzzy set (LFS) and an upper fuzzy set (UFS) in \(\Gamma\), respectively. For easy it can be written as \(\hat{\varphi} = [\hat{\varphi}_-^\prime, \hat{\varphi}_+^\prime]\). We also show the definitions of the symbols \(\leq^\prime\) and \(=^\prime\) on any \(\hat{\varphi}_1, \hat{\varphi}_2 \in [I]^{\Gamma}\) as follows:

\[
\begin{align*}
    \hat{\varphi}_1 \leq \hat{\varphi}_2 \iff \hat{\varphi}_1(\omega) \leq \hat{\varphi}_2(\omega), \forall \omega \in \Gamma, \\
    \hat{\varphi}_1 = \hat{\varphi}_2 \iff \hat{\varphi}_1(\omega) = \hat{\varphi}_2(\omega), \forall \omega \in \Gamma. \tag{8}
\end{align*}
\]

The complement of any \(\hat{\varphi} \in [I]^{\Gamma}\) is symbolized by \(\hat{\varphi}^\prime\) and defined as:

\[
\hat{\varphi}^\prime(\omega) = \hat{\varphi}(\omega)^\prime, \forall \omega \in \Gamma. \text{ That means } \hat{\varphi}^\prime(\omega) = [1 - \hat{\varphi}_+^\prime(\omega), 1 - \hat{\varphi}_-^\prime(\omega)], \forall \omega \in \Gamma. \tag{9}
\]
If \( \{ \hat{C}_a \in [I] \mid \alpha \in \Psi \} \) is a family of (IVFs). We define “\( \cup \) ” and “\( \cap \)” on \( [I] \) as follows:

\[
(\bigcup_{a \in \Psi} \hat{C}_a)(\omega) = r \sup_{a \in \Psi} \hat{C}_a(\omega), \forall \omega \in \Gamma, \quad (10)
\]

\[
(\bigcap_{a \in \Psi} \hat{C}_a)(\omega) = r \inf_{a \in \Psi} \hat{C}_a(\omega), \forall \omega \in \Gamma.
\]

**Definition 2.4:** ([19]) Assume \( \Gamma \) is an initial universe set and \( \Omega \) is a set of parameters. We say \((\beta, \theta)\) is a soft set (over \( \Gamma \)) where \( \beta \) is a multivalued function \( \beta : \theta \rightarrow P(\Gamma) \), where \( P(\Gamma) \) is the power set of \( \Gamma \) and \( \theta \subseteq \Omega \).

**Definition 2.5:** ([2]) A cubic set \( \lambda \) (CS) in universe set \( \Gamma \) is defined as:

\[
\lambda = \{ \langle \omega, \nu(\omega), \eta(\omega) \rangle / \omega \in \Gamma \} \quad (11)
\]

where \( \nu \) and \( \eta \) are (IVFS) and (FS), respectively. For easy we can symbolize \( \lambda \) as \( \langle \nu, \eta \rangle \).

**Definition 2.6:** ([2]). Assume \( \lambda_1 = \langle \nu, \gamma \rangle \) and \( \lambda_2 = \langle \nu, \eta \rangle \) are cubic sets (CSs) in a universe \( \Gamma \). We define “\( \subseteq_p \)”, “\( \subseteq_R \)” and “=” as follows:

(i) \( (P\text{-order}) \lambda_1 \subseteq_p \lambda_2 \Leftrightarrow \nu \subseteq \nu \) and \( \gamma \leq \eta \).

(ii) \( (R\text{-order}) \lambda_1 \subseteq_R \lambda_2 \Leftrightarrow \nu \subseteq \nu \) and \( \gamma \geq \eta \).

(iii) \( (Equality) \lambda_1 = \lambda_2 \Leftrightarrow \nu = \nu \) and \( \gamma = \eta \).

**Definition 2.7:** ([2]). Let \( \{ \lambda_a = \{ \langle \omega, \nu_a(\omega), \eta_a(\omega) \rangle / \omega \in \Gamma \} \}_{a \in \Psi} \) be a family of (CSs) in \( \Gamma \). The symbol “\( \bigcup_p \)” (resp., “\( \bigcap_p \)”, “\( \bigcup_R \)” and “\( \bigcap_R \)” ) is called \( (P\text{-union}) \) (resp., \( P\text{-intersection}, R\text{-union} \) and \( R\text{-intersection} \)) and defined as follows:

\[
(1) \quad \bigcup_{a \in \Psi} \lambda_a = \{ \langle \omega, \bigcup_{a \in \Psi} \nu_a(\omega), \bigvee_{a \in \Psi} \eta_a(\omega) \rangle / \omega \in \Gamma \},
\]

\[
(2) \quad \bigcap_{a \in \Psi} \lambda_a = \{ \langle \omega, \bigcap_{a \in \Psi} \nu_a(\omega), \bigwedge_{a \in \Psi} \eta_a(\omega) \rangle / \omega \in \Gamma \},
\]

\[
(3) \quad \bigcup_{a \in \Psi} \lambda_a = \{ \langle \omega, \bigcup_{a \in \Psi} \nu_a(\omega), \bigwedge_{a \in \Psi} \eta_a(\omega) \rangle / \omega \in \Gamma \},
\]

\[
(4) \quad \bigcap_{a \in \Psi} \lambda_a = \{ \langle \omega, \bigcap_{a \in \Psi} \nu_a(\omega), \bigvee_{a \in \Psi} \eta_a(\omega) \rangle / \omega \in \Gamma \}.
\]

**Remarks 2.8:** ([31])

(1) The complement of \( \lambda = \langle \nu, \eta \rangle \) is defined as:

\[
\lambda^c = \{ \langle \omega, \nu(\omega)^c, 1 - \eta(\omega) \rangle / \omega \in \Gamma \}. \quad (12)
\]
\begin{equation}
(\mathcal{L}^c)^c = \mathcal{L}.
\end{equation}

(3) Let \( \{ \mathcal{L}_a = \{ (\omega, \delta_a(\omega), \sigma_a(\omega)) / \omega \in \Gamma \} \}_{a \in \mathcal{C}} \) be a family of (CSs) in \( \Gamma \). We have:
\[
\left( \bigcup_{a \in \mathcal{C}} \mathcal{L}_a \right)^c = \bigcup_{a \in \mathcal{C}} (\mathcal{L}_a)^c, \quad \left( \bigcap_{a \in \mathcal{C}} \mathcal{L}_a \right)^c = \bigcap_{a \in \mathcal{C}} (\mathcal{L}_a)^c, \quad \left( \bigcup_{a \in \mathcal{C}} (\mathcal{L}_a)^c \right)^c = \bigcap_{a \in \mathcal{C}} (\mathcal{L}_a)^c
\]
and \( \left( \bigcap_{a \in \mathcal{C}} (\mathcal{L}_a)^c \right)^c = \bigcup_{a \in \mathcal{C}} \mathcal{L}_a \).

In what follows, a (CS) \( \mathcal{L} = \{ (\omega, \bar{\delta}_a(\omega), \bar{\sigma}_a(\omega)) / \omega \in \Gamma \} \) is clearly referred as \( \mathcal{L} = (\bar{\delta}, \bar{\sigma}) \). The set of all (CSs) in \( \Gamma \) is symbolized by \( \mathcal{L}^\Gamma \).

**Definition 2.9:** ([31]) Assume \( \Gamma \) is an initial universe set and \( \Omega \) is a set of parameters. We say \( (\Theta, T) \) is a cubic soft set (CSS) over \( \Gamma \), where \( \Theta : T \rightarrow \mathcal{L}^\Gamma \) is a mapping and \( T \subseteq \Omega \). Also, \( (\Theta, T) \) can be written as:
\[
(\Theta, T) = \{ (\Theta(e) / e \in T) \}, \text{ where } \Theta(e) = (\bar{\delta}_e(e), \bar{\sigma}_e(e)).
\]

The set of all cubic soft sets (CSSs) is symbolized by \( \mathcal{L}^\Gamma \).

**Definition 2.10:** ([31]) Let \( (\Theta, T), (L, H) \in \mathcal{L}^\Gamma \). The R-union of \( (\Theta, T) \) and \( (L, H) \) is a (CSS) \( (N, Z) \) and is symbolized by \( (N, Z) = (\Theta, T) \cup^R (L, H) \), where \( Z = T \cup H \) and
\[
N(e) = \begin{cases}
\Theta(e), & \text{if } e \in T \setminus H \\
L(e), & \text{if } e \in H \setminus T, \quad \forall e \in Z \\
\Theta(e) \cup^R L(e), & \text{if } e \in T \cap H
\end{cases}
\]

**Definition 2.11:** ([31]) Let \( (\Theta, T), (L, H) \in \mathcal{L}^\Gamma \). The P-union of \( (\Theta, T) \) and \( (L, H) \) is a (CSS) \( (N, Z) \) and is symbolized by \( (N, Z) = (\Theta, T) \cup^P (L, H) \), where \( Z = T \cup H \) and
\[
N(e) = \begin{cases}
\Theta(e), & \text{if } e \in T \setminus H \\
L(e), & \text{if } e \in H \setminus T, \quad \forall e \in Z \\
\Theta(e) \cup^P L(e), & \text{if } e \in T \cap H
\end{cases}
\]

**Definition 2.12:** ([31]) Let \( (\Theta, T), (L, H) \in \mathcal{L}^\Gamma \). The P-intersection of \( (\Theta, T) \) and \( (L, H) \) is a (CSS) \( (N, Z) \) and is symbolized by \( (N, Z) = (\Theta, T) \cap^P (L, H) \), where \( Z = T \cup H \) and
\[
N(e) = \begin{cases}
\Theta(e), & \text{if } e \in T \setminus H \\
L(e), & \text{if } e \in H \setminus T, \quad \forall e \in Z \\
\Theta(e) \cap^P L(e), & \text{if } e \in T \cap H
\end{cases}
\]
Definition 2.13: ([31]) Let \((\Theta,T),(L,H) \in \Omega\Gamma\). We say \((\Theta,T)\) is an R-cubic soft subset of \((L,H)\) if

1. \(T \subseteq H\),
2. \(\Theta(e) \subseteq_{R} L(e), \forall e \in T\) \hspace{1cm} (19)

Definition 2.14: ([31]) Let \((\Theta,T),(L,H) \in \Omega\Gamma\). We say \((\Theta,T)\) is a P-cubic soft subset of \((L,H)\) if

1. \(T \subseteq H\),
2. \(\Theta(e) \subseteq_{P} L(e), \forall e \in T\) \hspace{1cm} (20)

3. Cubic Soft \(\rho\) – Subalgebras in \(\rho\) – Algebras

Definition 3.1: Assume \((\Gamma,\circ, \ell)\) is \((\rho – A)\) and \((\Theta,T)\) is \((CSS)\) over \(\Gamma\). We say \((\Theta,T)\) is a cubic soft \(\rho\)-subalgebra over \(\Gamma\) based on a parameter \(e\) [briefly, \( \rho – CS\rho – SA\) over \(\Gamma\)] if there exists a parameter \(e \in \Theta\) such that

\[
\hat{\epsilon}_{\Theta(e)}(\omega \circ \emptyset) \geq r \min\{\hat{\epsilon}_{\Theta(e)}(\omega), \hat{\epsilon}_{\Theta(e)}(\emptyset)\}, \forall \omega, \emptyset \in \Gamma. \hspace{1cm} (21)
\]

\[
\eta_{\Theta(e)}(\omega \circ \emptyset) \leq \max\{\eta_{\Theta(e)}(\omega), \eta_{\Theta(e)}(\emptyset)\}, \forall \omega, \emptyset \in \Gamma. \hspace{1cm} (22)
\]

If \((\Theta,T)\) is an \( \rho – CS\rho – SA\) over \(\Gamma\), \(\forall e \in T\), we say \((\Theta,T)\) is a cubic soft \(\rho\)-subalgebra \(\rho – CS\rho – SA\) over \(\Gamma\).

Theorem 3.2: Suppose that \((\Theta,T)\) and \((L,H)\) are \((CS\rho – SA)\) over \(\Gamma\). Then the R-union of \((\Theta,T)\) and \((L,H)\) is a \((CS\rho – SA)\) over \(\Gamma\), if \(T\) and \(H\) are disjoint.

Proof. From Definition (2.10), we have \((N,Z) = (\Theta,T) \bigcup_{R} (L,H)\), where \(Z = T \cup H\) and

\[
N(e) = \begin{cases} 
\Theta(e), & \text{if } e \in T \setminus H \\
L(e), & \text{if } e \in H \setminus T, \forall e \in Z \\
\Theta(e) \cup_{R} L(e), & \text{if } e \in T \cap H 
\end{cases} \hspace{1cm} (23)
\]

Therefore, either \(e \in T \setminus H\) or \(e \in H \setminus T\), \(\forall e \in Z\) (since \(T \cap H = \emptyset\)). If \(e \in T \setminus H\), then \(N(e) = \Theta(e)\) is a \(CS\rho\)-subalgebra over \(\Gamma\). Also, if \(e \in H \setminus T\), then \(N(e) = L(e)\) is a \((CS\rho – SA)\) over \(\Gamma\). So \((N,Z) = (\Theta,T) \bigcup_{R} (L,H)\) is a \((CS\rho – SA)\) over \(\Gamma\).

Remark 3.3: If \(T\) and \(H\) are not disjoint, then above theorem is not valid in general.

Example 3.4: Let \(\Gamma = \{h_1, h_2, h_3, h_4, h_5\}\) be a universe set of houses and \(\circ\) be defined as Table (1):
Then, \((\Gamma, \rho, \ h_1)\) is a \(\rho\)–algebra. Now, let \(\Omega = \{Cheap \ (e_1), \ Old \ (e_2), \ Modern \ (e_3), \ Big \ (e_4), \ with \ Garden \ (e_5)\}\) be a set of parameters. That each member in \(\Omega\) give us the description for these houses that somebody want to buy one of them based on his opinion of what he like of these descriptions. Take \(T = \{e_1, e_2, e_3, e_4\}\) and \(H = \{e_3, e_4, e_5\}\), then from Table (2) and Table (3) we consider that \((\Theta, T)\) and \(CS\rho - \Gamma\), respectively.

| Table 1: \((\Gamma, \rho, \ h_1)\) is a \(\rho\)–algebra |
|-----------------------------------------------|
| \(\circ\) | \(h_1\) | \(h_2\) | \(h_3\) | \(h_4\) | \(h_5\) |
| \(h_1\) | \(h_1\) | \(h_1\) | \(h_1\) | \(h_1\) | \(h_1\) |
| \(h_2\) | \(h_2\) | \(h_1\) | \(h_1\) | \(h_1\) | \(h_1\) |
| \(h_3\) | \(h_3\) | \(h_5\) | \(h_4\) | \(h_4\) | \(h_5\) |
| \(h_4\) | \(h_4\) | \(h_4\) | \(h_4\) | \(h_4\) | \(h_5\) |
| \(h_5\) | \(h_5\) | \(h_5\) | \(h_5\) | \(h_5\) | \(h_1\) |

| Table 2: \((\Theta, T)\) is \((CS\rho - S4)\) |
|-----------------------------------------------|
| \(e_1\) | \(e_3\) | \(e_5\) |
| \(h_1\) | \([0.7, 0.6], 0.2\) | \([0.6, 0.5], 0.1\) | \([0.5, 0.8], 0.5\) |
| \(h_2\) | \([0.3, 0.6], 0.5\) | \([0.3, 0.4], 0.4\) | \([0.3, 0.8], 0.7\) |
| \(h_3\) | \([0.7, 0.5], 0.8\) | \([0.5, 0.4], 0.8\) | \([0.5, 0.7], 0.9\) |
| \(h_4\) | \([0.6, 0.4], 0.7\) | \([0.5, 0.2], 0.6\) | \([0.2, 0.4], 0.6\) |
| \(h_5\) | \([0.3, 0.5], 0.3\) | \([0.3, 0.4], 0.8\) | \([0.5, 0.7], 0.5\) |

| Table 3: \((L, H)\) is \((CS\rho - S4)\) |
|-----------------------------------------------|
| \(e_1\) | \(e_2\) | \(e_3\) | \(e_4\) |
| \(h_1\) | \([0.4, 0.7], 0.4\) | \([0.6, 0.9], 0.6\) | \([0.6, 0.5], 0.1\) | \([0.4, 0.5], 0.3\) |
| \(h_2\) | \([0.2, 0.7], 0.6\) | \([0.4, 0.8], 0.8\) | \([0.5, 0.4], 0.3\) | \([0.3, 0.1], 0.5\) |
| \(h_3\) | \([0.4, 0.6], 0.8\) | \([0.6, 0.8], 0.7\) | \([0.3, 0.2], 0.6\) | \([0.4, 0.3], 0.4\) |
| \(h_4\) | \([0.1, 0.3], 0.5\) | \([0.3, 0.5], 0.7\) | \([0.5, 0.3], 0.8\) | \([0.1, 0.4], 0.8\) |
Here $T$ and $H$ are not disjoint. Also, the $R$-union $(N,Z) = (\Theta,T) \cup_R (L,H)$, of $(\Theta,T)$ and $(L,H)$ is
given by Table (4).

| $h_5$ | $[(0.4, 0.7], 0.4)$ | $[(0.5, 0.8], 0.6)$ | $[(0.3, 0.2], 0.8)$ | $[(0.3, 0.2], 0.3)$ |
|-------|---------------------|---------------------|---------------------|---------------------|

Table 4: $(N,Z)$ is (CSS)

| $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|-------|-------|-------|-------|-------|
| $h_1$ | $[(0.6, 0.7], 0.2)$ | $[(0.6, 0.9], 0.6)$ | $[(0.6, 0.5], 0.1)$ | $[(0.4, 0.5], 0.3)$ | $[(0.5, 0.8], 0.5)$ |
| $h_2$ | $[(0.7, 0.7], 0.5)$ | $[(0.4, 0.8], 0.6)$ | $[(0.5, 0.4], 0.3)$ | $[(0.3, 0.1], 0.5)$ | $[(0.3, 0.8], 0.7)$ |
| $h_3$ | $[(0.3, 0.6], 0.8)$ | $[(0.6, 0.8], 0.7)$ | $[(0.5, 0.4], 0.6)$ | $[(0.4, 0.3], 0.4)$ | $[(0.5, 0.7], 0.9)$ |
| $h_4$ | $[(0.3, 0.4], 0.5)$ | $[(0.3, 0.5], 0.7)$ | $[(0.5, 0.3], 0.6)$ | $[(0.1, 0.4], 0.8)$ | $[(0.2, 0.4], 0.6)$ |
| $h_5$ | $[(0.3, 0.7], 0.3)$ | $[(0.5, 0.8], 0.6)$ | $[(0.3, 0.4], 0.8)$ | $[(0.3, 0.2], 0.3)$ | $[(0.5, 0.7], 0.5)$ |

We have $\bar{\tau}_{N(e_5)}(h_2 \circ h_3) = \bar{\tau}_{N(e_5)}(h_5) = [0.3, 0.4] \subset [0.5, 0.4]$ = $r \min \{[(0.5, 0.4), (0.5, 0.4)\} =$

$r \min \{\bar{\tau}_{N(e_5)}(h_4), \bar{\tau}_{N(e_5)}(h_3)\}$ \hspace{1cm} (24)

and/or

$\eta_{N(e_5)}(h_2 \circ h_3) = \eta_{N(e_5)}(h_5) = 0.8 > 0.6 = \max(\eta_{N(e_5)}(h_2), \eta_{N(e_5)}(h_3))$ \hspace{1cm} (25)

**Theorem 3.5:** Assume $(\Gamma, \sigma, \ell)$ is $(\rho - A), (\Theta,T) \in \Omega [\Gamma \Gamma]$ and $e \in T$. Then $(\Theta,T)$ is $(e - CS\rho - SA)$

over $\Gamma$ if and only if the sets $\bar{\tau}_{\theta(e)}[\delta_1, \delta_2] = \{\omega \in \Gamma / \bar{\tau}_{\theta(e)}(\omega) \bar{\geq} [\delta_1, \delta_2]\}, \eta_{\theta(e)}(\sigma) = \{\omega \in \Gamma /

$\eta_{\theta(e)}(\omega) \leq \sigma\} \hspace{1cm} (26)
are $\rho$-subalgebras of $\Gamma$, \( \forall[\delta_1, \delta_2] \in [I] \) and \( \sigma \in [0, 1] \).

**Proof.** Assume that a (CSS) \((\Theta, T)\) is \((e - CS\rho - SA)\) over $\Gamma$, let \( \omega, \vartheta \in \Gamma \). If \( \omega, \vartheta \in \overline{\tau_{\Theta(c)}}[\delta_1, \delta_2] \), \( \forall[\delta_1, \delta_2] \), then \( \tau_{\Theta(c)}(\omega) \geq [\delta_1, \delta_2] \) and \( \tau_{\Theta(c)}(\vartheta) \geq [\delta_1, \delta_2] \). It follows from (21) that

\[
\hat{\tau}_{\Theta(c)}(\omega \circ \vartheta) \geq r \min \{\hat{\tau}_{\Theta(c)}(\omega), \hat{\tau}_{\Theta(c)}(\vartheta)\} \geq r \min \{[\delta_1, \delta_2], [\delta_1, \delta_2]\} = [\delta_1, \delta_2]
\]

(27)

Thus \( \omega \circ \vartheta \in \overline{\tau_{\Theta(c)}}[\delta_1, \delta_2] \). Also, if \( \omega, \vartheta \in \overline{\eta_{\Theta(c)}}(\sigma) \) \( \forall \sigma \in [0, 1] \), so \( \eta_{\Theta(c)}(\omega) \leq \sigma \) and \( \eta_{\Theta(c)}(\vartheta) \leq \sigma \).

By (22), we have \( \eta_{\Theta(c)}(\omega \circ \vartheta) \leq \max\{\eta_{\Theta(c)}(\omega), \eta_{\Theta(c)}(\vartheta)\} \leq \sigma \), and hence \( \omega \circ \vartheta \in \overline{\eta_{\Theta(c)}}(\sigma) \).

Hence \( \overline{\tau_{\Theta(c)}}[\delta_1, \delta_2] \text{ and } \overline{\eta_{\Theta(c)}}(\sigma) \) are \( \rho \)-subalgebras of $\Gamma$.

Conversely, assume \( \overline{\tau_{\Theta(c)}}[\delta_1, \delta_2] \text{ and } \overline{\eta_{\Theta(c)}}(\sigma) \) are \( \rho \)-subalgebras of $\Gamma$, \( \forall[\delta_1, \delta_2] \in [I] \) and \( \sigma \in [0, 1] \) If for some \( \omega, \vartheta \in \Gamma \) satisfy:

\[
\hat{\tau}_{\Theta(c)}(\omega \circ \vartheta) < r \min \{\hat{\tau}_{\Theta(c)}(\omega), \hat{\tau}_{\Theta(c)}(\vartheta)\}
\]

(28)

Let \( \hat{\tau}_{\Theta(c)}(\omega) = [q_1, q_2], \hat{\tau}_{\Theta(c)}(\vartheta) = [u_1, u_2] \) and \( \hat{\tau}_{\Theta(c)}(\omega \circ \vartheta) = [\delta_1, \delta_2] \). Hence

\[
[\delta_1, \delta_2] < r \min \{[q_1, q_2], [u_1, u_2]\} = [\min \{q_1, u_1\}, \min \{q_2, u_2\}]
\]

(29)

Therefore, \( \delta_1 < \min \{q_1, u_1\} \) and \( \delta_2 < \min \{q_2, u_2\} \). Take

\[
[f_1, f_2] = \frac{1}{2} \left[ \hat{\tau}_{\Theta(c)}(\omega \circ \vartheta) + r \min \{\hat{\tau}_{\Theta(c)}(\omega), \hat{\tau}_{\Theta(c)}(\vartheta)\} \right]
\]

(30)

Hence, we have:

\[
[f_1, f_2] = \frac{1}{2} \left[ \delta_1, \delta_2 \right] + \left[ \min \{q_1, u_1\}, \min \{q_2, u_2\} \right] = \frac{1}{2} (\delta_1 + \min \{q_1, u_1\}) \quad \frac{1}{2} (\delta_2 + \min \{q_2, u_2\})
\]

Also, we consider the following:

\[
\min \{q_1, u_1\} > f_1 = \frac{1}{2} (\delta_1 + \min \{q_1, u_1\}) > \delta_1,
\]

\[
\min \{q_2, u_2\} > f_2 = \frac{1}{2} (\delta_2 + \min \{q_2, u_2\}) > \delta_2.
\]

(31)

Thus, we have:

\[
\left[ \min \{q_1, u_1\}, \min \{q_2, u_2\} \right] \supseteq [f_1, f_2] \supseteq [\delta_1, \delta_2] = \hat{\tau}_{\Theta(c)}(\omega \circ \vartheta).
\]

(32)

And hence \( \omega \circ \vartheta \not\in \overline{\tau_{\Theta(c)}}[f_1, f_2] \). In other side, we have
\( \hat{\xi}_{\Theta(e)}(\omega) = \{q_1, q_2\} \supseteq \min \{q_1, u_1\}, \min \{q_2, u_2\} \supseteq [f_1, f_2] \) 

(33)

\( \hat{\xi}_{\Theta(e)}(\vartheta) = \{u_1, u_2\} \supseteq \min \{q_1, u_1\}, \min \{q_2, u_2\} \supseteq [f_1, f_2] \)

Then \( \omega \circ \vartheta \in \hat{\xi}_{\Theta(e)}[f_1, f_2] \). But this contradiction, therefore we get that:

\( \hat{\xi}_{\Theta(e)}(\omega \circ \vartheta) \supseteq r \min \{\hat{\xi}_{\Theta(e)}(\omega), \hat{\xi}_{\Theta(e)}(\vartheta)\}, \forall \omega, \vartheta \in \Gamma \)  

(34)

Now, if \( \eta_{\Theta(e)}(\omega \circ \vartheta) > \max \{\eta_{\Theta(e)}(\omega), \eta_{\Theta(e)}(\vartheta)\} \) for some \( \omega, \vartheta \in \Gamma \). Hence, \( \exists h \in (0, 1) \) satisfies the following:

\( \eta_{\Theta(e)}(\omega \circ \vartheta) > h \supseteq \max \{\eta_{\Theta(e)}(\omega), \eta_{\Theta(e)}(\vartheta)\} \).  

(35)

Then \( \omega, \vartheta \in \overline{\eta_{\Theta(e)}}(h) \), however \( \omega \circ \vartheta \notin \overline{\eta_{\Theta(e)}}(h) \). But this contradiction, so we get that:

\( \eta_{\Theta(e)}(\omega \circ \vartheta) \leq \max \{\eta_{\Theta(e)}(\omega), \eta_{\Theta(e)}(\vartheta)\}, \forall \omega, \vartheta \in \Gamma \)  

(36)

Hence \( (\Theta, T) \) is \( (e - \text{CS} \rho - \text{SA}) \) over \( \Gamma \).

**Proposition 3.6:** Assume \( (\Gamma, \Theta, \ell) \) is \( (\rho - A), (\Theta, T) \in \ell \Gamma \) and \( e \in T \). If \( (\Theta, T) \) is \( (e - \text{CS} \rho - \text{SA}) \) over \( \Gamma \), then \( \hat{\xi}_{\Theta(e)}(\ell) \supseteq \hat{\xi}_{\Theta(e)}(\omega) \) and \( \eta_{\Theta(e)}(\ell) \leq \eta_{\Theta(e)}(\omega), \forall \omega \in \Gamma \).

**Proof:** \( \forall \omega \in \Gamma \); we consider that:

\[
\hat{\xi}_{\Theta(e)}(\ell) = \hat{\xi}_{\Theta(e)}(\omega \circ \omega) \supseteq r \min \{\hat{\xi}_{\Theta(e)}(\omega), \hat{\xi}_{\Theta(e)}(\omega)\} = r \min \{[\hat{\xi}_{\Theta(e)}(\omega)^{-}, \hat{\xi}_{\Theta(e)}(\omega)^{+}]\}
\]

\[
[\hat{\xi}_{\Theta(e)}(\omega)^{-}, \hat{\xi}_{\Theta(e)}(\omega)^{+}] = [\hat{\xi}_{\Theta(e)}(\omega)^{-}, \hat{\xi}_{\Theta(e)}(\omega)^{+}] = \hat{\xi}_{\Theta(e)}(\omega)
\]

(37)

and \( \eta_{\Theta(e)}(\ell) = \eta_{\Theta(e)}(\omega \circ \omega) \leq \max \{\eta_{\Theta(e)}(\omega), \eta_{\Theta(e)}(\omega)\} = \eta_{\Theta(e)}(\omega) \).

**Theorem 3.7:** Let \( (\Gamma, \Theta, \ell) \) be \( (\rho - A) \) and \( (\Theta, T) \) be \( (e - \text{CS} \rho - \text{SA}) \) over \( \Gamma \). If \( \lim_{\omega \to \omega_{n}} \hat{\xi}_{\Theta(e)}(\omega) = [1, 1] \) and \( \lim_{\omega \to \omega_{n}} \eta_{\Theta(e)}(\omega) = 0 \), then \( \hat{\xi}_{\Theta(e)}(\ell) = [1, 1] \) and \( \eta_{\Theta(e)}(\ell) = 0 \).

**Proof:** Since \( \hat{\xi}_{\Theta(e)}(\ell) \supseteq \hat{\xi}_{\Theta(e)}(\omega_{n}), \forall n \in N \), and \( \eta_{\Theta(e)}(\ell) \leq \eta_{\Theta(e)}(\omega_{n}), \forall n \in N \), we have

\( \hat{\xi}_{\Theta(e)}(\ell) \supseteq \hat{\xi}_{\Theta(e)}(\omega_{n}), \forall n \in N \),  

(38)

\( \eta_{\Theta(e)}(\ell) \leq \eta_{\Theta(e)}(\omega_{n}), \forall n \in N \).
However, \( [1,1] \geq \hat{\varepsilon}_{\Theta(e)}(\ell) \geq \lim_{n \to \infty} \hat{\varepsilon}_{\Theta(e)}(\omega_n) = [1,1] \). Also, \( 0 \leq \eta_{\Theta(e)}(\ell) \leq \lim_{n \to \infty} \eta_{\Theta(e)}(\omega_n) = 0 \).

Therefore \( \hat{\varepsilon}_{\Theta(e)}(\ell) = [1,1] \) and \( \eta_{\Theta(e)}(\ell) = 0 \).

**Theorem 3.8:** Let \((\Gamma, e, \ell) \) be \((p-A), (\Theta, T) \in \Omega \) and \( e \in T \). If \((\Theta, T) \) is \((e-CSP - S)\) over \( \Gamma \), then \( \tilde{\varepsilon}_{\Theta(e)}(\omega) = \hat{\varepsilon}_{\Theta(e)}(\ell) \) and \( \tilde{\eta}_{\Theta(e)}(\omega) = \hat{\eta}_{\Theta(e)}(\ell) \) are \( \rho \)– subalgebras of \( \Gamma \).

**Proof:** Assume \( \omega, \theta \in \Gamma \) and \( \omega, \theta \in \tilde{\varepsilon}_{\Theta(e)}(\omega) \), then \( \tilde{\varepsilon}_{\Theta(e)}(\omega) = \hat{\varepsilon}_{\Theta(e)}(\ell) = \tilde{\varepsilon}_{\Theta(e)}(\theta) \). Hence,\( \tilde{\varepsilon}_{\Theta(e)}(\omega \circ \theta) \geq r \min \{ \tilde{\varepsilon}_{\Theta(e)}(\omega), \tilde{\varepsilon}_{\Theta(e)}(\theta) \} = r \min \{ \tilde{\varepsilon}_{\Theta(e)}(\ell), \tilde{\varepsilon}_{\Theta(e)}(\ell) \} = \tilde{\varepsilon}_{\Theta(e)}(\ell). \)

Also, let \( \omega, \theta \in \tilde{\eta}_{\Theta(e)}(\omega) \). Then we have \( \tilde{\eta}_{\Theta(e)}(\omega \circ \theta) \leq \max \{ \tilde{\eta}_{\Theta(e)}(\omega), \tilde{\eta}_{\Theta(e)}(\theta) \} = \max \{ \tilde{\eta}_{\Theta(e)}(\ell), \tilde{\eta}_{\Theta(e)}(\ell) \}. \)

By Proposition (3.7), we get \( \tilde{\varepsilon}_{\Theta(e)}(\omega \circ \theta) = \tilde{\eta}_{\Theta(e)}(\omega \circ \theta) \). Therefore \( \omega \circ \theta \in \tilde{\varepsilon}_{\Theta(e)}(\omega \circ \theta) \). Then \( \tilde{\varepsilon}_{\Theta(e)}(\omega \circ \theta) \) and \( \tilde{\eta}_{\Theta(e)}(\omega \circ \theta) \) are \( \rho \)– subalgebras of \( \Gamma \).

**Corollary 3.9:** If \((\Theta, T) \) is \((e-CSP - S)\) over \( \Gamma \), then the set \( \tilde{\varepsilon}_{\Theta(e)}(\omega) \cap \tilde{\eta}_{\Theta(e)}(\omega) \) is a \( \rho \)– subalgebra of \( \Gamma \).

**Proof:** The proof is straightforward.

**Theorem 3.10:** Let \((\Theta, T), (L, H) \in \Omega \) be a \((CSP - S)\). Then the \(R\)-intersection of \((\Theta, T)\) and \((L, H) \) is \((CSP - S)\) over \( \Gamma \).

**Proof:** Assume \((\Theta, T), (L, H) \in \Omega \) are \((CSP - S)\) and \((N, Z) = (\Theta, T) \cap_{R} (L, H), \) where \( Z = T \cup H \) and \( N(e) = \begin{cases} \Theta(e), & \text{if } e \in T \setminus H \\ L(e), & \text{if } e \in H \setminus T \\ \Theta(e) \cap \bigcup_{e \in Z} L(e), & \text{if } e \in T \cap H \end{cases} \)

Now, \( \forall e \in Z \), There are three cases: (i) \( e \in T \setminus H \), (ii) \( e \in H \setminus T \), (iii) \( e \in T \cap H \).

In case (i), we have

\[ \hat{\varepsilon}_{\Theta(e)}(\omega \circ \theta) = \hat{\varepsilon}_{\Theta(e)}(\omega \circ \theta) \geq r \min \{ \hat{\varepsilon}_{\Theta(e)}(\omega), \hat{\varepsilon}_{\Theta(e)}(\theta) \} = r \min \{ \hat{\varepsilon}_{\Theta(e)}(\omega), \hat{\varepsilon}_{\Theta(e)}(\theta) \}. \]
\[ \eta_{N(e)}(\omega \circ \mathcal{G}) = \eta_{\Theta(e)}(\omega \circ \mathcal{G}) \leq \max\{ \eta_{\Theta(e)}(\omega), \eta_{\Theta(e)}(\mathcal{G}) \} = \max\{ \eta_{N(e)}(\omega), \eta_{N(e)}(\mathcal{G}) \}, \forall \omega, \mathcal{G} \in \Gamma. \]

In case (ii), we have
\[ \hat{\eta}_{N(e)}(\omega \circ \mathcal{G}) = \hat{\eta}_{L(e)}(\omega \circ \mathcal{G}) \geq r \min\{ \hat{\eta}_{\Theta(e)}(\omega), \hat{\eta}_{L(e)}(\mathcal{G}) \} = r \min\{ \hat{\eta}_{N(e)}(\omega), \hat{\eta}_{N(e)}(\mathcal{G}) \}, \quad (43) \]
\[ \eta_{N(e)}(\omega \circ \mathcal{G}) = \eta_{L(e)}(\omega \circ \mathcal{G}) \leq \max\{ \eta_{L(e)}(\omega), \eta_{L(e)}(\mathcal{G}) \} = \max\{ \eta_{N(e)}(\omega), \eta_{N(e)}(\mathcal{G}) \}, \forall \omega, \mathcal{G} \in \Gamma. \]

In case (iii), we have
\[ \hat{\eta}_{N(e)}(\omega \circ \mathcal{G}) = (\hat{\eta}_{\Theta(e)} \cap_{R} \hat{\eta}_{L(e)})(\omega \circ \mathcal{G}) = r \min\{ \hat{\eta}_{\Theta(e)}(\omega), \hat{\eta}_{L(e)}(\mathcal{G}) \} \]
\[ = r \min\{ r \min\{ \hat{\eta}_{\Theta(e)}(\omega), \hat{\eta}_{L(e)}(\mathcal{G}) \}, \hat{\eta}_{L(e)}(\mathcal{G}) \} \]
\[ = r \min\{ \hat{\eta}_{\Theta(e)}(\omega), \hat{\eta}_{L(e)}(\mathcal{G}) \} \]
\[ = r \min\{ \hat{\eta}_{\Theta(e)}(\omega), \hat{\eta}_{L(e)}(\mathcal{G}) \}. \quad (44) \]
\[ \eta_{N(e)}(\omega \circ \mathcal{G}) = (\eta_{\Theta(e)} \cap_{R} \eta_{L(e)})(\omega \circ \mathcal{G}) = \eta_{\Theta(e)}(\omega \circ \mathcal{G}) \lor \eta_{L(e)}(\omega \circ \mathcal{G}) \]
\[ = \max\{ \eta_{\Theta(e)}(\omega), \eta_{L(e)}(\mathcal{G}) \} \]
\[ \leq \max\{ \max\{ \eta_{\Theta(e)}(\omega), \eta_{L(e)}(\mathcal{G}) \}, \max\{ \eta_{L(e)}(\omega), \eta_{L(e)}(\mathcal{G}) \} \} \quad (45) \]
\[ = \max\{ \max\{ \eta_{\Theta(e)}(\omega), \eta_{L(e)}(\mathcal{G}) \}, \max\{ \eta_{L(e)}(\omega), \eta_{L(e)}(\mathcal{G}) \} \}
\[ = \max\{ \eta_{N(e)}(\omega), \eta_{N(e)}(\mathcal{G}) \}, \forall \omega, \mathcal{G} \in \Gamma. \]

Hence \((N, Z) = (\Theta, T) \cap_{R} (L, H)\) is a \((\text{CSP} - \text{SA})\) over \(\Gamma\).

**Corollary 3.11:** Let \(\mathcal{M} = \{(L, H)_{\alpha} \in \Omega \mid \alpha \in \Psi\}\) be a collection of cubic soft \(\rho - \)subalgebras over \(\Gamma\).

Then the \(R\)-intersection \(\cap_{R} \{(L, H)_{\alpha} \}_{\alpha \in \Psi}\) is a \((\text{CSP} - \text{SA})\) over \(\Gamma\).

**Proof:** The proof is straightforward from (2.7) and (3.10).

4. **Conclusion and Future Work:**

In this paper, we investigated new notions of cubic soft subalgebras like \((e - \text{CSP} - \text{SA})\) and \((\text{CSP} - \text{SA})\). Moreover, some of their basic characteristics are given. In future work, we will ask and discuss some questions of our notions as following:

1. Let \((\Theta, T)\) be a \((\text{CSP} - \text{SA})\) and \((L, H)\) is an \(R\)-cubic soft subset of \((\Theta, T)\), is \((L, H)\) a \((\text{CSP} - \text{SA})\) ?

2. Let \((\Theta, T)\) be a \((\text{CSP} - \text{SA})\) and \((L, H)\) is an \(P\)-cubic soft subset of \((\Theta, T)\), is \((L, H)\) a \((\text{CSP} - \text{SA})\) ?

3. Is the \(P\)-union (resp. \(P\)-intersection) of two a cubic soft \(\rho - \)subalgebras is \((\text{CSP} - \text{SA})\)?
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