Radio emission of magnetars driven by the quasi-linear diffusion.

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ABSTRACT
In this paper we study the possibility of generation of electromagnetic waves in the magnetospheres of radio magnetars by means of the quasi-linear diffusion (QLD). Considering the magnetosphere composed of the so-called beam and the plasma components respectively, we argue that the frozen-in condition will inevitably lead to the generation of the unstable cyclotron modes. These modes, via the QLD, will in turn influence the particle distribution function, leading to certain values of the pitch angles, thus to an efficient synchrotron mechanism, producing radio photons. We show that for three known radio magnetars the QLD might be a realistic mechanism for producing photons in the radio band.

Key words: plasmas - radiation mechanisms: non-thermal -

1 INTRODUCTION
Pulsars since their discovery deserve a great attention and many aspects of their nature still remain unclear. One of the peculiar class of the pulsars are characterized by long period of rotation, which in turn leads to very strong magnetic fields exceeding the so-called Schwinger limit, $B_{cr} \approx 4.41 \times 10^{13}$G. For such a huge magnetic field they are called magnetars. Despite some predictions, that magnetars must be dark in the radio band (e.g. Baring & Harding 1998), it is now observationally evident that some of the magnetars are detected in the radio band as well (Olausen & Kaspi 2014). According to the McGill catalogue, there are six magnetars that exhibit radio spectra. In particular, the following is the list of the objects seen in the indicated radio band: 1E 1547.0-5408: [18.6; 18.5; 18.5; 45]GHz, PSR J1622-4950: [14.0; 17; 24]GHz, and SGR J1745-2900: [12.8; 14.620; 22]GHz. There are also three more magnetars, for which the radio emission has been announced: 4U 0142+61: 0.11GHz, XTE J1810-197: [0.06; 0.35; 9.42; 88.5; 144]GHz and SGR 1916-0143: [0.06; 0.11]GHz (Malofeev et al. 2010), although the detections have not yet been confirmed by another Observatory.

In this paper, we focus on the radio-loud pulsars to see the possible role of the quasi-linear diffusion (QLD) in generation of radio emission. For explaining the radiation in the radio band, we account for the synchrotron mechanism. Without dissipation the pitch angles very soon become zero, leading to the one dimensional distribution function of particles and as a result the synchrotron mechanism completely vanishes. In this paper we rely on the pulsar emission model developed by Machabeli & Usov (1979); Lominadze et al. (1979). According to this approach, in the pulsar magnetospheres the cyclotron instability appears (Kazbegi et al. 1992), which during the quasi-linear stage, causes a diffusion of particles along and across the magnetic field lines, leading to the required balance.

The mechanism of QLD was applied to pulsars and active galactic nuclei in a series of papers: (Malov & Machabeli 2001; Osmanov 2013; Osmanov & Chkheidze 2013; Chkheidze et al. 2013; Chkheidze & Osmanov 2013). In the framework of the mechanism, the synchrotron radiation appears by means of the feedback of the cyclotron waves on relativistic particles due to the diffusion, and as a result, the pitch angles are arranged according to the aforementioned balance. Generally speaking, during the QLD, the physical system will be characterized by two processes: (a) generation of the cyclotron waves and b) the synchrotron mechanism.

In this paper we consider the magnetospheric parameters of radio-loud magnetars to investigate the role of the QLD in generation of the radio waves. Thus the paper is organized as follows: in Section 2 we introduce the mechanism of the QLD, in Sect. 3 we apply the method to magnetars and obtain results, and in Sect. 4 we summarize them.
2 MAIN CONSIDERATION

We assume that the pulsar’s magnetosphere is composed of the so-called primary beam with the Lorentz factor, $\gamma_b$ and the bulk component with the Lorentz factor, $\gamma_b$ (Osmanov & Chkheidze 2013; Chkheidze et al. 2013; Chkheidze & Osmanov 2013). By Kazbegi et al. (1992) it was shown that in the pulsar magnetospheric plasmas, which satisfy the frozen-in condition, the anomalous Doppler effect induces resonance unstable cyclotron waves

$$\omega - k_z c - k_x u_x - \omega_B \gamma_b = 0$$

with the corresponding frequency (Malov & Machabeli 2001)

$$\nu = \frac{\omega_p}{2\pi \delta \gamma_b}$$

$$\delta = \frac{\omega_p^2}{4\omega_B^2 \gamma_b^2}$$

where $k_z$ is the longitudinal (along the magnetic field lines) component of the wave vector, $u_x \approx c^2 \omega_b/(\rho \omega_p)$ is the so-called curvature drift velocity, $c$ is the speed of light, $\rho$ is the magnetic fields’ curvature radius, $k_x$ is the wave vector’s component along the drift, $\omega_p \equiv eB/mc$ is the cyclotron frequency, $e$ and $m$ are the electron’s charge and the rest mass respectively, $\omega_p \equiv \sqrt{4\pi n_p e^2/m}$ is the plasma frequency, $n_p$ is the plasma number density and $B$ is the magnetic induction. Inside the magnetosphere of magnetars the induction of magnetic field on any lengthscale is given by

$$B \approx 3.2 \times 10^{14} \times \left( \frac{P}{10^8}\right)^{1/2} \times \left( \frac{\rho}{10^{-11}\text{g cm}^{-3}}\right)^{1/2} \times \left( \frac{R_{\star}}{R}\right)^3 G,$$

where $R_{\star} \approx 10^6\text{cm}$ is the magnetar’s radius, $R$ is the distance from the star’s centre, $P$ is the rotation period and $\rho$ - the slow down rate of the magnetar. As it is clear from equation $3$, we have normalized $P$ and $\rho$ on their average values and as a result, the magnetic field becomes higher than the Schwinger limit. It is worth noting that the aforementioned expression is derived from the standard assumption that the spin-down rate is caused by the magneto-dipole emission. Although based on observations of two anomalous pulsars Malofeev et al. (2006) suggest to revise the magnetar model itself. According to this approach magnetars are young pulsars with relatively low magnetic field.

In order to study the development of the QLD, one should note that two major forces control dissipation. When particles emit in the synchrotron regime, they undergo the force perpendicular to the magnetic field lines. The wave excitation leads to a redistribution process of the particles via the QLD, which is described by the following kinetic equation (Machabeli & Usov 1974)

$$\frac{df}{dt} + \frac{1}{p_{\perp}} \frac{\partial}{\partial p_{\perp}} \left( p_{\perp} [F_{\perp} + G_{\perp}] f \right) =$$

$$= \frac{1}{p_{\perp}} \frac{\partial}{\partial p_{\perp}} \left( p_{\perp} D_{\perp} \frac{\partial f}{\partial p_{\perp}} \right),$$

where $f$ is the distribution function of the zeroth order, $D_{\perp} = D_0 |E_k|^2$, is the diffusion coefficient, $|E_k|^2$, is the energy density per unit of wavelength and $D = e^2/8c$ (Chkheidze et al. 2010). For estimating $|E_k|^2$, it is natural to assume that half of the plasma energy density, $mc^2 n_p \gamma_b/2$ converts to the energy density of the waves $|E_k|^2 k$ (Machabeli & Usov 1971), then for $|E_k|^2$ we obtain

$$|E_k|^2 = \frac{mc^2 n_p \gamma_b}{4\pi \nu},$$

where $n_p$ is the number density of the beam.

It is evident from the model that it depends on relativistic effects of electrons. Therefore, one has to estimate the maximum possible values of particles’ Lorentz factors. For this purpose we apply the method of centrifugal acceleration (Osmanov et al. 2007), where, based on the fact that plasma particles in strong magnetic field are in the frozen-in condition, they follow magnetic field lines. On the other hand, if these field lines are corotating, the particles will undergo centrifugal force, efficiently amplified close to the light cylinder surface (a hypothetical zone, where the linear velocity of rotation exactly equals the speed of light), leading to extremely efficient acceleration in the mentioned area. It has been shown by Rieger & Mannheim (2000) that the Lorentz factors of centrifugally accelerated electrons depend on the radial distance as \( \gamma \approx 1 \left( 1 - R^2/R_{lc}^2 \right)^{-1/2} \), where $R_{lc} \equiv c P/(2 \pi)$ is the light cylinder radius. As it is evident from this expression, by reaching the light cylinder the energy must tend to infinity. In real astrophysical situations, particles, apart from the centrifugal force undergo other forces reducing the acceleration. In particular, the electrons encounter soft photons and by means of the inverse Compton (IC) scattering they lose energy. Initially energy losses are negligible and acceleration is dominant, but in due course of time energy gain and energy losses will balance each other and further acceleration will be stopped.

One can show that the maximum Lorentz factor provided by the IC losses is given by (Osmanov et al. 2007)

$$\gamma_{IC} \approx 10^{14} \left[ \frac{6 \Omega}{U_{rad}(R_{lc})} \right]^2 \approx$$

$$\approx 1.05 \times 10^{15} \times \left( \frac{P}{10^8}\right)^2 \times \left( \frac{10^{33} \text{ergs s}^{-1}}{L} \right)^{1/2},$$

where $\Omega = 2 \pi / P$ is neutron star’s angular velocity of rotation, $U_{rad} = L/(4 \pi R_{lc}^2 c)$ is the radiation energy density on the light cylinder lengthscales and $L$ is the luminosity of the magnetar.

Another mechanism, responsible for limiting the maximum attainable energies is the breakdown of the bead on the wire (BBW) approximation. In strong magnetic fields particles follow the field lines, also gyrating around them. Apart from the Lorentz force, that is responsible for gyration the electrons also undergo the force perpendicular
to the magnetic field lines, \( F_\perp = m\Omega (2\gamma \mathbf{\hat{e}}_\parallel \times \mathbf{B} + \mathbf{B}_\perp) \). Unlike this force, the Lorentz force, \( \mathbf{F}_L = \frac{e}{c} \mathbf{v} \times \mathbf{B} \), in due course of motion changes its direction with respect to \( F_\perp \). Since the Lorentz force is responsible for binding the particle close to the field lines, the electrons follow them until \( F_\perp \) exceeds \( F_L \). When this happens, the head-on-the-wire approximation is no longer valid and acceleration continues, limiting the maximum attainable energies of particles. As it has been shown in (Rieger & Mannheim 2004) and generalized by Osmanov et al. (2007) the corresponding maximum Lorentz factor is given by

\[
\gamma_{BBW} \approx \left( \frac{B_0 e}{2m c} \right)^{2/3} \approx 1.19 \times 10^5 \times \left( \frac{P}{10^8} \right)^{-3/2} \times \left( \frac{\dot{P}}{10^{-11} \text{s}^{-1}} \right)^{1/3}. \quad (9)
\]

We see from the aforementioned expressions that maximum Lorentz factor governed by the IC mechanism is greater than that of the BBW process. Therefore, the mechanism responsible for particle acceleration in the magnetospheres of magnetars is the BBW and the corresponding value of the Lorentz factor for the mentioned parameters is of the order of \( 10^5 \). This is the primary beam with the Goldreich-Julian number density (Goldreich & Julian 1969)

\[
n_{GJ} = \frac{B}{P c e}. \quad (10)
\]

It is worth noting that curvature emission also can limit the maximum energies of electrons, but like the IC mechanism it is also small compared to the BBW.

Turning to equations (10) and by taking into account the relations \( \psi \equiv p_\perp/p_\parallel, \ t = m c \gamma_\perp \), one can estimate the following ratio

\[
\frac{F_\perp}{G_\perp} \approx 3.2 \times 10^{-3} \gamma_\perp^2 \times \left( \frac{P}{10^8} \right)^{-11/2} \times \left( \frac{\dot{P}}{10^{-11} \text{s}^{-1}} \right)^{4/3}. \quad (11)
\]

It is clear from equation (11) that for physically reasonable parameters, one can neglect the transversal component of the radiation reaction force. This regime differs from that of consideration in our previous study (Chkheidze et al. 2013, 2011), because, as it is evident from the above equation, the ratio is very sensitive to the period of rotation and for long period neutron stars, unlike the previously studied millisecond pulsars, the radiative force becomes small compared to \( G_\perp \).

Therefore, equation (6) reduces to

\[
\frac{\partial f}{\partial t} + \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} (p_\perp G_\perp f) = \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} \left( p_\perp D_\perp f \right). \quad (12)
\]

Equation (12) makes evident that two major factors compete in this "game". On the one hand, the force responsible for conservation of the adiabatic invariant attempting to decrease the transversal momentum (thus the pitch angle), whereas the diffusion process, by means of the feedback of the cyclotron waves, attempts to increase the transversal momentum. Dynamically this process saturates when the aforementioned factors balance each other. Therefore, it is natural to study the stationary regime, \( \partial f / \partial t = 0 \) and examine a saturated state of the distribution function. After imposing the condition \( \partial f / \partial t = 0 \) on Eq. (12) one can straightforwardly solve it

\[
f(p_\perp) = C \exp \left( \int \frac{G_\perp}{D_\perp} dp_\perp \right) = C e^{\left( \frac{\psi}{\tau_{\gamma_0}} \right)^2}, \quad (13)
\]

where \( C = \text{const} \) and

\[
p_{\gamma_0} \equiv \left( \frac{2p D_\perp}{c} \right)^{1/2}. \quad (14)
\]

Since \( f \) is a continuous function of the transversal momentum, it is natural to examine an average value of it and estimate the corresponding mean value of the pitch angle,

\[
\bar{\psi} = \frac{1}{p_{\parallel}} \int_0^\infty f(p_\perp) dp_\perp \approx \frac{1}{\sqrt{\pi}} \frac{p_{\gamma_0}}{p_{\parallel}}. \quad (15)
\]

As we see, the QLD leads to a certain distribution of particles with the pitch angles, which will inevitably result in the synchrotron radiation mechanism with the following frequency (e.g. Rybicki & Lightman 1979)

\[
\nu_{\text{syn}} \approx 2.9 \times 10^{-3} B_0^{-2} \sin \psi \text{GHz}. \quad (16)
\]

### 3 DISCUSSION

In this section we will apply the mechanism of the QLD to the long period pulsars for studying the possibility of generation of radio waves. In the framework of the proposed model, the QLD is provided by the unstable cyclotron waves. Therefore, it is important to estimate the growth rate of the cyclotron instability. Kazbegi et al. (1992) have shown that for \( \gamma_\perp/(2 \omega_\parallel) \ll \delta \) (which is the case) the increment characterizing amplification of the cyclotron waves is given by

\[
\Gamma = \frac{\omega_b^2}{2 \gamma_\perp}, \quad (17)
\]

where \( \omega_b = \sqrt{4 \pi n_e e^2 / m} \) is the plasma frequency corresponding to the beam component. By considering mildly relativistic particles of the plasma component with \( \gamma_p = 2 \) and the beam component with \( \gamma_\perp = [10^2 - 10^5] \), by taking into account that the energy is uniformly distributed, \( n_\gamma \approx n_p \gamma_\perp \approx \gamma_{GJ}/\gamma_{BBW} \), one can show that for the aforementioned parameters, the growth rate is very high \(~ 10^6 - 10^9 \text{s}^{-1} \). Therefore, the corresponding time-scale, \( \tau \approx 1/\Gamma \), will be in the following interval \(~ 10^{-8} - 10^{-5} \text{s} \). On the other hand, the kinematic, or escape time-scale, \( \tau_{\text{esc}} \approx R_{\text{BB}}/c \) is of the order of \(~ 1 \text{s} \). As we see, the instability time-scale exceeds by many orders of magnitude the kinematic time-scale, which means that the process is extremely efficient and therefore, physically feasible.

As we have shown in the previous section, the cyclotron waves inevitably influence the particle distribution via diffusion (feedback mechanism) leading to certain pitch angles (see equation (13)), which in turn leads to the synchrotron radiation. In Fig. 1 we show the dependence of synchrotron photon frequency on the values of Lorentz factors for three radio magnetars: SGR J1745-2900; PSR J1622-4950 and 1E 1547.0-5408. The set of parameters is: \( \gamma_\perp = 2, R_{\text{BB}} \approx 10^8 \text{cm} \),
The main aspects of this work can be summarized as follows:

(i) In this paper we examined the role of the QLD in producing radio emission in the magnetospheres of three known radio magnetars: SGR J1745-2900; PSR J1622-4950 and 1E 1547.0-5408.

(ii) Considering the anomalous Doppler effect, which leads to the unstable cyclotron waves, we have studied the feedback of these waves on a distribution of relativistic particles. Solving the equation governing the QLD, the corresponding expression of the average value of the pitch angle is derived and analysed for physically reasonable parameters.

(iii) We have shown that for appropriate parameters \( \gamma_p = 2, \gamma_h = [3.5 \times 10^2 - 1.7 \times 10^3] \) the QLD might provide a generation of radio emission in plasmas placed on the light cylinder distances and might explain the observed frequencies in the interval \( \sim [1 - 10] \text{GHz} \).

The present investigation shows that the QLD is a mechanism that can explain a generation of radio waves in three confirmed radio magnetars (Olausen & Kaspi 2014). The aim of this paper was to examine only one part of the problem, although a complete study requires to investigate the spectral pattern of emission as well. In the standard theory of the synchrotron emission it is assumed that due to the chaotic character of the magnetic field lines (Bekefi & Barrett 1977; Ginzburg 1981), the pitch angles lie in a broad interval (from 0 to \( \pi/2 \)). In our model the distribution function of particles is strongly influenced by the process of the QLD and as a result the pitch angles are restricted by the balance of dissipative and diffusive factors. This will inevitably lead to a spectral pattern, different from that of Bekefi & Barrett (1977); Ginzburg (1981). Therefore, we will investigate this problem in future studies.

In the framework of the model the synchrotron mechanism is maintained by means of the induced cyclotron instability. On the other hand, it is well known that in certain cases unstable Cherenkov-drift modes might do the same work as well. In particular, Osmanov & Chkheidze (2013), examining the generation of synchrotron radiation by means of the feedback of Cherenkov-drift modes on particle distribution in magnetospheres of AGN, have shown high efficiency of the process. Therefore, sooner or later we are going to study this particular problem in the context of magnetars.

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REFERENCES

Baring, M. G. & Harding, A. K., 1998, ApJ, 507, L55
Bekefi, G., & Barrett, A. H. 1977, Electromagnetic vibrations, waves and radiation. MIT Press, Cambridge, MA

Figure 1. Synchrotron photon frequency as a function of \( \gamma_h \) for three radio magnetars: SGR J1745-2900; PSR J1622-4950 (top panel) and 1E 1547.0-5408 (bottom panel). The set of parameters is: \( \gamma_p = 2, R_s \approx 10^6 \text{cm}, P = 3.76 \times 10^{-12} \text{s}^{-1} \) for SGR J1745-2900; \( P = 4.33 \times 10^{-11} \text{s}^{-1} \) for PSR J1622-4950 and \( P = 2.07 \times 10^{-11} \text{s}^{-1} \) for 1E 1547.0-5408. It is clear from the plot that \( \nu_{\text{syn}} \) is a continuously increasing function of \( \gamma_h \)’s. This is a direct consequence of equations (14,15). In particular, according to equation (15) the photon frequency behaves as to be \( \nu_{\text{syn}} \sim \gamma_h^2 \psi \). On the other hand, by taking into account the relation \( D_{\perp,\parallel} = D \delta|E_k|^2 \), one can see from equations (14,15) that \( \nu \sim \gamma_h^{3/2} \), which by combining with equation (16) leads to the following dependence \( \nu_{\text{syn}} \sim \gamma_h^{3/2} \).

As it is clear from the plots, for physically reasonable parameters, the cyclotron instability can lead to generation of radio waves in the observed interval of frequencies. We show only relatively low range of frequencies: \( \sim [1 - 10] \text{GHz} \) and it is quite straightforward to find physical parameters for higher frequencies of radio emission. Therefore, the present investigation shows that the contribution of QLD in generation of radio waves in radio magnetars might be important.
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Chkheidze, N., Machabeli, G. & Osmanov, Z., 2013, ApJ, 773, 143
Chkheidze, N., Machabeli, G. & Osmanov, Z., 2010, ApJ, 721, 318
Chkheidze, N. & Osmanov, Z., 2013, MNRAS, 419, 2391
Ginzburg V.L., 1981, "Teoreticheskaya Fizika i Astrofizika", Nauka M.
Goldreich, P. & Julian, W. H, 1969, ApJ, 157, 869
Kazbegi, A.Z., Machabeli, G.Z & Melikidze, G.I., 1992, in Proc. IAU Colloq. 128, The Magnetospheric Structure and Emission Mechanisms of Radio Pulsars, ed. T.H. Hankins, J:M: Rankin & J:A: Gil (Zielona Gora), 232
Landau L.D. & Lifshitz E.M., 1971, The Classical Theory of Fields. Pergamon Press, London
Rybicki G.B. & Lightman A. P., 1979, Radiative Processes in Astrophysics. Wiley, New York
Lominadze, J.G., Machabeli, G.Z. & Mikhailovsky, A.B., 1979, J. Phys. Colloq., 40, Phenomena in Ionized Gases., p. 713
Machabeli G.Z. & Usov V.V., 1979, Pis'ma Astron. Zh., 5, 445
Malov, I.F. & Machabeli, G.Z., 2001, ApJ, 554, 587
Malofeev et al., 2005, Astron. Rep., 49, 242
Malofeev, V. M., Teplykh, D. A. & Logvinenko, S. V., 2012, Astron. Rep., 56, 35
Olausen, S. A. & Kaspi, V. M., 2014, ApJ, 507, L55
Osmanov, Z., 2013, Int. J. Mod. Phys. D, 22, 1350081
Osmanov, Z. & Chkheidze, N., 2013, ApJ, 764, 59
Osmanov, Z., Rogava, A.S. & Bodo, G., 2007, A&A, 470, 395
Rieger, F.M. & Mannheim, K., 2000, A&A, 353, 473

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