Numerical research of placement problem on lines with forbidden zones and routing communications

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Abstract. This paper discusses of placement problem of connected equipment (facilities) on parallel lines with forbidden zones. You can’t place equipment inside of forbidden zones. The facilities are connected among themselves and with zones by some kind of communications. The communications (routing of communications) between the facilities and between facilities and zones located on adjacent lines pass through the vertical equipment (viaduct). The target is to find the facilities placement on lines outside zones. Facilities do not must intersect among themselves. It is necessary to minimize the total cost of communications facilities among themselves and with zones. Practical usage of this problem is found, for example, in technology in the design of engineering devices. We consider cases when the viaduct placement is fixed and when its optimal placement is found. The goal function’s value for a fixed viaduct with a specified placement step is compared with the goal function for optimal placement of the viaduct. The results of numerical experiments are presented.

1. Introduction

Placement problems are well-known class problems in operations research. Variants of the problems have many applications, for example, in projection of electronic devices and placement of equipment. The classification of such problems is given, for example, in [3, 4, 7]. Urgency of these problems requires the construction of new mathematical models and of productive solution algorithms. This article discusses optimal placement of facilities that are connected by certain communications on lines with forbidden zones and a special way of laying communications.

Proper placement of equipment is the main link in the organization of safe operation of the production site and shop. When placing equipment, you must observe the minimum gaps between the machines, between the machines and individual elements of the building, and correctly determine the width of passages and driveways. Failure to comply with the rules and regulations for placing equipment leads to cluttering of premises and injuries. The placement of equipment on the floor area of a shop or site is determined mainly by the technological process and local conditions. In automated production (complex automatic plants or workshops, automatic lines, in-line production), the equipment is placed in a single chain accordingly the technological process, observing the distances between the equipment and the structural elements of the building. On automatic and long-distance production lines, transition bridges (viaducts) are arranged for switching from one line to another or for laying communications between lines.

Optimal placement of interconnected facilities is an important class of placement problems. The problem of placing interconnected point facilities in metric space with fixed facilities often
is called the Weber (Fermat) problem [1, 5]. The key parameter of the specified problem is the presence of connections between facilities. The connections can be, for example, the flows any product. In the general statement, the problem can be formulated as follows. Some area has fixed facilities. There are new facilities that are connected to each other and to fixed facilities by some connections (communications). It is need to place new facilities in the specified area and to minimize the total cost of connections facilities among themselves. The classical Weber problem without restrictions on their placement [9] is well studied in the literature. If rectangular metric is used, the problem in the plane is decomposed into two independent linear programming problems. Also a number of streaming algorithms are proposed to solve this problem. Weber’s problem with various constraints was studied in [4].

Designing the placement of technological equipment in production shops is an important aspect that allows you to take into account the effective use of production space. One of the features of the problems under consideration is the need to take into account various restrictions. For example, when developing shop floor plans, equipment should be placed regularly, often along lines. This makes it possible to better zone the territory and for convenient operation and maintenance of equipment. One–Dimensional Space Allocation Problem (ODSAP) is an optimal placement’s problem of interconnected rectilinear facilities on a line. It is necessary to minimize the total cost of communications facilities among themselves. There are many algorithms to solve ODSAP, for example, dynamic programming, branch and bounds, and others [6, 8, 10]. Problems with forbidden zones (areas where facilities can’t be placed) are a subclass of placement problems. These problems for large facilities and with the conditions for regular placement are not well understood. Problems on line without other restrictions on placement more often are researched [2, 6, 8, 10]. The problem of optimal placement of rectangular interconnected facilities on the line with forbidden zones was explored in [11, 13, 15]. The original problem was decomposed into a series of discrete subproblems of lower dimension. For the exact solution of the subproblems the method of branch and bound was used [12], and for the approximate solution polynomial heuristic procedure was applied [11]. An overview of the results of research on the problem, taking into account combinatorial methods and integer programming, was provided in [13]. In [15], a generalization of the One–Dimensional Space Allocation Problem (ODSAP) with forbidden zones was considered. The structure of relations between facilities was defined as a directed acyclic graph. To find a local optimum, a polynomial procedure was developed when the graph of connections between facilities is a composition of rooted trees and parallel–serial graphs. In [13] were describes examples of models, methods and algorithms for exact and approximate solutions, and results of computational experiments for a generalization of the ODSAP.

In this work, the problem of placing connected rectilinear facilities in plane on parallel lines with rectilinear forbidden zones is investigated. The positions of the lines are fixed. Straight passages between the lines for easy maintenance of facilities are provided. Communications connect the centers of the facilities and the centers of zones. The laying of communications between facilities, as well as between facilities and zones located on adjacent lines, passes through a vertical viaduct. We need to place the facilities on lines so that the total cost of connecting facilities to each other and to the zones is minimal. A mathematical model of nonlinear integer programming for finding a local minimum of the problem is constructed. The solution of the problem was found using the built model and IBM ILOG CPLEX package. We consider cases when the viaduct placement is fixed and when its optimal placement is found. For a problem with a fixed viaduct, the step of its placement was set. The value of the goal function and the running time for a fixed viaduct are compared with the goal function and running time for the case of optimal placement of the viaduct. The results of numerical experiments are shown.
2. Problem statement

We present a meaningful statement of the problem and describe its mathematical model [14]. Several segments of the same length $LS$ parallel to the axis $OX$ are located on the plane. The segments contain rectangular areas with sides parallel to the coordinate axes (forbidden zones). For example a forbidden area is some technological equipment. For simplicity’s sake we will refer to the segments as lines. We assume that the left borders of all lines have coordinates zero. In addition, the lines are arranged in such a way that the distance between adjacent lines allows you to install straight passages for ease of maintenance of technological equipment and safety of its operation. There are new rectangular facilities. Centers of the facilities are connected among themselves and with the centers of forbidden zones by some communications. Communications the facilities among themselves and with forbidden zones located on adjacent lines are laid through special technological equipment (viaduct). The goal is to find how to place the facilities among themselves and with zones if they are located on different lines. Let $LI_i$ be the facilities that need to be placed on lines and forbidden zones, respectively. Using $(x_i, y_i)$ and $(b_{1j}, b_{2j})$ we denote the coordinates of the centers of facilities and zones, respectively. Also using $w_{ij} \geq 0$, $u_{ik} \geq 0$ we denote the costs of communications between $X_i$ and $F_j$, $X_i$ and $X_k$ for $i, k \in I$, $j \in J$, and $i < k$. Using $JL_i$ we denote the set of numbers of zones on the line with number $t$, $t = 1, 2$. So, $J = JL_1 \cup JL_2$. If $F_j$ is placed on the line with number $t$, then $b_{2j} = Ly_t$. It is need to place the facilities $X_1, \ldots, X_n$ on the lines. They do not must intersect between themselves and with zones $F_1, \ldots, F_m$. The total cost of communications must be minimal.

Next, we introduce Boolean variables: $z_{it}$ for $i \in I$, $t = 1, 2$, such that $z_{it} = 1$ if $X_i$ is on the line with the number $t$, otherwise $z_{it} = 0$. Then alternative conditions for placing facilities on lines can write as shown below.

Using $V$ we denote the viaduct through which communications are laid that connect facilities with each other and with zones if they are located on different lines. Let $\delta y$ be the height of the viaduct $V$. Then $\delta y = Ly_2 - Ly_1$ and it is a constant value. Using $x_0$ we denote the abscissa of the viaduct placement.

An example of the problem for two facilities and four zones is shown in Figure 1. Forbidden zones are painted as shaded rectangles. Communications between $X_1$ and other facilities and zones are shown as segments.

Then we have a mathematical model of nonlinear programming as shown:

$$G(x) = \sum_{t=1}^{2} \sum_{i \in I} \sum_{j \in JL_i} w_{ij} \left( z_{it} |x_i - b_{1j}| + (1 - z_{it})(|x_i - x_0| + |b_{1j} - x_0| + \delta y) \right) +$$

$$+ \sum_{t=1}^{2} \sum_{i \in I} \sum_{k \in I, i \neq k} u_{ik} \left( z_{it} z_{kt} |x_i - x_k| + z_{it}(1 - z_{kt})(|x_i - x_0| + |x_k - x_0| + \delta y) \right) \rightarrow \min,$$

under constraints

$$|x_i - b_{1j}| \geq z_{it} \frac{l_i + p_j}{2}, \quad i \in I, \quad j \in JL_t, \quad t = 1, 2; \quad (2)$$

$$|x_i - x_k| \geq (z_{it} + z_{kt} - 1) \frac{l_i + l_k}{2}, \quad i, k \in I, \quad i < k, \quad t = 1, 2; \quad (3)$$

$$\frac{l_i}{2} \leq x_i \leq LS - \frac{l_i}{2}, \quad i \in I; \quad (4)$$
\[ y_i = \sum_{t=1}^{2} z_{it} L y_t, \quad i \in I; \]  
(5) ∑
\[ \sum_{t=1}^{2} z_{it} = 1, \quad i \in I; \]  
(6) \[ z_{it} \in \{0, 1\}, \quad i \in I, \quad t = 1, 2, \]  
(7)  

where (1) is the total cost of communications facilities with each other and with zones. The goal function \( G(x) \) depends only on the \( x \) coordinates.

\[ y = \sum_{t=1}^{2} z_{it} L y_t, \quad i \in I; \]  
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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The example of the problem.}
\end{figure}

Note that if the facilities \( X_i \) and \( X_k \) are placed on one line, the cost of communication between them is determined by the expression \( u_{ik}(|x_i - x_k|) \) and it is included to the goal function \( G(x) \). If these facilities are placed on different lines, the cost of communication between them is \( u_{ik}(|x_i - x_0| + |x_k - x_0| + \delta y) \). For the facility \( X_i \) and the zone \( F_j \) the contribution to \( G(x) \) is set analogously. The expressions (2) are disjoint conditions between facilities and zones. The expressions (3) are disjoint conditions between facilities among themselves. The expressions (4) and (5) are the conditions for placing of facilities on the lines. The expression (6) is the condition for placing a facility on one line.

In [14] the problem of placement of the facilities on the parallel lines taking into fixed viaduct was studied. Some properties of the problem were provided. Let's briefly note the main of these properties. Namely a feasible area of the problem consists of \( r \) disjoint segments \( B_k \) (blocks) with lengths \( L_k, k = 1, \ldots, r \). It is necessary to place the facilities \( X_i, i \in I, \) in the blocks. Problem (1)–(7) is NP–hard, just as the problem for one line is NP–hard. Any valid solution to the problem can be obtained as a solution to the problem of one–dimensional packing of facilities \( X_i \) with lengths \( l_i, i \in I, \) into containers \( B_k \) with sizes \( L_k, k = 1, \ldots, r \).

Let \( x = (x_1, \ldots, x_n) \) is a feasible placement of the facilities. Using \( I_k(x) \) we denote the set of numbers of the facilities in \( B_k, I = \bigcup_{k=1}^{r} I_k(x) \). Let \( n_k \) be the capacity of set \( I_k(x) \). A remainder in the block \( B_k \) is a dummy segment. Using \( H_k(x) \) we mark the remainders in \( B_k \) for \( x \), then \( |H_k(x)| \leq n_k + 1 \).

For a feasible solution \( x \) of a problem for one line, we can find a feasible solution \( x' \) such that \( |H_k(x')| \leq 1, k = 1, \ldots, r \). The goal function’s value for \( x' \) will be no worse than for \( x \), so
\[ G(x') \leq G(x) \]. Thus, we can consider at most one remainder in arbitrary block \( B_k \). As a result, the original continuous problem with an infinite number of feasible solutions can be reduced to the discrete problem [14].

Many of the noted properties of the problem under consideration are based on the use of the relative order of the facilities between themselves and in relation to zones. This idea is the basis for proving the validity of the problem properties. The cases, when the viaduct was placed to the right and to the left from the block, were considered [14]. As noted above it was shown that for these cases the initial continuous problem is reduced to a finite number of discrete problems with fewer variables and the same structure. Also it was shown that algorithms for solving of a problem without the viaduct with rectangular metric can be used for solving of the problem under consideration with fixed viaduct for the specified cases of the viaduct’s placement.

Let’s now consider the problem for the case when the viaduct is not fixed and its optimal placement is founded. Important to note that the placement of the viaduct can be found if you declare \( x_0 \) as a variable in the model described above. The problem can be solved for example using this model and a package CPLEX.

Consider an arbitrary block \( B_k \) with the left \( LB_k \) and right \( RB_k \) borders. We will call a feasible solution \( x \) of problem (1)–(7) a local minimum of the problem if \( G(x) \leq G(x') \) for every \( x' : I_k(x') = I_k(x), k = 1, \ldots, r \).

Let a partition of facilities into blocks is fixed. Then the variables \( z_{it} \) and \( y_t, i \in I, t = 1, 2, \) are uniquely defined and the mathematical model for finding the local optimum of the problem formulated above can be written as follows:

\[
G(x) = \sum_{t=1}^{2} \sum_{i \in I} \sum_{j \in J_i} w_{ij} \left( z_{it} |x_i - b_{1j}| + (1 - z_{it})(|x_i - x_0| + |b_{1j} - x_0|) \right) + \\
+ \sum_{t=1}^{2} \sum_{i \in I} \sum_{k \in I, i < k} u_{ik} \left( z_{it} z_{kt} |x_i - x_k| + z_{it} (1 - z_{kt})(|x_i - x_0| + |x_k - x_0|) \right) + C \rightarrow \min,
\]

under constraints
\[
LB_k + \frac{l_i}{2} \leq x_i \leq RB_k - \frac{l_i}{2}, \quad i \in I_k(x), \quad k = 1, \ldots, r,
\]
\[
|x_i - x_j| \geq \frac{l_i + l_j}{2}, \quad i, j \in I_k(x), \quad i < j, \quad k = 1, \ldots, r,
\]
where \( C \) is a constant value for each partition of facilities into blocks.

Note that this model has less restrictions than in the previous one. There are also less variables, since the values of Boolean variables \( z_{it}, i \in I, t = 1, 2, \) are already known when the partition of facilities into blocks is fixed. This allows to speed up the process of searching for a problem’s local optimum. So, it can be solved using this model and, for example, the package CPLEX.

3. Numerical experiment

The partition of facilities into blocks is fixed. Local optimum is found by applying the model (8)–(10) and IBM ILOG CPLEX (12.10) package. Two cases are considered. The first case is when a series of problems with a fixed placement of the viaduct is solved. The viaduct is fixed sequentially with a certain step \( hv \), for example, \( hv = LS/3 \). Each problem is considered for fixed viaduct placements \( x_0 = hv, x_0 = 2 \cdot hv, \) and \( x_0 = 3 \cdot hv \), respectively. Then the average value of the goal function \( G_m \) and the running time \( t_m \) were found from the obtained values.

In the second case, the optimal placement of the viaduct is found, considering it as a variable in the model (8)–(10). The value of a variable \( x_0 \), goal function and the running time is
determined for the optimal placement of the viaduct. The results of experiments are shown in the table 1. Namely, comparison the goal function’s values and the running time for a fixed viaduct and for its optimal placement are presented.

| No | n  | m  | r  | \(G_m\), sec. | \(x_0\) | \(G\), sec. | \(|G_m - G|/G\), % | \(|t_m - t|\), sec. |
|----|----|----|----|----------------|-------|------------|------------------|------------------|
| 1  | 3  | 4  | 2  | 1035,33        | 4     | 802        | 0,03             | 29               |
| 2  | 5  | 5  | 3  | 2452           | 0,02  | 2070       | 0,13             | 18               |
| 3  | 10 | 4  | 2  | 6257           | 0,09  | 5166       | 403,39           | 21               |
| 4  | 16 | 11 | 9  | 25518,17       | 5,49  | 20543,5    | 167,3            | 24               |
| 5  | 12 | 5  | 3  | 9556           | 0,39  | 7997       | 1141,58          | 19               |
| 6  | 17 | 12 | 10 | 24729,17       | 2,54  | 20526,5    | 332,5            | 2                |
| 7  | 15 | 10 | 8  | 19652,67       | 1,74  | 17543      | 185,89           | 12               |
| 8  | 20 | 15 | 13 | 28402          | 8,97  | 24280      | 815,5            | 17               |
| 9  | 22 | 17 | 15 | 35861          | 12,31 | 30871      | 370,19           | 16               |
| 10 | 25 | 20 | 18 | 60193,67       | 13,12 | 54861      | 1345             | 1                |

Based on the results, we can conclude that with a fixed viaduct, the CPLEX finds a local optimum significantly faster. Note, that the deviation from the optimal solution is insignificant. Thus, by setting a fairly small step of the viaduct placement, you can get a solution that is close to the optimal one in less time. For example, for the problem of placement 15 facilities, you can find an approximate solution that deviates from the exact one by 12% in a time 105 times less than when the problem of finding the optimal placement of the viaduct is solved.

4. Conclusion
This work touches upon the placement of connected facilities on parallel lines with forbidden zones. Route communications between facilities, as well as between facilities and zones located on different lines, pass through a vertical viaduct. A mathematical model of mixed programming for searching a local optimum of the problem is constructed. Using the mathematical model and the IBM ILOG CPLEX package, an optimal placement of the viaduct was found. Calculations were also performed for a fixed viaduct placement with a specified step. According to the calculations with the fixed viaduct, the solution was found very quickly. This can be applied to find an approximate problem’s solution when the optimal placement of the viaduct is located. Practical usage of this problem is found, for example, in science and technology when designing engineering devices, computer-aided design systems.

Acknowledgments
The work was supported by the program of fundamental scientific research of the SB RAS No. I.5.1., project No. 0314-2019-0019.

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