Atmospheric and Solar Neutrino Masses from Horizontal $U(1)$ Symmetry

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Abstract

We study the neutrino mass matrix in supersymmetric models in which the quark and charged lepton mass hierarchies and also the suppression of baryon or lepton number violating couplings are all explained by horizontal $U(1)_X$ symmetry. It is found that the neutrino masses and mixing angles suggested by recent atmospheric and solar neutrino experiments arise naturally in this framework which fits in best with gauge-mediated supersymmetry breaking with large $\tan \beta$. This framework highly favors the small angle MSW oscillation of solar neutrinos, and determine the order of magnitudes of all the neutrino mixing angles and mass hierarchies.

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The fermion mass problem consists of understanding the flavor mixing structure among quarks or leptons as well as the hierarchy of their mass eigenvalues. It has been suggested that these hierarchical structures can be explained by a horizontal $U(1)_X$ symmetry whose spontaneous breaking is described by $\lambda \approx$ Cabbibo angle $[^{1,4}]$. Recent experimental data on atmospheric and solar neutrinos suggest non-vanishing neutrino masses and mixing $[^{5,6}]$. If spontaneously broken $U(1)_X$ is the origin of the quark and charged lepton mass spectrum, it is expected to have implications for the neutrino masses also. It has been noted that when implemented in supersymmetric (SUSY) models, $U(1)_X$ can explain not only the quark and lepton mass spectrum, but also the smallness of dangerous baryon/lepton number ($B/L$) violating interactions $[^{4}]$. This framework is interesting since renormalizable $L$-violating couplings are small enough to satisfy the current experimental bounds, but still nonvanishing and thus can generate neutrino masses. In this paper, we wish to examine the possibility that the neutrino masses and mixing angles suggested by recent atmospheric and solar neutrino experiments arise naturally in the framework of SUSY models in which the quark and charged lepton mass hierarchies and also the suppression of $B/L$-violating couplings are all explained by horizontal $U(1)_X$ symmetry. Combining the neutrino oscillation data with the informations from the quark and charged lepton sector and also the constraints on $B/L$-violating couplings, we find the $U(1)_X$ charge assignments producing all the fermion masses and mixing angles correctly. This framework fits in best with gauge-mediated SUSY breaking models with large $\tan \beta$, favors the small angle MSW oscillation of solar neutrinos over the large angle just-so oscillation, and determines the order of magnitudes of all the neutrino mixing angles and mass eigenvalues. In this framework, $m_2/m_3 \approx 4 \times 10^{-2}$ is essentially due to the loop to tree mass ratio, while $m_1/m_2 \approx U_{e2}^2 \approx \lambda^4$ is due to the $U(1)_X$ selection rule where $m_A$ ($A = 1, 2, 3$) denote the neutrino mass eigenvalues and $U_{iA}$ the mixing matrix.

To proceed, let us briefly summarize the $U(1)_X$ selection rule estimating the size of couplings $[^{1,4}]$. The Kähler potential and superpotential of the model are generically given
by

\[ K = Z_{IJ}(\lambda, \bar{\lambda})\Phi^I \Phi^* J + [X_{IJ}(\lambda, \bar{\lambda})\Phi^I \Phi^J + \text{h.c.}] + ..., \]

\[ W = \frac{1}{2} \tilde{\mu}_{IJ}(\lambda) \Phi^I \Phi^J + \frac{1}{3!} \tilde{Y}_{IJK}(\lambda) \Phi^I \Phi^J \Phi^K + ..., \]  

(1)

where \( \Phi^I \) denote light chiral superfields and the ellipses stand for the terms of higher order in \( \Phi^I \). The \( U(1)_X \)-breaking order parameter \( \lambda \) corresponds to the VEV of a chiral superfield \( \phi \) with the \( U(1)_X \) charge \( X(\phi) = -1 \): \( \lambda = \langle \phi \rangle / M \approx 1/5 \) for the fundamental mass scale \( M \) which is presumed to be of order the Planck scale \( M_P \). The \( U(1)_X \) selection rule states that the hierarchical structures among the coefficients are due to the insertion of \( \lambda = \langle \phi \rangle / M \) or of \( \bar{\lambda} = \langle \phi^* \rangle / M \) to make the corresponding operators to be \( U(1)_X \)-invariant. This leads to \( Z_{IJ} \approx \lambda^{x_I - x_J} \), \( X_{IJ} \approx \lambda^{x_I + x_J} \), \( \tilde{\mu}_{IJ} \approx \tilde{\mu}\lambda^{x_I + x_J} X(\bar{\mu}) \), \( \tilde{Y}_{IJK} \approx \lambda^{x_I + x_J + x_K} \theta(x_I + x_J + x_K) \) where \( x_I \equiv X(\Phi^I) \), i.e. the \( U(1)_X \) charge of \( \Phi^I \), \( \tilde{\mu} \) denotes the representative component of \( \tilde{\mu}_{IJ} \) whose operator has the \( U(1)_X \) charge \( X(\bar{\mu}) \), and \( \theta(x) = 1 \) when \( x \) is a non-negative integer, while \( \theta(x) = 0 \) otherwise. The overall size of dimensionful \( \tilde{\mu}_{IJ} \) depends upon the mechanism generating the corresponding bilinear terms and can differ from the fundamental mass scale \( M \) in general.

After integrating out supersymmetry breaking fields while taking into account supergravity effects, one can redefine the chiral superfields in the resulting effective theory to have a canonical Kähler metric: \( \Phi^I \rightarrow R_{IJ}\Phi^J \) where \( R_{IJ} \) obeys \( R_{IJ}Z_{JK}R_{KL}^* = \delta_{IL} \). The order of magnitude estimate of \( Z_{IJ} \) above implies \( R_{IJ} \approx \lambda^{x_I - x_J} \). Then for the redefined \( \Phi^I \) with canonical kinetic term, the bilinear and trilinear couplings of the effective superpotential are given by

\[ \mu_{IJ} \approx \tilde{\mu}_{IJ} + R_{IK} \tilde{\mu}_{KL} + m_{3/2} X_{IJ} \]

\[ Y_{IJK} \approx \tilde{Y}_{IJK} + R_{IL} R_{JM} R_{KN} \tilde{Y}_{LMN}, \]  

(2)

including first the contribution from the bare superpotential \( W \), second the effects of superfield redefinition \( \Phi^I \rightarrow R_{IJ}\Phi^J \), and finally the supergravity contribution from the Kahler potential which is proportional to the gravitino mass \( m_{3/2} \).
The most general $SU(3)_c \times SU(2)_L \times U(1)_Y$-invariant superpotential of the MSSM superfields is given by

$$W_{\text{MSSM}} = \mu H_1 H_2 + Y_{ij}^u H_2 Q_i U^c_j + Y_{ij}^d H_1 Q_i D^c_j + Y_{ij}^e H_1 L_i E^c_j$$

$$+ \Lambda_{ijk} U^c_i D^c_j D^c_k + \Lambda_{ijk} L_i Q_j D^c_k + \Lambda_{ijk} L_i L_j E^c_k$$

$$+ \frac{1}{M_S} \Gamma_{ij} L_i H_2 L_j H_2 + ..., \quad (3)$$

where $(H_1, H_2)$, $(L_i, E^c_i)$, $(Q_i, U^c_i, D^c_i)$ denote the Higgs, lepton, and quark superfields, respectively. Among the possible non-renormalizable operators, we include only the $d = 5$ see-saw operator which is presumed to be induced by the physics at the scale $M_S$. Here we have rotated away the possible bilinear term $\mu_i L_i H_2$ through the unitary rotation of superfields, which does not alter the order of magnitude estimates of couplings.

The $U(1)_X$ charges denoted by the small letters $q_i, u_i, \text{e.t.c.}$ for the superfields $Q_i, U^c_i, \text{e.t.c.}$ are well constrained by the experimental data. The quark Yukawa couplings $Y_{ij}^{u,d}$ are determined by the $U(1)_X$ charges of the operators $H_2 Q_i U^c_j$ and $H_1 Q_i D^c_j$. The large $m_t \approx \langle H_2 \rangle$ suggests first of all $q_3 + u_3 + h_2 = 0$, while $m_b \tan \beta \approx \lambda^x m_t$ where $x = q_3 + d_3 + h_1$ and $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$. Then the observed pattern of the CKM matrix and the quark mass eigenvalues $(m^u) \approx m_t(\lambda^8, \lambda^4, 1)$ and $(m^d) \approx m_b(\lambda^4, \lambda^2, 1)$ contain enough informations to reconstruct the Yukawa matrices $Y^u$ and $Y^d$, leading to

$$\begin{align*}
(q_{13}, q_{23}, u_{13}, u_{23}, d_{13}, d_{23}) &= (3, 2, 5, 2, 1, 0) \quad \text{or} \quad (-3, 2, 11, 2, 7, 0), \quad (4)
\end{align*}$$

where $q_{ij} = q_i - q_j$, e.t.c., and $x < 3$ for the second case. The charged lepton mass hierarchy $(m^e) \approx m_\tau(\lambda^5, \lambda^2, 1)$ suggests $(e_{13}, e_{23}) = (5 - l_{13}, 2 - l_{23})$ (Case I) or $(9 - l_{13}, -2 - l_{23})$ (Case II) where $e_{ij} = e_i - e_j$, e.t.c. In order to have $Y^{e}_{12,11} \lesssim \lambda^{2+x}$ and also a non-singular $Y^{e}_{ij}$, we need more constraints: $l_{13} \geq l_{23}$ or $l_{13} < l_{23} - (x + 2)$ for Case I, $-2 \leq l_{23} \leq 0$, $x < 2$, $l_{13} \geq l_{23} + 4$ for $l_{23} > x - 2$ for Case II. As we will see, the neutrino oscillation data imply $l_{23} = 0$ and $l_{13} = 2$, which is compatible only with Case I. We thus have
\[(e_{13}, e_{23}) = (5 - l_{13}, 2 - l_{23}) = (3, 2). \]

The lepton flavor mixing is usually read off from the non-trivial neutrino mass matrix \(m_{ij}^\nu\) in the basis of the diagonal charged lepton mass matrix. Denoting the unitary mixing matrix by \(U_{ij}^A\), the mass eigenvalues are given by \(m_A = U_{ij}^A m_{ij}^\nu (A = 1, 2, 3)\). Recent Super-Kamiokande and other experiments on the atmospheric and solar neutrino oscillations suggest the following neutrino oscillation parameters:

\[
\begin{align*}
\Delta m_{32}^2 &\approx \Delta m_{31}^2 \approx 2.2 \times 10^{-3} \text{eV}^2, \quad \theta_{\text{atm}} \approx 1, \\
\Delta m_{21}^2 &\approx 5 \times 10^{-6} \text{eV}^2, \quad \theta_{\text{sol}} \approx 3.7 \times 10^{-2} \quad \leftarrow \text{MSW} \\
\Delta m_{21}^2 &\approx 6.5 \times 10^{-11} \text{eV}^2, \quad \theta_{\text{sol}} \approx 0.52 \quad \leftarrow \text{just-so}
\end{align*}
\]

(6)

where \(\Delta m_{AB}^2 = m_A^2 - m_B^2\), \(\theta_{\text{atm}} = \theta_{\mu\tau}^{32}\) or \(\theta_{\mu\tau}^{31}\), and \(\theta_{\text{sol}} = \theta_{e\mu}^{21}\) or \(\theta_{e\tau}^{21}\) for the effective mixing angle: \(\sin^2 2\theta_{\alpha\beta}^{ij} = 4|U_{\alpha i}^U U_{\alpha j}^U U_{\beta i}^U U_{\beta j}^U|\).

In our framework, \(L\)-violating couplings are suppressed by having \(l_i\) significantly bigger than \(h_1\) and \(-h_2\). As will be shown explicitly later, the covariance under \(U(1)_X\) suggests that the neutrino mass matrix in our framework takes the form:

\[
(m^\nu)_{ij} = m_3 \lambda^{l_{i3} + l_{j3}} A_{ij},
\]

(7)

where \(m_3\) is the largest mass eigenvalue, \(l_{i3} = l_i - l_3\), and all \(A_{ij}\) are of order unity. This form of \(m^\nu\) leads first of all to \(U_{i3} \approx \lambda^{l_{i3}}\). The large atmospheric \(\nu_\mu-\nu_\tau\) mixing unambiguously implies \(U_{\mu3} \approx 1\), and thus

\[
l_2 = l_3.
\]

(8)

For \(l_2 = l_3\), the mass matrix (7) implies also \(U_{i2} \approx \lambda^{l_{i3}}\). Combined with the unitarity, this determines the mixing matrix to take the form:

\[
U \approx \begin{pmatrix}
1 & \lambda^{l_{i3}} & \lambda^{l_{i3}} \\
\lambda^{l_{i3}} & 1 & 1 \\
\lambda^{l_{i3}} & 1 & 1
\end{pmatrix}.
\]

(9)
Given the above form of $U$, the MSW mixing angle $\theta_{\text{sol}} \approx \lambda^2$ implies

$$l_1 = l_2 + 2 = l_3 + 2,$$

(10)

while the just-so oscillation leads to $l_1 = l_2 = l_3$. Recent Super-Kamiokande and CHOOZ data [3,4] indicates that $\nu_\mu$ rarely if ever oscillates into $\nu_e$, which can be interpreted as excluding $l_1 = l_2 = l_3$ [5].

We have seen that all $U_{iA}$ resulting from the mass matrix (7) are determined essentially by the $U(1)_X$ charges $l_i$. As it will become clear later, although all $A_{ij}$ are of order unity, the corresponding matrix is naturally \textit{approximately singular} and thus gives a mass hierarchy $m_3 \gg m_2 \gg m_1$. Then the oscillation data (6) implies

$$m_3 \approx 5 \times 10^{-2} \text{eV}, \quad m_2/m_3 \approx 4 \times 10^{-2}.$$  

(11)

Let us now discuss how the neutrino mass matrix (7) with approximately singular $A_{ij}$ arises in SUSY models with $U(1)_X$. Although not a unique possibility, an attractive scheme to suppress dangerous $L$-violating couplings in our framework is to have $l_i$ significantly bigger than $h_1$ and $-h_2$. In this scheme, one can easily arrange the physics at $M_S$, e.g. the $U(1)_X$ charges of the superheavy singlet neutrinos, to make the resulting see-saw coefficients $\Gamma_{ij}$ in (3) suppressed by $\lambda^{l_i+l_j+2h_2}$ [6]. If $M_S$ is the string scale $M_{\text{string}} \approx 5 \times 10^{17} \text{GeV}$ or the unification scale $M_{\text{GUT}} \approx 2 \times 10^{16} \text{GeV}$, which is perhaps the most plausible possibility, this would result in

$$m_{ij}^{\text{see-saw}} \approx (10^{-3} \sim 10^{-4}) \times \lambda^{l_i+l_j+2h_2} \text{ eV},$$

(12)

which is too small to be relevant for the atmospheric and solar neutrino masses for $l_i$ significantly bigger than $-h_2$. In fact, the two representative models that we found in this paper have $l_i + h_2 \geq 8$ and thus a completely negligible see-saw contribution.

Once the see-saw contribution is negligibly small, the atmospheric and solar neutrino masses arise from the renormalizable interactions in the superpotential (3) and also the following soft SUSY breaking terms [10,12]:

6
\[ V_{\text{soft}} = m_{L_i H_1}^2 L_i H_1^* + B_i L_i H_2 + A_{ij}^d H_1 Q_i D_j^c + A_{ij}^c H_1 L_i E_j^c + C_{ij}^d L_i Q_j D_c + C_{ij}^c L_i L_j E_c^c + \text{h.c.} \]  

where now all field variables denote the scalar components of the corresponding superfields.

The \( L \)-violating \( B_i \) or \( m_{L_i H_1}^2 \) (in the basis where \( \mu_i L_i H_2 \) in the superpotential are rotated away) results in the tree-level neutrino mass \([13,14]\):

\[ m_{\text{tree}}^{ij} \approx \frac{g_2 a}{M_a} \langle \tilde{\nu}_i^* \rangle \langle \tilde{\nu}_j^* \rangle, \]

where \( M_a \) denote the \( SU(2) \times U(1) \) gaugino masses and the sneutrino VEV’s are given by

\[ \langle \tilde{\nu}_i^* \rangle \approx \frac{2M_Z (m_{L_i H_1}^2 \cos \beta + B_i \sin \beta)}{m_i^2 + \frac{1}{2} M_Z^2 \cos 2\beta}, \]

for the \( Z \)-boson mass \( M_Z \) and \( m_i \) denoting the slepton soft mass which is assumed to be (approximately) flavor-independent. There are also the contribution from the finite 1-loop graph involving squark or slepton exchange in the \( \mu_i = 0 \) basis:

\[ m_{\text{loop}}^{ij} = \frac{1}{16\pi^2} \left[ \frac{\Lambda_{ijk} \Lambda_{jnk} Y_{ik}^* \langle H_1^* \rangle \left( A_{im}^d \langle H_1^* \rangle + \mu Y_{im}^e \langle H_2^* \rangle \right)}{m_i^2} \right. 
\]

\[ + \left. \frac{\Lambda_{ijk} \Lambda_{jnk} Y_{ik} L_i^* \left( C_{ijk}^e \langle L_i^* \rangle + \mu \Lambda_{ijk}^e \langle H_2^* \rangle \right)}{m_i^2} \right] + (i \leftrightarrow j), \]

where the Greek indices \( (\alpha, \beta, ...) \) run from 0 to 3 with \( L_0 \equiv H_1 \), while the Roman indices \( (i, j, ...) \) run from 1 to 3. Here \( \Lambda_{ijk} = -\Lambda_{jik}^e Y_{ik}^e, C_{ijk}^e = -C_{jik}^e \equiv A_{ik}^e \), and \( m_i^2 \) and \( m_0^2 \) denote the squark and slepton soft masses which are assumed to be (approximately) flavor-independent. Note that all the parameters in \((14)\) and \((16)\) are renormalized at the weak scale. The contribution involving the sneutrino VEV \( \langle L_i \rangle \equiv \langle \tilde{\nu}_i \rangle \) in the loop mass has been overlooked so far, but turns out to be crucial to fit \( m_2/m_3 \approx 4 \times 10^{-2} \) in our framework.

If \( U(1)_X \) is anomalous, which is the most interesting possibility, the quadratically divergent Fayet-Iliopoulos coefficient \( \lambda^2 M_P^2 \) naturally yields \( \langle \phi \rangle / M_P \approx \lambda \). It also leads to a nonvanishing \( U(1)_X \) D-term \([13,16]\):

\[ D_X \approx |F|^2/M_X^2 \approx |F|^2/g^2 \lambda^2 M_P^2, \]
where $F$ denotes the SUSY-breaking $F$-term, $M_X \approx g \langle \phi \rangle$ the $U(1)_X$ gauge boson mass. The soft scalar mass of $\Phi^I$ then receives a $D$-term contribution $\delta m^2_I = X(\Phi^I)D_X$. In gravity-mediated SUSY-breaking models, this $D$-term contribution dominates over the standard $F$-term contribution $|F|^2/M^2_P$. If the $U(1)_X$ charge $X(\Phi^I)$ is flavor-independent, the scalar masses dominated by the $D$-term contribution would be (approximately) degenerate, thereby avoid the dangerous flavor violation \[13,17\]. However in our framework, $X(\Phi^I)$ are flavor-dependent to explain the fermion mass hierarchy. When the $D$-term contribution is important, the requirement to avoid dangerous flavor violation while explaining the quark and charged lepton mass spectrum through flavor-dependent $X(\Phi^I)$ severely constrains the possible $U(1)_X$ charge assignment \[18\], and actually leads to the so-called “more” minimal supersymmetry \[19\]. However the resulting $X(\Phi^I)$ do not fit in with our framework explaining the small $B/L$-violating couplings by means of $U(1)_X$. It thus appears that gravity-mediated models with $U(1)_X$ do not fit in well with our framework.

The bothersome flavor-dependent $U(1)_X$ $D$-term contribution becomes negligible in gauge-mediated SUSY-breaking models with a messenger scale $M_m \ll \frac{\alpha}{4\pi} M_X$ for which $\sqrt{D_X} \ll m_{\text{soft}} \approx \frac{\alpha}{4\pi} F/M_m$. To discuss the neutrino mass matrix in gauge-mediated case, let $\mu_a L_a H_2$ and $B_a L_a H_2$ denote the bilinear terms in the superpotential and soft scalar potential in generic basis, and $m^2_{\alpha\beta} L_\alpha L^\beta$ the soft scalar masses of $L_\alpha = (H_1, L_i)$. If the messenger gauge interactions do not distinguish $H_1$ from $L_i$, $B_\alpha$ is naturally aligned to $\mu_\alpha$ and also $m^2_{\alpha\beta} = m^2_0 \delta_{\alpha\beta}$ at $M_m$. In this case, $B_i$ and $m^2_{L_i H_1}$ can be simultaneously rotated away as $\mu_i$ at $M_m$, i.e. $B_i(M_m) = m^2_{L_i H_1}(M_m) = 0$ in the basis of $\mu_i = 0$, and their low energy values at $M_Z$ are determined by the RG evolution which is governed by the $\Delta L = 1$ Yukawa couplings $\Lambda_{ij}^{d,e}$ and generic $L$-conserving couplings. The $L$-violating trilinear soft scalar couplings $(C_{ijkl}^{d,e})$ at $M_Z$ are also determined by the RG evolution. Thus in gauge-mediated models, all renormalizable $L$-violating couplings at $M_Z$ are calculable in terms of $\Lambda_{ij}^{d,e}$ and also of generic $L$-conserving couplings.

Soft parameters in gauge-mediated models \[20\] typically satisfy: $M_a/\alpha_a \approx m_4/\alpha_3 \approx \frac{\alpha}{4\pi} F/M_m \approx \frac{\alpha}{4\pi} m_{\text{soft}}$.
$m_i/\alpha_{1,2}$ at the gauge messenger scale $M_m$ where $M_a$, $m_l$, and $m_i$ denote the gaugino, squark and slepton masses, respectively, and $\alpha_a = g_a^2/4\pi$ for the (SU(5)-normalized) standard model gauge coupling constants $g_a$ ($a = 1, 2, 3$). Trilinear scalar coefficients do vanish at $M_m$ and thus their low energy values are determined by the RG evolution. The size of the bilinear term $B H_1 H_2$ in the scalar potential depends upon how $\mu$ is generated. An attractive possibility in this regard is $B(M_m) = 0$ for which all CP-violating phases in soft parameters at $M_Z$ are automatically small enough to avoid a too large electric dipole moment \cite{22,23}. In this case, the RG-induced low energy value of $B$ yields a large $\tan \beta \approx (m_{H_1}^2 + m_{H_2}^2 + 2\mu^2)/B(M_Z) = 40 \sim 60$ which corresponds to $x = 0$ in view of $\tan \beta \approx \lambda x m_t/m_b$. In fact, a careful analysis of the neutrino mass matrix implies that when $x \geq 1$ it is rather difficult to fit $m_2/m_3 \approx 4 \times 10^{-2}$ for reasonable range of soft parameters in gauge-mediated models \cite{12} without a sizable cancellation \cite{21}, and thus here we concentrate on $x = 0$.

Analyzing the neutrino masses (14) and (16) determined by the RG evolution of couplings with the boundary conditions that trilinear soft scalar couplings, $B$, $B_i$ and $m_{L_iH_1}$ are all vanishing at $M_m$, and also $M_a/\alpha_a \approx m_l/\alpha_3 \approx m_i/\alpha_{1,2}$ at $M_m$, it is straightforward to find that (for $x = 0$ and thus $\tan \beta = 40 \sim 60$)

$$m_{ij}^{\text{tree}} = 10^{-1}\xi_{1 ij} a_i a_j \left( \frac{\mu^2 M_Z^2}{m_i^2} \right),$$

where $a_i \approx Y_b \Lambda_{33}^d \approx \chi_{i - h_1}$, $t = \ln(M_m/m_i)/\ln(10^3)$ and $\xi_{1}$ is the coefficient of order unity summarizing the uncertainty of our estimate. Among various terms in the loop mass (16), the leading contribution to the loop mass comes from the piece involving $\langle L_i \rangle \langle H_2 \rangle$ for large $\tan \beta$, because $Y^e \langle L_i \rangle / \Lambda^e \langle H_1 \rangle \approx \tan \beta \gg 1$. We then have

$$m_{ij}^{\text{loop}} \approx 10^{-2}\xi_2 t^2 Y_b Y^3 \Lambda_{33}^d (\delta_{i3} \Lambda_{33}^e + \delta_{j3} \Lambda_{33}^e) \left( \frac{\mu^2 M_Z^2}{m_i^3} \right),$$

where the smaller contributions are ignored and again the coefficient $\xi_2$ of order unity is introduced to take into account the uncertainty of our estimate. Here the powers of $t \propto \ln(M_m/m_i)$ are from $\langle \tilde{\nu}_i \rangle \propto B_i(m_i) \propto t^2$ under the boundary condition $B_i(M_m) = 0$. The above neutrino mass matrices are derived in the basis for which $Y_{ij}^e$ and $Y_{ij}^d$ are diagonal, and $Y_b = Y_{33}^d$ and $Y_{e} = Y_{33}^e$. At any rate, from (18) and (19), we find
\[
\frac{m_{23}^{\text{loop}}}{m_{23}^{\text{tree}}} \approx 10^{-2} \left( \frac{\xi_2 \Lambda_{233}^e}{\xi_1 \Lambda_{233}^d} \right) \left( \frac{\tan \beta}{50} \right)^2 \left( \frac{\ln 10^3}{\ln \frac{M_Z}{m_l}} \right)^2,
\]

where we have used \( Y_\tau \approx \tan \beta / 95, \ Y_b \approx \tan \beta / 50 \). Since \( \Lambda_{233}^e \) and \( \Lambda_{233}^d \) are comparable to each other, the above result shows that \( m_{ij}^{\text{tree}} \) gives a dominant contribution.

Obviously \( m_{ij}^{\text{tree}} \) is a rank 1 matrix, and thus the total neutrino mass matrix takes the form (18) with an approximately singular matrix \( A_{ij} \) when \( m_{ij}^{\text{tree}} \) dominates. We then find from (18) and (19) the following mass hierarchies:

\[
\begin{align*}
m_3 & \approx U_{i3} m_{ij}^{\text{tree}} U_{j3} \approx 10^{-1} \eta M_Z \lambda^2 (l_3 - h_1), \\
m_2 & \approx U_{i2} m_{ij}^{\text{loop}} U_{j2} \approx m_3 \frac{m_{23}^{\text{loop}}}{m_{23}^{\text{tree}}}, \\
m_1 & \approx U_{i1} m_{ij}^{\text{loop}} U_{j1} \approx m_2 \lambda^4,
\end{align*}
\]

where \( \eta = \xi_1 (\ln \frac{M_Z}{m_l}/\ln 10^3)^4 (M_Z \mu^2/m_l^3) \). For \( m_l \approx 200 \sim 400 \text{GeV} \) and \( \mu \approx 2 m_l \) which has been suggested to be the best parameter range for correct electroweak symmetry breaking \[23\], \( \eta \) is roughly of order unity and then the experimentally favored \( m_3 \approx 5 \times 10^{-2} \text{eV} \) can be obtained for

\[
7 \lesssim l_3 - h_1 \lesssim 9.
\]

Note that in our framework small \( m_2/m_3 \) is essentially due to the loop to tree mass ratio, while the other small mass ratios \( m_1/m_2 \approx \lambda^4 \) and \( m_3/M_Z \approx 10^{-1} \lambda^2 (l_3 - h_1) \) are from the \( U(1)_X \) selection rule.

We found many possible \( U(1)_X \) charge assignments producing all fermion masses and mixing discussed so far, while satisfying all the bounds on \( B/L \)-violating couplings \[24\] through the \( U(1)_X \) selection rule under the condition that the maximum \( U(1)_X \) charge is not unreasonably large for \( X(\lambda) = -1 \). For more detailed discussions, see \[21\]. In this paper, we pick two representative solutions: Model 1 and Model 2 which are listed in Table 1.

To conclude, we have studied the neutrino mass matrix in supersymmetric models in which the observed quark and charged lepton masses and also the suppression of \( B/L \)
violating couplings are all explained by horizontal $U(1)_X$ symmetry. A particular attention was paid for the possibility that the neutrino masses and mixing angles suggested by recent atmospheric and solar neutrino experiments arise naturally in this framework. It is found that our framework fits in best with gauge-mediated SUSY breaking models with large $\tan \beta \approx 50$, and favors the small angle MSW oscillation of solar neutrinos over the large angle just-so oscillation. Combining the informations from neutrino oscillation experiments with those from the quark and charged lepton sector and also the constraints on $B/L$-violating couplings, we find the $U(1)_X$ charge assignments producing all the fermion masses and mixing angles correctly. This framework determines the order of magnitudes of the neutrino mixing matrix elements and mass eigenvalues to be: $U \approx U^T$ with $U_{e2} \approx U_{e3} \approx \lambda^{\ell_3-\ell_1} = \lambda^2$, $U_{\mu 3} \approx \lambda^{\ell_3-\ell_2} \approx 1$ and $m_1/m_2 \approx \lambda^{2(\ell_3-\ell_1)} \approx \lambda^4$, $m_2/m_3 \approx (\text{Loop}/\text{Tree}) \approx \mathcal{O}(10^{-2})$ for $m_3 \approx \mathcal{O}(10^{-1}) \times M_Z \lambda^{2(\ell_3-\ell_1)} \approx 5 \times 10^{-2} \text{ eV}$.

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TABLE I. \( U(1)_X \) charge assignments for the MSSM fields of two representative models.

| Model | \( q_1, q_2, q_3 \) | \( u_1, u_2, u_3 \) | \( d_1, d_2, d_3 \) | \( l_1, l_2, l_3 \) | \( e_1, e_2, e_3 \) | \( h_1, h_2 \) |
|-------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 1     | 8,7,5             | -3,-6,-8          | -1,-2,-2          | 7,5,5             | 1,0,-2            | -3,3              |
| 2     | \( 7/2, 5/2, 1/2 \) | -1/2,-7/2,-11/2   | 11/2, 9/2, 9/2    | 5,3,3             | 5,4,2             | -5,5              |
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