Avoiding dark states in open quantum systems by tailored initial correlations

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We study the transport of excitations on a V-shaped network of three coupled two-level systems that are subjected to an environment that induces incoherent hopping between the nodes. Two of the nodes are coupled to a source while the third node is coupled to a drain. A common feature of these networks is the existence of a dark-state that blocks the transport to the drain. Here we propose a means to avoid this state by a suitable choice of initial correlations, induced by a source that is common to both coupled nodes.

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I. INTRODUCTION

The transport dynamics of energy or charge in many physical systems can be described by considering transport on a network of coupled 2-level systems. Depending on the physical system, the transport can range from being purely coherent to being purely incoherent. In the former case the dynamics can be described by continuous-time random walks, while in the latter case the dynamics follows from Schrödinger’s equation, which for complex systems and certain choices of the Hamiltonian defines so-called continuous-time quantum walks. Coupling of a quantum system with purely coherent dynamics to a bath of harmonic oscillators (for instance phonons) can lead to a mixture of coherent transport and incoherent hopping induced by the environment. The dynamics (of the reduced density matrix) can be described by a quantum master equation of Lindblad type, where a certain choice of Lindblad operators defines so-called quantum stochastic walks.

In this work we consider transport on a V-shaped trimer configuration, resembling for instance a system of coupled quantum dots. The transport will be generated by connecting a (incoherent) single source to the end nodes of the V-shaped trimer and a (incoherent) drain to the middle node, see below. Most theoretical works have focused on the case where the source creates an excitation on a single node of the network. One can also, however, consider the possibility where a source is connected to multiple nodes of the network. Such a source can then create an excitation that is in a superposition of these nodes, leading to initial correlations between these nodes.

A common feature of these V-shaped, or circular, trimer configurations is the existence of a dark state. Such a state causes the excitation to become trapped in the network and therefore leads to a blocking of the transport in the purely coherent case. To overcome this problem, one can either introduce an energetic disorder on the nodes or couple the system to a suitable environment where the decoherence process destroys the interference effects that lead to the dark state. Here we propose a third method: a suitable choice of initial correlations that are induced by a single source which is coupled to the end nodes of the network creates an initial state which is orthogonal to the dark state. This then causes the absence of the dark state in the transport even in the purely coherent case, leading to a complete transport to the drain.

The paper is organized as follows. In Sec. II we introduce our model and provide a detailed discussion on the exact mathematical implementation of a source that induces initial correlations. In Sec. III we discuss the transport efficiency of different initial configurations with the help of both analytical and numerical computations.

II. MODELLING THE TRANSPORT

1. Coherent and incoherent dynamics

We consider the dynamics of excitations on a trimer network. The coherent (quantum) dynamics on this network is described by the Schrödinger equation, or equivalently by the Liouville-von Neumann equation, with the general Hamiltonian

$$H_0 = \begin{pmatrix} E_1 & V_{12} & V_{13} \\ V_{12} & E_2 & V_{23} \\ V_{13} & V_{23} & E_3 \end{pmatrix}. \quad (1)$$

Here, $E_k$ is the site energy of node $|k\rangle$ and $V_{kl}$ are the transfer rates between nodes $|k\rangle$ and $|l\rangle$. Note that for certain choices of the site energies and the couplings there exists an eigenstate $|D\rangle = (|1\rangle - |2\rangle)\sqrt{2}$ of $H_0$, having only an overlap with nodes 1 and 2.

Now, if the system is in contact with an external environment, the total Hamiltonian takes the form $H_{\text{tot}} = H_0 + H_E + H_{\text{int}}$, where $H_0$ is the Hamiltonian of the network, $H_E$ is the Hamiltonian of the environment and $H_{\text{int}}$ specifies the interactions between the network and the environment. When the environmental correlation time is small compared to the relaxation time of the system, one can describe the dynamics on the network by a master equation in Lindblad form:

$$\frac{d\rho_N(t)}{dt} = -i[H_0, \rho_N(t)] + \sum_{k,l=1}^{3} \lambda_{kl} D(L_{kl}, \rho_N(t)), \quad (2)$$
known as the quantum stochastic walk

\[ L_{kl} \] incoherent transfer between the nodes and the genera-

\[ \Lambda \] tion (for the diagonal elements of

\[ \sum_k a_k \left| k \right> \] of the network can be phenomenologically modelled by the Lindblad operator \( L_s = \left| \psi \right> \left< 0 \right| \), leading to the follow-

\[ \text{FIG. 1} \] in the purely incoherent limit we assume simple hop-

\[ D(L_{kl}, \rho_N(t)) = L_{kl} \rho_N(t) L_{kl}^\dagger - \frac{1}{2} \left\{ L_{kl}^\dagger L_{kl}, \rho_N(t) \right\} \]. (3)

For our model we assume the Lindblad operators \( L_{kl} \) to be given by \( L_{kl} = \left| k \right> \left< l \right| \). The term in Eq. (2) corre-

\[ \lambda_{kl} \] sponding to the operator \( L_{kl} \) models the incoherent ex-

\[ \lambda_k = \Lambda |V_k|^2 \]citation dynamics, induced by the environment, between the

\[ \Lambda \] nodes \( l \) and \( k \) with rate \( \lambda_{kl} \).

In the purely incoherent limit we assume simple hopping dynamics to be described by a Pauli master equation (for the diagonal elements of \( \rho(t) \) only). The corre-

\[ \lambda_{kk} = \lambda |V_k|^2 \]sponding transition rates \( \lambda_{kl} \), for \( k \neq l \), can be phe-

\[ \lambda_{kk} = \lambda \]nomenologically estimated with Fermi’s golden rule, e.g.

\[ \lambda_{kl} \]sponding to the operator \( L_{kl} \) models the incoherent ex-

\[ \Lambda \] citation dynamics, induced by the environment, between the

\[ \lambda_{kk} = \lambda \]nodes \( l \) and \( k \) with rate \( \lambda_{kl} \).

Since the dissipative terms in the Lindblad master equation induce incoherent hopping between the nodes, we can introduce a parameter \( \alpha \), with \( 0 \leq \alpha \leq 1 \), that allows us to interpolate between purely coherent dynamics (\( \alpha = 0 \)) and purely incoherent dynamics (\( \alpha = 1 \)):

\[ \frac{d \rho_N(t)}{dt} = (1 - \alpha) L_{\text{coh}}(\rho_N(t)) + \alpha L_{\text{env}}(\rho_N(t)) \], (4)

with \( L_{\text{coh}}(\rho_N(t)) = -i [H_0, \rho_N(t)] \) and \( L_{\text{env}}(\rho_N(t)) = L_{\text{incoh}}(\rho_N(t)) + L_{\text{deph}}(\rho_N(t)) \). This approach is also known as the quantum stochastic walk. The genera-

\[ \left| \psi \right> \left< 0 \right| \text{t} \]tor \( L_{\text{incoh}} \) corresponds to the terms in (2) that generate incoherent transfer between the nodes and the generator \( L_{\text{deph}} \) corresponds to the terms that generate pure dephasing.

2. Sources and drains

A source is included in our system as an extra node \( |0\rangle \) that is incoherently coupled to the nodes of our network in order to make sure that there is only transport from the source to the network and not back. In general, transitions from the source to a general state \( |\psi\rangle = \sum_k a_k |k\rangle \) of the network can be phenomenologically modelled by the Lindblad operator \( L_s = |\psi\rangle \langle 0| \), leading to the follow-

\[ \mathcal{L}_{\text{source}}(\rho_S(t)) = \Gamma [D(L_s, \rho_N(t)), \rho_N(t)] \], (5)

with \( \Gamma \) representing the rate at which the excitation flows into the network. Note that for a \( N \)-dimensional network with a source, the reduced density matrix \( \rho_S \) is repre-

\[ \rho_S(0) = |0\rangle \langle 0| \]sented by a \( (N + 1) \times (N + 1) \) matrix. For an initial preparation in the source node, i.e. \( \rho_S(0) = |0\rangle \langle 0| \), one can show that the density matrix can be written in the form:

\[ \rho_S(t) = \begin{pmatrix} \rho_{00}(t) & 0 \\ 0 & \rho_N(t) \end{pmatrix} \], (6)

where \( \rho_N(t) \) is the density matrix corresponding to the network nodes.

In a similar fashion, we include a drain by coupling the state \( |N + 1\rangle \) to the network. The incoherent transition from a state \( |\psi\rangle \) of the network to the drain with rate \( \gamma \) can then be modelled by the term

\[ \mathcal{L}_{\text{drain}}(\rho_{SND}(t)) = \gamma [D(L_d, \rho_{SND}(t)), \rho_{SND}(t)] \], (7)

in the master equation, with \( L_d = |N + 1\rangle \langle \psi| \). Thus the final reduced density operator \( \rho_{SND}(t) = \rho(t) \) is repre-

\[ \rho(t) \]sented by a \( (N + 2) \times (N + 2) \) matrix.

3. Creating initial correlations with a source

There are now two interesting ways in which we can connect the source to the end nodes \( |1\rangle \) and \( |2\rangle \) of the trimer network:

(I) The source can either feed node 1 with rate \( \Gamma/2 \) or node 2 with rate \( \Gamma/2 \). We assume these processes to be independent of each other. We can model this with two dissipators representing the two independent processes:

\[ \mathcal{L}_{\text{source}}^{(1)}(\rho(t)) = \frac{\Gamma}{2} [D(|1\rangle \langle 0|, \rho(t)), \rho(t)] + \frac{\Gamma}{2} [D(|2\rangle \langle 2|, \rho(t)), \rho(t)] \]

\[ = \frac{\Gamma}{2} \rho_{00}(t) \left( |1\rangle \langle 1| + |2\rangle \langle 2| - 2 |0\rangle \langle 0| \right) \], (8)

(II) The source can also feed a superposition state \( |\psi\rangle \) between node 1 and node 2, which can in general be
written as \(|\psi⟩ = \left( |1⟩ + e^{i\phi} |2⟩ \right) / \sqrt{2}\). We can model this process with one dissipator:

\[
L_{\text{source}}^{(2)}(\rho(t)) = \Gamma D(|\psi⟩ \langle 0|, \rho(t)) = \frac{\Gamma}{2} \rho_{00}(t) \left[ |1⟩ ⟨1| + |2⟩ ⟨2| - 2 |0⟩ ⟨0| \right]
+ e^{-i\phi} |1⟩ ⟨2| + e^{i\phi} |2⟩ ⟨1|.
\]

(10)

See Fig. 1 for an illustration of these two configurations. The key difference between these two choices is that \(L_{\text{source}}^{(2)}(\rho(t))\) creates initial correlations, depending on the phase \(\phi\), between the two nodes, while \(L_{\text{source}}^{(1)}(\rho(t))\) does not. How these initial correlations affect the transport properties will be addressed in the following section.

III. TRANSPORT WITH AND WITHOUT INITIAL CORRELATIONS

In previous work we used the expected survival time (EST) \(\eta\) as a measure for the transport properties of the excitation in the network. Here we use it specifically to study the effects of initial correlations on the transport. This EST is defined as the average time it needs for the excitation to move completely from the source to the drain:

\[
\eta(\alpha) = \int_0^\infty dt \left( 1 - \rho_{N+1,N+1}(t, \alpha) \right).
\]

(11)

The following representation of the EST in terms of the Laplace transforms \(\hat{\rho}_{kk}(s)\) of the components of the density matrix, allows for a more convenient way to obtain analytical expressions for the EST.

\[
\eta(\alpha) = \lim_{s \to 0} s \sum_{k=0}^N \hat{\rho}_{kk}(s, \alpha).
\]

(12)

We denote the EST corresponding to a source feeding independently the two nodes 1 and 2, Eq. (5), as \(\eta_I(\alpha)\) and \(\eta_{II}(\alpha)\) for \(\phi = 0\) and \(\phi = \pi/2\), together with the result for \(\eta(\alpha)\).
and the EST corresponding to the source feeding into an entangled state, Eq. (12), as \( \eta_{11}(\alpha) \). To illustrate the key effects, we assume, for simplicity, that \( E_1 = E_2 = E_3 = 1 \), \( V_{13} = V_{23} = 1 \), \( \gamma = 1 \) and \( V_{12} = 0 \). That is, we focus on the situation when there is no bond between nodes 1 and 2. For these parameters, it follows from Eq. (12) that the ESTs \( \eta_{1}(\alpha) \) and \( \eta_{11}(\alpha) \) take the form

\[
\eta_{1}(\alpha) = 1/\Gamma + f(\alpha)/g(\alpha) \quad (13)
\]

\[
\eta_{11}(\alpha) = 1/\Gamma + [f(\alpha) - h(\alpha) \cos \phi]/g(\alpha), \quad (14)
\]

with \( h(\alpha) = 4(1 - \alpha)^2 \) and

\[
f(\alpha) = 4 + \alpha(17 + 13\lambda) + 2\alpha^2(\lambda(\lambda - 8) - 19) + 3\alpha^3(11 + \lambda(9 + 2\lambda))
\]

\[
g(\alpha) = 4\alpha(2 + \lambda) - \alpha^2(15 + 7\lambda) + \alpha^3(11 + \lambda(9 + 2\lambda)).
\]

The dependence of \( \eta_{11}(\alpha) \) on the phase \( \phi \) is therefore proportional to \( \cos \phi \). Its amplitude \( -h(\alpha)/g(\alpha) \) is a monotonically decreasing function of \( \alpha \) and vanishes when \( \alpha = 1 \). Therefore \( \eta_{11}(\alpha) \) converges to \( \eta_{1}(\alpha) \) when \( \alpha \to 1 \), where they both reach the value \( 4 + 1/\Gamma \). This also shows that for \( \phi \in [0, \pi/2] \) and \( \phi \in (3\pi/2, 2\pi] \), \( \eta_{11}(\alpha) < \eta_{1}(\alpha) \) and that the converse result holds for \( \phi \in (\pi/2, 3\pi/2) \). In the limit \( \alpha \to 0 \) we find that \( \eta_{1}(0) = \infty \), while

\[
\lim_{\alpha \to 0} \eta_{11}(\alpha) = \begin{cases} 
1/\Gamma + 25 + 13\lambda /8 + 4\lambda & \text{for } \phi = 0 \\
\infty & \text{for } \phi \neq 0.
\end{cases} \quad (16)
\]

To illustrate these analytical results we show in Fig. 2 for \( \lambda = 1 \), the dependence of the EST \( \eta_{11}(\alpha) \) on the phase \( \phi \) and compare it to \( \eta_{1}(\alpha) \). One clearly observes the \( \cos \phi \)-dependence of \( \eta_{11}(\alpha) \), see left panel, and an infinite EST for \( \alpha = 0 \) and \( \phi \neq 0 \). The infinite EST can be understood by noting that for these values of \( \alpha \) and \( \phi \) the dark state is not influenced by the drain and is a stationary state of the system, causing both EST’s to diverge when \( \phi \neq 0 \). For \( \phi = 0 \) the state \( |\psi\rangle \) is orthogonal to \( |D\rangle \), causing the absence of the dark state in the full dynamics and leading to complete transfer to the drain.

In Fig. 3 we show for \( \phi = 0 \) the dependence on the dephasing rate \( \lambda \). We observe that for increasing values of \( \lambda \) the EST \( \eta_{1}(\alpha) \) decreases, leading to faster transport. This resembles noise-assisted transport found in many other systems.\footnote{N. V. Kampen, *Stochastic Processes in Physics and Chemistry* (North Holland, Amsterdam, 1990)} The EST \( \eta_{11}(\alpha) \), in contrast, increases for larger values of \( \lambda \), leading to slower transport to the drain. Thus, here the optimal transport efficiency is obtained in the purely coherent case \( (\alpha = 0 \text{ and } \lambda = 0) \). However, \( \eta_{11}(\alpha) \) is always smaller than \( \eta_{1}(\alpha) \). We further observe that \( \eta_{11}(\alpha) \) increases until a certain \( \alpha_c \), after which it follows the curve of \( \eta_{1}(\alpha) \). The value of \( \alpha_c \) becomes smaller with increasing dephasing rates. This happens because the dephasing process destroys the coherences between nodes 1 and 2. Therefore after this point, the initial correlations do not significantly influence the transport properties anymore and \( \eta_{11}(\alpha) \approx \eta_{1}(\alpha) \). When \( \phi = 0 \), a tractable analytical expression for \( \alpha_c \) is possible:

\[
\alpha_c = \frac{4}{3 + 2\lambda} \left[ \frac{(2 + \lambda^2)}{11 + 9\lambda + 2\lambda^2} - \frac{1}{4} \right]. \quad (17)
\]

Our results show that initial correlations, induced by the source feeding the superposition state \( |\psi\rangle = (|1\rangle + |2\rangle)/\sqrt{2} \), leads to faster transport than for feeding any other state of the form \( |\psi\rangle = (|1\rangle + e^{i\phi}|2\rangle)/\sqrt{2} \) with \( \phi \neq 0 \), even in the presence of dephasing. Additionally, if \( \cos \phi > 0 \) one always has \( \eta_{11}(\alpha) < \eta_{1}(\alpha) \), see Eqs. (13) and (11). Therefore, when initial correlations are present, a smaller coupling to the environment (smaller values of \( \alpha \)) is sufficient to avoid the dark state, compared to the situation without initial correlations.

### IV. SUMMARY

In conclusion, we have shown that by connecting a source to the two end nodes of the V-shaped network it is possible to induce initial correlations between the coupled nodes and that these initial correlations can overcome the detrimental effects of the dark state, leading to complete transfer to the drain. The source can also feed the two end nodes independently, i.e., without initial correlations between the two nodes. Then, the dark state inhibits complete transfer. When increasing the coupling to the environment, the differences in the expected survival times between the two types of sourcing processes diminish. We expect that the results obtained here also hold for larger networks that exhibit invariant subspaces, as for example described in\footnote{E. Farhi and S. Gutmann, *Phys. Rev. A* **58**, 915 (1998)} for fully connected networks, in\footnote{O. Mülken and A. Blumen, *Phys. Rep.* **502**, 37 (2011)} for larger ring-like structures or in\footnote{Lenz for useful discussions.} for Erdős-Rényi graphs. Furthermore, our results are also related to the study of electrical currents through a network of two-level systems, since the current is related to the long-time limit of the time derivative of the EST.

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