ANALYSIS OF RAYLEIGH TAYLOR INSTABILITY IN NANOFLUIDS WITH ROTATION

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Abstract. This article focuses on the hidden insights about the Rayleigh-Taylor instability of two superimposed horizontal layers of nanofluids having different densities in the presence of rotation factor. Conservation equations are subjected to linear perturbations and further analyzed by using the Normal Mode technique. A dispersion relation incorporating the effects of surface tension, Atwood number, rotation factor and volume fraction of nanoparticles is obtained. Using Routh-Hurtwitz criterion the stable and unstable modes of Rayleigh-Taylor instability are discussed in the presence/absence of nanoparticles and presented through graphs. It is observed that in the absence/presence of nanoparticles, surface tension helps to stabilize the system and Atwood number has a destabilizing impact without the consideration of rotation factor. But if rotation parameter is considered (in the absence/presence of nanoparticles) then surface tension destabilizes the system while Atwood number has a stabilization effect (for a particular range of wave number). The volume fraction of nanoparticles destabilizes the system in the absence of rotation but in the presence of rotation the stability of the system is significantly stimulated by the nanoparticles.

1. Introduction. The Rayleigh Taylor instability is the subject of interest for the researchers due to its wide implications in the field of planetary interiors, stellar, astrophysics (such as supernova), deep convection in oceans, coastal upwelling, geophysics, vortex control and plasma physics etc. As the continuous stratification (instabilities) is a very common phenomenon and Lord Rayleigh [19] was the protagonist to contemplate and discover the instability (which was originated due to denser fluid accelerated into the less dense fluid) by studying the case of inviscid fluid and drew the mathematical expression of the conditions of linear stability. Sir Geoarry Taylor [26] showed that when two superposed fluids are accelerated in the
direction perpendicular to the plane, then system experiences the Rayleigh Taylor instability with accelerated fluids. He also found the relationship between the rate of development of instability and the length of wave like perturbations. Chandrasekhar [9] added the condition of viscosity to Rayleigh’s work and extracted the onset of Rayleigh Taylor instability’s condition. The Rayleigh Taylor instability in different kind of fluids are also studied by some researchers in the past. Bhatia [5], was one of the pioneer researchers who articulated Rayleigh Taylor instability for viscous compressible plasma. Using variational principle, exact solutions for semi-infinite plasma in the presence of magnetic field have been obtained and interpreted that both the viscosity and magnetic field have a stabilizing influence while compressibility is found to have destabilizing influence. Afterwards, Rayleigh Taylor instability of Rivlin-Ericksen elastico-viscous fluids in the presence of magnetic field is studied by Sharma et al. [23] in porous medium where he quoted that the absence of magnetic field hastens the instability for all wave numbers whereas the presence of this parameter is able to stabilize certain wave number band.

Hide [15] studied the impact of rotation on Rayleigh Taylor instability by assuming the angle of rotation (in z-direction) zero and reported that the variational principal is attained not only for continuously stratified but for two-layer system also. In this connotation, Chakraborthy [7][8] made a successful attempt to record the upper bound of the growth rate of instability of the system by taking the vertical rotation field under consideration. After a while, Dávalos-Orzoco [13] estimated the upper bound of the growth rate in the presence of horizontal component of rotation and also able to expose the concept of overstability by predicting the complex number as the eigen value of the growth rate. Considering the axis of rotation normal to acceleration of interface, Tao et al. [25] succeeded to demonstrate that the nonlinear Rayleigh Taylor instability could be reduced at arbitrary Atwood number in two uniform inviscid fluids. In his theoretical analysis he provided the fact that Coriolis force in uniform rotation always helps the configuration to get a retarding amplitude of growth rate at any disturbance wavelength present on the interface.

Following the remarkable work reported in literature, the combined effects of rotation and magnetic field on Rayleigh Taylor instability of two superposed fluids through porous media was analyzed for stability analysis by Sharma [22]. An insightful deduction came into the light when Kumar [16] stated that rotation does not impart any effect on stability or instability of configuration of rotating Oldroydian viscoelastic fluids with variable magnetic field through porous medium. Rotation factor is also studied in stratified plasma by Sharma et al. [20] where they churned out the fact that the rotation (angular velocity) and magnetic field fasten Rayleigh Taylor instability by pouring stabilizing influence on configuration. Ansary et al. [14] made a break through by taking two and three fluid systems. He revealed that in two-fluid or three fluid system the rotation is not imparting the effect on critical value of stability rather surface tension plays a crucial role in stabilizing the unstable mode of configuration.

Some very useful experimental work is also reported in the literature on Rayleigh Taylor instability. Baldwin et al. [3] presented a new experimental technique for studying the effect of rotation in Rayleigh Taylor instability. They found that rotation retards the growth rate of instability and also recorded the expression of critical growth rate involving Atwood number and aspect ratio of the system for stabilizing the long wavelength modes. The other experimental work conducted on heavy diamagnetic and light paramagnetic fluids is produced by Scase et al. [21].
They compared the theoretical prediction and experimental results and concluded that the instability can be suppressed up to a maximum rate of rotation but it cannot be suppressed indefinitely.

Thus, through this literature review, one may conclude that the enormous structure of qualitative research is dedicated to explore the hidden facts and features of Rayleigh Taylor instability considered under different kind of fluid structure. But here it is important to mention that Rayleigh Taylor instability is still unmarked in field of nanofluids though nanofluids are known for the revolutionary breakthrough in the industries like biotechnology, pharma, fuel cells, microelectronics and water treatment etc. and well documented in research [1], [4], [6], [11],[12], [17], [18] and [27].

But the Rayleigh Taylor instability in nanofluids is still unexplored. Therefore, the objective of the current study is pristine and original to unveil the qualitative insights about Rayleigh Taylor instability along with the rotation in non-porous medium. This study involves elaborative mathematical calculations to obtain a modified dispersion relation using normal mode techniques on the linear perturbed configuration. For better understanding of the theory and results of rotation factor, the comparison is drawn between two cases i.e. with and without considering rotation parameter. Further the final mathematical expression is evaluated among the following cases for recording the effect of different parameters like surface tension, Atwood number, rotation and volume fraction of nanoparticles as

(i) While the nanoparticles are not present and effect of surface tension and Atwood number is shown in the presence/absence of rotation factor considering only the base fluids in the both of layers. Even the effect of rotation is also captured through graphs.

(ii) When two same kinds of nanoparticles are considered in both of the layers of fluid and then the influence of surface tension, Atwood number and volume fraction of nanoparticles is presented through graphs with or without rotation parameter for predicting the stabilization/destabilization and effect of rotation along with nanoparticles is also noticed.

(iii) On this platform, different kind of nanofluids are taken in both of the layers and the effect of rotation is analyzed along with behavior of different parameters for the stability analysis of the configuration for the unstable mode that is projected due to Rayleigh Taylor instability.

Thus, this study will help to draw the fair conclusion about the effect of rotation parameter on Rayleigh Taylor instability in nanofluids.

2. Mathematical modeling. This model is considered with two infinite horizontal layers of homogeneous nanofluids with the region $z < 0$ as a lower layer of nanofluid with density $\rho_1$, viscosity $\mu_1$, and $\phi_1$ respectively and $z > 0$ as an upper layer of nanofluid with density, viscosity and volume fraction of nanoparticle as $\rho_2$, $\mu_2$, and $\phi_2$ respectively. These layers are superimposed on each other with the assumption that heavier fluid is overlying on lighter fluid and are separated at the interface $z = 0$ where the surface tension is effective. The system is accelerated due to gravity $g(0, 0, -g)$ and rotation parameter is supposed to act in vertical direction as $\Omega (0, 0, \Omega)$. It is important to mention that nanoparticles are having higher densities than those of the base fluids. The nanoparticles having uniform shape and size are distributed homogeneously among both of the layers. Temperature is supposed to be constant in the configuration.
Basic relevant equations of the problem (Chandrasekhar [10], Buongirono [6] and Tzou [27]) are

\[
\nabla \cdot \mathbf{v} = 0, \quad (1)
\]

\[
\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = \nabla \cdot \left[ D_B \nabla \phi + D_T \frac{\nabla T_p}{T_p} \right], \quad (2)
\]

\[
\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right] = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho g + 2\rho (\mathbf{v} \times \Omega) + \Sigma \left[ T \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) z_s \right] \delta(z - z_s), \quad (3)
\]

\[
\frac{\partial \rho}{\partial t} = -wD\rho, \quad (4)
\]

where \( \mathbf{v} \), \( p \), \( T_p \), \( \rho \), \( \mu \), \( \phi \), \( \delta(z - z_s) \), \( T \) and \( w \) denote the velocity, pressure, temperature, density, viscosity, volume fraction, Dirac function, surface tension of interfacial surface and \( z \)-component of the velocity. Here, \( z_s(s = 1, 2, 3, \ldots) \) is considered the point of discontinuity for \( \rho \) at the surface. The total density of the fluid is defined as

\[
\rho = \rho_1 + \rho_2, \quad (5)
\]

and

\[
\rho = \phi \rho_p + (1 - \phi) \rho_f, \quad (6)
\]

and the densities of nanofluids in lower and upper layer are \( \rho_1 \) and \( \rho_2 \) which are defined as

\[
\rho_1 = \phi_1 \rho_{p1} + (1 - \phi_1) \rho_{f1}, \quad (7)
\]

\[
\rho_2 = \phi_2 \rho_{p2} + (1 - \phi_2) \rho_{f2}, \quad (8)
\]

where base fluids having density \( \rho_{f1} \) and \( \rho_{f2} \) and nanoparticles with density \( \rho_{p1} \) and \( \rho_{p2} \) in lower layered fluid and upper layered fluid respectively.

3. Perturbation Analysis. Consider the fluid with linear perturbation on the interface where \( \delta \rho, \delta p, \mathbf{v}' \) and \( \delta \phi \) denote the perturbations in density, pressure, velocity and volume fraction of nanoparticles respectively. Assume that, at the interface i.e. \( z = 0 \) density of nanoparticles as well as of base fluids remains unchanged, then Eqs.(1)-(4) are transformed into

\[
\nabla \cdot \mathbf{v}' = 0, \quad (9)
\]

\[
\frac{\partial \delta \phi}{\partial t} = \nabla \cdot \left[ D_B \nabla \delta \phi \right] = D_B \nabla^2 (\delta \phi), \quad (10)
\]

\[
\rho \left[ \frac{\partial \mathbf{v}'}{\partial t} \right] = -\nabla \delta p + \mu \nabla \mathbf{v}' + (\delta \rho) \mathbf{g} + 2\rho (\mathbf{v}' \times \Omega) + \Sigma \left[ T \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) z_s \right] \delta(z - z_s), \quad (11)
\]

\[
\frac{\partial \delta \rho_f}{\partial t} = -wD\rho_f, \quad (12)
\]

\[
\frac{\partial \delta \rho_p}{\partial t} = -wD\rho_p. \quad (13)
\]

Expressing Eq. (11) in component of velocity \( u, v \) and \( w \) (in \( x, y \) and \( z \) directions)

\[
\rho \frac{\partial u}{\partial t} = -\frac{\partial (\delta p)}{\delta x} + \mu \nabla^2 u + 2\rho \Omega v, \quad (14)
\]
\[ \rho \frac{\partial v}{\partial t} = -\frac{\partial (\delta p)}{\partial y} + \mu \nabla^2 v - 2\rho \Omega u, \] 

(15)

\[ \rho \frac{\partial w}{\partial t} = -\frac{\partial (\delta p)}{\partial z} + \mu \nabla^2 w - (\delta p)g + \sum T \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) z_s \delta(z - z_s). \] 

(16)

And from Eq.(6)

\[ \delta p = (\rho_p - \rho_f)\delta \phi + (1 - \phi)\delta \rho_f + \phi \delta \rho_p. \] 

(17)

Using Eq. (17) into Eq. (16),

\[ \rho \frac{\partial w}{\partial t} = -\frac{\partial (\delta p)}{\partial z} + \mu \nabla^2 w - g[(\rho_p - \rho_f)\delta \phi + (1 - \phi)\delta \rho_f + \phi \delta \rho_p]. \] 

(18)

Thus, Eqs. (9)-(10) (12)-(15) and (18) constitute the perturbation equations. These equations are further analysed by using normal mode technique.

4. Analysis Using Normal Mode Technique. All above perturbation equations are analyzed through normal modes \[10\] in the independent variables \( x, y \) and \( t \) expressed as

\[ f(z) \exp(ik_x x + ik_y y + nt), \] 

(19)

where \( n \) is a growth parameter and \( k_1 = \sqrt{k_x^2 + k_y^2} \) is wave number. Here, it is worth mentioning that the instability in considered system will be checked through the values of \( n \) as it is the symbol of growth rate parameter. Applying Eq. (19) in Eqs. (14), (15) and (18)

\[ npu = -ik_x(\delta p) + \mu(D^2 - k_1^2)u + 2\rho \Omega v, \] 

(20)

\[ npv = -ik_y(\delta p) + \mu(D^2 - k_1^2) v - 2\rho \Omega u, \] 

(21)

\[ npw = -D(\delta p) + \mu(D^2 - k_1^2) w - g[(\rho_p - \rho_f)\delta \phi + (1 - \phi)\delta \rho_f + \phi \delta \rho_p] + \sum T \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) z_s \delta(z - z_s). \] 

(22)

Using Eq. (19) in Eqs. (9) - (10) and (12)-(13), we obtain

\[ ik_x u + ik_y v + Dw = 0, \] 

(23)

\[ n\delta \phi = D_B(D^2 - k_1^2)\phi, \] 

(24)

\[ n\delta \rho_f = -wD\rho_f, \] 

(25)

\[ n\delta \rho_p = -wD\rho_p, \] 

(26)

where \( D = d/dz \). Now from Eq.(20) and Eq.(21) and using Eq. (23)

\[ n\rho \zeta = \mu(D^2 - k_1^2)\zeta + 2\rho \Omega Dw, \] 

(27)

and

\[ n\rho Dw = -k_1^2 \delta \rho + \mu(D^2 - k_1^2) Dw - 2\rho \Omega \zeta, \] 

(28)

where \( \zeta \) is defined as

\[ \zeta = ik_x v - ik_y u. \] 

(29)

Now eliminating \( \zeta \) from Eq. (27) and Eq. (28),

\[ \zeta = \frac{n\rho Dw + k_1^2 \delta \rho + \mu(D^2 - k_1^2)}{2\rho \Omega}. \] 

(30)
Using the Eq. (30) in Eq. (27) we get
\[
\left[(n\rho - \mu(D^2 - k_1^2))^2 + 4\Omega^2\rho^2\right] = -k_1^2 \left(n\rho - \mu(D^2 - k_1^2)\right) \delta p. \tag{31}
\]
Eliminating the pressure term from Eq. (31) and Eq. (22), the expression will be
\[
(n\rho - \mu(D^2 - k_1^2))^2 (D^2 - k_1^2) w + 4\Omega^2\rho^2 = g \left(n\rho - \mu(D^2 - k_1^2)\right)
\]
\[
[(\rho_p - \rho_f)\delta \phi + (1 - \phi)\delta \rho_f + \phi \delta \rho_p] + Tk_1^4 \left(n\rho - \mu(D^2 - k_1^2)\right) \delta (z - z_s). \tag{32}
\]
Using the Eq. (24)-(26) in Eq. (31) it becomes as
\[
(n\rho - \mu(D^2 - k_1^2))^2 (D^2 - k_1^2) w + 4\Omega^2\rho^2 = g \left(n\rho - \mu(D^2 - k_1^2)\right)
\]
\[
\left[(\rho_p - \rho_f) D_B(D^2 - k_1^2) \phi + (1 - \phi)\left(-\frac{w}{n}\right) D\rho_f + \phi \left(-\frac{w}{n}\right) D\rho_p\right]
\]
\[
+ Tk_1^4 \left(n\rho - \mu(D^2 - k_1^2)\right) \delta (z - z_s). \tag{33}
\]
Without taking the rotation ($\Omega = 0$), the above mathematical expression is reduced to
\[
(n\rho - \mu(D^2 - k_1^2))^2 (D^2 - k_1^2) w = g \left(n\rho - \mu(D^2 - k_1^2)\right)
\]
\[
\left[(\rho_p - \rho_f) D_B(D^2 - k_1^2) \phi + (1 - \phi)\left(-\frac{w}{n}\right) D\rho_f + \phi \left(-\frac{w}{n}\right) D\rho_p\right]
\]
\[
+ Tk_1^4 \left(n\rho - \mu(D^2 - k_1^2)\right) \delta (z - z_s), \tag{34}
\]
which is further simplified as after taking density, viscosity and volume fraction of nanoparticles as constant
\[
n \left(\rho D^2 w - \rho k_1^2 w\right) = \mu(D^2 - k_1^2) D^2 w - k_1^2 \mu(D^2 - k_1^2) w
\]
\[
+ g k_1^2 \left[(\rho_p - \rho_f) D_B(D^2 - k_1^2) \frac{\phi}{n}\right]. \tag{35}
\]
Eq. (35) coincides with the dispersion relation of Sharma and Bhardwaj [24] without taking the nanoparticles, effect of porosity and the effect of viscoelastic fluid under consideration which validates our results. Thus, the presence of nanoparticles, rotation and surface tension has modified the dispersion relation in the current study and this is shown through the Eq. (33).

5. Boundary Conditions and Two Superimposed Nanofluids of Uniform Densities. The dispersion relation in Eq. (33) is expressed in term of rotation, density, surface tension and in presence of nanoparticles. Here, let us consider the case of the uniform nanofluids with densities $\rho_1$ in lower layered nanofluid and $\rho_2$ in upper layered nanofluid separated at the interface $z = 0$. Treating density as constant in two regions and in the absence of the surface tension, Eq. (33) becomes
\[
(n\rho - \mu(D^2 - k_1^2))^2 (D^2 - k_1^2) w + 4\Omega^2\rho^2 D^2 w = g \left(n\rho - \mu(D^2 - k_1^2)\right)
\]
\[
\left[(\rho_p - \rho_f) D_B(D^2 - k_1^2) \phi\right]. \tag{36}
\]
For getting C.F. of ordinary differential equation; Eq. (36) must be written as
\[
\left[(D^2 - q^2)(D^2 - k_1^2) + \frac{4\Omega^2\rho^2}{\mu^2}\right] w = 0, \tag{37}
\]
where \( q \) is defined as \( q^2 = k_1^2 + \frac{n\rho}{\mu} \). Equation (37) gives two real roots and four complex roots. In order to satisfy boundary conditions, only real roots are considered and complex roots are ignored. As \( w \) must vanish both as \( z \to -\infty \) (in lower nanofluid) and \( z \to \infty \) (upper nanofluid) which suggests the general solution of Eq. (37) can be written as
\[
\begin{align*}
  w_1 &= B_1 e^{Pz}, \text{for } z < 0, \\
  w_2 &= B_2 e^{-Pz}, \text{for } z > 0,
\end{align*}
\] (38) (39)
where \( P \) is defined as
\[
P = \frac{(l_1 + l_2)^{1/2}}{(p_1 + p_2)^{1/6}},
\] (40)
where \( l_1, l_2, p_1 \) and \( p_2 \) are defined as
\[
\begin{align*}
  l_1 &= -2q^2\mu^2k_1^2 + \mu^2q^4 - 12\Omega^2\rho^2 + \mu^2k_1^4 + (p_1 + p_2)^{2/3}, \\
  l_2 &= (2q^2\mu + \mu k_1^2)(p_1 + p_2)^{1/3}, \\
  p_1 &= 3q^4\mu^3k_1^2 - \mu^3q^6 - 18\Omega^2\rho^2k_1^2 - 36\Omega^2\rho^2q^2 - 3\mu^2k_1^4q^2 + \mu^3k_1^6, \\
  p_2 &= 6\Omega \rho \\
  (9q^4\mu^4k_1^4 - 9\mu^4q^6k_1^2 + 60\mu^2\Omega^2\rho^2k_1^2q^2 + 24\mu^2\Omega^2\rho^2q^4 - 3\mu^2k_1^4q^2 + 3\mu^4q^8 + 48\Omega^4\rho^4)^{1/2}.
\end{align*}
\] (41) (42) (43) (44)
Now for ensuring the continuity of \( w \) (Chandrasekhar [25]) at \( z = 0 \), the boundary condition must be
\[
B_1 = B_2 = B_0.
\] (45)
In view of continuity of \( Dw \) and \( D^2w \), the result of integrating the Eq. (33) and using \( z_s = \frac{w_s}{n} \), we obtain
\[
4\Omega^2\Delta_0(\rho^2 Dw) = gk_1^2 (n\rho + \mu k_1^2) (\rho_p - \rho_f)\Delta_0(D_B\phi) - \frac{gk_1^2 (n\rho + \mu k_1^2)}{n} (1 - \phi)\Delta_0(\rho_f)w_0 - \frac{gk_1^2 (n\rho + \mu k_1^2)}{n} \phi\Delta_0(\rho_p)w_0 + \frac{Tk_1^4 (n\rho - \mu(D^2 - k_1^2))}{n} w_0,
\] (46)
where \( \Delta_0 \) denotes the jump that a quantity experience at the interface \( z = 0 \) while substituting Eq. (38) - (39) and Eq. (45) into the Eq. (46). The following dispersion relation is obtained for the analysis of Rayleigh Taylor instability comprising the parameters rotation, surface tension and concentration of nanoparticles as
\[
-4n\Omega^2B_0P(\rho^2 + \rho_f^2) = \rho^2n (n\rho + \mu k_1^2) (\rho_p - \rho_f)(D_B\phi_0) - 2B_0gk_1^2
\]
\[
(n\rho + \mu k_1^2) (1 - \phi)(\rho_f2 - \rho_f1) - 2B_0gk_1^2 (n\rho + \mu k_1^2) \phi(\rho_f2 - \rho_p) + 2B_0Tk_1^4 (n\rho - \mu(D^2 - k_1^2)),
\] (47)
which further can be simplified by taking \( C = \frac{D_0}{B_0} \) and introducing Atwood number as \( A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \), the Eq.(47) becomes

\[
\begin{align*}
&n^2 g k_1^2 \rho (\rho_p - \rho_f) \phi_0 C \\
&+ n \left[ 4 \Omega_2 P (\rho_2 + \rho_2^2) + g \mu k_1^2 (\rho_p - \rho_f) \phi_0 C + 2 k_1^2 \rho f \left( g A - \frac{T k_1^2}{\rho} \right) \right] \\
&+ 2 \mu \rho k_1^4 \left( g A - \frac{T k_1^2}{\rho} \right) = 0.
\end{align*}
\]

On substituting the expression of \( P \) from Eqs. (40)-(44) into Eq. (48), then Eq. (48) takes the form

\[
\lambda_0 n^{16} + \lambda_1 n^{15} + \lambda_2 n^{14} + \lambda_3 n^{13} + \lambda_4 n^{12} + \lambda_5 n^{11} + \lambda_6 n^{10} + \lambda_7 n^9 + \lambda_8 n^8 \\
+ \lambda_9 n^7 + \lambda_{10} n^6 + \lambda_{11} n^5 + \lambda_{12} n^4 + \lambda_{13} n^3 + \lambda_{14} n^2 + \lambda_{15} n + \lambda_{16} = 0.
\]

Eq. (49) gives the expression in degree 16 and the due to complexity of the expression of \( P \), coefficients \( \lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_{16} \) become quite lengthy to write upon here. Now obtained dispersion relation is further analyzed to study the stability of the configuration in the presence/absence of nanoparticles while the system is already experiencing the Rayleigh Taylor instability.

6. Stability Analysis and Discussion. In order to examine the stability of the system, the dispersion relation is put forth under investigation by substituting the numerical values of different parameters. These numerical calculations have been done nicely by using the software Mathematica and the numerical results for stable and unstable modes of the configuration are discussed by considering all the aspects of different parameters.

**Stable Mode** \( (\rho_1 > \rho_2) \): Equation (48) does not go under any variation as far as the sign of coefficients of all terms are concerned for \( \rho_1 > \rho_2 \). Therefore, the configuration is supposed to experience stability in absence and presence of nanoparticles for the present case. It is worth mentioning that the stability is counted on the virtue of base fluids bearing higher density than those of nanoparticles.

**Unstable Mode** \( (\rho_2 > \rho_1) \): For \( \rho_2 > \rho_1 \), Routh -Hurtwitz’s criterion ensures the instability when at least one variation in the sign of coefficient is experienced by the dispersion relation for the case of whenever upper layered nanofluid bears higher than that of lower layer nanofluid. This Routh -Hurtwitz’s criterion is considered for all cases in unstable mode (which are reported here in different cases like absence/presence of nanoparticles). Here, the last term in Eq. (48) is capable of ensuring one variation of sign with the condition

\[
k_n = \left[ \frac{g (\rho_2 - \rho_1)}{T} \right]^{1/2}.
\]

The above condition is validated with the result discovered by Sharma and Bhardwaj [24]. Therefore, in this case presence and absence of different parameters can cause the hastening or fastening the onset of Rayleigh Taylor instability. For better analysis, the study is further categorized on three platforms which are described as follows:
6.1. **Absence of nanoparticles.** Here, the behavior of different parameters present in system is pondered over in the absence of nanoparticles i.e. taking $\rho_p = 0$, $\phi_0 = 0$ and $\phi = 0$ in the Eq. (48). For the analysis, water is chosen as the base fluid in lower layered fluid and glycerol as a base fluid in upper layered fluid. For tracing the curves using Eq. (48), numerical values of physical quantities of these base fluids are described as $\rho_f_1 = 1000\text{kg/m}^3, \rho_f_2 = 1260\text{kg/m}^3, \mu = 1.41089\text{Pa.s, } \rho = 2260\text{kg/m}^3$ [Volk and Kahler [28]].

To study the effect of a particular parameter, variation in that parameter is taken while keeping others fixed and graph is plotted between growth rate of unstable mode named $n$ and wave numbers by $k = (k_1/10)$. Behavior of surface tension and Atwood number are notified through the figure 1 and figure 2 respectively and further a fair comparison is drawn between the presence and the absence of rotation with the reference of work of Ahuja and Girotra [2] for a certain range of wave number. On varying surface tension, it is depicted through figure 1 that surface tension destabilizes the system of two superimposed nanofluid layers in the presence of rotation parameter whereas it stabilizes the system in absence of rotation in the considered range of wave number. Further, it is noticed that rotation does not help the growth rate parameter to achieve its critical value while in the absence of rotation growth rate first increases, achieves its critical value then it starts decreasing. Ahuja and Girotra [2] has shown the critical values of the growth rate for varying surface tension are experiencing a great fall but due to presence of rotation parameter, for certain range of wave number, surface tension is pouring destabilization. Figure 2 signifies the impact of Atwood number on the system of superimposed nanofluid layers in the presence/absence of rotation. It is noted that (for a particular range of wave number) in the absence of rotation Atwood number has destabilizing impact on the system [2] while adding rotation in the system its starts stabilizing the configuration but with insignificant rate. Thus, it is concluded that rotating the system of two superimposed nanofluid layers in vertical direction suppresses the contribution of Atwood number in Rayleigh Taylor instability. Impact of rotation is studied by keeping all other parameters fixed and varying rotation parameter as $\Omega = 0.1, 0.2, 0.3$ only. It is recorded through the figure 3 that rotation parameter increases the stability of the system with a significant rate.

6.2. **Presence of same kind of nanoparticles in both layers of nanofluids.** In this case instability is studied when same kind of nanoparticles are present in both of the layers of fluids. Base fluids are considered as water and glycerol in lower and upper layer respectively and nanoparticles as aluminium with density $\rho_p_1 = \rho_p_2 = 2700\text{mg/m}^3$ [Volk and Kahler [28]] with volume fraction i.e. $\phi = 0.01$. Other physical quantities are having numerical values as $\mu = 1.41089\text{Pa.s, } \rho_f_1 = 1000\text{mg/m}^3, \rho_f_2 = 1261\text{mg/m}^3$.

Behavior of surface tension (for $T=0.0001, 0.0002$) and Atwood number (for $A=0.11, 0.19$) in the presence of rotation and nanoparticles are interpreted through figures 4 and 5 respectively. Figure 4 depicts that presence of rotation in two superimposed nanofluid layers sets surface tension into destabilizing mode (for a certain range of wave number) when both layers of fluids are having same type of nanoparticles. While considering rotation to be nullified in the presence of nanoparticles as taken by Ahuja and Girotra [2], the curves represented for surface tension are
Figure 1. Effect of surface tension on growth rate parameter of Rayleigh Taylor instability in the absence of nanoparticles with rotation and without rotation for varying $T=0.1,0.2$.

Figure 2. Effect of Atwood number on growth rate parameter of Rayleigh Taylor instability in the absence of nanoparticles with rotation and without rotation for varying $A=0.11,0.119$.

imparting the significant impact on decelerating the onset of Rayleigh Taylor instability. On the other hand, the curves in figure 5 for Atwood number under the influence of rotation are so overlapped to describe their contribution to stability as the rate of stabilization is very slow. And when the effect of rotation is not taken
under consideration (for a certain range of wave number), Atwood number starts destabilizing the configuration[2].

Further, if the volume concentration of nanoparticles is kept on increasing from 0.01 to 0.05, it is observed from figure 6 that growth rate parameter decreases. Thus, it creates the delaying process of Rayleigh Taylor instability in the presence of rotation whereas the volume fraction of nanoparticles in the absence of rotation[2] is expressing the destabilization impact. Thus, it is established that whenever
presence of rotation takes place, volume fraction of nanoparticles throws significant stabilization influence but the absence of rotation creates entirely opposite scenario for the configuration.

Now in figure 7, the entire observation is made on the behavior of rotation with same kind of nanoparticles present in both of the layers of nanofluids. In figure 7, while keeping the value of rotation as (in increasing order), growth rate is decreasing. Thus, it is concluded that rotation parameter is exhibiting the stabilization in the
6.3. Presence of different kind of nanoparticles in both layers of nanofluids. In this case, different kind of nanoparticles are uniformly distributed in both of layers of base fluids. Here, the base fluids as water and glycerol in lower and upper layer respectively and nanoparticles as aluminium and magnesium in lower and upper base fluids with density \( \rho_1 = 2700 \text{mg/m}^3, \rho_2 = 1738 \text{mg/m}^3 \) (Volk and Kahler [28]) with volume fraction i.e. \( \phi = 0.01 \). Other physical quantities are having numerical values as \( \mu = 1.41089 \text{Pa.s}, \rho_f = 1000 \text{mg/m}^3, \rho_f = 1261 \text{mg/m}^3 \).

The observation is recorded on the behavior of surface tension and concentration of nanofluids in the presence of rotation when both layers of fluid are having different kind of nanoparticles. The curves shown in figure 8 (for surface tension) demonstrate that presence of rotation on two superimposed nanofluid layers is successful in inducing destabilization for the case of presence of different nanoparticles in both of layers of fluid for a particular range of wave number shown in figure 8. And the same figure for surface tension in the absence of rotation, is embarking the stability with a fast pace. When figures 1, 4 and 8 are compared, then it throws the light on the fact that the presence of rotation parameter is reversing the impact of surface tension on the Rayleigh Taylor instability. As per the discussion of Ahuja and Girotra [2], without having rotation in the system for the same range of wave number, surface tension is drawing a great impact to stabilize the unstable mode of configuration and the rate of stabilization is increasing in the presence of nanoparticles as compared to the absence of nanoparticles. But the presence of rotation factor influences it to fasten the onset of Rayleigh Taylor instability in the presence of nanoparticles.

To enlist the behavior of Atwood number, one needs to pay attention to figure 9. In figure 9, even though curves are overlapping for different values of Atwood number due to negligible difference between the corresponding values, still it clearly entitles to pour stabilization in the presence of rotation. While the absence of rotation...
Figure 8. Effect of surface tension on growth rate parameter of Rayleigh Taylor instability in the presence of different kind of nanoparticles with rotation and without rotation for varying $T=0.0001, 0.0002$.

Figure 11 gives totally opposite observation about Atwood number (for same range of wave number) that Atwood number enhances the destabilization in the unstable mode of configuration for the presence of different nanoparticles as it is clear from the table of critical values of growth rate parameter for the Atwood number depicted in [2]. Through the figures 2, 5 and 9, it can be observed that Atwood number along with rotation parameter (in absence/ presence of nanoparticles) becomes the tranquil agent to induce stability while in absence of rotation parameter it is still pouring destabilization.

Now, when figure 10 is observed, then it is notified that as the value of volume concentration of nanoparticles is increased from 0.01 to 0.05, the corresponding curves are getting lower and contributes to delay of Rayleigh Taylor instability in the presence of rotation factor. While without considering the impact of rotation (10), the curves of concerned case are capable of rousing the Rayleigh Taylor instability for unstable mode of configuration. By comparing the figures 6 and 10 one may analyze that the absence of rotation factor with the presence of nanoparticles, the volume fraction of nanofluids stimulates the onset of Rayleigh Taylor instability as portrayed in the figures (10-11) plotted in [2]. On the other hand, as the rotation factor is introduced, the volume fraction of nanofluids becomes an active participant to damp the instability quickly and pave the path to stability.

Figure 11 gives the chance to tell the behavior of rotation in presence of different kind of nanofluids. Again, the variations are taken as $\Omega = 0.1, 0.2, 0.3$ where as the value of volume fraction is 0.01 and surface tension and Atwood number are fixed. Figure 11 cascades the stabilization influence on the configuration and controls the onset of Rayleigh Taylor instability in a significant way. It is analyzed through figures 3, 7 and 11 that the rotation factor seems to be dominant to recreate the stability zone by dampening the effect of Rayleigh Taylor instability.
7. **Conclusion.** This study produces a quintessential sketch of the Rayleigh Taylor instability of nanofluids superimposed on one another in non porous medium by considering the effect of different parameters. Here, two base fluids water and glycerol are considered as lower layered and upper layered fluids. Conservation equations are interpreted as a mathematical model and subjected for linear perturbation and
further analyzed by using Normal mode technique. A modified dispersion relation is sorted and the impact of different parameters like surface tension, Atwood number, rotation and volume fraction of nanoparticles among two nanofluids are observed through the graphs between the wave number and growth rate by keeping one parameter variable and others fixed. The following observations are analyzed in this paper:

i) The effect of surface tension is recorded by considering all the aspects of other parameters. It is comprehended that presence of rotation helps surface tension to stimulate destabilization (for a certain range of wave number) while absence of rotation factor enhances the impact of surface tension to induce stabilization. In the absence of rotation factor, surface tension stands for depressing the onset of Rayleigh Taylor instability in absence/presence of nanoparticles but as rotation factor comes into the existence, surface tension does accelerate instability in the presence of same/different kind of nanoparticles suspension.

ii) Atwood number also interprets different behavior with or without rotation factor for a particular range of wave number. It is a destabilizing agent for the configuration whenever system is set to produce Rayleigh Taylor instability in all cases where the rotation factor is missing. But whenever the rotation factor shows its presence, then sudden change in behavior is observed as it starts to stabilize configuration at very slow pace in such a way the curves appears to be overlapped and tough to separate.

iii) It is valuable disclosure that volume fraction of nanoparticles enhances the impact of Atwood number to destabilize the system more rapidly and it also helps surface tension to stabilize at good rate in all cases whenever the rotation factor being absent. And for higher volume fraction of nanoparticles the more instability at the interface is recorded in the absence of rotation factor. But as the rotation factor is applied even if in small ratio the whole observation is turn out be the contrary to prior observation. So, here it is worth mentioning that presence of rotation factor...
helps the volume fraction of nanofluids in delaying the onset of Rayleigh Taylor instability.

iv) In the last, when the rotation factor is considered in all of the cases, it is contemplated that not only it dominates over the instability and also acts as a tranquil substance at the interface to delay the process of Rayleigh Taylor instability. But also is a reason to get the attribute of different parameters changed from stabilization to destabilization or vice versa.

After summarizing all valuable facts and analysis about the study, this is relevant to say that presence of nanoparticles along with the rotation do pour a significant influence for accelerating or choking the effects of Rayleigh Taylor instability on the interface. These facts indeed create the needful attention for future scope of research by merging the majorly contributing research fields of modern era C nanofluids, Rayleigh Taylor instability and rotation.

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