Modeling a Propagating Sawtooth Flare Ribbon Structure as a Tearing Mode in the Presence of Velocity Shear

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Abstract

On 2014 April 18 (SOL2014-04-18T13:03), an M-class flare was observed by IRIS. The associated flare ribbon contained a quasi-periodic sawtooth pattern that was observed to propagate along the ribbon, perpendicular to the IRIS spectral slit, with a phase velocity of $\sim 15$ km s$^{-1}$. This motion resulted in periodicities in both intensity and Doppler velocity along the slit. These periodicities were reported by Brannon et al. to be approximately $\pm 0.5$ in position and $\pm 20$ km s$^{-1}$ in velocity and were measured to be $\sim 180^\circ$ out of phase with one another. This quasi-periodic behavior has been attributed to others by bursty or patchy reconnection and slipping occurring during three-dimensional magnetic reconnection. Though able to account for periodicities in both intensity and Doppler velocity, these suggestions do not explicitly account for the phase velocity of the entire sawtooth structure or the relative phaseing of the oscillations. Here we propose that the observations can be explained by a tearing mode (TM) instability occurring at a current sheet across which there is also a velocity shear. Using a linear model of this instability, we reproduce the relative phase of the oscillations, as well as the phase velocity of the sawtooth structure. We suggest a geometry and local plasma parameters for the April 18 flare that would support our hypothesis. Under this proposal, the combined spectral and spatial IRIS observations of this flare may provide the most compelling evidence to date of a TM occurring in the solar magnetic field.

Key words: instabilities – magnetic reconnection – magnetohydrodynamics (MHD) – Sun: flares

1. Introduction

Chromospheric flare ribbons, observed in many flares, are believed to provide indirect evidence of magnetic reconnection occurring in the corona above. The most widely cited interpretation of these elongated ribbons of chromospheric emission is the CSHKP model (Carmichael 1964; Sturrock 1968; Hirayama 1974; Kopp & Pneuman 1976). It holds that an eruption has opened magnetic field lines in regions of opposite sense, and the resulting regions are temporarily separated by a current sheet (CS). Reconnection at this CS creates closed field lines that subsequently retract downward to form the post-flare arcade. The reconnection and retraction energizes the field lines that have been reconnected, and that energy is deposited into the corona to produce the observed emission. The elongated ribbon-like structure thereby maps out, in the chromosphere, the locus in the corona at which reconnection is occurring; it is an image of the CS.

The structure and motions of flare ribbons have provided great insight into the manner in which magnetic reconnection occurs in solar flares. Spreading motion perpendicular to the CS has been used to infer the progress of reconnection and to measure the reconnection electric field (Forbes & Priest 1984; Poletto & Kopp 1986; Qiu et al. 2002, 2004; Isobe et al. 2005). Other motions, either parallel to the ribbon or less orderly, provide evidence of reconnection more complicated than that in simple two-dimensional models (Warren & Warshall 2001; Fletcher et al. 2004; Qiu 2009; Li & Zhang 2015). The detailed, fine structure of the flare ribbons has also been used as evidence of complex structure within the CS itself (Nishizuka et al. 2009). Recent UV spectral and imaging observations of flare ribbons at extremely high spatial resolution made by Interface Region Imaging Spectrograph (IRIS) (De Pontieu et al. 2014) have revealed distinctive quasi-periodic, sawtooth patterns in certain flare ribbons (Brannon et al. 2015; Brosius & Daw 2015; Li & Zhang 2015; Brosius et al. 2016). These may offer a new clue to some facet of magnetic reconnection. At the moment, however, there are several competing hypotheses about the cause of these small-scale sawtooth patterns in the flare ribbon. The pattern was first reported by Li & Zhang (2015) based on IRIS observations of the ribbons of the 2014 September 10 flare (SOL2014-09-10T17:45). They interpreted it as a signature of quasi-periodic slipping generated by a mode of three-dimensional reconnection (Aulanier et al. 2006). A similar pattern was observed by Brannon et al. (2015) and Brosius & Daw (2015) in the ribbons of the 2014 April 18 flare (SOL2014-04-18T13:03). Brosius & Daw (2015) and subsequently Brosius et al. (2016) noted that individual pixels on the IRIS slit exhibited quasi-periodic brightenings and Doppler shifts at $\sim 3$ minute intervals in a number of spectral lines observed by IRIS, as well as by Hinode EUV Imaging Spectrometer (EIS). They attributed this quasi-periodic behavior to a bursty or patchy mode of magnetic reconnection.

Brannon et al. (2015, hereafter called BLQ) studied the same flare as Brosius & Daw (2015) but reached a different conclusion about the cause of the quasi-periodic phenomena. They studied both the 1394 and 1403 Å spectral lines of Si IV ($T \approx 80,000$ K) over a range of positions along the spectral slit, as well as the sequence of 1400 Å slit-jaw images (SJIs) showing mostly the same Si emission. The SJIs clearly revealed the sawtooth pattern in the flare ribbon with a spatial period of approximately $\lambda_{\text{SL}} = 1.8$ Mm (roughly $2\pi$) shown in Figure 1. They also reported that the pattern seemed to propagate along the ribbon with an apparent speed of $v_p \approx 15$ km s$^{-1}$ (denoted in Figure 1 as $v_p/2$). This parallel pattern speed was superimposed on the much slower, $1–2$ km s$^{-1}$, outward motion of the ribbon itself (southward in Figure 1). As the pattern moved across the slit, it caused the
point of peak intensity to move up and down at ~20 km s\(^{-1}\) with the ~3 minute period. This apparent motion of the bright ribbon along the slit resulted in the quasi-periodic brightening of a fixed pixel position reported by Brosius & Daw (2015).

BLQ successfully fit each Si iv spectral line with two Gaussians in about 80% of the ribbon’s pixels. The central positions of each Gaussian component oscillated in time, redward and then blueward. The bluer of the two components oscillated about a small mean velocity and therefore showed redshifts followed by blueshifts of ±20 km s\(^{-1}\). The measurement of this secondary bluer component, as well as the phase velocity of the sawtooth feature, is the fundamental difference between the interpretation in this work and those of previous studies. Brosius & Daw (2015) calculated Doppler velocities by finding the centroid of various line profiles, a procedure that might mask the Doppler velocities of the separate components. The redder component oscillated about a central value of ~40 km s\(^{-1}\), presumably from explosive chromospheric evaporation (Fisher et al. 1985). In both cases, the velocity shift was approximately 180° out of phase with the position shift: it was bluest when the pattern had reached its northernmost point on the slit. This phase relation is depicted in Figure 11 of BLQ, which we reproduce here in Figure 2. It is especially clear in a single oscillation, highlighted in the bottom panel, that the velocity-versus-position curve forms a diagonal line (180° phase difference) rather than an ellipse (90° phase difference).

Since the April 18 flare occurred some 40° from disk center, plasma motion that is horizontal at the solar surface would produce Doppler shifts, as well as motion along the slit (BLQ). Kinematics of linear motion dictate that position and velocity oscillate 90° out of phase from one another. To explain the observed phasing, BLQ proposed that the plasma was moving in horizontal ellipses, so its two orthogonal components were ~90° out of phase with one another. In that case, the velocity of one component (the line of sight (LOS)) would be ~180° out of phase with the position of the other (position along the slit), in agreement with the observations.

BLQ went on to observe that elliptical fluid motions occur in a number of well-known, large-scale instabilities occurring at some kind of surface or interface. The classic Kelvin–Helmholtz (KH) instability (Chandrasekhar 1961) and the tearing mode (TM; Furth et al. 1963) both occur at interface surfaces (a velocity shear layer and a CS, respectively) and therefore have eigenmodes that are periodic along the interface and decay exponentially away from it. Such a structure in the velocity stream function naturally produces elliptical fluid motions. Indeed, a simple surface gravity wave (an f-mode) is well-known for its elliptical motions. BLQ therefore proposed that the quasi-periodic oscillation they observed in the flare ribbon was a manifestation of some form of surface-confined instability such as the KH or TM.

Both the KH and TM instabilities have been the subjects of extensive study and previously invoked in numerous roles in solar phenomena. Si iv line profiles with bright cores and broad wings have been associated with TM island formation, “plasmoids,” during small-scale reconnection and transition region explosive events (Innes et al. 2015). Plasma blobs imaged by Atmospheric Imaging Assembly (AIA; Lemen et al. 2012) have been observed to occur in the CS above a flare arcade (Takasao et al. 2012, 2016) and in coronal bright points (Zhang et al. 2016). These plasma blobs are thought to be the result of magnetic islands generated by the TM instability. Wavelike structures observed on the edge of erupting coronal structures (Foullon et al. 2011, 2013; Öfman & Thompson 2011) with various phase velocities have been attributed to the KH instability. Most attributions to the TM and KH instability rely on appearance alone. While we trust that those attributions are correct, we hope to show that the spatial and spectral IRIS observations of April 18 provide a more quantitative link between observation and model.

Either instability, in its distinct, traditionally studied form, faces significant challenges in explaining the quasi-periodic pattern observed in the April 18 flare ribbon (BLQ). For shear in a magnetized plasma to undergo KH instability, the velocity difference across the shear layer must exceed the component of Alfvén speed parallel to the flow difference (Chandrasekhar 1961). The magnetic field strength near the ribbon was ~150 G, leading to an Alfvén speed \(v_{A} \sim 10,000\) km s\(^{-1}\), some 500 times greater than any of the observed flow speeds. While only a fraction of that field strength will be directed...
parallel to the flow, it seems extremely unlikely that the fraction could be as small as 0.2%.

TMs, in contrast, are well-known features in CSs. They will be unstable if the plasma resistivity is low enough that the Lundquist number, computed with respect to the thickness of the CS, exceeds \( S \sim 10^8 \) (Biskamp 1986). Classical resistivity and a Sweet–Parker CS result in values as large as \( S \sim 10^5 \). The instability will produce elliptical motions but at speeds lower than the Alfvén speed by an inverse fractional power of \( S \). Depending on the values of \( S \) and the inverse power (this depends on other factors as well), that might be well below the 20 km s\(^{-1}\) flow speeds observed. Furthermore, the traditional TM analysis, set in an otherwise stationary CS, does not predict a phase velocity with which we might associate the observed pattern speed of \( V_\phi \sim 15 \text{ km s}^{-1} \).

In the present work, we show that when a steady sub-Alfvénic velocity shear occurs across a CS, the modified TM instability can achieve higher flow speeds and exhibit a significant phase velocity. Indeed, the two speeds will generally be comparable, as they were observed to be in the April 18 flare. We find that such a combined KH/TM instability is compatible in many respects with the observations reported by BLQ. We therefore propose that the flare of 2014 April 18 provides evidence of a TM-like instability occurring at its CS.

While many previous studies of the TM have been set at a static CS (Furth et al. 1963; Steinolfson & van Hoven 1983; Velli & Hood 1989; Pucci & Velli 2014), several notable exceptions did include an equilibrium shear flow (Einaudi & Rubini 1986; Ofman et al. 1991, 1993; Chen et al. 1997; Li & Ma 2010; Zhang et al. 2011; Doss et al. 2015). Those analyses focused their attention on the effects of the shear on the growth rates of the instabilities. We repeat these previous analyses here in order to compute the amplitude and structure of the eigenmode’s velocity field. A simple linear model allows us to easily generate flow structures while minimizing free parameters, thereby permitting us to explore a wide swath of parameter space. From this we are able to synthesize the position and Doppler signature of a plasma element. We find that these reproduce the aspects of the April 18 observation described above reasonably well. We propose that there was a horizontal bulk plasma motion of \( \nu_0 \sim 37 \text{ km s}^{-1} \) on the open-field (unreconnected) side of the flare ribbon and no flow on the other (postreconnection) side, as indicated in Figure 1. In this configuration, any TM in the CS will propagate with a phase speed of \( \nu_0/2 \sim 18.5 \text{ km s}^{-1} \), the average of the far-field equilibrium shear flows, as observed (Doss et al. 2015). The fluid elements on the stationary side of the CS will then undergo elliptical motion with a similar velocity about a fixed central point. Since this is the postreconnection side, it harbors the energized ribbon and is the plasma whose emission we observe. This, we believe, was what occurred and was observed by IRIS on 2014 April 18. If so, it provides a novel observational characterization of a TM and the CS hosting it. It may offer the most compelling evidence to date of a TM occurring in the solar magnetic field.

We present our analysis and support our conclusion as follows. In the next section, we reprise the linear eigenmode calculation for TMs at a CS at which there is also an equilibrium shear flow (Einaudi & Rubini 1986; Ofman et al. 1991); they are a form of combined TM/KH mode. We forgo discussion of growth rates and focus instead on the structure of the velocity fields of the eigenmodes that were not thoroughly presented by previous investigators. In Section 3, we propose a geometry in which such an instability might have occurred in the 2014 April 18 flare. We synthesize the observations that would result from such a scenario. The synthesized observation compares favorably to the observations reported by BLQ and to our Figure 2 in particular. Finally, in Section 4, we discuss the significance of our proposed explanation.

**2. Linear TM Instability with Shear Flow**

Since the first publication on the linear TM instability in resistive plasmas (Furth et al. 1963 hereafter FKR), many investigators have found solutions to the eigenvalue problem numerically. These numerical studies tested the validity of the assumptions used to find the instability growth rates and scaling relations analytically (Steinolfson & van Hoven 1983). They also examined the effects of extra terms in the governing equations, different coordinate systems, and boundary conditions on instability growth (Velli & Hood 1989; Ofman et al. 1991). We repeat this analysis in order to verify the observational...
effects of the TM instability with an asymmetric shear flow. The following solution method has been used in many of those previous investigations (Killeen 1970; Steinolfson & van Hoven 1983; Ofman et al. 1991) and is included here for completeness.

For our analysis, we use the two-dimensional, incompressible, resistive magnetohydrodynamic equations. The TM instability is governed by the resistive induction equation,

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) - \nabla \times (\eta \nabla \times B),$$

(1)

vorticity equation,

$$\frac{\partial (\nabla \times u)}{\partial t} = \nabla \times \left\{ \frac{1}{4\pi \rho_0} (\nabla \times B) \times B - (\nabla \times u) \times u \right\},$$

(2)

and condition of incompressibility,

$$\nabla \cdot u = 0.$$  

(3)

We also take resistivity, $\eta$, and density, $\rho_0$, to be uniform.

We linearize these equations about an initial magnetic field, $B_0(y)$, given by the Harris CS (Harris 1962), along with an initial velocity shear profile $u_0(y)$. For simplicity, we give the shear profile the same functional form as the magnetic field. In Section 3, we consider an asymmetric velocity shear; however, to simplify the analysis in this section, we move to a reference frame in which the velocity shear is symmetric: the flow is equal and opposite across the CS. In this reference frame, for sufficiently small shear velocity $V$, the phase velocity of any instability will be zero. The instability will then be given a phase velocity when, in Section 3, we return to the stationary frame of reference. For further simplicity, the thickness, $a$, of the CS and velocity shear are taken to be the same,

$$B_0(y) = B_0 \tanh(y/a) \hat{x},$$

(4)

$$u_0(y) = V \tanh(y/a) \hat{x},$$

(5)

where $B_0$ and $V$ are the far-field magnitudes of the field and velocity, respectively.

We perturb all quantities with first-order terms of the form $f_i(x, y, t) = f_i(y, t) e^{i\omega t}$. Using incompressibility to eliminate the $\hat{x}$ component of $u$ and the solenoidal magnetic field condition to eliminate the $\hat{x}$ component of $B_i$ and taking only the $\hat{y}$ component of the vorticity equation gives a coupled set of second-order linear differential equations. We nondimensionalize the equations by scaling the magnetic field to the asymptotic field strength, $B_0$, velocity to the asymptotic Alfvén speed, $v_0 = B_0/\sqrt{4\pi \rho_0}$, and lengths to the thickness of the CS, $a$. The resulting nondimensionalized quantities are denoted with over bars:

$$\bar{u} = u_1/y, \quad \bar{B} = B_1/y, \quad \bar{t} = tv_0/a,$n

$$\bar{k} = ka, \quad \bar{S} = a\bar{v}_0/\eta, \quad \bar{y} = y/a.$$  

(6)

(7)

The nondimensionalized profiles are denoted

$$F(y) = \frac{B_{0y}}{B_0} = \tanh(y), \quad G(y) = \frac{u_{0y}}{v_0} = \bar{V} \tanh(y),$$

(8)

In terms of these, the nondimensional linearized equations are

$$\frac{\partial \bar{B}}{\partial \bar{t}} = i\bar{k}(\bar{F}\bar{u} - \bar{G}\bar{B}) + S^{-1}L\bar{B}$$

(9)

and

$$\frac{\partial \bar{u}}{\partial \bar{t}} = i\bar{k} (F\bar{B} - \bar{G}\bar{u} + L^{-1}(G^n\bar{u} - F^n\bar{B})),$$

(10)

where primes denote derivatives with respect to $\bar{y} \equiv y/a$, and we have introduced the linear operator

$$L = \frac{\partial^2}{\partial \bar{y}^2} - \bar{k}^2,$$  

(11)

whose inverse is denoted $L^{-1}$.

Our velocity variable, $\bar{u}$, differs from other presentations of TM eigenfunctions (Velli & Hood 1989; Ofman et al. 1991) by a factor of $i$. Their definition is preferable in the absence of shear flow because our version of $\bar{u}$ becomes purely imaginary in that case. When shear is present ($V \neq 0$), we find the more symmetric definition of $\bar{B}$ and $\bar{u}$, given in Equation (6), to be preferable.

Considering separately the real and imaginary parts of Equations (9) and (10) gives four equations to advance four independent fields: the real and imaginary parts of $\bar{u}$ and $\bar{B}$. We solve these equations numerically by implicitly integrating all four components forward in time until the fastest-growing unstable mode dominates and pure exponential growth sets in at a single, real growth rate $\gamma = \gamma_0/v_0$. At every time step, the eigenfunctions are rescaled to make the largest value of the largest field (typically the real part of $\bar{B}$) unity. The growth rate for that time step is calculated to be

$$\gamma = \frac{\ln(\max(|\bar{B}, \bar{u}|))}{\Delta \bar{t}},$$

(12)

prior to the solutions being rescaled.

The solution is said to have converged on the fastest-growing mode when

$$\frac{\sum_i |\bar{a}_{ij} - \bar{a}_{ij-1}|^2}{\sum_i |\bar{a}_{ij-1}|^2} < 1 \times 10^{-9},$$

(13)

where $i$ is a spatial index and $j$ is the time step. The velocity eigenfunction, $\bar{u}$, is used to test convergence because it is found to change more over the course of simulation and therefore takes longer to converge. At the point of convergence, the last calculated value of $\gamma$ is the growth rate for that mode.

A purely real $\gamma$ and odd equilibrium functions $F$ and $G$ mean that the real parts of $\bar{B}$ and $\bar{u}$ will be even in $\bar{y}$ while the imaginary parts will be odd:

$$\text{Re}[\bar{u}(y)] = \text{Re}[\bar{u}(-y)], \quad \text{Im}[\bar{u}(y)] = -\text{Im}[\bar{u}(-y)],$$

(14)

$$\text{Re}[\bar{B}(y)] = \text{Re}[\bar{B}(-y)], \quad \text{Im}[\bar{B}(y)] = -\text{Im}[\bar{B}(-y)].$$

(15)

To enforce these symmetries, we set to zero the values of the imaginary parts of the real parts of both $\bar{u}$ and $\bar{B}$ at $\bar{y} = 0$. This permits us to solve in the half space $\bar{y} \geq 0$. Any normal modes with a complex growth rate (i.e., nonvanishing phase velocity) will not have these symmetries. In this case, all four components of the solution will have both odd and even parts, and a solution in full space will be necessary. We do not attempt this in the current study.
Spatial derivatives are computed using a second-order finite differencing scheme on a nonuniform grid. We use a grid from Steinolfson & van Hoven (1983) with spacing prescribed as

\[ D = \frac{D_{\text{max}} - D_{\text{min}}}{1 + \frac{D_{\text{min}}}{D_{\text{max}}}} \]

For our purposes, we use a minimum spacing near the origin, \( D_{\text{min}} = 0.004 \), that increases to a maximum spacing, \( D_{\text{max}} = 0.5 \), at grid point \( i_n = 300 \). Beyond this point, a spacing of \( D_{\text{max}} \) is maintained to the outer boundary \( y = 1000 \).

Our outer boundary condition is set to match the solutions onto the asymptotic analytic solutions to Equations (9) and (10). There are four asymptotic solutions to these equations: two exponentially growing and two exponentially decaying. One exponential decay solution has a decay rate that is proportional to \( S \) and therefore decays much faster than the other for the large Lundquist number. We assume that this mode is absent far from the CS and impose the more slowly decaying asymptotic solution as a boundary condition. This gives

\[
\lim_{y \to \infty} u(y) \propto e^{-\gamma y},
\]

\[
\lim_{y \to \infty} B(y) \propto e^{-\gamma y}.
\]

We implement this numerically by enforcing

\[
f(\bar{y}_{n+1}) = f(\bar{y}_{n-1})e^{-\bar{k}(\bar{y}_{n+1} - \bar{y}_{n-1})}
\]

at the outermost grid point, \( n \), for both \( \bar{u} \) and \( \bar{B} \).

To obtain a single solution, we fix \( S \), \( \bar{k} \), and \( V \) and integrate forward in time using a backward Euler method. The time evolution operator for this method does not have explicit time dependence and therefore only needs to be inverted once for every run. Since we are not interested in the time-dependent behavior of these solutions, a low-order integration scheme can be used without loss of accuracy. The stability that comes from integrating implicitly allows for large time steps, \( \Delta t \sim 100 \), and leads to very fast convergence in roughly 100 time steps. The choice of time step and convergence time depends mostly on the choice of \( S \). Larger time steps can be taken when the growth rate is slower or for larger \( S \).

Figure 3 shows an example of the eigenfunctions for the TM instability with an equilibrium shear flow \( V = 1.5 \times 10^{-3} \), \( \bar{k} = .0234 \), and \( S = 1 \times 10^7 \). Here \( \bar{k}_{\text{max}} \) is the wavenumber leading to a maximum growth rate for the given value of \( S \). The magnetic eigenmode is fairly uniform (\( \bar{B} \geq 0.03 \)) out to about \( \bar{y} \geq 0.03 \). The velocity eigenmode has structure outside this region and thus does not conform to the so-called constant-\( \psi \) approximation (FKR) frequently used to solve for eigenmodes analytically. This is consistent with the findings of Steinolfson & van Hoven (1983), who found that the maximum growth rate for a given \( S \) always occurs in an area of parameter space where this approximation is not valid.

To visualize these eigenfunctions, we contour the flux function, \( A \), and stream function, \( \Phi \), defined to produce the magnetic field and velocity fields:

\[
B = \nabla \times A(x, y)\hat{\xi},
\]

\[
u = \nabla \times \Phi(x, y)\hat{\xi}.
\]
The full value of each consists of an equilibrium and a perturbed contribution,

\begin{align*}
A_0 &= \ln(\text{sech}(y)), \\
A_1 &= \frac{1}{k} \{ \text{Re}(\bar{B})\sin(\bar{k}x) + \text{Im}(\bar{B})\cos(\bar{k}x) \}, \\
\Phi_0 &= \bar{V} \ln(\text{sech}(y)), \\
\Phi_1 &= \frac{1}{k} \{ \text{Re}(\bar{u})\sin(\bar{k}x) + \text{Im}(\bar{u})\cos(\bar{k}x) \}.
\end{align*}

The full flux function and stream function consist of the equilibrium contributions added to these perturbations times a small amplitude \(\alpha\),

\begin{align*}
A(x, y) &= A_0 + \alpha A_1, \\
\Phi(x, y) &= \Phi_0 + \alpha \Phi_1.
\end{align*}

These functions are contoured in Figure 4 for \(\alpha = 0.0025\) and three different values of \(V = 0\), \(V = 1.5 \times 10^{-3}\), and \(V = .1\) and \(\bar{V} = 0.1\). We choose these model parameters to illustrate the transition of the eigenmodes from zero shear flow to shear flow that is a significant fraction of the Alfvén speed. In the figure, \(\Phi\) is contoured in red and \(A\) in black. Magnetic field lines follow the black contours, and velocity flow follows the red. Contour spacing is greatly exaggerated and chosen to highlight key features. The magnetic structure in all three cases consists of large magnetic islands separated into two lobes each, with O-points at the lobe centers. These double-lobed islands are separated from one another by X-points located at \(\bar{k}x = (4n + 3)\pi/2\). There is also an X-point at the center of each island, \(\bar{k}x = (4n + 1)\pi/2\), separating the two lobes and their central O-points. Remarkably, the flux functions are virtually indistinguishable in these three cases in spite of their very different shear flows. This is because the imaginary part of \(\bar{B}\) is extremely small compared to its real part for all values of \(\bar{V}\) we have considered.

With no background shear velocity (Figure 4(a)), flow is into the magnetic X-point on either end of the island along straight vertical paths and out along straight horizontal paths. The horizontal flows then converge on the X-points inside each island. They exit these along vertical paths, filling the separated lobes composing each island. This flow pattern brings new flux into the X-points, where it reconnects and then moves into the island. Once in the island, the lines are reconnected a second time to form the two interior island lobes. This flow pattern forms closed stream lines above and below the midplane of the CS.

In Figure 4(c), \(\text{Im}(\bar{u})_{\text{max}} \ll \bar{V} \ll v_a\). Shear flow of this speed is incapable of perturbing the magnetic field and is therefore forced to trace the perturbed magnetic field lines over most of the eigenmode. This behavior comes out of Equations (9) and (10) in the limit \(\text{Im}(\bar{B}) \ll 1\) and \(\bar{V} \gg \text{Im}(\bar{u})_{\text{max}}\), where \(\text{Re}(\bar{u}) \rightarrow (G/F) \text{Re}(\bar{B})\).

Figure 4(b) shows an intermediate regime, \(\bar{V} \approx \text{Im}(\bar{u})\), in which stream lines occur in two distinct classes. Far enough from the CS, stream lines form open contours following the perturbed field lines and not crossing the CS. Closer stream lines, in contrast, are closed and circulate through magnetic X-points in distorted versions of the closed vortices in the shear-free case (Figure 4(a)). The distance from the CS where the stream lines transition from open to closed depends on the magnitude of the perturbation \(\alpha\).

The linear model of the TM/KH mode instability outlined above is not new. However, we hope that a more detailed view of the instability eigenmodes for various values of background shear flow help illuminate the nature of this combined instability. Repeating the analysis has also allowed us to confirm our conjecture in the following section that the combined TM/KH instability can reproduce the behavior observed by Brannon et al. (2015).

3. Modeling Observations

3.1. Geometry

During the flare observed by IRIS on 2014 April 14, BLQ observed that a periodic sawtooth pattern propagated with a phase velocity of \(15 \pm 2\) km s\(^{-1}\) parallel to the ribbon and therefore perpendicular to the IRIS slit. LOS Doppler measurements of the fluid composing this sawtooth pattern found oscillations with an amplitude of \(\approx 20\) km s\(^{-1}\). These velocities were found to be \(\approx 180^\circ\) out of phase with the position on the slit. The observed behavior is best captured in Figure 2. It can also be seen in that figure that the amplitude of the oscillations along the slit are roughly \(1''\). The TM/KH modes described above can produce features consistent with described observation.

We propose that a TM occurred within a secondary CS that had formed along the separatrix legs of the flare arcade. A schematic version of the proposed geometry is shown in Figure 5. This differs from the geometry first put forth by BLQ.
who expected the TM to occur within the main CS beneath the erupting flux rope. We find that our geometry is able to better match all aspects of the observation.

The secondary CS separates newly reconnected arcade field lines from still-open flux connected to the flux rope above. We propose that horizontal motion in this open flux, perhaps persisting from the eruption, creates the velocity shear, as well the magnetic shear that is the CS itself. The flux on the other side of this layer, consisting of newly closed arcade field lines, is expected to be stationary and undetected. These are the field lines in which energy has been deposited, so the ribbon itself should occur within this stationary flux, rather than at the secondary CS.

The current in the secondary sheet flows vertically, so the TM in it will produce a horizontal chain of islands, as illustrated in Figure 5. These will deform the magnetic field outside the sheet and therefore the ribbon. If the TM propagates along the sheet, so will this deformation. We propose that it is this propagating deformation that produces the moving sawtooth pattern in the ribbon.

### 3.2. Introducing Dimensions

To help constrain parameters, we assume that the linear instability has had sufficient time for ~10 e-foldings during the typical flare timescale of $t \simeq 100$ s. We therefore set $\tilde{\gamma} \times 10$ to obtain the constraint

$$\tilde{\gamma} \geq \frac{10 \ a}{(100 \ s) v_{\alpha}} = \frac{\tilde{k} \lambda}{2 \pi v_{\alpha} (10 \ s)}. \tag{26}$$

Here $\lambda$ is the wavelength of the sawtooth pattern from Figure 1 along the solar surface. We have no direct measurement of the sheet’s thickness, $a$, but we can use the observed wavelength, $\lambda$, to dimensionalize our expressions. It is thus, in Equation (26), that we use $\tilde{k} = k a = 2 \pi a / \lambda$ to formally eliminate $a$.

We assume that the horizontal motion of the open flux has deflected it by some small angle, say $10^\circ$, relative to the stationary and undetected arcade flux. This angular difference, $\pm 5^\circ$, produces the tangential discontinuity that is the secondary CS. The field strength in the vicinity of the flare ribbon region was estimated by BLQ to be 150 G. The single-sided $10^\circ$ deflection will therefore produce a reconnecting component of $150 \sin (5^\circ) = 13$ G. Assuming a typical electron density of $3 \times 10^9$ cm$^{-3}$ gives the value $v_{\alpha} = 500$ km s$^{-1}$, which was used in our nondimensionalization.

The sawtooth pattern is observed to propagate with phase velocity perpendicular to the IRIS slit of $V' = 15$ km s$^{-1}$. This was found in the plane of the sky (primed coordinates) but applied to horizontal motion some $40^\circ$ from disk center. Foreshortening effects will reduce the velocity $V$ in the horizontal $x$-direction to an apparent velocity

$$V' = V \cos \theta_c, \tag{27}$$

where $\theta_c \simeq 33^\circ$ is the horizontal distance from disk center. The observed phase velocity of 15 km s$^{-1}$ thus corresponds to a real horizontal velocity of $V \simeq 18.5$ km s$^{-1}$. Scaling this to the reconnection-component Alfvén speed, $v_{\alpha} = 500$ km s$^{-1}$, yields $\tilde{V} = 0.037$.
3.3. The Saturated State

We use these dimensions to construct a synthetic observation from the velocity structure of our model. We do so by first using the velocity structure given by the linear eigenmode, \( \Phi_l(x, y) \), discussed above. We construct the stream function by superposing this using the amplitude, \( \alpha = 1 \), in Equations (24) and (25). Figure 6 shows the magnetic field lines (black) and stream lines (red) of the resulting construction. This specific eigenfunction results from \( S = 10^4 \) and \( \tilde{k} = 0.1315 \), which has a growth rate of \( \gamma = 0.00928 \) in agreement with Equation (26). The green contour shows the trajectory of a fluid element that begins at \([x, y] = [86, 18]\).

Because the growing linear eigenmode has been used in the nonlinear regime, the stream lines (red) cross the field lines (black) in this construction. The magnetic field evolution implied by continuing island growth requires this cross-field flow in the regions outside the diffusion region. This cross-field flow will naturally cease when the linear phase ends and the island growth ends.

Fully nonlinear treatments of the TM, in which island saturation occurs self-consistently, have found magnetic structures that are very similar to those produced by the linear modes with \( \alpha = 1 \) (Ofman et al. 1993; Chen et al. 1997; Li & Ma 2010; Zhang et al. 2011; Doss et al. 2015). However, these treatments find a velocity field very different from the linear eigenmode. In particular, the velocity must align with the magnetic field outside the diffusion region, which is not the case with the unsaturated linear eigenmode, as seen in Figure 6. Nonlinear studies of island saturation in the presence of external shear have shown that the saturated stream function instead assumes a form \( \Phi(x, y) \approx (V_0/B_0)A(x, y) \). In this case, the flow follows the magnetic field lines exactly and matches the external shear flow asymptotically. Such a structure is clear in the nonlinear simulation of Ofman et al. (1993). Using this instead of the linear eigenmode results in the stream lines (red) shown in Figure 7. A particular fluid element is traced in green, as it was in Figure 6. While the interrelations of the magnetic field and stream functions differ significantly between these two constructions, it is noteworthy how similar the green flow lines appear. In both cases, the flow is made to oscillate by the magnetic field distortions produced by the islands.

3.4. Transformation to Observed Quantities

The analysis of the previous section was performed in a reference frame moving so as to make the velocity shear symmetric, thereby endowing the TM with zero phase velocity. In our proposed model, one side of the CS is stationary, while the other is moving horizontally at some speed \( v_0 \) (see Figures 1 and 5). This means that the analysis reference frame is translating at \( V = v_0 / 2 = 18.5 \text{ km s}^{-1} \) to produce the phase velocity observed in the flare ribbon. Solutions for an asymmetric shear flow are found through a Galilean transformation of the horizontal dimension,

\[
\tilde{X} = \tilde{x} - \tilde{V} \tilde{t}.
\]

Note again that \( \tilde{V} \) is not the observed speed but rather the speed of propagation along the solar surface. The transformation is such that flow below the CS is \( 2\tilde{V} = 0.074 \) and zero above, as in Figure 1. In this reference frame, an observer would see the train of islands propagating in the direction of the nonzero shear flow, positive \( \tilde{X} \), with a velocity equal to \( \tilde{V} \). A fluid element following the green trajectories in either Figure 6 or 7 loses most of its net velocity along the CS. Instead, it undergoes elliptical motion (Figures 8(a) and (c), respectively) as the islands move past.

To construct the flows in Figure 8, we move from our nondimensional model to physical units and observed quantities in the plane-of-sky (POS) frame. Model quantities are foreshortened through the following transformations to
observed quantities (denoted by primes):

$$y' = y \cos(\theta_y),$$

$$X' = X \cos(\theta_x),$$

where the viewing angles are

$$\theta_x = \sin^{-1}(y_{\text{POS}}/R_x) = 33^\circ, \quad \theta_y = \sin^{-1}(y_{\text{POS}}/R_y) = -11^\circ,$$

for our region located at $$x_{\text{POS}} = 553''$$, $$y_{\text{POS}} = -188''$$, when the apparent semi-diameter is $$R_x = 956''$$.

Spatial dimensions are scaled to arcseconds through CS thickness $$a$$. For each run of the model, we specify $$\lambda$$, which, with a fixed $$\lambda$$, also specifies the CS thickness, $$a$$. This is done in the following way:

$$k = ka = \frac{2\pi a}{\lambda}, \quad \lambda = \frac{\lambda'}{\cos(\theta_x)}, \quad \text{and} \quad \lambda' = 2''3.$$ (32)

In this tilted frame, a portion of the motion in both the $$x$$- and $$y$$-dimensions are along the IRIS LOS:

$$V_{\text{LOS}} = \sin(\theta_x) \frac{dX}{dt} + \sin(\theta_y) \frac{d\bar{y}}{dt}. \quad \text{(33)}$$

This velocity is scaled to physical units through $$v_a = 500 \text{ km s}^{-1}$$. These transformations and scalings allow for a direct comparison between Figures 2 and 8.

Figures 8(b) and (d) show our model’s reproduction of Figure 2 using the parameters above. These use the fluid trajectories from Figures 6 and 7, respectively, and obtain very similar results for reasons already mentioned. For both constructions, we find that we can produce similar results using a wide range of other parameters.

It is worth noting that in Figures 8(b) and (d), as well as in the observations of Figure 2, LOS velocities oscillate from roughly $$-15$$ to $$+25 \text{ km s}^{-1}$$. The non-zero mean velocity in the model versions come from the fact that the oscillations are not perfectly elliptical, as seen in Figures 8(a) and (c). The observational data is probably consistent with this behavior within the measurement uncertainties. It is also worth noting that the Doppler velocity is not perfectly $$180^\circ$$ out of phase with the relative $$y$$-position. This is due to the fact that motion in the $$\bar{y}$$-direction has a component of velocity along the LOS in the tilted reference frame of the observation. The measured amplitude of the oscillation along the slit is harder to reproduce. This we can attribute to the fact that our linear model is incapable of generating islands that are truly saturated in height.

The measured amplitude of the sawtooth pattern is roughly $$1''$$, which is larger than $$\lambda/2$$. Therefore, our linear model would never be able to exactly match the observation. Regardless, we are able to report that our simple model gets us within a factor of 5 of the true measured values.

The amplitudes of both position and velocity depend on how far the fluid element is from the sheet, which is the starting value of $$\bar{y}$$. Elliptical motions in the solar frame are produced from stream lines that are open in the analysis frame. Closed stream lines will produce motion moving with the analysis frame. Moving closer to the CS gives higher amplitudes in position and velocity but also deviates from the mostly elliptical trajectory. Regardless, the elliptical motion and resulting phase portrait of Figure 8 are achievable for a variety of initial conditions. Doppler velocities on the order of $$20 \text{ km s}^{-1}$$ and elliptical trajectories with sufficient amplitude have been produced for $$S = 10^4–10^7$$ and $$\bar{k} = 0.01–0.5$$. The most limiting factor, and the reason we chose $$S = 10^4$$ for producing Figures 6–8, was getting a growth rate high enough to satisfy Equation (26).

4. Discussions and Conclusions

We have used a simple model of the TM instability, defined in Section 2, to provide evidence that a TM with asymmetric
shear flow could produce the flare ribbon oscillations observed by Brannon et al. (2015). In our model, asymmetric shear flow provides a net phase velocity to the instability, a key element of the findings. It also produces elliptical horizontal motions of fluid elements of the kind inferred by BLQ on the basis of the phase relationship between oscillations in velocity and position. We propose a scenario in which a secondary CS develops at the separatrix of the flaring field. A TM in that secondary CS produces fluid motions similar to those observed. We rely on the measurement of the sawtooth pattern wavelength, $\lambda' = 2''3$, and phase velocity of 15 km s$^{-1}$, as well as a typical flare timescale of 100 s, to redimensionalize our model. Under these constraints, we are able to reproduce the observed behavior of Figure 2 in Figures 8(b) and (d) to a satisfactory level of accuracy.

Our model is constrained by observations of the parameters $\lambda$, $\Delta y$, $\Delta v_{\text{LOS}}$, and a phase velocity of 15 km s$^{-1}$, as well as achieving a minimum growth rate to satisfy Equation (26). These constraints limit our solution to a small area of parameter space. Assuming that $k = k_{\text{max}}$, the only free parameters are $S$ and the distance $y$ from the CS. We can reasonably fit the observations for $S = (1\text{--}1.25) \times 10^{4}$ or, subsequently, $k = 1.22\text{--}0.1315$. This range of $k$, along with a measured wavelength $\lambda = 2''3$, yields a CS thickness $a \approx 39\text{--}42$ km. This is considerably smaller than CSs reported through other observations. Savage et al. (2010) observed a long, thin, bright structure above an arcade with a thickness of several times 10$^3$ km. Others have measured CS thicknesses of similar magnitude (Seaton et al. 2017). Models have shown that these large observed thicknesses may be the result of a layer of hot dense plasma surrounding the CS rather than the CS itself (Yokoyama & Shibata 1998; Seaton & Forbes 2009; Reeves et al. 2010). This leads us to believe that a CS on the order of 10 km is a realistic estimate for the April 18 flare.

Figure 8. Panels (a) and (b) show the green trajectory from Figure 6 after undergoing the Galilean transformation in Equation (28). Panels (c) and (d) show the same for the green trajectory in Figure 7. All displacements have been foreshortened to account for off-disk observation and scaled to physical units. The green dot indicates the particle’s starting location.
Resistivity sets a lower limit on the CS thickness given by the steady Sweet–Parker model: \( a = LS^{-1/2} \), where \( L \) is the length of the CS. The entire flare ribbon, in this case, has a length \( L \sim 50 \text{ Mm} \). Classical Spitzer resistivity yields \( S \sim 10^{12} \), from which we would predict a lower limit of \( a = 50 \text{ m} \). It is possible that the resistivity in the vicinity of the CS is enhanced, perhaps by turbulence or instability (Strauss 1986), to a level where \( S \sim 10^{9} \) for which a Sweet–Parker CS would have the thickness we infer. It is also possible that the sheet has not fully collapsed, and its thickness is instead set by the dynamics of its formation. It is not, after all, the flare CS and is only formed as a secondary consequence of the flare.

The geometrical scenario proposed in Figure 5, necessary for this observation, does not follow from the standard CSHKP model and is not inevitable in a two-ribbon flare. The lack of sawtooth reports from the many other flare ribbon observations made by IRIS suggests that sheared, tearing-unstable secondary CSs are not common to all flares. It would seem that they have occurred at least twice—in the flares of 2014 April 18 and again on 2014 September 10. A survey of several other seemingly suitable IRIS ribbon observations has shown that sawteeth are indeed uncommon, but other candidates seem to have occurred (Roegge & Brannon 2017). Shear flows responsible for deflecting the overlying magnetic field must be fast enough and sustained long enough for a secondary CS to develop and become unstable. Furthermore, if the wave-number or the amplitude were smaller than that of April 18, the pattern may be too small to resolve with IRIS. Indeed, the pattern observed in that flare is marginally detectable in coincident AIA images (BLQ).

The proposed shear flow, embedding the CS, is essential for explaining the observed phase velocity of the sawtooth. It is plausible that the eruption, initially traveling at many hundreds of \( \text{km s}^{-1} \), left a slower but persistent flow in its wake. We have not, however, seen any clear evidence for this proposed flow, aside from the propagation of the sawtooth itself, in any of the data we examined. It is possible that the shear flow and the TM occur higher up the CS, away from the chromospheric ribbon. The line-tying effect of the lower boundary could suppress the flow while still permitting the form of the instability to be imprinted on the chromosphere. A more complicated three-dimensional geometry of this kind would need to be explored in a model more sophisticated than our two-dimensional version of the standard TM model. Our model scenario calls for the horizontal shear flow just discussed, as well as a CS flowing vertically along the separatrix—this is the secondary CS. Such a secondary CS was reported by Janvier et al. (2014) in vector magnetogram data from a much larger flare. In spite of the favorable orientation of that flare, measurement of this weak secondary CS proved extremely difficult. It is nevertheless possible that a similar careful analysis of Solar Dynamics Observatory/Helioseismic and Magnetic Imager (SDO/HMI) data could reveal the presence of the secondary CS in the 2014 April 18 flare.

We have used a simple two-dimensional analytic model of a TM in order to demonstrate, as simply as possible, the viability of our explanation of the observation. An analytic model allows us to explore parameter space and to easily escalate the results for comparison to observations. The simplified analytic model is, however, subject to limitations. We are unable to self-consistently describe nonlinear phases, including saturation. Instead, we construct velocity fields intended to represent two extreme limits. Figure 6 shows an unsaturated state produced from the velocity field of the growing linear eigenmode. Figure 7 approximates the fully saturated state, where flow must follow the magnetic field. Remarkably, both these approximate limiting cases yield observational signatures, compared side by side in Figure 8, that are virtually indistinguishable from one another. This is no coincidence. The basic feature revealed by the observation is flow near the CS being modulated by a moving chain of magnetic islands. This produces elliptical motion of the kind reported by BLQ, and our model implements the explanation originally given in that work. A self-consistent nonlinear model is beyond the scope of our exploratory study, but we believe it would produce velocity structures falling between the two extreme approximations we do consider. Observations synthesized from it would therefore probably resemble the indistinguishable versions produced by the extreme cases. It would therefore match the observations for the same reasons they do.

If our hypothesis stands up under further scrutiny, the April 18 observation provides novel evidence for a TM in the solar corona. Plasmoid observation and the presence of a TM instability provide insight into the onset of fast magnetic reconnection. It is also noteworthy that while the TM occurs during a flare, it does not occur in the flare’s primary CS. The sawtooth would thus seem to be a secondary effect offering only limited insight into the flare itself.

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