Research Article

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Exploration of the dynamics of hyperbolic tangent fluid through a tapered asymmetric porous channel

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Abstract: The present physical problem has a significant number of applications in intra-uterine fluid motion with tiny particles in a nonpregnant uterus, and this situation of fluid motion is very important in examining the embryo motion in a uterus. Due to these real-life applications, in the current investigation, a perturbation-oriented numerical investigation has been performed to describe the characteristics features of velocity, pressure rise, and trapping bolus through streamlines in a tapered channel under a porous medium. The present physical model results in the governing two-dimensional coupled nonlinear flow equations under low Reynolds number and long-wavelength approximations. A suitable equation for stream function is derived and a regular perturbation scheme is employed to produce the numerical solutions in terms of pressure rise, velocity, and streamlines for various values of physical parameters. The current investigation depicts that the enhancing Darcy parameter upsurged the pressure field, and the increasing power-law index suppressed the pressure field in the flow regime. The increasing channel width significantly diminished the velocity field at the central portion of the channel. The size of the trapping bolus suppressed for the enhancing values of Weissenberg number. In addition, the size of the trapping bolus increased for the magnifying values of wave amplitudes. Finally, current numerical solutions reasonably agree with the previously published results in the literature, and this fact confirms the accuracy of the present problem.

Keywords: peristaltic flow, tangent hyperbolic fluid, porous medium, tapered channel, perturbation method

Nomenclature

\[(X, Y)\] dimensional coordinates in the laboratory frame (m)
\[(\bar{X}, \bar{Y})\] dimensional coordinates in the wave frame (m)
\[U, V\] velocity field (m s\(^{-1}\))
\[H_1, H_2\] dimensional wall surface in the laboratory frame (m)
\[h_1, h_2\] non-dimensional wall surface
\[\bar{t}\] dimensional time (s)
\[c\] wave speed (m s\(^{-1}\))
\[d\] channel width (m)
\[n\] power-law index
\[Re\] Reynolds number
\[\lambda\] Wavelength (m)
\[\bar{d}\] channel width (m)
\[a, b\] wave amplitudes (m)
\[\bar{P}\] dimensional pressure (Pa)
\[P\] Non-dimensional Pressure
\[\eta_\infty\] viscosity at the infinite shear rate (Pa s)
\[\eta_0\] viscosity at the zero shear rate (Pa s)
\[We\] Weissenberg number
\[Q\] dimensionless mean flow
\[Da\] Darcy number
\[k\] permeability of the porous medium
\[\delta\] dimensionless wave number
\[\rho\] fluid density (kg m\(^{-3}\))
\[\psi\] dimensionless stream function

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1 Introduction

Peristaltic mechanism is an instinctive process of asymmetrical progression of wave motion and contraction along the elastic walls of the tube filled with the biofluid. The peristaltic fluid flow can be found in many living organisms, such as blood flow in small vessels, food passing through the esophagus, fluids motion through lymphatic vessels, in cilia movement, sperm pumping, and a few other biomedical applications such as heart-lung machine, trapping phenomena, pumping, dialysis equipment, reflux, machine that pumps blood, finger pumps, and so on. These physical processes are studied by considering well-known non-Newtonian fluid flow models. However, in this direction, the hyperbolic tangent fluid flow model is one of the most significant liquid models in the class of non-Newtonian liquids. From the experimental point of view, it is noticed that the tangent hyperbolic flow model assumes the shear thinning behaviour very exactly. Hyperbolic tangent liquids are mostly used in laboratory experiments, R&D industries, medicine, and engineering fields for various purposes. Following are a few examples of hyperbolic tangent fluid in the field of biology and industry: blood, solutions, whipped cream, ketchup, polymers, melts, nail polish and paints.

Latham [1] first investigated the peristaltic motion of various fluid in a peristaltic pump. Akram et al. [2] discussed the characteristic features of velocity and thermal distribution of Maxwell fluid passing through the porous medium under the action of peristaltic flow conditions. Jyothi and Rao [3] discussed the peristaltic flow characteristics of Williamson fluid through the symmetric channel under the influence of the porous medium. However, the porous boundaries have a number of applications in the field of biomedicine such as the mechanism of blood dialysis in the artificial kidneys, gaseous diffusion process, and transpiration cooling. Hayat et al. [4] studied the magnetized peristaltic flow mechanism of Sisko fluid through the asymmetric tapered channel under the influence of thermal radiation. Their investigation shows that, the non-linearity behaviour unaltered under long wavelength and small Reynolds number assumptions. Jyothi et al. [5] described the peristaltic motion of tangent hyperbolic liquid through an inclined porous channel under the influence of unsteady conditions. Jyothi and Rao [6] discussed the magnetized peristaltic motion of hyperbolic tangent fluid in a planar channel under the action of the porous medium. Further, Reddy et al. [7] studied the peristaltic motion of hyperbolic tangent liquid through the porous medium. Reddy and Reddy [8] investigated the vertical channel flow of tangent hyperbolic liquid in the presence of the porous medium in the flow direction.

Prakash et al. [9] discussed the characteristic features of thermal and viscous distribution in peristaltic motion of nanoliquids through the porous tapered channel under the action of the thermal radiation effect. The results of their study reveal that, for various values of amplitude and phase, the porous channel preserves the wave speed. Further, the produced solutions may have significant applications in practical and theoretical experiments such as drugs transport and synthesis of nanoparticles and haemodialysis. Vaidya et al. [10] studied the impact of magnetic field on the peristaltic flow properties of Jeffrey fluid in an asymmetric tapered porous channel. The investigation presented in the literature [10] reveals that the increasing Brinkman parameter raises the thermal field. Vaidya et al. [11] described the effects of heterogeneous and homogenous chemical reactions on peristaltic motion of Ree-Eyring non-Newtonian fluid through the permeable channel, and this study has a number of applications in the field of hydrodynamic fluid flows. Ashraf et al. [12] numerically analysed the peristaltic motion of nanofluids by considering blood as nanoparticles via a generalized differential quadrature scheme. Their investigation shows that the velocity field and thermal profile of nanofluid suppressed by enhancing the non-Newtonian fluid parameter.

Elmaboud et al. [13] studied the effect of variable viscosity on the peristaltic flow of Newtonian incompressible liquid producing sinusoidal wave propagation on the finite tube. Raza et al. [14] investigated the impact of thermal radiation on Williamson fluid flow over an elongating surface under the influence of the thermal transport process. Elmaboud et al. [15] demonstrated the impact of the magnetic field on the flow of generalized Burger’s fluid through two concentric tubes by considering DC- and AC-operated micro-pumps. Koumy et al. [16] analysed the impact of porous and Hall boundary conditions on the channel flow of the Maxwell fluid under the action of the porous medium. It is observed that the velocity field diminished the increasing values of the magnetic field. Eldesoky et al. [17] described the influence of thermal transport on the magnetized peristaltic flow of Newtonian fluid through the channel under the porous medium. It is noted that liquid suspension suppressed the fluid temperature in the flow regime. Abdelsalam and Zaher [18] investigated the impact of electro-osmotic forces on the thermal transport properties of the Rabinowitsch fluid suspension in elastic walls. It is noticed that electro-osmotic forces magnify the shear stress on the walls of the channel. Abdelsalam et al. [19] demonstrated the effect of
electro-magnetic forces on the biomechanics study of swimming of the human sperms in a cervical tube in the female reproductive system. It is observed that the size of the trapping bolus enhanced throughout the fluid motion in the cervical canal. Eldesoky et al. [20] depicted the temperature impact on the peristaltic motion of specific liquid mixture in a cathereterized tube under the long wavelength and low Reynolds number approximations. Bhatti and Abdelsalam [21] numerically investigated the impact of induced and applied magnetic field on the peristaltic flow of Carreau liquid through a symmetric channel. Also, the influence of hybrid nanofluid on the peristaltic motion in the channel with tantalum (Ta) and gold (Au) nanoparticles is noted. Mekheimer et al. [22] studied the synovial liquid movement in an ultra-viscous plasma filtration that lubricates the joint motion in humans. This study has significant advantages in the cartilage repair surgical involvement; further, the blood is injected into the joint and mixed with the variable volumes of synovial liquid. Thumma et al. [23] analysed the influence of nonlinear thermal source/sink and temperature variation on the 3D flow of Maxwell nanofluid about an elongating sheet. It is observed that the Biot parameter amplifies the thermal and concentration fields in the flow regime.

Mahmoud et al. [24] discussed the peristaltic movement of Jeffery fluid in a porous channel under the influence of magnetic parameters via the ADM scheme numerically. The literature [24] shows that, at the central portion of the channel, axial flow is insignificant when compared with the hystromagnetic fluid. Ramesh and Devakar [25] analysed the flow and thermal behaviour of magnetized Williamson fluid under the influence of the peristaltic condition through the inclined asymmetric porous channel with long wavelength and low Reynolds number approximations via the perturbation scheme. Also, the literature [25] predicts that the velocity field enhanced from the porous medium to the non-porous medium in the direction of fluid motion. Elma-boud and Mekheimer [26] discussed the characteristics flow features of nonlinear two-dimensional second-grade liquid through the porous channel with oscillating surfaces under peristaltic conditions. The outcome of their study depicts that the peristaltic motion of the considered fluid is influenced by the weak non-Newtonian character of the fluid. The asymmetric channel flow of hyperbolic tangent fluid under the peristalsis flow condition was illustrated by Nadeem and Akram [27]. The flow and thermal motion of hyperbolic tangent liquid in a vertical channel under the action of magnetic number with peristaltic flow conditions was discussed by Nadeem and Akram [28]. Akbar et al. [29] analysed the slip effects on the peristaltic movement of hyperbolic fluid in an inclined channel with respect to flow and thermal characteristics. Nadeem and Akram [30] discussed the influence of slip parameters on the peristaltic movement of hyperbolic fluid through the asymmetric channel. Kavitha et al. [31] discussed the asymmetric channel motion of Williamson liquid under the influence of peristaltic conditions through the porous medium by using the perturbation scheme.

Wakif et al. [32] analysed the impact of Buongiorno’s model on the flow of micropolar nanofluid about a stretching sheet under the action of various physical effects. It is observed that the enhanced volume fraction results in the dual flow behaviour for the velocity profile in the flow regime. Xia et al. [33] investigated the effect of the oscillatory magnetic field on the flow of the second-grade fluid between two concentric Darcy–Forchheimer porous cylinders with thermal radiation under the influence of convective conditions. It is observed that the increasing magnetic field diminished the velocity profile in the flow regime. Saeed et al. [34] numerically investigated the thermal and concentration diffusion flow properties of magnetized unsteady Oldroyd-B fluid under the influence of ramped conditions. It is found that the increasing Prandtl number suppressed the velocity field. Kumbinarasaiyah and Raghunatha [35] studied the impact of the porous medium on the heat and mass transport properties of two-dimensional rotating micropolar liquid through the porous channel. Suresha et al. [36] numerically investigated the influence of magnetic field and thermal radiation on the flow of the couple stress fluid over a vertical cylinder with viscous dissipation impact. Sharma et al. [37] demonstrated the action of temperature jump, partial slip, and Joule dissipation on the time-dependent flow of nanofluid containing ethylene glycol with graphene nanoparticles over a linear stretching sheet under the influence of the porous medium. Shah et al. [38] studied the thermodynamic flow properties of unsteady second-grade ternary nanofluid flow about a stretching sheet under the action of generalized Fourier’s law. Vedavathi et al. [39] numerically investigated the impact of thermal radiation on nanofluid flow about an elongating sheet under the influence of convective Nield conditions and energy activation with non-Darcy medium. Dawar et al. [40] numerically discussed the impact of magnetized three-dimensional Jeffery nanofluid motion over a stretching sheet under the influence of velocity slip conditions. Afridi et al. [41] studied the thermal transfer and entropy generation in an incompressible two-dimensional viscous fluid over a thin needle under the influence of temperature-dependent conditions. Lund et al. [42] numerically discussed the hybrid nanofluid flow over a moving sheet under the influence of viscous dissipation and thermal radiation effects.

Akram et al. [43] discussed the influence of the half-breed effect on the nanofluid flow under the pseudoplastic
conditions with the applied magnetic field. The outcome of the study describes that the increasing magnetic number enhances the trapping bolus. Javed and Naz [44] described the impact of the elastic surface on the tapered channel flow of realistic liquid in an acquiescent channel. The literature [44] shows that the reverse fluid motion is observed near the walls of the geometry. Bhatti et al. [45] discussed the movement of Jeffrey liquid in an acquiescent tapered channel, and this investigation has a rich set of advantages in intra-uterine studies. The analysis presented in the literature [45] shows that the velocity profile takes a parabolic trend near the walls of the channel. Khan et al. [46] discussed the impact of Soret and Dufour numbers on Johnson–Segalman liquid motion in an asymmetric channel under the influence of Hall currents. It is determined that the increasing Hall parameter raises the velocity field on the surface of the walls. Saleem et al. [47] analyzed the action of secondary velocity slip effects on two-dimensional incompressible Jeffrey liquid motion in an asymmetric channel under the influence of the inclined constant magnetic number. It is observed from the literature [47] that the magnitude of trapped bolus enhanced with the increase in the Hall number. Also, the enhancing Soret parameter decayed the concentration distribution field in the peristaltic flow region.

The current literature review depicted that the peristaltic motion of hyperbolic tangent liquid in an asymmetric tapered channel is not yet attempted. The current physical situation discussed in this article has a significant number of applications in an intra-uterine fluid motion with tiny particles in a non-pregnant uterus, and this situation is very significant in inspecting the embryo motion in the uterus. Further, in this article, the perturbation scheme is employed to produce the semi-numerical solutions in the tapered channel under Darcy flow parameters. Flow patterns are shown in terms of channel width, wave amplitudes and phase, power-law index, and Darcy and Weissenberg parameters.

2 Rheological relation of hyperbolic tangent fluid

The transient viscous two-dimensional conservation equations under the consideration of Cauchy’s tensor for hyperbolic tangent fluid are defined as follows [27–30]:

\[ \text{div } \mathbf{V} = 0, \]
\[ \rho \frac{d\mathbf{V}}{dt} = \text{div } \mathbf{S} + \rho \mathbf{f}. \]

In Eqs. (1) and (2), \( \rho \) is the density of hyperbolic fluid, \( \mathbf{V} \) is the velocity vector, \( \mathbf{S} \) is the Cauchy stress tensor, \( \mathbf{f} \) indicates the body force term, and \( \text{d} t / \text{d} t \) is the substantial derivative. The value of the stress tensor \( \mathbf{S} \) in Eq. (2) is obtained as follows [27–30]:

\[ \mathbf{S} = -\Pi \mathbf{l} + \boldsymbol{\tau}, \]
\[ \boldsymbol{\tau} = -[\eta_\infty + (\eta_0 + \eta_\infty)\tanh(\Gamma \tilde{\gamma})^n] \hat{\gamma}. \]

In Eqs. (3) and (4), \( -\Pi \mathbf{l} \) is the spherical component of stress, \( \boldsymbol{\tau} \) is the extra stress tensor, \( \eta_\infty \) is the rate of shear stress at infinite viscosity, \( \eta_0 \) is the viscosity at rate of zero shear, \( \Gamma \) is the time constant, \( n \) is the power-law index, and \( \tilde{\gamma} \) is defined as follows [27–30]:

\[ \tilde{\gamma} = \sqrt{\frac{1}{2} \sum_i \sum_j \eta_0 \gamma_{ij}^s} = \sqrt{\frac{1}{2} \Pi}. \]

The various terms existing in Eq. (5) are defined as follows:

\[ \Pi = \frac{1}{2} \text{trace}(\nabla \mathbf{V} + (\nabla \mathbf{V})^T)^2. \]

In Eq. (6), the second invariant strain tensor is depicted with \( \Pi \). Also, assume that the values of \( \eta_\infty \) and \( \tilde{\gamma} \) in Eq. (4) are taken as follows:

\[ \eta_\infty = 0 \quad \text{and} \quad \tilde{\gamma} < 1. \]

Under these assumptions, the value of tensor \( \boldsymbol{\tau} \) is obtained as follows:

\[ \boldsymbol{\tau} = -\eta_0 [(\Gamma \tilde{\gamma})^n] \hat{\gamma} = -\eta_0[(1 + \Gamma \tilde{\gamma} - 1)^n] \hat{\gamma} = -\eta_0[1 + n(\Gamma \tilde{\gamma} - 1)] \hat{\gamma}. \]

3 Mathematical statement of the problem

Present numerical investigation deals with the flow and pressure rise features of asymmetric hyperbolic tangent liquid in a tapered channel of width \( 2d \) under the porous medium. To describe the current peristaltic flow mechanism well, a two-dimensional rectangular coordinate system is employed under the porous effect. Also, the current flow process is studied by considering long wavelength and low Reynolds number assumptions. Owing to the progress of sin waves on the walls of the porous tapered channel, the hyperbolic tangent fluid flow generated and sinusoidal wave travels with the velocity \( c \) in the asymmetric channel. Figure 1 clearly portrays the current physical situation of hyperbolic tangent fluid flow under all necessary
conditions. In view of the above flow assumptions, the present flow mechanism is defined as follows [27–30]:

\[ H_1 = -d - m'\bar{X} - a_1\sin \left( \frac{2\pi}{\lambda} (\bar{X} - c\bar{t}) + \phi \right), \] (9)

\[ H_2 = d + m'\bar{X} + a_2\sin \left( \frac{2\pi}{\lambda} (\bar{X} - c\bar{t}) \right). \] (10)

In Eqs. (9) and (10), wave amplitudes are denoted by \( a_1 \) and \( a_2 \), wave length as \( \lambda \), porous channel width is \( 2d \), \( c \) is the phase of wave speed, time is denoted with \( \bar{t} \), path of sine wave progression is depicted by \( \bar{X} \), and the parameter \( m' \) is assumed to be non-uniform with value \( m' \ll 1 \). The symbol \( \phi \) indicates the phase difference and whose value lies in \([0, \pi]\) and \( \phi = 0 \) states the symmetric channel with waves out of phase, i.e., both walls move towards inward or outward simultaneously. However, the physical parameters \( a_1, a_2, d, \) and \( \phi \) identically satisfy the following relation:

\[ a_1^2 + a_2^2 + 2a_1a_2 \cos \phi \leq (2d)^2. \] (11)

With all these assumptions and based on the current geometry, the governing conservation equations in terms of two-dimensional form are defined as follows [27–30]:

\[ \frac{\partial \psi}{\partial \bar{X}} + \frac{\partial \nu}{\partial \bar{Y}} = 0, \] (12)

\[ \rho \left( \frac{\partial \psi}{\partial \bar{t}} + \bar{U} \frac{\partial \psi}{\partial \bar{X}} + \bar{V} \frac{\partial \psi}{\partial \bar{Y}} \right) = -\frac{\partial P}{\partial \bar{X}} - \frac{\partial \bar{r} \bar{X}}{\partial \bar{X}} - \frac{\partial \bar{r} \bar{Y}}{\partial \bar{Y}} - \frac{\eta_0 \bar{U}}{\bar{k}}, \] (13)

In Eqs. (13) and (14), \( \rho \) is the hyperbolic fluid density and \( k \) is the porous medium permeability. The fluid flow equations are simplified by adopting the following transformations in which the wave frame \((\bar{x}, \bar{y})\) is moving with velocity \( c \) far from the fixed wave frame \((\bar{X}, \bar{Y})\) and is defined as follows:

\[ \bar{x} = \bar{X} - c\bar{t}, \quad \bar{y} = \bar{Y}, \quad \bar{u} = \bar{U} - c, \]

\[ \bar{v} = \bar{V}, \quad \text{and} \quad \bar{p}(\bar{x}) = \bar{P}(\bar{X}, \bar{t}). \] (15)

Along with these transformations and assumptions, the following dimensionless quantities are utilized to reduce the governing dimensional equations into their dimensionless form [27–30].

\[ \begin{align*}
\bar{x} &= \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{\lambda}, \quad \bar{u} = \frac{u}{c}, \quad \bar{t} = \frac{t}{\lambda}, \quad \bar{v} = \frac{\bar{v}}{c\delta}, \quad \bar{h}_1 = \frac{h_1}{\delta}, \\
\bar{h}_2 &= \frac{h_2}{\bar{d}_1}, \quad \bar{r}_u = \frac{r_u}{\bar{d}_1}, \\
\bar{r}_y &= \frac{dy}{\eta_0 c^2}, \quad \bar{r}_{xy} = \frac{d_{xy}}{\eta_0 c^2}, \quad \delta = \frac{\delta}{\lambda}, \quad \text{Re} = \frac{\rho c d_{\bar{d}}}{\eta_0}, \\
\text{We} &= \frac{c}{\bar{d}_1}, \quad \bar{p} = \frac{d_{\bar{d}}}{\delta}, \quad \bar{y} = \frac{\bar{y}}{c}, \quad \text{Da} = \frac{k}{\bar{d}_1}.
\end{align*} \] (16)

Letting Eq. (16) into Eqs. (13) and (14) gives the following flow expressions in view of stream function

\[ \psi \left( \frac{\partial \phi}{\partial \bar{y}}, \nu = -\frac{\partial \bar{\phi}}{\partial \bar{x}} \right) \] and are shown as follows:

![Figure 1: Flow mechanism of current physical problem.](image-url)
The equation of motion is clearly nonlinear in nature. The general solution in closed form for Eq. (25) seems to be impossible to obtain for arbitrary values of all parameters occurring in this equation of motion. However, the authors are showing their interest in the class of liquids that are shear thinning (hyperbolic tangent fluid) but show weak normal stresses. Thus, authors noticed that the terms containing Reynolds number (Re) and other material coefficients vanish automatically under low Reynolds number and long-wavelength approximation. Hence, we seek the results of the current problem in terms of power series expansion in small parameters such as Weissenberg number (We). Thus, for perturbation solutions, the authors expanded the functions \( \psi \), \( F \), and \( P \) in terms of power series as follows [27–30]:

\[
\psi = \psi_0 + \text{We} \psi_1 + O(\text{We}^2),
\]

\[
F = F_0 + \text{We} F_1 + O(\text{We}^2),
\]

\[
P = P_0 + \text{We} P_1 + O(\text{We}^2).
\]

Letting Eqs. (31)–(33) into Eqs. (23), (25), and (29) and on re-adjusting the power of We results in the below simplified flow equations in Sections 4.1–4.4.


4.1 System of order \( \text{We}^0 \)

\[
(1 - n) \frac{\partial^3 \psi_0}{\partial y^3} - \frac{1}{\text{Da}} \left( \frac{\partial^3 \psi_0}{\partial y^2} \right) = 0,
\]

\[
\frac{\partial P_0}{\partial x} = (1 - n) \frac{\partial^3 \psi_0}{\partial y^3} - \frac{1}{\text{Da}} \left( \frac{\partial^3 \psi_0}{\partial y} \right) + 1.
\]

\[
\psi_0 = -\frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = 0 \quad \text{for} \quad y = h_1(x),
\]

\[
\psi_0 = \frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = 0 \quad \text{for} \quad y = h_2(x).
\]

4.2 System of order \( \text{We}^1 \)

\[
(1 - n) \frac{\partial^3 \psi_1}{\partial y^3} - \frac{1}{\text{Da}} \left( \frac{\partial^3 \psi_1}{\partial y^2} \right) = -n \frac{\partial^2}{\partial y^2} \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2,
\]

\[
\frac{\partial P_1}{\partial x} = (1 - n) \frac{\partial^3 \psi_1}{\partial y^3} + n \frac{\partial^3 \psi_0}{\partial y^2} \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 - \frac{1}{\text{Da}} \left( \frac{\partial \psi_1}{\partial y} \right).
\]

\[
\psi_1 = -\frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial y} = 0 \quad \text{for} \quad y = h_1(x),
\]

\[
\psi_1 = \frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial y} = 0 \quad \text{for} \quad y = h_2(x).
\]

4.3 Solutions for the system of order \( \text{We}^0 \)

The simplified results of the flow equation (34) under the conditions given in Eqs. (36) and (37) are obtained as follows:

\[
\psi_0 = C_1 + C_2 y + C_3 \cosh(M y) + C_4 \sinh(M y).
\]

The dimensionless pressure gradient along the axial flow path is obtained as follows:

\[
\frac{dP_0}{dx} = -\frac{1}{\text{Da}} \left( 1 + C_2 + M_1 \left( 1 + \text{Da} M_1^2 \right) \left( 1 - n \right) \right) \left( C_3 \cosh(M y) + C_4 \sinh(M y) \right).
\]

Eq. (43) is integrated for one wavelength and results in terms of \( \Delta P_{h_0} \) as follows:

\[
\Delta P_{h_0} = \int_0^1 \frac{dP_0}{dx} dx.
\]

4.4 Solution for the system of order \( \text{We}^1 \)

By letting the 0th-order result presented in Eq. (42) into Eq. (38), the reduced semi-numerical results of the final equation satisfying the necessary boundary conditions are shown as follows:

\[
\psi_1 = C_5 + C_6 y + C_7 \cosh(M y) + C_8 \sinh(M y) + R_{33} e^{-2M y} + R_{44} e^{2M y}.
\]

However, the pressure difference in the axial flow path is described as follows:

\[
\frac{dP_1}{dx} = M_1^2 \left( 1 - n \right) \left( -8e^{-2M y} R_{13} + 8e^{2M y} R_{44} + C_6 \cosh(M y) \right)
\]

\[
+ C_8 \sinh(M y))
\]

\[
= -\frac{1}{\text{Da}} \left( C_6 - 2e^{-2M y} M_1 R_{13} + 2e^{2M y} M_1 R_{44} \right)
\]

\[
+ C_6 M_1 \cosh(M y) + C_8 M_1 \sinh(M y))
\]

\[
+ \left( C_7^2 + C_8^2 \right) \sinh(2M y)\right).
\]

Further, the integration of Eq. (46) gives the value of \( \Delta P_h \) and is obtained as follows:

\[
\Delta P_h = \int_0^1 \frac{dP_1}{dx} dx.
\]

Finally, all the numerical solutions obtained for low Weissenberg numbers are summarized, and the expression of \( \psi \) and pressure difference are listed as follows:

\[
\psi = \psi_0 + \text{We} \psi_1.
\]

\[
\frac{dP}{dx} = \frac{dP_0}{dx} + \text{We} \frac{dP_1}{dx},
\]

\[
\Delta P = \Delta P_{h_0} + \text{We} \Delta P_{h_1}.
\]

Further, the quantities like \( \psi_0, \psi_1, \frac{d P_0}{dx}, \frac{d P_1}{dx}, \Delta P_{h_0}, \) and \( \Delta P_{h_1} \) are presented through the Eqs. (42)–(47). Also, the various physical parameters occurring in Eqs. (42)–(47) are obtained and listed as follows:

\[
M_1 = \frac{1}{\sqrt{\text{Da}(1 - n)}}, \quad a_{13} = \cosh(M_1 h_2),
\]

\[
a_{14} = \sinh(M_1 h_2), \quad a_{33} = \cosh(M_1 h_3), \quad a_{56} = \sinh(M_1 h_4),
\]

\[
b_1 = \frac{F_1}{2} - \left( R_{33} e^{-2M_1 h_1} + R_{44} e^{2M_1 h_1} \right),
\]

\[
b_2 = -\left( R_{33} e^{-2M_1 h_1} + R_{44} e^{2M_1 h_1} \right),
\]

\[
b_3 = \frac{F_1}{2} - \left( R_{33} e^{-2M_1 h_1} + R_{44} e^{2M_1 h_1} \right),
\]

\[
b_4 = -\left( R_{33} e^{-2M_1 h_1} + R_{44} e^{2M_1 h_1} \right),
\]
R_{33} = \frac{n(C_3 - C_0)^2M_2^2}{12(n - 1)}, \quad R_{44} = \frac{n(C_3 + C_0)^2M_2^2}{12(n - 1)}.

C_1 = \frac{(F_0(-a_1^2 + a_1^2 + a_1^2 - a_1^2 - M_1a_1a_3(h_1 + h_2) + M_1a_1a_3h_2(h_1 + h_2))}{(2(a_1^2 - (a_1 - a_3)^2 + a_1^3(-2a_3 + M_1a_3(h_1 - h_2)) + a_1^3(a_33 + M_1a_3h_1 - h_2)))},

C_2 = \frac{(M_1(-a_3a_3 + a_3a_3)F_0)}{(-(a_1 - a_3)^2 - a_1^3(a_3 + M_1a_3h_1 - h_2) + M_1a_3a_3h_1 - h_2))},

C_3 = \frac{(a_1 - a_3)F_0}{((a_1 - a_3))F_0},

C_6 = \frac{\left(\frac{M_1a_1^3(b_1 - b_1h_1) + M_1a_1^3(-b_1 - b_1h_1) + M_1a_1^3 - a_1^3)(b_1 - b_1h_1) + a_1^3(-2a_3 + M_1a_3h_1 - h_2)) + a_1^3(a_33 + M_1a_3h_1 - h_2)))}{(M_1(-a_1^2 - a_1^2 + a_1^2(a_3 + M_1a_3h_1 - h_2) + a_1^3(a_33 + M_1a_3h_1 - h_2)))},

C_7 = \frac{\left(\frac{M_1a_1^3(b_1 - b_1h_1) + M_1a_1^3(-b_1 - b_1h_1) + M_1a_1^3 - a_1^3)(b_1 - b_1h_1) + a_1^3(-2a_3 + M_1a_3h_1 - h_2)) + a_1^3(a_33 + M_1a_3h_1 - h_2)))}{(M_1(-a_1^2 - a_1^2 + a_1^2(a_3 + M_1a_3h_1 - h_2) + a_1^3(a_33 + M_1a_3h_1 - h_2)))},

C_8 = \frac{\left(\frac{M_1a_1^3(b_1 - b_1h_1) + M_1a_1^3(-b_1 - b_1h_1) + M_1a_1^3 - a_1^3)(b_1 - b_1h_1) + a_1^3(-2a_3 + M_1a_3h_1 - h_2)) + a_1^3(a_33 + M_1a_3h_1 - h_2)))}{(M_1(-a_1^2 - a_1^2 + a_1^2(a_3 + M_1a_3h_1 - h_2) + a_1^3(a_33 + M_1a_3h_1 - h_2)))}.

5 Results and discussion

The numerical investigation presented in this article describes the impact of the porous medium on the incompressible two-dimensional flow of hyperbolic tangent fluid in a tapered asymmetric channel. The obtained results are expressed in terms of pressure rise and velocity for various physical parameters such as wave amplitude numbers \((a, b)\), channel height \((d)\), Darcy number \((Da)\), hyperbolic tangent fluid power-law index \((n)\), phase difference \((\phi)\), and Weissenberg number \((We)\).

5.1 Flow profiles

Figures 2–9 show the influence of various dimensionless physical parameters such as \(n, Da, a, b, m, \phi, We,\) and \(d\) on the pressure field \((\Delta P)\) versus mean flow rate \((Q)\). Further, it is assumed that the conditions \(\Delta P > 0, Q > 0\) indicate the peristaltic pumping region, \(\Delta P < 0, Q < 0\) indicate the enhanced pumping, and \(\Delta P > 0, Q < 0\) indicate the backward pumping.

Figure 2 elucidates the impact of power-law index \((n)\) on the pressure rise against \(Q\). This plot shows that the increasing \(n\) suppressed the pumping rate in the wave frame. Figure 3 illustrates the influence of Darcy parameter \((Da)\) on the pressure rise and it is observed that enhancing \(Da\) magnified the peristaltic pumping and backward pumping in the second half region, and inverse behaviour is noticed in the first half region for the augmented pumping. Figures 4 and 5 illustrate the impact of wave amplitudes \((a, b)\) on the pressure rise against \(Q\). It is found from these figures that enhancing \(a, b\) increased the pressure rise in the first half region and

![Figure 2: Effect of \(n\) on \(\Delta P\).](image-url)
correspondingly dual behaviour is recorded in the pressure regime. Figures 6 and 7 show the influence of non-uniform parameter \( m \) and wave phase \( \phi \) on the pressure rise versus mean flow rate \( Q \). It is observed from these graphs that enhancing \( m \) and \( \phi \) suppressed the pressure rise in the first half region and upsurge in the second half region. Figure 8 illustrates the influence of the Weissenberg number (We) on the pressure rise in the two-dimensional wave frame. Figure 8 portrays that the increasing Weissenberg number has an insignificant effect on the...
pumping rate in the pressure regime. Similarly, Figure 9 shows the effect of the channel height ($d$) on the pressure rise against the mean flow rate $Q$. It is found from Figure 9 that enhancing the channel height increases the backward pumping and enhances the augmented pumping flow in the pressure regime.

Figures 10–17 show the influence of $n$, $Da$, $a$, $b$, $m$, $\phi$, $We$, and $d$ on the axial velocity field in the flow regime in the
produces the dual behaviour in the flow regime. Also, a diminished velocity is noticed in the first region, and an enhanced velocity is observed in the second half region. Further, Figure 11 portrays the impact of the Darcy number (Da) on the axial velocity profile. It is observed from Figure 11 that the velocity profile enhanced for the increasing values of the Darcy parameter at the central region of the tapered porous channel. Further, the velocity field diminished in the first and second half regions. Figures 12 and 13 depict the influence of wave amplitudes (a, b) on the axial flow field in the two-dimensional flow regime. It is observed from Figures 12 and 13 that the increasing wave amplitudes suppressed the main flow in the central region of the two-dimensional wave frame of reference and velocity enhanced in the other two regions. The influence of nonlinear parameter (m) and phase difference (ϕ) on the velocity field is illustrated in Figures 14 and 15. Further, Figures 14 and 15 reveal that the increasing nonlinear parameter and wave phase suppressed the velocity field at the central region of the channel. Figure 16 describes the impact of the Weissenberg number (We) on the velocity profile in the flow regime. It is noticed from Figure 16 that the increasing Weissenberg number gives dual flow behaviour for the velocity field in the flow regime. Similarly, Figure 17 portrays that the enhancing channel height suppressed the flow field at the central region of the tapered channel.

Figures 18–22 show the effect of wave amplitudes (a, b), Darcy number (Da), wave phase (ϕ), and Weissenberg number (We) on the streamline visualization of the hyperbolic tangent fluid in the two-dimensional wave frame of reference. Figures 18(i–iv) and 19(i–iv) describe the influence of wave amplitudes (a, b) on the streamline flow of tangent hyperbolic fluid. It is noted from Figure 18 that the enhancing wave amplitudes deviate the streamlines from the central region towards the outward direction. Further, Figure 19 portrays that the magnifying wave amplitudes enhance the magnitude of the trapped bolus in the flow domain. Figures 20(i–iv) and 21(i–iv) depict the influence of Darcy parameter (Da) and mean flow rate (Q) on the streamline flow of hyperbolic tangent fluid in the flow regime. Figures 20 and 21 shows that the enhancing Darcy parameter and mean flow rate grouped the streamlines at the central region of the channel. It is also noted that the magnitude of the trapping bolus suppressed for the increasing values of Da and Q. Similarly, Figure 22(i–iv) illustrates the impact of the Weissenberg number (We) on the streamlines in the two-dimensional wave frame. It is revealed from Figure 22 that the wave frame of the reference. Figure 10 illustrates the influence of power law number (n) on the axial velocity field. Figure 10 shows that the increasing power-law index (n)
enhancing Weissenberg number deviates the motion streamlines from the central region towards the outward direction. Further, the size of the trapped bolus enhanced in the flow region for the increasing values of the Weissenberg number.

5.2 Validation of current solutions

The guarantee and accuracy of the current peristaltic flow model is obtained by validating current solutions with the former results of Nadeem and Akram [27] for the fixed
numerical values of parameters $a = 0.5$, $We = 0.03$, $n = 0.04$, $\phi = \frac{\pi}{4}$. Figure 23 clearly portrays that the current solutions obtained based on the perturbation scheme reasonably agree with the available results of Nadeem and Akram [27]. This validation assures the guarantee and accuracy of the present solutions. Further, Figure 23 shows that the increasing channel width diminished the pressure rise in the two-dimensional flow regime.

**Figure 19:** Streamlines for (i) $b = 0.3$, (ii) $b = 0.5$, (iii) $b = 0.7$, and (iv) $b = 0.9$. 
6 Conclusions

A perturbation-oriented numerical investigation has been performed to determine the characteristic features of velocity, pressure rise, and streamline for the viscous incompressible two-dimensional hyperbolic tangent fluid in a tapered asymmetric porous channel under the action of low Reynolds numbers and long-wavelength approximations. The resultant nonlinear system of equations is solved by employing a regular perturbation technique. The resultant flow patterns are discussed for various values of physical parameters in the flow regime. However, based on the

Figure 20: Streamlines for (i) $Da = 0.5$, (ii) $Da = 1.3$, (iii) $Da = 1.5$, and (iv) $Da = 2.0$. 
current peristaltic flow mechanism under the porous medium, the following central findings in the limiting sense are obtained, which are listed as follows:

- Pressure field increased with upsurge in the Weissenberg parameter.
- Enhancing channel height suppressed the pressure rise in the flow regime.
- Increasing Darcy parameter enhances the pressure rise.
- Axial velocity diminished for the increasing wave amplitudes.

Figure 21: Streamlines for (i) $Q = 0.3$, (ii) $Q = 0.5$, (iii) $Q = 0.7$, and (iv) $Q = 0.9$. 
- Dual flow behavior is recorded for the increasing Darcy parameter in the flow regime.
- Size of trapping bolus enhanced for the increasing values of the Weissenberg number.

- Enhanced Darcy parameter and mean flow rate, suppressed the size of trapping bolus.
- Size of the trapping bolus increased for the increasing values of wave amplitudes.

Figure 22: Streamlines for (i) \( \text{We} = 0.6 \), (ii) \( \text{We} = 0.8 \), (iii) \( \text{We} = 1.0 \), and (iv) \( \text{We} = 1.2 \).
Figure 23: Validation of current solutions with the available results of Nadeem and Akram [27].

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