Entanglement-Enhanced Quantum Ranging in Near-Earth Spacetime

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A quantum ranging protocol to determine the distance between an observer and a target in the near-Earth curved spacetime is proposed. Unlike quantum illumination, the quantum ranging scheme utilizes multiple quantum hypothesis testing to simultaneously determine the presence and location of the target. The interest is in how the Earth’s spacetime curvature influences photon propagation as well as the performance of the quantum ranging. It is found that the maximum potential advantage of the quantum ranging strategy in the curved spacetime outperforms its flat spacetime counterpart. It is shown that the number of transmitted modes can enhance the maximum potential advantage of the quantum ranging tasks. In contrast, the maximum potential advantage of quantum ranging cannot be significantly increased by dividing the range into multiple slices in the curved spacetime.

1. Introduction

Quantum illumination (QI)[1–6] is an entanglement-assisted target detection scheme that uses an optimal quantum receiver and has a significant advantage over the optimal classical strategy in terms of error exponent. Additionally, the employment of entanglement can solve the disadvantage of the rapid attenuation of traditional radar signals. Recently, many efforts have been made to make the theoretical advantage of QI schemes practical,[7–9] and some of these schemes have been experimentally demonstrated.[10–12] However, the QI scheme is limited to querying a single spatiotemporal resolution bin at a time, which hinders the realization of quantum radar in the real world.[13] Fortunately, recent research on quantum ranging protocols has overcome this limitation.[14] In such proposal, the transmitter sends signal pulses to the target region and performs continuous measurements at the receiver side to determine the reflection of the target at the line of sight.[14,15] The main advantage of quantum ranging is the employment of the multiple quantum hypothesis testing scenario[16–18] instead of the binary hypothesis to determine the existence and location of the target at once. The quantum ranging task can be formulated as a multiple hypothesis testing problem,[14] where each hypothesis corresponds to the conclusion that the target exists in a specific distance slice.

On the other hand, the novel field of relativistic quantum information seeks to understand the preparation, manipulation, and transmission of quantum information in a relativistic setting.[15–32] Many experimental and theoretical proposals have been put forward to measure gravitationally induced decoherence of a quantum state.[25,26] The study of quantum information within the framework of relativity is expected to provide new insights into some fundamental questions in quantum mechanics and relativity, including nonlocality, causality and the information paradox of black holes. Furthermore, it is of practical significance to clarify the roles of relativistic effects in realistic quantum information tasks when the parties are separated by far in the near-Earth curved spacetime.

Quantum entanglement plays a critical role in quantum ranging protocol.[14] However, it is a fragile quantum resource that is easily corrupted by noise and loss. It is important to note that the propagation pulse is affected via changing their frequency distribution in center and shape in the near-Earth curved spacetime.[30–38] The Earth’s gravity has been found to make observable effects on entanglement and fidelity of quantum communication,[30,31] the accuracy of quantum metrology,[33–35] and the reliability of quantum clock synchronization.[32] Therefore, it is crucial to consider the gravitational field of the Earth in practical quantum ranging tasks.

In this paper, we propose a quantum ranging protocol that takes into account the nonmaskable gravity of the Earth. We investigate the influence of the Earth’s gravity on the detection performance of the protocol. Specifically, we assume that one component of the entangled signal-idler photon pair is sent from the Earth to a spatial target region to perform the quantum ranging task. During the propagation, the wave packet overlap and the reflectivity of the photons are deformed by the Earth’s...
spacetime curvature. To assess the performance of the protocol, we use the maximum potential quantum advantage,\(^\text{[39]}\) which represents the maximum possible improvement of quantum ranging compared to classical ranging. It is shown that the maximum potential advantage of the quantum ranging strategy in the curved spacetime has distinct superiority over its flat spacetime counterpart. This highlights the importance of considering the gravitational field of the Earth in practical quantum ranging tasks.

This paper is organized as follows. In Section 2, we introduce the propagation of the photons under the background of the Earth. In Section 3, we briefly introduce the quantum ranging tasks and calculate the potential maximum quantum advantage in different spacetime. Finally, the conclusions are drawn in Section 4.

2. Light Wave Packets Propagating in Earth

In this section, we discuss the transmission of light wave packets from the ground to the target region.\(^\text{[30,31,33]}\) The Earth’s spacetime can be approximately described by the Kerr metric, which provides a good approximation for a rotating spherical planet. In this paper, we restrict our analysis to the equatorial plane \(\theta = \frac{\pi}{2}\) and obtain the reduced Kerr metric in the Boyer-Lindquist coordinates \((t, r, \phi)\)^\(^\text{[40]}\)

\[
\begin{align*}
ds^2 &= -(1 - \frac{2M}{r})dt^2 + \frac{1}{\Delta}dr^2 + \left(r^2 + a^2 + \frac{2Mr}{r}\right)d\phi^2 \\
&\quad - \frac{4Ma}{r}dt\,d\phi
\end{align*}
\]

\(\Delta = 1 - \frac{2M}{r} + \frac{a^2}{r^2}\), Kerr parameter \(a = \frac{J}{M}\), and \(r_s, M, J\) are the radius, mass, angular momentum of the Earth. Throughout this paper we set \(\hbar = c = 1\).

We consider only the radial propagation case, which is reasonable because the angular velocity of the Earth is negligible, and the frequencies of the observers on the Earth are much smaller than the characteristic frequencies involved. In this scenario, the evolution of the quantum field is a 1+1 dimensional problem. As the uncharged scalar field is a good approximation of the Maxwell electromagnetic field in longitudinal or transverse modes,\(^\text{[32,41]}\) we can restrict our analysis based on the solutions of the massless Klein-Gordon equation. A photon can be simulated by a wave packet of electromagnetic fields with a distribution \(F^{(2)}_{\Omega, \delta}\) of modes peaked around the frequencies \(\Omega_{K, \delta}\).\(^\text{[41,42]}\) From the perspective of an observer at a different location, the annihilation operator for photons takes the form

\[
\hat{a}_{\Omega,K} (r_K) = \int_{0}^{\infty} d\Omega_k e^{-i\Omega_k t_K} F^{(2)}_{\Omega,\delta} (\Omega_k) \hat{a}_{\Omega_k}
\]

where \(\Omega_k\) are the physical frequencies as measured in the corresponding labs, and \(r_K\) is the proper time relating the Schwarzschild coordinate time \(t_K\) with \(r_K = \sqrt{f(r_K)} t_K\). \(K = A, B\) labels either Alice or Bob, each observer satisfies the canonical bosonic commutation relations \([\hat{a}_{\Omega_k}, \hat{a}^\dagger_{\Omega'_k}] = \delta(\Omega_k - \Omega'_k)\).

If Alice at location \(r_A\) and time \(\tau_A\) sends a wave packet \(F^{(A)}_{\Omega_{A},\delta}\) to the observer Bob, the wave packet that propagates radially will be modified when it reaches the location \(r_B\) at the time \(\tau_B = \Delta \tau + \sqrt{f(r_B)} \tau_A\), where \(f(r)\) is the gravitational frequency shifting factor at different heights and \(\Delta \tau\) represents the propagation time of the light. Due to the curvature effects, the modified wave packet is denoted by \(F^{(B)}_{\Omega_{B},\delta}\). The time evolution of the modes takes the form \(i\hbar \partial_{\tau_B} \phi^{(A)}_{\Omega_B} = \Omega_B \phi^{(A)}_{\Omega_B}\), and this equation defines the physical frequency \(\Omega_B\) measured by the observer at height \(r_B\) as \(\Omega_B = \frac{\omega}{\sqrt{f(r_B)}}\). If Alice sends a sharp frequency mode with \(\Omega_A\), Bob will receive a mode with frequency \(\Omega_B = \sqrt{\frac{\omega_{A}}{f(r_B)}} \Omega_A\), which is known as the gravitational redshift.\(^\text{[36,37,43]}\) It is feasible to utilize the relation between the annihilation operator to derive the relation between the wave packet before and after propagation.\(^\text{[30,33]}\)

\[
F^{(B)}_{\Omega_{B},\delta} (\Omega_B) = \sqrt{\frac{f(r_B)}{f(r_A)}} F^{(A)}_{\Omega_{A},\delta} \sqrt{\frac{f(r_A)}{f(r_B)}} (\Omega_A)
\]

\(\text{(3)}\)

From the above, the observers Alice and Bob have different peak frequencies and shapes. These changes are due to the gravitational field of the Earth and cannot be corrected simply by linear shifting frequencies. We can decompose the mode \(\hat{a}\) received by Bob into the mode \(\hat{a}\) prepared by Alice and the orthogonal mode \(\hat{a}^\perp\).\(^\text{[31,44–46]}\)

\[
\hat{a}^\perp = \Theta \hat{a} + \sqrt{1 - \Theta^2} \hat{a}^\parallel
\]

\(\text{(4)}\)

where \(\Theta\) is the mode overlap between the two distribution \(F^{(A)}_{\Omega_{A},\delta}\) and \(F^{(B)}_{\Omega_{B},\delta}\).

\[
\Theta = \int_0^{\infty} d\Omega_B F^{(B)*}_{\Omega_{B},\delta} (\Omega_B) F^{(A)}_{\Omega_{A},\delta} (\Omega_B)
\]

\(\text{(5)}\)

Then, the fidelity of the quantum channel is employed to quantify the similarity between the information transmitted and the information received. For a lossy channel, the fidelity of the channel is \(F = |\Theta|^2\). For a perfect channel, one finds \(F = 1\).

It is assumed that the wave packet is a normalized Gaussian wave packet

\[
F_{\Omega,\delta}(\omega) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{\omega^2 - \omega_0^2}{2\sigma^2}}
\]

\(\text{(6)}\)

with wave packet width \(\sigma\). The wave packet overlap is obtained by using Equations (3) and (6)

\[
\Theta_{\Omega,\delta} = \sqrt{\frac{2(1 + \delta)}{1 + (1 + \delta)^2}} e^{-\frac{\omega^2}{2\sigma^2} - \frac{\omega_0^2}{2\sigma^2}}
\]

\(\text{(7)}\)

where the parameter \(\delta = |\sqrt{\frac{\omega_{B}}{f(r_B)}} - 1|\), and \(\Theta_{\Omega,\delta}\) and \(\Theta_{\Omega,\delta}\) are respectively corresponding to the upward (i.e. \(r_B > r_A\)) and downward (i.e. \(r_A > r_B\)) processes. The explicit expression of the frequency ratio for the photon propagated between Alice and Bob has been
Figure 1. The entanglement-assisted ranging protocol under the action of the Earth’s gravitational field. The signal and idler modes are created from the two mode squeezed vacuum (TMSV) state. The signal \( \hat{a}_S \) is transmitted toward a target region that may contain a target of effective reflectance \( \eta \). The idler modes \( \hat{a}_I \) are stored and later measured jointly with the modes \( \hat{a}_R \) that are reflected form the target region. The target is hidden in a bright thermal noise bath, regardless of whether the target is detected or not, there will be a contribution of the thermal states in the return signal.

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\[
\delta = \delta_{\text{Sch}} + \delta_{\text{rot}} + \delta_h
\]

where \( \delta_{\text{Sch}} \), \( \delta_{\text{rot}} \), and \( \delta_h \) represent the first order Schwarzschild term, the lowest order rotation term, and the higher order correction term, respectively. The parameter \( R \) is the height difference between Bob and Alice, and \( \omega \) denotes the Earth’s equatorial angular velocity. If the target is located at the height \( R \approx \frac{\Delta}{2} \), the received photon frequency at this height will not experience any frequency shift. Here the \( \delta_{\text{Sch}} \) differs from the \( \delta \) presented in the Schwarzschild scenario papers. This is because we take into account the effects of both special and general relativity in expanding the total frequency shift \( \delta \). In contrast, in refs. [30, 33], the \( \delta \) only extends to the gravitational frequency shift. The wave packet overlap parameter is \( \Theta = 1 - \frac{\delta^2}{2 \Omega_{\text{b}}^2} \) in the regime \( \delta \ll \frac{\Omega_{\text{b}}^2}{2 \omega^2} \), which occurs for typical communication where \( \Omega_{\text{b}} = 700 \text{ THz} \) and Gaussian bandwidth \( \sigma = 1 \text{ MHz} \). Accordingly, both the reflected mode and the wave packets overlap are related to the range \( R \) of the target. We assume that the signal photon propagation has additional bright light interference, which accumulates the advantage of the ideal QI radar in the optical band.

3. Quantum Target Ranging in Curved Spacetime

Unlike the QI scheme, quantum ranging tasks can determine not only the existence of a target but also the distance between the observer and the target along the line of sight. As shown in Figure 1, a low reflectance \( \eta \) object is embedded in the thermal background, and we assume that the distance between the object and the observer can be divided into \( m \geq 2 \) discrete intervals. Thus, we have \( m \) hypotheses, with each hypothesis corresponding to one of the \( m \) range slices. The observer on the ground transmits a signal pulse \( \hat{a}_s \) to the target region and continuously collects the returned photons \( \{ \hat{a}_l \} \) at the receiver side. The returned signal photon reaches the receiving side after time \( t = \frac{2R}{c} \), where \( \Delta \) is the precision of the ranging task.

In this model, the curvature of Earth’s spacetime affects the frequency and shape of signal photons transmitted through it. In hypothesis \( R \), the target is located on the distance with \( R \Delta \), and the reflected mode arrives at the receiver after a time of \( t = \frac{2R}{c} \). Therefore, the ranging problem is modeled as the determination of the reflected mode \( \hat{a}_h \) among the continuously collect modes. It is noteworthy that the effects resulting from the gravitational redshift and the gravitational blueshift on the illuminating signal \( \hat{a}_s \) cancel each other out in the entire process (upward plus downward), but the gravity has the effects on the signal coming from the thermal background which is transmitted by the target to the receiver. The annihilation operator of the reflected mode at time \( t \) is...
\[ \hat{a}_R = \sqrt{n}\hat{a}_S + \sqrt{1 - \eta} \left( \Theta_\Delta \hat{a}_R + \sqrt{1 - \Theta_\Delta^2} \hat{a}_{R\perp} \right) \]  

(9)

where \( \hat{a}_R \) is in a thermal state. In the flat space limit, the wave packets overlap \( \Theta_\Delta = 1 \) is attained, which yields \( \hat{a}_R = \sqrt{n}\hat{a}_S + \sqrt{1 - \eta^2} \hat{a}_{R\perp} \).\(^{[2,14]} \) The overall reflectivity \( \eta \) can provide the ratio between received power and transmitted power\(^{[6]} \) in the short range scenario.\(^{[6]} \) This means that gain is ideally given\(^{[6]} \) by fixing the receiving antenna collecting area \( A_R = 0.1 \text{ m}^2 \), we can obtain a correspondence between the reflectivity \( \eta \) and the range \( R \). If the returned signal fails to reach the receiver side at time \( t_0 \), the collected photon \( \hat{a}_{i\perp R} \) is in a thermal state

\[ \hat{a}_{i\perp R} = \Theta_\Delta \hat{a}_R + \sqrt{1 - \Theta_\Delta^2} \hat{a}_{R\perp} \]  

(12)

In the entanglement ranging protocol, we prepare an idler-signal photon pair where one part is emitted to the space target region as the signal photon \( \hat{a}_i \), and the other part is retained in the local laboratory as the idler signal \( \hat{a}_i \). The wave function of the TMSV state can be expressed as

\[ |\phi^{\text{TMSV}}\rangle = \sum_{n=0}^{\infty} \left( \frac{N_S!}{(N_S + 1)n!} \right) |n\rangle_S |n\rangle_1 \]  

(13)

where \( N_S \) is the average photon number per mode. In the phase space representation, \( |\phi^{\text{TMSV}}\rangle \) is a zero mean Gaussian state whose corresponding covariance matrix \( \Lambda_{\text{SI}} \) is denoted by\(^{[48]} \)

\[ \Lambda_{\text{SI}} = \begin{pmatrix} (2N_S + 1)I_2 & 2S_2 S_2 & (2N_S + 1)I_2 \\ 2S_2 S_2 & (2N_S + 1)I_2 & (2N_S + 1)I_2 \end{pmatrix} \]  

(14)

where \( X_i = \frac{1}{\sqrt{2}}(\hat{a}_i + \hat{a}_i^\dagger) \), \( P_i = \frac{1}{\sqrt{2}}(\hat{a}_i - \hat{a}_i^\dagger) \), \( X_2 = \frac{1}{\sqrt{2}}(\hat{a}_i + \hat{a}_i^\dagger) \), \( P_2 = \frac{1}{\sqrt{2}}(\hat{a}_i - \hat{a}_i^\dagger) \), and \( S_2 = \sqrt{N_S(N_S + 1)} \). In this expression, \( I_2 = \text{diag}(1, 1) \), \( Z_2 = \text{diag}(1, -1) \).

In the quantum ranging protocol, we assume that the target position is located at the center of the \( R\Delta \) slice. Then, the overall return-idle state at the receiver side is given by\(^{[14,15]} \)

\[ \hat{\rho}_E^{\text{SI}} = |\phi^{\text{TMSV}}\rangle_{\hat{a}_i} \otimes |\hat{\sigma}_{\hat{a}_R}^{(T)}\rangle \otimes |\tilde{\sigma}_{\hat{a}_R}^{(T)}\rangle \]  

(15)

where \( \hat{\sigma}_{\hat{a}_R}^{(T)} \) is a set of \( M \) mode thermal signals, and the average number of photons of each thermal signal is \( N_b \). Moreover, \( \hat{\sigma}_{\hat{a}_R}^{(T)} \) is the joint measurement state of \( M \) mode signal-idler photon pairs returned at \( R\Delta \), where the returned mode is Equation (9). Each pair in the state is described by the covariance matrix

\[ \Lambda_{\hat{a}_R} = \begin{pmatrix} (1 + 2\eta N_b + 2\Theta_\Delta^2 N_b)I_2 & 2\sqrt{\eta}S_2 S_2 (2N_b + 1)I_2 \\ 2\sqrt{\eta}S_2 S_2 (2N_b + 1)I_2 & (2N_b + 1)I_2 \end{pmatrix} \]  

(16)

In the classical strategy, the position of the target is on the center of the \( R\Delta \) slice, and the state of the output signal at the receiving side can be written as

\[ \hat{\rho}_E^{\text{SI}} = \left( |\phi^{\text{TMSV}}\rangle_{\hat{a}_i} \otimes |\hat{\sigma}_{\hat{a}_R}^{(T)}\rangle \otimes |\hat{\sigma}_{\hat{a}_R}^{(T)}\rangle \right) \]  

(17)

The target state \( \hat{\rho}_{\hat{a}_R}^{(T)} \) represents the \( M \) mode reflect signal embedded in a local thermal background noise, generated through the thermal loss channel Equation (9). The covariance matrix of the target state is \( [1 + 2\Theta_\Delta^2 N_b]I_2 \).

The performance of the quantum ranging strategy is measured by the bound of error probability of the hypotheses. In both detection schemes, the input state \( \hat{a}_i \) is assumed to have a positive \( P \) function. For mixed states, it is difficult to obtain a general bound for the Helstrom limit \( P_{\text{H}}([\rho_{\hat{a}_R}, \rho_{\hat{a}_R}]) \). However, an upper bound can be obtained from the pretty good measurement (PGM)\(^{[49–51]} \) described by the positive operator-valued measure. The error probability can be expressed as

\[ p_{\text{PGM}} = 1 - \sum_{n=0}^{m-1} p_n \text{tr} \left( \Pi_n^{\text{PGM}} \rho_{\hat{a}_R} \right) \geq P_{\text{H}}([\rho_{\hat{a}_R}, \rho_{\hat{a}_R}]) \]  

(18)

Fortunately, the form of error probability of ranging protocol based on the fidelity is easier to calculate\(^{[15,51,52]} \)

\[ p_{\text{H}} \leq P_{\text{H,UB}} := 2 \sum_{n'} \sqrt{p_{n'} p_n F(\rho_{\hat{a}_R}, \rho_{\hat{a}_R})} \]  

(19)

and

\[ P_{\text{H}} \geq P_{\text{H,LB}} := \sum_{n'} \left( p_{n'} p_n F(\rho_{\hat{a}_R}, \rho_{\hat{a}_R}) \right) \]  

(20)

where the Bures fidelity \( F(\rho, \sigma) = \sqrt{\text{tr}\sqrt{\rho \sigma} \sqrt{\rho \sigma}} = \text{tr} \sqrt{\rho \sigma} \sqrt{\rho} \) is computed asymptotically.

It is assumed that all slices may occur with equal probability, so we get \( p_n = \frac{1}{m} \) for any \( n \). Then, we get the simplified boundary

\[ P_{\text{H,UB}} := (m - 1) F(\rho_{\hat{a}_R}, \rho_{\hat{a}_R}) \]  

(21)

\[ P_{\text{H,LB}} := \frac{m - 1}{2m} F^2(\rho_{\hat{a}_R}, \rho_{\hat{a}_R}) \]  

(22)

Combining the symmetry of the entire range slice and the fidelity of the classical strategy, the lower bound of the optimal performance of the classical detection strategy can be calculated using the input state with a positive \( P \) function.\(^{[14]} \) The lower bound of the classical strategy under the gravitational field of the Earth is
In the quantum ranging protocol, conditioned on the target range being $R\Delta$, the overall state at the receiver is described by Equation (15). Owing to the symmetry and structure of the overall return-idler states, the error probability of the multiple hypothesis testing problem is equivalent to the error exponent of discriminating two three-mode zero-mean Gaussian states $(\hat{\sigma}^{(1)}_{a_i} \otimes \hat{\sigma}^{(2)}_{b_1} \otimes \hat{\sigma}^{(3)}_{b_1})$. Specifically, the returned idler modes are differentiated under two different quantum ranging hypotheses.

\[
P_{C_{LB}}' = \frac{m - 1}{2m} P_{LB}^{2M}(\sigma^{(1)}, \sigma^{(2)})
\]

\[
\simeq \frac{m - 1}{2m} \exp\left[-\frac{2\eta N_s}{1 + 2\Theta_0^2 N_B}\right]
\]

(23)

The lower bound\(^{[51]}\) for the error probability of the entanglement enhanced ranging protocol in curved spacetime can also be obtained

\[
P_{E_{LB}}' = \frac{m - 1}{2m} P_{LB}^{2M}(\Lambda_{[1]}^{(1)}, \Lambda_{[2]}^{(1)})
\]

\[
\simeq \frac{m - 1}{2m} \exp\left[-\frac{2\eta N_s}{1 + \Theta_0^2 N_B}\right]
\]

(26)

The corresponding bounds $P_{C_{LB}}$ and $P_{E_{LB}}$ in flat space can be obtained by setting $\Theta_0 = 1$ in the Equations (23)–(26). In curved spacetime, the wave packet overlap parameter is less than 1. Thus, we can conclude that the Earth’s spacetime effects can decrease the error probability of ranging tasks. This finding is similar to the performance of QI in the Earth’s gravitational field.\(^{[34]}\)

We now proceed to give a physical interpretation of this phenomenon. When the quantum ranging task is operating in a curved spacetime, the gravitational effects deform both the signal $\hat{a}_s$ and the local thermal state $\hat{a}_r$ in the present scheme. The gravitational redshift on the signal $\hat{a}_s$ emitted by Alice is designed to be eliminated by an opposite gravitational blueshift factor $\sqrt{\frac{f_{\text{ref}}}{f_{\text{ref}}}}$, since the signal is sent downward from the satellite to Earth. However, the local thermal state $\hat{a}_r$ returned from the target region is only affected by the gravitational blueshift because it only experiences a one-way transmission process that satisfies Equation (4). Comparing our final state with the flat space case\(^{[2]}\) it is evident that the original $1 + 2N_s$ is replaced with $1 + 2\Theta_0^2 N_B$ in the present model. Therefore, the spacetime effects decrease the contribution of the thermal noise to the received signal, leading to an increase in the efficiency of the quantum ranging protocol.

The above calculations yield bounds for the error probability, which demonstrate that the quantum protocol outperforms the optimal classical protocol. We are interested in examining the impact of Earth’s gravitational field on the performance of quantum ranging tasks. To do so, we determine the maximum potential advantage of the quantum ranging protocol in both flat ($f\cdot\Delta P_{\text{max}}$) and near-Earth curved ($c\cdot\Delta P_{\text{max}}$) spacetime, respectively. The maximum potential quantum advantage is defined as the difference between the classical and quantum lower bounds\(^{[39]}\)

\[
f\cdot\Delta P_{\text{max}} = P_{C_{LB}} - P_{E_{LB}}
\]

\[
c\cdot\Delta P_{\text{max}} = P_{C_{LB}}' - P_{E_{LB}}'
\]

(27)

(28)

The difference between the maximum potential quantum advantage in the curved spacetime and its flat counterpart can be denoted by

\[
D(\Delta P_{\text{max}}) = (c\cdot\Delta P_{\text{max}}) - (f\cdot\Delta P_{\text{max}})
\]

(29)

In Figure 2, we present the maximum potential quantum advantage with respect to the number of copies $M$ for different spacetime backgrounds. It is shown that the maximum potential advantage of quantum ranging protocol increases with the number of copies $M$, both in the curved spacetime and in the flat spacetime. As mentioned above, the maximum potential quantum advantage in the curved spacetime is higher than that in the flat spacetime, suggesting that gravity promotes the maximum possible advantage for the quantum ranging strategy.

Figure 3 shows the difference $D(\Delta P_{\text{max}})$ between the maximum potential quantum advantage in the curved spacetime and its flat counterpart. The results show that the $D(\Delta P_{\text{max}})$ is always positive, indicating that the maximum potential quantum advantage in the curved spacetime is higher than that in flat spacetime. Moreover, the difference increases as the number of copies of transmitted modes $M$ increases. It is concluded that increasing the number of copies of transmitted mode can enhance the maximum potential advantage of the quantum ranging tasks.

The quantum ranging protocol divides the range between the ground observer and the target into $m \geq 2$ length slices, where each hypothesis corresponds to the target being in one of the slices. To gain a better understanding of the influence of the Earth’s gravitational field on the performance of quantum range-
Figure 3. The difference of the maximum quantum potential advantage in curved spacetime and the flat spacetime case. The number of range slices is fixed as $m = 10$, the signal brightness is fixed as $N_S = 0.01$ and the environmental noise is $N_B = 20$.

Figure 4. The maximum potential advantage of quantum ranging strategy in near-Earth spacetime versus the number of range slices $m$. The environmental thermal background noise $N_B = 20$, the transmitter-target range $R = 30$ m and the number of modes $M = 10^5$.

Figure 4 displays the maximum potential quantum advantage in the curved spacetime as a function of the number of distance slices $m$ for various numbers of emitted signal photons $N_S$. The results reveal that the maximum potential advantage of the quantum strategy increases rapidly for a small number of range slices in curved spacetime. However, $c - \Delta_{\text{max}}$ cannot be raised sharply by dividing the range into more slices. Moreover, increasing the emitted signal photons of the $N_S \ll 1$ can enhance the maximum potential advantage of quantum strategy in the curved spacetime. By selecting suitable signal energies and number of range slices, we can achieve the maximum potential advantage for the quantum ranging protocol in the near-Earth curved spacetime.

4. Conclusions

In this paper, we propose a near-Earth quantum ranging protocol for measuring the distance between an observer and a target, in which the binary quantum hypothesis in QI is replaced by $m$ hypotheses. The performance of quantum ranging is affected by the curvature of the Earth’s spacetime since the wave packet overlap and overall reflectivity of the photons are deformed by the spacetime effects. We show that the maximum potential advantage of the quantum strategy in curved spacetime outperforms its counterpart in flat spacetime, indicating that spacetime curvature can reduce the error probability of quantum ranging tasks. This is due to the effects of gravitational redshift and blueshift on the entangled signal beam can cancel each other, while gravity always reduces the signal coming from the thermal background. Furthermore, we find that increasing the copies of transmitted modes can promote the maximum potential advantage of the quantum ranging tasks. The maximum potential advantage of the quantum strategy increases rapidly for a small number of range slices. Therefore, one can choose the appropriate signal energies and the number of range slices to obtain a better maximum potential advantage for the quantum ranging protocol in the near-Earth curved spacetime.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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[1] S. Lloyd, Science 2008, 321, 1463.
[2] S. H. Tan, B. I. Erkmen, V. Giovannetti, S. Guha, S. Lloyd, L. Maccone, S. Pirandola, J. H. Shapiro, Phys. Rev. Lett. 2008, 101, 253601.
[3] Z. Zhang, S. Mouradian, F. N. C. Wong, J. H. Shapiro, Phys. Rev. Lett. 2015, 114, 110506.
[4] Q. Zhuang, Z. Zhang, J. H. Shapiro, Phys. Rev. A 2017, 96, 020302(R).
[5] S. Pirandola, B. R. Bardhan, T. Gehring, C. Weedbrook, S. Lloyd, Nat. Photonics 2018, 12, 724.
[6] A. Karsa, G. Spedalieri, Q. Zhuang, S. Pirandola, Phys. Rev. Res. 2020, 2, 023414.
[7] S. Guha, B. I. Erkmen, Phys. Rev. A 2009, 80, 052310.
[8] Q. Zhuang, Z. Zhang, J. H. Shapiro, Phys. Rev. Lett. 2017, 118, 040801.
[9] S. Barzanjeh, S. Guha, C. Weedbrook, D. Vitali, J. H. Shapiro, S. Pirandola, Phys. Rev. Lett. 2015, 114, 080503.
