Path Planning of Serial Two Link Planar Manipulator for Avoidance of Steam Generator Plugged Tube

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Abstract. Prototype Fast Breeder Reactor (PFBR) has 8 Steam Generators (SGs). Inspection of SGs requires a remote tooling to reach each of the 547 tubes. In PFBR SG, Sodium flows through the shell side and water/steam through the tube side. Degradation of the SG tubes are caused by corrosion, pitting, wear due to flow induced vibration; so it is necessary that degraded SG tube must be plugged at both ends in order to increase the availability of plant. So the plugged tubes will have some protrusion above tube sheet. As robotic device for inspection is two-axis Selective Compliant Assembly Robotic Arm (SCARA), so during movement from one tube to another tube, plugged tube has to be avoided for safe operation. In this paper path planning technique using vector algebra has been used to find out via point in the Cartesian space. The via point is selected such that robot can avoid plugged tube & ensures healthy operation of device.

1. Introduction
The PFBR has 8 numbers of once through Steam Generators (SG) with 4 numbers in each loop [1]. Each SG generates 156 MWt capacity of thermal power [2]. The steam generator is a shell and tube heat exchanger with secondary sodium in the shell side and the water/steam in the tube side. There are 547 numbers of tubes connecting the top and the bottom headers. Due to geometrical constraint of SG, a 2 axes robotic arm of SCARA configuration has been developed to inspect the healthiness of each tube as shown in Fig.1. So in order to ensure the smooth motion of end effector, forward and inverse velocity kinematic has been discussed in this paper. In Steam generators, degradation of the SG tubes are caused by stress corrosion cracking, denting, pitting, Fatigue cracking, Fretting, Inter-granular corrosion, tube wear. The degraded steam generator tube must be plugged at both ends so that it is separated from the rest of heat exchanger tubes to increase plant availability so, it is mandated to have the SG tubes inspected through periodical in-service inspection (PSI). So during PSI inspection campaign, robot has avoid the plugged tube for smooth operation. In this regard via point & path planning algorithm development & simulation has been done & same has been presented in the paper.
2. Kinematics

Robot arm kinematics deals with the analytical study of the motion of the robot arm with respect to the reference coordinate system as a function of time without regard to forces/moments that caused the motion.

2.1. Forward Position Kinematics

The forward kinematics maps the joint space \((\theta_1, \theta_2)\) coordinates to Cartesian coordinates. The end effector position of two link robotic arm is given by Eq. (1-2) [1].

\[
x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) \\
y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)
\]

\[\delta \theta_1 \quad \text{Shoulder Arm Length (L_1)} \] \[\delta \theta_2 \quad \text{Elbow Arm Length (L_2)} \] \((\delta x, \delta y)\)

2.2. Inverse Position kinematics

The inverse kinematics relates the incremental change in Cartesian position \((\delta x, \delta y)\) to the incremental change in joint space coordinates \((\delta \theta_1, \delta \theta_2)\) by a function which is known as Jacobian [1] which is given in Eq. (3).

\[
\begin{bmatrix}
\delta x \\
\delta y
\end{bmatrix} = J
\begin{bmatrix}
\delta \theta_1 \\
\delta \theta_2
\end{bmatrix}
\]

Here \(\delta x, \delta y\) represents the small displacements of robotic arm end effector. \(\delta \theta_1, \delta \theta_2\) represents the small angular displacement of shoulder and elbow arm joint for corresponding change in \(\delta x, \delta y\).
Where  \[ J = \begin{bmatrix}
\frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\
\frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2}
\end{bmatrix} \]  
(4a)

\[ J = \begin{bmatrix}
-l_1 \sin(\theta_1) & -l_1 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\
l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2)
\end{bmatrix} \]  
(4b)

Each element of the Jacobian matrix describes how a differential change in \( \theta_1 \) & \( \theta_2 \) affects the differential change in x & y. The Jacobian matrix is also a function of \( \theta_1 \) & \( \theta_2 \) and hence it is dependent on configuration of arm. [5].

The incremental joint angle of robotic arm can be determined based on incremental change in the robotic end effector position which is given by Eq. (5a)

\[ \delta \theta = J^{-1} \delta e \]  
(5a)

\[ \begin{bmatrix}
\delta \theta_1 \\
\delta \theta_2
\end{bmatrix} = \begin{bmatrix}
-l_1 \sin(\theta_1) & -l_1 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\
l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2)
\end{bmatrix}^{-1} \begin{bmatrix}
\delta x \\
\delta y
\end{bmatrix} \]

\((\delta x, \delta y)\)

\(\delta \theta_1\)

Shoulder Arm Length (L₁)

Elbow Arm Length (L₂)

\(\delta \theta_2\)

2.2.1. Forward Velocity Kinematics

The forward velocity kinematics relates the joint velocity \((\dot{\theta}_1, \dot{\theta}_2)\) to end effector Cartesian space velocity \((v_x, v_y)\). The forward velocity of end effector is obtained by differentiating the forward position kinematics which is given in Eq. (5b)

\[ V = J \dot{\theta} \]  
(5b)

2.2.2. Inverse Velocity Kinematics

The velocity inverse kinematics deals with the relationship between the given end effector Cartesian space linear velocity \((v_x, v_y)\) to joint space angular velocity \((\dot{\theta}_1, \dot{\theta}_2)\)

which is given in Eq. (6)

\[ \begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix} = [J]^{-1} \begin{bmatrix}
v_x \\
v_y
\end{bmatrix} \]  
(6)
3. Path planning of two link serial manipulator for avoidance of plugged tube

As already discussed that degraded tube has to be plugged from both ends to ensure plant availability. So in PSI campaign robot has to avoid the plugged zones for smooth inspection. The collision avoidance trajectory planning is normally carried out in two steps. In first stage path planning of two link robotic manipulator to avoid collision with plugged tube (i.e., the identification of paths that do not intersect obstacle). In this regard path planning which avoid the obstacle (plugged tube) is specified by a number of via points in the Cartesian space. The conventional path planning approach to avoid obstacle is detailed in [11-12]. Then each of these via points will be mapped into a set of joint angles with the help of inverse kinematics algorithm shown in Fig.3. In the second stage mainly consists of the interpolation of joint angle obtained through inverse kinematics. The second stage is referred to as motion planning or trajectory planning. The tube sheet with plugged tube position is shown in Fig.4.

Figure 3. Velocity Inverse Kinematics Algorithm for SG Inspection.

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3.1 Via-Point Selection for Plugged Tube Avoidance

The via point selection algorithm is shown in Fig. 5. In Fig. 5, \( \vec{OA} \) represents the obstacle distance from the origin, \( \vec{OB} \) represents the projected length of the obstacle vector along the link and \( \vec{BA} \) is the shortest length from the link to obstacle. The projected length (OB) is given by Eq. (7a)

\[
\vec{OB} = \left( \frac{\vec{OC}}{||\vec{OC}||^2} \right) \cdot \left( \vec{OC} \cdot \vec{OA} \right)
\]

(7a)

From the triangle of \( \angle OBA \), \( \vec{BA} \) can be calculated by applying the vector algebra given by Eq. (7b)

\[
\vec{OA} = \vec{OB} + \vec{BA} \quad \vec{BA} = \vec{OA} - \vec{OB}
\]

(7b)

The via point selection algorithm is shown in Fig. 6.
Figure 5a. Shortest Distance Calculation

Figure 5b. Via Point Selection of Plugged Tube
Figure 6. Flowchart of Via Point Selection.

START

Get the obstacle distance from the origin

Obstacle Distance > L₁

Yes

Obstacle is closer to Link2

No

Obstacle is closer to Link1

Calculate obstacle shortest distance from Link1

Obstacle Distance > L₁

Calculate obstacle shortest distance from Link2

Via point distance from the obstacle = max(obstacle1 shortest distance, obstacle2 shortest distance, obstacle3 shortest distance, ............)
3.2. Trajectory generation for Plugged Tube avoidance

In the earlier section path planning algorithm for avoiding the plugged tube is discussed. In this section, planning of joint angle obtained through inverse kinematics has been interpolated through polynomial. Motion planning in joint space is computationally very easy due to no inverse kinematics transformation. The end effector of the robotic arm obtained through joint angle interpolation is not easily visualizable. Motion planning in Cartesian space is computationally very difficult. The tracking of end effector path is achieved by Cartesian space motion planning. The only problem associated with Cartesian space motion planning is singularity. The path which avoids the plugged tube is specified by a number of points in the Cartesian space. Then each of these via points will be mapped into a set of joint angles with the help of inverse kinematics algorithm. The joint angle is in discrete form. For smooth motion of end effector, joint angle has to be interpolated by means of some function [3-10]. Many functions are available for joint interpolation; fifth order polynomial has been selected. The position, velocity, acceleration obtained by the fifth order polynomial is very smooth. Piecewise quintic polynomial [3, 8] (fifth order) will provide continuous position and velocity and acceleration at the intermediate via points. The jerk value at starting and ending will have some finite value. The quintic polynomials are the best choice to obtain both local smoothness and $c^2$ continuity.

\[
\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5
\]
\[
\theta(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4
\]
\[
\theta(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3
\]
\[
\theta(t) = 6a_3 + 24a_4 t + 60a_5 t^2
\]

Where $a_0, a_1, a_2, a_3, a_4, a_5$ are polynomial coefficient, $t$ - is the time of motion

3.3. Formulation of Quintic Interpolation Joint Trajectory

The discrete joint angles for the joint 1 ($\theta_1$) and joint 2 ($\theta_2$) of a two link serial robotic arm is given in Table 1. The piecewise polynomial is being used to interpolate two points. To interpolate 4 points 3 segments are used. The joint trajectory for the two link robotic arm for the joint 1 and joint 2 is interpolated through fifth order polynomial as given Eq. (9-10)

\[
\begin{align*}
\theta_1(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \\
t_1 &< t \leq t_2 \\
\theta_2(t) &= a_6 + a_7 t + a_8 t^2 + a_9 t^3 + a_{10} t^4 + a_{11} t^5 \\
t_2 &< t \leq t_3 \\
\theta_3(t) &= a_{12} + a_{13} t + a_{14} t^2 + a_{15} t^3 + a_{16} t^4 + a_{17} t^5 \\
t_3 &< t \leq t_4
\end{align*}
\]
The continuity for position, velocity, acceleration, jerk and snap are given by Eq. (11)

\[
\begin{align*}
\theta_{21}(t) &= b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5, \\
&\quad t_1 \leq t \leq t_2 \\
\theta_{22}(t) &= b_6 + b_7 t + b_8 t^2 + b_9 t^3 + b_{10} t^4 + b_{11} t^5, \\
&\quad t_2 < t \leq t_3 \\
\theta_{23}(t) &= b_{12} + b_{13} t + b_{14} t^2 + b_{15} t^3 + b_{16} t^4 + b_{17} t^5, \\
&\quad t_3 < t \leq t_4
\end{align*}
\]  

(10)

The continuity for position, velocity, acceleration, jerk and snap are given by Eq. (11)

\[
\begin{align*}
\theta_{11}(t = t_2) &= \theta_{1v1} \\
\theta_{12}(t = 0) &= \theta_{1v1} \\
\theta_{12}(t = t_3) &= \theta_{1v2} \\
\theta_{13}(t = 0) &= \theta_{1v2} \\
\theta_{11}(t = t_2) &= \theta_{1v2} (t = 0) \\
\theta_{13}(t = t_3) &= \theta_{1v2} (t = 0)
\end{align*}
\]  

(11)

The initial and final boundary conditions are given by Eq. (12)

\[
\begin{align*}
\theta_{11}(t = t_1) &= \theta_{1i} \\
\theta_{13}(t = t_4) &= \theta_{i} \\
\dot{\theta}_{11}(t = t_1) &= 0 \\
\dot{\theta}_{13}(t = t_4) &= 0 \\
\ddot{\theta}_{11}(t = t_1) &= 0 \\
\ddot{\theta}_{13}(t = t_4) &= 0
\end{align*}
\]  

(12)

The three segment quintic spline each having 6 unknown, totally 18 unknowns. The 18 unknowns are to be calculated by applying conditions given in Eq. (11-12)

3.4. Case Study

The movement from the tube (11,11) to (9,52) in between plugged tubes (10,7),(5,4),(10,2) are there. In order to avoid the plugged tube for smoother operation of two link robotic arm, via points have been selected as shown in Table 1. The corresponding joint angles for the Cartesian points is obtained through the inverse kinematics algorithm shown in Fig. 3.

| Table 1. Cartesian Coordinates(X, Y) and Joint angles (θ1, θ2). |
|-----------------|-----------------|---------|---------|
| start point(11,11) | 177.1 | 306.75 | 27.49 | 65.01 |
| via point(7,8) | 80.5 | 195.20 | 7.77 | 119.64 |
| via point(6,1) | 177.1 | 27.89 | -55.78 | 129.46 |
| end point(9,52) | 257.6 | -55.77 | -63.35 | 102.26 |
Figure 7. Joint Position, Velocity, Acceleration Profile for Link 1.

Figure 8. Joint Position, Velocity, Acceleration Profile for Link 2.
The joint 1 angular position, angular velocity, angular acceleration of the given position of two link robotic arm shown in Fig.7. The acceleration profile obtained through fifth order interpolation is very smooth compared to third order interpolation. The joint 2 angular position, angular velocity, angular acceleration of two link robotic arm is shown in Fig.8.

4. Conclusions
In this paper, position, velocity kinematics of a two link robotic arm for steam generator inspection is discussed. The path planning algorithm for using vector algebra for avoidance of plugged tube has been proposed. The vector algebra technique is used to locate the via point. The joint angle obtained through inverse kinematics is interpolated by quintic spline (fifth order polynomial). The case study for the movement from tube (11, 11) to tube (9, 52) is discussed.

5. References
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