Study of nuclear dynamical deformation in the synthesis of super-heavy nuclei

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Abstract. The nuclear dynamical deformations in the fusion reactions for the production of superheavy nuclei (SHN) are studied within the framework of the di-nuclear system model with dynamical potential energy surface (DNS-DynPES model). The development of the dynamical deformation for the incident channel is investigated and the maximum dynamical deformations for different configurations are also calculated. The influences of the dynamical deformations on the potential energy surface (PES) and the fusion probability are also calculated. The results indicate that the structure of the PES changes significantly and the fusion for synthesizing SHN is also hindered for the inclusion of the nuclear dynamical deformation. The fusion probability as a function of angular momentum is also investigated.

1. Introduction

In recent years, the synthesis of superheavy nuclei (SHN) has already obtained much progress in both experimental and theoretical aspects. On the experimental aspect, the elements from 107 to 113 were synthesized in cold fusion reactions with Pb or Bi as target and the elements from 112 to 118 were synthesized in hot fusion reactions with the neutron-rich projectile ⁴⁸Ca [1-5]. Several theoretical models were put forward, such as the macroscopic dynamical model [6], the fusion-by-diffusion (FBD) models [7-12], the nuclear collectivization model [13,14], the di-nuclear system (DNS) model [15-25] and some others, e.g., Ref. [26], to understand the fusion process and predict the evaporation residue cross sections.

The theoretical investigation about the fusion of SHN is of vital importance for the experimental research. The evaporation residue (ER) cross sections are decreasing significantly as increasing the charge number of the SHN. After SHN with Z=116, the ER cross sections drop exponentially. Because of the very low ER cross sections and very narrow excitation functions for synthesizing SHN, the SHN could be synthesized only at the optimal
projectile-target combination and the optimal bombarding energy. The predicted optimal reaction condition and the expected maximum ER cross section are anticipated by experimentalists. However, the predictions of ER cross sections for SHN are very difficult because the fusion for producing SHN is a very complicated process where the relaxations of the nuclear deformations, angular momentum, and radial kinetic energy are coupled with the mass transfer. Although much progress has been made in understanding the fusion reaction, there are still many open problems to be explored. As mentioned above, the nuclear deformation relaxation plays an important role in the heavy-ion reactions. This kind of dynamical deformations [27-29] developed in the fusion process should be taken into consideration for the investigations in the synthesis of SHN. In this paper, the development of the dynamical deformation is investigated and its effects on the potential energy surface and fusion probability will be investigated with the di-nuclear system model with dynamical potential energy surface (DNS-DynPES model) [30]. The paper is organized as follows. In section 2, an introduction to the theoretical model is given. In section 3, some results about the dynamical deformation and fusion probability are studied. Finally, a summary is given.

2. DNS-DynPES model

In Ref. [30], we developed a DNS-DynPES model. In this section we briefly mention the formalism for the theoretical model. Generally, the production of SHN can be separated into three stages: (1) the capture process in which the system overcomes the Coulomb barrier, (2) the fusion process in which an excited compound nucleus (CN) is formed, (3) the survival process in which the emission of one or more neutron(s) leads the excited CN to an SHN with expected Z. The evaporation residue cross section in a heavy-ion fusion reaction can be written as the summation over all partial waves J as

$$\sigma_{\text{ER}} (E_{\text{c.m.}}) = \frac{\pi \hbar^2}{2 \mu E_{\text{c.m.}}} \sum J (2J +1) T(E_{\text{c.m.}}, J) P(E_{\text{c.m.}}, J) W_{\text{sur}} (E_{\text{c.m.}}, J)$$  \hspace{1cm} (1)$$

with \( E_{\text{c.m.}} \) being the incident energy in the center of mass (c.m.) frame. Here \( T(E_{\text{c.m.}}, J) \) is the transmission probability and the survival probability \( W_{\text{sur}} (E_{\text{c.m.}}, J) \) is calculated with a statistical model.

The DNS concept was proposed by Volkov [31] and later used to study the competition between complete fusion and quasi-fission and to calculate the fusion probability in fusion reactions [32-34]. The basic idea of the DNS model is that after the capture process, a DNS in the entrance channel is formed. Then the DNS evolves via nucleon transfer along the mass asymmetry coordinate \( \eta = \frac{A_1 - A_2}{A_1 + A_2} \) instead of the direction of the relative distance \( R \) between the projectile and target. If the nucleon transfer occurs from the light nucleus to the heavy one, the mass asymmetry increases. Otherwise, if the nucleon transfers from the heavy nucleus to the light one, the mass asymmetry decreases and the quasi-fission rate increases, and that hinders the fusion. Therefore, when \( \eta \) is close to 1, a CN is formed with a high probability, while the decrease of \( \eta \) away from 1, the CN formation is hindered and it may be formed with much lower probability.

We calculate the formation probability \( P_{\text{CN}} \) of a CN based on the DNS concept. During the nucleon transfer process, any configuration of DNS \( (A_1, A_2) \) with \( A_1 = 0, 1, \ldots, A_P + A_T \) and \( A_2 = A_P + A_T - A_1 \) can be formed. The evolution of the distribution function of each DNS with time can be described by a master equation
\[
\frac{dP(A_1,t)}{dt} = \sum W_{A,A'}(t) \left[ d_A P(A_1',t) - d_A P(A_1,t) \right] - A_{sf}^A(t) P(A_1,t)
\]  

(2)

Since \( A_1 + A_2 = A_1 + A_1 \), only \( A_1 \) is explicitly included in the above equation. \( d_A \) is the microscopic dimension for a DNS with a local excitation energy \( E_{DNS}^* \) defined in Eq. (3) which is shared by the two nuclei in this DNS.

For each nucleus, a valence space is opened due to the excitation and those nucleons in the states within the valence space are active for the transfer between the two nuclei. \( d_A = C_{m_1}^N \cdot C_{m_2}^N \) where \( N_k \) is the number of valence states and \( m_k \) is that of valence nucleons. \( A_{sf}^A(t) \) is the quasi-fission rate of the DNS(A1, A2) and \( W_{A,A'} = W_{A',A} \) is the mean transition probability between the DNSs (A1, A2) and (A1', A2').

In Eq. (2), the microscopic dimension, the quasi-fission rate and the mean transition probability are all related to the local excitation energy of the DNS which reads

\[
E_{DNS}(A_1,t) = E_{total} - E_{DNS}(A_1,t) - E_{DNS}^T(A_1,t)
\]

where

\[
E_{total} = E_{c.m.} + (M_T + M_P)c^2,
\]

\[
E_{DNS}(A_1,t) = V_{DNS}(A_1,t) + (M_1 + M_2)c^2,
\]

\[
E_{DNS}^T(A_1,t) = \frac{J(J+1)}{2I_{DNS}(A_1,t)},
\]

\[
V_{DNS}(A_1,t) = V_N(A_1,t) + V_C(A_1,t).
\]

Here \( V_N(A_1,t) \) is the nuclear potential calculated with a double-folding method [35] and \( V_C(A_1,t) \) is the Coulomb interactions potential from the Wong's formula [36]. In this work a tip-tip orientation of the two deformed nuclei is assumed. The potential energy in the mass asymmetry degree of freedom (the driving potential) at \( t=0 \), is defined as

\[
V_{PES}(A_1,t) = V_N(A_1,t) + V_C(A_1,t) + (M_1 + M_2 - M_P - M_T)c^2
\]

(8)

The interaction potential between the two nuclei is related to the distance between two centers and in this paper the interaction potential takes the minimum value in the pocket. Because of the attractive nuclear force and the repulsive Coulomb force, both nuclei in a DNS are distorted and dynamical deformations develop during the process of nuclear reactions [27-29]. This results in a time-dependence of the potential energy surface (PES). The dynamical deformations of the two nuclei satisfy \( \delta \beta_i C_1 / A_i = \delta \beta_i^C C_2 / A_2 \) [13] with the stiffness parameter \( C_i \) (i=1 and 2) calculated from a liquid drop model [37]. The total dynamical deformation is the average of the dynamical deformations of the two nuclei \( \delta \beta = (\delta \beta_1 + \delta \beta_2) / 2 \). Following Refs. [27-29], we assume that the dynamical deformation evolves in an over-damped motion,

\[
\delta \beta_i(t) = \delta \beta_{max} (1 - e^{-t/\tau_{def}}) .
\]

(9)

In Eq. (9), the relaxation time \( \tau_{def} = 40 \times 10^{-22} \) s and the maximal dynamical deformation is found when the total "intrinsic" energy reaches the minimum,

\[
E_{int}(A_1, \delta \beta) = V_N(A_1; \beta_1, \beta_2) + V_C(A_1; \beta_1, \beta_2) + \sum_{i=1,2} C_i \delta \beta_i^2
\]

(10)

where the quadrupole deformation \( \beta_i = \beta_i^0 + \delta \beta_i \) (i = 1 or 2) with the static deformation parameter \( \beta_i^0 \) taken from Ref. [38].
3. Results and discussion

The intrinsic energy of the di-nuclear system $^{48}\text{Ca} + ^{244}\text{Pu}$ as a function of nuclear dynamical deformation is shown in Fig.1. For the configuration above, the maximum dynamical deformation for the system is about 0.31, where the minimum value for the intrinsic energy could be found and the maximum dynamical deformation $\delta \beta_{\text{max}}$ is indicated as dashed line in the figure. The intrinsic energy increases as increasing the dynamical deformation when it is larger than $\delta \beta_{\text{max}}$. As explained in Ref. [39], the phenomenon can be attributed to the rise of the nuclear energies due to their distortions. The interaction potential for the di-nuclear system $^{48}\text{Ca} + ^{244}\text{Pu}$ as a function of dynamical deformation is also indicated in Fig. 1 and it is found that the interaction potential decreases as increasing the dynamical deformation. The interaction potential at the maximum dynamical deformation is about 145 MeV.

![Fig. 1 Intrinsic energy and potential energy as functions of the dynamical deformation for the reaction. The maximal dynamical deformation is indicated with the vertical dashed line.](image1)

The dynamical deformations for the two fragments are shown in Fig.2. The solid and dashed lines are for the dynamical deformations of $^{48}\text{Ca}$ and $^{244}\text{Pu}$, respectively. The dynamical deformations are found to be increasing in the beginning $120\times10^{-22}$ seconds and then gradually get saturated. The heavier fragment may get more significant dynamical deformation. The saturated dynamical deformation parameters for $^{244}\text{Pu}$ and $^{48}\text{Ca}$ are 0.42 and 0.18, respectively.

![Fig. 2 Dynamical deformations for $^{48}\text{Ca}$ and $^{244}\text{Pu}$ as a function of the reaction time. Taken from Ref. [39](image2)
The maximum dynamical deformations for the incident channel $^{48}\text{Ca}^{+}^{244}\text{Pu}$ as a function of mass asymmetry are shown in Fig. 3. It can be seen that the maximum dynamical deformations are relatively large for the symmetric configurations with the mass asymmetry ranging between $-0.5$ and $0.5$ and are decreasing as increasing the mass asymmetry of the di-nuclear configuration. The maximum dynamical deformation for the incident channel is about 0.3 as the dashed line indicates. For some very asymmetric cases, the maximum dynamical deformations are nearly zero.

The potential energy surface, namely the driving potential, for the reaction $^{48}\text{Ca}^{+}^{244}\text{Pu}$ is shown in Fig. 4. The di-nuclear configurations from the incident one evolve along the mass asymmetry degree of freedom. To form a CN, the di-nuclear configuration must overcome the BG point on the driving potential and the barrier between the BG point and the incident channel is defined as inner fusion barrier. The dashed line is for the case when the dynamical deformation is fully developed and the solid line is for the case without dynamical deformation. It is found that the potential energy surface changes significantly due to the nuclear dynamical deformations. The potential energies for the incident channel are $-0.808$ MeV and $-16.392$ MeV for the cases when $r=0$ (solid line) and the dynamical deformations are fully developed (dashed line), respectively. In Fig. 4, the inner fusion barrier for the two cases are found to be 15.4 MeV and 26.8 MeV, respectively. It can be seen that the inner fusion barrier increases significantly due to the inclusion of the dynamical deformation and the fusion probability may be hindered for the increase of the inner fusion barrier.
The fusion probability for the reaction $^{48}$Ca + $^{244}$Pu at zero angular momentum as a function of the incident energy is depicted in Fig. 5(a). The solid line and dashed line are for the cases with and without dynamical deformation, respectively. It is found that the fusion probabilities increase with the incident energy for both the cases. When the incident energy is below 210 MeV, the fusion probabilities increase rapidly. After that, the fusion probabilities increase slowly with increasing the incident energy. It is also found that the fusion probability decreases significantly due to the dynamical deformation. For the case of $^{48}$Ca+$^{244}$Pu, the fusion probability with dynamical deformation is about one order of magnitude smaller than that for the case without dynamical deformation. The inclusion of the dynamical deformation leads to the decrease of the fusion probability.

The fusion probabilities for the reaction $^{48}$Ca + $^{244}$Pu as a function of the angular momentum (in unit of $\hbar$) at the incident energies $E_{c.m.} = 210$ MeV, 200 MeV, and 190 MeV are also depicted in Fig.5(b). The solid, dashed, and dash dotted lines are for the cases with incident energy being 210 MeV, 200 MeV, and 190 MeV, respectively. The $Q$ value for the reaction is about 161 MeV. It is found that the fusion probability decreases as increasing the angular momentum. In the region with small angular momentum, the fusion probability decreases slowly while it drops rapidly when the angular momentum is larger than about $40\hbar$.
4. Results and discussion

The nuclear dynamical deformation effects in the fusion reactions for the production of superheavy nuclei are studied within the framework of the di-nuclear system model with dynamical potential energy surface (DNS-DynPES model). The development of the dynamical deformation for the incident channel is investigated and the maximum dynamical deformations for different configurations are also calculated. The influences of the dynamical deformations on the potential energy surface (PES) and fusion probability are studied. The calculation results indicate that the structure of the PES changes significantly because of the dynamical deformations. The fusion for synthesizing SHN is also hindered and it is found the fusion probabilities decrease by nearly one order of magnitude because of the inclusion of the dynamical deformations. The fusion probability as a function of angular momentum is found to decrease with increasing the angular momentum.

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