A comparative study of hybrid simulation and dynamical substructuring system schemes for shake table tests

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Abstract.
In shaking table experiments, the dynamics of the shake table is strongly affected by a specimen to be tested, particularly when the specimen is very heavy. This is also the case when substructuring experiments are implemented by the shaking tables. This study numerically examines two substructuring schemes applied in shaking table experiments. Hybrid simulation scheme, which is commonly used in substructuring experiments, directly uses the output of the numerical substructure as the input to the physical substructure. Dynamical substructuring system (DSS) scheme, which requires more complicated control design, determines the control input to minimize control error obtained by outputs of numerical and physical substructures. This study has found that, in shaking experiments, the stability of the hybrid simulation scheme is very sensitive to a pure time delay. It is also found that DSS scheme can provide much higher stability margin against the pure time delay than the hybrid simulation scheme. The effort to rather complicated controller design of DSS scheme will be rewarded at actual control practices.

1. Introduction
A dynamical substructuring experiment is a technique which physically excites the core part of the specimen and simulates the rest of the specimen in numerical computations with a real-time interaction. In this technique, the original specimen is referred to as an emulated system, the core of the specimen to be tested is a physical substructure and the part in the computations is a numerical substructure. This technique is becoming a possible choice for experimentations of huge specimens (e.g. architectural buildings or civil infrastructures) which exceed the capacity of experimental apparatus or loading specifications. Its application to shaking table tests is a major interest in earthquake and structural engineering. This study overviews two control approaches in dynamical substructuring experiments and examines their applicability to shaking table tests.

Hybrid simulation (HS) scheme, known as the most common method for substructuring experiments, exists from the birth of its concept in structural engineering [1]. In HS scheme, the output of the numerical substructure directly becomes the input of the physical substructure [2,3]. The controller in this scheme is designed to cancel the dynamics of a transfer system, which physically realises the demand signal from the numerical simulation onto the physical substructure. For this purpose, the inversion of the dynamics of the transfer system is commonly employed in this scheme. Additional controllers are possible choices to deal with the error caused by the inaccurate modelling of the table dynamics [4,5,6].
The dynamical substructure system (DSS) scheme was developed from control engineering point of view in the early 2000s, [7] together with its basic control synthesis of linear substructuring control (LSC) [7]. Nonlinear substructuring control (NLSC) [8,9] was developed as its direct extension to nonlinear systems by employing nonlinear signal-based control [10]. Controllers in DSS scheme are designed by considering the entire dynamics of substructures and shake table dynamics to minimize the control error of the outputs of the numerical and physical substructures. Thus, DSS scheme basically requires more complicated controller design than HS scheme. However, the effort at the controller design will be rewarded at the control practice as the enhancement of robustness: its stability margin becomes much larger than that of HS scheme [11].

Shaking table tests, which are performed for structural buildings or civil infrastructures, are driven by actuations systems. As a distinct feature of the shaking table tests, the dynamics of the table is significantly affected by the specimen on the table. This study examines the performance and stability of two basic control schemes (HS without any additional controllers and DSS with LSC) via substructuring shaking table test targeting on a linear 2DOF system in Fig. 1.

2. Substructuring shake table tests

The modelling of shaking table is discussed before the substructuring shake table tests. The shake table dynamics for the displacement control is commonly expressed by a second order transfer function: 

$$ \frac{1}{s^2 + a_s s + a_0} $$

When the table mass \(m_0\) is known and the pure time delay is \(\tau = 0.0\), its dynamics equivalently expressed by

$$ m_0 \ddot{y}_0(t) + c_0 \dot{y}_0(t) + k_0 y_0(t) = k_0 u(t), $$

where \(m_0, c_0, k_0\) and \(y_0\) are the mass, damping, stiffness and absolute displacement of the shake table and \(u\) is the input motion to the table.

When an SDOF system is placed on the table as the specimen and the pure time delay is zero, its
The equation of motion is expressed by

\[
\begin{align*}
    m \ddot{y}(t) + c (\dot{y}(t) - \dot{y}_0(t)) + k (y(t) - y_0(t)) &= 0, \\
    m_0 \ddot{y}_0(t) + c_0 \dot{y}_0(t) + k_0 y_0(t) - c (\dot{y}(t) - \dot{y}_0(t)) - k (y(t) - y_0(t)) &= k_0 u(t),
\end{align*}
\]

(1)

where \( m, c, k \) and \( y \) are the mass, damping, stiffness and absolute displacement of the SDOF system on the table, respectively. Eq. (1) explicitly describes the mechanism of the dynamical interaction between the shake table and its specimen.

Substructuring shake table tests in this study are discussed on the base of Eq. (1). The emulated system in this study is a linear 2DOF system subjected to a seismic acceleration \( \ddot{d}(t) \) as shown in Fig. 1. The equation of motion for the emulated system can be written as

\[
\begin{align*}
    m_i \ddot{y}_{c2}(t) + c_2 (\dot{y}_{c2}(t) - \dot{y}_{c1}(t)) + k_2 (y_{c2}(t) - y_{c1}(t)) &= 0, \\
    m_i \ddot{y}_{c1}(t) + c_1 (\dot{y}_{c1}(t) - \ddot{d}(t)) + k_1 (y_{c1}(t) - d(t)) &= c_2 (\dot{y}_{c2}(t) - \dot{y}_{c1}(t)) + k_2 (y_{c2}(t) - y_{c1}(t)),
\end{align*}
\]

(2)

where \( m_i, c_i, k_i \) and \( y_{c1}, y_{c2} (i = 1,2) \) are the mass, damping, stiffness and absolute displacement of the \( i \)th storey in the emulated system, respectively and \( d \) is the seismic displacement. For a substructuring shake table test, this emulated system is divided into physical and numerical substructures. The physical part to be tested consists of the shaking table and the physical substructure. Its equation of motion is expressed by

\[
\begin{align*}
    m_i \ddot{y}_1(t) + c_1 (\dot{y}_1(t) - \ddot{d}(t)) + k_1 (y_1(t) - d(t)) - c_2 (\dot{y}_2(t) - \dot{y}_0(t)) - k_2 (y_2(t) - y_0(t)) &= k_0 u(t),
\end{align*}
\]

(3)

The rest part of the emulated system becomes the numerical substructure which is expressed by

\[
\dot{m}_i \ddot{y}_1(t) + k_1 y_1(t) - c_2 \ddot{y}_2(t) - k_2 y_2(t) = c_1 \dot{d}(t) + k_0 d(t) - c_2 \dot{y}_0(t) - k_2 y_0(t).
\]

(4)

Based on Eqs. (3) and (4), the dynamics of table accounting for the interaction with the specimen becomes

\[
G_0(s) = \frac{1}{u(s)} = \frac{k_0}{m_0 s^2 + (c_0 + c_2) s + k_0 + k_2 - (c_2 s + k_2) G_2(s)},
\]

(5)

where \( G_2(s) = \frac{\ddot{y}_2(s)}{\ddot{y}_0(s)} = \frac{c_2 s + k_2}{m_0 s^2 + c_2 s + k_2} \). Based on the table dynamics in Eq. (5), the output of the shake table at \( \tau \neq 0.0 \) becomes

\[
y_0(s) = G_0(s) e^{-\tau} u(s)
\]

The controllers for both two control schemes are designed to achieve \( y_0 = y_1 \) in HS scheme and \( e = y_1 - y_0 = 0.0 \) in DSS scheme.
2.1 Hybrid simulation scheme

In HS scheme where the input signal is determined as \( u(s) = K_{HS}(s)y_1(s) \), the outputs of the controlled points in substructures are expressed by

\[
y_0(s) / y_1(s) = \frac{\dot{y}_0(s)}{\dot{y}_1(s)} = G_0(s)K_{HS}(s).
\]

Since its control purpose is to achieve \( y_0(t) = y_1(t) \) or \( \dot{y}_0(t) = \dot{y}_1(t) \), an appropriate controller transfer function is found to be

\[
K_{HS}(s) = G_0(s)F_0(s),
\]

where \( F_0 \) is a filter to realise a proper transfer function \( K_{HS} \). When the relative degree of the transfer function \( G_0 \) is two, \( F_0 \) need to be the second order lowpass filter with very high cut-off frequency.

In Fig. 2 for \( \tau \neq 0.0 \), the output of the numerical substructure becomes

\[
\begin{align*}
\dot{y}_1(s) &= U_d(s)d(s) - U_y(s)y_0(s) = U_y(s)(G_d(s)d(s) - y_0(s)) \\
\dot{y}_0(s) &= U_d(s)d(s) - U_y(s)\dot{y}_0(s) = U_y(s)(G_d(s)d(s) - \dot{y}_0(s)).
\end{align*}
\]

Based on Eq. (7), the closed-loop transfer function becomes

\[
\frac{y_1(s)}{d(s)} = \frac{U_d(s)}{1 + U_y(s)G_0(s)K_{HS}(s)e^{-\tau s}}.
\]

Now, its closed-loop characteristic equation (CLCE) \( L_{HS} \) becomes

\[
L_{HS}(s) = 1 + H_{HS}(s)e^{-\tau s},
\]
where  \( H_{HS}(s) = \frac{(c_2s + k_2)m_s s^2}{m_s s^2 + c_1s + k_1 (m_2 s^2 + c_2s + k_2)} F_0(s) \). The open-loop characteristic equation (OLCE) \( H_{HS}(s) \) can be rewritten as

\[
H_{HS}(s) = \frac{(2 \zeta_2 \omega_2 s + \omega_2^2) s^2}{(\gamma_m s^2 + 2 \zeta_2 \omega_2 s + \omega_2^2 \gamma_k)} \left( s^2 + 2 \zeta_2 \omega_2 s + \omega_2^2 \right) F_0(s),
\]

(8)

where \( m_1 / m_2 = \gamma_m, \ c_1 / c_2 = \gamma_c, \ k_1 / k_2 = \gamma_k, \ \omega_1 = \sqrt{k_1 / m_1}, \ \omega_2 = \sqrt{k_2 / m_2}, \ \zeta_1 = \frac{c_1}{2 \omega_1 m_1}, \ \zeta_2 = \frac{c_2}{2 \omega_2 m_2} \).

According to Eq. (8), the stability of HS scheme is governed by the parameters of the numerical and physical substructures. The Nyquist stability criterion with \( H_{HS}(s): \left| H_{HS}\left(j \omega\right) e^{-\tau \omega}\right| = 1.0 \) and \( \angle H_{HS}\left(j \omega\right) e^{-\tau \omega} = \pm \pi \) provides the critical pure time delay, which is the maximum allowable delay for the stability.

Critical pure time delays for the physical structures having a different natural frequency \( \omega_2 \) and the fixed-damping ratio of the physical substructure \( \zeta_2 = 0.05 \) is depicted in Fig. 3, which is calculated for varying parameters \( \gamma_c = 0.5, 1.0, 1.5 \) and \( \gamma_k = 0.5, 1.0, 1.5 \) as well as the fixed \( \gamma_m = 1.0 \). Here, the filter \( F_0 \) is the second order lowpass filter with the cut-off frequency 500.0 Hz, so that the gains of the filter near the frequencies of the interest become near 1.0.

Fig. 3 shows curvatures which are the border between stable and unstable region for different conditions. The below of the curvature is the stable region, while its above is the unstable region. This stability plot provides only the assessment of stability and it does not provide the control performance. Thus, control performance may not be sufficient for the testing conditions near the border.

According to Fig. 3, the critical pure time delay is found to be greatly influenced by \( \omega_2 \), which is the natural frequency of the physical specimen. For example, in the substructuring experiment for \( \omega_2 = 20.0 \) rad/s, its critical pure time delay becomes about 0.01 s. This indicates that substructuring experiments for the physical substructure of \( \omega_2 = 20.0 \) rad/s cannot be achieved by the experimental system having a pure time delay over 0.01 s. In addition, according to the stability plot, the physical substructure with higher natural frequency requires smaller critical pure time delays for the implementation of the substructuring tests.
2.2 Dynamical substructuring system scheme

In Fig. 4 for $\tau \neq 0.0$, the output of the numerical substructure in DSS scheme becomes

$$y_1(s) = U_d(s)d(s) - U_{y1}(s)y_0(s) = U_{y1}(s)(G_d(s)d(s) - y_0(s)).$$

Then, the error signal becomes

$$e(s) = y_1(s) - y_0(s) = U_d(s)d(s) - U_a(s)e^{-\tau}u(s),$$

where $U_a(s) = \left[ \frac{(c_s s + k_2)}{(m_is^2 + c_is + k_1)}(1 - G_2(s)) + 1 \right]G_0(s)$ and $U_d(s) = \frac{c_s s + k_1}{m_is^2 + c_is + k_1}$. In DSS scheme based on LSC, the input signal is determined to be $u(s) = K_d(s)d(s) + K_e(s)e(s)$ and the error signal can be rewritten as

$$e(s) = \frac{G_d(s) - G_a(s)K_d(s)e^{-\tau}}{1 + G_a(s)K_e(s)}d(s). \quad (9)$$

In order to achieve zero-error in Eq. (9) for the case of $\tau = 0.0$, the feedforward controller needs to be

$$K_d(s) = G_a(s)^{-1}G_d(s)F_d(s).$$

According to Eq. (9), its CLCE can be

$$L_{DSS}(s) = 1 + H_{DSS}(s)e^{-\tau s},$$

where $H_{DSS}(s) = G_a(s)K_e(s)$, which corresponds with the OLCE of Eq. (9).

The error feedback controller is designed to maintain the stability conditions of $H_{DSS}$. When $K_e(s) = G_a(s)^{-1}F_e(s)$ is employed, the OLCE becomes $H_{DSS}(s) = F_e(s)$ and the stability condition is determined only by $F_e(s)$. When the filter is designed to be $F_e(s) = \frac{\omega_e^2}{s^2 + 2\zeta_\omega \omega_e s}$, the critical pure time delay can be obtained by solving the Nyquist stability criterion of $|H_{DSS}(j\omega)e^{-\tau s}| = 1.0$ and $\angle H_{DSS}(j\omega)e^{-\tau s} = \pm \pi$.
\[
\begin{align*}
\omega_c &= \omega_c \sqrt{1 + 4\zeta_e^2 - 2\zeta_e^2} \\
\tau_c &= \frac{1}{\omega_c} \tan^{-1}\left(\frac{2\zeta_e}{\omega_c}\right)
\end{align*}
\]

where \( \tau_c \) is the critical pure time delay and \( \omega_c \) is the critical circular frequency. When \( \zeta_e = 1.0 \) is employed as its damping ratio, the critical values become \( \omega_c = \omega_c \sqrt{5} - 2 \approx \omega_c / 2 \) and \( \tau_c = 2.74/\omega_c \). This indicates that the stability of DSS scheme is tuneable from the cut-off frequency in the error feedback filter. The stability plot in Fig. 3 visually provides the influence of the damping ratio and circular frequency assigned in \( F_e(s) \) to the critical pure time delay. According to Fig. 5, the filter with the high frequencies and low damping ratios result in small critical pure time delays.

3. Numerical simulations

In order to examine the performance of two control schemes, numerical simulations are conducted for the substructure experiment shown in Fig.1. The table properties in the simulations are \( m_0 = 100.0 \text{ kg}, c_0 = 12.6 \text{ kNsm/m}, k_0 = 394.8 \text{ kN/m} \), which results in \( \omega_0 = 10.0 \times 2\pi \text{ rad/s} \) and \( \zeta_0 = 1.0 \) when any specimen is not mounted on the table. The parameters of the emulated system are \( m_1 = m_2 = 100 \text{ kg}, c_1 = c_2 = 0.126 \text{ kNsm/m}, k_1 = k_2 = 15.8 \text{ kN/m} \), which result in the first and second natural frequencies \( \omega_1 = 1.2 \times 2\pi \text{ rad/s} \) and \( \omega_2 = 3.2 \times 2\pi \text{ rad/s} \), respectively. The parameters on the first and second storey are identical and the ratios of mass, damping, stiffness become \( \gamma_m = 1.0 \), \( \gamma_c = 1.0 \), \( \gamma_k = 1.0 \), respectively.

Fig. 5. Stability plot of DSS scheme.

Fig. 6. Emulated response: (a) time history of the Kobe-earthquake acceleration data, (b) each storey’s response of the emulated system.
Since shake table tests are mainly performed for structures under some seismic motion, in this study, an acceleration data recorded at Kobe earthquake 1995 in Japan, which is shown in Fig. 6(a), is employed as the seismic acceleration $\ddot{d}(t)$. Note that examinations of structures for some steady state are available by changing the seismic acceleration to some external disturbance. The response of the emulated system under the seismic acceleration is obtained by solving Eq. (2) with the above-mentioned parameters. As shown in Fig. 6(b), the maximum displacements on the first and second storeys of the emulated system reach to 258.5 mm and 420.1 mm, respectively. Its first-storey response is referred to as emulated response, which will be compared with the output of the substructures in HS and DSS schemes.

3.1 Hybrid simulation scheme

Numerical simulations of HS scheme are conducted for different pure time delays of $\tau = 5.0, 10.0, 20.0$ ms, to examine its effect to the control performance. The controller design directly follows the procedure mentioned in 2.1 and the filter $F_0$ is the second order lowpass filter with the cut-off frequency 500.0 Hz. According to the stability plot in Fig. 3, the stability can be maintained up until 17.2 ms. Then, the simulations for $\tau = 5.0, 10.0$ ms are expected to maintain the stability, while the simulation for $\tau = 20.0$ ms is anticipated to be unstable. The numerical results are shown in Table 1 and Fig. 7.

| $\tau$ (ms) | $y_e - y_p$ (mm) | $y_e - y_n$ (mm) | $y_p - y_n$ (mm) |
|------------|------------------|------------------|------------------|
| 5          | 28.5             | 26.5             | 11.6             |
| 10         | 50.8             | 47.3             | 21.2             |
| 20         | NaN              | NaN              | NaN              |

NaN: The maximum value is not measurable because of instability.

![Fig. 7. Numerical results of HS scheme: (a) $\tau = 5.0$ ms, (b) $\tau = 10.0$ ms, (c) $\tau = 20.0$ ms.](image)
The simulations for $\tau = 5.0$ ms and 10.0 ms have maintained stability, while the instability has been observed at $\tau = 20.0$ ms, as the stability analysis suggested. Even in the simulations with stability, those control accuracies are not very high: the maximum error at $\tau = 5.0$ ms becomes over 10.0% of the emulated response (258.5 mm) and the maximum error at $\tau = 10.0$ ms becomes near 20.0% of the emulated response. In the simulations, the error between the substructures’ outputs: $y_p - y_n$ is found to become smaller than the error obtained from the comparisons with the emulated response: $y_e - y_p$, or $y_e - y_n$.

### 3.2 Dynamical substructuring system scheme

Numerical simulations of DSS scheme are also conducted for different pure time delays of $\tau = 5.0, 10.0, 20.0$ ms, to examine its effect to the control performance. The controller design directly follows the procedure mentioned in 2.2 and the error filter $F_e$ is the recommended filter with $\omega_e = 20.0 \times 2\pi$ rad/s and $\zeta_e = 0.05$. According to the stability plot in Fig. 5, the stability can be maintained up until 21.8 ms and the simulations for $\tau = 5.0, 10.0, 20.0$ ms are expected to produce stable results. The numerical results are shown in Table 2 and Fig. 8.

|       | $y_e - y_p$ (mm) | $y_e - y_n$ (mm) | $y_p - y_n$ (mm) |
|-------|-----------------|-----------------|-----------------|
| 5 ms  | 1.3             | 1.5             | 0.9             |
| 10 ms | 2.6             | 3.0             | 1.7             |
| 20 ms | 5.2             | 6.1             | 3.6             |

Fig. 8. Numerical results of DSS scheme: (a) $\tau = 5.0$ ms, (b) $\tau = 10.0$ ms, (c) $\tau = 20.0$ ms.
As stability analysis suggested, numerical simulations for $\tau = 5.0, 10.0, 20.0$ ms have obtained stable results, as shown in Fig.8. In this case, the control error obtained in DSS scheme is ten times smaller than that of HS scheme. In addition, DSS scheme has achieved stable control for the pure time delay of $\tau = 20.0$ ms, which caused instability at the practice of HS scheme. In an additional simulation with the pure time delay of $21.9$ ms, instability has happened, as the stability analysis expected.

4. Conclusions

This study has examined basic stabilities of HS and DSS schemes for a linear system and its substructures. Then, the followings are found:

- The stability of HS scheme heavily relies on the properties of substructures and its stability margin is simply not tuneable.
- The stability of DSS scheme is determined by the error feedback controllers, which users can more flexibly design. A proper controller design enables the stability of DSS scheme to be insensitive to the properties of substructures.

This study has focussed on the linear controlled system with an idealised control conditions that all parameters of the controlled system is accurately known and its information can be reflected into the controller design. Studies considering nonlinearity and modelling error are required to examine the control schemes for more realistic control conditions in practice. An application of NLSC [8,9], which was developed for nonlinear systems, to shaking table tests is also important further work.

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