Triple Higgs coupling in the most general 2HDM at SM-like scenario

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Abstract We consider the triple Higgs coupling for \( h(125) \) Higgs boson within the most general 2HDM. At moderate values of parameters of model, allowing by modern data, noticeable deviation of this coupling from its SM value is improbable. This deviation can be sizable only if some measurable parameters of the model are exotic.

1 Introduction

The recent discovery of a Higgs boson with \( M \approx 125 \text{ GeV} \) at the LHC [1–6] suggests that the spontaneous electroweak symmetry breaking is most probably brought about by the Higgs mechanism. The simplest realization of the Higgs mechanism introduces a single scalar isodoublet \( \phi \) with the Higgs potential \( V_H = -m^2(\phi^\dagger \phi)/2 + \lambda(\phi^\dagger \phi)^2/2 \). This model is usually called the Standard Model (SM).

The mentioned data do not rule out the possibility of realization of beyond SM models (BSM) which include both neutral Higgs scalars \( h_a \) (generally without definite CP parity) and charged Higgs scalars \( H^\pm_a \) with masses \( M_a \) and \( M_{a\pm} \), respectively.

In the discussion that follows we use the relative couplings for each neutral Higgs boson \( h_a \) (for the case with single charged Higgs boson \( H^\pm \)):

\[
\begin{align*}
\chi^P_a & = \frac{g_a^P}{g^P_{\text{SM}}} \quad [P = V (W, Z), \ q = t, b, \ldots, \ \ell = \tau, \ldots], \\
\chi^{\pm}_a & = \frac{g(H^+H^-h_a)}{2M^2_{a\pm}/v}, \quad \chi^{H^+W^-}_a = \frac{g(H^+W^-h_a)}{M_W/v}, \\
\chi_{abc} & = \frac{g(h_ah_bh_c)}{g(hhh)_{\text{SM}}}. 
\end{align*}
\]

The quantities \( \chi^P_a \) are the ratios of the couplings of \( h_a \) with the fundamental particles \( P \) to the corresponding couplings for the would-be SM Higgs boson with \( M_h = M_a \). The other relative couplings describe the interaction of \( h_a \) with a charged Higgs boson. The couplings \( \chi^V_a \) and \( \chi^{\pm}_a \) are real due to the Hermiticity of the Lagrangian, while the other couplings are generally complex.

We omit the adjective “relative” below.

1.1 SM-like scenario

Current data allow us to suggest that Nature realizes an SM-like scenario\(^1\): The observed particle with mass \( M \approx 125 \text{ GeV} \) is a Higgs boson, and we denote it \( h_1 \). It interacts with the gauge bosons and \( t \)-quarks with coupling strengths that are close to those predicted by the SM within experimental accuracy (see e.g. [7–10]). In particular, for coupling with the gauge bosons

\[
\epsilon_V = |1 - (\chi^V_1)^2| \ll 1. 
\]

In the estimates we have in mind \( \epsilon_V \lesssim 0.1. \)

1.2 Two Higgs doublet model (2HDM)

The 2HDM presents the simplest extension of the standard Higgs model [22]. It offers a number of phenomenological scenarios with different physical contents in different regions of the model parameter space, such as a natural mechanism for spontaneous CP violation, etc. [22–24] For example, the Higgs sector of the MSSM is a particular case of 2HDM. Some variants of 2HDM have interesting cosmological consequences [25,26].

\(^1\) The term SM-like scenario was introduced in [11–18], the term alignment limit was introduced recently for this very situation, see e.g. [19–21], the decoupling limit is the particular case of this scenario.
In the most general 2HDM the couplings (1) obey the following sum rules [27–30]:

\[
\sum_{a} (\chi_{a}^{V})^{2} = 1, \\
|\chi_{a}^{f}|^{2} + |\chi_{H}^{\pm}W^{\mp}|^{2} = 1, \\
\sum_{a} (\chi_{a}^{f})^{2} = 1.
\]  

We have constructed in [30] the minimal complete set of measurable quantities ("observables") which determine all parameters of the 2HDM. This set contains

v.e.v. of Higgs field \( v = 246 \text{ GeV} \),
masses of Higgs bosons \( M_{a} \), \( M_{\pm} \) \( (a = 1, 2, 3) \),
two out of three couplings \( \chi_{a}^{V} \),
3 couplings \( H^{\pm}H^{-}h_{a} \) (quantities \( \chi_{a}^{\pm} \) Eq. (1)),
quartic coupling \( g(H^{+}H^{-}H^{+}H^{-}) \).

In the most general 2HDM, these observables are independent of each other. In some particular variants of 2HDM, additional relations between these parameters may appear (for example, in the CP-conserving case we have \( \chi_{3}^{0} = 0 \), \( \chi_{3}^{\pm} = 0 \)).

1.3 Limitations for parameters

The values of parameters \( \lambda_{a} \) of 2HDM (and therefore the mentioned basic parameters) obey two groups of constraints (see e.g. [23, 24]).

**Positivity constraints** are conditions for the stability of Higgs potential at large quasi-classical values of fields. They do not restrict the parameters from above.

**Perturbativity (and unitarity) constraints** make it possible to use the first non-vanishing approximation of perturbation theory for description of physical phenomena with reasonable accuracy – a *perturbative description*. (This is a tree approximation for most of the phenomena and a one-loop approximation for the phenomena which are absent at tree level, e.g. decays \( h \to \gamma\gamma, h \to Z\gamma, h \to gg \).) The starting point in obtaining of these constraints is the observation that the effective parameter of the perturbative expansion is not \( \lambda_{i} \) \( (i = 1, 2, \ldots 7) \) but \( \lambda_{i}/\Delta \) with \( \Delta = 8\pi \) or \( 4\pi \). The perturbativity condition is written usually in the form \( |\lambda_{i}| \ll \Delta \).

At \( |\lambda_{i}| \approx \Delta \) a perturbative description of the physical phenomena is incorrect even at low energies. In particular, the equations, expressing masses and couplings via the parameters of the Lagrangian, become invalid. Good examples provide the one-loop radiative corrections (RC) to the triple Higgs coupling [31–38]. In the SM-like scenario these RC reach 150 \( \div \) 200\% at \( |\lambda_{i}| \approx \Delta \). (Reference [38] presents an example with clear details. The authors consider the Inert Doublet Model, i.e. 2HDM with exact \( Z_{2} \) symmetry in the SM-like case, at \( \lambda_{4} = \lambda_{5} = 0 \) and \( \lambda_{1} = \lambda_{SM} \). The one-loop corrections to \( g(h_{1}h_{1}h_{1}) \) are described by the single parameter \( \lambda_{3} \), and they reach 180\% at \( |\lambda_{3}| \approx \Delta \).

The first non-vanishing approximation of perturbation theory describes physical phenomena with relative inaccuracy \( k \) only at

\[ |\lambda_{i}| < k\Delta \quad (k < 1). \]  

In particular, in the region of the parameters, the provided accuracy of the standard description in 30\% one should have \( k = 0.3 \). In this region of parameters the value of RC, discussed in [31–38], does not exceed 20\%.

The second limitation on the parameter \( \lambda_{i} \) is due to the perturbativity constraints, leading to additional restrictions. In particular, according to Eq. (23) from Ref. [30], the perturbativity constraints (5) impose the limitation on the couplings of \( h_{1} \) to charged Higgs bosons: \( |\chi_{1}^{\pm}| \ll 1 \) at \( M_{\pm} > 500 \text{ GeV} \).

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In particular, in the region of the parameters, the provided accuracy of the standard description in 30\% one should have \( k = 0.3 \). In this region of parameters the value of RC, discussed in [31–38], does not exceed 20\%. Below we will have in mind this very limitation with \( k \approx 0.3 \).

The realization of the SM-like scenario imposes additional restrictions on the parameters. Because of the sum rules (3), in the SM-like scenario the couplings of the other neutrals \( h_{a} \) with gauge bosons \( \chi_{a}^{V} \) are small. Besides, the absolute value of the non-diagonal coupling with EW gauge bosons for the observed Higgs boson \( \chi_{1}^{H^{+}H^{-}} \) is small,\(^2\) while similar couplings for the other neutrals \( \chi_{2,3}^{W^{\pm}W^{\mp}} \) are close to their maximal possible values:

\[ |\chi_{1}^{V}| < 2 \epsilon_{V} \ll 1, \quad a = 2, 3, \]  

\[ |\chi_{1}^{H^{\pm}W^{\mp}}| < 2 \epsilon_{V} \ll 1; \quad |\chi_{2,3}^{W^{\pm}W^{\mp}}|^{2} \approx 1. \]

In the SM-like scenario the perturbativity constraints lead to additional restrictions. In particular, according to Eq. (23) from Ref. [30], the perturbativity constraint (5) imposes the limitation on the coupling of \( h_{1} \) to charged Higgs bosons:

\[ |\chi_{1}^{\pm}| < 1 \text{ at } M_{\pm} > 500 \text{ GeV}. \]  

It means that the heavy charged Higgs boson gives only a small contribution to the two-photon width of the observed Higgs boson \( h_{1} \).

Next, we consider heavy neutral Higgs bosons \( h_{a} \) \( (a = 2, 3) \) in the SM-like scenario. The couplings \( \chi_{a}^{V} \) are small (see (6)), while Eq. (23) from Ref. [30] allows us to have big values of \( \chi_{a}^{\pm} (\ll 1/\sqrt{\epsilon_{V}}) \). Therefore, the two-photon width of the boson \( h_{a} \) is strongly different from a similar width as calculated for the would-be SM Higgs boson with the mass \( M_{a} \).

The last statement is illustrated by a simple calculation for the toy case \( M_{2} = 600 \text{ GeV}, M_{\pm} = 300 \text{ GeV} \). We present in the table the total Higgs width \( \Gamma_{\text{tot}} \) in GeV, and its two-photon width \( \Gamma(\gamma\gamma) \) in MeV for different scenarios, assuming the partial width \( h_{2} \rightarrow h_{1}h_{1} \) to be small.

\(^2\) The calculations of \( H^{\pm} \rightarrow W^{\pm}h_{1} \) decay at LHC in [66, 67] are made in the CP-conserving 2HDM and with not very small \( \epsilon_{V} \).
2 Triple Higgs vertex

The observation of $hh$ production and the extraction of the triple Higgs vertex $g(hhh)$ from the future experiments is scheduled at the LHC and other colliders. This is a necessary step in the verification of the Higgs mechanism. Hopefully, these observations will allow us to see the effects of BSM.3

The studies of triple Higgs coupling have long history; for recent reviews see e.g. [39–42]. There are two major problems. The first one is whether it is possible to use these observations for the extraction of New Physics effects beyond SM.3

The accuracy in the extraction of a triple Higgs vertex $g(hhh)$ from the future data cannot be high, since in each case the corresponding experiments deal with interference of two channels – an independent production of two Higgses and production of Higgses via $hhh$ vertex. This interference is mainly destructive [43,44]. For example, for a 100 TeV hadron collider with total luminosity $3\text{ab}$ one can hope to reach an accuracy of 40% in this vertex from future data [45–48]; at ILC the accuracy in the extraction of $g(hhh)$ will be better than 80% only after 10 years of operation [49]. Therefore, the effects of New Physics will be distinguishable in the data of $g(hhh)$ in the realistic future only if the deviation of this coupling from its SM value is high enough,

$$|\chi_{111} − 1| \gtrsim 1. \quad (9)$$

One of the approaches in the description of the SM violations is to add in the SM Lagrangian terms with anomalous interactions of Higgs boson. It was found for many reasonable benchmark points that these anomalous interactions are difficult to observe [50–54].

The other approach is to consider some special form of BSM. The review of the whole variety of possible BSM mod-

3 In the models containing additional heavy Higgs bosons $h_a$ with $M_a > 2M_h$, the resonant $h_1h_1$ production like $pp \rightarrow (h_2 \rightarrow h_1)h_1 \cdots$ becomes possible. In this paper we discuss only non-resonant $h_1h_1$ production, without intermediate $h_a \neq h_1$.
the value $\chi_{1}^{\pm} \approx 1$. In view of Eq. (8), it can happen if this
coupling is either very small or negative.

The term $R_{2} \sim 3 + \varepsilon_{V}(M_{h}^{2} / M_{Z}^{2})$ can give $|\chi_{111} - 1| \gtrsim 1$
only if at least one of the other Higgs bosons $h_{2,3}$ is heavy
enough, $M_{2,3} > 1$ TeV. Direct discovery of such Higgs
bosons seems to be a difficult task. Therefore the value of
$g(h_{1}h_{1}h_{1})$ might become an important source of knowledge
as regards such heavy neutrals for a long time.

The term $R_{3}$ contains small factors $\chi_{2,3}^{3}, \chi_{1}^{V}W^{-}$ and fac-
tors $\chi_{2,3}^{\pm}$ which can be large (up to $1 / \sqrt{\varepsilon_{V}}$). The term $R_{3}$
may not be small if $H^{\pm}$ is heavy.

Certainly, the real range of possible values of discussed
parameters is restricted by other observations. Better esti-
mates are possible only after measuring of $\varepsilon_{V}$ with reason-
able accuracy. In particular, at $\varepsilon_{V} \ll 0.1$ we cannot expect
sizable effects in the triple Higgs vertex.

3 Summary

- Measuring $hh$ production at various colliders is a neces-
sary step in the verification of the Higgs mechanism
of EWSB. Within the SM-like scenario in the 2HDM,
these measurements can give information as regards New
Physics beyond SM only at exotic values of the param-
eters listed above. The enlargement of the field of param-
eters of 2HDM at the transition from CP-conserved softly
$Z_{2}$ broken potential to the most general case gives no new
essential opportunities in the deviation of triple Higgs
coupling from its SM value.

In our conclusions we limit ourselves to perturbative lim-
itations in the form (5) with $k \sim 0.3$. These limitations
guarantee us applicability of first orders of perturbation
theory for a description of model (including the expres-
sions of the masses and couplings via the parameters of
the Lagrangian) and a small value of the quantum (loop)
corrections.

- In other models the deviation of the triple Higgs coupling
from its SM value can be stronger than that in 2HDM at
moderate values of the parameters; see [68,69]. In the
particular case of the nMSSM (2HDM +Higgs singlet)\nvalues $\chi_{111}$ can range from $-5$ to $20$ [61–64].

- If the mass $M_{2}$ of the heavier Higgs boson $h_{2}$ lies within
the interval $\langle 250 \div 400 \rangle$ GeV and $|\chi_{2}^{2}| > 1$ (in the SM-like scenario
for $h_{1}$), the following interesting phenomenon takes place.
The boson $h_{2}$ becomes relatively narrow and the cross
section of gluon fusion $gg \rightarrow h_{2}$ can be larger
than that for the would-be SM Higgs boson with mass $M_{2}$.

The process $gg \rightarrow h_{2} \rightarrow h_{1}h_{1}$ can be seen as a resonant
production of the $h_{1}h_{1}$ pair. In principle, it allows us to
discover the mentioned $h_{2}$ at LHC (see the examples in
[55–60,70,71] for special sets of parameters).

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References

1. ATLAS Collaboration, Observation of a new particle in the search
for the Standard Model Higgs boson with the ATLAS detector at
the LHC. Phys. Lett. B 716, 1 (2012). arXiv:1207.7214 [hep-ex]

2. ATLAS Collaboration, ATLAS-CONF-2014-010 (2014)

3. CMS Collaboration, Observation of a New Boson at a Mass of 125
GeV with the CMS Experiment at the LHC

4. CMS Collaboration, Phys. Lett. B 716, 30 (2012). arXiv:1207.7235
[hep-ex]

5. CMS Collaboration, CMS-PAS- HIG-14-009 (2014)

6. The ATLAS and CMS Collaborations, Measurements of the Higgs
boson production and decay rates and constraints on its couplings
from a combined ATLAS and CMS analysis of the LHC pp colli-
sion data at $\sqrt{s} = 7$ and 8 TeV. arXiv:1606.02266 [hep-ex]

7. G. Belanger, B. Dumont, U. Ellwanger, J.F. Gunion, S. Kraml,
Higgs Couplings at the End of 2012. JHEP1302, 053 (2013).
arXiv:1212.5244 [hep-ph]

8. A. Pilaftsis, Symmetries for SM Alignment in Multi-Higgs Doublet
Models. arXiv:1602.02017 [hep-ph]

9. B. Dumont, Higgs Couplings After Moriond. arXiv:1305.4635
[hep-ph]

10. P.M. Ferreira, R. Guedes, J.F. Gunion, H.E. Haber, M.O.P. Sam-
paio, R. Santos, The Wrong Sign limit in the 2HDM.
arXiv:1410.1926 [hep-ph]

11. I.F. Ginzburg, M. Krawczyk, P. Osland, Distinguishing Higgs Mod-
els at Photon Colliders, 2nd ECFA/DESY Study 1998–2001, pp.
997–1001

12. I.F. Ginzburg, M. Krawczyk, P. Osland. arXiv:hep-ph/9909455

13. I.F. Ginzburg, M. Krawczyk, P. Osland. Potential of photon collid-
ers in resolving SM-like scenarios. Nucl. Instrum. Meth. A 472,
149–154 (2001)

14. I.F. Ginzburg, M. Krawczyk, P. Osland. arXiv:hep-ph/0101229

15. I.F. Ginzburg, M. Krawczyk, P. Osland, AIP Conf. Proc. Ser. 578,
304 (2001). IFT-2001-2, 304

16. I.F. Ginzburg, M. Krawczyk, P. Osland, Standard-Model-
Like Scenarios in the 2HDM and Photon Collider Potential.
arXiv:hep-ph/0101331

17. I.F. Ginzburg, M. Krawczyk, P. Osland, Two-Higgs-doublet models
with CP violation. In: Proc. SUSY02 Conference, pp. 703–706

18. I.F. Ginzburg, M. Krawczyk, P. Osland. arXiv:hep-ph/0211371

19. M. Carena, I. Low, N.R. Shah, C.E.M. Wagner, Impersonating
the Standard Model Higgs Boson: Alignment without Decoupling.
JHEP 1404, 015 (2014). arXiv:1310.2248 [hep-ph]

20. P.S.B. Dev, A. Pilaftsis, Maximally symmetric two higgs doublet
model with natural standard model alignment. JHEP 1412, 024
(2014)

21. P.S.B. Dev, A. Pilaftsis, JHEP 1511, 147 (2015). arXiv:1408.3405
[hep-ph]
