Time-Continuous Energy-Conservation Neural Network for Structural Dynamics Analysis

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Abstract

Fast and accurate structural dynamics analysis is important for structural design and damage assessment. Structural dynamics analysis leveraging machine learning techniques has become a popular research focus in recent years. Although the basic neural network provides an alternative approach for structural dynamics analysis, the lack of physics law inside the neural network limits the model accuracy and fidelity. In this paper, a new family of the energy-conservation neural network is introduced, which respects the physical laws. The neural network is explored from a fundamental single-degree-of-freedom system to a complicated multiple-degrees-of-freedom system. The damping force and external forces are also considered step by step. To improve the parallelization of the algorithm, the derivatives of the structural states are parameterized with the novel energy-conservation neural network instead of specifying the discrete sequence of structural states. The proposed model uses the system energy as the last layer of the neural network and leverages the underlying automatic differentiation graph to incorporate the system energy naturally, which ultimately improves the accuracy and long-term stability of structures dynamics response calculation under an earthquake impact. The trade-off between computation accuracy and speed is discussed. As a case study, a 3-story building earthquake simulation is conducted with realistic earthquake records.

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1. Introduction

With the explosive growth of data and cloud computing resources, recent advances in machine learning have yielded transformative results across diverse scientific disciplines, including computer vision [1], natural language processing [2], genomics [3], and business intelligence [4]. The main objective of machine learning is to grant the machine with strong generalization power using as few parameters as possible [5]. The structural dynamics and physics law are not well-explored with modern machine learning approaches. While there are some attempts to simulate the wave propagation using neural networks [6,7], it is unclear if the neural network follows the basic physics laws. In addition, if the long sequence of discrete data is trained with neural network naively, the neural network becomes a very deep network, which suffers from the vanishing or exploding gradient problems [8,9]. Many tricks were studied to alleviate the problem, such as non-saturating activation functions [10], batch normalization [11], gradient clipping [12], and faster optimizers [13]. Nevertheless, it is still tricky for a neural network to predict a long-term time-series sequence.

The conventional approaches for structural dynamics problems, like finite element methods, have the following difficulties.

- Finite element analysis always involves complicated modeling and meshing processes. The created mesh needs to capture the input domain geometry with high-quality and well-shaped cells. In addition, the mesh should be adaptively refined in areas that are important for the subsequent calculations. The complicate pre-processing and post-processing procedure are cumbersome burdens for the users.

- Besides, when the material inelasticity or geometry nonlinearity plays a critical role in the dynamic response, the convergence of conventional
approaches becomes an issue. Many of the conventional numerical approaches require extensive computer simulations to achieve solid results.

- Last but not least, in finite elements analysis, the discretization errors in the meshing and integration process are hard to control or estimate. The finite element method usually gives the final results anyway regardless of the mesh quality in the preprocessing steps. Therefore, users of the finite element analysis must be a domain expert to justify the correctness and accuracy of the results [14].

To avoid the problems above, machine learning techniques, specifically neural networks, are explored to solve the structural dynamics problems in this paper.

- Machine learning approaches operate on the data directly without the complicated geometrical preprocessing steps.

- The activation function in the neural network supports the nonlinearity natively, which can learn the material nonlinearity and geometrical nonlinearity directly from the experimental data.

- The machine learning has a training dataset and a testing dataset, which lets the users control the error and the tolerance explicitly. The simplicity of straightforward learning from the data democratizes the analysis of structural dynamics problems.

While a real-world structure is complicated, this paper focuses on the fundamental structural dynamics problem in single-degree-of-freedom (SDOF) first and then generalizes to a multiple-degrees-of-freedom (MDOF) application. The energy-conservation neural network is designed to solve the structural dynamics problem in a physically informative way.

For any machine learning task, the quality, volume, and variety of the training data are crucial to the accuracy of the prediction results. In earthquake engineering, accessing to earthquake ground motion was hindered by the large-scale of the data and the inconsistency in how the data were gathered and
stored. Fortunately, nowadays the Pacific Earthquake Engineering Research (PEER) Center ground motion database [15] and United States Geological Survey (USGS) National Strong Motion Program [16] provide large-scale ground motion records for the next generalization of ground motion prediction research. The consistent and reliable worldwide ground motion records built a cornerstone for this paper. Specifically, the PEER strong motion database contains over 174,000 records from 486 earthquake events and 1379 recording stations [16]. The earthquake magnitude covers a range between 3.4 and 7.9, and a distance range between 0.05 and 1533 km. For each earthquake record, the available spectra are available for periods 0.01 to 20 seconds and the damping ratios ranges from 0.1 to 30%.

Performance-based seismic design requires an effective and efficient structural dynamics analysis approach. Historically, the primary focus of building design is to ensure life safety and collapse prevention. Continued operation of the structure and reduction of economic losses associated with earthquake damage to the structure are secondary considerations if they are considered at all. Performance-based seismic design allows the engineer to choose the appropriate levels of ground motion objectives and the level of protection for those ground motions. Multiple levels of ground shaking are evaluated with a different level of performance specified for each level of ground shaking. Nonlinear dynamic analysis of the structure response plays a crucial role in seismic response prediction. The proposed neural network in this paper provides a novel approach to estimate seismic demands at the operational performance level. The details of the contributions in this paper are as follows.

- A versatile neural network framework with continuously-defined dynamics. The framework is able to naturally exploit observed data that arrives at arbitrary times. After the training, it is also convenient to predict model states at any arbitrary times. In addition, the prediction to the derivative can be computed in parallel, which has a better scalability.

- A novel energy conservation neural network that incorporate the system
energy natively. The proposed model is physically informed, which predicts the structural dynamics response realistically.

- An efficient machine learning approach for structural dynamics problems from a fundamental single-degree-of-freedom system to a complicated multiple-degrees-of-freedom system. The proposed model potentially opens the path for the data-driven computation of structural dynamics problems.

2. Background and Related Work

Among various machine learning techniques, the conventional neural network for predicting time series is recurrent neural network (RNN) \cite{17}. RNNs process a time series step-by-step, maintaining an internal state summarizing the information they have seen so far. Basically, the output of the neuron is fed as a part of the input to the same neuron again and again. To train an RNN on long sequences, the neuron needs to run over many time steps, making the unrolled RNN a very deep network, which may have prohibitive time complexity in practical situations. To fix the issues in RNNs, various types of cells with long-term memory have been introduced. The popular types are long short term memory (LSTM) \cite{18} cell, gated recurrent unit (GRU) \cite{19} cell, and the attention mechanism in the Transformer model \cite{20}. Transformer uses 12 separate attention mechanism without using sequence aligned convolution, which is convenient for the large-scale parallelism on GPU. Although Transformer achieves state of the art performance in natural language processing (NLP), the time series data won’t have exactly the same events in NLP. And since the time series data lack repeated tokens, Transformer may not be applied to time series prediction directly.

Besides the conventional neural network for time series forecasting, customized neural network have been designed and applied to solve structural dynamics and wave propagation problems. These works are classified into four categories.
The first category of works focus on wave physics. For example, Hughes [21] built a mapping between the dynamics of wave physics and the computation in the recurrent neural network. The mapping leveraged physical wave systems to learn complex features in temporal data, which is in the opposite direction of this work. Zhu [22, 23] applied the neural network to predict the propagation of wavefront which contains complex wave-dynamics phenomena like velocity anomalies, reflection, and diffraction. Sorteberg [24] built a predictive neural network comprising of autoencoder and LSTM (long short term memory) cells. The neural network can predict at most 80 time-steps of wave propagation.

The second category of works relies on a large-scale of observed data to extract the dynamics features. For instance, Mousavi [25] trained the CNN-RNN earthquake detector based on deep neural network. A large-scale of seismograms were feed into the neural network to detect the earthquake signals. Broggin [26] proposed a data-driven wave-field focusing which retrieves the Green’s function recorded at the acquisition surface due to a virtual source located at depth. Besides, Cao [27] applied a seismic data-driven rock physics model to estimate the partial saturation effects in tight rocks [28].

The third category of works explores the relation between neural network and wave differential equation. Lu [29] proposed a new linear multi-step (LM) architecture inspired by the linear multi-step method solving ordinary differential equations. The new architecture was used to achieve higher accuracy on image classification. Zhu [30] introduced Runge-Kutta method to build a convolutional neural network and achieved superior accuracy. Chen [31] demonstrated the neural ordinary differential equations in continuous-depth residual networks and continuous-time latent variable models.

The fourth category of works applied end-to-end machine learning models to specific applications. Bagriacik [32] investigated the earthquake damage to water pipelines using statistical and machine learning approaches. Kiani [33] applied machine learning techniques for deriving seismic fragility curves. Song [34] predicted structural responses accurately and identified salient attributes using the generalized additive model.
While the existing researches focus on wave propagation, signal detection, and differential equations. This research focuses on the structural dynamics problem in earthquake engineering. This paper built a novel time-continuous energy-conservation neural network for structural dynamics analysis.

3. Baseline Time-Continuous Neural Network

Instead of training the discrete signal directly using RNN, the neural network in this paper parameterizes the derivative of structural states with respect to time. For a classic single-degree-of-freedom (SDOF) mass-spring-damper system, the equilibrium equation using Newton’s second law of motion is

\[ m\ddot{u} + c\dot{u} + f_S(u, \dot{u}) = p(t) \]  

where \( m \) is the mass, \( u \) is the displacement, \( c \) is the viscous damping coefficient, \( f_S \) is the elastic or inelastic resisting force from the structure, and \( p(t) \) is the external force. In Equation 1, \( m\ddot{u} \) is the inertial force and \( c\dot{u} \) represents the damping force. The left side of Equation 1 is the summation of all the internal forces, which is equal to the right side, the external force \( p(t) \).

Instead of building a neural network solving the equation above, we describe the motion in an incremental perspective

\[ u_{t+1} = u_t + f(u_t, \theta_t) \]  

where \( \theta_t \) is the state variable at step \( t \), and \( f(u_t, \theta_t) \) is the displacement increment.

The dynamics can be parameterized as an ordinary differential equation (ODE) specified by a neural network:

\[ \frac{du(t)}{dt} = f(u(t), \theta) \]  

The neural network in this paper focuses on the approximation of the time-continuous function \( f(u(t), \theta) \) in Equation 3. The computation graph of the neural network is shown in Figure 1.
A classic neural network is trained to predict the time series derivative from the initial state of the SDOF problem. The initial state $u(0)$ is considered as the input layer $u(0)$. The following states $u(T)$ is defined as the output layers, which can be computed by the neural network. Defining and evaluating the time series model using differential equation has the following benefits:

**Continuous time-series models** In contrast to RNNs, which require discrete observed data in fixed time steps, continuously-defined dynamics can naturally incorporate data that arrives at arbitrary times. This method makes it easier to collect observation data. After the training, the method is also easier to predict model states at an arbitrary time.

**Scalable Inference** While the time-continuous equation must be integrated sequentially, the inference from the displacement to the derivative can be computed in parallel. This feature ensures the accessibility of this method for a large-scale dynamics system.

4. Theory of proposed Neural Network

On top of the baseline time-continuous neural network in the previous section, the energy conservation neural network is proposed. The idea of the pro-
posed neural network is inspired by Hamiltonian mechanics [20, 35].

**Hamiltonian Mechanics**

Hamiltonian mechanics describes the Lagrangian mechanics in a different set of mathematical formulation. In Hamiltonian mechanics, the physical system is described in canonical coordinates \( r = (q, p) \). The vector \( q \) are the generalized coordinates, which represents the positions of the targeting objects. The vector \( p \) denotes the corresponding momentum. The time evolution of the system is defined by Hamilton’s equations:

\[
\frac{dp}{dt} = -\frac{\partial H}{\partial q}, \quad \frac{dq}{dt} = +\frac{\partial H}{\partial p} \tag{4}
\]

where \( H = H(q, p, t) \) is the Hamiltonian, which corresponds to the total energy of the system in this paper. The Hamiltonian equation establishes the framework to relate the position and the moment vectors \((p, q)\) to the total energy. So the motivation to use Hamiltonian mechanics is to describe the vibration system in which energy changes from kinetic to potential and back again over time.

**Energy Conservation Neural Network**

In this paper, an energy conservation neural network (ECNN) is proposed to learn the structural dynamics function. The network is able to learn exactly conserved quantities from data. During the forward pass, the model consumes the displacement and velocity and outputs the energy. Next, the in-graph gradient of the energy with respect to the input coordinates are automatically computed. Then, with respect to the gradient, the \( L_2 \) loss is computed and optimized.

\[
\mathcal{L}_{\text{ECNN}} = \left\| \frac{dp}{dt} + \frac{\partial E}{\partial q} \right\|_2 + \left\| \frac{dq}{dt} - \frac{\partial E}{\partial p} \right\|_2 \tag{5}
\]

The architecture comparison between the baseline neural network and ECNN is shown in Figure 2.

For an undamped free vibration, the energy input to an SDF system is controlled by the initial displacement \( u(0) \) and initial velocity \( \dot{u}(0) \). At any instant of time, the total energy in a freely vibrating system is made up of two parts, kinetic energy \( E_K \) of the mass and potential energy equal to the strain...
Figure 2: Architecture comparison between baseline neural network and energy conservation neural network. In the figure, the blocks represent layers of neural network (NN). u and v represents the earthquake wave displacement and velocity respectively. du and dv represent the derivative of u and v with respect to time.

energy $E_S$ of deformation in the spring. Considering the input energy in an undamped system, the total energy is

$$E_K(t) + E_S(t) = E_{tot}(t) = \frac{1}{2}k[u(0)]^2 + \frac{1}{2}m[\dot{u}(0)]^2$$  \hspace{1cm} (6)

Equation 6 tells us that the displacement and velocity in structural dynamics keep the total energy exactly constant over time. The mathematical framework relates the displacement and velocity of a system to its total energy $E_{tot} = G(u, \dot{u})$. This is a powerful approach because it engravels the total energy to the neural network. The model can predict the motion in an SDOF system, but its true strengths only become apparent when tackling systems with multiple degrees of freedom (MDOF).

The illustrative comparison result between ECNN and the baseline neural network is shown in Figure 3 and Figure 5. The procedures in ECNN allow the conserved quantities of total energy to be learned from the data. With the physics laws, ECNN establishes the perfect mapping from the dynamics states to their corresponding derivatives. Besides, the conservation law can be manipulated to control the total energy in the structural dynamics system.
Figure 3: Comparison of displacement and system energy between baseline neural network and energy-constant neural network.
5. Single Degree of Freedom

5.1. Damping Effects

Besides the energy conservation properties, in this section, we explore if the energy conservation properties can be manipulated to simulate the damping effects in a structural dynamics system. The exact damping effects are hard to be quantized in conventional structural dynamics [36]. The reasons are as follows.

First, the damping matrix cannot be determined by individual structural elements like the stiffness matrix because the damping properties of various construction materials are usually not well defined.

Secondly, a significant part of the damping effect is from the energy dissipated in the opening and closing of micro-cracks in concrete [37], friction at steel connections, and nonstructural elements, like heavy equipment and indoor decoration. Traditionally, classical damping, like Rayleigh or Caughey damping, are used when there are similar damping mechanisms distributed throughout the structure [38]. However, when the dynamics system consists of multiple parts with different levels of damping, classical damping is no longer appropriate. For example, in a soil-structural interaction system, the damping of the structural part is usually only one-tenth of the soil part.

ECNN provides another approach to quantize the damping effects. The damping effects are applied by controlling the system energy in ECNN, as shown in Figure 4. For each node, the predicted derivative of displacement is modified in Equation 7 with the consideration of damping effects.

\[
\frac{du}{dt} = e^{-\xi t} \left( \frac{du}{dt} \right)_{\text{original}}
\]

where \( \xi \) represents the damping effect and a bigger \( \xi \) represents a larger damping effect.

As shown in Figure 5, ECNN has 50% less phase lag error in damping effects calculation compared to the baseline neural network.
Figure 4: Damping effects in energy conservation neural network. In the figure, the system energy decreases with time due to the damping effects. In addition, the maximum displacement magnitude $u_t$ also decreases with time.

5.2. External Forces

While the damping force dissipates energy, the external force injects energy to the system. The energy dissipated by viscous damping in one cycle of harmonic vibration is

$$E_D = \int_{0}^{2\pi/\omega} f_D du = \int_{0}^{2\pi/\omega} (c\dot{u}) \dot{u} \, dt = \int_{0}^{2\pi/\omega} c\dot{u}^2 \, dt$$

$$= c \int_{0}^{2\pi/\omega} [\omega u_0 \cos(\omega t - \phi)]^2 \, dt = \pi c \omega u_0^2 = 2\pi \xi \omega_n ku_0^2$$

The energy dissipated is proportional to the square of the amplitude of motion. The energy dissipated increases linearly with excitation frequency. On the other side, an external force $p(t)$ injects energy to the system, which is

$$E_I = \int_{0}^{2\pi/\omega} p(t) \dot{u} \, dt$$

$$= \int_{0}^{2\pi/\omega} \left[ p_0 \sin \omega t \right] [\omega u_0 \cos(\omega t - \phi)] \, dt = \pi p_0 u_0 \sin \phi$$

for each cycle of vibration.
Figure 5: Comparison of displacement and phase lag between baseline neural network and energy-constant neural network.
Figure 6: The relationship between the input energy $E_I$ and the damping energy $E_D$ in a dynamic system. The system energy becomes stable when the input energy $E_I$ equals to the damping energy $E_D$.

In a steady-state vibration, the energy input to the system matches dissipated energy in viscous damping, namely $E_D = E_I$, which gives

$$u_0 = \frac{p_0}{c\omega_n}$$

The relation between the input energy and the dissipated energy is plotted in Figure 6.

Leveraging the constant energy condition in this steady-state, the ground-truth motion is as follows.

$$u_t = \frac{p_0}{k} \frac{1 - (\omega/\omega_n)^2}{[1 - (\omega/\omega_n)^2]^2 + [2\zeta(\omega/\omega_n)]^2} \sin \omega t$$

$$+ \frac{p_0}{k} \frac{-2\zeta\omega/\omega_n}{[1 - (\omega/\omega_n)^2]^2 + [2\zeta(\omega/\omega_n)]^2} \cos \omega t$$

where $p_0$ is the maximum value of the periodic force, whose frequency $\omega$ is called exciting frequency. Frequency $\omega_n$ is the natural frequency of the SDOF system, and $k$ represents the stiffness of the system, $\zeta$ represents the viscous damping of the system.

In numerical experiments, the predicted displacements using the baseline model and ECNN are shown in Figure 7. The baseline model results drift away after 3 cycles of prediction, but the ECNN model prediction stays stable across the entire experiment.
Figure 7: Comparison of displacement and error under a periodic external force between baseline neural network and energy-constant neural network.
6. Multiple Degrees of Freedom

In previous sections, structural dynamics problem is formulated for structures discretized as an SDOF system. However, in real-world applications, most buildings and structures are actually MDOF systems. Generalization of the neural network from the SDOF system to MDOF system is crucial for real-world applications.

In an MDOF system, structure states are composed of dynamics responses at all components. These responses contain both essential and redundant information about the system. To reduce the computation complexity, a selection of important targets is predicted using the baseline neural network and energy conservation neural network. The objective function of neural networks is in Equation 12.

\[
    u_{\text{target}}(\omega) = g(u_{\text{source}}(\omega), \theta(\omega))
\]

where \( \omega \) is the frequency of the motion, \( \theta \) represents the structure states, \( u_{\text{source}} \) represents the strong motion records, and \( u_{\text{target}} \) represents the target motion to predict.

The computation graph of MDOF system is illustrated in Figure 8.

Practically, in earthquake engineering, the acceleration at the top of the
structure is essential to the performance-based seismic design. The dynamics response of floors are also important to reduce the economic cost due to earthquakes. Engineers can choose the specific targets to estimate the dynamics responses at different performance levels. Compared to finite element simulation, a neural network allows engineers to explicitly control the trade-off between accuracy and performance. In most scenarios, with a few targets to predict, neural network works much faster than a full scale finite element analysis. Although the finite element method is used to generate the ground truth data in this work, neural networks can also work and be calibrated with experimental data directly without any modification.

7. Application

The application is an illustrative 3-story upper structure resides on top of the foundation and soil layers. The earthquake excitation is applied at the bottom of the model. The material types and geometries of the model are shown in Figure 10.

Domain Reduction Methods

Domain reduction method (DRM)[39] is one of the best methods to apply 3-D seismic motions to seismic numerical models. DRM has a two-stage strategy for the complex, realistic 3-D earthquake engineering simulation. The first stage is the generation of free field models with correct geology. And the second stage is the application of the generated free-field motion to the structure of interest. The DRM layer here is modeled as a single layer of elastic soil. The damping layers adjacent to DRM layer were modeled to prevent the reflected waves.

Structure Model

The upper structure consists of a 3-story building and a shallow foundation. The 3-story building consists of 2 floors of 0.3 m thickness, ceiling floor of 0.5 m thickness, the exterior wall of 0.8 m thickness, and interior walls of 0.2 m thickness. The exterior and interior walls are embedded down to the depth of the foundation. The foundation is a square shallow footing of size 160 m².
and thickness of 5.0 m. The upper building is modeled as shell elements, and foundation as linear brick elements, both having the properties of concrete of elastic Young’s modulus 30 GPa, Poisson ratio 0.22 and density 2100 kg/m$^3$. The upper structure has a natural frequency of 3.47 Hz.

**Soil Model**

The depth of the soil modeled below the foundation was 30 m, which is also the depth of DRM layer. It is assumed that within this range the soil will have plasticity because of its self-weight, structure, and seismic motions. The soil is assumed to be a stiff saturated-clay with undrained behavior having a shear velocity of 450 m/s, unit weight of 20 kPa, and the Poisson ratio of 0.15. The soil material model applies the nonlinear kinematic hardening of Armstrong Frederick [40] type. The nonlinear inelastic model was calibrated for yield strength achieved at 0.01% shear strain. The stress-strain response for the nonlinear material model is shown in Figure 9.

**Energy Dissipation**

In this MDOF application, the energy dissipation in decoupled elastic plastic material under isothermal condition [41, 42] is given by

\[
\dot{\Phi} = \sigma_{ij}\dot{\epsilon}_{ij} - \sigma_{ij}\epsilon_{ij}^{el} - \rho\dot{\psi}_{pl} \geq 0
\]  

(13)

where $\dot{\Phi}$ is the change rate of energy dissipation per unit volume, $\sigma_{ij}$ and $\epsilon_{ij}$ are the stress and strain tensors respectively, $\epsilon_{ij}^{el}$ is the elastic part of the strain tensor, $\rho$ is the mass density of the material, and $\psi_{pl}$ is the plastic free energy
Figure 10: Materials types and geometries of the simulation model.

per unit volume. Note that Equation 13 is derived from the first and second laws of thermodynamics, which represents the conditions of energy balance and nonnegative rate of energy dissipation, respectively [43, 44]. Considering all possible forms of energy, the energy balance between input mechanical work $W_{\text{Input}}$ and the combination of internal energy storage $E_{\text{Stored}}$ and energy dissipation $E_{\text{Dissipated}}$ can be expressed as

$$W_{\text{Input}} = E_{\text{Stored}} + E_{\text{Dissipated}} = E_{\text{KE}} + E_{\text{SE}} + E_{\text{PF}} + E_{\text{PD}}$$

(14)

where $E_{\text{KE}}$ is the kinetic energy, $E_{\text{SE}}$ is the elastic strain energy, $E_{\text{PF}}$ is the plastic-free-energy, and $E_{\text{PD}}$ is the energy dissipation due to material plasticity. Note that the plastic dissipation term $E_{\text{PD}}$ includes energy dissipated in both elastic plastic solids (soil) and contact elements.

Simulation and Results

The illustration results of the simulation at a time step are shown in Figure 12. For this application, the neural network is trained using earthquake records from PEER database [45]. The testing data are untrained new earth-
quake records. The training data consist of 100 earthquake records from PEER database and each record has around 2000 timesteps. In this work, the total number of earthquake timesteps used in training is around 225,000. The test data consist of 5 untrained earthquake records with around 10,000 timesteps in total. The prediction target is the dynamics response at the top of the structure, which plays an important role in the performance-based seismic design.

The detailed parameter settings of the neural network is as follows. Firstly, the input layer transforms the number of degrees of freedom to the number of hidden dimensions. Secondly, multiple fully connected hidden layers process the input data consecutively. In addition, identity shortcut connections are added that skips one or more hidden layers. Finally, the output layer transforms the data to a single scalar, which represents the system energy of the building structure. The architecture of the energy-conservation is visualized in Figure 11. Besides, orthogonal initialization is used as the initial weights. Across each layer, ReLU (rectified linear unit) \[46\] is used as the activation function. During the back-propagation, Adam is used as the optimizer with learning rate at \(10^{-3}\) and weight decay at \(10^{-4}\). To demonstrate the efficacy of the proposed model, the parameter settings and performance are extensively compared with different number of hidden layers and dimensions, as listed in Table 1 and Table 2.

Furthermore, as shown in Figure 13 and Figure 14, the training error and test error decrease with the increasing number of dimensions and layers. The increased accuracy is at the cost of a longer training time, illustrated in Figure 15.
| # Layers | # Dimensions | # Params | Baseline Error | ECNN Error |
|----------|--------------|---------|----------------|------------|
| 2        | 50           | 5k      | 2.77e-4        | 9.45e-5    |
| 2        | 200          | 82k     | 2.22e-4        | 1.03e-4    |
| 2        | 350          | 248k    | 1.13e-4        | 5.31e-5    |

Table 1: The number of model parameters and test errors with different dimensions.

| # Layers | # Dimensions | # Params | Baseline Error | ECNN Error |
|----------|--------------|---------|----------------|------------|
| 2        | 200          | 82k     | 2.22e-4        | 1.03e-4    |
| 4        | 200          | 162k    | 1.18e-4        | 3.15e-5    |
| 6        | 200          | 243k    | 1.07e-4        | 2.41e-5    |

Table 2: The number of model parameters and test errors with different number of hidden layers.

Figure 12: Illustrative deformation results of the simulation model at a time step.
Figure 13: Test error over the entire training set at the end of each epoch. Wider dimensions tend to have smaller testing error.

Figure 14: Test error over the entire training set at the end of each epoch. Testing error decreases with the increasing number of hidden layers.
8. Discussion

Energy conservation neural networks (ECNN) were proposed in this work. Instead of training on the structural dynamics output directly, ECNN uses system energy as the last layer of the neural network. The gradient of the system energy is leveraged to calculate the structure states in the next step. The proposed model takes advantage of the underlying automatic differentiation graph directly without adding extra parameters. While conventional neural networks describes the real world from data, ECNN incorporates physics principles to the neural network. The resulting method showcase a series of promising results from a fundamental single-degree-of-freedom system to a complicated multiple-degrees-of-freedom system. Statistically, the physics rule is engraved into the neural network and the solution space of the neural network is reduced. The proposed model opens the path for limiting the solution space of neural network in a physically feasible world and demonstrates the possibility to endow machine learning with the powerful capacity of mathematical physics rules in the real-world. As machine learning technology is continuing to grow fast both in terms of methodological development and hardware capability, the proposed model
can benefit practitioners across a wide range of scientific domains. However, the proposed model should not be viewed as replacements of the classical numerical methods for solving structural dynamics problems. Such methods have matured over years, which have met the robustness and efficiency requirements in practice. Nevertheless, this work favors implementation simplicity and the rapid development and testing of new ideas, potentially opening the path for the data-driven computation of structural dynamics problems.

9. Scope and Limitations

Uniqueness

Since the structural dynamics problem can be abstracted into an initial value problem, the uniqueness of the structural dynamics problem is equivalent to the uniqueness of the solution to an initial value problem. According to Picard’s existence theorem [47], the solution is unique if the differential equation is uniformly Lipschitz continuous. This theorem holds for the energy-conservation neural network, which has finite weights and uses Lipshitz nonlinearities with ReLU as the activation function.

Time-series prediction

In the SDOF system, the time-series prediction works well, as shown in Figure 3 and Figure 5. In the MDOF system, the acceleration and displacement amplitude is predicted in the frequency domain, which are the essential parameters for structural design in earthquake engineering. A proper prediction of the time-series in the MDOF system can be achieved along with structural geometry.

Error and tolerance

Keeping the error within tolerance is crucial to structural dynamics in earthquake engineering. In conventional numerical methods of structural dynamics, the error comes from time-series discretization and geometry discretization. So, conventionally, the error is controlled indirectly by time step-size and mesh size. With the neural network in this work, the error is controlled directly during
the training stage. While training error evaluates how well the neural network matches the training data, testing error shows if the pattern learned by the neural network fits new data. Besides the direct control over the training error and tolerance, the neural network allows the user to trade off speed for precision, which means that the user can choose a good enough error tolerance for the best performance.

10. Conclusion and Future Work

The classic and energy-conservation neural networks are investigated to predict the structural dynamics response for both SDOF and MDOF systems. With the learned physics laws, the predicted structural dynamics response is more accurate. Besides accuracy, the proposed neural network predicts the derivative of the structural states instead of the discrete sequence of structural states. With the predicted structural states derivatives, the actual structural response can be integrated with arbitrary time step-size, which provides the time-continuous solution. Furthermore, unlike the discretization errors in the conventional numerical methods for structural dynamics, the proposed neural network model allows the explicit control of the trade-off between computation speed and accuracy. In addition, for MDOF systems, the acceleration and displacement are predicted in the frequency domain.

For future works, although the predicted frequency-domain results in the MDOF system are essential for structural design, a time-domain prediction in the MDOF system is able to give more detailed information for the structural dynamics design. The time-domain prediction in the MDOF system can be achieved with structural geometry. In addition, the proposed approach is a new method in structural dynamics analysis. There are many opportunities to improve the performance and to solve specific application problems. On the performance side, dimensionality reduction or reduced order modeling is a popular technique in both the machine learning and structural dynamics community. Using reduced order modeling to accelerate the simulation is worth further ex-
ploration. On the application side, performance-based building design is a new engineering approach to design elements of a building upon performance goals and objectives. The proposed method in this paper can be naturally customized to fit various performance indicators as needed. Besides, from the machine learning perspective, the possibility of transfer learning on different building structures is also a promising opportunity for future study.

Last but not least, although the training data for the neural network in this paper are from finite element simulation, training data from the realistic experiments will work as well without any modification to the neural network.

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