CHROMO- AND ELECTRODYNAMICS OF HEAVY UNSTABLE PARTICLES

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Abstract

In this talk I attempt to survey some selected physics issues on radiative interference phenomena in the production of heavy unstable particles. A special emphasis is placed on the reactions $e^+e^- \rightarrow W^+W^- \rightarrow 4 \text{jets}$ and $e^+e^- \rightarrow tt \rightarrow bW^+bW^-$. 

A transparent recipe is given for quantifying the level of suppression of the interference effects in the inclusive production processes. The influence of the $W$ width on the Coulomb corrections to the threshold $W^+W^-$ production is briefly addressed.
1. Introduction

There is a common belief that in particle physics "tomorrow belongs" to the detailed studies of heavy unstable objects. Firstly, we anticipate the exciting discoveries of new heavy particles (Higgs boson(s), SUSY particles, $W'$, $Z'$,...) at increasingly higher energies. Secondly, for the precision tests of the Standard Model one needs the high accuracy determination of the parameters of the $W$ boson and of the top quark, primarily their masses.

Let us briefly address the latter point. This year we have witnessed several very important developments in precision electroweak tests. Besides the record achievements of LEP1 and SLC in measurements of the $Z^0$ parameters$^{[1-3]}$, one of the most sensational news was the direct evidence reported by the CDF collaboration for production of the top quark$^{[4]}$. Meanwhile, all the dominant loop radiative corrections to the $Z^0$-boson physics are now available$^{[5,6]}$. One can consider as an impressive success of the Standard Model the fact that the top mass $m_t$ predicted from the electroweak data agrees within the stated errors with the tentative direct CDF observation

$$m_t = 174 \pm 16 \text{ GeV}. \quad (1)$$

There has also been further progress with the determination of the $W$ boson mass $m_W$ and width $\Gamma_W$.$^{[7-9]}$ from the analysis of $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ events in $\bar{p}p$ collisions. The combined $W$ mass result gives

$$m_W = 80.23 \pm 0.18 \text{ GeV}. \quad (2)$$

The CDF collaboration has also performed a direct measurement of $\Gamma_W$.$^{[9]}$. The result is in good agreement with the indirect measurements and a Standard Model prediction. At the same time, so far no strong indication of physics beyond the Standard Model has been found, and no significant new constraint on the Higgs mass has been obtained. To fully exploit the remarkable accuracy of exploring the $Z^0$ physics the other precise measurements have to be performed. The key role here belongs to the precise determination of $m_W$ and $m_t$. This could open a new era in the analysis of electroweak data. One may hope to pin down the Higgs mass or/and to look for possible manifestations of physics beyond the Standard Model.

What are the prospects of the experimental studies? Future improvements in measurements of $m_W$ are expected from the CDF and DØdetectors at the Tevatron$^{[7,8]}$. There are hopes to reduce the $W$ mass uncertainty up to $\pm 100$ (or even $\pm 50$) MeV. The precise determination of $m_W$ is one of the main objectives of the upgrade of LEP by a factor 2 in energy, called the LEP2 program$^{[10,11]}$. With the foreseen integrated luminosity of 500 pb$^{-1}$ a statistical error of about 60 MeV per experiment is anticipated.

Let us turn to the top mass determination. The future studies at the Tevatron could increase an accuracy of $m_t$ measurements up to $\pm 5$ GeV. A unique precise
determination of $m_t$ (with an accuracy of a few hundred MeV) will be one of the most attractive physics topics at future linear $e^+e^-$ colliders\cite{12-15}.

An obvious requirement for success of these precise studies is that the accuracy of the theoretical predictions should match or better exceed the experimental errors. This requires a detailed understanding of production and decay mechanisms and, in particular, of the effects arising from the large width, $\Gamma \sim 0$ (1 GeV). Recall that in production processes of heavy unstable particles it is natural to separate the production stage from the decay processes. In general these stages are not independent and may be interconnected by radiative interference effects. Particle(s) (e.g. gluon(s) and/or photon(s)) could be produced at one stage and absorbed at another; we speak of virtual interference. Real interference will occur as well since the same real particle can be emitted from the different stages of the process.

Many observations rely on a clear understanding of the role of these interference effects. Indeed there is a long list of examples where a detailed knowledge of interferences can be important for the interpretation of experimental data (see [16-24]).

In this talk I concentrate mainly on the QCD interconnection phenomena that may occur when two unstable particles (W bosons, top quarks) decay and hadronize close to each other in space and time. The word ‘interconnection’ is here introduced to cover those aspects of final-state particle production that are not dictated by the separate decays of unstable objects, but can only be understood in terms of the joint action of the two.

Finally, a transparent recipe is presented for quantifying the level of suppression of the radiative interferences in the inclusive production of heavy unstable particles. Some particular attention is paid to the effects of the $W$ width on the QED Coulomb corrections to $W^+W^-$ production.

2. Colour Reconnection in Hadronic $W^+W^-$ Events

The preferable approach to determine the $W$ mass at LEP2 is the method based on the direct kinematical reconstruction of $m_W$ in fully hadronic events\cite{10}. QCD interconnection effects between the $W^+$ and $W^-$ are of special practical interest here because they undermine the traditional meaning of a $W$ mass in the process

$$e^+e^- \rightarrow W^+W^- \rightarrow q_1\bar{q}_2q_3\bar{q}_4.$$ \hfill (3)

Specifically, it is not even in principle possible to subdivide the final state into two groups of particles, one of which is produced by the $q_1\bar{q}_2$ system of the $W^+$ decay and the other by the $q_3\bar{q}_4$ system of the $W^-$ decay: some particles originate from the joint action of the two systems. Therefore, it is important to understand how large the ambiguities can be.\footnote{Despite its evident shortcomings (large missing momentum) the mixed leptonic-hadronic channel is free from the potential reconnection-induced ambiguities.} A systematic analysis of QCD rearrangement phenomena in hadronic $W^+W^-$ events has been performed in [23] which we follow here.
The perturbative aspects of QCD interference phenomena could be complex but, in principle, are well controllable. However, a complete description of these effects is not possible because of the lack of any deep understanding of the non-perturbative hadronization process. Here one has to rely on the model predictions rather than on exact calculations. The concept of colour reconnection (rearrangement/recoupling) is useful to quantify the interference effects (at least in a first approximation). In a reconnection two original colour singlets (such as $q_1\bar{q}_2$ and $q_3\bar{q}_4$) are transmuted into two new ones (such as $q_1\bar{q}_4$ and $q_3\bar{q}_2$). Subsequently each singlet system is assumed to hadronize independently according to the standard algorithms, which have been so successful in describing e.g. $Z^0$ decays. Depending on whether a reconnection has occurred or not, the hadronic final state is then going to be somewhat different.

The colour reconnection effects were first discussed by Gustafson, Pettersson and Zerwas\cite{25}, but their results were mainly qualitative and were not targeted on what might actually be expected at LEP2. Their picture represents an example of the so-called instantaneous reconnection scenario, where the alternative colour singlets are immediately formed and allowed to radiate perturbative gluons. Further detailed analysis was performed in Refs. [23,26] with the emphasis on LEP2 studies.

In order to understand which QCD interference effects can occur in hadronic $W^+W^-$ decays, it is useful to examine the space-time picture of the process. Consider a typical c.m. energy of 170 GeV, a $W$ mass $m_W = 80.2$ GeV and width $\Gamma_W = 2.07$ GeV. Then a mean separation of the two decay vertices in space and in time is of order 0.1 fm. A gluon with an energy $\omega \gg \Gamma_W$ therefore has a wavelength much smaller than the separation between the $W^+$ and $W^-$ decay vertices, and is emitted almost incoherently either by the $q_1\bar{q}_2$ system or by the $q_3\bar{q}_4$ one\cite{17}. Only fairly soft gluons, $\omega \lesssim \Gamma_W$, feel the joint action of all four quark colour charges. On the other hand, the typical distance scale of hadronization is about 1 fm, i.e. much larger than the decay vertex separation. Therefore the hadronization phase may contain significant interference effects. In the following, we will first discuss perturbative effects and subsequently non-perturbative ones.

Until recently, perturbative QCD has mainly been applied to systems of primary partons produced almost simultaneously. The radiation accompanying such a system can be represented as a superposition of gauge-invariant terms, in which each external quark line is uniquely connected to an external antiquark line of the same colour. The system is thus decomposed into a set of colourless $q\bar{q}$ antennae/dipoles\cite{27}. One of the simplest examples is the celebrated $q\bar{q}g$ system, which (to leading order in $1/N_C^2$, where $N_C = 3$ is the number of colours) is well approximated by the incoherent sum of two separate antennae, $\bar{q}g$ and $\bar{q}\bar{q}$. These dipoles radiate gluons, which within the perturbative scenario are the principal sources of multiple hadroproduction.

Neglecting interferences, the $e^+e^- \rightarrow W^+W^- \rightarrow q_1\bar{q}_2 q_3\bar{q}_4$ final state can be subdivided into two separate dipoles, $\hat{q}_1\bar{q}_2$ and $\hat{q}_3\bar{q}_4$. Each dipole may radiate gluons from a maximum scale $m_W$ downwards. Within the perturbative approach, colour transmutations can result only from the interference between gluons (virtual as well
as real) radiated in the $W^+$ and $W^-$ decays. A colour reconnection then corresponds to radiation, e.g. from the dipoles $q_1\bar{q}_4$ and $q_3\bar{q}_2$. The emission of a single primary gluon cannot give interference effects, by colour conservation, so interference terms only enter in second order in $\alpha_s$.

Thus, at least two primary gluons, real or virtual, should be emitted to generate a colour flow rearrangement, see Figs. 1-3. Note that the diagrams of Figs. 1a and 1b do not interfere with each other, and that the diagrams of Fig. 2 could interfere with those with single gluon emission, thus inducing a colour transmutation. The infrared divergences in the virtual pieces are cancelled by the corresponding real emissions. For the case of decay-decay radiative interference the soft emissions are cancelled in the inclusive cross section up to at least $\mathcal{O}(\Gamma_W/m_W)$ (see Refs. 19,20 and Section 4).

The main qualitative results for the reconnection effects appearing in $\mathcal{O}(\alpha_s^2)$ are not much different for various decay-decay interference samples. We shall examine below one example corresponding to the diagrams of Fig. 1a. Let us label the momenta of the final state quarks by $e^+ e^- \rightarrow q_1(p_1)\bar{q}_2(p_2)q_3(p_3)\bar{q}_4(p_4)$ with $Q_1 = p_1 + p_2$ and $Q_2 = p_3 + p_4$. In the limit $k_1, k_2 \ll p_i$ after summing over colours and spins, the interference term may be presented in the form

$$\frac{1}{\sigma_0} d\sigma^\text{int} \sim \frac{d^3k_1 d^3k_2}{\omega_1 \omega_2} \left( \frac{C_F \alpha_s}{4\pi^2} \right)^2 \frac{1}{N^2_C - 1} \chi_{12} H(k_1)H(k_2),$$

where $C_F = (N^2_C - 1)/(2N_C) = 4/3$. We proceed to comment on the non-trivial factors in this expression.

The radiation pattern $H(k)$ is given by

$$H(k) = \hat{q}_1\hat{q}_4 + \hat{q}_3\hat{q}_2 - \hat{q}_1\hat{q}_3 - \hat{q}_2\hat{q}_4,$$

where the radiation antennae are\[^{[27]}\]

$$\hat{ij} = \frac{(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)}.$$

The so-called profile function $\chi_{12}$\[^{[16,17]}\] controls decay-decay radiative interferences

$$\chi_{12} = \left( \frac{m_W \Gamma_W}{\pi} \right) \mathcal{R} \int dQ_1^2 dQ_2^2 D(Q_1 + k_1)D^*(Q_1 + k_2)D(Q_2 + k_2)D^*(Q_2 + k_1).$$

Here $D(Q)$ is the propagator function

$$D(Q) = \frac{1}{Q^2 - m_W^2 + i\epsilon},$$

$D^*$ is the complex conjugate of $D$ and $\mathcal{R}$ represents the real part. The profile function has the formal property that $\chi_{12} \rightarrow 0$ as $\Gamma_W \rightarrow 0$ and $\chi_{12} \rightarrow 1$ as $\Gamma_W \rightarrow \infty$. 

The interference is suppressed by $1/(N_C^2 - 1) = 1/8$ as compared to the total rate of double primary gluon emissions. This is a result of the ratio of the corresponding colour traces,

$$\frac{Tr(T^aT^b) \cdot Tr(T^aT^b)}{Tr(T^aT^a) \cdot Tr(T^bT^b)} = \frac{(C_F N_C)/2}{(C_F N_C)^2} = \frac{1}{N_C^2 - 1}.$$  (9)

Such a suppression takes place for any decay-decay radiative interference piece, real as well as virtual, as is clear from Fig. 3.

Near threshold and in the limit of massless quarks the interference contribution to the radiation pattern is

$$F_{int}^a = \frac{2\chi_{12}}{\omega_1^2 \omega_2^2 \sin\theta_1 \sin\theta_3 \sin\bar\theta_1 \sin\bar\theta_3},$$  (10)

where $\theta_i(\bar\theta_i)$ is the angle between the $q_i$ and the gluon $k_1(k_2)$, and $\phi_{13}(\bar\phi_{13})$ is the relative azimuth between $q_1$ and $q_3$ around the direction of the $k_1(k_2)$. The expression in Eq. (10) evidently contains a dependence on the relative orientation of the decay products of the two $W$’s. (The interference is maximal when all the partons lie in the same plane, $\phi_{13} = \bar\phi_{13} = 0$.) Therefore one might expect that the decay-decay interferences would induce some colour-suppressed reconnection effects in the structure of final states in process (3).

It is the profile function $\chi_{12}$ that cuts down the phase space available for gluon emissions by the alternative quark pairs (or by any accidental colour singlets) and thus eliminates the very possibility for the reconnected systems to develop QCD cascades. That the $W$ width does control the radiative interferences can be easily understood by considering the extreme cases.

If the $W$-boson lifetime could be considered as very short, $1/\Gamma_W \to 0$, both the $q_1\bar q_2$ and $q_3\bar q_4$ pairs appear almost instantaneously, and they radiate coherently, as though produced at the same vertex. In the other extreme, $\Gamma_W \to 0$, the $q_1\bar q_2$ and $q_3\bar q_4$ pairs appear at very different times $t_1, t_2$ after the $W^+W^-$ production,

$$\tau_p \sim \frac{1}{m_W} \ll \Delta t = |t_1 - t_2| \sim \frac{1}{\Gamma_W}.$$  (11)

The two dipoles therefore radiate gluons and produce hadrons according to the no-reconnection scenario.

The crucial point is the proper choice of the scale the $W$ width should be compared with. That scale is set by the energies of primary emissions, real or virtual 17,20. Let us clarify this supposing, for simplicity, that we are in the $W^+W^-$ threshold region. The relative phases of radiation accompanying two $W$ decays are then given by the quantity

$$\omega_i \Delta t \sim \frac{\omega_i}{\Gamma_W}.$$  (12)

When $\omega_i/\Gamma_W \gg 1$ the phases fluctuate wildly and the interference terms vanish. This is a direct consequence of the radiophysics of the colour flows 27 reflecting the wave
dynamics of QCD. The argumentation remains valid for energies above the $W^+W^-$ threshold as well.

An instructive Gedanken experiment to highlight the filtering role of $\Gamma$ can be obtained by comparing the emission of photons in the eV to MeV range for the two processes

$$\gamma\gamma \rightarrow W^+W^- \rightarrow \mu^+\nu_\mu\mu^-\bar{\nu}_\mu, \quad (13)$$

$$\gamma\gamma \rightarrow K^+K^- \rightarrow \mu^+\nu_\mu\mu^-\bar{\nu}_\mu, \quad (14)$$

near threshold, in the extreme kinematical configuration where the $\mu^+$ is collinear with the $\mu^-$. For the first process, $\omega \ll \Gamma_W$, and one expects hardly any radiation at all, because of the complete screening of the two oppositely charged muons. For the second process, $\omega \gg \Gamma_K$, the parent particles have long lifetimes and the $\mu^+$ and $\mu^-$ appear at very different times. The photon wavelength is very small compared with the size of the $\mu^+\mu^-$ dipole and, therefore, the $\mu^+$ and $\mu^-$ radiate photons independently, with no interference.

Suppression of the interference in the case of radiation with $\omega_i \gg \Gamma_W$ can be demonstrated also in a more formal way.

One can perform the integration over $dQ_1^2$ and $dQ_2^2$ in eq. (7) by taking the residues of the poles in the propagators. This gives

$$\chi_{12} = \frac{m_W^2\Gamma_W^2(\kappa_1\kappa_2 + m_W^2\Gamma_W^2)}{(\kappa_1^2 + m_W^2\Gamma_W^2)(\kappa_2^2 + m_W^2\Gamma_W^2)}, \quad (15)$$

with

$$\kappa_{1,2} = Q_{1,2} \cdot (k_1 - k_2). \quad (16)$$

For the interference between the diagrams of Fig. 1b, the corresponding profile function is given by the same formula with $k_2 \rightarrow -k_2$. Near the $W^+W^-$ pair threshold Eq. (15) is reduced to

$$\chi_{12} = \frac{\Gamma_W^2}{\Gamma_W^2 + (\omega_1 - \omega_2)^2}. \quad (17)$$

From Eq. (17) it is clear that only primary emissions with $\omega_{1,2} \ll \Gamma_W$ can induce significant rearrangement effects: the radiation of energetic gluons (real or virtual) with $\omega_{1,2} \gg \Gamma_W$ pushes the $W$ propagators far off their non-radiative resonant positions, so that the propagator functions $D(Q_1 + k_1)$ and $D(Q_1 + k_2)(D(Q_2 + k_1)$ and $D(Q_2 + k_2))$ corresponding to the same $W$ practically do not overlap. We can neglect the contribution to the inclusive cross section from kinematical configurations with $\omega_1, \omega_2 \gg \Gamma_W$, $|\omega_1 - \omega_2| \ll \Gamma_W$ since the corresponding phase-space volume is negligibly small. The possibility for the reconnected systems to develop QCD cascades is thus reduced, i.e. the dipoles are almost sterile. Other interferences (real or virtual) are described by somewhat different expressions, e.g. with $\omega_1 - \omega_2 \rightarrow \omega_1 + \omega_2$, but have the same general properties. Eq. (15) clearly shows that $\chi_{12}$ vanishes if any of the
scalar products $Q_i \cdot k_j (i, j = 1, 2)$ well exceeds $m_W \Gamma_W$. Again accidental kinematics with $\kappa_1, \kappa_2 \ll m_W \Gamma_W$ is suppressed because of phase space reasons. Hence all our arguments concerning cutting down the QCD cascades induced by the alternative systems remain valid above the threshold as well. The smallness of the decay-decay radiative interference for energetic emission in the production of a heavy unstable particle pair, at the threshold and far above it, proves to be of a general nature. For the case of $e^+e^- \to bW^+\bar{b}W^-$ this was explicitly demonstrated in [16] (see also Section 3).

From the antenna pattern given by Eq. (5) one immediately sees that, in addition to the two dipoles $\hat{q}_1\bar{q}_4$ and $\hat{q}_3\bar{q}_2$, which may be interpreted in terms of reconnected colour singlets, two other terms, $\hat{q}_1\bar{q}_3$ and $\hat{\bar{q}}_2\bar{q}_4$ appear. These terms are intimately connected with the conservation of colour currents. Moreover, the $\hat{q}_1\bar{q}_3$ and $\hat{\bar{q}}_2\bar{q}_4$ antennae come in with a negative sign. In general, QCD radiophysics predicts both attractive and repulsive forces between quarks and antiquarks, see [27-29]. Normally the repulsion effects are quite small, but in the case of colour-suppressed phenomena they may play an important role.

One can elucidate the physical origin of the attraction and repulsion effects by examining the photonic interference pattern in

$$\gamma\gamma \to Z^0 Z^0 \to e^+e^-\mu^+\mu^-, \quad (18)$$

(see [30]). In addition to the attractive forces between opposite electrical charges ($e^-\mu^+$ and $e^+\mu^-$ QED-antennae) there is a negative-sign contribution ($e^-\mu^-$ and $e^+\mu^+$ QED-antennae) corresponding to the repulsive forces between two same-sign charges.

It should be emphasized that within the perturbative picture, analogously to other colour-suppressed interference phenomena [27-29], rearrangement can be viewed only on a completely inclusive basis, when all the antennae are simultaneously active in the particle production. The very fact that the reconnection pieces are not positive-definite reflects their wave interference nature. Therefore the effects of reconnected almost sterile cascades should appear on top of a dominant background generated by the ordinary-looking no-reconnection dipoles $\hat{q}_1\bar{q}_2$ and $\hat{\bar{q}}_3\bar{q}_4$.

Summing up the above discussion, it can be concluded that perturbative colour reconnection phenomena are suppressed, firstly because of the overall factor $\alpha_s^2/(N_C^2 - 1)$, and secondly because the rearranged dipoles can only radiate gluons with energies $\omega \lesssim \Gamma_W$. Only a few low-energy particles should therefore be affected, $\Delta N_{\text{recon}}/N_{\text{no-recon}} \lesssim \mathcal{O}(10^{-2})$.

Having demonstrated that perturbative rearrangement is very small we now turn to the possibility of reconnection occurring as a part of the non-perturbative fragmentation phase. Since hadronization is not well understood, this requires model building. In [23] the Standard Lund string fragmentation model [31] was used as a starting point, but it was considerably extended. \footnote{\textsuperscript{3}This choice, by no means, was dictated by a tendency to ignore the other successful hadroniza-}
Recall that the string description is entirely probabilistic, i.e. any negative-sign interference effects are absent. This means that the original colour singlets $q_1\bar{q}_2$ and $q_3\bar{q}_4$ may transmute to new singlets $q_1\bar{q}_4$ and $q_3\bar{q}_2$, but that any effects, e.g., of the $q_1q_3$ and $\bar{q}_2\bar{q}_4$ dipoles (cf. Eq. (5)) are absent. In this respect, the non-perturbative discussion is more limited in outlook than the perturbative one above. This does not necessarily mean that there is a physics conflict between the two pictures: one should remember that the perturbative approach describes short-distance phenomena, where partons may be considered free to first approximation, while the Lund string picture is a model for the long-distance behaviour of QCD, where confinement effects should lead to a subdivision of the full system into colour singlet subsystems (ultimately hadrons) with screened interactions between these subsystems.\(^4\)

The imagined time sequence is the following (for details see [30]). The $W^+$ and $W^-$ fly apart from their common production vertex and decay at some distance. Around each of these decay vertices, a perturbative parton shower evolves from an original $q\bar{q}$ pair. The typical distance that a virtual parton (of mass $m \sim 10$ GeV, say, so that it can produce separate jets in the hadronic final state) travels before branching is comparable with the average $W^+W^-$ separation, but shorter than the fragmentation time. Each $W$ can therefore effectively be viewed as instantaneously decaying into a string spanned between the partons, from a quark end via a number of intermediate gluons to the antiquark end. The strings expand, both transversely and longitudinally. They eventually fragment into hadrons and disappear. Before that time, however, the string from the $W^+$ and the one from the $W^-$ may overlap. If so, there is some probability for a colour reconnection to occur in the overlap region. The fragmentation process is then modified.

The Lund string model does not constrain the nature of the string fully. At one extreme, the string may be viewed as an elongated bag, i.e. as a flux tube without any pronounced internal structure. At the other extreme, the string contains a very thin core, a vortex line, which carries all the topological information, while the energy is distributed over a larger surrounding region. The latter alternative is the chromo-electric analogue to the magnetic flux lines in a type II superconductor, whereas the former one is more akin to the structure of a type I superconductor. One can use them as starting points for two contrasting approaches, with nomenclature inspired by the superconductor analogy.

In scenario I, the reconnection probability $P_{recon}$ is proportional to the space-time volume over which the $W^+$ and $W^-$ strings overlap, with strings assumed to have

\(^4\text{However, in my view, the lack of understanding of how to handle the negative sign antenna pieces casts some shadow on the reliability of the probabilistic descriptions. I prefer to consider the latter as a reasonable qualitative guide allowing one to estimate the size of the interconnection effects rather than the complete predictions.}\)
transverse dimensions of hadronic size. In scenario II it is assumed that reconnections can only take place when the core regions of two string pieces cross each other. This means that the transverse extent of strings can be neglected, which leads to considerable simplification.

Both scenarios were implemented in a detailed simulation of the full process of $W^\pm$ production and decay, parton shower evolution and hadronization\cite{32}. It is therefore possible to assess any experimental consequences for an ideal detector.

The reconnection probability is predicted in scenario II without any adjustable parameters, although with the possibility to vary the baseline model in a few respects. Scenario I contains a completely free strength parameter $k_1$. It was chosen to give an average $P_{\text{recon}} \approx 0.35$ at 170 GeV, as is predicted in scenario II.

An instructive issue is related to the energy dependence of the reconnection phenomena. At first glance, one might expect that these “undesirable” effects “go away” with an increase of the threshold energy. However, as was demonstrated in [23] the resulting c.m. energy dependence of $P_{\text{recon}}$ in the whole LEP2 region is very slow: between 150 and 200 GeV the variation is less than a factor of 2. Here it is useful to remember that the $W^\pm$ are never produced at rest with respect to each other: the naive Breit-Wigner mass distributions are distorted by phase-space effects, which favour lower $W$ masses.\footnote{Similar effects appear in the momentum distribution of the top quarks in the threshold region (see [33]). Here they are additionally modified by the final state QCD interactions.}

For 150-200 GeV the average momentum of each $W$ is therefore in the range 22-60 GeV, rather than in the range 0-60 GeV, see Fig. 4. It is largely this momentum that indicates how fast the two $W$ system are flying apart, and therefore how much they overlap in the middle of the event. Also the energy variation in the perturbative description is very small. If we want to call colour reconnection a threshold effect, we have to acknowledge that the threshold region is very extended.

Let us turn now to the $W$ mass determination at LEP2. Experimentally, $m_W$ depends in a non-trivial fashion on all particle momenta of an event. Errors in the $W$ mass determination come from a number of sources\cite{10,23} which we do not intend to address here. Therefore, we only study the extent to which the average reconstructed $W$ mass is shifted when reconnection effects are added, but everything else is kept the same. Even so, results do depend on the reconstruction algorithm used. In [23] we have tried a few different ones, which, however, all are based on the same philosophy: a jet finder is used to define at least four jets, events with two very nearby jets or with more than four jets are rejected, the remaining jets are paired to define the two $W$’s, and the average $W$ mass of the event is calculated. Events where this number agrees to better than 10 GeV with the input average mass are used to calculate the systematic mass shift.

In scenario I this shift is consistent with being zero, within the 10 MeV uncertainty in our results from limited Monte Carlo statistics (160,000 events per scenario). Scenario II gives a negative mass shift, of about -30 MeV; this also holds for several
variations of the basic scheme. A simpler model, where reconnections are always assumed to occur at the centre of the event, instead gives a positive mass shift: about $+30$ MeV if results are rescaled to $P_{\text{recon}} \approx 0.35$. We are, therefore, forced to conclude that not even the sign of the effect can be taken for granted, but that a real uncertainty of $\pm30$ MeV does exist from our ignorance of non-perturbative reconnection effects.

To estimate the size of perturbative rearrangement, one can use a scenario where the original $q_1\bar{q}_2$ and $q_3\bar{q}_4$ dipoles are instantaneously reconnected to $q_1\bar{q}_4$ and $q_3\bar{q}_2$ ones, and these are allowed to radiate gluons with an upper cut-off given by the respective dipole invariant mass$^{[25]}$. This gives a mass shift by about $+500$ MeV. We have above argued that real effects would be suppressed by at least a factor of $10^{-2}$ compared to this, and thus assign a 5 MeV error from this source. Finally, the possibility of an interplay between the perturbative and non-perturbative phases must be kept in mind. We believe it will not be much larger than the perturbative contribution, and thus assign a further 5 MeV from this source. The numbers are added linearly to get an estimated total uncertainty of 40 MeV.

In view of the aimed-for precision, 40 MeV is non-negligible. However, remember that as a fraction of the $W$ mass itself it is a half a per mille error. Reconnection effects are therefore smaller in the $W$ mass than in many other observables, such as the charged multiplicity. This is good news. Otherwise, LEP2 would not have significant advantages in the measurements of $m_W$ over hadronic colliders where the accuracy is steadily improving.

In total, our conservative estimate of the systematic uncertainty on the $W$ mass would thus be a number roughly like 40 MeV from the reconnection phenomenon alone. It may well be the largest individual source of systematic error for $W$ mass determinations in doubly hadronic $W^+W^-$ decays.

We cannot today predict what will exactly come out of the forthcoming studies and the results of [23] might form only a starting point for future activity$^6$. First of all, it is important to study how sensitive experimental mass reconstruction algorithms are, and not just rely on the numbers in [23]. We believe that the uncertainty can be reduced by a suitable tuning of the algorithms, e.g. with respect to the importance given to low-momentum particles (for which the detection efficiency may be limited) and with respect to the statistical treatment of the wings of the $W$ mass distribution.

There is another challenging reason to study the phenomenon of colour rearrangement in hadronic $W^+W^-$ events. As was first emphasised in [25], it could provide a new laboratory for probing the non-perturbative QCD dynamics. The very fact that different models for the colour string give different predictions means that it might be possible to learn about the structure of the QCD vacuum. For example, one may hope to distinguish scenarios I and II by exploiting the difference in the sensitivity of the reconnection to the event topology$^{[23,26]}$.

Unfortunately to make any progress at all, we have had to rely on models and

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$^6$A new analysis based on these results is now performed by the DELPHI collaboration$^{[11]}$. 

10
approximations that are far from perfect. There is a true limit to our current physics understanding. One unresolved problem concerns an evident breakdown of the exclusive probabilistic interpretation of the colour-suppressed interference effects (see [23] and the discussion above). Another open question addresses the role of an interplay between the perturbative and non-perturbative phases\[30\].

Let us now come to the issue of experimental observability of the reconnection. An analysis in [23] concerned mainly the standard global event measures where effects seem to be very small. The change in the average charged multiplicity is expected at the level of a percent of less, and similar statements hold for rapidity distributions, thrust distributions, and so on. This is below the experimental precision one may expect at LEP2, and so may well go unobserved.

There are some other potentially promising approaches, e.g. comparison of the event properties in fully hadronic and mixed leptonic-hadronic decays. An interesting vista on the reconnection problem is connected with Bose-Einstein effects\[30\].

A high-statistics run above the $Z^0$ threshold would allow an unambiguous determination of any systematic mass shift, given that the $Z^0$ mass is already known from LEP1 with a record accuracy ($\pm 4$ MeV). If the various potential sources of systematic error could be disentangled, it could also imply a direct observation of reconnection effects. More generally, $Z^0$ events from LEP1 can be used to predict a number of properties for $Z^0 Z^0$ events, such as the charged multiplicity distribution. Any sign of deviations would provide important information on the reconnection issue.

A new proposal attempting to disentangle the recoupling phenomenon is discussed in [26].\[6\] It is advocated there that the dynamical effects could enhance the reconnection probability for configurations which correspond to so-called short strings producing few hadrons. It is predicted that with 10% probability for recoupling the reconnected events can be experimentally identified.

In my view, in order to pin down the reconnection in real-life experiments some further efforts are needed. Besides all the theoretical uncertainties, the background issue should be addressed more carefully. For example, the short string states could be generated also by the conventional $e^+ e^- \rightarrow 4$ parton events (e.g. the so-called rapidity gap events\[34\]). This could provide a natural lower limit for a recoupling-type signal.

Finally, one may anticipate that the reconnection-type effects might appear also in $Z^0$ decay events. They would be induced, e.g. by some subsets of gluons in the colour singlet states. It could be quite instructive to test the predictions of different models for recoupling in hadronic $Z^0$ events, where one can benefit from the huge statistics of LEP1.

3. Correlations of Particle Flow in Top Events

One of the main objectives of a future linear $e^+ e^-$ collider will be to determine...
the top mass $m_t$ with high accuracy. Besides the traditional measurements of the $tt$ excitation curve, several other approaches are discussed\cite{21,22,35,36}. One method is to reconstruct the top invariant mass event by event, another is to measure the top momentum distribution\cite{33,35}. In either case, the QCD interconnection effects could introduce the potentiality for a systematic bias in the top mass determination.

It is not my intention to go here through all the details of the problem. As a specific topical example, we consider the production and decay of a $tt$ pair in the process

$$e^+e^- \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$$

(19)

and concentrate on the possible manifestations of the interconnection effects in the distribution of the particle flow in the final state (for details see \cite{24}). For simplicity we assume that the $W$’s decay leptonically, so the colour flow is generated only by the $t$ and $b$ quarks. Further, we restrict ourselves to the region a few GeV above the $tt$ threshold to exemplify the size of effects. The radiation pattern is especially simple in this region.

Recall that the dominance of the $t \rightarrow bW^+$ decay mode leads to a large top width $\Gamma_t$, which is about 1.3 GeV for a canonical mass $m_t \simeq 170$ GeV. This width is larger than the typical hadronic scale $\mu \sim 1 \text{ fm}^{-1}$, and the top decays before it have time to hadronize\cite{37,38}. It is precisely the large width that makes top physics so unique. Firstly, the top decay width $\Gamma_t$ provides an infrared cut-off for the strong forces between the $t$ and $\bar{t}$\cite{39-41}. The width $\Gamma_i$ acts as a physical ‘smearing’\cite{42}, and the top production becomes a quantitative prediction of perturbative QCD, largely independent of non-perturbative phenomenological algorithms. Secondly, $\Gamma_t$ controls the QCD interferences between radiation occurring at different stages of the $tt$ production processes\cite{16-18}. These interferences affect the structure of the colour flows in the $tt$ events and may provide a potentially serious source of uncertainties in the reconstruction of the final state.

The interplay of several particle production sources is reminiscent of the colour rearrangement effects we have studied for process (3), but there are important differences. From the onset, $W^+W^-$ events consist of two separate colour singlets, $q_1\bar{q}_2$ and $q_3\bar{q}_4$, so that there is no logical imperative of an interplay between the two. Something extra has to happen to induce a colour rearrangement to $q_1\bar{q}_4$ and $q_3\bar{q}_2$ singlets, such as a perturbative exchange of gluons or a non-perturbative string overlap. This introduces a sizeable dependence on the space-time picture, i.e. on how far separated the $W^+$ and $W^-$ decay vertices are. The process (19) only involves one colour singlet. Therefore an interplay is here inevitable, while a colour rearrangement of the above kind is impossible. Recall also that, contrary to the $WW$ case, there are no purely leptonic channels which could provide a reconnection-free environment. Analogously to the $W^+W^-$ case we expect that the perturbative restructuring is suppressed because of the space-time separation between the decays of the $t$ and $\bar{t}$ quarks. However, a priori there is no obvious reason why interconnection effects have to be small in the fragmentation process. Moreover, the $b$ and $\bar{b}$ coming from the top decays carry
compensating colour charges and therefore have to ‘cross-talk’ in order to produce a final state made up of colourless hadrons\textsuperscript{[24,43]}.

Let us start from the perturbative picture. In the process (19) the standard parton showering can be generated by the systems of quarks appearing within a short time scale, namely the $\hat{t}\bar{t}$, $\hat{t}b$ and $\hat{b}\bar{t}$ antennae/dipoles. In the absence of interferences these antennae do not interact and the particle flows are not rearranged.

As was discussed in [16,17], the energy range of primary gluons, real or virtual, generated by the alternative quark systems of the type $\hat{t}\bar{b}$, $\hat{b}t$ and $\hat{b}\bar{b}$ is strongly restricted. Not so far from the $t\bar{t}$ threshold one expects $\omega \lesssim \omega_{\text{max}}^{\text{int}} \sim \Gamma_t$. Therefore the would-be parton showers initiated by such systems are almost sterile, and can hardly lead to a sizeable restructuring of the final state. In other words, the width of an unstable particle acts as a kind of filter, which retains the bulk of the radiation (with $\omega > \Gamma_t$) practically unaffected by the relative orientation of the daughter colour charges.

The general analysis of soft radiation in process (19) in terms of QCD antennae was presented in [16]. Here we focus on the emission close to the $t\bar{t}$ threshold.

The primary-gluon radiation pattern can be presented as:

\[
dN_g \equiv \frac{d\sigma_g}{\sigma_0} = \frac{d\omega}{\omega} \frac{d\Omega}{4\pi} C_F\alpha_s \pi I,
\]

where $\Omega$ denotes the gluon solid angle; $I$ is obtained by integrating the absolute square of the overall effective colour current over the virtualities of the $t$ and $\bar{t}$.

Near threshold the contribution from the $\hat{t}\bar{t}$ antenna may be neglected, and the $\hat{t}b$ and $\hat{b}\bar{t}$ antennae are completely dominated by the emission off the $b$ quarks. The distribution $I$ may then be presented in the form

\[
I = I_{\text{indep}} + I_{\text{dec-dec}}.
\]

Here $I_{\text{indep}}$ describes the case when the $b$ quarks radiate independently and $I_{\text{dec-dec}}$ corresponds to the interference between radiation accompanying the decay of the top and of the antitop

\[
I_{\text{dec-dec}} = 2\chi(\omega) \frac{\cos\theta_1 \cos\theta_2 - \cos\theta_{12}}{(1 - v_b \cos\theta_1)(1 - v_b \cos\theta_2)}.
\]

Here $\theta_1(\theta_2)$ is the angle between the $b(\bar{b})$ and the gluon, $\theta_{12}$ is the angle between the $b$ and $\bar{b}$ and $\chi(\omega)$ is the profile function [16], which controls the radiative interferences between the different stages of process (19).

Near threshold

\[
\chi(\omega) = \frac{\Gamma_t^2}{\Gamma_t^2 + \omega^2}
\]

(cf. Eq. (17)).
Analogously to the $W^+W^-$ case, the profile function $\chi(\omega)$ cuts down the phase space available for emissions by the alternative quark systems and, thus, suppresses the possibility for such systems to develop QCD cascades. As $\Gamma_t \to \infty$, the top lifetime becomes very short, the $b$ and $\bar{b}$ appear almost instantaneously, and they radiate coherently, as though produced directly. In particular, gluons from the $b$ and $\bar{b}$ interfere maximally, i.e. $\chi(\omega) = 1$. At the other extreme, for $\Gamma_t \to 0$, the top has a long lifetime and the $b$ and $\bar{b}$ appear in the course of the decays of top-flavoured hadrons at widely separated points in space and time. They therefore radiate independently, $\chi(\omega) = 0$. Thus a finite top width suppresses the interference compared to the naive expectation of fully coherent emission. The same phenomena appear for the interference contributions corresponding to virtual diagrams. The infrared divergences induced by the unobserved gluons are cancelled when both real and virtual emissions are taken into account.

The bulk of the radiation caused by primary gluons with $\omega > \Gamma_t$ is governed by the $\hat{t}b$ and $\hat{\bar{t}}\bar{b}$ antennae. It is thus practically unaffected by the relative orientation of the $b$ and $\bar{b}$ jets. In particular, the $\hat{b}\bar{b}$ antenna is almost inactive. The properties of individual $b$ jets are understood well enough, thanks to our experience with $Z^0 \to b\bar{b}$ at LEP1.

Because of the suppression of energetic emission associated with the interferences, the restructuring could affect only a few particles.

Interconnection phenomena could affect the final state of $t\bar{t}$ events in many respects, but multiplicity distributions are especially transparent to interpret. As a specific example, we examined in Ref. [24] the total multiplicity of double leptonic top decays as a function of the relative angle between the $b$ and $\bar{b}$ jets. Let us make some comments concerning the basic ideas of these studies:

1. As usual, one needs to model the fragmentation stage and study quantities accessible at the hadron level. This offers one advantage: the fragmentation has many similarities with the $\omega \to 0$ limit of the perturbative picture, and thus tends to enhance non-trivial angular dependences.

2. A complication of attempting a full description is that it is no longer enough to give the rate of primary-gluon emission, as in Eq. (20): one must also allow for secondary branchings and specify the colour topology and fragmentation properties of radiated partons. It is then useful to benefit from the standard parton shower plus fragmentation picture for $e^+e^- \to \gamma^*/Z^0 \to q\bar{q}$, where these aspects are understood, at least in the sense that much of our ignorance has been pushed into experimentally fixed parameters.

3. The relation between $\gamma^*/Z^0 \to q\bar{q}$ and $t\bar{t} \to bW^+\bar{b}W^-$ is most easily formulated in the antenna/dipole language. The independent emission term corresponds to the sum of two dipoles, $I_{\text{indep}} \propto \hat{t}b + \hat{\bar{t}}\bar{b}$, while the decay-decay interference
one corresponds to $I_{\text{dec-dec}} \propto \chi(\omega)(\hat{b}\bar{b} - \hat{t}\bar{b} - \hat{b}\bar{b})$. In total, therefore,

$$I \propto (1 - \chi(\omega))\hat{t}\bar{b} + (1 - \chi(\omega))\hat{b}\bar{b} + \chi(\omega)\hat{b}\bar{b}. \quad (24)$$

Each term here is positive definite and can be translated into a recipe for parton shower evolution: starting from the respective normal branching picture, each potential primary branching $q \rightarrow qg$ or $\bar{q} \rightarrow \bar{q}g$ is assigned an additional weight factor $1 - \chi(\omega)$ or $\chi(\omega)$. This factor enters the probability that a trial branching will be retained. For the rest, the same evolution scheme can be used as for $\gamma^* / Z^0 \rightarrow q\bar{q}$, including the choice of evolution variable, $\alpha_s$ value, and so on. To first approximation, this means that the $\hat{t}\bar{b}$ and $\hat{b}\bar{b}$ dipoles radiate normally for $\omega \gtrsim \Gamma_t$ and have soft radiation cut off, with the opposite for the $\hat{b}\bar{b}$ dipole.

4. The top quarks are assumed to decay isotropically in their respective rest frame, i.e. we do not attempt to include spin correlations between $t$ and $\bar{t}$. Close to threshold this gives an essentially flat distribution in $\cos\theta_{\text{parton}}$, defined as the angle between the ‘original’ $b$ and $\bar{b}$ directions before QCD radiation effects are considered. Breit-Wigner distributions are included for the top and $W$ masses.

On the phenomenological side, the main conclusions of the analysis in [24] are:

- The interconnection should be readily visible in the variation of the average multiplicity as a function of the relative angle between the $b$ and $\bar{b}$ (see Fig. 5).
- A more detailed test is obtained by splitting the particle content in momentum bins. The high-momentum particles are mainly associated with the $\hat{t}\bar{b}$ and $\hat{b}\bar{b}$ dipoles and therefore follow the $b$ and $\bar{b}$ directions, while the low-momentum ones are sensitive to the assumed influence of the $\hat{b}\bar{b}$ dipole.
- A correct description of the event shapes in top decay, combined with sensible reconstruction algorithms, should give errors on the top mass that are significantly less than 100 MeV.

The possibility of interference reconnection effects in $t\bar{t}$ production is surely not restricted to the phenomena discussed here. They could affect various other processes/characteristics.

One topical example concerns the top quark momentum reconstruction. As was emphasised in Ref. [33], the momentum measurement combined with the threshold scan could significantly improve the overall precision in determination of $m_t$ and $\Gamma_t$. As a supplementary bonus, the top momentum proves to be less sensitive to the beam effects.

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8Experimental aspects are summarized, e.g., in Ref. [35]. It is anticipated that $m_t$ and $\Delta\Gamma_t$ will be measured with accuracies of $\approx 350$ MeV and 0.07 respectively. (I am grateful to K. Fujii for discussion of these results.)
In order to reconstruct the top momentum we need at least one of the secondary 
$W$’s decaying hadronically. So the final state configurations are either a lepton plus 
four quark jets or six quark jets.

QCD interconnection may efface the separate identities of the top and antitop 
systems and, thus, could produce a potential source of the systematic error in the top 
momentum determination. The interference pattern here is more complicated than 
in the case of double leptonic decays because of an additional cross-talking between 
the hadronically decaying $W$ and the $b\bar{b}$ products.

It is clear that the problem of these interconnection-related uncertainties deserves 
theoretical studies. It may well happen that these uncertainties are non-
negligible in view of the aimed-for high precision of measurements.

Finally, let us make some comments concerning the definition of $m_t$. It is obvious 
that the high aimed-for precision of the top mass determination at a future linear col-
lider ($\Delta m_t \approx \Lambda_{QCD}$) requires clear theoretical understanding of this issue. After all, a 
quark is a colour object surrounded by the (long-distance) colour fields and its phys-
ical mass cannot be unambiguously defined in the full theory once non-perturbative 
QCD effects are taken into account.

To avoid confusion, I would like to stress that it is the so-called physical or pole 
mass, $m^p_t$, that is observable experimentally, see also the discussion in [47]. The pole 
mass is related to the location of the singularity of the renormalized quark propagator 
and appears naturally in the perturbative calculations. Unlike many other notions, 
$m^p_t$ can be defined in a gauge invariant way.

The pole mass is related to the running $\overline{MS}$ mass evaluated at the $m^p_t$ scale, 
$\hat{m}_t \equiv \hat{m}_t(m^p_t)$ as

$$m^p_t = \hat{m}_t \left[ 1 + C_F \frac{\alpha_s(m^p_t)}{\pi} + K_t \left( \frac{\alpha_s(m^p_t)}{\pi} \right)^2 + \ldots \right] \quad (25)$$

with the two-loop coefficient $K_t$ calculated in Ref. [48]. Numerically, $K_t \approx 11$. Let 
us point attention to the fact that the difference between these two masses is quite 
large (about 10 GeV).

Distinguishing between various masses is an important issue in the higher-order 
calculations of various characteristics of the top quark physics. Each time one needs to 
understand clearly what mass definition is the most appropriate to the phenomenon in 
question. The principal ambiguity in the definition of the pole mass of a quark$^{[44,45]}$ 
is deeply rooted in the divergence of perturbation theory at high orders (the so-
called infrared-renormalon problem$^{[49]}$). Because of the asymptotic explosion of the 
perturbation series in Eq. (25), truncating it at some optimal order leads to an in-
trinsic uncertainty of order $\Lambda_{QCD}$ assuming $\hat{m}_t$ as given$^{[44,45]}$. This imposes a limit

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9The reconstruction of the $W$-momentum could be affected as well. The QCD interferences can 
have some impact also on the top momentum distribution itself. The estimates show that there is, 
indeed, a small effect$^{[20,21]}$.

10For an intensive recent discussion of this old (but still hot) topic see, e.g., Refs. [44-46].
\(O(\Lambda_{QCD})\)) on the precision of the definition of the quark pole mass. On the other hand, the running mass \(m_t(\mu)\) does not suffer from the infrared-renormalon disease and, in principle, can be found to any accuracy at a high scale \(\mu \gg \Lambda_{QCD}\). Because the top quarks contributing to the electroweak loops have high virtualities, the QCD effects are characterized by momenta of order of \(m_t\) and the mass parameter most relevant to the electroweak corrections (like \(\Delta \rho\)) is \(m_t(m_t)\)\(^{[46,51]}\). The various theoretical expectations for the top quark mass (e.g. in SUSY GUTs, supergravity or superstring models\(^{[52]}\)) are also given in terms of this running mass.

An experienced reader may well wonder what is the possible impact of the very rapid top decay on the issue of its mass. Recall that the intrinsic uncertainty in the pole mass of a heavy quark results from the infrared effects associated with the gluon momenta \(k \sim \Lambda_{QCD}\), and measuring of this mass requires a very long time, \(t_{\text{meas}} \sim \frac{1}{\Lambda_{QCD}}\)\(^{[44]}\). This time is of the same order as the infrared gluon formation time, \(t_f\). The top quark decays well before these gluons can be formed and its physics should not, in principle, suffer from the long-distance problems. The infrared renormalon disease is inherited by the daughter \(b\)-quarks.

An instructive example comes from examining the soft gluon emission in the process (19), see [16]. At \(k < \Gamma_t\) the radiation decouples from the top quarks and the infrared effects become associated with the \(b\bar{b}\) system only.

The presence of the width provides further options for the definition of the on-shell mass (see [47] and references therein). It appears to be quite challenging to find a way to reduce the long-distance uncertainties in the physical mass of the top quark. We are working now in this direction.

**4. Radiative Interferences in the Inclusive Production of Heavy Unstable Particles**

As we have already discussed, in production processes of heavy unstable particles it is natural to separate the production stage from the decay stages. The various production-decay and decay-decay radiative interferences for a given process might, in principle, be expected to produce a complicated effect. Fortunately, as was shown explicitly in [19,20] such interference corrections are cancelled in the inclusive cross-sections up to terms of relative order \(\alpha_{\text{int}} \frac{\Gamma}{M}\) (where \(\alpha_{\text{int}} = \alpha\) or \(\alpha_s\) as appropriate) or better. The only exception is the contribution arising from the universal Coulomb interaction between slowly moving produced objects.

In [20] an explicit study was performed with the help of soft-insertion techniques and, in each case, we identified the particular degree of inclusiveness that is required for the interference effects to be suppressed. Of course, the real and virtual interference contributions, taken separately, are not suppressed but are infrared divergent.

\(^{11}\)In terms of the non-relativistic description of the quark-antiquark system this long-distance uncertainty is related to the ambiguity in the additive constant term in the QCD potential, see Refs. [36,50].
Clearly the infrared divergent parts have to cancel for physically meaningful values. This is not the issue. Rather the crucial question is to what level does this cancellation occur?

Here, following [19], we present a general theorem which states that the effects of radiative interferences are each suppressed by $O(\Gamma/M)$ in the totally inclusive production. This proof does not rely on specific assumptions, like soft-insertion factorization, and the resulting approach is applicable to any order in $\alpha_{\text{int}}$.

It may be useful to express the resulting recipe in a symbolic form. To be specific let us consider the production of $N$ heavy unstable particles $A_1, \ldots A_N$ with masses $M_i$ and widths $\Gamma_i$. For reference purposes we first consider the production in the absence of the decays. Then the inclusive cross section may be written in the form

$$\sigma_{\text{stable}}(A_1, \ldots A_N) = \sigma_0(M_i^2)(1 + \delta(R, C)), \quad (26)$$

where $\sigma_0(M_i^2)$ is the production cross section in the Born approximation and $\delta(R, C)$ represents the radiative corrections. Here we have separated Coulomb corrections $C$ from the remaining radiative corrections $R$. The Coulomb effects $C$, which are associated with large space-time intervals, are only important if two charged (or coloured) particles are slowly moving in their c.m. frame. Note that the separation of Coulomb effects can only be done uniquely near threshold, but this is the very region where the instability effects are most important. As was demonstrated in [39-41,54] these effects are especially important for process (19) where they drastically modify the threshold cross section. We shall briefly address below the issue of the QED Coulomb corrections to $e^+e^- \rightarrow W^+W^-$, see Refs. [55,56].

The question is “how is (26) modified when we allow the particles $A_i$ to decay?” We will show

$$\sigma_{\text{unstable}}(A_1, \ldots A_N) = \int \prod_i (ds_i \rho(s_i)) \sigma_0(s_i)(1 + \delta(R, \bar{C})) + \sum_n O\left(\alpha_{\text{int}}^n \frac{\Gamma_i}{M_i}\right) \quad (27)$$

with

$$\rho(s_i) = \frac{\sqrt{s_i \Gamma_i(s_i)}}{\pi[(s_i - M_i^2)^2 + s_i \Gamma_i^2(s_i)]} \quad (28)$$

where $\Gamma_i(s_i)$ is “running” physical width which incorporates the radiative effects associated solely with the decay of $A_i$.

In other words, the theorem for the production of heavy unstable particles says that, apart from the two modifications explicitly shown in the formula, the introduction of the widths gives rise to no new corrections up to order $\alpha_{\text{int}}^0 \Gamma_i/M_i$, where the $n = 0$ term corresponds to the standard (non-radiative) non-resonant backgrounds and the $n \geq 1$ terms to interference induced by $n$ radiated quanta. The two modifications in going from (26) to (27) are, first, the natural kinematic effect leading to the integrations over $\rho(s_i)$ and, second, the modification (symbolically $C \rightarrow \bar{C}$) of the Coulombic interaction between particle pairs which are non-relativistic in their
c.m. frame. In particular, the theorem says the remaining radiative corrections are unchanged (see also Ref. [57]).

The proof\cite{19} relies on two facts. First, the suppression of interferences between the various production and decay stages arising from energetic photons/gluons with $|k^0| \gg \Gamma$, and, second, the absence of infrared divergences in the total or inclusive cross-section. Both facts have a simple physical interpretation. Let us consider, without loss of generality, the case when the total energy of the process is of the order of the masses $M_i \sim M$, see previous sections. Then the typical time, $\tau_p$, of the duration of the production stage, as well as of the decay stages, is of order $1/M$. This time $\tau_p$ is much less than the characteristic time $\Delta t$ between the various stages

$$\Delta t \sim \max \left( \frac{1}{\Gamma_i}, \frac{1}{\Gamma_j} \right) \sim \frac{1}{\Gamma}.$$  \hspace{2cm} (29)

As a consequence the relative phases of photon/gluon emissions (or between emission and absorption) at the different stages of the process are approximately equal to $|k^0|\Delta t \sim |k^0|/\Gamma$. When $|k^0| \gg \Gamma$ this phase shift is large and therefore the radiative interference effects are suppressed.

The second basic fact, the absence of infrared divergences in totally inclusive cross-sections, has also a clear physical interpretation. Infrared divergences appear when we use states containing a definite number of photons/gluons. Now the acceleration of charge/colour leads to radiation with finite spectral intensity at zero frequency and so the scattering or the creation of charged/coloured particles is accompanied by the emission of an infinite number of photons/gluons. To obtain a physically meaningful cross section we must include arbitrary numbers of emitted photons or gluons (in the case of gluons we need also to average over the initial colour states).

For the proof we only need the absence of those infrared divergences in the total cross-section which are connected with the radiative interference between the various production and decay stages. To show this we first note that the total cross-section is proportional to the imaginary part of the forward scattering amplitude. When this amplitude is expressed as the sum of Feynman diagrams, all particles except the initial particles, appear as internal lines. Therefore the interference photon/gluon lines must be attached to internal lines, at least at one end. But it is well known that infrared divergent contributions only arise from photons/gluons with lines which couple at both ends to external lines corresponding to on-mass-shell particles. Because of the absence of infrared divergences the contribution from the region of small photon/gluon energies $|k^0| \lesssim \Gamma$ is small due to the lack of phase space. On the other hand, as shown above, for $|k^0| \gg \Gamma$ the interference effects are small due to the large time separations between the various stages. Therefore radiative interference between the production and decay stages is suppressed by at least a factor $\Gamma/M$. For multiple exchanges the conclusion remains valid. In the case of multiple exchange large energies $k_i^0$ of individual photons/gluons are allowed, since the constraint that the invariant mass
of the unstable particle must not be shifted far from its resonant value only requires

$$| \sum_i k_i^0 | \lesssim \Gamma$$  \hspace{1cm} (30)

where the sum is performed over photons or gluons emitted ($k_i^0 > 0$) and absorbed ($k_i^0 < 0$) at one of the decay stages. But because of the absence of infrared divergences there is again a suppression of at least one factor of $\Gamma/M$ due to the restriction of the phase space imposed by (30).

Finally let us consider the Coulomb radiative effects. Now there are infrared singularities connected with the Coulomb interaction of charged/coloured particles which are special in the sense that they are not cancelled by real emissions, but rather they appear in matrix elements as a phase factor with an infinite phase. Therefore they do not appear in the expression for the cross section and are not seen at all in the approach where we express the cross section in terms of the forward scattering amplitude.

However there is a Coulomb interaction which is connected with small photon/gluon frequencies, but which is not infrared divergent. For two charged/coloured particles, with reduced mass $\mu$ and momentum $q$ in their c.m. frame, the essential energies $k^0_c$ of the exchanged Coulombic photons/gluons are typically $|k^0_c| \sim q^2/\sqrt{(q^2 + \mu^2)}$. When $q^2 \lesssim \mu \Gamma$ these energies are $|k^0_c| \lesssim \Gamma$. Thus for two slowly moving charged/coloured particles in their c.m. frame there is an important Coulombic radiative interaction coming from the region of small photon/gluon energies. At first sight this appears to violate our previous statement, that the contribution from the region $|k^0| \lesssim \Gamma$ is small. But that statement referred to interference photons/gluons and it remains correct for them. The reason is that for interference between the different production and decay stages, the only important Coulomb interactions are those between a charged/coloured decay product of one of the unstable particles and some other particle (e.g. another unstable particle or one of its decay products) with the interacting pair slowly moving in their c.m. frame (that is $q^2 \lesssim \mu \Gamma$). This corresponds to a very small region of the available phase space and so these Coulomb effects are also suppressed by at least a factor $\Gamma/M$. This concludes the proof of the theorem.

Some words should be added to explain the modifications $\delta(R, C) \rightarrow \delta(R, \bar{C})$ in going from formula (26) for “stable” heavy particles to the realistic formula (27) for unstable particles. Away from the heavy particle production threshold the typical heavy particle interaction time is $1/\sqrt{s} \lesssim 1/M$, i.e. much smaller than their lifetimes. Thus the influence of instability on the Coulomb corrections at the production stage gives effects of relative order $\Gamma/M$ or less. Thus $\delta(R, \bar{C}) \approx \delta(R, C)$.

The situation is different for heavy unstable particle production near threshold. Then the typical Coulomb interaction time $\tau_c$ can be comparable to, or even larger than, the particle’s lifetime $\tau$

$$\tau_c \sim \frac{1}{k^0_c} \sim \frac{\mu}{q^2} \gtrsim \frac{1}{\Gamma} = \tau.$$  \hspace{1cm} (31)
Therefore the Coulomb part $C$ of the radiative correction $\delta$ shown in (26) will be considerably modified by instability, and hence it is denoted by $\bar{C}$ in (27). The calculations of the modified contribution $\bar{C}$ close to the threshold can be best done using old-fashioned non-relativistic perturbation theory. The diagrams are the same as in the stable particle case, but we require the momentum $p_i = (\epsilon_i, \mathbf{p}_i)$ of $A_i$ to satisfy $p_i^2 = s_i$, and we must replace the energies of the unstable particles by

$$\epsilon \rightarrow \epsilon + \frac{i M \Gamma}{2 \sqrt{p^2 + M^2}}$$

in the energy denominators of all intermediate states.

Concluding this section, let us make some comments concerning the effect of the $W$ width on the Coulomb corrections to the process $e^+ e^- \rightarrow W^+ W^-$ [55,56]. Accurate knowledge of these corrections is of importance for the precise energy scan of $W^+W^-$ production through the threshold region. In principle, this provides us with an “interconnection-free” method of measurement of the $W$ boson mass and width. To my knowledge, modification of the Coulomb interaction induced by the $W$ width effects was first studied in Ref. [55]. In this paper the non-relativistic technique was used with the virtuality of the $W$'s properly taken into account.

Consider the Coulomb attraction between the slowly moving $W$ bosons. Because the underlying Coulomb physics is different from the other radiative corrections it is possible to treat the Coulomb corrections separately from the short-distance effects (see [58] and later Refs. [40,41]). It was originally discovered in QED [55] that when oppositely charged particles have low relative velocity, $v \ll c$, Coulomb effects enhance the cross section by a factor which, to leading order in $\alpha/v$, is $(1 + \alpha \pi/v)$, provided the particles are stable. Now it has been shown [39,40] that the Coulomb effects may be radically modified when the interacting particles are short-lived rather than stable. This is the case for $W$ bosons. We would anticipate the modification to be significant when the characteristic distance of the Coulomb interaction ($d_C \sim \frac{1}{p}$) is greater than the typical spatial separation when the diverging $W$ bosons decay ($d_\tau \sim \frac{p}{m_W \Gamma_W}$). The more that $d_\tau$ is less than $d_C$ the more we expect the Coulomb attraction to be suppressed. If we note that $p \approx \sqrt{E m_W}$ then we see that the condition $d_\tau \lesssim d_C$ translates into $E \lesssim \Gamma_W$ where $E = \sqrt{s - 2m_W}$ is the non-relativistic energy of the $W$ bosons.

Recall here that the interplay between the Breit-Wigner propagators and the phase space factor (see Section 2) leads to larger values of $\langle p \rangle$ and, thus, induces an additional suppression of the Coulomb effects.

Allowing for the virtuality and the finite width of the $W$ bosons we find the following expression for the Coulomb correction in the non-relativistic approximation [55]:

$$\frac{\alpha}{v} \delta_{Coul} \simeq 2 \text{Re} \left\{ \int \frac{d^3 k}{(2\pi)^3 k^2} \cdot \frac{4\pi \alpha}{[(p + k)^2/M_W - E - i\Gamma_W]} \right\},$$

where $p$ is the momentum of a virtual $W$, and $v = \frac{4p}{\sqrt{s}}$ is the relative velocity of the
virtual $W$ bosons,

$$v = 2\left(\hat{s} - s_1 - s_2\right)^2 - 4s_1s_2)^{\frac{1}{2}}/s.$$  \hfill (34)

On integrating (34) over $d^3k$ we find

$$\delta_{\text{Coul}} = 2\text{Re}\left\{-i\ln\left(\frac{p_1 - ip_2 + ip}{p_1 - ip_2 - ip}\right)\right\},$$  \hfill (35)

where $p_1 - ip_2 = \sqrt{M_W(-E - i\Gamma_W)}$, that is

$$p_{1,2} = \left[M_W\left(\sqrt{E^2 + \Gamma_W^2} \mp E\right)/2\right]^{\frac{1}{2}}.$$  \hfill (36)

The result (35) giving the Coulomb correction for unstable particles should be contrasted with the leading order value $(\delta_{\text{Coul}})^{\text{(1)}}_{\text{st}} = \pi$ for stable particles. To calculate the Coulomb corrections for interactions between stable particles it becomes increasingly necessary to include higher order terms in $\alpha/v$ as the threshold is approached, that is as $v \to 0$. Summation of all the order $(\alpha/v)^n$ terms gives the famous Sommerfeld-Sakharov result\cite{53}

$$\langle \delta_{\text{Coul}} \rangle_{\text{st}} = \frac{\pi}{1 - e^{-Z}},$$  

$$Z = \frac{2\alpha\pi}{v}. $$  \hfill (37)

It is worth noting that at the very threshold $(\delta_{\text{Coul}})_{\text{st}} = 2(\delta_{\text{Coul}})^{\text{(1)}}_{\text{st}}$\footnote{This old result seems to have been completely forgotten in many recent publications.} However, as was first indicated in [55] in the finite width case the instability of the particles prevents the large distance effects from contributing and so the effects of the higher-order Coulomb corrections are small. One can roughly estimate that in the presence of the width the expansion parameter in the Coulomb series is

$$\frac{\Delta\sigma_{\text{Coul}}}{\sigma} \sim \frac{\alpha\pi}{\sqrt{(E^2 + \Gamma_W^2)^{\frac{1}{2}}/M_W}} < 0.15$$

instead of $Z$ in the stable case.

Note that the contribution of the higher-order Coulomb effects can be exactly calculated (if necessary) using the non-relativistic Green’s function formalism of Refs. [20,40,59].

Finally I would like to mention a claim of Ref. [56] that the non-relativistic approach is not sufficient for the precise description of the Coulomb effects in the whole
LEP2 region. My personal feeling is that we need some further studies of this important issue.

4. Summary and Outlook

The large width \((\Gamma \sim O(1 \text{ GeV}))\) of heavy unstable particles controls the radiative interferences between emission occurring at different stages of the production processes. The QCD interferences may efface the separate identities of these particles and produce hadrons that cannot be uniquely assigned to either of them. Here we concentrated mainly on two topical problems, namely the QCD interconnection phenomena in events of the type \(e^+e^- \rightarrow W^+W^- \rightarrow q_1\bar{q}_2q_3\bar{q}_4\) and \(e^+e^- \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-\). We have shown\(^{[23,24]}\) that, on the perturbative level, these interference effects are suppressed and we have applied hadronization models to help us estimate the non-perturbative effects.

One of the favoured methods of the \(W\) and top mass determination is to reconstruct them event by event. The total contribution to the systematic error on the \(W\) mass reconstruction was estimated conservatively as 40 MeV and the uncertainty in the top mass was found on the level of less than 100 MeV. We believe that with sophisticated analysis methods these uncertainties can be reduced. Therefore, one may expect that in the foreseeable future the precise determinations of \(m_W\) and \(m_t\) are not jeopardized by the QCD interconnection effects.

In some sense, the interconnection effects discussed here could be considered as only the tip of the iceberg. Colour reconnection can occur in any process which involves the simultaneous presence of more than one colour singlet. Many of the techniques developed in Refs. \([20,23,24]\) could be directly applied to these problems.

Among other examples of practical importance are \(e^+e^- \rightarrow Z^0H^0, e^+e^- \rightarrow Z^0Z^0, pp/\bar{p}p \rightarrow W^+W^-, pp/\bar{p}p \rightarrow t\bar{t}, pp/\bar{p}p \rightarrow tb, pp/\bar{p}p \rightarrow W^\pm H^0,\) etc. One could discuss also interferences with beam jets. The problem with these processes is that there are too many other uncertainties which make systematic studies look very difficult.

QCD interconnection is interesting in its own right, since it potentially provides a laboratory for a better understanding of the hadronization dynamics. With a lot of hard work (and good luck), LEP2 could probably be in a position to discriminate between some hadronization models.

The other methods of the \(W\) mass measurements\(^{[10,11]}\) also require a clear understanding of the role of the \(W\) width, this time because of the QED radiative phenomena. Thus, the width effects modify the QED Coulomb corrections to the cross-section of the process \(e^+e^- \rightarrow W^+W^-\), which should be known with a high accuracy for the measurements scanning across the \(WW\) threshold region\(^{[55,56]}\). The \(W\) width can have an impact on the measurements of \(m_W\) from the shape of the lepton spectrum (the lepton end-point method). To my knowledge this study has never been systematically addressed.

At last, a direct determination of the \(W\) width itself using the transverse mass
distribution of $W \rightarrow e\nu$ decays\cite{9} requires, in principle, a careful analysis of the QED interconnection effects.

We cannot today predict what will come out of the forthcoming systematic studies, e.g., within the working groups for LEP2 and a future $e^+e^-$ linear collider. The results reviewed here can be considered as a starting point for more refined and detailed investigations. We believe that radiative interference phenomena will be of topical interest for many years to come.

Acknowledgements

It is a pleasure for me to thank Risto Orava and Masud Chaichian for creating such a stimulating atmosphere and for the warm hospitality in Lapland. I wish to thank V.S. Fadin, C.J. Maxwell, W.J. Stirling, N.G. Uraltsev and especially T. Sjöstrand for fruitful discussions. This work was supported by the United Kingdom Particle Physics and Astronomy Research Council.

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