Atomic-molecular effects in geophysical hydrodynamics

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Abstract. To calculate the dynamics and structure of flows, a system of fundamental equations of fluid mechanics with equations of state for the Gibbs potential and density of an inhomogeneous medium is applied. The complete solution of the system describes ligaments, waves, vortices, jets, wakes, and other types of flows. Calculations of flow patterns around obstacles are consistent with the experiment. Observations of the processes of merging a freely falling drop with a target fluid revealed that the finest components are formed during the direct generation of ligaments by atomic-molecular processes. The involvement of a scaled and parametrically invariant system of fundamental equations permits the study of unsteady energetic flows and more accurately describes their dynamics and structure in the whole range of scales from microscopic to global.

1. Introduction

Developed techniques of optical observations helped to establish that all flows of fluids or gases are characterized by a "fine structure" in which relatively thick components are separated by thin high-gradient interfaces. The fine structure of liquids and gases is recorded over the entire observable range of scales. A striking example of the structures of outer space are photographs of the "light echo" of the V838 Monoceros star flash. Fibers, deformed rings, thin streaky structures have been observed for several years in interstellar space on spatial scales of several light-years [1].

Wavy, vortex, annular, and jet elements are expressed in the atmospheres of stars and planets, including the Earth. The very fact of the existence of structures is paradoxical since dissipative factors should lead to the smoothing of large gradients. The prevalence of the fact of the very existence of fine structures in fluid flows of various scales, from galactic to microscopic, indicates the universality of the continuously operating mechanism of their formation.

Theories of fluid flows are mostly constructed to describe the phenomena of only one selected class that are jets, wakes, waves, vortices, etc. in a limited range of parameters (exist subcritical and supercritical conditions), as well as in their own temporal and spatial scales. The only scale-invariant system of equations is a set of equations for the transfer of density, matter, momentum, and energy, given back in the middle of the last century in the first edition of the treatise [2] and later reproduced in modern monographs [3, 4]. The system has not yet been completely investigated as the closed system of connected equations taking into account the condition of compatibility.

The theory of fluid flows based on the "continuous medium" concept, although, as is known, all bodies are composed of atoms, molecules, clusters, and other elementary structural formations. Considering the controversy of two approaches that are continuous description in macroscopic mechanics and discrete interactions in atomic-molecular physics, it is natural to search for an "agent" – a flow component that connects processes at opposite boundaries of the scale range. These flow components would be limited by two scales that are a small thickness specified by atomic-molecular
processes and a large length determined fluid volume as well as by a characteristic velocity and the time of evolution (existence) of the system.

The framework of macroscopic hydrodynamics is the system of fundamental equations, the symmetries of which correspond to the basic principles of physics expressed in conservation laws. Balance transport equations [2-4] in relation to the theory of flows determine the basic physical quantities that characterize a fluid as a continuous medium and its flows caused variations of the fluid parameters. The relationships between physical quantities are determined by equations of state, which were first found empirically [5] however later were derived from the principles of thermodynamics [6, 7]. The rational theory takes into account the requirements of engineering mathematics that is the axiomatic science for the principles of choosing the content of symbols, rules of operations, and setting accuracy control criteria. The accuracy concept here means the degree of compliance of the results obtained to the basic axioms of the theory.

2. The system of governing equations and definition of a fluid flow

The set of fundamental quantities for which the conservation laws are formulated includes the following variables – mass (or the specific mass – density), as well as specific momentum and energy. The free mobility of atoms and molecules in the undisturbed state of fluid forbids introducing the tensor of inertia (the mutual position of particles is continuously changing) and predetermines the special role of energy. The energy – a scalar quantity which is a universal parameter applied at all range of scales of physical phenomena extending from sub-nuclear scales to cosmic distances – needs to be used in describing the fluid state and dynamics of its flows.

Energy, including kinetic, potential, and internal components, is the basis of the modern definition of the equation of state of a continuous medium and, at the same time, one of the measures of motion, plays a special and insufficiently studied role in fluid mechanics. The non-mechanical part of the fluid energy is characterized by thermodynamic potentials that are proper internal energy, enthalpy (thermal function), free energy, and free enthalpy (Gibbs potential) [6]. All potentials related by Maxwell's relations are equivalent. However, in the theory of flows, the Gibbs potential the derivatives of which explicitly determine the equilibrium density of the medium and entropy, and indirectly - all other thermodynamic quantities, is most widely used [7]. All thermodynamic quantities that temperature, pressure, the concentration of impurities can be expressed through derivations of the Gibbs potential.

The concept of energy gives room to take into account all types of the mechanism of its transport in fluids as fast resulting of the direct action of atomic-molecular processes, as common transport with a flow velocity or with waves with their intrinsic group velocity and the slowest dissipative processes. The mobility and the combining (clustering) atoms and molecules of a continuous medium mean that the whole or only some of the energy transfer mechanisms can act in fluid flows. The fastest direct atomic-molecular interactions manifest themselves in process of releasing available potential internal energy in the chemical reactions or eliminating the free surface in coalescing fluids. Independent transport of energy takes places with the velocity of a macroscopic flow and with the group velocity of waves (acoustic, capillary, gravitational surface and internal, inertial, and hybrid), as well as slow dissipative transport, affecting both the dynamics and the spatial structure of the fields of physical quantities [9]. Taking into account its special role, energy is included in the definition of the state of a liquid, gas, or plasma in the theory of flows.

To describe flows in the approximation of a continuous medium in fluid mechanics, the balance equations for the transfer of matter (the continuity equation), momentum (Navier-Stokes equation), and energy are postulated [2-4]. The intrinsic properties of an inhomogeneous medium include flow induced by diffusion on topography [10]. Forced flows are created by the action of gradients of pressure (force), temperature (convective), concentration (gravitational), the joint action of some or all types of disturbances (multicomponent or double diffusive convection), the motion of solids and gas volumes.

The dynamics of the fields of physical quantities that are density, momentum, temperature, and concentration of dissolved substances in a liquid with equations of state for density $\rho$ as derivatives of
the Gibbs potential $G = G(x, t)$ in a field of uniform gravity with acceleration $g$ is described by solutions of a system of fundamental equations, which are presented here. The simplest form of the system without taking into account the global rotation of the liquid and the action of sources of density $\rho$, temperature $T$, and concentration of impurity $S$ was selected [2–4]

$$\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho P}{\partial x_i} &= 0, \\ \rho = \rho(P(x, t), S_i(x, t), T(x, t)), \\ G = G(x, t)
\end{align*}$$

(1)

Here symbols $\nabla$ and $\Delta$ are the Hamilton and Laplace operators, $\nu_T$, $\nu_S$ are the coefficients of kinematic viscosity, temperature and concentration diffusivities. The fluid velocity is a local kinematic parameter $v_i = p_i / \rho$, which is defined as the ratio of two invariant quantities, one of which is momentum $p$ and the second is density $\rho$.

The form of equations of state for thermodynamic potentials, density, and other physical quantities is selected taking into account the composition of the medium and the energetics of the flows under study. In the equations of system (1), momentum dissipation is taken into account, the appropriate choice of source functions allows one to take into account the external fluxes of energy and matter. In this case, most models use the assumption that the density of the medium is constant. However, in real conditions, the density and other physical characteristics of the state of a continuous liquid (gas, plasma) are non-uniformly distributed due to the action of gravity and other factors causing the movement and stratification of liquids. System (1) itself serves as the definition of physical quantities characterizing both the state and the flow of fluids.

The physical properties of a continuous medium (liquid, gas, plasma) determine the equations of state - thermodynamic potentials and their derivatives - density, pressure, entropy, temperature), as well as kinetic coefficients in the equations of molecular transfer. The special role of density in mechanics as a measure of inertia, gravitation, and thermodynamic processes predetermines the choice of the form of the basic equation of state in the form of the density $\rho = \rho(P, T, S_i)$ dependence on pressure $P$, temperature $T$, the concentration of solutes, and suspended particles $S_i$.

A fluid flow is defined as an intrinsic or forced self-consistent transfer of momentum $p$, matter with the density $\rho$, and the total energy $E$, which causes changes in the pattern of physical quantities $p, E, \rho, P, T, S_i$ fields in space, ensures the interactions of the flow components and the entire flow with external bodies (gaseous, liquid or solid).

3. Classification of dynamic components of flows

Taking into account the variability of the density permits to find solutions of system (1) in a three-dimensional setting and to classify flow components on the basis of complete solutions of the linearized system. The solutions are constructed by methods of the theory of singular perturbations [8], taking into account the compatibility condition that determines the rank of the complete system, the order of its linearized version, and the degree of algebraic (dispersion) equation. Periodic fluid flows with real frequency $\omega$ and complex wave vectors $k$ are considered. Substitution of the solution in the form of plane waves transforms system (1) into algebraic equations and construction of dispersion relations. In a medium with stable unperturbed vertical profiles of temperature $T_0(z)$, salinity $S_0(z)$, and density
\[ \rho_0(z) \], which are characterized by the intrinsic scales of buoyancy \( \Lambda_T = -d\ln T_0(z)/dz \), \( \Lambda_S = -d\ln S_0(z)/dz \), \( \Lambda_\rho = -d\ln \rho_0(z)/dz \).

The dispersion equation for system (1), which takes into account the action of all dissipative factors, has the form [11]

\[ D_\rho(k, \omega) \cdot F(k, \omega) = 0 \]

\[ F(k, \omega) = -D_\rho(k, \omega)D_{\kappa_T}(k, \omega)D_{\kappa_S}(k, \omega) \left( k^2 + i\frac{k_z(\Lambda_T + \Lambda_\rho)}{\Lambda_T\Lambda_S} \right) + \\
+ D_{\kappa_T}(k, \omega) \left( \frac{\omega k^2}{\Lambda_S} D_\rho(k, \omega) - N^2 k_\perp^2 \right) + D_{\kappa_S}(k, \omega) \left( \frac{\omega k^2}{\Lambda_T} D_\rho(k, \omega) - N^2 k_\perp^2 \right) \]  

\[ D_\rho(k, \omega) = -i\omega + v k^2 \]

Equation (3) is simplified and takes the form

\[ \kappa_S v (k^2 + k_y^2)^3 - i\omega \left( v + \kappa_S \right) \left( k^2 + k_y^2 \right)^2 - \omega^2 \left( k^2 + k_y^2 \right) + N^2 k^2 = 0 \]  

The complete solution of the system (2) for periodic flows of viscous fluids always contain two types of perturbations that are thin ligaments, and more long waves [8]. They are described by families of singular and regularly perturbed solutions, respectfully. In general, other (non-periodic) components can also be form in the flow patterns.

**Ligaments** (ligands, interfaces, shells, envelopes, fascia, fibers, threads, filaments, ...) are thin and extended components of flows. In linear models, they are described by the group of singularly perturbed solutions of the system of fundamental equations [11]. The transverse scales of ligaments \( \delta \) are determined by the dissipative properties of the medium (coefficients of kinematic viscosity \( v \), temperature \( \kappa_T \) or salinity diffusivities \( \kappa_S \), or their combinations) and the characteristic values of temporal variability - the time of the process formation \( \Delta t \), the frequency of the wave \( \omega \) or the velocity of momentum and energy transfer \( U \) : \( \delta_T = \sqrt{v/\Delta t} \), \( \delta_\rho = \sqrt{v/\omega} \), \( \delta_S = v/U \). The length of the ligaments is determined by the lifetime of the process under study. There are many types of ligaments in fluids, their number of which is determined by the form of the equation of state and is equal to four if only the variability of density and viscosity is taken into account. When heat conduction effects are taken into account there are six ligaments. And if the equation for salinity diffusion is additionally included in the analysis there are eight different ligaments [11]. In the experiment, the ligaments correspond to thin shells, interfaces, and fibers that visualize the structure of the flows.

**Wave** is a process in which the parameters of local temporal variability of physical fields (frequency \( \omega \) ) and of instantaneous spatial structure (wavenumber \( k \) or wavelength \( \lambda \) ) are related by a functional (dispersion) relationship \( \omega = \omega(k, kA, ...) \), which may include the amplitudes of disturbances \( A \). Waves are described by a group of regularly perturbed solutions of the fundamental system.

A **vortex** is an unsteady flow component with a relatively high vorticity value \( \omega = \text{rot} u \). In a vortex, free solids in the bulk or on the surface of the fluid are transported by the flow and simultaneously twist around their own axis. The compact continuous "liquid particles" are broken down by the ligaments into distinguishable fibers in fluid flows.

The structure of the solution to equation (2) and system (1) depends on the shape of the source. In the case of matching the symmetries of the wave source and the pattern of a conical beam of periodic internal waves, which is emitted by a horizontal disk with a radius \( R \) oscillating in the vertical direction, neglecting the effects of diffusion, equation (2) is simplified and takes the form

\[ \kappa_S v (k^2 + k_y^2)^3 - i\omega \left( v + \kappa_S \right) \left( k^2 + k_y^2 \right)^2 - \omega^2 \left( k^2 + k_y^2 \right) + N^2 k^2 = 0 \]
which for the given problem takes the form $\delta_N^y, \delta_N^k << \lambda \sim R$. Regular roots describe the structure of a beam of conical internal waves

$$k_1 \approx -k \cot \theta + \frac{i(v + \kappa_S)k^3}{2\omega \sin^3 \theta \cos \theta}$$

(4)

where $\theta = \arcsin(\omega N)$ is the angle of inclination of the wave cone to the horizon (as well as the direction of the group velocity of the waves). The regular root $k_1$ in this problem have an analogue in an ideal fluid.

Singular solutions $k_2$ and $k_3$ have a different nature. One of them depends only on the coefficient of kinematic viscosity and characterizes the components of the flow with a transverse length scale

$$\delta_\omega^v = \sqrt{2v/\omega} \quad [11]$$

$$k_2 \approx \frac{1 + i}{\delta_\omega^v}$$

(5)

The corollary to the assumed assumption of azimuthal decreases in the order of the dispersion equation (3.6) and the loss of one solution describing the isopycnic (Stokes) singular component. In this geometry, the intrinsically inherent component is transformed to the form of the Stokes periodic flow, since only on the horizontally placed moving source motion in the periodic boundary layer is isopycnic.

In the general case, when the radiating plane is inclined at an arbitrary angle to the horizon, the determining dispersion equation has a higher (eighth) order. Its roots describe a larger number of singular flow elements, which include internal, Stokes velocity and diffusion boundary layers. Their parameters depend on the characteristics of the medium ($v, \kappa_S, N$), the wave frequency $\omega$ and the geometry of the problem (the angle of inclination of the emitting surface to horizon $\varphi$). The next root of equation (3.6) defines a singular solution with thickness

$$\delta_\omega^{k_S} = \sqrt{2\kappa_S/\omega}$$

(6)

Thus, the viscous and diffusion boundary layers in this problem completely split.

The imaginary parts of the roots (4) - (6) of equation (5.8) are chosen from the condition for the damping of perturbations at infinity $\text{Im} k_j > 0 \quad [11]$.

The intrinsic scales of the structural components determine the requirements for the experimental techniques. The choice of the size of the observation field, the number of recorded parameters are defined by spatial and temporal characteristics of large flow components that are by sizes and periods of waves, vortices, wakes, jets and so on. The spatial and temporal resolutions of instruments are defined by parameters of the smallest components that are ligaments. These conditions were taken into account when developing the stands of the USF "HPC IPMech RAS" for studying the dynamics and structure of stratified flows, waves and vortices [12].

Calculated and observed images of periodic internal waves generated by an oscillating horizontal disc and sphere are presented in figure 1. Fine ligaments form envelopes of the wave beams. They are almost invisible in the fields of basic variables like velocity or density but clear manifest themselves in fields of their derivatives. In the experiment, the schlieren instrument helps to visualize these flow components.

Experimentally observed waves of finite amplitude, vortices, and some other structural components of real flows are products of nonlinear interactions of solutions of the linearized model. Moreover, all components of the flows directly interact with each other: waves with waves, waves with ligaments, as well as ligaments with ligaments [13]. As a result of interactions, new groups of all components of complete solutions arise with their own temporal and spatial characteristics.
Figure 1. Calculated pattern of flow in the central cross section of the conical periodic wave beam produced by vertically oscillating disc ($R = 4 \text{ cm}$, $\omega = 1 \text{ s}^{-1}$, $T_b = 5.2 \text{ s}$): a, b) – the horizontal component of velocity and its second derivative at $t = 0$; c, d) – Schlieren images of periodic flows produced by oscillating sphere $D = 4.5 \text{ cm}$, $T_b = 11.2 \text{ s}$

Nonstationary ligaments, by virtue of the nature of their formation, connect processes at the atomic-molecular level with large structural components, such as waves, vortices, jets, the formation time of which is determined by their eigenvalues (for a wave it is the period).

4. Calculation and schlieren visualization of stratified flow patterns around obstacles

When studying certain types of flows, system (1) is simplified while preserving its basic properties. In particular, when analyzing the flows of an isothermal fluid with a high heat capacity flow around bodies, the effects of compressibility and thermal diffusivity are usually neglected and the following system of equations is used [14].

\[
\begin{align*}
\text{div } \mathbf{v} &= 0, \\
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} &= -\frac{1}{\rho_0} \nabla P + \mathbf{v} \Delta \mathbf{v} - s \cdot \mathbf{g}, \\
\frac{\partial s}{\partial t} + \mathbf{v} \cdot \nabla s &= \kappa_s \Delta s + \frac{v_z}{\Lambda}, \\
\rho &= \rho_0 \exp \left( -\frac{z}{\Lambda} + s \right).
\end{align*}
\] (7)

Here $S(x, t) = S_0(z) + s(x, t)$ is the total salinity (stratifying component), including the coefficient of salt contraction, $s(x, t)$ is its perturbation, $\mathbf{v} = (v_x, 0, v_z)$ is the induced velocity, $P$ is the pressure minus the hydrostatic part. The equation of state of the medium takes into account only the dependence of density on salinity (stationary undisturbed vertical profile is $\rho_0 S_0(z)$), $\rho_0$ is the density on the horizon of the body center) with the boundary conditions of no-slip for velocity and no-flux for salinity [9]. In an unperturbed stratified medium, thin layered diffusion-induced flows start to form at the edges of the fixed body. They are transformed into extended ligaments with the beginning of self-motion of a body of neutral buoyancy.

The compensation flow induced by diffusion on the plate is further taken as the initial conditions

\[
\begin{align*}
\mathbf{v}\big|_{t=0} &= \mathbf{v}_i(x, z), \\
\mathbf{s}\big|_{t=0} &= \mathbf{s}_i(x, z), \\
\mathbf{P}\big|_{t=0} &= \mathbf{P}_i(x, z), \\
\mathbf{v}_z\big|_E &= 0, \\
\frac{\partial \mathbf{s}}{\partial \mathbf{n}}\big|_E &= \frac{1}{\Lambda} \frac{\partial z}{\partial \mathbf{n}}, \\
\mathbf{v}_z, \xi, \zeta \rightarrow \infty &= (U, 0, 0),
\end{align*}
\] (8)

where $U$ is the flow velocity, $(\xi, \zeta)$ are directed along and the outward normal to the obstacle surface.

The system of equations (7) and boundary conditions (8) characterizes a set of parameters with the dimension of length, including the scale of buoyancy $\Lambda$, length $L$ or height $h$ of the body. The set of temporal scales includes the period of buoyancy $T_b$ and intrinsic transit time for distance equal the size
of the body \( T_U^L = L / U \) marking the change in the sign of the pressure gradient in the laboratory coordinate system.

The set contains intrinsic microscales determined by the properties of the medium – dissipative coefficients (kinematic viscosity \( \nu \) or diffusion \( \kappa_S \)) and the buoyancy frequency \( N \) that are \( \delta^v_N = \sqrt{\nu / N} \) and \( \delta^S_N = \sqrt{\kappa_S / N} \). The structure of the formula for \( \delta^v_N \) is similar to the thickness in the Stokes flow \( \delta^v_{N_0} = \sqrt{\nu / \omega} \) over an oscillating plane with frequency \( \omega \) [2]. Here, in the expressions for microscales instead of frequency \( \omega \) the buoyancy frequency \( N \) that is the frequency of natural oscillations of a particle of a stratified fluid displaced from the equilibrium horizon, was included. The group of large scales of dynamic nature includes the length of the attached internal waves \( \lambda = U T_b \) and \( \Lambda_v = \frac{g \nu}{N} = \frac{\sqrt{\nu}}{N} \), which is the combined viscous-wave scale. The transverse dimensions of the ligaments characterize the Prandtl \( \delta^P_U = v / U \) and Peclet scales \( \delta^P_S = \kappa_S / U \), which are determined by the dissipative coefficients and the obstacle velocity.

Modern computer technology has resources to construct numerical solutions of systems (1) and (2) in a wide range of parameters, in particular for the Reynolds number \( 1 < \text{Re} < 100000 \). Calculations of perturbations and a schlieren photograph of the flow pattern near a free wedge, self-propelled under the action of diffusion-induced flows [10], are shown in figure 2 (local the excess is blue). The wedge moves from right to left at a velocity of about 0.8 cm/hour.

![Figure 2. Calculation and schlieren photograph of the patterns of disturbances near a free wedge on the horizon of neutral buoyancy: a) - the field of disturbances of the vertical component of the density gradient \( (L = 10, h = 2 \text{ cm}, T_b = 6.3 \text{ s}); b) - photo of a self-propelled wedge in the tank filled with solution of common salt \( (T_b = 7 \text{ s}, \text{“vertical slit - Foucault knife”}) \)](image)

The results of the numerical solution of system (2) with boundary conditions (3) with uniform motion of the vertical band are shown in figure 3 [14]. In a creeping flow with a slow motion of the body, both leading (inclined rays) and internal waves attached behind the body are expressed with ligaments limiting the density trace and contacting with the back side of the plate (figure 3 a).

In the regime of intense wave disturbances, both waves and an extensive family of ligaments are presented, forming a thin-layered wake (figure 3 b). Moreover, both in the calculations and in the experiments, the fine structure of the anticipatory disturbance is visualized, indicating the existence of ligaments both behind and in front of the body. In the experiments, the phase surfaces of the attached internal waves are deformed by the wake flow somewhat more strongly than in the calculations; the split pattern of interfaces is more pronounced.

The set contains intrinsic microscales determined by the properties of the medium - dissipative coefficients (kinematic viscosity or diffusion) and buoyancy frequency. The structure of the formula is similar to the estimation of the thickness in the Stokes flow over a plane oscillating with frequency [1]. Here, instead of the buoyancy frequency, that is the frequency of natural oscillations of a particle of a stratified fluid displaced from the equilibrium horizon, was included in the expressions for microscales. The group of large scales of dynamic nature includes the length of the attached internal waves and the viscous-wave scale. The transverse dimensions of the ligaments characterize the Prandtl and Peclet scales, which are determined by the dissipative coefficients and the obstacle velocity and.
With a further increase in the band velocity, the general structure of the flow is somewhat transformed (the number of visible phase surfaces of internal waves, which deviate in the direction of body motion, change, the geometry of interfaces changes) and the degree of expression of individual components is disturbed. The strongest structural changes are observed in the wake behind the body.

![Schlieren images of the flow pattern (“vertical slit - Foucault knife” method) and calculations of the horizontal component of the density gradient fields around a moving band (h = 2.5 cm, T_b = 12.5 s): a – c) – U = 0.03, 0.18, 0.75 cm/s; Re = U/h / ν = 7.5, 45, 187.5](image)

**Figure 3.** Schlieren images of the flow pattern (“vertical slit - Foucault knife” method) and calculations of the horizontal component of the density gradient fields around a moving band (h = 2.5 cm, T_b = 12.5 s): a – c) – U = 0.03, 0.18, 0.75 cm/s; Re = U/h / ν = 7.5, 45, 187.5

The length of the attached internal waves increases proportionally the band velocity \( \lambda = U T_b \). In the central part of the density field pattern in figure 3 b, in the regime of intense generation of internal waves (Froude number \( Fr = U / Nh = 0.14 < Fr_{cr} \), \( Fr_{cr} = 1 \)), thin layered ligaments are expressed, separating the attached waves in antiphase in the upper and lower half-spaces. They can be traced along the entire length of the track, starting from the bottom of the plate, in the form of a wavy structure. The phase
surfaces of the attached waves become inclined, their shape noticeably differs from that calculated within the framework of the linear approach.

The flow structure changes radically with a further increase in velocity, as shown in figure 3 c. Since in this case the conditions are subcritical \( \text{Fr} = 0.6 < \text{Fr}_{cr} \), both the amplitude and the length of the attached waves continue to increase with velocity grows. The structure of the wake radically changes - an oscillating bottom vortex is adjacent to the trailing edge of the plate, generating elongated vortex structures in a thin density wake.

In the patterns of the density gradient fields shown in figure 3, all the structural components of the stratified flow are expressed that are ligaments, drawing the fine structure of the flow, waves, compact vortices. Calculations show that two types of upstream waves can form in front of the body. Long waves ahead the body coupled with attached waves. Short waves are placed at the edges of the band.

Only long waves are represented in the schlieren image, which is due to the shading of some of the structures in the Foucault method. In the experiment in figure 3 c, the bottom vortex, which forms a meandering density wake, moves up and down the rear side of the band. The shape of a tortuous fine-structured wake with a vortex tip adjacent to the bottom of the plate reflects the pattern of displacement of the region of separation of ligaments from the bottom vortex in this regime. Due to the superposition of different-scale components, the patterns of all flows are unsteady; however, some parts of the flow structure evolve especially rapidly.

All illustrations shown in figure 4 were obtained by solving system (7) with physically justified boundary conditions (8) without invoking additional hypotheses, equations, or constants.

Both systems of equations, complete (1) and reduced (2) ones, are parametrically and scale-invariant. However, since the complete solutions of these systems contain a large number of independent multi-scale functions, their overlays, which are represented in the configuration space as a "flow pattern", have different forms in remote ranges of parameters. Limitations of the sensitivity and resolution of instruments, mesh sizes, and time intervals form natural tools for denoting the "boundaries of mode regions" of a system of equations. However, the system of fundamental equations does not contain internal boundaries and does not need the introduction of additional scales, which was noticed by G.G. Stokes.

The differences in kinetic coefficients provide a significant difference in the transverse scales of ligaments of different nature associated with the effects of viscosity, thermal and salinity diffusivities, and, consequently, the difference in the patterns of physical fields on the micro- and macroscales. In a number of phenomena, ligaments provide the formation of stable boundaries between individual elements of a complex flow structure. The ligaments are most clearly expressed in drop impact flows, in which a fast conversion of the available potential energy into other forms during the coalescence of liquids and the elimination of the free surface take place. The elimination can be accompanied by a slow transformation of mechanical energy of a fluid motion into surface potential energy during the formation of a new free surface of the liquid.

**Hydrodynamics of a drop impact**

In a fluid with a free surface, the density and other characteristics of the medium in the bulk of the liquid and in the near-surface layer with a thickness of the order of the molecular cluster size \( \delta_{\alpha} \sim 10^{-6} \) cm differ markedly. The anisotropy of atomic-molecular interactions is manifested in the existence of available potential surface energy, chemical and other types of internal energy, which can be transformed into thermal, the mechanical energy of fluid flows, and into the work of creating a new free surface. The differential of the Gibbs potential \( dG_{\alpha} \) in a near-surface layer with an area \( S_{\alpha} \) and thickness of the order of the molecular cluster size \( \delta_{\alpha} \), cm has the form [9]

\[
dG_{\alpha} = -sdT + VdP - S_{\alpha}d\sigma,
\] (9)
where $s$ is entropy, $V = 1/\rho$ is specific volume, $S_\alpha$ is area of the free surface and $\sigma$ is surface tension coefficient.

Even greater changes in the atomic-molecular structure of matter are observed directly at the liquid-gas interface, where water decomposes into ionic clusters [9]. Here, in a layer with a thickness of several molecular sizes $\delta_\alpha \sim 10^{-7}$ cm, the thermodynamic potential includes terms that depend on the chemical potential $\mu_i$ and the concentration differential of the components

$$dG_\alpha = -sdT + VdP - S_\alpha d\sigma + \mu_i dS_i,$$

(10)

The inhomogeneity of the distribution of thermodynamic potentials and the action of various mechanisms of energy transfer (fast direct atomic-molecular, with a flow of matter, with the group wave velocity and slow diffusion) are clearly manifested in the evolution of the flow pattern of a freely falling drop plunging into a liquid.

The photographs of the flow patterns illustrate the complex texture of the surface of the cavity bottom and crown in the process of merging transparent liquids with water. The coalescence of the water drop with water is given in figure 4a or a yellowish concentrated aqueous solution of ferrous sulfate drop with water shown in figure 4b.

**Figure 4a.** Drop of water coalesces with target water ($D = 0.43$ cm, $U = 3.7$ m/s, $\rho = 1$ g/cm$^3$, $\sigma = 73$ g/cm$^2$, $\mu = 0.01$ g/(c·cm), $E_\alpha = 4$ $\mu$J, $E_k = 266$ $\mu$J

**Figure 4b.** Drop of saturated solution of ferrous sulfate coalesces with target water ($D = 0.43$ cm, $U = 3.5$ m/c, $\rho = 1.18$ g/cm$^3$, $\sigma = 75$ g/cm$^2$, $\mu = 0.02$ g/(c·cm), $E_\alpha = 4.2$ $\mu$J, $E_k = 313$ $\mu$J

Designations: 1 is coalescing drop, 2 is the apex of a crown with teeth, 3 is a wake of a spray droplet impact on the surface of a drop 1, 4 is the boundary of the region of fluid confluence, 5 is annular capillary waves at the bottom of the cavity, circling the confluence region, 6 is the bottom of the cavity, 7 is boundary the cavity and the crown, 8, 9 are wall and upper edge of the crown, 10 is veil, 11 are thorns, 12 are small droplets (sprays), 14 are fine jets (ligaments, trickles) at the bottom of the cavity, 14 is 3D texture of the crown wall, 15 are teeth on the apex of the crown. The marker length is 1 cm.
The contact line \(4\) of the coalescing drop 1 with the target fluid is not smooth and consists of smooth depressions and pointed ridges [15]. The surface of the cavity wall is distorted by narrow radial jets 13 (ligaments) and diverging annular capillary waves 5 \(\lambda = 0.03\) cm long, which are emitted by a moving line \(4\).

The minimum width of ligaments \(\Delta w \approx 0.012\) cm observed near their source, which is tips of ridges on the contact line and it monotonically, increases with the distance from which (figure 4). Small ring structures 3 on the surface of droplet 1 in figures 4, 4 are wakes of fallen droplets escaping from the tips of thorns 11 impacts [15]. Separate sections of veil 10, adjacent to the inner wall of crown 2, are inclined towards the center (in figure 4 at 4 and 7 o'clock), the main part directed outward (figure 4). Accordingly, the thorns 11 are mainly directed outward, as well as the droplets 12 (sprays) flying out from their tips [8].

Examination of the videos shows that thin fast fine jets 13, forming line structures at the bottom of the cavity 6 (figure 4), emerge from the crests of the contact line 4, pass the boundary of flow components 7, form a line texture 13 of the crown wall 8, penetrate the veil 10 and protrude in the form thorns 11. The flow pattern is complicated by inhomogeneities 14 that are three-dimensional capillary waves traveling from the upper edge of the crown 2 to the bottom of the cavity 6.

The formation of fast fine jets (trickles) is supported by the conversion of the available potential surface energy into other forms when the free surfaces of the coalescing liquids are eliminated. Transformed energy creates large positive perturbations in the thin layer where two fluids are merged rather fast, accelerate this layer and induced its decay on separated trickles. Forming fine fast jets are rather stable. They propagate along the bottom of the cavity, wall of the crown, and form thorns on the rim of the external veil, from the tip of which small droplets are successfully pushed out.

The complexity and fast restructuring of the droplet impact flow patterns indicates the influence of all mechanisms of energy transfer - with a fluid flow, with the group velocity of various waves, with diffusion momentumless transfer, and fast direct atomic-molecular interactions (for example, in the conversion of the available potential surface energy during the coalescence of fluids), on the dynamics and structure of flows.

5. Conclusion

The dynamics and structure of liquid and gas flows are characterized by a system of fundamental equations that is analogs of conservation laws. The physical quantities included in the system are observable, that is the error can be estimated during measurements. The involvement of a scaled and parametrically invariant system of fundamental equations has room for the study of unsteady energetic flows and more accurately describes their dynamics and structure in the whole range of scales from microscopic to global.

The sets of functions obtained taking into account the compatibility condition of the system of equations, which is characterized by the rank, order of the linearized version, or the degree of the characteristic (dispersion) equation, constitute a complete solution, containing regular and singular perturbed functions describing wave, vortices, and family of ligaments.

In an adequate hydrodynamic experiment, it is necessary to register the field of flows containing basic large-scale components that are waves, vortices, wakes, jets and resolve the microstructural components of the processes described by ligaments, control the completeness of the method, and evaluate the data errors.

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References

[1] Light Echo of V838 Monoceros Star flash http://www.astronet.ru/db/msg/1254974
[2] Landau L D and Lifshitz E M 1987 Fluid Mechanics vol 6 Course of Theoretical Physics (Oxford: Pergamon Press) p 593
[3] Müller P 2006 The Equations of Oceanic Motions (Cambridge: CUP) p 291
[4] Vallis G K 2017 *Atmospheric and Oceanic Fluid Dynamics* 2nd ed (Cambridge: CUP) p 946

[5] Mendeleeff D I 1875 *On elasticity of gases* (Saint Petersburg) p 262 (in Russian)

[6] Gibbs J W 1902 *Elementary Principles in Statistical Mechanics* (New York: Charles Scribner's sons) p 207

[7] Feistel R 2018 Thermodynamic properties of seawater, ice and humid air: TEOS-10, before and beyond *Ocean Sciences* 14 471

[8] Nayfeh A H 2011 *Introduction to Perturbation Techniques* (Wiley-VCH) p 533

[9] Chashechkin Y D 2019 Evolution of the fine structure of the matter distribution of a free-falling droplet in mixing liquids *Atmospheric and Oceanic Physics* 55(3) 285

[10] Dimitrieva N F and Chashechkin Y D 2018 Fine structure of stratified flow around a fixed and slow moving wedge *Oceanology* 58(3) 340

[11] Chashechkin Y D 2018 Singularly perturbed components of flows – linear precursors of shock waves *Math. Model. Nat. Phenom.* 13(2) 1

[12] Unique Science Facility “Hydrophysical complex for modeling hydrodynamic processes in the environment and their impact on underwater technical objects, as well as the transport of impurities in the ocean and atmosphere (USF “HPC IPMech RAS”)”

http://www.ipmnet.ru/uniquequip/gfk/#equip

[13] Chashechkin Y D 2021 Conventional partial and new complete solutions of the fundamental equations of fluid mechanics in the problem of periodic internal waves with accompanying ligaments generation *Mathematics* 9(6) 586

[14] Chashechkin Y D and Zagumennyi I V 2020 Visualization of stratified flows around a vertical plate: laboratory experiment and numerical simulation *Int. J. Comp. Meth. Exp. Meas.* 8(2) 148

[15] Chashechkin Y D and Ilinykh A Y 2021 Drop decay into individual fibers at the boundary of the contact area with the target fluid *Doklady Physics* 66(2) 20