ESTIMATING AND ANALYZING NEURAL FLOW USING SIGNAL PROCESSING ON GRAPHS

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ABSTRACT

Neural communication is fundamentally linked to the brain’s overall state and health status. We demonstrate how communication in the brain can be estimated from recorded neural activity using concepts from graph signal processing. The communication is modeled as a flow signals on the edges of a graph and naturally arises from a graph diffusion process. We apply the diffusion model to local field potential (LFP) measurements of brain activity of two non-human primates to estimate the communication flow during a stimulation experiment. Comparisons with a baseline model demonstrate that adding the neural flow can improve LFP predictions. Finally, we demonstrate how the neural flow can be decomposed into a gradient and rotational component and show that the gradient component depends on the location of stimulation.

Index Terms— network neuroscience, neural communication, graph diffusion, edge flows

1. INTRODUCTION

Analyzing communication and flow processes in the brain has received much attention in past years \cite{1}. Understanding and modeling such processes is thought to be crucial for developing better treatments for neurological diseases \cite{2}. Especially using graphs to study neural processes is a promising direction as they can naturally model structural and functional brain connectivity \cite{3,4}. While most research has focused on analyzing the graph that is associated with a certain pattern of neural activity using concepts from graph theory, more recently, graph signal processing has been proposed as a way to study neural signals that are observed at the nodes of an underlying brain network \cite{5,6,7}. The central idea of graph signal processing is that, rather than focusing on the graph itself, we are interested in processing signals indexed by the nodes of the graph, for example, by finding a graph spectral representation of the signal, or performing filtering in the graph domain \cite{8}.

Graph signal processing in neuroscience has, to the best of our knowledge, exclusively focused on signals defined on the nodes of an underlying brain graph. Our goal is to shift the focus from signals at the nodes to flow signals that live on the edges of a graph and that characterize the communication flow in the brain. Specifically, we will address the following questions: 1) How can we leverage methods from graph signal processing to estimate the neural flow in the brain? 2) How can theory of signal processing on higher order networks \cite{9,10} be used to analyze neural flow signals?

This new perspective allows for modeling neural flow on a finer temporal scale than other techniques used in neuroscience such as Granger causality \cite{11,12,13}. To test our theoretical findings, we will apply our model to estimate neural flow from electrophysiological recordings obtained from monkeys during a stimulation experiment.

The remainder of the paper is outlined as follows: In Sec. 2 we describe the model that turns neural recordings into a flow signal. In Sec. 3 the model is used to estimate the neural flow in two non-human primates during a stimulation experiment. Sec. 4 describes ways to analyze the neural flow signal by decomposing it into different components. Finally, Sec. 5 summarizes the main findings and points towards future research directions.

2. ESTIMATING NEURAL FLOWS FROM TIME SERIES OF NEURAL ACTIVITY

In this section, we describe how a graph diffusion model can be used to estimate neural flow signals from vector time series of neural activity. Diffusion models have been used in the graph signal processing community to describe the dynamic behavior of network data \cite{8,14,15}, as well as in the neuroscience community to model functional connectivity \cite{15}. Instead of focusing on functional connectivity, here we demonstrate how neural activity on the nodes of a graph can be transformed into an edge flow signal using a diffusion process.

We start by assuming a graph with \( N \) nodes and \( E \) edges. Furthermore, time series of neural activity are measured at the nodes of the graph. We will use \( s[t] \in \mathbb{R}^{N \times 1} \) to denote the neural activity at time \( t \) across all nodes. An example of this is shown on the left of Fig. 1 with the graph topology (nodes and edges) in black and the node time series \( s_i[t] \) in blue. Our goal is to use the given graph topology and observed node time series to estimate the time dependent information flow along the edges of the graph. This flow is illustrated on the right in Fig. 1.

2.1. 1st Order Diffusion Model

To estimate the flow, we use a parameterized diffusion model that naturally gives rise to an edge flow and whose parameters can be estimated from the observed node time series. In a nutshell, for each node the model expresses the node signal at the current time step as its weighted own past (memory) plus the inflow minus the outflow from all neighboring nodes. To describe this mathematically, we first encode the graph topology using the node-to-incidence matrix \( B \in \mathbb{R}^{N \times E} \), where each column represents an edge \( e = (i,j) \) with tail node \( i \) and head note \( j \). \( B \) is defined such that \( B_{ie} = -B_{je} = -1 \) and \( B_{ke} = 0 \) otherwise. For every edge, it is thereby arbitrary which of the incident nodes is the tail and head node. For more details and examples for \( B \), we refer the reader to \cite{9}. Using \( B \), our diffusion
To describe the neural flow of information induced by the laser stimulation, first the μ-ECoG array is used to construct a sparsely connected graph, where each node is connected approximately to its 8 nearest neighbors. The LFP time series measured by each electrode are demeaned and treated as the node signals $s[t]$. The defined graph topology along with $s[t]$ starting at the time of stimulation of the first laser until 30 ms after stimulation from the second laser (first laser if only one is used) are used to estimate the diffusion flow model parameters $M_k$ and $W_k$. The flow can now be computed according to (4). Examples of neural flow for 5 ms and 10 ms after stimulation are shown in Fig. 2 for a model order of $K = 5$. The location of stimulation is indicated by the cross. One can see that 5 ms after stimulation the flow magnitude is mainly located near the stimulation site, whereas at 10 ms the flow has spread further into the network.

To assess the importance of the flow part of the model, we demonstrate that the estimated flow can improve LFP predictions over a no-flow baseline model. The no-flow model is obtained by using (3) with the constraint $W_k = 0$ ∀ $k$. That is the no-flow model reduces to a simple $K^{th}$ order autoregressive model with no connections between the nodes. Both the flow and no-flow model are fitted to all the $K^{th}$ lag. The node signals can now be transformed into the edge flow domain via

$$f[t] = \sum_{k=1}^{K} W_k B^T s[t - k]. \quad (4)$$

### 2.3. Estimating the Diffusion Model Parameters

The parameters of the $K^{th}$ order diffusion model $M_k$ and $W_k$ can be estimated from neural recordings by minimizing the least squares prediction error

$$e = \sum_{t \in T_e} ||s[t] - \hat{s}[t]||^2; \quad (5)$$

where $s[t]$ is the predicted neural activity according to (3) and $\hat{s}[t]$ the observed neural activity. $T_e$ denotes the set of all time points $t$ used for fitting the model. Note that this set does not need to contain successive indices.
Finally, we compute by how much percent the LFP predictions over the flow set. For each stimulation block, the model is independently fitted and predicted LFPs $\hat{s}$ as a one-step-ahead prediction on the remaining 20%. Then, for each session, the estimated model parameters are used for 80% of the trails (training set) in each session. For all 63 sessions, our proposed flow model improves the LFP predictions over the no-flow model (%)

$$I(s) = 100 \cdot \text{median}_t \left\{ \frac{\text{RMSE}_{s,\text{no flow}}[t] - \text{RMSE}_{s,\text{flow}}[t]}{\text{RMSE}_{s,\text{no flow}}[t]} \right\}.$$ (6)

The distribution of $I(s)$ for all 63 sessions is shown in Fig. 3 for $K = 1$, and 9. It is notable that, for $K = 9$, the flow model always predicts the LFPs better than the no-flow model. This is in fact true for all model orders greater than 1 (for $K = 1$ the flow model predicts the LFPs better than the no-flow model for 62 out of 63 sessions). With increasing model order, the average improvement across all sessions increases from 0.77% for $K = 1$ (statistical significant; p-value: $2.2 \cdot 10^{-3}$; Student’s t-test) to 3.42% for $K = 9$ (statistical significant; p-value: $2.3 \cdot 10^{-12}$; Student’s t-test).

4. ANALYZING NEURAL FLOW SIGNALS

So far we have illustrated how neural flow arises as the result of a graph diffusion process. In this section, we will show some preliminary results demonstrating how we can analyze the obtained flow signals and how they depend on the location of laser stimulation.

4.1. Decomposing Flow into Gradient and Rotational Component

Only very recently, efforts have been made to develop a signal processing theory, for signals supported on edges and higher order networks [9,10,18]. This includes defining a spectral representation based on the Hodge-Laplacian that can be used to decompose a flow signal into different gradient and rotational modes, as well as defining flow filters based on this notion [19]. Here we will demonstrate how the neural flow can be decomposed into its gradient and rotational component.

Any flow signal defined on the edges of a graph can be decomposed into a gradient and rotational component [13]:

$$f = f_{\text{grad}} + f_{\text{rot}},$$ (7)

where $f_{\text{grad}}$ and $f_{\text{rot}}$ have the following properties:

$$B f_{\text{grad}} > 0$$ (8)

$$B f_{\text{rot}} = 0.$$ (9)

Recall that $B$ applied to a flow signal computes its divergence, i.e., the sum of all inflow minus outflow at each node. That is, the rotational component $f_{\text{rot}}$ is flow preserving at each node, whereas the gradient flow has non-zero divergence at least for some of the nodes causing those nodes to act more as sources (outflow > inflow) or sinks (inflow > outflow).

The gradient flow can be further decomposed into $N - 1$ orthogonal modes

$$f_{\text{grad}} = f_{\text{grad},1} + \ldots + f_{\text{grad},N-1}$$

with $f_{\text{grad},i} \perp f_{\text{grad},j}$ for $i \neq j$, (10)

where $N$ is the number of nodes in the graph. These gradient flow modes can be ordered in increasing amount of divergence. That is

$$B f_{\text{grad},1} \| f_{\text{grad},1} \|_2 \leq B f_{\text{grad},2} \| f_{\text{grad},2} \|_2 \leq \ldots \leq B f_{\text{grad},N-1} \| f_{\text{grad},N-1} \|_2.$$ (11)

Gradient modes with small divergence can be interpreted as smooth or low-frequency gradient flows on the graph, whereas modes with high divergence are said to be high-frequency gradient flows. Based on this notion, we can filter out the high-frequency gradient modes to obtain a smooth gradient flow [19].
The decomposition of the neural flow signal \( f \) 10 ms after stimulation into \( f_{\text{grad}} \) and \( f_{\text{rot}} \) for the same experimental session as in Fig. 2 and model order \( K = 5 \) is illustrated in Fig. 3. For a better visual representation, we have filtered out the gradient flow modes with high divergence (3 dB cutoff approximately at \( f_{\text{grad},14} \)) and only plotted the filtered (smooth) gradient flow in Fig. 2(left). We expect that the neural activity spreads from the stimulation location into other parts of the network. This is in agreement with the gradient flow signal in Fig. 4 showing flow spreading away from the stimulation site (black cross).

The behavior of the rotational flow illustrated in Fig. 4(right) is less intuitive and a decomposition into different modes is not straightforward. Nevertheless, the rotational flow may be of value as it can be studied in the edge flow domain. In the future, we plan to analyze the rotational flow and its dependence on the location of stimulation and other experimental parameters.

### 4.2. Relation Between Gradient Flow and Stimulation Location

Fig. 4(a) indicates that the gradient flow depends on the location of stimulation. To quantify that, we can use the gradient flow to determine the location of the global broadcaster \( x_b \) in the network and compare it to the stimulation location \( x_s \). We hypothesize that \( x_b \) is significantly closer to \( x_s \) than a randomly placed broadcaster. The steps to compute the location of the global broadcaster are as follows:

1. Treat the entries of the gradient flow component as edge weights of a directed graph and construct the adjacency matrix \( A \) of this directed graph. If we denote the entry of \( f_{\text{grad}} \) corresponding to the edge between node \( i \) and \( j \) as \( f_{\text{grad}}^{(i,j)} \), \( A(i,j) = f_{\text{grad}}^{(i,j)} \) if there is a positive flow from node \( i \) to node \( j \), \( A(j,i) = -f_{\text{grad}}^{(j,i)} \) if there is a positive flow from \( j \) to \( i \), and \( A(i,j) = 0 \) else.

2. Compute the aggregate downstream reachability (ADR) for each node according to [20]. For a given node \( n \), the ADR denotes how well any other node in the network can be reached from \( n \) following the direction of the flow. That is, a high ADR for a node indicates flow spreading away from this node.

One can clearly see that, as expected, the distance of the global broadcaster \( x_b \) is significantly smaller than \( x_s \) (p-value: \( 3.1 \times 10^{-22} \); Student’s t-test), which supports the hypothesis that the gradient flow depends on the stimulation location.

### 5. CONCLUSION

We have illustrated how neural flow naturally arises as the result of a graph diffusion process. Furthermore, this diffusion process can be extended to higher orders to allow for variable flow delays between different nodes of the graph. To validate our diffusion model, we have applied it to neural recordings from monkeys obtained during a stimulation experiment and have shown that the flow part of the model improves the prediction of LFPs. Finally, we have demonstrated how the neural flow can be decomposed into a gradient and rotational component. The gradient component depends on the location of stimulation and generally shows flow spreading away from the stimulation site.

In the future, we plan to investigate the relation between our diffusion model and vector autoregressive models that are widely used in the neuroscience community. Additionally, we want to study the flow signals obtained from the diffusion model in more detail by further analyzing their spatiotemporal patterns and better incorporating the rotational component.
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