Effects of an external magnetic field on the fluctuations of quark number, fluctuations and correlations of conserved charges, including baryon number, electric charge and strangeness, are studied in the 2+1 flavor Polyakov–Nambu–Jona-Lasinio model. We find that magnetic field increases fluctuations and correlations in the regime of chiral crossover. It makes the transition of quadratic fluctuations more abrupt, and the peak structure of quartic fluctuations more pronounced. Our calculations indicate that $\chi^B_1/\chi^B_2$, $\chi^Q_1/\chi^Q_2$, and $\chi^{BQ}_{11}$ are very sensitive to the external magnetic field and maybe can be used for probes for the strong magnetic field produced in the early stage of noncentral collisions.

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I. INTRODUCTION

Studies of influences of strong magnetic fields on hot QCD matter have attracted lots of attentions in recent years. It is estimated that the strength of magnetic fields produced in noncentral relativistic heavy ion collisions can reach up to $eB \sim 0.1m^2_\pi$ for SPS, $eB \sim m^2_\pi$ for RHIC, and $eB \sim 15m^2_\pi$ for LHC \[1\]. Furthermore, The strength of magnetic fields produced in the early universe where the electroweak phase transition happened, may even reach up to $eB \sim 200m^2_\pi$ \[2\]. An interesting phenomenon related to magnetic fields in noncentral heavy ion collisions is the electric charge separation along the system’s orbital angular momentum axis \[3, 4\]. The observed electric charge separation can be explained as the chiral magnetic effect, where an electric current is induced along the direction of magnetic field and positive charges are separated from negative ones in parity-odd domains \[5–10\].

Effects of strong magnetic fields on the chiral and deconfinement phase transitions have been studied within the frameworks of effective models and lattice-QCD simulations. In a two-flavor Polyakov–Nambu–Jona-Lasinio (PNJL) model, it was observed that the external magnetic field works as a catalyzer of dynamical symmetry breaking and the critical temperature increases with the strength of $B$ \[11\]. Calculations of a linear sigma model coupled to quarks and to the Polyakov loop indicated that the chiral and deconfinement lines split and both chiral and deconfining critical temperature increase with $B$ \[12\]. Furthermore, it was also found within effective models that the transition strength increases when the magnetic field strength is increased \[13\]. Based on a renormalization group (RG) analysis, Fukushima and Pawlowski found a reason for the magnetic catalysis for the dynamical chiral symmetry breaking \[14\], which was observed in various effective model calculations: when a strong magnetic field is applied, the transverse dynamics of charged particles is frozen and the dimensional reduction takes place. In this case the RG flow of the dimensionless coupling results in a divergent coupling, no matter how small the initial coupling is. However, this happens only when the initial coupling exceeds a critical value in the case $B = 0$.

Two flavor lattice-QCD simulations in Ref. \[15\] found that both the deconfinement and chiral restoring critical temperatures increases with $B$, but they did not find the splitting of the chiral phase transition. They also found that the transition strength increases with increasing $B$. However, another state-of-the-art lattice simulations with 2+1 flavors of stout smeared staggered quarks with physical masses found the transition temperature decreases
with increasing magnetic field, which is inconsistent with various model calculations, but they also found the transition strength increasing mildly with $B$ [16]. With the framework of QCD effective potential for the homogeneous Abelian gluon field, it was found that the strong magnetic field catalyze the deconfinement transition [17].

Within the framework of 2+1 flavor Polyakov-loop improved NJL model [18], in this work we study the effects of the magnetic field on the fluctuations and correlations of quark number and conserved charges, e.g., baryon number, electric charge, and strangeness. Fluctuations and correlations of conserved charges are sensitive to the degrees of freedom of the thermal strongly interacting matter and behave quite differently between the hadronic and quark gluon plasma (QGP) phases [19, 20]. Fluctuations and correlations are usually enhanced near the QCD phase transitions, and are related to the critical behavior of the QCD thermodynamics [21–25]. Furthermore, fluctuations and correlations of conserved charges can be measured with event-by-event fluctuations in heavy ion collision experiments [20, 23, 26, 27], and so they are valuable probes of the deconfinement and chiral restoring phase transitions [28–31].

The fluctuations of conserved charges as well as the correlations among them without external magnetic field have been studied in the 2+1 flavor PNJL model [32, 33]. The calculated results were compared with lattice simulations performed with an improved staggered fermion action with almost physical up and down quark masses and a physical value for the strange quark mass. It was found that the calculated results of effective model are well consistent with those obtained in lattice simulations [32], which indicates that the 2+1 flavor PNJL model is well suitable for the calculations of the cumulants of conserved charge multiplicity distributions. This computation of effective model was also extended to study the fluctuations and correlations near the QCD critical point and many interesting results are obtained [33]. Since the fluctuations and correlations of conserved charges can be observed in heavy ion collision experiments, it is expected that effects of the strong magnetic fields produced in the early evolution stage of noncentral collisions are imprinted onto these observables. In another word, maybe we can employ the fluctuations and correlations to infer the presence or information of the magnetic field. But before this idea comes true, we have to study the influences of an external magnetic field on the fluctuations and correlations of quark number and conserved charges, which is our focus in this work.

The paper is organized as follows. In section II we introduce the effective model and
the fluctuations and correlations of conserved charges. In Sec. III we show the calculated results of the deconfinement and chiral restoring phase transitions. In Sec. IV we give our calculated results of fluctuations of light quarks (up and down quarks) and strange quarks in an external magnetic field. Section V shows the fluctuations of conserved charges and correlations among them in an external magnetic field. In Sec. VI we present our summary and conclusions.

II. EFFECTIVE MODEL

In this work, we employ the 2+1 flavor Polyakov-loop improved NJL model to study the fluctuations and correlations of conserved charges in an external magnetic field. We begin with the 2+1 flavor PNJL model (for more details about the PNJL model, see Ref. [18] and references therein), whose Lagrangian density reads

\[ \mathcal{L}_{\text{PNJL}} = \sum_{f=u,d,s} \bar{\psi}_f (i \gamma_\mu D^\mu_f + \gamma_0 \mu_f - m_{0f}) \psi_f + G \sum_{a=0}^{8} \left[ (\bar{\psi} \tau_a \psi)^2 + (\bar{\psi} i \gamma_5 \tau_a \psi)^2 \right] \\
- K \left[ \text{det}_f (\bar{\psi} (1 + \gamma_5) \psi) + \text{det}_f (\bar{\psi} (1 - \gamma_5) \psi) \right] - U(\Phi, \Phi^*, T), \]  

(1)

where the covariant derivative \( D_{\mu f} = \partial_{\mu} + i q_f e a_{\mu} - i A_{\mu} \) couples the quark field \( \psi_f \) to the electromagnetic field (here we denote it as \( a_{\mu} \)) and the background gluon field \( A_{\mu} \). \( q_f \) \((f = u, d, s)\) is the electric charge in unit of elementary electric charge \( e \) for the quark of flavor \( f \). Usually in the Polyakov-loop improved effective models, we only keep the temporal component of the background gluon field, i.e., \( A^\mu = \delta^\mu_0 A_0 \), with

\[ A^0 = g A^0_a \frac{\lambda_a}{2}, \]  

(2)

where \( \lambda_a \) are the Gell-Mann matrices in color space. \( m_{0f} \) and \( \mu_f \) in Eq. (1) are the current quark masses and the quark chemical potentials, respectively. We choose \( m_{0u} > m_{0d} = m_{0s} \equiv m_{0l} \) throughout this work, which breaks the \( SU(3)_f \) symmetry. In addition to the quark chemical potentials, we will encounter chemical potentials for conserved charges in the following discussions, e.g., \( \mu_B, \mu_Q, \) and \( \mu_S \), which are the chemical potentials for the baryon number, electric charge, and strangeness, respectively. They are related with the quark chemical potentials through the following relations:

\[ \mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q, \quad \mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q, \quad \text{and} \quad \mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S. \]  

(3)
$U(\Phi, \Phi^*, T)$ in Eq. (1) is the Polyakov-loop effective potential, which is usually a function of an order parameter of the deconfinement phase transition, the traced Polyakov loop $\Phi = (\text{Tr}_c L)/N_c$, and its conjugate $\Phi^* = (\text{Tr}_c L^\dagger)/N_c$. The Polyakov loop is linked to the background gluon field through

$$L(\vec{x}) = \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right] = \exp \left[ i\beta A_4 \right], \quad (4)$$

where $\mathcal{P}$ denotes path ordering; $\beta = 1/T$ is the inverse of temperature and $A_4 = iA^0$. The functional form and parametrization of the Polyakov-loop effective potential in the PNJL model are determined phenomenologically by fitting the thermodynamical behavior of the Polyakov dynamics for the pure gauge field. In this work, we employ the Polyakov-loop effective potential which is a polynomial in $\Phi$ and $\Phi^*$ [34], given as

$$\frac{U(\Phi, \Phi^*, T)}{T^4} = -\frac{b_2(T)}{2} \Phi^* \Phi - \frac{b_3}{6} (\Phi^3 + \Phi^*^3) + \frac{b_4}{4} (\Phi^* \Phi)^2, \quad (5)$$

with

$$b_2(T) = a_0 + a_1 \left( \frac{T_0}{T} \right) + a_2 \left( \frac{T_0}{T} \right)^2 + a_3 \left( \frac{T_0}{T} \right)^3. \quad (6)$$

Parameters in the effective potential are $a_0 = 6.75$, $a_1 = -1.95$, $a_2 = 2.625$, $a_3 = -7.44$, $b_3 = 0.75$, $b_4 = 7.5$, and $T_0 = 270$ MeV, which are fixed according to lattice simulations. Furthermore, five parameters in the quark sector of the model are determined to $m_0^l = 5.5$ MeV, $m_0^s = 140.7$ MeV, $G\Lambda^2 = 1.835$, $K\Lambda^5 = 12.36$, and $\Lambda = 602.3$ MeV. They are fixed by fitting the properties of low energy mesons: $m_\pi = 135.0$ MeV, $m_K = 497.7$ MeV, $m_{\eta'} = 957.8$ MeV and $f_\pi = 92.4$ MeV [35].

We consider a homogeneous magnetic field $B$ along the $z$-direction. In the mean field approximation, one can obtain the thermodynamical potential density of the PNJL model in an external magnetic field. Details of this calculation can be found in Ref. [10]. Here we
just give the final result as follows

\[ \Omega = -N_c \sum_{f=u,d,s} \frac{|q_f|eB}{2\pi} \sum_{n=0}^{\infty} \alpha_n \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \left( f_\Lambda^2(p_f)E_f + \frac{T}{3} \ln \left\{ 1 + 3\Phi^* \exp \left[ -\left( E_f - \mu_f \right)/T \right] + 3\Phi \exp \left[ -2\left( E_f - \mu_f \right)/T \right] + \exp \left[ -3\left( E_f - \mu_f \right)/T \right] \right\} \right) + \frac{T}{3} \ln \left( 1 + 3\Phi \exp \left[ -\left( E_f + \mu_f \right)/T \right] + 3\Phi^* \exp \left[ -2\left( E_f + \mu_f \right)/T \right] + \exp \left[ -3\left( E_f + \mu_f \right)/T \right] \right) + 2G(\phi_u^2 + \phi_d^2 + \phi_s^2) - 4K\phi_u \phi_d \phi_s + U(\Phi, \Phi^*, T), \] (7)

where we have

\[ p_f = \sqrt{2n|q_f|eB + p_z^2} \] (8)

and

\[ E_f = \sqrt{2n|q_f|eB + p_z^2 + M_f^2}. \] (9)

The constituent mass is

\[ M_i = m_{0i} - 4G\phi_i + 2K\phi_j \phi_k, \] (10)

where \( \phi_i \) is the chiral condensate \( \langle \bar{\psi}\psi \rangle_i \). Since charged particles in the lowest order Landau level are polarized by the external magnetic field, the spin-degeneracy factor \( \alpha_n \) in Eq. (7) is 1 for \( n = 0 \) and 2 otherwise. Furthermore, we should mention that when the magnetic field is strong, the sharp cutoff usually used for the vacuum part in the PNJL model has a problem to introduce cutoff artifact. To avoid this problem, we use a smooth cutoff instead of the sharp one, which is realized by introducing a cutoff function \( f_\Lambda(p) \) in the vacuum part as Eq. (7) shows. We adopt the form of the cutoff function in Ref. [11], as given by

\[ f_\Lambda(p) = \sqrt{\frac{\Lambda^{2N}}{\Lambda^{2N} + p^{2N}}}. \] (11)

\( N = 10 \) is chosen in our numerical calculations. As one can see, \( f_\Lambda(p) \) is in fact the sharp cutoff function \( \theta(\Lambda - p) \) in the limit \( N \to \infty \).

From stationary conditions, we obtain a set of equations of motion by Minimizing the thermodynamical potential in Eq. (7) with respect to three-flavor quark condensates, Polyakov
loop $\Phi$ and its conjugate $\Phi^*$. These equations of motion can be solved as functions of temperature $T$, strength of magnetic field $B$, three-flavor quark chemical potentials $\mu_u$, $\mu_d$, and $\mu_s$ or conserved charge chemical potentials $\mu_B$, $\mu_Q$, and $\mu_S$.

Substituting solutions of the equations of motion into Eq. (7), one can also obtain the thermodynamical potential density and the pressure ($P = -\Omega$) of a thermodynamical system in the mean field approximation. Then we can calculate the derivatives of the pressure with respect to three conserved charge chemical potentials, i.e.,

$$\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k}(P/T^4)}{\partial(\mu_B/T)^i \partial(\mu_Q/T)^j \partial(\mu_S/T)^k},$$

which generalizes the quark number susceptibility to a more general expression. In fact, the generalized susceptibilities $\chi$’s in Eq. (12) are related to the cumulants of the conserved charge multiplicity distributions, which can be observed in heavy ion collision experiments. For example, the relations between the second and higher order susceptibilities and the fluctuations of conserved charges are given by

$$\chi_X^2 = \frac{1}{VT^3} \langle \delta N_X^2 \rangle,$$

$$\chi_X^4 = \frac{1}{VT^3} \left( \langle \delta N_X^4 \rangle - 3 \langle \delta N_X^2 \rangle^2 \right),$$

$$\chi_X^6 = \frac{1}{VT^3} \left( \langle \delta N_X^6 \rangle - 15 \langle \delta N_X^4 \rangle \langle \delta N_X^2 \rangle - 10 \langle \delta N_X^3 \rangle^2 + 30 \langle \delta N_X^2 \rangle^3 \right),$$

where $\delta N_X = N_X - \langle N_X \rangle$ ($X = B, Q, S$) and $\langle N_X \rangle$ is the ensemble average of the conserved charge number $N_X$. $V$ is the volume of the system. In the same way, the mixed cumulants of conserved charge distributions, i.e., the correlations among conserved charges, can also be expressed as their corresponding generalized susceptibilities, e.g.,

$$\chi_{11}^{XY} = \frac{1}{VT^3} \langle \delta N_X \delta N_Y \rangle.$$

In this work, we only consider the cases with $\mu_{B,Q,S} = 0$, in which the generalized susceptibilities are nonvanishing only when $i + j + k$ is even.

### III. CHIRAL AND DECONFINEMENT PHASE TRANSITIONS

In this section we focus on the chiral and deconfinement phase transitions of the 2+1 flavor PNJL model with an external magnetic field. Figure 1 shows the chiral phase transition, where the constituent quark masses are plotted as functions of the temperature at
FIG. 1. (color online). Constituent masses of quarks as functions of the temperature calculated in the PNJL model with several values of $eB$ in unit of $m_N^2$. Left panel is for the $u$ and $d$ light quarks, and right panel is for the strange quarks. $\mu_{B,Q,S} = 0$ is chosen throughout our work.

several values of the magnetic field strength. We find that the pseudocritical temperature for the chiral restoring phase transition, which is defined by the position of the peak of $|dM_{u,d}/dT|$ as a function of $T$, increases from 224 to 240 MeV, when $eB$ is increased from 0 to $20m_N^2$. It is also found that the constituent quark masses and the chiral condensates increase with $B$ at a given temperature, which is more pronounced at low temperature as Fig. 1 shows. The dependence of the chiral phase transition temperature on the strength of an external magnetic field obtained in the 2+1 flavor PNJL model, are consistent with former effective model computations of two flavor systems [11–13], and the expected magnetic field-temperature phase diagram of QCD (as shown in Fig.1 in Ref. [12]) where the chiral critical temperature increases with $B$, while the deconfining one decreases with increasing $B$. Like various effective models, our result is in conflict with that of the state-of-the-art lattice simulations [16], where it was predicted that the chiral critical temperature decreases with increasing magnetic field. The left panel of Fig. 1 also shows an interesting result for light quarks: there is a split between the $u$ and $d$ constituent masses when the magnetic field is nonvanishing, and this split becomes more prominent with the increase of $B$. The split between light quarks at finite $B$ indicates that the $SU(2)$ symmetry between $u$ and $d$ quarks is broken under the influence of an external magnetic field, since the $u$ and $d$ quarks have different electric charges. But this broken effect is not significant even $eB$ is increased...
to the maximal value $20m^2_{\pi}$ of our calculations, as Fig. 1 shows.

![Graph showing Polyakov loop as a function of temperature for different values of $eB$.]

**FIG. 2.** (color online). Polyakov loop as a function of $T$ at several values of $eB$.

In Fig. 2 we show the deconfinement phase transition at several values of $eB$, which is characterized by the dependence of the deconfining order parameter, i.e., the Polyakov loop, on the temperature. One can see that the impact of the magnetic field on the Polyakov loop dynamics is much smaller than that on the chiral phase transition, in particular, when the temperature is high, where there is almost no difference among the several curves in Fig. 2. However, since the Polyakov loop dynamics is entangled with the chiral one, especially at low temperature, the pseudocritical temperature for the deconfinement phase transition increases a little with $B$ as well, which is inconsistent with lattice simulations [16] and the expected magnetic field-temperature phase diagram of QCD. This problem also appears in former effective model calculations [11, 12]. This insufficiency of these effective models is due to the fact that in these models, the Polyakov-loop effective potential which governs the Polyakov-loop behavior is introduced by hand. The dynamical couplings between the gluon field and the external magnetic field at large $B$, are not included in these models [14, 17].

Figure 3 shows the derivatives of the $u$ quark constituent mass and the Polyakov loop with respect to $T$ at several values of $eB$. We observe that the peak value of $|dM_u/dT|$ increases with $B$, which indicates that the chiral phase transition becomes sharper when $B$ is increased. In another word, the strength of the chiral phase transition increases with $B$. This result agrees with two flavor effective model calculations [12, 13] and lattice simulations [16].
Furthermore, we also find that the peak value of $d\Phi/dT$ increases a little with $B$.

To summarize the results in this section, we should mention that effective models and the state-of-the-art lattice simulations give different results on how the chiral and deconfining critical temperatures are influenced by an external magnetic field, and the interplay between the external magnetic field and the gluonic dynamics and that between the magnetic field and the chiral dynamics are not fully understood. However, effective models and lattice simulations all consistently predict that the strength of the phase transition increases with $B$. We mainly focus on fluctuations and correlations of quarks and conserved charges in an external magnetic field in this work. We will show below that our conclusions are mainly based on the fact that the transition strength of the crossover increases with $B$. Therefore, we expect that results about the fluctuations and correlations obtained in the 2+1 flavor PNJL model are reliable.

**IV. FLUCTUATIONS OF QUARKS**

In this work we employ the method of Taylor expansion to calculate the fluctuations and correlations given in Eq. 12. First of all, we focus on the influences of an external magnetic field on the fluctuations of quarks. We plot quadratic, quartic and sixth-order fluctuations of $u$ quarks as functions of $T$ at several values of $eB$ in Fig. 4. One can see that the quadratic fluctuations $\chi_u^2$ above the pseudocritical temperature increase with $B$. 

FIG. 3. (color online). $|dM_u/dT|$ (left panel) and $d\Phi/dT$ (right panel) versus temperature at several values of $eB$. 

FIG. 4. Quadratic, quartic and sixth-order fluctuations of $u$ quarks as functions of $T$ at several values of $eB$. 

To summarize the results in this section, we should mention that effective models and the state-of-the-art lattice simulations give different results on how the chiral and deconfining critical temperatures are influenced by an external magnetic field, and the interplay between the external magnetic field and the gluonic dynamics and that between the magnetic field and the chiral dynamics are not fully understood. However, effective models and lattice simulations all consistently predict that the strength of the phase transition increases with $B$. We mainly focus on fluctuations and correlations of quarks and conserved charges in an external magnetic field in this work. We will show below that our conclusions are mainly based on the fact that the transition strength of the crossover increases with $B$. Therefore, we expect that results about the fluctuations and correlations obtained in the 2+1 flavor PNJL model are reliable.
FIG. 4. (color online). Quadratic (top), quartic (middle), and sixth-order (bottom) fluctuations of $u$ quarks as functions of the temperature calculated in the PNJL model with several values of $eB$.

Furthermore, the evolution of $\chi_u^2$ with $T$ during the chiral crossover becomes sharper as $B$ is increased. Similar results are also found in the computations of $\chi_u^4$ and $\chi_u^6$. We find that the peak value of quartic fluctuations and the amplitude of oscillations of $\chi_u^6$ during the chiral crossover are greatly enhanced with the increase of $B$. As we have found in the last section that the transition strength of the crossover increases with $B$, it is natural to expected that the fluctuations increase with $B$ as well, since fluctuations are closely related with the strength of a crossover. When the crossover becomes an exact second-order phase transition, the fluctuations should be divergent.

We show the mixed susceptibility between $u$ and $d$ quark numbers, i.e., the second-
order correlation between light quarks, at several values of $eB$ in Fig. 5. Our calculated result indicates that the value of $\chi_{11}^{ud}$ is negative, and there is an inflection point at the pseudocritical temperature in the curve of $\chi_{11}^{ud}$ as a function of $T$ when there is no magnetic field. With the increase of $B$, the inflection point develops a complicated dependent behavior on $T$, where $\chi_{11}^{ud}$ oscillates during the crossover. In order to compare $\chi_{11}^{ud}$ with $\chi_{2}^{u}$, we plot their ratio in the right panel of Fig. 5. One can observe that $\chi_{11}^{ud}$ is comparable to $\chi_{2}^{u}$ when the temperature is below the pseudocritical temperature. However, when the temperature is high, $\chi_{11}^{ud}$ can be neglected compared with $\chi_{2}^{u}$, since $\chi_{11}^{ud} \to 0$ in the Stefan-Boltzmann limit at high temperature.

Figure 6 shows the evolution of quadratic and quartic fluctuations of $s$ quarks with $T$ during the chiral crossover. In the same way, we choose several values of $eB$. The influence of the magnetic field on the fluctuations of $s$ quarks are much smaller than that on light quarks, in particular for the low order fluctuations $\chi_{2}$. One can see that there is almost no difference among the several curves corresponding to different values of $eB$ in the left panel of Fig. 6. But with the increase of the order of fluctuations, the impact of magnetic field becomes more pronounced as shown in the right panel of Fig. 6. One observes that there are two peaks on the curves of $\chi_{4}^{u}$ as a function of $T$, which correspond to chiral restorations for light quarks and strange quarks, respectively. More detailed discussions about this can be found in Ref. [18].

It is usually believed that the ratio of the quartic to quadratic fluctuations of quarks is a
FIG. 6. (color online). Quadratic (left panel) and quartic (right panel) fluctuations of $s$ quarks as functions of the temperature at several values of $eB$.

FIG. 7. (color online). Ratio of the quartic to quadratic fluctuations for $u$ quarks (left panel) and $s$ quarks (right panel) as functions of $T$ at several values of $eB$.

valuable probe of the deconfinement and chiral phase transitions [28–31]. Because there is a pronounced peak in the curve of the ratio at the critical temperature. In Fig. 7 we show this ratio for $u$ and $s$ quarks with several values of $eB$. One observes that both the peak in $\chi_u^4/\chi_u^2$ and that in $\chi_s^4/\chi_s^2$ increase with the strength of magnetic field. We should emphasize that since the ratio deducts the influence of the phase space change resulting from the Landau levels in an external magnetic field, the increase of the ratio is not due to the phase space, but to the increase of the transition strength of the crossover. More discussions about the ratio $\chi_4/\chi_2$ can be found in Refs. [25, 32]. It should be mentioned that the ratio in Ref. [25]
is not $\frac{\chi_4}{\chi_2}$ calculated here, but is more closely related to $\frac{\chi_4^B}{\chi_2^B}$ which will be discussed in the following (there is a difference of factor 9).

V. FLUCTUATIONS AND CORRELATIONS OF CONSERVED CHARGES

In this section we discuss fluctuations and correlations of conserved charges, e.g., baryon number, electric charge, and strangeness. These charges are conserved throughout the evolution of a fire ball produced in relativistic heavy ion collisions. Therefore, it is expected that fluctuations of these conserved charges and correlations among them, which can be extracted from event-by-event fluctuations in experiments [20, 23, 26], carry information about the properties of the fire ball during its early evolution stage, including the strong and quickly decaying magnetic field produced in noncentral collisions.

Figure 8 and figure 9 shows fluctuations of baryon number and electric charge, respectively. The quadratic and quartic fluctuations and their ratios are plotted as functions of $T$ at several values of $eB$. Like the quadratic fluctuations of quarks, we find that the transition of $\chi_2^B$ ($\chi_2^Q$) with $T$ during the chiral crossover becomes sharper when the strength of magnetic field is increased. The quartic fluctuations of baryon number and electric charge present a pronounced peak at the pseudocritical temperature, and the peak value increases with increasing $B$. In the same way, we also show $\chi_4^B/\chi_2^B$ and $\chi_4^Q/\chi_2^Q$ versus temperature. Both the peak values of these two ratios increase with the magnetic field strength, which demonstrates that since the transition strength of the crossover increases with $B$, the fluctuations of conserved charges during the chiral crossover are enhanced when the strength of magnetic field is increased. Comparing Fig. 9 with Fig. 8 we find that the magnitude of electric charge fluctuations is larger than that of baryon number fluctuations.

In Fig. 10 we show the correlations $\chi_{11}^{BQ}$, $\chi_{11}^{BS}$, and $\chi_{11}^{QS}$ as functions of $T$ at several values of $eB$. We find that the impact of the external magnetic field on $\chi_{11}^{BS}$ and $\chi_{11}^{QS}$ is small, like quadratic fluctuations of conserved charges discussed above. However the correlations between baryon number and electric charge are much more sensitive to the magnetic field. One can observe that there is a bump on the curve of $\chi_{11}^{BQ}$ versus temperature. The bump becomes sharper with increasing $B$, and the height of the bump increases rapidly with $B$. Since the mass of strange quarks is much larger than that of light quarks, the response of strange quark fluctuations and correlations to the external magnetic field is less sensitive.
FIG. 8. (color online). Quadratic (top) and quartic (middle) fluctuations of baryon number, and their ratio (bottom) as functions of $T$ at several values of $eB$.

than that of light quarks, which is also presented in the calculations of $\chi_2^B$ and $\chi_4^B$ in Fig. 6. Therefore, the dependence of $\chi_{11}^{BQ}$ on $B$ is more pronounced than those of $\chi_{11}^{BS}$ and $\chi_{11}^{QS}$. Another reason is that $\chi_{11}^{BQ}$ is vanishing in the Stefan-Boltzmann limit at high temperature, while $\chi_{11}^{BS}$ and $\chi_{11}^{QS}$ have finite values in this limit. So the dependence of $\chi_{11}^{BS}$ and $\chi_{11}^{QS}$ on $B$ may be polluted by the background, while $\chi_{11}^{BQ}$ has no such problem.

VI. CONCLUSIONS

In this work, we have studied influences of an external magnetic field on the deconfinement and chiral restoring phase transitions in the 2+1 flavor PNJL model. Calculations
FIG. 9. (color online). Quadratic (top) and quartic (middle) fluctuations of electric charge, and their ratio (bottom) as functions of $T$ at several values of $eB$.

Effects of the external magnetic field on the fluctuations of quark number are studied in the effective model. We find that the magnetic field makes the transition of the quadratic fluctuations with respect to $T$ during the chiral crossover sharper, since the transition strength of the crossover increases with $B$. The peak structure in quartic fluctuations and the oscillation in sixth-order fluctuations become more and more prominent when the 2+1 flavor PNJL model indicate that magnetic field catalyze the dynamical chiral symmetry breaking, and the chiral pseudocritical temperature increases with increasing $B$. The deconfinement pseudocritical temperature increases a little with $B$ as well. We find that the transition strength increases when $B$ is increased.
FIG. 10. (color online). Correlations between baryon number and electric charge (top), baryon number and strangeness (middle), electric charge and strangeness (bottom) as functions of the temperature with several values of $eB$.

magnetic field strength is increased. With the increase of $B$, the inflection point in the curve of $\chi_{11}^{ud}$ develops a complicated dependent behavior on $T$, where $\chi_{11}^{ud}$ oscillates during the crossover. Comparing the fluctuations of strange quarks with those of light quarks, we find the influences of an external magnetic field on strange quark fluctuations are smaller than those on light quarks.

Special attentions are paid on the fluctuations and correlations of conserved charges, including baryon number, electric charge and strangeness. Since they can be measured with event-by-event fluctuations in heavy ion collision experiments. In the same way, we find that
the transition of quadratic fluctuations of conserved charges becomes more abrupt when $B$ is increased. The peak value of quartic fluctuations of conserved charges increases with $B$. We should emphasize that the peak structure in the ratios $\chi_{B}^{4}/\chi_{B}^{2}$ and $\chi_{Q}^{4}/\chi_{Q}^{2}$ become more and more pronounced with the increase of $B$, which indicates that $\chi_{B}^{4}/\chi_{B}^{2}$ and $\chi_{Q}^{4}/\chi_{Q}^{2}$ may be useful probes for the strong magnetic field produced in early noncentral collisions. We also study the correlations of conserved charges. We find that the impact of the external magnetic field on $\chi_{11}^{BS}$ and $\chi_{11}^{QS}$ is small, but $\chi_{11}^{BQ}$ are sensitive to the magnetic field.

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