Status estimator using Kalman filter for a permanent magnet synchronous machine

F Mesa\textsuperscript{1}, Andrés Camelo\textsuperscript{1}, and Carlos Ramírez V\textsuperscript{1}
\textsuperscript{1} Universidad Tecnológica de Pereira, Pereira, Colombia
E-mail: femesa@utp.edu.co

Abstract. In this work, the problem of estimating the states of a nonlinear dynamic system is exposed, where it is necessary to implement algorithms that treat said linearity and allow to estimate in an appropriate way the future states of said system. One of the techniques that is responsible for the estimation task for non-linear models is the extended Kalman filter which will be developed in this work for the estimation of the states of the synchronous machine of permanent magnets with its dynamic model determined in the framework reference \(d_q\).

1. Introduction

The estimation of states is a technique that consists in finding the unknown values of the state variables of a dynamic system through certain measurements that may be imperfect. This technique is based on statistical criteria where the minimization of the sum of the squares of the errors is one of the most common criteria used to estimate the real value of the unknown variables. Some cases where the estimation of the state variables is applied, is the location of space objects, missiles and airplanes, estimation of control parameters and even in energy systems [1]. In power systems, imperfect measurements of voltage, angle, active and reactive power, current flows, etc. are the inputs for the state estimators in each of the nodes of the system. Where the objective is to give reliability to the energy system through said measurements, for this control algorithms must be applied that allow to bring the system to the desired state.

One of the main techniques for the estimation of dynamic states is the Kalman filter [2], which is an algorithm that consists of determining precise values for the unknown variables of the system operating over time and observing the input data for so find an optimal estimate of the original system. For the operation of the classic Kalman filter it is required that the studied system must be linear, a quality that is not present in power electrical systems, therefore, it is necessary to use the Kalman filter for non-linear systems. These non-linear systems are described mathematically by the following Equations (1) and Equation (2):

\[ x_{i+1} = f_i(x_i, u_i) + n_i \]  
\[ y_i = h_i(x_i, u_i) + v_i, \]

where \(x_i \in \mathbb{R}^{n \times 1}\) are the states of the dynamic system in discrete time \(i\), \(u_i \in \mathbb{R}^{m \times 1}\) are the system inputs, and \(y_i \in \mathbb{R}^{p \times 1}\) are the outputs or measurements. \(f_i: \mathbb{R}^{n \times 1} \times \mathbb{R}^{m \times 1} \rightarrow \mathbb{R}^{n \times 1}\) and \(h_i: \mathbb{R}^{n \times 1} \times \mathbb{R}^{m \times 1} \rightarrow \mathbb{R}^{p \times 1}\)
$\mathbb{R}^{mx1} \rightarrow \mathbb{R}^{px1}$ are functions that describe the dynamics of the system. $n_i$ and $v_i$ are system noises and in the measurement with zero mean and covariances $Q_i$ and $R_i$ respectively.

2. Estimation methods for power electrical systems

The state estimate is used to find an approximation of the state variables that are unknown in the power electrical system through imperfect measurements. This is achieved by means of measurement instruments known as phasor measurement unit (PMU) that sends the measurement signals to the control center which by means of the supervisory control and data acquisition systems (SCADA) and using the state estimation techniques is in charge to process the measurements, perform an adequate filtering, detect and eliminate erroneous data, and determine the best estimate of the state variables in each node of the system, said state variables are the voltages and angles. The technique developed in this document deals with the Kalman filter for non-linear systems [3].

2.1. Classic Kalman filter

The Kalman filter provides an optimal recursive solution to the problem of linear filtering, its application consists in the estimation or prediction of the current and/or future states of the hidden variables in stationary and non-stationary linear dynamic systems. For a linear dynamic system, whose mathematical model is represented by Equations (3) and Equation (4). Equations (1) and Equation (2) can be rewritten as follows [4,5]:

$$x_{i+1} = F_ix_i +$$

$$y_i = H_ix_i + v_i,$$

where $F_i$ is known as the transition matrix, which carries the state $x_i$ from instant $i$ to instant $i+1$, $H_i$ is the measurement matrix. The noise of the system $n_i$, is assumed to be a Gaussian white noise with zero mean and covariance defined in Equation (5):

$$E\{n_in_j^T\} = \begin{cases} Q_i & \forall \ i = j \\ 0 & \forall \ i \neq j \end{cases}$$

In the same way, the noise in the measurement $v_i$ is assumed as a white Gaussian noise with zero mean and covariance defined in Equation (6):

$$E\{v_iv_j^T\} = \begin{cases} R_i & \forall \ i = j \\ 0 & \forall \ i \neq j \end{cases}$$

Assuming that a measurement for the linear dynamic system described by Section 2.1 and Section 2.2 was made at the time instant $i$. The idea is to use the information contained in the new measurement $y_i$ to update the estimate of the hidden state $x_i$, for this we must have $\hat{x}_i^-$ which is considered as the estimate of the a priori hidden state (which is already available at time $i$). The Kalman filter algorithm estimates $x_i$ as a linear combination of $\hat{x}_i^-$ and the new measurement $y_i$ as follows Equation (7):

$$\hat{x}_i = \hat{x}_i^- + G_i(y_i - H_i\hat{x}_i^-),$$

where $G_i$ is known as the Kalman gain matrix. The algorithm for the detailed Kalman filter can be reviewed in [6,7], a summary of the algorithm is presented in Table 1:

For the implementation of the algorithm, the matrices $P_i^-$ and $R_i$ are used, which are the covariance matrix a priori and a posteriori respectively.
Table 1. Algorithm for the Kalman filter.

| Algorithm: Classic Kalman Filter |
|----------------------------------|
| Initialization: For i = 0 \( \hat{x}_0 = E(x_0) \) |
| \( P_0 = E((x_0 - E(x_0))(x_0 - E(x_0))^T) \) |
| For i=1, 2, ..., #ite |
| Estimated hidden status |
| \( \hat{x}_i = F_i \hat{x}_{i-1} \) |
| Propagation of the covariance error |
| \( P_i^{-} = F_i P_{i-1} F_i^T + Q_{i-1} \) |
| Kalman gain matrix |
| \( G_i = P_i^{-} H_i^T [H_i P_i^{-} H_i^T + R_i]^{-1} \) |
| Update of the hidden state estimate |
| \( \hat{x}_i = \hat{x}_{i-1} + G_i (y_i - H_i \hat{x}_{i-1}) \) |
| Covariance update |
| \( P_i = (I - G_i H_i) P_{i-1} \) |

2.2. Kalman filter for non-linear systems

When dealing with non-linear systems, that is, specifically the functions \( f_i \) and \( h_i \), are non-linear, there are versions of the Kalman Filter algorithm, such as the Extended Kalman Filter (EKF), which calculates an estimated solution through The linearization by Taylor series for the functions \( f_i \) and \( h_i \) (Equation (8) and Equation (9)), supposing that there are no external inputs \( u_i \) is the linearization of the phase space, remains [8,9]:

\[
F_i = \frac{\partial f(x_i)}{\partial x_i} \bigg|_{x_i = \hat{x}_{i-1}} \tag{8}
\]

\[
H_i = \frac{\partial h(x_i)}{\partial x_i} \bigg|_{x_i = \hat{x}_{i}} \tag{9}
\]

Once the phase space of the dynamic system is linearized, the previous algorithm is applied with the following variations (Equation (10) and Equation (11)).

\[
\hat{x}_{i-1} = f_i(\hat{x}_{i-1}) \tag{10}
\]

\[
\hat{x}_{i} = \hat{x}_{i-1} + G_i (y_i - h_i(\hat{x}_{i-1})) \tag{11}
\]

3. Numerical results

In this section we analyze the results obtained through the implementation of the EKF algorithm, in Figure 1 to Figure 3 each of the estimates of the PMSM states is shown. Figure 1 shows the currents \( i_a \) and \( i_b \) respectively in amperes with respect to time in seconds [10,11]. Then the angular velocity of the rotor is shown in \( \omega \) in rad/s and finally the angular position \( \theta \) in rad in Figure 2.

![Figure 1. Estimation \( i_a \) and \( i_b \) (X axis: time (s)).](image-url)
The green lines represent the actual states, while the fuchsia line dots the relevant estimates. It can be seen that the state estimator the first 0.3 seg has a considerable error, however before the 0.5 seg manages to reach the real states.

![Figure 2. Estimation ω and θ (X axis: time (s)).](image)

The error graphs allow to visualize in a better way the difference between the real values of the states and the estimated states, in Figure 3 each one of the differences between the real states and the estimated ones is shown, where it is observed that during the first 0.3 seg and the can be considerable as in the variable ω that gets to take a mistake of up to 6 rad nevertheless, in each one of the estimators these errors have to disappear with the passage of time [12].

![Figure 3. Estimation errors (X axis: time (s)).](image)

4. Conclusion
The state estimation plays a very important role in the control and monitoring of the electrical power systems, allowing to give reliability to the network through the SCADA systems. State estimators can be static or dynamic, however, when dealing with state estimation of energy systems the best approach is the use of dynamic estimators, as developed in this document.

In this document the algorithm of the Extended Kalman Filter was implemented in order to estimate the states of the permanent magnet synchronous machine (PMSM), which is a highly non-linear dynamic system, but at the same time quite simple. It was possible to estimate each of the state variables treated with an estimation error that became much smaller as time progressed. On the other hand, the use of linearization to apply the EKF algorithm causes a loss of precision when you have large systems, therefore, an improvement that can be implemented later is the use of filters that do not depend on the calculation of the Jacobian matrix for obtain better computational performance and greater accuracy in the estimation.
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