Improving actuator disk wake model

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Abstract. The wind energy industry has traditionally relied on simple wake models for estimating Wind Turbine (WT) wake losses. Despite limitations, low requirements in terms of detailed rotor information makes their use feasible, unlike more complex models, such as Blade Element Method (BEM) or Actuator Line. Froude’s Actuator Disk (AD) does not suffer the simpler model’s limitation of prescribing the wake through a closed set of equations, while sharing with them the low rotor data requirements. On the other hand they require some form of parametrization to close the model and calculate total thrust acting on the flow. An Actuator Disk model was developed, using an iterative algorithm based on Froude’s one-dimensional momentum theory to determine the WT’s performance, proving to be successful in estimating the performance of both machines in undisturbed flow and in the wake of an upstream machines. Before Froude’s AD limitations compared to more complex rotor models, load distributions emulating those of a BEM model were tested. The results show that little impact is obtained at 3 rotor diameters downstream and beyond, agreeing with common definition of a far-wake that starts at 1-2 diameters downstream, where rotor characteristics become negligible and atmospheric flow effects dominate.

1. Introduction
Solutions to model an operating Wind Turbine (WT) in atmospheric flow simulations range [1, 2] from the simplest empirical models to the most complex, with Direct Rotor Modelling (DRM). Increased complexity is accompanied by increases in computational resource cost and input data, but also produces a better representation of rotor flow dynamics, so agreement between flow prediction and real flow should naturally be improved. The ideal engineering model combines features from opposite ends of this spectrum, being a challenge striking a balance between the model’s realizability and the precision and consistency of its predictions.

Complex forms of WT rotor modelling, such as DRM [3] or Actuator Line (AL) [4], model individual blades and their movement over time, allowing the prediction of intricate details of the wake dynamics, such as stalled flow and tip vortices. Using blade geometry and rotor control algorithms as inputs, they are able to evaluate the WT’s effect on the flow autonomously, by calculating blade loads according to local flow. This type of rotor information is unavailable for most commercial WTs, limiting their applicability.

On the other end are simpler solutions, that trade complexity for low cost and robustness. These can be, at the crudest possible level, empirical relations for velocity deficit [5]. Parabolized Navier-Stokes (NS) solvers such as the Ainslie [6] and UPMWAKE [7] models allow a dynamical solution to the WT wake, solving velocity deficit and, in some cases, turbulence intensity distributions at a moderate computational cost. The simplifications assumed allow these models
to be autonomous solutions to wake flow, but unables them to take into account local flow effects, such as blockage (other WTs) or flow acceleration (terrain). The wind energy industry has learned to tackle these limitations by modelling wake-terrain and wake-wake interactions [8], by assuming they combine linearly. However, these solutions still cannot account for complex flow effects and their need sensitivity to calibration makes them susceptible to error, limiting their flexibility.

The Actuator Disk (AD) models [9] fall midway in the spectre: like DRM and AL, the AD models the rotor and allows the wake to be solved by flow governing equation, but simplifies it to a disk covering the span of the blades, loosing the detail of flow over individual blades. They are simpler to deploy and allow savings in computational power, by permitting lower spatial resolutions and solution as a steady-state problem. Glauert’s Blade-Element Momentum (BEM) theory [10] for AD is similar in many aspects to DRM and AL models, since rotor loads depends on local velocity and blade geometry. The classic AD model of Froude [11] simplifies the rotor a step further, by assuming rotor load uniformity. Their most defining characteristic is that they disregard rotor geometry entirely, parametrizing the rotor/flow interaction and rotor load through an equivalent thrust coefficient, a simple solution but one that requires user prescription or some form of parametrization [12].

While for WT research purposes DRM, AL and BEM models are widely used, the same cannot be done at the engineering level for wind resource evaluation, where blade geometry information is unavailable and the computational cost makes their use difficult to justify. The wind energy industry uses parabolized NS solver models for these purposes, but they too are limited in their application. The solution attempted in this work was to produce an AD model for wind engineering purposes with as little input as possible, by building on Froude’s AD model, taking advantage of its realizability and flexibility. A modification to Froude’s AD model is proposed to enable its use as a tool in wind energy engineering context, with the available information, usually consisting only of power and thrust coefficient curves, by enabling the model to estimate the WT’s thrust autonomously. Additionally, recognizing Froude AD’s limitations in terms of rotor description, different rotor load distributions emulating those obtained through BEM models are tested, to see how wake dynamics are affected and whether they are of interest for wind resource assessment purposes.

2. Applying basic AD model

Froude’s AD was developed as a form of parametrization of propellers and turbines induced forces in numerical flows models. The concept suffers from several limitations [9] but offers many advantages, the main being that by loosing all pretensions of accurately describing the near wake, it is solvable at course spatial resolutions with reasonable results in the far wake region. To do so the rotor is simplified to a permeable disk covering the rotor span, which acts as a momentum sink equivalent to that of an operating WT. To do so is not at all unreasonable, considering how scarce information on any given WTs blade geometry is available. What little information exist comes by reverse engineering from rotor flow measurements [13].

Considering this disk to be uniformly loaded and acting only on the direction normal to the flow, one-dimensional momentum theory allows the total thrust of the disk to be determined by:

$$T = \frac{1}{2} \rho C_T A_D U_\infty^2$$  \hspace{1cm} (1)

This equation, paired with a thrust coefficient $C_T$ curve (supplied by the WT manufacturer) and the undisturbed velocity at the hub $U_\infty$, is sufficient to deploy the AD model in a CFD flow solver by distributing it uniformly over the AD area $A_D$, with:

$$dT = \frac{T}{A_D} dA$$  \hspace{1cm} (2)
The AD model was implemented in a finite-volume Reynolds averaged Navier-Stokes (RaNS) solver for atmospheric flows, using standard k-ε model for turbulence closure. This form of turbulence closure is not ideal for wake modelling applications, as pointed out by [11] and [14], but its robustness and versatility still make it a favourite in atmospheric flow modelling. More information on the RaNS code used can be found in [15].

The rotor is modelled using a virtual cylindrical rotor mesh where the AD is described exactly, from which the actuator surface area is extrapolated to the domain’s structured horizontally orthogonal mesh. All domain meshes have a region of uniform spacing in all directions, in the area covering the span of the rotor(s) and between D/2 upstream of the first and downstream of the last WT.

The basic Froude AD implementation was tested for different spatial resolutions, with the thrust coefficient set to $C_T = 0.701$ and free-stream velocity set to $U_\infty = 8.06$ m/s, the results of which are presented in Figure 1. Velocity deficit prediction seems to be insensitive to resolutions finer than $D/5$, both along the WT axis line as well as across the wake at hub height; even a resolution of $D/2$ seems sufficient to reasonably predict the wake width at 3 and 7 rotor diameters downstream. On the other hand, the added Turbulence Intensity (TI) prediction is strongly sensitive to mesh resolution, with a small but visible step in TI even between the two finer meshes.

Quantitatively, the cross-wake velocity deficit distributions found have the expected inverted U shape. The literature shows that this is reproducible irrespective of using RaNS [11] or LES [16] flow solvers or modelling the rotor by AD or AL [4]. Turbulence on the other hand does not do so well: 3 diameters downstream of the WT there are no visible signatures of blade tip vortices, and turbulence in the centre of the wake seems to have been exaggerated [16, 14]. One-dimensional momentum theory is valid under the assumption of inviscid flow, so unnatural results are expected when applying the same conclusions to significantly different conditions.

**Figure 1.** Velocity ratio and turbulence along rotor axis line and across wake at hub height, for four different mesh resolutions.
3. Estimating $U_∞$

To close equation 1 and determine the total thrust of the AD, an estimation of the free-stream velocity $U_∞$ is necessary. A precursor simulation is the obvious solution, because it provides the exact $U_∞$ for a single WT. In the case of multiple WTs, wake interaction can mean multiple precursor simulations (in the correct order) are necessary to reach a complete solution. Politis [17] progressively switches on WTs over the course of one RaNS simulations, as their respective undisturbed hub velocities converge. In the case on-shore presented by the author and in most off-shore wind farms, application of this model is feasible, as determining the order in which the WTs are deployed is simple, but on most on-shore cases this would not be straight-forward.

To avoid the need for precursor simulations, WT performance estimation is done updated continuously as the flow solution converges, akin to BEM, AL and DRM models. The proposed model uses the conclusions of Froude’s one-dimensional momentum theory’s, regarding momentum conservation for inviscid 1-D flow and the thrust coefficient $C_T$ curve available from the manufacturer (see Figure 2). Since $U_∞$ and $C_T$ pairing is fixed, the estimation of the disk velocity $U_{disk}$ is key to estimate the total thrust of the operating WT [12]. It finds a $U_∞/C_T$ pair that obeys to both the manufacturer’s thrust coefficient curve and momentum theory’s equation relating thrust coefficient and induction factor $a = 1 - U_{disk}/U_∞$:

$$C_T = 4a(1 - a)$$

For that, the model considers that the velocity at the hub position $U_{hub}$ is a good approximation to 1-D momentum theory’s $U_{disk}$. After prescribing an initial guess for $U_∞$, iterating the following algorithm yields a converged $U_∞/C_T$ pair:

(i) interpolate $C_T$ from manufacturer’s curve, using most recent $U_∞$ estimate;
(ii) interpolate velocity at hub position $U_{hub}$;
(iii) update $U_∞$ estimation using Equation 3 and induction factor $a = 1 - U_{hub}/U_∞$.

The free-stream velocity estimate provided by the proposed method over-estimated the undisturbed velocity at the hub position in the lower part of the the WT’s operating range (Fig. 2), peaking at +2% at 5 m/s. Reasons for error could be many, but the results seen in Section 2 seem to indicate issues with the turbulence modelling used in the RaNS solver: over-estimation of $k$ in the centre of the rotor seems to be a signature of exaggerated turbulence production [18] in areas of flow blockage, where stream-wise shear is high. The resulting excess in $k$ combined with the turbulent viscosity hypothesis leads to over-enhanced turbulent mixing thus a smoothed stream-wise velocity deficit distribution and an over-estimated $U_{hub}$. This would agree with the result in Fig. 2, where peak estimation error and peak $C_T$ coincide: when the momentum sink is largest so will be the stream-wise velocity gradient and strongest the turbulent diffusion in the vicinity of the disk.

Moving upwards in the operating range, above 10 m/s the error level drops to small negative values. In this region $C_T$ is lower, diminishing the effect described above, and convection begins to strongly overpower diffusion as the driving factor in momentum transport in the stream-wise direction, which should explain the sharp drop in error levels between 5 and 10 m/s.

Approximating $U_{disk}$ to the velocity at the hub position $U_{hub}$ can lead to large $U_∞$ estimation errors in flows with strong lateral inhomogeneities, such as inside large wind farms where WTs operate in undisturbed, partially and fully-waked flow. To gauge the error induced by this approximation, the $U_∞$ estimation of a WT in waked flow was studied. Figure 3 shows the error in method’s estimation error for a WT T2 when in the wake of an upstream turbine T1, by comparing: the velocity at T2’s hub position without modelling T2 itself (T2’s ‘undisturbed’ hub velocity) and the model’s $U_∞$ estimation for T2 (when modelling both T1 and T2), both as velocity ratios relative to the inflow’s hub height velocity. By doing this over a range wind
directions it is possible to see how the model responds from fully waked flow (0°, when flow is aligned with the WT row) to partially and undisturbed flow (misalignment of ±20°). Cases were run for two different WT spacings, and undisturbed inflow velocity was 8.06 m/s.

Overall, the method proposed was able to predict $U_\infty$ with an error level comparable to that found in the case of an isolated WT (Fig. 2), but proximity to the centre of the wake and to T1 increases the estimation error. As velocity in the wake drops, estimation errors are expected to increase (error for an isolated WT more than doubles from 8 to 6 m/s), so part of the error seen in Figure 3 is inherent to every individual WT. Still T2’s error peaks at +3%, above the maximum found for an isolated machine, which shows that the method is affected by T1’s wake, and T2’s $U_\infty$ over-estimation results moderately aggravated. The culprit for this is should be the same excessive turbulent mixing behind the single WT case error, since the exaggerated $k$ travelling in T1’s wake centre accumulates with that produced by T2. With enough spacing between WT’s, diffusion and dissipation lowered $k$ and this effect is attenuated.
Figure 4. Force distribution scaling factors along dimensionless rotor radius, based on the results by of Sørensen and Shen [19].

4. Improve wake dynamics

By simulating the blade’s heterogeneous aerodynamic proprieties, BEM models are able to mimic the non-uniform loads real WT rotors see. The classic Froude AD model is not able to do that, but is able to model the acting rotor thrust without any blade geometry information, at the cost of rotor detail and potential wake effects. These can be broken down to two parts: rotation induced by forces acting tangentially to the AD and variation of force along the rotor’s radius. To determine the impact of those losses different force distributions were tested, to assess their individual and combined effects.

With the total thrust acting on the flow known (Equation 1), different forms of distributing that total force over the AD area were applied through by scaling the uniform load distribution of Equation 2, using factors that depended on the dimensionless rotor radius $r/R$ for normal $(f_n)$ and tangential $(f_t)$ directions:

$$dF_n = dT \times f_n \left( \frac{r}{R} \right)$$  \hspace{1cm} (4)

$$dF_t = dT \times f_t \left( \frac{r}{R} \right)$$  \hspace{1cm} (5)

The following forms of distributing force were tested:

- **Base** - classic Froude AD: $f_n \left( \frac{r}{R} \right) = 1$, $f_t \left( \frac{r}{R} \right) = 0$;
- **Base+rot.** - Froude AD with added rotation terms: $f_n \left( \frac{r}{R} \right) = 1$, $f_t \left( \frac{r}{R} \right) = 0.2$;
- **Var.** - variable normal force: $f_n \left( \frac{r}{R} \right)$ as in Figure 4, $f_t \left( \frac{r}{R} \right) = 0$;
- **Var.+rot.** - variable normal and tangential forces: $f_n \left( \frac{r}{R} \right)$ and $f_t \left( \frac{r}{R} \right)$ as in Figure 4.

Note that when using variable force, hub drag was taken into account.

Changing the force distribution seems to yield little to no overall effect in wake behaviour, judging by the results shown in Figure 5. The impact of introducing radially variable force is visible in the velocity profiles along the rotor axis line, but after 2-3 rotor diameters downstream velocity is largely unaffected. What added shear the different force distributions introduce leads only to a small increase in turbulence.

Looking deeper into the near wake, at 0.5 diameters downstream of the rotor: the top half of Figure 6 shows that shifting force distribution towards the end of the blades has a strong effect on the location of the velocity deficit peak, and leads to an increase of turbulence in the area of the rotor edge. Rotation on the other hand seems to have a negligible impact. Outside the near-wake, at 3.0 diameters downstream of the rotor (bottom half of Figure 6), the roles appear
to reversed, and shifting force weighting towards the edge of the rotor has no visible effect, and
effect of tangential force terms remain in the form of an increase turbulence, along with a small
shift in the wake centreline position, despite tangential flow velocity being near-zero.

The excessive mixing described in Sections 2 and 3 contribute to this result, as the wake is
artificially smoothed the turbulence model. While these conclusions are made in the context of
RaNS modelling with $k-\varepsilon$ closure, the work of Porté-Agel and Wu [4, 16, 20] on LES (which
does not suffer from the limitations described regarding turbulence modelling) seems to point
to roughly the same conclusion of a far-wake largely unaffected by rotor description.

5. Conclusion

An AD model was deployed in a RaNS flow solver to simulate the presence and wake of a
WT in atmospheric flow. Results show that the characteristic U-shaped velocity distribution
is reproduced, but turbulence behaviour doesn’t appear as successfully predicted: $k$ is very
high at the centre of the wake and no trace remains in the far wake of the annular shear layer
from the rotor edge. This is thought to be tied to limitations of the turbulence model, and its
repercussions are felt throughout the study. Finally, Momentum theorys conclusions are valid
in the assumption of one-dimensional in-viscid flow, conditions different to those in which the
model is being applied.

The method proposed for estimating $U_\infty$ shows overall moderate results. Peak error of of an
isolated WT’s undisturbed velocity prediction was +2% at 5 m/s. This is thought to be largely
due to excessive turbulent mixing, exaggerated by the $k-\varepsilon$ model in the WT vicinity due to
its inability to correctly model turbulence in areas with strong blockage effects. The velocity
distribution becomes artificially smoothed, leading to an over-estimation of the local velocities
and consequently of $U_\infty$. This effect appears to propagate to downstream turbines through the
$k$ transported in the wake.

Attempts at improving force distribution to bridge the gap to BEM models showed that
little evidence of rotor description changes remain in the far-wake. Inside the near-wake, flow
Figure 6. Snapshots in plane perpendicular to the flow of velocity ratio, tangential velocity vectors and turbulence for the four different force distributions tested. The rotor tip and hub are represented by black circumference and circle, respectively. Top set of figures represents flow inside near-wake, at 0.5D downstream, bottom set of figures is of flow outside the near-wake, at 3.0D downstream.
dynamics are changed only by moving a bigger share of the normal load towards the rotor tip, resulting in a re-arranged velocity ratio distribution and significantly increasing turbulence near the rotor tip. Moving outside the near wake, turbulent mixing appears to have blended these effects, with almost no clues of the different force distributions visible in the flow. At 3.0D downstream of the WT, only the introduction of rotation by tangential force terms seems to have a lasting effect, in the form of a small increase in turbulence. Again the excessive turbulent mixing predicted by the $k-\epsilon$ closure is thought to take a large part in this result, by smoothing what heterogeneities appear in the near wake, with little to no traces of them being found in the far wake. Even if exaggerated by this fact, the conclusions still seem to agree with the the common definitions of the near and far wake: inside the former, rotor flow dynamics dominate, whereas in the latter the dominating forces are those from atmospheric flows.

For wind resource assessment purposes, where the near wake characteristics are not important, there seems to be little to gain in increasing model complexity over Fröde’s AD model. While this seems to be in agreement with the literature, introducing flow rotation on the rotor seems to produce a change in wake dynamics that could be interesting to explore. A solution was found to close one-dimensional momentum theory’s thrust calculation, allowing the closure of the equation and continuous updating of the AD thrust during the calculation of a flow solution. While the results are encouraging, the proposed model seems to be hampered by the limitations of the turbulence closure used here, with the turbulent viscosity hypothesis and the $k-\epsilon$ model. Future developments will focus on this subject, with LES as a primary development tool and source for comparison and validation, along with wind tunnel wake data. At a latter stage validation will proceed with comparison full scale wind farm data. This framework makes possible the development a tool for wind turbine placement and wind resource studies, both for on and off-shore project, using reasonable computational resources and only what limited WT characteristics are available for most commercial machines.

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