Gravitational-Recoil Effects on Fermion Propagation in Space-Time Foam

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Abstract

Motivated by the possible experimental opportunities to test quantum gravity via its effects on high-energy neutrinos propagating through space-time foam, we discuss how to incorporate spin structures in our \(D\)-brane description of gravitational recoil effects \textit{in vacuo}. We also point to an interesting analogous condensed-matter system. We use a suitable supersymmetrization of the Born-Infeld action for excited \(D\)-brane gravitational backgrounds to argue that energetic fermions may travel slower than the low-energy velocity of light: \(\delta c/c \sim -E/M\). It has been suggested that Gamma-Ray Bursters may emit pulses of neutrinos at energies approaching \(10^{19}\) eV: these would be observable only if \(M \gtrsim 10^{27}\) GeV.

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1 Introduction

It has recently been pointed out that the constancy of \( c \), the velocity of light, can be tested stringently using distant astrophysical sources that emit pulses of radiation, such as Gamma-Ray Bursters (GRBs) \([1, 2, 3]\), Active Galactic Nuclei (AGNs) \([1, 4, 2]\) and pulsars \([5]\). So far, this idea has been explored by comparing the arrival times of photons of different energies \( E \) (frequencies \( \nu \)). It has been suggested \([1, 6, 3]\) that certain quantum theories of gravity might cause variations in \( c \) that increase with \( E \) (or \( \nu \)), possibly linearly: \( (\delta c/c) \sim (E/M) \), or quadratically: \( (\delta c/c) \sim (E^2/\tilde{M}^2) \), where \( M \) or \( \tilde{M} \) is a high mass scale characterizing quantum fluctuations in space-time foam \([6, 8, 9, 6]\). Such a linear or quadratic dependence would enable any such conjectured quantum-gravity effects to be distinguished easily (in principle) from the effects of conventional media on photon propagation and the effects of a possible photon mass, both of which would decrease with increasing energy. It is clear that, in order to probe quantum-gravity effects by putting the strongest possible lower limits on \( M \) and \( \tilde{M} \), there is a premium on distant pulsed sources that emit quanta at the highest available energy.

Unfortunately, from this point of view, the distance over which high-energy photons can travel through the Universe is limited by scattering on photons in the intergalactic medium. Therefore, one is led to consider the emissions of other ultra-energetic particles, such as neutrinos, protons and neutrons. These also scatter in the intergalactic medium, resulting in energy cutoffs as functions of distance, e.g., the Greisen-Zatsepin-Kuzmin cutoff for protons \([10]\). Because of their low interaction cross sections, the best prospects for the highest-energy quanta from the largest distances may be provided by neutrinos.

As yet, no ultra-high-energy neutrinos have been detected, but the sensitivity of neutrino telescopes is planned to increase dramatically in the coming years \([11]\). There may well be diffuse sources such as ultra-heavy relics in the galactic halo, and one cannot expect that all discrete sources will exhibit useful time structures. However, calculations suggest that both GRBs and AGNs may be observable pulsed sources of high-energy neutrinos. If GRBs do indeed emit pulses of neutrinos at energies up to \( 10^{19} \) eV, as recently suggested \([12]\), they might provide ideal opportunities to probe quantum gravity (see also \([13]\)), since GRBs have measurable cosmological redshifts: \( z \sim 1 \), and exhibit short time structures: \( \lesssim 1 \) s. We return later to a discussion of the sensitivities to the quantum-gravity parameters \( M, \tilde{M} \) that such GRB neutrino bursts might provide.

The bulk of this paper is devoted to a formal discussion of the interaction of high-energy fermions with space-time foam. We extend our previous D-brane model of quantum-gravitational fluctuations \textit{in vacuo} \([2]\) to include fermions, by developing the appropriate supersymmetric Born-Infeld (BI) effective action. This enables us to demonstrate that a high-energy fermion scattering off a D-brane defect in space-time induces a linear deformation of the background metric \( G_{0x} \sim E/M \), analogous to that induced by a high-energy boson, \textit{if gravitational recoil effects are taken into account}. Section 2 contains a review of our previous BI treatment of D-particle recoil, and points to an interesting analogous condensed-matter system \([14]\). Section 3 discusses the supersymmetrization of the BI action, and the consequences for fermion propagation are stressed in Section 4.

The potential phenomenological implications for the observability of high-energy neu-
trino pulses from GRBs and other sources are discussed in Section 5. This may be read without ploughing through the earlier sections, if the reader is not concerned with the formal underpinnings of the phenomenological analysis. As we discuss there, high-energy neutrino pulses from GRBs could provide sensitivity to $M \sim 10^{27}$ GeV or $\tilde{M} \sim 10^{19}$ GeV. There is also an Appendix where certain group-theoretical aspects of the breaking of Lorentz invariance are developed.

2 Space-Time Distortion due to $D$–Particle Recoil

We first review in more detail the theoretical foundation underlying any such phenomenological probes of quantum gravity. We have argued that virtual $D$ branes provide one possible model for space-time foam, and that the recoil of a $D$ brane struck by a bosonic closed-string particle would induce an energy-dependent modification of the background metric ‘felt’ by an energetic quantum: $G_{0x} \sim u_x \sim E/M_D$. Here $u_x$ is the average recoil velocity of a generic $D$-brane excitation of mass $M_D$ when struck by a boson, such as a photon, moving in the $x$ direction with energy $E$. Such a change in the background metric would clearly break Lorentz invariance, but in a relatively simple one-dimensional manner that is symmetric about the $x$ axis. The remaining aspects of Lorentz symmetry along directions transverse to the direction of motion are preserved.

As preparation for our subsequent extension of this discussion to propagating fermions, such as neutrinos, we first review briefly the relevant $D$-brane formalism \[15\]. We consider the recoil induced by a closed (super)string state (representing some conventional matter particle) when it strikes a $D$-particle defect in space-time. We assume that the defect is very massive: $M_D = M_s/g_s$, where $M_s$ is the string mass scale: $M_s \equiv (\alpha')^{-1/2} \equiv 1/\ell_s$, with $\alpha'$ is the Regge slope, and $g_s$ is the string coupling, which we assume to be weak: $g_s << 1$.

It has been suggested in \[16\]  that this $D$-particle recoil process be described by a logarithmic conformal field theory on the string world sheet \[19\]. Dynamics on the world sheet is described by a $\sigma$-model formalism, and the pertinent $\sigma$-model perturbations are given by world-sheet boundary deformations in the Neumann picture, of the form:

$$\frac{1}{2\pi \alpha'} \int_{\partial \Sigma} A_M(X^0) \partial_\tau X^M, \quad M = 0, \ldots 9$$

where the background gauge field $A_M(X^0)$ has the following structure:

$$A_M(X^0) = \left( A_0(X^0), -\frac{1}{2\pi \alpha'} Y^i(X^0) \right); \quad Y_i(X^0) = \left( \epsilon Y_i + u_i X^0 \right) \Theta_{\epsilon}(X^0)$$

where the suffix 0 denotes temporal (Liouville) components, $Y_i, u_i, \ i = 0, \ldots 9$ denotes the collective coordinates and recoil velocity of the $D$-particle, and $\Theta_{\epsilon}(X^0)$ denotes a Heaviside step function regulated by a cutoff parameter $\epsilon \rightarrow 0^+$ \[18\], which is related to the world-sheet scale: $\epsilon^{-2} \sim \ln|L/a|$, ensuring consistency of the logarithmic conformal algebra \[19\] satisfied by the pair of operators (with couplings $Y_i$ and $u_i$) appearing in \[4\] \[10\].
The recoil velocity of the D brane is given in terms of the energy/momentum transferred between the incident and the outgoing (low-energy) massless particle \[17, 18\]:

\[ u_i = \ell_s g_s (k_i^1 + k_i^2) \] (3)

It can be shown \[18\] that this energy-momentum conservation relation survives the summation over world-sheet topologies, and hence is an exact conservation law of the full quantum system. A technical point, which is however important for our purposes here, is that the physical recoil velocity and string couplings are \(\epsilon\)-regularized quantities \[18\]. In particular, the renormalized physical recoil velocity and string coupling (denoted by overlines) are:

\[ \overline{u}_i = \epsilon u_i; \quad \overline{g}_s = \epsilon g_s \] (4)

This renormalization becomes important after the identification of the change in the scale \(\epsilon\) with a target-time translation \[17, 18\], as we discuss later. Upon this identification, the renormalization (4) yields the correct exactly-marginal \(\sigma\)-model couplings for the recoil-velocity deformation. The conservation of energy-momentum (3) implies that this deformation is of order \(E/M_D\).

It was shown in \[18\] that the target-space dynamics of the recoil phenomenon described by the logarithmic algebra on the world sheet may be represented in ten-dimensional target space by a Born-Infeld (BI) Lagrangian for the gauge fields \(A_M\). This picture should be contrasted with the Dirichlet picture, in which the target-space Lagrangian of a \(Dp\) brane, although of BI type, is actually its world-volume Lagrangian. In our Neumann picture, the pertinent Lagrangian is of the form:

\[ \mathcal{L}_{BI} = \sqrt{\eta_{MN} + F_{MN}} \] (5)

where \(\eta_{MN}\) is a flat Minkowski metric, and \(F_{MN} = \partial_M A_N - \partial_N A_M\) is the Maxwell tensor for the gauge field \(A_M\) (2) describing the recoil. We note for future reference that the only non-vanishing components of the Maxwell tensor \(F_{\mu\nu}\), corresponding to the recoiling background \(2\), are \(F_{0i} = u_i\), which represents a constant electric-field background.

The propagation of ordinary photons has been discussed using this Lagrangian in \[1\], which we follow here. In the Neumann picture, we simply add a similar perturbation to the \(\sigma\) model, but now in terms of a full-fledged quantum \(U(1)\) field \(a_\mu\) with a field strength \(f_{MN}(a)\). The corresponding Lagrangian is then a straightforward extension of (5):

\[ \mathcal{L}_{BI} = \sqrt{\eta_{MN} + f_{MN} + F_{MN}} \] (6)

where, we repeat, \(F_{MN}\) corresponds to the background \(2\), and \(f_{MN}\) is the Maxwell tensor of the dynamical photon field.

As discussed in \[17, 20\], in addition to the world-sheet boundary deformations, the perturbed theory describing the recoil of the \(D\) particle also has a bulk graviton deformation, due to the fact that recoil is not a conformal process and hence requires Liouville dressing. Writing the boundary operator \(2\) as a bulk world-sheet (total derivative) operator, and taking into account \[16\] that it has a world-sheet anomalous dimension \(-\epsilon^2/2\), one dresses the bulk theory with a Liouville mode \(\phi\) \[21\]. Then one identifies the Liouville
field $\phi$ with the target time $X^0$ [17], which results in a metric $\sigma$-model perturbation of the form:

$$G_{ij} = \delta_{ij}, G_{00} = -1, G_{0i} = \epsilon(\epsilon Y_i + \epsilon u_i t)\Theta(\epsilon)(t), \ i = 1, \ldots D - 1$$

(7)

The presence of the (regularized) $\Theta$ function indicates that the induced space-time is piecewise continuous [1]. An important aspect of the approach of [16, 17, 18] is the identification of the parameter $\epsilon$ with the target time, for asymptotically long times $t \gg 0$ after the collision:

$$\epsilon^{-2} \sim t$$

(8)

The above relation should be understood as implying that the changes in both quantities coincide in the limit $\epsilon \to 0^+, t \to \infty$.

In view of (8), one observes that the metric (7) becomes to leading order for $t \gg 0$:

$$G_{ij} = \delta_{ij}, G_{00} = -1, G_{0i} \sim u_i, \ i = 1, \ldots D - 1$$

(9)

and thus is constant in space-time. However, the metric depends on the energy content of the low-energy particle that scattered off the $D$ particle, as a result of momentum conservation during the recoil process [18]. We shall concentrate on the flat asymptotic metric (9) in what follows.

The energy dependence of the metric is the main deviation from space-time Lorentz invariance induced by the $D$ particle recoil. As a result, the space-time group symmetry is reduced to rotations in the space-like plane perpendicular to the direction of motion and Lorentz boosts along the direction of motion, as discussed in the Appendix. The residual group of transformations is a subgroup of $SL(2, C)$. Upon diagonalization of the perturbed metric, one finds a retardation in the propagation of an energetic photon: $(\delta c/c) \sim (E/M_D)$. The fact that propagation is subluminal, rather than superluminal, is linked to the underlying BI action (11) for electromagnetism, which underlies the dynamics of massless photons in the background of a recoiling brane [18, 13].

To conclude this review section, we would like to draw a comparison between the above results and some condensed-matter systems such as $d$-wave superconductors or superfluid $^3He$. It was observed in [14] that relativistic fermionic quasiparticle excitations appear near the nodes of such systems, with a spin-triplet pairing potential

$$V_{\vec{p}, \vec{p}'} \propto \vec{p} \cdot \vec{p}'$$

(10)

and an energy gap function $\Delta(\vec{p}) \sim c p_x$ in the polar phase, where $p_x$ denotes the momentum component along, say, the $x$ direction, and $c$ denotes the effective ‘speed of light’ in the problem. This is, in general, a function of the superflow velocity $w$: $c(w)$, that is determined self-consistently by solving the Schwinger-Dyson-type equations that minimize the effective action.

This system was considered in the context of $^3He$ in a container with stationary rigid walls and a superflow velocity $w$ taken, for simplicity, also along the $x$ direction. The

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1The important implications for non-thermal particle production and decoherence for a spectator low-energy field theory in such a space-time were discussed in [20, 17].
Doppler-shifted energy of the fermions in the pair-correlated state with potential (10) is given by

$$E(p_x, \epsilon_p) = \sqrt{\epsilon_p^2 + c^2 p_x^2} + w p_x,$$

(11)

where $$\epsilon_p = (p^2 - p_F^2)/2m$$ is the energy of the fermion in the absence of the pair correlation, $$p_F$$ is the Fermi momentum and $$m$$ is the mass of a Helium atom. The term $$w p_x$$ appearing in the quasiparticle energy spectrum (11), as a result of the motion of the superfluid, yields an effective off-diagonal (1+1)-dimensional metric $$G_{\mu\nu}$$ with components

$$G^{00} = -1, \quad G^{01} = w, \quad G^{11} = c^2 - w^2$$

(12)

The off-diagonal elements of the induced metric (12) are analogous to those of our metric (1). In this analogy, the role of the recoil velocity $$\vec{u}$$ in our quantum-gravitational case is played by the superflow velocity field $$w$$.

However, an important difference between our case and that of superfluid $^3$He is that, in our case, the spatial elements of the metric (1) are free from the horizon problem that characterizes the metric (12). This arises when the superflow velocity $$w = c$$, in which case the metric element $$G^{11}$$ in (12) crosses zero, leading to a signature change for superluminal flow $$w > c$$. In fact, as shown in [14] by an analysis of the gap equation, the superluminal flow branch is not stable, because it corresponds to a saddle point rather than a minimum of the effective action. This suggests that the intactness of the analogy with our problem, in which the BI action that governs the recoil dynamics [18] keeps the photon velocity subluminal, may be maintained, as we now discuss 3.

### 3 Supersymmetric Born-Infeld Action

It is not immediately apparent from the BI action (1) that a fermion such as a neutrino will also propagate subluminally, and (if so) experience the same retardation as a photon of the same energy. To see whether this is the case, one should consider the recoil of a D brane when struck by an energetic fermion. It is the technical analysis of this problem that is the next objective of this paper. Because of the symmetries of the scattering problem, one would expect any recoil to be (on average) along the $$x$$ axis, with a velocity $$\vec{u}_x = E/M_D$$ as before. This in turn would induce a modification $$G_{0x}$$ to the metric of form similar to that derived in the bosonic case, and hence a corresponding modification of the velocity of propagation: $$\delta c/c \sim E/M_D$$. To see this more mathematically, we now study a supersymmetric extension of the above model. This enables us, formally, to describe the propagation of a photino, rather than a neutrino, but we expect the conclusions to be the same. The breaking of supersymmetry is an issue, because the distortion of space-time induced by D-brane recoil itself breaks supersymmetry [22]. Nevertheless, if one ignores gravity effects, supersymmetry still constrains the relevant dynamics, especially the form of the boson-fermion interactions. For this reason, as we now show, particles in a supermultiplet induce identical recoil distortions to leading order.

2Such condensed-matter analogues of fermions moving in non-trivial space times may be a useful tool for analyzing quantum-gravitational problems, that might also be interesting to those working in the context of the loop-gravity approach to quantum gravity [8, 9, 13].
A complete analysis should involve superstrings and supermembranes, and an appropriate supersymmetric extension of the analysis of [18] to a logarithmic superconformal algebra on the world sheet would be necessary, but this lies beyond the scope of this work. As we now discuss, a relevant first step towards the introduction of fermions is to consider the scattering process directly in target space-time, in the heuristic context of a supersymmetric version of the \((d \lesssim 10)\)-dimensional \(U(1)\) BI theory. We recall that a supersymmetric version of BI theory in flat \((d = 10)\)-dimensional Minkowski space-time was considered in [23], and is particularly simple:

\[
\mathcal{L}_{SBI} \sim \int d^{10}x \sqrt{-\det \left( \eta_{MN} + F_{MN} - 2 \Xi_M \partial_N \lambda + \Xi_P \partial_M \lambda \Xi_P \partial_N \lambda \right)} \tag{13}
\]

This model was used in [23] to study \(D\) branes in the Dirichlet picture. In this sense, the ten-dimensional Lagrangian \([13]\) was applied to the world volume of a nine-brane. In that case there were two supersymmetries, one of which was spontaneously broken by the presence of the \(D\) brane, with the photino \(\lambda\) the corresponding goldstino particle of spontaneously-broken Poincare symmetry. The second supersymmetry is more subtle, but its appearance is explained in [23].

In a conventional string-theoretic approach, in order to obtain the form of the \((d < 10)\)-dimensional BI action relevant for our purposes here, one needs to implement dimensional reduction of the above action, which leads to extended supersymmetries. However, in our approach, one may obtain directly a four-dimensional BI action, by choosing the recoil background deformation \([2]\) appropriately, i.e., restricting oneself to \(u_i\) with non-trivial components only for \(i = 1, 2, 3\). In such a case, one may simply discuss a \(N = 1\) target-space supersymmetrization of the four-dimensional BI action \([24]\). This is what we do below, using it as a toy model for the discussion of fermion propagation in our recoiling \(D\)–brane framework.

We start with the bosonic part of the four-dimensional BI Lagrangian:

\[
L_{BI} = \beta^2 \left( 1 - \sqrt{-\det \left( g_{\mu\nu} + \frac{1}{\beta} F_{\mu\nu} \right)} \right) \tag{14}
\]

where the signature of the metric is assumed to be \((+, -, - -)\). We have in four space-time dimensions the identity

\[
\det \left( g_{\mu\nu} + \frac{1}{\beta} F_{\mu\nu} \right) = -1 - \frac{1}{2\beta^2} F_{\mu\nu}^2 + \frac{1}{16\beta^4} \left( F_{\mu\nu} \tilde{F}^{\mu\nu} \right)^2 , \quad \tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \tag{15}
\]

which allows the BI action to be expressed in terms of quadratic structures of the Maxwell tensor. An important ingredient is the appearance of the CP-violating term \(F \tilde{F}\), which is a characteristic feature of the gauge action in four dimensions. In the approach to \(D\)–brane recoil of [18], the \(U(1)\) field is a background \([2]\) associated with the collective coordinates of the \(D\)–brane soliton. The quantity \(\beta\) is related to the string coupling and the string length by:

\[
\beta = 1/\left( \ell_s \sqrt{g_s} \right) \tag{16}
\]
where $\sigma_s$ is the physical string coupling, renormalized (4) in the sense of [18].

We next consider treating [6] the interactions of photons with the background of recoiling D branes through (6), as appropriate for the Neumann picture [18]. Due to the identity (15), it is evident that if one ignores the gravity effects (9), the leading corrections to photon propagation will come from Lorentz-invariant terms of the form $f^2_{\mu\nu} \times \mathcal{O}(u_i^2)$, i.e., quadratic in the small recoil velocity $u_i$, and hence corrections are suppressed by quadratic powers of $M_s$, as expected due to Lorentz invariance. Our key step is to go beyond this, by treating gravitational recoil effects.

For the purposes of supersymmetrization, we treat the $U(1)$ gauge field $A_\mu$ as a full-fledged quantum field, and not simply as a background related to the collective coordinates of the $D$ brane. The $N = 1$ supersymmetric version of (14) can be constructed in a compact form if one uses superfields [24]:

$$L_{\text{BI\ susy}} = \frac{1}{4} \left\{ \int d^2 \theta W^2 + \int d^2 \bar{\theta} \overline{W}^2 \right\} + \sum_{s,t=0}^\infty a_{1st} \int d^4 \theta W^2 \overline{W}^2 X^s Y^t$$  \hspace{1cm} (17)

where $W_\alpha$ is the field-strength chiral supermultiplet, related to the vector superfield $V_\alpha$ in the usual way, and $X,Y$ are appropriate superfields, whose bosonic components read:

$$X_{|\theta=0} = -\beta^{-2} D^2 - \frac{1}{2} \beta^{-2} F^2_{\mu\nu} - i\bar{\lambda} \overline{\gamma} \lambda - i\bar{\varphi} \gamma \lambda,$$

$$Y_{|\theta=0} = \frac{1}{2} \beta^{-2} F^{\mu\nu} \overline{F}^{\mu\nu} + \lambda \overline{\gamma} \lambda - \bar{\lambda} \gamma \varphi \lambda$$  \hspace{1cm} (18)

where $D$ is an auxiliary field and $\lambda$ the photino field, which is a two-component Majorana spinor [1]. The expansion coefficients $a_{1st}$ are expressed in terms of inverse powers of the coupling $\beta^2$ [24]. For our purposes, we note that the supersymmetric extension (17) yields three kinds of terms: (i) pure bosonic terms, which yield the bosonic BI Lagrangian (14) when one uses the equations of motion for the auxiliary field $D$ (which also yield $D = 0$), (ii) self-interacting fermion terms $L_f$, and (iii) boson-fermion interactions, $L_{fb}$, which include the kinetic term for the fermions. The latter are the terms needed for our purposes, and we concentrate on them henceforth. Their detailed structure is given in [24], and will not be given here. It is sufficient for our purposes of describing recoil induced by fermion scattering to restrict ourselves to a background of the form (2), whilst keeping the photino field a full-fledged quantum field.

Combining the two-component fermions $\lambda_\alpha, \overline{\lambda}_\dot{\alpha}$ into a four-component Majorana spinor

$$\Lambda = \begin{pmatrix} \lambda_\alpha \\ \overline{\lambda}_{\dot{\alpha}} \end{pmatrix}$$  \hspace{1cm} (19)

one observes that the relevant $N = 1$ supersymmetry transformations can be expressed in the form:

$$\delta^S A_\mu = -i \varepsilon \gamma_\mu \Lambda,$$

$$\delta^S \Lambda = -i \left( \Sigma^{\mu\nu} F_{\mu\nu} + \gamma^5 D \right) \varepsilon,$$

$$\delta^S D = i \varepsilon \overline{\gamma}^5 \varphi \Lambda$$  \hspace{1cm} (20)

\textsuperscript{3} It is interesting to note that the action (17) yields pairing interactions similar to (10).
where the upper index $S$ in $\delta^S$ denotes a supersymmetry transformation, $\varepsilon$ is the appropriate (infinitesimal) supersymmetry parameter, $\Sigma_{\mu\nu} \equiv \frac{i}{4}[\gamma_\mu, \gamma_\nu]$, and $\gamma^5 \equiv i\gamma^1\gamma^2\gamma^3\gamma^0$.

We are now in a position to discuss the compatibility of the background (2) with $N = 1$ supersymmetry. As is obvious from the form of the supersymmetry transformation (20) and from the form of the bosonic background (2), compatibility with supersymmetry can be achieved for 'photino' fields $\Lambda$ which are independent of space and depend linearly on time $X^0$. A generic form for the Majorana spinor $\Lambda$ would then be:

$$\Lambda = \varepsilon \Lambda_1 + \Lambda_2 X^0$$

The quantities $\Lambda_i, i = 1, 2$ are quantized as a result of the summation over genera in a world-sheet framework [17, 18]. Although, rigorously, one should first explicitly check that supersymmetry survives such a resummation over higher world-sheet topologies, i.e., there are no anomalies associated with its quantization, here we simply assume this is the case. The $N = 1$ supersymmetry transformation (20) would then imply:

$$\delta^S Y_i = -i\varepsilon\gamma_i \Lambda_1, \quad \delta^S u_i = -i\varepsilon\gamma_i \Lambda_2, \quad \delta^S \Lambda_1 + \delta^S \lambda_2 X^0 = -i\Sigma^{0i} u_i \varepsilon + i\gamma^5 D\varepsilon, \quad \delta^S D = i\varepsilon\gamma^5 \partial_0 \Lambda_2,$$

from which it is clear that the $N = 1$ supersymmetric partner of the background (2) is the one with $\Lambda_2 = 0$, implying $D = 0$ and $\delta^2 D = \delta^S u_i = 0$, which is compatible with on-shell supersymmetry [1]. Thus, in flat target space times, the background is compatible with $N = 1$ supersymmetry.

We now study spinor propagation in the background (2). We notice first that this background conserves CP, since $\tilde{F}_{\mu\nu} = 0$, and then make a derivative expansion of the fermion-boson interactions. Restricting ourselves to the leading order in this expansion, we obtain the terms:

$$L_{fb} \ni -\frac{i}{2} \nabla\Lambda - \frac{i}{8} \nabla\Lambda \left( D^2 + \frac{1}{2} F^2_{\mu\nu} \right) - \frac{i}{4} \nabla\gamma^\mu \partial^\nu \Lambda F_{\nu\rho} F^\rho_{\mu} + \ldots$$

where the $\gamma^\mu$ are $4 \times 4$ Dirac matrices, and the $\ldots$ denote subleading derivative terms.

Using the background (2), then, we obtain from (23):

$$L_{fb} \ni -\frac{i}{2} \left( 1 - \frac{1}{8} u_i^2 \right) \nabla\Lambda + \frac{i}{4} u_i u_j \nabla\gamma^i \partial^j \Lambda - \frac{i}{4} u_i^2 \nabla\gamma_0 \partial^0 \Lambda + \ldots$$

We observe that, in flat space-times, supersymmetrization of the BI action implies non-trivial propagation of the massless ‘photino’ field in the recoil background (2). Moreover, as with the bosonic counterparts, the effects are suppressed by quadratic inverse powers of $M_D = M_s/g_s$, with $M_s = \ell_s^{-1}$. This may be traced back to the Lorentz-invariant form of the flat-space BI action (14), (15). As we discuss below, it is only after coupling to gravity, which manifestly breaks Lorentz invariance, that the modification of the propagation becomes linear. Such linear terms arise from the kinetic term of the photino field after coupling to gravitational backgrounds.

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4 We recall that the background (2) is a solution of the classical equations of motion.
Before analyzing this issue, we first comment on the extended supersymmetries that characterize BI actions when the latter are viewed as world-volume actions of $D3$ branes, which is different from the picture described above. Such supersymmetric formulations are obtained by appropriate dimensional reductions of the ten-dimensional flat Minkowski space-time. Six of the coordinates give rise to scalar fields, $y^i, i = 4, 5, \ldots, 9$ in the four-dimensional world-volume theory. In that case there is an extended supersymmetry of the $N = 4$ Yang-Mills type, as discussed in [25]. The spectrum of the gauge-fixed supersymmetric formulation of the $D3$-brane action in a flat space-time background consists of the world-volume Abelian gauge field $A_\mu, \mu = 0, \ldots, 3$, four four-component $d = 4$ Majorana spinors (extended ‘photinos’) $\Lambda^I_\alpha, I = 1, \ldots, 4$, where $\alpha$ is a superfield spinor index:

$$\Lambda^I = 2 \left( \frac{\lambda^I_\alpha}{\Lambda_{I,\bar{\alpha}}} \right)$$

and the scalar fields obtained from the dimensional reduction of the ten-dimensional theory, which are conveniently written as $s^{IJ} = \frac{1}{2} (\tilde{\sigma}_t)^{IJ} y^t, I, J = 1, \ldots, 4, t = 4, \ldots, 9$, where the $\tilde{\sigma}_t$ are $4 \times 4$ matrices appearing in the chiral representation of the Dirac matrices in six dimensions. In this way, there is a manifest $SU(4)$ symmetry, which makes the problem analogous to $N = 4$ supersymmetric Yang-Mills theory, in terms of expressed in terms of $d = 4$-dimensional ‘Yang-Mills’ variables ($A_\mu, \Lambda^I, s^{IJ}$).

The above construction is potentially useful, in that it incorporates four species of Majorana fermions, including those that become members of chiral supermultiplets when $N = 4$ supersymmetry is eventually broken down to $N = 1$. Hence it may be closer to providing a toy model for neutrino propagation in space-time foam.

The formalism of [18] applies intact to the description of the recoil of the $D$ particle after scattering by low-energy supersymmetric matter, i.e., photons and photinos, on the world-volume $D3$ brane. The recoil appears as a background contribution to the four-dimensional world-volume gauge potential of the form (2). The pertinent interaction terms can be read easily from the component form of the supersymmetric Lagrangian given in [23], and again, one arrives at similar conclusions (24) as above. The advantage of the above world-volume formalism is that one may combine two Majorana neutrino species into a Dirac one, and thus discuss formally the propagation of massless Dirac spinors as well.

4 Fermion Propagation in a Space-Time Metric Distorted by Gravitational Recoil

So far, using the recoil formalism of [18], and the appropriate supersymmetrization of the BI lagrangian, we have discussed the propagation of $U(1)$ vector particles and the corresponding ‘photinos’, ignoring the effect of gravitational recoil on the background. In this simplified case, the velocities of both photons and photinos differ from the naive low-energy value $c$ by amounts that are suppressed quadratically by two inverse powers of the string or $D$–brane scale, which are assumed in four-dimensional models to be near the
Planck scale $\sim 10^{19}$ GeV. Now it is time to explore the effect on fermions of the distorted gravitational background given by the metric (9).

As was discussed in section 2, the appearance of such a metric has been proven in the bosonic part of the world-sheet $\sigma$ model of the string, using an appropriate Liouville dressing on the world-sheet, and identifying the Liouville field as the target time, as explained in [17]. A similar procedure should be valid for superstrings, providing a formal arguments that fermions and bosons should create similar metric backgrounds when they scatter off a $D$ particle. A complete proof of this would involve extending the Liouville analysis of [17] to a world-sheet superfield Liouville formalism. We do not present such a proof here, but limit ourselves to heuristic arguments why fermions should induce a modified metric analogous to (9).

Physically, one expects a high-energy incident fermion to induce a $D$-brane recoil which is similar at least parametrically to that induced by an incident boson, since the most important kinematic constraint is that of energy-momentum conservation (3). Just as in the bosonic case, the $D$-brane recoil velocity $\tilde{u}_i$ should be of order $E/M$, where $M$ is of order the Planck or string scale. The only possible difference might be in the angular distribution of the recoil induced by fermion scattering. This order-of-magnitude argument would be strengthened in the limit of supersymmetry. As mentioned earlier, the recoil process itself violates supersymmetry, e.g., because it causes a deviation from the ground-state energy. However, we expect this breaking of supersymmetry to be negligible at high energies. In any case, we know that supersymmetry is not exact even in the ground state, so any argument based on exact supersymmetry should be treated with caution, except at high energies much larger than the supersymmetric mass splitting. This is actually the case for the main application we make at the end of this paper, namely to fermions with energies approaching $10^{19}$ eV. However, even at lower energies we expect the basic kinematic argument concerning the magnitude of the recoil velocity to be valid. Since the metric perturbation (9) is directly related to this recoil velocity, we also assume that the metric deformation induced by an energetic fermion is also of the generic form (1).

The next step is to consider the velocity of fermion propagation in such a deformed metric. In [3] there is a simple description of the propagation of electromagnetic waves in such a background, and the corresponding induced refractive index, based on an elementary analysis of Maxwell’s equations. We now carry out a similar analysis using the massless Dirac equation to calculate the fermion propagation.

The Dirac equation in an external gravitational field can be written using the spin connection given by Fock-Ivanenko coefficients:

$$
\Gamma_\mu = \frac{i}{4} \cdot \gamma^\nu \cdot \gamma_{\nu,\mu} = -\frac{1}{4} \cdot e^\nu_m \cdot e_{\nu m;\mu} \cdot \sigma^{mn} \tag{26}
$$

where $\sigma^{mn} \equiv -\frac{1}{2} \cdot [\tilde{\gamma}^m, \tilde{\gamma}^n]_-$, where we use the usual relations between the general relativity $\gamma^\nu$ and Lorentz $\tilde{\gamma}^m$ matrices:

$$
\gamma^\nu = e^\nu_m \cdot \tilde{\gamma}^m, \quad \gamma_\nu = e^m_\nu \cdot \tilde{\gamma}^m \tag{27}
$$
and
\[ \{ \gamma^\mu, \gamma^\nu \} = 2 \cdot g^{\mu\nu}, \quad \{ \tilde{\gamma}^m, \tilde{\gamma}^n \} = 2 \cdot \eta^{mn}, \]
(28)
as usual.

Assuming the small metric perturbation (9), about flat Minkowski space time, with \(|\vec{u}| << 1\), one has get the following expression for the vierbeins:

\[ e^\nu_m = e^m_\nu = \begin{pmatrix} -1 & 0 & 0 & -u_1 \\ 0 & -1 & 0 & -u_2 \\ 0 & 0 & -1 & -u_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]
(29)

The general form of the Dirac equation can be written in the next form:

\[ \{ \gamma^\nu \cdot (\nabla_\nu - \Gamma_\nu) \} \psi = 0 \]
(30)
where the covariant derivative \( \nabla_\nu \) is derived from the connection \( \Gamma^\alpha_\mu_\nu \):

\[ \nabla_\nu = \partial_\nu + \Gamma^\mu_\nu_\mu \]
(31)
In our metric (9), the Dirac equation (30) becomes

\[ \{ \tilde{\gamma}^m \cdot \partial_m - \tilde{\gamma}^0 \cdot (\vec{u} \cdot \vec{\nabla}) \} \Psi = 0. \]
(32)

In order to derive the dispersion relation for the massless fermion, we act on (32) from the left with the operator \( \tilde{\gamma}^\mu \cdot \partial_\mu \) to obtain:

\[ \{ \partial_0^2 - \vec{\nabla}^2 - 2 \cdot (\vec{u} \cdot \vec{\nabla}) \partial_0 \} \cdot \Psi = 0. \]
(33)
It is then easy to see that the correct dispersion relation between the energy \( E \) and momentum \( p \) of the massless fermion in the metric background (9) is:

\[ E^2 = p^2 - 2 \cdot E \cdot (\vec{p} \cdot \vec{u}) : \quad |\vec{u}| \sim E/M. \]
(34)
which is similar to that obtained previously for a massless boson (photon), as we expected.

The naturalness of this conclusion can also be seen by considering the group theory of the perturbation (9), as discussed in the Appendix. The metric is invariant under a subgroup of the \( SL(2,C) \) transformations which leave the magnitude of the vector \( \vec{u} \) invariant, so one expects the dispersion relation to have the same invariance properties, which leaves (34) as the unique possibility.

We also comment that similar quantum-gravity corrections to the dispersion relations of bosons and fermions in space-time foamy backgrounds has also been observed in the loop approach to quantum-gravitational space-time foam, an a study of massive spin 1/2 fields [13]. The corrections in that case result [9] from the discrete (cellular) structure of space-time at Planckian distances, which is a characteristic feature of the loop-gravity approach [9]. The temptation to take (34) as a serious possibility and explore its phenomenological consequences can only be enhanced by this convergence of two very different approaches to the description of space-time foam.
5 The Phenomenology of High-Energy Neutrino Pulses

We now apologize to any astrophysical readers for the previous formal excursion, and now consider the possible observational implications of the modification (34) of the propagation of massless fermions in the gravitational background (9) induced by gravitational recoil: $|\vec{u}| \sim E/M$.

As we have pointed out previously [1, 3] in analyses of possible deviations of photon velocities from the naive velocity of light, the differences in travel times $t$ of particles with different energies are given to leading order by

$$\delta t \sim -L \cdot \delta c$$

in units where $c = 1$, where $\delta c \sim -E/M$, resulting in

$$\delta t \sim L \cdot E/M$$

so there is a premium on observing astrophysical sources at large distances $L$ that emit high-energy pulses with narrow time structures, so as to be sensitive to the largest value of $M$.

Possible examples might include AGNs, at typical redshifts $z \sim 0.03$, and GRBs at $z \sim 1$, though we emphasize that no energetic neutrinos from such distant sources have yet been observed. Concerning AGNs, neutrino energies comparable to the maximum observed $\gamma$ energies of around $10^{12}$ eV could be envisaged, whereas it has recently been proposed that GRBs might emit neutrinos with energies as high as $10^{19}$ eV [12]. AGN $\gamma$-ray fluxes are known to exhibit time variations on scales down to 300 s, and one might conjecture a similar time scale for possible $\nu$ emissions. On the other hand, a typical GRB time scale is 1 s, and even much shorter time scales have been observed, though not (yet) in the subsample of GRBs known to have cosmological redshifts.

Because of their similar $\nu$ and $\gamma$ energies, the sensitivity to deviations from the low-energy velocity of light $c$ obtained from observations of AGN neutrinos would be comparable to that from photons, namely approaching $M \sim 10^{17}$ GeV, as discussed elsewhere. On the other hand, to estimate the possible sensitivity that could be obtained from an observation of a GRB $\nu$ pulse, we assume a distance of 3000 Mpc, $\nu$ energies $E \sim 10^{19}$ eV and a pulse resolution of 3 s, leading to a sensitivity to $M \sim 10^{27}$ GeV! This is many orders of magnitude beyond what could be achieved with photons, because of their much shorter mean free path at high energies, and is far beyond the normal Planck scale $M_P \sim 10^{19}$ GeV. If our proposal of a linear deviation $\delta c \sim -E/M$ of the velocities of high-energy particles from the canonical low-energy velocity of light $c$, with $M \lesssim M_P$ is correct, no such pulse should ever be seen. Conversely, if such a high-energy neutrino pulse were to be seen, it would cast doubt on our expectation of a linear deviation of the velocities of high-energy particles from the canonical low-energy velocity of light $c$.

As in the case of photons, our medium effects can easily be distinguished from those of a neutrino mass.

6 For the record, we comment that even if the deviation from $c$ were only quadratic: $\delta c \sim -(E/M)^2$, which we repeat that we do not expect, such a high-energy GRB $\nu$ pulse would be sensitive to $\dot{M} \sim 10^{19}$ GeV!
6 Summary and Prospects

High-energy neutrinos from astrophysical sources may offer an unparalleled observational opportunity to study whether the velocity of light is universal, since they can propagate across the entire universe essentially unimpeded. Encouraged by this possibility, we have given in this paper heuristic arguments extending our previous suggestion that $\delta c \sim -E/M$ for energetic photons to the analogous case of fermions. Our arguments have been based on a supersymmetric extension of our previous $D$-brane approach to modelling the medium properties of the quantum-gravitational vacuum, and on an analysis of fermion propagation in the perturbed gravitational background metric (9) suggested by these arguments. We re-emphasize that the suggested linear effect would be due to gravitational recoil, and that the neglect of this possibility would lead to a quadratic deviation $\delta c \sim -(E/M)^2$. A linear deviation has also been motivated by studies within the loop approach to quantum gravity [13].

Much work remains to be done, even within the framework of the $D$-brane approach espoused here. For example, the development of the appropriate logarithmic superconformal algebra would be of formal interest. More practically, it would be good to rederive the gravitational-recoil effect for fermions without appealing at all to supersymmetry, which is certainly broken, both in the ground state and by the recoil process itself [22]. At a more profound level, it is desirable to establish more firmly the theoretical foundations of the $D$-brane approach, and to relate it more directly to alternatives such as the loop approach to quantum gravity.

Despite these theoretical lacunae, we believe that we have provided in Sections 2 to 4 sufficient motivation from fundamental physics to take an active interest in the observational opportunity that may be provided by distant high-energy neutrino sources. Moreover, as seen in Section 5, the sensitivity these could offer to possible deviations of high-energy particle velocities from the canonical low-energy velocity of light are very impressive: plausible GRB parameters could provide sensitivity to $\dot{M} \sim 10^{27}$ GeV in the (favoured) case of a linear dependence on energy, and even $\dot{M} \sim 10^{19}$ GeV in the (non-gravitational) case of a quadratic dependence. Let us hope that Nature obliges us by providing these or other such distant and pulsed sources of high-energy neutrinos.

Appendix: Reduced Lorentz Symmetry

We recall that there is a homomorphism: $A \to L_A$ which relates elements $A$ of the universal covering group $SL(2, \mathbb{C})$ to elements $L_A$ of the connected component $L^+_0$ of the Lorentz group $L$ with positive determinant and time-like coefficients $L_0^0$. We also recall that any $A \in SL(2, \mathbb{C})$ can be decomposed as

$$A = B \cdot \Omega,$$

where the Hermitian matrix $B$ and the unitary matrix $\Omega$ determine the boost and the 3-space rotation, respectively:

$$B = \exp(\omega/2)(\vec{n} \cdot \vec{\sigma}),$$

where $\vec{n}$ is a unit vector.
\[ \Omega = \exp \left( -i \cdot \frac{\varphi}{2} \right) \langle \vec{n} \cdot \vec{\sigma} \rangle. \quad (38) \]

where \( \vec{\sigma} \) denotes the \( 2 \times 2 \) Pauli spin matrices, and the operator \( L_B \) is a pure \( \vec{n} \)-boost with a velocity

\[ \frac{\vec{V}}{c} = \tanh(\omega) \cdot \vec{n}. \quad (39) \]

For example, for a pure Lorentz boost in the direction \( x_1 \) the corresponding matrix \( A \in SL(2, \mathbb{C}) \) has the following simple form: \( A = a_0 \cdot \sigma_0 + a_1 \cdot \sigma_1 \) with the following constraint on the real parameters: \( a_0 = \cosh \omega/2 \) and \( a_1 = \sinh \omega/2 \), such that \( a_0^2 - a_1^2 = 1 \).

Let us now consider the interval in our modified metric (9):

\[ ds^2 = g_{\mu \nu} \cdot dx_\mu dx_\nu = -dx_1^2 - dx_2^2 - dx_3^2 + dx_0^2 - 2 \cdot \vec{u} \cdot \vec{dx} \cdot dx_0, \quad (40) \]

where

\[ g^{\mu \nu} = \begin{pmatrix} -1 & 0 & 0 & -u_1 \\ 0 & -1 & 0 & -u_2 \\ 0 & 0 & -1 & -u_3 \\ -u_1 & -u_2 & -u_3 & 1 \end{pmatrix} \quad (41) \]

and \( x_0 = c \cdot t = -i \cdot x_4 \).

One can introduce a group of transformations that leave this metric invariant:

\[ \hat{\Lambda} = \hat{O}^{-1} \cdot L \cdot \hat{O}, \quad (42) \]

where \( L \) is an ordinary Lorentz transformation, such as a boost \( \omega \) in the \( x \) direction. The operator \( \hat{O} \) diagonalizes the metric in the form:

\[ g^D_{mn} = diag(-r^2, -1, -1, +r^2), \quad (43) \]

where \( r = \sqrt{1+u^2} \). In the special case, \( u_1 \neq 1, u_2 = u_3 = 0 \) and \( r = \sqrt{1+u_1^2} \), one finds:

\[ O = \begin{pmatrix} \frac{1}{r} & 0 & 0 & \frac{\sqrt{r^2-1}}{r} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (44) \]

and the general case is also easily found.

It is also possible to define transformed Pauli matrices:

\[ \hat{\sigma}(x) = \begin{pmatrix} x_0 - \sqrt{r^2-1} \cdot x_1 + x_3 \\ r \cdot x_1 + ix_2 \\ r \cdot x_1 - ix_2 \\ x_0 - \sqrt{r^2-1} \cdot x_1 - x_3 \end{pmatrix}, \quad (45) \]

whose determinant reproduces the deformed metric, where

\[ \hat{\sigma}_i = \sigma_i, \quad i = 0, 2, 3 \quad (46) \]

and

\[ \hat{\sigma}_1 = r \cdot \sigma_1 - \sqrt{r^2-1} \cdot \sigma_0 = \begin{pmatrix} -\sinh \varpi & \cosh \varpi \\ \cosh \varpi & -\sinh \varpi \end{pmatrix} \quad (47) \]
where \( \cosh \varpi = r \) and \( \sinh \varpi = \sqrt{r^2 - 1} \).

The modified Lorentz transformations \( \hat{\Lambda} \) can be expressed by the following \( SL(2,C) \) matrices:

\[
\begin{align*}
\hat{\Lambda}_0 &= \frac{1}{2} \cdot \left\{ \text{Tr} (A\tilde{\sigma}_n A^+) + \tanh \varpi \text{Tr} (\sigma_1 A\tilde{\sigma}_n A^+) \right\} \\
\hat{\Lambda}_1 &= \frac{1}{2} \cdot \frac{1}{\cosh \varpi} \cdot \left\{ \text{Tr} (\sigma_1 A\tilde{\sigma}_n A^+) \right\} \\
\hat{\Lambda}_2 &= \frac{1}{2} \cdot \left\{ \text{Tr} (\sigma_2 A\tilde{\sigma}_n A^+) \right\} \\
\hat{\Lambda}_3 &= \frac{1}{2} \cdot \left\{ \text{Tr} (\sigma_3 A\tilde{\sigma}_n A^+) \right\}.
\end{align*}
\]

\( \text{(48)} \)

A boost in the \( x_1 \) direction can be expressed by the following type of modified symmetric and unimodular Lorentz operator:

\[
\hat{\Lambda}_{B_1}^T = \hat{\Lambda}_{B_1} = O^{-1} \cdot L_{B_1} \cdot O, \quad \text{det}(\hat{\Lambda}_{B_1}) = 1.
\]

\( \text{(49)} \)

which forms a one-parameter subgroup. The composition of two boosts \( \Lambda_{B_1}(\omega_1|\varpi) \) and \( \Lambda_{B_1}(\omega_2|\varpi) \), is itself a boost \( \Lambda_{B_1}(\omega|\varpi) \) in the same direction, with parameter \( \omega = \omega_1 + \omega_2 \):

\[
\Lambda_{B_1}(\omega_1|\varpi) \cdot \Lambda_{B_1}(\omega_2|\varpi) = \Lambda_{B_1}(\omega_1 + \omega_2|\varpi).
\]

\( \text{(50)} \)

There is also a one-parameter group symmetry connected to pure rotations in the \((x_2-x_3)\) plane:

\[
\hat{\Lambda}_{\Omega_{23}} = O^{-1} \cdot L_{\Omega_{23}} \cdot O.
\]

\( \text{(51)} \)

Thus, for finite \(|u|\), there exists a two-parameter symmetry group, generated by the above boosts and rotations:

\[
\hat{\Lambda} = \hat{\Lambda}_{B_u} \cdot \hat{\Lambda}_{\Omega_u}
\]

\( \text{(52)} \)

surviving from the full \( SL(2,C) \) group.

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