Thin liquid film entrainment by moving solids

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Abstract
The thin liquid film entrained by a continuous immersed solid pulled through a tank is of the main interest in many industrial processes. In order to appraise the state of art, it has been reviewed the experimental and theoretical studies of a solid being withdrawn horizontally from a tank. This review has revealed that the open literature is scarce. The main objective of this paper is to study the above stated problem. To this end, it has been performed a series of experiences of horizontal withdrawal of wires from a tank of water, and an approximate theoretical model has been developed.

Key words : Liquid film entrainment, Horizontal moving wires, Coating, Film thickness, Horizontal withdrawn

1. Introduction

There are many industrial processes in which a continuous solid – filaments, yarns, wires, pipes, sheets and many others of different materials: polymer, wool, plastic, glass and steel- is moving immersed through one or several tanks containing liquid for impregnation, flushing, cleaning or coating operation. A solid body, withdrawn at a constant velocity from a tank containing liquid, appears covered by a layer of liquid which thickness becomes uniform after certain distance from the exit of the tank. The knowledge of this film thickness is one of the main interests from many practical applications. The manufacture of magnetic information storage systems by coating a solid with a layer of solute can be an example. In this process, the final film thickness is required to be very thin and must be highly accurate at high-speed applications. In other cases, when solids pass from a tank containing liquid to another, the concomitant fluid retention on them causes bath concentration to change.

With regard to the general problem of entrainment viscous liquids by solids vertically withdrawn from a tank containing liquid, one has carried out a brief summary of the existing knowledge. This knowledge is necessary in order to have a reference that help us to place and analyse the experimental results obtained in the present study because the entrainment of water by cylindrical solids (wires), horizontally withdrawn at high velocity from a tank, clearly shows performance differences in comparison to the same entrainment process when the wires are vertically withdrawn.

2. Brief review of previous works on thin liquid film entrainment by moving solids

When a solid is carry away at velocity $U$ through a reservoir containing a pure Newtonian fluid, the thin liquid film $h$ entrained by the solid, for instance a cylindrical solid of radius is $a$ must be a function of the capillary number, the Reynolds number and the Bond number:

$$\frac{h}{a} = f\left(\frac{\mu U}{a}, \frac{\rho U a}{\mu}, \rho g \frac{a^2}{\sigma}\right)$$  \hspace{1cm} (1)
When the solid is moving horizontally, the gravitational effect is negligible. In addition, if the velocity of the solid is low, the coating thickness is solely the result of the balance between viscous forces and capillary pressure forces. Then:

$$\frac{h}{a} = f\left(\frac{\mu u}{\sigma}\right)$$  \hspace{1cm} (2)

In England, Research Staff of General Electric Company Ltd (1922) raised the interesting problem of viscous lifting of a liquid by withdrawal of a plate. Unfortunately, their theory failed down because was based on the negligible effect of the surface tension of the liquid. Afterwards, Bindham and Young (Bindham and Young, 1922), Jeffreys (Jeffreys, 1930) and Gaucher and Ward (Gaucher and Ward, 1922) published experimental and theoretical studies of liquid film entrainment by both plate and fibre coating. The cited authors concluded that the final thickness of the entrained film was the result of a competition between the viscous liquid driving the solid and the capillary resistance of the surface to strain, neglecting the effect of gravity. Thus, they proposed a solution of the form $h = Cte_f (Ca)$. Landau and Levich (Landau and Levich, 1942) and Derjaguin (Derjaguin, 1943) proposed the first theory for the thickness of the film for capillary numbers smaller than unity, which relevance Block and Van Rossum (Block and Van Rossum, 1948) proved later empirically.

This theory starts from the Navier-Stokes equation for the flow of thin liquid films in laminar motion, which, written in the form of boundary layer equation, is:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -1 \frac{\partial p}{\rho \partial x} + \nu \frac{\partial^2 u}{\partial y^2} + f_x$$  \hspace{1cm} (3)

If the film thickness is small, this equation may be simplified because all the velocity derivatives across the film are large compared to those along the film. The Landau and Levich theory examines the thickness of the liquid layer remaining on the surface, a flat plate withdrawn from a tank of liquid, from Eq. (3),

$$\frac{\sigma}{\rho} \frac{d^2 h}{dx^2} + \nu \frac{\partial^2 u}{\partial y^2} + g = 0$$  \hspace{1cm} (4)

To this equation, they arrive assuming, among others, the following conditions: (i) The plate is lift out the tank of liquid at constant low velocity, (ii) The capillary pressure is, according to the Laplace’s equation, the pressure gradient in the liquid mass flow entrained, and (iii) The mass of liquid brings out the surface of the tank is set up as two regions: one, in which the plate entrains a liquid film; the other where the meniscus is stationary. According to these assumptions, and using the simplified form of Eq. (3), the limiting thickness of the liquid layer entrained by the plate is,

$$h_0 = 0.93 \left(\frac{\mu v_0}{\sigma \rho g}\right)^{\frac{2}{3}}$$  \hspace{1cm} (5)

Equation (3) is written, after being transformed, as

$$h_0 = 1.34 \frac{a C_a}{Ca}^{\frac{2}{3}}$$  \hspace{1cm} (6)

Levich (Levich, 1962) concludes that the procedure followed to obtain the Eq. (5) is valid provided that holds the following inequality, $Ca \ll 1$. On the other hand, the Landau and Levich model it is not valid to calculate the thickness of the thin liquid layer entrained by solids, cylindrical in shape (fibres, wires, etc.) at high velocity.

By knowing the important implications of this phenomenon in industrial processes, it has been subject of a large number of research works. For instance, the studies carried out by White and Tallmadge (White and Tallmadge, 1965), Dussan (Dussan, 1979), de Ryck and Queré (Ryck and Queré, 1993, 1996), Queré & De Ryck (Queré & De Ryck, 1999) and Rebouillat, et al. (Rebouillat, et al., 1999, 2002), among many others.

In the case of liquid entrainment by moving solids at capillary number tending to unity, some experimental works have shown that the Landau equation is no longer applicable. With the purpose to overcome this limitation, White &
Tallamdege (Tallamdege, 1966) proposed the following modified form of that law:

\[ h_0 = \frac{1.34\alpha \rho g}{\mu^2} \]  

(7)

Another modification of the Landau equation is required when the withdrawal velocity is high. Based on dimensional arguments, Quéré & de Rick (Quéré & De Ryck, 1999) proposed an expression that introduces the Weber number in the form:

\[ h_0 = \frac{1.34\alpha \rho g}{\mu^2} \]  

(8)

In Eq. (8) \( \beta \) is a constant that can be determined through curve fitting of experimental data. In the general case of an arbitrary withdrawal velocity of a plate, Levich (Levich, 1962) proposes the equation:

\[ h_0 = \left( \frac{\mu v_0}{\rho g} \right)^{\frac{1}{3}} f \left( \frac{\mu v_0}{\sigma} \right) \]  

(9)

In the Eq. (9), the function \( f \) takes the form,

\[ f \left( \frac{\mu v_0}{\sigma} \right) \approx 0.93 \left( \frac{\mu v_0}{\sigma} \right)^{\frac{1}{3}} \]  

(10)

Deryagin & Titijevskaya (Deryagin & Titijevskaya, 1945) verified experimentally the Eq. (9) over a wide range of withdrawal velocities. The experiments, carried out on fibres rapidly withdrawn from a bath by Koulago, et al. (Koulago, et al., 1995) deserve particular attention. The results displayed in a plot show the relative thickness of film entrained by the fibres, referred to the thickness defined by the Landau equation, as a function of the dimensionless number \( S \): the Weber number. This data plot shows that the deviation occurs around a value of \( S \) equal to one for the different radius of fibres. By defining the parameter \( T_0 = h_0(\rho g/\mu U)^0.5 = 0.944\alpha \rho g \), all the results tend to concure to one curve for a value of \( S \) approximately 0.3. The asymptotic value of \( T_0 \) given by Ruschak (Ruschak, 1985) is 0.68. It seems that the surface tension is no longer important beyond a value of \( S \), approximately 0.3.

In the process of vertical withdrawal of solids, the liquid film entrained drains by the simultaneous action of the forces due to the Laplace pressure gradient and the gravity field. For this reason all the theoretical models include the term of gravity, \( g \). However, gravity force can be neglected if it remains smaller than the capillary force: \( \rho g \ll \sigma/(\alpha + h)^2 \). For capillary numbers smaller than unity, that condition is properly if the value of Bond number is much smaller than one.

2.1 Summary

The previous review concludes that nearly all of experimental and theoretical studies of liquid entrainment by moving solids concern to flat plate and fibres withdrawn vertically at low velocity from a large reservoir of viscous liquid. On one hand, although several researchers, namely Cerro and Scriven (Cerro and Scriven, 1980) and Kheshgi, et al. (Kheshgi, et al., 1992), analysed coating flows at high capillary and Reynolds numbers by using approximated methods, no systematic experimental studies have been performed in the past under those conditions.

On the other hand, the moving solid always drags the liquid from a mass of fluid limited by walls sufficiently far from the solid. This condition is of great importance when cylindrical solids are horizontal withdrawn at high velocity at the end of a tank containing water. This problem does not appear explicitly in the open literature. Only the experiments conducted by A. de Ryck and D. Quéré (Ryck and D. Quéré, 1996) have some interest as an approximation to the problem concerning to the present work; however, the experimental set-up characteristics, the liquid properties (silicone oil) and the withdrawn velocity differ considerably from the experiences described in this work.

Therefore, since there are many important industrial processes, such as high turbulence pickling systems, cleaning and quenching of steel wire, etc, the study of the liquid film entrainment by moving horizontally solids submerged into
water at high speed is of great interest, and can be very useful for the designing of equipment that guarantees reliability and efficacy. By these reasons, we think that the carry out of experiences about that particular flow phenomenon and the developing of an approximate theoretical model in order to explore the underlying physical principles in the obtained results, can contribute to fill the existing gap on the matter.

3. Experimental study
3.1 Materials and method

Experimental tests on four steel wires have been carried out, three of them galvanized. Their diameters are 4.4, 3.0, 2.7, and 1.6 mm, being the first one the wire no galvanized. To characterise the surface of each wire, it was measured its roughness in a Palm-top Roughness tester RT-10. According to standard ISO 4287, the most important parameters characterizing the surface irregularities of a solid body are $Ra$, $Rq$, $Rt$, $Rz$, $Rc$ and $Rsm$. Table 1 summarizes the mean value of these parameters, five times measured, corresponding to the wires 4.4 mm and 2.7 mm diameter, respectively. By observing the results obtained, it is obvious that the galvanized wire is the wire with less roughness.

Table 1  Roughness parameters of the rough 4.4 mm diameter wire and the 2.7 mm diameter galvanized wire, according to standard ISO 4287

| d (mm) | $Ra$ | $Rq$ | $Rt$ | $Rz$ | $Rc$ | $Rsm$ |
|-------|------|------|------|------|------|-------|
| 4.4   | 1.21 | 1.48 | 7.97 | 6.23 | 3.35 | 142.80 |
| 2.7   | 0.56 | 0.69 | 4.14 | 2.86 | 1.67 | 366.00 |

3.2 Experimental method

Figure 1 shows the sketch of the experimental set rig.

![Figure 1 Sketch of the experimental set-up](image)

The dimensions of the tank are 2.5 m length, 0.057 m width and 0.075 m depth. The tank has two orifices of 0.010 m diameter, one at each end. The wires crossed over the tank submerged into the water at a depth of 0.010 m from the water free surface. In order to determine the mass of water entrained by each wire, the experimental set up sketched in Fig. 1 was fitted with two absorbing plain pads and an attached small receiver. The mass of water carried by each wire in every experience is the sum of the increased weight of the absorbing pads maintained in contact with the wire for 60 s, and the weight of water in the small receiver, collecting the falling drops from the entry to pads. The length of the pans was enough to assure that the wire becomes fully dry at its exit.

4. Experimental results, comments and remarks
4.1 Results and comments

Table 2 shows the data and the experimental results for all the experiences carried out.
Table 2  Data and experimental results

| $a$ (mm) | $\dot{m}$ (g/s) | $h$ (mm) | $\dot{m}$ (g/s) | $h$ (mm) | $\dot{m}$ (g/s) | $h$ (mm) | $\dot{m}$ (g/s) | $h$ (mm) |
|---------|-----------------|---------|-----------------|---------|-----------------|---------|-----------------|---------|
| 2.2     | 0.900           | 0.1955  | 1.5             | 0.130   | 0.0552          | 1.35    | 0.750           | 0.1608  |
| 1.5     | 1.450           | 0.2098  | 1.35            | 0.2050  | 0.1673          | 0.8     | 0.1570          | 0.1683  |
| 1.35    | 2.173           | 0.1887  | 0.8             | 2.860   | 0.1517          | 120     | 2.150           | 0.1521  |
| 0.8     | 2.587           | 0.2004  | 160             | 3.470   | 0.1381          | 200     | 3.610           | 0.1277  |
|         | 3.500           | 0.1266  |                 |         |                 |         |                 |         |

Figure 2 renders evidence that the mass flow of water entrained by the moving wire, $\dot{m}$, is, for the same values of liquid viscosity and surface energy, an increasing function of the its radius and its velocity.

The equation fitting the experimental results with $r^2 \geq 0.999$ for every wire is:

$$ \dot{m} = a + bV^c $$  

(11)

In the Eq. (11), the value of the exponent $c$ belongs to the interval $0.48 \pm 8\%$ for the wires of smooth surface, and $0.3$ for the rough one. The continuity equation for the water mass flow entrained by wire is:

$$ \dot{m} = \rho \pi ((h + a)^2 - a^2)V $$

(12)

where $\rho$ is the liquid density, $a$ the wire radius, $V$ the wire velocity, and $h$ the liquid film thickness entrained. The equation $\dot{m} \approx \rho 2\pi a h V$ simplifies the Eq. (12) provided that $h \ll a$ is satisfied. Hence, entrained water mass flow for which the film thickness $h$ fulfills $h \ll a$, should be a $1/aV$ function. The Fig. 3 shows this relationship.
Figure 4 shows the mass of water entrained by unit surface of wire in function of wire velocity and its radius, as parameter. This plot clearly shows the occurrence of a change of behaviour in the entrainment of liquid by the wire when it reaches a speed that we call critical. This wire critical speed is 0.75 m/s for the smooth wire and 0.5 m/s for the rough one, approximately. The corresponding capillary numbers to these velocities are 0.007 and 0.00997, respectively. The last value is close to the limit 0.01, quoted by many authors for the Landau-Levich basic theory for the vertical withdrawal of plate surface, when the effect of gravity is taking into account.

The evidence of change of behaviour appears also in Fig. 5 showing the liquid film thickness entrained by the wires, calculated by means of Eq. (12) with the experimental values of $\dot{m}$ and of $V$, in function of wire velocity. Figure 6 shows the non-dimensional water film thickness versus the Reynolds number, both scaled by the wire radius $a$. The change of $h/a$ trend appears between 600 and 1200 Reynolds numbers defined in terms of wire radius $a$, or between 1200 and 2400 Reynolds numbers defined in terms of wire diameter $d$, values for which the flow regime in pipes is transition.

Considering the experimental results and the negligible effect of gravity because the movement of the wire is horizontal, Eq. (1) it is substituted by the more simple $(h/a) = f(S)$, since $g(Ca, Re) = f(S)$. Figure 7 shows the non-dimensional liquid film thickness $h/a$ versus $S$ for different wires, and the Fig.8 the liquid film thickness versus $S$ for $Re(d) \geq 2000$.

In summary, it can be concluded that the phenomenon of liquid entrainment by wires moving horizontally through a tank of water shows two different behaviours depending on the velocity that are being pulled out the tank. Up to a critical velocity, the thickness of the liquid layer entrained by the wire is an increasing function of the pulling velocity; to velocities greater than the critical, is decreasing. This change of behaviour occurs when the flow generated by the moving wire becomes turbulent.
Fig. 4 Entrained water mass by unit surface of wire versus wire velocity

Fig. 5 Liquid film thickness versus wire velocity
Fig. 6 Non-dimensional water film thickness $h/a$ versus Reynolds number defined in terms of wire radius

Fig. 7 Non-dimensional liquid film thickness versus $S$
5. A plausible approximate theoretical model for the TLF entrainment phenomenon

In order to develop a theoretical model approaching the studied phenomenon, helping us to explain the obtained experimental results in terms of the main physical magnitudes and their dimensionless numbers, is useful to divide the solid-fluid interaction process in two stretches. The travel of the continuous cylindrical solid along the water tank, moving submerged in it, is the first stretch. The second stretch extends from the orifice at the exit of the tank up to where the moving solid, with the entrained thin liquid film, is horizontally withdrawn.

At the first stretch, the flow promoted by the movement of the solid into a mass of water at rest it is supposed to be a stationary two-dimensional boundary layer type. At the solid surface, the fluid moves in the direction of the moving solid with an equal velocity, whereas at increasing distance from the surface the velocity of the fluid approaches zero.

The approximate theoretical model starts from the following conditions and hypothesis:

(i) The wire is moving horizontally into a tank containing water, and the promoted flow regime is stationary
(ii) The capillary numbers and the Bond numbers are smaller than unity; hence, the gravity force can be neglected and the liquid film remains axisymmetric during the withdrawal
(iii) The flow induced by the moving solid into the tank is defined by the Navier-Stokes equations
(iv) The thin liquid layer entrained by the moving solid starts from the stationary meniscus at the exit of the tank, whereby the pressure gradient is defined by the Laplace pressure
(v) The entrainment of the thin liquid layer by the solid occurs there where the equilibrium between the flow stresses and capillary pressure is broken
(vi) The velocity distribution of the flow entrained by the moving solid in the tank is defined by the equation:

\[ u = U_w \left( 1 - \beta \ln \left| \frac{x}{\Delta} \right| \right) \] (13)

A previous matter that it is necessary to analyze is if the boundary layer thickness of the flow arriving to the hole is larger than the gap between the edge of the orifice and the surface of the wire or not, for what it is needed to know the boundary layer thickness of the flow arriving to the orifice.

The study of the boundary layer flow promoted by a submerged solid moving horizontally into a liquid at rest it was first undertaken theoretically by Sakiadis (Sakiadis, 1961) and by Koldenhof (Koldenhof, 1963) experimentally. Later on, several researches have approached the problem, namely Crane (Crane, 1972), Rao (Rao, 1972), and very recently Tutty (Tutty, 2008), Jordan (Jordan, 2014) and Xu (Xu, 2014), among others. With regard to the boundary layer thickness, Tutty assumes that close to the orifice the boundary layer is small in thickness and is considerably smaller than the radius of the cylinder. This means that the flow close to the orifice can be bear in mind as the flow over a moving flat sheet issuing from a slit with \( U_0 \) velocity. This problem was considered by Sakiadis, deriving the parameter \( 4v/L/u_0a^2 \), where \( x \) is the distance from the orifice at inlet of the tank.

This flat sheet solution will clearly be a good approximation when the boundary layer thickness is much smaller than the radius of the cylinder, i.e. \( 4v/L/u_0a^2 \) \( \ll 1 \). For large \( x \), Crane assumes that the boundary layer is much thicker than the cylinder. In this case, this approximation to take into account the thickness, or the displacement thickness, is no suitable because the displacement of the streamlines away of the surface decay inversely with \( r \) due to the expansion in area with \( r \). Hence, the utilization of the displacement area parameter \( \Delta \) is better than the boundary layer thickness. For the set of wires and velocities involved in the experiences, the \( 4v/L/u_0a^2 \) value, where \( x \) is the tank length and \( U_0 \) the wire velocity, is much greater than the unity. For this condition, the non-dimensional displacement area, defined by Crane by the expression:

\[ \frac{\Delta}{\pi a^2} = \frac{4vL}{U_0a^2} \left[ 1.63 + 0.21 \left( \frac{a}{\sqrt{r}} \right) \right] \] (14)

is always much greater than the unity whichever be the values \( (a, U_0) \) in the experiences. Therefore, the entrained flow inside the tank and the free surface of the mass of liquid surrounding the wire after the tank exit, it can be assumed that approximately take the shape depicted in Fig. 9.
On the other hand, the flow of water coming out the tank must satisfy the Navies-Stokes equations with an adverse pressure gradient due to capillary pressure without body forces. These equations are:

\[
\rho \frac{d\vec{V}}{dt} = -\Delta p_c + \nabla \tau ; \quad \nabla \vec{V} = 0
\]  

(15)

For the water flow coming out of the tank, the dynamic boundary conditions at the free surface with curvature are much more difficult to write and explain than for static conditions. Physically, one boundary condition states that (for the case of gas-liquid interfaces) there is no tangential shear stress to the interface. The other dynamic condition states that the stress arising from the surface tension acting normal to the surface of liquid (described by the Young-Laplace equation) has to balance the stress within the liquid, normal to the interface. According to the above conditions, it is possible to accept that there where the liquid film entrained by the wire is withdrawn the following condition must be satisfied:

\[
\Delta p_c = \nabla \tau
\]  

(16)
In cylindrical coordinates, the component of the Eq. (16) in the \( x \) direction is,

\[
\frac{\partial}{\partial x} p_c = \frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) - \frac{1}{r} \frac{\partial}{\partial r} \left( \rho r \mu \frac{u^2}{r} \right) \tag{17}
\]

In Eq. (17), the capillary pressure is:

\[
p_c = \frac{\sigma}{R}, \text{ in which } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \tag{18}
\]

For axis-symmetric surfaces, the free surface curvature radius involved in the Young-Laplace equation is given by the following equation:

\[
\frac{1}{R} = \frac{d^2 r / dx^2}{[1+(dr/dx)^2]^{3/2}} + \frac{1}{r[1+(dr/dx)^2]^{1/2}} \tag{19}
\]

However, it is possible to simplify the Eq. (19) in the case of a thin cylinder. In fact, the first term of the second member of Eq. (19) expresses the axial curvature contributing to the equation of motion with a term \( \sigma (d^3 r / dx^3) \), while the radial curvature does it with the \( \sigma (d / dx) (1/r) \) term. Comparing the order of magnitude of both curvatures, the radial curvature is dominant provided \( h \ll a \) is fulfilled.

In fact, \( \sigma \frac{d^2 r}{dx^2} \sim \sigma \frac{a+h}{r^2} \) and \( \sigma \frac{d}{dx} \left( \frac{2}{r} \right) \sim \sigma \left( \frac{a+h}{a} \right) \), where \( l \) is the length of the dynamic meniscus; hence, the ratio of both magnitudes gives \( \sigma \frac{d^2 r}{dx^2} / \sigma \frac{d}{dx} \left( \frac{2}{r} \right) = \left( \frac{a+h}{l} \right)^2 < 1 \) for \( a+h < l \). Consequently, Eq. (19) becomes \( 1/R \approx 1/r \) since \( (dr/dx)^2 \ll 1 \). Therefore, it is possible to assume that:

\[
\frac{\partial}{\partial x} \left[ \frac{\sigma}{R} \right] = \frac{d r / dx}{[1+(dr/dx)^2]^{3/2}} + \frac{1}{r[1+(dr/dx)^2]^{1/2}} \tag{20}
\]

On one hand, the shearing stress for the considered flow regime has the value:

\[
-\rho \overline{u'v'} = \frac{1}{2} c_I^f \rho U_0^2 \tag{21}
\]

where \( c_I^f \) is the friction coefficient. By replacing Eq. (20) into Eq. (17), results

\[
\sigma \left( \frac{d}{dx} \left[ \frac{1}{r} \right] \right) = \mu \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) - \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho \overline{u'v'} \right) \tag{22}
\]

The integration of Eq. (22) with the boundary conditions at:

\[
r = a + h_0 \rightarrow \frac{\partial u}{\partial r} = 0 \quad , \quad \overline{u'v'} = 0 \quad , \quad \frac{\partial}{\partial x} \left( \frac{1}{r} \right) = 0 \tag{23}
\]

yields,

\[
r^2 \left( \frac{\partial}{\partial x} \left[ \frac{1}{r} \right] \right) = \mu r \frac{\partial u}{\partial r} - r \frac{\rho \overline{u'v'}}{\sigma} \tag{24}
\]

According to Eq. (13),

\[
r \frac{\partial u}{\partial r} = \beta U_w \tag{25}
\]

And, on the other hand,

\[
- r \frac{\rho \overline{u'v'}}{\sigma} = \frac{1}{2} c_I^f \rho U_0^2 \tag{26}
\]

\[
\left( \frac{\partial}{\partial x} \left[ \frac{1}{r} \right] \right) = - \frac{1}{r^2} \frac{dr}{dx} \tag{27}
\]

By replacing Eq. (25), Eq. (26) and Eq. (27) in Eq. (24), and rearranging the terms, it is obtained:
\[ \frac{r}{a} = \frac{-dr}{dx} \frac{2\beta Ca}{c_f S} \quad \text{or} \quad \frac{h}{a} = \frac{-dr}{dx} \frac{2\beta Ca}{c_f S} - 1 \]  

(28)

where \(-\frac{dr}{dx}\) is the tangent to the flow free surface line at \(r = h + a\); i.e., where the stationary meniscus is matching the dynamic one. Hence, you can write as well:

\[ -\frac{dr}{dx} = \tan \theta_d \]  

(29)

where \(\theta_d\) is the dynamic contact angle. By replacing Eq. (29) in Eq. (28), is obtained:

\[ \frac{h}{a} = \frac{\tan \theta_d - 2\beta Ca}{c_f S} - 1 \]  

(30)

### 6. Discussion

The interpretation of the experimental results in the light of those obtained by other authors on TLF entrainment should take into account not only the substantial differences in the properties of the fluid and velocity of solid outgoing of the bath, but also the geometrical parameter that is used to define the Reynolds and Weber numbers.

It should also be taken into account the great difference in scale between the value of the radius of the wires used in the experiences carried out for Quéré and other researchers (at the order of tenths of microns), to those used in the presented experiences (of several millimeters). This difference of size reaches greater importance when is related to the tank of liquid from which the solid is withdrawn. In the work of Quéré and other researchers, the size relationship, distance from the tank wall to wire radius, is defined as infinite, while this relationship is finite in the present work.

In the authors’ opinion, the water mass flow entrained by wires is function of its radius, surface roughness and velocity in addition to the fluid properties (density, viscosity and surface tension). Moreover, the dependence of the mass flow entrained with regard to wire velocity presents three different characteristics. In a first interval of velocities values, \(0.0 \leq V \leq 0.18 \text{ m/s}\), it is not possible to collect the mass of water entrained by wire with the technique used, because correspond to a liquid film thickness value of 100\(\mu\) approximately (it is adsorbed water). In a second interval, \(0.18 \leq V \leq 1.3 \text{ m/s}\), the mass flow of water entrained is an increasing function of the capillarity number. For the last interval, \(V \geq 1.3 \text{ m/s}\), the function defining the relation between the entrained water mass flow and the wire velocity is nearly linear.

The graph that represents the values of entrained water mass by square meter of wire surface versus wire velocity clearly show that there is a total change of the phenomenon trend at a critical velocity. The value of Reynolds number by this critical velocity corresponds to the onset of turbulent flow regime.

A clear evidence of the turbulent flow regime at the exit of the tank is that the rough wire reaches turbulent regime at lower velocity than that of the galvanized wires. Moreover, the reduction of the \((h/a)\) value is much pronounced. Both differences it can be attributed to the fact that the surface roughness of the wire causes the Reynolds’ stresses to be greater and they reveal nearer at the wire surface.

In this regard, Grass (Grass, 1971) comments that two well-defined intermittent features of the turbulent structure in the boundary layer on rough surface, visually identified close to the boundary, are fluid ejection phases, corresponding with ejection of low momentum fluid outwards from the boundary, and fluid inrush phases associated with the transport of high momentum fluid inwards towards the boundary. Both inrush and ejection sequences correlated with an extremely high contribution to Reynolds stress and hence turbulent production close to the boundary.

On the other hand, Schlichting (Schlichting, 1962) had already discussed that transition occurs at a lower Re on a rough wall than on a smooth wall. That this should be so follow clearly from the theory of stability: the existence of roughness elements give rise to additional disturbances on the laminar stream which have to be added to those generated by turbulence and already present in the boundary layer.

With regard to the influence of surface roughness on the boundary layer structure and the formation of the liquid film as a result of its entrainment by the moving wire, several papers have been recently published, among them, Antonia and Krogstad (Antonia and Krogstad, 2000), Jiménez (Jiménez, 2004) and Akinlade (Akinlade, 2005).
The experiences carried out with the rough wire show that the surface roughness of the wire has a significant influence; however, it should be convenient to verify it by carrying out more experiences with other wires of different diameters and roughness degrees. This influence has been studied by Krechetnikov and Homsy (Krechetnikov and Homsy, 2005) who have carried out experimental work concerning the substrate roughness effects on the Landau-Levich’ law, but a lower velocities that the critical one.

Their results show that for an interval of Ca values inferior to the critical value (value limit of validity of Landau-Levich’ law corresponding to the force equilibrium of gravity, viscous and surface tension as stability criterion of a liquid film vertically withdrawn) the film thickness on a rough surface is greater than that on a smooth surface. This behaviour also appears in our experiences with the rough 4.4 mm diameter wire in the interval of $V$ values inferior to the critical one as is depicted in Fig. 5. However, there is a lack of information about the influence of surface roughness on liquid entrainment by wires moving horizontally under great inertial forces where the film stability requires a unique equilibrium between Reynolds stresses and capillary pressure, owing to both gravity and viscous forces are negligible in front of inertial forces.

It is known since the work of Wenzel (Wenzel, 1936) that the presence of roughness amplifies wetting, either increasing the contact angle in a smooth surface if $\theta_0 > \pi/2$, or decreasing it if $\theta_0 < \pi/2$. At Ca $\leq 8 \times 10^{-4}$, Krechetnikov y Homsy observed a substantial deviation of the film thickness, which they attribute to roughness effects and could be explained by the substitution of the non-slip condition with a partial slipping condition. To conclude, Krechetnikov and Homsy expose that even though the importance of surface roughness in coating processes is well recognized, its many aspects are not understood yet.

In order to simulate the values of the liquid film thickness entrained by wires, an approximate theoretical model it has been developed from the experimental evidence that the genesis of the liquid film must be the result of a balance between the flow stresses and the surface tension. The evaluation by the model of the liquid film thickness calls for the knowledge of following three parameter values: the curvature gradient in the free liquid surface where the thin film is dragged by the moving wire, the parameter $\beta$ used to take into account the influence of the wire radius on the velocity distribution equation, and the $c_f$ coefficient.

The $c_f$ coefficient is equivalent to the hydrodynamic resistance of a fully submerged object moving at a constant velocity. Longitudinal drag coefficients for cables and long cylinders have been extensively studied by Hoerner (Hoerner, 1960), Berteaux, et al. (Berteaux, et al., 1979). Published values vary from 0.02 for rough cylinders to 0.0025 for smooth cylinders, being a decreasing function of Reynolds number and the relative roughness. A good approximation of the value of the $c_f$ coefficient it is the equation $c_f = 0.3164/Re^{0.25}$ owed to Maron (Maron, 1972).

![Fig. 10 Comparison of the experimental and model results of h/a](image-url)
7. Conclusions

This study on the thin liquid film entrained by continuous wires moving horizontally a high velocity submerged into water it is, in our opinion, the first carried out on the subject. The experimental results, and the approximate theoretical model developed, show that the entrainment phenomenon has a change of behavior when the wire velocity reaches a critical value from which the liquid film thickness decreases as the wire velocity increases.

This change of physical behavior has been quoted as validity limit of Landau-Levich’ theory and also of the several modifications of this theory later do by other researchers studying the subject, in particular by A. de Ryck and D. Quéré (Quéré 1993, 1996) in relation to the visco-inertial regime; but, at the level of information available, none has attributed this limit to the onset of a turbulent flow regime.

A clear evidence of the turbulent flow regime at the exit of the tank is that the rough wire reaches turbulent regime at a lower velocity than the galvanized wires. Moreover, the reduction of the \( h/a \) value is much pronounced. Both differences it can be attributed to the fact that the surface roughness of the wire causes the Reynolds’ stresses to be greater and they itself reveal closer to the wire surface.

In our opinion, the removal of the liquid layer entrained by the wire in horizontal movement at high speed from the mass of water contained by the stationary meniscus at the output of the tank, it is the result of a breakdown in the balance between the tensions of Reynolds and the capillary pressure in the flow.

In order to discern the suitability of the theoretical model for the evaluation of the liquid film thickness, we have compared the experimental results of \( h/a \) for the wire 1.5 mm radius with the results given by the mathematical model from Re>2000. The value of physical magnitudes and parameters applied to the Eq. (30) are the followings:

Fluid properties (water at 20°C), \( \rho = 10^3 \text{ kg/m}^3 \), \( \mu = 10^{-3} \text{ Pa.s} \), \( \sigma = 72.74 \times 10^{-3} \text{ N/m} \) and Flow parameters, \( \beta = 0.8, \ c_f = 0.3164/\text{Re}^{0.25} \). Geometrical boundary condition: \( \tan \theta_d, \) with \( \theta_d = c (Ca^{0.3}) \), similar to the Voinov’ law (Voinov, 1976). The coefficient c takes a value, in proportion to Ca value, in the range \( 2.5 < c < 4 \).

Figure 10 shows the comparison between the experimental \( h/a \) values and the calculated by the Eq.(30) for the wire 1.5 mm radius and \( Re(d) \geq 2000 \). The qualitative agreement between the experimental results and those provided by the model is very good, but not the quantitative. At Reynolds numbers next to 2000, the error in the value of \( h \) is of the order of 25%

Finally, it is clear that the theoretical model developed requires verification by applying proven experimentally values of the \( \beta, \ c_f, \) and \( \theta_d \).

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Nomenclature

\( a \) \quad wire radius

\( Bo \) \quad Bond number, \( Bo = a^2 \rho g / \sigma \)

\( Ca \) \quad capillary number, \( Ca = \mu U_w / \sigma \)

\( c_f \) \quad fluid friction coefficient

\( d \) \quad wire diameter

\( E \) \quad functional \( h/a \)

\( h \) \quad liquid film thickness from \( m = \rho V \pi [(h + a)^2 - a^2] \)

\( h_0 \) \quad limiting thickness of the liquid layer entrained by the plate

\( h_s \) \quad film thickness at the exit of the tank of water

\( g \) \quad gravity

\( l_c \) \quad capillary length, \( l_c = (\sigma / \rho g)^{1/2} \)

\( L \) \quad length of the tank of water
$R$ curvature radius of the free surface
$Re$ Reynolds number, $Re = \frac{\rho Ud}{\mu}$
$S$ Weber number, $S = \frac{\rho U^2 a}{\sigma}$
TLF Thin liquid film
$-\rho u \overline{vv}$ Reynolds’ stress
$u$ liquid velocity at $r$
$U_w$ liquid film velocity at wire surface
$U$ wire velocity
$We$ Weber number $S$

Greek letters
$\rho$ liquid density
$\nu$ kinematics viscosity
$\mu$ dynamics viscosity
$\sigma$ surface tension
$\tau$ shear stress
$\nabla$ gradient
$\theta_{ap}$ apparent dynamic contact angle
$\theta_d$ dynamic contact angle
$\alpha$ coefficient
$\beta$ coefficient
$\gamma$ coefficient
$\Delta$ displacement area parameter

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