Lepton Family Symmetry and the Neutrino Mixing Matrix

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Abstract. I review some of the recent progress (up to September 2005) in applying non-Abelian discrete symmetries to the family structure of leptons, with particular emphasis on the tribimaximal mixing ansatz of Harrison, Perkins, and Scott.

1. Introduction
Using present data from neutrino oscillations, the $3 \times 3$ neutrino mixing matrix is largely determined, together with the two mass-squared differences [1]. In the Standard Model of particle interactions, there are 3 lepton families. The charged-lepton mass matrix linking left-handed ($e, \mu, \tau$) to their right-handed counterparts is in general arbitrary, but may always be diagonalized by 2 unitary transformations:

$$\mathcal{M}_l = U_L^\dagger \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} (U_R^\dagger).$$  \hspace{1cm} (1)

Similarly, the neutrino mass matrix may also be diagonalized by 2 unitary transformations if it is Dirac:

$$\mathcal{M}_\nu^D = U_L^\dagger \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} (U_R^\dagger),$$  \hspace{1cm} (2)

or by just 1 unitary transformation if it is Majorana:

$$\mathcal{M}_\nu^M = U_L^\dagger \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} (U_L^\dagger)^T.$$  \hspace{1cm} (3)

Notice that whereas the charged leptons have individual names, the neutrinos are only labeled as 1, 2, 3, waiting to be named. The observed neutrino mixing matrix is the mismatch between $U_L$ and $U_R$, i.e.

$$U_{\nu} = (U_L^\dagger) U_R^\dagger \approx \begin{pmatrix} 0.85 & 0.52 & 0.053 \\ -0.33 & 0.62 & -0.72 \\ -0.40 & 0.59 & 0.70 \end{pmatrix} \approx \begin{pmatrix} \sqrt{2}/3 & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}.$$  \hspace{1cm} (4)
This approximate pattern has been dubbed tribimaximal by Harrison, Perkins, and Scott [2]. Notice that the 3 vertical columns are evocative of the mesons ($\eta_8, \eta_1, \pi^0$) in their $SU(3)$ decompositions.

How can the HPS form of $U_{\ell\nu}$ be derived from a symmetry? The difficulty comes from the fact that any symmetry defined in the basis ($\nu_e, \nu_\mu, \nu_\tau$) is automatically applicable to ($e, \mu, \tau$) in the complete Lagrangian. To do so, usually one assumes the canonical seesaw mechanism and studies the Majorana neutrino mass matrix

$$M_\nu = -M_D^T M^{-1}_N (M_D^T)^T$$

in the basis where $M_l$ is diagonal; but the symmetry apparent in $M_\nu$ is often incompatible with a diagonal $M_l$ with 3 very different eigenvalues.

In this talk, I will discuss first the pitfall of $\mu \leftrightarrow \tau$ symmetry based on maximal $\nu_\mu - \nu_\tau$ mixing. I will show how it can be done properly with the permutation symmetry $S_3$. I will then spend most of the rest of my time on the tetrahedral symmetry $A_4$ and a little on the permutation symmetry $S_4$. These are examples of how exact and approximate tribimaximal mixing may be obtained.

2. Maximal $\nu_\mu - \nu_\tau$ Mixing
Consider just 2 families. Suppose we want maximal $\nu_\mu - \nu_\tau$ mixing, then we should have

$$M_\nu = \begin{pmatrix} a & b \\ b & a \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a + b & 0 \\ 0 & a - b \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$ (6)

This seems to require the exchange symmetry $\nu_\mu \leftrightarrow \nu_\tau$, but since ($\nu_\mu, \mu$) and ($\nu_\tau, \tau$) are $SU(2)_L$ doublets, we must also have $\mu \leftrightarrow \tau$ exchange. Nevertheless, we still have the option of assigning $\mu^c$ and $\tau^c$. If $\mu^c \leftrightarrow \tau^c$ exchange is also assumed, then

$$M_l = \begin{pmatrix} A & B \\ B & A \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} A + B & 0 \\ 0 & A - B \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$ (7)

Hence $U_{\ell\nu} = (U_L^\dagger)^{-1} U_L^\nu = 1$ and there is no mixing. If $\mu^c$ and $\tau^c$ do not transform under this exchange, then

$$M_l = \begin{pmatrix} A & B \\ A & B \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2}(A^2 + B^2) & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix},$$ (8)

where $c = A/\sqrt{A^2 + B^2}$, $s = B/\sqrt{A^2 + B^2}$. Again $U_{\ell\nu} = (U_L^\dagger)^{-1} U_L^\nu = 1$.

3. Permutation Symmetry $S_3$
To overcome the difficulty of obtaining maximal $\nu_\mu - \nu_\tau$ mixing, consider the non-Abelian discrete symmetry $S_3$, i.e. the permutation group of 3 objects, which is also the symmetry group of the equilateral triangle. It has 6 elements divided into 3 equivalence classes, with the irreducible representations $\frac{1}{3}, \frac{1}{3}'$, and $\frac{2}{3}$. Let

$$\omega = \exp \left( \frac{2\pi i}{3} \right) = -\frac{1}{2} + \frac{i\sqrt{3}}{2},$$ (9)

then the 6 matrices of the $\frac{2}{3}$ representation may be chosen as follows.

$$C_1 : \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C_2 : \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \quad C_3 : \begin{pmatrix} 0 & 1 \\ \omega & 0 \end{pmatrix},$$ (10)
Table 1. Character table of $S_3$.

| class | $n$ | $h$ | $\chi_1$ | $\chi_1'$ | $\chi_2$ |
|-------|-----|-----|-----------|------------|----------|
| $C_1$ | 1   | 1   | 1         | 1          | 2        |
| $C_2$ | 2   | 3   | 1         | 1          | -1       |
| $C_3$ | 3   | 2   | 1         | -1         | 0        |

where $C_i$ refer to the 3 equivalence classes in the character table shown. The fundamental multiplication rule is then

$$2 \times 2 = 1(12 + 21) + 1'(12 - 21) + 2(22, 11).$$

(11)

Let $(\nu_i, l_i) \sim 2$, $l'_i \sim 2$, $(\phi_i^0, \phi_i^-) \sim 1$, $(\phi_i^0, \phi_i^-) \sim 1'$, then

$$\mathcal{M}_l = \begin{pmatrix} 0 & f v_1 + f' v_2 \\ f v_1 - f' v_2 & 0 \end{pmatrix} = \begin{pmatrix} m_\mu & 0 \\ 0 & m_\tau \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$  

(12)

Let $\xi_i = (\xi_i^+, \xi_i^+, \xi_i^0) \sim 2$ and $\xi_0 \sim 1$, then

$$\mathcal{M}_\nu = \begin{pmatrix} h u_1 & h_0 u_0 \\ h_0 u_1 & h u_2 \end{pmatrix} = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

(13)

for $u_1 = u_2$. Thus

$$U_{\nu \nu} = (U_1^{\nu})^\dagger U_2^{\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$  

(14)

i.e. maximal $\nu_\mu - \nu_\tau$ mixing may be achieved, despite having a diagonal $\mathcal{M}_l$ with $m_\mu \neq m_\tau$.

4. Tetrahedral Symmetry $A_4$

For 3 families, we should look for a group with a 3 representation, the simplest of which is $A_4$, the group of the even permutation of 4 objects, which is also the symmetry group of the tetrahedron.

Table 2. Character table of $A_4$.

| class | $n$ | $h$ | $\chi_1$ | $\chi_1'$ | $\chi_2$ |
|-------|-----|-----|-----------|------------|----------|
| $C_1$ | 1   | 1   | 1         | 1          | 3        |
| $C_2$ | 4   | 3   | 1         | $\omega$   | $\omega^2$ 0 |
| $C_3$ | 4   | 3   | $\omega^2$ | $\omega$   | 0        |
| $C_4$ | 3   | 2   | 1         | 1          | -1       |

The fundamental multiplication rule is

$$3 \times 3 = \frac{1}{2}(11 + 22 + 33) + 1'(11 + \omega^2 22 + \omega 33) + 1''(11 + \omega 22 + \omega^2 33) + 3(23, 31, 12) + 3(32, 13, 21).$$  

(15)

Note that $3 \times 3 \times 3 = 1$ is possible in $A_4$, i.e. $a_1 b_2 c_3 +$ permutations, and $2 \times 2 \times 2 = 1$ is possible in $S_3$, i.e. $a_1 b_1 c_1 + a_2 b_2 c_2$. 


Table 3. Perfect geometric solids.

| solid    | faces | vertices | Plato | group |
|----------|-------|----------|-------|-------|
| tetrahedron | 4     | 4        | fire  | $A_4$ |
| octahedron  | 8     | 6        | air   | $S_4$ |
| icosahedron | 20    | 12       | water | $A_5$ |
| hexahedron  | 6     | 8        | earth | $S_4$ |
| dodecahedron | 12    | 20       | quintessence | $A_5$ |

The tetrahedron is one of five perfect geometric solids known to the ancient Greeks. In order to match them to the 4 elements (fire, air, water, and earth) already known, Plato invented a fifth (quintessence) as that which pervades the cosmos and presumably holds it together. Since a cube (hexahedron) may be embedded inside an octahedron and vice versa, the two must have the same group structure and are thus dual to each other. The same holds for the icosahedron and dodecahedron. The tetrahedron is self-dual. Compare this first theory of everything to today’s contender, i.e. string theory. (A) There are 5 consistent string theories in 10 dimensions. (B) Type I is dual to Heterotic $SO(32)$, Type IIA is dual to Heterotic $E_8 \times E_8$, and Type IIB is self-dual.

4.1. Exact HPS

Following the original papers [3, 4] on $A_4$, let $(\nu_i, l_i) \sim 3$, but $l_i'^c \sim 1, 1', 1''$, then with $(\phi_i^0, \phi_i^-) \sim 3$, let

$$M_l = \begin{pmatrix} h_1 v_1 & h_2 v_1 & h_3 v_1 \\ h_1 v_2 & h_2 v_2 \omega & h_3 v_2 \omega^2 \\ h_1 v_3 & h_2 v_3 \omega^2 & h_3 v_3 \omega \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{pmatrix} \sqrt{3} v, \quad (16)$$

for $v_1 = v_2 = v_3 = v$. Let $\xi_1 \sim 1, \xi_2 \sim 1', \xi_3 \sim 1'', \xi_{4,5,6} \sim 3$, with $\langle \xi_5 \rangle = \langle \xi_6 \rangle = 0$, then [5]

$$M_\nu = \begin{pmatrix} a + b + c & 0 & 0 \\ 0 & a + b \omega + c \omega^2 & d \\ 0 & d & a + b \omega^2 + c \omega \end{pmatrix}. \quad (17)$$

In the $(\nu_e, \nu_\mu, \nu_\tau)$ basis, it becomes

$$M_\nu^{(e,\mu,\tau)} = \begin{pmatrix} a + 2d/3 & b - d/3 & c - d/3 \\ b - d/3 & c + 2d/3 & a - d/3 \\ c - d/3 & a - d/3 & b + 2d/3 \end{pmatrix}. \quad (18)$$

If $b = c$, then the eigenvalues of this matrix are simply

$$m_1 = a - b + d, \quad m_2 = a + 2b, \quad m_3 = -a + b + d, \quad (19)$$

and

$$U_{\nu} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \quad (20)$$

i.e. tribimaximal mixing is obtained as desired. If $b \neq c$, then $U_{\nu_3} \neq 0$, and $|U_{\nu_3}| < 0.16$ implies $0.5 < \tan^2 \theta_{12} < 0.52$, whereas experimentally, $\tan^2 \theta_{12} = 0.45 \pm 0.05$. 

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The above pattern involves 4 parameters \((a, b, c, d)\). If a model can be constructed for which \(b = c\) naturally, then the HPS ansatz of tribimaximal mixing will be realized. Of course, the three masses are not predicted, as shown in Eq. (19). If \(b \neq 0\) and \(c \neq 0\), it is difficult, if not impossible, to find an auxiliary symmetry which will enforce their equality. On the other hand, they can both be zero, and thus equal to each other, if impossible, to find an auxiliary symmetry which will enforce their equality. On the other hand, they can both be zero, and thus equal to each other, if \(\xi_2\) and \(\xi_3\) are absent in the above. This is the essence of how the problem is first solved by Altarelli and Feruglio [6]. In that case,

\[
m_1 = a + d, \quad m_2 = a, \quad m_3 = -a + d.
\]  

(21)

The requirement \(\Delta m_{12}^2 \simeq \Delta m_{sol}^2 < < \Delta m_{atm}^2 \simeq \Delta m_{23}^2\) implies

\[
|d| \simeq -2|a| \cos \phi, \quad |m_{1,2}|^2 \simeq |a|^2, \quad |m_3|^2 \simeq |a|^2 (1 + 8 \cos^2 \phi),
\]

(22)
i.e. normal ordering of neutrino masses with the sum rule [7]

\[
|m_{\nu e}|^2 \simeq |m_{ee}|^2 + \Delta m_{atm}^2 / 9,
\]

(23)

where \(|m_{\nu e}|\) is the kinematic \(\nu_e\) mass measured in beta decay and \(|m_{ee}|\) is the effective Majorana neutrino mass measured in neutrinoless double beta decay.

Another 2-parameter tribimaximal scenario [7] is to choose \(a = 0, b = c\). In that case,

\[
m_1 = -b + d, \quad m_2 = 2b, \quad m_3 = b + d.
\]

(24)

Here both normal and inverted ordering of neutrino masses are possible with the sum rule

\[
|m_{\nu e}|^2 \simeq 3|m_{ee}|^2 - (2/3)\Delta m_{atm}^2.
\]

(25)

More recently, exact HPS was obtained by Babu and He [8] with \(A_4\), using the canonical seesaw mechanism. Their solution may be considered the “inverse” of Ref. [6]. Another example of exact HPS was obtained by Grimus and Lavoura [9] with \(S_3\) plus 1 commuting and 6 noncommuting \(Z_2\) symmetries.

### 4.2. Approximate HPS

An alternative \(A_4\) assignment [10] is to let \((\nu_i, l_i), l_i^c \sim 3\) with \((\phi_i^\prime, \phi_i^-) \sim 1, 1', 1''\), then \(\mathcal{M}_l\) is diagonal with

\[
\begin{pmatrix}
 m_e \\
 m_\mu \\
 m_\tau
\end{pmatrix} =
\begin{pmatrix}
 1 & 1 & 1 \\
 1 & \omega & \omega^2 \\
 1 & \omega^2 & \omega
\end{pmatrix}
\begin{pmatrix}
 h_1 v_1 \\
 h_2 v_2 \\
 h_3 v_3
\end{pmatrix}.
\]

(26)

For the neutrino mass matrix, let \(\xi_1 \sim 1, \xi_2 \sim 1', \xi_3 \sim 1'', \xi_{4,5,6} \sim 3\), with \(\langle \xi_4 \rangle = \langle \xi_5 \rangle = \langle \xi_6 \rangle\), then

\[
\mathcal{M}_\nu =
\begin{pmatrix}
 a + b + c & d \\
 d & a + b + c & d \\
 d & d & a + b + c + \omega^2
\end{pmatrix}.
\]

(27)

Let \(b = c\) and rotate to the basis \([\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2}, (-\nu_\mu + \nu_\tau)/\sqrt{2}]\), then

\[
\mathcal{M}_\nu =
\begin{pmatrix}
 a + 2b & \sqrt{2}d & 0 \\
 \sqrt{2}d & a - b + d & 0 \\
 0 & 0 & a - b - d
\end{pmatrix},
\]

(28)
i.e. maximal \(\nu_\mu - \nu_\tau\) mixing and \(U_{e3} = 0\). The solar mixing angle is now given by \(\tan 2\theta_{12} = 2\sqrt{2}d / (d - 3b)\). For \(b << d\), \(\tan 2\theta_{12} \rightarrow 2\sqrt{2}\), i.e. \(\tan^2 \theta_{12} \rightarrow 1/2\), but \(\Delta m_{sol}^2 << \Delta m_{atm}^2\) implies \(2a + b + d \rightarrow 0\), so that \(\Delta m_{atm} \rightarrow 6bd \rightarrow 0\) as well. Therefore, \(b \neq 0\) is required, and \(\tan^2 \theta_{12} \neq 1/2\), but should be close to it, because \(b = 0\) enhances the symmetry of \(\mathcal{M}_\nu\) from \(Z_2\) to \(S_3\). Here \(\tan^2 \theta_{12} < 1/2\) implies inverted ordering and \(\tan^2 \theta_{12} > 1/2\) implies normal ordering.
5. Permutation Symmetry $S_4$

In the above application of $A_4$, approximate tribimaximal mixing involves the \textit{ad hoc} assumption $b = c$. This problem is overcome by using $S_4$ in a supersymmetric seesaw model proposed recently [11], yielding the result

$$M_\nu = \begin{pmatrix} a + 2b & c & c \\ c & a - b & d \\ c & d & a - b \end{pmatrix}. \tag{29}$$

Here $b = 0$ and $c = d$ are related limits. The permutation group of 4 objects is $S_4$. It contains both $S_3$ and $A_4$. It is also the symmetry group of the hexahedron (cube) and the octahedron.

| Table 4. Character table of $S_4$. |
|-----------------|----------|-------|-------|-------|
| class           | $n$      | $h$   | $\chi_1$ | $\chi_1'$ | $\chi_2$ | $\chi_3$ | $\chi_3'$ |
| $C_1$           | 1        | 1     | 1       | 1       | 2       | 3       | 3         |
| $C_2$           | 3        | 2     | 1       | 1       | 2       | -1      | -1        |
| $C_3$           | 8        | 3     | 1       | 1       | -1      | 0       | 0         |
| $C_4$           | 6        | 4     | 1       | -1      | 0       | -1      | 1         |
| $C_5$           | 6        | 2     | 1       | -1      | 0       | 1       | -1        |

The fundamental multiplication rules are

$$3 \times 3 = (1(11 + 22 + 33) + 2(11 + \omega^222 + \omega 33, 11 + \omega 22 + \omega^233) + 3(23 + 32, 31 + 13, 12 + 21) + 3'(23 - 32, 31 - 13, 12 - 21), \tag{30}$$

$$2' \times 2' = 1' + 2' + 2s + 2'A, \tag{31}$$

$$3 \times 3' = 1' + 2 + 2s + 3A. \tag{32}$$

Note that both $3 \times 3 \times 3 = 1$ and $2 \times 2 \times 2 = 1$ are possible in $S_4$. Let $(\nu_i, l_i^c, N_i) \sim 3$ under $S_4$. Assume singlet superfields $\sigma_{1,2,3} \sim 3$ and $\zeta_{1,2} \sim 2$, then

$$M_N = \begin{pmatrix} A + f(\langle \zeta_2 \rangle + \langle \zeta_1 \rangle) & h(\sigma_3) & h(\sigma_2) & h(\sigma_1) \\ h(\sigma_3) & A + f(\langle \zeta_2 \rangle \omega + \langle \zeta_1 \rangle \omega^2) & h(\sigma_2) & h(\sigma_1) \\ h(\sigma_2) & h(\sigma_1) & A + f(\langle \zeta_2 \rangle \omega^2 + \langle \zeta_1 \rangle \omega) \end{pmatrix}. \tag{33}$$

The most general $S_4$-invariant superpotential of $\sigma$ and $\zeta$ is given by

$$W = M(\sigma_1\sigma_1 + \sigma_2\sigma_2 + \sigma_3\sigma_3) + \lambda \sigma_1\sigma_2\sigma_3 + m \zeta_1\zeta_2 + \rho(\zeta_1\zeta_1\zeta_1 + \zeta_2\zeta_2\zeta_2) + \kappa[\sigma_1\sigma_1 + \sigma_2\sigma_2\omega + \sigma_3\sigma_3\omega^2] + \langle \sigma_1\sigma_1 + \sigma_2\sigma_2\omega^2 + \sigma_3\sigma_3\omega \rangle \zeta_1]. \tag{34}$$

The resulting scalar potential has a minimum at $V = 0$ (thus preserving supersymmetry) only if $\langle \zeta_1 \rangle = \langle \zeta_2 \rangle$ and $\langle \sigma_2 \rangle = \langle \sigma_3 \rangle$, so that

$$M_N = \begin{pmatrix} A + 2B & C & C \\ C & A - B & D \\ C & D & A - B \end{pmatrix}. \tag{35}$$

To obtain a diagonal $M_1$, choose $\phi_{1,2,3}^{l} \sim 1 + \frac{1}{2}$. To obtain a Dirac $M_{\nu,N}$ proportional to the identity, choose $\phi_{1,2,3}^{N} \sim 1 + \frac{1}{2}$ as well, but with zero vacuum expectation value for $\phi_2^N$. This allows $M_\nu$ to have the form of Eq. (29), and thus approximate tribimaximal mixing.
6. Conclusion and Outlook
Since my talk on finite groups in Dubrovnik exactly two years ago (which was itself exactly two years after my talk at the Gran Sasso Laboratory on that fateful day), much progress has been made.

With the application of the non-Abelian discrete symmetry $A_4$, a plausible theoretical understanding of the HPS form of the neutrino mixing matrix has been achieved, i.e. $\tan^2 \theta_{23} = 1$, $\tan^2 \theta_{12} = 1/2$, $\tan^2 \theta_{13} = 0$.

Another possibility is that $\tan^2 \theta_{12}$ is not 1/2, but close to it. This has theoretical support in an alternative version of $A_4$, but is much more natural in $S_4$.

In the future, this approach to lepton family symmetry should be extended to include quarks, perhaps together in a consistent overall theory, such as $SU(3)^3$ finite unification [12].

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