ON THE MACHIAN ORIGIN OF INERTIA

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Abstract

We examine Sciama’s inertia theory: we generalise it, by combining rotation and expansion in
one unique model, we find the angular speed of the Universe, and we stress that the theory is
zero-total-energy valued. We compare with other theories of the same null energy background.

We determine the numerical value of a constant which appears in the Machian inertial force
expression devised by Graneau and Graneau[2], by introducing the above angular speed. We point
out that this last theory is not restricted to Newtonian physics as those authors stated but is, in
fact, compatible with other cosmological and gravitational theories. An argument by Berry[7] is
shown in order to ’’derive” Brans-Dicke relation in the present context.

Keywords: Einstein; Brans-Dicke; Newton; Gravitation; Graneau and Graneau; Mach.

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Section I - Introduction

The advancement of scientific knowledge has produced, in the aftermath of Newtonian theory, the surge of General Relativity and alternative versions of it. On a different line of thought, Sciama[1] has put forward a gravity theory based on an electrodynamical analogy. The research on the origin of Inertia is a problem that involved passionately a number of physicists especially from the crisis of Classical Physics and from the birth of General Relativity. Einstein himself underlined the importance of his meditation on this argument in the development of his theory of gravitation. Moreover he formulated precisely his reflections on inertia origin in the Mach’s Principle - that in its simpler form says that inertia properties of matter are determined in some manner by the other bodies of the Universe. Even though with the development of General Relativity Einstein rejected explicitly his first considerations on Inertia, the Principle represented, in a compact form, a research project that guided many scientists in the development of gravitational theories alternative to General Relativity (for example Brans-Dike Theory) - with the following cosmological implications - and the reformulations of Classical Mechanics based on Mach’s reflections on Inertia (see for example the model proposed by Shrödinger).

This short introduction has only the simple task to set in his historical background the work submitted by the present author, that moves on these lines of thoughts. This letter, has a double structure: in the beginning part we present changes in the Sciama cosmological model (linked to gravitational theories alternative to General Relativity) with the introduction of the expansion and rotation of the Universe, and in the second part of the article we use the results just obtained (the angular velocity of the Universe) in the calculation of a constant of Graneau and Graneau theory. Moreover at the end of this letter, I present in a simple form an argument proposed by Berry, where the Machian analysis of inertia and the alternative cosmological model (like Sciama and Brans-Dike model) shows some common aspects.

The suggestion of a Universe in rotation - certainly the most interesting and the most problematic - is better understood in my prior papers ([3]-[8]), so leaving a small gap in the
presentation. Also the Sciama cosmological model is presented in a manner that I myself consider a bit concise.

Another critical point, a key argument to link Machian Universe to Graneau and Graneau Theory, is the extremely concise - but I think extremely interesting - presentation of my interpretation of the Mach Principle as the mathematical representation of the zero-total-energy of the Universe and the consequent description of the Pioneer effect [3] - [8].

This letter also creates interesting links among research programs, like the cosmological model based on Field Theory and Newtonian Machian Physics, that are often considered in competition.

I consider this letter as an original contribution to the debate on the origin of inertia and the connected argument of the cosmological model: so I think the arguments proposed in this manuscript are very suitable.

In Section II, it is shown that Sciama’s paper can be generalised, by including, in the same model, a rotating and expanding Universe; this also implies that the total energy density of the Universe is null, at each point of space; on summing for the whole Universe, we obtain zero-total energy. A possible angular speed proportional to the inverse of the scale-factor is found, which is of the same form of that found by Berman(2007; 2007a; 2007b; 2008; 2008a; 2008b).

In Section III, we review a theory with the otherwise ”arbitrary” constant $B$ in the inertia formula, by Graneau and Graneau[2]. The numerical value was in fact, found by us, in accordance with a possible rotation of the Universe. In Section IV, a Machian argument by Berry[9] is shown to be related with Sciama’s theory.

Section II - Sciama’s Inertia Model

In order to fulfill a theoretical need for accounting the inertia properties of matter, Sciama supposes that gravitation is analogous to electrodynamics. The working hypothesis is that inertial and gravitational forces cancel each other at any point of space, so that the total field is null. We combine first the calculations of expansion and rotation of the Universe; each one was treated by Sciama, isolated.
For a rest particle, the "electric" potential contribution from the whole Universe, as observed in time \( t \) is given by:

\[
\Phi = - \int \rho r dV . \quad (1)
\]

If the density is uniform, we have from (1):

\[
\Phi \approx -2\pi \rho c^2 \tau^2 , \quad (2)
\]

while, by symmetry, the "vector" potential is null:

\[
\vec{A} = 0 . \quad (3)
\]

In formula (2), \( \tau \) is associated with \( H^{-1} \), where \( H \) stands for Hubble’s parameter.

If the particle moves with a linear velocity \( \vec{v} \), and we add Hubble’s expansion, its total velocity is \( [\vec{v} + \vec{r}H] \), at any point in the Universe. In the first approximation, \( \Phi \) keeps approximately the same form as above. However, the vector potential \( \vec{A} \), when \( \vec{v} \) does not depend on \( \vec{r} \), will be given by:

\[
\vec{A} \approx - \int \frac{\rho \vec{v}}{cr} dV \approx \frac{\Phi \vec{v}(t)}{c} . \quad (4)
\]

The "electric" field is given by the usual electromagnetic formula:

\[
\vec{E} = -\nabla \Phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t} \approx -\frac{1}{c^2} \Phi \frac{\partial \vec{v}}{\partial t} . \quad (5)
\]

The above is called in this case, a gravitoelectric field.

On the other hand, we would have a null gravitomagnetic field,

\[
\vec{B} = \nabla \times \vec{A} \approx 0 . \quad (6)
\]

Now, let us have a body of mass \( M \), placed in the Universe. In the rest frame of a particle at rest, the total "electric" field will be given by:

\[
E_{TOT} \approx -\frac{1}{c^2} \Phi \frac{d\vec{v}}{dt} + \left[ \frac{M}{r^2} + \phi \frac{d\vec{v}}{dt} \right] . \quad (7)
\]

In the above, \( \phi \) is the usual potential of the body with mass \( M \), on a test particle, i.e.
\[
\phi = -\frac{M}{r} \tag{8}
\]

Now, we write the total field as:

\[
E_{\text{TOT}} \approx \frac{M}{r^2} + \frac{1}{c^2} \left[ \phi - \Phi \right] \vec{a} \tag{9}
\]

where \( \vec{a} \) is the total acceleration (of the Universe plus the body), relative to the test particle. Put it in other frame: the particle accelerates towards the rest body, relative to the whole Universe.

As this theory is of the electromagnetic type, it is a linear one. We superimpose, then, the cases of radial expanding, and rotating pictures.

In the first place, consider a non-rotating Universe; now we set a reference frame with origin at the body; in relativistic units, near the origin, we shall have the scalar and vector potentials given by:

\[
\vec{A} = 0 \tag{10}
\]

and,

\[
\Phi \approx -1 \tag{11}
\]

In a rotating Universe, however, if the axis of rotation is the Z, then near the origin, we shall have:

\[
A_x \approx \omega y \tag{12}
\]

\[
A_y \approx -\omega x \tag{12}
\]

\[
A_z \approx 0 \tag{12}
\]

\[
\Phi^{\text{rot}} \approx -\left[ 1 + \omega^2 r^2 \right]^{1/2} \tag{12}
\]

In this case, the total field, will be given by:

\[
E_{\text{TOT}}^{\text{rot}} = -\nabla \Phi^{\text{rot}} - \frac{\partial \vec{A}}{\partial t} \approx -\frac{\omega^2 r}{\left[ 1 + \omega^2 r^2 \right]^{1/2}} + \frac{M}{r^2} \tag{13}
\]
Now, we equate, on the test particle, inertia to gravitation, one balancing the other, so that the total field is zero, where this total is not the above only but added to $\vec{E}_{TOT}$, which equate to zero:

$$-\frac{\omega^2 r}{[1+\omega^2 r^2]^{1/2}} + \frac{2M}{r^2} - 2\pi \rho H^{-2} \approx 0$$

(14)

A particular solution, is composed by the two equalities below, whose sum retrieves the above one:

$$M \rho H^{-2} \approx 2\pi a$$

(15)

(However, the Newtonian acceleration, in the above, is given by $a = -\frac{M}{r^2}$).

$$\frac{\omega^2 r}{[1+\omega^2 r^2]^{1/2}} \approx \frac{M}{r^2}$$

(16)

Equality (15) is satisfied by the Whitrow-Randall expression,

$$G \rho H^{-2} \approx 1$$

(17)

The above is equivalent, because $\rho \approx \frac{M}{\frac{4\pi}{3} r^3}$, to the Brans-Dicke form,

$$\frac{GM}{r} = \gamma \approx 1$$

(18)

Notice that with the above, mass and radius are directly proportional with each other. If we now go to (16), and solve it with the approximation (18), and we take the angular speed, as given by the Machian expression,

$$\omega \approx \frac{\alpha}{r}$$

(19)

where $\alpha$ is a constant that must be found later, and then, the total spin of the rotating Universe will be proportional to $r^2$, i.e.:

$$L \approx Mr^2\omega = \gamma \alpha r^2 \propto r^2$$

(20)

We see, that from (16), after some algebra, if $\gamma \approx 2$, then $\alpha \approx c$, and thus, we make contact with Berman models [3]-[8].

We notice that the local effect at each point of space, makes the total energy density equal to zero; it is given, in the electromagnetic case, by a Poynting-like electric field density,
\[ \rho_{TOT} = E_{TOT}^2 / 8\pi \cong 0 \] (22)

When we sum for all points of space, we obviously find that the total energy of the Universe is zero-valued.

**Section III - Graneau and Graneau’s Theory**

Graneau and Graneau[2], discuss a version of inertia theory, that would be originated in a Machian Newtonian theory. According to those authors, it is an instantaneous action-at-a-distance theory. The consequence of such theory, is the "new" force law for inertia,

\[ \Delta F_i = -\frac{a}{\pi^2 B} \left[ \frac{m_0 m_x}{r^2} \right] \] (23)

where, \( \Delta F_i \) is the inertial force between two particles with masses \( m_0 \) and \( m_x \), separated by a radial distance \( r \), while \( B \) is a universal constant, relating \( \Delta F_i \) with a universal relative acceleration \( a \) between the two objects.

As I have shown elsewhere, Mach’s principle can be thought of, as the mathematical representation of the zero-total-energy of the Machian Universe. In another paper, Berman[4] calculated the existence of an anomalous universal acceleration which acts relative to each pair of "observer" and "observed" objects, due to the same Machian principle. The numerical value of the expected relative acceleration, acting in a radial direction from the "observed" to the "observer", is about \( 8.0 \times 10^{-8} cm/sec^2 \). This result was shown by Berman to agree and explain the so-called Pioneers' anomalous acceleration relative to the Earth, which is affecting both spaceships that travel in the outskirts of the Solar system, in two opposite directions relative to the Earth or approximately towards the Sun.

Graneau and Graneau[2], have failed to determine the numerical value of their constant \( B \). By comparing with Newtonian law of gravitation, we may write, on the assumption that the acceleration to be met is the Pioneers’ anomalous one,

\[ \Delta F_i = -\frac{a}{\pi^2 B} \left[ \frac{m_0 m_x}{r^2} \right] = -G \left[ \frac{m_0 m_x}{r^2} \right] \] (24)

so that,
\( a = \pi^2 BG \), \( (25) \)

where, \( G \) stands for Newton’s gravitational constant.

When the numerical values above are plugged, we find:

\[ B \approx 1.0 \ m^{-2}kg^{-1} \] \( (26) \)

This constant is the result of interactions \( B_i \) from all other masses in the Universe, in the treatment of Graneau and Graneau, where the \( B_i \)'s are determined by the mass of each "cause", i.e., each mass in the Universe, divided by its distance to the given local point of space where they cause the inertia force.

Though we have found the otherwise undetermined numerical value for \( B \), we point out that we do not agree with Graneau and Graneau, when they discard General Relativity in favor of Newtonian gravitation. In fact, it has been shown earlier, that cosmological models obeying Brans-Dicke-Whitrow-Randall-Sciama relation,

\[ \frac{GM}{c^2R} \approx 1, \quad (26) \]

which is derived from the Machian zero-total-energy hypothesis, and which models are based on General Relativity theory or alternative generalizations of such theory, should be regarded as fulfilling the Machian property.

**Section IV - On Berry’s Machian Argument**

Berry [9] has posed a Machian query. Consider a body of mass \( m \), acted on by a large one \( M \) located at a distance \( r \), while the large mass has an acceleration \( \vec{a} \) relative to the small one. In order to satisfy Mach’s principle, the force exerted on the small mass by the larger, must contain a part proportional to \( m \vec{a} \). By means of dimensional analysis, we find that the correct force should be proportional to \( m \vec{a} \) and also to other terms: \( M \), \( r \), \( G \) and \( c \), at some powers. According to Newton’s third law, the power of \( M \) and \( m \) must be the same, i.e., they occur symmetrically in the force equation. The solution is,

\[ \vec{F} = -GM \frac{m}{c^2r} \vec{a} \] \( (28) \)
This looks like the force whose acceleration measures mutually accelerated charges. For the gravitational case, Sciama has given a name to it: law of inertial induction. For one thing, we may understand from the analogy, that if electromagnetic radiation is possible, then we would also have gravitational radiation.

If law (28) is to be applied to the distant masses of the Universe, in the Machian picture, and if Newton’s second law should be valid, we need the following Brans-Dicke relation to be valid:

\[
\frac{GM}{c^2 r} = 1, \quad (29)
\]

so that,

\[
\vec{F} = -GM \frac{m}{c^2 r} \vec{a} = -m \vec{a} \quad . \quad (30)
\]

This section was a digression on a Berry’s argument. It must be said that from formula (30), we have the same kind of zero-total force applied to each and all particles in the Universe, (inertial force plus gravitational force, equals zero) so that we retrieve a zero-total energy of the Universe.

Section V - Conclusions

Sciama’s linear theory, was generalised by including in the same model a rotating and at the same time expanding Universe. The resultant equations, are equivalent to the Machian treatment by Berman[3]-[8].

We have also found the constant \( B \) numerical value, but, nevertheless, we guess that we need not exclude other theories in favor of Newton’s one. Section IV shows the deep value of Machian ideas. Indeed, inertia has been treated as Machian-originated.

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References

[1]. Sciama, D.W. (1953) - M.N.R.A.S., 113, 34.

[2]. Graneau, P.; Graneau, N. (2006) - In the Grip of the Distant Universe - the Science of Inertia, World Scientific, Singapore, pages 256-267.

[3]. Berman, M.S. (2007) - Introduction to General Relativity and the Cosmological Constant Problem, Nova Science, New York.

[4]. Berman, M.S. (2007a) - Introduction to General Relativistic and Scalar-Tensor Cosmologies, Nova Science, New York.

[5]. Berman, M.S. (2007b) - The Pioneer Anomaly and a Machian Universe - Astrophysics and Space Science, 312, 275.

[6]. Berman, M.S. (2008) - A General Relativistic Rotating Evolutionary Universe, Astrophysics and Space Science, 314, 319-321.

[7]. Berman, M.S. (2008a) - A General Relativistic Rotating Evolutionary Universe - Part II, Astrophysics and Space Science, 315, 367-369.

[8]. Berman, M.S. (2008b) - A Primer in Black Holes, Mach’s Principle and Gravitational Energy, Nova Science, New York.

[9]. Berry, M.V. (1989) - Principles of Cosmology and Gravitation, Adam Hilger, Bristol.