Research Article

Allocation of Optimal PMUs for Power System Observability Using PROMETHEE Approach

Shubhrajyoti Kundu, Mehebub Alam, Biman K. Saha Roy, and Siddhartha Sankar Thakur

Department of EE, NIT, Durgapur, India

Correspondence should be addressed to Shubhrajyoti Kundu; sk.17ee1102@phd.nitdgp.ac.in

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1. Introduction

To operate the power system in a secure and economical manner, it should be monitored continuously. With the increase in the size of the power system, monitoring and operation have become more complex. With the advent of synchrophasor technology, power system monitoring has gained a new revolution, thus paving the way for the effective implementation of wide-area monitoring, protection, and control. The prerequisite of accurate and reliable state estimation is to make the system completely observable through PMUs [1]. PMU has seen widespread popularity among power engineers owing to its time synchronized complex voltage and current measurements [2].
this problem has been termed as optimal PMU placement (OPP) problem.

Several techniques and methods have been proposed by various researchers for solving the problem of OPP. Preliminary research work on the development and implementation of PMU technology has been proposed by Phadke et al. [2–4]. To find the optimal set of PMUs, the authors [5, 6] applied graph theory and simulated annealing-based approaches. In [7–9], the topological observability analysis has been carried out considering the construction of a full rank spanning tree. However, this approach is complex and suffers from a computational burden. Chen and Abur [10] allocated PMUs strategically to enhance the bad data processing capability of state estimation. Some other authors [11, 12] have implemented orthogonal transformation-based state estimation for observability analysis. The authors in [13–15] have also performed observability analysis using numerical approaches. In [16], the authors have considered a combination of topological and numerical approaches for observability analysis and power system state estimation. In [17], the authors formulate the problem as a binary semi-definite programming model and solve it by applying a binary integer linear programming-based approach. A mixed-integer nonlinear programming-based approach has been applied by the authors [18] for the optimal allocation of PMUs. The optimal allocation of PMUs has been performed in [19] based on the sum of the variance of robust estimators. A numerical approach considering non-Gaussian measurement noise statistics is presented in [20]. The Groebner-based algorithm for the optimal allocation of PMU is discussed in [21]. Several metaheuristic techniques have also been proposed by various researchers for solving the OPP problem. A binary search algorithm has been applied in [22] for determining optimal locations of PMU for interconnected power systems. Babu et al. [23] placed PMUs optimally by applying binary particle swarm optimization (BPSO). In [24], the authors applied intelligent search techniques for solving the OPP problem in the integrated power network to make the entire system observable. In [25, 26], the genetic algorithm has been applied to determine the optimal locations of PMU. Jamuna and Swapup [27] proposed multi-objective biogeography-based optimal PMU placement. The recursive Tabu search method has been suggested in [28] for the optimal allocation of PMUs. A multi-objective PMU placement method based on a binary gravitational search algorithm has been proposed in [29]. In [30], the PMU allocation problem is solved as an optimization problem with the application of branch and bound approach and binary-coded genetic algorithm. Some other researchers implemented BPSO [31], modified BPSO [32], and exponential BPSO [33] for solving the OPP problem. Meenakshi Devi and Geethanjali [34] obtained optimal allocation of PMUs by combining the genetic algorithm approach with the minimum spanning tree method. In [35], the authors applied non-dominated sorting genetic algorithm II (NSGA II) for determining optimal locations of PMUs. A realistic method that implements a modified whale optimization algorithm [36] has been suggested with the dual objective of minimizing the required number of PMUs and enhancing the power system reliability. However, these optimization techniques suffer from a high computational burden and large memory requirements. Rational random walk-based PMU allocation has been performed in [37]. In [38], a three-stage algorithm is proposed in which PMUs are eliminated in successive stages from less important buses retaining them at strategically important locations. Alumnif and Fan [39] implemented a mathematical programming method for optimal PMU placement. A security-based observability approach for optimal PMU sensor placement is suggested in [40]. A globalized optimization approach, i.e., generalized pattern search algorithm (GPSA) [41], has been implemented for determining the PMU locations in a bounded nonconvex nonlinear framework. The GPS algorithm shows significant improvement in measurement redundancy over other approaches. A robust local search procedure based on the interior point approach along with branch and bound algorithm is applied in [42] for solving the problem of OPP. The problem of placing PMUs can also be considered as a multi-criteria decision-making (MCDM) problem. However, only a handful of researchers have focused on the MCDM approach for solving the OPP problem. Sodhi et al. [43] and Sadanandan Sajan et al. [44] have applied the analytical hierarchy process (AHP) for the multi-phasing of PMUs. However, for solving OPP, both the authors have applied integer linear programming (ILP) instead of the MCDM approach. The authors in [45–47] also applied ILP for solving the problem of OPP. The main drawback of the ILP-based approach is that it does not guarantee maximum measurement redundancy [22]. To optimize the measurement redundancy, the authors in [48] implemented a novel model to represent the redundancy level of each bus. In [49], the graph theory and AHP approach have been applied for solving the OPP problem. However, the authors obtained the OPP solution in two steps; i.e., pruning operation has been performed to obtain the final OPP solution. The authors in [50] implemented AHP and intelligent search technique (IST) for determining the minimum number of PMUs while monitoring the power system components such as generators, transformers, loads, and synchronous condensers. However, in both approaches, the authors have found a suboptimal solution using AHP in the process of finding the optimal locations. Recently, Kumar et al. [51] applied AHP for multi-phasing of PMUs, considering the effect of voltage stability and reliability. However, for the OPP solution, they have considered linear programming (LP)-based branch and bound algorithm. The LP-based algorithm has drawback as mentioned above. Further, the authors have not considered the effect of ZIB in their approach.

From the existing literature, it is clear that all the above-mentioned MCDM approaches have implemented AHP for solving the OPP problem. Some literature has considered the MCDM approaches only for multi-phasing of PMUs. Though some literature has implemented AHP considering MCDM approaches for obtaining the optimal PMU location, the authors have obtained the OPP solution in two
steps. In the 1st step, they have found a suboptimal solution. From the suboptimal solution, they have found the optimal solution utilizing pruning operation in the 2nd step. Thus, there is a scope for further improvement in achieving optimal PMU placement solution in one shot by discarding pruning operation utilizing better criteria and advanced MCDM approach. PROMETHEE is also a quite simple ranking and efficient method in terms of concept and application compared with available multi-criteria decision-making (MCDM) approaches [52]. The advantage of PROMETHEE is that it requires fewer inputs; it takes into account the preference function of each criterion determined by the decision-makers. In this way, each criterion is evaluated on a different basis and it is possible to make better decisions. The PROMETHEE approach has seen extensive applications in various MCDM problems such as the selection of calcium milk products [53], regional tourism competitiveness [54], and material selection problem [55]. PROMETHEE can also be applied to solve the OPP problem. In this study, the PROMETHEE approach is used for the first time for solving OPP problem that has been considered forming five different criteria for the decision matrix. Unlike the previous OPP approaches, the proposed technique achieves the OPP solution in a single step. The performance of the proposed approach is verified by applying it on IEEE 14-, 30-, 57-, and 118-bus test systems. The proposed method is further implemented on the Indian practical NRPG 246-bus system and larger Polish 2383-bus system to verify its effectiveness in solving the OPP problem for large-scale systems.

The contribution of the study is manifold, which are given as follows:

(i) Novel criteria for determining the minimum number of PMUs for complete system observability while providing maximum measurement redundancy are formulated.

(ii) The proposed approach also considers the effect of ZIB to further reduce the required number of PMUs.

(iii) The condition of single PMU outage and presence of conventional measurements have also been considered while solving the OPP problem.

(iv) An improved MCDM method called PROMETHEE is proposed for solving OPP problem to get the optimal solution directly discarding pruning operation.

The rest of the article is organized as follows. In Section 2, the problem formulation of the optimal PMU allocation is presented. Section 3 discusses the proposed methodology for solving OPP. In this section, several criteria along with the PROMETHEE approach have been discussed. The results and discussions are presented in Section 4. Finally, Section 5 concludes the article.

2. Problem Formulation

Optimal PMU placement problem for a system having no. of buses \( n_{bus} \) is mathematically given as follows:

\[
\min \sum_{i=1}^{n_{bus}} W_i X_i \quad i = 1, 2, \ldots, n_{bus},
\]

Subject to \( BX \geq I, \)

where \( I \) is a \((n_{bus} \times 1)\) matrix and mathematically expressed as follows:

\[
[I]_{n_{bus}} = [1 1 1 1 \ldots 1 1]^T,
\]

where \( W \) represents the installation cost of PMU and \( X \) is a binary decision variable. The entries of \( X \) are given as follows:

\[
x_i = \begin{cases} 
1, & \text{if PMU installed at } i^{th} \text{ bus}, \\
0, & \text{otherwise}, 
\end{cases} \quad i = 1, 2, \ldots, n_{bus}.
\]

With the assumption of installation cost same for all buses, \( W_i \) has been considered unity. \( B \) is a binary connectivity matrix defined as follows:

\[
B_{ij} = \begin{cases} 
1, & \text{if } i = j \text{ and } i^{th} \text{ bus is connected to } j^{th} \text{ bus}, \\
0, & \text{otherwise}, 
\end{cases}
\]

3. Proposed Methodology

In this study, graph theory knowledge and PROMETHEE-based MCDM technique have been used for solving the OPP problem. A decision matrix (DM) is required for determining the best alternative. A decision matrix (DM) has been constructed by representing the power system as a graph model \( \hat{G}(V, E) \). An interconnected power system consists of different nodes and branches. In graph theory, the nodes and branches of the power system are termed as vertices and edges, respectively. Vertices \( V \) of graph \( \hat{G} \) are connected to each other through a set of edges \( E \).

3.1. Important Definitions. Some important definitions corresponding to the criteria used to form the DM are provided as follows:

(i) Degree of Vertex: the degree of a vertex is defined as the total number of edges connected to that vertex.

(ii) Pendant Vertex: a pendant vertex is that vertex that is connected to only one other vertex, and the degree of pendant vertex is 1.

(iii) Extra Bus Observability Index (EBOI): PMU placed at a vertex makes that vertex along with its connected vertices observable. EBOI of a particular vertex is defined as the total number of new vertices that can be made observable by allocating PMU at that vertex. For example, consider the sample bus system shown in Figure 1.

If PMU is allocated at vertex 5, then vertices 4 and 5 will become observable. Now, for vertex 3, the EBOI will become three because if a PMU is allocated at vertex 3 then it will make vertices 1, 2, and 3 observable. Here, vertex 4 is not
considered as it is already observable through PMU placed at vertex 5.

3.2. Decision Matrix Criterion. In this study, five different criteria used for the development of the decision matrix (DM) have been evaluated for all the vertices of the system and a final priority score has been computed to obtain the priority ranking. The criteria are selected to maximize system observability in each iteration. The five different criteria are discussed below.

### 3.2.1. Criterion 1
PMU placement is not encouraged at the pendant vertices. The pendant vertices should be made observable by allocating PMUs at the vertex attached to pendant vertices. In criterion 1, the $i^{th}$ vertex connected to pendant vertices has been given maximum priority and is expressed as follows:

$$ C_1^i = \begin{cases} 2, & \text{if } i^{th} \text{ vertex connected to pendant vertices,} \\ 1, & \text{otherwise.} \end{cases} $$

### 3.2.2. Criterion 2
In this criterion, the unobservable $i^{th}$ vertex, which is connected to the maximum number of vertices having minimum EBOI, is given the highest priority. Mathematically, this criterion is given as follows:

$$ C_2^i = \begin{cases} 2, & \text{if } i^{th} \text{ vertex is unobservable and attached to maximum no. of vertices having minimum EBOI,} \\ 1, & \text{otherwise.} \end{cases} $$

### 3.2.3. Criterion 3
In this criterion, the observable $i^{th}$ vertex that is connected to the maximum no. of vertices having minimum EBOI is given the maximum priority. The mathematical expression of this criterion is given as follows:

$$ C_3^i = \begin{cases} 2, & \text{if } i^{th} \text{ vertex is observable and attached to maximum no. of vertices having minimum EBOI,} \\ 1, & \text{otherwise.} \end{cases} $$

### 3.2.4. Criterion 4
In this criterion, if the unobservable $i^{th}$ vertex that is attached to a single vertex has minimum EBOI and if that $i^{th}$ vertex has maximum EBOI among all the vertices connected to the single vertex having minimum EBOI, then it is prioritized. This criterion is described in Figure 2. In Figure 2, a portion of a complete graph is shown. It is assumed that vertices 2 and 3 have minimum EBOI among all the vertices of the entire graph in a particular iteration. It is assumed further that vertices 1 and 6 are the unobservable vertices connected to vertices 2 and 3, respectively, whereas the other connected vertices, i.e., 4, 5, and 7, are assumed observable. Now, if vertex 1 has maximum EBOI among all the vertices connected to vertices 2 and 3 then vertex 1 will be given higher priority for placing PMU.

This criterion can be expressed as follows:

$$ C_4^i = \begin{cases} 2, & \text{if } i^{th} \text{ vertex satisfy the criterion 4,} \\ 1, & \text{otherwise.} \end{cases} $$

### 3.2.5. Criterion 5
In this criterion, the observable $i^{th}$ vertex is prioritized if it is attached to a vertex that has minimum EBOI and if the observable $i^{th}$ vertex has maximum EBOI among all the observable vertices which are connected to minimum EBOI vertex. This criterion can be explained through the same Figure 2. In this figure, vertices 2 and 3 have minimum EBOI. Here, it is assumed that vertex 4 is observable and it has the maximum EBOI among all the vertices connected to vertices 2 and 3 then vertex 4 will be given higher priority for PMU placement.

This criterion can be expressed as follows:

$$ C_5^i = \begin{cases} 2, & \text{if } i^{th} \text{ vertex satisfy the criterion 5,} \\ 1, & \text{otherwise.} \end{cases} $$

### 3.3. Calculation of DM and Weights
DM is calculated by combining alternatives and criteria as shown in Table 1. The dimension of the matrix is $MM \times NN$, where $MM$ and $NN$ are the no. of alternatives and criteria, respectively. The individual entries of DM are represented through "pp.”

Priority or weights are computed from a pairwise weight matrix (PWM), and these weights have been assigned to various criteria based on their importance. PWM shows the significance of the pairwise evaluation of the criteria. The
weightage of comparison implies how one criterion is important with respect to another. (%_he weightage of comparison can be defined as 1–4 weak to moderate weightage, 5–7 high to very high weightage, and 8-9 very high to extreme high weightage. If the weightage of comparison, i.e., comparison score, is specified as 1, then it implies that both the criteria are equally important to achieve the goal. If the comparison score is 2, then it means that one criterion is slightly important than the other criterion. Similarly, if the comparison score is 9, then it indicates that one criterion is extremely important compared with the other criterion. (%_he dimension of the PWM matrix is $K \times L$. $K$ and $L$ represent the numbers of criteria. (%_he PWM can be formulated as follows:

$$\text{PWM} = \begin{bmatrix} w_1/w_K & \cdots & w_K/w_K \\ \vdots & \ddots & \vdots \\ w_1/w_L & \cdots & w_K/w_L \end{bmatrix}.$$ (10)

To compute the normalized values of criterion weight, all the row elements are summed up, and thereafter, each row sum is divided by the total sum of all rows.

3.4. PROMETHEE Approach. The PROMETHEE outranking approach is one of the most recent MCDM techniques that has been developed by Brans [52] in 1982; however, the concept has been extended by Vincke along with Brans and named as PROMETHEE II [56]. The application and concept of PROMETHEE are quite simple in comparison with other MCDM approaches. PROMETHEE requires much fewer inputs and considers a preference function for each criterion decided by the decision-makers.

The PROMETHEE II establishes a complete preorder among the alternatives and offers a methodology for building a valid outranking relation. The basic idea of PROMETHEE II is to make a pairwise comparison among alternatives. To implement PROMETHEE II, the following information is required, i.e.:

(i) The Weights of Criterion: choice of weights is a vital step in most multi-criteria methods. PROMETHEE II assumes that the decision-maker is able to weigh the criteria appropriately.

(ii) The Preference Function: for each criterion, the preference function converts the difference between evaluations obtained by two alternatives into a preference degree ranging from 0 to 1. This function is applied to calculate the degree of preference associated with the best action in the case of pairwise comparisons.

These two steps are performed in a simultaneous manner. PROMETHEE II considers any one of the six types of preference functions depending on the nature of the problem as proposed in [56], i.e., (a) usual, (b) U-shape, (c) V-shape, (d) level, (e) V-shape with indifference, and (f) Gaussian preference functions. The mathematical definition of various preference functions is discussed as follows.

3.4.1. Usual Preference Function. The usual preference function can be mathematically expressed as (11) follows:

$$P(d) = \begin{cases} 0, & d \leq 0, \\ 1, & d > 0, \end{cases}$$ (11)

where $d$ is the difference between evaluations of two alternatives calculated from (17). The usual preference function will assign a value of either 0 or 1 depending on the value of $d$.

3.4.2. U-Shape Preference Function. The U-shape preference function can be presented as follows:

$$P(d) = \begin{cases} 0, & d \leq q, \\ 1, & d > q. \end{cases}$$ (12)

3.4.3. V-Shape Preference Function. The V-shape preference function is given as follows:

$$P(d) = \begin{cases} 0, & d \leq 0, \\ d/p, & 0 \leq d \leq p, \\ 1, & d > p. \end{cases}$$ (13)

3.4.4. Level Preference Function. The mathematical form of level preference function is depicted as follows:
3.4.5. V-Shape with Indifference Preference Function. The V-shape with indifference preference function is expressed as follows:

\[
P(d) = \begin{cases} 
0, & d \leq q, \\
1/2, & q < d \leq p, \\
1, & d > p.
\end{cases}
\]  

(14)

3.4.6. Gaussian Preference Function. The Gaussian preference function is mathematically given as follows:

\[
P(d) = \begin{cases} 
0, & d \leq 0, \\
1 - e^{-(d^2/2q^2)}, & d > 0.
\end{cases}
\]  

(16)

The indifference threshold \( q \) denotes the highest deviation, which is negligibly taken into account by the decision-makers. The preference threshold \( p \) represents the lowest deviation, which is enough to form a full preference. \( s \) is the specified intermediate value between \( p \) and \( q \). The selection of the preference function is a vital step in this procedure. The various steps of the PROMETHEE II approach are discussed as follows.

Step 1: the first step is initialized by calculating the deviations based on pairwise comparisons. This is given as follows:

\[
d_j^{(m,n)} = c^m_j - c^n_j,
\]  

(17)

where \( d_j^{(m,n)} \) is the deviation of criterion \( c_j \) over two alternatives \( m \) and \( n \).

Step 2: in the next step, a suitable preference function, i.e., the usual preference function, has been adopted for every criterion as provided as follows:

\[
p_{f_j}^{(m,n)} = \text{func}(d_j^{(m,n)}), \quad j = 1, 2, \ldots, N,
\]  

(18)

where \( N \) is the total number of criteria.

Step 3: global preference index \( \pi(m,n) \), i.e., preference of alternative \( m \) over another alternative \( n \) is calculated in this step. \( \pi(m,n) \) can be computed as follows:

\[
\pi(m,n) = \sum_{j=1}^{N} p_{f_j}^{(m,n)} \omega_j, \quad \forall m,n \in A,
\]  

(19)

where \( \pi(m,n) \) can be defined as the summation of weight multiplied preference function of each criterion. \( \omega_j \) is the weight of \( j^{th} \) criterion. \( A \) is the set of all alternatives.

Step 4: in this step, leaving and entering flow for each alternative has been computed, and partial ranking has been obtained. The leaving flow determines how the alternative \( m \) is outperforming the other alternatives. The larger the leaving flow, the better will be the alternative. The entering flow represents how the alternative \( m \) is outperformed by other alternatives. The lower the entering flow, the better will be the alternative.

The leaving flow \( \Psi_m^l \) is computed as follows:

\[
\Psi_m^l = \frac{1}{n-1} \sum_{a \in A} \pi(m,a).
\]  

(20)

The entering flow \( \Psi_m^e \) is calculated as follows:

\[
\Psi_m^e = \frac{1}{n-1} \sum_{a \in A} \pi(a,m).
\]  

(21)

Step 5: finally, the net outranking flow for each alternative is obtained in this step. The net outranking flow (NOF) is calculated as follows:

\[
\Psi_m = \Psi_m^l - \Psi_m^e.
\]  

(22)

Higher the \( \Psi_m \), better will be the alternative.

3.5. Optimal Allocation of PMU considering Zero Injection Bus (ZIB). Optimal PMU placement has been solved with and without considering ZIB effects. ZIBs are those buses that do not inject current into the power system. ZIBs are not attached to any loads or generations. The optimal no. of PMUs required for complete system observability is reduced when ZIBs considered pseudo-measurements compared with the case when the effect of ZIB is ignored. Some of the important ZIB rules used for the observability analysis are given as follows:

Rule 1: if all but one bus associated with a ZIB is observable, then that single bus can also be made observable by applying KCL at ZIB.

Rule 2: if total “z” no. of ZIBs attached to each other and all its incident buses form a set of “y” candidate buses, then at least (“y-z”) buses should be observable to make all the “y” buses observable.

The steps for the optimal allocation of PMUs considering ZIB are presented as follows:

(1) Construct the DM using all the criteria discussed in Section 3.2. While constructing the DM, ignore the vertices having the least EBOI if those are ZIBs or connected to ZIBs provided there exist non-ZIBs.

(2) Compute NOF (\( \Psi_m \)) for all the alternatives using (22). Consider the alternative corresponding to non-ZIB that gives the maximum value of \( \Psi_m \) as the most preferred location for PMU placement.

(3) Allocate the PMU to the most preferred location. Check the system observability.

(4) If the system is already observable, print the optimal locations. Otherwise, update the DM and Goto Step 2 where NOF (\( \Psi_m \)) is calculated for all the alternatives except the PMU allocated vertices.
The flowchart of the proposed approach considering the ZIB effect is given in Figure 3.

4. Results and Discussions

The effectiveness of the proposed approach has been verified by applying it on IEEE 14-, 30-, 57-, 118-bus test systems, a practical Indian utility system of NRPG 246-bus, and larger Polish 2383-bus system. Optimal locations of PMUs are obtained with and without considering ZIB and considering cases of single PMU outage and existence of conventional measurements.

4.1. OPP Solution Using PROMETHEE. The OPP solution for all the test systems using the PROMETHEE approach is provided as follows.

4.1.1. Efficacy of the Proposed Approach on IEEE 14-Bus System. IEEE 14-bus system comprises 14 vertices and 20 edges. The DM has been formed by integrating all the criteria described in Section 3.2. The priority and the normalized weights for each criterion obtained by (10) are listed in Table 2.

It can be seen that criterion 1, i.e., the radial bus observability criterion, is given a maximum priority of 4.5. The subsequent criteria have been assigned with decremetal priority; i.e., criteria 2, 3, 4, and 5 are assigned with values 3, 1.5, 0.5, and 0.343, respectively. The same priority and normalized weights used for IEEE 14-bus system are adopted for determining the OPP solution for other systems.

Table 3 presents the DM along with the net outranking flows for all the alternatives at iteration 1 for normal condition, i.e., without considering the ZIB effect. From Table 3, it is noted that bus 7 has a maximum NOF value of 0.4930. Therefore, PMU is placed initially at bus 7.

The NOF value for IEEE 14-bus system at various iterations for the normal condition is presented through bar charts in Figures 4(a)–4(d). It is observed from these figures that NOF values for IEEE 14-bus system at iterations 1, 2, 3, and 4 are maximum at buses 7, 9, 6, and 2, respectively. At iteration 1, it is observed that the maximum NOF value is 0.4930. The maximum NOF values for iterations 2, 3, and 4 are 0.0562, 0.0315, and 0.3315, respectively. Considering the maximum NOF value, the OPP solution obtained for IEEE 14-bus system at normal condition is {7, 9, 6, 2}. The NOF for IEEE 14-bus system at various iterations considering ZIB is presented in Figures 5(a)–5(c).

Bus 7 has the maximum NOF value at the initial iteration. Bus 7, being a ZIB, is not considered for PMU allocation by Step 2 of the proposed algorithm. The bus that has the next highest NOF at iteration 1 is 9. Thus, the initial PMU is allocated at bus 9. PMUs are allocated next at buses 6 and 2, as they have the maximum NOF for the subsequent iterations. The DM and NOF at iteration 3, considering the effect of ZIB, are further presented in Table 4. From this table, it is seen that bus 2 has a maximum NOF value of 0.3315. This result justifies the allocation of PMU at bus 2 at iteration 3. With the placement of PMUs at buses 9, 6, and 2, the system becomes completely observable considering the ZIB effect.

4.1.2. Efficacy of the Proposed Approach on IEEE 30-, 57-, and 118-Bus System. The NOF for IEEE 30-, 57-, and 118-bus systems at iteration 1 without considering ZIB is depicted in Figures 6(a)–6(c).
Figure 4: Net outranking flow for IEEE 14-bus system at (a) iteration 1, (b) iteration 2, (c) iteration 3, and (d) iteration 4 at normal condition.

Figure 5: Net outranking flow for IEEE 14-bus system at (a) iteration 1, (b) iteration 2, and (c) iteration 3 considering the inclusion of ZIB.
For IEEE 30-bus system, the maximum NOF is obtained at iteration 1 for buses 9, 12, and 25. In case more than one bus possesses a maximum NOF value at an iteration, PMU is allocated at any one of those buses. Thus, PMU is allocated at bus 9 for iteration 1. The other buses have been chosen for optimal location based on the maximum NOF value for the subsequent iterations. The OPP solution for IEEE 30-bus system using the PROMETHEE approach is presented in Table 5, where it is noted that 10 PMUs need to be placed at normal conditions. The required no. of PMUs while considering ZIB is 7.

For IEEE 57-bus system, it is noted that bus 32 has a maximum NOF value at iteration 1. Therefore, PMU is placed initially at bus 32. The subsequent PMUs are placed based on the highest value of NOF for subsequent iterations. The required number of PMUs for the IEEE 57-bus system is 17 for normal condition and 11 considering ZIBs, as provided in Table 5. For IEEE 118-bus system, bus nos. 9, 12, 68, 71, 86, and 110 give the highest NOF at the first iteration. However, a single PMU is placed at bus 9 at iteration 1. The other PMUs are placed based on the highest NOF value during subsequent iterations till the observability is
Achieved. The optimal no. of PMUs required for complete observability of the IEEE 118-bus test system is 32 for normal operating condition. However, only 28 PMUs are needed to be placed for making the system observable while considering the ZIB effect, as provided in Table 5.

### 4.2. Comparative Assessment of the Proposed Method

#### 4.2.1. Comparative Performance Analysis of the Proposed Method under Normal Condition

Comparative performance of the proposed approach with some established methods at normal condition is presented in Table 6.

From this table, it is noted that the proposed approach is at par with the other methods. This is to be noted that the proposed approach successfully provides the optimal solutions for all the test systems directly in one shot. Thus, the proposed approach discards the necessity of pruning operation used by the existing MCDM-based approach [49] for achieving optimal solution from suboptimal one. The comparative results of maximum redundancy (MR) obtained by the proposed method and some other methods under normal condition are provided in Table 7.

From Table 7, it is understood that the MR obtained by the proposed method for smaller systems is not appreciable compared with the available methods. However, for larger systems, the proposed approach provides higher redundancy in comparison with other methods. For IEEE 118-bus system, the MR obtained through the proposed approach is 40.7, which is the maximum among the MR obtained by other methods.

The MR obtained through the proposed approach is further compared with methods such as Babu and Bhattacharyya [18], which provides multiple OPP solution. This comparison is shown in Table 8, where multiple OPP solution along with their MR is provided. From the results presented, it is clearly noticed that the proposed approach gives better MR, especially in the case of larger systems.

#### 4.2.2. Comparative Performance Analysis of the Proposed Approach considering ZIB Effect

The comparative performance of the proposed approach with some existing methods considering ZIBs is presented in Table 9. The enhancement in MR utilizing the proposed approach is depicted in Table 10.

It is revealed that the proposed approach does not provide appreciable MR for smaller systems such as IEEE 30 bus, but for large systems such as IEEE 118 bus the MR obtained through the proposed approach is higher compared with approaches [29, 31, 38, 49] and [50].

In Table 11, MR of the proposed method is compared with multiple OPP solution provided by Babu and Bhattacharyya [18] considering ZIB effects. The presented results prove the supremacy of the proposed method, especially in the case of larger system.

### 4.3. Efficacy of the Proposed Approach considering Single PMU Outage

Table 12 provides the optimal no. of PMUs and their locations considering single PMU outage.

Comparative performance of the proposed approach with other methods considering single PMU outage is presented in Table 13. From the results, it is clear that the proposed approach provides lesser or same no. of PMUs for assuring system observability while considering the condition of single PMU outage.

### 4.4. Implementation of the Proposed Approach considering Conventional Measurements

Table 14 provides the optimal location of PMUs considering the existence of conventional power flow and power injection measurements.

For IEEE 14-bus system, it is seen that 3 PMUs have to be placed at buses 2, 6, and 9 for making the system observable in the presence of bus power injections and flow measurements at the desired locations mentioned in Table 14. Similarly, for other test systems, the optimal number of PMUs has been reduced considering the existence of conventional measurements.

### 4.5. Implementation of the Proposed Approach on an Indian Practical Utility, i.e., NRPG 246-Bus System

NRPG 246-bus system comprises 246 nodes and 376 edges. In Table 15, optimal no. of PMUs and their locations for the NRPG 246-bus system have been presented, considering with and without ZIB effects. It is seen that 70 PMUs are required for entire system observability without considering ZIB. It is also noted that 49 PMUs are enough to make the system completely observable in the presence of ZIBs.

To show the effectiveness of the proposed approach, the comparative performance for NRPG 246-bus system is presented in Table 16, considering with and without ZIB effects. It is noted that the proposed approach needs 70 PMUs, while Venkatesh and Jain [24] reported 77 no. of PMUs required for complete system observability without considering ZIBs.
Table 6: Comparative performance of the proposed approach with other methods at normal condition.

| Methods          | IEEE 14 bus no. of PMUs | IEEE 30 bus no. of PMUs | IEEE 57 bus no. of PMUs | IEEE 118 bus no. of PMUs |
|------------------|-------------------------|--------------------------|--------------------------|--------------------------|
| Saha Roy et al. [38] | 4                       | 10                       | 17                       | 32                       |
| Singh and Singh [29] | 4                       | 10                       | —                        | 32                       |
| Chatterjee et al. [50] | 4                       | 10                       | 17                       | 32                       |
| Ahmadi et al. [31]   | 4                       | 10                       | 17                       | 32                       |
| Ghosh et al. [49]    | 4                       | 10                       | 17                       | 32                       |
| Proposed            | 4                       | 10                       | 17                       | 32                       |

Table 7: Comparison of MR (%) with other methods at normal condition.

| Methods          | IEEE 14 bus | IEEE 30 bus | IEEE 57 bus | IEEE 118 bus |
|------------------|-------------|-------------|-------------|--------------|
| Saha Roy et al. [38] | 35.7        | 66.7        | 19.3        | 31.4         |
| Singh and Singh [29] | 35.7        | 73.3        | —           | 38.9         |
| Chatterjee et al. [50] | 35.7        | 73.3        | 24.6        | 35.6         |
| Ahmadi et al. [31]   | 35.7        | 73.3        | 24.6        | 35.6         |
| Ghosh et al. [49]    | 35.7        | 73.3        | 24.6        | 35.6         |
| Proposed            | 35.7        | 46.7        | 24.6        | 40.7         |

Table 8: Comparison of MR (%) of the proposed approach with multiple OPP solution-based approach of Babu and Bhattacharyya [18].

| Bus system | OPP solution | MR (%) | MR (%) (proposed approach) |
|------------|--------------|--------|----------------------------|
| IEEE 14    | 2, 6, 7, 9   | 35.7   | 35.7                       |
|            | 2, 7, 11, 13 | 14.3   |                            |
|            | 2, 7, 10, 13 | 14.3   |                            |
|            | 2, 6, 8, 9   | 0      |                            |
|            | 1, 5, 9, 10, 12, 18, 24, 25, 28, 29 | 16.7 |                           |
|            | 1, 2, 6, 9, 10, 12, 15, 19, 25, 27 | 66.7 |                           |
|            | 2, 4, 6, 9, 10, 12, 15, 19, 25, 27 | 73.3 |                           |
|            | 3, 5, 8, 9, 10, 12, 18, 23, 26, 30 | 23.3 |                           |
|            | 2, 4, 6, 10, 11, 12, 18, 23, 25, 27 | 60   | 46.7                       |
|            | 1, 6, 7, 9, 10, 12, 19, 24, 25, 27 | 56.6 |                           |
|            | 3, 5, 8, 10, 11, 12, 18, 23, 25, 30 | 23.3 |                           |
|            | 1, 7, 8, 9, 10, 12, 18, 24, 26, 30 | 26.6 |                           |
|            | 1, 6, 7, 9, 10, 12, 19, 24, 26, 27 | 50.0 |                           |
|            | 3, 6, 7, 9, 10, 12, 15, 18, 25, 27 | 60.0 |                           |
| IEEE 30    | 3, 5, 9, 12, 15, 17, 20, 23, 28, 30, 35, 40, 43, 46, 50, 51, 54, 62, 64, 68, 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114 | 31.4 | 31.4                       |
|            | 3, 7, 9, 11, 12, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 63, 68, 71, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114 | 32.2 | 32.2                       |
|            | 1, 5, 9, 12, 15, 17, 21, 23, 28, 30, 35, 40, 43, 46, 50, 51, 54, 62, 64, 68, 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114 | 24.6 | 24.6                       |
|            | 1, 5, 9, 12, 15, 17, 21, 23, 28, 30, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114 | 38.1 | 38.1                       |
|            | 3, 5, 10, 12, 15, 17, 21, 23, 28, 30, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 114 | 38.1 | 38.1                       |
| IEEE 57    | —            | —        | 24.6                      | 24.6                      |
| IEEE 118   | 3, 5, 9, 12, 13, 17, 21, 25, 29, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 76, 79, 84, 86, 89, 92, 96, 100, 105, 110, 114 | 31.4 | 31.4                       |
|            | 1, 5, 9, 12, 13, 17, 21, 25, 29, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 75, 77, 80, 85, 86, 91, 94, 102, 105, 110, 114, 116 | 33.1 | 33.1                       |
|            | 1, 5, 9, 12, 15, 17, 21, 23, 28, 30, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114, 116 | 35.6 | 35.6                       |
|            | 1, 5, 9, 12, 15, 17, 21, 25, 29, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 78, 84, 86, 89, 92, 96, 100, 105, 110, 114, 118 | 35.6 | 35.6                       |
|            | 1, 5, 9, 12, 13, 17, 21, 25, 29, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 78, 84, 86, 89, 92, 96, 100, 105, 110, 114, 118 | 33.1 | 33.1                       |
### Table 9: Comparative performance of the proposed approach with other methods considering ZIB.

| Methods             | IEEE 14 bus no. of PMUs | IEEE 30 bus no. of PMUs | IEEE 57 bus no. of PMUs | IEEE 118 bus no. of PMUs |
|---------------------|-------------------------|-------------------------|-------------------------|--------------------------|
| Saha Roy et al. [38]| 3                       | 7                       | 11                      | 28                       |
| Singh and Singh [29]| 3                       | 7                       | —                       | 28                       |
| Chatterjee et al. [50]| 3                    | 7                       | 11                      | 28                       |
| Ahmadi et al. [31]  | 3                       | 7                       | 13                      | 29                       |
| Ghosh et al. [49]   | 3                       | 7                       | 11                      | 28                       |
| Proposed            | 3                       | 7                       | 11                      | 28                       |

### Table 10: Comparison of MR (%) with other methods considering ZIB.

| Methods             | IEEE 14 bus | IEEE 30 bus | IEEE 57 bus | IEEE 118 bus |
|---------------------|-------------|-------------|-------------|--------------|
| Saha Roy et al. [38]| 14.3        | 3.3         | 7.0         | 16.9         |
| Singh and Singh [29]| 7.1         | 20.0        | —           | 24.6         |
| Chatterjee et al. [50]| 14.3      | 26.7        | 7.0         | 26.3         |
| Ahmadi et al. [31]  | 7.1         | 3.3         | 15.7        | 25.4         |
| Ghosh et al. [49]   | 14.3        | 26.7        | 7.0         | 26.3         |
| Proposed            | 14.3        | 20.0        | 7.0         | 33.1         |

### Table 11: Comparison of MR (%) of the proposed approach with multiple OPP solution-based approach of Babu and Bhattacharyya [18] considering ZIB effects.

| Bus system | OPP solution | MR (%) | MR (%) (proposed approach) |
|------------|--------------|--------|----------------------------|
| IEEE 14    | 2, 6, 9      | 14.3   | 14.3                       |
|            | 1, 2, 10, 12, 19, 23, 27 | 20.0   |                            |
|            | 3, 5, 10, 12, 18, 23, 27 | 10.0   |                            |
|            | 1, 2, 10, 12, 19, 23, 27 | 23.3   |                            |
|            | 1, 2, 10, 12, 18, 23, 27 | 20.0   |                            |
|            | 1, 2, 10, 12, 18, 24, 27 | 23.3   | 20.0                       |
|            | 3, 5, 10, 12, 19, 23, 27 | 10.0   |                            |
|            | 1, 5, 10, 12, 18, 23, 27 | 10.0   |                            |
|            | 3, 7, 10, 12, 19, 24, 27 | 16.7   |                            |
|            | 3, 7, 10, 12, 19, 23, 27 | 13.3   |                            |
| IEEE 30    | —            | —       | 7.0                        |
|            | 1, 8, 11, 12, 15, 17, 21, 25, 29, 34, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 114 | 27.1   |                            |
|            | 2, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110 | 31.4   |                            |
|            | 3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 91, 101, 105, 110 | 32.2   |                            |
|            | 1, 8, 11, 12, 17, 21, 25, 29, 33, 34, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 114 | 27.9   |                            |
|            | 1, 9, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 87, 90, 94, 101, 105, 110 | 32.2   |                            |
| IEEE 57    | —            | —       | 7.0                        |
|            | 1, 8, 11, 12, 15, 17, 21, 25, 29, 34, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110 | 27.1   |                            |
|            | 2, 8, 11, 12, 15, 17, 21, 25, 29, 34, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110 | 28.8   |                            |
|            | 3, 9, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110 | 25.4   |                            |
|            | 3, 9, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110 | 30.5   |                            |
Table 12: Optimal no. and locations of PMU considering single PMU outage.

| Test system | Optimal no. | Optimal locations |
|-------------|-------------|-------------------|
| IEEE 14     | 9           | 2, 4, 5, 6, 7, 8, 9, 11, 13 |
| IEEE 30     | 21          | 1, 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 18, 19, 21, 23, 25, 26, 27, 29 |
| IEEE 57     | 33          | 1, 3, 4, 6, 9, 12, 15, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 43, 45, 46, 47, 50, 51, 53, 54, 56, 57 |
| IEEE 118    | 68          | 2, 3, 5, 7, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 26, 27, 29, 31, 32, 34, 36, 37, 40, 42, 44, 45, 46, 49, 52, 53, 56, 57, 58, 59, 62, 64, 65, 67, 68, 70, 71, 73, 75, 77, 79, 80, 84, 85, 86, 87, 89, 91, 92, 94, 96, 100, 102, 105, 107, 109, 110, 111, 112, 115, 116, 117, 118 |

Table 13: Comparative performance of the proposed approach with other methods considering single PMU outage.

| Methods | IEEE 14 bus no. of PMUs | IEEE 30 bus no. of PMUs | IEEE 57 bus no. of PMUs | IEEE 118 bus no. of PMUs |
|---------|-------------------------|-------------------------|-------------------------|-------------------------|
| Saha Roy et al. [38] | — | — | — | — |
| Singh and Singh [29] | 9 | 21 | — | 68 |
| Chatterjee et al. [50] | 9 | 21 | 33 | 68 |
| Ahmadi et al. [31] | — | — | — | — |
| Ghosh et al. [49] | 9 | 21 | 33 | 68 |
| Proposed | 9 | 21 | 33 | 68 |

Table 14: Optimal PMU placement considering conventional measurements.

| Bus system | Flow measurements | Injection measurements | PMU location |
|------------|-------------------|------------------------|--------------|
| IEEE 14    | 5-6, 13-14, 10-11, 6-12, 4-9, 7-9 | 8, 11, 13 | 2, 6, 9 |
| IEEE 30    | 2-5, 18-19, 4-12, 6-28, 6-8, 25-27, 15-23, 23-24 | 2, 13, 16, 26 | 1, 10, 12, 15 |
| IEEE 57    | 2-3, 3-4, 7-8, 6-7, 14-15, 34-35, 22-23, 37-38, 11-41, 42-56, 23-24, 14-46, 9-55, 46-47, 10-51 | 5, 21, 26, 43, 45, 48, 50 | 1, 9, 20, 25, 29, 32, 38, 54, 56 |
| IEEE 118   | 5-8, 4-5, 5-6, 2-12, 19-20, 23-22, 25-27, 51-52, 49-54, 54-55, 55-59, 50-57, 57-66, 70-71, 82-83, 89-90, 90-91, 91-92, 94-96, 100-106, 92-102, 27-115, 12-117 | 7, 11, 16, 21, 26, 32, 36, 41, 43, 53, 58, 64, 73, 87, 93, 95, 101, 114 | 1, 10, 12, 15, 17, 24, 25, 29, 45, 49, 62, 75, 77, 80, 85, 86, 100, 105, 110, 115 |

Table 15: Optimal PMU no. and locations for NRPG 246 bus considering with and without ZIB effects.

| No. | PMU location |
|-----|--------------|
| Without considering ZIB |
| 3, 6, 14, 19, 23, 29, 34, 40, 45, 48, 50, 54, 57, 60, 61, 62, 63, 64, 65, 69, 74, 75, 80, 84, 88, 92, 96, 98, 101, 103, 106, 109, 117, 122, 125, 126, 128, 129, 132, 133, 134, 140, 141, 142, 147, 157, 158, 160, 167, 168, 174, 181, 185, 187, 190, 191, 193, 194, 199, 201, 202, 203, 215, 216, 219, 229, 234, 235, 243, 245 |
| Considering ZIB |
| 3, 6, 14, 19, 23, 29, 34, 40, 45, 48, 50, 54, 57, 60, 61, 62, 63, 64, 65, 69, 74, 75, 80, 84, 88, 92, 96, 98, 101, 103, 106, 109, 117, 122, 125, 126, 128, 129, 132, 133, 134, 140, 141, 142, 147, 157, 158, 160, 167, 168, 174, 181, 185, 187, 190, 191, 193, 194, 199, 201, 202, 203, 215, 216, 219, 229, 234, 235, 243, 245 |

Table 16: Comparative performance for NRPG 246-bus system with and without considering ZIB effects.

| Methods | Without ZIB no. of PMUs | With ZIB no. of PMUs |
|---------|-------------------------|----------------------|
| Singh and Singh [29] | 70 | 51 |
| Venkatesh and Jain [24] | 77 | 57 |
| Sodhi et al. [43] | 70 | — |
| Proposed | 70 | 49 |
5. Conclusions

In this study, the optimal PMU placement problem has been solved by an improved MCDM approach named PROMETHEE for power system observability. The proposed PROMETHEE-based approach has unique property for achieving one-shot OPP solution, which helps to overcome the drawback of the previous MCDM approach used for solving OPP. Thus, the proposed approach discards the necessity of pruning operation used by the existing MCDM-based approaches for achieving optimal solution from suboptimal one. Some advanced criteria have been developed utilizing the concept of graph theory to find the optimal PMU location. Selected criteria and the PROMETHEE-based proposed MCDM approach not only determine the minimum number of PMUs for complete system observability but also try to achieve maximum measurement redundancy in the process. The efficacy of the proposed approach is verified by applying it on various IEEE standard test bench systems and on an Indian practical utility named NRPG 246-bus system and larger system of Polish 2383 bus. The OPP problem has been solved at normal conditions, considering ZIB effects, and for cases such as single PMU outage, and existence of conventional measurements. The obtained results have been compared with some other popular existing techniques, and it has been found that the proposed method provides better results, especially in the case of larger bus systems.

Data Availability

The data used to support the findings of this study are available at https://matpower.org/docs/ref/matpower5.0/case2383wp.html.
Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

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