Students’ misconception on equal sign

N F Kusuma¹, S Subanti¹, B Usodo¹
¹Mathematics Education ¹Departement, Universitas Sebelas Maret, Surakarta, Indonesia

Email: niyankusuma@gmail.com

Abstract. Equivalence is a very general relation in mathematics. The focus of this article is narrowed specifically to an equal sign in the context of equations. The equal sign is a symbol of mathematical equivalence. Studies have found that many students do not have a deep understanding of equivalence. Students often misinterpret the equal sign as an operational rather than a symbol of mathematical equivalence. This misinterpretation of the equal sign will be label as a misconception. It is important to discuss and must resolve immediately because it can lead to the problems in students’ understanding. The purpose of this research is to describe students’ misconception about the meaning of equal sign on equal matrices. Descriptive method was used in this study involving five students of Senior High School in Boyolali who were taking Equal Matrices course. The result of this study shows that all of the students had the misconception about the meaning of the equal sign. They interpret the equal sign as an operational symbol rather than a symbol of mathematical equivalence. Students merely solve the problem only single way, which is a computational method, so that students stuck in a monotonous way of thinking and unable to develop their creativity.

1. Introduction
Mathematical equivalence concept has been taught to students since elementary school, specifically in material arithmetic. Learning mathematics in junior high school is a transition from learning arithmetic to algebra. An example of the current discontinuity between elementary school arithmetic and the algebra learnt in upper grades, relates to understanding the concept of an equal sign. The equal sign is a symbol of mathematical equivalence. The equal sign should be understood as a relational symbol, indicating that a balanced relationship exists between numbers on the two sides of the equivalence symbol [6].

Students who defined the equal sign as a symbol of mathematical equivalence were more likely to recognize not only that the equations $2 \times \square + 15 = 31$ and $2 \times \square + 15 - 9 = 31 - 9$ had the same solutions, but this also could be determined without actually solving the two equations [7]. Students who already hold a relational view of the equal sign, successfully solve $84 + 104 = \square + 103$ by discourage computation and encourage looking for relationships across the equal sign. In this case, a student who is thinking relationally might notice that 103 is one less than 104 and that the number to be placed in the box must therefore be one more than 84.

Studies have found that many students do not have a deep understanding of equivalence. Based on a research of [8], in the exploratory study of the mathematical thinking of a selection of Year 7 and Year 8 students in Brazil, they found that when students were asked to solve numerical expressions using four arithmetic operations, most students opted for computational methods. Students often
misinterpret the equal sign as an operational rather than a symbol of mathematical equivalence [1, 4, 6, 7]. This misinterpretation of the equal sign will be label as a misconception. Misconception is mistaken idea or view resulting from a misunderstanding of something. It arises from the fact that before knowing the correct concept they already have their own concept which is formed from reasoning, intuition, culture, or the other [5].

This misconception can lead to the problems of students’ understanding if not resolve immediately, for example students who define the equal sign as an operational found difficulty in solving algebraic related problems. When students were asked to solve the equation 5 + 3\(x\) = 14, students tend to move 5 to the right side into 3\(x\) = 14 − 5, then students moved to the right side into \(x\) = \(\frac{9}{3}\) without knowing the reasons why students do this. Students who view the right-hand side of an equation as the answer and who prefer to solve equations by transposing lack an understanding of the balance between the right and left-hand sides of the equation [2].

There are five factors that causes misconceptions in students namely students, teachers, textbooks, context, and way of teaching [9]. Arithmetic algorithms are teachable as procedures for solving algebraic equations, by focusing on relations rather than calculating the answer [3]. In the Indonesian curriculum, there is a serious discontinuity between the arithmetic taught in elementary school and the algebra taught in upper grades. The elementary school arithmetic syllabus focuses upon computational performance, without attending to relations and fundamental properties of arithmetic operations. As a result, students hold the misconception about mathematical equivalence and experience difficulty when they begin to study algebra in upper grades. It also influences the student who is finding an answer without understanding the equation concept and merely find it only single way, which is a computational method. The consequence is students stuck in a monotonous way of thinking and unable to develop their creativity. Creativity is one of important skills required for 21st century education besides critical thinking, communication, and collaboration.

The concept of equivalence is also taught to students in High School, specifically in material equal matrices. A matrix is a group of numbers arranged in a square-shaped range or a rectangle made up of rows and columns. Two matrices \(A\) and \(B\) are equal \((A = B)\) if and only if they are of the same size and their corresponding entries are equal [10]. The equal matrices can often be use to determine the value of the variable or variables on the entries of a matrix.

2. Method
This research uses a qualitative method with descriptive research type. The strategy used in this research is a case study. This research was conducted at one of senior high school in Boyolali Central Java Province, Indonesia. There are 5 students as participants. The subjects of the study are students of grade 12 with relatively the same scores and with good communication skills. They are SR, SD, SW, SA, and SL. This qualitative study was carried out in the following procedures: Firstly, researchers collected data from an individual written test. Secondly, data collected from an individual interview. Thirdly, data analyzed by comparing data that were collected from the individual written test with data from the individual interview. Fourthly, sum up for conclusion.

3. Result and Discussion
According to data analyzing, the results classified into two sections. The sections are seen from students’ test result and students’ interview data results. Three questions are given by researcher to identify students’ misconception on the equal sign. Students’ misconception will be described in the following section.

Question 1:

\[
\begin{pmatrix}
1 & -1 \\
4 & 5
\end{pmatrix}
= 
\begin{pmatrix}
0 & -\frac{1}{3} \\
\frac{16}{4} & \frac{10}{2}
\end{pmatrix}
\]
(a) The arrow points to a symbol. What is the name of this symbol?
(b) What does this symbol mean?”

Figure 1 below is the student SR answer of question 1.

![Figure 1](image)

**Figure 1.** The student SR answer of question 1

According to the student’s test result, SR defined the equal sign as a symbol that indicates the existence of an equivalence between the right side and the left side. On the other hand, student SD, SW, SA, and SL defined the equal sign as a symbol that indicates corresponding entries on both matrices are equal.

Question 2:
“Given the matrices as mentioned below:

\[
A = \begin{pmatrix} 74 & p \\ 3 & 8 \end{pmatrix} \quad B = \begin{pmatrix} q \\ -78 \end{pmatrix} \quad C = \begin{pmatrix} 75 + 23 & 1000 - 86 \\ 923 - 778 & 37 + 14 \end{pmatrix}
\]

If \( A + B = C \), determine the value of \( p, q, r, \) and \( s \)”

According to student’s test result, all of the students solve the problem in the same way. Figure 2 below is an example of student SW answer to question 2.

![Figure 2](image)

**Figure 2.** The student SW answer of question 2

First, students SW wrote the matrix in the form of known sum operation which was \( A + B = C \). On the right side, student SW calculated the sum or subtraction of numbers on matrix C. On the left side, student SW summed every corresponding entries on matrices A and B. According to the calculation result on both sides, student SW use the definition of equal matrices to identify the unknown value. Based on the definition, matrix \( A + B \) and matrix \( C \) are equal \( (A + B = C) \) if and only if they are of the same size and their corresponding entries are equal [10].

Furthermore, student SW given four new equations, which were \( 74 + p = 98; -900 + q = 113; -78 + s = 14,5; \) and \( 3,8 + r = 5,1 \). Student SW moves the numbers on both sides in every equation to determine the value of \( p, q, r, \) and \( s \). The result of the researcher’s interview with the student SR on the first question listed in Table 1 below:
Table 1. Result of interview with student SW of question 2

| Code | Interview Result |
|------|------------------|
| R    | “Just look at the sheet on you, what is the value of p?” |
| SW1  | “This is mom. The value of p is 1013.” |
| R    | “How do you get the value of p?” |
| SW2  | “Move the numbers -900 on left side to right side.” |
| R    | “Why are you moving the numbers -900? How does the negative sign may change into positive sign?” |
| SW3  | “Because we will find the value of p, then the numbers -900 is moved to the right side, so the only remaining variable p on the left side. A sign of negative numbers 900 can be turned into a positive because if a number of the moved side, then the mark will change.” |

The interview result with student SW indicates that student SW experienced misconception concerning on equal sign. It was proved by code SW3, in which student SW moves the segment on equation without knowing the right reason for doing so. Student SW did not use the concept of equivalence at all thus students SW could not explain why number 900 appeared on the right side and how number -900 which located on the left side at first, then left only with variable p. This also happens to identify the value of q, r, and s. In the same way, student SW got the right value of q, r, s, although the concept that being used was not exact.

Question 3:
“Given matrices as mentioned below:

\[ P = \begin{pmatrix} a \times 10 & b \div 6 \\ c \times 4 & d \div 8 \end{pmatrix} \]

\[ Q = \begin{pmatrix} 27 \times 30 & 90 \div 18 \\ 2400 \times 0.4 & 12 \div 0.8 \end{pmatrix} \]

If \( P = Q \), determine the value of \( a \), \( b \), \( c \), and \( d \)!”

The student’s test result shows that all of the students solve the problem in the same way. Figure 3 below is an example of student SA answer to question 3 on first activity.

![Figure 3](image-url)

Figure 3. The student SA answer of question 3 on first activity

The question above indicates \( P = Q \). Student SA uses the definition of equal matrices to determine the unknown value. Matrix \( P \) and matrix \( Q \) are equal \( (P = Q) \) if and only if they are of the same size and their corresponding entries are equal [10]. The figure 3 shows that student SA given four new equations, consist of \( a \times 10 = 27 \times 30 \) and \( c \times 4 = 2400 \times 0.4 \). In order to determine the value of \( a \) and \( c \), student SA divides the numbers on the right side with the numbers on the left side in every equation. The result of the researcher’s interview with the student SA on the first question listed in Table 2 below:
**Table 2.** Result of interview with student SA on question 3 at first activity

| Code | Interview Result |
|------|------------------|
| R    | “Just look at the sheet on you, what is the value of a?” |
| SA1  | “This is mom. The value of a is 81.” |
| R    | “How do you get the value of a?” |
| SA2  | “Yes like that. The numbers 27 \times 30 divided by 10.” |
| R    | “Oke. Then, where the number 10 on the left side?” |
| SA3  | “The number 10 on the left side does not exist because it has been used to divide the numbers 27 \times 30 on the right side.” |
| R    | “Is there another way to get the value of a?” |
| SA4  | “hmmm… I think there is no, mom. I only know the way.” |

The interview result with student SA explains that student SA experienced misconception concerning equal sign. It was proved by code SA2 and SA3, in which student SA divides number 27 \times 30 on the right side with number 10 on the left side. Student SA did not use the concept of equivalence. Student SA unable to understand that number on the left side also should be divided with number 10. The same case also carried out in finding the value of c as student SA obtain the right value of c, even though the concept that used was not accurate. Defining the value of b and d, all of the students also solve the problem in the same way. Figure 4 below is an example of student SR answer to question 3 on second activity.

**Figure 4.** The student SR answer of question 3 on second activity

Figure 4 informs that student SR got two new equations, which were \( \frac{b}{6} = 5 \) and \( \frac{d}{8} = 15 \). In finding the value of b and d, student SR uses cross multiplication on that equation. The result of the researcher’s interview with the student SR on the second question listed in Table 3 below:

**Table 3.** Result of interview with student SR on question 3 at second activity

| Code | Interview Result |
|------|------------------|
| R    | “Just look at the sheet on you, what is the value of b?” |
| SR1  | “This is mom. The value of a is 30.” |
| R    | “How do you get the value of b?” |
| SR2  | “The number 6 on left side multiplied by the number 5 on the right side.” |
| R    | “Do you mean the cross product?” |
| SR3  | “Yes mom. That I mean.” |
| R    | “Why do you do multiplication? In other words, why are you not doing the addition, substraction or division against the number?” |
| SR4  | “No mom, because on the left side that is forms division. So, if the form division, how to find the unknown number is multiply the denominator on the left side with the number on the right side. If the form multiplication, how to find the unknown number is divide the number that is in the right side with coefficients on the left side, such as finding the value of a and c.” |
| R    | “Ok. Then, where is the number 6 in the left side?” |
The number 6 are already multiplied by the number 5, so there is no number 6 on the left side.

Is there another way to get the value of a?

No mom. From I used to work on the problem in a way like it.

According to interview result, student SR also experienced misconception regarding the equal sign. It was proved by code SR2-SR5, as student SR multiplies number 6 on the left side with number 5 on the right side. Student SR did not use the concept of equivalence so that student SR unable to understand that on the left side should also multiply with number 6. In other words, both sides multiplied by number 6. The same case applied in finding the value of d. In the same way, students SR got the right value of d, even though the concept that being used was not appropriate.

4. Conclusion
Based on the analysis of the result, the researcher concluded that all of the students hold the misconception about the meaning of the equal sign. They recognize that the equal sign as a symbol of mathematical equivalence, but did not have a deep understanding of equivalence. The students move the numbers on both sides when faced with forms of the equation with an addition or subtraction as the operation. On the other hand, the students conducted cross multiplication on that equation with multiplication or division as the operation. Students merely solve the problem only single way, which is a computational method, so that students stuck in a monotonous way of thinking and unable to develop their creativity. The solution offered to reduce that misconception, teacher can develop learning method that demands the student to more focus on creative thinking and often several kinds of question in order significantly increase the students’ relational thinking.

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