$J/\psi$-meson production within improved color evaporation model
with the $k_T$-factorization approach for $c\bar{c}$ production

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Abstract

We use a new approach to color evaporation model (CEM) for quarkonium production. The production of $c\bar{c}$ pairs is performed within $k_T$-factorization approach using different unintegrated gluon distribution functions (UGDF) from the literature. We include all recent improvements to color evaporation model. We cannot describe simultaneously mid and forward rapidity data measured at the LHC when using the KMR UGDF with the same normalization parameter. Furthermore we get somewhat too hard distribution in $J/\psi$ transverse momentum. Correcting the standard KMR distributions for saturation effects at small values of $x$ improves $J/\psi$ rapidity distributions, while taking correction for emissions hidden in the KMR UGDF in the evaluation of $x$ values improves a bit $J/\psi$ transverse momentum distributions. The modifications improve also description of the LHCb $D$-meson data.

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I. INTRODUCTION

Inclusive production of quarkonia is one of the most actively studied topics at the LHC. The $J/\psi$, $\Upsilon', \Upsilon, \Upsilon'$ and $\Upsilon''$ are the usually measured quarkonia. The production of $J/\psi$ is a model case. There was (still is) a disagreement related to the underlying production mechanism. There are essentially two approaches. The first one is the so-called non-relativistic QCD (NRQCD) approach \[1–4\]. There are two versions of such an approach based on collinear or $k_T$-factorization approaches. It was shown recently that the LHC data can be explained within the NRQCD $k_T$-factorization approach with a reasonable set of parameters \[5\].

Another popular approach is color evaporation model \[6, 7\]. In this approach one is using perturbative calculation of $c\bar{c}$. The $c\bar{c}$-pair by emitting a soft radiation goes to color singlet state of a given spin and parity. The emission is not explicit in this approach and everything is contained in a suitable renormalization of the $c\bar{c}$ cross section when integrating over certain limits in the $c\bar{c}$ invariant mass. It was proposed recently how to improve the original color evaporation model \[8–10\]. The original color evaporation model is based on calculation in collinear approach. The leading-order (LO) approach is known to be not sufficient to describe the inclusive charm data. The next-to-leading order (NLO) approach is much better in this respect \[11–13\]. There is even next-to-next-to-leading-order (NNLO) approach \[14\] but only for inclusive single charm distributions. One needs to calculate correlation distributions ($c\bar{c}$ invariant mass) in the context of the color evaporation model so the NNLO calculation cannot be used in this context. On the other hand the $k_T$-factorization with the KMR unintegrated distributions turned out to be successful in the inclusive production of $D$ mesons \[15, 16\] as well as for some correlation observables \[15, 17\] at the LHC. It seems therefore interesting, and valuable, to apply the $k_T$-factorization approach for $c\bar{c}$ production in the context of applying the color evaporation model for $J/\psi$ meson production.

In the present paper we wish to study whether such a combination of elements can allow to describe the world data for $J/\psi$ production\[^1\].

\[^1\] When our analysis was almost finished we found a new preprint \[10\] which discusses exactly the same process. The main difference is a use of different unintegrated gluon distributions.
II. THEORETICAL FRAMEWORK

In the basic step of our approach, i.e. calculation of the cross section for $c\bar{c}$-pair production, we follow the $k_T$-factorization approach. This framework was shown many times by different authors to provide very good description of heavy quark production in proton-proton collisions at different energies. Some time ago it was successfully used for theoretical studies of $pp \rightarrow c\bar{c}X$ reaction at the LHC, including open charm meson $[15, 17]$, as well as $\Lambda_c$ baryon production $[16]$. Very recently, this approach was also applied e.g. for $pp \rightarrow c\bar{c}$+jet $[18]$, $pp \rightarrow c\bar{c}+2$jets $[19]$ and $pp \rightarrow c\bar{c}c\bar{c}X [20]$.

According to this approach, the transverse momenta (virtualities) of both partons entering the hard process are taken into account and the sum of transverse momenta of the final $c$ and $\bar{c}$ no longer cancels. Then the differential cross section at the tree-level for the $c\bar{c}$-pair production reads:

$$\frac{d\sigma(pp \rightarrow c\bar{c}X)}{dy_1dy_2d^2p_{1,t}d^2p_{2,t}} = \int \frac{d^2k_{1,t}}{\pi} \frac{d^2k_{2,t}}{\pi} \frac{1}{16\pi^2(x_1x_2s)^2} |M_{\text{off-shell}}|^{2} \times$$

$$\delta^2\left(\vec{k}_{1,t} + \vec{k}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t}\right) F_8(x_1, k_{1,t}^2) F_8(x_2, k_{2,t}^2),$$

where $F_8(x_1, k_{1,t}^2)$ and $F_8(x_2, k_{2,t}^2)$ are the unintegrated gluon distribution functions (UGDFs) for both colliding hadrons and $M_{\text{off-shell}}$ is the off-shell matrix element for the hard subprocess. The extra integration is over transverse momenta of the initial partons. We keep exact kinematics from the very beginning and additional hard dynamics coming from transverse momenta of incident partons. Explicit treatment of the transverse part of momenta makes the approach very efficient in studies of correlation observables. The two-dimensional Dirac delta function assures momentum conservation. The unintegrated (transverse momentum dependent) gluon distributions must be evaluated at:

$$x_1 = \frac{m_{1,t}}{\sqrt{s}} \exp(y_1) + \frac{m_{2,t}}{\sqrt{s}} \exp(y_2), \quad x_2 = \frac{m_{1,t}}{\sqrt{s}} \exp(-y_1) + \frac{m_{2,t}}{\sqrt{s}} \exp(-y_2),$$

where $m_{i,t} = \sqrt{p_{i,t}^2 + m_i^2}$ is the quark/antiquark transverse mass. In the case of charm quark production at the LHC energies, especially in the forward rapidity region, one tests very small gluon longitudinal momentum fractions $x < 10^{-5}$.

The matrix element squared for off-shell gluons is taken here in the analytic form proposed by Catani, Ciafaloni and Hautmann (CCH) $[21]$. It was also checked that the CCH
expression is consistent with those presented later in Refs. [22, 23] and in the limit of $k_{1t}^2 \to 0, k_{2t}^2 \to 0$ it converges to the on-shell formula.

The calculation of higher-order corrections in the $k_T$-factorization is much more complicated than in the case of collinear approximation. However, the common statement is that actually in the $k_t$-factorization approach with tree-level off-shell matrix elements a dominant part of real higher-order corrections is effectively included. This is due to possible emission of extra soft (and even hard) gluons encoded in the unintegrated gluon densities. More details of the theoretical formalism adopted here can be found e.g. in Ref. [15].

In the numerical calculation below we apply the Kimber-Martin-Ryskin (KMR) unintegrated gluon distributions [24, 25] that has been found recently to work very well in the case of charm production at the LHC [15]. As discussed also in Ref. [18] the $k_T$-factorization approach with the KMR UGDF gives results consistent with collinear NLO approach. For the calculation of the KMR distribution we used here up-to-date collinear MMHT2014 gluon PDFs [26]. For completeness of the present studies, we also use the CCFM-based JH2013 UGDFs [27] that were applied in the same context in Ref. [10].

The renormalization and factorization scales $\mu^2 = \mu_R^2 = \mu_F^2 = \frac{m_{1t}^2 + m_{2t}^2}{2}$ and charm quark mass $m_c = 1.5$ GeV are used in the present study. The uncertainties related to the choice of these parameters were discussed, e.g. in Ref. [15] and will be not repeated here.

Having calculated differential cross section for $c\bar{c}$-pair production one can obtain the cross section for $J/\psi$-meson within the framework of improved color evaporation model (ICEM) [8, 9]. The $c\bar{c} \to J/\psi$ transition can be formally written as follows:

$$\frac{d\sigma_{J/\psi}(P_{J/\psi})}{d^3P_{J/\psi}} = F_{J/\psi} \int d^2P_{c\bar{c}} dM_{c\bar{c}} \frac{d\sigma_{c\bar{c}}(M_{c\bar{c}},P_{c\bar{c}})}{dM_{c\bar{c}} d^2P_{c\bar{c}}} \delta^3(\vec{P}_{J/\psi} - \frac{M_{J/\psi}}{M_{c\bar{c}}} \vec{P}_{c\bar{c}}),$$ (2.2)

where $F_{J/\psi}$ is the probability of the $c\bar{c} \to J/\psi$ transition which is fitted to the experimental data, $M_{J/\psi}$ (or $M_D$) is the mass of $J/\psi$ (or $D$) meson and $M_{c\bar{c}}$ is the invariant mass of the $c\bar{c}$-system. Using the momentum relation

$$\vec{P}_{J/\psi} = \frac{M_{J/\psi}}{M_{c\bar{c}}} \vec{P}_{c\bar{c}}, \quad \text{where} \quad \vec{P}_{c\bar{c}} = \vec{p}_c + \vec{p}_{\bar{c}},$$ (2.3)

one can easily calculate also rapidity of $J/\psi$-meson.
III. NUMERICAL RESULTS

Let us start our presentation of numerical results with the discussion of open charm meson production. Here, we concentrate ourselves on the LHCb open charm data at $\sqrt{s} = 7$ TeV in $pp$-scattering [28]. A good description of these data will be a starting point for a construction of a theoretical framework for $J/\psi$-meson production within the color evaporation model at the given collision energy. In both cases, in the numerical calculations here we follow the $k_T$-factorization approach.

![Distribution of D meson in rapidity and transverse momentum](image)

**FIG. 1:** Distributions in rapidity and transverse momentum of $D$ meson for $\sqrt{s} = 7$ TeV obtained within the $k_T$-factorization realization of the color evaporation model with the KMR and the JH2013 UGDFs.

In Fig. 1 we present results of our calculations for charged $D$-meson production in the LHCb kinematical range of $0 < p_T < 8$ GeV and $2 < y < 4.5$. The left and right panels show the meson rapidity and transverse momentum distributions, respectively. The $p_T$-distributions are shown for different rapidity bins as specified in the figure. The
top panels present the default $k_T$-factorization result for the three different UGDFs: KMR (solid lines), JH2013 set1 (dotted lines) and JH2013 set2 (dashed lines). Each of them leads to overestimated rapidity distribution with respect to the LHCb data. The overestimation is related to small-$p_T$ region (see the right-top panel). The same conclusion is relevant for the two first bins in transverse momentum. This is the region which is very sensitive to the small-$k_T$ behavior of the UGDF. This uncertain nonperturbative regime of the UGDFs is still not under theoretical control and can be treated only in a phenomenological way. Different models follow various theoretical assumptions and lead to quite different results e.g. for charm production cross sections (see Ref. [15]). The situation looks much better for intermediate and larger transverse momenta of $D$ meson, where all of the applied UGDFs give a very good description of the data points in the considered $p_T$-range. A small discrepancy between the data and the result of the KMR UGDF appears at larger $p_T$’s when moving to the forward rapidity region. This can be related to a special construction of the model of UGDF which will be discussed shortly in the end of this section. We observe that in this kinematical domain the JH2013 set2 of unintegrated gluon density leads to a better behavior of the predicted cross section. It might be related to the fact that set2 of the JH2013 UGDF includes the LHC charm data in the fitting procedure, while set1 does not. It is set1 which was used in recent calculation of $J/\psi$ distributions in Ref. [10].

In the bottom panels of Fig. 1 we illustrate how the description of the LHCb charm data can be improved within the $k_T$-factorization approach by inclusion of the saturation effects in the unintegrated gluon density in proton. We follow here an useful and pragmatic prescription of these effects as presented in Ref. [5]. We correct the default KMR unintegrated gluon distribution function by assuming its saturation as follows: $F_g(x,k_t^2) = F_g(x,Q_s^2)$ for small initial gluon transverse momenta $k_t^2 < Q_s^2$, where the saturation scale $Q_s^2(x) = Q_0^2 \cdot (x_0/x)^{\lambda}$. The three free parameters $Q_0, x_0$ and $\lambda$ are fitted to the LHCb charm data. As a result, we get much better description of the LHCb data for both, the rapidity and the transverse momentum distributions. The saturation effect is visible in the region of $0 < p_T < 2$ GeV and seems to be of similar size in each of the considered bins of meson rapidity.

Let us now go to the main results of the present paper, i.e. to theoretical predictions for $J/\psi$-meson production within the improved color evaporation model in the version based
on the $k_T$-factorization approach. For consistency, in the numerical calculations here we keep all the details and parameters as in the case of $D$-meson discussed above.

![Graphs showing distributions in rapidity and transverse momentum of $J/\psi$ meson](image)

**FIG. 2:** Distributions in rapidity and transverse momentum of $J/\psi$ meson for $\sqrt{s} = 7$ TeV obtained within the $k_T$-factorization realization of the color evaporation model for different UGDFs.

In the left and right panel of Fig. 2 we show the $J/\psi$-meson rapidity and transverse momentum distributions, respectively, together with the ALICE [29, 30] and the LHCb data [31]. Here we present results for the default KMR (solid lines) and for the JH2013 set2 (dashed lines) UGDFs. In this calculation the model parameter $P_{J/\psi}$ was fixed to 0.031 to describe the ALICE data at midrapidities. Our model rapidity distribution is broader than the experimental one. Similar situation was observed in Ref. [5] where standard approach for $J/\psi$-meson production cross section was used. There it was interpreted as due to gluon saturation at small values of longitudinal momentum fraction. As already discussed this effect is not explicitly included when using the default KMR or JH2013 UGDFs. The same idea can also be the explanation for the color evaporation model approach. There are visible differences between the predictions of the KMR and the JH2013 set2 UGDFs. In principle, one could fit both rapidity distributions with the same quality taking different values of $P_{J/\psi}$. However, the calculated transverse momentum distributions differs very strongly and the LHCb data prefers the result with the modified KMR UGDF. The distributions obtained with the JH2013 set2 have completely different $p_T$-slope than the experimental one and falls down much faster. This observation is consistent with the results presented in Ref. [10]. There, this behavior of the $p_T$-distributions was, in our opinion, correctly recognized as a consequence of treatment of the $k_t > \mu$
region in the UGDF. The KMR model includes this contribution explicitly.

It was shown in Ref. [15] that the KMR and CCFM-based UGDFs lead to significant differences in correlation observables for $c\bar{c}$-pair, e.g. in $D\bar{D}$ invariant mass and/or azimuthal angle distributions. In the case of the CCFM-based unintegrated gluon distributions, an improved description of correlation observables can be obtained once the higher-order process of gluon-splitting is taken into account in an explicit way [32]. Just to illustrate the differences we show in Fig. 3 two-dimensional distributions as a function of transverse momentum of initial gluon $k_t$ and transverse momentum of the meson $p_T^{J/\psi}$. The latter observable, within the color evaporation model, in fact represents a modified transverse momentum of the $c\bar{c}$-pair. This variable is very strongly correlated with the gluon transverse momentum $k_t$. We observe, that the KMR and the JH2013 UGDFs provide very different results. In the case of the KMR, we get long tails in gluon $k_t$ that allows for larger $p_T^{J/\psi}$’s (see the left panel). On the other hand, in the JH2013 model, $k_t$ distribution drops down very quickly and therefore production of $J/\psi$-meson with larger $p_T$’s is strongly limited (see the right panel).

![FIG. 3: The double-differential cross section as a function of transverse momentum of initial gluon $k_t$ and transverse momentum of the meson $p_T^{J/\psi}$ for the KMR (left) and for the JH2013 set2 (right) UGDFs.](image-url)

Now let us take into account the conclusions from the discussion of $D$-meson production from the beginning of this section to the case of $J/\psi$-meson production. As we can see from Fig. 4 the inclusion of the saturation effects improves the shape of the rapidity distribution. Adjusting the normalization via modifying $P_{J/\psi}$ allows to nicely describe
the LHC data. It also improves the shape of the transverse momentum distributions in the region $0 < p_T^{J/\psi} < 4$ GeV but still does not lead to a perfect description of the data points at larger-$p_T$’s, especially in the region of the forward rapidities.

Finally, we want to shortly discuss our idea how one could try to improve the shapes of the $p_T$-distributions of $D$-meson (and in consequence also of $J/\psi$-meson) at larger rapidities. We propose that one should include in the kinematics the fact that with the KMR UGDF one do has an additional hidden hard emissions (jet or two-jets) by its construction. What we suggest is rather pragmatic: we add into the calculations a special condition $x_1 + x_2 + \tilde{x}_1 + \tilde{x}_2 < 1$, where $\tilde{x}_1 = \frac{k_T}{\sqrt{s}} \exp(y_{\text{max}} + \delta y) + \frac{k_T}{\sqrt{s}} \exp(y_{\text{min}} - \delta y)$ and $\tilde{x}_2 = \frac{k_T}{\sqrt{s}} \exp(-y_{\text{max}} - \delta y) + \frac{k_T}{\sqrt{s}} \exp(-y_{\text{min}} + \delta y)$. Above $y_{\text{min}} = \min(y_1, y_2)$ and $y_{\text{max}} = \max(y_1, y_2)$ where $y_1$ and $y_2$ are rapidities of $c$ and $\bar{c}$, respectively. The free parameter $\delta y$ is fitted to the LHCb open charm data. It provides a separation in the rapidity space between the central system and additional hard emissions from the UGDF. Unfortunately, this parameter is energy-dependent. Correcting effectively $x$-values for extra hidden emissions, not taken explicitly in the $k_T$-factorization, improves the shapes a bit. It works in the way as in the LHCb data (see Fig. 5) but it looks that this correction is, however, not sufficient. We need a bigger effect for the $J/\psi$-meson than in the case of open charm (please, compare the left and the right panel).
FIG. 5: Transverse momentum distribution of $D$ (left) and $J/\psi$ (right) mesons for the LHCb experiment for $\sqrt{s} = 7$ TeV. The standard $k_T$-factorization calculations with the KMR UGDF (solid) are shown together with the results of the calculations with the special condition for longitudinal momentum fractions.

IV. CONCLUSIONS

In the present paper we have discussed how to extend color evaporation model for production of $J/\psi$ meson to be used in the framework of $k_T$-factorization approach for production of $c$ and $\bar{c}$ pairs. The same was done independently very recently in [10]. We have included recent developments proposed recently in the literature. In our calculations we have used the KMR unintegrated gluon distributions which allows to describe the single $D$-meson distributions as well as meson correlation observables.

Rapidity and transverse momentum distributions of $J/\psi$ mesons have been calculated and the normalization factors, being a probability of $c\bar{c}$ soft transition to color singlet $S$ wave quarkonium, have been obtained. We could not describe all the world data with exactly the same normalization.

Inspired by our earlier work [5] we have tried to include saturation effects that modify the KMR UGDF for small values of longitudinal momentum fractions. Including such an effect improves agreement of the $J/\psi$ distributions as well as those for $D$ meson production.

A still better agreement for the transverse momentum distributions can be achieved by including explicitly hidden, in the $k_T$-factorization with the KMR UGDF, emissions in calculating $x$ values for UGDFs.
In summary, a reasonable description of the data for $J/\psi$ can be achieved in the color evaporation model supplemented for the $k_T$-factorization approach for production of $c\bar{c}$ pairs. Simultaneously a rather good description of the $D$ meson production is achieved with the same UGDF.

A relatively good description obtained within the improved color evaporation model does not proof that we have understood the underlying mechanism. A similar quality of agreement with experimental data can be obtained within the $k_T$-factorization npQCD approach applied directly to $J/\psi$ production [5]. Production of pairs of quarkonia may provide further tests.

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