SCALING REGION MESON PHOTOPRODUCTION
AND ELASTIC FORM FACTORS OF HADRONS

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ABSTRACT

First, I define generalized Bloom-Gilman duality and Bjorken-like scaling for inclusive photoproduction of pions, relating Quark-Parton Model description in the deep-inelastic region to the properties of exclusive resonance excitation. Secondly, connection between inclusive and exclusive processes is established in the formalism of Nonforward Parton Densities used here to predict deviations from dimensional scaling for a variety of observables: nucleon and pion elastic form factors, and pion Compton scattering.

1. Extending Bloom-Gilman Duality to Meson Photoproduction

For inclusive lepton nucleon scattering, $N(l,l')X$, Bloom-Gilman duality is a relation between the deep-inelastic scattering region and the nucleon resonance region. It may be formulated as follows: At large $Q^2$, photoproduction cross sections of hadronic states with a fixed invariant mass may be adequately described in two physically equivalent ways, a) as an integral of structure functions continued from the scaling region and b) as a sum of squared form factors of nucleon resonance excitation. In other words, scaling structure functions on average give a true curve of resonance excitation, and this relation remains valid as $Q^2$ increases. Duality shows that the fundamental single quark QCD process is decisive not only in the deep-inelastic regime, but also in setting the scale of the reaction in the resonance region.

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Let us extend duality studies to the process of inclusive pion (or other meson) photoproduction, $\gamma + N \rightarrow \pi + X$, where the hadronic state $X$ may be either within or above the resonance region. In the latter case and high transverse momenta of the produced pion, one may apply a Quark-Parton Model for the single quark process, see Ref.\textsuperscript{2} and references therein.

For high enough photon energies, isolated pions directly produced via the mechanism of Fig.1 dominate as one approaches the kinematic upper limit, i.e., the resonance region.

![Fig. 1. Direct pion photoproduction off a quark via gluon exchange. There are four diagrams total, corresponding to the four quark lines the photon may couple to.](image)

For this process, one may define a scaling variable $x$ in terms of the Mandelstam variables,

$$x = \frac{-t}{s + u - 2m_N^2}, \quad 0 \leq x \leq 1. \quad (1)$$

This new variable is a precise analog of $x_{\text{Bjorken}}$ in lepton scattering, i.e., a fraction of the target momentum carried by the struck quark. The value $x = 1$ corresponds to the exclusive limit of pion photoproduction ($m_X = m_N$).

The modified scaling variable, a necessary step in derivation of duality, is defined by analogy with Bloom’s and Gilman’s proposal, with $-t$ replacing $Q^2$:

$$\omega' = \frac{1}{x'} = \frac{1}{x} + \frac{m_X^2}{-t}. \quad (2)$$

The duality relation then may be written as an $\omega'$-integral of the differential cross section $d\sigma/dxdt(\gamma N \rightarrow \pi X)$ and in the region of the direct process (Fig.1) dominance reads

$$\int_{1-m_N^2}^{1-x^2} d\omega' \sum_q G_{q/N}(\omega') \frac{d\sigma(\gamma q \rightarrow \pi q')}{dt} \propto \sum_R \frac{d\sigma}{dt}(\gamma + N \rightarrow \pi + R) \quad (3)$$
Summation in the r.h.s. of Eq.(3) is done over all resonances $R$ with masses $m_R \leq M$, with the nucleon final state included. If the parton distribution function of the nucleon is $G_{q/N}(\omega') \approx (\omega' - 1)^3$ at $\omega' \to 1$ and the subprocess $\gamma q \rightarrow \pi q'$ cross section is determined by the one-gluon exchange mechanism of Fig.1, then duality (3) requires that the resonance excitation cross section $d\sigma/dt(\gamma + N \rightarrow \pi + R) \propto 1/s^7$ at fixed $t/s$ - the result known from the constituent counting rules.

Presence of hard gluon exchange (Fig.1) indicates that one needs sufficiently high energies to apply pQCD formalism here. However, since only one pion distribution amplitude is involved for the direct process, if the photon attaches to the produced $qq'$ pair in Fig.1, average virtuality of the gluon in question corresponds to the one determining pion electromagnetic form factor at $Q^2 \approx 20(35)$ GeV$^2$ scale, for the asymptotic (Chernyak-Zhitnitsky) pion distribution amplitude assuming CEBAF energy of 12 GeV, pion emission angle of $22^\circ$ and $m_X=2$ GeV (see Ref. for details). Therefore one may hope to observe a single-gluon exchange, which is a higher-twist effect, in inclusive photoproduction of pions even at CEBAF energies generally considered not high enough to reach the perturbative QCD domain. Indications for presence of this direct pion production off a quark were obtained in $\pi N$ scattering (see Ref. for references and discussion). Experimental observation of this mechanism at CEBAF would be very important for our understanding of underlying mechanisms of exclusive and semi-exclusive reactions.

Testing scaling and duality in inclusive photoproduction of mesons requires coverage of the large-$x$ region, where the cross sections are rapidly falling as $x$ approaches 1. Therefore such a uniquely designed high-luminosity machine as CEBAF can do an excellent job in these duality studies.

2. Elastic Form Factors and Nonforward Parton Densities

Another way to relate inclusive and exclusive reactions may be realized through Nonforward Parton Densities introduced by Radyushkin (NFPD). For related studies, see Refs. Recently Radyushkin used the fact that NFPD satisfy certain sum rules giving elastic nucleon form factors (both vector and axial-vector) and produced an NFPD-based model of elastic nucleon form factors. Using standard parton densities measured in deep-inelastic lepton scattering and a Gaussian dependence of the proton light-
cone wave function on the transverse momentum, he obtained a good
description of the proton Dirac form factor adjusting only one parameter:
average transverse momentum carried by the quarks. The model, however
has anomalous behavior in the low-$Q^2$ region because massless current (not
constituent) quarks are considered in the calculations, therefore we will dis-
cuss here predictions for the form factors at $Q^2 \geq 1$ GeV$^2$, except for the
$Q^2 = 0$ point where normalization of NFPD is fixed.

In order to analyze behavior of the Pauli form factor, $F_2(Q^2)$ one needs
to construct a model of spin-flip NFPDs ($K_a(x,t)$). Assuming that spin-flip
NFPD has similar dependence on the transverse momentum as spin-nonflip
NFPD of Ref.\cite{Ref} and making an additional assumption that the anomalous
magnetic moment of the nucleon is determined only by valence quarks , we
are left only with one unknown ingredient of the model: the forward ($t = 0$)
limit of spin-flip NFPD, $k_a(x)$.

$$K_a(x,t) = k_a(x)e^{(1-x)t/\lambda_k}, \quad (4)$$

where $\lambda_k$ is a mass parameter. The distribution $k_a(x)$ cannot be observed in
deep-inelastic scattering, but may be accessed in Deeply Virtual Compton Scattering.

To build a model for $k_a(x)$, I make a pQCD-based assumption that
spin-flip deep-inelastic structure functions are suppressed at $x \to 1$ by an
extra factor of $(1 - x)^{2\delta h}$ compared to spin-nonflip ones, where $\delta h$ is the
magnitude of helicity flip. It results in an extra factor of $(1 - x)$ for spin-flip
NFPD. Another constraint comes about in the $x \to 0$ limit if one recalls that
spin-flip Regge amplitudes are suppressed at high energies, giving spin-flip
NFPD an extra factor of $x$. Based on these arguments, I studied two choices
of spin-flip NFPD: a) $k_a(x) = (1 - x)f_a(x)$ and b) $k_a(x) = x(1 - x)f_a(x)$.
Adjusting the parameter $\lambda_k$ to fit the data on the proton $F_2/F_1$ ratio, the
best description of experimental data from SLAC\cite{Ref} is obtained with the
choice (b) (at $\lambda_k^2 = 0.3$ GeV$^2$), confirming the Regge argument in the low-$x$
region.

Both magnetic and electric proton form factors appear to be quite close
to the phenomenological dipole fit. Swapping up- and down-quark dis-
tributions yields neutron elastic form factors without additional parameters
(Fig.3) and with reasonable agreement with experiment\cite{Ref}. The available
neutron form factor data provide even stronger evidence in favor of addi-
tional suppression of $k_a(x)$ at low $x$. Predictions for the electric neutron
form factor are also presented in Fig. 3. One can see that the predictions for the elastic nucleon form factors are very sensitive to the models of nucleon NFPD, namely, $F$ and $K$. Therefore Jefferson Lab experimental program of precise measurements of nucleon form factor is very important for obtaining constraints on NFPD.

The same method also allows one to compute axial-vector form factor of the nucleon, $F_A(Q^2)$. Assuming the same value of the transverse momentum parameter $\lambda^2 = 0.7 \text{ GeV}^2$ for polarized quarks as for unpolarized ones, and using a phenomenological parametrization of polarized parton distributions from deep-inelastic experiments, I found $F_A(Q^2)$ close to the dipole form with a dipole mass $M_A \approx 1.1 \text{ GeV}$, in reasonable agreement with neutrino-nucleon elastic scattering data.

Using NFPD for the pion in the form $\mathcal{F}_\pi(x,t) \propto \frac{(1-x)}{\sqrt{x}} \exp\left[\frac{(1-x)t}{4x\lambda^2}\right]$, 

Fig. 2. Proton form factors from NFPD.
where the $t \to 0$ limit is justified by the analysis of all available Drell-Yan and prompt photon $\pi N$ data, and taking $\lambda^2 = 0.7$ GeV$^2$, I obtained the result for the pion elastic form factor shown in Fig.4. It is close to the one predicted in QCD Sum Rules but has a different shape due to different $x$-dependence at low $x$, ($\propto 1/\sqrt{x}$ as required by Regge constraint vs. $\propto x$ in the QCD Sum Rules). Both models predict deviations from the constituent counting rules.

With this model of pion NFPD, I calculated cross sections for the pion Compton scattering, $\gamma\pi \to \gamma\pi$ (the method of calculation was similar to the one used in Ref.). The results, same for the neutral and charge pions (since both initial and final photons are assumed to couple to the same quark, which is justified at high energies), are shown in Fig. 5. One can see deviation from constituent scaling behavior as a function of the c.m.
scattering angle ($\theta_{c.m.}$) with a general trend: the smaller is $\theta_{c.m.}$, the slower the cross section falls with $s$. Only around $\theta_{c.m.} = 45^\circ$, the cross section falls approximately as $1/s^4$, as predicted by perturbative QCD, but this is accidental since in our model the reaction proceeds through the overlap of initial and final pion wave functions (described by NFPD), not caused by hard gluon exchange.

Both pion form factor and pion Compton scattering may be studied on a fixed nucleon target at CEBAF in a peripheral process such that a pion from a meson cloud surrounding the nucleon absorbs the most of the momentum of the incident (either real or virtual) photon. Pion form factor measurements are being analyzed after the first (lower energy) run of JLab-E-93-021. Pion (both real and virtual) Compton scattering may be probed at CEBAF with radiative pion photoproduction, $\gamma p \to \gamma\pi N$, taking advantage of the third-arm photon spectrometer to be installed at Hall A.

Future experiments at Jefferson Lab will provide precise tests of deviation from constituent scaling behavior of cross sections of exclusive electromagnetic processes involving pions and nucleons.
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