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Study of nucleonic matter with a consistent two- and three-body perturbative chiral interaction

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Abstract. We calculate perturbatively the energy per nucleon in infinite nuclear matter with a chiral N\textsuperscript{3}LO (next-to-next-to-next-to-leading order) two-body potential plus a N\textsuperscript{2}LO three-body force (3BF). The 3BF low-energy constants which cannot be constrained by two-body observables are chosen such as to reproduce the $A = 3$ binding energies and the triton Gamow-Teller matrix element. This enables to study the nuclear matter equation of state in a parameter-free approach.

1. Introduction

The study of infinite nuclear matter is a very interesting topic for its connections with the properties of other physical systems. As a matter of fact, for instance, the equation of state (EOS) of pure neutron matter (PNM) may be related to the features of supernova explosions and neutron stars, while the compressibility and the symmetry energy of symmetric nuclear matter (SNM) are linked to giant dipole excitations and to the neutron-proton radii in atomic nuclei, respectively. This is why nowadays a great effort is currently being made to theoretically study infinite nuclear matter using high-precision nucleon-nucleon (NN) potentials based on chiral perturbation theory (ChPT) [1–3], that may provide a clear link between nuclear physics and quantum chromodynamics (QCD) (see for instance [4–11]).

In the framework of ChPT, nuclear two- and many-body forces are generated on an equal footing [3, 12, 13]. As a matter of fact, most interaction vertices that appear in the three-nucleon forces (3NF) and in the four-nucleon forces (4NF) are present already at the two-body level. In this context, a major issue is the choice of the low-energy constants (LECs) related to the interaction vertices. The LECs corresponding to the two-nucleon-force (2NF) vertices are fitted...
to two-nucleon data, and consistency requires that these values are not changed for the same vertices appearing in the 3NF, 4NF, ... As regards the other LECs, it is worth to note that the calculation of the EOS for pure neutron matter, with chiral 3NF up to N^2LO, depends only on constants that are fixed in the two-nucleon system. This comes out since, in this case, the contact interaction, \( V_E \), and the 1π-exchange term, \( V_D \), that appear in the N^2LO three-body force, vanish [14]. This is not the case in SNM, whose EOS calculation is influenced by the intermediate-range 1π-exchange component \( V_D \) and the short-range contact interaction \( V_E \) of the 3NF. Therefore, the LECs of \( V_E \) and \( V_D \) (known as \( c_E \) and \( c_D \)), which are not constrained by two-body observables, affect the calculation of the ground-state energy of SNM. Their values should be fixed taking into account only \( A \leq 3 \) observables, to avoid biases induced by additional many-body contributions. Since the reproduction of the observed \( A = 3 \) binding energies is not sufficient to fully constrain both \( c_D \) and \( c_E \), recently, the attention has been focused on the fitting of the triton half-life, specifically the Gamow-Teller matrix element [15, 16], as first suggested by Gärdestig and Phillips [17].

In the present paper we study the EOS of both SNM and PNM employing two- and three-nucleon chiral forces with consistent LECs, aiming to ascertain the possibility to obtain realistic nuclear matter predictions with chiral interactions constrained by the properties of the two- and the three-nucleon systems without any additional adjustment. We employ a chiral potential with a cutoff parameter \( \Lambda = 414 \) MeV [18] and calculate, including 3NF effects, the energy per nucleon for infinite nuclear matter at nuclear densities in the framework of many-body perturbation theory. It is worth pointing out that this \( NN \) potential has been already employed in perturbative many-body calculations for finite nuclei providing a successful reproduction of their spectroscopic properties [18, 19]. The effects of the N^2LO 3NF are taken into account via a density-dependent two-body potential \( V_{NNN} \) obtained by summing one nucleon over the filled Fermi sea [20, 21], and that is added to the chiral N^3LO potential \( V_{NN} \). The LECs \( c_D \) and \( c_E \) of the N^2LO chiral three-nucleon force are fixed so to reproduce to the binding energies of \( A = 3 \) nuclei and the \(^3\text{H}-^3\text{He} \) Gamow-Teller matrix element.

The paper is organized as follows. In section 2, we briefly describe the procedure we have followed to choose the LECs of the 1π-exchange term \( V_D \) and contact interaction \( V_E \), and the perturbative calculation of the properties of infinite nuclear matter is outlined. Our results and some concluding remarks are presented in sections 3 and 4, respectively.

2. Outline of calculations

2.1. Chiral potential

In the last decade \( NN \) potentials that are able to reproduce accurately the \( NN \) data have been derived in the framework of chiral perturbation theory (ChPT) [1–3]. In this work, we employ a chiral N^2LO \( NN \) potential with a cutoff \( \Lambda = 414 \) MeV (published in Ref. [18]), and using \( n = 10 \) in the regulator function \( f(p', p) = \exp[(-(p'/\Lambda)^2n - (p/\Lambda)^2n)] \), i.e., a smooth, but rather steep cutoff function is applied.

In addition to this 2NF, we consider the contributions of a N^2LO 3NF. At this order the chiral 3NF is built up by three terms: a two-pion exchange term, a one-pion exchange plus a 2N-contact interaction, and a pure 3N-contact interaction. The two-pion exchange 3NF contains only LECs that are already present in the \( NN \) potential, while the last two terms

\[
V_D = -\frac{c_D}{f^2\Lambda} \frac{g_A}{8f^2} \sum_{i\neq j \neq k} \frac{\delta_j \cdot \vec{q}_j}{q_j^2 + m^2} (\tau_i \cdot \tau_j)(\delta_i \cdot \vec{q}_i)
\]

and

\[
V_E = \frac{c_E}{f^2\Lambda} \frac{1}{2} \sum_{j \neq k} \tau_j \cdot \tau_k .
\]

\( V_E \)

\( V_D \)
involves two new parameters $c_D$ and $c_E$, which do not appear in the 2N problem. There are many ways to fix these two parameters. In the present work we have adopted a procedure that has been recently introduced to constrain $c_D$ and $c_E$ [15–17]. This procedure is based on the observation that the LEC $c_D$ appears also in a two-nucleon contact term in the $NN$ axial current operator derived in chiral EFT up to $N^2$LO. Therefore, $c_D$ can be fixed to reproduce the accurate experimental value of the triton $\beta$-decay half-life, and in particular of its Gamow-Teller component (GT). More precisely, we have first calculated the $^3$H and $^3$He wave functions within the hyperspherical harmonics method (see Ref. [22] for a review), using the chiral 2NF plus 3NF presented above. The LECs $c_D$ and $c_E$ are then determined by fitting the $A = 3$ experimental binding energies and the observed triton GT value. The values obtained are: $c_D = -0.40$ GeV$^{-1}$ and $c_E = -0.07$ GeV$^{-1}$.

2.2. Nuclear matter calculations

We calculate the ground-state energy (g.s.e.) per nucleon of infinite nuclear matter within the framework of many-body perturbation theory, expressing the energy as a sum of Goldstone diagrams up to third order in the interaction.

In order to take into account the effects of the $N^2$LO 3NF, a density-dependent two-body potential $V_{NNN}(k_F)$ is obtained by summing one nucleon over the filled Fermi sea and added to the chiral $N^3$LO potential $V_{NN}$. It is worth pointing out that, in the evaluation of the diagrams, the matrix elements of $V_{NNN}(k_F)$ have been multiplied by a factor 1/3 in the first-order Hartree-Fock (HF) diagram, and by a factor 1/2 in the calculation of the self-consistent single-particle energies. This is done to take care of the correct combinatorial factors of the normal-ordering at the two-body level of the 3NF [14].

In figure 1 we report the diagrams we have included in our calculation with $V_{NN}$ and $V_{NNN}(k_F)$ vertices. The contribution of the third-order particle-hole (ph) diagram (diagram (e) in figure 1), whose calculation is cumbersome, has been included, at present, only for the SNM EOS.

![Diagram](image)

**Figure 1.** First-, second-, and third-order diagrams of the Goldstone expansion included in our calculations.

The analytic expressions of first-, second-, and third-order particle-particle ($pp$) and hole-hole ($hh$) contributions may be found in Ref. [23], while the implicit expression of the third-order $ph$ diagram is reported in Ref. [24].

Padé approximants give an estimate of the value to which a perturbative series may converge [25], therefore we have calculated the [2|1] Padé approximant [25]

$$E_{[2|1]} = E_0 + E_1 + \frac{E_2}{1 - E_3/E_2},$$

(3)
\( \mathcal{E}_i \) being the \( i \)th order energy contribution in the perturbative expansion of the g.s.e., in order to study its convergence properties.

### 3. Results

As mentioned in the previous section, we calculate the energy per particle of infinite nuclear matter in the framework of many-body perturbation theory, including contributions up to third-order in the interaction. Therefore, it is worth studying the convergence of the g.s.e. perturbative expansion.

**Figure 2.** (Color online) PNM energy per particle. The first, second, and third order in the perturbative expansion and the Padé approximant \([2|1]\) are shown as a function of density.

In figure 2 and 3 we show, respectively, the PNM and SNM EOS as a function of density, calculated at various orders in the perturbative expansion. From figure 2 it can be seen that the PNM energy per nucleon calculated at second order is almost indistinguishable from the one computed at third order and consequently from the \([2|1]\) Padé approximant for the whole range of density considered. As regards the SNM, we see from figure 3 that the EOS calculated at second order does not differ much from the one computed at third order, the latter being placed almost on the top of the \([2|1]\) Padé approximant. This is a clear indication that the adopted chiral N\(^3\)LO \(NN\)+N\(^4\)LO \(NNN\) potential has a satisfactory perturbative behavior for both PNM and SNM calculations.

The calculated PNM and SNM EOS and the corresponding symmetry energy are shown in figure 4 and 5, respectively. The inspection of these figures shows how our results reproduce well the empirical SNM saturation point and the value of the symmetry energy at saturation density, the latter quantity being related to the isospin dependence of nuclear forces.
**Figure 3.** (Color online) Same as in figure 2, but for the SNM energy per particle.

**Figure 4.** (Color online) Results obtained for the PNM (red line) and the SNM (black line) energy per nucleon at third-order in perturbation theory.
Two other interesting physical quantities related to PNM and SNM at saturation density are the incompressibility $K_0$ and the slope of the symmetry energy $L$.

The empirical value of the SNM incompressibility is determined from experimental data on Giant Monopole Resonance in finite nuclei, at present its accepted value is $K_0 = 230 \pm 30$ MeV [26]. From our calculations we obtain a compressibility $K_0 = 279$ MeV, a value that is very close to the empirical one.

The quantity $L$ determines most of the behavior of the symmetry energy in the proximity of saturation. Its empirical determination - centered at $L = 70$ MeV - is indirect and mainly based on the analysis of heavy-ion collisions at intermediate energies and nuclear structure measurements [27]. Our calculated value for $L$ is 68 MeV, in good agreement with the empirical value.

4. Concluding remarks
In this paper we have studied the properties of infinite nuclear matter employing a chiral potential. This has been done within the framework of the perturbative Goldstone expansion, using a chiral $N^3$LO $NN$ and $N^2$LO $NNN$ potential with a sharp cutoff $\Lambda = 414$ MeV. The LECs involved in the potential have been chosen consistently for the two- and three-body components, and in particular the 3NF LECs $c_D$ and $c_E$ have been fixed as to reproduce the experimental $A = 3$ binding energies and Gamow-Teller matrix element in triton $\beta$-decay. Our results for PNM and SNM EOS turn out to be in good agreement with the empirical properties of infinite nuclear matter. The ability to provide realistic nuclear matter predictions employing (consistent) two- and three-body interactions whose LECs are constrained by the properties of the two- and the three-nucleon systems is the main outcome of our study, and this is certainly a very important point that should be further investigated.

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