TRANSIT AND RADIAL VELOCITY SURVEY EFFICIENCY COMPARISON
FOR A HABITABLE ZONE EARTH

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ABSTRACT

Transit and radial velocity searches are two techniques for identifying nearby extrasolar planets to Earth that transit bright stars. Identifying a robust sample of these exoplanets around bright stars for detailed atmospheric characterization is a major observational undertaking. In this study we describe a framework that answers the question of whether a transit or radial velocity survey is more efficient at finding transiting exoplanets given the same amount of observing time. Within the framework we show that a transit survey’s window function can be approximated using the hypergeometric probability distribution. We estimate the observing time required for a transit survey to find a transiting Earth-sized exoplanet in the habitable zone (HZ) with an emphasis on late-type stars. We also estimate the radial velocity precision necessary to detect the equivalent HZ Earth-mass exoplanet that also transits when using an equal amount of observing time as the transit survey. We find that a radial velocity survey with $\sigma_{rv} \sim 0.6 \, m \, s^{-1}$ precision has comparable efficiency in terms of observing time to a transit survey with the requisite photometric precision $\sigma_{phot} \sim 300 \, ppm$ to find a transiting Earth-sized exoplanet in the HZ of late M dwarfs. For super-Earths, a $\sigma_{rv} \sim 2.0 \, m \, s^{-1}$ precision radial velocity survey has comparable efficiency to a transit survey with $\sigma_{phot} \sim 2300 \, ppm$.

Key words: eclipses – methods: statistical – planetary systems – surveys – techniques: photometric – techniques: radial velocities

1. INTRODUCTION

Transiting extrasolar planets4 of bright host stars provide a unique opportunity for physical characterization in ways impossible for non-transiting planets that are also too close to their host stars for direct imaging. Currently, the transit and radial velocity methods dominate the growth in planet discoveries both overall and for planets suitable for detailed physical characterization.5

This paper addresses the question, “of these two methods, transits or radial velocities, which is more efficient at discovering planets that transit bright stars?” This is a complex question given the astrophysical effects (e.g., planet mass–radius relation, stellar mass–radius relation, stellar mass–luminosity relation, radial velocity jitter, stellar activity, etc.) and technical challenges (e.g., instrumentation, telescope aperture, available observing time, costs of operation, available funding opportunities, etc.) that influence the planet yield of a survey. To compare the performance of a radial velocity survey to that of a transit survey, we adopt a parametric model for each. With some simplifying assumptions, we quantify the precision of a radial velocity survey that will achieve the same level of search completeness as a transit survey using the same amount of observing time.

For any transit survey, the vast majority of observations of any particular star cannot contribute directly to the detection of a transit, simply because they are made when the planet is not transiting. To be successful, a transit survey must make a sufficiently large number of independent observations in order that the probability $p_{\geq k}$ of witnessing $k$ or more transit(s) matches or exceeds a design requirement. For example, the 4 yr Kepler mission was designed to witness $k \geq 3$ transits of Earth analogs with orbital periods $\leq 1$ yr (Borucki et al. 2010). For a specific set of observation times of a transit survey, the window function, $p_{\geq k}(P)$, describes the visibility of the repetitive transit signal of any given period, i.e., the planet’s orbital period, $P$ (Gaudi 2000). A survey’s window function can be computed numerically with high fidelity, either before or after the actual observations, by Monte Carlo simulations (Burke et al. 2006; von Braun et al. 2009).

However, analytic approximations to the window function are quicker and more convenient to compute than Monte Carlo simulations (Deeg et al. 2004; Beatty & Gaudi 2008; Fleming et al. 2008). In addition, an analytic approximation is readily inverted to predict the minimum observing time necessary to exceed a given probability $p_{\geq k}(P)$ of witnessing $k$ or more transits of a given $P$. We show in the Appendix that the hypergeometric distribution provides a high fidelity statistical approximation to the window function and its derivation gives physical insight into the design of a transit survey or interpretation of results from them. Most importantly we show how the transit survey window function is primarily dependent upon the number of independent observing epochs, which enables us to compare the performance of a transit survey to a radial velocity survey for detecting transiting planets.

For a radial velocity survey, the situation is quite different: nearly every measurement may contribute to the planet detection because the star’s radial velocity can be measured at any time and is constantly changing in response to the orbiting planet. (We write “nearly” every measurement because multiple observations made at the same epoch could be redundant if one would suffice at that particular orbital phase.) To estimate the detection sensitivity of a radial velocity survey, we adopt the prescription outlined by Cumming (2004).

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4 Hereafter, “planets” means “extrasolar planets.”

5 http://exoplanetarchive.ipac.caltech.edu
Our analysis ends at the phase of a survey when the significance of the detection reaches a suitable level such that follow-up observations to refine the ephemeris and confirm the detection are warranted. Thus, we neglect budgeting for the additional observational effort required to follow up on a significant signal as the effort required depends very strongly on the nature of the signal and remaining ambiguities of the signal that must be ruled out. In the case of a transit survey a detection from a single to few events leads to ambiguity in the orbital period (Stevenson et al. 2012; Berta et al. 2013) that can be difficult to reconcile with low signal-to-noise detections. The numerous sources of astrophysical false positives to the transiting planet signal need to be investigated (Brown 2003; Torres et al. 2004; O’Donovan et al. 2006b) along with statistical validation (Torres et al. 2011; Fressin et al. 2011; Barclay et al. 2013) or mass measurement confirmation (Charbonneau et al. 2009; Queloz et al. 2009; Batalha et al. 2011). In the case of a radial velocity detection, in addition to the photometric confirmation of a transit signal (Henry et al. 2000; Charbonneau et al. 2000), typically additional radial velocity observations are required to refine the ephemeris to a suitable level for scheduling the photometric observations (Gillon et al. 2007; Kane et al. 2009; Winn et al. 2011). Also, radial velocity detections must contend with the astrophysical false positives induced by stellar variability (Saar & Donahue 1997; Boisse et al. 2011; Gomes da Silva et al. 2012).

We apply our prescription to address two representative case studies. First, we compare our models of the performance of a particular transiting-planet finder, MEarth and its southern extension, MEarth-South (Nutzman & Charbonneau 2008; Irwin et al. 2011), to that of a particular radial velocity instrument, HARPS (Bonfils et al. 2013) and its northern twin, HARPS-N (Cosentino et al. 2012; Covino et al. 2013), for detecting a super-Earth transiting at the inner edge of the habitable zone (HZ) as a function of stellar mass. In the second case, we study the relative performances of these planet finding techniques for finding Earth-sized planets transiting at the inner edge of the HZ of M dwarf stars. In each case, we use the inner edge of the HZ simply because planets there will have shorter periods than any other planets in their respective HZs and hence will be more easily detected by either method.

This paper is organized as follows. In Section 2 the transit survey window function is introduced, and in the Appendix we derive analytic approximations to it. This leads to simple approximations for the observing time required to reach a specified level of completeness of a transit survey. The detection sensitivity of a radial velocity survey is inverted in Section 3 in order to determine the observing time required for it to reach an appropriate level of completeness to compete with the transit survey. The implications of this study as applied to the detection of a transiting super-Earth and an Earth analog at the inner edge of the HZ in Section 4 and its conclusions are given in Section 5.

2. TRANSIT SURVEY PERFORMANCE

We can approach the statistical modeling of a transit survey window function in two ways. We can ask

(A) each time the planet transits, were we observing?
or

(B) each time we observe, is the planet transiting?

For our statistical model, we want independent trials, so to form a trial, we group observations over an appropriate time interval. Previous work (e.g., Equation (A2) of Deeg et al. 2004) adopted approach A, where each independent planetary orbital period interval is a binomial trial with probability of our witnessing it equal to the average fraction of time spent observing $f_{\text{cov}}$. In approach B (see the Appendix), the appropriate independent time interval is the transit duration,

$$t_{\text{dur}} = 0.058 \left( \frac{M_\star}{M_\odot} \right)^{-1/3} \left( \frac{R_\star}{R_\odot} \right) \left( \frac{P}{1 \text{ day}} \right)^{1/3} \text{[day]},$$

for arbitrary stellar mass, $M_\star$, radius, $R_\star$, and orbital period, $P$. We have assumed the average relative chord length, $\pi/4$ and eccentricity $e = 0$ (Gilliland et al. 2000; Seager & Mallén-Ornelas 2003). Effects of non-zero eccentricity on transit survey yield are addressed by Barnes (2007) and Burke (2008). Approach B has the advantage of being readily related to individual observations parcelled into lengths of the transit duration. The window function is determined by the number of unique observing epochs, $n$, of duration $t_{\text{dur}}$. A set of observation exposures at a cadence shorter than the transit duration will improve the signal-to-noise ratio of the detection, but count as a single parcel increment to $n$ for the purposes of the window function. Thus, $n$ is not necessarily the number of exposures (see Section A.3).

Regardless of approach (A or B), the physical situation is the same: in the time span, $T_{\text{obs}}$, between the first and last observations of a particular star with a transiting planet of orbital period, $P$, the expected number of transits is\footnote{6 We are explicitly separating out the dependence of transit detection on the window function and signal-to-noise ratio (Gaudi 2000).
7 There are either $M = \lfloor T_{\text{obs}}/P \rfloor$ or $M = \lfloor T_{\text{obs}}/P \rfloor + 1$ transits where $\lfloor \cdot \rfloor$ is the truncate to nearest integer operation. For uniform and random relative phase of the planetary transits with respect to the set of observation epochs, the expectation value of $(M) = T_{\text{obs}}/P$.}

$$M = \frac{T_{\text{obs}}}{P}. \quad (2)$$

From the probability, $p_k$, of witnessing exactly $k$ transits (derived in the Appendix), the probability, $p \geq k$, of witnessing $k$ or more transits is given by

$$p \geq k = 1 - \sum_{i=0}^{k-1} p_k. \quad (3)$$

A transit survey may require detection of $k \geq 2$ transits in order to know the period and hence predict future transits, or it may require $k \geq 3$ transits in order to validate the detection, i.e., confirm that the times separating the transits are all consistent with a particular orbital period.

In the Appendix we have isolated the mathematics of approximating the window function analytically. Figure 1 shows the window function for a hypothetical transit survey that operates in a mode dedicated to a single field for 30 consecutive days with observations for 8 hr each night. The specifications for the hypothetical transit survey are the same as presented in Figure 1 of Beatty & Gaudi (2008) to demonstrate their analytical representation of a transit survey and is typical for the first generation of ground-based hot-Jupiter transit surveys (Hungarian Automated Telescope (HAT)—Bakos et al. 2007; Kilodegree Extremely Little Telescope (KELT)—Beatty et al. 2012; Optical Gravitational Lensing Experiment—Konacki et al. 2003; Qatar Exoplanet Survey (QES)—Alsubai et al. 2011; Trans-Atlantic Exoplanet Survey (TrES)—Alonso et al. 2004; Wide Angle Search for Planets (WASP)—Collier Cameron et al. 2007; XO—McCullough et al. 2006; and others).
Figure 1. Left: exact numerical window function for a hypothetical ground-based transit survey that observes for 30 consecutive days with observations for 8 hr each night and requires two transits for detection (dashed black line). Average of the numerical window function using a one day moving average (solid black line) compared to the hypergeometric model (solid red line) from this study along with its Poisson model limit (dash blue line). The window function approximation of Beatty & Gaudi (2008; solid blue line) is also shown. Right: same as left panel, but requiring three transits for detection.

The window function presented in this study and others from the literature (e.g., Deeg et al. 2004; Beatty & Gaudi 2008) provide convenient approximations to the average performance of a transit survey. These approximations may be useful for design purposes and for real time updates to a transit survey in progress. Also, this study articulates that transit survey completeness, in terms of the window function, derives primarily from the number of distinct observing epochs, \( n \), and only secondarily from the time span \( T_{\text{obs}} \) (see Figure 2 and discussion in Section A.5).\footnote{In general, \( n \leq \) the number of exposures (see Section A.3).}

That is, observations can be spread over an observing season or years rather than concentrated in a single observing run. A consequence is that a longitudinally distributed network (e.g., HAT-Net) is not necessarily more observationally efficient at finding planets per unit of observing time than a single site (e.g., WASP), all other factors being equal. However, a longitudinally distributed network of \( N \) telescopes can retire a particular field of view of targets in a smaller number of days than could be accomplished by the same \( N \) telescopes located at a single site at the same latitude as the telescopes of the network. Also, continuous observations avoid the ambiguities (e.g., of orbital period) inherent in interpreting observations with many gaps where transits can hide.

Figure 3 shows the window function for a transit survey that operates in a mode cycling between targets such as employed...
by the MEarth project (Nutzman & Charbonneau 2008; Berta et al. 2013). MEarth surveys the brightest M dwarfs by cycling between targets to obtain several observation per target per night. The observation span is an observing season for each target. We simulate this mode of operation through specifying a time series with \( n = 200 \) observations that is expected for a target from a single observatory over the course of an observing season including losses due to weather. As a proxy for an observing time series we begin with a continuous precipitable water vapor (PWV) time series downloaded from the NOAA Earth System Research Laboratory Ground-Based GPS Meteorology program\(^9\) (Wolfe & Gutman 2000) for 9 months starting 2008 September 1 with a 30 minute cadence from Flagstaff, AZ. We use the celestial coordinates R.A. \(7^h\) decl. \(+30^\circ\), optimized for the middle of the observing season and observatory latitude, to find the airmass for each PWV observations. Only values when the airmass \(<1.8\), PWV \(<10\) mm (to crudely simulate when the weather conditions would be too poor for observations), and Sun altitude \(<-5^\circ\) are used to simulate times when observations could be made. The window function assumes stellar parameters appropriate for a mid M dwarf star \((M_\odot = 0.2\ M_\odot, [M_\odot]; R_\star = 0.26\ [R_\odot])\).

In addition to the window functions assuming at least \( k \geq 1, 2, 3 \), and 5 transit events are observed (solid lines from top to bottom, respectively), Figure 3 also shows the window function (solid orange line) which corresponds to the triggering mode of operation adopted by the MEarth project (Nutzman & Charbonneau 2008). Real-time analysis of the data identifies a potential flux decrement for a target, and the observing scheduler interrupts cycling through targets and enters a high cadence follow-up and stare mode in order to get a high sensitivity detection of the transit egress. This has the potential of detecting a planet with a single transit and relaxed photometric precision, which extends the detectability to longer orbital periods and enables sampling more targets. However, in practice for M dwarfs, the transit duration \(<\sim1\) hr and the transit must be observed during the first portion of the transit event to allow sufficient time to commence the high cadence staring mode. Thus, \( \tau_{\text{dur}} \) must be reduced to account for the triggering mode delay. Throughout, we reduce \( \tau_{\text{dur}} \) by \( \tau_{\text{neg}} = 30 \) minutes when simulating the triggering mode. For the mid M dwarf example shown in Figure 3, the triggering mode window function is superior to the \( k = 2 \) window function for \( P > 4 \) days.

### 3. RADIAL VELOCITY PERFORMANCE

In this section we summarize our metrics for the performance of a radial velocity survey. We follow the procedure by Cumming (2004) to determine the limiting semi-amplitude threshold, \( K \), of a radial velocity survey for a given \( n \) observations, each of which has radial velocity precision, \( \sigma_{rv} \). We use the performance of the Lomb–Scargle periodogram for detecting planetary orbits. For a given completeness probability, \( p_c \), we invert Equation (24) of Cumming (2004) to determine the argument of the error function,

\[
\Gamma = \text{erf}^{-1}(2p_c - 1). \tag{4}
\]

The error function argument, \( \Gamma \), relates the expectation of the periodogram in the presence of a signal,

\[
\langle z_s \rangle = \frac{(v/2)(K^2/2\sigma_{rv}^2)}{\nu}, \tag{5}
\]

where \( \nu = n - 5 \) is the degrees of freedom, to the detection threshold of the experiment,

\[
z_d = \frac{(v/2)(m/F)^{2/\nu} - 1}. \tag{6}
\]

where \( m \) is the number of independent frequencies searched and \( F \) is the false alarm probability of the experiment. The relationship between Equations (4), (5), and (6) is given by

\[
\langle z_s \rangle - z_d = 2\Gamma\sqrt{\langle z_s \rangle}. \tag{7}
\]

Equations (4)–(7) are taken directly from Cumming (2004). We numerically solve Equation (7) to derive the limiting \( K/\sigma_{rv} \) that a radial velocity achieves for a given \( p_c \) and \( n \). We adopt parameters following Cumming (2004) that the radial velocity survey has a \( F = 0.001 \) and that \( m \approx n \) (Horne & Baliunas 1986). An additional assumption for the validity of these results is that the baseline of observations is longer than the orbital period of the planet. In this study we ignore the impact of uncorrected stellar activity for setting a noise floor on radial velocity precision (Saar & Donahue 1997; Boisse et al. 2011; Gomes da Silva et al. 2012), and the additional observations necessary to refine the orbital parameters with sufficient precision to make an accurate prediction for the epoch of transit (Kane et al. 2009). When analyzing the identification of transiting planets from a radial velocity detected sample of planets in this paper, we assume a purely geometric prior transit probability rather than the potentially 10%–25% higher posterior transit probability for planets first detected with radial velocities (Stevens & Gaudi 2013).

### 4. DISCUSSION

With the results from the previous sections, we can evaluate a transit survey designed to detect a transiting planet and quantify the equivalent radial velocity survey that is expected to have the same detection capability for the same total amount of observing time. With the framework described in this paper, we assume both the radial velocity survey and the transit survey achieve \( n \) observations. We then determine which method using

\(^9\)http://www.gpsmet.noaa.gov
n (independent of how long it takes to achieve n observations) achieves a higher survey completeness level for the planet of interest. One opportunity provided by the James Webb Space Telescope (JWST) will be an unprecedented capability for measuring the atmospheric constituents of potentially habitable planets (Valenti et al. 2005; Deming et al. 2009). Taking advantage of the JWST opportunity requires identifying the most viable transiting planets for follow up by pushing the limits of photometric and radial velocity precision. We first compare the performances of the MEarth transiting survey (Nutzman & Charbonneau 2008; Berta et al. 2013) and the HARPS M dwarf radial velocity survey (Bonfils et al. 2013) for detection of super-Earth planets orbiting the brightest M dwarfs. Then we show general results for the design of future surveys pushing the detection limits down to Earth-sized planets. The steps for the comparison begin with determining the n necessary for a transit survey to reach a given percent completeness for an inner HZ planet. We then calculate the radial velocity precision required to detect the same planet with the same amount of observing time.

### 4.1. Bright M Dwarfs Hosting A Transiting HZ Super-Earth

An example we calculate the performance for a survey modeled after the MEarth transit survey (Nutzman & Charbonneau 2008; Berta et al. 2011). The MEarth survey is focused on identifying transiting super-Earths orbiting the brightest M dwarfs employing the triggering mode of detection along with ex post facto period-folding search for multiple transit events. Since MEarth is observing the brightest M dwarfs, which are far apart on the sky, it typically can observe only a single target per telescope pointing. In order to specify n required for a transit survey, we outline the properties of the planets and their stellar hosts in Table 1. For a given stellar mass, $M_\star$, the stellar radius, $R_\star$, stellar surface gravity log g, and stellar effective temperature, $T_{\text{eff}}$, are estimated by a spline fit to the calibration of MK spectral types Table 15.7 and 15.8 of Cox (2000). For the given stellar parameters, we determine the orbital period corresponding to the inner edge of the HZ, $P_{\text{HZ}}$, from the results of Kasting et al. (1993). Empirically for M dwarfs, we find that an equilibrium temperature $T_{\text{eq}}$ equal to 273 K occurs at the inner edge of the Kasting et al. (1993) HZ for a planetary bond albedo, $a = 0.3$, and full redistribution of the stellar flux, $f = 1$. Although a warmer $T_{\text{eq}} = 288$ K is a better match to the inner HZ edge of Kasting et al. (1993) for higher mass stars, $M_\star > 0.4 M_\odot$, we use the cooler $T_{\text{eq}} = 273$ K to define the inner HZ for all stellar types throughout this work.

With the stellar parameters and $P_{\text{HZ}}$ given, the additional quantities follow ($\Delta t_{\text{dur}}$, transit probability, $p_t$, radial velocity semi-amplitude, $K$, and transit depth, $\Delta$) for an Earth-mass and Earth-radius analog as well as a super-Earth mass and radius analog to GJ 1214 (Charbonneau et al. 2009; Berta et al. 2011).

### Table 1

Stellar, Transit, and Radial Velocity Parameters

| $M_\star$ ($M_\odot$) | $R_\star$ ($R_\odot$) | $T_{\text{eff}}$ (K) | $P_{\text{HZ}}$ (day) | $p_t$ | $\Delta$ | $\Delta S_{\text{eq}}$ (ppm) | $\Delta t_{\text{dur}}$ (hr) | $n^2$ | $n_{\text{avg}}$ | $n/p_t$ | $K_{\text{S\text{\textend{symbol}}}S_{\text{eq}}}$ (m s$^{-1}$) | $\sigma_{\text{rv}} S_{\text{eq}}$ b ($K_{\text{S\text{\textend{symbol}}}S_{\text{eq}}}$ (m s$^{-1}$)) | $K_{\text{S\text{\textend{symbol}}}S_{\text{eq}}}$ (m s$^{-1}$) | $\sigma_{\text{rv}} S_{\text{eq}}$ a (m s$^{-1}$) |
|----------------------|----------------------|---------------------|----------------------|------|---------|--------------------------|----------------|------|------|-------|---------------------------------|------------------|----------------|----------------------|
| 0.15 | 0.20 | 3077 | 10.47 | 0.0188 | 2167 | 15801 | 1.18 | 492 | 73 | 26139 | 6.83 | 2.69 | 1.035 | 0.81 |
| 0.17 | 0.22 | 3109 | 11.62 | 0.0184 | 1724 | 12568 | 1.29 | 472 | 76 | 25606 | 6.07 | 2.29 | 0.920 | 0.70 |
| 0.20 | 0.26 | 3155 | 13.61 | 0.0179 | 1266 | 9233 | 1.46 | 449 | 82 | 25086 | 5.17 | 1.85 | 0.783 | 0.58 |
| 0.22 | 0.28 | 3185 | 15.13 | 0.0176 | 1052 | 7671 | 1.59 | 436 | 87 | 24825 | 4.68 | 1.62 | 0.709 | 0.51 |
| 0.25 | 0.32 | 3230 | 17.75 | 0.0171 | 818 | 5967 | 1.82 | 421 | 96 | 24655 | 4.08 | 1.35 | 0.618 | 0.44 |
| 0.27 | 0.35 | 3261 | 19.75 | 0.0168 | 703 | 5127 | 1.99 | 413 | 102 | 24635 | 3.74 | 1.21 | 0.566 | 0.40 |
| 0.30 | 0.38 | 3310 | 23.14 | 0.0163 | 572 | 4171 | 2.26 | 404 | 114 | 24849 | 3.30 | 1.04 | 0.501 | 0.34 |
| 0.35 | 0.44 | 3404 | 29.86 | 0.0154 | 426 | 3110 | 2.75 | 404 | 139 | 26278 | 2.74 | 0.86 | 0.415 | 0.28 |
| 0.40 | 0.50 | 3520 | 37.76 | 0.0144 | 336 | 2449 | 3.26 | 418 | 170 | 29071 | 2.32 | 0.76 | 0.351 | 0.25 |
| 0.45 | 0.55 | 3661 | 46.32 | 0.0133 | 278 | 2025 | 3.69 | 442 | 204 | 33257 | 2.00 | 0.70 | 0.303 | 0.22 |
| 0.50 | 0.59 | 3811 | 55.12 | 0.0123 | 239 | 1745 | 4.06 | 473 | 240 | 38559 | 1.76 | 0.67 | 0.267 | 0.20 |
| 0.55 | 0.63 | 3952 | 64.07 | 0.0114 | 213 | 1555 | 4.38 | 503 | 276 | 44098 | 1.57 | 0.63 | 0.238 | 0.19 |
| 0.60 | 0.66 | 4105 | 74.87 | 0.0106 | 192 | 1402 | 4.75 | 538 | 319 | 50889 | 1.41 | 0.61 | 0.213 | 0.18 |
| 0.70 | 0.75 | 4591 | 117.39 | 0.0085 | 148 | 1078 | 5.95 | 587 | 489 | 77718 | 1.09 | 0.56 | 0.166 | 0.15 |
| 0.80 | 0.86 | 5198 | 189.37 | 0.0066 | 114 | 834 | 7.49 | 827 | 775 | 125410 | 0.85 | 0.53 | 0.129 | 0.14 |
| 0.90 | 0.90 | 5599 | 231.34 | 0.0059 | 103 | 749 | 8.15 | 924 | 941 | 157421 | 0.73 | 0.49 | 0.112 | 0.13 |
| 1.00 | 1.03 | 5794 | 307.04 | 0.0053 | 80 | 581 | 9.78 | 1011 | 1235 | 190520 | 0.63 | 0.44 | 0.095 | 0.11 |
| 1.10 | 1.16 | 6063 | 404.58 | 0.0048 | 63 | 459 | 11.76 | 1097 | 1613 | 226371 | 0.54 | 0.40 | 0.081 | 0.10 |

Notes:

a Single transit triggering mode of operation and $p_t = 0.75$ completeness level.

b Assumes $X = 24$ (see Section 4.1).

Assumes $X = 10$ (see Section 4.2).

d The bold face row is discussed in Section 4.1.
the southern hemisphere) or observing fainter stars with multiple targets per field. Of course, if the fraction of stars with such a planet is less than unity, the required observing time will be more.

It is interesting to note that $n$ is not monotonically increasing with $M_*$ because the increasing HZ period is offset by the increase in $t_{dur}$ with increasing $M_*$. In addition, the trigger mode delay becomes an appreciable fraction of $t_{dur}$ for the lowest mass stars. For $M_* < 0.6 M_\odot$, $n \sim 500$, thus in principle similar number of observations are needed to find a HZ planet orbiting a late K as late M star in the triggering mode of operation. However, the minimum number of clear nights necessary to achieve the requisite number of observations $n_{\text{ngt}} = \frac{t_{dur} H}{t_{ngt}}$, where $t_{ngt}$ is the uninterrupted observing available in a single night, is a monotonically increasing function of $M_*$. The numerical result for $n_{\text{ngt}}$ in Table 1 assumes $t_{ngt} = 8$ hr. In addition, the depth of the transit for fixed planet size is $\sim$six times smaller for late K hosts than late M hosts placing more stringent requirements on the photometric precision for observations of late K hosts.

For an individual target we determine the radial velocity precision necessary to reach the same level of survey completeness for making a super-Earth $M_p \sin i$ detection at $P_{\text{HZ}}$ given the same amount of observing epochs as the transit survey. In this case the goal of the radial velocity survey is to make a sufficient number of $M_p \sin i$ detections in order to identify at least one that also transits. For the example at 0.2 $M_*$, the circular orbit semi-amplitude of a super-Earth at $P_{\text{HZ}}$ is $K \sim 5.2$ m s$^{-1}$. To complete the analysis, we must relate $n$ of a transit survey to an equivalent $n_{\text{obs, rv}}$ for a radial velocity survey. The MEarth survey cycles between targets with a five minute cadence on average and employs eight independent telescopes, and the HARPS M dwarf survey had a fixed exposure time of 15 minutes. Thus, for the super-Earth survey comparison, we assume $n = X n_{\text{obs, rv}}$, where $X = 24$, implying 24 observing epochs over independent targets are obtained in the transit survey for each radial velocity measurement. In the case of a transit survey with multiple targets per field, the conversion factor can be significantly larger ($X \geq 4$ orders of magnitude larger for a survey like Kepler; Borucki et al. 2010). Using the same completeness probability $p_r = 0.75$ for the radial velocity survey with a false alarm probability of $F = 0.001$, $n_{\text{obs, rv}} = n/24$, and $m = n_{\text{obs, rv}}$, we use Equation (7) to estimate the limiting semi-amplitude precision ratio, $K/\sigma_r = 2.8$. Hence, for the 0.2 $M_*$ case, $\sigma_r = 1.85$ m s$^{-1}$. To summarize, a radial velocity survey will have reached the same survey completeness probability for a super-Earth orbiting at the inner HZ for a late M dwarf as a simulated trigger-mode transit survey if it obtains $n_{\text{obs, rv}}$ with a precision of $\sigma_r = 1.85$ m s$^{-1}$. If the radial velocity precision is $\sigma_r > 1.85$ m s$^{-1}$, then the radial velocity survey will not have reached the same level of survey completeness as the transit survey and is not as efficient for surveying $M_p \sin i$ detections that also transit their host.

For comparison, the mode of the radial velocity precision from the HARPS M dwarf survey sample is approximately equal to $\sigma_{\text{HARPS}} = 3.0$ m s$^{-1}$ (Bonfils et al. 2013). In the above example of a super-Earth at the inner HZ of $M_* = 0.2 M_\odot$ host star, the inequality, $\sigma_{\text{HARPS}} \gtrsim \sigma_r$, implies that formally the efficiency for transiting planet detection for MEarth is slightly greater than that for transiting planet detections among the HARPS M dwarf survey $M_p \sin i$ detections. The transiting planet yield not only depends upon relative efficiency, but also total observing time. The advantage for MEarth may even be larger given that the MEarth telescopes are dedicated to the survey whereas HARPS is a shared instrument.

The above results depend on the definition of the inner HZ (Selsis et al. 2007; Kopparapu et al. 2013; Zsom et al. 2013). To first order, the results of Section A.4 show that for a transit survey for fixed detection completeness $n \propto \left( P/t_{dur} \right) \propto P^{2/3}$. The expected scaling with $P$ for a radial velocity survey maintaining the same detection completeness $n_{\text{obs, rv}} \propto K^{-2} \propto P^{2/3}$ (see Equation (26) of Cumming 2004) is the same. Numerical calculations confirm this expected weak dependence on $P$. For the 0.2 $M_*$ case, $\sigma_r = 1.9$ m s$^{-1}$ and 1.7 m s$^{-1}$ for a cooler, $T_{eq} = 240$ K, and hotter, $T_{eq} = 300$ K, inner HZ definition, respectively.

The discovery of GJ 1214 did not rely on the triggering mode for detection, but was detected after phasing two potential transit events (Charbonneau et al. 2009). Figure 4 summarizes the radial velocity precision required in order to reach the same level of completeness for a single target using the same amount of observing time for the transit survey design of requiring $k \geq 1, 2, 3$, and 5 events as the solid lines from bottom to top, respectively. The dash line shows results for the triggering mode of operation. The achieved precision of the HARPS M dwarf survey $\sigma_{\text{HARPS}} = 3.0$ is also shown (horizontal dotted line) demarcates the stellar hosts where a eight-telescope system like MEarth is more (below) and less (above) efficient. In the triggering discovery mode (dashed line), $M_* > 0.15 M_\odot$ is the stellar host regime where the eight-telescope MEarth system in the triggering discovery mode is more efficient than HARPS for detecting transiting super-Earth planets. For a three-transit requirement detection mode, the detection efficiency equivalency is at $M_* \sim 0.22 M_\odot$.

The results shown in Figure 4 and the previous discussion assume the goal of the survey is to find a HZ planet that actually transits. Given the small probability to transit, the radial velocity survey will achieve a large number of detections $\propto 1/p_r$ for every single HZ planet that transits. If the goal of the radial
radial velocity precision is predicted to be \( \sigma_{rv} \approx 0.87 \, \text{m s}^{-1} \) for M dwarf precision radial velocities was \( \sigma_{rv} = 1.1 \, \text{m s}^{-1} \) for the HARPS M dwarf survey (Bonfils et al. 2013). Results for additional transit survey designs of two, three, or five transits for detection are shown in Figure 6, and the precision needed to detect the shallower Earth-analog transit depth. In principle, the requisite photometric precision for enabling the trigger mode of detection for Earth-size planets orbiting M dwarfs has been demonstrated from the ground. Scintillation limited performance using 4 m class telescopes has achieved \( \sigma_{p} = 250 \, \text{ppm minute}^{-1} \) photometric noise rate on \( V \sim 13 \) late K and early M dwarfs with a 2 m class telescope with an innovative target cycling approach. Thus, the practical limitation of a scaled up MEarth survey is available observing time on larger telescopes rather than a physical limit imposed by the Earth’s atmosphere.

Carrying through the above calculation for an Earth analog at the inner HZ, we find \( \sigma_{rv} = 0.58 \, \text{m s}^{-1} \) is needed for comparable efficiency to the triggering mode transit survey. For this result we assume \( X = 10 \), which would be expected given the efficiency difference between a high-resolution spectrograph for precision radial velocity measurements compared to single-target photometry. Since radial velocity instrumentation is typically on larger aperture telescopes, assuming more efficient radial velocity observations, \( X = 5 \), results in \( \sigma_{rv} = 0.87 \, \text{m s}^{-1} \). The noise floor for M dwarf precision radial velocities was \( \sigma_{rv} = 1.1 \, \text{m s}^{-1} \) for the HARPS M dwarf survey (Bonfils et al. 2013). Results for additional transit survey designs of two, three, or five transits for detection are shown in Figure 6, and the precision needed to reach comparable efficiency of an \( M_p \sin i \) detection is shown in Figure 7.

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10 The multi-telescope radial velocity concept (Bottom et al. 2013) is another possibility; http://www.astro.caltech.edu/minerva/Home.html.
5. CONCLUSION

Pushing the ground-based photometric and radial velocity techniques provides an opportunity for detecting true Earth-sized planets at the inner HZ orbiting the brightest late M dwarfs. For example a scaled-up MEarth survey could detect HZ Earth-sized transiting planets among the brightest late M dwarfs. We predict that for the same amount of observing time, a radial velocity survey requires precision $\sigma_v < 0.6$ m s$^{-1}$ in order to more efficiently detect a $M_P \sin i = 1 M_\oplus$ transiting planet at the inner HZ for mid M dwarf. From space, the TESS mission (Ricker et al. 2014) plans to survey early M and K dwarfs for such Earth analogs. There is great impetus for finding the rare Earth analogs that also transit among the brightest stars to take full advantage of the follow-up potential of the Hubble Space Telescope and the upcoming opportunities provided by the JWST. The JWST will usher in an unprecedented capability for measuring the atmospheric constituents of potentially habitable planets (Valenti et al. 2005; Deming et al. 2009).

We demonstrate that the window function for a transit survey can be expressed in terms of the hypergeometric probability distribution. An analytic approximation to the window function simplifies the design of a transit survey and provides an accurate estimate to the observing time necessary to achieve a given level of completeness for detection of a transiting planet. Furthermore, to first order, the resulting window function for a transit survey depends only on the total number of independent observations and secondarily depends upon the distribution of observation times or baseline over which the observations are obtained. We employ these results regarding the transit survey window function model in order to determine the comparable radial velocity survey precision that achieves the same completeness level given the same number of observations. For the brightest M dwarfs, $V < 12$, we predict HARPS and the MEarth surveys have approximately the same efficiency for detecting super-Earth planets that transit and orbit at the inner HZ. However, at the median brightness of the MEarth sample, $V = 15$, the MEarth survey is predicted to have much higher efficiency at detecting transiting super-Earths than HARPS due to the degraded radial velocity precision for fainter stars. The higher efficiency is predominately driven by the small geometric transit probability at the inner HZ for late M dwarfs. HARPS is predicted to have much higher efficiency in $M_P \sin i$ detections than MEarth has for transiting planets, thus making HARPS efficient at determining super-Earth $M_P \sin i$ population statistics. However, complementary planet population studies are possible from wide-field transit surveys that observe many targets per observing epoch ($X \sim 10^5$ in the case of Kepler; Youdin 2011; Howard et al. 2012; Dong & Zhu 2013; Fressin et al. 2013; Dressing & Charbonneau 2013; Petigura et al. 2013).

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APPENDIX

ANALYTIC APPROXIMATIONS TO THE WINDOW FUNCTION

A.1. Binomial

Prior work (e.g., Equation (A2) of Deeg et al. 2004) adopted approach A of Section 2, “each time the planet transits, were we observing?” For this statistical model of the window function each orbital period time interval represents an independent trial. Specifically, the orbital period time interval represents a binomial trial with probability of our witnessing it equal to the average fraction of time spent observing $f_{\text{cov}}$. Thus, the probability of witnessing exactly $k$ transits is

$$p_k = \binom{M}{k} f_{\text{cov}}^k (1 - f_{\text{cov}})^{M-k}, \quad k = 0, 1, 2, ..., M, \tag{A1}$$

recalling that the expectation value for the number of trials $M = T_{\text{obs}} / P$ (Equation (2)) and that the probability of witnessing $k$ or more transits $p \geq k$ is given by Equation (3).

A.2. Poisson

Approach A also can be addressed, in appropriate limits, specifically large $M$ and small $f_{\text{cov}}$, using the Poisson approximation to the binomial distribution,

$$p_k = \frac{\lambda^k}{k!} e^{-\lambda}, \quad k = 0, 1, 2, ..., M, \tag{A2}$$

where $\lambda = M f_{\text{cov}}$. Even though the Poisson distribution is defined for all $k \geq 0$, the maximum number of transits in time span $T_{\text{obs}}$ is $M = \lfloor T_{\text{obs}} / P \rfloor + 1$. The physical interpretation of the Poisson process would be that transits are observed at random times at an average rate of $\lambda$ transits in $T_{\text{obs}}$, which approaches $f_{\text{cov}} / P$ in the appropriate limit of large $M$. Of course, transits do not occur at random times, but the approximation works if observations are sparsely and randomly distributed throughout the time span $T_{\text{obs}}$. Deeg et al. (2004) recommend that the Poisson approximation gives good results so long as $f_{\text{cov}} \lesssim 0.5$ and $M$ is large. While it is the least accurate, the Poisson approximation is the simplest to compute; we recommend it for hand calculations (see Section A.4) and to validate more complicated window function implementations.

A.3. Hypergeometric

In approach B, “each time we observe, is the planet transiting?,” the statistical model of the window function adopts the
transit duration time interval as the relevant independent trial. We make an analogy between searching for a transiting planet and randomly drawing colored balls from an urn without replacement (Figure 8). Each ball represents a time interval of the transit duration. Between the first and last observations, there are $N = 12$ time intervals total: $n = 5$ intervals (solid circles) each with sufficient observational sensitivity to detect a transit if it had occurred during the given interval, and seven intervals (dashed circles) each with insufficient or no observations during the given interval. Red balls (labeled) correspond to $M = 2$ intervals in which a transit occurs; all other balls are white (no label).

![Figure 8](image_url)

**Figure 8.** Planetary transit observability can be modeled as drawing balls without replacement. A schematic of a transit light curve, with $P = 6$ units and with depth and width exaggerated for clarity, is above a set of colored balls. Each ball in the contiguous line of balls corresponds to a time interval equal to the transit duration. Between the first and last observations, there are $N = 12$ time intervals total: $n = 5$ intervals (solid circles) each with sufficient observational sensitivity to detect a transit if it had occurred during the given interval, and seven intervals (dashed circles) each with insufficient or no observations during the given interval. Red balls (labeled) correspond to $M = 2$ intervals in which a transit occurs; all other balls are white (no label).

A limitation common to the binomial and hypergeometric distributions is that their input parameters formally are required to be integers, because of the combinatorics operator “choose.” For example, each equation contains $M$ choose $k$; that $k$ is an integer is not a problem, but if $M$ is forced to also, then the window function can have a stair-step appearance. This is readily mitigated by using implementations of the operators that can handle real-valued inputs via the gamma function, e.g.,

$$p_k = \binom{M}{k} \frac{(N-M)^{n-k}}{N^n}, \quad k = 0, 1, 2, ..., M, \quad (A3)$$

which is appropriate for this case of sampling without replacement. In the nomenclature of statistics,

$$M = \text{subpopulation size}, \quad (A4)$$

$$N = \text{total population size}, \quad \text{and} \quad (A5)$$

$$n = \text{sample size}. \quad (A6)$$

$$p_1 = e^{-\lambda} \quad \text{Poisson}, \quad (A8)$$

$$= 1 - (1 - f_{\text{cov}})^M \quad \text{binomial, or} \quad (A9)$$

$$= 1 - \frac{\binom{N-M}{n}}{\binom{N}{n}} \quad \text{hypergeometric.} \quad (A10)$$

Associating $f_{\text{cov}} = n \tau_\text{dur}/T_\text{obs}$, Equation (A8) is readily inverted: $n = -\ln(1 - p_1)/(P/\tau_\text{dur})$. This provides a convenient benchmark for planning purposes as well checking how probable a potential planetary discovery is in a transit search.

The probability of witnessing at least two transits is

$$p_{\geq 2} = e^{-\lambda}(1 - \lambda) \quad \text{Poisson}, \quad (A11)$$

$$= 1 - (1 - f_{\text{cov}})^M - M f_{\text{cov}}(1 - f_{\text{cov}})^{M-1} \quad \text{binomial, or} \quad (A12)$$

$$= 1 - \frac{\binom{N-M}{n}}{\binom{N}{n}} - M \frac{\binom{N-M}{n-1}}{\binom{N}{n}} \quad \text{hypergeometric.} \quad (A13)$$
A.5. Comparisons to Numerical Simulations

In Figure 1 we compare the resulting window function given by Equation (3) to the exact numerical window function for a hypothetical ground-based transit survey that observes for 30 consecutive days with observations for 8 hr each night. The specified transit survey is the same as presented in Figure 1 of Beatty & Gaudi (2008) to demonstrate their analytical representation of a transit survey (solid blue line). The exact numerical window function (dashed black line) shows the substantial alias structure that is typical of ground-based transit surveys due to the day–night observing cycle, and which is not modeled by the simpler analytic approximations.

The analytic expressions approximate well the average window function. To demonstrate this we show in Figure 1 a moving average of the exact numerical function using a one day window (solid black line). The left and right panels provide results for requiring \( k \geq 2 \) and \( k \geq 3 \) transit events for detection, respectively. We show the hypergeometric window function model from this study (Equation (A3)) as the solid red line. Solar values for the stellar parameters are assumed. The Poisson window function model (Equation (A2), dash red line) also does a reasonable job in this scenario with \( f_{\text{ov}} = 0.33 \).

Figure 3 shows the window function for a transit survey that operates in a low duty cycle mode obtaining several observations per night over the course of an observing season. We simulate this mode of operation through specifying a time series with \( n = 200 \) that is expected for a target from a single observatory over the course of an observing season including losses due to weather and stellar parameters appropriate for a mid M dwarf star. The exact numerical window functions assuming at least \( k \geq 1, 2, 3 \), and 5 transit events are observed (solid lines from top to bottom, respectively). The hypergeometric model (solid red line) and its Poisson model approximation (dashed blue line) agree extremely well as expected for a low duty cycle transit survey.

Figure 9 shows the window function for two typical targets observed by Kepler using Quarters 1 through 16 (∼3.6 yr) of nearly continuous coverage (Tenenbaum et al. 2014). The rightmost set of solid black, red solid, and blue solid lines correspond to the coverage of Kepler -11, host to six transiting planets (Lissauer et al. 2011). This target was observed for all 16 Kepler Quarters. For the time series, we employ barycentric Julian date mid-cadence times stamps that have valid data as determined by the Kepler pipeline (Smith et al. 2012; Wu et al. 2010). The hypergeometric model (solid red line) requiring \( k \geq 3 \) transit events being observed agrees well with the exact numerical calculation (solid black line) also requiring \( k \geq 3 \). As expected the analytical window model function for continuous observations with no gaps of (Table 2 of Fleming et al. 2008), overestimates the window function due to the reality of gaps in Kepler data. The leftmost set of solid black and solid red lines correspond to the coverage of the host to TrES-2b (a.k.a. Kepler-1b; O’Donovan et al. 2006a; Barclay et al. 2012). This target is impacted by the loss of a detector in Quarter 4 (Batalha et al. 2013), causing missing data every fourth Kepler Quarter. The detector loss impacts ∼19% of Kepler targets having reduced amount of data available. The dash lines are the same as above, but they require \( k \geq 6 \) transits for detection.

The hypergeometric model does not fully capture the details present in the exact numerical window functions. The assumptions of the hypergeometric model begin to break down in observing scenarios consisting of high duty cycle blocks interspersed with long periods of no data. In addition to the sharp, isolated aliases at the diurnal cycle and its harmonics prevalent in ground-based transit survey window functions (which we readily average over), there are “broadband” alias structures present across all orbital periods that can result in the exact numerical window function to deviate from the hypergeometric model.

In Figure 2 we demonstrate the principal dependence of a transit survey window function on \( n \) and the weak dependence on \( T_{\text{obs}} \). We show the hypergeometric model (solid lines) and the moving average of the exact numerical window function (dash lines) for three scenarios that have the same \( n \) but vary in \( T_{\text{obs}} \). The left and right panels vary by requiring \( k \geq 2 \) and 3 transit events for detection, respectively. The baseline transit survey consists of 7 hr of observations each day for 30 consecutive days (black lines). We simulate a longitudinally distributed network survey by considering a transit survey with half \( T_{\text{obs}} \) (15 consecutive days) of the baseline case while doubling the observing time each day to conserve \( n \) (red lines). The window function does show an increase in detection probability at intermediate periods at the cost of removing the low detection probability tail toward longer periods of the baseline scenario. The third \( n \)-conserving scenario consists of splitting the 30 consecutive days of observations from the baseline case into two 15 consecutive day groups separated by a year (blue lines) giving a \( T_{\text{obs}} = 380 \) days. In contrast to the \( n \)-conserving cases, we show a transit survey window function resulting from doubling \( n \) while keeping \( T_{\text{obs}} \) the same as the baseline case (magenta lines). These scenarios demonstrate that the detection probability of a transit survey are very sensitive to \( n \).

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