The contribution of pseudoscalar and axial-vector mesons to hyperfine structure of muonic hydrogen

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Abstract. In the framework of the quasipotential method in quantum electrodynamics we calculate the contribution of light pseudoscalar (PS) and axial-vector (AV) mesons to the interaction operator of a muon and a proton in muonic hydrogen atom. The coupling of mesons with the muon is via two-photon intermediate state. The parametrization of the transition form factor of two photons into PS and AV mesons, based on the experimental data on the transition form factors and QCD asymptotics, is used. Numerical estimates of the contributions to the hyperfine structure of the spectrum of the S and P levels are presented. It is shown that such contribution to the hyperfine splitting in muonic hydrogen is rather important for a comparison with precise experimental data.
1. Proton radius puzzle

Precise investigation of the Lamb shift (LS) and hyperfine structure (HFS) of light muonic atoms is a fundamental problem for testing the Standard model and establishing the exact values of its parameters, as well as searching for effects of new physics. Recently, the CREMA (Charge Radius Experiments with Muonic Atoms) Collaboration from PSI by using the laser spectroscopy method measured with unprecedented accuracy the transition frequencies between the 2P and 2S states in muonic hydrogen ($\mu p$) [1, 2]

$$\nu_t \left( 2P_{3/2}^F - 2S_{1/2}^F \right) = 49881.35(65) \text{ GHz}, \quad h\nu_t = \Delta E \left( 2P_{3/2}^F - 2S_{1/2}^F \right)$$

$$\nu_s \left( 2P_{3/2}^F - 2S_{1/2}^F \right) = 54611.16(1.05) \text{ GHz}, \quad h\nu_s = \Delta E \left( 2P_{3/2}^F - 2S_{1/2}^F \right).$$

From these two transition measurements, both the Lamb shift $\Delta E_{LS}$ and the 2S-HFS $\Delta E_{HFS}$ was determined independently with the result [2]

$$\Delta E_{LS}^{\text{exp}} \left( 2P_{1/2}^F - 2S_{1/2}^F \right) = 202.3706(23) \text{ meV},$$

$$\Delta E_{HFS}^{\text{exp}} \left( 2S_{1/2}^F - 2S_{1/2}^F \right) = 22.8089(51) \text{ meV}.\quad (3)$$

From theory side these quantities calculated in the frame work of bound-state QED are expressed in terms of the charge $r_E^2$ and Zemach $r_Z$ radii of the proton as [3]

$$\Delta E_{LS}^{\text{th}} = 206.0668(25) - 5.2275(10) r_E^2 \text{ meV},$$

$$\Delta E_{HFS}^{\text{th}} = 22.9843(30) - 0.1621(10) r_Z \text{ meV}.\quad (5)$$

Remind that the charge RMS radius $r_E$ is defined via the normalized proton charge distribution $\rho_E$, while the Zemach radius is correlated with the proton magnetic moment distribution $\rho_M$ as

$$r_E^2 = \int d^3r r^2 \rho_E (r), \quad r_Z = \int d^3r \int d^3r' \rho_E (r) \rho_M (r - r').$$

The first term on the right side of (5) accounts for radiative, relativistic, and recoil effects, while the first term on the right side of (6) is the Fermi energy arising from the interaction between the muon and the proton magnetic moment and different corrections to it. It is also important, that like for the problem of the anomalous magnetic moments of leptons [4, 5], the coefficients in front of the radius terms in (5), (6) are much stronger enhanced for the muonic system relative to the electronic system, and thus much more sensitive in extraction of the radii parameters from the experimental data.

The comparison of (5), (6) with (3), (4) provides [2, 3]

$$r_E^{\text{CREMA}} = 0.84087(39) \text{ fm}, \quad r_Z^{\text{CREMA}} = 1.082(37) \text{ fm}.\quad (8)$$

This should be compared with the values recommended by CODATA [6] and based on the electron-proton scattering and electronic hydrogen spectroscopy

$$r_E^{\text{CODATA}} = 0.8751(61) \text{ fm}.\quad (9)$$

Thus we see two basic results of the laser spectroscopy experiment for $\mu p$. First, from the $\mu p$ spectroscopy the value of $r_E$ is determined with precision 10 times higher than from electronic data. Second, and more striking, that there is the large discrepancy between $r_E^{\text{CREMA}}$ and $r_E^{\text{CODATA}}$ at the level of 5.6 $\sigma$ (or the muonic hydrogen value is 4% smaller than the CODATA
value). This is so-called the proton size puzzle, not explained up to now! Later on, the similar problem was detected for the deuteron radius [7].

At present, several experimental groups plan to measure HFS of various muonic atoms with more high precision [8, 9, 10]. This will make it possible to better understand the existing "puzzle" of the proton charge radius, to check the Standard Model with greater accuracy and, possibly, to reveal the source of previously unaccounted interactions between the particles forming the bound state in QED. One of the ways of overcoming the crisis situation arises from a deeper theoretical analysis of the fine and hyperfine structure of muonic atom spectrum, with the verification of previously calculated contributions and the more accurate construction of the particle interaction operator in quantum field theory, the calculation of new corrections whose value for muonic atoms can increase substantially in comparison with electronic atoms. The expected results will allow to get also a new very important information about the forces which are responsible for the structure of atoms.

2. Light meson exchange contributions to HFS of $\mu p$

From the theory side it is urgently needed to study the possible effects of exchanges between muon and proton which can contribute to HFS of $\mu p$. Some of such effects was considered in recent papers [11, 12, 13, 14, 15]. Below, we discuss the effects of exchanges between muon and proton which can contribute to HFS of $\mu p$ coming from the light pseudoscalar (PS) and axial-vector (AV) meson exchanges between muon and proton induced by meson coupling to muon through two photons (see Fig. 1).

The leading contribution to HFS of $\mu p$ is coming from one-photon exchange and has the following form [16, 17, 18]:

$$\Delta V_{\mu p}^{hfs} = \frac{8\pi\alpha\mu_p}{3m_\mu m_p} (S_p S_\mu) \delta(r) - \frac{\alpha \mu_p(1 + a_\mu)}{m_\mu m_p r^3} [(S_p S_\mu) - 3(S_p n)(S_p n)]$$ (10)

where $m_\mu$, $S_\mu$ and $m_p$, $S_p$ are masses and spins of muon and proton, correspondingly, $\mu_p$ is the proton magnetic moment. The potential (10) gives the main contribution of order $\alpha^4$ to the HFS of muonic atom. Precision calculation of the HFS of the spectrum, which is necessary for comparison with experimental data, requires the consideration of various corrections accounting for the vacuum polarization, nuclear structure and recoil, and relativistic effects [16, 20, 19, 21, 22, 23].

We calculate further the contribution to HFS coming from the pion and axial-vector $f_1(1285)$, $a_1(1260)$ and $f_1(1420)$ meson exchanges shown in Fig. 1. The effective vertices of the interaction of the PS and AV mesons and virtual photons can be expressed in terms of the transition form factors as follows:

$$V^{\mu\nu}(k_1, k_2) = i\varepsilon^{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \frac{\alpha}{F_\pi} F_{\pi \gamma^* \gamma^*}(k_2^2, k_2^2),$$ (11)

$$T^{\mu\nu\alpha} = 8\pi i \varepsilon_{\mu\nu\alpha\tau} k^\tau F_{\text{AV} \gamma^* \gamma^*}(k_1^2, k_2^2),$$ (12)

where $k_1$, $k_2$ are four-momenta of virtual photons. For small values of the relative momenta of particles in the initial and final states and small value of transfer momentum $t$ between muon and proton, the transition amplitude takes a simple form

$$T^{\mu\nu\alpha} = 8\pi i \varepsilon_{\mu\nu\alpha\tau} k^\tau F_{\text{AV} \gamma^* \gamma^*}(t^2, k_2^2, k_2^2),$$ (13)
where \( k = k_1 = -k_2 \).

The final result for the HFS potential is equal to

\[
\Delta V_{hfs}^{PS}(t) = \frac{\alpha^2}{6\pi^2 m_p} \frac{g_p}{t^2 + m_{\pi}^2} A(t^2),
\]

\[
\Delta V_{hfs}^{AV}(t) = -\frac{32\alpha^2 g_{AVPP} F_{AV\gamma\gamma}(0,0)}{3\pi^2(t^2 + M_A^2)} I\left(\frac{m_\mu}{\Lambda_A}\right),
\]

where

\[
A(t^2) = \frac{2}{\pi^2 t^2} \int \frac{id^4k [k^2 - (tk)^2]^2}{k^2(k - t)^2(k^2 - 2kp_1)} F_{PS\gamma\gamma^*}(k^2, (k - t)^2),
\]

\[
I\left(\frac{m_\mu}{\Lambda_A}\right) = \int \frac{id^4k (2k^2 + k_0^2)}{k^2(k^2 - 2m_\mu k_0)} F_{AV\gamma\gamma^*}(k^2, k^2).
\]

The integral \( A \) in (16) is well studied in connection to the problem of interpretation \[24, 25\] of the KTeV (FermiLab) data on the pion decay into \( e^+e^- \) pair. In order to fix the transition form factors in the most model independent way we used corresponding data from CLEO \[26\] and L3 \[27, 28, 29\] collaborations for the PS and AV mesons, respectively (see for details \[11, 12\]).

We would like to point out, that one can expect the important contribution of the AV exchange to spin dependent part of muon-proton interaction because the exchange particle has the spin one. Furthermore, it is also well known that in the channel with quantum number 1\(^{++} \) axial anomaly effects can play an important role and, in particularly, these effects might be considered as a cornerstone to solve so-called "proton spin crisis" \[31, 32\]. The other phenomenological input, the meson-nucleon couplings, were determined by using the Regge approach analysis of the deep-inelastic scattering, \( f_1 \) and \( a_1 \) trajectories contributing to the polarization of quarks in the nucleon \[33, 34\].

Calculating the matrix elements with wave functions of 1\(^S\), 2\(^S\) and 2\(^P\) 1/2 states, we obtain the corresponding contributions to the HFS spectrum

\[
\Delta E^{hfs}(1S) = \frac{\mu^3\alpha^5 g_A}{6F_{\pi}^2 \pi^3} \left\{ A(0) \frac{4W(1 + \frac{W}{m_\pi})}{m_\pi(1 + 2W/m_\pi)^2} - \frac{1}{\pi} \int_0^{\infty} ds \frac{ds}{s} \text{Im} A(s) \times \right\}
\]

\[1\] The contribution of the \( \eta \) and \( \eta' \) mesons is negligible \[11\].

\[2\] In recent paper \[30\] within NJL model the estimation for form factor \( F_{f_1(1285)}^{(NJL)} \gamma\gamma^* \approx 0.276 GeV^{-2} \) was obtained. This value is in the agreement with L3 data which was used in our paper \[12\] (see Table 1.).
where $W = \mu \alpha$ and $\mu$ is reduced mass.

### Table 1. Pion and axial-vector mesons exchanges contribution to HFS of muonic hydrogen.

| Mesons       | $I^G(J^{PC})$ | $\Lambda_A$ | $F^{(0)}_{AV \gamma\gamma^*}(0,0)$ | $\Delta E^{hfs}(1S)$ | $\Delta E^{hfs}(2S)$ |
|--------------|---------------|-------------|----------------------------------|----------------------|----------------------|
| $f_1(1285)$  | $0^+(1^{++})$ | 1040        | 0.266                            | -0.0093 ± 0.0033     | -0.0012 ± 0.0004     |
| $\sigma(1260)$ | $1^-(1^{++})$ | 1040        | 0.591                            | -0.0437 ± 0.0175     | -0.0055 ± 0.0022     |
| $f_1(1420)$  | $0^+(1^{++})$ | 926         | 0.193                            | -0.0013 ± 0.0008     | -0.0002 ± 0.0001     |
| $\pi^0$      | $1^-(0^{-+})$ | 776         | -0.0017 ± 0.0001                 | -0.0002 ± 0.0000     |
| Sum          |               |             | -0.0560 ± 0.0178                 | -0.0071 ± 0.0024     |

In Table 1 our results for contribution of the pion and AV mesons exchanges to HFS are presented. For the case of 2S state the summary contribution from pion and AV meson exchanges is equal to (-0.0071) meV, which is quite important to obtain the total value of HFS with high precision.

### 3. Conclusion

A new important contribution to the muon-nucleon interaction is discovered. It is coming from effective pion and AV mesons exchanges induced by anomalous meson vertices with two photon state. The contribution of this exchange to the HFS of $\mu p$ is calculated in framework of the quasipotential method in QED and the use of the technique of projection operators on the states of two particles with a definite spin. It is shown that this contribution is rather large and should be taking into account for the interpretation of the new data on HFS in this atom.

As has been mentioned the CREMA Collaboration measured two transition frequencies in muonic hydrogen for the 2S triplet state ($2P_{3/2}^{F=2} - 2S_{1/2}^{F=1}$) and for 2S singlet state ($2P_{3/2}^{F=1} - 2S_{1/2}^{F=0}$) [2]. From these measurements it is possible to extract the value of HFS for 2S level. Obtained value [4] allows to get the value of the Zemach radius [8] with accuracy 3.4 % with help of relation (9). This is in the agreement with another numerical values $r_Z = 1.086(12)$ fm [35], $r_Z = 1.045(4)$ fm [36], $r_Z = 1.047(16)$ fm [37], $r_Z = 1.037(16)$ fm [38] obtained from electron-proton scattering and from H and muonium spectroscopy. We should emphasize that
the changing of the theoretical value of HFS on 0.001 meV leads to the changing of the Zemach radius on 0.006 fm. Therefore, our contribution coming from pion and AV meson exchange leads to the new value of this radius

$$r_Z = 1.040(37) \text{ fm},$$

which is smaller in the comparison with most listed results but still agree with them within errorbars.

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