Working Mechanism and Numerical Simulation of a Novel 6-DOF Robotic Crusher

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Abstract. Based on the inter-particle breakage theory and characteristics of oscillating motion, a novel 6-DOF robotic crusher is proposed which combines the chamber structure of the cone crusher with the high flexibility of the parallel robot. According to the method of analytical geometry, equations of the inverse and forward models are deduced. Then, considering the periodicity of eccentric rotation, the trajectory model of the oscillating motion is established by using an eccentric simulation. Based on the derived model, the motion variation of the actuators can be solved by ADAMS. Finally, the liner wear of the concave is deduced by a numerical example which can provide a theoretical foundation for the development of the intelligent crusher.

1. Introduction
The crushing of materials is a widely used process in the metallurgy, mining, construction, chemical, and other industrial sectors [1]. It is urgently necessary to continuously improve the crushing operations with the shortage of energy, such as the development of new efficient crushing equipment and the improvement of existing crushing equipment, which are of great significance for achieving high quality, high production and low energy consumption. However, the economic efficiency of the departments is seriously affected due to the conservative design, heavy machinery and low efficiency.

Based on the inter-particle breakage theory and population balance theory, scholars have conducted a lot of research on the efficient and energy saving of cone crusher. Evertsoon put forward the crushing process consists of a series of crushing events, and the feed of each crushing event is the product of the previous crushing event [2]. Huang proposed a multi-objective optimization model in which the crushing chamber, output, and size distribution are coupled to each other [3]. Zhang presented an improved model of inter-particle breakage [4] In this model, a relative particle size ratio is introduced for describing the breakage characteristics of particles with different sizes. Cleary proposed the discrete element method to study the cone crusher [5]. Estimates of power, product size, throughput rate and crusher wear are calculated. Quist used the discrete element method to predict the power and particle size distribution, which was supplemented with the experimental verification [6]. However, the above researches are based on the existing cone crusher structure and retained the traditional form of power transmission.

The novel 6-DOF robotic crusher combines the characteristics of cone crusher and parallel robot, which has the crushing chamber structure and large rigidity. Based on the geometric analysis, equations of the inverse and forward models are established which provides a mathematical basis for the dynamics
and control. According to the eccentric simulation, the model of the oscillating motion of the mantle is established. Then, the kinematics of the 6-DOF robotic crusher can be calculated. Considering the distribution of crushing pressure on the liner surface, the model of liner wear is derived which can be solved by MATLAB.

2. Kinematic modeling

2.1. Inverse kinematic model

As shown in Figure 1, the 6-DOF robotic crusher consists of a crusher fixed unit (CFU) and a crusher drive unit (CDU). CFU is fixed to the ground, and CDU is used to drive the mantle to crush the particles in the crushing chamber. The final product is discharged from the open side setting (OSS) due to gravity. A reference frame O-XYZ is fixed on the ground, and the mantle frame O₁-X₁Y₁Z₁ is connected with the mantle. The position and orientation of the mantle frame relative to the reference frame can be described by a matrix \( Q = [α, β, γ, x, y, z]^T \). According to the geometric analysis of the structure, the homogeneous coordinate of the spherical joint \( s_A \) and \( s_B \) (i=1,2,...,6) in the reference frame can be represented as [7, 8]:

\[
C = [TA]_{6×6}, B = \begin{bmatrix} b_x \ b_y \ b_z \end{bmatrix}_{6×1}
\]

where the matrix \( A \) denotes the homogeneous coordinate of the upper spherical joints in the mantle frame and \( T \) is the rotation matrix, which can be expressed as:

\[
T = \begin{bmatrix}
\cos β \cos γ & -\cos α \sin γ + \sin α \sin β \cos γ & \sin α \sin γ + \cos α \sin β \cos γ & x \\
\cos β \sin γ & \cos α \cos γ + \sin α \sin β \sin γ & -\sin α \cos γ + \cos α \sin β \sin γ & y \\
-\sin β & \sin α \cos β & \cos α \cos β & z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Inverse kinematic of the 6-DOF robotic crusher is used to solve the displacements of the actuators when the position and orientation of the mantle are given. Then, the length variation of each actuator can be derived as:
\[ \Delta L_j = L_j - L_0 = \sqrt{\sum_{i=1}^{3} \left( c_{ij} - b_{ij} \right)^2} - L_0 \quad (j = 1, 2, \ldots, 6) \] (4)

where \( L_j \) represents the length of each actuator, \( L_0 \) denotes the initial length.

2.2. Forward kinematic model

Forward kinematic is used to solve the position and orientation of the mantle when the length of each actuator is given. Using (4), the following equation can be obtained as:

\[ f_j(Q) = L_j^2 - \sum_{i=1}^{3} \left( c_{ij} - b_{ij} \right)^2 = 0 \] (5)

Taylor's formula for multivariate is used to expand the equation \( f_j(Q) \) when \( Q^{(m)} = [\alpha_m, \beta_m, \gamma_m, \xi_m, \eta_m, \zeta_m]^T \). \( Q^{(n)} (n=1, 2, \ldots, 6) \) denotes the \( n \)th element of the matrix \( Q^{(m)} \), and \( Q_n (n=1, 2, \ldots, 6) \) is the \( n \)th element of the matrix \( Q \). Then, the linear part of \( f_j(Q) \) can be represented as:

\[ f_j(Q) = f_j(Q^{(m)}) + \sum_{n=1}^{6} \frac{\partial f_j(Q^{(m)})}{\partial Q_n} (Q_n - Q_n^{(m)}) \] (6)

where (6) can be rewritten as:

\[
\begin{cases}
\sum_{n=1}^{6} \frac{\partial f_1(Q^{(m)})}{\partial Q_n} (Q_n - Q_n^{(m)}) = -f_1(Q^{(m)}) \\
\vdots \\
\sum_{n=1}^{6} \frac{\partial f_6(Q^{(m)})}{\partial Q_n} (Q_n - Q_n^{(m)}) = -f_6(Q^{(m)})
\end{cases}
\] (7)

According to (5), the following equation can be given by:

\[ \frac{\partial f_j}{\partial Q_n} = 2 \left[ c_{ij} - b_{ij} \right] = \left[ \frac{\partial c_{ij}}{\partial Q_n} \frac{\partial c_{ij}}{\partial Q_n} \frac{\partial c_{ij}}{\partial Q_n} \right]^T \] (8)

Using (1), the following equation can be obtained as:

\[ \frac{\partial C}{\partial Q_n} = \frac{\partial T}{\partial Q_n} A \] (9)

Substituting (3) into (9) yields \( (n=1) \):

\[
\frac{\partial T}{\partial Q_1} = \begin{bmatrix}
0 & \sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma & \cos \alpha \sin \gamma - \sin \alpha \sin \beta \cos \gamma & 0 \\
0 & -\sin \alpha \cos \gamma + \cos \alpha \sin \beta \sin \gamma & -\cos \alpha \cos \gamma - \sin \alpha \sin \beta \sin \gamma & 0 \\
0 & \cos \alpha \cos \beta & -\sin \alpha \cos \beta & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\] (10)

Similarly, the partial derivatives \( (n=2, 3, \ldots, 6) \) can be derived. The solution accuracy is represented as \( \delta \).

3. Compressive Behavior modeling

3.1. Model of oscillating motion

Compared with the cone crusher, the transmission form of the 6-DOF robotic crusher is more concise. The oscillating motion of a cone crusher can be considered as a special form of the 6-DOF robotic crusher. In order to obtain the trajectory model of the mantle, the model in ADAMS is established which can be shown in Figure 2 [9]. According to the extracted data, nonlinear regression is used to obtain the trajectory model which can be expressed as:
\[
\begin{align*}
\alpha &= 0.044\sin(1.483t) \\
\beta &= 0.078\sin(1.483t) \\
\gamma &= 0.044\cos(1.483t) - 0.042 - 14.361\sin(1.483t)
\end{align*}
\]

\[
\begin{bmatrix}
\alpha \\
\beta \\
\gamma
\end{bmatrix} = \begin{bmatrix}
0.044\sin(1.483t) \\
0.078\sin(1.483t) \\
0.044\cos(1.483t) - 0.042 - 14.361\sin(1.483t)
\end{bmatrix}
\]

In order to ensure that the eccentricity is 2.5°, the initial length of each actuator should be adjusted to a special position. Based on (11), the motion variation of each actuator is calculated by ADAMS. Figure 3 shows the velocity of actuator 1 is comparatively close to the actuator 2. The velocities of actuators 3 and 6 are approximately the same except for their peaks. Compared with these four actuators, the velocities of actuators 4 and 5 have larger peaks, and they are always in opposite directions. Positive values indicate the elongation of the actuators, and negative values denote the contraction. Figure 4 represents the accelerations of actuators which are basically similar to the characteristics of the velocities.

3.2. Liner wear of the mantle
The crushing chamber is crucial for the crushing of materials. As the final product increases, the amount of liner wear increases. Liner wear is mainly related to the liner material and the distribution of crushing force on the crushing chamber surface. Lindqvist proposed that the crushing pressure is related to the compression ratio and particle size distribution [10]. Based on the particle mechanics experiment of RMT-150B, Dong presented the crushing force model according to the experimental data, which can be expressed as [11]:

\[
p = \begin{cases} 
\varepsilon \exp(2.16\sigma + 2.65), & \varepsilon < 0.1266 \\
\exp(7.9\varepsilon + 2.16\sigma - 0.42), & \varepsilon \geq 0.1266
\end{cases}
\]

where \( \varepsilon \) represents the compression ratio, \( \sigma \) denotes particle size distribution. They can be derived as:

\[
\begin{align*}
\varepsilon &= \frac{s}{b} \\
\sigma &= \left[ \sum_{j=1}^{m} p_j \left( \overline{d}_j - \overline{d} \right)^2 \right]^{\frac{1}{2}} / \overline{d}
\end{align*}
\]

where \( b \) represents the initial height, \( s \) denotes the stroke, and \( m \) is the total numbers of size ranges. \( p_j \) is the mass percentage of the particles in size range \( j \), \( \overline{d}_j \) denotes the average size of the particles in size range \( j \), and \( \overline{d} \) represents the average size of the particles.

Archard proposed that the wear model is proportional to sliding distance and pressure [12]. Lindqvist suggested that liner wear is generated even if there is no macroscopic sliding motion between the particles and liner. Then, the model for liner wear can be given by [13]:

Figure 3. Velocities of six actuators.  
Figure 4. Accelerations of six actuators.
\[ \Delta \omega = \frac{P}{W} \quad (14) \]

where \( W \) is the wear resistance coefficient.

The discharge outlet should be adjusted periodically due to the liner wear. But the position of the adjusting ring is difficult to set because the liner wear is generated in the crushing chamber. Compared with the cone crusher, the 6-DOF robotic crusher can automatically compensate the liner wear. According to the structure of the crushing chamber, the geometrical relationship between the liner wear \( \Delta \omega \) and the height \( Y \) is obtained which can be shown in Figure 5. Then, the wear amount of the liner can be calculated. Figure 6 shows that the maximum wear amount is generated at the starting point D.

![Cross section of the liner.](image1)

![Wear amount of the liner.](image2)

**Figure 5. Cross section of the liner.**

**Figure 6. Wear amount of the liner.**

## 4. Conclusion

This paper proposed a novel 6-DOF robotic crusher which combines the characteristics of cone crusher and parallel robot. The particles are squeezed and crushed by the oscillating motion of the mantle, which is accomplished by the elongation and contraction of actuators. Based on the analytical geometry, kinematic models were derived. Then, the trajectory model of the mantle was established according to an eccentric simulation. The velocities and accelerations of the actuators vary periodically. The maximum values were generated on the actuators 4 and 5 due to the initial position. Finally, the liner wear model of the concave was derived which can be used for the automatic compensation.

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