The Bose-Einstein correlation function $C_2(Q)$ from a Quantum Field Theory point of view

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(July 3, 2018)

We show that a recently proposed derivation of Bose-Einstein correlations (BEC) by means of a specific version of thermal Quantum Field Theory (QFT), supplemented by operator-field evolution of the Langevin type, allows for a deeper understanding of the possible coherent behaviour of the emitting source and a clear identification of the origin of the observed shape of the BEC function $C_2(Q)$. Previous conjectures in this matter obtained by other approaches are confirmed and have received complementary explanation.

PACS numbers: 25.75.Gz 12.40.Ee 03.65.-w 05.30.Jp

In this work we would like to focus attention on two specific features of Bose-Einstein correlations (BEC) clearly visible when BEC are presented in the language of some specific (thermal) version of Quantum Field Theory (QFT) supplemented by operator-field evolution of Langevin type proposed recently [1,2]. Because the importance of BEC and their present experimental and theoretical status are widely known and well documented (see, for example, [3] and references therein), we shall not repeat it here. The features we shall discuss are: (i) how the possible coherence of the hadronizing system (modelled here by some external stationary force occurring in the Langevin equations describing hadronization process [2]) influences the BEC function $C_2(Q)$ and (ii) what is the true origin of the experimentally observed $Q$-dependence of the $C_2(Q)$ correlation function in the approach used here.

In what concerns the first point we have obtained identical expression as derived in [4] by means of a quantum statistical (QS) approach with novel interpretation of the chaoticity parameter $p$ introduced there. On the other hand, our results (and that of [4]) differ from the formula recently obtained in [5]. We shall argue that the origin of this difference lies in different ways of introducing the concept of coherence in both approaches.

In the second point we demonstrate that in order to obtain a given (experimentally observed) shape of the BEC correlation function $C_2(Q)$ (i.e., its $Q$-dependence, where $Q = |k_\mu - k'_\mu| = \sqrt{(k_\mu - k'_\mu)^2}$) one has to account somehow for the finiteness of the space-time region of the particle production (i.e., of the hadronizing source). In our approach this means the necessity of smearing out of some generalized functions (delta functions: $\delta(Q_\mu = k_\mu - k'_\mu)$) appearing in the definition of thermal averages of some operators, which is characteristic feature of QFT approach used here. The freedom in using different types of smearing functions to perform such a procedure allows us to account for all possible different shapes of hadronizing sources apparently observed by experiment. [6].

Referring to [1] for details, let us recapitulate here the main points of our approach. The collision process produces a lot of particles out of which we select one (we assume for simplicity that we are dealing only with identical bosons) and describe it by operator $b(\vec{k}, t)$ (the notation is the usual one: $b(\vec{k}, t)$ is an annihilation operator, $\vec{k}$ is 3-momentum and $t$ is a real time). The rest of the particles are then assumed to form a kind of heat bath, which remains in equilibrium characterized by a temperature $T = 1/\beta$ (which will be one of our parameters). All averages $\langle \ldots \rangle$ are therefore thermal averages of the type: $\langle \ldots \rangle = Tr \left[ \ldots e^{-\beta H} \right] / Tr \left( e^{-\beta H} \right)$. We shall also allow for some external (to the above heat bath) influence to our system. Therefore we shall represent the operator $b(\vec{k}, t)$ as consisting of a part corresponding to the action of the heat bath, $a(\vec{k}, t)$, and also of a part describing action of these external factors, $R(\vec{k}, t)$:

$$b(\vec{k}, t) = a(\vec{k}, t) + R(\vec{k}, t).$$

(1)

The time evolution of such a system is then assumed to be given by a Langevin equation [2]

$$i\partial_t b(\vec{k}, t) = F(\vec{k}, t) - A(\vec{k}, t) + P$$

(2)

and a similar conjugate equation for $b^+(\vec{k}, t)$). These equations are supposed to model all aspects of the hadronization process. The combination $F(\vec{k}, t) - A(\vec{k}, t)$ represents the so called Langevin force and is therefore responsible for the internal dynamics of hadronization in the following manner: $A$ is related to stochastic dissipative forces and is given by [2,1]

$$A(\vec{k}, t) = \int_{-\infty}^{+\infty} d\tau K(\vec{k}, t - \tau) b(\vec{k}, \tau),$$

(3)

with the operator $K(\vec{k}, t)$ being a random evolution field operator describing the random noise and satisfying the usual correlation-fluctuation relation for the Gaussian noise [8]. The operator $F(\vec{k}, t)$ describes the influence of heat bath,

$$F(\vec{k}, t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \psi(k_\mu) \tilde{c}(k_\mu)e^{-i\omega t}.$$
Our heat bath is represented by an ensemble of damped oscillators, each described by operator \( \hat{c}(k_\mu) \) such that

\[
[\hat{c}(k_\mu), \hat{c}^+(k'_\mu)] = \delta^4(k_\mu - k'_\mu),
\]

and characterized by some function \( \psi(k_\mu) \) [9]. Finally, the constant term \( P \) (representing the external source term in Langevin equation) denotes the possible influence of some external force (assumed here to be constant in time). This force would result, for example, in a strong ordering of phases, leading therefore to the coherence effect in the sense discussed in [4]. Out of many details (for which we refer to [1]) what is important in our case is the fact that the 2-particle correlation function for like-charge particles, is defined as

\[
C_2(Q) = \xi(N) \cdot \frac{\tilde{f}(k_\mu, k'_\mu)}{\tilde{f}(k_\mu)} \cdot \tilde{f}(k'_\mu) = \xi(N) \cdot [1 + D(k_\mu, k'_\mu)],
\]

where

\[
\tilde{f}(k_\mu, k'_\mu) = \int d\omega \frac{\omega}{2\pi} |\omega|^2 n(\omega) e^{i\omega(t-t')}.
\]

This defines \( D(k_\mu, k'_\mu) \) by definition commute with themselves and with any other operator considered here:

\[
\tilde{f}(k_\mu, k'_\mu) = \tilde{f}(k_\mu) \cdot \tilde{f}(k'_\mu) + \langle \hat{a}^+(k_\mu) \hat{a}^+(k'_\mu) \hat{a}(k'_\mu) \hat{a}(k_\mu) + \langle \hat{a}^+(k'_\mu) \hat{a}(k_\mu) \hat{R}^+(k'_\mu) \hat{R}(k_\mu) + \langle \hat{a}^+(k_\mu) \hat{a}(k'_\mu) \hat{R}(k'_\mu) \hat{R}(k_\mu),
\]

\[
\tilde{f}(k_\mu) = \langle \hat{a}^+(k_\mu) \hat{a}(k_\mu) + |\hat{R}(k_\mu)|^2.
\]

This defines \( D(k_\mu, k'_\mu) = \tilde{f}(k_\mu, k'_\mu)/[\tilde{f}(k_\mu) \cdot \tilde{f}(k'_\mu)] - 1 \), which in our case are equal to:

\[
\hat{a}(k_\mu) = \frac{\tilde{F}(k_\mu)}{K(k_\mu) - \omega} \quad \text{and} \quad \hat{R}(k_\mu) = \frac{P}{K(k_\mu) - \omega}.
\]

where \( n(\omega) = \{\exp[(\omega - \mu)\beta] - 1\}^{-1} \) is the number of (by assumption - only bosonic in our case) damped oscillators of energy \( \omega \) in our reservoir characterized by parameters \( \mu \) (chemical potential) and inverse temperature \( \beta = 1/T \) (both being free parameters) [10]. Notice that with only delta functions present in (9) one would have a situation in which our hadronizing system would be described by some kind of white noise only. The integrals multiplying these delta functions and depending on \( (a) \) momentum characteristic of our heat bath \( \psi(k_\mu) \) (representing in our case, by definition, the hadronizing system) and \( (b) \) assumed bosonic statistics of produced secondaries resulting in factors \( n(\omega) \) and \( 1 + n(\omega) \), respectively, bring the description of our system closer to reality.

It should be stressed at this point that, contrary to the majority of discussions of BEC [3,4], we are working here directly in phase space [11], so far no space-time considerations were used. It is easy to realize now that the existence of BEC, i.e., the fact that \( C_2(Q) > 1 \), is strictly connected with nonzero values of the thermal averages (9). However, in the form presented here, they differ from zero only at one point, namely for \( Q = 0 \) (i.e., for \( k_\mu = k'_\mu \)). Actually, this is the price one pays for the QFT assumptions tacitly made here, namely for the finite spatial extension and for the uniformity of our reservoir. But we know from the experiment [3] that \( C_2(Q) \) reaches its maximum at \( Q = 0 \) and falls down towards its asymptotic value of \( C_2 = 1 \) at large of \( Q \) (actually already at \( Q \sim 1 \text{ GeV}/c \)). To reproduce the same behaviour by means of our approach here, one has to replace delta functions in eq. (9) by functions with supports larger than limited to a one point only. This means that such functions should not be infinite at \( Q_\mu = k_\mu - k'_\mu = 0 \) but remain more or less sharply peaked at this point, otherwise remaining finite and falling to zero at small, but finite, values of \( |Q_\mu| \) (actually the same as those at which \( C_2(Q) \) reaches unity):

\[
\delta(k_\mu - k'_\mu) \implies \Omega_0 \cdot \sqrt{\Omega(q = Q \cdot r)}.
\]

Here \( \Omega_0 \) has the same dimension as the \( \delta \) function (actually, it is nothing else but 4-dimensional volume restricting the space-time region of particle production) and \( \Omega(q) \) is a dimensionless smear function which contains the \( q \)-dependence we shall be interested in here. In this way we are tacitly introducing a new parameter, \( r_\mu \), a 4-vector such that \( \sqrt{(r_\mu)^2} \) has dimension of length and which makes the product \( Q \cdot r = Q_\mu r_\mu = q \) dimensionless. This defines the region of nonvanishing density of oscillators \( \hat{c} \), which we shall identify with the space-time extensions of the hadronizing source. The expression (11) has to be understood in a symbolic sense, i.e., that \( \Omega(Q \cdot r) \) is a function which in the limit of \( r \to \infty \) becomes strictly a \( \delta \) function. Making such replacement in eq. (9) one must also decide how to accordingly adjust \( n(\omega) \) occurring there because now, in general, \( \omega \neq \omega' \). In what follows we shall simply replace \( n(\omega) \to n(\bar{\omega}) \) with
\[ \tilde{\omega} = (\omega + \omega')/2 \] (which, for classical particles would mean that \( n(\omega) \rightarrow \sqrt{n(\omega)n(\omega')} \)).

In such way \( \tau \) becomes new (and from the phenomenological point of view also the most important) parameter entering here together with the whole function \( \Omega(Q \cdot r) \), to be deduced from comparison with experimental data [12]. With such a replacement one now has

\[ D(k_{\mu}, k'_{\mu}) = \frac{\sqrt{\tilde{\Omega}(q)}}{(1 + \alpha)(1 + \alpha')} \left[ \sqrt{\tilde{\Omega}(q)} + 2\sqrt{\alpha \alpha'} \right] \] (12)

where

\[ \tilde{\Omega}(q) = \gamma \cdot \Omega(q), \quad \gamma = \frac{n^2(\tilde{\omega})}{n(\omega)n(\omega')}, \quad \alpha \alpha' \propto \frac{P^2}{|\psi(k_{\mu})|^2 n(\omega)} \] (13)

with \( n(\omega) \) the same as defined above. The parameter \( \alpha \) is another very important parameter, which summarizes our knowledge of other than space-time characteristics of the hadronizing source (given by \( \Omega(q) \) introduced above). In particular it contains the external static force \( P \) present in the evolution equation (2). It is combined (in multiplicative way) with information on the momentum dependence of the reservoir (via \( |\psi(k_{\mu})|^2 \)) and on the single particle distributions of the produced particles (via \( n(\omega) = \mu_T \cosh y \) where \( \mu_T \) and \( y \) are, respectively, the transverse mass and rapidity). Notice that \( \alpha > 0 \) only when \( P \neq 0 \). Actually, for \( \alpha = 0 \) one has

\[ 1 < C_2(Q) < 1 + \gamma \Omega(Q \cdot r), \] (14)

i.e., it is contained between limits corresponding to very large (lower limit) and very small (upper limit) values of \( P \). Because of this \( \alpha \) plays the role of the coherence parameter [3,4]. For \( \gamma \simeq 1 \), neglecting the possible energy-momentum dependence of \( \alpha \) and assuming that \( \alpha' = \alpha \) one gets the expression

\[ C_2(Q) = 1 + \frac{2\alpha}{(1 + \alpha)^2} \cdot \sqrt{\tilde{\Omega}(q)} + \frac{1}{(1 + \alpha)^2} \cdot \Omega(q), \] (15)

which is formally identical with what has been obtained in [4] by means of QS approach. It has precisely the same form, consisting two \( Q \)-dependent terms containing the information on the shape of the source, one being the square of the other, each multiplied by some combination of the chaoticity parameter \( p = 1/(1 + \alpha) \) (however, in [4] \( p \) is defined as the ratio of the mean multiplicity of particles produced by the so called chaotic component of the source to the mean total multiplicity, \( p = \langle N_{ch} \rangle / \langle N \rangle \)). In fact, because in general \( \alpha \neq \alpha' \) (due to the fact that \( \omega \neq \omega' \) and therefore the number of states, identified here with the number of particles with given energy, \( n(\omega) \), are also different) one should rather use the general form (5) for \( C_2 \) with details given by (12) and (13) and with \( \alpha \) depending on such characteristics of the production process as temperature \( T \) and chemical potential \( \mu \) occurring in definition of \( n(\omega) \).

Notice that eq. (15) differs from the usual empirical parameterization of \( C_2(Q) \) [3],

\[ C_2(Q) = 1 + \lambda \cdot \Omega(Q \cdot r), \] (16)

with \( 0 < \lambda < 1 \) being a free parameter adjusting the observed value of \( C_2(Q = 0) \), which is customary called "incoherence", and with \( \Omega(Q \cdot r) \) represented usually as Gaussian. Recently eq. (16) has found strong theoretical support expressed in great detail in [5]. The natural question arises: which of the two formulas presented here is correct? The answer is: both are right in their own way. This is because each of them is based on different ways of defining coherence of the source. In [5] one uses the notion of coherently and chaotically produced particles or, in other words, one divides hadronizing source into coherent and chaotic subsources. In [4] one introduces instead the notion of partially coherent fields representing produced particles, i.e., one has only one source, which produces partially coherent fields. Our approach is similar as we describe our particle by operator \( b(\hat{k}, t) \), which consists of two parts, cf. eq. (1), one of which depends on the external static force \( P \). The action of this force is to order phases of particles in our source (represented by the heat bath). The strength of this ordering depends on the value of the external force \( P \). In any case, for \( P \neq 0 \), it demonstrates itself as a partial coherence [13].

Let us return to the problem of \( Q \)-dependence of BEC. One more remark is in order here. The problem with the \( \delta(k_{\mu} - k'_{\mu}) \) function encountered in two particle distributions does not exist in the single particle distributions, which are in our case given by eq. (7) and which can be written as \( \tilde{f}(k_{\mu}) \propto \langle \tilde{a}^\dagger(k_{\mu}) \tilde{a}(k_{\mu}) \rangle + |\tilde{R}(k_{\mu})|^2 \sim (1 + \alpha') \langle \tilde{a}^\dagger(k_{\mu}) \tilde{a}(k_{\mu}) \rangle [14] \). To be more precise

\[ \tilde{f}(k_{\mu}) = (1 + \alpha) \cdot \Xi(k_{\mu}, k_{\mu}), \] (17)

where \( \Xi(k_{\mu}, k_{\mu}) \) is one-particle distribution function for the "free" (undistorted) operator \( \tilde{a}(k_{\mu}) \) equal to

\[ \Xi(k_{\mu}, k_{\mu}) = \Omega_0 \cdot \left| \frac{\psi(k_{\mu})}{K(k_{\mu}) - \omega} \right|^2 n(\omega). \] (18)

Notice that the actual shape of \( \tilde{f}(k_{\mu}) \) is dictated both by \( n(\omega) = n(\omega; T, \mu) \) (calculated for fixed temperature \( T \) and chemical potential \( \mu \) at energy \( \tilde{\omega} \) as given by the Fourier transform of random field operator \( \tilde{K} \) and by shape of the reservoir in the momentum space provided by \( \psi(k_{\mu}) \)) and by external force \( P \) in parameter \( \alpha \). They are both unknown, but because these details do not enter the BEC function \( C_2(Q) \), we shall not pursue this problem further. What is important for us at the moment is that both the coherent and the incoherent part of the source have the same energy-momentum dependence (whereas in other approaches mentioned here they were usually assumed to be different). On the other hand it is clear from (17) that \( \langle N \rangle = \langle N_{ch} \rangle + \langle N_{coh} \rangle \) (where...
\( \langle N_{ch} \rangle \) and \( \langle N_{coh} \rangle \) denote multiplicities of particles produced chaotically and coherently, respectively) therefore justifying definition of chaoticity \( p \) mentioned above.

For an illustration we plotted in Fig. 1 the correlation function \( C_2(Q) \) as given by eq. (15) for different choices of \( \Omega(q) \) corresponding to different hadronizing sources discussed in the literature [15–17] (here \( r = |r_\mu| = \sqrt{r_\mu^2} \):

- Gaussian: \( \Omega(q) = \exp \left( -Q^2 r^2 \right) \);
- exponential: \( \Omega(q) = \exp(-Qr) \);
- Lorentzian: \( \Omega(q) = 1/ (1 + qr)^2 \);
- given by Bessel function [16]: \( \Omega(q) = [J_4(qr)/(qr)]^2 \).

All curves are drawn for the same values of the size parameter \( r = 1 \text{ fm} \) and assuming for simplicity constant and equal values of \( \alpha \) and \( \alpha' \) parameters (i.e., using simplified eq. (15)), which have been put equal \( \alpha = 0.2 \) here, just for illustrational purpose (it corresponds to \( p = 0.8 \) in [4]). Fig. 2 shows in detail (using Gaussian shape of \( \Omega(q) \) function) the dependence of \( C_2(Q) \) on different values of \( \alpha = 0, 0.25, 1, 4 \) (again, used in the same approximate way as before and corresponding to \( p = 1, 0.8, 0.5, 0.2 \)) and compare it to the case when the second term in eq. (15) is neglected, as is the case in majority of phenomenological fits to data.

To summarize: using a specific version of QFT supplemented by Langevin evolution equation (2) to describe hadronization process [1,2] we have derived the usual BEC correlation function in the form explicitly showing the origin of both the so-called coherence (and how it influences the structure of BEC) and the \( Q \)-dependence of BEC represented by correlation function \( C_2(Q) \). The dynamical source of coherence is identified in our case with the existence of a constant external term \( P \) in the Langevin equation. Its influence turns out to be identical with the one obtained before in the QS approach [4] and is described by eq. (12). Its action is to order phases of the produced secondaries. Therefore for \( P \to \infty \) we have all phases aligned in the same way and \( C_2(Q) = 1 \).

This is because both here and in [4] the coherence has already been introduced on the level of a hadronizing source, as property of fields (in [4]) or operators describing produced particles. Dividing instead the hadronizing source itself into coherent and chaotic subsources leads to results obtained in [5] and given by eq. (16). The controversy between results given by [4] and [5] is therefore explained: both approaches are right, one should only remember that they use different descriptions of the notion of coherence. It is therefore up to the experiment to decide which proposition is followed by nature: the simpler formula (16) or rather the more involved (5) together with (12). From Fig. 2 one can see that differences between both forms are clearly visible, especially for larger values of coherence \( \alpha \), i.e., for lower chaoticity parameter \( p \).
From our presentation it is also clear that the form of \( C_2 \) reflects distributions of the space-time separation between the two observed particles rather than the distribution of their separate production points \([18]\) (i.e., it is Fourier transform of two-particle density profile of the hadronizing source, \( \rho(r_1, r_2) = \rho(r_1 - r_2) \), without approximating it by the product of single-particle densities, as in \([3]\)).

Finally, we would like to stress that our discussion is so far limited to only a single type of secondaries being produced. It is also aimed at a description of hadronization understood as kinetic freeze-out in some more detailed approaches. So far we were not interested in the other (highly model dependent) details of the particle production process. This is enough to obtain our general goals, i.e., to explain the possible dynamical origin of coherence in BEC and the origin of the specific shape of the correlation \( C_2(Q) \) functions as seen from the QFT perspective. Actually, our source of coherence should be regarded as being only one possibility \([4, \text{[8]}]\), the others were discussed in detail in \([5]\). It is then plausible that in general description of the BEC effect they should be somehow combined, especially if experimental data would indicate such necessity. But to do so our approach should first be generalized to allowing, as is the case in \([5]\), for production of different types of secondaries and allow also for resonance production and final state interactions (both of strong and Coulomb origin). This is, however, outside the scope of the present paper.

We would like to acknowledge support obtained from the Bogolyubov-Infeld program in JINR and partial support of the Polish State Committee for Scientific Research (KBN), grants 621/E-78/SPUB/CERN/P-03/DZ4/99 and 2P03B05724.

To obtain such level of description we would have to specify in more detail all ingredients of eq. (2), which would lead us out of the scope of this note.

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[6] Actually, careful inspection of all previous approaches to BEC using QFT (cf., for example, \([7]\) ) shows that this was always the procedure used, though never expressed so explicitly as is done here.
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[8] This means that one demands that \( \langle K^+(\vec{k}, t)K(\vec{k}', t) \rangle = 2\sqrt{\pi} \rho \delta(\vec{k} - \vec{k}') \) where \( \kappa \) and \( \rho \) are parameters defining the effect on the particle evolution in thermal environment caused by the assumed (here) Gaussian noise \([2]\).
[9] The only condition function \( \psi(k_\mu) \) is subjected to is a kind of normalization involving also dissipative forces represented by Fourier transformed operator \( \vec{K}(k_\mu) \), namely: \( \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\psi(k_\mu)}{\vec{R}(k_\mu)} = 1 \).
[10] The origin of the two parameters occurring at this point, the temperature \( \beta = 1/T \) and chemical potential \( \mu \) is the Kubo-Martin-Schwinger condition that \( \langle a(\vec{k}', t) a^+(\vec{k}, t) \rangle = \langle a^+(\vec{k}, t) a(\vec{k}', t - i\beta) \rangle \exp(-\beta \mu) \), see [1,2]. This form of averages reflects the corresponding averages for the \( \vec{c}(k_\mu) \) operators, namely that \( \langle \vec{c}^+(k_\mu) \vec{c}(k'_\mu) \rangle = \delta (k_\mu - k'_\mu) \cdot n(\omega) \) and \( \langle \vec{c}(k_\mu) \vec{c}^+(k'_\mu) \rangle = \delta^2 (k_\mu - k'_\mu) \cdot [1 + n(\omega)] \).
[11] As, for example, in T.Osada, M.Maruyama and F.Takagi, Phys. Rev. D59 (1998) 014024 or O.V.Utyuzh, G.Wilk and Z.Włodarczyk, Phys. Lett. B522 (2001) 273.
[12] The opposite line of reasoning has been used in paper by K.Zalewski, Lecture Notes in Physics 539, (2000), Springer, p.291 where at first a kind of our \( \Omega(q) \) function was constructed for a finite source function and it was demonstrated that in the limit of infinite, homogeneous source one ends with a delta function.
[13] It is of the same type as that considered in \([4]\). When comparing with \([5]\) one should notice that although our operators \( \vec{a}(k_\mu) \) and \( \vec{R}(k_\mu) \) look similar to operators defined in eqs. (4) and (5) of \([5]\) they differ in the following. Our \( \vec{R}(k_\mu) \) describes essentially the action of constant force \( P \) and as such it commutes with all other operators (including themselves). So it only introduces a partial ordering of phases of particles decreasing the \( C_2 \) correlation function, i.e., acting as a coherent component, albeit we do not have coherent particles as such. It is also seen when realizing that in eq. (7) the two last terms contain only one pair of operators \( a \). This in the language of \([5]\) translates to only one Wigner function, \( f_{\text{coh}} \), to be present here. The operators \( R \) cannot form the second Wigner function \( f_{\text{coh}} \) in \([5]\). This is the technical origin of the three terms present in (15) (and in \([4]\) ) in comparison to two terms in (16) and obtained in \([5]\).
[14] Notice that it is normalized to the mean multiplicity, i.e., \( \int d^4k \, f(k_\mu) = \langle N \rangle \).
[15] Cf., for example, R.Shimoda, M.Biyajima and N.Suzuki, Prog. Theor. Phys. 89 (1993) 697; T.Mizoguchi,
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