Mass Spectra and Semileptonic Decays of Doubly Heavy $\Xi$ and $\Omega$ Baryons

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Abstract

In the framework of a non-relativistic quark model, the mass spectra of the ground and excited states of doubly heavy $\Xi$ and $\Omega$ baryons are calculated. We estimate the mass difference between the $\Omega$ and corresponding $\Xi$ baryons as $M_{\Omega} - M_{\Xi} \simeq 178$ MeV for all the states containing $cc$, $bc$, or $bb$ quarks. A simple form of the universal Isgur-Wise function, as the transition form factor between the doubly heavy baryons, is introduced. Working in the close-to-zero recoil limit, we investigate the $b \to c$ semileptonic decay widths and branching fractions of the doubly heavy baryons. The obtained results are compared with other theoretical predictions.

1 Introduction

The investigation of doubly heavy baryons is of great interest in understanding quantum chromodynamics (QCD) inspired potential model, the non-relativistic QCD factorization theory, etc., at the hadronic scale. During the last few years many theoretical progresses have been achieved in doubly heavy baryon spectroscopy [1-5]. In 2002, the lightest doubly charmed baryon $\Xi_{cc}^+$ was observed with a statistical significance of 6.3$\sigma$ with a measured mass of $(3519 \pm 1)$ MeV in the decay mode $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ by the SELEX collaboration [6]. This state was subsequently confirmed by the same collaboration in another decay mode $\Xi_{cc}^+ \to p D^+ K^-$ [7]. However, negative results in searching for the $\Xi_{cc}^+$ were reported by FOCUS [8], BaBar [9], or Belle [10] collaborations. In 2017, the doubly charmed baryon $\Xi_{bc}^{++}$ was first observed by the LHCb collaboration via the decay mode $\Xi_{bc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ with a measured mass $(3621.40 \pm 0.72 \pm 0.14 \pm 0.27)$ MeV [11], where the uncertainties are statistical, systematic, and from the limited knowledge of the $\Lambda_c^+$ mass, respectively, and confirmed in another decay mode $\Xi_{cc}^{++} \to \Xi_c^{++} \pi^+$ [12]. The measured mass is about 100 MeV higher than that of $\Xi_{cc}^+$ determined by the SELEX collaboration [6]. The lifetime of the $\Xi_{cc}^{++}$ was measured to be $\tau(\Xi_{cc}^{++}) = (0.256^{+0.024}_{-0.022} \pm 0.014)$ ps [13]. The $\Xi_{cc}^{++}$ observation demonstrates how really the LHC is a powerful discovery machine, stimulating the theoretical studies of mass spectra of doubly heavy baryons. Therefore, the predictions for doubly heavy baryon masses have become a subject of renewed interest.

Other than the double-charm $\Xi_{cc}$ baryons, the beauty-charm and also double-beauty baryons are the different kinds of doubly heavy baryons, which have not been found yet. Recently, several experimental efforts have been made on the exclusive channels $\Xi_{bc}^0 \to D^0 p K^-$ [14] and $\Xi_{bc}^+ \to \Xi^+ \pi^-$ [15] to search for the beauty-charm baryons, but no signals were observed. The $\Xi_{bc}$ baryons are expected to have smaller sizes than the $\Xi_{cc}$ states. In comparison to the $\Xi_{cc}$, searching for the $\Xi_{bc}$ are more complicated and fewer states

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could be produced at the LHC. To overcome this difficulty, an inclusive decay channel $\Xi_{bc} \to \Xi_{cc}^{++} + X$ has been proposed \[16\], where $X$ stands for all the possible particles.

For the singly charmed baryons, some semileptonic decays have been calculated in theory \[17, 18, 19, 20, 21, 22\] and measured in experiments \[23, 24, 25, 26, 27\]. However, for the doubly heavy baryons, no experimental data on semileptonic decays are reported and only a limited number of theoretical calculations are available. As there are more doubly heavy baryons that may be discovered in the future, proposing a theoretical model for their structures is essential. The investigations of the inclusive and exclusive semileptonic decays of doubly heavy baryons are of special interest for two basic reasons: they play an important role in the calculation of fundamental parameters of the electroweak standard model and provide a useful tool to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{cb}$. Some characteristics of semileptonic decays, such as the transition form factors and exclusive decay rates, also provide information about the internal structures of heavy baryons and the strong and weak interactions inside them.

Some theoretical calculations to the semileptonic weak decays of doubly heavy baryons have been provided based on the QCD sum rules (QCDSR) \[28, 29\], covariant confined quark model \[30\], Bethe-Salpeter equation \[31\], heavy quark spin symmetry (HQSS) \[32\], relativistic quark model (RQM) \[33, 34\], non-relativistic quark model (NRQM) \[35\], or heavy diquark effective theory (EFT) \[16, 36\]. However, the calculations of semileptonic decays presented by different approaches lead to essentially different values for the decay widths.

In this paper, we present a description of the properties of heavy baryons containing two heavy quarks (charm or beauty quark) within the framework of non-relativistic quark model proposed in Ref. \[37\]. Firstly, we calculate the mass spectra of the ground and orbitally excited states of doubly heavy $\Sigma$ and $\Omega$ baryons, including $\Sigma_{cc}$, $\Sigma_{bc}$, $\Sigma_{bb}$, $\Omega_{cc}$, $\Omega_{bc}$, and $\Omega_{bb}$ states, in the hypercentral constituent quark model. Then, we focus on the studies of $b \to c$ semileptonic decays of the ground states of doubly heavy $\Sigma$ and $\Omega$ baryons. We proceed our model close to the zero recoil point in which the form factors of these transitions can be expressed by a few universal functions \[38\]. Considering the doubly baryon states can be produced sizably at LHC, especially LHC has started Run3 data taking, our results can be tested in the near future.

This paper is structured as follows. In Section 2 we present our predictions for the ground and also orbitally excited states of the doubly heavy baryons using the results obtained in our previous work. In Section 3 we introduce a universal function as the transition form factor to study the semileptonic decays of doubly charm and bottom heavy baryons close to the zero recoil point and present our numerical results for semileptonic decay widths and branching fractions of specific modes. Section 4 includes a short summary.

2 Doubly heavy baryon spectra

We start with a brief review of our previous work \[37\] in which the mass spectra and radiative transitions of $\Sigma_b$ and $\Lambda_b$ baryons are studied within the hypercentral constituent quark model \[39, 40, 41, 42, 43, 44\]. The hypercentral quark model contains a few free parameters, whose values are obtained by fitting the baryon spectrum. Once the parameters are obtained, the model is completely determined and we can make our predictions for the baryon properties. In Ref. \[37\] we introduced a phenomenological potential model, and solved the baryonic three-body equation in a non-relativistic limit by choosing the Killingbeck potential $V(x) = ax^2 + bx - \frac{c}{x}$. The potential parameters $a$, $b$, and $c$ are constant.

We apply the Ansatz method \[45, 46, 47\] to solve the Schrödinger equation. In the hypercentral approach, the hyperradial part of the wave function is determined by

$$\frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \frac{\gamma(\gamma + 4)}{x^2} \psi_\gamma(x) = -2m[E_\gamma - V(x)]\psi_\gamma(x).$$

(1)

Assuming the transformation $\psi(x) = x^{-5/2} \phi(x)$ we get

$$\phi''(x) + \left[\varepsilon - a_1 x^2 - b_1 x + c_1 x - \frac{(2\gamma + 3)(2\gamma + 5)}{4x^2}\right] \phi_\gamma(x) = 0.$$  

(2)

We make use of the following Ansatz
\[ \phi_\gamma(x) = \exp[-\frac{1}{2}(ax^2 - \beta x + \delta \ln x)], \]  

(3)

where \( \alpha, \beta \) and \( \delta \) parameters are determined in terms of the potential parameters. By substitution of equation 3 into equation 2 and comparing the coefficient of \( x \) on both sides of the new equation we can evaluate the energy eigenvalues and normalized eigenfunctions for the baryon states as follows (for details see Refs. [37, 47]):

\[ E_\gamma = (2\gamma + 6) \frac{w}{2} - \frac{2mc^2}{(2\gamma + 5)^2} \]

(4)

and

\[ \psi_\gamma(x) = N_\gamma x^{-\frac{3}{2}} \phi_\gamma(x) = N_\gamma x^\gamma \exp(-\frac{mw}{2}x^2 - \frac{2mc}{(2\gamma + 5)}x), \]

(5)

where \( x \) is the hyperradius and \( w = \sqrt{\frac{2m}{m}} \) is the oscillating frequency. \( m, \gamma \) and \( N_\gamma \) are the reduced mass, angular quantum number and normalization constant, respectively. Baryon mass is obtained by summing the quark masses, energy eigenvalues, and hyperfine interaction potential treated as a perturbation:

\[ M_{baryon} = \sum_{i=1}^{3} m_i + E_\gamma + \langle H_S \rangle. \]

(6)

Here, \( \langle H_S \rangle \) is the expectation value of the hyperfine spin-spin interactions given as [37, 46]

\[ H_S = \Sigma_{i<j} A_S \left( \frac{1}{\sqrt{\pi} \sigma_S} \right)^3 \exp(-\frac{x^2}{\sigma_S^2})(\vec{s_i} \cdot \vec{s_j}), \]

(7)

where \( \vec{s_i} \) is the spin operator of the \( i^{th} \) quark, and \( \sigma_S \) and \( A_S \) are constant. The isospin values of the strange and heavy quarks are zero and therefore, in the case of doubly heavy baryons the isospin dependent terms are not included in the hyperfine interactions. The spin-orbit interaction has values smaller than 0.01 GeV in the hyperfine contributions and therefore, we neglect it in our calculations. All of the model parameters listed in Table 1 are taken from our previous work [37]. We take the experimentally measured mass of the \( \Xi_{cc}^{++} \) to determine the mass of the charm quark. Our calculated results of the masses of the ground and excited baryon states including \( P \)-wave, \( D \)-wave, and \( F \)-wave are listed in Tables 2 and 3 and compared with the ones obtained from different models of RQM, NRQM, QCDSR, HQSS, and EFT [48, 49, 50, 51, 52, 53, 54, 55]. Table 2 shows that our results for the masses of the \( \Xi_{cc}, \Xi_{bc}, \Omega_{cc}, \) and \( \Omega_{bc} \) are in good agreement with those reported in Ref. [48]. For the masses of the \( \Xi_{bb} \) and \( \Omega_{bb} \) states, our predictions are close to the results from Refs. [50, 51, 52].

Table 1: Quark-model parameters, where \( q \) refers to the light quarks.

| Parameter | Value     | Parameter | Value     |
|-----------|-----------|-----------|-----------|
| \( m_q \) | 320 MeV   | \( w \)   | 1.557 fm$^{-1}$ |
| \( m_s \) | 440 MeV   | \( b \)   | 5.79 fm$^{-2}$  |
| \( m_c \) | 1360 MeV  | \( A_s \) | 67.4 fm$^2$     |
| \( m_b \) | 4670 MeV  | \( \sigma_S \)| 4.76 fm      |
Table 2: Masses of the ground states of doubly heavy baryons (in GeV). The sign “∗” refers to the \( s = \frac{3}{2} \) baryons.

| Baryon | Content | Our results | RQM \[48\] | HQSS \[49\] | QCDSR \[50\] | NRQM \[51\] | NRQM \[52\] |
|--------|---------|-------------|-------------|-------------|-------------|-------------|-------------|
| \( \Xi_{cc} \) | qcc     | 3.620       | 3.620       | 3.613       | 3.676       | 3.685       |
| \( \Xi^*_{cc} \) | qcc     | 3.653       | 3.727       | 3.707       | 3.690       | 3.753       | 3.754       |
| \( \Xi_{bc} \) | qbc     | 6.958       | 6.933       | 6.928       | 7.020       |
| \( \Xi^*_{bc} \) | qbc     | 6.991       | 6.980       | 6.996       | 7.250       | 7.078       |
| \( \Xi_{bb} \) | qbb     | 10.322      | 10.202      | 10.198      | 10.340      | 10.314      |
| \( \Xi^*_{bb} \) | qbb     | 10.355      | 10.237      | 10.237      | 10.400      | 10.367      | 10.339      |
| \( \Omega_{cc} \) | cc      | 6.958       | 6.933       | 6.928       | 7.020       |
| \( \Omega^*_{cc} \) | cc      | 6.991       | 6.980       | 6.996       | 7.250       | 7.078       |
| \( \Omega_{bc} \) | bc      | 10.322      | 10.202      | 10.198      | 10.340      | 10.314      |
| \( \Omega^*_{bc} \) | bc      | 10.355      | 10.237      | 10.237      | 10.400      | 10.367      | 10.339      |
| \( \Omega_{bb} \) | bb      | 10.322      | 10.202      | 10.198      | 10.340      | 10.314      |
| \( \Omega^*_{bb} \) | bb      | 10.355      | 10.237      | 10.237      | 10.400      | 10.367      | 10.339      |

Using the masses from our calculations shown in Table 2, we can get the mass differences \( \Delta M \) between \( \Omega \) and corresponding \( \Xi \) doubly heavy baryons:

\[
\begin{align*}
M_{\Omega_{bb}} - M_{\Xi_{bb}} &= 178 \text{ MeV}, \\
M_{\Omega_{cc}} - M_{\Xi_{cc}} &= 178 \text{ MeV}, \\
M_{\Omega_{bc}} - M_{\Xi_{bc}} &= 179 \text{ MeV}.
\end{align*}
\]
We obtain $\Delta_M \sim 178$ MeV for all of the $M_{\Omega} - M_{\Xi}$ splittings, compared with $\Delta_M = 100 \pm 10$ MeV reported in Ref. [28] and $\Delta_M = 155 \sim 158$ MeV predicted in Ref. [48]. In other works referenced in Table 2 $\Delta_M$ has different values for the different $M_{\Omega} - M_{\Xi}$ splittings.

3 Semileptonic decays of doubly heavy baryons

To study the semileptonic transitions of baryons we need the form factors, which can be parameterized in different approaches. Some earlier works [38, 50, 57, 58] simplified the transition form factors using different methods. The authors of Ref. [38] studied the form factors and semileptonic decays of doubly heavy baryons using a relativistic covariant quark model (RCQM). According to Refs. [38, 59, 60], in the heavy quark limit, the expressions for the decay rates can be simplified and the weak transition form factors between doubly heavy baryons can be expressed through the single Isgur-Wise (IW) function

$$ F_1(\omega) = G_1(\omega) = \eta(\omega), $$

$$ F_2(\omega) = F_3(\omega) = G_2(\omega) = G_3(\omega) = 0. $$

In Ref. [38] a closed-form expression has been derived for the IW function $\eta(\omega)$ using a Gaussian Ansatz for the three-quark correlation function in the heavy quark limit and close to zero recoil point. In the present work we take the universal function $\eta(\omega)$ calculated by Ref. [38] which depends on the kinematical parameter $\omega$ and is given by

$$ \eta(\omega) = \exp(-3(\omega - 1)\frac{m_{cc}}{\Lambda_B^2}) $$

with $m_{cc} = 2m_c$ for the $bc \rightarrow cc$ weak transitions [38]. The parameter $\Lambda_B$ characterizes the size of the given baryon and represents the extension of the distribution of the constituent quarks in the baryon. For the doubly heavy baryons the size of parameter $\Lambda_B$ is allowed to vary in the range $2.5 \leq \Lambda_B \leq 3.5$ GeV [38]. The values of parameter $\Lambda_B$ are fixed using data on fundamental properties of mesons and baryons such as leptonic decay constants, magnetic moments, and radii. The dependence of the universal $\eta(\omega)$ function on $\omega$ with $\Lambda_B = 3$ for the $bb \rightarrow bc$ transitions is shown in Fig. 1. For the slope $\rho^2$ of the $\eta(\omega) = 1 - \rho^2(\omega - 1) + ...$, one obtains

$$ \rho^2 = -\frac{d\eta(\omega)}{d\omega} |_{\omega=1} = \frac{3m_{cc}^2}{\Lambda_B^2}. $$

By replacing $m_{cc}$ with $m_{bb}$ in the IW function one can obtain the results for the $bb \rightarrow bc$ transitions. Accordingly, the slope of the IW function for the $bb \rightarrow bc$ transitions is obtained from Eq. (12) by replacing $m_{cc}$ with $m_{bb}$ if one uses the same size of parameter $\Lambda_B$ in both cases [38]. Close to zero recoil, the IW functions for $bb \rightarrow bc$ and $bc \rightarrow cc$ transitions explicitly contain the flavor factors $m_{cc}$ and $m_{bb}$, and there exists only spin symmetry. There is no dependence on the light quark masses. At zero recoil ($\omega = 1$) there exists a spin-flavor symmetry and $\eta(1) = 1$ means that $bb \rightarrow bc$ and $bc \rightarrow cc$ transitions are identical.

According to Ref. [38], in zero recoil limit, the expressions for the semileptonic decay widths can be simplified considerably and we can get the decay rates using the IW function $\eta(\omega)$ and the following relations:

$$ \Gamma_{\frac{1}{2} \rightarrow \frac{1}{2}} = \frac{G_F^2 |V_{bc}|^2 M_{cc}^2 r^4}{12\pi^3} \int_{\omega_1}^{\omega_{\text{max}}} d\omega \sqrt{\omega^2 - 1} \left( l_1(\omega) \eta^2(\omega) + l_2(\omega) \eta(\omega) \right), $$

$$ \Gamma_{\frac{3}{2} \rightarrow \frac{1}{2}} = \frac{G_F^2 |V_{bc}|^2 M_{cc}^2 r^4}{12\pi^3} \int_{\omega_1}^{\omega_{\text{max}}} d\omega \sqrt{\omega^2 - 1} l_2(\omega) \eta^2(\omega), $$

$$ \Gamma_{\frac{1}{2} \rightarrow \frac{3}{2}} = \frac{G_F^2 |V_{bc}|^2 M_{cc}^2 r^4}{24\pi^3} \int_{\omega_1}^{\omega_{\text{max}}} d\omega \sqrt{\omega^2 - 1} l_3(\omega) \eta^2(\omega), $$

$$ \Gamma_{\frac{5}{2} \rightarrow \frac{1}{2}} = \frac{G_F^2 |V_{bc}|^2 M_{cc}^2 r^4}{24\pi^3} \int_{\omega_1}^{\omega_{\text{max}}} d\omega \sqrt{\omega^2 - 1} \left( l_4(\omega) \eta^2(\omega) + l_5(\omega) \eta(\omega) \right). $$
The parameter $G_F$ is the Fermi Coupling constant, $V_{bc}$ is the CKM matrix element and its value is $V_{bc} = 0.04$, $M_1$ and $M_2$ are the initial and final state baryon masses, respectively. We take the following values of $\omega_{\text{max}}$ for different transitions:

$\omega_{\text{max}}[\Xi_{bb} \to \Xi_{bc}] = 1.07$, $\omega_{\text{max}}[\Omega_{bb} \to \Omega_{bc}] = 1.07$, $\omega_{\text{max}}[\Xi_{bc} \to \Xi_{cs}] = 1.22$, $\omega_{\text{max}}[\Omega_{bc} \to \Omega_{cc}] = 1.20$.

Using the obtained masses listed in Table 2 and Eqs. (14)-(21), the semileptonic decay rates of doubly heavy $\Xi$ and $\Omega$ baryons are calculated. The $\omega$ dependence of the semileptonic decay widths for $\Omega_{bb} \to \Omega_{bc}\ell\bar{\nu}_\ell$, $\Xi_{bb} \to \Xi_{bc}\ell\bar{\nu}_\ell$, $\Omega_{bc} \to \Omega_{cc}\ell\bar{\nu}_\ell$, and $\Xi_{bc} \to \Xi_{cc}\ell\bar{\nu}_\ell$ transitions is shown in Fig. 2 and 3, respectively, where we take $\Lambda_B = 3$ GeV and neglect the mass difference between the $u$ and $d$ quarks. Regarding $\omega = \omega_{\text{max}}$, the $\Lambda_B$ dependence of $\eta$ functions for $\Omega_{bb} \to \Omega_{bc}$ and $\Xi_{bb} \to \Xi_{bc}$ transitions is shown in Fig. 4. Note that a smaller value of $\Lambda_B$ gives smaller decay rates and vice versa.
Figure 2: Behavior of semileptonic decay widths versus $\omega$ for $\Omega_{bb} \rightarrow \Omega_{bc}\ell\bar{\nu}_\ell$ and $\Xi_{bb} \rightarrow \Xi_{bc}\ell\bar{\nu}_\ell \ (\ell = e \text{ or } \mu)$ transitions ($\Lambda_B = 3$ GeV).

Figure 3: Behavior of semileptonic decay width versus $\omega$ for $\Omega_{bc} \rightarrow \Omega_{cc}\ell\bar{\nu}_\ell$ and $\Xi_{bc} \rightarrow \Xi_{cc}\ell\bar{\nu}_\ell \ (\ell = e \text{ or } \mu)$ transitions ($\Lambda_B = 3$ GeV).
by fitting to the experimental data [37] can be neglected. A comparison between our results and those

The calculated semileptonic decay widths of doubly heavy baryons and their variations are summarized in Table 4. The uncertainties in decay widths are due to the parameter \( \Lambda_B \), which can take a value in the range \( 2.5 \leq \Lambda_B \leq 3.5 \) GeV. The uncertainties from the quark masses and potential parameters determined by fitting to the experimental data [37] can be neglected. A comparison between our results and those derived by Refs. [32, 38, 61, 62] is also presented. For \( s = \frac{1}{2} \rightarrow s = \frac{1}{2} \) transitions, including \( \Xi_{bb} \rightarrow \Xi_{bc} \), \( \Omega_{bb} \rightarrow \Omega_{bc} \), and \( \Omega_{bc} \rightarrow \Omega_{cc} \), our results are in good agreement with those of Ref. [38]. In the case of \( \Xi_{bb} \rightarrow \Xi_{bc} \) and \( \Omega_{bb} \rightarrow \Omega_{bc} \) transitions our results are close to those of Ref. [48].

Table 4: Semileptonic decay widths of doubly heavy baryons in units of \( 10^{-14} \) GeV and their variations.

| Decay       | Our results | RCQM [38] | NRQM [61] | RQM [48] | HQSS [32] | LFQM [62] |
|-------------|-------------|-----------|-----------|-----------|-----------|-----------|
| \( \Xi_{bb} \rightarrow \Xi_{bc} \ell \bar{\nu}_\ell \) | 1.75 ± 0.73 | 1.33 ± 0.61 | 1.37      | 1.63      | 1.93 ± 0.25 | 3.30      |
| \( \Xi_{bc} \rightarrow \Xi_{cc} \ell \bar{\nu}_\ell \) | 4.39 ± 0.83 | 4.01 ± 1.21 | 5.07      | 2.30      | 2.57 ± 0.26 | 4.50      |
| \( \tilde{\Xi}_{bc} \rightarrow \Xi_{bc} \ell \bar{\nu}_\ell \) | 0.49 ± 0.18 | 0.25 ± 0.10 | 0.66      | 0.28      | 0.35 ± 0.03 | 0.59      |
| \( \tilde{\Xi}_{bc} \rightarrow \Xi_{cc} \ell \bar{\nu}_\ell \) | 1.00 ± 0.16 | 0.58 ± 0.14 | 1.16      | 0.38      | 0.43 ± 0.06 | 0.63      |
| \( \Xi_{bc} \rightarrow \Xi_{bc} \ell \bar{\nu}_\ell \) | 1.07 ± 0.43 | 0.61 ± 0.15 | 1.45      | 0.53      | 0.61 ± 0.04 | 0.87      |
| \( \Xi_{bc} \rightarrow \Xi_{cc} \ell \bar{\nu}_\ell \) | 2.82 ± 0.52 | 1.39 ± 0.34 | 3.32      | 0.72      | 0.75 ± 0.06 | 1.09      |
| \( \Xi_{bc} \rightarrow \Xi_{bc} \ell \bar{\nu}_\ell \) | 1.10 ± 0.48 | 1.62 ± 0.73 | 1.92      | 2.09 ± 0.16 | 2.39      |
| \( \Xi_{bc} \rightarrow \Xi_{cc} \ell \bar{\nu}_\ell \) | 2.62 ± 0.62 | 4.63 ± 1.23 | 2.69      | 2.63 ± 0.40 | 2.94      |

| Decay       | Our results | RCQM [38] | NRQM [61] | RQM [48] | HQSS [32] | LFQM [62] |
|-------------|-------------|-----------|-----------|-----------|-----------|-----------|
| \( \Omega_{bb} \rightarrow \Omega_{bc} \ell \bar{\nu}_\ell \) | 1.87 ± 0.76 | 1.92 ± 1.15 | 2.48      | 1.70      | 2.14 ± 0.20 | 3.69      |
| \( \Omega_{bc} \rightarrow \Omega_{cc} \ell \bar{\nu}_\ell \) | 4.70 ± 0.83 | 4.12 ± 1.10 | 5.39      | 2.48      | 2.59 ± 0.20 | 3.94      |
| \( \Omega_{bb} \rightarrow \Omega_{bc} \ell \bar{\nu}_\ell \) | 0.53 ± 0.19 | 0.26 ± 0.10 | 0.69      | 0.29      | 0.35 ± 0.04 | 0.59      |
| \( \Omega_{bc} \rightarrow \Omega_{cc} \ell \bar{\nu}_\ell \) | 1.09 ± 0.16 | 0.59 ± 0.13 | 1.23      | 0.40      | 0.44 ± 0.06 | 0.67      |
| \( \Omega_{bb} \rightarrow \Omega_{bc} \ell \bar{\nu}_\ell \) | 1.14 ± 0.45 | 0.57 ± 0.23 | 1.53      | 0.55      | 0.67 ± 0.08 | 0.99      |
| \( \Omega_{bc} \rightarrow \Omega_{cc} \ell \bar{\nu}_\ell \) | 2.99 ± 0.52 | 1.78 ± 0.64 | 3.52      | 0.74      | 0.76 ± 0.13 | 1.09      |
| \( \Omega_{bb} \rightarrow \Omega_{bc} \ell \bar{\nu}_\ell \) | 1.72 ± 0.50 | 1.72 ± 0.77 | 2.00      | 2.29 ± 0.31 | 2.59      |
| \( \Omega_{bc} \rightarrow \Omega_{cc} \ell \bar{\nu}_\ell \) | 3.32 ± 0.62 | 4.95 ± 1.26 | 2.88      | 2.79 ± 0.60 | 3.21      |

The absolute branching fractions of semileptonic decays of doubly heavy baryons can be easily derived
by using the following relation

\[ B = \Gamma \times \tau, \]  

(24)

where \( \tau \) is the lifetime of the initial baryon. Taking \( \tau_{\Xi_{bc}} = 370 \times 10^{-15} \text{s} \), \( \tau_{\Xi_{bb}} = 244 \times 10^{-15} \text{s} \), \( \tau_{\Omega_{bc}} = 220 \times 10^{-15} \text{s} \), and \( \tau_{\Omega_{bb}} = 800 \times 10^{-15} \text{s} \) as input values, the calculated branching fractions of semileptonic decays of doubly heavy baryons are summarized in Table 5.

| Process               | Our results | NRQM [61] | LFQM [62] |
|-----------------------|-------------|-----------|-----------|
| \( \Xi_{bc} \to \Xi_{cc} \ell \bar{\nu}_\ell \) | 1.63 \times 10^{-2} | 1.11 \times 10^{-2} | 1.67 \times 10^{-2} |
| \( \Xi_{bb} \to \Xi_{bc} \ell \bar{\nu}_\ell \) | 0.98 \times 10^{-2} | 0.28 \times 10^{-2} | 1.86 \times 10^{-2} |
| \( \Omega_{bc} \to \Omega_{cc} \ell \bar{\nu}_\ell \) | 1.57 \times 10^{-2} | 1.10 \times 10^{-2} | 1.32 \times 10^{-2} |
| \( \Omega_{bb} \to \Omega_{bc} \ell \bar{\nu}_\ell \) | 2.27 \times 10^{-2} | 4.49 \times 10^{-2} |

4 Summary

In this paper we present a phenomenological study of the mass spectra and semileptonic decays of doubly heavy \( \Xi \) and \( \Omega \) baryons. The three-body problem of these baryons is considered in the hypercentral approach. Applying an Ansatz method introduced in our previous work, the mass spectra of the ground and excited states of doubly heavy \( \Xi \) and \( \Omega \) baryons are obtained. By introducing a simple form for the universal IW function, we also investigate the semileptonic decay rates and branching fractions of the \( bb \to bc \) and \( bc \to cc \) baryonic transitions near to zero recoil point. We present a comparison between our results and other available theoretical calculations, and find that the results are acceptable. Note that the triplet doubly heavy baryons with \( s = \frac{3}{2} \) are dominated by the strong or electromagnetic decays. If these excited states are the initial ones, due to the smallness of the weak coupling, the weak decays can not be observed in the experiments. Therefore, one can shelve the calculations for the semileptonic decays of \( s = \frac{3}{2} \) doubly heavy baryons, since it is hard to perform relevant measurements in the experiments. In the current work, we have performed the related calculations for all the ground states of doubly heavy baryons as done in Refs. [38, 48, 32]. We hope our results are useful to extract the value of the CKM matrix element \( V_{cb} \) from future experiments via the semileptonic decays of doubly heavy baryons.

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