Anomalous Transient Current in Nonuniform Semiconductors

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Nonequilibrium processes in semiconductors are considered with highly nonuniform initial densities of charge carriers. It is shown that there exist such distributions of charge densities under which the electric current through a sample displays quite abnormal behaviour flowing against the applied voltage. The appearance of this negative electric current is a transient phenomenon occurring at the initial stage of the process. After this anomalous negative fluctuation, the electric current becomes normal, i.e. positive as soon as the charge density becomes more uniform. Several possibilities for the practical usage of this effect are suggested.

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\section*{I. INTRODUCTION}

The study of electric processes in semiconductor materials plays an important role in understanding the physics of semiconductor devices as well as in their design and development \cite{1,2}. One of the most difficult problems is the consideration of strongly nonequilibrium effects in essentially nonuniform semiconductors. At the same time the latter can display quite interesting specific features caused by nonuniform distributions of charge carriers \cite{3–6}.

For instance, electric current through a semiconductor device can display rather abnormal behaviour, with transient fluctuations corresponding to the flow of the current against the applied voltage \cite{5,6}. In the short communications \cite{5,6} a model case of a unipolar semiconductor was considered, with numerical analysis not including the relaxation parameters and diffusion coefficients.

The aim of the present paper is to study nonequilibrium processes, with a strongly nonuniform initial distribution of charge carriers, for realistic semiconductor materials. We consider the general case of a semiconductor with two kinds of charge carriers, positive and negative. Numerical analysis takes account of relaxation and diffusion effects. The total current through a semiconductor device is considered, together with the currents across the left and right surfaces of this device. And also the role of the generation–recombination noise is analysed.

In Sec.\textsuperscript{2} we collect the main equations defining the problem and needed for the following analysis. An approximate analytical solution of these equations is presented in Sec.\textsuperscript{3}, which permits to show explicitly the motion of charge carriers under an applied voltage. The conditions for the occurrence of the negative electric current, directed against this applied voltage, are derived in Sec.\textsuperscript{4}. This anomalous transient effect is illustrated by numerical solutions. Sec.\textsuperscript{5} contains conclusion and discussion with several suggestions for the possible practical usage of the considered effect. Appendix contains the proof that the approximate regular solution asymptotically coincides with the regular stable solution of the exact equations.

\section*{II. BASIC EQUATIONS}

The charge carriers are characterized by the densities $\rho_1 > 0$ and $\rho_2 < 0$ that are functions of the space position $\vec{r}$ and time $t$, i.e. $\rho_i = \rho_i(\vec{r}, t)$ with $i = 1, 2$. Carrier transport can be described in terms of a semiclassical approach called the drift–diffusion approximation which is the basis of the majority of semiconductor device models \cite{1,2}. This approach is, of course, phenomenological since it does not concern the microscopic derivation of the used parameters, such as mobilities or relaxation widths. The values of these parameters depend on a number
of different underlying causes. For instance, the lattice structure of the considered semiconductor is important for defining the values of these parameters. Thus, the scattering of carriers on phonons influences both mobilities as well as damping parameters. However, the calculation of such parameters is a separate problem that is not the aim of the present paper. In the semiclassical approach all parameters are assumed to be given a priori. With the given phenomenological parameters, the drift–diffusion approximation is known to give a very good description of realistic semiconductor devices, which explains why this approximation is so widely used [1,2]. The first set of equations in this approach consists of the set of continuity equations

\[
\frac{\partial \rho_i}{\partial t} + \nabla \cdot j_i + \frac{\rho_i}{\tau_i} = \xi_i \tag{1}
\]

for each kind of carriers. The relaxation term, with the relaxation time \(\tau_i\), is taken in the simplest form since, as will be clear from what follows, it does not play an essential role in transient processes occurring at times \(t \ll \tau_i\). The right–hand side of (1) is the generation–recombination noise [7] which is always present in semiconductor devices. Not to overcomplicate the problem we do not include other types of noise [7–9] assuming that they are of less importance.

Another equation is a Maxwell equation

\[
\varepsilon \nabla \cdot E = 4\pi(\rho_1 + \rho_2), \tag{2}
\]

with the dielectric permittivity \(\varepsilon\).

The electric–current density in (1) is

\[
\vec{j}_i = \mu_i \rho_i \vec{E} - D_i \nabla \rho_i, \tag{3}
\]

where \(\mu_1 > 0\) and \(\mu_2 < 0\) are the carrier mobilities and \(D_i \equiv (\mu_i/e_i)k_BT\) are the diffusion coefficients in which \(e_1 > 0\) and \(e_2 < 0\) are the carrier charges, \(k_B\) is the Boltzmann constant, and \(T\) is temperature. The first and second terms in (3) are the drift and diffusion current densities, respectively. Adding here the displacement current, one has the total current density

\[
\vec{j}_{\text{tot}} = \vec{j}_1 + \vec{j}_2 + \frac{\varepsilon}{4\pi} \frac{\partial \vec{E}}{\partial t}. \tag{4}
\]

If the device is biased with an externally applied voltage \(V_0\) along a path \(\vec{l}\), then

\[
\int_{\vec{l}} E(\vec{r}, t) \, d\vec{l} = V_0. \tag{5}
\]

For concreteness, we consider a positive bias, that is, \(V_0 > 0\). It is assumed that metal contacts supplying the external voltage are nondamaging, that is, do not induce in their vicinity incubation effects.

Consider a plane device of the width \(L\) and area \(A\). Then instead of \(\vec{r}\) we are to deal with one space variable \(x \in [0, L]\).

An important characteristic is the transit time

\[
\tau_0 \equiv \frac{L^2}{\mu V_0}, \tag{6}
\]

where \(\mu = \min\{\mu_1, |\mu_2|\}\). Usually, \(\mu_1 < |\mu_2|\). This time is to be compared with the relaxation times \(\tau_i\). It is often more convenient to deal with inverse times called widths. For instance,

\[
\gamma_i \equiv \frac{1}{\tau_i}, \tag{7}
\]

with \(i = 1, 2\), define the relaxation widths.

Another convenience is to deal with dimensionless quantities, which will be done in what follows. To return to dimensional quantities, we shall imply that \(x\) is measured in units of \(L\); \(t\) and \(\tau_i\), in units of \(\tau_0\); and other physical quantities, in the corresponding units listed below:

\[
D_0 \equiv \mu V_0, \quad E_0 \equiv \frac{V_0}{L}, \quad Q_0 \equiv \varepsilon A E_0,
\]
\[
\rho_0 = \frac{Q_0}{AL}, \quad \xi_0 = \frac{\rho_0}{\tau_0}, \quad J_0 = \frac{Q_0L}{\tau_0}.
\] (8)

For the case considered, Eq. (1) reduces to
\[
\frac{\partial \rho_i}{\partial t} + \mu_i \frac{\partial}{\partial x} (\rho_i E) - D_i \frac{\partial^2 \rho_i}{\partial x^2} + \gamma_i \rho_i = \xi_i,
\] (9)

and Eq. (2) becomes
\[
\frac{\partial E}{\partial x} = 4\pi (\rho_1 + \rho_2),
\] (10)

where
\[
0 < x < 1, \quad t > 0.
\] (11)

Condition (5) for an applied voltage reads
\[
\int_0^1 E(x, t) dx = 1.
\] (12)

These equations are to be supplemented by initial conditions
\[
\rho_i(x, 0) = f_i(x) \quad (i = 1, 2).
\] (13)

Electric field can be expressed, from Eq. (10), as a functional
\[
E(x, t) = 1 + 4\pi \left[ Q(x, t) - \int_0^1 Q(x, t) dx \right]
\] (14)

of the charge densities, so that
\[
Q(x, t) = \int_0^x [\rho_1(x', t) + \rho_2(x', t)] dx'.
\] (15)

And the total density of current (4) can be written as
\[
j_{tot} = \mu_1 \rho_1 E - D_1 \frac{\partial \rho_1}{\partial x} + \mu_2 \rho_2 E - D_2 \frac{\partial \rho_2}{\partial x} + \frac{1}{4\pi} \frac{\partial E}{\partial t},
\] (16)

where \( j_{tot} = j_{tot}(x, t) \).

The quantities that can be measured and that we are going to study in what follows are the current across the left surface
\[
J(0, t) \equiv j_{tot}(0, t),
\] (17)

the current across the right surface
\[
J(1, t) \equiv j_{tot}(1, t),
\] (18)

and the total current through the device
\[
J(t) = \int_0^1 j_{tot}(x, t) dx.
\] (19)

The latter, using Eqs. (16) and (12), can be presented as
\[
J(t) = \int_0^1 \left[ \mu_1 \rho_1(x, t) + \mu_2 \rho_2(x, t) \right] E(x, t) dx +
\]
\[
+ D_1 [\rho_1(0, t) - \rho_1(1, t)] + D_2 [\rho_2(0, t) - \rho_2(1, t)].
\] (20)

Our aim is to study the peculiarities in the time dependence of the electric current when the initial conditions (13) correspond to a strongly nonuniform charge distribution. Such nonuniform distributions can be prepared in different ways. For example, one can organize a nonuniform distribution in the process of growing of a semiconductor sample. Another way is to irradiate semiconductor by narrow laser beams [10]. One more possibility is by forming heavily doped layers by ion irradiation [11]. This method makes it possible to form narrow layers of positive carriers with a density of \( 10^{20} cm^{-3} \).
III. CARRIER DENSITIES

To understand better the physics of processes resulting from a nonuniform initial distribution of charge carriers, it would be useful to find an analytical, though approximate, solution to the system of equations (9) and (10). This can be done by means of the method of scale separation [12,13], whose mathematical foundation is based on the Krylov–Bogolubov averaging method [14].

The first step in the method of scale separation [12,13] is to classify the solutions onto fast and slow. In our case this can be done as follows. The electric field, as is seen from Eq. (14), is the functional of the charge densities, averaging the latter over the space variable . This results in that varies in space slower than . On the other hand, the voltage integral (12) shows that the electric field, being averaged over space, does not depend on time. This means that can be treated as a slow function in time. Therefore, the electric field can be regarded as a slow solution, as compared to the charge densities, with respect to both space and time. This permits us to consider the equation (9) for a fast solution keeping there as a space–time quasi–integral. After solving Eq. (9), the found is to be substituted into Eq. (14) giving an equation for which can be solved iteratively.

In solving Eq. (9), it is convenient to continue outside the region of by defining as zero for and . Then we may invoke the Fourier transforms with respect to . Finally, we obtain

\[
\rho_i(x, t) = \rho_{i,reg}(x, t) + \rho_{i,ran}(x, t) \tag{21}
\]

the first term being the regular solution

\[
\rho_{i,reg}(x, t) = \int_{-\infty}^{+\infty} G_i(x - x', t) f_i(x') dx' \tag{22}
\]

induced by the initial condition (13), while the second term being the random solution

\[
\rho_{i,ran}(x, t) = \int_{0}^{t} \int_{-\infty}^{+\infty} G_i(x - x', t - t') \xi_i(x', t') dx' dt' \tag{23}
\]

generated by the noise. The Green function in Eqs. (22) and (23) is

\[
G_i(x, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp\{ikx - i\omega_i(k)t\} dk \tag{24}
\]

with the spectrum

\[
\omega_i(k) = \mu_i E_k - iD_i k^2 - i\gamma_i. \tag{25}
\]

Function (24) has the properties

\[
G_i(x, 0) = \delta(x), \quad \int_{-\infty}^{+\infty} G_i(x, t) dx = e^{-\gamma_i t}.
\]

In the case considered, the integration in (24) can be realized explicitly resulting in

\[
G_i(x, t) = \frac{1}{2\sqrt{\pi D_i t}} \exp \left\{ -\frac{(x - \mu_i E t)^2}{4D_i t} - \gamma_i t \right\} . \tag{26}
\]

As the initial condition in Eq. (13) it is reasonable to accept the physically realistic case of the Gaussian distribution

\[
f_i(x) = \frac{Q_i}{Z_i} \exp \left\{ -\frac{(x - a_i)^2}{2b_i} \right\}, \tag{27}
\]

in which and

\[
Q_i = \int_{0}^{1} f_i(x) dx, \quad Z_i = \int_{0}^{1} \exp \left\{ -\frac{(x - a_i)^2}{2b_i} \right\} dx.
\]

With the initial condition (27), the regular solution (22) becomes
\[ \rho_{i}^{\text{reg}}(x,t) = \frac{Q_i b_i}{Z_i \sqrt{b_i^2 + 2D_i t}} \exp \left\{ -\frac{(x - \mu_i Et - a_i)^2}{2b_i^2 + 4D_i t} - \gamma_i t \right\}. \] (28)

The regular and random solutions satisfy the initial conditions
\[ \rho_{i}^{\text{reg}}(x,0) = f_i(x), \quad \rho_{i}^{\text{ran}}(x,0) = 0. \] (29)

The regular solution, as time increases, moves with the velocity \( \mu_i E \), becomes wider and smaller, so that
\[ \lim_{t \to \infty} \rho_{i}^{\text{reg}}(x,t) = 0. \] (30)

The behaviour of the random solution depends on that of the noise. It is customary to treat the latter as the white noise with the averaging properties
\[ \langle \xi_i(x,t) \rangle = 0, \quad \langle \xi_i(x,t)\xi_j(x',t') \rangle = \gamma_{ij}\delta(x-x')\delta(t-t'). \] (31)

Accepting (31), one has
\[ \langle \rho_{i}^{\text{ran}}(x,t) \rangle = 0, \] (32)
and, consequently,
\[ \lim_{t \to \infty} \langle \rho_i(x,t) \rangle = 0. \] (33)

Since the electric field, according to (14), is a linear functional in \( \rho_i \), we find
\[ \lim_{t \to \infty} \langle E_i(x,t) \rangle = 1. \] (34)

Although the limiting values (33) and (34) have been obtained by analysing the approximate solutions, it is possible to show (see Appendix) that the limits (33) and (34) are stable stationary solutions of the exact equations.

The random solution (23) influences the electric current (2) through the correlator
\[ \langle \rho_i^{\text{ran}}(x,t) \rho_j^{\text{ran}}(x',t) \rangle = \gamma_{ij} \int_0^t G_{ij}(x-x',t')dt', \] (35)
in which
\[ G_{ij}(x-x',t) = \int_{-\infty}^{+\infty} G_i(x-x'',t)G_j(x'-x'',t)dx''. \] (36)

With the Green function (26), this gives
\[ G_{ij}(x,t) = \frac{1}{2\sqrt{\pi(D_i+D_j)t}} \exp \left\{ -\frac{(x - \mu_i Et + \mu_j Et)^2}{4(D_i+D_j)t} - \left( \frac{\gamma_i + \gamma_j}{4} \right)t \right\}. \] (37)

At large time, Eq.(37) decays by the law
\[ G_{ij}(x,t) \approx \frac{1}{2\sqrt{\pi(D_i+D_j)t}} \exp (-\gamma_{eff}t), \] (38)
as \( t \to \infty \), with the effective attenuation
\[ \gamma_{eff} = \frac{(\mu_i - \mu_j)^2 E^2}{4(D_i+D_j)} + \gamma_i + \gamma_j. \]

Consequently, the correlator (35) tends to a time constant.

At small time, one has
\[ G_{ij}(x,t) \approx \frac{1}{2\sqrt{\pi(D_i+D_j)t}} \exp \left\{ -\frac{x^2}{4(D_i+D_j)t} \right\}, \] (39)
as \( t \to 0 \). Therefore, the correlator (35) behaves as
\[ \langle \rho_i^{\text{ran}}(x,t) \rho_j^{\text{ran}}(x',t) \rangle \approx \frac{\gamma_{ij}\sqrt{t}}{2\sqrt{\pi(D_i+D_j)}} \exp \left\{ -\frac{(x - x')^2}{4(D_i+D_j)t} \right\}, \] (40)
when \( t \to 0 \). Equation (40) shows that the influence of noise at small times is exponentially suppressed. This conclusion is of high importance for the following analysis.
IV. ELECTRIC CURRENT

We have now enough information about the behaviour of the system in order to answer the question: Is it possible that a negative electric current could appear, directed against the applied voltage?

The first evident necessary condition for such a possibility is the space nonuniformity of the charge densities. Really, if \( \rho_i(x,t) \) is uniform in \( x \), then from Eq. (20) it follows immediately that the electric current is the positively defined quantity \( \mu_1 \rho_1 + \mu_2 \rho_2 > 0 \).

The properties of the carrier densities, studied in the previous section, are such that, even, if at the initial time \( \rho_i(x,t) \) is nonuniform in space, it tends to become uniform with time. Consequently, if a negative electric current would appear, this could happen only at the initial stage of the process, when \( t \ll 1 \).

In this way, the occurrence of negative electric current, if any, can arise only as a principally transient effect, when the charge densities are yet nonuniform. The processes of diffusion and relaxation need some time to make \( \rho_i \) uniform. Therefore, there always can be found such a time \( t \ll 1 \) when diffusion and relaxation are yet not important. But these processes shorten the time of a negative–current fluctuation, if it appears. Thus, the conditions favoring the longer lifetime of such a fluctuation would be \( D_i \ll 1 \) and \( \gamma_i \ll 1 \).

The influence of noise, according to Eq. (40), is exponentially small at the initial stage. Thus, even a strong noise would not kill the effect, although it, of course, would shorten the negative–fluctuation lifetime. So, the condition favoring the longer lifetime is weak noise, when \( \gamma_{ij} \ll 1 \).

After understanding the necessary and favoring conditions for the transient effect of a negative–current fluctuation, let us elucidate sufficient conditions for the inequality

\[
J(t) < 0
\]

occurring at \( t \ll 1 \). As a limiting case we may take \( t = 0 \) and the maximally nonuniform initial density

\[
\rho_i(x,0) = f_i(x) = Q_i \delta(x - a_i)
\]

following from condition (27) under \( b_i \to 0 \). Then from expression (20) we readily get

\[
J(0) = \mu_1 Q_1 E(a_1,0) + \mu_2 Q_2 E(a_2,0).
\]

This emphasizes once again that for such a nonuniform initial charge density the diffusion, relaxation, and noise do not influence much the value of the electric current \( J(0) \). The corresponding electric field, defined by Eq. (14), is

\[
E(x,0) = 1 + 4\pi Q_1[a_1 - \Theta(a_1 - x) + 4\pi Q_2[a_2 - \Theta(a_2 - x)],
\]

where \( \Theta(x) \) is the unit step function.

Combining Eqs. (41), (43), and (44), we have the inequality

\[
\mu_1 Q_1 \left\{ 1 + 4\pi Q_1 \left[ a_1 - \frac{1}{2} \right] + 4\pi Q_2 \left[ a_2 - \Theta(a_2 - a_1) \right] \right\} + \\
+ \mu_2 Q_2 \left\{ 1 + 4\pi Q_2 \left[ a_2 - \frac{1}{2} \right] + 4\pi Q_1 \left[ a_1 - \Theta(a_1 - a_2) \right] \right\} < 0
\]

(45)

for the parameters of the system allowing the appearance of a negative current.

There can be a number of different cases satisfying inequality (45). To show that such situations do really exist, consider a particular example when \( a_1 = a_2 = a \). Then Eq.(41) reduces to

\[
(\mu_1 Q_1 + \mu_2 Q_2) E(a,0) < 0.
\]

(46)

From equality (44) we get

\[
E(a,0) = 1 + 4\pi Q \left( a - \frac{1}{2} \right) \quad (Q \equiv Q_1 + Q_2).
\]

Recall that \( \mu_1 \) and \( Q_1 \) are positive, while \( \mu_2 \) and \( Q_2 \) are negative; so that \( \mu_i Q_i > 0 \). Thence, inequality (46) can be hold only if

\[
E(a,0) < 0.
\]

(47)
As follows from solution (28), the quantity $\mu_i E_i$ plays the role of the effective velocity of motion for the corresponding charge packet. In the case of inequality (47), we have $\mu_1 E < 0$ and $\mu_2 E > 0$. This means that the positive carriers effectively move against the applied voltage; and the negative carriers, along the latter; that is, they move oppositely to what one would expect. Hence, the negative electric current is related to the anomalous drift of charge carriers.

Substituting into Eq. (47) the electric field, we find

$$4\pi Q \left(\frac{1}{2} - a\right) > 1. \quad (48)$$

Depending on whether $Q$ is positive or negative, inequality (48) yields

$$a < \frac{1}{2} - \frac{1}{4\pi Q} \quad (Q > 0),$$

$$a > \frac{1}{2} + \frac{1}{4\pi Q} \quad (Q < 0). \quad (49)$$

Taking also into account that $0 < a < 1$, we obtain from inequalities (49) the condition

$$|Q| > \frac{1}{2\pi}. \quad (50)$$

Equations (49) and (50) are sufficient conditions for the appearance of a negative electric current at the initial stage of the process. Similarly, it is easy to show that the currents (17) and (18) can also become negative in the transient regime.

To study in more detail the behaviour of the electric current as a function of time, we have solved Eqs.(9) and (10) numerically. In accordance with the above analysis, the case favoring the negative–current fluctuation is accepted, when $\gamma_{ij} \ll 1$. The initial conditions are given by the Gaussian form (27). The voltage integral (12) plays the role of the boundary condition for the electric field. For the charge densities one may take the Neumann or Dirichlet boundary conditions [1]. We have tried both and found that the general picture does not change much, with the only difference that the calculational procedure is less stable for the Dirichlet conditions. To achieve the best stability, we opted for the Neumann boundary conditions.

For the characteristic parameters we accept the values typical of semiconductors [1,2], such as $Si$. Then the diffusion coefficients are $D_1 \sim 10 \, \text{cm}^2/\text{s}$, $D_2 \sim 30 \, \text{cm}^2/\text{s}$. The mobility of positive carriers $\mu_1 \sim 500 \, \text{cm}^2/\text{Vs}$ for the average concentration $10^{13} - 10^{14} \, \text{cm}^{-3}$ and $\mu_1 \sim 200 \, \text{cm}^2/\text{Vs}$ for the concentration $10^{17} - 10^{18} \, \text{cm}^{-3}$. The mobility of electrons $\mu_2 \sim 1500$ for the average concentration $10^{13} - 10^{14}$ and $\mu_2 \sim 400$ for the concentration $10^{16} - 10^{18}$. The recombination time $\tau_1 \sim \tau_2 \sim 10^{-12} - 10^{-10} \, \text{s}$, hence the relaxation width $\gamma_1 \sim \gamma_2 \sim 10^{10} - 10^{12} \, \text{s}^{-1}$. We consider a plane device of the size $A \sim 1 \, \text{cm}^2$, $L \sim 0.1 - 1 \, \text{cm}$, with the applied voltage $V_0 \sim 10^3 - 10^5 \, \text{V}$. For the calibration parameters in Eq.(8) we get $\mu \sim 10^6 \, \text{cm}^2/\text{Vs}$, $\rho_0 \sim 10^5 - 10^7 \, \text{V} \, \text{cm}$, $\rho_0 \sim 10^4 \sim 10^7 \, \text{V}/\text{cm}$, $\rho_0 \sim 10^4 \sim 10^7 \, \text{V}/\text{cm}^2$.

The results of our numerical calculations are presented in the Figs.1 to 6, where we show the time dependence of the electric current across the boundaries as well as the behaviour of the total current through the device. These figures demonstrate that the appearance of negative electric current is really a transient effect occurring at dimensionless times $t \ll 1$, which in dimensional units means that $t \ll \tau_0$. All values in the figure captions are given in dimensionless units employing the calibration parameters from Eq. (8). Also, for shortness, we write $a_1 \equiv a$ and $b_1 \equiv b$.

Fig.1 shows that the electric current through the left boundary of the semiconductor sample, through its right boundary, and the total electric current are different. This difference is not merely quantitative but can be qualitative, so that the negative current may happen at the right surface and on average through the sample, but may be absent on the left surface. Such a difference depends on semiconductor characteristics as mobilities and relaxation widths. This suggests the possibility of employing the principal difference in the behaviour of the currents for extracting information on the semiconductor characteristics. For example, in Fig. 2 it is seen that changing the electron mobility mainly influences the current across the left surface. The transient negative current becomes more pronounced when increasing the absolute value of the electron mobility, as is seen in Fig. 3. The lifetime of the negative–current fluctuation strongly depends on the relaxation width, which is illustrated in Fig. 4. Increasing the relaxation width shortens the fluctuation lifetime. Figure 5 demonstrates the role of the total initial charge on the occurrence of the negative current, and Fig. 6 shows the role of the initial distribution of charge carriers. The importance of special conditions for the initial charge and its location has been discussed in detail above. The amplitude of the transient negative–current fluctuation becomes smaller when the charge layer at initial time is shifted farther from the left surface of the semiconductor sample.
V. DISCUSSION

We have considered electric processes in nonequilibrium nonuniform semiconductors. The transport equations are taken in the standard drift–diffusion approximation that is widely used for describing realistic semiconductor devices. Both analytical and numerical solutions of these equations are accomplished.

It is shown that under special circumstances an unusual transient phenomenon appears displaying negative electric current. The necessary condition for such an anomalous current is nonuniformity of carrier densities at the initial stage. A general sufficient condition (45) is derived and its particular forms (49) and (50) are analysed in detail. We studied the influence of diffusion, relaxation, and of generation–recombination noise and showed that these processes, even being strong, do not destroy the effect although may shorten the lifetime of a negative–current fluctuation. Therefore such an anomalous electric current can really be observed in semiconductor devices.

An important physical question is how one could use the considered effect for practical applications. Several possibilities of using this effect can be suggested:

(i) The appearance of the transient negative–current fluctuation is rather sensitive to the characteristic parameters of semiconductor, such as the carrier mobilities and relaxation widths. Therefore one could use the observation of the current fluctuation for defining these parameters. This could be done in the following way. Assume that we know all parameters except one, say a mobility or a relaxation coefficient. Comparing the time–dependence of the measured current with that of the calculated one, we may try to find such a value of the sought parameter that the measured and the calculated behaviour of the electric currents be as close as possible to each other, in the optimal case, be almost coinciding. Then the so fitted quantity would give the value of the sought parameter.

(ii) When the layer of charge carriers is formed by irradiating semiconductor with an ion beam, the stuck ions are distributed approximately in the Gaussian law centered at the mean free path of the ions. A necessary condition for the occurrence of the negative current is that the charge layer is located at a particular distance from the semiconductor surface, as e.g. in Eq. (49). Thence, this effect is very sensitive to the initial location of charge carriers and, thus, could be used for measuring the mean free path of ions in specific semiconductor materials.

(iii) The value of the total charge in the initial nonuniform layer is also crucially important for the occurrence of the negative current, as is seen from Eqs. (48)–(50). Hence, studying this current, we could measure the initial charge. The latter may be unknown when the initial distribution of charge carriers is formed by irradiating semiconductor with narrow laser beams whose influence on the generation of carriers in not precisely known.

(iv) Semiconductor devices often work in the close vicinity of radiation sources, such as atomic reactors, or under the influence of other strong radiation, as cosmic rays. In the presence of radiation, the functioning of semiconductor devices can be drastically disturbed because of the arising carrier nonuniformities. This can lead not only to the malfunctioning of semiconductor devices but even to dramatic accidents. In order to prevent from these, one could employ controlling schemes reacting to the appearance of the negative current, signaling by this that the level of the carrier nonuniformity induced by irradiation has become dangerous.

(v) As we have shown, the generation–recombination noise does not destroy the effect of the negative–current fluctuation. However, there exist other types of noise [7–9] whose influence on the supression of this effect can be different. Therefore, analysing the peculiarity of the electric negative–current fluctuation, one could judge what kind of noise dominates the process in the studied semiconductor.

It is certainly not possible to enumerate all feasible applications of the considered effect. But we hope that the examples listed above do demonstrate that the specific unusual features of the negative–current transient effect could provide us several interesting physical applications. Three types of such applications are, generally, admissible. One type is for investigating the characteristics of semiconductor materials. Another type can be used for studying the properties of irradiating beams. And, finally, this effect can be employed for the practical purpose of creating special controlling instruments.

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Appendix. Stationary Solution

Here we prove that Eqs. (33) and (34) are the stable stationary solutions of the exact equations (9) and (10) under the voltage condition (12). To this end, because of equality (32), it is sufficient to prove that

\[
\lim_{t \to \infty} \rho_{i}^{reg}(x, t) = 0, \quad \lim_{t \to \infty} E^{reg}(x, t) = 1,
\]

where \( E^{reg} \) implies the functional (14) including the dependence on only \( \rho_{i}^{reg} \). In what follows we shall write for brevity \( \rho_{i} \) and \( E \) keeping in mind \( \rho_{i}^{reg} \) and \( E^{reg} \). The proof will be based on the method of multipliers [15].

Write Eq. (9) in the form

\[
\frac{\partial}{\partial t} \rho_{i}(x, t) = v_{i}(x, \rho, t),
\]

with the velocity field

\[
v_{i}(x, \rho, t) = D_{i} \frac{\partial^{2} \rho_{i}}{\partial x^{2}} - \mu_{i} \frac{\partial}{\partial x}(\rho_{i} E) - \gamma_{i} \rho_{i}.
\]

Define the multiplier matrix

\[
M_{ij}(x, x', t) = \frac{\delta \rho_{i}(x, t)}{\delta \rho_{j}(x', 0)}
\]

and the Jacobian matrix

\[
L_{ij}(x, x', \rho, t) = \frac{\delta v_{i}(x, \rho, t)}{\delta \rho_{j}(x', t)}.
\]

The latter, with the given velocity field, consists of the elements

\[
L_{ii}(x, x', \rho, t) = \left( D_{i} \frac{\partial^{2}}{\partial x^{2}} - \mu_{i} \frac{\partial}{\partial x} - \gamma_{i} \right) \delta(x - x') -
\]

\[
-4\pi \mu_{i} (2 \rho_{i} + \rho_{j}) \delta(x - x') - \mu_{i} \frac{\partial}{\partial x} \frac{\delta E(x, t)}{\delta \rho_{j}(x', t)}.
\]

\[
L_{ij}(x, x', \rho, t) = -4\pi \mu_{i} \rho_{i} \delta(x - x') - \mu_{i} \frac{\partial \rho_{i}}{\partial x} \frac{\delta E(x, t)}{\delta \rho_{j}(x', t)},
\]

where \( i \neq j \). Here, the variational derivative of the electric field can be found from expression (14) giving

\[
\frac{\delta E(x, t)}{\delta \rho_{i}(x', t)} = 4\pi \left[ \frac{\delta Q(x, t)}{\delta \rho_{i}(x', t)} + x' - 1 \right], \quad \frac{\delta Q(x, t)}{\delta \rho_{i}(x', t)} = \Theta(x - x').
\]

Varying the evolution equation for \( \rho_{i} \), we obtain the equation

\[
\frac{\partial}{\partial t} M_{ij}(x, x', t) = \sum_{k} \int_{0}^{1} L_{ik}(x, x'', \rho, t) M_{kj}(x'', x', t) dx''
\]

for the multiplier matrix, with the initial condition

\[
M_{ij}(x, x', 0) = \delta_{ij} \delta(x - x'),
\]

following from the variation of condition (13).

For the case \( \rho_{i} = 0 \) and \( E = 1 \), the Jacobian matrix is

\[
L_{ij}(x, x', 0, t) = \delta_{ij} \left( D_{i} \frac{\partial^{2}}{\partial x^{2}} - \mu_{i} \frac{\partial}{\partial x} - \gamma_{i} \right) \delta(x - x').
\]
This leads to the equation
\[
\frac{\partial M_{ij}}{\partial t} = \left( D_i \frac{\partial^2}{\partial x^2} - \mu_i \frac{\partial}{\partial x} - \gamma_i \right) M_{ij}
\]
for the multiplier matrix. From the latter equation, invoking the Fourier transform
\[
M_{ij}(x, x', t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} M_{ij}(k, t) e^{ik(x-x')} dk,
\]
we find
\[
M_{ij}(k, t) = \delta_{ij} \exp \left\{ -(i\mu_i k + D_i k^2 + \gamma_i) t \right\}
\]
As is seen,
\[
|M_{ij}(k, t)| < 1
\]
for all \(k \in (-\infty, +\infty)\) and \(t > 0\). Hence the motion in the vicinity of the stationary solutions \(\rho_i = 0\) and \(E = 1\) is stable. It is also asymptotically stable since
\[
\lim_{t \to \infty} M_{ij}(k, t) = 0
\]
for all \(k \in (-\infty, +\infty)\). This completes the proof.

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Figure Captions

**Fig.1** The time behaviour of the electric current (17) across the left boundary (solid line), of the current (18) across the right boundary (long–dashed line), and of the total current (19) through the semiconductor device (short–dashed line). The used parameters, in dimensionless units, are $Q_1 = 1$, $Q_2 = -\frac{1}{2}$, $\mu_1 = 1$, $\mu_2 = -10$, $\gamma_1 = \gamma_2 = 10$, $a = 0.25$, $b = 0.1$, $D_1 = 10^{-3}$, $D_2 = 3D_1$.

**Fig.2** The same as in Fig.1, but for $\mu_2 = -3$ and $a = 0.05$. The smaller absolute value of the electron mobility influences mainly the current across the left boundary.

**Fig.3** Electric current (19) through the semiconductor device as a function of time for different mobilities. The fixed parameters are $Q_1 = 1$, $Q_2 = -\frac{1}{2}$, $\gamma_1 = \gamma_2 = 1$, $a = 0.25$, $b = 0.1$, $D_1 = 10^{-3}$, $D_2 = 3D_1$, $\mu_1 = 1$. The curves correspond to the varying mobility: $\mu_2 = -10$ (solid line), $\mu_2 = -5$ (long–dashed line), and $\mu_2 = -3$ (short–dashed line). Increasing the absolute value of the electron mobility makes the negative current more pronounced.

**Fig.4** The same as in Fig.3, except for $\mu_2 = -3$ and for varying relaxation widths: $\gamma_1 = \gamma_2 = 25$ (solid line), $\gamma_1 = \gamma_2 = 10$ (long–dashed line), and $\gamma_1 = \gamma_2 = 1$ (short–dashed line). Increasing relaxation width suppresses the negative current.

**Fig.5** The time dependance of the electric current (19) for the following parameters: $\mu_1 = 1$, $\mu_2 = -3$, $\gamma_1 = \gamma_2 = 1$, $a = 0.25$, $b = 0.1$, $D_1 = 10^{-3}$, $D_2 = 3D_1$, $Q_1 = 1$, and for varying $Q_2 = 0$ (solid line), $Q_2 = -\frac{1}{4}$ (long–dashed line), $Q_2 = -\frac{1}{2}$ (short–dashed line), $Q_2 = -\frac{3}{4}$ (dotted line), and $Q_2 = -1$ (dash–dotted line).

**Fig.6** The same as in Fig. 5, but for the fixed $Q_2 = -\frac{1}{4}$ and for a varying location of the initial distribution of carriers: $a = 0.1$ (solid line), $a = 0.2$ (long–dashed line), $a = 0.3$ (short–dashed line), $a = 0.4$ (dotted line), and $a = 0.5$ (dash–dotted line).