Spatial distribution of stars and brown dwarfs in $\sigma$ Orionis

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Accepted 2007 October –. Received 2007 October –; in original form 2007 August 13

ABSTRACT

I have re-visited the spatial distribution of stars and high-mass brown dwarfs in the $\sigma$ Orionis cluster ($\sim$3 Ma, $\sim$360 pc). The input was a catalogue of 340 cluster members and candidates at separations less than 30 arcmin to $\sigma$ Ori AB. Of them, 70% have features of extreme youth. I fitted the normalised cumulative number of objects counting from the cluster centre to several power-law, exponential and King radial distributions. The cluster seems to have two components: a dense core that extends from the centre to $r \approx 20$ arcmin and a rarified halo at larger separations. The radial distribution in the core follows a power-law proportional to $r^{1}$, which corresponds to a volume density proportional to $r^{−2}$. This is consistent with the collapse of an isothermal spherical molecular cloud. The stars more massive than 3.7 $M_\odot$ concentrate, however, towards the cluster centre, where there is also an apparent deficit of very low-mass objects ($M < 0.16 M_\odot$). Last, I demonstrated through Monte Carlo simulations that the cluster is azimuthally asymmetric, with a filamentary overdensity of objects that runs from the cluster centre to the Horsehead Nebula.

Key words: open clusters and associations: individual: $\sigma$ Orionis – stars: formation – stars: low mass, brown dwarfs.

1 INTRODUCTION

The $\sigma$ Orionis region in the Ori OB 1 b association is finally becoming recognised as one of the most important young open clusters, with an age of only about 3 Ma. In the discovery paper, Garrison (1967) used the term “clustering” to refer to an agglomeration of fifteen B-type stars surrounding and including the multiple star $\sigma$ Ori. Afterwards, Lyngå (1981) tabulated $\sigma$ Orionis in his catalogue of open clusters. Since the rediscovery of the cluster by Wolk (1996) and its subsequent study in depth, which has revealed the most numerous and best known substellar population (Béjar et al. 1999; Zapatero Osorio et al. 2000, 2002; Caballero et al. 2007), only a few authors have investigated the $\sigma$ Orionis spatial distribution. In particular, Béjar et al. (2004) and Sherry et al. (2004) analysed the radial distribution of $\sigma$ Orionis cluster members and candidates in annuli of width $\Delta r$ as a function of the separation $r$ to $\sigma$ Ori AB. To maximise the number of objects per annulus and minimise the Poissonian errors, $\Delta r$ must be wide. This leads to have few annuli (no more than 12 in the $r = 0$–30 arcmin interval) to fit to a suitable radial profile (exponential decay – Béjar et al. 2004; King – Sherry et al. 2004). Both studies agree that the cluster may extend only up to $\sim$25–30 arcmin. The low surface density of cluster members at larger separations, the sharp increase of extinction due to the nearby Horsehead Nebula-Flame Nebula-IC 434 complex and the closeness to (or even overlapping with) other stellar populations in the Orion Belt surrounding Alnitak ($\zeta$ Ori) and Alnilam ($\epsilon$ Ori) prevent from suitably broaden the radial distribution analysis (Caballero 2007a). At the canonical heliocentric distance to $\sigma$ Orionis of 360 pc (e.g. Brown, de Geus & de Zeeuw 1994), the cluster would have an approximate radius of 3 pc.

In spite of the agreement on the size of $\sigma$ Orionis, the fits and the profiles in Béjar et al. (2004) and Sherry et al. (2004) seem to be rather incomplete and inappropriate, respectively. On the one hand, the King models were designed for tidally truncated globular clusters (King 1962, 1966; Meylan 1987), and have also been satisfactorily used for describing galaxies (e.g. Kormendy 1977; Binggeli, Sandage & Tarenghi 1984). These systems have had enough time to be isothermal, on the contrary to very young open clusters like $\sigma$ Orionis, where only gravitational relaxation by initial mixing may have occurred (King 1962). On the other hand, Béjar et al. (2004) exclusively focused on the cluster substellar population. Besides, the exponential fit in Béjar et al. (2004) only accounted for the five innermost annuli, which leded to a high uncertainty in the derived parameters. Last, in the works by Sherry et al. (2004) and Béjar et al. (2004), the input list of cluster members and candidates came from...
VRI/IZ optical surveys. Many sources in both analysis had no near-infrared or spectroscopic follow-up.

For a correct study of the spatial distribution in σ Orionis, it is therefore necessary to use new fitting radial profiles and an input catalogue as comprehensive as possible. It must cover a wide mass interval. Maximum completeness and minimum contamination of the catalogue are also desired. These requirements are verified by the Mayrit catalogue, which tabulates 339 σ Orionis members and candidates in a 30 arcmin-radius circular area centred on σ Ori AB (Caballero 2007c). Of them, 241 display features of extreme youth (e.g. OB spectral types, Li i in absorption, Hα in strong emission, spectral signatures of low gravity, near- and mid-infrared excesses due to discs). The catalogue covers three orders of magnitude in mass, from the ∼18+12M⊙ of the O9.5V+B0.5V binary σ Ori AB to the ∼0.033M⊙ of the brown dwarf B05 2.03–617 (Caballero & Chabrier, in prep.). Accounting for σ Ori A and B as different objects separated by ∼0.25 arcsec, then the equatorial coordinates of 340 young stars, brown dwarfs and cluster member candidates are available. I will use this input catalogue to investigate the radial and azimuthal distribution of objects in the σ Orionis region.

2 ANALYSIS AND RESULTS

2.1 Cluster centre and radial gradient

Caballero (2007a) showed that ∼46% of the mass in the σ Orionis stars with M ≥ 1.2M⊙ is contained in the quintuple Trapezium-like system that gives the name to the cluster, and ∼29% only in the AB components. This suggests using the binary as the cluster centre (r = 0). From the masses for the objects in the Mayrit catalogue derived in Caballero & Chabrier (in prep.), one third of the total cluster mass is encircled in the innermost 5 arcmin. If the mass were homogeneously distributed within the survey area, the innermost 5 arcmin would contain only 2.8% of the total cluster mass [(5/30)² ≈ 0.028]. I will consider σ Ori AB as the origin of coordinates because of: (i) simplicity (the coordinates of the binary are well determined by Hipparcos; the actual coordinates of the cluster barycentre may change when a different input list of cluster members and individual masses is used); (ii) reflection of the geometry of the Mayrit survey in Caballero (2007c), which was centred on σ Ori AB; and (iii) uniformity with previous works (especially with Béjar et al. 2004 and Sherry et al. 2004, who also used σ Ori AB as the coordinate origin). There might be an additional reason: the largest mass aggregation is probably associated to the densest region of the original molecular cloud where the fragmentation and star formation initiated (assuming that the origin of the reference frame is locked to the moving-cluster barycentre). This reason may be unconvincing, because highly turbulent (and fractal-like?) molecular clouds probably do not have a “centre” that can be defined in any sensible way, as shown in the simulations of Bonnell, Bate & Vine (2003). See Section 2.1 in this work and fig. 1 in Caballero (2007a) for pictorial views of the spatial distribution of confirmed and candidate cluster members in the σ Orionis region, and Caballero (2007b) for a description of the cluster centre. The old-fashioned plot of the surface density is shown in Fig. 1; see similar plots in Sherry et al. (2004) and Béjar et al. (2004) for comparison.

There exists a Q-parameter which quickly, and simply, shows if a distribution of cluster members is smooth (a large-scale radial density gradient) or clumpy (a multiscale fractal subclustering; Cartwright & Whitworth 2004). The Q-parameter is defined by:

$$Q = \frac{m}{s}$$

where m and s are the edge length of the Euclidean minimum spanning tree and the separation between cluster mem-
bers, respectively. A minimum spanning tree is a network (“graph”) of \(N_{\text{max}}-1\) lines (“edges”) that connect the \(N_{\text{max}}\) objects (“nodes”) in the shortest possible way under the condition no closed loops allowed. See, e.g., Graham, Cloves & Campusano (1995) for an application of minimum spanning trees in Astrophysics. The normalization factors are \((N_{\text{max}} \pi r_{\text{max}}^2)^{1/2}/(N_{\text{max}} - 1)\) and \(r_{\text{max}}\) for \(m\) and \(s\) \((N_{\text{max}})\) is the total number of cluster members and \(\pi r_{\text{max}}^2\) is the area of the circular survey.

For the 340 \(\sigma\) Orionis cluster member and candidates, I have measured \(\bar{\sigma} = 0.589\) and \(\overline{\sigma} = 0.668\). Therefore, the Cartwright & Whitworth (2004) parameter is \(Q \approx 0.88\). This value is larger than 0.80, which distinguishes \(\sigma\) Orionis as a cluster with a smooth large-scale radial density gradient and a moderate degree of central concentration. This concentration is larger than in \(\rho\) Ophiuchi, other cluster with a radial density gradient, but less than in IC 348 (Cartwright & Whitworth 2004). Other sparse clusters and star-forming regions, like IC 2391, Taurus and Chamaeleon, have \(Q\)-parameters in the interval 0.47–0.67, which indicates that they have, on the contrary, substructure with fractal dimensions between 1.5 and 2.5.

### 2.2 Surface density and cumulative number of objects

I present an innovative, accurate, simple method to derive the actual expression of the surface density as a distance from the cluster centre, \(\sigma(r)\). It can be applied to other open and globular clusters and galaxies. The normalised cumulative number of objects counting from the cluster centre, \(f(r)\), is:

\[
f(r) = \frac{N(r)}{N(r_{\text{max}})}
\]

where \(N(r)\) is the total number of stars in projection within a distance \(r\) of the centre. If there is azimuthal symmetry, \(N(r)\) is related to the surface density through the following expression:

\[
N(r) = 2\pi \int_0^r r' \sigma(r').
\]

The relatively high value of the \(Q\)-parameter of \(\sigma\) Orionis supports the hypothesis of azimuthal symmetry in this cluster in particular. For systems without azimuthal symmetry (e.g. elliptical galaxies), use instead:

\[
N(r) = \int_0^r \int_0^{2\pi} d\theta \ r' \sigma(r', \theta).
\]

The surface \(\sigma(r)\) and volume \(\rho(r)\) densities are linked through the simple relation \(\sigma(r) = 2r\rho(r)\). This equality comes from:

\[
N(r) = 4\pi \int_0^r d' r' \sigma(r') = 2\pi \int_0^r d' r' \sigma(r') \sqrt{r'}.
\]

1 The first algorithm for finding a minimum spanning tree was developed to find an efficient electrical coverage of Czech Moravia (Borůvka 1926)

assuming again azimuthal symmetry. The function \(f(r)\) varies from 0 at \(r = 0\) to 1 at \(r = r_{\text{max}}\). In the Magrit survey, \(r_{\text{max}} = 30\) arcmin and \(N(r_{\text{max}}) \equiv N_{\text{max}} = 340\). In the discrete approximation, \(N(r) \approx N^*(r)\) and:

\[
f(r) \approx f^*(r) = \frac{\sum_{i=1}^{\infty} N_{i}}{\sum_{i=1}^{\infty} N_{i} (r_{\text{max}})_{i}} = \frac{N^*(r)}{N_{\text{max}}}.
\]

I have investigated several functional expressions of \(\sigma(r)\) that fit in more or less degree the observed normalised cumulative number of objects, \(f^*(r)\). A general expression for a power-law surface density of index \(\delta - 2\) is:

\[
\sigma(r, \delta) = \frac{\delta N_{\text{max}}r^{\delta-2}}{2\pi r_{\text{max}}^2},
\]

which, after integration, leads to a simple expression for \(f(r)\):

\[
f(r, \delta) = \left(\frac{r}{r_{\text{max}}}\right)^{\delta}.
\]

In this approach, the objects are uniformly distributed in a circular area if \(\delta = 2\) (\(\sigma = \text{constant}\)). Surface densities with parameter \(\delta < 0\), that predict a lower number of objects close to the centre, were obviously not considered.

Following Béjar et al. (2004), I have also studied two expressions of exponential decay of the surface density:

\[
\sigma(r, \epsilon) = \sigma_0 e^{-\epsilon r}
\]

\[
\sigma_0 = \frac{N_{\text{max}}}{2\pi} \left(\frac{1}{\epsilon} - e^{-\epsilon r_{\text{max}}} \left(\frac{1}{\epsilon} + \frac{r_{\text{max}}}{\epsilon}\right)\right)
\]

\[
f(r) = \frac{1 - e^{-\epsilon r}}{1 - e^{-\epsilon r_{\text{max}}}}
\]

and:

\[
\sigma(r, \epsilon) = \sigma_0 e^{-\epsilon r^2}
\]

\[
\sigma_0 = \frac{e N_{\text{max}}}{\pi} \left(\frac{1}{1 - e^{-\epsilon^2 r_{\text{max}}^2}}\right)
\]

\[
f(r) = \frac{1 - e^{-\epsilon^2 r^2}}{1 - e^{-\epsilon^2 r_{\text{max}}^2}}
\]

Finally, I have also investigated the King (1962) profile for gravitationally relaxed globular clusters. Close to the centre, the surface density can be expressed by:

\[
\sigma(r) \approx \sigma_c(r) = \frac{\sigma_0}{1 + (r/c_0)^2},
\]

where \(c_0\) is the core radius and \(\sigma_0\) is the central surface density. In the limit of the cluster, the surface density is:

\[
\sigma(r) \approx \sigma_\infty(r) = \sigma_1 \left(\frac{1}{r} - \frac{1}{r_\infty}\right)^2,
\]

where \(r_\infty\) is the tidal radius (the value of \(r\) at which \(\sigma_\infty(r)\) reaches zero) and \(\sigma_1\) is a constant. The overall normalised cumulative number of objects that embodies \(\sigma_c(r)\) and \(\sigma_\infty(r)\) is, following the nomenclature by King (1962):
where \( x = (r/r_c)^2 \), \( x_t = (r_t/r_c)^2 \) and \( x_{\text{max}} = (r_{\text{max}}/r_c)^2 \).

Fig. 3 illustrates the fits of \( f(r) \) to \( f^*(r) \) to evaluate the most suitable expression for \( \sigma(r) \). The best match for a simple power-law density is acquired for \( \delta = 0.9 \). Power-laws with \( \delta \gg 1 \) and \( \delta \ll 1 \) provide inaccurate fits. Likewise, the exponential profiles cannot predict the large actual surface density close to the cluster centre (the binned surface density profile in Fig. 1 when plotted in logarithmic scale, also shows that the innermost bin deviates from the exponential profile). The overall King profile has the same problem. I performed intensive computations, not shown here, to cover the \((r_t, r_c)\) parameter space of the King profile. No clear minimum of the \( \chi^2 \) exists for the \( \sigma \) Orionis radial distribution when fitted to the King empirical density law. The best solutions were found for all the combinations that satisfy \( r_c = 8-12 \) arcmin and \( r_t \gg r_c \). The excesses of light at large radii of young massive clusters with respect to King (and Elson, Fall & Freeman 1987) profile(s), attributed to gas expulsion by Goodwin & Bastian (2006), cannot explain the poor fitting for the King profile at small radii in \( \sigma \) Orionis.

The best general fit is obtained for a composite power-law, as shown in Fig. 4. The cumulative number of \( \sigma \) Orionis objects grows proportional to \( r \) (i.e. \( \delta = 1.0 \)) up to \( \sim 20 \) arcmin. This size translates into a physical radius of \( \sim 2 \) pc. At larger separations, \( f(r) \) increases with a lower slope, indicating an exponent \( \delta \approx 0.7 \). In reality, at such separations, an exponential or a limit King \([\sigma(r)]\) profile would also fit the data. From the extrapolation of the \( f(r) \) law up to the radius of 30 arcmin, there is deficit of 30–40 objects in the outermost annulus. As a result, the \( \sigma \) Orionis cluster may be spatially described as a central \((r \lesssim 20 \) arcmin\), dense region –the “core”– and an outer \((r \gtrsim 20 \) arcmin\), more rarified region –the “halo”–. From these data, the power-law index transition between the core and the halo is quite smooth. However, the relative drop of \( f(r) \) at \( r \gtrsim 20 \) arcmin might be more or less abrupt because of our poor knowledge of the \( \sigma \) Orionis stellar population at large separations from the cluster centre. As an example, while more than 90% of cluster members and candidates of the Mayrit catalogue in the innermost 10 arcmin have known features of extreme youth, this ratio is about 50% in the halo (past spectroscopic, mid-infrared and X-ray analysis have been naturally focused on the cluster centre and its surroundings). Going deep into this subject seems to be meaningless because of the relatively small amount of investigated cluster objects in the halo, contamination by young neighbouring stellar populations in Orion (Jeffries et al. 2006; Caballero 2007a) or fore-/background stars (e.g. Caballero, Burgasser & Klement, in prep.), variable extinction to the northeast of the survey area, incompleteness of the Mayrit catalogue (Caballero 2007c) and the “anomalous” radial distribution discussed next.

The radial distribution of the cluster core is, on the contrary to the halo, free of possible systematic errors. The \( f(r) \propto r \) law in the core is translated to surface and volume densities \( \sigma(r) \propto r^{-1} \) and \( \rho(r) \propto r^{-2} \). Radial profile investigations carried out in other very young star-forming regions

\[ f(r) = \frac{\log(1 + x)}{\log(1 + x_{\text{max}})} = \frac{4(1 + x_t^{1/2}) + x_t}{4(1 + x_{\text{max}}^{1/2}) + x_{\text{max}}}, \]

\[ (17) \]
have found similar distributions. Not far away from $\sigma$ Orionis, Bate, Clarke & McCaughrean (1998) noticed that the stars of the Orion Nebula Cluster are distributed with a core of uniform volume density and radius $r_{\text{core}} = 0.5$ arcmin and a volume density profile $\rho(r) \propto r^{-2}$ at larger separations. Alike power-law volume density distributions have been found in other star-forming regions, like Taurus (Fuller et al. 1992; Ward-Thompson et al. 1994), or low-mass cold dark molecular clouds, like Barnard 68 (Alves, Lada & Lada 2001). Furthermore, the power-law index 2 is an “often-used initial condition for numerical calculations of star formation” (Burkert, Bate & Bodenheimer 1997). The $\rho(r) \propto r^{-2}$ distribution corresponds to a singular, self-gravitating, (rotating) isothermal sphere. Finally, from fig. 5 in Cartwright & Whitworth (2004) and the $Q$-parameter value for $\sigma$ Orionis derived in Section 2.1 I estimate that the volume density in the cluster varies as $\rho(r) \propto r^{-1.7 \pm 0.4}$. The $1.7 \pm 0.4$ index is intermediate between those in $\rho$ Ophiuchus ($-2$) and IC 348 ($-2.2$) and consistent with $\rho(r) \propto r^{-2}$.

The use of the power-law function $f(r) \propto r$, whose corresponding $\sigma(r)$ diverges at $r = 0$, has the drawback of an incorrect fit in the innermost 1 arcmin of the cluster. Characterising the very centre of the cluster is out of the scope of this work, since it can be only accomplished with high spatial-resolution facilities (e.g. adaptive optics or mid-infrared instruments – van Loon & Oliveira 2003; Caballero 2005). Moreover, the Bate et al. (1998)’s value of $r_{\text{core}}$ can be suitable applied to $\sigma$ Orionis. There are only seven stars at less than 0.5 arcmin to $\sigma$ Ori AB (Caballero 2007b), so the central surface density is $\sigma_0 \approx 9$ arcmin$^{-2}$. This value matches well with the actual value of $\sigma(r_{\text{core}})$ for $f(r) \propto r$. The observational parallelism between the radial distributions of the Orion Nebula Cluster and other star-forming regions and of $\sigma$ Orionis evidences that the collapse of an isothermal cloud to form a star cluster might be universal.

2.3 Mass-dependent radial distribution

I have also investigated the radial profile of $\sigma$ Orionis cluster members and candidates for different mass intervals. I have separated the 340 stars and brown dwarfs in four mass groups that are equally spaced in logarithmic scale. The boundaries between the groups are at about 3.7, 0.77 and 18 $M_\odot$. The groups contain, from the most to the least massive, 10, 103, 155 and 72 elements. Figs. 5 to 8 show the normalised cumulative number of objects for the four groups compared to the average distribution. It is manifest that the star groups more massive than 3.7 $M_\odot$ depart from the general trend $f(r) \propto r$, while the brown dwarfs and stars below this mass roughly follow it. The massive stars seem to obey the King profile close to the cluster centre (with a surface density $\sigma(r) = \frac{\rho(r)}{1 + \left(\frac{r}{r_{\text{core}}}\right)^4}$) rather than a power-law, whose
On the one hand, there are only two representatives of brown dwarfs. Note the deficit of very low-mass cluster members at radii between \(0.16\) and \(0.03\) \(M_\odot\).\(^3\)

The range defined by the OB-type stars of the \(\theta^1\) Ori multiple system. This result shows again the resemblance between both Orion clusters.

There is no other characteristic feature in the mass-dependent distribution in Figs. 5 to 8 except for a remarkable deficit of very low-mass stars and high-mass brown dwarfs \((0.16\, M_\odot \leq M \leq 0.03\, M_\odot)\) in the innermost 4 arcmin together with a steep raise of \(f^*(r)\) at 6–7 arcmin (Fig. 8).

On the one hand, there are only two representatives of this mass interval within 4 arcmin: Mayrit 36273 and the Class I object Mayrit 111208. Both of them are the faintest sources with \(I\)-band photometry identified in the near-infrared/optical/X-ray survey in the centre of \(\sigma\) Orionis by Caballero (2007b). On the other hand, there are almost 20 low-mass stars and brown dwarfs in the narrow annulus 5–8 arcmin. An 0.16 \(M_\odot\)-mass object in the cluster, in the upper limit of low-mass interval, has typical magnitudes \(I \sim 16.0\) mag, \(J \sim 14.0\) mag. These values are far brighter than the DENIS and 2MASS completenesses, even in the innermost region affected by the glare of the multiple system \(\sigma\) Ori. Caballero (2007b) failed to confirm or refute this absence of very low-mass stars and high-mass brown dwarfs. The raise of \(f^*(r)\) at 6–7 arcmin indicates, on the contrary, a larger density of very low-mass objects in this annulus. One can think of many ways which could give rise to the deficit of low-mass objects at small radii and excess at intermediate radii. Firstly, low-mass objects could actually form in the 6–7 arcmin annulus and not in the inner regions (maybe core masses were higher in the centre, or competitive accretion caused central brown dwarfs to grow into stars). Alternatively, they could form in the cluster centre, but were ejected via dynamical interactions with the massive stars. Low-mass objects do not have enough energy, however, to move further away from the deep \(\sigma\) Ori gravity well of \(M > 50\, M_\odot\). The deficit-excess needs to be explained by theory, but this particular set of observations does not give any indication of a preferred formation scenario [see Whitworth et al. (2007) for a review on the theory of formation of brown dwarfs and very low-mass stars]. Further and innovative observations, able to avoid the extensive glare of the OB system, are required to determine if the peculiar distribution of very low-mass objects in the cluster centre are due to an observational bias or to an actual consequence of the formation mechanism. Some observational efforts on this topic have been carried out by Caballero (2005, 2007b) and Sherry, Walter & Wolk (2005).

### 2.4 Azimuthal asymmetry

Top window in Fig. 9 shows that the distribution of confirmed cluster members and candidates is not whole radially symmetric, with an evident lower density to the west of \(\sigma\) Ori AB with respect to the east. An elongated subclustering is manifest to the east-northeast of the cluster centre, just in the direction to the Horsehead Nebula. On the contrary, Béjar et al. (2004) derived that the variation of the radial distribution of their very-low mass stars and brown dwarfs over their best exponential law fit was Poissonian, implying no evidence of subclustering. The object sample presented in this paper surpasses Béjar et al.’s one and allows to corroborate or invalidate their statement.

I have looked for an azimuthal asymmetry in the \(\sigma\) Orionis cluster in several steps. First, I have generated 1000 simulated distributions\(^3\) following the power law \(f(r) = \left(\frac{r}{r_{\text{max}}}\right)^{-\delta}\). Ten of them (one is highlighted) are shown in the bottom window in Fig. 9. The resemblance with the actual distribution, in the top window of the Figure, is evident. Second, I have divided the survey area in nine regions:

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\(^3\) In the general power-law distribution case with index \(\delta\), if \(r\) is a vector of length \(N_{\text{max}}\) of uniformly distributed pseudo-random radii between \(r = 0\) and \(r_{\text{max}}\), then the distribution of the vector \(r^{1/\delta}\) follows \(f(r) = \left(\frac{r}{r_{\text{max}}}\right)^{-\delta}\).
Spatial distribution in $\sigma$ Orionis

I have measured the asymmetry factor for the actual $\sigma$ Orionis distribution: $\zeta^* = 0.1853$. This value deviates 6.1 times the $\sigma_\zeta$ from the $\bar{\zeta}$ and is significantly larger than the $\zeta_{\text{max}}$ among 1000 Monte Carlo simulations. Considering as a first order of approximation that the values $\zeta_i$ (i = 1...1000) are distributed following a standard normal distribution with parameters $\bar{\zeta}$ and $\sigma_\zeta$, then there is a probability $p = 1 - \text{erf}(6.1/\sqrt{2}) \approx 10^{-9}$ (where "erf" is the error function) that the actual value $\zeta^*$ follows such distribution. Even accounting for generous systematic errors or biases, it is highly probable that the radial distribution of objects at more than 10 arcmin from the centre of $\sigma$ Orionis is azimuthally asymmetric.

The most populated segments of annulus of the actual distribution are, counting anti-clockwise from 12 hours, the second (coinciding with the elongated subclustering in the direction to the Horsehead Nebula) and the fifth ones. Both two segments and the cluster centre spatially coincide with a filamentary region of maximum emission at the 12 $\mu$m IRAS passband (see fig. 2 in Oliveira & van Loon 2004). This “warmer” region has not been identified in works based on recent observations with IRAC and MIPS onboard Spitzer (e.g. Hernández et al. 2007) or with the Spatial Infrared Imaging Telescope on the Midcourse Space Experiment satellite (Kraemer et al. 2003). From my data, I cannot postulate whether the largest surface density of $\sigma$ Orionis objects originally arises from an hypothetical larger density of warm dust in the region or, inversely, the filamentary region is a consequence of both the low spatial resolution imaging capabilities of IRAS and the largest surface density of objects (i.e. the red $\sigma$ Orionis objects, many with mid-infrared excesses due to discs –Oliveira et al. 2006; Hernández et al. 2007; Caballero et al. 2007–, generate a smooth background at 12 $\mu$m that IRAS was not able to resolve).

The accumulation of stars and brown dwarfs in a filamentary pattern in $\sigma$ Orionis strongly supports some star formation scenarios of collapse and fragmentation of a large-scale turbulent molecular cloud, especially those that predict burst of star formation in filamentary gas (e.g. Bate, Bonnell & Bromm 2003). It is stimulating to notice that these simulations assumed the contraction of an isothermal, spherical molecular cloud with $\rho(r) \propto r^{-2}$ (see Section 2.2). The filamentary accumulation in $\sigma$ Orionis is, however, peculiar, because no similar arrangements have been found in other star-forming regions. For example, Gómez et al. (1993) and Larson (1995) found that the subclustering in Taurus-Auriga is in the form of star clumps of $\sim$15 components, while Bate et al. (1998) showed that in the Orion Nebula Cluster there is no subclustering at all. It is obvious that further investigations are needed; percolation or two-point correlation function of stars are different approaches that can be used.

3 SUMMARY

The $\sim$3 Ma-old $\sigma$ Orionis cluster is a perfect laboratory of star formation. I have investigated the radial distribution of 340 cluster members and candidates in a 30 arcmin-radius area centred on $\sigma$ Ori AB, taken from Caballero (2007c). The analysis has covered a mass interval from the 13+12 $M_\odot$ of $\sigma$ Ori AB to the $\sim0.03 M_\odot$ of the faintest brown dwarf detectable by DENIS. The cluster shows a clear radial density...
gradient, quantified by the Q-parameter, that accounts for
the mean separation between members and the Euclidean
minimum spanning tree of the cluster. I have calculated the
functional relations between normalised cumulative numbers
of objects counting from the cluster centre, f(r), and surface
densities, σ(r). Cumulative distribution functions as these
avoid many problems associated with binning. Among the
studied radial (power-law, exponential and King) profiles,
the best fit is for a composite power-law distribution of clus-
ter members with a core and a rarified halo. The core extends
up to ∼20 arcmin from the cluster centre and is nicely mod-
eled by a surface density σ(r) ∝ r−2, that corresponds to a
volume density ρ(r) ∝ r−3, this volume density matches, in
its turn, the radial profile in a cluster formed from the col-
apse of a self-gravitating, isothermal sphere. The most mas-
tive σ Orionis stars deviate, however, from the general trend
and are much more concentrated towards the cluster centre.
There is also an apparent deficit of very low-mass stars and
high-mass brown dwarfs (0.16 M⊙ ≳ M ≳ 1 M⊙) in the
innermost 4 arcmin and an excess in the annulus at 6–
7 arcmin to the central Trapezium-like system. Last, there is
a significant azimuthal asymmetry due to a filament-shape
overdensity of objects that connects the cluster centre with
a part of the Horsehead Nebula. This discovery supports the
formation scenarios that predict burst of star formation in
filamentary gas.

ACKNOWLEDGMENTS

I thank the anonymous referee for helpful comments. J.A.C.
was formerly an Alexander von Humboldt Fellow at the
MPIA, and is currently an Investigador Juan de la Cierva
at the UCM. Partial financial support was provided by the
Universidad Complutense de Madrid and the Spanish Min-
isterio Educación y Ciencia under grant AyA2005–02750 of
the Programa Nacional de Astronomía y Astrofísica and by
la Comunidad Autónoma de Madrid under PRICIT project
S–0505/ESP–0237 (AstroCAM).

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