Spin Vortex in Magnon BEC of Superfluid $^3$He-B

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Received 12 June 2005

Abstract

The phenomenon of the spontaneous phase-coherent precession of magnetization in superfluid $^3$He and the related effects of spin superfluidity are based on the true Bose-Einstein condensation of magnons. Several different magnon BEC states have been observed: homogeneously precessing domain (HPD); BEC condensation in the spin-orbit potential trap ($Q$-balls); coherent precession with fractional magnetization; and two modes of the coherent precession in squeezed aerogel. The spin superfluidity effects, like spin Josephson phenomena, spin current vortices, spin phase slippage, long distance magnetization transport by spin supercurrents have been observed.

Key words: BEC, Spin current, Nuclear magnetic resonance, Quantum spin liquid

PACS: 67.57.-z 67.80.Jd 76.50.+g

Bose-Einstein condensation (BEC) is a phenomenon of formation of collective quantum state, in which the macroscopic number of particles is governed by a single wave function. The phenomenon of Bose-Einstein condensate was predicted by Einstein in 1925. For a review see, for example Ref. [1]. The almost perfect BEC state was observed in ultra cold atomic gases. In Bose liquids, the BEC is strongly modified by interactions, but still remains the key mechanism for the formation of a coherent quantum state in Bose systems, which exhibits the phenomenon of superfluidity characterized by non-dissipative superfluid mass currents discovered first in $^4$He by P.L. Kapitza [2]. Superfluidity proved to be a more general phenomenon: superfluid mass current has been found in Fermi liquid $^3$He; superfluidity of electric charge – superconductivity – is known in metals; superfluidity of chiral charge is discussed in quantum chromodynamics; color superfluidity – in quark matter and baryonic superfluidity – in neutron stars; etc. Here we discuss magnon BEC with spin current superfluidity.

Strictly speaking, the theory of superfluidity and Bose-Einstein condensation is applicable to systems with conserved $U(1)$ charge or particle number. However, it can be extended to systems with a weakly violated conservation: it can be applicable to a system of sufficiently long-lived quasiparticles – discrete quanta of energy that can be treated as condensed matter counterpart of elementary particles. In magnetically ordered materials, the corresponding propagating excitations are magnons – quanta of spin waves. Under stationary conditions the density of thermal magnons is small, but they can be pumped by resonance radio-frequency (RF) field (magnetic resonance). One may expect that at very low temperatures, the non-equilibrium gas of magnons could live a relatively long time, sufficient for formation of coherent magnon condensate.

Recently there appeared a number of articles, where authors claimed the observation of BEC of quasiparticles: excitons [3] and magnons [4]. To
claim the observation of BEC one should demonstrate the spontaneous emergence of coherence \cite{5}, and, even better, to show interference between two condensates. Since the spontaneous coherence has not been observed directly, the observation of BEC in the above articles is still under question.

In superfluid $^3$He-B, the formation of a coherent state of magnons was discovered about 20 years ago \cite{6}. In pulsed NMR experiments the spontaneous formation of a domain with fully phase-coherent BEC of magnons has been observed even in the presence of inhomogeneous magnetic field. The main feature of this Homogeneously Precessing Domain (HPD) is the induction decay signal, which rings in many orders of magnitude longer, than prescribed by inhomogeneity of magnetic field. This means that spins precess NOT with a local Larmor frequency, but precess coherently with common frequency and phase. This BEC can be also created and stabilized by continuous NMR pumping. In this case the NMR frequency plays a role of magnon chemical potential, which determines the density of magnon condensate. The interference between two condensates has also been demonstrated. It was shown that HPD exhibits all the properties of spin superfluidity (see Reviews \cite{7,8}). The main property is the existence of spin supercurrent, which transports the magnetization on a macroscopic distance more than 1 cm long. This spin supercurrent flows separately from the mass current, in contrast to the spin-polarized $^3$He-A$_1$, where spin is transported by the mass current. Also the related phenomena have been observed: spin current Josephson effect; phase-slip processes at the critical current; and spin current vortex – a topological defect which is the analog of a quantized vortex in superfluids and of an Abrikosov vortex in superconductors; etc.

The spin-orbit coupling due to dipole-dipole interaction between the spins of $^3$He atoms is responsible for the interaction between magnons. This interaction is relatively small, as a result HPD represents almost pure BEC of magnons \cite{9,10}. In typical $^3$He experiments, the critical temperature of magnon condensation is 3 orders of magnitude higher, than the temperature of superfluid transition; i.e. magnons undergo the condensation as soon as chemical potential and spin-orbit coupling allow for this process. The superfluid $^3$He is a very unique complex macroscopic quantum system with broken spin, orbital and gauge symmetries, where the structure of spin-orbit coupling can be varied experimentally. This leads to experimental realization of different types of magnon BEC in $^3$He-B \cite{11}; non-topological solitons in $^3$He-B called $Q$-balls in high energy physics \cite{12}; and also magnon BEC in $^3$He-A \cite{13}.

As distinct from the static equilibrium magnetic states with broken symmetry, the phase-coherent precession is the dynamical state which experiences the off-diagonal long-range order:

$$\left\langle \hat{S}_+ \right\rangle = S_+ = S \sin \beta e^{i\omega t + i\alpha}. \quad (1)$$

Here $\hat{S}_+$ is the operator of spin creation; $S_+ = S_x + i S_y$; $S = (S_x, S_y, S_z = S \cos \beta)$ is the vector of spin density precessing in the applied magnetic field $H = H\hat{z}$; $\beta$, $\omega$ and $\alpha$ are correspondingly the tipping angle, frequency and the phase of precession. In the modes under discussion here, the magnitude of the precessing spin $S$ equals to an equilibrium value of spin density $S = \chi H / \gamma$ in the applied field, where $\chi$ is spin susceptibility of $^3$He-B, and $\gamma$ the gyromagnetic ratio of the $^3$He atom (the coherent state with half of magnetization $S = (1/2) \chi H / \gamma$ has been also observed in bulk $^3$He-B \cite{14}). Similar to the conventional mass superfluidity which also experiences the off-diagonal long-range order, the spin precession in Eq. (11) can be rewritten in terms of the complex scalar order parameter $\psi$ \cite{7,9,10}

$$\left\langle \hat{\psi} \right\rangle = \psi = \sqrt{2S/\hbar} \sin \beta e^{i\omega t + i\alpha}, \quad (2)$$

If the spin-orbit interaction is small and its contribution to the spectrum of magnons is neglected in the main approximation (as it typically occurs in $^3$He), then $\psi$ coincides with the operator of the annihilation of magnons, with the number density of magnons being equal to condensate density:

$$n_M = \left\langle \hat{\psi}^\dagger \hat{\psi} \right\rangle = |\psi|^2 = \frac{S - S_z}{\hbar}. \quad (3)$$

This implies that the precessing states in superfluid $^3$He realize the almost complete BEC of magnons. Spin-orbit coupling produces a weak interaction between magnons and leads to the interaction term in corresponding Gross-Pitaevskii equation for the BEC of magnons (further we use units with $\hbar = 1$):

$$\frac{\delta F}{\delta \psi^*} = 0, \quad (4)$$

$$F = \int d^3 r \left( \frac{|\nabla \psi|^2}{2m_M} - \mu |\psi|^2 + \hat{E}_D(|\psi|^2) \right), \quad (5)$$

Here the role of the chemical potential $\mu = \omega - \omega_L$ is played by the shift of the precession frequency from the Larmor value $\omega_L = \gamma H$. In coherent states, the
precession frequency $\omega$ is the same throughout the whole sample even in the nonuniform field; it is determined by the number of magnons in BEC, $N_M = \int d^3r n_M$, which is conserved quantity if the dipole interaction is neglected. In the regime of continuous NMR, $\omega$ is the frequency of the applied RF field, $\omega = \omega_{RF}$, and the chemical potential $\mu = \omega_{RF} - \omega_L$ determines the magnon density. Finally, $m_M$ is the magnon mass; and $E_D$ the dipole interaction averaged over the fast precession. The general form of $E_D(|\Psi|^2)$ depends on the orientation of the orbital degrees of freedom described by the unit vector $\mathbf{l}$ of the orbital momentum, see Ref. [15].

In the coherent precession in bulk $^3$He-B the spin-orbit coupling orients $\mathbf{l}$ along $\mathbf{H}$. In this case the interaction term $E_D$ has the form different from conventional 4-th order term in dilute gases [9]:

$$E_D = 0 \ , \ |\Psi|^2 < \frac{5}{4} S \ , \ (6)$$

$$E_D = \frac{8}{15} \chi \Omega_L^2 \left( \frac{|\Psi|^2}{S} - \frac{5}{4} \right)^2 \ , \ |\Psi|^2 > \frac{5}{4} S \ . \ (7)$$

Here $\Omega_L \ll \omega_L$ is the Leggett frequency which characterizes the dipole interaction. If the chemical potential $\mu$ is negative, i.e. $\omega < \omega_L$, the minimum of the Ginzburg-Landau (GL) energy $E_D(|\Psi|^2 - \mu|\Psi|^2)$ corresponds to $\Psi = 0$, i.e. to the static state with equilibrium magnetization ($\beta = 0$). For $\mu > 0$ ($\omega > \omega_L$) the minimum of the GL energy corresponds to $|\Psi|^2/S = (5/4) + (15/32)\tilde{\mu}$, where $\tilde{\mu} = (\omega^2 - \omega_L^2)/\Omega_L^2$. The consequence of such a peculiar profile of the interaction term is that as distinct from the dilute gases the formation of the magnon BEC starts with the finite magnitude $|\Psi|^2 = (5/4) S$. This means that the coherent precession starts with a tipping angle equal to the magic Leggett angle, $\beta = 104^\circ$, and then the tipping angle increases with increasing frequency shift. This coherent state called the HPD persists indefinitely, if one applies a small RF field to compensate the losses of magnons caused by small spin-orbit interaction.

In conventional magnetic systems, magnetization precesses in the local field with the local frequency shift and thus experiences dephasing in the inhomogeneous field. In the case of magnon BEC, the rigidity of the order parameter (the gradient term in Eq. 5) plays an important role. The spatial dephasing leads to the gradient of chemical potential. This in turn excites the spin supercurrents, which finally equilibrate the chemical potential. In the steady state of magnon BEC the gradient of a local field is compensated by small gradient of magnon density $|\Psi|^2$ in such a way, that $\omega$ and $\alpha$ remain homogeneous throughout the whole sample.

In a pulsed NMR experiment, magnetization is deflected by a strong RF pulse. The typical induction signal after the pulse in the cell with a large gradient of magnetic field along the axis of the cell is shown in Fig. 1. Due to the field gradient the induction signal should dephase and disappear in about 1 ms. Instead, after a transient process of about 2 ms, the induction signal acquires an amplitude corresponding to a 100% coherent precession of the deflected magnetization with the spontaneously emerging phase. This coherent state lives 500 times longer than the dephasing time caused by inhomogeneity.

What happens with the magnon system during the transient period? As soon, as at the higher field end of the cell the deflection of magnetization becomes smaller than $104^\circ$, the spin-orbit interaction cannot anymore compensate the gradient of magnetic field. The cell splits into two domains, in one of them the magnetization is stationary, while in the other one magnons condense with the density close to $|\Psi|^2 = (5/4) S$, i.e. the magnetization precesses with $\beta$ slightly above $104^\circ$. In the subsequent process of relaxation caused by the non-conservation of magnon number, the volume of the BEC condensate (HPD) decreases. During the relaxation, the BEC does not loose the phase coherence, but its chemical potential (precession frequency) changes. The frequency of HPD corresponds to Larmor frequency at

![Fig. 1.](image-url)
the boundary of the domain and slowly changes in time with relaxation as the boundary moves down (see Fig. 1; the amplitude of the signal exactly corresponds to the record of the frequency).

The small RF field can compensate the HPD relaxation by pumping additional magnons. In this steady state many phenomena of spin superfluidity have been observed: Josephson effect; magnetization transport by spin supercurrent; critical spin current velocity, phase slip; etc. Here we will discuss the observation of a spin vortex [10].

In HPD with a single vortex in the central part of cylindrical cell the phase $\alpha$ of precession changes by $2\pi$ around the center, i.e. $\alpha$ is opposite on the opposite sides of the cell. In the central part of the cell, i.e. in the vortex core, the magnetization remains vertical and does not precess. We created HPD with a spin vortex by applying the quadrupole RF field. For this purpose we connected two parts of the saddle NMR coil in opposite directions, so that the phase of RF field (and consequently the phase $\alpha$) was opposite at the opposite sides of the cell. By these NMR coils we observed practically the same HPD signal as in the conventional arrangement with the parallel connection of the coils, though with a slightly reduced amplitude. This shows that we created HPD with opposite $\alpha$ on opposite sides of the cell. To verify this we installed a pair of small pick-up coils at the top of the cell connected in usual way. When we switched off the RF field, the pick up coils received a very small RF signal from HPD, while the frequency of this signal corresponded to the full HPD signal. This means that HPD generated the signal with the opposite phase at the two sides of the pickup coils, which nearly compensated each other (see Fig. 2).

This corresponds to HPD with a circular gradient of $\alpha$, as shown in inset of Fig. 2. The magnetization is oriented vertically in the vortex core. On the periphery of the cell it precesses with tipping angle $104^\circ$, and with $2\pi$ phase winding around the center. This type of HPD should radiate at frequency, which corresponds to the Larmor field on the boundary of HPD, but should not produce any signal in the pick-up coil. However, a small signal appears due to asymmetry of the pick-up coil; oscillations of this signal may correspond to mutations of the vortex core.

Of course, the HPD has been observed, studied and explained on the basis of theory of spin superfluidity and non-linear NMR long time ago [6]. However, the consideration of this phenomenon in terms of magnon BEC not only demonstrates the real system with the BEC of excitations, but also allows us to simplify the problem and to study and search for the other types of magnon BEC in $^3$He, such as $Q$-balls [13]; HPD$_2$ state found in a deformed aerogel [11]: coherent precession in $^3$He-A also found in the deformed aerogel [14]; etc.

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