Simple potential model for interaction of dark particles with massive bodies

Nurgali Takibayev

al-Farabi Kazakh National University, Almaty 050040, Kazakhstan
E-mail: takibayev@gmail.com

Abstract. A simple model for interaction of dark particles with matter based on resonance behavior in a three-body system is proposed. The model describes resonant amplification of effective interaction between two massive bodies at large distances between them. The phenomenon is explained by catalytic action of dark particles rescattering at a system of two heavy bodies which are understood here as the big stellar objects. Resonant amplification of the effective interaction between the two heavy bodies imitates the increase in their mass while their true gravitational mass remains unchanged. Such increased interaction leads to more pronounced gravitational lensing of bypassing light. It is shown that effective interaction between the heavy bodies is changed at larger distances and can transform into repulsive action.

1. Introduction

It is well known, that the dark matter mystery emerged at the analysis of astronomical data when considerable discrepancy between orbital velocities of stellar objects at periphery of large galaxies compared to velocities calculated for usual gravitational fields has been found. This phenomenon was explained by the dark matter action that resulted in increased mass of such stellar objects. Discovered effect of gravitational lensing supported such assumption [1, 2].

In this presentation a model for resonance amplification of interaction between two stellar objects is proposed. The amplification is stimulated by additional interaction of the massive objects with dark matter particles (hereinafter dark particles). Our consideration is conducted in the framework of non-relativistic quantum theory of scattering in the three-body system, more exactly, for describing the interactions of two heavy bodies in presence of a dark particle of a very small mass. The solutions distinctly demonstrate that the effective interaction of these massive objects has the resonance behavior with the changing of distance between them.

So, at relatively small distances the catalytic amplification is negligibly small and the effective interaction here coincides with the direct (gravitational) interaction between the heavy bodies. At larger distances, the catalytic action of the dark particles increases and the effective interaction between the heavy bodies also increases. At even larger distances, the catalytic action becomes smaller and then negative; so instead of attraction between the heavy bodies, it may cause their repulsion. The last to a certain extent would imitate the effects of dark energy.

We assume that the dark particles do not interact with each other; the massive bodies interact not only via gravity, but also through exchange of dark particles. For simplicity, we would not consider the three-body type forces.
2. Three-body quantum-mechanical scattering problem

Solutions for scattering amplitudes in the three-body problem are based on solutions for scattering amplitudes in two-body subsystems [3, 4]. The i-pair potential of interaction between the dark particle and a heavy body could be taken in the separable form: $V_i = |\nu_i > \lambda_i < \nu_i|$, where $< \nu_i|k >$ is the form-factor of the potential, $\lambda_i$ is the coupling constant. Then the two-body $t_i$-matrix equals: $t_i = |\nu_i > \eta_i < \nu_i|$, where $\eta_i^{-1} = \lambda_i^{-1} - < \nu_i|G_0|\nu_i >$, $G_0 = (E_0 - H_0)^{-1}$ is the free Green’s function, $E_0 = k_0^2/2m$.

For illustration, we consider the simple potential $< \nu_i|k > = Nk/(\beta^2 + k^2)$, where constants $N$ and $\beta$ have been taken to obtain the virtual level with the extremely small energy $E_v = -k^2/2m$, when $\eta_i^{-1}(E_0) = 0$ for $k_0 = -i\kappa_v$. We use the units $\hbar = 1$ and $c = 1$.

Another simplification is related to the Born-Oppenheimer approximation which leads to the simple solutions of the three-body problem [4]. The task therefore is reduced to determination of $T$-matrix for interactions in two-body subsystems [3, 4]. The solutions for scattering amplitudes in the three-body problem are based on solutions for interactions in two-body subsystems [3, 4].

The effective potential as well as the amplitude $M_{ij}(r)$ would depend resonantly on both the distance between the heavy particles and the energy of the dark particle [5]. The enhancement factor could be written as $\xi = < V_{ef} > / < V_i >$, where $< V >$ means taking the average values with consideration of the density of dark matter particles. Assuming that the density of number of dark matter particles is comparable with density of massive particles, one can write:

$$\xi = 1 + (1 - \eta_{dh}/\lambda_{dh})[1 + (M_{ih}^+ + M_{ih}^-)\eta_{dh}] .$$ (8)
Figure 1. Real and imaginary parts of $\xi(t_0, \rho)$; $\lambda_{dh} = -0.95$; the resonance at $\rho \approx 2.65$.

Let us assess the characteristic distances. For example in our model, resonance takes place at the following parameters: $\lambda_{dh} = -0.95$, $\rho = r \cdot \beta = 2.65$. In the case of the distance between the massive bodies is about $r \approx 2.65 \cdot 10^{22} \text{cm}$, then $k_0 \approx 10^{-23} \text{cm}^{-1}$, $t_0 = k_0/\beta$.

Taking into account the resonant behavior of the effective interaction at such distances, one can get from (8) the following multiplication of the interaction potential for the factor: $\xi \approx 100$ at $\rho = 2.65$.

This important outcome demonstrates that the interactions with the dark particles can substantially change the character of forces acting between the heavy bodies (Fig.1).

The orbital velocity of the peripheral body becomes higher than that at normal gravity owing to the enhancement factor. Moreover, this flux would glue up these two heavy bodies and, with respect to other particles and fields (gamma quanta, for instance) such a system would appear as a single object. The system would have the effective mass much higher than its own mass.

Such mechanism can contribute to gravitational lensing of electromagnetic radiation. Really, we can include in the equation (6) the additional interactions with gammas and obtain the additional enhancement of these interactions.

It is remarkable that the enhancement factors can be negative at certain values in the case of two-body attractive potentials. For example, in the case of $\lambda_{dh} = -0.95$ we have $\xi \approx -80 - i \cdot 50$ at $\rho = 2.85$. Then the resulting potential between two heavy bodies looks like a repulsive force.

Conclusion
It is important to note that the catalytic impact of dark particles on the interaction between the two heavy bodies becomes very strong only at large distances between these bodies. This effect has to be much stronger in the case the energy of two-body (dark particle and heavy body) virtual state of the subsystem is situated closer to the physical scattering region ($\lambda_{dh} = -1 + \delta$, when $\delta \to 0$). It is remarkable that if the distance between two heavy bodies is increasing, then the attractive forces can transform to the repulsive ones. This changing can lead to the formation of structures, for example, star clusters. If we assume the existence of several types of dark particles, they can produce many different structures such as galaxies, galaxy clusters, etc.

In reality, the interactions between dark particles and a heavy body can be more complex and the catalytic action may happen to be far from simple.

References
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