Control Strategy for the Pseudo-Driven Wheels of Multi-wheeled Mobile Robots Based on Dissociation by Degrees-of-Freedom

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ABSTRACT The tractive capability of Multi-wheeled mobile robots (WMRs) depends heavily on the performance of the driving control of their steerable wheels. The recently proposed concept of a pseudo-driven wheel (PDW) provides a simple, versatile, and effective way to improve the tractive capability. However, its focus is to eliminate the internal forces between the wheels and save energy; therefore, it cannot provide sufficient driving force to the body on rough terrains. This paper proposes a control strategy to adjust the power output of a PDW along the directions of two degrees of freedom, an approach that improves the trajectory tracking control of WMRs, especially on rough terrains. To achieve the desired active following control of the PDW, an active disturbance rejection controller is used to compensate the disturbances arising from the wheel-terrain interaction. The Adams and Simulink are used for a joint simulation of which results show that the proposed control strategy is necessary for accurate trajectory tracking, for which the feasibility and effectiveness were verified by the experimental results. In summary, the trajectory tracking accuracy of the WMR and its tractive capabilities on rough, sandy terrain were improved.

INDEX TERMS Active disturbance rejection controller (ADRC), active following control, control strategy, pseudo-driven wheel (PDW), wheeled mobile robot (WMR).

I. INTRODUCTION

The terrains on the surface of planets and other astronomical bodies such as Mars and the Moon are mostly rough and deformable [1], [2], and thus present considerable challenges to the trajectory tracking control of wheeled mobile robots (WMRs). Rovers with steerable wheels are widely used in such contexts, because of their high maneuverability and flexibility [3], which improve their bodies’ steering ability. However, these robots are over-actuated [4], and the actuators have to be synchronized in order to minimize wheel slippage [5], which increases the complexity of the control scheme. Determining the proper control strategy for steerable wheels in Multi-wheeled mobile robots is therefore an important aspect of immediate practical impact.

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Trajectory tracking on WMRs is usually done with both universal and conventional wheels [6]. Universal wheels (e.g., mecanum wheels) are composed of a small number of passive rollers mounted on the periphery of the wheel [7], which allow the rover to move both linearly and rotationally. The holonomic features and omnidirectional mobility of robots with universal wheels have been widely studied in the context of trajectory tracking. A trajectory tracking approach for an omnidirectional mobile robot based on trajectory linearization and a nonlinear controller has been presented in [8], but the authors did not consider the effect of wheel slippage in the direction of the traction force. A differential sliding-mode controller to control three-wheeled omnidirectional mobile systems in the presence of disturbances and friction has also been proposed in [9]. Additionally, a nonlinear-model-based predictive control algorithm for online motion planning and tracking of an omnidirectional autonomous robot...
with moving-obstacle avoidance was presented in [10]. These studies have shown that robots with universal wheels have simpler mechanical structures than robots with conventional wheels, which makes them simpler to control. However, the capabilities of their wheels when moving on rough terrains are limited by the diameter of the rollers [11]. Furthermore, this type of wheel creates some vertical and horizontal vibrations on the body [12], which can cause extra body sinkage. Therefore, these wheels have not been widely adopted in the field of planetary exploration.

Conventional wheels do not present many of the shortcomings of universal wheels; they are more efficient when driving on a straight line, and can successfully go over higher bumps [13]. They can also achieve near-omnidirectional motion by adding a wheel steering mechanism [7]. Steering wheels have also been the subject of much research. Yun has shown that increasing the number of steerable wheels from one to two can reduce the complexity of the robot steering control and increase its intuitiveness [14], but the studied WMR had a very simple structure. The problem of coordinated rotation and steering angle on Multi-wheeled mobile robots has also been studied in [15], [16], where the current state of motion of the robot is represented by its instantaneous center of motion (ICM), for which a valid trajectory is derived. However, this method suffers from some numerical drawbacks when \( N \) inputs (steerable wheels) must satisfy \((N-1)\) nonholonomic constraints. To address these numerical problems, an alternative ICM representation, based on spherical coordinates, was proposed in [17]. However, this approach does not consider constraints such as steering or driving speeds.

Overall, robots with multiple wheels can effectively average the driving load, but the resulting system’s numerical drawbacks and redundant control present big challenges to trajectory tracking control. Recently, Qi et al. [18] proposed the concept of a new universal-steerable wheel, the pseudo-driven wheel (PDW), which is driven by the body’s motion. It is based on a model-free control approach, and is capable of reducing the system model’s dimension, thus removing the difficulties associated with redundant control. In [19], the authors designed a new control system with an online sequential extreme learning machine combined with a proportional-integral-derivative controller to improve the performance of the PDW for working in unknown environments. However, the focus of the previous researches was on the control of the PDW, i.e., how the active following control is better respectively achieved by adjusting the drawbar pull \( F_x \) and lateral force \( F_y \), rather than the coordinate control between the forces to the PDW. In other words, the wheel can move both in heading \( (x) \) and in the lateral \( (y) \) direction at the same time. However, uncoordinated movements of the PDW in these two directions will cause the WMRs to deviate from the track when the WMR moves on rough terrains. Therefore, in this paper, we propose a control strategy to improve the tracking control of WMRs using PDWs, and then proceed to analyze its application in different contexts.

The proposed PDW control strategy is designed to improve the tractive capability of WMRs. It enhances the driving power between the PDW and the body on complicated sandy terrains, and optimizes energy consumption on flat, sandy terrain. Its application on three-, four- and six-wheeled mobile robots is discussed. This control strategy can adjust the open-loop weights along the directions of the two degrees of freedom (DOFs) of the PDW, based on the force measured at the wheel. To successfully achieve the desired active following control, an active disturbance rejection controller (ADRC) was also introduced in [18], which can compensate the disturbances resulting from wheel-terrain interactions. The proposed control strategy is experimentally validated on the PDW of a three-wheeled mobile robot.

The main contributions of this work are:

1) Proposing a PDW control strategy that improves the tractive capabilities of WMRs on rough terrains.
2) Effectively and efficiently enhance the trajectory tracking accuracy of WMRs.
3) Study and analyze the application of the proposed control strategy on different Multi-wheeled mobile robots.

The rest of this paper is organized as follows. Section II describes the problem that is solved by the control strategy. Section III proposes the PDW control strategy and discusses its application on WMRs. Section IV introduces the model-free controller used to achieve the desired active following control. Section V verifies the necessity for the control strategy. Section VI describes the proposed control strategy’s performance through physical experimentation. Finally, the main conclusions to be extracted from this study are summarized in Section VII, which concludes the paper.

II. PROBLEM DESCRIPTION

PDW has free movements in 2 DOFs of the horizontal plane simultaneously to reduce the internal force between the wheels of WMRs largely and optimize energy consumption [18]. However, this control method decreases the driving force from the PDW to the body, causing side effects (especially on complex terrains), high slip of wheels, and even breakdown. We were inspired by the fish tail movement, and the characteristic of a robotic fish tail is shown in Figure 1 [20].

The swing of the fish tail generates the reaction force of water as well as helps in traversing along the objective trajectory. A phenomenon was discovered in which the swing strength is discrepant in different situations [21]. Because the swing of the fish tail adds additional leading-edge vortex [22], it causes extra burden on movement and deviation from the trajectory. Therefore, the fish continuously adjusts its swing according to the state of tracking the trajectory.

Applications of the PDWs on soft terrains are considered b the WMRs are equipped with mechanical tails, and the soft sand is similar to the surface of water. Therefore, we believe that the wheel’s motion in the 2 DOFs should be constrained actively according to the roughness level of terrain to ensure...
the sufficient driving force from the PDWs to the body in trajectory tracking. In rough terrains, saving energy should become a secondary concern. Therefore, the control strategy for PDWs based on dissociation by degree of freedom is proposed.

III. CONTROL STRATEGY FOR PSEUDO-DRIVEN WHEELS

This section first proposes a control strategy that changes the power output levels of a PDW along the wheel’s longitudinal and lateral DOFs. Applications of PDWs on three-, four-, and six- (or more) wheeled mobile robots are then discussed.

A. CONTROL STRATEGY

Motions of a WMR’s wheel can be simplified into straight and steering movements. These two different motion states are shown in Figure 2. As we found, when the rover moves straight (linear motion) over an obstacle in rough terrain, the lateral power output should be turned off, as shown in Figure 2(a). On the other hand, when the rover is steering on rough terrain, the heading power output should be adjusted, as shown in Figure 2(b). The proposed control strategy is based on this; the power output levels along the two DOFs are adjusted after considering the force being sensed by the wheel. In other words, the terrain on which the rover is moving is characterized by the forces it creates on the PDW, which in turn determines the power output of the PDW in both DOF directions.

For the remainder of this article, the DOF of a PDW along the heading direction will be represented by $DOF_x$, whereas $DOF_y$ will represent the DOF corresponding to the lateral direction. Furthermore, open-loop weights corresponding to those two DOFs will be introduced as $\alpha$ and $\beta$, respectively. The state of total PDW power output $w_o$ is given by Eq. (1).

$$w_o = \alpha DOF_x + \beta DOF_y,$$

(1)

where $\alpha$ and $\beta \in [0, 1]$, and $DOF_x = DOF_y = \{0, 1\}$ throughout the paper. Moreover, $DOF_x$ and $DOF_y$ are used to express the subjective intentions of the controller. $\alpha$ and $\beta$ can be adjusted adaptively in future works.

To ensure the wheel’s tractive capability, the power output along $DOF_x$ must always be different from zero, irrespective of whether the WMR is involved in straight or steering movement. On the other hand, the power output along $DOF_y$ is an essential part of all steering motions. The focus of the discussion will therefore be on adjusting the power along $DOF_y$ when steering. These adjustments are key factors in determining whether the WMR can or cannot accurately track the trajectory. In straight motion tracking, the wheels should adjust the open-loop weight $\beta$ corresponding to $DOF_y$ according to the magnitude and fluctuations of the lateral force $F_y$. For example, if the value of $F_y$ increases suddenly without the WMR having initiated a steering maneuver, indicating that the wheel may have hit an obstacle, what should be done is to roll over that obstacle, instead of attempting to steer and avoid it. That is: decrease $\beta$ and increase $\alpha$.

Assumption 1: the PDW will not encounter a major obstacle that cannot be overcome (e.g., an obstacle larger than the distance from the body to the terrain) because it is a universal-steerable wheel adaptively driven by the body motion. It will not be placed in the front of the body.

On the other hand, when the PDW is subjected to lateral forces with small values and smooth changes, the lateral power output should be adjusted to reduce the internal

FIGURE 2. Pseudo-driven wheel tracking the motion of the body on rough terrain.
conflicting forces between the wheels, thereby improving their tractive capability and reducing energy consumption. In fact, in this case, the other wheels will also be affected by lateral forces, leading to a different velocity for each wheel, which will then create some internal conflicting forces between the wheels. The two rules for reducing the internal conflicting forces by adjusting the power output along $DOF_y$ are shown in Figure 3. Where $F_1$ denotes the internal force between wheels, and $F_2$ denotes the driving force from PDW to the body. The PDW can be driven so as to resist the lateral force by adjusting the power output along $DOF_y$ as depicted in Figure 3(a), or it can be driven so as to follow the lateral force, as shown in Figure 3(b), an approach that can also effectively eliminate the internal conflicting forces, but will cause the body to deviate from the trajectory.

B. USING PDWs ON WMRs

Given that the PDWs use an active following control approach [18] (and the proposed control strategy), they should be placed in the secondary driving position of the WMR, such as the middle or the rear. Furthermore, in order to reduce the additional internal forces resulting from the necessary adjustments, PDWs should appear in pairs or be placed on the body’s axis of symmetry.

Applying PDWs to Multi-wheeled mobile robots—such as three-, four-, or six-wheeled mobile robots—can help reduce the dimension of the system’s model and remove the problem of redundant control. Therefore, operation of the WMR becomes simpler, and the double objective of a redundant mechanism with a non-redundant control system is achieved. Furthermore, when the robot’s intended motion encounters a difficult challenge to overcome, researchers can easily convert the PDWs into ordinary steerable wheels through the control terminal, thus restoring full human control.

In three- or four-wheeled mobile robots, considering their driving characteristics [23], [24], the PDWs should be used in the symmetry and rear, so as to support and steer the body’s motion. Because these wheels can sense changes in the terrain and adaptively adjust to them, they not only increase the flexibility of the WMR’s motion, but also allow more timely adjustments when the body is about to deviate from the trajectory, when compared to human control. Application examples are shown in Figure 4(a) and (b).

Rough terrains pose severe difficulties in tracking steering motions, because the power output of the PDW may not be compliant, given the driving characteristics resulting from the active following control. For example, when the WMR is steering on sloping terrain, the power output along $DOF_x$ must be adjusted. Otherwise, the wheel will cause the body to deviate from the trajectory, because an excessive power output in downhill terrain or an insufficient power output in uphill terrain will result in additional movements of the WMR along $DOF_y$. Therefore, when the WMR is steering, the PDWs should increase or decrease their open-loop weight $\alpha$ (on uphill and downhill terrain, respectively), and increase the open-loop weight $\beta$ to preserve the trajectory tracking performance.
pseudo-driven wheels are more suitable for being placed in the middle positions, as shown in Figure 4(c). Generally speaking, the use of PDWs in Multi-wheeled mobile robots should present greater advantages as the number of wheels increases, a notion that requires further exploration.

**IV. ACTIVE FOLLOWING CONTROL DESIGN**

This section first introduces the problem of wheel-terrain interaction, and then presents the design of a model-free ADRC controller to adjust the power output along the two DOFs of a PDW. This control system can accomplish the desired active following control and compensates the resistance and disturbances resulting from terrain deformation.

**A. WHEEL-TERRAIN INTERACTION**

The wheel-terrain interaction of a wheel moving on deformable terrain is shown in Figure 5, where $W$ is the equivalent vertical load on the wheel; $F_R$, $F_N$, and $F_{DP}$ are the horizontal resistance, normal force, and drawbar pull, respectively; $\tau$ is driving torque; $\omega$ and $v$ are the angular and linear velocities of the wheel, respectively; $\theta_1$, $\theta_2$, and $\theta_m$ are the entrance angle, leaving angle, and angular position of the maximum stress, respectively; $\sigma(\theta)$ and $\tau(\theta)$ are the continuous normal stress and the shearing stress, respectively; $z$ and $l$ are wheel sinkage and the vertical distance between the wheel and the ground equivalent force acting point and the wheel center $O$; and $r_w$ is the wheel radius [26].

![Figure 5. Forces and torque acting on a driving wheel.](image)

Based on the results presented in [27], [28] concerning the wheel-terrain interaction, the torque of the wheel is given by:

$$
\begin{align*}
\tau &= F_{DP}r + F_Nr(aR + cR - RC), \\
\tau_R &= aR F_Nr + cR F_Nr,
\end{align*}
$$

where $\tau_R$ is the resistance torque; $RC$ is the resistance torque coefficient; $s$ is the slip rate; and $aR, bR,$ and $cR$ are the wheel parameters.

Applying the momentum moment theorem to the wheels, their dynamic characteristics can be expressed as follows:

$$
I \dot{\omega} = \tau - \tau_R,
$$

where $I$ denotes the rotational inertia.

Considering (2) and (3), a dynamic equation expressing the mechanics of the wheel-terrain interaction can be obtained as:

$$
I \dot{\omega} = F_{DP}r - F_Nr(aR + cR - RC),
$$

where $\omega$ is the control input, and the optimized drawbar pull force $F_{DP}$ is the system output, which is tracked by an improved active disturbance rejection controller (ADRC). The simplified dynamic model [18] can be written as:

$$
\ddot{q} = f(F_N, s, \Delta) + Bu,
$$

where $q$ denotes the desired angular displacement, $B$ is a constant, $u$ denotes the force on the wheel measured by a force sensor, and $f(F_N, s, \Delta)$ is an additional state variable, and $\Delta$ represents the uncertain part of the model. The system has order two.

**B. MODEL-FREE ADRC CONTROLLER DESIGN**

Currently, the widely used model-free control method is PID control. Furthermore, to consider special circumstances, iterative learning control [29] and repetitive control [30] were developed mainly. However, in the case of frequent large disturbances and unknown working environment, the active disturbance rejection controller (ADRC) [31] is a more suitable method, because it has stronger adaptive control ability for disturbances than traditional methods.

When performing velocity tracking based on active following control [18], the control objective is achieved by controlling the drawbar pull and the lateral force (forcing $F_x = 0$ and $F_y = 0$, respectively). When the focus is on trajectory tracking, the control objective will be achieved by adjusting the open-loop weights $\alpha$ and $\beta$.

The overall control system uses two ADRC subsystems to reduce the drawbar pull $F_x$ and the lateral force $F_y$ of the PDW, as shown in Figure 6(a). Where $F_{yd}$ is the desired drawbar pull, and $F_{yd}$ is the desired lateral force. $\omega$ and $\theta$ are

![Figure 6. Active following control decoupling scheme.](image)
the objective angular velocity of the PDW and steering angle, respectively. There is a strong structure similarity between these two controllers, and the parameters of one of them can be used in the other with slight adjustments.

The decoupled control system can independently adjust the output power along the two PDW DOFs (DOF₁ and DOF₂) based on an active following control approach. The design procedure for the lateral force $F_y$ of the PDW will be described next, and the structure of the respective controller is shown in Figure 6(b). Moreover, each part can be designed independently.

The overall ADRC control law can be divided into three parts: tracking differentiator (TD), nonlinear state error feedback law (NLSEF), and extended state observer (ESO).

1) TRACKING DIFFERENTIATOR
To smooth out rapid changes of the input signal and avoid noise amplification, it is necessary to add a tracking differentiator at the early stages of the controller. The desired signal and its differential signal are obtained by transient profile generation. An improved approximate calculation to obtain the differential signal was proposed in Han [32], and is shown in (6).

\[ \dot{\eta}(t) = \frac{\eta(t - T_1) - \eta(t - T_2)}{T_1 - T_2}, \]  

where $\eta$ is the desired signal, and $T_1$ and $T_2$ represent two different moments.

The time-optimal control comprehensive function $f_{han}$ $(x_1, x_2, r_0, h_0)$ was defined by Han [33] as:

\[
\begin{cases}
  d = r_0 h_0^2, \\
  a_0 = h_0^2 x_2, \\
  y = x_1 + a_0, \\
  a_1 = \sqrt{d(2d + 8y)}, \quad a_2 = a_0 + \text{sign}(y)(a_1 - d)/2, \\
  f_{sg}(x, d) = (\text{sign}(x + d) - \text{sign}(x - d))/2, \\
  a = (a_0 + y)f_{sg}(y, d) + a_2(1 - f_{sg}(y, d)), \\
  f_{han} = -r_0 \frac{d}{d}f_{sg}(y, d) + r \text{sign}(a)(1 - f_{sg}(a, d)),
\end{cases}
\]

where $r_0$ and $h_0$ are controller parameters; $x_1$ and $x_2$ are the desired input and its differential signal, respectively.

The desired input and its approximate differential signal are then tracked by the tracking differentiator based on the time-optimal control comprehensive function [32], as per (8), where $h$ is the sampling period.

\[
\begin{cases}
  f_{hh} = f_{han}(x_1 - x_4, x_2, r_0, h_0), \\
  x_1(k + 1) = x_1(k) + h x_2(k), \\
  x_2(k + 1) = x_2(k) + h f_{hh}.
\end{cases}
\]

2) NONLINEAR EXTENDED STATE OBSERVER
The control system performance can be seriously affected by unknown disturbances and the unmodeled part of the WMR; the linearization feedback of the dynamic system is therefore a critical aspect. The core ADRC functions are the estimation and compensation of total disturbances. Treating $f(\cdot)$ as an additional state variable, $\xi_3 = f(\cdot)$, the original plant (5) can be rewritten as follows [18]:

\[
\begin{align*}
  \xi_1 &= q, \\
  \xi_2 &= Bu, \\
  \xi_3 &= f(\cdot), \\
  \dot{\xi}_1 &= \xi_3 + \xi_2, \\
  y &= \xi_1.
\end{align*}
\]

A nonlinear ESO for a two-order system, in discrete form, was designed by Gao [34] and is shown in (10).

\[
\begin{align*}
  e(k) &= z_1(k) - y(k), \\
  fe_1 &= fal(e_{12}, y_{12}, \delta_{12}), \\
  fe_2 &= fal(e_{12}, y_{12}, \delta_{12}), \\
  fe_3 &= fal(e_{13}, \gamma_{13}, \delta_{13}), \\
  z_1(k + 1) &= z_1(k) + h(z_2(k) - \beta_1 fe_1), \\
  z_2(k + 1) &= z_2(k) + h(z_3(k) - \beta_2 fe_2) + Bu, \\
  z_3(k + 1) &= z_3(k) + h(-\beta_3 fe_3),
\end{align*}
\]

where $z_1 \rightarrow \xi_1$, $z_2 \rightarrow \xi_2$, and $z_3 \rightarrow \xi_3$ denote the observed values; $y$ is the regulated output of the system; $\beta_1$, $\beta_2$, and $\beta_3$ are tuning parameters.

3) NONLINEAR ERROR STATE FEEDBACK
A more efficient nonlinear combination method is used here to replace the weighted-sum linear combination of the PID controllers. According to Han [33], the efficacy of the controller with nonlinear feedback will be significant. Therefore, nonlinear error state feedback was used to improve the obtained results using the following function proposed by Han [32]:

\[
fal(e, \gamma, \delta) = \begin{cases} 
  e, & |x| < \delta \\
  \frac{1}{\delta^2 - \gamma^2}, & |x| \geq \delta
\end{cases}
\]

where $e$ denotes the error between observer outputs and the desired value of the controller, and $\gamma$ and $\delta$ are parameters.

The nonlinear feedback is calculated with (12):

\[
\begin{align*}
  u_0(k + 1) &= u_0(k) + kp fal(e_1, \gamma_{21}, \delta_{21}) \\
  &+ k_I \sum_{i=1}^{n} fal(e_1, \gamma_{21}, \delta_{21}) \\
  &+ kp fal(e_2, \gamma_{22}, \delta_{22}),
\end{align*}
\]

where $kp$, $k_I$, and $k_D$ are the proportional, integral and differential gains of the controller, respectively.
4) OVERALL CONTROL LAW

Combining the above three components, the overall control law can be derived as (13):

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= a_2 x_1 + a_1 x_2 + Bu, \\
y &= x_1,
\end{align*}
\]

where \(a_1\) and \(a_2\) are the gain coefficients of the system.

The desired force is zero in this system; therefore, all the outputs of TD are zeros.

Consider the controller output \(u\) of Eq. (13) and Eqs. (14), (15). Moreover, letting \(X = [x_1, x_2]^T\) and \(Z = [z_1, z_2]^T\) as follows:

\[
\dot{X} = A_{11}X + A_{12}Z + A_{13}Z_3
\]

where \(A_{11} = \begin{bmatrix} 0 & 1 \\ a_2 & a_1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 0 \\ -\lambda_1(z_1)k_p - \lambda_2(z_2)k_D \end{bmatrix}, \quad A_{13} = [0, -1]^T.\)

Combining Eqs. (17) and (18), we get

\[
\dot{Z} = A_{21}Z + A_{22}u
\]

where \(A_{21} = \begin{bmatrix} 0 & 1 \\ -\lambda_1(z_1)k_1 - \lambda_2(z_2)k_2 \end{bmatrix}, \quad A_{22} = [\beta_1 \phi_1(e) \beta_2 \phi_2(e)]^T; \quad k_1 = k_p \) and \(k_2 = k_D.\)

Combining Eqs. (17) and (18), we obtain

\[
\dot{Y} = A_{11}Y + A_{11}A_{12}Z + A_{13}\beta_3 \phi_3(e) \tilde{u}
\]

\[
\dot{Z} = A_{21}Z + A_{22}\tilde{u}
\]

The observation and state variables of the system are combined to further get

\[
\dot{x} = A\bar{x} + b\tilde{u}
\]

\[
e = c^T x + \rho\tilde{u},
\]

where \(\bar{x} = [Y \ Z]^T, \ A = \begin{bmatrix} A_{11} & A_{11}A_{12} \\ 0 & A_{21} \end{bmatrix}, \ b = A_{11}A_{11}A_{12}, \ c = [c_1^TA_{11}^{-1} c_2^T], \) and \(\rho = -c_1^TA_{11}^{-1}A_{13}\beta_3 \phi_3 = -\beta_3^T \phi_3(e).\)

Taking the derivative of the second formula of Eq. (21), we get

\[
\dot{\tilde{u}} = c^T A\tilde{y} + (\rho + c^T b)\tilde{u},
\]

which we rewrite by substituting \(\tilde{y}\) for \(\tilde{x}\) as follows:

\[
\dot{\tilde{y}} = A\tilde{y} + \tilde{b}
\]

\[
\dot{\tilde{e}} = c^T A\tilde{y} + (\rho + c^T b)\tilde{u}.
\]
Referring to [36], the following linear transformation is derived by setting \( \hat{x} = \ddot{y}, \)
\[
\begin{cases}
\ddot{y} = A\dot{x} + b\xi \\
e = c^T A\dot{x} + (\rho + c^T b)\xi \\
\dot{\xi} = \ddot{u}.
\end{cases}
\tag{24}
\]
and we have
\[
\begin{cases}
\ddot{x} = \ddot{A}\dot{x} + \dot{b}\xi \\
e = c^T A\ddot{x} + (\rho + c^T b)\xi.
\end{cases}
\tag{25}
\]
According to Eq.(25), \( \dot{\xi} \) is eliminated by using the two formulas. Then, the derivative of the error equation is derived, and \( \ddot{x} \) is substituted in the transformed first formula, as follows:
\[
\begin{cases}
\ddot{x} = \ddot{A}\dot{x} + \dddot{b}e \\
\dot{e} = \dddot{c}^T \dddot{A}\dot{x} + \dddot{\rho}e + \dddot{\rho}\dddot{u}.
\end{cases}
\tag{26}
\]
where
\[
\dddot{A} = A - \frac{1}{\rho + c^T b} b c^T A, \quad \dddot{b} = \frac{1}{\rho + c^T b} b c^T A, \quad \dddot{c}^T = c^T A (A - \frac{1}{\rho + c^T b} b c^T A), \quad \dddot{\rho} = \frac{1}{\rho + c^T b} b c^T A, \quad \dddot{\rho} = \rho - c^T b.
\]
Letting \( l = [\dddot{x} \quad e]^T \), as follows:
\[
\dot{l} = Wl + O(\dot{e})f e_1,
\tag{27}
\]
where \( W = [\dddot{A} \quad \dddot{b}] \quad O = [0 \cdots 0 \quad \rho]^T \).

Consider system (27), and \( Q(l) = Wl + O\phi(e) \) and \( L = \frac{\partial Q}{\partial l} \mid_{l=0} \); then, we have the following Theorem

**Theorem**: The zero solutions of Eq.(27) are asymptotically stable for the plant (14) when the matrix \( L \) is Hurwitz.

**Proof**: Letting \( \dot{l} = 0 \), we obtain that \( l = 0 \) is a disequilibrium state of Eq.(27). Then, linearizing Eq.(27) at the equilibrium points, the coefficient matrix \( L \) [36] of the linearized system \( \dot{l} = Ll \) is obtained, i.e., (28), as shown at the bottom of the page, where \( \kappa_1 = \delta_{11}^{(1)} - 1 \), \( \kappa_2 = \delta_{22}^{(1)} - 1 \), \( \kappa_3 = \delta_{13}^{(1)} - 1 \), \( \kappa_4 = \delta_{12}^{(1)} - 1 \), \( \kappa_5 = \delta_{11}^{(1)} - 1 \).

The matrix \( L \) is Hurwitz, which is the condition for the asymptotic stability of the Eq.(27) based on the Lyapunov’s indirect method. Furthermore, the stability results are calculated by simulation and experiment, respectively.

\[
L = \frac{\partial Q}{\partial l} \bigg|_{l=0} =
\begin{bmatrix}
0 & 1 & -k_1 \kappa_1 & -k_2 \kappa_2 & 0 \\
-\beta_3 \kappa_3 & a_1 & a_1 k_1 \kappa_1 & -\beta_3 \kappa_3 - a_1 k_2 \kappa_2 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & -k_1 \kappa_1 & -\beta_2 \kappa_4 - k_2 \kappa_2 & -\beta_2 \kappa_4 & 0 \\
0 & 0 & -\beta_2 \kappa_4 & -\beta_2 \kappa_4 - \beta_1 \kappa_5 & 0
\end{bmatrix}
\tag{28}
\]

**V. SIMULATION**
To obtain the force variations in a PDW when the WMR has a straight motion, we use the designed control system to test the PDW of a three-wheeled mobile robot moving on a simulated planetary terrain. The desired conclusion of the simulation is that the PDW would continue to have clarity and large force variations over a single stone or several stones when moving on a soft terrain under the closed-loop control system pertaining to force. The simulation environment is shown in Figure 7, where the rear wheel is a PDW, and the front wheels are used as driving wheels driven by the order speed. The outer diameter of the front and rear wheels of the rover is 700 mm.

**A. SIMULATION SETTING**
The simulation environment can be divided into three parts: designing the robotic machinery using Solidwork, modeling the planetary terrain using Ansys, developing contract forces and motions of the WMR using Adams, and implementing the control system via Simulink (MATLAB). Furthermore, the environment and control algorithm were used to run the joint simulation on a computer with an Intel(R) Core (TM) i9-9880H CPU, AMD Radeon Pro 5500M 4 GB GPU, and 16 GB RAM. A schematic diagram of the simulation system and the flowchart of the control system are shown in Figure 8.

The characteristics of the simulated planetary terrain are summarized in Table 1. In this table, \( \Phi \) is the internal friction angle, \( c_0 \) is the cohesion stress, \( k_c \) is the pressure-sinkage modulus for the internal friction angle, \( k_\Phi \) is the pressure-sinkage modulus for the cohesion stress, \( k \) is...
TABLE 1. Nominal parameters of planetary soil [37], [38].

| Symbol | Value | Unit  |
|--------|-------|-------|
| \(\Phi\) | 9.80665 | N/kg |
| \(c_0\) | 170 | Pa |
| \(k_a\) | 814.4 | kN/m\(n+2\) |
| \(k_b\) | 1379 | N/m\(n+1\) |
| \(k_c\) | 0.0178 | m |
| \(n\) | 1.0 | - |
| \(c_1\) | 0.4 | - |
| \(c_2\) | 0.15 | - |

The deformation modulus, \(n\) is the pressure-sinkage ratio, and \(c_1\) and \(c_2\) are the normal stress coefficients of the considered Distribution 1 and Distribution 2, respectively.

The simulated terrain is obtained by inputting these parameters into software Ansys and then applying them to the software Adams (Matlab) to build the entire simulation environment. Furthermore, parameters of control system in Simulink (Matlab) are as follows:

\[
\begin{align*}
\beta_1 & = \frac{1}{h}, \beta_2 = \frac{1}{3h^2}, \beta_3 = \frac{1}{32h^3} \\
\gamma_{11} & = 0.5, \gamma_{12} = 0.5, \gamma_{13} = 0.25, \\
\gamma_{21} & = 0.5, \gamma_{22} = 0.25, \\
\delta_{11} & = \delta_{12} = \delta_{13} = 25h, \\
\delta_{21} & = \delta_{22} = h, \\
k_p & = 1.0, k_l = 9.0, k_D = 0.2.
\end{align*}
\]

The frequency of the joint control system is 100 Hz; however, the parameter \(h = 0.02\) in the joint simulation. Other parameters can be calculated by Eq. (31). A gravitational constant of 9.80665 N/kg was used in the simulation.

Setting \(a_1 = a_2 = 0\) and substituting relevant parameters into (28), we have

\[
L = \begin{bmatrix}
0 & 1 & -0.7 & -3.8 & 0 \\
-92.9 & 0 & 0 & 92.9 & -92.9 \\
1 & 0 & 0 & 0 & 1 \\
16.7 & 0 & -0.7 & -20.46 & 16.7 \\
16.7 & -1 & 0 & -16.7 & -53.85
\end{bmatrix}
\]

For the matrix \(L\), which is a Hurwitz matrix, all eigenvalues have negative real parts. Therefore, the designed ADRC control system is asymptotically stable.

B. PERFORMANCE

The designed controller was used in a straight motion on flat, deformable terrain with stones. In the terrain, we placed a single large stone under the soil at three places. In addition, a gravel area was set up on the soil surface. The velocity of the WMR is set to 60 cm/s. We set \(\alpha = 1\) and \(\beta = 0\) based on \(\text{DOF}_x = 1\) and \(\text{DOF}_y = 0\) in the control system. The movement of the WMR has two stages: moving through a rock under the sand and over the gravel terrain. In the last stage, there are three processes: transition from the soft to gravel terrain and from the gravel to soft terrain, and running on the gravel terrain between front processes.

During \(t = 0 \sim 1.5\) s, the WMR moves in the first stage. On soft sandy terrain, the velocity generation \(v\) of the PDW is stable at \(v = 20 \sim 50\) cm/s, as shown in Figure 9 (a). Meanwhile, the drawbar pull \(F_x\) and lateral force \(F_y\) approach 0 N, as shown in Figure 9 (b), (c). This shows that the designed control system is efficient. At \(t = 0.21\) s, 0.95 s, and 1.22 s, the wheel moved on a rock under the sand. Then, the velocity generation \(v\) is clearly changed to eliminate the variation in forces. The drawbar pull \(F_x\) increased instantly and was then eliminated. Their values are different because of the different sizes of rocks. During this stage, the wheel does not directly touch the stone in a straight motion, and the lateral force \(F_y\) does not change significantly.

After \(t = 1.5\) s, the wheel first transitions from the sandy to gravel terrain (T1), then moves on the gravel terrain (T2), and then transitions from the gravel to sandy terrain (T3). In T1, the control system quickly adapted to the new environment and operated in a new state. Because the slip of the wheel changes from high to low, the adaptability of the system can be fully demonstrated. In T2, the lowest velocity generation \(v\) increased from 20 cm/s to 30 cm/s, because the slip of the wheel was reduced, as shown in Figure 9 (a). The drawbar pull \(F_x\) is clearly changed because the size of each stone in the rocky terrain is different. Therefore, as the controller always dynamically adjusts the force while the force fluctuates significantly, the wheel can move steadily, as shown in Figure 9 (b). The lateral force \(F_y\) is not adjusted by the closed-loop system in this simulation; therefore, it varies significantly and has a single direction as shown in Figure 9 (c).

It is worth mentioning that the PDW has a single left support arm in the designed mechanism, which also has an impact on the direction change of the lateral force \(F_y\). In T3, i.e.,
transition from the gravel terrain to soft sand, the velocity and forces of the wheel caused violent vibration under the effect of gravity because of the wheel has undergone a process of turning out of a small pit due to the different terrain in simulation. More details, the PDW went through a rapid downhill and uphill movement, which caused a dramatic change in force. Next, according to the changes in drawbar pull $F_x$ and lateral force $F_y$, as shown in Figure 9(b), (c), the wheel quickly recovers stability under the closed-loop system.

While the closed-loop control system has fast response capabilities, the force change during the transition process is large. Therefore, it is not suitable to adjust the lateral force $F_y$ in the situation considered above. The violent swing of the PDW will negatively impact the body. In contrast, it is known that the lateral force $F_y$ of the wheel changes significantly when it is driven on a hard terrain in a straight motion. Therefore, it can be concluded from the simulation experiment that it is necessary to design a control strategy to adjust the output power of the PDW motion in 2 DOFs.

VI. EXPERIMENTAL EVALUATION

This section verifies the performance of the proposed strategy through experimentation. First, the experimental setup is introduced. Then, the effect of different levels of power output in two different contexts (straight motion on sloping, sandy terrain, and steering motion on flat, sandy terrain) is presented and discussed. Additionally, we discuss the conditions under which the power output along $DOF_y$ needs to be brought to zero, using an interesting experiment in which the WMR is moving straight ahead on gravelled terrain. The obtained conclusions are analyzed and discussed.

A. EXPERIMENTAL SETUP

The experimental setup was designed to physically validate the proposed control strategy using a velocity-controlled three-wheeled mobile robot, which is depicted in Figure 10.

Each one of its two front wheels has one brush DC motor and one six-axis force sensor, whereas the rear wheel (the PDW) has two brush DC motors (FAULHABER—3257G024CR) and one six-axis force sensor (SRI 3705C). Each motor has an absolute encoder (HEDM-5540). As shown in Figure 10, the PDW has a single support arm on its right side. One of the PDW motors works in velocity control mode to provide driving power, whereas the other works in position control mode, to provide steering power. The front motors work in velocity control mode. The industrial computer (BECKHOFF CX2020) realizes the closed loop of signal transmission, as shown in Figure 11.

High-gain parameters are especially advantageous in this system [18]. Setting the parameters based on a system bandwidth approach [34] leads to the parameters given
in (31):

\[
\begin{align*}
    \gamma_{11} &= 0.60, \gamma_{12} = 0.30, \gamma_{13} = 0.50, \\
    \gamma_{21} &= 0.50, \gamma_{22} = 0.25, \\
    \delta_{11} &= \delta_{12} = \delta_{13} = h, \\
    \delta_{21} &= \delta_{22} = 2h, \\
    k_p &= 0.58, k_I = 0.25, k_D = 0.30,
\end{align*}
\]

(31)

The controllers were implemented with a sample period

\( h = 0.01 \). The remaining parameters were chosen in accordance with (31). The forces measured on the PDW are eliminated by the active following controller, with a slight phase delay caused by the ESO. It should also be pointed out that the flatness of the loose terrain in the experiments was only approximate; it was not strictly enforced.

Setting \( a_1 = a_2 = 0 \) and substituting relevant parameters into (28), we have

\[
L = \begin{bmatrix}
0 & 1 & -4.10 & -5.64 & 0 \\
-312.5 & 0 & 0 & 312.5 & -312.5 \\
1 & 0 & 0 & 0 & 1 \\
132.6 & 0 & -4.10 & -138.24 & 132.6 \\
132.6 & -1 & 0 & -132.6 & -498.4 
\end{bmatrix}
\]

(32)

For the matrix \( L \), which is a Hurwitz matrix, all eigenvalues have negative real parts. Therefore, the designed ADRC control system is asymptotically stable.

**B. EXPERIMENTAL RESULTS**

The product of the \( DOF_y \)-related open-loop weight by the generated velocity (\( \alpha \cdot v \)) and the product of the \( DOF_y \)-related open-loop weight by the steering angle (\( \beta \cdot \theta \)) are the inputs for the driving and steering motors of the PDW, respectively. The generated velocity \( v \) and steering angle \( \theta \) are collected by the driving and steering motors, respectively. The drawbar pull \( F_x \) and lateral force \( F_y \) are collected by the six-axis force sensor, and are the inputs to the ADRC 1 and ADRC 2 controllers, respectively.

1) EXPERIMENTAL RESULTS COMPARISON

To evaluate the applicability of the proposed strategy, two tracking tasks were performed, in different contexts: straight motion on a terrain with a slope of 10 degrees (Task 1) and steering motion (Task 2). We set the open-loop weight (\( \alpha, \beta \)) based on \( DOF_x = DOF_y = 1 \) with respect to the boundary value to compare and evaluate the performance of the strategy in these experiments, by considering the pairs (1, 1), (1, 0), and (0, 1). The tracking objective in Task 1 was set to a linear body velocity of \( v_b = 110 \) mm/s, whereas the tracking objective for Task 2 was defined as the maintenance of an angular velocity of \( \omega_b = 0.2 \) rad/s. These motions of the WMR are shown in Figs. 13(a) and (b), respectively. The dotted lines in the last frames of each task allow a better appreciation of the trajectory tracking performance under different strategies. Further details will be provided below.

The generated velocity \( v \), steering angle \( \theta \), drawbar pull \( F_x \), and lateral force \( F_y \) in Task 1 are shown in Figure 12. As shown in Figure 12(a), state (1, 0) provides a more stable generated PDW velocity \( v \) and a faster response than state (1, 1). Because of the effect of the wheel-slip on soft terrain [39], the actual velocity of the body is lower than intended. Therefore, the PDW has a low velocity objective to track. Additionally, adjusting the lateral force \( F_y \) in rough terrain not only has resulted in a larger lateral resistance in state (1, 1) when compared with (1, 0), as shown in Figs. 12(b) and (d), but also reduces the heading driving power that the wheel should provide to the body, as shown in Figure 12(c).

These two strategies therefore show that, even though the PDW is only supposed to provide support to the body [18],
it should provide some adjustment along $DOF_y$ even in the straight motion case, so as to provide greater driving power to the body on these terrains. As shown in Figure 13(a), in state (1, 0) the WMR moved 1400 mm and saved 5001 ms relative to state (1, 1). Additionally, the dotted lines in the last frames show that the WMR had a straighter trajectory under the (1, 1) policy. The average drawbar pull $|F_x|$, average lateral force $|F_y|$, average PDW generated velocity $v$, and average linear velocity of the body $v_b$ are shown in Table 2. The value of $v_b$ was measured with a program timer and a driving ruler. The other values were calculated as:

$$\overline{X} = \frac{1}{n} \sum_{j=0}^{n} |X_j|, \quad X = F_x, F_y.$$ (33)

**TABLE 2.** Average drawbar pull, average lateral force, average PDW generated velocity, and average linear velocity of the body.

| Strategy state | $|F_x|$ (N) | $|F_y|$ (N) | $v$ (mm/s) | $v_b$ (mm/s) |
|----------------|------------|------------|------------|--------------|
| (1, 0)         | 6.42       | 5.01       | 19.83      | 27.99        |
| (1, 1)         | 2.87       | 5.85       | 21.11      | 25.51        |

From the obtained experiment results we can therefore conclude that when the WMR is moving straight ahead on rough and sandy terrain, reducing the open-loop weight $\beta$ corresponding to $DOF_y$ can improve both the tractive capability of the PDW and the accuracy of its trajectory tracking.

In Task 2, the trend and behavior of the generated velocity $v$ are similar to those of Task 1, as shown in Figure 14(a). We inhibited the adjustments along $DOF_x$ to...
achieve state (0, 1). To complete the steering movement, we used the kinematic model of the PDW (rear wheel) of a three-wheeled mobile robot proposed in [18] to calculate a constant velocity generation of $v = -71.5$ mm/s using (34).

$$\begin{align*}
\omega_3 &= \frac{v_x}{r} & \omega_1 &= \omega_2 \\
\omega_3 &= \frac{R(|\omega_2 - \omega_1|)}{2D} & \omega_1 \neq \omega_2,
\end{align*}$$

(34)

where $R = 140$ mm is the radius of the circular motion of the rear wheel, $D = 250$ mm is the distance between the geometric center of the WMR and the wheel center, and $v_x$ is the body’s velocity in the $x$ (heading) direction.

During this task the drawbar pull $F_x$ and lateral force $F_y$ with strategy (1, 1) stabilized (after a large adjustment) within a small range, as shown in Figs. 14(c) and (d). In contrast, the forces were larger and with larger variations under strategy (0, 1). Because all adjustments along $DOF_x$ were inhibited, some excessive power output led to additional deflection and increased lateral force $F_y$, a consequence of the inflexible output of the controller in this instance. The trajectory tracking ability was diminished, and the energy consumption increased. As shown in Figure 13(b), the angular velocity of the body $\omega_b$ can, in practice, be calculated with the following equation:

$$\omega_b = \frac{(360 - \alpha_1 - \alpha_2) \times 2\pi}{360t},$$

(35)

where $\alpha_1$, $\alpha_2$ are the body’s initial and final angles relative to the $x$-axis, as shown in Figure 13(b).

The angular velocities under (1, 1) and (0, 1) were, respectively, $\omega_b = 0.21$ rad/s and $\omega_b = 0.22$ rad/s during the whole process. We can therefore conclude that the trajectory tracking performance under strategy (1, 1) was 50 % more accurate than under (0, 1), a result calculated with the following equation:

$$\delta_i = \frac{\bar{\epsilon}(1.1) - \bar{\epsilon}(1.0)}{\bar{\epsilon}(1.0)} \times 100\%.$$  

(36)

The obtained average drawbar pull $|F_x|$, average lateral force $|F_y|$, average PDW generated velocity $v$ and average body angular velocity $\omega_b$ were calculated with (33), and are shown in Table 3.

We can therefore conclude from the experiments that when the WMR is steering on flat and sandy terrain, increasing the open weight $\alpha$ of the $DOF_x$ can improve the accuracy of trajectory tracking of the WMR and optimize energy consumption.

2) FURTHER EXPERIMENTAL RESULTS

To further discuss the contexts where adjustments along $DOF_y$ should not be applied, we will discuss another case requiring $\alpha = 1$ and $\beta = 0$ based on $DOF_x = 1$ and $DOF_y = 0$: when the WMR is moving straight on graveled terrain with many small ups and downs. This last experiment was only performed under state (1, 0). We set a linear body velocity of $v_b = 55$ mm/s as the tracking objective for the PDW. Using this strategy, the robot was capable of smoothly tracking linear movements on the considered complex terrain, and do it with high precision. It should be noted that there was no other external closed-loop adjustments to complete this experiment, something that can only be achieved by using PDWs. The resulting motion is shown in Figure 15(a).

![FIGURE 15. WMR on straight motion on graveled terrain. *Symbol △ indicates the moment when the PDW entered the graveled terrain. To better show the control effect, the start time in subpanel (a) is different from that of the other subpanels.]

During the period $t = 0 \sim 9$ s, the WMR moves on flat and sandy terrain. The generated velocity $v$ and drawbar pull $F_x$ show a stable trend, and the lateral force is stabilized at $F_y = -25$ N because the PDW has a single support arm on its right side.

After $t = 9$ s, the PDW starts moving on graveled terrain. The lateral force $F_y$ suddenly changes by more than 10 N, and starts showing a large, intense jitter, as shown in Figure 15(d). Because of the lack of adjustments along $DOF_y$, neither the

### Table 3: Average drawbar pull, average lateral force, average PDW generated velocity, and average angular velocity of the body.

| Strategy state | $|F_x|$ (N) | $|F_y|$ (N) | $v$ (mm/s) | $\omega_b$ (rad/s) |
|----------------|-----------|-----------|-------------|-----------------|
| (0, 1)         | 7.99      | 11.34     | 71.51       | 0.22            |
| (1, 1)         | 5.59      | 5.76      | 50.66       | 0.21            |
drawbar pull $F_x$ nor the velocity generation of the wheel $v$ are considerably affected, as shown in Figs. 15(b) and (c). We also note that, as the wheel-slip decreases on this terrain, the PDW generated velocity $v$ increases slightly. We tried to also perform this experiment under state (1,1), but when the PDW entered the gravel terrain, the wheel turned sharply, causing the body to lose control.

The obtained experimental results therefore support the conclusion that, on rough terrains with large undulations, the open-loop weight of the power output along $DOF_x$ should be substantially reduced or even brought to zero, thus increasing the trajectory tracking ability of the WMR.

VII. CONCLUSION

This work presented and discussed a control strategy for the PDW of Multi-wheeled mobile robots, based on adjusting the power outputs along the directions of the two degrees of freedom of such wheels, $DOF_x$ and $DOF_y$, an approach that was shown to improve the trajectory tracking ability of the WMRs. The boundary values of such strategy were studied and evaluated through experimentation. The obtained experimental results demonstrate that, when the WMR is moving in straight motion on rough and sandy terrain, the open-loop weight $\beta$ corresponding to $DOF_x$ should be decreased. On the other hand, increasing the open-loop weight $\alpha$ corresponding to $DOF_x$ when the WMR is steering on flat and sandy terrain can increase the WMR’s trajectory tracking ability. The control strategy can efficiently increase the driving force of the PDW imparted to the body on rough terrains.

In future work, we will determine an intelligent control strategy to adaptively choose the open-loop weights $\alpha$ and $\beta$ according to the different terrains. More importantly, even though the basic rules of the proposed control strategy have been presented here, we will design and produce more experiments in multiple scenarios of different characteristics, to explore the possibility of defining more complete laws.

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