Critical frequency of metasurfaces on dielectric half-space

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In this letter, the critical frequency of the periodic metasurface in a dielectric half-space is studied. First, according to the mode expansion theory, the wave of the periodic metallic elements can be decomposed into different eigen-modes. Then, as the frequency increases, partial high-order modes are excited by the periodic metallic elements that may propagate into the substrates as spics. Finally, the critical frequency is analytically derived for a homogenized surface without spics. This study shows that the reflected and transmitted spics can co-exist, but there is a distinct difference between their corresponding critical frequencies. Thus, the actual critical frequency of homogenized metasurface in a dielectric half-space is not only related to the periodicity and incident angle, but also to the substrates.

Introduction: Metasurface is usually composed of periodic metallic elements at the interface of different substrates [1]. Especially, the metallic elements are always designed to be a sub-wavelength that allows an average boundary condition [2]. Based on its characteristics, the metasurface is widely applied to the high impedance surface (HIS), frequency selective surface (FSS), and so on [3–5]. However, the sub-wavelength condition cannot always be satisfied, especially at higher frequencies. In this case, the spics from higher-order modes can propagate along the interface, and an average boundary condition not sufficient to describe the periodic metallic elements. Thus, it is necessary to quantitatively evaluate what factors affect the critical frequency of the spics, especially for the general situation where the periodic metallic elements are placed on a dielectric half-space.

In this work, we study the critical frequency of the periodic metallic elements that can be regarded as homogenized metasurface. When the wave obliquely impinges on the metallic elements, the reflected and transmitted waves can be regarded as the sum of different modes of the periodic metallic elements, with the following tangential wave vectors [6]:

\[ k_1 = n_1 k_0 \sin \theta_1 + \frac{2\pi}{p} = n_2 k_0 \sin \theta_2 + \frac{2\pi}{p}, \]

(1)

where \( k_0 \) is the wavenumber in vacuum, \( n_1 \) and \( n_2 \) are the index of refraction of Substrate 1 and 2, respectively; \( p \) is the period of the metallic elements; \( l \) and \( g \) are arbitrary integers. It should be noted that all modes result from the traditional Fresnel reflection (refraction) and the secondary radiation of the excited metallic elements. Especially, the traditional Fresnel reflection (refraction) can fully be characterized by the zero-order mode \( l = 0 \) (\( g = 0 \)), while partial zero-order mode \( l = 0 \), \( g \neq 0 \) and all high-order modes \( l \neq 0 \), \( g \neq 0 \) are generated from the secondary radiation of the metallic elements.

Furthermore, the radiation characteristics of the periodic metallic elements are strongly related to the ratio between the period \( p \) and the operation wavelength \( \lambda \). Generally, \( p \) is designed to be much smaller than \( \lambda \) so that all high-order modes \( l = 0 \) \( g = 0 \) are strongly bounded as surface waves, and the periodic metallic elements can be regarded as a homogenized surface. However, as the frequency increases, \( p \) gradually approaches \( \lambda \). In this case, the partial high-order modes generated from the excited periodic metallic elements may propagate into the substrate as spics. Thus, it is necessary to solve the critical frequency of the spics for homogenized metasurface in a dielectric half-space.

The critical frequency of the periodic metallic elements (homogenized surface) can be solved as follows. As shown in Figure 1a, one transmitted spics from the excited periodic elements is assumed to propagate at the angle of \( \theta_{1p} \) in Substrate 2. According to the antenna theory, the phase difference \( 2\pi g \) exists between the neighbouring metallic elements, where \( g \) is an integer. Thus, the following equation is satisfied:

\[ k_0 p (n_1 \sin \theta_1 - n_2 \sin \theta_{1p}) = 2\pi g. \]

(2)

In (2), \( \theta_{1p} \) can be a positive or negative value. Taking Figure 1a as a reference, the former means that the spics and incident waves are located at two sides of the normal to the interface (right and left of the normal), while the latter means that the spics and incident waves are located on the same side (both in the left side). Furthermore, Equation (2) can be rewritten as follows:

\[ f_{1p} = \frac{gc}{p(n_1 \sin \theta_1 - n_2 \sin \theta_{1p})}. \]

(3)

where \( c \) is the speed of light in vacuum. Obviously, when \( \theta_{1p} = -90^\circ \) and \( g = 1 \) are satisfied, the minimum value of Equation (3) reaches the following critical frequency:

\[ f_{11} = \frac{c}{p(n_1 \sin \theta_1 + n_2)}. \]

(4)

Equation (4) ensures that all high-order modes in Substrate 2 propagate along the interface at the frequency \( f_{1p} \) exactly. Thus, \( f_{11} \) is the critical frequency of the transmitted spics in Substrate 2.

After solving the critical frequency for the transmitted spics, the critical frequency for the reflected spics remains to be solved. As shown in Figure 1b, the critical frequency of the reflected spics in Substrate 1 can be similarly solved as follows:

\[ f_{11} = \frac{c}{n_1 p(\sin \theta_1 + 1)}. \]

(5)

Finally, considering that the reflected and transmitted spics co-exist for the wave from Substrate 1 to Substrate 2, the actual critical frequency \( f_{21} \) should be the smaller value of \( f_{11} \) and \( f_{1p} \), that is,

\[ f_{21} = \min (f_{11}, f_{1p}) = \left\{ \begin{array}{ll} f_{11}, & (n_1 \geq n_2) \\ f_{1p}, & (n_1 < n_2) \end{array} \right. \]

(6)

Equation (6) indicates that the critical frequency \( f_{21} \) is strongly related to the substrates.

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Fig. 1 Schematic diagrams a Transmitted and reflected waves b Metallic elements
To verify the theory, we design a metallic element (unit cell) with geometric parameters $d_1 = 1.8$ mm and $q = 0.2$ mm, as shown in Figure 1b. The unit cell is then used to construct a periodic metasurface. The commercial software HFSS is used to carry out the simulations.

The four boundaries of the unit cell are set to be periodic; the metallic elements are placed on Substrate 2 with a backed radiation boundary to void multiple reflections. For convenience, Substrate 1 is fixed as air. When the perpendicular polarized (TE) or parallel polarized (TM) wave obliquely impinges on the periodic metallic elements [7], the co-polarized reflection coefficients $R_{TM-TE}$ and $R_{TE-TM}$ can be used to verify the theory (Note that the cross-polarized reflection coefficients are zero due to the symmetry of the metallic element.)

Figures 2–4 show the simulated $R_{TM-TE}$ and $R_{TE-TM}$ for various relative permittivity $\varepsilon_r$, incidence angle $\theta_1$, and periodicity $p$. It can be found that, no matter how these parameters change, the simulated $R_{TM-TE}$ and $R_{TE-TM}$ always happen to vary rapidly with strong jitter at a specific frequency. By plotting a vertical blue line as the theoretical, critical frequency $f_{c1}$, the consistency demonstrates that the strong jitter is precisely caused by the spics from the high-order modes, as discussed above. Besides, the critical frequency $f_{c1}$ is indeed independent of the polarization status, which conforms to the theory above. Therefore, the simulations verify the correctness of the critical frequency related to the substrates, incidence angle, and periodicity.

**Conclusion:** This letter studied the critical frequency of spics for periodic metasurfaces in a dielectric half-space. When the frequency is smaller than a particular value, the periodic metallic elements can be regarded as a homogenized surface. The study showed that the critical frequency for homogenized metasurface could be theoretically determined from the periodicity, angle of incidence, and substrate’s permittivity. It has been shown that there is a distinct difference between the critical frequencies of the reflected and transmitted spics.

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**References**

1. Yu, N.F., Capasso, F.: Flat optics with designer metasurfaces. Nat. Mater. **13**, 139–150 (2014)
2. Kuester, E., et al.: Averaged transition conditions for electromagnetic fields at a metafilm. *IEEE Trans. Antennas Propag.* **51**(10), 2641–2651 (2003)
3. Luukkonen, O., et al.: Simple and accurate analytical model of planar grids and high-impedance surfaces comprising metal strips or patches. *IEEE Trans. Antennas Propag.* **56**(6), 1624–1632 (2008)
4. Khan, M., Fraz, Q., Tahir, F.: Ultra-wideband cross polarization conversion metasurface insensitive to incidence angle. *J. Appl. Phys.* **121**(4), 045103 (2017)
5. Ameen, M., Chaudhary, R.: Metamaterial-based wideband circularly polarised antenna with rotated V-shaped metasurface for small satellite applications. *Electron. Lett.* **55**(7), 363–366 (2019)
6. Xiao, S. et al.: Mode-expansion theory for inhomogeneous meta-surfaces. *Opt. Express* **21**(22), 27219–27237 (2013)
7. Liu, X., et al.: Babinet principle for anisotropic metasurface with different substrates under obliquely incident plane wave. *IEEE Trans. Microw. Theory Techn.* **66**(6), 2704–2713 (2018)