Implications for CP asymmetries of improved data on $B \to K^0\pi^0$

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The decay $B^0 \to K^0\pi^0$, dominated by a $b \to s$ penguin amplitude, holds the potential for exhibiting new physics in this amplitude. In the pure QCD penguin limit one expects $C_{K\pi} = 0$ and $S_{K\pi} = \sin 2\beta$ for the coefficients of $\cos \Delta m t$ and $\sin \Delta m t$ in the time-dependent CP asymmetry. Small non-penguin contributions lead to corrections to these expressions which are calculated in terms of isospin-related $B \to K\pi$ rates and asymmetries, using information about strong phases from experiment. We study the prospects for incisive tests of the Standard Model through examination of these corrections. We update a prediction $C_{K\pi} = 0.15 \pm 0.04$, pointing out the sensitivity of a recent theoretical prediction $S_{K\pi} \approx 1$ to the branching ratio for $B^0 \to K^0\pi^0$ and to other observables.

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One of the most challenging CP asymmetry measurements in $B$ meson decays has involved the coefficients $C_{K\pi}$ and $S_{K\pi}$ in the time-dependent asymmetry measured in $B^0 \to K_S\pi^0$ [1]

$$A(t) = \frac{\Gamma(B^0(t) \to K^0\pi^0) - \Gamma(B^0(t) \to K^0\pi^0)}{\Gamma(B^0(t) \to K^0\pi^0) + \Gamma(B^0(t) \to K^0\pi^0)} = -C_{K\pi} \cos(\Delta m t) + S_{K\pi} \sin(\Delta m t). \quad (1)$$

The parameter $C_{K\pi}$ is related to the direct CP asymmetry by $C_{K\pi} \equiv -A_{CP}(B^0 \to K^0\pi^0)$. The decay $B^0 \to K^0\pi^0$ is expected to be dominated by the $b \to s$ penguin amplitude and thus is a good place to look for any new physics that may arise in this amplitude [2–4]. In the pure QCD penguin limit one expects $C_{K\pi} = 0$ and $S_{K\pi} = \sin 2\beta$, respectively, where $\beta = (21.5 \pm 1.0)^\circ$ [5] is an angle in the unitarity triangle. Accounting for small non-penguin contributions leads to corrections to these expressions, which are calculable in terms of isospin-related $B \to K\pi$ decay rates and asymmetries. In this Letter we study the prospects for incisive tests of the Standard Model through examination of these corrections. We update a prediction $C_{K\pi} = 0.15 \pm 0.04$ and point out the sensitivity of a recent theoretical prediction $S_{K\pi} \approx 1$ [6] to the branching ratio for $B^0 \to K^0\pi^0$ and to other observables.

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1 On sabbatical leave from the Physics Department, Technion, Haifa 32000, Israel.
Table I: Measurements of $C_{K\pi}$ and $S_{K\pi}$.

| Ref.       | $C_{K\pi}$          | $S_{K\pi}$          |
|------------|---------------------|---------------------|
| BaBar [7]  | 0.24 ± 0.15 ± 0.03  | 0.40 ± 0.23 ± 0.03  |
| Belle [8]  | 0.05 ± 0.14 ± 0.05  | 0.33 ± 0.35 ± 0.08  |
| Average [5]| 0.14 ± 0.11         | 0.38 ± 0.19         |

Table II: CP-averaged branching ratios and CP rate asymmetries for $B \to K\pi$ decays and $B^+ \to \pi^+\pi^0$, based on averages in Ref. [5].

| Mode                   | Branching ratio ($10^{-6}$) | $A_{CP}$          |
|------------------------|-----------------------------|-------------------|
| $B^0 \to K^+\pi^-$     | 19.4 ± 0.6                  | −0.097 ± 0.012    |
| $B^0 \to K^0\pi^0$     | 9.8 ± 0.6                   | −0.14 ± 0.11      |
| $B^+ \to K^0\pi^+$     | 23.1 ± 1.0                  | 0.009 ± 0.025     |
| $B^+ \to K^+\pi^0$     | 12.9 ± 0.6                  | 0.050 ± 0.025     |
| $B^+ \to \pi^+\pi^0$   | 5.59$^{+0.44}_{-0.40}$      | 0.06 ± 0.05       |

The current status of measurements of $C_{K\pi}$ and $S_{K\pi}$ is summarized in Table I. The value of $C_{K\pi}$ is consistent with the pure-penguin value of zero, while that of $S_{K\pi}$ is $1.6\sigma$ below the pure-penguin value of $\sin 2\beta = 0.681 ± 0.025$.

A sum rule for direct CP asymmetries in $B \to K\pi$ decays has been derived purely on the basis of the $\Delta I = 0$ property of the dominant penguin amplitude, using an isospin quadrangle relation among the four $B \to K\pi$ decay amplitudes which depend also on two $\Delta I = 1$ amplitudes [9, 10]:

$$A(B^0 \to K^+\pi^-) + \sqrt{2}A(B^0 \to K^0\pi^0) = A(B^+ \to K^0\pi^+) + \sqrt{2}A(B^+ \to K^+\pi^0) .$$  \hspace{1cm} (2)

In its most precise form the sum rule relates the four CP rate differences [11],

$$\Delta(K^+\pi^-) + \Delta(K^0\pi^+) = 2\Delta(K^+\pi^0) + 2\Delta(K^0\pi^0) ,$$  \hspace{1cm} (3)

where one defines

$$\Delta(f) \equiv \Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f) .$$  \hspace{1cm} (4)

This sum rule includes interference terms of the dominant penguin amplitude with all small non-penguin contributions. A few very small quadratic terms representing interference of tree and electroweak penguin amplitudes vanish in the SU(3) and heavy quark limits [11].

Using the decay branching ratios and CP asymmetries summarized in Table II [5] and the known lifetime ratio $\tau(B^+)/\tau(B^0) = 1.071 ± 0.009$ [5], one can use this relation to solve for the least-well-known quantity $\Delta(K^0\pi^0)$, implying

$$A_{CP}(K^0\pi^0) = -0.148 ± 0.044 .$$  \hspace{1cm} (5)
The error on the right-hand-side is dominated by the current experimental errors in $A_{CP}(K^0\pi^0)$ and $A_{CP}(K^+\pi^0)$. The prediction (5) following from (3) involves a smaller theoretical uncertainty at a percent level from quadratic terms describing the interference of small non-penguin amplitudes. Verification of this prediction would provide evidence that non-penguin amplitudes behave as expected in the Standard Model. [If one uses the corresponding sum rule for CP asymmetries,

$$A_{CP}(K^+\pi^-) + A_{CP}(K^0\pi^-) = A_{CP}(K^+\pi^0) + A_{CP}(K^0\pi^0),$$

one predicts $A_{CP}(K^0\pi^0) = -0.138 \pm 0.037$. Using this relation with $A_{CP}(K^0\pi^+) = 0$, as expected since $B^+ \rightarrow K^0\pi^+$ should be dominated by a penguin amplitude with only a very small annihilation contribution [12], one predicts $A_{CP}(K^0\pi^0) = -0.147 \pm 0.028$.]

Non-penguin amplitudes are generally agreed to increase $S_{K\pi}$ from its pure-penguin value of $\sin 2\beta = 0.681 \pm 0.025$ by a modest amount, generally to 0.8 or below [13–16]. Model-independent bounds using flavor SU(3) [17, 18] also favor at most a deviation of 0.2 from the pure-penguin value. An exception is noted in the treatments of Refs. [19] and [20], and most recently in Ref. [6], where a relation between $C_{K\pi}$ and $S_{K\pi}$ was studied implying a value $S_{K\pi} = 0.99$ for the central value measured for $C_{K\pi}$. A geometrical construction is performed which illustrates the way in which such a large value arises.

An aspect of the prediction of $S_{K\pi} \simeq 0.99$ which has not been sufficiently stressed is its extreme sensitivity to the branching ratio ($B^0 \rightarrow K^0\pi^0$). In the present Letter we analyze the sensitivity of $S_{K\pi}$ to this and other observables within the Standard Model, and highlight those measurements which would shed light on the presence of new physics. In order to restrict the range allowed for $S_{K\pi}$ in the Standard Model one needs certain information about strong phases. Theoretical calculations of strong phases in $B \rightarrow K\pi$ based on $1/m_b$ expansions are known to fail, most likely because of long distance charming penguin contributions [21, 22]. We propose to obtain the necessary information about strong phases directly from experiments. Somewhat different but not completely independent arguments were presented in Ref. [6].

The $B \rightarrow K\pi$ amplitudes may be decomposed into contributions from various amplitudes as follows [23, 24]:

$$A_{++} \equiv A(B^0 \rightarrow K^+\pi^-) = -(p + t),$$
$$A_{00} \equiv \sqrt{2}A(B^0 \rightarrow K^0\pi^0) = p - c,$$
$$A_{0+} \equiv A(B^+ \rightarrow K^0\pi^+) = p + A,$$
$$A_{++} \equiv \sqrt{2}A(B^+ \rightarrow K^+\pi^0) = -(p + t + c + A),$$

$$t \equiv T + P_{EW}^C, \ c \equiv C + P_{EW}, \ p \equiv P - \frac{1}{3}P_{EW}^C.$$ (7)

The terms $T, C$ and $A$ represent color-favored and color-suppressed tree amplitudes and a small annihilation term, while $P$ stands for a gluonic penguin amplitude. Color-favored and color-suppressed electroweak penguin amplitudes are represented by $P_{EW}$ and $P_{EW}^C$. The sums of the first two and last two amplitudes in Eq. (7) are equal [see Eq. (2)] and both correspond to an amplitude $A_{3/2}$ for a $K\pi$ state with isospin $I_{K\pi} = 3/2$ [9, 10]:

$$A(B^0 \rightarrow K^+\pi^-) + \sqrt{2}A(B^0 \rightarrow K^0\pi^0) = A(B^+ \rightarrow K^0\pi^+) + \sqrt{2}A(B^+ \rightarrow K^+\pi^0)$$
which fix analytically by solving simple quadratic equations for the central values of the parameters
ratio $\tau$ range (5) for values of the rates and CP asymmetries in Table II, are
triangle relation, amplitudes as their square roots. (We first divide $B$ and a similar relation for the CP-conjugate amplitudes in which the sign of central values of $|\xi$ and a smaller uncertainty from CKM factors. We neglect a potential small strong phase $P$ and Wilson coefficients, ($P_{EW}$ the ratio of these two amplitudes is given numerically in terms of ratios of CKM factors $1$ includes an uncertainty from SU(3) breaking, which we will take as $\pm 0.017$.

Solutions for the amplitude triangle (12) and its CP-conjugate may be obtained numerically in terms of ratios of CKM factors

\begin{align}
|T + C| &= \sqrt{2} \frac{V_{us} f_K}{V_{ud} f_\pi} |\xi_{T+C}| A(B^+ \rightarrow \pi^+ \pi^0) |. \\
A_{00} + A_{++} &= A_{3/2} = -|T + C| \left( e^{i\gamma} - 0.66 \xi_{EW} \right) ,
\end{align}

and a similar relation for the CP-conjugate amplitudes in which the sign of $\gamma$ is reversed.

In order to visualize the geometric construction of the triangle [12] and its CP-conjugate, as proposed in Ref. [6] but with realistic quantities including the restricted range $\pm$ for $A_{CP}(K^0\pi^0)$, we express all branching ratios in units of $10^{-6}$, and take amplitudes as their square roots. (We first divide $B^+$ branching ratios by the lifetime ratio $\tau(B^+)/\tau(B^0) = 1.071 \pm 0.009 [5]$ to compare them with $B^0$ branching ratios.) The central values of $|T + C|$ for $\xi_{T+C} = 1$ and the squares $|A_{ij}|^2$ and $|\tilde{A}_{ij}|^2$, based on central values of the rates and CP asymmetries in Table I, are

\begin{align}
|T + C| &= 0.900 , \\
|A_{00}|^2 &= 2(9.8)(1 + 0.14) = 22.3 , \\
|A_{++}|^2 &= (19.4)(1 + 0.097) = 21.3 , \\
|A_{00}|^2 &= 2(9.8)(1 - 0.14) = 16.9 , \\
|\tilde{A}_{++}|^2 &= (19.4)(1 - 0.097) = 17.5 .
\end{align}

Solutions for the amplitude triangle [12] and its CP-conjugate may be obtained analytically by solving simple quadratic equations for the central values of the parameters which fix $A_{3/2}$ in [12], $\xi_{EW} = 1$, $\gamma = 65^\circ$. The quadratic equation for each triangle has

$$-(t + c) = -(T + C + P_{EW}^C + P_{EW}) = A_{3/2} .$$

The contribution $-(T + C)$ to $A_{3/2}$ has a magnitude which can be obtained from the decay $B^+ \rightarrow \pi^+ \pi^0$ via flavor SU(3) [25],

$$|T + C| = \sqrt{2} \frac{V_{us} f_K}{V_{ud} f_\pi} |\xi_{T+C}| A(B^+ \rightarrow \pi^+ \pi^0) | .$$

SU(3) breaking in this amplitude is often assumed to be given by the factor $f_K/\pi = 1.193 \pm 0.006 [26]$. Here we introduce a parameter $\xi_{T+C} = 1.0 \pm 0.2$ which represents an uncertainty in this factor. The weak phase of $T + C$ is $\text{Arg}(V_{ub}^* V_{us}) = \gamma$, where $\gamma = (65 \pm 10)^\circ [27]$. We take its strong phase to be zero by convention. All other strong phases will be taken in the range $(-\pi, \pi)$. The penguin amplitude $P$ dominating $B \rightarrow K\pi$ decays carries the weak phase $\text{Arg}(V_{tb}^* V_{ts}) = \pi$. Its strong phase relative to that of $T + C$ will be denoted $-\delta_c$ [28]. Thus

$$T + C = |T + C| e^{i\gamma} , \\
P = -|P| e^{-i\delta_c} .$$

The electroweak penguin contribution $P_{EW}^C + P_{EW}$ was shown in Refs. [29] and [30] to have the same strong phase as $T + C$ in the SU(3) symmetry limit. In this limit the ratio of these two amplitudes is given numerically in terms of ratios of CKM factors and Wilson coefficients, $(P_{EW} + P_{EW}^C)/(T + C) = -0.66 \xi_{EW} e^{-i\gamma}$. The parameter $\xi_{EW}$ includes an uncertainty from SU(3) breaking, which we will take as $\xi_{EW} = 1.0 \pm 0.2$, and a smaller uncertainty from CKM factors. We neglect a potential small strong phase of $\xi_{EW}$ which has a negligible effect on our analysis below. Thus we have an amplitude triangle relation,
Figure 1: Triangles relating amplitudes for $B^0 \rightarrow K^0\pi^0$ and $B^0 \rightarrow K^+\pi^-$ to the amplitude $A_{3/2}$, and triangles for the corresponding charge-conjugate processes.

two solutions, which can be visualized by flipping the triangle around the side $A_{3/2}$ or $\bar{A}_{3/2}$ which is kept fixed. One thus obtains a total of $2 \times 2 = 4$ solutions, of which two are illustrated in Fig. 1. The other two solutions correspond to flipping one triangle but not the other.

We have chosen to express the triangles with $A_{00}$ or $\bar{A}_{00}$ emanating from the origin, in order to illustrate the relative phase of $A_{00}$ and $\bar{A}_{00}$ which will be important in the evaluation of $S_{K\pi}$. This relative phase vanishes in the limit of pure penguin dominance and is expected to be smaller than $\pi/2$ when including small color-suppressed tree and electroweak penguin contributions in $A_{00}$. This feature holds true for the two illustrated solutions but excludes the two solutions with one triangle flipped, for which the relative phase between $A_{00}$ and $\bar{A}_{00}$ is larger than $\pi/2$.

The expected value of $S_{K\pi}$ is related to the magnitudes and phases of $A_{00}$ and $\bar{A}_{00}$ in the following manner:

$$S_{K\pi} = \frac{2|A_{00}\bar{A}_{00}|}{|A_{00}|^2 + |\bar{A}_{00}|^2} \sin(2\beta + \phi_{00}).$$

(14)

The correction $\phi_{00} \equiv \text{Arg}(A_{00}\bar{A}_{00})$ to $2\beta$ is found to be positive for both of the displayed solutions. It is quite large, $\phi_{00} = 42.6^\circ$ corresponding to $S_{K\pi} = 0.99$, for the solution (1) with negative real values of the amplitudes $A_{00}$ and $\bar{A}_{00}$ and smaller, $\phi_{00} = 16.1^\circ$ corresponding to $S_{K\pi} = 0.85$, for the solution (2) with positive real values. Since $A_{00}$ is dominated by the penguin amplitude, $P = -|P|\exp(-i\delta_c)$, solution (1) corresponds to $\cos \delta_c > 0$ ($|\delta_c| < \pi/2$) while solution (2) involves $\cos \delta_c < 0$ ($|\delta_c| > \pi/2$).

In order to exclude solution (2) one would have to show unambiguously that $\cos \delta_c > 0$ or $|\delta_c| < \pi/2$, where $\delta_c$ is the strong phase difference between $T + C$ and $P$. A most
direct proof for $\cos \delta_\text{c} > 0$ would need an observation of destructive interference between $P$ and $T + C$ in the CP-averaged decay rate of $B^+ \rightarrow K^+\pi^0$ normalized by that of $B^+ \rightarrow K^0\pi^+$. However, this interference is cancelled by constructive interference of $P$ and $P_{EW} + P_{EW}^C$ [31]. Arguments for small strong phase differences including $\delta_\text{c}$ have been presented in studies of $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$ based on a heavy quark expansion [32]. These arguments failed, however, when predicting a very small phase $\text{Arg}(C/T)$. This would imply $A_{CP}(K^+\pi^0) < A_{CP}(K^+\pi^-)$, contrary to the two asymmetries quoted in Table 11 which show that this phase is not very small and must be negative (see argument below [31].) A small value of $\delta_\text{c}$ ($|\delta_\text{c}| < 30^\circ$) was obtained in global flavor SU(3) fits to decay rates and CP asymmetries measured for $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$ [13,33]. Within these fits it is difficult to pinpoint a small subset of $B \rightarrow K\pi$ measurements forcing a small value for $\delta_\text{c}$. The purpose of the subsequent discussion is to prove $\cos \delta_\text{c} > 0$ using a series of arguments based on specific measurements, stressing the minimal use of untested assumptions about flavor SU(3).

A strong phase which is more directly accessible to experiment than $\delta_\text{c}$ is $\delta$, the strong phase of $T$ relative to that of $P$. This phase occurs in the amplitude for $B^0 \rightarrow K^+\pi^-$. Its cosine term appears in the ratio $R$ of CP-averaged decay rates for this process and $B^+ \rightarrow K^0\pi^+$ [34,35]. Neglecting $P_{EW}^C$ and $A$ terms in these amplitudes, one would expect $R$ to be smaller than one for $\cos \delta > 0$ and larger than one for $\cos \delta < 0$. The current value $R = 0.899 \pm 0.048$, obtained from branching ratios in Table 11 and the above-mentioned ratio of $B^+$ and $B^0$ lifetimes, favors $\cos \delta > 0$ over $\cos \delta < 0$. This evidence is statistically limited and may suffer from $P_{EW}^C$ corrections in $B^0 \rightarrow K^+\pi^-$. The negative asymmetry $A_{CP}(K^+\pi^-) = -0.097 \pm 0.012$ proves unambiguously that $\delta$ is positive.

An argument proving $|\delta| < \pi/2$ unambiguously is based on the time-dependent CP asymmetry parameter $S_{\pi^+\pi^-}$ in $B^0 \rightarrow \pi^+\pi^-$. Assuming flavor SU(3), the ratio of penguin and tree amplitudes and their relative phase are equal in this process to those in $B^0 \rightarrow K^+\pi^-$, up to CKM factors defining the ratios of amplitudes. Neglecting small $W$-exchange and penguin annihilation contributions (the resulting systematic uncertainty introduced by this approximation is taken as part of an uncertainty due to SU(3) breaking mentioned below), one has [36]

$$S_{\pi^+\pi^-} = \frac{\sin 2\alpha + 2r \cos \delta \sin(\beta - \alpha) - r^2 \sin 2\beta}{1 - 2r \cos \delta \cos(\beta + \alpha) + r^2} ,$$

(15)

where $\alpha = \pi - \beta - \gamma$ and $r$ is the ratio of penguin and tree amplitudes in $B^0 \rightarrow \pi^+\pi^-$. In the absence of a penguin amplitude one has $S_{\pi^+\pi^-} = \sin 2\alpha$, and to first order in the ratio $r$ one finds [37]

$$S_{\pi^+\pi^-} = \sin 2\alpha + 2r \cos \delta \sin(\beta + \alpha) \cos 2\alpha .$$

(16)

BaBar [38] and Belle [39] find the same value for this quantity; the average is large and negative [5], $S_{\pi^+\pi^-} = -0.61 \pm 0.08$. Since $\alpha = \pi - \beta - \gamma \simeq \pi/2$ [27] one has $\sin 2\alpha \simeq 0$ and $\cos 2\alpha \simeq -1$, while $\sin(\beta + \alpha) > 0$, which implies $\cos \delta > 0$.

A detailed analysis using the exact expression (15) and measurements of $S_{\pi^+\pi^-}$ and a second asymmetry $C_{\pi^+\pi^-} \equiv -A_{CP}(\pi^+\pi^-)$ confirmed this conclusion obtaining
Figure 2: Illustration of relative strong phases of $T$, $C$, and $P$ in $B \to K\pi$ decays and the construction leading to Eq. (17). Here $\delta = \text{Arg}(T/P)$; $\delta_c = \text{Arg}[(T+C)/P]$. 

A value $\delta = (33 \pm 7_{-10}^{+8})^\circ$ [37]. The first error is experimental, while the second is associated with a systematic uncertainty in flavor-SU(3) breaking. The positive sign of $\delta_c$ follows from the negative averaged $C_\pi + \pi^-$. The two CP rate asymmetries are equal within experimental errors and have opposite signs [40, 41]. Expressed in units of $10^{-6}$ they are $\Delta(K^+\pi^-) = -1.88 \pm 0.24 = -\Delta(\pi^+\pi^-) = -1.96 \pm 0.37$ [5]. This confirms the flavor SU(3) assumption for equal ratios of penguin and tree amplitudes and equal relative strong phases in these two processes. A difference of 180$^\circ$ between the two phases, which would not affect the equality of CP rate asymmetries, is extremely unlikely. The property $|\delta| < \pi/2$ implies constructive (destructive) interference between $T$ and $P$ in the CP averaged rate for $B^0 \to \pi^+\pi^-$ ($B^0 \to K^+\pi^-$).

In order to constrain $\delta_c$ (the strong phase difference between $T+C$ and $P$), using the above range for $\delta$ (the strong phase difference between $T$ and $P$), one needs information about the strong phase of the ratio $C/T$. The observation $A_{CP}(K^+\pi^0) > A_{CP}(K^+\pi^-)$ implies that $\text{Arg}(C/T)$ is negative and larger in magnitude than $\delta$ [31]. A simple proof of this behavior, for terms in the two asymmetries which are linear in $|T+C|/|P|$ and $|T|/|P|$, respectively, follows from the geometrical identity

$$|T+C|\sin\delta_c = |T|\sin\delta + |C|\sin(\delta + \text{Arg}(C/T))$$

illustrated in Fig. 2. The amplitudes $T+C$ interfere constructively in $B^+ \to \pi^+\pi^0$. This follows from the observation that $2(B^+ \to \pi^+\pi^0) > (B^0 \to \pi^+\pi^-)$ [5], and the above-mentioned constructive interference of $T$ and $P$ in $B^0 \to \pi^+\pi^-$. Thus $-\pi/2 < \text{Arg}(C/T) < -\delta < 0$ which implies geometrically $-\pi/2 < \delta_c < \delta < \pi/2$, without making any assumption about the magnitude $|C/T|$. This concludes the proof of $\cos\delta_c > 0$ which excludes solution (2) in Fig. 1.

It is the large value of $\phi_{00} \equiv \text{Arg}(A_{00}\bar{A}_{00}^*)$ in solution (1) in Fig. 1 which is thus responsible for boosting the expected value of $S_{K\pi}$ from its penguin-dominated value of $\sin 2\beta \approx 0.68$ to a value very close to 1. We now explore the sensitivity of this effect to small changes in experimental inputs.

We find the greatest sensitivity of $S_{K\pi}$ is to variations of the branching ratio $B(K^0\pi^0) \equiv (B^0 \to K^0\pi^0)$. In Fig. 3(a) we plot $\phi_{00}$ and $S_{K\pi}$ versus $B(K^0\pi^0)$ for nominal values of the parameters noted in the text. We note that $S_{K\pi}$ drops from a value of 0.99 at the central value of $B(K^0\pi^0)$ to 0.91 and 0.72 at $-1\sigma$ and $-2\sigma$ below the central value.
Figure 3: Dependence of $\text{Arg}(A_{00}/\bar{A}_{00})$ and $S_{K\pi}$ on $B(K^0\pi^0) \equiv (B^0 \to K^0\pi^0)$. Vertical dashed lines in top panel show central value and $\pm 1\sigma$ errors of $B(K^0\pi^0)$. The plotted point on the lower panels shows the experimental values. (a) All parameters as in text; (b) same as (a), but $\gamma = 55^\circ$; (c) same as (b), but $(\bar{B}^0 \to K^0\pi^-) = 20 \times 10^{-6}$.

Table III: Comparison of sensitivity of $\phi_{00} \equiv \text{Arg}(A_{00}/\bar{A}_{00})$ (in degrees) and $S_{K\pi}$ to various parameters.

| Parameter | $-1\sigma$ $\phi_{00}$ | $+1\sigma$ $\phi_{00}$ | $S_{K\pi}$ | $S_{K\pi}$ |
|-----------|----------------|----------------|--------------|--------------|
| $(\bar{B}^0 \to K^0\pi^0)$ | 23.9 | 60.6 | 0.911 | 0.963 |
| $\gamma$ | 24.3 | 59.4 | 0.913 | 0.967 |
| $(B^0 \to K^0\pi^-)$ | 52.0 | 33.3 | 0.986 | 0.962 |
| $\xi_{T+C}$ | 41.0 | 44.4 | 0.985 | 0.989 |
| $\xi_{EW}$ | 26.3 | 58.0 | 0.926 | 0.972 |
We next vary $\gamma$ within its 1$\sigma$ limits to 55° [Fig. 3(b)]. The experimental values become considerably more compatible with the Standard Model predictions, and even more so if $(B^0 \to K^+\pi^-)$ is increased by 1$\sigma$ to $20 \times 10^{-6}$ [Fig. 3(c)]. In Figs. 3 the quantity $\phi_{00}$ is more sensitive than $S_{K\pi}$ to variations in $(B^0 \to K^0\pi^0)$, $\gamma$, and $(B^0 \to K^+\pi^-)$. For the central value of $\phi_{00}$, $S_{K\pi}$ is very close to its maximum value, so it is only for considerably lower values of $\phi_{00}$ that $S_{K\pi}$ becomes sensitive to these parameters.

In Table IV we summarize the effects on $\phi_{00}$ and $S_{K\pi}$ of varying $(B^0 \to K^0\pi^0)$, $\gamma$, and $(B^0 \to K^+\pi^-)$ by $\pm 1\sigma$ around their central values. (See Table III; we are taking $\gamma = (65 \pm 10)^\circ$.) A possible effect combining these three errors is seen in Fig. 3(c). We also include the effects of $\pm 1\sigma$ variations of $\xi_{T+C} = 1.0 \pm 0.2$ and $\xi_{EW} = 1.0 \pm 0.2$. For nominal values of the parameters, one has $\phi_{00} = 42.6^\circ$ and $S_{K\pi} = 0.987$. Table III indicates the greatest sensitivity of $\phi_{00}$ to $(B^0 \to K^0\pi^0)$, followed by $\gamma$ and $\xi_{EW}$. There is relatively little sensitivity to $\xi_{T+C}$.

Other variations are found to have a negligible effect on $S_{K\pi}$. This includes the asymmetry $A_{CP}(B^0 \to K^+\pi^-)$, which involves a very small experimental error, and $A_{CP}(B^0 \to K^0\pi^0) \equiv -C_{K\pi}$, which is predicted in (5) with a small uncertainty. A large variation in this asymmetry would in any case have little effect on $S_{K\pi}$, as a geometric construction similar to that in Fig. 1 illustrates. The phases of $A_{00}$ and $\bar{A}_{00}$ are found to shift nearly together, so that the correction to $\sin 2\beta$ in Eq. 14 changes very little. This insensitivity to $C_{K\pi}$ is displayed for the favored $S_{K\pi}$ solution in Ref. [6], where $C_{K\pi}$ is left unconstrained disregarding the sum rule (3).

Thus the possibility that the above calculation of $S_{K\pi}$ in the Standard Model differs both from its penguin-dominated value of $\sin 2\beta \simeq 0.68$ and from the data remains intriguing. However, for it to become a robust conclusion about the presence of new physics, accuracies of measurements of the $B^0$ branching ratios to $K^0\pi^0$ and $K^+\pi^-$ and of the CKM angle $\gamma$ need to be improved. We look forward to such advances in future data, and to more precise measurements of the two asymmetries $C_{K\pi}$ and $S_{K\pi}$ in $B^0 \to K^0\pi^0$.

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Note added: The measurements of $C_{K\pi}$ and $S_{K\pi}$ given in Table III have been updated very recently by the BaBar and Belle collaborations. New results and their averages are summarized in Table IV. The averaged value of $C_{K\pi}$ agrees with the prediction

Table IV: Updated measurements of $C_{K\pi}$ and $S_{K\pi}$.

| Ref.     | $C_{K\pi}$         | $S_{K\pi}$         |
|----------|--------------------|--------------------|
| BaBar    | $0.13 \pm 0.13 \pm 0.03$ | $0.55 \pm 0.20 \pm 0.03$ |
| Belle    | $-0.14 \pm 0.13 \pm 0.06$ | $0.67 \pm 0.31 \pm 0.08$ |
| Average  | $0.00 \pm 0.10$ | $0.58 \pm 0.17$ |
within 1.4σ, while $S_{K\pi}$ is now consistent with $\sin 2\beta$ and somewhat larger values. Recent updates by BaBar of the branching ratio for $B^0 \to K^0\pi^0$ and the CP asymmetry in $B^0 \to K^+\pi^-$ [44] do not affect significantly the corresponding two averaged values in Table II.

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