PAPER

Simple empirical correction functions to cross sections of the photoelectric effect, Compton scattering, pair and triplet production for carbon radiation shields for intermediate and high photon energies

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Abstract
The general review of cross section formulas for the photoelectric effect, Compton scattering, pair and triplet production is presented in the intermediate and high photon energy spectrum for carbon ($Z = 6$). Using the classical Gaussian regression method, the empirical correction functions for the theoretical cross section formulas are calculated based on the existing experimental data. The simple formulas can be used to calculate the cross sections of four main radiation effects with a good accuracy. The proposed formulas are enough for practical calculations in the excel spreadsheet in e.g. radiation protection applications or radiation shields designing.

1. Introduction
Electromagnetic ionizing radiation (gamma and X rays) is of crucial importance to radiation protection worldwide. Such type of radiation interacts with matter in different ways, from which four are the most important: the photoelectric effect, Compton scattering, pair production and triplet production (Fornalski 2013). Thus, the mentioned four effects and their cross section formulas for carbon ($Z = 6$) will be discussed within the presented paper. Other effects, like the photonuclear absorption, Thomson or Rayleigh (coherent) scattering were omitted because of their non-significant contribution in the analyzed range of energies.

One can easily find in the literature the proper physical formulas for all four mentioned effects (Hubbell et al 1980, Patrignani et al 2016). However, for simple calculations of cross section values, e.g. by a radiation protection officer for shield design, one needs to use approximate but not precise formulas or use a dedicated software, e.g. XCOM (Berger et al 2010).

The presented paper discussed the simple-in-use analytic formulas of cross sections for the photoelectric effect, Compton scattering, pair and triplet production, together with simple correction functions which makes all the formulas more precise in practical applications. Thanks to that approach the potential user, e.g. a radiation protection officer or a nuclear technician, can easily calculate the values of cross section with a good accuracy for a wide range of photon energies, using a simple excel software without the necessity of complicated integral equations calculations.

The practical purpose of the presented paper is the carbon-composite radiation shield designing which is a part of the research project being conducted in Ex-Polon Laboratory (Poland).

2. Methods
All presented formulas of cross sections are related to the unit of barn (b) per atom. The energy of photon is presented as $k = E_\gamma/E_e$, where $E_\gamma = h\nu$ is the photon energy given in eV and $E_e = m_e c^2 \approx 5.11 \times 10^5$ eV is the electron rest mass energy.
In the used convention the related cross section is presented as

$$\sigma = \epsilon \cdot (f + f_{corr})$$  (1)

where \( \epsilon \) is the constant usually related to \( Z \) (atomic number) and \( \alpha = e^2/(\hbar c) \approx 1/137 \) (fine structure constant) and \( r_e^2 = e^4/E_e^2 \approx 0.079 \) \( 41 \) b (square of the classical electron radius), \( f \) is the classical cross section relation for each process (Hubbell et al 1980) and \( f_{corr} \) is the correction function.

The correction function to cross section, \( f_{corr} \), is calculated using the classical Gaussian regression method (Wolberg 2005) which finds the best curve fit to all \( F \) points determined for each \( k \):

$$F(k) = \frac{1}{\xi} \sigma_{real}(k) - f(k)$$  (2)

where \( \sigma_{real} \) is the precise value of the cross section received in experimental way or, if experimental data are not available, \( \sigma_{real} \) is obtained by detailed and precise calculations using the quantum electrodynamics formalism (Hubbell et al 1980). Thus, \( \epsilon \cdot f \) is the cross section value calculated by using popular simplified equations (see next chapters) which need correction functions to be equal to \( \sigma_{real} \).

The \( f_{corr} \) function is the best fit to all \( F \) points. Due to the very low scatter of \( F \)s, the Gaussian regression method is sufficient and more complicated regression methods are not necessary (Fornalski et al 2010).

The physical background for \( f_{corr} \) function is an empirical comprehensive expression of many additional corrections and effects, like the Coulomb correction, screening correction and radiative correction (Hubbell et al 1980) which are usually too complex for calculation by a radiation protection officer or nuclear technician.

3. Results

The presented results of \( f_{corr} \) functions forms are divided into four parts according to the effects mentioned earlier: the photoelectric, Compton, pair and triplet production. \( f_{corr} \) functions for each effects are presented in the following subsections and their applications are discussed.

All presented correction functions, \( f_{corr} \), improve calculations of cross sections to be more accurate to real values. However, some differences between calculated cross sections with \( f_{corr} \) and real values can be found but they are lower than:

- 0.55% for the photoelectric effect (figure 1(a)),
- 0.1% for the Compton scattering (figure 1(b)),
- 0.6% for the pair production (figure 1(c)),
- 0.35% for the triplet production (figure 1(d)).

Such accuracy of each \( f_{corr} \), used, which is presented in figure 1, is calculated as the ratio of precise cross sections (Hubbell et al 1980) and cross sections from the presented method (with appropriate \( f_{corr} \) functions from table 1). Table 1 presents all recommended cross section formula numbers (see next subsections for exact functions) with their \( f_{corr} \) function numbers selected to be the most appropriate due to their accuracy (figure 1).

3.1. Photoelectric effect

For low energies \( (k < 0.9) \) the cross section of the photoelectric effect can be approximated by the simplified equation of (Davisson 1965)

$$\sigma_{ph} = \frac{16}{3} \sqrt{2} \pi r_e^2 \alpha^4 \frac{Z^2}{E_e^{3.5}} \approx 3 \cdot 10^{12} \frac{Z^4}{E_e^{3.5}}$$  (3)

where \( Z = 6 \) for carbon. However, the equation (3) do not cover the resonance changes for the specific \( k \) and \( Z \) (ionization edges), thus the accuracy of equation (3) is relatively low.

For higher energies \( (k > 0.9) \) the cross section of the photoelectric effect is described by much more precise equation proposed by Pratt and Scofield (Pratt 1960, Scofield 1973, Hubbell et al 1980)

$$\sigma_{ph} = Z^2 \left[ \sum_{n=1}^{4} \frac{a_n + b_n Z}{1 + c_n Z} k^{-n} \right] + f_{corr,ph}$$  (4)

and modified by adding \( f_{corr} \). The values of parameters in equation (4) are presented in table 2. Using the method described in the previous chapter, one can find for \( 0.9 < k < 10^6 \):
Figure 1. Accuracy of the presented method calculated as the ratio of precise cross sections (Hubbell et al. 1980) and cross sections from the presented method for (a) the photoelectric effect, (b) Compton scattering, (c) pair and (d) triplet production (see table 1 for equations).
Table 1. The quotes of recommended equations of cross section formulas and their best correction functions (see next subsections for exact formulas) for various energy ranges (k).

| Energy range, k | Photoelectric effect | Compton scattering | Pair production | Triplet production |
|----------------|----------------------|--------------------|-----------------|-------------------|
|                | Cross section formula, $\sigma_{ph}$ | correction function, $f_{corr,ph}$ | Cross section formula, $\sigma_{C}$ | correction function, $f_{corr,C}$ | Cross section formula, $\sigma_{pair}$ | correction function, $f_{corr,pair}$ | Cross section formula, $\sigma_{trip}$ | correction function, $f_{corr,trip}$ |
| 0.2–0.9        | equation (3)         | —                  | equation (7)    | —                 | n.a.               | n.a.               | n.a.               | n.a.               |
| 0.9–2          | equation (4)         | equation (5)       | equation (7)    | —                 | n.a.               | n.a.               | n.a.               | n.a.               |
| 2–4            | equation (4)         | equation (5)       | equation (7)    | equation (10)     | equation (15)      | equation (17)      | n.a.               | n.a.               |
| 4–5.9          | equation (4)         | equation (5)       | equation (7)    | equation (10)     | equation (15)      | equation (17)      | n.a.               | n.a.               |
| 5.9–9.8        | equation (4)         | equation (5)       | equation (7)    | equation (10)     | equation (15)      | equation (17)      | equation (19)      | equation (23)      |
| 9.8–12         | equation (4)         | equation (5)       | equation (7)    | equation (10)     | equation (15)      | equation (19)      | equation (19)      | equation (23)      |
| 12–30          | equation (4)         | equation (5)       | equation (7)    | equation (11)     | equation (15)      | equation (19)      | equation (19)      | equation (23)      |
| 30–100         | equation (4)         | equation (5)       | equation (7)    | equation (12)     | equation (15)      | equation (19)      | equation (19)      | equation (23)      |
| 100–300        | equation (4)         | equation (5)       | equation (7)    | equation (12)     | equation (15)      | equation (19)      | equation (19)      | equation (23)      |
| 300–4000       | equation (4)         | equation (5)       | equation (7)    | equation (12)     | equation (15)      | equation (19)      | equation (19)      | equation (23)      |
| 4000–4·10^5    | equation (4)         | equation (5)       | equation (7)    | equation (13)     | equation (15)      | equation (19)      | equation (19)      | equation (23)      |

Symbol '—' represents the situation when the correction function is not necessary. Symbol 'n.a.' represents the situation when the function does not exist due to the energy lower than the effect threshold.
One has to note, that the number precision in equation (5) is high. However, for less precise results one can simplify the equation (5), or even use the fourth order log-polynomial only.

The equation (5) shape is presented in figure 2.

For intermediate and high energies the cross section of the photoelectric effect can be simply neglected due to its very low and non-significant values. In this paper, however, the Pratt-Scofield equation (4) is applied for all energies.

3.1.1. Photoelectric effect—simplification for high energies

For higher energies \( k > 30 \) the equation (5) can be substituted by the simpler form:

\[
 f_{\text{corr,ph}} = 10.71 \cdot 10^{-10} k^{-1.00795} \tag{6}
\]

but the precision of equation (6) in relation to equation (5) is lower by maximally 2.5% within the range \( 50 < k < 2.3 \cdot 10^5 \) and 7% within the range \( 30 < k < 3.5 \cdot 10^5 \), while reaches 40% for \( k = 10^6 \).

3.2. Compton scattering

The total cross section of the Compton (incoherent) effect is given by the Klein-Nishina equation (Hubbell et al 1980) for \( k > 0.2 \)

\[
 \sigma_C = 2\pi q r_e^2 \left\{ \frac{1 + k}{k^2} \left[ \frac{2(1 + k)}{1 + 2k} - \frac{\ln(1 + 2k)}{k} \right] + \frac{\ln(1 + 2k)}{2k} - \frac{1 + 3k}{(1 + 2k)^2} + f_{\text{corr,C}} \right\} \tag{7}
\]

where \( q \) is the average number of electrons per atom. For neutral atoms \( q = Z \), which equals 6 for carbon.

For lower energies \( k < 0.2 \) the equation (7) shall be substituted by (Hubbell et al 1980):

\[
 \sigma_C = \frac{8}{3} \pi q r_e^2 \left( \frac{1}{1 + 2k} \left[ 1 + 2k + \frac{6}{5} k^2 - \frac{1}{2} k^3 + \frac{2}{7} k^4 - \frac{6}{35} k^5 + \frac{8}{105} k^6 + \frac{4}{105} k^7 + \ldots \right] \right) \tag{8}
\]
The correction function \( f_{\text{corr},C} \) for \( 2 < k < 4 \cdot 10^5 \) can be presented as:

\[
f_{\text{corr},C} = \exp(-8.7029 \cdot 10^{-3} \ln^6 k + 3.8767 \cdot 10^{-3} \ln^5 k - 6.63785 \cdot 10^{-2} \ln^4 k \\
+ 0.544758 \ln^3 k - 2.221839 \ln^2 k + 3.924915 \ln k - 10.0088) \tag{9}
\]

However, better precision (approximately 4.5 times lower average standard deviation value) can be received when equation (9) will be substituted by four different equations, the first one for \( 2 < k < 12 \):

\[
f_{\text{corr},C} = -1.8391 \cdot 10^{-4} \ln^4 k + 0.00132 \ln^3 k - 0.00354 \ln^2 k + 0.004157 \ln k - 0.0013 \tag{10}
\]

For \( 12 < k < 30 \):

\[
f_{\text{corr},C} = -4.914 \cdot 10^{-5} \ln^3 k + 5.268 \cdot 10^{-4} \ln^2 k - 0.001927 \ln k + 0.0027 \tag{11}
\]

For \( 30 < k < 4000 \):

\[
f_{\text{corr},C} = \exp(4.18575 \cdot 10^{-3} \ln^6 k - 0.142581 \ln^5 k + 1.963767 \ln^4 k \\
- 13.982136 \ln^3 k + 54.257221 \ln^2 k - 109.141022 \ln k + 81.2282) \tag{12}
\]

and for \( k > 4000 \):

\[
f_{\text{corr},C} = 0.101442 \cdot e^{-0.905914 \ln k} \tag{13}
\]

The curve given by equation (9) is presented in figure 3. For that energies the binding energy process for the Compton scattering can be omitted.

3.2.1. Compton absorption

The cross section of photon energy absorption in the Compton effect (the energy transfer coefficient) can be described as (Attix 1986):

\[
\sigma_{\text{abs}} = 2\pi qr^2 \left[ \frac{2(1 + k)^2}{k^2(1 + 2k)} - \frac{1 + 3k}{(1 + 2k)^2} - \frac{(1 + k)(2k^2 - 2k - 1)}{k^2(1 + 2k)^2} \right. \\
- \left. \left( \frac{1 + k}{k^3} - \frac{1}{2k} + \frac{1}{2k^4} \right) \ln (1 + 2k) \right] \tag{14}
\]

which is often used in radiation protection calculations. The accuracy of equation (14) does not require implementing of the correction function (for \( k > 2 \cdot 10^{-4} \)). For very high energies the equation (14) can be substituted by equation (7).

3.3. Pair production

The cross section for the pair production effect, namely the electron-positron (\( e^- e^+ \)) creation in the nuclear field after photon interaction, is usually described by the Maximon equation (Maximon 1968, Hubbell et al 1980):
\[ \sigma_{\text{pair}} = Z^2 \alpha \frac{\pi^2}{5} \left( \frac{28}{9} \ln 2k - \frac{218}{27} + \left( \frac{2}{k} \right)^3 \ln 2k - \frac{7}{2} + \frac{2}{3} \ln^2 2k - \ln^2 2k \right. \\
- \left. \frac{1}{3} \pi^2 \ln 2k + 2 \zeta(3) + \frac{\pi^2}{6} - \left( \frac{2}{k} \right)^3 \frac{3}{16} \ln 2k + \frac{1}{8} - \left( \frac{2}{k} \right)^6 \right) \times \left[ \frac{29}{9} - \frac{77}{27} \frac{\ln 2k}{512} \right] + f_{\text{corr, pair}} \]

where \( \zeta(3) \approx 1.2020569 \) is the Riemann zeta function and \( Z = 6 \) for carbon. It is important to note, that the energy threshold for the pair production effect is \( k = \sqrt{m_e^2} \) (the positron and electron rest mass energy).

It is difficult to find the universal correction function for the whole range of energies. However, a good approximation for \( 10 \leq k \leq 4 \cdot 10^5 \) can be presented as:

\[ f_{\text{corr, pair}} = -7.68798 \cdot 10^{-5} \ln^6 k + 2.8555 \cdot 10^{-3} \ln^5 k - 0.0372044 \ln^4 k + 0.190339 \ln^3 k - 0.415673 \ln^2 k + 0.37442 \ln k - 0.1097 \]

The equation (16) was presented in figure 4 (lower curve).

However, for low energies (especially for \( k \leq 10 \)) other but simpler forms are more likely due to the average standard deviation value is approximately 15 times lower. For example for \( k < 5.9 \):

\[ f_{\text{corr, pair}} = -0.0055747 k^2 + 0.004904 k - 0.00898 \]

For \( 5.9 < k < 9.8 \):

\[ f_{\text{corr, pair}} = -7.5 \cdot 10^{-5} k^2 + 0.001117 k - 0.00455 \]

For \( 9.8 < k < 100 \):

\[ f_{\text{corr, pair}} = -0.037145 \ln^3 k + 0.293299 \ln^2 k - 0.72204 \ln k + 0.5629 \]

3.3.1. Pair production—simplifications for high energies

For higher energies the equation (16) can be substituted by simpler forms, like (for \( k > 100 \)):

\[ f_{\text{corr, pair}} = 0.026375 \ln^3 k - 0.84578 \ln^2 k + 5.9531 \ln k - 12.252 \]

For \( k > 1000 \):

\[ f_{\text{corr, pair}} = -0.08704 \ln^2 k - 1.1864 \ln k + 9.7094 \]

For extreme energies (\( k > 15000 \)) the correction function is practically log-linear:

\[ f_{\text{corr, pair}} = -3.04997 \ln k + 19.572 \]
The differences between the results obtained by equation (15) with (20–22) and the results presented by Hubbell et al (1980) do not exceed 1%.

### 3.4. Triplet production

The triplet production effect, where positron and electron is produced in the field of other electron, is similar to the pair production, with the threshold at $k = 4$. However, the cross section, similarly to the Compton effect, is not related to $Z$ but to the number of electrons per atom ($q$), which do not need to be always equal to each other (Mork 1967, Haug 1975). The most popular form of the triplet cross section was proposed by Borsellino and Ghizzetti (Hubbell et al 1980):

$$
\sigma_{\text{trip}} = \frac{q \alpha r_e^2}{k} \left[ \frac{28}{9} \ln 2k - \frac{218}{27} + \frac{1}{k} \left( -\frac{4}{3} \ln^3 2k + 3 \ln^2 2k - \frac{60 + 16a}{3} \ln 2k \right) + \frac{123 + 12a + 16b}{3} \right] + \frac{1}{k^2} \left( \frac{8}{3} \ln^3 2k - 4 \ln^2 2k + \frac{51 + 32a}{3} \ln 2k \right) - \frac{123 + 32a + 64b}{6} + \frac{1}{k^3} \left( -\frac{49}{18} \ln 2k - \frac{115}{432} \right) + \frac{1}{k^4} \left( -\frac{77}{36} \ln 2k - \frac{10831}{8640} \ln 2k + \frac{2915 - 288a}{216} \right) - \frac{64573}{36000} + \frac{1}{k^5} \left( -\frac{4423}{1800} \ln 2k - \frac{394979}{216000} \ln 2k \right) + f_{\text{corr,trip}} \right] \tag{23}
$$

where $a = -2.4674$ and $b = -1.8031$. The universal correction function ($k < 4 \cdot 10^5$) can be presented as:

$$
f_{\text{corr,trip}} = -10.5675 \cdot 10^{-3} \ln^6 k + 4.3638 \cdot 10^{-3} \ln^5 k - 0.06687965 \ln^4 k + 0.46089 \ln^3 k - 1.54884 \ln^2 k + 2.51739 \ln k - 1.6113 \tag{24}
$$

The equation (24) is presented in figure 4 (upper curve). One has to note, however, that the accuracy of equation (24) for $k < 300$ is acceptable but relatively low. For more precise results for $k < 300$ one shall use the correction function given by:

$$
f_{\text{corr,trip}} = -10.4005 \cdot 10^{-4} \ln^5 k + 0.018729 \ln^4 k - 0.15862 \ln^3 k + 0.70326 \ln^2 k - 1.4446 \ln k + 1.0475 \tag{25}
$$

#### 3.4.1. Triplet production—Haug’s simplifications for low and intermediate energies

The equation (23) is quite long, so Haug proposed simpler analytical forms of triplet cross section (Haug 1981). Especially for the lowest energies $4 < k < 4.6$:

$$
\sigma_{\text{trip},I} = q \alpha r_e^2 \left[ 5.6 + 20.4(k - 4) - 10.9(k - 4)^2 - 3.6(k - 4)^3 + 7.4(k - 4)^4 \right] \times 10^{-3}(k - 4)^2 \tag{26}
$$

For $4.6 < k < 6$:

$$
\sigma_{\text{trip},I} = q \alpha r_e^2 \left( 0.582814 - 0.29842k + 0.04354k^2 - 0.0012977k^3 + f_{\text{corr,trip},I} \right) \tag{27}
$$

For $6 < k < 18$:

$$
\sigma_{\text{trip},I} = q \alpha r_e^2 \left( \frac{3.1247 - 1.3394k + 0.14612k^2}{1 + 0.4648k + 0.016683k^2} + f_{\text{corr,trip},I} \right) \tag{28}
$$

For $k > 14$ Haug proposed to use a shorter form of Borsellino equation:

$$
\sigma_{\text{trip},I} = q \alpha r_e^2 \left[ \frac{28}{9} \ln 2k - \frac{218}{27} + \frac{1}{k} \left( -\frac{4}{3} \ln^3 2k + 3.863 \ln^2 2k - 11 \ln 2k + 27.9 \right) + f_{\text{corr,trip},I} \right] \tag{29}
$$

The correction functions for the Haug formulas for $4.6 < k < 15$ can be presented as:

$$
f_{\text{corr,trip},I} = 0.00863907 \ln^2 k - 0.02550985 \ln k + 0.01872 \tag{30}
$$

For $15 < k < 100$:

$$
f_{\text{corr,trip},I} = -0.01805017 \ln^4 k + 0.257819 \ln^3 k - 1.38472 \ln^2 k + 3.329071 \ln k - 2.9987 \tag{31}
$$
For 100 < \( k < 200 \):

\[
f_{\text{corr,trip,H}} = -0.0429185 \ln^3 k + 0.476846 \ln^2 k - 1.735 \ln k + 2.096
\] (32)

For \( k > 200 \):

\[
f_{\text{corr,trip,H}} = 0.02896 \ln^3 k - 0.9744 \ln^2 k + 7.882 \ln k - 18.829
\] (33)

For higher energies (\( k > 2000 \)) the correction function can be approximated by the simpler formula:

\[
f_{\text{corr,trip,H}} = -0.11462 \ln^2 k - 0.5137 \ln k + 8.1297
\] (34)

while for extreme energies (\( k > 15000 \)) is the log-linear one:

\[
f_{\text{corr,trip,H}} = -2.997 \ln k + 21.455
\] (35)

3.4.2. Triplet production—simplifications for high energies

The correction function to equation (23) for extreme energies (\( k > 15000 \)) can be presented as the log-linear equation:

\[
f_{\text{corr,trip,H}} = -3.012 \ln k + 21.629
\] (36)

which is almost equivalent to equation (35).

4. Application

The presented method of correction functions calculations is universal and can be applied for all elements from \( Z = 1 \) to \( Z = 100 \). However, the exact values presented in this paper are appropriate for carbon only (\( Z = 6 \)) and, with a lower accuracy, for surrounding elements (from \( Z = 5 \) to \( Z = 9 \)). For other elements one shall apply the presented method to their corresponding data to get the necessary correction functions.

The table 1 shows how to calculate the total cross section components (\( \sigma_{\text{total}} = \sigma_{\text{ph}} + \sigma_{\text{C}} + \sigma_{\text{pair}} + \sigma_{\text{trip}} \)) in a wide range of energies precisely. Such results are presented in figure 5 altogether. They can be easily transferred into the linear attenuation coefficient

\[
\mu = \sigma_{\text{total}} N
\] (37)

which is very useful in radiation protection and radiation shield designing (\( N \) is an atomic density).

The total cross section in figure 5, which is the sum of cross sections of the photoelectric, Compton, pair and triplet effects (see table 1), can be also approximated by one simpler empirical equation:

\[
\sigma_{\text{total}} \approx \exp(-3.2534 \cdot 10^{-5} \ln^4 k + 1.376 \cdot 10^{-3} \ln^3 k - 2.216 \cdot 10^{-2} \ln^2 k + 0.1641 \ln k - 0.4961 \ln^2 k + 0.0624 \ln k + 0.3802)
\] (38)

which is useful in quick but less precise calculations (for \( 1.5 < k < 10^5 \)).

The presented results are useful for light carbon (\( Z = 6 \)) radiation shields which are under scientific investigations in the Ex-Polon Laboratory (Poland). Especially, the process of material charging, where \( q > Z \), is
of crucial importance due to its total cross section increase. This phenomena can be easily deduced from equations (7) and (23) when the higher value of q is simply used (Fornalski 2017).

5. Discussion and conclusions

The presented paper describes the methodology of cross sections calculations using simple empirical correction functions which can be applied e.g. in the Excel spreadsheet. This approach omits detailed calculations which can be too difficult for e.g. radiation protection inspectors, nuclear technicians or radiation shield designers.

The accuracy of the proposed functions, described in table 1, is very high and generally maximal deviation from the real value does not exceed 0.6% (figure 1). One can find, that the most accurate results were obtained for the Compton scattering (figure 1(b)). However, to receive more precise cross section values one needs to use the complete methodology described by Hubbell et al. (1980), but for simple shield designing the methodology presented in this paper is completely sufficient.

The paper discusses various correction functions, $f_{corr}$, regarding their accuracy and corresponding energy range. The most accurate results are presented in table 1 and their relative goodness is shown in figure 1. However, one can use some other functions as well as discussed in section no. 3. Especially figures 2–4 contain very general correction functions, which can be applied to a wide range of energies but with a lower accuracy. The decision which function shall be selected—simpler and less accurate or more complicated but more accurate—depends on the user and his/her needs and expectations. In addition one has to note, that all presented correction functions cover many physical effects connected with additional corrections for Coulomb interactions, screening or radiative influences etc.

To conclude one should note, that the presented paper can substantially help some group of nuclear specialist and technicians in accurate cross section calculations, especially for carbon radiation shields (figure 5) which became more and more popular in specialized medical and scientific research programs.

Acknowledgments

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