A Gumbel-based activation function for imbalanced datasets

Yuexin Wu  
Shanghai Ocean University  
Shanghai, China

Tianyu Gao  
Sichuan University  
Chengdu, China

Hongtao Liu  
Tianjin University  
Tianjin, China

ABSTRACT

Rating prediction is a core problem in recommender systems to quantify user’s preferences towards different items. Due to the imbalanced rating distributions in training data, existing recommendation methods suffer from the biased prediction problem that generates biased prediction results. Thus, their performance on predicting ratings which rarely appear in training data is unsatisfactory. In this paper, inspired by the superior capability of Extreme Value Distribution (EVD)-based methods in modeling the distribution of rare data, we propose a novel Gumbel Distribution-based Rating Prediction framework (GRP) which can accurately predict both frequent and rare ratings between users and items. In our approach, we first define different Gumbel distributions for each rating level, which can be learned by historical rating statistics of users and items. Second, we incorporate the Gumbel-based representations of users and items with their original representations learned from the rating matrix and/or reviews to enrich the representations of users and items via a proposed multi-scale convolutional fusion layer. Third, we propose a data-driven rating prediction module to predict the ratings of user-item pairs. It’s worthy to note that our approach can be readily applied to existing recommendation methods for addressing their biased prediction problem. To verify the effectiveness of GRP, we conduct extensive experiments on eight benchmark datasets. Compared with several baseline models, the results show that: 1) GRP achieves state-of-the-art overall performance on all eight datasets; 2) GRP makes a substantial improvement in predicting rare ratings, which shows the effectiveness of our model in addressing the bias prediction problem.

CCS CONCEPTS
• Computer systems organization → Embedded systems; Redundancy; Robotics; • Networks → Network reliability.

KEYWORDS
rating prediction, gumbel distribution, recommender system

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1 http://jmcauley.ucsd.edu/data/amazon/

Figure 1: The rating distribution of five categories in Amazon datasets.

1 INTRODUCTION

With the popularization of the network applications, the recommender systems are playing important roles in the age of information overload, and have been widely used in many online platforms such as Amazon [1], YouTube [2], etc. The goal of recommender systems is to provide recommendations from various optional items or products according to users’ preferences and needs. Within all the recommendation methods, the rating prediction methods are commonly utilized to measure whether a user likes an item.

To predict the ratings given by users, many of the existing methods are based on Collaborative Filtering (CF), which models users and items with their historical interaction records (e.g., ratings, clicks, etc.) [3–5]. These methods are classified as rating-based prediction methods that usually utilize matrix factorization to obtain the latent feature of users and items, and then predict the users’ ratings for different items. However, the rating matrix is always of natural sparsity [6], which would make it difficult for the rating-based models to learn the accurate latent features [7].

Hence, to tackle the limitations of rating-based methods, many works start to learn representations of users and items from the reviews, which contain rich semantic information about users’ preferences and item characteristics [7–9]. Although these review-based methods have achieved good performance in real-world recommendations, most of them focus on the representations of users and items, but omit the following rating prediction process, which results in biased prediction results. We analyze the rating distributions of some training datasets from Amazon, the results are shown in Figure 1. It can be observed that the rating distribution is imbalanced that the low ratings (1, 2, 3) are quite rare in these real-world datasets compared with the ratings greater than 3. The imbalanced characteristics of datasets can be commonly found in real-world datasets, and it makes the existing models difficult to predict the rare ratings. As shown in Table 1, we report the results of several recently proposed baseline methods to verify their performance on different rating levels (the decimal ratings are rounded into 1-5). We can find that these methods always generate extremely biased prediction results and are incapable of predicting the rare
ratings, even if they can still achieve a good overall performance on the evaluation metric (e.g., MSE/MAE). This phenomenon can be described as the biased prediction problem. The prediction results of the existing methods could not match well with the original rating distributions in training datasets, and it will make the recommender system to ignore the preference of specific groups of users in real-world applications. Therefore, this problem should arouse the extensive concern of both academia and industry.

Based on the aforementioned observations, inspired by the superior capability of Extreme Value Distribution (e.g., Gumbel distribution) in modeling the imbalanced data [10], we propose a Gumbel-based Rating Prediction (GRP) framework in this paper to effectively address the biased prediction problem and further improve the overall prediction performance. As the first attempt in this field for addressing this problem, our proposed framework composed of three main modules. First of all, we propose a Gumbel-based feature learning module with different Gumbel distributions to model each rating and obtain a score feature vector for each user and item. Within this module, a novel dynamic approach is proposed to model the parameters of the Gumbel distributions with the historical rating statistics, which helps to estimate the rating preference of each user and item. Moreover, this approach can continuously update the parameters during the training process, which enables our method to better model the input data and gain robustness. Secondly, we employ the multi-scale convolutional fusion layer to summarize the feature information of the score features and the interactive features obtained from baseline methods. In this proposed module, we utilize three different sizes of convolution filters to capture the underlying information for each type of feature, the interaction information between different features, and the overall information across all the features, respectively. Thirdly, we design a data-driven rating prediction module to output the predicted ratings with the highly-compressed features and the original rating statistics of users and items. Note that the historical statistics of users and items can help our model to make better personalized recommendations. In conclusion, we jointly use the aforementioned three modules in GRP to achieve a better prediction performance and effectively address the biased prediction problem.

The extensive experiments on eight real-world datasets show that GRP significantly outperforms all the compared baselines in overall prediction performance. Compared with the performance achieved by baselines on each rating, our model achieves a significant improvement on rare ratings while ensuring the optimal overall performance on all ratings, which demonstrates the effectiveness of our model in addressing the biased prediction problem. Moreover, we verify the necessity of utilizing three proposed modules by conducting an ablation experiment. The proposed dynamic approach to model the parameters of Gumbel distribution obtains substantial performance gain over the traditional approach, which proves this is an alternative for modeling the probability density function in probability distribution-based methods. Furthermore, according to the experimental results in Section 6, our method GRP can be readily applied to other existing recommendation methods (e.g., rating-based and review-based methods) to improve their prediction performance and alleviate the biased prediction problem.

In summary, we conclude the contributions in this paper as follows:

- We propose a novel rating prediction method based on Gumbel distribution to estimate the rating preference for each user and item. The key parameters of the Gumbel distribution are obtained by utilizing a proposed dynamic modeling approach. As a general framework, any existing review-based and rating-based methods can be applied in GRP to significantly improve their overall performance and alleviate the biased prediction problem.
- A multi-scale convolutional fusion layer is designed to summarize different feature information via different sizes of convolution kernels. Moreover, we propose a data-driven rating prediction module to further exploit the rating distribution information and enables our model to make better personalized recommendations.
- We conduct extensive experiments to evaluate our method on eight real-world datasets, and our method significantly outperforms all the compared baselines. According to the comparison results, our model achieves the best performance in addressing the biased prediction problem.

2 RELATED WORKS
The existing rating prediction methods can be classified into rating-based methods and review-based methods. In this section, we introduce three different areas that are highly relevant to our work, which includes two types of the aforementioned rating prediction methods, and the class imbalance problem.

2.1 Rating-based Prediction Methods
The rating-based prediction method aims to learn latent factors base on the rating matrix between users and items. The most common rating-based methods of the recommender system are Factorization Machine (FM) [11], Latent Factor Model (LFM) [12], and fully connected layers. FM models the interactive feature vectors and further seek the high-level representation of the user-item interactions. LFM makes a prediction via summing the dot-product of multiple calculated results, including the user and item feature vector, the user and item bias, and the global bias. Fully connected layers utilize deep neural networks to learn a high-order characteristic representation of the interaction records. He et al. [13] used Multilayer Perceptron (MLP) to fit the interaction function which shows a reasonable improvement compared with traditional methods such as matrix factorization. The rating-based methods suffer from a key limitation, which is the sparsity of the data. Specifically, when collecting data from real-world platforms, the summarized rating matrix is sparse in most cases, which will make the performance of recommender systems deteriorate to a different extent [14, 15].

| Method   | 1-score | 2-score | 3-score | 4-score | 5-score |
|----------|---------|---------|---------|---------|---------|
| DeepCoNN | 0       | 0       | 0       | 30251   | 0       |
| NARRE    | 0       | 0       | 0       | 30199   | 52      |
| NRPA     | 0       | 0       | 0       | 29530   | 721     |

Table 1: Prediction results of three baselines on each rating level of the Grocery Gourmet dataset
2.2 Review-based Prediction Methods

The review-based methods [16–29] were proposed to alleviate the aforementioned issue of sparseness existing in rating-based methods. DeepCoNN [29] was proposed to represent a user’s rating preference with all the reviews written by users. The item’s features are characterized by a similar method. Then, the user and item documents are encoded with CNNs and fed into an FM for prediction. Atttn+CNN [30] and D-Attn [31] extend DeepCoNN with an attention mechanism on top of the text reviews, it improves the prediction performance and allows the model to explain predictions by highlighting significant words.

Considering the varying aspect-level importance between users and items, the development of attention mechanisms for modeling relationships between two sequences helps to capture the dynamic aspect-level importance of user-item pair [25]. ALFM [27] proposes an aspect-aware topic model on the review text to model the user preferences and learn different aspects of user and item. ANR [32] is an end-to-end aspect-based method with a co-attention mechanism. It emphasizes that different parts of the reviews may have different contributions to the overall rating. Both NARRE [18] and MPCN [33] argue that not all reviews are of the same importance. NARRE selects useful reviews by employing an attention mechanism to the reviews from different user-item pairs. MPCN uses a co-attentive multi-pointer to select few important reviews and combines multiple extracted user-item interactions. CMLE [28] integrates the word embedding model with a standard matrix factorization model to better utilize the local context information of reviews.

ACNN-FM [25] jointly utilizes a word-level attention mechanism and a phrase-level attention mechanism to extract feature information in historical texts. Those review-based methods can capture the aspect-level information to represent the historical preference of users. However, they are incapable of telling whether a user likes or dislikes an aspect of an item according to what viewpoint the user holds and to what extent. CARP [19] utilizes a capsule network architecture [34] and infer the sentiments of each aspect. Although the review-based methods can achieve relatively better performance compared with rating-based methods, they still face the main limitation that the users and items with rare ratings will be ignored because they are relatively few in the datasets. Therefore, it’s necessary to tackle this problem by proposing a novel method that can better models the users and items with different ratings.

2.3 Class Imbalance

In machine learning (ML)-related applications, the class imbalance problem has always been a focus issue and attracts the widespread attention of both industries and academia [35–37]. When the sample sizes of some classes are significantly different from others, it may cause the biased prediction problem, which will let the ML algorithms to obtain unsatisfactory results on the minority classes (i.e., classes with fewer samples). Two mainstream solutions for addressing the biased prediction problem mainly focus on data resampling heuristics and cost-sensitive learning. Data resampling heuristics are designed to achieve a balanced data distribution by randomly over-sampling or under-sampling the original imbalanced data [38–40]. However, sampling heuristics may not be the optimal strategy. For example, over-sampling suffers from overfitting while under-sampling may make it difficult for models to learn the comprehensive features of the majority classes.

Cost-sensitive learning aims at assigning different weights of loss to their classes [41]. However, these methods depend on professional knowledge heavily, and a lot of human effort is needed to obtain the best setting of all the hyper-parameters (i.e., the different weights).

The aforementioned methods have been applied in many fields such as the object detectors [42], the medical decision [43] and the E-mail classifier [44]. Some of the existing methods investigated the impact of rating characteristics like rating density, rating frequency distribution, and value distribution, on the accuracy of popular collaborative filtering techniques [45]. Due to the overwhelming percentage of some ratings (e.g., the high ratings), the rating-based and review-based models often suffer from biased prediction problem. Zhao et al. [46] proposed MLR to learn multiple latent representations for low rating and high rating respectively based on NeuMF [47]. However, MLR has two main limitations: 1) it suffers from the sparseness of the rating matrix; 2) without leveraging the features from the reviews, it’s incapable of achieving good results on different rating levels. Therefore, it cannot be considered as an ideal solution to the biased prediction problem. In this paper, we propose a transferable recommendation framework, which can be readily applied in the existing rating prediction methods for addressing their biased prediction problem. Our method is an end-to-end model that considers both features of historical ratings and reviews for improving the prediction performance.

3 PRELIMINARY

3.1 Problem Definition

The rating prediction task in this paper is to predict the ratings of different items given by different users with the historical ratings and the review texts. In general, the historical ratings of user \( u \) can be denoted as \( r_{u,i} \), which is a numerical rating that defines the user \( u \)’s satisfaction towards item \( i \). Moreover, all the reviews written by the user \( u \) can be represented as \( D_u = \{d_{u,1}, d_{u,2}, \ldots, d_{u,N}\} \), where \( d_{u,i} \) denotes user \( u \)’s review on item \( i \). The feature expression of item \( i \) is obtained similarly, denoted as \( D_i \). Therefore, each user-item interaction can be represented as a tuple \((u, i, r_{u,i}, d_{u,i})\). The primary objective of this work is to build an end-to-end model to estimate the rating \( \hat{r}_{u,i} \) for any user-item pair.

3.2 Extreme Value Distribution

The Extreme Value Distribution (EVD) [48] is utilized to model the probability for extreme events, and it was widely used in many disciplines (e.g., finance applications [49], earth science [50]). Inspired by the excellent performance of the EVD in modeling the minimum value, we introduce Minimum Gumbel distribution into our method to address the problem that traditional rating prediction models are difficult to predict the low rating data (e.g., 1-score, and 2-score) due to the imbalanced rating distribution in the training set.

The Gumbel Minimum distribution [10] is one of the most commonly utilized EVDs and it can effectively model the minimum
values in a series of data. Given a data $x = a$, $a \in R$, we can calculate its probability of being the minimum value in a series of data by using the Probability Density Function (PDF) of Gumbel Minimum distribution, this process is described as follows,

$$P(x) = \frac{1}{\beta} \exp \left\{ \frac{x - \alpha}{\beta} - \exp \left( \frac{x - \alpha}{\beta} \right) \right\}, \beta > 0$$

(1)

where $\alpha$ is the location parameter of the Gumbel distribution, indicating the value which has a maximum likelihood to be the minimum value of a series of data in Gumbel distribution. $\beta$ is the scale parameter for stretching out the PDF graph. Gumbel PDF is negative skew and the function graph is shown in Figure 2. $\alpha$ and $\beta$ in Eq. (1) can be calculated using the existing data samples. In details, given many series of data, denoted as $X = \{x_1, x_2, ..., x_n\}$, we can obtain a series of minimum values of each series, denoted as $x_{min} = \{x_{1min}, x_{2min}, ..., x_{nmin}\}$, where $x_{min}$ is the minimum value of the series data $x_i$. Then, $\alpha$ and $\beta$ can be obtained by:

$$\bar{x} - \frac{\sum_{i=1}^{n} x_i \exp \left( x_i / \beta \right)}{\sum_{i=1}^{n} \exp \left( x_i / \beta \right)} - \beta = 0$$

(2)

$$-\beta \log \left( \frac{1}{n} \sum_{i=1}^{n} \exp \left( x_i / \beta \right) \right) - \alpha = 0$$

(3)

4 METHOD

In this section, we will introduce our proposed framework GRP in detail. As shown in Figure 4, our approach contains three major components, which are: 1) the Gumbel-based score feature learning module to learn score features from both items and users, which is the main concern of this work; 2) the feature fusion layer that using the multi-scale convolutional fusion layer to combine score features and the interactive features between users and items as the final representation; 3) the data-driven rating prediction module to predict the rating scores based on the aforementioned feature representation and the original rating ratio feature.

4.1 Gumbel-based Score Feature Learning

As different users tend to have different rating schema towards items, e.g., some users tend to give some items a higher rating even though they are not satisfied with them, and some items may get a higher overall rating more easily than others, etc. These phenomena will increase the uncertainty of the rating prediction task and further influence the performance of the prediction models. Since each rating from the same user or item is closely related to their rating preference, we introduce the rating ratio from the rating history of each user and item to model score features. Specifically, given all the historical ratings of the user, denoted as $r_u = \{r_{u1}, r_{u2}, ..., r_{un}\}$, where $n$ is the number of ratings, we define the statistical rating ratio feature of user $u$ according to $r_u$, as follows:

$$q_u = \left\{ \frac{n_1}{n}, \frac{n_2}{n}, ..., \frac{n_c}{n} \right\}^T$$

(4)

where $\{n_1, n_2, ..., n_c\}$ is the number of each rating level in $r_u$, $c$ is the number of ratings (e.g., $c = 5$ denotes that there are 5 optional ratings for each item), $q_u \in R^{c \times 1}$ is the obtained rating ratio feature.

Similarly, we can obtain the rating ratio feature of the item, denoted as $q_i$. Next, we define the Gumbel distribution base on the rating ratio feature $q_u$ and $q_i$, respectively. We adopt the fully connected layer to obtain the key parameters of Gumbel distribution (i.e., the location parameter $\alpha$ and the scale parameter $\beta$). Take $q_u$ as an example, the aforementioned calculation process is as follows,

$$\alpha = W_1 q_u + b_1$$

(5)

$$\beta = \text{ReLU}(W_2 q_u + b_2) + \Delta \delta$$

(6)

where $W_1, W_2 \in R^{c \times 1}, a, b \in R^{c \times 1}$ are the trainable parameters, $b_1, b_2 \in R^{c \times 1}$ are the bias, ReLU denotes the Rectified Linear Unit [51], which is a non-linear activation function, $\Delta \delta$ is a threshold to avoid gradient explosion when $\beta$ is close to 0. We jointly use ReLU and $\Delta \delta$ to ensure that the value of $\beta$ is always greater than 0.

As a result, we project the ratio of each rating into Gumbel PDF parameters. According to Section 3.2, the traditional approach to calculate the key parameters $\alpha$ and $\beta$ of the Gumbel distribution is utilizing the minimum value of each input rating feature sequence. However, in the rating prediction task, there is hidden correlation information between different ratings from the same input sequence that needs to be extracted and learned. When using the minimum value to calculate $\alpha$ and $\beta$, we only obtain the feature information of the minimum rating without the statistic of other rating levels, which makes the model incapable of learning enough feature information for improving its prediction performance. Moreover, the ratios of different ratings from the same user or item affect each
other, and this dependency should be considered into the calculation process of \( \alpha \) and \( \beta \). Therefore, using the minimum value will lead to insufficient feature extraction results, and it’s necessary to utilize all the ratings of the input sequence to calculate \( \alpha \) and \( \beta \).

To obtain better feature extraction results than traditional approach, we propose a novel dynamic approach to calculate the key parameters \( \alpha \) and \( \beta \), which leverage the full input feature vectors of users and items, as shown in Eq. (5) and (6). After that, the users or items that with similar rating ratio features would have a similar Gumbel PDF, which better fit the real-world situation.

To better extract the feature information of the aforementioned feature vectors (i.e., the user score feature \( p_u \), the item score feature \( p_i \), and the user-item interaction feature \( p_{ui} \) that can represent the rating features of the users or items. Next, we consider how to represent the score features of users or items by \( c \) Gumbel PDF obtained by the previous step. For each Gumbel PDF, we sample a fixed score from the rating ratio feature and we take its probability as the corresponding value in the score feature. Thus, we can obtain \( c \) feature values from \( c \) Gumbel PDF, denoted as \( p_u, p_i \in \mathcal{R}^{c \times 1} \), which represents the score feature of a user and item, respectively. Take the rating ratio feature of user \( u \) as an example, its score feature \( p_u \) can be obtained by:

\[
p_u = \frac{1}{\beta_u} \exp \left( \frac{x_u - \alpha_u}{\beta_u} - \exp \left( \frac{x_u - \alpha_u}{\beta_u} \right) \right) \tag{7}
\]

where \( x_u \in \mathcal{R}^{c \times 1} \) denotes the sampling scores, \( \alpha_u, \beta_u \in \mathcal{R}^{c \times 1} \) are the obtained parameters of \( c \) Gumbel distributions.

In practice, we set sampling scores as \( \{1, 2, 3, 4, 5\} \) which are equal to the practical optional ratings in our experiments. Moreover, we find that the sampling score has little effect on the model’s performance, so it’s unnecessary to fine-tune the sampling scores in our method. As described in this section, the approach that using Gumbel distribution to represent the score feature of the users and items can be readily applied to any other rating prediction methods to obtain the useful features to improve their performance, especially for prediction tasks with extreme imbalanced datasets.

### 4.2 Feature Fusion Layer

We have obtained the score feature for each user and item based on Gumbel distribution, and as shown in Figure 4, the proposed GRP can incorporate the representations learned from other methods. To better extract the feature information of the aforementioned feature vectors (i.e., the user score feature \( p_u \), the item score feature \( p_i \), and the user-item interaction feature \( p_{ui} \) learned from existing models, denoted as \( s_{ui} \)), it’s necessary to fuse them effectively. Inspired by the excellent feature extraction results obtained by the multi-scale convolutional neural network (CNN) on many other NLP-related tasks [52–54], in this paper, we introduce a multi-scale CNN-based fusion layer into the rating prediction task to obtain the final user-item features. This method can not only extract the feature information from the single input vector but also effectively learn the hidden correlation across different input vectors.
Firstly, we project the feature \( s_{u,i} \in \mathbb{R}^{c_1 \times 1} \) into the same feature dimension as \( p_u \) and \( p_i \):

\[
t_{u,i} = W_3 s_{u,i} + b_3
\]

where \( W_3 \in \mathbb{R}^{c \times n} \), \( n \) is the dimension of the interaction feature, which depends on other review-based or rating-based models. Then, we stack \( p_u \), \( p_i \) and \( t_{u,i} \) into the same feature map, denoted as \( x_{u,i} \in \mathbb{R}^{c \times 3} \). This operation is shown as follows,

\[
Z_{u,i} = \text{stack} \left[ p_u, p_i, t_{u,i} \right]
\]

Our multi-scale convolutional fusion mechanism consists of three types of convolution filters, which are: \( f_1 \in \mathbb{R}^{c \times 1} \), \( f_2 \in \mathbb{R}^{c \times 2} \), and \( f_3 \in \mathbb{R}^{c \times 3} \). \( f_1 \) can capture the underlying information for each feature vector in \( Z_{u,i} \); \( f_2 \) captures the correlation information between \( p_u \) and \( t_{u,i} \); \( f_3 \) is utilized to learn the overall interaction across all the three feature vectors in \( Z_{u,i} \). Then, the max-pooling operation is applied after each convolutional layer, and in this way, we can obtain the comprehensive features from both the user and the item, denoted as \( h_{u,i} \in \mathbb{R}^{K \times 1} \), where \( K \) is the number of all filters. Finally, a fully connected layer is stacked to transform the feature dimension of \( h_{u,i} \) as follows:

\[
h_{u,i} = W_4 h_{u,i} + b_4
\]

where \( W_4 \in \mathbb{R}^{c \times K} \) is the weight matrix, \( b_4 \in \mathbb{R}^{c \times 1} \) is the bias. the obtained \( m_{u,i} \in \mathbb{R}^{c \times 1} \) is the final feature vector for the given user-item pair, which incorporate the score feature, the interactive feature, and their hidden correlation.

### 4.3 Data-driven Rating Prediction

Since we have obtained the final feature vector \( h_{u,i} \) for the user-item pair, and considering that each user has their unique rating preference, we design a personalized data-driven method (i.e., integrating the features contained in the rating history information of users and items) to calculate the prediction results.

Firstly, we obtain the weight for each optional rating based on the historical rating ratio feature of users and items (i.e., \( q_u \) and \( q_i \) in Section 4.1) via a fully connected layer:

\[
\begin{pmatrix}
\hat{o}_u \\
\hat{o}_i
\end{pmatrix} = W \begin{pmatrix}
q_u \\
q_i
\end{pmatrix} + b
\]

where \( \hat{o}_u \) and \( \hat{o}_i \) are the weights for optimal ratings of users and items, respectively, \( W \) is the weight matrix and \( b \) is the bias. Considering the different contributions of historical information of users and items, the final weights \( o \) can be obtained as:

\[
o = \xi \hat{o}_u + \phi \hat{o}_i
\]

where hyper-parameters \( \xi \) and \( \phi \) are leveraged to quantify the different contributions. Afterward, the final rating that user would score towards the item is denoted as the weighted summation:

\[
\hat{r}_{ui} = \sum_{\ell=1}^{c} q_{\ell} h_{u,i}
\]

Since the rating prediction task is a regression problem, we utilize the square loss function to train our model as follows:

\[
L_{sqr} = \sum_{u,i \in \Omega} (\hat{r}_{u,i} - r_{u,i})^2
\]

where \( \Omega \) denotes the set of instances (i.e., users and items) for training, \( r_{u,i} \) is the ground truth rating assigned by the user \( u \) to the item \( i \), \( L_{sqr} \) is the obtained loss value.

### 5 EXPERIMENTS

In this section, we present the datasets, comparison baseline methods, experimental setup, the evaluation metric, and the overall performance of the proposed GRP.

#### 5.1 Experimental Setting

**Datasets.** We use eight benchmark datasets selected from Amazon Product Reviews to test and verify the overall prediction performance of our proposed method GRP. We explore the biased prediction problem with respect to the different rare rating (i.e., rating of 1, 2, and 3) ratio of the different datasets. The detailed ratio information of each rating level is listed in Table 2.

All the datasets are preprocessed in a 5-core fashion to ensure that each user and item has at least 5 reviews. We randomly split each dataset into training and testing sets using the ratio 80:20. Note that, all reviews belonging to interactions are not included in both the train and test stages of our experiments to prevent the rating prediction problem from a sentiment analysis task.

**Baselines.** We select recent competitive methods as comparison baselines, which can be concluded into two categories:

1. **Rating-based methods:**
   - **PMF** [4]: PMF is a standard matrix factorization to model users and items from rating matrix.
   - **NeuMF** [13]: NeuMF characterizes a interactive feature of user-item pairs by concatenating the results learned from both FM and MLP.

2. **Review-based methods:**
   - **DeepCoNN** [29]: DeepCoNN obtains the convolutional representations of user’s and item’s reviews and passes the concatenated embedding results into a FM model.
   - **NRPA** [23]: NRPA uses a hierarchical personalized attention to learn personalized representation for users and items.
   - **NARRE** [18]: NARRE utilizes two unparalleled neural networks with attention mechanism to assign weights to reviews, and automatically select useful reviews.
   - **ANR** [33]: ANR is a neural recommendation system which employs attention mechanism to capture the fine-grained interactions between users and items at an aspect-level.

| Score: Rating Level | 1   | 2   | 3   | 4   | 5   |
|---------------------|-----|-----|-----|-----|-----|
| Musical Instrument  | 1.97| 2.31| 7.29| 20.37| 68.06|
| Automotive          | 2.65| 2.95| 6.61| 18.48| 69.31|
| Office Product      | 2.26| 3.15| 8.94| 26.85| 58.80|
| Tool Home           | 3.90| 3.69| 7.87| 20.60| 63.94|
| Toys Game           | 2.91| 3.80| 9.47| 21.66| 62.16|
| Grocery Gourmet Food| 3.96| 5.06| 10.62| 20.08| 60.28|
| Ratio               | 3.97| 4.98| 12.28| 24.08| 54.69|

Table 2: The ratio of each rating level (%).
Table 3: Performance comparison (MAE) on 8 benchmark datasets. The best results of all eight datasets are in bold. $\Delta$% denotes the percentage of relative improvement compared with the origin models. 1$^*$ represents the difference between the performance of the original model on the Musical Instrument and its performance on other datasets. 2$^*$ represents the difference between the performance of the proposed model on the Musical Instrument and its performance on other datasets. Note that the aforementioned two differences can be denoted as the relative differences.

| Method   | Musical Instrument | Automotive | Office Product | Tool Home | Toys Game | Digital Music | Grocery | Gourmet Food | Patio | Average |
|----------|--------------------|------------|----------------|-----------|-----------|--------------|---------|--------------|-------|---------|
| Rating-based |                   |            |                |           |           |               |        |              |       |         |
| PMF      | 0.744              | 0.904      | 0.749          | 0.885     | 0.788     | 0.895        | 0.879   | 0.953        | 0.850 |
| PMF(G)   | **0.639**          | **0.608**  | **0.584**      | **0.651** | **0.566** | **0.662**    | **0.678** | **0.741**    | **0.641** |
| $\Delta$% | 14.11              | 32.74      | 22.03          | 28.17     | 26.03     | 22.87        | 22.25   | 24.33        |
| 1$^*$    | -                  | -0.160     | -0.005         | -0.141    | -0.044    | -0.151       | -0.135  | -0.209       | -0.106 |
| 2$^*$    | -                  | 0.031      | 0.055          | 0.073     | 0.023     | -0.039       | -0.102  | -0.002       |
| NeuMF    | 0.655              | 0.705      | 0.669          | 0.818     | 0.723     | 0.778        | 0.793   | 0.798        | 0.742 |
| NeuMF(G) | **0.652**          | **0.631**  | **0.622**      | **0.672** | **0.552** | **0.689**    | **0.750** | **0.778**    | **0.668** |
| $\Delta$% | 0.46               | 10.50      | 7.03           | 23.65     | 11.44     | 5.42         | 2.51    | 9.86         |
| 1$^*$    | -                  | -0.050     | -0.014         | -0.163    | -0.068    | -0.123       | -0.138  | -0.143       | -0.087 |
| 2$^*$    | -                  | 0.021      | 0.03           | 0.02       | 0.037     | 0.098        | -0.126  | -0.087       |
| Review-based |                |            |                |           |           |               |        |              |       |         |
| DeepCoNN | **0.636**          | 0.664      | 0.662          | 0.745     | 0.664     | 0.769        | 0.768   | 0.766        | 0.709 |
| DeepCoNN(G) |            | **0.606**  | **0.600**      | **0.584** | **0.538** | **0.667**    | **0.665** | **0.732**    | **0.629** |
| $\Delta$% | -0.31             | 8.73       | 9.37           | 21.61     | 18.98     | 13.26        | 13.28   | 4.40         | 11.17 |
| 1$^*$    | -                  | -0.028     | -0.026         | -0.109    | -0.028    | -0.133       | -0.132  | -0.130       | -0.073 |
| 2$^*$    | -                  | 0.032      | 0.038          | 0.054     | 0.100     | -0.029       | -0.027  | -0.094       | 0.009 |
| NRPA     | 0.684              | 0.685      | 0.690          | 0.758     | 0.692     | 0.806        | 0.821   | 0.818        | 0.744 |
| NRPA(G)  | **0.616**          | **0.597**  | **0.609**      | **0.602** | **0.570** | **0.664**    | **0.708** | **0.728**    | **0.637** |
| $\Delta$% | 11.04              | 14.74      | 13.30          | 25.91     | 21.40     | 21.39        | 15.96   | 12.36        | 17.01 |
| 1$^*$    | -                  | -0.001     | -0.006         | -0.074    | -0.008    | -0.122       | -0.137  | -0.134       | -0.060 |
| 2$^*$    | -                  | 0.019      | 0.007          | 0.014     | 0.046     | -0.048       | -0.092  | -0.112       | -0.021 |
| NARRE    | 0.771              | 0.782      | 0.705          | 0.804     | 0.710     | 0.802        | 0.787   | 0.851        | 0.781 |
| NARRE(G) | **0.665**          | **0.516**  | **0.592**      | **0.601** | **0.534** | **0.669**    | **0.669** | **0.780**    | **0.628** |
| $\Delta$% | 13.75              | 34.02      | 16.03          | 25.25     | 24.79     | 16.58        | 14.99   | 8.34         | 19.22 |
| 1$^*$    | -                  | -0.011     | 0.066          | -0.033    | 0.061     | -0.031       | -0.016  | -0.080       | -0.005 |
| 2$^*$    | -                  | 0.149      | 0.073          | 0.064     | 0.131     | -0.004       | -0.004  | -0.115       | 0.037 |
| ANR      | 0.623              | 0.645      | 0.660          | 0.732     | 0.657     | 0.766        | 0.780   | 0.771        | 0.704 |
| ANR(G)   | **0.611**          | **0.608**  | **0.604**      | **0.604** | **0.510** | **0.675**    | **0.754** | **0.716**    | **0.635** |
| $\Delta$% | 1.93               | 5.74       | 8.48           | 17.49     | 22.37     | 11.88        | 3.33    | 7.13         | 9.79  |
| 1$^*$    | -                  | -0.022     | -0.037         | -0.109    | -0.034    | -0.143       | -0.157  | -0.148       | -0.093 |
| 2$^*$    | -                  | 0.003      | 0.007          | 0.007     | 0.101     | -0.064       | -0.143  | -0.105       | -0.028 |

It can be observed that the comparison baselines of our experiments include several commonly-utilized rating prediction methods. In our framework GRP, these baselines are used to extract the interactive feature of the input data (i.e., the gray rectangle in Figure 4).

Experimental Setup. We set all the parameters in the baseline methods the same as mentioned in their original papers. The dropout rate and learning rate are fine-tuned for all baselines to achieve the best performance. All the models utilize 300-D word embedding trained on Wikipedia using GloVe \(^2\). The gradient-based optimizer Adam [56] is utilized to update the trainable parameters. We set GRP with a learning rate among \([0.005, 0.007, 0.09]\). In practice, we found that using the dropout method [57] may lead to extremely unstable training process, thus the dropout rate is set to 0. Moreover, the learning rate is decayed by 60% every epoch, and the batch size is set to 32. We train all the models for a maximum of 10 epochs and report the best result on the corresponding test set. For

\(^2\)https://nlp.stanford.edu/projects/glove/
the preprocessing part for reviews, we keep the maximum length of a single review text and the maximum number of history reviews to cover 85% of the total review text length and the total number of history reviews, respectively. While the final weights can include the rating statistics of both users and items as $o = \xi o_u + \phi o_i$, we found that the historical ratings of items do not improve performance and the weights of users’ statistics have little effect on all eight datasets, so we set $\xi$ to 1 and $\phi$ to 0.

Evaluation Metric. In our comparison experiments, including the overall performance of all the methods, the ablation study, the prediction performance on rare data, and the further analysis, we use the Mean Absolute Error (MAE) as the evaluation metric:

$$M = \frac{1}{m \sum_{u,i \in \Omega}} \left| r_{u,i} - \hat{r}_{u,i} \right|$$  \hspace{1cm} (15)$$

where $m$ denotes the number of the user-item pairs in the instance set $\Omega$, $M$ denotes the performance of the model on $\Omega$. In this paper, we use MAE (also used in related work [58]) to evaluate the predicting results of the rare data, the reason is that the MAE is more robust and won’t be easily influenced by a few exceptional results.

5.2 Performance Evaluation

In this section, we report the overall performance of the original baselines and the baselines that using along with GRP, e.g., NARRE(G) denotes a model that using NARRE to extract the interactive feature as shown in Figure 4.

As shown in Table 3, the overall prediction performance of our method significantly outperforms all the original rating-based and review-based methods, and it achieves a substantial improvement of about 0.46% to 34.02% on all eight datasets, which proves the extensive effectiveness of GRP. The only experiment that achieves slightly better performance than our method is using DeepCoNN on the Musical Instrument dataset. However, DeepCoNN(G) significantly improves the original prediction performance on all other seven datasets, which illustrates the excellent overall performance of GRP. Moreover, the most significant improvement is on PMF, up to 24.33% on average. The reason is that PMF is composed of matrix factorization and a global bias, which is a relatively simple architecture that may cause the underfitting problem and further affect the prediction performance.

The improvements obtained by GRP is related to the ratio of different datasets. It’s worthy to note that the ratio of rare ratings (i.e., ratings of 1, 2, and 3) of the eight datasets in Table 3 keeps increasing from left to right. With the increasing ratio of rare ratings, the prediction performance of all the models in on the decline, which is because that the models need to predict more samples with rare ratings, and the increased number of samples is too small to provide adequate feature learning capability for the models. The relative differences in Table 3 (i.e., the 1* and 2* rows) show the influence of different rating ratio on the prediction performance of different models. Ideally, the average relative differences should approximate to zero for superior overall prediction performance and biased prediction results. According to all the relative differences, with the increasing ratio of rare ratings, our methods can achieve more stable prediction results. Especially for the relative differences obtained by PMF and DeepCoNN, our method can achieve an average relative difference of -0.002 and 0.009, respectively, which shows that our model can significantly improve these two models on datasets with different ratio of rare ratings.

6 MODEL ANALYSIS

In this section, we visualize the performance of GRP over rare ratings to highlight the main concern of this work. Note that the aforementioned experiments are conducted using four out of eight datasets in Section 5.2, i.e., Musical Instrument, Office Product, Toys Game, and Grocery Gourmet Food, where their respective rare rating ratio increases from 11.57% to 19.64% from left to right.

6.1 Prediction for Rare Rating

To verify the effectiveness of GRP in addressing the biased prediction problem, we conduct the comparison experiments between the original baselines and the baselines with GRP (i.e. the (G) method).

On the other hand, we argue that the mean prediction value obtained by the models should approximate to the ground-truth rating for superior prediction accuracy. Thus, the average prediction made by models should be close to the corresponding rating levels to ensure a reasonable prediction. As shown in Figure 5, the color lines indicate the mean prediction result of the corresponding methods on different ratings. We can observe that the baselines with GRP can achieve better mean prediction result over the original ones, especially on the rare ratings. Moreover, the inclination of different lines reflects the ability of models to distinguish between different rating levels. Ideally, a straight line with an angle of 45 degrees from the horizontal represents the optimal prediction results. However, the original baselines tend to have a relatively flatter line than the Gumbel-based methods with a lower variance, which is due to their insufficient ability to extract features of rare ratings, so the rare ratings are ignored to some extent. When using the Gumbel-based method to model the imbalanced data, we can alleviate the biased prediction results by changing the data distribution to gain robustness. As a result, the comparison of both the prediction error and mean prediction result confirms the effectiveness of the GRP in addressing the biased prediction problem in rating prediction tasks.

6.2 Ablation Study of GRP

We conduct ablation studies to quantify the influence of three main components (i.e., the Gumbel-based feature learning module, the fusion layer, and the data-driven rating prediction module) in the GRP on four datasets. The comparison models are defined as follows: 1) -G: This model removes the Gumbel-based feature learning module, which means the original rating ratio feature is directly fed into the following fusion layer as the score features; 2) -F: A model with the infusion layer. We use the average of user score feature, item score feature, and interactive feature as the final representation, instead of using the original feature fusion layer (i.e., multi-scale convolutional neural network) to summarize the aforementioned features; 3) -D: A model that removes the data-driven prediction module, and the prediction result is obtained by using $\hat{r}_{u,i} = \sum_{l=1}^{L} h_{u,i}$ to sum the score representation calculated by Eq. (10). The comparison results in Table 4 show that the methods with GRP outperforms most of the ablation models. The performance of
Figure 5: Prediction error (MAE) and the mean prediction result for each rating on four datasets. The histogram denotes the prediction error and the line graph represents the mean prediction result. The numbers on horizontal axis represent different ratings (e.g., 1 represents the rating level is 1)
models that removing the Gumbel-based feature learning module drops significantly compared with all other ablation models on all four datasets, which shows the effectiveness of the proposed Gumbel-based module. Moreover, comparing with all the original models without applying any components of GRP, the -G models marginally improve their overall performance, which demonstrates the necessity of utilizing the proposed multi-scale feature fusion layer and the data-driven rating prediction module.

Several -D and -F methods can obtain slightly better performance on some datasets than those with complete GRP architecture (i.e., (G) methods). For example, NARRE has modest performance improvement after removing the data-driven rating prediction module on the Musical Instrument dataset; PMF achieves the best performance on the Toys Game dataset without utilizing the feature fusion layer; Using the fusion layer and the data-driven prediction module, ANR can obtain slight better performance on the Grocery Gourmet Food dataset. However, in most cases, the best results can be obtained by using the complete GRP architecture, which further demonstrates the necessity of jointly using the proposed three components for rating prediction. We conclude that the Gumbel-based module plays a major role in improving the overall performance of the rating prediction models. Moreover, the multi-scale feature fusion layer and the data-driven rating prediction module can be regarded as the auxiliary components which further improves the performance of the proposed model. While the aforementioned auxiliary components are developed for alleviating the biased prediction problem, it can be considered to readily apply to other rating prediction tasks for improving their overall performance.

| Method | Musical Instrument | Office Product | Toys Game | Grocery Food |
|--------|--------------------|----------------|-----------|--------------|
| PMF(G) | 0.639              | 0.584          | 0.566     | 0.678        |
| -G     | 0.694              | 0.703          | 0.630     | 0.744        |
| -F     | 0.790              | 0.631          | 0.554     | 0.754        |
| -D     | 0.640              | 0.581          | 0.594     | 0.709        |
| DeepCoNN(G) | 0.636           | 0.662          | 0.664     | 0.768        |
| -G     | 0.711              | 0.644          | 0.576     | 0.719        |
| -F     | 0.677              | 0.637          | 0.539     | 0.737        |
| -D     | 0.637              | 0.607          | 0.557     | 0.676        |
| NRPA(G) | 0.684             | 0.690          | 0.692     | 0.821        |
| -G     | 0.660              | 0.701          | 0.605     | 0.779        |
| -F     | 0.766              | 0.664          | 0.648     | 0.771        |
| -D     | 0.679              | 0.580          | 0.595     | 0.732        |
| NARRE(G) | 0.806             | 0.705          | 0.710     | 0.787        |
| -G     | 0.665              | 0.592          | 0.534     | 0.669        |
| -F     | 0.790              | 0.680          | 0.586     | 0.726        |
| -D     | 0.677              | 0.738          | 0.572     | 0.726        |
| ANR(G) | 0.623              | 0.660          | 0.657     | 0.780        |
| -G     | 0.733              | 0.676          | 0.563     | 0.706        |
| -F     | 0.649              | 0.650          | 0.550     | 0.711        |
| -D     | 0.638              | 0.627          | 0.545     | 0.723        |

| Method | Musical Instrument | Office Product | Toys Game | Grocery Food |
|--------|--------------------|----------------|-----------|--------------|
| PMF(G) | 0.639              | 0.584          | 0.566     | 0.678        |
| -G     | 0.692              | 0.700          | 0.594     | 0.748        |
| -F     | 0.683              | 0.618          | 0.560     | 0.684        |
| -D     | 0.740              | 0.684          | 0.587     | 0.753        |
| PMF(W) | 0.696              | 0.590          | 0.578     | 0.697        |
| PMF(F) | 0.686              | 0.607          | 0.551     | 0.703        |
| NeuMF(G) | 0.655             | 0.669          | 0.723     | 0.793        |
| -G     | 0.679              | 0.735          | 0.704     | 0.778        |
| -F     | 0.735              | 0.722          | 0.719     | 0.828        |
| -D     | 0.715              | 0.722          | 0.713     | 0.834        |
| NeuMF(F) | 0.744              | 0.740          | 0.735     | 0.828        |
| DeepConn(N) | 0.628           | 0.648          | 0.562     | 0.749        |
| DeepConn(W) | 0.702             | 0.650          | 0.564     | 0.688        |
| DeepConn(E) | 0.660             | 0.624          | 0.547     | 0.694        |
| DeepConn(F) | 0.653             | 0.618          | 0.552     | 0.712        |
| NRPA(G) | 0.684              | 0.690          | 0.692     | 0.821        |
| -G     | 0.691              | 0.675          | 0.646     | 0.781        |
| -F     | 0.673              | 0.713          | 0.638     | 0.744        |
| -D     | 0.677              | 0.668          | 0.547     | 0.699        |
| NARRE(G) | 0.806             | 0.705          | 0.710     | 0.787        |
| -G     | 0.665              | 0.592          | 0.534     | 0.669        |
| -F     | 0.666              | 0.689          | 0.600     | 0.710        |
| -D     | 0.678              | 0.675          | 0.548     | 0.680        |
| ANR(G) | 0.623              | 0.660          | 0.657     | 0.780        |
| -G     | 0.611              | 0.604          | 0.510     | 0.754        |
| -F     | 0.629              | 0.655          | 0.553     | 0.698        |
| -D     | 0.631              | 0.600          | 0.523     | 0.665        |
| ANR(E) | 0.636              | 0.653          | 0.626     | 0.742        |
| ANR(W) | 0.628              | 0.572          | 0.541     | 0.678        |
| ANR(F) | 0.735              | 0.624          | 0.517     | 0.704        |
6.3 Comparison of Different Probability Distributions

In this section, we present the reasons for using Gumbel distribution instead of other probability distributions. Combining Table 5 and the “-G” rows of Table 4, we first conclude that it’s an effective approach to use the probability distributions to model the distribution of rare ratings for addressing the biased prediction problem.

The other two commonly used EVDs (i.e., Weibull and Fréchet distribution) both have a shape parameter (a kind of numerical parameter of a parametric family of probability distributions [55]), thus we need an extra set of trainable parameters to obtain this parameter, which will increase the uncertainty of the training process and deteriorate the performance to some extents. According to our experiments, using the probability distribution with shape parameter may make the model difficult to converge due to the instability of the gradient change. Moreover, as shown in Table 5, the performance drop with models using the other two EVDs in most cases. Therefore, the Gumbel distribution can be considered as the optimal EVD for modeling the imbalanced data.

Next, we consider using other commonly utilized probability distributions that don’t have the shape parameter, which are the Poisson distribution (P), the Normal distribution (N), and the Exponential distribution (E). As shown in Table 5, the models with Gumbel distribution significantly outperforms other models in most experiments, which demonstrate the effectiveness of using Gumbel distribution to model the distribution of rare ratings. It can be observed that the only comparable method is to use the Normal distribution. However, the shape of the Normal distribution is symmetric, which ignores the bipolarity of the data distribution and significantly change the original distribution. According to our observation, the Gumbel distribution often performs best improvements, we infer the reason behind is that Gumbel distribution has a negative skewness, which is more similar to the actual distribution of the imbalanced training datasets.

6.4 Comparison of Methods for Modeling Key Parameters in Gumbel Distribution

As demonstrated in Section 4.1, we propose a novel dynamic approach for modeling the parameters (i.e., location parameter $\alpha$ and scale parameter $\beta$) in Gumbel distribution. To quantify its effectiveness, we compare the Gumbel distribution using the proposed approach and the traditional approach (i.e., model with minimum values) on four datasets. Note that the (G) and (M) models in Table 6 are all using GRP but with different approaches to model the parameters of Gumbel distribution. As shown in Table 6, we can observe that the dynamic approach significantly outperforms the traditional approach in most cases. Using only the minimum values of the input data, the traditional approach cannot sufficiently extract feature information to make the probability distribution continuously fit the samples along the training process. For the Musical Instrument dataset, PMF and NARRE achieve relatively higher performance using the traditional approach, which is because the sample size of this dataset is too small that our approach is incapable of learning enough feature information. For the rest three larger datasets, our approach benefits from the massive training samples that result in modeling the parameters with relatively better robustness. Since the neural networks are usually trained batch-wise, there is a certain change in the data distribution in each batch. When modeling the parameters with the dynamic approach, we can obtain different Gumbel distributions that can better represent the data distribution in their corresponding batches, which enables our model to better fit the data. While this method is developed for modeling the parameters of the Gumbel distribution, it can be extended to other tasks that using probability distributions to model the input data.

7 CONCLUSION

In this paper, we propose a novel Gumbel-based Rating Prediction (GRP) framework to address the biased prediction problem. Our method is inspired by the ability of Gumbel distribution in modeling the rare data. We define different Gumbel distributions for each rating, and this distribution can estimate the score features of users or items based on historical rating statistics. The parameters of the Gumbel distributions are learned from the historical rating statistic of users and items, and keep updated during the training process. Moreover, we propose a multi-scale convolutional fusion layer to summarize and extract feature information from the obtained feature vectors. Then, a data-driven rating prediction is applied to make better personalized recommendations based on the highly-compressed feature vectors and the original rating ratio features. Extensive experiments on eight benchmark datasets show that the overall performance of GRP significantly outperforms all compared baselines. As a result, GRP can achieve relatively higher improvement on rare ratings while ensuring the optimal performance on all ratings, which demonstrates the effectiveness of using GRP to address the biased prediction problems.

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