Refractive effects in the scattering of loosely bound nuclei

Florin Carstoiu\textsuperscript{1,2,3}, Livius Trache\textsuperscript{1}, Robert E. Tribble\textsuperscript{1}, Carl A. Gagliardi\textsuperscript{1}

\textsuperscript{1} Cyclotron Institute, Texas A\&M University, College Station, TX 77843-3366, USA
\textsuperscript{2} Laboratoire de Physique Corpusculaire, IN2P3-CNRS, ISMRA et Université de Caen, F-14050 Caen cedex, France
\textsuperscript{3} National Institute for Physics and Nuclear Engineering “Horia Hulubei’, P.O. Box MG-6, 76900 Bucharest-Magurele, Romania

(Dated: October 30, 2018)

A study of the interaction of loosely bound nuclei \( ^{6,7}\)Li at 9 and 19 A MeV with light targets has been undertaken. With the determination of unambiguous optical potentials in mind, elastic data for four projectile-target combinations and one neutron transfer reaction \( ^{12}\)C\( ^{7,8}\)Li\( ^{12}\)C have been measured on a large angular range. The kinematical regime encompasses a region where the mean field (optical potential) has a marked variation with mass and energy, but turns out to be sufficiently surface transparent to allow strong refractive effects to be manifested in elastic scattering data at intermediate angles. The identified exotic feature, a “plateau” in the angular distributions at intermediate angles, is fully confirmed in four reaction channels and interpreted as a pre-rainbow oscillation resulting from the interference of the barrier and internal barrier far-side scattering subamplitudes.

PACS numbers: PACS number(s): 25.70.Bc, 24.10.Ht, 25.70.Hi, 27.20.+n

\section{I. INTRODUCTION}

The study of nucleus-nucleus elastic scattering has a long history and remains of interest due to both successes and failures that mark it (see for example, Refs. \textsuperscript{1,2} and references therein). It is an important subject per se, and is also important as a tool for the description of a series of phenomena that involve the distorted waves given by optical model potentials (OMP). We are searching here for reliable ways to predict optical model potentials for reactions with radioactive nuclear beams (RNB). In particular our interest focuses on finding reliable descriptions for transfer reactions involving relatively light, loosely bound nuclei, which are used in indirect methods in nuclear astrophysics. A range of RNB studies were made at energies around 10 MeV/nucleon, where the reactions are peripheral, with the intent to obtain information about the surface of the nuclei involved. These reactions use DWBA techniques to extract nuclear structure information. However, the well known existence of many ambiguities in the OMPs extracted from elastic scattering can raise questions about the accuracy of these determinations. Therefore, we are searching for ways to reduce these ambiguities and to predict OMP for reactions with RNBs. Experimental studies using RNBs have, heretofore, not been suitable for detailed elastic scattering analyses. The closest we can get using stable beams is by studying the elastic scattering of loosely bound nuclei. We chose here to study the elastic scattering of \( ^{6,7}\)Li projectiles, because they are fragile (loosely bound), with a pronounced cluster structure and with low Z and can, therefore, exhibit a range of phenomena, involving absorption, diffraction and refraction, mostly of nuclear nature.

Earlier we have carried out a study of elastic scattering around 10A MeV for a range of projectile-target combinations involving p-shell nuclei \( ^{\text{\textsuperscript{\textit{3}}}7}\)Li. We found a relatively simple method to predict OMP for loosely bound nuclei, based on the renormalization of the independent real and imaginary terms obtained from a double folding procedure using the JLM nucleon-nucleon (NN) effective interaction. The procedure successfully described the data for all the projectile-target combinations and the energies in the study for most of the angular ranges measured. In one single case (\( ^{7}\)Li at 99 and 130 MeV on \( ^{22}\)Ne) the folding potentials failed to describe well the large angle data. Later, the results were used to describe elastic scattering angular distributions measured in a series of experiments with RNBs at or around 10 MeV/nucleon: \( ^{7}\)Be on \( ^{10}\)B and melamine targets \( ^{7}\)Be, \( ^{11}\)C \( ^{7}\)Be, \( ^{13}\)N \( ^{7}\)Be and \( ^{17}\)F \( ^{7}\)Be on \( ^{12}\)C and \( ^{14}\)N targets. We return to that study here with new data extending the angular ranges for the \( ^{7}\)Li scattering and adding data for \( ^{6}\)Li scattering and with a refined analysis.

Recent work \( ^{5}\) \textsuperscript{5} has established that elastic scattering of light tightly bound heavy ion systems such as \( ^{16}\text{O}+^{12}\text{C} \) and \( ^{16}\text{O}+^{16}\text{O} \) show sufficient transparency for the cross section to be dominated by the far-side scattering. Intermediate angle structures appearing in the elastic scattering distributions at angles beyond the Fraunhofer diffractive region have been identified as Airy minima of a nuclear rainbow, i.e. a destructive interference between two far-side trajectories which sample the interior of the potential. A number of high order Airy minima have been identified by observing that such structures are largely insensitive to an artificial reduction of the absorption in the optical potential, and therefore they appear as a manifestation of the refractive power of the nuclear potential. While at high energy \( ^{10}\) \textsuperscript{10} this picture was well substantiated by a semiclassical nonuniform decomposition of the scattering function \( ^{11}\) \textsuperscript{11}, at lower energies the situation is more difficult to understand. It has
been shown by Anni [12], that such structures could be explained by the interference of two amplitudes appearing in different terms of a multireflection uniform series expansion of the scattering amplitude and therefore the interpretation using rainbow terminology is not appropriate.

For loosely bound nuclei the situation is even more uncertain. When a nucleon or a group of nucleons has small separation energy, the wave function penetrates well beyond the potential range. The corresponding components in the optical potential are expected to be more diffuse as compared to normal nuclei, leading to a competition between the increased refractive power of the real potential and the increased absorption at the nuclear surface. The small separation energy implies also that the dynamic polarization potential (DPP) arising from the coupling to breakup states may be strong and with a complicated energy and radial dependence. It follows that for loosely bound nuclei the DPP cannot be treated as a small perturbation and the usual phenomenological procedure in renormalizing the folding potential form factor for loosely bound nuclei the DPP cannot be treated as a small perturbation and the usual phenomenological procedure in renormalizing the folding potential form factor may be questioned. It has been estimated that the DPP is strongly repulsive at the nuclear surface in the case of $^6\text{Li}$ [14] and this prompted Mahaux, Ngo and Sat切尔 [15] to conjecture that for loosely bound nuclei the barrier anomaly may be absent due to the cancellation between the repulsive (DPP) and attractive (dispersive) components of the optical potential.

In the specific case of $^6,7\text{Li}$ scattering on light targets a large body of data have been accumulated in the range 5-50 MeV/nucleon. At high energy, Nadasen and his group [10,15] have been able to derive a unique optical potential which was essential to assess the quality of the folding model. At lower energies, ambiguities found in the analysis of data prevented any definite conclusion about the strength and energy dependence of the optical potential. A study by Trcka et al. [18] on $^6\text{Li}+^{12,13}\text{C}$ elastic scattering at 50 MeV, found an exotic feature (“plateau”) in the angular distribution of the elastic scattering at intermediate angles which resembles similar structures found in more bound systems. They interpreted the structure as a diffractive effect arising from an angular momentum dependent absorption. There are experimental hints that such structures also appear in neighboring systems, $^6\text{Li}+^{16}\text{O}$ and $^6\text{Li}+^{9}\text{Be}$, as a possible manifestation of the average properties of the interaction potential.

In this paper we present a precision measurement of elastic scattering of $^6,7\text{Li}$ on $^{12,13}\text{C}$ and $^9\text{Be}$ targets at 9 and 19 MeV/nucleon. The lower energy was chosen in view of our systematic studies of nuclear reactions for astrophysics. The higher energy is close to the saturation energy for these projectiles, i.e. the energy where almost all reaction channels are open. The ”plateau” feature is confirmed in four projectile-target combinations at 9 MeV/nucleon. The high selectivity induced by this structure allowed the derivation of an almost unique Woods-Saxon optical potential. A folding model analysis using the complex, density and energy dependent NN interaction of Jeukenne, Lejeune and Mahaux (JLM) [19], where corrections due to the strong DPP have been included, confirmed that our elastic distributions could be described using deep and extremely transparent potentials. The remaining ambiguities have been eliminated using an accurate dispersion relation analysis. The intermediate angle structures have been discussed using the semiclassical uniform approximation for the scattering function of Brink and Takigawa [20]. We explain the intermediate angle structure as a coherent interference effect of two subamplitudes corresponding to trajectories reflected at the barrier and interfering with trajectories which sample the nuclear interior. Thus, this refractive effect appears as a signature of a highly transparent interaction potential.

The paper is structured in the following way: after this introduction, the experimental methods are discussed in Sect. III, the analysis of the elastic scattering data using phenomenological and microscopic optical model potentials is discussed in Sect. III and the implications of this analysis for the transfer reaction ($^7\text{Li},^3\text{He}$) is discussed in Sect. IV. In Sect. V the dispersion relation is used to put additional constrains on the potentials extracted, followed by a discussion of the decomposition of the far-side scattering amplitude into barrier and internal barrier components responsible for the ”plateau” structure at intermediate angles (Sect. VI), and the conclusions (Sect. VII).

II. THE EXPERIMENTS

The experiments were performed using $^6\text{Li}$ and $^7\text{Li}$ beams of 9 and 19 MeV/nucleon from the Texas A&M University K500 superconducting cyclotron and the Multipole Dipole Multipole (MDM) magnetic spectrometer [21]. A list of the measurements is given in Table I. The measurements with $^7\text{Li}$ were done to extend the angular range that was effectively covered in earlier work. The experimental setup and the data reduction procedures were similar to those used in Ref. [3]. The beams were prepared using the beam analysis system [22], which allows for the control of the energy spread ($\Delta E/E$ up to 1/2500) and angular spread (0.1°) of the beam. Self-supported $^9\text{Be}$ (200 µg/cm² thick), $^{12}\text{C}$ (200 µg/cm²) and $^{13}\text{C}$ (390 µg/cm²) targets were placed perpendicular to the beam in the target chamber of the MDM. The magnetic field of the MDM spectrometer was set to transport the fully stripped Li ions to the focal plane where they were observed in the modified Oxford detector [23]. In the detector, the position of the particles along the dispersive direction was measured with resistive wires at four different depths, separated by about 16 cm each. For particle identification we used the specific energy loss measured in the ionization chamber and the residual energy measured in a NE102A plastic scintillator located behind the output window of the detector. The input and output windows of the detector were made

...
of 1.8 and 7.2 mg/cm² thick Kapton foils, respectively. The ionization chamber was filled with pure isobutane at 40 torr. The entire horizontal acceptance of the spectrometer, $\Delta \theta = \pm 2^\circ$, and a restricted vertical opening, $\Delta \phi = \pm 0.5^\circ$, were used in the measurements at forward angles, whereas at the largest angles the vertical opening of the acceptance window was raised to $\Delta \phi = \pm 1.0^\circ$. Raytracing was used to reconstruct the scattering angle. For this purpose, position calibration of the detector was performed by using the scattering on a thin Au target ($212 \mu g/cm^2$) and an angle mask consisting of five openings of $\pm 0.1^\circ$, located at $-1.6^\circ$, $-0.8^\circ$, $0^\circ$, $+0.8^\circ$ and $+1.6^\circ$ relative to the central angle of the spectrometer. In addition to RAYTRACE calculations, angle calibration data were obtained at several angles by using the angle mask. Typically the spectrometer was moved by $2^\circ$ or $3^\circ$ at a time, allowing for an angle overlap that provided a self-consistency check of the data. Normalization of the data was done using current integration in a Faraday cup. Focal plane reconstruction was done at each angle using the position measured with the signals in the wire nearest to the focal plane and using the detector angle obtained from the position measured at two of the four wires (typically the first and last). The angular range, $\Delta \theta = 4^\circ$, covered by the acceptance slit was divided into 8 bins, resulting in 8 points in the angular distribution being measured simultaneously, with each integrating over $\Delta \theta_{lab} = 0.5^\circ$.

The measurements with the angle mask showed that the resolution in the scattering angle (laboratory) was $\Delta \theta_{res} = 0.18^\circ - 0.25^\circ$ full-width at half maximum (FWHM). This includes a contribution from the angular spread of the beam of about $0.1^\circ$. The best energy resolution obtained at forward angles was 150 keV FWHM. It degraded as we advanced to larger angles due to the spread of the beam of about $0.1^\circ$ FWHM. This includes a contribution from the angular distribution being measured simultaneously, with each integrating over $\Delta \theta_{lab} = 0.5^\circ$.

The measurements for the elastic peak, even in the case of the $^7$Li experiments where the first excited state of the projectile is only 477 keV away. The active length of the focal plane allowed us to cover a total excitation energy of about 7 MeV, centered around the elastic peak. Thus we were able to measure inelastic scattering to the lowest excited states of the projectile-target systems at the same time. These inelastic scattering data were used as additional information to check the experimental procedures. In one of the runs we have also measured the neutron transfer reaction $^{13}$C($^7$Li,$^8$Be)$^{12}$C at E($^7$Li)=63 MeV, which was discussed elsewhere in detail and is used here to check the sensitivity of observables in other channels to the OMP extracted from the elastic scattering data.

To obtain accurate absolute values for the cross sections, target thickness and charge collection factors were determined by a two-target method as described in Ref. 24. We also determined the target thickness by measuring the energy loss of alpha particles from a $^{228}$Th source and the accuracy in normalization is 9%. Combining the results of these independent determinations, we conclude that we have an overall normalization accuracy of 7% for the absolute values of the cross sections.

### III. OPTICAL-MODEL ANALYSIS

The measured elastic data at 9 MeV/nucleon, shown in Fig. 1 as the ratio to the Rutherford cross section, extend to a larger angular range than previously measured. These data show complex forms with characteristic rapid oscillations at small angles followed by a marked change in shape at intermediate angles: a plateau is developed at $\theta = 50^\circ - 70^\circ$ followed by a deep minimum at $\theta \approx 80^\circ$. Assuming pure Fraunhofer scattering at forward angles, we extract a grazing angular momentum $l_g \approx 15$ from the angular spacing $\Delta \theta = \pi/(l_g + 1/2)$. The striking fact is that the same pattern emerges for all four projectile-target combinations, including that for the $^9$Be target where a much stronger absorption is expected. (We remind the reader that $^9$Be is a perfect black disc target since it has very low thresholds for breakup into the neutron and alpha channels $S_n=1.66$ MeV and $S_\alpha=2.47$ MeV, and there are no bound excited states. These values should be compared with $S_n=1.47$ MeV in $^6$Li and $S_\alpha=2.47$ MeV in $^7$Li.)

Similarities seen in the differential cross sections shown in Fig. 1 indicate general wave-mechanical characteristics of the scattering process and average systematic properties of the nuclear interaction. Specific structure effects can be isolated only as small deviations from the normal behavior. Therefore the data are analyzed using optical potentials with conventional Woods-Saxon (WS) form factors for the nuclear term, supplemented with a Coulomb potential generated by a uniform charge distribution with a reduced radius fixed to $r_c=1$ fm. No preference has been found for volume or surface localized absorption and throughout the paper only volume absorption is considered. In the absence of any spin dependent observables, spin-orbit or tensor interactions have been ignored. Ground state reorientation couplings also have been neglected. The potential is defined by six parameters specifying the depth and geometry of the real and imaginary terms

$$U(r) = -(Vf_V(r) + iWf_W(r))$$

where

$$f_x(r) = \left[1 + \exp \left(\frac{r - r_x(A_1^{1/3} + A_2^{1/3})}{a_x}\right)\right]^{-1}$$

and $x=V,W$ stands for the real and imaginary parts of the potentials, respectively. The number of data points per angular distribution exceeds N=100 points and therefore the usual goodness of fit criteria ($\chi^2$) normalized to N has been used. A source of bias was the finite angular acceptance of the detectors (the $0.5^\circ$ bins, in the present
FIG. 1: (Color online) Woods-Saxon optical model analysis (full lines) of elastic scattering data (open points) at 9 MeV/nucleon (Table II). Far-side/near-side cross sections are also shown by dashed and dotted lines, respectively. The depth of the real potential is shown to identify the particular WS potential parameters used in the calculations.

The averaging associated with this finite angular resolutions has most effect on the depth of sharp minima. A few exploratory calculations showed that allowing the normalization to vary did not result in any qualitative changes and did not indicate that any renormalization by more than a few percent would be preferred. Optical parameter sets collected from the literature were used as starting values for the search procedure. In particular the potential OM1 of Trcka et al. [18] has been extensively tested. Guided by these potentials and by our earlier analysis a number of some \(10^6\) potentials with real volume integrals in the range \(J_V = 200 - 600\) MeV fm\(^3\) have been generated for each reaction channel, thus exploring the functional Woods-Saxon space in full detail. Local minima were identified and a complete search on all six parameters determined the best fit potentials. The plateau feature at the intermediate angles and the sharp decrease in the cross section near \(\theta = 80^\circ\) could be fitted only with deep potentials with real volume integrals (per nucleon) exceeding a critical value \(J_{V \text{crit}} \approx 300\) MeV fm\(^3\). There is a consistent preference for potentials with relatively weak imaginary parts with values of \(W\) around \(15\) MeV except for \(^7\)Li scattering where somewhat larger values are needed to fit the data. We systematically find \(r_V < r_W\) and large diffuseness parameters \(a_V \approx a_w \approx 0.8\) fm in agreement with theoretical expectations for loosely bound nuclei [27, 28]. A grid search procedure on the real depth of the potential allowed to identify discrete ambiguities. Parameters for the first two discrete families are given in Table II. These are identified by a jump of \(\Delta J_V \approx 100\) MeV fm\(^3\) from one family to another and almost constant imaginary volume integral. As a con-
sequence, the total reaction cross section seems to be a well determined observable. Gridding on other WS parameters revealed a continuous ambiguity of the form $J V R_V \approx$ const, where $R_V$ is the $rms$ radius of the potential. The larger the volume integral, the smaller the radius that is required to fit the data. This is a clear manifestation of a complicated radial dependence of the dynamic polarization potential (DPP) which may lead to radii much smaller than the minimal value implied by the folding model (e.g., $R^2 = R^2_1 + R^2_2$, for a zero range NN effective interaction). However, for each discrete family rather precise values of the $rms$ radii were required to fit both forward and intermediate angle cross sections.

Sometimes more subjective criteria may be used to choose between various ambiguous potentials based upon general theoretical expectations. For example one may require consistency with the results of analyses of other data for the same system at nearby energies with the expectation that the potential should not change rapidly with mass and energy. Individual elastic data sets possess individual idiosyncrasies which facilitate the inference of a single local potential. We note that, seemingly, there is a compatibility between all data sets: an optimum potential found for one data set gives already a good fit to the other. In fact, potentials given as first entry in Table II were obtained by iterating several times this procedure in an attempt to find a single potential which would simultaneously fit all data at 9 MeV/nucleon. A compromise could be obtained with transparent deep potentials close to $V_0 \approx 225$ MeV having a strongly refractive core at small radii, surrounded by a weakly absorptive halo. In fact, examining the ratio $w(r) = W(r)/V(r)$ as a function of the radial distance, we found that our potential may by qualified as having internal $(r \sim 0 - 4$ fm) and surface $(r > 8$ fm) transparency $(w \approx 0.1)$ but with a pronounced maximum $(w \approx 0.8)$ near the empirical strong absorption radius $(R_s \approx 6$ fm) in agreement with the systematics found in other more bound systems.

The surface localized absorption suggests that the reaction mechanism is dominated by direct reactions. The relatively large radius of the absorption required by the data is an indication that fusion already sets in the region of the barrier and that fusion is a large component of the total reaction cross section.

A variety of notch tests have been performed to determine the radial sensitivity of the potential. One test was done using a Gaussian spike superimposed on the real potential at a given radius. The resulting influence on the $\chi^2$ of the fits is displayed in Fig. 2. It shows that there is a relatively high sensitivity for radial distances as low as 4-6 fm, well inside the strong absorption radius. Deeper inside this radial range, the refractive index, defined as $n = \sqrt{1 - \frac{V}{V_0}}$ is almost real and reaches values as high as $n = 2.6$, comparable to that of diamond.

As mentioned already, it was shown in Refs. 8, 9, 10 that the elastic scattering of light heavy ion systems such as $^{16}$O+$^{12}$C and $^{18}$O+$^{16}$O shows sufficient transparency for the cross section to be dominated by far-side scattering. Structures appearing in the elastic scattering angular distributions at intermediate angles have been identified as Airy minima of a nuclear rainbow, due to a destructive interference between two far-side trajectories which sample the interior of the potential. At 19 MeV/nucleon the $^7$Li scattering data show rapid, diffractive Fraunhofer oscillations at small angles due to the strong near-far amplitude interference (Fig. 3). Beyond the crossover the near-side amplitude makes a negligible contribution to the cross section. The shoulder and the deep minimum seen at 9 MeV/nucleon (Fig. 1) are washed out in the far-side amplitude and only a broad, less pronounced minimum survives, followed by a broad Airy maximum and an exponential, structureless decay of the cross section at large angles. Clearly, both the data at 9 and 19 MeV/nucleon (Figs. 1 and 5) show far-side dominance as a possible manifestation of refractive effects. However, this simple dominance does not explain, by itself, the difference in the angular distributions seen at these energies, suggesting a difference in the reaction mechanism. In fact the above picture has been already challenged by Anni 12 and by Michel et al. 31 for the simple reason that the far-side amplitude has never been decomposed in subamplitudes which would explain the quoted interference. We come back to this topic in Section VI. For the moment we adopt the interpretation of Michel et al. 31 and denote the complex structure at intermediate angles in our data as pre-rainbow oscillations.

In the remainder of this section we discuss the ability of the folding model to describe the pre-rainbow oscillations.

![Image of graph showing $\chi^2/\nu$ versus $r$ for $^6$Li+$^12$C 54 MeV Pot 225]
where \( v \) is the (complex) NN interaction, \( \rho_1(2) \) are the single particle densities of the interacting partners, calculated in a standard spherical Hartree-Fock procedure using the energy density functional of Beiner and Lombard with the surface term adjusted to reproduce the total binding energy [31, 32], \( s = \vec{r}_1 + \vec{R} - \vec{r}_2 \) is the NN separation distance between interacting nucleons and \( \rho \) is the overlap density. The effective NN interaction contains an isovector component which gives a negligibly small contribution for \( p \)-shell nuclei but is included here for convenience in conjunction with appropriate single particle isovector densities. The smearing function \( g(s) \) is taken as a normalized Gaussian [3, 10, 33],

\[
g(s) = \frac{1}{t^2} \exp(-s^2/t^2) \tag{4}
\]

which tends to a \( \delta \)-function for \( t \to 0 \), while for finite values of the range parameter \( t \) it increases the \( r_{\text{rms}} \) radius of the folding form factor by \( r_{\text{rms}}^2 = (3/2)t^2 \), leaving unchanged the volume integral. Inclusion of a smearing function with a varying range parameter, greatly increases the ability of the folding form factor to simulate the radial dependence of DPP.

The geometric or arithmetic mean of the overlapping densities has been used to define the overlap density \( \rho \) in Eq. 6

\[
\rho = \frac{1}{2} \rho_1(\vec{r}_1 + \frac{1}{2}\vec{s})\rho_2(\vec{r}_2 - \frac{1}{2}\vec{s})^{1/2} \tag{5}
\]

and

\[
\rho = \frac{1}{2} [\rho_1(\vec{r}_1 + \frac{1}{2}\vec{s}) + \rho_2(\vec{r}_2 - \frac{1}{2}\vec{s})]. \tag{6}
\]

The former was introduced by Campi and Sprung in density-dependent Hartree-Fock calculations [34]. It is physically appealing since the overlap density tends to zero when one of the interacting nucleons is far from the bulk, and to the nuclear matter saturation value at complete overlap. The approximation in Eq. 6 is similar to that used in folding calculations with density-dependent M3Y effective interactions [35], except for the factor 1/2 which has been introduced here because JLM interaction is defined only up to the nuclear matter saturation value \( \rho \leq \rho_0 \). It has been suggested to us [36] that the drawbacks seen in our earlier analysis of the scattering of \( ^7\text{Li} \) at 19 MeV/nucleon (see Fig. 6a in Ref. [3]) may be due to the weak density dependence introduced by Eq. 6 and thus rainbow patterns could not be reproduced. However, the optical model analysis presented above showed clearly that the pre-rainbow oscillations (at 9 MeV/nucleon) and rainbow patterns (at 19 MeV/nucleon) could be described if and only if the potentials have the proper \( r_{\text{rms}} \) radius. It turns out that the smearing procedure described above is essential in simulating the complicated radial dependence of the dynamic polarization potential.

In the earlier analysis [3], fixed values for the range parameters \( t_V = 1.2 \) fm and \( t_W = 1.75 \) fm, found from a global analysis of the data were used. Only the renormalization factors \( N_V \) and \( N_W \) were left free in the fits for each case. In the present analysis with double folded potentials, all four parameters: two strength parameters (\( N_V \) and \( N_W \)) and two range parameters (\( t_V \) and \( t_W \)),
have been searched simultaneously to fit the data for each case

\[ U_{DF}(r) = N_V V(r, t_V) + i N_W W(r, t_W) \]  

(7)

to obtain a phenomenological representation of the DPP as a uniform renormalization of the depths and radii of the folding potentials. The calculations using Eqs. (5) and (6) are dubbed JLM1 and JLM2 respectively. As these give very similar results only JLM1 parameters are listed in Table III and the results of the calculations are shown in Figs. 4 to 7. At 9 MeV/nucleon (Fig. 4) the same pattern emerges as with Woods-Saxon form factors. The pre-rainbow oscillation is carried entirely by the dominant far-side component. Some other high order structures appear at angles near 180° as the result of near/far amplitude interference. At most forward angles this interference produces an inner Fraunhofer crossing which give rise to a deep minimum in the cross section.

For \(^7\text{Li}^+\text{Be}\) at 63 MeV, JLM1 calculation failed to describe the oscillation near \(\theta = 80°\) for the simple reason that data required a \(r_{ms}\) radius for the real potential \(R_V = 3.4\) fm, while the bare JLM interaction predicts a minimal \(R_Y = 3.6\) fm for \(t_R \approx 0\). This once again reflects the critical role played by the radial behavior of DPP. This is also illustrated in the upper right quadrant of Fig. 5 where two JLM solutions for the reaction \(^{10}\text{B}^+\text{Be}\) at 10 MeV/nucleon are indicated (see also Table III). The solution with smaller real volume integral which better fits the forward angles predicts a smooth, exponentially decaying cross section beyond \(\theta \approx 60°\). The second solution with a real volume integral close to the critical value \(J_{V_{crit}} \approx 300\) MeVfm\(^3\) gives rise to a shallow pre-rainbow
oscillation at these angles (not covered by experiments). The high selectivity of the pre-rainbow oscillations to the optical potentials is also illustrated in Fig. 6 where other $^6$Li scattering data from literature, at somewhat lower energies, are explored. The $^6$Li+$^12$C data at 50 MeV [18] could be described in the whole angular range only with potentials exceeding the critical value of the real volume integral found before. In Fig. 7 we show $^6$Li+$^12$C elastic scattering data at 7 energies between 15 and 50 MeV/nucleon. Now, even at high energy (Fig. 7) the JLM1 description of the rainbow patterns is exemplary (to be compared with Figs. 6 a) and c) of Ref. [8]). This suggests that the geometrical details of the optical potential rather than the density dependence are essential for a correct description of $^6$,$^7$Li elastic scattering at low and intermediate energies.

A close examination of the parameters in Table III reveals an erratic variation of the range parameters $t_{V(W)}$ from one energy to another and from system to system. As mentioned above, this largely reflects the mass and energy dependence of DPP. The other parameters are more stable. The strength parameter $N_V$ decreases slowly from 5 to 16.5 MeV/nucleon and then increases again up to 53 MeV/nucleon, the highest energy at which reliable data exist. This may suggest that DPP reaches its maximum amplitude at energies around 16 MeV/nucleon. On average the $N_V$ values in Table III are somewhat larger than in our earlier analysis [8] reflecting the need for stronger refractive effects, but again $N_W$ approaches unity, on average.

FIG. 5: (Color online) Comparison of JLM1 folding model calculations with $^7$Li scattering data at 19 MeV/nucleon. The data are taken from [8]. Two JLM1 solutions are indicated for $^{10}$B+$^9$Be reaction. The parameters are given in Table III. Far-side (dashed) and near-side (dotted) cross sections are indicated in ratio to Rutherford cross sections.
FIG. 6: (Color online) Comparison of the JLM1 folding model calculations with $^6$Li scattering data on light targets at 30 and 50 MeV laboratory energy. For the $^{12}$C target at 50 MeV, only a solution with a real volume integral exceeding the critical value $J_V=300$ MeV fm$^3$ (Table III) is able to reproduce both forward and intermediate angles (right bottom panel). Far-side (dashed lines) and near-side (dotted) cross sections are indicated in ratio to Rutherford cross sections.

IV. TRANSFER REACTION

As already mentioned, in one experimental run we have also measured the neutron transfer reaction $^{13}$C($^7$Li,$^8$Li)$^{12}$C at E($^7$Li)=63 MeV. The purpose of the study was to determine the ANC for the ground state of $^8$Li, and then, using charge symmetry to relate it with that in its mirror nucleus $^8$B. The ANC was then used to calculate the astrophysical factor $S_{17}$ that gives the rate of the proton capture reaction $^7$Be(p, $\gamma$)$^8$B, of crucial importance for the solar neutrino problem. The major advantage of the neutron transfer reaction over its mirror proton transfer reaction is that it involves a stable beam, and, therefore, a much more precise and detailed angular distribution could be measured. That allowed the determination of the admixture of the minor component $1p_{1/2}$ in the wave function of the ground state of $^8$Li (and $^8$B, respectively), dominated by the $1p_{3/2}$ orbital. The results of this experiment were reported in Ref. [23]. In that study we paid particular attention to the dependence of the results on the optical model potentials used in the entrance and exit channels.

Eleven different combinations of entrance/exit potentials were used to show that the resulting values for $C_{p_{3/2}}^2$ and $C_{p_{1/2}}^2$ are very stable, when the potentials are reasonable. The potentials used were either volume Woods-Saxon forms with the parameters from similar projectile-target combinations at similar energies, or were obtained from the double folding procedure with the renormaliza-
tion coefficients from the previous paper \textsuperscript{3}. Calculations done after the publication with the new (deeper) potential "227" in Table \textsuperscript{III} in both entrance and exit channels lead to minor (~5\%) variations in the results. The very good agreement between the experimental data and the DWBA calculations and between the results of present and previous calculations (Figure \textsuperscript{5}) shows that the region of the potential contributing to transfer (the surface) is well and uniquely described. This simultaneous description of elastic and transfer data is also an argument for the complete determination of the optical potentials.

V. DISPERSION RELATION

The dispersion relation is a fundamental property of the optical potential (see for example \textsuperscript{12}) and a selection between ambiguous potentials can be performed by studying the dispersive properties of these potentials, provided accurate analyses of experimental data are available over a large energy range.

The threshold anomaly which manifests itself as a sharp increase of the real optical potential for energies close to the Coulomb barrier, has been explained by Nagarajan, Mahaux and Satchler \textsuperscript{42} as due to the opening of reaction channels with increasing energy. An application of the dispersion relation for elastic scattering of \textsuperscript{16}O on \textsuperscript{208}Pb at energies around 80 MeV accounted well for this effect. Later it was conjectured by Mahaux, Ngo and Satchler \textsuperscript{15} that for loosely bound nuclei, this anomaly may be absent. For these nuclei, the strong coupling with breakup channels gives rise to a repulsive DPP which compensates the strong attractive component. According to Sakuragi \textsuperscript{14} this effect would explain the large renormalization needed by most of the effective interactions used in the folding model for elastic scattering of \textsuperscript{6\textit{Li}}. The coupling with inelastic channels alone has been invoked by Gomez-Camacho et al. \textsuperscript{13} to explain this reduction. The earlier analysis of Kailas \textsuperscript{44} found strong dispersive effects for \textsuperscript{6\textit{Li}}+\textsuperscript{12}C scattering. Recent studies by Tiede et al. \textsuperscript{45} and by Pakou et al. \textsuperscript{46} of \textsuperscript{6\textit{Li}}+\textsuperscript{28}Si at near barrier energies found that the strength of the real part of the folding potential using the M3Y interaction remains almost independent of energy, suggesting a cancellation between the attractive (dispersive) component and the strong repulsive dynamic polarization potential arising from the coupling to continuum states.

Another study of \textsuperscript{6\textit{Li}}+\textsuperscript{208}Pb near the Coulomb barrier \textsuperscript{47} found that at low energies the DPP is of opposite sign for the two projectiles and there is a threshold anomaly for \textsuperscript{7\textit{Li}} but none for \textsuperscript{6\textit{Li}}. No significant fusion hindrance caused by breakup effects was found in the fusion reaction of \textsuperscript{6\textit{Li}} on a \textsuperscript{59}Co target near the Coulomb barrier \textsuperscript{48}, thus leading the authors to conclude that breakup suppression above the barrier appears to be a common feature of \textsuperscript{6\textit{Li}} induced reactions.
Therefore, the energy dependence of the $^6\text{Li}$ optical potential is far from clear and the competition between dispersive (attractive) and coupling to continuum (repulsive) effects need to be studied more carefully. An earlier study showed that the total reaction cross section for $^6\text{Li}$ scattering saturates at energies around 20 MeV/nucleon and therefore dispersive effects could be identified by accumulating good optical potentials in this energy range. The real and imaginary volume integrals for the optical potentials obtained in the previous sections are plotted in Fig. These are supplemented with values derived from the smooth OM1 potential of Trcka et al. [15].

We assume that the local optical potential may be written as $V = V_0 + \Delta V(E)$ where $V_0$ is independent of energy and $\Delta V(E)$ is the energy dependent DPP. We ignore the spurious energy dependence of $V_0$ arising from non locality which is expected to be weak for heavy ions. We use the dispersion relation connecting the imaginary and real volume integrals in the subtracted form,

$$J_{\Delta V,E_s}(E) = (E - E_s) P \frac{d}{dE'} J_W(E') \exp(-\alpha E) \left(\frac{E'}{E - E_s}\right)$$

where $E_s$ is a reference energy and $P$ is the principal value of the integral. In principle the evaluation of this equation requires the knowledge of $J_W$ values at all energies. The above subtracted form takes advantage of the fact that the energy dependence of $J_W$ far from saturation energy is not very important and the unknown contributions are absorbed by normalizing to the empirical value at a convenient reference energy,

$$J_{\Delta V,E_s}(E) = J_{\Delta V}(E) - J_{\Delta V}(E_s)$$

Two schematic models have been employed here to estimate the energy dependence of the imaginary volume integral. A first one approximates this energy dependence by straight line segments [15], which makes the evaluation of Eq. analytical. A more realistic energy dependence is given by

$$J_W(E) = J^0_W(1 - \beta \exp(-\alpha E))$$

where the parameters $J^0_W=170 \text{ MeV fm}^3$, $\alpha=0.023 \text{ MeV}^{-1}$ and $\beta=0.95$ describe better the energy dependence in the important range 0-20 MeV/nucleon. In both calculations the reference energy was set at $E_s=156 \text{ MeV}$, an energy where the JLM folding model gives precise values for volume integrals. In general, the calculated dispersion contributions get more repulsive as the energy increases, and the corresponding real potentials get shallower, in qualitative consistency with phenomenology. An empirical logarithmic dependence of the form $J_V = -785 + 95 \ln(E)$ has been found in Ref. mostly based on unique OM potentials determined from 35 and 53 MeV/nucleon $^6\text{Li}$ scattering on light targets. This matches perfectly the dependence obtained with the dispersion relation for $E>10 \text{ MeV/nucleon}$, but disagrees at lower energies. In fact, this logarithmic dependence is physically meaningful and can be understood on the basis of the dispersion relation with a schematic (line segments) approach for the imaginary volume integral.

A relatively strong localized energy variation is predicted by the linear model in the range 0-20 MeV/nucleon, while the exponential model predicts a smooth dependence on the entire range of energies. This last calculation is much closer to the data and seem to confirm $J_V=320 \text{ MeV fm}^3$ as the most realistic value at 9 MeV/nucleon, in surprising agreement with values found for the more bound system $^{16}\text{O}+^{16}\text{O}$ (see e.g. Fig 6 in ref. [51]). Most probably the phenomenological values found at 5 MeV/nucleon are due to the erratic variation in the WS parameters due to the rapidly changing elastic scattering angular distributions near the resonance energy region around 20 MeV.

**VI. SEMICLASSICAL BARRIER AND INTERNAL BARRIER AMPLITUDES**

Once we have established the main features of the average OM potential, we turn now to study the reaction mechanism in the elastic scattering of $^6,^7\text{Li}$ on light targets at 9 MeV/nucleon using semiclassical methods. The far-side dominance observed in the angular distributions...
at 9 and 19 MeV/nucleon is not able to explain the differences in the reaction mechanism at these energies. The reason is of course that the far/near (F/N) decomposition method does not perform a dynamic decomposition of the scattering function, but merely decomposes the scattering amplitude into traveling waves. The intermediate angle structures, such as those observed in our angular distributions, have been repeatedly interpreted as arising from the interference of two ranges in angular momenta \( \ell_1 < \ell_2 \) contributing to the same negative deflection angle. However, the corresponding cross sections \( \sigma_{F<} \) and \( \sigma_{F>} \) cannot be isolated because their dynamic content (S-matrix) is not accessible.

The semiclassical uniform approximation for the scattering amplitude of Brink and Takigawa [20] is well adapted to describe situations in which the scattering is controlled by at most three active, isolated, complex turning points. An approximate multireflection series expansion of the scattering function can be obtained, the terms of which have the same simple physical meaning as in the exact Debye expansion for the scattering of light and define the effective potential as,

\[
V_{\text{eff}}(r) = V(r) + \frac{\hbar^2 \lambda^2}{2\mu r^2}, \quad \lambda = \ell + \frac{1}{2}
\]

where the Langer prescription has been used for the centrifugal term. This guarantees the correct behavior of the semiclassical wave function at the origin [53]. Then we calculate the deflection function,

\[
\Theta(\lambda) = \pi - 2 \int_{r_1}^{\infty} \frac{\sqrt{\frac{h^2}{2\mu} \lambda dr}}{r^2 \sqrt{E_{\text{c.m.}} - V_{\text{eff}}}}
\]

where \( r_1 \) is the outer zero of the square root, i.e. the radius of closest approach to the scatterer and \( \mu \) is the reduced mass. Note that with the replacement \( \hbar \lambda = b\sqrt{2 \mu E} \), Eq. 12 becomes identical with the classical deflection function \( \Theta(b) \), where \( b \) is the impact parameter.

The results are shown in Fig. 10. The behavior of \( \Theta(\lambda) \) is the one expected for a strong nuclear potential in a near orbiting kinematical situation in which the c.m. energy approximately equals the top of the barrier for some specific angular momentum. The deflection functions exhibit no genuine minimum, but rather a pronounced cusp close to an orbiting logarithmic singularity. Therefore any interpretation of structures in angular distributions in terms of Airy oscillations can be discarded. Rather we need an interpretation appropriate for orbiting, a well documented situation in classical physics [54]. We identify the cusp angular momenta as orbiting momenta (\( \lambda_o \)) since they are related with the coalescence of two (barrier) turning points and the innermost turning point given by the centrifugal barrier become classically accessible. There are two branches that can be distinguished, an internal branch, for low active momenta \( \lambda < \lambda_o \) related to semiclassical trajectories which penetrate into the nuclear pocket and a less developed external (barrier) branch (\( \lambda > \lambda_o \)) related to trajectories deflected at the diffuse edge of the potential.

However this simple calculation cannot determine the relative importance of these branches and provides no information about the interference effects of the corresponding semiclassical trajectories. To clarify these points it is best to go into the complex \( r \)-plane and look for complex turning points, i.e. the complex roots of the quantity \( E_{\text{c.m.}} - V_{\text{eff}} - iW \). This is an intricate numerical problem, because, for a WS optical potential, the turning points are located near the potential singularities and there are an infinite number of such poles. The situation for integer angular momenta is depicted in Fig. 11 for the reaction \( ^6\text{Li}+^{12}\text{C} \) at 54 MeV using the potential "225" in Table 11. Only turning points nearest the real axis are retained and we observe an ideal situation with three, well isolated turning points for each partial wave. Even small absorption plays an essential role in the motion of turning points. Removing the imaginary part \( W \), the barrier turning points (\( r_{1,2} \)) become complex conjugates while the internal turning point is purely real (open symbols in Fig. 11).

The multireflection expansion of the scattering func-
where,\( \delta \) correspond to the turning point \( r \) with potential poles is avoided. Each term in Eq. 13 is the WKB (complex) phase shift \( 1 \) retains contributions from trajectories reflected at the barrier, not penetrating the internal region. The \( q \)th term corresponds to trajectories refracted \( q \) times in the nuclear interior with \( q \)-1 reflections at the barrier turning point \( r_q \). Summation of terms \( q \geq 1 \) can be recast into a single term,

\[
S_I = \frac{\exp[2i(S_{32} + S_{21} + \delta_1)]}{N(S_{21}/\pi)^2} \frac{1}{1 + \exp[2iS_{32}/N(S_{21}/\pi)]}
\]

and is known as the internal barrier scattering function. When the absorption in the nuclear interior is large, the second factor in the above equation reduces to one and we are left with the expression used in [30]. Since the semiclassical scattering function is decomposed additively, \( S_{WB} = S_B + S_I \), the corresponding total scattering amplitude is decomposed likewise as \( f_{WB} = f_B + f_I \) and conveniently the corresponding barrier and internal barrier angular distributions are calculated as \( \sigma_{B,I} = |f_{B,I}|^2 \), using the usual angular momentum expansion of the amplitudes.

The accuracy of the semiclassical calculation has been checked by comparing the barrier and internal barrier absorption profiles with the exact quantum-mechanical result in Fig. 12. First, one observe that the semiclassical B/I expansion is an exact decomposition of the quantum result. They are virtually identical at the scale of the figure. The internal component gets significant values up to the grazing angular momentum \( \ell_g = 15 \) and is negligibly small beyond this value. The barrier component

FIG. 11: (Color online) Complex turning points (full symbols) for the potential "225" shown in Table III at integer angular momenta. Open symbols denote turning points for the real potential alone. Stars indicate complex poles of the potential.

\[
S_{WB}(\ell) = \sum_{q=0}^{\infty} S_q(\ell)
\]

where,

\[
S_0(\ell) = \frac{\exp(2i\delta_1^0)}{N(S_{21}/\pi)}
\]

and for \( q \neq 0 \),

\[
S_q(\ell) = (-)^{q+1} \frac{\exp[2i(qS_{32} + S_{21} + \delta_1^q)]}{N^{q+1}(S_{21}/\pi)}
\]

In these equations \( \delta_1^q \) is the WKB (complex) phase shift corresponding to the turning point \( r_1 \), \( N(z) \) is the barrier penetrability factor,

\[
N(z) = \frac{\sqrt{2\pi}}{\Gamma(z + \frac{1}{2})} \exp(z \ln z - z)
\]

and \( S_{ij} \) is the action integral calculated between turning points \( r_i \) and \( r_j \),

\[
S_{ij} = \int_{r_i}^{r_j} dr \left\{ \frac{2\mu}{\hbar} [E_{c.m.} - V_{eff} - iW] \right\}^{1/2}
\]

FIG. 12: (Color online) Semiclassical decomposition of scattering function for the WS potential of Fig. 11. Barrier (open circles) and internal barrier components (squares) are indicated. The exact total quantum \( S \)-matrix is indicated by small dots. The line is a cubic spline interpolation of the total semiclassical scattering function for the same potential.

In these equations \( \delta_1^q \) is the WKB (complex) phase shift corresponding to the turning point \( r_1 \), \( N(z) \) is the barrier penetrability factor,

\[
N(z) = \frac{\sqrt{2\pi}}{\Gamma(z + \frac{1}{2})} \exp(z \ln z - z)
\]

and \( S_{ij} \) is the action integral calculated between turning points \( r_i \) and \( r_j \),

\[
S_{ij} = \int_{r_i}^{r_j} dr \left\{ \frac{2\mu}{\hbar} [E_{c.m.} - V_{eff} - iW] \right\}^{1/2}
\]
resembles a strong absorption profile and this justifies the
interpretation that it corresponds to that part of the flux
not penetrating into the nuclear interior. For values near
the orbiting angular momentum \( \ell_o \approx 12 \), the two com-
ponents interfere and a downward spike appears in the
total profile, in complete agreement with the quantum
result. Second, the B/I components are almost decou-
pled in the angular momentum space and therefore they
will contribute in different angular ranges.

Semiclassical cross sections are compared with the data
in Fig. 13 for the reaction \( ^6 Li + ^{12} C \) at 54 MeV. Better in-
sight into this technique is obtained by further decom-
posing the B/I components into far and near (BF/BN and
IF/IN) subcomponents. Clearly, the barrier com-
ponent dominates the forward angle region. Fraunhofer
diffra ctive oscillations appear as the result of BF and BN
interference. At large angles, the internal contribution
accounts for the full cross section. As both B/I contribu-
tions are dominated by the far-side component (Fig. 13
bottom panels), we show in Fig. 14 the angles at which
the phase difference of the BF and IF amplitudes passes
through an odd multiple of \( \pi \), i.e. where minima should
be expected. Since the crossing angle (where \( \sigma_B \approx \sigma_I \)) is
about \( \theta \approx 75^\circ \) and lies just in between predicted minima,
the coherent interference around this angle gives rise to
the "plateau" (constructive) and the deep minimum (de-
structive) at \( \theta \approx 80^\circ \). Similar consideration apply to the
other three reactions.

Thus, the intermediate angle exotic structure in an-
gular distributions for the elastic scattering of \(^6,7 Li\) on
light targets can be understood as a result of coherent
interference of two far-side subamplitudes generated by
different terms in the uniform multireflection expansion
of the scattering function (terms \( q=0 \) and \( q=1 \) in Eq.
13), corresponding to the scattering at the barrier and
the internal barrier. This interference effect appears as
a signature of a surprisingly transparent interaction po-
tential for loosely bound nuclei \(^6,7 Li\) which allows part
of the incident flux to penetrate the nuclear interior and reemerge with significant probability.

VII. CONCLUSIONS

We have performed precise measurements on extended
angular ranges of the elastic scattering of loosely bound nuclei \(^6,7 Li\) on \(^{12,13} C\) and \(^9 Be\) in four projectile-target
combinations at 9 MeV/nucleon and reanalyzed previous
data for the scattering of \(^7 Li\) at 19 MeV/nucleon in an ef-
fort to obtain systematic information on the interaction of
p-shell nuclei with light targets. Optical potentials
for these nuclei are needed for studies in which highly
peripheral transfer reactions involving radioactive nuclei
are used as indirect methods for nuclear astrophysics and
are an important factor in the accuracy and reliability of
these methods. At the present status of the experimental
techniques, the best information on the optical potentials
for radioactive nuclei can be obtained only by extrapo-
lation from adjacent less exotic nuclei. Our intention is
to narrow the ambiguities in the optical model potentials
by systematic studies of the scattering of loosely bound
projectiles over a large range of angles and energies, and
extract information that can be used for systems involv-
ing radioactive projectiles, for which elastic scattering
data of very good quality are not easily available. We
demonstrate this procedure by reanalyzing the one neu-
tron transfer reaction \(^{13} C(^7 Li,^8 Li)^{12} C\) using optical
potentials obtained in the present study.

The present data, which extend over a much larger an-
gular range than previously measured, confirm the exis-
tence of an exotic intermediate angle structure, observed
previously by Trcka et al. It was interpreted in Ref.
15 as a diffractive effect arising from an angular mo-
mentum dependent absorption. We adopt an opposite
point of view and interpret these structures as refrac-
tive effects arising from a fine balance between the real
and imaginary components of the optical potential. We
have performed a traditional analysis of our data in terms
of Woods-Saxon and microscopic JLM folded potentials.
Both approaches lead to the conclusion that the optical
potential is deep and surprisingly transparent, in line
with findings for more bound systems. Folding model
form factors have been renormalized in the usual way in
order to account for the energy and radial dependence of
the dynamic polarization potential. It is suggested that
DPP attains its maximum amplitude at approximately
16 MeV/nucleon for these systems. The intermediate
angle structures could be reproduced only with poten-
tials exceeding a critical volume integral of about 300
MeV fm\(^3\) and, consequently, are severely selective, lim-
iting the ambiguities in the determination of the OMP.
The remaining discrete ambiguities could be removed by
a dispersion relation analysis. Based on a good estima-
tion of the absorption at low energy (5-20 MeV/nucleon),
this analysis allowed us to extract a smooth energy de-
pendence of the optical potential. Our analysis did not
find any spectacular anomaly near the Coulomb barrier
and seems to confirm, to some extent, the conjecture of a
canceling effect between the repulsive dynamic polariza-
tion potential due to the coupling with breakup channels
and the attractive, dispersive component of the optical
potential.

In our previous study 8 we found a simple recipe
to obtain OMP for loosely bound \( p \)-shell nuclei from a
double folding procedure using the JLM effective NN
interaction. The already independent real and imagi-
ary parts were smeared with constant, but different
ranges \( t_V = 1.2 \) fm and \( t_W = 1.75 \) fm, which ac-
counted for the well known need for a wider imaginary
potential to describe the experimental data. We found
that a considerable renormalization of the real part was
needed \( N_V = 0.37 \pm 0.02 \) (leading to volume integrals
\( J_V \approx 220 \) MeV fm\(^3\)), but not for the imaginary part
\( N_W = 1.00 \pm 0.09 \). That recipe was already successfully
applied to predict the elastic scattering angular distribu-
tions of RNBs on light targets in a number of cases at
energies around 10 MeV/nucleon. The present analysis shows that in order to reproduce the structures observed at intermediate angles in the same cases measured, one needs to allow for a more complicated radial dependence of the dynamic polarization potential, energy and target dependent, and require deep real potentials with volume integrals larger than a critical value of $J_{V \text{crit}} \approx 300 \text{ MeV fm}^3$. This is a conclusion of the phenomenological analyses and is supported by the dispersion relation analysis. However, the elastic scattering data in the angular range of the Fraunhofer oscillations and the transfer reactions can be equally well described by the previous potentials produced by the folding procedure with fixed smearing ranges for the effective NN interaction and the simple renormalization of Ref. 3, showing that the potentials are well described in the surface region.

In an effort to clarify the reaction mechanism responsible for the intermediate angle structures found at 9 MeV/nucleon, we performed extensive semiclassical calculations within the uniform multireflection expansion of the scattering function of Brink and Takigawa. It has been shown that using complex trajectories, the (external) barrier/internal barrier expansion is an exact realization of the dynamic decomposition of the quantum result into components responsible for that part of the incident flux reflected at the barrier and the part of the flux which penetrates into the nuclear interior and reemerges with significant probability. By combining the B/I decomposition with the usual far-side/near-side expansion, we explain the intermediate angle structure as a coherent interference effect of two subamplitudes (BF and IF). Thus, this refractive effect appears as a signature of a
highly transparent interaction potential.

Acknowledgments

This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-93ER40773 and by the Robert A. Welch Foundation. One of the authors (F.C.) acknowledges the support of the Cyclotron Institute, Texas A&M University and of the IN2P3 including that provided within the framework of the NIPNE-HH–IN2P3 convention.

[1] G. R. Satchler and W. G. Love, Phys. Rep. 55, 183 (1979).
[2] M. E. Brandan and G. R. Satchler, Phys. Rep. 285, 143 (1997).
[3] L. Trache, A. Azhari, H. L. Clark, C. A. Gagliardi, Y.-W. Lui, A. M. Mukhamedzhanov, R. E. Tribble and F. Carstoiu, Phys. Rev. C 61, 024612 (2000).
[4] A. Azhari, V. Burjan, F. Carstoiu, C. A. Gagliardi, V. Kroha, A. M. Mukhamedzhanov, F. M. Nunes, X. Tang, L. Trache and R. E. Tribble, Phys. Rev. C 60, 055803 (1999).
[5] X. Tang, A. Azhari, C. A. Gagliardi, A. M. Mukhamedzhanov, F. Pirlepesov, L. Trache, R. E. Tribble, V. Burjan, V. Kroha and F. Carstoiu, Phys. Rev. C 67, 015804 (2003).
[6] X. Tang, A. Azhari, Changbo Fu, C. A. Gagliardi, A. M. Mukhamedzhanov, F. Pirlepesov, L. Trache, R. E. Tribble, V. Burjan, V. Kroha and F. Carstoiu, Phys. Rev. C, accepted for publ. (2004).
[7] J. Blackmon et al., Nucl. Phys. A, in press.
[8] A. A. Ogloblin et al., Phys. Rev. C 62, 044601 (2000).
[9] S. Szilner et al., Phys. Rev. C 64, 064614 (2001).
[10] E. Stiliaris et al., Phys. Lett. B223, 291 (1989).
[11] J. Knoll and R. Schaeffer, Ann. Phys. (N.Y.) 97, 307 (1976).
[12] R. Anni, Phys. Rev. C 63, 031601R (2001).
[13] H. Feshbach, Ann. Phys. (N.Y.) 5, 357 (1958); 19, 287 (1962).
[14] Y. Sakuragi, Phys. Rev. C 35, 2161 (1987).
[15] C. Mahaux, H. Ngo and G. R. Satchler, Nucl. Phys. A449, 354 (1986).
[16] A. Nadasen et al., Phys. Rev. C 37, 132 (1988), and priv. comm.
[17] A. Nadasen et al., Phys. Rev. C 47, 674 (1993), and priv. comm.
[18] D. E. Trcka, A. D. Frawley, K. W. Kemper, D. Robson and E. G. Myers, Phys. Rev. C 41, 2134 (1990).
[19] J. Jeukenne, A. Lejeune and C. Mahaux, Phys. Rev. C 16, 80 (1977).
[20] D. M. Brink and N. Takigawa, Nucl. Phys. A279, 159 (1977).
[21] D. M. Pringle, W. N. Catford, J. S. Winfield, D. G. Lewis, N. A. Jelley, K. W. Allen, and J. H. Coupland, Nucl. Instr. Meth. A245, 230 (1986).
[22] D. H. Youngblood and J. B. Bronson, Nucl. Instr. and Meth. A361, 37 (1995).
[23] D. H. Youngblood, Y.-W. Lui, H. L. Clark, P. Oliver, G. Simler, Nucl. Instr. and Meth. A361, 539 (1995).
[24] S. Kowalski and H. A. Enge, computer code RAYTRACE, 1986, unpublished.
[25] L. Trache, A. Azhari, F. Carstoiu, H. L. Clark, C. A. Gagliardi, Y.-W. Lui, A. M. Mukhamedzhanov, X. Tang, N. Timofeyuk and R. E. Tribble, Phys. Rev. C 67, 062801(R) (2003).
[26] A. M. Mukhamedzhanov et al., Phys. Rev. C 56, 1302 (1997).
[27] M. S. Hussein and K. W. McVoy, Nucl. Phys. A445, 123 (1985).
[28] A. Bonaccorso and F. Carstoiu, Nucl. Phys. A706, 322 (2002).
[29] M. E. Brandan and K. W. McVoy, Phys. Rev. C 55, 1362 (1997).
[30] F. Michel, G. Reidemeister and S. Ohkubo, Phys. Rev. Lett. 89, 152701 (2002); ibidem, Phys. Rev. C 63, 034620 (2001).
[31] M. Beiner and R. J. Lombard, Ann. Phys. 86, 262 (1974).
[32] F. Carstoiu and R. J. Lombard, Ann. Phys. 217, 279 (1992).
[33] E. Bauge, J. P. Delaroche and M. Girod, Phys. Rev. C 58, 1118 (1998).
[34] X. Campi and D. W. L. Sprung, Nucl. Phys. A194, 401 (1972).
[35] Dao T. Khoa, G. R. Satchler and W. von Oertzen, Phys. Rev. C 56, 954 (1997).
[36] W. von Oertzen, priv. comm.
[37] M. F. Vineyard, J. Cook, K. W. Kemper, M. N. Stephens, Phys. Rev. C 30, 916 (1984) and private communication.
[38] Yu. Gluchov et al., preprint IAE-2989, 1978 and priv. comm.
[39] D. P. Stanley, F. Petrovich, P. Schwandt, Phys. Rev. C 22, 1357 (1980), and priv. comm.
[40] K. Katori et al., Nucl. Phys. A480, 323 (1988).
[41] J. Cook, H. J. Gils, H. Rebel, Z. Majka, H. Klewe-Nebenius, KfK 3233, 1981.
[42] M. A. Nagarajan, C. C. Mahaux and G. R. Satchler, Phys. Rev. Lett. 54, 1136 (1985).
[43] J. Gomez-Camacho, M. Lozano and M. A. Nagarajan, Phys. Lett. B161, 39 (1985).
[44] S. Kailas, Phys. Rev. C 41, 2943 (1990).
[45] M. A. Tiede, D. E. Trcka and K. W. Kemper, Phys. Rev. C 44, 1968 (1991).
[46] A. Pakou et al., Phys. Lett. B556, 21 (2003).
[47] N. Keeley, S. J. Bennett, N. M. Clarke, B. R. Fulton, G. Tungate, P. V. Drumm, M. A. Nagarajan and J. S. Lilley, Nucl. Phys.A571, 326 (1994).
[48] C. Beck et al., Phys. Rev. C 67, 054602 (2003).
[49] F. Carstoiu and M. Lassaut, Nucl. Phys. A597, 269 (1996).
[50] A. Nadasen, T. Stevens, J. Farhat, J. Brusoe, P. Schwandt, J. S. Winfield, G. Yoo, N. Anantaraman, F. D. Becchetti, J. Brown, B. Hotz, J. W. Jänecke, D. Roberts, R. E. Warner, Phys. Rev. C 47, 674 (1993).
[51] M. M. González and M. E. Brandan, Nucl. Phys. A693, 603 (2001).
[52] M. F. Vineyard, J. Cook and K. W. Kemper, Phys. Rev. C 31, 879 (1985).
[53] P. Fröbrich and R. Lipperheide, Theory of Nuclear Reactions, Clarendon Press, Oxford, 1996.
[54] K. W. Ford and J. A. Wheeler, Ann. Phys. (N. Y.) 7, 259 (1959).
TABLE I: List of the elastic scattering experiments presented in this paper.

| No. | Reaction  | $E$ [MeV] | $\theta_{lab}$ [deg.] |
|-----|-----------|-----------|----------------------|
| 1   | $^6$Li + $^{12}$C | 54        | 2 - 56               |
| 2   | $^6$Li + $^{13}$C | 54        | 2 - 56               |
| 3   | $^7$Li + $^9$Be  | 63        | 4 - 52               |
| 4   | $^7$Li + $^{13}$C | 63        | 4 - 56               |
| 5   | $^7$Li + $^9$Be  | 130       | 4 - 47               |
| 6   | $^7$Li + $^{13}$C | 130       | 4 - 47               |

TABLE II: Best fit Woods-Saxon parameters. Reduced radii are defined in the heavy ion convention. All lengths are given in fm, depths and energies in MeV, cross sections in mb and volume integrals in MeV fm$^3$. Coulomb reduced radius is fixed to $r_c=1$ fm. $R_V$ and $R_W$ are the rms radii of the real and imaginary potentials, respectively.

| Reaction  | $E$ [MeV] | $V_0$ [MeV] | $W_0$ [MeV] | $r_V$ [fm] | $r_W$ [fm] | $a_V$ [fm] | $a_W$ [fm] | $\chi^2$ | $\sigma_R$ [MeV fm$^3$] | $J_V$ [mb] | $R_V$ [fm] | $J_W$ [fm] | $R_W$ [fm] |
|-----------|-----------|-------------|-------------|------------|------------|------------|------------|--------|------------------------|------------|-----------|------------|-----------|
| $^6$Li + $^{12}$C | 54 | 225.47 | 15.75 | 0.503 | 1.157 | 0.900 | 0.737 | 17.71 | 1309 | 338 | 3.70 | 121 | 4.59 |
|           |         | 371.31 | 17.70 | 0.439 | 1.109 | 0.856 | 0.777 | 13.60 | 1322 | 419 | 3.47 | 125 | 4.56 |
| $^6$Li + $^{13}$C | 54 | 225.28 | 14.75 | 0.502 | 1.181 | 0.916 | 0.707 | 14.62 | 1327 | 327 | 3.76 | 114 | 4.63 |
|           |         | 364.46 | 16.95 | 0.443 | 1.133 | 0.871 | 0.744 | 14.24 | 1338 | 403 | 3.53 | 119 | 4.58 |
| $^7$Li + $^9$Be  | 63 | 225.85 | 24.74 | 0.536 | 0.941 | 0.828 | 0.980 | 10.14 | 1456 | 369 | 3.49 | 146 | 4.66 |
|           |         | 368.34 | 29.38 | 0.478 | 0.882 | 0.790 | 1.004 | 11.85 | 1470 | 464 | 3.28 | 153 | 4.62 |
| $^7$Li + $^{13}$C | 63 | 227.94 | 15.37 | 0.529 | 1.186 | 0.932 | 0.669 | 20.09 | 1367 | 328 | 3.87 | 107 | 4.64 |
|           |         | 278.86 | 24.19 | 0.594 | 1.050 | 0.789 | 0.721 | 20.04 | 1334 | 411 | 3.53 | 126 | 4.38 |
| $^7$Li + $^{13}$C | 130 | 149.11 | 29.73 | 0.636 | 0.932 | 0.885 | 0.929 | 2.61 | 1403 | 282 | 3.90 | 132 | 4.62 |
|           |         |         |         |         |         |         |         |         |         |         |         |         |         |
| $^7$Li + $^9$Be  | 130 | 143.41 | 33.64 | 0.581 | 0.829 | 0.892 | 1.094 | 3.03 | 1446 | 295 | 3.76 | 169 | 4.80 |
| $^{13}$C + $^9$Be | 130 | 159.85 | 24.43 | 0.674 | 0.983 | 0.868 | 0.914 | 13.69 | 1552 | 280 | 3.96 | 104 | 4.79 |

$^a$ uniform 10% errors.
TABLE III: Best fit JLM1 parameters. The notations are those from the text. Lengths are given in fm, energies in MeV, cross sections in mb and volume integrals in MeV fm$^3$.

| Reaction | Energy | $t_v$ | $t_w$ | $N_v$ | $N_w$ | $\chi^2$ | $\sigma_R$ | $J_v$ | $R_v$ | $J_W$ | $R_W$ |
|----------|--------|-------|-------|-------|-------|---------|------------|-------|-------|-------|-------|
| $^6$Li$^+$+$^{12}$C | 30$^a$ | 0.30 | 2.45 | 0.60 | 0.46 | 14.8 | 1371 | 396 | 3.66 | 72 | 4.93 |
| | 50$^b$ | 0.08 | 2.78 | 0.56 | 0.78 | 12.0 | 1315 | 373 | 3.64 | 120 | 4.42 |
| | 54 | 0.08 | 2.76 | 0.54 | 0.77 | 21.4 | 1556 | 351 | 3.64 | 116 | 5.11 |
| | 90$^b$ | 0.70 | 2.70 | 0.52 | 1.24 | 18.4 | 1591 | 313 | 3.73 | 173 | 4.96 |
| | 99$^c$ | 0.60 | 1.75 | 0.47 | 1.01 | 4.21 | 1225 | 277 | 3.69 | 145 | 4.27 |
| | 124$^d$ | 0.60 | 1.75 | 0.51 | 1.09 | 3.96 | 1243 | 292 | 3.69 | 168 | 4.28 |
| | 156$^e$ | 0.50 | 1.50 | 0.50 | 0.94 | 7.98 | 1146 | 271 | 3.66 | 154 | 4.19 |
| | 168$^d$ | 0.60 | 1.75 | 0.58 | 1.11 | 5.87 | 1231 | 305 | 3.68 | 185 | 4.28 |
| | 210$^f$ | 0.20 | 1.35 | 0.56 | 0.93 | 23.5 | 1062 | 276 | 3.59 | 161 | 4.05 |
| | 318$^g$ | 0.80 | 1.95 | 0.60 | 0.85 | 9.00 | 1069 | 251 | 3.69 | 148 | 4.35 |
| $^6$Li$^+$+$^{13}$C | 54 | 0.08 | 2.76 | 0.54 | 0.77 | 21.4 | 1556 | 351 | 3.64 | 116 | 5.11 |
| $^6$Li$^+$+$^{16}$O | 50$^h$ | 0.50 | 2.81 | 0.55 | 0.60 | 13.4 | 1643 | 346 | 3.64 | 91 | 5.24 |
| $^7$Li$^+$+$^9$Be | 63 | 0.09 | 1.20 | 0.46 | 0.98 | 19.5 | 1538 | 274 | 3.64 | 152 | 4.80 |
| $^7$Li$^+$+$^{13}$C | 63 | 0.12 | 2.59 | 0.52 | 0.78 | 19.0 | 1652 | 335 | 3.74 | 113 | 5.07 |
| $^7$Li$^+$+$^{13}$C | 130$^i$ | 0.13 | 1.97 | 0.48 | 1.02 | 4.58 | 1392 | 280 | 3.73 | 146 | 4.50 |
| $^7$Li$^+$+$^{13}$Be | 130$^i$ | 0.12 | 2.34 | 0.50 | 1.23 | 7.98 | 1404 | 304 | 3.62 | 183 | 4.65 |
| $^{14}$N$^+$+$^{13}$C | 162 | 1.44 | 1.82 | 0.39 | 0.73 | 33.1 | 1563 | 220 | 4.29 | 89 | 4.66 |
| $^{10}$B$^+$+$^{9}$Be | 100 | 1.89 | 1.02 | 0.30 | 1.01 | 6.9 | 1266 | 185 | 4.33 | 146 | 4.08 |
| | 0.47 | 2.28 | 0.48 | 0.93 | 29.6 | 1558 | 298 | 3.75 | 133 | 4.79 |

Data from $^a$[37], $^b$[38], $^c$[39], $^d$[40], $^e$[41], $^f$[16], $^g$[17], $^h$[18], $^i$uniform 10% errors.