Viscoelastic fluid flow driven by non-propagative membrane contraction

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Abstract. A viscoelastic fluid flow model is developed to study the flow analysis under the application of membrane contraction followed by the contraction and relaxation. This analysis is carried out subject to slip boundary condition. In particular, study of the kinematic of membrane motion in the insect’s tracheal tube that induces the fluid flow. The impact of a single tracheal tube’s membrane contraction has been computed mathematically, which regulated as a wall channel geometrically. Jeffrey fluid model is considered to see the viscoelastic behaviour of the fluids. The lubrication theory is adopted at a low Reynolds number for the non-linear differential equations. The pertinent influences of critical parameters on the pressure distribution have been graphically depicted. It is observed that an increase in the Jeffrey fluid parameter causes the reduction in pressure difference. This model is a highly bioinspired pumping mechanism. This model’s outcomes can be very useful in the designs of smart pumps for various biomedical applications.

Keywords: Viscoelastic fluid, Membrane propagation, Trachea, Pressure difference.

1. Introduction
The transport/distribution of the non-Newtonian fluids in the physiological system by the pumping mechanisms are quite prevalent in nature. For illustration, the blood distributes in arteries and veins, the urine flow through ureters, the chyme moves in the gastrointestinal tract, the food transports through the oesophagus. Moreover, the gas exchange phenomena through the tracheal tube in the physiological system due to membrane contraction and relaxations mechanism have been a topic of immense interest for analysts. Its prominent applications in diverse areas like physical and biological sciences are reported in literature.

The selective membrane mechanism of the physical process of the insect respiratory system for the fluid exchange is an innovative novel pumping flow mechanism in the bioscience and industrial application. The physical process of the cockroach’s tracheal tube was visualized by Socha et al. [1] using an X-ray synchrotron image. The progression of fluid-transport proteins in the insects’ circulatory system has been scrutinized by Holler et al. [2]. It is clear from the survey that the rhythmic tracheal compression (RTC) tube performed both diffusion and convection transport during the process of gas exchange. Analysts experimentally found that the internal pressure gradient induces by the RTC which plays a vital role for enhancing both direct and tidal flow. These studies provided an idea to develop a novel pumping mechanism inspired by the ground beetle. In this regard, many mathematical models have been evolved [3-6]. Bhandari et al. [7] have theoretically studied the behavior of non-Newtonian fluid propagated by membrane contraction. Motivated by this model Tripathi et al. [8] addressed the electroosmosis flow of the Newtonian fluid in the presence of membrane motion in a microchannel. Continue in this way, Bhandari et al. [9] have scrutinized the influence of the magnetic field on the fluid flow with rhythmic membrane contraction. The evolution of this type of novel genetic pumping mechanisms in insect rhythmic tracheal system is addressed by
Hanna & Popadic [10]. In the last two decades, there have been developed various pumping models by considering different channel geometry and multiple types of fluids to construct better efficient actuators on control of flow [11-17].

The non-Newtonian fluid is also an essential topic for researchers to understand the rheological behavior of the fluids. Since, most of real fluids are non-Newtonian nature such as blood, plasma, larva, toothpaste, honey, paint, oil, fuel, etc. The viscoelastic characteristic of the fluid attracted the attention of the researchers to explore and develop a consistent model in the membrane filtration process. In view of this, Jeffrey fluid model is considered as fluid with viscoelastic properties. At the specific value of Jeffrey fluid parameter, viscoelastic fluid model reduces to the Newtonian fluid model. The fundamental characteristics of the Jeffrey fluids are related to relaxation and retardation time, which easily simulate non-Newtonian fluid. In view of this, Pandey and Tripathi [18] for the flow of Jeffrey fluid through the channel are presented by considering long wavelength. Nallapu et al. [19] have exhibited the two-fluid blood approach model to comprehend the blood circulation in arteries. They have considered Jeffrey-Newtonian fluids induced by the transverse magnetic field. Some other non-Newtonian fluid models have been developed to explore the bioscience and engineering applications [20-25].

The viscoelastic fluid flow induced by the single tracheal tube with the slip boundary condition is studied in this mathematical model. As per the literature available for the membrane pumping flow models, viscoelastic fluid with Jeffrey model has not been reported yet. The analytical approach with the aid of in-house MATLAB are utilized to explore the graphical results under the effects of viscoelastic parameter and slip parameter. Finally concluding remarks of the analysis have been presented.

2. Mathematical Formulation
2.1 Problem Definition
The membrane motion is assumed to produce the flow throughout the channel (see figure 1) which is mathematically expressed as:

\[
w_2(x, t) = \frac{1}{2} \left( 1 + \tanh(\alpha(x - x_1)) - \tanh(\alpha(x - (x_1 + d_1))) \right) (1 - \cos 2\pi t), \quad 0 \leq t \leq 1
\]

where \( w_2(x, t) \) is the dimensionless membrane contraction parameter represent the spatial and temporal variation of the channel geometry. Here, \( \alpha = \frac{2\pi}{\delta}, \quad x_1 \) denotes the starting point of contraction and \( d_1 \) is the longitude of the contraction. \( L, \ H \) define the channel length and height with \( \delta \equiv H/L \ll 1 \).

![Figure 1: The physical sketch of membrane motion in the channel](image)

2.2 Governing equations
The membrane contraction in the tracheal tube of the insect respiratory system is a kinematic propagation. In addition, this membrane motion induce fluid flow within slip flow regime in the two-dimensional microchannel. The velocity vector is assumed to be \( \vec{V} = (\vec{u}, \vec{v}, 0) \) corresponding to the axial and transverse direction. The continuity equation in the vector form can be written as:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0.
\]  

(2)

\[
\frac{\partial (\rho \vec{V})}{\partial t} = \nabla \cdot (\vec{\tau}) + \vec{b},
\]

(3)

where \( \rho \) is the density of the fluid, \( \vec{\tau} \) is the stress tensor and \( \vec{b} \) is the body force per unit volume is considered as negligible, \( t \) is the time, \( \vec{V} \) is the velocity. The constitutive equation of the Jeffrey fluid model is given as

\[
\vec{\tau} = -p\vec{I} + \vec{S},
\]  

(4)

\[
\vec{S} = \frac{\mu}{1 + \lambda_2 \frac{\partial}{\partial t}} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \vec{\gamma}
\]

(5)

\[
\lambda_1 = \frac{\text{relaxation}}{\text{retardation time}},
\]

(6)

\[
\vec{\gamma} = \nabla \vec{V} + (\nabla \vec{V})' - \frac{2}{3} \nabla \cdot \vec{V},
\]

(7)

where \( p \) is the pressure, \( \vec{I} \) is the identity tensor, and \( \vec{S} \) is the extra stress, \( \lambda_2 \) is the retardation time, \( \vec{\gamma} \) is the shear rate and the dot denotes the quantities indicate differentiation w. r. t. time, \( \mu \) is the dynamic viscosity. We further assume that the fluid is incompressible, unsteady with constant viscosity in the 2-dim channel. Using Eqs. (4-6), equations (2-3) can be rewritten as:

\[
\frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{v}}{\partial y} = 0,
\]

(8)

\[
\rho \left( \frac{\partial}{\partial t} + \vec{u} \frac{\partial}{\partial x} + \vec{v} \frac{\partial}{\partial y} \right) \vec{u} = -\frac{\partial \rho}{\partial x} + \frac{\partial \vec{S}_{xx}}{\partial x} + \frac{\partial \vec{S}_{xy}}{\partial y},
\]

(9)

\[
\rho \left( \frac{\partial}{\partial t} + \vec{u} \frac{\partial}{\partial x} + \vec{v} \frac{\partial}{\partial y} \right) \vec{v} = -\frac{\partial \rho}{\partial y} + \frac{\partial \vec{S}_{yx}}{\partial x} + \frac{\partial \vec{S}_{yy}}{\partial y},
\]

(10)

Rescaling the factors by introducing dimensionless parameters

\[
x = \frac{x}{l}, y = \frac{y}{h}, w_1 = \frac{w_1}{h}, w_2 = \frac{w_2}{h}, t = \frac{t}{c}, u = \frac{u}{c}, v = \frac{v}{c}, S = \frac{h \vec{S}}{\mu c}, \rho = \frac{\rho c h^2}{\mu}, \delta = \frac{H}{l}
\]

where, \( x, \ y, W, t, u, v, S, p \), stand counterparts of the dimensional parameters. \( Re = \frac{\rho c H}{\mu} \ll 1 \) is the Reynolds number based on wall amplitude. The governing equations after the non-dimension parameter can be written as:

\[
\frac{\partial S_{xy}}{\partial t} \bigg|_{\delta = 0} = \frac{1}{1 + \lambda_1 \frac{\partial}{\partial t} c^2} \frac{\partial^2 u}{\partial x^2},
\]

(11)

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \frac{\partial p}{\partial x} + \frac{\partial S_{xy}}{\partial y} = 0, \quad \frac{\partial p}{\partial y} = 0
\]

(12)

where the pressure \( p = p(x, t) \) distributes along with the axial direction. The slip boundary condition is employed as:

\[
u \big|_{y = w_1} = \frac{\beta}{\delta} \frac{\partial u}{\partial t}, \quad u \big|_{y = w_2} = -\frac{\beta}{\delta} \frac{\partial u}{\partial t}, \quad \nu \big|_{y = w_1} = 0, \quad \nu \big|_{y = w_2} = \frac{\partial w_2}{\partial t}
\]

\[
\frac{1}{1 + \lambda_1} \frac{\partial^2 u}{\partial x^2} = \frac{\partial p}{\partial x}.
\]

(13)
The slip flow parameter is defined by \( \beta = kn^{\frac{2-\sigma}{\sigma}} \), where \( \sigma \) is the tangential momentum accommodation coefficient and \( Kn = \lambda/L \) is the dimensionless Knudsen number. \( \lambda = \frac{RT}{\sqrt{2\pi}d^2NAp} \) is the mean free path of a fluid molecule, where \( R, T, d, N_A, p \) is the universal gas constant, temperature, diameter fluid molecule, Avogadro’s number and pressure respectively. Axial velocity profiles can inscribe the typical induced flow field.

\[
u = \frac{1}{2}(1 + \lambda) \frac{\partial p}{\partial y}(y^2 + \beta(w_z - w_y) - \frac{1}{2}(w_x^2 + w_y^2)).
\] (14)

The Volumetric flow rate is expressed as:

\[
Q = \int_{w_y}^{w_z} \int_0^{y} u dy
\]

which implies that

\[
\frac{\partial p}{\partial x} = 12 \frac{Q}{1 + \lambda)((w_1 - w_2)^3 - 6\beta(w_1 - w_2)^2)
\] (16)

The non-dimensional pressure rise per wavelength are defined, respectively, as

\[
\Delta p(t) = \int_0^1 \frac{\partial p}{\partial x} dx,
\] (17)

3. Numerical results and discussion

In this paper, our main objective is to observe the effect of the non-Newtonian fluid parameter and slip flow parameter. The influence of various parameters on the pressure gradient and pressure difference is numerically computed and illustrated through figures 3-7. The kinematics of non-propagative membrane motion by a generic expression \( w_2(x, t) \) is presented. The contraction and relaxation of the membrane profiles are composed over a complete cycle is shown in figure 2(a). The highest amplitude of the membrane contraction at time \( t=0.5 \) second is referred to as the transition phase. The pressure gradient and pressure can be calculated efficiently by the volume flow rate \( Q \) see in Eqs. 16. The structure of the membrane motion at various times in an inelastic channel is seen in figure 2. The present mechanism produces a small volume of fluid during a complete contraction cycle. Here, the various parameter values (like \( x_1 = .40, d_1 = .25, A = -0.352, \) and \( \delta = 0.2 \) ) are fixed to certifies the successive propagation at a different time instant. Figure 2(a) depicted the channel geometry of the membrane \( w_2(x, t) \) that ensures the continuous contraction over a periodic contraction cycle. This result confirms that the fluid distribution can be used for pumping flow. The rate of change of the membrane \( \partial w_2/\partial t \) at different time snapshots are shown in figure 2(b). These sketches illustrate that the membrane profile behavior is well-expressed & continuous and actuate as a “peristaltic” mode. Figures 2(a-b) characterizes the rhythmic membrane contraction, which generates a pumping mechanism in term of pressure gradient.
Figure 2: The membrane contraction at various time point using \( x_1 = .40, \ d_1 = .25, \ A = -0.352, \ \delta = 0.2 \) (a) non-propagation of membrane motions in a contraction domain (b) The rate of change of the membrane motion.

Figure 3: variation of the pressure gradient \( dp/dx \) at \( x_1 = .40, \ d_1 = .25, \ A = -0.352, \ \delta = 0.2, \beta = 0 \) (a) for Newtonian fluid (b) for Jeffrey fluid

The pressure difference can comprehend the pumping characteristic. There are two possibilities to observe the pressure difference. The first one is the internal pumping force that is produced by the membrane motion in the microchannel. Another one is, external pumping force applied on the microchannel. Pressure difference plays a significant role in fluid transport throughout the channel. Figures 3-7 illustrate the characteristic of channel geometry with the help of pressure gradient \( (\partial p/\partial x) \) and pressure difference \( (\Delta p) \) of the Jeffrey fluid model produced by the membrane motion. Here, we have considered \( \lambda_1 \) which represents the viscoelasticity of Jeffrey fluid. As \( \lambda_1 \equiv 0 \), the present Jeffrey model change its nature into the Newtonian fluid model. From this conclusion, the comparative correlation between the Newtonian fluid model and Jeffrey fluid model is shown in figure 3. The pressure gradient at the various time is also observed to understand the rheological behavior of the fluids. We have observed that the pressure gradient reduces as \( \lambda \) more than zeros. Moreover, a small amount of fluid transport can be achieved for the Jeffrey fluid model.

Another important parameter is slip parameter \( \beta \), which is applied on the upper wall of the channel due to membrane kinematic movement. As the slip parameter \( \beta = 0 \), the present slip flow model reduces to the no-slip flow model. Figure 4 illustrates that the distribution of the pressure gradient for different values of slip parameter. This result concludes that the pressure gradient accelerates as the slip parameter is increased. The pressure difference induces by the periodic membrane contraction which is demonstrated through figure 5. The maximum pressure has occurred for the Newtonian fluid
model. As $\lambda_1$ increases the pressure difference is demoted which characterizes that the mechanism is consistent for the Jeffrey fluid model.

Furthermore, the effect of the membrane on the axial pressure rise is depicted through figure 6. There is a sharp peak attained by the axial pressure rises throughout the channel. In pressure rise difference, $\lambda_1$ physically interprets that the highest value of the $\lambda_1$ leading to reduce pressure rise difference. Therefore, $\lambda_1$ is responsible for the fluid flow deceleration in the channel. The influence of the slip parameter on the pressure rise is depicted in figure 7. The minimum pressure rise is observed for the no-slip parameter $\beta = 0$. The fluctuation of the pressure rise due to the slip parameter with accelerate mode. These results conclude that the fluid pressure is increased with the increments of the slip parameter and demotes with the Jeffrey fluid parameter.

![Figure 4: Variation of the pressure gradient $dp/dx$ at $x_1 = 0.40$, $d_1 = 0.25$, $A = -0.352$, $\delta = 0.2$, $\lambda_1 = 0.5$ for various value of $\beta$.](image1)

![Figure 5: Pressure distribution $(p(x, t) - p(0, t))$ at $x_1 = 0.40$, $d_1 = 0.25$, $A = -0.352$, $\delta = 0.2$ for various value of $\lambda_1$.](image2)
Figure 6: The pressure rise distribution at $x_1 = .40$, $d_1 = .25$, $A = -0.352$, $\delta = 0.2$, $\beta = 0$ for various value of $\lambda_1$.

Figure 7: The pressure rise distribution at $x_1 = .40$, $d_1 = .25$, $A = -0.352$, $\delta = 0.2$, $\lambda_1 = 0.5$ for various value of $\beta$.

4. Conclusion

The present pumping flow model induced by internal membrane mechanisms is studied. The influence of Jeffrey fluid parameter, slip parameter on the pressure distribution are investigated in great detail. Some key observations are also recorded as:

- The contraction and relaxation of the membrane profiles are composed in an inelastic channel over a complete cycle.
- The continuous membrane profile actuates as a “peristaltic” mode to transport a small amount of the fluid.
- The pressure gradient is reduced as $\lambda_1$ increased.
- The pressure gradient accelerates as the slip parameter is increased at a fixed time snapshot.
- $\lambda_1$ is responsible for the fluid flow deceleration in the channel.
- There is a sharp peak attained by the axial pressure rises throughout the channel for $\lambda_1$.
- The minimum pressure rise is observed for the no-slip parameter $\beta = 0$.
- The fluctuation of the pressure rises due to increment in the slip parameter.

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