Effect of temperature variations on interfacial debonding of FRP-plated beams: A coupled mix-mode cohesive zone modeling

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Abstract. The use of fiber-reinforced polymer (FRP) plates for the strengthening of structural members involves their exposure to service temperature variations (i.e., thermal loadings). Such temperature variations may lead to thermal stress and the interfacial debonding of the FRP-plated beam as a result of the thermal mismatch between the FRP plate and the substrate beam. There is lacking research related to the effect of temperature variations on the interfacial debonding of FRP-plated beams. This paper presents the first-ever analytical study on the behavior of the FRP-plated beam under combined thermal and mechanical loadings. The interfacial stress and slip evolutions as well as the debonding process of the FRP-plated beam are estimated based on a coupled mix-mode cohesive zone modeling. In addition, a finite element (FE) approach is proposed to verify the analytical solution in which the analytical results are compared with the FE model predictions, indicating that a good agreement was achieved between the analytical and FE results.

Keywords: Fiber reinforced polymer (FRP); Interfacial stress; Cohesive zone modeling; Mixed mode; Temperature variations; Plate end debonding.

1. Introduction

Fiber-reinforced polymer (FRP) composites have been increasingly used for the repairing and strengthening of infrastructure elements. The main advantages of the FRP composites include their high strength-to-weight ratio, excellent corrosion resistance, and ease of application [1-2]. One of the most common strengthening methods involves the flexural strengthening of deficient structural members with externally bonded FRP laminates or sheets [3-4] (hereafter referred to as plates for simplicity of presentation). The FRP-plated beams in service are likely to experience significant temperature variations (i.e., thermal loadings) due to the seasonal ambient temperature changes. Such temperature variations may induce the thermal stresses at the FRP-to-substrate interface due to the thermal mismatch between the FRP plate and the substrate (e.g., steel, concrete, or timber), which may further influence the debonding process of the FRP-plated beam under combined thermal and mechanical loadings [5-6]. However, limited research studies are available on the effect of temperature variations on the debonding failure of FRP-to-substrate interfaces [7-10]. Among the limited studies, most of them were focused on the bond behavior of the FRP-to-substrate bonded joints under combined thermal and mechanical...
loadings [7-9]. This paper presents the first-ever analytical study on the effect of temperature variations on the interfacial debonding of FRP-plated beams. In the existing literature, the cohesive zone model was usually used for predicting the interfacial stresses and debonding failure of FRP-plated beams under mechanical loading solely [11-12]. This paper follows the research initiated in [12] and moves forwards the understanding of the behavior and debonding process of the FRP-plated beam under combined thermal and mechanical loadings. Details of the closed-form analytical solution using a coupled mixed-mode cohesive zone modeling approach are provided in the following sections.

2. Research significance
Although some analytical studies in the literature were conducted on the interfacial debonding of FRP-plated beams under mechanical loading only [11-14], there is lacking research on the effect of temperature variations on the debonding process of such beams. This paper aims to provide an analytical solution for the first time on the debonding process of the FRP plated beams under combined mechanical and thermal loadings. Within the analytical solution, the interfacial thermal stress induced by the differences in the thermal expansion coefficients between the FRP plate and the substrate beam is considered. Such an analytical solution is expected to be useful to obtain an in-depth understanding of the thermal stress and the debonding failure of FRP-plated beams when they are subjected to seasonal ambient temperature changes.

3. Theoretical background and analytical solution
The analytical solution and the nomenclature parameters for a FRP-plated beam subjected to three-point bending as presented in this paper, is shown in figure 1. As the analysis is related to the plate end debonding process, all the materials involved at the interface are assumed as linear elastic following the assumption adopted by De Lorenzis et al. [12]. The cohesive zone (CZ) model defined for the bond interface is illustrated in figure 2, in which the coupled damage CZ model proposed by previous studies [12-13, 15] is adopted. Bilinear bond-slip models were implemented as they best suitable for capturing the nonlinear characteristics of the bond interface [7-8, 16-18].

Figure 1. Simply supported FRP-plated beam under three-point bending load.

Figure 2. Mixed-mode cohesive zone laws. (a) Mode I. (b) Mode II.
The bond-slip relationships considered for the normal and shear directions of the bond interface under a coupled mix-mode loading can be defined as follows:

$$\sigma = \begin{cases} 
K_N \delta_N & \delta_M \leq \delta_M^0 \\
(1-d)K_N \delta_N & \delta_M^0 < \delta_M < \delta_M^f \\
K_N(\delta_N) & \delta_M > \delta_M^f 
\end{cases}$$

$$\tau = \begin{cases} 
K_T \delta_T & \delta_M \leq \delta_M^0 \\
(1-d)K_T \delta_T & \delta_M^0 < \delta_M < \delta_M^f \\
0 & \delta_M > \delta_M^f 
\end{cases}$$

The strains of the bottom fiber of the beam and of the top fiber of the plate can be obtained from the equilibrium equations:

$$\varepsilon_1 \equiv \frac{d \varepsilon_1}{dx} = \frac{1}{E_1} M_1(x) - \frac{1}{E_{11}} N_1(x) + \alpha_{T1} \Delta T$$

$$\varepsilon_2 \equiv \frac{d \varepsilon_2}{dx} = \frac{1}{E_2} M_2(x) + \frac{1}{E_{22}} N_2(x) + \alpha_{T2} \Delta T$$

The strain solutions presented in equation (3) and equation (4) are different from those in done by De Lorenzis et al. [12] as they include the thermal strains due to the thermal mismatch between the FRP and the substrate beam. Due to the space limitation, the detailed process on how to obtain the solution for the three stages (namely elastic stage, elastic-softening stage, and elastic-softening-debonding stage) is not given in the conference paper; the results from the analytical solution are presented herein to illustrate the effect of thermal loading on the plate end debonding of FRP-plated beams.

### 3.1. Interfacial stress and slip components for the elastic stage (E-stage).

The shear stress and slip components can be obtained as follows:

$$\tau_E(x) = B_1 \cosh \lambda x + B_2 \sinh \lambda x + m_1 F$$

$$\delta_{TE}(x) = \frac{1}{K_T} (B_1 \cosh \lambda x + B_2 \sinh \lambda x + m_1 F)$$

where the constants in the above equations are determined as follows:

$$B_1 = \frac{1}{\lambda} \left[ \frac{m_2 a}{2} F - K_T (\alpha_{T2} - \alpha_{T1}) \Delta T \right] \tanh \lambda L - m_1 F \frac{E_{22}}{2 \cosh \lambda L}$$

$$B_2 = \frac{1}{\lambda} \left[ \frac{m_2 a}{2} F + K_T (\alpha_{T2} - \alpha_{T1}) \Delta T \right]$$

Similarly, for the normal stress and slip components, the equations are

$$\sigma_E(x) = (C_1 \cos \beta x + C_2 \sin \beta x) \exp(-\beta x) - \eta_1 \lambda (B_1 \sinh \lambda x + B_2 \cosh \lambda x)$$

$$\delta_{NE}(x) = \frac{1}{K_N} \left[ (C_1 \cos \beta x + C_2 \sin \beta x) \exp(-\beta x) - \eta_1 \lambda (B_1 \sinh \lambda x + B_2 \cosh \lambda x) \right]$$

where the constants in the above equations are determined as follows:

$$C_1 = \frac{K_N F}{4 \beta^3 E_{11}} \left( 1 + \beta a \right) + \eta_2 \lambda \frac{m_1 F}{2 \beta^2} + \frac{K_T (\alpha_{T2} - \alpha_{T1}) \Delta T}{2 \beta^2}$$

$$C_2 = \frac{K_N F}{2 \beta^3 E_{11}} \left( \frac{m_2 a}{2 \beta^3} + (\alpha_{T2} - \alpha_{T1}) \Delta T \right) - \eta_1 \lambda \frac{B_2 F}{2 \beta^3}$$

where $\tau_E(x)$, $\sigma_E(x)$, $\delta_{TE}(x)$, and $\delta_{NE}(x)$ relates to the shear and normal stresses in addition to their respective relative slip within the elastic range. $K_T$ and $K_N$ are the elastic interfacial stiffness in the shear and normal direction respectively, while $G_a$ and $E_a$ are the shear and elastic modulus of the adhesive.
3.2. Determination of the load corresponding to the onset of softening.
When considering the mixed mode effects, the onset of softening occurs upon the fulfilment of the quadratic stress failure criterion which can be described in equation (13). Assuming that the maximum stresses (at \( x = 0 \)) are reached and therefore causes the onset of softening, the maximum values for shear and normal stresses would produce equation (14). After substituting the relevant constants and terms, equation (14) could be simplified to equation (15), where \( K_1 \) to \( K_6 \) are all constants and the force corresponding to the onset of softening could be easily computed.

\[
\left( \frac{x}{\tau_p} \right)^2 + \left( \frac{\sigma}{\sigma_p} \right)^2 = 1 \tag{13}
\]

\[
\left( \frac{B_1 + m_1 \tau}{\tau_p} \right)^2 + \left( \frac{C_1 - \eta_1 \lambda B_2}{\sigma_p} \right)^2 = 1 \tag{14}
\]

\[
\frac{K_1 F^2 + K_2 F + K_3}{\tau_p^2} + \frac{K_4 F^2 + K_5 F + K_6}{\sigma_p^2} = 1 \tag{15}
\]

3.3. Interfacial stress and slip components for the elastic-softening stage (E-S stage).
If the applied load is greater than the load to cause the onset of softening, \( \delta_{T_e, \text{max}} \) is larger than the tangential relative slip at the onset of mixed-mode softening (\( \delta_{\text{mm}}^0 \)) and/or \( \delta_{N_e, \text{max}} \) is larger than the normal relative slip at the onset of mixed-mode softening (\( \delta_{\text{mm}}^0 \)), then the equations for the elastic stage would not be valid due to the length of softening, denoted as (\( \overline{x} \)). Therefore, the equations given below are for the analytical solution of the E-S stage.

Step 1. By defining the displacement-based mode-mixity ratio [equation (16)] according to De Lorenzis et al. [12], a value for \( \delta_{T_e, \text{max}} \) is firstly selected, and the corresponding \( \delta_{N_e, \text{max}} \) can be calculated by equation (16). Also, since \( \delta_{T_e, \text{max}} \) is known, the value can be applied to its definition as observed in equation 17.

\[
\gamma_{e, \text{min}} = \gamma_e(x) = \gamma_e(0) = \frac{\delta_{T_e, \text{max}}}{\delta_{N_e, \text{max}}} = \frac{k_m (B_1 + m_1 \frac{\tau}{2})}{k_f (C_1 - \eta_1 \lambda B_2)} \tag{16}
\]

\[
\delta_{T_e, \text{max}}(x) = \frac{1}{k_f} (B_1 + m_1 \frac{F}{2}) \tag{17}
\]

Step 2. Rearranging equation (17) and by substituting \( B_1 \) into the equation, the expression for the force component can be found. This component is then substituted into the shear stress equation for the elastic stage [equation (5)] to produce equation (18). After repeating these steps to obtain the normal stress force component (\( F_{\text{nmax}} \)), this expression is then also substituted into the normal stress equation for the elastic stage [Equation (9)].

Shear stress force component, \( F_{te} = \frac{2k_f \delta_{T_e, \text{max}}(x) + k_{\text{inh}T}(F_{t2} - a_{T1}) \Delta T}{(m_{\text{inh}T} + m_{\text{T1}})} \) \tag{18}

Step 3. After having all known terms in equations (5) and (9), the shear and normal stress equations would be inserted into equation (13) would now take the form of equation (19) and the length of softening (\( \overline{x} \)) could now be computed.

\[
\left( \frac{x}{\tau_p} \right)^2 + \left( \frac{\sigma}{\sigma_p} \right)^2 = 1 \tag{19}
\]

Step 4. Having \( \overline{x} \), \( \delta_T \), \( \delta_N \), total mixed-mode relative displacements (\( \delta_m^0 \)), mode-mixity ratio (\( \gamma \)), total mixed-mode relative displacement at the onset of softening (\( \delta_{\text{mm}}^0 \)), mixed-mode critical total displacement (\( \delta_m^f \)) and the damage evolution (\( d \)) can be all calculated as functions of \( x \). Further information on these equations can be found in the previous study conducted by De Lorenzis et al. [12] to analyze the FRP-plated beam under mechanical loading solely.
3.4. Determination of the ultimate load.

The approximate ultimate load which includes the thermal stress effects is given in the following equation. For the load which corresponds to the length of softening, equation (20) can be used in its current form. For obtaining the ultimate load, equation (20) should be maximized with respect to \( \bar{x} \).

\[
F = \frac{2}{\cos \lambda \bar{x}} \left[ \frac{\tau_p (\tan \lambda \bar{x} + \coth (\lambda (1 - \bar{x})))}{\alpha_1 (\alpha_2 - \alpha_1)} \Delta T \left( \frac{\sin \lambda \bar{x}}{\cos \lambda \bar{x}} \right) \right]
\]

(20)

4. Finite element (FE) model

A 3D FE model using the general purpose software ABAQUS was proposed to verify the analytical solution as stated above. A CFRP-plated steel beam was taken as an example using the geometry, material and interface data which were taken from the previous study [12]. The specimen is a 127×76×13 steel beam, which had a 1.2m long and a clear span of 1.1m. A 3-mm-thick carbon FRP (CFRP) plate was bonded to the bottom of the steel beam with an epoxy bonding adhesive (1mm thick). More details of the geometrical dimensions of the steel beam are listed in Table 1, while the mechanical and bond properties are depicted in Table 2. As previously stated, the cohesive zone model was implemented within the FE model and could be observed in figure 3. In the FE model, the steel beam and the CFRP plate were modeled using shell elements. The bond interface between them was modelled using a cohesive interface element. The coefficient of thermal expansion for the CFRP plate and steel beam was taken as 0/°C [7] and 12×10^{-6} /°C [8], respectively.

**Table 1.** Details of the geometry for FRP-strengthened steel beam

| \( b_2 \) (mm) | \( y_1 \) (mm) | \( y_2 \) (mm) | \( A_1 \) (mm²) | \( A_2 \) (mm²) | \( I_{1v} \) (mm⁴) | \( I_2 \) (mm⁴) | \( t_a \) (mm) | \( l \) (mm) | \( a \) (mm) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 76             | 63.5           | 1.5            | 1602           | 228            | 4588602        | 171            | 1              | 150            | 400            |

**Table 2.** Details which consists of the material properties and interfacial data

| \( E_1 \) (N/mm²) | \( \tau_p \) (N/mm²) | \( \delta_0^t \) (mm) | \( \delta_f^t \) (mm) | \( G_{IIc} \) (N/mm²) | \( \alpha_1 \) (°C) | \( E_2 \) (N/mm²) | \( \sigma_f \) (N/mm²) | \( \delta_0^r \) (mm) | \( \delta_f^r \) (mm) | \( \alpha_2 \) [°C] |
|-------------------|----------------------|------------------------|------------------------|------------------------|----------------------|-------------------|-----------------------|------------------------|------------------------|----------------------|
| 205000            | 26.73                | 0.0526                 | 0.1197                 | 1.60                   | 12×10^{-6}          | 212000           | 29.70                  | 0.0037                 | 0.0044                 | 0                    |

**Figure 3.** The FE model.
5. Comparison with FE results
Since the stresses vary across the interface, the FE results examined include the interfacial stresses at a distance of 5 mm from the side and at the mid-width of the beam, respectively (the curves are labeled as “side” and “mid-width” in figure 4). A very close agreement between the analytical solutions and the FE results can be observed in Figures 4a and 4c when the FRP-plated beam under mechanical loading only (i.e., ΔT=0°C). As the temperature variation of 50 °C is taken into account, the results presented in figures 4b and 4d are still satisfactory in addition to the lengths of softening being close to each other. In figure 4a, the applied load is less than the softening load (i.e., the loading corresponding to the onset of softening) and hence the interface is within the elastic stage. However, due to the presence of thermal loading, the stresses shown in figure 4b indicate that the interface enters into the elastic-softening stage at the same applied load. Figures 4c and 4d show the stress responses of the interface (i.e., the elastic-softening stage) under a much higher applied load. It is clearly seen that the thermal stresses can not only influence the interface stress responses but also result in the reductions in the softening load.

![Figure 4](image_url)

**Figure 4.** Normalized interfacial stresses ς/ς₀ and σ/σ₀ along the CFRP length. (a) F = 85kN, ΔT = 0°C. (b) F = 85kN, ΔT = 50°C. (c) F = 121kN, ΔT = 0°C. (d) F = 121kN, ΔT = 50°C.

6. Parametric Study
In this section, the verified analytical solution is used to conduct a parametric study by examining the thickness and length of the FRP plate on the softening load of the FRP-plated beam under combined thermal and mechanical loadings. Figure 5 presents the effects of FRP thickness and length on the softening load as the temperature changes. From this parametric study, it is seen that a thinner and longer
FRP plate has experienced more load changes with the temperature variations. In addition, the softening load has changed from around 140-170 kN at a minus 50 °C variation to around 60-70 kN at a positive 50 °C variation.

![Figure 5](image)

**Figure 5.** Effect of temperature variations on the softening load. (a) Effect of CFRP plate thickness. (b) Effect of CFRP plate length.

7. Concluding remarks
This paper presented an analytical solution for predicting the interfacial stress and slip evolutions as well as the debonding process of the FRP-plated beam under combined thermal and mechanical loadings. In the analytical model, a coupled mix-mode cohesive zone model with bilinear traction-separation relationships was used to describe the bond behavior of the FRP-to-substrate interface. The thermal stress induced by the different thermal expansion coefficients of the FRP plate and the substrate beam was the primary focus of the present study. Closed-form expressions for the interfacial stresses, the interfacial slips, the load corresponding to the onset of softening and the ultimate load were determined by the analytical solution. The analytical results indicated that the thermal loading had significant effects on the interfacial stress and slip evolutions as well as the loading behavior of the FRP-plated beam. Besides, an FE model was proposed to verify the analytical solution. The comparisons between the analytical results and the FE model predictions indicated that a good agreement was achieved between them, which further demonstrated the accuracy and reliability of the analytical solution. In addition, a parametric study was conducted to investigate the effects of two main design parameters (i.e., thickness and length of the CFRP plate) have on the performance of the FRP-plated beams under combined thermal and mechanical loadings. The results showed that the thermal loading had a significant effect on the softening loads of the FRP-plated beams. More research work is needed to propose a predictive model for the load capacity of the FRP-plated beam under combined thermal and mechanical loadings, which will be expressed as a function of various design parameters.

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