Magnon Trap by Chiral Spin Pumping

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Chiral spin pumping is the generation of a unidirectional spin current in ferromagnetic films by dynamic dipolar stray fields from close-by nanomagnets. We formulate the theory of long-range chiral interactions between magnetic nanowires mediated by unidirectional spin waves in a magnetic film. We predict that two magnetic nanowires, of which one is actuated by microwaves, can trap spin waves. When both nanomagnets are excited by a uniform microwave, the interaction induced by the film magnons creates an imbalance in their magnon numbers.

Introduction.—Unidirectional propagation of quasiparticles is an interesting fundamental phenomenon with practical interest for information processing such as logic devices [1–6]. Similar to electrons and photons, magnons carry an intrinsic angular momentum that can transport spin information [7–10]. A unidirectional spin current carried by magnons can be generated in a ferromagnetic film by dynamic dipolar stray fields of ferromagnetic nanostructures in its proximity [11–14]. Magnons can propagate over centimeters [15] in magnetic insulators such as yttrium iron garnet (YIG) without Joule heating. While electron states are easily controlled or even trapped by electric gates, electric control of magnons on a small length scale is difficult. Magnons are trapped by inhomogeneous magnetic fields in, e.g., a spin-polarized atomic hydrogen gas [16] and superfluid 3He-B [17, 18]. Existing magnon transistors [19, 20] cannot fully trap magnons in the film because of inefficient gates.

In this Letter, we propose unique functionalities of chiral spin pumping of spin waves [21] in a device consisting of two magnetic transducers (nanowires) on top of a high-quality magnetic insulator such as YIG (Fig. 1). By exciting one of the nanowires by a local (evanescent) microwave source, spin waves actuated by the first wire interact with the second one (which does not see the microwaves) and excite its magnetization, which in turn emits spin waves as well. The relative phase shift of the magnetizations in the two wires is \( \pi + \phi_k \), where \( \phi_k \) is the transmission phase of the spin waves in the film. The phase shift \( \pi \) is caused by twice the dissipative phase shift at the resonance of two identical nanowires. When the spin waves from both sources interfere destructively outside the two wires, the nanowires form a magnonic cavity that traps the spin waves by geometrical interference, irrespective of the distance \( L \) or phase \( \phi_k \). Since spin waves between the two wires do not form energy-consuming standing waves, this mechanism implies nearly perfect spin and energy transfer between the wires.

Model and formalism.—We consider the effectively one-dimensional model in Fig. 1 with two sufficiently long magnetic nanowires (thickness \( d \) and width \( w \)) on top of a thin YIG film of thickness \( s \). The latter is of the order of tens of nanometers, such that the excited magnetization is distributed uniformly across the film [12, 13]. The distance between the nanowires is \( L \gg w \). Magnons in the nanowires are excited and detected by local metal stripline antennas on top of the nanowires that have a small rectangular cross section and uniform AC electric current distributions [21]. The interlayer exchange interaction between the wire and film has been found to be smaller than the dipolar one in the antiparallel configuration [12, 13], and can be further suppressed by a spacer without affecting the longer-ranged dipolar coupling.

The magnetization \( \mathbf{M}_l \) in the \( l \)-th magnetic wire couples to the dipolar field \( \mathbf{h} \) emitted by the magnetization \( \mathbf{M} \) of the film via the Zeeman interaction [22]

\[
\hat{H}_{\text{int}} = -\mu_0 \int_0^s dx d\rho \tilde{M}_{l,\beta}(x, \rho) h_\beta(x, \rho),
\]

where \( \mu_0 \) is the vacuum permeability and we use the summation convention over repeated Cartesian indices \( \beta = \{ x, y, z \} \). The equilibrium magnetizations \( \mathbf{M}_l \) in the

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$-\hat{z}$ direction are anti-parallel to $\mathbf{M}$. We focus on the linear regime at temperatures far below the critical one. To leading order, the magnetization operators in the magnetic wires and film may be expanded \[23, 24\]

\[
\hat{M}_\alpha(r) = -\sqrt{2M_s\gamma\hbar} \sum_k \left( m_{\alpha}^\kappa(x) e^{ik\cdot\hat{r}} + \text{H.c.} \right),
\]

\[
\hat{M}_{\alpha,l}(r) = -\sqrt{2M_s\gamma\hbar} \left( m_l^\kappa_{\alpha}(r) \hat{\beta}_l + \text{H.c.} \right),
\]

where $M_s$ and $M_{s,l}$ are the saturated magnetizations of film and nanowire, $-|\gamma|$ is the electron gyromagnetic ratio, $m_{\alpha}^\kappa(x)$ and $m_l^\kappa_{\alpha}(r)$ represent, respectively, the amplitudes of the spin waves in the film and the Kittel modes in the wires, while $\hat{\alpha}_k$ and $\hat{\beta}_l$ denote the magnon operators in the film and nanowire, and $k$ denotes $k_y$. The $l$-th nanowire centered at $r_l$ = $R_l\hat{y}$ feels the dipolar field $\hat{h}_\beta(r) = 1/(4\pi)\hat{\partial}_\beta\partial_\alpha \int \text{d}r' \hat{M}_\alpha(r')/|r' - r|$ from the film. It is chiral in the sense that, when generated by the right (left) propagating (exchange) spin waves, it vanishes below (above) the film \[14\]. Its precession direction is opposite to that of the magnetization. The magnetic nanowires on top of the film therefore couple only to the spin waves propagating in one direction (see below), i.e., the coupling is chiral \[13\].

The total Hamiltonian of our system then reads \[14\]

\[
\hat{H}/\hbar = \sum_l \omega_{K,l} \hat{\beta}_l^\dagger \hat{\beta}_l + \sum_k \omega_{\alpha} \hat{\alpha}_k^\dagger \hat{\alpha}_k \\
+ \sum_l \sum_k \left( g_{k,l} e^{-ikR_l} \hat{\beta}_l \hat{\alpha}_k^\dagger + g_{k,l}^* e^{ikR_l} \hat{\beta}_l^\dagger \hat{\alpha}_k \right),
\]

where the coupling constants

\[
g_{k,l} = -F_l(k) \left( m_{x}^{(k)*}, m_{y}^{(k)*} \right) \left( \begin{array}{c} |k| \\ i\hbar \\ -|k| \end{array} \right) \left( \begin{array}{c} \tilde{m}_{x}^\kappa_{l,}\alpha \\ \tilde{m}_{y}^\kappa_{l,}\alpha \end{array} \right),
\]

are real and the form factor

\[
F_l(k) = \frac{2\mu_0\gamma}{k^3} \sqrt{M_s M_{s,l}} \frac{\mathcal{L}}{\hbar} (1-e^{-|k|d})(1-e^{-|k|s}) \sin \left( \frac{kw}{2} \right).
\]

$\mathcal{L}$ is the (sufficiently large) length of the magnetic nanowire. By tuning the magnetic field to change the resonant momentum $k$ of the spin waves to the Kittel mode, the factor $\sin(kw/2)$ allows for tuning of the dipolar coupling strength. The normalized amplitudes and energy dispersions of the film $\omega_{\alpha}$ and nanowire magnons $\omega_{K,l}$ are addressed in Refs. \[3, 14\]. Chiral coupling is reflected by $g_{-|k|} = 0$ for the circularly polarized spin waves with $m_{y}^{(k)} = i m_{x}^{(k)} (M \parallel \hat{z})$ \[14\]. The general quantum description between harmonic oscillators is readily applied to other chiral coupled systems, such as magnetic spheres and waveguide photons \[32\], or electric dipoles and surface plasmons \[3\].

We now consider two identical magnetic nanowires located at $r_1 = R_1\hat{y}$ and $r_2 = R_2\hat{y}$, which act as transducers for microwaves that are emitted or detected by local microwave antennas. They communicate by exciting and absorbing magnons in the film. Hereafter, $g_{k,1} = g_{k,2} = g_k$ and $\omega_{K,1} = \omega_{K,2} = \omega_K$. Expressing the local magnon operators at $R_1$ and $R_2$ by $\hat{\beta}_1$ and $\hat{\beta}_2$, the equations of motion of the coupled nanowires and film read \[25, 26\]

\[
i \frac{d\hat{\beta}_1(t)}{dt} = \omega_K \hat{\beta}_1(t) + \sum_k g_k e^{ikR_1} \hat{\alpha}_k(t) - i \frac{\kappa + \kappa_p(1)}{2} \hat{\beta}_1(t)
\]

\[
- i \sqrt{\kappa_p(1)\hat{P}_m(1)}(t),
\]

\[
i \frac{d\hat{\beta}_2(t)}{dt} = \omega_K \hat{\beta}_2(t) + \sum_k g_k e^{ikR_2} \hat{\alpha}_k(t) - i \frac{\kappa + \kappa_p(2)}{2} \hat{\beta}_2(t),
\]

\[
- i \sqrt{\kappa_p(2)\hat{P}_m(2)}(t),
\]

\[
i \frac{d\hat{\alpha}_k(t)}{dt} = \omega_k \hat{\alpha}_k(t) + g_k e^{-ikR_1} \hat{\beta}_1(t) + g_k e^{-ikR_2} \hat{\beta}_2(t)
\]

\[- i (\kappa_k/2) \hat{\alpha}_k(t).
\]

Here, $\kappa = 2\alpha_G\omega_K$ is the intrinsic damping of the Kittel modes in the nanowires parameterized by the Gilbert coefficient $\alpha_G$, $\kappa_p(i)$ is the additional radiative damping induced by the microwave photons $\hat{P}_m(i)$, i.e., the coupling of the nanowire with the microwave antennas, and $\kappa_k$ denotes the intrinsic Gilbert damping of magnons with momentum $k$ in the films.

We can understand the long-range chiral interaction between the two wires by integrating out the magnon dynamics in the film. In the Markov approximation in Eq. \[4\] we arrive at the equations of motion for the coherent amplitudes $\hat{\beta}_1$ and $\hat{\beta}_2$ mediated by the spin waves in the film,

\[
\frac{d\hat{\beta}_1}{dt} = -i \left( \omega_K - i\frac{\kappa}{2} \right) \hat{\beta}_1 - \Gamma(\omega) \hat{\beta}_1 - \Gamma_{12}(\omega) \hat{\beta}_2 + \hat{P}_1,
\]

\[
\frac{d\hat{\beta}_2}{dt} = -i \left( \omega_K - i\frac{\kappa}{2} \right) \hat{\beta}_2 - \Gamma(\omega) \hat{\beta}_2 - \Gamma_{21}(\omega) \hat{\beta}_1 + \hat{P}_2.
\]

Here, we have disregarded the radiative damping $\kappa_p$ that is usually much smaller than $\kappa$. \[\hat{P}_l \equiv -\sqrt{\kappa_p(1)\hat{P}_m(1)}\] represent the input terms from the local antennas $\hat{P}_m(i)$ to the Kittel magnons. With $R_2 > R_1$ in mind, the couplings between wires read

\[
\Gamma_{12}(\omega) = \frac{1}{v(k_\omega)} |g_{k_\omega}|^2 e^{i k_\omega (R_1 - R_2)},
\]

\[
\Gamma_{21}(\omega) = \frac{1}{v(k_\omega)} |g_{-k_\omega}|^2 e^{i k_\omega (R_1 - R_2)},
\]

and the self-interaction

\[
\Gamma(\omega) = \frac{1}{2v(k_\omega)} (|g_{k_\omega}|^2 + |g_{-k_\omega}|^2)
\]
is the additional damping for a single nanowire due to the chiral spin pumping [14]. Here, \( v(k) \) is the group velocity of the spin waves in the film and \( k_\omega \) is the positive root of \( \omega_k = \omega_k \) at the ferromagnetic resonance (FMR), \( |\Gamma_{12}(\omega)| \neq |\Gamma_{21}(\omega)| \) since \( g_{|k|} \neq g_{-|k|} \), implying the (partially) chiral dissipative coupling [27, 39]. In the fully chiral limit with, e.g., \( g_{-k} = 0 \), \( |\Gamma_{12}(\omega)| = 2\Gamma(\omega) \), i.e., twice the magnon broadening by chiral pumping Eq. (7). When one of the couplings is exactly zero, one wire can influence the other wire but without backaction. This breaks the reciprocity of the interaction and promises new functionalities as addressed below.

**Magnon trap by magnetic field.**—We now only turn on \( B_{in}^{(1)} \) and calculate the excited magnetization in the film. From Eq. (4), in frequency space and the chiral limit we have

\[
\hat{\alpha}_k(\omega) = G_k(\omega) g_k \left( e^{-ikR_1} \hat{\beta}_1(\omega) + e^{-ikR_2} \hat{\beta}_2(\omega) \right),
\]

\[
\hat{\beta}_2(\omega) = -\frac{i}{\Gamma(\omega)} \sum_k g_k^2 G_k(\omega) e^{ik(R_2-R_1)} \hat{\beta}_1(\omega),
\]

where \( G_k(\omega) = 1/[(\omega - \omega_K) + ik/2] \) is the magnon Green’s function. Equation (8) gives the phase relation between \( \hat{\beta}_2 \) and \( \hat{\beta}_1 \) when the left nanowire is excited. At the FMR,

\[
\hat{\beta}_2(\omega_K) = \eta(\omega_K) e^{i\pi + ik_r(R_2-R_1)} \hat{\beta}_1(\omega_K),
\]

where \( k_r \) is the positive root of \( \omega_k = \omega_K \), and

\[
\eta(\omega_K) = \frac{2\Gamma(\omega_K)}{\kappa/2 + \Gamma(\omega_K)}
\]

modulates the magnitude of the excited magnon amplitude. This corresponds to a phase shift

\[
\Delta \phi = \pi + k_r(R_2 - R_1)
\]

between the two nanowires. \( k_r(R_2 - R_1) \) is the phase delay by the spin wave transmission between the two wires. The phase shift of \( \pi \) reflects the doubled phase shifts \( \pi/2 \) between magnons in the nanowire and film that is the key for the magnon trap addressed below. We have recently reported observation of this phase shift by microwave spectroscopy [21].

The phase relation Eq. (9) implies trapping of magnons at the FMR when \( \eta(\omega_K) \to 1 \), i.e., when the additional damping and the intrinsic damping are comparable. At the FMR, the excited magnon amplitude with momentum \( k_r \) reads

\[
\langle \hat{\alpha}_{k_r}(\omega_K) \rangle = G_{k_r}(\omega_K) g_{k_r} e^{-ik_r R_1} \langle \hat{\beta}_1(\omega_K) \rangle (1 - \eta(\omega_K)),
\]

which indicates suppression of the right-propagating waves when \( \eta(\omega_K) \to 1 \). Meanwhile, the left-propagating waves are not excited due to the nature of chiral coupling. Therefore, the excited spin waves are trapped in the right nanowire. By tuning \( \eta \) via the magnetic field one can modulate the transport of spin waves in the film as well (see below).

We now calculate the excited magnetization in real space. At the FMR \( \omega \to \omega_K \), the magnetization in Eq. (2) is the real part of

\[
\tilde{M}_\alpha(x, y) = -2\sqrt{2m_s\gamma \hbar \beta_1(\omega_K)} \sum_k m_{\alpha}^{(k)}(x) G_k(\omega_K) g_k
\]

\[
\times \left( e^{-ik(R_1-y)} - \eta(\omega_K)e^{ik_r(R_2-R_1)} e^{-ik(R_2-y)} \right),
\]

in which the \( k \)-integral can be carried out by closing the contour in the complex plane. Since \( R_2 > R_1 \), the singularities in the denominator of \( G_k \) are \( k^*_\gamma = \pm (k_r + i\epsilon) \), where \( \epsilon \) is the inverse of the propagation decay length. When \( y < R_1 < R_2 \), the integral path is chosen in the lower half plane that selects the singularity \( k^{*+}_\gamma \), leading to

\[
\tilde{M}_\alpha^L(x) = \frac{2i}{v_{kr}} \sqrt{2m_s\gamma \hbar \beta_1(\omega_K)m_{\alpha}^{(k)}(x)} g_{k_r}
\]

\[
\times \left( e^{ik_r(R_1-y)} - \eta(\omega_K)e^{ik_r(2R_2-R_1-y)} \right),
\]

which vanishes when \( \eta(\omega_K) \to 1 \). Finally, when \( R_1 < y < R_2 \),

\[
\tilde{M}_\alpha^R(x) = \frac{2i}{v_{kr}} \sqrt{2m_s\gamma \hbar \beta_1(\omega_K)m_{\alpha}^{(k)}(x)} g_{k_r}
\]

\[
\times e^{-ik_r(R_1-y)} (1 - \eta(\omega_K)),
\]

which vanishes when \( \eta(\omega_K) \to 1 \). Finally, when \( R_1 < y < R_2 \),

\[
\tilde{M}_\alpha^M(x) = \frac{2i}{v_{kr}} \sqrt{2m_s\gamma \hbar \beta_1(\omega_K)m_{\alpha}^{(k)}(x)} g_{k_r} e^{-ik_r(R_1-y)}
\]

is a right-propagating wave in the chiral limit. We note that the trapped magnetization is not a standing wave as there are no reflections. This helps to focus the magnetization to a small region of micrometers and efficiently transport the spin information directly from one wire to the other. The energy is trapped in the right magnet, which can be seen by the factor \( \eta \to 2 \) when the intrinsic magnetic broadening vanishes.

We illustrate the concept by calculating the additional damping and magnon trapping under chiral pumping of spin waves for Co nanowires of thickness 30 nm and width 100 nm on top of a YIG film with \( s = 20 \) nm. We use the magnetizations \( \mu_0 M_s = 0.177 \) T for YIG and \( \mu_0 M_s = 1.62 \) T for Co [13]. The intrinsic Gilbert damping coefficient of Co wire is taken to be \( \alpha_G = 0.01 \) [13, 21, 31]. Figure 2 is the plot of the magnetic-field
dependence of the additional broadening $\Gamma$ and intrinsic one $\kappa/2 = \alpha_G \omega_K$ of the wire Kittel mode, which can be measured in terms of the broadening of the wire FMR. For particular magnetic fields $\mu_0 H \approx 31.8$ and 139.7 mT, the additional damping equals the intrinsic one, at which the trapping becomes perfect.

In Figs. 3(a) and (b) we plot a snapshot of $M_x$ in real space for magnetic fields $\mu_0 H \approx 31.8$ and 50 mT. We choose Co wires of $d = 30$ nm and $w = 100$ nm, centered at $R_1 = 0$ and $R_2 = 2 \mu m$. At the critical field 31.8 mT, the excited magnetization is very well confined between the two wires [(a)], while magnetization is allowed to leak into the right half-space otherwise [(b)]. This device therefore functions as a magnon valve/switch/transistor that can be opened and closed by weak magnetic fields with characteristics far superior to previous realizations that operate by very different principles [19, 20].

\begin{align}
\rho_{ij} = \sum_{l=1}^{2} \Omega_l (\beta_i \beta_l^\dagger) + [\tilde{\rho}, \frac{\Gamma_2}{2} \beta_1 \beta_2 - \frac{\Gamma_2}{2} \beta_1^\dagger \beta_2^\dagger] + \sum_{l} \tilde{\mathcal{L}}_{l} \rho + \frac{\Gamma_2}{2} \tilde{\mathcal{L}}_{21} \rho,
\end{align}  

where $\tilde{\Gamma} = \kappa/2 + \Gamma$. The Heisenberg equation for this Hamiltonian recovers the equations of motion in Eq. 9. The coupling between magnons has both coherent and dissipative components. The dissipative coupling between magnons is responsible for the collective damping. With the same coherent driving, we may expect that chirality causes different magnon populations in the two wires. We set up the master equation of the density operator $\tilde{\rho}$ to calculate the dynamics driven by the microwaves, $\hat{P}_l(t) = \hat{P}_l(0)(e^{-i\omega t} + e^{i\omega t})$ with frequency $\omega_d$. In the chiral limit $\Gamma_{12} = 0$ and at the FMR, the master equation in the rotating frame becomes [33]

\begin{align}
\partial_t \tilde{\rho} = i \left[ \rho, \rho, \gamma \Omega \tilde{\rho} \right] + \left[ \rho, \frac{\Gamma_2}{2} \beta_1 \beta_2 - \frac{\Gamma_2}{2} \beta_1^\dagger \beta_2^\dagger \right] + \sum_{l} \gamma \tilde{\mathcal{L}}_{l} \rho + \frac{\Gamma_2}{2} \tilde{\mathcal{L}}_{21} \rho,
\end{align}  

Again, magnon $\tilde{\beta}_2$ is affected by $\tilde{\beta}_1$ but without any backaction. When the excitation microwave is uniform with the same amplitude $\tilde{\langle P_1 \rangle} = \tilde{\langle P_2 \rangle} = i \Omega$ and identical magnetic nanowires with small intrinsic damping $|\Gamma| \rightarrow |\Gamma_{21}|/2$. The ratio of the steady-state magnon populations in the two magnetic nanowires

\begin{align}
\langle \tilde{\beta}_1 \langle \tilde{\beta}_2 \rangle / \langle \tilde{\beta}_1^\dagger \tilde{\beta}_1 \rangle = 5 - 4 \cos (k_r (R_2 - R_1)),
\end{align}  

and hence can be on the order of ten and is tunable over a wide range by changing their separation or $k_r$ by the Kittel frequency. This amplification is caused by the chiral dissipative coupling between magnets, through which one magnet can input energy to another without backaction. This effect can be enhanced by adding more magnets [32].

Discussions.—In conclusion, we propose a method to control spin wave transport by weak magnetic fields based on the theory of chiral pumping of spin waves. By
exploiting two nanowires that communicate by unidirectional spin waves, we achieve new functionalities such as magnon trapping, amplification and a valve/transistor effect. The spatial distribution of magnons can be detected inductively via microwave emission of a third magnetic wire (supposing weak disturbance on the magnonic cavity) [13], NV center magnetometry [34], Brillouin light scattering [35], and electrically by the inverse spin Hall effect with a normal metallic wire such as Pt [36]. Replacing the nanowires by other objects such as magnetic spheres or qubits, and the unidirectional spin waves by other propagating quasiparticles such as waveguide photons, surface plasmons, electrons or phonons, we envision our mechanism to be extended to other fields including optomagnonics, nano-optics, plasmonics [2], quantum optics, spintronics, and spin mechanics. T. Y. and M. A. S. acknowledge funding through the DFG Emmy Noether program (SE 2558/2-1). H. W. and H. Y. are supported by NSF China under Grants No. 674020 and No. U1801661. G.B. is supported by JSPS KAKENHI Grant No. 19H006450.

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