Affect of Simulated Atmospheric Turbulence on Three Entangled Photons

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The thorough experimental study for the measure of sustainability of the atmospheric turbulence of the entangled photons for two qubit has been analysed by G. Puentes et al. [Phys. Rev. A 75, 032319 (2007)]. Here in this paper we propose a scheme to measure the effect of entanglement in three entangled photon GHZ state with the help of simulated atmospheric turbulence. We simulate the result of affected entangled GHZ state by introducing turbulence in three entangled photon path alternatively. We observe, by introducing the turbulence in photon path we are loosing the entanglement property, because of the increase in entropy. We compare our result with Werner curve and observe that the simulated result perfectly fit with the theoretical result.

INTRODUCTION

The emergence of quantum information theory made quantum entanglement an interesting subject to study. It has been the backbone for various quantum protocols. Quantum entanglement has various applications in the field of information theory. It is generally utilized to study quantum logical gates [3] [4], to carry-out computation of algorithms more efficiently [5] [6]. Even it is used for sharing information with more security [7] [8] in quantum computers and networks [9] [11]. Hyper-dense coding was the first application [11] [2] that was experimentally studied. In the resent past, the bipartite entangled state has been explored immensely, whereas the development of multiparticle entangled states is quite challenging. Recently, due to the development of quantum state fusion [12] [13] and expansion [14] [16] technology, large scale of multiparticle entangled states can be created. It has attracted much attention in recent times.

After the thorough exploration of two-photon entanglement we encounter a enormous demand for the production of multiphoton entangled states such as Greenberger-Horne-Zeilinger (GHZ) states [17]. Till date, spontaneous parametric down-conversion (SPDC) is the most prevailing method by which we can produce photonic entangled states. In the process of producing entangled photons using SPDC, we realize that it produces photons in pairs which causes them to entangle through various degrees of freedom [18]. As far as the best quantum channels are of concern, we can consider GHZ state for this purpose. It can be used for teleportation [19] [20], quantum key distribution [21], and many other application. To fabricate multiparticle entangled GHZ state, various methods have been proposed, such as cavity QED system [22] [23], optical system [24], ion trap system [25] and quantum dot system [26] [27].

In the work [28], they have studied the effect of multiple scattering for single-photon. In the work [29] [33], they have verified that the anticipation of maintaining correlation for arbitrary distance is authentic. They have performed free-space propagation with polarized entangled photons over distances of up to 144 km. The robustness of polarized entangled states can be proved by implementing scattering processes like [34], entangled mixed-state generation by twin-photon scattering [35], effect of polarization entanglement in photon-photon scattering [36]. Now we know that polarized entangled photons are not perturbed by the scattering process as long as they are linear (that can be described by a scattering matrix) and as long as we detect the photons in a single spatial mode [37] [38]. Therefore, we would like to characterize entanglement decay by using linear scattering processes along with multi-mode detection.

In this paper we investigate the impact of the turbulence on the polarized entangled GHZ state by creating turbulence in one arm. Then we have created the turbulence in two arms of GHZ state in all combinations. We finally crated the turbulence in all the three arms of the GHZ state of the generated photons. The experimental and simulation showed the same output.

EXPERIMENTAL SET UP

A schematic of the source of direct generation of three entangled photon based on the work [19] is shown in Fig. 1. A laser diode (404 nm) is used to pump the first entangled photon source (EPS1) which produces entangled photons. To produce the entangled photon pairs we have to tune the temperature when there is a phase matching in the crystal that produces this photon pairs. The produced photons are approximately 776 nm and 842 nm. We pump EPS2 which acts as source for second entangled photon by 776 nm photons, transferring the entanglement property to two new photons to produce a GHZ state. The EPS2 similar to EPS1 are also temperature controlled, and they are phase matched to produce photons centered nearly 1530 nm and 1570 nm. The photons 842nm at 1530 nm and 1570 nm are measured by the analysers APD1, APD2 and APD3 respectively. To measure the combined effect of coupling and detection efficiency of the 842 nm photons, 1530 nm and 1570 nm photons we have to measure the ratio of photon detections to coincident photon detections.

For this experiment they have used two types of detectors. Here in this experiment, they detect the 842 nm photons with silicon avalanche photodiodes (Si-APD). The photons at 1530 nm and 1570 nm are detected by tungsten silicide superconducting nanowire single-photon detectors (SNSPD).

Here in Fig. 1 we have triggered the turbulence in the first...
Where the two photon entanglement is well established [40, 41]. The experimental verification cal and theoretical aspect is how much noise ad-mixture pure- to analyse the chaos in the system. We have preferred this method as it is concurrence[32]. We have approached the concurrence of quantum entanglement. For instance, one could quan- in all the three arms of the generated GHZ state.

RESULTS

Mathematical analysis for entanglement measure

There are many methods available for the measurement of quantum entanglement. For instance, one could quanti- fy logarithmic negativity [30], entanglement distillation[31], concurrence[32]. We have approached the concurrence method in this paper. We have preferred this method as it is based on the measure of entropy which is the standard method to analyse the chaos in the system.

An important question which is interesting for both practi- cal and theoretical aspect is how much noise ad-mixture pure-state entanglement can sustain. The experimental verification for the two photon entanglement is well established [40,41].

The Werner state for the two qubit system as

$$\rho_{ws}(p) = p|\phi\rangle\langle\phi| + \frac{1-p}{4}I_4,$$  \hspace{1cm} (1)

where $\phi$ are the Bell State.

We are constructing the three photons system using the same principle as proposed by [40]. For three qubit state we con- sider the Greenberger-Horne-Zeilinger (GHZ) states. The Werner state for the three qubit system considering the GHZ state is defined as:

$$\rho_{ws}(p) = p|GHZ\rangle\langle GHZ| + \frac{1-p}{8}I_8,$$  \hspace{1cm} (2)

Where $|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ and $p \in [0, 1]$.

So for a $N$ qubit the mathematical form for the Werner state can be expressed as[39],

$$\rho_{ws}(p) = p|\phi ME\rangle\langle\phi ME| + \frac{1-p}{2^N}I_{2^N}.$$  \hspace{1cm} (3)

For $N = 2$ the state $\phi ME$ represents the Bell states.

Currently we can measure the degree of entanglement by various method like the entanglement distillation [42], the relative entropy of entanglement [43]. Here we will measure the entanglement by using linear entropy [44] which is given by

$$S_L = \frac{4}{3}\{1 - Tr[\rho^2]\},$$  \hspace{1cm} (4)

which range from 0 (for pure state) to 1 (for a maximally- mixed state). Here $\rho$ is the density state of the system that is under study. Tangle which is just the square of concurrence [43] is given by

$$\tau = C^2 = [max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}]^2.$$  \hspace{1cm} (5)

Here the $\lambda$’s are the square roots of the eigenvalue of the matrix $\rho \rho = \rho \sigma_y \otimes \sigma_y \sigma_y \otimes \sigma_y$ where $\sigma_y$ is the Pauli matrix and is denoted as $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$. For a maximally-entangled pure state $\tau = 1$, while we have $\tau = 0$ for non-entangled state.

The generic state of the three-qubit system(i.e, A, B and C) in the standard basis, where each index takes the values 0 and 1 is defined as

$$|\eta\rangle = \sum_{i,j,k} a_{ijk}|ijk\rangle,$$  \hspace{1cm} (6)

where $a_{ijk}$ are the co-efficients of the states and $|ijk\rangle = |i_A\rangle \otimes |j_B\rangle \otimes |k_C\rangle$.

The tangle for three qubit i.e. tangle of A with BC is the sum of tangle AB, AC, and the three-way tangle. It’s mathematical representation stands as [46],

$$\tau_{ABC} = \tau_{AB} + \tau_{AC} + \tau_{AB} + \tau_{AB}.$$

Here $\tau_{ABC} = 4|d_1 - 2d_2 + 4d_3|$, where

$$d_1 = a_{000}a_{111}^2 + a_{001}a_{110}^2 + a_{010}a_{101}^2 + a_{100}a_{011}^2,$$
$$d_2 = a_{000}a_{111}a_{010} + a_{000}a_{111}a_{101} + a_{000}a_{111}a_{001} + a_{000}a_{111}a_{110},$$
$$d_3 = a_{000}a_{110}a_{011} + a_{111}a_{001}a_{010}.$$

We can calculate $\tau_{AB}$ and $\tau_{AC}$ by using Eq. (5).

We want to measure the endurance of the entanglement due to the effect of the turbulence. For turbulence or noise, we have considered atmospheric turbulence which affects the po- larization of the entangled photons that are generated by using BBO crystals. The mathematical form of the noise [47] is given by

$$A = \begin{bmatrix} \cosh \theta & \sinh \theta \\ \sinh \theta & \cosh \theta \end{bmatrix},$$

where $\theta \in [0, \pi]$. We have considered Eq. (9) as a noise which affects the polarization of the generated photon. This noise is the general form of the noise which counts all the angles that changes the polarization of the photons [47].
It is well known from the theoretical perspective that if entropy increase due to the atmospheric turbulence the entanglement of the photons reduces. In this paper we are going to generate entangled GHZ state using three photon. Here we have done a simulation of the experimental set up.

The photons that are generated from the experimental set up as shown in Fig. 1 are polarized entanglement photon. We preferred this type of atmospheric turbulence (Eq. (9)) as this affects the polarization property of the entangled photon.

**Simulation results**

For the computational purpose we considered the density state as $\rho = \frac{1}{2}|GHZ\rangle\langle GHZ|$. We apply turbulence first at one photon path and measure how the atmospheric turbulence affect the entanglement of the generated photon. The density state after applying the noise stands as

$$
\rho = |0_1\rangle\langle 0_1| |0_2\rangle\langle 0_2| |0_3\rangle\langle 0_3| A_1 A_2 A_3 \\
+ |0_1\rangle\langle 0_2| |0_3\rangle\langle 1_1| |1_2\rangle\langle 1_3| A_1 A_2 A_3 \\
+ |1_1\rangle\langle 1_2| |1_3\rangle\langle 0_1| |0_2\rangle\langle 0_3| A_1 A_2 A_3 \\
+ |1_1\rangle\langle 1_2| |1_3\rangle\langle 1_1| |1_2\rangle\langle 1_3| A_1 A_2 A_3. $$

(11)

Here the noise is placed in the first photon path which is denoted by $A_1$. Here $A_1$ is the turbulence defined as in Eq. (9). Similarly we applied the noise individually to the second photon path and on the then on the third photon path. The outcome of the effect in all the cases are the same.

Similarly we apply turbulence to the two of the photon path in all permutation and the outcome for noise at two photon are always the same which is shown in Fig. 4 by the delta scattered plot. The mathematical form for the density state when turbulence is applied at two photon path is

$$
\rho = |0_1\rangle\langle 0_1| |0_2\rangle\langle 0_2| |0_3\rangle\langle 0_3| A_1 A_2 A_3 \\
+ |0_1\rangle\langle 0_2| |0_3\rangle\langle 1_1| |1_2\rangle\langle 1_3| A_1 A_2 A_3 \\
+ |1_1\rangle\langle 1_2| |1_3\rangle\langle 0_1| |0_2\rangle\langle 0_3| A_1 A_2 A_3 \\
+ |1_1\rangle\langle 1_2| |1_3\rangle\langle 1_1| |1_2\rangle\langle 1_3| A_1 A_2 A_3. $$

(12)

Here $A_1$ and $A_2$ is applied to the first photon path and second photon path. We can develop all the permutation similarly as described in Eq. (11).

Following the same procedure we apply the noise to all the three photon path to measure how turbulence affect the entanglement. The simulation result of this case is shown in Fig. 1 by the circle scattered plot. The mathematical form for the density state when turbulence is applied at two photon path is

$$
\rho = |0_1\rangle\langle 0_1| |0_2\rangle\langle 0_2| |0_3\rangle\langle 0_3| A_1 A_2 A_3 \\
+ |0_1\rangle\langle 0_2| |0_3\rangle\langle 1_1| |1_2\rangle\langle 1_3| A_1 A_2 A_3 \\
+ |1_1\rangle\langle 1_2| |1_3\rangle\langle 0_1| |0_2\rangle\langle 0_3| A_1 A_2 A_3 \\
+ |1_1\rangle\langle 1_2| |1_3\rangle\langle 1_1| |1_2\rangle\langle 1_3| A_1 A_2 A_3. $$

(12)

Here $A_1$, $A_2$, $A_3$ is applied to the first photon, second photon and the third photon path, where $A_1 = A_2 = A_3 = A$.

![FIG. 2. Plot of entropy versus turbulence parameter for the GHZ state that are generated by photon scattering. The blue line represent the theoretical Werner State (Eq. (2)). The pink scattered plot is the simulation of the entangled photon with noise at three of the photon path. Similarly, the green is for noise at two photon path and the orange for the noise at one photon path.](image)

![FIG. 3. Plot of tangle versus turbulence parameter for the GHZ state that are generated by photon scattering. The blue line represent the theoretical Werner State (Eq. (2)). The pink scattered plot is the simulation of the entangled photon with noise at three of the photon path. Similarly, the green is for noise at two photon path and the orange for the noise at one photon path.](image)
To summarize our results, we see from the plot, that due to the turbulence there is a loss in entanglement property. From the Fig. 2 we can infer that as we inject the turbulence in one photon path in all subsequent permutations there is a lateral shift from the theoretical Werner curve. Next, we insert the turbulence to the two photon path in all permutation. Similar to the result that we obtained in the previous case (i.e turbulence in one photon path) we get a lateral shift from Werner curve but more than that of the previous situation. For the turbulence in three photon path the shift was even more than the second case where turbulence was present in two of the photon path.

From the result we infer that as we insert turbulence in photon path, the disorder in the system increases which reduces the entanglement property between the photons. It increase sequentially if we insert the turbulence in two and three photon path respectively.

This work has a wide range of application in the field of quantum information. Generation of three photon entangled state ensure us that it has a use for multi-party secure communication, further we will use this scheme for development of quantum cheque.

METHOD

A schematic representation of the source of direct generation of three entangled photon is shown in Fig 1, we have triggered the turbulence in the first arm of the setup then following the same procedure we have triggered the turbulence on second arm and finally on the third arm. Similarly, we triggered the turbulence at two arms simultaneously. Finally we triggered turbulence in all the three arms.

We have made a numerical analysis using the state as shown in Eq. (10), where we have simulated the different turbulence scenarios, discussed above, by introducing a turbulence matrix in the photon path in each case. Here we have considered those turbulence which affects the polarization of the entangled photon which is shown in Eq. (9). For turbulence in two photon we have considered the state as Eq. (11) and for turbulence in three photon path it is shown in Eq. (12).


discussion and conclusion

To summarize our results, we see from the plot, that due to the turbulence there is a loss in entanglement property. From the Fig. 2 we can infer that as we inject the turbulence in one photon path in all subsequent permutations there is a lateral shift from the theoretical Werner curve. Next, we insert the turbulence to the two photon path in all permutation. Similar to the result that we obtained in the previous case (i.e turbulence in one photon path) we get a lateral shift from Werner curve but more than that of the previous situation. For the turbulence in three photon path the shift was even more than the second case where turbulence was present in two of the photon path.

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\[ \text{Entropy} \]

\[ \text{Tangle} \]

\[ \text{Three photon entanglement generation using GHZ state} \]

\[ \text{Werner State} \]

\[ \text{Turbulence at one photon path} \]

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