Linear response function in the presence of elastic scattering:
plasmons in graphene and the two-dimensional electron gas

M. Bahrami$^1$ and P. Vasilopoulos$^1$

$^1$Department of Physics, Concordia University, 7141 Sherbrooke Ouest, Montreal, Quebec, H4B 1R6, Canada

Within linear-response theory we derive a response function that thoroughly takes into account the influence of elastic scattering and is valid beyond the long-wavelength limit. We apply the theory to plasmons in graphene and the two-dimensional electron gas (2DEG), in the random-phase approximation, and for the former take into account intraband and interband excitations. The scattering-modified dispersion relation shows that below a critical scattering strength $\gamma_c$, no plasmons are allowed. The critical strengths $\gamma_c$ and the allowed $(\omega, q)$ plasmon spectra for intraband and interband transitions in graphene are different. In both graphene and the 2DEG the strength $\gamma_c$ falls rapidly for small $\omega$ but much more slowly for large $\omega$.

I. INTRODUCTION

Physical properties of a system such as conductivity, susceptibility, etc., can be determined by studying its response to external probes, such as electric or magnetic fields. Many properties in the classical regime could be explained by classical equations, such as the Boltzmann equation [1]. One powerful tool to deal with many-body systems, which provides us with a wealth of information, is linear response theory (LRT). A classical LRT example is the Brownian motion [2], [3]. To describe such properties of a system quantum mechanically one can use the quantum master equation or the quantum linear response theory (QLRT) introduced by Kubo in the middle of last century [4]. QLRT could provide us with vital information about system properties at zero and finite temperature [5]. For instance, static screening, effective interactions, collective modes, electron energy-loss spectra and Raman spectra of a system can be obtained from its density-density response function (DDRF) [6]. One major motivation in developing LRT was finding the connection between fluctuations of a quantity about equilibrium and dissipation, which is the content of the fluctuation-dissipation theorem. Obviously it was of interest to have a quantum mechanical version of this theorem.

Kubo’s QLRT has been often criticized, see, e.g., Ref. [1] and references cited therein. In particular, in developing QLRT Kubo assumed an adiabatic switching-on of the perturbation to avoid a divergence in an integration over time and satisfy causality. The relevant parameter though is regarded in the literature as a dissipative factor, see Eq. (3.42) in Ref. [6]. Moreover, to retrieve Drude’s result for the conductivity in the long-wavelength limit usually one enters this parameter in the final result phenomenologically, see, e.g., Refs. [8–10]. As mentioned in Ref. [1], the dissipation should originate from some randomness in the system and the latter should appear in the Hamiltonian. This randomness term, for example, could represent one-body or two-body randomizing collisions [1, 7]. In Ref [7] the authors applied the van Hove limit, see below, to all relevant operators. This gave explicit expressions for, e.g., the electrical conductivity and the scattering-independent part of the response function. However, the theory developed is valid only for homogeneous systems and the scattering-dependent part of the response function was not evaluated. In this paper we develop a QLRT for inhomogeneous systems and investigate the influence of this randomness term (or scattering) on plasmons in graphene and the two-dimensional electron gas (2DEG).

The paper is organized as follows. In Sec. II we present some general QLRT expressions and obtain the DDRF as a sum of two terms, one of which is the usual term that is independent of scattering and one that depends on it. In Sec. III we investigate plasmons in graphene and in Sec. IV plasmons in the 2DEG. In Sec. IV we also contrast its results with those for graphene. A summary follows in Sec. V and some important results of Sec. II are detailed in the appendix.

II. FORMALISM

We consider a system whose Hamiltonian is [1],

$$H = H_0 - AF(t) + \lambda V,$$

where $H_0$ is the system’s Hamiltonian in the absence of external stimuli and randomness. The external time-dependent probe $F(t)$ couples to the operator $A$ and randomness or scattering is represented by $\lambda V$; this could be, for instance, electron-impurity or electron-phonon interaction, and $\lambda$ is strength of this interaction. Be in the linear-response regime is expressed by the inequality

$$\langle A F(t) \rangle, \langle V \rangle \ll \langle H_0 \rangle.$$

Many optical and transport properties can be obtained from the knowledge of the density-density response function (DDRF) and we focus on obtaining it. In this case
The general expression for the DDRF is
\[ \chi^0(r, t; r', t') = -\frac{i}{\hbar} \Theta(t-t') \left[ \rho(r, t), \rho(r', t') \right] \big|_0, \]
where \((...)_0\) denotes an average over a thermal equilibrium ensemble. \(\chi^0(r, t; r', t')\) is proportional to the probability of finding an electron at position \(r'\) and time \(t'\) knowing its position \(r\) at time \(t\). The density operator is
\[ \rho(r, t) = \psi^\dagger(r, t) \psi(r, t) = \sum_{ij} c_i^\dagger(t) c_j(t) \phi_i^\dagger(r) \phi_j(r), \]
with \(\phi\) being the single particle wave function. In the absence of the randomness term, \(\lambda V\), the time evolution of an operator in the Heisenberg picture is
\[ c(t) = e^{iH_0t/\hbar} ce^{-iH_0t/\hbar}. \]
We now rename \(\chi\) as \(\chi_{\text{non}}\) to distinguish it from the part that depends on scattering, see below. Employing Eqs. (5) and (6), the DDRF in the absence of randomness in the frequency domain is given by
\[ \chi^0_{\text{non}}(r, r', \omega) = \lim_{\nu \to 0} \sum_{i,j} \Delta_{ij} \phi_i^\dagger(r) \phi_j(r') \phi_j(r') \phi_i(r), \]
where
\[ \Delta_{ij} = \frac{f_i - f_j}{E_i - E_j + \hbar(\omega + i\nu)}. \]
and \(f\) stands for the Fermi-Dirac distribution function. The van Hove limit, described in Refs. [1], is given by
\[ \lambda \to 0, t/\tau_t \to \infty, \chi^2 t = \text{finite}, \]
where \(\tau_t\) is the transition time. This limit alters the time evolution of an operator \(\hat{c}\) in the manner
\[ c(t) = e^{-\Lambda c} c_0 e^{iH_0t/\hbar} c_0 e^{-iH_0t/\hbar}, \]
where \(\Lambda\) is a super-operator defined by
\[ \Lambda = \sum_{i,j} \langle i | [W_{ji}(j) | c_j(j) - W_{ij}(i) | c_i(i)] \rangle; \]
the transition rate \(W_{ij}\) given by Fermi’s golden rule
\[ W_{ij} = \left(2\pi \lambda^2/\hbar^2 \right) |\langle i | v | j \rangle|^2 \delta(E_i - E_j). \]
Note that in Ref. [1] the calculation has been done in a representation in which all operators have, in principle, a diagonal and a nondiagonal part. Here we exploit the approach used in Refs. [8] and [17]. It is worth mentioning that Eq. (10) satisfies the equation of motion, \(i\hbar o_c(t) = [c(t), H_0 + \lambda V] \) in the van Hove limit. Applying Eqs. (5) and (10) the DDRF takes the form
\[ \chi^0(r, r', \omega) = \chi_{\text{im}}^0(r, r', \omega) + \chi_{\text{non}}^0(r, r', \omega), \]
with
\[ \chi_{\text{im}}^0(r, r', \omega) = \sum_{i,j} M_{ij} \phi_i^\dagger(r) \phi_j(r) \phi_j(r') \phi_i(r'), \]
and
\[ M_{ij} \equiv \frac{1}{\hbar(\omega + i(\Lambda_i + \Lambda_j))} \langle f_i - f_j \rangle; \]
here \((...)_b\) denotes an average over the boson bath states. The details of the calculation are given in appendix A.

### III. PLASMONS IN GRAPHENE

From the DDRF many properties such as plasmons, reflection and transmission amplitudes can be evaluated [10]. Below we investigate graphene plasmons in the random-phase approximation (RPA) [20]. For low energies graphene’s energy spectrum is given by [21]
\[ E_{\zeta}(\mathbf{k}) = \zeta \hbar v_F k, \]
where \(\zeta = -1(+1)\) indicates the valence (conduction) band. The corresponding single-particle wave function is
\[ \psi_{\zeta \mathbf{k}s}(\mathbf{r}) = \frac{1}{\sqrt{2A}} \left( e^{-i\theta(\mathbf{k})} \right) \zeta e^{i\mathbf{k} \cdot \mathbf{r}} X_s \]
with \(\theta(\mathbf{k}) = \tan^{-1}(k_y/k_x)\), \(X_s\) the spin-dependent part of the wave function, and \(\mu = 1(-1)\) the valley index for \(K (K')\). In the long-wavelength limit, at zero temperature, and for single-band (SB) transition, also known as intraband transition in the literature, \(\chi_{\text{non}}^{0,SB}\) for an electron-doped system is given by [21]
\[ \chi_{\text{non}}^{0,SB}(q', \omega') = \frac{k_F}{\pi \hbar v_F} \frac{q'^2}{\omega'^2}. \]
As for \(\chi_{\text{im}}^{0,SB}\), by employing Eqs. (16), (17), and (14) we obtain
\[ \chi_{\text{im}}^{0,SB}(q', \omega') \equiv \frac{k_F}{2\pi \hbar v_F} C(\omega', \gamma') (1 - \delta \gamma' s), \]
where we used the dimensionless parameters \(q', \omega'\), and \(\gamma' (q' \equiv q/k_F, \omega' \equiv \hbar \omega/E_F, \gamma' \equiv \hbar \gamma/E_F)\) to simplify all expressions. \(C(\omega', \gamma')\) is given below. Although \(\langle \Lambda_b \rangle\) is a function of momentum, for simplicity in the derivation of Eq. (20) it has been replaced by \(\tau/2\) where \(\tau\) is the
relaxation time and \( \gamma = 1/\tau \); this is valid only for elastic scattering. For two-band (TB) transitions, known as interband transitions, \( \chi_{non}^{0,TB} \) is given by

\[
\chi_{non}^{0,TB}(q', \omega') = \frac{k_F}{\pi \hbar v_F} \frac{q'^2}{2 \omega'} \left[ A(\omega') - \frac{i}{2} \Theta(\omega' - 2) \right]
\]  

(20)

and \( \chi_{im}^{0,TB} \) by

\[
\chi_{im}^{0,TB}(q', \omega') = \frac{k_F}{\pi \hbar v_F} C(\omega', \gamma')(1 - \delta_{\gamma',0}).
\]  

(21)

where

\[
A(\omega') = \frac{2}{\omega'} + \frac{1}{2} \ln \left| \frac{2 - \omega'}{2 + \omega'} \right|, \quad C(\omega', \gamma') = \frac{\omega' - i \gamma'}{\omega' + \gamma^2}.
\]  

(22)

The DDRF in momentum and energy space, \( \chi(q', \omega') \), characterizes the probability to find an electron which its final and initial states differing in momentum and energy. For example, the zeros of its real part determine the DDRF in real space, and the positions of its poles determine the DDRF in momentum space. In contrast, scattering is the DDRF in energy space, and the poles of its real part determine the DDRF in momentum space. The upper panels are for \( \omega' \) increasing \( \gamma' \) makes \( \chi_{im}^{0,TB} \) weaker since increasing \( \gamma' \) leads to shorter scattering time and length. Therefore, the probability for an electron to reach the final desired momentum is reduced by strengthening the interaction with impurity.

The real part of the total TB DDRF (Re\( \chi \)), containing both \( \chi_{non} \) and \( \chi_{im} \), is shown in Fig. 1(a) versus \( \omega' \) for a typical value of \( q' \) in the long-wavelength limit. The solid black curve represents the TB DDRF without inclusion of scattering whereas the coloured curves are for several different \( \gamma' \). To make clearer its dependence on \( \omega' \) in Fig. 2(b) we blow up the part of Fig. 2(a) for \( \omega' \leq 0.4 \). As seen, the TB DDRF for small \( \omega' \) decreases dramatically because in this energy range the principal contribution to it emanates from \( \chi_{non} \) as we mentioned in the justification of Fig 1.

FIG. 1: Real parts of \( \chi_{non} \) and \( \chi_{im} \) versus \( \omega' \) where \( \chi_{im} \) is displayed for several \( \gamma' \).

FIG. 2: (a) Real part of the TB DDRF (Re\( \chi \)) for several values of \( \gamma' \). (b) A part of (a) for \( \omega' \leq 0.4 \).

FIG. 3: (a) Imaginary part of the TB DDRF, \( \Im \chi \), for \( q' = 0.01 \) and several \( \gamma' \). (b) A segment of \( \Im \chi \) for three \( \gamma' \).

The imaginary part of the TB DDRF (Im\( \chi \)), versus \( \omega' \), is shown in Fig. 3(a) for several \( \gamma' \). To make more transparent its dependence on \( \omega' \), in Fig. 3(b) we show it for three different \( \gamma' \) on an expanded scale. Notice though that this makes the cusps or maxima of Fig. 3(a) invisible.

Figures 3(a) and (b) show the real and imaginary parts of the TB DDRF versus \( q' \) for several different values of \( \gamma' \) and a typical frequency \( \omega' = 0.001 \). As shown in Fig. 3(a), the dependence of Re\( \chi \) is approximately parabolic because only \( \chi_{non} \) has a term that contains \( q' \) and contributes to the TB DDRF. In addition, it is clear that for \( q' \) fixed Re\( \chi \) increases as \( \gamma' \) decreases.

Since the logarithmic and step function terms alter the behaviour of Re\( \chi \) and Im\( \chi \), respectively, in the vicinity of \( \omega' = 2 \), which is not clear in Figs. 2(a) and 3(a) due to the large difference between their values with and without scattering, in Fig 4 we display them separately. The upper panels are for \( \gamma' = 0 \) and the lower ones for \( \gamma' = 10 \). Notice i) how including scattering, \( \gamma' \neq 0 \), strengthens the behaviour of the results without it near \( \omega' = 2 \) and ii) without scattering (\( \gamma' = 0 \) Im\( \chi \), shown in Fig. 5(b), vanishes for \( \omega' \leq 2 \) and that there is no dissipation in the system. In contrast, when scattering is included Im\( \chi \) has approximately a constant slope.

The permittivity \( \epsilon \) of a system has a wealth of information. For example, the zeros of its real part determine
its plasmon dispersion. One of the approximations that takes into account the electron-electron interaction is the RPA. The Fourier transformed, in space and time, RPA permittivity is given by

$$\epsilon^{RPA}(q, \omega) = 1 - V(q)\chi^0(q, \omega),$$  \hspace{1cm} (23)

where $V(q)$ is the 2D Fourier transform of the Coulomb potential. In terms of $q'$ we rewrite $V(q)$ as

$$V(q') = \eta / q',$$  \hspace{1cm} (24)

with $\eta \equiv 8\alpha^2\hbar c/\epsilon_b k_F$ and $\epsilon_b$ the background permittivity. For SB transitions the plasmons can be derived by combining Eqs. (15) and (22). The result is

$$1 - \frac{\beta}{q'} \left[ \frac{q'^2}{\omega'^2} + \frac{\omega'}{2\omega'^2 + \gamma'^2} \right] = 0.$$  \hspace{1cm} (25)

One solution of this quadratic equation is

$$q' = \frac{\omega'^2}{2\beta} \left( 1 + \sqrt{1 - \frac{2\beta^2 (1 - \delta_{\gamma', \omega})}{\omega'^2 (\omega'^2 + \gamma'^2)}} \right)^{1/2},$$  \hspace{1cm} (26)

with $\beta \equiv \eta k_F / \pi \hbar v_F$, gives the dispersion relation. Notice that Eq. (26) gives the well-known dispersion $\omega \propto q^{1/2}$ for $\gamma' = 0$. The other solution, with $1+\gamma'$ in Eq. (26) replaced by $1-$, is unphysical and therefore rejected.

For TB transitions combining Eqs. (20), (21), and (25) gives the plasmon spectrum as

$$q' = \frac{\omega'}{\beta A(\omega')} \left[ 1 + \left\{ 1 - \frac{2\beta^2 A(\omega') (1 - \delta_{\gamma', \omega})}{(\omega'^2 + \gamma'^2)} \right\}^{1/2} \right].$$  \hspace{1cm} (27)

Again the second solution for $q'$, with $1+\gamma'$ in Eq. (27) replaced by $1-$, is unphysical and rejected. In Figs. 6 (a) and 7 (a) we show the dispersion relations, resulting from Eqs. (26) and (27), for several values of $\gamma'$. To make these graphs more clear in Figs. 6 (b) and 7 (b) we show their windows for $0.03 \leq q' \leq 0.05$ and different values of $\gamma'$, respectively. In both cases the frequency increases with $q'$ while the momentum plasmon range, i.e., the lower acceptable value of $q'$, decreases with increasing $\gamma'$. Judging from the results as $\gamma$ increases in Figs. 6 and 7, we see that the SB plasmon frequency is larger than that of the TB one, whereas the SB plasmon momentum range is a bit shorter than the TB one. Note that the plasmon group velocity $d\omega/dq$ is approximately constant and independent of the scattering strength $\gamma'$.
in dissipation by single-particle excitations. In other words, a single-particle excitation competes with the collective one: if the mean-free path related to the single-particle excitation is of the order of the wavelength of the collective one, there would be no plasmon. In Figs. 6 (b) and 7 (b) we can see that there is a critical plasmon momentum below which there is no plasmon spectrum for a typical $\gamma'$. This can be explained as follows. The mean-free path decreases with increasing $\gamma'$. By the uncertainty principle then its momentum increases with $\gamma'$ and so does the critical plasmon momentum. Physically, if the wavelength of the collective oscillation, the displacement from equilibrium, is smaller than the mean-free path, the system supports plasmons.

We also see, in both figures, that for fixed plasmon momentum the plasmon frequency increases with decreasing $\gamma'$. This can be justified as follows. At fixed plasmon momentum the coherent collective dipole momenta generate the plasmon electromagnetic field (EM) whose energy is determined by the displacement from the equilibrium positions and the number of available coherent dipole momenta. Higher impurity density, that is larger $\gamma'$, increases the elastic scattering probability which reduces the number of coherent dipole momenta. Therefore, the plasmon EM field resulting from them will have lower energy as $\gamma'$ increases.

As emphasized above, there are critical values $\gamma'_c$ below which there are no SB or TB plasmons. To find them we set the factors $[\ldots]^{1/2}$ and $\{\ldots\}^{1/2}$ in Eqs. (26) and (27), respectively, equal to zero. This gives

$$\gamma'_c^{SB} = \left[2\beta^2/\omega' - \omega'^2\right]^{1/2},$$

$$\gamma'_c^{TB} = \left[2\beta^2 A(\omega') - \omega'^2\right]^{1/2}. \quad (29)$$

Figure 8 shows $\gamma'_c$ versus the plasmon frequency $\omega'$ for SB and TB plasmons. In the former case the high value for low frequencies decreases fast for $\omega'$ small but much more slowly for $\omega' > 0.5$, while in the latter its value falls down very fast.

![FIG. 8: SB and TB critical values $\gamma'_c$ versus $\omega'$.](image)

It is worth observing that setting $[\ldots]^{1/2} = 0$ in Eq. (26) leads, for $\gamma \neq 0$ fixed, to a simple cubic equation for $\omega', \omega'^3 + \gamma'^2 \omega' - 2\beta^2 = 0$. It’s acceptable solution $\omega_c$ is given below in Eq. (30). This then can be used to find analytically the lowest limit for $q_c = \omega_c^2/2\beta$, shown in Figs. 6 and 7, from Eq. (26). The explicit results for $\gamma'_c \geq \gamma'_c$ are

$$\omega_c = \left[\beta^2 + \sqrt{Z}\right]^{1/3} + \left[\beta^2 - \sqrt{Z}\right]^{1/3}, \quad Z = \beta^4 + \gamma'^6/27. \quad (30)$$

Unfortunately, for TB transitions this is not possible due to the factor $A(\omega')$ in Eq. (27).

The plasmon spectrum in Figs. 6 and 7 involves only a few discrete values of $\gamma'$. For a continuous $\gamma'$ we show it in Fig. 9 as a contour plot. Plasmons are not allowed outside the coloured regions. For the same plasmon frequency and momentum, we easily see that the corresponding $\gamma'$ differ drastically.

### IV. PLASMONS IN A 2DEG

For a 2DEG the single-particle wave function is given by

$$\psi_{k,s}(\vec{r}) = \frac{1}{\sqrt{A}} e^{i\vec{k}.\vec{r}} X_s. \quad (31)$$

Employing Eqs. (31) and (27) we obtain the long wavelength limit $\chi^0_{non}$ in the form

$$\chi^0_{non}(q', \omega') = \frac{n\hbar^2 k_F^2 q'^2}{m E_F^2 \omega'^2}, \quad (32)$$

where $n$ and $m$ are the charge density and electron mass, respectively. As for $\chi^0_{im}$, with Eqs (31) and (14) and the assumption that the scattering is elastic and $\tau$ independent of the wave vector, we obtain

$$\chi^0_{im}(q', \omega') = \frac{k_F^2}{2\pi E_F} C(\omega',\gamma'). \quad (33)$$

In the long-wavelength limit the plasmon spectrum can be evaluated by utilizing Eqs. (23), (52), and (33). The result is similar to graphene’s plasmon spectrum, namely,

$$q' = \frac{\omega'^2}{2\beta'} \left(1 + \left[1 - \frac{2\beta'^2 (1 - \delta_{\gamma',0})}{\omega' (\omega'^2 + \gamma'^2)}\right]^{1/2}\right) \quad (34)$$
with $\beta' = 4/k_F a_B$ and $a_B$ the Bohr radius. In Fig. 10 we show this 2DEG plasmon spectrum in the absence and presence of impurities for various values of $\beta'$. To contrast it with that of graphene we give $\beta'$ in "units" of $\beta$. In Fig. 10(a) we can see that for a fixed $q'$ the plasmon energy increases; this can explained as follows. If the number of plasmon dipole-momenta increases we expect the energy of plasmon EM field to increase as well. Note that in Eq. (34) the plasmon momentum is inversely proportional to $\beta'$. In addition, $k_F$ in a 2DEG is proportional to the square root of the electron density, $k_F = \sqrt{2\pi n}$. Then one finds that the plasmon momentum $q'$ is likewise proportional to $\sqrt{n}$. In contrast, in graphene the dimensionless plasmon momentum is independent of the electron density $n$. In Fig. 10(b) we show the plasmon dispersion in a 2DEG in (a) the absence and (b) the presence of impurity scattering; in (b) we took $\gamma' = 25$. It can be seen that for fixed plasmon momentum and decreasing $\beta'$ the plasmon energy decreases due to the reduction of the plasmon dipole momenta. Furthermore, compared to the graphene case $\beta' = \beta$, we can see that for the lower value of $\beta'$ the critical plasmon momentum becomes smaller due to the fact that scattering by impurities weakens with decreasing electron density.

Further, as in the case of graphene, in a 2DEG the critical $\gamma'$, below which no plasmons are allowed, is obtained in the same way. It is given by

$$\gamma_{c2DEG}' = \left[2\beta'^2/\omega' - \omega'^2\right]^{1/2}. \tag{35}$$

We show it in Fig. 11 $\gamma_{c2DEG}'$ for several values of $\beta'$. It can be seen that for fixed $\omega'$ and increasing $\beta'$ the value of $\gamma_{c}'$ increases as well. We further remark that, similar to graphene for SB transitions, with $\gamma_{c}' \neq 0$ fixed Eq. (33) allows an analytic evaluation of the allowed $\omega'$ which in turn determines the lower value of $q'$ below which no plasmons are allowed. One simply has to replace $\beta$ with $\beta'$ in Eq. (30) to obtain the corresponding $\omega_c$ and $q_c$.

**V. SUMMARY**

We evaluated the linear-response function to an external stimulus, obtained an expression that is valid for elastic scattering, and applied it to plasmons in graphene and the 2DEG in the random-phase approximation. This was achieved by applying the van Hove limit to all operators and by utilizing appropriate super-operators of the literature. The resulting linear-response function, or DDRF, has two terms, $\chi_{non}$ which is independent of the scattering, and $\chi_{im}$ who does depend on it and produces results that are qualitatively and quantitatively different from those of $\chi_{non}$. In graphene the term $\chi_{non}$ dominates the response in the long wavelength limit, i.e., for very low frequencies, while the term $\chi_{im}$ dominates for all other frequencies.

The main result of the term $\chi_{im}$ is that introduces scattering-dependent wavevector limits below which no plasmons are allowed. It is also valid for all values of the wave vector, that is, it is not limited to the long-wavelength limit as $\chi_{non}$ is. Another nice feature is that it simply explains and retrieves the Drude model results in the long-wavelength limit. We reported new plasmon results for i) graphene and ii) the 2DEG. In i) we distinguished between intraband (SB) and interband (TB) transitions. In both i) and ii) we obtained the scattering-induced limits referred to above, analytical dispersion relations, and their well-known long-wavelength limit in the absence of scattering. An important difference between i) and ii) is that in the dimensionless units used the plasmon wave vector for graphene is independent of the electron density whereas in a 2DEG it is proportional to its square root, $q' \propto \sqrt{n}$. As discussed, depending on the scattering strength $\gamma'$ the single-particle excitations due to scattering drastically modify the frequency and wave vector domains ($\omega, q$) of the collective excitations. The latter are suppressed below a critical $\gamma'$.

Finally, it’s worth emphasizing that our results for both $\chi_{non}$ and the scattering-dependent $\chi_{im}$ are both fully quantum mechanical. This is different from the widespread practice of shifting the frequency $\omega$ in the classical Drude result to $\omega + i\tau$ and relate $\tau$ to scattering. This is why our results may appear strikingly dif-

---

**FIG. 10:** 2DEG plasmon dispersion (a) in the absence and (b) presence of impurity scattering. In (b) $\gamma' = 25$ is used.

**FIG. 11:** $\gamma_{c2DEG}'$ versus $\omega'$ for several values of $\beta'$. 

---
ferent than the usual one. Similar results were obtained in [7] for homogeneous systems, see Eq. (2.35) in there, but, as stated in Sec. I, the corresponding $\chi_{im}$ term was not explicitly evaluated.

ACKNOWLEDGMENTS

This work was supported by the Canadian NSERC Grant No. OGP0121756
Electronic addresses: $^1$m_bahra@live.concordia.com, $^1$p.vasilopoulos@concordia.ca

Appendix A: Scattering-dependent part of the DDRF

In order to simplify derivation we rewrite Eq. (110) as

$$c_i(t) = c_i^m(t) + c_i^{\text{non}}(t), \quad (A1)$$

with

$$c_i^m(t) = e^{-\Lambda_i t} c_i^d \quad (A2)$$

and

$$c_i^{\text{non}}(t) = e^{iH_0 t/\hbar} c_i e^{-iH_0 t/\hbar}. \quad (A3)$$

Notice that $c_i^{\text{non}}(t)$ is the time evolution of the operator in the absence of impurity. There is an advantage in splitting time evolution of operator as in Eq. (35): applying it to the density operator we have

$$\rho(\vec{r}, t) = \rho_{im}(\vec{r}, t) + \rho_{\text{non}}(\vec{r}, t). \quad (A4)$$

Substituting (A3) into expression (4) and exploiting the algebra of Hamiltonians, we obtain

$$\chi^*(\vec{r}, \vec{r'}, t) = \chi_{im}(\vec{r}, \vec{r'}, t) + \chi_{\text{non}}^{*}(\vec{r}, \vec{r'}, t), \quad (A6)$$

where $\chi_{\text{non}}^*$ is represented in the frequency domain by Eq. (7). The result for $\chi_{im}$ is

$$\chi_{im}(\vec{r}, \vec{r'}, t) = -\frac{i}{\hbar} \Theta(t) \left< \left[ \rho_{im}(\vec{r}, t), \rho_{im}(\vec{r'}) \right] \right>_0. \quad (A7)$$

Using Eqs. (A2) and (A4), Eq. (A7), and integrating over time we obtain Eq. (A8) in the frequency domain as

$$\chi_{im}^0(\vec{r}, \vec{r'}, \omega) = -\frac{i}{\hbar} \int_{-\infty}^{\infty} dt \Theta(t) e^{i\omega t} \left< \chi_{im}(\vec{r}, \vec{r'}) \right>_b \times \sum_{i,j} \sum_{m,n} \phi_i^*(\vec{r}) \phi_j(\vec{r}) \phi_m^*(\vec{r'}) \phi_n(\vec{r'}) (\delta_{i,j} \delta_{m,n}). \quad (A8)$$

where we defined $\Lambda_{ij} = \Lambda_i + \Lambda_j$. Evaluating the integral in Eq. (A8) we obtain Eq. (15).

References:

[1] C. M. Van Vliet, “Equilibrium and Non-Equilibrium Statistical Mechanics” (World Scientific Press, Singapore, 2008).
[2] A. Einstein, Ann. der Physik 17, 549 (1905).
[3] Gertruida L. de Hass-Lorentz, “Die Brvonsche Bewegung und einige anverwandte Erscheinungen”, Vieweg Verlag, Braunschweig, Germany, 1912.
[4] R. Kubo, J. Phys. Soc. Jpn. 12, 570 (1957).
[5] L. Pitaevskii and S. Stringari, Bose-Einstein Condensation and Superfluidity (Oxford University Press, New York, 2003).
[6] G. F. Giuliani and G. Vignale, Quantum theory of the electron liquid (Cambridge University Press, Cambridge, 2005).
[7] M. Charbonneau, K. M. Van Vliet, and P. Vasilopoulos, J. Math. Phys. 23, 2 (1982).
[8] A. Principi, G. Vignale, M. Carrega, and M. Polini, Phys. Rev. B. 121405, 88 (2013).
[9] G. X. Ni, H. Wang, J. S. Wu, Z. Fei, M. D. Goldflam, F. Keilmann, B. zyilmaz, A. H. Castro Neto, X. M. Xie, M. M. Fogler, and D. N. Basov, Nature Materials. 1217, 14 (2015).
[10] D.N. Basov, M.M. Fogler, A. Lanzara, Feng Wang, and Yuanbo Zhang, Rev. Mod. Phys. 959, 86 (2014).
[11] Alessandro Principi, Matteo Carrega, Mark B. Lundeberg, Achim Woessner, Frank H. L. Koppens, Giovanni Vignale, and Marco Polini, Phys. Rev. B. 165408, 90 (2014).
[12] I. J. Luxmoore, C. H. Gan, P. Q. Liu, F. Valmorra, P. Li, J. Faist, and G. R. Nash, ACS Photonics. 1151, 1 (2014).
[13] F. Javier Garcia de Abajo, ACS Photonics. 1151, 1 (2014).
[14] T. Low and P. Avouris, ACS Nano. 13537, 14 (2014).
[15] M. Bagheri and P. Vasilopoulos, Optics Express. 1217, 14 (2014).
[16] Q. Bao, H. Zhang, B. Wang, Z. Ni, C. H. Y. X. Lim, Y. Wang, D. Y. Tang and K. P. Loh, Nature Photonics. 5, 411 (2011).
[17] H. Bruus and K. Flensberg, Introduction to many-body quantum theory in condensed matter physics (Oxford University Press, United Kingdom, 2004).
[18] M. S. Kushwaha, Surf. Sci. Reports. 41, 161 (2001).
[19] M. Bagheri and M. Bahrami, J. Appl. Phys. 115, 174301 (2014).
[20] M. Bahrami and P. Vasilopoulos, Optics Express. 14, 16842 (2017).
[21] B. Wunsch, T. Stauber, F. Sols, and F. Guinea, New Journal of Physics. 8, 318 (2006).