γ-ray bursts are produced by the dissipation of the kinetic energy of a highly relativistic fireball, via the formation of a collisionless shock. When this happens, Ultra High Energy Cosmic Rays up to ≈ 10^{20} eV are produced. I show in this paper that these particles produce, via synchrotron emission as they cross the acceleration region, photons up to 300 GeV which carry away a small, ≈ 0.01, but non-negligible fraction of the total burst energy. I show that, when the shock occurs with the interstellar medium, the optical depth to photon–photon scattering, which might cause energy degradation of the photons, is small. The burst thusly produced would be detected at Earth simultaneously with the parent γ-ray burst, although its duration may differ significantly from that of the lower energy photons. The expected fluences, ≈ 10^{-5} – 10^{-6} erg cm^{-2}, are well within the range of planned detectors. A new explanation for the exceptional burst GB 910217 is discussed.

PACS numbers: 98.70.Rz, 98.70.Sa

It can be argued compellingly that a model for extragalactic γ-ray bursts (GRBs) must involve highly relativistic motions [1]. This model (fireball) involves the acceleration of matter to large Lorenz factors (\( \gamma \approx 100–1000 \)) from a compact object, and the formation of a highly relativistic shock. The shock converts the directed kinetic energy (mostly of the baryons) into internal energy, equally distributed between electrons and protons. Electrons then promptly radiate their share of the internal energy, giving rise, through synchrotron radiation in a suitably generated magnetic field, to the observed GRB.

The relativistic environment surrounding the above-mentioned shock seems suitable for the acceleration of protons to high energies [2]. In non-relativistic shocks, particle acceleration is a painfully slow process: particles shuffle diffusively from downstream back to upstream and vice versa, each time increasing their speed infinitesimally (this is the modern version of the Fermi mechanism, [3]). Instead, in relativistic shocks, the distribution function of non-thermal particles in the fluid frame is strongly collimated in the direction perpendicular to the shock, and they suffer deflections which differ little from forward/backward scattering [4]. Furthermore, at each cross-shock, their energies are multiplied by the factor \( \gamma^2 \); for the large Lorenz factors for the shocks in the GRBs’ scenarios described above, the largest known in the Universe, just two or three cycles suffice to propel protons to energies \( \approx 10^{20} \) eV. In GRBs, a large part of the energy loss must go through this channel, but the mechanism seems so fast and powerful that it has been suggested that the whole flux of Ultra High Energy Cosmic Rays (UHECRs) observed at Earth is generated in GRBs [5].

The major rival to acceleration of UHECRs in GRBs is acceleration in blazars, or in the hot spots of radio galaxies [6]. This mechanism is however hampered by the paucity of blazars and radio galaxies inside the Greizen–Zatsepin–Kuzmin limit \( \eta < 100 \) Mpc. Thus it has problems explaining the rough isotropy of the directions of arrival of the UHECRs, and the lack of any suitable candidate as the site of acceleration around the direction of arrival of the highest energy cosmic ray observed so far [7], which are not a problem for the GRB theory [8].

However, the GRB scenario ought to find independent confirmation. One possible test has been proposed already [9]: it involves the fact that the spectrum of cosmic rays emitted by a single source, as observed at the Earth, is wide (a power-law) if the source is a continuous emitter, but is much narrower if the source is explosive, because of energy–dependent time–delays in the propagation of cosmic rays in the magnetic field of intergalactic regions. In this Letter, I propose a different test which may also reveal something about the details of the acceleration process.

Synchrotron emission by UHECRs. In models for cosmological GRBs, two different scenarios for the generation of the shock have been envisioned so far: in the first [10], a shock is generated when the ejecta crash into the interstellar medium, much like a SuperNova. In the second [12], the compact object generates two expanding relativistic shells, endowed with slightly different Lorenz factors; when the second, faster shell overcomes the first one, collisionless shocks propagate through both shells. It has been argued [13] that the first mechanism may be responsible for bursts with smooth lightcurves, and the second one for spiky lightcurves. For reasons to be explained later, I concentrate on the first one.

In this scenario, a total energy release \( E_{\text{GRB}} = E_{\text{GRB}}^{\text{51}} \) erg is contaminated with a baryon mass \( M_b \) such that

\[
\eta \equiv E_{\text{GRB}}/M_b c^2 \approx 10^3. \tag{1}
\]

The baryons, and the Coulomb–dragged electrons, are
accelerated to a Lorenz factor $\gamma \approx \eta$, and then start a coasting phase which is terminated with the formation of a shock with the interstellar medium, of number density $n_1 \approx 1 \text{ cm}^{-3}$, for typical ISM environments. The shock forms approximately when a total ISM matter $\approx M/\eta$ has been collected. This occurs at a radius

$$r_d = 10^{18} \text{ cm} E_{51}^{1/3} n_1^{-1/3} \eta^{-2/3}. \quad (2)$$

At this moment, the just-formed shock splits: a forward shock has been collected. This occurs at a radius

$$r_{sh} = r_d/\eta$$

in the shell frame, where the post-shock magnetic field is $B = 1 \text{ G} n_1^{1/2} \eta^{1/2}$, with $\xi$ parametrizing departures from exact equipartition (corresponding to $\xi = 1$); for this GRB model to work properly, it is necessary that that $\xi \approx 1$, which I shall assume henceforth.

It was argued in [2] that the distribution of non-thermal protons extends from $E_i \approx \eta^2 m_p c^2 \approx 10^{15} \text{ eV}$ to $E_u = 10^{19} \text{ eV} \eta^{1/3} E_{51}^{1/3} n_1^{1/6} \xi^{1/2}, \quad (3)$

all energies measured at the Earth. In the shell frame, the lower and upper Lorentz factors become $\gamma_l \approx \eta$ and $\gamma_u \approx 10^8 \eta/10^3)^{-2/3} E_{51}^{1/3} n_1^{1/6} \xi^{1/2}$.

The spectrum of non-thermal particles behind non-relativistic shocks is a power-law with index $p \approx 2$. Albeit less is known about relativistic shocks, a similar conclusion holds [3]. This is consistent with observations: Waxman [15] has computed the injection spectrum at GRBs, such that, after inclusion of photopion losses [7] and cosmological effects, these particles fit the cosmic ray spectrum observed at Earth, for observed energies exceeding $3 \times 10^{18} \text{ eV}$. He finds good agreement for any index such that $1.8 < p < 2.3$, consistent with production of UHECRs in GRBs. I shall thus take, in the fluid frame, $d\eta = N_\text{UHECR} \gamma^{-p} d\gamma$, for $\gamma_i < \gamma < \gamma_u$.

For a power-law energy distribution of non-thermal particles, the spectrum of synchrotron emission follows a power-law $\propto \nu^{-s}$ with index $s = (p - 1)/2 \approx 0.5$ in our case. Thus synchrotron emission is heavily dominated by the high-energy end of the spectrum. Each non-thermal particle, in the fluid frame, emits at a typical frequency given by $\nu_c = 3e^2 \epsilon B / 2m_\text{p} c^2$, where I dropped the inessential dependence on the angle between the field and the particle mean velocity, and $\epsilon$ is the particle energy in the fluid frame $\approx E/\eta$. The high-energy end of the photons’ spectrum is cutoff at the typical emission frequency $\nu_{uc}$ of the highest energy particles, $E_u$, which, scaling to the values given above, and transforming the emission frequency to the Earth’s reference frame, is

$$E_{co} = \eta \nu_{uc} = 1 \text{ GeV} \eta^{2/3} E_{51}^{2/3} n_1^{-1/6} \xi^{3/2}; \quad (4)$$

for the values favoured by observations of $\eta = 10^3$ and $E_{51} = 4$ [10], the cutoff energy becomes $E_{co} \approx 300 \text{ GeV}$.

While the spectral shape and the cutoff energy are easy to derive, the overall normalization is trickier. In non-relativistic shocks [7], the total energy emitted as radiation is of the same order of magnitude as that channeled into non-thermal particles; while for relativistic shocks no equivalent computation exists, still the arguments given above make it clear that relativistic shocks ought to be, if anything, more efficient than non-relativistic ones in accelerating protons. Assuming rough equipartition, the total flux of UHECRs at Earth is explained by the GRBs’ scenario [7]. Thus I will equate the total energy released in cosmic rays $E_{\text{CR}}$ to that released by the burst in radiation, both, of course, in the shell frame:

$$E_{\text{CR}} = N_{\text{UHECR}} m_p c^2 \int_{\gamma_l}^{\gamma_u} \gamma^{-1} d\gamma = E_{\text{GRB}} / \eta, \quad (5)$$

obtaining $N_\text{UHECR} = E_{\text{GRB}} / \eta m_p c^2 \ln \gamma_u / \gamma_l$.

The total mass in non-thermal particles $M_{\text{CR}}$ thusly determined is reassuringly small: we have

$$M_{\text{CR}} = N_{\text{UHECR}} m_p \int_{\gamma_l}^{\gamma_u} \gamma^{-2} d\gamma = N_{\text{UHECR}} m_p \frac{\gamma_l}{\gamma_u}, \quad (6)$$

which, using Eqs. [2] [3] and $\gamma_l \approx \eta$, gives

$$\frac{M_{\text{CR}}}{M_\text{b}} = \frac{1}{\eta \ln \gamma_u / \gamma_l} \ll 1. \quad (7)$$

The synchrotron energy loss per particle per unit time is given by $\dot{\epsilon} = -2 e^4 B^2 \epsilon^2 / (3 m_\text{p} c^2)$. Most of the energy is lost by the highest energy non-thermal particles; for those with energy $\approx E_u$, it can be shown that the ratio of synchrotron deceleration time to shell crossing time is [2]:

$$\frac{t_{sy}}{t_{sc}} = 90.0 n_1^{4/3} E_{51}^{1/3} \xi^{-3/2}, \quad (8)$$

which depends weakly upon all parameters except the efficiency $\xi$ with which equipartition magnetic fields are built up behind the shocks.

The total energy loss through synchrotron emission by non-thermal protons is obtained by integrating the energy loss rate $\dot{\epsilon}$ over the particle spectral distribution, with the normalization given above, and multiplying times the flight time across the shell thickness, which is the region over which the magnetic field is appreciable. Transforming then to the Earth frame, the total energy radiated is

$$E_{sy} = 3.0 \times 10^{49} \text{ erg} \eta^{-1/3} n_1^{-7/6} E_{51}^{5/3} \xi^{3/2}. \quad (9)$$

For the favoured values $\eta = 10^3$ and $E_{51} = 4$, I find $E_{sy} = 3.0 \times 10^{49} \text{ erg}$. For $p = 2.3$, the limit of the range allowed by fitting the UHECRs’ spectrum at Earth [15],
the above computation yields \( E_{sy} \approx 3 \times 10^{48} \text{ erg} \). The reduction of the yield in GeV photons is due to the fact that steep particle spectra have less energy in the high-energy region, where synchrotron losses are stronger. In summary, I expect a fraction \( \approx 0.01 \) of the total photon energy release to end up in the GeV region.

The spectrum can now be rewritten in terms of the photon energy as observed at the Earth \( E_{ph} \) in units of the cutoff energy, Eq. (3), \( x \equiv E_{ph}/E_{co} \), as

\[
dE_{s\gamma}^{(sp)} = \frac{E_{sy}}{s} x^{-s} \, dx
\]

with \( E_{sy} \) given above. Since detectors in this energy range are obviously photon counting devices, it is convenient to give also the particle spectrum:

\[
dN_{s\gamma}^{(sp)} = \frac{E_{sy}}{s E_{co}} x^{-(s+1)} \, dx \approx \frac{1.0 \times 10^{50}}{s} x^{-(s+1)} \, dx .
\]

The last loose end left is to check whether the optical depth for pair creation against the photons emitted by the electrons exceeds unity; in this case, the GeV–range photons would be degraded in energy, and no flux near the photon cutoff energy might reach the Earth. I follow in this ref. [18]. The total optical depth is \( \tau_{\gamma\gamma} = r_{sh}/\ell_{\gamma\gamma} \), where the mean free path for photon–photon pair creation is \( \ell_{\gamma\gamma} = \sigma_T U_\gamma \epsilon_{ph}/(4m_c c^2)^2 \). Here \( \sigma_T \) is the Thompson cross section, \( U_\gamma = E_{GRB}/(4\pi r_0^2) \) is the photon energy density in the shell frame, and \( \epsilon_{ph} \) the test photon energy also in the shell frame. The above formula approximates the photon/photon pair creation cross–section as a constant, \( \approx 3\sigma_T/16 \), neglecting its decrease with increasing energies. It is thus, strictly speaking, an upper limit to the optical depth, which is adequate here. It can easily be seen with the values provided above that

\[
\tau_{\gamma\gamma} = 1.2 \times 10^{-5} E_{51}^{1/3} \eta^{1/3} \epsilon_{ph}^{1/3} \frac{m_c c^2}{300} .
\]

For the usual favoured values \( E_{51} = 4, \eta = 10^3 \), I find that \( \tau_{\gamma\gamma} = 1 \) only for photons which, as seen from Earth, exceed \( 3 \text{ TeV} \). Thus there will be no energy degradation in situ because of photon/photon pair creation.

This contrasts with the opposite result obtained in ref. [18], but this is because they considered the other scenario for the generation of the shock, i.e., where two relativistic shells collide with each other. In their scenario the shock occurs at smaller radii, so that the photon energy density is much higher than in the scenario adopted here. This is the reason why I concentrated on ISM shock scenario. It should be stressed that UHECRs are accelerated in both scenarios, the only difference being that no GeV photons can come out of one of them, and that, potentially, detection of these high energy photons may distinguish between the two GRB scenarios.

**Observability.** I now consider the observability of the highest energy photons, those with energies close to \( E_{co} \). High energy photons produce pairs by collisions with photons of the Infra Red or Microwave Background, thus losing energy efficiently. This limits the range from which 300 GeV photons can reach us to \( D_m \approx 300 \text{ Mpc} \). The flatness of the log \( N \) – log \( S \) relation for bursts [14] implies that we are already seeing the edge of the GRBs’ distribution; thus the rate of GRBs deduced from observations, \( \approx 30 \text{ yr}^{-1} \text{ Gpc}^{-3} \), is not likely to be very incomplete. From this, I deduce a rate of GRBs inside \( D_m \) of \( 3 \text{ yr}^{-1} \). For these distances, the expected fluence from Eq. (4) is \( 3 \times 10^{-6} \text{ erg cm}^{-2} \).

There are currently no experiments which can detect showers initiated by primaries with energies around \( E_{co} \) with the required sensitivity, but the next generation of high–altitude (> 4000 m a.s.l.) detectors currently being planned will have relevant detection thresholds of \( \approx 10 \text{ GeV} \). Since I argued that the spectrum is expected to be very flat, most energy, though not most counts, will be deposited above this detection threshold, so that the limiting factor will be the experiments’ detection surface. As an example, ARGO [20] can detect signals down to flux levels of \( \approx 3 \times 10^{-7} \text{ erg cm}^{-2} \) for spectra extending out to 300 GeV, provided the bursts last 1 s, and the spectra are flatter than \( s \approx 1.5 \). However, with this low detection threshold, a burst with fluence \( 3 \times 10^{-6} \text{ erg cm}^{-2} \) could be detected over background noise even if it lasted 100 s.

**Implications.** High energy photons will be emitted as long as protons are accelerated to very high energies, which only requires the presence of large magnetic fields. There is no obvious reason why the field should decay on the timescale of the burst. In fact, first, as seen from Earth, the shell remains relativistic for about a month after the burst [21], which implies that relativistic electrons will be available in the post–shock region to generate a non–negligible magnetic field. Second, after the burst, the electrons exchange energy with protons through a variety of processes. So long as electrons are thusly kept relatively hot, their chaotic motion may maintain an appreciable magnetic field. Since processes coupling electrons to protons are relatively inefficient in transferring energy, some magnetic field may be left for some time after the burst. So observations of the secondary burst discussed in this paper may reveal burst durations appreciably longer than those of their lower energy counterparts.

Another important issue is whether this emission may be masked by Inverse Compton effects: it was shown [13] that photons with energies as high as \( 10 (\eta/100)^6 \) GeV, very uncertain because of the steep dependence upon \( \eta \), can be produced this way. In this case, the expected spectrum is simply that of lower energy, seed photons, which get a kick to higher energies: typically the photon number spectrum is \( \propto \nu^{-2} \), different from Eq. (10).
furthermore implies that the energy released per photon energy decade is constant, and thus exceeds the estimate of Eq. 9 by about two orders of magnitude. Thus, spectral steepness and fluence allow an easy discrimination between the two mechanisms.

The above argument finds an interesting application at intermediate energies, ≈ 1 GeV. One may in fact wonder whether the low energy tail of the emission derived in Eq. 10 has been observed already, since some bursts do show detectable fluxes at these energies [22]. This can be certainly excluded for GB 930131, which has comparable fluences in the low–energy BATSE spectrum (≈ 1 MeV) and in the GeV region, with nearly identical spectral slopes [23]. The situation is however less clearcut for other bursts, like GB 940217, GB 910503, which have fluences, at their respective highest energies, much lower than those emitted at lower energies, and thus in keeping with the estimate of Eq. 9. It should also be noticed that all these bursts show, at the highest energies, burst duration much longer than at low energies, a fact naturally accounted for in this model, as discussed above.

A mechanism for the production of delayed GeV/TeV photons from GRBs based upon the presence of UHECRs has been proposed [24], but it differs greatly from the present one: there UHECRs produce photons through photopion processes off CMB photons in flight from the site of the burst to our detectors, while in this model the emission mechanism is synchrotron in situ.

Summary. Physical conditions at the relativistic shocks which give origin to GRBs are surely favorable to acceleration of high energy particles. If the energy in UHECRs is comparable to that released in photons (and thus if UHECRs from GRBs account for the whole flux of UHECRs at Earth), the fluxes are those predicted above. Otherwise, the observed flux of GeV photons will allow measurement of the fraction of energy channeled into non–thermal particles.

I have shown that the synchrotron emission spectrum from protons extends to $E_{\gamma} \approx 300$ GeV (Eq. 10) and should be rather flat (Eq. 10), that the total energy release is $E_{\gamma} \approx 0.01E_{GRB}$ (Eq. 3), and that a few such events per year will surely become observable with the next generation of high altitude, small size air shower detectors. I have also argued that all GeV emission observed to date from GRBs is compatible with this model.

Acknowledgements. Thanks are due to G.C. Perola for a critical reading of the manuscript.

[1] T. Piran, to appear in Some unsolved problems in Astrophysics, Princeton 1995, eds. J. Bahcall and J.P. Ostriker, astro–ph n. 9507114 (1995); M.J. Rees, astro–ph n. 9701162 (1997).
[2] M. Vietri, Astrophys. J. 453, 883 (1995).
[3] E. Fermi, Phys. Rev. 75, 1169 (1949).
[4] J.J. Quenby and R. Lieu, Nature 342, 654 (1989).
[5] E. Waxman, Phys. Rev. Lett. 75, 386 (1995).
[6] For an enthusiastic review, see R.J. Protheroe, to appear in Towards the Millennium in Astrophysics: Problems and Prospects, Erice 1996, eds. M.M. Shapiro and J.P. Wefel (World Scientific, Singapore); astro–ph n. 9612212 (1996).
[7] High energy cosmic rays ($E > 10^{19}$ eV) lose energy very efficiently through photopion production off Cosmic Microwave Background photons; at $E \approx 10^{20}$ eV, the energy loss distance is < 20 Mpc, K. Greisen, Phys. Rev. Lett. 16, 748 (1966); G.T. Zatsepin and V.A. Kuz’m in, JETP Lett. 4, 78 (1966); R.J. Protheroe and P.A. Johnson, Astroparticle Phys. 4, 253 (1995) and erratum 5, 215, 1996.
[8] J.W. Elbert and P. Sommers, Astrophys. J., 441, 151 (1995).
[9] E. Waxman, to appear in Proc. ICRR Symposium on Extremely High Energy Cosmic rays, Ed. M. Nagano (Tokyo 1996); astro–ph n. 9612061.
[10] J. Miralda–Escudé and E. Waxman, Astrophys. J. Lett. 457, 11 (1996).
[11] P. Mézéras, P. Laguna and M.J. Rees, Astrophys. J. 415, 181 (1993).
[12] M. Rees, and P. Mézéras, Mon. Not. Roy. Astron. Soc. 258, 41P (1994).
[13] R. Sari and T. Piran, subm. to Astrophys. J. Lett., astro–ph n. 9701002.
[14] A.R. Bell, Mon. Not. Roy. Astron. Soc. 225, 615 (1987); F.C. Jones and D.C. Ellison, Space Sci. Rev. 58, 259 (1991); J.G. Kirk and P. Schneider, Astroph. J. 315, 425 (1987).
[15] E. Waxman, Astrophys. J. Lett. 452, 1 (1995).
[16] T. Piran, Astrophys. J. Lett. 389, 45 (1992).
[17] H. Völk, L.O.C. Drury and J. McKenzie, Astron. & Astrophys. 130, 19 (1984).
[18] E. Waxman and J. Bahcall, to appear in Phys. Rev. Lett. 78, astro–ph n. 9701231 (1997).
[19] G.N. Pendleton et al., Astrophys. J. 464, 606 (1996).
[20] B. D’Ettorre–Piazzolla et al., in Proceedings of the 24th ICRC Meeting, Rome 1996, ed. N. Iucci, 3, 508 and 504; B. D’Ettorre–Piazzolla, in Astroparticle Physics with ARGO, unpublished, Rome (1996), and personal communication (1997).
[21] M. Vietri, Astrophys. J. Lett. 478, 1 (1997).
[22] They are GB 910503, GB 910601, GB 930131, GB 940217 and GB 940301, for a discussion see B. Dingus, Astrophys. and Space Sci. 230, 187 (1995).
[23] C. Kouveliotou et al., Astrophys. J. Lett. 422, 59 (1994); D. Sommer et al., Astrophys. J. Lett., 422, 63 (1994).
[24] E. Waxman and P. Coppi, Astrophys. J. Lett. 464, 75 (1996).