Laser plasma acceleration with a negatively chirped pulse: all-optical control over dark current in the blowout regime

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Abstract. Recent experiments with 100 terawatt-class, sub-50 femtosecond laser pulses show that electrons self-injected into a laser-driven electron density bubble can be accelerated above 0.5 gigaelectronvolt energy in a sub-centimetre-length rarefied plasma. To reach this energy range, electrons must ultimately outrun the bubble and exit the accelerating phase; this, however, does not ensure high beam quality. Wake excitation increases the laser pulse bandwidth by red-shifting its head, keeping the tail unshifted. Anomalous group velocity dispersion of radiation in plasma slows down the red-shifted head, compressing the pulse into a few-cycle-long piston of relativistic intensity. Pulse transformation into a piston causes continuous expansion of the bubble, trapping copious numbers of unwanted electrons (dark current) and producing a poorly collimated, polychromatic energy tail, completely dominating the electron spectrum at the dephasing limit. The process of piston formation can be mitigated by using a broad-bandwidth (corresponding to a few-cycle transform-limited duration), negatively chirped pulse. Initial blue-shift of the pulse leading edge compensates for the nonlinear frequency red-shift and delays the piston formation, thus significantly suppressing the dark current, making...
the leading quasi-monoenergetic bunch the dominant feature of the electron spectrum near dephasing. This method of dark current control may be feasible for future experiments with ultrahigh-bandwidth, multi-joule laser pulses.

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1. Introduction

Plasma-based laser wake field accelerators (LWFAs) [1, 2] have recently produced electron beams in the 0.5–1.5 gigaelectronvolt GeV energy range [3–11]. The introduction of short-pulse, compact petawatt (PW) laser systems [12–17] opens up possibilities beyond the GeV-energy frontier [18–23], enabling further progress towards practical high-brightness x-ray sources [24–35] and compact high-energy physics particle colliders [36, 37]. Femtosecond (fs)-scale duration and multi-kiloampere current of beams of electrons trapped and accelerated by the laser wake fields [38, 39] are very well suited to these applications. However, much work is yet to be done to meet the stringent requirements of high collimation and low energy spread. Currently, GeV-scale beams produced by non-channelled LWFAs have polychromatic energy distributions, sometimes with a quasi-monoenergetic (QME) feature at the high-energy end [5–9, 30]. In this paper, we explore the relationship between the quality of the electron beam and the nonlinear evolution of the accelerating structure, and present methods for controlling electron energy spread and for suppressing the polychromatic background.

Most LWFA experiments work in the blowout regime [21]. The ponderomotive force of a focused laser pulse produces full cavitation of the surrounding electron fluid. Nearly all plasma electrons in the path of the pulse are expelled by the radiation pressure, whereas fully stripped ions remain immobile. The fields due to this charge separation attract electrons from the bulk to the axis, creating closed cavity of electron density (the ‘bubble’ [40, 41]). The cavity, surrounded by a dense shell (sheath) of relativistic electrons, encompasses the pulse and guides it until depletion [21, 42]. The bubble is a high-quality three-dimensional electromagnetic (3D EM) accelerating structure. Its focusing gradient, uniform longitudinally and linear radially, implies strict conservation of normalized transverse emittance. In addition, the accelerating field is radially uniform, which helps mitigate longitudinal emittance dilution [43, 44]. The bubble propagates with the group velocity of the laser pulse, which, in a linear approximation, can be expressed as \( v_g \approx c (1 - \gamma_g^{-2})^{1/2} \). Here, \( c \) is the speed of light in vacuum, \( \gamma_g = \omega_0 / \omega_{pe} \gg 1 \), \( \omega_0 \) is the laser frequency, \( \omega_{pe} = (4 \pi e^2 n_0 / m_e)^{1/2} \) is the electron Langmuir frequency, \( m_e \) is
the electron rest mass, \( n_0 \) is the background electron density and \( e \) is the electron charge. Even with the Lorentz factor \( \gamma_g \) approaching 100, the bubble is a ‘slow’ structure capable of capturing and accelerating initially quiescent electrons of the ambient plasma [22, 23, 45–47]. Optical diagnostics directly correlate the generation of a collimated electron beam with bubble formation [48–52]. While other (e.g. all-optical) injection schemes are currently being explored [53–57], electron self-injection has its own advantages: it greatly reduces the technical complexity of the experiment, preserving flexibility in parameters and enabling a single-stage acceleration of nano-Coulomb (nC) charge [23, 47].

Accelerated electrons eventually outrun the slow bubble. They exit the accelerating phase within a time interval \( \tau_d = L_d/c \), gaining the maximum possible energy, \( E_{\text{max}} \approx 85 \gamma_g^2 (k_p R_b)^2 \text{keV} \) [21]; here, \( L_d = (2/3) \gamma_g^2 R_b \) is the dephasing length, \( R_b \) is the bubble radius and \( k_p = \omega_{pe}/c \). In strongly rarefied plasmas, where \( \gamma_g \gg k_p R_b \), dephasing takes many Rayleigh lengths\(^5\). Propagation of the pulse over this distance relies on a combination of relativistic and ponderomotive self-guiding [58–60]. Upon entering the plasma, the pulse, with \( P \gg P_{\text{cr}} \) and duration \( \tau_L < 2\pi/\omega_{pe} \), self-focuses until full electron cavitation is achieved, and the charge-separation force balances the radial ponderomotive force; the pulse is then guided until depletion (here, \( P_{\text{cr}} = 16.2 \gamma_g^2 \text{GW} \) is the critical power for relativistic self-focusing [61]). As this force balance is approached, the bubble size oscillates, causing electron injection during a brief time interval. A QME electron bunch thus forms early [45, 62, 63]. However, transient dynamics before the onset of self-guiding [23], as well as laser evolution during self-guiding [5, 7, 22, 23, 63–65], may cause unwanted additional electron injection (dark current), degrading the beam quality. The lack of balance between the light pressure and charge-separation force makes the pulse spot size oscillate [22, 23, 64, 65]; self-steepening [22, 66–72] and depletion [73] gradually turn the pulse into a relativistic piston [63]; and relativistic filamentation distorts the transverse profile of the pulse [23, 74–76]. Any of these phenomena can cause the undesirable deformations of the bubble and bring about a large dark current [5, 7, 22, 23, 63]. Novel ultrafast optical diagnostics can capture these deformations in a single shot [38, 50, 77], providing feedback for real-time optimization and yielding raw experimental data for validation of new numerical models. Controlling laser evolution thus provides an avenue to suppress dark current without complicating the experimental design.

To reduce nonlinear effects and stabilize the bubble, it is necessary to work with optically perfect, matched PW-class pulses in low-density plasmas, where \( \gamma_g > 35 \) [21]. At even lower densities, \( \gamma_g > 65 \), one can enforce and control injection by properly mismatching initial conditions for the pulse [45], producing dark-current-free, nC-charge, multi-GeV electron beams [46, 47]. At higher plasma densities, \( \gamma_g < 20 \), longitudinal deformation of the pulse becomes the dominant source of dark current [63]. This situation is typical of recent LWFA experiments with single-jet plasma targets and 100 terawatt (TW)-class, 30–60 fs laser pulses [5–7, 28, 30, 78]. The leading edge of the pulse rapidly red-shifts in frequency, and anomalous (or negative) group velocity dispersion of plasma compresses the pulse [59, 63, 66–72]. This physical process is the same as the soliton effect occurring in optical fibres [79, 80]. Depletion of the leading edge further enhances the self-steepening effect [73]. An initially smooth driver thus turns into a relativistically intense optical ‘piston’ or a ‘snow-plow’ that pre-accelerates and compresses the initially quiescent electron fluid. A large

\(^5\) This estimate implies that the laser pulse spot size is roughly the same as the bubble radius [21].
charge separation immediately behind this piston results in sheath electrons receiving a strong longitudinal kick, increasing their inertia and delaying their return to the axis. As a result, the bubble constantly elongates, and massive, uninterrupted self-injection follows [5, 7, 63]. Notably, as the pulse self-compresses, and continuous injection begins halfway through the plasma, transverse matching the pulse for self-guiding (following [21]) does not help suppress the dark current or improve the beam characteristics [7, 22]. Even though terminating acceleration after the first oscillation of the laser spot size may still result in the production of a few hundreds of MeV QME beam [45, 62], the electron beam energy and, hence, the accelerator efficiency cannot be maximized this way.

It is commonly assumed [5, 6, 22, 78, 81, 82] that, owing to nC-scale injected charge, repulsive transverse EM fields of the trapped electron beam, acting on the sheath electrons, resisting their return to the axis (viz the effect of beam loading [83–85]), play a key role in the bubble expansion, and are thus chiefly responsible for continuous injection. Were that true, dark current would be impossible to control, making broad-bandwidth electron beams the only possible outcome of experiments with 100 TW-class lasers [5–7, 28, 30]. Fortunately, simulations with the beam loading ‘turned on’ (full 3D PIC) and ‘off’ (quasi-static PIC) reveal very little difference in the bubble evolution and mechanics of beam formation. As is shown in [63] and section 3 of this paper, the effect of beam loading becomes noticeable only in the proximity of the dephasing limit. Beam loading is thus a much less important factor in the generation of dark current than is commonly believed. As a corollary, a judicious choice of the phase profile of the driving laser pulse mitigates piston formation, suppressing expansion of the bubble and, hence, reducing the dark current.

We demonstrate suppression of dark current using a broad-bandwidth, negatively chirped laser pulse. The initial frequency up-shift of the leading edge compensates for the gradually accumulating red-shift, delaying the formation of the relativistic piston and reducing the amount of dark current. To elucidate the connection of electron injection with the laser pulse evolution, we use two complementary simulation approaches. Using the quasi-static, cylindrically symmetric, fully relativistic PIC code WAKE with 3D test-particle tracking [45, 86, 87], we explain the physics and develop the conceptual framework of the problem. WAKE simulations link the details of the self-injection process to the paraxial evolution of the driving pulse and quasi-static evolution of the bubble, separating these details from the effects caused by beam loading. Physical scenarios predicted using WAKE are reproduced and verified in fully explicit quasi-cylindrical PIC simulations using the code CALDER-Circ [88]. In section 2, we give a summary of the numerical models and specify the tasks for which different codes are used. In section 3, we examine self-injection in an LWFA driven with a strongly overcritical, transform-limited Gaussian pulse under conditions similar to those of recent experiments [5–7, 28, 30, 78]. We find that, contrary to the earlier interpretation [78], the contribution of beam loading to the bubble expansion is not dominant, and we associate the origin of dark current with a nonlinear optical effect—laser pulse self-compression. In section 4, we show how this effect can be mitigated and controlled using a broad-bandwidth, negatively chirped driver. In section 5, we summarize the results and point out directions for future work. The appendix demonstrates that our method of dark current control helps improve the beam characteristics even under the extremely nonlinear conditions of experiments with the Astra Gemini laser [5].
2. Simulation methods

We examine various scenarios of electron self-injection and acceleration until dephasing and relate them to nonlinear dynamics of the laser pulse. The quasi-static nature of the bulk plasma response [44] makes it possible to elucidate the physics of self-injection in a conceptually simple and computationally efficient way. Incorporating 3D test-particle tracking into the cylindrically symmetric, time-averaged (over $\omega_0^{-1}$) quasi-static PIC code WAKE [86] provides significant physical insight into the self-injection process [63]. WAKE models the laser pulse propagation using an extended paraxial solver for a slowly varying laser pulse envelope. WAKE preserves group velocity dispersion in the vicinity of the carrier frequency and calculates precisely radiation absorption due to the wake excitation. The plasma response to the time-averaged ponderomotive force is calculated assuming that all plasma electrons (macroparticles) and ions (treated as a non-relativistic fluid) eventually fall behind the laser pulse and the bubble. This approach, termed the quasi-static approximation [89], greatly speeds up the simulation, enabling serial runs and parameter scans using small workstations. On the other hand, neglecting the long-term contribution of macroparticles travelling with the structure prohibits self-consistent modelling of electron injection and trapping. To simulate self-injection without compromising computational efficiency, we use test particles, which are fully dynamic and evolve in Cartesian 3D space. In particular, we take into account the interaction of test electrons with the non-averaged, linearly polarized laser field with non-paraxial corrections [87, 90]. To capture the laser pulse interaction with non-quasi-static background electrons (and thus to model self-injection into non-stationary quasi-static wake fields), a group of quiescent test electrons is placed before the laser pulse at each time step. In this way, electron self-injection associated with bubble and driver evolution is separated from the effects brought about by the collective fields of the trapped electron bunch, i.e. from effects due to the beam loading [83–85].

Collective fields of the electron beam, neglected in the test-particle treatment, are known to change the shape of the sheath and thus reduce the accelerating gradient, eventually terminating self-injection [83]. We verify the test-particle results with fully explicit 3D PIC simulations using the identical initial conditions. We use the quasi-cylindrical code CALDER-Circ [88], which preserves realistic geometry of interaction, and accounts for the axial asymmetry by decomposing EM fields (laser and wake) and currents into a set of poloidal modes (whereas the particles retain full 3D dynamics). Well-preserved cylindrical symmetry during the interaction enables us to use just the two lowest-order modes and thus reduce the 3D problem to essentially a 2D one. Ion species are treated as either movable macroparticles or as a frozen positive background. We have verified that, under the conditions of our case study, the ion response to the laser and wake fields does not change the results in any noticeable way, and we present the results of simulations made with frozen ions. CALDER-Circ is thus perfectly suited for rapid, high-fidelity parameter scans [63], and for computationally inexpensive large-scale production runs [23, 47] using medium-size clusters.

All the reported simulations use the same set of initial conditions. A linearly polarized Gaussian laser pulse, either transform-limited or chirped, with a full-width at half-maximum (FWHM) in intensity $\tau_L = 30 \text{ fs}$, central wavelength $\lambda_0 = 0.805 \mu \text{m}$ and 70 TW power, is focused at the plasma border ($z = 0$) into a spot size $r_0 = 13.6 \mu \text{m}$, and propagates in the positive $z$-direction. The peak intensity at focus is $2.3 \times 10^{19} \text{ W cm}^{-2}$, corresponding to the normalized vector potential $a_0 = |e|A_\perp/m_e c^2 = 3.27$. The plasma density has a 0.5 mm linear entrance ramp followed by a plateau. The density in the plateau region, $n_0 = 6.5 \times 10^{18} \text{ cm}^{-3}$,
corresponds to $\gamma_g \approx P/P_{cr} \approx 16.3$, and dephasing length $L_d \approx 1.7$ mm. All WAKE simulations use the grid $\Delta r \approx 0.1 k_p^{-1} \approx 210$ nm, with 30 macroparticles per radial cell, $\Delta \xi = \Delta r/3$ (where $\xi = z - ct$), and time step $\omega_0 \Delta t \approx 1.325$. In the CALDER-Circ simulations, we suppress the sampling noise by using a large number of macroparticles (45 per cell) and high resolution in the direction of propagation, $\Delta z = 0.125 c/\omega_0 \approx 16$ nm. The aspect ratio $\Delta r/\Delta z = 15.6$, and the time step $\omega_0 \Delta t = 0.1244$. As we shall see in the following sections, despite a much coarser grid, larger time step and underlying approximations, WAKE correctly captures all the relevant physics of plasma wake evolution and self-injection dynamics. In addition, CALDER-Circ having fully self-consistent macroparticle dynamics yields the complete electron phase space, and thus allows the determination of the injected charge and beam emittance.

3. Origin of dark current

3.1. Two stages of laser and bubble evolution

The simulations with an unchirped driving pulse show that electrons are accelerated until dephasing in two distinct stages, each characterized by entirely different laser dynamics [63]. Transverse evolution of the laser pulse is the hallmark of stage I. The pulse spot size oscillates, first causing expansion and then contraction of the bubble. Bubble expansion enforces self-injection of electrons from the sheath; stabilization and contraction extinguish injection, limiting the beam charge to a fraction of nC. Phase space rotation rapidly creates a well-collimated QME bunch long before dephasing. Further acceleration (stage II) is dominated by longitudinal/temporal self-compression of the pulse, leading to gradual elongation of the bubble and continuous injection, producing a polychromatic, poorly collimated energy tail with a few-nC charge. This two-stage evolution has been noticed in earlier simulations [5, 7], but without examining its physical origin. Bubble evolution and its connection to the self-injection process are presented in figure 1. The length of the accelerating phase on the axis, i.e. the length of the region inside the bubble where the longitudinal electric field is negative, is plotted against propagation distance in figure 1(a). This plot of $L_{acc.ph}(z)$ precisely identifies the intervals of bubble expansion, and highlights the differences in bubble evolution brought about by beam loading. Tracking initial positions of beam particles associates their injection with the intervals of bubble expansion. Figure 1(b) shows the longitudinal ‘collection phase space’ (CPS), namely electron momenta at the dephasing point versus their initial longitudinal positions in plasma. Figure 1(c) shows the initial positions of electrons reaching the dephasing point, i.e. the collection volume in the coordinate space. Details of the local structure of the bubble, electron phase space, and energy spectra can be seen in figures 2 and 3.

3.1.1. Stage I: injection into the oscillating bubble and the formation of quasi-monoenergetic bunch. As the laser pulse enters the plasma, it rapidly self-focuses, reaching the highest intensity near the beginning of the density plateau ($z \approx 0.8$ mm). Full blowout is maintained over the entire propagation distance. The pulse is longer than one-half of the plasma period, and its head and tail evolve differently. The head, residing in an incompletely evacuated channel, retains its local spot size, whereas the tail confined within the bubble is mismatched and flaps radially [45]. The resulting expansion and contraction of the bubble, clearly seen in progression from 0.64 to 1.24 mm in figure 1(a), cause electron self-injection and the formation of a QME bunch. Electron density, longitudinal phase space and energy spectrum from the CALDER-Circ
Figure 1. Electron self-injection during the two stages of bubble evolution. (a) Length of the accelerating phase versus propagation distance in CALDER-Circ (red) and WAKE simulations (black). Both simulations show that expansion and contraction of the bubble (stage I) are followed by continuous expansion (stage II). Red (black) dots in panels (b) and (c) are the CALDER-Circ macroparticles (WAKE test particles) reaching the dephasing point, $z_{\text{deph}} \approx 2.2$ mm (only particles with $\gamma > \gamma_g$ are shown). (b) Collection phase space (longitudinal momenta, $p_z(z = z_{\text{deph}})$, versus initial longitudinal positions $z_{\text{in}}$). A QME bunch forms during stage I and retains low energy spread through dephasing. Groups of electrons encompassed by ellipses were injected and pre-accelerated in the second bucket, and then captured by the expanding first bucket. Continuous injection during stage II creates a polychromatic energy tail. (c) Collection volume: $R_{\text{in}} = \sqrt{x_{\text{in}}^2 + y_{\text{in}}^2}$ versus $z_{\text{in}}$. In both simulations, injection initiates, terminates and resumes at exactly the same positions along the laser pulse path. All accelerated electrons have impact parameters corresponding to sheath electrons, and are injected during the intervals of bubble expansion.

In both simulations, the bubble starts expanding at $z \approx 0.64$ mm, and fully expands at $z \approx 0.99$ mm (cf figure 1(a)). In spite of larger expansion given by the CALDER-Circ ($\Delta L_{\text{acc.ph.}} \approx 2.5 \mu$m against 1.5 $\mu$m given by WAKE), beam loading is unable to delay the bubble stabilization and enforce further expansion. According to the semi-empirical injection condition (2) of [63], this expansion is sufficient to re-phase the sheath electrons situated near the base of the bubble into the bubble. The resulting uninterrupted injection produces a large spread in longitudinal momentum and energy, shown in figures 2(d), (e), (g) and (h). As soon as the bubble fully expands, injection terminates. Figures 2(b) and (e) show that the bubble remains closed, and the longitudinal phase spaces of CALDER-Circ macroparticles and WAKE test particles are nearly identical. Beam loading thus plays no role in stabilization of...
Figure 2. Formation of a QME electron bunch during stage I (CALDER-Circ simulation). (a)–(c) Electron density in the laser polarization plane; (d)–(f) electron density in the longitudinal phase space (in arbitrary units); yellow dots are test electrons from the WAKE simulation with identical initial conditions; (g)–(i) electron energy spectra. Panels (a), (d) and (g) show the physical quantities at the beginning of the bubble expansion; (b), (e) and (h) correspond to the fully expanded bubble; and (c), (f) and (i) correspond to the contracted bubble. At the end of stage I, electrons trapped within the bubble form a bunch with the central energy $E = 247\text{ MeV}$; injection into the second bucket gives a diffuse spectral peak near $150\text{ MeV}$.

The bubble and termination of injection. Contraction of the bubble between $z = 1$ and $1.24\text{ mm}$ truncates the bunch: electrons injected between $z = 0.825$ and $0.985\text{ mm}$ are expelled. These particles do not reach dephasing and are thus missing from both CPS and collection volume in figures 1(b) and (c). Electrons injected between $x = 0.64$ and $0.825\text{ mm}$ remain in the bucket and are further accelerated. This well-separated group is clearly visible in figures 1(b) and (c). At the point of full contraction, the accelerating phase shrinks to the same size in both simulations, $L_{\text{acc.ph.}} \approx 9.5\mu\text{m}$. The transverse self-fields of the bunch are thus unable to resist the bucket contraction and prevent truncation of the bunch. During the contraction interval, the tail of the bunch, exposed to the highest accelerating gradient, equalizes in energy with the earlier injected electrons, resulting in a characteristic ‘U’-shape of distribution in the longitudinal phase space.
Figure 3. Continuous injection in quasi-static (WAKE with test particles) and full 3D PIC (CALDER-Circ) simulations during stage II. (a)–(c) Electron density from CALDER-Circ (top) and WAKE (bottom) runs. Yellow dots are the test electrons with $\gamma > \gamma_g$. (d)–(f) Electron energy spectrum (CALDER-Circ). (g)–(i) Longitudinal phase space (colourmap—CALDER-Circ; test electrons—yellow dots). Reprinted with permission from Kalmykov et al [63]. Copyright (2011), American Institute of Physics.

This feature, clearly visible in figure 2(f), was also observed in the similar situation in [21]. Bubble expansion, followed by stabilization and/or contraction, thus creates a QME bunch long before dephasing [45]. The bunch generated in CALDER-Circ simulation at the end of stage I has 0.214 nC charge and 247 ± 8 MeV energy (the spread estimated as FWHM of the spectral peak in figure 2(i)). This energy is consistent with the earlier 3D Cartesian PIC simulation [45], and is roughly 60 MeV (or 19%) lower than the test-particle prediction (cf figure 2(f)). At the dephasing limit, according to figure 1(b), the relative difference in energy between QME components of the test-particle and macroparticle beams decreases to 15.5%. The spectrum in figure 2(i) also reveals a diffuse feature near 150 MeV, corresponding to the electrons from the second bucket. These particles dephase early and are not accelerated effectively. During stage II, they are partly captured by the expanding first bucket and thus contribute to the dark current.
3.1.2. Stage II: continuous injection into the growing bubble. Although the QME bunch forms early, the general experimental trend is to push the accelerator efficiency to the limit and use the entire dephasing length. Beam quality, however, can be compromised in this pursuit. The laser pulse and bubble evolve continuously, maintaining a constant threat of dark current. By the end of stage I, the pulse leading edge, constantly staying on the descending slope of the nonlinear index at the front edge of the bubble, accumulates considerable red shift. During stage II, group velocity dispersion slows down the red-shifted spectral components, etching the pulse front and compressing the pulse into a relativistically intense, few-cycle-length optical piston \[63, 68–70\]. The bubble simultaneously elongates, trapping copious numbers of electrons. At the end of stage II, the beam consists of two distinct components clearly visible in figure 1(b): the leading bunch (created during stage I) and a polychromatic tail (the product of stage II). Electron density and longitudinal phase space from CALDER-Circ and WAKE simulations, as well as energy spectra of CALDER-Circ macroparticles, are presented in figure 3 at the beginning, in the middle and at the end of stage II.

Even though the tail accumulates an nC-scale charge, beam loading contributes little to the bubble expansion during most of stage II. From figure 1(a), we see that the bubble starts expanding around \(z \approx 1.2\) mm, with \(L_{\text{acc,ph}} \approx 9.5\) \(\mu\)m. Around \(z \approx 1.8\) mm (or 60\% of stage II), the bubble expands by 2.1 \(\mu\)m in the WAKE simulation, and by 2.85 \(\mu\)m in the CALDER-Circ simulation. This sub-micron difference is barely noticeable in figure 3(b); quasi-static and fully kinetic bubbles are almost indistinguishable, and test particle and macroparticle phase space distributions in the beam tail are nearly identical (cf figure 3(h)). Since the bubble shape remains virtually unaltered in the presence of the electron beam, beam loading as the cause of continuous injection is ruled out.

The bubble never stops expanding and capturing electrons. At the dephasing point, \(z \approx 2.2\) mm, the energy tail in the WAKE simulation \((\gamma_t < \gamma < 1100)\) accumulates a factor of 7.9 more test particles than the leading bunch. Beam loading reduces this ratio by roughly 18\%: CALDER-Circ generates the tail \((\gamma_t < \gamma < 900)\) with a factor of 6.5 higher charge than the head; namely, 1.4 nC against 0.214 nC. At this point, elongation of the quasi-static (WAKE) bubble, estimated from figure 1(a), is 3.5 \(\mu\)m, whereas beam loading in the CALDER-Circ simulation adds another 2.75 \(\mu\)m. Beam loading at last becomes noticeable, and yet not dominant. Bubble evolution thus remains predominantly quasi-static over the entire dephasing length. This conclusion is supported by analytic estimates \[83\], showing that the charge trapped before \(z \approx 2.2\) mm is insufficient to fully load the bucket \[63\].

In spite of consistently larger bubble expansion, beam loading leaves the collection volume unaltered. Figure 1(c) shows that electrons are collected from the cylindrical shell with the radius somewhat smaller than the local bubble size \([23, 63, 81, 91]\); the radius of the shell is prescribed by the local spot size of the laser pulse head \([23, 63, 92]\). Just a few particles are injected from the near-axis region. Thus, in agreement with earlier studies \([23, 63, 92]\), only electrons with impact parameters such that they enter the sheath are collected and accelerated at all stages of laser and bubble evolution.

These results highlight the usefulness of reduced models for understanding the physics of self-injection. In spite of the great difference in the algorithms and physics content, WAKE and CALDER-Circ predict the same correlation between the evolution of the bubble and electron self-injection. Common features of the two simulations help isolate and illuminate the most critical elements of the injection process. Most importantly, the collection volume of electrons reaching the dephasing limit is unaffected by the beam loading. In both simulations, injection
begins, terminates and resumes again at exactly the same spatial locations. Beam loading is unable to counteract the stabilization and contraction of the bucket. Furthermore, a careful examination of quasi-static and fully kinetic results indicates that the bubble expansion during stage II is predominantly a quasi-static effect. Thus, the qualitative picture of trapping is largely insensitive to beam loading, and the essential physics of the self-injection mechanism can be understood entirely in terms of test-particle dynamics. Even though beam loading reduces the accelerating gradient and slows down phase space rotation, it does not preclude the formation of the QME bunch at the end of stage I (namely before continuous injection begins). Even more importantly, beam loading is not responsible for continuous injection producing the massive energy tail during stage II. Overestimation of the QME bunch energy in test-particle simulation appears to be modest; here, it ranges from 19% at the end of stage I to 16% at the dephasing limit. Therefore, in practical terms, certain features of accelerator performance (electron beam duration; roughly, mean energy and energy spread) can be assessed without recourse to computationally intensive 3D PIC simulations, providing rapid feedback to the laboratory experiment and sufficiently motivating subsequent 3D PIC investigations.

The features of the electron beam evidently most sensitive to sheath conditions at the base of the bubble are the transverse emittance and total injected charge. Without self-fields of injected electrons, predictive modelling of these quantities seems to be impossible. Simulations with test particles usually underestimate the transverse emittance by a factor 5 to 10 [23, 47], and overestimate the total charge by more than an order of magnitude [63, 93] (one important corollary is that the absence of test electron injection rules out self-injection in full PIC simulations with identical initial conditions). Therefore, we do not attempt to estimate these quantities on the basis of test particle simulations, relying on full 3D PIC simulations for this purpose.

3.2. Physical origin of the dark current

Asymmetric growth (elongation) of the bubble displayed in figures 3(a)–(c) is accompanied by compression of the driver from 25 to roughly 5.5 fs, with a simultaneous 2.5-fold increase in the normalized vector potential [63]. The compressed pulse now acts as a snow-plow. As seen in figures 3(b) and (c), the ponderomotive push of its front pre-accelerates plasma electrons to \( \gamma > \gamma_g \), creating a strongly compressed electron slab an order of magnitude denser than the ambient plasma. The formation of this electron density 'pancake' enhances the charge separation, nearly doubling the longitudinal electric field acting on the sheath electrons immediately behind the driver (cf figures 4(a)–(d)). Figure 4(d) shows that electrons entering the sheath become exposed to a positive electric field a factor of 2.25 higher than in the case of a smooth driver (cf figure 4(c)). Consequently, as one can see in figure 4(f), the sheath electrons passing the piston receive much stronger kick in the backward direction, and quickly become relativistic with \( p_{\text{min}} \approx -1.65m_e c \) (in contrast to \(-0.55m_e c\) in the smooth driver case of figure 4(e)). Hence, once the piston forms, it takes nearly twice as long for the sheath electron to reach the point of return, \( p_z = 0 \), and to start getting accelerated; reaching the axis also takes a longer time, which explains the elongation.

The above argument suggests that the pulse self-steepening, causing elongation of the quasi-static bubble, is the root cause of continuous injection. Pulse energy depletion in the WAKE simulations is quite low, reaching merely \( \sim 33\% \) at \( z = 2.165 \) mm. Therefore, the formation of the relativistic piston is chiefly a nonlinear optical effect [68–72] unrelated to the
Figure 4. Bubble elongation due to pulse transformation into a relativistic piston (WAKE simulation). Physical quantities are shown before (left column) and after (right column) the formation of the optical piston. Panels on the left correspond to the bottom half of figure 3(a), and those on the right to the bottom half of figure 3(c). (a), (b) Longitudinal electric field $E_z$ (in GV cm$^{-1}$). Black lines are the trajectories $r_{in}(\xi)$ of the innermost sheath electrons (quasi-static macroparticles) plotted until the point of return, $p_z = 0$ (note the difference in longitudinal scales in left and right columns). The blue lines are iso-contours of laser intensity at $5 \times 10^{18}$ W cm$^{-2}$. (c), (d) Longitudinal electric field in the points of the innermost electron trajectories, $E_z(r_{in}(\xi), \xi)$. (e), (f) Longitudinal momentum of the innermost electron along its trajectory, $p_z(r_{in}(\xi), \xi)$.

energy transfer to the wake. We characterize the self-steepening process by expressing the complex envelope of the laser vector potential as $a = |a| \exp(i\phi)$, and presenting axial lineouts of the normalized intensity, $|a|^2$; normalized local frequency shifts, $\Delta \omega/\omega_0 \equiv -\omega_0^{-1} \partial \phi/\partial t = (c/\omega_0) \partial \phi/\partial \xi$; and frequency spectra in figure 5. Figures 5(a) and (c) indicate that, at the beginning of stage II, the pulse head is down-shifted by $\Delta \omega \approx -0.3 \omega_0$, whereas the tail, travelling inside the bubble, remains unshifted. Thus, in agreement with estimates of [68] significant spectral broadening occurs before the amplitude compression begins. Anomalous group velocity dispersion in the plasma slows down the red-shifted leading edge, building up the field amplitude first in the pulse head (figure 5(a)), and later near the pulse centre (figure 5(b)). As the frequency spectrum further expands toward $\omega = \omega_{pe}$, the low-frequency components, generated in the region of index down-ramp, slip behind the intensity peak, etching away the pulse front [68], and fill the bubble with mid-IR radiation [72]. As a result, the entire pulse becomes red-shifted with $\Delta \omega \leq -0.5 \omega_0$ with the lowest-frequency components concentrated in the tail area (cf figure 5(d)). The large final bandwidth, in combination with weak depletion, enables the pulse compression to roughly two cycles, reaching $\omega_{peak} \approx 16.5$. As we show in the next section, negative chirp of the incident pulse (i.e. phase modulation with higher frequencies temporally leading lower frequencies) compensates for the gradually accumulating frequency red shift, thus delaying the pulse contraction and suppressing continuous injection.
Figure 5. Pulse frequency red-shift and the formation of the optical piston (WAKE simulation). (a) Axial lineouts of normalized intensity (blue) and nonlinear refractive index (black) corresponding to figure 4(a). (b) Lineouts of intensity (red) and index (black) corresponding to figure 4(b). (c), (d) Local frequency shifts corresponding to panels (a) and (b). (e) Laser frequency spectra (radially integrated); blue corresponds to panels (a) and (c), red to panels (b) and (d) and black to the incident pulse. The co-moving index gradient causes spectral broadening, enabling self-compression of the pulse to approximately two cycles.

4. Suppression of continuous injection using a negatively chirped pulse

Ploughing through the electron fluid significantly reduces the frequency of the pulse leading edge. Simultaneous self-steepening of the pulse causes gradual expansion of the bubble, bringing about massive continuous injection, degrading beam quality. This prevents effective use of the entire dephasing length for high-energy QME electron acceleration. We propose to solve this problem using a negatively chirped pulse. Given a sufficiently large frequency bandwidth (corresponding to a few-cycle transform-limited pulse duration), the initial blue shift of the leading edge must partly compensate for the gradually accumulating red shift, thus mitigating self-steepening, and delaying the relativistic piston formation. Elongation of the bubble should thus be reduced, and continuous injection partly suppressed.

We first verify this scenario in a WAKE simulation using test particles. We take the 30 fs, 70 TW Gaussian pulse with the parameters specified at the end of section 2 and introduce a linear chirp, temporally advancing higher frequencies. The centre of this negatively chirped pulse corresponds to the carrier frequency $\omega_0$, and the bandwidth corresponds to a transform-limited 5 fs duration (namely duration of the relativistic piston of the last section); the frequency thus varies by $\pm \omega_0/8$ within an FWHM in intensity. Multi-joule amplification systems delivering such broad-bandwidth pulses are not available yet, but their development and application to the plasma-based electron acceleration are being actively pursued [38, 94–96]. We present the results of runs with the chirped and transform-limited (unchirped) drivers in figure 6. Figures 6(a) and (b) indicate that relativistic self-focusing is not very sensitive to the chirp.
Figure 6. Suppression of optical piston formation and bubble expansion using negative chirp (WAKE simulations). (a) Peak intensity, (b) length of the accelerating phase and (c) pulse energy versus propagation distance in the simulations with unchirped (blue) and chirped (red) driver. Positions (1)–(3) in panel (b) are where the density snapshots of figures 3(a)–(c) were taken. (d.1)–(d.3) Electron density and test electrons with $\gamma > \gamma_g$ at positions (1)–(3) (labelled accordingly). Top (bottom) sub-panels correspond to the runs with unchirped (chirped) pulses. Greyscale is linear with a cutoff at $n_e = 3.25 \times 10^{19} \text{ cm}^{-3}$. (e.1)–(e.3) Normalized laser intensity, $|a|^2$, at positions (1)–(3). Top (middle) sub-panels correspond to the runs with unchirped (chirped) pulses. Bottom sub-panels show the axial lineouts of intensity; blue (red) curves correspond to the unchirped (chirped) pulse. Negative chirp prevents rapid self-compression of the pulse, slowing down bubble expansion and reducing the number of injected test electrons.
stage I \((z < 1.24 \text{ mm})\) thus remains almost unaltered: maximal bubble expansion around \(z = 1 \text{ mm}\) appears to be 25% smaller than in the unchirped driver case, with a proportionally smaller number of injected test particles.

It is during stage II that the negative chirp changes the pulse behaviour and dynamics of self-injection most drastically. Figure 6(a) shows that the chirped pulse stabilizes and propagates between \(z \approx 1.2\) and 2.1 mm with almost unchanging intensity. Front steepening becomes noticeable only at the end of the run (cf figure 6(e.3)). Frequency spectra of the pulses and their complex phases at the end of the run are compared in figures 7(c)–(e). Even though the frequency reduction in the area of peak intensity is nearly the same in both cases, the tail of the initially chirped pulse remains almost unshifted, and its frequency spectrum (figure 7(e)) rapidly decays for \(\omega < \omega_0/2\), in sharp contrast to the unchirped case. Initial up-shift of the pulse head thus effectively suppresses the generation of low-frequency components, reducing the average frequency shift, suppressing front etching and mitigating pulse steepening. The snow-plow effect, causing strong charge separation, is thus avoided. The absence of strong charge separation in the chirped-driver case reduces the decelerating field acting on sheath electrons more than twice. As a result, sheath electrons passing the driver receive a much
Figure 8. Effect of negative chirp on electron acceleration and suppression of dark current (WAKE simulations). The data are taken at position (3) of figure 6(b). Panels (a), (c) correspond to the unchirped case, and panels (b), (d) to the chirped-driver case. (a), (b) Phase space of test electrons. Continuous injection produces a tail in the longitudinal phase space, containing electrons with (a) $p_z < 0.5 \text{ GeV/c}$ and (b) $p_z < 0.7 \text{ GeV/c}$. Negative chirp reduces the flux in the tail (number of particles per MeV) by nearly half. (c), (d) Accelerating gradient. Shaded area covers the accelerating phase, $E_z < 0$. Negative chirp reduces expansion of the bubble and delays dephasing, increasing the leading bunch energy to 780 MeV (compared to 610 MeV in the unchirped case).

According to figure 6(b), the bubble driven by the chirped pulse expands by roughly 12% between positions (1) and (3), in contrast to 35% in the unchirped case. Figure 6(c) indicates that the chirped pulse transfers energy to the plasma wake less effectively, showing 22% depletion in contrast to 33% in the unchirped case. Thus, not only are the frequency red shift and pulse self-compression (nonlinear optical effects unrelated to the pump depletion) compensated for, but the pulse front etching due to the local depletion is reduced as well. As a result, the chirped pulse propagates with a higher group velocity and, as seen in figure 6(e.3), outruns its unchirped counterpart. The larger velocity of the structure extends the dephasing length and increases final electron energy. Comparison of figure 8(b) with 8(a) shows that, in contrast to dephasing of the leading bunch in the unchirped case, electrons accelerated with the negatively chirped driver over the same distance are still located deep inside the accelerating phase. Owing to much slower bubble expansion during stage II, these particles have been exposed to a higher gradient, gaining additional 170 MeV energy. Despite all promising tendencies, continuous injection is not completely shut down, producing continuous energy tails in both cases; the beam statistics, however, shows that chirp reduces the electron flux in the tail (the number of particles per MeV) by nearly half.

We quantify the effect of dark current suppression more precisely, taking into account all kinetic effects, with a CALDER-Circ simulation using the same initial conditions and incident pulse chirp. Bubble snapshots, longitudinal phase space and energy spectra of electrons from the CALDER-Circ simulations with negatively chirped and unchirped drivers are compared in figures 9–11. Comparison of the bubble snapshots in figure 9 with the quasi-static bubbles in
Figure 9. Electron density in the CALDER-Circ runs with transform-limited (top) and negatively chirped pulse (bottom). Initial conditions are identical to those of figure 6. Panels (a)–(c) ((d), (e) and (h)) are counterparts of top (bottom) subpanels of figures 6(d.1), (d.2) and (d.3), respectively. Strong compression of the electron component before the bubble in panel (c) is a signature of the relativistic piston formation; conversely, such a compression is barely noticeable in panel (f). The negative chirp thus prevents the formation of the piston, reducing the expansion of the bubble in exactly the same manner as in the quasi-static WAKE simulation of figure 6.

Figures 6(d.1)–(d.3) reveals the same reduction of expansion rate in the chirped-driver case, supporting the quasi-static scenario of the bubble evolution. Snapshots of the longitudinal phase space in figure 10 display two-component beams. In the chirped-driver case, the dark current is noticeably reduced, and the leading QME bunch is accelerated to higher energy, in qualitative agreement with the test-particle simulation of figure 8. This observation is further quantified in figure 11 showing the energy spectra. We find that the beam components—high-energy QME feature and a polychromatic background—are affected differently by the chirp. The QME components in figures 11(c) and (f) look similar. However, in the chirped-driver case, the increase of the bubble velocity delays dephasing and slightly reduces the injected charge (thus reducing beam loading). As a result, the QME bunch accelerates to 637 MeV with 4.7% FWHM spread (in contrast to 515 MeV with 8.5% spread in the unchirped case). At the same time, the tail is affected profoundly. When the driver is negatively chirped, the build-up of massive dark current is almost completely avoided; the high-energy QME component remains dominant over the entire acceleration length (cf figures 11(d)–(f)).

Final characteristics of the beam (mean energy, energy spread, charge and emittances) are summarized in tables 1 and 2. The normalized transverse emittance is calculated using the usual definition $\varepsilon_{N,i} = (m_e c)^{-1}[(\langle p_i^2 \rangle - \langle p_i \rangle^2)(\langle r_i^2 \rangle - \langle r_i \rangle^2) - (\langle p_i r_i \rangle - \langle r_i \rangle \langle p_i \rangle)^2]^{1/2}$, where $i = x$ and $y$ correspond to the emittance in and out of the laser polarization plane. We assess the effect of the chirp on the high-energy QME feature, and evaluate the suppression of the polychromatic background using flux and average brightness, the quantities used in beam physics to characterize the quality of the particle source [25]. The flux is defined as the number of electrons in a given spectral interval per bunch per MeV; we estimate it for the QME component using the formula

$$\langle F \rangle \approx |e|^{-1} Q_{qm} / \Delta E_{qm},$$

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Figure 10. Electron density in longitudinal phase space (in arbitrary units) corresponding to the density snapshots of figure 9 (panels labelled accordingly). Panels (c) and (f) are the counterparts of figures 8(a) and (b). Chirping the driver reduces the dark current and accelerates the leading bunch to higher energy.

where $Q_{qm}$ is the total charge of this component, and $\Delta E_{qm}$ is the absolute energy spread (FWHM of the energy peak). For the energy tail, $Q_{qm}$ is replaced with a total charge between 50 MeV and $E_{max}$ (where $E_{max}$ is the upper limit to which the tail extends) and $\Delta E_{qm}$ with $E_{max} - 50$ MeV. The average brightness of the QME feature is estimated using the formula

$$\langle B \rangle \approx \langle F \rangle \gamma_{qm}^2 (\epsilon_{N,x}^2 + \epsilon_{N,y}^2)^{-1},$$

where $\gamma_{qm} = E_{qm}/(m_e c^2)$ is the Lorentz factor corresponding to the energy peak.

One can see from table 1 that, even though individual characteristics of the leading bunch (total charge, mean energy, emittance and flux) are quite insensitive to the chirp and change within a $\sim 20\%$ margin (only relative energy spread drops nearly twice), in combination all these changes boost the brightness by a factor of 2.5.

Parameters of the polychromatic background in table 2 tell us that not only does the charge in the tail drop by 60%, but the flux $\langle F \rangle_{tail}$ also drops by nearly a factor of 3 when the chirp is introduced (we recall that WAKE with test particles predicts only twofold reduction of the flux). Additionally, the flux ratio $\langle F \rangle_{qm}/\langle F \rangle_{tail}$ is only 1.6 in the unchirped case; introduction of chirp improves this to 5.9. The poorly collimated tail electrons, $\langle \Delta \alpha \rangle_{tail} \approx 3 \langle \Delta \alpha \rangle_{qm}$, which are distributed over an energy range 17 times broader than the absolute energy spread of the QME bunch, can be effectively dispersed in vacuum using miniature magnetic quadrupole lenses, further improving the beam collimation and reducing the energy spread [97, 98].

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Figure 11. Normalized electron energy spectra corresponding to the phase space snapshots of figure 10 (panels labelled accordingly); normalization is the same for all panels. Chirping the driver reduces the amount of continuously injected charge by nearly 60%, whereas the leading bunch is accelerated to 23.5% higher energy, with nearly twice lower relative energy spread (cf tables 1 and 2). Electrons in the second bucket dephase early and are never accelerated beyond 250 MeV.

Table 1. Parameters of the leading QME component for panels (f) and (c) of figures 10–11. \( Q_{qm} \) is the charge in nC; \( E_{qm} \) is the central energy in MeV; \( \Delta E_{qm} \) is the absolute energy spread (FWHM) in MeV; \( \langle \Delta E/E \rangle_{qm} \) is the normalized energy spread; \( \varepsilon_{N,x} \) and \( \varepsilon_{N,y} \) are the root mean square (rms) normalized transverse emittances (in mm mrad) in and out of the laser polarization plane; \( \langle \Delta \alpha \rangle_{qm} \) is the rms divergence in mrad; \( \langle F \rangle_{qm} \) is the flux in \( 10^7 \) electrons MeV\(^{-1} \); \( \langle B \rangle \) is the average brightness in \( 10^{11} \) electrons MeV\(^{-1} \) mm\(^{-2} \) mrad\(^{-2} \).

|       | \( Q_{qm} \) | \( E_{qm} \) | \( \Delta E_{qm} \) | \( \langle \Delta E/E \rangle_{qm} \) | \( \varepsilon_{N,x} \) | \( \varepsilon_{N,y} \) | \( \langle \Delta \alpha \rangle_{qm} \) | \( \langle F \rangle_{qm} \) | \( \langle B \rangle \) |
|-------|--------------|-------------|-----------------|-----------------|----------------|----------------|----------------|----------------|-------------|
| Chirp | 0.181        | 637         | 30              | 0.047           | 8.10           | 7.4            | 10.6           | 3.77           | 5.1         |
| No chirp | 0.214       | 515         | 44              | 0.085           | 8.74           | 8.5            | 12.9           | 3.04           | 2.1         |

More details of electron acceleration in the wake field of the negatively chirped pulse can be found in figure 12. Comparison of figures 12(c) and (d) indicates that the chirped driver accelerates the leading bunch to a given energy (here, 515 MeV) in a shorter plasma, and with much weaker background (cf figure 12(a)). Figures 12(b) and (e) demonstrate that negatively chirping the driver improves the LWFA efficiency, increasing the dephasing length and final electron energy without compromising the beam quality. The leading bunch with 181 pC charge, energy 660 MeV and 5.8% relative energy spread remains the dominant spectral feature in the energy range \( E > 100 \) MeV. In contrast, figures 12(b) and (f) show that electron beam accelerated in the wake of the unchirped driver is completely dominated by the dark current at this point.

In summary, the dark current produced in the course of electron acceleration until dephasing in the blowout regime can be suppressed using a broad-bandwidth, negatively chirped driving laser pulse. Even though complete elimination of low-energy tails is hard to achieve in high-density plasmas (\( \gamma_b = 10–15 \)), strong reduction of the charge in these poorly collimated
Table 2. Parameters of the energy tail, $50 \text{ MeV} < E < E_{\text{max}}$, for panels (f) and (c) of figures 10–11. $Q_{\text{tail}}$ is the charge in nC; $E_{\text{max}}$ is the high-energy cutoff (in MeV); $\langle \Delta \alpha \rangle_{\text{tail}}$ is the rms divergence in mrad; and $\langle F \rangle_{\text{qm}}$ is the flux in $10^7$ electrons MeV$^{-1}$.

|            | $Q_{\text{tail}}$ | $E_{\text{max}}$ | $\langle \Delta \alpha \rangle_{\text{tail}}$ | $\langle F \rangle_{\text{tail}}$ |
|------------|-------------------|-------------------|---------------------------------------------|---------------------------------|
| Chirp      | 0.46              | 500               | 33                                          | 0.644                           |
| No chirp   | 1.06              | 400               | 36                                          | 1.89                            |

Figure 12. Suppression of the dark current and increasing the accelerator efficiency using the negatively chirped driver (CALDER-Circ simulations). Electron energy spectra plotted with solid lines in panels (a) and (b) and longitudinal phase spaces in panels (c) and (e) are from the chirped-driver simulation. Energy spectra plotted with dashed lines in (a) and (b) and phase spaces in (d) and (f) are from the unchirped simulation. (a), (c) and (d) Acceleration of the leading QME bunch to the same energy ($\approx 515$ MeV) is achieved in the chirped-driver simulation over 20% shorter distance, and with the amount of continuously injected charge reduced by nearly 77%. (b), (e) and (f) Acceleration until dephasing in the chirped-driver case preserves the leading bunch as the most prominent feature in the phase space. The QME feature with the energy 660 MeV and 5.8% relative spread is the dominating spectral signal in the energy range $E > 100$ MeV. In the unchirped case, the phase space and energy spectrum are dominated by dark current.
tails is advantageous for many applications. Subsequent manipulations of the beam using permanent magnets may further improve its quality.

5. Conclusions

A time-varying electron density bubble created by the radiation pressure of a tightly focused laser pulse guides the pulse through a uniform, rarefied plasma, traps ambient plasma electrons and accelerates them to GeV-level energies. Nonlinear focusing and self-compression of the pulse cause slow variations of the bubble shape and potentials, playing a key role in the initiation and termination of electron self-injection from the bubble sheath. Expansion of the bubble injects electrons, with the amount of charge trapped over a given time interval determined by the expansion rate. Stabilization and contraction of the bubble terminate injection; simultaneously, the longitudinal non-uniformity of the accelerating gradient causes rotation of the electron beam phase space, reducing the beam energy spread. Even though beam loading reduces the accelerating gradient and slows down phase space rotation, a well-collimated QME electron bunch forms long before dephasing. It is important in many applications to preserve the quality of this bunch and maintain it as a dominant feature of the energy spectrum until dephasing, accelerating it to the highest possible energy and maximizing accelerator efficiency. Achieving this goal in modern LWFA experiments with high-density plasmas ($\gamma_0 = 10–15$) is not straightforward. The natural evolution of the driver causes further deformations of the accelerating bucket, increasing the possibility for additional injection (dark current) and degrading the beam quality. In particular, wake excitation red-shifts the frequency of the pulse leading edge, thus increasing the pulse bandwidth up to $\Delta\omega \sim \omega_0$ long before dephasing. Negative group velocity dispersion compresses the pulse, turning it into a few-cycle duration, ultrarelativistic-intensity optical piston. The resulting elongation of the quasi-static bubble leads to continuous secondary injection, producing polychromatic, poorly collimated tail totally dominating the electron energy spectrum near dephasing. A combination of reduced and fully self-consistent 3D PIC simulations shows that the role of beam loading in the early stage of continuous injection is insignificant.

We have demonstrated that a negative chirp of the incident laser pulse can compensate for the nonlinear frequency red shift and delay the piston formation. Using broad-bandwidth, viz corresponding to a few-cycle transform-limited duration, negatively chirped pulse, we have shown a nearly threefold reduction of flux (the number of particles per beam per MeV) in the polychromatic background produced by continuous injection under conditions accessible with existing 100 TW-class lasers. In addition, higher group velocity and lower depletion of the negatively chirped pulse boost the average brightness of the high-energy QME bunch by a factor of 2.5. This approach is promising for the control of electron beam quality in future experiments with multi-joule, ultra-high-bandwidth short laser pulses. In particular, as shown in the appendix, large negative chirp dramatically improves the beam quality and flexibility of beam parameters under the strongly nonlinear conditions of recent experiments [5], needing no manipulations with the target. In this situation, shaping the longitudinal density profile (viz gradually increasing the plasma density along the pulse propagation direction) may further suppress bubble expansion, potentially leading to dark current-free acceleration. Exploring this possibility will be the subject of our future work.
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Appendix. Dark current control in the strongly nonlinear interaction regime

Using a negatively chirped pulse helps improve the beam characteristics over a broad parameter range, particularly in the regimes with much stronger nonlinear interaction, such as the recently reported GeV-scale LWFA experiment with a cm-scale dense gas jet [5]. We carry out a CALDER-Circ simulation with a transform-limited pulse and other initial conditions identical to those of [5] (except that the pulse temporal profile is Gaussian, rather than polynomial, with $\tau_L = 55$ fs FWHM in intensity). A 200 TW pulse with the central wavelength $\lambda_0 = 0.8\, \mu$m, linearly polarized in the x-direction, is focused at the plasma border ($z = 0$) into a spot size $r_0 = 18.7\, \mu$m, and propagates in the positive z-direction. The peak intensity at the focus is $3.3 \times 10^{19} \, \text{W cm}^{-2}$ (normalized vector potential $a_0 = 3.9$). The plasma density has a trapezoidal profile, with a 5 mm plateau and 0.65 mm-length entrance and exit ramps. The density in the plateau region, $n_0 = 5.7 \times 10^{18} \, \text{cm}^{-3}$, corresponds to $\gamma_g \approx 17.5$, $P/P_{cr} \approx 40$, and $\omega_{pe}\tau_L \approx 7.4$.

We compare it with another simulation with identical initial conditions except that the pulse is negatively chirped, having the bandwidth equivalent to 5 fs transform-limited duration, with the carrier frequency corresponding to the pulse centre.

Initial evolution of laser and plasma is qualitatively the same in both cases. The overcritical and rather long ($\omega_{pe}\tau_L > 2\pi$) laser pulse rapidly overfocuses upon entering the plasma, showing signs of filamentation and Raman sidescatter, as well as strong oscillations of intensity, leading to multiple injections into the first bucket during the first 2.5 mm of propagation. With a transform-limited pulse, front steepening begins even before the end of this transient stage. As a result, the bunches of early injected electrons are never distinct from the polychromatic background produced by continuous injection. In the chirped-pulse simulation, however, the piston formation is delayed until $z \approx 4$ mm. As a result, bubble expansion is considerably reduced, and its axial symmetry much improved, as is seen from a comparison of figures A.1(a) and (c). Figure A.1(d) indicates that two electron bunches injected during the transient stage equalize in energy and remain well separated from the background at their dephasing point, $z \approx 4$ mm. The electron energy spectrum from the chirped-pulse run, presented in figure A.2(a) (red curve), shows that the dominating high-energy feature ($E = 800 \pm 100 \, \text{MeV}$, $Q_{\text{mono}} \approx 0.4 \, \text{nC}$, $\varepsilon_{N,x} \approx 13.4 \, \text{mm mrad}$, $\varepsilon_{N,y} \approx 8.9 \, \text{mm mrad}$) is well separated from the weak background produced by incompletely suppressed continuous injection ($50 < E < 600 \, \text{MeV}$, $Q_{\text{cont}} \approx 1.6 \, \text{nC}$, electrons from the second and third buckets contributing to the range between 50 and 200 MeV). The QME feature has average brightness, $B \approx 10^{11} \, \text{MeV}^{-1} \, \text{mm}^{-2} \, \text{mrad}^{-2}$. Conversely, the spectrum retrieved from the run with a transform-limited driver is totally dominated by the polychromatic background (cf figure A.2(a), black curve). Electrons making up multiple peaks in the range $E > 250 \, \text{MeV}$ have total charge 3.7 nC and normalized transverse emittances $\varepsilon_{N,x} \approx \varepsilon_{N,y} \approx 41 \, \text{mm mrad}$.

Furthermore, the negative chirp improves the beam characteristics even well beyond dephasing, when phase-space rotation generates a QME beam consisting of continuously
(a), (b) Simulation with a transform-limited 200 TW, 55 fs pulse of [5]. (c), (d) Simulation with the same initial conditions except that the pulse is negatively chirped, having a bandwidth equivalent to 5 fs transform-limited duration. Snapshots of electron density ((a), (c)) and longitudinal phase space ((b), (d)) are taken at the dephasing point for electrons in the chirped-pulse simulation. Chirp delays the piston formation and reduces bubble elongation, thus increasing electron energy near dephasing. The same simulation with a transform-limited driver yields the beam totally dominated by the dark current (panel (b)).

Figure A2. Electron energy spectra from the simulations with transform-limited and negatively chirped pulse under parameters of [5]. Black and red curves correspond to the unchirped and chirped-driver runs, respectively. (a) Energy spectra corresponding to the phase space snapshots of figures A.1(b) and (d) ($z \approx 4$ mm). Energy spectrum from the chirped-driver run reveals a high-energy feature, $E = 800 \pm 100$ MeV, separated from the weak background. (b) Energy spectra obtained in acceleration beyond dephasing, $z \approx 5.75$ mm. Phase space rotation reduces the energy spread of continuously injected electrons, creating QME features. The beam obtained with the chirped pulse has three times lower energy spread and seven times higher brightness.

injected electrons [81, 82]. At the end of the density plateau, $z \approx 5.75$ mm, electrons at the front of the beam dephase and lose energy, whereas electrons originally from the energy tail continue gaining energy. Figure A.2(b) shows the resulting QME energy distributions.
black curve, corresponding to the unchirped run, shows a distribution similar to that of [5],
with the broad feature in the range 600 < E < 800 MeV. This beam contains charge 
\( Q(E > 500 \text{ MeV}) \approx 3.1 \text{nC}, \) and is rather poorly collimated, having emittances 
\( \varepsilon_{N,x} \approx \varepsilon_{N,y} \approx 60 \text{ mm mrad}. \) The corresponding average brightness is 
\( B \approx 0.25 \times 10^{11} \text{ MeV}^{-1} \text{ mm}^2 \text{ mrad}^{-2}. \) On the other hand, the QME feature obtained with the chirped driver (red curve), although peaked at 
lower energy, \( E_{\text{peak}} \approx 640 \text{ MeV}, \) has three times lower FWHM energy spread, slightly higher 
charge, \( Q(E > 500 \text{ MeV}) \approx 3.4 \text{nC}, \) 35% lower emittances \( (\varepsilon_{N,x} \approx \varepsilon_{N,y} \approx 38.4 \text{ mm mrad}), \) and, therefore, seven times higher brightness, \( B \approx 1.75 \times 10^{11} \text{ MeV}^{-1} \text{ mm}^2 \text{ mrad}^{-2}. \) Thus, the negatively chirped driver makes even continuous injection much quieter, producing a beam with reasonably good characteristics in acceleration beyond the dephasing limit without any manipulations with the target.

References

[1] Tajima T and Dawson J M 1979 Phys. Rev. Lett. 43 267
[2] Esarey E, Schroeder C B and Leemans W P 2009 Rev. Mod. Phys. 81 1229
[3] Leemans W P, Nagler B, Gonsalves A J, Tóth Cs, Nakamura K, Geddes C G R, Esarey E, Schroeder C B and Hooker S M 2006 Nature Phys. 2 696
[4] Nakamura K, Nagler B, Tóth Cs, Geddes C G R, Schroeder C B, Esarey E, Leemans W P, Gonsalves A J and Hooker S M 2007 Phys. Plasmas 14 056708
[5] Kneip S et al 2009 Phys. Rev. Lett. 103 035002
[6] Kneip S et al 2011 Plasma Phys. Control. Fusion 53 140008
[7] Froula D H et al 2009 Phys. Rev. Lett. 103 215006
[8] Clayton C E et al 2010 Phys. Lett. 105 105003
[9] Liu J S et al 2011 Phys. Rev. Lett. 107 035001
[10] Pollock B B et al 2011 Phys. Rev. Lett. 107 045001
[11] Lu H et al 2011 Appl. Phys. Lett. 99 091502
[12] Aoyama M, Yamakawa K, Akahane Y, Ma J, Inoue N, Ueda H and Kiriyama H 2003 Opt. Lett. 28 1594
[13] Hein J, Kaluza M C, Bödefeld R, Siebold M, Podleska S and Sauerbrey R 2006 Lect. Notes Phys. 694 47
[14] Hooker C J et al 2006 J. Physique IV 13 673
[15] Saul E W et al 2010 Appl. Opt. 49 1676
[16] Sung J H, Lee S K, Yu T J, Jeong T M and Lee J 2010 Opt. Lett. 35 3021
[17] Korzhimannov A V, Gonoskov A A, Khazanov E A and Sergeev A M 2011 Phys.—Usp. 54 9
[18] Gorbunov L M, Kalmykov S Yu and Mora P 2005 Phys. Plasmas 12 033101
[19] Lifschitz A F, Faure J, Glinec Y, Malka V and Mora P 2006 Laser Part. Beams 24 255
[20] Kalmykov S Y, Gorbunov L M, Mora P and Shvets G 2006 Phys. Plasmas 13 113102
[21] Lu W, Tzoufras M, Joshi C, Tsung F S, Mori W B, Vieira J, Fonseca R A and Silva L O 2007 Phys. Rev. Spec. Top. Accel. Beams 10 061301
[22] Martins S F, Fonseca R A, Lu W, Mori W B and Silva L O 2010 Nature Phys. 6 311
[23] Kalmykov S Y et al 2010 New J. Phys. 12 045019
[24] Rousse A et al 2004 Phys. Rev. Lett. 93 135005
[25] Rousse A Ta, Phuc K, Shah R, Fitour R and Albert F 2007 Eur. Phys. J. D 45 391
[26] Kneip S et al 2008 Phys. Rev. Lett. 100 105006
[27] Kneip S et al 2010 Nature Phys. 6 980
[28] Fourmaux S et al 2011 New J. Phys. 13 033017
[29] Kneip S et al 2011 Appl. Phys. Lett. 99 093701
[30] Cipiccia S et al 2011 Nature Phys. 7 867
[31] Grüner F et al 2007 Appl. Phys. B 86 431

New Journal of Physics 14 (2012) 033025 (http://www.njp.org/)
[70] Faure J, Glinec Y, Santos J J, Ewald F, Rousseau J P, Kiselev S, Pukhov A, Hosokai T and Malka V 2005 Phys. Rev. Lett. 95 205003
[71] Vieira J, Fiuza F, Silva L O, Tzoufras M and Mori W B 2010 New. J. Phys. 12 045025
[72] Pai C H et al 2010 Phys. Rev. A 82 063804
[73] Decker C D, Mori W B, Tzeng K C and Katsouleas T 1996 Phys. Plasmas 3 2047
[74] Andreev N E, Gorbunov L M, Mora P and Ramazashvili R R 2007 Phys. Plasmas 14 083104
[75] Thomas A G R et al 2007 Phys. Rev. Lett. 98 095004
[76] Thomas A G R et al 2009 Plasma Phys. Control. Fusion 51 024010
[77] Li Z, Zgadzaj R, Wang X, Reed S, Dong P and Downer M C 2010 Opt. Lett. 35 4087
[78] Malka V, Faure J, Glinec Y, Pukhov A and Rousseau J P 2005 Phys. Plasmas 12 056702
[79] Mollenauer L F, Stolen R H and Gordon J P 1980 Phys. Rev. Lett. 45 1095
[80] Mollenauer L F and Stolen R H 1984 Opt. Lett. 9 13
[81] Tsung F S, Lu W, Tzoufras M, Mori W B, Joshi C, Vieira J M, Silva L O and Fonseca R A 2006 Phys. Plasmas 13 056708
[82] Joshi C 2007 Phys. Plasmas 14 055501
[83] Tzoufras M, Lu W, Tsung F S, Huang C, Mori W B, Katsouleas T, Vieira J, Fonseca R A and Silva L O 2009 Phys. Plasmas 16 056705
[84] Rechatin C, Davoine X, Lifschitz A F, Ben Ismail A, Lim J, Lefebvre E, Faure J and Malka V 2009 Phys. Rev. Lett. 103 194804
[85] Rechatin C et al 2010 New J. Phys. 12 045023
[86] Mora P and Antonsen T M Jr 1997 Phys. Plasmas 4 217
[87] Malka V et al 2001 Phys. Plasmas 8 2605
[88] Lifschitz A F, Davoine X, Lefebvre E, Faure J, Rechatin C and Malka V 2009 J. Comput. Phys. 228 1803
[89] Ting A, Esarey E and Sprangle P 1990 Phys. Fluids B 2 1390
[90] Quesnel B and Mora P 1998 Phys. Rev. E 58 3719
[91] Pukhov A, van der Brugg D and Kostyukov I 2010 Plasma Phys. Control. Fusion 52 124039
[92] Wu H C, Xie B S, Liu M P, Hong X R, Zhang S and Yu M Y 2009 Phys. Plasmas 16 073108
[93] Morshed S, Antonsen T M Jr and Palastro J P 2010 Phys. Plasmas 17 063106
[94] Fülöp J A et al 2007 New J. Phys. 9 438
[95] Herrmann D, Veisz L, Tautz R, Tavella F, Schmid K, Pervak V and Krausz F 2009 Opt. Lett. 34 2459
[96] Major Zs, Klingebiel S, Skrobel C, Ahmad I, Wandt C, Trushin S A, Krausz F and Karsh S 2010 AIP Conf. Proc. 1228 117
[97] Weingartner R et al 2011 Phys. Rev. Spec. Top. Accel. Beams 14 052801
[98] Wiggins S M et al 2010 Plasma Phys. Control. Fusion 52 124032

New Journal of Physics 14 (2012) 033025 (http://www.njp.org/)