Controlling vortex chirality in hexagonal building blocks of artificial spin ice

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Abstract. We exploit dipolar coupling to control the magnetic states in assemblies of single-domain magnetic nanoislands, arranged in one, two and three adjacent hexagonal rings. On tailoring the shape anisotropy of specific islands, and thus their switching fields, we achieve particular target states with near perfect reliability, and are able to control the chirality of the vortex target states. The magnetic states are observed during magnetization reversal with x-ray photoemission electron microscopy and our results are generally in excellent agreement with a numerical model based on point dipoles and realistic values of disorder. We conclude with a quantitative discussion of how our results depend on disorder and the chosen bias in shape anisotropy.

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1. Introduction

Topological defects in ferromagnetic nanostructures including domain walls, vortices or skyrmions are attractive candidates for information storage due to their inherent stability [1]. However, for magnetic storage applications it is also essential to be able to set and access such topologically stable states reproducibly. Considerable interest has been attracted by nanodiscs and nanorings since their shape anisotropy favours formation of accessible vortex-like configurations in elements of sufficient diameter [2–5]. Magnetic configurations in nanodiscs can be probed in detail via photoemission electron microscopy (PEEM) [6, 7] or other techniques such as scanning tunnelling microscopy. The latter technique reveals that near the centre of a vortex-like configuration the potential singularity of an in-plane magnetization is circumvented by the escape of the magnetization out of the film plane thus forming a polarized vortex core [8]. Such a vortex-like defect with a polarized core constitutes an example of a ‘meron’-topological defect [1].

The removal of material near the centre of a disc yields a ring-shaped nanostructure where such a high energy vortex core is avoided. Consequently, at remanence such ring structures support robust coreless vortices where the magnetic flux circulates around the ring in a right- or left-handed (counterclockwise or clockwise) fashion with respect to the substrate normal, often referred to as ‘vortex chirality’. These vortex states can be accessed by an applied field [9]. Beside their robust nature, vortex states in ring shaped samples are well suited for information storage as they also exhibit negligible long range magnetostatic interactions due to the divergenceless nature of the coreless vortex magnetization. Unfortunately, both discs and rings suffer from the fact that at sufficiently small scales, the vortex states give way to uniform magnetization states [10]. Thus at the smallest scales the advantage of ring structures is lost, namely the negligible dipolar interaction between individual memory elements.

In this paper we follow a different approach and demonstrate that vortex states may be reliably accessed within a design based on single-domain islands [11–13] in simple hexagonal ring configurations. We achieve high reliability via controlled modification of specific islands [12] and our new design has the advantage of being scalable to the smallest sizes without the detrimental transitions to uniform magnetization which are typical of ring or disc structures [10]. Our proposal takes advantage of the insight gained from the study of artificial spin ice systems [14–16], where individual single-domain nanoscale magnetic islands are coupled by dipolar interactions. For a square lattice geometry, experiments have demonstrated [17, 18] that a ground state consisting of a vortex lattice is realized and as a consequence of its two-fold degeneracy large domains are observed. Here we focus on vortices in the hexagonally shaped building blocks of artificial kagome spin ice, consisting of hexagonal ring-like structures built from individual elements which are themselves already in a single domain state. We demonstrate the reliable and robust creation of vortices in such structures at the same length scale as that of vortex states in continuous rings [4, 19, 20], but in contrast to rings our structures can be scaled down, and maintaining the geometric aspect ratio of all islands the structures remain operative at the smallest length scales given sufficiently low temperatures. Previously, such arrays of dipolar coupled elements [14, 21, 22] have been proposed for logic devices [23, 24] including individual addressing of islands [25], but in general typical disorder yielded substantial error rates [24]. Here we show that this error rate can be reduced to close to zero by replacing selected islands with narrower ones having higher switching fields. The structures we investigated consist of elongated single domain elements arranged to form one, two and three adjacent hexagons, the latter in figure-eight and trefoil geometries, respectively (cf figure 1).
**Figure 1.** Schematics of symmetric one-, two- and three-ring dipolar coupled elements which consist of wide, low anisotropy (grey) and narrow, high anisotropy (white) islands. The left column (red background) summarizes structures with identical islands while the middle column (yellow background) indicates structures where approximately one island per ring has been replaced by a narrow island. The right column (blue background) refers to structures where the number of narrow islands has been chosen to maximize the probability for achieving a given target state. The magnetization orientation of the considered target states is indicated by inscribed (red) circles with arrows. The direction of the arrows refers to the ascending branch of the hysteresis; in the descending branch all arrows are reversed. The target states consist of the zero-field low energy states. Experimental (simulated) percentages of achieved target states are indicated in black (green). The multiplicities (red numbers) reflect the degeneracy of the target states, and in case 3c, \( \pm 1 \) indicates all states satisfying the ‘ice rule’ at the central vertex with two-in/one-out or one-in/two-out orientations of the magnetic moments.

| homogeneous | modified | chirality controlled |
|-------------|----------|----------------------|
| ![Diagram](image1) | ![Diagram](image2) | ![Diagram](image3) |
| ![Diagram](image4) | ![Diagram](image5) | ![Diagram](image6) |
| ![Diagram](image7) | ![Diagram](image8) | ![Diagram](image9) |

| Case | Homogeneous | Modified | Chirality Controlled |
|------|-------------|----------|----------------------|
| 1    | 31% 10%     | 63% 53%  | 100% 99%             |
| 2    | 3% 1%       | 34% 5%   | 94% 71%              |
| 3    | 5% 1%       | 3% 2%    | 94% 80%              |

(configurations shown in 3a-3c plus those symmetry related)
We demonstrate that the introduction of tailored islands allows us to switch these structures into unique target states via an external magnetic field with a reliability of close to 100%. The target states that we consider here either correspond to dipolar ground states or to energetically low lying flux-closure states [13]. Note that in the case of three connected rings, it is no longer possible for the ground state to accommodate all three rings simultaneously in a chiral ground state. This is a manifestation of the frustrated nature of the hexagonal building blocks of artificial spin ice. The two lowest energy states correspond to states with at least two rings in their chiral ground state or an external flux closure state [13] as indicated at the bottom of figure 1.

For all structures considered, the presence of tailored islands allows us to selectively access precisely one of the energetically degenerate states with a specific chirality. In achieving such high reliability the dipolar interaction between individual elements plays a crucial role. This is confirmed by our numerical simulations which demonstrate that reliable switching cannot be achieved in the absence of dipolar interaction for realistic values of disorder.

2. Sample fabrication and x-ray imaging

The arrays were fabricated from sputtered permalloy (80% Ni/20% Fe) thin films using electron beam lithography in combination with a lift-off process [13]. The standard elements are elongated islands of width 170 nm and length 470 nm. The modified narrow islands have the same length and a reduced width of 130 nm and exhibit higher shape anisotropy than the standard islands. Figure 1 presents the schematics of the structures considered in this work: standard islands with low anisotropy are shown in grey while narrow islands with high anisotropy are shown in white. The statistics of the magnetization configurations was collected based on 200 different one-ring structures and 128 of the two- and three-ring structures as shown in figure 1. Scanning electron micrographs of selected structures are shown in figure 2. Three types of structures are distinguished in figure 1: ‘homogeneous’ ring structures with all islands 170 nm wide, ‘modified’ structures with approximately one narrow island per ring and ‘chirality controlled’ structures with up to three narrow islands per ring. The latter terminology refers to the fact that these structures can be switched into chiral target states with the highest reliability. These target states are the lowest dipolar energy states in the absence of an applied magnetic field as determined in [13]. In general, the ground states correspond to configurations with the maximally attainable number of vortex configurations compatible with a certain structure. For example in a single ring, the ground state is a vortex state as indicated in red in the first row of figure 1. For the two and three ring structures, the lowest energy state contains a vortex pair of opposite chirality, as indicated with inscribed red circles in panels 2a, 2b, 2D, 3a, 3b, 3D of figure 1. Slightly higher in energy for the two- and three-ring structures is a state which involves a flux closure along their periphery (panels 2c, 2E, 3c, 3E). As a consequence of frustration, it is no longer possible to realize a configuration with three complete vortices in the case of a three-ring structure [13]. Note also that the chiralities indicated by arrows in figure 1 refer to the ascending branch of the hysteresis after the samples have been saturated to the left and a field is applied to the right. For a given structure, the magnetization state of opposite chirality (all arrows reversed) may be accessed in the descending branch of the hysteresis.

Using PEEM together with x-ray magnetic circular dichroism (XMCD) [26], the magnetic states of the structures were characterized during in situ magnetization reversal. XMCD–PEEM offers the advantage of being able to image the spatial distribution of the sample magnetization. Of course, other methods such as magnetic force microscopy or
Figure 2. Scanning electron microscopy (SEM) images of selected structures and XMCD–PEEM images displaying the magnetization state at a selected field. (a) SEM image of a modified single ring structure (1B) indicating the difference between standard islands and narrow, higher anisotropy islands. (b) SEM image of the modified two ring structure (2E). PEEM images for various chirality controlled designs during magnetization reversal with the schematics in the insets indicating the magnetic configurations. (c) Counterclockwise and (d) clockwise vortex configuration of a single ring. (e) Double vortex configuration and (f) external flux closure configuration for a two-ring structure. Panel (g) shows one of the degenerate zero-field ground states [13] while (h) shows an external flux closure state. For the corresponding percentages of achieved target states based on all observed samples we refer to figure 1.

magnetoresistance measurements in combination with multilayer stacks may also be used for the detection of the various states in our structures. In XMCD–PEEM, by tuning the circularly polarized x-rays to the Fe L$_3$-edge, the single-domain permalloy islands appear light or dark depending on the orientation of the in-plane magnetic moments with respect to the x-ray propagation direction [13]. As shown in figure 2, light islands indicate magnetic moments pointing to the left, whereas dark islands have moments pointing towards the right.
The magnetic configurations were observed during magnetization reversal on applying an in-plane magnetic field of strength up to 70 mT, ensuring that the permalloy islands are fully saturated. Since the secondary electrons used for imaging are deflected away from the imaging axis by the electromagnet’s field, it was necessary to record images at remanence after each field step. Images taken at remanence indicate that all islands in the ring structures are single domain with an island’s magnetization pointing along its long axis.

3. Use of modified islands to achieve target state

We now demonstrate that vortex states can be addressed with high accuracy. This is achieved by selective modification of the shape anisotropy of individual islands in the various ring structures as shown in figure 1. PEEM images were taken during several steps of magnetization reversal. Initially, all structures were saturated in a large magnetic field pointing to the left, which resulted at remanence to a white contrast for horizontally oriented islands and a light grey contrast for islands oriented at 60° from the horizontal axis corresponding to the x-ray propagation direction. Subsequently a reverse field was applied to the right in steps up to a maximal field value of 18 mT, and images were taken at zero field. For each structure detailed in figure 1 we recorded the observed percentage (shown in black) of successfully achieved target states at an optimally chosen field of 13 mT. Figures 2(c)–(h) show the PEEM images of target states of the chirality controlled structures in the ascending branch of the hysteresis. States with reversed chirality are reached in the descending branch but are not shown here explicitly.

The observed percentages of achieving the low energy target states as shown in figure 1 are highly sensitive to the position of the narrow islands in the two and three-ring structures. Comparison of structures 1B and 1B’ also shows that the placement of the narrow islands dictates the chirality of the target state during magnetization reversal: after application of a saturating field to the left and a reversal field to the right, a one-ring structure with narrow islands at the top (figure 2(c)) reverses through an anticlockwise vortex state whereas narrow islands at the bottom of the ring (figure 2(d)) enforce a clockwise state.

Comparison of the observed percentages for target states between homogeneous structures on the one hand, and chirality controlled structures containing islands with modified anisotropy on the other hand reveals the striking consequence of the introduction of selected narrow islands. For the homogeneous one-ring samples the percentage of observed low energy states during magnetization reversal is below 35% and it is even less than 10% for the two- and three-ring structures. A single narrow island with larger shape anisotropy per ring results in a higher success rate and for the one-ring sample significantly increases the percentage of successfully reached target states to 63%. Note, however, that there is still a decrease of the percentage of observed target states as the number of rings increases. In the two-ring structures 2a, 2b, and 2c, the success of achieving target states for most configurations with one narrow island per ring is 34–41%. In contrast, replacing up to three of the islands per ring with narrow islands drives the reversal mechanism through a single pathway from the fully saturated state through target states of chosen chirality in the one-, two- and three-ring structures. The reliability to reach a target state is now close to 100%, independent of the number of rings, and is achieved at a single field value of 13 mT.

The observed trend that the target states were achieved more frequently with an increasing number of narrow islands per ring was reproduced in numerical simulations (percentage shown in green in figure 1). For the simulations we used a ratio of the mean switching fields between narrow and wide islands of 2:1. For an individual configuration as shown in figure 1, the
switching fields were taken from a Gaussian distribution around these values with $\sigma = 0.13$ (cf [27–29]). The percentages of figure 1 were obtained after averaging over 200 realizations of disorder (‘samples’) with random switching fields of individual islands. Taking advantage of the small system size, the simulations themselves were performed in a fully deterministic fashion, unlike the Monte Carlo approach of [27] adapted to large systems. After each small field step, it is uniquely determined which island will be next to switch its magnetization. For the ascending branch of the hysteresis, switching occurs for a particular island $i$ with switching field $H_{K_i}$, when the sum of the dipolar fields exerted by the instant configuration of all remaining islands and the external field, i.e. $H_{\text{eff},i} = H_{\text{dip},i} + H_{\text{ext}}$ is such that $H_{\text{eff},i} \cdot \mathbf{n}_i > H_{K_i}$ [27]. Here $\mathbf{n}_i$ denotes the unit vector along the $i$th island chosen such that the $x$-component is positive. For an individual sample, the switching history in a hysteresis is thus unique and determined by the particular realization of disorder. This strategy is not only computationally advantageous to a Monte Carlo simulation, but also more accurate since for the small systems studied here, switching the ‘wrong’ island in a Monte Carlo strategy could drastically alter the sequence of magnetic configurations visited during a hysteresis and hence induce an unphysical distortion of the hysteresis behaviour.

The distinct effect of the dipolar interactions in stabilizing a desired target state is illustrated in figure 3. While all moments on the periphery in a flux-closure configuration experience large
Figure 4. Numerical results for the percentage of successful switching into a vortex state for a ring structure of type 1B (cf figure 1) as a function of switching field bias $\Delta H \equiv H_{K2} - H_{K1}$. Here $H_{K2}$ ($H_{K1}$) denotes the switching fields of the hard (soft) islands, respectively. The various curves refer to different values of the disorder in the individual elements which is characterized by the standard deviation of the Gaussian distributions of the respective switching fields (cf inset). The experimental results of the paper have been reproduced with a value of $\Delta H / H_{K1} = 1$ and disorder $\sigma = 0.13$. The results clearly show that lower values of disorder require a less pronounced difference between hard and soft islands. The numerical results have been obtained by averaging over 1000 rings whose elements have switching fields taken from Gaussian ensembles with standard deviation $\sigma$ as indicated.

stabilizing dipolar fields (shown as blue arrows in panels b and d), the state of the central island is ‘frustrated’ as the moment experiences no net dipolar field as a consequence of the conflicting interactions. This frustration leads to a strong disorder dependence of the switching of the central island, which results in an ambiguity in the target state depending on the particular realization of disorder in a sample. This is confirmed in the PEEM image in figure 3(a), where half of the samples have the central island moment pointing to the right and half of them to the left as evidenced by the corresponding dark/bright contrast. This ambiguity of the target state, indicated by a double arrow in the schematic of figure 3(a), can be eliminated by increasing the switching field of the central island by giving it a narrow shape. Such controlled modification keeps the island in the configuration set by the original saturating field, removes the unwanted degeneracy and provides a method for a unique target as shown in figure 3(c).

4. Role of switching field bias, disorder and structure separation

So far we have shown how different structures can be reliably switched into a well defined target state for a specific set of experimental parameters such as the switching fields, $H_{K1}$, $H_{K2}$ of soft and hard islands, respectively, and the disorder $\sigma$ associated with the standard deviation of the...
Figure 5. Numerical results showing the reliability of vortex switching as a function of distance between two rings. The switching probability per ring is shown for a system consisting of two rings of type 1B, interacting via dipolar interactions and separated by a distance $d$ which is measured in units of the side length $l$ of a hexagon. Note that for sufficiently large switching field bias $\Delta H$, the maximal switching reliability is maintained down to very small ring separations of approximately one third of an island length. The simulations have been performed on 1000 samples of two neighbouring single rings, each with different realizations of the random switching fields which are taken from Gaussian ensembles with standard deviation $\sigma = 0.13$.

distribution of the corresponding switching fields. In order to put our results into a wider context and to motivate future experiments, it is interesting to see how our results depend on these experimental parameters. To this end, we use our numerical approach described in the previous section and investigate how the switching probability depends on (i) the chosen switching field bias $\Delta H \equiv H_{K_2} - H_{K_1}$ and disorder $\sigma$, and (ii) on the separation between different structures. For simplicity we shall restrict our attention to the rings of type 1B which exhibit an observed switching reliability of 100%.

In figure 4, the switching success is plotted as a function of $\Delta H / H_{K_1}$ for different values of disorder $\sigma$. For small disorder, high switching success already occurs for values around $\Delta H / H_{K_1} \approx 0.3$. From the figure it is evident that decreasing $\Delta H$ or increasing $\sigma$ from the experimentally given values of $\Delta H / H_{K_1} \approx 1$ and $\sigma = 0.13$ decreases the switching success. On the other hand increasing $\Delta H$ or decreasing $\sigma$ from these values would ensure a reliable switching arbitrarily close to 100%.

In order to understand how the interaction between rings affects the reliability of their switching, the switching success of two neighbouring single rings is plotted in figure 5 as a function of the distance $d$ between the outer edges of two rings of type 1B (cf figure 1). For disorder characterized by $\sigma = 0.13$ as realized in our systems, figure 5 shows the probability that either of the two rings switches into the vortex state thus allowing easy comparison with the results of figure 4. Indeed, for larger values of the ring separation $d$, the results of figure 4 for $\sigma = 0.13$ are reproduced (within statistical errors due to different ensemble choices). The
results shown in figure 5 strikingly demonstrate that the two rings do not influence each other unless their distance becomes of the order of 1/3 of the length of an individual island. This demonstrates that in practice such elements can be packed very closely, thus proving our assertion made in the introduction that our memory elements provide an attractive combination of the flux closure of continuous rings and the scalability of single-domain islands.

5. Conclusion

In conclusion, we have demonstrated reliable control of vortex states of both chiralities in discrete hexagonal ring structures. Since the individual islands are in a single domain state, our setup does not suffer from the cross-over to vortex-free or uniform states that exists for sufficiently small dimensions in continuous rings [10], and hence our structures are scalable down to the smallest length scales. Via controlled modification of the anisotropy of selected islands we are able to achieve a reliability of close to 100% for the switching into a vortex state of given chirality. We thus have shown that it is possible to produce structures which combine the robustness of topological defects with the accessability needed for storage applications. This reliable control of vortex chirality in dipolar coupled island structures thus opens a new route to future spintronic applications, either for memory devices or to perform logic operations [23, 24]. The use of islands with different shape anisotropy has also implications beyond the small structures studied here. In quasi-infinite arrays of kagome ice modified islands help to control avalanches [27, 29] or they can help to attain states of minimal dipolar energy [30, 31].

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