Cosmological Gravimetry Using High-Precision Atomic Clocks

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In this paper, a hypothesis that the cosmological gravitational potential can be measured with the use of high-precision atomic clocks is proposed and substantiated. It is assumed that the cosmological potential is formed by all matter of the Universe (including dark matter and dark energy) and that it is spatially uniform on planet scales. It is obvious that the cosmological potential, $\Phi_{CP}$, is several orders of magnitude greater than Earth’s gravitational potential $\varphi_E$ (where $|\varphi_E/c^2| \sim 10^{-9}$ on Earth’s surface). In our method, the tick rates of identical atomic clocks are compared at two points with different gravitational potentials, i.e. at different heights. In this case, the information on $\Phi_{CP}$ is contained in the cosmological correction $\alpha \neq 0$ in the relationship $\Delta \omega/\omega = (1 + \alpha)\Delta \varphi/c^2$ between the relative change of the frequencies $\Delta \omega/\omega$ (in atomic clocks) and the difference of the gravitational potential $\Delta \varphi$ at the measurement points. The consideration is made both in the case of a quasi-classical description of the gravitational shift and within general relativity theory. It is shown that using a modern atomic clock of the optical range it is possible to measure $\alpha$ in earth-based experiments if $|\alpha| > 10^{-5}$. The obtained results, if confirmed experimentally, will open up new unique possibilities for investigation of the Universe: measurement of the components of the cosmological metric tensor, determination of the size and age of the Universe, and testing of various cosmological models. These results will also increase the measurement accuracy in relativistic geodesy, chronometric gravimetry, global navigation systems, and global networks of atomic clocks.

PACS numbers: 06.30.Ft, 32.10.-f

Introduction

Atomic clocks are currently the most precise physical devices. By now, unprecedented relative uncertainty at a level of $10^{-18}$ has been achieved \cite{1}. The problem of attaining a level of $10^{-19}$ is on the agenda. Frequency measurements of such accuracy can have a great impact on the further development of fundamental and applied physics (see, for example, review \cite{2}). In particular, atomic clocks play a special role in astrophysics and cosmology. In this context, the drift of fundamental constants \cite{3-8} and the detection of dark matter \cite{9-11} are of the greatest interest.

In this paper, we develop an idea of cosmological gravitation measurements with the use of high-precision atomic clocks. According to the general theory of relativity, the clock tick rate in the gravitational field is slowed. As a result, the frequency of atomic transition experiences a gravitational redshift depending on the value of the gravitational potential. In the case of a spatially non-uniform gravitational potential, this effect leads to the following well-known relationship:

$$\frac{\omega(\mathbf{r}_1) - \omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = 1 - \frac{\varphi(\mathbf{r}_1) - \varphi(\mathbf{r}_2)}{c^2},$$

(1)

which describes the measured relative difference of the frequencies $\omega(\mathbf{r}_1)$ and $\omega(\mathbf{r}_2)$ for the same clock transition at two different spatial points $\mathbf{r}_1$ (observer) and $\mathbf{r}_2$ (emitter) with different gravitational potentials, $\varphi(\mathbf{r}_1)$ and $\varphi(\mathbf{r}_2)$ ($c$ is the light speed) \cite{13, 14}. In particular, formula (1) allows using high-precision atomic clocks to measure the difference of the gravitational potentials for various Earth’s surface points, which can form a basis for the so-called chronometric geodesy (relativistic geodesy, chronometric leveling) \cite{13}.

In this paper, we propose and substantiate a hypothesis according to which the following relationship [which is more exact than Eq. (1)] can be used:

$$\frac{\omega(\mathbf{r}_1) - \omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = (1 + \alpha)\frac{\varphi(\mathbf{r}_1) - \varphi(\mathbf{r}_2)}{c^2},$$

(2)

where the parameter $\alpha$ does not depend on the type of atomic clock and has the meaning of a cosmological correction, which contains information on the cosmological gravitational action of the Universe on our planetary system. In particular, this correction can be interpreted as the value that is proportional to the cosmological gravitational potential $\Phi_{CP}$ at Earth’s location point. We assume that the cosmological potential is formed by all matter of the Universe (including dark matter and dark energy) and that it is spatially uniform on planet scales. We also assume that the value of the gravitational-cosmological “background” $\Phi_{CP}$ is several orders of magnitude greater than the gravitational potential of our planet $\varphi_E$ (where $|\varphi_E/c^2| \sim 10^{-9}$ on Earth’s surface). Thus, Eq. (2) can be used as a basis for cosmological gravimetry.

Below we develop two alternative approaches: one of them is quasi-classical, and the other is based on general relativity. Despite some mathematical and methodolog-
I. QUASI-CLASSICAL CONSIDERATION OF CHRONOMETRIC GRAVIMETRY

In this section, we will use the results of methodologically simplest approach (e.g., see in Refs. [16, 17]), which explains the gravitational redshift of arbitrary atomic transition as resulting from the “mass defect” (for quantum atomic states) in the presence of the classical (Newtonian) gravitational potential $\varphi(r)$, that is not within the framework of general relativity. In particular, the frequency at the point $r$ is expressed by the following formula (e.g., see in Ref. [16]):

$$\omega(r) = \omega_0 \left(1 + \frac{\varphi(r)}{c^2}\right), \quad (3)$$

where $\omega_0$ is the unperturbed frequency of the atomic transition (clock transition) in the absence of gravitation. Therefore, from the methodological point of view it makes sense at first to consider chronometric gravimetry using this “quasi-classical” approach, which can make it much easier to understand our method of cosmological gravimetry. For simplicity, we will analyze the stationary case in the absence of motion of celestial bodies.

First of all, let us show an important feature of the formula (3) which leads to the absoluteness of the gravitational potential as a uniquely specified function $\varphi(r)$ in the entire space. To prove this, we consider the ratio of two frequencies at two different points, $r_1$ and $r_2$:

$$\frac{\omega(r_2)}{\omega(r_1)} = 1 - \frac{1}{1 + \varphi(r_1)/c^2} \frac{\varphi(r_1) - \varphi(r_2)}{c^2} \equiv fixed, \quad (4)$$

which is some fixed value measured experimentally. This seemingly obvious and trivial fact leads, nevertheless, to far-reaching consequences. Indeed, if we consider gravitation only from the viewpoint of gravitational force $F_{grav} = -M \nabla \varphi(r)$ (where $M$ is the mass of a body), the gravitational potential is determined up to an arbitrary constant $C$, because the transformation $\varphi(r) \rightarrow \varphi(r) + C$ does not affect the force. However, the transformation $\varphi(r) \rightarrow \varphi(r) + C$ is absolutely inadmissible from the viewpoint of formula (4), because the result becomes dependent on $C$ and an arbitrariness of the constant $C$ causes total uncertainty of the ratio $\omega(r_1)/\omega(r_2)$ in a non-uniform gravitational potential for any atomic transition. This does not agree with the experiments and defies common sense. Thus, formula (4), surprisingly, leads to the fact that the gravitational potential is described by some uniquely defined function $\varphi(r)$ for the Universe. In summary, we believe that the expression (4) can be considered as an independent physical principle, which we call the nonlocal chronometric principle. This phenomenological postulate imposes strong restrictions on the gravitational potential. It is, in essence, a change of the boundary conditions if the function $\varphi(r)$ is considered as the solution of the differential equation for infinite space (i.e., for the Universe).

Now let us consider the expression:

$$\frac{\omega(r_1) - \omega(r_2)}{\omega(r_1)} = \frac{1}{1 + \varphi(r_1)/c^2} \frac{\varphi(r_1) - \varphi(r_2)}{c^2}, \quad (5)$$

which follows from Eq. (3). Within the framework of Newtonian theory of gravitation, the potential $\varphi(r)$ can be structured as follows:

$$\varphi(r) = \varphi_{loc}(r) + \Phi_{CP}(r), \quad (6)$$

$$\varphi_{loc}(r) = \varphi_E(r) + \varphi_S(r) + \varphi_M(r) + \ldots, \quad (7)$$

where the local potential $\varphi_{loc}(r)$ contains the Newtonian potentials of all bodies of the solar system: the Earth $\varphi_E(r)$, the Sun $\varphi_S(r)$, the Moon $\varphi_M(r)$, etc. The second contribution in Eq. (6) corresponds to the cosmological potential, $\Phi_{CP}(r)$, including the potentials of all other bodies of the Universe (located very far from the Earth), and the gravitational effect of dark matter and dark energy. If we describe experiments on Earth’s surface or in the near-Earth space, the distance between the atomic clock positions, $|r_1 - r_2|$, can be at the level of $1 - 10^6$ m. It is obvious that at such small distances we can absolutely neglect the spatial variation of the cosmological potential $\Phi_{CP}(r)$, i.e., $\Phi_{CP}(r) = const$ and it can be considered as gravitational-cosmological background. Thus, the potential difference in Eq. (5) is fully described by the local potential:

$$\varphi(r_1) - \varphi(r_2) = \varphi_{loc}(r_1) - \varphi_{loc}(r_2) \approx \varphi_E(r_1) - \varphi_E(r_2). \quad (7)$$

Let us estimate the various contributions in Eq. (6). Near Earth’s surface we have an estimate of the Earth potential $|\varphi_E/c^2| \approx 0.7 \times 10^{-9}$, and the estimate $|\varphi_S/c^2| \approx 10^{-8}$ corresponds to the Sun potential in Earth’s orbit. For a lower estimate of the cosmological potential $\Phi_{CP}$, we consider the orbital motion of the solar system around the center of the Galaxy at a speed $v_S \approx 240$ km/s. Consequently, we have a rough lower estimate for the cosmological potential, $|\Phi_{CP}/c^2| > v_S^2/c^2 \approx 10^{-6}$. Moreover, because our Galaxy is only an insignificant part of the Universe, we can expect, in reality, a much stronger value, $|\Phi_{CP}/c^2| \gg 10^{-6}$. In any case, near Earth’s surface the following condition is valid: $|\Phi_{CP}| > |\varphi_{loc}(r)|$. Therefore, for the denominator in the right-hand side of formula (4) we can use approximation: $\varphi(r_1) \approx \Phi_{CP}$. Thus, taking into account Eq. (7), the expression (5) can be rewritten in the following form:

$$\frac{\omega(r_1) - \omega(r_2)}{\omega(r_1)} \approx \frac{1}{1 + \Phi_{CP}/c^2} \frac{\varphi_{loc}(r_1) - \varphi_{loc}(r_2)}{c^2} \quad (8)$$

which clearly shows that the cosmological potential $\Phi_{CP}$ can be measured using high-precision atomic clocks.
Comparing formulas (8) and (2), we find the following interrelation:
\[ \alpha = -\frac{\Phi_{CP}/c^2}{1 + \Phi_{CP}/c^2} \Leftrightarrow \Phi_{CP}/c^2 = -\frac{\alpha}{1 + \alpha}. \] (9)

In the case of \(|\Phi_{CP}/c^2| \ll 1\), we have the following approximation:
\[ \alpha \approx -\Phi_{CP}/c^2. \] (10)

Because the Newtonian gravitational potential is negative (the attractive potential), the cosmological potential is also negative, \(\Phi_{CP}(r) < 0\) (see comment [18]). This means that the cosmological correction \(\alpha\) in Eq. (2) has to be positive (at least within the quasi-classical consideration), \(\alpha > 0\).

In spite of the fact that \(|\Phi_{CP}| \gg |\varphi_{loc}|\), the existence of a great constant gravitational-cosmological background \(\Phi_{CP}\) practically does not affect the relative motion of bodies in the solar system, because mechanical motion is determined by the gravitational force \(F_{\text{grav}} = -M\nabla \varphi(r)\), which is invariant relative to the transformation \(\varphi(r) \rightarrow \varphi(r) + C\). Thus, in our case the gravitational force describing the relative motion of bodies in the solar system can be considered only as a consequence of the local potential: \(F_{\text{grav}} = -M\nabla \varphi_{loc}(r)\).

The above reasonings show that chronometric investigations involve more fundamental aspects of gravitation in comparison to study of only mechanical motion of bodies in the gravitational field. In particular, it is shown that the concept of cosmological gravitational potential \(\Phi_{CP}(r)\) taken into account in Eqs. (5) and (8) is justified, despite a great distance of objects of deep space to the local planetary system. In our opinion, there are no reasons to believe that \(\Phi_{CP}(r_E) = 0\) (where \(r_E\) is the coordinate of Earth’s center). Moreover, we have shown (see Eqs. (2), (8)-(10)) how the value of \(\Phi_{CP}(r)\) can be experimentally measured in the case of \(\Phi_{CP}(r) \gg |\varphi_{loc}(r)|\).

In the essence, cosmological correction \(\alpha\) in Eq. (2) corresponds to the nonlinear theory on the total gravitational potential \(\varphi(r)\) [see Eq. (8)].

## II. COSMOLOGICAL GRAVITY FROM THE VIEWPOINT OF GENERAL RELATIVITY

Let us consider the problem of the cosmological gravitation correction \(\alpha\) from the viewpoint of general relativity [12]. In this case, the physical picture must be described in a four-dimensional curved space-time when the relation between two infinitely near spatio-temporal events is described by the interval:
\[ ds^2 = g_{jk}(\vec{x})dx^jdx^k \quad (j, k = 0, 1, 2, 3), \] (11)

where \(g_{jk}(\vec{x}) = g_{jk}(t, \vec{r})\) is the metric tensor depending on the coordinates of the four-vector \(\vec{x} = \{x^i = t, x^1, x^2, x^3\} = \{t, \vec{r}\}\). If gravitation is absent, the tensor \(g_{jk}\) describes the Minkowski space-time metric for a special theory of relativity:
\[ g_{jk}^{(M)} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \] (12)

According to the well-known standard approaches, we write the ratio of frequencies \(\omega(r_1)\) and \(\omega(r_2)\) for the same atomic transition at two different spatial points, \(r_1\) and \(r_2\), in the presence of spatially non-uniform gravitation:
\[ \frac{\omega(r_2)}{\omega(r_1)} = \sqrt{\frac{g_{00}(r_2)}{g_{00}(r_1)}} = 1 - \frac{g_{00}(r_1) - g_{00}(r_2)}{g_{00}(r_1)}. \] (13)

Considering this expression from the viewpoint of the nonlocal chronometric principle according to which this ratio is unique and can be measured experimentally (see also Eq. (11)), we see that the transformation \(g_{00}(r) \rightarrow g_{00}(r) + B\) (where \(B\) is an arbitrary constant) is inadmissible, because the result becomes dependent on \(B\). This means that if we determine the metric tensor \(g_{jk}(r)\) as some solution of differential equations (for example, Einstein’s equations), the choice of a solution and boundary (or initial) conditions cannot be arbitrary (for example, it cannot be made for the sake of mathematical simplicity). Therefore, it is necessary to initially take into account the gravitation of the Universe.

Let us consider the limit of a weak Newtonian potential \(\varphi: |\varphi/c^2| \ll 1\). In this case, the metric tensor has the following well-known form based on Einstein’s equations (for example, see Ref. [19]):
\[ ds^2 = -c^2\left(1 + \frac{2\varphi}{c^2} + \frac{2\varphi^2}{c^4} + O(c^{-6})\right)dt^2 + \left(1 - \frac{2\varphi}{c^2} + O(c^{-4})\right)(dr)^2. \] (14)

In the near-Earth space it is usually believed that the space-time metric is determined only by the Newtonian potential formed by the solar system bodies: \(\varphi \rightarrow \varphi_{loc}(r)\) [see in Eq. (4)]. However, if we will use the total gravitational potential \(\varphi \rightarrow |\varphi_{loc}(r) + \Phi_{CP}(r)|\) including the cosmological contribution in accordance with Eq. (6), then up to linear corrections of \(\varphi_{loc}(r)\) the expression (14) can be presented in the following form:
\[ ds^2 = -c^2\left[G_{00} + \frac{2\varphi_{loc}(r)}{c^2}(1 + K^{(1)}_{00})\right]dt^2 \] (15)
\[ + \left[G_{11} - \frac{2\varphi_{loc}(r)}{c^2}(1 + K^{(1)}_{11})\right](dr)^2, \]
where the four parameters depend on the cosmological
gravitational potential $\Phi_{CP}(r)$:

$$G_{00} = 1 + \frac{2\Phi_{CP}(r)}{c^2} + \frac{2\Phi_{CP}^2(r)}{c^4} + O(c^{-6}),$$
$$G_{11} = 1 - \frac{2\Phi_{CP}(r)}{c^2} + O(c^{-4}),$$
$$K_{00}^{(1)} = \frac{2\Phi_{CP}(r)}{c^2} + O(c^{-4}), \quad K_{11}^{(1)} = O(c^{-2}),$$

in the case of $|\Phi_{CP}(r)/c^2| \ll 1$.

Note that formulas (16) were derived within the model of weak Newtonian potential $\varphi$ using the conventional Einstein’s equations for the metric tensor $g_{jk}$. However, the expression (15) can also be considered in a more general context, as universal phenomenological generalization independent of the basic equations for $g_{jk}$. Thus, the parameters $G_{00}$ and $G_{11}$ should be interpreted as components of the cosmological metric tensor $G^{(\text{Cosm})}$:

$$G_{jk}^{(\text{Cosm})}(r) = \begin{pmatrix}
-\frac{c^2 G_{00}}{1} & 0 & 0 & 0 \\
0 & G_{11} & 0 & 0 \\
0 & 0 & G_{11} & 0 \\
0 & 0 & 0 & G_{11}
\end{pmatrix},$$

which describes the Universe as a whole and is an object of study of various cosmological models, including models with dark energy and even models not including standard Einstein’s equations (see, for example, review [20]). Possible presence of nondiagonal components $G_{0m}^{(\text{Cosm})}(r) \neq 0$ and $G_{m0}^{(\text{Cosm})}(r) \neq 0 (m = 1, 2, 3)$ connected with cosmological rotations is not discussed in this paper. Two other parameters, $K_{00}^{(1)}$ and $K_{11}^{(1)}$, in Eq. (15) should be considered as coefficients of cosmological-local coupling, which can also depend on the choice of a global cosmological model. In this case, the inequalities $K_{00}^{(1)} \neq 0$ and $K_{11}^{(1)} \neq 0$ are associated with nonlinearity of the cosmological equations being used for the metric tensor.

Although $\{G_{00}(t, r), G_{11}(t, r), K_{00}^{(1)}(t, r), K_{11}^{(1)}(t, r)\}$ depend on the coordinates and time (due to the evolution of the Universe), within the local planetary system these parameters can be considered as constants. From the global viewpoint they are determined by the spatio-temporal location of the planetary system being considered (for example, the solar system in our case) within the framework of the general cosmological picture of the evolving Universe. As shown below, the parameters $\{G_{00}(t, r), G_{11}(t, r), K_{00}^{(1)}(t, r), K_{11}^{(1)}(t, r)\}$ can be, in principle, measured experimentally. The numerical information about cosmological gravitation is contained in the values: $(G_{00} - 1), (G_{11} - 1), K_{00}^{(1)}$, and $K_{11}^{(1)}$.

Using the expression for $g_{00}(r)$ in Eq. (15) and taking into account the condition $|\varphi_{\text{loc}}(r)/c^2| \ll G_{00}$, we find the ratio of frequencies in Eq. (19):

$$\frac{\omega(r_2)}{\omega(r_1)} \approx \sqrt{1 - \frac{1 + K_{00}^{(1)} \varphi_{\text{loc}}(r_1) - \varphi_{\text{loc}}(r_2)}{G_{00}}},$$

which leads to the formula

$$\frac{\omega(r_1) - \omega(r_2)}{\omega(r_1)} \approx \frac{1 + K_{00}^{(1)} \varphi_{\text{loc}}(r_1) - \varphi_{\text{loc}}(r_2)}{G_{00}} \cdot \frac{c^2}{1},$$

Comparing this expression with Eq. (2), it would seem that we find the following relationship:

$$\alpha = \frac{1 + K_{00}^{(1)} \varphi_{\text{loc}}(r_1) - \varphi_{\text{loc}}(r_2)}{G_{00}} - 1.$$

However, in our opinion, the real situation is different from this. Indeed, in Eqs. (15), (18) and (19) we use the value of the gravitational potential $\varphi_{\text{loc}}(r)$ in a geometrical (global) space-time in which the metric tensor including both the local and cosmological contributions is described. However, in formula (2) it is necessary to use the value of the gravitational potential $\varphi_{\text{loc}}^{(\text{lab})}(r)$ measured by the non-chronometric method in a laboratory (physical) space-time.

The interrelation between the geometrical and laboratory space-time is described as follows. If we ignore the insignificant contribution of the local potential $\varphi_{\text{loc}}(r)$ into Eq. (15), the metrics within the near-Earth space can be described with a good accuracy by the following expression:

$$ds^2 = \frac{c^2}{G_{00}} (dr)^2 + G_{11}(dr)^2,$$

which corresponds to a plane geometrical space-time depending on the cosmological parameters $G_{00}$ and $G_{11}$. In this case, change to a laboratory space-time is made by a simple transformation of linear compression/stretching:

$$dt_{\text{lab}} = \sqrt{G_{00}} \, dt, \quad d\mathbf{r}_{\text{lab}} = \sqrt{G_{11}} \, d\mathbf{r}.$$

As a result, the metric of the laboratory space-time practically corresponds to the ideal Minkowski space-time:

$$ds^2 = \frac{c^2}{(dt_{\text{lab}})^2} + (d\mathbf{r}_{\text{lab}})^2,$$

in which the information about the cosmological parameters $G_{00}$ and $G_{11}$ is deeply “hidden” from the observer.

Note that real physical measurements of the difference of the gravitational potentials $\Delta \varphi^{(\text{lab})}$ at two points, $r_1$ and $r_2$, are made within the laboratory space-time using mechanical measurement methods (for example, by quantum gravimeters based on atomic interferometers or classical ballistic gravimeters). It is obvious that there is a linear interrelation between the potential $\varphi_{\text{loc}}^{(\text{lab})}$ measured in the laboratory space-time and the potential $\varphi_{\text{loc}}$ determined (calculated) in the geometrical space-time:

$$\varphi_{\text{loc}}(r) = A \varphi_{\text{loc}}^{(\text{lab})}(r),$$

where the coefficient $A$ somehow depends on the transformation coefficients in Eq. (21), that is, on $G_{00}$ and $G_{11}$. Substituting the expression (23) into Eq. (19), we obtain:

$$\frac{\omega(r_1) - \omega(r_2)}{\omega(r_1)} \approx A \frac{1 + K_{00}^{(1)} \varphi_{\text{loc}}^{(\text{lab})}(r_1) - \varphi_{\text{loc}}^{(\text{lab})}(r_2)}{G_{00}} \cdot \frac{c^2}{1}.$$
As a result, we have the following general expression for the cosmological correction:

\[ \alpha = A \left( 1 + \frac{\kappa_{00}^{(1)}}{G_{00}} \right) - 1, \]

which is universal for all types of atomic clocks.

From the physical viewpoint, we have two approaches to determining the coefficient \( A \). One of them is associated with the dimensionality of the gravitational potential corresponding to that of the squared speed, \( \text{m}^2/\text{s}^2 \). Thus, we can assume that the coefficient \( A \) corresponds to the law of transformation of squared speed in the laboratory space-time to the squared speed in the geometrical space-time. According to formula (21), this transformation law is determined as follows:

\[ v_{\text{lab}} = \frac{d r_{\text{lab}}}{d t_{\text{lab}}} = \frac{\sqrt{G_{11}}}{G_{00}} \frac{d r}{d t} = \sqrt{\frac{G_{11}}{G_{00}}} v \Rightarrow v^2 = \frac{G_{00}}{G_{11}} (v_{\text{lab}})^2, \]

that is, \( A = G_{00}/G_{11} \) in Eq. (23). As a result of this “geometrical” approach we find a following estimate in the case of \( |\Phi_{\text{CP}}/c^2| \ll 1 \):

\[ \alpha \approx 4\Phi_{\text{CP}}/c^2 < 0, \]

which differs in value and sign from the quasi-classical estimate (10). Note that in Eq. (29) we assume an universality of the definitions of the mass \( M_j \) and gravitational constant \( \gamma \) for both geometrical and laboratory space-time. In the case of other scenarios, the coefficient \( A \) will be different from \( \sqrt{G_{11}} \).

Thus, within general relativity theory we have shown that a combination of high-precision nonlocal chronometric and mechanical-gravimetric measurements allows us to extract unique information on cosmological gravitation, namely, on the components of the cosmological metric tensor, \( \{G_{00}, G_{11}, K_{00}\} \). It should be emphasized that the obtained general expressions [see Eqs. (25), (27) and (30)] can be used not only for the model of weak Newtonian cosmological potential, but also for any other cosmological models (including, for example, dark matter and dark energy).

It is of interest that formulas (27) and (30) have different signs for the cosmological correction \( \alpha \) [see (25) and (31)]. Therefore, the results of the experimental measurements will allow us to make an unambiguous choice between the expressions (27) and (30).

Note also that in the present paper we have considered the cosmological gravimetry in the stationary case (i.e., in the absence of motion of celestial bodies). However, the motion (e.g., different types of relatively local cosmic rotations: Earth around of Sun, Solar system around of Galaxy’s center, and so on) can lead to the some additional contributions (relatively small, we assume) for the parameters \( \{G_{00}, G_{11}, K_{00}\} \) in Eq. (15), which determine the cosmological correction \( \alpha \).

### III. Prospects for Experimental Measurements of the Cosmological Correction \( \alpha \)

It should be noted that formula of the type (2) has been used for many decades in the relativistic theory of gravitation and in the processing of some gravimetric experiments (see, for example, review [20]). However, a hypothetic difference from zero (\( \alpha \neq 0 \)) was interpreted as violation of the principle of local position invariance when \( \alpha \) depends on the type (nature) of atomic clock (see Ref. [20]). In contrast, we predict universality of formula (2), which should not be associated with the violation of local position invariance. In our approach, the parameter \( \alpha \) does not depend on the type of atomic clock and contains information on cosmological gravitation.

In this context, it is of interest to analyze some experiments that have been performed for the last several decades (starting from 1960). These results are presented in Fig. 3 in Ref. [20]. Note that in this figure we should not take into account so-called “null redshift experiments” (denoted by red arrows in Fig. 3 in Ref. [20]). In “null redshift experiments” it is assumed that \( \alpha \neq 0 \) [in formula (2) depends on the type of atomic clocks. Then, comparing the behavior of two different clocks based on
different atomic transitions (with frequencies $\omega^{(1)}$ and $\omega^{(2)}$), one can measure with high accuracy the difference between the corresponding coefficients, $(\alpha_1 - \alpha_2)$:

$$\frac{\Delta \omega^{(1)}}{\omega} - \frac{\Delta \omega^{(2)}}{\omega} = (\alpha_1 - \alpha_2) \frac{\Delta \varphi}{c^2}. \tag{32}$$

The logic of “null redshift experiments” is following: if the experiments show that $(\alpha_1 - \alpha_2) \to 0$, this means that $\alpha_{1,2} \to 0$, that is, the principle of local position invariance is not violated. However, as shown above, the presence of $\alpha \neq 0$ in formula (1) can be associated with cosmological gravitation, that is, $\alpha$ is universal and does not depend on the type of atomic clock. Thus, “null redshift experiments” in Fig. 3 in Ref. [14] cannot be used to determine the universal cosmological correction $\alpha$. Therefore, if now we analyze the data of all other experiments in Fig. 3 in Ref. [20], we see that these experiments allow, in principle, a value of $\alpha > 10^{-3} - 10^{-4}$ (see also comment [21]).

It is also of great interest to analyze experiments in the recent paper [14]. Indeed, if we look at Fig. 3a in Ref. [14], we clearly see some imbalance between geodetic measurements of the geopotential difference (see red shading in Fig. 3a in Ref. [14]) and measurements of frequency differences between clocks (see blue shaded region in Fig. 3a in Ref. [14]). If we interpret this imbalance as the presence of $\alpha \neq 0$ in Eq. (2), then we can admit that $\alpha > 10^{-3}$ ($\alpha \approx 3 \times 10^{-3}$ for the midline of the blue shaded region in Fig. 3a in Ref. [14]). It corresponds to the cosmological gravitational potential $\Phi_{CP}$, which is three orders of magnitude greater than the gravitational potential of the Galaxy at the Sun’s orbit ($|\varphi_{Galaxy}/c^2| \sim 10^{-6}$).

However, in our opinion, experiments on measuring of $\alpha$ in formula (2) should be made with two identical atomic clocks located at different heights, but at the same geographical point. This allows us: a) to minimize the time-dependent influence of the gravitational potentials of the Sun and Moon; b) to measure $[\omega(r_1) - \omega(r_2)]/\omega(r_1)$ and $[\varphi(r_1) - \varphi(r_2)]$ with maximal accuracy.

Let us consider the case where the height difference between two identical atomic clocks is $h$, that is, for points $z_1$ and $z_2 = z_1 + h$. For this we rewrite Eq. (2) in the following form:

$$\frac{\Delta \omega}{\omega} - \frac{\Delta \varphi}{c^2} = \alpha \frac{\Delta \varphi}{c^2}. \tag{33}$$

The change in the gravitational potential between the two atomic clocks is defined as:

$$\Delta \varphi = \varphi(z_1) - \varphi(z_2) = - \int_{z_1}^{z_1 + h} g(z)dz, \tag{34}$$

where $g(z)$ is the local free fall acceleration at a point with the vertical coordinate $z$. In the case of small $h$, we can use approximation: $\Delta \varphi \approx -hg(z_1)$.

Let us estimate the possibility of measuring $\alpha$ using Eq. (33) if the relative uncertainty of the atomic clocks is $10^{-18}$ and the height difference between the two identical atomic clocks is $h = 10$ m when $\Delta \varphi/c^2 \approx 1.1 \times 10^{-15}$. It is obvious that the measurement error of $\Delta \varphi/c^2$ corresponds to the uncertainty of the atomic clocks, that is, $10^{-18}$. Because the free fall acceleration, $g$, can be measured by modern gravimeters (for example, quantum gravimeters based on atomic interferometry or classical ballistic gravimeters) with relative accuracy much better than $10^{-6}$, the main error in determining $\Delta \varphi/c^2$ is connected with the measurement of the height $h$. If the relative position of work zones (i.e., places where atoms or ions are localized) of the interrogated atomic clocks is known with an accuracy of 1 cm, for $h = 10$ m this results in an uncertainty of $10^{-18}$ for $\Delta \varphi/c^2$ in Eq. (33).

As a result, with the expression (33), $\alpha$ can be measured if $|\alpha \Delta \varphi/c^2| > 10^{-18}$, that is, if $|\alpha| > 10^{-3}$. The sensitivity of the measurement method will increase ten times if the atomic clocks have a relative uncertainty of $\sim 10^{-19}$ and a measurement accuracy of $h$ of 1 mm. In this case, we can measure $|\alpha| > 10^{-4}$ (for $h = 10$ m).

Using similar estimates for the height $h = 100$ m, we find that for atomic clocks with a relative uncertainty of $10^{-18}$ and a measurement accuracy of $h$ of 1 cm (that is, $\Delta h/h \sim 10^{-4}$), one can measure $\alpha$ if $|\alpha| > 10^{-4}$. For clocks with an uncertainty of $10^{-19}$ and a measurement accuracy of $h$ of 1 mm, one can measure $\alpha$ if $|\alpha| > 10^{-5}$.

In the same way, the reasoning can be continued for $h = 1000$ m when $|\alpha| > 10^{-5} - 10^{-6}$ can be measured. Schemes for measuring $\alpha$ with the use of spacecrafts can also be developed.

Note that the validation criterion for measurements of the cosmological correction $\alpha$ is obvious: experiments at various geographical points and for clocks with various atomic transitions should show the same results.

**Conclusion**

In this paper, a hypothesis that the cosmological gravitational potential can be measured with the use of high-precision atomic clocks has been proposed and substantiated. It was assumed that the cosmological potential is formed by all matter of the Universe (including dark matter and dark energy). The information on cosmological gravitation is contained in the nonzero cosmological correction $\alpha$ in Eq. (2). If our hypothesis will be confirmed experimentally, this will open up new unique possibilities for investigation of the Universe. In particular, if $|\alpha| > 10^{-5}$, we can assert that the cosmological gravitational contribution is much greater than the gravitational potential of the Galaxy. This will provide unprecedented possibilities for determining the global curvature of the space-time and estimating the size and age of the Universe, as well as a new objective criterion for the selection of admissible cosmological models. Further continuous monitoring for the purpose to detect possible variations of $\alpha$ can lead (if such variations exist) to the discovery of new cosmological processes and detec-
tion of dark matter (see comment [22]) and gravitational waves. These results will also allow us to increase the accuracy of measurements in relativistic geodesy, chronometric gravimetry, global navigation systems, the global network of atomic clocks, etc.

It should be noted that modern atomic clocks with a relative uncertainty of $10^{-18}$ make it possible to find, in earth-based experiments, the cosmological parameter if $|\alpha| > 10^{-4}$ (for $h \sim 100$ m). Analysis of some well-known experimental results indirectly shows the possibility of $\alpha > 10^{-3}$. Thus, the problem of verifying experiments becomes very important.

Acknowledgements — We thank C. Oates, C. Tamm and U. Sterr for useful discussions and comments. This work was supported by the Russian Science Foundation (project no. 16-12-00052).

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[18] Note that from the formal mathematical viewpoint under determination of the general gravitational potential in Eq. (4) we can add some constant $C_0$ (universal for all Universe), which can be incorporated in cosmological potential: $\Phi_{CP}(r) + C_0 \rightarrow \Phi_{CP}(r)$. In this case, cosmological potential can be positive: $\Phi_{CP}(r_E) > 0$ if $C_0 > 0$. However, from the physical viewpoint the existence of $C_0 \neq 0$ looks quite artificially and groundlessly, because it leads to the existence of gravitational potential even for empty space in the absence of a matter.
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[21] Moreover, if we believe that $\alpha = 0$ in reality and nonzero results in Fig. 3 in Ref. [20] are caused by only errors (uncertainties) of experiments, then it is unclear why in all these experiments the errors always lead only to the positive sign: $\alpha > 0$?
[22] Indeed, it is possible to assume that due to the motion of the Sun in the Galaxy the trajectory of this motion will cross areas with spatially non-uniform distribution of dark matter, and, respectively, with spatially non-uniform gravitational potential. In this case, there will be some variations of the cosmological correction $\alpha$, which can be experimentally detected.