Renormalization of the Fayet-Iliopoulos Term
in Softly Broken SUSY Gauge Theories

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Abstract

It is shown that renormalization of the Fayet-Iliopoulos term in a softly broken SUSY gauge theory, in full analogy with all the other soft terms renormalizations, is completely defined in a rigid or an unbroken theory. However, contrary to the other soft renormalizations, there is no simple differential operator that acts on the renormalization functions of a rigid theory and allows one to get the renormalization of the F-I term. One needs an analysis of the superfield diagrams and some additional diagram calculations in components. The method is illustrated by the four loop calculation of some part of renormalization proportional to the soft scalar masses and the soft triple couplings.

1 Introduction

In our previous publications [1, 2, 3], we gave a complete set of the rules needed for writing down the RG equations for the soft SUSY breaking terms in an arbitrary non-Abelian N=1 SUSY gauge theory. Our main statement is that all the renormalizations in a softly broken SUSY theory are completely defined by the rigid, or unbroken, theory and may be evaluated by the use of simple differential operators [4, 5, 1, 3] or by expansion over the Grassmannian parameters [2]. However, in the Abelian case, there exists an additional gauge invariant term, the so-called Fayet-Iliopoulos or the D-term [6]

$$L_{F.I.} = \xi D = \int d^4 \theta \xi V,$$

which requires special consideration. In Ref. [7], it has been shown that in the unbroken theory this term is not renormalized provided the sum of hypercharges and their cubes equals zero. These requirements guarantee the absence of chiral and gravity anomalies and are usually satisfied in realistic models.

In case of a softly broken Abelian SUSY gauge theory, the F-I term happens to be renormalized even if anomalies are cancelled. The RG equation for $\xi$ depends not only on itself, but on the other soft breaking parameters (the soft mass of chiral scalars $m^2$, the soft triple coupling $A_{ijk}$ and the gaugino masses $M_i$). Recently, the renormalization of $\xi$ has been performed up to three loops [8, 9] using the component approach and/or superfields with softly broken Feynman rules. Here, following our main idea that renormalizations of a softly broken SUSY theory are completely
defined by a rigid one, we suggest the method to get the renormalization of the F-I term directly from the unbroken theory\textsuperscript{1}.

## 2 Renormalization of the Fayet-Iliopoulos Term

Consider an arbitrary N=1 SUSY gauge theory with the rigid Lagrangian

\[ \mathcal{L}_{\text{rigid}} = \int d^2 \theta \frac{1}{4g^2} \text{Tr} W^a W_a + \int d^2 \bar{\theta} \frac{1}{4g^2} \text{Tr} \bar{W}_\alpha \bar{W}^\alpha \]

\[ + \int d^2 \theta d^2 \bar{\theta} \; \Phi^i \Phi^j \Phi_k \Phi_l + \int d^2 \theta \; W + \int d^2 \bar{\theta} \; \bar{W}, \]

where

\[ \Phi^i = \frac{1}{2} g^2 \bar{D}_2 e^{-V} D_\alpha e^V \]
\[ \bar{W}_\dot{\alpha} = \frac{1}{2} g^2 \bar{D}_2 e^{-V} \bar{D}_\dot{\alpha} e^V, \]

are the gauge field strength tensors and the superpotential \( W \) has the form

\[ W = \frac{1}{6} y^{ijk} \Phi^i \Phi^j \Phi_k + \frac{1}{2} M^{ij} \Phi^i \Phi^j. \]

Soft SUSY breaking terms can be written as

\[ -L_{\text{soft--breaking}} = \left[ \frac{M}{2 g^2} \lambda + \frac{1}{6} A^{ijk} \Phi^i \Phi^j \Phi_k + \frac{1}{2} B^{ij} \Phi^i \Phi^j + h.c. \right] + (m^2)^i_j \bar{\Phi}^i \Phi^j. \]

In the case of the Abelian gauge group the addition of the F-I term leads to the modification of the Lagrangian in components. The relevant part of the Lagrangian is as follows:

\[ \mathcal{L} = \frac{1}{2 g^2} D^2 + \xi D + D \bar{\phi}^j \mathcal{Y}^j \phi_i - \bar{\phi}^j (m^2)^j_i \phi_i + ... \]

where \( \mathcal{Y}^j_i \) is the hypercharge matrix of the chiral supermultiplet, and \( (m^2)^j_i \) is a soft scalar mass. After eliminating the auxiliary field \( D \) this becomes

\[ \mathcal{L} = - \bar{\phi}^j (\bar{m}^2)^j_i \phi_i - \frac{1}{2} g^2 (\bar{\phi}^j \mathcal{Y}^j \phi)^2 + ..., \]

where

\[ (\bar{m}^2)^j_i = (m^2)^j_i + g^2 \xi \mathcal{Y}^j_i. \]

From eqs.(6) and (7) it follows that the F-I term gives an additional contribution to the renormalization of the soft scalar mass \( (m^2)^j_i \)

\[ [\beta_{m^2}]^j_i = [\beta_{m^2}]^j_i + g^2 \xi \mathcal{Y}^i_j + g^2 \beta_\xi (m^2, ... \mathcal{Y}^j_i = [\beta_{m^2}]^j_i + g^2 \beta_\xi (\bar{m}^2, ... \mathcal{Y}^j_i. \]

The last equality follows from the fact that eq.(6) does not contain \( \xi \) explicitly and, hence, \( \xi \) should be dropped from all the expressions.

There are four different types of contributions to the renormalization of the F-I term in a softly broken theory: those proportional to \( (m^2)^j_i \), \( M \bar{M} \), \( A^{ijk} \bar{A}_{lmn} \) and \( M \bar{A}_{lmn} \). Consider them separately.

\textsuperscript{1}We have used the program DIANA \[\text{[10]}\] for the generation of Feynman diagrams and the package MINCER \[\text{[11]}\] for the evaluation of three and four loop diagrams
2.1 The contribution proportional to \((m^2)^i_j\)

We start with the contribution proportional to the soft scalar mass. To find it in a superfield formalism, it is necessary to calculate the diagrams shown in Fig.1, where the dot on the line in Fig.1a means a softly broken superpropagator of a chiral superfield and the vertex without external line means the vertex proportional to \((m^2)^i_j\theta^2\bar{\theta}^2\). Compare these diagrams with those giving contribution to the Abelian vector superfield renormalization. In terms of superfields, one has the diagrams shown in Fig.2.

It is obvious that all the fields in a supermultiplet are renormalized similarly. Consider the diagrams in components with external lines being the D-components\(^2\). In this case, the vertex in diagram 2.a has the form

\[
\mathcal{Y}_j^i D\phi^j \phi_i = \int d^4\theta \mathcal{Y}_j^i D\theta^2\bar{\theta}^2\Phi^j \Phi_i. \quad (9)
\]

Inserting in this equation \((m^2)^i_j\) instead of \(\mathcal{Y}_j^i D\), one gets

\[
\int d^4\theta (m^2)^i_j \theta^2\bar{\theta}^2\Phi^j \Phi_i = (m^2)^i_j \phi^j \phi_i, \quad (10)
\]

which is nothing else but insertion of the soft scalar mass from eq.(4) into the scalar propagator. Hence, the contribution of diagram 2.a to the renormalization of the vector superfield is the same as the contribution of diagram 1.a to that of \(\xi\) from eq.(1) with the replacement of the hypercharge \(\mathcal{Y}_j^i\) by the soft mass \((m^2)^i_j\).

Analogously, for diagram 1.b with a vertex with more than one vector superfield, one can relate it to the diagram 2.b

\[
\int d^4\theta (m^2)^i_j \theta^2\bar{\theta}^2\Phi^j \Phi_i = (m^2)^i_j \phi^j \phi_i, \quad (10)
\]

\[
\mathcal{Y}_{\alpha k}^i \mathcal{Y}_{\beta j}^k D_{\alpha} \phi^j \phi_i C_{\beta} = \int d^4\theta \mathcal{Y}_{\alpha k}^i \mathcal{Y}_{\beta j}^k D_{\alpha} \theta^2\bar{\theta}^2 \phi^j \phi_i C_{\beta} = \int d^4\theta \mathcal{Y}_{\alpha k}^i \mathcal{Y}_{\beta j}^k D_{\alpha} \theta^2\bar{\theta}^2 \Phi^j \Phi_i V_{\beta} \Rightarrow \int d^4\theta (m^2)^i_j \mathcal{Y}_{\alpha k}^i \mathcal{Y}_{\beta j}^k \theta^2\bar{\theta}^2 \Phi^j \Phi_i V_{\beta} = \int d^4\theta (m^2)^i_j \mathcal{Y}_{\alpha k}^i \mathcal{Y}_{\beta j}^k \theta^2\bar{\theta}^2 \phi^j \phi_i C_{\beta} = (m^2)^i_j \phi^j \phi_i C_{\beta}, \quad (11)
\]

\(^2\)Since we make our calculations in superfields, in the corresponding component diagrams one has to take into account all the fields from a vector supermultiplet (see Appendix).
where $C_\beta$ stands for the lowest component of the vector superfield $V_\beta$.

![Figure 3: An example of the Abelian vector superfield propagator diagram](image)

One can get the same rule of correspondence in a different way. Consider the diagram shown in Fig. 3. Take the part of it from the vertex 1 to the vertex 3

$$\int d^4(\theta_1 \theta_2 \theta_3) \frac{1}{p_1^2} \bar{D}_{1,p_1} D_{2,-p_1}^2 \delta_{12} \mathcal{Y}_j^i V(2,p) \frac{1}{p_2^2} \bar{D}_{2,p_2} D_{3,-p_2}^2 \delta_{23},$$

and integrate by parts along the matter field propagator $(2 \rightarrow 3)$. This gives

$$\int d^4(\theta_1 \theta_2 \theta_3) \left( D_{2,p_2}^2 \bar{D}_{2,p_2}^2 \frac{1}{p_1^2 p_2^2} \mathcal{Y}_j^i V(2,p) \bar{D}_{1,p_1} D_{2,-p_1}^2 \delta_{12} \right) \delta_{23},$$

Integration over $\theta_3$ and the substitution $V(2,p) \rightarrow D \theta_2^2 \bar{\theta}_2^2$ give $(p = 0, p_1 = p_2)$

$$\int d^4(\theta_1 \theta_2) \frac{1}{p_1^2 p_2^2} D_{2,p_2}^2 \bar{D}_{2,p_2}^2 \mathcal{Y}_j^i D \theta_2^2 \bar{\theta}_2^2 D_{2,-p_1}^2 \bar{D}_{2,-p_1}^2 \delta_{12},$$

which coincides with the softly broken matter field superpropagator with the substitution $\mathcal{Y}_j^i D \rightarrow (m^2)^i_j$.[12]

Hence, in a general case, one has to calculate the self-energy diagrams for the vector superfield and in the resulting expression to replace the hypercharge $\mathcal{Y}_j^i$, corresponding to the external line, by the soft scalar mass $(m^2)^i_j$.

Using the results of Ref.[13] and the above formulated rule, after some algebraic manipulations and taking into account the gauge invariance of the Lagrangian (2.3)

$$y^{i j n} \mathcal{Y}_n^k + y^{i n k} \mathcal{Y}_n^j + y^{n j k} \mathcal{Y}_n^i = 0, \quad (m^2)^i_k \mathcal{Y}_j^k = \mathcal{Y}_k^i (m^2)^i_j,$$

one can quickly get the contribution of the soft mass $(m^2)^i_j$ to the renormalization of the F-I term, which coincides with that from Ref. [9].

### 2.2 The contribution proportional to $A^{ijk} \bar{A}_{lmn}$

There is another possibility of getting the desired result from the unbroken theory, namely, to consider the propagator of the matter superfield. It is known that, provided the anomalous dimension of the matter field, one can get the beta function for the soft mass $(m^2)^i_j$ acting on anomalous dimension by the differential operator $D_2$[4,5] (or $O[6]$). The action of this operator means that...
in the self energy diagram one has to replace each pair of Yukawa couplings of opposite chirality $y^{ijk}$ ($\bar{y}^{ijk}$) by the soft triple couplings $A^{ijk}$ ($\bar{A}^{ijk}$) with the same indices (the term proportional to $A^{ijk}$ ($\bar{A}^{ijk}$)), or to insert in each line the soft mass term $(m^2)^i_j$ and contract the indices (the term proportional to $(m^2)^i_j$). Consider how this procedure works in components.

In one loop order in the unbroken theory there is only one superfield diagram shown in Fig.4a. In a softly broken theory, it leads to the following beta function for the soft mass:

$$[\beta_{m^2}]^i_j^{(1)} = \frac{1}{2} A^{ikl} \bar{A}_{jkl} + y^{ind}(m^2)^i_j \bar{y}_{jkl} + \frac{1}{4}(m^2)^i_j y^{ntk} \bar{y}_{jkl} + \frac{1}{4} y^{ikl} \bar{y}_{nk}(m^2)^n_j. \quad (16)$$

In components, the same result comes from the three diagrams (Fig.4b). The first two diagrams give the contribution corresponding to the first and second terms in (16), respectively. However, the second diagram is the tadpole and this is the same tadpole which gives the contribution to the renormalization of the F-I term! One has only to make the replacement $y^{ind} \bar{y}_{jkl} \to \gamma^m_k \delta^i_j$ in the second term of eq.(16) to get the contribution to the renormalization of the F-I term proportional to the soft mass $(m^2)^i_j$. Analogously, in two loops a single superfield diagram containing only the Yukawa couplings is shown in Fig.5a, and the corresponding soft beta function reads (the contribution proportional to $(m^2)^i_j$)

$$[\beta_{m^2}]^i_j^{(2)} = \frac{1}{2} y^{ikl}(m^2)^m_l \bar{y}_{mst} y^{nst} \bar{y}_{jkn} + \frac{1}{2} y^{ikm} \bar{y}_{mst} y^{nst}(m^2)^i_j \bar{y}_{jkl} + y^{ikm} \bar{y}_{mlt}(m^2)^l_s \bar{y}_{nst} \bar{y}_{jkn} + \frac{1}{4} y^{ikm} \bar{y}_{mst} y^{nst} \bar{y}_{jkn} + \frac{1}{4} y^{ikm} \bar{y}_{mst} y^{nst} \bar{y}_{lkn}(m^2)^l_j. \quad (17)$$

Again in components, this result comes from the three diagrams in Fig.5b. The first two terms in (17) come from the first diagram and only from it. The rest diagrams give a contribution to

\[\text{Footnote 3: In one and two loops, there are only terms proportional to the soft scalar mass $(m^2)^i_j$ due to the requirement of anomaly cancellation. All the other contributions appear starting from the three loop level.}\]
the other terms. Again, like in one loop order, the first diagram is the same tadpole as in the D-term renormalization. Besides, one can notice, that if being interested in this analogy with the D-term renormalization, one may consider only those superfield diagrams where external lines are connected by a single matter superfield line.

In three loops, one has two diagrams like that which shown in Fig.6. The contribution from

$$\left[ \beta_{m^2} \right]_j^{(3)} = \frac{1}{3 \epsilon} y^{ikm} y_{mpr} y^{rst} y_{qst} y^{npq} y_{jkn} + \frac{1}{12 \epsilon} y^{ikm} y_{mst} y^{lnt} y_{lps} y^{npq} y_{jkn}$$

Figure 6: The three loop superfield diagrams containing only Yukawa couplings

these diagrams to the soft beta function is (only the one proportional to $A^{ijk} \bar{A}_{lmn}$)

$$\left[ \beta_{m^2} \right]_j^{(3)} = y^{ikm} \bar{A}_{mpr} A_{rst} y_{qst} y^{npq} y_{jkn} + y^{ikm} y_{mpr} y^{rst} \bar{A}_{qst} A_{npq} y_{jkn} + y^{ikm} \bar{A}_{mpr} y^{rst} y_{qst} A_{npq} y_{jkn} + ...$$

The corresponding contributions in components come from the diagrams shown in Fig.7. Again, it

$$\left[ \beta_{m^2} \right]_j^{(3)} = -\frac{1}{4} y^{ikm} \bar{A}_{mst} A_{lst} y_{lpq} y^{npq} y_{jkn} + ...$$

Figure 7: The component diagrams corresponding to Fig.6 and contributing to $[\beta_{m^2}]_j^{(3)}$

is easy to see that the first and the third lines in (18), when calculated in components, come from the tadpole diagrams while all the rest from the other diagrams. Taking these lines and performing the replacement

$$y^{ikm} \bar{y}_{jkn} \rightarrow \mathcal{Y}_m^m \delta^i_j,$$

after making use of (15), valid also for replacement of all the Yukawa vertices $y^{ijk}$ by the corresponding soft triple couplings $A^{ijk}$, one obtains the contribution to the renormalization of the F-I term proportional to $A \bar{A} y \bar{y}$ coinciding with that of Ref. 9.

Unfortunately, this procedure does not work always. To see this, consider three loop diagrams, contributing to the matter superfield propagator, with one internal vector line as shown in Fig.8. Consider the first diagram. The simple pole is equal to

$$-4g^2 \frac{1}{3 \epsilon} y^{i kl} \mathcal{Y}_l^m y_{orst} y_{npq} y_{j kp}$$

(19)
and gives the following contribution to the soft scalar mass beta function (only the term proportional to $AA$)

$$[\beta_{m^2}]_j^{(3)} = -4g^2y^{ikl}\gamma^m_lA_{nst}\gamma^{np}\bar{y}_{jkp} - 4g^2A^{ikl}\gamma^m_l\bar{A}_{nst}\gamma^{np}\bar{y}_{jkp} + ...$$

(20)

In components one has three different diagrams shown in Fig. 8. All three diagrams give a contribution to the first term in (20). So one cannot directly extract the contribution from the tadpole graphs. However, to get the renormalization of the D-term proportional to $g^2AA$ it is sufficient to calculate only two diagrams, namely 8.b and 8.c. In the rest of the diagrams in Fig. 8 (except the first) the contribution from the tadpole graphs can be figured out from the superfield diagrams. The simple poles for diagrams 8.b and 8.c omitting the tensor structure are, respectively,

$$9.b = \frac{2}{3\varepsilon}, \quad 9.c = \frac{2}{\varepsilon}.$$

Subtracting these expressions from (19) one gets the result for the diagram 8.a

$$-4g^2\gamma^{ikl}\gamma^m_l\bar{y}_{nst}\gamma^{np}\bar{y}_{jkp},$$

which, after the replacement $y^{ikl}\bar{y}_{jkp} \rightarrow \gamma^p_i\gamma^j_l$, gives the contribution to the beta function of $\xi$ equal to $-12g^2\gamma^{ikl}\gamma^m_l\bar{A}_{nst}\gamma^{np}\bar{y}_{jkp}$. This term together with the results for the tadpole graphs obtained from the superfield diagrams of Fig. 8, after reducing to the same tensor structure gives the renormalization of the F-I term proportional to $g^2AA$ coinciding with that of Ref. 3.

The same way one can determine the contributions proportional to $A^{ijkM}(\bar{A}^{ijkM})$. As an illustration of efficiency of this method, we present below the calculation of the four loop contribution to the renormalization of the F-I term proportional to $A\bar{A}y\bar{y}y\bar{y}$.

2.3 The contribution proportional to $A\bar{M}(\bar{A}M)$

This contribution to the F-I term renormalization can be calculated in a way similar to the previous one. In this case, the analysis of the component diagrams shows that one should consider the
following four graphs and in all these diagrams the contribution of the tadpole graphs to the soft scalar mass renormalization can be easily determined without any use of the component calculations.

Notice, however, that in the Wess-Zumino gauge the first two diagrams do not give any contribution. Still we have to take into account all the fields from the vector supermultiplet. The results for these diagrams are, respectively, (simple pole):

\[
\begin{array}{cccc}
a & b & c & d \\
-\frac{1}{6\epsilon} & -\frac{2}{3\epsilon} & \frac{1}{\epsilon}\zeta(3) & \frac{2}{3\epsilon}
\end{array}
\]

which, after the replacement of the Yukawa vertices with an external lines by a hypercharge in the proper part of the soft scalar mass beta function and the reduction of tensor structures, give the answer coinciding with that of Ref. [9].

2.4 The contribution proportional to \( M\bar{M} \)

The contribution to the D-term renormalization proportional to \( M\bar{M} \) may come either from one of the vector lines (\( M\bar{M} \)), or from two different lines (\( M \) and \( \bar{M} \)). In the first case, to get the result, one has to calculate in a rigid theory the superfield diagrams shown in Fig.11b, where in the triple vector vertex the external line does not contain supercovariant derivatives. This diagram corresponds to the tadpole graph \( \text{[1]} \)a with a softly broken vector superpropagator proportional to \( M\bar{M} \)

\[
M\bar{M} \frac{D^\alpha\bar{D}^2\theta^2\bar{\theta}^2D^2\bar{\theta}^2\bar{\theta}^2\bar{D}^2\bar{D}^2\delta p_{\alpha\beta}}{p^4(p^2-MM)},
\]

and gives the same result as diagram Fig.11a after the replacement \( Y_j^i \rightarrow M\bar{M}\delta_j^i \) for the hypercharge corresponding to the external vertex.

![Figure 10: Three loop superfield diagrams giving contribution proportional to \( A\bar{M} \)](image)

Figure 11: The tadpole graph (a) giving contribution proportional to \( M\bar{M} \) and the corresponding self-energy vector superfield diagram (b)

In the second case, one can proceed as follows. Consider the superfield diagram of the vertex type (the interaction of a vector superfield with matter) shown in Fig.12a.
Rewriting the superfield diagram 12.a in components, one finds four types of diagrams: 12.b and 12.c when cutting the external legs are the same component diagrams which give a contribution to the renormalization of the D-term proportional to $A\bar{A}y\bar{y}$; analogously, 12.d gives the contribution proportional to $g^2yAM$ (with the replacement of the fermion matter field propagator by that of gaugino) and 12.e is analogous to 12.f, which we are looking for, but with the insertion of mass into the gaugino lines. Thus, subtracting from the superfield result 12.a the already known component expressions 12.b-d, we get the answer for diagram 12.e which, after the replacement of the external Yukawa vertices by the gaugino mass, gives the desired result for diagram 12.f.

One may have an impression that the calculations in components are simpler. However, first of all, in our approach we practically does not have to calculate anything new and, second, we just want to emphasize that all the information on the renormalizations in a softly broken theory can be extracted from the rigid one, though for the case of the F-1 term in a rather tricky way.

### 3 Calculations in Four Loops

#### 3.1 The contribution proportional to $(m^2)^i_j$

To calculate the contribution proportional to $(m^2)^i_j$, we follow the recipe of Sect.2.1 and use the results of Ref.[13]. After reduction of tensor structures one finds

$$
\Delta\beta^{(4)}_\xi = -F_1^{(m^2)} - \frac{13}{2}F_2^{(m^2)} - \frac{10}{3}F_3^{(m^2)} - \frac{5}{24}F_4^{(m^2)} + \frac{1}{2}\left(F_5^{(m^2)} + \bar{F}_5^{(m^2)}\right) + \frac{3}{2}\left(F_6^{(m^2)} + \bar{F}_6^{(m^2)}\right)
+ \left(\frac{7}{6} - 2\zeta(3)\right)F_7^{(m^2)} + (1 - 2\zeta(3))F_8^{(m^2)} - \left(\frac{1}{2} + \zeta(3)\right)F_9^{(m^2)} - 3\zeta(3)F_{10}^{(m^2)},
$$

where

\begin{align*}
F_1^{(m^2)} &= y^{ijkl}\gamma^i_j\gamma^m_{klm}\gamma^npq\gamma^{qrst}(m^2)^{r}_{sp}\gamma_{pq} \\
F_2^{(m^2)} &= y^{ijkl}\gamma^i_j\gamma^m_{klm}\gamma^npq\gamma^{qrst}\gamma_{pq} \\
F_3^{(m^2)} &= y^{ijkl}\gamma^i_j\gamma^m_{klm}\gamma^npq\gamma^{qrst}\gamma_{pq} \\
F_4^{(m^2)} &= y^{ijkl}\gamma^i_j\gamma^m_{klm}\gamma^npq\gamma^{qrst}\gamma_{pq} \\
F_5^{(m^2)} &= y^{ijkl}\gamma^i_j\gamma^m_{klm}\gamma^npq\gamma^{qrst}\gamma_{pq} \\
F_6^{(m^2)} &= y^{ijkl}\gamma^i_j\gamma^m_{klm}\gamma^npq\gamma^{qrst}\gamma_{pq} \\
F_7^{(m^2)} &= y^{ijkl}\gamma^i_j\gamma^m_{klm}\gamma^npq\gamma^{qrst}\gamma_{pq} \\
F_8^{(m^2)} &= y^{ijkl}\gamma^i_j\gamma^m_{klm}\gamma^npq\gamma^{qrst}\gamma_{pq} \\
F_9^{(m^2)} &= y^{ijkl}\gamma^i_j\gamma^m_{klm}\gamma^npq\gamma^{qrst}\gamma_{pq} \\
F_{10}^{(m^2)} &= y^{ijkl}\gamma^i_j\gamma^m_{klm}\gamma^npq\gamma^{qrst}\gamma_{pq} \\
\end{align*}
3.2 The contribution proportional to $A\bar{A}$

To calculate the contribution proportional to $A\bar{A}y\bar{y}y\bar{y}$, according to our method, one has first of all to take the results of the four loop calculation of the self-energy diagrams for the matter superfield where the external lines are connected by a single propagator of the matter superfield. There are six diagrams of this sort. The results of their calculations are (single pole)

\[
\begin{align*}
(a) & : \frac{3}{8\epsilon} \zeta(3) + \frac{3}{16\epsilon} \\
(b) & : \frac{5}{8\epsilon} \\
(c) & : -\frac{5}{48\epsilon} + \frac{1}{8\epsilon} \zeta(3) \\
(d) & : -\frac{5}{96\epsilon} \\
(e) & : -\frac{1}{6\epsilon} \\
(f) & : \frac{1}{32\epsilon} - \frac{1}{16\epsilon} \zeta(3)
\end{align*}
\]

The sum coincides with the proper part of Ref.[14].

Acting on this sum by an operator $A^{ijk}\bar{A}_{lmn} \frac{\partial}{\partial y^{ijk}} \frac{\partial}{\partial \bar{y}_{lmn}}$, one obtains the part of the beta function for the soft mass $(m^2)_j$ proportional to $A\bar{A}y\bar{y}y\bar{y}y$.

Comparing the results with the corresponding component diagrams containing two soft vertices of opposite chirality ($A^{ijk}$ and $\bar{A}_{ijk}$), one finds that to extract the contribution of the tadpole diagrams one need to calculate three diagrams

\[
\begin{align*}
(a) & : \frac{3}{8\epsilon} \\
(b) & : -\frac{1}{24\epsilon} \\
(c) & : \frac{1}{48\epsilon}
\end{align*}
\]

The single poles of these diagrams are

After subtraction of these diagrams from the corresponding superfield ones and replacing the Yukawa vertices with the external lines by a hypercharge, we get after reduction of tensor structures

\[
\Delta \beta_{\xi}^{(4)} = \frac{17}{6} F_1(A\bar{A}) + \frac{7}{3} F_2(A\bar{A}) + 2 F_2(A\bar{A}) + \frac{7}{6} F_3(A\bar{A}) + \frac{1}{8} F_4(A\bar{A}) + \frac{7}{3} F_5(A\bar{A}) + \frac{1}{2} F_5(A\bar{A})
\]
where
\[ F_1^{(AA)} = A^{ijkl} \bar{y}_i y_{jkm} y^{mnp} A_{lnq} y^{gst} y_{pst}, \]
\[ F_2^{(AA)} = A^{ijkl} \bar{y}_i y_{jkm} y^{mnp} A_{lnq} y^{gst} y_{pst}, \]
\[ F_3^{(AA)} = A^{ijkl} \bar{y}_i y_{jkm} y^{mnp} A_{lnq} y^{gst} y_{pst}, \]
\[ F_4^{(AA)} = y^{ijkl} A_{ikm} y^{mnp} A_{lnq} y^{gst} y_{pst}, \]
\[ F_5^{(AA)} = y^{ijkl} A_{ikm} y^{mnp} A_{lnq} y^{gst} y_{pst}, \]
\[ F_6^{(AA)} = y^{ijkl} A_{ikm} y^{mnp} A_{lnq} y^{gst} y_{pst}, \]
\[ F_7^{(AA)} = y^{ijkl} A_{ikm} y^{mnp} A_{lnq} y^{gst} y_{pst}, \]
\[ F_8^{(AA)} = y^{ijkl} A_{ikm} y^{mnp} A_{lnq} y^{gst} y_{pst}, \]
\[ F_9^{(AA)} = y^{ijkl} A_{ikm} y^{mnp} A_{lnq} y^{gst} y_{pst}. \]

Notice that a direct calculation of the tadpoles in components requires the evaluation of nearly thirty different diagrams. The same proportion is valid for the other four loop contributions to the D-term renormalization.

### 4 Conclusion

We have found that all the information about the renormalizations of the soft SUSY breaking terms in the N=1 SUSY gauge theory is contained in a rigid, unbroken theory. In the case of the non-Abelian gauge group, the RG equations for the soft terms are obtained from the anomalous dimensions of the matter and vector superfields by acting of the differential operators \([1, 3, 5]\). In the presence of the Abelian gauge group, to calculate the renormalization of an additional Fayet-Iliopoulos term, one needs an analysis of superfield diagrams. To find the contribution proportional to the soft scalar mass \((m^2)_{ij}\) (the square of gaugino mass \(M\bar{M}\)), one needs to take the self-energy diagrams for the vector superfield and replace one of the external vertices with the hypercharge \(Y_j\) by \((m^2)_{ij}\) \((M\bar{M}\delta^j_i)\). In this case, there is no need to do any calculations except in superfields.

The other contributions (proportional to \(A\bar{A}\) and \(M\bar{A}\)) can be found from the analysis of the matter superfield propagator diagrams in a rigid theory and the corresponding component diagrams in a softly broken theory extracting from the latter the contribution of the tadpole graphs. In this case, one needs to calculate additionally some component diagrams the number of which is essentially reduced compared to a direct component calculation.

### Appendix

Throughout the paper we use the standard metric \(g_{\mu\nu} = diag(1, -1, -1, -1)\). In this metric the chiral matter superfield and the vector superfield can be written as
\[
\Phi = \phi + i \theta \sigma^\mu \bar{\theta} \partial_\mu \phi - \frac{1}{4} \theta \theta \bar{\theta} \bar{\sigma} \partial_\mu \phi + \sqrt{2} \theta \psi + \frac{i}{\sqrt{2}} \theta \theta \bar{\theta} \bar{\sigma} \partial_\mu \psi + \theta \theta F, \tag{23}
\]
\[
V = C + i \theta \chi - i \bar{\theta} \bar{\chi} + \frac{i}{2} \theta \theta N - \frac{i}{2} \bar{\theta} \bar{\theta} \bar{N} - \theta \sigma^\mu \bar{\theta} \nu_\mu + i \theta \theta \bar{\theta} \bar{\lambda} - \frac{1}{2} \theta \theta \bar{\theta} \bar{\sigma} \partial_\mu \chi - i \theta \theta \bar{\theta} \lambda
+ \frac{1}{2} \theta \theta \bar{\theta} \bar{\sigma} \partial_\mu \bar{\chi} + \frac{1}{2} \theta \theta \bar{\theta} \partial D - \frac{1}{4} \theta \theta \bar{\theta} \bar{\sigma} \partial_\mu \partial_\nu \phi. \tag{24}
\]
The interaction of a matter superfield with an Abelian superfield is given by eq.(2) and for the triple vertex has the form (in arbitrary gauge, not the Wess-Zumino one)

\[
\int d^2\theta d^2\bar{\theta} \Phi^i(e^V)i\Phi_j = \frac{i}{\sqrt{2}} \bar{\phi} \lambda \psi - \frac{i}{\sqrt{2}} \bar{\psi} \lambda \phi + \frac{1}{2} \bar{\phi} D \phi - \frac{i}{2} \bar{\phi} v_{\mu} \partial^\mu \phi + \frac{i}{2} \bar{\phi} v_{\mu} \phi + \frac{1}{2} \bar{\psi} \bar{\sigma}^\mu v_{\mu} \psi \\
+ \partial^\mu \bar{\phi} C \partial^\mu \phi - \frac{1}{2} \bar{\phi} \partial^\mu \partial_\mu C \phi - \frac{1}{\sqrt{2}} \bar{\phi} \bar{\sigma}^\mu \partial_\mu \phi - \frac{1}{\sqrt{2}} \partial_\mu \bar{\psi} \bar{\sigma}^\mu \phi + \frac{i}{\sqrt{2}} \bar{\psi} \bar{\chi} F \\
- \frac{i}{\sqrt{2}} F \chi \psi + FCF + \frac{i}{2} F N \phi - \frac{i}{2} \bar{\phi} N F - \frac{i}{2} \bar{\psi} C \bar{\sigma}^\mu \partial_\mu \psi + \frac{i}{2} \partial_\mu \bar{\psi} C \bar{\sigma}^\mu \psi.
\]

In the Wess-Zumino gauge only the first line is left. However, for our analysis one needs all the vertices and therefore some new diagrams arise.

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