Research Article

Top Quark Pair-Production in Noncommutative Standard Model

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The differential cross-section of the top quark pair production via the quark-antiquark annihilation subprocess in hadron collision is calculated within the noncommutative standard model. A pure NC analytical expression for the forward-backward asymmetry at the tree level is obtained. Moreover, using recent Tevatron results from the full RUN2 data, a new lower bound on the noncommutative geometry parameter is deduced.

1. Introduction

In 2012, the validity of the standard model of particle physics has been confirmed, following the announcement of both research groups Atlas [1] and CMS [2] on their detection of the Higgs boson, which is the last missing piece of this model. The standard model could explain any collision process studied in different research laboratories and was also able to explain the structure of the known particles up to now. Despite all of these successes, the model does not represent a theory of everything since it remains many unsolved open questions. Indeed, there are many arbitrary parameters, and the model is unable to explain the origin of dark matter, matter-antimatter asymmetry in the universe, the hierarchy problem, etc. To solve these problems, many theories have been proposed (generally qualified as beyond the standard model); the most important are those based on the space-time noncommutativity, unparticle, technicolor, and supersymmetry. In the last two decades, the physicists were interested in building a standard model in a noncommutative space-time. This is particularly clear from the works of Calmet et al. [3], Chaichian et al. [4], Alboteanu et al. [5], and others in the context of noncommutative QED or standard model [6–8]. The Feynman law was obtained in studying certain processes (like fermion pair production and Compton scattering [9] e⁺e⁻ collision [10] e⁺e⁻ → μ⁺μ⁻ [11]).

In this work, we study the top-antitop quarks production in proton-antiproton collisions at the Tevatron energy within NCSM, obtain another correction to the forward-backward asymmetry which could give information about this new physics and derive a new lower limit of the noncommutativity parameter. The motivation is due to the huge quantity of available data from Tevatron and LHC and the fact that these data are among the most accurately available ones. In Section 2, we give the basics of noncommutative standard model and present the analytical expressions of the cross-section for the top-antitop production in hadronic collisions within NCSM. In Section 3, we discuss the numerical results concerning the azimuthal angle and the transverse momentum dependence of the noncommutative differential cross-section as well as deriving a new lower bound of the noncommutative parameter. Finally, in Section 4, we draw our conclusions.

2. Top-Antitop Quarks Pair-Production in NCSM

The original idea of taking space-time coordinates to be noncommutative (NC) goes back to the pioneering work of Snyder [12], arising interest in the noncommutative (NC) field theory. However, this idea was ignored for some time ago because of the success of the renormalization program in quantum field theory QFT which has been revived by the
work of Seiberg and Witten on string theory. These authors have shown that the noncommutative gauge theory appears as a low energy limit in the presence of D-branes with non-zero B-field background.

Contrary to most other beyond standard models where new particles are introduced, there are no new massive degrees of freedom are included here, but the standard model interactions get modified due to the space-time noncommutativity. So new interactions that are forbidden in ordinary SM become allowed and therefore new phenomena appear.

In the noncommutative standard model (general properties of noncommutative field theories can be found in ref. [13]), the ordinary coordinates are represented by operators which no longer commute:

\[ [x^\alpha, x^\beta] = i\theta^{\alpha\beta}. \]  

(1)

where \( \theta^{\alpha\beta} \) is a real antisymmetric matrix and not a tensor because it is frame-independent (so NC theories violate Lorentz invariance). By convention, we write \( \theta^{\alpha\beta} \) as:

\[ \theta^{\alpha\beta} = \frac{\epsilon^{\alpha\beta}}{\Lambda^2}, \]  

(2)

where the parameter \( \Lambda \) describes the NC scale at which one expects to see the effect of space-time noncommutativity and \( \epsilon_{\mu\nu} \) are antisymmetric elements.

The noncommutativity may be separated into two classes: (1) space-space noncommutativity with \( \theta^{ij} \neq 0 (i, j = 1, 2, 3) \), and (2) space-time noncommutativity with \( \theta^{\mu i} \neq 0 \). It is worth to mention that the latter suffer from the unitarity and causality problems. In order to construct the so-called noncommutative standard model (NCSM), one has to use (1) the Weyl-Moyal star product instead of the ordinary product defined as:

\[ (f \star g) = \exp \left( \frac{1}{2} \theta^{\alpha\beta} \partial_{\alpha} \partial_{\beta} \right) f(x) g(y) |_{x=y}, \]  

(3)

where \( f \) and \( g \) are regular functions. We remind that the replacement of the dot product by the star one leads to the so-called UV/IR mixing. (2) The Seiberg-Witten maps [14] where both the spinor and gauge field in noncommutative space are expanded in terms of that of the standard model as:

\[ \tilde{\psi} [V] = \psi - \frac{1}{2} \theta^{\alpha\beta} V_{\alpha} \partial_{\alpha} \psi + \frac{i}{8} \theta^{\alpha\beta} [V_{\alpha}, V_{\beta}] \psi + O(\theta^2), \]  

\[ \tilde{V}_{\mu} = V_{\mu} + \frac{1}{6} \theta^{\alpha\beta} [\partial_{\alpha} V_{\mu} + F_{\alpha\nu}, V_{\beta}] + O(\theta^2). \]  

(4)

It is very important to mention that if fields are assumed to be Lie algebra valued and allow for the closure of the Lie algebra valued noncommutative transformation gauge parameters, constraints that only U(N) structure groups are conceivable as well as the corresponding gauge transformations must be in the fundamental representa-

tion of this group. The matching of the noncommutative action to the ordinary one requires that the noncommutative fields are mapped to commutative ones by means of the Seiberg-Witten maps. The latter has the remarkable property that ordinary gauge transformations induce noncommutative ones. In this case, the low energy action is local in the sense that there is no UV/IR mixing. However, the basic assumption is that the noncommutative fields are not Lie algebra valued but are in the enveloping algebra and allows them to consider SU(N) groups. Despite the nice mathematical properties of noncommutative gauge theories, at high energies (where the theory is relevant), one can have the violation of Lorentz invariance. Contrary to theories beyond the standard model, no new particle degrees of freedom are introduced in NCSM but rather standard model (SM) interactions are modified as well as the presence of new interactions (forbidden in SM) and thus a new phenomenology will take place due to the noncommutativity of the space-time.

In SM or NCSM, the dominant diagrams involved in the physical elastic cross-section of top-antitop quarks production in hadronic collisions are those coming from quark-antiquark and gluon fusion subprocesses (see Figure 1). The main production subprocess at the LHC is different from the one at the Tevatron. In fact at the Tevatron, the dominant production channel is the quark-antiquark annihilation (~85%) while at the LHC the gluon-gluon is the most relevant (~85%). The corresponding physical differential cross-section (using the factorization theorem) can be written in the form [15, 16]:

\[ \sigma_{HH} \rightarrow t\bar{t} = \sum_{a,b=q,g} \int_{x_{\text{min}}}^{x_{\text{max}}} dx_1 dx_2 f_{aHH}(x_1, \mu_F) f_{bHH}(x_2, \mu_F) \cdot \delta_{ab-\mu}(x_1, x_2, \alpha_s(\mu_R)), \]  

(5)

where \( f_{aHH} \) and \( f_{bHH} \) are the partons distribution functions (PDF) of the partons \( a \) and \( b \) with momentum fraction \( x_1 \) and \( x_2 \) inside the hadrons \( H \) and \( H' \), respectively, and \( \delta_{ab-\mu} \) is the subprocess differential cross-section. Here, \( x_{\text{min}}, x_{\text{max}} \) and \( \mu_F, \mu_R, \) and \( \alpha_s \) stand for the kinematical limits of the momentum fraction \( x_1 \), factorization and renormalization scales, and strong running coupling, respectively. In what follows, we take the physical choice \( \mu_F = \mu_R = m_t \) and use the MSTW2008 PDF’s. Taking into account the minimal NCSM Feynman rules of ref. [17], the Born scattering amplitude for the dominant subprocess \( q + \bar{q} \rightarrow t + \bar{t} \) in the Tevatron reads:

\[ M = \bar{v}(p_3) c_{\mu(NC)}^{\gamma} V_{\mu(NC)}^{\gamma h}(p_1) \left[ -i\gamma^{\nu} g^{ab} \frac{q^\nu}{q^2} \right] \bar{u}(p_2) c_{\mu}^{\gamma h} V_{\gamma(NC)}^{\alpha \gamma}(p_4), \]  

(6)

where \( q \) is the momentum transfer and \( V_{\mu(NC)} \) is the NC \((q \bar{q})\) vertex which is given at first order in \( \theta \) by [17]:
Here, $c_{ij}$ are color component elements, $u(p)$ and $v(p)$ are the quark and antiquark Dirac spinors, $\lambda_a = T^a/2$ and $g_a$ are the Gell-Mann matrix and strong coupling constant. The momentum $p_{\text{in}}$ and $p_{\text{out}}$ denote the incoming and outgoing quarks, respectively. In what follows, we denote by $\eta$ the NC parameter and use the parametrization choice (of course our general arguments and conclusions are not affected by this choice):

$$\Theta^{\nu\mu} = \begin{pmatrix} 0 & \eta & \eta & \eta \\ -\eta & 0 & 0 & 0 \\ -\eta & 0 & 0 & 0 \\ -\eta & 0 & 0 & 0 \end{pmatrix},$$

where $\eta = 1/\Lambda^2$. In the centre of mass system of $t$ and $\bar{t}$, the 4-momentum of the initial and final particles denoted by $(p_1, p_2)$ and $(p_3, p_4)$, respectively, are written as [18]:

$$p_1 = \frac{\sqrt{s}}{2} (1, 0, 0, 1),$$

$$p_2 = \frac{\sqrt{s}}{2} (1, 0, -1),$$

$$p_3 = (m_T \cos y, p_T \cos \phi, p_T \sin \phi, m_T \sinh y),$$

$$p_4 = (m_T \cosh y, -p_T \cos \phi, -p_T \sin \phi, -m_T \sinh y),$$

where $y = y_t - y_{\bar{t}}$ is the difference between the top and antitop quark rapidities and $p_T = |p_T| \cos \theta$ is the transverse momentum of the final particle.

The scattering amplitude $\mathcal{M}$ can be written as

$$\mathcal{M} = \mathcal{M}_1 - \mathcal{M}_2,$$

with

$$\mathcal{M}_1 = i g^2 C_F \left[ 1 - \frac{1}{4} (p_2 \cdot \partial p_1)(p_2 \cdot \partial p_4) + \frac{i}{2} (p_2 \cdot \partial p_1 + p_2 \cdot \partial p_4) \right] + \left[ \bar{v}(p_2) \gamma^\mu u(p_1) \right] \left[ \bar{u}(p_3) \gamma^\nu v(p_4) \right],$$

$$\mathcal{M}_2 = \frac{g^2}{2q^2} C_F \left[ 1 - \frac{i}{2} (p_2 \cdot \partial p_1)(p_2 \cdot \partial p_4) \right] v(p_2) \gamma^\nu u(p_1) \bar{u}(p_3) (p_4 - m_t) v(p_4),$$

where $C_F = \lambda_3^a \lambda_3^a / 4$ is a QCD color factor.

After straightforward but tedious calculations, the spin and color-averaged and summed square amplitude is given by

$$|\mathcal{M}|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 - 2 \text{Re}(\mathcal{M}_1^{\dagger} \mathcal{M}_2),$$

with:

$$|\mathcal{M}_1|^2 = \frac{8 g^4}{9} \eta^2 \left[ 1 + \frac{F^2}{16} \left( 5\lambda^2 + 2F^2 \right) \right] m_T^2 + m_T^2 (1 + 2 \sinh^2 y),$$

$$|\mathcal{M}_2|^2 = \frac{16}{9} \eta^2 m_T^2 \frac{g^4}{s} \left[ 1 + \frac{F^2}{16} \left( 5\lambda^2 + 2F^2 \right) \right] p_T^2 + m_T^2 \sinh^2 y \cdot \left[ p_T (\sin \phi + \cos \phi) + m_T \sinh y \right]^2 - 2 m_T^2 \cosh^2 y,$$

$$2 \text{Re}(\mathcal{M}_1^{\dagger} \mathcal{M}_2) = \frac{8 g^4}{9} \eta^2 m_T^2 m_T p_T \frac{g^4}{s} \left[ s - F \right] \cdot (\cos \phi + \sin \phi) \cosh y,$$

where $s = x_1 x_2 s$, and

$$m_T = \sqrt{m_t^2 + p_T^2},$$

$$F = 2 m_T (\cos \phi + \sin \phi) p_T + m_T \sinh y \cosh y.$$

After some simplifications, the noncommutative differential cross-section is shown to have the form:

$$\frac{d^4 \sigma_{NC}}{dp_T dy d\phi} = \left( 1 + \frac{\eta^2}{16} \left( 16 m_T^4 \cosh^4 y + 4 F^2 \right) \right) \frac{d^4 \sigma_{SM}}{dp_T dy d\phi}.$$

3. Numerical Results and Discussions

In this section, we study numerically the effect of noncommutativity on the processes $q + \bar{q} \rightarrow t + \bar{t}$ at the Tevatron energy collision $\sqrt{s} = 1.96 \text{ TeV}$.

3.1. Azimuthal Distribution. Figure 2 shows $d\sigma / d\phi$ as a function of the azimuthal angle $\phi$ (in units where $h = c = 1$). We fix the top quark mass to $m_t = 173 \text{ GeV}$. The lowest horizontal curve represents the standard model contribution which is independent of $\phi$, while the three other curves correspond to $\Lambda = 0.9, 0.8$ and $0.7 \text{ TeV}$. Notice that the $d\sigma / d\phi$ is a
such an azimuthal distribution clearly reflects the asymmetry (denoted by $A_{FB}$) as a function of the transverse momentum $p_T$ for various values of the transverse momentum $p_T$ and maxima. The largest maxima for each of the three processes is peaked at $\phi = 0.78$ rad, and the minimum is picked up at $\phi = 2.42$ rad and $5.42$ rad. It is important to note that such an azimuthal distribution clearly reflects the nature of the noncommutative geometry which is rarely to be found in other classes of new physics models.

3.2. Forward-Backward Asymmetry. The forward-backward asymmetry (denoted as $A_{FB}$) is defined as [19]:

$$A_{FB} = \frac{N_i(\cos \theta) - N_i(-\cos \theta)}{N_i(\cos \theta) + N_i(-\cos \theta)},$$

where $N_i(\cos \theta) = d\sigma/d\Omega(\cos \theta)$ is the number of $i$ quarks this observable was measured by the D0 [20] and CDF [21] experiments at the Fermilab collider (Tevatron). In the ordinary standard model, the forward-backward asymmetry is expected at proton-antiproton collision and interpreted as a consequence of initial state asymmetry, where the produced top (resp. antitop) quark tends to emerge in the same direction of initial quark (resp., antiquark) coming from the proton resp. antiproton) and the produced antitop tends to emerge in the same direction of initial antiquark which is not the case for the subprocess $gg \rightarrow t\bar{t}$ which is important at the LHC proton-proton collider. NCSM is shown to have the form:

$$A_{FB} = \frac{2\eta^2 \alpha m_t^2(\cos \phi + \sin \phi) \cosh y \sin y}{1 + \eta^2 m_t^2[m_t^2(\cosh y + \sinh y) + p_T^2(\cos \phi + \sin \phi)^2] \cosh y}.$$ 

As we can see, this contribution is proportional to $\eta^2$, which means that at the tree level, $A_{FB}$ is a pure noncommutative effect. Figure 1 show the NC $A_{FB}$ as a function of the azimuthal angle for various values of the transverse momentum and final particle state rapidity at the Tevatron energy and for the NC parameter $\Lambda = 1$ TeV. Notice that the NC $A_{FB}$ has a negative value in the region where $2.35 \leq \phi \leq 5.5$ degrees. Therefore, the number of the top quark is less than that of the antitop. In the remaining region, the opposite occurs.

3.3. Constraining the NC Parameter $\Lambda$. The CDF and D0 $\sigma_{\bar{t}t}$ measurements are combined using the BLUE method [22] yielding to a Tevatron average of $\sigma_{\bar{t}t} = 7.6$ pb with a statistical and systematic uncertainty of about 0.41 pb assuming $m_t = 172.5$ GeV. The Tevatron combination has a $\chi^2$ of 0.01 for one degree of freedom corresponding to a probability of 92%. The maximum value of the cross-section is about 8.01 pb. Now using our predicted numerical expression

$$\sigma_{q\bar{q}\rightarrow\bar{t}t} = 1.715 \times 10^{-8} \text{ GeV}^{-2} + 95.37 \eta^2,$$

one can derive a lower limit of the noncommutativity parameter. By taking into account the fact that the quark-antiquark subprocess contribution is about 85% of that of the total top-antitop pair production, [23], one gets a new NC lower bound of approximatively $\Lambda \geq 0.7$ TeV. It is worth mentioning that

1. there is no physical principle or mathematical formalism which can constrain the order of magnitude of the noncommutativity parameter nor the scale for which the NC model in consideration is relevant

2. The related energy scale could be as low as a few TeV, the same order of magnitude of energies employed in collider experiments (LHC, ILC, etc.), and it is a process dependent [24–27] or Planck scale, as it is the case of string or quantum gravity [27, 28]. A softer lower-bound on the NC parameter $\Lambda$ within the NCSM formalism which is $\Lambda \geq 0.2731$ TeV was obtained by imposing the convergence of the perturbative expansion of the differential cross-section with respect to the noncommutativity parameter $\eta^2$. Notice that it is a lower (not an upper) limit. Thus, it is not in contradiction with the value used in the literature (see some added references in the manuscript) which is approximately of the order of TeV energy obtained by many authors using the various collider experimental data such as LEP, LHC, and Tevatron. For the present case of the differential angular distribution and the forward-backward asymmetry, we have taken $\Lambda \in [0.7, 1]$ TeV (see Figures 2 and 3).

Figure 4 display the cross-section $\sigma$ vs. $\Lambda$ for SM and NCSM.

4. Conclusion

In this work, we have investigated the effect of the Seiberg-Witten space-time noncommutativity on the top-antitop pair production at the Tevatron. In fact, we have considered the NC forward-backward asymmetry $A_{FB}$ and obtain an
analytical expression. We have shown that this observable defined from the angular distribution is a pure NC effect at the tree level and can be considered as a signal and signature of the NCG. Thus, such a nontrivial azimuthal angle distribution dependence of $A_{FB}$ is a unique feature of NCG. Moreover, comparing our obtained total cross-section including the NC effects with the combined CDF and D0 $\sigma_t/C_2^2$ measurements, a new lower bound for the NC parameter is derived. In conclusion, the top pair production is an excellent process to study NCSM.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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