Conceptualizing mathematical knowledge for teaching of Indonesian teacher in teaching number sense to early childhood

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Abstract. This paper discusses about Mathematical Knowledge for Teaching (MKT) of Indonesian teacher’s in teaching number sense to early children, reviewed from Subject Matter Knowledge and Pedagogical Content Knowledge. The subjects of this research were three kindergarten teachers in Depok, West Java, Indonesia. The research method was qualitative method while the data was collected by observation, study of documentation, and interview. The result showed that the respondents had not understood well the number sense, teaching strategy and the early childhood level of achievement. However, in regard to the pedagogical ability, the respondents had conducted the teaching process well. The respondents were able to arrange a proper plan and strategy for the teaching process; able to know the characteristic of the children; and able to utilize the teaching evaluation which was beneficial for the stakeholder and themselves.

1. Introduction

Mathematical Knowledge for Teaching (MKT) is a skill that should be owned by teachers in order to improve their teaching quality in Math. [1] states that MKT is the mathematical knowledge used to carry out the work of teaching mathematics. The concept of MKT emerged from studying records of mathematics teaching practice, and identifying the recurrent tasks; teachers do those require mathematical knowledge, reasoning, and insight [1],[2].

Mathematical knowledge for teaching (MKT) consists of Pedagogical Content Knowledge (PCK) and Subject Matter Knowledge (SMK) [3] [4] [5]. The relationship between PCK and SMK in the construct of MKT can be seen in Figure. 1 [6]. A framework of mathematical knowledge for teaching according to this knowledge comprises four domains: common content knowledge (school curriculum mathematical knowledge), specialized mathematical knowledge (mathematical knowledge that teachers use in practice but goes beyond the school curriculum), knowledge of students and content (that lies at the intersection of knowledge about students and about mathematics, for example, knowledge about the most common misconceptions related to a given concept) and knowledge of teaching and content (lies at the intersection of knowledge about teaching and about mathematics, for example, developing an adequate sequence for introducing a given concept) [3] [7][8].
Figure 1. Domains of mathematical knowledge for teaching.

There are many scientists and researchers who reveal the importance of MKT for a teacher. Hill [9] says that a Math teacher should improve their MKT. Mathematical knowledge for teaching (MKT) does not only need knowledge of the content but also the pedagogical knowledge [3], [10]. The work that teachers are expected to do: use new curriculum materials, open their classrooms to wider mathematical participation or students, help students to succeed on more challenging assessments, demand substantial mathematical skills [9]. On the other side, there are also many researchers who explain that mathematical teaching for early childhood has a big advantage for their logical intellectual development. The research has demonstrated that early mathematics skills predict both mathematics skills and literacy skills in the future [11], [12].

Based on the interview result, teachers have difficulties in teaching number sense to early childhood. Number sense is one of learning materials for early childhood. Number sense is the knowledge that can be demonstrated by identifying written numbers, performing counting activities, organizing numbers in sequence, and making decisions about magnitudes (i.e., comparisons between quantities). It gradually develops skills as the result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms [13], [14]. Number sense plays an important role in understanding the further concept of Math. Number sense is one of the most fundamentally important concepts to be developed in early mathematics [13]–[17]. Number sense skills developed in pre-K and kindergarten are not only foundational but also correlated with first grades mathematics achievement [16], [18].

2. Method
The subjects of this research were three kindergarten teachers in Depok, West Java, Indonesia. Those three subjects were teachers of kindergarten whose students are in the age of 5-6 years old. The age range was chosen because children in 5-6 years old are expected to be able to name numbers when being shown the numbers’ symbol. The following is the table of the identity of those teachers.

Table 1. Identity of respondents.

| No | Name       | Age          | Teaching Experience                                      | Latest education background      |
|----|------------|--------------|---------------------------------------------------------|----------------------------------|
| 1  | Yeti       | 49 years old | 15 years teaching in Al-Quran education school; 6 years teaching in kindergarten. | S1 Religion Education           |
| 2  | Santi      | 40 years old | 16 years teaching in kindergarten.                       | S1 Early Education Childhood    |
| 3  | Rina       | 34 years old | 8 years teaching in kindergarten.                       | S1 Early Education Childhood    |
This research examined the phenomenon of teacher’s MKT about number sense which was analyzed qualitatively. Data collection was done by observation in class. In this case, the researcher conducted the meeting of observations twice for analyzing how was the learning process of number sense, especially in learning number addition and subtraction. In addition, the researcher conducted an interview to the respondents and study of documentation toward the needed documents for this research. Research instrument is based on the table of trajectory interpretation learning for MKT categories [5] as follows Table 2. MKT categories.

| MKT Domain | MKT category | MKT definition |
|------------|--------------|----------------|
| SMK | Common content Knowledge | Knowledge of mathematics is not particular to teaching; knowledge of the mathematics is being taught, including mathematical terms and notation |
| | Specialized content Knowledge | Knowledge of mathematics is particular to teaching; knowledge of the mathematics behind student errors and nonstandard approaches; the ability to unpack mathematical ideas in order to make particular points salient |
| PCK | Knowledge of content and students | Knowledge of students in relation to mathematics; knowledge of what students typically find challenging or motivating; knowledge of common conceptions and misconceptions |
| | Knowledge of content and teaching | Knowledge of the design of instruction including sequencing content, useful representations and models, and instructional tasks |

In order to dig more about pedagogical content knowledge, instrument is adjusted to the pedagogic competence on Regulation of The Minister of Education and Culture of the Republic of Indonesia No. 137 of 2014.

3. Result and discussion

3.1. Subject matter knowledge

Number sense of early childhood was not a difficult material, but a teacher needed to understand the concept correctly to avoid any mistake. In order to identify whether the teacher mastered the number sense or not, the respondents were given several questions related to number sense. Yeti did not really aware what number sense was since her background Academy was not Education for Early Childhood. Yeti said “Number is like numeral. An early childhood use to say numbers in the form of the object, so it cannot be abstract but through some media such as pictures, toys, or any others”. Santi and her companion teacher were smiling and added, “Regarding number sense for the kindergarten student, basically, they need to know and recognize the numbers. So, 5 years-old-child is expected to able to write numbers. Yet, we do it step by step by playing or using media.” Meanwhile, Rina said that she did not really understand about number sense.

Next, the respondents were asked to explain their level of learning achievement for early childhood based on the applicable law. Every teacher must know the level of learning achievement for children in 0-6 years old in the scope of logical and symbolic development. It was aimed to help the teacher in making teaching plan with the right goals. Those three respondents had the same answer when they were asked about children’s level of learning achievement. They thought that children in Playgroup (3 years old) were able to know digit 1 to 5, children in the 1st class of kindergarten (4-5 years old) were able to know 1 to 10, and children 2nd class of kindergarten was able to know 1 to 20. Santi explained that 5 years-old-child was expected to be able to write numeral 1 to 5 and to understand how to count object in 1 to 20. In details, Rina explained that in teaching 5 years-old-child, a teacher used the object to
introduce numbers, for example; pencil for 1, duck for 2, chair for 4 and so on. Yeti stated that even there were 5 years-old children who were already able to count up to 100. Yet, they were still confused by its meaning.

As early childhood teacher, they surely have to master the concepts of material that will be taught to children. When the teacher explains the concept of numbers, a teacher must master the whole concept of numbers. If one is going to teach mathematics, then, first of all, one must know mathematics’ [8]. Teachers’ Subject Matter Knowledge (SMK) effects to children’ achievement. [19] states that there is much less correlation between teacher subject knowledge and student achievement than one might reasonably expect. The expert teachers can make use their subject knowledge to organize and use content knowledge more effective to make their students understand.

3.2. Pedagogical content knowledge
In regard to the learning plan, all three respondents had worked very well. Each teacher had arranged the annual, semester, weekly, and daily program form according to the applicable curriculum. The format and contents of the design were clear and detailed. Santi stated that the draft was arranged and discussed by colleagues in Teachers Working Group (KKG). The draft was guided by the government-developed 2013 curriculum adapted to the characteristics of the school. In contrast to Santi, Yeti says “I make the whole learning plan, but still, it is based on the applicable curriculum. Sometimes it is changed every year, because I always make it based on the evaluation of the previous year's learning.” Meanwhile, Rita said that she did not make any change on her plan from the previous year.

All respondents had been ready with the learning plan as teaching guidance. The research showed that teachers with more teaching preparation were more confident and successful in teaching their students than those who had less training [20], [21], [22], [23]. Guidelines should be based on available research and expert practice, elaborate and focused on the big ideas of mathematics, and represented a range of expectations for child outcomes that were developmentally appropriate [24]–[26].

In the learning process, Yeti had already aware of her students’ characteristic so that she could easily recognize children who was fast learner and who was not. Besides, she always inspected whether the children had already understood or not. Based on the interview, Yeti had strategy to know whether the children had already understood or not by sorting the number backward or eliminating some digit in the number sequence. “Usually, the children only memorize it, so I tried them by sorting the number backward or eliminating some digit in the number sequence. If they are able to answer it correctly, it means that they have already understood about the numbers,” she said. However, the children who can memorize or count the number in sequence cannot be ascertained that they really understood the number concept.

Related to strategy for introducing numbers sense to children, the three respondents applied the concept of study by playing. Since the school was near a garden, Yeti invited the children to the garden in order to introduce kinds to the plants. She asked the children to look for some leaves and summed up all leaves gathered. They looked very enthusiastic. In the end of the class, the teacher gave ‘star’ as a reward for student who had done the task.

For the learning media, three respondents chose media around the children, such as clamp clothes, rocks, grains, buttons, bottle caps, beams, lego, sticks, candles, paper and other game tools. However, among the other media, children loved color paper most. Children tore the color paper to the desired amount. Rina also added that “Song can also be a medium to introduce various themes about numbers; like ‘Balonku Ada Lima’ song. Yet, we could use creativity to create our own simple songs while demonstrating our finger as a counting tool, so that children slowly caught the abstract concept in numbers”.

In order to give motivation to students, Yeti had special strategy, i.e. knowing the tools the children like. For instance, if a boy liked a ball, then he was asked to take the ball and count it. Without realizing it, they had learned while playing and teachers did not force them to study. This was similar to Rina; she knew a child who did not like to study if there are many people around. When the child took a rest and play by himself, Rina approached the child and invited him to play while studying. In playing a
game, teacher persuaded children to play together with others in class. It helped children to slowly understand the material and to gather with others at the same time.

In the learning strategy, all respondents applied the concept of learning by playing. The children will have teaching experience when they do it with pleasure and enjoy the lesson. Children’ learning experience is an effective thing in learning Mathematics ([27]–[29][30]. Such experience might impress teachers with pedagogical images and habits that did not contribute to their effectiveness with young students [28].

Unfortunately, the three respondents did not use technology in the learning process. Respondents stated that there was no time to search for learning resource from internet. In fact, nowadays technology was very important to improve the teachers’ quality. However, they used additional props like VCD and computer to children. There was not much information about technology from the three teachers.

The three respondents always evaluated their students. The evaluation was done every day and the reporting was done after the learning process. The evaluation was very useful for teachers. Through the evaluation, teachers know the childrens’ development and know the right strategy for the next meeting. A report was created for every evaluation of early children’ process and shared the result to the stakeholders including children’ parents and headmaster.

4. Conclusion
Mathematical Knowledge for Teaching is an ability that must be owned by teachers, especially in teaching Mathematics. Related to this research, respondents have to improve their ability especially related to conceptual understanding. The subject matter knowledge of teachers needs to be improved so that there will be no concept mistake when teaching. Teachers should continue to improve their knowledge by reading and studying appropriate references. For aspects of Pedagogical Content Knowledge, respondents had shown good results. Teachers were able to make designs, implemented appropriate strategies and evaluated them well. However, the researcher suggests making more focused and programmed teaching strategy, so that children can understand the number sense easily.

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