New $U(1)'$ model with natural quark mass structure

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Abstract

We propose a new non-universal $U(1)'$ extension of the standard model with the addition of three exotic quark singlets, two scalar singlets and one additional scalar doublet. By introducing discrete symmetries and mixing couplings between ordinary and exotic fermions, we obtain predictable mass relations in the quark sector compatible with the phenomenological values without large fine tuning of the Yukawa couplings and with few free parameters. We obtain nontrivial constraints between Yukawa constants and mass parameters. For example, the model exhibit a "natural" scenery (in the sense of symmetry) where a large ratio between the top and charm quarks can be obtained by providing Yukawa couplings with a nearly symmetric structure, consistent with a flavor symmetry of the Yukawa couplings.

1 Introduction

The Standard Model (SM) is a theory based on the gauge group $G_{sm} = SU(3)_c \times SU(2)_L \times U(1)_Y$ that successfully explain most of the particle physics observations [1, 2, 3]. However, there is a wide interest to study alternative extensions consistent with the current experimental energies, and that provide more fundamental answers to some theoretical questions that the SM does not explain in a satisfactory form. For example, the SM flavor puzzle is one of the longstanding problem, where regardless that all the fermions acquire masses at the same scale $\nu = 246$ GeV, experimentally they exhibit very different mass values, as shown in Fig. 1 for the quark sector.

In the SM, the fermion masses are provided by Yukawa interactions consistent with the gauge symmetry and with the inclusion of only one scalar doublet. Taking into account that $q^i_L$ represents the usual left-handed quark doublet, $U^i_R$ and $D^i_R$ the right-handed up- and down-type singlets, and $i = 1, 2, 3$ the three phenomenological families, the Yukawa Lagrangian for the quark sector reads:

\[ L_{Yukawa} = \sum_{i=1}^{3} \left( y_{tq}^i \bar{q}^i_L \gamma^5 t R + y_{cq}^i \bar{c}^i_L \gamma^5 q^i R + y_{tq}^i \bar{q}^i_L t_R \right) + \text{h.c.} \]
\[- \mathcal{L}_Q = q_L^i (\phi h^U)_{ij} U_R^j + q_L^i (\bar{\phi} h^D)_{ij} D_R^j + h.c., \] (1)

where the scalar doublet \( \phi \) acquire a vacuum expectation value (VEV) \( \nu = 246 \text{ GeV} \). After the symmetry breaking, the above Lagrangian leads to the mass matrices. Since the original symmetry \( G_{\text{sm}} \) does not impose any restriction in the family indices, there arise off-diagonal (mixing) terms in the Yukawa couplings \( h_Q^i \), leading to mass matrices of the form:

\[
M_{U,D} = \frac{\nu}{\sqrt{2}} h_{U,D} = \begin{pmatrix}
* & * & * \\
* & * & * \\
* & * & *
\end{pmatrix}, \] (2)

where the symbol * indicates components proportional to the same scale \( \nu \) and the Yukawa components \( h_{ij}^Q \). However, the phenomenological masses range from MeV (2.5 MeV) to GeV scales (173 GeV), where the heaviest quark (top quark) is about 5 order of magnitude larger than the lightest one (up quark). This require ”unnatural” fine tuning of the Yukawa parameters without any apparent fundamental reason. On the other hand, many physicists believe that there must exist some sort of naturalness mechanisms that explain why some parameters have smaller values than others. For example, it is possible that small numbers are low energy manifestations of broken symmetries which become exact above some energy scale, where the small parameters are exactly cancelled. This idea has been implemented into the Yukawa structure using non-universal \( U(1)' \) symmetry (the Froggatt-Nielsen mechanism) \cite{4}, non-Abelian symmetries \cite{5}, discrete symmetries \cite{6}, among other models \cite{7}. In this work we propose a new model where the combination of an extra \( U(1)' \) symmetry and a \( Z_2 \times U(1)_{T_3} \) global symmetry generate mass matrices of the form:

\[
M_U = \frac{\nu_1}{\sqrt{2}} h^U = \begin{pmatrix}
0 & 0 & 0 \\
* & * & * \\
* & * & *
\end{pmatrix}, \quad M_D = \frac{\nu_2}{\sqrt{2}} h^D = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
* & * & *
\end{pmatrix}. \] (3)

where \( \nu = \sqrt{\nu_1^2 + \nu_2^2} \) is the SM breaking scale. The matrix \( M_U \) exhibits vanishing determinant with one massless particle (i.e, the symmetries suppress the up quark mass) and two massive quarks at the electroweak scale GeV (for the charm and top quarks), while \( M_D \) exhibits two massless particles (the down and strange quarks) and one massive quark also at the GeV scale (the bottom quark). In addition, the extra gauge symmetry implies a new neutral gauge boson (\( Z' \)). However, experimentally we do not see any massless quark and no such second weak neutral current has been detected, which suggests that i.) the extra \( U(1)' \) gauge symmetry must be spontaneously broken and ii.) the new breaking scale must be at a large scale \( \nu' \gg \nu \) in order to obtain a heavy \( Z' \) boson above current experimental limits. In addition, an extra symmetry \( U(1)' \) may induce new triangle anomalies \cite{8} which must be cancelled by introducing an extended quark spectrum. With these new elements, our model includes additional terms in the Yukawa Lagrangian which lead to extended mass matrices compatible with the symmetries of the form:
where the symbols $\ast$ and $\bullet$ indicate terms at the electroweak scale $\nu$, and $\times$ terms at the large scale $\nu'$. The above extended matrices have non-vanishing determinant, providing masses to all quarks. In particular, after diagonalization, the quarks up (u), down (d) and strange (s) acquire masses at the scale $(\ast \bullet / \times) \sim \nu^2 / \nu' \sim \text{MeV}$ as a result of the new mixing terms $\bullet$ and the large scale $\times$, which is consistent with Fig. [1]

2 The Model

The proposed model belongs to the class of models with one extra non-universal family $U(1)'$ symmetry which we label as $U(1)_X$. The particle content is composed by the ordinary SM particles and new exotic non-SM particles, as shown in Tabs. [1] and [2] respectively, where column $G_{sm}$ indicates the transformation rules under the SM gauge group ($SU(3)_c, SU(2)_L, U(1)_Y$), the column $U(1)_X$ are the values of the new quantum number $X$, and in the column labeled as Feature we describe the type of field. This spectrum exhibits the following properties:

- The $U(1)_X$ symmetry is non-universal in the left-handed SM quark sector: the quark family $i = 1$ have $X_1 = 1/3$ while families $i = 2, 3$ have $X_{2,3} = 0$. However, the corresponding right-handed singlets are universal.

- The SM leptons are family universal but with nontrivial charges $X$.

- The scalar doublet $\phi_1$ also has a nontrivial charge $X$.

- The three extra singlets $T$ and $J^n$ are new up- and down-like quarks, respectively, where $n = 1, 2$. They are quasi-chiral, i.e. chiral under $U(1)_X$ and vector-like under $G_{sm}$.

- We include new neutrinos $(\nu_R^n)^c$ and $N_R^i$ which may generate see-saw neutrino masses in order to obtain a realistic model compatible with oscillation data.
- The spectrum includes an additional scalar doublet $\phi_2$ identical to $\phi_1$ under $G_{sm}$ but with different $U(1)_X$ charges, where the electroweak scale is related to the VEVs by $\nu = \sqrt{\nu_1^2 + \nu_2^2}$. There arises a $\beta$-like parameter defined by
\[
\tan \beta = \frac{\nu_1}{\nu_2}.
\] (5)

- An extra scalar singlet $\chi_0$ with VEV $\nu_\chi$ is required to produce the symmetry breaking of the $U(1)_X$ symmetry. We assume that it happens at a large scale $\nu_\chi \gg \nu$.

- Another scalar singlet $\sigma_0$ is introduced. Since it is not essential for the symmetry breaking mechanisms, we may choose a small VEV $\langle \sigma_0 \rangle = \nu_\sigma \lesssim \nu$.

- A fundamental condition of the model is the cancelation of chiral anomalies. Since the new symmetry introduces an additional gauge boson, there arise new couplings that induce the following nontrivial triangle anomalies:

\[
\begin{align*}
[SU(3)_c]^2 & U(1)_X \rightarrow A_1 = \sum_Q X_{Q_L} - \sum_Q X_{Q_R} \\
[SU(2)_L]^2 & U(1)_X \rightarrow A_2 = \sum_\ell X_{\ell_L} + 3 \sum_Q X_{Q_L}, \\
[U(1)_Y]^2 & U(1)_X \rightarrow A_3 = \sum_{\ell, Q} [Y_\ell^2 X_{\ell_L} + 3Y_Q^2 X_{Q_L}] - \sum_{\ell, Q} [Y_{\ell_R}^2 X_{\ell_R} + 3Y_Q^2 X_{Q_R}] \\
U(1)_Y \ [U(1)_X]^2 & \rightarrow A_4 = \sum_{\ell, Q} [Y_{\ell_L} X_{\ell_L}^2 + 3Y_Q X_{Q_L}^2] - \sum_{\ell, Q} [Y_{\ell_R} X_{\ell_R}^2 + 3Y_Q X_{Q_R}^2] \\
[U(1)_X]^3 & \rightarrow A_5 = \sum_{\ell, Q} [X_{\ell_L}^3 + 3X_{Q_L}^3] - \sum_{\ell, Q} [X_{\ell_R}^3 + 3X_{Q_R}^3] \\
[Grav]^2 & \otimes U(1)_X \rightarrow A_6 = \sum_{\ell, Q} [X_{\ell_L} + 3X_{Q_L}] - \sum_{\ell, Q} [X_{\ell_R} + 3X_{Q_R}] 
\end{align*}
\] (6)

where the sums in $Q$ run over all the quarks ($u^i, d^i, T, J^n$), while $\ell$ runs over all leptons with nontrivial $U(1)_X$ values (i.e. $e^i, \nu_{L\ell}^i, (\nu_{R\ell}^i)^c$). The parameter $Y$ are the corresponding weak hypercharges. It is a matter of arithmetic to show that the $U(1)_X$ values given in Tabs. [1] and [2] are possible solutions that cancel the above anomaly equations.

3 Yukawa Lagrangian

With the above particle spectrum, we find the Yukawa Lagrangians compatible with the $G_{sm} \times U(1)_X$ symmetry. For the quark sector we find:
\[ -\mathcal{L}_Q = \overline{q}_L^i \left( \bar{\phi}_2 h_2^U \right)_{ij} U^i_R + \overline{q}_L^i \left( \bar{\phi}_1 h_1^U \right)_{ij} U^i_R + \overline{q}_L^i \left( \phi_1 h_1^D \right)_{ij} D^i_R + \overline{q}_L^i \left( \phi_2 h_2^D \right)_{aj} D^j_R \]

\[ + \quad \overline{q}_L^i \left( \bar{\phi}_1 h_1^I \right)_{im} J^m_R + \overline{q}_L^i \left( \phi_2 h_2^I \right)_{am} J^m_R + \overline{q}_L^i \left( \bar{\phi}_2 h_2^T \right)_{1} T_R + \overline{q}_L^i \left( \phi_1 h_1^T \right)_{a} T_R \]

\[ + \quad \overline{T}_L \left( \sigma^a h^U_\sigma + \chi^a h^U_\chi \right) U^i_R + \overline{T}_L \left( \sigma^a h^T_\sigma + \chi^a h^T_\chi \right) T_R \]

\[ + \quad \overline{J}_L^i \left( \sigma^a h^D_\sigma + \chi^a h^D_\chi \right)_{nj} D^j_R + \overline{J}_L^i \left( \sigma^a h^J_\sigma + \chi^a h^J_\chi \right)_{nm} J^m_R + h.c., \quad (7) \]

where \( \bar{\phi}_{1,2} = \imath \sigma_2 \phi^I_{1,2} \) are conjugate fields, \( a = 2, 3 \) is the index that labels the second and third quark doublets and \( n(m) = 1, 2 \) is the index of the exotic \( J^{n(m)} \) quarks. For the leptonic sector we obtain:

\[ -\mathcal{L}_\ell = \overline{\ell}_L^i \left( \bar{\phi}_1 h_1^\nu \right)_{ij} \nu_j^i + \overline{\ell}_L^i \left( \bar{\phi}_2 h_2^N \right)_{ij} N_j^i \]

\[ + \quad (\nu_R^i)^T \left( \sigma^a h^N_\sigma + \chi^a h^N_\chi \right)_{ij} N_j^i + \frac{1}{2} MN (N_i^R)^{\epsilon} N_j^\epsilon \]

\[ + \quad \overline{\ell}_L^i \left( \phi_1 h_1^e \right)_{ij} e_j^i + h.c. \quad (8) \]

In particular, we can see in the quark Lagrangian in Eq. (7) that due to the non-universality of the \( U(1)_X \) symmetry, not all couplings between quarks and scalars are allowed by the gauge symmetry, which lead us to specific zero-texture Yukawa matrices. To see this, we write (7) in a shorter form as:

\[ -\mathcal{L}_Q = \overline{q}_L^i \left( \bar{\phi}_1 h_1^U + \bar{\phi}_2 h_2^U \right)_{ij} U^i_R + \overline{q}_L^i \left( \phi_1 h_1^D + \phi_2 h_2^D \right)_{ij} D^i_R \]

\[ + \quad \overline{q}_L^i \left( \phi_1 h_1^I + \phi_2 h_2^I \right)_{im} J^m_R + \overline{q}_L^i \left( \phi_1 h_1^T + \phi_2 h_2^T \right)_{1} T_R \]

\[ + \quad \overline{T}_L \left( \sigma^a h^U_\sigma + \chi^a h^U_\chi \right) U^i_R + \overline{T}_L \left( \sigma^a h^T_\sigma + \chi^a h^T_\chi \right) T_R \]

\[ + \quad \overline{J}_L^i \left( \sigma^a h^D_\sigma + \chi^a h^D_\chi \right)_{nj} D^j_R + \overline{J}_L^i \left( \sigma^a h^J_\sigma + \chi^a h^J_\chi \right)_{nm} J^m_R + h.c., \quad (9) \]

where the Yukawa matrices exhibit the following zero-textures:

\[ h_1^U = \begin{pmatrix} 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad h_2^U = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \]

\[ h_1^D = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad h_2^D = \begin{pmatrix} 0 & 0 & 0 \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{pmatrix}, \]
\[ h_{\sigma,\chi}^D = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \end{pmatrix}, \quad h_{\sigma,\chi}^U = (c_1, c_2, c_3), \]

\[ h_1^J = \begin{pmatrix} j_{11} & j_{12} \\ 0 & 0 \end{pmatrix}, \quad h_2^J = \begin{pmatrix} 0 & 0 \\ i_{21} & i_{22} \end{pmatrix}, \quad h_{\sigma,\chi}^J = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}, \]

\[ h_1^T = \begin{pmatrix} 0 \\ w_2 \\ w_3 \end{pmatrix}, \quad h_2^T = \begin{pmatrix} y_1 \\ 0 \\ 0 \end{pmatrix}, \quad h_{\sigma,\chi}^T = h_T. \] (10)

After the symmetry breaking, we obtain the following mass terms:

\[
- \langle L_Q \rangle = \overline{U_L} (\nu_1 h_1^U + \nu_2 h_2^U) U_R i_j D_R^j + \overline{D_L} (\nu_1 h_1^D + \nu_2 h_2^D) i_m J_R^m \\
+ \overline{U_L} (\nu_1 h_1^U + \nu_2 h_2^U) J_R^j U_R i_j + \overline{D_L} (\nu_1 h_1^D + \nu_2 h_2^D) J_L^m U_R i_m \\
+ \overline{T_L} (\nu_1 h_1^U + \nu_2 h_2^U) J_R^j U_R i_j + \overline{T_L} (\nu_1 h_1^D + \nu_2 h_2^D) J_L^m U_R i_m + h.c. \] (11)

On the other hand, since (9) exhibits terms where \( \phi_1 \) and \( \phi_2 \) or \( \sigma_0 \) and \( \chi_0 \) couple simultaneously, the zero structures of the Yukawa matrices in (10) does not imply zero-texture mass matrices. Thus, the extra \( U(1)_X \) symmetry is not sufficient to explain the mass spectrum. In this work, we assume the existence of two types of global symmetries. They are:

- **\( Z_2 \) symmetries**: We restrict the couplings of scalar fields by requiring the discrete symmetries

  \[ \phi_2 \rightarrow -\phi_2, \quad \sigma_0 \rightarrow -\sigma_0, \quad D_R^j \rightarrow -D_R^j, \quad T_{L,R} \rightarrow -T_{L,R}. \] (12)

- **\( U(1)_{T_3} \) symmetry**: The \( U(1)_X \) symmetry distinguishes the quark family \( q_1^L \) from the others two \( q_{2,3}^L \), while the right-handed components are universal. Thus, in the absence of the Yukawa couplings, the model has the following global symmetry:

  \[ G_{global}(h^Q = 0) = SU(2)_{q^a} \times SU(3)_U \times SU(3)_D. \] (13)

In particular, the \( SU(2)_{q^a} \) symmetry in the left-handed sector remains in the model even after the gauge symmetry breaking. However, the experimental observation shows that this symmetry does not remain if the quark masses are taken into account. Let us assume that the Yukawa interactions break the \( SU(2)_{q^a} \) global symmetry, but an \( U(1)_{T_3} \) symmetry remains only in the left-handed down sector, under which

\[ D_R^2 \rightarrow -D_R^2, \quad D_L^3 \rightarrow D_L^3. \] (14)
Thus, by requiring the symmetries (12) and (14), the mass Lagrangian (11) becomes:

\[-\langle \mathcal{L}_Q \rangle = \begin{align*}
&\overline{U}_L (\nu_1 h_1^U)_{ij} U^T_R i_j + \begin{bmatrix}
D^U_L (\nu_2 h_2^U)_{ij} + D^D_L (\nu_1 h_1^D)_{ij} + D^D_L (\nu_2 h_2^D)_{ij}
\end{bmatrix} D^R_i R

&+ \overline{U}_L (\nu_2 h_2^U)_{ij} \nu^T_R + \begin{bmatrix}
D^U_L (\nu_1 h_1^D)_{1m} + D^D_L (\nu_2 h_2^D)_{1m} + D^D_L (\nu_1 h_1^D)_{1m}
\end{bmatrix} J^R m

&+ T_L (\nu_3 h_3^U)_{ij} U^T_R + \begin{bmatrix}
J^U_L (\nu_3 h_3^D)_{nj} D^R_i R + J^F_L (\nu_3 h_3^D)_{nm} J^R_n R + h.c.
\end{bmatrix}

&-\langle \mathcal{L}_Q \rangle = \begin{align*}
&\overline{U}_L (M_U)_{ij} U^T_R i_j + \begin{bmatrix}
D^U_L (M_D)_{ij} D^D_i R
\end{bmatrix} + \overline{U}_L (k) \nu^T_R + \begin{bmatrix}
D^U_L (s)_{nm} J^R m
\end{bmatrix}

&+ T_L (K)_{ij} U^T_R + \begin{bmatrix}
J^U_L (M_D)_{nj} D^R_i R + J^F_L (M_D)_{nm} J^R_n R + h.c.
\end{bmatrix}
\end{align*}
\]

where the mass matrices are:

\[M_U = \frac{\nu_1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad M_D = \frac{\nu_2}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ B_{31} & B_{32} & B_{33} \end{pmatrix},\]

\[M_J = \frac{\nu_3}{\sqrt{2}} \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}, \quad M_T = \frac{\nu_4}{\sqrt{2}} h^T \chi,\]

\[S = \frac{\nu_5}{\sqrt{2}} \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \end{pmatrix}, \quad K = \frac{\nu_6}{\sqrt{2}} \begin{pmatrix} c_1 & c_2 & c_3 \end{pmatrix},\]

\[s = \frac{\nu_1}{\sqrt{2}} \begin{pmatrix} j_{11} & j_{12} \\ 0 & 0 \end{pmatrix} + \frac{\nu_2}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ i_{21} & i_{22} \end{pmatrix}, \quad k = \frac{\nu_2}{\sqrt{2}} \begin{pmatrix} y_1 \\ 0 \end{pmatrix}.\]

We see that in absence of mixing between the ordinary and the exotic sector, the matrices of the SM quarks $M_{U,D}$ have the same form as (3) written in the introduction, which exhibit three massless ($m_{a,d,s}^0 = 0$) and three massive ($m_{c,b,t}^0 \sim \nu_1 \sim \text{GeV}$) quarks, while the non-SM quarks acquire heavy mass values ($m_{f,t}^0 \sim \nu_\chi \gg \nu$). However, if we consider the contributions due to the mixing mass matrices $s, k, S$ and $K$, the extended mass matrices are

\[M'_U = \begin{pmatrix} M_U & s \\ k & M_T \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \nu_1 \nu_{12} & \nu_{13} & 0 \\ \nu_{11} & \nu_{12} & \nu_{13} & 0 \\ \nu_{21} & \nu_{22} & \nu_{23} & 0 \\ \nu_{31} & \nu_{32} & \nu_{33} & 0 \\ \nu_{41} & \nu_{42} & \nu_{43} & 0 \end{pmatrix},\]

\[M'_D = \begin{pmatrix} M_D & s \\ K & M_J \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \nu_1 \nu_{12} & \nu_{13} & 0 \\ \nu_{11} & \nu_{12} & \nu_{13} & 0 \\ \nu_{21} & \nu_{22} & \nu_{23} & 0 \\ \nu_{31} & \nu_{32} & \nu_{33} & 0 \\ \nu_{41} & \nu_{42} & \nu_{43} & 0 \end{pmatrix}.\]
which exhibit the same form as the matrices (4), except that the mixing components marked as • in $M_D'$ have two columns (rows), which is essential to obtain a hierarchy between the d- and s-quark masses, as we will see below. Thus, due to the mixing components, the mass matrices $M_{U,D}'$ exhibits three eigenvalues at the scale $m_{u,d,s} \sim \text{MeV}$, three at the scale $m_{c,b,t} \sim \text{GeV}$ and three at the scale $m_{T,J} \sim \text{TeV}$.

4 Mass schemes

To explore the consequences of the above mass scheme, we consider an specific structure of the matrices in (17) from the naturalness criterion that the Yukawa couplings have similar values to each other. To achieve this without spoil the mass structures, we assume the following scenery: $M_U'$ have identical Yukawa components except the top coupling (i.e. $a_{ij} = Y_U$ for $ij \neq 33$ and $a_{33} = Y_t$); $M_D'$ have identical components (i.e. $B_{31} = B_{32} = B_{33} = Y_D$); the $2 \times 2$ matrix $M_J$ is diagonal (i.e. $k_{12} = k_{21} = 0$); the mixing matrices $K$ and $k$ have the same zeros (i.e. $c_2 = c_3 = 0$); the sub matrix $s$ is diagonal with identical components (i.e. $j_{12} = i_{21} = 0$ and $\nu_{1j_{11}} = \nu_{2i_{22}} = h_D$), and $S$ exhibits the same zeros as $s$ with identical components (i.e. $C_{ij} = 0$ for $ij \neq 11, 22$ and $C_{11} = C_{22} = \Gamma_D$). Thus, the matrices in (17) become

$$M_U' = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & \nu_{2Y_1} \\ \nu_1 Y_U & \nu_1 Y_U & \nu_1 Y_U & 0 \\ \nu_1 Y_U & \nu_1 Y_U & \nu_1 Y_U & 0 \\ \nu_{\sigma} c_1 & 0 & 0 & \nu_{\chi} h^T_X \end{pmatrix},$$

$$M_D' = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & h_D & 0 \\ 0 & 0 & 0 & 0 & h_D \\ \nu_2 Y_D & \nu_2 Y_D & \nu_2 Y_D & 0 & 0 \\ \nu_{\sigma} \Gamma_D & 0 & 0 & \nu_{\chi} k_{11} & 0 \\ 0 & \nu_{\sigma} \Gamma_D & 0 & 0 & \nu_{\chi} k_{22} \end{pmatrix}. \quad (18)$$

The above matrices are diagonalized through bi-unitary transformation of the form $m_Q = (O^Q_L)^\dagger M_Q'O_R^Q$, with $m_Q$ a diagonal matrix with real and positive values. In order to guarantee real values, we calculate the squared matrices $M_Q'^2 = M_Q'O_Q'^\dagger$ which diagonalize as $m_Q'^2 = (O^Q_L)^\dagger M_Q'O_Q'^\dagger$. We find for the up sector the following approximate eigenvalues:
\[
\lambda_1^U = m_u \approx \left( \frac{y_1 c_1 \nu_2 \nu_2}{\sqrt{2 h_\chi^2 \nu_\chi}} \right) = y_1 c_1 \left( \frac{\nu_2 \nu_\sigma}{2 m_T} \right)
\]
\[
\lambda_2^U = m_c \approx \frac{\nu_1}{2 \sqrt{2}} (Y_U + Y_t) \left[ 1 - \sqrt{1 + 4 y_U t \epsilon_U t} \right]
\]
\[
\lambda_3^U = m_t \approx \frac{\nu_1}{2 \sqrt{2}} (Y_U + Y_t) \left[ 1 + \sqrt{1 + 4 y_U t \epsilon_U t} \right]
\]
\[
\lambda_4^U = m_T \approx \frac{1}{\sqrt{2}} h_\chi T \nu_\chi,
\]

where we consider that \(\nu_\chi \gg \nu_2, \nu_\sigma\) and define the parameters

\[
y_{Ut} = \frac{Y_U}{Y_U + Y_t}, \quad \epsilon_{Ut} = \frac{Y_U - Y_t}{Y_U + Y_t}.
\]

The parameter \(\epsilon_{Ut}\) "measures" the level of asymmetry of Yukawa interactions between the top quark and the lighter ones (charm and up). It is interesting to note that if \(\epsilon_{Ut} = 0\) (i.e. \(Y_U = Y_t\)), we obtain interactions with an exact flavor symmetry between flavors \(a = 2, 3\). As a consequence, the c-quark becomes massless. Furthermore, the ratio between the mass of the c- and t-quark is sensible to the asymmetry parameter according to:

\[
\frac{m_c}{m_t} \approx \frac{-y_{Ut} \epsilon_{Ut}}{1 + y_{Ut} \epsilon_{Ut}}.
\]

Thus, there is a scenery where the closer are the Yukawa values \(Y_U\) and \(Y_t\), the larger is the difference between \(m_c\) and \(m_t\).

For the down sector we find:

\[
\lambda_1^D = m_d \approx \left( \frac{\Gamma_D h_D \nu_\sigma}{\sqrt{2 k_{11} \nu_\chi}} \right) = j_{12} \Gamma_D \left( \frac{\nu_1 \nu_\sigma}{2 m_{J1}} \right)
\]
\[
\lambda_2^D = m_s \approx \left( \frac{\Gamma_D h_D \nu_\sigma}{\sqrt{2 k_{22} \nu_\chi}} \right) = j_{12} \Gamma_D \left( \frac{\nu_1 \nu_\sigma}{2 m_{J2}} \right)
\]
\[
\lambda_3^D = m_b \approx \frac{1}{\sqrt{2}} Y_D \nu_2
\]
\[
\lambda_4^D = m_{J1} \approx \frac{1}{\sqrt{2}} k_{11} \nu_\chi,
\]
\[
\lambda_5^D = m_{J2} \approx \frac{1}{\sqrt{2}} k_{22} \nu_\chi.
\]

In this case, the ratio between the masses of the down and strange quarks gives:

\[
\frac{m_d}{m_s} = \frac{m_{J2}}{m_{J1}}.
\]

Thus, the ratio between the lightest quarks is determined only by the mass splitting of the heavy quarks \(J^1\) and \(J^2\). Regarding \(m_u\) and \(m_b\), we find that:
\[ \frac{m_u}{m_b} = \left( \frac{y_1 c_1}{\sqrt{2} Y_D} \right) \frac{\nu_\sigma}{m_T}. \]  

(24)

Considering the central values, the experimental masses of the phenomenological quarks are [2]:

\[
\begin{align*}
    m_u & = 2.3 \text{ MeV}, \\
    m_d & = 4.8 \text{ MeV}, \\
    m_s & = 95 \text{ MeV}, \\
    m_c & = 1.275 \text{ GeV}, \\
    m_b & = 4.65 \text{ GeV}, \\
    m_t & = 173.5 \text{ GeV}
\end{align*}
\]  

(25)

Using the above values, the relations (21), (23) and (24) leads to:

\[
y_{Ut} \epsilon_{Ut} = \frac{-m_c/m_t}{1 + m_c/m_t} \approx -7.3 \times 10^{-3} \]  

(26)

\[
m_{J1} \approx 20 m_{J2} \]  

(27)

\[
\frac{\nu_\sigma}{m_T} \approx (5 \times 10^{-4}) \frac{\sqrt{2} Y_D}{y_1 c_1}. \]  

(28)

Fig. 2 shows the top quark coupling \( Y_t \) as function of the light-quark coupling \( Y_U \) according to (26), which exhibits two possible solutions which lead to two different mass schemes. First, the traditional scheme, where \( Y_t/Y_U \approx 135.05 \) is required to fit the experimental masses, and where small variations of \( Y_t \) imply large variations of \( Y_U \). This ratio implies an asymmetry factor \( \epsilon_{Ut} \approx -0.985 \). Second, we obtain a ”natural scheme” where \( Y_t/Y_U \approx 1.03 \), which is consistent with a symmetry where degenerated up-type Yukawa couplings is favored, with \( \epsilon_{Ut} \approx -0.015 \). Fig. 3 shows \( Y_D \) as function of the ratio \( \nu_\sigma/m_T \) according to (28), where we assume for simplicity that \( y_1 \sim c_1 \sim Y_D \). We see that large values of \( Y_D \) require small values of the VEV \( \nu_\sigma \) in relation to the mass of the \( T \)- quark. For example, if \( m_T \sim 1 \text{ TeV} \), to obtain \( Y_D \sim 0.01 \) and 0.1, we require that \( \nu_\sigma \sim 70 \) and \( 7 \text{ GeV} \), respectively. Finally, we find from (23) that the large splitting between \( m_d \) and \( m_s \) is consequence of the existence of non-degenerated heavy massive quarks according to (27). For example, if \( M_{J2} \sim 1 \text{ TeV} \), then \( M_{J1} \sim 20 \text{ TeV} \) to obtain the observable \( m_d/m_s \) ratio.

5 Conclusions

Extensions with abelian non-universal \( U(1)' \) symmetry are very well-motivated models which involves a wide number of theoretical aspects. In this work, by requiring non-universality in the left-handed quark sector, we proposed a new \( G_{sm} \times U(1)_X \) gauge model with Yukawa interactions invariant under a global \( Z_2 \times U(1)_{T_3} \) symmetry. With the addition of three exotic quark singlets, one scalar doublet and two scalar singlets, we obtained a free-anomaly theory which predict zero-texture mass matrices for the ordinary SM quarks with three massless quarks (u, d, s), and three massive quarks (c, b, t) at the electroweak scale (GeV). The mixing between SM quarks and the new exotic quarks consistent with the symmetries and the cancelation of the chiral anomalies, provided masses to the lightest quarks (u, d, s) at the
MeV scales. Predictable mass ratios were obtained with few free parameters, and hierarchical structures arose "naturally" without large tuning of the Yukawa couplings. These hierarchies can be understood as follows:

- \( m_{u,d,s}/m_{c,b,t} \): Due to the non-universal \( U(1)_X \) gauge symmetry and a global \( Z_2 \) symmetry, the Yukawa interactions among ordinary matter lead to zero-texture mass matrices of the up- and down-type quarks. However, since an additional \( U(1)_{T_3} \) global symmetry is required only for the down sector, the mass structure between up- and down-type quarks is not equivalent: while in the up sector the c- and t-quarks acquire masses at the scale \( \nu_1 \sim \text{GeV} \), in the down sector only the b-quark obtains mass at this scale through \( \nu_2 \). Thus, the three quarks (u, d, s) remain massless. On the other hand, to cancel the chiral anomalies, three exotic quarks \( T, J_1, J_2 \) (and one neutrino singlet \( \nu_R^c \)) are introduced. The interactions between the massless and the exotic quarks, provide masses to the former. These masses are inverse in the heavy quark masses, obtaining a see saw-type mechanisms: \( m_{u,d,s} \sim \nu \nu_{\sigma}/\nu_X \sim [\text{GeV}^2]/[\text{TeV}] \sim [\text{MeV}] \).

- \( m_d/m_s \): Since the d- and s-quarks acquire MeV scale masses through interactions with two non-degenerate heavy quarks \( J^{1,2} \), the ratio \( m_d/m_s \) is not one according to (23). If \( m_{J_1} > m_{J_2} \), then \( m_d/m_s < 1 \).

- \( m_c/m_t \): Although both c- and t-quarks acquire masses at the scale of GeV, they are non-degenerate due to the likeness between Yukawa constants in the natural scheme. Thus, a large ratio \( m_t/m_c \) arises naturally if the top coupling \( Y_t \) is slightly different to the other up-type quark coupling \( Y_U \). Indeed, if the flavor symmetry between families 2 and 3 is exact in the up sector (\( \epsilon_{t:1} = 0 \)), the charm quark becomes massless and only the top quark acquire mass.

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Table 1: Ordinary SM particle content, with $i = 1, 2, 3$

| Spectrum | $G_{sm}$ | $U(1)_X$ | Feature |
|----------|----------|----------|---------|
| $q_L^i = \left( \begin{array}{c} U^i \\ D^i \end{array} \right)_L$ | $(3, 2, 1/3)$ | 1/3 for $i = 1$ 0 for $i = 2, 3$ | chiral |
| $U_R^i$ | $(3^*, 1, 4/3)$ | 2/3 | chiral |
| $D_R^i$ | $(3^*, 1, -2/3)$ | -1/3 | chiral |
| $\ell_L^i = \left( \begin{array}{c} \nu^i \\ e^i \end{array} \right)_L$ | $(1, 2, -1)$ | -1/3 | chiral |
| $e_R^i$ | $(1, 1, -2)$ | -1 | chiral |
| $\phi_1 = \left( \frac{1}{\sqrt{2}}(\nu_1 + \xi_1 + i\phi^0_1) \right)$ | $(1, 2, 1)$ | 2/3 | Scalar Doublet |

Table 2: Exotic non-SM particle content, with $n = 2, 3$

| Spectrum | $G_{sm}$ | $U(1)_X$ | Feature |
|----------|----------|----------|---------|
| $T_L$ | $(3, 1, 4/3)$ | 1/3 | quasi-chiral |
| $T_R$ | $(3^*, 1, 4/3)$ | 2/3 | quasi-chiral |
| $J_L^n$ | $(3, 1, -2/3)$ | 0 | quasi-chiral |
| $J_R^n$ | $(3^*, 1, -2/3)$ | -1/3 | quasi-chiral |
| $(\nu_R^n)^c$ | $(1, 1, 0)$ | -1/3 | Majorana |
| $N_R^n$ | $(1, 1, 0)$ | 0 | Majorana |
| $\phi_2 = \left( \frac{1}{\sqrt{2}}(\nu_2 + \xi_2 + i\phi^0_2) \right)$ | $(1, 2, 1)$ | 1/3 | Scalar doublet |
| $\chi_0 = \frac{1}{\sqrt{2}}(\nu_\chi + \xi_\chi + i\zeta_\chi)$ | $(1, 1, 0)$ | 1/3 | Scalar singlet |
| $\sigma_0 = \frac{1}{\sqrt{2}}(\nu_\sigma + \xi_\sigma + i\zeta_\sigma)$ | $(1, 1, 0)$ | 1/3 | Scalar singlet |
Figure 1: Mass scales for the up-type (charge $2/3$) and down-type (charge $-1/3$) phenomenological quarks. The dashed line shows the electroweak breaking scale $\nu = 246$ GeV.

Figure 2: Yukawa coupling relation of top ($Y_t$) and light ($Y_U$) quarks compatible with the experimental ratio $m_c/m_t = 7.35 \times 10^{-3}$. Both lines show two schemes: the traditional scheme with $Y_t/Y_U \approx 135.05$ and the natural scheme with $Y_t/Y_U \approx 1.03$. 
Figure 3: Yukawa coupling of down-type quarks $Y_D$ as function of the ratio $r = \nu_\sigma / m_T$ compatible with the experimental ratio $m_u/m_b = 5 \times 10^{-4}$. 