Revisiting the genome-wide significance threshold for common variant GWAS

Zhongsheng Chen, Michael Boehnke, Xiaquan Wen, and Bhramar Mukherjee

Department of Biostatistics and Center for Statistical Genetics, University of Michigan School of Public Health, Ann Arbor, MI 48109-2029, USA

*Corresponding author: Department of Biostatistics and Center for Statistical Genetics, University of Michigan School of Public Health, 1415 Washington Heights, Ann Arbor, MI 48109-2029, USA. boehnke@umich.edu

Abstract

Over the last decade, GWAS meta-analyses have used a strict P-value threshold of $5 \times 10^{-8}$ to classify associations as significant. Here, we use our current understanding of frequently studied traits including lipid levels, height, and BMI to revisit this genome-wide significance threshold. We compare the performance of studies using the $P = 5 \times 10^{-8}$ threshold in terms of true and false positive rate to other multiple testing strategies: (1) less stringent P-value thresholds, (2) controlling the FDR with the Benjamini–Hochberg and Benjamini–Yekutieli procedure, and (3) controlling the Bayesian FDR with posterior probabilities. We applied these procedures to re-analyze results from the Global Lipids and GIANT GWAS meta-analysis consortia and supported them with extensive simulation that mimics the empirical data. We observe in simulated studies with sample sizes $\approx$20,000 and $>120,000$ that relaxing the $P$-value threshold to $5 \times 10^{-7}$ increased discovery at the cost of 18% and 8% of additional loci being false positive results, respectively. FDR and Bayesian FDR are well controlled for both sample sizes with a few exceptions that disappear under a less stringent definition of true positives and the two approaches yield similar results. Our work quantifies the value of using a relaxed $P$-value threshold in large studies to increase their true positive discovery but also show the excess false positive rates due to such actions in modest-sized studies. These results may guide investigators considering different replication strategies and downstream work such as gene-set enrichment or pathway analysis. Finally, we demonstrate the viability of FDR-controlling procedures in GWAS.

Keywords: multiple testing; FDR; family-wise error rate; Bonferroni correction; Benjamini–Hochberg; Bayesian false discovery probability

Introduction

There has been recent discussion in the statistical community on changing the standard $P$-value significance threshold for a single test from 0.05 to 0.005 (Benjamin et al. 2018; Amrhein et al. 2019; Wasserstein et al. 2019). Although the authors of the corresponding paper (Benjamin et al. 2018) commended human geneticists for using very stringent $P$-value thresholds to help ensure reproducibility, the cost of this strategy in current genetic studies is that many true genetic signals are not identified. The benefit is, of course, rigorous control of false positives.

To account for multiple testing in genome-wide association studies (GWAS), a fixed $P$-value threshold of $5 \times 10^{-8}$ is widely used to identify association between a common genetic variant and a trait of interest. Risch and Merikangas (1996) suggested this strict $P$-value threshold for studying the genetics of complex diseases due to the many false positive discoveries reported by candidate gene studies at that time. Later, the International HapMap Consortium (Altshuler and Donnelly 2005), Dudbridge and Gusnanto (2008), and Pe’er et al. (2008) independently suggested near-identical thresholds for common variant (minor allele frequency [MAF] >5%) GWAS. Each group of investigators sought to control the family-wise error rate (FWER) through Bonferroni correction for the effective number of independent tests given the linkage disequilibrium (LD) structure of the genome; they used different approaches to estimate the effective number of independent tests. Based on these studies and reinforced by widespread use, the $P = 5 \times 10^{-8}$ threshold soon became standard for common variant GWAS. Using this threshold has been remarkably successful in limiting false positive association findings, leading to robust and reproducible results in a field that prior to GWAS had reported many nonreplicable results.

Since the acceptance of the $P = 5 \times 10^{-8}$ threshold a decade ago, there have been substantial experimental and methodological advances that have allowed study of many more common variants in much larger samples. The construction of denser genotype arrays (Burdick et al. 2006), development of genotype imputation (Li et al. 2009, 2010), and increasing sizes of imputation reference panels (McCarthy et al. 2016) now allow assay of nearly all common human genetic variants. Development of tools for meta-analysis (Willer et al. 2010; Winkler et al. 2014) has facilitated the aggregation of results across GWAS and contributed to the increasing sample sizes of genetic studies. With this changing landscape, it is worthwhile to revisit (Panagiotou and Ioannidis 2012) the common variant genome-wide threshold of $P = 5 \times 10^{-8}$ considering the knowledge and data acquired in the last decade.

Received: June 18, 2020. Accepted: November 5, 2020
© The Author(s) 2021. Published by Oxford University Press on behalf of Genetics Society of America. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted reuse, distribution, and reproduction in any medium, provided the original work is properly cited.
Instead of controlling the FWER, an inherently conservative metric, an alternative approach to multiple testing corrections is to use adjusted $P$-values to control the false discovery rate (FDR) or to use posterior probabilities to control the Bayesian FDR (Efron et al. 2001). Although using the Benjamini–Hochberg (B–H) procedure (Benjamini and Hochberg 1995) is the standard practice in expression quantitative trait locus (eQTL) studies and several Bayesian counterparts has also been proposed (Tang et al. 2007; Bogdan et al. 2008; Wen 2017), FDR-controlling procedures have not been widely used in GWAS. In the case of B–H, this may be due to concerns about excess estimates of FDR under the LD structure observed in genetic data (Schwartzman and Lin 2011). Recently, Brzyski et al. (2017) proposed a blocking strategy that groups tested variants into clusters based on LD before applying B–H and showed that this adapted procedure controlled the FDR at their target threshold of 5%. However, their analysis was limited to 364,590 variants in 5,402 samples and it is unclear how their procedure applies to meta-analysis of multiple studies. There is a need to evaluate this adapted B–H and the more conservative Benjamini–Yekutieli (B–Y) procedure (Benjamini and Yekutieli 2001) as well as other procedures that control the Bayesian FDR over a broad range of FDR thresholds at the current scale of common variant GWAS with larger samples and millions of variants.

Here, we use knowledge gathered from current studies to re-evaluate earlier common variant GWAS meta-analyses and assess the impact of different multiple testing procedures on true and false positive rate. Along with varying the $P$-value threshold which controls the FWER, we evaluate the B–H and B–Y procedures to control the FDR, and the Bayesian false discovery probability (BFDP) (Wakefield 2007) procedure to control the Bayesian FDR. We apply the multiple testing procedures to earlier common variant meta-analyses from the Global Lipids (GLGC) and GIANT GWAS consortia on several frequently studied traits: lipid levels, height, and body mass index (BMI). For the lipid traits which are correlated, we also consider the performance of the multiple testing procedures in multivariate analysis of multiple traits and compare it with multiple univariate analyses. Since the true set of causal variants for each trait is unknown, we use the largest and largest meta-analyses for each trait as the approximate “truth” to evaluate the performance of the multiple testing procedures in our empirical datasets. We supplement this analysis with simulation studies where the truth is known. Our results demonstrate that the standard $5 \times 10^{-8}$ $P$-value threshold is the best multiple testing procedure for limiting false positives and is appropriate for modest-sized studies or for resource-intensive follow-ups such as constructing animal models where the cost of follow-up for each locus is high. In contrast, a less stringent $P$-value threshold of $5 \times 10^{-7}$ [as first suggested by the Wellcome Trust Case Control Consortium (2007)] or the adapted B–H procedure at target FDR thresholds of 5% increases power to detect true positives in large studies and can be viable for follow-ups where the cost of including a more modestly greater set of false positives is low, such as gene set enrichment, pathway analysis, or high-throughput functional follow-ups. This in-depth examination provides useful guidance to investigators who are currently conducting GWAS.

### Materials and methods

#### Introduction

We first consider an additive genetic model for a single continuous trait $Y$ and the genotype $G_j$ at variant $j = 1, \ldots, m$.

$$Y = X^T \beta + G_j \theta_j + \epsilon_j,$$

(1)

where $X$ is a $p \times 1$ vector of covariates including the intercept, $\beta$ is a $p \times 1$ vector of covariate effects, $\theta_j$ is the effect of variant $j$, and $\epsilon_j$ is the normally distributed error with mean 0 and variance $\sigma_j^2$. This model can be applied to binary traits using a logit link function.

In a sample of $n$ individuals, we wish to test the null hypotheses $H_{0j} : \theta_j = 0$ against the alternatives $H_{1j} : \theta_j \neq 0$ for each variant $j$. Table 1 summarizes the possible outcomes for the $m$ tests of which $m_0$ null hypotheses are true. For studying multiple testing procedures, we focus on the first row of the table: $R$ is the total number of rejected null hypotheses, $V$ the number of null hypotheses incorrectly rejected (false positives), and $S$ the number of null hypotheses correctly rejected (true positives). The proportion of false positives $Q$ among all rejected hypotheses is then equal to $V/R$ for $R > 0$ and set to 0 for $R = 0$.

Several procedures can be used to address the issue of controlling false positives when testing multiple hypotheses. In the remainder of this section we describe four such procedures, their extension to joint analysis of multiple traits, and application and assessment of these procedures in empirical and simulation studies in the context of common variant GWAS.

### FWER control

The standard procedure to correct for multiple testing in GWAS is to control the FWER, the probability of rejecting at least one true null hypothesis:

$$\text{FWER} = P(V > 0) = P(Q > 0).$$

Fixed $P$-value thresholds often control the FWER by using the Bonferroni procedure which provides control of FWER at level $\alpha$ by rejecting any null hypothesis $H_{0j}$ for variant $j = 1, \ldots, m$ with $P$-value:

$$p_j \leq \frac{\alpha}{m}.$$

The Bonferroni criterion is conservative in two senses. One, by definition it tries to protect against making at least one mistake/false positive under the global null. The second is regarding the conservative behavior, with FWER often falling far below the desired nominal level due to correlated test statistics. This leads to loss of power. When the variants are in LD and the corresponding test statistics are correlated, one can increase the power of the Bonferroni procedure by adjusting for the effective number of independent tests (Altshuler and Donnelly 2005; Dudbridge and Gusnanto 2008; Pe’er et al. 2008) $m' \leq m$ that takes into account LD.

#### FDR control

Although FWER procedures control the probability of incorrectly rejecting at least one true null hypothesis, FDR procedures control the expected proportion of incorrectly rejected true null hypotheses. At equal values of $\alpha$, control of FDR is less conservative.

| True hypothesis | Total |
|-----------------|-------|
| $H_0$ | $H_1$ |
| Test | $H_0$ rejected | $V$ | $S$ | $R$ |
| $H_0$ not rejected | $U$ | $T$ | $m-R$ |
| Total | $m_0$ | $m-m_0$ | $m$ |
than control of FWER (Goeman and Solari 2014). In the context of Table 1,

\[ FDR = E[Q] = \begin{cases} \frac{E[V]}{R} & \text{if } R > 0 \\ 0 & \text{if } R = 0 \end{cases} \]

The B–Y procedure controls the FDR at level \( \alpha \) under any dependency structure by ordering the P-values for the \( m \) variants from smallest to largest: \( P_{[1]}, \ldots, P_{[m]} \) and rejecting all null hypotheses \( H_{0j} \), \( j = 1, \ldots, k \) where \( k \) is the largest value for which:

\[ P_{[k]} \leq \frac{k}{m} \left( \frac{1}{m} \right) \]

and

\[ c(m) = \sum_{i=1}^{m} \frac{1}{i}. \]

The B–H procedure, a commonly used FDR procedure that is valid when test statistics are positively correlated, is a special case of B–Y where \( c(m) \) is instead defined to be equal to 1. It requires an assumption of positive regression dependence on a subset (PRDS) among the test statistics as formally defined in Benjamini and Yekutieli (2001). In GWAS, the PRDS assumption means that a variant with a less significant P-value than another must also be more likely to have no effect on the trait (i.e. truly null). For the complete set of GWAS results, PRDS is likely to be violated due to correlation among tested variants. We detail below a modification of the B–H procedure proposed by Brzyski et al. (2017) that satisfies the PRDS assumption by filtering the full set of tested variants into independent variants using LD.

Applying the B–H or B–Y procedure to GWAS can be challenging because discoveries are counted in units of loci (clusters of nearby variants that are correlated due to LD) rather than by each individual variant. Thus, FDR-controlling procedures need to control for a subset of tested variants, typically the most strongly associated (lead) variant at each locus. Since FDR-control does not extend to a subset of the rejected null hypotheses (Goeman and Solari 2014), we adapt the B–H and B–Y procedures to GWAS by applying an approach proposed by Brzyski et al. (2017) We first cluster the \( m \) null hypotheses into \( m^* < m \) loci by performing LD clumping on the \( m \) variants using a LD threshold of \( r^2 > 0.1 \) and a maximal variant distance of 1 Mb (e.g. Fritsche et al. 2019). This procedure can be done using Swiss (https://github.com/statgen/swiss). We then form a set of \( m^* \) P-values using the lead variant from each locus and apply the B–H or B–Y procedures on these \( m^* \) P-values.

**Bayesian approach to multiple testing**

A Bayesian approach to multiple testing involves calculating the posterior probability of the null hypotheses of no association given the data. For a single variant \( j \), let the likelihood of the observed data \( D = (Y, X, G_{j}) \) given the null hypothesis \( H_{0j} \) be \( p(D|H_{0j}) \). Then by Bayes’ theorem, the probability of the null hypothesis given the data is:

\[ P(H_{0j}|D) = \frac{p(D|H_{0j})P(H_{0j})}{p(D|H_{0j})P(H_{0j}) + p(D|H_{1j})(1 - P(H_{0j}))} = \frac{BF \times PO}{BF \times PO + 1}, \]

where \( BF = p(D|H_{0j})/p(D|H_{1j}) \) is the Bayes factor and \( PO = P(H_{0j})/(1 - P(H_{0j})) \) is the prior odds of no association. Here, we make the commonly accepted exchangeability assumption that every tested variant has the same prior probability of being associated with the trait, i.e. \( 1 - P(H_{0j}) = \pi \) and then conservatively estimate \( \pi \), as the proportion of tested variants with \( P < 5 \times 10^{-8} \) in the observed summary statistics. This assumption can be easily relaxed, allowing for different priors among tested variants based on their functional annotations (Yang and Wang 2015).

For calculating an approximation of the posterior probability called the Bayesian false discovery probability (BFDP), Wakefield (2007) proposed using an approximate Bayes Factor (ABF) based on the maximum likelihood estimator (MLE) \( \hat{\theta} \) of the variant effect \( \theta \), as a succinct summary of the observed data. Following Wakefield, we approximate the BF by \( P(\hat{\theta}|H_{0j})/P(\hat{\theta}|H_{1j}) \). Further assuming the sampling distribution of \( \theta \) is normal with mean \( \theta_{j} \) and variance \( V_{j} \) and that \( \theta_{j} \) has a prior normal distribution with mean 0 and variance \( W_{j} \), we calculate the ABF as a ratio of prior predictive densities \( \theta_{j}|H_{0j} \sim N(0, V_{j}) \) and \( \theta_{j}|H_{1j} \sim N(0, V_{j} + W_{j}) \) and use it to approximate the BFDP:

\[ ABF_{j} = \frac{1}{\sqrt{1 - r_{j}}} \exp \left( -\frac{Z_{j}^{2}}{2} \right) \]

\[ BFDP_{j} = \frac{ABF_{j} \times PO}{ABF_{j} \times PO + 1}. \]

where \( Z_{j} \) is the test statistic and \( r_{j} = W_{j}/(V_{j} + W_{j}) \) is the ratio of the prior variance to the total variance where the prior variance \( W_{j} \) is specified based on the study (Wakefield 2007) and \( V_{j} \) is the variance of \( Z_{j} \). Calculating the approximate BFDP requires effect size or standard error estimates. These may not be included in publicly available GWAS results, which often are limited to P-values and/or Z statistics. If necessary, we can reliably estimate the effect size and standard error for each variant from its Z statistic and estimated MAF (Zhu et al. 2016).

Bayesian FDR is the expected proportion of false positives among all discoveries conditional on the observed data, whereas the traditional FDR is the average Bayesian FDR over many hypotheses (Wen 2017). Controlling the Bayesian FDR (Müller et al. 2004; Wen 2017) is similar to controlling the FDR except we use BFDP in the procedure in place of P-values. To control the Bayesian FDR in multiple hypotheses testing at level \( \alpha \), we order the BFDPs for \( m \) variants from smallest to largest: \( BFDP_{[1]}, \ldots, BFDP_{[m]} \) and reject all null hypotheses \( H_{0j} \), \( j = 1, \ldots, k \) where \( k \) is the largest value for which:

\[ \sum_{j=1}^{k} BFDP_{j} \leq \alpha. \]

As with the B–H and B–Y procedures, we first cluster the tested variants into loci and then apply the BFDP procedure on the lead variant for each locus.

**Joint analysis of multiple traits**

In studies of \( L \) correlated traits, there is potentially more power to detect association if the traits are analyzed together (Diggie et al. 2002). One approach is to conduct \( L \) parallel univariate tests and correct for testing multiple traits simultaneously (i.e. divide the P-value thresholds by \( L \)); an alternative is to jointly analyze the \( L \) traits using multivariate test statistics and then apply the usual multiple testing procedures to the \( m \) resulting tests.

Consider joint testing of the association between genetic variant \( j \) and the \( L \) traits under an extension of (1):

\[ Y_{L \times 1} = X_{L \times p_{L}}^\top \beta_{L \times 1} + G_{L \times 1} + \epsilon_{L \times 1}, \]

where \( \epsilon_{L \times 1} \) is normally distributed with mean \( 0_{L \times 1} \) and variance \( \Sigma_{L \times L} \) representing the covariance matrix of the trait residuals. In (3),
we test the m null hypotheses of no association with any trait: $H_{0,j}: \theta_{ij} = \cdots = \theta_{nj} = 0$ for each variant $j = 1, \ldots, m$.

For Bonferroni and $B-H/B-Y$, we jointly analyzed all traits with metaMANOVA (Bolormaa et al. 2014; Ray and Boehnke 2018) using the test statistic:

$$t_{\text{metaMANOVA}} = Z^T \hat{\Omega}^{-1} Z,$$

where $Z$ is the vector of test statistics for the $L$ traits, $\hat{\Omega}$ is the estimated correlation matrix for the $L$ traits, and $t_{\text{metaMANOVA}}$ follows an approximate chi-squared distribution with $L$ degrees of freedom. We then apply the Bonferroni and $B-Y$ procedures to the multivariate test statistics using the same approach as for the univariate study. To control BFDP, we use an extension (Wakefield 2007) of the ABF in (2) to multiple traits (Appendix 1 of this paper).

**Empirical studies**

We evaluated the performance of the multiple testing procedures in the context of common variant GWAS by using publicly available meta-analysis results from the GLGC and the GIANT consortia. For each procedure, we calculated the empirical false discovery rate (eFDR) as the number of false positive loci in the test set ($V$ in Table 1) divided by the total number of significant loci identified in the test set ($R$ in Table 1). Since $V$ is unknown as we do not know the truth, we assume that the largest, most recent GWAS represents “truth.” We clustered variants declared significant by each procedure into loci using LD clumping. First, we ordered the significant variants by $P$-values and then using the variant with the smallest $P$-value (i.e., most significant) as the lead variant, we grouped all other variants that had LD threshold of $r^2 > 0.1$ with the lead variant and within ±1 Mb of the lead variant into one locus. Next, we repeated this step on the remaining ungrouped variants until all significant variants were clustered into loci. For the adapted $B-H$ and BFDP procedures, we first used LD clumping on all tested variants and then applied the procedures on the lead variant from each locus to obtain a set of significant loci. In the test set, we labeled loci whose lead variants had high LD (defined as $r^2 > 0.80$) with a variant in the truth set with $P < 5 \times 10^{-8}$ as true positives; the remaining loci we considered false positives. We also performed sensitivity analyses on a relaxed definition of true positive which lowered the $r^2$ threshold to 0.60, corresponding to moderate LD between test and truth variants.

Out of four GWAS meta-analyses (Willer et al. 2008; Kathiresan et al. 2009; Teslovich et al. 2010; Willer et al. 2013) sequentially carried out for plasma high-density lipoprotein cholesterol (HDL), low-density lipoprotein cholesterol (LDL), and triglycerides (TG) levels, we picked the largest meta-analysis (Willer et al. 2013) with $n = 188,577$ to serve as the truth set and the second smallest meta-analysis (Kathiresan et al. 2009) with $n = 19,840$ to serve as the test set. We do not present results for the other two meta-analyses in the main text because one (Willer et al. 2008) ($n = 8,816$) had limited power and detected few significant variants and the other (Teslovich et al. 2010) ($n = 100,184$) had very substantial overlap in samples with the truth set so that there was insufficient sample size differences for the truth set to well approximate the truth. Of the 2,373,282 variants analyzed in both the truth and test sets, we analyzed the 2,120,069 (89%) with MAF >5% in both sets and imputed to the HapMap 3 reference panel (Altshuler et al. 2010).

To evaluate the multiple testing procedures over a wider range of sample sizes and genetic architectures, we also applied the procedures to meta-analyses for height and body mass index (BMI) from the GIANT consortium (Lango Allen et al. 2010; Speliotes et al. 2010; Yengo et al. 2018). We present results for these meta-analyses from a larger set of sequential meta-analyses (Lango Allen et al. 2010; Speliotes et al. 2010; Wood et al. 2014; Locke et al. 2015; Yengo et al. 2018) using the same rationale as described above for GLGC: the largest, most recent meta-analyses (Yengo et al. 2018) for height and BMI ($n = 694,529$ and $n = 681,275$, respectively) served as the truth sets and the smallest meta-analyses (Lango Allen et al. 2010; Speliotes et al. 2010) for each trait ($n = 133,653$ and $n = 123,865$) served as the test sets. Of the 2,282,242 variants analyzed in both meta-analyses for height, we analyzed the 2,036,404 (89%) with MAF > 5%. Of the 2,282,195 variants in both meta-analyses for BMI, we analyzed the 2,035,656 (89%) with MAF > 5% in both sets and imputed to the HapMap 3 reference panel (Altshuler et al. 2010).

For univariate analysis of each lipid and anthropometric trait, we used published meta-analyses results. Detailed descriptions of the statistical analyses for each of the results can be found in their respective papers (Kathiresan et al. 2009; Lango Allen et al. 2010; Speliotes et al. 2010; Willer et al. 2013; Yengo et al. 2018). For multivariate analysis of the three lipid traits together, we combined the univariate results using the appropriate multivariate extension for each of the procedures as described above.

**Simulation studies**

To evaluate the multiple testing procedure when truth is known, we generated 1000 replicate datasets based on a simplified version of the empirical association structure observed in the latest GWAS for each of the five traits. Since we are only using the significant variants from the empirical study as causal variants in our simulation study, the number of true and false positives may differ between the two studies. We assessed the true and false positive rate of each procedure using the same method as described for our empirical studies.

To mimic the GLGC test set which consisted of European cohorts, we randomly sampled 19,840 individuals from 276,791 unrelated individuals of white British ancestry in the UK Biobank dataset. For each replicate, we used the genotypes of these individuals to generate outcomes on $n = 19,840$ individuals for each lipid trait following model (1). We assumed the trait value $Y$ is inverse normal to maintain consistency with the empirical studies, we chose the causal variant effect sizes $\theta$ from the estimated values for variants with $P < 5 \times 10^{-8}$ latest GLGC GWAS (the truth set), and the error term is normally distributed with mean 0 and variance equal to 1 minus the proportion of trait variance explained by the simulated causal variants. We ran association analysis with each replicate dataset using a linear regression model with no additional covariates. We took a similar approach for simulating height and BMI based on the GIANT dataset using separate generation models for the two traits. An estimated numbers of causal variants that can be detected in simulation for the five traits at different $P$-value thresholds of can be found in Supplementary Table S1.

**Data availability**

Meta-analyses results for the Global Lipids Genetics Consortium are available at http://lipidgenetics.org. Meta-analyses results for the GIANT Consortium are available at https://portals.broadinstitute.org/collaboration/giant/index.php/GIANT_consortium_data_files. UK Biobank genotype data are available at http://biobank. ctsu.ox.ac.uk/crystal/label.cgi?id=263.
Supplementary material is available at figshare DOI: https://doi.org/10.25387/g3.13211507.

Results

We applied the multiple testing procedures to HDL, LDL, TG, height, and BMI to assess their performances for different sample sizes and genetic structures.

P-value threshold

Applying various fixed P-value thresholds to the empirical GLGC and GIANT test sets, we observed as expected that the empirical false discovery rate (eFDR) generally increased as the P-value threshold increased (Tables 2 and 3). The lone exception (for HDL) likely reflected statistical noise due to the small number of identified loci.

For height and BMI, we identified substantially more loci by relaxing the threshold from nearly all these new loci being true positives (Table 2 and Table 3). Further relaxing the threshold from P = 5 × 10^{-7} to P = 5 × 10^{-6} maintained high proportions of true positives among the additional loci: 95 of 96 (99%) for height, 18 of 19 (95%) for BMI. For the lipid traits in the GLGC test set, relaxing the threshold increased (Tables 2 and 3); the inconsistency described before the additional loci: 95 of 96 (99%) for height, 18 of 19 (95%) for BMI (Table 3) and more extreme (1–25%; Supplementary Table S2) being true positives. Further relaxing the threshold from P = 5 × 10^{-7} to P = 5 × 10^{-6} resulted in ≤ 56% of the additional loci being true positives for the lipid traits.

We observed in the GLGC- and GIANT-based simulated data-sets that the average eFDR increased as the P-value threshold increased for all traits (Table 3); the inconsistency described before for the empirical HDL test set disappeared when we averaged over 1,000 simulation replicates. Consistent with the empirical results, there was a clear difference in the proportion of true positives between the lipid and anthropometric traits in the simulated results (Table 3). Relaxing the threshold from P = 5 × 10^{-8} to P = 5 × 10^{-7} in the simulated datasets resulted in an average of 77% to 87% of the additional loci being true positives for the lipid traits and 93% and 90% for height and BMI. Further relaxing the threshold from P = 5 × 10^{-7} to P = 5 × 10^{-6} resulted in 47% to 60% of the additional loci being true positives for lipids, and 84% and 79% for height and BMI.

To address whether the higher rates of true positives we observed when relaxing the P-value threshold for height and BMI compared to those for lipids were the result of differences in sample sizes, we simulated test sets for height and BMI at the same sample sizes (n = 8,816 and n = 19,840) as the GLGC meta-analyses. For both traits, an increase in sample size generally led to higher proportion of true positives gained from relaxing the P-value threshold (Table 4), suggesting a better yield of true positives by using relaxed thresholds in larger samples than in smaller ones.

Benjamini–Hochberg and Benjamini–Yekutieli procedures

As expected, empirical results for the two FDR controlling procedures showed B–Y was conservative, resulting in eFDR far below the target FDR threshold for all traits at commonly used (5-15%; Table 5) and more extreme (1–25%; Supplementary Table S2) thresholds. B–H controlled the eFDR at the target thresholds (Table 6 and Supplementary Table S3) for height and BMI but not lipid traits, likely because the number of lipid trait discoveries was modest (≤ 26 loci for B–H) so that even a small change in the proportion of true positives could produce a substantial increase in the eFDR.

Table 2 Empirical and simulation results for all traits and commonly used thresholds

| Trait   | Empirical | Simulation |
|---------|-----------|------------|
|         | Positives | eFDR | Positives | eFDR (SE) |
|         | False     | True  |          | False     | True    |          |
|         |           |       |           |           |         |
| HDL     |           |       |           |           |         |
| P = 5 × 10^{-8} | 1   | 16   | 5.9%     | 0.28      | 9.5    | 2.9% (0.33%) |
| n_{truth} = 188,577 | 1   | 18   | 5.3%     | 0.89      | 12     | 6.7% (0.45%) |
| LDL     |           |       |           |           |         |
| P = 5 × 10^{-9} | 0   | 14   | 0%       | 0.70      | 12     | 5.6% (0.43%) |
| n_{truth} = 188,577 | 0   | 14   | 0%       | 0.17      | 8.2    | 2.0% (0.31%) |
| TG      |           |       |           |           |         |
| P = 5 × 10^{-8} | 2   | 8    | 11%      | 0.41      | 10     | 4.0% (0.37%) |
| n_{truth} = 188,577 | 2   | 8    | 11%      | 0.19      | 13     | 1.5% (0.19%) |
| Height  |           |       |           |           |         |
| P = 5 × 10^{-8} | 0   | 157  | 0%       | 0.71      | 16     | 4.2% (0.27%) |
| n_{truth} = 693,529 | 0   | 157  | 0%       | 0.67      | 16     | 4.0% (0.27%) |
| BMI     |           |       |           |           |         |
| P = 5 × 10^{-8} | 0   | 22   | 0%       | 0.13      | 12     | 1.1% (0.17%) |
| n_{truth} = 681,275 | 0   | 22   | 0%       | 0.37      | 14     | 2.5% (0.22%) |

* Number of loci in truth set for HDL: 89, LDL: 72, TG: 60, height: 1100, BMI: 724.
* eFDR is calculated as number of false positives divided by sum of true and false positives.
numbers of true and false positives substantially influenced estimated eFDR. When the total number of discoveries is small, it is more useful to assess control of FDR averaged over a large number of datasets in simulation since the variance of the eFDR based on a single empirical study can be quite large.

Simulation results for B–Y were consistent with empirical results in showing that B–Y is overly conservative for all five traits and all target FDR thresholds (Table 5 and Supplementary Table S2). For example, the observed eFDR for target threshold of 15% is < 3.4% for all traits and the equivalent P-value threshold for that target threshold is 10 times more stringent for B–Y than B–H (Supplementary Table S4). Compared to the empirical results, B–H did a better job of controlling eFDR at the commonly used thresholds (Table 6) for all traits; only for height at 5% and BMI at 5% did B–H show noticeable excess in eFDR (6.8% for height, 7.9% for BMI). When we relaxed our criterion for defining a true positive (see below), excess eFDR for height and BMI decreased (eFDR = 5.5% and 5.2%) (Supplementary Table S5). eFDR was well-controlled at high thresholds 20% and 25% for all five traits but poorly-controlled at low thresholds 1% and 3% (Supplementary Table S3).

We investigated whether FDR control for B–H and B–Y extended across sample sizes by using simulated datasets for height and BMI at n = 8,816, n = 19,840 and n = 133,653 (height) or 123,865 (BMI). Both procedures controlled eFDR at the target FDR thresholds 5-15% for height (Supplementary Tables S6 and S7); BMI showed excess eFDR under B–H for all test sets which disappeared under the relaxed definition of true positives (Supplementary Table S5; data not shown for smaller sample sizes).

**Bayesian false discovery probability**

For the BFPD procedure, we estimated the prior probability of association at a variant site (n1) separately for each test set using the proportion of tested variants with \( P < 5 \times 10^{-8} \). Empirical results showed that eFDR was well controlled for height and BMI at target Bayesian FDR thresholds 1-25% but poorly controlled for lipid traits (Table 7 and Supplementary Table S8), again likely due to the smaller number of discoveries for lipid traits (≤ 24 loci for BFPD).

Simulation results for BFPD showed that eFDR was generally well controlled at target Bayesian FDR thresholds 5-15% (Table 7) for all traits except height (eFDR = 8.1%, 13%, and 17%). For more extreme thresholds (Supplementary Table S8), eFDR was controlled at 1 and 3% for lipid traits, albeit with excess eFDR for HDL at 1%; eFDR was controlled at 20% and 25% for all traits.

**Multi-trait analysis results for lipids**

In empirical results (Supplementary Table S9), the \( P = 5 \times 10^{-8} \) threshold had the lowest eFDR for the parallel univariate tests both adjusted (Bonferroni corrected threshold of \( 1.67 \times 10^{-5} \)) and unadjusted (\( 5 \times 10^{-8} \)) for testing three traits. For the multivariate tests, the \( P = 5 \times 10^{-8} \) and \( P = 5 \times 10^{-7} \) thresholds had identical eFDR of 0%. Between the three sets of thresholds, the multivariate analysis had the lowest eFDR as well as the highest proportion of true positive discoveries when relaxing the P-value thresholds. For both the B–H and BFPD procedures, multivariate tests had lower eFDR than the univariate tests but only the multivariate B–H procedure controlled the eFDR at target thresholds 5-15%.

In simulation results (Supplementary Table S10), the \( P = 5 \times 10^{-8} \) threshold had the lowest eFDR for all three sets of tests. Consistent with empirical results, multivariate tests had the lowest eFDR at all three P-value thresholds and better true positive rate for relaxing thresholds compared with the univariate tests. For the B–H and BFPD procedure, both univariate and multivariate tests controlled the eFDR at target thresholds 5-15%.

**Sensitivity analyses**

We defined true positives in the test set strictly as loci whose lead variants had LD threshold of r² > 0.80 with a genome-wide significant \( (P < 5 \times 10^{-8}) \) variant in the truth set. We chose this strict criterion to avoid underestimating the number of false positives in our analysis but it likely led to overestimation of eFDR. To assess the impact of this, we repeated our simulation analyses using a relaxed definition of true positives by lowering the LD threshold requirement to 0.60 and the P-value requirement to \( 5 \times 10^{-7} \) (Supplementary Table S5). As expected, under this relaxed definition we observed fewer false positives and occurrences of excess eFDR largely disappeared as well. For example, simulation results for height using the BFPD procedure at FDR

---

**Table 3: Empirical and simulation results for P-value thresholds**

| Trait   | Threshold (P-value) | Positives | eFDR[^a] | Δ in # of sig. loci (% true) | Simulation | Positives | eFDR[^aa] | Δ in # of sig. loci (% true) |
|---------|---------------------|-----------|----------|--------------------------------|------------|-----------|----------|--------------------------------|
|         |                     | False     | True[^a] |                                 | False      | True[^a] | False    | True[^a] |
| HDL     | 5 × 10^{-8}         | 1         | 16       | 5.9%                            | 0.28       | 9.5      | 2.9%     |                                 |
|         | 5 × 10^{-7}         | 1         | 18       | 5.3% +2 (100%)                  | 0.89       | 12       | 6.7% +3 (83%) |                                 |
|         | 5 × 10^{-6}         | 8         | 21       | 28% +10 (30%)                   | 4.2        | 17       | 20% +8.2 (60%) |                                 |
| LDL     | 5 × 10^{-8}         | 0         | 14       | 0%                              | 0.19       | 13       | 1.5%     |                                 |
|         | 5 × 10^{-7}         | 3         | 16       | 16% +5 (40%)                    | 0.71       | 16       | 4.2% +3.9 (87%) |                                 |
|         | 5 × 10^{-6}         | 10        | 19       | 34% +10 (30%)                   | 4.6        | 22       | 17% +9.1 (58%) |                                 |
| TG      | 5 × 10^{-8}         | 1         | 8        | 11%                             | 0.11       | 9        | 1.2%     |                                 |
|         | 5 × 10^{-7}         | 2         | 10       | 17% +3 (67%)                    | 0.54       | 10       | 4.9% +1.9 (77%) |                                 |
|         | 5 × 10^{-6}         | 6         | 15       | 29% +9 (56%)                    | 3.8        | 13       | 22% +6.1 (47%) |                                 |
| Height  | 5 × 10^{-8}         | 0         | 157      | 0%                              | 1.6        | 181      | 0.89%    |                                 |
|         | 5 × 10^{-7}         | 1         | 217      | 0.46% +61 (98%)                 | 4.9        | 223      | 2.2% +46 (93%) |                                 |
|         | 5 × 10^{-6}         | 2         | 312      | 0.64% +96 (99%)                 | 16        | 283      | 5.4% +72 (84%) |                                 |
| BMI     | 5 × 10^{-8}         | 0         | 22       | 0%                              | 0.62       | 39       | 1.6%     |                                 |
|         | 5 × 10^{-7}         | 0         | 37       | 0% +15 (100%)                   | 2.7        | 58       | 4.4% +22 (90%) |                                 |
|         | 5 × 10^{-6}         | 1         | 55       | 1.8% +19 (95%)                  | 11        | 90       | 11% +41 (79%) |                                 |

[^a] Number of loci in truth set for HDL: 89, LDL: 72, TG: 60, height: 1100, BMI: 724.
[^aa] eFDR is calculated as number of false positives divided by sum of true and false positives.
Figure 1: Manhattan plot of empirical P-value thresholds for variants with \( P \geq 5 \times 10^{-8} \). Plots of different P-value thresholds applied to empirical test sets for HDL, BMI, and height. Colored variants depict true positive loci (blue) and false positive loci (red) for variants with \( P \geq 5 \times 10^{-8} \). Lead variants for true and false positive loci are represented by large blue circles and large triangles, respectively.

Table 4: Effect of sample size on simulation results for P-value thresholds

| Trait   | Threshold (P-value) | \( n = 8,816 \) | \( n = 19,840 \) | \( n = 133,653 \) or 123,865 |
|---------|---------------------|-----------------|-----------------|---------------------------|
|         | Positives eFDR \( \Delta \) sig. loci (\% True positive) | Positives eFDR \( \Delta \) sig. loci (\% True positive) | Positives eFDR \( \Delta \) sig. loci (\% True positive) | Positives eFDR \( \Delta \) sig. loci (\% True positive) |
|         | False True \( ^a \) | False True \( ^a \) | False True \( ^a \) | False True \( ^a \) |
| Height  | \( 5 \times 10^{-8} \) | 0.04 0.90 43\% | 0.03 0.27\% | 1.6 181 89\% | \( 5 \times 10^{-7} \) | 0.40 2.2 15\% +1.7 (78\%) | 0.32 18 1.7\% +7.4 (96\%) | 4.9 223 2.2\% +46 (93\%) |
| BMI     | \( 5 \times 10^{-9} \) | 0.30 5.9 34\% +6.3 (58\%) | 3.6 30 11\% +15 (79\%) | 16 283 5.4\% +72 (84\%) | \( 5 \times 10^{-7} \) | 0.09 1.5 5.7\% | 0.62 39 1.6\% | 7.4 22 (96\%) | \( 5 \times 10^{-6} \) | 0.34 0.20 17\% +0.51 (41\%) | 0.46 2.4 16\% +1.3 (72\%) | 2.7 58 4.4\% +22 (90\%) | 5.8 4.6 41\% +5.0 (44\%) | 11 90 11\% +41 (79\%) |

\( ^a \) Number of loci in truth set for HDL: 89, LDL: 72, TG: 60, height: 1100, BMI: 724.

\( ^b \) eFDR is calculated as number of false positives divided by sum of true and false positives.
thresholds of 10% and 15% showed eFDR of 13% and 17% under the strict definition and 10% and 14% under the relaxed definition.

In addition to our LD-based definitions, we used physical distance to define loci and true positives. We grouped variants within 61Mb of the lead variants into loci and defined true positives as loci whose lead variants were within 650kb of a genome-wide significant variant in the truth set. The analyses results (Supplementary Table S11) showed that the distance-based definitions led to smaller numbers of true and false positives for all traits and multiple testing procedures.

**Discussion**

In this paper, we leverage the sequentially growing nature of GWAS meta-analyses to evaluate true and false positive rate of P-value thresholds and other multiple testing procedures. Although the standard procedure for identifying significant associations in common variant GWAS is to use a $P$-value threshold of $5 \times 10^{-8}$, relaxing the significance criteria, whether through use of less stringent $P$-value thresholds or controlling for alternative error rate measures such as FDR (depending on the target threshold) increases the number of identified loci. We demonstrated that a substantial proportion of the additional loci identified by relaxed $P$-value thresholds are true positives, with larger proportions of true positives in analysis of larger samples.

**Application to downstream analyses**

GWAS identify trait-associated variants and loci based on association analysis of millions of variants. The identified loci are often further validated in replication studies before being used for statistical and functional analyses to identify causal genes, variants, and mechanisms. Although relaxed $P$-value thresholds are often used to generate the list of loci for replication, the expected true

| Table 5 Empirical and simulation results for Benjamini–Yekutieli procedure |
|-----------------------------|-----------------------------|
| Trait | Threshold (FDR) | Empirical | Simulation |
| | | Positives | eFDR$^b$ | \(\text{False} \quad \text{True}^a\) | eFDR | \(\text{False} \quad \text{True}\) |
| | | \(\text{False} \quad \text{True}^a\) | eFDR | False | True |
| HDL | (\(n_{\text{true}} = 19,840\)) | 5% | 0 | 14 | 0% | 0.17 | 8.2 | 2.0% |
| LDL | (\(n_{\text{true}} = 188,577\)) | 10% | 1 | 16 | 5.9% | 0.25 | 9.1 | 2.7% |
| | (\(n_{\text{true}} = 19,840\)) | 15% | 1 | 16 | 5.9% | 0.31 | 9.6 | 3.1% |
| TG | (\(n_{\text{true}} = 19,840\)) | 5% | 0 | 14 | 0% | 0.13 | 12 | 1.1% |
| | (\(n_{\text{true}} = 188,577\)) | 10% | 0 | 14 | 0% | 0.19 | 13 | 1.5% |
| Height | (\(n_{\text{true}} = 133,653\)) | 15% | 0 | 15 | 0% | 0.26 | 13 | 1.9% |
| BMI | (\(n_{\text{true}} = 693,529\)) | 5% | 0 | 8 | 0% | 0.05 | 8.5 | 0.58% |
| (\(n_{\text{true}} = 19,840\)) | 10% | 0 | 8 | 0% | 0.06 | 8.7 | 0.68% |
| (\(n_{\text{true}} = 123,865\)) | 15% | 0 | 8 | 11% | 0.11 | 9.0 | 1.2% |
| (\(n_{\text{true}} = 681,275\)) | 0 | 26 | 0% | 4.3 | 217 | 2.0% |

\(a\) Number of loci in truth set for HDL: 89, LDL: 72, TG: 60, height: 1100, BMI: 724. \(b\) eFDR is calculated as number of false positives divided by sum of true and false positives.

| Table 6 Empirical and simulation results for Benjamini–Hochberg procedure |
|-----------------------------|-----------------------------|
| Trait | Threshold (FDR) | Empirical | Simulation |
| | | Positives | eFDR$^b$ | False | True$^a$ | eFDR | False | True |
| | | \(\text{False} \quad \text{True}^a\) | eFDR | False | True |
| HDL | (\(n_{\text{true}} = 19,840\)) | 5% | 1 | 18 | 5.3% | 0.70 | 12 | 5.6% |
| LDL | (\(n_{\text{true}} = 188,577\)) | 10% | 1 | 16 | 22% | 1.3 | 13 | 8.5% |
| | (\(n_{\text{true}} = 19,840\)) | 15% | 6 | 20 | 23% | 1.7 | 14 | 11% |
| TG | (\(n_{\text{true}} = 19,840\)) | 5% | 3 | 16 | 16% | 0.67 | 16 | 4.0% |
| | (\(n_{\text{true}} = 188,577\)) | 10% | 4 | 16 | 20% | 1.1 | 18 | 6.0% |
| Height | (\(n_{\text{true}} = 133,653\)) | 15% | 7 | 17 | 29% | 1.7 | 18 | 8.6% |
| BMI | (\(n_{\text{true}} = 693,529\)) | 5% | 2 | 9 | 18% | 0.32 | 10 | 3.1% |
| (\(n_{\text{true}} = 123,865\)) | 10% | 5 | 10 | 33% | 0.63 | 11 | 5.6% |
| (\(n_{\text{true}} = 681,275\)) | 15% | 5 | 10 | 33% | 0.96 | 11 | 8.0% |

\(a\) Number of loci in truth set for HDL: 89, LDL: 72, TG: 60, height: 1100, BMI: 724. \(b\) eFDR is calculated as number of false positives divided by sum of true and false positives.
and false discovery rates under different thresholds have not been quantified. We showed by simulation for common variant GWAS with sample size > 100,000 that 90-93% of additional discoveries with P-values between $5 \times 10^{-8}$ and $5 \times 10^{-7}$ were true positives, representing true associations that would be lost under a more stringent threshold. However, for more modest sample sizes (~20,000), our simulation showed that only 77-87% of additionally discovered loci with P-values between $5 \times 10^{-8}$ and $5 \times 10^{-7}$ were true positives. Here, investigators should exercise caution when relaxing the significance threshold for replication studies as the increase to replicated associations may not outweigh the excess false discovery rate.

For follow-up studies such as constructing animal models where the per-locus cost of follow-up is high, a stringent P-value threshold of $5 \times 10^{-8}$ is appropriate in both large and modest-sized studies to generate a highly accurate list of associated loci. However, such threshold may be unhelpfully conservative for analyses where including (many) more true loci at the cost of (a few) more false positives is acceptable such as gene-set enrichment or pathway analysis. In these situations, a relaxed threshold of $5 \times 10^{-7}$ may be better served to prioritize GWAS results for downstream analyses.

### Table 7: Empirical and simulation results for BFDP procedure

| Trait     | Threshold (Bayesian FDR) | Empirical | Simulation |
|-----------|--------------------------|-----------|-----------|
|           | $\pi_1^a$ | Positives | eFDR$^c$ | $\pi_1^d$ | Positives | eFDR |
|           | False | True$^b$ |          | False | True |          |
| HDL       | 5%     | $1.3 \times 10^{-4}$ | 1 | 17 | 5.6% | $8.7 \times 10^{-5}$ | 0.41 | 10 | 4.0% |
|           | 10%    | 4 | 17 | 19% | 1.2 | 13 | 8.4% |
|           | 15%    | 6 | 18 | 25% | 0.37 | 14 | 2.5% |
| LDL       | 5%     | $1.3 \times 10^{-4}$ | 2 | 17 | 11% | $9.6 \times 10^{-5}$ | 0.83 | 16 | 4.9% |
|           | 10%    | 5 | 17 | 23% | 1.3 | 18 | 7.0% |
|           | 15%    | 6 | 18 | 25% | 0.36 | 9.8 | 3.6% |
| TG        | 5%     | $2.1 \times 10^{-4}$ | 1 | 10 | 9.1% | $1.6 \times 10^{-4}$ | 1.0 | 11 | 8.4% |
|           | 10%    | 4 | 10 | 29% | 1.6 | 12 | 12.2% |
| Height    | 5%     | $2.0 \times 10^{-3}$ | 2 | 338 | 0.59% | $2.9 \times 10^{-3}$ | 28 | 317 | 8.1% |
|           | 10%    | 7 | 406 | 1.7% | 51 | 356 | 13% |
|           | 15%    | 9 | 468 | 1.9% | 76 | 385 | 17% |
| BMI       | 5%     | $3.6 \times 10^{-4}$ | 0 | 35 | 0% | $5.2 \times 10^{-4}$ | 3.9 | 67 | 5.4% |
|           | 10%    | 0 | 43 | 0% | 7.2 | 82 | 8.0% |
|           | 15%    | 0 | 50 | 0% | 11 | 93 | 10% |

| a | $\pi_1$ is the estimated prior probability of association at a variant site equal to the proportion of tested variants with $P < 5 \times 10^{-8}$. |
| b | Number of loci in truth set for HDL, LDL, TG: 60; height: 1100; BMI: 724. |
| c | eFDR is calculated as number of false positives divided by sum of true and false positives. |
| d | Average $\pi_1$ in 1000 replicate datasets. |

For BFDP, a Bayesian alternative to B–H, we estimated the proportion of trait-associated variants $\pi_1$ using the proportion of tested variants with $P < 5 \times 10^{-8}$ and found the Bayesian FDR to be reasonably well controlled at thresholds of 5-25%. For comparison, when we estimated $\pi_1$ as the number of loci with lead variant $P < 5 \times 10^{-8}$ divided by 1 million [an estimate for the total number of independent common variants in the genome (Altshuler and Donnelly 2005; Pe’er et al. 2008)], the resulting lower $\pi_1$ estimates led to conservative results (Supplementary Table S12).

### Comparison between procedures

A P-value threshold has the advantages of familiarity, simplicity, and ease of implementation, whereas B–H and BFDP control the eFDR across a range of sample sizes. A stringent P-value threshold is needed if our primary goal is to limit the number of false positives as both B–H and BFDP struggled to control the eFDR at low target thresholds 1% and 3%. In addition, the P-value threshold can also be used to control the Per Family Error Rate (PFER) as discussed in Gordon et al. (2007) since the Bonferroni procedure can be trivially extended to control PFER.

### Limitations

Our analysis is based on five anthropometric and lipid traits which obviously do not fully reflect the wide range of phenotypes studied in GWAS. However, our simulated traits do represent different levels of polygenicity ranging from HDL with 89 causal variants and some larger effect sizes (Supplementary Figure S1) to height with 1100 causal variants and generally smaller effect sizes, covering a wide range of genetic architectures for quantitative phenotypes.

Our analysis is also limited to studies of European-origin individuals. Further work using test and truth sets with non-European samples would be welcome to confirm that our findings are applicable more generally.

We relied on publicly available meta-analyses results from the GLGC and GIANT consortia for our empirical and simulation studies. However, imputation qualities were not provided for these datasets, which raises the concern that our analyses would...
have missed the effects of poorly imputed variants. Since we restricted our analyses to ~2 million HapMap3 common variants with consistently high imputation r-squared across past studies, we believe our results were not substantially affected by variation in imputation quality.

We defined true and false positive discoveries in our empirical analysis using the largest, most recent common variant GWAS which we called the truth set. Since our empirical test set came from the same sequential set of meta-analyses as the truth set, the samples in our test set were a complete subset of those in the truth set. Although this procedure did not guarantee the (unknown) list of loci truly associated with each tested trait, the truth sets served as reasonable approximations when there were considerable sample size differences between the truth and test sets. When we repeated our analysis for smaller test sets ($n=8,816$), we observed well-controlled eFDR for the B–H procedure as expected. However, in larger test sets ($n=253,288$), we observed noticeable excess in eFDR which disappeared in simulations. This is likely because the sample size differences between the larger empirical test sets and the approximate truth set is small enough that the “truths” may no longer be accurate. When this occurs, false positive discovery in the test set can become biased since a false positive in the test set may be discovered (and thus become a true positive) in a subsequent study larger than the truth set. In simulated datasets where the truth was known, the eFDR was well-controlled.

Association results on the same samples may differ owing to changes in quality control and/or statistical analysis. For example, some studies may choose not to apply genomic control which can then lead to inflated P-values due to confounding from population stratification. In this scenario, relaxing the significance threshold may not be appropriate since the number of false positives would already be higher than expected. We did not observe substantial genomic inflation in our analyses (average lambda GC of 1.04, 95% CI: 1.02, 1.05) and so could not easily investigate how our results would have changed depending on whether GC is or is not applied.

Although our results showed only modest differences between multivariate and univariate tests in terms of multiple testing corrections, we only considered the case of three correlated traits as opposed to the tens to thousands of traits that might simultaneously be tested in metabolomics or imaging studies. With many traits, combining univariate tests but not adjusting for multiple traits will surely lead to excess false positive discoveries (as even our limited results demonstrated), whereas adjusting for hundreds of traits may be overly conservative. Here, multivariate tests may represent an attractive option.

In this study, we focused on common variants (MAF > 5%). This allowed us to construct likely highly accurate truth sets of loci based on serial common variant GWAS for anthropometric and lipid traits to facilitate the evaluation of different procedures for multiple testing corrections. We plan next to consider multiple testing in the context of rare variant analysis (Pulit et al. 2017; Lin 2019) which necessarily will increase the number of hypothesis tests for which we need to correct and introduce the consideration of gene- and more generally set-based association tests.

Summary
In this study, we evaluated the performance of four procedures for multiple testing corrections in the context of common variant GWAS: P-value thresholds, B–H and B–Y for FDR control, and BFDP for Bayesian FDR control. We have shown that for studies based on large samples, using a less stringent P-value threshold of $5 \times 10^{-7}$ or use of FDR-controlling procedure (B–H) at target threshold of 5% substantially increases the number of true positive discoveries that can be used in downstream analyses while only modestly increasing false positives compared with the commonly used $5 \times 10^{-8}$ P-value threshold. The latter threshold remains the preferred choice for modest-sized studies or when a stringently curated list of loci is desired. Finally, we show that FDR-control largely extends across sample sizes with a few exceptions that disappear under a relaxed definition of true positives and FDR-controlling procedures can be similarly applied to large and modest-sized studies.

Acknowledgments
We thank Professor Laura J. Scott for helpful discussions.

Funding
This research was supported by the National Institutes of Health grants HG009976, CA046592, and the National Science Foundation grant DMS 1712933.

Conflicts of interest: None declared.

Literature cited
Altschuler D, Donnelly P. 2005. The international HapMap consortium: a haplotype map of the human genome. Nature. 437:1299–1320. doi:10.1038/nature0226.
Altschuler DM, Gibbs RA, Peltonen L, Altschuler DM, Gibbs RA, et al. 2010. Integrating common and rare genetic variation in diverse human populations. Nature. 467:52–58. doi:10.1038/nature09298.
Amrhein V, Greenland S, McShane B. 2019. Scientists rise up against statistical significance. Nature. 567:305–307. doi:10.1038/d41586-019-00857-9.
Benjamin DJ, Berger JO, Johannesson M, Nosek BA, Wagenmakers E-J, et al. 2018. Redefine statistical significance. Nat Hum Behav. 2:6–10. doi:10.1038/s41562-017-0189-z.
Benjamini Y, Hochberg Y. 1995. Controlling the false discovery rate: a practical and powerful approach to multiple testing. J R Stat Soc Ser B (Methodol). 57:289–300.
Benjamini Y, Yekutieli D. 2001. The control of the false discovery rate in multiple testing under dependency. Ann Statist. 29:1165–1188.
Bogdan M, Ghosh JK, Tokdar ST. 2008. A comparison of the Benjamini-Hochberg procedure with some Bayesian rules for multiple testing. 211–230:arXiv:08052479 [math, stat]. doi:10.1214/1993403070000000158.
Bolormaa S, Pryce JE, Reverter A, Zhang Y, Barendse W, et al. 2014. A multi-trait, meta-analysis for detecting pleiotropic polymorphisms for stature, fatness and reproduction in beef cattle. PLoS Genet. 10:e1004198. doi:10.1371/journal.pgen.1004198.
Brzeski D, Peterson CB, Sobczyk P, Candes EJ, Bogdan M, et al. 2017. Controlling the rate of GWAS false discoveries. Genetics. 205:61–75. doi:10.1534/genetics.116.193987.
Burdick JT, Chen W-M, Abecasis GR, Cheung VG. 2006. In silico method for inferring genotypes in pedigrees. Nat Genet. 38:1002–1004. doi:10.1038/ng1863.
Diggle P, Liang K-Y, Heagerty PJ, Zeger S. 2002. Analysis of Longitudinal Data. Oxford: OUP.
Dudbridge F, Gusnanto A. 2008. Estimation of significance thresholds for genomewide association scans. Genet Epidemiol. 32:227–234. doi:10.1002/gepi.20297.
Efron B, Tibshirani R, Storey JD, Tusher V. 2001. Empirical bayes analysis of a microarray experiment. J Am Stat Assoc. 96: 1151–1160. doi:10.1198/016214501753382129.

Fritsche LG, Beesley LJ, VandeHaar P, Peng RB, Salvatore M, et al. 2019. Exploring various polygenic risk scores for skin cancer in the phenomes of the Michigan genomics initiative and the UK Biobank with a visual catalog. PLoS Genet. 15:e1008202. doi: 10.1371/journal.pgen.1008202.

Goeman JJ, Solari A. 2014. Multiple hypothesis testing in genomics. Statist Med. 33:1946–1978. doi:10.1002/sim.6082.

Gordon A, Glazko G, Qiu X, Yakovlev A. 2007. Control of the mean number of false discoveries, Bonferroni and stability of multiple testing. Ann Appl Stat. 1:179–190. doi:10.1214/07-AOAS102.

Kathiresan S, Willer CJ, Peloso GM, Demissie S, Musunuru K, et al. 2009. Common variants at 30 loci contribute to polygenic dyslipidemia. Nat Genet. 41:56–65. doi:10.1038/ng.291.

Lango Allen H, Estrada K, Lettre G, Berndt SI, Weedon MN, et al. 2014. Multiple hypothesis testing in genomics. Statist Med. 33:1946–1978. doi:10.1002/sim.6082.

Li Y, Willer CJ, Sanna S, Abecasis GR. 2009. A simple and accurate method to determine genome-wide significance for association tests in sequencing studies. Genet Epidemiol. 34:816–834. doi: 10.1002/gepi.20533.

Lin D. 2019. A simple and accurate method to determine genome-wide significance for association tests in sequencing studies. Genet Epidemiol. 43:365–372.

Leaving H, Estrada K, Berndt SI, Weedon MN, et al. 2010. Hundreds of variants clustered in genomic loci and biological pathways affect human height. Nature. 467:832–838. doi: 10.1038/nature09410.

Li Y, Willer C, Sanna S, Abecasis GR. 2009. Genotype Imputation. Annu Rev Genom Hum Genet. 10:387–406. doi:10.1146/annurev-genom.9.081307.164242.

Li Y, Willer CJ, Ding J, Scheet P, Abecasis G. 2009. Genotype Imputation. Annu Rev Genom Hum Genet. 10:387–406. doi:10.1146/annurev-genom.9.081307.164242.

Locke AE, Kahali B, Berndt SI, Justice AE, Pers TH, The LifeLines Cohort Study, et al. 2015. Genetic studies of body mass index yield new insights for obesity biology. Nature. 518:197–206. doi:10.1038/nature14177.

McCarthy S, Das S, Kretzschmar W, Delaneau O, Wood AR, Haplotype Reference Consortium, et al. 2016. A reference panel of 64,976 haplotypes for genotype imputation. Nat Genet. 48: 1279–1283. doi:10.1038/ng.3643.

Müller P, Parmigiani G, Robert C, Rousseau J. 2004. Optimal sample size for multiple testing: the case of gene expression microarrays. J Am Stat Assoc. 99:990–1001.

Panagiotou OA, Ioannidis JPA, for the Genome-Wide Significance Discovery in Genetic Epidemiology Studies. Nat Genet. 48:208–227. doi:10.1038/ng.1924.

Pe’er I, Yelensky R, Altshuler D, Daly MJ. 2008. Newly identified loci that influence lipid concentrations and risk of coronary artery disease. Nat Genet. 40:161–169. doi:10.1038/ng.76.

Schwartzman A, Lin X. 2011. The effect of correlation in false discovery rate estimation. Biometrika. 98:199–214. doi:10.1093/biomet/asq075.

Speliotes EK, Willer CJ, Berndt SI, Monda KL, Thorleifsson G, MAGIC, et al. 2010. Association analyses of 249,796 individuals reveal 18 new loci associated with body mass index. Nat Genet. 42: 937–948. doi:10.1038/ng.686.

Wasserstein RL, Schirm AL, Lazar NA. 2018. Moving to a world beyond “p < 0.05.” Am Statist. 73:1–19. doi:10.1080/00031305.2019.1583913.

Wen X. 2017. Robust Bayesian FDR control using Bayes factors, with applications to multi-tissue eQTL discovery. Stat Biosci. 9:28–49. doi:10.1007/s12561-016-9153-0.

Willer CJ, Li Y, Abecasis GR. 2010. METAL: fast and efficient meta-analysis of genome-wide association scans. Bioinformatics. 26:2190–2191. doi:10.1093/bioinformatics/btq430.

Winkler TW, Day FR, Croteau-Chonka DC, Wood AR, Locke AE, The Genetic Investigation of Anthropometric Traits (GIANT) Consortium, et al. 2016. Quality control and conduct of genome-wide association meta-analyses. Nat Protoc. 9:1192–1212. doi:10.1038/nprot.2014.071.

Yang H, Wang K. 2015. Genomic variant annotation and prioritization with ANNOVAR and WANNOVAR. Nat Protoc. 10:1556–1566. doi:10.1038/nprot.2015.105.

Yengo L, Sidorenko J, Kemper KE, Zheng Z, Wood AR, the GIANT Consortium, et al. 2016. Meta-analysis of genome-wide association studies for height and body mass index in ~700,000 individuals of European ancestry. Hum Mol Genet. 25:3641–3649. doi:10.1093/hmg/ddy271.

Zhu Z, Zhang F, Hu H, Bakshi A, Robinson MR, et al. 2016. Integration of summary data from GWAS and eQTL studies predicts complex trait gene targets. Nat Genet. 48:481–487. doi:10.1038/ng.3538.
Appendix 1: Multivariate BFDP

Consider joint testing of the association between a genetic variant and $L$ traits under model (3). To match our analysis, we set $L = 3$ for the rest of this section but this multivariate extension can be applied to any number of traits.

As in the univariate case described in Materials and Methods and in Wakefield (2007), we approximate the multivariate Bayes’ factor by $P(\hat{\theta}|H_1)/P(\hat{\theta}|H_0)$ where $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3)$ is the estimated vector of variant effect sizes for the three traits. We assume the sampling distribution of $\hat{\theta}$ is multivariate normal with mean $\theta$ and variance $V$ and that $\theta$ has a prior multivariate normal distribution with mean 0 and variance $W$. If $q$ is the $3 \times 3$ matrix of correlation between the traits and $q_{ij}$ is the correlation between traits $i$ and $j$, then:

$$V = \begin{bmatrix} V_1 & \rho_{12} \sqrt{V_1 V_2} & \rho_{13} \sqrt{V_1 V_3} \\ \rho_{21} \sqrt{V_2 V_1} & V_2 & \rho_{23} \sqrt{V_2 V_3} \\ \rho_{31} \sqrt{V_3 V_1} & \rho_{32} \sqrt{V_3 V_2} & V_3 \end{bmatrix}$$

$$W = \begin{bmatrix} W_1 & \rho_{12} \sqrt{W_1 W_2} & \rho_{13} \sqrt{W_1 W_3} \\ \rho_{21} \sqrt{W_2 W_1} & W_2 & \rho_{23} \sqrt{W_2 W_3} \\ \rho_{31} \sqrt{W_3 W_1} & \rho_{32} \sqrt{W_3 W_2} & W_3 \end{bmatrix},$$

where $V_k$ is the sample variance and $W_k$ is the prior variance for trait $k$. Finally, the multivariate approximate Bayes’ factor (ABF) can be calculated as a ratio of prior predictive densities $\hat{\theta}|H_0 \sim \text{MVN}(0, V)$ and $\hat{\theta}|H_1 \sim \text{MVN}(0, V + W)$ and used to approximate the BFDP:

$$ABF_{\text{multi}} = V^{-1} + W^{-1} \exp \left[ \theta^T \left( -V^{-1} + (V + W)^{-1} \right) \theta \right].$$

$$BFDP_{\text{multi}} = \frac{ABF_{\text{multi}} \times PO}{ABF_{\text{multi}} \times PO + 1}.$$