Simulating heat and mass transfer processes during water film evaporation in a horizontal channel

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Abstract. This paper is devoted to the numerical study of heat and mass transfer at evaporative cooling of a stream of a moist air in the horizontal channel, taking into account the final thickness of a film of water. Heat and mass transfer simulation is based on solving a system of differential equations for the laminar flow regime of air flow in a two-dimensional setting in the boundary layer approximation. Numerical studies are performed in a wide range of input parameters changes: temperature $T_0 = 10 \pm 50°C$, relative humidity $q_0 = 0 \pm 100\%$, and Reynolds number $Re = 50 \div 2000$. The geometric dimensions of the channel are taken for height $H = 6$ mm and length $L = 50H$. Calculations are performed at atmospheric pressure.

1. Introduction

One of the simplest and most effective ways to reduce the temperature of the air flow is evaporative cooling. With this method of cooling, the moist air flow is cooled by evaporation at the interface with the water film. A large number of papers have been devoted to the study of heat and mass transfer processes during convective evaporation of water film in channels [1-7]. At the same time, due to the multi-factor nature of these processes, many issues remain under-investigated. Forced convection in laminar flow was studied in [1-4] with adiabatic evaporation in channels, as well as in the presence of additional heat flux on the surface. Research in this direction is being intensively developed today; however, this problem is far from full understanding due to its complexity and multi-parametricity.

In most works on evaporative cooling, the influence of thermal resistance of the film on the evaporation intensity is usually neglected. Obviously, this approximation cannot be performed in all cases, so determining the degree of influence of the liquid film on heat and mass transfer and evaporation rate is of great interest. This work is devoted to the study of conjugate heat and mass transfer in the cocurrent flow of a liquid film and a gas stream at the bottom of a horizontal channel with parallel walls infinitely extended in the transverse direction.

2. Numerical simulation and analysis

The channel under investigation (Figure 1) consists of two horizontal plates, the lower plate is wetted with a film of water, and the upper one remains dry. The channel height is assumed to be $H = 6$ mm and length $L = 50H$. The liquid is fed at the inlet through a flat slot with height $\delta_0$ and the same temperature as that of the gas flow. The Reynolds number of the film is constant and equals $Re_x = 0.5$. 
Figure 1. Flow diagram.

The assumptions made when creating the mathematical model are as follows.

– The flow modes of the water film and wet air are two-dimensional and laminar. Wave effects on the surface of the liquid phase are not taken into account.

– The channel walls are adiabatic.

– The vapor-gas mixture is an ideal gas.

– The surface tension of the water film is not taken into account. The interface is in thermodynamic equilibrium, and the conditions of equality of temperatures, velocities, heat fluxes, and tangential stresses are performed there.

– Radiation heat transfer and viscous dissipation are not taken into account.

– Longitudinal thermal conductivity and diffusion are negligible.

The problem is solved by approximating the boundary layer for the gas and liquid phases.

For a water film, the system of differential equations takes the form:

– continuity equation:

\[ \frac{\partial (\rho_L u_L)}{\partial x} + \frac{\partial (\rho_L v_L)}{\partial y} = 0; \]  

(1)

– equation of motion:

\[ \rho_L u_L \frac{\partial u_L}{\partial x} + \rho_L v_L \frac{\partial u_L}{\partial y} = \frac{\partial}{\partial y} \left( \mu_L \frac{\partial u_L}{\partial y} \right) - \frac{\partial P}{\partial x}; \]  

(2)

– energy equation:

\[ \rho_L c_p u_L \frac{\partial T_L}{\partial x} + \rho_L c_p v_L \frac{\partial T_L}{\partial y} = \frac{\partial}{\partial y} \left( \lambda_L \frac{\partial T_L}{\partial y} \right); \]  

(3)

For the gas phase (moist air), the system of equations is similar to (1) – (3), taking into account the diffusion equation for the vapor phase:

– continuity equation:

\[ \frac{\partial (\rho_A u_A)}{\partial x} + \frac{\partial (\rho_A v_A)}{\partial y} = 0; \]  

(4)

– equation of motion:

\[ \rho_A u_A \frac{\partial u_A}{\partial x} + \rho_A v_A \frac{\partial u_A}{\partial y} = \frac{\partial}{\partial y} \left( \mu_A \frac{\partial u_A}{\partial y} \right) - \frac{\partial P}{\partial x}; \]  

(5)

– energy equation:

\[ \rho_A c_p u_A \frac{\partial T_A}{\partial x} + \rho_A c_p v_A \frac{\partial T_A}{\partial y} = \frac{\partial}{\partial y} \left( \lambda_A \frac{\partial T_A}{\partial y} \right) + \rho_A D (c_p \gamma - c_p) \frac{\partial T_A}{\partial y} \frac{\partial K_A}{\partial y}; \]  

(6)
\[ \rho_A u_A \frac{\partial K_A}{\partial x} + \rho_A v_A \frac{\partial K_A}{\partial y} = \frac{\partial}{\partial y} \left( \rho_A D \frac{\partial K_A}{\partial y} \right). \] (7)

In this case, the boundary conditions are written as:

- at the channel inlet (\( x = 0 \)), the parameters of the water film and air are constant in cross-section:
  \[ u_L = u_{0,L}, \ T_L = T_{0,L}, \ u_A = u_{0,A}, \ T_A = T_{0,A}, \ K_A = K_{0,A}; \] (8)

- on the surface of the lower wall (\( y = 0 \)):
  \[ u_L = 0, \ v_L = 0, \ q_{w1} = -\lambda_L \left( \frac{\partial T_L}{\partial y} \right)_{y=0}; \] (9)

- on the surface of the upper wall (\( y = H \)):
  \[ u_A = 0, \ v_A = 0, \ q_{w2} = -\lambda_A \left( \frac{\partial T_A}{\partial y} \right)_{y=H} = 0, \ \frac{\partial K_A}{\partial y} = 0; \] (10)

- at the interface of the liquid and gas phases (\( y = \delta \)) there is an equality of velocities
  \[ u_L = u_A \] (11)

and tangential stresses \( \tau_L = \tau_A \), respectively,

\[ \mu_L \left( \frac{\partial u_L}{\partial y} \right)_{y=\delta} = \mu_A \left( \frac{\partial u_A}{\partial y} \right)_{y=\delta}. \] (12)

The transverse component of the steam flow velocity on the film surface is written as:

\[ v_y = -\frac{D}{1-K_r} \left( \frac{\partial K}{\partial y} \right)_{y=\delta}, \] (13)

and the surface temperature of the evaporating water film is found from the heat balance equation:

\[ -\lambda_L \left( \frac{\partial T_L}{\partial y} \right)_{y=\delta} = -\lambda_A \left( \frac{\partial T_A}{\partial y} \right)_{y=\delta} - \rho_A D \frac{\partial K_A}{\partial y} \left( \frac{\partial K_A}{\partial y} \right)_{y=\delta}, \] (14)

where \( r \) is the latent heat of water vaporization.

Water vapor on the film surface is in thermodynamic equilibrium, and its concentration is related to the temperature corresponding to the saturation curve \( K_v = f(T_v) \) and is determined according to Dalton’s law [4-7] for an ideal gas:

\[ K_v = \frac{m_v}{m_L m_A / m_A}, \] (15)

where \( m_v = 18 \) and \( m_A = 29 \) are the molecular masses of water vapor and air, respectively, and \( P_v \) is the saturation pressure.

The specified system of equations (1) – (7) together with the boundary conditions (8) – (15) is solved numerically using the finite difference method. The system of equations obtained in this way is written in the form of tridiagonal matrices and solved by the Thomas method (sweep method). The non-linearity of the differential equations is eliminated by simple iterations at each integration step with an accuracy of \( 10^{-3} \% \). The step along the \( x \) axis is assumed to be uniform. The grid is
compressed evenly along the \( y \) axis with a compression ratio of 1.05. Test computational experiments have shown that the optimal grid size was 400x120x70 cells in the longitudinal and transverse (for the water film and air) directions, respectively.

The results of numerical studies are the main thermodynamic and thermohydraulic parameters of the liquid and gaseous phases, such as: velocity fields, temperatures, and concentrations, as well as local characteristics (heat and mass fluxes, and Nusselt and Sherwood numbers).

The dependences of changes in average mass temperatures of air, water film, and concentrations along the channel length for different values of the input relative air humidity are shown in Figure 2. Constant in these calculations are the Reynolds number of the gas (\( \text{Re}_A = 200 \)) and liquid (\( \text{Re}_L = 0.5 \)) phases, as well as the temperature of the vapor-gas mixture at the channel inlet.

![Figure 2](image-url)

**Figure 2.** Changes in temperature and concentration of vapor along the length of the channel \((T_0 = T_{0,A} = T_{0,L} = 30^\circ\text{C}, \text{Re}_A = 200, \text{Re}_L = 0.5)\).

Due to evaporation processes, the average mass (bulk) temperature of moist air decreases along the length of the channel. In this case, the average temperature of the liquid film decreases more intensively, and at a certain distance from the inlet reaches a saturation state. This area of thermal stabilization is due to the non-adiabatic nature of the evaporation, and part of the thermal energy is spent on cooling the film.

As the inlet humidity increases, as seen in Figure 2, the average mass (bulk) values of the temperatures of both phases increase and in the limit of \( \varphi_0 = 100\% \) heat and mass transfer between the saturated vapor-gas mixture and the film stops. At the same time, the length of the stabilization area is reduced, and the saturation mode occurs earlier since the share of thermal energy consumed for cooling the film also becomes smaller.

The concentration of water vapor due to evaporation continuously increases along the length of the channel. The rate of change in \( K_{\text{sat}} \) is proportional to the intensity of liquid evaporation. The most intensive evaporation processes expectedly occur when dry air is supplied to the inlet. If the humidity at the inlet increases, the overall level of vapor concentration also increases, but its distribution along the length becomes flatter, which indicates the suppression of evaporation processes.

Changes in the thermodynamic parameters of the flows of moist air and water film at the outlet of the channel with variations in the input temperatures and Reynolds numbers are shown in Figure 3. The values varied within the temperature range \( T_0 = T_{0,A} = T_{0,L} = 10\pm50^\circ\text{C} \) and Reynolds number \( \text{Re}_A = 50\pm2000, \text{Re}_L = 0.5 \).

From the graphical dependences, it follows that an increase in the input temperature leads to an intensification of the evaporation process from the surface of the water film, hence, the concentration
of water vapor at the outlet of the channel increases. An increase in the Reynolds number leads to enhancement of evaporation processes, as a result of which the liquid film cools more intensively. At the same time, the cooling of the vapor-gas mixture at high Reynolds numbers worsens due to a significant increase in the mass flow of the gas phase, the value of which is proportional to the Reynolds number. For the same reason, the average mass concentration of water vapor also decreases at the channel outlet.

Thus, to obtain the lower temperatures of moist air, it is necessary to reduce the flow rate in the channel. For the inverse problem, namely, for film cooling, the more efficient modes are achieved for large Reynolds numbers.

**Conclusions**
The developed mathematical model describing the heat and mass transfer processes during the evaporative cooling of air in a horizontal channel allows analyzing a complex multiparametric problem. Based on numerical calculations, the dependences of the main parameters of coolants, which include temperature-humidity and thermal-hydraulic characteristics, on the initial parameters have been revealed. The influence of high humidity of the input air on the overall level of vapor concentration and its distribution along the channel length has been established quantitatively. It is shown that an increase in the Reynolds number of the moist air flow leads to an intensification of evaporation processes, as a result of which the liquid film cools the plate more intensively than the gas flow.

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