Production and delivery batch scheduling with a common due date and multiple vehicles to minimize total cost

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Abstract. This paper studies production and delivery batch scheduling problems for a single-supplier-to-a-single-manufacturer case, with multiple capacitated vehicles wherein different holding costs between in-process and completed parts are allowed. In the problem, the parts of a single item are first batched, then the resulting batches are processed on a single machine. All completed batches are transported in a number of deliveries in order to be received at a common due date. The objective is to find the integrated schedule of production and delivery batches so as to satisfy its due date and to minimize the total cost of associated in-process parts inventory, completed parts inventory and delivery. It should be noted that both holding costs constitute a derivation of the so-called actual flow time, and the delivery cost is proportional to the required number of deliveries. The problem can be formulated as an integer non-linear programming and it is solved optimally by Lingo 11.0 software. Numerical experiences show that there are two patterns of batch sizes affected by the ratio of holding costs of in-process and completed parts. It can be used by practitioners to solve the realistic integrated production and delivery batch scheduling problem.

1. Introduction
High expectations of customers and high competition in today’s global market force companies to decrease cost without reducing manufacturing service level. It can be achieved by integrating decisions of different functions such as outsourcing, procurement, production planning, inventory and distribution [1]. However, due to the complexity of the problems, it is not easy to build a model which includes the decisions of all functions. This could be the reason of increasing the models of research pieces in integrating production and distribution, so called IPODS (Integrated Production and Outbound Distribution Scheduling). The most IPODS models consider three performance measures represented by cost-based, time-based and revenue-based[2].

This paper discusses batch scheduling problems for a single supplier produces parts and then the completed parts are delivered to a single manufacturer within a short lead time. In order to satisfy on time delivery, the supplier links both production and delivery stages with minimum inventories. It will lead to decrease cost. Thus, the integrated schedule of production and delivery enables to decrease cost without reducing manufacturing service level.
In a practical situation, the holding cost of work-in-process is smaller than the holding cost of finished good. This paper explicitly distinguishes the holding costs of in-process as work-in-process and completed parts as finished good due to the added value of the parts from the production stage to the delivery stage. In the proposed problem, a single supplier produces parts of a single item. The parts are first batched considered as production batches, and then the resulting batches are processed on a single machine at the production stage. All completed batches are transported as delivery batches using multiple capacitated vehicles to a single manufacturer in order to be received at a common due date. The problem is to find the integrated schedule of production and delivery batches so as to satisfy its due date and to minimize the total cost including holding cost and delivery cost. It should be noted here that both holding costs constitute a derivation of the so-called actual flow time (AFT) and the delivery cost is proportional to the required number of deliveries.

In the literature, much IPODS research focuses on cost-based performance but differs from this research. For example, IPODS research considers total shipping cost under committed delivery dates [3] and [4], includes joint inventory cost of buyer and supplier in the case of deteriorating item in JIT environment [5], considers stage-dependent inventory holding costs (work-in-process and finished-good inventory costs) and delivery cost [1], [6], [7], and [8].

Prasetyaningsih et al. [8] study the same problem as ours but a single capacitated vehicle is used to deliver the batches. In order to meet the due date with minimum total cost, Prasetyaningsih et al. [8] combine the AFT performance and backward scheduling approach. The model is applied to perfect/imperfect matching situations whether the transportation time from supplier to manufacturer and vice versa is shorter or longer than the production time. However, Prasetyaningsih et al. [8] have not considered yet the impact of the ratio of the holding costs between in-process and completed parts, whereas the model shows that the batch decisions are affected by this ratio. This paper aims to investigate the impact of this ratio to the batch decisions which affect the batch scheduling. We restrict to study the imperfect matching situation with the transportation time is longer than the production time.

This paper is organized as follows. Section 2 explains the problem formulation, Section 3 describes solution method and numerical experiences, and Section 4 contains some concluding remarks.

2. Problem formulation

Let there be $n$ parts of a single item processed on a single machine by a supplier, then the completed parts are delivered using $m$ vehicles and should be received by a manufacturer at a common due date, $d$. In each delivery of $R$ deliveries, the parts are batched into $k$ production batches, $b_{h_g[i]}^p(i = 1, ..., k; g = 1, ..., m; h = 1, ..., R)$, with batch sizes of $Q_{h_g[i]}^p$. Each production batch is moved to the production stage by a bounded container with capacity of $c$. The ready time of batches to be processed is assumed to be able to coincide with the starting time of their batch, $B_{h_g[i]}^p$. The processing time of a part, $t$, is given and fixed, and the setup time, $s$, required for a batch before processing is assumed constant, regardless the batch sizes and the batch positions. The completed batches leave the production stage at their completion time, $C_{h_g[i]}^p$.

A number of $k$ production batches must be held in the delivery stage until the delivery time, $B_{h_g}^d$. Each delivery batch, $b_{h_g}^d$, is then transported from supplier to manufacturer within a transportation time, $v$. The delivery batches arrive at the manufacturer location at $C_{h_g}^d$, and the batches must wait during $(d - C_{h_g}^d)$, since the batches should be received by the manufacturer at the common due date. We will find the integrated schedule of production and delivery batch which is a derivation of AFT. Hence, we will define the AFT of the proposed problem.
Let us define the following constant and variables.

- $i$ The index identifying the position, counted from the due date position on a time scale, of a batch on a production schedule.
- $h$ The index identifying the delivery batch number.
- $g$ The index identifying the vehicle number.
- $b_{hg}^i$ The symbol for delivery batch number $h(h = 1, ..., R)$ transported by vehicle $g(g = 1, ..., m)$.
- $b_p^i$ The symbol for production batch sequenced in position $i(i = 1, ..., k)$ in the delivery batch number $h(h = 1, ..., R)$ and transported by vehicle $g(g = 1, ..., m)$.
- $B_{hg}^d$ The departure time of batch $b_{hg}^i$.
- $B_{hg[i]}^p$ The starting time of batch $b_{hg}^i$ to be processed.
- $C_{hg}^d, C_{hg[i]}^p$ The completion times of batch $b_{hg}^i$ and of batch $b_{hg[i]}^p$ respectively.
- $c$ The container’s capacity.
- $c^c, c^t, c^v$ The procurement cost of container, the transportation cost per delivery, and the rental cost of vehicle during the scheduling period respectively.
- $c^{fs}, c^{fm}$ The holding cost per unit time for a completed part at supplier location and at manufacturer location respectively.
- $c^w$ The holding cost per unit time for an in-process part.
- $d$ The due date.
- $F_a$ The total AFT of parts processed in both production and delivery stages.
- $F_{hgi}^d$ The AFT of batch $b_{hg}^i$.
- $F_{ab}^d$ The AFT of total batch $b_{hg}^i$.
- $H_{hg[i]}^{fs}, H_{hg[i]}^{fm}$ The holding time of completed parts in batch $b_{hg}^i$ at supplier and at manufacturer locations respectively.
- $H_{hg[i]}^{w}$ The holding time of in-process parts in batch $b_{hg}^i$.
- $k$ The vehicle’s capacity.
- $n$ The demand rate.
- $Q_{hg}^p$ The size of batch $b_{[h]g}^i$.
- $Q_{hg}^d$ The size of batch $b_{hg}^i$.
- $R$ The number of deliveries.
- $s$ The setup time of a batch.
- $t$ The processing time of a part.
- $v$ The transportation time from supplier to manufacturer or vice versa.
- $W_{hg}$ The delay time of batch $b_{[h]g}$ in the delivery number $h(h = 1, ..., R)$ transported by vehicle $g(g = 1, ..., m)$.
- $X_{hg[i]}$ The binary variable of the procurement cost of container, that is 1 if batch $b_{hg}^i$ needs a container, and 0 for otherwise.
- $\alpha_{hgi}$ The idle time of production facility in the delivery number $h(h = 1, ..., R)$.
- $\beta_a, \gamma_a, \delta_a$ The lower bound, the upper bound and the midpoint of interval $[\beta_a, \gamma_a]$ respectively.

### 2.1. The AFT

The AFT performance in the production stage is introduced by Halim and Ohta [9] which define AFT as the time the batch spends in the shop from its starting time until its due date. If the arrival time of batch $b_{[i]}$ is assumed coincide with the starting time to process the batch, $B_{[i]}$, and all completed batches
must leave the shop at a common due date, \( d \), then the AFT, \( F_{b_{\lceil i \rceil}} \), of batch \( b_{\lceil i \rceil} \) can be formulated as follows.

\[
F_{b_{\lceil i \rceil}} = d - B_{\lceil i \rceil}
\] (1)

The AFT performance of integrated production and delivery batch problems for a single-supplier-to-a-
single manufacturer is extended by Prasetyaningsih et al. [8] which define AFT as the interval time
between respective batch arrival time and its due date, i.e. the time when the batch should be received
by the manufacturer. If delivery batch \( b^d_i \) consisting of \( k \) production batches arrive at the production
stage correspond with the starting time, \( B^p_{\lceil k \rceil} \), of production batch \( b^p_{\lceil k \rceil} \), and all delivery batches should
be received at a common due date, \( d \), then the AFT of batch \( b^d_i \), i.e. \( F^{d}_{b_i} \), can be formulated as follows.

\[
F^{d}_{b_i} = d - B^p_{\lceil k \rceil}
\] (2)

If we use \( m \) vehicles to deliver all batches, we should add a new index, i.e. \( g \), to identify the vehicle
number and the AFT of batch \( b^d_{h \lceil g \rceil} \), \( F^{d}_{b_{h \lceil g \rceil}} \), can be formulated as the following equation.

\[
F^{d}_{b_{h \lceil g \rceil}} = d - B^p_{h \lceil g \rceil}
\] (3)

Let us consider the following illustration to describe AFT. Suppose there are three delivery batches
transported by two vehicles. The batches are expressed by \( b^d_{11}, b^d_{22}, \) and \( b^d_{32} \), where
\( b^d_{11} \) is the first delivery batch transported by vehicle 1,
\( b^d_{22} \) is the second delivery batch transported by vehicle 2, and
\( b^d_{32} \) is the third delivery batch transported by vehicle 2.

If each delivery batch consists of three production batches, then the AFT of delivery batches can be
illustrated by figure 1.

Figure 1. The AFT of batch \( b^d_{h \lceil g \rceil} \)
In figure 1, we can see that the AFT of a delivery batch includes holding time of completed parts at manufacturer and at supplier locations, transportation time from supplier to manufacturer and holding time of in-process parts, so that the AFT of each delivery batch, $F_b$, is formulated as follows.

$$F_b = d - B_p + v + 3 \left( B_p - C_p \right) + t Q_p$$  \hspace{1cm} (4)$$

$$F_b = d - B_p + v + 3 \left( B_p - C_p \right) + t Q_p$$  \hspace{1cm} (5)$$

We can then formulate the AFT of 3 deliveries using 2 vehicles with 3 production batches for each delivery as follows.

$$F_b = (d - C_p) + v + 3 \left( B_p - C_p \right) + 3 t Q_p$$  \hspace{1cm} (6)$$

The AFT of $R$ deliveries using $m$ vehicles with $k$ production batches for each delivery can be generalized to be the following formula.

$$F_b = \sum R \sum m \sum k (d - C_p) + vR + \sum R \sum m \sum k B_p - C_p$$ + $\sum R \sum m \sum k t Q_p$  \hspace{1cm} (7)$$

The AFT of total parts can be calculated by multiplying the AFT of a batch by the number of parts in the batch, since each part in the batch must wait in the batch until all parts in the batch completed [9]. Hence, the AFT of total parts, $F_{ab}$, can be written as follows.

$$F_{ab} = \sum R \sum m \sum k Q_p (d - C_p) + vR + \sum R \sum m \sum k Q_p B_p - C_p$$ + $\sum R \sum m \sum k t Q_p^2$  \hspace{1cm} (8)$$

2.2. Model Formulation

The discussed problem is called as a single-machine multi-vehicles common-due-date (SMMVCD) problem. The assumptions of the model are as follows.

- All parts are divisible,
- The total processed parts are equal to the total demand,
- The transportation time includes packing of some production batches to be a delivery batch, loading the delivery batches to the vehicle and unloading the delivery batches from the vehicle,
The production batch sizes may not exceed the container’s capacity,
The delivery batch sizes may not exceed the vehicle’s capacity,
The homogeneous capacitated vehicles are used to transport delivery batches,
The opportunity cost of vehicle due to idleis neglected,
The delivery cost is proportional to the required number of deliveries.

The objective of the problem is to minimize total cost that can be formulated as follows.

\[ TC = \text{holding cost of completed parts at manufacturing location} + \text{transportation cost} + \text{rental cost of vehicles} + \text{holding cost of completed parts at supplier location} + \text{procurement cost of container} + \text{holding cost of in-process parts} \quad (10) \]

We can be rewritten Equation (10) as follows.

\[ TC = c^f m \sum_{h=1}^{R} \sum_{g=1}^{m} \sum_{i=1}^{k} H_{h,g}^{f_{m}} + c^v R + c^w n^w + c^{f_s} \sum_{h=1}^{R} \sum_{g=1}^{m} \sum_{i=1}^{k} H_{h,g}^{f_{s}} + c^c N^c \]

\[ + c^w \sum_{h=1}^{R} \sum_{g=1}^{m} \sum_{i=1}^{k} H_{h,g}^{w} \quad (11) \]

Applying Equation (9) into Equation (11) yields a formulation of SMMVCD problem as the following INLP model

\[ TC = c^f m \sum_{h=1}^{R} \sum_{g=1}^{m} Q_{h,g}^{d} (d - c_{h,g}^{d}) + c^v R + c^w n^w + c^{f_s} \sum_{h=1}^{R} \sum_{g=1}^{m} \sum_{i=1}^{k} Q_{h,g}^{p} \left( B_{h,g}^{d} - c_{h,g}^{p} \right) \]

\[ + c^w \sum_{h=1}^{R} \sum_{g=1}^{m} \sum_{i=1}^{k} t \left( Q_{h,g}^{p} \right)^2 + c^c X_{h,g} \quad (12) \]

Subject to:

\[ Q_{h,g}^{p} X_{h,g} V_{h,g} t = t batch_{h,g} \quad h = 1, \ldots, R; \quad g = 1, \ldots, m; \quad i = 1, \ldots, k \quad (13) \]

\[ \sum_{i=1}^{k} Q_{h,g}^{p} X_{h,g} V_{h,g} = Q_{h,g}^{d} \quad h = 1, \ldots, R; \quad g = 1, \ldots, m; \quad (14) \]

\[ Q_{h,g}^{p} X_{h,g} V_{h,g} \leq c, \quad h = 1, \ldots, R; \quad g = 1, \ldots, m; \quad i = 1, \ldots, k \quad (15) \]

\[ \sum_{h=1}^{R} \sum_{g=1}^{m} Q_{h,g}^{d} = n, \quad h = 1, \ldots, R; \quad g = 1, \ldots, m; \quad (16) \]

\[ B_{h,g}^{d} + v = d, \quad h = 1; \quad g = 1, \ldots, m \quad (17) \]

\[ B_{h,g}^{p} + t batch_{h,g} - B_{h,g}^{d} = 0, \quad i = 1; \quad h = 1; \quad g = 1, \ldots, m \quad (18) \]
\[ B_{hg[i]}^p - B_{hg[i-1]}^p + tbatch_{hg[i]} + sV_{hg} = 0, \quad i = 2, ..., k; h = 1, ... R; g = 1, ..., m \] (19)

\[ B_{(h-1)g[k]}^p - sV_{(h-1)g} - \alpha_{(h-1)g1} - tbatch_{hg[i]} - B_{hg[i]}^p = 0, \quad i = 1; h = 2, ..., R; g = 1, ..., m \] (20)

\[ B_{hg[i]}^p + tbatch_{hg[i]} - C_{hg[i]}^p = 0, \quad i = 1, ... k; h = 1, ... R; g = 1, ..., m \] (21)

\[ B_{hg}^d - B_{(h-1)g}^d + 2vV_{(h-1)g} = 0, \quad h = 2, ..., R; g = 1, ..., m \] (22)

\[ B_{hg}^d + vV_{hg} - C_{hg}^d = 0, \quad h = 1, ... R; g = 1, ..., m \] (23)

\[ B_{(h-1)g[i]}^p - sV_{(h-1)g} - \alpha_{hg1} - B_{hg1}^d \leq 0, \quad i = k; h = 2, ..., R; g, g_1 = 1, ..., m \] (24)

\[ C_{hg[1]}^p - B_{hg}^d \leq 0, \quad h = 2, ..., R; g = 1, ..., m \] (25)

\[ C_{hg[1]}^p - B_{(h-1)g1[k]}^p - sV_{(h-1)g1} < 0, \quad i = 1; h = 2, ..., R; g, g_1 = 1, ..., m \] (26)

\[ \sum_{g=1}^{m} V_{hg} = 1, \quad h = 1, ... R \] (27)

\[ X_{hg[i]} \in \{0, 1\}, \quad h = 1, ... R; g = 1, ..., m; i = 1, ... k \] (28)

\[ Q_{hg[i]}, Q_{hg[i]}, B_{hg[i]}^p, B_{hg[i]}^d, C_{hg[i]}^p, C_{hg[i]}^d, \alpha_{hg1} \geq 0, \quad h = 1, ... R; g, g_1 = 1, ..., m; i = 1, ... k \] (29)

Equation (13) shows the processing time of each production batch. Equations (14) and (15) enforce the batch restrictions. Equation (16) accomplishes a material balance in both stages. Equation (17) guarantees that the first delivery batch will be received at the due date. Equations (18) to (20) show the starting time of processing the production batches. Equation (21) shows the completion time of the production batches. Equation (22) implies the departure time of vehicles unless for the first delivery, while Equation (23) represents the completion time of the delivery batches. Equation (24) defines the idle time. Equation (25) ensures that the vehicles depart after \( k \) production batches are completed. Equation (26) represents the completion time of the first production batch of each delivery. Equation (27) restricts that each delivery batch will be transported by one vehicle at most. Equation (28) establishes the binary restriction, while Equation (29) specifies the non-negative constraint.

3. Solution method and numerical experiences

The SMMVCD problem is solved by a relaxation of \( R \) variable to be a parameter by calculating \( R = \lceil n/c_k \rceil \). The problem is then solved using general optimization software. The following numerical experiences will show how the model will be solved.

**Example 1.** Consider \( n = 150; \quad v = 20; \quad s = 2; \quad t = 0.5; \quad d = 200; \quad k = 3; \quad c = 20; \quad g = 2; \quad c^e = 25; \quad c^l = 50; \quad c^r = 40; \quad c^{fs} = 20, \) while \( c^w \) and \( c^{fm} \) are stated as a ratio of \( c^{fs} \), i.e. \( c^w = 0.75c^{fs}; \quad c^{fm} = 1.5c^{fs} \). The SMMVCD problem is then solved using Lingo 11.0 software run by PC processor Intel Core i3-3240 CPU @ 3.40GHz with 4 GB of RAM.
In Example 1 we find $R = 3$, since $n = 150$. Lingo reports a global optimum solution where $\text{TC}=151,775$ with the production batch sizes of the first delivery are \{$q_{12[1]}^p = 20; q_{12[2]}^p = 20; q_{12[3]}^p = 20$\} of the second delivery are \{$q_{21[1]}^p = 20; q_{21[2]}^p = 20; q_{21[3]}^p = 20$\}, and of the third delivery are \{$q_{31[1]}^p = 19; q_{31[2]}^p = 11; q_{31[3]}^p = 0$\}. Figure 2 shows the Gantt Chart of Example 1.

![Gantt Chart of Example 1](image_url)

Figure 2 shows that for $n = 150$, there are 3 delivery batches, i.e. $b_{12}^{d1}, b_{21}^{d1}$ and $b_{31}^{d1}$. Batches $b_{12}^{d1}$ and $b_{21}^{d1}$ depart together at $t = 180$ and they arrive at the manufacturer location at $t = 200$. Batch $b_{31}^{d1}$ depart at $t = 140$ and it arrives at the manufacturer location at $t = 160$. Batch $b_{31}^{d1}$ must wait during 40 time unit, since all batches must be received by the manufacturer at the due date, i.e. $t = 200$.

Figure 2 also shows that batch $b_{12}^{d2}$ is loaded by 3 production batches, i.e. $b_{12}^{p1}, b_{12}^{p2}$ and $b_{12}^{p3}$. The production batches start to process at $t = 170$, $t = 158$ and $t = 146$, while the completion times of all production batches are $t = 180, t = 168$ and $t = 156$ respectively. Batch $b_{21}^{d1}$ also contains 3 production batches, i.e. $b_{21}^{p1}, b_{21}^{p2}$ and $b_{21}^{p3}$ which are started to process at $t = 134$, $t = 122$ and $t = 110$ respectively. All production batches of the second delivery are ready to deliver at $t = 144$, but they must wait during $W_{21} = 36$ until vehicle 1 delivers the batches. Batch $b_{31}^{d1}$ contains 2 production batches only, i.e. $b_{31}^{p1}$ and $b_{31}^{p2}$, the remaining batches. Batches $b_{31}^{p1}$ and $b_{31}^{p2}$ are ready to deliver at $t = 108$, but they must wait during $W_{31} = 32$ until vehicle 1 delivers the batches.

Differ from Example 1, we use another ratio of ($c^w/c^{fs}$) in Example 2 to show how this ratio affects the batch sizes.

**Example 2.** Consider the problem-instance of Example 1, but change the ratio with $c^w$ and $c^{fs}$ to be ($c^w/c^{fs}$) = 0.25. Computing the model using Lingo 11.0 software run by PC processor Intel Core i3-3240 CPU @ 3.40GHz with 4 GB of RAM yields a global optimum solution with $\text{TC}=137,280$, where the production batches of the first, second and third delivery are \{$q_{12[1]}^p = 20; q_{12[2]}^p = 20; q_{12[3]}^p = 20$\}, \{$q_{21[1]}^p = 20; q_{21[2]}^p = 20; q_{21[3]}^p = 20$\}, and \{$q_{31[1]}^p = 20; q_{31[2]}^p = 10; q_{31[3]}^p = 0$\}, respectively.

Examples 1 and 2 show a different pattern of the batch sizes, i.e. the third delivery batch. It shows that changing the ratio of the holding costs between in-process and completed parts can change the pattern of the batch sizes. Solving the model with different ratios of ($c^w/c^{fs}$), for more than 50 numerical examples yield solutions with two patterns of batch sizes. If the ratio is low, then $(kR-1)$ production batches will be fulfilled first and the remaining parts are then loaded into the last production batch (so called batch size pattern I). If the ratio is high, the production batches are fulfilled from the first to the
either (R-1) delivery or previous delivery (so called batch size pattern II). Hence, we should find the critical ratio changing the pattern of the batch sizes.

The critical ratio of \(c^w/c^{fs}\) will be approximated by evaluating a midpoint, \(\delta_a\), of an interval \([\beta_a, \gamma_a]\) which minimizes the maximum of \((\delta_a - \beta_a)\) and \((\gamma_a - \delta_a)\). Notation \(\beta_a\) and \(\gamma_a\) define the lower and the upper bounds of ratio \((c^w/c^{fs})\) in the interval. If evaluation of \(\delta_a\) yields batch size pattern I, then set \(\beta_a = \delta_a\), else set \(\gamma_a = \delta_a\). The iteration will be stopped when the differences either \((\delta_a - \beta_a)\) or \((\gamma_a - \delta_a)\) is equal to 0.5. We will show five numerical examples of the different demand rates to show how the critical point will be approximated. Table 1 shows an example of calculating the critical ratio \((c^w/c^{fs})\) of a demand rate, while of the other demand rates can be seen in Table 2.

**Table 1. Approximation of critical ratio of \((c^w/c^{fs})\).**

| Iteration | \(\beta_a\) | \(\gamma_a\) | \(\delta_a\) | Batch size pattern |
|-----------|--------------|--------------|--------------|-------------------|
| 1         | 0.2          | 0.9          | 0.5          | I                 |
|           | \(Q_{12}^p[1] = 20; Q_{12}^p[2] = 20; Q_{12}^p[3] = 20; Q_{21}^p[1] = 20; Q_{21}^p[2] = 20; Q_{21}^p[3] = 12\) |             |
| 2         | 0.5          | 0.9          | 0.7          | I                 |
|           | \(Q_{12}^p[1] = 20; Q_{12}^p[2] = 20; Q_{12}^p[3] = 20; Q_{21}^p[1] = 20; Q_{21}^p[2] = 20; Q_{21}^p[3] = 12\) |             |
| 3         | 0.7          | 0.9          | 0.8          | II                |
|           | \(Q_{12}^p[1] = 20; Q_{12}^p[2] = 20; Q_{12}^p[3] = 20; Q_{12}^p[4] = 20; Q_{21}^p[1] = 20; Q_{21}^p[2] = 20; Q_{21}^p[3] = 12.7\) |             |
| 4         | 0.7          | 0.8          | 0.75         | I                 |
|           | \(Q_{12}^p[1] = 20; Q_{12}^p[2] = 20; Q_{12}^p[3] = 20; Q_{12}^p[4] = 20; Q_{21}^p[1] = 20; Q_{21}^p[2] = 20; Q_{21}^p[3] = 12\) |             |

Table 1 shows that the critical ratio occurs when \((c^w/c^{fs}) = 0.75\). It means that for \(n=112\), if \((c^w/c^{fs}) \leq 0.75\), then \((Kr-1)\) production batches should be fulfilled first, while the remaining parts should be filled into the last production batch (batch size pattern I). If \((c^w/c^{fs}) > 0.75\), batch size pattern II is occurred, i.e. the production batches of the first delivery are fulfilled, while the last two production batches of the second delivery are not fulfilled.

**Table 2. Critical ratios of some demand rates.**

| \(n\) | \((c^w/c^{fs})_{critical}\) |
|-------|--------------------------|
| 112   | 0.75                     |
| 105   | 0.60                     |
| 95    | 0.65                     |
| 67    | 0.65                     |
Table 2 shows that $n=95$ and $n=67$ have a same critical ratio, i.e. 0.65. It means that for $n=67$ and $n=95$ similar batch size pattern occurs.

Based on numerical experiences, we can state that practitioners can use the proposed model to solve the realistic integrated production and delivery batch scheduling problems, since solution of the model yields a global optimum decision. The solution of the model is affected by ratio of holding costs between in-process and completed parts. In general, if the ratio $(c^w/c^f)$ of a certain demand is less than the critical ratio $(c^w/c^f)$, then the batch sizes can be determined using pattern I directly, i.e. $(kR-1)$ production batches should be fulfilled first, while the remaining parts should be filled into the last production batch.

4. Concluding Remarks
This paper deals with production and delivery batch scheduling problems for a single-supplier-to-a-single-manufacturer case, with multiple capacitated vehicles considering a common due date (SMMVCD) wherein different holding costs of in-process and completed parts are allowed. The problems can be formulated as an INLP model with the objective of minimizing $TC$ constitutes a derivation of the AFT. The SMMVCD model can be solved using Lingo 11.0 software to find a global optimum solution. Then, numerical experiences prove that combining AFT and backward scheduling can satisfy on-time delivery with minimum inventory. In addition, there are two patterns of batch sizes affected by the ratio of holding costs between in-process and completed parts. In this research we also approximate a critical ratio represents a changing point of the batching pattern.

This research uses Lingo 11.0 software to solve the model. Fortunately, Lingo’s computation capacity is limited, so that it is important to develop a heuristic solution for further research. In addition, the common due date case can be extended into multiple due dates.

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