Vortex charge in mesoscopic superconductors

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The electric charge density in mesoscopic superconductors with circular symmetry, i.e. disks and cylinders, is studied within the phenomenological Ginzburg-Landau approach. We found that even in the Meissner state there is a charge redistribution in the sample which makes the sample edge become negatively charged. In the vortex state there is a competition between this Meissner charge and the vortex charge which may change the polarity of the charge at the sample edge with increasing magnetic field. It is shown analytically that in spite of the charge redistribution the mesoscopic sample as a whole remains electrically neutral.

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I. INTRODUCTION

Recently it was predicted that the core of an Abrikosov vortex in bulk type-II superconductors is charged. This effect occurs because of the difference of the chemical potential in the superconducting versus normal state. Such a change in chemical potential results in a redistribution of the electrons in the region near the vortex core and culminates in a charging of the vortex core when the superconductor is in the mixed state.

In the present paper we investigate this phenomenon in mesoscopic superconductors. A mesoscopic sample has a typical size which is comparable to the coherence length ($\xi$) and the magnetic field penetration length ($\lambda$). The behaviour of such structures in an external magnetic field ($H$) is strongly influenced by the sample shape and may lead to various superconducting states and different phase transitions between them. Jumps in magnetization were observed when the applied magnetic field or temperature ($T$) are varied.

A number of earlier works studied the geometry dependent magnetic response of mesoscopic superconductors: i) disk shape samples; ii) infinitely long cylinders; iii) ring-like structures and more complicated geometries. Theoretical studies of mesoscopic superconductors are based on the phenomenological Ginzburg-Landau (GL) theory, which successfully describes mesoscopic samples in a wide $H$-$T$ region. In particular, it has been shown that in mesoscopic samples (disks or cylinders) surrounded by a vacuum or an insulator two kinds of superconducting states can exist. Firstly, there is a circular symmetric state with a fixed value of angular momentum, called giant vortex. The observed magnetization jumps correspond to first order phase transitions between giant vortices with different angular momentum. Secondly, in samples with a sufficiently large radius multi-vortex structures can nucleate, which are the analogue of the Abrikosov flux line lattice in a bulk superconductor. These states can be represented as a mixture of giant vortex states with different angular momentum. For multi-vortex states it is also possible to introduce an effective total angular momentum, which is nothing else then the number of vortices in the disk, i.e. the vorticity. With changing the magnetic field there is a second order phase transition between the multi-vortex and the giant vortex states.

It is expected, that the charge distribution in such mesoscopic samples may be appreciably altered due to the presence of a boundary. Furthermore, screening currents near the boundary of the sample will also lead to a redistribution of charge and consequently even in the Meissner state there will be a non-uniform distribution in the sample. In a certain sense, this case can be viewed as a vortex turned inside out, i.e. with its core at infinity. In the presence of vortices there will be an interplay between the Meissner charge and the previously studied vortex charge.

The present paper is organized as follows. In Sec. II we give the necessary theoretical formalism on which our numerical results are based. In Sec. III we investigate the charge distribution in both the Meissner state and the giant vortex states. Then the charge distribution in the multi-vortex state is discussed for thin disks (Sec. IV). Our results are summarized in Sec. V. In the Appendix we present the proof of electrical neutrality in a mesoscopic superconductor of general shape.
II. THEORETICAL APPROACH

We consider a mesoscopic superconducting sample of circular symmetry with radius \( R \) and thickness \( d \) surrounded by an insulating medium. The external magnetic field \( \mathbf{H} = (0, 0, H) \) is uniform and directed normal to the superconductor plane. The starting point of our analysis is that the rotating motion of Cooper pairs around the vortex core leads to a spatial redistribution of charge carriers, which generate the electrostatic potential

\[
\varphi (\mathbf{r}) = \varphi_0 \left( |\psi (\mathbf{r})|^2 - 1 \right),
\]

where \( \psi (\mathbf{r}) \) is the dimensionless superconducting order parameter normalized so that \( |\psi (\mathbf{r})|^2 \) is measured in units of the Cooper pair density in a bulk superconductor. The amplitude \( \varphi_0 \) is different in different approaches. We use \( \varphi_0 = |\alpha| / 2e \) as proposed in Ref.\(^a\). A three-times smaller value \( \varphi_0 = |\alpha| / 6e \) was used in Ref.\(^b\). From the theory in Ref.\(^c\) it approximately follows \( \varphi_0 = (|\alpha| / 2\pi \varepsilon) (dT_c/d\ln \epsilon_F) \), where \( dT_c/d\ln \epsilon_F \approx \ln (\hbar \omega_D/k_BT_c) \sim 1 \) to 10. Thus all approaches yield an electrostatic potential of a similar magnitude.

The distribution of the corresponding charge density \( q (\mathbf{r}) \) is obtained from the Poisson equation\(^d\)

\[
4\pi q (\mathbf{r}) = -\nabla^2 \varphi (\mathbf{r}).
\]

The Cooper pair density \( |\psi (\mathbf{r})|^2 \) is determined from a solution of the system of two coupled non-linear GL equations for the superconducting order parameter, \( \psi (\mathbf{r}) \), and the magnetic field (or vector potential \( \mathbf{A} (\mathbf{r}) \))

\[
\left( -i \nabla - \mathbf{A} \right)^2 \psi = \psi - |\psi|^2 \psi,
\]

\[
\kappa^2 \nabla \times \nabla \times \mathbf{A} = \mathbf{j},
\]

where the density of the superconducting current \( \mathbf{j} \) is given by

\[
\mathbf{j} = \frac{1}{2i} \left( \psi^* \nabla \psi - \psi \nabla \psi^* \right) - |\psi|^2 \mathbf{A}.
\]

Here \( \mathbf{r} = (\mathbf{r}, z) \) is the three-dimensional position in space. Due to the circular symmetry of the sample we use cylindrical coordinates: \( \rho \) is the radial distance from the disk center, \( \theta \) is the azimuthal angle and the \( z \)-axis is taken perpendicular to the disk plane, where the disk lies between \( z = -d/2 \) and \( z = d/2 \). For \( d \to \infty \) we obtain the cylinder geometry. All distances are measured in units of the coherence length \( \xi = \hbar / \sqrt{2m^* |\alpha|} \) \( (m^* = 2m \) is the mass of the Cooper pair), the vector potential in \( ch/2e \xi \), the magnetic field in \( H_c = ch/2e \xi^2 = k\sqrt{2}H_c \), where \( H_c \) is the thermodynamical critical field, the superconducting current in \( j_0 = cH_c / 2\pi \xi \), and \( \kappa = \lambda/\xi \) is the GL parameter.

Eqs. (3, 4) has to be supplemented by boundary conditions for \( \psi (\mathbf{r}) \) and \( \mathbf{A} (\mathbf{r}) \). For the superconducting condensate it can be written as \( \psi = 0 \) and \( \mathbf{A} = 0 \), respectively.

\[
\mathbf{r} \cdot \left( -i \nabla - \mathbf{A} \right) |\psi|_{s = \infty} = 0,
\]

where \( \mathbf{r} \) is the unit vector normal to the sample surface. The boundary condition for the vector potential has to be taken far away from the superconductor where the magnetic field becomes equal to the external applied field \( H \)

\[
\mathbf{A} |_{r \to \infty} = \frac{1}{2} H \rho \mathbf{e}_\theta,
\]

where \( \mathbf{e}_\theta \) denotes the azimuthal direction.

The free energy of the superconducting state, measured in \( F_0 = H^2 \pi / 8 \) units, is determined by the expression

\[
F = \frac{2}{V} \left\{ \int dV \left[ -|\psi|^2 + \frac{1}{2} |\psi|^4 + \left| -i \nabla \psi - \mathbf{A} \psi \right|^2 + \kappa^2 \left( \mathbf{H} (\mathbf{r}) - \mathbf{H} \right)^2 \right] \right\}
\]

with the magnetic field

\[
\mathbf{H} (\mathbf{r}) = \nabla \times \mathbf{A} (\mathbf{r}).
\]
Here we will consider three different geometries for the superconductor: I) an infinitely long cylinder; II) a thin disk with finite thickness \( d \ll \lambda, \xi \); and III) an infinitely thin disk, i.e. \( d \to 0 \). In all three cases the superconducting order parameter does not depend on \( z \). In case I it obviously follows from the sample geometry. In cases II and III it was found\(^1\) that the dependence of \( \psi(\vec{r}) \) on \( z \) is very slow. This allows us to average \( \psi(\vec{r}) \) over the sample thickness and to solve the problem for the two-dimensional problem for the order parameter \( \psi(\rho, \theta) \). However, the magnetic field in case II has a \( z \)-dependence, which is responsible for the demagnetization effect. For both cases I and II we solve the problem numerically by the method proposed in Ref\(^2\). For thin mesoscopic disks we use the results of Refs\(^3,4\) which allowed us to solve the problem semi-analytically.

### III. CHARGE IN THE MEISSNER AND THE GIANT VORTEX STATES

Firstly, we consider the situation with a fixed value of the vorticity \( L \). The giant vortex state has cylindrical symmetry and consequently the order parameter can be written as \( \psi(\vec{r}) = f(\rho) \exp(iL\theta) \). For a thin disk (case III) the order parameter is\(^2\)

\[
\psi(\rho, \theta) = \left( -\Lambda \frac{I_2}{I_1} \right)^{1/2} f_L(\rho) \exp(iL\theta),
\]

where

\[
f_L(\rho) = \left( \frac{H\rho^2}{2} \right)^{L/2} \exp\left( -\frac{H\rho^2}{4} \right) M\left( -\nu, L + 1, \frac{H\rho^2}{2} \right),
\]

\[
I_1 = \int_0^R \rho d\rho \ f_L^1(\rho), \quad I_2 = \int_0^R \rho d\rho \ f_L^2(\rho),
\]

\[
\Lambda = H (1 + 2\nu) - 1.
\]

Here \( M(a, b, y) \) is the Kummer function\(^2\) and the value of \( \nu \) is determined by the non-linear equation, which results from the boundary condition\(^3\)

\[
\left( L - \frac{\Phi}{2} \right) M\left( -\nu, L + 1, \frac{\Phi}{2} \right) - \frac{\nu\Phi}{L + 1} M\left( -\nu + 1, L + 2, \frac{\Phi}{2} \right) = 0.
\]

Here \( \Phi = HR^2 \) is the magnetic flux through the disk in the absence of any flux expulsion. Using\(^3,4\) we can derive explicitly the charge distribution

\[
q(\rho) = 4\Lambda \frac{I_2}{I_1} \left( \frac{H}{2} \right)^L \rho^{2(L-1)} \exp\left( -\frac{H\rho^2}{2} \right)
\]

\[
\times \left\{ L^2 M^2 \left( -\nu, L + 1, \frac{H\rho^2}{2} \right) + \frac{H\rho^2}{2} (2L + 1) M\left( -\nu, L + 1, \frac{H\rho^2}{2} \right) \right\},
\]

where the charge density is measured in units of \( q_0 = \hbar^2 / (16\pi m^* e \xi^4) = (a_B/32\pi \xi^4) e \) (\( a_B = \hbar^2 / me^2 \) is the Bohr radius). For Al we have \( \xi = 250 \text{ nm} \), which results into \( q_0 / e \approx 1.3 \cdot 10^{-13} \text{ nm}^{-3} \).
A. MEISSNER STATE

First, we study the Meissner state, i.e. \( L = 0 \). The radial dependences of the Cooper pair density (or the corresponding distribution of the potential \( \varphi(\rho) \) (see Eq. (1))) and the screening current \( j(\rho) \) are shown in Figs. 1(a-c) for the cases of cylinder, finite disk and very thin disk, respectively, for different values of the applied field. Fig. 1(d) shows these dependencies for a cylinder at the field \( H = 0.52H_c \), but for different \( \kappa \) values. Due to the finite radial size of the samples all distributions are inhomogeneous along the radius of the sample. The Cooper pair density is maximum at the center and decays towards the sample edge. As a result, in the center of the sample there is a region of positive charge while near the edge a negative "screening" charge is created. To avoid confusion let us note that for simplicity we write "positive charge" instead of "charge of the same sign as is the sign of the dominant charge carriers". In the cylinder the Cooper pair density decreases with increasing field, while both the screening superconducting current and the charge polarization monotonously increase. The behavior for the disks is more complicated. For small fields the picture is very similar to the one of the cylinder. But for fields where the Meissner state becomes metastable (i.e., \( H/H_c > 0.32 \) for the parameters used in Fig. 1(b)) the screening charge becomes maximal. The positive charge is pulled further towards the center of the disk and the screening charge region expands. Notice that now the maximum of the positive charge decreases with field and also at the surface its absolute value decreases. For a cylinder, at fixed field but with increasing \( \kappa \) (Fig. 1(d)) its charge distribution behaves similar as for fixed \( \kappa \) and increasing magnetic field (see Fig. 1(a)).

Notice that even for \( L = 0 \) when no vortex is present inside the superconductor, there is still a non-uniform charge distribution. This charge redistribution can be characterised by two quantities: i) the distance \( \rho^* \) which separates the positive and negative charge regions, and ii) the total screening charge \( Q_{-,scr} \) which is defined by the integral over the region \( V_- \) occupied by the negative charge:

\[
Q_{-,scr} = 2\pi d^2 \int_{\rho^-}^{\rho^*} \rho q_{scr}(\rho) d\rho.
\]

The dependences of \( \rho^*(H) \) are shown in Fig. 2(a-c) for the cylinder, the finite disk and the thin disk, respectively. In Fig. 2(c) the open squares refer to the magnetic field region where the \( L = 0 \) state is metastable (the crossed circles will be explained below). Notice that \( \rho^* \) decreases with increasing field and this decrease is more pronounced for thinner disks. The magnetic field dependences of the absolute value \( |Q_-| \) of the screening charge is shown in Fig. 3(a-c) for the same geometries. The screening charge \( Q_{scr} \) increases with magnetic field, but for the disk cases a local maximum is reached. This local maximum is reached for fields where \( \rho^* \) starts to decrease more strongly. Taking \( \xi = 250 \) nm and \( \alpha_B = 0.05 \) nm we obtain that for the characteristic value of screening charge \( |Q_{-,scr}| \approx 3 \cdot 2\pi^2 q d \xi \approx 8 \cdot 10^{-6} e \) at the field \( H = 0.3H_c \) for the Al disk with \( d/\xi = 0.2 \) (the case II). For strongly type-II superconductors which have a very small coherence length \( \xi \) this induced charge can be made orders of magnitude larger.

By direct integration of \( q(\rho) \) over the sample surface one can convince ourselves that the total charge per unit of sample length \( Q = \oint d\theta \oint_0^R \rho q(\rho) d\rho = 0 \), i.e. there is charge neutrality over the whole sample. But this fact can be easy generalized analytically for states with any vorticity. The proof is given in the Appendix.

B. GIANT VORTEX STATE

The same dependences as in Figs. 1(a-d) are shown in Figs. 4(a-d) for samples in the \( L = 1 \) vortex state. In this state \( |\psi(\rho)|^2 = 0 \) in the center of the vortex core located in the centre of the sample. Notice, that for a cylinder the charge distribution almost does not change with magnetic field. The reason is that the external field affects the Cooper pair density only near the sample edge region. This is different for the disk geometry, where large demagnetization effects strongly influences the penetration of the magnetic field in the disk which changes the vortex structure. The vortex core is negatively charged and at small fields the positive charge outside the vortex extends up to the border of the sample. The dependences of the position \( \rho^*(H) \) and the size of the charge pile-up \( Q_-|V^*| \) for the \( L = 1 \) state are shown in Figs. 2(a-c) and 3(a-c).

With increasing external magnetic field the screening current at the sample surface increases and makes the region near the border of the sample negatively charged. In this case there exist two \( \rho^* \), where \( q(\rho^*) = 0 \). The core of the vortex is negatively charged with total charge \( Q_{-,v} \). Around this core there is a ring of positive charge which compensates the charge of the vortex core and the surface charge. Near the surface a ring of negative screening charge exists with total charge \( |Q_{-,scr}| \). The size of the latter increases with increasing magnetic field. For disks, \( |Q_{-,scr}| \) reaches a local maximum after which it decreases in the large magnetic field region; this is the region where the \( L = 1 \) state is unstable (see circles with crosses in Fig. 3(c)).
Next we investigated the charge distribution for the vortex states with \( L > 1 \) and we limited ourselves to the thin disk case. From Eq. (14) one finds immediately that \( q = 0 \) in the center of the disk when \( L \geq 2 \). Consequently, the charge distribution in the vortex core has a ring shape. To illustrate this we show in Figs. 3(a,b) the same dependences as in Fig. 2(c) but now for \( L = 2 \) and 3, respectively. With the exception of the core region the charge distribution for the giant vortex states is qualitatively similar to the case \( L = 1 \), and the charge on the sample surface changes sign with increasing external magnetic field. Notice also that the number of areas where the charge changes its sign does not increase with \( L \); it equals 2 for small fields and increases to 3 for higher magnetic fields.

### IV. VORTEX CHARGE IN THE MULTI-VORTEX STATE

For sufficiently large radial size of the superconductor the giant vortex state can break up into multi-vortices. To explain the physics we limit ourselves to the case of a thin disk. It was shown that the order parameter of the multi-vortex state in general can be viewed as a superposition of giant vortex states with different \( L_j \)

\[
\psi(\overrightarrow{\rho}) = \sum_{L_j=0}^{\infty} C_{L_j} f_{L_j}(\rho) \exp(iL_j\theta),
\]

where \( L \) is now the value of the effective total angular momentum which equals the number of vortices in the disk. For disks with not so large radius (3 \( \xi \)) the order parameter of the multi-vortex state is the superposition of only two states and is described by the expression

\[
\psi(\overrightarrow{\rho}) = C_{L_1} f_{L_1}(\rho) \exp(iL_1\theta) + C_{L_2} f_{L_2}(\rho) \exp(iL_2\theta),
\]

where

\[
C_{L_1} = \left( -\Lambda_{L_1} A_{L_1} B_{L_1} + 2\Lambda_{L_2} A_{L_1} L_1 B_{L_2} \right)^{1/2}, \quad C_{L_2} = \left( -\Lambda_{L_2} A_{L_2} B_{L_2} + 2\Lambda_{L_1} A_{L_2} L_1 B_{L_1} \right)^{1/2},
\]

\[
A_{L_1} = \frac{2\pi d}{V} \int_0^R \rho d\rho \ f_{L_1}(\rho), \quad A_{L_1, L_2} = \frac{2\pi d}{V} \int_0^R \rho d\rho \ f_{L_1}^2(\rho) \ f_{L_2}(\rho), \quad B_{L_1} = \frac{2\pi d}{V} \int_0^R \rho d\rho \ f_{L_1}''(\rho),
\]

and \( f_{L_i}(\rho) \) and \( \Lambda_{L_i} \) are determined by Eqs. (10) and (12), respectively. The charge density distribution is then given by the expression

\[
q(\rho, \theta) = 2C_{L_1}^2 \left[ f_{L_1}^2(\rho) + f_{L_1}(\rho) f_{L_1}''(\rho) + \frac{1}{\rho} f_{L_1}(\rho) f_{L_1}'(\rho) \right] + 2C_{L_2}^2 \left[ f_{L_2}^2(\rho) + f_{L_2}(\rho) f_{L_2}''(\rho) + \frac{1}{\rho} f_{L_2}(\rho) f_{L_2}'(\rho) \right] + 2C_{L_1} C_{L_2} \cos[(L_1 - L_2)\theta] \left[ f_{L_1} f_{L_2}(\rho) + 2f_{L_1}'(\rho) f_{L_2}(\rho) + f_{L_1}(\rho) f_{L_2}'(\rho) \right] + \frac{1}{\rho} \left( f_{L_1}(\rho) f_{L_2}(\rho) + f_{L_1}(\rho) f_{L_2}'(\rho) \right) - \frac{(L_1 - L_2)^2}{\rho^2} f_{L_1}(\rho) f_{L_2}(\rho),
\]

where the prime denotes the derivative with respect to \( \rho \). The explicit expression is rather lengthy and is therefore not given here.

Earlier analyses have shown that there exist two kinds of multi-vortex states: i) stable configurations which correspond to a minimum of the free energy; and ii) states which correspond to saddle points of the free energy. The latter ones correspond to the energy barrier states between states with different vorticity \( L \) and describe the penetration of flux into the disk. Due to the transitions between the different \( L \) states with increasing (or decreasing) external field some giant vortex states are never realised (for example, such states for \( L = 0 \) and 1 correspond to the crossed circles in Figs. 2(c) and 3(c)).
As an example, we consider a thin disk with \( R/\xi = 4.0 \). The charge density \( q(x, y) \) distribution over the disk for the different kinds of multi-vortex states are shown in Figs. 6 and 7. In Fig. 6 this distribution is given for the saddle point state (1:2) at the field \( H = 0.32H_{c2} \) and the contour plot of the distribution of the Cooper pair density \( |\psi|^2 \) for this state is shown at the bottom of the figure. The dark regions on the \( |\psi|^2 \) contour plot correspond to low Cooper pair density. The same distributions for the stable multi-vortex state (0:4) at the field \( H = 0.75H_{c2} \) are shown in Fig. 7. One can see regions of negative charge located at the vortex core and positive charge near the edge of the sample. Using Eqs. (17-19) it is easy to prove that also in the multi-vortex state the disk is electrically neutral as a whole (see also the Appendix).

V. CONCLUSIONS

We studied theoretically the redistribution of electrical charge in circular mesoscopic superconducting samples with different shape, i.e. disks and cylinders. The theory applies for intermediate temperatures where the Ginzburg-Landau theory still gives reasonable results while the share of the superconducting electrons is already of order unity so that the screening by normal particles may be neglected. Previously, it was predicted that the vortex core in bulk type-II samples is negatively charged. Here we found that even in the Meissner state with no vortices inside the sample there exists a non-uniform charge distribution. Due to the finite radial size a region near the sample edge becomes negatively charged while the interior of the sample has a corresponding positive charge. This charge redistribution is a consequence of the screening currents near the sample edge which makes it behave like a vortex which is turned inside out. When vortices are inside the sample there is a superposition of the vortex charge and this Meissner charge. Because of this interplay between vortex charge, which is positive near the sample surface, and the Meissner charge, which is negative at the sample surface, the charge at the sample edge changes sign as a function of the applied magnetic field. We also proved analytically that the there is only a redistribution of charge and that the total sample charge is neutral as long as the boundary condition (6) is satisfied.

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VII. APPENDIX: PROOF OF ELECTRICAL NEUTRALITY IN A MESOSCOPIC SAMPLE

Using Eqs. (12) and Gauss theorem the total charge \( Q \) can be expressed as

\[
Q = -\varepsilon \frac{|\alpha|}{2\varepsilon} \int_V \nabla^2 |\psi(\vec{r})|^2 \, dV
\]

\[
= -\varepsilon \frac{|\alpha|}{2\varepsilon} \int_S \nabla |\psi(\vec{r})|^2 \, d\vec{S}
\]

\[
= -\varepsilon \frac{|\alpha|}{2\varepsilon} \int_S \vec{n} \cdot \left[ \psi^*(\vec{r}) \nabla \psi(\vec{r}) + \psi(\vec{r}) \nabla \psi^*(\vec{r}) \right] \, dS
\]

\[
= -\varepsilon \frac{|\alpha|}{2\varepsilon} \int_S \vec{n} \cdot \left[ \psi^*(\vec{r}) \left( \nabla - i\vec{A} \right) \psi(\vec{r}) + \psi(\vec{r}) \left( \nabla + i\vec{A} \right) \psi^*(\vec{r}) \right] \, dS.
\]

From the boundary condition (1) and its complex conjugate it follows immediately that \( Q \equiv 0 \). Notice, that this result is very general, it is valid for any vortex configuration (the giant vortex states and the multi-vortex ones) and arbitrary shape of the superconducting sample as long as the boundary condition (1) is satisfied.

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FIG. 1. The radial dependences of the Cooper pair density $|\psi(\rho)|^2$, the charge density $q(\rho)$ and the supercurrent density $j(\rho)$ in the Meissner state for (a) an infinite long cylinder with $\kappa = 1.0$, (b) finite thickness disk with $d/\xi = 0.2$, $\kappa = 1.0$, (c) very thin disk for different magnetic fields, and (d) for an infinite long cylinder with different Ginzburg-Landau parameter $\kappa$ at the magnetic field $H = 0.52H_c2$. All samples have the same radius $R/\xi = 4.0$.

FIG. 2. The position of the boundary between the regions of negative and positive charges inside the sample as a function of the external magnetic field for: (a) the infinite long cylinder with $H = 0$, (b) the finite thickness disk with $d/\xi = 0.2$, $\kappa = 1.0$, and (c) the thin disk. The $\rho^*(H)$ dependences for the Meissner state are shown by the solid line, and for the vortex state - by the dashed lines. The dash-dotted curves represent the position of zero current.

FIG. 3. The absolute value of the negative charge inside the sample as a function of the external magnetic field for: (a) the infinite long cylinder with $\kappa = 1.0$, (b) the finite thickness disk with $d/\xi = 0.2$, $\kappa = 1.0$, and (c) the thin disk. The dependences for the Meissner state are shown by the solid line, and for the vortex state - by the dashed lines.

FIG. 4. The same as in Fig. 3 but for the single vortex state (with $L = 1$).

FIG. 5. The radial dependences of the Cooper pair density $|\psi(\rho)|^2$, the charge density $q(\rho)$ and the supercurrent density $j(\rho)$ in the giant vortex state with (a) $L = 2$ and (b) $L = 3$ for a thin disk with $R/\xi = 4.0$ and for different applied magnetic fields.
FIG. 6. The charge density distribution for the (1:2) saddle-point state at the field $H = 0.32H_{c2}$ for the thin disk with $R/\xi = 4.0$. The bottom contour plot shows the distribution of the Cooper pair density.

FIG. 7. The same as in Fig. 6 but for the (0:4) multivortex state in the disk with $R/\xi = 4.0$ at the field $H = 0.75H_{c2}$. 
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