The explanation of matter-antimatter asymmetry of the universe requires new origins of CP-violation beyond the Standard Model (BSM). One of the interesting CP-violating operators that can be induced in the QCD Lagrangian due to BSM physics is the Weinberg operator.\(^1\) In this report I point out a novel relation between the hadronic matrix element of the Weinberg operator and a certain twist-four correction in polarized Deep Inelastic Scattering (DIS). Such a relation suggests an exciting possibility that polarized DIS experiments can provide useful information to the physics of the nucleon electric dipole moment (EDM), or more generally, BSM-origins of hadronic CP violations.

The Weinberg operator is a dimension-six purely gluonic operator

\[
O_W = g f_{abc} E^a_{\mu \nu} F^b_{\mu \sigma} F^c_{\nu \sigma}.
\]

This operator violates CP and can be induced in the QCD Lagrangian by physics beyond the Standard Model. It is considered as one of the candidate operators to generate a large EDM of the nucleons and nuclei.

The key observation is the following exact operator identity

\[
O_W = - \partial^\mu (\tilde{F}_{\mu \nu} D^\nu - \frac{1}{2} \tilde{F}_{\mu \nu} D^2 F^{\mu \nu}) = O_4 + O_D,
\]

Eq. (2) shows that one can choose \(O_W\) and \(O_4\) as the independent basis of operators and study their mixing. Due to the equation of motion, one can write

\[
O_4 \approx \partial^\mu (\bar{\psi} g \tilde{F}_{\mu \nu} \gamma^\nu \psi),
\]

to linear order in partial derivative \(\partial^\mu\). Such mixing is usually neglected in the literature because \(O_4\) is a total derivative and hence does not contribute to the CP-violating effective action \(\int d^4 x O_4 = 0\). However, when it comes to hadronic matrix elements, mixing becomes crucial because only the nonforward matrix element is nonvanishing. Specifically, their RG equation takes the form

\[
\frac{d}{d \ln \mu^2} \left( \frac{O_W}{O_4} \right) = - \frac{\alpha_s}{4\pi} \left( \begin{array}{cc} \gamma_W & 0 \\ 0 & \gamma_{12} \end{array} \right) \langle O_W \rangle \quad \langle O_4 \rangle
\]

where

\[
\gamma_W = \frac{7}{3} N_c + \frac{2}{3} n_f
\]

is the anomalous dimension of the Weinberg operator.\(^2\) The anomalous dimension of \(O_4\) is the same as that of the undifferentiated, twist-four operator \(\bar{\psi} g \tilde{F}_{\mu \nu} \gamma_\mu \psi\) and is known to be\(^3\)

\[
\gamma_4 = \frac{7}{3} \alpha_s + \frac{2}{3} n_f.
\]

To determine the off-diagonal component \(\gamma_{12}\), I evaluate the following three-point Green’s function

\[
\langle 0 | T \{ \bar{\psi} (-k) A_\mu^a (q) \bar{\psi} (p) O_W \} | 0 \rangle
\]

with off-shell momenta and nonzero momentum transfer \(k = p - q \neq 0\). The result is

\[
\gamma_{12} = -3 N_c.
\]

It immediately follows that the following linear combination is the eigenstate of the RG evolution

\[
O_W + \frac{\gamma_{12}}{\gamma_W - \gamma_4} O_4 = O_W - \frac{9 N_c^2}{3 N_c^2 + 4} O_4.
\]

Since this operator has a rather large anomalous dimension \(\gamma_W \sim 10\), in particular larger than \(\gamma_4\) by a factor of about 2, at high enough renormalization scales \(\mu^2\) one has

\[
\langle O_W \rangle \approx \frac{9 N_c^2}{3 N_c^2 + 4} \langle O_4 \rangle \approx 2.61 \langle O_4 \rangle.
\]

In terms of the nucleon matrix elements

\[
\langle P | \bar{\psi} g \tilde{F}_{\mu \nu} \gamma_\mu \psi | P^\prime \rangle = -2 f_0 M^2 S^a
\]

\[
\frac{1}{M^2} \langle P^\prime | g f_{abc} E^a_{\mu \nu} F^b_{\mu \sigma} F^c_{\nu \sigma} | P \rangle = 4 E \bar{u} i \gamma_5 u.
\]

I get

\[
E \approx \frac{9 N_c^2}{2(3 N_c^2 + 4)} f_0 \approx 1.3 f_0.
\]

Therefore, one can evaluate the matrix element \(E\) of the Weinberg operator through the measurement of the \(f_0\) parameter relevant to the twist-four corrections in polarized DIS.\(^4,5\) \(f_0\) can be extracted from the \(g_1(x)\) structure function measured at the future Electron-Ion Collider (EIC) in the U.S. This is a new connection between the EIC and physics beyond the Standard Model. It will demonstrate the EIC’s unique capability to address low-energy nucleon observables in a high energy collider.

References
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