Heavy baryon spectrum on lattice with NRQCD bottom and HISQ lighter quarks

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We determine the mass spectra of heavy baryons containing one or more bottom quarks along with their hyperfine splittings and various mass differences on MILC 2+1 Asqtad lattices at three different lattice spacings. NRQCD action is used for bottom quarks whereas relativistic HISQ action for the lighter up/down, strange and charm quarks. We consider all possible combinations of bottom and lighter quarks to construct the bottom baryon operators for the states $J^P = 1/2^+$ and $3/2^+$.

I. INTRODUCTION

Lattice QCD has been extensively employed to study $B$ quark phenomenology and especially the decay constants, mixing parameters needed for CKM matrix elements and mass differences in the meson sector [1]. The $B$ mesons spectroscopy and mass splittings has undergone thorough investigation on lattice, see [1] references therein, with increasing impact on heavy flavor phenomenology, see for instance [2]. However, studying heavy baryons with bottom quark(s) on lattice is relatively a recent pursuit. Some of the early studies of heavy baryons on lattice can be found in [3, 4]. Of late, a slew of low lying $J^P = 1/2^+$ bottom baryons – $\Lambda_b$, $\Sigma_b$, $\Xi_b$ and $\Omega_b$ – have made entries in PDG [3]. Possibilities of discoveries of $J^P = 3/2^+$ are rather high, whereas doubly and triply bottom baryons are right now beyond the reaches of present experiments. In this state, lattice QCD can provide an insight into the masses, mass splittings and other properties of such bottom baryons from the first principle. To this end, quite a few lattice investigations of heavy baryons containing one, two or three bottom quarks have been undertaken with a range of light quark actions [9–11]. For an extensive list on contemporary lattice literature on heavy baryon see [11].

These studies on heavy hadrons with bottom quark(s) is largely made possible by the use of nonrelativistic QCD action proposed and formulated in [12, 13] because of the well-known fact that the current lattice spacings, even for as low as 0.045 fm, render $a m_b \gtrsim 1$. Although in almost all of the above studies employed different heavy quark actions for charm quark, HISQ action [14] is becoming an increasingly popular choice for charm quark. This approach of simulating bottom quark with NRQCD and the rest of the quarks i.e. charm, strange and up/down with HISQ for calculation of bottom baryon spectra has been adopted in this work.

In this paper we present our lattice QCD results of heavy baryons involving one and two bottom quarks. We consider all possible combinations of bottom quark(s) with charm, strange and up/down lighter quarks of the form $(lb)$, $(lb)$ and $(l_1l_2b)$, where $b$ is the bottom quark and $l$ are the lighter charm, strange and up/down quarks. We are addressing the charm quark as “light” quark in the sense that we have used relativistic action for it. The action for the lighter quarks is HISQ [14] and NRQCD [12, 13] for the bottom. We discuss these actions in Section II. The propagators generated using nonrelativistic and relativistic actions are required to be combined to construct baryon states of appropriate quantum numbers. A discussion to achieve this combination is spread over both Section II and III. The bottom baryon operators are described in details in Section II. In the following Section IV, we present the simulation details including the lattice ensembles used, various parameters and tuning of different quark masses. The lattice calculations are carried out at three different lattice spacings with fixed $m_u/d/m_s$ value and several quark masses. We assemble our bottom baryon spectrum results along with hyperfine and various other mass splittings in the Section V. Finally we conclude and summarize in Section VI which also includes a comparison of our results to the existing ones.

II. QUARK ACTIONS

As of now the bottom ($b$) quark masses are not small i.e. $am_b \neq 1$ in units of the lattice spacings available. The use of improved NRQCD is the action of choice for the $b$-quarks. We have used $O(a^0)$ NRQCD action in this paper. The charm ($c$) quark is also similarly heavy enough for existing lattices, but Fermilab proposal [15], made it possible to work with relativistic actions for $c$-quark, provided we trade the pole mass with the kinetic mass. Subsequently, HISQ action [14] became available for relativistic $c$-quark. In this paper, we choose HISQ action for the $c$-quark along with $s$ and $u/d$ quarks. In this work, as because we use the same relativistic HISQ action for all quarks except the bottom, we use the word light quarks to refer to $c$, $s$ and $u/d$ quarks. This is similar to what has been done in [24] for $B$ meson states calculation. Besides, one of the big advantages of this choice of action is the ability to use the MILC code [16] for this bit.

A. NRQCD Action and $b$ quark

In order to perform lattice QCD computation of hadrons containing bottom quarks in publicly available relatively coarse lattice spacings, NRQCD [12, 13] is per-
haps the most suitable and widely used quark action for bottom. As is understood, the typical velocity of a b quark inside a hadron is nonrelativistic. Comparison of masses of bottomonium states to the mass of b quark supports the fact that the velocity of b quark inside hadron \( (v^2 \sim 0.1) \) is much smaller than the bottom mass. For example \( M_T = 9460 \text{ MeV} \) whereas \( 2 \times m_b = 8360 \text{ MeV} \) in the \( \overline{MS} \) scheme. For bottom hadrons containing lighter valence quarks, the velocity of the bottom quark is even smaller. This allows us to study the b quark with nonrelativistic effective field theory. NRQCD will remain action of choice for b quark until finer lattices with \( a m_b < 1 \) become widely available.

In NRQCD, the upper and lower components of the Dirac spinor decouple and the b quark is described by two component spinor field, denoted by \( \psi_b \). NRQCD Lagrangian has the following form

\[
\mathcal{L} = \bar{\psi}_b(x,t)[U(x)\psi_b(x,t+1) - \psi_b(x,t) + aH\psi_b(x,t)]
\]

where \( a \) is the lattice spacing and \( U(x) \) is the temporal gauge link operator. \( H = H_0 + \delta H \) is the NRQCD Hamiltonian where,

\[
H_0 = -\frac{\Delta^2}{2m_b} - \frac{a}{4n} \left( \frac{\Delta^2}{2m_b} \right)^2 \quad \text{and} \quad \delta H = \sum_i \delta H^{(i)}
\]

The \( H_0 \) is the leading \( \mathcal{O}(a^2) \) term, the \( \mathcal{O}(a^4) \) and \( \mathcal{O}(a^6) \) terms are in \( \delta H \) with coefficients \( c_1 \) through \( c_7 \).

\[
\begin{align*}
\delta H^{(1)} &= -c_1 \left( \frac{\Delta^2}{2m_b} \right)^2 \\
\delta H^{(2)} &= c_2 \frac{ig}{8m_b^3} \left( \tilde{\Delta}^\pm \cdot \tilde{E} - \tilde{E} \cdot \tilde{\Delta}^\pm \right) \\
\delta H^{(3)} &= -c_3 \frac{g}{8m_b^3} \sigma \cdot \left( \tilde{\Delta}^\pm \times \tilde{E} - \tilde{E} \times \tilde{\Delta}^\pm \right) \\
\delta H^{(4)} &= -c_4 \frac{g}{2m_b} \sigma \cdot \tilde{F} \\
\delta H^{(5)} &= -c_5 \frac{g}{8m_b} \left\{ \tilde{\Delta}^2, \sigma \cdot \tilde{B} \right\} \\
\delta H^{(6)} &= -c_6 \frac{3g}{64m_b} \left\{ \tilde{\Delta}^2, \sigma \cdot \left( \tilde{\Delta}^\pm \times \tilde{E} - \tilde{E} \times \tilde{\Delta}^\pm \right) \right\} \\
\delta H^{(7)} &= -c_7 \frac{ig^2}{8m_b^3} \sigma \cdot \tilde{E} \times \tilde{E}
\end{align*}
\]

The \( b \) quark propagator is generated by the time evolution of this Hamiltonian,

\[
G(\tilde{x}, t+1; 0, 0) = \left( 1 - \frac{aH_0}{2m_b} \right)^n \left( 1 - \frac{a\delta H}{2} \right) U_4(\tilde{x}, t) + G(\tilde{x}, t; 0, 0)
\]

with \( G(\tilde{x}, t; 0, 0) = \left\{ \begin{array}{ll}
0 & \text{for } t < 0 \\
\delta \tau_0 & \text{for } t = 0
\end{array} \right. \)

The tree level value of all the coefficients \( c_1, c_2, c_3, c_4, c_5, c_6 \) and \( c_7 \) is 1. Here \( n \) is the factor introduced to ensure numerical stability at small \( a m_b \) [13], where \( n > 3/2m_b \). The symmetric derivative \( \Delta^\pm \) and Laplacian \( \Delta^2 \) in terms of forward and backward derivatives are

\[
\begin{align*}
\Delta^\pm &= \frac{1}{2}(\Delta^+ + \Delta^-) \\
\Delta^2 &= \sum_i \Delta^+_i \Delta^-_i = \sum_i \Delta^-_i \Delta^+_i
\end{align*}
\]

By Taylor expanding the symmetric derivative and the Laplacian operator, we can find their forms corrected up to \( \mathcal{O}(a^4) \) that are used in the above Equation [12]

\[
\begin{align*}
\Delta^\pm &= \Delta^\pm - \frac{a^2}{6} \Delta^+_i \Delta^-_i \\
\Delta^2 &= \Delta^2 - \frac{a^2}{12} \sum_i [\Delta^+_i \Delta^-_i]^2.
\end{align*}
\]

In the same way, the gauge fields are improved to \( \mathcal{O}(a^4) \) using cloverleaf plaquette,

\[
g\tilde{F}_{\mu \nu}(x) = gF_{\mu \nu}(x) - \frac{a^4}{6} \left[ \Delta^+_\mu \Delta^-_\mu + \Delta^+_\nu \Delta^-_\nu \right] gF_{\mu \nu}(x)
\]

The chromo-electric \( \tilde{E} \) and chromo-magnetic \( \tilde{B} \) fields in \( \delta H^{(3)} \) and \( \delta H^{(4)} \) of Equation [3] are therefore \( \mathcal{O}(a^4) \) improved.

\section*{B. HISQ charm and lighter quarks}

For the lighter quarks – charm, strange and up/down – relativistic HISQ action [14] is used. Apart from anything else, from practical point of coding the bottom-light operators (\( lbb, llb, l_1l_2b \), using the same relativistic action for all lighter quarks offers a great degree of simplification. The HISQ action is given by,

\[
S = \sum_x I(x) (\gamma^\mu D^{\text{HISQ}}_\mu + m) I(x)
\]

with \( D^{\text{HISQ}}_\mu = \Delta_\mu(W) - \frac{a^2}{6} (1 + \epsilon) \Delta^3_\mu(X) \)

with \( W_\mu(x) = F^{\text{HISQ}}_\mu U_\mu(x) \) and \( X_\mu(x) = UF_\mu U_\mu(x) \).

The \( F^{\text{HISQ}}_\mu \) has the form

\[
F^{\text{HISQ}}_\mu = \left( F_\mu - \sum_{\rho \neq \mu} \frac{a^2 \delta_{\rho \nu}}{2} \right) UF_\mu
\]

Here the \( U \) is the unitarizing operator, it unitarizes whatever it acts on and the smearing operator \( F_\mu \) is given by

\[
F_\mu = \prod_{\rho \neq \mu} \left( 1 + \frac{a^2 \delta_{\rho \nu}}{4} \right).
\]
The $\delta_p$ and $\delta_p^{(2)}$ in the Equations (10) and (11) are covariant first and second order derivatives. Because HISQ action reduced $\mathcal{O}(\alpha_s a^2)$ discretization errors found in Asqtad action, it is well suited for $s$ and $u/d$ quarks. The parameter $\epsilon$ in the coefficient of Naik term can be appropriately tuned to use the action for $c$ quark. For $s$ and $u/d$ quarks, the $\epsilon = 0$. Later, in the Table (14) we listed the parameters used for HISQ quarks. We have taken the values of $\epsilon$ from [24] and used MILC subroutines for generating HISQ propagators.

Since HISQ action is diagonal in spin, propagators obtained do not have any spin structure. The full $4 \times 4$ spin structure can be regained by multiplying the propagators by Kawamoto-Smit multiplicative phase factor [19],

$$ \Omega(x) = \sum_{\mu=1}^{4} (\gamma_\mu)_{\mu\nu} = \gamma_1 \gamma_2 \gamma_3 \gamma_4 \cdot \quad (12) $$

MILC library uses a different representation of $\gamma$ matrices than the ones used in NRQCD. However, $\gamma$ matrices of these two representations are related by the unitary transformation of the form

$$ S^{\text{MILC}}_{\gamma_{\mu}} S^\dagger = S^\text{NR}_{\gamma_{\mu}} \quad \text{where,} \quad S = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} -\sigma_y & \sigma_y \\ -\sigma_y & \sigma_y \end{array} \right) \quad (13) $$

### III. TWO-POINT FUNCTIONS

In this section we discuss the construction of the bottom baryons by combining spin and color indices of the appropriate quark fields to form necessary baryon operators i.e. two-point functions. The $b$ quark field is universally represented with $Q$, which is defined later in the Equation (17), throughout the paper.

#### A. Bottom-bottom meson two-point function

After the $b$ quark propagators are generated according to (4), we calculate the masses of bottomonium states from the exponential fall-off of two-point functions i.e. correlators of the state with quantum numbers of interest. The meson creation operator are constructed from two component quark and anti-quark creation operators $\psi_h^\dagger$ and $\chi_h^\dagger$ [17,18]. As antiquarks transform as 3 under color rotation, we rename the antiquark spinor as $\chi_h^\dagger \equiv \chi_h^\dagger$ [13]. The meson creation operator is thus,

$$ \mathcal{O}_{hh}(x) = \psi_h^\dagger(x) \Gamma \chi_h(x). \quad (14) $$

Heavy-heavy i.e. bottom-bottom meson two-point function is then given by [13,20],

$$ C_{hh}(\vec{p}, t) = \sum_{x} \langle \mathcal{O}_{hh}^\dagger(x) \mathcal{O}_{hh}(0) \rangle = \sum_{x} e^{i\vec{p}\cdot \vec{x}} \text{Tr} \left[ G^\dagger(x,0) \Gamma_{\text{sink}} G(x,0) \Gamma_{\text{src}} \right] \quad (15) $$

$\Gamma_{\text{sink}} = \Gamma_{\text{src}} = I$ and $\sigma_i$ for the pseudoscalar and vector mesons respectively. Heavy-heavy propagator $G(x,0)$ is a $2 \times 2$ matrix in spin space. If we think $G(x,0)$ as a $4 \times 4$ matrix with vanishing lower components then we can rewrite the above equation (15) as [9]

$$ C_{hh}(\vec{p}, t) = \sum_{\vec{x}} e^{i\vec{p}\cdot \vec{x}} \left( \gamma_5 G^\dagger(x,0) \gamma_5 \Gamma_{\text{sink}} G(x,0) \Gamma_{\text{src}} \right) \quad (16) $$

where $\Gamma$ matrices now changed to $\Gamma_{\text{sink}} = \Gamma_{\text{src}} = \gamma_5$ and $\gamma_i$ for pseudoscalar and vector mesons respectively. In Equation (16) we have used the non-relativistic Dirac representation of $\gamma$ matrices. In the Equations (15), (16) and (19) the trace is taken over both the spin and color indices.

#### B. Heavy-light meson two-point function

As discussed above, $b$ quark field $\psi_h$ has only two spin components. We convert it to a 4-component spinor having vanishing lower components

$$ Q = \left( \begin{array}{c} \psi_h \\ 0 \end{array} \right) \quad (17) $$

This helps us to combine the $b$ and light quark fields in the usual way,

$$ \mathcal{O}_{hl}(x) = \bar{Q}(x) \Gamma \ell(x) \quad (18) $$

where $l(x)$ stands for the light quark fields, $\bar{Q} = Q^\dagger \gamma_4$ and depending on pseudoscalar and vector mesons $\Gamma = \gamma_5$ and $\gamma_i$ respectively. Note that in the Dirac i.e. NR representation of $\gamma$-matrices $\gamma_4 Q = Q$. The zero momentum bottom-light two-point function becomes [10,27],

$$ C_{hl}(t) = \sum_{\vec{x}} \langle \mathcal{O}_{hl}^\dagger(x) \mathcal{O}_{hl}(0) \rangle = \sum_{\vec{x}} \text{Tr} \left[ \gamma_5 M^\dagger(x,0) \gamma_5 \Gamma G(x,0) \Gamma \right] \quad (19) $$

where $M(x,0)$ is the light quark propagator. It has the usual full $4 \times 4$ spin structure. As before, $G(x,0)$ is the $b$ quark propagator having vanishing lower components. However, before implementing Equation (19), $G(x,0)$ has to be rotated to the MILC basis.

#### C. Bottom baryon two-point functions

The bottom quark field $Q$ has vanishing lower components and hence can be projected to positive parity states only. Besides, the use of $\Gamma = C \gamma_5$ in a diquark operator made from same flavor i.e. $I C \gamma_5 l$ is not allowed by Pauli exclusion principle. In other words, the insertion of $C \gamma_5$ between two quark fields of same flavor creates a combination which is antisymmetric in spin indices, while the presence of $\epsilon_{abc}$ makes the combination
antisymmetric in color indices. This makes the overall operator become symmetric under the interchange of the same flavored quark fields. Keeping this in mind, the constructions of various bottom baryon two-point functions are described below.

**Triply bottom baryon:** Triply bottom baryon operator is defined by

\[
(O^{hhb}_k)_\alpha = \epsilon_{abc} \left( Q^{aT} C \gamma_k Q^b \right) Q^c
\]

where \( C = \gamma_4 \gamma_2 \). Here, \( a, b, c \) are the color indices, \( \alpha \) is the spinor index and \( k \) is the Lorentz index which runs from 1 to 3. The zero momentum two-point function reads \[1\]

\[
C^{hhb}_{ijk;\alpha;\delta}(t) = \sum_x \langle x | O^{hhb}_j(x) \gamma_k O^{hhb}_i(0) | 0 \rangle
\]

\[
= \sum_x \epsilon_{abc} \epsilon_{fgh} C^{hhb}_\alpha(0) \times \text{Tr} \left[ C \gamma_j G^{bg}(x, 0) \gamma_k \gamma_2 G^{afT}(x, 0) \right]
\]

The corresponding baryon correlators are

\[
C^{hhb}_{ijk;\alpha;\delta}(t) = \sum_x \langle x | O^{hhb}_j(x) \gamma_k O^{hhb}_i(0) | 0 \rangle
\]

\[
= \sum_x \epsilon_{abc} \epsilon_{fgh} \left[ M^{ch}(x, 0) \gamma_4 \right]_{\alpha;\delta} \times \text{Tr} \left[ \gamma_4 \gamma_2 \gamma_j G^{bg}(x, 0) \gamma_k \gamma_2 G^{afT}(x, 0) \right]
\]

The propagators \( M(x, 0) \) and \( G(x, 0) \) are required to be converted to MILC or NR basis using the unitary matrix \( S \) defined in Equation (13) whenever needed.

An additional spin-\( \frac{3}{2} \) operator can be defined for the \( O^{hhb} \) type operator as

\[
(O^{hhb}_5)_\alpha = \epsilon_{abc} \left( Q^{aT} C \gamma_5 Q^b \right) Q^c
\]

The two-point function for this operator is obtained by replacing \( \gamma_j \) and \( \gamma_k \) by \( \gamma_5 \) in Equation (21). We cannot have a \( C \gamma_5 \) between two \( Q \) in diquark and hence no \( O^{hhb}_5 \).

**Bottom-bottom-light baryon:** Interpolating operator for baryons, having two \( b \) quarks and a light quark, can be constructed in two ways based on how the diquark component is formed \[2\].

\[
\langle O^{hhb}_k \rangle_\alpha = \epsilon_{abc} \left( Q^{aT} C \gamma_k Q^b \right) Q^c
\]

\[
\langle O^{hhb}_k \rangle_\alpha = \epsilon_{abc} \left( Q^{aT} C \gamma_k Q^b \right) Q^c
\]

In this paper, we use these projections to separate the different spin states. We would like to point out that the spin-\( \frac{1}{2} \) state of triply bottom baryon is not a physical state as it violates Pauli exclusion principle even though we can take the projection in practice.

In Table I we tabulated the full list of triple and double bottom baryon operators that are used in this work. We have broadly followed the nomenclature adopted in \[11\] but with certain modifications as needed for this work. The baryons having the same quark content and \( J^P \) are obtained in two different ways, as mentioned above. The operators with “tildes”, for instance \( \Omega_{bb} \) (1/2+), are obtained by projecting the relevant \( (Q C \gamma_k l) Q \) operator with \( P^{3/2} \). The operators with “prime”, such as \( \Omega'_{bb} \) (1/2+), are obtained from

| Baryon | Quark content | \( J^P \) | Operator |
|--------|--------------|----------|----------|
| \( \Omega_{bb} \) | \( bbb \) | \( \frac{1}{2}^+ \) | \( \epsilon_{abc}(Q^{aT} C \gamma_k Q^b) Q^c \) |
| \( \Omega'_{bb} \) | \( bbb \) | \( \frac{1}{2}^+ \) | \( \epsilon_{abc}(Q^{aT} C \gamma_k Q^b) Q^c \) |
| \( \Omega_{bb} \) | \( cbb \) | \( \frac{1}{2}^+ \) | \( \epsilon_{abc}(Q^{aT} C \gamma_k Q^b) Q^c \) |
| \( \Omega'_{bb} \) | \( cbb \) | \( \frac{1}{2}^+ \) | \( \epsilon_{abc}(Q^{aT} C \gamma_k Q^b) Q^c \) |
| \( \Xi_{bb} \) | \( ubb \) | \( \frac{1}{2}^+ \) | \( \epsilon_{abc}(Q^{aT} C \gamma_k Q^b) Q^c \) |
| \( \Xi'_{bb} \) | \( ubb \) | \( \frac{1}{2}^+ \) | \( \epsilon_{abc}(Q^{aT} C \gamma_k Q^b) Q^c \) |
| \( \Xi_{bb} \) | \( ubb \) | \( \frac{1}{2}^+ \) | \( \epsilon_{abc}(Q^{aT} C \gamma_k Q^b) Q^c \) |
| \( \Xi'_{bb} \) | \( ubb \) | \( \frac{1}{2}^+ \) | \( \epsilon_{abc}(Q^{aT} C \gamma_k Q^b) Q^c \) |

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and the corresponding two-point function is

\[
C^{h_1 l_2}_{j k; \alpha \delta}(t) = \sum_{\vec{x}} \left\langle [O^{h_1 l_2}_{j k}(x)]_{\alpha} [O^{h_1 l_2}(0)]_{\delta} \right\rangle 
\]

and the corresponding two-point function is

\[
C^{h_1 l_2}_{k j; \alpha \delta}(t) = \sum_{\vec{x}} \left\langle [O^{h_1 l_2}(x)]_{\alpha} [O^{h_1 l_2}(0)]_{\delta} \right\rangle 
\]

\[
\text{Tr} \left[ \gamma_4 \gamma_2 \gamma_5 M_{l_2}^{b g}(x, 0) \gamma_5 \gamma_2 \gamma_4 M_{l_1}^{a T}(x, 0) \right] \]  

(29)

\[
(O^{h_1 l_2})_{\alpha} = \epsilon_{abc} (l^a T C^{\gamma_5 l_2}) Q^{c}_{\alpha} \tag{30}
\]

The two-point function for this operator has the same form as in Equation (31) with \( \gamma_j \) and \( \gamma_k \) replaced by \( \gamma_5 \).

### Table II. Operators for single bottom baryons

| Baryon | Quark content | \( J^P \) | Operator |
|--------|---------------|-----------|----------|
| \( \Omega^{c c b} \) | ccb | \( \frac{5}{2}, \frac{3}{2} \) | \( \epsilon_{abc} (Q^{a T} C^{\gamma_5 c c}) Q^{c} \) |
| \( \Omega^{c b b} \) | ccb | \( \frac{3}{2}, \frac{1}{2} \) | \( \epsilon_{abc} (Q^{a T} C^{\gamma_5 b c}) Q^{c} \) |
| \( \Omega^{c c b} \) | ccb | \( \frac{3}{2}, \frac{1}{2} \) | \( \epsilon_{abc} (s^{a T} C^{\gamma_5 c c}) Q^{c} \) |
| \( \Omega^{c b b} \) | ccb | \( \frac{3}{2}, \frac{1}{2} \) | \( \epsilon_{abc} (Q^{a T} C^{\gamma_5 b c}) s^{c} \) |
| \( \Xi^{c c b} \) | ucb | \( \frac{3}{2}, \frac{1}{2} \) | \( \epsilon_{abc} (Q^{a T} C^{\gamma_5 c c}) u^{c} \) |
| \( \Xi^{c b b} \) | ucb | \( \frac{3}{2}, \frac{1}{2} \) | \( \epsilon_{abc} (Q^{a T} C^{\gamma_5 b c}) u^{c} \) |
| \( \Xi^{s c b} \) | ucb | \( \frac{3}{2}, \frac{1}{2} \) | \( \epsilon_{abc} (s^{a T} C^{\gamma_5 c c}) u^{c} \) |
| \( \Xi^{s b b} \) | ucb | \( \frac{3}{2}, \frac{1}{2} \) | \( \epsilon_{abc} (Q^{a T} C^{\gamma_5 b c}) s^{c} \) |
| \( \Xi^{b c b} \) | ucb | \( \frac{3}{2}, \frac{1}{2} \) | \( \epsilon_{abc} (Q^{a T} C^{\gamma_5 c c}) s^{c} \) |
| \( \Xi^{b b b} \) | ucb | \( \frac{3}{2}, \frac{1}{2} \) | \( \epsilon_{abc} (Q^{a T} C^{\gamma_5 b c}) s^{c} \) |
| \( \Lambda^{c c b} \) | ucb | \( \frac{3}{2}, \frac{1}{2} \) | \( \epsilon_{abc} (Q^{a T} C^{\gamma_5 c c}) u^{c} \) |

In Table II we tabulate our full list of single bottom baryon operators that we made use of in this work. The “tilded” and “prime” states that appear in the table have been explained before in the context of multi bottom baryon operators.

**Light baryon:** We occasionally need charmed baryon states like \( qcc \) or \( qqc \), where \( q \) is any of \( s \) or \( u/d \) quarks or both, hence we include a discussion on charmed baryon operators. The \( c \)-quark in the present case is relativistic. For reason discussed above, with HISQ action for lighter quarks we can define only the spin-\( \frac{1}{2} \) operators. Consider a \((l_1 l_2 l_3)\)-baryon where at least two quarks are differently flavored, say \( l_1 \neq l_2 \). The spin-\( \frac{1}{2} \) operator and

Because the light quark propagators \( M_1(x, 0) \) and \( M_2(x, 0) \) are proportional to each other, the relative positions of the quark fields \( l_1 \) and \( l_2 \) in Equation (33) is irrelevant. Interpolating operator defined in Equation (33) has overlap with both spin-\( \frac{1}{2} \) and \( \frac{3}{2} \) states and can be projected out by appropriate projection operators \( I_{l j}^{1/2}, 3/2 \).
the corresponding two-point function in such case is
\[ (C^I_{\mu_1 \mu_2})_a = \epsilon_{abc} (\bar{l}^I_1 T C \gamma_\mu l^I_2) \bar{\ell}_{3a} \] (36)
\[ C^l_{55, a\bar{a}}(t) = \sum_{x} \epsilon_{abc} \epsilon_{fgh} \left[ M^b_{3g}(x, 0) \gamma_4 \gamma_3 \alpha_\partial \times \right. \]
\[ \left. \text{Tr} \left[ \gamma_4 \gamma_2 \gamma_5 M^b_{4g}(x, 0) \gamma_5 \gamma_2 \gamma_4 M^{aT}_I(x, 0) \right] \right] \] (37)

The two light baryon states \((J^P = 1/2^+)\) that we are interested in this work are,
\[ \Sigma_c \ (ucc) : \quad \epsilon_{abc} (\bar{c} T C \gamma_3 u^h) u^c \]
\[ \Xi_{cc} \ (ucc) : \quad \epsilon_{abc} (\bar{c} T C \gamma_3 u^h) c^c \] (38)

IV. SIMULATION DETAILS

We calculated the bottom baryon spectra using the publicly available \(N_f = 2 + 1\) Asqtad gauge configurations generated by MILC collaboration. Details about these lattices can be found in [21]. It uses Symanzik-improved Lüscher-Weisz action for the gluons and Asqtad action [22] [23] for the sea quarks. The lattices we choose have a fixed ratio of \(am/\alpha_s = 1/5\) and lattice spacings ranging from 0.15 fm to 0.09 fm corresponding to the same physical volume. We have not determined the lattice spacings independently but use those given in [21]. In Table III we listed the ensembles used in this work.

| TABLE III. MILC configurations used in this work. The gauge coupling is \(\beta\), lattice spacing \(a\), \(u/d\) and \(s\) sea quark masses are \(m_t\) and \(m_s\) respectively and lattice size is \(L^3 \times T\). The \(N_{cfg}\) is number of configurations used in this work. |
| \(\beta = 10/g^2\) | \(a(\text{fm})\) | \(am_t\) | \(am_s\) | \(L^3 \times T\) | \(N_{cfg}\) |
|-----------------|-------------|---------|---------|----------------|---------|
| 6.572           | 0.15        | 0.0097  | 0.0484  | 16^3 \times 48 | 400     |
| 6.76            | 0.12        | 0.01    | 0.05    | 20^3 \times 64 | 400     |
| 7.09            | 0.09        | 0.0062  | 0.031   | 28^3 \times 96 | 300     |

In NRQCD the rest mass term does not appear in Equation 43 and therefore we cannot determine hadron masses from their energies at zero momentum directly from the exponential fall-off of the correlation functions. Instead, we calculate the kinetic mass \(M_k\) of heavy-heavy mesons from its energy-momentum relation, which to \(O(p^2)\) is [12],
\[ E(p) = E(0) + \sqrt{p^2 + M_k^2} - M_k \]
\[ \Rightarrow E(p)^2 = E(0)^2 + \frac{E(0)}{M_k} p^2 \] (39)

We calculate the \(E(p)\) at different values of lattice momenta \(p = 2\pi n/L\) where, \(n = (0,0,0), (1,0,0), (1,1,0), (1,1,1), (2,0,0), (2,1,0)\) and (2,1,1).

\(m_b\) tuning: The \(b\) quark mass is tuned from the spin average \(\Upsilon\) and \(\eta_b\) masses
\[ M_{bb} = \frac{3M_\Upsilon + M_{\eta_b}}{4} \] (40)

using kinetic mass for both \(\Upsilon\) and \(\eta_b\). The experimental value to which \(M_{bb}\) is tuned to is not 9443 MeV, as obtained from spin averaging \(\Upsilon\) (9460 MeV) and \(\eta_b\) (9391 MeV) experimental masses, but to an appropriately adjusted value of 9450 MeV [24], which we denote as \(M_{bb}\) later in the Equation (41). The reasons being, firstly electromagnetic interaction among the quarks are not considered here. Secondly, the disconnected diagrams while computing two-point function are also not considered thus not allowing \(b\), \(b\) quarks to annihilate to gluons. And finally, we do not have sea \(c\) quarks in our simulation. For a more detailed discussion on this, see [24].

The \(b\) quark mass \(m_b\) and the coefficient \(c_4\) in Equation 41 are then tuned to obtain modified spin average mass and the hyperfine splitting of \(\Upsilon\) and \(\eta_b\) (~ 70 MeV). In order to achieve the desired hyperfine splittings, we tuned only \(c_4\) since at \(O(1/m_b)\) this is the only term that contains Pauli spin matrices and therefore it allows the mixing of spin components of \(\psi_b\). This term contributes maximally to the hyperfine splitting compared to the others that contain Pauli spin matrices.

We set stability factor \(n = 4\) and \(c_4 = 1.9\) throughout our simulation. All other coefficients \(c_i\) in Equation 41 are set to 1. The Table IV lists the values of \(m_b\) used in this work.

| TABLE IV. Tuned \(b\), \(c\) and \(s\) quark bare masses for lattices used in this work. For \(s\)-quark mass, we mentioned the particle states to which it is tuned to. The values of \(c\)-parameter used for \(c\)-quark are given in the last column. |
| \(a\) | \(am_b\) | \(am_c\) | \(am_s\) | \(am_s\) | \(\epsilon\) |
| \(\text{fm}\) | \((\eta_b)\) | \((B_s)\) | \([24]\) |
| 0.15 | 2.76 | 0.850 | 0.065 | 0.215 | -0.34 |
| 0.12 | 2.08 | 0.632 | 0.049 | 0.155 | -0.21 |
| 0.09 | 1.20 | 0.452 | 0.0385 | 0.114 | -0.115 |

\(m_c\) tuning: The \(c\)-quark mass is tuned pretty much in the same way as \(m_b\), except that \(M_{cc}\) is tuned to the spin average of \(J/\psi\) and \(\eta_c\) experimental masses. In this case, however, the adjustment to spin averaged value due to the absence of electromagnetic interaction, \(c\)-quarks in sea and disconnected diagrams are very small and hence neglected. The bare \(c\)-quark masses used in this work are given in Table IV.

\(m_s\) tuning: The \(s\)-quark mass is tuned to two different values. In the first case, we tune to the mass of fictitious \(s\) pseudoscalar meson \(\eta_s\) while in the second case to the mass of \(B_s\). The \(\eta_s\) is a fictitious meson that is not allowed to decay through \(s\bar{s}\) annihilation. Hence
no disconnected diagrams arise in the two-point function calculation. From chiral perturbation theory its mass is estimated to be $m_{\eta_8} = \sqrt{2m_{K}^2 - m_{\pi}^2} = 689$ MeV. [25, 26]. The $s$-quark mass thus tuned is checked against $D_s$ meson, making use of the $c$-quark mass obtained above, and it agrees well with the experimental $D_s$ (1968 MeV).

TABLE V. $D$ and $B$ meson masses in MeV with the tuned $am_u, am_c$ and $am_s$.

| $L^3 \times T$ | $D_s$ | $B_s$ |
|----------------|-------|-------|
| $16^3 \times 48$ | 6260(8) | 1994(3) | 2197(2) |
| $20^3 \times 64$ | 6263(12) | 1977(4) | 2172(2) |
| $28^3 \times 96$ | 6255(10) | 1971(3) | 2167(2) |
| PDG [8]         | 6275   | 1968   |

Next we explore, tuning $m_s$ when $s$-quark is in a bound state with a heavy $b$ quark. Here we are assuming that the potential experienced by the $s$ quark in the field of $b$ quark in $B_s$ meson remains the same in other strange bottom baryons and there is no spin-spin interactions taking place between the quarks. In the infinite mass limit, the HQET Lagrangian becomes invariant under arbitrary spin rotations. [21]. Thereby, we can argue that for $sbb$ and $sbc$ systems the spin-spin interactions do not contribute significantly in spectrum calculation. However, this argument is perhaps not valid in systems like $bsb$ or $bsd$ but still with $s$ quark thus tuned, we possibly can obtain their masses close to their physical masses without resorting to any extrapolation.

In this paper, we will present our results obtained at these two different values of $m_s$. In Table (V) we listed these values of $s$-quark masses. In the Table (V) we calculate $B_c$ and $D_s$ mesons using tuned $b$, $c$ and $s$ masses. As is seen, when $m_s$ is tuned with $\eta_b$ the $D_s$ mass obtained is fairly close to PDG value whereas when tuned to $B_s$ we see an upward shift by an average 200 MeV. We have observed similar differences when $s$-quark appears together with $c$ in $(sbc)$-baryon masses.

For the valence $u/d$ quark mass, we used a range of bare quark masses varying from the lightest sea quark masses all the way to little above where $s$ mass is tuned to $B_s$. Whenever mass of a bottom baryon containing $u/d$ quark(s) is quoted, it will correspond to $u/d$ quark mass tuned at $B_s$ mass. In our calculation, since we are not including either electromagnetic or isospin breaking, we do not distinguish between $u$ and $d$ quarks and it is always $m_u = m_d$. This tuning of $u/d$ mass to $B_s$ works well in capturing the $b$-baryon states containing single $u/d$, such as $(usb)$ or $(ucb)$ baryons, when compared to either PDG or other works. In case of states containing two $u/d$ quarks when $b$ and $u/d$ form diquarks $(QC\gamma(k,5)u)$, as in $\Sigma_b$ ($usb$), the masses obtained are consistent with $\Sigma_b$'s but fails for $\Lambda_b$ ($ubd$) where the diquark part is $(uc\gamma d)$ (see Table II). For $\Lambda_b$ only, we have to resort to different tuning.

In Table (VI), we listed $u/d$ quark masses ($am_u$) against the lattice spacing. We show in the Figure I our strategy used to tune $m_u$ and $m_{u/d}$. The tuned $am_{u/d}$ for different lattices are

$16^3 \times 48 : 0.165 \quad 20^3 \times 64 : 0.115 \quad 28^3 \times 96 : 0.085$

V. RESULTS AND DISCUSSION

In order to extract the masses of the bottom baryons, we perform two-exponential uncorrelated fit to the two-point functions. We then cross-checked it with fitting the effective masses over the same range of time slices. However, this zero momentum energy does not directly give us the mass of the bottom baryons because of unphysical shift in zero of energy. To account for it, the mass is obtained considering energy splittings,

$$M_{\text{latt}} = E_{\text{latt}} + \frac{n_b}{2} \left(M_{\text{phys}}^{\text{mod}} - E_{\text{latt}}^{\text{mod}}\right)$$

where $E_{\text{latt}}$ is the lattice zero momentum energy in MeV, $n_b$ is the number of $b$-quarks in the bottom baryon. For bottom mesons, $n_b$ is obviously always 1. As discussed before, $M_{\text{phys}}^{\text{mod}}$ is the modified spin average mass of $Y$.
and \( n_b \) and is equal to 9450 MeV and \( M_{\text{latt}} \) is the lattice bottom baryon mass in MeV.

While calculating the mass splittings this shift in energies, however, is cancelled by subtraction among energies of hadrons having equal number of bottom quarks \((n_b)\) in them. In calculation of the mass splittings, we use jack-knifed ratio of the correlation functions for fitting [3].

\[
C^{Y-X}(t) = \frac{C^Y(t)}{C^X(t)} \sim e^{-(M_Y-M_X)t} \tag{42}
\]

Below in the Figure 2 we show a few correlators for single baryons containing exclusively either two \( c \) or \( s \) or \( u/d \).

![FIG. 2. \( \Sigma_b, \bar{\Omega}_b \) and \( \bar{\Omega}_{ccb} \) correlators in $28^3 \times 96$ lattice.](image)

The fitting range is typically chosen looking at positions of what we consider plateau in the effective mass plots. In the effective mass plot Figure 3 the zero momentum energies and the errors of the same three states as in the Figure 2 are represented as bands.

![FIG. 3. The effective mass plots corresponding to the states in Figure 2. The bands are placed over what we consider plateau.](image)

In these figures, we choose to present the data from $28^3 \times 96$ lattices but the data from $16^3 \times 48$ and $20^3 \times 64$ are similar. Just to remind, in order to obtain the masses in MeV from these, we need the Equation (41).

**Single bottom baryons:** A couple of baryon states containing one \( b \) quark have been listed in the PDG [8], such as \( \Lambda_b^0 \) \((u/db)\), \( \Omega_b^- \) \((ssb)\), \( \Xi_b^0 \) \((usb)\) etc. and they provide a good matching opportunity. We varied the \( u/d \)-quark mass to tune to \( B \)-meson, which is shown in the Figure 4 and obtained the spectra of these states against the same set of light quark masses. In the Figure 4 we show this change in the baryon masses.

![FIG. 4. Single bottom baryon masses in MeV against \( m_{u/d} \). \( m_l = 0.085 \) is the tuned \( u/d \)-quark mass indicated by dashed vertical line. The bands are the PDG values of the states considered, except for \( \Xi_{cb} \) which is compared with [11].](image)

We collect our results for single bottom baryon, not containing \( s \)-quark(s), in the Table VII and those with \( s \)-quark in Table VIII. For the \( m_{u/d} \), we state the results when the valence \( m_l \) gives physical \( B \) meson mass.

**TABLE VII.** Masses, in lattice unit, of baryons involving single \( b \) quark and no \( s \) quark. The bare \( u/d \)-quark masses are 0.165 for $16^3 \times 48$, 0.115 for $20^3 \times 64$ and 0.085 for $28^3 \times 96$.

| Baryons | $16^3 \times 48$ | $20^3 \times 64$ | $28^3 \times 96$ | Average (MeV) |
|---------|-----------------|-----------------|-----------------|--------------|
| \( \Omega_{ccb} \) | 2.954(5) | 2.497(4) | 2.088(3) | 7807(11) |
| \( \Omega_b \) | 2.933(5) | 2.482(3) | 2.078(3) | 7780(9) |
| \( \Omega'_{ccb} \) | 2.952(4) | 2.497(3) | 2.078(3) | 7797(11) |
| \( \Xi_{ccb} \) | 2.222(6) | 1.809(4) | 1.648(6) | 6835(20) |
| \( \Xi_b \) | 2.177(11) | 1.881(4) | 1.623(5) | 6787(12) |
| \( \Xi'_{ccb} \) | 2.199(8) | 1.886(4) | 1.631(4) | 6805(16) |
| \( \Xi'_{cb} \) | 2.227(6) | 1.904(4) | 1.653(6) | 6843(19) |
| \( \Lambda_{ccb} \) | 1.468(8) | 1.292(5) | 1.189(5) | 5836(22) |
| \( \Lambda_b \) | 1.460(7) | 1.290(3) | 1.174(6) | 5820(21) |
| \( \Sigma_{ccb} \) | 1.470(7) | 1.305(3) | 1.194(9) | 5848(18) |
| \( \Sigma_b \) | 1.322(7) | 1.208(6) | 1.109(9) | 5667(14) |
Since $s$ quark has been tuned in two different ways, we quote both i.e. the $b$-baryon masses at $B$, $B_s$ point.

TABLE VIII. Masses, in lattice unit, of baryons containing single $b$-quark and $s$-quark(s).

| Baryons | Tuning | $16^3 \times 48$ | $20^3 \times 64$ | $28^3 \times 96$ | Average (MeV) |
|---------|--------|------------------|------------------|------------------|--------------|
|         |        | (0.15 fm)        | (0.12 fm)        | (0.09 fm)        | (MeV)        |
| $\bar{\Omega}_b$ | $\eta_s$ | 2.035(5) | 1.782(5) | 1.542(3) | 6611(9) |
|         | $B_s$ | 2.297(2) | 1.957(6) | 1.693(4) | 6930(19) |
| $\Omega_b$ | $\eta_s$ | 2.010(8) | 1.754(5) | 1.532(3) | 6578(9) |
|         | $B_s$ | 2.248(11) | 1.937(7) | 1.684(2) | 6893(16) |
| $\bar{\Omega}_b$ | $\eta_s$ | 2.012(7) | 1.765(5) | 1.536(3) | 6587(10) |
|         | $B_s$ | 2.267(8) | 1.943(7) | 1.686(2) | 6906(17) |
| $\Omega_b$ | $\eta_s$ | 2.052(5) | 1.785(5) | 1.548(3) | 6625(8) |
|         | $B_s$ | 2.297(6) | 1.966(6) | 1.705(2) | 6946(17) |
| $\Xi_b$ | $\eta_s$ | 0.987(4) | 0.945(2) | 0.918(3) | 5237(8) |
|         | $B_s$ | 1.541(8) | 1.352(6) | 1.235(6) | 5935(22) |
| $\Xi_b$ | $\eta_s$ | 0.986(5) | 0.947(2) | 0.909(4) | 5231(11) |
|         | $B_s$ | 1.520(9) | 1.345(3) | 1.207(6) | 5901(20) |
| $\Xi_b$ | $\eta_s$ | 0.978(5) | 0.944(2) | 0.904(5) | 5222(13) |
|         | $B_s$ | 1.532(11) | 1.350(4) | 1.224(4) | 5921(19) |
| $\Xi_b$ | $\eta_s$ | 0.987(4) | 0.948(3) | 0.913(5) | 5235(11) |
|         | $B_s$ | 1.544(10) | 1.366(4) | 1.238(6) | 5946(16) |
| $\Omega_b$ | $\eta_s$ | 1.129(5) | 1.058(3) | 1.012(4) | 5430(11) |
|         | $B_s$ | 1.611(8) | 1.412(6) | 1.264(3) | 6019(20) |
| $\Omega_b$ | $\eta_s$ | 1.118(7) | 1.050(3) | 0.997(4) | 5410(10) |
|         | $B_s$ | 1.600(11) | 1.411(7) | 1.264(3) | 6014(17) |
| $\Omega_b'$ | $\eta_s$ | 1.131(9) | 1.057(3) | 1.007(2) | 5427(9) |
|         | $B_s$ | 1.615(8) | 1.425(7) | 1.295(2) | 6051(15) |

As is evident from our results, the numbers coming from $s$-quark tuned to $\eta_s$ are about 300 MeV smaller from those tuned to $B_s$. If we take $\Omega_b$ ($ssb$) and compare with PDG value 6071 MeV, it becomes obvious.

Next we determine mass differences between various single bottom sector including the hyperfine splittings.

The mass splittings are calculated using ratio of correlators as given in the Equation (12). As an example, in the Figure[5] we provide the plots for ratio of correlators, $\Omega_b' - \Lambda_b$ and $\Xi_b - \Xi_b$, for comfortable viewing because of their relatively large mass differences i.e. slopes are prominent and well separated. In case of smaller differences, for instance $\Omega_b' - \bar{\Omega}_b$ or $\Xi_b - \bar{\Xi}_b$, the slopes of the ratio correlators are rather small and not quite visible. In the Table[9] we collect the results of single $b$ baryon mass splittings.

TABLE IX. Single bottom baryons mass splittings in MeV. Averages and the statistical errors of the three lattices used in this work.

| Baryon splittings | $16^3 \times 48$ | $20^3 \times 64$ | $28^3 \times 96$ | Average (MeV) |
|-------------------|-----------------|-----------------|-----------------|--------------|
| $\Omega_b' - \Omega_b$ | 28(3) | 23(2) | – | 26(3) |
| $\Omega_b' - \Omega_b$ | 59(8) | 62(13) | 61(22) | 61(15) |
| $\Xi_b' - \Xi_b$ | 37(6) | 44(5) | 44(9) | 42(7) |
| $\Omega_b' - \bar{\Omega}_b$ | 29(5) | 28(11) | 29(4) | 29(7) |
| $\Omega_b' - \bar{\Omega}_b$ | 396(4) | 391(9) | 406(10) | 398(9) |
| $\Xi_b' - \bar{\Xi}_b$ | 138(20) | 122(38) | 138(46) | 133(36) |
| $\bar{\Xi}_b - \bar{\Xi}_b$ | 170(9) | 166(11) | 163(6) | 166(9) |
| $\Lambda_b - B$ | 391(20) | 431(20) | 397(22) | 406(21) |
| $\bar{\Xi}_b - \bar{\Xi}_b$ | 30(9) | 30(8) | 29(8) | 30(8)(1) |
| $\bar{\Xi}_b - \Lambda_b$ | 224(13) | 203(12) | 175(13) | 201(13) |

The heavy quark basically acts as a static color source, and therefore, we expect that the hyperfine splittings between states containing single or multiple $s$ and $u/d$ quark(s) not to depend on the tuning of $m_s$ and $m_{u/d}$. For $m_{u/d} \leq 0.85$ and two values of $m_s$ we show this pattern for a couple of hyperfine splittings in Figure[6].

FIG. 5. Ratio of correlators for the calculation of the two splittings shown in Table IX. The bands overlaid on the data points represent single exponential fits.

FIG. 6. Hyperfine splittings at various $m_s$ and $m_{u/d}$ for a selected few bottom baryons on $28^3 \times 96$ lattice. The horizontal bands are the average values of the splittings and is used to guide the eye.
Double bottom baryons: For the heavier baryons, i.e. those involving more than one $b$-quark, the data are relatively less noisy than the baryons containing single $b$-quark. The effective mass plots in the Figure shown for only $16^3 \times 48$ lattices but similar for two other lattices, is an evidence for this.

![Effective mass plots](image)

**FIG. 7.** $\Omega_{bb}^*$, $\tilde{\Omega}_{bb}^*$ and $\tilde{\Omega}_{cc}^*$ effective masses.

The plot for $\Omega_{bb}^*$ appears counter intuitive since being the heaviest, it is showing lower mass compared to the other two. However, it receives large correction because of shift in rest mass of three $b$-quarks.

We tabulate our results for double bottom non-strange baryons in the Table X while those containing $s$ quark in Table XI.

| Baryon | $16^3 \times 48$ | $20^3 \times 64$ | $28^3 \times 96$ | Average |
|--------|-----------------|-----------------|-----------------|---------|
| $\Omega_{bb}^*$ | 1.983(4) | 2.031(3) | 2.154(4) | 14403(7) |
| $\Omega_{bb}$ | 1.974(4) | 2.023(5) | 2.148(4) | 14390(8) |
| $\Omega_{bb}^*$ | 2.429(16) | 2.259(4) | 2.117(3) | 11081(21) |
| $\Omega_{bb}$ | 2.409(16) | 2.246(5) | 2.110(3) | 11060(23) |
| $\Omega_{bb}^*$ | 2.431(8) | 2.255(4) | 2.113(3) | 11077(14) |
| $\Omega_{bb}$ | 2.432(10) | 2.251(4) | 2.113(3) | 11075(13) |
| $\Omega_{bb}^*$ | 2.433(8) | 2.250(4) | 2.114(4) | 11076(12) |
| $\Sigma_{bb}$ | 1.721(12) | 1.643(10) | 1.666(5) | 10103(24) |
| $\Sigma_{bb}$ | 1.700(12) | 1.640(7) | 1.664(5) | 10091(17) |
| $\Sigma_{bb}^*$ | 1.720(10) | 1.635(8) | 1.668(3) | 10100(27) |
| $\Sigma_{bb}$ | 1.703(16) | 1.634(8) | 1.661(4) | 10087(22) |
| $\Sigma_{bb}$ | 1.704(16) | 1.635(10) | 1.672(3) | 10096(24) |

It is to note that $\Omega_{bb}^*$ is a spin-3/2 state having no spin-1/2 counterpart. But in practice we can take a spin-1/2 projection to get such a fictitious state. Therefore, we label the physical $(bbb)$ spin-3/2 state with $\Omega_{bb}^*$ to keep consistency with our remaining notation. In this case none of the states have PDG entries.

**TABLE XI.** Double bottom strange baryon spectra.

| Baryon | Tuning | $16^3 \times 48$ | $20^3 \times 64$ | $28^3 \times 96$ | Average |
|--------|--------|-----------------|-----------------|-----------------|---------|
| $\Omega_{bb}^*$ | $\eta_s$ | 1.545(11) | 1.536(6) | 1.576(4) | 9902(12) |
| $\Omega_{bb}$ | $B_s$ | 1.791(12) | 1.703(11) | 1.716(3) | 10203(22) |
| $\Omega_{bb}^*$ | $\eta_s$ | 1.533(9) | 1.527(7) | 1.570(4) | 9896(13) |
| $\Omega_{bb}$ | $B_s$ | 1.768(12) | 1.699(8) | 1.715(3) | 10190(17) |
| $\Omega_{bb}^*$ | $\eta_s$ | 1.542(9) | 1.529(7) | 1.575(4) | 9896(12) |
| $\Omega_{bb}$ | $B_s$ | 1.791(12) | 1.693(10) | 1.717(3) | 10199(28) |

We would like to point out that the variation of the $\Xi_{bb}^*$ ($ubb$) masses with $m_{u/d}$ is almost absent as almost the entire contribution to these baryons are coming from the two $b$ quarks. Similarly, from the Table XI we see that the different tuning of $s$ quark has significantly less influence on the double bottom baryon masses, a situation unsurprisingly similar to double bottom baryons with a $u/d$ quark.

The splittings in double bottom sector is tabulated in the Table XII.

**TABLE XII.** Double bottom baryon mass splittings in MeV. None of the splittings have PDG entries.

| Baryon | $16^3 \times 48$ | $20^3 \times 64$ | $28^3 \times 96$ | Average |
|--------|-----------------|-----------------|-----------------|---------|
| $\Omega_{bb}^* - \Omega_{bb}$ | – | 25(5) | 35(2) | 30(5) |
| $\Omega_{bb} - \Xi_{bb}$ | 34(5) | 25(8) | 37(9) | 32(7) |
| $\Xi_{bb} - \Xi_{bb}$ | – | 25(4) | 39(7) | 32(5) |

In the double bottom sector, the splittings between the spin-3/2 and 1/2 states are particularly interesting because HQET relates this mass differences with hyperfine splittings of heavy-light mesons, which in the heavy-quark limit [34]

$$\frac{\Delta M_{b\text{baryon}}}{\Delta M_{b\text{meson}}} \approx \frac{3}{4}$$

This behavior is consistent with our results within errors as can be seen in Table XIII.

A few GMO mass relations involving $b$-quark are provided in the reference [33], which we try to verify in this work,

$$M_{\Omega_{cc}^*} - M_{\Omega_{cc}} \approx M_{\Omega_{bb}^*} - M_{\Omega_{bb}}$$

$$M_{\Xi_{b}^*} - M_{\Xi_{b}} \approx M_{\Xi_{bb}^*} - M_{\Xi_{bb}}$$
TABLE XIII. Ratio of hyperfine splittings of doubly heavy baryons to heavy mesons in the heavy quark limit in $28^3 \times 96$ lattice.

| Baryon splittings | Our results (MeV) | Meson splittings (MeV) | Ratio |
|-------------------|----------------|------------------------|-------|
| $\bar{\Omega}_{bb}-\bar{\Omega}_{bb}$ | 35(2) | $B_s^* - B_c$ | 46(4) | 0.76(4) |
| $\Omega_{bb} - \Omega_{bb}$ | 37(9) | $B_s^* - B_s$ | 45(9) | 0.82(9) |
| $\Xi_{bb} - \Xi_{bb}$ | 39(7) | $B^* - B$ | 47(7) | 0.83(8) |

For the GMO relation [11], the both sides are expected to be approximately 31 MeV. In our case for $20^3 \times 64$ lattice, for which we have data for both the sides, they are approximately equal but is around 24 MeV as against 31 MeV given in [33]. Our lattice data is also consistent with the approximate GMO relation [15], where each side is about 30 MeV against 20 MeV calculated in [33].

VI. SUMMARY

In this paper we presented lattice QCD determination of masses of the baryons containing one or more $b$ quark using NRQCD action for the $b$-quark and HISQ action for the $c$, $s$ and $u/d$ quarks. This combination of NRQCD and HISQ has previously been employed in [24] for bottom mesons, however, the exact implementation was rather different. In this work, we converted the one component HISQ propagators to $4 \times 4$ matrices by the Kawamoto-Smit transformation and the two component NRQCD propagators to $4 \times 4$ matrices using the prescription suggested in [11].

We have discussed the construction of one and two bottom baryon operators in details and pointed out the difficulty for constructing operators motivated by HQET for single bottom baryons. Consequently, we modified them accordingly. For some the baryons, we have multiple operators for the same state i.e. baryons having the same quantum numbers. It would be natural in such cases to construct correlation matrices and obtain lowest lying i.e. ground states by solving the generalized eigenvalue method.

Single bottom baryons can have isodoublets with the same overall quantum numbers $J^P$. For instance, there exist three isodoublets of $\Xi_b$ which are not radially or orbitally excited states [33]. These states have been categorized by the spin of the $us$ or $ds$ diquark denoted by $j$ and the spin-parity of the baryon. These baryons are referred to as $\Xi_b(j = 0, J^P = \frac{3}{2}^+)$, $\Xi_b(j = 1, J^P = \frac{1}{2}^+)$ and $\Xi_b(j = 1, J^P = \frac{3}{2}^+)$.

This pattern is observed in $\Xi_c$ states [8]. The mass difference between $\Xi_b$ and $\Xi_b$ is about 150 Mev. So depending upon the choice of the wave function having the same overall quantum numbers we can have different baryon states. If we choose $(s^T C \gamma_5 q)Q$ as our baryon operator then we will be simulating $\Xi_b$ state and if we project out the spin-$1/2$ state of $(s^T C \gamma_5 d)Q$ then we will get the $\Xi'_b$. For reason discussed before we can not define $j = 1$ light-light diquark here. In our case the wave function that corresponds to $\Xi_b$ is $(Q^T C \gamma_5 s) d$. Constructing operator in this way allows the $s$ and $u/d$ quarks to have parallel spin configurations. We expect this operator to have a good overlap with $\Xi_b$ state and is also supported by our result. As the choice of wave function having the same overall quantum number leads to different states we do not calculate the correlation matrix of these operators.

FIG. 8. Comparison of our single bottom baryon spectra with Brown et al. [11], Burch [10], Mathur et al. [6] and PDG [8] where available.

FIG. 9. Comparison of our triple and double bottom baryon spectra with Brown et al. [11] and Burch [10].

The $b$ mass has been tuned to modified $\Upsilon - \eta_c$ spin averaged mass while $c$ quark to $J/\psi - \eta_c$ spin averaged mass. The $s$ quark required to be tuned to both the fictitious $\eta_s$ and $B_s$ mass since we expect the bottom-strange bound state to be more appropriate than $s - \bar{s}$ bound state in bottom baryons. For the light $u/d$ quarks, we have considered a wide range of bare masses and tune it using $B$ meson. We demonstrated the variation of bottom baryons as well as hyperfine splittings against varying $m_s$.
and \( m_u/d \). We showed that the hyperfine splittings are almost independent of \( s \) and \( u/d \) quark masses.

We compare our bottom baryon results with other works where available in the Figures \( \text{[8]} \) and \( \text{[9]} \). Various calculations of bottom baryon spectra appear to agree quite well with each other.

The comparison of the hyperfine splittings is shown in the Figure \( \text{[11]} \).

Apart from the hyperfine splittings, a few other mass splittings calculated in this work are assembled in the Table \( \text{XIV} \).

The bottom baryon spectra and various mass splittings reported in this paper and those appearing in \( \text{[8]} \) \( \text{[11]} \) are well comparable given the wide choice of actions and tuning employed in achieving them.

### VII. ACKNOWLEDGEMENT

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### TABLE XIV. Bottom baryon mass differences in MeV. PDG values without error is simply the differences of the two states.

| Mass splittings | This work | Brown et al. | PDG |
|-----------------|-----------|--------------|-----|
| \( \Omega^\star_b - \Lambda_b \) | 398(9) | – | 426.4(2.2) |
| \( \Xi^\prime_b - \Xi_b \) | 133(36) | 189(20) | 155.5 |
| \( \Xi_b - \Lambda_b \) | 166(9) | – | 172.5(0.4) |
| \( \Lambda_b - B \) | 406(21) | – | 339.2(1.4) |
| \( \Sigma_b^\star - \Lambda_b \) | 201(13) | 251(46) | 213.5 |

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