The calculation of the rates in the swirling flow for the confuser section

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Abstract. The advantages of using a confuser section in the outlet part of the vortex spillway are shown in the paper; they may be more rational in terms of ensuring cavitation-free operation of the flow swirler. An analytical method for calculating the aerated swirling flow in the confuser section of the vortex spillway is presented, as well as its correspondence to experimental results, which allow us to quickly and accurately conduct the distribution analysis of axial and tangential rates in the swirling flow along the length of the confuser at different taper angles.

1. Introduction
In reservoir water systems with a pressure of 200-300 m, the water rate in the outlet tunnel is 50-70 m/s. Such high-speed flows have a number of specific features: aeration processes, cavitation phenomena and cavitation erosion occur; banks wash-out in the downstream is observed, flow inhomogeneity can cause other phenomena in hydro-technical structures, a disruption in the reliable operation of water systems may occur, and there are problems of extinguishing excess energy. The introduction of vortex mine spillways using swirling flows makes it possible to solve the above problems [1,2,3].

2. Methods of research
In studies, the outlet part of the vortex spillway was considered as a confuser section with a quenching chamber. The confuser section, as well as the cylindrical water conduit, operates under the conditions of a swirling flow passage. Visual observations showed that along the length of the confuser the pattern of a swirling flow is aligned and at the end its flow is more transparent and stable compared to the flow in a cylindrical water conduit. The swirling flow with the core emerging from the confuser, then goes into the quenching chamber, made in the form of a cylindrical section or a section of a channel-shaped cross section. The presence of a quench chamber and a pressure-free tunnel behind the confuser allows us to quickly convert the swirl flow to the axial one. The decrease in energy along the length of the confuser is not so sharp compared to the piezometric pressure, which is associated with an increase in the kinetic energy of the flow due to an increase in the axial flow rate along the length of the confuser. An increase in the compression of the confuser end leads to a decrease in discharge.
capacity of the spillway, but the transformation of the swirling flow into the axial flow occurs with a sufficiently high quenching efficiency in a short section. In addition, the study showed that the use of the confuser section may be more rational in terms of ensuring the cavitation-free operation of the flow swirler, Fig. 1 [4,5,6,7].

Figure 1. Scheme of the confuser section of the vortex spillway

3. Research results.
In [8, 9, 10], an analytical method is given to calculate the aerated swirling flow in a cylindrical water conduit; the results of a comparison of experimental and theoretical studies are also presented. To study the aeration process and determine the hydrodynamic and hydraulic parameters, along with diffusion mixing at intensive mass transfer, the Kh.A.Rakhmatulin theory of multiphase interpenetrating and interacting media [11] is used.

After numerous transformations for the vector components of the dispersed mixture particle velocity the equation is obtained in the form [12,13]:

\[
2\hat{u}_{\theta,\theta} \frac{\partial \hat{u}_{\theta,\theta}}{\partial z} + \frac{\partial^2 \hat{u}_{\theta,\theta}}{\partial r \partial z} = \frac{1}{\text{Re}_s} \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \hat{u}_{\theta,\theta}}{\partial r} \right) \right) + \frac{\partial}{\partial r} \left( r \frac{\partial \hat{u}_{\theta,\theta}}{\partial r} \right) \right] \]

\[
\hat{u}_{\theta,\theta} \frac{\partial \hat{u}_{\theta,\theta}}{\partial z} = \frac{1}{\text{Re}_s} \left[ 1 + \varepsilon \left( \frac{\partial^2 \hat{u}_{\theta,\theta}}{\partial r^2} + \frac{1}{r} \frac{\partial \hat{u}_{\theta,\theta}}{\partial r} \right) - \left( 1 - \varepsilon \right) \frac{1}{r} \frac{\partial \hat{u}_{\theta,\theta}}{\partial r} \right] \]

In order to obtain an exact solution of equations (1), introduce a self-similar variable in the form

\[
y = \frac{\text{Re}_s \hat{r}^2}{4 \hat{z}} \]

(2)

To reduce equation (1) to an ordinary differential equation, determine the derivatives on \( y \) :

\[
\frac{\partial y}{\partial \hat{r}} = \frac{2y}{\hat{r}} , \quad \frac{\partial y}{\partial \hat{z}} = -\frac{y}{\hat{z}} .
\]

Considering these equalities, equations (1) are reduced to ordinary differential equations for the vector components of the dispersed mixture rate \( \hat{u}_{\theta,\theta} \) and \( \hat{u}_{\theta,\theta} \) (tangential and axial components):

\[
-\hat{y} \left[ \hat{u}_{\theta,\theta} \frac{d\hat{u}_{\theta,\theta}}{dy} + \frac{d}{dy} \left( \hat{y} \frac{d\hat{u}_{\theta,\theta}}{dy} \right) \right] = \frac{d}{dy} \left[ \frac{d}{dy} \left( \hat{y} \frac{d\hat{u}_{\theta,\theta}}{dy} \right) + \left( 1 + \varepsilon \right) \frac{d}{dy} \left( \hat{y} \frac{d\hat{u}_{\theta,\theta}}{dy} \right) \right]
\]

(3)
\[ (1 - \varepsilon^*) \frac{d \hat{u}_{\theta \theta}}{dy} + \frac{d}{dy} \left( y \frac{d \hat{u}_{\theta \theta}}{dy} \right) = 0 \]  

(4)

where \( \varepsilon^* = \frac{\varepsilon_i}{1 + \varepsilon_i} \).

Solution of equation (4) has the form

\[
 u_{n0} = \frac{\Gamma n_0}{r} \left( \text{erf} \left( \sqrt{y} \right) - \frac{2 \sqrt{y} e^{-y}}{\sqrt{\pi}} \right)
\]

Equation (3) may be written as:

\[
 -\frac{1}{2} \frac{d u_{c,\theta}^2}{dy} = \frac{d}{dy} \left( y \frac{d u_{c,\theta}}{dy} \right) + \varepsilon_i \frac{d}{dy} \left( y \frac{d u_{c,\theta}}{dy} \right) = 0
\]

Introduce function \( \Phi_0(y) \) into equation (4) in the form

\[
 \Phi_0(y) = y \frac{d \hat{u}_{\theta \theta}}{dy}
\]

(5)

Then we get the equation for the introduced function \( \Phi(y) \):

\[
 -\frac{1}{2} \frac{d u_{c,\theta}^2}{dy} = \frac{d}{dy} \left( y \frac{d \Phi_0(y)}{dy} \right) + \varepsilon_i \frac{d}{dy} \left( y \frac{d \Phi_0(y)}{dy} \right) = 0
\]

(6)

Integrating by \( y \) we get at \( y = 0 \) \( \Phi_0(0) = 0 \).

Considering the conditions for axial velocity and equality (5), we obtain the following expressions for axial velocity of the mixture:

for laminar flow

\[
 \frac{d \Phi_0(y)}{dy} = \frac{\hat{u}_{\theta \theta} - \hat{u}_{\theta \theta}}{y}
\]

or

\[
 \Phi_0(y) = \int_0^y \left( \frac{\hat{u}_{\theta \theta} - \hat{u}_{\theta \theta}}{y} \right) dy.
\]

for turbulent flow the equations are reduced to equations (4) and their solution under the above conditions will have the form [14]:
\[ u_{ac} = \frac{1-r_0^2}{x^2(1-r_c^2)} \{ 1 - (1 + D \sqrt{\frac{\lambda}{8}})[\exp(-x_0) - \exp(-x_c^2) + x_0(1-r_c^2)\exp(-x_r^2)] + \\
+ \beta \sqrt{\frac{\lambda}{8}} [1 + x_0 \exp(-x_0) - (1 + x_c^2)\exp(-x_r^2)] - \frac{1}{2}\sqrt{\frac{\lambda}{8}}[\exp(x_0) - \\
- \exp(-x_r^2) - 2(1 + x_c^2)\exp(-x_r^2)\ln(r_c) - Ei(-x_c) + Ei(-x_r^2)] - \\
- \frac{1}{2}\sqrt{\frac{\lambda}{8}}[\exp(-x_c^2) - \exp(-x_r^2)] - \exp(-y_0)(1 - (1 - r_0^2)(1 + D \sqrt{\frac{\lambda}{8}})(1 - r_c^2) + \beta \sqrt{\frac{\lambda}{8}} r_c^2 - \\
\frac{1}{2}\sqrt{\frac{\lambda}{8}} r_c^2 \ln(r_c)] + \Gamma_n^2 \frac{1}{2(1 - r_0^2)} \left[ \frac{4}{\pi} \left[ (1 + x_0)\exp(-x_0) - (1 + x_c^2)\exp(-x_r^2) \right] - \\
- \frac{2}{\pi}[\exp(-2x_0) - \exp(-2x_r^2)] - (1 + \frac{1}{\pi})\exp(-x_0)[2 - \exp(-x_0) + 2\ln(x_0) - 2Ei(-x_0)] + \\
+ (1 + \frac{1}{\pi})\exp(-x_r^2)[2 - \exp(-x_r^2) + 2\ln(x_r^2) - 2Ei(-x_r^2)] + \\
+ [3 + \frac{2}{\pi} - 2x_0(1 + \frac{1}{\pi})\left[ Ei(-x_0) - Ei(-2x_0) \right] - [3 + \frac{2}{\pi} - 2x_c^2(1 + \frac{1}{\pi})\left[ Ei(-x_0^2) - Ei(-2x_0^2) \right] + \\
+ \exp(-x_r^2)[Ei(x_0) - 2\ln(x_0) + Ei(-x_0)] - \exp(-x_r^2)[Ei(x_r^2) - 2\ln(x_r^2) + Ei(-x_r^2)] \right] - \\
- \frac{\Gamma_n^2}{2r^2} \left[ 1 - \exp(-y_0) \right] + \frac{2\Gamma_n^2}{\pi} x_c \exp(-y_0)[y_0 - \exp(-y_0)] + \Gamma_n^2 x_c (1 + \frac{1}{\pi})\left[ Ei(-y_0) - Ei(-2y_0) \right] - \\
- \Gamma_n^2 x_c \exp(-y_0)(1 + \frac{1}{\pi})[\ln(y_0) - Ei(-y_0)] + \frac{\Gamma_n^2}{2} x_c \exp(-y_0)[Ei(y_0) - 2\ln(y_0) + Ei(-y_0)] \right]\]

Function \( \hat{u}_{m \theta}^2(y) \) is determined from the solution of differential equation (4), at boundary conditions:

\[ u_{ac}^0(0) = u_{ac} = \frac{\Gamma_0}{r} = r \omega_0 \]

(7)

where \( \Gamma_0 \) is the velocity circulation and \( \omega_0 \) is the angular velocity of the dispersed mixture particles.

Integrating equation (4) under condition (7) we will have

\[ \epsilon^* \left( \hat{u}_{m \theta} \right) \right\}_{y=0} = 0. \]

Hence:

\[ \frac{d\hat{u}_{m \theta}}{dy} = 1 + \epsilon^* \hat{u}_{m \theta} \]

Solution of this equation is:

\[ \hat{u}_{m \theta} = \hat{u}_{m \theta} \exp(-\epsilon^*_1 y) \]

\[ \epsilon^* = \frac{\epsilon_1}{1 + \epsilon_1}, \quad \epsilon^*_1 = 1 + \epsilon^* \]

Then, the equation is written in the form:
\[
\frac{1}{y} \frac{d(y\Phi_0(y))}{dy} = \left(-2\varepsilon \bar{u}_{\text{edge}} \exp(-2\varepsilon y)\right)
\]

or

\[
\frac{d(y\Phi_0(y))}{dy} = 2\varepsilon \bar{u}_{\text{edge}} y \exp(-2\varepsilon y)
\]

Integrating the last equation, we determine the sought for function \(\Phi_0\)

\[
\Phi_0(y) = \int_0^y \exp(-y^*)dy^*
\]

Integrating \(\Phi_0\) in parts we will have:

For tangential velocity:

\[
\bar{u}_{\text{zt}} = \left(-\frac{y}{z}\right) \frac{d\bar{u}_{\text{zt}}}{dy} = \frac{4y}{\text{Re}_t r^2} \left(1 + \frac{\varepsilon_t}{\varepsilon}\right) \left[\frac{d}{dy} \left(y \frac{d\bar{u}_{\text{zt}}}{dy}\right)\right]
\]

To integrate this function, introduce the following function

\[
F(y) = y \frac{d\bar{u}_{\text{zt}}}{dy}
\]

Then the differential equation is reduced to Euler equation:

\[
\left(1 + \frac{\varepsilon_t}{\varepsilon}\right) \frac{dF(y)}{dy} + F(y) = 0
\]

The solution is written as [15,16]

\[
F(y) = e^{\lambda y}
\]

\[
\lambda \left(1 + \frac{\varepsilon_t}{\varepsilon}\right) e^{\lambda y} + e^{\lambda y} = 0
\]

Then, from equation (9) the following expression is obtained to determine parameter \(\lambda\):

\[
\lambda = -\frac{\bar{u}_{\text{edge}}}{1 + \frac{\varepsilon_t}{\varepsilon}}.
\]

\[
F(y) = \exp\left(-\frac{y\bar{u}_{\text{edge}}}{1 + \frac{\varepsilon_t}{\varepsilon}}\right) + c
\]

Considering boundary conditions (9) the constants of integration are determined as [17,18]

\[
\text{At } y = 0 \quad \frac{d\bar{u}_{\text{zt}}}{dy} << \infty, \quad y \to \infty, \quad c = 1
\]

So, the sought for function with account for (10) is
\[ F(y) = 1 - \exp\left(-\frac{yu_{ume}}{1 + \frac{\varepsilon_t}{\varepsilon}}\right) \]

From equality (8) the following equation for tangential velocity is obtained

\[ 1 - \exp\left(-\frac{yu_{ume}}{1 + \frac{\varepsilon_t}{\varepsilon}}\right) = \frac{d\hat{u}_{\theta\|}}{dy} = \frac{1}{y} \]

After integration the expression of tangential velocity is

\[ \hat{u}_{\theta\|} = \ln y - \frac{1}{u_{ume}} \left[ \int \frac{y u_{ume}}{1 + \frac{\varepsilon_t}{\varepsilon}} \right] + c_2 \]

The coefficient \( c_2 \) is determined from condition, \( \hat{z} \rightarrow 0 \), \( \hat{u}_{\theta\|}(r,0) = \hat{u}_{ume} \).

For the confuser section, the axial and tangential velocities are determined according to the above equations considering the taper angle; the radius at any point of the swirling flow thickness corresponds to the confuser taper angle or \( R_1 = R_1/cos \phi \), \( R_2 = R_2/cos \phi \). According to our computing program, the distribution graphs of axial and tangential velocities in a swirling flow along the length of the confuser at different taper angles were plotted. These graphs are shown in Figs. 2.3 at a taper angle of 60°.

**Figure 2.** The distribution of axial velocities in a swirling flow along the length of the confuser at a taper angle of 60°
4. Discussion

The calculations were performed for a pipe with a different diameters in sections located at distances of 1, 4, 16, 64 radii from the end of the pipe at Reynolds number \( Re = 6.5 \times 10^4 \). An analysis of the graphs (Figs. 2, 3) showed that at the beginning of the confuser there is an increase in tangential velocity values related to an increase in pressure, then there is a rapid attenuation of the swirl along the length of the confuser, that is, a decrease in tangential velocity values and an increase in axial velocities, which agrees with experimental results. So in the cylinder at a radius of 0.2 \( r \) ... 0.3 \( r \) there is a maximum of tangential velocities, and then in the central paraxial zone there is an intense attenuation and further along the radius there is a more smooth attenuation of velocities.

It should be noted that the use of the confuser section makes it possible for swirler to work cavitation-free. The tangential velocities in aerated swirling flows are greater than in a flow without aeration. This means that with an increase in the rate of air capture, the tangential velocities and the pressure at the wall increase, which leads to an increase in the cavitation safety of the tunnel wall [19, 20].

The distribution graph of axial component of velocity as a function of the radius and the distance from the inlet gate shows that the distribution of axial component is characterized by the fact that at \( r \rightarrow 1 \) (near the pipe walls), with increasing distance from the inlet gate, the velocity decreases, while in the paraxial zone axial velocities gradually increase along the length of the pipe. The axial velocities along the length of the confuser sharply increase in comparison with the cylinder, which leads to attenuation of the flow swirling, therefore, the flow energy is quenched in a short section of the tunnel, which agrees with experimental results.

5. Conclusions

1. The use of the confuser section in the outlet part of the vortex spillway is more rational in terms of ensuring the cavitation-free operation of the flow swirler.

2. The axial velocities along the length of the confuser sharply increase compared to the cylinder, which leads to attenuation of the flow swirl, therefore, the flow energy is quenched in a short section of the tunnel, which agrees with experimental results.
3. An analytical method for calculating aerated swirling flow in the confuser section of the vortex spillway is given.

4. The stated analytical calculation method is recommended for use in design of vortex spillways, it allows us to quickly and accurately perform the distribution analysis of axial and tangential velocities in the swirling flow along the length of the confuser at different taper angles.

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