Asymmetrical Large Extra Dimensions

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Abstract

We study scenarios in which there is a hierarchy of two sets of large compactified extra dimensions. One particularly interesting case has a single millimeter size extra dimension and five TeV\(^{-1}\) size dimensions. The Standard Model gauge bosons have Kaluza-Klein excitations with respect to one of the TeV scale dimensions. We discuss astrophysical constraints on this scenario, as well as prospects for signals at future high energy colliders.
1 Introduction

New developments in superstring theory [1] have led to a radical rethinking of the possible phenomenological implications of the existence of extra spatial dimensions. In superstring theory there are regions of moduli space where compactification radii become large while the string coupling, gauge couplings, and Newton’s constant remain fixed [2, 3]. The scale $R$ of these large extra dimensions determines the relation [4] between the usual Planck mass $M_P$ and an effective higher dimensional Planck scale $M_*$:

$$M_P^2 = M_*^{n+2} R^n,$$  \hspace{1cm} (1)

where for the moment we have taken all $n$ large extra dimensions to have the same size $R$. The scale $M_*$ is related to the string scale $M_S$ in a way which depends in general on the vacuum values of other moduli, including the size of other (smaller) extra dimensions. Roughly speaking, $M_S$ plays the role of the ultraviolet cutoff $\Lambda$ for the effective (nonrenormalizable) Kaluza-Klein theory.

Recently it was observed [4] that this scenario can be phenomenologically viable for $n \geq 2$, if we assume that the fields of the Standard Model are confined to a three-dimensional brane or intersection of branes in the larger dimensional space. Assuming further that the scale of the brane tension is of the order of the cutoff $\Lambda$ or larger, the resulting effective theory consists of $(3+1)$-dimensional Standard Model fields coupled to $4+n$ gravity and, perhaps, other $(4+n)$-dimensional “bulk” fields. With these assumptions the phenomenological constraints from gravity experiments, collider physics, and astrophysics are surprisingly weak [4]–[8], allowing $1/R$ scales as low as $10 \text{ MeV}$ to $10^{-3} \text{ eV}$, for cutoff scales $\Lambda$ in the range $1 - 100 \text{ TeV}$.

It has also been shown in superstring theory that it is possible to obtain $d = 4$ $\mathbb{N}=1$ supersymmetric chiral gauge theories confined to the world-volumes of stable configurations of intersecting D-branes [3]. The regions of string moduli space where such configurations have a perturbative description is not necessarily incompatible with the region where large extra dimensions may occur. Thus within our current knowledge (or ignorance) of superstrings it is not implausible to imagine that the Standard Model is confined to a brane configuration [10, 11, 12, 13], while large compactified dimensions are probed only by gravity and other bulk fields [4, 14].

It is equally possible that the Standard Model gauge theory is confined to a brane with more than three spatial dimensions, with the brane wrapped around one or more extra large dimensions. In this case collider limits constrain the compactification scale to be larger than about 1 TeV [15, 16]. The Standard Model gauge couplings will exhibit power law running above the compactification scale at which the gauge bosons begin to probe one or more extra dimensions [17]. If we equate the gauge coupling unification roughly with the string scale, this would imply that the string scale is no more than one or two orders of magnitude above the compactification scale. This scenario is not, however, compatible with the simple scaling relation Eq. (1), if we assume that no extra dimensions are larger than an inverse TeV. Thus it has been
generally assumed that the “millimeter” and “TeV” large extra dimension pictures are mutually exclusive.

The basic scaling relation Eq. (1) is the simplest one possible, and many researchers have observed that a variety of asymmetrical scenarios are also possible. With a total of 7 extra compactified dimensions at a generic point in superstring/M theory moduli space, one can hypothesize the existence of several separate compactification scales, all distinct from \( M_S \). In this paper we will consider the next simplest case, where there is a hierarchy between two sets of “large” extra dimensions. Thus:

\[
M_P^2 = M_{*}^{n+2} R^n, \quad (2)
\]

\[
= M^{n+m+2} R^n r^m, \quad (3)
\]

where \( n+m \leq 7 \), and we now call \( R \) the size of the “very large” extra dimensions, while \( r \) denotes the size of the merely “large” extra dimensions.

At this point we must decide which large dimensions, if any, are probed by Standard Model particles. We could assume that only the graviton and the right-handed neutrino probe any of the \( n+m \) extra dimensions. In this case the asymmetrical setup is useful for evading the rather stringent astrophysical bounds \([4]\) on the case \( n=2 \) in the symmetrical scenario. One also can obtain an attractive scenario for neutrino masses \([18]\).

Another possibility for these more general scenarios is that the brane volume containing the Standard Model is transverse to the very large dimensions of size \( R \), but does extend in one or more of the large extra dimensions of size \( r \). This has dramatic consequences for the evolution of the Standard Model gauge couplings, as noted earlier \([17]\). Above the energy threshold \( 1/r \), the logarithmic running of the gauge couplings will be replaced by (effectively) power law running; nevertheless gauge coupling unification may still occur, albeit at a rather lower energy scale. For example, we can assume that Standard Model chiral matter is confined to a 3-brane volume, while the gauge and Higgs fields are confined to a 4-brane with one dimension compactified at scale \( r \). Then the analysis of \([17]\) indicates that gauge coupling unification can still occur, at a scale which is no more than about 20 times \( 1/r \).

Roughly speaking, we ought to identify the gauge coupling unification scale with the string scale \( M_S \). Thus the scale \( 1/r \) is naturally one order of magnitude less than \( M_S \). However, we ought to allow ourselves at least one additional order of magnitude of stretch in this ratio. This is because extra matter with Standard Model charges can effect the running of the gauge couplings, and there may be large threshold effects at the string scale. On the other hand, it is certainly more difficult to arrange a hierarchy between the scale \( 1/r \) and the string scale if the Standard Model particles probe more than one extra dimension. New interpretations of gauge coupling unification also need to be considered \([19, 20]\).

A particularly dramatic case of this class of asymmetric scenarios occurs when
\( n = 1, \ m = 5 \). In this case (3) becomes:

\[
M_P^2 = M_\ast^3 R , \quad (4)
\]

\[
= M^8 R r^5 . \quad (5)
\]

We can assume that the compactification scale \( 1/R \) is \( 10^{-3} \) eV, which is in the range accessible to millimeter gravity experiments, as well as providing the appropriate neutrino mass scale for an explanation solar neutrino data [18]. For the compactification scale \( 1/r \), we choose 1 TeV, attempting to saturate the current collider lower bounds. With these inputs (3) implies:

\[
M = 137 \, \text{TeV} , \quad (6)
\]

\[
M_\ast = \left( \frac{M}{1 \, \text{TeV}} \right)^{8/3} = 5 \times 10^5 \, \text{TeV} . \quad (7)
\]

Note that \( M \sim 100 \) TeV is compatible with our argument above that we expect \( M_S \) to be not more than two orders of magnitude above \( 1/r \). In this scenario both \( M_P \) and \( M_\ast \) are parameters measuring the strength of gravitational couplings; they do not correspond to scales of new ultraviolet physics. \( M \), on the other hand, can be regarded as roughly equal to the ultraviolet cutoff \( \Lambda \).

This scenario is quite novel in that it predicts a single millimeter size extra dimension, as opposed to two such dimensions in the scenario based on (1). It also predicts TeV scale Kaluza-Klein (KK) thresholds for the Standard Model gauge bosons (and perhaps the Higgs), but probably not for quarks and leptons. Thus this scenario is constrained both by phenomenological considerations similar to those discussed in [4] for millimeter size extra dimensions, as well as those discussed in [16] for TeV scale Kaluza-Klein gauge bosons.

## 2 Astrophyiscal constraints

Here we review the constraints on a single mm size extra dimension, following the discussion of Arkani-Hamed et al [4] and later papers [5, 6, 8]. Note that in our scenario the effective ultraviolet cutoff for Kaluza-Klein mode sums is \( M \sim 100 \) TeV. At energy scales \( \sqrt{s} \) less than \( 1/r \), the effects of KK graviton emission are greatly suppressed in our scenario. Cross sections for real emission of KK gravitons scale like

\[
\sigma \sim \frac{\sqrt{s}}{M_\ast^3} . \quad (8)
\]

Above the scale \( 1/r \) but below the scale \( M \), KK graviton emission is less suppressed but still very soft; cross sections scale like:

\[
\sigma \sim \frac{s^3}{M_\ast^8} . \quad (9)
\]
Of course in this energy regime we also have the possibility of real emission of KK gauge bosons (or Higgs); these cross sections are proportional to powers of Standard Model couplings and do not have any extra suppression above the kinematic threshold.

Effects due to virtual KK exchanges are much less suppressed than real KK emission in the low energy region. The cross sections scale like $s^3/M^8$, or like $s/M^4$ in channels where there is interference with Standard Model diagrams.

Thus we expect that the effects of real KK graviton emission are completely negligible at energies below $1/r$, while virtual KK graviton effects are perhaps marginally observable. The remainder of this section is devoted to confirming these expectations.

Most important to check are the astrophysical constraints. Some constraints come from the fact that the energy loss due to the KK graviton emissions by the sun, red giants, and the supernova 1987A must not exceed certain upper bounds. The most stringent bound comes from the supernova, where the dominant process for the energy loss is nucleon–nucleon bremsstrahlung ($N+N\rightarrow N+N+\text{graviton}$). Following [6], the energy loss per gram per second for our scenario is given by

$$\dot{\varepsilon} = 3.9 \times 10^{22} \text{erg g}^{-1} \text{s}^{-1} \left( X_n^2 + X_p^2 + 4.3 X_n X_p \right) \rho_{14} T_{\text{MeV}}^{4.5} M_*^{-3}$$

(10)

where $X_n, X_p$ are the neutron and proton fractions, $\rho_{14}$ is the density measured in units of $10^{14}$ grams per $cm^{-3}$, $T_{\text{MeV}} = T_{SN}/1 \text{ MeV}$, and $M_*$ is in TeV units. Using $T_{\text{MeV}} = 30, X_n = 0, \rho_{14} = 3$, and requiring that the energy loss rate to gravitons, as given by Eq. (10), do not exceed $10^{19} \text{erg g}^{-1} \text{s}^{-1}$, we obtain

$$M_* \geq 3700 \text{ TeV}, \quad \rightarrow M \geq 22 \text{ TeV}.$$  

(11)

We now consider the cosmological bound arising from the absence of “MeV bumps” in the cosmic diffuse gamma (CDG) radiation background as set by the recent measurement using COMPTEL instrument, in the energy range of 0.8 to 30 MeV [7]. Neutrino-antineutrino, and photon-photon annihilation will produce massive KK gravitons, and their subsequent decays to two photons will produce such a bump. Recent measurement of the photon spectrum is fitted well by a continuous distribution,

$$\frac{d n_\gamma}{d E} = A \left( \frac{E}{E_0} \right)^a$$

(12)

where $A = (1.05 \pm 0.2) \times 10^{-4} \text{ MeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{ster}^{-1}, E_0 = 5 \text{ MeV}, a = -2.4 \pm 0.2$. Following the work of [8], the contribution to the energy distribution from the annihilation of two photons, and three flavors of neutrino-antineutrinos are given by

$$\left( \frac{d n_\gamma}{d E} \right)_{T_* = 1 \text{ MeV}} = 7 \alpha_n \left( \frac{M}{\text{TeV}} \right)^{-(n+2)} \text{MeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{ster}^{-1}.$$  

(13)

where

$$\alpha_n (E) = 4.6 \times 10^{-6(n-2)} \frac{2\pi^{n/2}}{\Gamma (n/2)} \frac{f_n (E, T_* = 1 \text{ MeV}) \left( \frac{E}{\text{MeV}} \right)^{1/2}}{(\text{MeV})^{n+5/2}}.$$  

(14)
with the expression for $f_n$ given in [8]. In our scenario this becomes
\[
\left( \frac{dn_\gamma}{dE} \right)_{T^*_\gamma = 1 \text{ MeV}} = 7 \alpha_1 (E) \left( \frac{M_*}{\text{TeV}} \right)^{-3} \text{cm}^{-2}\text{s}^{-1}\text{ster}^{-1}.
\] (15)

We have calculated $\alpha_1 (E)$ for the energy range $E = 1$ to 10 MeV. The bound on $M_*$ is given by
\[
\left( \frac{M_*}{\text{TeV}} \right) \geq 7 \alpha_1 (E) \left( \frac{d\theta}{dE} \right)_{\text{measured}}^{-1}
\] (16)

In the range of $E = 1$ to 10 MeV, the bound for $M$ as obtained from Eq. (16) is fairly flat, varying from 24 to 48 TeV. The most stringent bound for $M$ comes using $E$ equal to 3 or 4 MeV, for which the value of $\alpha_1 (E)$ is $1.55 \times 10^9$ and $7.1 \times 10^8$ respectively. Using these values, we obtain from Eq. (16):
\[
M_* \geq 30000 \text{ TeV}, \quad \rightarrow M \geq 48 \text{ TeV}.
\] (17)

Thus we find that our scenario evades even the extremely stringent astrophysical bounds on a single millimeter size extra dimension.

3 Collider constraints

In this section, we discuss the laboratory constraints for our model coming from high energy processes. In our model, the gauge and the Higgs bosons live on a brane which contain one or more large TeV size dimensions. Thus, their KK excitations will contribute to the effective 4-dimensional theory. Below the threshold $\mu_0 = r^{-1}$ these may be observable as off-shell contributions, while in future high energy colliders we may be able to produce some of the low lying KK states as resonances. The interesting point to note here is that the constraints from these processes will give bounds on the masses of these KK states, and hence on the scale $\mu_0$. A number of analyses already give interesting bounds. For example Marciano [21] has looked into the effect of the $W^*$, a KK excitation of the W boson, to low energy weak processes, and obtained a bound of $m_{W^*} \geq 3.7$ TeV for the case of only one extra dimension. The authors of Ref. [22] studied the effects for the processes $e^+ e^- \rightarrow \mu^+ \mu^-$ at LEP2, and the Drell-Yan process in hadronic colliders. They set lower bounds on $\mu_0$ in the range of 1 to 3 TeV.

As an example, we consider the effects of the KK excitations of gluons to top production at the Tevatron. The KK excitations of the gluons will contribute to both $q \bar{q} \rightarrow t \bar{t}$, and $gg \rightarrow t \bar{t}$ subprocesses via their exchanges in the appropriate s,t and u channels. In the light of the above bounds, it is unlikely that we will see such KK resonances directly in Tevatron dijet spectra, unless the gluon KK modes turn out to be somewhat light compared to the KK modes of the electroweak gauge bosons.
The effect of the KK states in the $q\bar{q}$ subprocess is to modify the propagator $1/s$ to $D(s)$ with (ignoring the appropriate width factor)

$$D(s) = \sum_{n=1}^{\infty} \frac{1}{s - m_n^2},$$

(18)

with similar modification in the t and u channels. Writing $m_n^2 = \mu_0^2 n^2$ for the KK excitations of the gluons, with one extra dimension Eq. (18) reduces to (for $s \ll \mu_0^2$):

$$D(s) \approx \frac{1}{s} - \frac{\pi^2}{6\mu_0^2}.$$

(19)

Thus, well below threshold, the net effect of the interference of the KK excitations is to reduce the subprocess cross sections by the factor $D(s)^2$. We have calculated the cross sections for $p\bar{p} \to t\bar{t}X$ at the Tevatron ($\sqrt{s} = 1.8$ TeV, 2 TeV) as functions of $\mu_0$. We included only the $q\bar{q}$ contribution, since $gg$ is small at the Tevatron. In Fig. 1, we show the ratio $R$ defined by

$$R \equiv \frac{\sigma_{\mu_0} (p\bar{p} \to t\bar{t})}{\sigma_{SM} (p\bar{p} \to t\bar{t})}$$

(20)

with $m_t = 175$ GeV for the Tevatron Run 2 ($\sqrt{s} = 2$ TeV) as a function of the compactification scale $\mu_0$. The cross section is expected to be measured to about five percent accuracy at the Tevatron Run 2. This can be used to set lower bound on the compactification scale, $\mu_0$, and is expected to be around 3 TeV. LHC experiments will raise this bound significantly, or discover the low lying KK excitations of the gluons.

### 4 New physics in hadron colliders

The smoking gun signature of our scenario is the existence of a single millimeter sized very large dimension, combined with dramatic new high $p_T$ physics in future colliders. Below we discuss some of these new physics signals relevant to hadron collider experiments near or above the KK threshold energy $\mu_0$.

At the hadronic collider, in our scenario, the main effect will come from $g_1^*$, the first KK excitation of the gluon which propagates into one of the TeV$^{-1}$ scale extra dimension. In our model, the quarks are localized in the usual 4 space-time dimensions. Their interactions do not conserve momenta in the extra dimensions, and hence the tree-level couplings $\bar{q}g_1^* g$ are allowed in the effective four dimensional theory. Since the gluons propagate in the extra dimension, fifth dimensional momentum conservation forbids $gg g_1^*$ couplings in the effective four dimensional theory. However, such an effective coupling is generated at the one loop level via a quark loop similar to the gluon-gluon-Higgs coupling ($ggH$) in the Standard Model. There are several
new interesting phenomena in our scenario related to these vertices, for high energy hadronic colliders, which could lead to new signals for the proposed TeV$^{-1}$ scale extra dimensions.

1. **Enhancement of high $p_T$ dijet production**

   The important subprocess relevant for the dijet productions are:
   
   \[ gg \rightarrow g_1^* \rightarrow gg, q\bar{q} \]
   \[ q\bar{q} \rightarrow g_1^* \rightarrow gg, q\bar{q} \]

   The first subprocess will be best studied at the upgraded Tevatron ($\sqrt{s} = 2$ TeV), since at this energy, about 90% of the luminosity is in the $q\bar{q}$ channel. At this energy, it is unlikely that we shall produce the $g_1^*$ resonance. However, we might see an enhancement in the high $p_T$ jet cross sections due to the off-shell effect of $g_1^*$. This will be similar to the single jet inclusive excess reported by the CDF collaboration \[23\]; thus, as in this case, it will important to have a firm handle on the parton distribution functions.

   At the LHC, resonance production of the $g_1^*$ may be energetically possible. This state will be very wide, with a width few tenths of its mass. Thus it will be very hard to observe the actual resonance structure. However, since the final states $gg$, $q\bar{q}$ are coming from the decay of a very massive $g_1^*$, they will carry very high $p_T$. Thus, we will see a large enhancement in the high $p_T$ jet cross sections compared to the usual QCD expectation.

2. **High $p_T$ trijet production**

   \[ gg \rightarrow g_1^* g, g_1^* \rightarrow gg, q\bar{q} \]

   In a very high energy hadronic collider, such as LHC, above the threshold of $g_1^*$ production, an onshell $g_1^*$ together with a gluon will be produced. $g_1^*$ will decay to $gg$ and $q\bar{q}$ giving rise to two very high $p_T$ jets. Thus, above this threshold, there will be anomalously large production of three jet events with two of the jets having very large $p_T$, and the third with somewhat lower $p_T$. Such events will be distinct from the usual QCD prediction. The decay $g_1^* \rightarrow ggg$ (which is somewhat suppressed compared to $g_1^* \rightarrow gg$ decay) will lead to four high $p_T$ jets in the final state with three of the jets having very large $p_T$.

3. **Pair productions of KK gluons and high $p_T$ four jet signals**

   The relevant subprocess for for the four jet signal is
   
   \[ gg \rightarrow g_1^* g_1^*, g_1^* \rightarrow gg, q\bar{q} \]

   This will be the most important signal in our scenario at the LHC energy ($\sqrt{s} = 14$ TeV). If the compactification scale for a “merely large” extra dimension is in the few TeV range, the first excitation of the gluon $g_1^*$, will be pair produced, and each one will decay dominantly into $q\bar{q}$, or $gg$. Since the quark, antiquark,
or the two gluons are coming from the decay of a very massive particle, they will have very high $p_T$, much higher than produced in the usual QCD process. Thus, above this $g_1^* g_1^*$ pair production threshold, there will be anomalous large production of high $p_T$ four jet events. These events will be very distinct from the usual QCD production, and will constitute the smoking gun signal for our model.

4. Top Production

In our scenario with TeV$^{-1}$ scale extra dimensions, there is a new source for the production of the top quark anti-top quark pairs. The relevant subprocesses are:

\begin{align*}
q\bar{q} & \rightarrow g_1^* \rightarrow t\bar{t} \\
gg & \rightarrow g_1^* g_1^*, g_1^* \rightarrow t\bar{t}
\end{align*}

The cross section for the top production at the hadronic colliders will be significantly altered due to the contributions from the low lying KK states. The first subprocess above is best explored at the upgraded Tevatron due to the high $q\bar{q}$ luminosity. As discussed in section 3, the $t\bar{t}$ cross section will be less than that expected from the SM below the scale of resonance production due to the negative interference effect. With the high luminosity, it may be possible to study such a deviation at Run 2. The second subprocess is best studied at the LHC energy. The four top (anti-top) production cross section will be significantly higher than expected from QCD. As in jet physics, the $p_T$ distributions of the top will be significantly altered compared to the SM expectations.

5. Caveats and outlook

We admit that we don’t know how to generate or stabilize the scales $R$ or $r$, and furthermore it is a little strange that one (or more) of the $r$ type dimensions has a brane extending in it while the other $r$ type dimensions don’t. We also admit that we haven’t shown why it is natural to have $M \sim 100\mu_0$; all we have really argued is that this may be consistent with a picture involving accelerated gauge coupling unification. At least $R$ and $r$ correspond to scales that already exist in physics (the cosmological constant and the electroweak scale, respectively).

Our scenario predicts a host of new phenomena that can be tested in upcoming high energy collider experiments. Of course these phenomena are common to many other scenarios with TeV KK scales. We hope to come back to the details of these phenomena in a future work.
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Figure 1: Ratio of the cross section for our model at scale $\mu_0$ over the SM cross section for $tt$ production.