Effect of isovector scalar meson on equation of state of dense matter within relativistic mean field model

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The effects of the isovector-scalar δ-meson field on the properties of finite nuclei, infinite nuclear matter and neutron stars are investigated within the Relativistic Mean Field (RMF) model which includes non-linear couplings. Several parameter sets (SRV’s) are generated to assess the influence of δ-meson on the properties of neutron star. These parametrizations correspond to different values of coupling constant of δ-meson to the nucleons with remaining ones calibrated to yield finite nuclei and infinite nuclear matter properties consistent with the available experimental data. It is observed that to fit the properties of finite nuclei and infinite nuclear matter, a stronger coupling between isovector-vector ρ meson and nucleons is required in the presence of δ field. Furthermore, the δ-meson is found to affect the radius of canonical neutron star significantly. The value of dimensionless tidal deformability, Λ for the canonical neutron star also satisfies the constraints from the waveform models analysis of GW170817 binary neutron star merger event. A covariance analysis is performed to estimate the statistical uncertainties of the model parameters as well as correlations among the model parameters and different observables of interest.

I. INTRODUCTION

Neutron stars are the densest objects in the observable universe and deep knowledge of the Equation of State (EoS) of the dense matter in beta-equilibrium is thus required to understand their behavior. It has been shown that the dense matter EoS must be treated relativistically [1,2]. For this reason, relativistic mean field (RMF) models have been widely used to obtain a realistic description of the properties of finite nuclei, bulk nuclear matter or the properties of neutron stars. Currently, many different variants of RMF models with various couplings are in use to study the finite nuclei and neutron star properties [3,4]. Accurate constraints are necessary to understand the limits of these different types of models. During the last decade, a wide range of astrophysical observations such as the precise measurement of massive millisecond pulsars using Shapiro delay technique [6,7], detection of gravitational wave generated by binary neutron stars in the GW170817 event by the LIGO-Virgo collaboration [8,9], or the joint mass radius measurement of neutron stars using X-ray timing technique by NICER collaboration [10,11] started to provide unprecedented new constraints on the dense matter EoS. They have triggered plethora of theoretical studies to look at the dense matter EoS from very different perspectives c.f. Ref. [12] and references therein. First of its kind model independent measurement of neutron skin thickness $\Delta r_{np}$ of $^{208}$Pb [12] and $^{48}$Ca [16] in the Jefferesen lab also inspired theoretical studies to take a fresher look at the isovector channel of the nuclear interaction [17,21].

Effective mass of nucleon quantifies the momentum dependence of nuclear force in the medium. It can be quoted for infinite nuclear matter at the Fermi surface. It is, however, necessary to realize that the concept of effective mass is different in non-relativistic [22,23] and relativistic formalism [24]. Nevertheless, it plays some crucial roles to determine various finite nuclear properties e.g. isoscalar giant quadrupole resonance (ISGQR) [25], nucleon nucleus scattering in optical potentials [26] or even in realizing various properties of nuclear matter and neutron stars [27,28]. Recently, a systematic study was performed using RMF models which assessed the impact of relativistic (Dirac) effective mass ($M^*$) on the properties of neutron star [29]. The isovector splitting of the effective mass, which measures the difference between neutron ($M^*_n$) and proton ($M^*_p$) effective mass, can influence greatly as well the physical properties of finite nuclei such as locating the drip-lines [30] or nucleon-nucleus scattering of asymmetric systems [26]. Its impact increases manifold in high density environment which can alter thermal and transport properties of asymmetric matter [31,32] or neutrino opacities of neutron star matter [33]. To settle its value, even at saturation, remains a persisting challenge both theoretically [34,38] and experimentally [26,39,40].

Appearance of isovector splitting of effective mass to the leading order in RMF models occurs through the isovector-scalar δ mesons. It impacts the proton fraction in neutron stars and hence the cooling process of
neutron stars after form[11,12]. It can also influence the global properties of neutron stars.[27,28,43–46]. A systematic study of RMF models with added freedom in the isospin channel through δ meson, optimized using well-constrained finite nuclear properties and extrapolating at high density to understand the properties of neutron stars and in general dense matter EoS, can enhance our knowledge on the density dependence of the isovector channel of nuclear interaction.

The present study is aimed towards investigating the effects of δ meson on the dense matter EoS within the framework of RMF model. We generate several parameter sets by varying the coupling strengths of the EOSs remain consistent with the available finite nuclei data and a few empirical properties of infinite nuclear matter evaluated at the saturation density. The properties of neutron stars obtained with these EOSs are then compared to assess the role of δ mesons.

The paper is organized as follows. In section II, the theoretical framework which is used to construct the EoS for neutron stars is discussed. We also discuss the procedure to optimize the coupling constants and the method to perform a covariance analysis in the same section. In section III, we present our results. We summarize and draw our conclusions in section IV.

II. FORMALISM

A. Theoretical model

The Lagrangian density for the RMF model used in the present study based on different non-linear, self and inter-couplings among isoscalar-scalar σ, isoscalar-vector ωμ, isovector-scalar δ and isovector-vector ρμ meson fields and nucleonic Dirac field Ψ, is given by

\[ \mathcal{L} = \sum_q \left[ \bar{\psi} \gamma^\mu \partial_\mu - (M - g_\sigma \sigma - g_\delta \delta \cdot \tau) \right] \psi - \left( g_\omega \gamma^\mu \omega_\mu + \frac{1}{2} g_\rho \gamma^\mu \rho_\mu \right) \psi + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - \mathcal{M}_\sigma^2 \sigma^2) \\
- \frac{3}{3!} g_\delta^3 \delta^3 - \frac{1}{4!} g_\delta^4 \delta^4 - \frac{1}{4} \omega_\mu \omega_\nu \omega_\rho \omega_\sigma - \frac{1}{2} m_\omega^2 \omega_\mu \omega_\nu \mu \nu \\
+ \frac{1}{6} \zeta g_\omega^2 (\omega_\mu \omega_\nu) \omega_\rho \omega_\sigma - \frac{1}{2} \partial_\mu \rho_\nu \rho_\mu \rho_\nu \\
+ \frac{1}{2} (\partial_\mu \delta_\rho \delta_\sigma - m_\delta^2 \delta^2) + \frac{1}{2} \sigma_\mu \sigma_\nu g_\rho^2 (\omega_\mu \omega_\nu \rho_\mu \rho_\nu). \quad (1) \]

The Dirac effective mass for the nucleons (q) appearing in the Lagrangian density above is specified as

\[ M_q^* = (M - g_\sigma \sigma - g_\delta \delta \cdot \tau), \quad (2) \]

where, \( \tau = 1(-1) \) for \( q = \text{neutron} (\text{proton}) \). Following the Euler-Lagrange formalism one can readily find the expressions for energy density \( \varepsilon \) and pressure \( P \) as a function of density from Eq. (1) [43].

B. Optimization and covariance analysis

In the present study, five new relativistic interactions SRV00, SRV01, SRV02, SRV03, and SRV04 have been generated for the Lagrangian density given by Eq. (1) to investigate the effect of δ meson on the properties of finite nuclei and neutron star matter. Here, SRV00, SRV01, SRV02, SRV03 and SRV04 parametrizations correspond to different value of the coupling of δ-meson to the nucleon i.e. \( g_\delta = 0, 1.0, 2.0, 3.0 \) and 4.0 respectively. As the effect of δ meson is predominantly important at suprasaturation densities, one can a-priori anticipate its insignificant impact in finite nuclei, which is primarily sensitive to the EoS at subsaturation densities. This is the reason why we kept fixed the \( g_\delta \) at aforementioned values optimizing the rest of the parameters in Eq. (1). This is not far from the strategy recently used by Li et. al. in Ref. [46]. The parameters of the model are obtained by fitting the experimental data [50] on binding energies \( (BE) \) and charge rms radii \( (r_{ch}) \) of some spherical nuclei [16,24]O, [40,48]Ca, [56,78]Ni, [88]Sr, [90]Zr, [100,116,132]Sn and [208]Pb. For the open shell nuclei, the pairing has been included using BCS formalism with constant pairing gaps [52,53] that are taken from the nucleon separation energies of neighboring nuclei [50]. Neutron and proton pairing gaps are evaluated by using fourth order finite difference mass formula (five point difference) [54]. The neutron and proton pairing gaps \( (\Delta_n, \Delta_p) \) in MeV for the open shell nuclei are \( [88]_{\text{Sr}}(0.0,1.284), [90]_{\text{Zr}}(0.0,1.239) \) and \( [116]_{\text{Sn}}(1.189,0.0) \). The neutron pairing gap for \( ^{24}O \) practically vanishes since the first unoccupied orbit \( 1d_{3/2} \) is almost 4.5 MeV above the completely filled \( 2s_{1/2} \) orbit [55,56]. The pairing correlation energies for a fix gap \( \Delta \) is calculated by using the pairing window of \( 2\hbar \omega \), where \( \hbar \omega = 45A^{-1/3} - 25A^{-2/3} \) MeV [48]. We also incorporated the recently measured neutron skin thickness of \( ^{208}Pb \) using the parity violating electron scattering experiment [15] in our fit data.

The optimization of the parameters (p) appearing in the Lagrangian (Eq. (1)) is done by using the simulated annealing method (SAM) [57,59] by following \( \chi^2 \) minimization procedure which is given as,

\[ \chi^2(p) = \frac{1}{N_d - N_p} \sum_{i=1}^{N_d} \left( \frac{M_i^{\text{exp}} - M_i^{\text{th}}}{\sigma_i} \right)^2, \quad (3) \]

where \( N_d \) is the number of experimental data points and \( N_p \) is the number of fitted parameters. The \( \sigma_i \) denotes adopted errors [60,61] and \( M_i^{\text{exp}} \) and \( M_i^{\text{th}} \) are the experimental and the corresponding theoretical values, respectively, for a given observable. The minimum value of \( \chi^2 \) corresponds to the optimal values \( p_0 \) of the parameters.

Once the optimized parameter set is obtained, the correlation coefficient between two quantities \( Y \) and \( Z \), can be calculated by covariance analysis [60,64] as

\[ r_{YZ} = \frac{\Delta Y \Delta Z}{\sqrt{\Delta Y^2 \Delta Z^2}}, \quad (4) \]
As the contribution of the δ binding due to the ρ higher value of the repulsion by the g parameter to a suitable set of on finite nuclei as described earlier. All parametrizations obtained in the present work give equally good fit to the properties of finite nuclei which were used for the optimization procedure. In Table I we display optimum values of the model parameters for all the five SRV parameter sets along with the uncertainties on them computed using Eq. (5). It can be seen that the parameter gρ increases with the increase in value of gδ. A larger value of gρ is required in the presence of the δ-field to fit the properties of finite nuclei. As the contribution of the δ-field is attractive, increased binding due to the δ-field has to be compensated by the higher value of the repulsion by the ρ-field. The parameter gσ has its lowest value for SRV00 parametrization (gδ=0). For any finite value of δ-coupling (gδ > 0) i.e. for SRV01, SRV02, SRV03 and SRV04 parametrizations, the strength of ρ-coupling (gρ) increases gradually. The cross-coupling between the ωμ and ρμ fields quantified by the term c1 decreases slightly from 4.6003 to 3.0107 as the value of coupling constant gδ increases from 0.0 to 4.0 corresponding to different SRV parametrizations.

In Table I different observables fitted in the present work, their experimental values, adopted errors σ on them, and along with the calculated values for different SRV parametrizations are displayed. The estimated uncertainties are also listed for the fitted observables. The fitted values of finite nuclei properties are quite close to their experimental counterparts. The root mean square (rms) errors on the BE are found to be in the range 1.50−1.86 MeV, and the ones for rch are found to be 0.02 fm for the different parameterizations. It is quite interesting to observe that even though g8 influences the coupling gρ, the isovector sensitive observable ∆rnp varies only slightly ∼ 0.01 fm across the different SRV models obtained in the present work. This observation is quite similar to the one obtained by Li et al. [46].

In Table III we present the results for the properties of Symmetric Nuclear Matter (SNM) such as binding energy per nucleon (E/A), incompressibility (K), the ratio of effective mass to the mass of nucleon (M*/M) along with symmetry energy coefficient (J), and its slope (L), all are evaluated at the saturation density (ρ0). We also quote the theoretically calculated error on them. The results are presented for all five SRV parametrizations.

### Table I. SRV parameter sets for the Lagrangian of RMF model as given in Eq. (1). The parameter ␥ is in fm⁻¹. The values of meson masses mω, mw, mρ and mδ are in MeV. The nucleonic mass (M) and meson masses (mω, mρ and mδ) are taken as 939, 782.5, 762.468 and 980 MeV, respectively. The values of ␥, ␥, and c1 are multiplied by 10⁵.

| Parameters | SRV00 | SRV01 | SRV02 | SRV03 | SRV04 |
|------------|-------|-------|-------|-------|-------|
| gσ         | 10.3109±0.1109 | 10.3442±0.0978 | 10.3344±0.1135 | 10.3723±0.0999 | 10.3735±0.1294 |
| gω         | 13.1772±0.1621 | 13.2508±0.1379 | 13.2137±0.1414 | 13.3113±0.1269 | 13.2984±0.1815 |
| gρ         | 10.8834±1.1730 | 11.2832±1.0110 | 11.5834±0.9676 | 12.4834±1.1274 | 13.1833±0.8043 |
| ␥           | 1.8509±0.0500 | 1.8624±0.0700 | 1.8780±0.0200 | 1.8242±0.0500 | 1.8490±0.0800 |
| ζ           | -0.0515±0.0700 | -0.0580±0.0600 | -0.0606±0.0600 | -0.0493±0.0800 | -0.0562±0.0800 |
| c₁          | 4.6003±2.3600 | 4.4921±1.9200 | 4.2085±1.5600 | 3.6616±1.4900 | 3.0106±1.8800 |
| mσ         | 0.0211±0.0017 | 0.02050±0.0013 | 0.0217±0.0011 | 0.02177±0.0012 | 0.02089±0.0027 |
| mρ         | 4.0800±0.0800 | 4.0800±0.0800 | 4.0800±0.0800 | 4.0800±0.0800 | 4.0800±0.0800 |
| mδ         | 501.9596±0.9230 | 501.0200±1.0071 | 501.6638±1.3141 | 500.7480±1.2629 | 501.0215±1.3663 |
The bulk nuclear matter properties at saturation density for SRV parametrizations are listed: $\rho_0$, E/A, K, J, L and $M^*/M$ denotes the saturation density, binding energy per nucleon, incompressibility coefficient, symmetry energy, the slope of symmetry energy, and the ratio of effective nucleon mass to the nucleon mass, respectively.

The value of E/A lies in the range -16.09 to -16.12 for the five parametrizations. The value of J and L obtained by our parametrizations are consistent with the constraints from observational analysis $J = 31.6 \pm 2.66$ MeV and $L = 58.9 \pm 16$ MeV [62, 63]. The value of K is also in agreement with the value of 240 \pm 20 MeV determined from isoscalar giant monopole resonance (IS-GMR) for $^{90}$Zr and $^{208}$Pb nuclei [62, 63]. It can also be seen from Table III that the mean values of the slope of symmetry energy (L) for SRV parametrizations decrease with the increase in the value of $g_\sigma$. The average value of L decreases from 65.23 MeV in SRV00 to 55.31 MeV for SRV04.

It can be noted that the isoscalar properties (E/A, K, $\rho_0$ and $M^*/M$) are well constrained for all SRV parametrizations. The only exception is the error on K in case of SRV04, where the error is almost 10% of its central value. But in the isovector sector, the percentage error on the slope of symmetry energy (L) is consistently on the larger side for all SRV parametrizations.

A great deal of importance to perform covariance analysis in theoretical studies has been pointed out recently [60, 62]. It not only enables one to quote statistical uncertainties on model parameters or any calculated observables, but also provides complementary information about the sensitivity of the parameters to physical observables, redundancies among fitted observables or interdependences among model parameters. As our primary objective is not to establish an ultimate model, for demonstrative purpose, we will discuss the results of covariance analysis as outlined in Section III, only for the model SRV02. The results for other parameter sets are quite similar (not shown here). In Fig. 1 the correlation coefficients between different model parameters appearing in Eq. 1 are outlined for SRV02 parametrization. A strong correlation is found between the several pairs of model parameters, like, $g_\sigma$ and $g_\omega$, $g_\rho$ and $c_1$ and $\lambda$.
and \( \zeta \) with correlation coefficients 0.99, 0.98 and 0.95, respectively. These interdependences mean that if one of these pairs are fixed at a particular value, the other must attain the precise value as suggested by their correlation to satisfactorily obtain the fit data. The results obtained for the correlations among model parameters presented in Fig. 1 are quite similar to the obtained in Refs. [19, 70]. Anticipating the strong correlation between L and \( \Delta r_{np} \) which is shown later (see Fig. 3), this may be attributed to the large experimental error on the \( \Delta r_{np} \) for \( ^{208}\text{Pb} \), which also led us choosing a rather large adopted error during optimization. The theoretical errors on the \( \Delta r_{np} \) of \( ^{208}\text{Pb} \) nucleus are found to be 0.032, 0.028, 0.026, 0.0353 and 0.029 fm for SRV00, SRV01, SRV02, SRV03, and SRV04 parametrizations, respectively. These are much smaller compared to the adopted error (0.071 fm, which is also the experimental error obtained in Ref. [15]).

We now display in Fig. 2 the correlation coefficients between the model parameters appearing in Lagrangian (Eq. 1) and the different properties of interest corresponding to the SNM and the density dependent symmetry energy as displayed in Table III and the \( \Delta r_{np} \) of \( ^{208}\text{Pb} \) for SRV02. A strong correlation is observed between the isovector parameter \( g_\rho \) with the symmetry energy coefficient \( J \), its slope \( (L) \) and \( \Delta r_{np} \) of \( ^{208}\text{Pb} \). The vector mixing parameter \( c_1 \) is also found to have a strong correlation with the \( J \) and \( L \). This strong correlation is anticipated, as \( c_1 \) and \( g_\rho \) are strongly correlated to each other, which was observed in Fig. 1. It is also realized from Fig. 2 that bulk properties of SNM like \( E/A \), \( K \), \( \rho_0 \) and \( M^*/M \) have strong correlations with isoscalar coupling parameters \( g_\sigma \), \( g_\omega \) and \( \kappa \). This study is quite consistent with previous calculations in the literature [20, 54, 70].

In Fig. 3 we display the correlation coefficients among the different observables in graphical form, particularly which were also studied in Fig. 2. In the isoscalar sector the only strong correlation observed is between binding energy per nucleon \( (E/A) \) and incompressibility coefficient \( (K) \). The \( K \) also shows some mild correlations with all other observables displayed in the figure. The symmetry energy \( J \) and its slope parameter \( L \) are found to be strongly correlated. As mentioned earlier in the discussion of Table III, we observe a strong correlation of the neutron skin thickness of \( ^{208}\text{Pb} \) with \( J \) and \( L \). These results are also in line with the earlier ones [70].
It is quite important to emphasize that we kept fixed the strength of the coupling of \( \delta \) meson to different values and optimized the rest. This might be partially responsible to impart a strong correlation among the isovector sensitive parameters \( g_{\rho c_1} \) to \( J, L \) or \( \Delta r_{nP} \) of \( ^{208}\text{Pb} \) (see Fig. 2) to reproduce the fitted data within bounds. It further gets clarified in the strong correlations among \( \Delta r_{nP} \) \( ^{208}\text{Pb} \), \( J \) and \( L \) in Fig. 3. Anticipating the results obtained for neutron stars which are discussed later, this strong correlation somewhat restricts the behavior of the matter at high densities in the isovector channel, resulting in a monotonic increasing in the radius and tidal deformability of 1.4 \( M_\odot \) (see Table IV) with the increase of the \( \delta \) meson coupling \( g_\delta \). A full optimization is thus needed with suitable data in the future including \( g_\delta \), to understand this behavior further. In the present work we have included the simplest form of \( \delta \)-meson coupling \( (g_\delta) \) to the nucleons. Furthermore, a higher order mixed scalar interactions of \( \delta \) meson has a large influence on the symmetry energy and its density dependence. This enables one to have flexibility to vary the behaviour of EoS at high density and gives a large influence on the properties of neutron stars \( ^7\text{Li}, ^7\text{Be} \). To study the effects of \( \delta \)-meson on nucleon mass, in Fig. 4 the effective mass of proton and neutron are plotted as a function of baryon density for three values of asymmetry parameter \( \alpha = 0, 0.5, 1 \) \((\alpha = \frac{\rho_p - \rho_n}{\rho_p + \rho_n})\) for SRV04 parametrization, which has the largest value of \( g_\delta \) amongst all SRV variants obtained in the present work. The asymmetry parameter \( \alpha = 0.0 \) represents the SNM and \( \alpha = 1 \) corresponds to the pure neutron matter (PNM). It is clear from the Eq. 2 that the presence of \( \delta \)-meson leads to splitting of nucleon mass. For SNM, there is no splitting of the nucleon mass. In Fig. 4 the solid (dashed) lines depict the effective mass of proton (neutron) for \( \alpha = 0, 0.5 \) and 1.0. One can observe from the figure that the effective proton mass is larger than the neutron effective mass. The splitting of the proton and neutron effective masses due to the \( \delta \)-meson can be important in the highly asymmetric system like a neutron star or supernova environment. At the center of a neutron star the density can reach \( \sim 5\times10^{14} \) \( \rho_0 \) and \( \alpha \sim 0.7-0.8 \). One can readily estimate the amount of splitting in the effective mass in this situation looking at Fig. 4. It can also affect the transport properties of neutron star matter \cite{74}. To assess the impact of \( \delta \)-meson on the global properties of neutron star, we plot the gravitational mass \( (M_G) \) of non-rotating neutron star as a function of radius for all SRV parametrizations in Fig. 5. The maximum mass \( (M_{\text{max}}) \) and the corresponding radius \( (R_{\text{max}}) \) of neutron star for all the models obtained here lie in the range 2.04 - 2.13 \( M_\odot \) and 11.48 - 12.11 km, respectively. This satisfies the recently measured radius of PSRJ0740+6620 with 12.45\(^{0.65}_{-0.65} \) km by NICER collaboration \cite{12,13}. The radius of neutron star of 2 \( M_\odot \) is also in accordance with the observational data of PSRJ0740+6620 by NICER \cite{12,13}. The maximum mass of the neutron star attained by various SRV parametrizations supports the constraint from PSRJ0740+6620 with the mass of 2.08 \( \pm 0.07 \) \( M_\odot \) \cite{75,76}. It is observed that the radius \( (R_{1.4}) \) of neutron star with mass 1.4 \( M_\odot \), can be significantly affected by the presence of \( \delta \)-meson as we move from parametrization set SRV00 to SRV04. The value of \( R_{1.4} \) increases by 7.27 \% and the maximum mass of neutron star changes by 4.4 \% from SRV00 to SRV04 parametrizations with the variation of coupling \( g_\delta = 0.0 \) to 4.0. This change in the neutron star properties may be attributed to the impact of \( \delta \)-meson, which affects high-density behavior of asymmetric nuclear matter.

Tidal deformability imparted by the companion stars on one another in a binary system can yield remarkable information on the EoS for neutron star \cite{77,80}. In Fig. 6 we show the results of dimensionless tidal deformability \( \Lambda \) defined as \( \Lambda = (2/3)k_2(R/M_G)^5 \), where \( k_2 \) is the love number, as a function of the neutron star mass \( M_G \) for different SRV models. The recent constraints on the tidal deformability \( \Lambda_{1.4} \) of 1.4 \( M_\odot \) neutron star including GW170817 \cite{8,81} is also given in the figure. The value of \( \Lambda_{1.4} \) lies in the range 484 - 783 for different SRV parametrizations, which satisfies the proposed limit are listed in Refs. \cite{8,78,82,83}. The value of \( \Lambda_{1.4} \) increases with the increase in the value of the coupling \( g_\delta \) corresponding to the SRV parametrizations as can be seen from Fig. 6. All these results are summarized in Table IV. The theoretical errors/uncertainties in neutron star properties for SRV parametrizations are also mentioned.

**FIG. 5.** (Color online) Mass-Radius relation of neutron star for SRV parametrizations.
TABLE IV. The properties of nonrotating neutron star for the various EOSs computed with SRV parameter sets are presented along with the theoretical errors on them. $M_G$ and $R_{\text{max}}$ denote the Maximum Gravitational mass and corresponding radius. The values for $R_{1.4}$ and $\Lambda_{1.4}$ denote radius and dimensionless tidal deformability at 1.4$M_\odot$.

| No. | EOS  | $M_G$ (M$_\odot$) | $R_{\text{max}}$ (km) | $R_{1.4}$ (km) | $R_{2.0}$ (km) | $\Lambda_{1.4}$ |
|-----|------|------------------|-----------------------|----------------|----------------|----------------|
| 1.  | SRV00 | 2.04±0.03        | 11.48±0.08            | 12.92±0.22     | 12.07±0.31     | 484.16±62.10  |
| 2.  | SRV01 | 2.06±0.02        | 11.55±0.13            | 12.99±0.20     | 12.24±0.21     | 504.95±58.02  |
| 3.  | SRV02 | 2.07±0.02        | 11.67±0.07            | 13.16±0.13     | 12.43±0.14     | 565.52±60.33  |
| 4.  | SRV03 | 2.08±0.02        | 11.81±0.11            | 13.41±0.21     | 12.60±0.16     | 652.60±52.01  |
| 5.  | SRV04 | 2.13±0.06        | 12.11±0.23            | 13.86±0.21     | 13.09±0.33     | 783.96±70.03  |

FIG. 6. (Color online) Variation of dimensionless tidal deformability ($\Lambda$) with respect to gravitational mass for SRV parametrizations.

in the table. The neutron star properties such as $M_{\text{max}}$, $R_{\text{max}}$, $R_{1.4}$, $R_{2.0}$ are relatively well constrained for all SRV parametrizations (at ≤ 3 %) whereas for $\Lambda_{1.4}$, the theoretical uncertainties are found to be ≤ 10%.

To this end, we may mention that the contribution of $\delta$ meson is considered only through its linear interaction with nucleons. However, inclusion of the self interaction of $\delta$ mesons and the mixed interactions with other mesons may alter the symmetry energy at supersaturation densities. This would further enhance the flexibility of EOS of dense matter and accordingly the properties of neutron stars [71-73]. Inclusion of the higher order contributions of $\delta$ meson thus enable one to model the properties of neutron stars in somewhat independent of the properties of finite nuclei. The optimization of effective Lagrangian that includes different terms involving $\delta$ meson field requires accurate knowledge of neutron star properties over a wide range of mass.

IV. SUMMARY AND CONCLUSIONS

The effect of the isovector-scalar field corresponding to $\delta$-meson in relativistic mean field theory is investigated. We have generated five sets of SRV parametrizations SRV00, SRV01, SRV02, SRV03 and SRV04 to explore the effects of $\delta$-meson on the properties of finite nuclei, infinite nuclear matter, and neutron stars. A covariance analysis to measure the accuracy of model predictions is also performed. This also enabled us to carry out a systematic study of correlations among model parameters and various finite nuclei and infinite nuclear matter properties of interest. The SRV parametrizations have been obtained in such a way that they reproduce the ground state properties of the finite nuclei and infinite nuclear matter properties quite convincingly. In turn, they satisfy the constraints on mass and radius of the neutron star and its dimensionless tidal deformability, $\Lambda$, from recent astrophysical observations [8, 66, 67, 78, 81].

It is observed that to fit the properties of finite nuclei and infinite nuclear matter, a stronger coupling between the $\rho$-meson and nucleons ($g_{\rho}$) is required in the presence of $\delta$-meson field. Furthermore, the $\delta$-meson significantly affects the radius of canonical neutron star. It is found that the contributions from $\delta$-meson is important and has some significant effects on the dense matter EoS. The value of $\Lambda_{1.4}$ for different SRV parametrization is also in line with the constraint obtained from GW170817 event. It is clear that the isovector splitting of effective mass of nucleon in the presence of $\delta$ in dense asymmetric matter, like the scenario present in the core of a neutron star, can be significant. It remains, however, an open question how to identify in future the signatures of isovector effective mass splitting from astrophysical observations.

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