Local Texture Estimator for Implicit Representation Function

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https://github.com/jaewon-lee-b/lte

Abstract

Recent works with an implicit neural function shed light on representing images in arbitrary resolution. However, a standalone multi-layer perceptron shows limited performance in learning high-frequency components. In this paper, we propose a Local Texture Estimator (LTE), a dominant-frequency estimator for natural images, enabling an implicit function to capture fine details while reconstructing images in a continuous manner. When jointly trained with a deep super-resolution (SR) architecture, LTE is capable of characterizing image textures in 2D Fourier space. We show that an LTE-based neural function achieves favorable performance against existing deep SR methods within an arbitrary-scale factor. Furthermore, we demonstrate that our implementation takes the shortest running time compared to previous works.

1. Introduction

Single image super-resolution (SISR) is one of the most fundamental problems in computer vision and graphics. SISR aims to reconstruct high-resolution (HR) images from its degraded low-resolution (LR) counterpart. Dominant approaches [3, 5, 14, 15, 18, 24, 34, 35] are to extract feature maps using a deep vision architecture and then upsample to HR images at the end of a network. However, we need to train and store several models for each scale factor when an upsampler is implemented by sub-pixel convolution [24]. In contrast, arbitrary-scale SR methods [4, 9] are promising since such ideas pave the way to restore images in a continuous manner with only a single network.

Recently, implicit neural functions parameterized by a multi-layer perceptron (MLP) achieved remarkable performance in representing continuous-domain signals, such as images [4], occupancy [19], signed distance [21], shape representation [11], and view synthesis [20, 26]. Such MLPs take coordinates as inputs and are trained in a framework of gradient descent optimization and machine learning. Inspired by recent progress in implicit representation, LIIF[4] replaced sub-pixel convolution with MLPs to accomplish arbitrary-scale SR, even at substantial scale factors.

One limitation of implicit neural representations is that a standalone MLP is biased towards learning low-frequency components [23] and fails to capture fine details [29]. Such phenomenon is referred to as spectral bias, and recent lines of research in resolving this problem are projecting input coordinates into a high-dimensional Fourier feature space [20, 29] or substituting a ReLU with a sinusoidal activation [25]. Motivated from previous works, we study arbitrary-scale SISR problems through the lens of Fourier analysis.

In this paper, we propose a Local Texture Estimator (LTE), a dominant-frequency estimator for natural images, to allow an implicit function to learn fine details while restoring images in arbitrary resolution. We assume that an implicit function prioritizes learning image textures when utilizing dominant frequencies of images, as described in...
that local region in HR representation shares the same on learning local features. Inspired by this, we hypothesize distributed over space, allowing an implicit function to focus 
mainly related to local implicit neural representation [4,11].

Such approaches assume that latent variables are evenly dis-

sion of data. Recently, meta-learned initialization [28] has been proposed, which provides a strong prior for rep-

dent neural network architecture designs, such as residual block [15], densely connected residual block [35], channel attention [5,34], or non-local neural networks [18].

inspired by the success of the self-attention mechanism in high-level vision tasks [6,16], general-purpose image processing transformers, such as IPT [3] or SwinIR [14], have been proposed. Even though transformer-based SR architectures using a large dataset surpass CNN-based architectures in performance, these approaches nevertheless need to utilize a specific upscale module for each upsampling rate.

Deep SR architecture After ESPCN [24] has proposed an efficient learnable upsampling module using pixel-shuffling, numerous CNN-based approaches [5,15,18,34,35] have been studied to improve the representational power of models. Such approaches have exploited more complicated neural network architecture designs, such as residual block [15], densely connected residual block [35], channel attention [5,34], or non-local neural networks [18].

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Arbitrary-scale SR Our work is highly related to SR tasks within an arbitrary-scale factor [4,9,27,31], which is convenient and efficient for practical benefits. Training and storing models for each specific scale factor [3,14,15,24,34,35] is unfeasible when considering limited memory resources. Since MetaSR [9] first proposed an arbitrary-scale SR method with a single model, various approaches have been explored. Recently, ArbSR [31] has been proposed as a general plug-in module using conditional convolutions. ArbSR conducts an SR with different scales along horizontal and vertical axes, respectively. More recently, SRWarp [27] successfully transformed an LR image into any shape in HR representation using a differentiable adaptive warping layer. Most related to ours is the model of [4].

Spectral bias Recent works [20,23,25,29] have shown that a standard MLP with ReLUs shows limited performance in representing high-frequency textures. Such a phenomenon is referred to as spectral bias, and various methods have been proposed to alleviate this problem. Recently, SIREN [25] used the sine layer as a non-linear activation instead of a ReLU, resulting in fast convergence and high data fidelity. Other approaches [20,29] are to map input coordinates into high-dimensional Fourier space by using position encoding or Fourier feature mapping before passing an MLP. Frequencies are fixed to the power of two [20] or randomly sampled from Gaussian distribution [29]. Unlike previous works, dominant frequencies from our LTE are data-driven and characterize high-frequency textures in 2D Fourier space.

2. Related Work

Implicit neural representation Based on the fact that neural networks are universal function approximators [8], coordinate-based MLP is widely applied to represent continuous-domain signals in computer vision and graphics. Such MLPs are a memory-efficient framework for HR data since the amount of memory to store MLPs is independent of data resolution. However, pre-trained MLPs show limited performance in representing unseen data, requiring training per each signal. To overcome this generalization issue, [25,26] trained representation with a hypernetwork, which maps latent variables to weights of an MLP as in [7]. In [19,21], an MLP takes not only coordinates but also latent variables as inputs to enable representation to be a function of data. Recently, meta-learned initialization [28] has been proposed, which provides a strong prior for representing signals, leading to fast convergence. Our work is mainly related to local implicit neural representation [4,11]. Such approaches assume that latent variables are evenly distributed over space, allowing an implicit function to focus on learning local features. Inspired by this, we hypothesize that the local region in HR representation shares the same image textures.

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3. Problem Formulation

In this section, we aim to represent $\mathbf{1}^{HR} \in \mathbb{R}^{r_y \times H \times W \times 3}$ from $\mathbf{1}^{LR} \in \mathbb{R}^{H \times W \times 3}$ given any fraction number $r_x, r_y$. We first review a continuous representation of an RGB image with a local implicit neural representation [4, 11]. Even though such approaches showed outstanding performance in representing continuous-domain signals, a standalone MLP fails to capture high-frequency details [23]. To overcome this spectral bias problem, we formulate a Local Texture Estimator (LTE), a dominant frequency estimator for natural images. Unlike prior arts [20, 29], estimated frequencies are data-driven and strongly correlated to image textures. Additionally, we introduce scale-dependent phase estimation and LR skip connection, which aid LTE in learning high-frequency textures.

**Local implicit neural representation** In local implicit neural representation [4, 11], a decoding function $f_{\theta}$ is shared by all images and is parameterized by an MLP with trainable weights $\theta$. A decoder $f_{\theta}$ maps both latent tensors and local coordinates into RGB values; $f_{\theta}(z, x) : (\mathcal{Z}, \mathcal{X}) \mapsto \mathcal{S}$. $z \in \mathcal{Z}$ is a latent tensor from an encoder $E_{\theta}$, $x \in \mathcal{X}$ is a 2D coordinate in the continuous image domain, $\mathcal{S}$ is a space of predicted values from $f_{\theta}$. For simplicity, we assume that a latent tensor $z \in \mathbb{R}^{H \times W \times C}$ has the same width and height as $\mathbf{1}^{LR}$. Then, predicted RGB values ($s \in \mathbb{R}^3$) at a coordinate $x \in \mathbb{R}^2$ are estimated as

$$s(x, \mathbf{1}^{LR}; \theta) = \sum_{j \in \mathcal{J}} w_j f_{\theta}(z_j, x - x_j), \quad (1)$$

where $z = E_{\theta}(\mathbf{1}^{LR})$. \(\theta = [\theta; \mathcal{J}], \mathcal{J} \in \mathbb{Z}^4\) is a set of indices for four nearest (Euclidean distance) latent codes around $x$, $w_j$ is the bilinear interpolation weight corresponding to the latent code $j$ (referred to as the local ensemble weight [4], $\sum_j w_j = 1$), $z_j \in \mathbb{R}^C$ is the $j$th nearest latent feature vector from $x$, and $x_j \in \mathbb{R}^2$ is the coordinate of the latent code $j$.

Given a series of $M$ data points from $N$ images such as $(x_m, \mathbf{1}^{HR}(x_m)), m = 1, \ldots, M$ and $n = 1, \ldots, N$, the learning problem is defined as follows:

$$\hat{\Theta} = \arg \min_{\Theta} \sum_{m,n} ||\mathbf{1}^{HR}(x_m) - s(x_m, \mathbf{1}^{LR}; \Theta)||_1. \quad (3)$$

In practice, $\mathcal{X}$ spans $[-H, H]$ and $[-W, W]$ for two dimensions. Note that a step size (Cell in Fig. 2) of an output grid ($\mathbb{R}^{r_y H \times r_x W}$) and an input grid ($\mathbb{R}^{H \times W}$) is different. In [4, 11], their decoding function ($f_{\theta}$) predicts continuous representation with a relative coordinate: $x - x_j$ ($|x - x_j| \leq 1$) known as Local grid. The same coordinate (Local grid) with [4, 11] is used for our work in order to represent a $r_x \times r_y$ local area in HR representation.

**Learning dominant frequency component** Recent works have shown that an MLP with ReLUs is biased towards learning low-frequency content [23]. To resolve this spectral bias problem of an implicit neural function, we propose a Local Texture Estimator (LTE), an essential Fourier information estimator for natural images. Inspired by position encoding [20] and Fourier feature mapping [29], LTE transforms input coordinates into the Fourier domain before passing an MLP. However, unlike [20, 29], estimated Fourier information is data-driven and reflects image textures in 2D Fourier space. The local implicit neural representation in Eq. (1) can be modified as follows:

$$s(x, \mathbf{1}^{LR}; \psi) = \sum_{j \in \mathcal{J}} w_j f_{\theta}(h_{\psi}(z_j, x - x_j)) \quad (4)$$

where $h_{\psi}(\cdot, \cdot)$ denotes the LTE, which is shift-invariant. LTE ($h_{\psi}(\cdot, \cdot)$) consists of three elements: (1) an amplitude
estimator \( (h_a(\cdot) : \mathbb{R}^C \rightarrow \mathbb{R}^{2K}) \), (2) a frequency estimator \( (h_f(\cdot) : \mathbb{R}^C \rightarrow \mathbb{R}^{K \times 2}) \), (3) a phase estimator \( (h_p(\cdot) : \mathbb{R}^2 \rightarrow \mathbb{R}^K) \). Thus, given a local-grid coordinate \( \delta = x - x_s \in \mathbb{R}^2 \), the estimating function \( h_\psi(\cdot, \cdot) : (\mathbb{R}^C, \mathbb{R}^2) \rightarrow \mathbb{R}^{2K} \) is defined as

\[
\begin{align*}
\quad h_\psi(z_j, \delta) &= A_j \odot \left[ \frac{\cos(\pi F_j \delta)}{\sin(\pi F_j \delta)} \right],
\quad (5)
\end{align*}
\]

where \( A_j = h_a(z_j), \quad F_j = h_f(z_j). \) \( A_j \in \mathbb{R}^{2K} \) is an amplitude vector for a latent code \( j \), \( F_j \in \mathbb{R}^{K \times 2} \) denotes a frequency matrix for a latent code \( j \), and \( \odot \) represents element-wise multiplication. We believe that the amplitude vector and the frequency matrix are extracted from the latent code \( j \) so as to represent \( s(x) \) as close as possible to original signals \( \mathbf{I}^{HR}(x) \). In this perspective, we understand that by observing pixels inside a receptive field (RF), LTE with the encoder \( (h_\psi \circ E_\varphi) \) estimates dominant frequencies and corresponding Fourier coefficients accurately. Here, the size of RF is decided by the encoder \( (E_\varphi) \). We visually demonstrate estimated frequencies and corresponding Fourier coefficients in Sec. 5.4.

In practice, to enrich the information in outputs of LTE, we apply the unfolding technique to a latent code \( j \) so as to represent \( s(x) \) as close as possible to original signals \( \mathbf{I}^{HR}(x) \). This is implemented with trainable convolutional filters \( (h_a(\cdot) : \mathbb{R}^{9C} \rightarrow \mathbb{R}^{2K}, \quad h_f(\cdot) : \mathbb{R}^{9C} \rightarrow \mathbb{R}^{K \times 2}) \).

**Scale-dependent phase estimation** Phase in Eq. (7) contains information about edge locations of features. For SR tasks, the location of edge changes within a small neighborhood in its HR domain when the scale factor changes. To address this issue, we redefine the estimating function as:

\[
\begin{align*}
\quad h_\psi(z_j, \delta, c) &= A_j \odot \left[ \frac{\cos(\pi F_j \delta + h_p(\hat{c}))}{\sin(\pi F_j \delta + h_p(\hat{c}))} \right],
\quad (7)
\end{align*}
\]

where \( c \) denotes the cell size. Inspired by the observation that MLPs with ReLUs are incapable of extrapolating unseen non-linear space [32], we use \( \hat{c} = \max(c, c_{tr}) \), where \( c_{tr} \) denotes the minimum cell size during training.

**LR skip connection** A long skip connection in local implicit representation enriches high-frequency components in residuals and stabilizes convergence [12]. In the Fourier domain, LTE tends to predict frequencies located near a low-frequency region (DC). To prevent LTE from learning the DC only, we add upsampled LR. Thus, local implicit neural representation with the proposed LTE can be formulated as follows:

\[
\begin{align*}
\quad \hat{s}(x) &= s(x, \mathbf{I}^{LR}; \Theta, \psi) + \mathbf{I}^{LR}(x)
\quad (8)
\end{align*}
\]

### 4. Method

#### 4.1. Network Detail

Our LTE-based arbitrary-scale SR network includes an encoder \( (E_\varphi) \), the LTE (a pink shaded area in Fig. 2), a decoder \( (f_0) \), and an LR skip connection. This section first describes a backbone structure (including encoder, decoder, and LR skip connection) and our LTE.

**Backbone** We use EDSR-baseline [15], RDN [35], and SwinIR [14] without their upsampling layers as an encoder \( (E_\varphi) \). Thus, an output of the encoder has the same width and height as an input LR image. We hypothesize that deep SISR networks [14, 15, 35] aid LTE in estimating crucial Fourier information by extracting features of natural images inside an RF. Our decoder \( (f_0) \), shared by all images as [4], is a 4-layer MLP with a ReLU activation, and its hidden dimensions are 256. Lastly, we add a bilinear upsampled LR image to the decoder output as in Eq. (8). We expect that such a long skip connection provides DC offsets; thus, LTE is biased toward learning dominant frequencies and corresponding essential Fourier coefficients.

**LTE** Our LTE contains an amplitude estimator \( (h_a) \), a frequency estimator \( (h_f) \), a phase estimator \( (h_p) \), and sinusoidal activations. An amplitude and a frequency estimator are designed with 3x3 convolutional layers having 256 (= 2K) output channels, respectively, identical to a fully connected layer when feature maps are unfolded. The phase estimator is a single fully connected layer with hidden dimensions of 128. Note that an amplitude and a frequency estimator take the same feature map while the phase estimator takes cell as an input. Let us assume that a \( r_x \times r_y \) local region in an HR domain shares amplitude and frequency information extracted from our LTE \( (h_\psi) \) as in Eq. (4). According to this, we upscale the extracted Fourier information using nearest-neighbor interpolation. Then, a predicted phase is added to an inner product between the predicted frequency and local grid before passing it through the sinusoidal activation layer, as in Eq. (7). Finally, we multiply the predicted amplitude and sinusoidal activation output.

#### 4.2. Training Strategy

We construct a minibatch with uniformly sampled scales from \( \times 1 - \times 4 \), dubbed in-scale, to teach the nature of bicubic degradation at various scales. Note that we evaluate our LTE for both in-scale and out-of-scale, which is an unseen scale (specifically \( \times 6 - \times 30 \)), to verify the generalization ability of our network.

Let \( r = r_x = r_y \) be a scale factor randomly sampled from \( \times 1 - \times 4 \) and \( H, W \) be a height, a width of training patch, respectively. We first crop \( rH \times rW \) patches from an HR image. When preparing training pairs, we randomly sample \( HW \) pixels from an HR patch for ground truth (GT) and downsample an HR patch by the scale factor \( r \) for an LR counterpart. When computing loss during training, we pick \( HW \) pixels from interpolation outputs to match the dimensions of prediction with GT.
Figure 3. Qualitative comparison to other arbitrary-scale SR. RDN [35] is used as an encoder for all methods.

Figure 4. Qualitative comparison with non-integer scale factors by LIIF [4] (top) and our LTE (bottom). RDN [35] is used as an encoder.

Figure 5. Qualitative comparison between various encoders with our LTE for ×6 SR.

Figure 6. Visual demonstration of SwinIR-LTE for ×32 SR.

5. Experiment

5.1. Training

Dataset We use a DIV2K dataset [1] of an NTIRE 2017 Challenge [30] for network training. For evaluation, we report peak signal-to-noise ratio (PSNR) results on the DIV2K validation set [1], Set5 [2], Set14 [33], B100 [17], and Urban100 [10].

Implementation detail We follow a prior implementation [15] and use 48 × 48 patches for inputs of our network. For arbitrary-scale down-sampling during training time, we follow [4] and use bicubic resizing in Pytorch [22]. We use an L1 loss [15] and an Adam [13] method for optimization. When we train LTE with CNN-based encoders, such as EDSR-baseline [15] or RDN [35], networks are trained for 1000 epochs with batch size 16. The learning rate is initialized as 1e-4 and decayed by factor 0.5 at [200, 400, 600, 800]. For a transformer-based encoder, specifically SwinIR [14], a model is trained for 1000 epochs with batch size 32. The learning rate is initialized as 2e-4 and decayed by factor 0.5 at [500, 800, 900, 950].
In this section, we demonstrate the effect of each component in LTE. Our LTE consists of an amplitude estimator, a frequency estimator, a phase estimator, and an LR skip connection. To support the significance of each component, we retrain the following models with EDSR-baseline [15]. (-A): LTE without an amplitude estimator. (-F): LTE with a frequency estimator that estimates only 128 frequencies (not 256). (-P): LTE without a phase estimator. (-L): LTE without an LR skip connection.

In Fig. 5, we show a qualitative comparison for ×6 SR. We remark that SwinIR [14] followed by LTE reconstructs the most visually pleasing image, faithful to the GT. It implies that LTE precisely extracts dominant frequencies and corresponding essential Fourier coefficients when jointly trained with a robust encoder. An empirical explanation using Fourier analysis is provided in Fig. 8 and Sec. 5.4.

As shown in Fig. 6, we visually demonstrate our LTE at an extremely large scale factor, specifically ×32. For the demonstration, we trained our LTE with SwinIR [14], and the width of an input image is 64px. We note that our LTE interpolates images with more sharp and natural edges compared to the bicubic method.

### 5.3. Ablation Study

In this section, we demonstrate the effect of each component in LTE. Our LTE consists of an amplitude estimator, a frequency estimator, a phase estimator, and an LR skip connection. To support the significance of each component, we retrain the following models with EDSR-baseline [15]. (-A): LTE without an amplitude estimator. (-F): LTE with a frequency estimator that estimates only 128 frequencies (not 256). (-P): LTE without a phase estimator. (-L): LTE without an LR skip connection.

Fig. 10 and Tab. 3 show the contributions of each LTE component on visual quality and performance. To verify the importance of each estimated frequency, we compare LTE to LTE (-A). We find that an amplitude estimator emphasizes dominant frequencies compared between LTE and LTE (-A). By comparing LTE and LTE (-P), disregarding a...
phase difference causes a significant performance drop. We see that an LR skip connection consistently enhances the quality of LTE when comparing LTE with LTE (-L).

5.4. Fourier Space

In this section (Figs. 7 to 9), we visualize extracted dominant frequencies with various textures. Furthermore, we investigate the contributions of each LTE component (specifically amplitude, frequency, phase, LR skip connection) through the lens of Fourier space.

**Setup** For visualization, we observe outputs of an amplitude estimator \((h_a)\) and a frequency estimator \((h_f)\). We first scatter dominant frequencies on 2D space and set color for each point with a magnitude. All scatter maps are defined on \([-1.5, 1.5]^2\), and the value range of each map is different from the other. In addition, 16-tap discrete Fourier transform (DFT) of GT images are provided to compare dominant frequencies of LTE and those of GT.

**Image texture and Fourier space** We choose three different textures: horizontal, vertical, diagonal textures, as illustrated in Fig. 7. Frequency maps from LTE in the bottom row are obtained from two-fold downsampled images. By comparing the middle row and the bottom row in Fig. 7, we observe that estimated dominant frequencies follow the dominant frequencies of GT. It indicates that LTE obtains dominant frequencies and corresponding Fourier coefficients by observing pixels inside an RF. Note that the size of RF is determined by deep SR encoders \((E_\phi)\), such as EDSR-baseline [15], RDN [35], and SwinIR [14].

**Encoder and Fourier space** Tab. 2 and Fig. 5 demonstrate that LTE accomplishes better performance when SwinIR [14] is used as an encoder. Fig. 8 supports the observation by visualizing Fourier space. We remark that SwinIR-LTE captures dominant frequencies on a diagonal axis while EDSR-baseline-LTE estimates only low-frequency components. From this observation, LTE with a powerful encoder extracts precise dominant frequencies.

**Ablation study on Fourier space** Fig. 9 shows Fourier spaces of LTE when each component is missing. We choose a diagonal texture in Fig. 7 for an ablation study. Please see a caption of Fig. 9 for definitions of -A, -F, -P, -L. LTE (-A) considers coefficients of all frequencies as equal since Fourier coefficients are not given from LTE. Therefore, LTE (-A) focuses on learning low-frequency content. LTE (-P) is incapable of estimating frequencies positioned on a diagonal axis. We suppose that without a scale-dependent phase estimation, LTE (-P) detects only scale-independent low the dominant frequencies of GT. It indicates that LTE obtains dominant frequencies and corresponding Fourier coefficients by observing pixels inside an RF. Note that the size of RF is determined by deep SR encoders \((E_\phi)\), such as EDSR-baseline [15], RDN [35], and SwinIR [14].

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information: Image signal is compactly supported by low-frequency regions. We validate that the deficiency of dominant frequencies fails to learn high-frequency details by comparing LTE and LTE (-F). On comparing LTE and LTE (-L), LTE (-L) is poor in capturing dominant frequencies.

6. Discussion

Advantage of LTE over DFT In DFT, Fourier information is represented with a linear combination of image intensity. However, the DFT of an LR image with aliasing (middle) is limited in capturing dominant frequencies. In contrast, LTE with a deep neural encoder (bottom), which is a multi-chain of linear combinations and non-linear activations, is capable of estimating accurate Fourier information for an HR image (top).

Effect of aliasing Fig. 8 shows that SwinIR-LTE is capable of estimating dominant frequencies of natural images. In addition, the middle row in Fig. 12 demonstrate that SwinIR-LTE extracts essential Fourier information under mild aliasing. However, such capability of SwinIR-LTE is limited when an LR image has severe aliasing. From Fig. 12, we observe that dominant frequencies (bottom right) are inconsistent with a GT spectrum (bottom right) when harsh aliasing artifacts occur in an LR image (bottom left). We can resolve such limitations by extending the size of an encoder’s RF, followed by increased computation and memory costs. Cost-effective architectures achieving robust performance even under severe aliasing will be investigated in future work.

Gibbs phenomenon When representing continuous signals with a finite sum of Fourier basis, function overshoots at discontinuities. Such observation is referred to in the literature as the Gibbs phenomenon or ringing artifacts in 2D images. From Fig. 13, we notice that LTE might cause overshoot at large scale factors, e.g., ×12. Further investigation of smoothing algorithms to alleviate such an issue is a promising direction for future work.

Computation time In practice, SR applications require short computation time. Moreover, reconstructing high-quality images, such as DIV2K, consumes extensive memory during evaluation. Tab. 4 compares the computation time of our LTE to other arbitrary-scale SR methods for both cases: memory-limited (top rows) and memory-consuming (bottom rows) on NVIDIA RTX 3090 24GB. To evaluate an HR image under a memory-limited condition, we compute 96 × 96 output pixels per query [4]. From the top rows of Tab. 4, we observe that our LTE takes the shortest computation time while increasing memory usage. LTE has 4-layer MLP for querying, two convolution layers for estimators, and LIIF has 5-layer MLP only for querying. When querying evaluation points only once, estimators become more dominant than querying, resulting in more computation time, as in the bottom rows of Tab. 4. To overcome such a limitation, we design an LTE+, which utilizes 1 × 1 convolution instead of a shared MLP for decoder implementation. Since 1 × 1 convolution has a GPU-friendly data structure, our LTE+ takes a shorter computation time and consumes less memory compared to previous works when all output pixels are queried at once.

Table 4. Memory consumption (GB) and computation time (ms) comparison to other arbitrary-scale SR methods for an ×2 SR task. We use a 624 × 624 sized input and EDSR-baseline [15] as an encoder. OOM denotes an out-of-memory, and our GPU memory was deficient in evaluating MetaSR [9].

7. Conclusion

In this paper, we proposed the Local Texture Estimator (LTE) to overcome the spectral bias problem of an implicit neural function. Our LTE-based arbitrary-scale SR method consists of three components: (1) Deep SR encoder (2) LTE (3) Implicit representation function. Firstly, a deep SR encoder extracts feature maps whose height and width are the same as an LR image. Then, LTE takes feature maps from the encoder and estimates dominant frequencies with corresponding Fourier coefficients for natural images. Scale-dependent phase and LR skip connection are further provided to allow LTE to be biased in learning high-frequency textures. Finally, the implicit function reconstructs an image in arbitrary resolution using estimated Fourier information. We showed that our LTE-based neural function outperforms other arbitrary-scale SR methods in performance and visual quality with the shortest computation time.

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