THE VARIATIONAL THEORY OF THE PERFECT
HYPERMOMENTUM FLUID

O. V. Babourova\textsuperscript{a} and B. N. Frolov\textsuperscript{b}

Department of Mathematics, Moscow State Pedagogical University,
Krasnoprudnaya 14, Moscow 107140, Russia

Abstract

The variational theory of the perfect hypermomentum fluid is developed. The
new type of the generalized Frenkel condition is considered. The Lagrangian
density of such fluid is stated, and the equations of motion of the fluid and
the Weyssenhoff-type evolution equation of the hypermomentum tensor are
derived. The expressions of the matter currents of the fluid (the canonical
energy-momentum 3-form, the metric stress-energy 4-form and the hypermo-
mentum 3-form) are obtained. The special case of the dilaton-spin fluid with
intrinsic spin and dilatonic charge is considered.

PACS number(s): 04.20.Fy, 04.40.+c, 04.25.+h
I. INTRODUCTION

The perfect hypermomentum fluid, a fluid element of which is endowed with an intrinsic hypermomentum, was considered in Refs. 1 and 2. The variational theory of such fluid in a metric-affine space \((L^4, g)\) (Ref. 3) was developed by various authors. This theory generalizes the variational theory of the Weyssenhoff-Raabe perfect spin fluid based on accounting the constraints in the Lagrangian density of the fluid with the help of Lagrange multipliers, which has been developed in case of a Riemann-Cartan space and in case of a metric-affine space. On the other variational methods of the perfect spin fluid in a Riemann-Cartan space see Refs. 23 and 24.

The theory of the perfect fluid with intrinsic degrees of freedom being developed, the additional intrinsic degrees of freedom of a fluid element are described by the four vectors \(\vec{l}_p (p = 1, 2, 3, 4)\), called directors, attached with the each element of the fluid. Three of the directors \((p = 1, 2, 3)\) are space-like and the fourth one \((p = 4)\) is time-like.

In Riemann and Riemann-Cartan spaces a fluid element endowed with directors moves according to the Fermi transport that preserves the orthonormalization of the directors. In a metric-affine space, in which a metric and a connection are not compatible it is naturally consider the directors to be elastic in the sense that they can undergo arbitrary deformations during the motion of the fluid. Nevertheless, in most theories it is accepted that the time-like director is collinear to the 4-velocity of the fluid element and the orthogonality of space-like directors to the 4-velocity is maintained.

The distinction of the variational machinery consists in using the generalized Frenkel condition, or the Frenkel condition in its standard classical form, where \(J^\alpha_\beta \) and \(S^\alpha_\beta := J^{[\alpha}_\beta \) are the specific intrinsic hypermomentum tensor and the specific spin tensor of a fluid element, respectively. Another possibility is so called “unconstrained hyperfluid” in which any type of Frenkel condition is absent.

In Ref. 12 it is mentioned that in case of generalized Frenkel condition (1.1) the dilatonic charge of a fluid element is expressed in terms of the shear tensor, \(J = (4/c^2)J^\alpha_\beta u_\alpha u^\beta\) (see decomposition (2.9) in the Sec. 11). In this case the hypermomentum fluid can not be of the pure dilatonic type with \(\tilde{J}^\alpha_\beta = 0\) and \(J \neq 0\). On the other hand, the Frenkel condition (1.2) leads to the unusual form of the evolution equation of the hypermomentum tensor which does not demonstrate the Weyssenhoff-type dynamics. As to the unconstrained hyperfluid, in Ref. 2 it is stated that such type of fluid does not contain the Weyssenhoff spin fluid as a particular case. Therefore all three kinds of the approaches mentioned are not satisfactory from physical point of view.

In this paper we consider the new type of the generalized Frenkel condition, which allows to construct the hypermomentum perfect fluid theory with the dilaton-spin fluid and the Weyssenhoff spin fluid as particular cases. In our approach it is also essential that all four directors are elastic. None of the orthogonality conditions of the four directors is maintained.
during the motion of the fluid. Besides, the time-like director needs not to be collinear to the 4-velocity of the fluid element. We use the exterior form variational method according to Trautman\cite{25,26} (see also Ref. \cite{3}).

II. THE DYNAMICAL VARIABLES AND CONSTRAINTS

In the exterior form language the material frame of the directors turns into the coframe of 1-forms \( l^p \) \((p = 1, 2, 3, 4)\), which have dual 3-forms \( l_q \), while the constraint

\[
l^p \wedge l_q = \delta^p_q \eta, \quad l^p \alpha_p = \delta^p_\alpha \, ,\]

being fulfilled, where \( \eta \) is the volume 4-form and the component representations are introduced,

\[
l^p = l^p \alpha \theta^\alpha, \quad l_q = l^q \eta_\beta \cdot (2.2)\]

Here \( \theta^\alpha \) is a 1-form basis and \( \eta_\beta \) is a 3-form defined as

\[
\eta_\beta = \bar{e}_\beta \| \eta = \ast \theta_\beta, \quad \theta^\alpha \wedge \eta_\beta = \delta^\alpha_\beta \eta \, ,\]

where \( \| \) means the interior product, \( \ast \) is the Hodge dual operator and \( \bar{e}_\beta \) is a basis vector, a coordinate system being nonholonomic in general.

Each fluid element possesses a 4-velocity vector \( \bar{u} = u^\alpha \bar{e}_\alpha \) which is corresponded to a flow 3-form \( u \) (Ref. \cite{26}), \( u := \bar{u} \| \eta = u^\alpha \eta_\alpha \) and a velocity 1-form \( \ast u = u_\alpha \theta^\alpha = g(\bar{u}, \cdot) \) with

\[
\ast u \wedge u = -c^2 \eta \, , \quad (2.4)\]

that means the usual condition \( g(\bar{u}, \bar{u}) = -c^2 \), where \( g(\cdot, \cdot) \) is the metric tensor.

A fluid element moving, the fluid particles number and entropy conservation laws are fulfilled,

\[
d(nu) = 0 \, , \quad d(nsu) = 0 \, , \quad (2.5)\]

where \( n \) is the fluid particles concentration equal to the number of fluid particles per a volume unit, and \( s \) is the the specific (per particle) entropy of the fluid in the rest frame of reference, respectively.

The measure of ability of a fluid element to perform the intrinsic motion is the quantity \( \Omega^q_p \) which generalizes the fluid element “angular velocity” of the Weyssenhoff spin fluid theory. It has the form

\[
\Omega^q_p \eta := u \wedge l^q_\alpha \mathcal{D} l^\alpha_p \, , \quad (2.6)\]

where \( \mathcal{D} \) is the exterior covariant differential with respect to a connection 1-form \( \Gamma^\alpha_\beta \),

\[
\mathcal{D} l^\alpha_p = dl^\alpha_p + \Gamma^\alpha_\beta l^\beta_p \, . \quad (2.7)\]

An element of the fluid with intrinsic hypermomentum possesses the additional “kinetic” energy 4-form,
\[ E = \frac{1}{2} n J^p_q \Omega^q_p \eta = \frac{1}{2} n J^p_u \wedge l^\alpha_p D^p_\alpha , \]  
(2.8)

where \( J^p_q := J^{\alpha\beta} l^p_\alpha l^q_\beta \) is the specific intrinsic hypermomentum tensor representing the new dynamical quantity which generalizes the spin density of the Weyssenhoff fluid.

The hypermomentum tensor \( J^p_q \) can be decomposed into irreducible parts,

\[
J^p_q = \hat{J}^p_q + \frac{1}{4} \delta^p_q J , \quad J := J^p_p , \quad \hat{J}^p_p = 0 ,
\]
(2.9)

\[
\hat{J}^p_q := S^p_q + \hat{J}^{(p)}_q , \quad S^p_q := J^{[p}_q , \quad \hat{J}^{(p)}_q = J^{(p)}_q - \frac{1}{4} \delta^p_q J .
\]
(2.10)

Here \( S^p_q \) is the specific spin tensor, \( J \) is the specific dilatonic charge and \( \hat{J}^{(p)}_q \) is the specific intrinsic proper hypermomentum (shear) tensor of a fluid element, respectively. We shall name the quantity \( \hat{J}^p_q \) as the specific \textit{traceless hypermomentum tensor}.

It is well-known that the spin tensor is spacelike in its nature that is the fact of fundamental physical meaning. This leads to the classical Frenkel condition, \( S^{\alpha\beta} u^\beta = 0 \). We shall suppose here that the full traceless part of the hypermomentum tensor \( \hat{J}^p_q \) (not only the spin tensor but also the tensor \( \hat{J}^{(p)}_q \)) has such property and therefore satisfies the generalized Frenkel conditions in the form,

\[
\hat{J}^p_q u_p = 0 , \quad u_p := u^\alpha l^\alpha_p ,
\]
(2.11)

\[
\hat{J}^p_q u^q = 0 , \quad u^q := u^\alpha l^\alpha_q ,
\]
(2.12)

which can be written in the following way,

\[
\hat{J}^p_q l_p \wedge u = 0 ,
\]
(2.13)

\[
\hat{J}^p_q l^q \wedge u = 0 .
\]
(2.14)

The Frenkel conditions (2.11), (2.12) are equivalent to the equality,

\[
\Pi^p_r \Pi^q_r \hat{J}^p_q = \hat{J}^p_q , \quad \Pi^p_r := \delta^p_r + \frac{1}{c^2} u^p u_r .
\]
(2.15)

Here \( \Pi^p_r \) is the projection tensor, which separates the subspace orthogonal to the fluid velocity.

The internal energy density of the fluid \( \varepsilon \) depends on the extensive (additive) thermodynamic parameters \( n, s, J^p_q \) and obeys to the first thermodynamic principle,

\[
d\varepsilon(n, s, J^p_q) = \frac{\varepsilon + p}{n} dn + nT ds + \frac{\partial \varepsilon}{\partial J^p_q} dJ^p_q ,
\]
(2.16)

where \( p \) is the hydrodynamic fluid pressure and \( T \) is the temperature.

We shall consider as independent variables the quantities \( n, s, J^p_q, u, l^\alpha, \theta^\sigma, \Gamma^\beta_\alpha \), the constraints (2.4), (2.5), (2.13), (2.14) being taken into account in the Lagrangian density by means of the Lagrange multipliers.

In what follows we need the variation,

\[
\delta \eta = \eta^\beta \delta g^{\alpha\beta} + \delta \theta^\sigma \wedge \eta_\sigma .
\]
(2.17)
As a result of the relation $\theta^\alpha \wedge u = u^\alpha \eta$ one has,

$$\eta \delta u^\alpha = -\delta u \wedge \theta^\alpha + \delta \theta^\alpha \wedge u - u^\alpha \delta \eta .$$  \hspace{1cm} (2.18)

The relation $\ast u = g_{\alpha\beta} u^\alpha \theta^\beta$ yields the variation,

$$\delta \ast u = g_{\alpha\beta} \theta^\alpha \delta u^\beta + u^\alpha \delta \theta^\alpha + u^\beta g_{\sigma\beta} \delta \theta^\sigma .$$  \hspace{1cm} (2.19)

As a result of the resolution of the constraints (2.1) and with the help of the relations (2.3), one can derive the variations,

$$\eta \delta l^p_{\alpha} = -\delta \theta^\sigma \wedge \eta \alpha l^p_{\sigma} + \delta l^p \wedge \eta^\alpha ,$$  \hspace{1cm} (2.20)

$$\eta \delta l^\alpha_p = \delta \theta^\alpha \wedge l^p - \delta l^q \wedge l^p_q \eta^r .$$  \hspace{1cm} (2.21)

**III. THE LAGRANGIAN DENSITY AND THE EQUATIONS OF MOTION OF THE FLUID**

The perfect fluid Lagrangian density 4-form of the perfect hypermomentum fluid should be chosen as the remainder after subtraction the internal energy density of the fluid $\varepsilon$ from the “kinetic” energy (2.8) with regard to the constraints (2.4), (2.5), (2.13), (2.14) which should be introduced into the Lagrangian density by means of the Lagrange multipliers $\lambda, \varphi, \tau, \chi^q, \zeta_p$, respectively. As a result of the previous section the Lagrangian density 4-form has the form

$$L_m = L_m \eta = -\varepsilon(n, s, J^p_q) \eta + \frac{1}{2} n J^p_q u \wedge l^q_p \mathcal{D}^\alpha_p + nu \wedge d\varphi + n\tau u \wedge ds + n\lambda (\ast u \wedge u + c^2 \eta) + n\chi^q J^p_q l^q_p \wedge u .$$  \hspace{1cm} (3.1)

The fluid motion equations and the evolution equation of the hypermomentum tensor are derived by the variation of (3.1) with respect to the independent variables $n, s, J^p_q, u, l^q$, and the Lagrange multipliers, the thermodynamic principle (2.16) being taken into account. We shall consider the 1-form $l^q$ as an independent variable and the 3-form $l^p$ as a function of $l^q$ by means of (2.1). As a result of such variational machinery one gets the constraints (2.4), (2.5), (2.13), (2.14) and the following variational equations,

$$\delta n : \ (\varepsilon + p) \eta - \frac{1}{2} n J^p_q u \wedge l^q_p \mathcal{D}^\alpha_p - nu \wedge d\varphi = 0 ,$$  \hspace{1cm} (3.2)

$$\delta s : \ T \eta + u \wedge d\tau = 0 ,$$  \hspace{1cm} (3.3)

$$\delta J^p_q : \ \frac{\partial \varepsilon}{\partial J^p_q} = \frac{1}{2} n \Omega^q_p - n(\chi^q u_p - \zeta_p u^q) + \frac{1}{4} n \delta^p_q(\chi^r u_r - \zeta_r u^r) ,$$  \hspace{1cm} (3.4)

$$\delta u : \ d\varphi + \tau ds - 2\lambda \ast u + \chi^q J^p_q \theta^\beta - \zeta_p J^p_q l^q_p + \frac{1}{2} J^p_q \mathcal{D}^\alpha_p = 0 ,$$  \hspace{1cm} (3.5)

$$\delta l^q : \ - \frac{1}{2} J^p_q l^q_p \eta^r - \chi^r J^p_q u^q l_p - \zeta_r J^p_q l^q_p = 0 .$$  \hspace{1cm} (3.6)

Here the “dot” notation for the tensor object $\Phi$ is introduced,
\[ \dot{\Phi}^\alpha_\beta := *(u \wedge D\Phi^\alpha_\beta) . \]  

Multiply the equation (3.5) by \( u \) from the left externally and using (2.5) and (3.2), one derives the expression for the Lagrange multiplier \( \lambda \),

\[ 2nc^2 \lambda = \varepsilon + p . \]  

As a consequence of the equation (3.2) and the constraints (2.4), (2.5), (2.13), (2.14) one can verify that the Lagrangian density 4-form (3.1) is proportional to the hydrodynamic fluid pressure, \( L_m = p\eta \), which corresponds to Ref. 17.

IV. THE EVOLUTION EQUATION OF THE HYPERMOMENTUM TENSOR

The variational equation (3.6) represents the evolution equation of the hypermomentum tensor. Multiplying the equation (3.6) by \( \bar{\nu}_\beta^\alpha \theta^\alpha \wedge \ldots \) from the left externally one gets,

\[ \frac{1}{2} \dot{J}^\alpha_\beta - \chi_r \dot{J}^\alpha_r u_\beta - \zeta_r \dot{J}^r_\beta u^\alpha = 0 . \]  

Contractions (4.1) with \( u^\alpha \) and then with \( u^\beta \) yield the expressions for the Lagrange multipliers,

\[ \zeta_r \dot{J}^r_\beta = -\frac{1}{2c^2} \dot{J}^r_\beta u_\gamma , \quad (4.2) \]

\[ \chi_r \dot{J}^\alpha_r = -\frac{1}{2c^2} \dot{J}^\alpha_r u^\gamma . \quad (4.3) \]

After the substitution of (4.2) and (4.3) into (4.1) one gets the evolution equation of the hypermomentum tensor,

\[ \dot{J}^\alpha_\beta + \frac{1}{c^2} \dot{J}^\alpha_\gamma u^\gamma u_\beta + \frac{1}{c^2} \dot{J}^r_\beta u^\gamma u^\alpha = 0 . \]  

This equation generalizes the evolution equation of the spin tensor in the Weyssenhoff fluid theory.

The equation (4.4) has the consequence,

\[ \dot{J}^\alpha_\beta u^\alpha u^\beta = 0 , \quad (4.5) \]

which permits to represent the evolution equation of the hypermomentum tensor (4.4) in the form,

\[ \Pi^\alpha_\sigma \Pi^\rho_\beta \dot{J}^\sigma_\rho = 0 , \quad (4.6) \]

where the projection tensor \( \Pi^\alpha_\sigma \) has been defined in (2.13). The evolution equation of the hypermomentum tensor in the form (4.6) was derived in Ref. 13.

The contraction (4.4) on the indices \( \alpha \) and \( \beta \) gives with the help of (4.3) the dilatonic charge conservation law,

\[ \dot{J} = 0 . \quad (4.7) \]
V. THE ENERGY-MOMENTUM TENSOR OF THE PERFECT HYPERMOMENTUM FLUID

With the help of the matter Lagrangian density (3.1) one can derive the external matter currents which are the sources of the gravitational field. In case of the perfect hypermomentum fluid the matter currents are the canonical energy-momentum 3-form $\Sigma_{\sigma}$, the metric stress-energy 4-form $\sigma^{\alpha\beta}$ and the hypermomentum 3-form $J^{\alpha_\beta}$, which are determined as variational derivatives.

The variational derivative of the explicit form of the Lagrangian density (3.1) with respect to $\theta^\sigma$ yields the canonical energy-momentum 3-form, 

$$\Sigma_{\sigma} := \frac{\delta L_m}{\delta \theta^\sigma} = -\varepsilon \eta_\sigma + 2\lambda n u_\sigma u + 2c^2 \lambda n \eta_\sigma - n \chi^r \hat{J}^r_{\rho}(g_{\sigma \rho} l^l_q u + u_\sigma l_q) + \frac{1}{2} n \dot{J}^\rho_{\sigma} \eta_\rho . \quad (5.1)$$

Using the explicit form of the Lagrange multiplier (3.8), one gets,

$$\Sigma_{\sigma} = p \eta_\sigma + \frac{1}{c^2}(\varepsilon + p) u_\sigma u + \frac{1}{2} n \dot{J}^\rho_{\sigma} \eta_\rho - n \chi^r \hat{J}^r_{\rho}(g_{\sigma \rho} u + l^l_q u_\sigma) . \quad (5.2)$$

On the basis of the evolution equation of the hypermomentum tensor (4.4) and with the help of (4.3) the expression (5.2) reads,

$$\Sigma_{\sigma} = p \eta_\sigma + \frac{1}{c^2}(\varepsilon + p) u_\sigma u + \frac{1}{c^2} n \dot{g}_{\alpha \beta} \hat{J}^\alpha_{\beta} u^\rho u . \quad (5.3)$$

After some algebra one can get the other form of the canonical energy-momentum 3-form,

$$\Sigma_{\sigma} = p \eta_\sigma + \frac{1}{c^2}(\varepsilon + p) u_\sigma u + \frac{1}{c^2} n \dot{g}_{\alpha \beta} \hat{J}^\alpha_{\beta} u^\rho u - \frac{1}{c^2} n J_{\alpha \beta} Q^{\alpha \beta \gamma} u^\gamma u^\alpha u , \quad (5.4)$$

where $Q_{\alpha \beta \gamma}$ are components of a nonmetricity 1-form,

$$Q_{\alpha \beta} := -\mathcal{D}g_{\alpha \beta} = Q_{\alpha \beta \gamma} \theta^\gamma . \quad (5.5)$$

The metric stress-energy 4-form can be derived in the same way,

$$\sigma^{\alpha\beta} := 2 \frac{\delta L_m}{\delta g_{\alpha \beta}} = T^{\alpha\beta} \eta ,$$

$$T^{\alpha\beta} = -\varepsilon g^{\alpha\beta} + 2n \lambda (u^\alpha u^\beta + c^2 g^{\alpha\beta}) - 2n \chi^r \hat{J}^r_{\nu}(g_{\alpha \nu} u + l^l_q u_\alpha)$$

$$= pg^{\alpha\beta} + \frac{1}{c^2}(\varepsilon + p) u^\alpha u^\beta + \frac{1}{c^2} n \dot{J}^\alpha_{\beta} u^\gamma . \quad (5.6)$$

For the hypermomentum 3-form one finds

$$J^{\alpha_\beta} := -\frac{\delta L_m}{\delta \Gamma_{\alpha \beta}^\gamma} = \frac{1}{2} \eta_\alpha \dot{J}^\alpha_{\beta} u . \quad (5.7)$$

The expressions of the canonical energy-momentum 3-form (5.3), the metric stress-energy 4-form (5.6) and the hypermomentum 3-form (5.7) are compatible in the sense that they satisfy to the Noether identity,
\( \theta^\alpha \wedge \Sigma_\beta = \sigma^\alpha_\beta + D J^\alpha_\beta, \)

that corresponds to the \( GL(n, R) \)-invariance of the Lagrangian density \((3.1)\).

Let us consider the special case of the perfect spin fluid with dilatonic charge, a fluid element of which does not possess the specific shear momentum tensor, \( \hat{J}^{(pq)} = 0 \), and is endowed only with the specific spin momentum tensor \( S^{pq} \) and the specific dilatonic charge \( J \). In this case the canonical energy-momentum 3-form \((5.3)\) reads,

\[
\Sigma_\sigma = \rho \eta_\sigma + \frac{1}{c^2} (\varepsilon + p) u_\sigma u + \frac{1}{c^2} n g_{\alpha[\sigma} \hat{S}^\alpha_\beta] u^\beta u ,
\]

where the specific energy density \( \varepsilon \) contains the energy density of the dilatonic interaction of the fluid. If the dilatonic charge also vanishes, \( J = 0 \), then the expression \((5.9)\) will describe the canonical energy-momentum 3-form of the Weyssenhoff perfect spin fluid in a metric-affine space.

**VI. CONCLUSIONS**

The essential feature of the constructed variational theory of the hypermomentum perfect fluid is the assumption that the frame realized by all four directors is elastic. The deformation of the directors during the motion of the fluid element, from one side, generates the space-time nonmetricity and, from the other side, allows nonmetricity of the space-time to be discovered. As the consequence of this fact the Lagrangian density \((3.1)\) does not contain the term maintaining the orthogonality of the directors. The time-like director needs not to be collinear to the 4-velocity of the fluid element. The essential feature of our variational approach is the using the Frenkel conditions \((2.11), (2.12)\), which do not coincide nor with their classical form, when the Frenkel condition was imposed on the spin tensor, \( S^{\alpha_\beta} u^\beta = 0 \), nor with its generalized form, when the Frenkel condition was imposed on the full intrinsic hypermomentum tensor, \( J^{\alpha_\beta} u^\beta = J^{\alpha_\beta} u_\alpha = 0 \).

We have derived the expression for the energy-momentum tensor of the fluid \((5.3)\), which coincides with one that has been obtained earlier. But our approach does not contain the shortcomings, which are inherent in the previous theories of the hypermomentum perfect fluid. First of all, our variational theory contains the Weyssenhoff spin fluid as the particular case that is important from physical point of view. Then, the evolution equation of the hypermomentum tensor \((1.4)\) demonstrates the Weyssenhoff-type dynamics. At last, the hypermomentum perfect fluid theory developed allows to describe as the special case the perfect spin fluid with dilatonic charge. It should be important to investigate the consequences of the employing the perfect fluid of such type as the gravitational field source in cosmological and astrophysical problems. For example, it is interesting to clarify whether the corresponding field equations have the regular solution with the upper limit for \( \varepsilon \) (the limiting energy density of the fluid).

**ACKNOWLEDGMENTS**

This paper is partly supported by the scientific programm “Univesitetey Rossii”.
REFERENCES

a) e-mail: babourova.physics@mpgu.msk.su
b) e-mail: frolovn.physics@mpgu.msk.su

1 O. V. Babourova, in Gravitaciya i Fundamental’nye vzaimodejstviya (UDN, Moscow, 1988), p. 119 [in Russian].
2 O. V. Babourova, “Variational theory of a perfect fluid with inrinsic degrees of freedom in modern gravitational theory”, Ph.d. thesis (VNICPV, Moscow, 1989) [in Russian].
3 F. W. Hehl, J. L. McCrea, E. W. Mielke and Yu. Ne´eman, Phys. Reports 258, 1 (1995).
4 O. V. Babourova, B. N. Frolov, M. Yu. Koroliov, in 13th Int. Conf. gen. rel. grav., Abstracts of contributed papers, edited by P. W. Lamberti and O. E. Ortiz (Cordoba, Argentina, 1992), p. 131.
5 O. V. Babourova, B. N. Frolov, M. Yu. Koroliov, in Materialy Nauchnoj sessii ... MPGU za 1991. Ser. Est. nauki (Moscow, “Prometey”, 1992), p. 4. [in Russian].
6 O. V. Babourova, B. N. Frolov, M. Yu. Koroliov, in Nauchnye trudy Mosk. Ped. Gos. Univ. Ser. Est. nauki (Moscow, “Prometey”, 1993), p. 170 [in Russian].
7 Y. N. Obukhov and R. Tresguerres, Phys. Lett. A 184, 17 (1993).
8 O. V. Babourova, B. N. Frolov, M. Yu. Koroliov, in Nauchnye trudy MPGU. Ser. Est. nauki (Moscow, “Prometey”, 1994), 1, p. 89 [in Russian].
9 L. L. Smally and J. P. Krisch, J. Math. Phys. 36, 778 (1995).
10 O. V. Babourova, B. N. Frolov, in 14th International Conf. on Gen. Rel. and Grav., Abstracts of Contributed Papers (Florence, Italy, 1995), p. A90.
11 O. V. Babourova and B. N. Frolov, “The variational theory of perfect fluid with intrinsic hypermomentum in space-time with nonmetricity”, LANL e-archive gr-qc/9509013 (1995).
12 Yu. N. Obukhov, Phys. Lett. A210, 163 (1996).
13 O. V. Babourova, B. N. Frolov, in Teoreticheskie i eksperimental’nye problemy gravitacii, Abstracts of Contr. Pap. 9 Russ. Grav. Conf., Novgorod (Moscow, 1996), Part 1, p. 44 [in Russian].
14 V. N. Tunyak, Dokl. Acad. Nauk BSSR 19, 559 (1975) [in Russian].
15 V. N. Tunyak, Izv. Vysš. Učebn. Zaved. Fizika, N 12, 11 (1977) [in Russian].
16 J. R. Ray and L. L. Smally, Phys. Rev. D 27, 1383 (1983).
17 R. de Ritis, M. Lavorgna, G. Platania, and C. Stornaiolo, Phys. Rev. D 28, 713 (1983). 
18 Yu. N. Obukhov and V. A. Korotky, Class. Quantum Grav. 4, 1633 (1987).
19 O. V. Babourova, Izv. Vysš. Učebn. Zaved. Fizika, N 10, 101 (1989) [in Russian].
20 O. V. Babourova, B. N. Frolov, in Abstr. cont. papers, 12th Intern. Conf. on Gen. Rel. and Grav. (USA, Boulder, 1989), p. 151 (A3:08).
21 O. V. Babourova, B. N. Frolov, in Gravitaziya i elektromagnetizm (Minsk, “Universitetskoe”, 1987), p. 3 [in Russian].
22 H. P. de Oliveira, Gen. Rel. Grav. 25, 473 (1993).
23 A. V. Minkevich and P. Karakura, J. Math. A: Math. Gen. 16, 1409 (1983).
24 W. Kopczyński, Phys. Rev. D 34, 352 (1986).
25 A. Trautman, Symp. Math. 12, 139 (1973).
26 A. Trautman, Bul. Acad. Pol. Sci. (Ser. sci. math., astr., phys.) 20, 895 (1972).