Radiative M1-decays of heavy-light mesons in the relativistic quark model

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Abstract

Radiative magnetic dipole decays of heavy-light vector mesons into pseudoscalar mesons $V \to P \gamma$ are considered within the relativistic quark model. The light quark is treated completely relativistically, while for the heavy quark the $1/m_Q$ expansion is used. It is found that relativistic effects result in a significant reduction of decay rates. Comparison with previous predictions and recent experimental data is presented.

Key words: radiative decays, bottom and charmed mesons, relativistic bound state dynamics, heavy quarks

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In this letter we consider radiative magnetic dipole (M1) transitions of the ground state vector ($V$) heavy-light mesons to the pseudoscalar ($P$) ones, $V \to P \gamma$ (in quark model notations, $1^3S_1 \to 1^1S_0 + \gamma$). For this purpose, we use the relativistic quark model based on the quasipotential approach in quantum field theory. Recently, this model has been successfully applied for the description of different properties of the heavy-light mesons, such as their mass spectra [1] and rare radiative decays [2]. Our analysis showed that the light quark in the heavy-light mesons should be treated completely relativistically, while for the heavy quark it is useful to apply the expansion in powers of the inverse heavy quark mass $1/m_Q$, which considerably simplifies calculations. The first and sometimes second order corrections in $1/m_Q$ are also important for the heavy-light meson decay description. Analogously, for the radiative decay calculations considered here, the $1/m_Q$ expansion is carried out up to...
the second order, and the light quark is treated completely relativistically (i.e. without the unjustified expansion in inverse powers of the light quark mass). It follows from the obtained results that the relativistic effects give substantial contributions to the calculated decay rates.

The radiative \( V \to P \gamma \) decay rate is given by [3]

\[
\Gamma = \frac{\omega^3}{3\pi} |\mathbf{M}|^2, \quad \text{where} \quad \omega = \frac{M_V^2 - M_P^2}{2M_V},
\]

\[ (1) \]

\( M_V \) and \( M_P \) are the vector and pseudoscalar meson masses. The matrix element of the magnetic moment \( \mathbf{M} \) is defined by

\[
\mathbf{M} = -i \left[ \frac{\partial}{\partial \Delta} \times \langle P | \mathbf{J}(0) | V \rangle \right]_{\Delta=0}, \quad \Delta = P - Q,
\]

\[ (2) \]

where \( \langle P | J_\mu(0) | V \rangle \) is the matrix element of the electromagnetic current between initial vector (\( V \)) and final pseudoscalar (\( P \)) meson states with momenta \( Q \) and \( P \) respectively.

We use the relativistic quark model for the calculation of the matrix element of the magnetic moment \( \mathbf{M} \) (2). In our model a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation of the Schrödinger type in the center-of-mass frame [1]:

\[
\left( \frac{b^2(M)}{2\mu_R} - \frac{P^2}{2\mu_R} \right) \Psi_M(p) = \int \frac{d^3q}{(2\pi)^3} V(p, q; M) \Psi_M(q),
\]

\[ (3) \]

where the relativistic reduced mass is

\[
\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3},
\]

\[ (4) \]

and \( b^2(M) \) denotes the on-mass-shell relative momentum squared

\[
b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}.
\]

\[ (5) \]

Here \( m_{1,2} \) and \( M \) are quark masses and a heavy-light meson mass, respectively.

The kernel \( V(p, q; M) \) in Eq. (3) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. An important role in this construction is played by the Lorentz-structure of the confining
quark-antiquark interaction in the meson. In constructing the quasipotential of the quark-antiquark interaction we have assumed that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of vector and scalar linear confining potentials. The quasipotential is then defined by [4]

\[ V(p, q; M) = \bar{u}_1(p) \bar{u}_2(-q) V(p, q; M) u_1(q) u_2(-q), \]  

(6)

with

\[ V(p, q; M) = \frac{4}{3} \alpha_s D_{\mu \nu}(k) \gamma_1^{\mu} \gamma_2^{\nu} + V_{\text{conf}}^V(k) \Gamma_1^{\mu} \Gamma_2;\mu + V_{\text{conf}}^S(k), \]

where \( \alpha_s \) is the QCD coupling constant, \( D_{\mu \nu} \) is the gluon propagator in the Coulomb gauge and \( k = p - q \); \( \gamma_\mu \) and \( u(p) \) are the Dirac matrices and spinors. The effective long-range vector vertex is given by

\[ \Gamma_\mu(k) = \gamma_\mu + \frac{i \kappa}{2m} \sigma_{\mu \nu} k^\nu, \quad k^\nu = (0, k), \]

(7)

where \( \kappa \) is the Pauli interaction constant characterizing the nonperturbative anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

\[ V_{\text{conf}}^V(r) = (1 - \varepsilon)(Ar + B), \quad V_{\text{conf}}^S(r) = \varepsilon(Ar + B), \]

(8)

reproducing

\[ V_{\text{conf}}(r) = V_{\text{conf}}^S(r) + V_{\text{conf}}^V(r) = Ar + B, \]

(9)

where \( \varepsilon \) is the mixing coefficient.

The quasipotential for the heavy quarkonia, expanded in \( p^2/m^2 \), can be found in Ref. [4] and for heavy-light mesons in [1]. All the parameters of our model, such as quark masses, parameters of the linear confining potential, mixing coefficient \( \varepsilon \) and anomalous chromomagnetic quark moment \( \kappa \), were fixed from the analysis of heavy quarkonium spectra [4] and radiative decays [3]. The quark masses \( m_b = 4.88 \text{ GeV}, m_c = 1.55 \text{ GeV}, m_s = 0.50 \text{ GeV}, m_u,d = 0.33 \text{ GeV} \) and the parameters of the linear potential \( A = 0.18 \text{ GeV}^2 \) and \( B = -0.30 \text{ GeV} \) have the usual quark model values. In Ref. [5] we have considered the expansion of the matrix elements of weak heavy quark currents between pseudoscalar and vector meson ground states up to the second order in inverse powers of the heavy quark masses. It has been found that the general structure of the leading, first, and second order \( 1/m_Q \) corrections in our relativistic model is in accord with the predictions of HQET. The heavy quark symmetry and QCD impose rigid constraints on the parameters of the long-range
Fig. 1. Lowest order vertex function $\Gamma^{(1)}$ corresponding to Eq. (12). Radiation only from one quark is shown.

Fig. 2. Vertex function $\Gamma^{(2)}$ corresponding to Eq. (13). Dashed lines represent the interaction operator $V$ in Eq. (6). Bold lines denote the negative-energy part of the quark propagator. As on Fig. 1, radiation only from one quark is shown.

potential in our model. The analysis of the first order corrections [5] fixes the value of the Pauli interaction constant $\kappa = -1$. The same value of $\kappa$ was found previously from the fine splitting of heavy quarkonia $3P_J$- states [4]. The value of the parameter characterizing the mixing of vector and scalar confining potentials, $\varepsilon = -1$, was found from the comparison of the second order $(1/m_Q^2)$ corrections in our model [5] with the same order contributions in HQET. This value is very close to the one determined from considering radiative decays of heavy quarkonia [3], especially the M1-decays (e. g. the calculated decay rate of $J/\Psi \rightarrow \eta_c \gamma$ can be brought in accord with the experiment only with the above value of $\varepsilon$).

In the quasipotential approach, the matrix element of the electromagnetic current $J_\mu$ between the states of a vector $V$ meson and a pseudoscalar $P$ meson has the form [6]

$$
\langle P|J_\mu(0)|V\rangle = \int \frac{d^3p d^3q}{(2\pi)^6} \bar{\Psi}_P(p) \Gamma_\mu(p, q) \Psi_V(q),
$$

where $\Gamma_\mu(p, q)$ is the two-particle vertex function and $\Psi_{V,P}$ are the meson wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame. The contributions to $\Gamma$ come from Figs. 1 and 2.
The contribution \( \Gamma^{(2)} \) is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function \( \Gamma^{(2)} \) explicitly depends on the Lorentz structure of the \( q\bar{q} \)-interaction. Thus the vertex function is given by

\[
\Gamma_\mu(p, q) = \Gamma^{(1)}_\mu(p, q) + \Gamma^{(2)}_\mu(p, q) + \cdots, \tag{11}
\]

where

\[
\Gamma^{(1)}_\mu(p, q) = e_1 \bar{u}_1(p_1)\gamma_\mu u_1(q_1)(2\pi)^3\delta(p_2 - q_2) + (1 \leftrightarrow 2), \tag{12}
\]

and

\[
\Gamma^{(2)}_\mu(p, q) = e_1 \bar{u}_1(p_1)\bar{u}_2(p_2)\left\{ V(p_2 - q_2) \frac{\Lambda^{(-)}(k'_1)}{\epsilon_1(k'_1) + \epsilon_1(q_1)} \gamma^0 \gamma_\mu + \gamma_\mu \right\} u_1(q_1)u_2(q_2) + (1 \leftrightarrow 2). \tag{13}
\]

Here \( e_{1,2} \) are the quark charges, \( k_1 = p_1 - \Delta; \quad k'_1 = q_1 + \Delta; \quad \Delta = P - Q; \)

\[
\Lambda^{(-)}(p) = \frac{\epsilon(p) - (m\gamma^0 + \gamma^0(\gamma P))}{2\epsilon(p)}, \quad \epsilon(p) = \sqrt{p^2 + m^2}.
\]

It is important to note that the wave functions entering the current matrix element (10) cannot be both in the rest frame. In the initial \( V \) meson rest frame, the final \( P \) meson is moving with the recoil momentum \( \Delta \). The wave function of the moving \( P \) meson \( \Psi_{P\Delta} \) is connected with the wave function in the rest frame \( \Psi_{P0} \equiv \Psi_P \) by the transformation [6]

\[
\Psi_{P\Delta}(p) = D^{1/2}_1(R_{L\Delta}^W)D^{1/2}_2(R_{L\Delta}^W)\Psi_{P0}(p), \tag{14}
\]

where \( R^W \) is the Wigner rotation, \( L_\Delta \) is the Lorentz boost from the rest frame to a moving one, and \( D^{1/2}(R) \) is the rotation matrix in the spinor representation.

We substitute the vertex functions \( \Gamma^{(1)} \) and \( \Gamma^{(2)} \) given by Eqs. (12) and (13) in the decay matrix element (10) and take into account the wave function transformation (14). To simplify calculations we note that the mass of the heavy-light mesons \( M_{V,P} \) is large (due to presence of the heavy quark \( M_{V,P} \sim m_Q \)) and carry out the expansion in inverse powers of this mass up to the second order. Then we calculate the matrix element of the magnetic moment operator (2) and get (a) for the vector potential.
\[ \mathcal{M}_V = \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_p(p) \frac{e_1}{2\epsilon_1(p)} \left\{ \sigma_1 + \frac{1 - \epsilon(1 + 2\kappa) [p \times [\sigma_1 \times p]]}{2\epsilon_1(p)[\epsilon_1(p) + m_1]} \right\} \Psi_V(p) + (1 \leftrightarrow 2), \]

(b) for the scalar potential

\[ \mathcal{M}_S = \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_p(p) \frac{e_1}{2\epsilon_1(p)} \left\{ \frac{\epsilon_1(p) + \epsilon_2(p) - M_V}{\epsilon_1(p)} \right\} \times \left\{ \sigma_1 - \frac{\epsilon_2(p)}{M_V} i \left[ p \times \frac{\partial}{\partial p} \right] - \epsilon \left[ p \times [\sigma_1 \times p] \right] \right\} \Psi_V(p) + (1 \leftrightarrow 2). \]

Note that the last terms in Eqs. (15), (16) result from the wave function transformation (14) from the moving reference frame to the rest one. It is easy to see that in the limit \( p/m \to 0 \) the usual nonrelativistic expression for the magnetic moment follows.

Since we are interested in radiative transitions of the vector mesons to the pseudoscalar mesons it is possible to evaluate spin matrix elements using the relation \( \langle \sigma_1 \rangle = - \langle \sigma_2 \rangle \). Then assuming one quark to be light \( q \) and the other one \( Q \) to be heavy and further expanding Eqs. (15), (16) in the inverse powers of the heavy quark mass \( m_Q \) up to the second order corrections to the leading contribution we get

(a) for the purely vector potential \( \epsilon = 0 \)

\[ \mathcal{M}_V = \frac{e_q}{2m_q} \left\{ \frac{m_q}{\epsilon_q(p)} - \frac{m_q^2}{6\epsilon_q(p)[\epsilon_q(p) + m_q]} \left( \frac{1}{\epsilon_q(p)} + \frac{1}{M_V} \right) \right\} \]

\[ + \frac{(1 + \kappa) m_q}{3} \left\{ \frac{\epsilon_q(p)}{\epsilon_q(p) + m_q} - \frac{1}{m_Q} \right\} - \frac{m_q}{6M_Vm_Q} \left\{ \frac{p^2}{\epsilon_q(p)} \right\}. \]

\[ - \frac{e_Q}{2m_Q} \left\{ 1 - \frac{2\langle p^2 \rangle}{3m_Q^2} + \frac{1 + \kappa}{3} \frac{\epsilon_q(p)}{m_Q} \left( \frac{1}{m_Q} - \frac{2}{\epsilon_q(p) + m_q} \right) \right\} \]

\[ - \frac{\langle p^2 \rangle}{6M_V} \left( \frac{1}{m_Q} + \frac{2}{\epsilon_q(p) + m_Q} \right). \]

\[ (17) \]
(b) for the purely scalar potential \((\varepsilon = 1)\)

\[
\mathcal{M}_S = \frac{e_q}{2m_q} \left\{ 2 \frac{m_q}{\epsilon_q(p)} \left[ \frac{m_q(M_V - m_Q)}{\epsilon_q(p)} \right] - \frac{m_q}{6M_Vm_Q} \frac{\langle \mathbf{p}^2 \rangle}{\epsilon_q(p)} + \frac{m_q}{2} \left[ \frac{\langle \mathbf{p}^2 \rangle}{\epsilon_q(p)} - \frac{1}{m_Q} - \frac{2}{3} \frac{M_V}{m_Q} \frac{\langle \epsilon_q(p) \rangle}{m_Q} + \frac{2}{3M_V} \right] \right\}
\]

\[
- \frac{e_Q}{2m_Q} \left[ \frac{\langle \mathbf{p}^2 \rangle}{m_Q} - \frac{\langle \epsilon_q(p) \rangle}{m_Q} + \frac{1}{M_V} \right] - \frac{1}{3M_V} \left[ \frac{\langle \mathbf{p}^2 \rangle}{\epsilon_q(p) + m_q} \right] \right\}
\]

\( (18) \)

Here \(\langle \cdots \rangle\) denotes the matrix element between radial meson wave functions. For these matrix element calculations we use the wave functions of heavy-light mesons obtained in Ref. [1]. It is important to note that in this reference while calculating the heavy-light meson mass spectra only the heavy quark was treated using the \(1/m_Q\) expansion but the light quark was treated completely relativistically.

The values of decay rates of mesons with open flavour calculated on the basis of Eqs. (1), (17), (18) are displayed in Table 1. In the second column (\(\Gamma^{NR}\)) we give predictions for decay rates obtained in the nonrelativistic approximation \((p/m \to 0)\) for both heavy and light quarks. In the third (\(\Gamma^V\)) and fourth (\(\Gamma^S\)) columns we show the results obtained for the purely vector and scalar confining potentials, respectively. And in the last column (\(\Gamma\)) we present predictions for the mixture of vector and scalar confining potentials (8) with the mixing parameter \(\varepsilon = -1\). As seen from this Table relativistic effects significantly influence the predictions. Their inclusion results in a significant reduction of decay rates \((\Gamma^{NR}/\Gamma = 2 \div 4.5)\). Both relativistic corrections to the heavy quark and the relativistic treatment of the light quark play an important role. The dominant decay modes of \(D^*\) mesons are the strong decay \(D^* \to D\pi\), which is considerably suppressed by the phase space, and the electromagnetic decay \(D^* \to D\gamma\). The corresponding branching ratios are known already for a long time and listed in PDG tables [7]. However, the total decay rates of \(D^*\) mesons were not measured until recently. In Ref. [8] CLEO collaboration reported the first measurement of the \(D^{**}\) decay width \(\Gamma(D^{**}) = 96 \pm 4 \pm 22\) keV. Combining this value with the measured \(BR(D^{**} \to D^+\gamma) = (1.6 \pm 0.4)\% \([7]\), the following experimental value of the decay rate can be obtained: \(\Gamma(D^{**} \to D^+\gamma) = (1.5 \pm 0.6)\) keV. Our model prediction is in agreement with this experimental value. However, the experimental errors are still large in order to discriminate the relativistic and nonrelativistic results. In the case of \(B\) mesons the pion emission is kinematically forbidden, so the dominant decay mode is electromagnetic decay \(B^* \to B\gamma\). None of \(B^*\) widths has been measured yet. We also present our predictions for the radiative decay rate of \(B^*_c\) meson,
Table 1
Radiative decay rates of mesons with an open flavour (in keV).

| Decay                  | $\Gamma^{NR}$ | $\Gamma^V$ | $\Gamma^S$ | $\Gamma$ |
|------------------------|--------------|------------|------------|----------|
| $D^{*\pm} \rightarrow D^{\pm}\gamma$ | 2.08         | 0.60       | 0.28       | 1.04     |
| $D^{*0} \rightarrow D^{0}\gamma$     | 37.0         | 14.3       | 17.4       | 11.5     |
| $D^{*}_{s}\rightarrow D^{s}\gamma$   | 0.36         | 0.13       | 0.08       | 0.19     |
| $B^{*\pm} \rightarrow B^{\pm}\gamma$ | 0.89         | 0.24       | 0.29       | 0.19     |
| $B^{*0} \rightarrow B^{0}\gamma$     | 0.27         | 0.087      | 0.101      | 0.070    |
| $B^{*}_{s}\rightarrow B^{s}\gamma$   | 0.132        | 0.064      | 0.074      | 0.054    |
| $B^{*}_{c}\rightarrow B^{c}\gamma$   | 0.073        | 0.048      | 0.066      | 0.033    |

which consists of two heavy quarks ($b$ and $c$). Therefore the expressions (17) and (18) can be further expanded in inverse powers of both quark masses up to the second order.

In Table 2 we compare our predictions for radiative decay rates of vector heavy-light mesons with other theoretical results. We show the predictions obtained in quark models [9–11], in the framework of heavy quark effective theory (HQET) combined with vector meson dominance (VMD) hypothesis [12] and in QCD sum rules [13–15]. These predictions vary quite significantly from each other. Our predictions are in rough agreement with the quark model calculations of Ref. [10], with HQET+VMD results of Ref. [12] and with some of the predictions of the QCD sum rules. \[1\] It is important to note that in our calculations we do not need to introduce the anomalous electromagnetic moment of the light quark as it is done in Ref. [11], where it was found that its value should be rather large ($\sim 0.5$) in order to get agreement with the experimental (CLEO) value for $D^{*+} \rightarrow D^{+}\gamma$ decay rate. \[2\] The large value of the anomalous electromagnetic moment is not justified phenomenologically (see e. g. Refs. [16,3]). The other differences of our calculations from those of Ref. [11] are the Lorentz structure of the confining potential and a more comprehensive account of relativistic effects. In particular, the relativistic transformation of the meson wave function from the rest frame to a moving one given by Eq. (14) is missing in Ref. [11].

In Table 3 we present the comparison of our results with the predictions of different quark models [17–19] for the rates of the radiative M1-transitions ($1^{3}S_{1} \rightarrow 1^{1}S_{0} + \gamma$) in the heavy-heavy $B_{c}$ meson. There we also give the predicted values of the photon energy, which is determined by the mass splitting.
Table 2. Comparison of different theoretical predictions for radiative decays of heavy-light mesons (in keV).

| Decay       | Quark models | HQET+VMD | QCD sum rules |
|-------------|--------------|----------|---------------|
| $D^{*\pm} \rightarrow D^{\pm}\gamma$ | 1.04 | 0.36 | 1.72 | 0.050 | 0.51 ± 0.18 | 0.09$^{+0.40}_{-0.07}$ | 1.5 | 0.23 ± 0.1 |
| $D^{*0} \rightarrow D^{0}\gamma$ | 11.5 | 17.9 | 7.18 | 7.3 | 16.0 ± 7.5 | 3.7 ± 1.2 | 14.4 | 12.9 ± 2 |
| $D_s^* \rightarrow D_s\gamma$ | 0.19 | 0.118 | 0.101 | 0.24 ± 0.24 | | | 0.13 ± 0.05 |
| $B^{*\pm} \rightarrow B^{\pm}\gamma$ | 0.19 | 0.261 | 0.272 | 0.084 | 0.22 ± 0.09 | 0.10 ± 0.03 | 0.63 | 0.38 ± 0.06 |
| $B^{*0} \rightarrow B^{0}\gamma$ | 0.070 | 0.092 | 0.064 | 0.037 | 0.075 ± 0.027 | 0.04 ± 0.02 | 0.16 | 0.13 ± 0.03 |
| $B_s^* \rightarrow B_s\gamma$ | 0.054 | 0.051 | 0.035 | | | | 0.22 ± 0.04 |
Table 3
Comparison of theoretical predictions for the radiative $B_c^* \to B_c \gamma$ decay.

| Photon Energy (MeV) | our [17] | [18] | [19] |
|---------------------|----------|------|------|
| $\Gamma(B_c^* \to B_c \gamma)$ (eV) | 33       | 135  | 60   | 59   |

of the vector and pseudoscalar ground states. In previous calculations [17–19] the nonrelativistic expression for the matrix element of the magnetic moment was used. We see that even in the heavy-heavy $B_c$ meson inclusion of the relativistic effects results in considerable reduction of the radiative M1-decay rate. As can be seen from Table 1, this reduction is evoked by significant contributions of relativistic effects for the $c$ quark, since it is not heavy enough, as well as by the special choice of the mixture of vector and scalar confining potentials in our model (8).

In summary we calculated radiative M1-decay rates of mesons with open flavour in the framework of the relativistic quark model. In our analysis the light quark was treated relativistically, while for the heavy quark the $1/m_Q$ expansion was carried out up to the second order. Relativistic consideration of the light quark, relativistic heavy quark corrections as well as Lorentz-structure of the confining potential considerably influence the predictions. We find that only the mixture of vector and scalar confining potentials (8), with the mixing coefficient fixed previously from quarkonium radiative decays [3] and weak decays of heavy-light mesons [5], is in agreement with recent CLEO data for the $D^{*+} \to D^{+} \gamma$ decay rate. More precise measurement of this decay rate and the measurement of radiative M1-decays of other heavy-light mesons will be crucial for testing the relativistic quark dynamics.

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