Research and empirical Analysis of Traffic flow Modeling based on fluid Mechanics

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Abstract. In this paper, the overtaking lane change flow is introduced, and the corresponding traffic flow continuity equation is established. Through the differential analysis of traffic flow parameters, the motion differential equation of road traffic flow is established. Compared with fluid mechanics, according to Newton's second law, the simple calculation of traffic pressure, viscous resistance coefficient and viscous resistance is put forward, and the interference from downstream traffic wave is defined as the viscosity of traffic flow, and the difference between wave velocity and maximum wave speed is the coefficient of viscous resistance along the way. the viscous resistance along the route is proportional to the length of the lane, the rate of change of flow along the direction of traffic flow and the coefficient of viscous resistance along the route. The model can obtain the relationship between traffic flow parameters. The simulation example shows that the model can reflect the basic characteristics of traffic flow.

Keywords: traffic engineering; traffic flow, hydrodynamics, dynamic model, traffic pressure, viscous resistance.

1. Introduction

In 1955, British scholars Lighthill and Whitham compared traffic flow to a kind of fluid [1]. For a long highway tunnel, the law of traffic flow under the condition of high traffic density was studied, and the hydrodynamics simulation theory was put forward, which is called Lmurw theory [2]. The theory is actually the continuity approximation of the car-following theory and is a first-order continuous model.

Because the first-order continuous model does not consider the influence of acceleration and inertia and assumes that the traffic flow speed is in equilibrium, it cannot truthfully reflect the dynamic characteristics of traffic flow in non-equilibrium state. nor can it describe the self-organizing phenomenon of stop-and-stop traffic with typical frequencies and wavelengths [3]. In order to solve these problems, many scholars have introduced acceleration expressions or momentum equations to form their own characteristic models.

Some of the models introduce the concepts of traffic pressure, viscosity coefficient and viscous resistance [4], but the physical meanings and definitions of these concepts are quite different, and the values of some parameters are not easy.

The relative viscosity hypothesis is introduced in reference [5], and the differential equation of motion of traffic flow is proposed, but it is not rigorous.
In this paper, the flow rate of overtaking lane change is introduced, and the corresponding traffic flow continuity equation is established; through the differential transformation of traffic flow parameters, the differential equation of traffic flow motion is established, which is compared with fluid mechanics and Newton's second law is applied. Traffic pressure, viscous resistance coefficient and viscous resistance with clear physical meaning and simple calculation are put forward. The dynamic model of traffic flow is composed of the continuity equation and the differential equation of motion including the flow rate of overtaking lane change. Taking the overtaking lane changing flow equal to zero as an example, the simulation calculation is carried out by using the model.

2. Continuity equation
The continuity equation in traffic flow is:

\[ \frac{\partial k}{\partial t} + \frac{\partial (ku)}{\partial x} = 0 \]  

(1)

Where \( u \) is the velocity; \( k \) is the density; \( t \) is the time; \( x \) is the position.

If there are vehicles in and out of the road section, the source and sink item (flow generation rate) \( s \) can be added to the right end of formula (1). For the section with no access ramp, \( s = 0 \); for the entrance ramp, \( s > 0 \); for the exit ramp, \( s < 0 \), that is:

\[ \frac{\partial k}{\partial t} + \frac{\partial (ku)}{\partial x} = s \]  

(2)

Formula (1) is only suitable for roads without ramps, and it is not suitable for traffic flow of roads with overtaking and changing lanes. Referring to the research method of hydrodynamics in reference [6], the traffic flow system and control domain are put forward, and the integral formula of some physical quantity to the total derivative of time in traffic flow system is deduced:

\[ \frac{dN}{dt} = \frac{\partial}{\partial t} \int_{cf} \eta F dF + \int_{q_i} \eta k u_i dL \]  

(3)

In the formula: \( N \) is the total amount of a certain physical quantity (such as the number of vehicles or momentum, etc.) possessed by the traffic flow in the t-instantaneous system; \( \eta \) is the physical quantity per unit number of vehicles; \( F \) is the area of the microelement; \( F \) is the integral of the control domain; \( u_i \) is the partial velocity along the normal direction of the microelement control line; \( L \) is the length of the microelement of the outflow part of the control line; \( L \) is the integral of the control line.

The law of conservation of mass is applied to the traffic flow, that is, the conservation of the number of vehicles. If the number of vehicles in the system is 1, then there is \( \frac{dN}{dt} = 0 \).

The derivative of the number of vehicles in the system to time can be obtained by using the formula (3), in which case \( \eta = 1 \), so the traffic flow continuity equation in integral form is obtained:

\[ \frac{\partial}{\partial t} \int_{cf} F k dF + \int_{c_1} k u_i dL = 0 \]  

(4)

For a single lane, if there are no vehicles entering or leaving the system from the lane demarcation line, that is, there is no vehicle entering or leaving the system, then formula (4) is:

\[ \frac{\partial}{\partial t} \int_{cf} k dF + ku - k_{1u} u = 0 \]  

Where: subscript 1 represents a section of the single lane control domain. If you take the derivative of \( x \), you can get the equation (1). Accordingly, if there is a vehicle entering and leaving the system from
the lane dividing line, that is, the vehicle entering and leaving the system, the incoming flow is set to r, and the distribution function along the lane dividing line is \( s = \frac{dr}{dx} \), \( r = \int_{x_1}^{x} sdx \).

The formula (4) is:

\[
\frac{\partial}{\partial t} \int_{x_1}^{x} kF + ku - k_1u_1 - \int_{x_1}^{x} sdx = 0
\]

Derive from x, get the formula (2) or

\[
\frac{\partial k}{\partial t} + \frac{\partial (ku)}{\partial x} = \frac{dr}{dx}
\]

(5)

In the same way, the continuity equation of the same form when a vehicle leaves can be obtained. In this way, Formula (2) or Formula (5) is the general formula of the continuity equation of traffic flow in the section with overtaking lane. When the vehicle leaves the lane, \( r < 0 \), \( r > 0 \), when the vehicle enters the lane, \( r > 0 \), and when there is no vehicle in and out, \( r > 0 \), \( r > 0 \), there is no one-to-one corresponding relationship between overtaking lane changing flow \( r \) and speed, density and flow, that is to say, there is no definite overtaking lane changing flow \( r \) for a certain speed, density or flow, \( r \) is an independent parameter.

3. Differential equation of motion

It can be seen from the formula (5) that the speed of traffic flow is a function of density \( k \) and overtaking lane change flow \( r \), that is, \( u = u(k, r) \). Through the differential transformation of the function, the result is obtained:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - \frac{u_w - u}{k} \frac{\partial q}{\partial x} + \frac{u_v - u}{s} + u \frac{\partial u}{\partial x}
\]

In the formula: \( u_w \) is the wave velocity. If \( u_I \) is the maximum wave velocity [6], then:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - \frac{1}{k} \frac{\partial q(u_I - u)}{\partial x} - \frac{u_w - u}{k} \frac{\partial q}{\partial x} + \frac{u_w - u}{s k}
\]

(6)

The differential equation of motion of traffic flow can be obtained by finishing formula (6):

\[
\frac{\partial}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{k} \frac{\partial q(u_I - u)}{\partial x} + \frac{u_w - u}{k} \frac{\partial q}{\partial x} = 0
\]

(7)

4. Relationship between parameters

The continuity equation and the differential equation of motion constitute the dynamic model of traffic flow.

\[
\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = s
\]

\[
\left\{ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{k} \frac{\partial (u_I - u)}{\partial x} \right\} + \frac{u_w - u}{k} \frac{\partial q}{\partial x}
\]

\[
\frac{u_v - u}{s} = 0
\]

Firstly, the coefficient of viscous resistance is analysed. Due to:
Velocity-density linear relation model for Greenshields:

\[ u = u_t(1 - k/k_f), u_t = u_f, u \leq u_t, \mu_s = 2(u - u_t) \]

By using the characteristic line method to solve the above model in this section, the theoretical relationship between speed, density and overtaking lane changing flow can be obtained as follows:

\[ u = \begin{cases} 
    \frac{1}{4}muf \frac{k_i}{e^{\frac{i}{\mu}}} & k_i e^{\frac{i}{\mu}} \leq k \leq 0.5 k_i e^{\frac{i}{\mu}} \\
    \frac{1}{4}muf(1 - \frac{k_i}{k} e^{\frac{i}{\mu}}) & 0.5 k_i e^{\frac{i}{\mu}} \leq k \leq k_i e^{\frac{i}{\mu}} 
\end{cases} \]

In the formula: uf is the free flow velocity, taking the design speed of the road; kf is the maximum density when traveling at the free flow speed, which is called the free flow density, which can be obtained by regression from the measured data; from the derivative of the flow to the density, the maximum wave velocity \( u_I = muf \), m is called the wave velocity coefficient, which can be obtained by regression. Kj is the congestion density with a speed of 00:00, and the density with a workshop distance of 00:00 is called the theoretical congestion density. If the average length of the vehicle is 6m, the theoretical congestion density (single lane, the same) is 166.7 pcukm kja, and the actual congestion density is less than this value. Through theoretical analysis and practical observation, it is advisable to take the density 111.1pcukm-1 when the workshop distance is equal to half of the average vehicle length as the actual congestion density kja:

\[ k = \frac{k_j}{4} - \frac{(1 - m)k_f}{m} \]

Based on the above analysis, the relationship between other parameters can be obtained.

5. Numerical simulation

In order to simplify the calculation, taking the overtaking lane change flow equal to zero as an example, the numerical simulation is carried out by using the model in section 4 with the measured traffic data of a certain period of time on the Guangzhou-Shenzhen expressway as the initial condition. The model is discretized by time forward difference, space forward difference (for velocity) and space back difference (for density).

\[ k^{i+1} = k^i + \frac{\Delta t}{\Delta x}(k^i u^i_i - k^i u^i_{i+1}) \\
\]

\[ u^{i+1} = u^i + \frac{\Delta t}{\Delta x}(k^i u^i_i - k^i u^i_{i+1}) \frac{u^i_i - u^i_{i-1} + \mu^i}{k^i} \]

In the formula: I is the calculation unit, I = 1, 2, 3, …; n is a time series. N = 0, 1, 2, … no, no, no. The calculation step size should satisfy \( \Delta t < \Delta x / u_I \).

The calculated length of the road section is L = 6 km, the design speed \( u_f = 120 \text{ km h}^{-1} \), and the congestion density \( k_{ja} = 111.1 \text{ pcu km}^{-1} \), the maximum wave velocity and free flow density are selected from the measured data of the second lane of Guangzhou-Shenzhen Expressway: \( u_I = muf = 84.1544 \text{ km h}^{-1} \), \( k_f = 1.771 \text{ pcu km}^{-1} \). The initial conditions are shown in Table 1.

### Table 1. Initial Speed and Density of Guangzhou-Shenzhen Expressway

| Section | 1 | 2 | 3 | 4 | 5 | 6 |
|---------|---|---|---|---|---|---|
| Density / (pcu km^-1) | 17.9 | 18.4 | 18.4 | 18.2 | 18.1 | 52.1 |
| Speed / (km h^-1) | 87.7 | 87.6 | 87.6 | 87.6 | 87.7 | 44.9 |
| Section | 7 | 8 | 9 | 10 | 11 | 12 |
| Density / (pcu km^-1) | 51.8 | 51.7 | 18.6 | 18.1 | 17.8 | 18.1 |
| Speed / (km h^-1) | 45.1 | 45.2 | 87.6 | 87.7 | 87.7 | 87.7 |
6. Conclusion

Through the differential analysis of traffic flow parameters, the differential equation of traffic flow motion is established. Compared with fluid mechanics, the traffic pressure, viscous resistance coefficient and viscous resistance with clear physical meaning and simple calculation are put forward by using Newton's second law. Defining the traffic pressure and viscous resistance in traffic flow will be helpful to the study of integral modeling method of traffic flow.

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