Theoretical Response to the Discovery of
Deeply Bound Pionic States in $^{208}$Pb(d, $^3$He) Reactions

H. Toki$^a$, S. Hirenzaki$^b$ and K. Takahashi$^b$

$a$) RCNP, Osaka University, Ibaraki, Osaka 567, Japan
$b$) Nara Women’s University, Nara 630, Japan

ABSTRACT

Recently, deeply bound pionic states were found experimentally in (d, $^3$He) reactions on $^{208}$Pb. They found an isolated peak structure in the bound region below the pion production threshold. We study theoretically these excitation functions in (d, $^3$He) reactions on $^{208}$Pb at $T_d=600$ MeV. We found very good agreement with the (d, $^3$He) excitation functions and could identify the underlying structures of the pionic states. We study the energy dependence of the (d, $^3$He) reactions and the change of the excitation functions with the incident energy.

The existence of deeply bound pionic atoms in heavy nuclei was predicted by Toki and Yamazaki as quasi-static states$^{1,2}$. Since then many efforts were made for detection of these states both experimentally and theoretically$^{3)−10)}$. In particular, it is worth mentioning that (n,d) reactions$^9$ and (p,pp) reactions$^{10}$ on the $^{208}$Pb target were able to identify some strength in the excitation function below the pion production threshold. It is, however, not yet convincing due to the lack of good resolution and good statistical accuracy. It was, hence, badly needed to perform the similar experiments with better resolution and statistical accuracy for identification of isolated peak structure in the bound region below the pion production threshold.

Very recently, Yamazaki et.al. performed such experiments with the use of (d, $^3$He) reactions on $^{208}$Pb with better resolution and statistical accuracy$^{11)}$. They, at last, succeeded to identify clearly a peak structure in the bound region. The results were presented in the recent publication with the theoretical predictions, which were made before the experiments were performed$^6$. The agreement of the theoretical predictions to the newly obtained experimental data is almost perfect.

In this paper, we would like to describe in detail the theoretical part of the (d, $^3$He) reaction on $^{208}$Pb. We should describe the underlying structures of the (d, $^3$He) excitation functions. At the same time, we would like to show the spectrum with better resolution to predict what should be expected when experiments were performed with better (feasible) resolution. In addition, we would like to show the energy dependence of the excitation function of the (d, $^3$He) reaction with the $^{208}$Pb target.

We describe the (d, $^3$He) reaction within the effective number approach. The (d, $^3$He) cross sections use the experimental cross section and the effective number of
nucleons exciting a pion in a definite orbit with inclusion of the distortion effect. The 
(d, $^3$He) cross section in the laboratory frame is expressed as

$$\frac{d\sigma}{d\Omega}_{d + A \rightarrow ^3He\pi} = \frac{d\sigma}{d\Omega}_{dn \rightarrow ^3He\pi} N_{eff}$$

Here, the effective number takes care of the initial neutron in a shell model orbit
specified by $j_n$ and the final pion in a pionic atom orbit specified by $l_\pi$ with the
initial deuteron and final $^3$He distortions. After manipulations, we find $N_{eff}$ for the
configuration $[l_\pi \otimes j_n^{-1}]J$ be written in the following form

$$N_{eff} = \frac{1}{2} \sum_{Lm} |S_{JL} \int \exp \left( i\mathbf{q} \cdot \mathbf{r} \right) D(b) \, R_\pi(r) \, R_n(r) Y_{Lm}(\hat{r}) d^3r |^2$$

with the statistical factor

$$S_{JL} = \langle l_\pi (l_\pi \frac{1}{2}) j_n; J \right| \langle l_\pi l_n \frac{1}{2}; J \rangle \sqrt{\frac{2J + 1}{2L + 1}} (-)^{l_n} \times \sqrt{\frac{(2l_\pi + 1)(2l_n + 1)}{4\pi(2L + 1)}} (l_\pi o l_n o | o)$$

The distortion factor $D(b)$ is estimated by the eikonal approximation,

$$D(b) = \exp \left( - \frac{1}{2} \int_{-\infty}^{\infty} \mathbf{\sigma} \rho(z', b) dz' \right)$$

As for the average cross section $\bar{\sigma}$, we take $\bar{\sigma} = \frac{1}{2}(\sigma_d N + \sigma_{^3He N})$, where $N$ denotes
nucleon. We calculate the cross section leading to a pion in the continuum in the
similar way as for the bound states. Essentially, we replace the bound pion wave
function by a scattering pion wave function, obtained by solving the distorted wave
optical model. All the details will be published in the near future\textsuperscript{12).

The important information for the pionic atom formation is the elementary dif-
fferential cross sections at the forward angle for $d + n \rightarrow ^3He + \pi^-$ in the laboratory
frame. We first discuss the incident energy dependence of the elementary differential
cross sections, which are shown in Fig.3 of Ref. 4. The cross section peaks around $T_d$
= 600 MeV. The momentum transfers for $(d, ^3He)$ reactions are provided in Fig.2 of
Ref.4. At $T_d = 600$ MeV, the momentum transfer is small. This small momentum
transfer and the large cross section at $T_d = 600$ MeV seem ideal for the study of
deeply bound pionic states in heavy nuclei.

We present here the numerical results for $^{208}$Pb(d, $^3$He) reactions for pionic atom
formation. As for the bound neutron, we take two major shells below the Fermi
surface; $50 < N < 126$. The separation energies of the neutron orbitals are $s_n =$
7.367 MeV for $p_{1/2}$, $s_n = 7.937$ MeV for $f_{5/2}$, $S_n = 8.264$ MeV for $p_{3/2}$ and $S_n = 9.000$
MeV for $i_{13/2}$. The pionic states are calculated by using the optical potential of Seki
and Masutani\textsuperscript{13) and the energies and the widths are given in Ref.2.
We show the calculated results on $^{208}\text{Pb}(d, \text{^3He})$ reactions at $T_d = 600$ MeV in Fig.1. In the upper part of Fig.1, we show the forward cross sections with the resolution of 500 keV in order to compare with the experimental spectra shown in the middle part of Fig.1. The agreement is almost perfect. We note that we have added the contributions from deeper neutron single particle orbits ($50 < N < 82$) in the calculation of the continuum cross sections as compared to the one in the discovery paper\textsuperscript{11}. We assume 20$\mu$b/sr/MeV constant background in the theoretical spectra.

In order to see the underlying structures and also the spectra with a better experimental resolution, we show in the lower part of Fig.1, the cross sections with 200 keV resolution. We can see now many peaks below the pion production threshold, which is denoted by the vertical dashed line. The peak seen experimentally consists of two states; $[l_\pi \otimes j_n^{-1}] = [1 \otimes p_{1/2}^{-1}]$ and $[1 \otimes p_{3/2}^{-1}]$. Since 4 neutrons are allowed in the $p_{3/2}$ state and 2 neutrons in the $p_{1/2}$ state, the ratio of the cross sections leading to $p_{3/2}^{-1}$ and $p_{1/2}^{-1}$ is 2. Those two states provide the peak and the shoulder structure at $Q = -136$ MeV (around 5 MeV binding energy). The pionic $s$ state seems to be only weakly excited due to the necessity of some momentum transfer. In addition, there is an appreciable amount of contribution from the neutron $f_{5/2}$ hole coupled with the pionic $s$ state to this small peak. Many more peaks are seen closer to the pion production threshold. The dominant contributions are again coming from the pionic atom states coupled with the $p_{3/2}$ and $p_{1/2}$ neutron holes. It is interesting to comment on the bump structure seen in the calculated spectra around $Q = -143$ MeV in the continuum. This bump structure is caused by exciting $[1s_\pi \otimes s_{1/2}^{-1}]$ state, where a $s_{1/2}$ neutron in the $50 < N < 82$ major shell is picked up and a pion is placed in the $1s$ orbit. In the calculation, the width of the $s_{1/2}$ neutron hole state is assumed zero.

We study now the energy dependence of the $(d, \text{^3He})$ cross sections. As could be guessed from the energy dependence of the elementary differential cross sections and the larger momentum transfers, the $(d, \text{^3He})$ cross sections at different energies should decrease. It is, however, interesting to see how the cross sections will change with the incident energy. For this purpose, we show the calculated results at $T_d = 500, 800$ and 1000 MeV with the resolution of $\Delta E = 200$ keV in Fig.2.

When we lower incident energy to $T_d = 500$ MeV, the peak structure at $E \sim 5$ MeV is enhanced relative to other structure due to smaller momentum transfer. The peak structures are almost exclusively produced by the $p$ neutron hole contribution. Note that the $1s$ pionic state is not appreciably excited at this energy. Those energy dependences are clearly explained by using the energy dependence figures; Fig.5-8 in Ref.4. As the energy is increased to $T_d = 800$ MeV, the cross sections are lowered. As indicated by the three kinds of curves, now the contributions from the $p$ neutron hole state become small, while the contributions from the $i_{13/2}$ and $f_{5/2}$ neutron hole states increase. Pionic states coupled with $f_{5/2}$ neutron hole state are excited appreciably. Further increased to $T_d = 1000$ MeV, the peak structures are completely dominated...
by the $i_{13/2}$ neutron hole state coupled with various pionic atom states.

It is amazing to observe that the calculated peak structures agree extremely well with the experimental results. Since various pionic atom - neutron hole states are excited by the (d, $^3$He) reactions, we expect some pion-hole residual interactions to mix these states. We ought to estimate this mechanism theoretically. We believe, however, that the residual interactions are much smaller than the case of nuclear particle hole states, since the overlaps between pionic states and the nuclear states are quite small.

We have studied theoretically the (d, $^3$He) reactions leading to pionic atom states in $^{208}$Pb at $T_d = 600$ MeV as the theoretical response to the recent discovery of deeply bound pionic atoms $^{11)}$. The peak structure seen around $E \sim 5$ MeV is due to $2\rho$ pionic state coupled with $p_{3/2}$ and $p_{1/2}$ neutron hole states. We have shown the excitation function of (d, $^3$He) reactions at the same incident energy with a better resolution, $\Delta E = 200$ keV, to see the peak structures clearly. In addition, we have presented the energy dependence of the excitation functions at various incident energies. At higher energy as $T_d = 1000$ MeV, the $i_{13/2}$ neutron hole state coupled with various pionic atom states dominate the excitation function.

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Fig. 1 Forward cross sections of the $^{208}$Pb(d, $^3$He) reaction at $T_d = 600$ MeV; (a) Theoretical results with the experimental resolution of 500 keV, (b) Observed spectrum by Yamazaki et. al.\textsuperscript{11}, and (c) Theoretical results with 200 keV experimental resolution. In (a) and (c) the flat background is assumed to be 20 $\mu$b/sr/MeV. The vertical dashed line denotes the $\pi^-$ emission threshold energy.

Fig. 2 Calculated cross sections of the $^{208}$Pb(d, $^3$He) reaction at (a) $T_d = 500$ MeV, (b) $T_d = 800$ MeV, and (c) $T_d = 1000$ MeV with 200 keV experimental energy resolution. Solid curves show the full spectra including all contributions. Dotted curves show the contributions from the neutron $2p$ states ($p_{3/2}$ and $p_{1/2}$) and dashed curve show those from the neutron $i_{13/2}$ state. Added is also the contribution from the neutron $1f$ states ($f_{5/2}$ and $f_{7/2}$) by dash-dotted curve in the middle figure.
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