Stable Learning via Causality-based Feature Rectification

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Abstract

How to learn a stable model under agnostic distribution shift between training and testing datasets is an essential problem in machine learning tasks. The agnostic distribution shift caused by data generation bias can lead to model misspecification and unstable performance across different test datasets. Most of the recently proposed methods are causality-based sample reweighting methods, whose performance is affected by sample size. Moreover, these works are restricted to linear models, not to deep-learning based nonlinear models. In this work, we propose a novel Causality-based Feature Rectification (CFR) method to address the model misspecification problem under agnostic distribution shift by using a weight matrix to rectify features. Our proposal based on the fact that the causality between stable features and the ground truth is consistent under agnostic distribution shift, but is partly omitted and statistically correlated with other features. We propose the feature rectification weight matrix to reconstruct the omitted causality by using other features as proxy variables. We further propose an algorithm that jointly optimizes the weight matrix and the regressor (or classifier). Our proposal can not only improve the stability of linear models, but also deep-learning based models. Extensive experiments on both synthetic and real-world datasets demonstrate that our proposal outperforms previous state-of-the-art stable learning methods. The code will be released later on.
1 Introduction

Machine learning models are typically following a consistency assumption that the training and testing datasets are generated from the same data distribution (i.e., i.i.d hypothesis). Under i.i.d hypothesis, a trained model can be directly applied to the testing dataset, and its performance should be equivalent to the training dataset. Although this approach is empirically proven to be highly successful in many public datasets, it is considered flawed in practice. In many real-world applications, there is no guarantee that the training data and the unknown testing data will have the same distribution due to the data generation bias. This agnostic distribution shift between training and testing datasets can result in inaccurate parameter estimation and unstable performance across different test datasets. Let us consider a case in the image classification task: why an image of a dog on the grass can be labeled as ‘dog’, but a cat on the grass should not. The reason is profound that there is a stable causality exist between certain essential stable features (features of the dog in this case) and the ground truth. This stable causality is consistent even if the unstable features (e.g., the feature of grass) are changed across different unknown environments. However, the data generation bias can lead to a statistical correlation between unstable features like grass and the ground truth ‘dog’. The model trained in such dataset can be easily trapped into this statistical correlation for yielding lower training loss, and resulting in unstable performance across testing datasets with different distributions.

Therefore, how to mitigate this model misspecification problem, and learn a stable model have attracted lots of attention from the community. (Pan et al., 2018; Wang et al., 2019; Shen et al., 2019; Kuang et al., 2020) One of the most straightforward thoughts to solve this model misspecification problem is to take advantage of the prior knowledge of the test data. Based on that, many domain adaption (Chen et al., 2020) and transfer learning methods (Pan et al., 2018; Wang et al., 2019) were proposed recently to correct the distribution shift by using the prior knowledge of the test dataset. However, in the agnostic distribution shift problem, it is unable to obtain the prior knowledge of the test data or the true distribution. Therefore, these works are not feasible when dealing with the agnostic distribution shift problem we focus on in this work.

More recently, many causality-based methods (Shen et al., 2018, 2019; Kuang et al., 2020) were proposed. Most of them are sample reweighting based methods. Shen et al. (2019) proposed a sample reweighting method to address the collinearity problem among input variables caused by agnostic distribution shift. Kuang et al. (2020) proposed a causality-based sample reweighting based feature decorrelation method. However, these sample reweighing based methods requires a sample reweighing matrix whose parameter number is proportional to the training sample number. Hence, these works are both computationally and memory intensive. This disadvantage limits the scalability of these methods in machine learning tasks with a large number of training data. Moreover, these works are limited to linear models, but not to deep-learning based nonlinear models.

In this work, we investigate the stable learning problem under the agnostic distribution shift. Specifically, we propose a novel Causality-based Feature Rectification (CFR) method to address the model misspecification problem caused by the agnostic distribution shift. Our proposal’s motivation is that the stable causality between the stable features and the ground truths is consistent even if it is partly omitted due to the agnostic distribution shift. Therefore, we can use the correlations between omitted stable features and other features as a proxy of these omitted causality. We propose to use a feature rectification weight to reconstruct the causality graph using these correlations as a proxy. We further propose an algorithm to jointly optimize the feature rectification weight and a regressor (or classifier), in which the feature rectification weight matrix is used to rectify the input features to aganist the impact of agnostic distribution shift. Our proposal is a feature pretreatment method, and it can integrate with most of the commonly used linear regression and image classification methods, including deep-learning based models. The experimental results demonstrate that the stability of our proposal outperforms all baselines and state-of-the-art stable learning methods in both synthetic and real-world datasets.

We summarize the contributions as following:

(1) We investigate the stable learning problem under the agnostic distribution shift in this work. Specifically, we focus on the model misspecification problem under agnostic distribution shift.

(2) We propose a novel Causality-based Feature Rectification (CFR) method to improve the stability of the model across different test datasets. Specifically, we propose a feature rectification weight to reconstruct the causality graph of features and help to learn the true underlying model under the
agnostic distribution shift. Unlike previous works, our proposal is not restricted to linear models, and can also be applied to deep-learning based models.

(3) We conduct linear regression and image classification experiments on both synthetic and real-world datasets to validate our proposal’s performance. The experimental results demonstrate that our proposal can achieve state-of-the-art performance on both synthetic and real-world datasets.

2 Related Works

2.1 Causality based Methods

Most recently proposed causality-based methods are based on sample reweighting, which not directly change the biased sample features, but shifting the training dataset’s distribution by varying the importance of the samples. Shen et al. (2018) proposed a Causally Regularized Logistic Regression model to address the agnostic distribution shift in Logistic Regression tasks. But it did not consider the model misspecification problem, and its algorithm was restricted to the predictive setting with binary predictors and binary response variable. Shen et al. (2019) proposed a sample reweighting method to address the collinearity among input variables caused by the agnostic distribution shift. Kuang et al. (2020) proposed a causality-based feature sample reweighting method, which shifts the features mutually independent with mean 0.

However, these works are restricted to regression tasks and binary classification tasks. Meanwhile, these sample reweighting based works require a large sample reweighting matrix, and its parameter number is proportional to the training data number. Hence, these works are not feasible for tasks with massive training data. Different from them, our proposal addresses the model misspecification problem by rectifying the correlation between features. Moreover, the parameter number of our proposal is proportional to the feature dimension. In most machine-learning tasks, the feature dimension is less than thousands. Hence, our proposal is more feasible than previous works for tasks with a large training dataset.

2.2 Non-causality based Methods

In addition to causality based methods, a variety of domain adaptation (Chen et al., 2020; Liu and Ziebart, 2014; Zadrozny, 2004) and transfer learning methods Pan et al. (2018); Wang et al. (2019) were proposed to address the non-i.i.d problem. Most of these methods handle the distribution shift between training and testing datasets by aligning the training dataset to the target dataset or vice versa. To achieve that, these methods require prior knowledge of the distribution of the target domain. However, in the agnostic distribution shift problem, there is no prior knowledge about the test datasets. Hence, these methods cannot be applied to the agnostic distribution shift problem we focused on in this work.

Except for domain adaptation and transfer learning methods, there are also some domain generalization (Muandet et al., 2013) and invariant causal prediction (Peters et al., 2016) methods were proposed recently to address the distribution shift problem. These works exploring the invariant structure between predictors and the response variables in multiple training data sets to make prediction (Kuang et al., 2020). However, these works cannot handle the distribution shifts that are not observed in the training data.

3 Causality-based Feature Rectification

3.1 Problem Formulation and Preliminaries

**Stable Learning.** Given a dataset $D^e = \{X^e, Y^e\}$ collected in environment $e \in \mathcal{E}$, where $X^e \in \mathcal{X}$ is the observed feature in environment $e$, $Y^e \in \mathcal{Y}$ is the corresponded ground truths. The environment $e$ is defined as a joint distribution $P(X, Y)$ on $\mathcal{X} \times \mathcal{Y}$ and $\mathcal{E}$ is the collection of all environments. The stable learning task is to learn a predictive model on training dataset $D^e$ which can achieve uniformly small error across unknown environments in $\mathcal{E}$ (Kuang et al., 2020).

**Notations.** In this work, we denote $p$ as the observed feature dimension, $n$ as the sample size. For a matrix $X \in \mathbb{R}^{n \times p}$, we let $X_i$ and $X_{:,j}$ represent the $i$-th row and the $j$-th column in $X$ respectively.
We then suppose in an environment $e$, and $X_{-i}$ and $X_{-j}$ denote the remaining matrix by removing the $i$-th row and $i$-th column respectively. For a vector $V = (V_0, V_2, ..., V_m)^\top$, we let $\|V\|_1 = \sum_{i=0}^{m} |V_i|$ and $\|V\|_2^2 = \sum_{i=0}^{m} V_i^2$.

### 3.2 Causality Stable Feature

According to the stability of the causality between ground truths $Y^e$ and features $X^e$, we can divide $X$ into stable features $S$ and unstable features $V$ under Assumption 1 and Assumption 2:

**Assumption 1.** (Kuang et al., 2020) There exists a stable function $f(s)$, so that in any environment $e \in E$ we have $f(s) = \mathbb{E}(Y^e|S^e = s, V^e = v) = \mathbb{E}(Y^e|S^e = s)$.

Assumption 1 can be guaranteed by $Y \perp V | S$. So the model misspecification problem under agnostic distribution shift can be addressed by learning the stable function $f(s)$ while ignoring the interference of unstable features. However, in real-world applications, there are no prior knowledge on which feature is stable and which is not. And there is no guarantee that no causality exits between features.

**Assumption 2.** All stable feature $S$ are observed.

Under Assumption 2, the stable learning problem can be addressed by rectifying the correlation between unstable features and the omitted causality.

### 3.3 Linear Regression under Agnostic Distribution Shift

We use linear regression task with model misspecification as an example to illustrate our method in this work. We assume the true underlying model is:

$$Y = S\beta + \epsilon = f(S) + \epsilon,$$

where $f(S)$ is the true stable causality function and $\epsilon$ obeys the Gauss–Markov assumptions (Henderson, 1975). Hence, $\epsilon$ is uncorrelated to each other, have equal variances and expectation value 0. Under this setting, an ordinary least squares (OLS) estimator is the best linear unbiased estimator (BLUE) of $\beta$.

We then suppose in an environment $e$, finite samples are sampled from the environment and formed dataset $D^e$. Due to the data generation bias, the distribution of $D^e$ is deviated from the true distribution $P(X, Y)$. Part of the causal relationship between the stable features and the ground truths being omitted due to this agnostic distribution shift. If a simple linear regression is applied, it may trapped in the statistical correlation between unstable features and the omitted causality. This statistical correlation between unstable features and the omitted part is the reason of model misspecification (Kuang et al., 2020). The model under environment $e$ is given by:

$$Y^e = f(S^e) + V^e \beta_V + \epsilon^e = S^e \beta_S + g(S^e) + V^e \beta_V + \epsilon^e,$$

where $g(S^e)$ is the respond of the omitted causality, which statistical correlated to $V^e$.

Since there is a strong correlation between $g(S^e)$ and $V^e$, then a proxy function can be learned to represented $g(S^e)$ using $V^e$ in environment $e$. Based on this idea, we propose the feature rectification regularizer.

### 3.4 Feature Rectification

We propose to learn a feature rectification weight matrix $W$ by reconstructing features using all other features. The weight matrix $W$ learns the correlations between features. By doing so, the omitted stable feature patterns correlated to the unstable features can be represented by the weight matrix. The objective function as following:

$$W_r = \arg \min_W \sum_{i=1}^{p} \|\mathbb{E}[X_{-i}] - \mathbb{E}[X_{i} W_{-i}]\|_2,$$

where $p$ is the feature dimension, $W \in \mathbb{R}^{p \times p}$, $X_{-i}$ means all the remaining feature by removing $i$-th feature, and $W_{-i}$ means the all the remaining weights by removing $i$-th column of weights.
Suppose a training dataset with sample size \( n \), we can denote the loss in Equation 3 as:

\[
\mathcal{L}_r = \sum_{i=1}^{n} \sum_{j=1}^{p} \| X_{i,-j} - X_{i,j} W_{j,-j} \|. \tag{4}
\]

### 3.5 Application in Linear Regression

To illustrate our method, we using an OLS estimator togther with our proposal to estimate the regression coefficients:

\[
\hat{\beta} = \arg \min_{\beta} \sum_{i=1}^{n} (Y_i - (X_i W_r) \beta - X_i \beta)^2. \tag{5}
\]

As we discussed above, the weight matrix \( W_r \) is trained by reconstructing the causality graph between features using the training dataset. The residual of the reconstruction is the independent part of the features which is independent from others. The OLS estimator is compelled to learn the residual.

We claim that the second term \( (X_i W_r) \beta \) helps the regressor to learn the true underlying model and improves the stability of the model across different test datasets. The experimental results have shown that our proposal worked when dealing with linear regression and image classification tasks.

The Loss function for optimizing Equation 5 is:

\[
\mathcal{L}_{CFR} = \sum_{i=1}^{n} (Y_i - (X_i W_r) \beta - X_i \beta)^2, \tag{6}
\]

where \( n \) is the sample number of the training dataset. As for image classification tasks with deep-learning based models, the loss function is:

\[
\mathcal{L}_{CFR} = \sum_{i=1}^{n} L_{CE}(z(f(X_i)) + z(f(X_i))) Y_1, \tag{7}
\]

where \( X_i \) denotes a training image sample, \( f(\cdot) \) denotes the output of the feature extractor, \( z(\cdot) \) denotes the output of the classifier and \( L_{CE} \) is a Cross-Entropy loss function.

Based on Equation 4 and Equation 6, we propose the Causality-based Feature Rectification algorithm to joint optimize feature rectification weight \( W_r \) and model parameters \( \beta \) as shown in Algorithm 1:

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**Algorithm 1: Causality-based Feature Rectification algorithm**

**Input:** Observed features \( X \), groud truths \( Y \), and maximum epoch number \( T \)

**Output:** Optimized \( \beta \) and \( W_r \)

1. Let \( t = 0 \)
2. Initialize parameters \( W_r^{(0)} \) and \( \beta^{(0)} \)
3. Calculate losses using Eq. 4 and 6 with \( (W_r^{(0)}, \beta^{(0)}) \)
4. repeat
5. \( t = t+1 \)
6. Update \( W_r^{(t)} \) with a stochastic gradient descent optimizer by fixing \( \beta^{(t-1)} \)
7. Update \( \beta^{(t)} \) with a stochastic gradient descent optimizer by fixing \( W_r^{(t-1)} \)
8. until \( t > T \) or Equation 4 and 6 coverage
9. return \( \beta^{(t)} \), \( W_r^{(t)} \)

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### 3.5.1 Complexity Analysis

The main time costs for our proposal are to calculate the value of loss function and update parameters \( W_r \) and \( \beta \). We used DWR (Kuang et al., 2020) as a reference to evaluate the complexity of our work. The computation complexity of Algorithm 1 is \( O(np) \), where \( n \) is the sample size and \( p \) is the feature dimension. The complexity of the state-of-the-art method DWR is \( O(np^2) \).
4 Experiments

We conduct extensive experiments to evaluate our proposal, comparing with several baselines and the state-of-the-art works on both synthetic and real-world datasets.

4.1 Datasets

4.1.1 Real-world Datasets

We use the benchmark dataset CIFAR-10 (Krizhevsky et al., 2009) to evaluate the performance of our proposal in image classification tasks. CIFAR-10 is an established computer-vision dataset used for object recognition. It is a subset of the 80 million tiny images dataset and consists of 60,000 32x32 color images containing one of 10 object classes, with 6000 images per class. The size of image in CIFAR-10 is 32 x 32 pixels. During training, images are random cropped and horizontal flipped with a probability 0.5.

4.1.2 Synthetic Datasets

Under Assumption 1, there are three kinds of relationship between $X = (S, V)$ and $Y$, including $S \perp V$, $S \rightarrow V$ and $S \leftarrow V$. The $S$ denotes stable features and $V$ denotes unstable features. We construct synthetic datasets following the setting of previous state-of-the-art work DWR (Kuang et al., 2020). This synthetic dataset mimic the model misspecification problem in which part of the causality is omitted due to the data generation bias. The data generation bias causes a statistical correlations between unstable features and the omitted causality.

We mimic the $S \perp V$ situation in this synthetic dataset. We generate input features $X = \{S, V\}$ with independent Gaussian distributions with the help of an auxiliary features $Z$: $V_1, ..., V_p \sim \mathcal{N}(0, 1)$, $Z_1, ..., Z_p \sim \mathcal{N}(0, 1)$, $S_i = 0.8 * Z_i + 0.2 * Z_{i+1}, i = 1, 2, ..., p_s$, where $p_s = 0.5 * p$ and $p_v = 0.5 * p$ is the number of stable features and unstable features respectively, $S_j$ denotes the $j$-th feature in $S$. The setting about $S \rightarrow V$ and $S \leftarrow V$ can be found in the supplementary materials.

To test the performance with different forms of missing nonlinear terms, we generate the ground truths $Y$ from a polynomial nonlinear function ($Y_{poly}$) and an exponential function ($Y_{exp}$):

$$Y_{poly} = f(s) + \epsilon = [S, V] \cdot [\beta_s, \beta_v] + S_1 \cdot S_2 \cdot S_3 + \epsilon$$
$$Y_{exp} = f(s) + \epsilon = [S, V] \cdot [\beta_s, \beta_v] + e^{S_1 \cdot S_2 \cdot S_3} + \epsilon,$$

where $\beta_s = \{1, \frac{1}{3}, \frac{2}{3}, 1, -\frac{1}{3}, \frac{2}{3}, 1, \ldots\}, \beta_v = \mathbf{0}$ and $\epsilon \sim \mathcal{N}(0, 0.3)$.

To test the stability, we generate a set of environments, each with a distinct joint distribution $P(X, Y)$, while preserving $P(Y|S)$. To achieve that, we generate environments by varying $P(V_b|S)$ on a subset of $V_b \in V$. Following the setting used by DWR (Kuang et al., 2020), we vary $P(V_b|S)$ via biased sample selection with a bias rate $r \in [-3, -1) \cup (1, 3]$. For each sample, the probability of being selected is defined as $P_r = \prod_{V_b \in V_b} |r|^{-5 * D_i}$, where $D_i = |f(s) - sign(r) * V_b|$. If $r > 0, sign(r) = 1$, otherwise $sign(r) = -1$.

4.2 Evaluation Metrics

We use RMSE, $\beta_{error}$, Average Error (AE) and Stability Error (SE) to evaluate the performance of our proposal. The $\beta_{error}$ between a learned coefficient $\hat{\beta}$ and the true coefficient $\beta$ is defined as $\beta_{error} = \frac{1}{p} \| \beta - \hat{\beta} \|_1$, where $p$ is feature dimension of $\beta$. We report both mean and variance of $\beta_{error}$ of 50 independent experiments. The AE and SE proposed by Kuang et al. (2020) is define as:

$$AE = \frac{1}{|E|} \sum_{e \in E} RMSE(D^e), SE = \sqrt{\frac{1}{|E| - 1} \sum_{e \in E} (RMSE(D^e) - AE)}$$ (8)
Figure 1: Experimental results with setting $S \perp V$ with $Y = Y_{poly}$. All models are trained with $n = 2000, p = 20, r_{train} = 1.7$.

4.3 Compared Methods

We use four methods as baselines in this work, including OLS, Lasso (Tibshirani, 1996), Ridge Regression (Hoerl and Kennard, 1970) and DWR (Kuang et al., 2020). DWR is the previous state-of-the-art causality-based sample reweighting method. We used the official implementation of DWR provided by the authors.

Our proposal has three advantages over the previous state-of-the-art works. First, the previous state-of-the-arts are restricted to the linear model and not to the deep nonlinear model. Our proposal can be used for both linear and depth models. Second, the parameter number of previous state-of-the-arts are proportional to the sample size of the training dataset, so it is not suitable for scenarios with a large training dataset. However, the parameter number of our proposal is invariant of sample size. Meanwhile, the performance of previous state-of-the-arts are sensitive to the sample size, but ours is not. Third, the previous state-of-the-arts require to calculated the loss of all training samples before optimizing parameters, which is both computational and memory intensive. Different from them, our proposal uses mini-batch training scheme, which is more practical.

4.4 Experiments on Synthetic Datasets

We evaluate the performance of all methods by comparing the accuracy on parameter estimation and stability on prediction across unknown test datasets. To evaluate the parameter estimation accuracy, we train all models on the same training dataset with a specific bias rate $r_{train}$. We repeat this training process for 50 times independently with different training data from the same bias rate $r_{train}$, and report the mean and variance of $\beta_v$ error on $V$ since the $V$ is the source of model misspecification in this synthetic dataset. To evaluate the prediction stability, we test all models on several test environments with various bias rate $r_{test} \in [-3, -1) \cup (1, 3]$. For each test bias rate, we generate 50 different test datasets and report the mean of RMSE. Using the RMSE results, we further calculate the AE and SE to evaluate the prediction stability across different test environments. As for hyper-parameters, we use random seed 47 for all experiments. The init learning rate for optimizing the weight matrix of DWR and our proposal is 0.005, and 0.001 for optimizing the linear regressor.

As shown in Figure 1, we visualized the results with setting $S \perp V$ with $Y = Y_{poly}$. We can notice that our algorithm can achieves the lowest parameter estimation error on $\beta_v$ compare with all baselines. It shows that our proposal can significantly mitigate the model misspecification caused by feature correlations. Meanwhile, our proposal can achieve the lowest SE across different $r_{test}$, which indicates the model trained by our proposal is more stable than all baselines including the state-of-the-art method.

As shown in Table 1, our method achieved the lowest $\beta_v$ error and SE in all experiments comparing with all baselines including state-of-the-art method DWR. It demonstrates that our proposal can effectively mitigate the model misspecification problem and improve the stability of the model across different test dataset. Meanwhile, we can notice that the performance of our proposal is stable when the sample size changing, but the performance of the state-of-the-art method DWR is affected by the sample size as shown in Table 1. It shows that our method is more robust when using different training sample size.
Table 1: Experimental results under setting $S \perp V$ with $Y = Y_{poly}$ when varying sample size $n$, feature dimension $p$ and training bias rate $r$. The smaller value in this table, the better. We use bold font to highlight the results of our proposal.

| Scenario 1: varying sample size $n$ | Methods | OLS | Lasso | Ridge | DWR | Our | OLS | Lasso | Ridge | DWR | Our | OLS | Lasso | Ridge | DWR | Our |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n=1000$, $p=10$, $r=1.7$ | $\beta_v$ _error | 0.099 | 0.102 | 0.099 | 0.066 | 0.027 | 0.097 | 0.101 | 0.097 | 0.080 | 0.025 | 0.097 | 0.101 | 0.097 | 0.057 | 0.016 |
| $n=2000$, $p=10$, $r=1.7$ | AE | 0.604 | 0.639 | 0.603 | 0.519 | 0.629 | 0.583 | 0.617 | 0.583 | 0.509 | 0.613 | 0.587 | 0.621 | 0.587 | 0.505 | 0.569 |
| $n=4000$, $p=10$, $r=1.7$ | SE | 0.254 | 0.285 | 0.254 | 0.103 | 0.086 | 0.236 | 0.267 | 0.236 | 0.110 | 0.071 | 0.236 | 0.267 | 0.236 | 0.114 | 0.089 |

| Scenario 2: varying feature dimension $p$ | Methods | OLS | Lasso | Ridge | DWR | Our | OLS | Lasso | Ridge | DWR | Our | OLS | Lasso | Ridge | DWR | Our |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n=2000$, $p=10$, $r=1.7$ | $\beta_v$ _error | 0.097 | 0.101 | 0.097 | 0.060 | 0.025 | 0.070 | 0.080 | 0.070 | 0.066 | 0.027 | 0.044 | 0.047 | 0.044 | 0.038 | 0.013 |
| $n=2000$, $p=20$, $r=1.7$ | AE | 0.583 | 0.617 | 0.583 | 0.509 | 0.613 | 0.612 | 0.720 | 0.612 | 0.550 | 0.546 | 0.538 | 0.618 | 0.538 | 0.519 | 0.471 |
| $n=2000$, $p=40$, $r=1.7$ | SE | 0.236 | 0.267 | 0.236 | 0.110 | 0.071 | 0.319 | 0.408 | 0.319 | 0.232 | 0.071 | 0.312 | 0.370 | 0.312 | 0.297 | 0.082 |

| Scenario 3: varying bias rate $r$ on training data | Methods | OLS | Lasso | Ridge | DWR | Our | OLS | Lasso | Ridge | DWR | Our | OLS | Lasso | Ridge | DWR | Our |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n=2000$, $p=20$, $r=1.5$ | $\beta_v$ _error | 0.059 | 0.067 | 0.059 | 0.060 | 0.010 | 0.070 | 0.080 | 0.070 | 0.066 | 0.027 | 0.079 | 0.091 | 0.079 | 0.077 | 0.023 |
| $n=2000$, $p=20$, $r=1.7$ | AE | 0.519 | 0.590 | 0.519 | 0.548 | 0.497 | 0.612 | 0.720 | 0.612 | 0.550 | 0.546 | 0.660 | 0.781 | 0.660 | 0.613 | 0.618 |
| $n=2000$, $p=20$, $r=2.0$ | SE | 0.220 | 0.297 | 0.220 | 0.197 | 0.031 | 0.319 | 0.408 | 0.319 | 0.232 | 0.071 | 0.364 | 0.447 | 0.364 | 0.303 | 0.119 |

Table 2: Results on image classification.

| Methods | CIFAR-10 |
|---|---|
| ResNet-50 | 94.98 |
| ResNet-50+DWR | 95.09 |
| ResNet-50+CFR | 95.59 |

These observations lead to the conclusion that our method can achieve better stability across different test datasets. Moreover, the experimental results also demonstrate that our proposal is more stable when dealing with limited sample numbers.

The experimental results of $S \rightarrow V$, $S \leftarrow V$ with $Y = Y_{poly}$, and results of $Y = Y_{exp}$ are reported in the supplementary materials.

### 4.5 Experiments on Real-world Datasets

We then evaluate the performance of our proposal in image classification tasks. We conduct experiments on the benchmark image classification dataset CIFAR-10. We use a ResNet-50 model as the feature extractor to extract feature embedding of the input images, and we then apply a classifier consists of one linear layer to predict the label of the input image. Similarly as Algorithm 1, we use a feature rectification weight matrix to learn the causality graph of the extracted feature embedding. The loss function for optimizing the weight matrix have shown in Equation 7.

As for baselines, we compare our proposal with the naive ResNet-50 and the ResNet-50 with DWR (Kuang et al., 2020). Note that the original DWR is restricted to linear regression tasks, so we applied the key idea of DWR to the image classification tasks. The original loss function of DWR requires to calculate loss over all training samples and then carry out backpropagate, which is not feasible in image classification tasks with using GPU accelerating. As a trade-off, we update part of the sample weight of DWR in each mini-batch.

As for hyper-parameters, we using init learning rate of 0.1 for optimizing all models and 5e-6 for optimizing the weight matrix of our proposal. All models are trained for 350 epochs. The learning rate is reduced after the 150 and 160 epoch by multiplying 0.1. The random seed is 47 for all experiments.

The results have shown in Table 2. Our proposal can improve the accuracy of the deep-learning based model ResNet-50 in all datasets. These experiments demonstrate that our proposal can be used in both linear regression and image classification tasks.
5 Conclusion

In this work, we address the model misspecification problem under the agnostic distribution shift by proposing a novel Causality-based Feature Rectification (CFR) method. Experiments on both synthetic and real-world datasets demonstrate that our proposal helps improve the performance of the baseline models, and outperforms the state-of-the-art stable learning methods. Our method is a general data pretreatment method, which can be seamlessly integrated into classical linear regression models and classification models. It provides a unified approach to alleviate the problem of model misspecification problem under the agnostic distribution shift. Unlike previous works, our proposal is not restricted to linear models, and can also be applied to deep-learning based models.

Broader Impact

Our proposal is a data pretreatment method which causes no explicit ethical problems. As for benefits, our proposal can help to improve the stability of the model on unknown test datasets, and reduce the data annotation costs for fine-tuning model into new scenarios. It can be applied to most deep-learning based models and linear models in many application scenarios like Quantitative financial analysis, social media and time series based forecasting tasks. Moreover, our proposal helps to leverages biases in the data. Thus the likely beneficiaries are companies that have business suffers the model misspecification problem. The consequence of the failure of the proposal is a poor classification or regression performance. Our work may affect the interests of companies whose primary business is data annotations.

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