Delving into Variance Transmission and Normalization: Shift of Average Gradient Makes the Network Collapse

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Abstract

Normalization operations are essential for state-of-the-art neural networks and enable us to train a network from scratch with a large learning rate (LR). We attempt to explain the real effect of Batch Normalization (BN) from the perspective of variance transmission by investigating the relationship between BN and Weights Normalization (WN). In this work, we demonstrate that the problem of the shift of the average gradient will amplify the variance of every convolutional (conv) layer. We propose Parametric Weights Standardization (PWS), a fast and robust to mini-batch size module used for conv filters, to solve the shift of the average gradient. PWS can provide the speed-up of BN. Besides, it has less computation and does not change the output of a conv layer. PWS enables the network to converge fast without normalizing the outputs. This result enhances the persuasiveness of the shift of the average gradient and explains why BN works from the perspective of variance transmission. The code and appendix will be made available on https://github.com/lyxzzz/PWSConv.

Introduction

We have witnessed the growth of deep learning in computer vision. With the maturity of theories for convolutional neural networks (CNNs) (LeCun et al. 1989; Krizhevsky, Sutskever, and Hinton 2012), several research directions, such as image classification (Simonyan and Zisserman 2015; He et al. 2016a; Szegedy et al. 2016; Huang et al. 2017a) and object detection (Redmon et al. 2016; Ren et al. 2015; Liu et al. 2016), can be explored in depth.

Batch Normalization (BN) (Ioffe and Szegedy 2015), benefiting from its ability to accelerate training speed and reduce the impact of complex initialization and variance transmission (Glorot and Bengio 2010; He et al. 2015), has been widely used in most of the state-of-the-art networks. With the help of normalization, the setting of learning rate (LR) is no longer scrupulous. Generally, the network (without BN) will collapse rapidly if a large LR is set for training. Intuitively, this is attributed to a large update caused by inappropriate LR. By contrast, the appearance of BN alleviates this phenomenon. Moreover, BN accelerates the convergence only by modifying the mean and variance of the output layer, which is puzzling.

Despite normalization’s pervasiveness, the real reason for why they are useful still needs to be studied. However, BN is not omnipotent. It tends to use large batch size and will crash by inaccurate batch statistics estimation when we use a miniature batch size. In Santurkar et al. 2018; Bjorck et al. 2018, BN was thought to make the optimization landscape significantly smoother. Many variants (Ba, Kiros, and Hinton 2016; Ulyanov, Vedaldi, and Lempitsky 2016; Wu and He 2018; Luo et al. 2019) of BN are proposed. Nonetheless, they may not replace BN entirely because of time cost, performance, and other factors. Inspired by those methods of conducting normalization on outputs, weight normalization (WN) (Salimans and Kingma 2016) and other variants (Qiao et al. 2019; Huang et al. 2017b; 2018) that conduct normalization on filters’ weights are proposed. Those methods which normalize the filters usually cooperate with BN to speed up convergence. Many of them may not work if the scale of the output is out of control. Although BN and other methods are effective, changing the outputs may block the variance transmission and lose some information. To better understand why normalizing the outputs is beneficial, we intend to analyze BN from the perspective of variance transmission. We attempt to find a method, which is equivalent to BN and simultaneously robust to mini-batch size. In this way, we may find out why BN works to some extent.

In this paper, we study the variance transmission to find out why BN can speed up convergence and keep the network stable when we use a large LR. We point out that the shift of the average gradient, which is reduced by BN and WN inadvertently, will hinder network training. Therefore, we present Parametric Weights Standardization (PWS) as an alternative to BN and WN. PWS solves the shift of the average gradient by normalizing outputs. It needs less computation and is robust to mini-batch size. Moreover, PWS does not directly control the output variance. PWS acts the same as normal conv operation. We have carried out object detection task in VOC and COCO (Lin et al. 2014) datasets, and image classification task in CIFAR10 and ImageNet (Russakovsky et al. 2015). All models are trained from scratch (Shen et al. 2017; Zhu et al. 2019) to exploit the power of normalization. We refocus on variance transmission to find the discrepancy among normalization operations. Our contributions are:

• Theoretically, we reveal how the shift of the average gradient makes a significant impact on variance transmission.
A huge variance will increase the scale of the gradient. Thus the network collapses.

- We propose Parametric Weights Standardization (PWS), a fast and robust to mini-batch size normalization operation, to solve the shift of the average gradient. Even if we use a large LR, PWS can still be trained without modifying the variance of the output layer. The variance transmits naturally. The relationship between BN and PWS may prove why BN is useful.

- The results indicate that normalizing outputs is not unique to get better results and faster convergence speed.

### The Shift of the Average Gradient

**Variance Transmission**

To understand the role of the shift of the average gradient, we first pay attention to the variance transmission and how the scale of variance influences on the gradient. For a conv layer, we can get

\[ Y_l = X_l \otimes W_l + b_l, \quad X_{l+1} = f(Y_l). \] (1)

Here, \( l \) represents the index of a layer. \( \otimes \) represents the convolutional operation. \( Y \) is the output feature map, which is a \( w \)-by-\( h \)-by-\( d \) tensor. \( w \) and \( h \) are the spatial width and height of the output feature map. \( d \) represents the number of output channels or the number of filters in that conv layer. \( X \) is the input feature map, which is a \( w'\times h'\times c \) tensor. \( w' \) and \( h' \) are the spatial width and height of the input feature map. \( c \) represents the number of input channels of that conv layer. \( W_l \) is a \( \hat{R}^{k\times k\times c\times d} \) tensor. \( k \) is the spatial filter size of that layer. We use \( n_l = k^2c \) to denote the number of units in one filter of that conv layer. \( b \) is a \( d \)-by-1 vector of biases. \( f \) is the activation function. In our experiments, \( f \) function is ReLU (Nair and Hinton 2010).

Generally, the elements in \( W_l \) will be initialized to be mutually independent of one another and have a symmetric distribution with a zero mean. As in (Glorot and Bengio 2010), we can assume that the elements in \( X_l \) are also mutually independent of each other and share the same distribution. The elements in \( W_l \) and the elements in \( X_l \) are independent of each other. For any tensor \( T \), we write \( \text{Var}[T] \) and \( \text{E}[T] \) for the shared scalar variance and mean of all elements in \( T \), respectively. \( \|T\| \) denotes the square root of the quadratic sum of all elements in \( T \). If we assume that \( b_l = 0, E[Y_l] = n_lE[X_l]E[W_l] = 0 \) and \( Y_l \) has a symmetric distribution around zero impacted by \( W_l \), this leads to:

\[ E[X_l^2] = E[\text{relu}(Y_{l-1})^2] = \frac{1}{2} \text{Var}[Y_{l-1}], \] (2)

\[ \text{Var}[Y_l] = \frac{1}{2} n_l \text{Var}[W_l] \text{Var}[Y_{l-1}]. \] (3)

For every layer, one suggestion is to initialize \( \text{Var}[W_l] \) as \( \frac{2}{n_l} \) (He et al. 2015) to transmit variance through ReLU.

Then we focus on the backward propagation case. We get a estimation from (Glorot and Bengio 2010; He et al. 2015):

\[ \text{Var}[^\nabla X_l L] = (\prod_{i=m}^{m-1} n_{i+1} \text{Var}[W_i]) \text{Var}[^\nabla X_m L], \] (4)

where \( m \) represents the number of layers in our networks. \( \theta \) is 1 (Glorot and Bengio 2010) when \( f'(0) = 1 \) and \( \theta \) is \( \frac{1}{\theta} \) (He et al. 2015) when \( f \) is ReLU. We use ReLU as function \( f \). The initialization for \( \text{Var}[W_l] \) directly impacts on the gradients for filters in all layers. \( \text{Var}[Y_l] \) and \( \text{Var}[^\nabla X_l L] \) may both explode due to an unreasonable initialization for \( W_l \). Since \( \nabla W_l L = X_l \otimes \nabla Y_l L, \nabla W_l L \) will be affected and the network may collapse. The influence of variance transmission will become significant if the outputs are not normalized. It deserves to research how to train the network without normalizing the output.

### Gradient Explosion Comes From the Shift

Obviously, an inappropriate initialization for weights will cause the gradient explosion. However, the gradient will also explode even though we use a suitable initialization method. We find that a slightly higher LR can also make networks collapse. Figure 1(a) displays \( \text{Var}[W_{o,l} - \Delta W_{o,l}]/\text{Var}[W_{o,l}] \) for every layer. Subscript \( o \) is used to index a filter in a layer. \( \Delta W_{o,l} \) represents \( LR \times \text{Var}_w L \). Here we use SSD (Liu et al. 2016) + VGG (Simonyan and Zisserman 2015) as the object detection model because it is sensitive to LR. If we set LR as 1e-2 and do not use BN after every conv layer, the network will collapse. It seems that using a large LR will not change the variance a lot (Figure 1(a)), but prompts the filter to have a large shift (Figure 1(b)). The changes of variance for different filters are insignificant (Figure 1(a)). Nevertheless, the network indeed collapses.

The collapse drives us to define a fine-grained variance propagation formula. Ignoring the bias, we can get \( Y_{o,l} = X_l \otimes W_{o,l} \) and \( E[Y_{o,l}] = n_l E[W_{o,l}] E[X_l] \) as mentioned before. For a conv layer, the number of output channels is equal to the number of filters. We use \( N_l = d \) to denote the number of filters in layer \( l \). According to Figure 1(b), we know \( E[\Delta W_{o,l}] \) for different filters in the same layer are distinct, especially for networks without BN. This shift has a great impact on variance transmission. \( Y_{o,l} \) is a part of \( Y_l \), where \( Y_l \) is a \( w \)-by-\( h \)-by-\( d \) tensor and \( Y_{o,l} \) is a \( \hat{w} \)-by-\( \hat{h} \) matrix. We can use \( \text{Var}[Y_{o,l}] \) and \( E[Y_{o,l}] \) to express \( \text{Var}[Y_l] \):

\[ \text{Var}[Y_{o,l}] = \frac{1}{2} n_l \text{Var}[W_{o,l}] \text{Var}[Y_{l-1}], \] (5)

\[ \text{Var}[Y_l] = \text{Var}[Y_{l-1}] \sum_o \frac{1}{N_l} \text{Var}[W_{o,l}] + \text{Var}[n_l E[W_{o,l}] E[X_l]]. \] (6)

A detailed proof is shown in Appendix. Some researches attribute the variance explosion to the change of input distribution and the amplification of the parameters \( W_{o,l} \). While Figure 1(a) shows that \( \text{Var}[W_{o,l}] \) are stable and not amplified too much, even in a collapsed network corresponding to Figure 1(a). We assume that parameters are initialized reasonably (such as \( n_l \text{Var}[W_{o,l}] = 2 \) when ReLU is the activation function) and \( n_l \text{Var}[W_{o,l}] \) are relatively stable at the beginning of the training. \( E[X_l] \) is independent of subscript \( o \) and can be regarded as a constant in \( \text{Var}[n_l E[W_{o,l}] E[X_l]] \).

Thus we can get:

\[ \text{Var}[Y_l] \approx \text{Var}[Y_{l-1}] + E^2[X_l] \text{Var}[n_l E[W_{o,l}]]. \] (7)
BN's Effect on the Shift of the Average Gradient

Generally, variance is transmitted through \( \frac{\sum_o n_o \text{Var}[W_{o,l}]}{N_{l}} \). \( \text{Var}[E[W_{o,l}]] \approx 0 \) when we initialize \( W_l \). Thus \( \text{Var}[Y_l] \) is in normal range transmitted by \( \text{Var}[Y_{l-1}] \). Everything has changed when \( \text{Var}[n_l E[W_{o,l}]] \) increases. It is worth mentioning that the shifts of \( E[W_{o,l}] \) are all coming from gradients because we initialize \( E[W_{o,l}] \) to 0. Thus the curves in Figure 1(b) imply an excessive shift on the weights when backward propagation. As Figure 1(b) has shown, the shift for filters with BN is reduced.

Detailed proof is shown in Appendix. \( \|\nabla_{Y_o} L\| \) will be restricted by the \( \sigma_o \) compared with \( \|\nabla_{Y_o} L\| \). Thus \( \sigma_o \) impacts on the gradients for \( W_o \). Moreover, \( E[\nabla_{Y_o} L] = 0 \) reduce the shift on the weights.

For every channel \( Y_o \) in the output feature map, which is computed by \( W_o \), BN will make it zero-mean. Therefore, the influence of the shift of the average gradient will be eliminated because BN's zero-mean operation will let \( \text{Var}[E[Y_{o,l}]] \) be zero, although it changes the output information. We speculate that we can get an analogous result without scaling if we keep a robust variance transmission. For Layer Normalization (LN) [Ba, Kiros, and Hinton 2016], Instance Normalization (IN) [Ulyanov, Vedaldi, and Lempitsky 2016], Group Normalization (GN) [Wu and He 2018], and Switchable Normalization (SN) [Luo et al. 2019], the zero-mean operation of those methods also reduces the shift of the average gradients.

**WN's Effect on Variance Transmission**

Instead of normalizing the outputs, WN divides the filter weights by its Frobenius norm. Formally, we can define a simplified WN of a filter \( W \) in a conv layer by \( W = \frac{g}{\|W\|} W \), where \( g \) is a scalar and \( W \) is a \( k \)-by-\( k \)-by-\( c \) tensor. \( c \) represents the number of input channels of that conv layer. \( k \) is the spatial filter size of that layer. We also use \( n_l = k^2 c \) to denote the number of units in a filter of that conv layer \( l \). For a conv operation, WN use \( W \) instead of \( W \) to compute the final output. If we set \( g \) to a small value, such as 1, instead of Data-Dependent Initialization [Salimans and Kingma 2016], networks with WN can be trained with a large LR. The reason might be the constrained \( \text{Var}[W] \). We can get \( \text{Var}[\hat{W}] \):

\[
\text{Var}[\hat{W}] = \frac{g^2 \text{Var}[W]}{\|W\|^2} = \frac{g^2}{n_l} \frac{\text{Var}[W]}{\text{Var}[W] + E^2[|W|]}.
\]

The elements in \( W \) will be initialized to a symmetric distribution with a zero mean. \( E[|W|] \approx 0 \) is established at the
beginning, leading $\text{Var}[\hat{W}]$ to $\frac{g^2}{n_l}$. Considering \( \alpha \), we know that the value of $\frac{1}{n_l}\text{Var}[\hat{W}] = \frac{g^2}{n_l}$ should be constrained. We assume that $E[W] \approx 0$ at the beginning. $E[\nabla W]L$ will be constrained when we set $g = 1$ due to:

$$E[\nabla W]L = \frac{g}{\|W\|}E[\nabla \hat{W}]L,$$  \hspace{1cm} (10)

$$\|\nabla W\|^2 = \frac{g^2}{\|W\|^2} - \frac{\langle \nabla W L, \hat{W} \rangle^2}{g^2}.$$  \hspace{1cm} (11)

The detailed proof is shown in Appendix. Filters will be initialized so that $n_l\text{Var}[W] = \|W\|^2 = 2$ as mentioned above. $\frac{g^2}{\|W\|^2}$ directly affect the gradients and the shift of the gradients. A suitable $g$ will prevent the gradient from overflow and shift. However, Data-Dependent Initialization sets $g$ to maintain an equivalent variance between the layer’s input and output. It may set $g$ to be relatively large, such as $\sqrt{2}$, which amplifies the shifts and becomes unstable.

Moreover, the gradient for filter $W$ will increase due to the weight decay, and finally numeric overflow if LR is large. \( \beta \) seems to be similar to BN’s. $\|\nabla W L\|^2$ will subtract an non-zero value and be constrained by $\|W\|^2$. The input’s variance is stable and will not descend a lot during the training. However, $\|W\|$ will be reduced due to the training steps and the weight decay. The descent of $\|W\|$ makes the gradients greater. When we use a large LR and maintain an equivalent variance, the network becomes sensitive and easy to collapse. The detailed illustration is shown in Appendix.

**Parametric Weights Standardization (PWS)**

To deal with the shift of the average gradient, we focus on how to solve the gradient shift elegantly without normalizing the outputs. Generally, normalizing the outputs requires more computation than normalizing the filters. However, many methods of normalizing filters do not take the output variance and the computation into account. Most of them should be used with BN, GN or other methods of normalizing the outputs. These methods increase the burden of the network to get a better result. To become robust to minibatch size, and become faster and stabler than those methods, we defined a PWS layer as follow:

$$\hat{W}_o = \sqrt{\frac{2}{n_l} \frac{W_o - E[W_o]}{\text{Var}[W_o] + \gamma}},$$

and $Y_o = \alpha_o \cdot X \otimes \hat{W}_o + \beta_o.$

Here $W_o$ is a $k$-by-$k$-by-$c$ tensor. $c$ represents the number of input channels of that conv layer. $k$ is the spatial filter size of that layer. Subscript $o$ denotes the index of a filter at that layer. $n_l = k^2c$ denotes the number of units in a filter of that conv layer $l$. $Y_o$ is a channel of the batch of output feature map as defined above. $X$ is a batch of input feature map. $\alpha_o$, $\beta_o$ and $\gamma$ are scalars. $\alpha_o$ and $\beta_o$ are trainable and $\gamma$ is fixed. $\text{Var}[W_o]$ and $E[W_o]$ represents the calculated variance and mean of all elements in $W_o$. It is crucial to multiply $\frac{W_o - E[W_o]}{\sqrt{\text{Var}[W_o] + \gamma}}$ by $\sqrt{2/n_l}$ to maintain a correct variance of $\hat{W}_o$. Most methods that normalize the filters do not adjust the filters’ variance and subsequently amplify the outputs. The variance will explode due to $\frac{\gamma}{\sqrt{2/n_l}}$. It is straightforward that we can eliminate the shift of the average gradient on different filters by letting $E[W_o] = 0$ due to:

$$E[\nabla W_o]L = \sqrt{\frac{2}{n_l} E[W_o] |\nabla \hat{W}_o L, \hat{W}_o |} = 0.$$  \hspace{1cm} (13)

The gradient here is similar to BN’s. However, this gradient will directly act on $W_o$. Thus the gradient for $W_o$ will not shift. Moreover, $E[W_o]$ and $E[Y_o]$ are correlated due to $E[Y_o] = n_l E[X |E[W_o]]$ at the beginning of the training, indicating that centralizing the filters is equivalent to centralizing the outputs from the perspective of variance transmission. PWS does not make a strong assumption of letting different inputs to have the same mean value of outputs, which is different from IN. The relationship between BN and PWS may be that they both solve the shift of the average gradient by making $E[Y_o]$ in different channels equal. This perspective helps us understand why BN works well and why we can use a large LR with BN. Moreover, PWS can be reset to a normal conv layer in the inference stage because PWS does not change the output variance.

**Trainable Parameter $\alpha$**

Think about BN’s case. In the inference stage, BN will apply constant $\sigma_o$ and $\mu_o$ on the output. Here $\sigma_o$ and $\mu_o$ are calculated by moving mean during the training steps, $\sigma_o$ and $\mu_o$ are fixed values when testing. If we let $P_o = W_o / \sigma_o$ and $Q_o = \beta_o - \mu_o / \sigma_o$, the complete operation will just seem like a regular convolution operation that $Y_o = X \otimes P_o + Q_o$.

However, the variance of $W_o$ is fixed if we use PWS without $\alpha$. Thus the output channels will not be attached to a weight and have the same variance. PWS will let the filters in the same layer to have a different weight with $\alpha$. Simultaneously, $\alpha$ will make PWS layer act the same as conv layer.

**Parameter $\gamma$**

The difference between normalizing outputs and normalizing filters is that the denominator $\sigma$ in BN is stabler than the denominator in those methods of normalizing filters. $\sqrt{\text{Var}[W_o]}$ will continuously decrease. Figure \[2\] shows the variance of $W_o$ during the training. The variance will be reduced due to weight decay. As the training goes on, the gradient for $W_o$ will increase. If we do not use a large LR, the network may benefit from this increase and converge faster. However, the network may collapse when we use a large LR. $\gamma$ in $\sqrt{\text{Var}[W_o] + \gamma}$ is not acting as $\epsilon$ in BN. As Figure \[2\] has shown, $\gamma$ can constrain the increase of the reciprocal. Note that with a large LR, even though we set $\gamma$ to 1e-3 ($\text{Var}[W_o]$ is less than 1.6e-3), the network may also converge. A suitable $\gamma$ adjusts the training gradient to an acceptable range when we use a large LR. We will show the results in the Experiment part.

**Experiments**

We conduct experiments in object detection and image classification tasks. To prove PWS’s usability, we use VGG.  

Object Detection in VOC07

All experiments are conducted on SSD [Liu et al. 2016] structure. The LR is set to 1e-2 and is multiplied by 0.1 after 90 and 170 epochs. For normal conv operation, it should be mentioned that the network **cannot be trained** without normalizing outputs when LR is 1e-2. The training finally stops at 240 epochs. We use stochastic gradient descent (SGD) to train the models, where the weight decay is set to 0.0005, and the momentum is set to 0.9. The image size is fixed to 320. We use the same Non-Maximum Suppression (NMS) operation and data augmentation with SSD. All models are trained from scratch in Pascal VOC 07+12 and tested in VOC07. The train set consists of 16,551 images, and the test set consists of 4,952 images. We do not use any tricks when we train our models and do not use a pre-trained model. γ is set to 1e-3 when LR is 1e-2, and 1e-5 when LR is 1e-3.

Comparison of other normalization methods  We experiment with different batch sizes to prove that PWS is robust to mini-batch size. Figure 3 shows the training condition with an LR of 1e-2 and a batch size of 16. The $f$-measure is calculated by the predicted object box and the ground truth object box. PWS has a faster convergence speed than other methods. Table 1 shows the training speed and the mAP result of different normalization methods in VOC07 test with an LR of 1e-2. These experiments are conducted on a single GPU. Training speed represents number of images that can be trained on an RTX 2080ti per second (batch size=8). IN, BN, GN, and SN are methods of normalizing the outputs. IN does not work well as others, and the possible reason is that IN eliminates the difference in different channels for different images. For every image, IN ensures the object box. PWS has a faster convergence speed than other methods.

Detailed settings for PWS refer to the configuration in our code repository. Class-wise scores and inference time will also be exhibited in our code repository.
Table 1: Sensitivity to batch size: mAP in VOC07, trained with 8, 4 and 2 images/GPU.

| Methods | Training speed (FPS) | Batch Size |
|---------|---------------------|------------|
|         |                     | 2   | 4   | 8   |
| BN      | 35.34               | 71.43| 72.69| 72.97|
| BN      | 45.66               | 72.97| 75.18| 76.74|
| SN      | 29.26               | 75.08| 75.26| 75.13|
| SN      | 28.56               | 76.29| 76.53| 76.86|
| WN      | 52.11               | NaN  | 76.70| 77.06|
| PWS     | 52.72               | 76.93| 77.11| 77.66|

Table 2: Ablation study in VOC07 test. Batch size is 16.

| Methods | LR  | Backbone  | Component | Trainable | mAP (%) |
|---------|-----|-----------|-----------|-----------|---------|
| Normal conv | 1e-3 | VGG-16    | \sqrt{2/n_l} in [12] | | 68.68   |
| BN      | 1e-3 | VGG-16    |                   |          | 72.80   |
| SN      | 1e-3 | VGG-16    |                   |          | 71.70   |
| PWS     | 1e-3 | VGG-16    | ✓           | ✓         | 72.99   |
| PWS     | 1e-3 | VGG-16    | ✓           | ✓         | 70.32   |
| BN      | 1e-2 | ResNet-50 | ✓          | ✓         | 75.83   |
| PWS     | 1e-2 | ResNet-50 | ✓          | ✓         | 77.19   |
| BN      | 1e-2 | VGG-16    |           |          | 77.00   |
| PWS     | 1e-2 | VGG-16    |           |          | 76.93   |
| BN      | 1e-2 | VGG-16    |           |          | 74.96   |
| PWS     | 1e-2 | VGG-16    |           |          | 75.22   |
| BN      | 1e-2 | VGG-16    |           |          | 74.96   |
| PWS     | 1e-2 | VGG-16    |           |          | 76.50   |
| BN      | 1e-2 | VGG-16    |           |          | 75.02   |
| PWS     | 1e-2 | VGG-16    |           |          | 75.22   |

Table 3: Detection result in COCO. †indicates the implementation by [Chen et al. 2019].

| Structure                  | Method | LR schedule | APbbox | AP50bbox | AP75bbox |
|----------------------------|--------|-------------|--------|----------|----------|
| Mask R-CNN†                | BN     | 1x          | 38.0   | 58.6     | 41.4     |
|                            | GN     | 1x          | 38.2   | 59.2     | 41.1     |
|                            | PWS    | 1x          | 38.9   | 59.6     | 42.5     |
| SSD                       | BN     | 1x          | 25.2   | 41.3     | 26.4     |
|                            | PWS    | 1x          | 25.3   | 41.2     | 26.6     |
|                            | BN     | 2x          | 26.6   | 42.7     | 28.1     |
|                            | PWS    | 2x          | 27.2   | 44.0     | 28.7     |

Object Detection in COCO

We experiment with Mask R-CNN [He et al. 2017] + FPN [Lin et al. 2017] and SSD structure for COCO dataset. The settings for SSD are the same as the experiments in VOC07 except for LR schedule. Batch size is 16 for SSD. LR is set to 1e-2. The standard LR schedule (1x) contains 80 epochs. LR is multiplied by 0.1 after 30, 50, and 70 epochs, respectively. 2x LR schedule contains 150 epochs. LR is multiplied by 0.1 after 60, 100, and 140 epochs, respectively. SSD uses VGG-16 as the backbone. The settings for Mask R-CNN are the same as in [Chen et al. 2019]. We use ResNet-50 as backbone. Mini-batch size is 4. The standard LR schedule contains 12 epochs. LR is multiplied by 0.1 after 8 and 11 epochs, respectively. For Mask R-CNN we use pretrained model to alleviate the impact of batch size on BN. The train set consists of 118k images, and the test set consists of 5k images. For PWS conv, we remove all BN layers and replace all conv layers in the backbone with PWS layer. γ is set to 1e-4. Details are available on our github.

Table 3 shows the results in COCO. For Mask R-CNN, BN gets an inferior result to GN and PWS due to the small batch size. PWS can provide a competitive result with BN and GN and be robust to mini-batch size. For SSD, the results for LR schedule 1x and LR schedule 2x imply that PWS can get similar results to BN under normal batch size. Moreover, PWS does not normalize the outputs and needs less computation, which is distinctive and promising.

Image Classification in CIFAR10

We experiment with PWS for image classification task in CIFAR10. This dataset consists of 60k images, with 50k training examples and 10k test examples. The LR for networks is multiplied by 0.1 after 100, 150, and 180 epochs, respectively. The training finally stops at 200 epochs. We use stochastic gradient descent (SGD), where the weight decay
In this experiment, be that the number of channels in a layer is not large enough. Appendix. GN does not work well. A possible reason may be that the shift of the average gradient is a problem, which causes the network to collapse. We propose a fast and robust to normalization strategy in ResNet-50. From Table 5, it seems that PWS may not work the best for classification task. However, PWS gets a similar result to GN. On the contrary, WN has degraded performance on ImageNet. PWS is the only method which normalize the filters and get a competitive result. Moreover, PWS requires less computation (Table 1) and converges faster (Figure 4 and Figure 5).

### Conclusion

PWS is similar to BN. However, it is robust to mini-batch size. The variance transmits naturally with PWS. PWS also has a faster training speed than other normalization operations. The experiment results show that the average gradient migration can affect the network training. Moreover, the results of PWS indicate that normalizing outputs is not the only way to get better results and faster convergence speed.

Nevertheless, it still has problems. We must pay more attention to variance transmission due to the decrease of $\sqrt{\text{Var}[W_o]} + \gamma$. The network can benefit from a proper $\gamma$, which is different from WN. Therefore, how to find a suitable $\gamma$ automatically remains a problem. Our suggestion is to use a large $\gamma$ when LR is large and use a small $\gamma$ when LR is suitable for normal conv. If your network is not deep, the choice of $\gamma$ will not have a huge impact. One suggestion for the choice of $\gamma$ is to set it as $0.1 \sqrt{n_l/2}$.

In this paper, we theoretically analyze the effect of BN from the perspective of variance propagation. We guess that the shift of the average gradient is a problem, which causes the network to collapse. We propose a fast and robust to mini-batch size method called PWS. It is proved that BN and PWS play the same role in the shift of the average gradient, indicating why BN is beneficial. PWS provides another way to speed up the network fitting. We refocus on variance transmission, which is critical to help us understand how networks work and how to make the network more robust.

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**Table 4:** The classification error for CIFAR-10

| Methods | Residual | Plain |
|---------|----------|-------|
| Normal  | 8.82     | 9.11  |
| BN      | 6.12     | 5.90  |
| GN      | 6.82     | 6.77  |
| SN      | 6.19     | 5.95  |
| WN      | 7.18     | 6.98  |
| PWS     | 6.50     | 6.16  |

**Table 5:** Comparison of top-1 error rates (%) of ResNet-50 in the ImageNet validation set.

| Measurement | BN | GN | LN | IN | WN | PWS |
|-------------|----|----|----|----|----|-----|
| val error (%) | 23.6 | 24.3 | 25.3 | 28.4 | 28.2 | 24.0 |

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**Figure 4:** Top-1 error for CIFAR-10 (LR:5e-2).

**Figure 5:** Top-5 error for ImageNet val set.
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Appendix

Proof of (6)

Proof. For (6), we give a detailed proof. We know $E[W_{o,l}] = 0$ when we initialize it. Thus we can get:

$$\text{Var}[Y_l] = E[Y_l^2] - E^2[Y_l]$$
$$= \sum_o \frac{E[Y_o^2]}{N_l} - E^2[Y_l]$$
$$= \sum_o \frac{\text{Var}[Y_o]}{N_l} + E[E^2[Y_o]] - E[E[Y_o]]^2$$
$$= \sum_o \frac{\text{Var}[Y_o]}{N_l} + \text{Var}[E[Y_o]]$$
$$= \sum_o \frac{\text{Var}[Y_o]}{N_l} + E^2[X_o] \text{Var}[n_t E[W_{o,l}]]$$
$$= \text{Var}[Y_{l-1}] \sum_o \frac{1}{2} n_t \text{Var}[W_{o,l}] + E^2[X_o] \text{Var}[n_t E[W_{o,l}]].$$

(14)

From Figure [a], we know that the variance of weights are stable. Assume that we set $n_t \text{Var}[W_{o,l}]$ as 2 (HE initializer). We may be able to get an approximation:

$$\sum_o \frac{1}{2} n_t \text{Var}[W_{o,l}] \approx \sum_o 1 = N_l.$$

Then we can get a transition:

$$\text{Var}[Y_l] \approx \text{Var}[Y_{l-1}] + E^2[X_o] \text{Var}[n_t E[W_{o,l}]]$$
$$= \text{Var}[Y_{l-1}] (1 + \frac{E^2[X_o]}{\text{Var}[Y_{l-1}]} \text{Var}[n_t E[W_{o,l}]]).$$

(15)

This completes the proof. □

The range of $E^2[X_o]$ and $E^2[X_o]/E[X_o^2]$ for every layer in the task of object detection

We know the variance will increase if the mean value for different channels is distinct. This increase does not come from the increase of the $\text{Var}[W_{i}]$. We speculate about the reasons for why the network collapses. We find that $\text{Var}[n_t E[\Delta W_{o,l}]]$ will be a large value at the beginning of the training especially for those networks without BN. To make that reason persuasive, we also measure the $E[X_o]$ to prove that $\text{Var}[n_t E[W_{o,l}]]$ may have a large impact on variance transmission.

As Figure 6 shows, the relationship between $E^2[X_o]$ and $E[X_o^2]$ (Figure 6(b)) will not have a large distinction for different networks (comparing standard and BN). BN divides the output variance by $\sigma$. Normally, $\sigma$ is larger than 1. Thus the inputs of the next layer may be smaller when we use BN (Figure 6(a)). $E^2[X_o]/E[X_o^2]$ may determine the influence of $\text{Var}[n_t E[W_{o,l}]]$. Assume that $E^2[X_o]/E[X_o^2]$ is stable at the beginning of the training. It is explicit that the scale of the shifts will have a large impact on the variance transmission.

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The result of all training data. The inputs $X_l$ at layer $l$, we use $E(x)$ and $E(x^2)$ denotes the expectation of all elements and the expectation of the quadratic of all elements in $x$, respectively. $E[X_l]$ denotes the range of $E(x)$ and $E[X^2_l]$ denotes the range of $E(x^2)$. We measure these values without updating steps, because the network without BN will directly collapse when we set $l$ as 1-2. For every batch, we calculate the forward process and the backward process. Then we calculate what happened when we update the filters’ gradients. We do not update the parameters in networks. Therefore, we can collect the result of all training data. The inputs $X_l$ of layers 3, 5, 8, and 11 came from max-pool layer.

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