Current noise, electron-electron interactions, and quantum interference

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It is shown that electron-electron interactions lead to a novel quantum-interference contribution to non-equilibrium current noise in mesoscopic conductors. The corresponding noise spectrum is obtained in detail for diffusive systems and found to be quadratic in the applied voltage and to exhibit a power-law dependence on the frequency $S_{\text{dc}}(\omega) \sim |\omega|^{-\alpha}$. The exponent $\alpha = (4 - d)/2$ depends on the dimensionality $d$ of the sample.

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Various sources of current noise in mesoscopic conductors have been investigated in recent years \[5,6\]. In thermal equilibrium, the noise power is related to the conductance via the fluctuation-dissipation theorem. However, this general relation does not hold at finite bias voltages in which case the noise power contains information beyond the conductance. For example, the quantization of the electronic charge manifests itself in shot noise at finite voltages. In mesoscopic conductors, shot noise is suppressed relative to its classical value due to Fermi statistics \[6\]. Quantum coherence leads to a weak-localization correction and mesoscopic fluctuations of the shot-noise power \[6\]. Sensitive measurements of noise in localizations \[7\] or the conductance via the fluctuation-dissipation theorem. In thermal equilibrium, the noise power is related to the sample-specific conductivity $\sigma$. Here we show that the fluctuating electric field \[8\], the coupling to non-electronic degrees of freedom such as fluctuating lattice impurities is believed to be the origin of $1/f$ noise \[9\].

It is the purpose of the present paper to show that electron-electron interactions lead to a novel quantum-coherence contribution to non-equilibrium noise in mesoscopic systems. This source of current noise is intrinsic to the electron system and distinct from equilibrium and shot noise. Its origin is the electromagnetic field experienced by an electron moving in a mesoscopic sample. This field, which mediates the interaction between the electrons, fluctuates because of the thermal fluctuations in the charge and current densities in the system. In a seminal paper, Altshuler, Aronov, and Khmelnitsky (AAK) \[8\] have studied how these field fluctuations suppress the weak-localization correction to the impurity-averaged conductivity. Here we show that the fluctuating field makes the sample-specific conductivity time dependent and thus produces current noise.

For a qualitative picture of the effect, we consider the current response to a uniform $dc$ electric field $E_{\text{dc}}$,

$$j(r,t) = \int_{-\infty}^{t} dt' \int dr' \sigma(r,t; r',t') E_{\text{dc}}. \quad (1)$$

The sample-specific conductivity $\sigma$ depends explicitly on two space-time points because the fluctuating field breaks both spatial and temporal translation invariance. In the semiclassical approximation the conductivity can be written as a double sum over classical paths $\alpha$ and $\beta$ from $(r',t')$ to $(r,t)$, namely

$$\sigma(r,t; r',t') \sim \sum_{\alpha,\beta} A_{\alpha}^* A_{\beta} \exp[i(S_{\alpha} - S_{\beta})/\hbar]. \quad (2)$$

Here $A_{\alpha}$ denotes a classical probability amplitude of path $\alpha$ and the phase factor is determined by the associated classical action $S_{\alpha}$. Quantum corrections to the conductivity arise from the interference of distinct paths $\alpha \neq \beta$ for which the phases do not cancel out. In the absence of the fluctuating field, the phases accumulated along the two diffusion paths due to the static disorder potential give rise to a random static interference pattern. In this way, sample-to-sample variations in the electrostatic disorder potential lead to the universal conductance fluctuations \[6\]. The fluctuations of the electromagnetic field lead, in addition, to a time-dependent potential landscape of a particular sample, thus making also the conductivity time dependent. Specifically, the fluctuating field leaves the classical paths essentially unchanged but adds time-dependent contributions to the phases. When measuring the $dc$ conductance, the resulting time-dependent interference pattern is effectively averaged out and the $dc$ conductance fluctuations are suppressed by the fluctuating field. In this sense, the fluctuating field leads to dephasing. Nevertheless, when measuring current noise, the interference is still effective because the time-dependent interference pattern results in temporal fluctuations of the conductivity.

The noise amplitude is characterized by the power spectrum

$$S(\omega) = \frac{1}{2} \int d\zeta e^{i\omega \zeta} \langle \{\delta I(t), \delta I(t + \zeta)\}\rangle_T, \quad (3)$$

where $\langle \ldots \rangle_T$ is the thermal expectation value in the presence of the bias voltage, $\delta I = I - I_{\text{dc}}$, and the curly brackets denote the anticommutator. According to the qualitative considerations given above, the largest possible equal-time current fluctuations $\langle \delta I^2(t) \rangle = \int d\omega S_{\text{ee}}(\omega)$ due to the electromagnetic field correspond to a situation
when the entire interference pattern becomes time dependent. In this situation, the current fluctuations would be of order $(\delta G^2)V^2$, in terms of the universal conductance fluctuations $(\delta G^2)$ and the applied voltage $V$. We find for one-dimensional systems that the current fluctuations do indeed reach this upper bound when the phase-coherence time $\tau_0$ is comparable to the diffusion time $\tau_D = L^2/D$ through the sample ($L$ is the sample size and $D$ the diffusion constant). We also show that the same is no longer true in higher dimensions.

The frequency spectrum of the noise is sensitive to the specific nature of the fluctuating field. Under realistic conditions, the field is essentially electric and may be characterized in terms of the correlator of the scalar potential $\varphi$. Using the fluctuation-dissipation theorem, one obtains for frequencies $\hbar \omega \ll T$:

$$\varphi(k, \omega)\varphi(-k, -\omega) = \frac{2T}{\sigma k^2}. \tag{4}$$

Phase-sensitive quantities are predominantly affected by the classical modes of the fluctuating field with $\hbar \omega \ll T$. This is a consequence of the infrared divergence in $\varphi$. In particular, AAK showed that the dephasing rate $\tau_0^{-1}$ is determined by these modes. By contrast, the energy-relaxation rate $\tau_{ee}^{-1}$ is determined by modes with $\hbar \omega \sim T$.

In one and two dimensions, AAK found that $\tau_0 \ll \tau_{ee}$ while in three dimensions the two times are of the same order. Eq. (4) shows that the temporal correlations of $\varphi(r, t)$ are short ranged with characteristic scale $\hbar/\tau_0$. On the other hand, the infrared divergence of the correlator $\varphi$ emphasizes long-range spatial correlations. This generates a second frequency scale much smaller than the temperature $T$: the inverse time of flight $\tau_D^{-1}$ along a typical diffusion path contributing to the time-dependent interference pattern. We find that the characteristic frequencies $\tau_D^{-1}$ and $T/\hbar$ enter as cutoffs on a power-law frequency spectrum $S_{ee}(\omega) \sim |\omega|^{-(4-d)/2}$.

In this paper we follow AAK and include electron-electron interactions by averaging over the fluctuating field within the impurity-diagram technique. It is also instructive to interpret the noise contribution discussed here in terms of many-body diagrams using an exact-eigenstate basis of the impurity potential. In the latter case, a typical diagram contributing to the noise power is shown in Fig. 1a. Averaging over the fluctuating field corresponds to an RPA-like treatment of the electron-electron interactions. This diagrammatic approach also clarifies the distinction between shot noise and the noise discussed in this paper. In the limit $eV < T$, shot noise is represented by diagrams in which the four external legs emanate from a single electron loop.

We now proceed to sketch the calculation of the interaction-induced noise power for diffusive conductors. Including electron-electron interactions by a (Gaussian) average over the fluctuating field, the general expression for the disorder-averaged noise power $\overline{S_{ee}}$ reduces to

$$S_{ee}(\omega) = \int d\zeta e^{i\omega \zeta} \overline{\left< I(t + \zeta) I(t) - I(t + \zeta) I(t) \right>}. \tag{5}$$

The overbar denotes the average over the fluctuating field and angular brackets the disorder average. We compute $S_{ee}(\omega)$ by the impurity-diagram technique, including the applied $dc$ voltage to quadratic order. In accordance with the qualitative discussion, the relevant diagram shown in Fig. 1b is that responsible for conductance fluctuations $\overline{I}$. This diagram represents the current correlations for a particular realization of the fluctuating field, which eventually needs to be averaged over, as specified in Eq. (5). While there are additional diagrams contributing to the conductance fluctuations, it turns out that only the one in Fig. 1b contributes to the noise power.

We employ the Keldysh technique to evaluate this diagram in space-time representation in the presence of a fluctuating electric field in the region $LT \ll L$. This is the most interesting region because the effect is largest for $\tau_D \sim \tau_0$ and the thermal length $LT = \sqrt{\hbar D/T}$ of metals is usually much smaller than the phase-coherence length $L_\phi$.

We find

$$\overline{\langle j(t + \zeta/2)j(t - \zeta/2)\rangle} = \frac{4\pi}{3} \left( \frac{e^2}{\hbar} \right)^2 \frac{\hbar/\tau_D}{T} \frac{D}{L^{2d-2}} \times \int dr dr' \int_{-\infty}^{t} dt' P_{t,t'}^\phi(r, r') P_{t,t'}^{\overline{\phi}}(r, r') E_{dc}^2 \tag{6}$$

in terms of the diffuson propagator $P_{t,t'}^\phi(r, r')$ in the presence of the electric potential. In the following, we will restrict attention to the region $LT < L < L_\phi$ where we can neglect the effects of processes with large energy transfers $\hbar \omega \sim T$.

Diagrammatically, the diffuson $P_{t,t'}^\phi(r, r')$ is represented by Fig. 2, and can be shown to satisfy the differential equation

$$\left\{ \partial_t + \frac{i e}{\hbar} \varphi(\mathbf{r}, t) + D \nabla^2 \right\} P_{t,t'}^\phi(\mathbf{r}, \mathbf{r'}) = \delta(t - t') \delta(\mathbf{r} - \mathbf{r'}). \tag{7}$$
The line last holds because the phases due to the fluctuating field are small for \( L_\phi > L \). The correlator \( \phi_+ \phi_- \) can be computed using the potential fluctuations \( \phi \). Due to the infrared divergence in \( \phi \), it decays as a function of the time difference \( \zeta \) on the scale of the diffusion time \( \tau_D \). We note that for large systems with \( L > L_\phi \) one may also obtain the suppression of the dc conductance fluctuations by the fluctuating field from the factorized average in \( \ref{eq:5} \) \[ \ref{eq:5} \]. In this way, one naturally recovers the expressions for the phase-coherence time \( \tau_\phi \) derived by AAK. In one dimension one finds \( \tau_\phi^{-1} = (4e^2\sqrt{2hD}/3\sqrt{\pi\hbar^2}c)^{2/3} \), in two dimensions \( \tau_\phi^{-1} = (e^2T/2\pi\sigma h^2)^{1/2} \log(T\tau_\phi/h) \), and in three dimensions \( \tau_\phi^{-1} = (2e^3/\pi^2 \hbar^3/\sqrt{6}D)^{1/2} \).

With these ingredients, we obtain for the noise power in the regime \( L_T < L < L_\phi \)

\[
S_{ee}(\omega) = \begin{cases} 
    (\frac{e^2}{h})^2 \left( \frac{L_T}{L} \right)^2 \left( \frac{\tau_D}{|\omega|^{1/2}} \right)^2 & \text{if } d = 1 \\
    (\frac{e^2}{h})^2 \left( \frac{L_T}{L} \right)^2 \left( \frac{\tau_D}{|\omega|^{1/2}} \right)^2 & \text{if } d = 2 \\
    (\frac{e^2}{h})^2 \left( \frac{L_T}{L} \right)^2 \left( \frac{\tau_D}{|\omega|^{1/2}} \right)^2 & \text{if } d = 3 
\end{cases}
\]

(12)

The numerical prefactors are \( c_1 \approx 0.01, c_2 \approx 0.03, \) and \( c_3 \approx 0.06 \). We also remark that in two dimensions \( S_{ee}(\omega) \) changes only by a factor \((L_\phi/L)^2\) in the regime \( L_\phi < L < L_{ee} \). These results are valid for voltages \( eV < h/\tau_D \) \[ \ref{eq:13} \]. The power-law frequency spectra extend over the interval \( 1/\tau_D < |\omega| < T/h \). Outside of this interval, \( S_{ee}(\omega) \) saturates for \( h|\omega| \ll T \) and vanishes for \( h|\omega| \gg T \). A sample has a reduced dimensionality if its transverse dimensions are smaller than \( L_T \). The dependence of the noise power on dimensionality \( d \) arises from two sources. The dimensional dependence of diffusion predominantly affects the prefactor, analogous to the universal conductance fluctuations \[ \ref{eq:5} \]. The characteristic dependence of the noise spectrum on the dimensionality \( d \) originates from the \( d \) dependence of the degree of divergence of the field correlator \[ \ref{eq:5} \]. Effectively, long-range correlations in space are emphasized less in higher dimensions. This leads to a faster decrease of \( \phi_+ \phi_- \) with \( \zeta \) and consequently to a weaker emphasis on small frequencies.

An important feature of the results in the coherent regime is that the noise power \( S_{ee}(\omega) \) does not depend explicitly on the temperature. Instead, the temperature enters only into the frequency scales and thereby determines the frequency range where these power laws can be observed. This insensitivity of the spectrum to the temperature is a result of the competition between the linear increase with temperature of the field fluctuations and the \( 1/T \) prefactor in \[ \ref{eq:13} \]. Of course, for fixed sample size \( L \), the range over which \( S_{ee}(\omega) \) is temperature independent is limited by the condition \( L_T < L < L_\phi \).

It is instructive to compare these results to the conduc-
tance fluctuations \[ \langle \delta G^2 \rangle \sim (e^2/h)^2(L_T/L)^2 f_d(L/L_T) \]
with \( f_1(x) = 1 \), \( f_2(x) = \ln x \), and \( f_3(x) = x \). In the introduction we argued that \( \langle \delta G^2 \rangle V^2 \) is an upper bound for the current fluctuations. In one dimension we find from Eq. (2) that \( \langle \delta I^2(t) \rangle \sim (e^2/h)^2(L_T/L)^3(T_\phi/\tau_D)^3/2 V^2 \). This shows that the current fluctuations do indeed reach the upper limit for \( \tau_D \sim T_\phi \). For \( \tau_D < T_\phi \), the reduction factor \( (\tau_D/T_\phi)^{3/2} \) is just the magnitude of the time-dependent phase accumulated along a typical path of length \( \tau_D \). By contrast, the current fluctuations do not reach the upper bound in higher dimensions. In two dimensions, one finds \( \langle \delta I^2(t) \rangle \sim (e^2/h)^2(L_T/L_\phi)^2(\ln(T_\phi/\tau_D)/\ln(T_\phi/\tau_D))V^2 \). Even for \( T_\phi = \tau_D \) the current fluctuations are smaller than the upper bound by a logarithmic factor. This is due to the fact that the weight of short paths in the noise power is reduced relative to their weight in the conductance fluctuations because the time-dependent phase accumulated by trajectories much shorter than \( T_\phi \) is small compared to one. The reduction is significant in two dimensions because of the logarithmic contribution of short trajectories of length \( \ln(T_\phi/\tau_D) \). Therefore, the current fluctuations \( \langle \delta I^2(t) \rangle \sim (e^2/h)^2(L_T/L_\phi)^2 V^2 \) are strongly reduced - by a factor \( (\ln(T_\phi/\tau_D))^{1/2} \) for \( \tau_D = T_\phi \) - relative to the upper bound.

We conclude by comparing the interaction-induced noise to other sources of noise. The origin of the 1/f-like frequency spectra found here is very different from the mechanism for standard 1/f noise. In the usual argument for 1/f noise, both in the absence and the presence of quantum coherence, the fluctuations of the impurity potential are supposed to have short-range spatial correlations and to vary slowly in time compared to electronic time scales. The 1/f noise spectrum arises because of the activated nature of impurity motion and the broad distribution of activation energies. By contrast, for the interaction-induced noise the field fluctuations are faster than the relevant electronic time scales but have long-range spatial correlations. The 1/f-like frequency spectra reflect the power-law divergence of the field correlator. Due to this difference, the lower cutoff frequency is vastly larger for the interaction-induced noise than for 1/f noise. Moreover, 1/f noise in the presence of quantum coherence is sensitive to weak magnetic fields since the conductance fluctuations decrease by a factor of four when applying a magnetic field. An analogous effect does not occur for the interaction-induced noise power.

The interaction-induced noise can be distinguished experimentally from equilibrium noise by its voltage dependence. Shot noise in the regime \( eV < T \), on the other hand, has the same voltage dependence as the interaction-induced noise, but its frequency spectrum, which is frequency independent for \( \hbar \omega < T \), is different. The shot-noise magnitude is reduced compared to its zero-temperature value by a factor \( eV/T \), \( S_{\text{shot}} \sim (e^2/h)g[(eV)^2/T] \) with \( g \) the dimensionless conductance. As discussed by de Jong and Beenakker, quantum-coherence contributions to shot noise are suppressed by \( 1/g \). By comparison, the interaction-induced noise is largest in one dimension for \( \tau_D \sim T_\phi \) and frequency \( \omega \sim 1/\tau_D \). From Eq. (12) one has for the corresponding noise power \( S_{\text{ne}}(\omega \sim 1/\tau_D) \sim (e^2/h)[(eV)^2/T] \). This is of the same order as the quantum-coherence corrections to shot noise. These estimates show that the interaction-induced noise is particularly important in almost localized systems with small dimensionless conductance \( g \sim 1 \).

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[1] G.B. Lesovik, JETP Lett. 49, 592 (1989); C.W.J. Beenakker and M. Büttiker, Phys. Rev. B 46, 1889 (1992).
[2] M.J.M. de Jong and C.W.J. Beenakker, Phys. Rev. B 46, 13400 (1992).
[3] M. Reznikov, M. Heiblum, H. Shtrikman, and D. Mahalu, Phys. Rev. Lett. 75, 3340 (1995); A. Kumar, L. Saninadayar, D.C. Glattli, Y. Yin, and B. Etienne, Phys. Rev. Lett. 76, 2778 (1996).
[4] S. Feng, P.A. Lee, and A.D. Stone, Phys. Rev. Lett. 56, 1960 (1986); A.A. Bobkov, V.I. Falko, and D.E. Khmel’nitskii, Zh. Eksp. Teor. Fiz. 98, 703 (1990) [JETP 71, 393 (1990)]; see also S. Feng, in Mesoscopic Phenomena in Solids, eds. B.L. Altshuler, P.A. Lee, and R.A. Webb (Elsevier, Amsterdam, 1991).
[5] R. Landauer, Phys. Rev. B 47, 16427 (1993).
[6] B.L. Altshuler, A.G. Aronov, and D.E. Khmel’ntskii, J. Phys. C 15, 7367 (1982); see also A. Stern, Y. Aharonov, and Y. Imry, Phys. Rev. B 41, 3436 (1990).
[7] B.L. Altshuler, JETP Lett. 41, 648 (1985); P.A. Lee, A.D. Stone, and H. Fukuyama, Phys. Rev. B 35, 1039 (1987).
[8] A. Schmid, Z. Phys. 271, 251 (1974); B.L. Altshuler and A. G. Aronov, Sol. State Comm. 38, 11 (1981).
[9] Details of the calculation will be published elsewhere, F. von Oppen and A. Stern (unpublished).
[10] For a review, see J. Rammer and H. Smith, Rev. Mod. Phys. 58, 323 (1986).
[11] A.I. Larkin and D.E. Khmel’ntskii, Sov. Phys. JETP 64, 1075 (1987).