The electron–hole superfluidity in two coaxial nanotubes

Oleg L Berman, Ilya Grigorenko and Roman Ya Kezerashvili

Physics Department, New York City College of Technology, The City University of New York, Brooklyn, NY 11201, USA

E-mail: rkezerashvili@citytech.cuny.edu

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Abstract

The superfluid phase and Coulomb drag effect caused by the pairing in a system of spatially separated electrons and holes in two coaxial cylindrical nanotubes are predicted. It is found that the drag resistance as a function of temperature experiences a jump at the critical temperature and can be used for the manifestation of the superfluid transition. It is demonstrated that at sufficiently low temperatures the order parameter and free energy density exhibit a kink due to the electron–hole asymmetry that is controlled by the radii of the nanotubes.

Keywords: superfluidity, Coulomb drag, coaxial nanotubes, electron–hole system, quantum size effect

1. Introduction

The superfluid state for a dense system of spatially separated electrons and holes in two parallel layers was predicted in [1], and physical properties of this state have been studied [2–4]. The peculiarity of such systems is that the pairing between the carriers occurs not via the usual weak electron–phonon interaction mechanism as in conventional superconductors, but through the much stronger electron–hole Coulomb attraction. When the strength of the mutual attraction in dense systems is sufficiently weak, a standard Bardeen–Cooper–Schrieffer (BCS) description [5] is applicable. Another limiting case is characterized by a sufficiently strong interaction in dilute systems, when fermions form bounded pairs, which can be described as composite bosons (indirect excitons), which undergo Bose–Einstein condensation (BEC). In both limits the Coulomb attraction between the electrons and holes can introduce Coulomb drag, that is a process in spatially separated conductors which enables a current flowing in one of the conductors to induce a voltage drop in the other one. In the case when the second conductor is a part of a closed microcavity, the exciton current can be used to detect superfluidity [10]. Theoretical predictions of the asymmetry of the drag processes in the system have been analyzed in [9]. If the external potential difference and temperature are applied to one of the layers, it will cause an electric current in the other layer. The current in the layer will be initiated due to the correlations between electrons and holes at temperatures below the critical one. Let us also mention that the theory of the drag effects in the system of spatially separated electrons and excitons in an optical microcavity developed in [9, 10] predicts that at low temperature an electron current induces the polariton flow, while the electron current dragged by the polariton flow is strongly suppressed below the polariton superfluid transition temperature. This demonstrates that the asymmetry of the drag processes in the system. In all above mentioned studies possible asymmetry between electron and hole contributions to the drag has been discussed.
hole excitation spectra is caused by the difference between the effective masses of the carriers. In this work we present the study of the asymmetry in the excitation spectra of electrons and holes and its effect on the Coulomb drag process due to the different radii of two coaxial nanotubes.

Let us consider a system of two coaxial cylindrical nanotubes separated by a dielectric with the spatially separated electrons and holes confined on each nanotube, as shown in figure 1. We study the formation of the superfluid phase resulting in the electron–hole Coulomb drag effect in this system. The electrons on the outer nanotube and holes on the inner one have different excitation spectra due to the different radii of the nanotubes. As a result, the conductivities for the inner and outer nanotubes can substantially differ from each other and also strongly depend on the radii of the nanotubes. By measuring the drag conductivity as a function of the temperature one can observe the superfluid transition in the system, and by measuring the jump in the drag coefficients one can obtain the critical temperature of the BCS phase transition causing the superfluidity. The paper is organized in the following way. In section 2 we discuss the particularities of the physical properties of the system originated from the cylindrical geometry of two coaxial nanotubes. The BCS state for the spatially separated electrons and holes in two coaxial nanotubes is described in section 3. In section 4 the calculation of the transconductivity coefficients is presented. Finally, the discussion of the results and the conclusions follow in section 5.

2. Two coaxial nanotubes

We consider a system of two coaxial cylindrical nanotubes and assume that the inner nanotube is doped by holes, while carriers on the outer nanotube are electrons. The geometry of two coaxial nanotubes leads to significant differences compared to a system of two parallel plane layers [1, 3, 4], and promises much richer physics compared to the plane geometry. Unlike the case of two plane layers, the conductivities for the inner and outer nanotubes can differ because of their different curvatures. The quantum confinement in the coaxial nanotube system may result in enhancement of the order parameter for smaller radii of the nanotubes. Similar confinement effects were studied in thin superconducting layers [11] and in metallic cylindrical superconducting nanowires [12].

For two coaxial cylindrical nanotubes there are two interesting cases: (i) the two nanotubes have equal numbers of electrons and holes; (ii) the two nanotubes have equal densities of electrons and holes. If the numbers of electrons and holes are equal, as shown in figure 1(a), the carrier concentrations and the chemical potentials for the electrons and holes on the outer and the inner nanotubes will be different. One would expect to observe a lower chemical potential in the outer nanotube. Even though at low temperatures all electrons and holes can be paired, it is expected that the mismatch between the chemical potentials \( \mu_e \neq \mu_h \) may significantly reduce the critical temperature of the superconducting transition. For sufficiently large mismatch between the chemical potentials, comparable to the order parameter, the BCS transition can be impossible. One has to note that even in the case of perfectly equal carrier concentrations \( \rho_e = \rho_h \) the chemical potentials may be slightly different due to the effect of the different curvatures of the nanotubes. This difference is expected to be noticeable for relatively small, up to several nanometers, but different radii of the nanotubes.

In the case of equal densities of the carriers on each nanotube, the total number of carriers will be higher on the outer nanotube than on the inner one, as shown in figure 1(b). At low temperatures this will result in a considerable number of carriers on the outer nanotube (electrons) which cannot find their pairs on the inner nanotube (holes). In this case, the conductivity in the system will be defined by the paired and unpaired components of the system. One has to note that we consider only the case of relatively weak imbalance between the carriers. A larger difference between the numbers of the electrons and holes may lead to a phase separation between paired and unpaired components [13].

Below we focus on the case of equal densities of the carriers on each nanotube when the system comprises three components: the ground state of electron–hole pairs (the superfluid component), the quasiparticle excitations above the ground state (the normal component) and the unpaired normal electron component on the outer nanotube. The contribution to the conductivity due to the unpaired component on the outer nanotube may be significant even at zero temperature. Let us consider a finite number of unpaired electrons on the outer nanotube with the conductivity \( \sigma_{ee}^{un} \). We assume that in the first approximation the unpaired electron component moves with the same average velocity as the normal component [14]. This will result in a greater current on the outer nanotube than on the inner one. The difference will correspond to the flow of the unpaired component. In this case, the effective electron conductivity coefficient (the conductivity of the outer nanotube) can be estimated as \( \sigma_{ee}^{eff} = \sigma_{ee} + \sigma_{ee}^{un} \). The conductivity for the unpaired component can be calculated as \( \sigma_{ee}^{un} = \frac{e^2}{m^* \tau} \), where \( m^* \) is the mass of an electron, and \( \tau \) is the characteristic scattering time. The density of the unpaired
electrons is simply $n_{\text{un}} = \rho \frac{2\text{max}-R_{\text{min}}}{2\text{max}}$. In the last expression $\rho$ is the concentration of the carriers on both nanotubes and $R_{\text{max}}, R_{\text{min}}$ are the radii of the outer and inner nanotube, respectively.

Note that if the carrier density is not very high the unpaired electrons may strongly interact with Cooper pairs. In two coaxial nanotubes in the dilute limit the electrons and holes form electron–hole bound states—indirect excitons [15]. Moreover, in the dilute limit, if the numbers of electrons and holes are different, in the quantum wells the carriers can even form bound states of trions [16–19]. In the dilute limit the trions can be formed in the system of two coaxial nanotubes as well. In this case, if the outer nanotube is doped by electrons we have to deal with negative trions $X^-$, while if it is doped by holes the positively charged trions $X^+$ are formed.

In the case of unequal electron and hole densities resulting in unbalanced chemical potentials, the system behavior becomes more complicated. The BCS superfluid phase is energetically unfavorable if the difference in the chemical potentials exceeds $\Delta_0/\sqrt{2}$, where $\Delta_0$ is the BCS gap function at $T = 0$ K and $\mu_e = \mu_h$. The system is expected to form inhomogeneous superfluid phases, where the Cooper pairs have nonzero total momentum. This is the case of the Larkin–Ovchinnikov–Fulde–Ferrell phase, discussed, for example, in [20–22]. In the present study we assume equal chemical potentials for the electrons and holes that also leads to close electron and hole densities.

3. The electron–hole BCS state in two coaxial nanotubes

Let us consider a system of two thin cylindrical nanotubes of radii $R_{\text{min}}$ and $R_{\text{max}}$, and assume that the nanotubes are doped in such a way that the electrons are the carriers on the outer nanotube, and holes are the carriers on the inner one. The effective Hamiltonian of this system can be written as [1]

$$H_{\text{eff}} = \sum_p \left( \xi_p - \mu_e \right) \alpha_p^\dagger \alpha_p + \sum_p' \left( \xi_{p'} - \mu_h \right) \alpha_p'^\dagger \alpha_{p'} + \sum_{p,p'} \left[ \Delta_{p,p'} \alpha_p^\dagger \alpha_{-p'} + \text{H.c.} \right],$$

(1)

where $\alpha_{p}$ is the operator of annihilation of a hole on the inner nanotube, and $b_p$ is the operator of annihilation of an electron on the outer nanotube. The single-particle eigenenergies of the electrons on the outer nanotube are given by

$$\xi_p = \frac{|p|^2}{2m_e} - \frac{\hbar^2}{2m_e} \left[ \frac{N^2}{d^2} + \frac{m^2}{R_{\text{max}}^2} + k_z^2 \right],$$

(2)

that correspond to the linear motion of an electron with the momentum $p_e = \hbar k_e$ and the angular motion with the angular momentum $L_e = R_{\text{max}} \rho_p = R_{\text{max}} \times \hbar \frac{m}{2\text{max}} = \hbar m, \ m = 0, \pm 1, \pm 2, \ldots$ Similarly, the single-particle eigenenergies of the holes confined on the inner nanotube are presented as

$$\xi_{p'} = \frac{|p'|^2}{2m_h} - \frac{\hbar^2}{2m_h} \left[ \frac{N^2}{d^2} + \frac{m^2}{R_{\text{min}}^2} + k_z^2 \right].$$

(3)

The angular momentum of a hole is $L_h = R_{\text{min}} \rho_p = R_{\text{min}} \times \hbar \frac{m}{2\text{min}} = \hbar m, \ m = 0, \pm 1, \pm 2, \ldots$ In equations (2) and (3) $m_e$ and $m_h$ are the effective masses of electrons and holes, and the expression $\frac{\hbar^2}{2m_{\text{eff}} d^2}$ in the energies corresponds to the radial confinement of the particles. We assume identical thickness $d$ for both nanotubes, and that $d$ is sufficiently smaller than the radii of the nanotubes, $d \ll R_{\text{min}}, R_{\text{max}}$. Within this assumption, below we use only the lowest energy state for the radial component of the eigenenergy $N = 1$. Note that asymmetry between the electron and hole excitation spectra exists in the system due to the different radii of the nanotubes, even for equal masses of the electrons and holes, $m_e = m_h$. The nonzero order parameter $\Delta_{p,p'}$ shows that the system is in the superfluid phase. One has to keep in mind that it is assumed that the electron–hole pairing occurs under the condition of the zero angular momentum of the pair. It results in canceling the linear momentum components for the electron and hole $hk_e +hk'_e = 0$, and also canceling their angular momenta: $L_e + L_h = 0$. The latter condition leads immediately to $m + m' = 0$.

Let us diagonalize equation (1) using Bogoliubov unitary transformations $\alpha_p = u_p \alpha^\dagger_{\downarrow, p} + v_p \beta^\dagger_{\uparrow, p}$ and $b_p = u_p \beta^\dagger_p - v_p \alpha_p$, with the amplitudes $u_p$ and $v_p$. Following [3], we assume that the order parameter is independent of the momentum: $\Delta_{p,p'} \equiv \Delta$. Thus, the self-consistency condition for the order parameter has the form [1]

$$\Delta = \frac{U}{S_{\text{eff}}} \sum_p u_p v_p \left( 1 - f(E_p^+) - f(E_p^-) \right),$$

(4)

where $U$ is the effective attractive interaction between electrons and holes, and $S_{\text{eff}} = \pi L \sqrt{R_{\text{min}} R_{\text{max}}}$. Here we use the notation $E_p^\pm = E_p \pm \eta_p, E_p = \sqrt{\epsilon_p^2 + \Delta^2}, \epsilon_p = (\xi_p + \xi_{p'}) - \mu_e - \mu_h)/2, \eta_p = (\xi_p - \xi_{p'} + \mu_e + \mu_h)/2$. In equation (4) the Fermi–Dirac distribution function is given by $f(x) = \exp(x/k_BT) + 1)^{-1}$, where $k_B$ is the Boltzmann constant, and $T$ is the temperature. The amplitudes $u_p, v_p$ are given by

$$u_p^2 = \frac{1}{2} \left( 1 + \frac{\epsilon_p}{E_p} \right),$$

$$v_p^2 = \frac{1}{2} \left( 1 - \frac{\epsilon_p}{E_p} \right).$$

(5)

Note that a dielectric with a high dielectric constant between the nanotubes can significantly reduce the effective interaction $U$, that will dramatically reduce the BCS transition temperature $T_c$. A similar effect is obtained in the case of the increase of the separation distance $D = R_{\text{max}} - R_{\text{min}}$ between the nanotubes. For two axial nanotubes with a relatively small separation distance $D \ll R_{\text{min}}, R_{\text{max}}$ one can use the result for the interaction potential obtained for the electron–hole pairing in two parallel plane layers, given by equation (7) in [1]. Assuming constant attraction of the paired particles, one can use the simplified expression $U = \frac{\epsilon^2_{\text{eff}}}{x_{\text{eff}} k_B^2} \exp(-Dk_F)$, where $k_F$ is the Fermi wavevector, and $\epsilon$ is the characteristic dielectric constant of the dielectric between the coaxial nanotubes. The cut-off in the sum given by equation (4) is set by the
characteristic plasma frequency \([1]\). The chemical potentials for electrons \(\mu_e\) and holes \(\mu_h\) are coupled self-consistently to the given surface densities of the carriers \(\rho_{e,h} = N_{e,h} / S_{e,h} \), \(N_e\) and \(N_h\) are the numbers of electrons and holes, respectively, while the surface of the outer nanotube is \(S_L = 2 \pi R_{\text{max}} L\), the surface of the inner nanotube is \(S_0 = 2 \pi R_{\text{min}} L\), and \(L\) is the length of each nanotube. Carrying out the statistical averaging for the operators \(\hat{N}_e = \sum_p n_p^+ c_p^\dagger\) and \(\hat{N}_c = \sum_p b_p^+ f_p\), one obtains

\[
\rho_e = 2 \frac{S_L}{N_e} \sum_p [f(E_+) u_p^2 + (1 - f(E_-)) v_p^2],
\]

\[
\rho_h = 2 \frac{S_0}{N_h} \sum_p [f(E_-) u_p^2 + (1 - f(E_+)) v_p^2].
\]

In our simulations we assume the electron surface density \(\rho_e\) is fixed on the outer nanotube and the chemical potentials \(\mu_e = \mu_{e0} = \mu_{e0}\) are equal. Then, \(\mu, \Delta\) and \(\rho_h\) are determined by the simultaneous solution of equations (4) and (6).

By many orders of magnitude than the order parameter.

Note that for metals the chemical potential is usually larger than 0 when the convergence condition is met. The chemical potential can reach up to 10% of the chemical potential, that requires the full self-consistent solution of the problem. The calculations stop when the convergence condition is met. The chemical potential and the order parameter have to satisfy the condition that for two consequent iterations the variations of their values are less than 0.1%. It usually takes about several dozens of iterations for convergence.

4. Calculation of the conductivities

For the system of two coaxial nanotubes we introduce \(\sigma_{ee}\) and \(\sigma_{hh}\) as the longitudinal quasiparticle conductivities for electron and hole nanotubes, and \(\sigma_{eh}\) and \(\sigma_{he}\) as the quasiparticle transconductivities. The quasiparticle conductivities can be obtained using the Gorkov–Nambu Green’s function formalism \([3, 5, 23]\), that gives the following expressions for the electron and hole conductivities respectively:

\[
\sigma_{ee} = \frac{e^2}{S_e} \sum_{p} \frac{\pi |p|^2}{m_e} \int_{-\infty}^{+\infty} \frac{\partial f(e)}{\partial e} A_{ee}(p, e) \, de,
\]

\[
\sigma_{hh} = \frac{e^2}{S_h} \sum_{p} \frac{\pi |p|^2}{m_h} \int_{-\infty}^{+\infty} \frac{\partial f(e)}{\partial e} A_{hh}(p', e) \, de.
\]

Note that the summations over the momenta \(p\) and \(p'\) are not equivalent, because of the different quantizations of the angular part for the inner and outer nanotubes. The transconductivities are obtained in a similar way:

\[
\sigma_{ij} = \frac{e^2}{S_i} \sum_{p,p'} \frac{\pi |p| |p'|}{m_i} \int_{-\infty}^{+\infty} \frac{\partial f(e)}{\partial e} A_{ij}^{ee}(p, e) \, de,
\]

where \((i, j) = (e, h)\) are the indices corresponding to the outer or inner nanotubes and the summation is done over the momenta with matching quantum numbers. Note that the momenta \(p\) and \(p'\) contain the continuous linear \(p_e, p_h\), and the discrete angular \(p_0, p'_0\) components in equation (9).

The matrix spectral function \(\hat{A}(p, e)\) is given by

\[
\hat{A}(p, e) = -\frac{1}{\pi} \text{Im} \hat{G}(p, e + i\delta),
\]

where \(\hat{G}\) is the Gorkov–Nambu matrix Green’s function, and \(\delta\) is an infinitesimal positive energy. In the absence of disorder we have \(\hat{G} = \hat{G}^0\), and \(\hat{G}^0\) determines the unperturbed spectral functions:

\[
A_{ee}^0(p, e) = \frac{h}{\pi} \delta (e - E_e) + \frac{8}{\pi} \delta (e + E_e),
\]

\[
A_{hh}^0(p, e) = \frac{h}{\pi} \delta (e - E_h) + \frac{8}{\pi} \delta (e + E_h).
\]

In the presence of weak impurities the spectral functions become \([23]\)

\[
A_{ee}(p, e) = \frac{h}{\pi} \delta (e - E_e) + \frac{8}{\pi} \delta (e + E_e),
\]

\[
A_{hh}(p, e) = \frac{h}{\pi} \delta (e - E_h) + \frac{8}{\pi} \delta (e + E_h).
\]

In the presence of impurities the conductivity for the outer nanotube \(\sigma_{ee}\) can be calculated as

\[
\sigma_{ee} = \frac{e^2}{S_e} \sum_{p,p'} \frac{p_e^2 + p_h^2}{m_e^2 k_B T} \int \left[ \frac{u^4}{\cosh^2 \left( \frac{E_e}{2 m_e k_B T} \right)} + \frac{t_e}{\cosh^2 \left( \frac{E_e}{2 m_e k_B T} \right)} \right].
\]

To evaluate this expression, \(p_e\) was quantized using \(p_e = \hbar / L\), where \(L = 0, \pm 1, \pm 2, \ldots\). In the simulations, the length of the nanotube \(L\) is set to be relatively large to simulate an infinite system, particularly \(L = 1000\) nm. Similarly, one can represent the conductivity for the inner nanotube \(\sigma_{hh}\):

\[
\sigma_{hh} = \frac{e^2}{S_h} \sum_{p',p''} \frac{p'_e^2 + p''_e^2}{m_h^2 k_B T} \int \left[ \frac{u^4}{\cosh^2 \left( \frac{E_h}{2 m_h k_B T} \right)} + \frac{t_e}{\cosh^2 \left( \frac{E_h}{2 m_h k_B T} \right)} \right].
\]

The interlayer transconductivity \(\sigma_{eh}\) can be similarly expressed as

\[
\sigma_{eh} = \frac{e^2}{S_e} \sum_{p,p',p_0,p'_0} \frac{p_e p'_e + p_0 p'_0}{m_e m_h k_B T} \int \left[ \frac{h_e}{\cosh^2 \left( \frac{E_h}{2 m_h k_B T} \right)} + \frac{t_e}{\cosh^2 \left( \frac{E_h}{2 m_h k_B T} \right)} \right] u^2_{p} v^2_{p'}.
\]
The corresponding transconductivity $\sigma_{te} = \sigma_{eh} R_{\text{max}}$, because in the derivation of the Kubo formula \cite{23} the induced current should be averaged over the area where it is flowing. Therefore, since the transconductivity $\sigma_{dh}$ describes the current of electrons on the outer nanotube induced by the hole current on the inner nanotube, the normalization area should be $S_z$, that is reflected in equation (9). Note that the asymmetry in the transconductivities $\sigma_{dh} \neq \sigma_{he}$ exists only for spatially separated electrons and holes in two coaxial nanotubes due to the difference between their radii, while the relation $\sigma_{eh} = \sigma_{he}$ holds for two parallel plane layers \cite{3}. Following \cite{3} the scattering lifetimes of quasiparticles can be evaluated as

$$t_h^{-1} = t_{en}^{-1} V_{pp} \frac{\partial E_{+}}{\partial \delta_{p}} \delta (\delta_{p} - \delta_{p}'),$$

$$t_e^{-1} = t_{en}^{-1} V_{pp} \frac{\partial E_{-}}{\partial \delta_{p}} \delta (\delta_{p} - \delta_{p}').$$

In equation (16), $t_{en}$ and $t_{he}$ are the electron and hole scattering times in the normal state. The electron and hole scattering times $t_{en}$ and $t_{he}$ are related to the hole $V_{pp}$ and electron $V_{pp'}$ impurity scattering potentials as follows:

$$t_{en}^{-1} = 4\pi \sum_{p}^{\text{imp}} (V_{pp}^2) \delta (\delta_{p} - \delta_{p}'),$$

$$t_{eh}^{-1} = 4\pi \sum_{p}^{\text{imp}} (V_{pp'}^2) \delta (\delta_{p} - \delta_{p}').$$

(17)

where $\langle \cdots \rangle$ denotes the statistical average over the impurities. It is assumed that the impurities on the inner and outer nanotubes are uncorrelated: $\langle V^e V^h \rangle = 0$. Using the chain rule one obtains $\partial E_{+}/\partial \delta_{p} = V_{pp} y + y_1$, $\partial E_{-}/\partial \delta_{p} = V_{pp'} y - y_1$, $\partial E_{+}/\partial \delta_{p} = V_{pp} y_2 + y_3$, and $\partial E_{-}/\partial \delta_{p} = V_{pp'} y_2 - y_3$. Here we used a notation $y = \frac{1}{2} (2 + x + w)$, $y_1 = \frac{1}{2} (x + w - 2)$, $y_2 = \frac{1}{2} (x - w - 1 + (x w)^{-1})$, $y_3 = \frac{1}{2} (x - 1 + (x w)^{-1}) - 2$, where $x = m_e/m_h$ and $w = R_{\text{max}}/R_{\text{min}}$. Note that for $R_{\text{min}} = R_{\text{max}}$ equation (16) reduces to the expression given by equation (16) in \cite{3}. Since we are interested to study the effects of the asymmetry between electrons and holes due to different radii of the nanotubes, in our simulations we assume that the electrons and holes have equal masses: $m_e = m_h = m$. We also assume that the scattering times of electrons and holes in the normal state are equal: $t_{en} = t_{he} = t_h$. Since the electron and hole scattering times enter the conductivities as linear weighting coefficients, we do not expect any qualitative change of our results in the case of unequal scattering times. A similar conclusion was also expressed in \cite{3}, where different electron and hole masses and lifetimes were used. Let us emphasize that the transconductivities $\sigma_{dh}$ and $\sigma_{eh}$ are negative \cite{3}, because for a system of spatially separated charges the Coulomb drag induces currents flowing in the opposite directions.

In general, the currents are carried by the BCS superfluid and quasiparticles \cite{3}. We characterize the quasiparticle contributions in the linear-response regime as follows:

$$\left( \begin{array}{c} J_h \\ J_e \end{array} \right) = \left( \begin{array}{cc} \sigma_{dh} & \sigma_{he} \\ \sigma_{eh} & \sigma_{ee} \end{array} \right) \left( \begin{array}{c} E_h \\ E_e \end{array} \right),$$

(18)

Figure 2. The order parameter $\Delta$ as a function of the inverse dielectric constant $\varepsilon^{-1}$. The radii of the nanotubes are $R_{\text{min}} = 6$ nm and $R_{\text{max}} = 8$ nm. The surface carrier concentration is $\rho = 10^{12}$ cm$^{-2}$ and the temperature is $T = 0$ K.

where $j_{e(h)}$ and $E_{e(h)}$ are the current flows of quasiparticles and electric field in electron (hole) nanotubes, correspondingly.

Let us mention that the electric fields in electron and hole nanotubes are identical \cite{3}: $E \equiv E_e = E_h$. Therefore, from equation (18) we obtain

$$E = \frac{j_e + j_h}{\sigma_{ee} + \sigma_{nh} + \sigma_{he} + \sigma_{eh}}.$$  

(19)

Let us consider the drag setup, where a fixed current $j_0$ flows through the drive nanotube and the voltage drop across the second nanotube, namely the drag nanotube, is measured, $j_{e0} = j_0$ and $j_{h0} = 0$. Using equation (19) for $j_e + j_h = j_0$, the voltage in the drag nanotube $V_{\text{drag}}$ can be written as

$$V_{\text{drag}} = LE = \frac{L}{\rho_{\text{ee}} + \rho_{\text{nh}} + \rho_{\text{he}} + \rho_{\text{eh}}}.$$  

(20)

5. Discussion and conclusions

For all presented calculations the surface electron concentration is set to $10^{12}$ cm$^{-2}$. We have also performed calculations using much higher density, $10^{14}$ cm$^{-2}$. We have found that the increase of the density does not lead to qualitative changes of the results presented in this work. The hole concentration is obtained self-consistently, and under the condition of equal chemical potentials it is very close to the concentration of electrons.

First, we study the role of electron–hole attraction in the system in the formation of the superfluid phase. It is known that in the case of the asymmetry caused by the difference between the masses of the electrons and holes there is a critical strength of electron–hole attraction, below which there is no BCS transition \cite{24}. In a system of two coaxial nanotubes the strength of interaction can be controlled, for example, by changing the dielectric constant of the dielectric between the nanotubes. To illustrate this we calculate the dependence of the
order parameter on the inverse dielectric constant presented in figure 2. From figure 2 one can see that there is a critical value for the interaction strength. For a weaker electron–hole interaction strength (or higher value of the dielectric constant) there is no BCS state. In our calculations the radii of the nanotubes are set to $R_{\text{min}} = 6 \text{ nm}$ and $R_{\text{max}} = 8 \text{ nm}$ and the temperature is $T = 0 \text{ K}$. The order parameter is normalized to its value $\Delta_c$ for the flat layer geometry, that corresponds to $R_{\text{min}}$, $R_{\text{max}} \rightarrow \infty$.

Second, we investigate how relatively large electron–hole asymmetry can destroy the superfluid phase in the system. The results of the calculations for the dependence of the normalized order parameter on the radius of the inner nanotube for different strengths of the electron–hole interaction and for different temperatures are presented in figures 3–5. According to the results of our calculations, the BCS state still can be formed in the case of moderate asymmetry between the electron and hole excitation spectra. By decreasing the radii $R_{\text{min}}$ and $R_{\text{max}}$ of the nanotubes while keeping a constant separation distance $D = R_{\text{max}} - R_{\text{min}}$ between the nanotubes, one can monotonically increase the asymmetry in the system. In the simulations shown in figures 3–5 the separation distance is set to $D = 2 \text{ nm}$ and the dielectric constant between the nanotubes is set to $\varepsilon = 27$. As seen from figure 3, for relatively large radii ($R_{\text{min}}$, $R_{\text{max}} > 25 \text{ nm}$) the order parameter in the system converges to its value for the plane layer system. At zero temperature, $T = 0 \text{ K}$, as the radii decrease, the order parameter has a kink at $\approx 25 \text{ nm}$ and then almost linearly drops to zero for the inner radius of the nanotube $12 \text{ nm} < R_{\text{min}} < 25 \text{ nm}$. One can suggest that for a fixed attraction strength between electrons and holes there is a critical asymmetry, and that for a higher degree of the asymmetry there is no BCS transition. The nature of the kink in figure 3 has to be studied in more detail. The oscillations in the dependence of the order parameter and critical temperature on the size of superconducting nanowires [25, 26], superconducting films [26], and superconducting metallic grains [27] were obtained in the framework of BCS and Bogoliubov–de Gennes theories caused by the quantum size effect. An experimental demonstration of the quantum size effects in the dependence of the critical temperature on the size of the superconducting film was observed in [28]. A similar kink to the one obtained in our paper for the ground state energy with respect to the coupling constant was treated as a first-order quantum phase transition in the orthogonal-dimer spin chain [29]. The possibility of the first-order quantum phase transition resulting in a point of nonanalyticity in the ground energy as a function of the nanotube radius will be studied in the future.

To further investigate the effect of the electron–hole asymmetry in figure 6 we show the normalized free energy density in the system as a function of the inner radius. At zero temperature the free energy is equal to the ground state energy of the system. For the calculations the difference $R_{\text{max}} - R_{\text{min}} = 2 \text{ nm}$ is kept constant. The dielectric constant and temperature are the same as for figure 3. The free energy density also has a kink at the radius $R_{\text{min}} \approx 25 \text{ nm}$, which is the same radius at which the kink in the order parameter is observed in figure 3. This should not be a surprise, since the ground state of a BCS superconductor contains a term proportional to $\Delta^2$ [5]. The oscillations in the dependence of the ground state energy on the nanotube radius $R_{\text{min}}$ for $R_{\text{min}} < 25 \text{ nm}$ seems to be caused by the quantum size effects.

For a stronger attraction between the electrons and holes, which is modeled by a dielectric with 1.5 times smaller dielectric constant $\varepsilon = 18$, the system can sustain a higher degree of Asymmetry, that is shown in figure 4. At $T = 0 \text{ K}$ the order parameter increases essentially with the decrease of the radii of the nanotubes. This effect is attributed to the quantum confinement and it was predicted for homogeneous [11] and inhomogeneous [30] superconducting nanowires. For radii less than the critical radius $R_{\text{min}} \approx 10 \text{ nm}$ the order parameter again shows an almost linear drop to zero.

A similar drop can be observed at the finite temperature $T = 0.5T_c$ presented in figure 5, where $T_c$ is the critical temperature for the plane layer system. In figure 5 the critical radius $R_{\text{min}} \approx 17 \text{ nm}$ is considerably bigger than the critical radius for $T = 0 \text{ K}$. One can also notice that the enhancement of the order parameter due to the quantum confinement does not take place at this sufficiently high temperature.

Finally, we study the temperature dependence of the transconductivity coefficients in the system. We have found that there are nonzero drag conductivities $\sigma_{\text{eh}} \neq 0$ between the two coaxial nanotubes at temperatures below the critical temperature of the BCS phase transition $T_c$. Since the drag conductivity is a signature of the BCS state, the measurements of the drag conductivity can be used to demonstrate the
existence of the BCS state, resulting in the electron–hole superfluidity in the system and superconductivity in each nanotube. The measurement of the temperature corresponding to vanishing of the drag conductivity can be treated as the measurement of \( T_c \). Note that in the limiting case of low carrier concentrations exciton pairs will be formed [15], that will result in a finite drag even at high temperatures due to the Coulomb attraction and formation of bounded states for electron–hole pairs. The results of calculation of the order parameter and the normalized conductivity coefficients as functions of the temperature are presented in figures 7 and 8. The conductivity coefficients are normalized to the conductivity of the outer nanotube in the normal state \( \sigma_{en} \), and therefore they do not depend on the scattering time of the carriers, which is assumed to be the same for electrons and holes.

To understand the dependence of the conductivities and the order parameter on temperature we considered two sce-
nanotubes, while for two plane electron and hole layers one has 

\( \sigma \) for an infinite two plane layer electron–hole system 

obtained for a system of two coaxial nanotubes with equal carrier 

centrations a contribution to the conductivity due to the 

unpaired component on the outer nanotube. The considered 

system has an intrinsic asymmetry due to different radii of 

the nanotubes. One can utilize this asymmetry in nanoscale 

devices. Unlike the ratio of the transconductivities \( \sigma_{\text{eh}} / \sigma_{\text{he}} \) for 

relatively small radii of the nanotubes the ratio \( \sigma_{\text{ee}} / \sigma_{\text{hh}} \) is not equal to the ratio of the radii, and thus the ratio of the 

numbers of the carriers on the nanotubes. Therefore, a situation 

is possible where the inner drive nanotube can induce a higher 

current in the outer drag nanotube. In other words, the drag 

current will be higher than the drive one. The potential drop 

will be lower on the outer nanotube than on the inner one. 

Therefore, the system functions as a step down transformer at 

zero frequency. In the case of using the outer nanotube as the 

drive, the system works as a step up transformer, where the 

drag current will be lower than the drive current. In the case of 

ideal drag, the transformation coefficient is given by the ratio 

of the radii of the nanotubes. One can possibly manufacture 

such systems using multiwall carbon nanotubes.

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Figure 8. The normalized quasiparticle conductivities \( \sigma_{\text{ee}} \) (line with the 

plus markers), \( \sigma_{\text{eh}} \) (dashed line), \( \sigma_{\text{eh}} \) (thin solid line) and \( \sigma_{\text{he}} \) 

(thick solid line) as functions of the normalized temperature \( T / T_c \). The radii of 

the nanotubes are \( R_{\text{min}} = 50 \text{ nm} \) and \( R_{\text{max}} = 52 \text{ nm} \); the surface 

carrier concentration is \( \rho = 10^{13} \text{ cm}^{-2} \). Note that the conductivities are 

normalized to the electron conductivity of the outer nanotube \( \sigma_0 \) in 

the normal state, and the order parameter is normalized to the order 

parameter \( \Delta_0 \) at \( T = 0 \text{ K} \). \( \sigma_{\text{ee}} \) and \( \sigma_{\text{eh}} \) are so close that they are 

indistinguishable on this figure.
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