Dynamical symmetry breaking, confinement with flat-bottom potential

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In this article, we calculate the dressed quark propagator with the flat bottom potential in the framework of the rainbow Schwinger-Dyson equation. Then based on the nonperturbative dressed quark propagator, we calculate the \( \pi \) decay constant and the quark condensate. The \( \pi \) decay constant is an important parameter in describing the interplay between dynamical symmetry breaking and confinement, while the quark condensate is an order parameter for dynamical chiral symmetry breaking. To implement confinement, we prove that the dressed quark propagator has no poles on the real timelike \( p^2 \) axial, the absence of Kallen-Lehmann spectral representation obviously precludes the existence of free quarks.

PACS numbers: 11.10.Gh, 12.38.Lg, 24.85.+p

Key Words: Schwinger-Dyson equation, Dynamical Chiral Symmetry Breaking, Quark Confinement

1. INTRODUCTION

The standard model gives highly successful descriptions of hadronic physics in terms of the basic force. Yet its key parameters, the masses of the quarks and leptons are specified by the coupling of the Higgs bosons, which is undetermined. It is possible that the dynamics of the fundamental gauge theories themselves generate the masses of all the matter fields, or in other word, dynamical chiral symmetry breaking (for quarks), which corresponding to the absence of the low mass scalar partners of the \( \pi \) [1]. Quantum chromodynamics (QCD) is the appropriate theory for describing the strong interaction at high energy. However, the lack of observation of free quarks and gluons leads to the assumption of colour confinement that only quantities which transform as colour singlets can be physical observable. One may expect that the nonperturbative infrared behaviour of gluon propagator which may lead to a linear or oscillator potential implements confinement, however, till now, all theoretical studies based on QCD can not give satisfactory explanation. Hence we take an equivalent statement that the propagator of a colored state should have no singularities on the real time-like \( p^2 \) axial, the absence of Kallen-Lehmann spectral representation precludes the existence of free quarks [2,3].

In fact, dynamical symmetry breaking and confinement are two crucial features of QCD, however, they correspond to two very different energy scales. The possible interplay of the two dynamics lies in the following two facts, the first one is that on large energy scale the chiral massive quark interacts with antiquark via a confining potential and creates a new quark mass, constituent quark mass; while the second one can be attributed to the double role of the \( \pi \), as both Nambu-Goldstone boson and \( q\bar{q} \) bound state.

The Schwinger-Dyson equation, in effect the functional Euler-Lagrange equation of quantum field theory, provides a natural framework for investigating the nonperturbative properties of quark and gluon Green’s functions. By studying the evolution behaviour and analytic structure of the dressed quark propagator, one can obtain valuable information about the dynamical symmetry breaking phenomenon and confinement.
Although lattice calculations are rigorous in view of QCD, they suffer from lattice artifacts and uncertainties, such as Gribov copies, boundary conditions and so on. Furthermore, current technique can not give reliable result below 1 GeV, where the most interesting and novel behaviour is expected to lie.

It would seem therefore that at the present time Schwinger-Dyson (SD) equation is the most reliable tool for studying the infrared behaviour of the quarks and gluons propagators in the continuum limit, while it has its own shortcomings.

The flat-bottom potential (FBP) is a sum of Yukawa potentials, which not only satisfies gauge invariance, chiral invariance and fully relativistic covariance, but also suppresses the singular point which the Yukawa potential has. It works well in understanding the meson structure, such as electromagnetic form factor, radius, decay constant [4].

Therefore, it is interesting to examine dynamical chiral symmetry breaking and quark confinement with the FBP.

The article is arranged as follows: in section 1, introduction; in section 2, infrared behaviour of the gluon propagator; in section 3, dynamical chiral symmetry breaking; in section 4, quark confinement; in section 5, flat bottom potential and Schwinger-Dyson equation; in section 6, conclusion and discussion.

II. INFRARED BEHAVIOUR OF GLUON PROPAGATOR

The infrared structure of the gluon propagator has important implication for quark confinement. One might expect that the behaviour of the quark interaction in region of small spacelike $p^2$ determines the long range properties of the $q\bar{q}$ potential and hence implements confinement.

From the discussion of previous section, we can see that the SD equation is the ideal tool for studying the infrared behaviour of the gluon propagator. Here we make a brief skeleton of the results based on Landau gauge and axial gauge studies.

It is impossible to solve an infinity series of coupled SD equations for gluons and quarks (ghosts), so the authors have to make truncations both in Landau gauge and axial gauge. In those studies, the quark loop contributions are neglected and higher order vertexes are taken as bare or dressed based on the analysis of corresponding Ward-Takahashi Identity.

In axial gauge studies, one makes further assumption that the dressed gluon propagator has the same tensor structure as the free one. That is in the following equations let $F_2 = 0$ [5],

$$D_{\mu\nu}(p, \gamma) = F_1(p, \gamma)M_{\mu\nu}(p, n) + F_2(p, \gamma)N_{\mu\nu}(p, n),$$

$$M_{\mu\nu}(p, n) = \delta_{\mu\nu} - \frac{p_\mu n_\nu + p_\nu n_\mu}{p \cdot n} + n^2 \frac{p_\mu p_\nu}{(p \cdot n)^2},$$

$$N_{\mu\nu}(p, n) = \delta_{\mu\nu} - \frac{n_\mu n_\nu}{n^2},$$

(1)

here $\gamma$ is gauge parameter and $n$ is an arbitrary four vector. After solving gluon SD equation one get the conclusion that the gluon propagator may have a double pole at the origin, implying an area law behaviour of the Wilson loop, which is often regarded as a signal of confinement [6]. Further more, the Fourier transform of the time-time component of the gluon propagator is related to the one-gluon exchange contribution to the potential between static colour charge and a $1/p^4$ behaviour for the gluon propagator in the origin. However, a general studies of the properties of the spectral representation of the gluon propagator in axial gauge show that the coefficient of the $\delta_{\mu\nu}$ term cannot be more singular then $1/n^2$. This suggests that neglecting the $F_2$ term is a poor approximation because there maybe cancellation in the infrared between $F_1$ and $F_2$.

In Landau gauge studies [7,9]. Suppose neglecting ghost loop will not remove the dominant infrared contributions in QCD also leads to a $1/p^4$ behaviour for the gluon propagator in the origin. Other studies suggest that fermion loop
may suppress the infrared singularity in the gluon propagator, the double pole found in the pure-gauge sector may be
removed by the nonperturbative fermion loop corrections \[10,11,17\]. Recently, the gluon SD equation studies including
ghost loop result in an infrared vanished gluon propagator and a strong infrared enhanced ghost propagator \[24,25\]
while lattice calculations support this type conclusion \[26\]. In fact, the truncations applied in these studies may
impair their persuasion. Furthermore, in some studies of this type, unphysical particle-like singularities occur in the
dressed ghost-gluon and 3-gluon vertices and the infrared vanished gluon propagator can neither produce dynamical
chiral symmetry breaking nor compliment confinement \[24,27\].

We can take a simple example by analyzing the renormalization group equation for the coupling constant. There
exists a competing mechanism between the self-interaction of gauge boson and the interaction of boson-fermion, which
accounts for the different evolution behaviours of the coupling constants with momentum for QCD and QED \[12\]. So
neglecting one another may lead to very different results.

All theoretical studies based on QCD are suffered from shortcomings in one or another way \[11\], so they can not
give satisfactory infrared behaviour for gluon propagator which may lead to a linear or oscillator potential implements
confinement.

Here we take an alternate description of confinement based on the analytic structure of the dressed quark prop-
agator, if the quark Green’s function is free of singularities on the real timelike \(p^2\) axial, then denies the existence of
a single particle spectrum and precludes free quarks. Although by no means a sufficient condition for the formulation
of finite size \(q\bar{q}\) states, it is an essential ingredient in describing of confinement based on a bold assumption that
interaction between dressed quarks which does not rise to infinity at large distance. The interaction between dressed
quarks may diminish at separation beyond the characteristic length scale at which point the quark Green’s function
approaches its vacuum values, but the vacuum’s repulsive force due to the absence of a mass pole in the dressed quark
propagator again compresses the quarks together.

So the traditional model of confinement based on an effective rising potential can be interpreted as the result
of combination of the repulsive interaction between quark-vacuum and the attraction of dressed quark-quark with
increasing separation. Although there maybe a long way to go before the repulsive quark-vacuum interaction can be
quantified, this description has proven successful in the formulation of a confining nontopological soliton model for
baryons \[13\]. In this article, the quark propagator calculated with the FBP does free singularities on the real timelike
\(p^2\) axial and implements quark confinement.

III. DYNAMICAL CHIRAL SYMMETRY BREAKING

The ultraviolet stability of QCD at large spacelike momentum makes a straightforward definition of the chiral limit
possible, in a manifest contrast to strong coupling QED, whose rigorous definition remains an instructive challenge.
The \(\pi\) decay constant \(f_\pi\) is an important parameter in describing the interplay between dynamical symmetry breaking
and confinement, while the quark condensate serves as an order parameter for dynamical chiral symmetry breaking.
In the chiral limit, the \(\pi\) plays a double role both as Nambu-Goldstone boson and \(q\bar{q}\) bound state, which lead to the
result that the scalar part of the inverse dressed quark propagator coincides with the approximated \(\pi\)’s Bethe-Salpeter
function. In the following, we calculate the \(\pi\) decay constant , the quark condensate and examine the dynamical chiral
symmetry breaking phenomenon.
A. π Decay Constant

Neglecting ghost, the Ward-Takahashi Identity (WTI) for quark-gluon vertex in the presence explicit quark masses can be written as \[23\]:

\[ k_\mu \Gamma^\mu_5(p', p) = \frac{\tau}{2} \left[ S^{-1}(p') \gamma_5 + \gamma_5 S^{-1}(p) \right] + 2im\Gamma_5(p', p). \]  

(2)

In the limit of exact chiral symmetry, the WTI reduces to

\[ k_\mu \Gamma^\mu_5(p', p) \xrightarrow{k \to 0} -\tau \gamma_5 B(p^2). \]  

(3)

Note that the usual perturbative isovector axial vector vertex is \( \frac{\tau}{2} \gamma_5 \gamma_5 \).

In the chiral limit and take \( k \to 0 \), the axial vector quark vertex becomes completely dominated by the pseudoscalar coupling of a massless \( \pi \) to the quark and the subsequent weak decay of the \( \pi \) into an axial vector current. In fact, the mass shell residue of the axial vector quark vertex \( \Gamma^\mu_5(p, p) \) is the \( \pi \) Bethe-Salpeter \( \Gamma_\pi \) amplitude, in the chiral limit can provide a direct connection to the quark propagator \[14\]. The function \( \Gamma^\mu_5(p + \frac{k}{2}, p - \frac{k}{2}) \) \( \Gamma_5 \) contains many Dirac structures, such as \( \gamma_5 \gamma_\mu \) \( (\gamma_5) \), \( \gamma_5 \gamma_\mu \gamma_\cdot k \) \( (\gamma_5 \gamma_\cdot k) \), \( \gamma_5 \gamma_\cdot pp_\mu \) \( (\gamma_5 \gamma_\cdot pp_\cdot k) \), \( \cdots \) and regular term, singular term. Exclude the singular term \( \Gamma_\pi \), the other terms are supposed to give small contributions and neglected. If we solve the Schwinger-Dyson equation for the vertex, we can see their relative weights. In the chiral limit, the \( \gamma_5 \) component of \( \Gamma_\pi \) makes a good approximation, though it has many Dirac structures, such as \( \gamma_5, \gamma_5 \gamma_\cdot k \) and so on. The \( \pi \) decay constant \( f_\pi \) is defined by the axial transition amplitude for the on-shell \( \pi \),

\[ \langle 0 | A_5^{\mu} \langle 0 | \pi^n \rangle(k) \rangle = \frac{if_\pi k^\mu \delta^{mn}}{2}. \]  

(4)

When \( k \to 0 \), we obtain

\[ i \Gamma^\mu_5(p', p) \xrightarrow{k \to 0} i \Gamma^\mu_5(p', p) \frac{i}{k^2} i \langle 0 | A_5^{\mu} | \pi^n \rangle(-k) \xrightarrow{k \to 0} -\Gamma_5(p', p) f_\pi \frac{k^\mu}{k^2}. \]  

(5)

Multiplying Eq.(5) with \( k_\mu \) on both sides,

\[ k_\mu \Gamma^\mu_5(p', p) = i f_\pi \Gamma_5(p', p). \]  

(6)

When combined with Eq.(3), we obtain the Goldberger-Treiman relation for the quark-pseudoscalar vertex,

\[ \Gamma_5(p, p) = i \tau \gamma_5 \frac{B(p^2)}{f_\pi} = i \tau \gamma_5 \frac{A(p^2) m(p^2)}{f_\pi}. \]  

(7)

The simplest generalization of Eq.(7) is

\[ \Gamma_5(p', p) = i \tau \gamma_5 \frac{B(p^2) + B(p^2)}{2f_\pi}. \]  

(8)

Based on above discussion, it is possible to obtain an expression for \( f_\pi^2 \) from the integral equation for the \( \pi \) decay constant,

\[ i \langle 0 | A_5^{\mu} \langle 0 | \pi^n \rangle(k) \rangle = \int \frac{d^4p}{(2\pi)^4} tr \{ iS(p + k) i \Gamma^\mu_5(p + k, p) S(p) i \tau^n \gamma_5 \}. \]  

(9)

As usual, the factor \(-1\) on the right hand side arises from the quark loop. We need to take the axial-vector vertex as a perturbative one in order to avoid double counting. The expression for \( f_\pi^2 \) can be written as

\[ f_\pi^2 = -12i \int \frac{d^4p}{(2\pi)^4} \left( \frac{B(p^2)}{A(p^2) p^2 - B(p^2)} \right)^2 \left( A(p^2) B(p^2) + \frac{p^2}{2} \left( B \frac{dA(p^2)}{dp^2} - A \frac{dB(p^2)}{dp^2} \right) \right). \]  

(10)
In the limit $A \equiv 1$, we can recover the Pagels and Stokar formula [18]. Our numerical results based on the quark propagator obtained from the SD equation studies with the FBP do give satisfactory result $f_\pi = 94\text{MeV}$, we can thus fit the parameters in the FBP.

There is another expression for the $\pi$ decay constant which determined from global colour model [16,29].

$$f_\pi = \frac{3}{8\pi^2} \int_0^\infty ds s B(s)^2 \left\{ \sigma_v^2 - 2[\sigma_s \sigma'_s + s \sigma_v \sigma'_v] - s[\sigma_s \sigma''_s - \sigma'_s] - s^2[\sigma_v \sigma''_v - \sigma'_v] \right\}$$  \hspace{1cm} (11)

where

$$\sigma_v = \frac{A(s)}{sA(s)^2 + B(s)^2}, \quad \sigma_s = \frac{B(s)}{sA(s)^2 + B(s)^2}. \hspace{1cm} (12)$$

In our calculation, the value of $A(p)$ is far from 1, we prefer Eq.(10) rather than Eq.(11), as Eq.(11) is exact only in the limit $A(p) \equiv 1$ [16,29].

**B. Quark Condensate**

The quark propagator is defined as

$$S(x) = \langle 0 | T[q(x)\bar{q}(0)]|0\rangle. \hspace{1cm} (13)$$

where $q(x)$ is the quark field and $T$ the time-ordering operator. For the physical vacuum consists of both perturbative and nonperturbative parts, so the quark propagator $S(x)$ can be divided into a perturbative and a nonperturbative part as the following:

$$S(x) = S_{PT}(x) + S_{NP}(x). \hspace{1cm} (14)$$

In the nonperturbative vacuum, the normal-ordered product $S_{NP}(x)$ does not vanish. The widely used nonlocal quark condensate $\langle 0 | :\bar{q}(x)q(0) : |0\rangle$ is given by the scalar part of the Fourier transformed inverse quark propagator,

$$\langle 0 | :\bar{q}(x)q(0) : |0\rangle_\mu = (-4N_c) \int_0^\mu \frac{d^4p}{(2\pi)^4} \frac{B(p^2)}{p^2A^2(p^2) + B^2(p^2)} e^{ipx}. \hspace{1cm} (15)$$

In the limit $x \to 0$, we can obtain the expression for the local quark condensate,

$$\langle 0 | :\bar{q}(0)q(0) : |0\rangle_\mu = -\frac{12}{16\pi^2} \int_0^\mu ds s \frac{B(s)}{A^2(s) + B^2(s)}. \hspace{1cm} (16)$$

In Ref. [19] the nonlocal condensate is put in the following form :

$$\langle 0 | :\bar{q}(x)q(0) : |0\rangle = g(x^2)\langle 0 | :\bar{q}(0)q(0) : |0\rangle,$$  \hspace{1cm} (17)

where $g(x^2)$ is the vacuum non-locality of the nonlocal quark condensate. The nonlocal properties of the vacuum condensates are of principal importance in the study of the distributions of quarks and gluons in hadrons [32]. Physically, it means that the vacuum condensates of quark and gluon can flow through the vacuum with nonzero momentum.

The effects from hard gluonic radiative corrections to the quark propagator are connected to a possible change of the renormalization scale $\mu$ at which the condensates are defined. Those effects are of minor importance for the nonperturbative effects in the low and medium energy regions, and neglected in our studies. In this article, we take the rainbow SD equation, so it is not a renormalizable interaction. As isolated closed quark loop integral, the
quark condensate has only one energy scale, the momentum cut off, \( \mu \), at which the quark condensate is defined. In our studies, the energy scale \( \mu \) is implicitly determined by the effective gluon propagator, we can use Eq.(16) as the definition for the quark condensate. However, presently, the nonperturbative technique can not prove the relation

\[
\int_0^\mu \frac{ds A(s)}{s A^2(s) + B^2(s)} \sim (\log(\mu^2/\Lambda_{QCD}^2))^d,
\]

(18)

at low energy scale. Here \( d \) is the anomalous dimension and takes the value \( d = \frac{12}{3n_f - 2n_s} \).

If we take the limit \( \mu \rightarrow \infty \) and assume ultraviolet dominating, the above relation \( \langle \bar{q}q \rangle_\mu \sim (\log(\mu^2/\Lambda_{QCD}^2))^d \) is indeed the case, although the quark condensate is only defined at low energy scale.

Operator product expansion has proven that at large Euclidean momentum, the effective quark mass evolves as

\[
m(-Q^2)_{Q \rightarrow \infty} = \frac{c}{Q^2} (\log(Q^2/\Lambda_{QCD}^2))^{d-1} + m(\mu) \left( \frac{\log(\mu^2/\Lambda_{QCD}^2)}{\log(Q^2/\Lambda_{QCD}^2)} \right)^d,
\]

(19)

here \( c = -\frac{4\pi d}{3} \frac{1}{[\log(\mu^2/\Lambda_{QCD}^2)]^2} \) is some constant independent of \( \mu \) \([30,31]\), this implies that \( \langle 0 | : \bar{q}q : | 0 \rangle_\mu \sim [\log(\mu^2/\Lambda_{QCD}^2)]^d \), but not the definition of Eq.(16).

On the other hand, the evolution of quark condensate with \( \mu \) can be determined from Gell-Mann-Oakes-Renner relation and renormalization group equation,

\[
\langle 0 | : \bar{q}q : | 0 \rangle_\mu = \left( \frac{\log(\mu^2/\Lambda_{QCD}^2)}{\log(\Lambda^2/\Lambda_{QCD}^2)} \right)^d \langle 0 | : \bar{q}q : | 0 \rangle_\Lambda.
\]

(20)

If we take into account the hard radiative corrections to the quark propagator, there is a more nature definition for the quark condensate (in Euclidean space) \([28]\):

\[
\langle 0 | : \bar{q}q : | 0 \rangle_\mu = \left( \frac{\log(\mu^2/\Lambda_{QCD}^2)}{\log(\Lambda^2/\Lambda_{QCD}^2)} \right)^d \frac{12}{16\pi^4} \int_0^\Lambda d^4p \frac{B(p^2)}{A(p^2)^2p^2 + B(p^2)^2},
\]

(21)

In the second definition, we can take the cut off be \( \Lambda \rightarrow \infty \), however, at large momentum, the quark propagator approximates its asymptotic free form which should be subtracted. From Fig.1, we can see that the cut off \( \Lambda = 2GeV \) makes a sound approximation.

In this article, \( \mu \) is taken to be 1 GeV. If we ignore the hard radiative corrections to the quark propagator, from Eq.(16), the numerical result gives the quark condensate \( \langle 0 | \bar{q}q | 0 \rangle_\Lambda^\dagger = -218MeV \). If we take into account the hard radiative corrections, from Eq.(21), we obtain:

\[
\begin{align*}
\Lambda_{QCD} &= 150MeV, & \langle 0 | \bar{q}q | 0 \rangle_\Lambda^\dagger &= -255MeV; \\
\Lambda_{QCD} &= 200MeV, & \langle 0 | \bar{q}q | 0 \rangle_\Lambda^\dagger &= -253MeV; \\
\Lambda_{QCD} &= 250MeV, & \langle 0 | \bar{q}q | 0 \rangle_\Lambda^\dagger &= -251MeV.
\end{align*}
\]

(22)

All our numerical results are compatible with the results of the QCD sum rule approach, and large enough to give dynamical chiral symmetry breaking. The definition of Eq.(21) is better.

**IV. QUARK CONFINEMENT**

To demonstrate that quark confinement arises from the absence of singularities on the real timelike \( p^2 \) axial, let us first make an observation on the large Euclidean time behaviour of the free fermion propagator \([20]\):

\[
S(x) = \int \frac{d^4k}{(2\pi)^4} e^{ik \cdot x} \frac{m - i\gamma \cdot k}{k^2 + m^2},
\]

(23)
The spatially averaged Schwinger function is a particularly insightful tool. Consider the fermion propagator and let $T = ix_4$ represent Euclidean time, then

$$\sigma_s(T) = \int d^3x \frac{1}{4} \text{tr}_\beta S(x, T) = \frac{1}{2} e^{-mT}. \quad (24)$$

Hence the free fermion’s mass can easily be obtained from the large T behaviour of the spatial average:

$$\lim_{T \to \infty} \log \sigma_s(T) = -mT. \quad (25)$$

This is just the approach used to determine the bound state masses in simulations of Lattice QCD.

In order to demonstrate that the confinement of quarks, we have to study the mass function of the quark and prove that there no poles on the real timelike like $p^2$ axial. So it is necessary to perform an analytic continuation of the dressed quark propagator from Euclidean space into Minkowski space $p_4 \to ip_0$. However, we have no knowledge of the singularity structure of quark propagator in the whole complex plane. One can take an alternative safe procedure and stay completely in Euclidean space avoiding analytic continuations of the dressed propagators [22]. Again we take the Fourier transform with respect to the Euclidean time $T$. It is sufficient to consider the scalar part $S_s$,

$$S_s^*(T) = \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4T} S_s,$$

$$= \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4T} \frac{B(q^2)}{q^2 A^2(q^2) + B^2(p^2)}. \quad (26)$$

If $S(p)$ had a pole at $p^2 = -m^2$, the Fourier transformed $S_s^*(T)$ would fall off as $e^{-mT}$ for large T or $\log S_s^* = -mT$. In our calculation, for large $T$, the values of $S_s^*$ is negative, except occasionally a very very small fraction positive values which can be safely neglected. We can express $S_s^*$ as $|S_s^*| e^{i\pi n}$, $n$ is an odd integer. $\log S_s^* = \log |S_s^*| + i\pi n = -mT$. If we neglect the imaginary part, we find that when the Euclidean time T is large, there indeed exists a crudely approximated linear function with respect to T, which is shown in Fig.2. However, such a behaviour is hard to acquire physical explanation. Here the word ‘crudely’ should be understand in the linearly fitted sense, to be exact, there is not linear function. This precludes the existence of free quarks.

V. FLAT BOTTOM POTENTIAL AND SCHWINGER-DYSON EQUATION

The FBP is a sum of Yukawa potentials which is an analogous to the exchange of a series of particles and ghosts with different masses (Euclidean Form),

$$G(k^2) = \sum_{j=0}^{n} \frac{a_j}{k^2 + (N + j\rho)^2}, \quad (27)$$

where $N$ stands for the minimum value of the mass, $\rho$ is their mass difference, and $a_j$ is their relative coupling constant. In Fig.3, we can see the curve of $G(p^2)$ is a gauss-like distribution. Due to the particular condition we take for the FBP, there is no divergence in solving the SD equation. In its three dimensional form, the FBP takes the following form:

$$V(r) = -\sum_{j=0}^{n} \frac{a_j e^{-(N+j\rho)r}}{r}. \quad (28)$$

In order to suppress the singular point at $r = 0$, we take the following conditions:
\[ V(0) = \text{constant}, \]
\[ \frac{dV(0)}{dr} = \frac{d^2V(0)}{dr^2} = \cdots = \frac{d^nV(0)}{dr^n} = 0. \] (29)

So we can determine \( a_j \) by solve the following equations, inferred from the flat bottom condition Eq.(29),

\[
\sum_{j=0}^{n} a_j = 0, \\
\sum_{j=0}^{n} a_j(N + j\rho) = V(0), \\
\sum_{j=0}^{n} a_j(N + j\rho)^2 = 0, \\
\text{...} \\
\sum_{j=0}^{n} a_j(N + j\rho)^n = 0. \] (30)

As in previous literature [4], \( n \) is set to 9.

In the rainbow approximation, the SD equation takes the following form:

\[ S^{-1}(p) = \gamma \cdot p - \tilde{m} + \frac{16\pi i}{3} \int \frac{d^4k}{(2\pi)^4} \gamma_\mu S(k)\gamma_\nu G^{\mu\nu}(k-p), \] (31)

where

\[ S^{-1}(p) = A(p^2)\gamma \cdot p - B(p^2) \equiv A(p^2)[\gamma \cdot p - m(p^2)], \] (32)
\[ G^{\mu\nu}(k) = -(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2})G(k^2), \] (33)

and \( \tilde{m} \) stands for an explicit quark mass-breaking term. With the explicit small mass term, we can preclude the zero solution for \( B(p) \) and in fact there indeed exist a bare current quark mass. Here we take Landau gauge. This dressing comprises the notation of constituent quarks by providing a mass \( m(p^2) = B(p^2)/A(p^2) \), which is corresponding to dynamical symmetry breaking. Because the form of the gluon propagator \( g^2G(p) \) in the infrared region is unknown (as we have discussed in section 2), one often uses model forms as input in the previous studies of the rainbow SD equation [29,17].

In this article, we assume that a Wick rotation to Euclidean variables is allowed, and perform a rotation analytically continuing \( p \) and \( k \) into the Euclidean region where them can be denoted by \( \bar{p} \) and \( \bar{k} \), respectively. Alternatively, one can derive the SD equation from the Euclidean path-integral formulation of the theory, thus avoiding possible difficulties in performing the Wick rotation [21]. As far as only numerical results are concerned, the two procedures are equal.

The Euclidean SD equation can be projected into two coupled integral equations for \( A(p^2) \) and \( B(p^2) \). For simplicity, here ignore the bar on \( p \) and \( k \). Numerical values for \( A(p^2) \), \( B(p^2) \) and \( m(p^2) \) are shown in Fig.[1].

VI. CONCLUSION AND DISCUSSION

In this article, we calculate the dressed quark propagator with the FBP in the framework of the rain-bow SD equation. The dressed quark propagator exhibits a dynamical symmetry breaking phenomenon and gives a constituent quark mass about 490 MeV \( (m(m^2)) \), which is larger then the value of commonly used constituent quark mass 350
MeV in the chiral quark model. Then based on dressed quark propagator, we obtain the quark condensate \( \langle 0 | \bar{q} q | 0 \rangle \), the \( \pi \) decay constant. The calculated \( f_\pi \) and \( \langle 0 | \bar{q} q | 0 \rangle \) are compatible with experimental and theoretical works respectively. Though the value of \( f_\pi \), we can fit the parameters: \( N = 1.0 \times 200\, MeV, V(0) = -27.0 \times 200\, MeV, \rho = 5.0 \times 200\, MeV, m_u = m_d = 8\, MeV \) and the large momentum cut-off \( L = 630 \times 200\, MeV \) in solving the SD equation.

The \( \pi \) decay constant \( f_\pi \) is an important parameter in describing the interplay between dynamical symmetry breaking and confinement, while the quark condensate serves as an order parameter for dynamical chiral symmetry breaking. In the chiral limit, the \( \pi \) plays a double role both as Nambu-Goldstone boson and \( q \bar{q} \) bound state, which lead to the result that the scalar part of the inverse dressed quark propagator coincides with the approximated \( \pi \)'s Bethe-Salpeter function.

Till now, all theoretical studies based on QCD are suffered from shortcomings in one or another way \[1\], so they can not give satisfactory infrared behaviour for gluon propagator which may lead to a linear or oscillator potential implements confinement. In this article, we take an alternative description by studying the analytic structure of the quark propagator. If the dressed quark propagator is free singularities on the real timelike \( p^2 \) axial, although the attraction between quarks may diminish at a characteristic distance, but the vacuum repulsive force due to the absence of mass poles compresses the quarks together. The traditional confinement based on the arising potential between quarks as the separation becomes long can be realized as combination of both the attractive and repulsive forces. In our calculation, we take a Fourier transform with respect Euclidean time \( T \), which is often used in lattice calculation to extrapolate physical masses, and find that at large time \( T \), there does not exist a physical quark mass, which may preclude the existence of physical free quark spectrum.

In calculation, the coupled integral equations for the quark propagator functions \( A(p^2) \) and \( B(p^2) \) are solved numerically by simultaneous iterations. The iterations converge rapidly to a unique stable solution of propagator functions and independent the initial guesses. The propagator functions \( A(p^2) \) and \( B(p^2) \) are shown in Fig.[1], at small \( p^2 \), \( A(p^2) \) differs from the value 1 appreciably, while it tends to 1 for large \( p^2 \). We find that at small \( p^2 \), \( m(p^2) \) is greatly re-normalized, while at large \( p^2 \), it takes asymptotic behaviour. For \( u \) and \( d \) quark, \( m(0) = 632\, MeV \), the connection of \( m(p) \) to constituent masses is somewhat less direct and is precise only for heavy quarks. For heavy quarks, \( m_{\text{constituent}}(p) = m(p = 2m_{\text{constituent}}(p)) \), for light quarks, it only makes a crude estimation \[30\]. At about \( p = 1\, GeV \), the mass function grows rapidly as the momentum decreases, that is an indication of dynamical symmetry breaking. There is a popularly used constituent quark definition, \( m(p^2) = p^2 \) \[29\]. Here \( p^2 \) is Euclidean momentum square and \( m(m^2) = 490\, MeV \). Our result is larger than the usually used constituent quark mass \( m = 350\, MeV \), however, it gives a good description of dynamical symmetry breaking.

The phenomenological FBP can be extended to other system, such as \( q \bar{q} \) system, \( qQ \) system, vector meson and \( qqq \) system or by applying the results to the calculation of quantities and processes requiring detailed knowledge of the quark propagator.

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Fig. 1 SD functions

SD functions

$B(p)/200\text{MeV}$

$A(p)$

$m(p)/200\text{MeV}$
Fig. 2: $-\log|S^*(T)|$ vs. $10^4 \text{GeV}^{-1}$
Fig. 3 Flat bottom potential