Magnetic field simulation of wiggler on LUCX accelerator facility using Radia

Y N Sutygina1*, A E Harisova1, D A Shkitov1

1 Tomsk Polytechnic University, 30, Lenin St., Tomsk, 634050, Russia

E-mail: yana.sutygina@mail.ru

Abstract. A flat wiggler consisting of NdFeB permanent magnets was installed on a compact linear electron accelerator LUCX (KEK) in Japan. After installing the wiggler on LUCX, the experiments on the generation of undulator radiation (UR) in the terahertz wavelength range is planned. To perform the detailed calculations and optimization of UR characteristics, it is necessary to know the parameters of the magnetic field generated in the wiggler. In this paper extended simulation results of wiggler magnetic field over the entire volume between the poles are presented. The obtained in the Radia simulation magnetic field is compared with the field calculated by another code, which is based on the finite element method.

1. Introduction

On a compact linear electron accelerator LUCX (KEK) in Japan in the end of 2015 was installed a compact planar wiggler [1] consisting of NdFeB permanent magnets. After the installation of this wiggler on LUCX, it is planned to conduct the experiments on the generation of undulator radiation in the terahertz (0.1 - 10 THz) wavelength range [2] on the basis of pre-bunched electron beam with a short distance between bunches which is in order of the bunch length. The electron energy after the electron gun is equal to 8.25 MeV and after the accelerating section is 30 MeV [3]. THz radiation is of a great interest due to the fact that this type of radiation does not ionize organic substances, for example, as X-rays, and therefore it can be used to study biological systems [4].

To perform the detailed calculations and optimize the characteristics of the undulator radiation, it is necessary to know the magnetic field parameters generated in the wiggler. One of the well-known programs that allow us to simulate the undulator radiation characteristics emitted by the electron beam based on freely definable data on a magnetic field is Spectra [5]. Moreover, knowledge of the magnetic field in the whole volume between the wiggler poles is required in order to make the evaluations of the changes and the displacement of the charged particles beam that moves in the wiggler. Those evaluations are necessary to adjust the magnetic system of the accelerator.

This paper presents the magnetic field calculations in the wiggler central plane and the field calculations along the electron beam trajectory (the central axis of the wiggler) for the several values of the gap width between the poles. Also paper presents an analysis of the wiggler rotation relative to it center around all three axes which is influenced to the magnetic field. The magnetic field is calculated on the basis of Radia code [6] which is an integrated module of Mathematica of Wolfram Research [7]. These simulations of magnetic field are compared to experimental data [1] and the data obtained in the simulation based on the finite element method.
2. Device geometry
The wiggler contains five periods consisting of permanent magnets, where the length of each period is equal to 60 mm, the gap width between the poles can be varied from 30 to 60 mm. Three-dimensional model of the wiggler, built in Radia, is shown in figure 1.

![Figure 1. Three-dimensional model of the wiggler.](image)

The magnets are in the shape of trapezoidal prism with the same edge angle $\phi$ (see figure 2). The magnetic blocks in each period have a horizontal or vertical magnetization direction that is shown in figure 2. The main parameters of the magnetic system are listed in table 1.

![Figure 2. Schemes of one period.](image)

| Parameters                      | Value       |
|---------------------------------|-------------|
| Period length, $\lambda_u$     | 60 mm       |
| Number of periods               | 5           |
| Magnet gap, $g$                 | 30 – 60 mm  |
| Edge angle, $\phi$              | 2.0 deg     |
| Permanent magnet                | Nd-Fe-B     |
| Nominal block size, $2b \times 2a \times 2c$ | 20×100×15 mm |
| Residual induction of magnet    | 1.32 Т      |

3. Radia description
The simulation was performed using three-dimensional magnetostatic code – Radia [8]. This software was developed at the ESRF for the magnetic solutions of problems occurring in the design of magnetic lenses, undulators and wigglers. This Radia code is optimized for the tasks of developing namely the undulators and wigglers consisting of permanent magnets and electromagnets. The basis for the calculations in Radia is the boundary element method [9]. During the simulation, the consequent algorithm is to be followed:

1. Construction of the element geometry of the magnetic system.
2. Setting the magnetic properties of the element materials created in step 1.
3. Grouping the elements into containers.
4. If necessary, the geometry segmentation of the system elements into smaller blocks.
5. If necessary, the implementation of an additional transformation to the established system (shifts and/or rotations).
6. The magnetic field calculation generated by the magnet system as a whole or it individual elements.

After constructing the geometry, it is necessary to make a decision on segmenting or non-segmenting of the magnet system. The calculation method used in Radia assumes a uniform
magnetization in each element. For the permanent magnet, the magnetic permeability is close to 1 and
the magnetization is homogeneous enough for the element volume. However, for materials with high
magnetic permeability, magnetization is distributed unevenly. To solve this problem it is necessary
to make the partition of the elements into smaller blocks, i.e. segmentation. As a result, the element
will be divided into a number of smaller blocks, in which the magnetization will be considered as a
homogeneous one. The current version of Radia code does not provide a mechanism for automatic
segmentation. The users must specify the segmentation parameters by themselves. The procedure
should be stopped when the magnetic field in a given area becomes constant at the desired level of
accuracy. The developers recommend firstly to find a sufficient number of blocks in the horizontal
direction with no or minimal partition in the longitudinal and vertical directions, and secondly to find
a sufficient number of blocks in the longitudinal direction with the previously found value of blocks for
the horizontal direction. Segmentation can be applied directly to the whole geometry (i.e. to the
container, which combines all the elements) or to different parts of the geometry separately. The
simulation has showed that the segmentation is not required.

4. Simulation results
Figure 3a shows a three-dimensional magnetic field distribution in the X axis projection ($B_x$) in the
central plane of the wiggler. As it can be clearly seen from figure 3b, the magnetic field is not
symmetrical with respect to the YZ plane. This asymmetry in the magnetic field distribution is caused
by the trapezoidal geometry of the magnets, which makes up the wiggler, and their magnetization
directions.

![Figure 3. 3D wiggler fields in the central plane with the gap 30 mm width (a), side view (b).](image)

The distribution maxima and minima correspond to the period centers and edges respectively.
These extrema of the magnetic field in the YZ plane have a practically constant value and the
maximum value spread in the range from -10 to 10 mm along Y-axis is equal to about 4%.

Figure 4 demonstrates the magnetic induction projection on the X axis ($B_x$) calculated for the gap
width values between the wiggler poles equal to 30, 40, 50 and 60 mm. The maximum value of the
magnetic induction obtained in the simulation in the wiggler center for the 30 mm gap width is 0.432
T, and the measured value is 0.421 T [1]. The difference between these values is 2.6%. The reason for
this discrepancy could be the systematic error of an edge angle in the magnetic block manufacture and
not complete accounting of the magnet material properties, in particular, in the simulation, the
permeability was assumed to be 1 though the actual value is 1.05 in the direction of the magnetization
axis.
Figure 4. Wiggler magnetic field along the axis for different gap widths between the poles.

Figure 5. Dependence of the magnetic induction projection on the X axis from the gap width.

Figure 5 shows the magnetic induction projection on the X axis at the wiggler center (0, 0, 0) as a function of the gap width. With the increase of the gap between the poles, the field value decreases exponentially according to the following expression: $a \cdot e^{-b \cdot g}$, where $a = 2.079 \pm 4 \cdot 10^{-4}$ T and $b = 0.052392 \pm 5 \cdot 10^{-6}$ 1/mm. The maximum calculated value of the magnetic field in the wiggler center varies from 0.09 to 0.432 T for the gap width from 60 to 30 mm respectively.

Figure 6 demonstrates the magnetic induction projection ($B_x$). It was calculated for wiggler with 30 mm gap width along the charged particle beam trajectory. The solid curve corresponds to the case when the beam path coincides with the wiggler axis and the dashed curve corresponds to the magnetic induction calculated for the wiggler rotated at $\alpha = 1/\gamma = 3.548$ degrees angle around the Y axis relative to its center (0, 0, 0) in a counterclockwise direction (see figure 1), where $\gamma = 16.14$ is Lorentz factor of the electron. For comparison of the magnetic field along the beam trajectory at different wiggler positions, the regions O, A and B indicated in figure 6 were chosen which correspond to the wiggler center, the center and the edge of the last period.

Figure 6. Dependence of the projection magnetic induction ($B_x$) along the Z axis for the two wiggler positions.

Figure 7. Dependence of the magnetic induction projection ($B_x$) from the wiggler rotation angle for three locations on the wiggler axis.

Figure 7 shows the dependences of the $B_x$ component, when wiggler turning around the Y axis in the angle range from 0 to $2\gamma$ in radians, where $2\gamma$ is the maximum angle at which the wiggler does not touch the beam trajectory.

Table 2 to presents the relation ($r$) of wiggler magnetic fields as a function of the rotation angle ($\alpha$) relative to the X, Y and Z axes in the regions O, A and B respectively. The parameter $r$ is the ratio of
the magnetic induction $B_\alpha$ for the wiggler rotated around an axis at an angle $\alpha$ to the magnetic induction $B_x$ for the wiggler at $\alpha = 0$.

Table 2. Relation of wiggler magnetic fields as a function of the rotation angle.

| Value   | Y  | Rotation axis |
|---------|----|---------------|
| $\alpha$,˚ | 1  | 1 | 1 | -2 | -2 | -2 | 2 |
| $r(0)$  | 1  | 1 | 1 | 1 | 1 | 1 | 0.994 | 0.994 |
| $r$(max A) | 1.02 | 1.046 | 1.087 | 1.01 | 1.09 | 0.988 | 0.978 | 0.994 | 0.994 |
| $r$(min B) | 1.048 | 1.1 | 1.2 | 0.969 | 0.985 | 1.013 | 1.026 | 0.994 | 0.994 |

The simulation has shown that parameter $r$ does not depend on the rotation direction around the Y axis. The parameter $r$ varies depending on the rotation direction around the X-axis due to the internal structure of the wiggler that is the geometry of its magnetic elements. The parameter $r$ varies insignificantly and does not depend on the rotation direction around the Z axis.

5. Method comparison

Figure 8 shows the graphs of the magnetic induction projection on the X axis calculated for 30 mm gap width between the wiggler poles by Radia and Femm. Femm is a program for simulation of the magnetic field in the 2D geometry, which implements the finite element method. It is quite easy to learn, has the graphic user interface and does not require knowledge of programming languages [4].

![Figure 8](image.png)

Figure 8. Comparison of the $B_x$ component of the magnetic field calculated in Radia (solid line) and Femm (dashed line).

The maximum value of the magnetic field in the wiggler center obtained by means of Radia is 0.432 T and Femm is 0.423 T. The difference between these values and the measured value is 2.6 and 0.5% respectively. Apparently, this discrepancy can not only be attributed to the various numerical methods implemented in these programs but also how to set the material magnetic properties, which are different from each other in these applications.

6. Conclusions

Thus, we can conclude that a wiggler model of the LUCX accelerator is developed. This model allows us to calculate the characteristics of the wiggler magnetic field depending on the following parameters of wiggler geometry: the gap width and the rotation angles relative to the beam trajectory. Moreover, on the basis of Radia functional, it is quite easy to move wiggler relative to the beam trajectory, which can be interpreted as a mistake of the wiggler installation or the beam shift relative to wiggler central axis.
The magnetic field is not symmetrical with respect to YZ plane due to the magnet geometry of the wiggler. The magnetic field varies exponentially with changing the gap width. Changing the magnetic field ($B_x$) in the range from 0.09 to 0.432 T undulator radiation fundamental frequency is changed from 14.5 to 4 THz.

The inaccuracy in the wiggler position adjustment for the rotation around the Y axis mostly affects the magnetic field value along the beam trajectory. The inaccuracies in the adjustment for the rotation around the X and Z axes do not noticeably affect the magnetic field along the beam trajectory. Wiggler rotation about axes X, Y and Z at the angle less than 0.5 degrees leads to the change in the magnetic field ($B_x$) not more than 1%.

The magnetic field ($B_x$) obtained by Radia based on the boundary element method, consistent with an accuracy of about 2% with the results obtained by Femm based on the finite element method. However, calculations carried out in Radia are preferred due to it wider range of options.

In the future we plan to simulate the trajectory of the electrons [11] passing through the wiggler at different state of the initial electrons with the help of Radia functionality.

7. References

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