Fine structure of the nuclear scissors mode  
(new type of collective motion)

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Abstract. The coupled dynamics of low-lying modes (including scissors) and various giant quadrupole resonances are studied with the help of Wigner Function Moments method generalized to take into account spin degrees of freedom. Equations of motion for collective variables are derived on the basis of Time Dependent Hartree-Fock equations in the model of harmonic oscillator with spin orbital interaction plus quadrupole-quadrupole residual interaction. Enlisting the spin allows one to introduce into consideration new types of nuclear collective motion.

1. Introduction  
The nuclear scissors mode was predicted [1, 2, 3] as the collective motion of two types of nucleons – all protons accomplish rotational vibrations with respect of all neutrons. Its theoretical and experimental aspects were discussed in many papers from various points of view (see the review [4] and references therein.) Here we will consider the Wigner Function Moments (WFM) method. On the one hand it is the pure microscopic method, because it is based on the Time Dependent Hartree-Fock (TDHF) equation. On the other hand the method works with average values (moments) of operators which have the direct relation to the considered phenomenon, that makes it the ideal instrument to describe the basic characteristics (energies and excitation probabilities) of collective excitations [5] and, in particular, the scissors mode [6]. Our investigations have shown that already the minimal set of collective variables (the nucleus angular momentum and nucleus quadrupole moments in the coordinate and momentum spaces) is sufficient to reproduce the most important properties of the scissors mode: its inevitable coexistence with the IsoVector Giant Quadrupole Resonance (IVGQR) and with the Fermi surface deformation.

Further improvement of WFM method, namely, the switch-over from TDHF to TDHFB equation, i.e. taking into account pair correlations, allowed us to improve considerably the quantitative description of the scissors mode: for rare earth nuclei the energies are reproduced with \(\sim 10\%\) accuracy and \(B(M1)\) factors decreased practically two times (though exceed nevertheless experimental values practically two times). The reason of the last discrepancy is hidden probably in spin degrees of freedom, which were ignored by WFM method until now. One can not exclude, that due to spin dependent interactions some part of the force of \(M1\) transitions is shifted to the energy region of 5-10 MeV, where \(1^+\) resonance of the spin nature is observed. The generalization of WFM method to take into account spin degrees of freedom is the goal of this paper. At a first step we include in the consideration only the spin orbital interaction, as the most important one among all possible spin dependent interactions because it...
enters into the mean field. It became clear already on the stage of formulation of the equations of motion for new collective variables, that we have met the new (at least not discussed until now) type of collective motion, or maybe several ones.

2. TDHF equation and model Hamiltonian

One starts with the time dependent HF equation in matrix form \[7\]

\[
i \hbar < \mathbf{r}, s | \hat{\rho} | \mathbf{r}''', s'''' > = \sum_{s'} \int d^3 \mathbf{r}' \left( < \mathbf{r}, s | \hat{h} | \mathbf{r}', s' > < \mathbf{r}', s' | \hat{\rho} | \mathbf{r}''', s'''' > - < \mathbf{r}, s | \hat{\rho} | \mathbf{r}', s' > < \mathbf{r}', s' | \hat{h} | \mathbf{r}''', s'''' > \right).
\]

By means of the Wigner transformation the following set of equations for the Wigner functions with specified spin indices is derived

\[
i \hbar \hat{f}^{\uparrow\uparrow} = i \hbar \{ \hat{f}^{\uparrow\uparrow}, f^{\uparrow\uparrow} \} + \hbar^{\uparrow\downarrow} f^{\downarrow\uparrow} f^{\uparrow\downarrow} - f^{\uparrow\downarrow} h^{\uparrow\uparrow} + \frac{i \hbar}{2} \{ \hat{h}^{\uparrow\downarrow}, f^{\uparrow\downarrow} \} - \frac{i \hbar}{2} \{ f^{\uparrow\downarrow}, h^{\uparrow\downarrow} \} - \hbar^{\uparrow\downarrow} \left( \hbar^{\uparrow\downarrow} - f^{\downarrow\uparrow} f^{\uparrow\downarrow} \right) + \frac{i \hbar}{2} \{ \hat{h}^{\uparrow\downarrow}, f^{\downarrow\uparrow} f^{\uparrow\downarrow} \} + \frac{i \hbar}{2} \{ f^{\downarrow\uparrow} f^{\uparrow\downarrow}, h^{\downarrow\uparrow} \} + \frac{i \hbar}{2} \{ h^{\downarrow\uparrow}, f^{\downarrow\uparrow} f^{\uparrow\downarrow} \} + \frac{i \hbar}{2} \{ \hat{h}^{\downarrow\uparrow}, \hbar^{\downarrow\uparrow} \} \] (2)

where \(\uparrow\) means \(s = 1/2\) and \(\downarrow\) means \(s = -1/2\), the functions \( f(\mathbf{r}, \mathbf{p}), \hat{f}(\mathbf{r}, \mathbf{p}) \) are the Wigner transforms of \( h, \hat{h} \) respectively, \( \{ f, h \} \) is the Poisson bracket of the functions \( f \) and \( h \) and \( \{ \{ f, h \} \} \) is their double Poisson bracket; the dots stand for terms proportional to higher powers of \( \hbar \). Hereinafter we omit \((\mathbf{r}, \mathbf{p})\)-dependence and isotopic indices of variables. Two more equations are obtained by the obvious change of arrows \(\uparrow\downarrow\).

We define the functions \( f^+ = f^{\uparrow\uparrow} + f^{\downarrow\downarrow} \) and \( f^- = f^{\uparrow\downarrow} - f^{\downarrow\uparrow} \) as spin-scalar and spin-vector ones respectively and introduce the notation \( f^u = f^{\uparrow\uparrow}, f^d = f^{\downarrow\downarrow}, h^z = h^{\uparrow\uparrow} \pm h^{\downarrow\downarrow} \). We get:

\[
i \hbar \hat{f}^+ = \frac{i \hbar}{2} \{ h^+, f^+ \} + \frac{i \hbar}{2} \{ h^-, f^- \} + \frac{i \hbar}{2} \{ h^{\uparrow\downarrow}, f^d \} + \frac{i \hbar}{2} \{ h^{\downarrow\uparrow}, f^u \} + \frac{i \hbar}{2} \{ \hat{h}^{\uparrow\downarrow}, f^d \} + \frac{i \hbar}{2} \{ h^{\downarrow\uparrow}, \hat{f}^+ \} + \frac{i \hbar}{2} \{ h^+, f^u \} + \frac{i \hbar}{2} \{ \hat{h}^{\uparrow\downarrow}, f^u \} + \frac{i \hbar}{2} \{ h^{\downarrow\uparrow}, f^d \} + \frac{i \hbar}{2} \{ \hat{h}^{\downarrow\uparrow}, f^u \} + \frac{i \hbar}{2} \{ h^+, f^d \} \] (3)

The microscopic Hamiltonian of the model is the harmonic oscillator with spin-orbital interaction plus separable quadrupole-quadrupole residual interaction

\[
H = \sum_{i=1}^{A} \left( \frac{P_i^2}{2m} + \frac{1}{2} m \omega^2 r_i^2 - \eta(r_i) I_i S_i \right) + \bar{\kappa} \sum_{\mu=-2}^{2} (-1)^\mu \sum_{i<j}^{Z} q_{2-\mu}(r_i) q_{2\mu}(r_j) \]
\[
+ \frac{1}{2} \bar{\kappa} \sum_{\mu=-2}^{2} (-1)^\mu \left\{ \sum_{i \neq j}^{Z} q_{2-\mu}(r_i) q_{2\mu}(r_j) + \sum_{i \neq j}^{N} q_{2-\mu}(r_i) q_{2\mu}(r_j) \right\},
\]

where \( N \) and \( Z \) are the numbers of neutrons and protons, respectively. The quadrupole operator \( q_{2\mu} \) can be written as the tensor product: \( q_{2\mu}(r) = \sqrt{16\pi/5} r^2 Y_{2\mu}(\theta, \phi) = \sqrt{6} \{ r \otimes r \}_{2\mu} \), where
\( \{ r \otimes r \} \lambda_{\mu} = \sum_{\sigma, \nu} C_{\sigma, \mu}^{\lambda} C_{1, \sigma, 1, \nu} r_{\sigma} r_{\nu}, \) \( C_{\sigma, \mu}^{\lambda} \) is the Clebsch-Gordan coefficient and \( r_1, r_0, r_{-1} \) are cyclic coordinates [8]. The mean field potential for protons (or neutrons) is
\[
V_\tau = \frac{1}{2} m \omega^2 r^2 + \sum_{\mu = -2}^{2} (-1)^\mu Z_{2-\mu}^\tau \{ r \otimes r \}_{2\mu} - \eta (r) \sum_{\mu = -1}^{1} (-)^{\mu} \hat{l}_\mu \hat{S}_{-\mu}, \quad \tau = n, p.
\] (5)

Here \( \hat{l}_\mu \) are angular momentum components, \( \hat{S}_\mu \) are spin matrices [8] and
\[
Z_{2\mu}^{n,+} = \chi R_{2\mu}^{p,+} + \bar{\chi} R_{2\mu}^{p,-}, \quad Z_{2\mu}^{p,+} = \chi R_{2\mu}^{p,+} + \bar{\chi} R_{2\mu}^{p,-}, \quad \chi = 6\kappa, \quad \bar{\chi} = 6\kappa,
\]
\[
R_{\lambda\mu}^{\tau}(t) = (2\pi\hbar)^{-3} \int d^3 p \int d^3 r \{ r \otimes r \}_{\lambda\mu} f^{\tau+}(r, p, t).
\] (6)

### 3. Equations of motion

Integrating the set of equations (3) over the phase space with the weights \( W = \{ r \otimes p \}_{\lambda\mu}, \{ p \otimes p \}_{\lambda\mu} \) and 1 one gets the dynamical equations for the following moments of second and zero orders – collective variables of the method:
\[
L_{\lambda\mu}^\sigma(t) = \int d(p, r) \{ r \otimes p \}_{\lambda\mu} f^\sigma(r, p, t), \quad R_{\lambda\mu}^\tau(t) = \int d(p, r) \{ r \otimes r \}_{\lambda\mu} f^\tau(r, p, t),
\]
\[
P_{\lambda\mu}^\sigma(t) = \int d(p, r) \{ p \otimes p \}_{\lambda\mu} f^\sigma(r, p, t), \quad F^\sigma(t) = \int d(p, r) f^\sigma(r, p, t),
\] (7)

where \( \int d(p, r) \equiv (2\pi\hbar)^{-3} \int d^3 p \int d^3 r \) and \( \sigma = +, -, u, d \). The required expressions for \( h^\pm, h^{\mp} \) and \( h^\pm \) are
\[
h^+ = \frac{p^2}{2m} + m \omega^2 r^2 + 2 \sum_{\mu} (-1)^\mu Z_{2-\mu}^{r+} \{ r \otimes r \}_{2-\mu}, \quad h^- = -\hbar \eta l_0, \quad h^{\mp} = -\hbar \sqrt{2} \eta l_1, \quad h^+= \frac{h}{\sqrt{2}} \eta l_1.
\]

We are interested in the scissors mode, the excitation with \( K^\pi = 1^+ \), therefore we need only the set of equations with \( \mu = 1 \). These equations are nonlinear and are solved in a small amplitude approximation.

Let us remind about isospin indices \( \tau = n, p \). One implies that all variables and equilibrium characteristics \( R_{20}^\pi(eq) \) and \( Z_{20}^\pi(eq) \) in equations have such indices. All difference between neutron and proton systems sits in the mean field characteristic \( Z_{20}^\pi(eq) \), which is different for neutrons and protons (see (6)). It is convenient to rewrite the equations in terms of the isovector \( X_{\lambda\mu} = X_{\lambda\mu}^n - X_{\lambda\mu}^p \) and isoscalar \( X_{\lambda\mu} = X_{\lambda\mu}^n + X_{\lambda\mu}^p \) variables. Isovector and isoscalar strength constants \( \chi_1 = (\chi - \bar{\chi})/2 \) and \( \chi_0 = (\chi + \bar{\chi})/2 \) are connected by the relation \( \chi_1 = \alpha \chi_0 \) [6]. Neglecting by the fourth order moments, generated by the term \( h^{\mp} f^{\tau+} - f^{\alpha+} h^\tau \) of (2) we get the closed set of 19 equations. As an example we demonstrate here only the small part of them, which in the case of \( \eta = 0 \) becomes closed:
\[
\hat{\mathcal{L}}_{21}^+ = \frac{1}{m} \hat{P}_{21}^+ - \left[ m \omega^2 - \frac{2}{3} \alpha \chi_0 R_{20}^\pi(eq) + \frac{1}{\sqrt{6}} (1 + \alpha) \chi_0 R_{20}^\pi(eq) \right] \hat{R}_{21}^+ - \hbar \eta \frac{2}{\sqrt{3}} \hat{L}_{22}^+ + 2 \sqrt{2} \hat{L}_{21}^+= \frac{m}{\sqrt{2}} \hat{R}_{21}^+ + 2 \hat{R}_{22}^+ + 2 \sqrt{2} \hat{L}_{20}^d,
\]
\[
\hat{\mathcal{L}}_{11}^+ = \frac{\sqrt{3}}{\sqrt{2}} (1 - \alpha) \chi_0 R_{20}^\pi(eq) \hat{R}_{21}^+ - \hbar \eta \frac{2}{\sqrt{3}} \hat{L}_{11}^+ + \sqrt{2} \hat{L}_{10}^d,
\]
\[
\hat{R}_{21}^+ = \frac{2}{m} \hat{L}_{21}^+ - \hbar \eta \frac{2}{\sqrt{3}} \hat{R}_{21}^- + 2 \hat{R}_{22}^u + 2 \sqrt{2} \hat{R}_{20}^d,
\]
\[
\hat{P}_{21}^+ = -[m \omega^2 + \sqrt{\frac{3}{2} \chi_0 R_{20}^\pi(eq)}] \hat{L}_{21}^+ + \sqrt{6} \chi_0 R_{20}^\pi(eq) \hat{L}_{11}^+ - \hbar \eta \frac{2}{\sqrt{3}} \hat{P}_{21}^- + \hat{P}_{22}^u + 2 \sqrt{2} \hat{P}_{20}^d.
\]
The isoscalar set of equations is easily obtained from (8) by taking $\alpha = 1$.

4. Energies and excitation probabilities

Imposing the time evolution via $e^{i\Omega t}$ for all variables one transforms dynamical equations into a set of algebraic equations. Eigenfrequencies are found as the zeros of its determinant. Excitation probabilities are calculated with the help of the linear response theory.

Table 1. Energies and excitation probabilities of the isovector system, calculated for $^{164}$Er with two values of the spin-orbital interaction constant $\eta$. The quantum numbers (including indices $+, -, u, d$) of variables responsible for the generation of the present level are shown in a first column of Table 1b.

| $E_{iv}$, MeV | $B(M1)$, $\mu^2_N$ | $B(E2)$, $B_W$ |
|---------------|------------------|----------------|
| 1.61          | 3.54             | 0.12           |
| 2.18          | 5.33             | 1.02           |
| 12.80         | 0.01             | 0.04           |
| 14.50         | 0.01             | 0.03           |
| 16.18         | 0.02             | 0.18           |
| 16.20         | 0                | 0              |
| 20.59         | 2.78             | 35.45          |

The results of calculations for the isovector system are demonstrated in the table 1a. They will be discussed in comparison with the results for $\eta = 0$ (table 1b). In this case the isovector set of equations splits into four independent subsets.

**The first one** (8) operates with spin-scalar variables $X^{+ \mu}_\sigma$ and describes the joint dynamics of the ”standard” nuclear scissors mode ($E=2.07$ MeV) and IVGQR ($E=20.55$MeV) [6].

**The second subset** operates with spin-vector variables $X^{- \mu}_\sigma$ and describes the relative motion of spin up nucleons with respect of spin down nucleons. The respective eigenfrequencies are

$$\Omega_\pm^2 = 2 \bar{\omega}^2 \left[ 1 + \delta/3 \pm \sqrt{(1 + \delta/3)^2 - \delta^2} \right].$$

The low-lying level has $E_{11}^{+} (low) = \hbar \Omega_- = 1.70$ MeV and the high-lying level has $E_{21}^{-} (high) = \hbar \Omega_+ = 14.50$ MeV. So, in addition to the well known ("standard") scissors mode with energy $E_{11}^{+} = 2.07$ MeV we get the new type of the scissors mode – spin scissors. The inclusion of the spin-orbital interaction only slightly repulses these levels. However, the main role of the spin-orbital interaction consists in the excitation of the new scissors mode. This mode is excited only via the coupling with the ”standard” scissors mode by means of the spin-orbital interaction. The new scissors mode is accompanied by the high lying excitation, which can be called the isovector spin-vector giant quadrupole resonance.

Third and fourth subsets operate with variables $X^u_\mu$ and $X^d_\mu$. They describe the spin-flip modes: giant quadrupole and monopole spin-flip resonances with energies $E_{20} = E_{22} = 16.20$ MeV and $E_{00} = 12.81$ MeV respectively.

The results of calculations for the isoscalar system are demonstrated in table 2.

The most interesting fact here is the appearance of the new low-lying mode with $E=0.39$ MeV which describes the relative motion of the orbital angular momentum and spin of the nucleus. Another new excitation with $E=1.73$ MeV corresponds to the rotational oscillations of all spin up nucleons out of phase with all spin down nucleons.

All high lying modes are just the isoscalar analogues of the respective isovector modes.
Table 2. The same as in table 1 for the isoscalar system.

| $E_{\text{in}}, \text{ MeV}$ | $B(M1)$, $\mu_N^2$ | $B(E2)$, $B_W$ |
|--------------------------|-----------------|-----------------|
| 1.73                     | 0.07            | 1.12            |
| 0.39                     | 0.24            | 117.19          |
| 12.83                    | 0               | 0.66            |
| 14.51                    | 0               | 0.12            |
| 16.20                    | 0               | 0               |
| 16.22                    | 0               | 0.20            |
| 10.28                    | 0               | 66.50           |

Table 2a. $\eta = 0.361$ MeV

Table 2b. $\eta = 0$ MeV

| $(\lambda, \mu)^{\sigma}$ | $E_{\text{in}}, \text{ MeV}$ | $B(M1)$, $\mu_N^2$ | $B(E2)$, $B_W$ |
|----------------------------|-------------------------------|-------------------|-----------------|
| (1,1)$^-$                  | 1.70                         | 0                 | 0               |
| (1,1)$^+$                  | 0                            | –                 | –               |
| (0,0)$^d$                  | 12.81                        | 0                 | 0               |
| (2,1)$^-$                  | 14.51                        | 0                 | 0               |
| (2,2)$^u$                  | 16.20                        | 0                 | 0               |
| (2,0)$^d$                  | 16.20                        | 0                 | 0               |
| (2.1)$^+$                  | 10.33                        | 0                 | 67.47           |

5. Conclusion

The WFM method is applied for a first time to solve the HF equation with spin. The model Hamiltonian is a harmonic oscillator with the spin-orbital interaction plus quadrupole-quadrupole residual interaction. Spin dependent collective variables are defined and the closed set of nonlinear dynamical equations for them is derived. Equations are solved in a small amplitude approximation. Two isovector and two isoscalar low-lying eigenfrequencies and five isovector and five isoscalar high-lying eigenfrequencies are found. Three low-lying levels correspond to the excitations of the new nature, which was not known earlier. For example the isovector level with energy $E=1.61$ MeV describes the rotational oscillations of all nucleons with the spin projection "up" with respect of all nucleons with the spin projection "down", i.e. one more nuclear scissors. It is necessary to note the curious feature of the new mode. Being isovector one it describes also the motion of protons with respect of neutrons and, therefore, the resulting motion can be characterized as the rotational oscillations of proton "spin" scissors with respect of neutron "spin" scissors. In the analogous isoscalar mode with the energy $E=1.73$ MeV the proton and neutron "spin" scissors move in phase.

One more new low lying mode with energy $E=0.39$ MeV is generated by relative motion of the orbital angular momentum and spin of the nucleus (they can change their absolute values and directions keeping the total angular momentum unchanged).

There are ten high-lying excitation, the two of them being really new: isovector and isoscalar "spin-vector" resonances with energies $E=14.5$ MeV. Another six high-lying modes, according to the physical sense of variables which generate them, are spin-flip ones.

The study of all mentioned excitations with pair correlations and proper residual interactions included will be the natural continuation of this work.

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