**A Comparison of different measures for dynamical event-mean transverse momentum fluctuation**

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Various measures for the dynamical event-mean transverse momentum fluctuation are compared with the real dynamical fluctuation using a Monte Carlo model. The variance calculated from the G-moment can reproduce the dynamical variance exactly provided the particles are emitted independently.

The realization of the subtraction procedure is that the dynamical transverse-momentum distribution in the event-by-event fluctuations of transverse momentum can also serve as an approximate measure after divided by the square root of mean multiplicity.

The so-called Φ(t) and Φ(qgp) on the theoretically predicted quark gluon plasma (QGP) phase transition will result in an anomalous behavior of the phase transition. It is expected that the appearance of the Φ_m can also serve as an approximate measure after divided by the square root of mean multiplicity.

\[ \Phi(t) = \int_\Delta p_m(p_t)dp_t = \frac{M}{m=1}(p_m)p_m, \]

\[ \Phi(qgp) = \sum_{m=1}^{M}(p_m)n_m/n. \]

Since the multiplicity in a single event is finite, there are inevitably statistical fluctuations (sf) in the event-by-event analysis of transverse momentum. Various methods have been proposed to get rid of the statistical fluctuations and measure the dynamical ones. Most of them are based on a subtraction procedure, i.e., first to estimate the variance of statistical fluctuations and then subtract it from that of the experimental data. The basic problem in this kind of methods is how to reliably estimate the statistical variance.

Another approach proposed recently is to eliminate the sf directly from the experimental data. This method can reproduce the dynamical variance exactly provided the sf are Poissonian, i.e., the particles are emitted independently.

The aim of this paper is to compare the different measures proposed in the market and, in particular, to examine the reliability of the estimation of statistical variance. The so-called Φ_m will also be discussed and a new measure Φ^{(r)}_m will be proposed, which can largely reduce the strong multiplicity dependence of Φ_m itself.

Let \( p(p_t) \) be the dynamical transverse-momentum distribution in single events and

\[ p_m = \int_{\Delta_m} p(p_t)dp_t, \quad (m = 1, 2, \ldots, M) \]

be the corresponding “coarse-grained” distribution. The realization of \( p(p_t) \) in experiment is the distribution of the total number \( n \) of particles in the \( p_t \) region \( \Delta \), and

\[ q_m = n_m/n \quad (m = 1, 2, \ldots, M) \]

is an evaluation of \( p_m \), where \( n_m \) is the number of particles falling into the \( m \)th bin. Thus the dynamical and experimentally-measured event-mean transverse momentum are, respectively,

\[ \tilde{p}_m^{\text{dyn}} = \int_{\Delta} p_m(p_t)dp_t = \frac{M}{m=1}(p_m)p_m, \]

\[ \tilde{p}_m^{\text{exp}} = \sum_{m=1}^{M}(p_m)n_m = \sum_{m=1}^{M}(p_m)n_m/n. \]

where \( (p_m)^n \) is the \( p_n \) value in the \( m \)th bin. The event-space moments of \( \tilde{p}_m^{\text{dyn}} \) and \( \tilde{p}_m^{\text{exp}} \) are

\[ C_p^{\text{dyn}}(\tilde{p}_m) = \langle (\tilde{p}_m)^p \rangle = \left\langle \left( \sum_{m=1}^{M}(p_m)n_m \right)^p \right\rangle, \]

\[ C_p^{\text{exp}}(\tilde{p}_m) = \langle (\tilde{p}_m)^p \rangle = \left\langle \left( \sum_{m=1}^{M}(p_m)n_m \right)^p \right\rangle, \]

respectively. These two are evidently unequal due to the existence of sf. In particular, the experimental moments \( C_p^{\text{exp}}(\tilde{p}_m) \) depend crucially on the particle multiplicity, which is mainly due to sf. Therefore, subtraction procedures are commonly used to extract the dynamical fluctuation of \( \tilde{p}_m \).

The basic idea of the subtraction procedure is that the variances \( \sigma^2(\tilde{p}_m) = C_2(\tilde{p}_m) - C_1(\tilde{p}_m) \) of dynamical and statistical fluctuations are additive

\[ \sigma^2_{\tilde{p}_m, \text{data}} = \sigma^2_{\tilde{p}_m, \text{dyn}} + \sigma^2_{\tilde{p}_m, \text{stat}}, \]

where \( \sigma^2_{\tilde{p}_m, \text{data}}, \sigma^2_{\tilde{p}_m, \text{dyn}} \) and \( \sigma^2_{\tilde{p}_m, \text{stat}} \) are the experimental, dynamical and statistical variances, respectively. This additivity is true if the DF and SF are independent, which

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is a reasonable assumption. Therefore, the main problem in the subtraction procedure for extracting the dynamical variance is to estimate the statistical ones. Various methods have been proposed for this purpose.

For example, in Ref. 4 the results from mixed events are considered as the baseline for the random distribution and the difference in the fluctuation from a random distribution defined as

\[ d = \omega_{\text{data}} - \omega_{\text{baseline}} \]  

is taken as a measure of the dynamical fluctuation. In Eq. (8)

\[ \omega = \frac{\sigma_{\bar{p}_t}^2}{\langle \bar{p}_t \rangle} \equiv \frac{\sqrt{\sigma_{\bar{p}_t}^2}}{\langle \bar{p}_t \rangle}, \]  

\( \bar{p}_t \) is the mean transverse momentum in a single event and \( \langle \cdot \rangle \) denotes the average over event sample.

Alternatively, in Ref. 5 the statistical variance of event mean \( p_t \), under the assumption of independent particle production, is estimated as

\[ \sigma_{p_t,\text{stat}}^2 = \frac{\sigma_{p_t,\text{incl}}^2}{N}, \]  

and, therefore, the dynamical variance is equal to

\[ \sigma_{p_t,\text{VKR}}^2 = \sigma_{p_t,\text{data}}^2 - \sigma_{p_t,\text{stat}}^2 = \frac{\sigma_{p_t,\text{data}}^2}{N} - \sigma_{p_t,\text{incl}}^2/n, \]  

(11)

A somewhat different expression is used in Ref. 6, where the experimental variance is weighted by event multiplicity \( N_j \):

\[ \sigma_{p_t,\text{CERES}}^2 = \frac{\sum_{j=1}^{N} N_j (\bar{p}_t^j - \langle \bar{p}_t \rangle)^2}{\sum_{j=1}^{N} N_j} - \sigma_{p_t,\text{incl}}^2/n, \]  

(12)

where \( N \) is the total number of events.

A widely used measure for the non-statistical mean \( p_t \) fluctuation is the \( \Phi_{p_t} \) proposed in Ref. 5

\[ \Phi_{p_t} \equiv \sqrt{\langle Z^2 \rangle/n} - \sqrt{\langle z^2 \rangle}, \]  

(13)

where \( z \) and \( Z \) are defined as \( z \equiv p_t - \langle p_t \rangle \) for each particle and \( Z \equiv \sum_{i=1}^{n} z_i = n(\bar{p}_t - \langle p_t \rangle) \) for each event, respectively. The second term for the r.h.s. of Eq. (13) is the square root of the inclusive variance \( \sigma_{p_t,\text{incl}}^2 = \langle (p_t - \langle p_t \rangle)^2 \rangle \). Assuming that the multiplicity fluctuation is uncorrelated with the \( p_t \) fluctuation, we get from Eq. (13)

\[ \Phi_{p_t} = \sqrt{(n^2)/n} \sigma_{p_t,\text{data}} - \sigma_{p_t,\text{incl}}. \]  

(14)

This equation is evidently similar to Eq. (11).

All of the above measures have the same structure, being based on a subtraction procedure, i.e. to subtract the variance of \( \bar{p}_t \) or a quantity related to it, that will be expected from a pure statistical system, from the same quantity obtained in experiment. These measures will, of course, vanish for a pure statistical system, and a non-vanishing value of them will indicate the existence of dynamical effect. Therefore, the measures based on the subtraction procedure, as those listed above, will at least qualitatively measures the effect of DF.

In order to have an idea on how the various subtraction methods work quantitatively, let us carry on a Monte Carlo simulation using a toy model. In this model the distribution of \( p_t \) is taken as

\[ p(p_t) = \frac{4}{a^2} p_t e^{-2p_t/a}, \]  

(15)

with a Gaussian distributed parameter \( a (\sigma^2(a) = 0.24) \). In total 20 \( \times \) 500,000 events are generated for 20 average multiplicities. The result of dynamical and experimental variances \( \sigma_{p_t,\text{dyn}}^2 \) and \( \sigma_{p_t,\text{exp}}^2 \) are plotted in Fig. 1 as dashed line and open triangles, respectively. For comparison, the \( \sigma_{p_t,\text{VKR}}^2 \) and \( \sigma_{p_t,\text{CERES}}^2 \) are also plotted in the same figure as upward and downward solid triangles, respectively.

The differences between the experimental and dynamical variances \( \sigma_{p_t,\text{exp}}^2 - \sigma_{p_t,\text{dyn}}^2 \) are due to SF. It can be seen from the figure that the \( \sigma_{p_t,\text{VKR}}^2 \) indeed reduces the SF, especially when the average multiplicity is not very low; while the \( \sigma_{p_t,\text{CERES}}^2 \) overshoots the influence of SF.

On the other hand, the open circles shown in the figure reproduce the dynamical variances very well. They are obtained through a totally different approach 7, which is based on the elimination of the SF directly from the experimental data.
For the first order moment the elimination of SF is straightforward,

\[ C_1^{(\text{dyn})}(\bar{p}_k) = G_1(\bar{p}_k) = \left\langle \sum_{m=1}^{M} (p_k)_m \frac{n_m}{\langle n \rangle} \right\rangle, \quad (16) \]

\[ C_2^{(\text{dyn})}(\bar{p}_k) = \left\langle \left( \sum_{m=1}^{M} (p_k)_m p_m \right)^2 \right\rangle = \left\langle \sum_{m=1}^{M} (p_k)_m^2 p_m^2 \right\rangle + \left\langle \sum_{m \neq m'} (p_k)_m (p_k)_{m'} p_m p_{m'} \right\rangle. \quad (17) \]

Using the formulae

\[ \left\langle \sum_{m=1}^{M} f_m p_m^2 \right\rangle = \left\langle \sum_{m=1}^{M} f_m \frac{n_m(n_m-1) \cdots (n_m-p+1)}{\langle n \rangle^p} \right\rangle, \quad (18) \]

which holds for Poissonian SF, we then get \( C_2^{(\text{dyn})}(\bar{p}_k) = G_2(\bar{p}_k) \), where

\[ G_2(\bar{p}_k) = \left\langle \sum_{m=1}^{M} (p_k)_m^2 \frac{n_m(n_m-1)}{\langle n \rangle^2} \right\rangle + \left\langle \sum_{m \neq m'} (p_k)_m (p_k)_{m'} \frac{n_m n_m}{\langle n \rangle^2} \right\rangle. \quad (19) \]

This guarantees the equality of \( G \)-variance \( \sigma_{\bar{p}_k G}^2 = G_2(\bar{p}_k) - G_1(\bar{p}_k)^2 \) to dynamical ones \( \sigma_{\bar{p}_k \text{dyn}}^2 = C_2^{(\text{dyn})}(\bar{p}_k) - C_1^{(\text{dyn})}(\bar{p}_k)^2 \).

On the contrary, the SF cannot be exactly gotten rid of using the subtraction procedure. The main problem lies in the estimation of statistical variance by Eq.\( (10) \). This provided the SF is Poissonian. The key point in eliminating SF in the second order moment is to expand the 2nd power in the definition Eq.\( (5) \) of \( C_2^{(\text{dyn})}(\bar{p}_k) \),

\[ \Phi_{\bar{p}_k} = \frac{\Phi_{\bar{p}_k}}{\sqrt{\langle n \rangle}} \approx \sigma_{\bar{p}_k \text{data}} - \frac{\sigma_{\bar{p}_k \text{incl}}}{\sqrt{\langle n \rangle}}. \quad (20) \]

cf. Eq.\( (11) \).

The Monte Carlo results of \( \Phi_{\bar{p}_k} \) are shown in Fig.3 as upward solid triangles together with \( \sigma_{\bar{p}_k \text{CERES}} = \sqrt{\sigma_{\bar{p}_k \text{CERES}}^2} \) (downward triangles) and \( \sigma_{\bar{p}_k \text{dyn}} \) (dashed...
FIG. 3: Variation of $\Phi_p$ and $\Phi_{r}$ with $\langle n \rangle$, compared with $\sigma_{\bar{p}CERES}$ and $\sigma_{\bar{p}dyn}$ line. It can be seen from the figure that $\Phi_{r}$ has a much weaker multiplicity dependence and is closer to $\sigma_{\bar{p}dyn}$ than $\Phi_p$. Therefore, $\Phi_{r}$ is a better measure of $df$ than $\Phi_p$ itself.

In this paper, various measures for the dynamical transverse momentum fluctuation in event-by-event analysis are compared with the dynamical fluctuation using a Monte Carlo model.

It turns out that the $\sigma^2_{\bar{p}G}$ calculated from the G-moments coincides with the dynamical variance $\sigma^2_{\bar{p}dyn}$, showing that the G-moment method is effective in eliminating the statistical fluctuations coming from the finite number of particle in a single event and thus provides a good measure for the dynamical fluctuations.

The $\sigma^2_{\bar{p}VKR}$ proposed by S. A. Voloshin, V. Koch and H. G. Ritter [5] and its revised version $\sigma^2_{\bar{p}CERES}$ [6] are found to be good approximate measures for the dynamical fluctuation when the multiplicity is not very low. Therefore, their application to relativistic heavy ion experiments is justified.

The $\Phi_{r}$ divided by the square root of average multiplicity ($\Phi_{r} = \Phi_p / \sqrt{\langle n \rangle}$) is another approximate measure for the dynamical fluctuation, which depends on multiplicity weaker than $\Phi_p$ and is closer to the square root of dynamical variance $\sigma_{\bar{p}dyn}$.

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