Underwater Acoustic Signal Prediction Based on Correlation Variational Mode Decomposition and Error Compensation

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Abstract

Underwater acoustic signal is highly complex and difficult to predict. To improve the prediction accuracy of underwater acoustic signal, a complex underwater acoustic signal prediction method combining correlation variational mode decomposition (CVMD), least squares support vector machine (LSSVM) and Gaussian process regression (GPR) is proposed. Aiming at the problem of sample partitioning, this paper proposes a method of obtaining the embedding dimension and time delay based on the extreme learning machine prediction model. By selecting the appropriate time delay and embedding dimension, the prediction accuracy has improved. Aiming at the $K$-value selection of variational mode decomposition (VMD), this paper proposes a CVMD decomposition method, which improves the adaptability of VMD algorithm by selecting $K$-value through the correlation coefficient. Firstly, CVMD is used to decompose the underwater acoustic time series into several different components. Then, LSSVM prediction models are established for each component. Finally, to further improve the prediction accuracy of the model, Gaussian process regression (GPR) is used to correct the prediction result. One-step and multi-step prediction of underwater acoustic time series is carried out in this paper. Simulation results show that the model proposed in this paper has high prediction accuracy and can be effectively used in underwater acoustic signal prediction.

Index Terms

Underwater acoustic signal, variational mode decomposition, least squares support vector machine, Gaussian process regression, prediction.

I. INTRODUCTION

Underwater acoustic signal prediction is the basis of underwater acoustic signal processing, which can be applied to many aspects such as noise reduction, detection and feature extraction of underwater target signal [1]–[4]. Underwater acoustic signal is highly complicated by the influence of marine noise and mechanical noise. The improvement of its prediction accuracy is an urgent problem. If the prediction accuracy can be improved, then underwater target signal with a lower signal-to-noise ratio can be detected, target signal features can be more accurately extracted, and a better noise reduction effect can be achieved. Therefore, researching the prediction of underwater acoustic signal is of great significance in underwater acoustic signal processing.

In recent years, the prediction models for underwater acoustic signal mainly include neural network prediction model, Volterra nonlinear prediction model, and combined prediction model. Zhou et al. [5] applied BP neural network and RBF neural network to underwater acoustic signal prediction, and achieved good prediction results. Sun et al. [6] and Fang et al. [7] used Volterra series theory to establish a nonlinear dynamic model of underwater acoustic signal, and realized the reduction of background noise and the suppression of reverberation interference through the local prediction of underwater acoustic signal. Yang et al. [8] used the fruit fly optimization algorithm to optimize the wavelet neural network to improve the prediction accuracy of underwater acoustic signal. Although the single prediction model has achieved good results, the single prediction model often cannot achieve higher prediction accuracy. The combined prediction model combined with decomposition method has excellent performance in improving accuracy and has been widely used in...
time series prediction [9], [10]. Ren et al. [11] combined empirical mode decomposition and support vector machine to realize wind speed prediction. Duan et al. [12] used EMD and support vector machines to realize short-term prediction of ocean waves. Li et al. [13] proposed a deep learning prediction model based on pole-symmetric mode decomposition and cluster analysis to predict the monthly sunspot mean value, and the proposed model has good prediction effect. Xiong et al. [14] and Xiang et al. [15] used a combination of wavelet decomposition and LSSVM to achieve short-term prediction of wind speed. Li et al. [16] proposed a combined underwater acoustic signal prediction method based on extreme-point symmetric mode decomposition (ESMD) and extreme learning machine (ELM). This method uses the idea of decomposition and integration to further improve the accuracy of underwater acoustic signal. If a decomposition method with a higher resolution is selected, the prediction accuracy of the underwater acoustic signal can be further improved. Among many decomposition methods, variational mode decomposition is a new decomposition method proposed by Dragomiretskiy and Zosso [17], which has been active in time series prediction in recent years. Li et al. [18] proposed a chaotic time series prediction model of monthly precipitation based on a combination of variational mode decomposition and extreme learning machine. This model can better predict the precipitation trend and improve the prediction accuracy. Meng and Zhang [19] introduced variational mode decomposition to ultra-short-term load forecasting, and combined extreme learning machine to achieve prediction of ultra-short-term load time series. Although variational mode decomposition is an effective decomposition method, the method needs to determine the number of decomposition modes $K$ before decomposition. Selecting a proper number of modes will effectively reduce the modal aliasing phenomenon and improve the resolution of the decomposition. The number of VMD decomposition levels is usually determined by observing the center frequency of each mode [20], [21]. This method is greatly affected by subjective factors and is difficult to be convincing. In recent years, Wang et al. [22] proposed a $K$-value determination method based on center frequency, which determines the optimal number of modes by judging the ratio of center frequencies between front and back modes. Wang et al. [23] proposed a method for determining the $K$-value based on energy conservation. This method determines the optimal $K$-value by judging the energy of each component and the energy of the original signal. This paper proposes a $K$-value selection method based on the correlation coefficient, that is, the best $K$-value is determined by judging the size of the correlation coefficient between the decomposed reconstructed sequence and the original sequence. Based on the selected $K$-value, underwater acoustic signal is decomposed into a series of relatively stable components, and then a prediction model is established for each component to effectively improve the prediction accuracy. Although the decomposition and integration prediction has achieved high prediction accuracy, there are still errors. If the error sequence can be predicted, the prediction accuracy can be further improved. Ding et al. [24] used neural network combined with error correction to realize the prediction of wind power. Huang et al. [25] adopted an error correction strategy based on the EEMD-LSTM prediction model to achieve a further improvement in wind speed prediction accuracy. The idea of error correction has not been applied in underwater acoustic signal prediction. This paper attempts to introduce the idea of error correction into underwater acoustic signal prediction to further improve the prediction accuracy of underwater acoustic signal. According to the central limit law, the probability density distribution of errors basically follows normal distribution [26]. Therefore, Gaussian process regression based on Gaussian model is adopted in this paper to predict the error sequence.

Based on the idea of decomposition integration and error correction, this paper proposes a complex underwater acoustic signal prediction method combining correlation coefficient variational mode decomposition (CVMD), least squares support vector machine (LSSVM) and Gaussian process regression (GPR), that is CVMD-LSSVM-GPR prediction method. Firstly, CVMD proposed in this paper is used to decompose underwater acoustic time series into multiple stable components. Then, an LSSVM prediction model is established for each component. Then GPR is used to predict the error sequence of each component. Finally, the predicted results and error predicted results of each component are summed up and reconstructed to obtain the final predicted results. Considering the compensation effect of error correction, this paper attempts to apply CVMD-LSSVM-GPR to the direct multi-step prediction of underwater acoustic time series after single-step prediction to improve the accuracy of direct multi-step prediction of underwater acoustic time series.

II. EXPERIMENTAL PRINCIPLES AND METHOD

A. VARIATIONAL MODE DECOMPOSITION

VMD can decompose the input signal into subsequences with different center frequencies and limited bandwidth $\{u_k(t)\}(k = 1, 2, \ldots, K)$. The decomposition process is a process of solving the variational problem. The constrained variational model is as follows:

$$\begin{align*}
\min_{\{u_k\}, \omega_k} \left\{ \sum_{k=1}^{K} \left| \sum_{k=1}^{K} \phi_k \left[ \delta(t) + \frac{j}{\pi t} \right] u_k(t) e^{-j\omega_k t} \right|^2 \right\},
\text{s.t.} \sum_{k=1}^{K} u_k(t) = f(t)
\end{align*}$$

(1)

where $\omega_k$ represents the center frequency of the $k$th modal function.

In order to solve the constrained variational problem (1), a penalty factor $C$ and a Lagrangian multiplication operator $\theta(t)$ are introduced to transform the constrained variational problem into an unconstrained variational
problem.

\[ L (\{u_k\}, \{\omega_k\}, \theta) = C \sum_{k=1}^{K} \left| \hat{u}_k \left[ \left( \delta(t) + \frac{j}{\pi t} \right) u_k(t) \right] e^{-j\omega t} \right|^2_2 \\
+ \left( f(t) - \sum_{k=1}^{K} u_k(t) \right)^2_2 + < \theta(t), f(t) - \sum_{k=1}^{K} u_k(t) > (2) \]

where \( \left( f(t) - \sum_{k=1}^{K} u_k(t) \right)^2_2 \) is the second penalty term and \(< \cdot, \cdot>\) is the inner product operation.

The unconstrained variational problem (2) is solved by the multiplication operator alternating direction method, and \( u_k(t), \omega_k \) and \( \theta(t) \) are alternately updated by the Equation (3)-(5).

\[ \hat{u}_{k}^{n+1} (\omega) = \frac{\hat{f}(\omega) - \sum_{k=1}^{K} \hat{u}_k (\omega) + \hat{\omega}(\omega)}{1 + 2C (\omega - \omega_k)^2} \] (3)

\[ \omega_{k}^{n+1} = \frac{\int_{0}^{\infty} \omega |\hat{u}_k (\omega)|^2 d\omega}{\int_{0}^{\infty} |\hat{u}_k (\omega)|^2 d\omega} \] (4)

\[ \hat{\theta}^{n+1} (\omega) = \hat{\theta}^{n} (\omega) + \tau \left[ \hat{f}(\omega) - \sum_{k=1}^{K} \hat{u}_{k}^{n+1} (\omega) \right] \] (5)

Performing an inverse Fourier transform on \( \hat{u}_k (\omega) \), the real part \( \{u_k(t)\} \) is the solution \( \{u_k(t)\}(k = 1, 2, \ldots, K) \).

The VMD algorithm introduces the original signal into the variational model, and then uses the process of finding the optimal solution of the constrained variational model to obtain the component. In this process, each component is alternately iteratively updated in the frequency domain, adaptively decomposing the frequency band of the signal. Finally, \( K \) narrowband components are got.

### B. A SELECTION METHOD OF MODAL NUMBER BASED ON CORRELATION COEFFICIENT

When performing VMD decomposition, selecting the proper number of decomposition modes can effectively reduce modal aliasing and improve the resolution of VMD decomposition. In order to select the proper modal number, this paper proposes a method for selecting the modal number of VMD based on the correlation coefficient, which is called the CVMD algorithm. The algorithm selects the modal number by calculating the correlation coefficient between the reconstructed sequence after VMD decomposition and the original sequence. The specific steps are as follows:

Step 1: Set the initial modal number \( K \) to 1, and perform VMD decomposition to obtain the decomposition component. All components are superimposed as a reconstructed sequence.

Step 2: Calculate the correlation coefficient between the reconstructed sequence and the original sequence to reflect the degree of correlation between them [27]. When the correlation coefficient reaches a threshold, the original signal is considered to be fully decomposed. The correlation coefficient between the reconstructed sequence and the original sequence is defined as follows:

\[ \rho_{xy}(k) = \frac{\sum_{n=0}^{\infty} \hat{y}(n)y(n)}{\sqrt{\sum_{n=0}^{\infty} \hat{y}^2(n) \sum_{n=0}^{\infty} y^2(n)}} \] (6)

where \( \hat{y}(n) \) is the reconstructed sequence, \( y(n) \) is the original sequence, \( \rho_{xy}(k) \) is the correlation coefficient when the number of decomposition layers is \( K \).

Step 3: Determine whether \( \rho_{xy}(k) \) is greater than \( \varepsilon \), when \( \rho_{xy}(k) \) is greater than \( \varepsilon \), then the corresponding \( k \) is the optimal modal number; when \( \rho_{xy}(k) \) is less than \( \varepsilon \), execute \( K = K + 1 \), and return to Step 1 again.

The experimental verification in this paper shows that the prediction effect is better when 0.997 is taken, so the threshold value selected in this paper is 0.997. The algorithm flow for solving the optimal \( K \)-value is shown in Figure 1.

### C. LEAST SQUARES SUPPORT VECTOR MACHINE

Support vector machine (SVM) is a machine learning method that follows the principle of minimizing structural risks. This method has high prediction accuracy and strong generalization ability. So it has been widely used in time series prediction [28], [29]. Least squares support vector machine (LSSVM) is an improvement of SVM by Suykens and Vandewalle [30]. Different from SVM, linear equations are used to solve the support vector in LSSVM, which greatly improves the solution speed of the support vector and reduces the
complexity of calculation. Therefore, LSSVM is more suitable for time series prediction research.

If a given sample is an n-dimensional vector, then m samples can be expressed as \((x_1, y_1), \ldots, (x_m, y_m) \in \mathbb{R}^n \times \mathbb{R}\).

The sequence \(x\) is the sample input, and the sequence \(y\) is the sample output. The regression function can be expressed as:

\[
f(x) = w^T x + c
\]

where \(w\) is the weight vector and \(c\) is the offset.

According to the structural risk minimization principle, when LSSVM makes predictions, the optimization objective function can be expressed as:

\[
\begin{align*}
\min_{w, c, \alpha} J_1(w, c) &= \frac{1}{2} w^T w + \frac{1}{2}\lambda \sum_{i=1}^{N} e_i^2 \\
\text{s.t. } y_i &= w^T \varphi(x_i) + c + e_i
\end{align*}
\]

where \(i = 1, \ldots, m, e_i\) is a relaxation variable, and \(\lambda\) is a regularization parameter.

To solve the equation (8), construct a Lagrange function to solve:

\[
L(w, c, e, \alpha) = \frac{1}{2} \|w\|^2 + \frac{1}{2}\lambda \sum_{i=1}^{m} e_i^2 - \sum_{i=1}^{m} \alpha_i (w^T x_i + c + e_i)
\]

where \(\alpha_i\) is a Lagrange multiplier.

According to Karush-Kuhn-Tucker conditions, we can get

\[
\frac{\partial L}{\partial w} = 0, \quad \frac{\partial L}{\partial e_i} = 0, \quad \frac{\partial L}{\partial \alpha_i} = 0
\]

Define a kernel function that meets the Mercer condition as \(K(x_i, x_j) = \phi(x_i)^T \phi(x_j)\), then the final LSSVM regression function can be expressed as:

\[
f(x) = \sum_{i=1}^{n} \alpha_i K(x_i, x_i) + c
\]

Among them, the kernel function in this paper uses a radial basis function:

\[
K(x_i, x_j) = \exp(-\frac{\|x - x_j\|^2}{2\sigma^2})
\]

D. GAUSSIAN PROCESS REGRESSION

Gaussian process regression (GPR) [31] is a finite set of random variables \(f(x_1), f(x_2), \ldots, f(x_n)\) that obey the joint Gaussian distribution. The specific expression is as follows:

\[
f(x) \sim \text{GPR}(m(x), k(x, x'))
\]

where \(m(x)\) is the mean function, \(k(x, x')\) is the covariance function.

Considering the existence of noise, the GPR prediction model can be expressed as:

\[
y = f(x) + \varepsilon
\]

where \(\varepsilon\) is a Gaussian white noise independent of \(f(x)\), that is, \(\varepsilon \sim N(0, \sigma^2)\).

A Gaussian process regression composed of the target output \(y\) can be expressed as:

\[
y \sim \text{GPR}(0, k(x, x') + \sigma^2)
\]

According to Bayesian principle, the joint Gaussian distribution of the target output \(y\) of the training sample and the target output \(y^*\) of the test sample is:

\[
\begin{bmatrix} y \\ y^* \end{bmatrix} \sim \begin{bmatrix} 0 \\ K(x^*, x) + \sigma^2 I_{m, n} \end{bmatrix} \begin{bmatrix} K(x, x) + \sigma^2 I_{n} & K(x, x^*) \\ K(x^*, x) & K(x^*, x^*) \end{bmatrix}^{-1}
\]

where \(K(x^*, x) = (x^*, x)^T\) is the covariance matrix between the training input \(x\) and the test input \(x^*, K(x^*, x^*)\) is the covariance matrix of the test input itself, and \(K(x, x)\) is the \(n \times n\)-order symmetric positive definite covariance matrix.

For test input \(x^*\), the posterior probability distribution of its predicted output \(y^*\) can be expressed as:

\[
p(y^*|x, y, x^*) \sim N(\tilde{y}^*, \text{cov}(y^*))
\]

where \(\tilde{y}^*\) and \(\text{cov}(y^*)\) are the mean and variance of the predicted values corresponding to the test input \(x^*\), respectively.

III. CONSTRUCTION OF CVMD-LSSVM-GPR PREDICTION MODEL

A. SAMPLE PARTITION BASED ON EXTREME LEARNING MACHINE

Underwater acoustic signal has not only nonlinear, non-Gaussian, non-stationary characteristics, but also typical chaotic, fractal and other characteristics [32], [33]. If appropriate delay and embedding dimension can be selected, the
original dynamical system can be restored in the topologically equivalent sense, and a better prediction effect can be achieved. For a given time series \( \{x_1, x_2, \ldots, x_N\} \), this one-dimensional time series can be extended into an \( m \)-dimensional phase space by delay \( \tau \) and the embedding dimension \( m \):

\[
X = \begin{bmatrix}
  x_1 & x_2 & \cdots & x_{n-h} \\
  x_{1+\tau} & x_{2+\tau} & \cdots & x_{n+\tau-h} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{1+(m-2)\tau} & x_{2+(m-2)\tau} & \cdots & x_{n+(m-2)\tau-h} \\
  x_{1+(m-1)\tau} & x_{2+(m-1)\tau} & \cdots & x_{n+(m-1)\tau-h}
\end{bmatrix}
\]  

(18)

where \( n = N - (m - 1) \tau \), \( h \) is the prediction step size.

The sequence to be predicted can be expressed as:

\[
dn = [x(1 + h + (m - 1)\tau), \ldots, x(N)]
\]  

(19)

\( X \) as an overall sample can be divided into training samples and test samples, and \( dn \) can be divided into corresponding training output and test output. When \( h = 1 \), it is a single-step prediction, and when \( h > 1 \), direct multi-step prediction can be achieved.

The selection of the embedding dimension and time delay plays a key role in improving the prediction accuracy. If the proper time delay and embedding dimension are selected, the prediction accuracy of the corresponding prediction model will also increase.

Extreme learning machine [34] has the advantage of fast training speed, so this paper proposes an embedding dimension and delay acquisition method based on the extreme learning machine prediction model.

The steps of the embedding dimension and time delay acquisition method based on the extreme learning machine prediction model can be summarized as follows:

Step 1: \( m \) and \( \tau \) are determined in the range of \( m \in [4, 10] \) and \( \tau \in [1, 10] \), and the time series \( \{x_1, x_2, \ldots, x_N\} \) is divided into training samples and test samples by equations (18) and (19). \( X_{in} = [X_1 \ X_2 \cdots \ X_r]^T \) is used as training input and \( d_{in} = [d_1 \ d_2 \cdots \ d_r] \) is used as training output. Take \( X_{out} = [X_{r+1} \ X_2 \cdots \ X_{n-h}]^T \) as the test input and \( d_{out} = [d_{r+1} \ d_2 \cdots \ d_{n-h}] \) as the test output.

Step 2: Importing the divided training input and output into ELM prediction model \( \sum_{i=1}^{m} \beta_i(f(a_i X_{in} + c_i)) = d_{in} \),
calculating the output weight $\beta$, determining the number of hidden layer nodes $L$, and then obtaining the trained prediction model. Where $f(x)$ is the activation function, $a_i$ is the input weight and $c_i$ is the offset.

Step 3: The output value $\hat{d}_{out} = [\hat{d}_{r+1} \hat{d}_2 \cdots \hat{d}_{n-h}]$ of the prediction model can be obtained by importing the test input into the trained prediction model. The RMSE value can be obtained by importing $\hat{d}_{out}$ and the real sample output $d_{out}$ into the root mean square error formula $\sqrt{\frac{1}{N} \sum_{i=1}^{N} (d_{out}(i) - \hat{d}_{out}(i))^2}$.

Step 4: $m$ and $\tau$ are determined in the range of $m \in [4, 10]$ and $\tau \in [1, 10]$ by enumeration method, and the above steps are repeated until a minimum RMSE value is obtained. $m$ and $\tau$ corresponding to the minimum RMSE value are the optimal embedding dimension and time delay.

The selection flow chart of $m$ and $\tau$ is shown in Figure 2.

### B. CVMD-LSSVM-GPR Prediction Model

Underwater acoustic signal is highly complicated due to the influence of marine noise and mechanical noise, and the prediction is difficult. Therefore, this paper uses CVMD to decompose the underwater acoustic time series into multiple relatively stable modal components to reduce the prediction difficulty of underwater acoustic signal. Suppose underwater acoustic time series is decomposed by CVMD to obtain $K$ relatively stable modal components, respectively, $u_1, u_2, \ldots, u_K$.

When using the least support vector machine (LSSVM) to predict each component, an appropriate regularization parameter $\lambda$ and a kernel function width $\sigma$ need to be selected. In this paper, the whale optimization algorithm (WOA) [35] is used to optimize the selection of $\lambda$ and $\sigma$. WOA is a new swarm intelligence optimization algorithm proposed by Mirjalili in 2016. This algorithm optimizes the search by simulating the predatory behavior of whales, such as enveloping and bubble attacks. Compared with the classic fruit fly algorithm [8], and ant colony algorithm [36], WOA has the advantages of fewer parameters and strong optimization ability. For the specific steps of the WOA algorithm, please refer to [37], [38].

When using WOA to optimize LSSVM parameters, the parameter settings of this article are shown in Table 1.

- **TABLE 1. WOA parameter settings.**

| Parameter Name                  | Parameter Value |
|---------------------------------|-----------------|
| Population size                 | 8               |
| The maximum number of iteration | 50              |
| Regularization parameter        | [1,500]         |
| Optimization range              | [1,100]         |
| Kernel function width optimization range | [1,100]         |

When analyzing the error of each component of underwater acoustic signal, it is found that the distribution of the error sequence obeys the normal distribution and has certain regularity. Therefore, it is possible to consider the error compensation for the prediction result, and the GPR prediction model is based on the Gaussian model. Based on the above, it is suitable for the prediction of underwater acoustic signal error sequences. Assuming that the error sequence prediction value obtained using the GPR prediction model is $e_1^*, e_2^*, \ldots, e_K^*$, the prediction result $u_1^*, u_2^*, \ldots, u_K^*$ of each component of
Underwater acoustic time series and the error sequence prediction value $e_1^*, e_2^*, \ldots, e_K^*$ are summed and reconstructed to obtain the final prediction value of underwater acoustic signal.

Underwater acoustic signal prediction model based on CVMD-LSSVM-GPR is shown in Figure 3.

C. EVALUATION INDEX

This paper uses the root mean square error (RMSE), the mean absolute error (MAE), the mean absolute percentage error (MAPE), and the determination coefficient $R^2$ to measure the prediction performance of the prediction model.

$$E_{RMSE} = \sqrt{\frac{N}{\sum_{i=1}^{N} (y(i) - \bar{y})^2}}$$  \hspace{1cm} (20)

$$E_{MAE} = \frac{1}{N} \sum_{i=1}^{N} |y(i) - y_d(i)|$$  \hspace{1cm} (21)

$$E_{MAPE} = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{y(i) - y_d(i)}{y_d(i)} \right|$$  \hspace{1cm} (22)

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (y(i) - \bar{y})^2}{\sum_{i=1}^{N} (y(i) - \bar{y})^2}$$  \hspace{1cm} (23)

where, $N$ is the number of predicted sample points, $y_d(i)$ is the actual output, $y(i)$ is the predictive output, $\bar{y}$ is the average of the time series. The closer the value of $R^2$ is to 1, the better the fit of the predicted output data to the original data.
IV. SIMULATION AND ANALYSIS

Underwater acoustic signal data used in this paper comes from the official website of the National Park Service (https://www.nps.gov/glba/learn/nature/soundclips.htm). The sampling frequency of data is 44.1kHz. In this paper, 1000 points are randomly selected as the total sample from 5000 points of sample data, and data is normalized. Based on the reconstruction of the phase space, extreme learning machine is used to optimize the selection of the delay and the dimension, so as to divide the training samples and test samples. This paper selects the last 150 points as test samples to test the predictive ability of the model. The time series of underwater acoustic signal is shown in Figure 4.

It can be seen from Figure 4 that the complexity of underwater acoustic signal is high and has strong fluctuation. To reduce the complexity of underwater acoustic signal and improve the prediction accuracy, CVMD is used to decompose the underwater acoustic signal before prediction, in which the balance parameter is set to 500. Firstly, $K$-value is selected using the correlation coefficient method, and the change trend of the correlation coefficient with the number of decomposition layers is shown in Figure 5.

It can be seen from Figure 5 that when $K = 8$, the correlation coefficient between the decomposed and reconstructed sequence and the original sequence reaches 0.9997. At this time, underwater acoustic time series is considered to have been sufficiently decomposed. Therefore, the $K$-value is set at 8. The CVMD decomposition results of underwater acoustic signal are shown in Figure 6.

A. SINGLE STEP PREDICTION OF UNDERWATER ACOUSTIC SIGNAL

Considering that the error sequence of the prediction result may have certain regularity, this paper analyzes the prediction error of the CVMD decomposition component by Q-Q plot, and finds that except for the IMF1 component, the quantiles of the remaining components and the quantile of the standard normal distribution exist obvious linear relationship and follow the normal distribution. This paper considers the use of GPR prediction models based on Gaussian models to predict error sequences. Figure 7 is a Q-Q diagram of a prediction error sequence.

In order to compare the prediction effect of the CVMD-LSSVM-GPR prediction model, ELMAN, SVM, LSSVM, EMD-LSSVM, and CVMD-LSSVM are used to predict underwater acoustic signal time series in one step. The prediction results of each model are shown in Figure 8, and the error distribution between the predicted output of each model and the true value is shown in Figure 9.
As can be seen from Figure 8(a), although one-step predicted output value of each model has a high degree of fitting with real value, and has achieved good prediction results, it can be seen from the local enlarged view of Figure 8(b) that EMD-LSSVM, CVMD-LSSVM, CVMD-LSSVM-GPR prediction models have obvious advantages over single prediction models. To distinguish the differences between models, Figure 9 shows prediction error distribution of each prediction model. It is obvious from the figure that the CVMD-LSSVM-GPR prediction model has the smallest error fluctuation and better prediction performance compared with other prediction models. In order to better reflect the prediction performance of CVMD-LSSVM-GPR, Figure 10 shows the boxplot of the one-step prediction of underwater acoustic signal. Table 2 shows the specific data of the RMSE, MAE, MAPE and $R^2$ of each prediction model.

As can be seen from Table 2, the CVMD-LSSVM-GPR prediction model has the minimum RMSE, MAE, MAPE values and the maximum determination coefficient, which shows that the prediction accuracy of CVMD-LSSVM-GPR is the best among the six models, and also shows that the prediction accuracy of CVMD-LSSVM-GPR prediction model with error compensation is improved to a certain extent compared with CVMD-LSSVM. As can be seen in Figure 10, the prediction error fluctuation of EMD-LSSVM, CVMD-LSSVM, CVMD-LSSVM-GPR is obviously smaller than that of a single prediction model, and the prediction error fluctuation of CVMD-LSSVM-GPR is the smallest. The above verifies that the CVMD-LSSVM-GPR prediction model proposed in this paper has a good effect in one-step prediction of underwater acoustic time series.

### B. DIRECT MULTI-STEP PREDICTION OF UNDERWATER ACOUSTIC SIGNAL

Each model has a high prediction performance in one-step prediction of underwater acoustic signal. In order to further
verify the effectiveness of the CVMD-LSSVM-GPR prediction model, this section attempts to make direct multi-step prediction of underwater acoustic signal.

Before direct multi-step prediction, in order to find the maximum predictable step size, this paper firstly uses Rosenstein method [39] to solve the Lyapunov exponent of the original data. For the delay and embedding dimension of the original underwater acoustic signal, the mutual information method [40] and Cao method [41] are used for selection. It is also necessary to know the average period of underwater acoustic signal to solve the Lyapunov exponent. This paper uses FFT to solve it. Figure 11 shows the delay, embedding dimension, average period, and maximum Lyapunov exponent of the original signal, respectively.

It can be seen from Figure 11 (a) that when $\tau = 4$, the mutual information amount reaches the minimum value for the first time. After this value, the mutual information amount basically stabilizes, so we take the delay to 4. It can be seen from Figure 11 (b) that when the embedding dimension is 7, the value of $E_1$ basically does not change and approaches 1. Therefore, we take the embedding dimension as 7. According to the fast Fourier transform (FFT) in Figure 11 (c), the main frequencies in the original signal can be extracted, and the average period of underwater acoustic signal can be obtained as 15. In Figure 11 (d), the red linear region is determined, and the slope is determined by fitting the regression line with the least square method, and the maximum Lyapunov exponent is 0.0375. The maximum predictable number of steps in the
H. Yang et al.: Underwater Acoustic Signal Prediction Based on CVMD and Error Compensation

underwater acoustic time series can be obtained from the reciprocal of the maximum Lyapunov exponent as 26.

This paper firstly uses direct 10-step prediction for underwater acoustic time series, and its prediction map and error distribution map are shown in Figure 12 and Figure 13.

It can be clearly seen from Figure 13 that in the direct 10-step prediction, the prediction ability of a single prediction model is significantly reduced, while the prediction performance of EMD-LSSVM in the combined prediction model is much lower than that of CVMD-LSSVM and CVMD-LSSVM-GPR. In order to further distinguish the prediction performance of CVMD-LSSVM and CVMD-LSSVM-GPR, the error boxplot and various prediction indexes of the prediction model are given below.

It can be seen from the prediction indexes in Table 3 that the RMSE, MAE and MAPE values of the CVMD-LSSVM-GPR prediction model is smaller than those of the CVMD-LSSVM prediction model, and the CVMD-LSSVM-GPR has a higher determination coefficient, which fully demonstrates that the CVMD-LSSVM-GPR prediction model is superior to the CVMD-LSSVM. As can be seen from Figure 15, CVMD-LSSVM-GPR prediction model has the smallest error fluctuation in the direct 10-step prediction and shows better prediction performance.

In order to explore the critical point of the maximum predictable step size, this paper continues to increase the prediction step size to predict the underwater acoustic time series. Because the single prediction model has poor results in the direct 10-step prediction, for the sake of analysis, this paper only uses EMD-LSSVM, CVMD-LSSVM, and CVMD-LSSVM-GPR for direct 20-step prediction and direct 26-step prediction.

In the direct 20-step prediction, the CVMD-LSSVM-GPR prediction model has a determination coefficient of 0.9108, which can also play a good prediction effect. But when the prediction step is the maximum prediction step 26, the prediction accuracy is significantly reduced, and the determination coefficient of CVMD-LSSVM-GPR is only 0.7328. It can be seen that when multi-step prediction is performed, with
the increase of the prediction step size, the prediction error becomes larger and larger. To verify this, Table 6 shows the prediction indexes corresponding to different prediction steps of CVMD-LSSVM-GPR prediction model.

As can be seen from Table 6, RMSE value, MAE value and MAPE value of CVMD-LSSVM-GPR prediction model gradually increase with the increase of prediction step size, while the determination coefficient gradually decreases. This fully shows that with the increase of the prediction step size, the prediction error is also increasing.

**C. DIEBOLD-MARIANO TEST**

In order to test the superiority of CVMD-LSSVM-GPR prediction model in prediction performance, this paper uses...
FIGURE 16. (a) Direct 26-step prediction; (b) Direct 26-step prediction of local magnification.

TABLE 6. CVMD-LSSVM-GPR prediction indicators.

| Prediction Step Size | RMSE  | MAE   | MAPE  | $R^2$  |
|----------------------|-------|-------|-------|--------|
| 1-step               | 0.0125| 0.0098| 0.0658| 0.9994 |
| 5-step               | 0.0503| 0.0400| 0.3579| 0.9735 |
| 10-step              | 0.0955| 0.0680| 0.3594| 0.9391 |
| 15-step              | 0.1113| 0.0871| 0.5769| 0.9553 |
| 20-step              | 0.1234| 0.0972| 0.6537| 0.9108 |
| 25-step              | 0.1818| 0.1419| 1.1174| 0.7569 |
| 30-step              | 0.2690| 0.2051| 1.6155| 0.2302 |

DM statistics [42] to test the above five comparative models and the proposed model based on 1-step prediction and 10-step prediction. The greater the value of DM statistic, the greater the difference between the prediction performance of CVMD-LSSVM-GPR and the comparison model. From 

Table 7 and Table 8, it can be seen that $P$-value of Diebold-Mariano test is less than 0.01, which indicates that the CVMD-LSSVM-GPR model has significant differences compared with other 5 comparison models at the significance level of 1%. This further shows that the prediction model proposed in this paper has significant advantages in prediction performance and can be effectively applied to underwater acoustic signal prediction.
TABLE 8. 10-step prediction of DM test result.

| CONTRAST MODEL | SVM | ELMAN | LSSVM | EMD-LSSVM | CVMD-LSSVM |
|----------------|-----|-------|-------|-----------|-----------|
| DM Statistics  | 6.855*| 6.851*| 6.821*| 6.190*    | 4.084*    |
| P-value        | 0   | 0     | 0     | 0         | 0.00002   |

* Indicates 1% confidence interval.

V. CONCLUSION

In this paper, a prediction model based on correlation variational mode decomposition and error compensation is proposed and applied to one-step and multi-step prediction of underwater acoustic signal. Through experiments, we can draw the following conclusions:

1) The CVMD decomposition method proposed in this paper has better decomposition effect than traditional EMD. CVMD can decompose complex underwater acoustic time series into multiple relatively stable components, thereby improving the prediction accuracy of underwater acoustic time series.

2) Using extreme learning machine prediction model to select embedding dimension and time delay can divide the sample space more reasonably and improve the prediction accuracy.

3) The WOA optimization algorithm is used to select the model parameters of LSSVM, which reduces the workload and improves the accuracy of LSSVM prediction model.

4) Adding error compensation for the basis of decomposing the prediction model CVMD-LSSVM can further improve the prediction accuracy of underwater acoustic signal, which provides a new idea of underwater acoustic signal prediction.

5) Through comparative analysis between models, it is found that CVMD-LSSVM-GPR prediction model has achieved good prediction results in both single-step prediction and multi-step prediction of underwater acoustic signal. However, with the increase of the prediction step, the prediction error of CVMD-LSSVM-GPR also increases, which shows error compensation can only improve the prediction accuracy to a certain extent, and cannot fully capture the regularity in error sequence.

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