ASSESSMENT OF TRITON POTENTIAL ENERGY

by

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Abstract

An assessment is made of the dominant features contributing to the triton potential energy, with the objective of understanding qualitatively their origins and sensitivities. Relativistic effects, short-range repulsion, and OPEP dominance are discussed. A determination of the importance of various regions of nucleon-nucleon separation is made numerically.
Substantial and numerous recent successes[1] in the area of few-nucleon physics indicate that it is appropriate to begin an assessment of our level of understanding of the physics underlying the dynamics of few-nucleon systems. In recent months and years, a variety of computational and theoretical techniques have been successfully applied to the bound (or low-lying) states of $A = 2, 3, 4, 5$, and $6$ and to the scattering states of the trinucleon systems. Attempts are underway to extend this program and are likely to be successful. These efforts have been very informative and generally produce results in excellent agreement with experiment except for a few small, but potentially very significant, discrepancies. A vital question that must be asked is whether we can now (or ever) understand the nuclear force well enough to accommodate all that we have learned and hope to learn from future progress. A systematic understanding of the structure of few-nucleon systems and unraveling the dynamics of these systems have long been considered the raison d'etre of the field of few-nucleon physics.

One recent calculation[2] underscores the recent and potential precision of the field. The Nijmegen group have constructed several new potential models, whose dominant feature is a charge-dependent one-pion-exchange potential (OPEP) [see below], by directly fitting the model parameters to the nucleon-nucleon (NN) scattering data. The partial-wave-dependent (Reid-like) versions of these potentials fit the NN data so well that they can be considered as alternative phase-shift analyses. Published and unpublished (preliminary) local versions of these potentials produce triton binding energies $\sim 7.62(3)$ MeV, while nonlocal versions are more bound by $\sim 100$ keV. The local result is therefore a benchmark for gauging the size of small components of the nuclear force. Since locality and nonlocality (which clearly exists and is conceptually important) cannot be determined from data analysis, only theoretical arguments can discriminate between them. Constructing a theoretically sound and compelling nuclear force model is therefore a requirement before we can discriminate between (small) three-nucleon forces and vagaries of the NN force model. Thus we must address the question: “Is this a realistic possibility?”

A major historical problem within the field, which has tainted the aforementioned successes, is the old perception that the nuclear force is too complicated to be understood in anything approaching fundamental terms and that perturbation theory, which underlies virtually all theoretical approaches, cannot be performed because the coupling constants are too large, implying that any systematic development is impossible. At least within the few-nucleon systems, where “exact” or “complete” nonrelativistic solutions of the Schrödinger equation are now routine and dynamical details can be tested, this pessimism is largely unfounded, which we argue below.

An appreciation of this problem begins with a dichotomy. On the one hand the
exchange of a pion (OPEP) has long been the archetype of the nuclear force, while on the other hand folklore (based entirely on properties of heavier nuclei) attributes most nuclear binding to correlated two-pion exchange. The first indication that the latter property did not hold for few-nucleon systems was given in Ref. [3], where it was shown that the tensor potential and the \(^3S_1 - ^3D_1\) partial wave of the nuclear force dominate the triton binding. Replacing the latter potential with a “pure” OPEP potential leads to nearly the same triton binding energy, demonstrating that OPEP dominates triton binding. This was more clearly documented[2,4] by subsequent calculations of \(\langle V_\pi \rangle\) for the triton, alpha particle and other systems: the fraction of the potential energy from OPEP, \(\langle V_\pi \rangle/\langle V \rangle\), is 70-80% for most realistic potentials. Not surprisingly, OPEP has an even more dominant effect in the deuteron, where an adequate (but not excellent) deuteron can be obtained using a pure regulated OPEP; ironically, OPEP is too strong at short distances to make a good deuteron without the regulation[3]. Details of the nuclear force are obviously required in order to guarantee excellent agreement with observables.

The importance of OPEP has been highlighted by the Nijmegen group[5], whose phase-shift analysis of nucleon-nucleon scattering data included an explicit OPEP tail for separations \(r \geq 1.4\) fm and a phenomenological treatment of the shorter-range interaction. Their study concluded that the NN data require the exchanges of a charged pion with mass \(m_{\pi^+} c^2 = 139.4(10)\) MeV, and a neutral pion with mass \(m_{\pi^0} c^2 = 135.6(13)\) MeV. Both results are consistent with the free-particle masses, while the very small error bars reflect the overall importance of OPEP. Although it has been long known that OPEP played a very significant role in some NN observables, this was the first direct quantitative measure of its global importance.

Thus, it is not unfair to characterize OPEP as the “Coulomb potential” of few-nucleon systems, although there remains a significant contribution from shorter-range forces. This is exceedingly fortunate, because the former physics is tractable, while the latter is exceedingly complex. These non-OPEP parts can be further subdivided into two convenient classes: two-pion-range (TPEP) and short-range forces. By two-pion exchange we mean the exchange of two uncorrelated (noninteracting) pions, rather than strongly correlated resonances such as the \(\rho\). Chiral symmetry is expected to play a major role in such a process, even in the long-range tail of the \(\rho\)-exchange channel. In addition, isobars contribute to this part of the force, although many potential models do not contain explicit isobar-induced components. Assessments of the role of the long-range two-pion-exchange force have been very disappointing. The literature on the topic is replete with model calculations but few definite conclusions. Many potentials contain a TPEP in one form or another, but some do not. Theoretical arguments based on chiral symmetry suggest that it is weak compared
to OPEP[6,7]. An urgent need of the field is an assessment of what (if any) long-range TPEP components are required by the NN data, rather than merely reflecting theoretical prejudice. Several recent developments suggest that such a study may be tractable[5].

The inner part of the (non-OPEP) potential contributes 20-30% of the triton potential energy, and a detailed understanding of its origin may be problematic unless the full power of QCD can be directly applied. A variety of folklore arguments have been constructed that suggest that an understanding is not possible. We address two of these arguments next: one concerning nonlocalities and the other Z-graphs.

The only known methodology for constructing a (potentially) testable dynamical framework in nuclear physics is field theory (from which OPEP follows). Field theory leads to powerful constraints on systems but, as conventionally formulated, produces complications that seem far removed from our understanding of nuclear dynamics. Doubts are frequently expressed (although seldom in print) about whether field theory is meaningful in the nuclear context. Nevertheless, if further progress is to be made, it will almost certainly be within the field theory context.

Given that QCD is the theory of strong interactions, the most natural description of any strongly interacting system would be in terms of the “simple” degrees of freedom (d.o.f.): quarks and gluons. Unfortunately, as has been emphasized many times, a perturbation expansion of this theory in the confinement region is not tractable[8,9]. Nucleons (and pions) are complicated composites of these d.o.f. and have extended structure. Because their mutual interactions are consequently nonlocal, it has been argued that field theory cannot be meaningfully applied to nuclear physics. All of these statements are correct except the last one.

The second (related) argument concerns the way a field theory of nucleons is organized, with manifest covariance playing a fundamental role. Nucleon and antinucleon d.o.f. are combined and treated together. The presence of nucleon-antinucleon pairs (Z-graphs, which have a very short range) has long been a thorn in the community’s side. They play a huge and unphysical role in some models of the nuclear force, and “pair suppression” was invoked long ago to remove their effect in an ad hoc manner. Subsequently, the argument against pairs has been updated to accommodate QCD and modern particle phenomenology. The latter follows from the observation that $N\bar{N}$ (pair) final states are not an important component in high-energy reactions, and therefore pairs should not be taken seriously in nuclear physics. The former argument is that nucleon Z-graphs should be viewed in terms of their constituents (quarks), and simultaneously reversing all three quark lines (in $N \rightarrow \bar{N}$) is highly improbable; therefore pairs shouldn’t be taken seriously in nuclear physics. These
arguments are both correct and irrelevant.

Degrees of freedom in physics are a choice, not an obligation. A poor choice hurts rather than helps. That a field theory of (composite) nucleons, pions, ..., can be established seems not to be in doubt\[8,9\]. This effective field theory can (and should) accommodate the chiral symmetry embedded in QCD, and indeed forms a surrogate for the underlying QCD. By choosing to “freeze out” d.o.f. corresponding to heavy mesons, etc., with a mass $\gtrsim \Lambda \sim 1$ GeV, the large-mass QCD scale, the complexities of the short-range physics (which surely require an appeal to QCD to understand fully) are reduced to complicated nonlocal structures (in the same way that freezing out the photon in QED leads to the retarded interaction between charged particles). The short range implies that at low energies, a nucleon in the act of exchanging a heavy meson doesn’t propagate very far, and consequently the nonlocal structures can be expanded in terms of an infinite (and hopefully convergent) series of local structures involving (derivatives of) delta functions\[10\]. Given that a sensible field theory is possible, at least for energies $\lesssim \Lambda$, the size of high-energy reactions producing $N\bar{N}$ pairs is irrelevant (since the energy is obviously greater than $\Lambda$). For the low energies appropriate to the nuclear domain, the $N\bar{N}$ pairs will be virtual and their effect is unphysical and manifestly unmeasurable. An analogous situation exists in electromagnetic interactions, where a change of gauge can modify or even eliminate the amount of “pairs” contributing to a given process, such as Compton scattering. In a theory of nucleons exhibiting chiral symmetry, a field redefinition (i.e., a change of variables) corresponding to a chiral rotation can mix pion and nucleon fields and can transform a PS type of coupling to PV coupling, for example\[11\]. This involves a massive change of the “pair” structure, from very large in magnitude to very small, and shows the unphysical nature of “pairs” at low energy. One can also easily freeze out the pairs (via a Foldy-Wouthuysen transformation) if one desires\[11\]. The results of freezing d.o.f. will always make the dynamics have a more complicated form. This is obvious in the nuclear physics context from the Feshbach (P,Q) theory of reactions\[12\], where such freezing is also performed and leads to operators with a much more complicated structure.

Finally, the calculability of reaction dynamics in few-nucleon systems currently requires a nonrelativistic approach, although this restriction is loosening. The successful Dirac approach to calculating heavy nuclei emphasizes the very strong (short-range) scalar- and vector-meson fields\[13\]. This leads to modest central forces (because of cancellation) and large spin-orbit forces. In any perturbative estimate, relativistic effects are large and the use of the Schrödinger equation very dubious. In few-nucleon systems, however, although a consensus and complete understanding do not yet exist, all estimates of these effects are small\[14\], though not negligible. Typically, changing
from $T_{NR} = p^2/2M$ to $T_R = \sqrt{p^2 + M^2} - M$ produces 5% changes in $\langle T \rangle$. Some care should be exercised, however. For example, the effect on the extreme tail of the momentum distribution is very large, because the shape is changed. Recently, two calculations of the interaction-boost corrections (a particular type of relativistic correction corresponding to an interacting pair of nucleons whose center-of-mass is not the nuclear center-of-mass) in the triton produced very similar results ($\sim$250-300 keV repulsion) using very different conceptual and calculational frameworks\cite{14,15}. In addition, triton binding differences between the Bonn-B potential and most other potential models ($\sim$300 keV) were shown recently\cite{16} to be due to a particular relativistic treatment of the tensor operator in the Bonn-B OPEP. Thus, there is every expectation that relativistic effects in few-nucleon systems are small and tractable, although the confirmatory calculations remain to be performed. Because OPEP dominates the nonrelativistic potential energy, it is \textit{a priori} reasonable to assume that it also dominates the relativistic corrections to the interaction energy.

Unfortunately, “exact” calculations are not tractable in heavier nuclei. It is therefore difficult to assess the importance of individual elements of their dynamics (as it was in few-nucleon systems prior to a decade ago). Nevertheless, it is safe to assume that the way in which calculations are organized can play a significant role in the size of relativistic effects in nuclei. The use of potentials seems to minimize or hide large relativistic corrections (i.e., large cancellations take place).

Another aspect is the strong short-range repulsion usually associated with the vector meson field. In an exact nonperturbative treatment, this leads to a barrier between two nucleons that must be penetrated in order to have small interparticle separations. This tends to strongly suppress the contribution of the short-range repulsion to the total energy. In other words, although the potential is very strong, the hole in the wave function that it produces overcompensates and produces a small net result (i.e., the hole always “wins”). This suppression will also tend to reduce the overall effect of relativity at small separations. The latter is known to produce contributions of $\sim$100 keV additional triton attraction in some nuclear force models\cite{2}. In any event, the short-range structures in the potential produce relatively little \textbf{net} effect, as we shall show.

Finally, we note that chiral perturbation theory (\chiPT) (the effective Lagrangian approach) suggests that the short-range potentials should be comparable to the OPEP contribution\cite{17}. In fact the former is suppressed due to the barrier, leading to overall OPEP dominance. Moreover, \chiPT suggests a weak three-nucleon (or many-nucleon) force, and leads to a suppression of higher-order perturbative contributions to the force (i.e., from loops). This suggests that a sufficient understanding of the nuclear force to accommodate recent calculational successes in the few-nucleon systems may
be possible, although a strategy to achieve this is not yet evident.

Figure 1: Percentages of accrual of kinetic energy (solid line), potential energy (short dashed line), and probability (long dashed line) within an interparticle separation, $x$, for any pair of nucleons.

Many of these facets of three-nucleon physics can be tied together using Figure (1). One can ask the question, “What fraction of the triton potential or kinetic energy accrues within an interparticle separation, $x$, between any two nucleons?” This corresponds to forming

$$\frac{\langle \Psi | \hat{O} \Theta(x - r_{12}) | \Psi \rangle}{\langle \Psi | \hat{O} | \Psi \rangle}, \quad (1)$$

where all three of the nucleon coordinates $r_1, r_2, r_3$ (and hence $r_{12}$) are integrated. For very large $x$, this quantity approaches 100%. Figure 1 shows the results for the original version of the Argonne $V_{18}$ potential[2] (although other models that were examined are virtually identical) for the kinetic energy ($\hat{O} = T$), the potential energy ($\hat{O} = V$), and the correlation function ($\hat{O} = 1$). The effect of the strong short-range repulsion is the dominant feature inside an interparticle separation of 1 fm. Although the net potential energy (tracked by the net kinetic energy) is repulsive inside 0.8 fm, the maximum accrual is quite small, $\sim 10\%$. The major accrual of attractive energy occurs between 1.0 and 2.0 fm, which is the domain of OPEP and the TPEP tail. A
much cruder study of relativistic corrections in the alpha particle produces a similar qualitative result[14].

This work demonstrates that some of the problems that we face are not as severe as supposed and interprets a number of disparate results previously found. Our conclusion is that OPEP dominance results from barrier impenetrability at small NN separations. This suppresses the short-range contributions relative to what one might expect, reduces relativistic effects at short distances where virtual momenta are very large, and suggests that relativistic corrections from one-pion-exchange might be the dominant such physics. It further suggests that an even better understanding of the origins of the observed triton binding energy may be possible, allowing a credible separation of two-nucleon- and three-nucleon-force mechanisms. We have contended that certain arguments against the use of field theory to explicate the properties of the few-nucleon systems are irrelevant to the nuclear domain, and that chiral-symmetry-based field theory is the best hope for future progress in this area.

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REFERENCES

1. J. L. Friar, Summary talk presented at XIV\textsuperscript{th} \textit{International Conference on Few-Body Problems in Physics}, Williamsburg, VA, May 31, 1994, ed. by F. Gross, ed. by F. Gross, AIP Conference Proceedings 334, 323 (1995). This talk discusses the status of the field and possible future directions.

2. J. L. Friar, G. L. Payne, V. G. J. Stoks, and J. J. de Swart, Phys. Lett. \textbf{B311}, 4 (1993).

3. J. L. Friar, B. F. Gibson, and G. L. Payne, Phys. Rev. C 30, 1084 (1984).

4. R. B. Wiringa, Phys. Rev. C 43, 1585 (1991).

5. R. A. M. Klomp, V. G. J. Stoks, and J. J. de Swart, Phys. Rev. C 44, R1258 (1991); V. Stoks, R. Timmermans, and J. J. de Swart, Phys. Rev. C 47, 512 (1993).

6. C. Ordóñez and U. van Kolck, Phys. Lett. \textbf{B291}, 459 (1992); C. Ordóñez, L. Ray, and U. van Kolck, Phys. Rev. Lett. \textbf{72}, 1982 (1994); U. van Kolck, Thesis, University of Texas, (1993).

7. J. L. Friar and S. A. Coon, Phys. Rev. C 49, 1272 (1994).

8. S. Weinberg, Physica \textbf{96A}, 327 (1979); S. Weinberg, in \textit{Proceedings of the XXVI International Conference on High Energy Physics, Volume I}, ed. by J. R. Sanford, AIP Conference Proceedings 272, 346 (1993).

9. J. Gasser and H. Leutwyler, Ann. Phys. (N. Y.) 158, 142 (1984); U.-G. Meißner, review talk at Third Workshop on High Energy Particle Physics, Madras, India, January 1994, Preprint CRN–94/04. See the text below equation (3) of the latter reference.

10. G. P. Lepage, in \textit{From Actions to Answers}, Proceedings of the 1989 Theoretical Advanced Studies Institute in Elementary Particle Physics, ed. by T. DeGrand and D. Toussaint, (World Scientific, Singapore, 1990), p. 483.

11. S. A. Coon and J. L. Friar, Phys. Rev. C 34, 1060 (1986).

12. F. S. Levin and H. Feshbach, \textit{Reaction Dynamics}, (Gordon and Breach, New York, 1973).

13. B. D. Serot, Repts. on Prog. in Physics \textbf{55}, 1855 (1992).
14. J. Carlson, V. R. Pandharipande, and R. Schiavilla, Phys. Rev. C 47, 484 (1993).

15. A. Stadler and F. Gross, Contributed paper presented at XIVth International Conference on Few-Body Problems in Physics, Williamsburg, VA, May 31, 1994, ed. by F. Gross, p. 922.

16. J. Adam, Invited talk presented at XIVth International Conference on Few-Body Problems in Physics, Williamsburg, VA, May 31, 1994, ed. by F. Gross, AIP Conference Proceedings (to appear); Y. Song and R. Machleidt, Contributed paper presented at XIVth International Conference on Few-Body Problems in Physics, Williamsburg, VA, May 31, 1994, ed. by F. Gross, p. 189; A. Amghar and B. Desplanques, *ibid.*, p. 547.

17. S. Weinberg, Nucl. Phys. B363, 3 (1991); Phys. Lett. B251, 288 (1990); Phys. Lett. B295, 114 (1992).