Band-gap tuning in 2D spatiotemporal phononic crystals

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Abstract

We investigate the effect of small spatiotemporal modulations in subwavelength-dimensioned phononic crystals with large band gaps on the frequency spectrum for elastic waves polarized in the plane of periodicity. When the radius of cylinders periodically placed inside a matrix with highly-contrasting elastic properties is time-varying, we find that due to the appearance of frequency harmonics throughout the spectrum, the notion of a band gap is destroyed in general, although with the appropriate tuning of parameters, in particular the modulation frequency, it is possible that some band-gap region is retained, making such systems possible candidates for tunable bandpass filters or phononic isolators, accordingly, and for sensor applications.

Keywords: elastic wave propagation, metamaterials, spatiotemporal

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I. INTRODUCTION

Phononic lattices are necessarily engineered structures made up of composite materials (metamaterials). The tunability of their design can permit novel behaviour not found in conventional materials, such as wide, complete band gaps and nonreciprocal propagation. In the present study, we investigate the effect of spatiotemporal modulation of material properties on the band-gap properties for a phononic-lattice prototype: cylinders embedded in a matrix of significantly different elastic properties in the subwavelength regime. This type of structure is well-known for yielding the large spectral gaps\textsuperscript{1–7} desirable in phononic lattices, enabling the transmission of one type of polarization or none at all.

Reciprocal propagation of sound or elastic waves in media, is a general principle which assures the interchangeability of source and observer: that waves originating from one of these points propagate in exactly the same manner as waves originating from the other point. It is only broken in special situations such as when nonlinearity is present, when time-reversal symmetry is broken such as by the inclusion of materials with gain or loss, chirality, angular-momentum bias, or in moving systems\textsuperscript{8}. It may also be broken when the system itself is not moving in time but the material parameters are varying in space and time, manifesting itself as asymmetric band gaps leading to directional propagation\textsuperscript{9–12}.

The rectification of the mechanical properties of phononic materials has been demonstrated in several experimental or numerical studies, based on phenomena such as electromagnetism\textsuperscript{13}, piezoelectricity\textsuperscript{14–17}, or magnetorheological polymers subjected to magnetic fields\textsuperscript{18}. For nonreciprocal propagation to be observed, apart from time-varying material properties, there must also be spatial variation present.

Some purely mechanical implementations of spatiotemporally-varying material properties include a time-varying effective acoustic capacitance achieved in simulations involving air-filled waveguides by varying the heights of the attached resonators\textsuperscript{19,20}, a metabeam comprised of multiple resonators with different relative orientations where the resulting stiffness changes in response to rotation\textsuperscript{21}. Frequency conversion without nonlinearity present can occur in media with spatiotemporally-modulated material properties. Depending on the direction of the incident wave in a beam with reflective ends or a beam containing an interface between a homogeneous material and one with time-space vary-
ing properties, the frequency can be either up or down-converted, a clear indicator of nonreciprocal propagation. The production of harmonics in time-space modulated media may be exploited for uses such as unidirectional acoustic isolation, or parametric amplification as a possible implementation of a gain medium. In an air-filled waveguide system with a vibrating membrane, time-varying the membrane tension leads to phenomena such as frequency conversion; adding spatial variation via the addition of a second membrane with a phase difference, is what leads to non-reciprocal propagation. Alternatively, it has been shown that spatial variation leads to the formation of band gaps while the addition of time-variation is what causes the band structure to display a directionality via a “tilting” of ω-k space, a phenomenon long-known for the case of electromagnetic waves propagating on a two-dimensional surface with spatiotemporally-modulated properties.

Given that phononic materials are a very technologically-promising area of research, our aim in this study is to examine whether time variation of the phononic crystals’ already spatially-varying material properties improves or degrades the performance of these systems for various objectives such as acoustic isolators or frequency sensors.

II. METHODS

We consider a composite composed of two homogeneous structures: a solid cylinder inside a background matrix of a different material, together comprising a square unit cell as shown in Fig. 1. This constitutes a simplified model of, for example, a split-ring structure where the two semicircular components have a different radius, mechanically modulated in time. Time-dependent material properties in this model are imposed implicitly, via a variation of the radius \( r = r(t) = r_0 + A(t) \) of the cylinder, resulting, due to the nature of the model chosen, in a modification of its density as well as the density of the background matrix, as explained further below. The radius of the cylinder \( r_0 \) corresponds to a filling factor on the order of 50%. Such large filling factors in periodic structures ideally lead to large band gaps. The function \( A(t) = A \sin (2\pi ft) \) is a slowly-varying sinusoid with small modulation amplitude \( A \) (up to 2% of \( r_0 \)), and modulation frequency \( f \).

The elastic wave equation in the resulting inhomogeneous elastic medium is derived
FIG. 1: Square unit cell in two dimensions with lattice parameter $a$ for the system under study: a composite comprised of a solid cylinder of radius $r_0$ inside a background of a different material. The filling factor is in the range 50-60%. The radius of the cylinder varies periodically in time, with a maximum amplitude of $A$, as depicted.

from a) momentum conservation,

$$\frac{\partial (\rho v_i)}{\partial t} = \sum_j \frac{\partial T_{ij}}{\partial x_j} \quad (1)$$

where the $i, j$ are the components in space, specifically $x, y, z$, of the various quantities, $x_i$ are the spatial coordinates, $v_i$ are the components of the velocity, i.e. $v_i = \partial u_i/\partial t$, for displacements $u_i$, $T_{ij}$ are the components of the stress tensor (defined below), and $\rho$ is the density.

b) Hooke’s law,

$$T_{ij} = \lambda u_{ij} \delta_{ij} + 2\mu u_{ij} \quad (2)$$

$\delta_{ij}$ is the Kronecker delta function,

$$u_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

are the components of the strain tensor $\bar{u}$ in terms of the displacement vector $u$ and the spatial coordinates $x$, and $\lambda$ and $\mu$ are the Lamé coefficients, and where $u$ as well as the variables $\rho$, $\lambda$, $\mu$ have implicit spatial and time dependence. In particular, the values assumed by $\lambda$ and $\mu$ at each instant are those of the species contained in the region bounded by the curve $r(t)$ while the value assumed by $\rho$ is explained below.
c) Mass conservation

In our model, the cylinder expands and contracts according to a sinusoidal function, but no mass is actually being added or removed from the system. Beginning with the conservation relation for each species

$$\frac{\partial \rho^s}{\partial t} + \nabla \cdot (\rho^s \mathbf{v}^s) = 0$$

where $\mathbf{v}^s = \frac{\partial \mathbf{u}^s}{\partial t}$ is the velocity of the species under consideration, we solve for the density at the next time step at all points in space in terms of the densities of the previous two timesteps, for both materials, with starting configuration of the densities corresponding to $r(t) = r_0$. This way, the total mass of each species is conserved separately. The same is obviously not true for the density, whether of each species, or the total, which varies, as seen from Eq. 3. We note that it is not possible to make use of any simplifying assumptions to Eq. 3, given that in our system, $\rho$ is not slowly varying in space or time but makes abrupt jumps at the boundary of the two materials even as $r(t)$ is slowly-varying in time.

The approximation of incompressibility, or the second term of Eq. 3 being set to zero, results in a particularly simple implementation of mass conservation: a simple scaling of the density of each species as a function of time, as $r$ varies. However, it has relatively limited accuracy, particularly over long time scales, especially when the densities of the two components are significantly different. In practice, this approximation yields the same qualitative results, but since the displacement trajectories become unstable after a period of time, the information which may be extracted from them is less useful.

The elastic parameters $\lambda$ and $\mu$ of the system are also dependent on $r(t)$ as the boundaries of the cylinder vary in time. Their values were taken as either those of the cylinder material or those of the background material. Total energy conservation was not implemented as the changes to the elastic energy resulting from this are second-order in strain. The displacement itself was slowly-enough varying in space in order for this to be an acceptable approximation and at the long timescales studied, the trajectories for the displacement were found to be stable.

After an initial disturbance was set in motion at a point inside the cylinder, the system was allowed to evolve. Using the finite-difference time-domain (FDTD) method, where
| Material | $\rho$ (g/cm$^3$) | $C_l$ (km/s) | $C_s$ (km/s) |
|----------|-----------------|-------------|-------------|
| Iron     | 7.69            | 5.9         | 3.2         |
| Epoxy    | 1.18            | 2.54        | 1.16        |
| Silicon  | 2.34            | 8.43        | 5.84        |

TABLE I: Densities $\rho$ and sound velocities ($C_l$: longitudinal and $C_s$: shear) for the materials utilized in the present study.

The spatial as well as the time domain is discretized, and applying periodic boundary conditions and Bloch’s theorem, we solve Eqs. 1-3 for the propagation of elastic waves in inhomogeneous media. In particular, the calculational approach is implemented by discretizing space on a square grid, where the displacements $u_x$, $u_y$, $u_z$ are defined in the centre of each grid cell and their derivatives are approximated by central-difference formulas in both space and time, resulting in second-order accuracy, while the derivatives for the material parameters $\rho$, $\lambda$, $\mu$ are approximated by finite differences between grid and time points. The displacements at the following time step are obtained in terms of their values, as well as those of other parameters, at previous steps, and Fourier-transformed in order to obtain a spectrum of resonant peaks corresponding to the bands in $k$ space.

We utilized a square grid of 90 by 90 points for the calculations of the band structure for the static model $A(t) = 0$ and 180 by 180 points or even denser, up to 360 by 360, for more-detailed calculations of the frequency responses of the $k = 0$ cases with and without time dependence because the denser grid let to less abrupt changes in the material parameters as a function of time, or led to the accessing of smaller amplitudes in $A(t)$.

All of the lengths are expressed in terms of the lattice parameter $a$. Frequency is expressed as a dimensionless quantity $\tilde{\nu} = \nu a/c$ where $c$ is taken as the speed of sound $c_b$ in the background material. The time step in the simulations was taken as $\Delta t = 7.698 \times 10^{-4} a/c_b$.

In Table I we list the densities and sound velocities of the materials we used for the composites in our study.
III. RESULTS

In this section we apply the formalism and methods outlined in the previous section to two different composite phononic material systems: iron (Fe) cylinders in epoxy, and silicon (Si) cylinders in epoxy.

![Band structure diagram](image)

**FIG. 2:** Band structure in two dimensions for elastic waves polarized in the plane of the unit cell (viz. Fig. 1) for a non-time-varying system with an Fe cylinder in an epoxy matrix and a filling factor of 50%. Also shown is the irreducible Brillouin zone. The frequency axis is dimensionless as explained in the text.

The calculated band structure for waves polarized in the plane of the unit cell, of mixed transverse-longitudinal character, for Fe cylinders in epoxy without time-variation of material properties is shown in Fig. 2, for a filling factor of 50%. The plot is shown for a dimensionless frequency $\tilde{\nu}$. The large band gaps are what make these types of materials ideal phononic materials. The ratio of the band-gap to the mid-gap frequency for the largest gap, $\Delta \omega / \omega_g$ is 0.86 while for a filling factor of 60% for the same materials, the ratio is 0.91.

As we time-vary the cylinder radius as outlined in the previous section, we obtain the spectrum shown in Fig. 3 for various maximum amplitude variations of a sinusoid and fixed modulation frequency, scaled respectively to be dimensionless as $A' = A/a$ and $f' = f/a$, at $k = 0$. The horizontal arrows denote the location of the absolute band gaps in the non-time-varying case (viz. Fig. 2). Most notable are the blueshifting of the main resonance peaks, as well as the accompanying sidebands, which are offset...
FIG. 3: Frequency spectrum for a point-source excitation for a system with an Fe cylinder in an epoxy matrix and a filling factor of 50% at $k = 0$. Shown is the spectrum without any time variation of cylinder radius ($A = 0$) as well as with a maximum amplitude modulation of 0.011 and 0.022, at a frequency of 0.00157 (dimensionless units). The horizontal arrows show the locations of the absolute band gaps from the non-time-varying case (Fig. 2). The insets show in detail the structure of the two lowest resonance peaks.

from the main resonances by the amount of the modulation frequency, and the existence higher harmonics fading with distance in intensity. The sidebands do not necessarily have a smaller amplitude than the main resonance peak. As the modulation amplitude is increased, the resonance peaks become increasingly blueshifted and the sidebands cover a wider frequency range, as can be seen clearly by the insets for the region around the resonance peaks corresponding to a frequency of 0.0681 and 0.1759 respectively for the non-time-varying case.

The localization character of the vibrations associated with each resonance remains unchanged; even up to the second harmonic, it remains constant with the only difference from the non-time-varying case being that the surrounding material is also disturbed. In Fig. 4 we show the displacement profile of the lowest resonance peak at the frequency $\tilde{\nu} = 0.0681$ in the non-time-varying case (point X1 in Fig. 2), and its counterpart for the case where the maximum amplitude is $A' = 0.022$ and frequency $f' = 0.00157$ where the main peak is now at a frequency of $\tilde{\nu} = 0.0689$. We also show the localization of the
FIG. 4: Displacement profile $u_x^2 + u_y^2$ in the plane of periodicity, at the lowest resonance peak for a Fe cylinder in epoxy at a 50% filling factor at $k = 0$ (at the point X1 in Fig. 2 and also Fig. 3) for a non-time-varying cylinder radius: (a), and for the time-varying case of $A' = 0.022$ and $f' = 0.00157$: (b)-(c). The excitation was performed by a plane wave with a frequency of 0.0681 for (a) while in (b) and (c) the excitation frequencies were 0.0689 and 0.0705 (in dimensionless units $\tilde{\nu}$), corresponding to the main resonance and first sideband, respectively. Shown are both the profile views as well as the plane projections of the displacements. The colour maps are arbitrary - the results depicted are not on the same scale.

In all of these cases, the vibrations were highly localized on the outer edge of the cylinder, albeit blurred in the case of the time-varying case. In all of the higher-frequency resonances, the vibrations were localized outside the cylinder. For example, for point X2 in Fig. 2 at a reduced frequency og $\tilde{\nu} = 0.2352$ without time-variation, we have Fig. 5a which is only slightly disturbed in Fig. 5b, by adding amplitude and frequency modulation of $A' = 0.022$ and $f' = 0.00157$ respectively, whereupon the resonance frequency shifts slightly to $\tilde{\nu} = 0.2361$. In all of the above situations, the displacement profile was generated by exciting with a plane wave at the frequency being investigated, and examining the displacement in the plane $u_x^2 + u_y^2$, at steady state once the excitation has long died out. During the steady state, there are of course vibrations but the localization character of each resonance remains unchanged.
FIG. 5: Displacement profile of the point labelled as X2 in Fig. 2 ($\tilde{\nu} = 0.2352$) at $k = 0$ for a Fe cylinder in epoxy at a 50% filling factor, for (a) a non-time-varying cylinder radius (plane projection) and (b) time-varying case (profile view) whereupon the resonance frequency is shifted to $\tilde{\nu} = 0.2361$. The red circle denotes the boundary of the cylinder.

For fixed amplitude modulation and varying frequency modulation of the cylinder radius, we obtain the results in Fig. 6, confirming that the sidebands are offset by an amount equal to the modulation frequency $f'$ and that the blueshift of the main resonance peak compared to the non-time-varying case is unaffected by the value of $f'$.

We repeated some of our calculations on a simplified model of a mechanically-modulated split-ring structure, that of a hollow cylinder, and the results were similar, with large band gaps for a non-time-varying radius and resonances with harmonics when the outer radius is periodically modulated in time.

Similarly, for Si cylinders in epoxy without time variation we also find a band structure with large gaps, as shown in Fig. 7 for a filling factor of 60%. The gap-width to midgap frequency ratio $\Delta \omega/\omega_g$ for the largest gap is 0.45, much smaller than for Fe in epoxy even at the same filling factor, a result explained by the smaller contrast in the densities between the cylinder and matrix for this case, a factor of greater significance in determining this ratio than the disparity between the elastic constants, which in our case, is about the same for both composites. Upon adding time modulation of the cylinder radius, the spectrum at $k = 0$ acquires sidebands and harmonics as shown in Fig. 8 for a fixed frequency modulation $f'$ and varying amplitude modulation $A'$, and Fig. 9 for a
FIG. 6: Frequency spectrum for a point-source excitation for a system with an Fe cylinder in an epoxy matrix and a filling factor of 50% at $k = 0$. Shown is the spectrum without any time variation of cylinder radius ($A' = 0$) as well as with a maximum amplitude modulation of 0.011, and frequency modulation 0.00157 or 0.00314 respectively. The horizontal arrows show the locations of the absolute band gaps from the non-time-varying case (Fig. 2). The inset shows in detail the structure of the lowest resonance peak.

FIG. 7: Band structure in two dimensions for elastic waves polarized in the plane of the unit cell (viz. Fig. 1) for a system with a Si cylinder in an epoxy matrix and a filling factor of 60%. The frequency axis is dimensionless as explained in the text.

fixed amplitude modulation and three different frequency modulations. As in the case of the Fe cylinder in epoxy, only the amplitude but not the frequency of modulation caused blueshifting of the resonance peak as compared to the non-time-modulated case $A' = 0$. 

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FIG. 8: Frequency spectrum for a point-source excitation for a system with a Si cylinder in an epoxy matrix and a filling factor of 60% at $k = 0$. Shown is the spectrum without any time variation of cylinder radius ($A' = 0$) as well as with a maximum amplitude modulation of 0.011 and 0.044, at a frequency of 0.00157 (dimensionless units). The horizontal arrows show the locations of the absolute band gaps from the non-time-varying case (Fig. 7). The inset shows in detail the structure of the lowest resonance peak.

In Fig. 8 the amplitude is twice as large as in Fig. 3 for Fe cylinders in epoxy and all of the band gaps are completely obliterated although it is clear that the harmonics destroy the band gaps in all cases - especially as a result of the contribution of other wavevectors given that the bands themselves are not flat. However, for small amplitude modulations, the harmonics do die off quickly and in principle, some region within the largest band gap would be left intact. We note that the bands for Fe in epoxy are flatter than for Si in epoxy, something which occurs even for the same filling factors and whose origin lies in the greater hybridization of the continuum bands with the localized resonances from rigid-body modes of the cylinder in the case of Fe, due to the sharper contrast in the densities.

In Fig. 10 we plot the displacement $u_x^2 + u_y^2$ in both a profile and a plane projection in order to see the localization of the vibrations for the lowest resonance peak at the frequency 0.1205 in the non-time-varying case (point Y1 in Fig. 7), and in order to compare it with its counterpart at the frequency 0.1217 in the time-varying case where $A' = 0.044$ and $f' = 0.00157$ (viz. inset in Fig. 9 in Fig. 10). In Fig. 10c we show the second harmonic of the lowest resonance at $\tilde{\nu} = 0.1248$. Similar to the case of Fe in epoxy,
FIG. 9: Frequency spectrum for a point-source excitation for a system with a Si cylinder in an epoxy matrix and a filling factor of 60% at $k = 0$. Shown is the spectrum without any time variation of cylinder radius ($A' = 0$) as well as with a maximum amplitude modulation of 0.011, at frequencies of 0.00157, 0.00314 or 0.00628 respectively. The horizontal arrows show the locations of the band gaps from the non-time-varying case (Fig. 7). The inset shows in detail the structure of the lowest resonance peak.

The vibrations are highly localized on the edge of the Si cylinder although it is markedly blurry in the time-varying case. There are some bulges near the corners of the unit cell but those are associated with the proximity of the neighbouring cylinders. They are not present in the case of the Fe cylinder in epoxy depicted earlier because there the filling factor was 50%, but they are seen in that system as well for a filling factor of 60%. As for the case of Fe in epoxy, for the higher resonances, the vibrations are localized in the matrix outside the cylinder. In Fig. 10d we show the localization of the vibrations for the point Y2 in Fig. 7 located at $\tilde{\nu} = 0.2169$. Due to the close proximity of the cylinders at this large filling factor in neighbouring unit cells, the vibrations of this resonance are preferentially localized around the corners of the unit cell, where the spacing between neighbouring cylinders is greatest.

IV. CONCLUSIONS

We have examined the effect of periodic structural modulations in two phononic materials composed of a cylinder inside a matrix of highly-contrastling elastic properties and
FIG. 10: Displacement profile at the lowest resonance peak for a Si cylinder in epoxy at a 60% filling factor at $k = 0$ (at the point Y1 in Fig. and also Fig. for a non-time-varying cylinder radius: (a), and for the time-varying case of $A' = 0.044$ and $f' = 0.00157$: (b)-(c). The excitation was performed by a plane wave with a frequency of $\tilde{\nu} = 0.1205$ for (a) while in (b) and (c) the excitation frequency was 0.1217 and 0.1248 respectively, corresponding to the main resonance and the second sideband respectively. Shown are profile views as well as plane projections of the vibrational displacements. In (d) the red circle denotes the outer boundary of the cylinder and we show a plane projection of the vibrational displacement for the second-lowest resonance for the non-time-varying case (point Y2 in Fig. occurring at an excitation frequency of $\tilde{\nu} = 0.2169$. The colour maps are arbitrary - as the results depicted are not on the same scale.

we have found that in general the large band gaps are destroyed by harmonics generated by the temporal modulation. In order to obtain stable trajectories over long time scales it was necessary to implement mass conservation in our constitutive equations. However, the resonances’ time-harmonics induced by structural modulations may be exploited in order to achieve tunable phononic isolation or alternatively, bandpass filtering given that in phononic systems with fairly flat bands, by employing a large-enough modulation frequency, it is feasible that small band gaps could be retained for some regions. The harmonics’ periodicity and wide range could also find implementation in a frequency sensor. Modulating the cylinders’ radius may be achieved relatively simply: such as by a mechanically-modulated split-ring structure where the two components have a different
radius.

1. Y. Tanaka, Y. Tomoyasu, and S. Tamura. Band structure of acoustic waves in phononic lattices: Two-dimensional composites with large acoustic mismatch. *Physical Review B*, 62:7387–7392, 2000.

2. M. M. Sigalas. Elastic wave band gaps and defect states in two-dimensional composites. *Journal of the Acoustical Society of America*, 101:1256–1261, 1997.

3. M. M. Sigalas and N. Garcia. Theoretical study of three dimensional elastic band gaps with the finite-difference time-domain method. *Journal of Applied Physics*, 87:3122–3125, 2000.

4. M. Sigalas, M. S. Kushwaha, E. N. Economou, M. Kafesaki, I. E. Psarobas, and W. Steurer. Classical vibrational modes in phononic lattices: theory and experiment. *Zeitschrift für Kristallographie*, 220:765–809, 2005.

5. M. Kafesaki, M. M. Sigalas, and E. N. Economou. Elastic wave band gaps in 3-d periodic polymer matrix composites. *Solid State Communications*, 96:285–289, 1995.

6. R. Sainidou, N. Stefanou, and A. Modinos. Formation of absolute frequency gaps in three-dimensional solid phononic crystals. *Physical Review B*, 66:212301, 2002.

7. J. O. Vasseur, B. Djafari-Rouhani, L. Dobrzynski, M. S. Kushwaha, and P. Halevi. Complete acoustic band gaps in periodic fibre reinforced composite materials: the carbon/epoxy composite and some metallic systems. *Journal of Physics: Condensed Matter*, 6(42):8759–8770, 1994.

8. M. A. Attarzadeh and M. Nouh. Elastic wave propagation in moving phononic crystals and correlations with stationary spatiotemporally modulated systems. *AIP Advances*, 8:105302, 2018.

9. G. Trainiti and M. Ruzzene. Non-reciprocal elastic wave propagation in spatiotemporal periodic structures. *New Journal of Physics*, 18:083047, 2016.

10. M. A. Attarzadeh and M. Nouh. Non-reciprocal elastic wave propagation in 2d phononic membranes with spatiotemporally varying material properties. *Journal of Sound and Vibration*, 422:264–277, 2018.

11. H. Nassar, X. C. Xu, A. N. Norris, and G. L. Huang. Modulated phononic crystals: Non-
reciprocal wave propagation and willis materials. *Journal of the Mechanics and Physics of Solids*, 101:10–29, 2017.

12 B. M. Goldsberry, S. P. Wallen, and M. R. Haberman. Non-reciprocal wave propagation in mechanically-modulated continuous elastic metamaterials. *Journal of the Acoustical Society of America*, 146:782–788, 2019.

13 Y. Wang, B. Yousefzadeh, H. Chen, H. Nassar, G. Huang, and C. Daraio. Observation of nonreciprocal wave propagation in a dynamic phononic lattice. *Physical Review Letters*, 121:194301, 2018.

14 G. Trainiti, Y. Xia, J. Marconi, G. Cazzulani, A. Erturk, and M. Ruzzene. Time-periodic stiffness modulation in elastic metamaterials for selective wave filtering: Theory and experiment. *Physical Review Letters*, 122:124301, 2019.

15 F. Casadei, T. Delpero, A. Bergamini, P. Ermanni, and M. Ruzzene. Piezoelectric resonator arrays for tunable acoustic waveguides and metamaterials. *Journal of Applied Physics*, 112:064902, 2012.

16 Y. Y. Chen, G. L. Huang, and C. T. Sun. Band gap control in an active elastic metamaterial with negative capacitance piezoelectric shunting. *Journal of Vibration and Acoustics*, 136:061008, 2014.

17 A. Merkel, M. Willatzen, and J. Christensen. Dynamic nonreciprocity in loss-compensated piezophononic media. *Physical Review Applied*, 9:034003, 2018.

18 K. Danas, S. V. Kankanala, and N. Triantafyllidis. Experiments and modeling of iron-particle-filled magnetorheological elastomers. *Journal of the Mechanics and Physics of Solids*, 60:120–138, 2012.

19 C. Shen, J. Li, Z. Jia, Y. Xie, and S. A. Cummer. Nonreciprocal acoustic transmission in cascaded resonators via spatiotemporal modulation. *Physical Review B*, 99:134306, 2019.

20 J. Li, X. Zhu, C. Shen, X. Peng, and S. A. Cummer. Transfer matrix method for the analysis of space-time-modulated media and systems. *Physical Review B*, 100:144311, 2019.

21 M. A. Attarzadeh, J. Callanan, and M. Nouh. Experimental observation of nonreciprocal waves in a resonant metamaterial beam. *Physical Review Applied*, 13:021001, 2020.

22 K. Yi, M. Collet, and S. Karkar. Frequency conversion induced by time-space modulated media. *Physical Review B*, 96:104110, 2017.
23. J. Li, C. Shen, X. Zhu, Y. Xie, and S. A. Cummer. Nonreciprocal sound propagation in space-time modulated media. *Physical Review B*, 99:144311, 2019.

24. H. Nassar, H. Chen, A. N. Norris, M. R. Haberman, and G. L. Huang. Non-reciprocal wave propagation in modulated elastic metamaterials. *Proc. R. Soc. A*, 473:20170188, 2017.

25. X. Zhu, J. Li, C. Shen, X. Peng, A. Song, L. Li, and S. A. Cummer. Non-reciprocal acoustic transmission via space-time modulated membranes. *Applied Physics Letters*, 116:034101, 2020.

26. E. Riva, M. Di Ronco, A. Elabd, G. Cazzulani, and F. Braghin. Non-reciprocal wave propagation in discretely modulated spatiotemporal plates. *Journal of Sound and Vibration*, 471:115186, 2020.

27. M.A. Attarzadeh, H. Al Ba’ba’a, and M. Nouh. On the wave dispersion and non-reciprocal power flow in space-time traveling acoustic metamaterials. *Applied Acoustics*, 133:210–214, 2018.

28. E. S. Cassedy and A. A. Olner. Dispersion relations in time-space periodic media: Part i - stable interactions. *Proc. Inst. Elect. Electronic Engrs*, 51:1342–1359, 1963.

29. E. S. Cassedy. Waves guided by a boundary with time-space periodic modulation. *Proc. IEE*, 112:269, 1965.

30. M. S. Kushwaha and P. Halevi. Bandgap engineering in periodic elastic composites. *Applied Physics Letters*, 64:1085–1087, 1994.

31. M. S. Kushwaha and B. Djafari-Rouhani. Giant sonic stop bands in two-dimensional periodic system of fluids. *Journal of Applied Physics*, 84:4677–4683, 1998.