Contact pressure in interference joint with modified grooves of shallow depth

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Abstract. The analytical method for determining the magnitude of the contact pressure in a cylindrical interference joint with shallow depth grooves is described in the publication. This method takes into account the geometric features and the relative position of the external members. The calculated dependences are based on the Lame’s formulas are given in the publication. They can be used to assess the bearing capacity of the joint at any macrogeometry of the joint, independently of the number, shape, size of grooves and their location. The method is different by simplicity and visibility. It provides sufficient accuracy of results for engineering calculations.

Key-words: interference joints, contact pressure, deformation, deformation wave, modification of the joint grooves, strength, bearing capacity

1. Introduction
Interference joint is one of the most common fixed joints. The strength and bearing capacity of such a connection depends on many factors, in particular, the interference (δ) [1, 2, 3, 4, 5], the size and shape of the fitting parts to be joined, their relative position [1, 4, 5, 6], the physicomechanical properties of the materials of the parts, the state of the contacting surfaces, their macro-and microgeometry [1, 4, 7, 8], etc. One of the method to increase the carrying capacity of these compounds is to modify the joint grooves of shallow depth. Which are made on one of the mating surfaces, as a rule, on the surface of the shaft. In this case, after assembly, the material external member is deformed differently in different parts of the joint. Due to the contact rupture within the grooves, the surface of the part receives less radial displacement than at the contact area, and the material partially or completely fills the grooves, forming a deformation wave. The strength of the joint is determined by the amount of friction force arising at the contact areas and the ability of each deformation wave to resist the relative displacement of parts. The value of contact pressure and, in particular, its average value should be known to assess the strength and bearing capacity of compounds modified grooves, the choice of rational geometry of the joint. Therefore, the development of new methods and improvement of known methods for calculating contact pressure are important and relevant.

2. Problem statement
Consider the engineering method for determining the average value of contact pressure in connection with the tension, modified grooves of shallow depth.

3. Theory
In real joints the contact between parts has discrete character [1, 7, 9]. Due to the complexity of determining the actual contact area and the actual contact pressure in engineering practice the Lame formulas are often used for the design and verification calculations [1, 3, 4, 5, 10]. So in [5] describes an engineering method that allows to calculate the average contact pressure \( q_{av} \) on the smooth cylindrical interference joint and takes into account the lengths of the connected parts and their relative position. According to the method, the value \( q_{av} \) depends on the number \( K_1 \) of protruding shaft ends and the size and number \( K_2 \) of cantilever elements of the external member. The formula for calculating the value \( q_{av} \) is

\[
q_{av} = q_0 + K_1 \Delta q_1 + K_2 \Delta q_2, \tag{1}
\]

where 
\[
q_0 = \frac{\delta}{d(C_1/E_1 + C_2/E_2)}, \quad C_1 = \frac{1+(d_1/d)^2}{1-(d_1/d)^2} - \mu_1, \quad C_2 = \frac{1+(d_2/d)^2}{1-(d_2/d)^2} - \mu_2; \quad d, d_1, d_2 \text{ are the sizes of parts to be joined (figure 1, a); } E_1, E_2, \mu_1, \mu_2 \text{ are the modulus of elasticity and Poisson's ratio materials of the shaft and sleeve, respectively. On figure 1, a in the joint the shaft is solid, respectively } d_1 = 0.
\]

\[\text{Figure 1.} \quad \text{I Interference joint modified by ring grooves:}
\]
\[\text{a} – \text{general view of joint; } \text{b} – \text{fragment of the joint;}
\]
\[1 \text{ is the shaft; } 2 \text{ is the external member; } 3 \text{ is the grooves; } 4 \text{ is the deformation wave.}
\]

Additional contact pressure \( \Delta q_1 \) appears due to one protruding end of the shaft. It is calculated by the formula

\[
\Delta q_1 = \frac{0.009 q_0 d}{L}, \tag{2}
\]

where \( L \) is the length of the joint, in the joint on figure 1, a \( L = l_2 \).

The value \( \Delta q_2 \) can be determined by the formula:

\[
\Delta q_2 = q_0 \frac{l_c}{L} \left(1 - \frac{2.68 d_2 l_c}{(1 - \mu_2)d^2 + (1 + \mu_2)d_2^2}\right)^2, \tag{3}
\]

where \( l_c \) is the length of the cantilever.

The algorithm for calculating the average value of the contact pressure in interference joint with modified grooves is considered in a specific example. On figure 1,a. to increase the bearing capacity of the axial force joint on the fitting surface of the shaft 1, ring grooves are provided with a width \( l_{gv} \). The contact areas are of equal width \( l \). Grooves reduce the geometric area \( S \) of the joint, but cause the
concentration of contact pressure and accordingly increase its average value ($q_{av}^D$). The average contact pressure may be different on different areas. In figure 1,a, in the connection the average value of contact pressure on the area located between the grooves is greater than on the areas at the mating surface boundaries.

To determine the contact pressure of the deformation wave (Figure 1,b) imagine it as a set of two cantilever, length $l_c = 0.5l_{gv}$. Each of them has an additional force effect $\Delta q_2$ on the contact area adjacent to the groove. The shaft within the groove has an additional force effect $\Delta q_1$ on adjacent areas. The formulas (2) and (3) are used for the calculation $\Delta q_1$ and $\Delta q_2$, respectively, provided that $L = l_a$.

Thus, at the areas located between the grooves, the average contact pressure $q_{av}$ will be calculated by the formula

$$q_{av} = q_0 + 2\Delta q_1 + 2\Delta q_2.$$  

According to the terminology adopted in [5], on the extreme area at the mating surface boundaries, an additional force effect is created by a deformation wave and two protruding shaft ends. The average value of the contact pressure can be determined by the formula

$$q_{av} = q_0 + 2\Delta q_1 + \Delta q_2.$$  

Knowing the average value of the contact pressure at each areas it is easy to calculate the average value $q_{av}$ at the joint. The average value $q_{av}^D$ of the contact pressure in the connection depends on the width of the grooves, their number, width of the contact areas. As the width of the grooves increases, the geometric area $S_D$ of the joint decreases. Since the volume of the deformable material remains the same, the value $q_{av}^D$ increases and the formula is used to determine it

$$q_{av}^D = q_{av}^S S_s / S_D,$$  

where $S_s$ is the geometric area of the corresponding smooth joint; $q_{av}^S$ is the average value of the contact pressure in the smooth joint.

4. Experimental results

The correctness of the proposed method for calculating the average contact pressure in the compounds modified by grooves and the correctness of the formula (4) tested by the finite element method (FEM). More than 40 joints with modified ring grooves in the fitting surfaces of the shaft were studied. Basic parameters of the joints: $d_1 = 0$, $d = 40$ mm, $d_2 = 70$ mm, $\delta = 0.06$ mm, $E_1 = E_2 = 2\cdot10^5$ MPa, $\mu_1 = \mu_2 = 0.28$. Joints of details of equal length, with longer shaft, with longer external member were considered. The width $l_{gv}$ of the grooves, their number ($n_{av}$) and location, the width $l_a$ of the contact areas were changed. When $n_{av} = 1$ the groove was located symmetrically with the respect to the mating surface boundaries. Some of the results are shown in figure 2 – 4 in graphic form. Figure 2 and 3 show graphs of the dependence of the average value of the contact pressure in the joints of details of equal length and with a longer shaft with modified by one groove, on its size $l_{av}$. 

Figure 2. The dependence graphic of the average contact pressure from the width of the groove. The results are obtained: 1 – FEM; 2 – by the proposed method; 3 – by the formula (4).

Figure 3. The dependence graphic of the average contact pressure from the size of the grooves in the joints with the longer shaft. The results are obtained: 1 – FEM; 2 – by the proposed method; 3 – by the formula (4).

Figure 4. The dependence graphic of $q_{av}^D$ from the width of the contact areas. The results are obtained: 1 – FEM; 2 – by the proposed method; 3 – by the formula (4).
Curve 1 is based on the results of the FEM calculation, curve 2 is based on using the proposed method, and curve 3 is based on using formula (4). Note that in all connections the width \( l_a \) of the contact areas remained constant and equal to 10 mm. The figures clearly show that the protruding ends of the shaft in the joints of details of different lengths increase the average contact pressure due to the concentration of \( q_{av}^D \) at the mating surface boundaries.

Figure 4 shows graphs of the dependence of the value \( q_{av}^D \) of the width of the contact areas in the joints of details of equal length modified by a single groove width of 8 mm.

5. Results discussion

Graphs in figure 2 and 3 show that the FEM and the proposed method of calculating the value \( q_{av}^D \) give nearly similar results. Their divergence increases with the increase of width of the groove. If \( l_{gv} = 0.2d \) there is a divergence of 2 \% (figure 2) and 1.5 \% (figure 3), when \( l_{gv} = 0.5d \) it is 5.6 \% (Figure 2) and 1.5 \% (figure 3).

The divergence between the calculation results obtained by the FEM and the formula (4) by \( l_{gv} = 0.2d \) is 4.5 \% (figure 2) and 3.2 \% (figure 3), and when \( l_{gv} = 0.5d \) increased to 30.3 \% and 20.9 \% respectively.

Curves 1 and 2 in figure 4 practically match. The divergence between the results obtained by the FEM and the proposed method was 3.3 \% at \( l_a = 0.1d \), and decreased to 1.5 \% at \( l_a = 0.25d \). The divergence between the values \( q_{av}^D \) calculated by FEM and the formula (4), at the same size \( l_a \) was 9 \% and 4.5\%, respectively. Thus, with the increase in the width of the areas with a constant width of the groove, the accuracy of the calculation increases.

At research of compounds with several grooves \( (2 \leq n_{av} \leq 4) \) width of the grooves and their position in the joint varied. In particular, the joints in which the grooves are located at the mating surface boundaries (or at one mating surface boundary) are considered. With this arrangement of grooves, intentional breakage of the contact can lead to the formation of cantilever elements and contact pressure concentration by of the external members.

The research found that regardless of the number of grooves, their position and the ratio of the lengths of the fitting joints by the FEM and the proposed method of calculation \( q_{av}^D \) give a good convergence of the results. Their maximum divergence is not more than 5.5 \%.

The divergence between the results \( q_{av}^D \) of the FEM calculation and the formula (4) at \( l_{gv} \leq 0.2d \) is no more than 7 %. With a larger width of the grooves and in the presence of cantilevering elements longer than 0.125d, the inaccuracy of the formula (4) in the external member increases to 16 \% at \( n_{gv} = 2 \), \( l_{gv} = l_a = 10 \text{ mm}, \ l_a = 10 \text{ mm} \) and up to 30 \% at \( n_{gv} = 1, \ l_{gv} = 20 \text{ mm}, \ l_a = 10 \text{ mm} \). Its maximum value does not exceed 10 \% in other variants. It should be noted that the use of wide grooves is impractical, since it can be dangerous for the strength of the joint and the concentration of contact pressure. For example, researches conducted by the FEM show that in joints having a size of \( L = 40 \text{ mm} \), with a groove width of 5 mm, the contact pressure varies in the range of 157 – 98.8 MPa, and with a groove width of 20 mm in the range of 239 – 101 MPa.

6. Conclusions

Obtained the results in the course of the research show that the proposed method of calculating the average contact pressure in the interference joints with modified by ring grooves provides sufficient accuracy for engineering calculations.
The average value of the contact pressure in the interference joints with modified grooves of shallow depth, increases with a decrease in the geometric contact area $S_D$. The dependence of the value $q_{av}^D$ on the value $S_D$ is nonlinear, when $l_{gv} \leq 0.25d$ the dependence is close to linear, which allows to use the formula (4) to determine the average contact pressure.

Formula (4) can be applied to any geometry grooves, regardless of the number and position of the grooves.

7. References

[1] Grechischev E S and Ilyashenko A A 1981 The interference joint (Moscow: Mashinostroenie)
[2] Zenkin A C 1981 Technological bases of assembly of the interference joints (Moscow: Mashinostroenie)
[3] Ponomarev S D 1958 Strength calculations in mechanical engineering: 3 tons (Moscow: Mashinostroenie)
[4] Ryazantseva I L 2015 Theory and Design of Interference Joints (Omsk: OmGTU)
[5] Ryazantseva I L Dyundik O S and Zgonnik I P 2019 The method of the calculation of the average contact pressure in the interference joints of the details of different length Journal of Physics: Conference Series 7 31–4
[6] Pérez Cerdán J C Lorenzo M and Blanco C 2012 Stress concentrations in interference fits joints with chamfered hubs Applied Mechanics and Materials 184 489–3
[7] Papshev D D Tyutikov G F and Mashkov A N 1981 The dependence of the strength of the interference joints on the methods of processing the mating surfaces Engineering Research 10 16–4
[8] Schneider J G and Zabrodin V A 1976 The strength of the fixed joints of parts with a regular microrelief Engineering Research 6 41-1
[9] Matlin M M Kazkin V A Kazankina E N and Mozgunova A I 2018 Prediction of the actual contact area in the interference joints taking into account the hardness ratio of the parts Engineering Research 8 8–3
[10] Camposa Urso A and David E Hallb 2019 Simplified Lamé’s equations to determine contact pressure and hoop stress in thin-walled press-fits Thin-Walled Structures 138199-8