Magnetic Field Effects in the Pseudogap Phase: A Competing Energy Gap Scenario for Precursor Superconductivity

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We study the sensitivity of $T_c$ and the pseudogap onset temperature, $T^*$, to low fields, $H$, using a BCS-based approach extended to arbitrary coupling. We find that $T^*$ and $T_c$, which are of the same superconducting origin, have very different $H$ dependences. This is due to the pseudogap, $\Delta_{pg}$, which is present at the latter, but not former temperature. Our results for the coherence length $\xi$ fit well with existing experiments. We predict that very near the insulator $\xi$ will rapidly increase.

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One of the central questions in understanding the underdoped cuprates is the extent to which the superconducting phase is described by BCS theory. Recent experiments indicate that the pseudogap persists below $T_c$ in the underlying normal density of states. Thus, the fermionic excitation gap $\Delta$ is to be distinguished from the order parameter $\Delta_{sc}$. These two energy gaps mirror a distinction between the two temperatures $T^*$ (the pseudogap onset), and $T_c$ (the superconducting transition), which behave differently as a function of hole concentration $x$ as well as of magnetic field, $H$. Indeed, $T^*$ and $T_c$ are, respectively, weakly and strongly dependent on $H$ in the well-established pseudogap regime. Moreover, the distinction between these temperature and energy scales has been frequently cited as evidence that they have different physical origins.

In this paper we provide a counter argument to this widely stated inference by demonstrating that these crucial magnetic field effects in the pseudogap phase, are entirely compatible with superconductivity as origin for both $T^*$ and $T_c$. Our approach is based on an extended version of BCS theory, in which the attractive coupling $g$ is contemplated to be strong enough so that pairs begin to form at a higher temperature $T^*$ than the $T_c$ at which they Bose condense. We have shown [4] that as a necessary consequence $\Delta \neq \Delta_{sc}$. Moreover, our work has emphasized [4] that a (pseudo)gap in the fermionic spectrum at $T_c$ is deleterious for superconductivity. Thus, as observed experimentally, as a function of decreasing $x$, $T_c$ decreases as the pseudogap or $T^*$ grows.

A calculation of the field dependence of $T_c$ (i.e., $H_{c2}$) is an important problem in its own right. (i) This is the only way to provide a precise interpretation of the “coherence length” $\xi$, which we demonstrate here is very different from that of BCS theory. (ii) An analysis of $H_{c2}$ is tantamount to arriving at a reformulation of the microscopically deduced Ginzburg-Landau (GL) free energy up to quadratic terms, which must necessarily incorporate the presence of a non-zero pseudo-gap at $T_c$. (iii) Because the “competing order parameter” scenario also addresses the observation that $\Delta \neq \Delta_{sc}$, as well as the competing $x$ dependences of $T_c$ and $T^*$, magnetic field effects may provide a unique testing ground for distinguishing between these two scenarios.

Indeed there is a rather close similarity in the structure of our zero-field theory to the phenomenology deduced from thermodynamical data by Loram et al. [11]. Our mean field calculations show that the gap equation for $T \leq T_c$ reduces to the usual BCS form, but with a new quasi-particle dispersion

$$E_k = \sqrt{\epsilon_k^2 + (\Delta_{sc}^2 + \Delta_{pg}^2)\phi_k^2} = \sqrt{\epsilon_k^2 + \Delta^2\phi_k^2}. \quad (1)$$

Here $\phi_k$ is associated with the pairing symmetry. It follows from this that the larger is $\Delta_{pg}(T_c)$, the lower the transition temperature $T_c$. In contrast to the work in Refs. [6] and [8] here $\Delta_{pg}(T)$, is determined self-consistently and derives from the presence of a strong pairing attraction. Moreover, this pseudogap, $\Delta_{pg}$, persists below $T_c$.

The conclusions of this paper are relatively straightforward and we begin with a simple intuitive argument to address $H_{c2}$. Consider the Ginzburg-Landau free energy functional near $T_c$ to quadratic order in $\Delta_{sc}$, in a finite field

$$F \sim \left(\tau_0(T) + \eta^2 \left(\frac{\nabla}{i} - \frac{2eA}{c}\right)^2\right)|\Delta_{sc}|^2. \quad (2)$$

Here $\tau_0$ describes how the system approaches the critical point with varying temperature, and $\eta^2$ is the stiffness against spatial variations of the order parameter. The mean-field behavior of $\tau_0$ near $T_c$ yields $\tau_0(T_c) = \tau_0(1 - T/T_c)$. It follows that

$$- \frac{1}{T_c} \frac{dT_c}{dH} \bigg|_{H=0} = \frac{2\pi}{\Phi_0} \frac{\xi^2}{\Phi_0 \tau_0} = \frac{2\pi \eta^2}{\Phi_0 \tau_0}. \quad (3)$$

where $\xi$ is the zero temperature coherence length. A rough extrapolation yields $H_{c2}(0) \approx \Phi_0/(2\pi c^2)$. In the small $g$ (i.e., BCS) case $\tau_0 = N(0)$, the density of states per spin at the Fermi surface, and $\eta^2 = N(0)\zeta(3/2)4\pi^2(v_F/T_c)^2$. The squared coherence length $\xi_{BCS}^2 = \frac{7\zeta(3)}{4\pi^2}(v_F/T_c)^2$ is determined by the stiffness $\eta^2$ with $\tau_0$ cancelling the density of states.

In contrast, in the strong coupling case the pseudogap modifies the fermionic quasiparticle dispersion through a replacement of $\epsilon_k$ by $E_k$ and thereby suppresses $\tau_0$. Moreover, the stiffness $\eta^2$ (which is relatively insensitive to the energy scale
of the pseudogap), decreases due to the diminishing pair size. There is, thus, a competition between the numerator and denominator in Eq. (10). The decrease in \( \tau_0 \) dominates at sufficiently large coupling \( g \), resulting in an extended flat region followed by an eventual growth of \( \xi^2 \) with increasing \( g \). The latter reflects the approach to the ideal “boson” limit, where \( T_c \) is suppressed to zero at any \( H \neq 0 \).

Next, we provide a microscopic derivation of the parameters in Eqs. (2) and (3) (and a related counterpart for \( T^* \)). A central theme in our paper is that both the field sensitivity of \( T_c \) and of \( T^* \) in small \( H \) can be studied through the zero-field normal state pair propagator, or the inverse \( t \)-matrix \( t^{-1}(Q) = 1/g + \chi(Q) \), where \( Q = (q, \Omega) \) is a four-vector. The coefficients in Eq. (2) will be shown to arise from an expansion of \( t^{-1} \) in the momentum components perpendicular to the field (indexed by \( i, j = x, y \) as

\[
\tau_0 = \frac{1}{g} + \chi(0, 0), \quad \eta^2 = \frac{1}{2} \sqrt{\left| \partial_q \partial_{q_j} \chi(Q) \right|}_{Q=0}, \quad (4)
\]

where we have generalized from Eq. (2) to include possible anisotropy.

A proper motivation for our choice of \( \chi(Q) \) is essential. The formalism in this paper combines a Green’s function decoupling scheme with a generalization of the BCS ground state wavefunction. This formalism allows for Fermi- and non-Fermi-liquid (i.e., \( \Delta_{pg}(T_c) \neq 0 \)) based superconductivity, according to the size of \( g \), with self consistently determined chemical potential \( \mu \). In the present paper all technical issues of this decoupling scheme can be simply by-passed, and the results obtained are not only intuitive, but rather general. All that is needed here is the observation that the pair susceptibility \( \chi(q, \Omega) = \chi(Q) = \sum_k G(K)G_0(Q - K)\varphi^2_{k - q/2} \). Thus

\[
\chi(q, 0) = \sum_k \varphi^2_{k - q/2} \times \left[ 1 - f(E_k) - f(\epsilon_{k - q}) \right] \frac{\varphi^2_k}{E_k + \epsilon_{k - q}} - f(E_k) - f(\epsilon_{k - q}) \right] \frac{\varphi^2_k}{E_k - \epsilon_{k - q}} \right], \quad (5)
\]

Here \( \varphi^2_k \) and \( \varphi^2_{k - q} \) are the usual BCS coherence factors, and \( \varphi^2_k = (1 + (k/k_0)^2)^{-1} \), or \( (\cos(k_x a) - \cos(k_y a))^2 \), for \( s \)-wave pairing in 3D jellium or \( d \)-wave pairing on a quasi- 2D lattice, respectively. That there is one full Green’s function (\( G \)) along with one bare Green’s function (\( G_0 \)), reflects the structure of the BCS gap equation, which introduces this \( \chi(Q) \) form, (with integrand proportional to the usual Gor’kov \( F \) function). All numerical calculations in this paper are based on Eqs. (1), (4), and (5), given \( \Delta_{pg} \equiv \Delta_{pg}(T_c) \). Although here we proceed more self consistently, our analytical scheme for computing the various energy scales can be by-passed, if \( \Delta_{pg} \) and \( T_c \) are pre-determined, e.g., fitted to cuprate experiments.

We next turn to \( T^* \), where the Fermi liquid begins to break down; this is associated with the onset of a resonance (5), as \( g \) becomes sufficiently large. Detailed numerical results based on the coupled Green’s function equations show that to a good first order approximation this resonance temperature can be deduced from the condition \( 1/g + \chi_0(0, 0) = 0 \), where \( \chi_0 \) is given by Eq. (6) with \( \epsilon_k \) substituted for \( E_k \). Indeed, quite generally, at \( T \geq T^* \), the \( t \)-matrix can be well approximated by using \( \chi_0(Q) \) in place of \( \chi(Q) \).

Magnetic field effects can be readily included into our formalism. We begin with a derivation of \( T^*(H) \) to linear order in \( H \). Our Hamiltonian consists of the field dependent kinetic energy term along with the usual two-body pairing interaction of strength \( V_{k,k'} = g\varphi_k^2\varphi^2_{k'} \), where we now include interactions between pairs of non-zero net momentum. Fluctuations of pairs with finite momenta \( \langle q, k \rangle \) are characterized by the correlation function

\[
D(q, k; q', k') = \langle T b(q, k; \tau)b^\dagger(q', k'; \tau') \rangle,
\]

where \( b(q, k, \tau) = e^{i(H - \mu N)\tau}c_{q/2-kq}c_{q/2+kq}e^{-(H - \mu N)\tau} \).

Summing ladder diagrams leads to a Dyson equation \( D = G_0G_0 - G_0G_0VD \) with solution

\[
\sum_{k, k'} D\varphi_k^*\varphi_{k'} = \frac{\widetilde{\chi}_0}{1 + \frac{\chi_0}{g}} \quad (6)
\]

where \( \widetilde{\chi}_0 = G_0G_0 \) is the counterpart pair susceptibility for \( H \neq 0 \), and the field-dependence of the bare electron propagator \( G_0 \) is implemented using the semiclassical phase approximation, elevating both \( \widetilde{\chi}_0 \) and \( \sum_{k, k'} D\varphi_k^*\varphi_{k'} \) to integral operators whose eigenvalues satisfy Eq. (6). This approximation allows the calculation of the eigenvalues of \( \chi_0 \) from the zero-field pair susceptibility \( \chi_0 \) in the regime \( T \gg \mu H/mc \). The field-induced relative phase shift between electrons in a pair renormalizes the interaction \( V_{k,k'} \), but the effect is quadratic in \( H \) and is therefore ignored here. The pairing resonance temperature \( T^* \) is defined by the appearance of an eigenvalue \( \Pi_0 = -g^{-1} \) of the \( \widetilde{\chi}_0 \), which causes Eq. (6) to diverge. We define parameters \( \eta^2 \) and \( \tau^0_0 = \tau^0(1 - T/T^*) \) analogous to those which appear in Eq. (4) and obtain

\[
g^{-1} + \Pi_0 = \tau^0_0 + \eta^2 \cdot \frac{2e}{c} H = 0 \quad (7)
\]

which defines \( T^*(H) \) to linear order in \( H \). The slope of \( T^*(H) \) at \( H = 0 \) is

\[
\left. \frac{1}{T^*} \frac{dT^*}{dH} \right|_{H=0} = \frac{\eta^2}{\tau^0_0} \frac{2\pi}{\Phi_0} \quad (8)
\]

which leads to the associated “coherence length” \( \xi^2 = \eta^2/\tau^0_0 \). The stiffness \( \eta^2 \) can be explicitly evaluated using the zero-field pair susceptibility, and

\[
\eta^2 = \sum_k \varphi^2_k \left[ -f'(\epsilon_k) + \frac{d\mu}{dt} \frac{T}{T^*} \right] = \left( 1 - \frac{2f'(\epsilon_k)}{2\epsilon_k} + f'(\epsilon_k) \right) \quad T = T^* \quad (9)
\]

Here we have included a contribution from the temperature dependence of \( \mu \). In the weak coupling case, the chemical
potential is pinned at \( E_F \), and we recover the (s-wave) BCS limit
\( \bar{\tau}_0 = -\sum_k f'(\epsilon_k) \phi_{\ell}^2 \approx N(0) \phi_{\ell}^2 \approx N(0) \).

The dashed line in Fig. 1 is a plot of the slope of \( T^* \) with \( H \), as a function of the coupling \( g \), for the case of s-wave jellium. This should be compared with the inset, from Ref. [3] which illustrates the suppression of \( T_c \) by \( \Delta_{pg} \) for all \( g \) in the fermionic regime; (ultimately, when \( \mu \) becomes negative, \( T_c \) starts to increase again). When pseudogap effects are weak, \( T^* \) is essentially the same as \( T_c \), and depends strongly on \( H \). A stronger pairing interaction \( g \) causes \( \Delta_{pg} \) to increase and the pair size to decrease; the latter effect diminishes the stiffness and causes \( T^* \) to be weakly field dependent.

We turn next to \( T_c(H) \) and note that a solution of the coupled equations of motion (as was done in the zero field case) appears prohibitively difficult. Nevertheless, based on the above observations that (i) the zero field \( T^* \) scales rather well with \( \Delta_{pg} \) (both theoretically \([13] \) and experimentally \([4] \)) and (ii) that \( T^* \) is very weakly field dependent in the well-established pseudogap regime, we infer that \( \Delta_{pg} \approx \Delta_{pg}(T_c) \) is weakly \( H \) dependent. This assumption, along with the semi-classical phase approximation for the full Green’s function, are the only essential assumptions made here. The weak \( H \) dependence in \( \Delta_{pg} \) appears compatible with experiment \([13] \) and underlies a GL formulation (Eq. 3) in which only the superconducting order parameter is coupled to the magnetic field. In this way, for the purposes of computing \( H_{c2} \), the pseudogap enters as a relatively rigid band structure effect, which is accounted for by introducing the full pair susceptibility into the standard \( H_{c2} \) formalism \([13] \). This approach necessarily yields the correct \( H = 0 \) result for \( T_c \).

It also leads naturally to \( \phi_{\ell}^2 \), from which one deduces a rather complicated expression (not shown) for \( \eta^2 \) along with

\[
\bar{\tau}_0 = -\sum_k \phi_{\ell}^2 f'(E_k),
\]

where we have omitted contributions from the temperature dependence of \( \mu \) and \( \Delta_{pg} \) near \( T_c \). Eq. (10) contains the essential physics introduced by the pseudogap. The summation essentially measures the \( E = 0 \) density of states which is depleted by the pseudogap at strong coupling, leading to \( \bar{\tau}_0 \sim e^{-\Delta_{pg}/T_c} \). Moreover, we have shown analytically that the neglected terms further suppress \( \bar{\tau}_0 \), and, therefore, do not qualitatively change our results.

FIG. 1. Coherence lengths \( \xi \) (for \( T_c \), solid) and \( \xi^* \) (for \( T^* \), dashed) with variable coupling \( g \) for s-wave jellium; here \( g_c = -4\pi/mk_0 \) units are \( k_F^{-1} \). See Eqs. (3) and (8). Inset shows the zero field behavior of \( T^* \), \( T_c \), and \( \Delta_{pg} \) from Ref. 2, where \( T \) and \( \Delta \) are in units of \( E_F \).

FIG. 2. Coherence lengths at \( T_c \) (solid) and \( T^* \) (dashed) versus variable coupling \( g \) for a d-wave lattice at density \( n = 0.85 \). \( \xi \) is in units of \( a \). The inset plots the zero field energy scales from Ref. 2, and \( T, \Delta \) are in units of \( 4t_|| \).
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The pseudogap is associated with precursor superconductivity. We presume in the absence of more microscopic information that \( g \) is independent, leaving one free parameter in our theory. As in experiment, \( T^* \) is found to be less field sensitive in the underdoped than overdoped regimes. As the insulator is approached, \( \xi \) rapidly increases, while \( \xi^* \) continues to decrease. We have thus demonstrated that the different observed field dependences of \( T^* \) and \( T_c \) (for both under- and over-doped cuprates) are contained in our theory, in which the pseudogap is associated with precursor superconductivity.

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Note Added. — After this work was completed we learned of related experimental studies by Shibauchi et al. which show strong similarities to our theoretical predictions.

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