Topological Descendants: DDK and KM Realizations

Beatriz Gato-Rivera and Jose Ignacio Rosado

Instituto de Matemáticas y Física Fundamental, CSIC,
Serrano 123, Madrid 28006, Spain

ABSTRACT

The ”minimal matter + scalar” system can be embedded into the twisted $N = 2$ topological algebra in two ways: à la DDK or à la KM. Here we present some results concerning the topological descendants and their DDK and KM realizations. In particular, we prove four ”no-ghost” theorems (two for null states) regarding the reduction of the topological descendants into secondaries of the ”minimal matter + scalar” conformal field theory. We write down the relevant expressions for the case of level 2 descendants.

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*e-mail addresses: bgato, jirs @cc.csic.es
1 Introduction

Two years ago it was shown [1], [2] that the "d ≤ 1 matter + scalar" system, extended by appropriate bc ghosts, provides two different realizations of the twisted $N = 2$ topological algebra: the DDK (David-Distler-Kawai) and the KM (Kontsevich-Miwa) realizations. The first one was a bosonic string construction with all the elements therein (the Liouville field and the $c = -26$ reparametrization ghosts) [3], while the second was related to the KP hierarchy through the Kontsevich-Miwa transform [2], [4] (for a short, pedagogical introduction to the subject see [5]).

In this letter we address the issue of the topological descendants (bosonic as well as fermionic) and their special features when they are realized à la DDK or à la KM. In particular, we prove four no-ghost theorems concerning the reduction of the topological descendants into secondaries of the "matter + scalar" conformal field theory (two of the theorems deal with null descendants). A key fact proves to be the $Q^m_0$ or $G^m_0$ invariance of the topological states ($Q^0_0$ or $G^0_0$ invariance of the topological null states). As an example, we write down the relevant results for the case of level 2 descendants, showing complete agreement with the no-ghost theorems.

2 Topological Descendants

The $N = 2$ twisted topological algebra reads [6], [7]

$$[\mathcal{L}_m, \mathcal{L}_n] = (m - n)\mathcal{L}_{m+n}, \quad [\mathcal{H}_m, \mathcal{H}_n] = \frac{c}{3}m\delta_{m+n,0},$$

$$[\mathcal{L}_m, \mathcal{G}_n] = (m - n)\mathcal{G}_{m+n}, \quad [\mathcal{H}_m, \mathcal{G}_n] = \mathcal{G}_{m+n},$$

$$[\mathcal{L}_m, Q_n] = -nQ_{m+n}, \quad [\mathcal{H}_m, Q_n] = -Q_{m+n}, \quad m, n \in \mathbb{Z}. \quad (2.1)$$

where $\mathcal{L}_m$ and $\mathcal{H}_m$ are the bosonic generators corresponding to the energy momentum tensor (Virasoro generators) and the topological $U(1)$ current respectively, while $Q_m$ and $\mathcal{G}_m$ are the fermionic generators corresponding to the BRST current and the spin-2 fermionic current respectively. The "topological central charge" $c$ is the true central charge of the $N = 2$ superconformal algebra [8].

In what follows we will restrict ourselves to descendants built on chiral primary states, that is, highest weight states that are also $Q^0_0$ and $G^0_0$ invariant.
2.1 Some Remarks

Let us consider the anticommutator \( \{Q_0, G_0\} = 2L_0 \) between the BRST charge \( Q_0 \) and \( G_0 \). Acting on chiral primary states this relation implies zero conformal dimension for the corresponding fields. Acting on secondary states, on the other hand, this anticommutator gives very useful information:

First, it shows that a \( Q_0 \)-closed (\( G_0 \)-closed) secondary state is also \( Q_0 \)-exact (\( G_0 \)-exact)

\[
|\Xi\rangle = \frac{1}{2\Delta}(Q_0G_0 + G_0Q_0)|\Xi\rangle
\]  
(2.2)

(\( \Delta \) is the conformal dimension of the corresponding secondary field).

Secondly, it tells us that bosonic and fermionic descendants are related in a peculiar way. Namely, \( Q_0 \)-invariant bosonic (fermionic) states are \( G_0 \) mapped onto \( G_0 \)-invariant fermionic (bosonic) states that in turn are \( Q_0 \) mapped onto the original \( Q_0 \)-invariant bosonic (fermionic) ones (up to a scale factor of \( 2\Delta \)). Similarly, \( G_0 \)-invariant bosonic (fermionic) states are \( Q_0 \) mapped onto \( Q_0 \)-invariant fermionic (bosonic) that in turn are \( G_0 \) mapped onto the original \( G_0 \)-invariant bosonic (fermionic) states.

In particular, bosonic and fermionic topological null vectors are connected in this way, since the \( G_0 \) and \( Q_0 \) actions on null vectors give null vectors as well.

2.2 DDK and KM Realizations of Topological Descendants

The DDK and KM realizations of the topological algebra (2.1) are the two possible twistings of the same \( N = 2 \) superconformal theory \[2\]. The field content in both realizations is very similar: \( d \leq 1 \) matter + scalar + \( bc \) system. However, the scalars differ by the background charge and the way they dress the matter (\( Q_s = Q_{\text{Liouville}}, \Delta = 1 \) in the DDK case versus \( Q_s = Q_{\text{matter}}, \Delta = 0 \) in the KM case), while the \( bc \) systems differ by the spin and the central charge (\( s = 2, c = -26 \) for the DDK ghosts versus \( s = 1, c = -2 \) for the KM ghosts).

In the DDK realization the generators of the topological algebra are

\[
\mathcal{L}_m = L_m + l_m, \quad l_m \equiv \sum_{n \in \mathbb{Z}} (m + n) :b_{m-n}c_n:
\]  
(2.3)

\[
\mathcal{H}_m = \sum_{n \in \mathbb{Z}} :b_{m-n}c_n: - \sqrt{\frac{3-c}{3}} I_m,
\]  
(2.4)
\[ \mathcal{Q}_m = 2 \sum_{p \in \mathbb{Z}} c_{m-p} L_p + \sum_{p,r \in \mathbb{Z}} (p-r) : b_{m-p-r} c_p c_r : + 2 \sqrt{\frac{3-c}{3}} \sum_{p \in \mathbb{Z}} c_{m-p} I_p + \frac{c}{3}(m^2 - m)c_m , \] (2.5)

\[ \mathcal{G}_m = b_m , \] (2.6)

and the chiral primary states can be written as \(|\Phi\rangle = |\Upsilon\rangle \otimes c_1 |0\rangle_{gh}^c\), where \(|\Upsilon\rangle\) is a primary state in the "matter + scalar" sector (for a spin-2 \(bc\) system \(c_1 |0\rangle_{gh}^c\) is the "true" ghost vacuum annihilated by all the positive modes \(b_n\) and \(c_n\)).

In the KM realization the generators read

\[ \mathcal{L}_m = L_m + l_m, \quad l_m = \sum_{n \in \mathbb{Z}} n : b_{m-n} c_n : \] (2.7)

\[ \mathcal{H}_m = - \sum_{n \in \mathbb{Z}} : b_{m-n} c_n : + \sqrt{\frac{3-c}{3}} I_m , \] (2.8)

\[ \mathcal{Q}_m = b_m , \] (2.9)

\[ \mathcal{G}_m = 2 \sum_{p \in \mathbb{Z}} c_{m-p} L_p + 2 \sqrt{\frac{3-c}{3}} \sum_{p \in \mathbb{Z}} (m-p)c_{m-p} I_p + \sum_{p,r \in \mathbb{Z}} (r-p) : b_{m-p-r} c_p c_r : + \frac{c}{3}(m^2 + m)c_m , \] (2.10)

and the chiral primary states split as \(|\Phi\rangle = |\Upsilon\rangle \otimes |0\rangle_{gh}^c\).

Now let us define the DDK and KM conformal field theories (CFT’s) as the theories given by the "matter + scalar" systems, without the ghosts. These theories are described by the commutation relations

\[ [L_m, L_n] = (m-n)L_{m+n} + \frac{D}{12}(m^3 - m)d_{m+n,0} , \]
\[ [L_m, I_n] = -nI_{m+n} - \frac{1}{2}Q_s (m^2 + m)d_{m+n,0} , \]
\[ [I_m, I_n] = -m\delta_{m+n,0} . \] (2.11)

where

\[ L_m = L^\text{matter}_m - \frac{1}{2} \sum_n : I_{m-n} I_n : + \frac{1}{2}Q_s (m+1)I_m \] (2.12)
and $D = 26$ ($D = 2$), $Q_s = \sqrt{26-d\over 3}$ ($Q_s = \sqrt{1-d\over 3}$) for the DDK (KM) realization. The states $|\Upsilon\rangle$ above are thus primaries of the DDK and KM CFT’s respectively.

We are now ready to establish four theorems concerning the DDK and KM realizations of topological descendants (two of them concern null vectors exclusively).

Let us take a topological descendant and split its topological generators into their DDK or KM components. The result is expected to be a sum of ghost-free terms, matter and/or scalar-ghost mixed terms, and pure-ghost terms. This is indeed what happens in the general case. However, there are special cases in which all the ghosts cancel out, so that the DDK or KM realization reduces the topological descendant to a secondary state of the DDK or KM CFT. Those special cases are considered in the following theorems.

**Theorem 1.** Let $|\Xi\rangle$ be a $G_0$-invariant topological descendant annihilated by all the positive modes $G_{n\geq 0}$, and with ghost number equal to the one assigned to $c_1|0\rangle_{gh}$ (whatever the convention). Then the DDK realization reduces $|\Xi\rangle$ to

$$|\Xi\rangle_{DDK} = |\Psi_{DDK}\rangle \otimes c_1|0\rangle_{gh} \quad (2.13)$$

where $|\Psi_{DDK}\rangle$ is a descendant of the DDK CFT.

**Proof.** Let $|\Xi\rangle$ be annihilated by $G_{n\geq 0}$. In the DDK realization this results in $b_{n\geq 0} |\Xi\rangle_{DDK} = 0$. Thus $|\Xi\rangle_{DDK}$ cannot contain any $c_{n\leq 0}$ ghost modes, since $\{b_m, c_n\} = \delta_{m+n}$. Nor can it contain any annihilation modes $c_{n>1}$ or $b_{n>-2}$. Therefore in the ghost sector only $c_1$ and $b_{n<-1}$ modes are possible. But $|\Xi\rangle$ is by assumption in the same ghost-number subspace as $c_1|0\rangle_{gh}$, so that $|\Xi\rangle_{DDK}$ must have the form (2.13).

**Theorem 2.** Let $|\Xi\rangle$ be a $Q_0$-invariant topological descendant annihilated by all the positive modes $Q_{n\geq 0}$, and with ghost number equal to the one assigned to $|0\rangle_{gh}$. Then the KM realization reduces $|\Xi\rangle$ to

$$|\Xi\rangle_{KM} = |\Psi_{KM}\rangle \otimes |0\rangle_{gh} \quad (2.14)$$

where $|\Psi_{KM}\rangle$ is a descendant of the KM CFT.

**Proof.** Let $|\Xi\rangle$ be annihilated by $Q_{n\geq 0}$. In the KM realization this results in $b_{n\geq 0} |\Xi\rangle_{KM} = 0$. Thus $|\Xi\rangle_{KM}$ cannot contain any $c_{n\leq 0}$ modes, nor can it contain any annihilation modes $c_{n>0}$ or $b_{n>-1}$ either. Thus, only $b_{n<0}$ modes are allowed in the ghost sector. But $|\Xi\rangle$ is by assumption in the same ghost-number subspace as $|0\rangle_{gh}$, so that $|\Xi\rangle_{KM}$ must have the form (2.14).

For the particular case of null vectors, it is now rather easy to prove the following.
Theorem 3.- Let $|\Xi\rangle$ be a $G_0$-invariant topological null vector at level $l$ with ghost number equal to the one assigned to $c_1|0\rangle_{gh}$. Then the DDK realization reduces $|\Xi\rangle$ to a level $l$ null vector of the DDK CFT.

Proof.- Applying Theorem 1 $|\Xi\rangle$ must be of the form (2.13). Then the DDK realization translates straightforwardly the topological highest weight conditions $L_{n>0}|\Xi\rangle = H_{n>0}|\Xi\rangle = G_{n>0}|\Xi\rangle = 0$ into the highest weight conditions of the DDK CFT $L_{n>0}|\Psi_{DDK}\rangle = H_{n>0}|\Psi_{DDK}\rangle = G_{n>0}|\Psi_{DDK}\rangle = 0$. In addition, from

$$(\mathcal{L}_0|\Xi\rangle)_{DDK} = L_0|\Psi_{DDK}\rangle \otimes c_1|0\rangle_{gh} + |\Psi_{DDK}\rangle \otimes L_0 c_1|0\rangle_{gh}$$

(2.15)

we obtain $\Delta_{|\Psi\rangle} = l + 1$. Since the conformal weight of all the primaries in the DDK CFT is equal to 1 (DDK dressing), we conclude that $|\Psi_{DDK}\rangle$ is a null vector at level $l$.

Theorem 4.- Let $|\Xi\rangle$ be a $Q_0$-invariant topological null vector at level $l$ with ghost number equal to the one assigned to $|0\rangle_{gh}$. Then the KM realization reduces $|\Xi\rangle$ to a level $l$ null vector of the KM CFT.

Proof.- Applying Theorem 2 $|\Xi\rangle$ must be of the form (2.14). The KM realization translates straightforwardly the topological highest weight conditions on $|\Xi\rangle$ into the highest weight conditions of the KM CFT on $|\Psi_{KM}\rangle$. Moreover, since

$$(\mathcal{L}_0|\Xi\rangle)_{KM} = L_0|\Psi_{KM}\rangle \otimes |0\rangle_{gh}$$

(2.16)

and the conformal weight of all the primaries in the KM CFT is zero (KM dressing), we conclude that $|\Psi_{KM}\rangle$ is a null vector at level $l$.

3 Level 2 Topological Descendants

Level 2 topological descendants of bosonic type have the generic form

$$|\Xi\rangle^B = (\alpha \mathcal{L}_2^2 + \theta \mathcal{L}_2 + \Gamma \mathcal{H}_2 \mathcal{L}_1 + \beta \mathcal{H}_1 \mathcal{L}_1 + \gamma \mathcal{H}_2 + \delta \mathcal{Q}_1 \mathcal{G}_1) |\Phi\rangle_h$$

(3.1)

where $|\Phi\rangle_h$ is a chiral primary state with $\mathcal{H}_0$ eigenvalue $h$.

From now on we will focus on the special cases of $G_{n\geq 0}$-invariant states and $Q_{n\geq 0}$-invariant states.

Let us start with the $G_{n\geq 0}$-invariant descendants $|\Xi\rangle^{BG}$. The condition $G_0|\Xi\rangle^{BG} = 0$ results in the equations
\[ 2\alpha - \Gamma + 2\delta = 0, \quad \Gamma + 2\delta - 2\beta = 0, \quad 2\theta + \Gamma - \beta - \gamma + 2\delta = 0 \] (3.2)

while \( G_1|\Xi\rangle^{BG} = 0 \) gives

\[ 2\alpha + 3\theta - \Gamma + \beta - \gamma + 2\delta (2 + h + \frac{c}{3}) = 0. \] (3.3)

The conditions \( G_{n>1}|\Xi\rangle^B = 0 \) are satisfied identically (on any \( |\Xi\rangle^B \)).

Now let us consider the \( Q_{n\geq 0} \)- invariant bosonic descendants \( |\Xi\rangle^{BQ} \). The condition \( Q_0|\Xi\rangle^{BQ} = 0 \) gives

\[ \beta = 0, \quad \gamma = 0, \quad \Gamma = 2\delta \] (3.4)

and \( Q_1|\Xi\rangle^{BQ} = 0 \) results in

\[ \theta = -2h\delta . \] (3.5)

The conditions \( Q_{n>1}|\Xi\rangle^B = 0 \) are satisfied identically.

As a first application of these results, notice that \( Q_0 \)- invariance (BRST- invariance) plus \( g_0 \)- invariance imply the vanishing of all the coefficients in \( |\Xi\rangle^B \). That is, a \( Q_0 \) and \( G_0 \)- invariant state must be a chiral primary necessarily (as we already knew).

Let us now move onto the fermionic descendants. Since \( G_n \) and \( Q_n \) have opposite \( \mathcal{H}_0 \) charges, there are two different types of fermionic descendants, mirrored under \( G_n \) versus \( Q_n \). The fermionic secondary state with \( \mathcal{H}_0 \) eigenvalue \( h + 1 \) has the form

\[ |\Xi\rangle^{FG} = (\delta \mathcal{L}_{-1} G_{-1} + G_{-2} + \beta \mathcal{H}_{-1} G_{-1})|\Phi\rangle_h \] (3.6)

and satisfies the conditions \( G_{n\geq 0}|\Xi\rangle^{FG} = 0 \) identically, in particular \( G_0 \)- invariance.

The fermionic secondary state with \( \mathcal{H}_0 \) eigenvalue \( h - 1 \) has the ”mirrored” form

\[ |\Xi\rangle^{FQ} = (\alpha \mathcal{L}_{-1} Q_{-1} + Q_{-2} + \gamma \mathcal{H}_{-1} Q_{-1})|\Phi\rangle_h \] (3.7)

and satisfies the conditions \( Q_{n\geq 0}|\Xi\rangle^{FQ} = 0 \) identically, in particular \( Q_0 \)- invariance.

Notice that fermionic descendants, at level 2, are either of the \( |\Xi\rangle^{FG} \) type or rather of the \( |\Xi\rangle^{FQ} \) type; in other words, not only there are not \( Q_0 \) plus \( G_0 \)-invariant fermionic descendants, like in the bosonic case, but not even fermionic descendants of the type \( |\Xi\rangle^F \) (neither \( G_{n\geq 0} \) nor \( Q_{n\geq 0} \)-invariant).
When the descendants are null states, one has to impose the complete set of highest weight conditions as well. We will present an analysis of level 2 and level 3 topological null states in [11].

### 3.1 DDK Realization of the Level 2 Topological Descendants

The general analysis of the DDK and KM realizations of topological descendants, with the resulting ghosts structures, etc...., is beyond the scope of this letter. Here we will consider only the cases met by Theorem 1 (for DDK) and Theorem 2 (for KM).

Let us start with the DDK realization of bosonic descendants. As we mentioned before, in the DDK realization the secondary states are built on chiral primaries of the form |Φ⟩_h = |Υ⟩ ⊗ |c|_1|0⟩_gh, where |Υ⟩ is a primary state of the DDK CFT. Then the DDK realization of any given |Ξ⟩^B results in the following terms:

a) Terms with the structure [ghost − free]|Υ⟩ ⊗ |c|_1|0⟩_gh, where [ghost − free] is

\[ \alpha L_{-1}^2 + (\theta + 2\delta)L_{-2} - \Gamma \sqrt{3 - c \over 3} I_{-1}L_{-1} + \beta \left( 3 - {c \over 3} I_{-1}^2 \right) - (\gamma + 2\delta) \sqrt{3 - c \over 3} I_{-2} . \]  

(3.8)

b) Terms with the structure [ghost − free]|Υ⟩ ⊗ |c|_0|0⟩_gh, where [ghost − free] is

\[ (2\alpha - \Gamma + 2\delta)L_{-1} - (\Gamma + 2\delta - 2\beta) \sqrt{3 - c \over 3} I_{-1} . \]  

(3.9)

c) Pure ghost terms with the structure |Υ⟩ ⊗ [ghost]|0⟩_gh, where [ghost] is

\[ (-2\theta - \Gamma + \beta + \gamma - 2\delta) b_{-2}c_0c_1 + (2\alpha + 3\theta - \Gamma + \beta - \gamma + 2\delta (2 + h + {c \over 3})) c_{-1} . \]  

(3.10)

Inspecting eqn’s (3.2) - (3.3) we see that by imposing G_0 and G_1 invariance all the coefficients in b) and c) vanish. Therefore, the structure of the G_{n≥0} - invariant bosonic descendants in the DDK realization is

\[ |Ξ⟩^{BG}_{DDK} = |Ψ_{DDK}⟩ ⊗ |c|_1|0⟩_gh \]  

(3.11)

where |Ψ_{DDK}⟩ is a secondary state of the DDK CFT, in agreement with Theorem 1.

For the G_{n≥0} - invariant fermionic descendants, it is not possible to meet the conditions of Theorem 1, so it is not of much use in this case. However, using relation (2.2) it is indeed possible to investigate the structure of the fermionic states directly connected,
through $Q_0$, to the given $G_{n\geq 0}$-invariant bosonic states. These fermionic descendants are of the $Q_{n\geq 0}$-invariant type. In the case at hand, relation (2.2) reads

$$|\Xi\rangle^{BG} = \frac{1}{4} G_0 |\Xi\rangle^{BG} = \frac{1}{4} G_0 |\Xi\rangle^{FQ} .$$

(3.12)

In the DDK realization this relation becomes

$$|\Xi\rangle^{BG}_{DDK} = \frac{1}{4} b_0 |\Xi\rangle^{FQ}_{DDK}$$

(3.13)

that shows that $|\Xi\rangle^{FQ}_{DDK} = 4 c_0 |\Xi\rangle^{BG}_{DDK}$. Therefore, in the DDK realization the $Q_{n\geq 0}$-invariant fermionic descendants reduce to the structure

$$|\Xi\rangle^{FQ}_{DDK} = |\Psi_{DDK}\rangle \otimes c_0 c_1 |0\rangle_{gh} .$$

(3.14)

### 3.2 KM Realization of the Level 2 Topological Descendants

In the KM realization the secondary states are built on chiral primaries of the form $|\Phi\rangle_h = |\Upsilon\rangle \otimes |0\rangle_{gh}$, where $|\Upsilon\rangle$ is a primary state of the KM CFT. As a result, the KM realization of a bosonic descendant $|\Xi\rangle^B$ is given by the following terms:

a) Terms with the structure $[\text{ghost} - \text{free}] |\Upsilon\rangle \otimes |0\rangle_{gh}$, where $[\text{ghost} - \text{free}]$ is

$$\alpha L_2^2 + \theta L_{-2} + \Gamma \sqrt{\frac{3-c}{3}} I_{-1} L_{-1} + \beta \frac{3-c}{3} I_{-1}^2 + \gamma \sqrt{\frac{3-c}{3}} I_{-2} .$$

(3.15)

b) Terms with the structure $[\text{ghost} - \text{free}] |\Upsilon\rangle \otimes b_{-1} c_0 |0\rangle_{gh}$, where $[\text{ghost} - \text{free}]$ is

$$(-\Gamma + 2\delta) L_{-1} - 2\beta \sqrt{\frac{3-c}{3}} I_{-1} .$$

(3.16)

c) Pure ghost terms with the structure $|\Upsilon\rangle \otimes [\text{ghost}] |0\rangle_{gh}$, where $[\text{ghost}]$ is

$$(\beta - \gamma) b_{-2} c_0 - (\theta + \beta + \gamma + 2\delta) b_{-1} c_{-1} .$$

(3.17)

Imposing $Q_0$ and $Q_1$ invariance, eqn’s (3.4) - (3.5), all the coefficients in b) and c) vanish. Therefore, the structure of the $Q_{n\geq 0}$-invariant bosonic descendants in the KM realization is

$$|\Xi\rangle^{BQ}_{KM} = |\Psi_{KM}\rangle \otimes |0\rangle_{gh}$$

(3.18)
where $|\Psi_{KM}\rangle$ is a secondary state of the KM CFT, in agreement with Theorem 2.

For the $Q_{n\geq 0}$ - invariant fermionic descendants it is not possible to satisfy the conditions of Theorem 2. However, we saw in the previous subsection that the DDK realization of those states actually reduces to the form $(3.14)$.

The same reasoning applies now to the $G_{n\geq 0}$ - invariant fermionic descendants, connected through $G_0$ to the given $Q_{n\geq 0}$ - invariant bosonic states. In this case, relation $(2.2)$ reads

$$|\Xi\rangle_{BQ} = \frac{1}{4} Q_0 G_0 |\Xi\rangle_{BQ} = \frac{1}{4} Q_0 |\Xi\rangle_{FG}$$

so that

$$|\Xi\rangle_{KM} = \frac{1}{4} b_0 |\Xi\rangle_{FG} .$$

Therefore $|\Xi\rangle_{FG} = 4c_0 |\Xi\rangle_{BQ}$ . As a result, in the KM realization the $G_{n\geq 0}$ - invariant fermionic descendants reduce to the structure

$$|\Xi\rangle_{KM} = |\Psi_{KM}\rangle \otimes c_0 |0\rangle_{gh} .$$

4 Final Remarks

We have analyzed the DDK and KM realizations of topological secondary states. In particular we have investigated the issue of the reduction of topological descendants to simple structures (ghost-free or quasi ghost-free structures). To this purpose four no-ghost theorems have been derived, two of them for null states. Finally, we have analyzed the corresponding results for the case of level 2 topological descendants, showing complete agreement with the no-ghost theorems.

The relation between the topological states considered in this letter and Lian-Zuckerman states \[9\] is under investigation \[10\].

The particular case of topological null descendants will be considered in \[11\].

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