‘Strange Stars’ - have they been discovered?

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Abstract

Recently there have been controversial claims about the nature of the isolated compact star RXJ1856.5−3754, with one group claiming it to be a strange star (Drake et al., 2002) while the other asserting it to be a normal neutron star (Walter & Lattimer, 2002). The controversy arises mainly due to the distance estimate, which in turn is used to resolve the measured angular diameter, and thus the radiation radius $R_{\infty}$. In this we discuss the theoretical constraints that appear from analysing the usual mass-radius relation along with the redshift factors arising from the strong gravity effects and possible lensing. Unless the distance estimate is confirmed independently, without any uncertainty, it is premature to come to any conclusion regarding the nature of this star.

Subject headings: star: RXJ1856.5−3754 – strange star – neutron star – equation of state

1 Introduction

One of the hot topics in astrophysics of late has been the possibility of the discovery of Strange Stars (stars composed of u, d and s quark matter),

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through X-ray observations of the candidate RX J1856.35−3754 from the Chandra satellite by Drake et al., (2002). Almost at the same time Walter & Lattimer (2002) have given a totally different picture of the same, claiming it to be a normal neutron star. Though discussions of the possibility of the existence of such stars dates back to the eighties (Witten, 1984 and later by Alcock et al., 1986, Haensel et al., 1986, etc.), with high resolution data from the X-ray observations, recently observers have improved the accuracy for ascertaining the dimensions of the compact objects associated with the observed emissions (Walter et al., 1996 Walter & Mathews 1997 and Pons et al., 2001). During the last few years, there have been several sources, which are considered neither as black holes nor neutron stars, like the Her-X1, 4U 1820−30 (Dey et al, 1998 and Bombaci, 1997), SAX J1808.4−3658 (Li et al., 1999a), 4U 1728−34 (Li et al., 1999b), PSR 0943+10 (Xu et al., 1999). The absence of spectral lines in the thermal components of the X-ray compact sources from the observations by Chandra and the XMM-Newton, led Xu (2002) to claim the existence of bare strange stars. While it is extremely difficult to come to a conclusion regarding a candidate being a black hole (for masses of the order of few solar masses), it is slightly better for neutron stars, once proper estimates of mass and radius are made along with the evidence for the existence of a hard surface which could produce bursts.

2 Determination of mass-radius relation

The method of obtaining the crucial parameters from observational data does depend upon the models adopted. One of the recent candidates which has evoked a lot of interest in this context is the source RXT1856.3-3754, an isolated compact star at a distance of ∼ 140 pc in the outskirts of the RCrA dark molecular cloud. Incidentally there are two conflicting claims regarding this source with Drake et al. (2002) claiming it to be a ‘strange star’ with radiation radius $R_\infty$ ranging from 3.8-8.2 kms, and mass $\sim 1.4 M_\odot$, whereas Walter & Lattimer (2002) claim it to be a neutron star with $R_\infty$ 15 km and $M \sim 1.7 M_\odot$. In this approach the radiation radius $R_\infty$ is defined through
the general relativistic relation

\[ R_\infty = \frac{R}{\sqrt{1 - \frac{2GM}{c^2R}}} \]  

wherein \( R \) and \( M \) are the actual radius and mass of the star. It is apparent that what is measured is \( R_\infty \), through observation of the angular diameter of the source, expressed as \( R_\infty/D \) where \( D \) is the distance of the source. Leaving aside the part of ambiguities and uncertainties in the measurement of \( D \), there seems to be very little consensus among observers about the distance measurement in this case.

However, what needs to be carefully looked into, is the crucial implications of the ‘measured’ value for determining the mass and radius of the star, which decides whether it is a neutron star or an exotic strange star. As the relevant formulae used takes into account the crucial redshift factor due to strong gravity effects, it is also important to consider other possible general relativistic effects for photon trajectories. In this context one should take into account ‘the self-lensing effect due to the sources’ own gravitational field’ of the emitted light rays close to the central star. Nollert et al. (1989) have discussed this effect while analyzing the relativistic ‘looks’ of a neutron star, who clearly points out the relevance of self-lensing while analyzing the data from a compact source. Accordingly, they point out that if \( I_v \) is the specific intensity of the observed radiation then \( I_{vs} \) and \( I_{v\infty} \) are related through the equation

\[ I_{v\infty} = I_{vs} \left( 1 - \frac{2GM}{Rc^2} \right)^{3/2} \]  

Hence, if one considers this relation for the specific intensity and evaluate the relation between the radii \( R_\infty \) and \( R_s \) for a black body emission one finds the relation to be

\[ R_{\infty}^2 = R_s^2(1 + Z) = R_s^2/ \left( 1 - \frac{2GM}{R_sc^2} \right)^{1/2} \]  

and not the one used by Drake et al. or Walter et al. Before proceeding further with the estimates one needs to consider few other aspects of general relativity. It is well known that \( R = 2MG/c^2 \) denotes the event horizon and further \( R = 3GM/c^2 \) corresponds to the circular photon orbit, as well as the radius at which the centrifugal force reversal occurs for particles on
circular geodesics (Abramowicz and Prasanna, 1990); Heyl, (2000), using this feature had obtained constraints on neutron star radii for the case of Type 1 X-ray burst sources. Before considering the authenticity of the constraint one ought to realize that the effect of centrifugal reversal occurs only for purely circular geodesics, whereas the presence of even a small radial velocity for the accreting particles would change its orbit to non-circular motion in which case the centrifugal reversal occurs only if the central star is rotating (Prasanna, 2001). In view of this, the constraint \( R_s > 3M \) may not be always effective, whereas \( R_\infty > 3M \) is indeed a must for any observation.

Considering now equation (3) which can be written as

\[
R_s^5 - R_\infty^4 R_s + 2M R_\infty^4 = 0
\]

a quintic equation in \( R \). Unlike in the case of a cubic, there is no way of expressing a general condition between \( R_\infty \) and \( M \) for the existence of real roots. However, one can find numerically that if \( R_\infty > 5.4954M \sim 3.75GM/c^2 \) then there are two positive real roots, for \( R_s \) one of which certainly lies outside \( 3GM/c^2 \), the photon circular orbit.

Table 1 yields the location of the highest real positive root \( R_s \) for given \( m \) and \( R_\infty \) chosen to be 9m. As these are the only consistent numbers, satisfying the defining equation one has to constrain M-R relation as given by this. For comparison we have also given the photon radius \( 3GM/c^2 \) for the corresponding mass, and one clearly sees that the actual radius of the star is greater than \( 3GM/c^2 \) for the chosen \( R_\infty \).

It may be seen from the table that for the mass \( M \sim 1.7\ M_\odot \), and \( R_\infty = 15.3 \) kms, the actual stellar radius \( R_s \approx 13.65 \), a value larger than that obtained by Walter and Lattimer. Another important thing to notice here is that the redshift factor \( 'Z' \) when calculated for the entire range of values of \( M \) and \( R_s \) as given in Table 1 yields a value \( \approx 0.256 \), which lies within the range as given by Pons et al. On the other hand, the ranges of \( M \) and \( R \) as obtained, after including the lensing factor takes the star away from the estimates of Drake et al.

If lensing is not taken into account then the two radii \( R \) and \( R_\infty \) are related through eq. (1), which may be re-written as

\[
R^3 - R_\infty^2 R + 2MR_\infty^2 = 0 \tag{5}
\]

As \( R \) represents the true radius of the star of mass \( M \) it is imperative
that $R_\infty$ has to be such that the equation will have real roots. As the last term is always positive, one of the roots is always negative. Considering the discriminant $4R_\infty^4(M^2 - R_\infty^2/27)$, it is clear that the other two roots, when real, are equal if $R_\infty = 3\sqrt{3}M$ and real but not equal for $R_\infty > 3\sqrt{3}M$.

Table 2 gives the locations of the outer real root for $R$ for a given $M$, and the photon orbit $R_P$ and the Radiation radius $R_\infty$ respectively, for this case.

Table 1: Solutions of the quintic equation (Eqn. 4) in $R$ for different values of masses, taking into account the effect of lensing.

| Mass $(M_\odot)$ | Radius of the star ($R_s$) (kms) | Photon Radius (kms) | Radiation Radius ($R_\infty$) (kms) |
|-----------------|----------------------------------|---------------------|-------------------------------------|
| 0.4             | 3.21224                          | 1.764               | 3.6                                 |
| 0.5             | 4.0153                           | 2.205               | 4.5                                 |
| 0.6             | 4.81836                          | 2.646               | 5.4                                 |
| 0.7             | 5.62142                          | 3.087               | 6.3                                 |
| 0.8             | 6.42448                          | 3.528               | 7.2                                 |
| 0.9             | 7.22754                          | 3.969               | 8.1                                 |
| 1.0             | 8.03059                          | 4.41                | 9.0                                 |
| 1.1             | 8.83365                          | 4.851               | 9.9                                 |
| 1.2             | 9.63671                          | 5.292               | 10.8                                |
| 1.3             | 10.4398                          | 5.733               | 11.7                                |
| 1.4             | 11.2428                          | 6.174               | 12.6                                |
| 1.5             | 12.0459                          | 6.615               | 13.5                                |
| 1.6             | 12.849                           | 7.056               | 14.4                                |
| 1.7             | 13.652                           | 7.497               | 15.3                                |
| 1.8             | 14.4551                          | 7.938               | 16.2                                |
| 1.9             | 15.2581                          | 8.379               | 17.1                                |
| 2.0             | 16.0612                          | 8.82                | 18.0                                |
Table 2: Solutions of Eqn. (5) in R for different values of masses, without taking into account the effect of lensing.

| Mass $\left( M_\odot \right)$ | Radius of the star $\left( R_s \right)$ (kms) | Photon Radius (kms) | Radiation Radius $\left( R_\infty \right)$ (kms) |
|-------------------------------|---------------------------------------------|---------------------|-----------------------------------------------|
| 0.4                           | 2.70758                                     | 1.764               | 3.6                                           |
| 0.5                           | 3.38448                                     | 2.205               | 4.5                                           |
| 0.6                           | 4.06138                                     | 2.646               | 5.4                                           |
| 0.7                           | 4.73827                                     | 3.087               | 6.3                                           |
| 0.8                           | 5.41517                                     | 3.528               | 7.2                                           |
| 0.9                           | 6.09206                                     | 3.969               | 8.1                                           |
| 1.0                           | 6.76896                                     | 4.41                | 9.0                                           |
| 1.1                           | 7.44586                                     | 4.851               | 9.9                                           |
| 1.2                           | 8.12275                                     | 5.292               | 10.8                                          |
| 1.3                           | 8.79965                                     | 5.733               | 11.7                                          |
| 1.4                           | 9.47654                                     | 6.174               | 12.6                                          |
| 1.5                           | 10.1534                                     | 6.615               | 13.5                                          |
| 1.6                           | 10.8303                                     | 7.056               | 14.4                                          |
| 1.7                           | 11.5072                                     | 7.497               | 15.3                                          |
| 1.8                           | 12.1841                                     | 7.938               | 16.2                                          |
| 1.9                           | 12.861                                       | 8.379               | 17.1                                          |
| 2.0                           | 13.5379                                     | 8.82                | 18.0                                          |
3 Neutron star and Strange star equation of states

There are many neutron star equation of states (EOS) which give mass-radius relation over a wide range when fed into the TOV equation. Almost all of the EOSs are calculated by considering either the relativistic Dirac-Brueckner-Hartree-Fock models, or the relativistic field theoretical models, or the non-relativistic potential models. Also, some models have been considered with the possibility of the stellar core possessing a Bose-Einstein condensate of the negative kaons (Kaplan & Nelson, 1986 and Thorsson et al., 1994). We have chosen only four neutron star EOSs just to compare them with the strange star models. Also we saw from literature that there are a wide variety of neutron star EOSs, but none of them come so close to the radius within 8 kms.

The curve labelled 7 in Fig. (1) is due to Lorentz, Ravenhall and Pethick (1993). They considered a microscopic Hamiltonian obtained by fitting a Skyrme-like energy density functional to the values of the employed microscopic two body potential V14 and the three body force TNI. These has been earlier used by Friedman and Pandharipande (1981) in hypernated chain techniques considering a range of densities and temperatures. Wiringa Ficks and Fabrocini (1988) calculated an EOS (labelled 5 in Fig. (1)) using a Hamiltonian where a two-nucleon potential that fits nucleon-nucleon scattering data and deuteron properties has been employed, and also possesses an explicit three-nucleon interaction term. They calculated the Hamiltonian using five combinations of the Argonne $v_{14}$, Urbana $v_{14}$ two-nucleon potentials and the Urbana VII three-nucleon potential. We have shown here only result with the AV14+UVII. Curve labelled 6 is the EOS as per the Bethe Johnson model (Bethe & Johnson, 1974) who used realistic potentials in their calculations, and curve labelled 8 is due to Pandharipande (1971).

The idea of existence of exotic stars such as the strange quark stars, has been a long term debate for astrophysicists and particle physicists too. The laboratory scale lifetime for deconfined $u$, $d$ and $s$ quarks is typically of the order of $\sim 10^{-24}$ sec, which is far away from the astrophysical scale of stellar lifetime. So, a stable strange quark star model raised questions about the existence of such stars. The scenario has changed very much with the coming up of new generation x-ray satellites. The analysis from observations
now supply us new constraints on the mass-radius relation of some of these compact objects. With the advent of time, some of the new data for some sources and their analysis (e.g., Haberl & Titarchuk (1995), Bombaci (1997), Dey et al. (1998), Li et al. (1999a & 1999b), Xu et al. (1999), Kapoor & Shukre (2001), etc.) proved that the stars developed from neutron star EOSs do not match with them, and they are more compact. This leaves the only choice of considering them as the strange quark stars.

Figure 1: The lines labelled A & B are the solutions of the roots of Eqn.(4) and Eqn.(5) respectively. It shows clearly that when the mass of the star is low (say around 1 M⊙), then the possibility of it being a neutron star is completely ruled out and it is a strange star. However if the mass is more than 1.5 M⊙, then it is a neutron star. So, the nature of the star cannot be established until an approximate value of the mass of the star is provided.

Ever since the strange star hypothesis was proposed by Witten (1984), various models have been developed. Among all of them, the most common one is that based on the MIT bag model. In this phenomenological model the
basic features of QCD, i.e., quark confinement and asymptotic freedom, are built in. However in this model, the deconfinement of quarks at high density is not obvious. Preliminary calculations of strange stars using the bag model has been done by Alcock et al. (1986) and Haensel et al. (1986). A result for the bag model EOS for strange stars is shown in the figure, where two cases for the EOSs are given, one for the $B = 94.9 \text{ MeV/fm}^3$ and strange quark masses of 0 MeV and 150 MeV for curves labelled 3 and 4 respectively.

Alternative to the MIT bag model, Dey et al., in 1998, derived an EOS for strange matter which has asymptotic freedom built in and describes deconfined quarks at high density and confinement at zero density. With a proper choice of the EOS parameters, this model gives absolutely stable strange quark matter. This EOS was used to calculate the structure of static strange stars and the mass-radius relations. Later, it was suggested (Li et al., 1999a) that the millisecond X-ray pulsar SAX1808.4–3658 is a strange star. This model which also explained the observed properties of some other compact objects like the analysis of semi-empirical mass-radius relations from the QPO observations in 4U 1728-34 (Li et al., 1999b) as also the RXTE observations of Her X-1 and 4U 1820-30 (Dey et al., 1998), leads to the suggestion that these objects host strange stars. In Li et al., (1999a), two sets of EOSs are used for two sets of parameters, namely SS1 and SS2. The maximum gravitational masses are $M_{\text{max}} = 1.437 \, M_\odot$ for SS1 (curve labelled 2) and $M_{\text{max}} = 1.325 \, M_\odot$ for SS2 (curve labelled 1). For different values of the mass parameter $\nu$ in the D98 model, the stars can have a sequence of masses. The calculations of Dey et al. (1998) has been done considering zero temperature of the strange matter. Ray et al. (2000) calculated the finite temperature effect on the quark stars developed by Dey et al. (1998) and found that it sustains even more mass for a particular radius of the star as compared to the case of cold star.

4 Discussions

Fig.1 shows the lines of $M - R$ relation for the cases with lensing (A) and without lensing (B) alongwith the curves depicting the various equations of state both for neutron stars (solid lines) and strange matter stars (dot dash lines). As is clear there can be possibilities of identifying a star as of either category depending upon the mass estimate and the corresponding radius,
once the measured radius $R_\infty$ is certain. Since the existence of the real roots for $R$ does depend upon the relation between $R_\infty$ and $M$, it is very important that both these parameters are estimated accurately and only then can one conclude about the nature of the star, provided the associated point in Fig. 1 overlaps the region covered by either of the types of equation of state. It is also equally important to realise, whether the strong gravity effect of lensing is to be taken into account or not. Either way, the regions of overlap between the real roots and that of reasonable equations of state are quite constrained and thus for a final diagnosis of the star in question RXJ1856.35−3754, one needs to have more unambiguous measurements of both $R_\infty$ and $M$. The theoretical argument put forth above is purely of mathematical and logistic in nature, which cannot be sidelined by any other preferences.

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