**Introduction.**—Measurements of the cosmic microwave background (CMB) \cite{1} are one of the observational pillars of the standard cosmological model (ΛCDM) and constrain its parameters to high precision (see most recently Ref. \cite{2}). This model extrapolates the Universe back to very high temperatures \( \gtrsim 10^{12} \text{K} \) and early times \( \ll 1 \text{s} \). Observations indicate that conditions at these early times are described by an almost uniform plasma with a nearly scale invariant spectrum of adiabatic density perturbations. However, ΛCDM itself offers
no explanation for how these conditions occurred. The
theory of inflation is an extension to the standard model,
which postulates a phase of exponential expansion at a
still earlier epoch ($\sim 10^{-35}\, \text{s}$) that precedes $\Lambda\text{CDM}$ and
produces the required initial conditions (See Ref. [3] for
a recent review and citations to the original literature.)

There is widespread support for the claim that existing
observations already indicate that some version of infla-
tion probably did occur, but there are also skeptics [4, 5].
As well as the specific form of the initial density perturba-
tions there is an additional relic which inflation predicts,
and which one can attempt to detect. Inflation launches
tensor mode perturbations into the fabric of space-time
which will propagate unimpeded as inflationary gravita-
tional waves (IGWs) to the present day. Their amplitude
is diminished with the expansion of the Universe, and de-
tection at the present epoch is not feasible with current
technology. The most promising potential method of de-
tection is to look for their signature written into the pat-
tern of the CMB at last scattering, 380,000 years after
the Universe entered the realm of fully known physics.
Inflationary theories generically predict that IGWs exist,
but many specific models have been proposed producing
a wide range of amplitudes—with some being unobserv-
ably small [3]. The size of the IGW signal is convention-
ally expressed as the initial ratio of the tensor and scalar
perturbation amplitudes $r$.

In the $\Lambda\text{CDM}$ standard model the CMB is polarized
by Thomson scattering of Doppler induced quadrupoles
in the local radiation field at last scattering. This natu-
urally produces a polarization pattern with direction par-
allel/perpendicular to the gradient of its intensity—this is
curl-free, or $E$-mode polarization, and was first de-
tected in Ref. [6]. Due to small gravitational deflections
of the CMB photons in flight by intervening large scale
structure, the initial purity of the $E$-mode pattern is dis-
turbed and a small lensing $B$-mode is produced at sub-
degree angular scales [7, 8].

IGWs are intrinsically quadrupolar distortions of the
metric and produce both $E$ and $B$-mode polarization de-
pending on their orientation with respect to our last scat-
tering surface. However, due to the large $\Lambda\text{CDM}$ $E$-mode
signal, the most promising place to search for an IGW sig-
na is in $B$-modes. Furthermore, since the IGW $B$-modes
have a much redder spectrum than the lensing $B$-modes,
the best place to look is at angular scales larger than a few
degrees (multipoles $\ell < 100$). Limits on IGW from non-
polarized CMB observations are now fully saturated at
cosmic variance limits [2] and it is generally agreed that
the best (only) way to make further progress is through
improved measurements of CMB $B$-modes.

The BICEP and $\text{Keck Array}$ telescopes are small aper-
ture polarimeters specifically designed to search for an
IGW signal at the recombination bump ($\ell \approx 80$). BI-
CEP1 operated from 2006 to 2008 and set a limit $r_{0.05} < 0.70$ at 95\% confidence [9]. BICEP2 operated from 2010 to 2012 at 150 GHz and in Ref. [10] reported a detec-
tion of a substantial excess over the lensed-$\Lambda\text{CDM}$ ex-
pectation in the multipole range $30 < \ell < 150$. Addi-
tional measurements at 150 GHz taken by the $\text{Keck Array}$
during 2012 and 2013 confirmed this excess [11]. How-
ever, new data from the $\text{Planck}$ space mission provided
evidence that emission from galactic dust grains could
be more polarized at high galactic latitudes than anticip-
pated [12, 13], a possibility emphasized by [14, 15]. Analy-
cis of the combined BICEP2 and $\text{Keck Array}$ 150 GHz
data in combination with data from $\text{Planck}$ (princi-
pally at 353 GHz) showed that a substantial part of the
150 GHz excess is due to polarized emission from galac-
tic dust grains, and that once this is accounted for, the
result becomes $r_{0.05} < 0.12$ at 95\% confidence [16].

BICEP2 was a simple 26 cm aperture all-cold refrac-
tor, and $\text{Keck Array}$ is basically five copies of this on a
single telescope mount [11, 17]. Both are sited at the
South Pole in Antarctica, taking advantage of the dry
atmosphere and stable observing conditions. In addition
to the all-cold optics these telescopes have two features
which aid greatly in the suppression and characterization
of instrumental systematics: i) they are equipped with
co-moving absorptive forebaffles resulting in extremely
low far side-lobe response, and ii) the entire instrument
can be rotated about the line of sight allowing modula-
tion of polarized signal.

$\text{Keck Array}$ was designed at the outset to observe in
multiple frequency bands—the 2012 and 2013 observa-
tions were all taken at 150 GHz because detectors for
other bands were not yet ready. Before the 2014 sea-
son two of the five receivers of $\text{Keck Array}$ were refitted
for operation in a band centered on 95 GHz (the other
three receivers remaining unchanged at 150 GHz). In
this paper we fold in this new data and perform a multi-
component, multi-spectral likelihood analysis similar to
our previous analysis [16].

This paper builds on the initial BICEP2 results pa-
ter [10, hereafter BK-I], the $\text{Keck}$ 2012+2013 results pa-
ter [11, hereafter BK-V], and the BICEP2/$\text{Keck}$/Planck
analysis paper [16, hereafter BKP].

$\text{Instrument and observations.}$—The $\text{Keck Array}$ instru-
ment is described in Sec. 2 of BK-V. (See also the BI-
CEP2 Instrument Paper [17] for further details.) Before
the 2014 observing season two of the receivers of $\text{Keck}$
Array were removed, the lenses and filters were replaced
with versions optimized for a band centered at 95 GHz,
and the focal planes were replaced with units loaded with
appropriately scaled versions of our antenna-coupled de-
tectors [18]. Because the physical size of these antennas
is larger each of the four tiles contains only a 6 \times 6 array
(rather than 8 \times 8 at 150 GHz). With two focal planes
at 95 GHz this gives 288 total detector pairs (576 total
detectors).

During the 2014 austral winter season the array was
operated exactly as for the previous seasons. A $\sim 1\%$
region of sky centered at RA $0h$, Dec. $-57.5^\circ$ was observed
from March until November over $\approx 4600$ fifty minute
“scansets”. Efficiency and yield was similar to previous
seasons. See Sec. 4 of BK-V for further details of the
observing strategy and data selection.

**BICEP2/Keck Maps.**—The processing from time stream to maps is identical to that described in Sec. III & IV of BK-I and summarized in Sec. 5 of BK-V. Relative gain calibration is applied between the two halves of each pair and the difference is taken. Filtering is then applied to remove residual atmospheric noise and any ground-fixed (scan-synchronous) pickup. The data are then binned into simple map pixels and, with knowledge of the polarization sensitivity directions, maps of Stokes parameters $Q$ and $U$ are formed. “Deprojection” is also performed to remove leakage of temperature to polarization due to beam systematics and this results in an additional filtering of signal.

Fig. 1 shows the 95 & 150 GHz $Q$ maps combining data from BICEP2 (2010–2012) and Keck Array (2012–2014)—we refer to these as the BK14 maps meaning that they contain all data up to and including that taken during the 2014 observing season. The 150 GHz maps add 3 more receiver years to the previous 13 in the BK13 based analysis of BKP, and modestly improves the $Q/U$ sensitivity from 57 nK deg to 50 nK deg (3.0 $\mu$ arcmin) over an effective area of 395 square degrees. These are the deepest maps of CMB polarization published to date. The 95 GHz maps contain only 2 receiver years of data and the $Q/U$ sensitivity is 127 nK deg (7.6 $\mu$ arcmin) over an effective area of 375 square degrees. (The survey weight is thus 310,000 (47,000) over an effective area of 395 square degrees. These are the deepest maps of CMB polarization published to date.

**External Maps.**—We use the Public Release 2 “full mission” maps available from the Planck Legacy Archive [19][21], noting that these are nearly identical to those used in BKP. For this analysis we also add the WMAP9 23 GHz (K-band) and 33 GHz (Ka-band) maps [20][23].

For each of these external maps we deconvolve the native instrument beam, reconvolve the Keck 150 GHz beam, and then process the result through an “observing” matrix to produce a map with the same filtering of spatial modes as the 150 GHz map. See Sec. II.A of BKP for further details of this process. For Planck we use the FFP8 simulations [24] and for WMAP we use simple inhomogeneous white noise simulations derived from the provided variance maps.

**Power Spectra.**—We convert the maps to power spectra using the methods described in Sec. VI of BK-I including the matrix based purification operation to prevent $E$ to $B$ mixing. We generate separate purification matrices to match the filtering of the 95 & 150 GHz maps.

We first subject the new 95 GHz data to our usual suite of “jackknife” internal consistency checks. The results are given in Appendix B and show empirically that the data are free of systematic contamination at a level greater than the noise. In addition, in Appendix C we investigate the stability of the previous 150 GHz spectrum when adding the new 2014 data—there is no indication of problems.

We now proceed to comparing the spectra and cross spectra of our 95 and 150 GHz maps—Fig. 2 shows the results. We use a common apodization mask as the geometric mean of the two (smoothed) inverse variance maps. The $EE$ spectra agree to within much better than the nominal error bar size because the uncertainty is dominated by sample variance and we are observing the same piece of sky. To make a rough estimate of the significance of deviation from lensed-$\Lambda$CDM, we calculate $\chi^2$ and $\chi$ (sum of normalized deviations) as shown on the plot. We see strong evidence for excess $BB$ power in BK14$_{150}\times$BK14$_{150}$ and BK14$_{150}\times$BK14$_{95}$ and moderate evidence in BK14$_{95}\times$BK14$_{150}$. Dashed lines for the lensed-$\Lambda$CDM+dust model derived in BKP are over-plotted and appear to be consistent with the new data.

Fig. 3 shows selected $BB$ cross spectra between the BK14 95 & 150 GHz maps and the Planck (P) and WMAP (W) bands. There is no strong evidence for detection of synchrotron emission—$W_{23}\times BK14_{95}$ and $W_{23}\times BK14_{150}$ are both mildly elevated but $P_{30}\times BK14_{150}$ has stronger nominal anticorrelation (as noted in the BKP paper). $W_{33}\times BK14_{95}$ and $W_{33}\times BK14_{150}$ are both consistent with null. The only strong detections of excess signal are in BK14$_{95}\times P_{353}$ and, at lower significance BK14$_{150}\times P_{217}$. See Appendix D for the full set of auto- and cross-spectra.

**Likelihood Analysis.**—We next proceed to a multicomponent, multi-spectral likelihood analysis which is an expanded version of that described in Sec. III of the BKP paper. We compute the likelihood of the data for any given proposed model using an extended version of the HL approximation [25] and the full covariance matrix of the auto- and cross-spectral bandpowers as derived from simulations (setting to zero terms whose expectation value is zero).

In this analysis we primarily use a lensed-$\Lambda$CDM+ dust+ synchrotron+ $r$ model and explore the parameter space using COSMOMC [26]. The COSMOMC module containing the data and model is available for download at [http://bicepkeck.org](http://bicepkeck.org). In this paper the “baseline” analysis is defined to:

- [ ] Use the BK14 maps as shown in Fig. 1 (all BICEP2/Keck data up to and including that taken during the 2014 observing season).
- [ ] Use all the polarized bands of Planck (30–353 GHz) plus the 23 & 33 GHz bands of WMAP.
- [ ] Use all possible $BB$ auto- and cross-spectra between these maps. This includes all the spectra shown in Figures 2 & 3—the complete set are shown in Appendix D. Spectra with no detection can, of course, still have constraining power—for instance
non-detection in $P_{30}\times P_{353}$ disfavors sync/dust correlation.

- Use nine bandpowers spanning the range $20 < \ell < 330$.

- Include dust with amplitude $A_{d,353}$ evaluated at 353 GHz and $\ell = 80$. As in the BKP analysis the frequency spectral behavior is taken as a simple modified black body spectrum with $T_d = 19.6$ K and $\beta_d = 1.59 \pm 0.11$, using a Gaussian prior with the given 1σ width. Analyzing polarized emission at intermediate galactic latitudes Fig. 11 of Ref. [27] shows that this model is accurate in the mean to within a few percent over the frequency range 100–353 GHz, while the patch-to-patch fluctuation is noise dominated. The spatial power spectrum is taken as a simple power law $D_\ell \propto \ell^{\alpha_d}$ marginalizing over the range $-1 < \alpha_d < 0$, where $D_\ell \equiv \ell (\ell + 1) C_\ell / 2\pi$.

- Include synchrotron with amplitude $A_{\text{sync},23}$ evaluated at 23 GHz (the lowest WMAP band) and $\ell = 80$, assuming a simple power law for the frequency spectral behavior $A_{\text{sync}} \propto \nu^{\beta_s}$ with a Gaussian prior $\beta_s = -3.1 \pm 0.3$ [28]. The spatial power spectrum is taken as a simple power law $D_\ell \propto \ell^{\alpha_s}$ marginalizing over the range $-1 < \alpha_s < 0$.

- Allow sync/dust correlation and marginalize over the correlation parameter $0 < \epsilon < 1$.

- Quote the tensor/scalar power ratio $r$ at a pivot scale of 0.05 Mpc$^{-1}$ and fix the tensor spectral index $n_t = 0$. 
The zero-to-peak likelihood ratio for \( \Lambda \text{CDM}+\text{dust+noise} \) simulations produces a similar number of degrees of freedom, we estimate that the probability in shape with mean/\( \sigma \) of 1.82/0.26. In Appendix E2 we investigate a variety of other variations from the baseline analysis and in Appendix E3 we perform some validation tests of the likelihood using simulations.

For the purposes of presentation we also run a likelihood analysis to find the CMB and foreground contributions on a bandpower-by-bandpower basis. The baseline analysis is a single fit to all 9 bandpowers across 66 spectra with 8 parameters. Instead we now perform 9 separate fits—one for each bandpower—across the 66 spectra, with 6 parameters in each fit. These 6 parameters are the amplitudes of CMB, dust and synchrotron plus \( \beta_d \), \( \beta_s \), and \( \epsilon \) with identical priors to the baseline analysis. Results for the additional parameters are shown in the upper right part of Fig. 4. The dust frequency spectral parameter \( \beta_d \) pulls weakly against the prior to higher values. The synchrotron frequency spectral parameter \( \beta_s \) just reflects the prior (as expected since synchrotron is not strongly detected). The data have a mild preference for values of \( \alpha_d \) close to the \(-0.42 \) found in Ref. [13], while \( \alpha_s \) is unconstrained. The data disfavor strong sync/dust correlation (due to the non detection of signal in spectra like \( W_{23}\times P_{353} \) —see Fig 3). As \( A_{\text{sync}} \) approaches zero \( \epsilon \) becomes unconstrained leading to an increase in the available parameter volume, and the “flare” in the \( A_{\text{sync}} \) constraints.

The maximum likelihood model (including priors) has parameters \( r_{0.05} = 0.026 \), \( A_{d,353} = 4.1 \mu K^2 \), \( A_{\text{sync},23} = 1.4 \mu K^2 \), \( \beta_d = 1.6 \), \( \beta_s = -3.1 \), \( \alpha_d = -0.19 \), \( \alpha_s = -0.56 \), and \( \epsilon = 0.00 \). This model appears to be an acceptable fit to the data—see Appendix D for further details.

In Fig. 4 we see that as compared to the primary BKP analysis the peak position of the likelihood curve for \( r \) has shifted down slightly. In Fig. 5 we investigate why. Although we have made extensive changes to the model, these make only a small difference. (See Appendix E1 for details of these changes.) The change from the BK13150 to the BK14150 maps causes some of the downward shift in the peak position. This may seem surprising given that only a relatively small amount of additional data has been added (~20%). However Appendix C shows that the shifts in the bandpower values are not unlikely and we should therefore accept the shift in the \( r \) constraint as simply due to noise fluctuation. Adding in the BK14a5 data produces an additional downward shift in the peak position, and also significantly narrows the likelihood curve.

Fig. 5 shows one additional variation. It turns out that the tight prior on \( \beta_d \) from Planck analysis of other regions of sky is becoming unnecessary. Removing the prior the peak position of the likelihood on \( r \) shifts up slightly and broadens so that \( r_{0.05} = 0.043_{-0.031}^{+0.036} \) & \( r_{0.05} < 0.11 \) (95%), while the likelihood curve for \( \beta_d \) is close to Gaussian in shape with mean/\( \sigma \) of 1.82/0.26. In Appendix E2 we investigate a variety of other variations from the baseline analysis and in Appendix E3 we perform some validation tests of the likelihood using simulations.

See Appendix E1 for a more detailed explanation of these choices.

Results of this baseline analysis are shown in Fig. 4 and yield the following statistics: \( r_{0.05} = 0.028_{-0.026}^{+0.026} \), \( r_{0.05} < 0.090 \) at 95% confidence, \( A_{d,353} = 4.3_{-1.9}^{+1.2} \mu K^2 \), and \( A_{\text{sync},23} < 3.8 \mu K^2 \) at 95% confidence. For \( r \) the zero-to-peak likelihood ratio is 0.63. Taking \( \frac{1}{2} (1 - f (\ln L_0 / L_{\text{peak}})) \), where \( f \) is the \( \chi^2 \) CDF (for one degree of freedom), we estimate that the probability to get a likelihood ratio smaller than this is 18% if, in fact, \( r = 0 \). Running the analysis on the lensed-\( \Lambda \text{CDM}+\text{dust+noise} \) simulations produces a similar number. The zero-to-peak likelihood ratio for \( A_d \) indicates that the detection of dust is now \( \sim 8 \sigma \).

Results for the additional parameters are shown in the
FIG. 4. Results of a multicomponent multi-spectral likelihood analysis of BICEP2/Keck+external data. The red faint curves are the primary result from the previous BKP paper (the black curves from Fig. 6 of that paper). The bold black curves are the new baseline BK14 results. Differences between these analyses include adding synchrotron to the model, including additional external frequency bands from WMAP & Planck, and adding Keck Array data taken during the 2014 observing season at 95 & 150 GHz. We see that the peak position of the tensor/scalar ratio curve $r$ shifts down slightly and the upper limit tightens to $r_{0.05} < 0.05$ at 95% confidence. The parameters $A_d$ and $A_{\text{sync}}$ are the amplitudes of the dust and synchrotron $B$-mode power spectra, where $\beta$ and $\alpha$ are the respective frequency and spatial spectral indices. The correlation coefficient between the dust and synchrotron patterns is $\epsilon$. In the $\beta$, $\alpha$ and $\epsilon$ panels the dashed red lines show the priors placed on these parameters (either Gaussian or uniform).

Combination with external maps produce $B$-mode based constraints on the tensor-to-scalar ratio $r$ which place an upper limit $r_{0.05} < 0.09$ at 95% confidence. The analysis of Planck full mission $TT$ data in conjunction with external data produces the constraint $r_{0.02} < 0.11$ ($r_{0.05} < 0.12$) at 95% confidence (“Planck TT+lowP+lensing+ext” in Equation 39b of Ref. [2]), and are saturated at cosmic variance limits. The BK14 result constitutes the first $B$-mode constraints that clearly surpass those from temperature anisotropies. In Fig. 7 we reproduce Ref. [2]’s result in the $r$ vs. $n_s$ plane, and show the effect of adding in our BK14 $B$-mode data. The allowed region tightens and the joint result is $r_{0.05} < 0.07$ (95%), although as emphasized in Ref. [2] the $TT$ derived constraints on $r$ are more model dependent than $BB$ ones.

Fig. 8 compares signal levels and current noise uncertainties in the critical $\ell \sim 80$ bandpower (updated from Fig. 13 of BKP). A second season of 95 GHz Keck Array data has already been recorded (in 2015) and will push the 95 × 95 point down by a factor of 2. During 2015 two receivers were also operated in a third band centered on 220 GHz, producing deep maps which will improve dust separation. This 2015 data is under analysis and will be reported on in a future paper. In addition, BICEP3 began operations in 2015 in the 95 GHz band.

In this paper, we have presented an analysis of all BICEP2/Keck data up through the 2014 season, adding, for the first time, 95 GHz data from the Keck Array. We have updated our multi-frequency likelihood analysis with a more extensive foreground parameterization and the inclusion of external data from the 23 & 33 GHz bands of WMAP, in addition to all seven po-
FIG. 5. Likelihood results on $r$ for several intermediate steps between the BKP (previous) and BK14 (current) analyses. See text for details.

The baseline analysis yields $r_{0.05} = 0.028_{-0.025}^{+0.026}$ and $r_{0.05} < 0.09$ at 95% confidence, constraints that are robust to the variations explored in analysis and priors. With this result, $B$-modes now offer the most powerful limits on inflationary gravitational waves, surpassing constraints from temperature anisotropies and other evidence for the first time. With upcoming multifrequency data the $B$-mode constraints can be expected to steadily improve.

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[1] A. A. Penzias and R. W. Wilson, Astrophys. J. 142, 419 (1965).
[2] Planck Collaboration 2015 XIII, ArXiv e-prints (2015), arXiv:1502.01589.
[3] M. Kamionkowski and E. D. Kovetz, ArXiv e-prints (2015), arXiv:1510.06042.
[4] A. H. Guth, D. I. Kaiser, and Y. Nomura, Physics Letters B 733, 112 (2014), arXiv:1312.7619.
[5] A. Ijjas, P. J. Steinhardt, and A. Loeb, ArXiv e-prints (2014), arXiv:1402.6980.
[6] J. M. Kovac, E. M. Leitch, C. Pryke, J. E. Carlstrom, N. W. Halverson, and W. L. Holzapfel, Nature 420, 772 (2002), astro-ph/0209478.
FIG. 7. Constraints in the $r$ vs. $n_s$ plane when using Planck plus additional data, and when also adding BICEP2/Keck data through the end of the 2014 season including new 95 GHz maps—the constraint on $r$ tightens from $r_{0.05} < 0.12$ to $r_{0.05} < 0.07$. This figure is adapted from Fig. 21 of Ref. [2]—see there for further details.

[7] Polarbear Collaboration, Astrophys. J. 794, 171 (2014), arXiv:1403.2369.

[8] R. Keisler, S. Hoover, N. Harrington, J. W. Henning, P. A. R. Ade, K. A. Aird, J. E. Austermann, J. A. Beall, A. N. Bender, B. A. Benson, L. E. Bleem, J. E. Carlstrom, C. L. Chang, H. C. Chiang, H.-M. Cho, R. Citron, T. M. Crawford, A. T. Crites, T. de Haan, M. A. Dobbs, W. Everett, J. Gallicchio, J. Gao, E. M. George, A. Gilbert, N. W. Halverson, D. Hanson, G. C. Hilton, G. P. Holder, W. L. Holzapfel, Z. Hou, J. D. Hrubes, N. Huang, J. Hubmayr, K. D. Irwin, L. Knox, A. T. Lee, E. M. Leitch, D. Li, D. Luong-Van, D. P. Marrone, J. J. McMahon, J. Mehl, S. S. Meyer, L. Mocanu, T. Natoli, J. P. Nibarger, V. Novosad, S. Padin, C. Pryke, C. L. Reichardt, J. E. Ruhl, B. R. Saliwanchik, J. T. Sayre, K. K. Schaeffer, E. Shirokoff, G. Smecher, A. A. Stark, K. T. Story, C. Tucker, K. Vanderlinde, J. D. Vieira, G. Wang, N. Whitehorn, V. Yefremenko, and O. Zahn, Astrophys. J. 807, 151 (2015), arXiv:1503.02315.

[9] BICEP1 Collaboration, Astrophys. J. 783, 67 (2014), arXiv:1310.1422.

[10] BICEP2 Collaboration I, Physical Review Letters 112, 241101 (2014), arXiv:1403.3985.

[11] Keck Array and BICEP2 Collaborations V, Astrophys. J. 811, 126 (2015), arXiv:1502.00643.

[12] Planck Collaboration Int. XIX, Astron. Astrophys. 576, A104 (2015), arXiv:1405.0871.

[13] Planck Collaboration Int. XXX, ArXiv e-prints (2014), arXiv:1409.5738.

[14] R. Flauger, J. C. Hill, and D. N. Spergel, J. Cosmol. Astropart. Phys. 8, 039 (2014), arXiv:1405.7351.

[15] M. J. Mortonson and U. Seljak, J. Cosmol. Astropart. Phys. 10, 035 (2014), arXiv:1405.5857.

[16] BICEP2/Keck and Planck Collaborations, Physical Review Letters 114, 101301 (2015), arXiv:1502.00612.

[17] BICEP2 Collaboration II, Astrophys. J. 792, 62 (2014), arXiv:1403.4302.

[18] BICEP2/Keck and Spider Collaborations, Astrophys. J. 812, 176 (2015), arXiv:1502.00619 [astro-ph.IM].

[19] Appendices are in Supplemental Material http://xxx. yyy which includes references [30, 31].

[20] See http://www.cosmos.esa.int/web/planck/pla.

[21] Planck Collaboration 2015 I, ArXiv e-prints (2015), arXiv:1502.01582.

[22] See http://lambda.gsfc.nasa.gov/product/map/dr5/m_products.cfm.

[23] C. L. Bennett, D. Larson, J. L. Weiland, N. Jarosik, G. Hinshaw, N. Odegard, K. M. Smith, R. S. Hill, B. Gold, M. Halpern, E. Komatsu, M. R. Nolta, L. Page, D. N. Spergel, E. Wollack, J. Dunkley, A. Kogut, M. Limon, S. S. Meyer, L. Page, G. S. Tucker, and E. L. Wright, Astrophys. J. Suppl. Ser. 208, 20 (2013), arXiv:1212.5225.

[24] Planck Collaboration 2015 XII, ArXiv e-prints (2015), arXiv:1509.06348.
[25] S. Hamimeche and A. Lewis, Phys. Rev. D 77, 103013 (2008), arXiv:0801.0554.
[26] A. Lewis and S. Bridle, Phys. Rev. D66, 103511 (2002), astro-ph/0205436.
[27] Planck Collaboration Int. XXII, Astron. Astrophys. 576, A107 (2015), arXiv:1405.0874.
[28] U. Fuskeland, I. K. Wehus, H. K. Eriksen, and S. K. Næss, Astrophys. J. 790, 104 (2014), arXiv:1404.5323.
[29] See http://lambda.gsfc.nasa.gov/product/map/dr5/mcmc_maps_info.cfm.
[30] S. K. Choi and L. A. Page, ArXiv e-prints (2015), arXiv:1509.05934.
[31] J. Dunkley, A. Amblard, C. Baccigalupi, M. Betoule, D. Chuss, A. Cooray, J. Delabrouille, C. Dickinson, G. Dobler, J. Dotson, H. K. Eriksen, D. Finkbeiner, D. Fixsen, P. Fosalba, A. Fraisse, C. Hirata, A. Kogut, J. Kristiansen, C. Lawrence, A. M. Magalhães, M. A. Miville-Deschenes, et al., AIP Conf. Proc. 1141, 222 (2009), arXiv:0811.3915.
Appendix A: Maps

Figures 9 & 10 show the full sets of $T/Q/U$ maps at 150 & 95 GHz. The right side of each figure shows realizations of noise created by randomly flipping the sign of data subsets while coadding the map—see Sec. V.B of BK-I for further details.

Appendix B: Keck Array 95 GHz Power Spectra and Internal Consistency Tests

A powerful internal consistency test are data split difference tests which we refer to as “jackknifes”. As well as the full coadded signal maps we also form many pairs of split maps where the splits are chosen such that one might expect different systematic contamination in the two halves of the split. The split halves are differenced and the power spectra taken. We then take the deviations of these from the mean of signal+noise simulations and form $\chi^2$ and $\chi$ (sum of deviations) statistics. In this section we perform tests of the new 95 GHz data set which are exactly analogous to the tests of the previous 150 GHz data sets performed in Sec. VII.C of BK-I and Sec. 6.3 of BK-V. Fig. 11 shows the signal spectra and a sample set of jackknife spectra. All the signal spectra are consistent with lensed-ΛCDM and the jackknife spectra with null.

Table I shows the $\chi^2$ and $\chi$ statistics for the full set of 95 GHz jackknife tests and Fig. 12 presents the same results in graphical form. Note that these values are partially correlated—particularly the 1–5 and 1–9 versions of each statistic. We conclude that there is no evidence for corruption of the data at a level exceeding the noise.

Appendix C: 150 GHz Spectral Stability

Questions were raised as to whether the BICEP2 and Keck Array 2012+2013 $BB$ spectra are mutually compatible. We investigated this in Sec. 8 of BK-V and concluded that they are. Here we perform a similar test on the difference of the BK13$_{150}$ and BK14$_{150}$ spectra—i.e. when adding the additional 150 GHz data from 2014. We compare the differences of the real spectra to the differences of simulations which share the same underlying input skies. Fig. 13 shows the results. While the bandpowers do shift around even when adding only $\sim 20\%$ of additional data these shifts are seen to be consistent with noise fluctuation. We go on to perform one more test—we instead take the difference of the BK13$_{150}$ and the 2014 only 150 GHz spectrum (which we refer to as K2014$_{150}$). Since we are measuring the bandpower differences in units of the expected shift given the degree of common data we expect, and find, similar results.

### Table I. Jackknife PTE values from $\chi^2$ and $\chi$ (sum of deviations) tests for Keck Array 95 GHz data taken in 2014.

| Jackknife         | Band powers 1–5 $\chi^2$ | Band powers 1–9 $\chi^2$ | Band powers 1–5 $\chi$ | Band powers 1–9 $\chi$ |
|-------------------|---------------------------|---------------------------|-------------------------|-------------------------|
| Deck jackknife    | 0.625                     | 0.591                     | 0.523                   | 0.569                   |
| BB                | 0.166                     | 0.192                     | 0.076                   | 0.020                   |
| EB                | 0.876                     | 0.539                     | 0.814                   | 0.445                   |
| Scan Dir jackknife| 0.439                     | 0.513                     | 0.760                   | 0.423                   |
| BB                | 0.944                     | 0.535                     | 0.565                   | 0.168                   |
| EB                | 0.539                     | 0.192                     | 0.912                   | 0.980                   |
| Tag Split jackknife| 0.543                    | 0.537                     | 0.810                   | 0.938                   |
| BB                | 0.768                     | 0.780                     | 0.687                   | 0.539                   |
| EB                | 0.313                     | 0.547                     | 0.407                   | 0.451                   |
| Tile jackknife    | 0.234                     | 0.477                     | 0.395                   | 0.709                   |
| BB                | 0.050                     | 0.072                     | 0.012                   | 0.046                   |
| EB                | 0.828                     | 0.902                     | 0.812                   | 0.822                   |
| Phase jackknife   | 0.862                     | 0.982                     | 0.577                   | 0.471                   |
| BB                | 0.944                     | 0.521                     | 0.639                   | 0.325                   |
| EB                | 0.691                     | 0.890                     | 0.204                   | 0.357                   |
| Mux Col jackknife | 0.084                     | 0.146                     | 0.182                   | 0.337                   |
| BB                | 0.172                     | 0.337                     | 0.012                   | 0.152                   |
| EB                | 0.541                     | 0.695                     | 0.956                   | 0.812                   |
| Alt Deck jackknife| 0.098                     | 0.076                     | 0.030                   | 0.036                   |
| BB                | 0.092                     | 0.126                     | 0.102                   | 0.140                   |
| EB                | 0.858                     | 0.842                     | 0.858                   | 0.741                   |
| Mux Row jackknife | 0.232                     | 0.289                     | 0.699                   | 0.918                   |
| BB                | 0.289                     | 0.267                     | 0.082                   | 0.014                   |
| EB                | 0.148                     | 0.130                     | 0.096                   | 0.098                   |
| Tile/Deck jackknife| 0.924                    | 0.956                     | 0.162                   | 0.399                   |
| BB                | 0.507                     | 0.034                     | 0.561                   | 0.343                   |
| EB                | 0.477                     | 0.361                     | 0.954                   | 0.994                   |
| Focal Plane inner/outer jackknife | 0.477 | 0.335 | 0.200 | 0.792 |
| BB                | 0.886                     | 0.437                     | 0.762                   | 0.569                   |
| EB                | 0.595                     | 0.876                     | 0.926                   | 0.780                   |
| Tile top/bottom jackknife | 0.261 | 0.519 | 0.998 | 0.990 |
| BB                | 0.756                     | 0.890                     | 0.415                   | 0.431                   |
| EB                | 0.850                     | 0.920                     | 0.377                   | 0.317                   |
| Tile inner/outer jackknife | 0.184 | 0.353 | 0.427 | 0.529 |
| BB                | 0.772                     | 0.772                     | 0.749                   | 0.707                   |
| EB                | 0.407                     | 0.038                     | 0.934                   | 0.667                   |
| Moon jackknife    | 0.569                     | 0.701                     | 0.228                   | 0.251                   |
| BB                | 0.305                     | 0.465                     | 0.978                   | 0.990                   |
| EB                | 0.349                     | 0.507                     | 0.677                   | 0.301                   |
| A/B offset best/worst | 0.635 | 0.267 | 0.104 | 0.431 |
| BB                | 0.407                     | 0.387                     | 0.677                   | 0.287                   |
| EB                | 0.321                     | 0.605                     | 0.860                   | 0.685                   |
FIG. 9. $T$, $Q$, $U$ maps at 150 GHz using all BICEP2/Keck data up to and including that taken during the 2014 observing season—we refer to these maps as BK14150. The left column shows the basic signal maps with 0.25° pixelization as output by the reduction pipeline. The right column shows a noise realization made by randomly assigning positive and negative signs while coadding the data. These maps are filtered by the instrument beam (FWHM 30 arcmin), timestream processing, and (for $Q$ & $U$) deprojection of beam systematics. Note that the horizontal/vertical and 45° structures seen in the $Q$ and $U$ signal maps are expected for an $E$-mode dominated sky.

Appendix D: Additional Spectra

Figures 2 & 3 show only a small subset of the spectra which are used in the likelihood analysis and included in the COSMOMC input file. We are using two BICEP2/Keck bands, two WMAP bands, and seven Planck bands resulting in 11 auto and 55 cross-spectra. In Fig. 14 we show all of these together with the baseline lensed-ΛCDM+dust and upper limit lensed-ΛCDM+synchrotron models. Note that, as expected from Fig. 8, several spectra contribute to constraining synchrotron.

Fig. 15 shows the distribution of the normalized deviations between the data and the maximum likelihood (ML) model (i.e. data minus expectation value divided by the square root of the diagonal of the bandpower covariance matrix). Since the bandpower distributions are not strictly Gaussian we overplot the same quantity from a set of lensed-ΛCDM+dust+noise simulations evaluated against their input model. (These simulations use the model $A_{d,353} = 3.75 \mu K^2$, $\beta_d = 1.59$ and $\alpha_d = -0.42$.) We see one nominally 4.0$\sigma$ point which is bandpower four of $P_{217} \times P_{217}$ (see Fig. 14)—comparing to the simulated distribution this event it not unlikely. Taking $\chi^2$ versus the ML model yields 654, which compared to the distribution from simulations has a PTE of $\sim 0.1$. We conclude that there is no evidence that the signal or noise models are an inadequate explanation of the data.

Appendix E: Likelihood Variation and Validation

1. Likelihood Evolution

In Fig. 5 some evolutionary steps were shown between the previous BKP analysis and the new BK14 analysis presented in this paper. Fig 16 shows some additional detail. The first step is to the alternate analysis including
FIG. 10. $T$, $Q$, $U$ maps at 95 GHz using data taken by two receivers of Keck Array during the 2014 season—we refer to these maps as $BK14_{95}$. These maps are directly analogous to the 150 GHz maps shown in Fig. 9 except that the instrument beam filtering is in this case 43 arcmin FWHM.

Synchrotron which was shown in Fig. 8 of BKP (solid red to dashed-red). This used the BK13 maps plus all of the polarized bands of Planck and set $\beta_s = -3.3$ and $\alpha_s = -0.6$. (In BKP the synchrotron pivot frequency was set to 150 GHz but since a fixed value of $\beta_s$ was used there we can simply transform the results to the pivot of 23 GHz used in this work.) Next we show the cumulative effects of model changes which we have made for this paper:

We extend the bandpower range from five ($20 < \ell < 200$) to nine ($20 < \ell < 330$) bandpowers—given that lensing is included in the model there is no real reason not to include these additional bandpowers (dashed-red to solid blue). We see that the $A_{\text{sync}}$ constraint tightens somewhat.

We switch from the use of Planck single-frequency split/split cross-spectra (in this case $Y_1 \times Y_2$) to full map auto spectra (blue to cyan). This is done for technical reasons—substituting in the cross-spectra causes numerical problems in the HL likelihood. The auto spectra have higher signal-to-noise and the constraint on $A_{\text{sync}}$ tightens further.

We include the WMAP 23 & 33 GHz bands and see that these have considerable additional power to constrain synchrotron (cyan to magenta).

In BKP we used $\beta_s = -3.3$ as this is the mean value within our field of the “model f” synchrotron spectral index maps available for download from the WMAP website [22]. However that analysis does not distinguish between the spectral behavior of temperature and polarization anisotropy. Ref. [28] analyzed the WMAP data and found a mean value of $\beta_s = -3.1 \pm 0.04$ for polarization at high galactic latitude. In this analysis we use a central value of $\beta_s = -3.1$, and since possible patch-to-patch variation is poorly constrained, to be conservative we marginalize over a Gaussian prior with width $\sigma = 0.3$. More recently Ref. [30] examined the same data and found $\beta_s \approx -3.0$ with considerable fluctuation. This change has very little effect (magenta to yellow).

Polarized synchrotron and dust emission can be spatially correlated—indeed they are guaranteed to be so on the largest scales. Ref. [30] reports a correlation of 0.2 for $30 < \ell < 200$. To be conservative in this analysis we marginalize over the range $0 < \epsilon < 1$. This causes
FIG. 11. Keck Array power spectrum at 95 GHz for signal (black points) and deck rotation jackknife (blue points). The solid black curves show the lensed-ΛCDM theory spectra. The error bars are the standard deviations of the lensed-ΛCDM+noise simulations and hence contain no sample variance on any additional signal component. The probability to exceed (PTE) the observed value of a simple $\chi^2$ statistic is given (as evaluated against the simulations). Note the very different $y$-axis scales for the jackknife spectra (other than $BB$). (Also note that the calibration procedure uses $EB$ to set the overall polarization angle so $TB$ and $EB$ as plotted above cannot be used to measure astrophysical polarization rotation.) This figure is analogous to Fig. 2 of BK-I and Fig. 4 of BK-V.

the constraint on synchrotron to tighten because of the non-detection of signal in spectra like $P_{30} \times P_{353}$ (yellow to green). We note that the data prefer the value $\epsilon = 0$ as seen in the upper-right panel of Fig. 4.

In BKP we used $\alpha_d = -0.42$ following the analysis of large regions of high latitude sky in Ref. [13], and $\alpha_s = -0.6$ taken from Ref. [31]. In this work we found that we can marginalize over generous ranges in these parameters $-1 < \alpha_d < 0$ & $-1 < \alpha_s < 0$ with only a tiny change in the bottom line results so we choose to do so (green to dashed-blue).

Finally we show the changes resulting from adding the new 150 GHz and 95 GHz data (dashed-blue to dashed-black and dashed-black to heavy-black). As already seen in Fig. 5 these are much more significant.

2. Likelihood Variation

In Fig. 17 we investigate several variations to the baseline analysis in terms of the model priors and input data sets. The first four of these loosen the priors and/or remove data, while the final three tighten the priors and/or add data.

First we repeat a variation already shown in Fig. 5—we remove the prior on the frequency spectral index of dust $\beta_d$ (black to cyan). The data then constrains $\beta_d$ to a well behaved, approximately Gaussian range (not shown) with mean/$\sigma$ of 1.82/0.26. The value of $A_{d,353}$ shifts up slightly but, with the steeper slope versus frequency, the $r$ constraint also shifts up slightly to $r = 0.043^{+0.033}_{-0.031}$ with a zero-to-peak likelihood ratio of 0.44 (10% likely if $r = 0$).

Second we relax the prior on the frequency spectral index of synchrotron to $-4 < \beta_s < -2$ and see that this has very little effect on any of the curves (black to green).

Third we remove all the Planck LFI bands from consideration (black to magenta). This causes the peak of the $r$ constraint to shift down a little and the $A_{\text{sync}}$ constraint to peak quite strongly away from zero, while the $A_d$ constraint is not significantly affected.

Fourth we drop the two bands of WMAP (black to
yellow). This slightly decreases the zero-to-peak ratio of the $r$ constraint and significantly tightens the $A_{\text{sync}}$ constraint.

We now progressively tighten the priors. For the fifth curve we switch from $0 < \epsilon < 1$ to the value preferred by Ref. [30] $\epsilon = 0.2$ (black to dashed-red). This makes almost no difference to any of the constraints, although we do note that the up-tick in the $A_{\text{sync}}$ curve approaching zero goes away.

In the sixth curve we also go back to the tight priors $\alpha_d = -0.42$ and $\alpha_s = -0.6$ which were used in the BKP analysis (dashed-red to blue). As expected from Fig. 16 this makes almost no difference to any of the constraints.

Finally in the seventh curve we also include all the $EE$ and $EB$ spectra under the assumption that the $EE/BB$ ratios for dust and synchrotron are exactly 2 (blue to dashed-black). For dust this ratio was found to apply when averaging over large areas of sky in Ref. [13]. Ref. [30] states that this ratio also applies on average for synchrotron. Assuming this fixed ratio leads to extra constraining power—the $r$ curve shifts up slightly, the $A_d$ curve narrows and the $A_{\text{sync}}$ curve peaks strongly away from zero. It is unclear how much patch-to-patch variation we should in fact allow in the $EE/BB$ ratio so this variation should not be over interpreted at this time.

3. Likelihood Validation

As already mentioned we run full timestream simulations of a lensed-$\Lambda$CDM+dust model ($A_{d,353} = 3.75 \mu K^2$, $\beta_d = 1.59$ and $\alpha_d = -0.42$). We would like to check that the HL likelihood as implemented is capable of recovering the input values of this model. However if we run the standard COSMOMC analysis on these we of course find that the ML values are biased, since only zero or positive values of $r$ and $A_{\text{sync}}$ are allowed. We therefore instead run a ML search on each sim realization where the values of $r$ and $A_{\text{sync}}$ are artificially allowed to go negative (as is $A_d$ although in practice it doesn’t). Fig. 18 shows the results—the input values are recovered in the mean as

![Fig. 12. Distributions of the jackknife $\chi^2$ and $\chi$ PTE values for the Keck Array 2014 95 GHz data over the tests and spectra given in Table I. This figure is analogous to Fig. 4 of BK-I and Fig. 6 of BK-V.](image)

![Fig. 13. Upper: Comparison of the 150 GHz BB auto-spectrum as previously published (BK13$_{150}$), for 2014 alone (K2014$_{150}$) and for the addition of the two (BK14$_{150}$). The inner error bars are the standard deviation of the lensed-$\Lambda$CDM+noise simulations, while the outer error bars include the additional fluctuation induced by a signal contribution matching the excess above lensing seen in the data. Note that neither of these uncertainties are appropriate for comparison of the band power values—for this see the lower panel. (For clarity both sets of points are offset horizontally.) Lower: The difference of the pairs of spectra shown in the upper panel divided by a factor of four. The error bars are the standard deviation of the pairwise differences of signal+noise simulations which share common input skies (the simulations used to derive the outer error bars in the upper panel). Comparison of these points with null is an appropriate test of the compatibility of the spectra and the PTE of $\chi$ and $\chi^2$ are shown. This figure is similar to Fig. 8 of BK-V.](image)
FIG. 14. **B**B auto- and cross-spectra between the BK14 95 & 150 GHz maps and bands of WMAP and Planck. In all cases the quantity plotted is \( \ell (\ell + 1) C_{\ell}/2\pi \) (\( \mu \)K\(^2\)), and the black curves show the lensed-ΛCDM theory spectrum. The error bars are the standard deviations of the lensed-ΛCDM+noise simulations and hence contain no sample variance on any additional signal component. The blue dashed lines show a baseline lensed-ΛCDM+dust model \( (A_{d,353} = 4.3 \mu \text{K}^2, \beta_d = 1.6, \alpha_d = -0.4) \). The red dashed lines show an upper limit lensed-ΛCDM+synchrotron model \( (A_{\text{sync},23} = 3.8 \mu \text{K}^2, \beta_s = -3.1, \alpha_s = -0.6) \).
FIG. 15. The normalized deviations of the bandpowers shown in Fig. 14 from the maximum likelihood model is shown as the blue histogram. The red curve is the same thing accumulated over 499 sims of a lensed-ΛCDM+dust model, and the green curve shows a Gaussian with unit width.

An additional piece of information which comes from this study is the standard deviation of the recovered ML $r$ parameter, $\sigma(r) = 0.024$. Unlike the width of the 68% highest posterior density intervals derived from the marginalized $r$ curve shown in Fig. 4 and quoted with our baseline results, this $\sigma(r)$ statistic is insensitive to where the peak value preferred by the data happens to lie, and is therefore a more robust measure of the intrinsic constraining power of the experimental data.
FIG. 16. Evolution of the BKP analysis to the “baseline” analysis as defined in this paper—see Appendix E 1 for details.

FIG. 17. Likelihood results when varying the data sets and the model priors—see Appendix E 2 for details.

FIG. 18. Results of validation tests running the likelihood on simulations of a lensed-ΛCDM+dust model ($A_d,_{353} = 3.75 \mu K^2$, $\beta_d = 1.59$ and $\alpha_d = -0.42$). The blue histograms are the recovered ML values with the red line marking their means. The black line shows the input value. In the left panel $\sigma(r) = 0.024$. See Appendix E 3 for details.