Dark matter in modified cosmologies

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Abstract. The IceCube experiment discovered a high-energy astrophysical neutrino flux with
energies of the order of PeV. To explain this phenomenon one may consider the minimal 4-
dimensional operator \( \sim y_{\alpha \chi} \bar{L}_\alpha H \chi \). An interesting possibility is to also explain the relic
abundance of DM in the Universe. Assuming that the cosmological background evolves
according to the standard cosmological model, this model fails since the rate of DM decay
\( \Gamma_\chi \sim |y_{\alpha \chi}|^2 \) needed to get the correct DM relic abundance differs by many orders of magnitude
with respect that one needed to explain the IceCube data. We show that such a discrepancy
can be solved if the Universe evolution is governed by a modified cosmology.

1. Introduction
In the recent years, the IceCube Collaboration [1, 2] has reported several neutrino-induced
cascade events with energies \( \sim 1 \) PeV [2]. Candidates for the generation of such neutrino high
energy events could be various astrophysical sources [3, 4, 5, 6], although till now there are no
clear correlations with the known astrophysical sources, such as supernova remnants or Active
Galactic Nuclei [7]. Other possibilities rely on the idea that neutrinos could arise from the decay
of PeV mass Dark Matter (DM) (see [8] and references therein).

An interesting possibility raised in Ref. [8] is whether it is possible to explain both the PeV
DM relic density and the decay rate required for IceCube with only one operator. The minimal
DM-neutrino 4-dimensional interaction is given by \( y \bar{L} \cdot H \chi \) [8]. Such a minimal dimension four
operator however fails to account for both the PeV dark matter relic abundance and the decay
rate required to explain IceCube. The DM models analyzed in [8] are based on the standard
cosmological. On the other hand, observations from Type Ia Supernovae [9], CMB radiation [10],
and the large scale structure [11, 12], have given evidence that the present cosmic expansion of the
Universe is accelerating, and that therefore a form of unknown energy/matter, generally called
Dark Energy (DE), is needed [13]. The latter could be also ascribed to an extension of Einstein’s
theory, and studies in this direction are till now under investigation [14, 15, 16, 17, 18, 19, 20].
From a cosmological point of view, one of the consequences of dealing with the cosmology based
on modified gravity is that the thermal history of particles gets modified and the expansion
rates \( H \) of the Universe can be written in terms of the expansion rate \( H_{GR} \) of General Relativity
(GR) [21, 22]

\[
H_{MC}(T) = A(T) H_{GR}(T),
\]

where \( A(T) \) is the so called amplification factor. To preserve the successful predictions of BBN,
one refers to the pre-BBN epoch since it is not directly constrained by cosmological observations.
where $a(T) \neq 1$ at early time, and $A(T) \to 1$ before BBN begins. Typically one has
\[ A(T) = \eta \left( \frac{T}{T_*} \right)^\nu, \]
where $T_*$ is a reference temperature, and $\{\eta, \nu\}$ free parameters that depend on the cosmological model under consideration [21, 22, 23]. The parameter $\nu$ labels cosmological models: $\nu = 2$ in Randall-Sundrum type II brane cosmology [24], $\nu = 1$ in kination models [25], $\nu = 0$ in cosmologies with an overall boost of the Hubble expansion rate [21], $\nu = -0.8$ in scalar-tensor cosmology [26, 21], $\nu = 2/n - 2$ in $f(R)$ cosmology, with $f(R) = R + \alpha R^n$ [27].

In this paper we consider generalizations of the standard cosmology with the aim to get a consistent minimal model of PeV DM and IceCube neutrinos. We calculate the freeze-in abundance of DM in a modified cosmology and show that the couplings required for obtaining the required relic abundance depend upon the modified gravity parameters [28]. In the following, we shall assume that the Universe is described by a flat Friedman-Robertson-Walker metric
\[ ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \]
where $a(t)$ is the scale factor. We refer to a Universe radiation dominated, so that the energy density is given by $\rho = \frac{\pi^4 g_* T^4}{30}$, $g_* = 106$, while the pressure is $p = \rho/3$ (the adiabatic index is $w = 1/3$). The equation of continuity $\dot{\rho} + 3H(\rho + p) = 0$ is assumed to hold (the dot will stand for the derivative with respect to the cosmic time $t$).

2. PeV neutrinos and Ice Cube data
The simplest 4-dimensional operator that allows to explain the IceCube high energy signal, is
\[ \mathcal{L}_{d=4} = y_{\alpha\chi} L_{La} H \chi, \quad \alpha = e, \mu, \tau, \]
where $\chi$ is the DM particle that transforms as $\chi \sim (1, 1, 0)$ of SM, $H \sim (1, 2, +1/2)$ is the Higgs doublet, $L_{La} \sim (1, 2, -1/2)$ is the left-handed lepton doublet corresponding to the generation $\alpha (= e, \mu, \tau)$, and finally $y_{\alpha\chi}$ are the Yukawa couplings. We consider the freeze-in production [29, 8], i.e. the DM particles are never in thermal equilibrium since they interact very weakly, but are gradually produced from the hot thermal bath. As a consequence, a sizable DM abundance is allowed until the temperature falls down to $T \sim m_\chi$ (temperatures below $m_\chi$ are such that DM particles phase-space is kinematically difficult to access).

Denoting with $Y_\chi = n_\chi/s$ the DM abundance, where $n_\chi$ is the number density of the DM particles and $s = 2\pi^2 g_*(T) T^3$ the entropy density, the evolution of the DM particle is governed by the Boltzmann equation
\[ \frac{dY_\chi}{dT} = -\frac{1}{HTs} \left[ \frac{g_\chi}{(2\pi)^3} \int C \frac{d^3 p_\chi}{E_\chi} \right], \]
where $H$ is the expansion rate of the Universe and $C$ the general collision term. The DM relic abundance is given by [8]
\[ \Omega_{DM} h^2 = \frac{2m_\chi^2 s_0 h^2}{\rho_{cr}} \int_0^\infty \frac{dx}{x^2} \left( -\frac{dY_\chi}{dT} \bigg|_{T = m_\chi} \right), \]
where $x = m_\chi/T$, $s_0 \simeq 2891.2/cm^3$ is the present value of the entropy density, and $\rho_{cr} = 1.054 \times 10^{-5}h^2$GeV/cm$^3$ the critical density. Equation (6) must reproduce the observed DM abundance [30]
\[ \Omega_{DM} h^2 \bigg|_{obs} = 0.1188 \pm 0.0010, \]
and at the same time, explain the IceCube data. Concerning the operator (4), the dominant contributions to DM production are a) the inverse decay processes $\nu_\alpha + H^0 \rightarrow \chi$ and $l_\alpha + H^+ \rightarrow \chi$, that occurs when $m_\chi > m_H + m_{\nu,l}$ proportional to factor $|y_{a\chi}|^2$, and b) the Yukawa production processes, such as $t + \bar{t} \rightarrow \nu_\alpha + \chi$ is proportional to $|y_{a\chi}|^2$, where $t$ represents the quark top. After solving the Boltzmann equation one gets

$$\Omega_{DM} h^2 |_{\text{inv.dec.}} = 0.1188 \sum_a |y_{a\chi}|^2 \frac{7.5 \times 10^{-25}}{10^4 m_\chi \text{sec}^{-1}}.$$ (8)

From (8) immediately follows that to have the correct DM relic abundance (7) one has to require

$$\sum_{\alpha=e,\mu,\tau} |y_{a\chi}|^2 = 7.5 \times 10^{-25}.$$ (9)

However, Eq. (9) is in conflict with the value of $\sum_{\alpha=e,\mu,\tau} |y_{a\chi}|^2$ needed to explain the IceCube data. To see that, first note that the DM lifetime $\tau_\chi = 1/\Gamma_\chi$ has to be larger that the age of the Universe, $\tau_\chi > t_U \approx 4.35 \times 10^{17} \text{sec}$. Moreover, IceCube spectrum sets a constraints on lower bounds of DM lifetime $\tau_\chi \geq 10^{28} \text{sec}$, i.e. $\tau_\chi \geq \frac{m_\chi}{\sqrt{2} \chi}$, which is (approximatively) model-independent (see [8]). Using (9) one obtains

$$\Gamma_\chi = \frac{\sum_{\alpha=e,\mu,\tau} |y_{a\chi}|^2}{8\pi} m_\chi \approx 4.5 \times 10^4 \frac{m_\chi}{\text{PeV}} \text{sec}^{-1} \rightarrow \tau_\chi \approx 2.2 \times 10^{-5} \frac{\text{PeV}}{m_\chi} \text{sec} \ll t_U.$$

The observations of IceCube require the DM decay lifetime $\tau_\chi \geq 10^{28} \text{sec}$ which implies

$$\sum_{\alpha} |y_{a\chi}|^2 \approx 10^{-58},$$ (10)

which is many order of magnitudes smaller than the value needed to explain the DM relic abundance, see (9). As a consequence, the IceCube high energy events and the DM relic abundance are not compatible with the DM production if the latter is ascribed to the 4-dimensional operator $\bar{L}_{L_a} H_\chi$.

### 3. PeV neutrinos in modified cosmologies

As seen in the previous Sections, the 4-dimensional operator fails to explaining both the IceCube data and DM relic abundance. We face this problem in terms of modified cosmology. Using (2), (5) and (6), it follows (for $\nu > -3$)

$$\Omega_{DM} h^2 = \frac{45 h^2}{1.66 \pi^2 g^{3/2}_c} \frac{s_0 M_p}{\rho_{\text{cr}}} \frac{\Gamma_\chi}{m_\chi} \frac{2^{2+\nu}}{\eta} \left( \frac{T_s}{m_\chi} \right)^{\nu} \Gamma \left( \frac{5 + \nu}{2} \right) \Gamma \left( \frac{3 + \nu}{2} \right) \approx 0.1188 \sum_{\alpha} |y_{a\chi}|^2 \Pi,$$

where $\Pi \equiv \frac{2^{3+\nu}}{3\pi \eta} \left( \frac{T_s}{m_\chi} \right)^{\nu} \Gamma \left( \frac{5 + \nu}{2} \right) \Gamma \left( \frac{3 + \nu}{2} \right)$.

$\Pi$ accounts for all corrections induced by modified cosmology. To explain the DM relic abundance and the IceCube data, we require

$$\sum_{\alpha} |y_{a\chi}|^2 \frac{7.5 \times 10^{-25}}{10^4 m_\chi \text{sec}^{-1}} \Pi \lesssim O(1).$$ (12)

The parameters $\{\eta, \nu, T_s\}$ in (11) are arbitrary and can be chosen such that the condition (12) is fulfilled. Therefore we may have $-3 < \nu < 0$ for $T_s < M_\chi$ or $\nu > 0$ for $T_s > m_\chi$.
4. Examples of modified cosmologies

In this Section we discuss the previous issues for two specific example of modified cosmologies.

4.1. Scalar Tensor Theories (STTs)

The total action of a STT of gravity is given by

\[ S_{STT} = \frac{1}{16\pi} \int d^4x \sqrt{-\tilde{g}} \left[ \Phi^2 \tilde{R}(\tilde{g}) + 4\omega(\Phi)\tilde{g}^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - 4\tilde{V}(\Phi) \right], \tag{13} \]

and \( S_m = S_m[\Psi, \tilde{g}_{\mu\nu}] \) is the matter action (the matter fields \( \Psi_m \) couple to the metric tensor \( \tilde{g}_{\mu\nu} \)). The action (13) encodes the Brans-Dicke theory of gravity for \( \omega(\Phi) = \omega = constant. \) In the form (13), the STT action is referred as Jordan frame. By means of the conformal transformation

\[ \tilde{g}_{\mu\nu} = A_C(\phi)g_{\mu\nu}, \]

\( A_C \) is the conformal factor that depends on \( \phi(x) \) and setting \( \Phi^2 = 8\pi M_s/A_C^2 \), \( V(\phi) = A_C^2(\phi)\tilde{V}(\phi)/4\pi \), and \( \alpha(\phi) = \frac{d\log A_C(\phi)}{d\phi} (= (\omega(\Phi) + 3)^{-1}) \), the action (13) can be casted in the so-called Einstein frame

\[ S_{STT} = \frac{M_s^2}{2} \int d^4x \sqrt{-g} \left[ R(g) + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2M_s^2 \tilde{V}(\phi) \right], \tag{14} \]

while the action of matter fields assumes the form \( S_m = S_M[\Psi, A_C^2(\phi)g_{\mu\nu}] \). \( M_s \) accounts for the fact that the gravitational constant may vary with the scalar field, \( M_s = M_{Pl}(\phi) = G^{-1}(\phi). \)

In the FRW flat Universe (3), the cosmological field equations read

\[ H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{1}{3M_s^2} \left[ \rho + \frac{M_s^2}{2} \dot{\phi}^2 + \tilde{V}(\phi) \right], \tag{15} \]

\[ \frac{\ddot{a}}{a} = -\frac{1}{6M_s^2} \left[ \rho + 3p + 2M_s^2 \dot{\phi}^2 - 2\tilde{V}(\phi) \right], \tag{16} \]

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{1}{M_s^2} \left[ \frac{\alpha(\phi)}{\sqrt{2}}(\rho - 3p) + V_\phi \right] = 0, \tag{17} \]

where \( V_\phi = \partial V/\partial \phi. \) The Bianchi identity (conservation of the energy momentum tensor)

\[ d(\rho a^3) + pd(\dot{a}^3) = (\rho - 3p)d\log A_C(\phi) \]

implies \( Ta = constant \) for \( p = \rho/3. \) From Eqs. (15)-(17), one gets \[ H^2 \equiv H^2_{MC} = \frac{A_C^2(\phi)\rho(1 + \alpha(\phi)\phi')^2}{1 - \phi'^2/6} H^2_{GR}. \tag{18} \]

The prime indicates the derivative with respect to \( N \equiv \ln a (\phi' \equiv \frac{d\phi}{dt} = \frac{d\phi}{\rho a} = -T\dot{\phi}T, \) while for the scalar field equation one gets (setting \( \lambda = V(\phi)/\rho \))

\[ \frac{2(1 + \lambda)}{3(1 - \phi'^2/6)} \phi'' + [(1 - w) + 2\lambda]\phi' + \sqrt{2}\alpha(\phi)(1 - 3w) + 2\lambda V_\phi \frac{\phi}{V} = 0. \tag{19} \]

The form of the factor \( A(T) \) in (2) for a STT follows from (18)

\[ A(T) \equiv \frac{A_C(\phi)(1 + \alpha(\phi)\phi')}{(1 - \phi'^2/6)^{1/2}}. \tag{20} \]

Assuming that \( A(T) \) is of the form (2), Eq. (20) can be rewritten in the form

\[ \left[ A_C^2 + \eta^2 T_s^2 \left( \frac{T}{T_s} \right)^{2(\nu + 1)} \right] \phi_T^2 = 2A_C^2 + \eta^2 \left( \frac{T}{T_s} \right)^{2\nu} = 0. \tag{21} \]
To solve Eq. (21) one has to specify the form of $A_C(\phi)$. Since the problem is quite involved, here we discuss the particular case $\alpha, A_C \ll 1$. Equation (21) reduces to \( \left( \frac{d\phi}{dz} \right)^2 = \frac{6}{z^2} \), where $z \equiv \frac{T}{T_*}$. The solution is $\phi(z) = \phi(0) + \sqrt{6} \log z \simeq \phi(1) - \sqrt{6} N$, with $\phi(0), \phi(1)$ constants. Noting that $\phi'' = 0$, Eq. (19) assumes the form $-\frac{\sqrt{6}}{T} + \frac{V}{T} (1 + \log V_\phi) = 0$. Writing the energy density in terms of the field $\phi$, $\rho = K_s e^{4\phi/\sqrt{6}}$ with $K_s = \frac{\pi^2 g_* T_*^4}{30}$, Eq. (19) allows to derive the potential $V$ (the integration constant is set equal to zero) $V(\phi) = \gamma^{-1} K_s e^{\gamma \phi}$, where $\gamma = \frac{\sqrt{6}}{\sqrt{4 + q}}$. The potential $V$ is suppressed for temperatures $T < T_*$ (see Fig. 1). Results are independent on $\nu$ and $\eta$. To obtain the correct DM relic abundance and explain the IceCube results, hence to fulfill the condition (12), the parameters $\{\eta, \nu, T_*\}$ must fine tuned. Setting $\eta \sim O(1)$ and $T_* = 10^8$GeV, and for $T_\ast \gg m_\chi$, Eq. (12) implies $\nu = \frac{34}{q - 6}$. For example, if the transition temperature occurs at $\sim 10^{12}$GeV, i.e. $q \sim 12$, then it follows $\nu \sim 5 - 6$.

\[ S_I = \frac{1}{16\pi G} \int d^4 x e [T + f(T)], \quad (24) \]

Figure 1. $\phi$ and $V(\phi)$ vs $T$.

4.2. $f(T)$ cosmology

Another interesting model is provided by the theory of gravity based on the Weitzenböck connection (instead of the usual Levi-Civita connection). Here the gravitational field is described by the torsion (instead of the curvature tensor). This model is referred as the Teleparallel Equivalent of General Relativity (TEGR), that is equivalent to GR at the level of field equations [31, 31, 32, 33, 34, 35]. In TEGR, one adopts the curvatureless Weitzenböck connection

\[ \hat{T}^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu} = e^\lambda_i (\partial_\mu e^i_\nu - \partial_\nu e^i_\mu), \quad (22) \]

where $e^i_\mu(x)$ are the vierbein fields defined as $g_{\mu\nu}(x) = e_{\mu j}(x) e^j_i(x)$. The action is given by

\[ S^0_I = \frac{1}{16\pi G} \int d^4 x e T, \quad \text{where} \quad T = S^\mu_{\rho\nu} T^\rho_{\mu\nu}, \quad \text{the torsion scalar}, \quad e = \text{det}(e^i_\mu) = \sqrt{-g}, \]

\[ S^\rho_{\mu\nu} = \frac{1}{2} \left[ \frac{1}{4} (T^\rho_{\mu\nu} - T^\nu_{\rho\mu} - T^\mu_{\rho\nu}) + \delta^\rho_{\rho} T^{\theta\nu}_{\theta\mu} - \delta^\nu_{\theta} T^{\theta\rho}_{\mu\nu} \right]. \quad (23) \]

We shall consider the simplest generalization of the action $S^0_I$, i.e.
Figure 2. For fixed values of $\eta = 1$ and $\beta_T$, the values of $\Pi$ needed to explain DM relic abundance and IceCube data follow for $n_T \lesssim 0.25$.

where in the present work we take $f(T) = \beta_T |T|^{\eta T}$ (in general $f(T)$ is a generic function of the torsion) [36, 37, 38].

Equations (22) and (23) imply $T = -6H^2$. The cosmological field equations are [34]

$$12H^2[1 + f_T] + [T + f] = 16\pi G\rho,$$

$$48H^2 f_{TT} H - (1 + f_T)[12H^2 + 4\dot{H}] - (T - f) = 16\pi Gp,$$

where $f_T = df/dT$. By rewriting (25) in the form $H^2 + H_T^2 = \frac{8\pi}{3M^2_{Pl}} \rho$, where $H^2_T \equiv \frac{f}{6} - \frac{T f_T}{3} = 6^{n-1} \beta_T (2n_T + 1) H^{2n}$, and assuming $H_T \gg H$, one gets $H_T \equiv H_{MC} = A(T)H_{GR}$, where $A(T)$ is of the form (2) with

$$T_* = \left( \frac{24\pi^3 g_*}{45} \right)^{\frac{1}{4}} \left( \frac{\beta_T}{v_{\text{GeV}}} \right)^{\frac{1}{4} (1-n_T)} \left( \frac{M_{Pl}}{\text{GeV}} \right)^{\frac{1}{2}} \text{GeV},$$

(26)

$\eta = 1$ and $\nu = \frac{2}{n_T} - 2$. For $a(t) = a_0 t^\delta$, i.e. $H = \frac{\delta}{\tau}$, it follows $T(t) a(t) = \text{constant}$. The transition temperature $T_*$ given in (26) has to be used into Eq. (12). In Fig. 2 is plotted (12) for the $f(T)$ model. The value of $\beta_T$ is obtained by fixing the transition temperature at $T_* \sim 10^{11} - 10^8 \text{GeV}$, that is $T_* \gg m_\chi$ (the parameter $\delta$ enters into the expression of $p$; we shall not present explicitly it being not relevant for our analysis).

5. Conclusions

In this paper we have studied the possibility to reconcile the current bound on DM relic abundance with IceCube data in terms of the 4-dimensional operator. Motivated by cosmological observations by Type Ia Supernovae, CMB radiation, and the large scale structure, according to which the present Universe is in an accelerating phase, new theories beyond GR have been proposed. We have shown that modified gravity models can explain the IceCube outputs and at the same time the DM relic abundance observed in a minimal particle physics model. This because the cosmological field equations based on modified gravity models change the thermal history of particles, so that the expansion rate of the Universe can be written in the form (1). As cosmological models, we have considered the STTs and models related to torsion $f(T)$. We have solved the Boltzmann equation to get the abundance of DM particles. The latter turns out to be modified by a quantity that does only depend on parameters of the modified cosmological models, and allows to explain, consistently, both the IceCube data and the correct DM abundance $\Omega_DM h^2 \sim 0.11$. 

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