Potential of Fuzzy Methodology for Investigation in Nanofluids Heat Transfer

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Abstract. In this paper, the Fuzzy Nanofluid Model (FNFM) used to develop a fuzzy analysis investigation on heat transfer optimal performance at different Nanofluids flow rate. The fuzzy Nanofluid model is applied to examine the effects of heat transfer parameters on heat transfer performance. Silicon Oxide SiO2 Nanofluid is used to explain their effects on heat transfer by two methods traditional and fuzzy (with two shapes of member ship function triangular and trapezoidal). This study evaluates the effects of nanoparticles SiO2 with different value of particle concentration PC (0.0-4.0%) using the water as a base fluid. This investigation covers a Reynolds number (Re) in the range of (100-500) as a flow rate (FR) for laminar flow. The main objective of present research, first one, compared a developed FNFM model with traditional model (TM) and determines how fuzzy model plays a significant role in prediction of Heat Transfer performance. Second one, to provide developed methodology for performance evaluation of heat transfer by connecting more than one parameter to a single output which is invaluable supplements relative to classical models. Third one, a developed FNFM can be used as a help tool for decision making to get the best judge (optimum) the performance of any system. The results of fuzzy model showed the heat transfer of SiO2/H2O Nanofluids significantly increased the PC compared with the increase in FR. However, however, using this method, there will be no need to resort to solving complex equations to arrive at a representation of the performance of any system. Finally, the study shows that fuzzy model plays significant role in prediction of heat transfer investigation without the complexity of mathematical tradition models. The correlations coefficients R2 between TM and FNFM models for heat transfer coefficient (0.97) and the average relative error (ε) is (4.4%). FNFM models can predict heat transfer characteristics with higher accuracy than that of the traditional model.

Keyword: Nanofluids, Heat Transfer, Flow Rate, Fuzzy theory, Decision Making.

1. Introduction
The Heat Transfer (HT) is usually used in several industrial applications such as power plant, chemical industries, food, cryogenic processes, manufacturing industry, environmental engineering, and air-conditioning etc. Nanofluids, in general, flow through the system. This Nanofluids flow creates the right amount of mixing to boost the HT coefficient. As Nanofluids enter the system, they induce significant heat transfer as compared to classical fluids at similar flow conditions and for specific Nanofluids with specific particle volume concentrations[1]. Many researchers have investigated turbulent heat transfer experimentally [2-4]. A Nanofluids is a traditional fluid containing nanometer-sized particles with an overall particle size of less than 100 nanometers. Conventional heat transfer fluids (HTFs) usually include ethylene glycol, water, and oil HTFs. Nanofluids have perfect heat transfer abilities as compared to conventional HTFs [5]. The thermal properties of fluids can be improved by introducing metallic particles to the fluid,
according to study of James Maxwell (1882)[6]. In the beginning, micrometer-sized metallic particles were used in HTFs, causing issues such as passage clogging, high-pressure decrease, sedimentation, and corrosion. The researchers used various types of Nanofluids and nanometer particles to establish nanotechnology in order to carry out Maxwell's concept. Dawood et al. [7] investigated the HT performance of several types of Nanofluids and found that when compared to other types of Nanofluids, SiO$_2$/H$_2$O Nanofluids with particles volume fraction had a higher HT coefficient. The fuzzy Nanofluids model (FNFM) is proposed in this paper to predict HT efficiency. Expert information is used in the Fuzzy Paradigm (FNFM). FNFM is a versatile model that improves the HT performance of high-level evolution capability [8]. It may play an important role in inaccurate calculations of modeled data. The data in input fuzzy sets differs from traditional data sets. Rather than numerical values, fuzzy sets use verbal expressions. As a result, FNFM is used to assess the efficiency of the HT coefficient and friction factor. Based on the previous literature, it is clear that the analysis of the effects of using SiO$_2$/H$_2$O on heat transfer and fluid flow characteristics based on fuzzy Nanofluids model has not been addressed previously (as far as researchers are aware from the research available to them), so it was an opportunity to expand on his study that had not been presented previously, and this has influenced the presentation.

As can be seen from the above literature review, HT efficiency of laminar flow using FMI in heat transfer needs further research and studies to be discussed, which is why the current study was undertaken. Furthermore, given the researchers' knowledge of the topic, more investigation and study analysis is needed to fill the gap related to Nanofluids, as many investigations focus on conventional fluids. However, studies involving the use of fuzzy model investigations of Nanofluids in heat transfer are desperately needed. The current research investigated laminar heat transfer in fuzzy Nanofluids models using a variety of Nanofluids, nanoparticle volume fractions (0.0–4.0%), and nanoparticle diameters (25.0–85.0 nm) dispersed in water as the base fluid. This research looks at laminar flow with Reynolds numbers (Re) ranging from 100 to 500. The thermal efficiency of SiO$_2$/H$_2$O Nanofluids flow is investigated analytically in this paper. Rather than fuzzy Nanofluids, many of the researchers used classical fluids and Nanofluids to conduct their study. The MATLAB program's fuzzy toolbox is used to apply fuzzy theory. The performance analysis of the FNFM was carried out by gathering data from the experimental calculations, which were made using known formulas from the literature, and then used to validate the fuzzy models.

### 1.1. Thermophysical Properties of Nanofluids

To do a calculation for Nanofluids, you must first measure the effective thermophysical properties of Nanofluids. The necessary properties, such as effective dynamic viscosity ($\mu_{\text{eff}}$), effective thermal conductivity ($k_{\text{eff}}$), effective coefficient of thermal expansion ($\beta_{\text{eff}}$), effective mass density ($\rho_{\text{eff}}$), and effective specific heat ($C_{p,\text{eff}}$), are listed in table 1 [9]. The mixing principle is used to measure the effective properties of coefficient of thermal expansion, real heat, and mass density. The effect thermal conductivity can be obtained by using Brownian motion of nanoparticles in a 3D horizontal concentric annulus and mean empirical correlation [10]:

$$k_{\text{eff}} = k_{\text{Brownian}} + k_{\text{Static}}$$  (1)

$$k_{\text{Brownian}} = 5 \times 10^4 \rho_{\text{bf}} \beta \rho_{\text{c,bf}} \sqrt{\frac{kT}{2k_{\text{np}}\rho_{\text{np}}}} f(T, \beta)$$  (2)

$$k_{\text{Static}} = k_{\text{bf}} \frac{2k_{\text{bf}}+k_{\text{np}}-2(k_{\text{bf}}-k_{\text{np}})\beta}{2k_{\text{bf}}+2k_{\text{np}}+(k_{\text{bf}}-k_{\text{np}})\beta}$$  (3)

$k$: is Boltzmann constant = 1.3807x10$^{-23}$ J/K, and $bf$ : is Base fluid;

$\beta = 1.37 \times 10^{-2}(1000)^{-0.8229}$ when $\beta < 1\%$, but $\beta = 1.1 \times 10^{-3}(1000)^{-0.7272}$ when $\beta > 1\%$;

$f(T, \beta) = ((3.917x10^{-3}) + (2.8217x10^{-2}\beta))(\frac{T}{\beta \rho_{\text{c,bf}}}) + (3.0699x10^{-2}\beta - 3.91123x10^{-3})$

Based on Brownian Motion of nanoparticles can be calculated the effective viscosity by[12]:
Viscosity:  
\[ \frac{\mu_{\text{eff}}}{\mu_f} = \frac{1}{1 - 34.87 \left( \frac{d_p}{d_f} \right)^{0.3}} \phi^{1.03} \]  
(4)

Diameter of base fluid:  
\[ d_f = \left[ \frac{6M}{N\pi \rho_f} \right]^{1/3} \]  
(5)

**Table 1** Thermophysical properties of pure H\(_2\)O and various types of Nanofluids [9].

| Thermophysical Properties | Al\(_2\)O\(_3\) | Water | SiO\(_2\) | TiO |
|---------------------------|----------------|-------|----------|-----|
| Dynamic Viscosity, \(\mu\) (Ns/m\(^2\)) | - | 2.01×10\(^{-3}\) | - | - |
| Density, \(\rho\) (kg/m\(^3\)) | 3970 | 998.203 | 2200 | 6500 |
| Specific Heat, \(c_p\) (J/kg.K) | 765 | 4182.2 | 703 | 535.6 |
| Thermal Conductivity, \(k\) (W/m.K) | 40 | 0.613 | 1.2 | 20 |
| Coefficient of Thermal Expansion, \(\beta\) (1/K) | 5.8×10\(^{-6}\) | 2.06×10\(^{-4}\) | 5.5×10\(^{-6}\) | 4.3×10\(^{-6}\) |

Density of the Nanofluid:  
\[ \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_{np} \]  
(6)

Where: \(\rho_f\): is mass densities of the base fluid; \(\rho_{np}\): is solid nanoparticles.

Effect Heat capacity in constant pressure of the Nanofluid:  
\[ (\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_{np} \]  
(7)

Where: \((c_p)_f\): is heat capacities of base fluid, \((c_p)_{np}\): is nanoparticles and \(nf\) Nanofluid.

Effect coefficient of thermal expansion of Nanofluid:  
\[ (\rho \beta)_{nf} = (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_{np} \]  
(8)

Where: \((\beta)_f\): is thermal expansion coefficients of base fluid and nanoparticles; is \((\beta)_{np}\)

The water and nanoparticles thermophysical properties are measured from previous literature at 293.15 K and listed in table 1 [9].

\(Nu\) and \(Re\) numbers are dimensionless parameters that are determined in the following way [13]: and numbers are dimensionless parameters which are calculated

\[ Nu = \frac{h D_h}{k} \]  
(9)

Where: \(k\): is the thermal conductivity; \(h\) average HT coefficient of fluid.

\[ Re = \frac{\rho u_m D_h}{\mu} \]  
(10)

Where \(\mu, \rho, u_m\), and : are dynamic-viscosity of fluid, density and mean fluid velocity over the cross-section respectively.

Additionally, a high \(Re\) will be used as an input parameter, and the findings were contrasted to equation (11) for \(Nu\) correlated by Dittus-Boelter correlation [14]:

\[ Nu = \frac{h}{k} D_h = 0.023 \left( \frac{\mu}{k} \right)^{0.8} Pr^{0.3} \]  
(11)

For the cooling system, the power of \(Pr\) (\(Pr = \frac{\mu c_p}{k}\)) was set to 0.3.

In the previous section, the complex calculation need to carry out the effective of Nanofluid parameters on the heat transfer in the system.

2. Methodology

2.1. Development of Fuzzy Nanofluid Model

Many modern techniques necessitate a comparatively precise mathematical calculation model that is adequate and acceptable. On the contrary, for a fuzzy model, an exact calculation mathematical model is not necessary. It does so because it relies on professional expertise. Many of the required processes
for developing a fuzzy model are included in the current methodology. Within FIS in the MATLAB Program, these processes are used to speed up complex mathematical calculations. Fuzzy models are used to represent the best results since they assist in making the best decisions. Since they are the most computationally effective methods, the centroid and weighted methods are most commonly used in fuzzy model applications [15,16]. The procedure of FNFM model is shown in figure 1. The figure 2 shows, the three diminution fuzzy set $\bar{V}$ is regarded as a two diminution MF on the plane (FR and PC). Therefore, the centroid operation shown in equation (12) is used to compact three diminution fuzzy set into two diminution that can describe the overall impact of the distribution data with respect to the input. $\bar{V}_G$ is carryout by the algebraic equation (12) is used the weighted in calculating of output (HT) [15]:

$$\bar{V}_G = \frac{\int_{v_{\min}}^{v_{\max}} v\mu(v)dv}{\int_{v_{\min}}^{v_{\max}} \mu(v)dv}$$

(12)

Where: $\bar{V}_G$: denotes center of gravity for $\bar{V}$ and $\mu(v)$ the membership degree of $v$ in the fuzzy set.

![Figure 1. Development model of fuzzy Nanofluid.](image1)

![Figure 2. The 2D Spatial output information translated from each crisp input. [15](image2)]

By representing human intelligence in an IF-THEN way, the fuzzy model used the fuzzy inference process. A collection of linguistic rules derived from human operators with extensive experiences (decision makers). Linguistic expression rules and creation of fuzzy sets, fuzzification of inputs, application of fuzzy operators, aggregation of outputs, and defuzzification of aggregated result are the five well-known stages of the Fuzzy Inference System (FIS).

The benefit of the proposed FNFM for calculating the efficiency of the complex coefficient of heat transfer is discussed in the following section (HT). The sensitivity of these criteria to different shapes of fuzzy MFs is investigated, as is the computational requirement for the implementation of the fuzzy model. The FIS in MATLAB 2016a was also used in this research.
2.2. Statistical Methods for Comparison

The traditional method (TM) was conducted to verify the results of proposed methodology FNFM according to statistical and mathematical methods. The performance of the TM and the FNFM proposed models were tested with the relative error ($\varepsilon$) with the correlation coefficient ($R^2$) by[17,18]:

$$\text{Error} = \varepsilon \% = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{HT_{TM} - HT_{FNFM}}{HT_{TM}} \right| \times 100\%$$  \hspace{1cm} (13)

The evaluated the performance of the proposed fuzzy model analysis and compared with the results of the experiment work through the evaluated the root mean square errors (RMSE) and coefficient of determination ($R^2$).

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(HT_{TM} - HT_{FNFM})^2}{\sum_{i=1}^{n}HT_{FNFM}^2}$$ \hspace{1cm} (14)

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(HT_{TM} - HT_{FNFM})^2}{n}}$$ \hspace{1cm} (15)

where $n$ is the number of data samples used in experimental work.

3. Case Study

The thermal efficiency of SiO$_2$/H$_2$O Nanofluids flow is investigated in this study using an analytical approach. The thermophysical properties of different nanoparticles were obtained from a previous literature review. Using H$_2$O as a base fluid, this research examines the effects of SiO$_2$ nanoparticles with varying $PC$ values (0.0 – 4.0 percent). This research looks at laminar flow with a Reynolds number ($Re$) ranging from 100 to 500. The Nanofluid flows are believed to be in an incompressible state.

The fuzzy set approach can be used to turn the Developed Model into a deterministic heat transfer problem, which can then be solved using the fuzzy rule-based Mamdani inference process. The fuzzy mathematical model formulation for heat transfer problem is based on the twenty-five simple linguistic rules for estimating the heat transfer. The two inputs are based on the Nanofluid Flow Rate (FR) and Nanoparticles concentration (PC) and the rules return a single output of the coefficient of heat transfer (HT).

For fuzzification of these factors the linguistic variables very slow (VS), slow (S), Quite (Q), fast rate (F) and very fast (V) are used for FR. Very low concentration (VLC), low concentration (LC), medium concentration (MC), high concentration (HC), very high concentration (VHC) are used for nanoparticles concentration (PC). Similarly, low heat transfer (L) medium heat transfer (M) and high heat transfer (H). Since these operators ensure a linear interpolation of the output between the laws, the max-min inference (Mamdani) method based on the center of gravity (CG) defuzzification approach was used in this paper. The selection of the MFs and their formations are based on the researcher's knowledge, experiment conditions and expert’s appraisals. For the single output and two inputs parameters, a fuzzy decision is formed as regulation rules by using If-Then rules. The twenty-five rules are developed and part of those rules are listed in table 2. For each of the linguistic term of this problem, there is a degree of MF that uses for input variables. Fuzzifications are made by using the functions as follows.

$$PC(i_1) = \begin{cases} i_1; & 0 \leq i_1 \leq 4 \\ 0; & \text{otherwise} \end{cases}$$ \hspace{1cm} (16)

$$FR(i_2) = \begin{cases} i_2; & 1 \leq i_2 \leq 5 \\ 0; & \text{otherwise} \end{cases}$$ \hspace{1cm} (17)

$$HT(O_1) = \begin{cases} O_1; & 800 \leq O_1 \leq 3600 \\ 0; & \text{otherwise} \end{cases}$$ \hspace{1cm} (18)
Table 2. Part of twenty-five rule base for FNFM.

| Rules | Input | output | Rules | Input | output | Rules | Input | output |
|-------|-------|--------|-------|-------|--------|-------|-------|--------|
| Rule 1 | VS   | VL     | Rule 9 | S     | LC     | Rule 17 | F     | MC     | M      |
| Rule 2 | VS   | VL     | Rule 10 | S    | VHC    | Rule 18 | F     | HC     | H      |
| Rule 3 | VS   | VL     | Rule 11 | Q    | MC     | Rule 19 | F     | not VHC | H      |
| Rule 4 | VS   | LC     | Rule 12 | Q    | MC     | Rule 20 | VF    | VL     | M      |
| Rule 5 | VS   | VHC    | Rule 13 | Q    | LC     | Rule 21 | VF    | not L  | H      |
| Rule 6 | S    | LC     | Rule 14 | Q    | MC     | Rule 22 | VF    | MC     | H      |
| Rule 7 | S    | LC     | Rule 15 | Q    | VHC    | Rule 23 | VF    | HC     | H      |
| Rule 8 | S    | HC     | Rule 16 | F    | LC     | Rule 24 | VF    | VHC    | H      |
| Rule 9 | S    | LC     | Rule 17 | F    | not L  | Rule 25 | VF    | not VL | L      |

The fuzzy MFs are assumed triangular and trapezoidal for illustrative simplicity. The triangular and trapezoidal MF shapes are governed by the range and value that has the greatest MF in each fuzzy set for each partition. The researchers selected these parameters based on their own preferences. Figure 3 depicts the fuzzy sets and their triangular and trapezoidal MF. Equation (19) yields triangular and trapezoidal fuzzy sets as well as MF values, which are depicted in Figure 2. For example, the triangular fuzzy number is a fuzzy number of FR parameter ($FR_{min}$, $FR_{middle}$, $FR_{max}$) with the following membership function:

$$
\mu(FR) = \begin{cases} 
0, & \text{if } FR < FR_{min} \\
\frac{FR - FR_{min}}{FR_{max} - FR_{min}}, & \text{if } FR_{min} \leq FR \leq FR_{max} \\
0, & \text{if } FR > FR_{max}
\end{cases}
$$

(19)

Figure 3. Partitions fuzzy number for FR parameter.

The development of MFs is based on a number of assumptions, including statistical data, human expertise, simulation of design parameters, and so on. Similarly, the MFs for the other parameters could be obtained. A figure 4 shows the MFs for the input PC and FR parameter with output HT for two shapes (Triangular and Trapezoidal) fuzzy membership functions. In table 3 listed a partition of value PC (0.0-4.0%) and so on for other parameters of case study.

Table 3. Percentage of value concentration PC data from FIS.

| Linguistic variable | Type    | C1 | C2 | C3 |
|---------------------|---------|----|----|----|
| Very low            | Z-shaped| 0  | 1  | -  |
| Low                 | Triangular | 0  | 1  | 2  |
| Medium              | Triangular | 1  | 2  | 3  |
| High                | Triangular | 2  | 3  | 4  |
| Very High           | S-shaped | 3  | 4  | -  |
4. Results and discussion

Fuzzy method used to simulate the heat transfer problems with the existing of Nanofluid such as SiO₂ to study the performance of heat transfer system. By traditional method to resolve Nanofluid heat transfer problem, the governing equations were solved by utilizing finite value method with certain assumptions and appropriate boundary conditions. The traditional calculated using equation (11) for the crisp output of HT (2580 W/m². K). However, certainly by employing Fuzzy models we will not need to solve very complex equations to get to characterize the performance of any system. Figures 5&6 display the surface results of fuzzy formed using FIS in MATLAB. In a 3D input-output vacuum, the figures depict the relationship between two input parameters (FR & PC) and their effect on a single output parameter, the coefficient of HT. As a result, if there are two input parameters, a conventional computational of Nanofluid approach makes it difficult to visualize the surface representation because the representation is constrained and complex.

The interpolation of rules based on 25 rules yields the surfaces representation. These guidelines have been used to improve the MF's accuracy using rules. If they are not correct, changes must be made in order to produce better results. If required, the set of rules is tweaked until the desired performance curve is achieved. This method has been applied to a variety of Nanofluids.

Comparisons have been done between the calculated of traditional Nanofluid method and FNFM developed method results for different both flow rates (FR) and particles value conditions (PC). The result shown in figure 5(A&B). From figure 5 (A,B) show an optimal performance of FR at (3-5), while an optimal performance of PC at (1-3) for both triangular and trapezoidal shape. It is clearly observed the optimal performance for HT at trapezoidal shape in yellow color. That is helped the decision maker and researchers to selected the optimal zone for correct and optimal decision for making his researches. That is reduce the time and cost for researches.

![Triangular MFs input](image1)
![Trapezoidal MFs input](image2)

![NanoParticles concentration (PC)](image3)

Flow Rate (FR)

Triangular MFs output

![Triangular MFs output](image4)

![Trapezoidal MFs output](image5)
Figure 4. shown two MFs shapes for input and output of case study

Figure 5. Output surfaces of FNFM for both triangle and trapezoidal MF

For both triangular and trapezoidal shapes of FNFM are found as \(2.2 \times 10^3\) and \(2.9 \times 10^3\), respectively (see figure 6 (A&B)). The small different between the calculated and FNFM fuzzy model in compared. The comparisons have been done between the TM and FNFM results by statistical and mathematical methods and equation (15). The correlation coefficient between TM and FNFM values of HT coefficient (HT, 14 values) in different FR and PC conditions have been shown in figure 6.

Figure 6. Output of FNFM for both triangle and trapezoidal MF
These results found the significant relationships for all parameters. By using equation (13) the correlation coefficients $R^2$ of heat transfer coefficient (HT) was obtained as 0.97 (see figure 7). The average relative errors ($\varepsilon$) between TM and FNFM for HT is found as 4.42%. The error ($\varepsilon$) presents the variation values between TM and FNFM and it is expected to attain zero. The error $\varepsilon$ of FNFM values are shown to be less than or the slightly over the allowable limits of 5%. FNFM goodness of fit values for HT are 0.97. The best goodness of fit value is one, which evaluates the capability of a fuzzy model. As anticipated, the values of the outcomes are close to one.

![Correlation Chart](image)

**Figure 7.** The Correlation between Traditional model and FNFM model.

5. Conclusion

In this analysis, a robust fuzzy model-based rule (developed model) was developed to predict heat transfer coefficients for water and $\text{SiO}_2/\text{H}_2\text{O}$ Nanofluids, and it was then compared to the results of traditional model (TM) calculations. The effects of both the measured and developed models are compared to make the decision-making process more easier and efficient. As a result of this inquiry, the following conclusions were reached:

1- The heat transfer coefficient increases as the value flow rate (FR) and nanoparticles PC value concentration rise, as seen in the performance figure 4 (A&B).

2- It shows that when the value FR and value concentration of nanoparticles PC both hit their relative maximum level, HT reaches its peak, and when the value flow rate and nanoparticles value concentration both decrease, the values of HT reverse. as depicted by the yellow color in the mesh of figure 5.

3- FNFM models are more accurate than standard models at predicting heat transfer. Eventually, it becomes clear that in order to achieve the highest heat transfer coefficient, it is important to keep the FR and PC values at their most desirable levels.

| Nomenclature | Ext.     |
|--------------|----------|
| $Al_2O_3$    | Aluminum Oxide          |
| $SiO_2$      | Silicon Oxide           |
| $C_p$        | Specific Heat, KJ/kg.K  |
| D            | Channel diameter, m     |
| H            | Heat transfer coefficient, W/m².K |
| K            | Thermal Conductivity, W/m.K |
| $TM$         | traditional model       |
| $\phi$       | concentration of nanoparticles (%) |
| $D_h$        | hydraulic diameter (m)  |
| $Pr$         | Prandtl number          |
| $PC$         | particle concentration  |
| $q$          | Cylinder Heat Flux, W/m² |
| T            | Temperature, K          |
| $U_n$        | Velocity inlet, m/s    |
| u            | Velocity               |
| $R_e$        | Reynolds number         |
| Nu           | Nusselt Number          |
| $R^2$        | correlations coefficients |
| $\varepsilon$| average relative error  |
| $f$          | friction factor         |
| FR           | flow rate               |
| $\mu$        | Dynamic viscosity, N.m/s |
| $\nu$        | Kinematic viscosity, m²/s |
| $B$          | coefficient of thermal expansion, 1/K |
| $\rho$       | Density, kg/m³         |
| $N$          | the membership degree of v in the fuzzy set. |

| Greek symbols | Ext.     |
|--------------|----------|
| $\beta$      | coefficient of thermal expansion, 1/K |
| $\phi$       | Nanoparticles value fraction (%) |
| $\rho$       | Density, kg/m³ |
| $\mu(v)$     | the membership degree of v in the fuzzy set. |

| Subscripts   | Ext.     |
|--------------|----------|
| $B_f$        | Base fluid |
| $Eff$        | Effective |
| F            | Fluid     |
| M            | Mean      |
| $nf$         | Nanofluid |
| $np$         | Nano particle |
| $o$          | Reference temperature |
| p            | Particles |
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