Graphical method for profiling hob mill that generate cycloid worms

V Teodor¹, S Berbinschi², N Baroiu¹ and N Oancea¹
¹ Department of Manufacturing Engineering, ”Dunărea de Jos” University of Galați, România
² Mechanical Engineering Department, ”Dunărea de Jos” University of Galați, România

E-mail: virgil.teodor@ugal.ro

Abstract. The hob mill for generating ordered curls of cycloid surface with non involute profiles may be profiled based on the fundamental theorems of surface enveloping – Olivier – as surface reciprocally enveloping with point like contact. In this paper, is proposed a methodology based on a complementary theorem of the surface enveloping in a graphical expression developed in a graphical design environment – CATIA. The graphical method presented in this paper is developed in two stages: determining of the rack gear model based on the solid model of the surface to be generated, using an original algorithm, following this, based on 3D modelling is determined the solid model of the primary peripheral surface of the hob mill. An application for a cycloid worm is presented – a central screw of helical pumps. In order to prove the quality of method, the analytical and graphical solutions are comparatively presented.

1. Introduction
In order to solve such a problem it is used the second theorem of Olivier – the case of conjugated surfaces, which depends by two independent parameters [1, 3]. It is also possible to use the Gohman theorem [1], when the surface to be generated, in relative motion regarding the peripheral primary surface of hob mill, depends by two independent parameters, usually the angular parameters of the revolution of blank and hob.

They are also knows and applied the fundamentals analytical theorems of surfaces enveloping, based on the method of intermediary surfaces (reference rack gear) [1], as so as the complementary theorems as: the method of “substitutive circles family” [3], the method of “minimum distance” [3], the method of “in-plane generating trajectories” [5] and others.

The development of graphical design environment as AutoCAD, CATIA etc. allows to create methods which use the capabilities of the AutoCAD [8] or CATIA [4] software.

Also, it is highlighted the concerning of the international researchers regarding the issue of profiling generating tools for involute teeth using the capabilities of graphical design environment [4, 9, 10, 11].

In this paper, is presented an application for profiling a hob mill which generates a cycloid worm – the central worm of a helical pump.

An algorithm presented by [4] is proposed. This algorithm made a methodology based on the theorems of helical movement composition and decomposition [2].
2. The geometry of rotor with cycloid profile

In reference system $XYZ$, see figure 1.a, the cycloid profile of the worm is given by equations:

$$X = 2R \cdot \sin(\alpha + \varphi) - R \cdot \sin(2\varphi + \alpha); \quad Y = 2R \cdot \cos(\alpha + \varphi) - R \cdot \cos(2\varphi + \alpha),$$

where:

$R$ is the base radius equal with the radius of the ruler, $R = 10$ mm; $\alpha = \pi/4$ - half-angle of the gap between two teeth; $R_e = 15$ mm – external radius of the worm; $z = 2$ – the rotor teeth number; $p_e = 100$ mm – helical pitch;

$$\varphi_{\text{min}} = 0; \quad \varphi_{\text{max}} = \arccos\left(\frac{-R_e^2 + 5R^2}{4R_e^2}\right).$$

The limits for $\varphi$ parameter are

![Figure 1.](image)

**Figure 1.** a). Cycloid rotor geometry; b). The 3D model of the cycloid worm.

In figure 1.b, is presented the 3D model of the cycloid worm, for the previously presented dimensions. The frontal profile of the rotor is composed from the $\overrightarrow{AB}$ arc, with the radius $R$ and the $\overrightarrow{BC}$ cycloid arc.

3. Generating rack-gear of cycloid worm

If it is accepted that the frontal profile of the cycloid worm is associated with a pair of rolling centrodes: $C_1$ is a circular centrode with radius $R_e$ associated with the cycloid worm; $C_2$ - linear centrode associated with the rack-gear, see figure 1.a.

In the movement assemblies:

$$x = \alpha \varphi \cdot X; \quad \xi = x - a; \quad a = \overrightarrow{-R_e - R_e \cdot \psi \cdot 0},$$

regarding the global reference system $xyz$, it is determined the family of cycloid profiles, in the reference system associated with the rack-gear, $\xi \eta \zeta$, see figure 1.a.

In movements (2), with $\psi$ is denoted the value of rotation angle around the $Z$ axe of the axode associated with the cycloid worm.

It is determined the cycloid profiles family and, after this, is determined the $\Sigma$ profiles families in the reference system of the rack-gear, in principle, in form:
The method of “minimum distance” allow to determine the enveloping of a profile associated with a pair of rolling centrode based on a specific theorem [3, 6]: The enveloping of an in-plane profile associated with a rolling pair of centrodes is the geometric locus of points belong to the profile for which, in various positions of rolling, the distance from the gearing pole is minimum.

In principle, the enveloping of the profiles family (3) is in form:

\[
\xi = \xi(\psi); \quad \eta = \eta(\psi); \quad \zeta = 0.
\]  

as result of the association between the equations (3) and the specific enveloping condition for minimum distance method [3].

The enveloping of the helical surfaces families \( \Sigma_{AB} \), \( \Sigma_{BC} \), \( \Sigma_{CD} \), represents the flanks of the generating rack-gear. The form of the rack-gear will be used, in following, to determine the peripheral primary surface of the future hob mill.

For problem solving it will be used the method of intermediary surface [1], (the rack-gear) based on the method of “minimum distance” [3].

3.1. Methodology in CATIA

In figures 2, 3 and 4 is shortly presented the method of “minimum distance” for generation of rack-gear conjugated with the crossing section of the cycloid worm, for profile segments profil \( \Sigma_{AB} \), \( \Sigma_{BC} \), \( \Sigma_{CD} \). On the \( C_1 \) centrode, associated with the worm’s profile, are considered equidistant points which are transferred onto the \( C_2 \) centrode of the rack-gear – the points \( P_1, P_2, \ldots, P_n \), representing the relative positions of the gearing pole, \( P \), in the process of rolling between the \( C_1 \) and \( C_2 \) centrodes.

![Figure 2](image2.png)  **Figure 2.** Determination of the rack-gear– the AB flank.

![Figure 3](image3.png)  **Figure 3.** Determination of the rack-gear profile – the BC flank.

![Figure 4](image4.png)  **Figure 4.** The rack-gear profile flank CD.

The distances measured onto the normals drawn to the \( \Sigma_{AB} \), \( \Sigma_{BC} \), \( \Sigma_{CD} \) profiles from the \( P_i \) (i=1, 2, ..., n) points, by points representing the feets of these normals are the points of the generating rack-gear.

In figure 4, is represented the frontal profile of the cycloid shaft, the crossing section of the rack-gear and the 3D surface of the generating rack – \( l \) (\( I_{AB}, I_{BC}, I_{CD} \)). The surface of the rack-gear is a
cylindrical surface having as directrix the enveloping of the $AB$, $BC$ and $CD$ profiles and the generatrix parallels with the tangent to the helix with radius $R_e$ of the cycloid worm - $t$. We have to notice that for the generating rack-gear form, in figure 6, as so as in following, we not take into account the changes due to the overlapping between the profiles as so as the possible passing curves due to the singular points onto the profile.

4. Helical movement decomposition
Knowing surface $I$, based on the theorem of helical motion decomposition is made an algorithm for determination of the characteristic curve at contact between the surface of the rack-gear and the helical peripheral primary surface of the tool, $S$, with axis $V_i$ and parameter $p_i$.

Based on the specific theorem of the helical movement decomposition, $V_i; p_i$, the helical motion is decomposes in two revolution movements with disjoint axis let $A$ and $B$ these axis. They are defined the $A$ and $B$ axis, the angular values $\alpha$ and $\beta$, as so as the $a$ and $b$ distances, measured along the common perpendicular between the axis $A$, $V_i$ and $B$.

The helical motion $(V_i, p_i)$ is decomposed in a sum of revolution movements with axis $A$ parallel with $V_i$ and translate along the direction of the cylindrical surface’s generatrix, $t$.

\[
(A, \alpha = 0) + T(t); (V_i, p_i).
\]

Figure 5. Generating rack-gear - $I_{AB}$; $I_{BC}$; $I_{CD}$; $t$ - unitary vector of the cylindrical surface’s generatrix; $V_i$ - unitary vector of the cycloid worm’s axis.

With $T(t)$ was denoted the translation motion along the generatrix $t$ of the cylindrical surface (the $I$ rack-gear).

If is denoted with $\theta$ the angle between $V_i$ and $t$ then, the position of $A$ axis regarding $V_i$ is given by the distance: $a = p_i \cdot \tan \theta$, here $\beta = \theta$, and the $A$ axis, parallel with $V_i$ is positioned at distance $a$ from this.

In this way, the $I$ surface of rack-gear in the helical motion $(V_i, p_i)$ is equivalent with the assembly of motion:- rotation around the $A$ axis, parallel with $V_i$ and the translation along the generatrix of the $I$ cylindrical surface, in the direction $t$.
The characteristic curve, namely the contact curve between $I$ and $S$ will depend only by this component of the motion in which the $I$ surface will not self-generate. How $t$ is the direction of generatrix of $I$ surface, the characteristic of rack-gear’s surface, in the (5) motion assembly will depend only by the rotation around the $A$ axis, previously defined ($a=0; a = p_{1} \cdot \tan \theta$).

As it is known, the characteristic curve of a surface, $I$, in rotation motion around an axis $A$ is the geometric locus of the points belongs to $I$ surface, which represent the projection of the rotation axis onto this surface [2].

These reasons allow determining a graphical methodology, developed in CATIA design environment, for profiling the hob mill generating the $I$ rack-gear and, as consequence, of the $\Sigma$ helical surface.

If the intermediary surface $I$ is defined, the rack-gear conjugated with the cycloid worm to be generated, see figure 6, through the $P$ pole, is drawn a line, $A^{*}$, parallel with the axis of the future hob mill $V$. The direction $A^{*}$ make with the $t$ generatrix the angle $\theta$, $\theta = \pi / 2$.

It is defined the profile of the generating rack-gear, in a plane normal to the rack-gear generatrix, the normal profile. The normal profile of the rack-gear is positioned in the plane determined by the directions $A^{*}$ and $V$, parallels, at the distance $R_{t}$ – the radius of the hob mill (technological value), in the plane normal to the $I$ surface’s generatrix.
By the helical motion of the normal profile around the $V_r$ axis (the hob mill's axis), as worm with sense contrary to this of the worm to be generated, but with the same inclination on the cylinder with the radius $R_t$ for the hob mill and $R_e$ for the cycloid worm, is generated the 3D model of the peripheral primary surface of hob mill, see figure 7, which is a complexes helical surface, with known normal pitch, from the intersection of the 3D model of the generating rack-gear with the plane normal to the $t$ direction of its generatrix (the projection of the $A$ axis on the intermediary surface $I$ – the rack-gear).

The angular parameters $\omega_p$ and $\omega_t$ are the angles presented in figure 7 and the axial section of the helical surface of the hob mill is presented in figure 8. To intersection of the characteristic curve $\Sigma - I$, see figure 6, with the characteristic $I - S_1$, figure 8, determines the characteristic point – the contact point between $\Sigma$ and $S_1$.

The geometric characteristics: cycloid worm $p_{f_{worm}} = \pi \cdot R_t$; normal pitch $p_n = p_{f_{worm}} \cdot \sin \omega_p$; $\tan \omega_p = p_{an} / (2 \pi R_t)$; rack-gear $p_{n-rack} = p_n$; hob mill $p_{an} = p_n / \cos \omega_t$; $\tan \omega_t = p_{an} / (2 \pi R_e)$.

In table 1 and figure 8, are presented the coordinates and axial profile of the hob mill and its form – the axial section of helical primary peripheral surface which is the peripheral surface of the future tool.

Obviously, this is the primary profile, which may be modified if we considered the issue linked to the presence of singular points and interference trajectories.

5. Conclusions
The graphical method for profiling the hob mill generating of the cycloid worm, based on the complementary theorem of the “minimum distance” was developed in graphical form in CATIA design environment.

The method use an intermediary surface – generating rack-gear conjugated with the surface to be generated.

The proposed method is intuitive and leads to rigorous results of the hob mill’s profile. The proposed principle can be applied for other types of teeth (ordered curls of surfaces associated with a centrode).

In this paper, it wasn’t approach the issue linked to singular points of the profile, nor does the interference process which always emerges in such a generating process
Acknowledgments
The work has been funded by the Sectoral Operational Program Human Resources Development 2007-2013 of the Ministry of European Funds through the Financial Agreement POSDRU/159/1.5/S/132397.

References
[1] Litvin F L 1984 Theory of Gearing Reference Publication 1212 NAS Scientific and Technical Information Division (Washington D C)
[2] Liukshin V S 1968 Theory of Screw Surfaces in Cutting Tool Machinostroyenie (Moscow)
[3] Oancea N 2004 Surfaces generation by enwrapping (I –II) Teoreme complementare (Galați: Editura Fundației Universitare „Dunărea de Jos”
[4] Berbinschi S, Teodor V and Oancea N 2013 3D graphical method for profiling gear hob tools The Int Journ of Adv Manuf Tech 64(1-4) pp 291-304
[5] Teodor V G 2010 Contribution to elaboration method for profiling tools which generate by enveloping Lambert Academic Publishing
[6] Oancea N and Oancea V G 1997 Geometrical modeling of surface generation through wraping Journal of Manufacturing Science and Engineering 119 pp 829-834
[7] Berbinschi S et al. 2011 A 3D Method for Profiling the Shaping Tool for Generation of the Helical Surfaces The Annals of „Dunărea de Jos” Univ of Galați Fasc V Tech in Machine Buildings
[8] Baicu I and Oancea N 2002 Profilarea sculelor prin modelare solidă (Chişinău: Tehnica Info Publishing House)
[9] Skoczylas M J 2007 Modelling the tooth flanks of hobbed gears in the CAD environment Springer – Verlag London Limited pp 746 – 751
[10] Dimitriou V and Antoniadis A 2008 CAD – based simulation of the hobbing process for the manufacturing of spur and helical gears Springer–Verlag London Limited pp 347–357
[11] Ott O S and Artyukin L L Shaping Involute Profiles by Means of a Disk Tool (London: Springer–Verlag London Limited)
[12] Liang Yao, Zhonghe Ye, Jian S Dai and Noiyi Cai 2005 Geometric Analysis and Tooth profiling of the Three-Lobe Helical Rotor of Roots Blower Journal of Materials Processing Technology 170 pp 259-267