Near-Zero-Sidelobe Optical Subwavelength Asymmetric Focusing Lens with Dual-Layer Metasurfaces

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1. Introduction

In recent years, with the in-depth research on metamaterial, 2D metamaterial with the same electromagnetic properties and designable characteristic—metasurface[1–3] was proposed, which rapidly became the research highlight owing to it controls light based on its subwavelength cell structures and can greatly improve the sensitivity of device performance. The mechanism of wavefront shaping by metasurface[4–10] is controlling the phase, amplitude, and polarization of the incident light wave. The optical characteristics of metasurface depend on the geometries and arrangements of its subwavelength cell structures, which have the superiority of controlling light waves. Recently, more and more attention was paid to its easy preparation and low insertion loss. In the fields of super-resolution focusing and imaging, the metasurface has become the mainstream[11–16] by replacing the photonic crystal[17–24] and traditional metamaterial[25] owing to its characteristics of ultrathin volume and high image quality. Meanwhile, various optical devices based on the metasurface have been widely proposed, including vortex beam generator,[26] holography device,[27] beam deflector,[28] and so on.

Asymmetric transmission[29,30] has attracted widespread attention owing to its potential value in integrated optics. In the past two decades, the field of asymmetric transmission has obtained important developments due to the rise of artificial microstructure materials, including the one-way transmission of light waves,[31,32] unidirectional invisible,[33,34] and unidirectional polarization surface plasmon.[35,36] Based on these characteristics, the micro photon devices such as optical isolators,[37] unidirectional polarizers,[38] and photodiodes[39–41] have been developed. Artificial microstructure materials, with their novel optical properties and flexible controllability, created conditions for asymmetric transmission of light. However, the above-mentioned artificial microstructure materials could not be widely used due to the limitation of the volume, i.e., their size was normally much larger than the operating wavelength. On the contrary, metasurfase, which could adjust the phase wavefront within the subwavelength thickness resulting from its controlling effect of phase wavefront was far greater than the phase accumulation effect of the traditional artificial microstructure materials, has become an ideal choice to achieve asymmetric transmission.[41–44] However, the existing metasurfaces for asymmetric transmission were often constructed by metal.[41,42] Thus, large material loss was unavoidable in the optical frequency region, which caused asymmetric transmission with low efficiency. At the same time, the existing optical metasurfaces could only realize the optical asymmetric transmission,[41–44] it could not realize the optical asymmetric transmission and focusing simultaneously. The simultaneous control of multiple light waves can be widely used in the fields of optical micromachining,[45] quantum communication,[46,47] optical micromanipulation,[48,49] and microimaging,[50,51] so it has a higher application value than that of single optical asymmetric transmission. Researchers have realized acoustic asymmetric focusing by using asymmetric focusing lens (AFL).[52] Nevertheless, the obtained focal point was accompanied by large sidelobe, i.e., the sidelobe intensity exceeded half of the intensity of the focusing. Therefore, this type of AFL could not be applied in the fields that need high quality of focusing such...
as acoustic micromachining, micromanipulation, and ultrasonic imaging.

In this paper, the dual-layer metasurfaces are designed by using impedance matching materials based on the generalized Snell's law to realize the asymmetric transmission. Then, to expand the application scope of the dual-layer metasurfaces, an AFL with dual-layer metasurfaces is constructed by making phase delays of metasurfaces interacted with each other. When light waves are incident from the left side, they can focus on the right side of the lens, but when light waves are incident from the right side, they cannot arrive at the left side of the lens. Finally, considering the drawbacks of the middle region of metasurface on the right side cannot be involved in focusing, as well as the side-lobe of optical asymmetric focusing is large, the design theory of AFL is optimized further, which can control the refraction of light waves as shown in Figure 1.

To control the refraction of light waves, the height and length of the cell structure remain constant, i.e., $h = 0.2\lambda$, and $l = 0.6\lambda$, where $\lambda$ is the incident wavelength, whose initial value is set to 5 $\mu$m, corresponding to 60 THz in air. The background of the metasurface is air. To reduce the reflection and multiple scattering of incident waves, the materials of the cell structure are set to different impedance matching materials, the refractive indices of the materials are $n_1 = 3.42$ and $n_2 = 1.42$, with the corresponding lengths of $a$ and $b$, respectively, and $a + b = l$. The phase delay is controlled by adjusting the values of $a$ and $b$.

The transmission characteristics and focusing of light waves are analyzed by the finite element method, and the light waves here we studied are all TM mode.

If the incident angle $\theta_i$ of the plane wave is 0, Equation (1) can be expressed as

$$\sin \theta_t = \sin \theta_i + \frac{1}{k} \frac{d\varphi}{dy}$$

where $\theta_i$ and $\theta_t$ are the incident and refractive angles, respectively, $\varphi$ is the phase delay generated by the metasurface, and $k$ is the wavenumber in the air. It is shown that in Equation (1), through properly designing the phase delay of the metasurface, the arbitrary refractive angle can be achieved. The cell structure of the metasurface is composed of two kinds of mediums with different refractive indexes. The phase delay is controlled by adjusting the ratio of the two mediums in the cell structure, which can control the refraction of light waves as shown in Figure 1.

2. Model and Theory

Based on the generalized Snell’s law,[4] when the background is air, the refractive angle when the plane wave passes the metasurface can be expressed as

$$\sin \theta_t = \sin \theta_i + \frac{1}{k} \frac{d\varphi}{dy}$$

Meanwhile, according to the relationship of the refractive index and phase delay, the phase delay caused by the cell structure of the metasurface becomes

$$\varphi = n_1ka + n_2kb$$
By binding of Equations (2) and (3), we can obtain

\[
\sin \theta_i = \frac{n_1 \Delta a + n_2 \Delta b}{h} \tag{4}
\]

where \(\Delta a\) and \(\Delta b\) are the length differences of the same refractive index parts of two adjacent cell structures. Meanwhile, owing to \(a + b = l, \Delta a = -\Delta b\). Therefore, Equation (4) can be simplified to

\[
\sin \theta_i = \frac{(n_1 - n_2) \cdot \Delta l}{h} \tag{5}
\]

where \(\Delta l = \Delta a = -\Delta b\). Thus, the control of the refractive angle can be realized by designing \(\Delta l\). The values of \(\Delta l\) in Figure 1a,d are set to 0.05\(\lambda\) and -0.10\(\lambda\), respectively. The intensity distributions of light waves in each cell structure of the metasurface in Figure 1a,d are shown in Figure 1b,e, respectively. The phase delay of light waves covers the whole 2\(\pi\) range, which means that the refraction of light waves can be controlled by the metasurface. Meanwhile, according to Equation (5), the refractive angle \(\theta_{11} = 30°\) (here we define the refractive angles are positive and negative when their directions are upward and downward, respectively) and \(\theta_{12} = -90°\) are realized by the metasurfaces corresponding to Figure 1b.e. These results are shown in Figure 1c,f, which indicates that the arbitrary refractive angle even surface wave can be realized by designing the phase delay of the metasurface.

3. Results and Discussion

3.1. The Asymmetric Transmission Realized by the Dual-Layer Metasurfaces

According to the generalized Snell’s law, the refractive light waves are no longer formed instead of becoming the surface wave that propagates along the surface when \(\theta_i = 0°\) and \(|\theta_i| = 90°\), where \(\theta_i\) is the incident angle and \(\theta_r\) is the refractive angle of light waves which caused by the metasurface. Moreover, if \(|\theta_i| = 90°\) when \(\theta_i = 0°\), then when the light waves are incident with certain angles and their signs are opposite to \(\theta_i\), they can pass the metasurfaces. Based on the above theory and the metasurfaces in Figure 1, if \(\theta_{12} = -90°\), then when \(\theta_{11} > 0°\), the asymmetric transmission can be realized by the dual-layer metasurfaces, as shown in Figure 2.

As shown in Figure 2a (light waves are incident from the left side) and Figure 2b (light waves are incident from the right side), the dual-layer metasurfaces are MS1 and MS2, whose parameters correspond to Figure 1a,d, respectively. The distributions of the phase delays of these two metasurfaces are symmetric about the \(x\)-axis. The phase delays \(\phi_1\) and \(\phi_2\), which respectively, belong to MS1 and MS2, can be obtained by Equations (2) and (3), as shown in Figure 2c,d, in which the blue solid line is the phase delays calculated by theory and the red balls are the phase delays of 79 cell structures in MS1 and MS2. Owing to the distribution of phase delay in MS1, the structure of MS1 is kept the same as Figure 2a, but the structure of MS2 is changed by associating with the phase delay of MS1. Meanwhile, due to MS1 and MS2 are placed in parallel, the refractive angle caused by MS1 is equal to the incident angle in MS2, i.e., \(\theta_{21U} = \theta_{11U}\) and \(\theta_{21D} = \theta_{11D}\). Similarly, \(\Delta l\) of upper and lower parts in MS2 are -0.10\(\lambda\) and 0.10\(\lambda\), respectively. Thus, the refractive angles of the upper and lower parts when light waves are normally incident on MS2 are \(\theta_{21U} = -90°\) and \(\theta_{21D} = 90°\), respectively, which means that both the upper and lower parts when light waves are incident on MS2 are 0.05\(\lambda\) and -0.05\(\lambda\), respectively. Therefore, the asymmetric transmission of light waves can be realized by the dual-layer metasurfaces. The corresponding simulation results of the field distribution are shown in Figure 2e,f.

3.2. The Asymmetric Focusing Realized by AFL

Although the asymmetric transmission can be realized by the dual-layer metasurfaces as shown in Figure 2, the single function limits its application in the more complex light field. To solve this problem, the structure of MS1 is kept the same as Figure 2a, but the structure of MS2 is changed by associating with the phase delay of MS1 (the specific theoretical derivation and explanation can
be seen below), which can realize the more complex asymmetric focusing. Its principle sketch maps are shown in Figure 3.

As shown in Figure 3a, to realize focusing when light waves are incident normally from the left side, the refractive angle caused by MS2 needs to maintain the distribution of focusing, i.e., $|\theta_{il2}|$ varies directly as the distance from the x-axis. To realize the goal, the structure of MS2 is changed. Owing to MS1 and MS2 are placed in parallel, the refractive angle caused by MS2 is equaled to the incident angle in MS2, i.e., $\theta_{il2} = \theta_{il1}$. Meanwhile, the change rule of incident angle $|\theta_{il1}|$ is contrary to that of $|\theta_{il2}|$, which means that the change rule of $|\theta_{il1}|$ is contrary to that of $|\theta_{il2}|$. As shown in Figure 3b, when light waves are normally incident on MS2, then $|\theta_{il2}| = 90^\circ$, light waves are converted into the rapidly decaying surface wave and propagate along the surface of MS2, and cannot arrive at the left side.

Owing to the distributions of phase delays of MS1 and MS2 are all symmetric about the x-axis, it only needs to analyze the phase delays when $y > 0$. The theoretical model of AFL is shown in Figure 4.

It can be seen from Figure 4, light waves are focused on the right side through AFL, so the transmission directions of light waves need to meet the transmission path shown by the red solid line in Figure 4, where $\theta_{il1}$ and $\theta_{il2}$ are the refractive angles caused by MS1 and MS2, respectively, when light waves are normally incident from the left side. The refractive angle $\theta_{il2}$ is caused by the MS2, when light waves are incident with $\theta_{il2}$, therefore, according to Equation (1) and $\theta_{il1} = \theta_{il2} = \theta_{il1}$, and $\theta_{il2}$ satisfy the following relation

$$\sin \theta_{il2} = \sin \theta_{il1} + \frac{1}{k} \frac{d\varphi_2}{dy}$$

(6)

where $d\varphi_2/dy$ is decided by $\varphi_2$, and $\varphi_2$ is the phase delay of MS2. Owing to the structure of MS2 coincides with that in Figure 2a, whose $\Delta l = -0.10\lambda$, and the phase delay of MS2 is irrelevant to the incident angle, then according to Equations (2), (5), and substitute the relevant values, Equation (6) can be expressed as

$$\sin \theta_{il2} = \sin \theta_{il1} - 1$$

(7)

meanwhile, according to the geometric relationship, $d_1$, $d_2$, $y_1$, and $y_2$ satisfy the following relations

$$\tan \theta_{il2} = -\frac{y_2}{d_2}$$

(8)

$$\tan \theta_{il1} = \frac{y_2 - y_1}{d_1}$$

(9)

where $d_1$ is the distance between MS1 and MS2, $d_2$ is the focal length, $y_1$ and $y_2$ are the positions where light waves are incident on MS1 and MS2, respectively. Through Equations (8) and (9), $\theta_{il1}$ satisfies the following relation

$$\tan \theta_{il1} = -\frac{d_2}{d_1} \tan \theta_{il2} - \frac{y_1}{d_1}$$

(10)

By combining Equations (1) and (2), the relationship between $\varphi_1$ and $y_1$ can be obtained

$$\varphi_1 = k \int \sin \theta_{il1} dy_1$$

(11)

where $\varphi_1$ is the phase delay of MS1. It can be seen from Equation (11), when the phase delays of MS1 and MS2 interact with each other, the distribution of phase delay of MS1 presents not generally periodic change but presents continuous change. If $d_1 = 5\lambda$ and $d_2 = 6\lambda$ are set, the distribution of phase delay of MS1 and asymmetric focusing field when the operating wavelengths $\lambda = 5 \mu m$ can be obtained by Equations (7), (10), and (11), as shown in Figure 5.

By combining the distribution of phase delay of MS1 (see Figure 5a) and Equations (2)–(5), $\Delta l$ of MS1 at different positions in the $y$-axis can be obtained, which can be applied to design MS1. It can be seen from Figure 5b,c, the focusing of the plane wave which is normally incident from the left side can be realized through AFL. However, its sidelobe is too large (the cause is analyzed and the result is optimized in Section 3.3), which is not conducive to practical application. As shown in Figure 5b,d,
asymmetric transmission and focusing are realized simultaneously by AFL, which can be used to the more complex light field.

3.3. NZS Asymmetric Focusing Realized by the Optimized AFL

Then the cause of large sidelobe of asymmetric focusing in Figure 5 is analyzed, and the structure of AFL is optimized to realize NZS asymmetric focusing.

Owing to the structure of the lens is symmetric about the x-axis and for the sake of the convenience in theoretical analysis, here only need to analyze the upper part of AFL which means \( y_1 \geq 0 \), according to Equation (9), i.e., \( y_1 \geq d_1 \tan \varphi_1 \). The minimal value of \( y_1 \) can be obtained when \( y_1 = 0 \), so \( y_{\text{min}} \approx 3.16 \lambda \). Meanwhile, the length of the upper part of MS1 is 7.9, which means that there is more than a third of the region in the middle of MS2 not involved in the focusing. Therefore, the components of light waves involved in focusing are less and cause large sidelobe. To eliminate sidelobe for better focusing, the middle region of MS2 has to be as far as possible to be involved in focusing. Therefore, the direction of the refractive angle caused by MS1 should be contrary to that in Figure 3 to make the light waves can pass the middle region of MS2. To this end, the mediums with refractive indexes \( n_1 \) and \( n_2 \), respectively, of MS1 in Figure 3 are switched positions. The theoretical models of focusing and transmission paths are as shown in Figure 6.

In Figure 6a, the structure of MS1 is consistent with that in Figure 3, but the refractive angle caused by MS1 is contrary to that in Figure 3. Therefore, the change rule is contrary to that in Figure 3 when light waves are normally incident from the left side, i.e., \( |\theta_{d12}| < |\theta_{d11}| \). Owing to the differences of MS1 between Figures 3 and 6 are only switching the position of mediums with different refractive indexes, a similar method can be used to derive the relationship satisfied by the lens in Figure 6

\[ \sin \theta_{d1m} - \sin \theta_{d1m} > \sin \theta_{d2m+1} - \sin \theta_{d1m+1} \geq 1 \]  \hspace{1cm} (13)

where integer \( m \in [1,7] \) represents the order number of refractive angles on different positions in MS2. According to Equation (6), \( \theta_{d1m} \) and \( \theta_{d2m} \) in this structure satisfy the following relation

\[ \sin \theta_{d2m} - \sin \theta_{d1m} = \frac{\sin \theta_{d2m+1} - \sin \theta_{d1m+1}}{\sin \varphi_{2m} / \sin \varphi_{1m}} \]  \hspace{1cm} (14)

Therefore, the phase delay of MS2 can be designed reasonably by combining Equations (13) and (14). Meanwhile, owing to \( |\theta_{d1}| \) in each position at the y-axis is identical, the parameter settings of metasurfaces can be determined by analyzing the value of \( |\theta_{d1}| \). In order to achieve NZS asymmetric focusing, when the light waves are incident from the left side and propagate the upper (lower) parts of MS1, they should be refracted as much as
Figure 7. The phase delays of the dual-layer metasurfaces of the optimized AFL and field diagram of NZS asymmetric focusing: a) the distribution of phase delay of MS$_1$; b) the distribution of phase delay of MS$_2$; c) the field diagram when the light waves are incident from the left side, inset figure is intensity curve of focal plane at focal point; d) the field diagram when the light waves are incident from the right side.

It can be seen from Figure 7a,b, the distributions of phase delays of MS$_1$ and MS$_2$ present periodic and gradient changes, respectively. From Figure 7c, the NZS focusing can be realized by optimized AFL when light waves are normally incident from the left side, and the results are superior to Figure 5a. From Figure 7d, the light waves are still transferred to surface wave and cannot pass the optimized AFL. Meanwhile, from the inset figure in Figure 7c, the sidelobe is far less than that in Figure 5c, which means the sidelobe of asymmetric focusing can be effectively reduced.

3.4. The Frequency Ranges of NZS Asymmetric Focusing Realized by the Optimized AFL

The frequency ranges, the focal spot size which can be defined as the full-width at half-maximum (FWHM)\cite{11,56,57} the focusing efficiency which can be defined as the ratio of the power within a line of three times the FWHM at the focal plane to the total power incident upon incident boundary of AFL,\cite{18} and field diagram of NZS asymmetric focusing at different frequencies can be obtained by changing the incident frequencies, the results are shown in Figure 8.

Figure 8a shows that the frequency ranges of NZS asymmetric focusing are $f \in (59.5, 67.5)$ THz. The NZS subwavelength focusing can be realized when plane waves are normally incident from the left side in these frequency ranges, as is depicted in Figure 8b. These results indicate that the optimized AFL can realize NZS asymmetric focusing in broadband, and the focusing efficiency of all frequencies is greater than 30%, which has great practical application values. From Figure 8c–g, all lights with different frequencies (59.7, 62.2, and 64.7 THz) can realize the NZS asymmetric focusing by the optimized AFL which further indicates that the NZS asymmetric focusing possesses broadband characteristics.

4. Conclusions

We design an AFL with dual-layer metasurfaces based on generalized Snell’s law for realizing optical asymmetric transmission and focusing simultaneously. Considering the initially designed AFL with drawbacks of the middle region of metasurface on the right side cannot be involved in focusing, as well as the sidelobe of optical asymmetric focusing is large, the design theory is optimized further, and the NZS asymmetric focusing on the subwavelength scale is realized. The NZS asymmetric focusing of the optimized AFL is effective in broadband, which has great application potential in micro/nano-optical devices.

Acknowledgements

J.X. and S.L. contributed equally to this work. This work was supported by the National Natural Science Foundation of China (grant nos. 61405058 and 61605166), the Natural Science Foundation of Hunan Province (grant nos. 2017JJ2048 and 2018J13514), and the Fundamental Research Funds for the Central Universities (grant no. 53118040112). The authors acknowledge Prof. J. Q. Liu for software sponsorship.
Conflict of Interest

The authors declare no conflict of interest.

Keywords

asymmetric focusing, asymmetric transmission, generalized Snell’s law, metasurfaces, sidelobes

Received: January 23, 2020
Revised: May 19, 2020
Published online: June 10, 2020