Simulation of Einstein-Podolsky-Rosen experiments in a local hidden variables model with limited efficiency and coherence.

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We simulate correlation measurements of entangled photons numerically. The model employed is strictly local. The correlation is determined by its classical expression with two decisive difference: we sum up coincidences for each pair individually, and we include the effect of polarizer beam splitters. We analyze the effects of decoherence, detector efficiency and polarizer thresholds in detail. The Bell inequalities are violated in these simulations. The violation depends crucially on the threshold of the polarizer switches and can reach a value of 2.0 in the limiting case. Existing experiments can be fully accounted for by limited coherence and non-ideal detector switches. It seems thus safe to conclude that the Bell inequalities are no suitable criterium to decide on the nonlocality issue.

The Einstein-Podolsky-Rosen (EPR) problem has long occupied a central place in the understanding of quantum mechanics. Bell’s inequalities in conjunction with correlation measurements seemed to prove that reality in microphysics is manifestly nonlocal. Furthermore, the experimental evidence seems to contradict even the notion of an independent reality. Both of these features, if true, are highly problematic. The former, because no field propagating with a velocity exceeding c has ever been observed. The latter, because without an independent reality there is no guarantee that theoretical models can at all be contradicted. And without the possibility of contradiction scientific progress follows no clear rules.

For these reasons the EPR problem is far more important than the experiments alone indicate. Consequently, a large amount of work has been devoted to this field. Two years ago, the standard reference on EPR - the book by Afriat and Selleri - more or less highlighted the dilemma. But in the same year Deutsch and Hayden could show, by an analysis of the information flow in such an experiment, that there is in fact no nonlocal connection between the two measuring devices. All information about the two angles of polarization, $\phi_1$ and $\phi_2$, is stored locally. Even though this information cannot locally be accessed. It is probably due to this new field of research, quantum information theory, that the problem is even more important today than it was ten years ago. Consequently, a number of papers in the last two years have analyzed the paradox from different angles, and the analysis brought two features into focus: (i) The validity and significance of Bell’s inequalities; and (ii) the relevance of a photon’s phase for the correlations. From the viewpoint of information flow a violation of Bell’s inequalities is no proof of nonlocality. From a formal point of view it seems that the standard inequalities cannot be derived without violating established notions about the measurement process. For a detailed discussion see Sica or Adenier. The notion of a phase seemed initially problematic because the phase e.g. of a wavefunction cannot be related to physical properties of a photon. But as shown later, the phase indicates the phase of a photon’s electromagnetic field. And it could be established that the existence of a phase connection between the two points of measurement, a connection which arises at the process of emission from a common source, is sufficient to explain correlations between two measurements in space-like separation. It was also emphasized that measurements cannot in general be factorized without losing the linearity of the fields between the two polarizers.

In this Letter we pursue a different strategy. We perform numerical simulations of actual experiments. We sought to include the features of the experimental situation as far as possible. For this reason we shall give not only the results of ideal measurements, but also measurements with limited efficiency, limited coherence, and under the condition of dead angles of our polarizers. It will be seen that all these effects have a bearing on the actual data.

Setup. - The setup of the experiment is shown in Fig. 1 (a). A source between two polarizers emits a pair of photons along the z axis. Two polarizers, at the positions $L_1$ and $L_2$, respectively, measure the angle of polarization. The angle of polarization one is varied by a half cycle, $\pi$, during the experiment. At every position of the polarizer a set of 1000 photon pairs is emitted and measured. The switch of the polarizer is shown in Fig. 1 (b). If $\cos^2(\phi_1 - \alpha)$ is larger than 0.5 $+ \Delta s$, then the event is recorded as a transmission ($+$). If it is less than 0.5 $- \Delta s$, it is recorded as an adsorption ($-$). No action is taken for values between these two boundaries. The same switch is applied to both measurement devices. The threshold $\Delta s$ formalizes three separate features of the experiments: (i) The transmission characteristics of the polarizer beam splitters; (ii) the threshold of the photodetectors; and (iii) the electronic evaluation of events, since double counts ($+$ and $-$) are excluded.

A single simulation run starts with the initialization of
the random number generator \(^{14}\). The generator is initialized only once, at the beginning of a full simulation cycle. The random number is mapped onto the initial phase from 0 to \(2\pi\) of the photon pair. Simulations are generally made with a phase difference of \(\pi/2\) between the angles of polarization of photon one and photon two. After covering the distance to polarizer one and two the photons are measured. We assumed, without lack of generality, that both distances are integer multiples of the wavelength. After a single pair has been measured, we record the coincidences (++,+-,-+,–). The procedure is repeated for all 1000 pairs, then the polarization angle of device one is changed by \(\pi/100\). A run ends, when all 1000 pairs at the final position of polarizer one have been measured (\(\pi\)). In all figures we only plot the coincidences \(N_{++}\).

We accounted for limited efficiency and decoherence in the following way. Limited efficiency means that not all pairs emitted are actually measured. In this case we simply did not evaluate all pairs, depending on the efficiency of the setup. 50 \% efficiency, for example, means that only every second pair is actually recorded. To simulate decoherence we created an independent random input for a certain fraction of a half cycle of \(\pi\). 100 \% decoherence here means that half a wavelength of the photon’s optical path is random. This translates into a polarization angle random in the interval \([0, \pi]\). Both effects reduce the maximum of the output measured, but it will be seen presently that they have very different effects.

**Ideal measurements.** - Initially we simulated an ideal measurement. The efficiency in this simulation is 100\%, the fields of the two photons are fully coherent throughout the distance between the two polarizers, and the experimental devices are supposed to have ideal characteristics. The result of this simulation is shown in Fig. 3. We did two separate simulations, one with a polarization difference between the two photons of 0 (full squares), the other with \(\pi/2\) (full circles). It can be seen that neither of the simulations comes close to the theoretical prediction of a \(\sin^2(\beta - \alpha)\). Instead, the curves representing ideal measurements would be of angular shape. However, the maximum of the correlation (\(N_0/2\), where \(N_0\) is the number of pairs) and the minimum (zero) are exactly obtained in the extreme cases.

It should be noted that the results given in these plots reflect the “classical” formulation of a coincidence, given by the equation \(^{17}\):

\[
P(\alpha, \beta) = \int d\lambda \cos^2(\lambda - \alpha) \cos^2(\lambda - \beta) \tag{1}
\]

with two decisive differences: first, the summation is performed over single pairs, as in the actual experiments, rather than over the two polarizers separately. The latter procedure, formalized in the given integral, includes not only photons of one pair, but also sums up contributions of different pairs. And second, we accounted for the digital output of the polarization devices. The integral is no suitable representation of the digital results in current experiments. If, for example, \(\lambda = \phi_1\) and \(\phi_2 = \phi_1 + \pi/2\), then for \(\alpha = \beta = 0\) the integral yields \(N_0/8\). But the actual count, under the condition that \(\cos^2 \phi_1 > 0.5\) and \(\cos^2(\phi_1 + \pi/2) > 0.5\) is zero. The digitalization, necessary to obtain formal agreement with spin measurements in quantum mechanics, is usually achieved by means of a polarizer beam splitter \(^{14}\).

**Dead angles.** - In our simulations we find that the curves obtained are not very sensitive to the threshold of the polarizer switches. We have performed simulations where \(\Delta s\) was varied from 0.00 to 0.20. Apart from a reduction of the absolute yield the increase of the threshold only affects the width of the minimum at the ultimate angles. This effect is equal to a retardation of the onset of the correlation function at its minimum position. The threshold therefore does not change the functional form of the correlations.

**Efficiency.** - The detection of photons is one of the problems experimenters are still confronted with. The efficiency is in fact so low (less than 10\% \(^{14}\)), that the correlations found in Aspect’s measurements \(^{3}\) were disputed on the grounds of an “efficiency loophole”. In our simulations such a conclusion would not be justified. Even though the shape of the curve changes somewhat and the statistical spread is dramatically increased in the low efficiency range, the maxima and minima are still clearly distinguishable. The minimum, moreover, remains zero. We included a dead angle of detectors by a threshold of 0.05. The same threshold will be used in subsequent simulations. The coincidence rates due to detector efficiency varying from 50\% to 10\% are shown in Fig. 4. From this result we conclude that efficiency is not the decisive issue to estimate the relevance of an experiment.

**Decoherence.** - The polarizers in current measurements are more than 400 m apart \(^{13}\). Furthermore, there is no vibration damping or cooling to very low temperatures involved in such a measurement. This feature of the measurements is bound to cause random motion of system components. From surface science the range of motion without damping can be estimated, it should be for an isolated surface no less than a few nanometer or more than one percent of the photon’s wavelength. Considering that we deal with three coupled components and optical paths in between it seems safe to increase this estimate by one order of magnitude. In this case we have to include random motion of our system in the range of about 5-10\% of the wavelength. This translates, in our simulation, into a rate of decoherence of 10-20\% (100\% means that half a wavelength of the photon’s optical path is random).

Simulation with a decoherence rate of 10\%, 50\% and 100\% are shown in Fig. 4. The interesting feature of decoherence is that it renders the resulting distribution more sinusoidal than the correlations of an ideal measurement. The fully decoherent simulation proves that correlations are independent of the setting as required.
but it also shows the noise due to a random distribution of the initial phase of the coupled system. In practice all effects analyzed will to a greater or lesser extent be present in any single measurement.

**Bell violations.** - Finally, we demonstrate the influence of the polarizer threshold on the violation of Bell’s inequalities. To this end we simulate the counts at four selected angular positions of the polarizers $\alpha$ and $\beta$ ($0^\circ$, $45^\circ$, $22.5^\circ$, $67.5^\circ$). These positions yield the maximum violation of Bell’s inequalities in the standard framework. We performed the simulations for varying threshold values from 0.0 (no threshold) to 0.2 (nearly half the photons remain undetected). For every setting we performed 10 separate runs, each with 10000 pairs of photons, the efficiency of the detectors was assumed to be 100%. Fig. 5 gives the result of our simulation. The violation (computed according to the version of Clauser et al. (CHSH) \[16\]) increases with increasing threshold. Furthermore, the limit of violation is close to 2.0 (CHSH value of 3.90) in the final setting. The actual threshold in the experiments can be estimated from the visibility of the correlation function. For decoherence rates of 10 - 20 %, as in the experiments, we can only obtain agreement with reported values (97% visibility \[4\]), if we increase the threshold $\Delta s$ to more than 0.10. Our simulations indicate that the violation depends crucially on the threshold. The maximum violation can reach a value of as much as 2.0 (the limiting case). It seems thus safe to conclude that the Bell inequalities are no suitable criterium to decide on the nonlocality issue.

**Summary.** - In summary we have presented a numerical simulation of EPR experiments under the assumption of strict locality and analyzed the effects of polarizer thresholds, limited efficiency, and decoherence in detail. We could show that Bell’s inequalities are violated in these simulation, and that the violation depends crucially on the threshold of the polarizer switches. The limit of violation in this model is about 2.0 (CHSH value of 4.0). We found that existing measurements can be fully accounted for by limited coherence and non-ideal polarizer switches.

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 Photon 2
Photon 1

FIG. 1. One dimensional model of EPR type experiments. (a) The measuring devices (1) and (2) are in opposite directions from the photon source S. The polarizers are set to the angles $\alpha$ and $\beta$, respectively. (b) The switches at both stations measure the polarization and, depending on the angle, either transmit 1 or 0 to the computer. Note that the dead angle of the polarizer is simulated by a threshold $\Delta s$ of the switch.

$\cos^2(\phi_1 - \alpha) > 0.5 + \Delta s +$

$\cos^2(\phi_1 - \alpha) < 0.5 - \Delta s -$

Switch 1

FIG. 2. Ideal measurement of correlations ($N_{++}$). Two simulations were performed with a difference of 0 (full squares) and $\pi/2$ (full circles) of the polarization angle at the origin. Neither of these curves is equal to the theoretical prediction, instead we obtain an angular shape for the correlations.

FIG. 3. Dependence of coincidence rates on the efficiency of detection. The shape of the distribution is similar to the ideal distribution, but the statistical spread is considerably larger. We include the $\sin^2(\alpha)$ function for reasons of comparison. This function has not been actually fitted to simulated data.
FIG. 4. Dependence of coincidence rates on the decoherence of photon beams. Due to decoherence the distribution becomes more sinusoidal. In the limit of full decoherence we obtain uncorrelated measurements.

FIG. 5. Violation of Bell’s inequalities depending on the threshold of the polarizer switches. (a) The inequality is violated in all cases where the threshold is not zero (full circles). We obtain a maximum CHSH value of 3.9 (threshold 0.2). The experiments of Weihs et al. indicate a threshold of 0.1 (empty circle). (b) Simulation of EPR experiments with a decoherence of 10% and a threshold of 0.1. It can be seen that the distribution differs only insignificantly from the square of a sinus.