Quark Masses from Gaugino Condensation in String Theories

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We present a mechanism able to generate the perturbatively absent up/down (10 · 10 · 5 H) quark Yukawa couplings of SU(5)/flipped SU(5) GUTS in Type II orientifold compactifications with D-branes. The mechanism works when there are Sp(N) gauge groups involved. The 5’s get charged under the Sp(N) gauge groups and the generation of quark masses proceeds via the generation of the fermionic Sp(N) singlet condensate (5 · 5 · 5 · 5) in the term (1/M 5) 10 · 10 · (5 · 5 · 5 · 5). Also non-chiral states charged under Sp gauge groups may become constrained by the requirement of Sp’s becoming strongly coupled.

I. INTRODUCTION

Non-perturbative mechanisms are very important in modern particle theories as they are used to solve various problems. For example, QCD instantons [1] are a famous solution in QCD for solving the η'-meson mass problem. [2] and SU(2) instantons in generating baryon number violating interactions. On the other hand in string compactifications worldsheet instantons have been associated with genus zero instantons that generate corrections to the spacetime superpotential [3]. Moreover gaugino condensation has been connected to the formation of hidden matter condensates [4] and supersymmetry breaking. Also in the context of intersecting brane worlds [8, 6] hidden sector confinement has been utilized to decouple hidden sector states at high energies [7]. On the other hand as intersecting brane models (IBM) [5, 6] fix some of the moduli either via supersymmetry conditions in intersections or fluxes, it is quite desirable to find a flipped SU(5) (FSU(5)) IBM string model which could mimic the phenomenological success of fermionic FSU(5) [8].

Quite recently, in the context of type II compactifications, D2-brane (E2) instantons wrapping special Lagrangian three-cycles of the internal Calabi-Yau [9] can generate perturbatively absent couplings like Majorana terms for right handed neutrinos, µ-terms in the MSSM [10, 11] or solving via instantons [12] the long standing problem of the missing 10 10 5 H. Also non-chiral states charged under Sp gauge groups may become constrained by the requirement of Sp’s becoming strongly coupled.

In this work we describe a new way of generating the missing 10 10 5 H in orientifolds of type IIA compactifications by utilizing the formation of strongly coupled condensates extending the work of [22]. In the first part of this work, we discuss the generation of down-quark masses in flipped SU(5)/FSU(5). In the second part we generate the up-quark masses in SU(5) GUTs via the same mechanism.

II. THE MISSING YUKAWA COUPLING TERM

A. FLIPPED SU(5) MODELS

Grand unified theories giving rise to SU(5) type gauge group structures [13, 14, 15, 16, 17, 18, 19, 20, 21] arising from 4D compactifications of type II theories on N=1 supersymmetric intersecting brane orientifolds are missing the Yukawa coupling 10 10 5 H, which is responsible for generating masses for the down quarks 15, 16, 17, 18, 19). In an SU(5) GUT the same coupling gives rise to a missing up-quark mass (e.g. see [14, 16, 17, 20, 21]).

In [22], a candidate solution for a perturbative term to the mass of the up-quarks in SU(5) GUTs was suggested where string models with gauge groups in the form SU(5) × U(1)n with n ∈ Z were used. We have argued that the form of the higher dimensional coupling that gives a mass to the missing 10 10 5 H SU(5) Yukawa’s appears from an operator in the form 1/M 5 10 10 5 5 5 5 5 1 H 1 H where the 1 H gauge singlets are used to cancel the surplus bifundamental charge. In this case, the up-quark mass may come from the condensation of 5 fermions Lets us consider the field theory operator 10 10 5 5 5 5 and rewriting this term in SU(5) notation as

\[
\frac{1}{M^5} \epsilon^{ijklm} 10^{ij} 10^{kl} \epsilon^{\mu\phi\chi\psi\omega} 5^\phi 5^\chi 5^\psi 5^\omega, \tag{1}
\]

we conclude that in components eq. (1) reads

\[
10^{ij} 10^{kl} 5^1 5^2 5^3 5^4 \equiv (u_d^c L) (d_L^c L) (d_L^c L) (e_L^c L). \tag{2}
\]
In [22] it was argued that, in the context of field theory [2], can only generate a mass as long as the fermions of the Higgs-like 5-plets condense. In this work, we will suggest that an alternative explanation for the appearance for such a fermion condensate exists, within in the context of SU(5) models constructed on Type IIA orientifolds with intersecting D6-branes where the SU(5) gauge symmetry is extended by Sp(N) gauge groups. The binding mechanism is easy to understand in terms of the gauge forces acting on the fermion condensate as the $\mathcal{F}$ condense and are charged under Sp(2). The order of the condensate is $(\bar{5}, 1, 1, 2)^4 \equiv \Lambda^6$, where $\Lambda$ is the scale the fermion condensate forms. The mass of the up-quarks is determined from (2) and for the present model it reads

$$M_{quarks}^U \approx \frac{\Lambda^6}{M_5^s}. \quad (3)$$

Let us also consider the flipped SU(5) model [20] FSU(5)-IV on Type IIA T6/($Z_2 \times Z_2$) orientifold that also includes metric, NSNS, and RR fluxes [24, 25]. In this flipped SU(5) model the complete gauge symmetry is $U(5) \times U(1)^{10} \times U(5) \times U(5)$ and $SU(5) \times U(5)$.

We have parametrized the number of non-chiral states

$$SU(3) \times SU(2)_{a_1} \times U(1)_{a_2} \cdots \times U(2)p(2). \quad (6)$$

We observe that only the Yukawa mass coupling term giving a mass to the up quarks is allowed (there is no obvious mass term for the leptons as well)

$$\{10_{(2,0)^{10}}, (\bar{5}_{(-1,1,0)^{10}})^H \}_{(-1, -1, 0)^{10}} \cdot \{\bar{5}_{(-1,1,0)^{10}}^H, 1_{(0,2,0)^{10}}^H \}_{(1,1,0)^{10}} \}$$

We used for convenience the notation $k^n \equiv (k, k, \ldots, k)$ with $k$ repeated $n$ times. Also as there is no mass term for the down-quarks of the form $10_{(2,0)^{10}} \cdot 10_{(2,0)^{10}} \cdot 5_{(1,1,0)^{10}}^H$ as this is not allowed by $U(1)_{a_1} \cdot U(1)_{b_1}$ charge conservation. This is the usual problem of any SU(5)/flipped SU(5) GUT from any type II orientifold compactifications e.g. [14, 15, 16, 17, 18, 19]. The down-quark masses may be generated by the expression

$$\frac{1}{M_5^s} \left( (10_{(1,1,1,2)^{2}(-1,0)^{10}} \right)^2 \cdot \left( \bar{5}_{(1,1,1,2)^{2}(-1,0)^{10}} \right)^4.$$

In order to show that the 5's condense we will adjoining the gauge group $U(5) \rightarrow U(3) \times U(2)$, so that the $U(5)$-D6-brane splits into a and all branes, each one associated with the $U(3)$, $U(2)$ brane stacks. The anomaly free hypercharge corresponding to the decomposition of table (1) is

$$Y = \frac{1}{6} U(1)_a - \frac{1}{2} U(1)_b + \frac{1}{2} U(1)_c + \frac{1}{2} U(1)_d$$

$$- \frac{1}{2} U(1)_e + \frac{1}{2} U(1)_f - \frac{1}{2} U(1)_g$$

$$- \frac{1}{2} U(1)_h + \frac{1}{2} U(1)_i - \frac{1}{2} U(1)_k$$

We have listed only the intersections of the branes with the Sp(2) generating brane.

| Intersection | Multiplicity | Y |
|--------------|--------------|---|
| a, 2         | 1            | $\{3, 1, 1, 1, 1, 2\}_{(-1,0)^{10}}$ |
| a_1, 2       | 1            | $\{1, 1, 1, 1, 1, 2\}_{(-1,0)^{10}}$ |
| b, 2         | $B'$         | $(1, 1, 1, 1, 2)_{(0,1,0,0,1,0)}$ |
| b, 2         | $B'$         | $(1, 1, 1, 1, 2)_{(0,1,0,1,0,0)}$ |
| c, 2         | $C'$         | $(1, 1, 1, 1, 2)_{(0,0,1,0,1,0)}$ |
| c, 2         | $C'$         | $(1, 1, 1, 1, 2)_{(0,0,0,1,0,1)}$ |
| d, 2         | 1            | $(1, 1, 1, 1, 2)_{(0,0,0,0,1,0)}$ |
| e, 2         | $E'$         | $(1, 1, 1, 1, 2)_{(0,0,0,0,1,0)}$ |
| e, 2         | $E'$         | $(1, 1, 1, 1, 2)_{(0,0,0,1,0,0)}$ |
| f, 2         | 6            | $(1, 1, 1, 1, 1, 2)_{(0,0,0,0,1,0)}$ |
| g, 2         | 3            | $(1, 1, 1, 1, 1, 2)_{(0,1,0,1,0,0)}$ |
| h, 2         | $H'$         | $(1, 1, 1, 1, 1, 2)_{(0,0,0,0,1,0)}$ |
| i, 2         | $I'$         | $(1, 1, 1, 1, 1, 2)_{(0,1,0,1,0,0)}$ |
| j, 2         | $I'$         | $(1, 1, 1, 1, 1, 2)_{(0,0,0,0,1,0)}$ |
| k, 2         | $K'$         | $(1, 1, 1, 1, 1, 1)_{(0,1,1,0,1,0)}$ |
| k, 2         | $K'$         | $(1, 1, 1, 1, 1, 1)_{(0,1,1,1,0,0)}$ |

in intersections where branes are parallel in at least one tori by $B'$, $C'$, $E'$, $H'$, $I'$, $K'$. The $\beta$-function for Sp(2) is $b_{Sp(2)} = 8 + B' + C' + E' + H' + I' + K' - 6$ and may become negative as long the number of non-chiral states is either 0 or if they decouple at a typical mass of order $M_5$, higher than the condensation scale $\Lambda_{cond}$. We may also need to give a mass $\Lambda_{cond}$ to the multiplets from the

\[\begin{array}{cccc}
\hline
\text{Intersector} & \text{Multiplicity} & \text{Y} \\
\hline
a_1, 2 & 1 & \{3, 1, 1, 1, 1, 2\}_{(-1,0,0)^{10}} & 0 \\
\hline
b, 2 & B' & (1 + 1, 1, 1, 2)_{(0,1,0,0,1,0)} & 0 \\
b, 2 & B' & (1 + 1, 1, 1, 2)_{(0,1,0,1,0,0)} & 0 \\
c, 2 & C' & (1 + 1, 1, 1, 2)_{(0,0,1,0,1,0)} & 0 \\
c, 2 & C' & (1 + 1, 1, 1, 2)_{(0,0,0,1,0,1)} & 0 \\
d, 2 & 1 & (1 + 1, 1, 1, 2)_{(0,0,0,0,1,0)} & 0 \\
e, 2 & E' & (1 + 1, 1, 1, 2)_{(0,0,0,0,1,0)} & 0 \\
e, 2 & E' & (1 + 1, 1, 1, 2)_{(0,0,0,1,0,0)} & 0 \\
f, 2 & 6 & (1 + 1, 1, 1, 1, 2)_{(0,0,0,0,1,0)} & 0 \\
g, 2 & 3 & (1 + 1, 1, 1, 1, 2)_{(0,1,0,1,0,0)} & 0 \\
h, 2 & H' & (1 + 1, 1, 1, 1, 2)_{(0,0,0,0,1,0)} & 0 \\
i, 2 & I' & (1 + 1, 1, 1, 1, 2)_{(0,1,0,1,0,0)} & 0 \\
j, 2 & I' & (1 + 1, 1, 1, 1, 2)_{(0,0,0,0,1,0)} & 0 \\
k, 2 & K' & (1 + 1, 1, 1, 1, 1)_{(0,1,1,0,1,0)} & 0 \\
k, 2 & K' & (1 + 1, 1, 1, 1, 1)_{(0,1,1,1,0,0)} & 0 \\
\hline
\end{array}
intersections (a,2), (a₁,2), (f,2), (g,2). The leptons may get a mass from
\begin{equation}
\frac{1}{M_s^2} \tilde{\sigma}^{(-1,1,0\beta)} (1_{(0,2,0\beta)} \tilde{\sigma}^{H}_{(1,1,0\beta)} ((1^{12};1,1,1,2)_{(0,-1,0\beta)})^4
\end{equation}
where the states (⋅⋅⋅) from the intersection (a₁,2) may condense and decouple. In Table III using eq.(3), we exhibit the condensation scale against the string scale given the phenomenological down-quark masses; 3.5 < m_d < 6 MeV, M_s ≈ 105 MeV, M_a ≈ 4.20 GeV [23]. Thus
\begin{table}
| Quark | M_s = 10^{16} | M_s = 10^{17} | M_s = 10^{18} |
|-------|--------------|--------------|--------------|
| d     | 5 × 10^{-3}  | 8.9 × 10^{-12}| 3.25 × 10^{-14}| 4.1 × 10^{-14}|
| s     | 105 × 10^{-7} | 1.5 × 10^{-13}| 1.0 × 10^{-14}| 6.9 × 10^{-14}|
| c     | 4.2          | 2.7 × 10^{-13}| 1.9 × 10^{-14}| 1.3 × 10^{-15}|
\end{table}

Table III: Scale vs down-quark using eq.(3).
a generic FSU(5) string model may therefore describe the masses of the down-quarks as long as its condensation scale is in the range
\begin{equation}
10^{12} \text{ GeV} < \Lambda_{\text{cond}}^{SU(5)} < 10^{15} \text{ GeV}
\end{equation}
If we consider the running gauge Sp(2) coupling following [7] with b_{Sp(2)} = -2 where have set, B', C', E', H', l', K' to zero and the states from intersections (a,2), (a₁,2), (g,2) decouple, \Lambda_{\text{cond}} is determined in table IX. M_s can be higher than 10^{18} GeV since its value depends on the number of non-chiral states which we will not determine here. Values of \Lambda_{\text{cond}} higher from \Lambda_{\text{run}} may be
\begin{table}
| \Lambda_{\text{cond}} | M_s = 10^{16} | M_s = 10^{17} | M_s = 10^{18} |
|---------------------|--------------|--------------|--------------|
| \Lambda_{\text{run}}| 2.6 × 10^{-9} | 2.6 × 10^{-12}| 2.6 × 10^{-15}| 2.6 × 10^{-18}|
\end{table}

Table IV: \Lambda_{\text{cond}} from the running Sp(2) gauge coupling.
understood (from the exact string amplitude of the associated fermionic correlator of eq.(2)) which involves a suppression factor lowering \Lambda_{\text{cond}} to \Lambda_{\text{run}}.

\section{SU(5) GUTs}

Let us consider for example the SU(5)-type D6-brane model I1.4. of [27]. It requires three stacks a, c and d of D6-branes giving rise to a U(5)_a × U(1)_c × U(1)_d gauge symmetry intersecting at angles in IIA orientifolds of Z_3 × Z_2 toroidal compactifications [26]. The U(5)_a splits into SU(5)_a × U(1)_a, where the anomalous U(1)_a gauge boson becomes massive via the generalized Green-Schwarz mechanism and U(1)_a appears as a global symmetry in the effective action. The matter transforming as 10 under SU(5)_a arises at intersections of stack a with its image a'; the matter fields transforming as 5 as well as Higgs fields 5_H and \bar{5}_H are located at intersections of stack a with c and c' or d and d'. The key input in the construction of the D-brane model is summarized in Tables VII and VIII. Table VII lists the candidate Higgs fields.

\begin{table}
| stack | U(5) \times U(1)_c \times U(1)_d \times USp(2) |
|-------|-----------------------------------------------|
| a     | (10, 0, -1, 0, 1) (1, 1, 0, 0, 0) (1, 1, 1, 1) |
| c     | (-1, -1, 0, 0, 0) (1, 1, 0, 0, 0) (1, 1, 1, 1) |
| d     | (0, -1, 0, 0, 0) (0, 0, 0, 0, 0) (1, 1, 1, 1) |

Table VII: Wrapping numbers of D6-branes.

As can be seen from tables VII and VIII, the only mass terms allowed are the ones that are associated with the Yukawa couplings giving masses to the down quarks and leptons respectively
\begin{equation}
\langle 10, 1, 1, 1, (2, 0, 0, 0, 0) \rangle + \langle 15, 1, 1, 1, (-2, 0, 0, 0) \rangle
\end{equation}
\begin{equation}
\langle 10, 1, 1, 1, (2, 0, 0, 0, 0) \rangle + \langle 15, 1, 1, 1, (-2, 0, 0, 0) \rangle
\end{equation}
and there is no mass term for the up-quarks of the form
\begin{equation}
\langle 10, 2, 0, 0, 0 \rangle \cdot \langle 10, 2, 0, 0, 0 \rangle \cdot 5_{H(1,1,0)}
\end{equation}
as this is not allowed by U(1)_a, U(1)_b charge conservation. This is the usual problem of any SU(5) GUT from any type II orientifold compactifications e.g. [14, 15, 16, 17]. Thus, working without loss of generality in the context of SU(5) model shown in Tables VII and VIII, a perturbative mass term for the up-quark masses of the form
\begin{equation}
\frac{1}{M_s^2} \langle 10, 1, 1, 1, (2, 0, 0, 0, 0) \rangle^2 \cdot \langle 5_H, 1, 1, 2 \rangle_{(-1,0,0,0)}^4
\end{equation}
exists. In order to demonstrate that the Sp(2) gauge group will become strongly coupled, we will use adjoint breaking (AB) to break the SU(5) down to the SU(3)_a × SU(2)_b × U(1)_Y, Y = (1/6)U(1)_a + (1/2)U(1)_b - (1/2)U(1)_c + (1/2)U(1)_d, which is equivalent to splitting the stacks on one torus. To establish notation, U(1)_a, U(1)_b are the U(1)'s within U(3), U(2) (of U(5)). Thus, we only present explicitly the states charged under Sp(2) in table VII. We have parametrized the appearance of an arbitrary number of non-chiral states becoming massless by A. Since b_{Sp(2)} = A = -2, the b-function becomes asymptotically free when the number of non-chiral multiplets becomes either 0 or 1.
The range of $\Lambda_{\text{run}}^{\text{cond}}$ is what one gets from the strong coupling of Sp is similar to that of FSU(5) (we have set $A = 0$, $\chi_C = 28/20$).

| $M_S = 10^{15}$ | $M_S = 10^{16}$ | $M_S = 10^{17}$ | $M_S = 10^{18}$ |
|----------------|----------------|----------------|----------------|
| $2.4 \cdot 10^9$ | $2.4 \cdot 10^{11}$ | $2.4 \cdot 10^{12}$ | $2.4 \cdot 10^{14}$ |

TABLE IX: $\Lambda_{\text{run}}^{\text{cond}}$ from the running Sp(2) gauge coupling.

We have presented a new perturbative mechanism to generate the masses of the up/down quarks through fermion condensates within N=1 supersymmetric SU(5)/flipped SU(5) constructions from intersecting D6-branes. We have also demonstrated that as $\Lambda_{\text{cond}}$ appears to be in the same range as that expected from the running of Sp $\Lambda_{\text{run}}^{\text{cond}}$ for up and down quarks (eqn’s 23,12), this indicates possible flavour independence. Also, from the $\Lambda^0$ dependence on the condensation scale, expected small differences in condensate values naturally may provide us with the fermion mass textures. It would be also important to further study this issue by calculating the relevant string amplitudes. We have also seen that a necessary condition for the existence of the fermion condensate giving a mass to the missing quark masses, is to simultaneously demand that the number of non-chiral states charged under the Sp(N) condensing gauge group may be fixed so that Sp(N) may have negative $\beta$-function.

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