We propose a physical mechanism for tuning the atom-atom interaction strength at ultra-low temperatures. In the presence of a dc electric field the interatomic potential is changed due to the effective dipole-dipole interaction between the polarized atoms. Detailed multi-channel scattering calculations reveal features never before discussed for ultra-cold atomic collisions. We demonstrate that optimal control of the effective atom-atom interactions can be achieved under reasonable laboratory conditions. Implications of this research on the physics of atomic Bose-Einstein condensation (BEC) and on the pursuit for atomic degenerate fermion gases will be discussed.

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The study of weakly interacting quantum gases has attracted significant attention since the initial success of Bose-Einstein condensation (BEC) [1]. Tremendous progress has been made over the last three years in both theory and experiment. One of the recent interests is the study of controlling the strength of atom-atom interaction. Several groups have discussed mechanisms for changing the scattering length of the atom-atom interaction using near resonant lasers [2], radio frequency fields [3], and Feschbach resonances induced by a magnetic field [4]. Indeed, very recently, Feschbach resonances have been observed by several experimental groups [5].

This letter concerns the physics of adjusting the effective low energy atomic interactions. We propose the use of an external dc electric field (dc-E) to influence the low energy atomic collisions by means of modifying the shape of the interaction potential between atoms. This letter is organized as follows: We start with a brief discussion of the effective interaction potential between two alkali-metal atoms in the presence of a dc-E, followed by an outline of the main results of multi-channel collision formalism for two polarized atoms at ultralow temperatures. Illustrative results are presented, using a typical model potential for the atom-atom interaction. For collisions between bosonic atoms, we show that: (i) the sign of the scattering length $a_{sc}$ may be determined by measuring the relative deviation of the total elastic cross section in the presence of a dc-E, (ii) it is possible to tune the value of $a_{sc}$ smoothly in a broad range, (iii) a dc-E can induce strong anisotropic interactions between the atoms, and (iv) a dc-E can induce zero energy resonances (these are shape resonances in contrast to Feschbach resonances recently observed [6]). For collisions between fermionic atoms we show that (v) the dc-E can induce strong interactions between atoms in spin symmetrized states even at zero temperature. We believe these results open the door for a new area of studies of quantum degenerate atomic gases with adjustable and anisotropic interactions.

In the usual treatment of the binary interaction between two spherically symmetric atoms in the ground state, the long-range interaction potential is given [in the London-van der Waals (LvW) formalism] by the following expression

$$V(R) = -\frac{C_6}{R^6} - \frac{C_8}{R^8} - \frac{C_{10}}{R^{10}} - \cdots,$$

where $C_6$, $C_8$, and $C_{10}$ are the dispersion coefficients, and $R$ is the internuclear distance. This is a “short-range” potential; therefore, the zero energy scattering is described essentially only by the S-wave scattering length, $a_{sc}$.

In the presence of a dc-E the spherical symmetry of the interacting atoms is distorted and consequently the long-range form of the interatomic potential [Eq. (1)] needs to be reconsidered. In the infinite separation limit, the ground state wave function of the atoms in a dc-E acquires a small P angular momentum component along the electric field, given by the dipole coupling between the S ground state and the P excited states of the unperturbed atom. To the leading order in the dc-E intensity $\mathcal{E}$, the LvW formalism generates an additional term,

$$V_E(R) = -\frac{C_E}{R^3}P_2(\cos \theta),$$

where $C_E = 2\mathcal{E}^2\alpha_A(0)\alpha_B(0)$ is the electric induced dipole interaction coefficient and $\alpha_A(0)$, $\alpha_B(0)$ are the static atomic dipole polarizabilities of atom A and B respectively. $P_2(\cdot)$ is the Legendre polynomial of order 2 and $\theta$ is the angle between the directions of the electric field and the internuclear axis.

We note that in a complete treatment of the long-range interactions, other terms induced by the dc-E are also present. The first order perturbation (in terms of the Coulomb interactions between the atomic charge distributions) generates an additional term proportional to $1/R^3$, and which is related to the quadrupole couplings between the P state components of the perturbed ground state of the atoms in a dc-E. This term is proportional to $\mathcal{E}^3$ and can be neglected for the $\mathcal{E}$ values of interest to us in this study (which are very weak in atomic units). The second order perturbation provides corrections to the dispersion coefficients $C_6$, $C_8$, and $C_{10}$ from Eq. (1), which are proportional to $\mathcal{E}^5$ (and higher powers of $\mathcal{E}$). These corrections are also neglected in the present analyses, again assuming a weak dc-E. Although quantitatively the values of the electric induced dipole
term from Eq. (2) is small (e.g. 100 kV/cm is equivalent to \(1.94401 \times 10^4\) a.u.), it provides a qualitatively different asymptotic behavior for the interaction potential (i.e. \(1/R^3\)) with significant implications for the scattering at very low energies. The complete long-range interatomic potential is therefore

\[ V(\vec{R}) = V_0(R) + V_E(R), \]

where \(V_0(R)\) is the usual long-range dispersion form Eq. (4) in the absence of dc-E.

The potential from Eq. (3) is anisotropic, and requires a multi-channel treatment of the scattering. The T-matrix elements, \(T_{lm}^{m'}\), can be extracted from the asymptotic conditions imposed on the partial wave channels. The total elastic cross section is given by

\[ \sigma_{B(F)} = 8\pi \sum_{l,m} \sum_{l',m'} |t_{lm}^{m'}|^2, \]

for bosons (B) and fermions (F) respectively, where \(t_{lm}^{m'} \equiv T_{lm}^{m'}/k\) are the reduced T-matrix elements.

The main analytical result obtained is this: the reduced T-matrix elements \(t_{lm}^{m'}\) are finite quantities in the limit of zero energy. This result may be intuitively understood as follows. If a potential approaches zero as \(1/R^n\) when \(R\) goes to infinity, then the phase shift \(\delta_l\) behaves in the limit of zero energy as \(k^{2l+1}\) if \(l < (n-3)/2\) and as \(k^{n-2}\) otherwise. Our problem is more complex since it involves a system of coupled equations. However, the coupling terms are essentially proportional to \(1/R^3\). Thus, the effective potential generated by the couplings, for each channel, behaves in the limit of large \(R\) as \(1/R^6\). Then, the character of the total effective potential for each partial wave equation is decided by the diagonal part of the potential. For \(l = 0\) the effective potential behaves as \(1/R^6\) and so \(\delta_0 \sim k\). For \(l \neq 0\) the effective potentials behave as \(1/R^3\) [generated by the diagonal part of \(V(E)\)] and so \(\delta_l \sim k\). From these assertions we infer that in our case all \(t_{lm}^{m'}\) are finite quantities in the limit of zero energy. A more rigorous proof of these assertions will be presented elsewhere.

Thus, the electric field induced part of the potential, Eq. (2), has a “quasi long-range” character in the sense that it generates a “short-range” contribution to the effective potential of the partial wave channel \(l = 0\) while it generates a “long-range” contribution (proportional to \(1/R^3\)) for all other partial wave channels.

To illustrate our results we present numerical results for a model interaction potential. This model contains all the features present in a real potential curve and it has been studied in the context of scattering length computations. Moreover, by slightly changing the cutoff radius \(R_c\), this model generates a broad range of values for the scattering length. We use it to simulate qualitatively the results which may be obtained for actual ground state potential curves of alkali-metal dimers.

Figure 1 presents the logarithm of \(\sigma_B\) in a.u. as a function of the electric field for six different potentials (defined by six different \(R_c\) values: 23.226, 23.171, 23.155, 23.146, 23.138, and 23.1245). The curves are labeled by the value of the corresponding scattering length in the absence of the external field. Since \(\sigma_B\) has a very weak dependency on the collision energy in the domain of sub-mK temperatures (regardless of the value of the electric field), these results may be interpreted as values at zero collision energy. Note that curves corresponding to potentials with a small scattering length contain a resonance peak at certain values of the electric field. The smaller the value of the scattering length, the smaller the value of the electric field where the resonance occurs. We note that for small values of the electric field \(\sigma_B\) tends to become smaller for potentials with a positive scattering length and to become larger for potentials with a negative scattering length. This behavior may be explained by the fact that the dc-E tends to lower the effective interatomic potential curves, eventually allowing the transformation of a virtual state into a new bound state. During this process the value of \(\sigma_{00}\) will increase with the electric field, which in turn yields a decrease of \(\sigma_B\) for potentials with an initial positive scattering length (negative \(-\sigma_{00}\)) and an increase of \(\sigma_B\) for potentials with an initial negative scattering length (positive \(-\sigma_{00}\)). Thus, by measuring the relative deviation of the \(\sigma_B\) in the presence of a dc-E, one may determine the sign of the scattering length. Given the extreme sensitivity of the dependence of \(\sigma_B\) on the interatomic potential as well as on the value of the electric field, one may be able to refine interatomic potentials by analyzing the values of the \(\sigma_B\) measured at different electric field intensities. One way to investigate such properties is to study the macroscopic effects of a dc electric field on degenerate atomic gases at ultralow temperatures.

Figure 2 presents \(\sigma_B\) in units of \(8\pi a_c^2\), i.e. \(\sigma_B = \sigma_B(E)/a_B(0)\) (solid line) and the asymmetry parameter \(\delta = 8\pi |\sigma_{00}|^2/\sigma_B\) (dashed line) as a function of the electric field for the potential with scattering length \(a_{\text{sc}} = 2470\) a.u. The asymmetry parameter \(\delta\) has a value close to 1 when \(\sigma_B\) is dominated by the S-wave contribution and has a value close to zero when \(\sigma_B\) is dominated by other partial wave contributions. In this case \(\delta\) decreases smoothly with the increase of the electric field. Moreover, \(\delta\) has values close to 1 in the region of maximum variation of \(\sigma_B\). Thus the effective scattering length, \(a_{\text{eff}} = -t_{00}^0\) is the only parameter needed to characterize the collision process. This fact suggests that for cases where the scattering length has a large positive value one may obtain a smooth variation of the \(\sigma_B\) at zero temperature by varying the external electric field. We envisage interesting applications in the study of atomic BEC by controlling the strength of the atomic interaction.

Figure 3 presents the values of the asymmetry parameter \(\delta\) as a function of the electric field for the case of \(a_{\text{sc}} = 32\) a.u. The zero of \(\delta\) is associated with the zero of \(t_{00}^0\) when it changes sign from minus to plus, which
in turn is the result of the balance between the repulsive effect of the last bound state and the attractive effect of the first virtual state. This occurs because when the last bound state is closer to the threshold than the first virtual state the sign of $a_{\text{sc}}$ is positive and when the first virtual state is closer to the threshold than the last bound state the sign of $a_{\text{sc}}$ is negative. In this region the scattering process has a strong anisotropic character. This result suggests that, by properly choosing the atomic species (with a moderate positive value for the scattering length) and the value of the electric field, one may study in a controlled fashion two particular interesting regimes of the atomic BEC. One is the atomic BEC with anisotropic atomic interactions and the other is the study of the collapse of the BEC when the value of the effective scattering length is changed from positive to negative.

In Fig. 4 we present the values of the reduced T-matrix element $t_{00}^{l_{\sigma}}$ (solid line) as a function of the electric field for the case of $a_{\text{sc}} = -2121$ a.u. As the figure suggests, $t_{00}^{l_{\sigma}}$ has an initial positive value and is increasing with the electric field until a new bound state is supported by the molecular system (i.e., a virtual state has been transformed into a bound state) and a jump to $-\infty$ occurs. Moreover, the asymmetry parameter $\delta$ (dashed line) has a value close to 1 in the region where the resonance occurs, which indicates that this resonance is an S-wave resonance. This result suggests that by properly choosing the value of the electric field one may study in a controlled fashion the zero energy resonances and their influence on the atomic BEC. Also it appears possible to obtain stable condensates for atomic species with a negative scattering length by tuning the effective scattering length to positive values.

Figure 5 presents the logarithm of $\sigma_F$ in a.u. for two colliding fermionic atoms as a function of the electric field. We note that the values of $\sigma_F$ are also insensitive to the collision energy in the domain of sub-mK temperatures. In the fermionic case only the T-matrix elements corresponding to odd partial wave channels for $l \geq 1$ contribute to the final expression of $\sigma_F$. Thus, the structure of the $\sigma_F$ from Fig. 5 is mainly a result of competition between the centrifugal potential $l(l+1)/R^2$ and the electric field induced part of the potential $V_E$, Eq. (2). No major effects are expected to come from the small details of the interatomic potential $V_0(R)$. The numerical values obtained for $\sigma_F$ are the same (in three digits) for all potentials discussed in the bosonic case. We also note that the increase in the value of $\sigma_F$ is significant. For a potential with $a_{\text{sc}} = 32$ a.u., the zero energy $\sigma_B$ in the absence of the external field is $2.574 \times 10^4$ a.u. This value is obtained for $\sigma_F$ (see Fig. 5) at 298 kV/cm. Thus, the value of $\sigma_F$ in the presence of a dc-E may be as large as the $\sigma_B$ for an equivalent bosonic system in the absence of dc-E. The fact that in the presence of a dc-E $\sigma_F$ is not zero is a direct consequence of the modified collision asymptotes: the reduced T-matrix elements are finite and non-zero quantities in the limit of zero energy. This result strongly indicates the possibility of studying interacting ultracold atomic fermion gases in a controlled field. The possibility of evaporative cooling of an atomic fermion gas in the presence of a dc-E is also suggested. It also opens the exciting new possibility of P-wave pairing of magnetically trapped fermionic atoms.

In conclusion, we have studied atomic collisions at ultralow temperatures in the presence of a dc electric field. We found interesting low energy behaviors strongly encouraging new directions of study for ultracold boson and fermion degenerate gases with controlled atomic interactions. We showed that for a bosonic system: (i) study of the total elastic cross section for small values of the electric field allows for determination of the sign of the scattering length, (ii) the effective scattering length may be smoothly changed in a broad range of values, (iii) for certain values of the parameters, the atomic interaction displays a strong anisotropic character, and (iv) in certain cases, the zero energy resonances are accessible under realistic experimental conditions. Similar possibilities exist for the control of the fermion interactions at ultralow temperatures. We have shown (v) that the interatomic interaction strength may be boosted to significantly large values by an external electric field.

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FIG. 1. The logarithm of the total elastic scattering cross section for two colliding boson-like atoms $\sigma_B$ as a function of the electric field for six different interatomic potentials. The curves are labeled by the value of the scattering length in the absence of the external field.

FIG. 2. Total elastic scattering cross section in units of $8\pi a_{sc}^2$, $\sigma_b$ (solid line), and the asymmetry parameter $\delta$ (dashed line) as a functions of the electric field for the potential with a scattering length $a_{sc} = 2470$ a.u.

FIG. 3. The values of the asymmetry parameter $\delta$ as a function of the electric field for the potential with a scattering length $a_{sc} = 32$ a.u.

FIG. 4. The values of the $t^{00}_{00}$ reduced T-matrix element (solid line) and the asymmetry parameter $\delta$ (dashed line) as functions of the electric field for the potential with a scattering length $a_{sc} = -2121$ a.u.

FIG. 5. The logarithm of the total elastic scattering cross section $\sigma_F$ for two colliding fermion-like atoms as a function of the electric field.
$a_{sc} = 2470$
\[ a_{sc} = 32 \]
Electric field (100kV/cm)

$a_{sc} = -2121$
\[ \log(\sigma_F) \]

Electric field (100kV/cm)