Study on Seismic Stability of Loess Landslide Based on Fuzzy Comprehensive Evaluation

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Abstract. Based on the characteristics of loess landslide under earthquake action, in order to more accurately and reasonably evaluate the stability of loess landslides under earthquake action, considering the systematicness and scientificity, a reasonable loess landslides under earthquake action stability evaluation index system has been set up, and a fuzzy comprehension evaluation model for the stability of the loess landslide under earthquake action was established to evaluate its stability. This model has been applied to Sanjiadi landslide in Gansu province to analyze the stability of loess landslides under earthquake action; the stability coefficient of the Sanjiadi landslide under earthquake action was calculated by use of the pseudo-static method. Both methods indicated that the Sanjiadi landslide under earthquake action was in an unstable state, and the results are reasonable and confirm to reality.

1. Introduction

The geological environment of loess landslide is a complex system, which includes both natural geological conditions and human activities. Its stability is affected by many factors. A large number of mechanical tests and engineering practices have shown that the boundaries of the landslide itself and its stability have a certain degree of ambiguity, and it is difficult to use a classical mathematical model to measure uniformly. Therefore, using fuzzy analysis method to analyze the stability of landslide has certain advantages. Since Zadeh proposed the concept of fuzzy sets in the 1960s, fuzzy mathematics has gradually applied to various fields of geotechnical engineering. Fuzzy mathematics transforms various uncertain geological data into fuzzy language, and expresses the membership degree of each factor in the form of membership function to comprehensively evaluate the landslide stability. The fuzzy comprehensive evaluation is to apply the fuzzy transformation principle and the principle of maximum membership degree, and comprehensively consider the relevant factors of the evaluated things or their attributes, and then carry out grade or category evaluation, it has the characteristics of judging the multi-factor and multi-level complex problems, and a series of results have been achieved [1-4]. However, the existing research results are basically based on the fuzzy evaluation of slope stability, but the research on the stability of loess landslide is rare, and the factors considered are not comprehensive enough. Therefore, this paper selects five categories of 16 influencing factors to establish a fuzzy comprehensive evaluation model for loess landslide stability under earthquake action. The model is used to evaluate the seismic stability of Sanjiadi landslide and compare with the calculation results of quasi-static method.

2. Fuzzy comprehensive evaluation principle
The so-called fuzzy comprehensive evaluation is to evaluate the things or phenomena affected by many factors. If the evaluation process involves fuzzy factors, it is called fuzzy comprehensive evaluation. An important mathematical means to achieve fuzzy comprehensive evaluation is fuzzy transformation. The fuzzy comprehensive evaluation method is formed by using fuzzy transformation to solve the practical comprehensive evaluation problem. The principle is as follows [5]:

There are two domains: Factor set \( U = \{u_1, u_2, \cdots, u_n\} \) (\( u_i \) is judging factor); Judging set \( V = \{v_1, v_2, \cdots, v_m\} \) (\( v_i \) is the rating level).

If each element \( u_i \) in \( U \) is judged separately \( f(u_i) \). It can be seen as a fuzzy mapping from \( U \) to \( V \). By fuzzy mapping \( f \), we can derive the fuzzy matrix \( R \):

\[
R = (r_{ij})_{n \times m} \quad 0 \leq r_{ij} \leq 1
\]  

(1)

In the formula: \( R \) is a single factor evaluation matrix from \( U \) to \( V \). If there is a fuzzy subset \( M = \{m_1, m_2, \cdots, m_n\} \) on a set \( U \), \( M \) is represented by a vector, and have:

\[
\sum_{j=1}^{m} a_j = 1
\]  

(2)

In the formula: \( x \) is the weight of the \( i \) factor, Then we can uniquely determine a fuzzy transformation \( N \) from \( U \) to \( V \), \( N \) is the fuzzy synthesis result:

\[
N = M \cdot R
\]  

(3)

\( B = \{b_1, b_2, \cdots, b_m\} \), \( b_j (j = 1, 2, \cdots, m) \) reflects the membership of the \( j \) criterion \( v_j \) and the fuzzy set \( N \).

According to the principle of maximum membership, choose the largest \( \max(n_1, n_2, \cdots, n_m) \) in \( N \). The corresponding level is the final result of the fuzzy comprehensive evaluation.

3. Fuzzy comprehensive evaluation model for stability of loess landslide under earthquake

3.1 Determine evaluation factors and grading standards

Based on a detailed field survey of a large number of loess landslides in the study area[6], combining the existing research results at home and abroad, and taking into account the actual project, we finally select 5 categories of 16 influencing factors. The internal factors mainly include two classification indicators: topography and geological conditions. Topography includes five basic indicators: average slope, slope height, gully development, slope deformation and vegetation cover. Geological conditions include six basic indicators: the friction angle of the soil in the sliding zone, the cohesive force of the slip zone soil, the relationship between the occurrence of the mudstone under the loess and the slope foot, the lithology grade of the underlying mudstone, the shape of the sliding surface, and the water permeability of the landslide. The external factors mainly include three classification indicators for earthquake, rainfall and other factors. Rainfall includes two basic indicators of rainfall intensity and annual rainfall, other factors include human factors and two basic indicators of slope construction load, earthquake has seismic intensity one basic indicator.

After the classification indicator and the basic indicator number, the 16 basic indicators affecting the seismic stability of loess landslide constitute 5 first-level factors:

\[
\begin{align*}
A_1 &= \{B_1\} \\
A_2 &= \{C_1, C_2, C_3, C_4, C_5\} \\
A_3 &= \{D_1, D_2\} \\
A_4 &= \{E_1, E_2, E_3, E_4, E_5, E_6\} \\
A_5 &= \{F_1, F_2\}
\end{align*}
\]  

(4)

Grouping 5 classification indicators into a secondary factor set:

\[
U = \{A_1, A_2, A_3, A_4, A_5\}
\]  

(5)
Dividing the stability of loess landslide into 5 levels, Stable (Class I), More stable (Level II), Basically stable (level III), Less stable (level IV), Unstable (V grade). And get the evaluation set V:

\[ V = \{V_1, V_2, V_3, V_4, V_5\} \quad (6) \]

According to relevant literature statistics, combined with the actual engineering analysis, the grading standard of single factor stability evaluation index is obtained.

### 3.2 Determination of membership function

In fuzzy mathematics, the fuzzy degree of the weight of things is described by membership degree. The determination of the membership function of each evaluation factor is the core problem of fuzzy mathematics, and it is also the key to the accuracy of fuzzy comprehensive evaluation results. The factors can be divided into two types: discrete factors (qualitative indicators) and continuous factors (quantitative indicators). Among them: Discrete factors use Delphi method to determine membership. According to Sun Jie the research results on the characteristics of each membership function and its distribution law. By comprehensively considering the distribution characteristics of various factors affecting the stability of loess landslide, the membership function of continuous factors is determined by the "half-trapezoid" distribution.

The discrete evaluation indicators include gully development, vegetation cover, slope deformation, relationship between loess and muddy slope, loess lithology grade, slip surface shape, landslide body permeability, Human factors and distribution of slope buildings. The degree of membership is determined by the Delphi method, and its value is shown in Table 1.

**Table 1. Discrete indicator membership degree table**

| level | Affiliation value |
|-------|-------------------|
|       | C3    | C4    | C5    | E3    | E4    | E5    | E6    | F1    | F2    |
| I     | 0.50  | 0.65  | 0.45  | 0.6   | 0.55  | 0.55  | 0.50  | 0.45  | 0.55  |
|       | 0.25  | 0.25  | 0.25  | 0.25  | 0.30  | 0.25  | 0.25  | 0.25  | 0.25  |
|       | 0.07  | 0.00  | 0.10  | 0.05  | 0.05  | 0.05  | 0.07  | 0.10  | 0.05  |
|       | 0.03  | 0.00  | 0.05  | 0.00  | 0.00  | 0.00  | 0.03  | 0.05  | 0.00  |
| II    | 0.25  | 0.30  | 0.20  | 0.30  | 0.20  | 0.20  | 0.25  | 0.20  | 0.20  |
|       | 0.45  | 0.40  | 0.40  | 0.45  | 0.40  | 0.40  | 0.45  | 0.40  | 0.40  |
|       | 0.15  | 0.15  | 0.15  | 0.15  | 0.15  | 0.15  | 0.15  | 0.15  | 0.15  |
|       | 0.10  | 0.10  | 0.12  | 0.07  | 0.10  | 0.10  | 0.10  | 0.12  | 0.10  |
|       | 0.05  | 0.08  | 0.03  | 0.05  | 0.05  | 0.05  | 0.05  | 0.08  | 0.05  |
| III   | 0.10  | 0.20  | 0.10  | 0.08  | 0.08  | 0.08  | 0.10  | 0.10  | 0.08  |
|       | 0.20  | 0.20  | 0.20  | 0.20  | 0.17  | 0.17  | 0.20  | 0.20  | 0.17  |
|       | 0.40  | 0.20  | 0.38  | 0.40  | 0.40  | 0.40  | 0.40  | 0.38  | 0.40  |
|       | 0.20  | 0.20  | 0.20  | 0.23  | 0.23  | 0.23  | 0.20  | 0.20  | 0.23  |
|       | 0.10  | 0.20  | 0.12  | 0.12  | 0.12  | 0.12  | 0.10  | 0.12  | 0.12  |
| IV    | 0.05  | 0.05  | 0.08  | 0.03  | 0.05  | 0.05  | 0.05  | 0.08  | 0.05  |
|       | 0.10  | 0.10  | 0.12  | 0.07  | 0.10  | 0.10  | 0.10  | 0.12  | 0.10  |
|       | 0.15  | 0.15  | 0.20  | 0.15  | 0.25  | 0.25  | 0.15  | 0.20  | 0.25  |
|       | 0.45  | 0.40  | 0.40  | 0.45  | 0.40  | 0.40  | 0.45  | 0.40  | 0.40  |
|       | 0.25  | 0.30  | 0.20  | 0.30  | 0.20  | 0.20  | 0.25  | 0.20  | 0.20  |
| V     | 0.03  | 0.00  | 0.05  | 0.00  | 0.00  | 0.00  | 0.03  | 0.05  | 0.00  |
|       | 0.07  | 0.00  | 0.10  | 0.05  | 0.05  | 0.05  | 0.05  | 0.07  | 0.10  | 0.05  |
|       | 0.15  | 0.10  | 0.15  | 0.10  | 0.15  | 0.10  | 0.15  | 0.15  | 0.15  |
|       | 0.25  | 0.25  | 0.25  | 0.25  | 0.30  | 0.25  | 0.25  | 0.25  | 0.25  |
|       | 0.50  | 0.65  | 0.45  | 0.60  | 0.55  | 0.55  | 0.50  | 0.45  | 0.55  |
For the continuous indicators, there are seismic intensity, flat slope, natural slope height, rainfall intensity, friction angle inside the sliding zone, etc. Using the "half-trapezoid" distribution, the functional relationship between the membership degree and the index value is established, that is the membership function.

3.3 Establish a single factor evaluation matrix
According to the actual value of the evaluation index, the appropriate membership function is selected to obtain the corresponding membership degree of each evaluation index to each evaluation level. Fuzzy evaluation of the basic evaluation indicators, and obtain a first-order fuzzy relation matrix \( R \) reflecting the fuzzy relationship between \( U \) and \( V \):

\[
R = \begin{pmatrix}
    r_{11} & r_{12} & \cdots & r_{1m} \\
    r_{21} & r_{22} & \cdots & r_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{n1} & r_{n2} & \cdots & r_{nm}
\end{pmatrix}
\]  

\( R \)

In the formula: The nth line reflects the possibility that each factor of the evaluated object takes the nth level in the evaluation set; The mth column reflects the mth factor of the evaluated object corresponding to the membership of each level in the evaluation set.

3.4 Determine the weight of the evaluation model
The weight of the evaluation index has a direct impact on the fuzzy comprehensive evaluation results, and must be objectively determined according to the importance of each indicator and its impact on stability. This paper uses AHP to determine the weight value of each evaluation index. It is characterized by a high degree of logic, system, simplicity and practicability. It combines the advantages of expert scoring legal analysis, and uses appropriate mathematical models for quantitative analysis. It is more suitable for both qualitative and quantitative indicators [6].

This paper compares the importance of the factors in each layer with respect to the previous level of objectives, and constructs the corresponding judgment matrix. Taking the weights of the basic indicators \( \{C_1, C_2, C_3, C_4, C_5\} \) as an example, constructing the judgment matrix \( A_2 \). Calculate its maximum eigenvalue and corresponding eigenvectors, and perform judgment matrix consistency test, When the matrix random consistency ratio \( CR<0.1 \) is judged, the judgment matrix is considered to have satisfactory consistency, indicating that the weight distribution is reasonable.

3.4.1 Construction judgment matrix

\[
A = \begin{pmatrix}
    1 & 3 & 2 & 2 & 5 \\
    1/3 & 1 & 3 & 2 & 3 \\
    1/2 & 1/3 & 1 & 2 & 2 \\
    1/2 & 1/2 & 1/2 & 1 & 2 \\
    1/5 & 1/3 & 1/2 & 1/3 & 1 \\
\end{pmatrix}
\]  

\( A \)

3.4.2 Calculate the maximum eigenvalue and eigenvector
Maximum eigenvalue \( \lambda_{\text{max}} = 5.172 \), The normalized eigenvector is \( (0.384, 0.252, 0.166, 0.131, 0.067) \)

3.4.3 Consistency test
Judging the matrix order \( n = 5 \), check the table to get the corresponding consistency index \( RI = 1.12 \), \( CR = 0.043 <0.1 \) meet the consistency test. Explain that the weight distribution is reasonable, After two levels of analysis, the weights of each evaluation index are shown in Table 2.
Table 2. Weighted allocation table for each evaluation index

| Classification indicator | Basic indicator                      | Weights | Classification indicator N | Basic indicator M |
|--------------------------|--------------------------------------|---------|---------------------------|-------------------|
| Earthquake               | Seismic intensity                    | 0.106   | 1.000                     |                   |
|                          | Average slope                        | 0.384   |                           |                   |
|                          | Slope height                         | 0.252   |                           |                   |
| Topography               | Gully development                    | 0.233   | 0.166                     | 0.131             |
|                          | Slope deformation                    | 0.252   |                           |                   |
|                          | Vegetation cover                     | 0.067   |                           |                   |
| Rainfall                 | Rainfall intensity                   | 0.155   | 0.654                     | 0.346             |
|                          | Average annual rainfall              | 0.654   |                           |                   |
| Geological condition     | Slipband internal friction           | 0.445   | 0.350                     | 0.209             |
|                          | Slippery soil cohesion               | 0.209   |                           |                   |
|                          | Direction of mudstone under loess    | 0.445   | 0.141                     | 0.102             |
|                          | Underlying mudstone lithology        | 0.141   |                           |                   |
|                          | Landslide permeability               | 0.108   |                           |                   |
|                          | Sliding surface shape                | 0.108   |                           |                   |
| Other factors            | Human Factors                        | 0.06    | 0.327                     | 0.673             |
|                          | Slope building distribution          | 0.327   |                           |                   |

3.5 Fuzzy comprehensive evaluation

The first-level comprehensive evaluation is obtained by the composite operation of the first-order fuzzy relation matrix R and the basic index weight matrix M.

\[ V' = R \times M = (\varphi'_1, \varphi'_2, \varphi'_3, \varphi'_4, \varphi'_5) \]  

\[ V' \] is the fuzzy relation matrix of the second-level evaluation and the second-order weight matrix N operation, that is, the maximum membership degree is obtained.

\[ V = V' \times N = \{V_1, V_2, V_3, V_4, V_5\} \]

In the formula: \( V_i \) is the membership degree of the factor \( i \) in the evaluation set \( V \), Principle of maximum membership, \( V_i = \max \{V_1, V_2, V_3, V_4, V_5\} \), The level corresponding to \( V_i \) is the stability level of the slope.

4. Project example

The Sanjiadi Land landslide is a loess-mudstone bedding landslide with a slope height of about 290m and an average slope of 13°. According to the "China Earthquake Parameter Zoning Map" (GB110386-2001) implemented in June 2008, the seismic intensity of the landslide area is VIII degrees. The landslide was affected by the “5.12” earthquake, and the front edge of the old landslide partially slipped, and the rear part produced a through crack. The rim of the landslide is eroded by the debris flow of Chen Li. The annual rainfall in the landslide area is small, but it is mainly concentrated in May to September. The frequency of heavy rain is high and the intensity is high. The landslide slip zone soil cohesion is 19 kPa and the internal friction angle is 12°.

Based on the fuzzy comprehensive evaluation model of loess landslide stability under the above earthquakes, the seismic stability of Sanjiadi landslide is graded and evaluated. The evaluation result is the degree of membership of the three landslides under the earthquake is \( V = (0.11, 0.074, 0.155, 0.358, 0.297) \), according to the principle of maximum membership, \( V_i = \max \{V_1, V_2, V_3, V_4, V_5\} = 0.358 \). The stability of the landslide is Grade IV, the Sanjiadi landslide is in a relatively unstable state under the action of earthquake.
Comparing the results of fuzzy comprehensive evaluation method for the stability of Sanjiadi landslides under earthquake action with the results of quasi-static method. The analysis result of fuzzy comprehensive evaluation method is that the stability of Sanjiadi Land landslide belongs to the IV grade, that is, the landslide is in a relatively unstable state under the action of earthquake. Compared with the landslide stability results calculated by quasi-static method, the practical feasibility of the fuzzy comprehensive evaluation model for loess landslide stability under earthquake action is verified.

5. conclusion
Fuzzy comprehensive evaluation is a very effective multi-factor decision method for comprehensive evaluation of things affected by many factors, suitable for landslides such as projects that control their stability by multiple factors. The fuzzy comprehensive evaluation method can transform the original fuzzy and subjective qualitative assessment into quantitative evaluation. The idea is clear and the evaluation result is intuitive, which is an effective method to improve the scientific results.

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