Exact Results in Renormalization of Softly Broken SUSY Gauge Theories

D.I. Kazakov

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141 980 Dubna, Moscow Region, RUSSIA, E-mail: kazakovd@thsun1.jinr.ru

Abstract

It is shown that softly broken theory is equivalent to a rigid theory in external spurion superfield. The singular part of effective action in a broken theory follows from a rigid one by a simple redefinition of the couplings. This gives an explicit relation between the soft and rigid couplings renormalizations. As an illustration the renormalization group functions in the MSSM have been calculated. The method opens a possibility to construct a totally all loop finite N=1 SUSY gauge theory, including the soft SUSY breaking terms. Explicit relations between the soft terms, which lead to a completely finite theory in any loop order, are given.

1 Talk presented at XXIX International Conference on High Energy Physics (Vancouver, Canada, July, 1998) and at XI International Conference "Problems of Quantum Field Theory" (Dubna, Russia, July, 1998)
1 Introduction

The gauge theory with softly broken supersymmetry has been widely studied. A powerful method which keeps supersymmetry manifest is the supergraph technique [1, 2]. It is also applicable to softly broken SUSY models by using the "spurion" external superfields [3, 4, 5]. As has been shown by Yamada [6] with the help of the spurion method the calculation of the $\beta$ functions of soft SUSY-breaking terms is a much simpler task than in the component approach.

In a recent paper [7] we have developed a modification of the spurion technique in gauge theories and have formulated the Feynman rules. We have shown that the ultraviolet divergent parts of the Green functions of a softly broken SUSY gauge theory are proportional to those of a rigid theory with the spurion fields factorized.

The main idea is that a softly broken supersymmetric gauge theory can be considered as a rigid SUSY theory imbedded into external space-time independent superfield, so that all couplings and masses become external superfields.

The main Statement:

**Softly Broken SUSY Theory $\approx$ Rigid SUSY Theory in External Field**

The Coupling $g \Rightarrow$ External Superfield $\Phi_0$

Consequence: Singular part of effective action depends on external superfield, but not on its derivatives:

$$S_{\text{eff}}^{\text{sing}}(g) \Rightarrow S_{\text{eff}}^{\text{sing}}(\Phi_0, D^2\Phi_0, \bar{D}^2\Phi_0, D^2\bar{D}^2\Phi_0)$$

This approach to a softly broken supersymmetric theory allows us to use remarkable mathematical properties of $N = 1$ SUSY theories such as non-renormalization theorems, cancellation of quadratic divergences, etc. We show that the renormalization procedure in a softly broken SUSY gauge theory can be performed in exactly the same way as in a rigid theory with the renormalization constants being external superfields. They are related to the corresponding renormalization constants of a rigid theory by the coupling constants redefinition. This allows us to find explicit relations between the renormalizations of soft and rigid couplings.

Throughout the paper we assume the existence of some gauge and SUSY invariant regularization and a minimal subtraction procedure.

As an application of the above mentioned relations we consider a possibility of constructing totally finite supersymmetric theories including the soft breaking terms. This problem has already been discussed several times [8, 9]. We show that by choosing the soft terms in a proper way one can reach complete all loop finiteness. Moreover, there is no new fine-tuning. The soft terms are fine-tuned in exactly the same way as the corresponding Yukawa couplings [10].
2 Softly Broken $N = 1$ SUSY Gauge Theory

Consider pure $N = 1$ SUSY Yang-Mills theory with a simple gauge group. The Lagrangian of a rigid theory is given by

$$L_{\text{rigid}} = \int d^2 \theta \frac{1}{4g^2} \text{Tr} W^\alpha W_\alpha + \int d^2 \bar{\theta} \frac{1}{4g^2} \text{Tr} \bar{W}^\alpha \bar{W}_\alpha.$$ (1)

To perform a soft SUSY breaking, one can introduce a gaugino mass term

$$- L_{\text{soft-breaking}} = \frac{m_A}{2} \lambda \lambda + \frac{m_A}{2} \bar{\lambda} \bar{\lambda},$$ (2)

where $\lambda$ is the gaugino field. To rewrite it in terms of superfields, let us introduce an external spurion superfield $\eta = \theta^2$, where $\theta$ is a Grassmannian parameter. The softly broken Lagrangian can now be written as

$$L_{\text{soft}} = \int d^2 \theta \frac{1}{4g^2} (1 - 2 \mu \theta^2) \text{Tr} W^\alpha W_\alpha + \int d^2 \bar{\theta} \frac{1}{4g^2} (1 - 2 \bar{\mu} \bar{\theta}^2) \text{Tr} \bar{W}^\alpha \bar{W}_\alpha.$$ (3)

In terms of component fields the interaction with external spurion superfield leads to a gaugino mass equal to $m_A = \mu$, while the gauge field remains massless. This external chiral superfield can be considered as a vacuum expectation value of a dilaton superfield emerging from supergravity, however, this is not relevant to further consideration.

As has been shown in [7] the Feynman rules corresponding to the Lagrangian needed for the calculation of the singular part of effective action are the same as in a rigid theory with the substitution:

$$g^2 \rightarrow \tilde{g}^2 = g^2 \left(1 + \mu \theta^2 + \mu \bar{\theta}^2 + 2 \mu^2 \theta^2 \bar{\theta}^2\right).$$ (4)

Then the vector propagator in a softly broken theory is:

$$\langle V(x_1, \theta_1) V(x_2, \theta_2) \rangle_{\text{soft}} = \frac{\tilde{g}^2}{g^2} \langle V(x_1, \theta_1) V(x_2, \theta_2) \rangle_{\text{rigid}} + \text{irrel. terms},$$ (5)

where by irrelevant terms we mean the ones decreasing faster than $1/p^2$ for large $p^2$. The same is true for the ghost fields

$$\langle G(z_1) \bar{G}(z_2) \rangle_{\text{soft}} = \left(\frac{\tilde{g}^2}{g^2}\right) \langle G(z_1) \bar{G}(z_2) \rangle_{\text{rigid}} + \text{irrel. terms},$$ (6)

where $G$ stands for any ghost superfield.

Hence, to perform the analysis of the divergent part of the diagrams in a soft theory, one has to use the same propagators as in a rigid theory multiplied by the factor $\tilde{g}^2/g^2$. It is also obviously true for any vertex of the ghost-vector interactions of the softly broken theory. Each vertex of this type has to be multiplied by the inverse factor $g^2/\tilde{g}^2$. The situation is less obvious with the vector vertices, however it happens to be true in this case as well.
Thus, we see that any element of the Feynman rules for a softly broken theory coincides with the corresponding element of a rigid theory multiplied by a common factor which is a polynomial in the grassmann coordinates.

Consider now a rigid SUSY gauge theory with chiral matter. The Lagrangian written in terms of superfields looks like

\[
\mathcal{L}_{\text{rigid}} = \int d^2 \theta d^2 \bar{\theta} \, \Phi^i (e^V)_i^j \Phi_j + \int d^2 \theta \, \mathcal{W} + \int d^2 \bar{\theta} \, \bar{\mathcal{W}},
\]

where the superpotential \( \mathcal{W} \) in a general form is

\[
\mathcal{W} = \frac{1}{6} \lambda^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} M^{ij} \Phi_i \Phi_j.
\]

The SUSY breaking terms which satisfy the requirement of ”softness” can be written as

\[
- \mathcal{L}_{\text{soft-breaking}} = \left[ \frac{1}{6} A^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} B^{ij} \phi_i \phi_j + \text{h.c.} \right] + (m^2)^i_j \phi^*_i \phi^j,
\]

Like in the case of a pure gauge theory the soft terms can be written down in terms of superfields by using the external spurion field. The full Lagrangian for the softly broken theory can be written as

\[
\mathcal{L}_{\text{soft}} = \int d^2 \theta d^2 \bar{\theta} \, \Phi^i (\delta^i_k - (m^2)^i_k \eta \bar{\eta}) (e^V)_k^j \Phi_j + \int d^2 \theta \, \frac{1}{6} (\lambda^{ijk} - A^{ijk} \eta) \Phi_i \Phi_j \Phi_k + \frac{1}{2} (M^{ij} - B^{ij} \eta) \Phi_i \Phi_j + \text{h.c.}
\]

The Lagrangian allows one to write down the Feynman rules for the matter field propagators and vertices in a soft theory.

\[
\langle \Phi(z_1), \Phi(z_2) \rangle_{\text{soft}} = (\delta^i_k + \frac{1}{2} (m^2)^i_k \bar{\eta} \eta) \langle \Phi(z_1), \Phi(z_2) \rangle_{\text{rigid}} (\delta^i_l + \frac{1}{2} (m^2)^i_l \bar{\eta} \eta) + \text{t.c.}
\]

The vector-matter vertices, according to eq.(10), gain the factor \((\delta^i_l - (m^2)^i_l \bar{\eta} \eta)\) so that if in a diagram one has an equal number of chiral propagators and vector-matter vertices the spurion factors cancel.

The chiral vertices of a soft theory, as it follows from eq.(10), are the same as in a rigid theory with the Yukawa couplings being replaced by

\[
\lambda^{ijk} \rightarrow \lambda^{ijk} - A^{ijk} \eta, \quad \bar{\lambda}^{ijk} \rightarrow \bar{\lambda}^{ijk} - \bar{A}^{ijk} \bar{\eta}.
\]

The structure of the UV counterterms in chiral vertices is similar to that of the vector vertices, but is simpler due to the absence of the covariant derivatives on external lines. Effectively the corrections to the propagator may be associated with the chiral vertices, which allows one to reduce all soft term corrections to the modification of the couplings.
3 Renormalization of Soft versus Rigid Theory: the General Case

The external field construction described above allows one to write down the renormalization of soft terms starting from the known renormalization of a rigid theory without any new diagram calculation. The following statement is valid [7]:

**Statement 1** Let a rigid theory \((\mathcal{L}, S)\) be renormalized via introduction of the renormalization constants \(Z_i\), defined within some minimal subtraction massless scheme. Then, a softly broken theory \((\mathcal{L}, \mathcal{S})\) is renormalized via introduction of the renormalization superfields \(\tilde{Z}_i\) which are related to \(Z_i\) by the coupling constants redefinition

\[
\tilde{Z}_i(g^2, \lambda, \bar{\lambda}) = Z_i(\tilde{g}^2, \tilde{\lambda}, \tilde{\bar{\lambda}}),
\]

where the redefined couplings are \((\eta = g^2, \bar{\eta} = \bar{g}^2)\)

\[
\tilde{g}^2 = g^2(1 + \mu \eta + \bar{\mu} \bar{\eta} + 2 \mu \bar{\mu} \eta \bar{\eta}),
\]

\[
\tilde{\lambda}^{ijk} = \lambda^{ijk} - A^{ijk} \eta + \frac{1}{2} (\lambda^{njk}(m^2)^i_n + \lambda^{ink}(m^2)^j_n + \lambda^{ijn}(m^2)^k_n) \eta \bar{\eta},
\]

\[
\tilde{\bar{\lambda}}^{ijk} = \bar{\lambda}^{ijk} - \bar{A}^{ijk} \bar{\eta} + \frac{1}{2} (\bar{\lambda}^{njk}(m^2)^i_n + \bar{\lambda}^{ink}(m^2)^j_n + \bar{\lambda}^{ijn}(m^2)^k_n) \eta \bar{\eta}.
\]

From eqs. (12) and (13-15) it is possible to write down an explicit differential operator which has to be applied to the \(\beta\) functions of a rigid theory in order to get those for the soft terms.

Consider first the gauge couplings \(\alpha_i\). One has

\[
\alpha_i^{\text{Bare}} = Z_{\alpha i} \alpha_i \Rightarrow \tilde{\alpha}_i^{\text{Bare}} = \tilde{Z}_{\alpha i} \tilde{\alpha}_i,
\]

where \(Z_{\alpha i}\) is the product of the wave function and vertex renormalization constants.

Though \(\tilde{\alpha}_i\) and \(\tilde{Z}_{\alpha i}\) are general superfields, one has to consider only their chiral or antichiral parts. The chiral part of eq. (16) is

\[
\alpha_i^{\text{Bare}} (1 + m_{\alpha_i}^{\text{Bare}} \eta) = \alpha_i (1 + m_{\alpha_i} \eta) Z_{\alpha i}(\tilde{\alpha})|_{\tilde{\eta} = 0}.
\]

Expanding over \(\eta\) one has

\[
\alpha_i^{\text{Bare}} = \alpha_i Z_{\alpha i}(\alpha),
\]

\[
m_{\alpha_i}^{\text{Bare}} \alpha_i^{\text{Bare}} = m_{\alpha_i} \alpha_i Z_{\alpha i}(\alpha) + \alpha_i D_1 Z_{\alpha i},
\]

where the operator \(D_1\) extracts the linear w.r.t. \(\eta\) part of \(Z_{\alpha i}(\tilde{\alpha})\). Due to eqs. (13-15) the explicit form of \(D_1\) is

\[
D_1 = m_{\alpha_i} \alpha_i \frac{\partial}{\partial \alpha_i}.
\]
Combining eqs. (17) and (18) one gets

\[ m_{Bare}^{A_i} = m_{A_i} + D_1 \ln Z_{\alpha i}. \tag{19} \]

To find the corresponding \( \beta \) functions one has to differentiate eqs. (17) and (19) w.r.t. the scale factor having in mind that the operator \( D_1 \) is scale invariant. This gives

\[ \beta_{\alpha i} = \alpha_i \gamma_{\alpha i}, \quad \beta_{m_{A_i}} = D_1 \gamma_{\alpha i}, \tag{20} \]

where \( \gamma_{\alpha i} \) is the logarithmic derivative of \( \ln Z_{\alpha i} \) equal to the anomalous dimension of the vector superfield in some particular gauges.

One can make also the transition from a rigid to a broken theory at the level of the renormalization group equation. Namely take

\[ \dot{\alpha} = \beta_{\alpha}(\alpha) \Rightarrow \dot{\tilde{\alpha}} = \beta_{\tilde{\alpha}}(\tilde{\alpha}), \tag{21} \]

and expand over \( \theta^2 \). Then one immediately reproduces eq. (20) for \( m_A \).

One can go even further and consider a solution of the RGE. Then one has in a rigid theory

\[ \int^\alpha \frac{d\alpha'}{\beta(\alpha')} = \log \left( \frac{Q^2}{\Lambda^2} \right). \tag{22} \]

Making a substitution \( \alpha \rightarrow \tilde{\alpha} \) one has

\[ \int^{\tilde{\alpha}} \frac{d\alpha'}{\beta(\tilde{\alpha}')} = \log \left( \frac{Q^2}{\tilde{\Lambda}^2} \right), \tag{23} \]

where \( \tilde{\Lambda} = \Lambda(1 + c\theta^2 + ...) \). Expanding over \( \theta^2 \) one finds

\[ \frac{m_{A\alpha}}{\beta(\alpha)} = \text{const.} \tag{24} \]

This result is in complete correspondence with that of Ref. [11].

The same procedure can be applied for the other soft terms. This needs the modification of the operator \( D_1 \) to include the \( A \) parameter from the chiral vertex. As for the mass square terms, they need the second order differential operator. Relations between the rigid and soft terms renormalizations in general case are summarized below. Similar results on the soft terms renormalization have been obtained in ref. [12].

4 Illustration

To make the above formulae more clear and to demonstrate how they work in practice, we consider the renormalization group functions in a general theory in one loop. We follow the notation of ref. [3] except that our \( \beta \) functions are half of those. Note that all the calculations in ref. [3] are performed in the framework of dimensional reduction and the \( \overline{MS} \) scheme.
The Rigid Terms & The Soft Terms

| $\beta_{\alpha i} = \alpha_i \gamma_{\alpha i}$ | $\beta_{m_{\alpha i}} = D_1 \gamma_{\alpha i}$ |
|---------------------------------------------|------------------------------------------|
| $\beta_{M}^{ij} = \frac{1}{2}(M^{il} \gamma_i^j + M^{lj} \gamma_i^l)$ | $\beta_{B}^{ij} = \frac{1}{2}(B^{il} \gamma_i^j + B^{lj} \gamma_i^l) - (M^{il} D_1 \gamma_i^j + M^{lj} D_1 \gamma_i^l)$ |
| $\beta_{y}^{ijk} = \frac{1}{2}(y^{ijkl} \gamma_k^j + y^{iljk} \gamma_i^j + y^{jikl} \gamma_i^j)$ | $\beta_{A}^{ijk} = \frac{1}{2}(A^{ijkl} \gamma_k^j + A^{ikjl} \gamma_i^j + A^{jikl} \gamma_i^j) - (y^{ijkl} D_1 \gamma_k^j + y^{iljk} D_1 \gamma_i^j + y^{jikl} D_1 \gamma_i^j)$ |
| $(\beta_{m^2})_j^i = D_2 \gamma_j^i$ |

Table 1: Relations between the rigid and soft term renormalizations in a massless minimal subtraction scheme

The gauge $\beta$ functions and the anomalous dimensions of matter superfields in a massless scheme are the functions of dimensionless gauge and Yukawa couplings of a rigid theory. In the one-loop order, the renormalization group functions of a rigid theory are (for simplicity, we consider the case of a single gauge coupling)$^2$

$$
\gamma_{\alpha}^{(1)} = \alpha Q, \quad Q = T(R) - 3C(G),
$$

$$
\gamma_{j}^{(1)} = \frac{1}{2} y^{ikl} y_{jkl} - 2 \alpha C(R)^{i},
$$

where $T(R)$, $C(G)$ and $C(R)$ are the Casimir operators. Using the formulae from the table we construct the renormalization group functions for the soft terms

$$
\beta_{m^1}^{(1)} = \alpha m_{A} Q,
$$

$$
\beta_{B}^{ij} (1) = \frac{1}{2} B^{il} \left( \frac{1}{2} y^{ikm} y_{jkm} - 2 \alpha C(R)^{j} \right)
$$

$$
+ M^{il} \left( \frac{1}{2} A^{ikm} y_{jkm} + 2 \alpha m_{A} C(R)^{j} \right) + (i \leftrightarrow j),
$$

$$
\beta_{A}^{ijk} (1) = \frac{1}{2} A^{ijkl} \left( \frac{1}{2} y^{kln} y_{jln} - 2 \alpha C(R)^{k} \right)
$$

$^2$ To simplify the formulas hereafter we use the following notation: $\alpha_i = g_i^2/16\pi^2$, $y^{ijk} = \lambda^{ijk}/4\pi$, $A^{ijk} = A^{ijk}/4\pi$. 

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\[ y_{ijl}(\frac{1}{2}A_{kmn}y_{mn} + 2\alpha m_A C(R)^i_k) + (i \leftrightarrow j) + (i \leftrightarrow k), \]
\[ [\beta_{m2}]^i_j^{(1)} = \frac{1}{2}A_{ijkl} - 4\alpha m_A C(R)^i_j \]
\[ + \frac{1}{4}y^{nkl}(m^2)^i_n y_{jkl} + \frac{1}{4}y^{ijkl}(m^2)^2_j y_{nkl} + \frac{4}{4}y^{isl}(m^2)^k_s y_{jkl}. \]

One can easily see that the resulting formulae coincide with those of ref. [9]. The same procedure works in higher orders of PT.

5 Soft Renormalizations in the MSSM

The general rules described in the previous section can be applied to any model, in particular to the MSSM. In the case when the field content and the Yukawa interactions are fixed, it is more useful to deal with numerical rather than with tensor couplings. Rewriting the superpotential (8) and the soft terms (9) in terms of group invariants, one has

\[ W_{SUSY} = \frac{1}{6} \sum_a y_a \lambda_a^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \sum_b M_b h_b^{ij} \Phi_i \Phi_j, \]

and

\[ - L_{soft} = \left[ \frac{1}{6} \sum A_a \lambda_a^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \sum B_b h_b^{ij} \phi_i \phi_j + \frac{1}{2} m_A \lambda_j \lambda_j + h.c. \right] + (m^2)^i_j \phi^{*i} \phi_j, \]

where we have introduced numerical couplings \( y_a, M_b, A_a \) and \( B_b \).

Usually, it is assumed that the soft terms obey the universality hypothesis, i.e. they repeat the structure of a superpotential, namely

\[ A_a = y_a A_a, \quad B_b = M_b B_b, \quad (m^2)^i_j = m_i^2 \delta_j. \]

The renormalization group \( \beta \) functions of a rigid theory are (for simplicity, we assume the diagonal renormalization of matter superfields)

\[ \beta_{\alpha_j} = \beta_j \equiv \alpha_j \gamma_{\alpha_j}, \]
\[ \beta_{y_a} = \frac{1}{2} y_a \sum K_{ai} \gamma_i, \]
\[ \beta_{M_b} = \frac{1}{2} M_b \sum T_{bi} \gamma_i, \]

where \( \gamma_i \) is the anomalous dimension of the superfield \( \Phi_i \), \( \gamma_{\alpha_j} \) is the anomalous dimension of the gauge superfield (in some gauges) and numerical matrices \( K \) and \( T \) specify which particular fields contribute to a given term in eq.(31).
Applying the algorithm of the previous section the renormalizations of the soft terms are expressed through those of a rigid theory in the following way:

$$\beta_{m_{A_j}} = D_1 \gamma_{\alpha_i},$$

$$\beta_{A_a} = -D_1 \sum_i K_{ai} \gamma_i,$$

$$\beta_{B_b} = -D_1 \sum_i T_{bi} \gamma_i,$$

$$\beta_{m_i^2} = D_2 \gamma_i,$$

and the operators $D_1$ and $D_2$ now take the form

$$D_1 = m_A \alpha_i \frac{\partial}{\partial \alpha_i} - A_a Y_a \frac{\partial}{\partial Y_a},$$

$$D_2 = (m_A \alpha_i \frac{\partial}{\partial \alpha_i} - A_a Y_a \frac{\partial}{\partial Y_a})^2 + m^2_{A_i} \alpha_i \frac{\partial}{\partial \alpha_i} + m_i^2 K_{ai} Y_a \frac{\partial}{\partial Y_a}. $$

where we have used the notation $Y_a \equiv y_a^2$.

To illustrate these rules, we consider as an example one loop renormalization of the MSSM couplings. Leaving for simplicity the third generation Yukawa couplings only, the superpotential is

$$W_{MSSM} = (y_t Q^i U^c H^i_2 + y_b Q^j D^c H^j_1 + y_e L^j \tilde{E}^c \tilde{H}^j_1 + \mu H^i_1 H^j_2) \epsilon_{ij},$$

where $Q, U, D', L$ and $E$ are quark doublet, up-quark, down-quark, lepton doublet and lepton singlet superfields, respectively, and $H_1$ and $H_2$ are Higgs doublet superfields. $i$ and $j$ are the $SU(2)$ indices.

The soft terms have a universal form

$$-\mathcal{L}_{soft-breaking} = \sum_i m_i^2 |\phi_i|^2 + \frac{1}{2} \sum_a \lambda_a \lambda_a$$

$$+ A_t y_t Q^i u^c h^i_2 + A_b y_b Q^j d^c h^j_1 + A_e y_e L^j \tilde{E}^c \tilde{H}^j_1 + B \mu h^i_1 h^j_2 + h.c.,$$

where the small letters denote the scalar components of the corresponding superfields and $\lambda_a$ are the gauginos. The $SU(2)$ indices are suppressed.

For illustration we calculate here the one loop soft term renormalizations and the gaugino mass renormalization at the three loop level out of a corresponding rigid $\beta$ functions.

Renormalizations in a rigid theory in the one loop order are given by the formulae

$$\gamma_{\alpha_i}^{(1)} = b_i \alpha_i, \quad i = 1, 2, 3, \quad b_i = \frac{33}{5}, 1, -3,$$

$$\gamma^{(1)}_Q = Y_t + Y_b - \frac{8}{3} \alpha_3 - \frac{3}{2} \alpha_2 - \frac{1}{30} \alpha_1,$$

$$\gamma^{(1)}_U = 2Y_t - \frac{8}{3} \alpha_3 - \frac{8}{15} \alpha_1,$$

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\[ \gamma_D^{(1)} = 2Y_b - \frac{8}{3} \alpha_3 - \frac{2}{15} \alpha_1, \]  
\[ \gamma_L^{(1)} = Y_\tau - \frac{3}{2} \alpha_2 - \frac{3}{10} \alpha_1, \]  
\[ \gamma_E^{(1)} = 2Y_\tau - \frac{6}{5} \alpha_1, \]  
\[ \gamma_{H_1}^{(1)} = 3Y_b + Y_\tau - \frac{3}{2} \alpha_2 - \frac{3}{10} \alpha_1, \]  
\[ \gamma_{H_2}^{(1)} = 3Y_\tau - \frac{3}{2} \alpha_2 - \frac{3}{10} \alpha_1. \]

Consequently, the renormalization group $\beta$ functions are

\[ \beta_{\alpha_i}^{(1)} = b_i \alpha_i^2, \]  
\[ \beta_{Y_i}^{(1)} = Y_i (6Y_i + Y_b - \frac{16}{3} \alpha_3 - 3\alpha_2 - \frac{13}{15} \alpha_1), \]  
\[ \beta_{Y_b}^{(1)} = Y_b (Y_b + 6Y_b + Y_\tau - \frac{16}{3} \alpha_3 - 3\alpha_2 - \frac{7}{15} \alpha_1), \]  
\[ \beta_{Y_\tau}^{(1)} = Y_\tau (3Y_\tau + 4Y_\tau - 3\alpha_2 - \frac{9}{5} \alpha_1), \]  
\[ \beta_{\mu^2}^{(1)} = \mu^2 (3Y_i + 3Y_b + Y_\tau - 3\alpha_2 - \frac{3}{5} \alpha_1). \]

This allows us immediately to write down the soft term renormalizations

\[ \beta_{A_1}^{(1)} = 6Y_i A_i + Y_b A_b + \frac{16}{3} \alpha_3 m_{A_1} + 3\alpha_2 m_{A_2} + \frac{13}{15} \alpha_1 m_{A_1}, \]  
\[ \beta_{A_b}^{(1)} = Y_i A_i + 6Y_b A_b + Y_\tau A_\tau + \frac{16}{3} \alpha_3 m_{A_1} + 3\alpha_2 m_{A_2} + \frac{7}{15} \alpha_1 m_{A_1}, \]  
\[ \beta_{A_\tau}^{(1)} = 3Y_b A_b + 4Y_\tau A_\tau + 3\alpha_2 m_{A_2} + \frac{9}{5} \alpha_1 m_{A_1}, \]  
\[ \beta_{B_1}^{(1)} = 3Y_i A_i + 3Y_b A_b + Y_\tau A_\tau + 3\alpha_2 m_{A_2} + \frac{3}{5} \alpha_1 m_{A_2}, \]  
\[ \beta_{m_{A_1}}^{(1)} = \alpha_1 b_i m_{A_i}, \]  
\[ \beta_{m_Q}^{(1)} = Y_i (m_Q^2 + m_U^2 + m_H^2 + A_i^2) + Y_b (m_Q^2 + m_D^2 + m_H^2 + A_b^2) \]  
\[ - \frac{16}{3} \alpha_3 m_{A_3}^2 - 3\alpha_2 m_{A_2}^2 - \frac{1}{15} \alpha_1 m_{A_1}^2, \]  
\[ \beta_{m_U}^{(1)} = 2Y_i (m_Q^2 + m_U^2 + m_H^2 + A_i^2) - \frac{16}{3} \alpha_3 m_{A_3}^2 - \frac{16}{15} \alpha_1 m_{A_1}^2, \]  
\[ \beta_{m_D}^{(1)} = 2Y_b (m_Q^2 + m_D^2 + m_H^2 + A_b^2) - \frac{16}{3} \alpha_3 m_{A_3}^2 - \frac{4}{15} \alpha_1 m_{A_1}^2, \]  
\[ \beta_{m_H}^{(1)} = Y_\tau (m_L^2 + m_E^2 + m_H^2 + A_\tau^2) - 3\alpha_2 m_{A_2}^2 - \frac{3}{5} \alpha_1 m_{A_3}^2, \]  
\[ \beta_{m_E}^{(1)} = 2Y_\tau (m_L^2 + m_E^2 + m_H^2 + A_\tau^2) - \frac{12}{5} \alpha_1 m_{A_3}^2, \]
\begin{align}
\beta_{m^2_{H_1}}^{(1)} &= 3Y_b(m_Q^2 + m_D^2 + m_{H_1}^2 + A_b^2) + Y_t(m_L^2 + m_E^2 + m_{H_1}^2 + A_t^2) \\
&- 3\alpha_2 m_{A_2}^2 - \frac{3}{5} \alpha_1 m_{A_1}^2, \tag{68}
\end{align}

\begin{align}
\beta_{m^2_{H_2}}^{(1)} &= 3Y_t(m_Q^2 + m_U^2 + m_{H_2}^2 + A_t^2) - 3\alpha_2 m_{A_2}^2 - \frac{3}{5} \alpha_1 m_{A_1}^2, \tag{69}
\end{align}

which perfectly coincide with those of ref. ([13]).

The RG \( \beta \) functions for the gauge couplings in the MSSM are

\[ \beta_{\alpha_i} = b_i \alpha_i^2 + \alpha_i^2 \left( \sum_j b_{ij} \alpha_j - \sum_f a_{if} Y_f \right) \]

\[ + \alpha_i^2 \left[ \sum_{jk} b_{ijk} \alpha_j \alpha_k - \sum_{jf} a_{ijf} \alpha_j Y_f + \sum_{fg} a_{ifg} Y_f Y_g \right] + \ldots, \tag{70} \]

where \( Y_f \) means \( Y_t, Y_b \) and \( Y_t \) and the coefficients \( b_i, b_{ij}, a_{ijf}, b_{ijk}, a_{ijf} \) and \( a_{ifg} \) are given in ref. [14].

For the gaugino masses we have

\[ \beta_{m_{A_i}} = b_i \alpha_i m_{A_i} + \alpha_i \left( \sum_j b_{ij} \alpha_j (m_{A_i} + m_{A_j}) - \sum_f a_{if} Y_f (m_{A_i} - A_f) \right) \]

\[ + \alpha_i \left[ \sum_{jk} b_{ijk} \alpha_j \alpha_k (m_{A_i} + m_{A_j} + m_{A_k}) - \sum_{jf} a_{ijf} \alpha_j Y_f (m_{A_i} + m_{A_j} - A_f) \right. \]

\[ + \sum_{fg} a_{ifg} Y_f Y_g (m_{A_i} - A_f - A_g) \right] + \ldots. \tag{71} \]

6 Finiteness of Soft Parameters in a Finite SUSY GUT

Consider now the application of the proposed formulae to construct totally finite softly broken theories.

For rigid N=1 SUSY theories there exists a general method of constructing totally all loop finite gauge theories proposed in refs. [13, 16, 17]. The key issue of the method is the one-loop finiteness. If the theory is one-loop finite and satisfies some criterion verified in one loop [18], it can be made finite in any loop order by fine-tuning of the Yukawa couplings order by order in PT. In case of a simple gauge group the Yukawa couplings have to be chosen in the form

\[ Y_a(\alpha) = c_a^0 \alpha + c_a^2 \alpha^2 + \ldots, \tag{72} \]

where the finite coefficients \( c_a^0 \) are calculated algebraically in the n-th order of perturbation theory.
Suppose now that a rigid theory is made finite to all orders by the choice of the Yukawa couplings as in eq.(72). This means that all the anomalous dimensions and the $\beta$ functions on the curve $Y_a = Y_a(\alpha)$ are identically equal to zero.

Consider the renormalization of the soft terms. According to eqs.(37-40) and (41,42) the renormalizations of the soft terms are not independent but are given by the differential operators acting on the same anomalous dimensions. One has either

$$\beta_{\text{soft}} \sim D_1 \gamma(Y, \alpha), \quad \text{or} \quad \beta_{\text{soft}} \sim D_2 \gamma(Y, \alpha),$$  

where $\gamma(Y_a, \alpha)$ is some anomalous dimension.

From the requirement of finiteness

$$\gamma_i(Y_a(\alpha), \alpha) = 0,$$  

the Yukawa couplings $Y_a(\alpha)$ are found in the form (72).

To reach the finiteness of all the soft terms in all loop orders, one has to choose the soft parameters $A_a$ and $m_i^2$ in a proper way. The following statement is valid: [10]

**Statement 2**  
The soft term $\beta$ functions become equal to zero if the parameters $A_a$ and $m_i^2$ are chosen in the following form:

$$A_a(\alpha) = -m_A \alpha \frac{\partial}{\partial \alpha} \ln Y_a(\alpha),$$  

$$m_i^2 = -m_A K^{-1}_{ia} \alpha A_a(\alpha)$$  

$$= m_A^2 K^{-1}_{ia} \alpha^2 \frac{\partial}{\partial \alpha} \ln Y_a(\alpha),$$

where the matrix $K^{-1}_{ia}$ is the inverse of the matrix $K_{ai}$.

This statement follows from the form of the operators $D_1$ and $D_2$. After substitution of solutions (73) and (76) into $D_1$ and $D_2$, $D_1$ becomes a total derivative over $\alpha$ and $D_2$ becomes a second total derivative.

Indeed, consider eq.(73). For $A_a$ chosen as in eq.(73) the differential operator $D_1$ takes the form

$$D_1 = m_A \alpha \frac{\partial}{\partial \alpha} - A_a Y_a \frac{\partial}{\partial Y_a} = m_A \frac{\partial \ln Y_a}{\partial \ln \alpha} \frac{\partial}{\partial \ln Y_a} + \frac{\partial}{\partial \ln \alpha} = m_A \frac{d}{d \ln \alpha}.$$  

Hence, since on the curve $Y_a = Y_a(\alpha)$ the anomalous dimension $\gamma(Y_a, \alpha)$ identically vanishes, so does its derivative

$$\frac{d}{d \ln \alpha} \gamma(Y_a(\alpha), \alpha) = 0.$$  

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The operator $D_2$ is the second derivative. Using eq. (76) one has

$$D_2 = (m_A\alpha \frac{\partial}{\partial \alpha} - A_aY_a \frac{\partial}{\partial Y_a})^2 + m_A^2\alpha \frac{\partial}{\partial \alpha} + m_A^2K_{ai}Y_a \frac{\partial}{\partial Y_a}$$

$$= (m_A\alpha \frac{\partial}{\partial \alpha} - A_aY_a \frac{\partial}{\partial Y_a})^2 - m_A\frac{\partial A_a}{\partial \ln \alpha} Y_a \frac{\partial}{\partial Y_a} + m_A(m_A\alpha \frac{\partial}{\partial \alpha} - A_aY_a \frac{\partial}{\partial Y_a})$$

$$= m_A^2 \frac{d}{d \ln \alpha} + m_A^2 \frac{d^2}{d \ln^2 \alpha}.$$

The term with the derivative of $A_a$ is essential to get the total second derivative over $\alpha$, since in the bracket the derivative $\alpha \partial / \partial \alpha$ does not act on $A_a$ by construction. Like in the previous case the total derivatives identically vanish on the curve $Y_a = Y_a(\alpha)$.

The solutions (75,76) can be checked perturbatively order by order. In the leading order one has

$$A_a = -m_A, \quad m_i^2 = \frac{1}{3}m_A^2,$$

since $\sum_a K_{ia}^{-1} = 1/3$. These relations coincide with the already known ones [8] and with those coming from supergravity [19] and superstring-inspired models [20]. There they usually follow from the requirement of finiteness of the cosmological constant and probably have the same origin. Note that since the one-loop finiteness of a rigid theory automatically leads to the two-loop one and hence the coefficients $c_i^2 = 0$, the same statement is valid due to eqs. (73,74) for a softly broken theory. Namely, relations (77) are valid up to two-loop order in accordance with [9]. In higher orders, however, they have to be modified.

This way one can construct all loop finite $N=1$ SUSY GUT including the soft SUSY breaking terms with the fine-tuning given by exactly the same functions as in a rigid theory.

7 Discussion

Our approach is based on a consideration of the soft theory as a rigid one embedded into the external $x-$independent superfields, that are the charges and masses of the theory. The Feynman rules together with the operator constructions $D_1$ and $D_2$ are just the technical consequences of this approach. One can see that all the essential information concerning the renormalization is actually contained in a rigid theory. There is no independent renormalization in softly broken SUSY theories.

There might be several applications of this approach. First, one can use it for finding of the soft terms RG functions in the MSSM and other theories. Second, one can try to consider the extended SUSY theories, like $N=2$ SUSY, where in unbroken case the exact results are known. Then one can break $N=2$ to $N=1$ or to $N=0$ in a soft way and consider the broken theory along the lines advocated above preserving the symmetry properties of the exact solution. In fact presumably any spontaneously broken theory may be treated this way.
Later there has been considerable activity exploring the renormalization group invariant relations between the soft term renormalizations \[21, 22, 23\]. They essentially use the explicit form of the differential operators \(D_1\) and \(D_2\) and their reduction to the total derivatives found in \[10\]. In particular the exact form for the correction due to the \(\epsilon\)-scalar mass in component approach in the renormalization scheme corresponding to the NSVZ \(\beta\) function has been obtained \[22\]. The exact \(\beta\) function for the scalar masses in NSVZ scheme for softly broken SUSY QCD has been also found \[23\].

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