Hopfion dynamics in chiral magnets

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Received 31 August 2021, revised 6 February 2022
Accepted for publication 9 February 2022
Published 31 March 2022

Abstract
Resonant spin dynamics of topological spin textures are correlated with their topological nature, which can be employed to understand this nature. In this study, we present resonant spin dynamics of three-dimensional topological spin texture, i.e., Neel and Bloch hopfions. Using micromagnetic simulations, we stabilize Bloch and Neel hopfions with bulk and interfacial Dzyaloshinskii–Moriya interaction, respectively. We identify the ground state spin configuration of both hopfions, effects of anisotropies, geometric confinements, and demagnetizing fields. To confirm topological nature, Hopf number is calculated for each spin texture. Then, we calculate the resonance frequencies and spin-wave modes of spin precessions under multiple magnetic fields. Unique resonance frequencies and specific magnetic field dependence can help to guide experimental studies to identify the three-dimensional topological spin texture of hopfions in functioning chiral magnets when imaging is not possible.

Keywords: magnetic texture, magnetization dynamics, solitons, hopfion

(Some figures may appear in colour only in the online journal)

1. Introduction

Topological spin textures attract considerable interest in condensed matter physics because of their non-trivial physical properties and promising applications in memory and storage technologies [1,2]. One well-known example of these spin textures is a magnetic skyrmion, which is a projection of three-dimensional hedgehog spins from a sphere to a two-dimensional surface [3,4]. When these three-dimensional hedgehog spins are projected on a torus with certain knotting, we end up having a hopfion, which is a three-dimensional topological spin texture. Although skyrmions have been intensely studied by theoretical, numerical, and experimental means [5–9], hopfions have not been studied thoroughly and started to receive attention very recently [1,10,11].

Hopfions have been initially observed in chiral liquid crystals and optical mediums [11–13]. More recently numerical studies of the magnetic version of hopfions [1,14–16] and experimental observation in chiral magnet Ir/Co/Pt multilayers have been reported [17]. Magnetic hopfions are stabilized in chiral magnets as a result of antisymmetric exchange interaction, commonly called the Dzyaloshinskii–Moriya interaction (DMI). Depending on the origin of DMI, i.e., bulk or interfacial DMI, Bloch or Neel hopfions are stabilized in magnetic nanodisks. The numerical studies of Bloch hopfions use interfaces with a strong perpendicular magnetic anisotropy (PMA) in addition to the bulk DMI in a B20FeGe chiral magnet. A recent experimental study by Kent et al. used Ir/Co/Pt multilayer nanodisks to stabilize magnetic hopfions and x-ray photoelectron emission microscopy and magnetic transmission x-ray microscopy to probe their internal spin texture [17].

For x-ray transmission, Ir/Co/Pt multilayer nanodisks were fabricated on Si3N4 membranes. Besides, observation of Bloch hopfions in B20 compounds with bulk DMI stays a challenge due to epitaxial growth of B20 compounds on Si3N4 membranes. The requirement of Si3N4 membrane is also present in other commonly used methods of Lorentz transmission electron microscopy imaging [18]. Scalable growth and device fabrication of hopfion systems require more stable substrates (e.g., Si), which will, however, make imaging studies more complicated.

Another important method of exploring nanoscale magnetic materials is studying their resonance spin dynamics [19–22]. For example, ferromagnetic resonance spectroscopy
In this article, we numerically study resonance spin precession dynamics of Bloch and Neel hopfions in chiral magnet nanodiscs utilizing micromagnetic simulations. We first introduce the geometry of the hopfion and hopfion charge. Then, we identify the ground state of the Bloch hopfion in the chiral magnet FeGe with bulk DMI and the Neel hopfion in magnetic multilayer with interfacial DMI. Particularly, we investigated the effect of the demagnetizing field in FeGe nanodisks, which was previously ignored in some studies [1, 30]. We found that the demagnetization field stabilizes a Bloch hopfion in small discs against the monopole–antimonopole pair (MAP) formation, which is another stable spin configuration due to strong PMA. Next, we excite the Bloch and Neel hopfions with a magnetic pulse and analyze the resulting precession frequencies and spin-wave modes, which then can be correlated with the hopfion’s real-space spin texture. Our results will provide the precession dynamics of three-dimensional spin-solitons and help their future experimental studies.

A hopfion is a knot on a three-dimensional torus consisting of continuous unit vector fields. In figure 1, we show three different hopfions with hopfion charges of 1, 2, and 3. The mathematical description of a hopfion charge is

\[ H = -\frac{1}{(8\pi)^2} \int F \cdot A \, d\mathbf{r}, \]

where \( F \) and \( A \) are vector fields calculated from the unit vector field \( n \) by

\[ F_i = \varepsilon_{ijk} n_j \cdot (\nabla n_k \times \nabla_k n) \quad \text{and} \quad \nabla \times A = F. \]

In our magnetic system, \( n \) corresponds to the unit microspin. Moreover, there is an indirect method to calculate the hopfion charge using the linking number of knots [31, 32]; as we demonstrate in figure 1, where we describe the torus by its toroidal (\( T \)) and poloidal (\( P \)) cycles and the hopfion charge \( H = TP \) [31, 32]. The left column of figure 1 shows these isosurfaces with a color-coding, whereas the right column shows one isosurface with a knot that is the point with the magnetization vector angles of \( \theta_i \) and \( \phi_i \) on a unit sphere. Isosurfaces knot the torus one, two, and three times in figures 1(a)–(c), respectively.

Hopfions being as spin textures in condensed matter systems have been previously introduced and explored theoretically as more abstract phenomena [33]. However, recent numerical studies have shown the stability of hopfions in the chiral magnets with the help of DMI [1, 14, 34]. There are two origins of DMI in chiral magnets, the bulk DMI from the broken crystal inversion symmetry (e.g., noncentrosymmetric B20 FeGe [25, 28, 35]) and the interfacial DMI from the broken interface symmetry (e.g., Ir/Co/Pt multilayers [17, 36]). The former facilitates Bloch hopfions and the latter facilitates Neel hopfions, similar to Bloch and Neel skyrmions [37]. Their spin profiles and dynamics are quite different; therefore, we first investigate Bloch hopfion in a FeGe nanodisk and then Neel hopfion in a magnetic multilayer.

2. Bloch hopfion

For a Bloch hopfion, we use the initial ansatz and nanodisc geometry from reference [1] to find its ground state using MuMax3 micromagnetic simulation [38]. The energy density of the system is given as

\[ \mathcal{E}(\vec{m}) = A_{ex}(\nabla \vec{m})^2 + D\vec{m} \cdot (\nabla \times \vec{m}) - K_u(\vec{u} \cdot \vec{m})^2 \]

\[ -\frac{1}{2} \vec{m} \cdot \vec{B}_{\text{demag}} - \vec{m} \cdot \vec{B}_{\text{ext}} \]

where \( \vec{m} \) is the magnetization vector, \( A_{ex} \) is the exchange constant, \( D \) is the DMI constant, \( K_u \) is the uniaxial anisotropy coefficient, \( \vec{u} \) is a unit vector along the anisotropy axis, \( \vec{B}_{\text{demag}} \) is the demagnetizing field, and \( \vec{B}_{\text{ext}} \) is the external magnetic field. Our simulation consists of a FeGe nanodisc with a diameter of \( d = 128 \text{ nm} \) and a height of \( h = 64 \text{ nm} \). We sandwiched the FeGe layer by two PMA layers with an anisotropy constant of \( K_u = 10^5 \text{ J m}^{-3} \), which can be experimentally created by oxide or heavy metal layers [39]. The thickness of the capping layer is 1 nm. Other parameters used for the simulations are the exchange stiffness \( A_{ex} = 6.65 \times 10^{-13} \text{ J m}^{-1} \), the bulk DMI strength \( D_{\text{bulk}} = 1.19 \times 10^{-4} \text{ (J m}^{-2} \rangle \), and the saturation magnetization \( M_{\text{sat}} = 1.5 \times 10^5 \text{ A m}^{-1} \). To find the ground state, we minimized the total energy of the system, including exchange, DMI, anisotropy, and demagnetizing energies by using micromagnetic simulation. Magnetic interactions and properties of FeGe are highly sensitive to temperature and we
Figure 2. Ground spin textures with and without the demagnetizing field. The simulation disc has $d = 128$ nm diameter and $h = 64$ nm height. (a), (c), and (e) are the magnetizations at $y = 64$ nm plane. (b), (d), and (f) are the magnetizations at $z = 32$ nm plane. (a) and (b) are the initial magnetization configuration to start the energy minimization. (c) and (d) and (e) and (f) are the MAP and hopfion ground-state spin textures without and with the demagnetizing field, respectively. (i) shows the color-coding for the magnetization. (g) and (h) show the exchange energies and demagnetizing energy densities at the $y = 64$ nm plane.

use these values near the skyrmion formation temperature of 276 K [28]. Besides, the skyrmion formation temperature in the FeGe is the highest among all the B20 components with a skyrmion phase, which makes the FeGe more promising for technological applications [37].

One surprising difference we observed in our simulations is that a hopfion in a nanodisc with a $d = 128$ nm diameter is stable, even though Tai and Smalyukh [1] found that the hopfion phase can only be stable in larger nanodiscs, i.e., with a diameter of $d > 3\lambda$, where $\lambda = 70$ nm (the helical period of FeGe). The difference between our micromagnetic simulation and Tai and Smalyukh [1]'s numerical method is that we account for the demagnetizing energy, which originates from the shape of the structure. In figure 2, we show the resulting spin textures by including and excluding the demagnetizing field in the total energy at zero external magnetic fields. First, we start the simulation with the initial ansatz of a hopfion [1, 33], as in figures 2(a) and (b), which shows the magnetization of the disc at $y = 64$ nm and $z = 32$ nm planes, respectively. The color coding for the magnetization maps is given in figure 2(i). When we exclude the demagnetizing field as in Tai and Smalyukh [1], the MAP ground state is reached (figures 2(c) and (d)), which is consistent with their observation. However, when the demagnetization field is included, the hopfion becomes a stable state as shown in figures 2(e) and (f). In addition, we calculated the exchange (symmetric + DMI) and demagnetization energy densities in the case of the hopfion state (figures 2(g) and (h)). Although the demagnetizing energy is quite smaller than the sum of the two exchange energies, it is still in the same order; and especially, it is strong at the center and at the edges of the disc, where the spins are aligned along $+z$ direction. Therefore, we found that the demagnetizing field plays
an important role and increases the stability of the hopfion phase, especially, in smaller diameter discs. This is particularly important because the smaller discs with a stable hopfion phase mean higher density for memory applications.

After finding the hopfion ground state in the FeGe nanodisc at zero magnetic fields, we study the stability of the hopfion phase at the external magnetic fields. A recent study by Liu et al [15] already discussed the effects of external fields; therefore, we leave the effects of the external fields to reference [15] and focus on its spin dynamics under various external magnetic fields. In our simulations, the hopfion is stable between 0 mT and −25 mT external magnetic fields in the direction of z-axis. At higher magnetic fields, the hopfion state turns into the MAP phase mean higher density for memory applications.

To find the resonance frequencies, we employ the ringdown method [40], in which we apply a magnetic field impulse in magnetic fields. In our simulations, the hopfion is stable between 0 mT and −25 mT external magnetic fields in the direction of z-axis. By using MuMax3 micromagnetic simulation, we found the ground state of a Neel hopfion as shown in figure 5. The multilayer under discussion is a FeGe nanodisc with a thickness of 0.5 nm was used throughout the Neel hopfion simulations.

Neel hopfion is also a knot on a three-dimensional torus consisting of continuous unit vector fields with the magnetization vector aligned with the z-axis, ρ direction on a unit sphere whereas the Bloch hopfion’s magnetization vector is aligned along with ϕ direction in the cylindrical coordinates. Previously, Neel hopfions were numerically [41] and experimentally [17] studied in systems with interfacial DMI. The explicit form of the interfacial DMI is given in reference [42]. The multilayer under discussion is a h = 8 nm thick ferromagnetic film.

We rotate the individual spins from the known hopfion ansatz [43] by 90 degrees around the z-axis, i.e., rotating magnetization along the axis with making coordinates as \( x \rightarrow y \) and \( y \rightarrow -x \). By using MuMax3 micromagnetic simulation, we found the ground state of a Neel hopfion as shown in figure 5. The magnetic energy parameters used in the simulations are the interfacial DMI = 1.15 × 10^{-3} J m^{-2}, the exchange stiffness \( A_{ex} = 1.1 \times 10^{-12} \) J m^{-1}, the uniaxial anisotropy constant \( K_u = 1 \times 10^{6} \) J m^{-3}, and the saturation magnetization \( M_{sat} = 3 \times 10^{5} \) A m^{-1}. The simulation size for the Neel hopfion is a disk with a \( d = 64 \) nm and a \( h = 8 \) nm. A cell size of 0.5 nm was used throughout the Neel hopfion simulations.

The hopfion charge for the Bloch hopfion is already extensively studied in [1, 15]; however, Neel hopfions are relatively new and we will confirm the topological nature of our Neel hopfions. The topological charge is calculated using reference

\[
\frac{\partial \mathbf{\hat{m}}}{\partial t} = \gamma_{LL} \frac{1}{1 + \alpha^2} \left( \mathbf{\hat{m}} \times \mathbf{\hat{B}}_{eff} + \alpha (\mathbf{\hat{m}} \times (\mathbf{\hat{m}} \times \mathbf{\hat{B}}_{eff})) \right),
\]

where \( \gamma_{LL} \) is the gyromagnetic ratio, \( \alpha \) is the damping parameter, and \( \mathbf{\hat{B}}_{eff} \) is the effective field. Next, by calculating the Fourier transform of the magnetization in the time domain, we find the spatially resolved spin dynamics in the frequency domain with a 5 MHz resolution up to 5 GHz in frequency. The averaged Fourier transforms of all spins are shown in figure 3 for \( B = -5, -10, -15, -20, \) and −25 mT magnetic fields. For clarity, the y positions of the spectrums are shifted by an offset.
Figure 4. Spatial distribution of the resonance amplitudes at 0.17 GHz (first and third rows) and 1.75 GHz (second and fourth rows) frequencies at $-15$ mT magnetic field. The top two rows show the amplitude at the middle plane of the hopfion at $z = 32$ nm and the bottom two rows show at $z = 16$ nm. The columns show the amplitude for the magnetization in $z$, $\rho$, and $\phi$ directions, respectively.

[41]'s method as follows:

$$H = \frac{1}{(2\pi)^2} \iiint_{x,y,z} (F_x \cdot A_x + F_z \cdot A_z) \, dx \, dy \, dz, \quad (5)$$

$$A_x = -\int_{-\infty}^{y} F_z \, dy, \quad (6)$$

$$A_z = \int_{-\infty}^{y} F_x \, dy, \quad (7)$$

where $F_i = \varepsilon_{ijk} n \cdot (\nabla \times \nabla_i n)$. We found that the Neel hopfion is stable in the field range of $-30$ mT to $-440$ mT, where the Hopf indices $H$ are above 0.9, indicating that all solitons are Neel hopfions. We show the magnetizations of hopfions at the fields of $-60$ mT, $-120$ mT, and $-300$ mT fields in figure 5, where the slices of $xy$ plane at $z = 4$ nm and $xz$ plane at $y = 32$ nm are shown. The color coding for the magnetization maps is as in figure 2(i). As the external field decreased to $-300$ mT, the size of the hopfion increases. Neel hopfion’s shape starts to change from a circular to a square due to the spin texture wrapping around the torus in the tangential plane. After we found the stable magnetic field range, we studied resonance dynamics from $-30$ to $-300$ mT magnetic field range.

Neel hopfion dynamics are studied using the method described in the previous section. We apply a magnetic field impulse and the magnetization of each unit cell is recorded with $\Delta t = 0.1$ ns time interval for $\tau = 200$ ns. Using the Fourier transform of the magnetization in the time domain, spatially resolved spin dynamics are calculated in the frequency domain with a 5 MHz resolution up to 5 GHz. The averaged Fourier transforms of all spins are shown in figure 6 for $B = -30, -60, -90, -120, -150, -180, -210, -240, -270, -300$ mT magnetic fields. For clarity, the $y$ positions of the spectrums are shifted by an offset.

As the magnetic field amplitude increases from $-30$ mT to $-60$ mT, the first resonance frequency $f_1$ at 3.2 GHz shifts to lower frequencies, e.g., $f = 2.19$ and 2.13 GHz at $B = -60$ and $-90$ mT fields, respectively. After 2.13 GHz, resonance frequencies shift to high frequencies, i.e., $f = 2.49, 2.9, 3.65, 3.98, 4.27$ and 4.51 GHz at $B = -120, -150, 180, 210, 240, 270$ and $-300$ mT fields,
Figure 5. Neel hopfion under magnetic fields of $B = -60$ mT, $-120$ mT, and $-300$ mT fields. The simulation disc has a diameter $d = 64$ nm and height of $h = 8$ nm. (b), (d), and (f) are the magnetizations at $z = 4$ nm plane and (a), (c), and (e) are the magnetizations at $y = 32$ nm plane under magnetic fields of $-60$ mT, $-120$ mT, $-300$ mT, respectively.

Figure 6. Resonance frequencies of the Neel hopfion at $B = -30$, $-60$, $-90$, $-120$, $-150$, $-180$, $-210$, $-240$, $-270$, and $-300$ mT fields in the direction of $z$-axis. The curves are the sum of the Fourier transform of all the individual spin dynamics. For clarity, the spectrums are shifted by an offset.

respectively. After, we investigate the spin-wave modes of 1.26 GHz, 2.19 GHz, and 4.39 GHz resonances by plotting spatial distributions of the magnetization components in $z$, $\rho$, and $\phi$ cylindrical coordinates at $B = -60$ mT field (figure 7). We show the spin-wave modes in $z = 4$ nm $xy$ plane (first three rows of figure 7) and $z = 2$ nm (fourth, fifth and sixth rows of figure 7) of micromagnetic simulations, which are the middle plane and one-fourth plane of the torus, respectively. Depending on the frequency of the applied magnetic pulse, we observe different hopfion modes. At lower frequencies, hopfion has a more rotational mode. At higher frequencies, it has a more breathing mode concentrated at the center of the hopfion. The breathing mode of the hopfion corresponds to the coupled oscillation of the hopfion size and its spin angles. The second and fourth rows show the uniformity of the spins along an easy axis in the breathing mode.

In summary, we numerically study the stability and spin dynamics of a three-dimensional spin texture of Bloch and Neel hopfions using micromagnetic simulations. Bloch hopfion’s stability under different external magnetic fields in the $z$-direction was analyzed. At lower magnetic fields, the Bloch hopfion turned into a field polarized state and at higher fields, into a MAP. We found that the demagnetizing field helps to stabilize Bloch hopfion with two times smaller disks, which is useful for higher density memory devices. By analyzing the transient dynamics of the Bloch hopfion spins, we found the resonance features, spin-wave modes, and the spatial distribution of spin dynamics at two different frequencies. In the second part of the paper, Neel hopfion is discussed. The stability of Neel hopfion was achieved under multiple magnetic fields pointing in the $z$-direction. At higher external magnetic fields, hopfion’s shape turned into a square. Then, resonance and spin waves were analyzed for varying external magnetic fields. Lastly, the spatial distribution of spin dynamics at three different frequencies is shown. Further investigation by varying thermal fluctuations and anisotropies by electric fields will help to further manipulate resonance features. These will lead to identifying the true nature of the spin texture when magnetic imaging or neutron scattering on functioning devices are not possible.
Figure 7. Spatial distribution of the resonance at $-60$ mT magnetic field. The top three rows show the amplitude at the middle plane of the hopfion at $z = 4$ nm $xy$ plane and the bottom three rows show at $z = 2$ nm.
Acknowledgments

This work was supported by the National Science Foundation Grant Number OIA-1929086. We also gratefully acknowledge NVIDIA Corporation with the donation of the Titan Xp GPU, which was used for this research.

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