Directivity of guided ultrasonic wave scattering at notches and cracks

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Abstract. Localized and distributed guided ultrasonic waves array systems offer an efficient way for the long-term monitoring of the structural integrity for large structures. The use of permanently attached sensor arrays has been shown to be applicable to detect simulated corrosion damage. However, the detection sensitivity for fatigue cracks depends on the location and orientation of the crack relative to the transducer elements. Crack-like defects have a directivity pattern of the scattered wave field depending on the angle of the incident wave relative to the defect orientation and on the ratio of the characteristic defect size to wavelength. From FE simulations it has been shown that for cracks and notches almost no energy is scattered in certain directions from the defect, i.e., the data processing algorithm must take into account that for some transducer combinations no change in the signal even for a significant defect will be detected. The directivity pattern of the scattered field for the A₀ Lamb wave mode is predicted from 3D Finite Element simulations and verified from experimental measurements at machined part-through and through-thickness notches using a laser interferometer. Good agreement was found and the directivity pattern can be predicted accurately. The amplitude of the scattered wave is quantified for a variation of the angle of the incident wave relative to the defect orientation, the defect depth, and the ratio of the characteristic defect size to wavelength. These results provide the basis for the quantification of the detection sensitivity for defects in plate structures using guided wave array sensors. A hybrid model has been developed, taking the different propagation distances and scattering characteristics into account, in order to predict the relative amplitudes of received pulses for given sensor locations. From a comparison with the signal to noise ratio of the array system, detection capabilities can be predicted for given defect size and orientation.

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1. Introduction
Low frequency guided ultrasonic waves typically have a wavelength that is large compared to the thickness of a structure and can propagate over long distances [1]. This provides an efficient way for the structural health monitoring (SHM) of large technical structures [2]. For structures containing plate-like components, different types of guided ultrasonic wave monitoring systems have been
developed. Localized phased array systems typically contain a group of sensor elements at one location of the structure and allow for the detection of defects in a structure based on the analysis of back-scattered waves. Systems have been developed employing the $S_0$ Lamb wave mode excited and measured using EMAT (electro-magnetic acoustical transducers) arrays [3], and the $A_0$ Lamb wave mode using circular [4] and rectangular [5] arrays of piezoelectric elements. Distributed array systems using sensor elements at different locations of the structure have been developed [6-8]. Due to the complexity of the received signals, quite often a baseline subtraction algorithm is used to identify the pulse scattered at a defect. Different concepts for temperature compensation of the signals have been implemented to overcome environmental changes between measurements for the long term monitoring of realistic structures [9].

Both monitoring concepts rely on an additional wave pulse scattered at the defect and were initially tested for circular defects. For the detection of crack-like defects with various, possibly unknown orientations, the scattering pattern must be taken into account to quantify the minimum defect sensitivity for such defects. The scattering of the $A_0$ Lamb wave mode at thickness changes in a plate and at notches has been studied in a 2D configuration through the plate thickness using FE simulations and experiments [10, 11]. Analytical models exist for the scattering of the $A_0$ mode at circular through holes [12], which have been verified from numerical simulations and experiments [13]. Models have been extended to take part-thickness circular defects into account and verified from FE simulations [14]. The change in the scattered field of a guided wave mode due to a fatigue crack emanating from a fastener hole has been reported previously [15]. Analytical models exist for the scattering of mechanical waves at crack-like defects [16], but apply restrictions on the geometry in order to allow for an analytical solution. More recently, FE modeling has been used to study the interaction of shear horizontal waves with a crack in plates [17]. Frequency domain FE models have been developed to investigate the scattering of bulk ultrasonic waves at different types of defects [18]. This allows for the efficient characterization of the defect by calculating the scattered ultrasonic field for all incident and scattering angles [19], called the scattering matrix or scattering amplitude [20]. Preliminary results for the extension of the frequency domain FE approach to 3D models for bulk and guided ultrasonic waves have been shown [21]. It would be beneficial to benchmark the computational efficiency and accuracy against experiments and conventional 3D FE scattering calculations [22, 23].

This contribution investigates the directivity pattern of the $A_0$ Lamb wave mode scattered at a crack-like defect (notch) in a plate, depending on the angle of the incident wave relative to the defect orientation, on the defect length relative to the wavelength, and on the defect depth. The scattered wave fields predicted from Finite Element (FE) simulations and measured experimentally are compared and show good agreement for part-through and through thickness notches. The obtained scattering characteristics for a variation of the geometric parameters are discussed with a view towards the detection of crack-like defects using guided ultrasonic waves.

Based on the FE results a hybrid model is developed to predict the relative amplitudes of guided wave pulses scattered at the defect received at given sensor locations. The model takes the wave propagation and scattering characteristics from the FE simulations for a given defect size into account. Damage maps can be calculated for each sensor pair, specifying the relative amplitude of the defect signal. Using the same data fusion algorithm as the guided wave array sensor evaluation, the defect detection sensitivity for a given defect size and orientation is predicted. This can be compared with the signal to noise ratio (SNR) of an implemented array design and used to optimize the sensor layout.
Figure 1. Finite Element simulation of $A_0$ Lamb mode wave propagation from $0^\circ$ excitation location in plate (wave crest propagating radially outwards), scattering at through thickness notch; $f = 100$ kHz; $\lambda = 19$ mm; $a = 20$ mm; different possible excitation locations shown.

2. Finite Element Simulations of Scattered Wave Field

The propagation and scattering of the first anti-symmetric Lamb wave mode $A_0$ in plates was simulated using three-dimensional (3D) Finite Element (FE) models with linear brick elements ($\Delta x = \Delta y = \Delta z = 1.25$ mm) in ABAQUS Explicit. The model of a 5 mm thick aluminum plate (size: 1 m x 1 m) with a notch of varying length and depth was implemented, similar to the model described in [22]. The notch was modeled by removing one row of elements to achieve the desired length and depth. This resulted in a notch with right angle corners and a width of 1.25 mm, matching the experimental configuration described below reasonably well. Explicit time integration was used, and the element size and time step were chosen to adhere to the usual stability criteria. The element size was not changed in the vicinity of the defect; therefore small errors might occur in the near field due to the inaccurate description of the stress concentration.

Point excitation of the $A_0$ Lamb wave mode was introduced at chosen node locations 300 mm from the defect location, allowing for a variation of the incidence angle between $0^\circ$ (perpendicular) and $90^\circ$ (parallel), relative to the normal of the notch (see Fig. 1). The excitation pulse was set as a 5 cycle toneburst with a center frequency $f$ of 100 kHz, resulting in a wavelength $\lambda$ for the $A_0$ mode of 19 mm, thus about 4 times the plate thickness. The amplitude of the $A_0$ mode was monitored as the out-of-plane displacement at the center (mid-plane) node on points in a square area of 100 mm by 100 mm around the notch center. The time trace at each monitoring node was time gated to remove reflections from the plate edges. Fast Fourier Transform (FFT) was used to extract the complex magnitude (amplitude and phase information) at the center frequency of 100 kHz for each monitoring node.
Figure 2. Normalized amplitude of scattered ultrasonic wave field (A₀ mode) at through thickness notch; f = 100 kHz; \( \lambda = 19 \text{ mm} \); a = 20 mm: a) view of combined field; b) scattered wave field only; amplitude of guided wave pulse color-coded (blue = low, red = high amp.).

This captures the combined wave field \( U_1 \) of the incident wave \( U_i \) and scattered wave \( U_s \) (Eq. 1), shown in Fig. 2a for a 0° incident wave. In the figure the amplitude of the guided wave pulse at each point is color-coded from low amplitude (dark blue) to high amplitude (red). The typical scattering pattern with large amplitude at the free surface of the defect and shadow area (low amplitude) behind the defect can be seen. Along the incident direction (positive x axis) a periodically alternating high and low amplitude pattern due to the positive and negative interference of the incident wave \( U_i \) and back scattered wave \( U_s \), according to their phase, can be observed.

\[
\begin{align*}
U_1 & = U_i + U_s \\
U_2 & = U_i \\
A_s & = |U_s| = |U_1 - U_2|
\end{align*}
\]

Additional simulations were run to capture the incident wave field \( U_i \) only, without a crack present (Eq. 2). The amplitude of the incident wave (not shown) is almost constant in the area of interest (vicinity of the defect). Taking the difference between the complex magnitudes for each point with and without a defect, the amplitude of the wave scattered at the notch (Fig. 2b) can be found [23]. It should be noted that the phase information contained in the complex magnitude is necessary for this subtraction (Eq. 3), as a simple subtraction of the measured amplitudes with and without a defect would not give the amplitude of the scattered wave only.

The scattered wave field (Fig. 2b) shows the expected behavior for a wave incident perpendicular on a long through-thickness defect (notch length a = 20 mm similar to wavelength \( \lambda = 19 \text{ mm} \)), with a significant shadow area of low amplitude behind the defect (negative x axis) and back scattered wave (positive x axis). Evaluating the scattered field on a radial line (x axis) more than a wavelength (\( \lambda = 19 \text{ mm} \)) away from the defect (length 20 mm) in Fig. 3a, the amplitude shows a decrease proportional to one over the square root of the distance, as would be expected for a point source of the guided wave mode on a plate. Therefore the amplitude of the scattered wave was extracted on a radius of 30 mm by interpolating between monitoring nodes to obtain an amplitude value every 5 degree. Shown in Fig. 3b is the polar plot of the normalized amplitude of the scattered A₀ mode, from which the shadow area behind the defect and the back scattered wave can be clearly identified. The amplitude of the scattered wave is normalized with respect to the amplitude of the incident wave at the defect center. Using the radial square root dependency relative to the monitoring radius of 30 mm, the amplitude of the scattered wave at any location in the plate can be calculated.
Figure 3. Scattered ultrasonic wave field (A₀ mode) at through thickness notch; orientation 90°-270°; incident wave from 0°; f = 100 kHz; \( \lambda = 19 \) mm; \( a = 20 \) mm: a) normalized amplitude (red, solid) along x axis (perpendicular to defect) and radial dependency (blue, dashed); b) polar plot of normalized amplitude (monitored at 30 mm radius).

The approach to use the local calculation of the scattering characteristics and the knowledge of the wave propagation to predict the ultrasonic wave field in the whole structure has been used in literature for bulk ultrasonic wave scattering [20], but with a different scattered amplitude scaling definition [19, 20].

3. Experimental Description

The scattered wave field around a 20 mm long notch was measured on a large, 5mm thick aluminum plate (size: 1.5 m x 1 m). The notch (width: 1 mm) was milled in steps to give a varying depth. Two cases, a part-thickness notch of 2.5 mm depth (1/2 thickness) and a through thickness notch are considered here. The notch had rounded edges due to the milling tool (diameter 1 mm) and a flat bottom for the part-through defect. Three excitation transducers were placed 300 mm away from the center of the notch location to give angles of the incident wave of 0° (perpendicular to notch), 45° and 90° (along notch orientation) relative to the normal of the notch (Fig. 4).

Figure 4. Experimental setup: aluminum plate (size: 1.5 m x 1 m) with 20 mm notch (zoom), 3 excitation transducers at 0°, 45°, and 90°, 300 mm from notch, and laser interferometer.
Figure 5. Typical experimental time signal, undamaged (black, solid), with notch (red, dashed), f = 100 kHz.

The transducers consist of a PZT disc (Ferroperm Pz27, diameter 5 mm, thickness 2 mm) and a brass backing mass (height 6 mm) and were permanently bonded to the plate using two-component epoxy glue. The transducers act in good approximation as point sources for the excitation of the first antisymmetric Lamb wave mode A₀. The excitation signal was a 5 cycle toneburst with a center frequency of 100 kHz modulated by a Hanning window. The signal was generated in a programmable function generator and amplified using a power amplifier. The velocity of the out-of-plane displacement was measured using a laser interferometer every 5 degrees on a circle around the notch with a radius of 30 mm. In the frequency-thickness range of interest the S₀ mode has a much smaller out-of-plane component than the A₀ mode. Therefore even for the part thickness defect case where mode conversion occurs, the laser measurement will mostly only detect the A₀ mode. The full time traces of the measured signals were bandpass filtered (75 kHz – 125 kHz) and averaged (20 averages) in a digital oscilloscope before being transferred to the measurement PC [23, 24].

Similar to the FE simulations, an initial measurement was done for each excitation transducer before the milling of the notch to capture the incident wave field, and then for each notch depth. The measured time traces (Fig. 5) were time gated and the amplitude and phase (complex magnitude) of the wave pulse at 100 kHz determined using FFT. Due to the re-positioning of the plate after each milling step relative to the laser, the repeatability of the phase measurement was somewhat limited. This leads to a noise level of about 5% of the amplitude of the incident wave when taking the difference between complex magnitudes to evaluate the scattered field.

Figure 6. Polar plot of normalized amplitude of A₀ mode scattered at notch (monitored at 30 mm radius); orientation 90°-270°; incident wave from 0°; f = 100 kHz; $\lambda = 19$ mm; a = 20 mm; experiment (black, dash-dotted), simulation (red, solid): a) $\frac{1}{2}$ thickness; b) through thickness.
Figure 7. Polar plot of normalized amplitude of A0 mode scattered at notch (monitored at 30 mm radius); orientation 90°-270°; incident wave from 45°; f = 100 kHz; λ = 19 mm; a = 20 mm; experiment (black, dash-dotted), simulation (red, solid): a) ½ thickness; b) through thickness.

4. Experimental Verification of FE Simulation Scattering Directivity Pattern
FE simulation and experimental results were compared for a 20 mm long notch half and fully through the thickness of the 5 mm aluminum plate. For the incident wave perpendicular to the notch (0° incidence angle) good agreement of the comparison between FE predictions and experimental results was found (Fig. 6). The noise level for the complex magnitude difference calculated from experimental results is about 5% as described in section 3, which is evident in Fig. 6a. For the ½ thickness a notch shadow effect behind the defect can be observed, as the path of the incident wave is blocked and mode conversion occurs. Only a small back-scattered amplitude is observed for the ½ thickness defect (about 15%), increasing to about 70% for a through thickness defect (Fig. 6b). This has a significant influence on the ability to detect a crack from pulse-echo type measurements. For a localized array of guided wave transducers with all sensors at one location of the structure, for shallow defects only limited energy is reflected back towards the array location. For the incident wave at a 45° angle relative to the defect orientation as well good agreement was found between experiments and the FE simulations (Fig. 7). For shallow defects (½ plate thickness) mostly the shadow effect behind the notch in the direction of wave propagation was observed (Fig. 7a), with a significant reflected wave only present for a through thickness defect (Fig. 7b). For this case, with the incident wave at a significantly larger angle of 45° relative to the defect orientation, the amplitude of all scattered wave lobes are lower than for the perpendicular incidence case discussed above. For the incident wave propagating along the defect orientation (90° angle), only a very small scattered wave is expected for a part-thickness notch of about 5% and such a defect could not be detected, even from a local laser measurement due to the noise level in the experimental evaluation. For the through thickness notch scattered amplitude of about 10% was predicted and could just about be measured.

5. Variation of the Scattered Field due to Defect Geometry and Orientation
The directivity pattern of the scattered wave field depends strongly on the geometric parameters, i.e., angle of the incident wave relative to the defect orientation, defect depth and length relative to the wavelength. As discussed above, a shadow area behind the notch develops and increases with increasing defect depth until most of the energy of the wave is blocked from propagating past the notch for a deep defect. However, for shallow defects only a small back-scattered wave exists. Only for deep defects a significant back scattered wave can be seen which increases in amplitude for a through thickness defect.
Figure 8. Left: polar plot of scattered wave amplitude at 20 mm through thickness notch for variation of incident angle from 0° (perpendicular, blue) in 15° steps (15° red; 30° green; 45° black; 60° brown, 75° yellow) to 90° (along defect, magenta); right: maximum scattered amplitude from polar plot; $f = 100$ kHz; $\lambda = 19$ mm.

Shown in Fig. 8 and Fig. 9 are the polar plots and maximum amplitude of the FE calculated amplitude of the $A_0$ mode scattered at a through thickness defect with lengths $a$ of 20 mm and 5 mm for a variation of the incident angle. It can be seen that the directivity pattern depends on the ratio between the wavelength and the defect size [24]. A pattern with lobes of the scattered field corresponding to the angle between the incident wave and the crack orientation develops (Fig. 8) when the defect length is comparable to the wavelength ($\lambda = 19$ mm). Significant maximum amplitude of the scattered wave field is found when the wave is incident on the notch at an angle up to about 45° and then decreases with increase of the incident angle. For defect lengths shorter than the wavelength, the scattered field shows a different directivity pattern with one or two main lobes approximately perpendicular to the crack orientation (Fig. 9). The maximum amplitude for this case is in the shadow area behind the defect and is significantly smaller than for a longer defect. Again, the amplitude of the scattered wave decreases with increasing incident angle and is very small when the incident wave direction is aligned with the notch orientation. Similar patterns can be observed for other variations of the geometric parameters. However, it can be clearly seen that the amplitude of the scattered wave shows a strong angular dependency and that along the orientation of the defect, almost no energy is scattered.

Figure 9. Left: polar plot of scattered wave amplitude at 5 mm through thickness notch for variation of incident angle from 0° (perpendicular, blue) in 15° steps (15° red; 30° green; 45° black; 60° brown, 75° yellow) to 90° (along defect, magenta); right: maximum scattered amplitude from polar plot; $f = 100$ kHz; $\lambda = 19$ mm.
6. Hybrid Model for SHM Sensitivity Prediction

From the results shown above it can be seen that the amplitude of the scattered wave shows a strong angular dependency and that there are directions in which almost no energy is scattered. This can be problematic for SHM applications as for given transducer locations on a structure, there exist possible defect (crack) positions and orientations, where even for a severe defect no signal from the defect would be detected due to the scattering directivity pattern of the defect. Shown in Fig. 10 is a typical geometrical configuration for a distributed array system with 2 sensor locations.

Based on the findings detailed above, some general recommendations for the placement of sensors on a structure can be given. Shallow defects show a significantly larger shadow effect than back scattered wave and only for deeper defects does a substantial back-scattered wave develop. Furthermore, the scattered wave field is very small when the incident wave direction is approximately aligned with the crack orientation or when measured along that direction for any incident angle. Often the likely crack orientation and location are known due the knowledge of the stress state in the structure, e.g. loading direction and areas of stress concentration. If possible, it is then advantageous to place the sensors in such a way that the incident wave is approximately perpendicular to the likely defect orientation. For a monitoring location placed behind the critical crack location in the shadow area of the incident wave, a significant amplitude change even for shallow defects can be expected. For other monitoring locations, smaller scattered signals are expected. However, this might have the advantage that an additional scattered wave pulse can be detected better than a change in amplitude of the wave transmitted past the defect location. For a distributed array system these conditions are often more easily met than for a localized array system placed at one location of the structure.

Furthermore, assuming all sensors to act in good approximation as point sources and receivers for the guided wave mode, a hybrid model can be developed to predict the amplitude of the wave scattered at a defect at any location on the structure and received at a given transducer location. Taking the decrease in amplitude due to the radial spread as one over the square root of the propagation distance into account, the incident amplitude at any given defect location can be calculated analytically. Using the knowledge of the defect orientation and scattering directivity the amplitude of the wave scattered in the direction of the receiving sensor element is obtained from the FE simulation results. Again correcting for the propagation distance, the amplitude of the received scattered wave is computed. For a distributed array system, the amplitude of the defect pulse relative to the direct pulse propagating from the transmitting to the receiving element is relevant to compare against the signal to noise ratio (SNR) of the system.

**Figure 10.** Typical geometric configuration for SHM application using distributed array system with 2 sensor locations on plate shown.
This calculation is shown in Fig. 11a for excitation and receiving transducers 500 mm apart horizontally on a square 1 m x 1 m plate, using the same parameters as described in the previous sections. For a 20 mm long, through thickness notch, aligned vertically, the predicted relative amplitude of the defect signal is shown color-coded in Fig. 11a. High received scattered amplitude is predicted for the defect located on and close to the line between sensors as the incident wave is approximately perpendicular to the crack orientation. Low received scattered amplitude can be seen for areas further away from the sensor locations and where the incident wave propagates along the crack orientation (x = 250 mm & x = 750 mm). For a distributed array system with multiple sensors, each acting in turn as excitation and receiving element, the same data fusion algorithm as for the measurements can be used to combine the predicted amplitude maps for each sensor pair. Shown in Fig. 11b is the predicted combined detection sensitivity for the same crack size and orientation using 4 sensors placed in a rectangular pattern 250 mm from the plate edges. Good detection sensitivity with relative amplitude above 20 dB is predicted for the center of the area covered by the 4 sensors. However, due to the considered vertical orientation of the crack, lower detection sensitivity can be seen between the vertically aligned transducers at x = 250 mm and x = 750 mm. Here the incident or scattered wave direction for a significant proportion of the considered transducer combinations is aligned along the crack and only very small scattered amplitude is expected.

Based on the knowledge of the guided ultrasonic wave propagation and scattering characteristics, accurate predictions of the sensitivity of a structural health monitoring system employing distributed arrays of sensors can be made. The placement of the sensors can be optimized for expected crack locations and orientations if the stress state in the structure is known. From a comparison to the SNR of the employed array system, the potential to detect crack-like defects can be predicted and the required number of sensors evaluated.

**7. Conclusions**

The scattering of the A\textsubscript{0} Lamb wave mode at part-through and through thickness crack-like in aluminum plates has been investigated experimentally and from FE simulations of the wave propagation and scattering. Good agreement was found between the experimentally measured scattered field directivity patterns and amplitudes and FE predictions. This allows for the further investigation of the influence of relative defect size and depth, wavelength, plate thickness and incidence angle from the FE simulations. For all investigated cases of incidence angle and defect size it was found that in certain directions there is only very little scattered wave energy. These findings
should be taken into account when developing permanently attached guided ultrasonic wave arrays for SHM, as the sensitivity for defect detection will depend strongly on the chosen geometric configuration in relation to the possible defect locations. Based on the investigation presented in this contribution, a hybrid model has been developed, taking the wave propagation and scattering into account, to predict and quantify the sensitivity for the detection of defects at likely locations and orientations.

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References
[1] Chimenti, D.E., *Appl. Mech. Rev.*, **50**(5), 247-284 (1997).
[2] Rose, J.L., *Mat. Eval.* **60**(1), 53-59 (2002).
[3] Wilcox, P.D., Lowe, M.J.S., Cawley, P., *IEEE Trans. on Ultrason. Ferroelec. and Freq. Cont.* **52**(4), 653-665 (2005).
[4] Fromme, P., Wilcox, P.D., Lowe, M.J.S., Cawley, P., *IEEE Trans. on Ultrason. Ferroelec. and Freq. Cont.* **53**(4), 777-785 (2006).
[5] Yu, L. and Giurgiutiu, V., *Ultrasonics* **48**(2), 117-134 (2008).
[6] Croxford, A.J., Wilcox, P.D., Drinkwater, B.W., Konstantinidis, G., *Proc. Roy. Soc. A* **463**(2007), 2961-2981 (2007).
[7] Michaels, J.E. and Michaels, T.E., *Wave Motion* **44**(6), 482-492 (2007).
[8] Fromme, P., in *Rev. Prog. QNDE, ed. by D.O. Thompson and D.E. Chimenti, AIP Conf. Proc.* 975, New York, **27A**, 78-85 (2008).
[9] Clarke, T. and Cawley, P., *IEEE Trans. on Ultrason. Ferroelec. and Freq. Cont.* **56**(12), 2666-2678 (2009).
[10] Lowe, M.J.S., Cawley, P., Kao, J.Y., Diligent, O., *J. Acoust. Soc. Am.* **112**(6), 2612-2622 (2002).
[11] Yeo, F. and Fromme, P., in *Rev. Prog. QNDE, ed. by D.O. Thompson and D.E. Chimenti, AIP Conf. Proc.* 820, New York, **25A**, 202-209 (2006).
[12] Pao, Y.H. and Chao, C.C., *AIAA J.* **2**(11), 2004-2010 (1964).
[13] Fromme, P. and Sayir, M.B., *J. Acoust. Soc. Am.* **111**(3), 1165-1170 (2002).
[14] Diligent, O., Grahn, T., Bostrom, A., Cawley, P., Lowe, M.J.S., *J. Acoust. Soc. Am.* **112**(6), 2589-2601 (2002).
[15] Fromme, P. and Sayir, M.B. *Ultrasonics* **40**(1-8), 199-203 (2002).
[16] Sih, G.C. and Chen, E.P., *in Plates and shells with cracks: a collection of stress intensity factor solutions for cracks in plates and shells, ed. by G.C. Sih*, Leyden: Noordhoff International (1977).
[17] Rajagopal, P. and Lowe, M.J.S., *J. Acoust. Soc. Am.* **124**(5), 2895-2904 (2008).
[18] Wilcox, P.D., and Velichko, A., *J. Acoust. Soc. Am.* **127**(1), 155-165 (2010).
[19] Zhang, J., Drinkwater, B.W., Wilcox, P.D., *IEEE Trans. on Ultrason. Ferroelec. and Freq. Cont.* **55**(10), 2254-2265 (2008).
[20] Schmerr, L.W., *Fundamentals of ultrasonic nondestructive evaluation: a modeling approach*, Plenum Press, New York (1998).
[21] Velichko, A. and Wilcox, P.D., in *Rev. Prog. QNDE, ed. by D.O. Thompson and D.E. Chimenti, AIP Conf. Proc.* 1211, New York, **29**, 57-64 (2010).
[22] Fromme, P., in *Proc. of SPIE, ed. by T. Kundu*, **6935**, W9350 (2008).
[23] Fromme, P., in *Rev. Prog. QNDE, ed. by D.O. Thompson and D.E. Chimenti, AIP Conf. Proc.* 1211, New York, **29**, 129-136 (2010).
[24] Fromme, P., in *Proc. of SPIE, ed. by T. Kundu*, **7650**, (2010).