A CRITICISM OF THE ARTICLE “AN EXPERIMENTAL TEST OF NON-LOCAL REALISM”

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Abstract. I make some critical comments on the article [11]. This article makes incorrect claims concerning Bell’s theorem [1]. Moreover, I point to the fact that a hypothesis referred to as “realism” in [11] is not used in the deduction of Leggett’s inequality.

“It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty.”

John Bell in [1 Introduction].

There is hardly a result that is more widely misunderstood in the scientific community than Bell’s theorem. In a nutshell, there is a widespread belief that in his celebrated article [1], Bell has shown that the assumption of locality, when taken together with an assumption of the existence of hidden variables, allows one to deduce a certain inequality (now known as Bell’s inequality) that contradicts quantum mechanical predictions. A crucial point that is usually overlooked is the fact that the existence of the hidden variables used in the deduction of Bell’s inequality is inferred from the assumption of locality using the EPR argument; it is not, as many physicists seem to think, an additional assumption that is necessary for proving the inequality. Therefore, violation of Bell’s inequality implies that locality has to be abandoned.

The misunderstanding regarding the meaning of Bell’s theorem seems to be widespread, but fortunately not universal. Indeed, attempts to clear up the confusion have been made by many authors (see, for instance, [6] pgs. 10—15, [10], [12], [14]), most notably Bell himself. For instance, in footnote number 10 of [3], Bell wrote:

1Bell explains his violation of locality argument in several of his writings (see [5] for the collection of Bell’s articles on the Foundations of Quantum Mechanics). In fact, one version of the argument goes directly from locality to an inequality that is violated by the quantum predictions (the Clauser–Holt–Shimony–Horne inequality) and it does not even have to mention any sort of hidden variables (see, for instance, [2] and [3 Section 4]). See also [4] for the (in my opinion) best of Bell’s articles on the subject.

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“[…] My own first paper on this subject (Physics 1, 195 (1965).) starts with a summary of the EPR argument from locality to deterministic hidden variables. But the commentators have almost universally reported that it begins with deterministic hidden variables.” (emphasis in the original)

The above noted misconception regarding the hypotheses necessary for the deduction of Bell’s inequality is not the only one; in addition, one encounters the now apparently popular habit of replacing the expression “hidden variables” with the word “realism”. This often leads to a confusion between the notion of “hidden variable” (usually understood as any parameter that represents more information about a given system than the wave function) with the several different possible meanings[] that can be given to the word “realism” in the Philosophy of Science (or Philosophy in general). For instance, right at the beginning of paper [11], in the abstract, we find the following statement:

“Most working scientists hold fast to the concept of ‘realism’ — a viewpoint according to which an external reality exists independent of observation. But quantum physics has shattered some of our cornerstone beliefs. According to Bell’s theorem, any theory that is based on the joint assumption of realism and locality (meaning that local events cannot be affected by actions in spacelike separated regions) is at variance with certain quantum predictions.”

So, the “realism” that appears in the abstract of [11] seems to be some sort of philosophical notion of realism (as we will see below, a different notion of “realism” is considered within the article). In fact, the citation above looks like a suggestion that working scientists should take solipsism seriously, or at least that they should consider the possibility that tables, mountains and planets do not exist at all in the absence of a conscious observer of some sort. There is no rational justification for such a radically counter-intuitive statement.

A rather extreme type of misunderstanding concerning Bell’s theorem is shown in the following passage from [11, pg. 871]:

“The logical conclusion one can draw from the violation of local realism is that at least one of its assumptions fails. Specifically, either locality or realism or both cannot provide a foundational basis for quantum theory. Each of the resulting possible positions has strong supporters and opponents in the scientific community. However, Bell’s theorem is unbiased with respect

[2]See [15] for a nice discussion of some possible meanings of the word “realism” and for more detailed criticism on the bad habit of saying that Bell has shown that we have to abandon “local realism”. For a nice discussion about the distinction between a realist and an instrumentalist interpretation of a given physical theory, see [7]. There is indeed a relation between the notion of “hidden variable” and some notions of “realism”, but confusion often arises when one takes their meanings to be identical or when one simply does not care to assign any precise meanings to those words at all.
to these views: on the basis of this theorem, one cannot, even in principle, favour one over the other. It is therefore important to ask whether incompatibility theorems similar to Bell’s can be found in which at least one of these concepts is relaxed.” (emphasis added)

In this passage, the authors of [11] give the impression that they think that the choice between abandoning locality or abandoning realism (whatever that turns out to mean) is a matter of subjective personal taste: some members of the scientific community prefer to abandon locality and others prefer to abandon realism. The statement that Bell’s theorem is “unbiased” reveals a great deal of misunderstanding regarding the hypotheses involved in its proof.

Let us now turn to the analysis of the hypotheses that the authors of [11] claim to be using in the proof of Leggett’s inequality ([11, pg. 872]):

“[…] The theories are based on the following assumptions: (1) all measurement outcomes are determined by pre-existing properties of particles independent of the measurement (realism); (2) physical states are statistical mixtures of subensembles with definite polarization, where (3) polarization is defined such that expectation values taken for each subensemble obey Malus’ law (that is, the well-known cosine dependence of the intensity of a polarized beam after an ideal polarizer).”

The notion of “realism” expressed in hypothesis (1) is different from the one considered in the paper’s abstract; here “realism” doesn’t seem to be taken as the opposite of solipsism or of some other radical form of philosophical idealism. It is not completely clear, though, what hypothesis (1) means. Below I will present a brief analysis of some possible meanings for hypothesis (1), but let me emphasize that my main point here is that hypothesis (1) is simply not used in the deduction of Leggett’s inequality. It is a bit odd, to say the least, that an article that claims to be doing “an experimental test of non-local realism” is apparently trying to accomplish its goal by verifying the violation of an inequality whose proof does not use the very hypothesis that the authors call “realism”! A more appropriate title would be “An experimental test of Leggett’s subensembles hypothesis”, which is precisely what the article presents.

Let us discuss some possible meanings for hypothesis (1). What exactly is supposed to determine the outcome of an experiment? One possible answer is: the outcome of an experiment is assumed to be completely determined by a complete description of the system of particles being observed and by the self-adjoint operator that would normally be used to compute the probability distribution on the set of possible outcomes for the experiment. I’ll call such a hypothesis operator realism. The problem with operator realism is that it is already well-known that it is inconsistent with quantum mechanical predictions: all those results that are usually called “no hidden variables
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Thus, it wouldn’t make much sense to take hypothesis (1) as being operator realism. An alternative meaning to hypothesis (1) is the following: the outcome of an experiment is assumed to be completely determined by a complete description of the system of particles being observed and by a sufficiently complete description of the experimental apparatus. For the purposes of the present discussion, I’ll call such a hypothesis determinism. It is very important to understand that determinism is very different from operator realism. Indeed, determinism is consistent with all quantum mechanical predictions: this is proven by the existence of Bohmian Mechanics, which is a completely deterministic theory that agrees with all quantum mechanical predictions. Therefore, any attempt to prove incompatibility between determinism and quantum mechanical predictions is completely pointless. It does make sense, however, to try to prove that determinism plus some additional hypotheses leads to some inequality that contradicts quantum mechanical predictions. Unfortunately, as it will be explained below, that is not what is done in [11]: hypothesis (1) (be it determinism or something else) is not used at all in the proof of Leggett’s inequality.

I will now present a derivation of Leggett’s inequality from hypotheses (2) and (3) of [11] alone. As usual, the experimenters will be called Alice and Bob. Let $A$ (resp., $B$) denote the result of Alice’s (resp., Bob’s) experiment, which can be either 1 or $-1$, and $a$ (resp., $b$) denote the setting of Alice’s (resp., Bob’s) experimental apparatus, which is an element of the unit sphere $S^2$. Hypotheses (2) and (3) amount to a “hidden variables” assumption, namely, that vectors $u, v \in S^2$ are associated to each run of the experiment (in addition to the usual quantum state vector). A precise mathematical statement of (2) and (3) can be formulated as follows:

\[ (*) \text{ for every } a, b \in S^2, \text{ there exists a probability distribution } P_{ab} \text{ on } \{-1, 1\} \times \{-1, 1\} \times S^2 \times S^2 \text{ representing the joint distribution of } (A, B, u, v) \text{ when the experiment is done with settings } a \text{ and } b. \]

These distributions are such that the (marginal) distribution $P_{uv}$ of

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3See, for instance, [6, pgs. 6—10] and [13]. A notable exception is the “no hidden variables theorem” due to von Neumann, which does not even allow the rejection of operator realism, since (in addition to operator realism) it assumes a hypothesis that is not justifiable solely on the basis of quantum mechanical predictions. See [13] for details.

4In the case of experiments with entangled particles located far away from each other, the “sufficiently complete description of the experimental apparatus” may include a description of parts of the experimental apparatus that are located far away from each other.

5See [8, 16]. Bohmian Mechanics is a formalism that is obtained from the standard quantum formalism by adding well-defined particle trajectories satisfying a simple first order ordinary differential equation involving the wave function. The addition of particle trajectories turns quantum mechanics into a precisely formulated physical theory, with a well-defined ontology. The privileged role played by “measurement” or “observers” in standard quantum mechanics simply disappears; in particular, Bohmian Mechanics solves the so called “measurement problem”. Bohmian Mechanics is obviously incompatible with operator realism (see [8]).
(\(u, v\)) does not depend on \((a, b)\), and the conditional expectations \(E_{ab}(A \mid u, v)\) and \(E_{ab}(B \mid u, v)\) are given by:

\[
(*) \quad E_{ab}(A \mid u, v) = u \cdot a, \quad E_{ab}(B \mid u, v) = v \cdot b.
\]

Here is how (\(\bullet\)) leads to Leggett’s inequality: equality (5) in [11], namely:

\[-1 + |A + B| = AB = 1 - |A - B|,
\]

is simply a consequence of the fact that \(A\) and \(B\) take values on \([-1, 1]\).

Taking expectations conditioned on \((u, v)\) and using the fact that the modulus of the expectation is less than or equal to the expectation of the modulus, we get:

\[-1 + |E_{ab}(A \mid u, v) + E_{ab}(B \mid u, v)| \leq E_{ab}(AB \mid u, v) \leq 1 - |E_{ab}(A \mid u, v) - E_{ab}(B \mid u, v)|,
\]

which is the same as inequality (8) in [11]. We now use (\(\star\)) and take expectations on both sides (with fixed \((a, b)\)), obtaining:

\[-1 + \int_{S^2 \times S^2} |u_0 \cdot a + v_0 \cdot b| \, dP_{uv}(u_0, v_0) \leq E_{ab}(AB) \leq 1 - \int_{S^2 \times S^2} |u_0 \cdot a - v_0 \cdot b| \, dP_{uv}(u_0, v_0).
\]

The rest of the proof of Leggett’s inequality involves only mathematical manipulations of the inequalities above and makes no further use of physical hypotheses of any kind.

To summarize, what can one conclude from the violation of Leggett’s inequality? The logical conclusion is that Leggett’s subensembles hypothesis (\(\bullet\)) is false, i.e., that a theory that contains the hidden variables \(u\) and \(v\) proposed by Leggett cannot be empirically viable. That doesn’t tell us anything about determinism or any type of philosophical realism. A title like “An experimental test of non-local realism” is severely misleading: it could, for instance, lead some readers into believing that the experiment reported by the article makes a theory like Bohmian Mechanics more implausible, while it is exactly the other way around: a prediction of Bohmian Mechanics has been experimentally verified and a class of alternatives to it has been shown not to be viable.

It should be observed, however, that the authors of [11] are careful enough to point out in their article that Bohmian Mechanics falls outside the class of theories that is falsified by their experiment. On the other hand, the authors do not present any arguments against Bohmian Mechanics itself and I’m very puzzled about why the authors make wild speculations about the violation of things like Aristotelian logic in the last paragraph of the article.
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