Predicting spin of compact objects from their QPOs: A global QPO model

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Abstract. We establish a unified model to explain Quasi-Periodic-Oscillation (QPO) observed from black hole and neutron star systems globally. This is based on the accreting systems thought to be damped harmonic oscillators with higher order nonlinearity. The model explains multiple properties parallelly independent of the nature of the compact object. It describes QPOs successfully for several compact sources. Based on it, we predict the spin frequency of the neutron star Sco X-1 and the specific angular momentum of black holes GRO J1655-40, GRS 1915+105.

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INTRODUCTION

The origin of Quasi-Periodic-Oscillation (QPO) and its properties are still ill-understood. The observed QPO frequencies of compact objects are expected to be related to the spin parameter of the compact object itself. In certain neutron star systems a pair of QPO forms and QPO frequencies appear to be separated either by the order of the spin frequency of the neutron star, apparently for slow rotators e.g. 4U 1702-429, or by half of the spin frequency, for fast rotators e.g. 4U 1636-53. Their frequency separation decreases with the increase of one of the QPO frequencies. The black hole systems, on the other hand, e.g. GRO J1655-40, XTE J1550-564, GRS 1915+105, exhibit the kHz QPO pairs which seem to appear at a 3 : 2 ratio [1].

Interaction between the surface current density in the disk and the stellar magnetic field generating warps and subsequent precession instability in the inner accretion disk produces motions at low frequencies. This is similar to the Lense-Thirring precession [2]. This can explain the mHz QPOs for strongly magnetized neutron stars [3]. The effect of nonlinear coupling between g-mode oscillations in a thin relativistic disk and warp has also been examined [4] for a static compact object. Recent observations [5] indicate a strong correlation between low and high frequency QPOs holding over mHz to kHz range which strongly supports the idea that QPOs are universal physical processes independent of the nature of the compact object. The correlation is also explained in terms of centrifugal barrier model [6]. Indeed earlier the correlation was shown in terms of the effective boundary wall created by the strong centrifugal force in the disk [7].

Accretion dynamics is a nonlinear hydrodynamic/magnetohydrodynamic phenomenon. It was already shown that QPOs may arise from nonlinear resonance phenomena in an accretion disk (e.g. [3, 9]) occurred due to resonance between...
epicyclic motions of accreting matter. However, the separation between vertical and radial epicyclic frequencies increases mostly as a function of either of the epicyclic frequencies contradicting the observed QPO feature.

We aim at establishing a global model based on higher order nonlinear resonance theory to describe black hole and neutron star QPOs together and then try to predict the spin parameter/frequency of compact objects. The model predicts, along with important QPO features, the spin parameter of black holes as well as spin frequency of a neutron star.

**MODEL**

A system of $N$ degrees of freedom has $N$ linear natural frequencies $\nu_i; i = 1, 2, \ldots N$ [10]. These frequencies have commensurable relations which may cause the corresponding modes to be strongly coupled and yield an internal resonance. If the system is excited by an external frequency $\nu_\ast$, then the commensurable relation exhibiting resonance might be

$$a \nu_\ast = \sum_{i=1}^{N} b_i \nu_i, \text{ with } a + \sum_{i=1}^{N} |b_i| = j,$$

(1)

apart from all the primary and secondary resonance conditions $c \nu_\ast = d \nu_m$, where $a, b_i, c, d$ are integers and $j = k + 1$ when $k$ is the order of nonlinearity.

Now we consider an accretion disk with a possible higher order nonlinear resonance. The resonance is driven by the combination of the strong disturbance created by the rotation of the compact object with spin $\nu_s$ and the existent (weaker) disturbances in the disk at the frequency of the radial ($\nu_r$) and vertical ($\nu_z$) epicyclic oscillations given by (e.g. [11])

$$\nu_r = \frac{\nu_o}{r} \sqrt{\Delta - 4(\sqrt{r} - a)^2}, \quad \nu_z = \frac{\nu_o}{r} \sqrt{r^2 - 4a\sqrt{r} + 3a^2},$$

(2)

where $\Delta = r^2 - 2Mr + a^2$, $a$ is the specific angular momentum (spin parameter) of the compact object and $\nu_o$ is the orbital frequency of the disk particle given by $2\pi \nu_o = \Omega = 1/(r^{3/2} + a)$.

We describe the system schematically in Fig. [1] It is composed of two oscillators due to radial and vertical epicyclic motion with different spring constants. The basic idea is that the mode corresponding to $\nu_s$ will couple to the ones corresponding to $\nu_r$ and $\nu_z$ igniting new modes with frequencies $\nu_{r,z} \pm p \nu_s$ [10], where $p$ is a number e.g. $L/2$ or $L$, when $L$ is an integer, if the effect is nonlinear or linear [10, 8] respectively. While $L = 1$ corresponds to the dominant interaction, modes with other $L$ are very weak to exhibit any observable effect. Now at a certain radius in the nonlinear regime where $\nu_s/2$ (or $\nu_s$ in the linear regime) is close to $\nu_z - \nu_r$ [10, 8], and $\nu_{z/2}$ (or $\nu_z$) is also coincidently close to the frequency difference of any two newly excited modes, a resonance may occur which locks the frequency difference of two excited modes at $\nu_s/2$ (or $\nu_s$).

As the neutron star has a magnetosphere coupled with the accretion disk, the mode with $\nu_s$ can easily disturb the disk matter. A black hole, on the other hand, with magnetic
field connecting to the surrounding disk, may transfer energy and angular momentum to the disk by the variant Blandford-Znajek process [12] what was already verified by XMM-Newton observation (e.g. [13]). This confirms possible ignition of new modes in a disk around a spinning a black hole.

Therefore, rewriting (1) for an accretion disk we obtain

\[ \nu_r + \frac{n}{2} \nu_s = \nu_z - \frac{m}{2} \nu_s \]  

with \(-b_1 = b_2 = 2\) and \(m,n\) are integers. Now we propose the higher and lower QPO frequency of a pair respectively to be

\[ \nu_h = \nu_r + \frac{n}{2} \nu_s, \quad \nu_l = \nu_z - \frac{\nu_s}{2}. \]  

Hence, we understand from (1) and (3) the order of nonlinearity in accretion disks exhibiting QPOs \( k = n - m + 4 \).

At an appropriate radius where \( \nu_z - \nu_r \sim \nu_s/2 \) and resonance is supposed to take place, \( \Delta \nu = \nu_h - \nu_l \sim \nu_s/2 \) for \( n = m = 1 \), and \( \Delta \nu \sim \nu_s \) for \( n = m = 2, n = 1 \) implies a nonlinear coupling between the radial epicyclic mode and the disturbance due to spin of the neutron star, which results in \( \Delta \nu \) locking in the nonlinear regime with \( m = 1 \). On the other hand, \( n = 2 \) implies a linear coupling resulting \( \Delta \nu \) locking in the linear regime with \( m = 2 \). For a black hole, however, in absence of its magnetosphere, disturbance and then corresponding coupling may not be strongly nonlinear and occurs with the condition \( \nu_z - \nu_r \sim \nu_s \) resulting the resonance locking at the linear regime with \( n = m = 2 \), which produces \( \Delta \nu \sim \nu_s \) (sometimes \( \sim 2\nu_s/3 \)). However, if we enforce the resonance strictly to occur at marginally stable circular orbit, then it occurs in the nonlinear regime with
\( \Delta \nu \lesssim \nu_s/2 \) (sometime \( \sim \nu_s/5 \)) and \( \nu_z - \nu_r \lesssim \nu_s \) for \( n = m = 1 \). In principle, there may be possible resonances with other combination of \( n \) and \( m \) (e.g. \( n = 2, m = 1 \)) which are expected to be weak to observe.

Once we know the spin frequency \( \nu_s \) of a neutron star from observed data we can determine specific angular momentum \( a \) (spin parameter) with the information of equatorial radius \( R \), spin frequency \( \nu_s \), mass \( M \), radius of gyration \( R_G \). If we consider the neutron star to be spherical in shape with equatorial radius \( R \), then the moment of inertia and spin parameter are computed as \( I = MR_G^2 \), \( a = I\Omega_s c/2GM^2 \), where \( \Omega_s = 2\pi\nu_s \), \( G \) is the Newton’s gravitation constant and \( c \) is speed of light. We know that for a solid sphere \( R_G^2 = 2R^2/5 \) and for a hollow sphere \( R_G^2 = 2R^2/3 \). However, in practice for a neutron star \( R_G^2 \) should be in between. Moreover, the shape of a very fast rotating neutron star is expected to be deviated from spherical to ellipsoidal. Hence, in our calculation we restrict \( 0.35 \leq (R_G/R)^2 \leq 0.5 \).

On the other hand, for a black hole QPO \( a \) is the most natural quantity what we supply as an input. The corresponding angular frequency of a test particle at the radius of marginally stable circular orbit \( r_{ms} \) in spacetime around it is then given by \[ \Omega_{BH} = 2\pi\nu_s = -\frac{g_{\phi r}(r = r_{ms})}{g_{\phi\phi}(r = r_{ms})} \frac{2a}{r_{ms}^3 + r_{ms}a^2 + 2a^2}, \]

where and light inside \( r_{ms} \) is practically not expected to reach us.

**RESULTS**

**Neutron stars**

The choice of mass \( M = 1.4M_{\odot} \) and \( (R_G/R)^2 = 0.4 \) does not suffice the observation mostly. Hence, we consider other values of parameters given in TABLE 1.

For 4U 1636-53, our theory has an excellent agreement with observed data in accordance with realistic EOS \[ \text{[14]} \] shown in Fig. 2a and given in TABLE 1. On the other hand, for 4U 1702-429, if the star is considered to be an ellipsoid and/or not to be a solid sphere and/or to have mass \( M < 1.4M_{\odot} \), then our theory has an excellent agreement with observed data with realistic \( R \) \[ \text{[14]} \] shown in Fig. 2b. Similarly for other neutron stars showing twin kHz QPOs, results from our model have good agreement with observed data (not discussed in the present paper but would be described elsewhere in future), which are the beyond scope to discuss in the present paper.

**Estimating spin of Sco X-1**

The spin frequency of Sco X-1 is not known yet. We compare in Fig. 2c the observed variation of frequency separation as a function of lower QPO frequency with that obtained from our model. We find that mass of Sco X-1 must be less than \( 1.4M_{\odot} \) and results with sets of inputs with smaller \( \nu_s \) and \( M \) fit the observed data better and argue that Sco X-1 is a slow rotator with \( \nu_s \sim 280 - 300 \).
FIGURE 2. Variation of QPO frequency difference in a pair as a function of lower QPO frequency for (a) 4U 1636-53, (b) 4U 1702-429, (c) Sco X-1. Results for parameter sets given in TABLE 1 from top to bottom row for a particular neutron star correspond to solid, dotted, dashed (4U 1702-429 and Sco X-1), long-dashed (Sco X-1) lines. The triangles are observed data points.

TABLE 1. \( \nu_s \) is given in unit of Hz, \( M \) in \( M_\odot \), \( R \) in km, radial coordinate where QPO occurs, \( r_{QPO} \), in unit of Schwarzschild radius.

| neutron star | \( \nu_s \) | \( M \) | \( (R_G/R)^2 \) | \( n, m \) | \( R \) | range of \( r_{QPO} \) |
|--------------|------------|--------|----------------|--------|--------|----------------|
| 4U 1636-53   | 581.75     | 1.4    | 0.5            | 1      | 16.5   | 6.7 – 7.7     |
| 4U 1636-53   | 581.75     | 1.1    | 0.4            | 1      | 14.3   | 8.2 – 9.3     |
| 4U 1636-53   | 581.75     | 1.2    | 0.35           | 1      | 16.8   | 7.7 – 8.7     |
| 4U 1702-429  | 330.55     | 1.0    | 0.5            | 2      | 18.8   | 10.2 – 11     |
| 4U 1702-429  | 330.55     | 0.83   | 0.35           | 2      | 18.5   | 11.7 – 12.5   |
| estimated \( \nu_s \) | | | | | | |
| Sco X-1     | 422.0      | 0.9    | 0.5            | 1      | 17.4   | 9.4 – 12.5    |
| Sco X-1     | 540.0      | 1.2    | 0.4            | 1      | 16.0   | 8.1 – 9.1     |
| Sco X-1     | 280.0      | 0.8    | 0.5            | 2      | 17.5   | 11.1 – 14.4   |
| Sco X-1     | 292.0      | 0.81   | 0.35           | 2      | 18.6   | 11.2 – 14     |

Black holes

The possible mass or range of mass of several black holes is already predicted from observed data. However, the spin of them is still not well established. By supplying predicted mass and arbitrary values of \( a \) our theory reproduces observed QPOs for GRO J1655-40 and GRS 1915+105 with their 3 : 2 ratio for \( n = m = 2 \) at a radius outside \( r_{ms} \) given in TABLE 2. However, if we enforce QPOs to produce at \( r_{ms} \) strictly, then they produce at a higher \( a \) for \( n = m = 1 \). Similarly, results from our model for other black holes showing twin kHz QPOs have good agreement with observed data (would be discussed elsewhere in future), which are the beyond scope to discuss in the present paper.
TABLE 2. \( v_l, h \) are given in unit of Hz, \( M \) in \( M_\odot \), \( r_{QPO} \) and its distance from marginally stable orbit, \( \Delta r \), are expressed in unit of Schwarzschild radius.

| black hole \( M \) | \( a \) | \( v_l \) | \( v_t \) | \( r_{QPO} \) | \( \Delta r \) | \( n.m \) |
|----------------------|--------|--------|--------|---------------|----------------|--------|
| GRO J1655-40         | estimated | theory/observation | theory/observation |             |               |        |
| 6 – 7                | 0.737 – 0.778 | 450/450  | 300/300  | 4.93 – 4.25  | 1.71 – 1.23    | 2      |
| 7.05                 | 0.95    | 451.31/450 | 299.04/300 | 1.94          | 0              | 1      |
| GRS 1915+105         | estimated | theory/observation | theory/observation |             |               |        |
| 10 – 20              | 0.606 – 0.797 | 168/168  | 113/113  | 7.38 – 3.9    | 3.58 – 0.98    | 2      |
| 18.4                 | 0.95    | 167.35/168 | 114.61/113 | 1.94          | 0              | 1      |

**SUMMARY**

We have prescribed a global QPO model based on nonlinear resonance mechanism in accretion disks. Based on this we have predicted the spin parameter/frequency of compact objects. The model has addressed, for the first time to best of our knowledge, the variation of QPO frequency separation in a pair as a function of the QPO frequency itself for neutron stars successfully. We argue that Sco X-1 is a slow rotator.

We have addressed QPOs of black holes as well and predict their spin parameter (\( a \)) which is not well established yet. According to the present model, none of them is an extremally rotating black hole. As our model explains QPOs observed from several black holes and neutron stars including their specific properties, it favors the idea of QPOs to originate from a unique mechanism, independent of the nature of compact objects.

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