An analytical model of subcritical and critical vapor-liquid flow through a granular bed

E A Tairov and P V Khan
Melentiev Energy System Institute, Lermontova 130, Irkutsk, Russia
tairov@isem.irk.ru

Abstract. This paper presents a computational and experimental justification of a theoretical model for the two-phase vapor-liquid flow through a fixed bed of solid particles. The theoretical description is based on the equations of gas dynamics of the granular bed and the homogeneous model of a single-component two-phase flow, given the difference between the velocities of liquid and vapor phases. Relationships between the phase slip ratio and polytropic coefficient of isenthalpic expansion of vapor-liquid flow were obtained using the multi-parameter nonlinear regression. We used the experimental data on the vapor-liquid flow with an inlet pressure, \( P_1 \), of 0.6-1.2 MPa and an inlet flow quality, \( x_1 \), of 0.016-0.178 through a \( H=50-355 \) mm bed consisting of steel spheres, 2 and 4 mm in diameter, \( d \). An expression for the critical pressure ratio in the granular bed is proposed. The developed model makes it possible to predict the values of mass velocity of the mixture over the entire range of experimental data with an average error of 3%. Based on the model equations, we obtained expressions for dimensionless mass velocity depending on the pressure ratio that can be applied to two-phase flows through both granular beds and other porous media.

1. Introduction

The study of a two-phase flow through porous media is of interest in such technical fields as design of chemical catalytic reactors, high-efficient heat exchangers, reactors with spherical coated fuel particles; analysis of nuclear reactor safety; technological advancement of oil and gas production. A major task of fluid mechanics and heat exchange is to establish relation between the flow rate and pressure drop. Most of the theoretical models of the two-phase flow are based on generalization of Darcy’s law [1, 2] and the binomial Forchheimer-Ergun equation [3, 4], as well as the equations including interfacial resistance [5, 6]. In all cases the two-phase flow is considered at the mass velocities below their maximum values. The maximum vapor-liquid flow rates, which lead to two-phase flow choking, are calculated, as a rule, as an independent problem. The main results were obtained on the basis of theoretical and experimental studies of critical two-phase flows through long and short pipes, orifices, nozzles, gates and safety valves. The applied theoretical models, which differ in the detail of phenomenon description, are presented in the state-of-the-art papers [7–9]. The slip models for rather long channels are noted to agree well with the experiment in the critical flow rate. The problem is reduced to selection of the method for determination of the specific mixture volume.

The problems of gas-dynamic flow choking in granular media were studied in the monograph by M.A. Gol’dshitik [10], who obtained an analytical expression for the maximum ideal gas flow rate. The flow to vacuum was considered as a condition for the critical flow rate. Generalization of Gol’dshitik’s model to a two-phase vapor-liquid flow is presented in [11]. It successfully described the only available experimental data on the critical flow through fixed beds of spherical particles [12].
This paper generalizes the results presented in [11] to describe subcritical and critical conditions of two-phase vapor-liquid flows through the close-packed beds of solid particles on the basis of the unified theoretical model.

2. Mathematical model of vapor-liquid flow

The critical and subcritical vapor-liquid flow through the bed of fixed spherical particles are described using the equations of gas dynamics of the granular bed [10]. The main equations comprise the motion equation

\[ \left[ 1 + \frac{4}{3} (1 - m) \right] \rho \frac{d \rho \nu_m}{dy} = - \frac{d P}{dy} - \frac{3}{2} m (1 - m) \rho \nu_m^2, \]

the continuity and polytropic equations

\[ \rho \nu_m = \rho_1 \nu_{1,m}, \]
\[ \frac{P}{P_1} = \left( \frac{\rho}{\rho_1} \right)^n. \]

Here \( m \) is the average porosity, \( \nu_m \) is the volume-averaged velocity of vapor-liquid flow in the pore space, \( d \) is the diameter of a spherical particle, \( \psi = 0.508 + 0.56(1 - m) \) is the relative minimum flow cross-section.

Integration of these equations by the bed height from 0 to \( H \) on the assumption that \( d \ll H \) results in the following expression for the mass velocity:

\[ \rho \nu_m = \left[ \frac{2n}{3(n+1)} \frac{\psi}{H} \frac{\nu_{1,m}}{P_1 \rho_1} \left( 1 - \left( \frac{P_2}{P_1} \right)^\frac{n-1}{n} \right) \right]^{0.5}. \]

Here \( P_1, \rho_1 \) are the pressure and density at \( y = 0 \); \( P_2 \) is the pressure at \( y = H \). The maximum velocity can be obtained by setting \( P_2 \) to zero.

In view of generalization of the presented model in [11] to a two-phase flow, we assume that the vapor-liquid flow through the bed is isenthalpic, and that the thermodynamic mixture state is subject to the polytropic equation. Phase temperatures are set equal and velocities of vapor and liquid are different. A vapor component moves faster than a liquid one, and the ratio of their velocities, \( s = \nu'_m / \nu_m \), called a slip ratio, has a significant effect on the void fraction:

\[ \varphi = \left[ 1 + s \frac{\rho'}{\rho} \frac{1-x}{x} \right]^{-1}, \]

which in turn determines the mixture density:

\[ \rho = \rho' (1 - \varphi) + \rho' \varphi. \]

The polytropic coefficient \( n \) and the slip ratio \( s \) in equations (3)–(5) are determined by generalization of experimental data obtained in [12] in the range of inlet pressures from 0.6 to 1.2 MPa, inlet flow quality \( x_1 \) from 0.016 to 0.178, and mass velocity from 150 to 1200 kg/(m²s). They are represented by the formulas:

\[ n = 0.43 + 0.57(1 - \exp(-x_1/0.16)), \]
\[ s = 1 + a_s(P_1) \exp \left( - \frac{b_s(P_1)}{x_1} + \frac{c_s(P_1)}{1-x_1} \right), \]

where

\[ a_s(P_1) = 16.0 + 39.5 P_1 - 42.4 P_1^2; \]
\[ b_s(P_1) = 0.0175 - 0.0123 P_1 - 0.00249 P_1^2; \]
\[ c_s(P_1) = -2.30 + 11.5 P_1 - 8.04 P_1^2. \]

As follows from equation (7), the polytropic coefficient does not depend on bed parameters and the inlet pressure, and is a function of the inlet flow quality solely.
3. Results and Their Discussion

In this work the model presented by equations (3)–(8) was adapted to description of both the critical and subcritical mass velocity values, which were calculated in a set of experiments with the beds of steel spheres, 2 and 4 mm in diameter and a bed height of 50, 100, 250, 355 mm. The pressure at the bed inlet was 0.6, 0.9 and 1.2 MPa, the inlet flow quality changed from 0.016 to 0.178, the measured mass velocity values were within the range from 150 to 1200 kg/(m²s). Dependence of the mass velocity on the pressure drop in the bed was determined based on 54 processed conditions. The critical mass velocity was obtained by gradual decrease of the outlet pressure to the air pressure at the fixed values of inlet pressure and inlet flow quality. With decrease in the ratio between the outlet and inlet pressures, β = P₂/P₁, below the critical pressure (or otherwise, with increase in the pressure drop, ΔP = P₁ − P₂, above the critical pressure) the mass velocity took its maximum value and became insensitive to further decrease of the pressure P₂ beyond the bed.

Analysis of the experiments reveals that the critical pressure ratio βcr = P₂cr/P₁, depending on experiment conditions, can take different values ranging from 0.15 to 0.36. Still, βcr does not practically depend on bed parameters, and decreases with increase of P₁ and inlet flow quality x₁. The least value, βcr = 0.145, is obtained at P₁ = 1.2 MPa and x₁ = 0.178. At the same pressure and lower flow qualities that are characterized by the values of x₁ = 0.033–0.055, βcr = 0.25. The decreasing effect of x₁ on the critical pressure ratio is also observed in the study of highly wet vapor flow rates through the Laval nozzle [13, 14]. As a result of processing available experimental data on reaching a critical flow through the beds the approximate formula for βcr is written as:

\[ β_{cr} = 0.145P_1^{-0.822}x_1^{-0.0813}. \] (9)

Formula (4) for critical conditions using βcr takes the form

\[ (\rho w_m)_{cr} = \left[ \frac{2n \cdot d \cdot \psi}{3(n+1) \cdot H \cdot m(1-m)} \cdot P_1 \cdot (1 - β_{cr}^{n+1}) \right]^{0.5}. \] (10)

Constitutive relations (7, 8) for the polytropic coefficient and the slip ratio are obtained by generalization of experimental results using formula (10). Note that formulas (7), (8) are related, as a rule, to the known mixture parameters at the bed inlet and applied in the range: 0.016 < x₁ < 0.178; 0.6 ≤ P₁ ≤ 1.2 MPa.

The developed mathematical model is used to generalize experimental results on the subcritical and critical vapor-liquid flows through different packed beds of spherical particles. A typical behavior of the mass velocity when reaching a critical condition with decrease in the pressure P₂ beyond the bed of particles is presented in figure 1. It indicates both experimental values and calculated relationships ρwₘ(ΔP) determined by formulas: (4) – in the subcritical condition, (10) – in the critical condition.

The dashed line in figure 1 shows critical pressure drops ΔPcr = P₁(1−βcr) corresponding to the predictions by formula (9). The theoretical model is seen to describe the studied relationships adequately enough.

Agreement between the calculation results and the experimental data at higher pressures are illustrated in figure 2. It is seen that the theoretical model generalizes experimental data obtained at different values of pressure P₁, particle diameter d and bed height H in the subcritical and critical conditions with high accuracy. The determination coefficient R² of the model is equal to 97% for the critical mass velocity, and 98% for the subcritical mass velocity.
Figure 1. Mass velocity depending on the pressure drop for \( P_1 = 0.6 \text{ MPa} \) (a) \( d = 2 \text{ mm}, H = 100 \text{ mm} \); (b) \( d = 4 \text{ mm}, H = 250 \text{ mm} \). Dots are experiment with the flow quality \( x_1 \): □ – 0.016; × – 0.022; ■ – 0.033; * – 0.055; Δ – 0.096; ▲ – 0.178. Solid lines show calculated values; dashed line shows critical condition boundary.

We introduce a relative dimensionless mass velocity via the ratio of expressions (4) and (10). For any fixed solid beds at equal parameters of the two-phase flow at the inlet this indicator depends only on \( \beta, n \) and \( \beta_{cr} \):

\[
PW_m = \left( \frac{\rho w_m}{\rho w_{m,c}} \right)_{cr} = \left( 1 - \frac{n+1}{\beta_n} \frac{1}{1 - \frac{\beta_{cr}}{\beta_n}} \right)^{0.5}.
\]

Figure 2. Calculated and experimental values of the mass velocity for \( d = 2 \) and 4 mm, \( H = 250 \) and 355 mm, \( 0.016 < x_1 < 0.178 \) (a) \( P_1 = 0.9 \text{ MPa} \); (b) \( P_1 = 1.2 \text{ MPa} \): + – subcritical values, □ – critical values.

Besides, \( n \) and \( \beta_{cr} \) at the fixed pressure \( P_1 \) at the bed inlet are functions of only the inlet mixture flow quality. In this case, formula (11) may be applied to generalize mass velocity data obtained for different types of beds at equal flow qualities \( x_1 \). Generalization of experimental results to all beds and some values of the inlet flow quality and the pressure is shown in figure 3. The left margin of the lines is limited by the corresponding value of \( \beta_{cr} \).
In the experiments, the critical pressure ratio is essentially lower than unity and decreases with the increasing inlet pressure $P_1$. The polytropic coefficient is also lower than unity. Both stipulate a very low value: $\beta_{cr}^{(n+1)/n} \ll 1$. Neglect of its low value in equation (11) yields the formula:

$$\left( \rho_{m,cr} \right) = \frac{\rho_{m}}{\left( 1 - \beta^{n} \right)^{0.5}}.$$  (12)

The value of $\left( \rho_{m,cr} \right)$ calculated by the approximate formula (12) is only by 0.3–2.3% higher than by formula (11). Availability of several measurements of $\rho_{m}(\Delta P)$ allows us to eliminate the critical condition parameters using formula (12) and to construct the entire curve for the mass velocity in the subcritical flow.

Formula (12) is suitable for the experimental study of two-phase flows in granular media with a complex internal structure, whose porosity and dominant diameter of grains are difficult to determine.

4. Conclusions
We developed a mathematical model of the two-phase vapor-liquid flow through the random bed of solid particles. The model is an extension of the theoretical model, proposed by M.A. Gol’dshlí to describe the ideal gas flow, to a two-phase flow. The average mixture density is calculated subject to the phase slip. The slip parameter and the polytropic coefficient are determined using the experimental data. The developed model allows generalizing the experimental results on the critical and subcritical vapor-liquid flows through different spherical beds with reasonable accuracy. The proposed approach may be applied to generalization of the experimental data on flows through granular beds with a complex internal structure.

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