Neutron star as a mirror for gravitational waves

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Abstract Gravitational wave (GW) has become one of the most active fields in physics and astronomy since the first direct detection of GW event in 2015. As is well known, multiple images of GW events are possible through the gravitational lenses. Here, we propose a novel mirror imaging mechanism for GW events different from the gravitational lens. In the literature, the superconductor was predicted to be highly reflective mirror for GWs. It is well known that neutron stars exhibit superconductivity and superfluidity. In this work, we predict that there are two types of GW mirror imaging phenomena caused by the neutron star located in Milky Way or the same host galaxy of GW source, which might be detected within a life period of man (namely the time delay $\Delta t$ can be a few years to a few tens of years). It is expected to witness this predicted GW mirror imaging phenomenon in the near future. In the long term, the observations of this novel GW mirror imaging phenomenon might help us to find numerous neutron stars unseen by other means, and learn more about the complicated internal structures of neutron stars, as well as their equations of state.

Keywords Neutron star · Gravitational wave · Mirror imaging phenomenon · Gravitational lens · Superconductor

Gravitational wave (GW) is a long-standing prediction of general relativity (GR) since 1916 (Einstein 1916; Einstein 1918; Einstein and Rosen 1937; Cervantes-Cota et al. 2016). The debate about the physical reality of GW was mainly settled during the Chapel Hill conference in 1957 (Cervantes-Cota et al. 2016; Saulson 2011). In 1974, Hulse and Taylor discovered the first indirect evidence for the existence of GW from the binary pulsar PSR B1913+16 (Taylor et al. 1979; Taylor and Weisberg 1982; Lorimer 2008), and hence they earned the 1993 Nobel Prize. In 2015, the LIGO/Virgo Collaboration made the first direct observation of GW event (GW150914) from a binary black hole merger (Abbott et al. 2016a; Abbott et al. 2016b), and then earned the 2017 Nobel Prize. From the first multi-messenger observations of a binary neutron star merger (GW170817) (Abbott et al. 2017a; Abbott et al. 2017b), the speed of GW was confirmed to be the speed of light $c$ (Abbott et al. 2017c). Nowadays, GW becomes one of the most active fields in physics and astronomy.

As is well known, most objects (e.g. earth) are nearly transparent to GWs. On the other hand, the gravitational lenses (e.g. massive galaxies) can deflect GW as well as light (Schneider et al. 1992; Schneider et al. 2006). Therefore, multiple images of GW events are possible through the gravitational lenses. Here, we propose a novel mirror imaging mechanism for GW events different from the gravitational lens.

We note that in Minter et al. (2010), Chiao et al. (2009), Chiao (2007), Chiao et al. (2017) the superconducting film was predicted to be highly reflective mirror for GWs (see also e.g. Quach 2015; Inan et al. 2017; Inan 2017). Following Minter et al. (2010), Chiao et al. (2009), Chiao (2007), Chiao et al. (2017), Quach (2015), Inan et al. (2017), Inan (2017), here we present a physical picture for the possible GW mirror reflection in a superconductor. As is well known, the main effect of a GW is making the particles follow the distortion in spacetime and then float (freely fall). In a superconductor, negatively charged Cooper pairs will be formed according to the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity. The Cooper
pairs in the ground state are in an exactly zero-momentum eigenstate. So, their positions are completely uncertain, i.e., their trajectories are completely delocalized, due to the well-known Heisenberg uncertainty relation for momentum and position. Note that the quantum delocalization of Cooper pairs is protected from the localizing effect of decoherence by the BCS energy gap. As a result, Cooper pairs cannot freely fall along with the positively charged ions and normal electrons. So, in the presence of GW, Cooper pairs undergo non-geodesic motion relative to the geodesic motion of its ionic lattice. In other words, Cooper pairs in a superconductor cannot respond at all to the passage of GW, in contrast to the positive ions. This non-geodesic motion leads to the existence of mass and charge supercurrents inside a superconductor. The generation of supercurrents by GW has an important consequence, i.e., the electrical polarization of the superconductor. The resulting separation of oppositely signed charges leads to a huge Coulomb force that strongly opposes the tidal force of the incoming GW. So, this GW will be expelled and then reflected. This effect in a superconductor is known as the “Heisenberg-Coulomb effect”. We refer to Minter et al. (2010), Chiao et al. (2009), Chiao (2007), Chiao et al. (2017), Quach (2015), Inan et al. (2017), Inan (2017) for more details.

We stress that the Heisenberg-Coulomb effect cannot occur in the normal matters (Minter et al. 2010; Chiao et al. 2009; Chiao 2007; Chiao et al. 2017; Quach 2015; Inan et al. 2017; Inan 2017), because no Cooper pairs can be formed in the normal matters. According to the equivalence principle, all particles in the normal matters freely fall along geodesics (namely decoherence-induced trajectories). As a result, the normal matters cannot energetically interact with GWs, i.e., they are almost transparent to GWs. The GW reflection is impossible for the normal matters. However, as mentioned above, the quantum delocalization of Cooper pairs (which undergo non-geodesic motion) is the key to make difference in a superconductor. Thus, the reflection of GWs becomes possible only for the superconductors.

Let us further discuss some key details following (Minter et al. 2010, Chiao et al. 2009, Chiao 2007, Chiao et al. 2017). As is well known, in the limit of weak gravitational field and non-relativistic matter, Einstein’s field equations can be approximately recast as the gravitational Maxwell-like equations (Braginsky et al. 1977; Wald 1984; Harris 1991; Clark and Tucker 2000). In such a formalism, one can consider the reflectivity of the incident GW, in close analogy with the electromagnetic (EM) case. The characteristic gravitational impedance $Z_G = \sqrt{\mu_G/\epsilon_G} = 4\pi G/c \sim \mathcal{O}(10^{-18})$ SI units from the gravitational Maxwell-like equations is much less than the EM counterpart $Z_0 = \sqrt{\mu_0/\epsilon_0}$. All normal (classical) matters have extremely high levels of dissipation compared to $Z_G$, and hence they are inevitably very poor reflectors of GWs. In contrast, superconductors are effectively dissipationless at temperatures near absolute zero because of their quantum mechanical nature. The fact that superconductor’s effectively zero impedance can be much less than the very small $Z_G$ allows it to reflect an incoming GW, similar to a low-impedance connection at the end of a transmission line can reflect an incoming EM wave. One can derive the reflectivity $R_G$ for an incoming GW in close analogy with the EM case. However, the Heisenberg-Coulomb effect makes a crucial difference, i.e., an enormously enhanced interaction between the incoming GW and superconductor. The magnitude of the enhancement is given by the ratio of the electrical force to the gravitational force between two electrons, i.e., $(q^2 Z_0)/(m^2 Z_G) = e^2/(4\pi \epsilon_0 G m^2) \simeq 4.2 \times 10^{42}$. It makes $\Sigma \equiv (\sigma_1 G/\sigma_2 G)^2 = (\omega/\omega_c)^2 \ll 1$ when the angular frequency $\omega \ll \omega_c$, where $\omega_c$ is a characteristic angular frequency associated with the modified plasma and resonance frequencies of a superconducting film. Typically, $\Sigma \sim 10^{-16}$ for $\omega_c \sim 10^{16}$ rad/s and $\omega \sim \mathcal{O}(1)$ rad/s. In this case, the GW reflectivity $R_G = [(1 + \Sigma)^2 + \Sigma]^{-1} \rightarrow 1$. So, the incoming GW at angular frequency $\omega \ll \omega_c$ will be almost 100% reflected. We refer to Minter et al. (2010), Chiao et al. (2009), Chiao (2007), Chiao et al. (2017) for the detailed derivations.

However, it was found in Inan et al. (2017), Inan (2017), Inan (2018) that some technical details of Minter et al. (2010), Chiao et al. (2009), Chiao (2007), Chiao et al. (2017) are faulty (we thank the referee for pointing out this issue). Actually, the reflectivity $R_G$ in Minter et al. (2010), Chiao et al. (2009), Chiao (2007), Chiao et al. (2017) was obtained by using vector field equations, vector coupling rule, and vector constituent equation, which do not apply to GWs. Thus, the gravitational impedance is associated with local oscillating gravito-electromagnetic fields rather than GWs (Inan et al. 2017; Inan 2017; Inan 2018). Instead, to obtain the impedance associated with GWs, one should use tensor field equations, tensor coupling rule, and tensor constituent equation (we thank the referee for pointing out this issue). They are given by (Inan et al. 2017; Inan 2017; Inan 2018)

$$- \frac{1}{c^2} \partial^2_{TT} h^{TT}_{ij} + \nabla^2 h^{TT}_{ij} = -2\kappa T^{TT}_{ij},$$

$$\pi^2 = p^2 + h^{TT}_{ij} p^i p^j,$$

$$T_{ij} = \mu_G E_{ij},$$

respectively, where $h^{TT}_{ij}$ is the transverse-traceless GW field, $T^{TT}_{ij}$ is the transverse-traceless stress tensor, $\pi^j$ is the kinetic momentum (please do not confuse it with the constant $\pi$), $p^i$ is the canonical momentum, $E_{ij} = -\partial_t h^{TT}_{ij}$ is a gravito-electric tensor field, $\mu_G$ is the gravitational conductivity, and $\kappa = 8\pi G/c^4$. In this formalism, one can find that the GW
impedance is given by (Inan et al. 2017; Inan 2017; Inan 2018)

\[
Z_G^{(SC)} = \frac{4\pi G/c}{\sqrt{1 - 2c^2\kappa_G/\omega^2}}.
\]

(4)

And then, the GW reflectivity reads (Inan et al. 2017; Inan 2017; Inan 2018)

\[
\mathcal{R}_G = \left[1 + \left(\frac{c^2}{2} \frac{\omega}{\mu_G Z_G^{(SC)} d}\right)^2\right]^{-1},
\]

(5)

where \(d\) is the thickness of a thin superconducting film (by “thin” we mean small relative to the wavelength of GW). We refer to Inan et al. (2017), Inan (2017), Inan (2018) for the detailed derivations.

Due to historical reasons, mainly the GWs at microwave frequencies were considered in Minter et al. (2010), Chiao et al. (2009), Chiao (2007), Chiao et al. (2017), Quach (2015), Inan et al. (2017), Inan (2017), far before the first direct GW detection by LIGO. Unfortunately, the experiment for laboratory-scale superconducting mirror of GWs is actually difficult to achieve on earth. For a usual superconductor, it was found by Inan et al. (2017), Inan (2017) that \(\mu_G \sim 10^8 \text{ J/m}^3\), \(Z_G^{(SC)} \sim Z_G^{(\text{vacuum})} = 4\pi G/c\), and then \(\mathcal{R}_G \sim 10^{-36}\), which implies effectively there is no reflection. Obviously, from Eq. (5), substantial reflection requires

\[
\frac{c^2}{2} \frac{\omega}{\mu_G Z_G^{(SC)} d} \ll 1.
\]

(6)

Further, one can take the limit when a superconductor behaves like classical matter, and then \(\mu_G = \rho v^2/2\) which is the kinetic energy density of the material. One can even use the upper bound \(\mu_G \sim \rho c^2\). From Eq. (6), the mass density required for substantial reflection of GWs at microwave frequency is \(\rho \sim 10^{29} \text{ kg/m}^3\). It is much greater than the mass density of neutron stars \(\rho \sim 10^{17} \text{ kg/m}^3\). So, the reflection of GWs at microwave frequency is completely negligible (we thank the referee for pointing out the above issues).

From Eq. (6), the way out is to consider a lower frequency \(\omega\) (less than the microwave frequency) and a very large mass density \(\rho\) as possible, which cannot be found in terrestrial laboratories.

Let us turn eyes to the sky. It is well known that neutron stars exhibit superconductivity and superfluidity (see e.g. Lattimer and Prakash 2004; Baym and Pethick 1979; Weber 2005; Page et al. 2011; Baldo et al. 2003). Due to the huge luminosity distances of GW sources (namely \(O(10^2)\) Mpc, or, \(O(10^8)\) light years, see e.g. Abbott et al. 2016a; Abbott et al. 2016b; Abbott et al. 2017a; Abbott et al. 2017b; LIGO 2019; Abbott et al. 2019) and the small size of neutron stars (namely \(O(10)\) kilometers, see e.g. Lattimer and Prakash 2004; Baym and Pethick 1979), it is safe to ignore the spherical shapes of neutron stars and their complicated internal structures, and hence one can simply regard them as superconducting films. For neutron stars with \(\rho \sim 10^{17} \text{ kg/m}^3\) and \(d \sim 10^4 \text{ m}\), one can find from Eq. (6) that substantial reflection is only possible for GWs at angular frequency \(\omega \ll 10^6 \text{ rad/s}\) (we thank the referee for pointing out this issue). Note that GWs could be detected by LIGO/Virgo are at frequencies 1 Hz to 100 Hz, and hence the angular frequency \(\omega \lesssim 700 \text{ rad/s} \ll 10^6 \text{ rad/s}\). In this case, the GW reflectivity \(\mathcal{R}_G \to 100\%\). Therefore, neutron stars can play the role of plane mirrors for GWs at low frequencies.

In principle, the positions of neutron stars will be perturbed when GWs go to them. But, such kind of perturbations can also be safely ignored with respect to the huge luminosity distances of GW sources.

By estimating the number of stars that have undergone supernova explosions, there are thought to be around \(10^8\) neutron stars in Milky Way (Camenzind 2007). But most of them are old, cold and hence almost undetectable. It is easy to imagine that numerous neutron stars are distributed over the universe. Although most of them cannot be seen by other means, neutron stars might manifest themselves in the mirror reflection of GWs.

At first, let us consider a simple and naive case given in Fig. 1 (not to scale). We suppose a transient GW event “S” which might be binary neutron star (BNS) or neutron star – black hole (NSBH).

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1Note that the neutron star “N” as a mirror for GWs is clearly not the one in the GW source “S” (which might be binary neutron star (BNS) or neutron star – black hole (NSBH)).
is an isosceles triangle SNO can reflect the GW from S going to it, and hence the observer O will detect a secondary GW signal later. Thus, O sees a mirror image “S′” of the GW source S. The luminosity distances satisfy $d_{OS} \cos \theta = d_{OS}$. The intensity of the image S′ divided by the intensity of S equals to $(d_{OS}/d_{OS})^{-2} = (\cos \theta)^2$. For example, $(\cos \theta)^2 = 3/4$ and $1/2$ even for $\theta = 30^\circ$ and $45^\circ$, respectively. Thus, the second GW signal is still detectable even if the angle $\theta$ is fairly large. Note that the allowed large angle $\theta$ between S and S′ makes the mirror phenomenon different from the tidal Loving lens whose Einstein angle is typically small. It is of interest to calculate the interval between the arrival times of GW signals S and S′, which is given by $\Delta t = t_S - t_S = (t_S/t_S - 1)t_S = (d_{OS}/d_{OS} - 1)t_S = ((\cos \theta)^{-1} - 1)t_S$. Unfortunately, for a not so small angle $\theta$, the arrival time interval $\Delta t$ is on the order of $t_S \sim O(10^5)$ years. This is far beyond the patience. If we hope to detect both GW signals S and S′, the angle $\theta$ must be very small. Noting that $\Delta t \simeq \theta^2 t_S/2$ for $\theta \rightarrow 0$, we find that $\theta \lesssim 10^{-3}$ or $10^{-4}$ is required by $\Delta t \lesssim 10^2$ years or 10 years, respectively. This means that the host galaxy of neutron star N lies in the middle of the line of sight from O to S. It will play the role of gravitational lens. Thus, the mirror imaging phenomenon will be destroyed by the host galaxy of neutron star N in this simple and naive case. Clearly, in the previous case it is too special that S, N, O form an isosceles triangle. Noting that any observer O on the extension line of S′N can always see the mirror image S′, we can consider the cases with $d_{ON} \ll d_{SN}$, i.e. the neutron star N is much closer to O, as shown in Fig. 2 (not to scale). In fact, N can be located in the same host galaxy of O (namely Milky Way). As is well known, the diameter of Milky Way is $1.5 \sim 2 \times 10^5$ light years (Lopez-Corredoira et al. 2018), and hence $d_{ON} \ll 2 \times 10^5$ light years. However, the neutron star N cannot be too close to earth (O), otherwise the humankind and other lives on earth will all be killed by it. To our best knowledge, the nearest neutron star to earth is claimed to be Calvera (1RXS J141256.0+792204) at the distance around $250 \sim 1000$ light years (Rutledge et al. 2008). Other two closest known neutron stars are RX J1856.5−3754 and PSR J0108−1431, which are both about 400 light years from earth (Posselt et al. 2009). So, we take $d_{ON} \gtrsim 250$ light years. There are mainly three types of configurations for O and N, as shown in Fig. 2. The case (2b) is fairly special, and it becomes possible because earth is nearly transparent to GWs. The mirror image S′ is in the opposite direction of the GW source S. In this case, the interval between the arrival times of GW signals S and S′ is $\Delta t = 2d_{ON}/c \gtrsim 500$ years. We turn to the case (2c) with the angle $\theta \geq 90^\circ$. In this case, $\Delta t = (d_{SN} + d_{ON} - d_{OS})/c > d_{OS}/c \gtrsim 250$ years. Thus, both the arrival time intervals $\Delta t$ of cases (2b) and (2c) are far beyond a life period of man (namely a few tens of years). The last hope lies in the case (2a) with the angle $\theta < 90^\circ$. Obviously, $\Delta t = (d_{SN} + d_{ON} - d_{OS})/c \rightarrow 0$ if $\theta \rightarrow 0$. However, we should avoid $\theta \rightarrow 0$, otherwise the GW from S might be reflected before it could reach the observer O. In fact, a large $\theta$ is possible in the case (2a). Using the law of cosines to the triangle SON, we have $\cos \theta = (d_{OS}^2 + d_{ON}^2 - d_{SN}^2)/(2d_{OS}d_{ON}) = (d_{OS}^2 + d_{ON}^2 - (d_{SN} + c\Delta t)^2)/(2d_{OS}d_{ON}) \simeq 1 - c\Delta t/d_{ON}$ for $d_{ON} \ll d_{OS}$. So, we obtain $\Delta t \simeq (1 - \cos \theta) d_{ON}/c$. If $d_{ON}$ is 250 light years, $\Delta t \simeq 0.04, 0.95, 3.8, 8.5, 15.1, 33.5, 47.7$ and 73.2 years for $\theta = 1^\circ, 5^\circ, 10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ, 35^\circ, 40^\circ, 45^\circ$. 

Fig. 2 The cases of neutron star as a mirror for GWs with luminosity distances $d_{ON} \ll d_{SN}$, namely the neutron star N is much closer to the observer O on earth. The plots are not to scale. See the text for details.
30°, 36° and 45°, respectively. If \( d_{SN} \approx 1000 \text{ light years} \), \( \Delta t \approx 0.15, 3.8, 15.2, 34.1 \text{ and 60.3 years for } \theta = 1°, 5°, 10°, 15° \text{ and 20°, respectively. Thus, we can see the mirror image } S' \text{ within a life period of man (namely the time delay } \Delta t \text{ can be a few years to a few tens of years) while the angle } \theta \text{ can still be fairly large. On the other hand, the intensity of the image } S' \text{ divided by the intensity of } S \text{ equals to } (d_{OS}/d_{OS})^{-2} = (1 + c \Delta t/d_{OS})^{-2} \approx 1 \text{ for } c \Delta t \ll d_{OS}. \) So, we will see two GW signals \( S \) and \( S' \) with almost same intensities and wave forms.

Similarly, we can also consider the cases with \( d_{SN} \ll d_{ON} \), namely the neutron star \( N \) is much closer to the GW source \( S \), as shown in Fig. 3 (not to scale). In fact, \( N \) can be located in the same host galaxy of \( S \). Noting that the remnants of most GW events are black holes which can also swallow GWs, in Fig. 3 there is no counterpart to the case (2b). Since we need not to care the alien lives, the neutron star \( N \) can be very close to \( S \), unlike the cases in Fig. 2. In fact, \( d_{SN} \) can be a few light years, or a few tens of light years. Because \( d_{SN} \ll d_{ON} \sim d_{OS} \), it is easy to see that the angle \( \theta \) must be very close to 0 (on the order of \( d_{SN}/d_{OS} \sim 10^{-8} \) to \( 10^{-7} \)). Thus, from the viewpoint of the observer \( O \) on earth, the mirror image \( S' \) is almost in the same direction of the GW source \( S \). But this does not mean that the neutron star \( N \) must lie in the line \( OS \) or its extension line, from the local viewpoints of \( S \) and \( N \), as will be shown below. In the case (3a) with the angle \( \phi \approx 90° \), following the similar derivations in the previous case (2a), we have \( \cos \phi \approx 1 - c \Delta t/d_{SN} \). Clearly, the angle \( \phi \) can be large if \( c \Delta t \) is comparable to \( d_{SN} \). So, the neutron star \( N \) does not lie in the line \( OS \) actually (although \( \theta \to 0 \) due to the huge \( d_{OS} \)). Because \( c \Delta t \approx (1 - \cos \phi) d_{SN} \), we can see the mirror image \( S' \) within a life period of man (namely \( \Delta t \) can be a few years to a few tens of years) if the neutron star \( N \) is close enough to the GW source \( S \) while the angle \( \phi \) can still be fairly large. Note that even if \( N \) is not so close to \( S \), \( \Delta t \) can still be a few years to a few tens of years for a suitable and large \( \theta \) (see the case (2a)). Similarly, in the case (3b) with the angle \( \phi \geq 90° \) (and hence the angle \( \psi \approx 90° \)), we have \( c \Delta t \approx (1 + \cos \psi) d_{SN} \). The angle \( \psi \) can be large, so that the neutron star \( N \) does not lie in the extension line of \( OS \) actually (although \( \theta \to 0 \) due to the huge \( d_{OS} \)). Again, we can see the mirror image \( S' \) within a life period of man (namely \( \Delta t \) can be a few years to a few tens of years) if the neutron star \( N \) is close enough to the GW source \( S \) while the angle \( \psi \) can still be fairly large. On the other hand, in both cases (3a) and (3b), the intensity of the image \( S' \) divided by the intensity of \( S \) equals to \( (d_{OS}/d_{OS})^{-2} = (1 + c \Delta t/d_{OS})^{-2} \approx 1 \) for \( c \Delta t \ll d_{OS} \). So, we will see two GW signals \( S \) and \( S' \) with almost same intensities and wave forms.

Besides the three cases shown in Figs. 1–3 (not to scale), the rest is the case with \( d_{SN} \) comparable to \( d_{ON} \). It is easy to see that this case is quite similar to the simple and naive case in Fig. 1, and hence is also inviable. It is worth noting that the main reason for the failure of the cases with \( d_{SN} \) comparable to \( d_{ON} \) (including the simple and naive case in Fig. 1) is either \( \Delta t \sim t_s \sim O(10^8) \) years if the angle \( \theta \) is not so small, or the host galaxy of neutron star \( N \) must play the role of gravitational lens to destroy the mirror imaging phenomenon if the angle \( \theta \) is very close to 0. In fact, the
latter might be evaded for an isolated neutron star N without a host galaxy, which might be a wandering star in the intergalactic space been kicked out of its original position due to some unknown reasons. However, this must be extremely rare, and hence we do not consider this possibility here.

In summary, we predict that there are two types of GW mirror imaging phenomena caused by the neutron star located in Milky Way (case (2a)) or the same host galaxy of GW source (cases (3a) and (3b)), which might be detected within a life period of man (namely the time delay $\Delta t$ can be a few years to a few tens of years). In both types of GW mirror imaging phenomena, we will see two GW signals $S$ and $S'$ with almost same intensities and wave forms. The separate angle $\theta$ between the GW source $S$ and its mirror image $S'$ can be fairly large in case (2a), while $\theta$ should be fairly close to 0 in cases (3a) and (3b). Noting that GWs became directly detectable since 2015, we hope to witness this predicted GW mirror imaging phenomenon in the near future.

In fact, the GW mirror imaging phenomenon predicted here is different from the well-known gravitational lenses (Schneider et al. 1992; Schneider et al. 2006) which can also image GWs. In the mirror imaging phenomenon, we will see the signal from the GW source $S$ directly, and after a time delay $\Delta t$ we will see a second GW signal from the unique mirror image $S'$. On the contrary, in the case of gravitational lens, one will see more than (or equal to) two images, or even rings and arcs. In the mirror imaging phenomenon might be detected within a life period of man, we will see two GW signals $S$ and $S'$ with almost same intensities and wave forms. On the contrary, in the case of gravitational lens, the images should be magnified (and/or distorted). Also, in the case of gravitational lens, the separate angle $\theta$ (Einstein angle) between images is very small (usually a few arcseconds, namely $\theta \sim 10^{-5}$), but in the mirror imaging phenomenon, the separate angle $\theta$ between the GW source $S$ and its mirror image $S'$ can be fairly large in case (2a). Finally, the time delay $\Delta t$ is usually short (a few days to a few years) in the case of gravitational lens, while the time delay $\Delta t$ could be relatively long (a few years to $O(10^5)$ years) in the mirror imaging phenomenon.

Note that the GW mirror imaging phenomenon predicted here is also different from other exotic phenomena like Poisson-Arago spot (Zhang and Fan 2018) or intensification (Halder et al. 2019) for GWs.

Clearly, the discussions in the present work are very preliminary. Many simplifications were made. For example, we ignored the spherical shape of neutron star and its complicated internal structures. Also, we have not calculated the realistic GW reflectivity $R_G$ for an impure superconducting solid sphere. On the other hand, the event rate of this GW mirror imaging phenomenon was not estimated. However, the present preliminary work has clearly shown the key idea of this GW mirror imaging phenomenon, and made clear predictions which might be detected in the near future. This is very important.

Actually, this GW mirror imaging phenomenon might not only be detected in the future, but also be used to understand the past. For example, it has been employed in Wei and Feng (2020) to explain the null result of searching the electromagnetic counterparts for the first high-probability neutron star–black hole (NSBH) merger LIGO/Virgo GW190814 (GCN 2019). It is nothing but a GW mirror image of the real NSBH merger before 14 September 2015. This could be regarded as a useful support to the GW mirror imaging mechanism.

We hope to construct some full and realistic models for this predicted GW mirror imaging phenomenon caused by neutron stars in the future works. In the long term, the observations of this novel GW mirror imaging phenomenon might help us to find numerous neutron stars unseen by other means, and learn more about the complicated internal structures of neutron stars, as well as their equations of state.

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