Novel Positioning Algorithm
Using A GNSS Satellite and Two Ground Receivers

Hironobu Onishi 1) Hiroyuki Hatano 2) Yoshihiko Kuwahara 3)
1) Shizuoka University, Graduate School of Engineering
3-5-1 Johoku, Naka-ku, Hamamatsu-shi, Shizuoka, 432-8561, Japan (E-mail: onishi@hatanolab.eng.shizuoka.ac.jp)
2)-3) Shizuoka University, Faculty of Engineering
3-5-1 Johoku, Naka-ku, Hamamatsu-shi, Shizuoka, 432-8561, Japan (E-mail: hatano@ieee.org)
Received on September 15, 2011

ABSTRACT: In this paper, we will present a novel positioning algorithm of ground receivers in urban area. Currently, position estimation algorithms using four GPS satellites are most popular. However, because of the influence of signals shaded by buildings and multipath, measurement error becomes large. For accurate estimation, we discuss the position estimation method which doesn’t depend on the GPS satellites too much. Our approach is to reduce the necessary number of the satellites. Moreover, our method can apply to Quasi-Zenith Satellite Systems effectively. In this paper, we introduce our proposal which can estimate by only one satellite and performance by computer simulations.

KEY WORDS: information, communication, and control, vehicle navigation system / communication system, portable navigation device / personal navigation device, navigation system / GPS, multipath, position estimation

1. Introduction

1.1. Personal movement for next generation and positioning

For next generation, worldwide interest in ITS (Intelligent Transport Systems) has been growing. ITS is good for the ecology, safe transportation, and convenient life. Especially, traffic problems such as car accidents, traffic jam can be solved. In Japan, ETC (Electronic Toll Collection) becomes a great success for common use. Moreover, integrated cars which have a radar or a stereo camera come to market. In the future, AHS (Automated Highway Systems) will be able to achieve safe driving technically. In this study, we consider personal transportation for next generation in the stream of ITS society.

Recently, electric vehicles and hybrid cars get the attention instead of cars using fossil fuel. In case of electric cars, the number of car parts can be greatly reduced. Because of possible to miniaturize components and save environment, personal small vehicles have been remarked. Segway and Winglet are famous as the examples (1). These vehicles are named as town mobility or city commuter. These vehicles can improve utility by relating own locations to the near information in case of considering transportation in urban area. For example, energy loss can be decreased by calculating suitable route from car location. Furthermore, efficient movement can be possible and local information services can be achieved by obtaining beneficial information of near their locations. Thus, advanced movements and support can be possible by connecting the location information with the vehicles in urban area. Therefore, it is important to discuss positioning algorithm for own accurate location.

1.2 Technology of positioning algorithm

The typical estimation method is GPS (Global Positioning System) (2). GPS estimates positions by calculating pseudorange based on signals from four GPS satellites. Navigation systems which are widely used for searching a route to a destination use the above GPS estimation. In urban area, estimation error becomes about 10m by positioning using GPS satellites. For accurate estimation, various technologies have been examined. For example, the following are enumerated.

(1) D-GPS (Differential GPS)
(2) RTK-GPS (Real Time Kinematic GPS)
(3) Map matching method

D-GPS and RTK-GPS are the methods of attempting the error reduction by known stations mounted on the ground. The position of the receiver is improved by the measured error at the known stations. The Map matching method is popular for car navigation systems. This method corrects the estimated position to the near road where the user lies (3). These techniques can suppress the error to several m or cm (4)(5).

1.3. Problem

The above accuracy improvement technologies can obtain good characteristic under good condition. However, in case of the lack of the map data and the reference points in correction data, it is difficult to correct estimated positions.
Especially, we assume personal small vehicles for urban area. Personal small vehicles are used conveniently in urban area. In urban area, it is difficult to catch adequate number of GPS satellites with LOS (line of site) which is needed to estimate the position. The reason is that the receivers are often enclosed by buildings in urban area. Additionally, the signals are also shaded by buildings. The signals from the GPS satellites which fly at various positions are affected by multipath. When quality of the received signals becomes degraded, estimation error of own locations becomes large. For these problems about positioning of the personal small vehicles, the dependence on the four GPS satellites under bad environment such as catching the satellites imperfectly should be solved. Therefore, a new positioning method that doesn't rely on the GPS satellites too much is needed.

1.4. Purpose

In this paper, we propose the positional estimation algorithm for the personal small vehicles in urban area. Especially, our assumed environment is that catching GPS satellite completely is difficult due to surrounded buildings. Our approach is to decrease the necessary number of catching satellites. Especially, we consider an extreme case, that is, the necessary number of satellite is only one. Of course, the possibility of catching one satellite with LOS is higher than that of catching four satellites completely. Moreover, in Japan, QZSS (Quasi-Zenith Satellite System) is always located in vicinity of the zenith in the plan of QZSS which is launched in September, 2010. Therefore, by our proposal, the accurate estimation will be achieved every time. In this paper, algorithm which can estimate the position by using one satellite and two receivers is presented.

2. QZS and GPS Satellite

2.1. QZSS

QZSS is hot topic as Japanese original satellites (6)(7). QZSS arranges the satellite to be always observed over Japan. The orbit of QZS is a semi-zenith. QZS is almost located in the zenith (right above) of Japan. At least, one QZS exists over Japan by using several QZSs. As a result, positioning systems with high accuracy are expected because of one QZS always can be observed in urban area with buildings where catching satellites is difficult. QZS has two roles on account of high catching capability. One is the GPS supplementation. Another is the GPS reinforcement. The GPS supplementation enables to regard QZS as one GPS satellite by using the compatible signals to the GPS satellites. As a result, spreading of usable area and the time zone that can catch the satellite becomes possible. On the other hand, in case of positioning by GPS, measurement error occurs by influence of a variety of error factors such as ionospheres and changing atmospheric pressure. The GPS reinforcement is a technique which reduces the measurement error by using the data which is measured by the reference points on the ground, and transmits correction data of the previous error factors to QZS beforehand.

Fig.1 Fixed true point and estimated points (example #1)

Fig.2 Fixed true point and estimated points (example #2)

As an original satellite in Japan, "MICHIBIKI" was launched as a first QZS in September, 2010 in the project of JAXA (Japan Aerospace eXploration Agency). Demonstrations of technologies have been examined. The second QZS will be launched after proof of MICHIBIKI, and finally the third will be scheduled to be launched. Our proposal needs one satellite which can be observed in LOS. QZS is a major candidate as our one satellite.

2.2. Estimation characteristic and problem of GPS satellites

In the urban area that includes the valley of buildings, the position estimation that uses the GPS satellites generates big error because of the influence of the multipath and the lack of the necessary number of observed satellites. To confirm the problems of the GPS positioning under the bad environment, we measured the positioning in the town that enclosed in the building. We used popular GPS logger (HOLUX, M-241). We estimated own position of a fixed-point for ten minutes. Fig.1 and Fig.2 show the fixed true point and estimated points in the map. The fixed points are surrounded by buildings. We measured on the road nearby the wall of the building. We plot on Google Map by using "Wadachi version3.44". From these figures, we can find that the estimated points vary about 20-30m.
compared with the true measurement point. Thus, the GPS measurement in urban area generates a large measurement error. Main reason is the influence of the multipath. In this paper, to reduce the position estimation error, we will try decreasing the necessary number of the catching satellites for the position estimation. Our goal is to estimate the position by only one satellite. By reducing the necessary satellite, it will be easy to receive the perfect signal in LOS.

3. Proposed Algorithm for Position Estimation

3.1. System model

In this paper, one satellite and two receivers on the ground are assumed (Fig.3). The satellite is assumed as GNSS (Global Navigation Satellite Systems) such as GPS and QZS. And we assume that the receivers are mounted on moving vehicles or walking pedestrians who have smart phones on the ground. On the receivers, we assume that a geomagnetism sensor is installed. The receiving terminal also has a moved distance sensor. The latest cellular phones or smart phones have variable sensors. The sensor information should be shared effectively. The moving direction to the north is derived from the geomagnetism sensor, and the moved distance is derived from wheel rotation. Moreover, we assume that the distance between the receivers can be recognized by wireless communication each other. Our positional estimation algorithm uses the above information.

Fig.4 shows a detailed system model. The coordinates of the two receivers at time $t_1$ are denoted as $(x_1,y_1,z_1)$ and $(x_2,y_2,z_2)$ respectively. The receivers move to the positions $(x_1',y_1',z_1')$ and $(x_2',y_2',z_2')$ at the next time $t_2=t_1+\Delta t$. The coordinates of the satellite is $(\alpha, \beta, \gamma)$. We define the distance from the satellite to the receiver #1 and receiver #2 is $r_1$ and $r_2$ respectively. After moving, time $t_2$, we also define $r_1'$, $r_2'$ respectively. Because the moved distance in $\Delta t$ is small, we assume that the z-positions of the receivers are not changed.

3.2. Proposed algorithm

In this section, we introduce the proposed algorithm analytically. First of all, we describe the sensing data by the above mentioned coordinates. The moved distance of receiving terminal is:

$$R_a = \sqrt{(x_a - x_a')^2 + (y_a - y_a')^2 + (z_a - z_a')^2}$$

, where variable $a$ is the index of the receivers. Then, the distance between the satellite and each of the receivers at $t_1$ and $t_2$ can be denoted as follows.

$$r_a = \sqrt{(x_a - x_a)^2 + (y_a - y_a)^2 + (z_a - z_a)^2}$$

$$r_a' = \sqrt{(x_a' - x_a')^2 + (y_a' - y_a')^2 + (z_a' - z_a')^2}$$

For basic study, in our assumption, the position of the satellite is not changed between the time $t_1$ and $t_2$ because the time interval is small. We can also have the moving direction $\theta_a$ as the sensing data.

Now we derive the receivers’ position analytically. We put a perpendicular down from the QZS to $xy$- plane. The intersection of the perpendicular and the $xy$- plane is denoted as $P(\alpha, \beta, z_a)$. The distance from the point $P$ to the each of receivers is denoted as $d_a=\sqrt{r_a^2-(\gamma-z_a)^2}$. At this time, the following relations can be realized.
\[ x_a^2 + y_a^2 = d_a^2, \quad x_a' = d_a' = \sqrt{d_a^2 - x_a^2} \]  

This expression is transformed to as follows.

\[ y_a = \pm \sqrt{d_a^2 - x_a^2}, \quad y_a' = \pm \sqrt{d_a'^2 - x_a'^2} \]  

Eq. (4) is substituted to Eq. (1).

\[ R_a^2 = (x_a - x_a')^2 + (d_a^2 - x_a^2) + (d_a^2 - x_a'^2) - R_a^2 \]

\[ = 2\sqrt{(d_a^2 - x_a^2)(d_a^2 - x_a'^2)} \]  

We square both sides of Eq. (6) and rearrange as follows.

\[ d_a^4 + 2d_a^2d_a'^2 + d_a'^4 \]

\[ -2(2d_a^2x_a + d_a^2R_a^2 + 2d_a^2x_a + d_a'^2R_a^3) \]

\[ + 4x_a^2x_a'^2 + 4R_a^2x_a + R_a^4 \]

\[ = 4(d_a^2d_a'^2 - d_a^2x_a'^2 - d_a'^2x_a^2 + x_a^2x_a'^2) \]  

By using the moving direction \( \theta_a \), the relation \( x_a' = R_a\sin\theta_a + x_a \) can be denoted. This is substituted to Eq. (7).

We rearrange in terms of the degree of \( x_a \).

\[ 4R_a^2x_a^2 \]

\[ + 4R_a\sin\theta_a(d_a^2 - d_a'^2 + R_a^2) + x_a \]

\[ +(d_a^2 - d_a'^2)^2 - 2R_a^2(d_a^2 + d_a'^2) \]

\[ + R_a^2(4d_a^2\sin^2\theta_a + R_a^2) = 0 \]  

From the above, the variable \( x_a \) can be solved.

\[ x_a = \frac{1}{4R_a^2} \left( 2R_a\sin\theta_a(d_a^2 - d_a'^2 + R_a^2) \right) \]  

When Eq. (9) is substituted to Eq. (4), \( y \) positions to each \( x_a \) can be obtained. Here, \( x_a \) has two patterns, that is \( \pm \), and \( y_a \) also has two patterns, respectively. Considering \( \pm \) of \( x_a \) and \( y_a \), four position candidates per one receiver can be obtained. The position at time \( t_2 \) can be calculated by Eq. (9) and the following relationship.

\[ x_a' = x_a + R_a\sin\theta_a, \quad y_a' = \pm \sqrt{d_a^2 - x_a'^2} \]  

Four position candidates of the receiver are also obtained respectively at time \( t_2 \). Then, from the position candidates obtained in Eq. (9), Eq. (4), and Eq. (10), we can calculate distances between the receiving terminals at time \( t_1, t_2 \), respectively:

\[ \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \]  

\[ \text{(Distance between the terminals at time } t_1 \text{)} \]  

\[ \sqrt{(x_1' - x_2')^2 + (y_1' - y_2')^2 + (z_1' - z_2')^2} \]  

\[ \text{(Distance between the terminals at time } t_2 \text{)} \]

Finally, the distances between the receivers derived from Eq. (11) and (12) are compared with the distance between the receivers which are obtained as the sensor information. The position candidate which is the closest value to the sensor information is decided as the estimation result of the receiver positions.

### 4. Numerical Examples and Performance Characteristics

#### 4.1 Simulation model

We assume the field environment as Fig. 5. There is a street on a 2D field, so we assume \( z \)-position as zero. The street has two lanes. The width of the street is 10m. We set \( x \)-axis across the street. The range is \(-5 \leq x \leq 5 [\text{m}]\). Moreover, we assume the length of the street as 100m. So we also set the \( y \)-axis along the street as \( 0 \leq y \leq 100 [\text{m}] \). The location of the satellite is \((x,y,z) = (0, 50, 35,000,000) [\text{m}] \). The \( z \)-position of the satellite is decided from the altitude of the QZS.
We simulate the following three cases. Each case is different in terms of the moving speed of the receivers.

(i) 5.55 m/s (20 km/h: personal small vehicle)
(ii) 11.1 m/s (40 km/h: general passenger vehicle)
(iii) 1.5 m/s (5.4 km/h: pedestrian)

In this simulation, we generate two receivers randomly. These positions are set as the locations at time $t_1$. And the moving direction is also set randomly. So, the locations of the next time $t_2$ are calculated from the position at time $t_1$, the moving direction, the moving speed which is set as the simulation parameter. The distances between the satellite and the receivers are derived from each position. The distances between the receivers are also calculated from the each location of the receivers.

In the simulations, the sensing data is generated from the above set parameters such as the moving directions, and the distances. The assumed sensing error of each sensor is described in Table 1. We consider two errors: (A) distance measurement error between receivers, (B) moving direction sensing error. Table 1 summarizes the specification of each error. The error range $\varepsilon$ of the distance measurement is set as $\pm 0.3$ m. The error range $\varepsilon$ of the moving direction is also set as $\pm 2^\circ$. This is the typical value of the accuracy of the geomagnetism sensors on market. These errors are modeled by normal distributions. The standard deviation $\sigma$ of the distribution is set as $2\sigma = \varepsilon$. Our proposal also uses the sensing data of the moving distance. In this simulation, we consider the error of the moving distance as zero. The reason is that the data of the moving distance can be gotten precisely from the number of wheel rotations in case of the vehicles.

In our simulations, the positions of the receivers are estimated from the sensing data by the proposed algorithm. The estimation performance is evaluated in terms of the following two performance measures.

- Estimation rate [%]
- Estimation error [m]

The estimation rate is defined as the following rate:

$$\text{Estimation rate} = \frac{\text{Estimation success times}}{\text{All trial times}} \times 100\%$$

Our algorithm can estimate the positions theoretically. However, if the sensing error is included in the sensing data, the estimation errors will be generated obviously. Especially, in case of the bad case, the position of the receivers cannot be derived. So the above estimation rate means the robustness of our proposal. Next, the estimation error is defined as the Euclidean distance between the true position of the receiver and the estimated position.

### 4.2 Simulation results

We present the simulation results in 3 cases which differ according to moving speed. The time interval $\Delta t$ is changed from 1 [sec] to 10 [sec]. To understand the influence of the error factors to the positional estimation characteristics, the results are derived in each error. The performance is evaluated from the average values of 10,000 trials. Occasionally, the proposed algorithm estimates the receiver position as a complex value. In the simulation, the imaginary part is disregarded, and only real part is used as the estimated position because the imaginary part is a very small value.

Firstly, the results of the case that the moving speed is set as personal small vehicles are shown in Fig.6. We assume the speed of the vehicles as 5.55 m/s (20 km/h). Fig. 6 shows results of both estimation rate and estimation error. The left-vertical axis means the estimation rate and the right-vertical axis means the
estimation error. From Fig. 6, the estimation rate becomes fine when the time interval $\Delta t$ increases. The estimation error also becomes fine when the time interval $\Delta t$ increases. The reason is that the positions between before moving and after moving become separate. We also find that the error of (A) Distance measurement between the receivers has big influence to both the estimation rate and error. So the performance of considering error (A)+(B) is also relatively low.

Fig. 7 and Fig. 8 show the performance in the case of general passenger cars and pedestrians, respectively. The moving speed of general passenger vehicles is set as 11.1m/s (40 km/h). In the case of the pedestrians, the speed is set as 1.5m/s (5.4km/h). The trend of the performance is the same to the case of the personal small vehicles. That is, the estimated performance of considering (A) the distance measurement error between the receivers is lower than that of considering (B) the moving direction sensing error. In case of the pedestrians, they do not have the distance sensor generally. The above simulation results are shown as just simple demonstration for obtaining basic performances. From a practical standpoint, in order to estimate the positions, we have to get the distance information from using alternative sensors. These sensors are typically used by passometers or acceleration gyro sensors.

In real environment, we must consider both the error (A) and (B). Then, the results of all speeds are summarized in Fig. 9 when the errors (A) and (B) are considered. From Fig. 9, the higher the speed is, the higher the estimation rate is. In the case of the estimation error, the lower the speed is, the larger the error is. That is, the estimation performance becomes fine in the case that the speed becomes fast.

In the case of the personal small vehicle, that is focused on in this paper, the estimation error is 2.52m and the estimation
rate is 72.9% when the time interval $\Delta t = 1$[sec]. The estimation rate is not so high. However, the position can be estimated two times every three times. And, the estimation error is 0.56m and the estimation rate is 78.7% in case that the time interval $\Delta t = 10$[sec]. Thus, it is not so important to extend the time interval $\Delta t$ for the improvement of the estimation rate. However, the estimated error decreases when the time interval $\Delta t$ increases. Therefore, we can improve the estimated error by extending the interval time $\Delta t$ between the measurements. As the estimation error, less than a few meters can be achieved when the time interval $\Delta t$ is larger than 3 seconds.

Note that the estimated rate is not 100% though there is no sensing error at all speeds. And the estimation error is also some value for no sensing error. The reason is that the computer simulations have the limitation of the significant digit. This limitation generates calculation errors. In localization algorithms, there is the difference of scales (dynamic range). The altitude of the satellite is very large value such as 35,000,000m. However the distance among the receivers and moved distances are small value (several meters). By the above difference of the scales and the calculations using the squares such as Eq. (1) – (12), numerical errors are generated.

And the above simulations do not consider the movement of the satellite position between the time $t_1$ and $t_2$ because of presentation of basic performance. So, the movement should be considered when the accurate performance in real situations is required.

As mentioned above Sec. 2.2, the estimation error of our GPS logger trials is about 20m in urban area. Thought our results are evaluated by simulations, our estimation algorithm can achieve the lower estimation error compared to our GPS experiments. Especially, it is meaningful to achieve the estimation by only one satellite but some problems of the estimation performance remain.

5. Conclusion

The personal small vehicles, such as town mobility or city commuter, are one of hot topics about next new transportation in ITS. If they know own locations, these vehicles will have advantages, such as ecology, safety, and convenience. To achieve these advantages, we consider a new estimation algorithm for own location. The estimation by four GPS satellites is popular. In order to avoid the influence of the poor signal propagation in urban area, such as multipath, we tried reducing the necessary number of satellites.

In this paper, we proposed the novel estimation algorithm which used only one satellite and two receivers on the ground. The receivers know the distance between the satellite and each of the receivers. And the receivers also know own sensing data such as the distance between the receivers, the moving direction, distance. Our algorithm can estimate from the above information.

By the computer simulations, we introduced the estimation performance of the proposal. In the simulations, we tried evaluating the case of the different moving speeds, that is (i) personal small vehicle, (ii) general passenger vehicle, (iii) pedestrian. Through the simulations, our algorithm had the estimation accuracy of less than a few meters in case that the measurement interval was larger than 3 seconds. These performances are better than the conventional GPS estimation we measured in the urban area.

In this paper, we evaluated the basic performance under some ideal situations for the prompt report. As future works, we should consider the estimation performance in more real situations. For example, in order to evaluate more precise performance, we will consider the movement of satellites. We will also consider the error characteristics of the receivers’ sensors which are recorded from real devices, such as geomagnetism and distance sensor.

For more advanced assistances to the vehicles, the more accurate estimation will be needed. So, we will also try improving the accuracy if the moving speed is slow. However the above problems remain, we will try resolving the problems because it is meaningful to achieve the estimation by only one satellite.

References

(1) Goto, T., & Yamaoka, M. (2010). Personal Mobility Robot. Journal of society of automotive engineers of Japan, 64(5), 75-78.
(2) Furuno, N., & Toshifumi K., & Kobayashi, K., & Watanabe, A. (2008). Mechanism and application operation of GPS. Transistor technology, 45(2), 100-163.
(3) Brakatsoulas, S., & Pfoser, D., & Salas, R., & Wenk, C. (2005). On MapMatching Vehicle Tracking Data. Very Large Data Base Conference (VLDB), Vol. 31, 853-864.
(4) Gonda, M., & Aiga, Y. (2003). On MapMatching Vehicle Tracking Data. Japan radio company, no.44, 35-38.
(5) Namie, H., & Hagiwara, N., & Nitta, N., & Shibahara, Y., & Saito, M., & Kanai, Y., & Kim, H., & Yasuda, A. (2001). Evaluation of RTK-GPS by Virtual Reference Station (VRS) Method. *IEICE transactions*, J84-B(12), 2160-2168.

(6) Japan Aerospace eXploration Agency (JAXA), Interface specification for QZS. (2010). Retrieved December 3, 2010. from http://qzss.jaxa.jp/is-qzss/

(7) Kawano, I. (2001). Satellite Positioning System using Quasi-zenith and Geostationary Satellites. *IEICE transactions*, J84-B(12), 2092-2100.