Degree-Depth Relation for Planetary Gravity Field Model Based on Wavelength

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Abstract In this paper we investigate the relation between the spherical harmonic (SH) degree of gravity field model and the corresponding depth of the source body. By investigating the gravity of a buried point mass, we find that a relationship can be found between the horizontal extent of the signal (which we assume to be indicative of the wavelength) and the depth of the source. We introduce a threshold to define the gravity anomaly wavelength of an isolated mass body. The region where the gravity anomaly of an isolated mass body is larger than the threshold defines its gravity anomaly wavelength. For an isolated point mass with fixed total mass, the wavelength is a function of the depth. The ratio between the maximum wavelength and the depth at which the maximum wavelength occurs (effective resolvable depth) is invariant provided that the mass and threshold are determined. When this maximum wavelength is larger than the minimum resolvable wavelength of n-degree gravity field, this isolated mass body is considered as a reliable density feature. Combining the relation between maximum wavelength and effective resolvable depth of the isolated mass body and the relation between minimum resolvable wavelength and SH degree of gravity field, we derived the degree-depth relation under the limiting resolvable condition: d = R / (n * sqrt(2)). This relation was validated with Gravity Recovery and Interior Laboratory gravity field data at lunar Orientale basin region, and provided insights for the planetary gravity field analysis.

Plain Language Summary The planetary gravity field models are usually expressed as a set of n-degree progression terms (spherical harmonic series), the low-degree terms represent gravity generated by deep and large-scale features, and the high-degree terms mainly stand for gravity signals from shallow-located and small-scale mass. In this paper, we explore the quantitative relation between the spherical harmonic degree of gravity data and the depth of mass body. A new expression for the degree-depth relation is given as d = pi*R / (n*sqrt(2)). The testing of the newly derived and two existing degree-depth relations (d = pi*R/n and d = R/(n-1)) is conducted with gravity data of the Moon. The filtered gravity data based on the degree-depth relation in this paper can better reflect the gravity signals within the lunar crust when compared with that based on the previous existed degree-depth relations.

1. Introduction

The relation between the spherical harmonic (SH) degree of gravity field model and the corresponding depth of source body is the basis for the band-filtered analysis of gravity data (i.e., expanding the spherical harmonic potential coefficients within particular bandwidth). The capacity of gravity data can be maximized through the filtering, especial in the planetary body with high-degree gravity field determined (e.g., the Moon). By removing the degree 1–5 potential coefficients in lunar gravity field model, the effect of the hemispheric asymmetry of the Moon and the South Pole-Aitken impact can be eliminated (e.g., Evans et al., 2016; Neumann et al., 2015). Featherstone et al. (2013) filtered the lunar gravity field based on Bowin (1983) equation (see Equation 1 below), and identified several impact basins on the Moon's farside. The Gravity Recovery and Interior Laboratory (GRAIL) mission measured the gravity field of the Moon in an unprecedented accuracy (Zuber, Smith, Lehman, et al., 2013, Zuber, Smith, Watkins, et al., 2013) and the lunar gravity field coefficients now up to 1500° (Park et al., 2015). With the help of the high-degree GRAIL gravity field model, removing the lower degree coefficients of gravity field model during the expanding is frequently utilized when the target features are shallow-located (e.g., Andrews-Hanna et al., 2018; Jansen et al., 2017; Jozwiak et al., 2017). The degree-dependent depth relation plays a huge role in selection of the filtering bandwidth.
A widely used degree-depth relation for planetary gravity field is the Bowin (1983) depth limiting relation (B1983), which is quantitatively given by:

\[ D = \frac{R}{(n - 1)} \]  

in which \( R \) is the planetary radius, \( n \) is the SH degree and \( D \) is the depth. In Featherstone et al. (2013), the gravity field model of the Moon is expanded begin at degree 18 and truncated at 70°. This filtered bandwidth is dedicated to enhance the data sensitive to source body no deeper than ~100 km. Nevertheless, Zuber et al. (2016) verified another empirical degree-depth relation (Z2016) in the range of lunar crustal thickness (<45 km) by gravity forward modeling:

\[ D = \frac{\pi R}{n} \]  

This degree-depended depth is based on the theoretical resolution of spherical harmonic function (Heiskanen & Moritz, 1967):

\[ s = \frac{1}{2} \lambda_{\min} = \frac{\pi R}{n} \]  

in which \( s \) is the spatial resolution scale and \( \lambda_{\min} \) is the minimum resolvable wavelength. A similar relation between wavelength (\( \lambda \)) and SH degree also can be found in Jeans (1923):

\[ \lambda = \frac{2\pi R}{\sqrt{n(n+1)}} \]  

For relative large SH degree \( n \), the wavelength indicated by Equation 4 is approaching that from Equation 3 (e.g., Dahlen & Simons, 2008; Wieczorek, 2015). The minimum resolvable wavelength (\( \lambda_{\min} \)) represent the ability of \( n \)-degree gravity field to resolve the gravity features. Mass body with its gravity anomaly wavelength smaller than the minimum resolvable wavelength should be considered as unreliable features theoretically. The degree-depended depth of B1983 is different to that of Z2016, but both of them are successfully implemented on the Moon.

In this work, we explore the degree-depth relation considering the lateral extension of the surface gravity anomaly of a sub-surface isolated mass body. The range occupied by the gravity anomaly can be regarded as its wavelength, which depends on the depth and mass (volume and density) of the mass body. For an isolated mass body, however, the extent range of its gravity anomaly is difficult to determine exactly as the gravity anomaly at the infinity distance is close to zero but not equal to zero. Therefore, we introduce a threshold (\( \sigma_T \)) to define the gravity anomaly wavelength (\( \lambda \)) of an isolated mass body. The region where the gravity anomaly of an isolated mass body larger than the threshold is used to calculate its gravity anomaly wavelength (\( \lambda \)). For an isolated point mass with fixed total mass, the wavelength (\( \lambda \)) is a function of the depth (\( d \)). Combining the relation between the wavelength and depth (\( \lambda \) and \( d \)) and the relation between minimum resolvable wavelength and SH degree of gravity field (\( \lambda_{\min} \) and \( n \), i.e., Equation 3), we derived the degree-depth relation under the limiting resolvable condition. This relation was validated with GRAIL gravity field data at lunar Orientale basin region.

2. Degree-Size-Depth Relation

To explore the relation between depth of mass body and the SH degree, we carry out our study using isolated point mass body. The wavelength of the isolated point mass, however, is hard to define accurately. Given that gravity data always have error, we introduce a gravity anomaly threshold (\( \sigma_T \)) to calculate the wavelength of gravity signal generated from an isolated body (Figure 1). In Figure 1, the black curve represents the gravity anomaly profile produced at the surface by an isolated sub-surface point mass body. The gravity anomaly is greatest directly above the buried point, and the farther from the mass body, the weaker the gravity anomaly. For the isolated point mass, given a gravity anomaly threshold we can circle the corresponding extended region of gravity anomaly and determine its wavelength (\( \lambda \)). In Figure 1, the point where the gravity anomaly of isolated mass body equal to the threshold (\( \sigma_T \)) is assumed to be \( P \). The distance from point \( P \) from the origin is defined as the half-wavelength (\( \lambda/2 \)) of the gravity anomaly signal. The half-wavelength (\( \lambda/2 \)) and burial depth (\( d \)) are used to calculate the distance from point \( P \) to the mass body (\( l \)). \( g \) is the gravitational acceleration vector generated by
the mass body, while $g_z$ is the vertical component of this gravitational vector. At point $P$, its vertical component $g_z$ equals to the threshold: $g_z(P) = \sigma_T$.

Gravitational vector $\mathbf{g}$ produced by an isolated sub-surface point mass at $P$ is:

$$\mathbf{g} = \frac{GM}{l^2} \tag{5}$$

and the vertical component $g_z$ is:

$$g_z = \frac{GM}{l^2} \cdot \frac{d}{l} \tag{6}$$

While this vertical component is equal to the gravity anomaly threshold, we obtain the relation between $d$ and $l$:

$$d = \frac{l^3}{k} \tag{7}$$

where $k$ is:

$$k = \frac{GM}{\sigma_T} \tag{8}$$

Under the condition that the gravity anomaly threshold ($\sigma_T$) and the total mass of the isolated point mass ($M$) are fixed, $k$ is considered as a constant. We have:

$$d^2 + \left(\frac{1}{2} \lambda \right)^2 = l^2 \tag{9}$$

From Equation 7 and Equation 9, we obtain the relation between $d$ and $\lambda$:

$$\lambda = 2 \cdot d \sqrt{\left(k^\frac{3}{2} - d^\frac{3}{2}\right)^2} \tag{10}$$
Figure 2. Example for the relation between burial depth \((d)\) and wavelength \((\lambda)\) as indicated by Equation 10. The total mass is assumed to be 2.09 \times 10^{14} \text{kg} (equivalent to a 5 km radius sphere with density of 400 kg/m^3) and the threshold value is assumed to be 4 mGal (1 mGal = 1 \times 10^{-5} \text{m/s}^2). According to Equation 8, we have \(\sqrt{k} = 18.7 \text{ km}\). As \(d_e\) is the value for which \(\lambda\) is maximal, we have \(d_e = 8.2 \text{ km}\) and \(\lambda_{\text{max}} = 23.2 \text{ km}\) from Equations 14 and 18. When \(d < d_e\) or \(d > d_e\), the wavelength obtained are to be less than the maximum wavelength \((\lambda_{\text{max}})\).

where \(d\) is in the range between 0 and \(\sqrt{k}\) as we have \(\frac{GM}{d^2} > \sigma_T\), which means that the gravity anomaly directly above the point mass should larger than the threshold. From the derivative of the \(\lambda-d\) relation (Equation 10) we have:

\[
\lambda'(d) = 2 \cdot \left[ \frac{1}{3} d_e^{-\frac{1}{2}} \cdot \left( k_e^\frac{1}{3} - d_e^\frac{1}{3} \right) + k_e^\frac{1}{2} \cdot \left( k_e^\frac{1}{3} - d_e^\frac{1}{3} \right)^{-\frac{1}{2}} \cdot \left( -\frac{4}{3} d_e^{-\frac{1}{2}} \right) \right]
\]

(11)

When \(\lambda'(d) = 0\), we obtain the depth that where the maximum wavelength \((\lambda_{\text{max}})\) is found and we refer this depth as the effectively resolvable depth \((d_e)\):

\[
\frac{1}{3} d_e^{-\frac{1}{2}} \cdot \left( k_e^\frac{1}{3} - d_e^\frac{1}{3} \right) = 2 \cdot d_e^\frac{4}{3} \cdot \left( k_e^\frac{1}{3} - d_e^\frac{1}{3} \right)^{-\frac{1}{2}}
\]

(12)

Therefore, we have:

\[
k_e^\frac{1}{3} - d_e^\frac{1}{3} = 2 \cdot d_e^\frac{4}{3}
\]

(13)

and we have

\[d_e = (27)^{-\frac{1}{4}} \cdot k_e^\frac{1}{2}\]

(14)

or

\[d_e^2 = \frac{1}{3 \sqrt{3}} k\]

(15)

This relation suggests that under the condition that the gravity anomaly threshold \((\sigma_T)\) and the mass \((M)\) is determined, that is, with fixed \(k\), at the effective resolvable depth \((d_e)\), the gravity anomaly of an isolated mass body exhibits its maximum wavelength defined by \(\sigma_T\). The existence of a maximum wavelength at \(d_e\) indicating that under the condition of fixed threshold, the maximum wavelength \((\lambda_{\text{max}})\) is not found at the closest point to the surface. The wavelength increases first and then decreases as it moves down from near the surface (Figure 2).

Substituting Equation 14 to Equation 7, we obtain l as.

\[l^2 = \frac{1}{\sqrt{3}} k\]

(16)

Substituting Equations 16 and 15 to Equation 9, we have the maximum wavelength \((\lambda_{\text{max}})\):

\[\lambda_{\text{max}}^2 = \frac{8}{3 \sqrt{3}} \cdot k\]

(17)

Equation 17 gives the relation between the maximum wavelength \((\lambda_{\text{max}})\) and \(k\). Combined with the relation between effectively resolvable depth \((d_e)\) and \(k\) (Equation 15), we have the relation between the effectively resolvable depth \((d_e)\) and the maximum wavelength \((\lambda_{\text{max}})\) under the condition that the threshold \((\sigma_T)\) and mass \((M)\) are determined:

\[\lambda_{\text{max}} = 2 \sqrt{2} \cdot d_e\]

(18)

Equation 18 indicates that the ratio between \(\lambda_{\text{max}}\) and \(d_e\) is fixed. In Figure 3, we show the vertical component of gravitational acceleration \((g_z)\) in the space generated by an isolated point mass. By defining the wavelength with a fixed threshold, the maximum value of its wavelength is identical to the maximum extending horizontal range of the gravity anomaly contour with the value of this threshold (yellow stars in Figure 3). The effective resolvable
depth ($d_e$) is thus the height where the maximum extending range was found (vertical coordinates of stars in Figure 3). The scaling relation between $\lambda_{\text{max}}$ and $d_e$ is invariant (red solid line).

In gravitational geophysical analysis, high-degree gravity field models can resolve smaller mass bodies. The theoretical resolvable ability of gravity field data in SH expression is inferred by Equation 3. According to Equation 3, in $n$-degree gravity field model, the mass body with its gravity anomaly wavelength ($\lambda$) smaller than the minimum resolvable wavelength ($\lambda_{\text{min}}$) is beyond the limit of resolution, thus considered to be unreliable. In the limiting condition that the mass body is just resolved by $n$-degree gravity field model, its maximum wavelength is equal to the minimum resolvable wavelength of the gravity data (Equation 3):

$$2\sqrt{2} \cdot d_e = \lambda_{\text{max}} = \lambda_{\text{min}} = \frac{2\pi R}{n}$$

Here, we give the relation between SH degree ($n$) and the effectively resolvable depth ($d_e$) (D2021):

$$d_e = \frac{\pi R}{\sqrt{2n}}$$

in which $R$ is the planetary radius.

### 3. Validation

GRAIL mission determined the gravity field model of the Moon up to 1500 SH degrees (Park et al., 2015). The high-degree GRAIL gravity field model of the Moon is an ideal example to test the applications for different degree-depth relations. Orientale basin is the best-preserved and youngest large impact basin on the Moon, formed about 3.8 Ga (Stöffler et al., 2006; Wilhelms, 1987). In the final phase of GRAIL mission, lower orbital altitudes were achieved when the spacecrafts fly over the Orientale basin region. This part of the data enhanced the sensitivity of the GRAIL models to the detailed gravity features in Orientale basin region. Therefore, the Orientale basin region is with the best quality data of GRAIL, making it the optimum location for the validation of different degree-depth relation for planetary gravity field. Moreover, with the development of basin formation

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**Figure 3.** The distribution of vertical gravitational acceleration in space generated by an isolated point mass. The maximum horizontal extending of the contour and the height where this maximum extending is found indicate the relation between $\lambda_{\text{max}}$ and $d_e$. The total mass of this point body is $2.09 \times 10^{14}$ kg (equivalent to a 5 km radius sphere with density of 400 kg/m$^3$). The point mass is located at the origin of coordinates, indicating by yellow circle. Yellow stars show maximum horizontal extending of each contours in the space. Their horizontal coordinates correspond to the maximum half wavelength ($\frac{1}{2} \lambda_{\text{max}}$) and their vertical coordinates correspond to $d_e$. The 4 mGal threshold case is shown as an example, where the calculated $\lambda_{\text{max}}$ and $d_e$ are 23.2 and 8.2 km, respectively. The function of $x = \sqrt{2z}$ is indicated by red solid line, representing the relations between $\lambda_{\text{max}}$ and $d_e$ (Equation 18).
model (e.g., Johnson et al., 2016, 2018; Melosh, 1989; Osinski & Pierazzo, 2013), the location and depth of basin-related sub-surface density structures at Orientale basin provide background geological evidence for the verification of these degree-depth relations.

Orientale basin exhibits multi-ring structures, named Inner Depression (ID, R ~ 160 km), Inner Rook Ring (IRR, R ~ 232 km), Outer Rook Ring (ORR, R ~ 310 km), and Cordillera Ring (CR, R ~ 462 km) from inside to the outside (Figures 4a and 4b). The free-air gravity anomaly in Orientale basin is strongly correlated with the...
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10.1029/2021EA002143

In the Bouguer gravity anomaly (Figure 4d), the positive anomaly in the center is suggested to arise from mantle uplift under the basin floor (Melosh, 1989). The annular negative anomaly outside the edge of mantle uplift is thought to be generated by thickening and subsiding of the crust, which possibly formed by the emplacement of impact eject (Andrews-Hanna, 2013; Byrne et al., 2015). Prior to GRAIL mission, Kattoum and Andrews-Hanna (2013) used gravity data to study the sub-surface density features beneath the multi-ring topography at Orientale basin, and initially revealed the ring faults from gravity. Using filtered GRAIL data, Andrews-Hanna et al. (2018) quantitatively gave the geometric parameters of the ring faults at Cordillera Ring and ring dikes at Outer Rook Ring. These ring faults and ring dikes are suggested to be located at the depth within the lunar crust (Figure 5).

In this section, we take the ring dikes and the thickened crust beneath ORR as example to study the applicability of different degree-depth relations. The revealing of ring dikes and the eliminating of the thickened crust in different filtered gravity anomaly maps can be used to validate these different degree-depth relations. The dikes are thought to be located within the crust (Nahm et al., 2013) while the thickened crust mainly changes the structure at lunar crust-mantle boundary.

The filtered gravity anomaly is derived through removing the SH coefficients lower than the corresponding degree, that is, expanding the SH coefficients begin at the filtered degree. Theoretically, removing the SH coefficients lower than the filtered degree will reduce the gravity signatures generated from density features located deeper than the corresponding depth and highlight the gravity features from the surface to the corresponding depth. Therefore, the applicability of each degree-depth relations can be judged by examining how the negative anomalies from thickened crust are removed.

Figure 6. Comparison between different degree-depth relations. Blue, red, and green lines indicate degree-depth relation proposed by Bowin (1983), this work, and Zuber et al. (2016), respectively. Gray area denotes the range of lunar crust-mantle boundary suggested by Wieczorek et al. (2013): 34–43 km. The 38.5 km crustal thickness corresponds to 46, 100, and 141° in B1983, D2021, and Z2016°-depth relations, respectively.
We take 38.5 km as the depth of lunar crust (34–43 km from Wieczorek et al., 2013) and the corresponding SH degree in different degree-depth relation is calculated (Figure 6). The Bowin (1983) limiting equation gives the SH degree of 46 by substituting the 38.5 km crust thickness. The corresponding SH degree from degree-depth relation by D2021 and the empirical equation of Z2016 are 100 and 141, respectively. The corresponding filtered gravity anomaly maps are then derived. As we are dedicated to remove the large-scale negative anomaly and highlight the small scale features, we use the complementary minimum amplitude filter (high-pass) suggest by Jansen et al. (2017) to filter the gravity data. This filter is designed based on the minimum amplitude filter (low-pass) \(w_L\) of Wieczorek et al. (2013) in the inversion of crustal thickness. We simplified the filter \(w_L\) of Wieczorek et al. (2013) as:

\[
w_L = \left\{ 1 + \lambda \left( \frac{M (2l + 1)}{4\pi \Delta \rho R^2} \right)^2 \right\}^{-1}
\]  

Figure 7. Complementary filter (\(c_L\), black solid line) used in this work and the minimum amplitude filter (\(w_L\), red solid line) from Wieczorek et al. (2013). At the filtered degree, that is, degree 46, the filtered value equals to 0.5. A cosine shape low-pass filter is applied in \(c_L\) from degree 350–600.

![Figure 7](image)

We take 38.5 km as the depth of lunar crust (34–43 km from Wieczorek et al., 2013) and the corresponding SH degree in different degree-depth relation is calculated (Figure 6). The Bowin (1983) limiting equation gives the SH degree of 46 by substituting the 38.5 km crust thickness. The corresponding SH degree from degree-depth relation by D2021 and the empirical equation of Z2016 are 100 and 141, respectively. The corresponding filtered gravity anomaly maps are then derived. As we are dedicated to remove the large-scale negative anomaly and highlight the small scale features, we use the complementary minimum amplitude filter (high-pass) suggest by Jansen et al. (2017) to filter the gravity data. This filter is designed based on the minimum amplitude filter (low-pass) \(w_L\) of Wieczorek et al. (2013) in the inversion of crustal thickness. We simplified the filter \(w_L\) of Wieczorek et al. (2013) as:

\[
w_L = \left\{ 1 + \lambda \left( \frac{M (2l + 1)}{4\pi \Delta \rho R^2} \right)^2 \right\}^{-1}
\]

Figure 8. Bouguer gravity anomaly maps at Orientale basin region under different filtered degree \(L_0\). \(L_0\) refers to the degree at which the value of \(c_L\) filter is 0.5. The 1200-degree Gravity Recovery and Interior Laboratory Bouguer model gggrx_1200l_bouguer is used (Goossens et al., 2020). (a) 46-degree filtering. (b) 100-degree filtering. (c) 141-degree filtering. The location of six linear gravity anomalies possible generated by ring dikes at Orientale Outer Rook Ring are shown (A-A’, B-B’, C-C’, D-D’, E-E’ and F-F’). The equidistant cylindrical projection is used for each panels, showing a longitude range of 245°–285° and a latitude range of 40°S to 0°.
in which \( \lambda \) is the filter constant, and \( M \) is the mass of the Moon. \( \Delta \rho \) is the density contrast between the crust and mantle (assumed to be 400 kg/m\(^3\)). \( R \) is the mean planetary radius (i.e., 1738 km). In Wieczorek et al. (2013), \( w_l \) equals to 0.5 at the filtered degree \( l \), thus the constant \( \lambda \) is determined. The complementary high-pass filter we used is written as:

\[
c_l = 1 - w_l
\]

(22)

Table 1

| Profile name | Starting point | Ending point | Local crustal thickness | Corresponding filtered SH degree \( (L_0) \) to local crustal depth |
|--------------|----------------|--------------|-------------------------|-----------------------------|
| A-A'         | -103/-17       | -106/-17     | ~50                     | 35, 77, 109                 |
| B-B'         | -96.8/-11.5    | -97.6/-8.7   | ~50                     | 35, 77, 109                 |
| C-C'         | -92.5/-11.5    | -92.5/-8.5   | 50-55                   | 33, 74, 104                 |
| D-D'         | -89.8/-12.5    | -88/-9.5     | 45-50                   | 37, 81, 115                 |
| E-E'         | -85.5/-19.5    | -82/-19.5    | 40-45                   | 41, 91, 128                 |
| F-F'         | -92/-27        | -91.5/-30    | 45-50                   | 37, 81, 115                 |

Note. \( L_0 \) refers to the degree where the \( c_l \) filter is 0.5.
In the expanding of lunar gravity field model, the filter $c_l$ is applied to the SH coefficients in each degree. At the high degree parts (350–600), we applied another cosine-shape filter to reduce the noise in the gravity data (e.g., Andrews-Hanna et al., 2018):

$$c_l = 0.5 \cdot \cos \left( \frac{(l - 350)}{250} \cdot \pi \right) + 0.5 \quad (23)$$

A 46-degree filter of $w_l$ and $c_l$ are shown as example in Figure 7.

Through the comparing of the gravity anomaly maps, we analyze how clearly the negative gravity anomalies from thickened crust are removed in each filtered gravity anomaly map. The range of color bar in each figures of gravity anomaly maps is determined by three times the standard deviation of gravity anomaly grids in the Orientale region. For example, in the 46-degree filtered gravity anomaly map (Figure 8a), the standard deviation of gravity anomaly in Orientale basin region is about 33 mGal, thus the color bar range is from −99 to 99 mGal in this figure.

We applied the $c_l$ filter with half-degree values of $L_0 = 46$, 100, and 141, respectively, to obtain the Bouguer maps (Figure 8). Here, I assume that $L_0$ in the caption refers to the degree at which the value of $c_l$ filter is 0.5. In the 46-degree filtered Bouguer gravity anomaly (Figure 8a), the linear shape positive gravity anomaly are hardly visible at the location of ORR (white solid line), which represent the dike structure. Nevertheless, the annular negative anomaly is clearly observed. This suggests that the 46-degree filtered of Bouguer gravity anomaly cannot entirely eliminate the gravity signal generated at crust-mantle boundary (the eject-induced crustal thickening structures). In the 100 and 141 filtered Bouguer gravity anomaly map (Figures 8b and 8c), there exist several obvious positive linear gravity features at the location of ORR. This positive linear anomaly represents the ring dike filling the space created by the fault at ORR. The degree-46 map will have the signal in there, but it is expressed clearer with the higher degree filters (100 and 141). The disappearance of large-scale negative anomaly at ORR in the 100 and 141 filtered gravity anomaly maps indicate that the degree-depth relations of Z2016 and D2021 precisely reflect the relation between the SH degree and the depth of source body. The degree-depth relations of Z2016 and D2021 are more applicable than the Bowin (1983) equation when the targeting density features are within in the depth of lunar crust.

To further comparing the revealing of ring dikes under negative gravity anomaly in the filtered gravity anomaly maps under different degree-depth relations, we choose six linear gravity anomalies at ORR as example and calculate their gravity profiles. Their locations are shown in Figure 8c and the local crustal thickness model (Wieczorek et al., 2013) is indicated in Figure 9. The dikes are under the negative anomaly ring. The point with the filtering is to get rid of that signal from depth, to highlight the shallower features. Referring to GRAIL crustal thickness model, we filtered the gravity data at SH degree corresponding to local crustal thickness (Figure 9) under different degree-depth relations (Table 1). Besides, the un-filtered Bouguer anomaly (expanding the gravity field coefficients within 2–600°) and the 50-degree filtered result suggested by Andrews-Hanna et al. (2018) are also calculated for comparison. Table 2 shows the maximum gravity anomaly in each profile under different filtered SH degree. Figure 10 includes the un-filtered profile, gravity profile filtered at $L_0 = 50$, and gravity profiles filtered under different degree-depth relations.

In the un-filtered Bouguer gravity anomaly profile, the negative anomaly generated by thickened crust is dominated (Table 2). Among all the gravity profile except for C-C', the negative gravity anomaly arisen from the thickened crust is clearly removed and the positive gravity anomaly from ring dikes is significant in the filtered gravity anomaly based on the degree-depth relation of D2021 and Z2016 (red line and purple line in Figure 10, Table 2). At profile C-C', the Z2016 filtered gravity anomaly is better than that of D2021 in the eliminating of negative gravity anomaly. The difference of the gravity profile between filtered SH degree based on Z2016 and D2021 relation is subtle in each location except for C-C'. This suggests that both Z2016 and D2021 empirical equation are applicable for the density structure in the depth range within the lunar crust, while the Bowin (1983) equation is less applicable.

### Table 2

| Profile name | Maximum gravity anomaly value in the profile (mGal) |
|--------------|--------------------------------------------------|
|              | Un-filtered $L_0 = 2$ | $L_0 = 50$ | $L_0 = 100$ | $L_0 = 141$ |
| A-A'         | −157.8                  | −4.0       | 14.7        | 24.1        | 24.2        |
| B-B'         | −148.4                  | −9.8       | 11.6        | 24.5        | 27.0        |
| C-C'         | −172.4                  | −50.1      | −15.8       | −4.1        | 12.5        |
| D-D'         | −104.6                  | −5.3       | 14.0        | 30.9        | 33.7        |
| E-E'         | −90.9                   | 0.1        | 8.6         | 19.3        | 18.1        |
| F-F'         | −173.6                  | −10.1      | 8.3         | 22.8        | 24.8        |

*Note.* The corresponding filtered SH degree $L_0$ are calculated with degree-depth relations of B1983, Z2016, and D2021.
Figure 10. Gravity anomaly profiles of six linear features at Orientale Outer Rook Ring in different filtered Bouguer gravity model. The corresponding filtered gravity anomaly profiles are calculated with local crustal thickness under the degree-depth relations of B1983, Z2016, and D2021, and are shown in orange, purple, and red lines, respectively. $L_0$ refers to the degree where the $c_e$ filter is 0.5. The unfiltered Bouguer gravity anomaly is indicated by black solid line and pink lines denote the 50-degree filtered profile suggested by Andrews-Hanna et al. (2018).
4. Conclusion

In this work, we explored the degree-depth relation considering the effects of depth variation of the mass body on the lateral extension of its surface gravity anomaly. We derived a degree-depth relation independent of the choice of gravity anomaly threshold and mass (density and volume). The surface gravity anomaly generated at this depth has the largest lateral extension. The degree-depth relation proposed in this study is: 

\[ d = \frac{x \cdot r}{\sqrt{2} a} \]

We test the applicable of degree-depth relation using GRAIL gravity data. At lunar Orientale basin, the ring dikes at ORR region are thought to be located at the depth within the crust. The SH degree corresponding to the depth of lunar crust under different degree-depth relations are also different. Based on the gravity signal of the ring dikes in different filtered Bouguer anomalies, we verified the three proposed degree-depth relations: B1983, Z2016, and D2021 (this work). The results show that Bowin (1983) limiting equation is not applicable when the targeting source bodies are at the depth of lunar crust. In contrast, the gravity signals of these ring dikes features are better reflected in the filtered gravity anomaly based on the empirical equations of Z2016 and D2021, indicating that these two degree-depth relations are more applicable. For the same depth, the corresponding filtered degree in D2021 relation is lower than that in Z2016 relation. Therefore, filtering the gravity data using D2021 relation can preserve more original gravity field information than using Z2016 relation.

Data Availability Statement

Data-The spherical harmonic gravity models of the Moon are available at (https://pds-geosciences.wustl.edu/grail/grail-l-lgrs-5-rdr-v1/grail_1001/shadr/). The derived science data of GRAIL is from Kahan (2013). LRO topography data we used here can be found at (https://pds-geosciences.wustl.edu/lro/lro-l-lola-3-rdr-v1/lrolol_1xxx/data/lola_gra/cylindrical/jp2). The crustal thickness model of the Moon (Wieczorek, 2012) we used is archived at Zenodo (https://doi.org/10.5281/zenodo.997347). Software-The authors are grateful to Mark Wieczorek for providing the open source software SHTools (Wieczorek et al., 2018), which can be found at Zenodo (https://doi.org/10.5281/zenodo.5255218). Several figures were created with the Generic Mapping Tools (GMT) software (Wessel & Smith, 1991).

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