Neutralino dark matter scattering and $B_s \to \mu^+ \mu^-$ in SUSY models

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Abstract

It is pointed out that there is a strong correlation between the neutralino dark matter scattering cross section $\sigma_{\tilde{\chi}p}$ and the branching ratio for $B_s \to \mu^+ \mu^-$ within minimal supergravity (mSUGRA) and its extensions. This correlation arises mainly from $\tan \beta$ and heavy neutral Higgs mass dependence, and shows a nice interplay between vastly different two observables within supersymmetric models. Current upper limit on $B(B_s \to \mu^+ \mu^-) < 5.8 \times 10^{-7}$ excludes substantial parameter space where $\sigma_{\tilde{\chi}p}$ is within the CDMS sensitivity region.
Minimal supersymmetric standard model (MSSM) is a well motivated candidate for physics beyond the standard model (SM). MSSM is consistent with precision electroweak data, and nicely complies with gauge coupling unification. Another nice feature of MSSM with $R$–parity conservation is the presence of natural candidates for cold dark matter (DM) of the universe. Recent data from WMAP collaboration indicates that $\Omega_{DM} h^2 \simeq (0.095 - 0.13)$, which could be dominated by the relic density of neutralino within SUSY models. In supergravity models, the LSP is the neutralino with $m_\chi \sim O(M_Z) - O(G_F^{-1/2})$ and could have suitable relic density.

There has been experimental progress in direct detection of neutralino DM through (in)elastic scattering on various nuclei. Such experiments can be sensitive to a neutralino DM with mass $O(100)$ GeV, which is usually the case in various supergravity scenarios.

Recently, the DAMA signal region [2] has been excluded by the CDMS cryogenic DM search experiment [3] in the range of

$$\sigma_{\tilde{\chi} p} = (10^{-6} - 10^{-5}) \text{ pb},$$

with the corresponding DM mass depends on galactic halo models. Since the CDMS experiment probes the DM scattering down to $3 \times 10^{-7}$ pb level in certain range of DM mass, it is important to calculate the DM scattering cross section within well defined and/or motivated SUSY models, in which the cross section can be in the CDMS sensitivity.

In Ref.s [4, 5], two of us considered a number of low energy phenomena such as $(g - 2)_{\mu}$, $B \to X_s \gamma$, $B \to X_s l^+ l^-$ and $B_s \to \mu^+ \mu^-$ within various SUSY breaking mediation mechanisms. In the present work, we extend our study to the neutralino DM scattering cross section $\sigma_{\tilde{\chi} p}$, its relic density $\Omega_{DM} h^2$, and $B(B_s \to \mu^+ \mu^-)$ in a class of (string inspired) supergravity models. We find that there is a strong correlation between $\sigma_{\tilde{\chi} p}$ and $B(B_s \to \mu^+ \mu^-)$ for a given $\tan \beta$. The origin of this correlation resides in $\tan \beta$ and neutral Higgs boson masses ($m_H, m_A$) within a given (string inspired) supergravity scenarios. In particular, a large $\sigma_{\tilde{\chi} p}$ implies a large $B(B_s \to \mu^+ \mu^-)$, which may exceed the current upper limit on this process. Before proceeding, let us mention that there is an important difference between our previous works and the present work. In Ref.s [4, 5], we did not assume the neutralino LSP since there are ways of avoiding problems with charged particle LSP. On the other hand, we assume that the LSP is the lightest neutralino in the present work, and consider their scattering with nuclei. Therefore SUSY contributions to various observables include the
process $B(B_s \to \mu^+\mu^-)$ considered in this work are generically smaller than those given in Refs. [4, 5].

If the LSP is a neutralino of mass around $O(M_Z) - O(v_{EW})$, one can detect the relic neutralino LSP through (in)elastic scattering with various nuclei. In the large tan $\beta$ limit, heavy neutral Higgs $H$ exchange contribution to the DM scattering becomes important because of its enhanced couplings to down type quarks such as strange or bottom quark. This is relevant to the heavy Higgs interaction with the strange quark contents inside nucleons, and the DM scattering cross section becomes enhanced. Therefore, the DM scattering amplitude increases linearly as $\tan \beta$ increases, and decreases as $m_A$ increases. Also the DM scattering amplitude is sensitive to the value of $\mu$, which determine the higgsino component of the neutralino because the neutralino-higgs coupling become significant when the neutralino is a mixed state of gaugino and higgsino. We use the code DARKSUSY [6] in order to calculate the DM scattering cross section and its relic density within minimal supergravity with (non)universal Higgs mass parameters, and string inspired scenarios including a $D$–brane model.

The decay $B_s \to \mu^+\mu^-$ can be an important probe of SUSY in the large tan $\beta$ limit, since its branching ratio grows like $\tan^6 \beta$ [7]. Unless the stop/charginos and neutral Higgs are too heavy, one can have a significant rate for this decay within SUSY models. If this decay is found at the level of $5 \times 10^{-7}$, then only gravity mediated SUSY breaking mediation (including string inspired scenarios) will survive (except for AMSB and no scale scenarios) [4, 5]. Also, one can get a useful lower bound on $\tan \beta$, once this decay mode is observed [8].

In more general SUSY models where gluino mediated FCNC can be important, one has to include their effects and the correlation between the DM scattering cross section and the $B_s \to \mu^+\mu^-$ branching ratio may be diluted. However gluino-mediated FCNC is not that important in the class of (string inspired) supergravity models we are considering, since the initial conditions for the soft parameters are universal or proportional to the Yukawa couplings, and $\delta$’s are generated mainly through RG evolution.

The minimal SUGRA (mSUGRA) is specified by 5 parameters,

$$m_0, \; m_{1/2}, \; A_0, \; \tan \beta, \; \text{sign}(\mu).$$

The nature of the neutralino LSP is determined by gaugino mass parameters $M_1, M_2$ and
$\tan \beta = 10, 35$ and $55$ (from the left to the right). Black dots for $\Omega_{\chi} h^2 \geq 0.13$, red dots for $0.095 \leq \Omega_{\chi} h^2 \leq 0.13$ and green dots for $\Omega_{\chi} h^2 \leq 0.095$.

the $\mu$ parameter. $|\mu|$ is determined by the electroweak symmetry breaking condition:

$$
\mu^2 = \frac{m_{H_u}^2 - m_{H_d}^2}{\tan^2 \beta - 1} \frac{\tan^2 \beta}{2 M_Z^2}.
$$

In the mSUGRA scenario, $|\mu|$ is naturally large, so that the LSP is binolike and the (pseudo)scalar Higgs bosons $H$ and $A$ are heavy. Therefore, the DM scattering cross section becomes small in this scenario, well below the CDMS sensitivity region, and $B(B_s \rightarrow \mu^+ \mu^-)$ is not so much enhanced. In Fig. 1 we show the correlation between $\sigma_{\tilde{\chi}p}$ vs. $B(B_s \rightarrow \mu^+ \mu^-)$ within mSUGRA with $\tan \beta = 10, 35$ and $55$, respectively. For large $\tan \beta$, there is a strong correlation between the two observables, as emphasized in the beginning of this work. After imposing the $B \rightarrow X_s \gamma$ branching ratio as well as the lower bounds on the lightest Higgs mass and SUSY particle masses, and assuming the neutralino LSP, we find that the DM scattering cross section is $\sigma_{\tilde{\chi}p} \lesssim 10^{-8}$ pb that is too small to be observed at the current or near-future DM search experiments, and $B(B_s \rightarrow \mu^+ \mu^-) \lesssim 2 \times 10^{-7}$, which is below the reach of Tevatron. In particular, the current upper limit $B(B_s \rightarrow \mu^+ \mu^-) < 5.8 \times 10^{-7}$ does not put any strong constraint on $\sigma_{\tilde{\chi}p}$ within the mSUGRA scenario with universal Higgs mass parameters.

The universal soft parameters are too restricted assumption without solid ground within supergravity framework. In order to consider more generic situation within supergravity scenario, let us relax the assumption of universal soft masses as follows:

$$
m_{H_u}^2 = m_0^2 (1 + \delta_{H_u}), \quad m_{H_d}^2 = m_0^2 (1 + \delta_{H_d}),
$$

FIG. 1: $\sigma_{\tilde{\chi}p}$ vs. $B(B_s \rightarrow \mu^+ \mu^-)$ within mSUGRA with universal Higgs mass parameters for $\tan \beta = 10, 35$ and 55 (from the left to the right). Black dots for $\Omega_{\chi} h^2 \geq 0.13$, red dots for $0.095 \leq \Omega_{\chi} h^2 \leq 0.13$ and green dots for $\Omega_{\chi} h^2 \leq 0.095$. The universal soft parameters are too restricted assumption without solid ground within supergravity framework. In order to consider more generic situation within supergravity scenario, let us relax the assumption of universal soft masses as follows:

$$
m_{H_u}^2 = m_0^2 (1 + \delta_{H_u}), \quad m_{H_d}^2 = m_0^2 (1 + \delta_{H_d}),
$$

(2)
whereas other scalar masses are still universal. Here $\delta$’s are parameters with $\lesssim O(1)$. This assumption is still too restrictive for the purpose of studying FCNC such as $B_s \to \mu^+\mu^-$ within supergravity framework. On the other hand, the nonuniversality in the squark masses is not so important to the DM scattering, since in DM scattering what matters is the nature of the LSP, whether it is bino like or Higgsino like. The strong correlation between $\sigma_{\tilde{\chi}p}$ and $B(B_s \to \mu^+\mu^-)$ could be diluted if we allow more general flavor structures in soft terms, which is visible in the $D$–brane models we consider in this work.

In order to emphasize the role of $B(B_s \to \mu^+\mu^-)$, we take the numerical values of $\delta$’s as in Refs. [10, 11]:

\begin{align}
(I) \quad & \delta_{H_d} = -1, \quad \delta_{H_u} = 0, \\
(II) \quad & \delta_{H_d} = -1, \quad \delta_{H_u} = 1.
\end{align}

In Fig. 2 (a) and (b), we show $\mu$ and the pseudoscalar mass $m_A$ as functions of $m_{1/2}$ for the case (II). For $\delta_{H_u} = +1$, $\mu$ becomes lower and the Higgsino component in the neutralino LSP increases so that $\sigma_{\tilde{\chi}p}$ is enhanced, as discussed in Ref. [10]. The change of $|\mu|$ also has an impact on the higgs masses because

$$m_A^2 = m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 \simeq m_{H_d}^2 + \mu^2 - M_Z^2/2$$

at weak scale. For $\delta_{H_d} = -1$, $m_A$ and $m_H$ becomes further lower, and both $\sigma_{\tilde{\chi}p}$ and $B(B_s \to \mu^+\mu^-)$ are enhanced compared with the mSUGRA case. Note that the $B(B_s \to \mu^+\mu^-) < 5.8 \times 10^{-7}$ provides a very significant constraint on the neutralino DM scattering cross section $\sigma_{\tilde{\chi}p}$, and removes the parameter space where the DM scattering is within the reach of CDMS experiment.

In Fig. 4 we show the scattered plot in the $(m_{\tilde{\chi}^0}, \sigma_{\tilde{\chi}p})$ for $\delta_{H_d} = -1, \delta_{H_u} = +1$ along with the CDMS data for (a) $\tan\beta = 35$ and (b) $\tan\beta = 50$. Note that the constraints from the CDMS experiment and the $B(B_s \to \mu^+\mu^-)$ are comparable for $\tan\beta = 35$. However, $B(B_s \to \mu^+\mu^-)$ becomes stronger for $\tan\beta = 50$. After imposing the $B(B_s \to \mu^+\mu^-) < 5.7 \times 10^{-7}$ constraint for the $\tan\beta = 50$ case, we find that $\sigma_{\tilde{\chi}p} \lesssim 2 \times 10^{-8}$ pb, which is well below the current or near-future DM search experiments.

We also considered nonuniversal gaugino masses, in which case the most important one is the gluino mass parameter via RG running. Therefore we considered a possibility that gluino mass can differ from the wino and bino masses ($M_1 = M_2 \neq M_3$). We find that the
FIG. 2: (a) \( \mu \) and (b) \( m_A \) vs. \( m_{1/2} \) in mSUGRA with nonuniversal Higgs mass parameters: \( \delta_{H_u} = 1 \) and \( \delta_{H_d} = -1 \).

FIG. 3: \( \sigma_{\tilde{\chi}p} \) vs. \( B(B_s \to \mu^+ \mu^-) \) in mSUGRA with nonuniversal Higgs mass parameters: (a) \( \delta_{H_u} = 1 \) and \( \delta_{H_d} = -1 \) and (b) \( \delta_{H_u} = 1 \) and \( \delta_{H_d} = 0 \).

FIG. 4: \( \sigma_{\tilde{\chi}p} \) vs. \( m_\chi \) in mSUGRA with nonuniversal Higgs mass parameters: \( \delta_{H_u} = 1 \) and \( \delta_{H_d} = -1 \) for (a) \( \tan \beta = 35 \) and (b) \( \tan \beta = 50 \). The red points (open circles) are excluded by \( B(B_s \to \mu^+ \mu^-) \) constraint.
qualitative feature is similar to the case with nonuniversal Higgs masses. In particular the current limit on $B(B_s \to \mu^+\mu^-)$ already puts a strong constraint on $\sigma_{\tilde{\chi}p}$ in the large $\tan\beta$ region.

Next, we consider a specific $D$ brane model where the SM gauge groups and 3 generations live on different $Dp$ branes [12]. In this model, scalar fermion masses are not completely universal and gaugino mass unification can be relaxed. Also the string scale is around $10^{12}$ GeV (the intermediate scale) rather than GUT scale.

Since there are now three moduli ($T_i$) and one dilaton superfields in this case, we use the following parametrization that is appropriate for several $T_i$ moduli:

$$F^S = \sqrt{3} (S + S^*) m_{3/2} \sin \theta,$$
$$F^i = \sqrt{3} (T_i + T_i^*) m_{3/2} \cos \theta \Theta_i \quad (4)$$

where $\theta$ and $\Theta_i (i = 1, 2, 3)$ with $\sum_i |\Theta_i|^2 = 1$ parametrize the directions of the goldstinos in the $S,T_i$ field space. The explicit expressions for the soft terms are given in Ref. [12]. Let us simply note that the scalar and the gaugino masses become nonuniversal for generic goldstino angles, and there could be larger flavor violations in the low energy processes as well as enhanced SUSY contributions to the $a^\mu_{\text{SUSY}}$.

Therefore the $D$ brane model considered in this work is specified by following six parameters:

$$m_{3/2}, \tan\beta, \theta, \Theta_{i=1,2}, \text{sign}(\mu).$$

Earlier phenomenological analysis of $D$ brane models can be found on the muon $(g-2)_\mu$ [13]. The discussion on $B \to X_s \gamma$, $B \to X_s l^+l^-$ and $B_s \to \mu^+\mu^-$ in this scenario is given in Ref.s [5]. Here, we combine the DM scattering and the branching ratio for $B_s \to \mu^+\mu^-$. We fix $\tan\beta = 50$ and scan over the following parameter space: $-\pi/4 \leq \theta \leq \pi/4$, $m_{3/2} \leq 1000$ GeV, and $\Theta_i$ in order to search the allowed parameter space. In this scenario again, it turns out that the current upper limit on $B(B_s \to \mu^+\mu^-)$ already puts a strong constraint on the parameter space in the $D-$brane scenarios. In Fig. 5 (a), we show the correlation between $B(B_s \to \mu^+\mu^-)$ and $\sigma_{\tilde{\chi}p}$. In Fig. 5 (b), we show the DM cross section as a function of the LSP mass $m_\chi$. Note that the upper limit on $B(B_s \to \mu^+\mu^-)$ makes a stringent constraint on the model, especially for light LSP mass $m_\chi \lesssim 150$ GeV. If we ignored the upper limit on $B(B_s \to \mu^+\mu^-)$, then the resulting DM scattering cross section could be well within the CDMS region with $\sigma_{\tilde{\chi}p} > 4 \times 10^{-7}$ pb. However, such a large DM scattering cross section
implies too large a branching ratio for $B(B_s \to \mu^+\mu^-) > 5.8 \times 10^{-7}$ for light LSP $m_\chi \lesssim 150$ GeV, and thus has to be discarded. For heavier LSP mass, both constraints have to be considered altogether, since they are complementary to each other.

The DM scattering in the AMSB scenarios is qualitatively similar to the previous cases. Although the LSP in the AMSB scenarios is winolike in this case, Higgs boson contribution to the DM scattering is still important. And there is a strong correlation between $\sigma_{\tilde{\chi}p}$ and $B(B_s \to \mu^+\mu^-)$. In the simplest version of the AMSB model, one adds a common scalar mass $m_0^2$ to scalar mass parameters in order to evade the tachyonic slepton mass problem.

In Fig. 6, we show the scattered plot for $\sigma_{\tilde{\chi}p}$ and $B(B_s \to \mu^+\mu^-)$ within such an AMSB scenario with $M_{\text{aux}} = 50$ TeV. The black dots are excluded by $B \to X_s\gamma$ constraint, and only the green points survive. The resulting predictions for the DM scattering and the branching ratio for $B_s \to \mu^+\mu^-$ is so small that this class of the AMSB model has no observable effects in the DM scattering or $B_s \to \mu^+\mu^-$. In the heterotic $M$ theory of Horava and Witten, we have the similar correlation between $\sigma_{\tilde{\chi}p}$ and $B(B_s \to \mu^+\mu^-)$ in the large tan $\beta$ region. However, after imposing direct search bounds on Higgs and SUSY particle masses as well as $B \to X_s\gamma$ constraint and the neutralino LSP condition, the resulting DM scattering cross section turns out very small: $\sigma_{\tilde{\chi}p} \lesssim 10^{-8}$ pb, which is well below the sensitivity of the current DM search experiments. Also we get $B(B_s \to \mu^+\mu^-) < 10^{-7}$ which is beyond the reach of Tevatron Run II.

In conclusion, we pointed out that there is a strong correlation between the neutralino dark matter scattering cross section with nuclei and the branching ratio for $B_s \to \mu^+\mu^-$ within a large class of supergravity models. This correlation arises mainly from tan $\beta$ and
FIG. 6: $\bar{\chi}_p$ vs. $B(B_s \rightarrow \mu^+\mu^-)$ within the AMSB model with $M_{aux} = 50$ TeV. Black dots are excluded by the upper limit on $B \rightarrow X_s\gamma$ branching ratio, whereas the green dots satisfy all the constraints.

heavy neutral Higgs masses ($m_H, m_A$). We have discussed mSUGRA with (non)universal gaugino masses and (non)universal scalar masses and supergravity scenarios derived from heterotic $M$ theory, and AMSB scenario. In the $D$ brane scenario considered in this work, the correlation is diluted because of nonuniversal scalar and gaugino mass parameters. Still the upper limit on $B(B_s \rightarrow \mu^+\mu^-)$ puts a very strong constraint on DM cross section, even stronger than the CDMS limit. Thus the decay $B_s \rightarrow \mu^+\mu^-$ could give invaluable informations not only on SUSY breaking mediation mechanisms as noticed in Refs. [4, 5], but also give a strong constraint on the neutralino DM scattering cross section within a large class of supergravity models in the large $\tan \beta$ region. This is another interesting example of complementarity between rare $B_s$ decays (indirect probe of SUSY) and DM scattering (direct probe of SUSY).

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