On the problem of two-tail heliosphere/astrospheres

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Abstract. We consider the effect of the solar/stellar wind collimation towards the solar/stellar wind rotation axis in the heliosheath between the termination shock and the heliopause/astropause. The collimation is due to the magnetic force produced by the toroidal component of the solar/stellar magnetic field. The collimation leads to formation of a two-jet structure and change of topology of the heliopause. Tube-like shape of the heliopause/astropause is formed instead of the commonly accepted sheet-like shape.

Three different situations are explored in this paper: (1) the Sun/star is at the rest with respect to local interstellar medium (LISM), (2) the Sun/star moves with respect to fully ionized LISM, (3) the Sun/star moves with respect to partially ionized LISM. 3D non-dissipative MHD model results have shown that the tube-like structure is formed in the first two cases. The thickness of the heliosheath depends on the model parameters strongly. The case of the partially ionized LISM is the most realistic one for the heliosphere. In this case the collimation towards the solar poles is also observed in the results of 3D kinetic-MHD modeling. However, the tube-like structure of the heliopause is not seen in the numerical results. We argue that this is due to charge exchange between proton and H atom components that results in displacement of the stagnation points in further downwind of the Sun. Note also that dissipative effects (e.g. reconnection at the current sheet in the non-stationary heliosphere) or instabilities could destroy the effects of the solar wind collimation.

1. Introduction

Starting from the pioneering papers \cite{1, 2} it was conventionally accepted that the heliopause (HP), i.e. tangential discontinuity separating the solar wind (SW) plasma from the plasma of the local interstellar medium (LISM), is topologically equivalent to a plane (see Figure 1a). The sheet-like topology remains to the distances $\sim 3000$ AU into the tail, where the heliopause - as a tangential discontinuity - disappears. This happens due to charge exchange with interstellar atoms of hydrogen, which interact with protons on both sides of the heliopause. Momentum and energy exchange between neutral and ionized components occurs due to the interaction. As a result, the solar wind and interstellar medium parameters on the both sides of the heliopause approach each other. As it has been shown in \cite{3, 4}, the jump in the plasma parameters across the heliopause disappears at $\sim 3000$ AU counted along the axis determined by the interstellar wind velocity vector.

Very recently, this ‘classical’ picture underwent a major revision. In 2015 \cite{5} and \cite{6} have shown that the heliopause may in fact have a tube-like shape (Fig. 1b). Such a shape has been obtained in \cite{5} in the frame of their numerical 3D multi-fluid MHD code for the case when the...
interstellar gas flows with respect to the star. Later this result has been discussed by [7] and [8] and will be discussed later in the paper.

A simpler case, when the solar wind flows into the homogeneous interstellar gas at rest, has been considered by [6] and later by [9, 10]. To understand the effects of the stellar (heliospheric) magnetic field qualitatively, let us start with the ‘classical’ stationary model of supersonic point source flow into the surrounding gas with non-zero pressure at rest (e.g. [1]). There is a shock transition in this solution (i.e. the termination shock) at

\[ R_{TS} \sim \sqrt{\frac{\dot{M}V_0}{4\pi p_\infty}}, \]

where \( R_{TS} \) is the heliocentric distance to the termination shock, \( \dot{M} \) is the stellar mass loss rate, \( V_0 \) is the terminal velocity of the supersonic stellar wind, \( p_\infty \) is the interstellar gas pressure. In the supersonic SW (for \( R < R_{TS} \)) the solution is \( V \sim V_0, \rho \sim 1/R^2 \) and \( p \sim 1/R^{2\gamma} \), where \( R \) is the distance to the Sun/star. In the subsonic region (\( R > R_{TS} \)) the gas may be considered incompressible and the solution is \( V \sim 1/R^2, \rho \sim \rho_\infty \) and \( p \sim p_\infty \).

This solution can be used to calculate the frozen-in magnetic field in the kinematic approximation. Solving \( \nabla \times [V \times B] = 0 \) and assuming that magnetic field is parallel to the velocity vector at the Sun:

\[ R < R_{TS} : B_R \sim 1/R^2, \quad B_\phi \sim (1/R) \sin \theta, B_\theta = 0 \quad (1) \]

The solution above for \( R < R_{TS} \) has been obtained in [11].

\[ R > R_{TS} : \quad B_R \sim 1/R^2, \quad B_\phi \sim R \sin \theta, B_\theta = 0. \quad (2) \]

Here \( \theta \) is the polar angle counted from the stellar rotational axis (x-axis), \( \phi \) is the azimuthal angle.

**Figure 1.** Schematic picture of the heliospheric interface with a sheet-like topology (a) of the heliopause (HP) and a tube-like topology (b).
Figure 2. Panels A and B present numerical results obtained in [9]: (A) the plasma streamlines and density isolines, (B) magnetic field contour plot. Panel C present contour plots of $R_{HP0}/R(TS)$ (red curves) and $r_{jet}/R_{TS}$ (blue plots) that can be obtained from the first integrals and ODE solution along the jet. Brown diamonds are the results of numerical solution.

In the subsonic wind the magnetic field grows proportionally to $r = R \sin \theta$, that is, the distance to the axis of stellar rotation. Alfvénic Mach number $A = \sqrt{(4\pi \rho V^2)/B^2} \sim 1/(R^3 \sin \theta)$ in the subsonic wind, so it decreases with distance rapidly. In the supersonic wind $A$ remains constant. For the Sun, for example, the constant is about 15. At the strong shock $A$ decreases by a factor of $((\gamma - 1)/(\gamma + 1))^{3/2}$ that is equal to 1/8 for $\gamma = 5/3$. Downstream of the TS $A \approx 15/8 \approx 1.9$ and then it decreases in the equatorial plane as $1/R^3$. The Alfvénic Mach number becomes on the order of unity at the distances of $\sim 1.23R_{TS}$. Therefore at these distances and further one can expect a strong influence of the magnetic field on the plasma flow.

Magnetic force $\mathbf{F}_{mag} = (\nabla \times \mathbf{B} \times \mathbf{B})/(4\pi)$ has the main component in $r$-direction (in the cylindrical $(x, r, \phi)$ coordinate system where $x$-axis is the axis of stellar rotation). As a result, the stellar wind deflects from the original radial direction and flows towards $x$-axis. Hence the two-jets structure of the flow is formed.

2. Two-jet structure of the stellar wind flow in the interstellar medium at rest
In [9, 10] we explored in detail the two-jet scenario for a simplified astrosphere in which 1) the star is at rest with respect to the local interstellar medium (LISM), 2) radial magnetic field is neglected as compared with the azimuthal component, 3) the stellar wind outflow is assumed to be hypersonic (both the Mach number and the Alfvénic Mach number are much greater than
unity at the inflow boundary).

It has been shown that the problem in its dimensionless form depends only on one parameter - the Alfvénic Mach number at the inflow boundary, \( A = \sqrt{(4\pi \rho V^2)/B^2} \). Sometimes, it is more convenient to use \( 1/A \) that is called as Alfvénic number and denoted as \( \varepsilon \) in [9, 10]. Numerical parametric studies have been performed for the parameter \( \varepsilon \) varying in the range from 0.01 to 0.5. The numerical results clearly show (Figure 2A,B) the formation of the tube-like heliopause and the two jets along the stellar rotation axis. According to the numerical results obtained in [9] the distances from the axis of stellar rotation to the heliopause at the equatorial plane (x = 0) and in the jets (x → ∞) are both proportional to \( \varepsilon^{-1/3} \) as long \( A \) is small enough (\( \varepsilon < 0.1 \)). Some analytical argumentation for such a dependence is given in [10].

Note that numerical results for \( \varepsilon < 0.01 \) and for \( \varepsilon > 0.5 \), were not presented in the papers because of complex vortex structure beyond the termination shock formed for these cases. Such solutions require more detailed studies.

Besides the numerical solution, analytical consideration of the three first integrals of the MHD equations allow to establish [9] analytical relations between three parameters – (1) the distance to the termination shock, \( R_{TS,0} \), (2) the distance to the heliopause, \( R_{HP,0} \) (both are in the stellar equatorial plane i.e. the plane perpendicular to the axis of the stellar rotation), and (3) the parameter \( \varepsilon \). Therefore, for a given value of \( R_{TS,0} \) one can obtain \( R_{HP,0} \) as a function of \( \varepsilon \).

The distribution of the plasma parameters in the jet as well as the size of the jet have been obtained in [9] as a solution of an ordinary differential equation obtained under assumptions of the hypersonic stellar wind outflow and the spherically symmetric termination shock.

These analytical considerations allow to estimate the magnitude of the stellar magnetic field from the geometric picture of a two-jet atmosphere. In particular, the knowledge (obtained, for example, from observations) of two ratios – \( R_{HP}/R_{TS} \) in the equatorial plane and \( r_{jet}/R_{TS} \) – allows (see Figure 2C) to determine the dimensionless distance to the termination shock \( \hat{R}_{TS} \) and \( \varepsilon \). Then, knowing \( \varepsilon \) and the actual dimensional distance to the TS one can derive the magnitude of the stellar magnetic field at any given distance from the star.

3. Two-jet structure of the stellar wind flow in the moving magnetized LISM (no interstellar H atoms)

In this section we demonstrate the first and preliminary results of the numerical 3D ideal MHD model of the SW/LISM interaction. In this model we assume that the interstellar medium is moving with respect to the Sun. The calculations were performed by employing the kinetic-MHD numerical model [8] and neglecting the effect of interstellar H atom on the plasma component. Therefore, the "kinetic" part of the model was abandoned for the present calculations.

Since in this paper we focus on qualitative picture, the results of the calculations will be presented in dimensionless form. In dimensionless formulation, the problem depends on the following 5 parameters: gas-dynamical and Alfvénic Mach numbers, \( M = v/\sqrt{\gamma p/\rho} \) and \( A = \sqrt{(4\pi \rho V^2)/B^2} \), in the supersonic solar wind at the inner boundary (\( M_s, A_s \)) and in the undisturbed interstellar wind (\( M_\infty, A_\infty \)), and the angle \( \alpha \) that determines the angle between the direction of the interstellar velocity vector and the direction of the interstellar magnetic field. Below we present the results for \( \alpha = 0 \) and \( \alpha = \pi/2 \).

The results of our calculations are presented in Figures 3 and 4 in the following coordinate system: axis \( Z \) is directed towards the interstellar flow direction, axis \( X \) coincides with the axis of stellar rotation, and axis \( Y \) is chosen to complete the right-handed coordinate system. Here we assume that the velocity vector of the pristine LISM is perpendicular to the axis of solar rotation.

Figure 3 presents the results of calculations with the following set of parameters: \( M_s = 10 \), \( A_s = 4.5 \), \( M_\infty = 1.5 \), \( A_\infty = 1.18 \), \( \alpha = 0 \). Panels A and C show the plasma streamlines and
density in the ZX- and ZY- planes, respectively. Panels B and D present the magnetic field lines and magnitude in the same planes. The discontinuities - the heliopause (HP), the termination shock (TS) and the bow shock (BS) - are shown as thick white curves. All parameters as well as heliocentric distances are shown in dimensionless units.

There is no doubt that the tube-like structure is present in this case. Note, however, that 1) the tube is strongly deflected towards the tail, 2) the downwind stagnation point is much (at least twice) further from the Sun than the upwind point. The stagnation points/lines are the points/lines at the heliopause at which the plasma velocity is zero.

The thickness of the heliosheath depend strongly on the orientation of the interstellar magnetic field. Panels A and B in Figure 3 show results of the calculations for the model with \( \alpha = \pi/2 \), i.e. when the interstellar magnetic field vector is perpendicular to the LISM velocity vector. In these calculations we assume that the magnetic field is in the XZ-plane. It is seen from the figure that the stagnation point in downwind is much further in this case as compared with the case of parallel magnetic field presented in Figure 3. Note, also, that the jumps of density and magnetic field at the heliopause are quite small in this case. So, despite the two-tail structure formally exists, the obtained picture is quite similar to the classical picture of the heliosphere.

Figure 4 demonstrates how the structure of the SW/LISM interaction depends on the other dimensionless parameters. Panels C and D demonstrate the results for \( A = 0.55 \) which corresponds to stronger interstellar magnetic field as compared with results presented in Figure 4A,B. It is seen that the stronger magnetic field makes the heliospheric tube more collimated. The heliopause jets become even more collimated (in dimensionless units) in the tail for smaller gas-dynamic Mach number as shown in Figure 4E,F.

Therefore, we conclude at the end of this section that our calculations in the frame of 3D ideal MHD model demonstrate, without any doubts, that the heliopause has tube-like structure. However, the degree of its collimation (or, in other words, the thickness of the heliosheath between the termination shock and the heliopause) strongly depend on model parameters. Detailed analysis of the dependence will be done elsewhere.

4. Are the jet features present in the model of Alexashov and Izmodenov (2015)?

The effect of solar wind collimation towards the solar rotation axis and formation of the two-jet structure is actually seen in the results of the 3D kinetic-MHD model developed in [8]. This model of the solar wind interaction with the two-component (plasma and H atoms) local interstellar medium takes into account both the interstellar and heliospheric magnetic fields. So the effects of the heliospheric magnetic field should be observed.

In order to explore these effects, the solar flux (\( \rho V \)) is shown in Figure 5 in projection on a closed surface located in the inner heliosheath. The surface has been constructed by adding a fixed distance to the TS distance in every direction. Panel A demonstrates the solar wind mass flux in the case when heliospheric magnetic field is not taken into account. It is seen that the mass flux reaches its maximum for \( \theta \sim 100^\circ-110^\circ \) and \( \phi \sim 180^\circ \). This nearly crosswind direction corresponds to the direction in which all of the solar mass flux from the upwind hemisphere passes into the tail. The maxima at \( \phi \sim 180^\circ \) correspond to that part of the (\( BV \)) plane where the distance between the TS and HP is minimal. Conversely, panel B demonstrates that there are two mass-flux maxima (i.e. jets) in the north and south directions. These jets are a clear result of the magnetic tension.

Therefore, in this respect, the effect of magnetic field collimation is clearly observed. Nevertheless, the conclusion on the tube-like shape of the heliopause proposed in [5] and confirmed by our MHD results presented in previous section is not supported in [8]. The most likely reason of the discrepancy is connected with the effect of charge exchange of solar protons and interstellar H atoms. Indeed, having large mean free path the interstellar H atoms penetrate
into the vicinity of the downwind/upwind stagnation points. There the atoms exchange charge with the protons. As a result of the charge exchange, momentum is transferred from the neutral component into the plasma component. The momentum transfer results in diminished heliocentric distance to the heliopause in the upwind side of the heliosphere. The effect is very well known for the traditional sheet-like heliosphere (see, e.g., Figure 2 in [12]). In the tube-like heliosphere, the same momentum-transfer effect should result in increase of the heliocentric distance to the heliopause in the downwind direction. The heliopause, in principle, could move further and further downwind, and so it can be treated as a sheet-like structure in the considered computational domain.

In principle, the effect of charge exchange is taken into account in the multi-fluid approach of [5]. However, as it has been shown in [13], the multi-fluid and kinetic approaches may have qualitatively different results. It is also shown in [7] that a multi-fluid treatment of charge exchange may create a region in the tail where charge exchange is very much suppressed as compared with the kinetic-MHD model results. This may explain different shape of the heliopause obtained in [5] and in [8].

5. Summary and conclusions
In this paper we present results of the 3D ideal-MHD model of SW/LISM interaction which demonstrate the effect of solar wind collimation in the region of inner heliosheath between the termination shock and the heliopause due to toroidal component of the heliospheric magnetic
Figure 4. Panels A, C, E show the plasma streamlines and density in the ZX-plane. Panels B, D, F show the magnetic field lines and magnitude. Panels A and B show results for $M_\infty = 1.5, A = 1.18, \alpha = \pi/2$. Panels C and D show results for $M_\infty = 1.5, A = 0.55$ and $\alpha = \pi/2$. Panels E and F show results for $M_\infty = 0.5, A = 1.77$ and $\alpha = 0$. The discontinuities are shown as white thick curves. All parameters are given in dimensionless units.

field. The collimation leads to the formation of the so-called tube-like shape of the heliopause (Figure 1b) contrary to the commonly accepted sheet-like shape.

The tube-like or two-jet structure of the heliosphere has been suggested in [5]. At present we do not have any doubt that such a structure exists when the solar wind interacts with fully ionized local interstellar medium. This conclusion has been made both (1) for the model with the Sun at the rest with respect to the interstellar medium and (2) for the model when the Sun moves in the LISM. In the latter case, the thickness of the heliosheath is 2-3 times larger downwind as compared to upwind. The thickness depends on the model parameters.

In the case of partially ionized local interstellar medium when interstellar H atoms interact with protons by charge exchange, the kinetic-MHD model of [8] does show the tube-like structure of the heliopause while the effect of solar wind collimation towards solar poles is clearly observed in the results. We explain the absence of the tube-like heliopause by momentum transfer between proton and H atom components at the downwind stagnation point. The momentum added to the plasma component in charge exchange moves the heliopause further away from the Sun into the downwind direction. Finally, although the tube-like structure could potentially exist in this case, the thickness of the heliosheath becomes so large in downwind of the Sun that locally there is no difference between the sheet-like or tube-like heliosphere.

Note that the explanation given above is qualitative and based on the results obtained in the cases examined. More detailed quantitative proof requires very computationally expensive parametric studies in the frame of 3D kinetic-MHD model.

It is also very important to note that all of the results presented in this paper as well as in
Figure 5. Solar wind mass flux ($\rho V$) projected on a closed surface located in the inner heliosheath at equal distances from the heliopause. Results of the model without heliospheric magnetic field are presented in panel A. Results of the model where the heliospheric magnetic field has been taken into account are presented in panel A. The angles are $\theta$ and $\phi$. Black curves are the projections of the streamlines into the surface. White curves are the projections of the heliospheric magnetic field lines. [8]

[5] have been obtained in the frame of the so-called unipolar approach for the solar magnetic field. This approach is based on the fact that the terms responsible for the influence of the magnetic field in the ideal MHD equations do not depend on the orientation of the magnetic field. Also, in the frame of the ideal MHD equations, we are not interested in the structure of the heliospheric current sheet assuming that it is a discontinuity where the magnetic field changes its orientation. Taking this into account the approach assumes unipolarity of the heliospheric magnetic field in MHD calculations that allows to avoid difficulties connected with requirements of numerical resolution of the current sheet structure. An MHD model with a current sheet would
either require very high resolution (impossible for modern computers) or have large numerical dissipation which could lead in unphysical results.

In addition, the reconnection of the heliospheric and interstellar magnetic field at the heliopause is not possible in the frame of the ideal MHD approach. We (in our model here and in [8]) consider the heliopause as a tangential discontinuity where \( B_n = 0 \) on both sides of the heliopause. This allows us to avoid numerical reconnection at the heliopause.

Overall, the solution of the considered problem does not depend on the orientation of the inner and outer magnetic fields along their magnetic field lines. Therefore, to perform numerical calculations, we can assume unipolar heliospheric magnetic field at the inner boundary: \( B_R = -B_E \left( \frac{R_E}{R} \right)^2 \). The actual polarity of the heliospheric magnetic field can be calculated after the self-consistent solution has been obtained. This unipolar magnetic field approach helps us to avoid problems with unphysical numerical reconnection and related unphysical dissipation.

The ideal MHD is not necessarily a valid approach in the heliosheath. Most likely, physical reconnection appears at the heliopause. In addition, the SW/LISM interaction region is not a stationary structure. Magnetic field change its polarity during the solar cycle. It has been argued in [7, 15] that the reconnection and solar cycle effects strongly reduce the solar wind collimation by the heliospheric magnetic field. It is also interesting to note the following results shown in [14, 15]: 1) the bent-tube structure of the heliopause disappears even within unipolar field, if solar-cycle effects are taken into account, 2) the development of kink instability obtained in numerical simulations qualitatively agrees with prediction made more than 40 years ago in [16]. So the kink instability could be another reason that reduces the solar wind collimation. Therefore, most likely that the heliopause has a ”classical” sheet-like topology rather than a tube-like.

Overall, we conclude that the effect of magnetic field collimation is certainly important and should be considered in the models of global structures of heliosphere and astrospheres. Nevertheless, in case of the heliosphere, there are effects that may potentially destroy the jet-like structure of the heliosphere in the tail. The main effect explored by us is charge exchange. Other effects such as reconnection at the heliopause and time-dependence have been explored in [7, 15]. Another important aspect of the considered problem that needs to be examined is the stability of the obtained two-jet solution.

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