Zero-Delay Gaussian Joint Source–Channel Coding for the Interference Channel

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Abstract—This letter studies zero-delay joint source–channel coding for transmission of correlated Gaussian sources over a Gaussian interference channel. We propose to adopt a delay-free hybrid digital and analog scheme, which is transmitting the superposition of scaled source and its quantized version after applying scalar quantization to the source at each transmitter. At the corresponding receiver, two kinds of estimators are presented. It is shown that both the schemes, when optimized, beat the uncoded transmission if the channel signal-to-noise ratio is higher than a threshold value for different correlation coefficients and interference values.

Index Terms—JSCC, Gaussian IC, zero-delay, HDA.

I. INTRODUCTION

In many emerging applications involving wireless sensor networks (WSN), low-cost sensors with limited processing capabilities and battery life are deployed which implies that encoders with low complexity are needed. Such applications usually require real-time monitoring and control of underlying physical systems, which impose strict delay constraints. As a result, novel coding methods instead of traditional long block codes for separate source and channel coding (SSCC) which exhibit high complexity and high delay are needed.

We consider the extreme of zero-delay problem, i.e., the transmission is to be done in a scalar fashion. It is well known that a zero-delay uncoded (i.e., scale-and-transmit) scheme can achieve the minimum squared distortion for a Gaussian source transmitted over an additive white Gaussian noise (AWGN) channel with an input power constraint. However, this is not the case for multi-terminal problems, in general. Also, in multi-terminal scenarios, the optimality of SSCC breaks down. Note that in WSN, measurements collected by the sensors close to each other exhibit statistical correlation. As the correlation of sources can be adopted to generate correlated channel inputs by JSCC, it is attractive to turn to JSCC. As a matter of fact, uncoded scheme is a special case of JSCC. In recent years, many works have been done to understand multi-user JSCC problems. For example, [1] derived the distortion lower bound when a bivariate Gaussian is transmitted over a Gaussian multiple access channel (GMAC). The necessary conditions for optimality of uncoded transmission for multi-user communications over Gaussian broadcast channel (BC) and GMAC were generalized in [2]. Reference [3] proposed two distributed delay-free JSCC schemes for a bivariate Gaussian on a GMAC. References [4] and [5] investigated zero-delay JSCC problems for a Gaussian source in the Wyner-Ziv Setting and over dirty-tape channel respectively.

In this letter, we consider zero-delay transmission of a pair of correlated Gaussian sources \(S_{1}, S_{2}\) over a two-user Gaussian interference channel (IC). Each of two separate transmitters observes a different component of the source pair and describes the observations to the corresponding destination over a Gaussian IC. Receivers \(i\) tries to recover the source \(S_{i}\), where \(i \in \{1, 2\}\) with the minimum average distortion. Reference [6] gave achievable distortion region for JSCC IC problem in the lossy setup. About JSCC Gaussian IC problem, [7] derived an outer bound on the achievable region when the interference is weak and showed the condition for optimality of uncoded transmission. For strong interference case, see [8]. Our goal here is to design low delay and low complexity JSCC techniques motivated by hybrid digital and analog strategies proposed in [6]. After applying scalar quantizer to each source, the source itself and the quantized value are scaled and superimposed to be channel inputs. At each receiver, we propose two reconstruction methods. We present numerical results to show that as long as CSNR is higher than a threshold value, the proposed schemes offer better performance than uncoded transmission for different correlation coefficients and interference values.

The remainder of the letter is organized as follows. Section II introduces the GIC problem while our scalar HDA encoding method and two kinds of estimation schemes are presented in Section III. Simulation results are given in Section IV. The supporting information for this letter can be found in [9, Appendices].

II. PROBLEM STATEMENT

We assume that a sequence of zero mean bivariate Gaussian source \(\{S_{1,j}, S_{2,j}\}_{j=1}^{\infty}\) is independent and identically distributed (i.i.d.) along time \(j\), and the covariance matrix for each \((S_{1}, S_{2})\) is

\[
\begin{bmatrix}
1 & \rho \\
\rho & 1
\end{bmatrix}
\]

with \(|\rho| < 1\). In other words, \(S_{i} \sim \mathcal{N}(0, 1)\) and

\[S_{ic} = \rho S_{i} + N_{i} \quad i, 1, 2, \quad i = \{1, 2\} \backslash i\]

with \(N_{i} \sim \mathcal{N}(0, \sigma^{2}_{N})\), where \(\sigma^{2}_{N} = 1 - \rho^{2}\) and \(N_{i}\) is not only independent of \(S_{i}\), but also of \(N_{ic}\).

At the \(i\)-th transmitter, an \(n\)-block of the \(i\)-th source \(\{S_{i,j}\}_{j=1}^{n}\) is to be mapped to channel input \(\{X_{i,j}\}_{j=1}^{n}\) which should satisfy individual average power constraints,
Each $X_i$ then goes through an additive Gaussian IC with i.i.d. noise $W_i \sim N(0, \sigma_i^2)$ whose output is $Y_i$, i.e.,

$$Y_i = X_i + c_i X_{i'} + W_i \quad i = 1, 2,$$

The source is recovered to be $\{\hat{S}_{i,j}\}_{j=1}^n$ as a function of $\{Y_i, Y_{j'}\}_{i=1}^n$. The quality of the reconstruction is measured by the mean-squared-error distortion, i.e.,

$$D_i = \frac{1}{n} \sum_{j=1}^{n} \mathbb{E}[(S_{i,j} - \hat{S}_{i,j})^2].$$

Then $D = \frac{1}{2}(D_1 + D_2)$ denotes average end-to-end distortion.

At the extreme of zero-delay, the block length $n$ equals to 1 and the encoding function is a sample-by-sample one. In the next section, we introduce our zero-delay JSCC encoding function and two kinds of corresponding decoding schemes.

### III. Proposed Schemes

Our proposed HDA encoder is depicted in Fig. 1. The digital information $T_i = Q(S_i)$ is produced by a midtread scalar quantizer with reconstruction levels $\{t_k\}_{k=-\infty}^{\infty}$, $k \in \mathbb{Z}$, which denotes the set of integers. We use $\Delta = t_k - t_{k-1}$ to denote quantization step. Then it holds that $t_k = k \Delta$. In parallel, the analog part is used to send the source itself. The scaled combination of $T_i$ and $S_i$, $X_i = f_i(S_i) = \delta_i T_i + \beta_i S_i$, is then transmitted through the channel. The average transmit power from encoder $i$ is

$$P_i = \mathbb{E}[(\delta_i T_i + \beta_i S_i)^2] = \delta_i^2 \mathbb{E}[T_i^2] + \beta_i^2 + 2\delta_i \beta_i \mathbb{E}[T_i S_i] \quad i = 1, 2,$$

where

$$\mathbb{E}[T_i S_i] = \sum_{k} t_k^2 P_i(s_i) ds_i = \sum_{k} \frac{t_k + \frac{\Delta}{2}}{\sqrt{2\pi}} \left[ \exp \left( -\frac{(t_k + \frac{\Delta}{2})^2}{2} \right) - \exp \left( -\frac{(t_k - \frac{\Delta}{2})^2}{2} \right) \right],$$

$$\mathbb{E}[T_i^2] = \sum_{k} t_k^2 \frac{1}{2} \left[ \text{erf} \left( \frac{t_k + \frac{\Delta}{2}}{\sqrt{2}} \right) - \text{erf} \left( \frac{t_k - \frac{\Delta}{2}}{\sqrt{2}} \right) \right].$$

As shown in Fig. 1, at $i$-th receiver, $(T_i, T_{i'})$ are firstly recovered by a joint estimator, then $S_i$ is reconstructed by utilizing the analog channel output $Y_i$ and the estimated digital information pair $(\hat{T}_i, \hat{T}_{i'})$ jointly. We propose two kinds of decoding schemes and our object is to find minimum average distortion $D$ with the average power constraint $P_1 + P_2 = 2P$.

A. Scheme A

Given the correlation $\rho$, we set the maximum distance between the quantization output of $S_i$ and the one of $S_{i'}$ as

$$M \Delta = \left[ 3\sqrt{1 - \rho^2} + \left[ \left(k_{\text{max}} - \frac{1}{2}\right) \Delta + \rho \left(k_{\text{max}} - \frac{1}{2}\right) \Delta \right] \right] \times \Delta,$$

$k$ denotes integer quantization index. Its absolute value is limited by $|k_{\text{max}} + \frac{1}{2}\Delta| = \kappa$, where $Pr(S_i \in [-\kappa, \kappa]) \approx 1$, that is, the overload distortion approximately equals to 0.

We apply maximum a posterior (MAP) estimator for recovery of digital information,

$$\hat{i}_i, \hat{i}_{i'} = \arg \max_{t_{i,k}, t_{i',k'}} P(T_i, T_{i'}, Y_i) \left( t_{i,k} - t_{i',k'} \right), y_i,$$

Herein,

$$P(T_i, T_{i'}, Y_i) \left( t_{i,k} - t_{i',k'} \right), y_i$$

$$= \int_{-\infty}^{t_{i,k} + \frac{\Delta}{2}} \int_{t_{i',k'} - \frac{\Delta}{2}}^{t_{i',k'} + \frac{\Delta}{2}} P_{S_i, S_{i'}}(s_i, s_{i'}) P_W(y_i) ds_i ds_{i'},$$

where

$$u_{i,k, t_{i'}, k'} = y_i - \delta_i t_i - \beta_i s_i + c_i (\delta_{i'} t_{i'} + \beta_{i'} s_{i'}).$$

After obtaining $(\hat{i}_i, \hat{i}_{i'})$, $\hat{S}_i$ is estimated as below,

$$\hat{S}_i = \mathbb{E}[S_i | (T_i, T_{i'})], Y_i$$

$$= \int_{-\infty}^{\infty} s_i P_{S_i | (T_i, T_{i'})} (s_i) ds_i \int_{-\infty}^{\infty} P_{S_i, S_{i'} | (T_i, T_{i'})} (s_i, s_{i'}) \cdot \int_{-\infty}^{\infty} P_{W} (y_i) ds_i ds_{i'}.$$

For Scheme A, it is hard to obtain analytical form for $D$.

B. Scheme B

Note that $X_i$ can be rewritten as the summation of quantized value and quantization error $R_i = S_i - T_i$,

$$X_i = \delta_i T_i + \beta_i (T_i + R_i) = \alpha_i T_i + \beta_i R_i,$$

where $\alpha_i$ denotes $\delta_i + \beta_i$. As $R_i$ is constrained in $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$, we propose a pseudo maximum likelihood (ML) estimator as follows,

$$\hat{i}_i, \hat{i}_{i'} = \arg \min_{t_{i,k} - t_{i',k'} \in [-\kappa, \kappa]} y_i - \alpha_i t_{i,k} - c_i' \alpha_i t_{i',k'}.$$

Fig. 1. Framework of our schemes.
As
\[ Y_i = \alpha_i T_i + \beta_i R_i + \epsilon_i \left[ \delta_i T_i^p + \beta_i (\rho (T_i + R_i) + N) \right] + W_i \]
\[ = (\alpha_i + c \beta_i \beta_i \rho) T_i + \epsilon_i (\alpha_i - \beta_i) T_i^p \\
+ (\beta_i + c \beta_i \beta_i \rho) R_i + \epsilon_i (\alpha_i - \beta_i) N + W_i, \]
the quantization error \( E_i \) is estimated linearly by
\[ \hat{E}_i = \sum \left[ y_i - (\alpha_i + c \beta_i \beta_i \rho) \hat{T}_i - \epsilon_i (\alpha_i - \beta_i) \hat{T}_i \right], \]
where \( \Gamma_i \) is linear coefficient. Finally, \( \hat{S}_i = \hat{T}_i + \hat{R}_i. \)

**Distortion Analysis:** For Scheme B, we can express the overall distortion \( D_i \) in analytical form. By definition,
\[ D_i = \mathbb{E}[(T_i + R_i - \hat{T}_i - \hat{R}_i)^2] \]
\[ = \mathbb{E} \left[ \left( \lambda + (1 - \Gamma_i (\beta_i + c \beta_i \beta_i \rho)) (T_i - \hat{T}_i) \\
- \Gamma_i c \beta_i (\alpha_i - \beta_i) (T_i - \hat{T}_i) \right)^2 \right] \]
\[ = \frac{2 \sigma^2 \lambda + \sigma^2 \sigma_N^2 + \sigma^2 \sigma_W^2 + 2 (1 - \Gamma_i (\beta_i + c \beta_i \beta_i \rho)) \sum_{j=1}^{i-1} \mathbb{E}[R_j (T_j - \hat{T}_j)]}{\sigma^2 (1 - \Gamma_i (\beta_i + c \beta_i \beta_i \rho))} \times \frac{2 (1 - \Gamma_i (\beta_i + c \beta_i \beta_i \rho)) \sum_{j=1}^{i-1} \mathbb{E}[R_j (T_j - \hat{T}_j)]}{\sigma^2 (1 - \Gamma_i (\beta_i + c \beta_i \beta_i \rho))}. \]
where
\[ \lambda = (1 - \Gamma_i (\alpha_i + c \beta_i \beta_i \rho)) (T_i - \hat{T}_i) \]
\[ - \Gamma_i c \beta_i (\alpha_i - \beta_i) (T_i - \hat{T}_i). \]
Next, we would analyze the components of (4) one by one.
\[ \mathbb{E} \left[ \lambda^2 \right] \]
\[ = \frac{2 \sigma^2 \lambda + \sigma^2 \sigma_N^2 + \sigma^2 \sigma_W^2 + 2 (1 - \Gamma_i (\beta_i + c \beta_i \beta_i \rho)) \sum_{j=1}^{i-1} \mathbb{E}[R_j (T_j - \hat{T}_j)]}{(1 - \Gamma_i (\beta_i + c \beta_i \beta_i \rho))} \times \frac{2 (1 - \Gamma_i (\beta_i + c \beta_i \beta_i \rho)) \sum_{j=1}^{i-1} \mathbb{E}[R_j (T_j - \hat{T}_j)]}{(1 - \Gamma_i (\beta_i + c \beta_i \beta_i \rho))}. \]
\[ \mathbb{E}[R_i | T_i] = \mathbb{E}[T_i S_i] - \mathbb{E}[T_i^2]. \]
\[ \mathbb{E}[R_i | T_i] \] can be obtained by substituting (2) and (3) into (6).
\[ \mathbb{E}[R_i | T_i] = \mathbb{E}[R_i | T_i \{ T_i = t_k \}] \]
\[ = \sum_k P_{T_i} (t_k) \mathbb{E}[R_i | T_i \{ T_i = t_k \}] \]
\[ = \sum_k P_{T_i} (t_k) \sum_m \int \frac{d m r_i}{m} \int m r_i P_{R_i, T_i, T_i \{ T_i = t_k \}} (r_i, t_m, t_k) d r_i \]
\[ = \sum_k \sum_m \int \frac{d m r_i}{m} \int m r_i P_{R_i, T_i, T_i \{ T_i = t_k \}} (r_i, t_m, t_k) d r_i \]
\[ = \sum_{k=m=m-M}^{k+M} \sum_{m=m-M}^{m+M} t_m \int \frac{d s_i - d s_i}{s_i - s_i} \int (s_i - t_k) P_{S_i, S_i \{ S_i = s_i \}} (s_i, s_i) d s_i d s_i. \]
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Substituting (10) into (5), and (11) into (8), (9), we obtain all the components needed to calculate (4).

IV. SIMULATION RESULTS

We compare our schemes with uncoded transmission and upper bounds. Though the schemes are proposed for general case, here we only present the symmetric interference case, that is, \( c_1 = c_2 = c \). The performance is measured by signal-to-distortion ratio \( SDR = \frac{\sigma_1^2}{D} \). In our experiments, \( |k|_{\text{max}} \), the maximum value of \( |k| \) is chosen to be \( |k|_{\text{max}} = \lceil \frac{6}{2} - \frac{1}{2} \rceil \). \( |k| \) denotes the absolute value of integer quantization index \( k \).

Fig. 2 (a), (b) show the comparison results when the interference is strong. Herein, the upper bound is obtained by applying the lower bound resulting from [8, Lemma 3]. The results while the interference is weak can be found in Fig. 2 (c), (d). The upper bound here is derived through [7, Lemma 1]. The markers ‘o’ represent the simulation results by Scheme B using optimal parameters obtained from minimizing the average end-to-end distortion after (4) is substituted, while the lines without marker show the calculation results by Scheme B. The effectiveness of (4) is demonstrated as markers ‘o’ basically stick to corresponding lines. From all these figures, it can be told that both Scheme A and Scheme B we proposed are superior to the uncoded transmission when CSNR is larger than a threshold value for different interference values and correlation coefficients. Fixing interference factor \( c \), the superiority of our schemes towards uncoded transmission becomes more obvious when \( \rho \) decreases, which is reflected by the fact that the threshold value of CSNR gets smaller.

Comparing the Fig. 2 (a), (b) with the figures (c), (d), the advantage of our schemes on uncoded transmission is more prominent for the cases with strong interference. In the scenarios with weak interference, the advantage decays with the decreasing of \( c \). This is reasonable as when \( c \rightarrow 0 \), the interference channel degrades to point to point channel, and as well known, uncoded transmission achieves optimal distortion in such scenario. In Fig. 2 (e), (f), we fix the correlation \( \rho \) and plot SDR as a function of \( c \) with varying CSNR. Note that the performances of proposed schemes enhance with the increasing of \( c \) in the presence of strong interference and with the decreasing of \( c \) for weak interference channel while the performance of uncoded transmission always deteriorates with the increasing of \( c \).

REFERENCES

[1] A. Lapidoth and S. Tinguely, “Sending a bivariate Gaussian over a Gaussian MAC,” IEEE Trans. Inf. Theory, vol. 56, no. 6, pp. 2714–2752, Jun. 2010.
[2] C. Tian, J. Chen, S. N. Diggavi, and S. Shamai (Shitz), “Matched multiuser Gaussian source channel communications via uncoded schemes,” IEEE Trans. Inf. Theory, vol. 63, no. 7, pp. 4155–4171, Jul. 2017.
[3] P. A. Floor, A. N. Kim, N. Wernersson, T. A. Ramstad, M. Skoglund, and I. Balasingham, “Zero-delay joint source-channel coding for a bivariate Gaussian on a Gaussian MAC,” IEEE Trans. Commun., vol. 60, no. 10, pp. 3091–3102, Oct. 2012.
[4] X. Chen and E. Tuncel, “Zero-delay joint source-channel coding using hybrid digital-analog schemes in the Wyner-Ziv setting,” IEEE Trans. Commun., vol. 62, no. 2, pp. 726–735, Feb. 2014.
[5] M. Varasteh, D. Gündüz, and E. Tuncel, “Zero-delay joint source-channel coding in the presence of interference known at the encoder,” IEEE Trans. Commun., vol. 64, no. 8, pp. 3311–3322, Aug. 2016.
[6] P. Minero, S. H. Lim, and Y.-H. Kim, “Joint source-channel coding via hybrid coding,” in Proc. IEEE Int. Symp. Inf. Theory Proc. (ISIT), Jul./Aug. 2011, pp. 771–785.
[7] I. E. Aguerri and D. Gündüz, “Correlated Gaussian sources over Gaussian weak interference channels,” in Proc. Inf. Theory Workshop, Jeju, South Korea, Oct. 2015, pp. 84–88.
[8] I. E. Aguerri and D. Gündüz, “Gaussian joint source-channel coding for the strong interference channel,” in Proc. Inf. Theory Workshop, Jerusalem, Israel, Apr./May 2015, pp. 1–5.
[9] X. Chen. (Jan. 2018). “Zero-delay Gaussian joint source-channel coding for the interference channel.” [Online]. Available: http://arxiv.org/abs/1801.07851