Quantum chromodynamics with various number of flavors

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The phase structure of QCD with various number of flavors is studied for Wilson quarks. For the case of \( N_F = 3 \) we find that the finite temperature deconfining transition is of first order in the chiral limit on an \( N_t = 4 \) lattice. Together with our previous results that the deconfining transition in the chiral limit is continuous for \( N_F = 2 \) and is first order for \( N_F = 6 \), the order of the transition is found to be consistent with a prediction of universality. The case of \( SU(2) \) QCD is also studied in the strong coupling limit and the phase structure is found to be quite similar to the case of \( SU(3) \): There exists a critical number of flavors \( N_F^{\star} \) and for \( N_F \geq N_F^{\star} \) the confinement is broken even in the strong coupling limit for light quarks. \( N_F^{\star} = 3 \) corresponding to 7 for \( SU(3) \).

1. INTRODUCTION

The deconfining transition of QCD is the last phase transition which our Universe has enjoyed in the past. To study the nature of this transition it is essential to include dynamical quarks. In this paper we study the phase structure of finite temperature lattice QCD with degenerate \( N_F \) Wilson quarks.

Our model has 4 parameters: \( N_F \) (the number of flavors), \( \beta \) (gauge coupling), \( K \) (hopping parameter) and \( N_t \) (lattice size in the temporal direction). For \( N_t = 4 \) – 8 we use a spatial \( \times \) lattice (the lattice size 10 is doubled for hadrons), while for \( N_t = 18 \) we use \( \times \) lattice. The method of simulation is similar to that described in [1,2].

1.1. \( N_F = 2 \) and 6

In a previous paper [1] we studied the deconfining transition \( K_T \) for the cases of \( N_F = 2 \) and 6 on an \( N_t = 4 \) lattice. Fig. 1 shows our result of the phase diagram for \( N_F = 2 \) near the chiral limit \( K_C \). (The result for \( N_F = 3 \) will be discussed later.) \( K_T \) is identified by a sudden change in the behavior of physical observables such as plaquette and \( m_\pi^2 \). \( K_C \) is determined by a linear extrapolation (in \( 1/K \)) of \( m_\pi^2 \) in the confining region on the lattice with the same \( N_t \). We find that \( N_t \) (as well as \( N_F \)) dependence of \( K_C \) is small for \( \beta \lesssim 4.5 \) compared with the difference of \( K_C \) determined by \( m_\pi^2 \) and \( m_q \); the difference is a result of the \( O(a) \) corrections concerning the chiral behavior of Wilson fermions. The location of the crossing point \( \beta_{CT} \) is identified by monitoring \( N_{\text{inv}} \), the number of CG iterations to invert the quark matrix, just on the \( K_C \)-line [1,2]. This can be done because of an empirical law [3] that \( N_{\text{inv}} \) becomes very large in the confining phase when we get close to \( K_C \), while \( N_{\text{inv}} \) remains small in the deconfining phase even at \( K_C \).

The nature of the transition at \( \beta_{CT} \) can be studied by measuring observables on the \( K_C \)-line [1]. We find that, when we decrease \( \beta \) toward...
To the case of 3 flavors. Performing simulations between 2 and 3, it is important to extend the study.

The transition temperature in the real world is between 2 and 3. Fig. 2 (a) is the result for $m^2$ on the $K_C$ line. We find that $m^2$ is large for $\beta > \beta_{CT} \simeq 3.0$. We also find a two-state signal at $\beta = 3.0$ (Fig. 2 (b)). We therefore conclude that the deconfining transition is of first order for $N_F = 3$ in the chiral limit, which is in accord with the prediction of universality by Pisarski and Wilczek [5].

3. MANY FLAVORS

3.1. SU(3) case

Now let us consider the case of much larger $N_F$. In our previous paper we have shown that, when $N_F \geq 7$, light quarks are not confining even in the strong coupling limit [2]. Extending the study to finite $\beta$ we find the following phase structure for $N_F \geq 7$ [3]. We have a $N_F$-independent deconfining transition line $K_D$, which starts from a finite $K$ at $\beta = 0$ and extends to larger $\beta$. In addition, we have a $N_F$-dependent deconfining transition/crossover line $K_T$, which reaches the $K_D$-line without crossing the $K_C$-line.

To understand this phase structure, let us recall the property of the perturbative $\beta$ function. Asymptotic freedom is broken for $N_F \geq 9$. This suggests the existence of an infrared fixed point (IRFP) for $16 \geq N_F \geq N_F^*$. We have a $N_F$-independent deconfining transition line $K_D$, which starts from a finite $K$ at $\beta = 0$ and extends to larger $\beta$. In addition, we have a $N_F$-dependent deconfining transition/crossover line $K_T$, which reaches the $K_D$-line without crossing the $K_C$-line.

To understand this phase structure, let us recall the property of the perturbative $\beta$ function. Asymptotic freedom is broken for $N_F \geq 9$. This suggests the existence of an infrared fixed point (IRFP) for $16 \geq N_F \geq N_F^*$ [3]. With an IRFP correlation functions scale with non-canonical powers at large distances and therefore there are no hadronic particles [3]. Our MC result [2] implies that $N_F^* = 7$.

3.2. SU(2) case

As an extension of the SU(3) case, we study an SU(2) QCD in the strong coupling limit. We note that the SU(2) $\beta$-function has the same characteristics as SU(3) except for that typical numbers for $N_F$ are generally smaller than those for SU(3): With SU(2), asymptotic freedom is lost when $N_F \geq 11$ and 2-loop result for $N_F^*$ is 6. This is natural because the confining force is weaker for SU(2). We may therefore expect that $N_F^*$ is
Performing a simulation for $N_F = 8$ and $N_t = 4$ we find that the deconfining transition appears at $K = 0.2 - 0.21$ and that the chiral limit at $K = 0.25$ is in the deconfining phase. Decreasing $N_F$ gradually with fixing $K = K_C$, we find that the chiral limit remains in the deconfining phase down to $N_F = 3$. When we further decrease $N_F$ to 2, $N_{inv}$ shows a rapid increase with the molecular-dynamics time and finally exceeds $10^4$ in clear contrast with small numbers of $O(10^2)$ for $N_F \geq 3$. The chiral limit is therefore in the confining phase for $N_F = 2$. Fig. 3 shows our results for physical observables for $K = K_C$. To study the $N_t$ dependence of the transition, we simulate on an $N_t = 8$ lattice for the critical case of $N_F = 3$. The stability of the result confirms that the deconfining transition we observe is a bulk transition. According to our experience with $SU(3)$ QCD, we expect that these properties in the strong coupling limit hold also for small $\beta$. We therefore conclude that $N_F^* = 3$ for $SU(2)$ QCD.

4. CONCLUSION

We studied the phase structure of QCD with dynamical Wilson quarks. Extending our previous study, we found that the finite temperature deconfining transition for $N_F = 3$ is of first order. We also studied an $SU(2)$ QCD and found that $N_F$-dependence of $SU(2)$ QCD is quite similar to that of $SU(3)$, but now $N_F^*$, above which confinement is lost for light quarks, is 3 instead of 7 for $SU(3)$.

We have studied the phase structure of QCD mainly on an $N_t = 4$ lattice. To study the implication to the continuum limit, the system should be investigated also at larger $\beta$ on a larger lattice. We are extending our study in this direction.

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REFERENCES
1. Y. Iwasaki, K. Kanaya, S. Sakai, and T. Yoshi´ e, Nucl. Phys. B (Proc. Suppl.) 30 (1993) 327; ibid. 26 (1992) 311.
2. Y. Iwasaki, K. Kanaya, S. Sakai, and T. Yoshi´ e, Phys. Rev. Lett. 69 (1992) 21.
3. Y. Iwasaki, K. Kanaya, S. Sakai, and T. Yoshi´ e, Phys. Rev. Lett. 67 (1991) 1494.
4. K. Rajagopal and F. Wilczek, Nucl. Phys. B399 (1993) 395.
5. R. Pisarski and F. Wilczek, Phys. Rev. D29 (1984) 338.
6. T. Banks and A. Zaks, Nucl. Phys. B196 (1982) 189.
7. C. Bernard, et al., Phys. Rev. D46 (1992) 4741; preprint AZPH-TH/93-29; D. Toussaint, this proceedings.
8. D. Zhu and N. Christ, this proceedings.