Reliability of swarming algorithms for mobile sensor network applications

Poster Summary for FFT 2012

Steven Senger
University of Delaware
senger@math.udel.edu

1 Abstract

There are many well-studied swarming algorithms (for example, [2], [4], [5], [6], [7], [8], and [9]) which are often suited to very specific purposes. As mobile sensor networks become increasingly complex, and are comprised of more and more agents, it makes sense to consider swarming algorithms for movement control. We introduce a natural way to measure the reliability of various swarming algorithms so a balance can be struck between algorithmic complexity and sampling accuracy.

The main idea is to utilize relatively well-developed tools (see [1]) from areas like frame theory to provide theoretical guidance for algorithm selection.
Frames, frame potential, and energy

Frames were first introduced in [3]. A frame is a set of vectors which, like a basis of a vector space, span a certain space of interest, but with perhaps (many) more vectors than necessary to span the space. This theory is immediately relevant for transmitting complex information through noisy or lossy media. More precisely, let $\mathbb{H}^d$ be a $d$-dimensional Hilbert space, then the set of vectors

$$\mathcal{F} = \{f_j\}_{j=1}^n \subset \mathbb{H}^d,$$

is a frame if there exist constants $A$ and $B$, called frame bounds, such that for any $y \in \mathbb{H}^d$, the following holds:

$$A \|y\|^2 \leq \sum_j |\langle y, f_j \rangle|^2 \leq B \|y\|^2,$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product associated to the Hilbert space.

The more control one has on the frame bounds, the more “well-behaved” a given frame is. One useful measure of how well a given frame behaves is the so-called frame potential, introduced in [1]. The lower the frame potential, the better the frame is suited to many tasks. Retaining the notation above, the frame potential of a given frame is defined to be

$$FP(\mathcal{F}) = \sum_{i=1}^n \sum_{j=1}^n |\langle f_i, f_j \rangle|^2.$$

The frame potential is related to a very physically motivated quantity used in many areas of mathematics called energy. Again, using the same notation as above, the energy of a collection of vectors is defined to be

$$E(\mathcal{F}) = \frac{1}{\binom{n}{2}} \sum_{i=1}^n \sum_{j>i} \frac{1}{\|f_i - f_j\|^2}.$$
3 Mobile sensor networks

As the number of agents in mobile sensor networks increases, it will be neces-
ssary to consider asymptotic analysis of the algorithms which control them.
It stands to reason that tools from frame theory could be usefully employed
to study mobile sensor networks by viewing them as frames with dynamic
vectors. Here is an example of such a result.

An excellent example would be a swarm of drones flying just above a
forest fire, regularly measuring quantities like temperature and humidity.
This swarm of sensors is moving coherently in a some direction. Each agent
of the swarm is taking samples of some quantity at regular intervals along
the way. The agents should be well-distributed, so that the the region under
consideration is sampled with minimum bias. Also, whatever quantities are
being sampled will be assumed to not change too much, on average, over
some small, fixed distance. Notice that the complexity of a given algorithm
can strongly influence the possible velocity for such applications.

Theorem 1. Let

\[ \mathcal{X}(t) = \{x_j(t)\}_{j=1}^n \subset \mathbb{R}^d \]

be the positions of agents in a swarm at time \( t \). Let \( v_j(t) \) denote the velocity
of the agent at position \( x_j \) at time \( t \), and let

\[ v_{\text{avg}} = \frac{1}{n} \sum_{j=1}^n v_j. \]

Suppose further that the desired resolution in space of the sampling is such
that each agent has a minimum separation of \( \delta > 0 \) from each other agent,
and that \( |v_j(t) - v_{\text{avg}}| < c\delta \), for some small constant \( c \), independent of \( n \). Fi-
nally, let \( \rho(t) \) denote the ratio of the area which is covered by \( \delta \)-balls centered
at the agents to the area of the smallest cube containing the swarm at time
\( t \). Then for all times \( t \) such that

\[ E(\mathcal{X}(t)) = \frac{1}{\binom{n}{2}} \sum_{i=1}^n \sum_{j>i}^n \frac{1}{\|x_i(t) - x_j(t)\|^2} \leq C, \]

for some value \( C \), independent of \( n \), \( \rho(t) \geq C' \), for some other value \( C' \),
which is also independent of \( n \).
4 Conclusion

Energy can be determined by simulation before any swarm is given a navigation algorithm, so that various parameter sets can be tested cheaply. If a given algorithm or parameter set has energy which depends on the number of agents, there could be an uneven distribution of sensors. However, the theorem above guarantees that the distribution of sensors will be asymptotically robust.

References

[1] Benedetto and Fickus, Finite normalized tight frames, Adv. Comput. Math. 18, pp. 357–385 (2003).

[2] Cui, Hardin, Ragade, Elmaghraby, A Swarm-based Fuzzy Logic Control Mobile Sensor Network for Hazardous Contaminants Localization, 2004 IEEE International Conference on Mobile Ad-hoc and Sensor Systems (2004).

[3] Duffin and Schaeffer, A class of nonharmonic Fourier series, Trans. Amer. Math. Soc. 72 (1952) pp. 341–366.

[4] Kadrovach and Lamont, A Particle Swarm Model for Swarm-based Networked Sensor Systems, Proceedings of the 2002 ACM symposium on Applied computing, New York, NY, (2002).

[5] Leonard, Paley, Lekien, Sepulchre, Fratantoni, and Davis, Collective Motion, Sensor Networks, and Ocean Sampling, Proceedings of the IEEE, Vol. 95, No. 1, January 2007.

[6] Miller, Kolpas, Juchem Neto, and Rossi, A continuum three-zone model for swarms, Bull Math Biol. 2012 Mar; 74(3):536–61. Epub 2011 Jul 29.

[7] Smith, Chao, Li, Caron, Jones, and Sukhatme, Planning and Implementing Trajectories for Autonomous Underwater Vehicles to Track Evolving Ocean Processes Based on Predictions from a Regional Ocean Model, International Journal of Robotics Research, 29(12) 1475–1497, (2010).

[8] Tillet, Rao, and Sahin, Cluster-Head Identification in Ad Hoc Sensor Networks using Particle Swarm Optimization, 2002 IEEE International Conference on Personal Wireless Communications (2002).

[9] Wang, Wang, Ma, An Improved Co-evolutionary Particle Swarm Optimization for Wireless Sensor Networks with Dynamic Deployment, Sensors 2007, 7, pp. 354–370.