A NEW METHOD OF VALIDATING THE CLUSTER VALIDITY INDICES FOR INTERVAL TYPE-2 FUZZY BASED CLUSTERING ALGORITHM

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Abstract

In recent years several classification techniques have been proposed which are classified into supervised and unsupervised classifications. In unsupervised classification, fuzzy clustering analysis is a most common technique since it never needs training data for fuzzy clustering algorithm. Nevertheless, different clustering algorithms have different initial conditions to generate different partitions and use different parameters in order to produce different results. Thus, the partitions generated by fuzzy clustering algorithm are in need to validate. Many cluster validity indices have been proposed in the last three decades for validating type-1 fuzzy based FCM algorithm. Recently many type-2 fuzzy based applications were presented due to its extract degree of fuzziness. But its computational complexity is very high, so interval type-2 fuzzy system is widely used in many applications. After the updation of cluster centroids in type-2 fuzzy based FCM algorithm, the type-2 fuzzy membership function is taken as unreliability of type-1 membership function. Therefore there is a need for a new method to validate the cluster validity index for interval type-2 fuzzy system based applications. In this paper, we have presented a new approach of validating the 14 cluster validity indices and performed extensive comparison of the mentioned indices in conjunction with various interval type-2 fuzzy c-means clustering algorithms. For experimental analysis we have taken the number of widely used datasets and Berkely image database.

Keywords: Cluster validity indices, IT2FCM, Extended IT2FCM, IT2FCMα
I. Introduction

Cluster analysis is a process of splitting the given dataset into similar groups. The partition should have two properties: homogeneity within clusters and heterogeneity between clusters. There are two types of clustering mechanisms such as hard clustering and fuzzy clustering available. In hard clustering, every point of dataset is assigned precisely into a single cluster, while in fuzzy clustering every point of dataset belongs to more than one cluster based on degree \([0, 1]\).

The clustering technique is used to done the partition and then it is validated using validation function. In clustering algorithm the validation function is used to know whether they present accuracy over the structures of the data set or not. If the users want to check the validity of the results, two dimensional data are needed whereas the data sets cannot visualize effectively increase large multidimensional data sets are present. Thus, in order to verify a high volume data, the objective of the cluster validity was fully brought into use as to discover an optimal cluster count which do the validation work more effectively on clustered data. Several validity indices of clusters have been presented in the last three decades for validating type-1 fuzzy based FCM algorithm. Recently many type-2 fuzzy based applications \([XIV, XLII, XLIX, XI, V]\) are proposed due to its extract degree of fuzziness. But its computational complexity is very high, so interval type-2 fuzzy system is widely employed in many applications \([XXVII, XLIV, XXXIV, XXIV, XV, XIII]\). Therefore, there is a need, a new method of validating the validity index of cluster for interval type-2 fuzzy system based applications. In this paper, we presented a novel technique of validating the validity indices of cluster and we have taken the experiment with widely used artificial data sets \([XLIII]\) and Berkeley image database. Review by Wang and Zhang \([XLIII]\) and Kim et al. \([XLVIII, III]\) on fuzzy cluster validity indices is plenty, which consequently resulted in proposing to distinctive measurement criteria to evaluate and select an optimal clustering design. They are namely identified as compactness and separation. Compactness brings the member the each cluster to be very close to the other whereas separation is meant to be separating the clusters widely. A general measurement of compactness is called as variance. The above said reviewers come out with three major and common approaches to calculate the distance of two clusters of different size, distance of closest and distant cluster members and distance of cluster centers.
Fuzzy c-mean is well known clustering algorithm, which is widely applied in many applications for the last three decades as it has many extended version. It also split into two types in which first type is to use to extend the measure of dissimilarity \( d(x_j, c_i) \) between cluster center \( c_i \) and data point \( x_j \) and the other is to add penalized term \([VI]\). At the same time we have to consider another significant factor which controls the efficiency of FCM is the weighting exponent \( \text{m} \). Pal and Bezdek\[XX\] and Kim et al. \[III\] have conducted many experiments for these components.

In past three decades count of cluster validity indices is proposed for fuzzy environment. Wang and Zhang \[XLVII\] and Kim et al. \[III\] have reviewed many fuzzy cluster validity indices. Validity indices in fuzzy environment can be split into three categories \[XLIII\]. (i) Indices including the membership measures alone which includes \( V_{PC} \[XXII,X\], V_{PE} \[XVII,XIX]\), \( V_{MPC} \[XL\] and \( V_{P} \[XXXI\]\) (ii) Indices including the data set and membership measures which includes \( V_{FS} \[XLVI\], V_{XB} \[XLV\], V_{K} \[XLI\], V_{FHV} \[XVI\], V_{PBMF} \[XXX\], V_{PCAES} \[XXV\], SC \[XXXIII\], V_{SCG} \[XXVIII\], V_{ECA} \[XXIX\] and DWSC \[XII\], \( V_{W} \[XLVII\] \) (iii) Other approaches for fuzzy cluster validity.

There are two techniques available to design fuzzy logic systems (FLSs)\[XXIII, XIV\]: Type-1 FLSs (T1FLSs) and Type-2 FLSs (T2FLSs). Type-2 fuzzy is an extended form of type-1 fuzzy by using two fuzziness parameters \( m_1 \) and \( m_2 \) to build FOU which corresponds to the upper and lower fuzzy clustering measures. The membership functions of type-1 fuzzy are fixed whereas the membership functions are allotted by themselves in type-2 fuzzy. The membership grade of type-1 fuzzy is a crisp number lies between \([0,1]\) and the membership grade of type-2 fuzzy is a subset lies in \([0, 1]\). This is known as primary membership. Moreover, every primary member contains a secondary membership measure. However, the measures are taken at the interval of \([0, 1]\) by the secondary membership functions and is known as generalized T2FLSs, but the uniform function value 1 is taken in the interval of T2FLSs. Since the computation process of common T2FLSs are much difficult and its type-reduction is computationally expensive. Therefore the interval T2FLSs is usually applied in many applications.

The section II explains the preliminaries of Type-2 fuzzy logic and outlines the IT2FCM\[II\], Extended IT2FCM \[XXXVIII\], IT2FCM\(\alpha \) \[XXXVIII\], IT2FPCM \[IX\], MKIT2FCM \[VII\] and GMK\(\alpha\)IT2FCM \[VI\]. The section III explains the new method of validating the cluster validity indices for IT2FCM based
algorithm, also the modified cluster validity indices are listed. Section 4 demonstrates the experimental results of proposed technique.

II. Preliminaries

FCM is a commonly accepted cluster algorithm which indeed permits a single piece of information that belongs to more than clusters as it is mainly employed in pattern recognition. In supervised learning, FCM is believed to be more powerful and an effective algorithm if it is meant to analyze a cluster.

The FCM algorithm, proposed by Dunn [XXI] and generalized by Bezdek [XXII], is used to depict the fuzzy classification for the pixels by computing the fuzzy membership measure. FCM is a data clustering algorithm where every data point in connection with a cluster, to a degree determined by a membership grade. The fundamental process of FCM is to split a finite collection of dataset $X = \{x_1, x_2, \ldots x_n\}$ into a collection of 'c' fuzzy clusters according to few given condition. When fuzzy clustering is attempted, fuzzy membership design comprises several uncertainties, such as distance measure, fuzzifier and prototype. It minimizes an objective function $J$, concerning fuzzy membership $U$,

$$J = \sum_{i=1}^{N} \sum_{j=1}^{C} u_{ij}^m \|x_i - c_j\|^2$$

where $m$ is any real number greater than 1, $u_{ij}$ is the membership degree of $x_i$ in the cluster $j$, $x_i$ is the $i^{th}$ of $d$-dimensional measured data, $c_j$ is the $d$-dimension center of the cluster, list of $c$ cluster centers $C$ and $\| * \|$ is any norm showing the resemblance between any evaluated data and the center.

The issues in FCM are computationally expensive and highly dependent on the initial choice of $U$. If data-specific experimental values are not available, then $m = 2$ is the usual choice. Extensions that exist, simultaneously estimate the intensity in homogeneity bias field while producing the fuzzy partitioning. In IT2 fuzzy set the primary membership $J_{x_i}$ of a pattern $x_i$ can be depicted as a membership interval with all secondary grades of the primary membership value equal to 1. The membership value for $x_i$ is illustrated by the interval between upper $\bar{u}(x_i)$ and lower $u(x_i)$ membership values. Thus every $x_i$ has primary membership interval as $[\bar{u}(x_i), u(x_i)]$

$$J_{x_i} = [\bar{u}(x_i), u(x_i)]$$

Therefore the primary membership extends the pattern $x_i$ by Interval type-2 fuzzy
set and turns as

\[
\bar{u}_j(x_i) = \begin{cases} 
\frac{1}{\sum_{k=1}^{c} \left( \frac{d_{ji}}{d_{ki}} \right)^{2/(m_1-1)}} & \text{if } \frac{1}{\sum_{k=1}^{c} \left( \frac{d_{ji}}{d_{ki}} \right)} < \frac{1}{C} \\
\frac{1}{\sum_{k=1}^{c} \left( \frac{d_{ji}}{d_{ki}} \right)^{2/(m_2-1)}} & \text{otherwise}
\end{cases}
\]

\[
u_j(x_i) = \begin{cases} 
\frac{1}{\sum_{k=1}^{c} \left( \frac{d_{ji}}{d_{ki}} \right)^{2/(m_1-1)}} & \text{if } \frac{1}{\sum_{k=1}^{c} \left( \frac{d_{ji}}{d_{ki}} \right)} \geq \frac{1}{C} \\
\frac{1}{\sum_{k=1}^{c} \left( \frac{d_{ji}}{d_{ki}} \right)^{2/(m_2-1)}} & \text{otherwise}
\end{cases}
\]

where

- \(d_{ji}\) is the Euclidean distance between the \(x_j\) and the centroid \(v_i\)
- \(d_{ki}\) is the Euclidean distance between the \(x_k\) and the centroid \(v_i\)
- \(C = \{c_1, c_2, ..., c_c\}\)

In equation (3) and (4) IT2 fuzzy set use the two unique measures of \(m\) ie. \(m_1\) and \(m_2\). \(m_1\) and \(m_2\) are fuzzifiers which represents the fuzzy degree. Using these fuzzifiers the objective function is to be reduced in IT2FCM as

\[
J_{m_1(u,v)} = \sum_{i=1}^{N} \sum_{j=1}^{c} u_j(x_i)^{m_1} d_{ji}^2
\]

\[
J_{m_2(u,v)} = \sum_{i=1}^{N} \sum_{j=1}^{c} u_j(x_i)^{m_2} d_{ji}^2
\]

**IT2FCM**

IT2 fuzzy sets possess upper and lower membership functions. The IT2FCM have two fuzzifiers \(m_1\) and \(m_2\) that depict several fuzzy degrees that provides unique objective functions which is to be reduced in IT2FCM, which are shown in equation (5) and (6), also the IT2 represent (3) and (4).
Step: 1 Compute $\mu_{ij}$ and $\overline{\mu}_{ij}$

Step: 2 Calculate cluster centroids using the following Equation (8). Each centroid of cluster is depicted by interval between $v^L$ and $v^R$.

$$v_i = \frac{\sum_i^N (u_{ik})^m x_k}{\sum_i^N (u_{ik})^m}$$

After attainment of $v^L_i , v^R_i$, type-reduction is employed to obtain cluster centroid using Equation (9)

$$v_i = \frac{v^L_i + v^R_i}{2}$$

Step: 3 Get membership grades:

$$u_j(x_i) = \frac{u_j^R(x_i) + u_j^L(x_i)}{2}, \quad j = 1, \ldots, C$$

$$u_j^R(x_i) = \frac{\sum_{i=1}^M u_j(x_i)}{M}$$

where $u_j(x_i) = \begin{cases} \overline{u}_j(x_i), & \text{if } x_i \text{ uses } \overline{u}_j(x_i) \text{ for } v_j^R \\ u_j(x_i), & \text{otherwise} \end{cases}$

$$u_j^L(x_i) = \frac{\sum_{i=1}^M u_j(x_i)}{M}$$

where $u_j(x_i) = \begin{cases} \overline{u}_j(x_i), & \text{if } x_i \text{ uses } \overline{u}_j(x_i) \text{ for } v_j^L \\ u_j(x_i), & \text{otherwise} \end{cases}$

Step: 4

If $\left( u_j(x_i) > u_k(x_i) \right)$, for $k = 1, \ldots, C$ and $j \neq k$ Then $x_i$ is assigned to cluster $j$.

Step: 5 If $\max_i \left\| v_i^{(l+1)} - v_i^{(l)} \right\| < \epsilon$ then stop; else $l = l+1$, and goto Step 2.

$\alpha$ - cut implemented IT2FCM (IT2FCM$\alpha$) for segmentation

A significant concept in fuzzy set theory and applications is the $\alpha$-cut decomposition theorem developed by Zadeh(1975). These cuts are crisp sets associated with certain $\alpha$ levels that represent distinct grades of membership. The idea behind the $\alpha$-cut representation is to define a useful special fuzzy set that is associated with each $\alpha$-cut. The equation (14) shows the $\alpha$-cut of Type-1 fuzzy...
set[XXVI]
\[
A_\alpha = \{ A \in X \mid A(x) \geq \alpha, \alpha \in [0,1]\}
\]  
(14)

The two-dimensional \(\alpha\)-plane[XXVI], denoted as \(\tilde{A}_\alpha\), the union of all primary membership shows secondary grades are greater than or equal to the special value \(\alpha\), i.e.,

\[
\tilde{A}_\alpha = \bigcup_{x \in X}(x, u) \mid \mu_{\tilde{A}}(x, u) \geq \alpha = \bigcup_{x \in X}(\mu_{\tilde{A}}(x))_\alpha
\]  
(15)

where \((\mu_{\tilde{A}}(x))_\alpha\) is the \(\alpha\)-cut of vertical slice \(\mu_{\tilde{A}}(x)\).

The membership values are represent in interval as the upper \(u\) and the lower \(u\), also each centroid of cluster is mentioned as the interval between \(v_L\) and \(v_R\). The centroid of an interval type-2 fuzzy set is represented in equation (16).

\[
v_c(\alpha) = Centroid(\tilde{A}(\alpha)) = \sum_{x \in x} \ldots \sum_{u \in x} \alpha / \sum_{i=1}^{N} x_i u_i / \sum_{i=1}^{N} u_i
\]  
(16)

Fuzzy clustering algorithm along with cluster cores includes more good clustering property; which won’t be able to specify the cluster cores for two overlapping areas. Therefore, to specify most common cluster cores for any overlapping areas in fuzzy partition, we have proposed \(\alpha\)-cut implemented IT2FCM clustering algorithm[XXXVII] denoted by ITFCM\(\alpha\). The main contribution of this algorithm is to utilize \(\alpha\)-plane representation concept in type-2 fuzzy for calculating centroid type reduction in IT2FCM and extended method presented in [II]. The idea of cluster core can be elaborated in more common manner that, if the given measure \(\alpha\) is much smaller than the membership measure \(\mu_{ij}\) at \(i^{th}\) cluster of data point \(x_j\), then it is known that this data point \(x_j\) will absolutely belongs to the \(i^{th}\) cluster along with the membership measure of 1 and \(\mu_{ij} = 0 \& \bar{\mu}_{ij} = 0\) for all \(i \neq i\). A basic rule to assure that any two of the \(c\) cluster cores will never overlap as \(0.5 \leq \alpha \leq 1\). The measure of a relative distance is employed to determine the cluster cores produced by IT2FCM\(\alpha\) is given as,
The cluster cores generated by IT2FCMα can be determined by a relative distance measure with

\[
\bar{u}_j(x_i) = \begin{cases} 
\frac{1}{\sum_{k=1}^{C}(d_{ji}/d_{ki})^{2/(m_1-1)}}, & \text{if } \frac{1}{\sum_{k=1}^{C}(d_{ji}/d_{ki})^{2/(m_2-1)}} < \frac{1}{c} \\
\frac{1}{\sum_{k=1}^{C}(d_{ji}/d_{ki})^{2/(m_2-1)}}, & \text{otherwise}
\end{cases}
\] (17)

\[
u_j(x_i) = \begin{cases} 
\frac{1}{\sum_{k=1}^{C}(d_{ji}/d_{ki})^{2/(m_1-1)}}, & \text{if } \frac{1}{\sum_{k=1}^{C}(d_{ji}/d_{ki})^{2/(m_1-1)}} \geq \frac{1}{c} \\
\frac{1}{\sum_{k=1}^{C}(d_{ji}/d_{ki})^{2/(m_2-1)}}, & \text{otherwise}
\end{cases}
\] (18)

Therefore, the cluster cores produced by the presented IT2FCMα algorithm ensure the cluster cores in IT2FCMα have different overlapping areas.

Step 1: Compute \( \mu_{ij} \) and \( \overline{\mu}_{ij} \)

Step 2: Calculate cluster centroids \( v_i \) in the same way of FCM. Every cluster centroid is illustrated by the interval between \( v_i^L \) and \( v_i^R \). Update cluster centers with equation (19)

\[
v_i(\alpha) = \text{Centroid} \left( A(\alpha) \right) = \sum_{u_{ij} \in A} \cdots \sum_{u_{Nj} \in A} \alpha / \sum_{i=1}^{N} u_i \sum_{i=1}^{N} u_i \]

\[
v_i(\alpha) = \frac{v_i^L + v_i^R}{2}
\] (20)

This center-updation contains type-reduction and defuzzification.

Step 3: Perform type reduction and defuzzification, to compute \( a_{v_i}^L \) and \( a_{v_i}^R \)

\[
u_j(x_i) = \frac{u^R_j(x_i) + u^L_j(x_i)}{2}, \quad j = 1, \ldots, C
\] (21)

\[
u^L_j(x_i) = \frac{\sum_{l=1}^{M} u_{lj}(x_i)}{M}
\] (22)
where \( u_{il}(x_i) = \begin{cases} \bar{u}_l(x_i), & \text{if } x_{il} \text{ uses } \bar{u}_l(x_i) \text{ for } v_i^R \\ u_l(x_i), & \text{otherwise} \end{cases} \) \hspace{1cm} (23)

And

\[
u_j^l(x_i) = \frac{\sum_{i=1}^{M} u_j(x_i)}{M}
\]

where \( u_j(x_i) = \begin{cases} \bar{u}_j(x_i), & \text{if } x_{ij} \text{ uses } \bar{u}_j(x_i) \text{ for } v_j^L \\ u_j(x_i), & \text{otherwise} \end{cases} \) \hspace{1cm} (24)

**Step: 4**

If \( u_j(x_i) > u_k(x_i) \), for \( k = 1, \ldots, C \) and \( j \neq k \), then \( x_i \) is allotted to cluster \( j \). \hspace{1cm} (26)

**Step: 5** If \( \max_i \frac{\alpha v_i^{(l+1)} - \alpha v_i^{(l)}}{\alpha v_i^{(l)}} < \epsilon \), then stop; else \( l = l + 1 \), and goto Step 2. \hspace{1cm} (27)

**Interval Type-2 Fuzzy Possibilistic C-Means Algorithm (IT2FPCM)**

The IT2FPCM algorithm [IX] is derived from FPCM algorithm and type-2 fuzzy logic approaches and it carries both fuzziness and possibility weights as \( \mu \) and \( m \), which are represented by a range instead of a accurate measure; that is \( m = [m_1, m_2] \), where \( m_1 \) and \( m_2 \) depict the weighting exponents of the lower limit and upper limit for fuzziness and \( \mu = [\mu_1, \mu_2] \) where \( \mu_1 \) and \( \mu_2 \) depict weighting exponents of the lower limit and upper limit of for probability.

**IT2FPCM algorithm comprises of the following steps:**

**Step 1:** \( c, m_1 \) and \( m_2 \) are initialized

**Step 2:** Centroids are initialized randomly for the lower bound and upper bound of the interval.

**Step 3:** By applying Equations (28) and (29) the fuzzy partition matrices are updated for lower bound and upper bound of the interval.

\[
\mu_j(x_k) = \min \left\{ \left( \sum_{j=1}^{C} \left( \frac{d_{ik}}{d_{jk}} \right)^{\frac{2}{m_1+1}} \right)^{-1}, \left( \sum_{j=1}^{C} \left( \frac{d_{ik}}{d_{jk}} \right)^{\frac{2}{m_2+1}} \right)^{-1} \right\}
\]

\hspace{1cm} (28)

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\[
\mu_i(x_k) = \max \left\{ \left( \sum_{j=1}^{c} \left( \frac{d_{jk}}{d_{jk}} \right)^{\frac{2}{m_2-1}} \right)^{-1}, \left( \sum_{j=1}^{c} \left( \frac{d_{jk}}{d_{jk}} \right)^{\frac{2}{m_1-1}} \right)^{-1} \right\} (29)
\]

**Step 4:** By applying Equations (30) and (31) the matrices of the possibilistic partition is updated for lower bound and upper bound of the interval.

\[
\tau_j(x_k) = \min \left\{ \left( \sum_{j=1}^{c} \left( \frac{d_{jk}}{d_{jk}} \right)^{\frac{2}{\eta_1-1}} \right)^{-1}, \left( \sum_{j=1}^{c} \left( \frac{d_{jk}}{d_{jk}} \right)^{\frac{2}{\eta_1-1}} \right)^{-1} \right\} (30)
\]

\[
\overline{\tau}_l(x_k) = \max \left\{ \left( \sum_{j=1}^{c} \left( \frac{d_{jk}}{d_{jk}} \right)^{\frac{2}{\eta_2-1}} \right)^{-1}, \left( \sum_{j=1}^{c} \left( \frac{d_{jk}}{d_{jk}} \right)^{\frac{2}{\eta_2-1}} \right)^{-1} \right\} (31)
\]

**Step 5:** Using Equations (32) & (33) number of centroids used for fuzzy partition matrix at lower and upper bounds are computed.

\[
v_j = \frac{\sum_{i=1}^{n} \left( \mu_i(x_j) + \tau_i(x_j) \right)^{m_1} x_j}{\sum_{i=1}^{n} \left( \mu_i(x_j) + \tau_i(x_j) \right)^{m_1}} (32)
\]

\[
\overline{v}_l = \frac{\sum_{i=1}^{n} \left( \mu_i(x_j) + \overline{\tau}_i(x_j) \right)^{m_1} x_j}{\sum_{i=1}^{n} \left( \mu_i(x_j) + \overline{\tau}_i(x_j) \right)^{m_1}} (33)
\]

**Step 6:** Using Equations (34) & (35) type reduction is performed at the centroids and fuzzy partition matrix.

\[
v_j = \frac{v_j + \overline{v}_l}{2} (34)
\]

\[
\mu_i(x_j) = \frac{\mu_i(x_j) + \overline{\mu}_i(x_j)}{2} (35)
\]

**Step 7:** Repeat step 3-5 until \( |f_{m,n}(t) - f_{m,n}(t - 1)| < \varepsilon \)

**Multiple Kernel Interval Type-2 Fuzzy c-Means algorithm (MKIT2FCM)**

Dzunget. al. presented the MKIT2FCM [VI]. The KIT2FCM employ a nonlinear map limited as \( \emptyset: x \rightarrow \emptyset(x) \in H, x \in X \in \mathbb{R}^d \). Where X refers the feature space or the dataset and H is known as a kernel space. The algorithm of KIT2FCM employs double fuzzifiers \( m_1 \) and \( m_2 \) alike with IT2FCM in order to deal with uncertainty. The lower and upper membership measures, \( \overline{\mu}_{ij} \) and \( \underline{\mu}_{ij} \) are determined as follows:
\[\bar{u}_{ij} = \begin{cases} 
\frac{1}{\sum_{i=1}^{c} (d_{ij}/d_{ij})^{2/(m_1-1)}} & \text{if } \frac{1}{\sum_{i=1}^{c} (d_{ij}/d_{ij})^{2/(m_2-1)}} < \frac{1}{c} \\
\frac{1}{
\sum_{i=1}^{c} (d_{ij}/d_{ij})^{2/(m_2-1)}} & \text{otherwise} 
\end{cases} \]  \hspace{1cm} (2)  \\
\[u_{ij} = \begin{cases} 
\frac{1}{\sum_{i=1}^{c} (d_{ij}/d_{ij})^{2/(m_1-1)}} & \text{if } \frac{1}{\sum_{i=1}^{c} (d_{ij}/d_{ij})^{2/(m_2-1)}} \geq \frac{1}{c} \\
\frac{1}{\sum_{i=1}^{c} (d_{ij}/d_{ij})^{2/(m_2-1)}} & \text{otherwise} 
\end{cases} \]  \hspace{1cm} (3)

in which \(i = 1, c\), \(j = 1, n\), \(d_{ij} = \|\varnothing(x_j) - \varnothing(v_i)\|\). If we employ the Gaussian kernel the \(k(x,x) = 1\) and \(\|\varnothing(x_j) - \varnothing(v_i)\|^2 = 2(1 - k(x_j, v_i))\). The main purpose of the MKIT2FCM is to obtain better results which are enhanced by uniting different kernels. The objective of the MKIT2FCM is to reduce the objective functions like KIT2FCM, i.e.,

\[J_{m1}((U, V)) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^{m_1} \|\varnothing_{\text{com}}(x_i) - v_i\|^2 \]  \hspace{1cm} (4)  \\
\[J_{m2}((U, V)) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^{m_2} \|\varnothing_{\text{com}}(x_i) - v_i\|^2 \]  \hspace{1cm} (5)

where, \(\varnothing_{\text{com}}\) is known as the transformation determined by the combined kernels.

\[k_{\text{com}}(x, y) = (\varnothing_{\text{com}}(x), \varnothing_{\text{com}}(y))\]

Here \(k_{\text{com}}\) is the composite kernel which is the integration of multiple kernels. Lower/Upper degrees of membership, \(\bar{u}_{ij}\) and \(u_{ij}\) are decided by the equation (36) and (37) along with \(d_{\varnothing}\) which is substituted by \(d_{\varnothing_{\text{com}}}\)

\[d_{\varnothing_{\text{com}}}(x_j, v_i)^2 = \|\varnothing_{\text{com}}(x_j) - v_i\|^2 = k_{\text{com}}(x_j, x_j) + 2 \sum_{h=1}^{n} (u_{ih})^{m_{\text{com}}} k_{\text{com}}(x_h, x_j) + \sum_{h=1}^{n} \sum_{i=1}^{n} (u_{ih})^{m_{\text{com}}} k_{\text{com}}(x_h, x_i) \]

\[\sum_{h=1}^{n} (u_{ih})^{m_{\text{com}}} \]

\[\hspace{1cm} (40)\]

Given a set of \(n\) data points = \(\{x_i\}_{i=1}^{n}\), a set of kernel functions \(\{k_{ij}\}_{i,j=1}^{n}\), parameters \(m_1, m_2\) and the suitable number of clusters c. Output of a membership matrix \(U = \{u_{ic}\}_{i,c=1}^{n}\) and the weights \(\{w_i\}_{i=1}^{n}\) for the kernels.

**Step 1:** Initiate the centroid matrix \(V^0 = \{v_i\}_{i=1}^{c}\) by selecting arbitrary value from
the membership matrix $U^0$ and the dataset is updated using the Equation (41).

$$u_{ij} = \frac{1}{\sum_{l=1}^{M} \left( \frac{d_{il}}{d_{ij}} \right)^2/m-3}$$  \hspace{1cm} (41)$$

where $m$ is a constant, $m>1$ and $d_{ij} = d(x_j - v_i) = \|x_j - v_i\|$.

**Step 2:** Perform the following steps until the termination condition is satisfied or maximum iterations are reached:

1. Calculate Interval membership values $\tilde{u}_{ij}$ and $u_{ij}$ using the Equation (36), (37) and (40).
2. Upgrade the centroid matrix adopted the iterative algorithm to discover centroids at KIT2FCM using Equation (42)

$$v_i = \left( v_i^R + v_i^L \right)/2$$  \hspace{1cm} (42)$$
3. Update the membership matrix using Equation (43)

$$u_i(x_k) = \left( u_i^R(x_k) + u_i^L(x_k) \right)/2, \quad j = 1, \ldots, C$$  \hspace{1cm} (43)$$

in which

$$u_i^L = \sum_{l=1}^{M} \frac{u_{il}}{M}, \quad u_i^L = \begin{cases} \tilde{u}_i(x_k) & \text{if } x_i \text{ uses } \tilde{u}_i(x_k) \text{ for } v_i^L \\ u_j(x_k) & \text{otherwise} \end{cases}$$

$$u_i^R = \sum_{l=1}^{M} \frac{u_{il}}{M}, \quad u_i^R = \begin{cases} \tilde{u}_i(x_k) & \text{if } x_i \text{ uses } \tilde{u}_i(x_k) \text{ for } v_i^R \\ u_j(x_k) & \text{otherwise} \end{cases}$$

4. Assign data $x_j$ to cluster $c_i$ if data $\left( u_j(x_i) > u_k(x_i) \right), k = 1, \ldots, C$ and $j \neq k$.

**Genetic Multiple Kernel Interval Type-2 Fuzzy c-Means algorithm (GMKIT2FCM)**

Dzunget al. [VI] proposed GMKIT2FCM cluster algorithm. Which consists two main stages:

(i) To discover the primary centroids and the count of the clusters using the coefficients average of the multiple kernel $w_i = \frac{1}{l}$.

(ii) To enhance the multiple kernel coefficients, Genetic Algorithm is used.

**Stage 1:** The objective of stage 1 is to discover the optimal count of clusters and primary centroids automatically. Here the average MKIT2FCM is used along with $w_{0} = w_{i} = \frac{1}{l}$ whereas the count is the count of kernels.

$$k_{com} = w_{0}^{b}k_{1} + w_{0}^{b}k_{2}^{a}+\cdots+w_{0}^{b}k_{l}$$
The stage 1 includes all the six steps to discover the optimal count of clusters and the stage 1 process returns the best chromosome with the optimal count of clusters and the primary centroids of the clusters.

Stage 2: The main objective of the stage 2 is to set the multiple kernel’s coefficients $w_i$ to produce the best clustering result. Similar to Stage 1, the Stage 2 process also uses the GA to get best results through the fitness value of 21 and it includes all the six main steps. After the completion of these two stages, the algorithm achieves better clustering performance with the cluster count $C_{\text{opt}}$, the cluster centroids $V(v_1, \ldots, v_{c_{\text{opt}}})$ and the multiple kernel’s coefficients $w_1, \ldots, w_i$ with the fitness measure $f_0$.

III. Validating the Cluster Validity Indices for Interval Type-2 Fuzzy Based Clustering Algorithms

The type-2 fuzzy set has the membership measures are attained as follow [XXXVI]:

$$a_{ij} = \mu_{ij} - \frac{1 - \mu_{ij}}{2}$$

Where $\mu_{ij}$ and $a_{ij}$ are the initial membership and type-2 membership respectively. After cluster center updation [XLIX], the function area of type-2 membership can be taken as the unreliability of type-1 membership. Therefore, in this paper it is proposed to apply the cluster validity indices for upper and lower centroids separately, after that the cumulative cluster validity index is calculated; because to perform accurate validation in interval type-2 fuzzy based clustering algorithm in type-2 fuzzy membership area. The cluster validity indices are defined either making use of these membership values or membership values with data set. The 14 cluster validity indices values have been calculated which are proposed for Type-1 based FCM clustering algorithm by various researchers.

Table 1 lists the cluster validation indices which are modified for IT2FCM, based on type-1 FCM cluster validity indices. Here $x_j$ is $j^{th}$ data point, $c_i$ is the cluster count, $\bar{v}_i$ is the center of the cluster, $\bar{\bar{v}}$ is the grand mean of given data and $\bar{u}_{ij}$ is the membership measure of data $x_j$ at cluster $c$. In Table 1, the first and second columns show the validity index of cluster and the cluster validity index description respectively. The last column indicates the cluster validity index should search for the resultant value of the function.
| Validity index                        | Function description                                                                                                                                                                                                 | Search for |
|--------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|
| **Validity involving only the membership values**                                                                                       |                                                                                                                                         |            |
| Interval Type-2 Partition Coefficient (PC)                                                                                                    | \( V_{PC} = \frac{1}{n} \sum_{i=1}^{c} \sum_{j=1}^{n} \bar{u}_{ij} \)                                                                                                                                             | Max        |
| Interval Type-2 Partition Entropy (PE)                                                                                                        | \( V_{PE} = -\frac{1}{n} \sum_{i=1}^{c} \sum_{j=1}^{n} \bar{u}_{ij} \log \bar{u}_{ij} \)                                                                                              | Min        |
| Interval Type-2 Modification of Partition Coefficient (MPC)                                                                                   | \( V_{MPC} = 1 - \frac{c}{c - 1} (1 - V_{PC}) \)                                                                                                                                            | Max        |
| Interval Type-2 Chen and Linkens. (Validity index P)                                                                                         | \( V_{P} = \frac{1}{n} \sum_{k=1}^{c} \max_{i} \bar{u}_{ik} - \frac{1}{c - 1} \sum_{k=1}^{c-1} \sum_{j=1}^{c} \left[ \frac{1}{n} \sum_{k=1}^{c} \min (\bar{u}_{ik}, \bar{u}_{jk}) \right] \) | Max        |
| **Validity including the dataset and the membership measures**                                                                               |                                                                                                                                         |            |
| Interval Type-2 Fukuyama and Sugeno (FS)                                                                                                        | \( V_{FS} = \sum_{i=1}^{c} \sum_{j=1}^{n} \bar{u}_{ij} \| x_j - \bar{v}_i \|^2 - \sum_{i=1}^{c} \sum_{j=1}^{n} \bar{u}_{ij} \| \bar{v}_i - \bar{v} \|^2 \) | Min        |
| Interval Type-2 Xie and Beni (XB)                                                                                                               | \( V_{XB} = \sum_{i=1}^{c} \sum_{j=1}^{n} \bar{u}_{ij} \| x_j - \bar{v}_i \|^2 \)                                                                 | Min        |
| Interval Type-2 Kwon (K)                                                                                                                     | \( V_{K} = \sum_{i=1}^{c} \sum_{j=1}^{n} \bar{u}_{ij} \| x_j - \bar{v}_i \|^2 + \frac{1}{c} \sum_{i=1}^{n} \| \bar{v}_i - \bar{v} \|^2 \) min \( \| \bar{v}_i - \bar{v} \|^2 \) | Min        |
| Interval Type-2 Pakhira-Bandyopadhyay-Maulik (PBMF)                                                                                           | \( V_{PBMF} = \left( \frac{1}{c} \times \frac{\sum_{i=1}^{n} \| x_j - \bar{v}_i \|^2}{\sum_{i=1}^{n} \sum_{j=1}^{n} \bar{u}_{ij} \| x_j - \bar{v}_i \|^2} \right) \times \max_{i=1}^{c} \| \bar{v}_i - \bar{v} \|^2 \) | Min        |
| Interval Type-2 Partition Coefficient and Exponential Separation (PCAES) | \( \tilde{\nu}_{\text{PCAES}} = \sum_{i=1}^{c} \sum_{j=1}^{n} \frac{\tilde{u}_{ij}}{\tilde{u}_M} \) | Max |
|---|---|---|
| \( \tilde{u}_M = \min_{1 \leq i < c} \left( \sum_{j=1}^{n} \tilde{u}_{ij}^2 \right) \) | \( \tilde{\beta}_T = \sum_{i=1}^{c} \sum_{j=1}^{n} \frac{\tilde{v}_i - \tilde{v}}{\beta} \) |

| Interval Type-2 Zahid. Separation and Compactness (SC) | \( \tilde{S}_C = \tilde{S}_C_1(c) - \tilde{S}_C_2(c) \) | Max |
|---|---|---|
| \( \tilde{S}_C_1(c) = \sum_{i=1}^{c} \frac{\sum_{j=1}^{n} \| \tilde{v}_i - \tilde{v} \|^2 / c}{\sum_{i=1}^{c} \sum_{j=1}^{n} \tilde{u}_{ij}^2 / \sum_{j=1}^{n} \tilde{u}_{ij}} \) | \( \tilde{S}_C_2(c) = \sum_{i=1}^{c} \sum_{j=i+1}^{n} \left( \sum_{j=1}^{n} \min \left( \tilde{u}_{ij}, \tilde{u}_{ij} \right) \right) / \sum_{j=1}^{n} \max \sum_{j=1}^{n} \tilde{u}_{ij} \) |

| Interval Type-2 W-index (W) | \( \tilde{\nu}_W = \left( \frac{\text{Var}(U, V)}{\text{Var}_{\text{max}}(U, V)} \right) / \left( \frac{\text{Sep}(c, U)}{\text{Sep}_{\text{max}}(c, U)} \right) \) | Min |
|---|---|---|
| \( \text{Var}(U, V) = \sum_{i=1}^{c} \sum_{j=1}^{n} \tilde{u}_{ij} \left( 1 - \exp \left( -\frac{\| x_j - \tilde{v}_i \|^2}{\tilde{\beta}} \right) \right) / n(i) \) | \( \tilde{\beta} = \left( \frac{\sum_{j=1}^{n} \| x_j - \tilde{v} \|^{-1}}{n} \right) \) |
| \( \text{Sep}(c, U) = \max_{i \neq j} \left[ \max_{x \in X} \min \left( \tilde{u}_{ik}, \tilde{u}_{jk} \right) \right] \) | \( \text{Var}_{\text{max}}(U, V) = \max_c \text{Var}(U, V) \) |

| Interval Type-2 Gath and Geva | \( \tilde{\nu}_{FHV} = \sum_{i=1}^{c} [\det(F_i)]^{1/2} \) | Min |

\( \tilde{n} \) and SC evaluate the separation ratio and compactness.

\( \text{Var}_{\text{max}}(U, V) = \max_c \text{Var}(U, V) \)

\( \text{Sep}_{\text{max}}(c, U) = \max \text{Sep}(c, V) \)

\( \text{Var}(U, V) = \max_c \text{Var}(U, V) \)

\( \text{Sep}(c, U) = \max_{i \neq j} \left[ \max_{x \in X} \min \left( \tilde{u}_{ik}, \tilde{u}_{jk} \right) \right] \)
\[
\tilde{F}_i = \frac{\sum_{j=1}^{n} (\tilde{u}_{ij})^m (x_j - \tilde{v}_i)(x_j - \tilde{v}_i)^T}{\sum_{j=1}^{n} (\tilde{u}_{ij})^m}
\]

| Fuzzy Hypervolume Validity (FHV) | \[
\tilde{V}_{SEG} (U, V, X) = \frac{\sum_{i=1}^{c} \sum_{j=1}^{n} (\tilde{u}_{ij}) (x_i - \tilde{v}_i)(x_i - \tilde{v}_i)^T}{\sum_{i=1}^{c} \sum_{j=1}^{n} (\tilde{u}_{ij}) (x_j - \tilde{v}_i)(x_j - \tilde{v}_i)^T / \sum_{j=1}^{n} (\tilde{u}_{ij})^m}
\] |
| --- | --- |
| Interval Type-2 Bouguessa and wang (Separation and Compactness as Global) | Max |
| Interval Type-2 Zarandi Exponential Compactness And Separation (ECAS) index. | Max |
| Exponential Separation (ESsep) | | |
| Exponential Compactness(ECcomp) | | |

\[
\tilde{V}_{ECAS} = \frac{EC_{comp}(c)}{\max(c) EC_{comp}(c)} - \frac{ES_{sep}(c)}{\max(c) ES_{sep}(c)}
\]

\[
EC_{comp}(c) = \sum \sum (\tilde{u}_{ij}) \exp \left( - \frac{1}{c+1} \right)
\]

\[
ES_{sep}(c) = \sum \exp \left( - \min \left( (c-1) \frac{\|\tilde{v}_i - \tilde{v}_k\|^2}{c} \right) \right)
\]

where \(x_j\) is \(j^{th}\) data point, \(c_i\) is the cluster count, \(v_i\) is the center of the cluster, 

\[
\tilde{\theta} = \frac{\sum_{i=1}^{n} \pi(x_i) + \pi(x_i)}{n} \text{ and } \tilde{u}_{ij} = \frac{(\tilde{u}_{ij} + u_{ij})}{2}
\]

**Methodology to calculate cluster validity indices**

The validation process of the interval type-2 fuzzy c-means is given as,

**Step 1:** Fix the clustering parameters depends upon the clustering algorithm for the given dataset X. Usually, except number of cluster \(c\), all the parameters are defined.

**Step 2:** Assign the minimal \((c_{\text{min}})\) and maximal \((c_{\text{max}})\) numbers.

**Step 3:** For each \(c = c_{\text{min}}\) to \(c_{\text{max}}\)

   - Initialize the centers of the cluster
   - Implement the interval type-2 fuzzy c-mean clustering algorithm.
     
     For each \(v_i\)
Compute the validity indices for upper and lower centorids separately.
Compute the cumulative validity index.
End for
Return the cumulative validity index.
End for

IV. Experimental Results

To test validity indices extensive comparisons have been made with various data sets with above mentioned validity indices in IT2FCM, Extended IT2FCM and IT2FCMα, IT2FPCM, MKIT2FCM,GMKIT2FCM. In all experiments the fuzzifier \( m_1 \) and \( m_2 \) are set into \( (6,8) \) and \( \alpha=.7 \), the test form convergence in the IT2FCM algorithm using \( \varepsilon=10^{-5} \) and \( \| \cdot \| \) is the function of distance specified as Euclidean distance. We have performed the experiment in two ways: (i) Calculating the cluster validity indices in interval type-2 fuzzy membership (i.e. calculating the validity indices upper and lower centroid separately) (ii) Calculating the cluster validity indices in interval type-1 fuzzy membership. For the experiments we have taken the widely used data set and Berkely image database.

Data set

We have evaluate the validity indices of cluster by eight common artificial datasets which are used by Wang and Zhang [XLIII] (Table 2). The eight data sets are Butterfly, Data_4_3, Data_6_2, Data_10_2, Data_15_2, Data_4 noise, Example_1, and Example_2. The dataset name itself represents the details of data set, for example Data_4_3 means there are four clusters and three dimensions. Also we have used Butterfly dataset comprises of 3 clusters and 15 two dimensional data-points. The datasets Example_1 and Example_2 comprises of 16 two dimensional data-points and 3 & 4 clusters respectively. Fig 1 shows these data sets.
Table 2. Common datasets with its characteristics

| Data set     | Number of clusters | Dimension |
|--------------|--------------------|-----------|
| Data_4_3     | 4                  | 3         |
| Data_6_2     | 6                  | 2         |
| Data_10_2    | 10                 | 2         |
| Data_15_2    | 15                 | 2         |
| Data_4noise  | 2                  | 4         |
| Butterfly    | 3                  | 2         |
| Example_1    | 3                  | 2         |
| Example_2    | 4                  | 2         |

**Image dataset**

Fig 2 demonstrates the sample images taken from the Berkely image database. Literature shows that HSV color space gives the best color differences among the different color spaces. Hence the images are converted into HSV color space. In HSV color space the color is presented in terms of three components: Hue (H), Saturation (S) and Value (V). So a color image of size m X n pixels corresponds to an array of size m X n and three dimensions.
Comparison of cluster Validity indices: Calculating the cluster validity indices in interval type-2 fuzzy membership

In Table 3 it is clearly seen that four unlike validity indices (including membership measures alone) are used to the eight common artificial datasets by implementing IT2FCM, Extended IT2FCM, IT2FCMα, IT2FPCM, MKIT2FCM and GMKIT2FCM respectively. In Table 3, the column \( c^* \) affords the expected clusters count for every dataset, and the rest of the columns indicates the optimal count of clusters obtained by applying every index. \( V_{IC} \) and \( V_{SPE} \) and \( V_{P} \) correctly identifies the optimum number of clusters in the Data_4_3, Data_6_2, Data_10_2, Data_15_2, Butterfly, Example_1 and Example_2 datasets except Data_10_2. \( V_{PE} \) fails to recognize \( c^* \) in the Data_10_2 dataset.

Table 4 to Table 6 covers the results achieved while ten unlike validity indices (including dataset and membership measures) are applied to the abovementioned 8 datasets using IT2FCM, Extended-IT2FCM, IT2FCMα, IT2FPCM, MKIT2FCM and GMKIT2FCM algorithm. Hence the ideal clustering algorithm is attained when the measure of \( V_{PCAES}, V_{SC}, V_{ECAS} \), these cluster validity indices are minimal. Also, \( \tilde{V}_{PCAES}, \tilde{V}_{SC}, \tilde{V}_{ECAS} \), the value of these cluster validity indices is maximum.

The observation also made from the obtained results that \( V_{PCAES}, V_{SC}, V_{ECAS} \), these cluster validity index values are minimum in IT2FCMα, Extended-IT2FCM, IT2FPCM, GMKIT2FCM algorithm compare with IT2FCM and MKIT2FCM algorithm. Also \( V_{PCAES}, V_{SC}, V_{ECAS} \), these cluster validity index values are maximum in IT2FCMα and Extended-IT2FCM IT2FPCM and GMKIT2FCM algorithm compare with IT2FCM and MKIT2FCM algorithm. Out of the experimental results it is shown that the proposed IT2FCMα and Extended-IT2FCM IT2FPCM and GMKIT2FCM algorithms identify the optimal cluster count for 8 datasets in a correct manner.

Table 7 outlines the results got while the four unlike validity indices (including the membership measures alone), are implemented to Berkely image database (shown in Figure 2) by implementing IT2FCM, Extended IT2FCM, IT2FCMα, IT2FPCM, MIT2FCM and GMIT2FCM respectively. In Table 7, the column \( c^* \) provides the exact cluster counts expected for every dataset, and the remaining columns demonstrate the optimal cluster count obtained by applying every index. Table 8 to Table 10 summarize the results got while the ten unlike validity indices (including the dataset and the membership measures) are used to the abovementioned 8 datasets using IT2FCM, Extended-IT2FCM, IT2FCMα, IT2FPCM, MIT2FCM and GMIT2FCM algorithm. The obtained results show that \( \tilde{V}_{PCAES}, \tilde{V}_{SC}, \tilde{V}_{ECAS} \), these cluster validity index values are minimum in IT2FCMα, Extended-IT2FCM, IT2FPCM and GMKIT2FCM algorithm compare with IT2FCM and MKIT2FCM algorithm. Also \( \tilde{V}_{PCAES}, \tilde{V}_{SC}, \tilde{V}_{ECAS} \), these cluster validity index values are minimum in IT2FCMα, Extended-IT2FCM, IT2FPCM and GMKIT2FCM algorithm compare with IT2FCM and MKIT2FCM algorithm. Also \( \tilde{V}_{PCAES}, \tilde{V}_{SC}, \tilde{V}_{ECAS} \), these cluster validity index values are maximum in IT2FCMα and Extended-IT2FCM, IT2FPCM and GMKIT2FCM algorithm.
and GMKIT2FCM algorithm compare with IT2FCM and MKIT2FCM algorithm. Therefore, the experimental results demonstrate that the proposed algorithms IT2FCMα and Extended-IT2FCM IT2FPCM and GMKIT2FCM correctly recognize the optimal cluster counts for Berkeley image database.

**Comparison of cluster Validity indices: Calculating the validity indices of cluster in interval type-1 fuzzy membership**

In order to gain a performance analysis of the proposed method of validating the validity index of cluster, a comparison is formulated using validity indices of the existing fuzzy cluster between the type-2 fuzzy membership and type-1 fuzzy membership. In Table 11, one can see the summary of the results obtained while the four unlike validity indices (including the membership measures alone), are applied to the Berkeley image database (shown in Figure 2) by implementing IT2FCM, Extended IT2FCM, IT2FCMα, IT2FPCM, MIT2FCM, and GMIT2FCM respectively.

In Table 11, the column $c^*$ provides the actual cluster count for every dataset, and the other columns show the optimal number of clusters obtained using each index. Table 12 to Table 14 summarize the results obtained while the ten unlike validity indices (including the dataset and membership measures) are applied to the abovementioned 8 datasets using IT2FCM, Extended-IT2FCM, IT2FCMα, IT2FPCM, MIT2FCM, and GMIT2FCM algorithm. Comparison between the data values of Table 7 to Table 10 and Table 11 to Table 14, calculating cluster validity indices in type-2 fuzzy membership is found more desirable for validating the interval type-2 fuzzy based clustering algorithm.

**V. Conclusion**

In this paper, we have applied the cluster validity indices for upper and lower centroids separately, after that the cumulative cluster validity index is calculated; because to perform accurate validation in interval type-2 fuzzy based clustering algorithm in type-2 fuzzy membership area. The cluster validity indices are defined either making use of these membership values or membership measures with dataset. The measure of 14 cluster validity indices has been calculated which are only proposed for Type-1 based FCM clustering algorithm by various researchers. Extensive comparisons have been done with artificial and Berkeley Image Database. The results show that the new way of validating the validity indices outer performs the IT2FCM based algorithms. Also the cluster validity indices are able to handle membership measures with datasets. From the experimental analysis it is noted that calculating cluster validity indices in type-2 fuzzy membership is more desirable for validating the interval type-2 fuzzy based clustering algorithm. Further experiment is required to evaluate the other overlapping clusters and noise points.
Table 3. Values of $c$ preferred by validity involving only the membership values for 8 dataset

|                | IT2FCM | Extended-IT2FCM | IT2FCM$\alpha$ | IT2FPCM | MKIT2FCM | GMKIT2FCM |
|----------------|--------|-----------------|----------------|---------|----------|-----------|
| $c^*$          | $\tilde{V}_P$ | $\tilde{V}_F$ | $\tilde{V}_M$ | $\tilde{V}_P$ | $\tilde{V}_F$ | $\tilde{V}_M$ | $\tilde{V}_P$ | $\tilde{V}_F$ | $\tilde{V}_M$ | $\tilde{V}_P$ | $\tilde{V}_F$ | $\tilde{V}_M$ | $\tilde{V}_P$ | $\tilde{V}_F$ | $\tilde{V}_M$ | $\tilde{V}_P$ | $\tilde{V}_F$ | $\tilde{V}_M$ | $\tilde{V}_P$ | $\tilde{V}_F$ | $\tilde{V}_M$ | $\tilde{V}_P$ | $\tilde{V}_F$ | $\tilde{V}_M$ |
| Data_4_3       | 4      | 4               | 4              | 4      | 4        | 4          | 4      | 4               | 4              | 4              | 4        | 4          | 4      | 4               | 4              | 4              | 4              | 4        | 4          | 4      | 4               | 4              | 4              | 4              | 4        | 4          | 4      | 4               |
| Data_6_2       | 6      | 6               | 6              | 6      | 6        | 6          | 6      | 6               | 6              | 6              | 6        | 6          | 6      | 6               | 6              | 6              | 6              | 6        | 6          | 6      | 6               | 6              | 6              | 6              | 6        | 6          | 6      | 6               |
| Data_1_0_2     | 1      | 0               | 0              | 5      | 0        | 5          | 0      | 0               | 0              | 0              | 0        | 0          | 0      | 0               | 0              | 0              | 0              | 0        | 0          | 0      | 0               | 0              | 0              | 0              | 0        | 0          | 0      | 0               |
| Data_4_5_2     | 1      | 5               | 5              | 6      | 5        | 6          | 5      | 5               | 5              | 5              | 5        | 5          | 5      | 5               | 5              | 5              | 5              | 5        | 5          | 5      | 5               | 5              | 5              | 5              | 5        | 5          | 5      | 5               |
| Data_4_noise   | 4      | 5               | 4              | 4      | 4        | 4          | 4      | 4               | 4              | 4              | 4        | 4          | 4      | 4               | 4              | 4              | 4              | 4        | 4          | 4      | 4               | 4              | 4              | 4              | 4        | 4          | 4      | 4               |
| Butterfly      | 2      | 2               | 2              | 2      | 2        | 2          | 2      | 2               | 2              | 2              | 2        | 2          | 2      | 2               | 2              | 2              | 2              | 2        | 2          | 2      | 2               | 2              | 2              | 2              | 2        | 2          | 2      | 2               |
| Example_1      | 3      | 3               | 3              | 3      | 3        | 3          | 3      | 3               | 3              | 3              | 3        | 3          | 3      | 3               | 3              | 3              | 3              | 3        | 3          | 3      | 3               | 3              | 3              | 3              | 3        | 3          | 3      | 3               |
| Example_2      | 4      | 4               | 4              | 4      | 4        | 4          | 4      | 4               | 4              | 4              | 4        | 4          | 4      | 4               | 4              | 4              | 4              | 4        | 4          | 4      | 4               | 4              | 4              | 4              | 4        | 4          | 4      | 4               |
Table 4. Values of c preferred by validity involving the membership values and data set for 8 dataset (IT2FCM, Extended-IT2FCM)

| Dataset | Data 4,3 | Data 6,2 | Data 10,2 | Data 15,2 | Data 4noisc | Butterfly | Example 1 | Example 2 |
|---------|---------|---------|-----------|-----------|-------------|-----------|-----------|-----------|
|          | \( \mu_{\text{FS}} \) | \( \mu_{\text{SB}} \) | \( \mu_{\text{ABA}} \) | \( \mu_{\text{FB}} \) | \( \mu_{\text{PCA}} \) | \( \mu_{\text{SCF}} \) | \( \mu_{\text{SFC}} \) | \( \mu_{\text{ESF}} \) |
| Data 4,3 | .1 | .1 | .2 | .7 | .8 | .1 | .3 | .1 |
|          | 2 | 1 | 9 | 24 | 2 | 2 | 1 | 8 |
| Data 6,2 | .1 | .1 | .2 | .8 | .8 | .1 | .1 | .3 |
|          | 2 | 0 | 2 | 6 | 6 | 1 | 2 | 8 |
| Data 10,2 | .1 | .1 | .2 | .8 | .3 | .1 | .2 | .3 |
|          | 2 | 1 | 2 | 1 | 1 | 2 | 1 | 1 |
| Data 15,2 | .1 | .1 | .2 | .7 | .9 | .1 | .2 | .1 |
|          | 3 | 2 | 2 | 1 | 1 | 2 | 1 | 1 |
| Data 4noisc | .1 | .1 | .2 | .8 | .8 | .1 | .2 | .1 |
|          | 4 | 3 | 3 | 2 | 2 | 2 | 1 | 3 |
| Butterfly | .1 | .1 | .2 | .8 | .8 | .1 | .2 | .1 |
|          | 1 | 0 | 1 | 1 | 1 | 2 | 1 | 1 |
| Example 1 | .1 | .1 | .2 | .8 | .9 | .1 | .2 | .1 |
|          | 1 | 0 | 2 | 1 | 1 | 2 | 1 | 1 |
| Example 2 | .1 | .1 | .2 | .8 | .9 | .1 | .2 | .1 |
|          | 2 | 1 | 2 | 1 | 1 | 2 | 1 | 1 |
Table 5. Values of c preferred by validity involving the membership values and data set for 8 dataset ((IT2FCM₀, IT2FPCM)

|       | IT2FCM₀ |       | IT2FPCM |
|-------|---------|-------|---------|
|       | $V_{FS}$ | $V_{SB}$ | $V_{PBA}$ | $V_{FH}$ | $V_{SC}$ | $V_{FCA}$ | $V_{FS}$ | $V_{SB}$ | $V_{PBA}$ | $V_{FH}$ | $V_{SC}$ | $V_{FCA}$ |
| Data  |         |         |         |         |         |         |         |         |         |         |         |         |
| _4_3  | .1      | .0      | .0      | .0      | .9      | .9      | .9      | .1      | .0      | .0      | .0      | .9      |
|       | 0       | 9       | 9       | 94      | 2       | 2       | 5       | 03      | 0       | 0       | 2       | 5       |
| Data  | .1      | .0      | .0      | .0      | .9      | .9      | .9      | .1      | .0      | .0      | .0      | .9      |
| _6_2  | 0       | 9       | 8       | 5       | .9      | .2      | 4       | 93      | 2       | 4       | 1       | 5       |
|       | .1      | .0      | .1      | .0      | .9      | .9      | .9      | .1      | .0      | .0      | .0      | .9      |
|       | 02      | 8       | 1       | 5       | .9      | .9      | .9      | .9      | .9      | .9      | .9      | .9      |
| Data  | .0      | .0      | .1      | .0      | .9      | .9      | .9      | .1      | .0      | .0      | .0      | .9      |
| _10_  | 0       | 9       | 9       | 1       | .09     | 5       | 4       | 1       | 0       | 0       | 3       | 5       |
|       | 2       | 2       | 2       | 2       | 2       | 2       | 2       | 2       | 2       | 2       | 2       | 2       |
| Data  | .1      | .0      | .1      | .0      | .9      | .9      | .9      | .1      | .0      | .0      | .0      | .9      |
| _15_  | 1       | 9       | 1       | 9       | .0      | 1.2     | 4       | 9       | 5       | 4       | 5       | 5       |
|       | 0       | 99      | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
| Data  | .1      | .1      | .1      | .0      | .9      | .9      | .9      | .1      | .1      | .1      | .1      | .1      |
| _4no  | 8       | 2       | 1       | 8       | .9      | .8      | .2      | 1       | 9       | 2       | 1       | 8       |
|       | 5       | 2       | 2       | 5       | 9       | 9       | 3       | 5       | 9       | 2       | 1       | 8       |
|       | .1      | .1      | .1      | .0      | .9      | .9      | .9      | .1      | .1      | .1      | .1      | .1      |
|       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
|       | .8      | 1       | 0       | 0       | .9      | .8      | .2      | 1       | .1      | .1      | .1      | .1      |
|       | 2       | 1       | 2       | 3       | 3       | 2       | 2       | 2       | 2       | 2       | 2       | 2       |
|       | .1      | .1      | .1      | .0      | .9      | .9      | .9      | .1      | .1      | .1      | .1      | .1      |
|       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
|       | .8      | 1       | 0       | 0       | .9      | .8      | .2      | 1       | .1      | .1      | .1      | .1      |
|       | 2       | 1       | 2       | 3       | 3       | 2       | 2       | 2       | 2       | 2       | 2       | 2       |
|       | .1      | .1      | .1      | .0      | .9      | .9      | .9      | .1      | .1      | .1      | .1      | .1      |
|       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
|       | 2       | 1       | 2       | 3       | 3       | 2       | 2       | 2       | 2       | 2       | 2       | 2       |
|       | .1      | .1      | .1      | .0      | .9      | .9      | .9      | .1      | .1      | .1      | .1      | .1      |
|       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
|       | 2       | 1       | 2       | 3       | 3       | 2       | 2       | 2       | 2       | 2       | 2       | 2       |
|       | .1      | .1      | .1      | .0      | .9      | .9      | .9      | .1      | .1      | .1      | .1      | .1      |
|       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
|       | 2       | 1       | 2       | 3       | 3       | 2       | 2       | 2       | 2       | 2       | 2       | 2       |
|       | .1      | .1      | .1      | .0      | .9      | .9      | .9      | .1      | .1      | .1      | .1      | .1      |
|       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
|       | 2       | 1       | 2       | 3       | 3       | 2       | 2       | 2       | 2       | 2       | 2       | 2       |
|       | .1      | .1      | .1      | .0      | .9      | .9      | .9      | .1      | .1      | .1      | .1      | .1      |
|       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
|       | 2       | 1       | 2       | 3       | 3       | 2       | 2       | 2       | 2       | 2       | 2       | 2       |

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Table 6. Values of c preferred by validity involving the membership values and data set for 8 dataset (MKIT2FCM, GMKIT2FCM)

|        | MKIT2FCM |                | GMKIT2FCM |                |
|--------|----------|----------------|-----------|----------------|
|        | \(V_{FS}\) | \(V_{SB}\) | \(V_{PB}\) | \(V_{FA}\) | \(V_{PC}\) | SC  | \(V_{EC}\) | \(V_{FS}\) | \(V_{SB}\) | \(V_{PB}\) | \(V_{FA}\) | \(V_{PC}\) | SC  | \(V_{EC}\) |
| Data   |          |                |           |                |          |     |           |          |                |           |                |          |     |           |
| 4_3    | .1       | .1             | .1        | 22             | .1       | 20  | .1        | .1       | 9               | .1       | 9               | .1       | 9   | 0         |
|        | 1        | 1              | 1         | 2               | 1        | 3   | 1         | 1        | 8               | 1         | 9               | 1        | 8   | 0         |
|        | 8        | 4              | 3         | 2               | 4        | 3   | 2         | 4        | 2               | 3         | 2               | 4        | 3   | 2         |
| Data   |          |                |           |                |          |     |           |          |                |           |                |          |     |           |
| 6_2    | .1       | .1             | .1        | 15             | .1       | 22  | .1        | .1       | 1               | .1       | 8               | .1       | 1   | 0         |
|        | 1        | 2              | 1         | 6               | 1        | 2   | 1         | 2        | 1               | 1        | 8               | 1        | 1   | 1         |
|        | 3        | 1              | 2         | 6               | 3        | 2   | 2         | 2        | 2               | 2        | 2               | 2        | 2   | 2         |
| Data   |          |                |           |                |          |     |           |          |                |           |                |          |     |           |
| 10_2   | .1       | .1             | .1        | 22             | .1       | 10  | .1        | .1       | 0               | .1       | 0               | .1       | 0   | 0         |
|        | 2        | 1              | 2         | 1               | 2        | 0   | 2         | 2        | 0               | 0        | 0               | 0        | 0   | 0         |
|        | 6        | 0              | 0         | 0               | 6        | 0   | 0         | 0        | 0               | 0        | 0               | 0        | 0   | 0         |
| Data   |          |                |           |                |          |     |           |          |                |           |                |          |     |           |
| 15_2   | .1       | .1             | .1        | 18             | .1       | 31  | .1        | .1       | 0               | .1       | 1               | .1       | 1   | 1         |
|        | 2        | 2              | 5         | 3               | 2        | 1   | 2         | 2        | 1               | 2        | 1               | 2        | 2   | 2         |
|        | 2        | 2              | 5         | 3               | 2        | 1   | 2         | 2        | 1               | 2        | 1               | 2        | 2   | 2         |
| Data   |          |                |           |                |          |     |           |          |                |           |                |          |     |           |
| 4nosc  | .1       | .1             | .1        | 24             | .1       | 7   | .1        | .1       | 0               | .1       | 1               | .1       | 1   | 1         |
|        | 3        | 2              | 3         | 1               | 3        | 7   | 1         | 1        | 1               | 1        | 2               | 1        | 1   | 1         |
|        | 3        | 1              | 1         | 1               | 3        | 1   | 1         | 1        | 1               | 1        | 2               | 1        | 1   | 1         |
| Buttery |          |                |           |                |          |     |           |          |                |           |                |          |     |           |
| 8_2    | .1       | .1             | .1        | 9               | .1       | 32  | .1        | .1       | 1               | .1       | 8               | .1       | 1   | 1         |
|        | 0        | 1              | 1         | 1               | 0        | 2   | 3         | 2        | 2               | 3        | 2               | 2        | 2   | 2         |
|        | 1        | 1              | 1         | 1               | 1        | 2   | 3         | 2        | 3               | 3        | 3               | 3        | 3   | 3         |
| Examp   |          |                |           |                |          |     |           |          |                |           |                |          |     |           |
| le 1   | .1       | .1             | .1        | 9               | .1       | 14  | .1        | .1       | 9               | .1       | 8               | .1       | 1   | 0         |
|        | 0        | 2              | 0         | 0               | 1        | 8   | 1         | 1        | 8               | 1        | 8               | 1        | 8   | 1         |
|        | 9        | 2              | 0         | 0               | 9        | 2   | 2         | 2        | 2               | 2        | 2               | 2        | 2   | 2         |
| Examp   |          |                |           |                |          |     |           |          |                |           |                |          |     |           |
| le 2   | .1       | .1             | .1        | 21             | .1       | 9   | .1        | .1       | 9               | .1       | 9               | .1       | 9   | 1         |
|        | 2        | 0              | 0         | 0               | 2        | 5   | 2         | 2        | 2               | 2        | 2               | 2        | 2   | 2         |
|        | 4        | 8              | 8         | 0               | 4        | 8   | 0         | 0        | 0               | 0        | 0               | 0        | 0   | 0         |

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Table 7. Values of $c$ preferred by validity involving only the membership values for 8 dataset

| Image 1 | IT2FCM | Extended-IT2FCM | IT2FCMα | IT2FCMβ | MKIT2FCM | GMKITTFCM |
|---------|--------|-----------------|---------|---------|----------|-----------|
| 𝑉      | 7      | 6               | 7       | 7       | 7        | 7         |
|         | 2      | 2               | 2       | 2       | 2        | 2         |
|         | 𝑉      | 3               | 4       | 3       | 2        | 2         |
|         | 2      | 1               | 1       | 2       | 1        | 1         |
|         | 𝑉      | 2               | 1       | 1       | 2        | 1         |
|         | 2      | 0               | 1       | 2       | 1        | 1         |
| Image 2 | 7      | 5               | 6       | 7       | 7        | 7         |
|         | 5      | 5               | 6       | 6       | 6        | 6         |
|         | 6      | 6               | 6       | 6       | 6        | 6         |
| Image 3 | 7      | 5               | 6       | 6       | 6        | 6         |
|         | 5      | 5               | 6       | 6       | 6        | 6         |
|         | 6      | 6               | 6       | 6       | 6        | 6         |
| Image 4 | 5      | 5               | 5       | 5       | 5        | 5         |
|         | 5      | 5               | 5       | 5       | 5        | 5         |
|         | 5      | 5               | 5       | 5       | 5        | 5         |
| Image 5 | 1      | 1               | 1       | 0       | 1        | 0         |
|         | 1      | 1               | 1       | 0       | 1        | 0         |
|         | 1      | 1               | 1       | 0       | 1        | 0         |
|         | 1      | 1               | 1       | 0       | 1        | 0         |
|         | 5      | 4               | 5       | 5       | 5        | 5         |
|         | 5      | 4               | 5       | 5       | 5        | 5         |
|         | 5      | 4               | 5       | 5       | 5        | 5         |
| Image 6 | 1      | 3               | 2       | 4       | 3        | 2         |
|         | 1      | 3               | 2       | 4       | 3        | 2         |
|         | 1      | 3               | 2       | 4       | 3        | 2         |
|         | 1      | 3               | 2       | 4       | 3        | 2         |
|         | 1      | 3               | 2       | 4       | 3        | 2         |
|         | 1      | 3               | 2       | 4       | 3        | 2         |
| Image 7 | 2      | 1               | 2       | 1       | 1        | 1         |
|         | 2      | 1               | 2       | 1       | 1        | 1         |
|         | 2      | 1               | 2       | 1       | 1        | 1         |
|         | 2      | 1               | 2       | 1       | 1        | 1         |
|         | 2      | 1               | 2       | 1       | 1        | 1         |
|         | 2      | 1               | 2       | 1       | 1        | 1         |
| Image 8 | 1      | 1               | 9       | 1       | 1        | 0         |
|         | 1      | 1               | 9       | 1       | 1        | 0         |
|         | 1      | 1               | 9       | 1       | 1        | 0         |
|         | 1      | 1               | 9       | 1       | 1        | 0         |
|         | 1      | 1               | 9       | 1       | 1        | 0         |
| Image 9 | 0      | 0               | 0       | 0       | 0        | 0         |
|         | 0      | 0               | 0       | 0       | 0        | 0         |
|         | 0      | 0               | 0       | 0       | 0        | 0         |
| Image 10| 2      | 2                | 3       | 2       | 2        | 2         |
|         | 2      | 2                | 3       | 2       | 2        | 2         |
|         | 2      | 2                | 3       | 2       | 2        | 2         |
|         | 2      | 2                | 3       | 2       | 2        | 2         |
|         | 2      | 2                | 3       | 2       | 2        | 2         |
|         | 2      | 2                | 3       | 2       | 2        | 2         |
Table 8. Values of $c$ preferred by validity involving the membership values and data set for 8 dataset (IT2FCM, Extended-IT2FCM)

|                | IT2FCM |                |                |                |                |                |                |                |                |                |                |                |
|----------------|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                | $\nu_{FS}$ | $\nu_{FB}$ | $\nu_{FH}$ | $\nu_{FC}$ | $\nu_{SC}$ | $\nu_{SC}$ | $\nu_{EC}$ | $\nu_{FS}$ | $\nu_{FB}$ | $\nu_{FH}$ | $\nu_{FC}$ | $\nu_{SC}$ |
| Image 1        | .1     | .1             | .1            | .1            | .1             | .1             | .1             | .1             | .1             | .1             | .1             | .1             |
|                | 7      | 8              | 9             | 6             | 45             | 5              | 4              | 8              | 62             | 2              | 5              |
| Image 2        | .1     | .1             | .1            | .1            | .1             | .1             | .1             | .1             | .1             | .1             | .1             | .1             |
|                | 6      | 5              | 1             | 8             | 46             | 3              | 2              | 9              | 68             | 4              | 8              |
| Image 3        | .1     | .1             | .1            | .1            | .1             | .1             | .1             | .1             | .1             | .1             | .1             | .1             |
|                | 8      | 7              | 2             | 6             | 52             | 5              | 4              | 2              | 72             | 1              | 1              |
| Image 4        | .1     | .1             | .1            | .1            | .1             | .1             | .1             | .1             | .1             | .1             | .1             | .1             |
|                | 5      | 6              | 5             | 9             | 55             | 5              | 3              | 6              | 69             | 2              | 3              |
| Image 5        | .1     | .1             | .1            | .1            | .1             | .1             | .1             | .1             | .1             | .1             | .1             | .1             |
|                | 6      | 5              | 4             | 9             | 48             | 4              | 3              | 5              | 70             | 1              | 1              |
| Image 6        | .1     | .1             | .1            | .1            | .1             | .1             | .1             | .1             | .1             | .1             | .1             | .1             |
|                | 5      | 4              | 2             | 9             | 41             | 3              | 9              | 2              | 72             | 5              | 1              |
| Image 7        | .1     | .1             | .1            | .1            | .1             | .1             | .1             | .1             | .1             | .1             | .1             | .1             |
|                | 6      | 4              | 0             | 1             | 55             | 5              | 3              | 4              | 62             | 2              | 2              |
| Image 8        | .1     | .1             | .1            | .1            | .1             | .1             | .1             | .1             | .1             | .1             | .1             | .1             |
|                | 6      | 6              | 5             | 2             | 53             | 4              | 9              | 1              | 72             | 1              | 1              |
| Image 9        | .1     | .1             | .1            | .1            | .1             | .1             | .1             | .1             | .1             | .1             | .1             | .1             |
|                | 9      | 6              | 5             | 3             | 62             | 5              | 2              | 4              | 62             | 5              | 2              |
| Image 10       | .1     | .1             | .1            | .1            | .1             | .1             | .1             | .1             | .1             | .1             | .1             | .1             |
|                | 6      | 5              | 4             | 6             | 56             | 4              | 6              | 5              | 72             | 1              | 1              |

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Table 9. Values of \( c \) preferred by validity involving the membership values and data set for 8 dataset ((IT2FCM\( \alpha \), IT2FPCM)

| IT2FCM\( \alpha \) | IT2FPCM |
|---------------------|---------------------|
| \( v_{FS} \) | \( v_{FX} \) | \( v_{FK} \) | \( v_{FPR} \) | \( v_{FH} \) | \( v_{FW} \) | \( \bar{v}_{FC} \) | \( \bar{v}_{SC} \) | \( \bar{v}_{SC} \) | \( \bar{v}_{ECAS} \) |
| \( v_{FS} \) | \( v_{FX} \) | \( v_{FK} \) | \( v_{FPR} \) | \( v_{FH} \) | \( v_{FW} \) | \( \bar{v}_{FC} \) | \( \bar{v}_{SC} \) | \( \bar{v}_{SC} \) | \( \bar{v}_{ECAS} \) |
| Ima ge1 | .1 | .1 | .1 | .1 | .80 | .9 | .9 | .9 | .1 | .1 | .1 | .1 | .6 | .3 | .8 | .8 | .1 | .1 | .1 | .61 | .7 | .7 | .8 | .8 |
| 8 | 1 | 2 | 5 | 9 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Ima ge2 | .1 | .1 | .1 | .1 | .89 | .9 | .9 | .9 | .1 | .1 | .1 | .1 | .6 | .3 | .8 | .8 | .1 | .1 | .1 | .67 | .7 | .7 | .8 | .8 |
| 5 | 1 | 2 | 9 | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Ima ge3 | .1 | .1 | .1 | .1 | .92 | .9 | .9 | .9 | .1 | .1 | .1 | .1 | .6 | .3 | .8 | .8 | .1 | .1 | .1 | .71 | .8 | .8 | .8 | .8 |
| 5 | 1 | 2 | 9 | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Ima ge4 | .1 | .1 | .1 | .1 | .84 | .9 | .9 | .9 | .1 | .1 | .1 | .1 | .6 | .3 | .8 | .8 | .1 | .1 | .1 | .68 | .7 | .7 | .8 | .8 |
| 2 | 1 | 2 | 9 | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Ima ge5 | .1 | .1 | .1 | .1 | .91 | .9 | .9 | .9 | .1 | .1 | .1 | .1 | .6 | .3 | .8 | .8 | .1 | .1 | .1 | .69 | .7 | .7 | .8 | .8 |
| 5 | 1 | 2 | 9 | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Ima ge6 | .1 | .1 | .1 | .1 | .91 | .9 | .9 | .9 | .1 | .1 | .1 | .1 | .6 | .3 | .8 | .8 | .1 | .1 | .1 | .66 | .7 | .7 | .8 | .8 |
| 1 | 2 | 2 | 9 | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Ima ge7 | .1 | .1 | .1 | .1 | .89 | .9 | .9 | .9 | .1 | .1 | .1 | .1 | .6 | .3 | .8 | .8 | .1 | .1 | .1 | .61 | .7 | .7 | .8 | .8 |
| 8 | 1 | 2 | 9 | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Ima ge8 | .1 | .1 | .1 | .1 | .92 | .9 | .9 | .9 | .1 | .1 | .1 | .1 | .6 | .3 | .8 | .8 | .1 | .1 | .1 | .71 | .8 | .8 | .8 | .8 |
| 3 | 1 | 2 | 9 | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Ima ge9 | .1 | .1 | .1 | .1 | .92 | .9 | .9 | .9 | .1 | .1 | .1 | .1 | .6 | .3 | .8 | .8 | .1 | .1 | .1 | .61 | .7 | .7 | .8 | .8 |
| 9 | 1 | 2 | 9 | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Ima ge1 | .1 | .1 | .1 | .1 | .89 | .9 | .9 | .9 | .1 | .1 | .1 | .1 | .6 | .3 | .8 | .8 | .1 | .1 | .1 | .71 | .8 | .8 | .8 | .8 |
| 0 | 1 | 2 | 9 | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |

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Table 10. Values of c preferred by validity involving the membership values and data set for 8 dataset (MKIT2FCM, GMKIT2FCM)

|                | MKIT2FCM | GMKIT2FCM |
|----------------|----------|-----------|
|                | $V_{FS}$ | $V_{KH}$  | $V_{PFB}$ | $V_{PH}$ | $V_{PC}$ | $SC$ | $V_{SC}$ | $V_{SC}$ | $V_{RCA}$ | $SC$ | $V_{RC}$ |
| I mage1        | .1       | .1       | .1       | .1       | .1      | .74     | .8      | .8       | .9       | 8       | .9     | 3       | .1   |
|                | 3        | 2        | 25       | 2        | 4       | 8       | 7       | 9        | 3        | 33      | 3      | 3      | 0   |
| I mage2        | .1       | .1       | .1       | .1       | .1      | .84     | .8      | .8       | .9       | 4       | 8      | 65     | .1   |
|                | 3        | 2        | 2       | 2       | 3       | 0       | 2       | 0        | 0        | 8       | 5      | 1      | 2   |
| I mage3        | .1       | .1       | .1       | .1       | .1      | .84     | .8      | .9       | .9       | 1       | 1      | 33     | .1   |
|                | 3        | 3        | 2        | 3       | 1       | 3       | 1       | 1        | 0        | 0      | 4      | 3      | 1   |
| I mage4        | .1       | .1       | .1       | .1       | .1      | .84     | .8      | .9       | .9       | 0       | 1      | 33     | .1   |
|                | 3        | 2        | 6        | 3       | 0       | 6       | 5       | 4        | 1        | 0      | 2     | 1      | 0   |
| I mage5        | .1       | .1       | .1       | .1       | .1      | .84     | .8      | .9       | .9       | 1       | 2      | 2      | .1   |
|                | 3        | 3        | 3        | 3       | 1       | 2       | 2       | 3        | 2       | 3      | 3     | 1      | 1   |
| I mage6        | .1       | .1       | .1       | .1       | .1      | .84     | .8      | .9       | .9       | 0       | 1      | 3        | .1  |
|                | 3        | 2        | 9        | 3       | 0       | 9       | 1       | 3        | 0        | 3      | 2      | 3     | 1   |
| I mage7        | .1       | .1       | .1       | .1       | .1      | .84     | .8      | .9       | .9       | 0       | 1      | 35     | .0   |
|                | 3        | 3        | 3        | 3       | 1       | 2       | 2       | 3        | 2       | 3      | 3      | 1      | 3   |
| I mage8        | .1       | .1       | .1       | .1       | .1      | .84     | .8      | .9       | .9       | 0       | 1      | 34     | .1   |
|                | 5        | 2        | 2        | 3       | 1       | 1       | 1       | 1        | 0        | 0      | 3      | 2      | 1   |
| I mage9        | .1       | .1       | .1       | .1       | .1      | .84     | .8      | .9       | .9       | 0       | 1      | 35     | .0   |
|                | 5        | 2        | 3        | 3       | 1       | 2       | 2       | 8        | 3       | 3      | 3      | 1      | 0   |
| I mage10       | .1       | .1       | .1       | .1       | .1      | .84     | .8      | .9       | .9       | 0       | 1      | 36     | .1   |
|                | 5        | 2        | 4        | 2       | 1       | 0       | 1       | 1        | 0        | 0      | 1      | 4      | 0   |

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Table 11. Values of $c$ preferred by validity involving only the membership values for 8 dataset -interval type-1 fuzzy membership

|       | IT2FCM | Extended-IT2FCM | IT2FCM$_a$ | IT2FCM$_h$ | MKIT2FCM | GMKIT2FCM |
|-------|--------|-----------------|-------------|-------------|-----------|-----------|
| Image 1 | 7      | 6               | 2           | 2           | 2         | 2         |
| Image 2 | 5      | 5               | 6           | 6           | 6         | 6         |
| Image 3 | 7      | 8               | 7           | 7           | 7         | 7         |
| Image 4 | 5      | 5               | 5           | 5           | 5         | 5         |
| Image 5 | 1      | 1               | 1           | 1           | 1         | 1         |
| Image 6 | 1      | 1               | 1           | 1           | 1         | 1         |
| Image 7 | 2      | 2               | 2           | 2           | 2         | 2         |
| Image 8 | 1      | 1               | 1           | 1           | 1         | 1         |
| Image 9 | 1      | 9               | 1           | 1           | 1         | 1         |
| Image 10| 2      | 2               | 2           | 2           | 2         | 2         |

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Table 12. Values of $c$ preferred by validity involving the membership values and data set for 8 dataset (IT2FCM, Extended-IT2FCM)- interval type-1 fuzzy membership

| Image | $V_{F_1}$ | $V_{X_1}$ | $V_{K}$ | $V_{PB}$ | $V_{FR}$ | $V_{W}$ | $V_{PC}$ | $S_C$ | $V_{SC}$ | $V_{EC}$ | $V_{F_3}$ | $V_{K}$ | $V_{PB}$ | $V_{FR}$ | $V_{W}$ | $V_{PC}$ | $S_C$ | $V_{SE}$ | $V_{E2}$ | $V_{E3}$ |
|-------|-----------|-----------|---------|----------|----------|--------|-------|-------|--------|-------|---------|-------|--------|--------|-------|-------|-------|--------|-------|-------|-------|
| e1    | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 91 | 5 | 6 | 8 | 7 | 0 | 1 | 1 | 8 | 0 | 8 | 8 | 7 |
| e2    | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 56 | 6 | 7 | 9 | 0 | 0 | 0 | 0 | 9 | 8 | 6 | 9 | 0 |
| e3    | 1 | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 91 | 6 | 7 | 8 | 0 | 1 | 1 | 2 | 9 | 1 | 2 | 9 | 1 |
| e4    | 1 | 1 | 1 | 2 | 5 | 2 | 0 | 0 | 69 | 6 | 9 | 4 | 0 | 1 | 9 | 8 | 9 | 1 | 5 | 9 | 1 |
| e5    | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 5 | 71 | 7 | 8 | 4 | 1 | 7 | 9 | 8 | 8 | 1 | 7 | 8 | 3 |
| e6    | 1 | 1 | 1 | 1 | 9 | 0 | 2 | 0 | 95 | 6 | 8 | 2 | 0 | 8 | 9 | 8 | 9 | 2 | 7 | 9 | 5 |
| e7    | 1 | 1 | 1 | 2 | 3 | 1 | 2 | 1 | 99 | 6 | 8 | 7 | 0 | 1 | 1 | 8 | 0 | 1 | 8 | 2 | 1 |

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Table 13. Values of c preferred by validity involving the membership values and data set for 8 dataset ((IT2FCM\(_\alpha\), IT2FPCM)- interval type-1 fuzzy membership

| Im age | \(V_{IP}\) | \(V_{XP}\) | \(V_{IK}\) | \(V_{IP}V_{XP}\) | \(V_{IP}V_{IK}\) | \(V_{IP}V_{IK}V_{WP}\) | \(V_{IP}V_{IK}V_{WP}V_{WC}\) | \(V_{IP}V_{IK}V_{WP}V_{WC}V_{CE}\) | \(V_{IP}V_{IK}V_{WP}V_{WC}V_{CE}V_{SC}\) | \(V_{IP}V_{IK}V_{WP}V_{WC}V_{CE}V_{SC}V_{BEAS}\) |
|--------|--------|--------|--------|----------------|----------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| e8     | 1      | 1      | 1      | 1              | 1              | 0.6                 | 0.6                  | 0.8                  | 0.8                  | 0.8                  |
| e9     | 1      | 1      | 1      | 1              | 1              | 0.5                 | 0.5                  | 0.8                  | 0.8                  | 0.8                  |
| e10    | 1      | 1      | 1      | 1              | 1              | 0.9                 | 0.9                  | 0.9                  | 0.9                  | 0.9                  |

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Table 14. Values of c preferred by validity involving the membership values and data set for 8 dataset (MKIT2FCM, GMKIT2FCM) -interval type-1 fuzzy membership

|       | MKIT2FCM | GMKIT2FCM |
|-------|----------|-----------|
|       | $\bar{\psi}_{FS}$ | $\bar{\psi}_{FH}$ |
| $\bar{\psi}_{FH}$ | $\bar{\psi}_{FH}$ |
| $\bar{\psi}_{FH}$ | $\bar{\psi}_{FH}$ |
| $\bar{\psi}_{FH}$ | $\bar{\psi}_{FH}$ |
| $\bar{\psi}_{FH}$ | $\bar{\psi}_{FH}$ |
| $\bar{\psi}_{FH}$ | $\bar{\psi}_{FH}$ |
| $\bar{\psi}_{FH}$ | $\bar{\psi}_{FH}$ |
| $\bar{\psi}_{FH}$ | $\bar{\psi}_{FH}$ |

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