New insight on the Sivers transverse momentum
dependent distribution function

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Abstract.

Polarised Semi-Inclusive Deep Inelastic Scattering (SIDIS) processes, \( \ell (S\ell) p(S) \rightarrow \ell h X \) allow to study Transverse Momentum Dependent partonic distributions (TMDs), which reveal a non trivial three dimensional internal structure of the hadrons in momentum space. One of the most representative of the TMDs is the so-called Sivers function that describes the distribution of unpolarized quarks inside a transversely polarized proton. We present a novel extraction of the Sivers distribution functions from the most recent experimental data of HERMES and COMPASS experiments. Using suitable parametrizations, within the TMD factorization scheme, and a simple fitting strategy, we also perform a preliminary exploration of the role of the proton sea quarks.

The Sivers distribution function \( \Delta N f_{q/p}^\uparrow(x,k_\perp) \) embodies the correlation between the nucleon spin and the quark intrinsic transverse momentum and gives the number density of unpolarized quarks \( q \) with intrinsic transverse momentum \( k_\perp \) inside a transversely polarized proton \( p^\uparrow \), with three-momentum \( P \) and spin polarization vector \( S \),

\[
\hat{f}_{q/p}^\uparrow(x,k_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2} \Delta N f_{q/p}^\uparrow(x,k_\perp) \vec{S} \cdot (\hat{P} \times \hat{k}_\perp). \tag{1}
\]

\( f_{q/p}(x,k_\perp) \) is the unpolarized \( x \) and \( k_\perp \) dependent parton distribution, and the mixed product \( S \cdot (\hat{P} \times \hat{k}_\perp) \) gives the parity invariant azimuthal dependence. The distribution of quarks in polarized hadrons is not axially symmetric, and shows a non trivial 3-D structure. The conceptual importance of the Sivers function, its relation with the color QCD dynamics and its gauge properties, are being more and more understood. Dedicated experiments are running, under construction or planned.

In Ref. \textsuperscript{1}, we presented an extraction of the Sivers distribution functions based on a fit of SIDIS experimental data from the HERMES and COMPASS collaborations. At that time, the

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only direct evidence of a non-zero Sivers effect came from HERMES experimental data on the azimuthal moment $A_{UT}^{\sin(\phi_h - \phi_S)}$, measured for pion and kaon production off a proton target [2]; the COMPASS collaboration, instead, found a Sivers effect compatible with zero for SIDIS off a deuteron target [3]. The extraction of the Sivers functions was made even more challenging by the fact that the $A_{UT}^{\sin(\phi_h - \phi_S)}$ measured by HERMES for $K^+$ was surprisingly large, about twice as much as the analogous asymmetry for $\pi^+$ production, which hinted at a possible important role of the sea quarks [1].

Since then, new experimental results have become available: COMPASS have released new data on SIDIS production of spinless hadrons off a proton target, showing a clear Sivers effect compatible with zero for SIDIS off a deuteron target [3]. The extraction of the Sivers functions was made even more challenging by the fact that the $A_{UT}^{\sin(\phi_h - \phi_S)}$ measured by HERMES for $K^+$ was surprisingly large, about twice as much as the analogous asymmetry for $\pi^+$ production, which hinted at a possible important role of the sea quarks [1].

Figure 1. The Sivers single spin asymmetry, $A_{UT}^{\sin(\phi_h - \phi_S)}$, as a function of $x$ for SIDIS production of $\pi^+$ (left panels) and $\pi^-$ (right panels) at HERMES. The upper (lower) panels show the results obtained from a fit which includes only valence (valence and sea) quark contributions.

The SIDIS transverse single spin asymmetry (SSA) $A_{UT}^{\sin(\phi_h - \phi_S)}$ measured by HERMES and COMPASS, in the $\gamma^* - p$ c.m. frame and at order $k_\perp/Q$, is given by [6, 7]:

$$A_{UT}^{\sin(\phi_h - \phi_S)} = \frac{\sum_q \int d\phi_S d\phi_h d^2k_\perp \Delta^N f_{q/p}(x, k_\perp) \sin(\phi - \phi_S) \frac{d\sigma_{\ell q \rightarrow \ell q}}{dQ^2} D_h^q(z, p_\perp) \sin(\phi_h - \phi_S)}{\sum_q \int d\phi_S d\phi_h d^2k_\perp f_{q/p}(x, k_\perp) \frac{d\sigma_{\ell q \rightarrow \ell q}}{dQ^2} D_h^q(z, p_\perp)}.$$  \hspace{1cm} (2)

$\phi_S$ and $\phi_h$ are the azimuthal angles identifying the directions of the proton spin $S$ and of the momentum of the outgoing hadron $h$ respectively, while $\varphi$ defines the direction of the incoming (and outgoing) quark transverse momentum, $k_\perp = k_\perp (\cos \varphi, \sin \varphi, 0)$; $\frac{d\sigma_{\ell q \rightarrow \ell q}}{dQ^2}$ is the unpolarized cross section for the elementary scattering $\ell q \rightarrow \ell q$. Finally, $D_h^q(z, p_\perp)$ is the fragmentation function describing the hadronization of the final quark $q$ into the detected hadron $h$ with momentum $P_h$; $h$ carries, with respect to the fragmenting quark, a light-cone momentum fraction $z$ and a transverse momentum $p_\perp$.

The Sivers function is parameterized in terms of the unpolarized distribution function, as in
Ref. [6], in the following factorized form:

$$\Delta N f_{q/p}(x, k_\perp) = 2 N_q(x) h(k_\perp) f_{q/p}(x, k_\perp) ,$$  \hspace{1cm} (3)

with

$$N_q(x) = N_q x^{\alpha_q} (1 - x)^{\beta_q} \left( \frac{\alpha_q + \beta_q}{\alpha_q + \beta_q} \right) \left( \frac{\alpha_q}{\beta_q} \right) ,$$

$$h(k_\perp) = \sqrt{2e} \frac{k_\perp}{M_1} e^{-k_\perp^2/M_1^2} ,$$  \hspace{1cm} (4)

where $N_q$, $\alpha_q$, $\beta_q$ and $M_1$ (GeV/c) are free parameters to be determined by fitting the experimental data. We adopt a Gaussian factorization for the unpolarized distribution and fragmentation functions with the Gaussian widths $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$ fixed to the values found in Ref. [7] by analysing the Cahn effect in unpolarized SIDIS: $\langle k_\perp^2 \rangle = 0.25$ (GeV/c)$^2$ and $\langle p_\perp^2 \rangle = 0.20$ (GeV/c)$^2$. For the unpolarized, $k_\perp$-integrated distribution and fragmentation functions we use the GRV98 [8] and DSS [9] sets.

We best fit the HERMES proton and COMPASS deuteron data from Refs. [3, 5] and exclude the COMPASS proton data [4] due to the presence of some experimental scale uncertainty which is difficult to account for in the present analysis. The new forthcoming data from COMPASS on a proton target will become very useful in the future.

In order to evaluate the significance of the sea-quark Sivers contributions we first perform a fit of the SIDIS data including only Sivers functions for $u$ and $d$ quarks. The results we obtain are rather satisfactory, with a total $\chi^2_{dof}$ of about 1.05. They are shown in the upper panels of Figs. [1] - [4]. The corresponding Sivers functions are plotted in the left panel of Fig. [5]. It is interesting to notice that the $M_1$ parameter, which fixes the Gaussian width of the Sivers function (i.e. its distribution in $k_\perp$) is rather well constrained and turns out to be between one half and two thirds of the unpolarized distribution function width.

We then perform a second fit, in which we allow for $\bar{u}$, $\bar{d}$, $s$, $\bar{s}$ sea contributions to the Sivers function: overall, we obtain similar results, shown in the lower panels of Figs. [1] - [4] but the total $\chi^2_{dof}$ improves to about 0.9. The corresponding Sivers functions are plotted in the right panel of Fig. [5].

At this stage, our results can only be considered as qualitative and preliminary, as the experimental data are not yet so well established and abundant to allow a precise definite
determination of the Sivers sea distribution functions. We can only try, by comparing the upper and lower panels of Figs. 1–4, to draw some general conclusions.

Adding the sea quark contributions to the Sivers functions does not induce any variation in the $A_{LT}^{\sin(\phi_h-\phi_S)}$ azimuthal moments for $\pi^+$ and $K^+$ production, which are confirmed to be essentially generated by the fragmentation of a valence $u$ quark from the proton; instead, the production of $\pi^-$ and mostly $K^-$ are more sensitive to the presence of the sea quark contributions, and might offer the chance to gain some preliminary information on the polarized proton sea.

We also learn that the $A_{LT}^{\sin(\phi_h-\phi_S)}$ asymmetries, as delivered by the presently available experimental data, are not very sensitive to the strange quark Sivers distributions, while they feel the presence of $\bar{u}$ and $\bar{d}$ contributions more strongly: we have actually checked that the $\chi^2_{dof}$ is not altered when adding or removing $s$ and $\bar{s}$.

When comparing the left and right plots in Fig. 5 one notices that, while the $\bar{u}$, $s$ and $\bar{s}$ Sivers functions are consistent with zero, the $\bar{d}$ Sivers function can be rather large and negative, definitely not consistent with zero. This originates from the small-$x$ behavior of the COMPASS data for $\pi^+$ production off a deuteron target, most sensitive to $\bar{d}$: as these data show a negative trend at small $x$ (and very tiny error bars, Fig. 3), the fit naturally tends to assign a negative value to the $\bar{d}$ Sivers function.

Further experimental measurements from present and future facilities are strongly needed to reach a better knowledge on the important Sivers distribution functions, in particular in the large and small $x$ regions where they are presently largely unconstrained. Some help might also come from the existing large SSAs in $p p \rightarrow h X$ processes, which can be interpreted within a TMD factorization scheme [10] [11].

References
[1] Anselmino M et al. 2009 Eur. Phys. J. A39 89–100 (Preprint arXiv:0805.2677)
[2] Diefenthaler M (HERMES) 2007 (Preprint arXiv:0706.2242 [hep-ex])
[3] Alekseev M et al. (COMPASS) 2009 Phys. Lett. B673 127–135 (Preprint 0902.2160)
[4] Alekseev M G et al. (The COMPASS) 2010 Phys. Lett. B692 240–246 (Preprint 1005.5609)
[5] Airapetian A et al. (HERMES) 2009 Phys. Rev. Lett. 103 152002 (Preprint 0906.3918)
[6] Anselmino M et al. 2005 Phys. Rev. D72 094007 (Preprint hep-ph/0507181)
Figure 4. The Sivers single spin asymmetry $A_{UT}^{\sin(\phi_h - \phi_S)}$, as a function of $x$ for SIDIS $K^+$ (left panels) and $K^-$ (right panels) production at COMPASS on a deuteron target. The upper (lower) panels show the results obtained from a fit which includes only valence (valence plus sea) quark contributions.

Figure 5. First $k_\perp$-moment of the Sivers distribution function, $x\Delta^N f^{(1)}_{q/p}$, as a function of $x$ as obtained by fitting SIDIS experimental data from HERMES and COMPASS. The left (right) panel shows the results obtained from a fit which includes only valence (valence plus sea) quark contributions.