HGKT: Introducing Hierarchical Exercise Graph for Knowledge Tracing

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ABSTRACT
Knowledge tracing (KT) which aims at predicting learner’s knowledge mastery plays an important role in the computer-aided educational system. In recent years, many deep learning models have been applied to tackle the KT task, which have shown promising results. However, limitations still exist. Most existing methods simplify the exercising records as knowledge sequences, which fail to explore rich information existed in exercises. Besides, the existing diagnosis results of knowledge tracing are not convincing enough since they neglect prior relations between exercises. To solve the above problems, we propose a hierarchical graph knowledge tracing model called HGKT to explore the latent hierarchical relations between exercises. Specifically, we introduce the concept of problem schema to construct a hierarchical exercise graph that could model the exercise learning dependencies. Moreover, we employ two attention mechanisms to highlight important historical states of learners. In the testing stage, we present a K&S diagnosis matrix that could trace the transition of mastery of knowledge and problem schema, which can be more easily applied to different applications. Extensive experiments show the effectiveness and interpretability of our proposed models.

CCS CONCEPTS
• Information systems → Data mining; • Social and professional topics → Student assessment.

KEYWORDS
intelligent education, knowledge tracing, hierarchical graph convolutional network, attention mechanism.

1 INTRODUCTION
Knowledge tracing is a fundamental task in computer-aided educational system, which can benefit both learning and teaching [1, 2, 18]. A typical knowledge tracing process is as follows: when a question is posted, a learner reads its text and applies knowledge to solve it. After getting a bunch of learners’ interactive items, corresponding knowledge sequences and interactive sequences of exercises are extracted to train a KT model which could predict the hidden knowledge states of a learner. However, there are two main problems shown in the traditional knowledge tracing workflow: (1) Exercise representation loss problem: traditional workflow simplifies the exercising records as knowledge sequences, which ignores the difficulty and semantic information contained in exercises. In other words, existing methods have an information loss in the process of representing exercises. (2) Insufficient diagnosis problem: it is also difficult to provide specific learning suggestions based on the diagnosis results. Specifically, suppose we know a learner has a weak knowledge (e.g., Coordinate Calculation), it is hard to decide which one fits him better (e.g., 𝑒7 or 𝑒8 in Fig. 1), since mappings between knowledge and exercises is too broad [15, 40].

In the literature, there are many efforts in knowledge tracing. Existing methods can be divided into two tracks: traditional knowledge based and exercise based track. The traditional knowledge based method converts learners’ exercising sequences into knowledge sequences without considering the text information of the exercises. The most popular one is Bayesian Knowledge Tracing (BKT) [5], which updates learner’s knowledge state by a Hidden Markov Model. Deep learning methods like Deep Knowledge Tracing (DKT) model the learning process as a recurrent neural network [26]. Dynamic Key-Value Memory Network (DKVMN) enhances the ability of the recurrent neural network by introducing two memory matrices to represent knowledge and learner’s mastery for each knowledge respectively [38]. Graph-based Knowledge Tracing (GKT) combines knowledge tracing with a graph neural network [25]. It encodes a learner’s hidden knowledge state as embeddings of graph nodes and updates the state in a knowledge graph. Those models have been proved effective but still have limitations. Most existing methods face the problem of exercise representation loss since they have not taken the texts of exercises into consideration. For the exercise based track, to the best of our knowledge, Exercise Enhanced Knowledge Tracing (EKT) is the first method to incorporate features of exercise text into the knowledge tracing model [16]. However, EKT extracts features of text by feeding the text of exercise directly into a bi-directional LSTM network [14], which fails to consider the latent hierarchical graph nature of exercises and brings in extra noise from embeddings of text.
we encode the information of hierarchical relations, we also hope with the same knowledge may also belong to different problem definitions of problem schema and indirect support relations remaining unknown. Second, the hierarchical exercise relations contain different levels of features of exercises, how to organically combine bringing in the following problems. First, the direct support relations exercises support relations into a KT task. However, it may also the prior support relation between exercises (i.e. \(e_2\) direct support \(e_7\), \(e_3\) indirect support \(e_8\)) into account, we can naturally speculate that the learner will more probably do \(e_7\) wrong and \(e_8\) right.

Our insight of solving the exercise representation loss problem and insufficient diagnosis problem hinges upon the idea of fully exploring latent hierarchical graph relations between exercises. Incorporating hierarchical relations between exercises could not only increase the accuracy of learner’s performance prediction, but also enhance the interpretability of knowledge tracing. Fig. 1 clearly illustrates how hierarchical relations affect knowledge diagnosis results and the advantages of our approach compared to traditional knowledge tracing methods. As researchers have demonstrated the effectiveness of exploiting prerequisite relations into KT tasks [3], we decompose the hierarchical graph relations between exercises into direct support and indirect support relations. The intuition of support relations is that they represent different types of exercise learning dependencies, which could act as constraints for a knowledge tracing task. Moreover, to learn more fine-grained representation of exercises and avoid single exercise noises, inspired by math word solving technique [11, 37], we introduce the concept of problem schema to summarize a group of similar exercises with similar solutions. The relation of two exercises is indirect support only if they belong to the same problem schema. It is worth mentioning that, we assume each exercise only has one main knowledge and one problem schema. Considering that exercises belonging to different knowledge could have similar solutions, and exercises with the same knowledge may also belong to different problem schemas due to their difficulty difference, we assume the relations between knowledge and problem schemas are many-to-many (as shown in Fig. 2(a)).

The above analysis shows the prospects of introducing prior exercises support relations into a KT task. However, it may also bring in the following problems. First, the direct support relations between exercises can be defined in multiple ways, but which is the most suitable one for KT tasks remains unknown. Second, the definitions of problem schema and indirect support relations require understanding of exercises from a semantic perspective. How to automatically understand and represent the information is still challenging. Third, the hierarchical exercise relations contain different levels of features of exercises, how to organically combine these different levels of features is still worth exploring. Last, after we encode the information of hierarchical relations, we also hope that the model can always draw on past key information during current prediction. As shown in Fig. 1, during making predictions for \(e_7\), our model needs easily look back to important historical information such as the learner’s wrong answer in \(e_2\) or the relation between \(e_2\) and \(e_7\).

Figure 1: A toy example of knowledge tracing. Given a learner who did exercise \(e_2\) with knowledge Pythagorean Theorem wrong and exercise \(e_3\) with knowledge Number Axis Properties right. Suppose we need to predict the learner’s performance for two exercises \(e_7\) and \(e_8\) which have the same knowledge Coordinate Calculation. Traditional knowledge tracing method (TKT) such as DKT, GKT would give the same prediction to them since both exercises belong to the same knowledge. However, if we take the prior support relation between exercises (i.e. \(e_2\) direct support \(e_7\), \(e_3\) indirect support \(e_8\)) into account, we can naturally speculate that the learner will more probably do \(e_7\) wrong and \(e_8\) right.

To address above challenges, we propose a hierarchical graph knowledge tracing framework named HGKT, which unifies the strengths of hierarchical graph neural network and recurrent sequence model with attention to enhance the performance of knowledge tracing. To summarize, the main contributions of this paper are as follows:

- We introduce the concept of hierarchical exercise graph consisting of direct and indirect support relations between exercises, which could serve as learning constraints for a knowledge tracing task. We also present several data-driven methods to model direct support relations and introduce a way to semantically model the indirect support relations.
- We propose the concept of problem schema for exercises and explore a novel approach called hierarchical graph neural network to learn a refined representation for problem schema. This approach could help to address the exercise representation loss problem.
- We present two attention mechanisms that can highlight the important states of learners and fully leverage the information learned in HEG.
- To make the diagnosis results detailed and convincing, we put forward a knowledge&schema (K&S) diagnosis matrix that can trace the mastery of both knowledge and problem.
We formulate the KT task in this paper as follows. The goal of knowledge tracing is to estimate learners’ hidden knowledge states based on learners’ exercise data. Suppose we have a learner set \( \mathcal{P} \), an exercise set \( \mathcal{E} \), a knowledge set \( \mathcal{K} \) and a problem schema set \( \mathcal{S} \). Each exercise associates with a knowledge \( k \in \mathcal{K} \) and a problem schema \( s \in \mathcal{S} \). We further record exercise sequence for each learner \( l \) as \( \mathcal{R}_l = \{(e_1, r_1), (e_2, r_2), \ldots (e_m, r_m)\} \), where \( e_i \in \mathcal{E} \) and \( r_i \in \{0, 1\} \). Here \( r_i = 1 \) indicates a correct answer and \( r_i = 0 \) indicates an incorrect one. To clearly explain the framework of HGKT, we first briefly introduce the definition of HEG.

### 2.0.1 Hierarchical Exercise Graph (HEG)

In an educational system, HEG is a hierarchical graph consisting of different levels of graphs that store prior direct and indirect support relations between exercises. Specifically, the HEG used in HGKT has two graphs: the bottom one is called direct support graph while the top one is called indirect support graph. Each node in the bottom graph corresponds to an exercise in our database while each node in top graph corresponds to a problem schema. The graph links in the bottom graph model the direct support relations while the relations between those two graphs model the indirect support relations. Formally, we represent a HEG graph as \((A, F, S_e)\), where \( A \in \{0, 1\}^{|\mathcal{E}| \times |\mathcal{E}|} \) is an adjacency matrix of the bottom graph, \( F \in \mathbb{R}^{|\mathcal{E}| \times |\mathcal{S}|} \) is the node feature matrix assuming each node has \( t \) features and \( S_e \in \mathbb{R}^{|\mathcal{E}| + |\mathcal{S}|} \) is an assignment matrix which denotes an assignment of each node from the bottom graph to a node in the top graph.

### 3 HGKT FRAMEWORK

#### 3.1 Framework Overview

The entire structure of the framework is shown in Fig. 3. Here, System 1 aims to learn hierarchical graph structure of exercises by a hierarchical graph neural network (HGNN) and provides those resources to System 2. System 2 then conducts the sequential processing, makes performance prediction and collects clues for System 1 to better refine the embeddings of each problem schema. To start training, we first need to generate the HEG from exercise database and interaction history. Thus, we introduce several methods to construct the direct support graph in section 3.2 and to extract indirect support relations from a semantic perspective in section 3.3. After the construction of HEG, exercising records and the problem schema embeddings are combined in a recurrent neural network (RNN) to predict the learner’s performance. It is worth mentioning that HGNN and RNN are trained in an end-to-end fashion.

#### 3.2 Direct Support Graph Construction

HGKT can leverage prior hierarchical relations among exercises for a KT task as shown in Fig. 1. However, the hierarchical relations of exercises are not explicitly provided in most cases. In this section, we first introduce several methods to explore the direct support relations among exercises, which are further used for learning the representations of problem schema. Many predecessors have proven the effectiveness of introducing graph structures into a KT task [25, 33], thus we propose several similarity rules based graph structures for exercises in section 3.2.1, 3.2.2, 3.2.3. Moreover, based on Bayesian statistical inference, we also present a method which could leverage the prior exercise support relations from the exercise interaction sequences in the training set and construct the direct support graph. To modeling the prior exercise relations as constraints, we first define the following properties about exercise support relations:

**Property 1:** We use \( \text{Sup}(e_1 \rightarrow e_2) \) to denote the support degree of two exercises \( e_1 \) and \( e_2 \). \( R_{e_1} \) and \( W_{e_2} \) denote an event that whether a learner give right or wrong answer to \( e_1 \). A large value of \( \text{Sup}(e_1 \rightarrow e_2) \) indicates the solutions of \( e_1 \) and \( e_2 \) have a strong support, which means if we know that a learner did \( e_1 \) wrong, then he has a high probability of doing \( e_2 \) wrong. In addition, if the learner is known to have done \( e_2 \) correctly, then the probability of him doing correct \( e_1 \) is also high.

\[
P(R_{e_1} | R_{e_2}) > P(R_{e_1} | W_{e_2}), P(W_{e_2} | R_{e_1}) > P(W_{e_2} | R_{e_1}, W_{e_1}),
\]

if \( \text{Sup}(e_1 \rightarrow e_2) > 0 \).

(1)

**Property 2:** Conversely, if \( \text{Sup}(e_1 \rightarrow e_2) \) is small, it means that there is no prior support relations between the content and the solutions of the two exercises. In other words, the performance of learners for the two exercises is two unrelated events. Thus, we could conduct the following formulas:

\[
P(R_{e_1} | R_{e_2}) = P(R_{e_1} | R_{e_2}, W_{e_2}), P(W_{e_2} | R_{e_1}) = P(W_{e_2} | R_{e_1}, W_{e_1}),
\]

if \( \text{Sup}(e_1 \rightarrow e_2) = 0 \).

(2)

Based on the above reasoning, we construct the value of support between exercises as follows. Here, \( \text{Count}((e_1, e_2)) = (r_1, r_2) \) counts the number of exercise sequences that reply \( e_1 \) with an answer \( r_1 \) before \( e_2 \) with an answer \( r_2 \). Besides, to prevent the denominator from being too small, we introduced the laplacian smoothing parameter \( \lambda_p = 0.01 [9] \) in Eq.(3) and Eq.(4).

\[
P(R_{e_1} | R_{e_2}) = \frac{\text{Count}((e_2, e_1) = (1, 1)) + \lambda_p}{\sum_{r_1 = 0}^{1} \text{Count}((e_2, e_1) = (1, r_1)) + \lambda_p},
\]

(3)

\[
P(R_{e_1} | R_{e_2}, W_{e_2}) = \frac{\sum_{r_2 = 0}^{1} \sum_{r_1 = 0}^{1} \text{Count}((e_2, e_1) = (r_2, r_1)) + \lambda_p}{\sum_{r_2 = 0}^{1} \sum_{r_1 = 0}^{1} \text{Count}((e_2, e_1) = (r_2, r_1)) + \lambda_p}.
\]

(4)

Similarly, we could also estimate the probability of \( P(W_{e_2} | R_{e_1}) \) and \( P(W_{e_2} | R_{e_1}, W_{e_1}) \). The value of support is defined as the sum of the following two components. Here, \( \max \) function is used to ensure the non-negativity of support value.

\[
\text{Sup}(e_1 \rightarrow e_2) = \max(0, \ln \frac{P(R_{e_1} | R_{e_2})}{P(R_{e_1} | R_{e_2}, W_{e_2})} + \max(0, \ln \frac{P(W_{e_2} | R_{e_1})}{P(W_{e_2} | R_{e_1}, W_{e_1})})).
\]

(5)
we adopt the hierarchical clustering [17] to cluster the BERT embeddings of exercises. Hierarchical clustering is an unsupervised clustering method using agglomerative or divisive strategies to build a hierarchy of clusters. We could set different clustering threshold \( \lambda \) to accommodate the cognitive nature of students at different levels, besides, to better obtain the clustering results with multiple levels of complexity, we use BERT [6] to encode all exercises in the database to obtain its semantic representation as many successes have demonstrated its semantic nature of problem schema, thus we learn representations of problem schema for each exercise, thus we extracted are used to compose the HEG. The goal of System 1 is to support relations between exercises. The hierarchical relations extracted are used to compose the HEG. The goal of System 1 is to learn representations of problem schema for each exercise, thus we also propose a way to fuse those hierarchical relations.

The essence of mining indirect support relations is to find the corresponding problem schema for each exercise (as shown in Fig. 1), which can be transformed into a problem of unsupervised clustering of exercises. Considering the semantic nature of problem schema, we use BERT [6] to encode all exercises in the database to obtain its semantic representation as many successes have demonstrated BERT’s ability to understand the semantic information behind texts. Besides, to better obtain the clustering results with multiple levels to accommodate the cognitive nature of students at different levels, we adopt the hierarchical clustering [17] to cluster the BERT embeddings of exercises. Hierarchical clustering is an unsupervised cluster analysis method using agglomerative or divisive strategies to build a hierarchy of clusters. We could set different clustering threshold \( \lambda \) to get different levels of clustering results, which could be used to find the most suitable level for problem schema. Moreover, in order to better combine the graph structures and the clustering results of the exercises, motivated by the assignment matrix proposed in DiffPool [36], we propose an exercise assignment matrix \( S_e \) that can provide an assignment of each exercise node in the direct support graph to a problem schema node in the indirect support graph. Here, each row of the \( S_e \) corresponds to one of exercise at bottom graph, and each column of \( S_e \) corresponds to one of problem schema at top graph. It is worth mentioning that the assignment matrix in DiffPool is learned by a separate GNN model which is computational expensive and hard to control hierarchical relations in our cases. Thus, we use hierarchical clustering to generate the assignment matrix to store indirect support relations.

After extracting the hierarchical relations from exercises, we now introduce the detailed strategie of fusing graph information in HEG. Here we propose a hierarchical graph neural networks (HGNN) to propagate semantic features of exercises into problem schema embedding in the HEG (as shown in Fig. 4). The HGNN consists of convolutional layers and pooling layers [20, 22, 28]. The key intuition of HGNN is that we stack a direct support graph with exercise information and a indirect support graph with problem schema information, and leverage assignment matrix to assign each node from direct support graph to indirect support graph. Formally, for a given HEG = \( (A, F, S_e) \), the HGNN propagates features using following equations. First of all, we build two GNN networks named \( GNN_{exer} \) and \( GNN_{sche} \) whose parameters are used to update exercise embedding and problem schema embedding correspondingly. The node feature matrix for the first \( k \) layers is one-hot embedding of each exercise \( F \) and a direct support graph \( A_e \). Here \( H_e \) and \( H_s \) correspond to the embedding of exercises and problem schemas. Note that \( H^0 = F \). At \( k \)-th layer, as shown in Eq. 7, we utilize a pool operation to coarsen the direct support graph to get a smaller indirect support graph \( A_s \). The linear transformation introduced in Eq. 8 aggregates the exercise representations to correspond problem schema representations. At last, \( GNN_{sche} \) updates each embedding of problem schema and sends this information to sequence processing stage of HGKT.

\[
H_e^{(l+1)} = GNN_{exer}(A_e, H_e^{(l)}), l < k
\]
This section mainly focuses on combining the representations of problem schemas with sequence information.

**Figure 4: The architecture of HGKT**

### 3.4 Sequence Modeling Process

#### 3.4.1 Sequence Propagation. The overall input for the sequence processing part is exercise interaction sequences. Each exercise interaction contains three components: knowledge \(q_t\), exercise result \(r_t\), problem schema \(s_t\). Here, \(q_t\) is a one-hot embedding of \(|\mathcal{K}|\) distinct knowledge. \(r_t\) is a binary value indicating whether the learner answers an exercise correctly. \(s_t\) is an embedding of problem schema generated in HEG. At each time step \(t\), to differentiate the exercise contribution to its corresponding knowledge hidden state, the input for the sequence model is a joint embedding \(x_t\) of \((q_t, r_t, s_t)\). In the propagation stage, we process \(x_t\) and the previous hidden state \(h_t\) using RNN network to get current learner’s hidden state \(h_{t+1}\) as shown in Eq. 10. Here we use LSTM as a variant of RNN since it can better preserve long-term dependency in the exercise sequences [14]. Eq. 11 shows the prediction of mastery for each problem schema \(m_{cur, t+1}^{s}\) in \(\mathbb{R}^{|\mathcal{S}|}\) at time \(t+1\). \(\{W_1, b_1\}\) are the parameters.

\[
\begin{align*}
A_s &= S^T \lambda A_s S_k, l = k, \\
H_{k}^{(t+1)} &= S^T H_{k}^{(t)}, l = k, \\
H_{k}^{(t+1)} &= \text{GNN}_{\text{sche}}(A_s, H_{k}^{(t)}), l > k.
\end{align*}
\]

**3.4.2 Attention Mechanism.** The HGKT leverages two kinds of attention mechanism, called sequence attention and schema attention, to enhance effects of typical states in the history of modeling. Learners may perform similarly on exercises with same problem schemas, thus we use a sequence attention shown in Eq. 12 to refer to previous results of similar exercises. Here, we assume the attention of historical problem schema mastery \(m_{att, t+1}^{s}\) is a weighted sum of previous mastery state. However, different from the attention used in [16], we set an attention window limit \(\lambda_d\) in HGKT for the following two reasons: (1) If the length of sequence attention is not limited, the computational cost would be intensive when the exercise sequence is extremely long. (2) Experiments prove that recent memory has a greater impact on knowledge tracing results than long past memories, which is consistent with educational psychology since learners start losing the memory of learned knowledge over time [7].

\[
m_{att, t+1}^{s} = \sum_{i=\max(t-\lambda_d,0)}^{t} \beta_i m_{cur, t+1}^{s}, \beta_i = \cos(s_{t+1}, s_t).
\]

The schema attention aims to focus learner’s attention on a given problem schema with \(s_t \in \mathbb{R}^{|\mathcal{S}|}\), which is the similarity to other problem schemas. As shown in Eq. 13, we propose an external memory \(M_{sc} \in \mathbb{R}^{k \times |\mathcal{S}|}\) collected from embeddings in the final layer of indirect support graph. Each column of \(M_{sc}\) corresponds to an embedding of problem schema. Here, \(k\) is the embedding dimension in HEG. The intuition for Eq. 13 is that the answer of an exercise would relate to exercises with similar problem schema, thus we could focus our attention on a certain problem schema. Notice that the value of memory \(M_{sc}\) changes over time during training.

\[
m_{f, t+1}^{s} = \alpha_{r, t+1} m_{cur, t+1}^{s}, \alpha_{t+1} = \text{Softmax}(s_{t+1}^T M_{sc}).
\]

To summarize, the states to predict the learner’s performance at time \(t+1\) consists of three components: current knowledge mastery \(m_{cur, t+1}^{s}\), historical related knowledge mastery \(m_{att, t+1}^{s}\) and focus problem schema mastery \(m_{f, t+1}^{s}\). As Eq. 14 shows, those states are concatenated together to predict a final result. \(\{W_2, b_2\}\) are the parameters.

\[
\hat{r}_{t+1} = \sigma(W_2 \cdot [m_{att, t+1}^{s}, m_{cur, t+1}^{s}, m_{f, t+1}^{s}] + b_2).
\]

#### 3.4.3 Model Learning. The goal of training is the negative log likelihood of the observed sequence of learner response. During training, both parameters, i.e., parameters in \(\text{GNN}_{\text{exer}}\) and \(\text{GNN}_{\text{sche}}\), parameters in sequence propagation \(\{W_1, b_1, W_2, b_2\}\) are updated. The loss for a learner is shown in Eq. 15. Specifically, the loss for a response log is defined as the cross entropy between the real answer \(r_t\) at time \(t\) and predicted score \(\hat{r}_t\) on exercise. The objective function is minimized using the Adam optimization [19]. More implementation details will be introduced in the experiments section.

\[
\text{loss} = - \sum_{t} (r_t \log \hat{r}_t + (1 - r_t) \log (1 - \hat{r}_t)).
\]

### 3.5 Prediction Output of HGKT

After modeling the exercising process of each learner from step 1 to \(t\), we now introduce our strategies for predicting his performance on the next exercise \(e_{t+1}\). Moreover, to diagnose a learner’s learning process in detail, we introduce a K&S diagnosis matrix to fully trace the mastery of both knowledge and problem schema dynamically.
3.5.1 K&S Diagnosis Matrix. Unlike the traditional knowledge tracing method, the input for HGKT in testing stage is the knowledge and problem schema of an exercise. Thus, we could trace the transition mastery of knowledge, problem schema, or their combination in HGKT. Specifically, at each time step $t$, we can predict the performance $r_j$ for each combination of $(k_i, s_j)$ $(k_i \in K, s_j \in S)$. Thus, we could use those results to generate a matrix $R_k^s$ called K&S diagnosis matrix whose vertical axis represents different knowledge and horizontal axis represents different problem schema. Next, the knowledge mastery $R_k^t$ at time $t$ is calculated by the weighted aggregation of mastery of each problem schema as shown in Eq. 17. Here $q_{i,j}$ denotes the number of exercises containing knowledge $k_i$ and problem schema $s_j$. Similarly, we could calculate the mastery of each problem schema $R_k^t$.

$$q_{i,j} = \left| \{ (k_i, s_j) \mid k_i \in K, s_j \in S \} \right|$$  \hspace{1cm} (16)

$$R_k^t = \frac{q_{i,j}}{\sum_j q_{i,j}}$$  \hspace{1cm} (17)

3.5.2 Interpretability of Problem Schema. The introduction of problem schema can effectively improve the effect of models in predicting learners’ performance. However, the interpretability of problem schema is unknown. Based on above challenges, we propose an unsupervised schema summarization algorithm. The core idea of this algorithm is to utilize TextRank [24] to extract meaningful condition descriptions and objective descriptions accordingly and use them to form a description for an exercise cluster based on certain rules. Table 1 shows an example of the summarization of a group of exercises. More details about the algorithm would be introduced in Appendix.

4 EXPERIMENTS

In this section, we evaluate the performance of the proposed framework HGKT from following three aspects: (1) The prediction performance of HGKT against baselines. (2) The analysis of effectiveness of hierarchical execution graph. (3) The effectiveness of sequence attention and schema attention mechanism used in HGKT.

4.1 Experiment Setup

4.1.1 Dataset and Preprocessing. Since there is no open dataset which could provide exercising records with text information, our experimental dataset is derived from a large real-world online education system. The Aixuexi online system could track exercising records of learners. Each record of a learner includes learner id, lecture id, class type, question information, answer time, learners’ answers and correction results. The question information includes the question id, question content, question type and the primary knowledge contained in the question. To avoid data distribution differences, we use the data obtained after 2018. In the data preprocessing stage, we group the exercising records with learner id and sort the records according to the response time. In total, we get 91,449,914 answer records from 132,179 learners. The detailed statistics of dataset are shown in Table 2.

4.1.2 Implementation Details. To set up the training of HGKT, we first need to use unsupervised data to generate the HEG. We leverage the exercising interaction data in the training set to construct the direct support graph and all exercise data involved in our experiment to construct the indirect support relations. Because the construction of the former one requires the learner’s exercising results while the latter one only needs the exercise information. Specifically, we utilize a public BERT-Base tool without any fine-tune to convert each exercise into an embedding vector with 768 dimensions, and the hierarchical clustering tool during the indirect support relations construction process. The framework of HGKT is joint training after the construction of HEG. During the testing stage, we use the same HEG to make predictions.

Table 1: An Example of Schema Summarization

| Exercise Cluster | Statistics |
|------------------|------------|
| Exercise 1 | If the ratio of lengths of triangle sides is 2 : 3 : 4 and its circumference is 18, the shortest side length is? |
| Exercise 2 | Given ratio of lengths of triangle sides is 2:4:4 and circumference is 20, what is the shortest side length? |
| Exercise 3 | If we know that ratio of lengths of three sides of a triangle is 3:4:5, and circumference of the triangle is 24, find the shortest side length? |

Table 2: The statistics of the Aixuexi dataset.

| Statistics                          | Value          |
|-------------------------------------|----------------|
| number of learners                  | 132,179        |
| number of response logs             | 9,144,914      |
| average of exercises correctly answered | 6,747,435     |
| number of different exercises       | 69             |
| number of knowledge related         | 11,410         |
| total number of knowledge           | 672            |

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1https://github.com/hanxiao/bert-as-service
2https://docs.scipy.org/doc/scipy/reference/cluster.hierarchy.html
optimizer with the learning rate 0.01 and set mini-batches as 32. We also use dropout [30] with probability 0.5 to prevent overfitting.

4.1.3 Comparison Baselines. To demonstrate the effectiveness of our framework, we compare HGKT with following state-of-art methods. Those methods are chosen from three aspects: (1) Traditional educational models: Bayesian Knowledge Tracing (BKT) (2) Deep learning models: Deep Knowledge Tracing (DKT), Dynamic Key-Value Memory Networks (DKVMN), Exercise-aware Knowledge Tracing (EKT), Graph Knowledge Tracing (GKT) (3) Variants of HGKT: HGKT-B, HGKT-S

- **BKT** is a traditional knowledge tracing model which is based on hidden Markov model. The knowledge states for each concept are a set of binary variables [5].
- **DKT** uses a recurrent neural network to model the learners’ exercising process [26]. It models question sequences as knowledge sequences. We follow the hyperparameters mentioned in [26], where the size of the hidden layer is 200 and we use a GRU for the RNN.
- **DKVMN** is a deep learning method using a key matrix to store knowledge representation and a value matrix for each learner to update the mastery for each concept [38]. In the experiment, we get best results when we set the size of the memory slot is 30, the size of embedding of key matrix is 50 and the size of embedding of value matrix is 200.
- **GKT** is a GNN-based knowledge tracing method which reformulate the knowledge tracing task as a time-series node-level classification problem in the GNN [25]. In experiments, we get the best results using the knowledge graph generated by transition-graph method.
- **EKTA** uses a bidirectional LSTM to encode exercise text and leverage an attention mechanism to enhance the prediction accuracy [16]. In experiments, we get best results when the shape of its hidden state is 672 and 30.
- **HGKT-B** is a variant of HGKT framework. Here, in the modeling process, we do not use problem schema embedding generated by HEG. Instead, we use BERT to encode text features.
- **HGKT-S** is another variant of HGKT framework. In the modeling process, we use the one-hot of problem schema generated by hierarchical clustering to replace embedding of problem schema generated by HEG.

4.1.4 Evaluation Setting. We first compare the overall performance of HGKT with baseline models. In the experiment, we randomly select 60%, 70%, 80%, 90% of the exercise records as training data, and the remaining as testing data. To qualify the effectiveness of the model, we compare the results of experiments from both regression and classification perspectives [10, 21, 34]. We use open-source platform to implement most comparison baseline models and searched the hyperparameters to find the best performance for each model. Our framework for HGKT is implemented by Torch on a Linux server with eight Intel Xeon Skylake 6133 (2.5 GHz) CPUs and four Tesla V100 GPUs. For all models, we test different parameters and experiments five times, respectively, and finally take the average of metrics as evaluation metrics.

4.2 Comparison

Fig. 5 shows the overall comparing results on this task. The results indicate that HGKT performs better than other baseline models. Besides, we draw several conclusions from the results: (1) All our proposed HGKT based models perform better than other baseline models. The result clearly demonstrates that HGKT framework could make full use of the information contained in exercising records and exercise texts, benefiting the prediction performance. (2) The result of HGKT-B outperforms EKTA, which shows that BERT could better extract the representation of text than a Bi-LSTM neural network. (3) The HGKT-S has better performance than HGKT-B, which proves that compared to directly using the text embedding of each exercise, summarizing a bunch of similar exercises with problem schemas would bring in less noise. (4) The HGKT performs better than HGKT-S, which reveals that the introduction of HEG for problem schema representation learning is also effective. The reason may be that in HEG, the representation of each exercise is propagated through a HGNN, so that problem schema embedding could aggregate the information of a certain kind of exercises.

4.3 Analysis

To understand the effects of the various components and hyperparameters in HGKT, we conduct ablation studies and parameter analysis on the Aixuexi dataset.

4.3.1 Ablation Study. We investigate the effects of different support relations and attention mechanisms in our model. The results of ablation study are shown in Table 4. To conduct a fair comparison experiment, we propose methods that have the same framework
Table 3: Model performance over different graph structure in HGKT. GS denotes the structure of graph. \(B - i\_T - j\) indicates \(i\) graph convolutional layers in \(GNN_{exer}\) and \(j\) graph convolutional layers in \(GNN_{sche}\).

| GS    | AUC (%) | ACC (%) | GS    | AUC (%) | ACC (%) |
|-------|---------|---------|-------|---------|---------|
| B-1\_T-1 | 81.4   | 81.5   | B-2\_T-2 | 82.5   | 82.4   |
| B-1\_T-2 | 82.3   | 82.4   | B-2\_T-3 | 81.7   | 81.6   |
| B-1\_T-3 | 81.8   | 81.9   | B-3\_T-1 | 82.8   | 82.6   |
| B-2\_T-1 | 82.4   | 82.4   | B-3\_T-2 | 82.4   | 82.4   |

Table 4: Ablation study of HGKT

| methods                  | attention | AUC (%) | ACC (%) |
|--------------------------|-----------|---------|---------|
| no support relations     | no        | 78.8    | 78.9    |
| only direct support      | no        | 79.3    | 79.5    |
| only indirect support    | no        | 80.4    | 80.2    |
| merge two support relations | no     | 80.3    | 80.5    |
| both support relations   | no        | 81.2    | 81.1    |
| (HGKT) both support relations | yes | 82.8    | 82.6    |

HGKT for other methods. A potential reason could be due to that the bayesian-based method could employ the information in exercise interactions history rather just use the exercise features like other methods. We also test impacts of different edge-to-node ratio for different Graph (as shown in Fig. 6(b)). The result shows the effect of graph convolution is the best when the ratio of edge-to-node is about 3-4.

The problem schema clustering level affects the number of nodes in an indirect support graph. As shown in Fig. 7(a), the number of problem schemas is more than 3430 when the threshold is 5, which results in intensive computational cost, and the generated problem schemas are not typical enough to represent a group of exercises. Thus, we test the overall performance of HGKT with thresholds between 5 to 20. Fig. 7(b) shows that the best AUC occurs when clustering threshold is 9 and the number of problem schema is 1136. Besides, from the overall trend of the curve, we could infer that there is a most suitable division of exercises for KT task, which means that using problem schema to represent a group of similar exercises is reasonable.

The numbers of graph convolutional layers used in the \(GNN_{exer}\) and \(GNN_{sche}\) are also tuneable hyperparameters. Thus, we investigate the effect of the numbers of graph convolutional layers on our model’s performance. Table 3 shows the result of experiments. We find that \(B - 3\_T - 1\) version achieves the best result for the KT task. The reason may be that the \(B - 3\_T - 1\) structure could achieve the optimal capacity of information aggregation.
4.3.3 Effectiveness of Attention. As we have clarified in Attention Mechanism section, the HGKT with attention could enhance the memory ability of the networks. The model could observe following information to predict the performance of a learner for the exercise: the performance of similar exercises in the past, learner’s current knowledge state, learner’s current focus. To confirm the effectiveness of those two attention mechanisms, we set up the following two experiments. Firstly, we compare the HGKT with the following three models: HGKT-noa, HGKT-sche and HGKT-seq. HGKT-noa is a no-attention version of HGKT. Similarly, HGKT-sche and HGKT-seq are variants of HGKT which only contain schema attention and sequence attention respectively. Fig. 8(a) shows the comparison results of them. From the figure, we could infer that HGKT outperforms other comparison methods. Besides, both HGKT-sche and HGKT-seq perform better than HGKT-noa, which proves that both attention mechanisms could contribute to the HGKT model. Moreover, we also conduct experiments to see the impacts of different window size of sequence attention. In the experiment, we set length of window sizes from 10 to 50 to see which size could better preserve the memory useful for KT. As Fig. 8(b) shows, the best parameter for window size is 20, which reveals that the results of nearly 20 exercises can best reflect their current learning status. This value provides a great reference for setting the number of exercises in learners’ proficiency test.

5 CASE STUDY

The introduction of problem schema and HEG can make diagnosis more accurate and improve the interpretability of diagnosis results. Moreover, the diagnosis results generated by HGKT could be easily applied to the following two applications: (1) Learner Diagnosis Report [27, 32] (2) Adaptive Question Recommendation [4, 23]. To make deep analysis about above statements, we visualize the predicted knowledge mastery of a learner during the exercising process in Fig. 9.

From the figure we could more intuitively understand the advantages of the HGKT compared to traditional knowledge tracing diagnosis (TKT). When \( t = 8 \), the learner did \( e_{12} \) wrong, the TKT diagnosis result shows that the color of the knowledge related to the \( k_7 \) (e.g., \( k_{29} \)) became darker. However, the reasons for the poor mastery of related knowledge remain unknown. In HGKT, it can be found that the transition of learners’ knowledge mastery is strongly correlated with the local HEG diagram in Fig. 9(b). Specifically, the learner’s mistakes in \( e_{12} \) led to the poor mastery of the knowledge \( k_7, k_{23} \) and \( k_{30} \), which demonstrates that incorporating the direct and indirect support graph do affect the results of the diagnosis. Moreover, the HEG information could also explain the change of knowledge mastery in a way, so the learner diagnosis report could be more reliable. The QA part in the figure shows the improvement of HGKT’s interpretability by comparing the explanations of HGKT and TKT for several questions. By K&S diagnosis matrix method, learners can not only know the underlying reasons for not mastering certain knowledge (e.g., \( k_1, k_2 \)), but also enhance the level of mastery of the knowledge through more targeted and accurate training advice, which reveals that HGKT framework can better be applied to adaptive question recommendation scenario.

6 RELATED WORK

6.0.1 Graph Neural Networks. Graph neural networks (GNN) that can model graph structured data such as social network data [13, 20, 31] or knowledge graph data [8, 29, 39] has attracted great attention recently. GNN learns node representation by transforming, propagating and aggregating node features and has been proved effective for most graph-structured data [12, 13]. However, the ability of current GNN architectures is limited since their graphs are inherently flat as they only propagate information across the edges of the graph and could not infer and aggregate the information in a hierarchical way [28, 35]. DiffPool [36] is a differentiable graph pooling method, which is first proposed to learn interpretable hierarchical representations of graph. However, it needs to train a separate cluster assignment matrix which is computation expensive. Thus, we present a variant of DiffPool to learn node representations in our paper.

7 CONCLUSION

In this article, we prove the importance of hierarchical relations between exercises for KT tasks. To make full use of text information in exercises, we propose a novel knowledge tracing framework...
HGKT which leverages the advantages of hierarchical exercise graph and sequence model with attention to enhance the ability of knowledge tracing. Besides, we come up with the concept of a K&S diagnosis matrix that could trace the mastery of both knowledge and problem schema, which has been proved more effective and useful in industrial applications than the traditional knowledge tracing methods. Moreover, we construct a large-scale knowledge tracing dataset containing exercise text information and conduct extensive experiments to show the effectiveness and interpretability of our proposed models.

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