Critical Crashes?

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Abstract

In this short note we discuss recent attempts to describe pre-crash market dynamics with analogies from theory of critical phenomena.

Stochastic dynamics is a commonly used approach to describe the time evolution of financial markets. The large number of agents, their complex mutual influence and obscure decision making - all of these leave the probabilistic picture as the only viable way to describe and, as a test, to predict financial markets. In the case of a stable market such a probabilistic description is on solid ground. Indeed, for the stable market the difference between the numbers of “buy” orders and “sell” orders is small and fluctuates around zero which results in corresponding fluctuations of the price of the traded asset. It means that peculiarities of individual decisions are mutually offset, the characteristic fluctuation time is less than the characteristic time of changes of external (fundamental) conditions and number of observations allows meaningful use of statistical procedures such as averaging. However the situation changes dramatically when the overwhelming number of market participants share the same view on the future and make the same orders (for example, order to sell) thus creating a market instability. The participants start to behave “coherently” and to create a realistic picture of such market one needs to model the decision making leading to such behaviour. The modelling of this decision making is the principle problem since the corresponding price movements are simple consequence of the demand-supply mechanism of price fixing. The “coherence” of such events and their extremely low frequency make a statistical description dubious and the whole problem even more challenging. Taking into account the immense importance of crashes as well as “bubbles” for financial systems and the world economy as a whole it is not surprising that the problem of description and prediction of the events attract a lot of efforts and attention. In this note we concentrate on the recently suggested analogy between critical phenomena and the financial crashes.

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The story started with empirical observations of so-called log-periodical oscillations prior the crashes which have been reported in physical literature by several authors [1, 2, 3, 4]. The empirics was questioned later in Ref. [5] where the stochastic nature of crashes was advocated (see also [6] on stochastic model of crashes). We carried out an extensive search for the patterns for Russian securities market and some other emerging markets and did not find convincing manifestations. Though this itself does not undermine the empirics and the developed theory suggested for well developed liquid markets, it cast a shadow on implied universality of the effect. Even with the most successful examples demonstrated in Ref. [7] (see figs. 1-3 therein) the log-periodic oscillations themselves generate such timing of the “sell” signals that would result in losses because of time lag (generally a few months). In fact, as it can be seen from the figures, the disciplined following of this strategy would result in up to 50% losses. This prompted a statistical (probabilistic) modification of the “sell” signals [7] which however significantly reduced the practical relevance of the method because of the lack of strict rules which are so important in technical trading. One can argue that similar or even better accuracy can be achieved by an investor with a common sense who observes the growing market “bubble”. It does not mean that the oscillation pattern is not worth to look at but rather that the real life of financial market once again is more complicated and we still have a long way to go to understand it.

It would be unfair to say that people never observed or used such patterns. The theory of cycles constitutes a valuable part of the Technical Analysis which aims to predict future market movements based on the price history. All reported technical rules are never 100% accurate and hence they are probabilistic in their nature. The belief is that they are right more often than wrong. Before using trading strategies they are to be thoroughly back tested which does not guarantee however the future performance. The environment changes all the time and this can kill the profitability of a particular strategy. Nevertheless practitioners tend to use technical predictions much more often than the recipes of canonic financial economics. For many years a huge number of people have been involved in pattern studies and many text books are available to cover the subject. No surprise then that it is possible to find log-periodic oscillations there! The chartists use another language and call it Log spirals to encapsulate both cyclic nature and the scaling property (see [8] for an introduction and further references on the subject) but the phenomena under study is essentially the same. There exists a speculation that the Log spirals can be applied not only to very large market movements such as crashes but also to less significant market movements. A recent paper [9] reported in physical literature a similar observation.

Now let us turn to the theory of the log-periodic oscillations suggested in Refs. [10, 11] (for recent developments see [12]). It was postulated that the microscopic origin of the phenomena is in mutual imitations by traders. This model and the market fitting show that the average number of interactions $\delta - 1$ felt by a typical trader is order of $2 \sim 3$ but the traders are so mixed up that ”long” connections can appear. This indeed resembles the well known mechanism of Long Range Order formation in spin systems which is the main attraction as well as the main danger for the application of the model to financial markets. The model is based on the assumption that the trader-trader imitation is the principle factor which governs the dynamics. Though the imitation is definitely an
important factor it is hard to believe that it is the principle one. Indeed, traders are clustered and it is not very realistic to imagine that the clusters are so intertwined. The market participants have very different time horizons (like minutes for speculators and years for fund managers) and the assumption that they watch each other for years prior the crashes does not seem to be quite motivated. At the same time there are several sources of information which influence most of the traders. Reuters news for institutional traders, the Financial Times or BullMarket reports for small investors play major role to form their expectations. The “coherence” might be a result of a common factor action rather than a result of a direct internal interaction such as the imitation of 2-3 fellow traders. It does not mean however that the news impact is something external since the news can reflect the market anticipations and, hence, represents some indirect impact. In any case it is more realistic to expect that common information resources will be more relevant to define investment decisions and create the “coherence” than the obscure network of fellow traders.

The microscopic theory gave the probability per unit time that the crash will happen in the next moment if it has not happened yet (hazard rate) $h(t)$ as defined by the "mean field" equation:

$$\frac{dh}{dt} = \text{const} \cdot h^\delta .$$

Furthermore, it was assumed that the corresponding price $p$ follows the stochastic process

$$dp = \mu(t)p(t) - k[p(t) - p_1]dj$$

where $k, p_1$ are some constants and $j$ is a jump process whose value is zero prior the crash and one afterwards and which is characterized by the hazard rate. To obtain an equation for the return $\mu(t)$ authors postulate the following “fair” game (or martingale) hypothesis:

$$E_t(p(t')) = p(t), \quad t < t'.$$

Here $E_t$ denotes an expectation conditional on information revealed up to time $t$. This is the last equation which we discuss now and show that it cannot be used in the situation of a financial “bubble” prior to the crash.

The martingale property for the price is associated with the market efficiency. It would be wrong to say however they are equivalent. Let us remind that the Efficient Market Hypothesis is defined as a superposition of the Rational Expectation Hypothesis and Orthogonality property [12]. The Rational Expectation Hypothesis states that:

1. Agents are rational, i.e. use any possibility to get more than less if the possibility occurs.

2. There exists a perfect pricing model and all market participants know this model.

3. Agents have all relevant information to incorporate into the model.

Using the model and the information the rational agents form an expectation value of the future return $E_t R_{t+1}$. This expectation value can differ from the actual value of the return $R_{t+1}$ on a estimation error $\epsilon_t = R_{t+1} - E_t R_{t+1}$. The Orthogonality property implies that:
1. \( \epsilon_{t+1} \) is a random variable which appears due to coming of new information.

2. \( \epsilon_{t+1} \) is independent on full information set \( \Omega_t \) at time \( t \) and
\[
E_t(\epsilon_{t+1}|\Omega_t) = 0.
\]

The martingale property for the price then appears from the Orthogonality property under the additional assumption that the perfect pricing model gives zero return as a best prediction:
\[
0 = E_t(\epsilon_{t+1}|\Omega_t) = E_t(R_{t+1}|\Omega_t) = E_t(p_{t+1} - p(t)|\Omega_t) \iff E_t(p_{t+1}|\Omega_t) = p(t). \quad (1)
\]

This means that the model of crashes we discussed above assumes a zero return as a best prediction for the market. No need to say that this is not what one expects from a perfect model of market “bubble”! Buying shares traders expect the price to rise and it is reflected (or caused) by their prediction model. They support the “bubble” and the “bubble” support them! This is not a coin tossing as Eqn (1) suggests. In this situation the whole market on average makes money rather than redistributes them as it is in fair game. These expectations have to be incorporated in the prediction model and indeed there exist several models of so-called rational “bubbles” which are models to describe “bubbles” under of the Efficient Market Hypothesis. We address to the textbook [12] (Chapter 7) for a review or to the original papers [13, 14] for further reading.

It is possible to look at the problem with Eqn (1) from other side. The equation postulates that the average price tomorrow is equal to the price today, i.e. the average rate of return is zero. It is well-known that, in general, investors are risk averse and require some additional risk premium to enter a bet. Otherwise they simply stay out and do not take the risk. The unrealistic feature of Eqn (1) is that it takes the risk premium equal to zero. One can improve the situation substituting Eqn (1) by the following equation:
\[
\nu(t)E_t(p_{t+1}|\Omega_t) = p(t). \quad (2)
\]
where \( \nu(t) \) is an appropriate discount factor which reflects the risk premium required by market participants. A common and often used approximation, \( \nu(t) \approx \text{const} \), does not work well for the problem of “bubbles” and crashes (see also discussion in [13]). This is due to the fact that the risk premium is defined by risk perceptions which are constantly changing in the course of the “bubble” according to a prediction model adopted by investors. Furthermore, to model the discount factor dynamics one needs once again a prediction model. At this point we return to the previous consideration.

It is an open question how to construct a perfect model or how to model traders decisions. Computer agents simulations give one of possible approaches. This is a new developing area and the last decade many research papers addressed the question [16]. It is difficult to say how accurate is the modelling and, hence, how reliable are the models. They mostly consider a market as a mixture of technical and fundamental traders and prescribe the chartists a set of technical trading rules. It is important to emphasize that there is no contradiction in using the technical analysis as a proxy of the perfect model under EMH and the Market Efficiency itself (see [17] for more
discussion). All relevant effects such as risk aversion, existence of common news source, the mentioned earlier common sense and imitations, even the log-periodic oscillations themselves can be included in the model.

In conclusion, the discussed above log-periodic oscillations or Log spirals can be considered as one of technical instruments to predict market movements. As any market phenomena they are worth to study both empirically and theoretically. To make the method more practically applicable it is important to combine it with other technical methods to validate the trading signals which is a common practice in Technical trading since the performance of the method alone does not produce convincing results. Finding a realistic theoretical model (not necessarily triggered by physical analogies) is definitely a difficult and challenging problem where the only the first steps have been made.

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