Robust Cognitive Beamforming With Partial Channel State Information

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Suggested Editorial Areas:
Cognitive radio, MISO, Partial channel state information, Power allocation, Wireless networks.

*The work is supported by the National University of Singapore (NUS) under start-up grants R-263-000-314-101 and R-263-000-314-112, by a NUS Research Scholarship, and by the U.S. National Science Foundation under Grants ANI-03-38807 and CNS-06-25637. This work was done while Y. Xin was visiting Princeton University.
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Abstract

A spectrum sharing based cognitive radio (CR) communication system, which consists of a secondary user (SU) having multiple transmit antennas and a single receive antenna and a primary user (PU) having a single receive antenna, is considered. The channel state information (CSI) on the link of the SU is assumed to be perfectly known at the SU transmitter (SU-Tx). However, due to loose cooperation between the SU and the PU, only the channel mean and/or covariance (partial CSI) of the link between the SU-Tx and the PU is available at the SU-Tx. With the partial CSI and a prescribed transmit power constraint, our design objective is to determine the transmit signal covariance matrix that maximizes the rate of the SU while keeping the interference power to the PU below a threshold with high probability. This problem, termed the robust cognitive beamforming problem, can be naturally formulated as a semi-infinite programming (SIP) problem with infinitely many constraints. The SIP problem is transformed into a finite constrained optimization problem and yields to a simple analytical solution, which is developed from a geometric perspective. As an alternative, a much more complex approach based on an interior point algorithm is provided. Simulation examples are presented to validate the effectiveness of the proposed algorithms.

Keywords: Cognitive radio, interference constraint, multiple-input single-output (MISO), partial channel state information, power allocation, rate maximization.

I. INTRODUCTION

One of the fundamental challenges faced by the wireless communication industry is how to meet rapidly growing demands for wireless services and applications with limited radio spectrum. Cognitive radio (CR) technology has been proposed as a promising solution to tackle such a challenge [1]–[8]. In a spectrum sharing based CR network, the secondary users (SUs) are allowed to coexist with the primary user (PU), subject to the constraint, namely the interference constraint, that the interference power from the SU to the PU is less than an acceptable value. Evidently, the purpose of the imposed interference constraint is to ensure that the quality of service (QoS) of the PU is not degraded due to the SUs. To be aware of whether the interference constraint is satisfied, the SUs must obtain knowledge of the radio environment cognitively.

In this paper, we consider a spectrum sharing based CR communication scenario, in which the SU uses a multiple-input single-output (MISO) channel and the primary user (PU) has one receive antenna.
We assume that the channel state information (CSI) about the SU link is perfectly known at the SU transmitter (SU-Tx). However, owing to loose cooperation between the SU and the PU, only the mean or/and covariance of the channel between the SU-Tx and the PU is available at the SU-Tx. With this CSI, our design objective is, for a given transmit power constraint, to determine the transmit signal covariance matrix that maximizes the rate of the SU while keeping the interference power to the PU below a threshold with high probability. We term this design problem the robust cognitive beamforming design problem.

In non-CR settings, the study of multiple antenna systems with partial CSI has received considerable attention in the past [9], [10]. Specifically, the paper [10] considers the case in which the receiver has perfect CSI but the transmitter has only partial CSI (mean feedback or covariance feedback). It was proved in [10] that the optimal transmission directions are the same as those of the eigenvectors of the covariance matrix. However, the optimal power allocation solution was not given in an analytical form. A universal optimality condition for beamforming was explored in [11], and quantized feedback was studied in [12].

In CR settings, power allocation strategies have been developed for multiple access channels [13] and for point-to-point multiple input multiple output (MIMO) channels [14]. Particularly, the solution developed in [14] can be viewed as cognitive beamforming since the SU-Tx forms its main beam direction with awareness of its interference to the PU. However, the papers [13] and [14] assume that perfect CSI about the link from the SU-Tx to the PU is available at the SU-Tx. Due to loose cooperation between the SU and the PU, it could be difficult or even infeasible for the SU-Tx to acquire accurate CSI between the SU-Tx to the PU. We naturally formulate the robust cognitive beamforming design problem as a semi-infinite programming (SIP) problem, which is difficult to solve directly. We therefore first transform the SIP problem into a finite constrained optimization problem, and further obtain its analytical solution by using a simple geometric approach. We also propose an alternative approach that is much more complex than the analytical approach. In the alternative approach, the original SIP problem is first transformed into a second order cone programming (SOCP) problem and a standard interior point algorithm is applied to solve the SOCP problem.

The rest of this paper is organized as follows. Section II describes the SU MISO communication
system model, and the problem formulation of the robust cognitive beamforming design. Section III presents several important lemmas which are used in the sequel to develop the algorithms. Two different algorithms, the analytical solution and the SOCP based solution, are developed in Section IV and Section V, respectively. Section VI presents simulation examples, and finally, Section VII concludes the paper.

The following notation is used in this paper. Boldface upper and lower case letters are used to denote matrices and vectors, respectively, $(\cdot)^H$ and $(\cdot)^T$ denote the conjugate transpose and transpose, respectively, $I$ denotes an identity matrix, $\text{tr}(\cdot)$ denotes the trace operation, and $\text{Rank}(A)$ denotes the rank of the matrix $A$.

II. SIGNAL MODEL AND PROBLEM FORMULATION

With reference to Fig. 1, consider a point-to-point SU MISO communication system, where the SU has $N$ transmit antennas and a single receive antenna. The signal model of the SU can be represented as $y = h_s^H x + n$, where $y$ and $x$ are the received and transmitted signals respectively, $h_s$ denotes the $N \times 1$ channel response from the SU-Tx to the SU-Rx, and $n$ is additive independent and identically distributed (i.i.d.) Gaussian noise with zero mean and unit variance. Suppose that the PU has one receive antenna. The channel response from the SU-Tx to the PU is denoted by an $N \times 1$ vector $h$. Further, assume that the SU-Tx has perfect CSI for its own link, i.e., it knows $h_s$. However, due to the loose cooperation between the SU and the PU, only partial CSI about $h$ is assumed to be available at the SU-Tx. It is also assumed that the SU-Tx knows that the channel $h$ is distributed according to a complex Gaussian distribution $\mathcal{CN}(h_0, R)$, where $h_0$ and $R$ are the mean and covariance of $h$, respectively. In previous work [10], [15]–[17], partial CSI has been considered in two extreme cases in a non-CR setting. One is the mean feedback case, $R = \sigma^2 I$, where $\sigma^2$ can be viewed as the variance of the estimation error; and the other is the covariance feedback case, where $h_0$ is a zero vector. In this paper, we study the case where the SU-Tx knows the mean and/or covariance of $h$ in a CR setting.

The objective of this paper is to determine the optimal transmit signal covariance matrix such that the information rate of the SU link is maximized while the QoS of the PU is guaranteed under a robust design scenario, i.e., the instantaneous interference power for the PU should remain below a given
threshold with high probability. Mathematically, the problem is formulated as follows:

**Robust design problem (P1):**

\[
\max_{S \geq 0} \log(1 + h_s^H Sh_s) \\
\text{subject to: } \text{tr}(S) \leq \bar{P}, \text{ and } h_s^H Sh_s \leq P_t \text{ for } (h - h_0)^H R^{-1}(h - h_0) \leq c,
\]

where \( S \) is the transmit signal covariance matrix, \( \bar{P} \) is the transmit power threshold, \( P_t \) is the interference threshold, and \( c \) is a positive constant. The parameter \( c \) defines the probability with which the interference constraint is satisfied; the larger the value of \( c \), the lower the probability that the interference constraint is violated. Thus, problem \( P1 \) can be treated as a robust design problem. Since the interference constraint contains a compact set with infinite cardinality, the problem is an SIP problem [18]. Generally speaking, there is no universal algorithm for solving such problems. The basic idea to approach an SIP problem is to transform the problem into an equivalent finite constraint problem. The key challenge, however, is to determine a finite constraint set that is equivalent to the original infinitely many constraints. To do so, we need to study the properties of the optimal solution.

**Remark 1:** An important observation is that the objective function in problem \( P1 \) remains invariant when \( h_s \) undergoes an arbitrary phase rotation. Without loss of generality, we assume, in the sequel, that \( h_s \) and \( h_0 \) have the same phase, i.e., \( \text{Im}\{h_s^H h_0\} = 0 \).

**III. Properties of the Optimal Solution**

The maximization problem \( P1 \) is a convex optimization problem, and thus has a unique optimal solution. The following lemma presents a key property of the optimal solution of problem \( P1 \) (see Appendix A for the proof).

**Lemma 1:** The optimal covariance matrix \( S \) for problem \( P1 \) is a rank-1 matrix.

**Remark 2:** Lemma 1 indicates that beamforming is the optimal transmission strategy for problem \( P1 \), and the optimal transmit covariance matrix can be expressed as \( S_{opt} = p_{opt} v_{opt}^H v_{opt}^H \), where \( p_{opt} \) is the optimal transmit power and \( v_{opt} \) is the optimal beamforming vector with \( \|v_{opt}\| = 1 \). Therefore, the ultimate objective of problem \( P1 \) is to determine \( p_{opt} \) and \( v_{opt} \).

Relying on Lemma 1, a necessary and sufficient condition for the optimal solution of problem \( P1 \) is presented as follows (refer to Appendix C for the proof).
Lemma 2: A necessary and sufficient condition for $S_{\text{opt}}$ to be the globally optimal solution of problem $P1$ is that there exists an $h_{\text{opt}}$ such that

$$S_{\text{opt}} = \arg \max_{S,p} \log(1 + h_s^H S h_s), \text{ subject to: } \text{tr}(S) \leq p, \ 0 \leq p \leq \bar{P}, \ h_{\text{opt}}^H S h_{\text{opt}} \leq P_t, \ (2)$$

where

$$h_{\text{opt}} = \arg \max_{h} h^H S_{\text{opt}} h, \text{ for } (h - h_0)^H R^{-1} (h - h_0) \leq c. \ (3)$$

Remark 3: The vector $h_{\text{opt}}$ is a key element for all $h : (h - h_0)^H R^{-1} (h - h_0) \leq c$, in the sense that, for the optimal solution, the constraint $h_{\text{opt}}^H S h_{\text{opt}} \leq P_t$ dominates the whole interference constraints, i.e., all the other interference constraints are inactive. Thus, if we can determine $h_{\text{opt}}$, the SIP problem $P1$ is transformed into a finite constraint problem (2). In [14] a similar MISO CR optimization problem is considered, in which the CSI on the link of the SU and the link between SU-Tx and PU are perfectly known at the SU-Tx. Hence, the problem in [14] has the same form as the problem (2). However, unlike the problem in [14], $h_{\text{opt}}$ in (2) is a parameter that is not known. If the key parameter $h_{\text{opt}}$ is determined, the solution of problem $P1$ can be obtained by solving (2).

In the following lemma (see Appendix D for the proof), the optimal beamforming vector $v_{\text{opt}}$ is shown to lie in a two-dimensional (2-D) space spanned by $h_0$ and the projection of $h_s$ into the null space of $h_0$. Define $\hat{h} = h_0/\|h_0\|$ and $\hat{h}_\perp = h_\perp/\|h_\perp\|$, where $h_\perp = h_s - (\hat{h}^H h_s) \hat{h}$, and we have $h_s = a_{h_s} \hat{h} + b_{h_s} \hat{h}_\perp$ with $a_{h_s}, b_{h_s} \in \mathbb{R}$.

Lemma 3: The optimal beamforming vector $v_{\text{opt}}$ is of the form $a_v \hat{h} + b_v \hat{h}_\perp$ with $a_v, b_v \in \mathbb{R}$.

Remark 4: According to Lemma 3, we can search for the optimal beamforming vector $v_{\text{opt}}$ on the 2-D space spanned by $\hat{h}$ and $\hat{h}_\perp$ which simplifies the search process significantly. The optimal $v_{\text{opt}}$ found on this 2-D space, is also the globally optimal solution of the original problem $P1$. As depicted in Fig. 2, problem $P1$ is transformed into the problem of determining the beamforming vector $v_{\text{opt}}$ and the corresponding power $p_{\text{opt}}$ in the 2-D space. Combining Lemma 2 and Lemma 3, it is easy to conclude that $h_{\text{opt}}$ lies in the space spanned by $\hat{h}$ and $\hat{h}_\perp$. 
IV. AN ANALYTICAL SOLUTION

In this section, we present a geometric approach to problem $P_1$. We begin by studying a special case, the mean feedback case, i.e., $R = \sigma^2 I$, for which a closed-form solution is developed in Subsection IV-A. Based on the closed-form solution derived in the mean feedback case, the analytical solution to problem $P_1$ with a general form of a covariance matrix $R$ is presented in Subsection IV-B.

A. Mean Feedback Case

Relying on the observation in Lemma 1 and the definition of the mean feedback, the special case of problem $P_1$ with mean feedback can be written as follows.

**Mean feedback problem (P2):**

$$\begin{align*}
\max_{p \geq 0, \|v\|=0} & \log(1 + ph_s^H vv^H h_s) \\
\text{subject to :} & \quad p \leq \bar{P}, \quad ph_s^H vv^H h \leq P_t, \quad \text{for} \quad \|h - h_0\|^2 \leq c\sigma^2.
\end{align*}$$

(4)

Problem P2 has two constraints, i.e., the transmit power constraint and the interference constraint. Similar to the idea in [13], the two-constraint problem is decoupled into two single-constraint subproblems:

**Subproblem 1 (SP1):**

$$\begin{align*}
\max_{p \geq 0, \|v\|=0} & \log(1 + ph_s^H vv^H h_s) \\
\text{subject to :} & \quad p \leq \bar{P}.
\end{align*}$$

(5)

**Subproblem 2 (SP2):**

$$\begin{align*}
\max_{p \geq 0, \|v\|=0} & \log(1 + ph_s^H vv^H h_s) \\
\text{subject to :} & \quad ph_s^H vv^H h \leq P_t, \quad \text{for} \quad \|h - h_0\|^2 \leq c\sigma^2.
\end{align*}$$

(7)

In the sequel, we first present the algorithm to obtain the optimal power $p_{opt}$ and the optimal beamforming vector $v_{opt}$ for both subproblems in subsection IV-A.1, and then describe the relationship between the subproblems and the problem P2 in subsection IV-A.2.

1) Solution to subproblems: For SP1, the optimal power is constrained by the transmit power constraint, and thus $p_{opt} = \bar{P}$. Moreover, since there does not exist any constraints on the beamforming direction, it is obvious that the optimal beamforming direction is equal to $h_s$, i.e., $v_{opt} = h_s / \|h_s\|$. Thus, the optimal covariance matrix $S_{opt}$ for SP1 is $\bar{P} h_s h_s^H / \|h_s\|^2$. In the following, we focus on the solution to SP2.
SP2 has infinitely many interference constraints, and thus is an SIP problem too. Follow a similar line of thinking as in Lemma 2, SP2 can be transformed into an equivalent problem which has finite constraints (refer to Appendix E for the proof) as follows.

**Lemma 4:** SP2 and the following problem:

\[
\max_{p \geq 0, \|v\|=1} \log(1 + p h_s^H v v^H h_s), \text{ subject to: } p h_{opt}^H v v^H h_{opt} \leq P_t,
\]

(9)

where \( h_{opt} = h_0 + \sqrt{c} \sigma v \), have the same optimal solution.

According to Lemma 4, problem (9) has the same optimal solution as SP2. Moreover, according to Lemma 3, the optimal solution \( v \) of problem (9) lies in the plane spanned by \( \hat{h} \) and \( \hat{h}_\perp \). In the sequel, we will search for the optimal solution geometrically, i.e., by restricting our search space to a 2-D space. As shown in Fig. 3, assume that the angle between \( v \) and \( h_0 \) is \( \beta \), and the angle between \( h_s \) and \( h_0 \) is \( \alpha \). Since \( v \) lies in a 2-D space, \( v \) can be uniquely identified by the angle \( \beta \). Hence, we need only to search for the optimal angle \( \beta_{opt} \). By exploiting the relationship between \( p, v, \) and \( \beta \), the two-variable optimization problem (9) can be further transformed into an optimization problem with a single variable \( \beta \), which is readily solved.

By observing Fig. 3, the angle between \( h_s \) and \( v \) is \( \beta - \alpha \), and hence the objective function of (9) can be expressed as

\[
\max \log(1 + p h_s^H v v^H h_s) = \max_{\beta} \log \left(1 + p \|h_s\|^2 \cos^2(\beta - \alpha) \right).
\]

(10)

Thus, the maximum rate is achieved if

\[
f(\beta) := p \|h_s\|^2 \cos^2(\beta - \alpha)
\]

(11)

is maximized.

Moreover, it can be proved by contradiction that the interference constraint is satisfied with equality, i.e., \( h_{opt}^H S h_{opt} = P_t \). Thus, we have

\[
p h_{opt}^H v v^H h_{opt} = p (h_0 + \sqrt{c} \sigma v)^H v v^H (h_0 + \sqrt{c} \sigma v) = p \|h_0\| \cos^2(\beta + \sqrt{c} \sigma) = P_t.
\]

(12)

\(^1\)We assume that \( 0 \leq \alpha \leq \pi/2 \), since if \( \alpha \geq \pi/2 \), we can always replace \( h_s \) by \( -h_s \) without affecting the final result, and the angle between \(-h_s \) and \( h_0 \) is less than \( \pi/2 \).
Hence, the interference constraint is transformed into

\[ p = \frac{P_t}{(\|h_0\| \cos \beta + \sqrt{c\sigma})^2}. \tag{13} \]

By substituting (13) into (11), we have

\[ f(\beta) = p\|h_s\|^2 \cos^2(\beta - \alpha) = \frac{\|h_s\|^2 P_t \cos^2(\beta - \alpha)}{(\|h_0\| \cos(\beta) + \sqrt{c\sigma})^2}. \tag{14} \]

Thus, by computing the maximum of \( f(\beta) \), the optimal \( \beta_{\text{opt}} \) is obtained, i.e.,

\[ \beta_{\text{opt}} = \arg \max f(\beta) = \arg \max \frac{\|h_s\|^2 P_t \cos^2(\beta - \alpha)}{(\|h_0\| \cos(\beta) + \sqrt{c\sigma})^2}. \tag{15} \]

The problem of (15) is a single variable optimization problem, and it is easy to observe that the feasible region for \( \beta \) is \([\alpha, \pi/2]\). According to the sufficient and necessary condition for the optimal solution of an optimization problem, \( \beta_{\text{opt}} \) lies either on the border of the region (\( \alpha \) or \( \pi/2 \)) or on the point which satisfies \( \partial f(\beta)/\partial \beta = 0 \). Since

\[ \frac{\partial f(\beta)}{\partial \beta} = \frac{2\|h_s\|^2 P_t \cos(\beta - \alpha) \left( \sin \alpha - \sin(\beta - \alpha) \sqrt{c\sigma}/\|h_0\| \right)}{\|h_0\|^2 \left( \cos \beta + \sqrt{c\sigma}/\|h_0\| \right)^3}, \tag{16} \]

by solving the equation

\[ \sin \alpha = \sin(\beta - \alpha) \sqrt{c\sigma}/\|h_0\|, \text{ where } \beta \in [\alpha, \pi/2], \tag{17} \]

we can obtain a locally optimal solution \( \beta_1 = \sin^{-1} \left( \frac{\|h_0\| \sin \alpha}{\sqrt{c\sigma}} \right) + \alpha \) for (17). In the case when \( \|h_0\| \sin \alpha /\sqrt{c\sigma} > 1 \), \( f(\beta) \) is a non-decreasing function. Hence, the optimal solution must lie at \( \pi/2 \), and we define \( f(\beta_1) = -\infty \) for this case. Therefore, the globally optimal solution is

\[ \beta_{\text{opt}} = \arg \max(f(\alpha), f(\pi/2), f(\beta_1)). \tag{18} \]

Relying on \( \beta_{\text{opt}} \), the optimal power \( p_{\text{opt}} \) can be obtained by substituting \( \beta_{\text{opt}} \) into (13), and according to the definition of \( \beta \) and Lemma 3, we have

\[ v_{\text{opt}} = a_v \hat{h} + b_v \hat{h}_\perp, \tag{19} \]

where \( a_v = \cos(\beta_{\text{opt}}) \) and \( b_v = \sin(\beta_{\text{opt}}) \). In summary, SP2 can be solved by Algorithm 1 which is described in Table I.
2) Optimal solution to problem P2: In the preceding subsection, we presented the optimal solutions for the two subproblems. We now turn our attention to the relationship between the problem P2 and subproblems, and present an overall algorithm to solve problem P2. Since the convex optimization problem P2 has two constraints, according to the activeness of the constraints, the optimal solution can be classified into three cases: 1) only the transmit power constraint is active; 2) only the interference constraint is active; and 3) both constraints are active. Relying on this classification, the relationship between the solutions of problem P2 and the two subproblems is described as follows (refer to Appendix F for the proof).

Lemma 5: If the optimal solution $S_1$ of SP1 satisfies the constraint of SP2, then $S_1$ is the optimal solution of problem P2. If the optimal solution $S_2$ of SP2 satisfies the constraint of SP1, then $S_2$ is the optimal solution of problem P2. Otherwise, the optimal solution of problem P2 simultaneously satisfies the transmit power constraint and $h_{opt}^H Sh_{opt} \leq P_t$ with equality.

Remark 5: To apply Lemma 5, we need to test whether $S_1$ and $S_2$ satisfy both constraints. The condition that $S_1$ satisfies the interference constraint is

$$P_{int} \leq P_t,$$

where $P_{int}$ can be obtained by the method discussed in Appendix B. The condition that $S_2$ satisfies the transmit power constraint is $\text{tr}(S_2) \leq P.$

We next discuss the method for finding the solution in the case where neither $S_1$ nor $S_2$ is the optimal solution of problem P2. Similarly to the method in the preceding subsection, we solve this case from a geometric perspective. According to Lemma 5, in the case in which neither $S_1$ nor $S_2$ is the feasible solution, the optimal covariance $S_{opt}$ must satisfy both constraints with equality, i.e.,

$$p_{opt} = \bar{P}, \text{ and } p_{opt} h_{opt}^H v_{opt} v_{opt}^H h_{opt} = P_t.$$

Combining these two equalities, we have

$$\bar{P}(\|h_0\| \cos(\beta) + \sqrt{c} \sigma)^2 = P_t.$$

Thus,

$$\beta_{opt} = \arccos \left( \frac{\sqrt{P_t/P} - \sqrt{c} \sigma}{\|h_0\|} \right).$$
Based on $\beta_{\text{opt}}$, we can obtain $v_{\text{opt}}$ through (19). We summarize the procedure to solve the case where both constraints are active for problem $P2$ as Algorithm 2 in Table II. Furthermore, we are now ready to present the complete algorithm to solve problem $P2$ as Algorithm 3 in Table III.

In Algorithm 3, we obtain the optimal solutions to $SP1$ and $SP2$ and the optimal solution to the case where both constraints are active separately. According to Lemma 5, the final solution obtained in Algorithm 3 is thus the optimal solution of problem $P2$.

**Proposition 1:** Algorithm 3 obtains the optimal solution of problem $P2$.

**B. The Analytical Method for Problem $P1$**

In the preceding subsection, the mean feedback problem $P2$ is solved via a closed-form algorithm. Unlike problem $P2$, problem $P1$ has a non-identity-matrix covariance feedback. To exploit the closed-form algorithm, we first transform problem $P1$ into a problem with the mean feedback form as follows.

**Equivalent problem ($P3$):**

\[
\max_{p, \bar{v}} \log(1 + p\bar{h}_s^H \bar{v}\bar{v}^H \bar{h}_s) \quad (24)
\]

subject to: \[p\|\Delta^{1/2} \bar{v}\|^2 \leq \bar{P}, \quad p\bar{h}_s^H \bar{v}\bar{v}^H \bar{h}_s \leq P_t, \quad \text{for} \quad \|\bar{h} - \bar{h}_0\|^2 \leq c; \]

where $R^{-1} := U^H \Delta U$ is the eigenvalue decomposition, $\bar{h} := \Delta^{1/2} U h$, $\bar{h}_0 := \Delta^{1/2} U h_0$, $\bar{h}_s := \Delta^{1/2} U h_s$, and $\bar{v} := \Delta^{-1/2} U v$. Problem $P3$ is equivalent to problem $P1$ in the sense that the optimal values of their objective functions are the same, i.e., the achieved rates are equal. Note that it is not necessary that $\|\bar{v}\| = 1$ in (24), and the optimal beamforming vector $\bar{v}_{\text{opt}}$ of problem $P3$ can be easily transformed into the optimal solution $v_{\text{opt}}$ for problem $P1$ by letting $v_{\text{opt}} = U^H \Delta^{1/2} \bar{v}_{\text{opt}}$.

In the preceding subsection, decoupling the multiple constraint problem into several single constraint subproblems facilitates the analysis and simplifies the process of solving the problem. For problem $P3$, it can also be decoupled into two subproblems as follows.

**Subproblem 3 ($SP3$):**

\[
\max_{p, \bar{v}} \log(1 + p\bar{h}_s^H \bar{v}\bar{v}^H \bar{h}_s) \quad (25)
\]

subject to: \[p\|\Delta^{1/2} \bar{v}\|^2 \leq \bar{P}. \quad (26)\]

**Subproblem 4 ($SP4$):**

\[
\max_{p, \bar{v}} \log(1 + p\bar{h}_s^H \bar{v}\bar{v}^H \bar{h}_s) \quad (27)
\]

subject to: \[p\bar{h}_s^H \bar{v}\bar{v}^H \bar{h}_s \leq P_t \quad \text{for} \quad \|\bar{h} - \bar{h}_0\|^2 \leq c. \quad (28)\]
It is easy to observe that SP3 is equivalent to SP1, and the optimal transmit covariance matrix of SP3 can be obtained the same way as that for SP1. Moreover, SP4 is the same as SP2, and thus it can be solved by Algorithm 1 discussed in Subsection IV-A.1.

The relationship between problem P3 and subproblems SP3 and SP4 is similar to the one between P2 and corresponding subproblems as depicted in Lemma 5, i.e., if either optimal solution of SP3 or SP4 satisfies both constraints, then it is the globally optimal solution; otherwise, the optimal solution satisfies both constraints with equalities. We hereafter need to consider only the case in which the solutions of both subproblems are not feasible for problem P3. For this case, the two equality constraints can be written as follows.

\[ \| \Delta^{1/2} \tilde{v} \| = 1, \ \text{and} \ \max (\tilde{h}^H \tilde{v} \tilde{v}^H \tilde{h}) = \frac{P_t}{P}, \ \text{for} \ \| \tilde{h} - \tilde{h}_0 \|^2 \leq c. \]  

(29)

Assume that the angle between \( \tilde{h}_0 \) and \( \tilde{v} \) is \( \tilde{\beta} \), and that \( \tilde{p} := \| \tilde{v} \| \). Similar to Lemma 3, the optimal \( \tilde{v} \) lies in a plane spanned by \( \tilde{h} \) and \( \tilde{h}_\perp \), where \( \tilde{h} = \tilde{h}_0/\| \tilde{h}_0 \|, \tilde{h}_\perp = \tilde{h}_\perp/\| \tilde{h}_\perp \|, \) and \( \tilde{h}_\perp = \tilde{h}_s - (\hat{\tilde{h}}^H \tilde{h}_s) \hat{\tilde{h}} \).

Thus, if we can determine \( \tilde{\beta} \) and \( \tilde{p} \) from (29), then the optimal \( \tilde{v} \) can be identified by

\[ \tilde{v} = \tilde{p} (\cos(\tilde{\beta}) \hat{\tilde{h}} + \sin(\tilde{\beta}) \hat{\tilde{h}}_\perp). \]  

(30)

Based on the new variables \( \tilde{\beta} \) and \( \tilde{p} \), the constraints (29) can be transformed as follows.

\[ \tilde{p} \| \Delta^{1/2} (\cos(\tilde{\beta}) \hat{\tilde{h}} + \sin(\tilde{\beta}) \hat{\tilde{h}}_\perp) \| = 1, \]  

(31)

\[ \text{and, } \tilde{p} \| \tilde{h}_0 \| + \sqrt{c} = \sqrt{\frac{P_t}{P}}. \]  

(32)

According to (31), we have

\[ \tilde{p} = \frac{1}{\| \Delta^{1/2} (\cos(\tilde{\beta}) \hat{\tilde{h}} + \sin(\tilde{\beta}) \hat{\tilde{h}}_\perp) \|}. \]  

(33)

Substituting (33) into (32), we have

\[ \sqrt{\frac{P_t}{P}} \| \Delta^{1/2} (\cos(\tilde{\beta}) \hat{\tilde{h}} + \sin(\tilde{\beta}) \hat{\tilde{h}}_\perp) \| = \cos(\tilde{\beta}) \| \tilde{h}_0 \| + \sqrt{c}. \]  

(34)

Hence, the optimal \( \tilde{\beta} \) can be obtained by solving (34), and \( \tilde{v}_{\text{opt}} \) can be obtained by substituting \( \tilde{\beta} \) into (30). In summary, the procedure to solve the case in which both constraints are active is listed as Algorithm 4 in Table IV. Moreover, we are now ready to present the complete algorithm for solving problem P1 as Algorithm 5 in Table V.
In Algorithm 5, we obtain the optimal solutions to SP3 and SP4 and the optimal solution to the case where both constraints are active separately. According to Lemma 5, the final result obtained in Algorithm 5 is thus the optimal solution of problem P1.

Proposition 2: Algorithm 5 achieves the optimal solution of problem P1.

V. An Alternative Approach: Second Order Cone Programming Solution

In this section, we solve problem P1 via a standard interior point algorithm [19]–[21]. We first transform the SIP problem into a finite constraint problem, and further transform it into a standard SOCP form, which can be solved by using a standard software package such as SeDuMi [22]. The infinite constraint part of problem P1 is the interference constraint which can be viewed as a worst-case constraint. Combining this observation with Lemma 1, problem P1 can be transformed as:

Equivalent problem (P4): \[ \max_{p \geq 0, \|v\| = 1} \log(1 + ph_s^H vv^H h_s) \]

subject to: \[ p \leq \bar{P}, \max_{h \in \mathcal{H}_1(c)} p h^H v v^H h \leq P_t, \]

where \( \mathcal{H}_1(c) := \{h|h = h_0 + h_1\} \), and \( h_1 \in \mathcal{H}_2(c) \) with \( \mathcal{H}_2(c) := \{h_1|h_1^H R^{-1} h_1 \leq c\} \). It is clear that maximizing \( \log(1 + ph_s^H vv^H h_s) \) is equivalent to maximizing \( |\sqrt{P} h_s^H v| \). By defining \( w = \sqrt{P} v \), the objective function can be rewritten as \( |h_s^H w| \). Similarly, the interference power can be expressed as \( |h_s^H w|^2 \). Thus, problem P4 can be further transformed as:

\[ \max_w |h_s^H w| \]

subject to: \[ \|w\| \leq \sqrt{P}, \max_{h \in \mathcal{H}_1(c)} |h^H w| \leq \sqrt{P}_t. \]

According to the definition of \( \mathcal{H}_1(c) \), we can rewrite the worst-case constraint in (36) as

\[ \max_{h \in \mathcal{H}_1(c)} |h^H w| = \max_{h_1 \in \mathcal{H}_2(c)} |(h_0 + h_1)^H w| \leq \sqrt{P}_t. \]

By the triangle inequality and the fact that \( \sqrt{c}\|Qw\| = \max |h_1^H w| \) for \( h_1 \in \mathcal{H}_2(c) \) (refer to Appendix B for details), the interference power can be transformed as follows.

\[ \|(h_0 + h_1)^H w\| \leq |h_0^H w| + |h_1^H w| \leq |h_0^H w| + \sqrt{c}\|Qw\|. \]
where $Q = \Delta^{-1/2}U$, $R^{-1} := U^H \Delta U$ is the eigenvalue decomposition. Moreover, since the arbitrary phase rotation of $w$ does not change the value of the objective function or the constraints, according to Remark 1 and Lemma 3, we can assume that $w$, $h_s$ and $h_0$ have the same phase, i.e.,

$$\text{Re}\{w^H h_s\} \geq 0, \text{Im}\{w^H h_0\} = 0, \text{ and } \text{Im}\{w^H h_s\} = 0. \quad (39)$$

Hence, the interference constraint can be transformed into two second order cone (SOC) inequalities as follows

$$\sqrt{c}\|Qw\| + h_0^H w \leq \sqrt{P_t}, \text{ and } \sqrt{c}\|Qw\| - h_0^H w \leq \sqrt{P_t}. \quad (40)$$

By combining (36), (40), with (39), problem $P1$ is transformed into the standard SOCP problem as follows

$$\max_w h_s^H w \quad \text{subject to: } \|w\| \leq \sqrt{P}, \text{Im}\{w^H h_0\} = 0, \sqrt{c}\|Qw\| + h_0^H w \leq \sqrt{P_t}, \sqrt{c}\|Qw\| - h_0^H w \leq \sqrt{P_t}. \quad (41)$$

Problem (42) can be solved by the standard interior point program SeDuMi [22]. In terms of the complexity, however, the preceding closed-form solution is better than the SOCP solution. In the next section, we compare the results obtained by the SeDuMi and the closed-form method in Section IV.

VI. SIMULATIONS

Computer simulations are provided in this section to evaluate the performance of the proposed algorithms. In the simulations, it is assumed that both the channels from the SU-Tx to the SU-Rx $h_s$, and to the PU $h_0$, are generated by independent circularly symmetric complex Gaussian (CSCG) random vectors with zero mean and unit variance on each element. Moreover, we denote by $l_1$ the
distance between the SU-Tx and the SU-Rx, and by $l_2$ the distance between the SU-Tx and the PU. It is assumed that the same path loss model can be used to describe the transmissions from the SU-Tx to the SU-Rx and to the PU, and the path loss exponent is 4. The noise power is chosen to be 1, and the transmit power and interference power are defined in dB relative to the noise power. For all cases, we choose $P_t = 0$ dB.

A. Comparison of the Analytical Solution and the Solution Obtained by the SOCP Algorithm

In this simulation, we compare the two results obtained by a standard SOCP algorithm (SeDuMi) and Algorithm 3. We consider the system with $N = 3$, $l_2/l_1 = 2$, and $\bar{P}$ ranging from 3 dB to 10 dB. In Fig. 4, we can see that the results obtained by different algorithms coincide. This is because both algorithms determine the optimal solution. Compared with the SOCP algorithm solution, Algorithm 3 can obtain the solution directly, and thus it has lower complexity. In Fig. 5, we compare the two results obtained by SeDuMi and Algorithm 5. We consider the system with $N = 3$, $\bar{P} = 5$ dB, and $l_2/l_1$ ranging from 1 to 10. The covariance matrix $R$ is generated by $R_1^H R_1$, where each element of $R_1$ follows Gaussian distribution with zero mean and unit variance. From Fig. 5, we can see that the results obtained by the two algorithms coincide. Moreover, we note that the achievable rate with $c = 0.2$ is always greater than or equal to the rate with $c = 0.3$, since a larger $c$ corresponds to the stricter constraints.

B. Effectiveness of the Interference Constraint

In this simulation, we apply Algorithm 3 to solve problem $P2$. In Fig. 6, we depict the achievable rate versus the ratio $l_2/l_1$ under different transmit power constraints. The larger of the ratio $l_2/l_1$ corresponds the less the interference power constraint. As shown in Fig. 6, with an increase of $l_2/l_1$, the achievable rate increases due to the lower interference constraint. Until the ratio $l_2/l_1$ reaches a certain value, the achievable rate remains unchanged, since the transmit power constraint dominates the result, and the interference constraint becomes inactive.
C. The Activeness of the Constraints

In this simulation, we compare the achieved rates of problem P1 with a single transmit power constraint, a single interference constraint and both constraints. In this simulation example, we choose $N = 3$, $c = 0.2$, and generate $R$ in the same way as in the first simulation example. Fig. 7 plots three achievable rates for different constraints, respectively. It can be observed from Fig. 7 that the rate under two constraints is always less than or equal to the rate under a single constraint. Of course, this is because extra constraints reduce the degree of freedom of the transmitter.

VII. Conclusions

In this paper, the rate maximization problem has been investigated, for the SU MISO communication system in which only partial CSI of the link from the SU-Tx to the PU is available at the SU-Tx. The problem can be naturally formulated as an SIP optimization problem. Two approaches have been proposed to obtain the optimal solution of the problem; one approach solves the problem in an analytical manner, while the other approach applies a standard interior point algorithm. Simulation examples were used to present a comparison of the two approaches as well as to study the effectiveness and activeness of imposed constraints.

Appendix

A. Proof of Lemma 1: Problem P1 involves infinitely many constraints. Denote the set of active constraints by $\mathcal{H}$, the cardinality of the set $\mathcal{H}$ by $K$, and the channel response related to the $k$th element of the set $\mathcal{H}$ by $h_k$. According to the Karush-Kuhn-Tucker (KKT) conditions for $P1$, we have:

$$h_s(1 + h_s^H S h_s)h_s^H + \Phi = \lambda I + \sum_{i=1}^{K} \mu_i h_i h_i^H,$$

(43)

$$\text{tr}(\Phi S) = 0,$$

(44)

where $\Phi$ is the dual variable associated with the constraint $S \geq 0$, and where $\lambda$ and $\mu_i$ are the dual variables associated with the transmit power constraint and the interference constraint, respectively.
First, we assume that $\lambda \neq 0$, and thus the rank of the right hand side of (43) is $N$. Since the first term on the left hand side of (43) has rank one, we have

$$\text{Rank}(\Phi) \geq N - 1. \tag{45}$$

Moreover, since $S \geq 0$ and $\Phi \geq 0$, from (44) we have $\text{tr}(\Phi S) = \text{tr}(U^H \Lambda U S) = \text{tr}(\Lambda S) = 0$, where $U^H \Lambda U$ is the eigenvalue decomposition of matrix $\Phi$, and $\tilde{S} := USU^H$. By applying eigenvalue decomposition to $\tilde{S}$, we have $\tilde{S} := \sum_i \tau_i s_i s_i^H$, where $\tau_i$ is the $i$th eigenvalue and $s_i$ is the corresponding eigenvector. We can show that $\text{Rank}(S) + \text{Rank}(\Phi) \leq N$ by contradiction. Since if $\text{Rank}(S) + \text{Rank}(\Phi) > N$, there exists an index $j$ such that the $j$th element of $s_i$ and the $j$th diagonal element of $\Lambda$ are non-zero simultaneously. Then, it is impossible that the equation $\text{tr}(\Lambda \tilde{S}) = 0$ holds. It follows that $\text{Rank}(S) + \text{Rank}(\Phi) \leq N$. Combining this with (45), we have $\text{Rank}(S) \leq 1$.

Second, we assume that $\lambda = 0$ in (43). In this case, $S$ must lie in the space spanned by $h_i$, $i = 1, \cdots, K$. Let the dimensionality of the space be $M$. Therefore, we can restrict $\text{Rank}(\Phi) \leq M$. Thus, the reminder of the proof is the same as that of the case $\lambda \neq 0$, and the proof is complete. ■

B. Lemma 6 and its proof:

Lemma 6: For the problem

$$\max_h p h^H vv^H h, \text{ subject to: } (h - h_0)^H R^{-1} (h - h_0) \leq c, \tag{46}$$

where $p$, $v$, and $h_0$ are constant, the optimal solution is

$$h_{\text{max}} = h_0 + \sqrt{\frac{c}{v^H R v}} \alpha R v, \text{ where } \alpha = v^H h_0 / |v^H h_0|. \tag{47}$$

Proof: The objective function $p h^H v v^H h$ is a convex function. The duality gap for a convex maximization problem is zero. The Lagrangian function is

$$L(h, \lambda) = p h^H v v^H h - \lambda \left( (h - h_0)^H R^{-1} (h - h_0) - c \right), \tag{48}$$

where $\lambda$ is the Lagrange multiplier. According to the KKT condition, we have $\frac{\partial L}{\partial h} = 2 p v v^H h - 2 \lambda R^{-1} (h - h_0) = 0$. Thus,

$$p(v^H h) v = \lambda R^{-1} (h - h_0). \tag{49}$$
We have $h_{\text{max}} = h_0 + b\alpha Rv$, where $b \in \mathbb{R}$, $\alpha \in \mathbb{C}$, and $|\alpha| = 1$. Since $(h - h_0)^H R^{-1}(h - h_0) = c$, we have $b = \sqrt{c}/\sqrt{v^H R^H v}$. Moreover, by observing (49), we have $\alpha = tv^H h = tv^H (h_0 + b\alpha Rv) = tv^H h_0 + tb\alpha v^H Rv$, where $t$ is a real scalar such that $|tv^H h| = 1$. Thus, we have $v^H h_0/|v^H h_0| = \alpha$.

The proof follows immediately.

C. Proof of Lemma 2:

First, we consider the sufficiency part of this lemma. We assume that there exist a covariance matrix $S_{\text{opt}}$ and an $h_{\text{opt}}$ that satisfy the conditions (2) and (3) simultaneously. Since $S_{\text{opt}}$ satisfies both the transmit power constraint and the interference constraint, $S_{\text{opt}}$ is a feasible solution for problem $P1$. Moreover, if we assume that there exists another solution $S_s$, which results in a larger achievable rate for the SU link, then a contradiction will be derived. Without loss of generality, we assume that the constraint set, which consists of all the active interference constraints for $S_s$, is denoted by $T$. We divide the set $T$ into two types: one type is $h_{\text{opt}} \in T$, and the other type is $h_{\text{opt}} \notin T$.

Assume that $C_s$ and $C_{\text{opt}}$ are the achievable rates for the covariance matrices $S_s$ and $S_{\text{opt}}$, respectively. In the case of $h_{\text{opt}} \in T$, we have $C_s \leq C_{\text{opt}}$, since $C_{\text{opt}}$ is obtained with fewer constraints. Since problem $P1$ is a convex optimization problem that has a unique optimal solution, $S_{\text{opt}}$ is indeed the optimal solution. In the case $h_{\text{opt}} \notin T$, we can observe that $S_{\text{opt}}$ satisfies the constraints in $T$, and $S_s$ satisfies the constraint $h_{\text{opt}}$. According to the lemma in [13], this case does not exist.

We next proceed to prove the necessity part. Suppose that $S_{\text{opt}}$ is the optimal solution of problem $P1$. According to Lemma 1, we have $S_{\text{opt}} = p_{\text{opt}} v_{\text{opt}} v_{\text{opt}}^H$. Thus, problem $P1$ is equivalent to

$$\max_{S \geq 0} \log(1 + h_s^H Sh_s)$$

subject to: $\text{tr}(S) \leq p_{\text{opt}}$, $h^H Sh \leq P_t$, for $(h - h_0)^H R^{-1}(h - h_0) \leq c$. (50)

According to Lemma 6, there is a unique

$$h_{\text{opt}} = h_0 + \sqrt{\frac{c}{v_{\text{opt}}^H R v_{\text{opt}}}} \alpha R v_{\text{opt}},$$

which is the optimal solution of $\max_{h \in \mathcal{N}_1(c)} h^H Sh \leq P_t$. Thus, for problem (50), only $\text{tr}(S) \leq p_{\text{opt}}$ and $h_{\text{opt}}^H Sh_{\text{opt}} \leq P_t$ are active constraints. Thus, it is obvious that problem (50) and problem (2) have the same optimal solution. Hence, the proof is complete.
D. Proof of Lemma 3: The proof of Lemma 3 is divided into two parts. The first part is to prove that $v_{opt}$ is in the form of $\alpha_v \hat{h} + \beta_v \hat{h}_{\perp}$, where $\alpha_v \in \mathbb{C}$ and $\beta_v \in \mathbb{C}$. The second part is to prove $\alpha_v \in \mathbb{R}$ and $\beta_v \in \mathbb{R}$. In the following proof, we assume that $\alpha_k \in \mathbb{C}$ are some proper complex scalars.

According to Lemma 2, and Theorem 2 in [14], we have

$$v_{opt} = \alpha_1 h_{opt} + \alpha_2 h_s.$$ (52)

According to Lemma 6, we have

$$h_{opt} = h_0 + \alpha_3 v_{opt} = h_0 + \alpha_3 (\alpha_1 h_{opt} + \alpha_2 h_s) = h_0 + \alpha_1 \alpha_3 h_{opt} + \alpha_2 \alpha_3 h_s.$$ (53)

According to (53), it can be observed that $h_{opt}$ can be expressed by the linear combination of $h_0$ and $h_s$, where the coefficients are complex. Combining this with (52), we have $v_{opt} = \alpha_4 h_0 + \alpha_5 h_s$, where $\alpha_4 \in \mathbb{C}$ and $\alpha_5 \in \mathbb{C}$. Moreover, since both $h_0$ and $h_s$ can be expressed as combinations of $\hat{h}$ and $\hat{h}_{\perp}$, we have $v_{opt} = \alpha_v \hat{h} + \beta_v \hat{h}_{\perp}$. Since rotating $v_{opt}$ does not affect the final result, we can assume $\alpha_v \in \mathbb{R}$.

We next prove that $\beta_v \in \mathbb{R}$ by contradiction. At first, we assume that $\beta_v = a + jb \notin \mathbb{R}$. Then we can find an equivalent $\hat{\beta}_v = \sqrt{a^2 + b^2} \in \mathbb{R}$ which is a better solution of problem P1 than $\beta_v$. Assume that $v_{opt} = \alpha_v \hat{h} + \beta_v \hat{h}_{\perp}$. It is clear that $\|v_{\text{opt}}\| = \|v_{opt}\|$, and the interference caused by $v_{\text{opt}}$ is

$$h_{opt}^H \hat{v}_{\text{opt}} \hat{v}_{\text{opt}}^H h_{opt} = p(h_0 + \frac{c}{\hat{v}_{\text{opt}}^H \hat{v}_{\text{opt}}^H} \alpha \hat{R} \hat{v}_{\text{opt}}^H \hat{v}_{\text{opt}}(h_0 + \sqrt{\frac{c}{\hat{v}_{\text{opt}}^H \hat{v}_{\text{opt}}^H} \alpha \hat{R} \hat{v}_{\text{opt}}^H \hat{v}_{\text{opt}}^H)} (54)$$

$$= p(\alpha_v \|h_0\| + \sqrt{\frac{c}{\hat{v}_{\text{opt}}^H \hat{v}_{\text{opt}}^H}} \alpha \|h_{\text{opt}}\|)$$ (55)

which is equal to that of $v_{opt}$. However, the corresponding objective function with $\hat{v}_{\text{opt}}$ is

$$\log(1 + p h_{opt}^H v_{opt} v_{opt}^H h_s) = \log(1 + p(a_h \hat{h} + b_h \hat{h}_{\perp})^H (\alpha_v \hat{h} + \beta_v \hat{h}_{\perp})(\alpha_v \hat{h} + \beta_v \hat{h}_{\perp})^H (a_h \hat{h} + b_h \hat{h}_{\perp}))$$

$$= \log(1 + p(a_h \alpha_v + b_h \beta_v)(a_h \alpha_v + b_h \beta_v^H)), \quad (56)$$

and the objective value with $v_{opt}$ is

$$\log(1 + p h_{opt}^H v_{opt} v_{opt}^H h_s) = \log(1 + p(a_h \hat{h} + b_h \hat{h}_{\perp})^H (\alpha_v \hat{h} + \beta_v \hat{h}_{\perp})(\alpha_v \hat{h} + \beta_v \hat{h}_{\perp})^H (a_h \hat{h} + b_h \hat{h}_{\perp}))$$

$$= \log(1 + p(a_h \alpha_v + b_h \beta_v)(a_h \alpha_v + b_h \beta_v^H)). \quad (57)$$

According to (56) and (57), we can conclude that $\hat{v}_{\text{opt}}$ is a better solution. The proof follows.
E. Proof of Lemma 4: Similar to the proof of Lemma 2, we can show that the problem

$$S_{\text{opt}} = \arg \max_{S, p} \log(1 + h_s^H S h_s) \text{ subject to }: h_{\text{opt}}^H S h_{\text{opt}} \leq P_t,$$  \hfill (58)

where $h_{\text{opt}} = \arg \max_h h^H S_{\text{opt}} h$, for $(h - h_0)^H R^{-1}(h - h_0) \leq c$, is equivalent to SP2.

Since $S_{\text{opt}}$ is a rank-1 matrix, according to Lemma 6, we have $h_{\text{opt}} = h_0 + \sqrt{c} \sigma v$. Combining this with (58), we have $S_{\text{opt}} = \arg \max_{S, p} \log(1 + h_s^H S h_s) \text{ s.t. } (h_0 + \sqrt{c} \sigma v)^H S (h_0 + \sqrt{c} \sigma v) \leq P_t$, which is equivalent to (9). The proof is complete.

F. Proof of Lemma 5: Assume that $S_{\text{opt}}$ is the optimal solution for problem P2. If $S_1$ satisfies the interference constraint, then $S_1$ is a feasible solution for problem P2. The optimal rate achieved by $S_{\text{opt}}$ cannot be larger than that of $S_1$, since the constraint of SP1 is a subset of problem P2. Similarly, we can prove the second part, and so we now consider the third part of this lemma. For problem P2, at least one of $\text{tr}(S) \leq \bar{P}$ and $h_{\text{opt}}^H S h_{\text{opt}} \leq P_t$ is an active constraint, since if neither of them is active, we can always find an $\epsilon$ such that $S_{\text{opt}} + \epsilon I$ is a feasible and better solution. Moreover, if only $\text{tr}(S) \leq \bar{P}$ is active, then $S_1$ is the optimal solution, which is contradicted with $h_{\text{opt}}^H S_1 h_{\text{opt}} \geq P_t$. Similarly, it is impossible that only $h_{\text{opt}}^H S h_{\text{opt}} \leq P_t$ is active. Therefore, both constraints are active constraints.

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TABLE I

THE ALGORITHM FOR SP2.

Algorithm 1

1. Compute $\beta_{\text{opt}}$ through (18),
2. Compute $p_{\text{opt}}$ according to (13),
3. Compute $v_{\text{opt}}$ according to (19),
4. $S_{\text{opt}} = p_{\text{opt}}v_{\text{opt}}v_{\text{opt}}^H$.

TABLE II

THE ALGORITHM FOR PROBLEM P2 IN THE CASE WHERE TWO CONSTRAINTS ARE SATISFIED SIMULTANEOUSLY.

Algorithm 2

1. Compute $\beta_{\text{opt}}$ through (23),
2. Based on (19), compute $v_{\text{opt}}$,
3. $S_{\text{opt}} = \bar{P}v_{\text{opt}}v_{\text{opt}}^H$.

TABLE III

THE COMPLETE ALGORITHM FOR PROBLEM P2.

Algorithm 3

1. Compute the optimal solution $S_1 = \bar{P}h_xh_x^H/\|h_x\|^2$ for SP1,
2. Compute the optimal solution $S_2$ for SP2 via Algorithm 1,
3. If $S_1$ satisfies the interference constraint, then $S_1$ is the optimal solution,
4. Elsif $S_2$ satisfies the transmit power constraint, then $S_2$ is the optimal solution,
5. Otherwise compute the optimal solution via Algorithm 2.
TABLE IV

The algorithm for problem P3 in the case where two constraints are satisfied simultaneously.

| Algorithm 4 |
|-------------|
| 1. Compute $\bar{\beta}$ via (34), and compute $\bar{v}$ via (30), |
| 2. Based on the relationship between $\bar{v}$ and $v$, compute $v_{\text{opt}}$, |
| 3. $S_{\text{opt}} = P v_{\text{opt}} v_{\text{opt}}^H$ |

TABLE V

The complete algorithm for problem P1.

| Algorithm 5 |
|-------------|
| 1. Compute the optimal solution $S_3 = Ph_s h_s^H / \|h_s\|^2$ for SP3, |
| 2. Compute the optimal solution $S_4$ for SP4 via Algorithm 4, |
| 3. If $S_3$ satisfies the interference constraint, then $S_3$ is the optimal solution, |
| 4. Elsif $S_4$ satisfies the transmit power constraint, then $S_4$ is the optimal solution, |
| 5. Otherwise compute the optimal solution through Algorithm 4. |

Fig. 1. The system model for the MISO SU network coexisting with one PU.
Fig. 2. The geometric explanation of Lemma 3. The ellipse is the projection of $h := \{(h - h_0)H R^{-1} (h - h_0) = c\}$ on the plane spanned by $\hat{h}$ and $\hat{h}_\perp$. 
Fig. 3. The geometric explanation of problem P2. The circle is the projection of \( h := \{ \| h - h_0 \|^2 = 0 \} \) on the plane spanned by \( \hat{h} \) and \( \hat{h}_\perp \).
Fig. 4. Comparison of the results obtained by the SOCP algorithm and Algorithm 3.
Fig. 5. Comparison of the results obtained by the SOCP algorithm and Algorithm 5.
Fig. 6. Effect of $l_2/l_1$ on the achievable rate of the CR network ($c = 1, N = 3$).
Fig. 7. Comparison of the rate under different constraints of problem $P1$. (i) the maximal rate subject to interference constraint and transmit power constraint simultaneously; (ii) the maximal rate subject to a single transmit power constraint; (iii) the maximal rate subject to a single interference constraint.