Ultra High Energy Behaviour

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Abstract

We reexamine the behaviour of particles at Ultra High energies in the context of the fact that the LHC has already touched an energy of $7\text{TeV}$ and is likely to attain $14\text{TeV}$ by 2013/2014. Consequences like a possible new shortlived interaction within the Compton scale are discussed.

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1 The ”two component” Klein-Gordon Equation

In relativistic Quantum Mechanics we encounter negative energies, unlike in Classical Physics. In the case of the Dirac electron, this lead to the postulation of the Hole theory. Let us now start with the Klein-Gordon equation. In the relativistic Klein-Gordon equation, the presence of the second time derivative and hence an extra degree of freedom for the wave function $\psi$ lead to interpretational problems, particularly in regard to negative energy solutions. Pauli and Weiskopf overcame these difficulties by treating the Klein-Gordon equation as describing a field and showed that the two degrees of freedom of $\psi$ would be distinctly charged states. Later Feshbach and Villars [1] showed that this interpretation could also be carried over to the case
of a single particle description. As has been shown in detail by Feshbach and Villars, we can rewrite the K-G equation in the Schrödinger form, invoking a two component wave function,
\[
\Psi = \left( \begin{array}{c} \phi \\ \chi \end{array} \right),
\]
(1)

The K - G equation then can be written as (Cf. ref. [1] for details)
\[
\begin{align*}
\hbar (\partial \phi / \partial t) &= (1/2m)(\hbar / i \nabla - eA/c)^2(\phi + \chi) + (eA_0 + mc^2)\phi, \\
\hbar (\partial \chi / \partial t) &= -(1/2m)(\hbar / i \nabla - eA/c)^2(\phi + \chi) + (eA_0 - mc^2)\chi
\end{align*}
\]
(2)

It will be seen that the components \(\phi\) and \(\chi\) are coupled in (2). In fact we can analyse this matter further, considering free particle solutions for simplicity. We write,
\[
\Psi = \left( \begin{array}{c} \phi_0(p) \\ \chi_0(p) \end{array} \right) e^{i\hbar(p \cdot x - Et)}
\]
\[
\Psi = \Psi_0(p)e^{i\hbar(p \cdot x - Et)}
\]
(3)

Introducing (3) into (2) we obtain, two possible values for the energy \(E\), viz.,
\[
E = \pm E_p; \quad E_p = [(cp)^2 + (mc^2)^2]^{1/2}
\]
(4)

The associated solutions are
\[
\begin{align*}
E &= E_p \quad \phi_0^{(+)}(p) = \frac{E_p + mc^2}{2(mc^2 - E_p)^{1/2}} \\
\psi_0^{(+)}(p) : \quad \chi_0^{(+)} &= \frac{mc^2 - E_p}{2(mc^2 - E_p)^{1/2}} \quad \phi_0^2 - \chi_0^2 = 1,
\end{align*}
\]
\[
\begin{align*}
E &= -E_p \quad \phi_0^{(-)}(p) = \frac{mc^2 - E_p}{2(mc^2 + E_p)^{1/2}} \\
\psi_0^{(-)}(p) : \quad \chi_0^{(-)} &= \frac{E_p + mc^2}{2(mc^2 + E_p)^{1/2}} \quad \phi_0^2 - \chi_0^2 = -1
\end{align*}
\]
(5)

It can be seen from this that even if we take the positive sign for the energy in (4), the \(\phi\) and \(\chi\) components get interchanged with a sign change for the energy. Furthermore we can easily show from this that in the non relativistic limit, the \(\chi\) component is suppressed by order \((p/mc)^2\) compared to the \(\phi\) component exactly as in the case of the Dirac equation [2]. Let us investigate
this circumstance further. It can be seen that (2) are Schrodinger equations and so solvable. However they are coupled. We have from them,

\[ \dot{\phi} + \dot{\chi} = (eA_0 + mc^2)(\phi + \chi) - 2mc^2\chi \quad (6) \]

In the case if

\[ mc^2 >> eA_0 \quad \text{(or } A_0 = 0) \quad (7) \]

(or in the absence of an external field) we can easily verify that

\[ \phi = e^{ipx-Et} \text{and} \chi = e^{ipx+Et} \quad (8) \]

is a solution.

That is \( \phi \) and \( \chi \) belong to opposite values of \( E(m \neq 0) \) (Cf. equation (5)). The above shows that the K-G equation mixes the positive and negative energy solutions.

If on the other hand \( m_0 \approx 0 \), then (6) shows that \( \chi \) and \( \phi \) are effectively uncoupled and are of same energy. This shows that if \( \phi \) and \( \chi \) both have the same sign for \( E \), that is there is no mixing of positive and negative energy, then the rest mass \( m_0 \) vanishes. A non vanishing rest mass requires the mixing of both signs of energy. Indeed it is a well known fact that for solutions which are localized, both signs of the energy solutions are required to be superposed [2, 3]. This is because only positive energy solutions or only negative energy solutions do not form a complete set. Interestingly the same is true for localization about a time instant \( t_0 \).

That is physically, only the interval \( (t_0 - \Delta t, t_0 + \Delta t) \) is meaningful. This was noticed by Dirac himself when he deduced his equation of the electron [4].

In any case both the positive and negative energy solutions are required to form a complete set and to describe a point particle at \( x_0 \) in the delta function sense. The narrowest width of a wave packet containing both positive and negative energy solutions, which describes the spacetime development of a particle in the familiar non-relativistic sense, as is well known is described by the Compton wavelength. As long as the energy domain is such that the Compton wavelength is negligible then our usual classical type description is valid. However as the energy approaches levels where the Compton wavelength can no longer be neglected, then new effects involving the negative energies and anti particles begin to appear (Cf.ref.[1]).
Further, we observe that from (8)

\[ t \rightarrow -t \Rightarrow E \rightarrow -E, \quad \phi \leftrightarrow \chi \quad (9) \]

(Moreover in the charged case \( e \rightarrow -e \)). This remark will be used later using the fact that we are dealing with a two state system (Cf.\( \text{[S]} \)). On the other hand we will show that the Schrodinger equation goes over to the Klein-Gordon equation if we allow \( t \) to move forward and also backward in \((t_0 - \Delta t, t_0 + \Delta t)\). Here we have done the reverse of getting the Klein-Gordon equation into two Schrodinger equations. This is expressed by (2).

In any case we would like to reiterate that the two degrees of freedom associated with the second time derivative can be interpreted, following Pauli and Weisskopf as positive and negatively charged particles or particles and anti particles.

We obtained from the Klein-Gordon equation a description in terms of two Schrodinger equations. We will now show how (9) with the Schrodinger equation leads to the Klein-Gordon equation briefly repeating an earlier result. We first define a complete set of base states by the subscript \( i \) and \( U(t_2, t_1) \) the time elapse operator that denotes the passage of time between instants \( t_1 \) and \( t_2, t_2 \) greater than \( t_1 \). We denote by, \( C_i(t) \equiv \langle i | \psi(t) \rangle \), the amplitude for the state \( |\psi(t)\rangle \) to be in the state \( |i\rangle \) at time \( t \). We have \([5, 6, 7]\)

\[ \langle i | U | j \rangle \equiv U_{ij}, U_{ij}(t + \Delta t, t) \equiv \delta_{ij} - \frac{i}{\hbar} H_{ij}(t) \Delta t. \]

We can now deduce from the super position of states principle that,

\[ C_i(t + \Delta t) = \sum_j [\delta_{ij} - \frac{i}{\hbar} H_{ij}(t) \Delta t] C_j(t) \quad (10) \]

and finally, in the limit,

\[ i\hbar \frac{dC_i(t)}{dt} = \sum_j H_{ij}(t) C_j(t) \quad (11) \]

where the matrix \( H_{ij}(t) \) is identified with the Hamiltonian operator. We have argued earlier at length that \([11]\) leads to the Schrodinger equation \([5, 7]\). In the above we have taken the usual unidirectional time to deduce the non relativistic Schrodinger equation. If however we consider a Weiner process.
in (10) to which we will return shortly, then we will have to consider instead of (11)

\[ C_i(t - \Delta t) - C_i(t + \Delta t) = \sum_j \left[ \delta_{ij} - \frac{i}{\hbar} H_{ij}(t) \right] C_j(t) \]  

(12)

Equation (12) in the limit can be seen to lead to the relativistic Klein-Gordon equation rather than the Schrodinger equation with the second time derivative [7, 8].

2 Remarks and Discussion

We summarize the following:
i) From the above analysis it is clear that a localized particle requires both signs of energy. At relatively low energies, the positive energy solutions predominate and we have the usual classical type particle behaviour. On the other hand at very high energies it is the negative energy solutions that predominate as for the negatively charged counterpart or the anti particles. More quantitatively, well outside the Compton wavelength the former behaviour holds. But as we approach the Compton wavelength we have to deal with the new effects.

ii) Let us remain in the realm of maximally localizable particles. The point is that if we approach distances of the order of the Compton wavelength, the negative energy solutions begin to dominate, and we encounter the well known phenomenon of Zitterbewegung [4]. This modifies the coupling of the positive solutions with an external field, particularly if the field varies rapidly over the Compton wavelength. In fact this is the origin of the well known Darwin term in the Dirac theory [2]. The Darwin term is a correction to the interaction of the order

\[ \left( \frac{p}{mc} \right)^4 \]  

and

\[ \left( \frac{p}{mc} \right)^2 \]  

(13)

for spin 0 and spin 1/2 particles respectively.

iii) To reiterate if we consider the positive and negative energy solutions given by \( + - E_p \), as in (5), then we saw that for low energies, the positive solution \( \phi_0 \) predominates, while the negative solution \( \chi_0 \) is \( \sim (\frac{v}{c})^2 \) compared to the positive solution. On the other hand at very high energies the negative solutions begin to play a role and in fact the situation is reversed with \( \phi_0 \) being suppressed in comparison to \( \chi_0 \). This can be seen from (5).
iv) There is another elegant way in which we could look at the considerations starting from (1). In analogy with the isospin formulation, we could think of the wave function \( \psi \) as having two possible states in a charge-spin (or particle-antiparticle) space. In this case introducing the Pauli spin matrices (Cf.[1]) given by

\[
\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad i\tau_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \\
\tau_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad 1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\] (14)

the Hamiltonian (2) can be written as

\[
H = (\tau_3 + i\tau_2)(1/2m)(p - eA/c)^2 + mc^2\tau_3 + e\phi
\] (15)

Now we would have

\[
\int \Psi^*\tau_3\Psi d^3x = \pm 1,
\] (16)

Let us proceed further.

v) We have already seen the symmetry given in (9): In case of a charged particle, in addition, \( e \rightarrow -e \) and vice versa (with complexification). Furthermore it can be seen that the coordinate \( \vec{x} \), as it were splits into the coordinate \( \vec{x}_1 \) and \( \vec{x}_2 \) which mimic the wave function in (1) at low and high energies, in the sense that the former dominates at low energies while the latter dominates at high energies, following the wave function as in (5). The fact that these go into each other following (9) as \( t \rightarrow -t \) can be explained in terms of the development of a two Weiner process see briefly above (Cf.[8]). In this case there are two derivatives, one for the usual forward time and another for a backward time given by

\[
\frac{d_+}{dt}x(t) = b_+, \quad \frac{d_-}{dt}x(t) = b_-
\] (17)

where we are considering for the moment, a single dimension \( x \). This leads to the Fokker-Planck equations

\[
\begin{aligned}
\frac{\partial \rho}{\partial t} + \text{div}(\rho b_+) &= V \Delta \rho, \\
\frac{\partial \rho}{\partial t} + \text{div}(\rho b_-) &= -U \Delta \rho
\end{aligned}
\] (18)

defining

\[
V = \frac{b_+ + b_-}{2} \quad ; U = \frac{b_+ - b_-}{2}
\] (19)
We get on addition and subtraction of the equations in (18) the equations

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho V) = 0$$  \hspace{1cm} (20)

$$U = \nu \nabla \ln \rho$$  \hspace{1cm} (21)

It must be mentioned that $V$ and $U$ are the statistical averages of the respective velocities and their differences. We can then introduce the definitions

$$V = 2\nu \nabla S$$  \hspace{1cm} (22)

$$V - iU = -2\nu \nabla (\ln \psi)$$  \hspace{1cm} (23)

We will not pursue this line of thought here but observe that the complex velocity in (23) can be described in terms of a positive or uni directional time $t$ only, but a complex coordinate

$$x \to x + ix'$$  \hspace{1cm} (24)

To see this let us rewrite (19) as

$$\frac{dX_r}{dt} = V, \quad \frac{dX_i}{dt} = U,$$  \hspace{1cm} (25)

where we have introduced a complex coordinate $X$ with real and imaginary parts $X_r$ and $X_i$, while at the same time using derivatives with respect to time as in conventional theory.

We can now see from (19) and (25) that

$$W = \frac{d}{dt}(X_r - iX_i)$$  \hspace{1cm} (26)

That is we can either use forward and backward time derivatives and the usual space coordinates as in (19) or we can use derivatives with respect to the usual uni directional time derivative to introduce the complex coordinate (24). (Cf.ref. [9].)

Let us now generalize (24), which we have taken in one dimension for simplicity, to three dimensions. Then we end up with not three but four dimensions,

$$(1, i) \to (I, \tau),$$

where $I$ is the unit $2 \times 2$ matrix and $\tau$s are the Pauli matrices. We get the special relativistic Lorentz invariant metric at the same time. (In this sense,
as noted by Sachs [10], Hamilton who made this generalization would have hit upon Special Relativity, if he had identified the new fourth coordinate with time).

That is,

\[ x + iy \rightarrow Ix_1 + ix_2 + jx_3 + kx_4, \]

(27)

where \((i, j, k)\) now represent the Pauli matrices; and, further,

\[ x_1^2 + x_2^2 + x_3^2 - x_4^2 \]

(28)

is invariant.

While the usual Minkowski four vector transforms as the basis of the four dimensional representation of the Poincare group, the two dimensional representation of the same group, given by the right hand side of (27) in terms of Pauli matrices, obeys the quaternionic algebra of the second rank spinors (Cf. Ref. [9, 11, 10] for details).

To put it briefly, the quaternion number field obeys the group property and this leads to a number system of quadruplets as a minimum extension. In fact one representation of the two dimensional form of the quaternion basis elements is the set of Pauli matrices as in (27). Thus a quaternion may be expressed in the form

\[ Q = -i \tau_\mu x^\mu = \tau_0 x^4 - i \tau_1 x^1 - i \tau_2 x^2 - i \tau_3 x^3 = (\tau_0 x^4 + i \vec{\tau} \cdot \vec{r}) \]

This can also be written as

\[ Q = -i \left( \begin{array}{cccc}
  ix_4 + x_3 & x_1 - ix_2 \\
  x_1 + ix_2 & ix_4 - x_3
\end{array} \right). \]

As can be seen from the above, there is a one to one correspondence between a Minkowski four-vector and \(Q\). The invariant is now given by \(\bar{Q}Q\), where \(\bar{Q}\) is the complex conjugate of \(Q\).

In this description we would have from (27)

\[ [x^i \tau^i, x^j \tau^j] \propto \epsilon_{ijk} \tau^k \neq 0 \]

(29)

In other words, as (29) shows, the coordinates no longer follow a commutative geometry. It is quite remarkable that the noncommutative geometry (29) has been studied by the author in some detail (Cf. [8]), though from the
point of view of Snyder’s minimum fundamental length, which he introduced to overcome divergence difficulties in Quantum Field Theory. Indeed we are essentially in the same situation, because as we have seen, for our positive energy description of the universe, there is the minimum Compton wave length cut off for a meaningful description [12, 3, 13].

Given (29), it has been shown that the energy momentum relation gets modified to

\[ E^2 = p^2 + m^2 - \alpha l^2 p^4 \quad (30) \]

The extra term in (30) can be related to the Darwin term (13) (which shows moreover that \( \alpha \sim 1 \)). In any case for high energies or if \( p >> 1 \), then

\[ E^2 \sim -\alpha l^2 p^4 E \]

becomes imaginary! This is true if

\[ \alpha l^2 p^4 > p^2 + m^2 \]

that is if

\[ p^2 > m^2 \quad \text{so that} \quad (\alpha \sim 1) p^2 \frac{1}{m^2} > 1 (l = \frac{1}{m}) \text{ or } p^2 > m^2 \]

which is true. All this happens when \( O(l^2) \neq 0 \) that is the noncommutative geometry (29) holds [14, 15].

Let us write (2) as (with \( \bar{\hbar} = 1 = c \))

\[ H\phi = H_{11}\phi + H_{12}\chi \quad (31) \]

and similarly we have

\[ H\chi = H_{21}\phi + H_{22}\chi \quad (32) \]

We now observe that in Quantum Field Theory, a sub space of the full Hilbert space can exhibit the complex or non Hermitian Hamiltonian.

Writing \( H = M - iN \) where \( M \) and \( N \) are real [16] we have

\[ M_{11} = -\frac{\hbar^2}{2m}\nabla^2 + \frac{e^2 A^2}{2mc^2} + (e\phi + mc^2) \]

\[ M_{21} = +\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2 A^2}{c^2} + (e\phi - mc^2) \]
\[ N_{11} = \frac{eA}{m} \hbar \nabla = N_{12} \]
\[ N_{21} = -N_{11} = N_{22} \] (33)

We can now treat \(|\phi, \chi >\) as a two state system and further it follows from the above that
\[ |\phi, \chi > (t) = \exp(-N_{12}t) \exp(-iM_{12}t) |\phi, \chi > (0) \] (34)

Equation (34) shows that the states \(|\phi >\) and \(|\chi >\) decay, but decay at different rates (Cf. also [16]).
Treating \(|\phi >\) and \(|\chi >\) as particle and anti particle, we have exactly this situation in \(B\) and \(K^0\) decay. The point here is that as in the case of the \(B\) or \(K^0\) mesons, the decay rates of the particles and antiparticles would be different, thus leading to a CPT violation. The above considerations provide an explanation. This is also the case when equation (30) holds: the foregoing considerations suggesting a similar explanation for the particle-anti particle asymmetry.

vii) We could now express the foregoing in the following terms: The Compton wavelength is the Quantum Mechanical analogue of the Einstein-Rosen bridge, in the sense that a penetration into this region leads to opposite charges and what to our description would be negative energy states and time going backwards. This "bridge" connects our "positive energy" universe with what may be called an anti universe that is one of negative energies. One of the puzzles has been the asymmetry between matter and anti matter – this could be explained in the above terms of the decay caused at very high energies.
In any case, ours is a universe that lies beyond the Compton scale (or above the minimum extension) where the negative energy states are irrelevant. We could think along the lines of \(SU(2)\) and consider the transformation [17]
\[ \psi(x) \to \exp[\frac{1}{2}i g \tau \cdot \omega(x)] \psi(x). \] (35)

This leads to a covariant derivative
\[ D_\lambda \equiv \partial_\lambda - \frac{1}{2} i g \tau \cdot W_\lambda, \] (36)
as in the usual theory, remembering that \(\omega\) in this theory is infinitessimal.
We are thus lead to vector Bosons \(W_\lambda\) and an interaction like the strong
interaction, described by

\[ W_\lambda \rightarrow W_\lambda + \partial_\lambda \omega - g\omega \Lambda W_\lambda. \]  

(37)

However we must bear in mind that all this would be valid only within the Compton time, inside this Compton scale Quantum Mechanical bridge. Further, as noted by Feshbach and Villars, the negative energy solutions are to be identified with anti particles. So this vector boson interaction would manifest itself as a new high-energy (non-electromagnetic) interaction between particles and anti-particles.

vi) Finally keeping in view the latest findings from LHC, it appears that in the high energy p-p collisions, the product particles display correlations. This has posed a puzzle. We could describe this in what may be called a genetic model. We have to think of the colliding particles as parents. They have genes or information. These are spacetime related properties including for example momenta, energy, relative locations, conservation laws and so forth. The product particles are children who carry away some of these properties in the form of correlations. In fact entanglement could also come in the same category. However these ”genetic bits of information” cannot be equated with hidden variables because the former are completely probabilistic.

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