Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
A singular value decomposition entropy approach to assess the impact of Covid-19 on the informational efficiency of the WTI crude oil market

G. Espinosa-Paredes a, E. Rodríguez b,⁎, J. Alvarez-Ramírez c

a Area de Ingeniería en Recursos Energéticos, Mexico
b Area de Computación, Mexico
c Area de Ingeniería Química, División de Ciencias Básicas e Ingeniería, Universidad Autónoma Metropolitana-Iztapalapa, Apartado Postal 55–534, Iztapalapa, CDMX, 09340, Mexico

A R T I C L E   I N F O

Article history:
Received 15 December 2021
Received in revised form 12 April 2022
Accepted 18 May 2022
Available online 23 May 2022

Keywords:
Crude oil market efficiency
Singular value decomposition entropy
Covid-19
Pandemic

A B S T R A C T

This work investigates the impact of the Covid-19 outbreak on crude oil market efficiency. The approach is based on the singular value decomposition (SVD) entropy. Iso-distributional surrogate data test was used to contrast the results against random patterns, and phase randomization based on Fourier transform was used to assess nonlinearities. The analysis considered the WTI market and focused on the Covid-19 pandemic period January 2020–November 2021 and contrasted with the long preceding period from January 2000 to date. It was found that the crude oil market was informationally efficient most of the time with small sporadic deviations from efficiency in the pre-Covid-19 years. The Covid-19 period exhibited the largest deviations from efficiency, mainly in the first months of the outbreak, accompanied by a marked reduction of nonlinear components. The analysis was conducted for different scales, and the results showed that the deviations from efficiency were more pronounced for quarterly scales. For the sake of comparison, the analysis was also carried out on the trading volume dynamics and the results showed that the market activity is not fully random. The dynamics of the trading volume exhibited significant deviations from the randomness behavior when the crude oil market was efficient, and a behavior that was consistent with nonlinear patterns. The opposite behavior was noted for stages when the crude oil market showed strong deviations from efficiency. Overall, the findings of this study suggest an increasing opportunity for crude oil price predictions and abnormal returns during the Covid-19 pandemic.

© 2022 Elsevier Ltd. All rights reserved.

1. Introduction

The Covid-19 pandemic has had a great impact on the social and economic activities in the recent 21 months. The outbreak started with the report of a cluster of cases of pneumonia with atypical symptoms by the Wuhan Municipal Health Commission on December 31, 2019 in Wuhan City, Hubei Province, China. Quickly, the infection spread to the whole world, and many countries took actions adjustments in human activity to reduce the impact on the population. By April 2020, about half of the world’s population was under some form of lockdown. More than 4.0 billion people in more than 100 countries, including US and Europe, were asked or ordered to stay at home by their governments. The lockdown had an immediate negative impact on the economic activity, adding uncertainty and risk to inversions and consumption. The Covid-19 outbreak damaged the economic activity, and in 2020 most economies fell to recession. In late February and March 2020, the stock markets crashed, with major indices dropping 20–30%. The shock in the different sectors of the economy has been long-lasting, with its adverse effects still perceived in the current labor and world trade.

1.1. Impact of Covid-19 on financial markets

The impact of the Covid-19 pandemic on financial markets has been documented in the recent two years [1]. For instance, Zhang et al. [2] showed that the shock induced by the Covid–19 outbreak has had a persistent effect on the long-range structure of global markets. Minil et al. [3] used multifractal detrended fluctuation analysis to show that the COVID-19 pandemic has impacted positively the cryptocurrency market efficiency. Lahmiri and Bekiros [4] used the largest Lyapunov exponent and approximate entropy computations to study the impact of the Covid-19 outbreak on the stability and sequential irregularity of equity and cryptocurrency markets. It was concluded that the Covid-19 pandemic imposed a severe shock on financial markets, such that investment in cryptocurrencies was highly risky. Choi [5] used detrended fluctuation analysis to analyze the efficiency of eleven sectors of the S&P-500 market. It was shown that the Covid-19 outbreak introduced a shift in the price return behavior, from anti-persistence to persistence. Recently, Assaf et al. [6] used a time-varying lifting method to estimate...
the Hurst exponent for cryptocurrencies. A marked change in the long-term memory behavior was observed during the Covid-19 period. Arouzet et al. [7] found that the volatility of cryptocurrency markets suffered a temporary shock in its long-range correlation structure in the Covid-19 outbreak by March 2020.

1.2. Impact of Covid-19 on the crude oil market

The uncertainty of the Covid-19 evolution and the induced risks in the financial and economic activities provoked the most severe downturn in the crude oil market in recent years. In April 2020, crude oil prices plunged to historic low levels as a consequence of a loss of 1/3 of global demand. April 20 saw WTI prices decrease from 17.85 to −37.63 $/bbl, being the largest one-day drop for US crude in history. Promptly, the impact of the Covid-19 pandemic was a matter of intensive research. Narayan [8] employed a threshold regression model and reported that the number of infections had a limited effect on the crude oil price in the first days of the Covid-19 outbreak. Gil-Alana and Monge [9] used the Whittle function in the frequency domain and concluded that the oil price series is mean reverting, implying that the shock will be transitory, albeit with long-lasting effects. Wang et al. [10] studied the cross-correlations between crude oil and agricultural futures markets (e.g., London Sugar and USA Cotton #2), concluding that the co-dependence of the agricultural futures with the crude oil market increased after the emergence of COVID-19. Zhang and Hamori [11] studied the return and volatility spillover between the COVID-19 pandemic in 2020, showing that the impact of Covid-19 on the volatility of the oil market exceeds that caused by the 2008 global financial crisis. Le et al. [12] considered autoregressive distributed lag bounds testing with a structural break to study the crude oil price behavior in the period January 17–September 14, 2020. The results revealed that increases in Covid-19 pandemic cases, US economic policy uncertainty, and expected stock market volatility contributed to the fall in the WTI crude oil price. Recently, Gharib et al. [13] applied log-periodic power-law singularity bubble indicators to explore the dynamic bubbles of oil prices and predict their crash times. The analysis showed that the WTI market experienced a negative financial bubble during the COVID-19 outbreak.

1.3. Informational efficiency of crude oil markets

Crude oil markets can be considered complex systems in the sense postulated by Kwapień and Drożdż [14]. Crude oil markets are open systems that interchange information with surrounding systems and adaptively modify their internal structure and behavior in the process of self-organization [15]. Flexibility and adaptation in the face of exogenous changing conditions are key characteristics of crude oil markets. A central issue in the operation of financial and commodity markets is the absence of exogenous bias and possible manipulations of the price outcomes. The efficient market hypothesis (EMH) is a central concept to deal with the informational fairness of financial markets [16]. In its weak form, the EMH indicates that price evolution follows a random walk, and as such, it cannot be predicted. In this way, the EMH is associated with the idea of a random walk, a concept used to characterize a price series where all subsequent price changes represent random departures from previous prices.

The study of the informational efficiency of crude oil markets is rich in results and controversy. Some salient works include Tabak and Cajueiro [17] which showed that the market is becoming more efficient with time, probably caused by market liberalization. Alvarez-Ramirez et al. [18] showed that the market efficiency was not uniform but depended on both scale and time over the period 1986–2009. In turn, this result was in line with the concept of the adaptive market hypothesis [15] that the market participants adapt their expectations and risks to the informational flow. Zhang et al. [19] utilized a time-varying CAR (1)-TGARCH (1,1) model and concluded that the market was extremely efficient up to 2014. Ghazani and Ebrahimi [20] used data covering the period 2003–2018 to analyze the crude oil market efficiency in the context of adaptive expectations. The results showed that the Brent and WTI markets possessed the highest efficiency levels. Kristoufek [21] used fractal rescaled range and detrended fluctuation analysis to characterize the crude oil market efficiency. The results, which were contrasted with the Geweke-Porter-Hudak estimator, showed that the market has been efficient most of the time with some deviations linked to the 2008 global financial crisis. Studies on the impact of the Covid-19 pandemic on the informational efficiency of the crude oil market are still scarce. Gil-Alana and Monge [9] used long-memory techniques to show that the crude oil market was consistent with the EMH in the pre-Covid-19 period and became inefficient in the pandemic outbreak. Mensi et al. [22] used asymmetric detrended fluctuation analysis to assess the effect of the oil price variability in the long memory and weak-form efficiency of stock markets. It was shown that asymmetric crude oil price dynamics is a key driver of the trend of stock dynamics. Wang et al. [23] used Hurst exponent computations to show that the informational efficiency of crude oil markets declined during the Covid-19 pandemic.

Okorie and Lin [24] pointed out that level of informational efficiency of markets is key to profiteering by strategic players. In this regard, exogenous shocks, such as the Covid-19 pandemic, play a significant role in the nature of informational efficiency. Out of the work by Gil-Alana and Monge [9] that focused on the first months of the pandemic, detailed studies of the impact of Covid-19 pandemic on the informational efficiency of the crude oil markets are lacking. The issue is important since Gil-Alana and Monge [9] hypothesized that the Covid-19 outbreak could have potential long-lasting effects. In this work, we try to fill the literature gap by examining the time-varying informational efficiency of the crude oil market, and in this way assess the impact of the Covid-19 pandemic. The methodology is based on computing the singular value decomposition (SVD) entropy of the log returns over the long period 2000–2022, with particular emphasis on the Covid-19 period from January 2020 to April 2022. The results are contrasted with an isodistributional surrogate data approach to determine the closeness to the fulfillment of the EMH, and also with phase randomized surrogate data to assess nonlinear behavior linked to the market dynamics.

2. Methodology

2.1. Entropy estimation

Entropy is an index associated with a state of disorder, randomness, or uncertainty of a system. In recent decades, the concept of entropy has been widely used for temporal complexity analysis of real-world signals. An important feature is that independent and identically distributed white noise is assumed to have maximal entropy and disorder. As a consequence, entropy is commonly used to assess the randomness of signals. The most used entropy measures include those introduced by Shannon [25], Renyi [26], Kolmogorov [27], and Eckmann and Ruelle [28]. These entropy concepts were proposed from strong theoretical grounds and their practical implementation presents some troubles. In this regard, the approximate entropy (ApEn) by Pincus [29], and the sample entropy (SampEn) by Richman and Moorman [30] were proposed as practical approaches to quantify the amount of regularity and the unpredictability of fluctuations over time-series data. The approaches rely on computing the occurrence and repeatability of patterns contained in a sequence. For real-time sequences, exact matching of patterns has a very low probability. Pincus [29] proposed to compute the approximate matching modulus of a given tolerance that should be adjusted to obtain reproducible results. ApEn and SampEn have been extensively used in many fields (e.g., physics, physiology and finance) to reveal the presence of complex patterns and their link to underlying dynamical mechanisms. In many instances, one is interested in computing the entropy over different time scales. The ApEn and SampEn computations are quite sensitive to the length of the sequence, such that establishing a scale for which the computations are
stable offers some troubles. Costa and Goldberger [31] addressed this issue by considering the computation of entropy over coarsely grained time series obtained by averaging over a prescribed scale factor. By doing this, the coarse-grained time series is a smoothed (i.e., low-pass filtered) version of the original time series. The multi-scale approach solved some problems presented by the ApEn and the SampEn, although the computations for relatively small time series can be inaccurate.

The analysis of the crude oil price dynamics carried out in the present work focuses on the relevant weekly, monthly and quarterly time scales. To achieve this task, the analysis will use entropy computations based on singular value decomposition (SVD). This approach allows a tight definition of the time scale and, in contrast to ApEn and SampEn, does not rely on a tolerance parameter for parameter matching [32]. Consider a time series given as $X(t_i)$, $i = 1, ..., N$. The problem under scrutiny is to decide whether the above time series contains serial correlations. To this end, consider a sequence of size $n$ with leading time $t_i$:

$$X(t_i;n)=\{X(t_{i-\tau};n)\}, ..., X(t_{i-1}), X(t_i)$$

(1)

If the time series $X(t_i)$ is affected by serial correlations, then the sequence Eq. (1) has similarities with past sequences of the same size. To test the similarity of the sequence Eq. (1) with past sequences, consider the following square matrix of $n$ lagged subsequences:

$$M_X(t_i;n)=\begin{bmatrix} X(t_i;n) \\ X(t_{i-1};n) \\ \vdots \\ X(t_{i-n+1};n) \end{bmatrix}$$

(2)

The similarities of the sequence Eq. (1) with past events should be reflected in the structure of the matrix $M_X(t_i;n)$. If similarities are not present at all, Eq. (2) corresponds to a random matrix. Although a matrix $M_X(t_i;n)$ containing correlations might be full rank (rank ($M_X(t_i;n)$) = $n$), the presence of correlation indicates that the information tends to be aggregated around a subspace. In contrast, the absence of correlations would imply that a dimensionality reduction might lead to important information loss. That is, all row vectors in a non-correlated matrix contain the same amount of information, such that no one-row vector can be discarded without important information loss.

Based on the above, the analysis focuses on how the information on the dynamics of a process is preferentially aggregated about a subspace. The singular value decomposition (SVD) is a suitable tool to address the question of whether the dimensionality of the matrix $M_X(t_i;n)$ can be reduced without a marked loss of information and hence to decide on the correlations contained in the sequence. The SVD is a factorization of real or complex matrices that generalizes the Eigen-factorization to the correlations contained in the sequence. The SVD is essentially a change of coordinates via rotations (of $U(t_i;n)$ and $V(t_i;n)$) and rescaling ($\mathcal{A}(t_i;n)$). In particular, the singular values of a matrix $M_X(t_i;n)$ can be seen as quantifying the geometry of the transformation $M_X(t_i;n)B$, where $B$ is the unit ball. In general, the transformed ball is an ellipsoid where the singular values correspond to the length of its principal axes.

The singular values of the matrix $M_X(t_i;n)$ recover the correlation information of the time series $X_i$ for a horizon of $n$ discrete times. The entropy is commonly used to define indices of the degree of interdependence of the row/columns of a matrix [36]. Sufficiently, the entropy is an index that reflects the average information contained in a process and is also a measure of the degree of randomness in the matrix. The higher the entropy, the higher the information required to reconstruct the underlying process dynamics. Entropy is estimated from the distribution of the singular values of the matrix $M_X(t_i;n)$. In a first step, the singular values are normalized as follows:

$$\lambda_j(t_i;n)=\frac{\lambda_j(t_i;n)}{\sum_{j=1}^n \lambda_j(t_i;n)}$$

(4)

Subsequently, the entropy of the matrix $M_X(t_i;n)$ is computed by

$$S_X(t_i;n)=-\frac{1}{\ln(n)} \sum_{j=1}^n \lambda_j(t_i;n) \ln \left(\lambda_j(t_i;n)\right)$$

(5)

For a perfectly non-correlated process (e.g., white noise), there are no preferential directions of information accumulation and $\lambda_j(t_i;n) = 1/n$, $j = 1, ..., n$, such that $S_X(t_i;n) = 1$. For a matrix containing correlations and reflecting preferential information directions, one should get that $S_X(t_i;n) < 1$.

The entropy value $S_X(t_i;n) = 1$ for uncorrelated sequences is a theoretical reference that holds asymptotically (i.e., for very long sequences). In practice, entropy analysis should deal with sequences of finite size. Also, one would like to explore the entropy for short sequences associated with relatively small scales (e.g., days for financial time series). In this way, the SVD entropy depends on the scale and should be smaller than one for sequences of finite size.

Severe events like the 2008 Great Recession, the 2015 Excess Production and the Covid-19 outbreak (Fig. 1a) exacerbated the volatility (Fig. 1b) in the crude oil market. Intuitively, large shocks are likely to disrupt the dynamic behavior of financial markets. Despite some insights from simple visual inspection, the time series is sufficiently complex to draw credible conclusions on the effects of shocks in, e.g., informational efficiency and the propagation of the effects over a wide range of time scales. Existing methods to analyze the complexity of time series (e.g., Hurst exponent estimators and approximate entropy computations) rely on relatively long sequences to obtain stable statistics. For instance, the well-known rescaled range (R/S) analysis and the detrended fluctuation analysis require a sequence of length non-smaller than 300 observations to analyze scales of 60 units [18]. The entropy computations based on SVD alleviate such limitations by requiring several observations that is only twice the size of the scrutinized scale. For instance, the analysis for scales of 5 and 22 business days (i.e., weekly and monthly scales) requires only 10 and 44 observations. In turn, this advantage provides a more detailed view of the variations of the entropy over a given period.

2.2. Randomness test

The SVD entropy will be computed to analyze the temporal variations of the informational efficiency of the crude oil market. Most approaches for characterizing the informational efficiency are based on testing the weak form of the EMH. In this way, one should test the hypothesis that the price dynamics is consistent with a random walk behavior [15]. The reader is referred to the recent critical review by Degitis and Novickýte [37] and references therein for a description of
the methods used to test informational efficiency. The existence of serial correlations via the Box-Pierce statistics is a widely used approach for testing the randomness of return time series. Run and variance ratio tests [38] remain among the most common approaches to checking market efficiency. The computation of the scaling (e.g., Hurst) exponent via the rescaled-range (R/S) [15] and DFA methods are increasingly used for a wider characterization (e.g., adaptive market hypothesis) of return time series.

In terms of the SVD approach described above, one should decide whether the entropy of a tested return sequence \( X(t; n) \) corresponds to the entropy of a random sequence. If the probability distribution \( P(X) \) that generated the values of the sequence \( X(t; n) \) is available, a suitable approach is to generate many random sequences of size \( n \) and to compute the statistics of the SVD entropy to obtain the corresponding confidence intervals (CI). However, the exact distribution is hardly available in practice for a given process. Bootstrapping estimates can be used by considering an approximate (i.e., empirical) distribution. The isodistributional surrogate data approach proposed by Theiler et al. [39] was used to test randomness. In this way, the following procedure is proposed to estimate the CI for randomness: a) Compute \( N_{in} \) shuffled sequences \( X_{sh}(t; n) \) from the original sequence \( X(t; n) \). In principle, shuffling destroys serial correlations while retaining the statistical distribution of values. That is, the sequences \( X_{sh}(t; n) \) and \( X(t; n) \) were generated from a common distribution \( P(X) \). b) Compute the SVD entropy for the shuffled sequences \( X_{sh}(t; n) \), which reflects the entropy of a random sequence. c) Carry out the statistical analysis of the \( N_{in} \) SVD entropy values to obtain the corresponding CI for randomness. The test used 5000 randomized sequences to compute the 80, 90 and 95% confidence intervals.

### 2.3. Nonlinearity test

The randomness test should provide insights on whether the crude oil market dynamics involve hidden predictable patterns. Diverse mechanisms may be responsible for deviations from random behavior. Watorek et al. [40] showed that the crude oil market is driven by nonlinear mechanisms and the effects are reflected as multifractal dynamics. Shuffling destroys all the temporal correlations, both the linear and nonlinear ones. The point was addressed by carrying out a test for assessing the nonlinear nature of the crude oil market dynamics. Phase randomization is a strategy that preserves the amplitude distribution while destroying the nonlinear ordering contained in the phase structure [41]. The method is widely used in the field of nonlinear data analysis for testing for weak nonlinearities. Fourier transform-based algorithms conserve the amplitude distribution in real space and reproduce the power spectrum of the original data set very accurately. The algorithm used in the present work is the following: a) given a sequence \( X(t; n) \), perform discrete Fourier transform to obtain the corresponding sequence representation in the frequency domain \( \hat{X}_f(j; n) = X_{re}(j; n) + iX_{im}(j; n) \), where \( f \) is the set of frequencies. b) The transformed sequence can be represented in polar coordinates given by the magnitude \( |X_f(j; n)| \) and the phase \( \phi[X_f(j; n)] \). The phase represents lag ordering of the different spectral components relative to a driving oscillatory signal. The underlying assumption is that the phase is the
structure behind the nonlinear behavior of a signal. In this way, the destruction of the phase ordering should imply the destruction of nonlinearities. In this way, the second step is to randomize the phase ordering relative to the frequencies $f_i$. This is done by shuffling the phase sequence $\phi \{X_f(f_i; n)\}$ to obtain the shuffled sequence $\phi_{sh} \{X_f(f_i; n)\}$. This procedure preserves the assignation of the magnitude $|X_f(f_i; n)|$ for each frequency (i.e., preserves the power spectrum) while destroying the original phase assignation $\phi \{X_f(f_i; n)\}$, c) Carry out the inverse Fourier transform for the phase shuffled sequence $\{|X_f(f_i; n)| \phi_{sh} \{X_f(f_i; n)\}\}$ to obtain the real-time sequence $X_{PhSh}(t; n)$. Apply SVD on $X_{PhSh}(t; n)$ to obtain the corresponding entropy. d) Apply the procedure for $N_{PhSh}$ phase randomization realizations and obtain the corresponding confidence intervals CI. The test was carried out for 5000 phase randomized sequences to compute the 80, 90 and 95% confidence intervals.

2.4. Implementation

The SVD entropy described above was implemented over an overlapping sliding window scheme for different values of the window size $n$. The sliding length was 15 observations for the analysis of the whole period 2000–2022. The sliding length was shortened to 5 observations to obtain a more detailed view of the COVID-19 period. The time scale was identified with the size of the sliding window $n$. In this way, the window sizes of 5, 22 and 66 business days corresponded to the weekly, monthly and quarterly scales, which are relevant for the market operation. In particular, the monthly scale is important since crude oil futures expire each month, on the third business day before the 25th calendar day of the month preceding the delivery month.

3. Data

The analysis of the crude oil market considered the West Texas Intermediate (WTI) prices and volume for the period from January 2000 to April 2022 (https://finance.yahoo.com/). The size of the time series is 5428 observations. Table 1 presents the results of the descriptive statistics for the price and volume log differences. Negative skewness and positive kurtosis were exhibited by the two differences. Also, the Shapiro-Wilk test rejects the normality of the log differences for a 1% level.

The scrutinized period contains several salient events, including the 2008 Great Recession triggered by the subprime mortgage crackdown and the recent price downturn provoked by the Covid-19 lockdown. Fig. 1a and b exhibit respectively the price and the log difference behaviors. In 2008:Q2, the WTI price reached its highest value of about 145 $/bbl. However, the advent of the subprime mortgage crisis that led to the Great Recession tumbled the price to very low values of about 35 $/bbl in 2008:Q4. The WTI spot prices down sharply in 2014:Q4. An excess of production relative to the aggregated demand caused the decline of the crude oil price. US increased 1.2 million barrels per day in 2014, up 16% from 2013, achieving 8.6 million bbl/d to be the highest production level in nearly 30 years. The Covid-19 lockdown in March 2020 was the more recent event that moved down the prices from about 70 to 20 $/bbl. The socio-economic panic induced by the uncertainty of Covid-19 pandemic led the price to negative values on April 20, 2020. The price recovered quickly from such a single-day event, achieving a positive rally to reach values of about 80 $/bbl in November 2021. The aforementioned events increased the market volatility (Fig. 1b), with the highest volatility levels associated with the Covid-19 lockdown. However, such high volatility had a short-lasting effect, in contrast to the volatility linked to the Great Recession that lasted about one year. The trading volume is an important technical indicator because it reflects the overall activity of the crude oil market. Fig. 1c shows the trading volume over the last 20 years. The trading volume was relatively low (about 0.06–0.1 million) in the first six years and increased to about 0.4 million in 2012. The subsequent two years witnessed a gradual decline in the trading volume. However, the increased US production elevated the trading volumes to values of 0.8–1.0 million. The Covid-19 lockdown induced a sharp reduction of the trading volume to achieve values as lower as 0.24 million in March–April 2020, which reflected the reduction of the worldwide economic activity. The volatility of the trading volume (Fig. 1d) has not shown serious shocks, even in the face of important market downturns.

4. Results and discussion

4.1. Price

Fig. 2a presents the variations of the SVD entropy for the weekly scale. The gray band denotes the 95% confidence interval for randomness computed from 5000 randomized versions of the window data. SVD entropy variations within the gray band indicate that the crude oil market meets the weak-form of the EMH with 95% confidence. Otherwise, the market is not efficient with a probability 5% since the sequence of log returns is not consistent with a random pattern. It should be stressed that the points located above the gray band still reflect random sequences since the band was computed for 95% confidence. Those points above the band should correspond to the right tail (e.g., extreme events) of the entropy distribution obtained from randomized sequences. Fig. 2a shows that the crude oil market was efficient most of the time in the recent 20 years. Short-lasted deviations from randomness were displayed in 2002:Q1, which were probably caused by political instabilities in Mid-East Asia region before the 2003 Iraq War. A large deviation from randomness was displayed in 2007:Q1, which can be attributed to a price run-up caused by rising aggregated demand confronting stagnating world production. Interestingly, the 2008 subprime mortgage financial crisis did not cause visible informational efficiency deviations for the crude oil market over weekly scales. A deviation from randomness was exhibited in 2014:Q4, which can be ascribed to an excess of crude oil production that led to a reduction in prices from about 100 to 50 $/bbl in 2015: Q1. Kristoufek [21] reported similar deviations from efficiency by using the rescaled range analysis. The deviation from randomness in 2020:Q3 coincided with the start of the Covid-19 second wave US and Europe. A more detailed view of the SVD entropy variations in this period is presented in Fig. 2b. The red arrows indicate two prominent peaks that occurred in 2020:Q3 and 2021:Q1. Interestingly, the entropy remained in the randomness band despite the decrease in its value. This means that the market dynamics have been unpredictable over weekly scales for the Covid-19 period. The lower bound of the confidence band can be seen as an index of the market complexity. In this way, the peaks visible in Fig. 2b suggest that the crude oil market complexity was reduced by the effect of, e.g., price trends, while retaining a certain degree of unpredictability over weekly time scales. Fig. 2c shows that the entropy of the price return remained most of the time in the 95% CI band of phase randomized sequences. This means that, except for short times, the dynamics of the crude oil price in the period 2000–2022 were consistent with a linear behavior. This is an expected result since

| Table 1 |
| --- |
| **Descriptive statistics of the WTI price and trading volume.** |
| **Price** | **Volume** |
| Observations 5302 | 5302 |
| Min $-0.1382$ | $-1.8431$ |
| Max 0.1226 | 1.7943 |
| Mean $1.2258 \times 10^{-4}$ | $1.1564 \times 10^{-4}$ |
| Median $2.4234 \times 10^{-4}$ | $6.9432 \times 10^{-3}$ |
| SD $1.4381 \times 10^{-2}$ | $0.1704$ |
| Skewness $-8.9923 \times 10^{-2}$ | $-0.8358$ |
| Kurtosis $18.2015$ | $14.9051$ |
| Shapiro-Wilk* 0.9136 | 0.9041 |

* Significant at 1% level.
the time scale of five business days reflects the dynamics for short horizons, which correspond to local patterns. The Covid-19 outbreak in February–April 2020 induced a short-lasted reduction of the low boundary of the 95% CI for the nonlinearity test (Fig. 2d). This departure might be attributed to the abrupt price reductions to historic low and even negative levels. By the end of March, the price bounced back to positive price values of about 20 $/bbl, restoring the linear behavior exhibited in the recent 22 years. Overall, the results in Fig. 2 showed that the price return dynamics were consistent with the EMH and a linear behavior over the weekly scale. That is, the price dynamics were driven by linear mechanisms and the evolution was not predictable most of the time.

Fig. 3a shows the variation of the SVD entropy for monthly time scales. Similar to the weekly scale, the crude oil market was informationally efficient most of the time in the last 21 years. Short duration deviations from efficiency were displayed in 2006:Q2, 2017:Q1 and 2018:Q4. A link between these deviations with socio-political and economic events is not clear at all. The red arrow in Fig. 3a indicates important deviations from randomness, meaning that in the corresponding periods the crude oil price contained some degree of predictability. Two marked peaks can be observed in 2020:Q1 and 2020:Q2. The larger one occurred in February–March 2020 (Fig. 3b) and coincided with the first Covid-19 lockdown, which led to a major disruption of the economic activity. As a consequence of the uncertain course of the Covid-19 pandemic, the WTI price dropped to historical minimums of negative levels. It is noted that the Covid-19 had a major negative effect on the informational efficiency for monthly scales in the last 21 years. Smaller deviations from the informational efficiency were displayed in 2021, which may be linked to subsequent Covid-19 waves and the disruption of the economic supply chains. The variation of the SVD entropy relative to the phase randomization is shown in Fig. 3c for the whole scrutinized period from 2000 to 2022. Except for a very small period at about March 2020 indicated by an arrow, the entropy of the price return was higher than the entropy of the phase randomized samples at a 95% CI. This means that the price return dynamics are driven by nonlinear mechanisms that incorporate the complexity to the market. Such nonlinear effects did not add predictability to the price dynamics but instead contributed to the fulfillment of the informational efficiency. Fig. 3d presents the details of the phase randomized entropy for a period around the COVID-19 outbreak. The marked deviation from the informational efficiency induced by the COVID-19 outbreak (Fig. 3b) was accompanied by a significant (p < 0.05) loss of the nonlinearity, which might be caused by the fast transient linked to the fast price decrease in historical, and even negative, low values. The presence of nonlinear dynamical components in the crude oil price dynamics is in line with recent results by He and Chen [42], reporting that the market efficiency is influenced mainly by a nonlinear temporal correlation mechanism instead of a non-Gaussian distribution. The results in Fig. 3 suggest that nonlinearity and informational efficiency are positively linked in the crude oil market over the monthly time scale.

The results of the SVD entropy for the quarterly scale are presented in Fig. 4a. As already shown for the monthly scale; the market has been informationally efficient most of the time for the longer quarterly horizon. The small deviation in 2009:Q2 reflects the price recovery in the aftermath of the 2008 Great Recession. The deviation is barely visible, indicating that the strong economic downturn in 2008 had only

---

**Fig. 2.** Behavior of the SVD entropy for the price log difference over the weekly scale. Panels (a) and (b) show the periods 2000–2022 and 2018–2022, respectively, and contrast the results with respect to the randomness test. Panels (c) and (d) show the periods 2000–2022 and 2018–2022, respectively, and contrast the results with respect to the nonlinearity (phase randomization) test. The gray band denotes the 90% confidence interval. Two peaks of low entropy in Panel (a) can be identified with the rise of the crude oil production by US and the Covid-19 outbreak. The two red arrows in Panel (b) indicate peaks that are coincident with the Covid-19 outbreak and the surge of the second Covid-19 wave.
price reductions. Gil-Alana and Monge [9] used the Whittle function in reduced a lot of economic uncertainties, leading to important crude oil severity of the second wave and the lack of an effective vaccine pro-
19 second wave that left a large number of infections and deaths. The similar to those of Gil-Alana and Monge [9] in the sense that the impact of break in the crude oil market are transitory, albeit with potentially long-lasting effects. The results derived from the present work are sim-
ilar to previous studies on the informational efficiency of crude markets. For instance, Tabak and Cajueiro [17] showed that the efficiency of the market was persistent to be visible its effects in the long run. Otherwise, the energy added by the sudden price drop is dissipated and the price varia-
tion is not propagated for relatively long times. The implementation of the SVD entropy computation on a sliding window aims to detect the propagation of sudden price drops. In this way, the sharp price drops in the Covid-19 outbreak were propagated over different scales, from weeks to quarters (Figs. 2 to 4). However, the sharp price drops exerted in the 2008 Great Recession did not affect the informational efficiency of the market (see Fig. 4b).

The behavior of the SVD entropy for the phase randomized samples (Fig. 4c) exhibited a pattern similar to that for the monthly scale (Fig. 3c). Except for the COVID-19 outbreak in the first semester of 2020, the entropy of the phase randomized samples was lower than the entropy of the price return series. This means that nonlinear mechanisms are involved in the formation of the crude oil price. The crude oil market was informationally efficient most of the time for the long period 2000–2022, and nonlinear mechanisms contributed to the informational efficiency over such a period. Similar results in this line were reported by Gozbasi et al. [44] showing that nonlinear dynamical mechanisms contributed to the informational efficiency of the Turkish stock market. A detail of the entropy behavior is shown in Fig. 4d, where the loss of informational efficiency in the COVID-19 outbreak coincided with a loss of nonlinear components in the price return dynamics.

The results described above showed that the WTI crude oil market has been in general informationally efficient in the last years prior-Covid-19 years. Short-term periods of deviations from randomness were exhibited, which were linked to the occurrence of financial and socio-economic events. Overall, the results are in line with previous studies on the informational efficiency of crude markets. For instance, Tabak and Cajueiro [17] showed that the efficiency of the market was increasing over time. Wang and Liu [45] showed that short-term, medium-term and long-term behaviors were generally turning into efficient behavior over time, which agrees with the results in the present study. Kristoufek [21] found that the crude oil markets have remained efficient except for some events until the outbreak of the Global Financial Crisis in 2008. Okoroafor and Leirvik [46] reported that the WTI market is persistently inefficient during financial crises, with the high

Fig. 3. Behavior of the SVD entropy for the price log difference over the monthly scale. Panels (a) and (b) show the periods 2000–2022 and 2018–2022, respectively, and contrast the results with respect to the randomness test. Panels (c) and (d) show the periods 2000–2022 and 2018–2022, respectively, and contrast the results with respect to the nonlinearity (phase ran-
domization) test. The gray band denotes the 90% confidence interval.
volatility of the efficiency in such periods. Our results suggested that this conclusion holds for the Covid-19 outbreak.

Mensi et al. [47] showed the negative impact of the Covid-19 pandemic on the crude oil market efficiency and, in line with our findings, that the market is sensitive to scales. Table 2 presents the percentage of the time the return dynamics is outside the randomness band for three different confidence intervals and the three scrutinized scales. The percentage decreased with the level of the confidence interval. More interestingly, the percentage depended on the scale [47], with the quarterly scale showing the highest percentage of deviations from randomness. In turn, this trend suggests that the Covid-19 outbreak has had the strongest effect over mid-term scales than over short-term scales (i.e., weeks). Recently, Sui et al. [48] showed that the Covid-19 epidemic is a key factor influencing oil price, and that the effect may spread from the stock market due to speculative investor behavior. On the other hand, Le et al. [12] determined that the increases in Covid-19 pandemic cases, US economic policy uncertainty, and expected stock market volatility had an important contribution to the fall in the WTI crude oil prices in the first semester of 2020. The Russia-Saudi Arabia crude oil price war in March 2020 could exacerbate the effect of the Covid-19 pandemic, driving the prices to historical lows. Our results showed that the price fall and subsequent recovery were accompanied by a marked reduction of the price dynamics complexity as well as of the informational efficiency, which was reflected for mid-term scales (i.e., monthly and quarterly).

4.2. Trading volume

The trading volume has been barely considered in the analysis of the predictability of the crude oil market. The trading volume can be seen as an indicator of the intensity of the market activity. The analysis of the predictability of the trading volume dynamics should provide valuable insights into the informational efficiency of the crude oil market.

The course of the trading volume SVD entropy for weekly scales in the last 22 years is shown in Fig. 5a. Similar to the case of crude oil prices, the trading volume was unpredictable most of the time, showing isolated short duration periods of low entropy. The peaks in 2006:Q2 and 2013:Q2 are not related to a particular market event. Interestingly, the 2008 Great Recession did not impact the trading volume predictability as the entropy remained in the randomness band. However, the Covid-19 had an important effect on the trading volume dynamics, which was reflected by the several peaks in Fig. 5b, with the largest

| Table 2 |
|-----------------|-------|-------|-------|
|                | Scale | 85    | 90    | 95    |
| Price           | Weekly| 13.47 | 6.30  | 4.12  |
|                 | Monthly| 14.51 | 7.93  | 5.62  |
|                 | Quarterly| 18.64 | 10.27 | 7.42  |
| Volume          | Weekly| 23.39 | 14.71 | 8.27  |
|                 | Monthly| 31.96 | 24.36 | 16.32 |
|                 | Quarterly| 89.76 | 86.68 | 77.17 |
peak in April 2021. Similarly, Zhang and Hamori\cite{11} reported that the impact of Covid-19 on the crude oil markets exceeded that of the 2008 financial crisis. Although the entropy was reduced to values of about 0.5, it remained in the randomness band. This means that the dynamics of the trading volume were unpredictable for the weekly scale, although the volume variations exhibited reduced complexity. Most of the time, the dynamics of the trading volume exhibited a linear behavior for weekly scales (Fig. 5c). Sporadic departures from the 95% CI band for the phase randomized samples were exhibited, with no salient deviations for the COVID-19 outbreak period (Fig. 5d).

The entropy for the monthly scale (Fig. 6a) showed important deviations from randomness in the period 2000–2006. The large peak at about 2006:Q3 represented a marked complexity reduction to make way for a behavior compatible with randomness. This transition coincided with an important increase in the trading volume (red arrow in Fig. 1c), which indicates that the WTI market reached a more intense and complex activity. The Covid-19 outbreak did not affect the trend exhibited in the last 13 years (Fig. 6b). The trading volume dynamics exhibited nonlinear dynamics over the period 2000–2022, except for a small period in 2004–2005 when the entropy decreased to values similar to those exhibited by the phase randomized samples (Fig. 7c). The COVID-19 period did not exhibit important variations of the behavior observed in the previous years (Fig. 7d), which suggests that the pandemic has added a weak effect on the dynamics of the trading volume.

It should be emphasized that, similar to the log price differences, the degree of unpredictability (i.e., distance to randomness) of the trading volume is sensitive to the scale. Table 2 shows that the percentage of the time the trading volume entropy is outside the confidence interval is visibly higher than that of the log price differences. The percentage values increased with the scale and can achieve values as high as 89.76% for 85% confidence interval and quarterly scale.

While crude oil price dynamics have received a lot of research attention, results on trading volume are scant. Abdullahi and Kouhy\cite{49} did not find evidence of a strong link between trading volume and returns, suggesting that trading volume and returns are not driven by the same information flow. Our results pointed out in a similar direction by indicating that price tends to be unpredictable while trading volume dynamics contain some regularities. In contrast, Ji and Zhang\cite{50}...
reported a significant Granger causal relationship from the return to trading volume (F-statistic = 3.61). The results presented in Figs. 5 to 7 showed that the Covid-19 outbreak has had a mixed impact on the predictability of the trading volume. While the entropy has decreased in the short term (i.e., weekly scale), it has been increased in the medium-term (i.e., quarterly scale). In turn, this would suggest that the Covid-19 outbreak has had a direct impact on short-term market activity through, for example, reduced trade volume, although its effects are spreading on longer scales.

5. Concluding remarks

This work investigated the impact of the Covid-19 outbreak on the informational efficiency of the crude oil market. The analysis was performed singular value entropy (SVD) with a randomness test on daily data from the WTI market. To gain a broad view of the impact of Covid-19, the analysis was conducted for the period from January 2000 to November 2021, with a particular emphasis on the Covid-19 period from January 2020 to date. The results of the analysis showed that the crude oil market was efficient most of the time in the years before Covid-19. However, market efficiency was not met in some prominent periods of the Covid-19 pandemic. The deviations were most prominent for the medium-term scales, from month to quarter, indicating that the pandemic likely induced lasting shocks in market dynamics. In contrast to price dynamics, the analysis revealed that trading volume dynamics have a certain degree of predictability as its behavior showed marked deviations from randomness. In general, the results reported in this work indicated that (a) in the last 21 years, the Covid-19 outbreak induced the largest deviations in information efficiency, mainly in the first months of the pandemic when economic activity was seriously affected; (b) the most severe effects of the Covid-19 pandemic on market efficiency were shown in the first months of 2020 and the impact spread over the following months in terms of fluctuations in market complexity; and (c) the market dynamics were dynamics for short time scales (e.g., week) and emerged for longer time scales (from month to quarter). Nonlinearities have had an important role in the prevalence of the informational efficiency of the crude oil market, mainly in the last ten years. The economic uncertainty brought on by the Covid-19 outbreak has likely led to incorrect pricing in the crude oil market and an increased likelihood of abnormal returns.

Some insights on the opposite pattern between price and trading volume during exogenous shocks (e.g., Covid-19 outbreak) can be drawn from the herding behavior effect [51]. Herding occurs in markets when investors follow the crowd instead of their analysis. Large, unfounded market rallies and sell-offs are often based on a lack of fundamental support. The fast sell-offs induce a strong market directionality, leading to sharp price drops. As a consequence, the market informational efficiency has deteriorated since the information flows are not uniformly distributed among the market participants. While price achieves a downturn directionality, the sell-off intensity, as reflected by the trading volume, increases its uncertainty as the market participants enter a phase of high uncertainty on when to sell their assets. Such uncertainty is reflected as an increased unpredictability of the trading volume.
CRediT authorship contribution statement

G. Espinosa-Paredes: Conceptualization, Writing - review & editing.
E. Rodriguez: Writing - original draft. J. Alvarez-Ramirez: Visualization, Investigation, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Li Y, Zhuang X, Wang J, Dong Z. Analysis of the impact of COVID-19 pandemic on G20 stock markets. North Am J Econ Finance. 2021;58:101530. https://doi.org/10.1016/j.najef.2021.101530.
[2] Zhang D, Hu M, Ji Q. Financial markets under the global pandemic of COVID-19. Finance Res Lett. 2020;36:101528. https://doi.org/10.1016/j.frle.2020.101528.
[3] Mnif E, Jarboui A, Mouakhar K. How the cryptocurrency market has performed during COVID 19? A multifractal analysis. Finance Res Lett. 2020;36:101647. https://doi.org/10.1016/j.frl.2020.101647.
[4] Lahmiri S, Bekiros S. The impact of COVID-19 pandemic upon stability and sequential irregularity of equity and cryptocurrency markets. Chaos Solit Fractals. 2020;138:109936. https://doi.org/10.1016/j.chaos.2020.109936.
[5] Choi SY. Analysis of stock market efficiency during crisis periods in the US stock market: differences between the global financial crisis and COVID-19 pandemic. Physica A. 2021;574:125988. https://doi.org/10.1016/j.physa.2021.125988.
[6] Assaf A, Bhandari A, Charif H, Demir E. Multivariate long memory structure in the cryptocurrency markets: the impact of COVID-19. Int Rev Financial Anal. 2022;102132. https://doi.org/10.1016/j.irfa.2022.102132. in press.
[7] Arouxet MB, Bariviera AF, Vampa V. Covid-19 impact on cryptocurrencies: evidence from a wavelet-based Hurst exponent. Physica A. 2022;596:127170. https://doi.org/10.1016/j.physa.2022.127170.
[8] Narayan PK. Oil price news and COVID-19— is there any connection? Energy Res Lett. 2020;1(1):13176. https://doi.org/10.46557/001c.13176.
[9] Gil-Alana LA, Monge M. Crude oil prices and COVID-19: persistence of the shock. Energy Res Lett. 2020;1(1):13200. https://doi.org/10.46557/001c.13200.
[10] Wang J, Shao W, Kim J. Analysis of the impact of COVID-19 on the correlations between crude oil and agricultural futures. Chaos Solit Fractals. 2020;136:109986. https://doi.org/10.1016/j.chaos.2020.109986.
[11] Zhang W, Hamori S. Crude oil market and stock markets during the COVID-19 pandemic: evidence from the US, Japan, and Germany. Int Rev Financial Anal. 2021;74:101702. https://doi.org/10.1016/j.irfa.2021.101702.
[12] Le TH, Le AT, Le HC. The historic oil price fluctuation during the Covid-19 pandemic: what are the causes? Res Bus Finance. 2021;58:101489. https://doi.org/10.1016/j.resourpol.2021.102392.
[13] Ghazani MM, Ebrahimi SB. Testing the adaptive market hypothesis as an evolutionary perspective on market efficiency: evidence from the crude oil prices. Finance Res Lett. 2019;30:60–8. https://doi.org/10.1016/j.frl.2019.03.032.
[21] Kristoufek L. Are the crude oil markets really becoming more efficient over time? Some new evidence. Energy Econ. 2019;82:253–63. https://doi.org/10.1016/j.econo.2018.03.019.

[22] Meni W, Lee YJ, Vo XV, Yoon SM. Does oil price variability affect the long memory and weak form efficiency of stock markets in top oil producers and oil Consumers? Evidence from an asymmetric MF-DFA approach. North Am J Econ Finance. 2021;57:101446. https://doi.org/10.1016/j.najef.2021.101446.

[23] Wang Q, Yang X, Li R. The impact of the COVID-19 pandemic on the energy market—a comparative relationship between oil and coal. Energy Strategy Rev. 2022;39:100761. https://doi.org/10.1016/j.esr.2021.100761.

[24] Okorie DI, Lin B. Adaptive market hypothesis: the story of the stock markets and market dynamics. Physica A. 2014;393:571–82. https://doi.org/10.1016/j.physa.2013.07.028.

[25] Costa M, Goldberger AL, Peng CK. Multiscale entropy analysis of complex physiologic time series. Phys Rev Lett. 2002;89(6):068102. https://doi.org/10.1103/PhysRevLett.89.068102.

[26] Kolmogorov AN. New metric invariant of transitive dynamical systems and endomorphisms of lebesgue space. Dokl Acad Sci USRR. 1958;119:861–4.

[27] Gozbasi O, Kucukkaplan I, Nazlioglu S. Re-examining the turkish stock market efficiency: evidence from informational entropy analysis. Energy Policy. 2012;41:365–73. https://doi.org/10.1016/j.enpol.2011.10.057.

[28] Deeg J, Novickytë Ž, Ži d Ž, Dzy z. Multifractality of gold and oil prices. Resour Policy. 2020;69:101829. https://doi.org/10.1016/j.resourpol.2020.101829.

[29] Ji Q, Zhang D. China crude oil futures markets: evidences from informational entropy analysis. Energy. 2020;181:109321. https://doi.org/10.1016/j.energy.2019.109321.

[30] Spyrou S. Herding in financial markets: a review of the literature. Rev Behav Finance. 2013;5(2):175–94. https://doi.org/10.1108/RFB-02-2013-0009.