Minimal hidden sector models
for CoGeNT/DAMA events

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Motivated by recent attempts to reconcile hints of direct dark matter detection by the CoGeNT and DAMA experiments, we construct simple particle physics models that can accommodate the constraints. We point out challenges for building reasonable models and identify the most promising scenarios for getting isospin violation and inelasticity, as indicated by some phenomenological studies. If inelastic scattering is demanded, we need two new light gauge bosons, one of which kinetically mixes with the standard model hypercharge and has mass < 2 GeV, and another which couples to baryon number and has mass $6.8 \pm 0.1$ GeV. Their interference gives the desired amount of isospin violation. The dark matter is nearly Dirac, but with small Majorana masses induced by spontaneous symmetry breaking, so that the gauge boson couplings become exactly off-diagonal in the mass basis, and the small mass splitting needed for inelasticity is simultaneously produced. If only elastic scattering is demanded, then an alternative model, with interference between the kinetically mixed gauge boson and a hidden sector scalar Higgs, is adequate to give the required isospin violation. In both cases, the light kinetically mixed gauge boson is in the range of interest for currently running fixed target experiments.

1. Introduction

Hints of direct detection of dark matter (DM) currently exist from two experiments. There is a long-standing observation of an annual modulation in the signal observed by DAMA [1], whose statistical significance is beyond question. Last year the CoGeNT
experiment reported excess events in their lowest electron energy bins [2], followed more recently by a 2.8σ detection of annual modulation in the signal [3]. Under the simplest assumptions about the nature of the dark matter interactions, these two observations appear to be incompatible with each other [4, 5] and with upper limits obtained by other experiments, especially CDMS [6, 7], Xenon10 [8] and Xenon100 [9]. Channeling of recoiling ions along the crystal planes in the detectors has been suggested as one loophole for reconciling the conflicts, but this has been argued to be too small an effect by ref. [10]. Uncertainties in quenching factors can also be used to help reconcile the two positive detections [11].

There are alternative microphysical ways of ameliorating the tensions that have been explored in recent papers [12]-[19]. One important modification is to allow for isospin-violating interactions of the dark matter with nucleons, since the coherent spin-independent matrix element is a sum over the interactions with protons and neutrons:

\[ \sigma_N \sim (f_p Z + f_n (A - Z))^2 \sigma_n \] (1)

where \( A, Z \) are the number of nucleons and protons in the nucleus, respectively, and \( \sigma_{n,N} \) are the respective cross sections for DM scattering on neutrons and the nucleus. If isospin is conserved then \( f_p = f_n \), but if \( f_n/f_p \approx -0.7 \), the limits placed on \( \sigma_n \) by the Xenon experiments are greatly relaxed [12, 13].

Moreover, several groups have indicated that inelastic scattering [14] of DM with a small mass splitting \( \delta \sim 10 \) keV can have a beneficial effect either for the the agreement between DAMA and CoGeNT [15], the conflict between CoGeNT and CDMS [12, 16], the goodness of fit to the CoGeNT modulated signal alone [18], or marginally the overall goodness of fit to all data [19].

In this work, we look for the simplest and most natural kind of models that could accommodate these two generalizations, isospin violation and inelasticity, with an emphasis on hidden sector models with Higgs or gauge kinetic portals to the standard model. (For previous model building efforts which do not focus on these aspects, see references [20]-[28].) A similar study to the present one was done in ref. [29], but considering more elaborate models than we examine here. Our inelastic model also has elements in common with that of ref. [30], although the latter incorporated neither isospin violation nor inelasticity. (See [31] for another recent isospin-violating model.) In ref. [32], isospin violating couplings of a single vector were obtained through a combination of kinetic mixing and mass mixing with the \( Z' \). We will argue that interference between two new vector mediators is an elegant way of getting isospin violation if one demands that only inelastic scattering takes place. It is well known that Dirac states coupling to vectors become off-diagonal in the mass eigenstate basis if small Majorana masses are introduced [14]. Thus it is not challenging to account for the inelastic nature of the couplings.

On the other hand, ref. [19] finds that inelasticity improves the global fit to the data only moderately, so that one might also contemplate models with no DM mass splitting.

\[ ^1 \text{Inelastic scattering increases the ratio of the modulation amplitude to the unmodulated rate, softening the discrepancy with CDMS [7], which reports no evidence of events that would be compatible with the CoGeNT signals.} \]
and purely elastic scattering. The overall fit is in fact not very good, indicating either
that some of the data have inconsistencies or that the dark matter interpretation is not
correct. In this work we will assume the former, in which case one might be motivated
to consider inelasticity as a secondary criterion, which might or might not survive as
the data improve. Accordingly, we consider both elastic and inelastic models in the
following and leave it to the reader to judge how strongly the latter is preferred by the
data. Elastic scattering naturally arises in alternative models where the isospin violation
comes about by interference between vector and scalar exchange. We will construct our
models for isospin-violating dark matter starting with this simpler possibility, showing
why it does not naturally accommodate inelastic couplings.

2. Scalar versus vector exchange

A natural way to induce scalar-mediated interactions between the dark sector and the
standard model (SM) is to introduce a new Higgs field \( \phi \) that is a singlet under the
SM gauge group, and communicates to the SM through the interaction \( \lambda \phi^2 h^2 \). If \( \phi \)
gets a VEV then the mass eigenstates are admixtures of \( \phi \) and \( h \) with mixing angle
\( \theta_s \), where \( m_{h,\phi} \) stand for the mass eigenvalues (we assume small
mixing, \( \theta_s \ll 1 \)) and \( v = 246 \) is the SM Higgs VEV. If the DM \( \chi \) has a Yukawa coupling
\( y \bar{\chi} \phi \chi \), then \( \phi \) mediates interactions with SM fermions \( f \) with the strength
\( y y_f \theta_s / m_\phi^2 \), assuming that \( m_\phi \ll m_h \). Here \( y_f \) is the SM Yukawa coupling of the Higgs to fermion \( f \).

In this scenario, \( \phi \) couples to SM matter proportionally to \( h \). Although these couplings
violate isospin, it is a very small violation when it comes to scattering on nuclei, because
nucleons get almost none of their mass from the valence quarks. The couplings of the
Higgs to nucleons are dominated by the sea-quark and gluon content, which are the
same for neutrons and protons (see for example [23, 29]). Therefore we get no significant
isospin violation from couplings mediated by the Higgs portal: it has \( f_p = f_n \).

If in addition we invoke a gauge kinetic mixing portal, \( (\epsilon / \cos \theta_W) F_{\mu \nu} Z_{\mu \nu}' \) (where \( F \)
and \( Z' \) are the respective field strengths of the SM hypercharge and the hidden sector
\( U(1)' \), and \( \theta_W \) is the Weinberg angle), then interference between the new vector and the
new Higgs does lead to tunable isospin violation. This is because the vector only mixes
significantly with the photon, leading to \( f_n / f_p = 0 \). Of course, this is not the ratio we
need for DAMA and CoGeNT. But interference between the vector and scalar exchange
allows for any desired value. If \( g' \) is the new \( U(1)' \) gauge coupling to the DM, then

\[
\frac{f_n}{f_p} \approx \frac{(yy_n \theta_s / m_\phi^2)}{(g' \epsilon / m_\phi^2) + (yy_n \theta_s / m_\phi^2)}
\]  

where \( y_n \) is the coupling of \( h \) to the nucleon: \( y_n \equiv 0.36 m_n / v \) in terms of the nucleon
mass \( m_n \) and the Higgs VEV \( v \) [33, 34].

The main objection to this scenario arises if we want the couplings to DM to also be
inelastic. Suppose there are two mass eigenstates \( \chi_\pm \) with masses \( M_\pm = M \pm \delta/2 \) split

3
by the small amount $\delta$. Let us first write down an effective potential:

$$V = \sum_\pm \bar{\chi}_\pm \chi_\pm + \frac{1}{2} m^2_{Z'} Z'^2 + \frac{1}{2} m^2_\phi \phi^2 + g\bar{\chi}_+ Z' \chi_- + y\bar{\chi}_+ \phi \chi_-$$

(3)

(It is also understood that $Z'$ couples to the electromagnetic current with strength $e\epsilon$ and $\phi$ to the SM fermions $f$ with strength $\theta_s y_f$.) The mass splitting can arise by spontaneous symmetry breaking through couplings of the form $\phi^* (y_L \chi_L \chi_L + y_R \chi_R \chi_R)$ where $\chi_{L,R}$ are the Weyl components in the original Lagrangian, and $\phi$ carries twice the U(1)$'$ charge of $\chi$.

The troublesome question is how the scalar interactions came to be purely off-diagonal in the mass eigenbasis, given that $\phi$ has a VEV which is needed in order to mix with the SM Higgs $h$. To arrive at (3), there must have been a bare mass term $\mu \bar{\chi}_+ \chi_- \phi$ here expressed in the mass eigenbasis) that was exactly canceled by $y \langle \phi \rangle \bar{\chi}_+ \chi_-$. In the absence of this tuning, $\phi$ will have diagonal plus off-diagonal couplings, and this presents a complication for getting the desired level of isospin violation, since the interference between $\phi$ and $Z'$ only occurs in the inelastic channel. This is because the Majorana vector couplings are purely off-diagonal (see the following subsection). We do not contemplate this finely-tuned situation any further.

However if instead of couplings of the form $\phi^* (\chi_L \chi_L + \chi_R \chi_R)$ in which $\phi$ must carry a compensating charge, we have the interactions

$$\bar{\chi} (M + y\phi) \chi$$

in terms of a Dirac fermion $\chi$, then $\chi$ remains Dirac after $\phi$ gets its VEV, and there is no mass splitting. $\phi$ can interfere with $Z'$ through the purely elastic couplings. Even though we must invoke an additional singlet Higgs $\tilde{\phi}$ to break the U(1)$'$ symmetry, since now $\phi$ is neutral under U(1)$'$, this is still an economical model, whose consequences we will consider. For simplicity we take $\phi$ to be real. We do not consider the case of a pseudoscalar ($i\gamma_5$) coupling because this leads to a nuclear scattering amplitude that is suppressed by the DM velocity, making it more difficult to interfere with the vector exchange contribution to get the desired isospin violation (since the latter has no such velocity suppression).

### 2.1. Nondiagonal gauge couplings

In contrast to the couplings of a scalar to DM, the purely off-diagonal coupling of the gauge field is natural if $\chi_{+}$ are Majorana fermions that originated from a Dirac particle before spontaneous symmetry breaking [14]. Consider the interactions (again in the model where $\phi$ carries two units of the $\chi$ charge)

$$V = \frac{1}{2} \bar{\chi}_L M \chi_R + \frac{y}{2} \phi^* (\bar{\chi}_L P_L \chi_L + \bar{\chi}_R P_R \chi_R) + \text{h.c.}$$

(5)

where now $\chi^T_L = (\psi_L, \sigma_2 \psi^*_L)$, $\chi^T_R = (-\sigma_2 \psi^*_R, \psi_R)$ denote Majorana-Dirac spinors constructed from the Weyl spinors, here renamed $\psi_{L,R}$ to avoid confusion. When $\phi$ gets a
VEV, the mass matrix becomes

\[
\begin{pmatrix}
\mu & M \\
M & \mu
\end{pmatrix}
\]

which is diagonalized by \(\chi_{L,R} = \frac{1}{\sqrt{2}}(\chi_+ \pm \chi_-)\) with mass eigenvalues \(|M_\pm| = M \pm \mu\), where \(\mu = y\langle\phi\rangle \ll M\). If the U(1)′ interaction was originally vector-like, then it becomes exactly off-diagonal in the mass basis because there is no vector current for a single Majorana state. This is a strong motivation to prefer vector mediators if we aim for both isospin violation and inelastic off-diagonal couplings.

3. Interfering vector exchanges

The previous discussion motivates us to build a model in which the interfering scalar current is replaced by another vector current. The new vector need only couple to isospin differently from the kinetically mixed U(1) that has \(f_n/f_p = 0\). Coupling to \(B - L\) is attractive from the point of view of anomaly cancellation, but such couplings are very strongly constrained because of the leptonic interactions (see for example [35]). The simplest possibility that avoids these constraints is coupling to \(B\) alone. \(U(1)_B\) is anomalous and it also has mixed anomalies with the SM gauge groups, that can be canceled by adding the appropriate exotic heavy particles [36]-[41]. We will not discuss the implications of these new particles further here, although they can provide complementary collider signatures to test the model. Our addition of a single vector-like DM particle coupling to \(B\) does not spoil the anomaly cancellation achieved in these models.

We refer to the \(U(1)_B\) gauge boson as \(B_\mu\) and for simplicity assume that it couples with equal strength \(g_B\) to the DM and to the SM baryons. It also couples with equal strength to protons and neutrons, just like the singlet Higgs of the previous section. Therefore it is clear that (2) is replaced by

\[
\frac{f_n}{f_p} \simeq \frac{(g^2_B/m^2_B)}{(g'\epsilon/m^2_Z') + (g^2_B/m^2_B)}
\]

This model is almost complete, but we have accounted for the breaking of only one linear combination of the two new U(1)’s through the VEV of \(\phi\). Notice that \(\phi\) must have charges \(-2(g', g_B)\) under \(U(1)' \times U(1)_B\) in order for (5) to be gauge invariant. To completely break the symmetry we need another field \(\tilde{\phi}\) with different charges. This means that the mass eigenstates for the gauge bosons are generally admixtures of the original fields, and that both will therefore kinetically mix with the SM hypercharge. We need to clarify the relation between the couplings appearing in (7) and the original Lagrangian parameters.

A simple way to ensure that the above relations are approximately correct despite mixing of the new gauge bosons is to assume that \(m^2_B \gg m^2_Z\), by assigning \(\tilde{\phi}\) the charges \((0, g_B)\) such that \(\tilde{g}^2_B\langle\phi\rangle^2 \gg g^2_B\langle\phi\rangle^2\). In that case the mixing is suppressed by the
Table 1: Best-fit values of the DM-neutron elastic scattering cross section, DM mass, mass splitting, and isospin violation, from ref. [19], appropriate to the given theoretical model.

| Model                  | $\sigma_n$ (cm$^2$) | $M$ (GeV) | $\delta$ (keV) | $f_p/f_n$ |
|------------------------|----------------------|-----------|----------------|-----------|
| vector $B_\mu$ exchange| $3 \times 10^{-38}$  | 8         | 9.3            | -1.53     |
| scalar $\phi$ exchange | $6 \times 10^{-39}$  | 7.5       | 0              | -1.54     |

The mixing angle is approximately $\theta \approx (g_B/g')(m_{Z'}/m_B)^2$. We will find that its value scales proportionally to the gauge kinetic mixing parameter $\epsilon$, such that $\theta \approx 4\epsilon$. (This relation follows from eq. (10) below and the relic density constraint, fig. 1(a).)

4. Determining the couplings

We must show that values for the parameters exist that can give the right cross section for CoGeNT and DAMA, and the right relic density for the dark matter. For the effective elastic cross section of DM on the neutron, the DM mass and mass splitting, and level of isospin violation, we will consider the three cases shown in table 1, which corresponds to the best-fit values found by ref. [19] for the cases of endothermic $\chi^-N \rightarrow \chi^+N$, and elastic scatterings, respectively. These are the ones appropriate to our two models. We will show that the exothermic reactions, $\chi^+N \rightarrow \chi^-N$, are possible when $\epsilon \gtrsim 10^{-3.5}$, whereas otherwise the excited state is depleted by $\chi^+\chi^+ \rightarrow \chi^-\chi^-$ downscatterings in the early universe.

The theoretical cross sections for DM-neutron scattering in our models, in the elastic limit, are

$$\sigma_n = \frac{\mu_n^2}{\pi} \times \left\{ \begin{array}{cc} \frac{g_B^4}{m_B^4}, & B_\mu \text{ exchange} \\ \frac{m_{Z'}^2}{m_\phi^2}, & \text{scalar } \phi \text{ exchange} \end{array} \right\} (9)$$

where $\mu_n$ is the reduced mass for the DM-nucleon system and $\vec{v}$ is the DM velocity. By equating the $\sigma_n$ values in table 1 to those in eq. (9) and using eqs. (2,7), we obtain

$$\frac{m_B}{g_B} = 232 \text{ GeV}, \quad \frac{m_{Z'}}{g' \epsilon} = -(79.9 \text{ GeV})^2 \quad (B_\mu \text{ exchange}) (10)$$

$$\frac{m_\phi}{\sqrt{yy_n\theta_s}} = 346 \text{ GeV}, \quad \frac{m_{Z'}}{g' \epsilon} = -(118.9 \text{ GeV})^2 \quad \text{(scalar } \phi \text{ exchange}) (11)$$

2If $\langle \phi \rangle = u$ and $\langle \tilde{\phi} \rangle = \tilde{u}$, then the gauge boson mass matrix in the basis $(B, Z')$ is

$$\begin{pmatrix} g_B^2/4g' & g_B g'/\sqrt{2}g' \epsilon \\ g_B g'/\sqrt{2}g' \epsilon & g'^2/4g'^2 \end{pmatrix}$$

3Since our DM is Dirac or quasi-Dirac, there is the interesting possibility for an asymmetry between $\chi$ and $\bar{\chi}$ being the origin of the relic density, which has been widely discussed in the recent literature (see for example [42]). For this work we will assume the asymmetry vanishes.
To get the correct mass spectrum for the model with $B_\mu$, we can set the bare Dirac mass directly to $M = 8$ GeV, and choose $y(\phi) = 4.7$ keV. Recall that only $Z'$ gets its mass primarily from $\phi$, in this model, so if $\epsilon'$ is sufficiently small, it is possible to have $\langle \phi \rangle \sim 10$ GeV or less. The Yukawa coupling still needs to be quite small in that case, $y \sim 0.5 \times 10^{-6}$. However this is only 4 times smaller than the electron Yukawa coupling in the standard model, so it is not unreasonable. Alternatively, for the purely elastic models with $\phi$ exchange, we need to set $M + y(\phi) = 7.5$ GeV.

4.1. Relic density

Next we consider the relic density. Starting with the $B_\mu$ vector exchange model, there are several possible annihilations into gauge bosons: $\chi\pm\chi\pm \to BB$, $\chi\pm\chi\pm \to Z'Z'$, $\chi\pm\chi\pm \to BZ'$. In addition there are coannihilations into quarks, $\chi\pm\chi^- \to q\bar{q}$ mediated by $B$ in the $s$-channel, and also $\chi\pm\chi^- \to f\bar{f}$ into all charged SM fermions $f$ except the kinematically inaccessible top, mediated by the $Z'$. Averaging over all the possibilities, we find the annihilation cross section

$$\langle \sigma_{ann} v \rangle = \frac{1}{32\pi M^2} S(y', g_B, \epsilon, x', x_B)$$

(12)

where $S$ is a dimensionless function of the couplings and the mass ratios $x' = m_{Z'}/M$, $x_B = m_{B}/M$, given by

$$S = \frac{1}{2} g'^4 f_1(x', x^c) + g_B^2 g^2 f_2(x', x_B) + \frac{1}{2} g_B^4 f_1(x_B, x_B) + g_B^4 \sum_{i=u,d,s,c,b} N_{c,i} f_3(x_B, x_i) + (g'\epsilon \epsilon)^2 \sum_{i=e,\mu,\tau} N_{c,i} Q_i^2 f_3(x', x_i)$$

(13)

Here $x_i = m_i/M$ for SM fermion $i$, with charge $Q_i$ and number of colors $N_{c,i}$, and the functions $f_i$ are given in the appendix.

For the the scalar exchange model, all the analogous processes to the previous case are present, with $B_\mu$ replaced by $\phi$. The $\chi\chi \to \phi\phi$ contribution is $p$-wave suppressed, as is $\chi\chi \to f\bar{f}$ by $\phi$ exchange, so we neglect them. The $\chi\chi \to Z'Z'$ and $Z'$-mediated $\chi\chi \to f\bar{f}$ contributions are the same as in (13). We find that (13) is replaced by

$$S \to \frac{1}{2} g'^4 f_1(x', x^c) + g^2 g^2 f_4(x', x_\phi) + (g'\epsilon \epsilon)^2 \sum_{i=e,\mu,\tau} N_{c,i} Q_i^2 f_3(x', x_i)$$

(14)

where $x_\phi = m_\phi/M$ and $f_4$ is defined in the appendix.

For each of the models, we equate (12) to the standard value of the cross section for the observed relic density, $\langle \sigma_{ann} v \rangle = 3 \times 10^{-26}$ cm$^3$/s. This gives the constraint

$$S = 1.7 \times 10^{-5}$$

(15)

To determine the parameters satisfying (15) in the vector exchange model, we assume several choices for $\epsilon$ that can be compatible with laboratory bounds on kinetic mixing (see next section), and use (10) to eliminate $g', g_B$ in favor of $m_{Z'}, m_B$. For the scalar
exchange model, we similarly use (11) to eliminate $y$ in favor of $m_{\phi}$. This case has the free parameter of the Higgs mixing angle $\theta_s$ to be varied in addition to $\epsilon$. For $m_{\phi} < 5$ GeV $\theta_s$ is constrained to be less than 0.01 from the width of the $Z$ boson due to decays $Z \to \phi f \bar{f}$ and from $B$ meson decays $B \to \phi f \bar{f}$. [43].

We thus obtain, for each value of $\epsilon$, the contour in the $m_{Z'}-m_B$ plane corresponding to the observed relic density for the vector exchange model, shown in fig. (1a). Similarly in the scalar model, for each pair $\{\epsilon, \theta_s\}$, we find a contour in the $m_{Z'}-m_{\phi}$ plane, shown in fig. (1b)-(d). It is clear from fig. (1) that $m_{Z'}$ and $m_{\phi}$ tend to be $< 1$ GeV and both scale as $\sqrt{\epsilon}$. Only the vector $B_{\mu}$ can remain somewhat heavier as we now explain.

Recall that we previously made a simplifying technical assumption, $m_B \gg m_{Z'}$, to ensure small mixing $\theta$ between the two gauge bosons. This assumption might be relaxed somewhat, but at the risk of increasing the highly constrained couplings of $B$ to leptons due to the gauge kinetic mixing. To the extent that $\theta$ is small, only the vertical part

Figure 1: (a): Contours in the $m_B$-$m_{Z'}$ plane that give the observed relic density in the vector exchange model, for gauge kinetic mixing parameter $\epsilon = 10^{-2}$, $10^{-3}$, and $10^{-4}$. The ellipse highlights the region where the gauge boson mixing angle $\theta$ is small as needed for consistency.

(b)-(d): Analogous contours in the $m_{\phi}$-$m_{Z'}$ plane for the scalar exchange model, for $\epsilon = 10^{-2}$ (b), $10^{-3}$ (c), $10^{-4}$ (d), and several values of the Higgs mixing angle $\theta_s$. 

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of the contours where \( m_B = 6.8 \text{ GeV} \) is relevant, giving a sharp prediction for the \( B_\mu \) mass, if \( M \) is known. Since the determination of \( M \) could well be uncertain by \( \pm 1 \text{ GeV} \), we find an uncertainty of \( \pm 0.1 \) GeV in \( m_B \) by varying \( M \). In this vertical branch, the \( \chi\chi \) annihilation cross section is dominated by the \( g_4^4 \) contributions in (13).

### 4.2. Relative abundance of excited state

In the vector exchange model where we have a small DM mass splitting, the process \( \chi^+\chi^+ \to \chi^-\chi^- \) mediated by the \( Z' \) (and the \( B \), although we find the former dominates) efficiently depletes the \( \chi^+ \) population in the early universe over part of the allowed parameter space. The downscattering cross section for a similar model was calculated in [26], which adapts to the present case as

\[
\langle \sigma v \rangle \cong \left( \frac{g'^2}{m_{Z'}^2 + 2M\delta} + \frac{g_B^2}{m_B^2 + 2M\delta} \right)^2 \frac{M^2}{4\pi} \sqrt{\frac{2\delta}{M}} \tag{16}
\]

These interactions freeze out at temperature \( T_f \) given by \( n\langle \sigma v \rangle = H \) where \( n \sim (7 \times 10^{-10}\text{GeV}/M)T_f^3 \) is the DM number density and \( H \sim T_f^2/M_p \) is the Hubble constant. If the DM remained in kinetic equilibrium with the SM down to \( T_f \) then the relative abundance of \( \chi^+ \) to \( \chi^- \) would be suppressed by \( \sim \exp(-\delta/T_f) \). However it is the kinetic temperature of the DM which is important here, and if kinetic decoupling occurs at a temperature \( T_d > T_f \), then the suppression is more severe, \( \sim \exp(-\delta T_d/T_f^2) \) (see [26] for a discussion of this issue.) The kinetic equilibrium is controlled by the scattering of DM on electrons, whose cross section is approximately

\[
\langle \sigma_{\chi e} v \rangle \cong \left( \frac{g'\epsilon e}{m_{Z'}^2 + 2M\delta} \right)^2 \frac{m_e^2}{\pi} \sqrt{\frac{2T}{M}} \tag{17}
\]

We find that this goes out of equilibrium at \( T_d \cong 10^3\delta \cong 10 \text{ MeV} \).

Using this methodology, we estimate that the relative abundance of \( \chi^+ \) is unsuppressed for \( \epsilon \gtrsim 10^{-3} \). The suppression turns on exponentially fast as a function of \( \epsilon \), with \( \epsilon = 10^{-4} \) giving a relative abundance of \( \sim \exp(-10^4) \), while \( \epsilon = 10^{-3} \) leads to almost no suppression, \( e^{-0.1} \). Therefore it is possible to realize the exothermic dark matter scenario suggested in ref. [15] over some part of the allowed parameter space. In these cases the scatterings will be an average over the endothermic and exothermic ones since both states are equally populated. We note that ref. [19] finds a moderate preference for exothermic reactions in their global fits.

### 5. Discussion

To recapitulate, we have investigated two hidden sector dark matter models that violate isospin in an optimal manner for reconciling the CoGeNT and DAMA signals with constraints from Xenon10 and Xenon100. The vector exchange model has two Majorana mass eigenstates \( \chi_{\pm} \) (or a single pseudo-Dirac particle) with masses \( M_{\pm} = M \pm \delta \cong 8 \)
GeV $\pm 5$ keV, and two light gauge bosons $Z'$ and $B$ with masses $m_{Z'} < 2 \text{ GeV}$, $m_B \approx 6.8$ GeV, and the couplings\footnote{There are also couplings to the $Z$ boson current $j_Z^\mu$ given by $-\epsilon \tan \theta_W \frac{m_Z^2}{\sqrt{2}} j_Z^\mu (m_{Z'}^2 Z'^\mu + m_B^2 B^\mu)$ that come from the mixing of $Z'$ with weak hypercharge; see for example [14]. Larger contributions to the kinetic mixing of the $B$ with $Z$ can be generated from SM loops below the scale of baryon symmetry breaking [37].}

$$\bar{\chi}^+(g' Z' + g B \bar{B}) \chi^- + g B j_B^\mu B^\mu + \epsilon j_{EM}^\mu (Z'_\mu + \theta B^\mu)$$  \hspace{1cm} (18)

Here $\epsilon < 10^{-2}$ is the gauge kinetic mixing parameter for the $Z'$, $\theta \approx 4\epsilon$ is the mixing angle between $B$ and $Z'$, and $j_{EM,B}^\mu$ are the respective electromagnetic and baryon number currents of the standard model. The couplings are adjusted to give the optimal isospin violation $f_p/f_n \approx -1.5$ \cite{19} via eq. (7). There must also be diagonal Yukawa interactions $y \chi_\pm \phi \chi_\mp$ to a singlet Higgs $\phi$ whose VEV leads to the small mass splitting, but $y \sim 10^{-5}$ is much smaller than the gauge couplings ($g_B = 0.029$) and therefore we have neglected these interactions. The Dirac mass $M = 8 \text{ GeV}$ appearing in the original Lagrangian is protected by chiral symmetry, and so does not introduce any new hierarchy problem. The small scale $\langle \phi \rangle \sim \text{GeV}$ on the other hand is unexplained unless one invokes supersymmetry in the hidden sector \cite{45} or some other UV completion.

The scalar exchange model has exactly Dirac dark matter with $M = 7.5 \text{ GeV}$ (hence only elastic scattering) and a real singlet $\phi$ that mixes with the SM through the Higgs portal $\lambda \phi^2 h^2$. Like the previous model these also have the kinetically mixed $Z'$ vector. The interactions are given by

$$\bar{\chi}(g' Z' + \phi) \chi + \theta \sum_i y_i \bar{f}_i \phi f_i + \epsilon j_{EM}\mu Z'_\mu$$  \hspace{1cm} (19)

where $f_i$ are the SM fermions with their Yukawa couplings $y_i$. We find that $m_\phi < 10$ GeV to satisfy the relic density constraint. If $m_\phi$ happens to be close to this upper limit, it could be discoverable at the LHC, while for the very light cases $m_\phi < 1 \text{ GeV}$, indirect discovery could come from rare decays such as $B \rightarrow \phi X$ followed by $\phi \rightarrow \mu^+ \mu^-$ [43].

One of the most exciting aspects of these models is that they predict new low-energy interactions mediated by the light $Z'$ with a strength relevant for detection in beam-dump experiments \cite{16, 17} such as APEX \cite{48} and the Mainz Microtron \cite{49} and the low-energy $e^+ e^-$ collider experiment KLOE [50]. The still-open window of parameter space in the $\epsilon - m_{Z'}$ plane corresponds roughly to that which we have identified in this paper as being compatible with the $\chi_\pm$ relic density. In our model, $m_{Z'}$ is only bounded from above, depending on the value of $\epsilon$, as shown in fig. 1 $m_{Z'} \gtrsim 2 \sqrt{\epsilon/10^{-2}}$ GeV in the vector exchange model.

On the other hand, the gauge boson $B$ of baryon number is extremely hard to detect due to its very weak coupling and relatively large mass. For example, constraints from new contributions to $\Upsilon$ decay into quarks are easily satisfied [51]. The Tevatron sets limits on $g_B \lesssim 0.6$ from the nonobservation of $p \bar{p} \rightarrow B^*_\mu \rightarrow \chi \bar{\chi} j$ where $j$ is a single jet [52], which is also satisfied by our model. Ref. [37] pointed out that the kinetic mixing of $B$ leads to weak Tevatron constraints from the Drell-Yan production of lepton pairs. It
may be interesting to update these constraints since [37] was written before the upgrade of the Tevatron.

The best indirect confirmation of its presence will be the discovery of an exotic extra family of quarks with baryon number $\pm 1$ [39]. In the simplest such models [37, 38], this fourth generation gets its mass through the usual couplings to the Higgs, requiring its mass to be at the electroweak scale and limited by large Yukawa couplings leading to a Landau pole near the TeV scale. But ref. [39] shows that this limitation can be removed using vector-like quarks (from the point of view of the SM SU(2) gauge symmetry) and giving mass to them through the VEV of the field which breaks baryon number, $\tilde{\phi}$ in our model. If the coupling $\tilde{g}_B$ in [39] is sufficiently small, for example $\tilde{g}_B \sim 0.007$, then $\langle \tilde{\phi} \rangle$ can be at the TeV scale.

Astrophysical constraints are rapidly closing in on light dark matter models. If $\sim 10$ GeV DM annihilates predominantly into $e^+e^-$ with the standard relic density cross section, it is ruled out by its distortions of the CMB [53, 54]. The $\mu^+\mu^-$ channel is still open since a large fraction of the muon energy is converted to neutrinos which have no effect on the CMB. Other channels have an intermediate effect between these two extremes [53]. In our model with two vector bosons, the annihilation is primarily into $B$'s followed by decay into light quarks, which appear to be still be allowed. On the other hand, models that produce too many neutrinos are constrained by SuperKamiokande limits on $\chi\chi \rightarrow \nu\bar{\nu}$ from the sun [53]. Even in optimally isospin violating models such as we have considered here, annihilation of $\sim 10$ GeV DM with the required cross section on nucleons for CoGeNT/DAMA is ruled out for the $b\bar{b}$ channel and marginally allowed for $c\bar{c}$ and lighter quarks as in our vector model. These limits can be improved in the future using data from IceCube/DeepCore [50]. Finally, annihilations of light DM in dwarf satellite galaxies of the Milky Way that produce too many gamma rays in association with charged particles have recently been severely constrained by Fermi data [57, 58] (see also [59]). Again, the $b\bar{b}$ channel is excluded but annihilation into light quarks is still allowed.

After the first version of this paper was posted, ref. [60] appeared, which considers a similar class of models.

Note added: After completing this work we became aware of refs. [61, 62] showing that PAMELA antiproton constraints are in conflict with a $B$ vector boson of mass greater than $2m_p$, which would favor the lower-$m_B$ parts of the contours of fig. 1(a). In addition, we discovered that the gauge boson mixing angle effects in the present model cannot be ignored even when $\theta$ is small, due to the occurrence of $1/m_B^2$ in the amplitudes, which scales as $1/\theta$. This can be overcome by introducing an additional contribution to $m_B$. We intend to address these issues in a forthcoming publication.

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A. Kinematic functions for annihilation cross section

The functions of mass ratios appearing in the annihilation cross sections are as follows.

They were computed using FeynCalc [63].

\[
\begin{align*}
  f_1(x_1, x_2) &= \frac{(1-x_2^2)^{3/2}}{(1-\frac{1}{4}x_1^2)^2} \Theta(1-x_2), \\
  f_2(x_1, x_2) &= \frac{(1-\frac{1}{2}(x_1^2 + x_2^2) + \frac{1}{16}(x_1^2 - x_2^2)^2)^{3/2}}{(1-\frac{1}{4}(x_1^2 + x_2^2))} \Theta(2-x_1 - x_2), \\
  f_3(x_1, x_2) &= \frac{(1+\frac{1}{2}x_2)(1-x_2^{1/2})}{(1-\frac{1}{4}x_1^2)^2} \Theta(1-x_2) \\
  f_4(x_1, x_2) &= \frac{(1+x_1^2 - \frac{1}{2}x_2^2 + \frac{1}{16}(x_1^2 - x_2^2)^2)(1-\frac{1}{2}(x_1^2 + x_2^2) + \frac{1}{16}(x_1^2 - x_2^2)^2)^{1/2}}{(1-\frac{1}{4}(x_1^2 + x_2^2))} \times \Theta(2-x_1 - x_2)
\end{align*}
\]

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