Small viscosity method and criteria for shock wave existence in relativistic magnetic hydrodynamics

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Abstract.

We obtain criteria for shock wave (SW) existence in relativistic magnetic hydrodynamics with no suppositions about convexity of the equation of state. Method of derivation involves consideration of a continuous SW profile in presence of Landau-Lifshitz relativistic viscosity tensor with both non-zero viscosity coefficients $\eta$ and $\zeta$. We point out that supposition of viscous profile existence with only one nonzero coefficient ($\eta=0$) appears to be too restrictive leading to losses of some physical solutions.

PASC classification codes
47.75.+f Relativistic fluid dynamics
52.35.Tc Shock waves

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1 Introduction

Relativistic shock waves (SW) arise in such powerful astrophysical phenomena as supernova explosions and gamma-ray bursts. Theoretical analysis of these processes involves criteria of existence and stability of discontinuous solutions that describe SW in superdense matter. Consideration of these criteria is complicated in case of general equation of state (EOS) (cf., e.g., the classical results [1] and the relativistic hydrodynamics [2, 3]).
Note that in case of a normal fluid we deal with convex EOS (this means the convexity of Poisson adiabats); therefore the only condition is needed to study discontinuous solutions of hydrodynamical equations: the well known entropy growth criterion (see [1, 4] for classical hydrodynamics and [5, 6] in relativistic MHD). However, in case of a general EOS the convexity condition may be violated, and neither customary entropy criterion, nor the evolutionarity criterion [4] are not sufficient to single out physical solutions in a correct way. Moreover, the rarefaction shocks and the compression simple waves, as well as complicated configurations of shocks and simple waves moving in the same direction are possible. This situation is well-known in classical hydrodynamics, it was first studied by H.Bethe [1]. In the relativistic theory such anomalous equations of state arise, e.g., when dealing with a super-dense matter in the neighborhood of phase transitions (see, e.g., [2, 3]).

One of the most effective methods to study the SW existence in case of the general EOS is investigation of the SW viscous profile. According to this method the generalized (discontinuous) solution is treated as a small viscosity limit of corresponding continuous solutions. The shock transition is admissible, if corresponding continuous solution (viscous profile) exists for any nonzero viscosity. In case of normal fluid (in the sense of Bethe and Weyl [1]) the results of this method are the same as that of the evolutionarity criterion. In the relativistic hydrodynamics the conditions for viscous profile existence in case of the general EOS have been derived and studied [3, 2] by using the Landau-Lifshits viscosity term in relativistic energy-momentum tensor [3]. This term involves two viscosity coefficients $\xi$ and $\eta$.

In relativistic magnetic hydrodynamics (MHD) [5, 6] investigation of the SW viscous profile becomes more complicated. Therefore this problem has been first considered [7, 8] in a restricted version with one of the viscosity coefficients put equal to zero ($\eta = 0$) under supposition that only one non-zero viscosity is sufficient to obtain the continuous SW profile. This was a technical supposition and it is not evident. At least, the results of [7, 8] for $\eta = 0$ cannot be considered as necessary conditions.

In the present paper we extend the results of [7, 8] to the case of arbitrary ratio of positive viscosity coefficients and prove conditions for existence of the SW viscous profile under less restrictive requirements. We consider stationary viscous flows of relativistic fluid with infinite conductivity. These solutions describe the MHD shock structure, the existence of SW being considered by means of corresponding continuous solutions with non-zero viscosity. Our treatment shows that we may relax the conditions of [7, 8] to
have a necessary and sufficient criteria.

2 Basic equations

The equations of motion of ideal relativistic fluid with infinite conductivity in magnetic field follow from the conservation laws involving the energy-momentum tensor [5, 6]

\[ T^{\mu\nu} = (p^* + \varepsilon^*)u^\mu u^\nu - p^* g^{\mu\nu} - \frac{\mu}{4\pi} h^\mu h^\nu, \]  

where \( u^\mu \) is the four velocity (Greek indexes run from 0 to 3), the flat space-time metric \( g^{\mu\nu} = g_{\mu\nu} = \text{diag}(1, -1, -1, -1) \) is used for raising and lowering the indexes, \( h^\mu = -\frac{1}{2} e^{\alpha\beta\gamma\delta} F_{\alpha\beta} u_{\gamma} \) is the magnetic field, \( e^{\alpha\beta\gamma\delta} \) is the absolutely antisymmetric symbol, \( F_{\mu\nu} \) is the tensor of electromagnetic field, \( p^* = p + \frac{\mu}{8\pi} |h|^2 \), \( \varepsilon^* = \varepsilon + \frac{\mu}{8\pi} |h|^2 \), \( |h|^2 = -h^\alpha h_\alpha > 0 \), \( \mu \) is the magnetic permeability that is supposed to be constant; \( p \) is the pressure and \( \varepsilon \) is the energy density (in the rest frame). We suppose EOS \( p = p(\varepsilon, n) \) to be a sufficiently smooth function.

Following the small viscosity method [1, 3, 7, 8] in order to study the SW structure we introduce dissipation effects that smear out discontinuities. Similarly to [2, 3, 7, 8], in case of relativistic problem we use the Landau-Lifshits viscosity tensor [4]

\[ \tau_{\mu\nu} = \eta(\nu_{\mu,\nu} + u_{\nu,\mu} - u_{\mu} u^{\alpha} u_{\nu,\alpha} - u_{\nu} u^{\alpha} u_{\mu,\alpha}) + (\xi - 2\eta/3) u^{\alpha}_{\nu,\alpha} (g_{\mu\nu} - u_{\mu} u_{\nu}), \]

the commas stand for derivatives.

Now the fluid motion is constrained by equations of energy-momentum conservation

\[ \partial_\mu (T^{\mu\nu} + \tau^{\mu\nu}) = 0, \]  
baryon charge conservation

\[ \partial_\mu (n u^{\mu}) = 0, \]  
and one more equation follows from the Maxwell’s equations [5, 6]

\[ \partial_\mu (u^\mu h^\nu - u^\nu h^\mu) = 0. \]
The discontinuous solutions follow from these equations in the limit, when $\xi$ and $\eta$ tend to zero. The questions is whether this limit depends on a relation between $\xi > 0$ and $\eta > 0$.

The viscous profile of stationary SW may locally be represented in proper reference frame of the shock front by a stationary continuous solution depending upon the only variable $x$; here $\tau^{\mu\nu} \to 0$ and all the parameters of this viscous flow tend to constant values as $x \to \pm \infty$.

Without loss of generality we suppose further that the limiting values of hydrodynamical parameters for $x \to -\infty$ correspond to the state ahead of the shock (denoted further by index “0”) and the values for $x \to +\infty$ correspond to the state behind the shock (denoted further by index ”1”), then we have $u^1 > 0$ behind and ahead of the shock.

Because all the values in (2),(3),(4) depend only upon variable $x$, we have from these equations

$$T^{1\nu} + \tau^{1\nu} = T^{1\nu}_{(0)}, \quad (5)$$

$$u^1 h^\nu - h^1 u^\nu = H^\nu \equiv u^1_{(0)} h^\nu_{(0)} - h^1_{(0)} u^\nu_{(0)}, \quad (6)$$

$$n u^1 = n_{(0)} u^1_{(0)}. \quad (7)$$

As a result of $\tau^{1\nu} \to 0$ for $x \to \pm \infty$, relations (5)-(7) must be fulfilled for corresponding asymptotic values $T^{\mu\nu}, n, h^\mu$, obtained from continuous solutions of the system (2)-(4). Similarly to classical hydrodynamics [1] we interpret the conditions for shock transition from the state $u^\mu_{(0)}, h^\mu_{(0)}, n_0, p_0$ (ahead of the shock) into the state $u^\mu_{(1)}, h^\mu_{(1)}, n_1, p_1$ (behind the shock), and as consequence of (5)-(7) these states must satisfy

$$T^{1\nu}_{(1)} = T^{1\nu}_{(0)}, \quad (8)$$

$$u^1_{(1)} h^\nu_{(1)} - h^1_{(1)} u^\nu_{(1)} = H^\nu \equiv u^1_{(0)} h^\nu_{(0)} - h^1_{(0)} u^\nu_{(0)}, \quad (9)$$

$$n_{(1)} u^1_{(1)} = n_{(0)} u^1_{(0)}. \quad (10)$$
3 Dynamical system for the shock structure

In this section we use some of the results of [7, 8]. Suppose that equations (8)-(10) are fulfilled.

Definition. We say that shock transition $u\mu(0), h\mu(0), n_0, p_0 \to u\mu(1), h\mu(1), n_1, p_1$ has viscous profile if there is a continuous solution of (5)-(7) having corresponding asymptotics for $x \to -\infty$ and $x \to +\infty$.

We use the reference frame such that $u_3 \equiv 0$ and $h_3 \equiv 0$; $u_1, h_1$ being normal components to the surface $x = const$, and $u_2, h_2$ being the tangential components; $u_2(0) = 0$.

Following [7, 8], due to (6) we represent $h\mu$ in terms of $u_1$ and $u_2$:

$$h\mu = \frac{1}{u_1}[H\mu - u\mu H^\alpha u_\alpha].$$

(Multiplying (5) by $u_\nu$ and taking into account that $\tau_{\mu\nu} u_\nu = 0$ we have

$$\varepsilon^* u_1 = T_{(0)}^{1\mu} u_\mu,$$

whence $\varepsilon$ and $h\mu$ can be expressed through $u_1$ and $u_2$.

Taking into account the explicit form of $\tau_{\mu\nu}$ we have from (5) for $\nu = 1, 2$

$$- (\xi + \frac{4}{3} \eta) \left[ 1 + (u_1^2) \right] \frac{\partial u_1}{\partial x} = T_{(0)}^{11} - T^{11},$$

$$- \eta \left[ 1 + (u_1^2) \right] \frac{\partial u_2}{\partial x} - (\xi + \frac{\eta}{3}) u_1^2 \frac{\partial u_1}{\partial x} = T_{(0)}^{12} - T^{12}$$

After eliminating of $\partial u_1/\partial x$ from (14) with the help of (13) the second equation transforms to

$$- \eta \left[ 1 + (u_1^2) \right] \frac{\partial u_2}{\partial x} = - \frac{u_1^2}{1 + (u_1^2)^2} \frac{\xi + \eta/3}{(\xi + 4\eta/3)} \left( T_{(0)}^{11} - T^{11} \right) + T_{(0)}^{12} - T^{12}$$

It is convenient to introduce a new variable $v = u_2/\sqrt{1 + (u_1^2)}$ in (13) and (14); this yields dynamical system with respect to $u_1$ and $v$

$$\left( \xi + \frac{4}{3} \eta \right) \frac{du_1}{dx} = F_1(u_1, v), \quad \eta \frac{dv}{dx} = F_2(u_1, v),$$

where
\[ F_1(u^1, v) = p - \frac{1}{1 + (u^1)^2} \left[ T_{(0)}^{11} + \frac{\mu}{4\pi} (H^\alpha u_\alpha)^2 - T_{(0)}^{1\mu} u_\mu u^1 \right] + \]
\[ + \frac{\mu}{8\pi (u^1)^2} [(H^\alpha u_\alpha)^2 - H^\alpha H_\alpha] \]  
\[ F_2(u^1, v) = \left( \frac{T_{(0)}^{10} u^0 - T_{(0)}^{12} u^2 u^1}{u^1 [1 + (u^1)^2]^{\frac{3}{2}}} \right) + \]
\[ + \frac{(\mu/4\pi) H^2(H^\alpha u_\alpha) - T_{(0)}^{12} u^1}{[1 + (u^1)^2]^{3/2} u^1} \]  
\[ (16) \]
\[ (17) \]

Continuous solutions of (15) describe the SW structure. It is important to note that \( p(\varepsilon, n) \) disappears from \( F_2 \).

\section{4 Existence conditions of SW viscous profile.}

Let the state parameters \( u^\mu_{(0)}, h^\mu_{(0)}, n_0, p_0 \) ahead of the shock and \( u^\mu_{(1)}, h^\mu_{(1)}, n_1, p_1 \) behind the shock satisfy the conservation laws (8)–(10) that relate hydrodynamic quantities on both sides of SW. We denote \( y = u^1, y_0 = u^1_{(0)}, y_1 = u^1_{(1)} \).

Consider now the curves on \((y, v)\) – plane where the right-hand sides of system (15) may change their signs. We denote \( V_1 \) a locus of points \((y, v)\) such that \( F_1(y, v) = 0 \) and \( V_2 \) is a locus of points \((y, v)\) such that \( F_2(y, v) = 0 \). We suppose that \( V_1 \) is represented by a smooth function \( v = V_1(y) \). From the results of [7, 8] it follows that \( V_2 \) is represented by a single-valued function \( y = Y_2(v) \); this function may be not monotonous so it is more convenient to use this function instead of the the inverse one. We suppose that "0" and "1" are connected by smooth components of \( V_1 \) and \( V_2 \) (see, e.g., Fig.1– Fig.2).

We consider a part of \((y, v)\) – plane between \( V_1 \) and \( V_2 \) such that there are two intersection points "0" and "1" corresponding to the states ahead and behind the shock: \( v_1 = V_1(y_1), y_1 = Y_2(v_1); v_0 = V_1(y_0), y_0 = Y_2(v_0) \), but the curves do not intersect between "0" and "1".

The points \((y_0, V_1(y_0)), (y_1, V_1(y_1))\) are the rest points of system (15).

We shall consider the following conditions.

A. The function \( v = V_1(y) \) is a single-valued function on \((y_1, y_0)\).
B. For all points of $V_2$, $v \in (v_1, v_0)$ the following inequality is valid (cf. [7, 8]):

$$(y_0 - y_1)F_1(Y_2(v), v) < 0. \quad (18)$$

Here we do not consider occurrence of the Chapman-Jouguet points, where the left hand side of (18) equals to zero but does not change its sign. This case may be studied by taking a corresponding limit in (18).

C. We suppose that $h_1h_2 \neq 0$ at the point "0".

This is a technical requirement. Otherwise we deal with much more simple situation of parallel or perpendicular MHD SW; this case is not considered here.

Let $u_{sl}$, $u_f$, $u_A$ stand for the speeds of relativistic slow, fast and Alfven waves [5, 6]; $u_{sl}$, $u_f$ being the roots of the polynomial $Q(y)$, where

$$Q(y) = (1 - c_S^2)(y^2 - u_f^2)(y^2 - u_{sl}^2) =$$

$$= (1 - c_S^2)y^4 - 2c_S^2y^2 + \mu|\mathbf{h}|^2 + \mu c_S^2(h_1)^2$$

$$c_S^2 = (\partial p/\partial \varepsilon)_S$$ is the speed of sound.

The relativistic Alfven speed $u_A$ is defined by the formula

$$u_A^2 = \frac{\mu(h_1)^2}{4\pi(p^* + \varepsilon^*)} \quad (20)$$

In [7, 8] one more parameter $u_A^*$ has been introduced, which is the positive root of the equation $R^*(y) = 0$, where

$$R^*(y) = (p + \varepsilon)y^2(1 + y^2) - \frac{\mu}{4\pi}(h_1)^2 \quad (21)$$

The above velocities satisfy the inequalities [8]

$$u_{sl} < u_A < u_A^* < u_f. \quad (22)$$

We introduce one more requirement (one can show that it is consistent with A,B).

D. Either

(DF): $u^1 > u_f$ ahead of the shock at the point "0", or

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(DS): \( u_A > u^1 > u_{sl} \) ahead of the shock at the point "0".

These inequalities correspond to the evolutionarity criteria of classical MHD for the velocities ahead of the shock [9]. The first inequality (DF) corresponds to the fast SW and the second one (DS) – to the slow SW.

By means of (10) – (12) the variables \( p \) and \( \varepsilon \) can be expressed in terms of the velocity components \( u^1 \) and \( u^2 \).

**Lemma 1.** If relations (7), (11) – (12) are satisfied, then at the point "0" we have

\[
\frac{\partial S}{\partial u^1} \bigg|_0 = \frac{\partial S}{\partial u^2} \bigg|_0 = 0,
\]

where \( S \) is the entropy per baryon.

The proof uses the thermodynamical relation \( T dS = pd(1/n) + d(\varepsilon/n) \), where \( T \) is the temperature. The statement of the lemma is obtained after direct calculation of differentials \( dn \) and \( dp \) using (10)–(12).

Using Lemma 1, after some calculations we have at the the point "0" \( (v_0 = 0) \):

\[
\frac{\partial F_1}{\partial u^1} \bigg|_0 = \frac{(p + \varepsilon)}{u^1(u^0)^4} D(u^1), \quad (23)
\]

where

\[
D(y) = (1 - c_s^2) y^4 - y^2 \left( 2c_s^2 + \frac{\mu |h|^2}{4\pi (p + \varepsilon)} - 1 \right) + \frac{\mu [(h^0)^2 - (h^2)^2]}{4\pi (p + \varepsilon)} - c_s^2.
\]

The other derivatives of the right hand sides (16), (17) of the dynamical system (15) at the point "0" are

\[
\frac{\partial F_1}{\partial v} \bigg|_0 = \frac{\mu}{4\pi} \frac{h^1 h^2}{u^0 u^1}, \quad (24)
\]

\[
\frac{\partial F_2}{\partial u^1} \bigg|_0 = \frac{\mu}{4\pi} \frac{h^1 h^2}{(u^0)^5 u^1}, \quad (25)
\]

\[
\frac{\partial F_2}{\partial v} \bigg|_0 = \frac{1}{(u^0)^4 u^1} R^* (u^1). \quad (26)
\]

Taking into account these relations in the vicinity of the point "0" we have on the curve \( V_2 \)
\[ F_1(y, V_2(y)) = \frac{(p + \varepsilon)^2}{u^1} \frac{Q(u^1)}{R^*(u^1)} (y - y_0). \] (27)

On the curve \( V_2 \) we have
\[ \left. \frac{dv}{dy} \right|_0 = -\frac{\mu h^1 h^2}{4\pi u^0 R^*(u^1)}. \] (28)

**Lemma 2.** In the case DF the rest point "1" of system (15) is a saddle point. In the case DS the rest point "0" is a saddle point.

Proof. Direct calculation yields
\[ \frac{\partial F_1}{\partial u^1} \frac{\partial F_2}{\partial v} - \frac{\partial F_1}{\partial v} \frac{\partial F_2}{\partial u^1} = \frac{(p + \varepsilon)^2}{(u^1)^2 (u^0)^4} Q(u^1) \]

In the case (DS) ahead of SW we have \( Q < 0 \) at "0" (see (19)) and this yields the required statement according to the properties of the saddle point [10]. In the case (DF) analogous result at "1" can be checked directly; however it is easier to use the same relations as (23)-(26) in case of "0" by using the Lorents transformation that preserves \( u^1(0) \) and transforms the transversal velocity component \( u^2(1) \) to zero at the point "1".

Further the coefficients of \( Q(y) \) and \( R^*(y) \) involving the magnetic field, energy density and baryon density are taken only at the point "0" ahead of the shock.

Now we proceed to prove the existence of viscous profile.

Consider first the case (DF) for \( y_1 < y_0 \), and put for definiteness \( h^1 h^2 > 0 \) at "0". Consideration of the opposite sign of \( h^1 h^2 \) is completely analogous.

First of all we note that the condition (18) guarantees that the trajectories of system (15) in the plane \((y = u^1, v)\) cross the curve \( V_2 \) from right to left (see Fig. 1).

On the curve \( v = V_1(y) \):
\[ F_2(y, V_1(y)) = \frac{4\pi (p + \varepsilon)^2}{\mu h^1 h^2 u^1(u^0)^4} (y - y_0) > 0 \] (29)

in the vicinity of the point "0", \( y < y_0 \).

Correspondingly trajectories of system (15) cross the curve \( V_1 \) bottom-up (Fig. 1). Evidently this is true not only for the neighborhood of "0", but for
all interval \((y_1, y_0)\); otherwise there must be additional rest points of system (15) between "0" and "1" , which contradicts to our suppositions.

Now we shall find out relative disposition of \(V_1\) and \(V_2\). Let at point "0"

\[
tg(\alpha_1) = \frac{dV_1}{dy} \bigg|_0, \quad tg(\alpha_2) = \frac{dV_2}{dy} \bigg|_0.
\]

Using (23)-(24) we obtain that the ratio of tangents at "0" equals

\[
\frac{tg(\alpha_1)}{tg(\alpha_2)} = \frac{16\pi^2(p + \varepsilon)^2(u_0^0)^2}{(\mu h^1 h^2)^2} Q(u^1) + 1 = \frac{16\pi^2(p + \varepsilon)}{(\mu h^1 h^2 u_0^0)^2} R^*(u^1) D(u^1). \tag{30}
\]

Then in case of fast SW \((Q(y_0) > 0)\) this ratio is > 1, that is \(V_1\) is above \(V_2\).

Therefore, the phase curves only can leave the domain between \(V_1\) and \(V_2\) (Fig. 1). Taking into account the sign of \(F_1\), it is easy to see that inside this domain all the phase curves come out from the point "0"; and there exists a phase curve of (15), that comes from "0" to "1". Because "1" is a saddle point (Lemma 2), this phase curve is unique. This conclusion does not depend upon relation between (positive) viscosity coefficients \(\xi, \eta\).

Now we proceed to the case (DS) (see Fig. 2); it is now convenient to put \(h^1 h^2 < 0\) at "0" (the opposite case is completely analogous). The sign of (29) remains the same as in the case (DF) and so is the direction of the phase curves crossing \(V_1\). The direction of the phase curves crossing \(V_2\) also remains due to (18). Taking into account (DS) and (22) we have \(Q(y_0) < 0\) and according to (30) we see that \(V_2\) is above \(V_1\) in the neighborhood of "0". As distinct from the case (DF) here the phase curves can only enter the domain between "1" and "0". Similarly to previous consideration there is a unique phase curve of system (15), passing from "0" to "1"; this is a separatrix of saddle point "0".

In the previous consideration we supposed that \(y_1 < y_0\); this corresponds to usual compression SW. In case of anomalous EOS the rarefaction shocks are also possible \([1, 3, 7, 8]\). The condition (18) is applicable in this case as well, and the consideration is completely similar.

Therefore we proved that equations (5)-(10) have a unique continuous solution that connects the states "0" and "1".

**Theorem.** Let the states \(u^\mu_{(0)}, h^\mu_{(0)}, n_0, p_0\) ahead of the shock and \(u^\mu_{(1)}, h^\mu_{(1)}, n_1, p_1\) behind the shock satisfy the conservation equations (8)-(10). If
the conditions (A–D) are satisfied, then the MHD shock transition "0"→"1" has a unique viscous profile satisfying equations (3)–(10).

Note that the analogous criteria for existence of SW viscous profile obtained in [7, 8] appear to be too restrictive. These criteria have been obtained under condition that one of the viscosity coefficients equals to zero (η = 0), and they rule out the shocks that satisfy the condition \( u_A < u^1 < u^*_A \) at "1" after the shock front. However these latter solutions are compatible with the criteria (A)-(D) of the present paper. The explanation of this inconsistency is as follows. If the function \( Y_2(v) \) is monotonous and η → 0, then the phase curve of system (15) that go from "0" to "1" tends to the curve \( V_2 \). However, it may happen that \( Y_2(v) \) is not monotonous; this just corresponds to \( u_A < u^1 < u^*_A \) at "1". In this case the above phase curve that begins at "0" smuggles down to \( V_2 \) only on some segment, and the corresponding limiting solution for η → 0 has a discontinuity (see Fig. 3). This explains why such solutions have been rejected in [7, 8], because the initial supposition of these papers was the existence of a regular viscous profile for η = 0.

5 Discussion

The SW existence conditions in relativistic MHD with general equation of state has been analyzed in [7, 8] in case of η = 0 in the Landau-Lifschits relativistic viscosity tensor. In the present paper we obtained the criteria for existence of viscous SW profile dealing with both nonzero viscosity coefficients (ξ > 0, η > 0) in this viscosity tensor. If additional limitations on EOS (e.g., convexity) are absent, our criteria are more restrictive than, e.g., evolutionarity conditions [9] or any other conditions that involve characteristics of the fluid only at the initial and final states. This is evident because condition (18) must be valid for the whole interval between the states "0" and "1". This situation is analogous to ordinary (non-magnetic) hydrodynamics [2, 3]; in this case our criteria reduce to the criteria of these papers.

On the other hand, the criteria of the present paper are less restrictive than that of [7, 8] obtained in case of η = 0. This is because the supposition of existence of a regular viscous profile used in [7, 8] does not always hold in case of η = 0 (even if ξ ≠ 0) and this rules out some physical solutions. It should be noted that such situation is specific just to relativistic MHD; this does not appear neither in non-relativistic case, nor in ordinary relativistic hydrodynamics.
Our criteria may be applied to arbitrary smooth EOS. However, we must note that the requirement for $V_1(y)$ to be a continuous (single-valued) function is not trivial and may not be fulfilled in case of certain equations of state (cf., e.g., [1]). Though consideration of a viscous profile seems to be rather effective for investigation of SW existence and stability, this method may not work in case of complicated EOS (cf. remarks in [11]) that require either modification of equations of the fluid motion or using additional physical information about solutions.

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Figure 1: The phase trajectories of system in case of DF; EOS is $p = \varepsilon/3$. The solid line ($V_1$) corresponds to $v = V_1(y)$, the dashed line ($V_2$) corresponds to $y = Y_2(v)$; the curve $VP$ connecting "0" and "1" describes the viscous profile of the fast shock transition "0" $\rightarrow$ "1". The arrows show the direction of the phase flow.
Figure 2: The phase trajectories of system in case DS with the same EOS. In this case the curve $V_1$ (solid) is below $V_2$ (dashed). The separatrix $VP$ of the saddle point "0" describing the viscous SW profile of slow shock transition "0" $\rightarrow$ "1" goes to the final state "1".
Figure 3: The example of viscous SW profiles for three ratios of $\eta/\xi$ in case of DF (fast SW) with non-monotonous dependence $y = Y_2(v)$; $p = \varepsilon/3$. Disposition of curves $V_1$ and $V_2$ is as on Fig. The curve "A" describes the profile with ratio $\eta/\xi = 1$, "B" corresponds to $\eta/\xi = 0.1$ and "C" - to $\eta/\xi = 0.01$. These curves corresponding to the smaller ratios snuggle down to $V_2$ (after they go out from "0") on some segment and then jump to "1".