QCD Prediction of $A_{TT}$ for Small $Q_T$ Dimuon Production in $pp$ and $p\bar{p}$ Collisions

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Abstract. We present QCD prediction of double-spin asymmetries ($A_{TT}$) in transversely polarized Drell-Yan process at small transverse momentum $Q_T$ of dimuon. Resummation of large logarithmic corrections, relevant in small $Q_T$ region, is performed up to next-to-leading logarithmic (NLL) accuracy. $A_{TT}$ at RHIC, J-PARC and GSI are studied numerically in the corresponding kinematic regions. We show that the large $A_{TT}$ is obtained for small $Q_T$ and moderate energies.

Keywords: Transversity, Drell-Yan process, Soft gluon resummation, Double-spin asymmetry

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The transversity $\delta q(x)$ is one of the twist-2 distribution functions of nucleon, which represents the distribution of transversely polarized quarks inside transversely polarized nucleon [1]. Although it is theoretically as important as other twist-2 distribution functions such as the unpolarized parton distributions ($q(x), g(x)$) and the helicity distributions ($\Delta q(x), \Delta g(x)$), very little has been known of it so far. This is because $\delta q(x)$ is a chiral-odd function, and should be always accompanied with another chiral-odd function in physical observable, and therefore cannot be measured in inclusive DIS. Transversely polarized Drell-Yan (tDY) process, $p^+ p^+ \to \ell^+ \ell^- X$, is one of the processes where we can access the transversity by measuring the double-spin asymmetry: $A_{TT} \equiv \frac{\Delta_T d\sigma}{d\sigma} \equiv \frac{d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow}} - \frac{d\sigma^{\downarrow\uparrow}}{d\sigma^{\uparrow\uparrow}}$. Since tDY is inclusive in the final state, it is in principle the cleanest process to access $\delta q(x)$. However, at the RHIC-Spin experiments, the asymmetries are likely to be quite small [2]. This comes from the fact that the DY process at $pp$ collider probes the sea distributions which are likely to be small for the transversity. Moreover, the rapid growth of unpolarized sea distributions enhances the denominator of $A_{TT}$ at low-$x$ which is typically probed by tDY at RHIC. On the other hand, much larger asymmetries are expected at the proposed spin experiments at J-PARC and GSI, which are to be performed at lower energies [3, 4, 5].

In this work, we study the asymmetries in $Q_T$ distribution of dimuon, especially at small $Q_T$ where the bulk of dimuon is produced. At small $Q_T$, the cross section is not described correctly by the fixed order calculations since large logarithmic corrections such as $\alpha_s^n \log^m (Q^2/Q_T^2)/Q_T^2$ ($m \leq 2n - 1$) appear at each order of perturbation series. These so-called “recoil logs” come from soft gluon emissions, and have to be resummed to all orders in $\alpha_s$ to make a reliable prediction of the cross section at small $Q_T$. 

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1 Deceased on Sep.16, 2006.
The resummation is carried out in the impact parameter $b$ space, conjugate to $Q_T$ space, to enforce transverse-momentum conservation, and the resummed cross section is expressed as the Fourier transform back to the $Q_T$ space. Here we perform the resummation at the next-to-leading logarithmic (NLL) accuracy, which corresponds to adding up the terms with $m = 2n - 1$ and $2n - 2$, respectively, for all $n$. At this level, the “resummed part” of the spin-dependent cross section of tDY, differential in invariant mass $Q$, transverse momentum $Q_T$ and rapidity $y$ of dimuon, and in the azimuthal angle $\phi$ of one of the outgoing leptons is given by \cite{7} ($\sqrt{S}$ is the CM energy of hadron system),

\[
\frac{\Delta_T d\sigma^{\text{NLL}}}{dQ^2dQ_T^2dyd\phi} = \cos(2\phi) \frac{\alpha^2}{3N_c\sqrt{S}Q^2} \sum_i e_i^2 \int_0^\infty db b J_0(bQ_T)e^{S(b,Q)} \int_0^\infty \frac{db}{2} \left[ (C_{qq} \otimes \delta q_i) \left( x_1^0, -\frac{b_0}{b} \right) (C_{\bar{q}\bar{q}} \otimes \delta \bar{q}_i) \left( x_2^0, -\frac{b_0}{b} \right) + (x_1^0 \leftrightarrow x_2^0) \right].
\] (1)

Here $J_0(bQ_T)$ is a Bessel function, $b_0 = 2e^{-\gamma_E}$ with $\gamma_E$ the Euler constant. The large logarithmic corrections are resummed into the Sudakov factor $e^{S(b,Q)}$ with $S(b,Q) = -\int_0^Q \frac{d\kappa^2}{\kappa^2} \left( \ln \frac{Q^2}{\kappa^2} A_q(\alpha_s(\kappa)) + B_q(\alpha_s(\kappa)) \right)$. Perturbatively calculable quantities, i.e., $A_q$, $B_q$ and the coefficient functions $C_{qq}(z)$ and $C_{\bar{q}\bar{q}}(z)$, are expressed as power series in $\alpha_s$, and the their explicit forms necessary at the NLL accuracy are found in Ref.\cite{7}. $x_{1,2}^0 = (Q/\sqrt{S})e^{\pm y}$, and $\delta q_i(x, \mu^2)$ is the transversity of $i$-th flavour quark at the $\overline{\text{MS}}$ scale $\mu$. The singularity in $b$-integration, due to Landau pole in $\alpha_s(\kappa)$, is taken care of by “contour deformation method” introduced in \cite{8}. Correspondingly, the nonperturbative effects are included by the replacement $e^{S(b,Q)} \rightarrow e^{S(b,Q)-\epsilon_N^2 b^2}$ in \cite{1} \cite{6,8,9}, with a non-perturbative parameter $\epsilon_N$, which (roughly speaking) parameterizes the intrinsic $k_T$-distribution of quarks inside nucleon. Then we combine the resummed part \cite{1}, which embodies the logarithmically enhanced contributions for small $Q_T$ to all orders, with the “residual part” of the fixed-order cross section, which is not associated with such logarithmic enhancement. To perform this consistently, we need the leading order (LO) tDY cross section at finite $Q_T$; this is of $O(\alpha_s)$ and is obtained as QCD prediction at large $Q_T$ \cite{7}. The matching of the NLL formula \cite{1} with the corresponding component in the LO cross section is performed at intermediate $Q_T$ following the formulation of Ref.\cite{6}, to ensure no double counting for all $Q_T$, and we obtain the “NLL+LO” prediction of the tDY cross section, $\Delta_T d\sigma^{\text{NLL}+\text{LO}}/(dQ^2dQ_T^2dyd\phi)$, which has a uniform accuracy over the entire range of $Q_T$ \cite{7}. The “NLL+LO” cross section of unpolarized Drell-Yan process, $d\sigma^{\text{NLL}+\text{LO}}/(dQ^2dQ_T^2dyd\phi)$, is obtained in the same way, utilizing the results in the literature \cite{6,10}.

Taking the ratio of these cross sections, we obtain the double-spin asymmetries:

\[
A_{TT} = \left[ \Delta_T d\sigma^{\text{NLL}+\text{LO}}/(dQ^2dQ_T^2dyd\phi) \right] / \left[ d\sigma^{\text{NLL}+\text{LO}}/(dQ^2dQ_T^2dyd\phi) \right].
\] (2)

In order to calculate $A_{TT}$ numerically, we need to assume a model of the transversity $\delta q_i(x)$ for the numerator. Here we take a model used in \cite{2}, which saturates the Soffer bound \cite{11} as $\delta q_i(x, \mu_0^2) = [q_i(x, \mu_0^2) + \Delta q_i(x, \mu_0^2)]/2$ at the low input scale $\mu_0 \sim 0.6\text{GeV}$ and is evolved to the higher $\mu^2$ with NLO DGLAP kernel \cite{12}; as the inputs, we
FIGURE 1. $A_{TT}$ at NLL+LO accuracy in $pp$ collision at RHIC kinematics: $\sqrt{S} = 200$ GeV, $Q = 3, 5, 8, 15$ GeV and $\phi = 0$ with $y = 0$ (left panel) and $y = 2$ (right panel).

FIGURE 2. $A_{TT}$ at NLL+LO accuracy. Left: $\sqrt{S} = 10$ GeV, $Q = 2, 3, 4$ GeV, $\phi = 0$, and $y = 0$ for $pp$ collision at J-PARC kinematics. Right: $\sqrt{S} = 14.5$ GeV, $Q = 2, 3, 4, 6$ GeV, $\phi = 0$, and $y = 0$ for $p\bar{p}$ collision at GSI kinematics.

use GRV98 [13] for $q_i(x, \mu_0^2)$ and GRSV2000 [14] for $\Delta q_i(x, \mu_0^2)$. Correspondingly, the GRV98 distributions are used for calculating the denominator of (2). The non-perturbative parameter $g_{NP}$ are taken to be common in the numerator and the denominator of (2), and we use $g_{NP} = 0.5$ GeV$^2$ which is consistent with the result of [15].

Fig. 1 shows $A_{TT}$ in $pp$ collision at RHIC kinematics: $\sqrt{S} = 200$ GeV, $Q = 3, 5, 8, 15$ GeV and $\phi = 0$, with $y = 0$ (left panel) and $y = 2$ (right panel). In this case, $A_{TT}$ are about 4-8% and rather flat in the small $Q_T$ region where the resummed cross sections are dominant in both the numerator and denominator of (2). We have smaller $A_{TT}$ for smaller $Q$ due to the growth of the sea distributions in the denominator of (2) at small $x$. We obtain slightly larger $A_{TT}$ for $y = 2$ compared with the $y = 0$ case, and it appears that generically the $y$-dependence of $A_{TT}$ is small.

The left panel of Fig. 2 is same as Fig. 1, but at J-PARC kinematics: $\sqrt{S} = 10$ GeV.

$\text{2}$ The $g_{NP}$-dependence of the polarized and unpolarized cross sections almost cancels in $A_{TT}$ of (2) in the range $g_{NP} = 0.3-0.8$ GeV$^2$. 
$Q = 2, 3, 4$ GeV, $\phi = 0$ and $\gamma = 0$. In this kinematics, the parton distributions at medium $x$ ($A_{1,2}^{0} = 0.2-0.4$) are probed, so that the transversity distributions in the numerator of (2) are larger than the RHIC case while the growth of the sea distributions in the denominator is not significant. Therefore we obtain much larger $A_{TT} \sim 15\%$ than the RHIC case of Fig. 1. $A_{TT}$ are again flat as functions of $Q_T$, and the dependence on $Q$ is also weak.

The right panel of Fig. 2 shows $A_{TT}$ in $p \bar{p}$ collision at GSI kinematics: $\sqrt{S} = 14.5$ GeV, $Q = 2, 3, 4, 6$ GeV, $\phi = 0$ and $\gamma = 0$, where $A_{TT}$ are dominated by valence distributions at medium $x$. The largest $A_{TT}$ of 15-30\% are obtained in this case. The results are extremely flat as functions of $Q_T$. Integrating the numerator and the denominator of (2) over $Q_T^2$, we reproduced the NLO asymmetries given by Barone et al.\cite{Barone}. We also calculated the $\gamma$-dependence of the results in Fig. 2, and it turns out to be small for both J-PARC and GSI cases.

To summarize, we have calculated the double-spin asymmetries $A_{TT}$ for small $Q_T$ DY pair production in high-energy $pp$ collisions at RHIC and in moderate-energy $pp$ and $p \bar{p}$ collisions at J-PARC and GSI. Our results demonstrate that $A_{TT}$ reach a finite value at $Q_T = 0$ through the flat behavior in the small $Q_T$ region, reflecting that the soft gluon resummation to the NLL level has universal structure for the polarized and unpolarized DY\cite{Kawamura}, and also reveal that $A_{TT}$ are large enough to be experimentally measured, especially in moderate energies.

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