Decaying cosmological parameter in the early universe from NKK theory of gravity

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Abstract

Using a formalism recently introduced we study the decaying of the cosmological parameter during the early evolution of an universe, whose evolution is governed by a vacuum equation of state. We use a stochastic approach in a nonperturbative treatment of the inflaton field from a Noncompact Kaluza-Klein (NKK) theory, to study the evolution of energy density fluctuations in the early universe.

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I. INTRODUCTION

Cosmological observations imply that there exists an extremely small upper limit on the vacuum energy density in the present state of our universe. This stands in sharp contradiction with theoretical predictions [1]. In fact, any mass scale in particle physics contributes to the vacuum energy density much larger than this upper bound [2]. According to modern quantum field theory, the structure of the vacuum is turned out to be interrelated with some spontaneous symmetry-breaking effects through the condensation of quantum scalar fields. This phenomenon gives rise to a non-vanishing vacuum energy density $\rho_{\text{vac}} \sim M_p^4$ ($M_p = G^{-1/2}$ is the Planckian mass and $G$ is the gravitational constant). The appearance of this characteristic mass scale may have an important effect on the cosmological constant because it receives potential contributions from this mass scale due to mass spectrum of corresponding physical fields in quantum field theory. By taking into account this contribution, an effective cosmological constant is defined as the sum of the bare cosmological constant $\lambda$ and $8\pi G\rho_{\text{vac}}$ [3]. This type of contribution gives rise to an immediate difficulty called the cosmological constant problem. There are some possible solutions to this problem rendering $\Lambda$ exactly or almost vanishing [4]. One of them consists to find some relaxation mechanism by which $\Lambda$ could relax to its present day small value [5,6]. A credible mechanism for obtaining such a decay already exists, which is to assume the existence of a scalar field

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presently relaxing towards the minimum of its potential. Scalar fields are not only predicted
to exist by some particle physics theories that go beyond the Standard Model, but are also
the most plausible engine behind a possible inflationary period in the very early universe
[8–11]. In this work we shall study a possible mechanism for a decaying cosmological pa-
rameter from the KK formalism, but by considering the extra (spatial-like) dimension $\psi$
oncompact [12,13]. This theory, also called induced-matter theory is, in its simplest form,
the basic KK theory in which the fifth dimension is not compactified and the field equations
of general relativity in 4D follow from the fact that the 5D manifold is Ricci-flat. Thus the
large extra dimension is responsible for the appearance of sources in 4D general relativity.
Hence, the 4D world of general relativity is embedded in a 5D Ricci-flat manifold. There
has recently been an uprising interest in finding exact solutions of the KK field equations
in 5D, where the fifth coordinate is considered as noncompact. This theory reproduces and
extends known solutions of the Einstein field equations in 4D. Particu-
lar interest revolves
around solutions which are not only Ricci flat, but also Riemann flat. This is because it
is possible to have a flat 5D manifold which contains a curved 4D submanifold, as implied
by the Campbell theorem. So, the universe may be “empty” and simple in 5D, but contain
matter of complicated forms in 4D [14].

In this work we use a stochastic approach to study the dynamics of the inflaton field in
the early universe, which is governed by a 4D vacuum equation of sta-
"state 
$p_{\text{vac}} = -\rho_{\text{vac}} = -\frac{\Lambda}{8\pi G}$, being $\Lambda$ the time dependent (decaying) cosmological parameter. To make it we shall use a
5D canonical metric which is Riemann flat ($R^A_{BCD} = 0$) and describes a 5D apparent vacuum ($G_{AB} = 0$). To describe the system we shall propose a 5D action for a purely kinetic inflaton
field which is minimally coupled to gravity.

II. 5D FORMALISM

We consider the 5D canonical metric

$$dS^2 = \psi^2 \frac{\Lambda(t)}{3} dt^2 - \psi^2 e^{2 \int \sqrt{\frac{4}{\psi}} dt} dr^2 - d\psi^2,$$

(1)

where $dr^2 = dx^2 + dy^2 + dz^2$, being $x, y, z$ dimensionless spatial coordinates. Furthermore,
t and $\psi$ has spatial units (in this paper we shall consider $c = \hbar = 1$. We shall assume in
what follows that the extra dimension is spacelike and that the universe is 3D spatially flat,
isotropic and homogeneous. The metric (1) is flat $R^A_{BCD} = 0$ and describes a 5D manifold
in apparent vacuum ($G_{AB} = 0$) and is a special case of the much-studied class of canonical
metrics $dS^2 = \psi^2 g_{\mu\nu} dx^\mu dx^\nu - d\psi^2$ [15–17].

To describe neutral matter in a 5D geometrical vacuum (1) we can consider the La-
grangian

$$(\mathcal{L}(\varphi, \varphi, A) = -\sqrt{\left|\frac{(5)g}{(5)g_0}\right|} \mathcal{L}(\varphi, \varphi, A),$$

(2)

where $\left|\frac{(5)g}{(5)g_0}\right| = \psi^8 (a/a_0)^6$, is the absolute value of the determinant for the 5D covariant metric
tensor with components $g_{AB}$ ($A, B$ take the values $0, 1, 2, 3, 4$) and $\psi_0$ is a constant
of dimensionalization determined by $\left|\frac{(5)g}{(5)g_0}\right|$ evaluated at $\psi = \psi_0$ and $a_0 = a(t = t_0)$. Here,
\(a(t)\) the scale factor of the universe such that \(\dot{a}/a = \sqrt{\Lambda}/3\). To describe the system we consider an action

\[
I = -\int d^4xd\psi \sqrt{\frac{(5)g}{(5)g_0}} \left[ \frac{(5)R}{16\pi G} + \mathcal{L}(\varphi, \varphi_A) \right],
\]

where \(\varphi\) is a scalar field minimally coupled to gravity. Furthermore, \((5)R = 0\) is the 5D Ricci scalar.

To describe the apparent vacuum, we shall consider the density Lagrangian \(\mathcal{L}\) in (2) must to be

\[
(5)\mathcal{L}(\varphi, \varphi_A) = \frac{1}{2} g^{AB} \varphi_A \varphi_B,
\]

for a free scalar field.

The dynamics for \(\varphi(t, \vec{r}, \psi)\) being given by the equation

\[
\dot{\varphi} + \left(3\sqrt{\frac{\Lambda}{3} - \frac{\dot{\Lambda}}{\Lambda}}\right) \varphi - \frac{\Lambda}{3} e^{-2f} \sqrt{\frac{3}{r}} \nabla^2 \varphi - \frac{\Lambda}{3} \left[ 4\psi \frac{\partial \varphi}{\partial \psi} + \psi^2 \frac{\partial^2 \varphi}{\partial \psi^2} \right] = 0.
\]

On the other hand, \(\varphi\) complies with the commutation expression

\[
[\varphi(t, \vec{r}, \psi), \Pi^i(t, \vec{r}, \psi, \psi')] = \frac{i}{a_0^2} g^{tt} \left| \frac{(5)g_0}{(5)g} \right| \left(\frac{\Lambda_0}{\Lambda}\right) \delta^{(3)}(\vec{r} - \vec{r}') \delta(\psi - \psi'),
\]

where \(\Pi^i = \frac{\partial \mathcal{E}}{\partial \varphi^i}\) and \(a_0\) is the scale factor of the universe when inflation starts. As can be demonstrated \(\varphi(t, \vec{r}, \psi) = e^{-\frac{i}{2} \int \left[ 3(\frac{\Lambda}{3})^{1/2} - \frac{\dot{\Lambda}}{\Lambda} \right] dt} (\psi_0/\psi)^2 \chi(t, \vec{r})\), so that \(\frac{\partial \varphi}{\partial \psi} = -\frac{\dot{\psi}}{\psi} \varphi\) and \(\chi\) can be written as a Fourier expansion

\[
\chi(t, \vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \int d\psi \left[ a_{k_r, \psi} e^{i(k_r \cdot \vec{r} + k_\psi \cdot \vec{\psi})} \xi_{k_r, \psi}(t, \psi) + a^\dagger_{k_r, \psi} e^{-i(k_r \cdot \vec{r} + k_\psi \cdot \vec{\psi})} \xi^{\dagger*}_{k_r, \psi}(t, \psi) \right],
\]

such that

\[
[\chi(t, \vec{r}), \chi(t, \vec{r}')] = \frac{i}{a_0^3} \delta^{(3)}(\vec{r} - \vec{r}') \delta(\vec{\psi} - \vec{\psi}'),
\]

and \(\xi_{k_r, \psi}(t, \psi) = e^{-i k_r \cdot \vec{r} + i k_\psi \cdot \vec{\psi}} \xi_{k_r, \psi}(t)\). The commutator (7) is satisfied for \(a_{k_r, \psi}, a^\dagger_{k_r, \psi}\) = \(\delta^{(3)}(k_r - k_{\psi}^r) \delta(k_\psi - k_{\psi}^\psi)\) and \(a^\dagger_{k_r, \psi}, a_{k_r', \psi}\) = \(a_{k_r, \psi}, a_{k_r', \psi}\) = 0, if the following condition holds:

\[
\bar{\xi}_{k_r, \psi}(t) \ddot{\xi}^*_{k_r, \psi}(t) - \dot{\xi}^*_{k_r, \psi}(t) \dot{\xi}_{k_r, \psi}(t) = \frac{i}{a_0^3}.
\]
III. 4D DYNAMICS

To describe the 4D dynamics we can make a foliation on \( \psi = \psi_0 \) on the line element (1), such that the effective 4D metric holds: \( dS^2|_{\text{eff}} = ds^2 \), where

\[
ds^2 = \psi_0^2 \frac{\Lambda(t)}{3} dt^2 - \psi_0^2 e^{2 \int \sqrt{\frac{3}{2}} \frac{\dot{t}}{t} dt} dr^2. \tag{9}
\]

In this section we shall study the dynamics described by the inflaton field \( \varphi \), making emphasis on the long wavelength section, which describes this field on cosmological scales.

A. 4D effective dynamics of \( \varphi \)

The 5D Lagrangian density (3) can be expanded as

\[
(4) \quad \mathcal{L} = \frac{1}{2} g^{\mu\nu} \varphi,_{\mu} \varphi,_{\nu} + \frac{1}{2} g^{\psi\psi} \varphi,_{\psi} \varphi,_{\psi}, \bigg|_{\psi_0}, \tag{10}
\]

such that the 4D potential is

\[
V(\varphi) = -\frac{1}{2} g^{\psi\psi} \varphi,_{\psi} \varphi,_{\psi} \bigg|_{\psi_0} = \frac{2}{\psi_0^2} \varphi^2(t, \vec{r}, \psi_0). \tag{11}
\]

Furthermore, from the equation (4), we obtain the effective 4D dynamics for \( \varphi(t, \vec{r}, \psi_0) \)

\[
\ddot{\varphi} + \left( 3 \sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{\Lambda} \right) \varphi - \frac{\Lambda}{3} e^{-2 \int \sqrt{\frac{3}{2}} \frac{\dot{t}}{t} dt} \nabla^2 \varphi - \frac{\Lambda}{3} \left[ 4 \psi \frac{\partial \varphi}{\partial \psi} + \psi^2 \frac{\partial^2 \varphi}{\partial \psi^2} \right] \bigg|_{\psi_0} = 0, \tag{12}
\]

where we can make the identification

\[
V'(\varphi) = -\frac{\Lambda}{3} \left[ 4 \psi \frac{\partial \varphi}{\partial \psi} + \psi^2 \frac{\partial^2 \varphi}{\partial \psi^2} \right] \bigg|_{\psi_0} = \frac{2 \Lambda}{3} \varphi(t, \vec{r}, \psi_0). \tag{13}
\]

After make the transformation \( \varphi(t, \vec{r}, \psi = \psi_0) = e^{-\frac{4}{3} \int \sqrt{\frac{3}{2}} \frac{\dot{t}}{t} dt} \chi(t, \vec{r}) \), we obtain the equation of motion for the redefined field \( \chi \)

\[
\ddot{\chi} - \frac{\Lambda}{3} e^{-2 \int \sqrt{\frac{3}{2}} \frac{\dot{t}}{t} dt} \nabla^2 \chi - \left[ \frac{\Lambda}{12} + 3 \frac{\dot{\Lambda}}{4 \Lambda} - \frac{\dot{\Lambda}}{2 \Lambda} \right] \chi = 0, \tag{14}
\]

which can be expanded on the hypersurface \( \psi = \psi_0 \), as

\[
\chi(t, \vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3 k_r \int d^3 k_\psi \left[ a_{k_r k_\psi} e^{i \vec{k_r} \cdot \vec{r}} \xi_{k_r k_\psi}(t) + a^*_{k_r k_\psi} e^{-i \vec{k_r} \cdot \vec{r}} \xi_{k_r k_\psi}^*(t) \right] \delta(k_\psi - k_\psi_0). \tag{15}
\]

Furthermore, the modes \( \xi_{k_r k_\psi}(t) \) are given by the equation of motion

\[
\ddot{\xi}_{k_r k_\psi} + \left[ \frac{k_r^2 \Lambda}{3} e^{-2 \int \sqrt{\frac{3}{2}} \frac{\dot{t}}{t} dt} - \left( \frac{\Lambda}{12} + 3 \frac{\dot{\Lambda}}{4 \Lambda} - \frac{\dot{\Lambda}}{2 \Lambda} \right) \right] \xi_{k_r k_\psi} = 0. \tag{16}
\]
B. 4D stochastic dynamics of $\chi$ on cosmological scales

In order to describe separately the long and short wavelength sectors of the field $\chi$ we can define the fields $\chi_L(t, \vec{r})$ and $\chi_S(t, \vec{r})$

$$\chi_L(t, \vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \int \theta(\epsilon k_0(t) - k_r) \left[ a_{k_r, k_{\psi}} e^{i\vec{k}_r \cdot \vec{r}} \xi_{k_r, k_{\psi}}(t) + c.c. \right] \delta(k_{\psi} - k_{\psi_0}),$$

$$\chi_S(t, \vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \int \theta(k_r - \epsilon k_0(t)) \left[ a_{k_r, k_{\psi}} e^{i\vec{k}_r \cdot \vec{r}} \xi_{k_r, k_{\psi}}(t) + c.c. \right] \delta(k_{\psi} - k_{\psi_0}),$$

where c.c denotes the complex conjugate and $k_0(t) = \sqrt{\Lambda^3 dt \left[ \frac{1}{4} + \frac{9 \lambda^2}{4 \Lambda^3} - \frac{3 \lambda}{2 \Lambda^2} \right]^{1/2}}$. The field that describes the dynamics of $\chi$ on the infrared sector ($k_r^2 \ll k_0^2$) is $\chi_L$. Its dynamics obeys the Kramers-like stochastic equation

$$\ddot{\chi}_L - \frac{k_r^2}{a^2} \chi_L = \epsilon \left[ \frac{d}{dt} \left( \dot{k}_0 \eta(t, \vec{r}) \right) + \dot{k}_0 \gamma(t, \vec{r}) \right],$$

where $\frac{k_r^2}{a^2} = \left( \frac{3}{\Lambda_0} \right)^2 \left[ \frac{1}{4} + \frac{9 \lambda^2}{4 \Lambda^3} - \frac{3 \lambda}{2 \Lambda^2} \right]^{1/2}$ and the stochastic operators $\eta$, $\kappa$ and $\gamma$ are

$$\eta = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \left[ \delta(\epsilon k_0 - k_r) \left[ a_{k_r, k_{\psi_0}} e^{i\vec{k}_r \cdot \vec{r}} \xi_{k_r, k_{\psi_0}}(t) + c.c. \right] \right],$$

$$\gamma = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \left[ \delta(\epsilon k_0 - k_r) \left[ a_{k_r, k_{\psi_0}} e^{i\vec{k}_r \cdot \vec{r}} \xi_{k_r, k_{\psi_0}}(t) + c.c. \right] \right].$$

This second order stochastic equation can be rewritten as two coupled stochastic equations

$$\dot{u} = \frac{k_r^2}{a^2} \chi_L + \epsilon \dot{k}_0 \gamma,$$

$$\dot{\chi}_L = u + \epsilon \dot{k}_0 \eta,$$

where we have introduced the auxiliary field $u = \dot{\chi}_L - \epsilon \dot{k}_0 \gamma$. The condition to can neglect the noise $\gamma$ with respect to $\eta$, is

$$\frac{\langle \dot{\xi}_{k_R} \dot{\xi}_{k_R}^* \rangle}{\langle \dot{\xi}_{k_R} \rangle^2} \ll \left( \frac{\dot{k}_0}{k_0} \right)^2.$$  

The Fokker-Planck equation for $P(\chi_L^{(0)}, u^{(0)}|\chi_L, u)$ is

$$\frac{\partial P}{\partial t} = -u \frac{\partial P}{\partial \chi_L} - \frac{k_r^2}{a^2} \frac{\partial P}{\partial u} + D_{11}(t) \frac{\partial^2 P}{\partial \chi_L^2},$$

where $D_{11}(t) = \frac{\epsilon \dot{k}_0 k_r^2}{4\pi^2} \langle \dot{\xi} \dot{\xi} \rangle$ and $P(\chi_L^{(0)}, u^{(0)}|\chi_L, u)$ describes the probability of transition from a configuration $(\chi_L^{(0)}, u^{(0)})$ to $(\chi_L, u)$. Furthermore, $\epsilon \simeq 10^{-3}$ is a dimensionless constant such that on cosmological scales holds $k_r/k_0 < \epsilon$. Hence, the equation of motion for $\langle \dot{\chi}_L^2 \rangle = \int d\chi_L du \chi_L^2 P(\chi_L, u)$ will be
\[
\frac{d}{dt} \langle \chi_L^2 \rangle = D_{11}(t). \tag{26}
\]

To describe the dynamics of the squared $\varphi_L$-expectation value we return to the original field $\varphi_L = e^{-\frac{1}{2} \int \left[ \sqrt{\Lambda/3 - \dot{\Lambda}/\Lambda} \right] dt} \chi_L$, so that the equation (26) can be rewritten as
\[
\frac{d}{dt} \langle \varphi_L^2 \rangle = -3 \sqrt{\frac{\Lambda}{3} - \frac{\dot{\Lambda}}{\Lambda}} \langle \varphi_L^2 \rangle + D_{11}(t)e^{-\int \left[ \sqrt{\frac{\Lambda}{3} - \frac{\dot{\Lambda}}{\Lambda}} \right] dt}, \tag{27}
\]

which has the following solution
\[
\langle \varphi_L^2 \rangle = e^{-\int \left[ \sqrt{\frac{\Lambda}{3} - \frac{\dot{\Lambda}}{\Lambda}} \right] dt} \left[ \langle \varphi_L^2 \rangle_0 + \int D_{11}(t) dt \right], \tag{28}
\]

where $\langle \varphi_L^2 \rangle_0$ is a constant of integration.

In order to understand better this result in the context of the inflaton field fluctuations $\phi(\vec{r}, t)$, we can make the following semiclassical approach:
\[
\varphi(\vec{r}, t) = \langle \varphi(\vec{r}, t) \rangle + \phi(\vec{r}, t), \tag{29}
\]

where $\langle \varphi \rangle = \phi_c(t)$ and $\langle \phi \rangle = 0$. With this representation one obtains
\[
\langle \varphi^2 \rangle = \phi_c^2 + \langle \phi^2 \rangle, \tag{30}
\]

where $\phi_c(t)$ is the solution of the zero mode equation in (12)
\[
\ddot{\phi}_c + 3 \sqrt{\frac{\Lambda}{3} - \frac{\dot{\Lambda}}{\Lambda}} \dot{\phi}_c + 2 \Lambda/3 \phi_c = 0. \tag{31}
\]

The solutions of physical interest for $\phi_c(t)$ should be decreasing with time, so that after inflation ends $\phi_c(t \to \infty) \to 0$. Hence, after inflation one obtains the following result:
\[
\langle \varphi^2 \rangle \bigg|_{t \to \infty} \approx \langle \phi_c^2 \rangle \bigg|_{t \to \infty}, \tag{32}
\]

which means that for $\frac{t}{t_0} \gg 1$ the following approximation is fulfilled:
\[
\langle \varphi_L^2 \rangle \bigg|_{t \to \infty} \approx \langle \varphi_L^2 \rangle \bigg|_{t \to \infty} \approx e^{-\int \left[ \sqrt{\frac{\Lambda}{3} - \frac{\dot{\Lambda}}{\Lambda}} \right] dt} \left[ \langle \varphi_L^2 \rangle_0 + \int D_{11}(t) dt \right]. \tag{33}
\]

Furthermore, we can estimate the amplitude of density energy fluctuations on cosmological scales
\[
\frac{\delta \rho}{\rho} \bigg|_{IR, (t/t_0) \gg 1} \approx \frac{\langle V'(\varphi) \rangle}{\langle V(\varphi) \rangle} \langle \phi_L^2 \rangle^{1/2}. \tag{34}
\]
IV. An Example for a Time Dependent \( \Lambda \)

As an example we consider a cosmological parameter \( \Lambda(t) = 3p^2t^{-2} \), which is related to a Hubble parameter \( H(t) = \left( \frac{\Lambda(t)}{3} \right)^{1/2} = p/t \). The effective 4D equation of state is \( p_{\text{vac}} = -\rho_{\text{vac}} = -\frac{\Lambda}{8\pi G} \). In this case the equation of motion (16) for the time dependent modes \( \bar{\xi}_{k,k_0}(t) \) is

\[
\ddot{\bar{\xi}}_{k,k_0} + \frac{1}{t^2} \left[ k^2 r^2 \left( \frac{t}{t_0} \right)^{-2p} - \frac{p^2}{4} \right] \bar{\xi}_{k,k_0} = 0. 
\]

The general solution for this equation is

\[
\bar{\xi}_{k,k_0}(t) = 2\nu \sqrt{t/t_0} \left[ \alpha \Gamma(\nu) \left( \frac{t}{t_0} \right)^{-\nu} \left( \frac{x(t)}{2} \right)^{-\nu} \mathcal{H}_\nu(x) + \beta \Gamma(-\nu) \left( \frac{t}{t_0} \right)^{\nu} \left( \frac{x(t)}{2} \right)^{\nu} \mathcal{J}_{-\nu}(x) \right],
\]

where \( \alpha \) and \( \beta \) are constants of integration, \( \nu = \sqrt{\nu^2 + 1} \) and \( x(t) = k_r (t_0/t)^p \). In this general case the solution is very difficult to be normalized by the expression (8) on the hypersurface \( \psi = \psi_0 \). However, for \( p = 2n \) (\( n \) integer positive), one obtains

\[
\bar{\xi}_{k,k_0}(t) = \sqrt{\frac{t}{t_0}} \left[ A_1 \mathcal{H}^{(1)}_\nu(x) + B_1 \mathcal{H}^{(2)}_\nu(x) \right],
\]

which can be normalized if we use the Bunch-Davies vacuum [18]. For \( A_1 = 0 \) and \( a_0 = \frac{t_0}{p} \), we obtain \( B_1 = i \frac{p}{t_0} \sqrt{\frac{\pi}{2}} \), so that for \( p = 2n \) the time dependent modes \( \bar{\xi}_{k,k_0}(t) \) holds

\[
\bar{\xi}_{k,k_0}(t) = i \frac{p}{2t_0} \sqrt{\frac{\pi}{2}} \mathcal{H}^{(2)}_\nu(x).
\]

On long wavelength modes \( (k_r \ll (t/t_0)^p) \), the second kind Hankel function \( \mathcal{H}^{(2)}_\nu(x) \) takes the asymptotic form \( \mathcal{H}^{(2)}_\nu(x) \big|_{x \ll 1} \simeq \frac{1}{\pi} \Gamma(\nu) \left( \frac{x}{2} \right)^{-\nu} \), so that the modes are

\[
\bar{\xi}_{k,k_0}(t) \big|_{k_r \ll (t_0)^p} \simeq \frac{p}{2t_0} \sqrt{\frac{t}{2\pi t_0}} \Gamma[\nu] \left[ \frac{k_r}{2} \left( \frac{t_0}{t} \right)^p \right]^{-\nu}.
\]

The diffusion coefficient \( D_{11}(t) \) in this particular case is

\[
D_{11}(t) = \frac{e^{3-2\nu} p^3 \Gamma^2[\nu] 2^{4(\nu-2)}}{\pi^3 t_0^3} \left( \frac{t}{t_0} \right)^{3p},
\]

so that, for late times (i.e., at the end of inflation) one obtains

\[
\left\langle \phi_L^2 \right\rangle_{IR, (t/t_0) \gg 1} \simeq \left( \frac{t}{t_0} \right)^{-(3p+2)} \left[ \left\langle \varphi_L^2 \right\rangle_0 + \frac{e^{3-2\nu} p^3 \Gamma^2[\nu] 2^{4(\nu-2)}}{\pi^3 (1+3p)t_0^2} \left( \frac{t}{t_0} \right)^{3p+1} \right],
\]

\[
\end{align}
\]
which is valid on cosmological scales. For late times one obtains $\langle \phi_L^2 \rangle_{IR} \sim t^{-1}$, independently of the power $p$. The evolution of $\phi_c(t)$ for this model is

$$\phi_c(t) = \phi_0 \left( \frac{t}{t_0} \right)^{-(\frac{1+3p}{2})} \left[ A \left( \frac{t}{t_0} \right)^{-\frac{\sqrt{1+6p+p^2}}{2}} + B \left( \frac{t}{t_0} \right)^{-\frac{\sqrt{1+6p+p^2}}{2}} \right],$$

(42)

where $\phi_0$ is $\phi_c(t = t_0)$ and $(A,B)$ are dimensionless constants such that $A + B = 1$. Note that $\phi_c$ is monotonically decreasing. Finally, the energy density fluctuations for late times are

$$\frac{\delta \rho}{\rho} \bigg|_{IR,(t/t_0)\gg1} \approx \frac{\phi_c(t)}{\langle \phi_L^2 \rangle} \left( \frac{\phi_L^2}{\langle \phi_L^2 \rangle} \right)^{1/2},$$

(43)

which for very large $p$ go as $\frac{\delta \rho}{\rho} \bigg|_{IR,(t/t_0)\gg1,p\gg1} \sim t^{-p}$.

V. FINNAL COMMENTS

We have studied a cosmological model for the early universe from a NKK theory of gravity with a space-like extra dimension, where the cosmological parameter decreases with time. We have worked a stochastic treatment for the effective 4D inflaton field without the hypothesis of a slow-roll regime. Hence, the dynamics the field on large scales is described by a second order stochastic equation. In this framework the long-wavelength modes of the inflaton field reduces to a quantum system subject to a quantum noise which is originated by the short-wavelength sector. In this approach, the effective 4D potential is quadratic in $\phi$ and has a geometrical origin. As in STM theory [14] of gravity 4D source terms are induced from a 5D vacuum and the fifth dimension (here a space-like one) is noncompact. In our theory the 5D vacuum is represented by a 5D globally flat metric (which describes a 5D apparent vacuum $G_{AB} = 0$) and a purely kinetic density Lagrangian for a quantum scalar field minimally coupled to gravity.

In the example here studied for $\Lambda = 3p^2t^{-2}$, we obtain that the energy density fluctuations decrease monotonically with time. In particular, we obtain that for very large $p$ these fluctuations go as $t^{-p}$.

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REFERENCES

[1] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[2] H. Salehi, Int. J. Theor. Phys. 26, 2035 (1997).
[3] W. Chen, Y-S. Wu, Phys. Rev. D41, 695 (1990).
[4] M. A. Jafarizadeh, et al, Phys. Rev. D60 063514 (1999).
[5] P. T. P. Viana, A. R. Liddle, Phys. Rev. D57, 674 (1998).
[6] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998).
[7] I. Dymnikova, M. Khlopov, Eur. Phys. J. C20, 139 (2001); K. A. Bronnikov, A. Dobosz, I. G. Dymnikova, Class. Quant. Grav. 20, 3797 (2003).
[8] A. H. Guth, Phys. Rev. D23, 347 (1981).
[9] A. Albrecht and P. Steinhardt, Phys. Rev. Lett. 48 1220 (1982).
[10] A. D. Linde, Phys. Lett. B108, 389 (1982).
[11] A. A. Starobinsky, Phys. Lett. B117, 175 (1982).
[12] P. Wesson, Gen. Rel. Grav. 22, 707 (1990); P. S. Wesson, Phys. Lett. B276, 299 (1992); P. S. Wesson and J. Ponce de Leon, J. Math. Phys. 33, 3883 (1992); P. Wesson, H. Liu and P. Lim, Phys. Lett. B298, 69 (1993); H. Liu and P. S. Wesson, J. Math. Phys. 33, 3888 (1992).
[13] T. Liko and P. S. Wesson, J. Math. Phys. 46, 062504 (2005).
[14] the reader can see, for example: P. S. Wesson, Space-Time-Matter, World Scientific, Singapore, 1999.
[15] J. Ponce de Leon, Gen. Relativ. Gravit. 20, 539 (1988).
[16] the reader can see, for example: M. Mashhoon, H. Liu, P. Wesson, Phys. Lett. B331, 305 (1994).
[17] D. S. Ledesma, M. Bellini, Phys. Lett. B581, 1 (2004); M. Bellini, Phys. Lett. B609, 187 (2005); J. E. Madriz Aguilar, M. Bellini, Phys. Lett. B619, 208 (2005).
[18] T. S. Bunch and P. C. W. Davies, Proc. Roy. Soc. A360, 117 (1978).