Spin nutation effects in molecular nanomagnet–superconductor tunnel junctions

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Abstract
We study the spin nutation effects of a molecular nanomagnet on the Josephson current through a superconductor–molecular nanomagnet–superconductor tunnel junction. We explicitly demonstrate that, due to the spin nutation of the molecular nanomagnet, two oscillatory terms emerge in the ac Josephson current in addition to the conventional ac Josephson current. Some resonances occur in the junction due to the interactions of the transported quasiparticles with the bias voltage and molecular nanomagnet spin dynamics. Their appearance indicates that the energy exchanged during these interactions is in the range of the superconducting energy gap. We also show that the spin nutation is able to convert the ac Josephson current to a dc current, which is interesting for applications.

1. Introduction
Molecular nanomagnets have recently attracted intense attention due to their applications in quantum information processing [1] and molecular spintronics [2], which is an emerging field combining the capabilities of molecular junctions and spintronics [3]. In molecular spintronics, magnetic molecules are employed to manipulate the spin degrees of freedom in order to exploit the various properties of molecular junctions. The long magnetization relaxation time of molecular nanomagnets [4, 5] and the small size of junctions consisting of them lead to interesting phenomena such as a negative differential conductance and complete current suppression [6, 7], which are crucial in high-density information storage [8], quantum computing [9] and nanoelectronics.

One of the powerful methods to probe the intrinsic properties of molecular nanomagnets is the use of transport measurements through junctions consisting of them [10–12]. An example is the direct observation of their magnetic states and their easy-axis orientations by three-terminal measurements of charge transport [13]. Electronic transport through magnetic molecules connected to metallic leads has been investigated by extensive theoretical works. Different aspects of molecular nanomagnets, such as the effect of the exchange coupling between spins of conduction electrons and the spin of the molecular nanomagnets [14], have been studied, as well as the Coulomb blockade in transport through a molecular nanomagnet weakly coupled to a magnetic and a nonmagnetic lead [15], the possibility of writing, storing, and reading spin information in memory devices [16], the Kondo effect in transport through a single molecular nanomagnet strongly coupled to two metallic electrodes [17], the effects of the spin Berry phase on electron tunneling [18], magnetic switching of the molecular nanomagnets’ spin by a spin-polarized current [19], the tunneling magnetoresistance [20] and the effect on the transport of a soft vibrating mode of the molecule [21]. The current through a precessing molecular nanomagnet connected to metallic contacts has been obtained by considering the molecular magnet as a...
It was shown that the spin precession causes modulation in the conductance with two frequencies \(\omega_L = 2\omega_A\), where \(\omega_A\) is the Larmor precession frequency. It has been recognized that the electron-spin-resonance scanning tunneling microscopy (ESR-STM) technique is capable of detecting the precession of a single spin through the modulation of the tunneling current [23]. The mechanisms underlying the ESR-STM technique are the spin–orbit coupling and exchange interactions between the localized spin and conduction electrons.

The coupling of molecular nanomagnets to superconducting leads changes their transport properties via the Andreev reflection at the contacts. The Josephson current through molecular nanomagnets connected to superconducting leads has been investigated experimentally in [24, 25]. Theoretical investigations of the Josephson current through an isotropic molecular nanomagnet revealed an asymmetric phase diagram in the exchange coupling [26]. The coupling of the spin dynamics of molecular nanomagnets and the Josephson current can reveal many interesting phenomena. It was found that the Josephson current through junctions consisting of two superconductors having equal spin-triplet pairing symmetry remains unmodulated [27]. It was also shown that a circularly polarized ac spin current with the Larmor precession remains unmodulated [27].

In this paper, we investigate the Josephson current through a junction consisting of a molecular nanomagnet connected to two spin-singlet superconductors via tunnel barriers. We present a theoretical study of the molecule’s spin nutation effects on the flowing supercurrent. The spin nutation can be generated by applying an external time-dependent magnetic field to the molecular nanomagnet which is a combination of a static and a rotating transverse rf field. Very recently, we have shown that the spin nutation of a molecular nanomagnet could pump the charge current through a Josephson junction [32]. Defining an anomalous Green’s function we obtain the Josephson current through the junction in the presence of a bias voltage. We show that the ac Josephson current depends strongly on the nutation. We explicitly demonstrate that, due to the molecular nanomagnet nutation, two oscillatory terms emerge in the ac Josephson current in addition to the conventional ac Josephson oscillation.

The outline of this paper is as follows. In section 2 we introduce our model, write the Hamiltonian of the Josephson junction and introduce the dynamics of the molecular nanomagnet. In section 3 we obtain the ac Josephson current. Results and discussions are presented in section 4. Finally, we summarize our results and give our conclusions.

2. Model and basic equations

The system under consideration is a Josephson tunnel junction with a nutating molecular nanomagnet sandwiched between two conventional spin-singlet superconductors, as shown in figure 1. The system is generally described by the Hamiltonian

\[
H(t) = H_L + H_R + H_T(t),
\]

where \(H_L\) and \(H_R\) are the BCS Hamiltonian of the left (L) and right (R) superconductors with the amplitude of the pair potential \(\Delta\) and phases \(\chi_L\) and \(\chi_R\):

\[
H_{\alpha} = \sum_{k,\sigma=\uparrow,\downarrow} \varepsilon_k c_{\alpha,k,\sigma}^\dagger c_{\alpha,k,\sigma} + \sum_k (\Delta_\alpha c_{\alpha,k,\uparrow}^\dagger c_{\alpha,-k,\downarrow} + \text{h.c.}),
\]

where \(c_{\alpha,k,\sigma}^\dagger\) is the creation (annihilation) operator of an electron in the lead \(\alpha = L, R\) with momentum \(k\) and spin \(\sigma\), and \(\varepsilon_k\) is the energy of a single conduction electron. The two leads are weakly coupled via the tunneling Hamiltonian:

\[
H_T(t) = \sum_{k,k',\sigma,\sigma'} (c_{R,k,\sigma}^\dagger T_{\sigma\sigma'}(t)c_{L,k',\sigma'} + \text{h.c.}),
\]

where \(T_{\sigma\sigma'}(t)\) is a component of the time-dependent tunneling matrix which transfers electrons through the system. When a local spin is embedded into the tunneling barriers the tunneling matrix can be written as

\[
\hat{T}(t) = T_0 \hat{1} + T_S \hat{S}(t) \cdot \hat{\sigma},
\]

where \(\hat{1}\) is a 2 \(\times\) 2 unit matrix, \(\hat{S}(t) = \frac{\hat{S}}{|\hat{S}|}\) is the unit vector along the molecular nanomagnet’s spin and \(\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)\) is the Pauli spin operator. The time dependence in equation (4) originates from the nutational motion of the molecular nanomagnet’s spin localized in the junction. The spin nutational motion shown in figure 1 is a combination of spin precession and oscillation. The parameter \(T_0\) is the spin-independent transmission amplitude and \(T_S\) is the spin-dependent transmission originating from exchange interactions between conduction electrons and localized spin [28].

![Figure 1. Schematic representation of the spin nutation of a molecular nanomagnet coupled to the spin-singlet superconductors via tunnel barriers.](image-url)
The corresponding equation of motion reads
\[ \frac{d\mathbf{S}}{dt} = -\gamma \mathbf{S} \times \mathbf{h}_{\text{eff}}. \] (5)
where \( \gamma \) is the gyromagnetic ratio and \( \mathbf{h}_{\text{eff}} \) is the effective time-dependent magnetic field, including the applied field as well as other contributions such as crystal anisotropy and demagnetization fields. To produce a nutational motion the effective magnetic field should contain two terms, a static magnetic field along the \( z \) axis and a rotating transverse \( rf \) field:
\[ \mathbf{h}_{\text{eff}}(t) = (-h_0 \sin \omega t \sin \Omega t, h_0 \sin \omega t \cos \Omega t, h_z). \] (6)
The solution of (5) is given by
\[ \mathbf{S}(t) = \mathbf{S}(0) \exp(i \gamma t \mathbf{h}_{\text{eff}}), \]
where \( \mathbf{h}_{\text{eff}} \) is the effective magnetic field.

These relations describe the nutational motion of the molecular nanomagnet. It is a combination of precession around the \( z \) axis with precession frequency \( \Omega = \gamma h_z \) and an axial tilt (angle between the spin and the \( z \) axis) oscillation about \( \theta_0 \) with amplitude \( \theta \) and frequency \( \omega = \gamma h_0/\theta \) (see figure 1). There are many studies in the literature which have focused on the properties of the system emerging from the interplay of transported particles and localized spin precession. However, none of them have investigated the effects of localized spin nutation. In this work we study the simultaneous effects of spin precession and spin oscillation (spin nutation) on the transported charge carriers. In a very recent work, we have shown that the spin nutation could serve as a pumping parameter in such a system and pump an ac Josephson current through the junction [32]. In the following, we investigate the effects of spin nutation on the Josephson current in the presence of a bias voltage.

3. Josephson current

In the tunneling limit and in the presence of a bias voltage \( V \), the Josephson current at lead \( \alpha \) is given by
\[ I_{\alpha}^d(t) = -e \int_{-\infty}^{t} dt' \left\{ \cos \chi(t,t') \left[ A_\alpha(t), A_\alpha(t') \right] \right\} + \text{h.c.}, \] (9)
where the operator \( A_\alpha(t) \) is
\[ A_\alpha(t) = \sum_{k,k',\sigma} \epsilon^{\alpha}_{k,k',\sigma}(t) T_{\sigma\sigma'}(t) c_{\alpha,k',\sigma}^\dagger(t). \] (10)
In the above equation \( \epsilon = L(R) \) and \( \alpha' = R(L) \). Let us define the following retarded potential
\[ X^{\alpha\beta}_{\text{ret}}(t,t') = -i \Theta(t-t') \left\{ \left[ a^{\alpha\beta}_{k,k',\sigma}(t), a^{\alpha\beta}_{k,k',\sigma}(t') \right] \right\}, \] (11)
where we have defined \( a^{\alpha\beta}_{k,k',\sigma}(t) = \epsilon^{\alpha}_{k,k',\sigma}(t) c_{\alpha,k',\sigma}(t) \). The retarded potential defined in equation (11) contains both triplet and singlet correlations.

In the presence of a spin-active junction including spin-flip processes between a spin-singlet superconductor and a ferromagnet, singlet correlations penetrating into the magnetic region convert to triplet correlations. These induced triplet correlations have the same magnitude as the singlet correlations at the interface, but they survive over a long range despite the singlet correlation. As a result, a nonzero Josephson current flows through strong ferromagnets (such as half metals) [33–35]. The triplet correlations also penetrate into the superconductors; however, their amplitudes are small in comparison to the bulk singlet component [36]. The triplet correlations are also generated by magnetization dynamics such as spin precession and magnonic excitations. The interplay of the magnetization dynamics and transported carriers results in the conversion of the spin-singlet to spin-triplet correlations, accompanied by the absorption/emission of magnons [37, 38]. In this work, the presence of the tunnel barriers indeed allows us to ignore the effects of the triplet correlations induced in the superconducting leads, and just retain the singlet to triplet conversion process [37]. Thus, the retarded potential (11) simplifies to \( X^{\alpha\beta}_{\text{ret}}(t,t') = \sigma \sigma' \delta_{\sigma,-\sigma} \delta_{\sigma',-\sigma} \chi_{\text{ret}}(t-t') \), where \( \sigma, \sigma' = \pm 1 \). The associated Matsubara potential reads
\[ \chi(\text{i}o\nu_n) = \frac{1}{\beta} \sum_{iq} \mathcal{F}^{\alpha}(k, iq) \mathcal{F}^{\alpha}(k', iq - \text{i}o\nu_n), \] (12)
where the \( \mathcal{F}^{\alpha}(k, iq) \) is the anomalous Green’s function [39]. By analytical continuation and at zero temperature, the real part of the retarded potential is obtained as follows:
\[ \text{Re} \left\{ \sum_{k,k'} X_{\text{ret}}^{\alpha\alpha}(x) \right\} = \begin{cases} \pi N_L N_R \Delta K(x) & x < 1 \\ \pi N_L N_R \Delta X(x) & x > 1 \end{cases}, \] (13)
where \( K(x) \) is the first kind of complete elliptic integral and \( N_{LR} \) are the normal state density of states in the left and right leads. Implementing the real part of the retarded potential and defining \( J(x) = \text{Re} \left\{ \sum_{k,k'} X_{\text{ret}}^{\alpha\alpha}(x) \right\} / \pi N_L N_R \), we find the scaled Josephson current \( \bar{J} = \frac{J}{\pi e N_L N_R} \) as
\[ \bar{J}(t) = J_0 \sin(\omega_0 t + \chi) + \bar{J}_+ \sin(\omega_+ t + \chi) + \bar{J}_- \sin(\omega_- t + \chi). \] (14)
where \( \chi = \chi_R - \chi_L \) is the phase difference between superconducting leads, \( \omega = 2eV/h \) is the Josephson frequency and \( \omega_{\pm} = \omega_0 \pm \omega \) represent the current oscillations due to molecular nanomagnet spin nutation. In the above calculations we have considered \( \theta/\theta_0 \ll 1 \). This approximation is fulfilled by the practical conditions of the system and does not impose any restriction on it. The coefficients \( J_0 \) and \( J_{\pm} \) are given by the following relations
\[ J_0 = 4t_0 \left\{ T^2_{II} - T^2_{0}\right\} J \left( \frac{eV}{2\Delta} \right) + 2T^2_2 \]
\[ \times \left[ J \left( \frac{eV + h\Omega}{2\Delta} \right) + J \left( \frac{eV - h\Omega}{2\Delta} \right) \right]. \] (15a)
where \( T_0 = T_S \cos \theta_0 \) and \( T_\perp = T_S \sin \theta_0 \) are spin-conserving and spin-flip transmission amplitudes, respectively. The first term of equation (14) is the usual ac Josephson current with oscillation frequency \( \omega_0 \) and the last two terms emerge due to the molecular nanomagnet spin nutation.

4. Results and discussions

We have obtained the Josephson current through the SC|molecular nanomagnet|SC junction by considering the spin nutation and applied bias voltage effects. In the absence of the tilt angle oscillations (\( \theta = 0 \) or equally \( h_0 = 0 \)) the last two terms in equation (14) vanish and the Josephson current is \( I_{ac} = J_0 \sin(\omega_0 t + \chi) \). This is nothing but the conventional ac Josephson current. The coefficient \( J_0 \) is nonzero whenever one of the spin-dependent amplitude \( (T_S) \) or spin-independent amplitude \( (T_0) \) is nonzero. The molecular nanomagnet spin nutation causes two \( \theta \)-dependent oscillatory terms to appear in the Josephson current. This effect strongly depends on the values of \( T_0/T_\perp = T_0^2 \sin 2\theta_0 \), the multiple of the spin-conserving and spin-flip transmission amplitudes. The coefficient \( T_0/T_\perp \) is nonzero whenever \( \theta_0 \neq n\pi/2 \) (\( n \) is an integer). Indeed, the spin nutation produces different potential energies for the two electron spin directions in the leads and the tunneling of the quasiparticles is mediated by a spin-flip process. These tunneling processes, which are accompanied by absorption/emission of the quantum of oscillations, result in the appearance of additional exotic oscillatory parts in the Josephson current. These additional terms are explained by implementing the Andreev level picture. The Andreev levels are sharp states in the superconducting gap produced by constructive Andreev reflections from the two superconducting interfaces. In [29, 30] it was shown that when a nanomagnet undergoes a precession due to an external magnetic field the Andreev levels are affected by the dynamics of the nanomagnet’s spin. The resultant Andreev levels depend on the tilt angle and precession frequency. In the adiabatic regime, when the tilt angle oscillation is slow in comparison to the characteristic time of the quasiparticle propagation, the Andreev level dynamics will follow the tilt angle oscillation. Thus, this oscillation modulates the Josephson current in addition to the modulation due to the nonzero bias voltage. In the tunneling regime, where the ac Josephson current is proportional to \( \sin(\omega_0 t + \chi) \), the sinusoidal time dependence of the tilt angle causes two additional terms to appear in the Josephson current with modulation frequencies \( \omega_0 \pm \omega \).

Numerical computations are needed to obtain the energy of the Andreev levels in the Josephson junction with a precessing spin and arbitrary transparency of the contacts. An analytical expression for the energy of Andreev levels in this junction has been provided in [30], apart from a term giving rise to the Zeeman splitting which has not been given an explicit expression. To give an illustration for the above discussions let us calculate the energy of Andreev levels by neglecting the tilt angle dependence of this term and replacing it by an effective Zeeman splitting. In the tunneling limit, \( T_0 \) and \( T_S \) are very small and the energy of Andreev levels is given by

\[
\epsilon_\pm = -\Delta_0 \sqrt{1 + \Phi(T_0, T_S, \Omega, \theta, \chi)} \pm \frac{\hbar \Omega}{2 \Delta},
\]

where

\[
\Phi(T_0, T_S, \Omega, \theta, \chi) = -2T_0 (1 + \cos(2\theta)) \sin^2(\chi/2) + \frac{\hbar \Omega}{2 \Delta}.
\]

The Josephson current at the zero temperature is given by \( dc/d\chi \). In the presence of an applied voltage we have \( \chi \rightarrow \chi + \omega_0 t \), and the Josephson current reads

\[
I(t) = \frac{e\Delta (T_0 - T_S)}{\hbar} \sqrt{1 + \left(\frac{\hbar \Omega}{2 \Delta}\right)^2} [1 + \cos(2\theta(t))] \sin(\omega_0 t + \chi).
\]

By expanding \( \cos(2\theta) \) in terms of the small amplitude of tilt angle oscillation \( \theta \), the above expression reduces to

\[
I(t) = \frac{e\Delta (T_0 - T_S)}{\hbar} \sqrt{1 + \left(\frac{\hbar \Omega}{2 \Delta}\right)^2} \cos^2\theta_0 + \theta \cos(\omega_0 t)
\times \sin\theta_0 \cos\theta_0 \sin(\omega_0 t + \chi).
\]

The term \( \cos(\omega_0 t + \chi) = (1/2) [\sin(\omega_0 + \omega)t + \chi] + \sin((\omega_0 - \omega)t + \chi)] \) justifies the appearance of the two terms in the ac Josephson current which are proportional to \( T_0/T_\perp \), as indicated in equation (15b).

The nanomagnet’s dynamics introduces two time-dependent parameters \( \varphi(t) = \varphi_0 + \Omega t \) and \( \theta(t) = \theta_0 - \Omega \cos(\omega t) \), specifying the time-dependent direction of the nanomagnet’s spin, in addition to the applied voltage, which causes a time-dependent phase difference across the junction, \( \chi(t) = \chi + \omega_0 t \). The transport properties of the Josephson junction depend on these time-dependent parameters. The combination of the Andreev reflection of quasiparticles from the superconducting surfaces and their interaction with the nanomagnet’s spin dynamics lead to transfer of the quasiparticles into sidebands with different energies given by \( \epsilon = \epsilon_0 + \hbar \omega_0 + m\hbar \Omega + n\hbar \omega_0 \), where \( \epsilon_0 \) is the energy of the quasiparticles in the absence of these time dependences and \( m = n = 0, \pm 1 \). Scattering of the quasiparticles to these sidebands leads to the appearance of different associated terms in the coefficients \( J_\pm \).

Because of the divergence of \( K(x) \) at \( x \approx 1 \), when each of the following nine values: \( eV, eV \pm \hbar \Omega, eV \pm \hbar \omega_0 \) and
$eV \pm h\Omega \pm \hbar \omega$ is equal to the superconducting energy gap $(2\Delta)$, the Josephson current diverges. These divergences are due to the inadequacy of the lowest order perturbation theory, and would disappear if we were able to include the higher orders of perturbation up to infinity, as has been discussed in [41]. More suitable approaches for calculating the current lead to singularities instead of divergences [28, 29]. These singularities are the consequence of the singular BCS density of states at the gap edges and are signatures of the contribution of an infinite number of extended states in the current. These resonances emerge in the junction whenever the absorbed/emitted energies of the transported quasiparticles, owing to their interactions with the bias voltage and molecular nanomagnet, are in the range of the superconducting energy gap $(2\Delta)$. The typical superconducting gap is of the order of $2\Delta \sim 1.0 \, \text{meV}$, and can be smaller in an atomic point contact. On the other hand, for a field $h \sim 200$ Gauss (typical value of magnetic fields produced in a lab) the Larmor frequency of the classical spin is about $560$ MHz; exactly in the range of the relaxation time of the real magnetic molecules. The energy associated with the precession energy is $\hbar \Omega \sim \hbar \omega \sim 10^{-6} \, \text{eV}$, which is much smaller than the typical superconducting gap. However, the dependence on the value of bias voltage provides the ability to simply tune the system towards a resonance condition.

It is worthwhile studying the Josephson current when the molecular nanomagnet oscillation frequency $\omega$ is the same as the bias-dependent frequency $\omega_j$. In this case $\hbar \omega = 2 \, \text{eV}$ and the Josephson current simplifies to

\[
\dot{\varphi}(t) = J_0 \sin(\omega_j t + \chi) + \vartheta \left[ J_+ \sin \chi + J_- \sin(2\omega_j t + \chi) \right].
\]

As can be seen, the current has two time-dependent oscillatory and one time-independent non-oscillatory parts. It means that at a special value of the molecular nanomagnet oscillation frequency, the spin nutation generates a time-independent dc Josephson current, $\vartheta J_+ \sin \chi$. In fact the effect of bias voltage is partially washed out by spin nutation of the molecular nanomagnet and a non-oscillatory part appears in the current. This is an interesting result, in which the spin nutation is able to convert the ac Josephson current to a dc current.

We have also studied the behavior of the Josephson current for an arbitrary value of the bias voltage. In figure 2 we show the density plot of $|J_-/J_+|$ in terms of $\Omega$ and $\omega$ for $eV = \Delta$. In the regions where $J_-/J_+ \sim 0$, the term oscillating with frequency $\omega_j$ has a dominant contribution. However, in the regions where $J_-/J_+$ tends to infinity, the term with a frequency of $\omega_j$ becomes important. For a fixed value of $\omega$ we have one or both of the second or third terms of the Josephson current, by varying $\Omega$.

5. Summary and conclusion

In this paper we have obtained the Josephson current through the junction superconductor/molecular nanomagnet superconductor in the presence of a bias voltage $V$. We have calculated the Josephson current through the junction by working in the tunneling limit and employing a Green’s function technique. We have investigated the effects of the spin nutation—the simultaneous effects of spin precession and spin oscillation—on the behavior of the Josephson current. The spin nutation is generated by applying a time-dependent magnetic field to the molecular nanomagnet. The interplay of the spin dynamics of the molecular nanomagnet and the Andreev reflection of the quasiparticles at the superconductor surfaces, which is accompanied by a spin-flip process while exchanging energy with the nutating spin, causes the generation of two extra terms in the Josephson current, with frequencies $\omega_j \pm \omega$, greater and smaller than the usual Josephson frequency by the tilt angle oscillation frequency. Depending on the values of the bias voltage, the precession frequency and the tilt angle oscillation frequency, some divergences emerge in the Josephson current. Such behavior appears whenever a resonance occurs at the junction. The resonance conditions arise when the superconducting energy gap is equal to the absorbed/emitted energy of the transported quasiparticles, due to their interactions with applied bias voltage ($V$) and with the dynamics of the molecular nanomagnet. We have also found that the spin nutation is able to convert the ac Josephson current to a dc current. This effect may be used in single-spin detection.

It has been shown that a spin current is generated in superconducting leads by spin nutation of a molecular nanomagnet [28–31]. The spin current induces a torque on the spin and changes its dynamics. This back-action effect leads to a change in the parameters of the nutational motion, i.e. a precession frequency and a tilt angle oscillation frequency [30, 40]. Thus, we can conclude that it does not change our main results. Incorporating a Josephson junction in a SQUID loop to control the phase difference of the junction makes it possible to measure the current phase relation. Recent advances in the fabrication of molecular Josephson junctions has made it possible to measure the current phase relation in an atomic point contact [43]. Tuning the bias voltage as $2eV = \hbar \omega$ causes a time-independent term
to appear in the Josephson current, whereas the other two terms oscillate with frequency \( \omega_j \). Taking the time average eliminates the effects of the oscillatory terms. Changing the magnetic flux penetrating into the loop changes the phase difference of the junction as well as the coefficient of the time-independent term \( J_\alpha \) without altering the value of \( \omega_0 \). Therefore, measuring the time average of the Josephson current will directly reveal the existence of this time-independent term.

We have considered an isolated molecular nanomagnet in our calculations. A real molecular spin \( S \) interacts with its environment [5]. Thus, its properties are different from those of the isolated spin considered in our model. The interaction of the spin with its environment, such as exchange field and magnetic anisotropy, can be considered by including a small Gilbert damping constant in the calculations [37]. In this case, the dynamics of a single spin is given by the Landau–Lifshitz–Gilbert equation:

\[
\frac{dS}{dt} = -\gamma S \times h_{\text{eff}} + \alpha S \times \frac{dS}{dt},
\]

where \( \alpha \) is the Gilbert damping constant. At a finite temperature the spin dynamics of a realistic nanomagnet is governed by the stochastic Landau–Lifshitz (SLL) equation [42]

\[
\frac{dS}{dt} = -\gamma S \times [h_{\text{eff}}(t) + b(t)] - \alpha[S \times [S \times h_{\text{eff}}(t) + b(t)]]],
\]

where \( b(t) \) is a stochastic field including the nonzero temperature effects with the following statistical properties

\[
\langle b_\alpha(t) \rangle = 0; \quad \langle b_\alpha(t)b_\beta(t') \rangle = 2D\delta_{\alpha\beta}\delta(t-t'),
\]

where \( \alpha \) and \( \beta \) are the Cartesian coordinates of the field and \( D \) is the strength of the thermal fluctuations. This equation leads to a reduction of the size of spin caused by excited states during the precession or reversal of the magnetization owing to weak coupling with a thermal bath. We will consider the effect of the temperature in subsequent works.

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