Axion Dark Matter and Cosmological Parameters

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We observe that photon cooling after big bang nucleosynthesis (BBN) but before recombination can remove the conflict between the observed and theoretically predicted value of the primordial abundance of $^7\text{Li}$. Such cooling is ordinarily difficult to achieve. However, the recent realization that dark matter axions form a Bose-Einstein condensate (BEC) provides a possible mechanism, because the much colder axions may reach thermal contact with the photons. This proposal predicts a high effective number of neutrinos as measured by the cosmic microwave anisotropy spectrum.

PACS numbers: 95.35.+d

INTRODUCTION

The agreement between observations and the BBN predictions for the primordial abundances of light elements is often touted as a triumph of the standard ΛCDM cosmological model. Under the assumption that there are three neutrino species, BBN as a theory requires essentially a single input: the baryon-to-photon ratio, usually given by the parameter $\eta_{10} = 10^{10} n_B / n_\gamma$ [1]. If one takes $\eta_{10}$ to be $6.190 \pm 0.145$, in accordance with the latest Wilkinson Microwave Anisotropy Probe (WMAP) results [2,3], the inferred primordial abundances of the majority of the light elements (D, $^4\text{He}$, $^3\text{He}$) are remarkably consistent with BBN predictions, save one exception: that of $^7\text{Li}$ is approximately two to three times less than what the theory predicts. The discrepancy is deemed statistically significant, and there is so far no widely accepted explanation for the anomaly. In the literature, this is referred to as the “Lithium Problem”.

One of the most difficult issues involved in testing BBN is how reliably to infer the primordial abundances of light elements from measurements that are available to us. Subsequent to BBN, the original relic abundances are all subject to further modification by complicated stellar processes. $^7\text{Li}$, for example, can be both depleted and synthesized in stars, as well as produced by cosmic-ray nucleosynthesis. As such, the abundance of $^7\text{Li}$ is inferred primarily from absorption lines in the atmosphere of galactic halo stars with low metallicity, since these stars are very old and have experienced very little nuclear processing (see [1,2,3] for details).

Although these post-BBN effects lead to considerable complication, they also open up many different avenues to explain the $^7\text{Li}$ anomaly. For many years, it has been hoped that better determination of nuclear parameters will gradually narrow the discrepancy, though it was eventually realized that does not seem achievable [3]. Quite the contrary, it was found in [1] that improved data on the neutron life-time and the cross sections $p(n,\gamma)d$ and $^3\text{He}(n,\gamma)^7\text{Be}$ increases the predicted abundance of $^7\text{Li}$, worsening the disagreement. Revisions to stellar evolution, as a consequence of systematic errors in the effective temperature of the metal-poor stars [7,8], and surface $^7\text{Li}$ depletion in the interior of stars due to some mixing or diffusive processes [9], have also been investigated as possible solutions, but are still considered controversial [1].

The fact that the nuclear reactions relevant to the production of both primordial and post-BBN $^7\text{Li}$ are now quite well understood has led to speculations that the anomaly might instead be caused by new physics. Many explanations have been proposed, such as the variation in time of the deuteron binding energy and of fundamental couplings [10,11], and the decay of a relatively long-lived particle in the context of supersymmetry [12]. At this point, none of these explanations have won general acceptance in the cosmology community.

In this paper, we propose a way to remove the conflict between data and theory for the abundance of $^7\text{Li}$, based on the cooling of photons between the end of BBN and decoupling. Processes that do this are difficult to come by. Indeed, typical processes arising from new physics tend to heat up the photons, modifying $\eta_{10}$ in the wrong direction [1]. However, the recent realization that dark matter axions form a BEC at approximately 500 eV photon temperature [13] provides a possible mechanism [14]. Essentially, the high occupation of axion modes with very low momenta greatly enhances the strength of their gravitational interactions, such that an exchange of energy between the photons and the much colder axions becomes possible. Photon cooling implies that $\eta_{10,\text{BBN}} < \eta_{10,\text{WMAP}}$, which has the effect of reducing the production of $^7\text{Li}$ [5]. If thermal equilibrium between the photons and axions is achieved, the $^7\text{Li}$ abundance is reduced by approximately a factor 2 (see below), alleviating the discrepancy and perhaps removing it altogether. However, our proposal predicts a higher abundance for D than present observations indicate and predicts that the effective number of thermally excited neutrino degrees of freedom is high: $N_{\text{eff}} = 6.77$.

Photon cooling by kinetic mixing with hidden photons was proposed in ref. [15].
DARK MATTER AXIONS

The axion was originally postulated to explain the absence of CP violation in the strong interactions \[16\]. It was later realized that the population of axions produced by the turn on of the axion mass during the QCD phase transition possesses the right properties to be cold dark matter (CDM) \[17\]. First, they have the measured CDM density if the axion mass \(m\) is of order \(10^{-5}\) eV/c\(^2\) \[18\]. Second, the axions thus produced are very cold: their average momentum is only of order the Hubble expansion rate \((3 \times 10^{-9}\) eV/h\) when the axion mass effectively turns on and has been redshifting ever since to a mere \(10^{-17}\) mc today. Third, axions in this mass range interact very weakly through all forces other than gravity. These properties make axions one of the leading candidates for CDM. The other main contenders are weakly interacting massive particles (WIMPs) and sterile neutrinos. Henceforth, we set \(\hbar = c = 1\).

Observationally it seems difficult to distinguish among the CDM candidates. However, it was recently realized that axions form a BEC through gravitational self-interactions at approximately a photon temperature of \(500\) eV \[13, 14\]. The relaxation rate of cold dark matter axions through gravitational self-interactions is of order

\[
\Gamma_a \sim 4\pi G n m^2 \ell^2
\]

where \(n\) is the number density of cold axions and \(\ell \sim t_1 a(t_1)\) their correlation length. \(t_1 \sim 2 \cdot 10^{-7}\) s is the time at which the axion mass effectively turns on during the QCD phase transition, and \(a(t)\) is the scale factor. Eq. (1) is appropriate when the energy dispersion of the particles is less than their relaxation rate. We refer to this case as ‘the condensed regime’, to distinguish it from the more commonly encountered ‘particle kinetic regime’ defined by the condition that the energy dispersion of the particles is large compared to the relaxation rate. Cold dark matter axions are in the condensed regime after \(t_1\). The ratio \(\Gamma_a(t)/H(t)\), where \(H(t)\) is the Hubble rate, is of order \(5 \cdot 10^{-7}\) at \(t_1\) but increases with time as \(a(t)^{-3} t\), and reaches one at approximately \(500\) eV photon temperature. At that time the axions thermalize and form a BEC. Almost all axions go to the lowest energy state available. The correlation length grows and becomes of order the horizon. In the linear regime of evolution of density perturbations and within the horizon, the lowest energy state is time independent and no rethermalization is necessary for the axions to remain in the lowest energy state. In that case, axion BEC and ordinary CDM are indistinguishable on all scales of observational interest \[13\]. However, beyond first order perturbation theory and/or upon entering the horizon, the axions rethermalize to try and remain in the lowest energy available state. Axion BEC behaves differently from CDM then and the resulting differences are observable.

The study of the catastrophe structure of the inner caustics of galactic halos provides evidence that the dark matter is an axion BEC \[13, 19\]. Briefly, there is a dichotomy in the classification of the inner caustics in terms of their catastrophe structure, depending on the angular momentum distribution of the infalling particles \[20\]. Axions in a BEC are in a state of net overall rotation and produce caustic rings, whereas ordinary CDM has an irrotational velocity field and produces tent-like caustics. There are several pieces of evidence for the existence of caustic rings at the predicted radii in various galaxies \[21, 22\]. It is shown in ref. \[19\] that the phase space structure implied by the evidence for caustic rings is precisely and in all respects that predicted by the assumption that dark matter is a rethermalizing axion BEC.

PHOTON COOLING

The Lithium Problem refers to the mismatch between the observed and predicted abundance of primordial \(^7\)Li by a factor 2 or 3 \[11, 12, 14\]. For \(\eta_\text{Li} \geq 2.7\), the predicted \(^7\)Li abundance increases with \(\eta_\text{Li}\). Hence, a cooling of the photons between the end of BBN and decoupling reduces the discrepancy.

The gravitational fields of the cold axion fluid cause transitions between momentum states of other particle species present. For particles which are bosons or non-degenerate fermions, the relaxation rate through gravitational interactions with the cold axions is of order \[14\]

\[
\Gamma \sim 4\pi G n m \ell \frac{\omega}{\Delta p}
\]

where \(\omega\) is the typical energy of the particles and \(\Delta p\) their momentum dispersion. Eq. (2) generalizes Eq. (1) to other species that are in the presence of the cold axions. [Eq. (1) follows from Eq. (2) by setting \(\omega = n\) and \(\Delta p = \ell^{-1}\) as is appropriate for the cold axions themselves.] For photons to cool substantially it is necessary that energy is transferred from the photons to the low momentum highly occupied axion states and from those to the relativistic axion states. For both relativistic axion states and for photons, \(\Delta p \sim \omega\) and hence their relaxation rate \(\Gamma_\text{r}\) through gravitational interactions with cold axions is of order \(4\pi G n m \ell\). Using the Friedmann equation, one finds that \(\Gamma_\text{r}/H \propto a(t)\) before equality between matter and radiation and remains constant after that. At equality, \(\Gamma_\text{r}/H|_{t_\text{eq}} \sim \ell(t_{\text{eq}})/t_{\text{eq}}\). If \(\ell/t\) is order one at equality, the photons reach thermal equilibrium with the axions and hence cool.

Gravitational interactions conserve particle number and therefore produce only kinetic (as opposed to chemical) equilibrium between the species involved. Also, after \(500\) eV photon temperature, the coupling between photons and baryons is in the kinetic, rather than chemical, equilibrium regime \[23\]. Upon cooling, the photons
that cannot be accommodated in thermally excited states enter the ground state, a plasma oscillation with zero wavevector. Since the photon chemical potential remains zero, the final photon spectrum is Planckian, consistent with observation.

Eq. (2) does not apply to degenerate fermions because of Pauli blocking. The cosmic neutrinos are semi-degenerate since they have a thermal distribution with zero chemical potential. Their thermalization rate is less than that $\Gamma_r$ of relativistic bosons. Since $\Gamma_r/H \propto t_\ell n_\ell \propto t^2 a^{-3}(t)$, that ratio does not grow after equality. Since the relativistic axions may only reach thermal contact with the cold axions at equality and the neutrinos are delayed relative to the relativistic axions, we believe it most likely that neutrinos remain decoupled from the axions, photons and baryons at all times.

It is straightforward to determine how much the photons cool if they reach thermal equilibrium with the axions. Energy conservation implies $\rho_{\gamma}\omega = \rho_{f,\gamma} + \rho_{f,a}$ because the contributions to the energy density of the initial axions and of the baryons are negligible. The ratio between the final and initial photon temperature is thus $(2/3)^{1/4}$. Since their number density is proportional to $T^3$, we find:

$$\eta_{10,\text{BBN}} = \left(\frac{2}{3}\right)^{3/4} \eta_{10,\text{WMAP}} = 4.57 \pm 0.11 \quad (3)$$

using $\eta_{10,\text{WMAP}} = 6.190 \pm 0.145 \ [2]$. Because the $^7\text{Li}$ abundance is proportional to $\eta_{10,\text{BBN}}^2$ in the range of interest, it is reduced by approximately the factor $(2/3)^{3/2} \simeq 0.55$.

A number of authors proposed earlier that the dark matter is a BEC [24]. The photon cooling described here may occur in those cases as well. If the particles are in the condensed regime, the relaxation rate is given by Eq. (1), calculated with appropriate values for $n$, $m$, and $\ell$. However, in many of these proposals, the cosmological history of the dark matter particle is not known, rendering the computation of the relaxation rate difficult.

**COSMOLOGICAL CONSEQUENCES**

**Effect on light element primordial abundances**

Whether photon cooling by axion BEC solves the Lithium Problem remains to be seen. The data have been time dependent in addition to the usual uncertainties. In Fig. 1, we plot the value of $\eta_{10,\text{BBN}}$ in the standard cosmological model, labeled ‘WIMP’, and in the scenario described here, labeled ‘axion’, along with the values inferred from the observed light element abundances according to the review by G. Steigman in 2005 [4], the review by F. Iocco et al. in 2008 [25] and a private communication from G. Steigman updating his 2005 estimates.

In the light of recent observations [26]. The error bars indicate the range of Steigman’s 2005 review ($\eta_{10,\text{BBN}}$ consistent with the estimated 1-$\sigma$ uncertainties in the observations.

Perhaps more problematic is that a smaller $\eta_{10,\text{BBN}}$ predicts an overproduction of D. Traditionally, D has been the prime choice as a baryometer among the light elements, due to its sensitivity to $\eta_{10,\text{BBN}}$ and simple post-BBN evolution (abundance monotonically decreasing). The major drawback with D is that its abundance is inferred from a very small set of (seven) spectra of QSO absorption line systems [27]. Worse yet, these few measurements have a large dispersion, and do not seem to correlate with metallicity, obscuring the expected deuterium plateau. Due to the various inadequacies in the D measurements mentioned, we have reservations about the common practice of attaching most significance on D in the comparison between data and BBN predictions. In comparison, $^7\text{Li}$ is inferred from a large number of measurements, which are more-or-less consistent. Also, since D is more easily destructible than $^7\text{Li}$, it is conceivable that unknown stellar processes further deplete D.

Finally the $^3\text{He}$ and $^4\text{He}$ inferred $\eta_{10,\text{BBN}}$ values have large error bars and hence carry less statistical weight. The $^4\text{He}$ inferred value has increased recently ($5.5 < \eta_{10,^4\text{He}} < 11$ according to ref. [24] and $7.5 < \eta_{10,^4\text{He}} < 20$ according to ref. [26]) compared to its accepted value a few years ago, creating additional uncertainty.
Effective number of neutrino species

After the axions are heated up and reach the same temperature as the photons, most of them are still in the ground state. The axions in the ground state behave as cold dark matter. The axions in the excited states contribute one bosonic degree of freedom to radiation. The radiation content of the universe is commonly given in terms of the effective number \( N_{\text{eff}} \) of thermally excited neutrino degrees of freedom, defined by

\[
\rho_{\text{rad}} = \rho_\gamma [1 + N_{\text{eff}} \left( \frac{7}{8} \left( \frac{4}{11} \right)^\frac{3}{2} \right)]
\]  

(4)

where \( \rho_{\text{rad}} \) is the total energy density in radiation and \( \rho_\gamma \) is the energy density in photons only. The standard cosmological model with ordinary cold dark matter predicts \( N_{\text{eff}} = 3.046 \), slightly larger than 3 because the three neutrinos heat up a little during \( e^+ e^- \) annihilation. Taking account of the fact that not only is there an extra species of radiation (thermally excited axions) but also the contribution of the three ordinary neutrinos is boosted because the photons have been cooled relative to them, the proposed scenario predicts

\[
\rho_{\text{rad}} = \rho_\gamma + \rho_\chi + \rho_\nu
\]

\[
= \rho_\gamma \left[ 1 + \frac{1}{2} + 3.046 \left( \frac{7}{8} \left( \frac{4}{11} \right)^\frac{3}{2} \right) \right],
\]

(5)

which yields \( N_{\text{eff}} = 6.77 \).

At present, the measured values are smaller than this prediction. The WMAP collaboration found \( N_{\text{eff}} = 4.34^{+0.86}_{-0.88} \) (68% CL) based on their 7 year data combined with independent data on large scale structure and the Hubble constant [2]. An analysis [28] using the Sloan Digital Sky Survey (SDSS) data release 7 halo power spectrum found \( N_{\text{eff}} = 4.8 \pm 2.0 \) (95% CL). The Atacama Cosmology Telescope (ACT) collaboration finds \( N_{\text{eff}} = 5.3 \pm 1.3 \) (68% CL) using only their CMB anisotropy data and \( N_{\text{eff}} = 4.56 \pm 0.75 \) (68% CL) when combining that data with large scale structure data. The tendency for the measured values to be larger than 3.046 has been taken sufficiently seriously to prompt proposals for new physics involving extra neutrino species or a neutrino asymmetry [30]. The Planck mission is expected to measure \( N_{\text{eff}} \) with much greater precision [31]. In so doing, it may shed light on the nature of dark matter.

We thank Gary Steigman for extended discussions, and our referees for numerous suggestions for improvement. This work was supported in part by the U.S. DOE under contract DE-FG02-97ER41029.

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