Superconducting wires under simultaneous oscillating sources: involved magnetic response, dissipation of energy and low pass filtering

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Numerical simulations of filamentary type II superconducting wires under simultaneous AC transport current and oscillating transverse magnetic fields are performed within the critical state approximation. The time dependences of the current density profiles, magnetic flux lines, local power dissipation and magnetic moment are featured. Noticeable non-homogeneous dissipation and field distortions are displayed. Also, significant differences between the obtained AC-losses and those predicted by regular approximation formulas are reported. Finally, an outstanding low pass filtering effect intrinsic to the magnetic response of the system is described.

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The practical configurations of type-II superconducting wires exhibit complicated non-linear and hysteretic behavior under oscillating electromagnetic fields. Thus, related to the design of power applications with these materials, considerable efforts have been made to comprehend the factors determining the AC losses under typical operating conditions. Major features of the macroscopic electromagnetic behavior have been captured by Bean in the phenomenological so-called critical state model (CSM). According to this, magnetization currents are induced at the periphery of the superconductor when external flux variations occur. Such currents distribute across the section of the sample with a density equal to so-called critical value at a given temperature and field, $J_c$. Although simple in idealized configurations, the Maxwell equation problem arising from the CSM statement becomes awkward when realistic systems are considered.

In this letter, we consider the coupling between simultaneous oscillating sources acting on a superconducting wire. In particular, the simultaneous action of an AC transport current and a transverse magnetic field will be studied. Notice that such a configuration is a basic

FIG. 1. (Color online) Sketch of one of the analyzed cases. Here, $B_y^{\text{max}} = 2$, $J_{tr}^{\text{peak}} = 0.75J_c$, and the time-step corresponds to $B_y = -0.64$, $I_{tr} = -0.24$. Units are $(\mu_0/4\pi)J_cR$ for B, $\pi R^2 J_c$ for $I_{tr}$, $J_cR^3$ for M, and $(\mu_0/4\pi)J_c^2 R^4$ for E-J. The experimental process $(B_y, I_{tr})$ is summarized in the central plot, and the physics behind this kind of system can be easily understood analyzing the attached video.
model for coil systems in which each wire is under the action of its neighbors. Apparently, the problem is 2D in nature and the main complication is as follows. When the external sources change in time, a sort of free-boundary problem has to be solved. Thus, the magnetic flux density variations penetrate from the sample’s surface and an evolutionary flux-front profile is defined. Determining such front (or the core within) is a main mathematical challenge, and has been tackled by several methods. As an important handicap, in many cases as the one studied here, the core does not remain static along the electromagnetic time-varying process, even for the case of a fully penetrated sample. It is apparent that the widely used concept of an effective radius description for the free-boundary can fail markedly if the core shifts within the sample (as may be observed in Fig.1) and a center of mass is not well defined. As a consequence the so-called front tracking methods cannot easily guarantee a unique physical solution for the problem.

Here, we adopt the most popular trend in the analysis of electromagnetic applications, i.e.: numerical simulations implementing finite-element methods. As an advantage, such techniques work without explicit inclusion of the free boundary. The whole superconducting region is involved in the calculation and the boundary is obtained as a part of the solution. Taking advantage of this methodology, we have performed a systematic investigation of the electromagnetic response of the superconducting wire. Some outstanding predictions have been obtained and are reported here. In particular, (i) we feature the relevance of localized power dissipation within the sample, (ii) as regards the averaged physical quantities, we will show that some standard approximations for the power dissipation per cycle have to be revised, and finally (iii) an intriguing low pass filtering effect is announced.

Going into detail about our calculations, we use a discrete formulation that solves Faraday’s law iteratively in a mesh of circuits that carry the macroscopic electric current. Calculations are performed under the material law restriction for the current density, that in this case reads $|\mathbf{J}| \leq J_c$. Equivalently, one may use conductivity law $\mathbf{E} = \rho \mathbf{J}$ with $\rho(J) = 0$ when $|\mathbf{J}| \leq J_c$ and $\rho(J) \to \infty$ if $|\mathbf{J}| > J_c$.

Under the assumption of translational symmetry, the finite-element implementation noticeably simplifies. Explicitly, we discretize the sample’s cross-section by a collection of straight infinite elementary wires, each of them carrying a current $I_i = J_i S_i$ with $J_i$ the current density and $S_i$ the cross sectional area of the element. As it has been shown in previous work, a variational formulation that suits the discrete modeling is possible for our electro-
magnetic problem. Thus, one can show that in quasi-steady regime (excellent approximation for the large scale application frequencies) the discrete form of Faraday’s law is obtained by minimizing a magneto-quasi-static field Lagrangian (of density $\mathcal{L} = (\mathbf{B}^t + \delta t - \mathbf{B}'^t)^2/2$), coupling successive time layers and under prescribed sources and material law. In our case, and by using standard electromagnetic manipulations, the quantity to be minimized becomes

$$\frac{1}{2} \sum_{i,j} I_i M_{ij} I_j - \sum_{i,j} \bar{I}_i M_{ij} \bar{I}_j + \mu_0 \sum_i I_i S_i (A_e - \bar{A}_e),$$

(1)

with $I_i$ the set of unknown currents for the collection of elements, $M_{ij}$ the mutual inductance matrix between such elements $i$ and $j$, $\bar{I}_i$ the solution at the previous time layer, and $A_e$ the vector potential related to external sources (applied transverse field). Optimization has to be performed under the restriction of applied transport current in the cross section $\Omega$, $\sum_{i \in \Omega} I_i = I_{tr}$, and for the critical state law $I_i \leq I_c$. This has been done by using specialized large scale constrained minimization algorithms as discussed before.2  

Some technical comments are worth of mention for the case under study. In general, the electromagnetic manipulations leading to Eq.(1) may be done as follows. By using that one can split the cross section of the cylinder into a high number of elements (wires), it is justified to assume that the elementary inner potential is created by a uniform current density and contributes to the energy as a constant, thus one can use $M_{ii} = \mu_0/4$ in Eq.(1). On the other hand, using the logarithmic expression for the two-dimensional Green’s function for the outer potential of the wires one gets $M_{ij}^{out} = (\mu_0/4\pi)\ln(r_{ij}^2)$ for the mutual inductances. These expressions have been obtained under the condition of continuity for the magnetic potential but are arbitrary save to a global constant (gauge invariance). In the absence of transport current, one can show that such constant may be obviated in the theory. The reason is that, when deriving Eq.(1), spatially constant terms from the potentials of the wires are multiplied by $\sum_i I_i (= 0)$. However, for problems with transport, unless one cares about such terms, some calculations, as the value of $E$ may be tampered. In order to show how this arises, we recall that, generally speaking, physically acceptable electric fields have to be expressed in the form $E = -\partial_t A - \nabla \phi$, including an electrostatic like term. For long wires, it can be argued that $\nabla \phi$ has to be constant in space, and thus one has $E = -[\partial_t A - C(t)] \hat{k} \equiv [\partial_t A'] \hat{k}$ where $A'$ works as a calibrated potential.4 Taking advantage of the fact that arbitrariness is only up to a constant, the situation may be tackled by progressively determining $C(t)$ according to the physical condition $E = 0$ at those points.
where the magnetic flux does not vary.

Simulations have been performed for the triangular oscillating process displayed in Fig. 1. The following quantities have been focused: (i) magnetic field lines derived from $B = \nabla \times A$, (ii) the sample’s magnetic moment (per unit length) $M = (1/2l) \int_\Omega r \times J$, (iii) the local density of power dissipation ($E \cdot J$), and the hysteretic AC losses per unit time and volume for cyclic excitations of frequency $\omega$ calculated as $L = (\omega/2\pi^2 R^2) \oint f_{c.c.} dt \int_\Omega E \cdot J$. Here, $f_{c.c.}$ denotes a full cycle of the time-varying electromagnetic sources. Our results, and conclusions are developed along the following paragraphs.

a. **Flux penetration profiles.** Fig. 1 shows some of the results obtained for one of the experimental processes considered along this work. The dynamics of the electromagnetic quantities can be followed in detail by means of the attached video. Several aspects to be noticed are: (i) the position of the flux free region (core) and the current density profiles are not axially symmetric by consumption of the magnetization currents. Related to this, (ii) strong distortions of the magnetic flux lines appear around the superconducting cable. This is especially notorious when the value of the applied field and the transport current approach zero. (iii) Outstandingly, the local density of power dissipated along the process also displays a strong localization. In fact, a higher heat production (and transfer for constant $T$) is always predicted for half cross-section of the superconducting wire. We argue that this phenomenon could increase the quench probability. Furthermore, the proper determination of the active area depends on the history of the first branch of the experimental process, e.g. in Fig. 1 a positive slope in both $B_y(t)$ and $I_{tr}(t)$ determines maximal power dissipation towards the positive x-axis.

b. **Analysis of AC losses.** Fig. 2 shows the calculated variation of the hysteretic AC losses for long wires in terms of the amplitude of the coupled electromagnetic sources, $L(B_{y,\text{max}}, I_{\text{peak}})$. Our results are compared to those obtained from several analytical approximations customarily applied for non-coupled periodic sources (see caption at Fig. 2) and Bean-like superconductors ($J_c = \text{constant}$). The main significance of our results is that linear superposition only makes sense for high magnetic fields and moderate (or low) currents. This complements previous work on the rectangular geometry and adds new perspectives on the validity of approximation formulas. We emphasize the failure of assuming simple linear superposition of the classical formulas. In fact, linear approximations as $L(B) + L(I_{tr}^{ac})$ and $L(B, I_{tr}^{dc})$ (see Ref. 6) can notoriously underestimate or overestimate
the real losses. Notice that, even for the best situation, $L(B) + L(I_{tr}^{(ac)})$, at low amplitudes of $B$ the differences between predicted losses can overcome 100% (e.g. at $L(1, \lesssim 0.25)$). Then, they decreases as the amplitude of $I_{tr}^{(ac)}$ increases ($\sim 15\%$ for $I_{tr}^{(ac)} \simeq 1$). On the other hand, if the amplitude of $B$ is as high as $B_p = 8$ (the magnetic full penetration field for zero transport current) the difference can oscillate between $\sim 20\%$ ($I_{tr}^{(ac)} \sim 0.25$) and $\sim 55\%$ ($I_{tr}^{(ac)} \sim 1$) or even higher for $I_{tr}^{(ac)} \sim 0.75$.

On average for the extensive set of experiments analyzed, the highest differences ($\sim 100\%$) are observed around the condition $(B_p/2, I_c/2)$. Specially at low values of $B$, the differences between analytical approaches and our numerical calculation can overcome 100%. In conclusion, for a proper determination of the hysteretic AC losses in systems with coupled electromagnetic sources $(B, I_{tr}^{(ac)})$ somehow sophisticated analysis resources are needed, even for relatively simple configurations as the one studied here.

c. Magnetic moment cycles (low pass filtering) Fig. 3 shows the dynamics of the magnetic moment component along the external field $M_y$ in terms of the evolution of the magnetic sources $B_y(t), I_{tr}(t)$. Notice that, only for small values of the applied transport current,
FIG. 3. (Color online) The renormalized magnetic moment $M_y/\text{Max}(M_y)$ as a function of the applied sources, $I_{tr}$ (left), $B_y$ (middle), and its temporal evolution (right). Several experiments are shown according to the maximal intensity of the applied sources as was sketched in Fig. 1. Indeed, each row corresponds to a singular choice of $I_{tr}^{\text{peak}}=0.25, 0.5, 0.75,$ and $1.0$. The amplitude of the oscillating magnetic field has been assumed also to change to consider the whole spectrum of possible experiments. Here we show the corresponding results for $B_{y}^{\text{max}}=1, 2, 4, 6,$ and $8$, as can be deduced directly from the plots.

nearly standard Bean-like loops are obtained. In particular, we recall that the flat saturation behavior of $M_y$ for high field values progressively disappears by increasing the value $I_{tr}^{\text{peak}}$. Remarkably, this phenomenon ends up with a symmetrization of the loop (both as a function of $I_{tr}$ and $B_y$), that exhibits a characteristic eye-shape. As a consequence of such process, an outstanding low pass filtering effect is predicted for the experimental situation described in this paper, which may lead to envisage new applications for superconducting wires. Thus, a plot of the induced magnetic moment as a function of time (rightmost column in Fig. 3) demonstrates that the triangular input excitation produces a nearly perfect sinusoidal output $M_y(t)$.

d. Conclusion. In this work, we have investigated the electromagnetic response of a superconducting circular wire under simultaneous AC sources through a numerical imple-
mentation of Bean’s critical state model. The main physical assumptions are an infinitely steep $E(J)$ law that goes from zero to infinity at the critical point $J_c$. In simple systems, these assumptions are known to lead to hysteretic (rate independent) losses with simple relations between monotonic and cyclic quantities. Here, we show that even for the 2D configuration studied, one can find intriguing phenomena, unexpected in simplified models that are based on plain linear superposition. Thus, we have obtained notorious localization effects in the density of power dissipation, strong field distortions, important failures of the customary approximation formulas for the bulk quantities, and also predict an outstanding low pass filtering effect in the magnetic response.

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FIG. 4. The video has been uploaded to the Data Conservancy Pilot Project of arXiv repositories. Also, this and more videos are available to download at:

http://www.unizar.es/departamentos/fisica_mat_condensada/people/hsruizr/

Movie Caption:

Sketch of one of the analyzed cases along this letter. In this case, \( B_{y}^{\text{max}} = 2 \), and \( J_{\text{tr}}^{\text{peak}} = 0.75J_{c} \) (Figure 1 is a frame of this video). Units are \( (\mu_{0}/4\pi)J_{c}R \) for \( B \), \( \pi R^{2}J_{c} \) for \( I_{\text{tr}} \), \( J_{c}R^{3} \) for \( M \), and \( (\mu_{0}/4\pi)J_{c}^{2}R^{4} \) for \( E \cdot J \). Here, we have assumed a quasistationary time-step defined by the experimental process \( (B_{y}, I_{\text{tr}}) \) which is summarized in the central plot. Top-Left. The magnetic field lines (projected isolevels of the vector potential over the wire cross-section) and their corresponding profiles of current are shown. The consumption of local magnetization currents and the generated field distortions around the superconducting wire are directly visualized. From time to time, straight isolines in zones free of current are plotted as a consequence of the number of isolevels which has been required to be plotted. They must be understanding only as a visual effect introduced by the graphical processing. Top-Right: Dynamics of the density of power dissipation along the cross-section of the superconducting wire. A clear non-symmetric distribution of the heat transfer in a cyclic process is always observed. Bottom-Left: The magnetization loop for the superconducting wire in terms of the renormalized electromagnetic sources \( (B_{y}/B_{y}^{\text{max}} \cdot 0.75, 0.75) \) is shown. Bottom-Right: The magnetic moment is shown in terms of the virtual time defined by the dynamics of the electromagnetic sources. A low pass filtering effect in the magnetic response as \( J_{\text{tr}} \to J_{c} \) is predicted.