Influence of the external magnetic field on hydrodynamic structure of the electrovortex flow in hemispherical container

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Abstract. Electrovortex flow is formed as a result of interaction of the non-uniform electric current passing through the liquid metal with the own magnetic field of this current. Numerical calculation and measurements of the flow velocity in liquid metal (indium-gallium-tin alloy) in hemispherical container were carried out. Results on influence of the axial external magnetic field on the flow are presented.

1. Introduction

Electrovortex flow (EVF) is formed as a result of interaction of the non-uniform electric current passing through the liquid metal with the own magnetic field of this current [1]. Such flows significantly affect many processes in mechanical engineering (electro-welding) and electrometallurgy (electroslag remelting, various electric melting furnaces). In particular, electrovortex flow determines the hydrodynamic structure in the baths of DC-arc furnaces, which are increasingly used in industry [2].

Here is the brief review of works about electrovortex flows. The first articles associated with investigation of electrovortex flow were about flow calculation at the small currents using Stokes approximation [3,4]. Article [5] was devoted to possible reasons of rotation of the EVF. Problems connected with the influence of the external axial magnetic field were started to consider in [6,7,8]. Questions related to the surface deformation were present in [9].

In this paper we investigate the influence of the external axial magnetic field (MF) on the EVF of liquid metal in the hemispherical container. Let us consider the system where electric current propagates from the small hemispherical electrode through the conductive media (liquid metal) to the big hemispherical electrode. Electromagnetic force $\mathbf{F} = \mathbf{J} \times \mathbf{B}$ (where $\mathbf{J}$ – current density and $\mathbf{B}$ – magnetic field) leads the liquid to a motion.

![Fig.1. Elecrovortex structure. 1 - small electrode, 2 - liquid metal, 3 - big electrode. a) Toroidal EVF; b) occurrence of the azimuthal swirl; c) occurrence of the secondary vortex; d) double-vortices rotating EVF.](image)

Fig.1. Elecrovortex structure. 1 - small electrode, 2 - liquid metal, 3 - big electrode. a) Toroidal EVF; b) occurrence of the azimuthal swirl; c) occurrence of the secondary vortex; d) double-vortices rotating EVF.
Usually it is assumed that in such system with the axially symmetric central electrode without the action of external magnetic fields EVF has the shape of a toroidal vortex (Fig. 1a). External axial magnetic field in this geometry leads to the azimuthal swirl of the flow (Fig. 1b). And due to the azimuthal rotation, the secondary vortex rotating in a vertical plane opposite main EVF (Fig. 1d) and pushing it to the periphery of the bath occurs.

2. Numerical technique
The motion of the liquid is described by the Navier-Stokes equation:

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{U} + \frac{1}{\rho} \mathbf{F},$$

where \( \mathbf{U} \) is the velocity of the liquid, \( \rho \) is the density of the liquid, \( \nu \) is the kinematic viscosity coefficient, \( p \) is the pressure and \( \mathbf{F} \) is the electromagnetic force.

$$\mathbf{F} = \mathbf{J} \times (\mathbf{B} + \mathbf{B}_{\text{ex}}),$$

where \( \mathbf{B} \) is the own magnetic field of the current and \( \mathbf{B}_{\text{ex}} \) is external field. We consider case with axial external magnetic field so \( \mathbf{B}_{\text{ex}} \) has the only \( z \)-component \( B_z \).

At the small magnetic Reynolds number we can use the so-called electrodynamic approximation, according to which we neglect the influence of velocity on electric current in the liquid, so the density of the current \( \mathbf{J} \) can be written in spherical coordinates as:

$$\mathbf{J} = \frac{l}{2\pi R^2} \mathbf{e}_R,$$

where \( l \) is the current. The magnetic field can be found from the Maxwell’s equation:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}.$$

This equation can be rewritten in projection to \( R \)-coordinate:

$$\frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} \left( B_\phi \sin \theta \right) = \frac{l}{2\pi R^2}.$$

So the magnetic field will be:

$$B_\phi = \frac{\mu_0 l (1 - \cos \theta)}{2\pi R \sin \theta},$$

and the forces:

$$F_\theta = -\frac{\mu_0 l^2 (1 - \cos \theta)}{4\pi^2 R^3 \sin \theta}; F_\phi = -\frac{l B_z \sin \theta}{2\pi R^2}.$$

Using following scales for generalization Navier Stokes equation:

**Table 1. Scales.**

| Length                     | \( R_2 \) |
|---------------------------|------------|
| Electric current density  | \( j_0 = l/R_2^2 \) |
| Magnetic field            | \( B_0 = \mu_0 l/R_2 \) |
| Electromagnetic force     | \( f_0 = \mu_0 l^2/R_2^3 \) |
| Velocity                  | \( u_0 = \sqrt{f_0 R_2/\rho} \) |
| Time                      | \( t_0 = R_2/v_0 \) |
we can rewrite Navier-Stokes equation in dimensionless form:

\[
\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}\nabla)\mathbf{U} = -\nabla p + \frac{1}{\sqrt{S}} \Delta \mathbf{U} + \frac{\cos \theta - 1}{4\pi^2 R^5 \sin \theta} \mathbf{e}_\theta - \frac{N \sin \theta}{S} \mathbf{e}_\phi
\]

To solve Navier–Stokes equation we use finite volume method on unstructured 2D-axisymmetric grid in cylindrical coordinates. In this coordinate system \((r, \phi, z)\) components of electromagnetic force have the following form:

\[
F_r = -\frac{z(\sqrt{r^2 + z^2} - z)}{4\pi^2 (r^2 + z^2)^{3/2}};
\]

\[
F_\phi = -\frac{N r}{2\pi (r^2 + z^2)^{3/2}};
\]

\[
F_z = -\frac{(\sqrt{r^2 + z^2} - z)}{4\pi^2 (r^2 + z^2)^{3/2}}.
\]

Unsteady calculations of the velocity were carried out for different ratio of the small and big electrode: 1:10, 1:37.6, and 1:100. To obtain boundary line dividing one- and two- vortices zones in parameters \(I - B_{\text{ext}}\) (or \(\alpha - S\)) the following algorithm was used: with some values of parameters \(S\) and \(\alpha\) calculation was being carried out to get and to observe streamlines (Fig. 2).

![Fig. 2. Calculated stream functions. a) single vortex system b) double vortex system](image-url)
3. The experimental technique

Experimental studies were carried out at the setup shown in Fig. 3. An eutectic indium-gallium-tin alloy (weight content: Ga-67%, In-20.55%, Sn-12.5%, physical properties: melting point +10.5°C, ρ = 6482 kg/m³, v=4.3e-7 m²/s, σ=3.3e6 Sm, [10]) was used as the working liquid in the experiments. Alloy filled copper hemispherical container with the radius $R_2=188$ mm which also served as a large electrode. Small electrode - copper or steel cylinder with the radius $R_1=2.5$ mm with hemispherical tip was immersed into the alloy in the middle of the working bath. Power source developed on the basis of three-phase AC rectifier (I≤1500 A) was used to supply an experimental setup. To create an external longitudinal magnetic field the coil consisting of 15 turns of hollow copper tube (cooled with pumped water) was used. Coil power was supplied from a stabilized power source, providing smooth control of DC in the range from 0 to 100 A. This system allowed to get the magnetic field with induction $B_z=5e^{-3}$ T in the middle of the working area at a current of ~100A.

The experiments were carried out with the fiber-optical transducers (Fig. 4) developed in our laboratory [11]. Transducer made of glass and is not sensitive to external electric and magnetic fields. The main characteristics of a typical transducer are in the table.

Table 2. Fiber-optical transducer main characteristics.

| Average velocity measurement range | 1÷50 cm/s |
|-----------------------------------|----------|
| Frequency range                   | 0÷200 Hz |
| Measuring volume                  | 0.04×0.04×1 mm³ |
| Temperature range                 | <100°C   |

![Fig. 3. Experimental setup. 1 - solenoid, 2 - small electrode, 3 - eutectic alloy In-Ga-Sn, 4 - hemispherical container, 5 - heat exchanger, 6 - programmable power supply.](image)

![Fig. 4. Fiber-optical transducer. 1 - sensor, 2 - pointer, 3 - light guides, 4 photodiode, 5 - photoreceiver.](image)

The second vortex appears when the value of the velocity read by the fiber-optical transducer changes its sign.

4. Results

Typical results on measurements and calculations of the velocity are presented below. There are the oscillograms of axial velocity showing the evolution of the velocity on time (Fig. 5). First part of graph shows that in beginning velocity rises then (when azimuthal rotation was developed) secondary upward flow suppresses main electrovortex flow and axial velocity degreases until zero and below.
Fig. 5. Oscillograms of axial velocity. $I=400\ A$, $B_{ext}=5\times10^{-4}\ T$ a) experiment, depth $z=10\ mm$ b) calculation, $z=1-10\ mm$, $2-30$, $3-80$.

Data on the time of the appearance of the second vortex are presented in Fig. 6. A bigger value of the external field leads to the fastest appearance of the second vortex.

Fig. 6. Dependence of the time of appearance of the second vortex on the external magnetic field. $I=400\ A$. 1 - Experiment, 2 - calculation.

Fig. 7 presents boundary line of zones with one and two vortices. With increasing of parameter S we need smaller $\alpha$ for formation of the second vortex. In dimensional form, with increasing of the electric current it requires stronger magnetic field.
5. Conclusions

Experiments and calculations of the axial velocity in electrovortex flow with presence of the external axial magnetic field were carried out. The boundary line separating one- and two-vortices zones and time of appearance of the second vortex were obtained. These results can be used for a priori estimates of the flow structure.

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7. References

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