Solution of the optimal power flow problem in direct current grids applying the hurricane optimization algorithm

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Abstract. The Colombian electrical power system is being transformed by the large-scale integration of energy storage systems and renewable energy resources to the power system. These technologies can be integrated using alternating current or direct current technologies via power electronic converters. Here we analyze the direct current paradigm by proposing a master-slave optimizer for solving the problem of optimal power flow considering nonlinear loads. The master stage covers the dispatch of all distributed generators using the hurricane optimization algorithm. In the slave stage, a power flow method based on successive approximations is used to determine the voltage variables and evaluate the objective function of the problem, which is the minimization of power loss. A 69-nodes direct current network is used as a test case to compare the numerical performance of the hurricane optimization algorithm with a nonlinear optimization package and a metaheuristic approach called black hole optimizer. All simulations are performed using MATLAB software version 2017a licensed by Universidad Tecnológica de Bolívar, Colombia.

1. Introduction
The last decade has seen a boom to reduce environmental pollution by redesigning production activities and energy generation patterns, and implementing policies towards decrease the carbon footprint. Particularly, one of the areas which these new technologies have focused of is the electricity generation, giving rise to what it is known as clean energy, based on renewable energy resources, whose objective is to replace conventional generation technologies such as gas and diesel [1]. In addition, renewable energy has allowed to supply and improve the electricity service in rural areas, through the implementation of direct current (DC) microgrids, providing great advantages in comparison with the traditional alternating current electrical systems, such as the ability to integrate into the grid different types of generation technologies, i.e., photovoltaic and wind [2,3]. Other advantages offered by DC microgrids are their low power losses, the absence of reactive power and harmonics, this latter due to the lack of frequencies [4,5]. In this sense, one of the main challenges to analyze DC networks corresponds to the optimal power flow (OPF) of the different energy resources that are part of the microgrid, with the aim to improve different technical and operative conditions, such as, the reduction of the power losses, voltage profile improvement, among others [6]. In this work the OPF problem is addressed from the metaheuristic point of view.
OPF problem have led to implement or conduct different investigations around the approach of optimization techniques, based on master-slave methods that combine different techniques of power flow and metaheuristic optimizers, as is the case of the techniques raised in [1, 6, 7]; being mainly used the reduction of power loss as objective function, in conjunction with the traditional constraints that represents the DC electrical networks. In this work, it is presented the implementation of a new metaheuristic technique to solve the OPF problem in DC grids, based on the hurricanes behavior, called as hurricane optimization algorithm (HOA), which is characterized for obtaining good results in the solution of large-scale complex problems [8], being a very promising technique in the field of metaheuristics. Here, HOA corresponds to a master stage responsible to fix the power supplied by the distributed generators (DGs) installed in the DC network. Conversely, HOA works in conjunction with a slave stage, which uses a technique named as successive approximations to solve the problem of power flow as proposed in [9]. Note that this optimization technique was not previously used for the solution of OPF problems in the specialized literature, which emerges as an opportunity of research that this paper tries to deal with.

In the context of the Colombian power system, this research is important as a current topic is addressed in the specialized literature associated with the large-scale deployment of renewable energy resources and energy storage systems into the current electrical network, which is one of the guidelines provided in the law 1715 approved by the Colombian Government in 2014 [10]. Here we try to contribute with a new methodology for the optimal generators dispatch when they are integrated in direct current networks. The main contribution of our approach is that it is purely algorithmic and it can be embedded into strategies that involve optimal selection and location of renewable energy resources and expansion planning projects to provide electrical service to new users [11]. All of these are under the DC paradigm, since the power losses are lower, voltage profiles are improved and easily controllable, due to the absence of reactive power and frequency components [5].

Regarding the above, section 2 presents the mathematical model of the optimal power flow problem in DC networks, which presents a nonlinear and non-convex structure; section 3 defines the proposed hybrid methodology based on the HOA and the method of successive approximations used for the power flow analysis; section 4 presents the characteristics of the test system and shows the numerical validation of the master-slave optimization algorithm proposed. Finally, section 5 presents the final observations derived from this research as well as some possible future works.

2. Mathematical model

The mathematical formulation of the optimal power flow problem in DC networks corresponds to an optimization model with nonlinear and non-convex characteristics [11–13]. The objective function is the minimization of the power losses in all the conductors of the network by defining the optimal dispatch of the distributed generators located along the network. Additionally, power balance constraints are solved numerically with an iterative procedure named successive approximation method that deals with the hyperbolic relation between voltages and powers at each node [11, 13]. The mathematical model of the OPF problem employed in this work, is presented in detail from Equation (1) to Equation (6). In Equation (1) is presented the objective function as follows.

$$\min P_{\text{loss}} = v^T G_L v,$$  \hfill (1)

where $P_{\text{loss}}$ is the function that allows to calculate the power loss along the branches of the network, $v \in \mathbb{R}^{n \times 1}$ is a vector that contains all the voltage profiles, $G_L \in \mathbb{R}^{n \times n}$ is the conductance matrix that contains the resistive effects on the branches (distribution lines), and $n$ is the number of nodes of the system. In addition, from Equation (2) to Equation (6) the set of constraints of the OPF model is presented.

$$P_g + P_{dg} - P_d = \text{diag}(v)[G_L + G_N]v,$$  \hfill (2)
\[ v_{\text{min}} \leq v \leq v_{\text{max}}, \quad (3) \]
\[ P_g^{\text{min}} \leq P_g \leq P_g^{\text{max}}, \quad (4) \]
\[ P_{dg}^{\text{min}} \leq P_{dg} \leq P_{dg}^{\text{max}}, \quad (5) \]
\[ 1^T (P_{dg} - \alpha P_d) \leq 0, \quad (6) \]

where \( P_g, P_{dg} \in \mathbb{R}^{n \times 1} \) and \( P_d \in \mathbb{R}^{n \times 1} \) are the powers at slack nodes (voltage controlled sources), distributed generation and the power demand associated to the loads, \( v_{\text{min}}, v_{\text{max}} \in \mathbb{R}^{n \times 1} \) are the upper and lower limits of nodal voltages; \( P_{dg}^{\text{min}}, P_{dg}^{\text{max}}, P_g^{\text{min}}, P_g^{\text{max}} \) correspond to the minimum and maximum power generation limits of slack nodes and the distributed ones. In this mathematical formulation, \( G_L \in \mathbb{R}^{n \times n} \) and \( G_N \in \mathbb{R}^{n \times n} \) are the conductance matrices that contain the resistance of the branches and the constant resistive loads connected to the DC network; \( \alpha \) represents the maximum penetration of generation, in such a way that \( 0 \leq \alpha \leq 1 \), being \( 1 \in \mathbb{R}^{n \times 1} \) a vector filled by ones.

The mathematical model presented from Equation (1) to Equation (6) has the following interpretation: the objective function that minimizes the power losses is given by Equation (1); Equation (2) is responsible for the power balance in all the nodes of the network. The voltage regulation bounds at each node of the network is defined by Equation (3). Equation (4) and Equation (5) define the power capacity bounds of the slack node and distributed generators, respectively. Finally, the maximum distributed generation penetration level is defined in Equation (6).

Here, it was adopted the a master-stage methodology to solve the OPF problem in DC grids, based on metaheuristic and numerical methods, due to the excellent results reported in the specialized literature for this type of methodologies [1, 6, 7].

3. Proposed methodology

To solve the DC OPF problem in this paper, we propose a master-slave methodology formed by the HOA [8] in conjunction with the successive approximation power flow method [9].

3.1. Master stage

HOA is an optimization method based on the nature of the hurricanes, which uses the pressure in the wind parcels to find the answer for the optimization problems, where the best solution represents the point of lowest pressure of the hurricane [14]. HOA leverages of the movement of the wind parcels, where most of the wind tends to go into the central zone of the hurricane. This zone is characterized to be that with the least pressure, where the hurricane eye is located.

3.1.1. Initial population. The HOA method initially proposes a set of possible solutions to the OPF problem. This set of solutions is known as the initial population, which is formed in this case, by the wind parcels that in context with the problem are the possible powers to generate by the distributed generators on the selected nodes. The creation and size of the population, is given with respect to the number of individuals (NI) and the number of generators (NG) used, as presented in Equation (7).

\[ X = \begin{pmatrix} x_{ij} & \ldots & x_{iNG} \\ \vdots & \ddots & \vdots \\ x_{NIj} & \ldots & x_{NING} \end{pmatrix}, \quad (7) \]

where \( X \) represents the set of possible solutions, \( x_i \) is an individual of the population. Note that \( x_{ij} \) corresponds to the power generation at node \( j \) for the individual \( i \). To generate all the components in Equation (7) we use a normal distribution of numbers as defined in Equation (8).

\[ X = P_{dg}^{\text{min}} \text{ones}(NI, NG) + (P_{dg}^{\text{max}} - P_{dg}^{\text{min}}) \text{rand}(NI, NG) \quad (8) \]
In this expression \( \text{ones}(NI, NG) \) represents a matrix filled by ones and \( \text{rand}(NI, NG) \), which is a matrix formed by random numbers that are generated between a range of 0 to 1 with a normal distribution. It should be noted that the best solution found in the initial population is contemplated as the initial position of the eye of the hurricane [8].

3.1.2. Wind movement. The movement of wind parcels is characterized to conserve a constant angular velocity \((w)\) and its displacement around the eye of the hurricane, to find the regions with lower atmospheric pressure [14]. This movement can be mathematically represented in two ways, due to the direction of rotation exerted by the winds, as presented in Equation (9) and Equation (10).

\[
x_{i+1} = \begin{cases} 
  r_i^t \sin(\varphi_i^0 + \varphi_i^t) + O_j^n, & \text{rand()} < 0.5, \\
  r_i^t \cos(\varphi_i^0 + \varphi_i^t) + O_j^n, & \text{rand()} \geq 0.5,
\end{cases}
\]  

(9)

\[
r_i^t = R_0 \exp(\text{rand()} \varphi_i^t),
\]

(10)

where \(x_{i+1}^t\) represents the offset of each wind parcel, \(\varphi_i^0\) is an angular coordinate that takes a random value between an interval \([\varphi_{\text{min}}, \varphi_{\text{max}}] = [0, 2\pi]\). This value is mathematically generated with Equation (11); the next angular coordinate (see \(\varphi_i^{t+1}\)) is also calculated with the rule presented in Equation (12), and \(O_j^n\) is the value that the eye initially receives (best solution obtained in the set \(X\)).

\[
\varphi_i^0 = \varphi_{\text{min}} \text{ones}(NI, NG) + (\varphi_{\text{max}} - \varphi_{\text{min}}) \text{rand}(NI, NG)
\]

(11)

\[
\varphi_i^{t+1} = \begin{cases} 
  \varphi_i^t + w; & r_i^t \leq R_{\text{max}}^0 \\
  \varphi_i^t + w(\frac{R_{\text{max}}^0}{r_i^t}); & r_i^t > R_{\text{max}}^0
\end{cases}
\]

(12)

The values of \(r_i^t\) and \(\varphi_i^t\) are in polar coordinates, which are rectangular and angular respectively. Initially \(\varphi_i^0\) is a vector composed of zeros, which allows to calculate \(r_i^t\) using Equation (10), like the position of wind plots. \(x_{i+1}^t\) must vary in each iteration, meeting the conditions proposed in Equation (13).

3.1.3. Eye evolution. With the updated positions of the \((x_{i+1}^t)\) set of individuals, it is crucial to determine whether new areas were found that can be considered as the new eye of the hurricane. To consider a change in the location of the eye it is necessary to fulfill the condition in which the population \(x_{i+1}^t\) finds a zone that with less pressure than the one that is in the eye (best solution), as described by Equation (13).

\[
O_j^{t+1} = \begin{cases} 
  x_{i+1}^{t+1}, & x_{i+1}^{t+1} < O_j^n \\
  O_j^n, & x_{i+1}^{t+1} \geq O_j^n
\end{cases}
\]

(13)

Note that this operation is made for all individuals \(x_i \in X\) at each iteration \(t\).

3.2. Slave stage

Successive approximations in power flow analysis is a method with the ability to work with power balance constraints described by Equation (2) [9]. Mathematically, this method is represented by Equation (14).

\[
P_{dg} - P_d = \text{diag}(v_d)[G_{ds}v_s + G_{dd}v_d],
\]

(14)

where \(P_{dg}\) and \(P_d\) are the powers of the distributed and demand generation nodes respectively; \(G_{ds}\) and \(G_{dd}\) are submatrices that are composed by elements of the conductance matrix, \(v_s\) contains the slack node voltages, and unknown voltages are contained in \(v_d\). Equation (15) allows the solution of non-linear problems, as the inverse of \(G_{dd}\) can always be found [9].
\[ v_d = G^{-1}_{dd} \left[ \text{diag}^{-1}(v_d)[P_{dg} - P_d] - G_{ds}v_s \right]. \] (15)

Equation (15) can be represented in an iterative form as shown in Equation (16), where \( m \) represents the number of the iteration, which encompasses a maximum number called \( m_{\text{max}} \). This method can determine that the desired solution was reached before the number of iterations is completed when it is met by Equation (17), being \( e \) a value of \( 1 \times 10^{-10} \) as recommended in [6, 9].

\[ v_d^{m+1} = G^{-1}_{dd} \left[ \text{diag}^{-1}(v_d^m)[P_{dg} - P_d] - G_{ds}v_s \right], \] (16)

\[ (|v_d^{m+1} - v_d^m|) \leq e. \] (17)

Note that the convergence of this power flow method can be guaranteed via fixed point theorems as reported in [5] for the Gauss-Seidel method taking the advantage of the positive definiteness of the admittance matrix.

4. Numerical validation
The 69-nodes test system is used for validation of the proposed master-slave optimization method and the complete information of this test feeder, this is, graph configuration, power consumption, and branch parameters can be consulted in [15]. This test feeder was previously used in the specialized literature [16], which is a DC adaptation of the Baran & Wu test feeder [7]. This system has an ideal voltage node located at the bus 1 and multiple constant power loads along the feeder. For this study, a penetration level was considered for the distributed generation at 60 % of the total power consumption in the system, this is, \( \alpha = 0.6 \) in Equation (6). The location of the distributed generators on the grid, was taken from previous works reported in the specialized literature [1]. Note that for simulation purposes, 12 p.u. was selected as the maximum power that can be injected by the distributed generators and these have been located at nodes 26, 66 and 69 of the test system [7].

Table 1 presents the results achieved by the proposed HOA, which is compared with the BHO [7] and the specialized software named as general algebraic modeling system (GAMS) [17] for the case of \( \alpha = 0.6 \). These data confirm that HOA has a superior performance in comparison with the black hole optimizer (BHO) [7], due to the the total system losses were 0.0702 p.u. with a reduction of 95.4331 % with respect to the total system losses without DGs. Similarly, the results achieved by GAMS were equalized with standard deviation of \( 7.58537 \times 10^{-9} \). This value was reached after 100 consecutive evaluations of the proposed method, demonstrating that the proposed optimization algorithm guarantees the overall optimum result obtained by GAMS with minimal (negligible) difference, despite the non-linear and convex nature of the OPF problems.

| Method | \( p_{dg}^{26} \) (p.u) | \( p_{dg}^{66} \) (p.u) | \( p_{dg}^{69} \) (p.u) | \( P_{loss} \) (p.u) | Reduction (%) |
|--------|------------------------|------------------------|------------------------|------------------------|----------------|
| BHO    | 4.2624                 | 11.6894                | 4.6787                 | 0.0836                 | 94.5661        |
| GAMS   | 3.8710                 | 12.0000                | 5.2263                 | 0.0702                 | 95.4331        |
| HOA    | 3.8710                 | 11.9999                | 5.2265                 | 0.0702                 | 95.4331        |

5. Conclusion and future works
A master-slave methodology to solve the optimal power flow problem in DC networks with constant power loads was presented. To solve this problem, it was proposed the hurricane optimization algorithm that defines the optimal dispatch of the distributed generators in master stage. To deal with the non-convexities of the power balance constraints the successive approximation power flow method was used. The main contribution of this paper was to propose a pure-algorithmic OPF approach for DC networks.
Additionally, the proposed method presented a great performance in comparison with the metaheuristic algorithm based on black holes behavior, which demonstrated a superiority in terms of results. Similarly, HOA managed to obtain the results achieved by GAMS, which is a specialized software in nonlinear optimization problems that presents a low dispersion of the results obtained in the test performed. As future works, it will be possible to use this proposed method to solve problems related to the economic dispatch in a 24 hours period and to determine the optimal locations of generators distributed in the electrical systems, or using the proposed approach as an slave stage in discrete optimization algorithms that deals with the problem of optimal location and sizing of DGs.

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