Numerical simulation of incompressible turbulent flows in presence of laminar to turbulent transition

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Abstract. Most of the widely used popular mathematical models of turbulence use a judicious combination of intuition, empiricism and the governing equations of instantaneous and mean motion-valid strictly for fully developed turbulence without any laminar region. In reality however, any wall bounded or free shear flow may consist of some laminar flow patches which eventually undergo transition over a finite length to grow into fully turbulent flows. Most of the turbulence models used in commercial CFD codes, are unable to predict the dynamics of turbulent flows with laminar patches. However, accurate prediction of transitional flows is often essential to estimate the pressure losses and/or heat transfer in industrial applications. The present paper implements two different transition models in an existing finite volume URANS-based code RANS3D, developed in house and validated against reliable measurement data for flow past flat plates with different free stream turbulence levels and flow past SD7003 aerofoil at a chord-based Reynolds number of 60,000.

1. Introduction

In the last few decades, various robust CFD codes have been developed to simulate fully turbulent flows in complex configuration, producing results, acceptable for engineering design applications. These codes employ wide range of semi-empirical turbulence models, maintaining a balance between the accuracy requirements, level of information required and the computational resources available. Unfortunately, none of these RANS equation based CFD algorithm, is capable of capturing the important effects of laminar-turbulent transition on the flow field. The underlying physics of the transition process has been explored by several researchers through careful measurements and/or powerful analyses based on complex theories to explain the phenomenon of intermittency. But none of these models are found compatible to be directly coupled to the Unsteady RANS Equation-based CFD codes, used by engineering design community, where turbulent stresses need to be evaluated in both structured and unstructured grid environment.

However, laminar to turbulent transition process is observed to have strong influence specially on the wall shear stress or heat transfer coefficients in wall shear layers for flow past solid surfaces like gas turbine blades. In spite of limited applicability, algebraic correlations based on very careful hot-wire measurements, proposed by Abu Ghanam et al [1], have been found to be quite useful for prediction of the inception and the extent of transition during the boundary layer development on a flat plate. Recently a more accurate transition model based on three strongly coupled transport equations viz., laminar kinetic energy ($k_L$), turbulent kinetic energy ($k_T$) and specific turbulent dissipation ($\omega$) is proposed by Walters et al [2] and validated for a few interesting test cases. In the present work, two different transition models,
v.i., one algebraic correlation based [1] and another transport equations based model [2] are directly coupled to the flow solver RANS3D, developed by the last author and his research associates at NAL and NMIT Bangalore [3], for prediction of transitional flows in complex configuration. The performance of the models are evaluated through comparison to reliable measurement data reported by ERCOFTAC [4] for flat plates and also for two dimensional flow past a cambered SD7003 aerofoil, operating at a chord-based Reynolds number of 60,000 for which computation data [5,6] on aerodynamic performance are available for validation.

2. Mathematical Modeling

The dynamics of incompressible flow is governed by the continuity equation (Eq.1) and the Reynolds Averaged Navier Stokes (RANS) equations (Eq.2) in non-orthogonal curvilinear coordinates.

\[
\frac{\partial}{\partial x_j} (U_k \beta_k^i) = 0
\]  

(Mass:)

\[
\frac{\partial U_i}{\partial t} + \frac{1}{J} \frac{\partial}{\partial x_j} \left( \left( U_i \beta_k^i k \right) \right) + \frac{P}{\rho} \beta_k^j = \frac{1}{J} \frac{\partial}{\partial x_j} \left[ \nu_{\text{eff}} \left( \frac{\partial U_i^l}{\partial x_m} B^l_{m} + \frac{\partial U_k^l}{\partial x_m} \beta_{m}^l \beta_k^j \right) \right]
\]  

(Momentum:)

\( p \) & \( U_i \) are the ensemble-averaged static pressure & cartesian velocity component along \( i \)-th direction respectively; \( \nu_{\text{eff}} \) is the effective kinematic eddy viscosity derived from the transition cum turbulence model, \( J \), the Jacobian and \( \beta_k^j \) & \( B^l_{m} \) are metric coefficients of transformation, \( \nu \) & \( \rho \) are kinematic viscosity & density of the fluid.

2.1 Turbulence Model for Fully Turbulent Flow

Standard \( k-\omega \) turbulence model computes the eddy viscosity, \( \nu \), from the solution of the scalar transport equations (Eqs. 3 & 4) for turbulence energy \( k \) and specific dissipation \( \omega \) respectively, where \( S_{ij} \) is the mean strain rate and \( P_k \) is the production of turbulence energy.

\[
D_i k = \frac{P_k}{k} - \nu + \nabla \left( \frac{\nu_{\text{eff}}}{\sigma_k} \nabla k \right) \tag{3}
\]

and

\[
D_i \omega = \alpha \frac{P_k}{k} \frac{\omega}{k} - \beta \omega^2 + \nabla \left( \frac{\nu_{\text{eff}}}{\sigma_\omega} \nabla \omega \right) \tag{4}
\]

The eddy viscosity is computed as \( \nu = k/\omega \) and the model constants are \( \alpha = 5/9, \beta = 3/40, \beta^* = 0.09, \sigma_\omega = 2 \) and \( P_k = 2
\nu \left| S_{ij} S_{ij} \right| \). However, in case of transitional flows, the updation of the effective eddy viscosity in the above equations is assumed to be a linear combination of the laminar viscosity \( \nu \) and the eddy viscosity \( \nu_{\text{eff}} \) given by the turbulence model for fully turbulent flow. The intermittency \( \gamma \) derived from the transition model, described in the next section, is used as the weighting function to compute \( \nu_{\text{eff}} \) as follows

\[
\nu_{\text{eff}} = \gamma \nu + (1-\gamma) \nu
\]  

(5)

2.2 Models for Transition Simulation

2.2.1 Algebraic Correlation based Transition Model

This empirical model [1] is based mainly on the transverse velocity profiles measured at different longitudinal stations, using Hot Wire Anemometers for flow over a smooth flat plate in a low speed wind tunnel. The important finding from these detailed measurements are the locations of the start and end of the transition along the plate. The non dimensional transition
length parameter is $\eta$ when $\eta = \frac{Re_s - Re_{ss}}{Re_s - Re_{ss}}$ and the intermittency $\gamma(x)$ is expressed as

$$\gamma(x) = 1 - \exp(-5\eta^3)$$

where $Re_s = \rho x U / \mu$ is the local Reynolds number based on the distance $x$ from the leading edge of the plate and free stream velocity $U$; $Re_{ss}$ is the Reynolds number based on $x_s$, the length at the start (onset) of transition, identified as the point where the local momentum thickness based Reynolds number $Re_s$ exceeds $Re_{ss}$ defined by the following empirical equation derived from measurement data as

$$Re_{ss} = 163 + \exp\left\{ F(\lambda_0) - \frac{F(\lambda_0) - Tu}{6.91} \right\} \quad (6)$$

where the function $F(\lambda_0)$ is expressed by Eq. 7, as function of the longitudinal pressure gradient $\lambda_0$ imposed on the boundary layer

$$F(\lambda_0) = 6.91 + 12.75\lambda_0 + 63.64\lambda_0^2 \quad \text{when } \lambda_0 < 0$$

$$F(\lambda_0) = 6.91 + 2.48\lambda_0 - 12.27\lambda_0^2 \quad \text{when } \lambda_0 > 0$$

where, $\lambda_0 = \frac{\partial^2 du}{\partial x^2}$ is the pressure gradient parameter and $U_c$ is velocity at the free stream edge of the boundary layer. Based on the inception of transition or onset Reynolds number, $Re_{ss}$ and the Reynolds number $Re_{xe}$ at the end of transition location $xe$ is determined using the following relationship:

$$Re_{xe} = Re_{ss} + 16.8 \frac{Re_{ss}^{0.8}}{Re_{xe}} \quad (8)$$

Once the longitudinal variation of the intermittency factor $\gamma$ is determined from the above set of correlations, one may use the linear formula (Eq. 5) to evaluate the effective eddy viscosity to be used for solving the transport equations for momentum & the relevant turbulence scalars.

### 2.2.2 Three Equation Eddy Viscosity based Model

The difficulty of correlation-based approach is that one needs to prescribe the location of onset and extent of transition along with some empirical relationship for the variation of intermittency. Specially for complex geometry where non-orthogonal structured or unstructured grid is used, it is difficult to compute the momentum thickness based Reynolds number which decides the onset of transition. In the RANS framework, the major assumption is to relate the effects of turbulence through the local eddy viscosity, represented by transport equations for the velocity and length scales of turbulence. The present three equation model [2] includes a third transport equation for laminar kinetic energy which in the pre-transition zone of laminar flow amplifies external disturbance leading to high amplitude streamwise fluctuations that eventually cause the transition through breakdown. Three scalar transport equations solved along with the mean momentum and continuity equations are as follows:

$$D_k T = P_k + R_{BP} + R_{NAT} - \omega k - D_T + \nabla \left[ \nu + \frac{\alpha_l}{\sigma_k} \right] \nabla k_T \quad (9)$$

$$D_L k_L = P_L - R_{BP} - R_{NAT} - D_L + \nabla \left[ \nu \nabla k_L \right] \quad (10)$$

$$D_t \omega = C_{\omega l} \frac{\omega}{k_T} P_k T - C_{\omega l} \omega^2 + C_{\omega l} f_0 \alpha_T f_w \frac{k_T}{d^3} + \nabla \left[ \nu + \frac{\alpha_l}{\sigma_k} \right] \nabla \omega \quad (11)$$

The first source term on the right hand side for all the equations represent the corresponding production term and the next two terms on the right side of the first two equations represent the production terms arising out of transition and the suffixes $BP$ and $NAT$ represent By Pass for high free stream turbulence and NATural transition for low free stream turbulence level.
respectively. The computation of production and dissipation terms of $k$, $l$, and $\omega$ and the closure coefficients based on intuition, empiricism and measurement data are discussed in details by Walters et al [2].

3. Finite Volume Method
All the computations in the present work use the solver RANS3D for unsteady incompressible flows. The code employs a finite volume procedure [3] in a multiblock structured grid environment, based on a pressure-velocity solution strategy where an implicit predictor-corrector scheme, similar to the SIMPLE algorithm has been used - but modified for cell-centered variable arrangement using Momentum Interpolation technique. Simple stretched Cartesian grids are used for the flat plate, whereas for the aerofoil, an appropriate curvilinear boundary fitted C-Grid topology is generated with geometric stretching near the wall, using a differential-algebraic procedure [4].

4. Results and Discussions
Two different test cases analysed in the present work are – (i) Flow over a smooth Flat Plate with sharp leading edge under zero pressure gradient and (ii) Flow past an isolated SD7003 aerofoil at a chord-based Reynolds number of 60,000 for different angles of attack.

4.1 Flow past a Flat Plate at Zero Pressure Gradient
The first test case considered is transitional flow past a flat plate for two different levels of free stream turbulence marked as T3A and T3B for which detailed measurement data on skin friction coefficient has been reported by ERCOFTAC [5].

Case T3A : Free Stream Turbulence Level = 3.3%  
Case T3B : Free Stream Turbulence Level = 6.5%

Figure 1. Variation of Skin Friction coefficient along the flat plate for zero pressure gradient

The present computation uses $k-\omega$ turbulence model coupled either to the algebraic correlation model of Abu Ghannam et al [1] or the three equation model of Walters et al [2] for transition. The variation of $C_t$ along the plate is shown in Fig. 1 for zero pressure gradient. In the laminar zone, reasonable agreement is observed between the present prediction and measurement data. However the skin friction coefficient in both the cases is observed to be overpredicted and this disagreement may be attributed to the gross approximations in linear weighting of the laminar and turbulent viscosity in the transition zone where the dynamics obviously is strongly non-linear. For both the cases of zero pressure gradient flow over a flat plate with a sharp leading edge, the computed skin friction coefficient value over the plate length is in reasonably good agreement with the corresponding measurement data.
4.2 Transitional Flow past an isolated SD -7003 Aerofoil

This test case computes flow past a SD-7003 aerofoil for different angles of attack at a relatively low value of the chord-based Reynolds number (Re=60,000). The farfield is assumed to be at a distance of 10C from the aerofoil trailing edge and a structured C-grid topology (516×103) is used for the computation. A zoomed view of the grid is shown in Fig. 2, and the wall-normal distance is maintained to be 8×10^{-4} times the chord length so that the value of the near wall $y^+$ is less than unity everywhere in the field. The oncoming flow is assumed to be uniform with constant values of the turbulent kinetic energy ($k$) and the specific dissipation rate ($\omega$), based on the assumption that the mean turbulence level is 0.1% and the eddy viscosity level is ten times the laminar viscosity of the fluid.

4.2.1 Surface Pressure and Skin Friction Coefficient

Fig 3 shows the pressure distribution on the aerofoil surface for two different angles of attack ($\alpha =2^0$ and $\alpha = 4^0$) at a chord-based Reynolds number of 60000, computed using the transport equation-based transition model [2]. In both the cases, the transition is manifested through a local jump (kink) of $Cp$ at some location on the upper suction surface of the aerofoil. Location of this kink shifts upstream from $x/C=0.3$ at $\alpha = 4^0$ to $x/C=0.3$ at $\alpha = 4^0$. Present prediction is compared to other [6,7] computation results – one is RANS computation [7], where the transition is assumed to occur when the initial disturbance amplitude is magnified by a factor of $e^8$, whereas the other computation [6] uses DNS for the same flow. It is interesting to observe that the location of the jump in $Cp$ is almost identical in all the three computations. However the discrepancies observed in amplitude of the pressure jump may be improved further by finer local resolution of the grid.

4.3.2 Aerodynamic Coefficients

Figure 2. Zoomed view of C Grid for SD 7003 aerofoil

![Figure 2. Zoomed view of C Grid for SD 7003 aerofoil](image)

Figure 3. Chordwise pressure distribution on SD7003 aerofoil surface (Re = 60,000)
In order to assess the accuracy level of the present computation, the computed aerodynamic coefficients, have been compared to the corresponding measurement data [9] and other computation results [7,8] in Fig. 4. The three-equation transition model is observed to marginally improve the lift coefficient, but considerable improvement is observed specially in the drag coefficient where the present computation is found to be quite close to the measurement data. The present three equation eddy viscosity-based transition-turbulence model is found to be capable of capturing the physics of transition for attached and separated boundary layer flows.

Figure 4. Variation of Aerodynamic Coefficients with angle of attack for SD-7003 aerofoil

5. Conclusion
The present simulation employing appropriate transition-turbulence model in the existing RANS3D code, successfully captures the transitional flow for attached boundary layers on flat plate and also for the separation bubble induced transition on an aerofoil surface. However the present model needs to be further validated for more complex transition problems like the drag crisis for flow around a circular cylinder.

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