Quantum Mechanics of Gravitational Waves
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Quantum Mechanics of Gravitational Waves

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Classical Gravity

Einstein's Theory of General Relativity:
Massive objects distort (or bend) spacetime around them.

- Gravitational waves: ripples in spacetime caused by moving objects.

Classical Hill Equation

\[ \ddot{\xi} = \frac{1}{2} \dot{h} \xi \]

- Metric perturbation
- Geodesic separation
The Quantization of Gravity

- **Graviton**: massless spin-2 particle that mediates gravitational interactions.

- Previous attempts to quantify gravity: string theory, loop quantum gravity

- Main idea: If gravity is quantized, how could we observe this?

**Question:** How does this equation of motion change if the spacetime metric is a quantum field?

\[ \ddot{\xi} = \frac{1}{2} \hbar \xi \]

- Metric perturbation
- Geodesic separation
Summary of results

• By treating the gravitational field quantum mechanically, we find that

\[ \ddot{\xi} = \frac{1}{2} \hbar \dot{\xi} \]

\[ \ddot{\xi} = \frac{1}{2} \left( \hbar + \tilde{N}_\Psi - \frac{m_0 G}{c^5} \frac{d}{dt} \frac{d^5}{dt^5} \xi^2 \right) \xi \]

• Characteristics of \( N \) depend on quantum state of the gravitational field
  • Calculated explicitly for coherent, thermal, and squeezed states

• Detection of noise \( \Rightarrow \) quantization of gravity, existence of gravitons!
Proposed method for detection of quantized gravity

• Detector is modeled as two free-falling masses
  • Separation is influenced by perturbations of metric (i.e., gravitational waves)

• Hill Equation:
  • acceleration of geodesic separation in presence of gravitational waves

\[ \dddot{\xi} = \frac{1}{2} \hbar \dot{\xi} \]

**Question:** How does this equation of motion change if the spacetime metric is a quantum field?
Coupling of falling masses with the gravitational field

- Action of the system
  - Einstein-Hilbert action coupled with free-falling masses;
  - Assumptions: small perturbation, weak field, one mass much smaller.
- Simplification: for a single mode and single polarization,

\[
S_\omega = \int dt \left( \frac{1}{2} m (q^2 - \omega^2 q^2) + \frac{1}{2} m_0 \dot{\xi}^2 - g \ddot{q} \ddot{\xi} \right) .
\]

Like harmonic oscillator coupled with a free particle

Quantization!

Energy = \hbar \omega
Probability of transitioning between detector states

• How does $\xi$ evolve in time, for any $|f\rangle$?
• Transition probability:

$$P_{\psi,\omega}(\phi_A \rightarrow \phi_B) = \sum_{|f\rangle} |\langle f, \phi_B | \hat{U}(T, 0) |\psi, \phi_A \rangle|^2.$$
Probability of transitioning between detector states

• Harmonic oscillator is the gravitational field mode
  • Energy = ħω.
• How does ξ evolve in time, for any |f⟩?
• Transition probability:

\[
P_{\psi_{\omega}}(\phi_A \rightarrow \phi_B) = \sum_{|f\rangle} |\langle f, \phi_B | \hat{U}(T, 0) |\psi_{\omega}, \phi_A \rangle|^2.
\]
Transition probability for all gravitational field modes

• Path integral to calculate transition probability:

\[ P_{\psi,\omega} (\phi_A \rightarrow \phi_B) \sim \int \mathcal{D}\xi \mathcal{D}\xi' e^{\frac{i}{\hbar} \int_0^T dt \frac{1}{2} m_0 (\dot{\xi}^2 - \dot{\xi}'^2) F_{\psi,\omega} [\xi, \xi'] } \]

• Several approximations allow us to evaluate this probability for different types of states.

• Generally,

\[ P_{\psi} (\phi_A \rightarrow \phi_B) \sim \int \mathcal{D}N_{\psi} e^{-\frac{1}{2} \int A_{\psi}^{-1} N_{\psi}^2 } \left| \int \mathcal{D}\xi e^{\frac{i}{\hbar} \int_0^T dt \left( \frac{1}{2} m_0 \dot{\xi}^2 + \frac{1}{4} m_0 (\ddot{\xi} + \dot{N}_{\psi}) \xi^2 \right) } \right|^2 \]

Additional fundamental fluctuation on the detector.
Quantum Geodesic Deviation Equation

\[ \dddot{\xi} = \frac{1}{2} \left( \dddot{h} + \dddot{N}_\Psi - \frac{m_0 G}{c^5} \frac{d^5}{dt^5} \xi^2 \right) \]
Quantum Noise Spectra

• Coherent State: Minimum Uncertainty / Classical Behavior

\[ S = \frac{4G\hbar\omega}{c^5} \]

• Thermal State: Temperature Effects / Hawking Radiation

\[ S = \frac{4G\hbar\omega}{c^5} \coth \left( \frac{\hbar\omega}{2k_B T} \right) \]

• Squeezed State: Concentrated Uncertainty / Early Universe Inflation

\[ S = \sqrt{\cosh 2r} \frac{4G\hbar\omega}{c^5} \]
Detecting Quantum Noise

• Coherent State:
  • Planck length / $10^{-35}$ m
  • 17 orders of magnitude beyond current LIGO sensitivity

• Thermal State:
  • 13 orders of magnitude beyond LIGO
  • Just 10 beyond planned LISA sensitivity

• Squeezed State:
  • Hard to detect
  • Authors unsure if there are realistic physical sources with sufficient squeezing
Paper summary

Question:
How does this equation of motion change if the spacetime metric is a quantum field?

\[ \ddot{\xi} = \frac{1}{2} \dot{h} \xi \]

Metric perturbation  Geodesic separation

\[ \ddot{\xi} = \frac{1}{2} \left( \ddot{h} + \ddot{N}_\Psi - \frac{m_0 G}{c^5} \frac{d^5}{dt^5} \xi^2 \right) \xi \]

Quantum Noise  Detection?

118th numbered equation of the extended paper!
Cited by...

Note: Previous essay announcing results (2020) cited by 15

Companion paper

Two independent citations

Proposed experiment

Estimation of spin contributions to gravitational wave signal
Paper impact

Recent acknowledgment of subject as of high interest/impact.

Also sparks theoretical interest.

Source: LIGO
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We would like to thank Jorge Noronha for extremely helpful discussions on the subject matter and the paper.

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The noise of gravitons
Maulik Parikh, Frank Wilczek and George Zahariade

https://doi.org/10.1142/S0218271820420018 | Cited by: 15
Thank You!
The Proposed Experiment

Indirect detection makes use of decoherence caused by noise of gravitons.

Cites the theory of this paper.

Reasonable because length of mirrors required are only one order of magnitude larger than LIGO.

FIG. 1. The proposed setup: an equal-arm Michelson interferometer for a single photon where there is a macroscopic suspended mirror at the end of each arm.