Stress and deformation analysis of double curvature arc dams using finite element method (FEM): A case of budhi gandaki hydropower project

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Abstract

High rise arc dams are widely used in the development of storage type hydropower project because of the economic advantage. Among different phases considered during the lifetime of dam, control of dam’s safety and performance becomes more concerned during the lifetime. This paper proposed the 3 – D finite element method (FEM) for stress and deformation analysis of double curvature arc dam considering the non – linearity of foundation rock following the Hoek – Brown Criterion. The proposed methodology is implemented through MATLAB scripting language and studied the double curvature arc dam proposed for Budhi Gandaki hydropower project. The stress developed in the foundation rock, compressive and tensile stress acting on the dam are investigated and analysed for the reservoir level variation. Deformation at the top of the dam and in the foundation rock is also investigated. In addition to that, stress and deformation variation in the foundation rock is analysed for various rock properties.

Keywords: Double curvature arc dams, Finite element method, Hoek – Brown Criterion.

1. Introduction

In storage type hydroelectric project, dams play important for capacity of hydroelectric project and its economic feasibility. Arch dams are economic and has advantage of higher height to base thickness ratio which differs from other though certain challenges of complexity in analysis and construction. There have been different methods developed for double curvature arch dam analysis [1]. Among different methods, finite element method (FEM) has advantages in handling material in-homogeneity and non-linearity [2]. Safety evaluation of rock foundation problems can be basically categorized in two types as represented by E.Hoek and P.Londe [3], which are

- Rupture of rock or resistance to failure
- Deformations and their effect on foundation rock and the structure

Rupture of rock considers failure of rock mass through over stress of either tensile compressive or shear stress which might be the effect of loads imposed after construction of structure. This can be evaluated using appropriate failure criterion of rock mass. Among different method of analysis of double curvature arch dam three dimensional Finite Element Method (FEM) is adopted for stress-strain analysis in double curvature arch dam along with its abutments and foundation.

2. Project Description and Considerations

The Budhi Gandaki Hydroelectric Project is a storage type of project located between boundary of Gorkha and Dhading district on the Budhi Gandaki River, which is estimated of 1200 MW. Double
curvature arch dam having total volume of dam will be 5753000 m$^3$ of concrete, with total height of dam between deepest foundation level (EL 280m) and the crest parapet (EL 543) is 263m. The dam width at the base is 80m and the developed dam crest length is 760m [4]. Dam is located over bedrock consisting of Nourpul formation. In finite element model, three types of rock are considered, all with similar E-W oriented strike and dip amount 75$^\circ$ to north. Apparent thickness of rock layers is considered as, Dandagaun phyllite 580m, Lower Nourpul Formation or Purebesi quartzite 225m, and Upper Nourpul Formation 595m [5].

During finite element analysis, required parameters of bed rock, three formation considered in Hoek–Brown failure criterion is considered as in table 1 and estimated according to Geological, Geotechnical and Seismological Investigations and Studies report of Budhi Gandaki HEP. [4,5]

Other parameters as Young’s modulus of elasticity (rock mass) $E_m$, Hoek and Brown reduced material constant ($m_b$), Hoek and Brown criterion constant ($s$ and $a$) and Poisson’s ratio of rock mass ($\nu_m$) are determined using following relations [6],

$$E_m = E_i \left(0.02 + \frac{1-D/2}{1 + e^{(60+1SD-GSI)/11}}\right)$$

$$m_b = m_i e^{(GSI-100)/(28-14D)}; s = e^{(GSI-100)/(9-3D)}$$

$$a = 0.5 + (e^{-GSI/15} - e^{-20/3})/6$$

$$V_m = 0.5 - \frac{0.2RMR}{RMR + 0.2(100 = RMR)}$$  

As dam body concrete of M20 grade is considered having modulus of elasticity 20000 MPa, Poisson’s ratio 0.2 and unit weight 24 kN/m$^3$ has been considered [4]. For study of variation in deformation and stress with change in bed rock properties, additional different seven properties of bed rock have been considered with variation in GSI values is considered as shown in table 1 [4,7].

### 3. Modelling and Analysis

#### 3.1 Finite Element Method

A problem with finite boundaries well, the FEM uses common technique of discretization beyond the zone of influence from the structure. It can be concluded that a foundation model of depth and width equal to 1.5 and 3.0 times the base width of the dam are sufficient to achieve accurate stress results [8].

Applying virtual work method [9], small increment of stress and strain in adopted element is governed by equation (2).

$$\int_{vol} \{\varepsilon\} \{\sigma\} dv = \int_{\text{area}} \{\varepsilon\} \{\sigma\} dv + \int_{\Gamma} \{\varepsilon\} \{\mathbf{t}\} d\Gamma + \sum \{\varepsilon\} \{p\}$$  

Where, $\{\varepsilon\}$ is the strain vector, $\{\sigma\}$ is the stress vector, $\{U\}$ is the displacements vector, $\{b\}$ is the body forces vector, $\{t\}$ is the traction forces vector, $\{P\}$ is the vector of concentrated forces applied, $dv$ is an element of volume, $d\Gamma$ is an element of the boundary of the element on which the traction forces $\{t\}$ are applied.

Using interpolation matrix which also is defined as shape function $[N]$ are represented in equation (3) can be represented as equation (5).
\[ [k] \{a\} = \{f\} \]  
(3)

Where \([k]\) is elemental stiffness matrix and \(\{f\}\) is elemental force matrix and given by,

\[
[k] = \int_{vol} [B][D][B] dv 
\]
(4)

\[
\{f\} = \int_{vol} [N]^T (b) dv + \int_{area} [N]^T (t) dA + \sum [N]^T \{P\},
\]
(5)

For numerical integration of equations (3), (4) and (5) Gauss quadrature integration is used.

### 3.2 Failure Criterion

For failure criterion of dam body simple Hoek’s law is adopted, whereas for bed rock elastic-plastic behaviour is adopted using Hoek and Brown criterion is adopted. In which plastic stress and strain are determined using return mapping method among two popular methods, Euler procedure and return mapping. As presented by Clausen, Johan Christian using Hoek and Brown criterion yield surface equation can be written in equation 5 where \(\sigma_1 > \sigma_2 > \sigma_3\) are principle stress tensile stress taken positive and rest of terms are as defined in equation (1) [10].

\[
f = \sigma_1 - \sigma_3 - \sigma_{ci}\left(S - m_b \frac{\sigma_1}{\sigma_{ci}}\right)^{a}
\]
(6)

Plastic potential function for non-associated material behaviour for perfectly plastic is chosen similar as yield function, so

\[
g = \sigma_1 - \sigma_3 - \sigma_{ci}\left(S - m_b \frac{\sigma_1}{\sigma_{ci}}\right)^{a}
\]
(7)

During return mapping stress after increment of load is obtained by constitutive relationship of deformable solid, and principle stress of stress obtained be \(\sigma^B\), if it lies out of yield surface i.e. value of gradient \(\frac{\partial f}{\partial \sigma}\) than 1 for \(\sigma^B\), then updated stress \(\sigma^C\) should be computed and slope of line connecting \(\sigma^B\) and \(\sigma^C\) should be equal to slope of plastic corrector which is slope of vector \(s = D \times \frac{\partial g}{\partial \sigma}\) then slope of plastic corrector \(\alpha_c = \frac{s_3}{s_1}\) and slope of line connecting \(\sigma^B\) and \(\sigma^C\) be \(\alpha\), return mapping is expressed in figure 1.[10]

\[
\alpha_r = \frac{\sigma_3^C - \sigma_3^B}{\sigma_1^C - \sigma_1^B}; h_r = \alpha_r - \alpha
\]
(8)

Above equation should value zero and is solved Newton-Raphson method and \(\sigma^C_1\) is obtained by iteration process and \(\sigma^C_3\); is obtained using equation gradient \(\left(\frac{\partial f}{\partial \sigma}\right)\), and

\[
\sigma_2^C = t_f s_2 + \sigma_2^B
\]
(9)

\[
t_f = \frac{\sigma_1^C - \sigma_1^B}{s_1}
\]
(10)

After updating stress constitutive matrix should also be modified for next increment of load, updated constitutive matrix \(D^{upc}\) is obtained using following relations [10].
Where $D^e$ is modified elastic stiffness matrix given by $D^e = DT$ and modification matrix $T$ is given by Plastic strain $\Delta \varepsilon^p$ can be obtained using constitutive matrix and plastic strain.

$$T = \left( I + \Delta \lambda D \frac{\partial^2 g}{\partial \sigma^2} \right)^{-1}$$

A schematic presentation of the Newton-Rapson in the elastic plastic finite element method is presented in the flow chart in figure 3 [10].

3.3 Mesh generation and loads

Hexahedral element with eight nodes is used with total 6807 number of nodes and 5409 number of hexahedral element were developed as shown in figure 2. Boundary conditions are considered by providing zero degree of freedom to bottom most nodes and two degree of freedom is provided to side faces, U/S and D/S faces of model, rest nodes are considered with three degrees of freedom. Total 1219 nodes are completely restrained out of 6807 number of nodes, rest of nodes were assigned 3 degree of freedom which resulted total 17070 degree of freedom in model.

3.4 Loads

Static loading is only considered to contribute for safety stability analysis of wedge in left abutment and Hoek and Brown criterion analysis of arch dam foundation, which are gravity, reservoir water, temperature changes, and silt, ice, uplift, and earthquake loads. Gravity loads due to weight of the material are computed from the unit weight and geometry of the finite elements.

Hydrostatic pressure: The surface loads are then applied to the structure as concentrated nodal loads. Therefore, hydrostatically varying surface pressure can be specified by using a reference fluid surface and a fluid weight density as input. Hydrostatic load is applied for incremental reservoir level as empty reservoir, water level RL 392m (reservoir depth 112m), water level RL 442m (reservoir depth 162m), water level RL 467m (MOL) (reservoir depth 187m), water level RL 492m (reservoir depth 212m) and water level RL 540m (FSL) (reservoir depth 260m). Silt. According to IS 6512-1984 silt load can be estimated by increasing unit weight of water in silt zone for horizontal pressure by 360 kg/m$^3$ and vertical pressure by 925 kg/m$^3$.
Initialization of load increments

- \( k = 0 \)
- \( \{ P_k \} = 0 \); Initial load
- \( \{ \sigma_k \} = 0 \); Initial stress
- \( \{ u_k \} = 0 \); Initial displacement
- \( \{ D_k^{epc} \} = \text{Elastic constitutive matrix}(E, \nu) \)

\[ \begin{align*}
& \text{Initialization of global equilibrium} \\
& \quad \bullet j = j + 1; \\
& \quad \bullet \text{Formulation of global stiffness matrix } [K_k^j] \text{ from } [D_k^{epc}] \\
& \quad \bullet \text{Calculation of incremental deflection: } \{ \Delta u_k^j \} = [K_k^j]^{-1} \times \{ \Delta P \}, \\
& \quad \bullet \text{Calculation of total deflection: } \{ u_k \} = \{ u_{k-1} \} + \{ \Delta u_k^j \}, \\
& \quad \bullet \text{Calculation of internal forces } \{ q \} \text{ from } \{ u_k \} \\
& \quad \bullet \text{Calculation of residual forces } \{ r \} = \{ P_k \} - \{ q \}, \\
& \quad \bullet \text{Calculation of deformation due to residual force: } \{ \delta u_k^j \} = [K_k^j]^{-1} \times \{ r \}, \\
& \quad \bullet \text{Updating displacement increment: } \{ \Delta u_k^j \} = \{ \Delta u_k^j \} + \{ \delta u_k^j \}, \\
& \quad \bullet \text{Calculating stress increment vector } \{ \Delta \sigma_k \} \text{ and strain increment vector } \{ \Delta \varepsilon_k \} \text{ from displacement increment vector } \{ \Delta u_k^j \}, \\
& \quad \bullet \text{Calculating total stress: } \{ \sigma_k \} = \{ \sigma_{k-1} \} + \{ \Delta \sigma_k \}, \\
& \quad \bullet \text{Calculating total strain: } \{ \varepsilon_k \} = \{ \varepsilon_{k-1} \} + \{ \Delta \varepsilon_k \}, \\
& \quad \bullet \text{Updating stress vector } \{ \sigma_k \}, \text{ strain vector } \{ \varepsilon_k \}, \text{ and } [D_k^{epc}] \\
\end{align*} \]

\[ \begin{align*}
\text{if } \{ r \} < 0.0005 & \quad \text{YES} \\
\text{if } \text{Load increment } \{ \Delta P \} & \quad \text{YES} \\
\text{Final output } \{ \sigma_k \} \text{ and } \{ u_k \} & \quad \text{NO} \\
\end{align*} \]

Figure 3. Flow chart for FEM procedure adopted
4. Results and Discussion

Horizontal deformation at top nodes of the dam at different reservoir level for Case I is presented in figure 4. It is found that D/S deformation of top dam due to gravitational load occurs towards U/S due to curved shape of dam with maximum magnitude of 5.8 cm at mid of dam, with increase in water level deformation gradually shifts towards D/S and maximum deformation is observed 19.4 cm at full supply level. Vertical displacement at top of dam as obtained in figure 5 due to gravitational load only is downward and with increase of water level in reservoir, due to curved shape of dam in vertical axis upward hydrostatic force also acts which reduces the vertical displacement due to gravitational load and with further increase in reservoir level upward displacement has occurred at top of dam, but at bottom of dam there is very less amount of upward displacement although which may produce tensile opening at contact of foundation and dam that should be minimized by increasing base width of dam. Over all deformed shape of dam at empty reservoir level (due to self-weight of dam) and at full supply level is observed as in figure 6 and 7. Total displacement is determined as resultant of vertical and horizontal deformation at different nodes of dam. Contour of total displacement full reservoir level is observed as in figure 8. Principle stress contour for full supply level (maximum static loading condition) are given in figures 9, 10, 11 and 12 which are obtained for case I foundation properties i.e. for actual foundation properties of site. It can be observed that maximum compressive stress developed in dam at full supply level is 11.5 MPa and, tensile stress is developed at contact with abutments (in US face). Which is developed due to bending action in dam due to hydrostatic load.

Results are compared with the report of ‘Feasibility Study and Detailed design of Budhi Gandaki HPP’ [9]. Variation of the deformation and stress are identical to the ‘Engineering guidelines for the evaluation of hydropower projects, chapter 11’ by Federal Energy Regulatory Commission Division of Dam Safety and Inspection (1999). Deformation in dam increases with decrease in foundation modulus of elasticity and the increase rate of deformation increases for lesser value of foundation modulus of elasticity, which is also observed. It was also observed that rate of increase in stress is less and found similar to the literature.

![Figure 4](image-url). D/S deformation at top of dam for different reservoir level (Case I properties of foundation)
Figure 5. Vertical displacement at top nodes of dam for different reservoir level (Case I properties of foundation)

Figure 6. Deformed shape of dam at empty reservoir level (Case I) due to self-weight

Figure 7. Deformed plot of dam at FSL (Case I)

Figure 8. Total displacement (m) contour on U/S face dam at FSL for Case I
Figure 9. Major principle stress contour on downstream face of dam at FSL

Figure 10. Major principle stress contour on upstream face of dam at FSL

Figure 11. Minor principle stress contour on upstream face of dam at FSL

Figure 12. Major principle stress contour on upstream face at empty reservoir

Figure 13. Modulus of elasticity of foundation rock vs Total deformation in dam body

Figure 14. Modulus of elasticity of foundation rock vs Total deformation in foundation
After finite element numerical model analysis results were obtained in terms of nodal displacement, nodal reaction forces, elemental stress, principle stress, strain and plastic strain. Some of these results are presented in previous chapter. At full supply level in reservoir, during this loading, maximum compressive stress in dam and foundation is 12 MPa and 4.2 MPa respectively. Also during full supply level maximum deformation obtained in dam and foundation is 12cm and 6.3cm respectively. From analysis it is observed that with decrease in the GSI value there is significant increase in the deformation in both dam and rock foundation, but there is only slight increment in the stress developed in them.

Rate of variation in deformation in both dam body and foundation not uniform, rate of increment in deformation is lesser for higher value of GSI and rate increases with decrease in the GSI value as represented in figure 15 and figure 16. Similarly, dam body and foundation deformation variation with respect to modulus of elasticity of foundation rock is presented in figure 13 and 14.

5. Conclusion

Figure 15. Total maximum deformation in dam vs GSI value of foundation rock

Figure 16. Total maximum deformation in foundation vs GSI value of foundation rock
Table 1. Different cases if bed rock properties used for analysis

| Case I (Actual Condition of Project) | Case II | Case III | Case IV |
|-------------------------------------|---------|----------|---------|
| S.N. Rock Properties | D.P. | P.Q. | N.P. | D.P. | P.Q. | N.P. | D.P. | P.Q. | N.P. | D.P. | P.Q. | N.P. |
| 1 GSI        | 40 | 60 | 50 | 30 | 50 | 40 | 30 | 35 | 40 | 40 | 80 | 50 |
| 2 σci(Mpa)  | 35 | 62 | 40 | 30 | 62 | 30 | 35 | 62 | 40 | 35 | 62 | 40 |
| 3 E(GPa)    | 40 | 52 | 46 | 40 | 52 | 46 | 40 | 52 | 46 | 40 | 52 | 46 |
| 4 Eia(GPa)  | 4.76 | 15 | 8 | 1 | 12 | 5.47 | 1 | 4.5 | 5.47 | 4.76 | 40 | 8 |
| 5 D       | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| 6 m0       | 0.92 | 4.907 | 1.37 | 0.62 | 3.3 | 0.63 | 0.62 | 1.82 | 0.63 | 0.92 | 10.85 | 1.37 |
| 7 s        | 0.0079 | 0.00855 | 0.0026 | 0.0024 | 0.0026 | 0.0079 | 0.0024 | 0.00436 | 0.0079 | 0.0079 | 0.0092 | 0.0026 |
| 8 a       | 0.511 | 0.503 | 0.506 | 0.522 | 0.5057 | 0.511 | 0.522 | 0.516 | 0.511 | 0.511 | 0.5 | 0.506 |
| 9 v       | 0.32 | 0.27 | 0.3 | 35 | 0.3 | 0.32 | 0.35 | 0.32 | 0.32 | 0.32 | 0.25 | 0.3 |
| 10 Ysk(kN/m^3) | 22 | 25 | 25 | 22 | 25 | 25 | 22 | 25 | 25 | 22 | 25 | 25 |

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