Constraining dark energy

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Abstract

In this paper we propose a mechanism that protects theories violating a holographic bound suggested in [arXiv:1203.5476] from developing accelerated expansion. The mechanism builds on work on transplanckian physics, and a non-trivial choice of vacuum states. If correct, it lends further support for detectable signatures in the CMBR signalling new physics.
1 Introduction

It is an intriguing and challenging problem to fit a cosmology undergoing accelerated expansion into a theory of strings or quantum gravity. It is also a problem of the utmost physical importance, since it is now an established fact that the universe not only underwent accelerated expansion during its early days, i.e. inflation, but is again entering into such a phase during the last few billion years. Apart from the challenges of detailed de Sitter model building in string theory, see [1] for a recent review and references, there are deep problems related to the concept of entropy in gravity and the possible application of holography. While black holes and anti de Sitter spaces are reasonably well understood from this point of view, de Sitter space remain elusive.

There has been numerous, more or less successful, attempts to apply holography to cosmology. In [2] an attempt was made to clear up some of the associated issues. In particular, it was argued that the de Sitter horizon, in analogue with a black hole horizon, should be thought as describing entropy of matter beyond the horizon. Or, to be more precise, matter that has passed out through the horizon. Hence, the horizon entropy can only indirectly be used to limit the property of matter inside of the de Sitter radius. This is achieved through the study of how the area of the horizon changes in response to the flow of matter.

Recently, it was suggested in [3] that holography actually do provide interesting constraints on models of dark energy and accelerated expansion. The idea is to count the number of initial field configurations that eventually develop into asymptotic de Sitter space, and compare this number with the holographic bound in de Sitter space. The relevant entropy obtained in this way is argued to be of the order

$$S \sim \frac{N a_{\text{min}}^{-2}}{H^2},$$

where $a_{\text{min}}$ is a cutoff associated with the light fields of the theory, and $N$ is the number of such fields. According to holography this number should be less than

$$S_{\text{dS}} = \frac{8\pi^2 M_{\text{pl}}^2}{H^2}.$$  \hfill (2)

There is an uncertainty when it comes to the actual numerical coefficient in the calculation leading to (1). One specific possibility is, according to [3], that the number coincides with the entanglement entropy. Using this, and for the sake of definiteness, the regularization proposed by [4], we find that

$$0.3 \frac{N a_{\text{min}}^{-2}}{H^2} < \frac{8\pi^2 M_{\text{pl}}^2}{H^2},$$

which leads to

$$N \frac{a_{\text{min}}^{-2}}{M_{\text{pl}}^2} \lessapprox 27\pi^2.$$  \hfill (4)
In [3] it was specifically argued that there exists in any typical string theory realization of accelerated expansion, a large number of axionic fields with periodicity

$$\phi_a \rightarrow \phi_a + f_a.$$ (5)

It was furthermore argued that the relevant cutoff in the entropy calculation is provided by the periodicity. To be precise,

$$a_{\text{min}} = \frac{1}{f_a \sqrt{2}}.$$ (6)

According to [3] these results provide highly nontrivial constraints on stringy realizations of inflation and dark energy.

While these results are quite intriguing, there are several questions that remain to be answered. In particular, it would be interesting to see how this limit is actually implemented by quantum gravity. What would happen if we pick a theory with the potential to violate the holographic bound, and force the theory into a state with a positive dark energy? How does the inconsistency show up? Perhaps there is some kind of physical mechanism that prevents the system from ending up in the holographically forbidden state? It turns out that the answer to this question can be found in a surprising place. Let us turn to the problem of transplanckian physics and its effects on the CMBR.

2 Transplanckian physics and the CMBR

As argued in, e.g., [5] and further discussed from various aspects in works such as [6-17] (this is just a selected few), it can be expected that physics beyond the string or Planck scale is magnified through the expansion of the universe, and affects phenomena at lower energies such as the spectrum of the CMBR. In inflationary model building it is usually assumed that the vacuum to be used for all the relevant fields is the Bunch-Davies vacuum. This is a reasonable choice if all modes can be traced back to infinitely small scales, where the expansion of the universe, and the resulting deviation from Minkowsky space, become negligible. However, in the presence of a fundamental energy scale such as the string or Planck scale there is no reason to expect the Bunch-Davies vacuum to be the preferred choice, [8]. New physics, connected with the string or Planck scale, can be expected to modify the vacuum. This can be conveniently modelled through a Bogolubov mixing linear in $\frac{H}{\Lambda}$, where there is a dependence on scale only through the Hubble constant $H$. $\Lambda$ is the energy scale of the new physics, which could be the string scale or the Planck scale.

As argued in [17], the effects propagating down to low energy will be of two types: a modulation of the CMBR spectrum and a back reaction on the expansion of the universe. The latter effect is the one that will be of relevance to us.
2.1 Effects on the CMBR

According to the analysis of [8], given a Bogolubov mixing as described above, the typical effect to be expected on the primordial spectrum is of the form

\[ P(k) = \left( \frac{H}{\dot{\phi}} \right)^2 \left( \frac{H}{2\pi} \right)^2 \left( 1 - \frac{H}{\Lambda} \sin \left( \frac{2\Lambda}{H} \right) \right), \]  

(7)

where we note a characteristic, relative amplitude of the correction given by \( \frac{H}{\Lambda} \), and a modulation sensitively depending on how \( \frac{H}{\Lambda} \) changes with \( k \). The claim is that whatever the nature of the high energy physics really is, a modulated spectrum of this form is what we should naturally expect. In [11] one can find an early discussion of the phenomenological relevance of the effect, and how its magnitude is related to the characteristic parameters describing the inflationary phase. Using the standard slow roll approximation, where an important parameter is

\[ \varepsilon = \frac{M_{pl}^2}{2} \left( \frac{V'}{V} \right)^2, \]  

(8)

with initial conditions imposed at some fundamental scale \( \Lambda = \gamma M_{pl} \), it is found that

\[ \frac{\Delta k}{k} \sim \frac{\pi H}{\varepsilon \Lambda} \sim 1.3 \cdot 10^{-3} \frac{1}{\gamma \sqrt{\varepsilon}}, \]  

(9)

and

\[ \frac{H}{\Lambda} \sim 4 \cdot 10^{-4} \frac{\sqrt{\varepsilon}}{\gamma}. \]  

(10)

These two relations are the key to estimating the expected magnitude of the effect. For instance, with a string scale a couple of order of magnitudes below the Planck scale, and a slow roll parameter \( \varepsilon \sim 10^{-2} \), we find an amplitude of \( \frac{H}{\Lambda} \sim 10^{-2} \) – comparable with cosmic variance – and a periodicity given by \( \frac{\Delta k}{k} \sim \mathcal{O}(1) \).

2.2 Back reaction

The presence of a non-standard vacuum, motivated by the presence of unknown high energy physics, raises the issue of backreaction. Focusing on the contribution to the vacuum energy coming form the non-standard vacuum, as compared with the Bunch-Davies vacuum, one finds an additional energy density naively given by \( \rho_\Lambda \sim \Lambda^2 H^2 \). To lowest order, as long as \( \Lambda < M_p \), we can ignore this contribution as was concluded in [18]. In [16] and [17], however, the discussion was taken a step further and it was noted that the presence of the background energy will change the effective slow roll parameters. In fact, there are regimes where these effects even will dominate.
To proceed with a quantitative analysis, we denote the slow roll parameter describing the time dependence of the Hubble constant by $\varepsilon$ and define it through

$$\varepsilon = \frac{\dot{H}}{H^2}. \quad (11)$$

In addition, we introduce a second slow roll parameter governing the rolling of the inflaton according to

$$\varepsilon_{\text{inf}} = \frac{\dot{\phi}^2}{2M_{\text{pl}}^2 H^2}, \quad (12)$$

where $\phi$ is a canonically normalized inflaton. In the slow role approximation, and in the absence of back reaction from the vacuum, we would have had $\varepsilon = \varepsilon_{\text{inf}}$, also coinciding with (8). With back reaction, as explained in [17], we find a decoupling of the expressions for the amplitude and the period according to

$$\frac{H}{\Lambda} \sim 4 \cdot 10^{-4} \frac{\sqrt{\varepsilon_{\text{inf}}}}{\gamma},$$

and

$$\frac{\Delta k}{k} \sim \frac{\pi H}{\varepsilon \Lambda} \sim 1.3 \cdot 10^{-3} \frac{\sqrt{\varepsilon_{\text{inf}}}}{\gamma \varepsilon}. \quad (14)$$

Let us now proceed with an estimate of $\varepsilon$, following [16].

The above estimate of the energy density is not good enough when we want to find an expression for $\varepsilon$. What we need to do is to take into account that $H$ will be changing with time, i.e. decrease. Modes with low momenta were created at earlier times when the value of $H$ were larger, and the contribution to the energy density from these modes will, as a consequence, be enhanced. We therefore find an energy density given by

$$\rho_{\Lambda} (a) = \frac{1}{2\pi^2} \int_0^\Lambda d\epsilon \left( \frac{H^2 (\frac{\epsilon}{\Lambda})}{\Lambda^2} \right) = \frac{1}{2\pi^2} \frac{\Lambda^2}{a^4} \int_0^a da a^3 H^2 (a), \quad (15)$$

where we have introduced a low energy cutoff corresponding to the energy at the time of observation of modes that started out at $\Lambda$ at some arbitrary initial scale factor $a_i$. If we take a derivative of the energy density with respect to the scale factor and use $\frac{d}{da} = \frac{1}{aH} \frac{d}{dt}$, we find

$$\dot{\rho}_{\Lambda} + 4H\rho_{\Lambda} = \frac{1}{2\pi^2} \Lambda^2 H^3, \quad (16)$$

from which we conclude that we must introduce a source term in the Friedmann equations. It was found in [16] that the evolution is governed by

$$\frac{d}{da} \left( a^5 H H' \right) = -\frac{\Lambda^2}{3\pi^2 M_{\text{pl}}^2} a^3 H^2 - \frac{1}{2M_{\text{pl}}^2} \frac{d}{da} \left( a^4 (aH\dot{\phi})^2 \right), \quad (17)$$

where we let $\dot{\cdot} = \frac{d}{da}$. The first term on the right hand side is due to the presence of the non-standard vacuum, while the second term is due to the presence of the inflaton.
potential. In a situation where the first term dominates, we find a slow roll governed by
\[ \varepsilon = \frac{\gamma^2}{12\pi^2}, \]  
(18)
for small \( \gamma = \frac{\Lambda}{M_{\text{pl}}} \). In the standard case, with no vacuum contribution, the only non-vanishing term is the second one, leading to a slow roll governed by (12).

While it is the slow roll of the inflaton that controls the overall amplitude of the primordial spectrum, one could easily imagine, as discussed in [17], that there are more fields in the non-standard vacuum. These would also contribute to the back reaction and enhance \( \varepsilon \) by a factor \( N \), where \( N \) is the number of participating fields. Hence, the expression becomes
\[ \varepsilon = \frac{N\gamma^2}{12\pi^2}. \]  
(19)
We will not discuss the further application of these results to the CMBR, but instead focus on their relevance to the problem of holographic constraints on dark energy.

3 The decay of dark energy

Let us now make the connection with the holographic constraint on dark energy. In [3] it was argued that theories with too many fields for a given cutoff break the holographic bound, and are inconsistent as models of de Sitter space. Using the results of the previous section we can now propose a mechanism for how this comes about.

The key is the decay of the dark energy that results if the vacuum of the theory is different from the Bunch-Davies vacuum. The calculated value of the slow roll parameter \( \varepsilon \) depends on the number of fields and the cutoff. If we choose the number of fields in the new vacuum, and the cutoff at \( \Lambda = 1/a_{\text{min}} \), such that the holographic bound is violated, we find that the slow roll parameter becomes
\[ \varepsilon > \frac{27\pi^2}{12\pi^2} \gtrsim 1. \]
This implies that the dark energy decays so fast that no accelerated expansion will take place, and, therefore, there cannot arise any conflict with the holographic de Sitter bound. It should be noted that theories that threaten to violate the bound are not necessarily inconsistent. It is just that these theories lack, when quantum gravity is taking into account, solutions corresponding to accelerated expansion.

The proposed mechanism relies on one assumption: \textit{in the presence of a fundamental scale due to stringy or quantum gravitational effects, there is no reason, in

\footnote{In this paper we consistently use the reduced Planck mass \( M_{\text{pl}} = 1/\sqrt{8\pi G} \sim 2.4 \cdot 10^{18}\text{GeV}. \) This should be kept in mind when comparing with the results in [17].}
an expanding cosmology, to prefer the Bunch-Davies vacuum. A much more natural candidate is a vacuum such as the instantaneous Minkowsky vacuum, as proposed in [8]. Given this, the theory automatically protects itself from de Sitter vacua in cases where the de Sitter bound may be violated. In theories where the bound is not violated, accelerated expansion is allowed but the decay of the dark energy due to the vacuum effect may still be physically relevant, and could play an important role during, e.g., inflation.

The argument can also be turned around. The inconsistency of accelerated expansion in certain models suggest the need for a general mechanism of vacuum decay. Such a mechanism is supplied if the natural vacuum choice of quantum gravity is not the Bunch-Davies. In fact, it turns out that the vacuum choice of [8] has precisely the desired effect. This, it can be argued, gives independent support for the idea of detectable, transplanckian signatures in the CMBR.

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