Eddington inspired Born Infeld Theory:  
A new look to the matter-coupling paradigm

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We discuss some consequences of changing the matter to gravity coupling without affecting the gravitational dynamics. The Einstein tensor is usually assumed to be proportional to the stress tensor due to the divergence free property of both object. This is not the only consistent way to couple matter to gravity; we explore some aspect of consistent modification to the matter/gravity coupling using the recently proposed Eddington inspired Born Infeld extension of gravity.

1. Motivations

The theory of General Relativity has raised questions soon after it was formulated. Even though it passed numerous experimental tests, it still poses problems at larger scales. For instance, already at the galactic scale, one has to invoke extra ingredient to understand the galactic rotational curve. On a more theoretical aspect, Gravity is a non renormalizable theory which is a major obstacle to a quantum formulation of gravity. All these reasons and many others call for a completion or modification to General Relativity. There are essentially three possibilities, either adding new degrees of freedom, either changing the space-time dynamics or the matter to gravity coupling. There might be relations between these approaches and they can be mixed, but we don’t discuss this aspect here. In this proceeding, we focus on the last possibility, namely the matter to gravity coupling. In particular, we use a recently proposed model to discuss the coupling, namely the Eddington inspired Born Infeld (EiBI) model.$^1$

2. EiBI model and the coupling paradigm

The Eddington inspired Born Infeld model of gravitation$^1$ is described by the following action

$$S_{EiBI} = \frac{1}{\kappa} \int \left( \sqrt{|g_{ab} + \kappa R_{ab}|} - \lambda \sqrt{|g_{ab}|} \right) d^4x + S_m,$$

where $g$ is the metric, $R$ is the Ricci tensor built from an independent connection, say $S^a_{bc}$, $S_m$ is a minimally coupled matter action, $\kappa$ is essentially the Newton constant and $\lambda - 1$ is the cosmological constant. Variation à la Palatini leads to the following set of equations$^1$

$$q_{ab} = g_{ab} + \kappa R_{ab}, \quad \sqrt{-q} q^{ab} = \sqrt{-g} \left( \lambda g^{ab} + \kappa T^{ab} \right),$$

$$S^a_{bc} = \frac{1}{2} q^{ad} \left( - \partial_d q_{bc} + \partial_b q_{dc} + \partial_c q_{bd} \right), \quad T^{ab} = \frac{1}{\sqrt{-g}} \frac{\delta S_m}{\delta g_{ab}},$$

(2)
where \( g \) and \( q \) are the determinant of \( g_{ab} \) and \( q_{ab} \) and where \( q^{ab} \), \( g^{ab} \) are the matrix inverses of \( g_{ab} \), \( q_{ab} \). Interestingly, this theory identically reduces to General Relativity in vacuum. In fact, it does not introduce additional degrees of freedom compared to General relativity. A direct consequence of these two ingredients is that the theory changes only inside matter. This leads to a bunch of interesting consequences, essentially regarding singularities in General Relativity. For example, it was shown that the Big-Bang singularity might be avoided with such a model. It was also argued that astrophysical singularities may be avoided or softened in black hole formation processes.

### 2.1. Reformulation of EiBI

The EiBi model of gravity can be reformulated in a way that makes the modification with respect to General Relativity very explicit. In particular, the case of perfect fluids matter field is quite enlightening. Indeed, equations (2) can be rewritten in the following form

\[
G^a_b[q_{cd}] = R^a_b - \frac{1}{2} \delta^a_b R = \gamma T^a_b - \Lambda \delta^a_b,
\]

where \( \tau = \sqrt{g/q} \) and where we defined the apparent stress tensor \( T^a_b \) defined as

\[
T^a_b = \tau T^a_b + \mathcal{P} \delta^a_b, \quad \gamma \mathcal{P} = \tau - 1 - \frac{1}{2} \tau \gamma \kappa T,
\]

and the cosmological constant \( \Lambda = (\lambda - 1)/\kappa \). \( \mathcal{P} \) plays the role of an isotropic pressure addition. For the case of a perfect fluid, the metric \( g \) can be eliminated explicitly, leaving a theory of pure General relativity coupled to a perfect fluid with a modified equation of state. More precisely, the metric determinant ratio is given by \( \tau = [\det (\delta^a_b - \kappa T^a_b)]^{-\frac{1}{2}} \).

This reformulation shows explicitly, at least in the case of a perfect fluid how the coupling between gravity and matter is changed in this theory. It should be noted however that in this reformulated case, matter is coupled (in a modified way) to the metric \( q \), which is an auxiliary metric in the original theory. However, since observations are usually performed from outside matter, since both metric are the same in vacuum, the effect of the modified coupling to \( q \) affects measurements performed with \( g \) in vacuum indirectly. Interestingly enough, all the nice features of the EiBI model can be understood in terms of the reformulated version and of the modification in the coupling. This motivates further studies in 'pure' coupling modification. It should be mentioned that the set of equivalent equations can be obtained from the following action principle:

\[
S_{equiv} = \frac{1}{\gamma} \int d^4 x \sqrt{-q} \left[ R[q] - 2 \frac{\lambda}{\kappa} + \frac{1}{\kappa} (q^{ab} g_{ab} - 2 \tau) \right] + S_M(g),
\]

\(^*\)it was recently proven that this formulation of EiBI suffers from specific singular behaviors for compact objects. In fact, because of the Palatini nature of the model, a antisymmetric part in the connection should be considered, introducing new degrees of freedom. In our work we assumed a symmetric connection, yet introducing no additional degrees of freedom.
where $S_M(g)$ is minimally coupled to $g$. This action is very similar to a bi-metric theory, apart from the fact that the metric $g$ is not dynamical.

This is the particular case of the EiBI theory of gravity, but it is tempting to use this action as a basis for investigating coupling modifications.

3. Conclusion

Theories effectively changing the coupling between matter and gravity pass most experimental tests of General Relativity, provided they are equivalent in vacuum. Note that most of the tests of Gravity are in fact testing General Relativity in vacuum. An interesting realization of such a theory is the Eddington Born Infeld theory of gravity. We made an explicit correspondence between the modification of the matter / Gravity coupling and the interesting features of the EiBI model. In particular we showed that in the case of a perfect fluid, it is possible to fully reexpress the theory as a pure General Relativistic model with a modification to the equation of state of the fluid. As a consequence, the fluid, keeps all its standard properties, except from a gravitational point of view where the geometry sees it differently. This results in an 'apparent' matter content.

It should be mentioned that the resulting apparent source displays interesting features towards energy conditions (see detailed discussion in⁶).

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