Three-party simultaneous quantum secure direct communication scheme with EPR pairs

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We present a scheme for three-party simultaneous quantum secure direct communication by using EPR pairs. In the scheme, three legitimate parties can simultaneously exchange their secret messages. It is also proved to be secure against the intercept-and-resend attack, the disturbance attack and the entangled-and-measure attack.

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I. INTRODUCTION

Quantum key distribution (QKD) has been one of the most promising applications of quantum mechanics, which provides unconditionally secure key exchange. The basic principle is that two remote legitimate users (Alice and Bob) establish a shared secret key through the transmission of quantum signals and use this key to encrypt or decrypt the secret messages. Since Bennett and Brassard [1] proposed the standard QKD protocol in 1984 (BB84), a good many QKD protocols have been advanced [2, 3, 4, 5, 6].

In recent years, a new concept, quantum secure direct communication (QSDC) has been proposed [7, 8, 9, 10, 11, 12, 13]. Different from quantum key distribution whose object is to establish a common random key between two parties, quantum secure direct communication is to transmit the secret messages directly without first establishing a random key to encrypt them. Boström and Felbinger’s protocol [8], which was called the ping-pong protocol, allows decoding the encoded bits instantaneously and directly in each respective transmission run. However, it was proved to be insecure against the disturbance attack [14]. Deng et al [9] have proposed a two-step quantum direct communication protocol by using EPR pairs. In this two-step scheme, the EPR pairs are divided into two sequences, which are sent by two steps, and the receiver needs to check the security of the channel twice (one for checking sequence and another for message sequence). In these schemes, QSDC is only one way communication. Based on the idea of ping-pong QSDC scheme, Nguyen [10] proposed a quantum dialogue scheme by using EPR pairs that enables both legitimate parties to exchange their secret messages in a direct way. However, it is not secure against the intercept-and-resend attack. An eavesdropper who adopts this attack strategy can steal the secret messages without being detected. More recently, Jin et al [12] proposed a three-party simultaneous QSDC scheme by using the GHZ states.

In this paper, we will introduce a three-party (Alice, Bob and Charlie) simultaneous QSDC scheme by using EPR pairs. In our scheme, each can obtain two other’s messages simultaneously. Moreover, it will be shown that this protocol is provably secure.

II. THE THREE-PARTY SIMULTANEOUS QSDC SCHEME WITH EPR PAIRS

Suppose that Alice, Bob and Charlie have a secret message respectively, without loss of generality we assume that the message length of three parties is the same.

\[ \text{Alice’s message} = \{i_1, i_2, i_3, \cdots, i_N\}, \]  
\[ \text{Bob’s message} = \{j_1, j_2, j_3, \cdots, j_N\}, \]  
\[ \text{Charlie’s message} = \{k_1, k_2, k_3, \cdots, k_N\}, \]

with \(i_n, j_n, k_n \in \{0, 1\}\).

Alice, Bob and Charlie agree on that Bob performs the two unitary operations

\[ C_{j_n} = \begin{cases} I, & \text{if } j_n = 0, \\ \sigma_x, & \text{if } j_n = 1. \end{cases} \]  

(4)

Charlie operates the two unitary operations

\[ C_{k_n} = \begin{cases} I, & \text{if } k_n = 0, \\ \sigma_z, & \text{if } k_n = 1. \end{cases} \]  

(5)

where

\( I = |0\rangle\langle 0| + |1\rangle\langle 1|, \sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|, \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|. \)  

(6)

To secretly exchange their messages, Alice first produces a large enough number of entangled pairs all in the state

\[ |\Psi_{ht}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle)_{ht}. \]  

(7)

Here \(h\) and \(t\) denote home particle remaining in the place of Alice, and transmitting particle being transmitted in the communication, respectively. Then Alice, Bob and Charlie proceed as follows:

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(S0) Protocol is initialized $n = 0$.

(S1) Set $n = n + 1$. Alice keeps qubit $h_n$ with her and sends qubit $t_n$ to Bob through the quantum channel.

(S2) When Bob received a qubit, he has two choices: one is to measure it, the other is to encode it.

2.1 If Bob decides to measure the qubit, that means to check the eavesdropping, he can complete it by the following procedure:

(a) Bob chooses randomly one of the two sets of measuring basis (MB), say $\{|0\rangle, |1\rangle\}$ and $\{|+, |−\rangle\}$, to measure the qubit $t_n$.

(b) Bob tells Alice the MB chosen by him and the outcome of his measurement.

(c) Alice chooses the same MB to measure the qubit $h_n$ and checks with the result of Bob. If no eavesdropper exists their results should be correlated, i.e., if Alice gets $|0\rangle$ ($|1\rangle$), Bob will get $|1\rangle$ ($|0\rangle$), when they choose their measurements along the Z-direction; or if Alice gets $|+\rangle$, $|−\rangle$, Bob will obtain $|+\rangle$ ($|−\rangle$) when they choose the MB $\{|+\rangle, |−\rangle\}$. If their results are correlated, set $n = n - 1$, go to step 1 and continue communication. Otherwise, they abort communication.

2.2 If Bob wants to encode his messages, he selects a running mode from message mode (MM) and control mode (CM).

1) If he selects MM, he encodes $j_n$ by performing the transformation $C_{j_n}$ on the qubit $t_n$.

2) If he selects CM, he does nothing.

Then Bob sends the qubit $t_n$ to Charlie.

(S3) Charlie confirms Bob that he received a qubit. Then Bob announces the running mode.

3.1 If it was run in CM, obviously the qubits $t_n$ and $h_n$ are in the state $|\Psi_{00}\rangle_{ht}$, Charlie and Alice check eavesdropper as procedure 2.1.

3.2 If it was run in MM, Charlie selects a running mode from his two modes: MM and CM.

1) If he selects MM, Charlie encodes $k_n$ by performing the transformation $C'_{k_n}$ on the qubit $t_n$, and then he sends it to Alice.

2) If he selects CM, Charlie randomly prepares a decoy qubit in one of states $\{|0\rangle, |1\rangle, |+\rangle, |−\rangle\}$ and sends it to Alice.

(S4) Alice confirms Charlie that she received a qubit. Then Charlie announces the running mode.

4.1 If the running mode was CM, Charlie publicly reveals the qubit state. Alice measures the decoy qubit using the corresponding basis. If the result is in accordance with Charlie’s result, set $n = n - 1$ and go to step 1, otherwise abort the communication.

4.2 If the running mode was MM, Alice measures $t_n$ and $h_n$ in the Bell basis. Suppose the result is $|\Psi_{r,n}\rangle_{ht}$, Alice encodes her message into $r_n, s_n$ by

$$r_n \oplus i_n = x_n, \quad s_n \oplus i_n = y_n.$$  \hspace{1cm} (8)

Then Alice announces publicly the values of $(x_n, y_n)$.

If $n < N$, go to step 1.

(S5) The three-party simultaneous QSDC has been successfully completed.

Now, we explicitly analyze the protocol described above. After Bob encodes his bit $j_n$ on the EPR-pair state $|\Psi_{00}\rangle_{ht}$, the pair state becomes

$$|\Psi_{00}\rangle_{ht} \rightarrow C_{j_n} |\Psi_{00}\rangle_{ht} = |\Psi_{j_n0}\rangle_{ht}.$$  \hspace{1cm} (9)

Furthermore, after Charlie encodes his message, the state is transformed as

$$|\Psi_{j_n0}\rangle_{ht} \rightarrow C'_{k_n} |\Psi_{j_n0}\rangle_{ht} = |\Psi_{j_nk_n}\rangle_{ht},$$  \hspace{1cm} (10)

where

$$|\Psi_{10}\rangle_{ht} = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle),$$  \hspace{1cm} (11)

$$|\Psi_{01}\rangle_{ht} = \frac{1}{\sqrt{2}}(|1\rangle|0\rangle - |0\rangle|1\rangle),$$  \hspace{1cm} (12)

$$|\Psi_{11}\rangle_{ht} = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle - |1\rangle|1\rangle).$$  \hspace{1cm} (13)

Therefore if no eavesdropper exists, Alice’s measurement result will be

$$r_n = j_n, \quad s_n = k_n.$$  \hspace{1cm} (14)

Clearly, if Alice publicly announces the values of $r_n, s_n$, the messages of Bob and Charlie will be revealed. For realizing the secure QSDC, Alice plays a trick: she encodes her message into $r_n$ and $s_n$, which was described in Eq. (8).

By Eqs. (8) and (14), we conclude easily that

$$j_n \oplus k_n = x_n \oplus y_n.$$  \hspace{1cm} (15)

Obviously, Alice, Bob and Charlie can obtain messages of other parties by Eqs. (8), (14) and (15). Decoding rules can be expressed as

$$\text{Alice: } j_n = x_n \oplus i_n, \quad k_n = y_n \oplus i_n,$$  \hspace{1cm} (16)

$$\text{Bob: } i'_n = x_n \oplus j_n, \quad k_n = x_n \oplus y_n \oplus j_n,$$  \hspace{1cm} (17)

$$\text{Charlie: } i_n = y_n \oplus k_n, \quad j_n = x_n \oplus y_n \oplus k_n.$$  \hspace{1cm} (18)

From the above decoding rules we may find that the messages $i_n, j_n$ and $k_n$ play two-fold role in the QSDC: on the one hand, they represent the messages of three parties; on the other hand, they play a role of the keys of three parties. However, it is worth pointing out that these keys are different from a private key which is prior shared by the parties of communication, but three parties encode these keys to the EPR pairs, so these keys hide in the entanglement.

III. SECURITY OF QSDC

In the following, we briefly discuss the security of the scheme.

Firstly, we consider the intercept-and-resend attack and disturbance attack. In both two attacks, Eve would destroy the entanglement of particles $h_n$ and $t_n$ in $A \rightarrow B$. 

line or \( B \rightarrow C \) line. In \( C \rightarrow A \) line, Eve’s attack will cause discord between Charlie’s decoy qubit state and Alice’s measurement outcome. Each of the above cases will make Eve be detected with a detection probability of 1/2. Thus our scheme is secure against the intercept-and-resend attack and disturbance attack.

Eve’s another attack is entangle-and-measure attack \([9]\). It acts in the following way. Eve prepares an ancilla in the initial state \( |\chi_0\rangle_e \) and waits in the route. When the qubit \( t_n \) passes by, Eve entangles his ancilla with it by performing a unitary operation \( \hat{E} \) defined as

\[
\hat{E}|0\rangle_t|\chi_e\rangle = \alpha|0\rangle_t|\chi_0\rangle_e + \beta|1\rangle_t|\chi_1\rangle_e, \tag{19}
\]
\[
\hat{E}|1\rangle_t|\chi_e\rangle = \alpha|1\rangle_t|\chi_0\rangle_e + \beta|0\rangle_t|\chi_1\rangle_e, \tag{20}
\]

with \( \alpha, \beta \) satisfying the normalization condition \( |\alpha|^2 + |\beta|^2 = 1 \) and \( \{|\chi_0\rangle_e, |\chi_1\rangle_e\} \) being the pure orthonormalized ancilla’s states uniquely determined by the unitary operation \( \hat{E} \). Obviously, Eve’s attack in \( A \rightarrow B \) or \( B \rightarrow C \) destroys the correlation of EPR pairs. The state of EPR pair and the ancilla becomes

\[
\hat{E}|\Psi_{00}\rangle_{ht}|\chi_e\rangle = \alpha|\Psi_{00}\rangle_{ht}|\chi_0\rangle_e + \beta|\Psi_{10}\rangle_{ht}|\chi_1\rangle_e, \tag{21}
\]

Eve conceals himself if his measurement ends up with \( |\chi_0\rangle_e \). However if the measurement outcome is \( |\chi_1\rangle_e \), he will be detected. If Eve’s attack is in the \( C \rightarrow A \) line, besides the above Eqs. (19) and (20), the attack on the decoy qubit in \( |+\rangle (|\rangle) \) is expressed as

\[
\hat{E}|+\rangle_t|\chi_e\rangle = |+\rangle_t(\alpha|\chi_0\rangle_e + \beta|\chi_1\rangle_e), \tag{22}
\]
\[
\hat{E}|\rangle_t|\chi_e\rangle = |\rangle_t(\alpha|\chi_0\rangle_e - \beta|\chi_1\rangle_e), \tag{23}
\]

In this case, the eavesdropper can not be detected. However the eavesdropper can be found by Charlie and Alice when the decoy qubit is in the state \( |0\rangle \) or \( |1\rangle \). So our scheme is also secure against this attack.

### IV. CONCLUSION

In summary, we have proposed a protocol for three-party quantum secure direct communication by using EPR states. Our scheme has its own advantages compared with the schemes proposed previously. Firstly, three legitimate parties can simultaneously exchange their secret messages. Secondly, compared with the preparation of GHZ states, obviously the EPR state will be more easily prepared in the experiment.

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