Fast polarimetry of the colliding proton beams based on the elastic $pp$ analyzer using the NICA detectors

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Abstract. It is shown that the existing data on analyzing power $A_n$ of the elastic $pp$ scattering could be successfully applied for polarimetry of the colliding proton beams using the NICA detectors. Performed calculations of the count rates of the elastic events have revealed that the polarimeter based on using $A_n$ for elastic $pp$ will have a high polarization measurement velocity.

1. Introduction.
The study of spin effects on modern colliders [1] is a complex methodical work. For the full reliability of the experimental results it is required to know the polarization values of colliding beams in their intersection points. The report analyzes opportunities of using the existing $A_n$ set [2] of elastic $pp$ data for the polarimetry of the NICA collider [1] polarized proton beams.

An example of using the existing $A_n(pp)$ and $d\sigma/d\Omega_{CM}(pp)$ data sets [2] for polarimetry of the proton beams at energies of the NICA collider is presented. It is shown that polarimetry with the elastic $pp$ process as polarization analyzer is possible only near the lower boundary of the NICA energy range at $T_{p,CM} \approx 2$ GeV. The event count rates have been estimated at the statistics accumulation during the beam polarization $p_Z$ measurements for a number of energy values of the colliding proton beams. It is shown that this polarimeter will provide fast polarization measurement approximately $\approx 20$ minutes.

2. The coordinate systems used for colliding beam polarimetry.
For the polarimetry, we have applied a convenient and often used projectile helicity frame (PHF) - $P(x_P, y_P, z_P)$, see Figure 1, in which we consider the polarizations $p_i$ and analyzing powers $A_i$, $A_{ij}$. The coordinate unit vectors of this frame are defined as:

$$z_P = \frac{k_{in}}{|k_{in}|}, \quad y_P = n = \frac{k_{in} \times k_{out}}{|k_{in} \times k_{out}|}, \quad x_P = n \times z_P = y_P \times z_P. \quad (2.1)$$

Under this choice of the coordinate system polarization observables $A_i$, $A_{ij}$ satisfy the following relations for the parity conserving interaction:

$$A_x = A_z = A_{xy} = A_{yz} = 0. \quad (2.2)$$

For the polarized beams produced by an ion source, a second coordinate system of interest for the projectile is that with its $Z$ axis along the axis of symmetry $S$ of the beam polarization.
In the PHF, the spin quantization axis, $\mathbf{S}$, of the colliding polarized beam is described by angles $\beta$, $\phi$, where $\beta$ is the angle between $\mathbf{S}$ and $\mathbf{k}_{in}$ and $\phi$ - the angle between the projection of $\mathbf{S}$ on $x_P$, $y_P$ plane and the $y_P$ -axis:

$$
\cos \beta = \mathbf{S} \cdot \mathbf{k}_{in}, \quad \cos \phi = (\mathbf{S} \times \mathbf{k}_{in}) \cdot (\mathbf{y}_P \times \mathbf{k}_{in}).
$$

The case of the $\mathbf{S}||\mathbf{y}_P$ is actually used, when the vector $\mathbf{S} \perp \mathbf{k}_{in}$, and the angle $\beta = 90^\circ$, and $\phi = \varphi$. But the angle $\phi$ is measured from the axis $\mathbf{y}_P$, and not from the $x_P$ axis to save a long-standing convention $\phi = 0$ for scattering to the left of the beam with spin up.

3. Spin formalism for elastic $pp$ scattering.

In the PHF, taking into account the relations (2.2), the general expression for the cross sections of elastic collision of two polarized protons is the following: (see ref. [3])

$$
I = I_o[1 + p_y^B A_y^B + p_y^B A_y^R + p_z^B p_y^R C_{xx} + p_z^B p_y^R C_{yy} + p_y^B p_z^R C_{yy} + p_z^B p_z^R C_{zz} + p_y^B p_z^R + p_z^B p_z^R C_{zz}],
$$

(3.1)

where the indices $B$ and $R$ refer to the parameters of blue and red colliding beams, respectively. This expression includes the polarization $p_y^{B,R}$ and the analyzing power $A_y^{B,R}$ of each proton, as well as the spin correlation coefficients $C_{ij}$.

To separate polarimetry of the $B$ or $R$ colliding beams, in the expression (3.1) we need to isolate a member $p_y^B A_y^B$ or $p_y^R A_y^R$ by means of all usual methods to resolve differences in the efficiencies of the detectors. The individual term in equation (3.1) can be distinguished by selecting combinations of the polarized - unpolarized beam, flip the orientation of the spins of the colliding particles, by special arrangement of detectors, etc. For example, to find the $p_y^B$ polarization of the blue beam, it is necessary to collide it with the unpolarized red beam $p_y^R = 0$ and vice versa. Then the expression (3.1) is reduced to the following:

$$
I = I_o[1 + p_y^B A_y^B]
$$

(3.2)

and allows one to determine $p_y^B$ at known $A_y^B$ and vice versa. In the PHF polarization $p_y^B$ of the colliding beam is associated with $p_z$ by means of the angles $\beta$ and $\phi$ as:

$$
p_y = p_z \sin \beta \cos \phi = p_z \cos \phi,
$$

(3.3)

where $p_z$ is the beam polarization in the ion source system.
4. How to measure the colliding proton beams polarization.

To measure the value and sign of the beam polarization \( p_Z \), it is necessary to use two detectors of the scattered proton - left (Lf) and right (Rg), placing them in diametrically opposite areas near \( \phi = 0 \), where \( p^L_{pp} = p_Z |\cos \phi| \) and near \( \phi = 180^\circ \) where \( p^R_{pp} = -p_Z |\cos \phi| \). The current value of the proton beam polarization \( p_Z \) from the polarized ion source is given by the following expression:

\[
p_Z = \left[ \frac{N^\text{pol}_{Lf}/N^0_{Lf} - N^\text{pol}_{Rg}/N^0_{Rg}}{2 |\cos \phi|} \right] A_n(T_p, \theta),
\]

where \( N^\text{pol}_{Lf} \) and \( N^0_{Lf} \) are the output of the elastic pp process with scattering to the left and \( N^\text{pol}_{Rg} \) and \( N^0_{Rg} \) are the output with scattering to the right polarized (pol) and unpolarized (o) beams, respectively. In these measurements it is necessary to use an unpolarized "target" beam.

In this formula, the current value of the angle \( \phi \) in PHF is calculated by using equation (2.3) and known coordinates of the vectors \( \mathbf{k}_n, \mathbf{y}_P \) and \( \mathbf{S} \) in the system of detectors \( D_c(x,y,z) \) or \( D_s(r,\theta,\varphi) \). The values of the analysing power \( A_n(T_p, \theta) \) of the elastic pp scattering at angle \( \theta \) with the energy of the colliding protons \( T_p \) are taken from the \( A_n(T_p, \theta) \) data set in the comp. [2].

5. Examples of existing \( A_n(T_p, \theta) \) data in the \( CM \) \( (D_{c,s}) \) system.

Figure 2 shows the experimental data of \( A_n(T_p, \theta) \) depending on the angle \( \theta_{p,CM} \).

![](image)

Table 1. The parameters for several sets of experimental data [2] on angular dependencies \( A_n(\theta_{CM}) \) for several values of the colliding proton energies \( T_{p,CM} \).

| \( T_{p,CM} \) (GeV) | \( T_{p,FT} \) (GeV) | \( \theta^o \) | \( -t_\ast \) (GeV/c) | \( A_n \) | \( dA_n \) |
|----------------------|----------------------|------------|-------------------|--------|--------|
| 1.106                | 7.027                | 37.89      | 43.87             | 1.39   | 1.84   | 0.154   | 0.008  |
| 1.505                | 10.849               | 29.05      | 36.49             | 1.28   | 2.00   | 0.146   | 0.006  |
| 1.712                | 13.093               | 23.90      | 30.50             | 1.05   | 1.70   | 0.129   | 0.020  |
| 2.005                | 16.587               | 22.65      | 28.14             | 1.20   | 1.84   | 0.075   | 0.031  |
| 2.483                | 23.080               | 18.34      | 24.18             | 1.10   | 1.90   | 0.042   | 0.010  |

It is seen that at energies of \( T_{p,CM} > 2.0 \) GeV, the \( A_n \) angular dependences are small and do not have the explicit extremum, which can be used for the polarimetry. We have also seen that in the range \( 20^\circ \leq \theta \leq 40^\circ \) of the working angle of the collider detectors the average \( A_n \) has a significant value \( A_n \approx 0.08 \) and acceptable precision \( dA_n \approx 0.03 \) only for the energies of the colliding protons \( T_{p,CM} \approx 2 \) GeV. Therefore polarimetry with the elastic pp process is possible only near the lower boundary of the NICA energy range.
6. Estimations of the elastic $pp$ output in the $CM$ ($D_{c,s}$) system.

According to the equations (4.1) the statistical accuracy of the polarization $p_Z$ measurements is determined by the value of accumulated statistics with polarized and unpolarized beams for scattering to the left and right: $N_{Lf}$, $N_{Lp}$, $N_{Rg}$, $N_{Rp}$. To estimate the count rate ($R = L \cdot \sigma_{int}$) of the elastic proton scattering in a given angular intervals $\Delta \theta$, $\Delta \varphi$ at a given energy $T_{p,Col}$, one needs to determine the aperture cross section:

$$\Delta \sigma(T_p, \Delta \theta, \Delta \varphi) = \int_{\Delta \theta} \int_{\Delta \varphi} d\sigma/d\Omega(T_p, \theta) \sin(\theta) d\theta d\varphi. \quad (6.1)$$

The angular $\theta_{p,CM}$ dependence of the $d\sigma/d\Omega_{CM}$ at given $T_{p,CM}$ energy can be determined from the existing data sets (see [2]) for the $\theta_{p,CM}$ dependencies of differential cross sections $d\sigma/d\Omega_{CM}(T_{p,CM}, \theta_{p,CM})$ for elastic $pp$ scattering. The experimental $d\sigma/d\Omega_{CM}$ data for a number of $T_{p,CM}$ energies have been approximated by the exponential type dependencies $d\sigma/d\Omega_{CM} = C \cdot \exp(-b \cdot \theta)$ with the parameter $C$ [cm$^2$] and a slope of $b$. With this approximation the aperture cross section $\Delta \sigma$ is defined as follows:

$$\Delta \sigma = \Delta \varphi \cdot C \int_{\theta_L}^{\theta_H} \exp(-b \cdot \theta) \sin \theta d\theta, \quad (6.2)$$

where $\Delta \varphi$ - is detector aperture in $\varphi$, and $\theta_L$ and $\theta_H$ are the border of integration.

![Figure 3](image-url)

Figure 3. Fitting results of the experimental $d\sigma/d\Omega_{CM}$ data [2]. For the fit, we have chosed the $d\sigma/d\Omega_{CM}$ sets with the values of the $T_{p,CM}$ energies close to the $T_{p,CM}$ values for which $A_n$ data are available. The region of the $\theta_{p,CM}$ angles for the $A_n$ for fitting has been selected to include the $\Delta \theta_{p,CM}$ region in which we are going to use the $A_n(\theta_{p,CM})$ values for the polarimetry purpose.

Figure 3 shows the $d\sigma/d\Omega_{CM}(T_{p,CM}, \theta_{p,CM})$ fitting results for $T_{p,CM} = 1.730$ and $2.080$ GeV.

**Table 2.** The results of $d\sigma/d\Omega_{CM}(T_{p,CM}, \theta_{p,CM})$ data fitting and calculation results of the aperture cross sections $\Delta \sigma$ and event count rate $R$.

| $T_{p,Col}$, GeV | $\theta^o_{p,CM}$ | $\theta_L$ | $\theta_H$ | $C$, mb/sr | $b$, | $\Delta \sigma$, mb | $R$, s$^{-1}$ | $A_n$, 10% | $A_n$, 5% |
|-----------------|-----------------|-----------|-----------|-------------|-----|----------------|-------------|-------------|-------------|
| 0.999           | 37.9            | 34.9      | 127.3     | 0.195       | 0.280| 280             | 0.154       | 0.4         | 1.6         |
| 1.128           | 37.9            | 34.9      | 583.2     | 0.254       | 0.119| 119             | 0.154       | 1.0         | 3.7         |
| 1.528           | 29.1            | 30.5      | 0.47      | 0.113       | 0.074| 74              | 0.146       | 1.8         | 6.5         |
| 1.730           | 32.9            | 30.5      | 3.72      | 0.206       | 0.068| 68              | 0.129       | 2.5         | 9.0         |
| 2.080           | 22.2            | 28.1      | 601.3     | 0.466       | 0.020| 20              | 0.075       | 24.0        | 92.0        |
| 2.483           | 18.3            | 24.2      | 6.047     | 0.295       | 0.041| 41              | 0.042       | 36.0        | 144.0       |

Table 2 presents the results of $d\sigma/d\Omega_{CM}(T_{p,CM}, \theta_{p,CM})$ data fitting - the parameters $C$ and $b$, the values of aperture cross sections $\Delta \sigma$, event count rate $R$ and necessary time $\Delta t$ for statistics.
accumulation to get the measured $p_Z$ result with the statistical precision of $\delta = 10\%$ and $\delta = 5\%$.

In the $\Delta \sigma$ and $R$ calculations it is supposed that the azimuthal $\Delta \varphi$ - capture of the polarimeter detectors in the right and left shoulders is equal to 90°. It has been also assumed that the luminosity $L$ of the colliding proton beams is equal to $L = 1 \times 10^{30}$ cm$^{-2}$ sec$^{-1}$.

The results of the $R$ calculations, presented in table 2, show a high count rate of elastic $pp$ events -tens events per second at the energies of the colliding protons $T_{p,CM} \approx 2$ GeV, that provides a short period of time of statistics accumulation $\sim 20$ minutes for obtaining $p_Z$ results with statistical precision of 10%.

The opportunities of registering and selecting of elastic $pp$ events by the collider detectors was considered by us earlier [4]. In this case, we have used the information about the expected parameters of the NICA colliding $pp$ beams and detectors surrounding the point of their intersection from [5,6].

7. Conclusion
Existing data sets [2] on the analysing power $A_n$ of elastic $pp$ scattering have been analyzed. It has been shown that in the working range of the proton scattering angle $20^\circ \leq \theta \leq 40^\circ$ the average $A_n$ values are significant $A_n \approx 0.08$ and have acceptable accuracy of $dA \sim 0.03$ only for the energies of the colliding protons: $T_{p,CM} \approx 2$ GeV. Therefore polarimetry with the elastic $pp$ process is possible only near the lower boundary of the energy range of the NICA collider.

The calculations of the count rates of the elastic events have been performed and it is shown that the polarimeter using $A_n$ for elastic $pp$ as polarization analyzer will have a high measurement velocity. A single polarization measurement will require $\sim 20$ minutes.

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