Quantum Cosmology and Grand Unification

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Abstract

Quantum cosmology may restrict the class of gauge models which unify electroweak and strong interactions. In particular, if one studies the normalizability criterion for the one-loop wave function of the universe in a de Sitter background one finds that the interaction of inflaton and matter fields, jointly with the request of normalizability at one-loop order, picks out non-supersymmetric versions of unified gauge models.

The investigations in modern cosmology have been devoted to two main issues. On one hand, there were the attempts to build a quantum theory of the universe with a corresponding definition and interpretation of its wave function [1,2]. On the other hand, the drawbacks of the cosmological standard model motivated the introduction of inflationary scenarios. These rely on the existence of one or more scalar fields, and a natural framework for the consideration of such fields is provided by the current unified models of fundamental interactions (see, for example, Ref. [3] and references therein). The unification program started with the proposal and the consequent experimental verification of the electroweak standard model $(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y)$, and has been extended to other simple gauge groups, like $SU(5)$, $SO(10)$ and $E_6$. All of them in fact, even if with different capability, unlike the electroweak standard model are able to allocate all matter fields in a few irreducible representations (IRR) of the gauge group, and require a small number of free parameters. However, since these enlarged gauge models predict new physics, a first source of constraints upon them is certainly provided by the experimental bounds on processes like proton decay, neutrino oscillations, etc. [4]. Further restrictions can be obtained from their cosmological applications, as discussed in Ref. [5].
One can say, however, that the majority of investigations, studying the mutual relations between particle physics and cosmology, leave quantum cosmology itself a bit aside, using it only as a tool to provide initial conditions for inflation. Meanwhile, one can get some important restrictions on particle physics models, using the general principles of quantum theory. In particular, we study the possible restrictions on unified gauge models resulting from a one-loop analysis of the wave function of the universe and from the request of its normalizability [6–10]. It is known that the Hartle-Hawking wave function of the universe [1], as well as the tunnelling one [2], are not normalizable at tree level [11]. In Ref. [6] it was shown that, by taking into account the one-loop correction to the wave function, jointly with a perturbative analysis of cosmological perturbations at the classical level, one can obtain a normalizable wave function of the universe provided that a restriction on the particle content of the model is fulfilled. Such a restriction is derived from the formula for the probability distribution for values of the inflaton field [6]

$$\rho_{HH,T}(\varphi) \approx \frac{1}{H^2(\varphi)} e^{\mp I(\varphi) - \Gamma_{1-loop}(\varphi)} ,$$

where the subscripts HH and T denote the Hartle-Hawking and tunnelling wave function, respectively, $H(\varphi)$ is the effective Hubble parameter, $\Gamma_{1-loop}$ is the one-loop effective action on the compact de Sitter instanton. One can show from (1), that the normalizability condition of the probability distribution at large values of the inflaton scalar field $\varphi$ is reduced to the condition

$$Z > -1 ,$$

where $Z$ is the total anomalous scaling of the theory. This parameter is determined by the total Schwinger-DeWitt coefficient $A_2$ in the heat-kernel asymptotics [12], and depends on the particle content.

In Ref. [7] the criterion (2) was used to investigate the permissible content of different models. It was noticed that the standard model of particle physics, as well as the minimal $SU(5)$ GUT model, does not satisfy the criterion of normalizability, while the standard supersymmetric model, the $SU(5)$ SUSY model and $SU(5)$ supergravity model do satisfy this criterion.

All the analysis in Ref. [7] was carried out in terms of physical degrees of freedom, e.g. three-dimensional transverse photons or three-dimensional transverse-traceless metric perturbations. However, over the last few years, the explicit calculations have shown that a covariant path integral for gauge fields and gravitation yields an anomalous scaling which differs from the one obtained from reduction to physical degrees of freedom. This holds both for closed manifolds and for manifolds with boundary [12–14].

Unfortunately, the reduction to physical degrees of freedom is not well defined on any curved Riemannian manifold [12]. Moreover, such a reduction does not take explicitly into account gauge and ghost terms in the path integral, and leads to a heat-kernel asymptotics which disagrees with the
well-known results of invariance theory. For all these reasons, we regard
the covariant version of the path integral as more appropriate for one-loop
calculations.

In Ref. [8] the investigation of the one-loop wave function was carried
out for a non-minimally coupled inflaton field with large negative constant
$\xi$. It was then shown that the behaviour of the total anomalous scaling $Z$
is
determined by interactions between the inflaton and remaining matter fields.

Here, we study normalizability properties of a wide set of unified gauge
models, with or without interaction with the inflaton field. The models stud-
ied are, as shown in Table I, the standard model of particle physics, $SU(5)$,
$SO(10)$ model in the 210-dimensional irreducible representation, $E_6$, jointly
with supersymmetric versions of all these models with or without supergravity.
The building blocks of our one-loop analysis are the evaluations of $A_2$
coefficients for scalar, spinor, gauge, graviton and gravitino perturbations.
All these coefficients (but one) are by now well-known, and are given by

\[
A_{2 \text{ scalar}} = \frac{29}{90} - 4\xi + 12\xi^2
\]
\[-\frac{1}{3}m^2 R_0^2 + 2\xi m^2 R_0^2 + \frac{1}{12}m^4 R_0^4 , \quad (3)
\]
\[
A_{2 \text{ spin-1/2}} = \frac{11}{180} + \frac{1}{3}m^2 R_0^2 + \frac{1}{6}m^4 R_0^4 , \quad (4)
\]
\[
A_{2 \text{ gauge}} = -\frac{31}{45} + \frac{2}{3}m^2 R_0^2 + \frac{1}{3}m^4 R_0^4 , \quad (5)
\]
\[
A_{2 \text{ gravitino}} = -\frac{589}{180} . \quad (6)
\]

It should be stressed that Eq. (3) only holds for scalar fields different from the
inflaton. With our notation, $m$, $\xi$ and $R_0$ represent effective mass, (dimen-
sionless) coupling parameter, and 4-sphere radius, respectively. Equation (4)
holds for a spin-1/2 field with half the number of modes of a Dirac field. Since
the results (5) and (6) rely on the Schwinger-DeWitt technique, they incor-
porate, by construction, the effect of ghost zero-modes. However, it has been
argued in Ref. [15] that zero-modes should be excluded to obtain an infrared
finite effective action which is smooth as a function of the de Sitter radius on
spherically symmetric backgrounds. On the other hand, the prescription
which includes ghost zero-modes makes the one-loop results continuous.
Strictly, we are considering small perturbations of a de Sitter background
already at a classical level (see [6–10]). There are also deep mathematical
reasons for including zero-modes, and they result from the spectral theory of
elliptic operators. Thus, we use the expressions (5) and (6).

Last, the contribution of gravitons to the total $Z$ should be calculated
jointly with the inflaton contribution. What happens is that the second-
order differential operator given by the second variation of the action with
respect to inflaton and metric is non-diagonal even on-shell, by virtue of a non-vanishing vacuum average value of the inflaton [16,17]. The resulting $A_2$ coefficient turns out to be independent of the value of $\xi$ and equal to [10]

$$A_{graviton+inflaton}^{2} = -\frac{171}{10}.$$  

(7)

In Table I, we report the total $Z$ for some relevant examples of GUT theories, whenever one neglects the mass terms. This ansatz is correct, if the interaction between inflaton and the other particles is not considered. In this case in fact, the term $m^2 R_0^2 \sim \varphi^{-2}$ is very small by virtue of the large value of $\varphi$. The analysis starts with the electroweak standard model (SM), which contains, in its non-SUSY version, 45 Weyl spinors (we neglect for simplicity right-handed neutrinos and their antiparticles), 24 gauge bosons and one doublet of complex Higgs fields. The particle content changes for the SUSY version of this model in its minimal form (MSSM) [18]. In this case, in fact, to the 45 Weyl leptons and quarks one has to add 4 higgsinos and 12 gauginos, whereas the scalar sector consists now of 90 sleptons and squarks plus 8 real scalar fields. A similar analysis is performed for the $SU(5)$ GUT model [19], which in its non-SUSY version, apart from the 24 gauge bosons, needs scalars belonging to $24 \oplus 5 \oplus \overline{5}$ IRR’s to accomplish the spontaneous symmetry breaking pattern. The matter content of the SUSY extension of the model [20] is obtained by doubling the number of Higgs IRR’s used, and by adding superpartners to any degrees of freedom. As far as $SO(10)$ gauge theories are concerned, we have considered the particular model containing $210 \oplus (126 \oplus 126) \oplus 10 \oplus 10$ IRR’s of Higgs fields, which is still compatible with the present experimental limit on the proton lifetime and neutrino phenomenology [4]. Furthermore, we have also considered the SUSY extension of $SO(10)$, which, to be consistent also with cosmological constraints, needs complex Higgs fields belonging to $1 \oplus 10 \oplus 10' \oplus 45 \oplus 45' \oplus 54 \oplus 54' \oplus 126 \oplus 126$ IRR’s [21]. Last, we have also considered $E_6$ GUT theories, for which fermions are allocated in three $27$ fundamental IRR’s, and scalars belong to two $(78 \oplus 27 \oplus 351)$ [22]. For the SUSY extension of this model, we have just added the superpartner degrees of freedom. Concerning the SUGRA versions of all the above models, they have been obtained from the supersymmetric ones, just by adding the gravitino contribution (i.e. subtracting the $A_2$ coefficient in Eq. (6), because of the fermionic statistics). Indeed, we have considered particular versions of $SO(10)$ and $E_6$ gauge models, but we expect that the qualitative features of the results should remain unaffected.

In Table I, we have assumed that one of the Higgs fields plays the role of the inflaton. The forbidden range denotes the range of values of $\xi$ for which the normalizability criterion (2) is not satisfied. Interestingly, conformal coupling (i.e. $\xi = 1/6$) is ruled out by all 12 models listed in Table I. Moreover, for the standard and $SU(5)$ models, minimal coupling (i.e. $\xi = 0$) is also ruled out. At this stage, supersymmetric models are hence favoured,
as well as non-supersymmetric models with a large number of scalar fields.

In the formulation of physical models, however, one has to move gradually from the original, simplified case, towards a more involved problem which is physically more realistic. In our investigation, this means having to deal with the interactions between the inflaton and remaining fields, since such interactions are responsible for the reheating in the early universe. This is a stage as important as the inflationary phase. Indeed, as shown in Refs. [8,10], for a scalar field with mass $m_\chi$ and constant $\xi_\chi$ of non-minimal interaction (which differs from $\xi$ in Eq. (3)), one finds on a de Sitter background

$$\zeta_\chi(0) = \frac{29}{90} - 4\xi_\chi + 12\xi_\chi^2 - \frac{1}{3} m_\chi^2 \frac{1}{H^2} + \frac{1}{12} m_\chi^4 \frac{1}{H^4}, \quad (8)$$

where $m_\chi^2 = \frac{\lambda_\chi \varphi_0^2}{2}$. Moreover, for a spin-1 gauge field with mass $m_A$, and a massive Dirac field with mass $m_\psi$, one finds [8,10]

$$\zeta_A(0) = 48\xi^2 \frac{g_A^2}{\lambda^2} \left[ 1 + \frac{(1 + 2\delta)}{4\pi} \frac{m_p^2}{|\xi| \varphi_0^2} + O(1/|\xi|) \right], \quad (9)$$

$$\zeta_\psi(0) = -48\xi^2 \frac{f_\psi^2}{\lambda^2} \left[ 1 + \frac{(1 + 2\delta)}{4\pi} \frac{m_p^2}{|\xi| \varphi_0^2} + O(1/|\xi|) \right], \quad (10)$$

where the coupling constants $g_A$ and $f_\psi$ are related to the masses by the formulas $m_A^2 = g_A^2 \varphi_0^2$, $m_\psi^2 = f_\psi^2 \varphi_0^2$, and the parameter $\delta$ is defined by $\delta \equiv -\frac{8\pi|\xi|m_\chi^4}{\lambda \varphi_0^2}$, $\lambda$ being the parameter of self-interaction for the inflaton. Thus, if one considers supersymmetry, jointly with a Wess-Zumino scalar multiplet interacting with the inflaton, the terms of order $m^4 R_0^4$ in Eqs. (3) and (4) cancel each other exactly after combining contributions proportional to [10]

$$\sum_\chi \lambda_\chi^2 + 16 \sum_A g_A^4 - 16 \sum_\psi f_\psi^4.$$

By contrast, terms of order $m^2 R_0^2$ have opposite signs, since they are proportional to

$$-8 \sum_\chi \lambda_\chi + 32 \sum_A g_A^2 - 32 \sum_\psi f_\psi^2.$$

At this stage, one has to bear in mind that, by virtue of cosmological perturbations, one can prove that $m^2 R_0^2$ is of order $10^4$ [23]. The effect of all these properties is hence a negative value of $Z$ which cannot be greater than $-1$ (cf. Eq. (2)). In other words, inflaton interactions revert completely the conclusions that, otherwise, would be drawn from Tab. I. In particular, our
analysis proves that the “pseudo-supersymmetric” combination of coupling constants considered in Refs. [8–10] does not improve the situation with respect to the criterion in Eq. (2).

Our investigation shows that the one-loop normalizability criterion for the wave function of the universe picks out non-supersymmetric versions of unified gauge models [24]. Despite this negative result, the investigation of supersymmetric cosmological models remains an important task, at least from the point of view of the general formalism of modern field theories [25,26]. Moreover, the problem remains of proving that our conclusion is not affected by higher-order effects in the semiclassical evaluation of the wave function of the universe. As far as we know, these effects cannot be studied with the help of ζ-function methods, and represent a fascinating problem in the quantum theory of the early universe.

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References

[1] J. B. Hartle and S. W. Hawking, Phys. Rev. D 28 (1983) 2960.

[2] A. Vilenkin, Phys. Rev. D 37 (1988) 888.

[3] A. D. Linde, Particle Physics and Inflationary Cosmology (Harwood Academic, New York, 1990).

[4] R. N. Mohapatra, Unification and Supersymmetry, The Frontiers of Quark-Lepton Physics (Springer-Verlag, Berlin, 1992); F. Acampora, G. Amelino-Camelia, F. Buccella, O. Pisanti, L. Rosa and T. Tuzi, Nuovo Cimento A 108 (1995) 375.

[5] R. N. Mohapatra and P. B. Pal, Massive Neutrinos in Physics and Astrophysics (World Scientific, Singapore, 1991).

[6] A. O. Barvinsky and A. Yu. Kamenshchik, Class. Quantum Grav. 7 (1990) L181.

[7] A. Yu. Kamenshchik, Phys. Lett. B 316 (1993) 45.

[8] A. O. Barvinsky and A. Yu. Kamenshchik, Phys. Lett. B 332 (1994) 270.

[9] A. O. Barvinsky and A. Yu. Kamenshchik, Phys. Rev. D 50 (1994) 5093.
[10] A. O. Barvinsky, A. Yu. Kamenshchik and I. V. Mishakov, Nucl. Phys. B 491 (1997) 387.

[11] S. W. Hawking and D. N. Page, Nucl. Phys. B 264 (1986) 185.

[12] G. Esposito, A. Yu. Kamenshchik and G. Pollifrone, *Euclidean Quantum Gravity on Manifolds with Boundary* (Fundamental Theories of Physics 85) (Kluwer, Dordrecht, 1997).

[13] P. A. Griffin and D. A. Kosower, Phys. Lett. B 233 (1989) 295.

[14] D. V. Vassilevich, Phys. Rev. D 52 (1995) 999.

[15] T. R. Taylor and G. Veneziano, Nucl. Phys. B 345 (1990) 210.

[16] Yu. V. Gryzov, A. Yu. Kamenshchik and I. P. Karmazin, Izv. VUZov, Fiz. (Russia) 35 (1992) 121.

[17] A. O. Barvinsky, A. Yu. Kamenshchik and I. P. Karmazin, Phys. Rev. D 48 (1993) 3677.

[18] P. Fayet, Phys. Lett. B 64 (1976) 159.

[19] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438.

[20] S. Dimopoulos and H. Georgi, Nucl. Phys. B 193 (1981) 150.

[21] R. Jeannerot, Phys. Rev. D 53 (1996) 5426.

[22] J. L. Hewett and T. G. Rizzo, Phys. Rep. 183 (1989) 193.

[23] D. S. Salopek, J. R. Bond and J. M. Bardeen, Phys. Rev. D 40 (1989) 1753.

[24] G. Esposito, A. Yu. Kamenshchik and G. Miele, “Unified Gauge Models and One-Loop Quantum Cosmology”, to appear in Phys. Rev. D (hep-th/9609178).

[25] P. D. D’Eath, *Supersymmetric Quantum Cosmology* (Cambridge University Press, Cambridge, 1996).

[26] P. R. L. V. Moniz, Int. J. Mod. Phys. A 11 (1996) 4321.
| Gauge group            | version | $Z$                                      | forbidden $\xi$ range |
|-----------------------|---------|------------------------------------------|-----------------------|
| $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ | non-SUSY | $36\xi^2 - 12\xi - \frac{543}{20}$     | $-.701 \leq \xi \leq .1035$ |
|                        | SUSY    | $1164\xi^2 - 388\xi + \frac{389}{180}$ | $.008 \leq \xi \leq .325$ |
|                        | SUGRA   | $1164\xi^2 - 388\xi + \frac{163}{5}$   | $.017 \leq \xi \leq .316$ |
| $SU(5)$                | non-SUSY | $396\xi^2 - 132\xi - \frac{103}{4}$    | $-.134 \leq \xi \leq .467$ |
|                        | SUSY    | $1884\xi^2 - 628\xi + \frac{1919}{180}$| $.020 \leq \xi \leq .314$ |
|                        | SUGRA   | $1884\xi^2 - 628\xi + \frac{209}{15}$  | $.026 \leq \xi \leq .308$ |
| $SO(10)$               | non-SUSY | $5772\xi^2 - 1924\xi + \frac{4678}{45}$| $.069 \leq \xi \leq .265$ |
|                        | SUSY    | $12444\xi^2 - 4148\xi + \frac{11321}{45}$| $.080 \leq \xi \leq .253$ |
|                        | SUGRA   | $12444\xi^2 - 4148\xi + \frac{5097}{20}$| $.082 \leq \xi \leq .252$ |
| $E_6$                  | non-SUSY | $10932\xi^2 - 3644\xi + \frac{39197}{180}$| $.078 \leq \xi \leq .255$ |
|                        | SUSY    | $12876\xi^2 - 4292\xi + \frac{42719}{180}$| $.070 \leq \xi \leq .263$ |
|                        | SUGRA   | $12876\xi^2 - 4292\xi + \frac{1203}{5}$ | $.072 \leq \xi \leq .262$ |