Abstract

The description of very high energy proton-proton cross sections in terms of a ‘black disc’ with an ‘edge’ allows of a simple generalization to highest energy proton-nucleus cross sections. This results in a leading $\ln^2 W$ term and a $\ln W$ term whose coefficient depends linearly on the radius of the nucleus ($W$ the c.m. energy). The necessary parameters are determined from the fits to p-p data. Since the coefficient of the $\ln W$ term is rather large, it is doubtful that the regime of $\ln^2 W$ dominance can be reached with available energies in accelerators or cosmic rays. However, the $\ln W$ term can be relevant for highest energy cosmic rays in the atmosphere, where a large increase for the cross section on nitrogen is expected. Tests of the theory should be possible by studying the coefficient of $\ln W$ at p-nucleus colliders.

1 Introduction

In recent years a simple picture for very high energy p-p cross sections has been developed, in good agreement with experimental information. A short review is presented in Ref. [1]. It consists of a ‘black disc’ [2] with a smooth ‘edge’ [3]. The radius of the ‘disc’ is growing as $\ln W$ with p-p center-of-mass energy $W$, while the ‘edge’ is constant with energy, with a thickness $t \approx 1.1 f$. The growing ‘disc’ contribution has however a small coefficient, with only a contribution $\sigma_{\text{disc}}^{\text{TOT}} \approx 1.1 \text{mb} \times \ln^2(W/m)$ to the total cross section [2]. Thus, although it finally dominates the more slowly growing $\sim \ln(W/m)$ ‘edge’ contribution, it does not do so until the $W \approx 10 \text{TeV}$ regime.

This situation is shown in Fig.1 taken from [3]. The radius $\sqrt{\sigma_{\text{TOT}}^{\text{TOT}}/2\pi}$ corresponding to the total cross section becomes larger than the ‘edge’ only around several TeV.
Figure 1: The “edge” and the “disc”. The dashed (blue) line is a plot of the quantity \( t \) from Ref. [3], representing, the effective thickness of the edge. Its constancy exhibits the energy independence of the edge. The dashed-dotted (red) line represents the radius inferred from the total cross section, \( \sqrt{\frac{\sigma_{TOT}}{2\pi}} \). The units are \( f = \text{fermi} = 10^{-13} cm \). From ref [3], which used data from fits for the total and elastic cross sections.

2 Nuclei

This simple geometric picture can be easily transferred to the case where, instead of another proton, a nucleus is the target for a very high energy proton.

At ‘high energy’ (\( W \sim \text{GeV} \)'s) the nucleus will absorb any incident proton up to the ordinary, low energy, radius of the nucleus \( R_A \), giving a ‘black disc’ of radius \( R_A \).

This radius is typically parameterized in terms of the mass number \( A \) as [4]

\[
R_A \approx 1.2 f A^{1/3},
\]

although the ‘1.2’ factor may vary somewhat with the author or the application.

At ‘very high energy’ (\( W \gtrsim 10 \text{TeV} \)) the growing size of the nucleon must be taken into account. For nucleons in the interior of the nucleus the increase in the cross section has little effect since the absorption is any case maximal,’black’, and cannot be sensibly increased. However it will affect interactions with the nucleons on the rim or outer edge of the nucleus, (seen in the plane transverse to the incoming proton). We take this increase to be governed by the parameters
determined from the data on p-p scattering, as illustrated in Fig. 1. At energies
where there is ‘disc dominance’ for the p-p interaction, one therefore has an
effective ‘black disc’ radius for the interaction on the nucleus \( R^{vhe}_A \), which is
\[
R^{vhe}_A \approx R_A + R_{pp}^{vhe}, \tag{2}
\]
For \( R_{pp}^{vhe} \) we use the fit values \( \sigma^{TOT} \approx 1.1 \, \text{mb} \times \ln^2(W/m) \) which implies
\[
R_{pp}^{vhe} = \sqrt{1.1 \, \text{mb} / 2\pi \times \ln(W/m)} \approx 0.13 \times \ln(W/m) \tag{3}
\]
for the radius of the ‘disc’ in p-p scattering, the asymptotic slope of the dashed-
dotted (red) line of Fig. 1.

3 Consequences

With these parameters established, we examine the consequences for nuclei. In
particular we consider the example of nitrogen (\( A = 14 \)) as representative of
cosmic ray interactions in the atmosphere.

We look at the inelastic cross section, \( \sigma = \pi R^{vhe}_A^2 \) as relevant for atmo-
spheric interactions. (Adding the very forward narrowly peaked elastic cross
section would give the total cross section, \( 2\pi R^{vhe}_A^2 \).)

\[
\sigma = \pi R^{vhe}_A^2 = \text{const.} + 0.26\pi R_A f \times \ln(W/m) + \sigma^{disc}_{pp} \tag{4}
\]
We thus arrive at an interesting situation with a small \( \ln^2(W/m) \) term and
a \( \ln(W/m) \) term with a relatively large coefficient, namely \( 0.26\pi f R_A \).

Evaluating this for nitrogen, \( A = 14 \) using Eq 4, one obtains
\[
0.26\pi R_A f \times \ln(W/m) = 24 \, \text{mb} \times \ln(W/m) \tag{5}
\]
Although the \( \ln^2(W/m) \) term is ultimately dominant, it will probably never
be directly observable, due to its small coefficient. With the value Eq 5 the \( \ln^2(W/m) \) term in Eq 4 becomes comparable to the \( \ln(W/m) \) term when
\( \ln(W/m) \approx 24/0.55 = 43 \), that is, at \( W \sim 10^{18} \text{GeV} \). This is much beyond the
highest energy at the LHC or the cosmic ray cutoff around \( W \approx \sqrt{10^{12} \text{GeV}^2} = 10^6 \text{GeV} \).

It thus appears that for conceivably available energies the cross sections will
be governed by the \( \ln(W/m) \) term, exhibiting effectively a \( \text{const.} + \ln(W/m) \) behavior. Despite this only linear behavior in the logarithm, the relatively
large coefficient induced by the nuclear radius means there can nevertheless be
significant effects, at very high energy. If we inquire as to at what energy the
growing Eq 5 becomes comparable to the low energy cross section \( \pi R_A^2 \), one has the condition
\[
\frac{\pi R_A 0.26f \times \ln(W/m)}{\pi R_A^2} = \frac{0.26f R_A}{\ln(W/m)} \sim 1, \tag{6}
\]
or when $\ln(W/m) \sim R_{2017}^3$. For nitrogen this gives $W \sim 6.3 \times 10^4 GeV \approx 60 TeV$, which is in the range considered for some future accelerators [5]. For cosmic rays this corresponds to a lab energy in the proton-proton system of $2.1 \times 10^{18} eV$, in the region of highest energy cosmic rays, which extends up to $\sim 10^{21} eV$. Taken at face value Eq[6] implies a doubling of the simple cross section $\pi R_A^2$ in this region. Such an increase would lead to air showers starting higher in the atmosphere and so resemble a change in the chemical composition of the cosmic rays [6].

4 Experimental Checks

Experimental checks of this theory could possibly be carried out at a p-nucleus collider.

At energies sufficiently high so that the ‘disc’ dominates the pp cross section, that is, beginning around $W \sim 10 TeV$, one may envision checking the following points

a) The cross sections vary with energy approximately as $const. + \ln(W/m)$

b) When comparing different nuclei the coefficient of the $\ln(W/m)$ is linearly proportional to the radius of the nucleus

These effects could be studied by direct measurement of the cross sections or by examining the narrowing of the elastic diffraction peak, corresponding to the increasing radius. Verification of these points would be a strong validation of our simple geometrical picture and provide an important input for analyzing air showers.

Finally, we note an interesting feature of these arguments. The leading $\ln^2$ term from p-p scattering is reproduced for nuclei in Eq[6] and with the same coefficient. This reinforces the impression (see [1]) that the coefficient of the $\ln^2$ term is a fundamental parameter, one that ought to be calculable from an underlying theory of hadron physics.

5 Acknowledgement

These ideas were stimulated by discussions at the Astropysics + MAGIC meeting, La Palma, June 2018. I would like to thank Razmik Mirzoyan and the other organizers for their invitation.

References

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[3] M. M. Block, L. Durand, F. Halzen, L. Stodolsky and T. J. Weiler, “Evidence for an energy-invariant ‘edge’ in proton-proton scattering at very high energies,” Phys. Rev. D 91, no. 1, 011501 (2015) doi:10.1103/PhysRevD.91.011501 [arXiv:1409.3196 [hep-ph]].

[4] See Elementary Nuclear Theory by H. A. Bethe and P. Morrison, John Wiley and Sons, p 11, second edition.

[5] CERN Courier May 2017 p34.

[6] A discussion of this issue for the Pierre Auger Collaboration is found in M. Plum [Pierre Auger Collaboration], “Measurement of the chemical composition of the ultra-high-energy cosmic rays with the Pierre Auger Observatory,” arXiv:1501.06325 [astro-ph.HE].