Numerical investigation of nanofluid convection performance in the fully developed flow regime of the pipe with constant wall temperature

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Abstract. In this work, single-phase dispersion model (SPD) is used to numerically study laminar force convection performance of Al₂O₃/water nanofluid in the fully developed flow regime of a constant wall temperature pipe, and a new average Nusselt number expression for Al₂O₃/water nanofluid flow in such a condition is put forward by modifying the classical average Nusselt number formula. In the numerical calculation, two-dimension equation is solved by using finite volume method, dispersion effects causing the increase in thermal conductivity and viscosity of nanofluid have been considered. It is shown that the numerical simulation results are in agreement with experimental data, the deviations are less than 7% in terms of Nusselt number and convective heat transfer coefficient. In addition, the modified average Nusselt number formula can predict heat transfer performance well for Al₂O₃/water nanofluid laminar flow in a constant wall temperature pipe.

1. Introduction
Nanofluid refers to a uniform and stable working fluid filled with the 10~100nm metal or non-metallic oxide nano powder [1]. As an innovation in heat transfer fluid medium, nanofluid has aroused the interest of many researchers. For the laminar flow of nanofluid, some similar conclusions of intensifying heat transfer were gotten from experiments [2-5]. Numerical simulations were also conducted, many of which focused on comparison and modification of single- and two-phase models of nanofluid flow under the constant heat flux condition [6-9].

There were few numerical investigations on nanofluid laminar full developed flow in a constant wall temperature pipe. Single-phase models in some existing literatures [10] are incomplete, because they don't take the effect of dispersion or Brownian into consideration on the thermal conductivity and viscosity of nanofluid, besides, the properties of nanofluid obtained was not based on actual experimental measurement. So it's necessary to predict nanofluid heat transfer performance more accurately by using more perfect model [8] for nanofluid laminar flow in a constant wall temperature pipe. Besides, the classical average Nusselt number formula also needs to be modified for Al₂O₃/water nanofluid flow in such a condition in order to calculate heat transfer performance better.

Present study aims to numerical investigation of Al₂O₃/water nanofluid laminar flow in the fully developed flow regime of a constant wall temperature pipe. For the sake of accuracy, the SPD model considering the impacts of dispersion on thermal conductivity and viscosity of nanofluid has been adopted in such a condition, instead of currently used sing-phase model(SPH) without any
modification of thermal conductivity and viscosity [10]. The simulation results are compared with experimental data available in literature [11] and that of SPH model.

2. Mathematical models

In single-phase dispersion model, the difference of velocity and temperature between nanoparticle and base fluid is ignored, nanofluid can be regarded as classical Newton fluid and nanofluid flow is assumed to be a steady incompressible flow. The continuity, momentum and energy equations therefore are expressed as follows:

\[ \nabla \cdot ( \rho_{nf} \vec{V} ) = 0 \] (1)

\[ \rho_{nf} ( \vec{V} \cdot \nabla \vec{V} ) = -\nabla P + \nabla \cdot ( \mu_{nf} \nabla \vec{V} ) \] (2)

\[ \nabla \cdot ( \rho_{nf} \nabla T ) = \nabla \cdot ( k_{nf} \nabla T ) \] (3)

where \( \rho_{nf} \), \( \mu_{nf} \), \( C_{p,nf} \), and \( k_{nf} \) are the density, effective viscosity, specific heat, and effective thermal conductivity of nanofluid, respectively.

\[ k_{eff} = k_{nf} + k_{disp} \] (4)

\[ k_{disp} = C(\rho C_{p}) u d_{p} R \varphi \] (5)

\[ \mu_{eff} = \mu_{nf} + \mu_{disp} \] (6)

\[ \mu_{disp} = \frac{k_{disp}}{C_{p,nf}} Pr_{nf} \] (7)

where \( k_{nf} \) is the thermal conductivity of nanofluid, \( k_{disp} \) is called the dispersion thermal conductivity [12], \( C \) is a constant which is determined by matching experimental data, \( u \) is mean velocity of nanofluid, \( d_{p} \) is particle diameter (40nm), \( R \) is the inner radius of pipe (5mm), \( \varphi \) is the particle volume fraction, \( \mu_{nf} \) is the viscosity of nanofluid, \( \mu_{disp} \) is the dispersion viscosity, and \( Pr_{nf} \) is the Prandtl number of nanofluid.

3. Problem statement and relevant parameters

A copper tube with 2m length and 5mm inner diameter is heated by water vapor and Al\(_2\)O\(_3\)/water nanofluid with 40 nm average nanoparticle size flow passes through a horizontal isolated copper tube with 50 cm length before it enters the heating zone, which is the same set-up as that in the experiment conducted by Heyhat [11].

Two dimensional axisymmetric formulation is considered in numerical simulation. Control volume technique is used to solve continuity, momentum and energy equations, and the residual is less than \(10^{-5}\).

Inlet velocity is determined by the Reynolds number adopted in experiment. The tube wall temperature is maintained at a constant temperature of 373K.

Relevant parameters such as average heat transfer coefficient, average Nusselt number, are calculated from results of simulation by following equations:

\[ h_{nf} = \frac{C_{p,nf} \rho_{nf} UA(T_{out} - T_{in})}{\pi DL(T_{w} - T_{i})_{LM}} \] (8)
\[ \overline{Nu_{nf}} = \frac{h_{nf} D}{k_{nf}} \]  

(9)

where \((T_W - T_i)_{LM}\) is the logarithmic mean temperature difference, \(U\) is the mean velocity of nanofluid, \(A\) is the cross-sectional area of the pipeline, \(T_{in}, T_{out}, \) and \(T_w\) are the inlet temperature, outlet temperature and wall temperature, respectively, \(D\) is the inner diameter and \(L\) is the length of tube.

Grid independence have to be checked, and 50 \(\times\) 5000 grid is selected as a suitable grid for present simulation calculation, the grid is concentrated near the wall. In order to verify the validity of current model and grid, the predicted Nusselt number for water in the same condition has been compared with experiment and theory (equation (10)) and good agreements have been seen, as is shown in figure 1.

\[ \text{Figure 1. Average Nusselt number changing with Reynolds number for water} \]

4. Results and discussion

The purpose of this work is to simulate nanofluid flow in the fully developed flow regime of a constant wall temperature pipe in terms of heat transfer by using SPD model. Dispersion coefficient \(C\) is calibrated by experimental data at Reynolds number 810 and 0.01 volume fraction.

As shown in figure 2, the results of SPD model are compared with the experiment and that of SPH model in terms of Nusselt number. The Nusselt number obtained from SPD model increases with the increase of Reynolds number, which is consistent with the experimental results and the largest deviation gotten is 7% at \(\varphi = 0.001, \text{Re} = 1900\). With the increase of nanoparticle volume fraction, the accuracy of simulation results of SPD is rapidly improved and much better than that of SPH, which shows that SPD model is more effective in predicting Nusselt number of nanofluid.
Figure 2. Average Nusselt number changing with Reynolds number at four kinds of nanoparticle volume fraction for Al₂O₃/water nanofluid

Figure 3 illustrates the change of average convective heat transfer coefficient with nanoparticle volume fraction at different Reynolds number. It is obvious that the results obtained from simulation are in good agreement with that of experiment. The convective heat transfer coefficient increases with the increase of nanoparticle concentration and the maximum deviation is 6.2% occurred at the \( \varphi = 0.001 \), Re=1900, the deviations of other results are less than 4.2%. The validity of SPD model is further proved.

Figure 3. Average heat transfer coefficient changing with nanoparticle volume fraction at four Reynolds number for Al₂O₃/water nanofluid

It was pointed out in literature [11] that the classical average Nusselt number formula of fluid flow in a pipe with constant wall temperature (equation (10)) failed to predict the average Nusselt number of Al₂O₃/water nanofluid flow in such a condition.
\[ Nu = 3.66 + \frac{0.0668(D/L) \text{Re}^{(14/13)} \text{Pr}_{nf}}{1 + 0.04[(D/L) \text{RePr}_{nf}]^{2/3}} \]  

(10)

So here, we propose a modified average Nusselt number correlation formula for Al$_2$O$_3$/water nanofluid laminar full developed flow in a constant wall temperature pipe by fitting of experiment data obtained.

\[ Nu = 3.66 + \frac{0.0668(D/L) \text{Re}^{(14/13)} \text{Pr}_{nf}}{1 + 0.04[(D/L) \text{RePr}_{nf}]^{2/3}} [1 + 3.5335 \text{Re}^{-0.3171}(1 + \phi)^{0.9258}] \]  

(11)

**Figure 4.** Comparison of correlated Nu with experiment and simulation

The average Nusselt number deviation between equation (11) and the experiment is depicted in figure 4(a) and it is about -1.1% ~ +2.9% in the range of $300 < \text{Re} < 2000$, $2 < \text{Pr}_{nf} < 5$, $0.001 \leq \phi \leq 0.015$, which validates the effectiveness of this correlation formula. The average Nusselt numbers obtained from equation (11) and SPD model are also compared. SPD model is an appropriate model in predicting heat transfer characteristics of nanofluid flow in a pipe, according to the analysis above, and, as is shown in figure 4(b), the average Nusselt number deviation between equation (11) and SPD model is about -4.1% ~ +6.8%, so, a comparatively good conformity is also seen.

5. Conclusion

Laminar full developed flow of Al$_2$O$_3$/water nanofluid in a pipe with constant wall temperature is simulated by using SPD model in terms of heat transfer and the results are compared with experimental data, which shows that the numerical simulation results are in agreement with that of experiment, heat transfer performance of nanofluid increases with the increase of Reynolds number and nanoparticle volume fraction, and the deviations of simulation are less than 7% in terms of Nusselt number and convective heat transfer coefficient. In addition, the classical average Nusselt number correlation formula for the convective heat transfer of fluid in a pipe has been modified for predicting Al$_2$O$_3$/water nanofluid laminar full developed flow in a pipe with constant wall temperature and its validity and good practicability are illustrated.

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