Mathematical model of propagation of liquid vibrations in a tubing string

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Abstract. Effectiveness of wave processes in tubing strings, wells and long pipelines may be limited by damping of hydrodynamic waves along the length of the channel. Violation of thermodynamic equilibrium in reservoir fluids during their rise to the surface and in ground pipelines may be accompanied by the release of asphalt-resin-paraffin components in the flow of crystals due to a decrease in temperature and pressure. Subsequent depositing of crystals takes place as they grow, due to adhesion to the wall of pipes, the surface of equipment. Deposits are most intensified in areas of local expansion and narrowing of the cross-section area of the flow (occurrence of vortices).

1. Introduction
Paraffin is a mixture of higher methane hydrocarbons. Refined paraffin is a colorless or white crystalline mass, odorless and tasteless, slightly greasy to the touch, the specific gravity of which ranges from 0.907 to 0.915 at 15 °C. The specific gravity of the crude paraffin is 0.881–0.905. The melting point of paraffin depends on the molecular weight and ranges from 49 to 60 °C. At lower temperatures, it is solid.

The depth of the onset of paraffin deposition during oil production from deposits varies widely. For example, during the exploitation of oil fields in the Perm region, the depth of the onset of paraffin deposition is 1000 m, in the Republic of Tatarstan - 1200-1250 m and lower, in the Republic of Bashkortostan - 400-1700 m, in the fields of South Mangyshlak - 1000-1100 m. At the Nizhnesortymsky oil field in West Siberia, paraffin deposits were observed in the depth interval of 300–1400 m, and at the Urengoyskoye gas condensate field - at a depth of 800–1600 m. These results were obtained by studying the deposits when lifting the tubing string from the wells. The significant depth of the onset of paraffin deposition is due to the crystallization of more refractory paraffins primarily. In this regard, during oil production in the tubing string, the most refractory paraffins accumulate in asphalt-resin-paraffin deposits (ARPD).

At a high resin content, solid paraffinic hydrocarbons crystallize in aggregate form in the form of small crystals. Being polar substances, resins are adsorbed on small crystals and cause paraffin aggregation. If at the initial moment many small crystalline nuclei are formed, then upon further cooling, solid paraffinic hydrocarbons released from the solution crystallize on these nuclei. This ultimately leads to the formation of small paraffin crystals in solution.
For oil with a high content of resins and asphaltenes, close to the temperature of its saturation with paraffin, the process of aggregate crystallization is more characteristic - a process in which crystals grown independently of each other from different crystallization centers turn out to be randomly cemented into a single system of high-molecular compounds.

With a deeper cooling of highly paraffinic oil, the process of aggregate-dendritic crystallization is possible, where the aggregate is formed not by single crystals of paraffin, but by individual dendrites. Thus, the ratio of paraffins and natural depressants has a significant effect on the crystallization process. With a small content of natural oil depressants, dendritic crystallization occurs with an external phase from molecular groups of paraffins, and with an excess of natural depressants, aggregate crystallization with an external phase from molecular groups of resins and asphaltenes occurs.

Decreases in temperature and pressure may be accompanied by the release of asphalt-resin-paraffin components in the flow of crystals when the thermodynamic equilibrium in layered fluids is disturbed. The problem of attenuation of the amplitude of hydrodynamic pressure waves in the tubing string and ground pipeline, taking into account the friction of the liquid against the pipe walls is covered in [1-11].

When using wave processes, reducing the intensity of asphalt-resin-paraffin deposits (ARPD) on the inner surface of pipe systems, as well as increasing the speed of their erosion, for example, by hot liquid (washing liquid), is provided by the following additional processes:

– increase in the total friction path, respectively, the intensity (speed) of destruction of the surface layer of the ARPD due to additional wave-like movement of the washout liquid relative to the average flow rate (increasing the total contact time);

– reduction of adhesion forces in the inner surface of pipes, due to the difference in densities of metal and the ARPD and, as a result, the radial components of inertial forces (in the limit, when the breaking forces from the flow of the extracted reservoir fluid exceed the adhesion forces) [12-14].

2. Materials and methods
When flushing, the efficiency of the process of dewaxing with a hot liquid (for example, water) of a tubing string in a well or a pipeline is limited by the distribution of hydrodynamic waves (HDW) along its length. Without external damping, the propagation of HDW is limited, mainly by the forces of internal friction in the fluid itself. When the fluid moves in the column or in the part of it narrowed by the ARPD, due to the friction forces on the inner surface of the channel, external damping forces arise, the influence of which increases with narrowing of the channel cross-sectional area and with an increase in the HDW power.

As a result, for real oilfield technological problems, characterized by a high intensity of HDW emitted by the hydrogenerators and their relatively low frequency (for the case under consideration, in the lower intervals of the sound frequency range), the effect of internal friction becomes negligible compared to the effect of flow friction against the inner surface of the column.

The effectiveness of the influence of HDW on the dewaxing process is directly related to the intensity of their decay. When moving in pipes, the main reason for the damping of waves is the absorption of their energy by the forces of friction against the channel walls and, to a much lesser extent, the forces of internal friction between the particles of the medium itself.

Therefore, we will initially consider the effect of friction on the damping of longitudinal waves of the medium against the walls of the waveguide. The influence on the damping of internal friction, in general, is considered in numerous educational literature and therefore is not presented in this article. Let us assess the influence of external friction on the damping of the HDW in more detail.

The propagation of flat longitudinal HDW in a cylindrical channel, taking into account external damping, i.e. due to the friction of the liquid against the walls, is characterized by a well-known nonlinear expression
reduced to a linearized one for harmonic waves, i.e. telegraphic equation
\[
\frac{\partial^2 u}{\partial t^2} + \frac{8\omega a}{3\pi} \frac{\partial u}{\partial x} = c^2 \frac{\partial^2 u}{\partial x^2},
\]
where \( u = \Delta P / (\rho c) \) – velocity in the considered cross section with the amplitude of vibration of the fluid \( U \); \( \Delta P = \rho a u^2 \) – hydrodynamic pressure; \( F \) – cross-sectional area of fluid flow; \( \chi \) – wetted perimeter; \( \omega = 2\pi \nu \) – circular frequency of the HDW; \( X = U / \omega \) – amplitude of vibration of the particles of the liquid; \( a = \lambda / 8\delta \); \( \delta = F / \chi \) – hydraulic radius of the flow cross-section; \( \delta = F / \chi \) – hydraulic radius of the flow cross-section (in the pipe flow) \( 4\delta = d_t \), where \( d_t \) – internal diameter of pipe; \( \lambda \) – coefficient of hydraulic resistance.

To determine the depth of the HDW propagation in the flow, we will reveal the coefficient of hydraulic resistance \( \lambda \) for turbulent flow in a cylindrical channel of Newtonian liquids.

3. Results and Discussion
At the turbulent flow of Newtonian liquids in cylindrical channels (corresponding to the Reynolds number \( Re > Re_{kr} = 2100–2320 \) – upper limit of \( Re_{kr} = 2320 \) – for technical water; the presence of a transient flow mode in the interval \( Re_{kr} = 2100–4000 \) is ignored in the given task), the coefficients of hydraulic resistance may decrease up to a multiple of a smaller value. This decrease is accompanied by a reduced degree of damping of liquid pressure HDW and, accordingly, an increase in the effectiveness of their influence on a particular technological process along the length of the tubing string in a well or a long pipeline.

In order to simplify the wave equation (1) as much as possible, the coefficient of hydraulic resistance \( \lambda \) in a cylindrical channel will be determined using the Blasius formula
\[
\lambda = 0.3164 Re^{-0.25}.
\]

In this case, the wave equation (1) will have the form
\[
\frac{\partial^2 u}{\partial t^2} + 0.3164 A \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}; \quad A=1.
\]

Thus, a strongly nonlinear wave equation is obtained even in the simplified version.

According to the work of B. I. Mitelman, in the turbulent regime in the area above the transition, i.e. in the area of quadratic friction with the lower boundary
\[
u / \nu = 2.4 \cdot 10^6; \quad m^{-1},
\]
where the velocity \( u \) and kinematic viscosity \( \nu \) are measured in m/s and m²/s, respectively, the coefficient of hydraulic resistance in pipes with a diameter \( d_t \) is equal to
\[
\lambda = 0.0121 d_t^{0.226},
\]
and instead of (4) by analogy with (2) we have a linearized equation
\[
\frac{\partial^2 u}{\partial t^2} + X \frac{\omega \lambda}{3\pi \delta} \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}
\]
or, for harmonic HDW, the expression
\[
\omega^2 U + i \frac{\omega \lambda}{3\pi \delta} U = c^2 \frac{\partial^2 u}{\partial x^2},
\]
the solution of which leads to Painleve transcendental function and after integration has the form
\[
\left( \frac{dU}{dx} \right)^2 - imU^3 + nU^2 + C_1 = 0,
\]
where
\[ n = \omega^2 / c^2; \quad m = 2\lambda \omega / (9 \pi \delta c^2); \quad \delta = d_s / 4. \] (10)

Extracting the square root from the resulting equation, we obtain the desired solution via incomplete elliptic integrals of the 1st kind, which is not given due to the bulkiness.

Under boundary conditions: at \( x = 0 \)
\[ U = U_0; \quad \frac{dU}{dx} = 0; \] (11)
in equation (9)
\[ C_1 = im U_0^3 - n U^2. \] (12)

Since we are primarily interested in a qualitative picture of the relative losses of hydrodynamic velocity or pressure, we will limit ourselves to a particular solution (9) at \( C_1 = 0 \). For a harmonic function, this physically corresponds to the case of placing the center of coordinates on the ridge of the HDW. In this case, instead of (9), we will consider the equation
\[ \left( \frac{dU}{dx} \right)^2 - imU^3 + nU^2 = 0 \] (13)
or in a transcendent form:
\[ \frac{dU}{U\sqrt{n-imU}} = dx, \] (14)
the solution of which corresponds to the formula:

\[
\begin{align*}
\text{when } n > 0 \\
&x = \frac{1}{\sqrt{n}} \ln \frac{\sqrt{n-imU} - \sqrt{n}}{\sqrt{n-imU} + \sqrt{n}}; \\
\text{when } n < 0 \\
&x = \frac{1}{\sqrt{-n}} \arctg \frac{\sqrt{n-imU}}{\sqrt{-n}}.
\end{align*}
\] (15)

The real part of the obtained complex expression corresponds to the desired hydrodynamic displacement velocity. In this case, after the transformations, we get: at \( n > 0 \)
\[ x = n^{-0.5} \ln \left( \sqrt{1+n^2U^2n^{-2}} -1 \right)^{-1} \left( 3+2\sqrt{2} \right)^{-1}. \] (16)

4. Conclusion
Stationary ground-based or borehole hydrogenerators should be used only (or primarily) in subinfra- or infrasound frequency ranges. With a pipeline length being small, the use of HDW in the lower areas of the sound range may also be satisfactory in special cases. High-frequency HDW should only be used when overlaying low-frequency ones.

At the lower end of the tubing string in the well, it is better to use a low-frequency pulsator.

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