Leaving no stone unturned in the hunt for SUSY naturalness: A Snowmass whitepaper

H. Baer\textsuperscript{1}, V. Barger\textsuperscript{2}, P. Huang\textsuperscript{2}, D. Mickelson\textsuperscript{1}, A. Mustafayev\textsuperscript{3}, W. Sreethawong\textsuperscript{4} and X. Tata\textsuperscript{3}

\textsuperscript{1}Dept. of Physics and Astronomy, University of Oklahoma, Norman, OK 73019, USA
\textsuperscript{2}Dept. of Physics, University of Wisconsin, Madison, WI 53706, USA
\textsuperscript{3}Dept. of Physics and Astronomy, University of Hawai‘i at Mānoa, Honolulu, HI 96822, USA
\textsuperscript{4}School of Physics, Suranaree University of Technology, Nakhon Ratchasima 3000, Thailand

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Imposing electroweak scale naturalness constraints (low $\Delta_{EW}$) on SUSY models leads to mass spectra characterized by light higgsinos $\sim 100 - 300$ GeV, highly mixed top-squarks and gluinos at the 1–5 TeV scale and allows for $m_h \sim 125$ GeV. First and second generation squarks can easily live at the 5-20 TeV scale, thus providing at least a partial solution to the SUSY flavor/CP problems. For such models at the LHC, gluino pair production is followed by cascade decays to $t$- and $b$-quark rich final states along with multileptons. The reach of LHC14 with $300 \, \text{fb}^{-1}$ is computed to be around $m_{\tilde{g}} \sim 1.8$ TeV. However, the small magnitude of the $\mu$-parameter – a necessary condition for naturalness – leads to a unique hadronically quite same-sign diboson ($W^\pm W^\pm$) signature from wino pair production. In low $\Delta_{EW}$ models with unified gaugino masses, this signature yields a somewhat higher reach up to $m_{\tilde{g}} \sim 2$ TeV. The smallness of $|\mu|$ implies that the ILC should be a higgsino factory in addition to a Higgs factory, and a complete search for SUSY naturalness seems possible for $\sqrt{s} \sim 600$ GeV. Since a thermal under-abundance of higgsino-like WIMP dark matter (DM) is expected, there is ample room for an axion DM contribution. A thorough search for higgsino-like WIMPs can be made by next generation WIMP detectors, such as those with ton-scale noble liquid targets.

1 Introduction

In previous studies \cite{1,2} and a companion contribution Ref. \cite{3}, we have argued that a necessary condition for the naturalness of SUSY models is the requirement that there are no large cancellations in the familiar one-loop effective potential minimization condition

$$\frac{M_Z^2}{2} = \frac{m^2_{H_u} + \Sigma_d - (m^2_{H_d} + \Sigma_u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2. \tag{1}$$

Here, Eq. (1) is implemented as a weak scale relation, even for SUSY theories purporting to be valid all the way up to scales as high as $M_{\text{GUT}} - M_P$. The quantities $\Sigma_u$ and $\Sigma_d$ are the one-loop corrections arising from loops of particles and their superpartners that couple directly to the Higgs doublets. The rationale for this, which has been presented in Ref. \cite{1,2,3}, led us to introduce the quantity $\Delta_{EW}$ which is determined essentially by the physical super-partner spectrum. Spectra that lead to low values of $\Delta_{EW}$ offer the possibility of being derivable from natural theories. Here, we summarize the phenomenological consequences of low $\Delta_{EW}$ models for LHC, ILC and dark matter searches.

The naturalness requirement of no large, uncorrelated contributions to $M_Z$ on the right-hand-side of Eq. (1) implies the following.

1. The magnitude of the weak scale $\mu$ parameter is not too far from $M_Z$: $|\mu| \sim 100 - 300$ GeV to enjoy better than 3% electroweak fine-tuning (lower $|\mu|$ gives less fine tuning).

2. The soft term $m^2_{H_d}$ is driven to small negative values with $|m^2_{H_d}| \sim 100 - 300$ GeV. This occurs in the FP region of mSUGRA/CMSSM but can occur at any $m_0$, $m_{1/2}$ values in less constrained models such as the two-extra-parameter non-universal Higgs model (NUHM2).
3. The top squarks are at the few-TeV scale but highly mixed. The large mixing suppresses the contributions $\Sigma_u^{(1,2)}$ to $\Delta_{EW}$ whilst raising $m_h$ up to the 125 GeV level\[^1\].

SUSY models which generate these conditions will automatically produce $Z$ and $h$ masses around the 100 GeV scale while respecting LHC constraints on $m_h$ and on sparticle masses, thus accommodating a Little Hierarchy (which in this case is no Problem). In SUSY models which are valid up to some high scale $\Lambda \sim M_{GUT}$ or $M_P$, the soft term $m_{H_u}^2$ can be radiatively driven to values $\sim -M_Z^2/2$; this class of models is called \textit{radiatively-driven natural SUSY}, or RNS. RNS with low electroweak fine-tuning $\Delta_{EW}$ can be realized within the two-parameter non-universal Higgs model (NUHM2)\[^6\], but not in more constrained models such as mSUGRA/CMSSM.

RNS spectra are characterized by the following features, with a typical spectrum shown in Fig. 1.

- Four light higgsinos $\tilde{Z}_1$, $\tilde{Z}_2$ and $\tilde{W}^{\pm}_1$ with mass $\sim 100-300$ GeV. In fact, the lighter end of this range (closer to $M_Z$) is preferred by naturalness since $\Delta_{EW} \gtrsim |\mu^2|/(M_Z^2/2)$.

- Highly mixed top- and bottom-squarks and also gluinos in the $1-5$ TeV range (this is significantly heavier than the range predicted by earlier natural SUSY models\[^4, 5\]).

- If gaugino mass unification is imposed, then $\tilde{Z}_3$ will be bino-like at $\sim 0.2-0.8$ TeV and $\tilde{Z}_1$ and $\tilde{W}_2$ will be wino-like at $\sim 0.4-1.6$ TeV.

- First/second generation squarks and sleptons may be at the $5-30$ TeV range, thus providing at least a partial decoupling solution to the SUSY flavor/CP problems.

\[\text{Figure 1: Typical mass spectrum from low } \Delta_{EW} \text{ models}\]

\[\text{Typical spectrum for low } \Delta_{EW} \text{ models}\]

\[\begin{array}{c}
\text{first/second generation matter scalars}\\
\text{stops, sbottoms, gluinos}\\
\text{winos}\\
\text{bino}\\
\text{Higgs, higgsinos}\\
\text{gauge bosons}
\end{array}\]

2 RNS at LHC

The phenomenological consequences of RNS models at LHC have been explored in Ref’s \[^7\] and \[^8\]. A brief summary of highlights includes the following.

\[^1\]In mSUGRA/CMSSM, $\mu \sim M_Z$ can be generated in the HB/FP region. However, in that region the top-squarks are so heavy that the $\Sigma_u$ radiative corrections dominate $\Delta_{EW}$ and lead to fine-tuning typically at the 0.3% level or worse.
• Gluino pair production can be substantial at LHC14 for \( m_{\tilde{g}} \sim 1-2 \) TeV. In RNS models, gluinos cascade decay dominantly to the three-body modes \( tt\tilde{Z} \) and \( tb\tilde{W} \), giving rise to the usual multilepton+multijet+\( E_T^{\text{miss}} \) final states. The LHC reach for gluino pair production at LHC14 with 300 fb\(^{-1}\) is estimated to be \( m_{\tilde{g}} \sim 1.8 \) TeV \[8, 9\].

• A qualitatively new SUSY signal for models with light higgsinos emerges: same-sign diboson (SSdB) production accompanied by modest jet activity. This signal arises from wino pair production \( pp \rightarrow \tilde{W}_2^+\tilde{Z}_4 \) followed by \( \tilde{W}_2 \rightarrow W\tilde{Z}_{1,2} \) and \( \tilde{Z}_4 \rightarrow W^{\pm}W^{\mp}_1 \). Half the time one arrives at the SSdB final state. The very compressed higgsino spectrum means the decay products of \( \tilde{W}_1 \) and \( \tilde{Z}_2 \) are very soft, so that accompanying hadronic activity emerges mainly from initial state QCD radiation. After cuts, the dominant background is found to be \( Wt\bar{t} \) production. The reach of LHC14 with 300 fb\(^{-1}\) is to \( m_{\tilde{g}} \sim 2.1 \) TeV \[8\], somewhat beyond the reach found from gluino pair production if the wino and gluino are related by gaugino mass unification. We emphasize that the SSdB signal from wino pair production is present in all models where winos are accessible and where higgsinos are light. As seen from Fig. 2, this signal can access \( M_2 \simeq 0.8m_{1/2} \) values as large as 800 GeV at luminosity upgrades of the LHC. Moreover, in low \( |\mu| \) models, this signal offers a larger reach than the gauginos \( \rightarrow WZ \rightarrow 3\ell+\not{E}_T \) trilepton signal.

**Figure 2:** Multilepton cross sections from wino pair production \( \tilde{W}_2^+\tilde{Z}_4 \) and \( \tilde{W}_2^+\tilde{W}_2^- \) after cuts vs. \( m_{1/2} \) along the RNS model line. The reach of LHC14 is indicated for various integrated luminosity choices. Solid curves represent the SSdB channel for several set of cuts optimized for light (blue) and heavy (red) winos, and for low integrated luminosity (green): see Ref. \[8\]. The corresponding situation for the gauginos \( \rightarrow WZ \rightarrow 3\ell+\not{E}_T \) channel is represented by the dashed magenta curve.

- The reaction \( pp \rightarrow \tilde{W}_2^+\tilde{Z}_4 \) and \( \tilde{W}_2^+\tilde{W}_2^- \) gives rise to \( WZ \) final states \[17\] which can be tagged via the much-studied trilepton signature. The reach of LHC14 with 300 fb\(^{-1}\) (1000 fb\(^{-1}\)) in this channel is to \( m_{\tilde{g}} \sim 1.3 \) TeV (1.65 TeV), well below the SSdB reach as already noted.

- The wino pair production reactions also lead to an observable rate for \( 4\ell+\not{E}_T \) events for integrated luminosity exceeding 100 fb\(^{-1}\). In fact, this channel yields a slightly better reach than via trilepton events just discussed, though not as high as via the SSdB search.
The reaction \( pp \rightarrow \tilde{W}_1 \tilde{Z}_2 \) gives rise to soft trilepton+\( E_T^{miss} \) events. These may be visible for the lower range of \( m_{1/2} \) where the \( \tilde{Z}_2 \) is mixed higgsino-bino with a larger mass gap \( m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \sim 30 - 40 \text{ GeV} \). As \( m_{1/2} \) (or \( M_2 \)) increases, the mass gap drops to the 10-20 GeV range and the OS/SF dilepton from \( \tilde{Z}_2 \rightarrow \ell^+\ell^-\tilde{Z}_1 \) decay gets lost under the \( W^*\gamma^* \) background.

### 3 RNS at ILC

The key prediction of RNS models and SUSY naturalness is the existence of four light higgsinos \( \tilde{Z}_1, \tilde{Z}_2 \) and \( \tilde{W}_1^\pm \) with mass \( \sim |\mu| \sim 100 - 300 \text{ GeV} \), the lower the better. Thus, we would expect the proposed International Linear e+e− Collider (ILC) – operating with \( \sqrt{s} > 2|\mu| \) – to be a higgsino factory in addition to a Higgs factory. The main production reactions include

- \( e^+e^- \rightarrow \tilde{W}_1^+\tilde{W}_1^- \) and \( e^+e^- \rightarrow \tilde{Z}_1\tilde{Z}_2, \tilde{Z}_2\tilde{Z}_2 \).

The \( \tilde{W}_1 \) decays dominantly through \( W^* \) into \( f'f'\tilde{Z}_1 \) (where \( f \) and \( f' \) are SM fermions) and \( \tilde{Z}_2 \rightarrow f'f'\tilde{Z}_1 \). Production cross sections for these reactions vs. \( \mu \) for \( \sqrt{s} = 500 \text{ GeV} \) and vs. beam polarization \( P_L(e^-) \) have been shown in Ref. [10]. For \( e^+e^- \) collisions at \( \sqrt{s} = 500 \text{ GeV} \) and not too close to threshold, \( \sigma(\tilde{W}_1^+\tilde{W}_1^-) \sim 400 \text{ fb} \) and \( \sigma(\tilde{Z}_1\tilde{Z}_2) \sim 150 \text{ fb} \).

Detailed studies of mixed higgsino-bino states from HB/FP SUSY with a mass gap of \( \sim 40 \text{ GeV} \) have been presented in Ref’s [11, 12]. The reaction \( e^+e^- \rightarrow \tilde{Z}_1\tilde{Z}_2 \rightarrow \ell^+\ell^- + E_T \) should easily yield the \( m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \) mass gap to sub-GeV precision. By examining \( e^+e^- \rightarrow \tilde{W}_1^+\tilde{W}_1^- \rightarrow (\ell\nu\tilde{Z}_1) + (q\bar{q}'\tilde{Z}_1) \) states, the \( m_{\tilde{W}_1} \) and \( m_{\tilde{Z}_1} \) masses could be measured to better than \( \sim 10\% \) assuming just 100 fb\(^{-1} \) of integrated luminosity. Then the weak scale Lagrangian parameters \( M_2 \) and \( \mu \) could be determined to 20% and 10% precision, respectively. A detailed study of higgsino pair production for the much more challenging case of the Brummer-Buchmueller (BB) model[13] where the \( \tilde{W}_1 - \tilde{Z}_1 \) mass gap is at the 1 GeV level[14] has been undertaken. Based on the early results of this analysis, together with those from the earlier analyses mentioned, we are optimistic that the RNS higgsino signal will be detectable at the ILC if \( \sqrt{s} > 2|\mu| \). A dedicated analysis should, however, be carried out to confirm this. Fig. 3 shows that ILC500 (ILC1000) will decisively be able to probe \( \Delta_{EW} \) values up to 15 (60) in this channel.

![Figure 3: A plot of \( \Delta_{EW} \) vs. \( m_{\tilde{W}_1} \) from a scan of NUHM2 models. We also show the projected reach of ILC with \( \sqrt{s} = 250, 350, 500 \) and 1000 GeV.](image-url)
4  Direct/indirect detection of higgsino-like WIMPs from RNS

The calculated thermal relic density of higgsino-like WIMPs from RNS has been calculated in Ref’s [2][15], and is typically found to be $\Omega_{h}^{TP} h^2 \sim 0.007 - 0.01$, i.e. a factor 10-15 below measured values. The authors of Ref. [2][15] suggest a cosmology with mixed axion/higgsino dark matter (two dark matter particles, an axion and a higgsino-like neutralino [16]). In such a cosmology, thermal production of axinos $\tilde{a}$ in the early universe followed by $\tilde{a} \rightarrow g\tilde{g}$, $\gamma\tilde{Z}_1$ leads to additional neutralino production. In the case where axinos are sufficiently produced, their decays may lead to neutralino re-annihilation at temperatures below freeze-out, which also augments the neutralino abundance. In addition, coherent-oscillation production of saxions s at high PQ scale $f_s > 10^{12}$ GeV followed by saxion decays to SUSY particles can also augment the neutralino abundance. Late saxion decay to primarily SM particles can result in entropy dilution of all relics (including axions) present at the time of decay, so long as BBN constraints are respected. The upshot is that, depending on additional Peccei-Quinn parameters, either the higgsino-like neutralino or the axion can dominate the dark matter abundance, or they may co-exist with comparable abundances, leading to possible detection of both an axion and a WIMP. In the case of the axion, it is straightforward to find acceptable CDM densities with PQ scale $f_s \sim 10^{12} - 10^{16}$ GeV, far beyond the usual acceptable range from non-SUSY axion theories.

In the case of mixed axion-WIMP dark matter, the local WIMP abundance might be well below the commonly accepted local abundance $\rho_{loc} \sim 0.3$ GeV/cm$^3$. Thus, current limits from experiments like Xe-100 and CDMS should be scaled up by a factor $\xi \equiv \Omega_{h}^{TP} h^2/0.12$ to be applicable. In the RNS model, the gauginos also cannot be too light so that the neutralino always has a substantial gaugino component even though it is primarily higgsino. This means that spin-independent direct detection rates $\sigma_{SI}(\tilde{Z}_1p)$ are never too small. Even accounting for the local scaling factor $\xi$, it is found in Ref. [15] that ton-scale noble liquid detectors such as Xe-1-ton should completely probe the model parameter space. One caveat is that if saxions give rise to huge entropy dilution after freeze-out while avoiding constraints from dark radiation and BBN, then the local abundance may be even lower than the assumed freeze-out value, and the dark matter would be highly axion dominated.

5  Conclusions

The main implication of natural SUSY models is that there should exist four light physical higgsino-like states $\tilde{Z}_1, \tilde{Z}_2$ and $\tilde{W}^\pm_i$ with mass $\sim 100 - 300$ GeV while $\tilde{Z}_1$ is the LSP which is dominantly higgsino-like (albeit with a non-negligible gaugino component). Due to the compressed spectrum amongst the various higgsino states (typically a 10-20 GeV mass gap in models with gaugino mass unification), their three-body decays yield only tiny visible energy release, making them very difficult to detect at LHC. On the other hand, the light higgsinos should be detectable at an ILC provided that $\sqrt{s} > 2|\mu|$.

The situation is summarized in Fig. 4 where we show the $\mu$ vs. $m_{1/2}$ plane from the RNS model, taking GUT-scale matter scalar masses $m_0 = 5$ TeV, $\tan\beta = 15$, $A_0 = -1.6 m_0$ and $m_A = 1$ TeV. The Higgs boson mass $m_h \sim 125$ GeV over the entire plane. We see that LHC8 has explored $m_{1/2} < 400$ GeV via the search for $g\tilde{g}$ production. The projected LHC14 reach with 300$^{-1}$ fb for $g\tilde{g}$ production [9] and for same-sign diboson production [11] extends to $m_{1/2} \sim 700 - 800$ GeV (corresponding to a reach to $m_{\tilde{g}} \sim 1.8 - 2.1$ TeV). The naturalness contours of $\Delta_{EW} = 30$ extend well beyond LHC14 reach to $m_{1/2} \sim 1200$ GeV. We see that ILC600 will probe the entire remaining parameter space with $\Delta_{EW} < 30$, thus either discovering higgsinos or ruling out SUSY electroweak naturalness.

6  Acknowledgments

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Figure 4: Contours of $\Delta_{EW} = 15$, 30, 50 and 75 in the $m_{1/2}$ vs. $\mu$ plane for the RNS model with parameters as shown. The blue vertical lines show the current reach of LHC8 and the projected reach of LHC14 with 300 fb$^{-1}$ via gluino pair searches (dashed line) and same-sign dibosons (dot-dashed). The reach of ILC with $\sqrt{s} = 250$, 500 and 1000 TeV is also shown. The green-shaded region has a thermal higgsino relic abundance $\Omega_{h}h^2 \leq 0.12$. 

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