Cerenkov generation of high-frequency confined acoustic phonons in quantum wells

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We analyze the Cerenkov emission of high-frequency confined acoustic phonons by drifting electrons in a quantum well. We find that the electron drift can cause strong phonon amplification (generation). A general formula for the gain coefficient \( \alpha \) is obtained as a function of the phonon frequency and the structure parameters. The gain coefficient increases sharply in the short-wave region. For the example of a Si/SiGe/Si device it is shown that the amplification coefficients of the order of hundreds of cm\(^{-1}\) can be achieved in the sub-THz frequency range.

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High-frequency lattice vibrations with a high degree of spatial and temporal coherence have been observed for a number of semiconductor materials and heterostructures. These include Si, Ge, GaAs as well as SiGe and AlGaAs superlattices. These studies provide information on excitation mechanisms for the coherent phonons, their dynamics, electron-phonon interaction, and other important phenomena, including phonon control of the ionic motion. Intense coherent phonon waves can be exploited for various applications: terahertz modulation of light, generation of high frequency electric oscillations, nondestructive testing of microstructures, etc. Usually, both optical and acoustic high-frequency coherent phonons are excited optically by ultrafast laser pulses. The development of electrical methods of coherent phonon generation is an important problem.

An electric current flowing through a semiconductor can produce high-frequency coherent acoustic phonons. Two distinct cases are possible. If the current results from transitions of carriers between bound electron states, coherent phonon generation can occur if there is a population inversion between these states. Hopping vertical transport in superlattices and three barrier structures provides examples of mechanisms for the establishment of a population inversion and for stimulated generation of terahertz phonons and plasmons. If the current is due to free electron motion in an electric field, phonon amplification (generation) can be achieved via the Cerenkov effect if the electron drift velocity exceeds the velocity of sound. This effect is well-known for bulk samples. High drift velocities and large densities of electrons are necessary for practical use of the Cerenkov effect. Advanced technology of semiconductor heterostructures opens new possibilities to employ this effect for high-frequency phonon generation. Indeed, such phenomena as high electron mobility at large electron density and phonon confinement in a quantum well can greatly facilitate achieving phonon amplification and generation by electron drift. In this letter, we analyze the generation of high-frequency confined acoustic phonons under the electron drift in a QW layer.

Consider a symmetric heterostructure shown in Fig. 1 (a) with electrons confined in the layer \( A \) of thickness \( 2d \). Assuming isotropic elastic properties for both semiconductors \( A \) and \( B \) one can introduce the longitudinal, \( V_{LA} \) and \( V_{LB} \), and transverse, \( V_{TA} \) and \( V_{TB} \), sound velocities. If \( V_{TA} < V_{TB}, V_{LA}, V_{LB} \), then localization of acoustic waves near the embedded layer will occur. These localized waves propagate along the layer and decay outside it. There are two classes of the localized waves: the shear-horizontal (SH) waves with the displacement vector \( \vec{u} = (0, u_y, 0) \), and the shear-vertical (SV) waves.
with the displacement vector: \( \vec{u} = (u_x, 0, u_z) \). Dispersive relations for each class of waves are represented by a set of branches \( \omega = \omega_\nu(q) \), with \( \omega \) and \( q \) being the wave frequency and wave vector, \( \nu \) is an integer. For a given \( \nu \), localization of the waves depends on \( q \). Let \( \vec{u}_{\nu,q}(x, z) = \vec{u}_{\nu,q}(z)e^{iqx-\omega t} \) be solutions of the elastic equation describing the localized waves. Solutions with different "quantum numbers" \( \{\nu, q\} \) are orthogonal. We normalize the solutions by imposing the condition that

\[
\int_0^L d\vec{r} \vec{u}_{\nu,q}(\vec{r})^\dagger \chi_\nu(z) = \delta_{\nu\nu'} \delta_{qq'}.
\]

Consider the interaction of a localized mode with electrons assuming that a) only the lowest two-dimensional electron subband is populated and b) presence of higher subbands can be ignored. Then, setting the area of the layer equal to 1, the electron wavefunctions have the form \( \Psi_k(\vec{r}, z) = e^{i\vec{k}\cdot\vec{r}} \chi(z) \), where \( \vec{k} \) is the two-dimensional electron wavevector. We suppose that electrons interact with phonons via the deformation potential (DP); thus, the energy of this interaction is \( H = b \nabla \vec{u} \cdot \vec{\kappa} \), where \( b \) is the DP constant. Then the probability of transition between electron states \( \vec{k} \) and \( \vec{k}' \) due to emission or absorption of a confined phonon \( \{\nu, q\} \) is

\[
P^{(\pm)}(k, k'|\nu, q) = \frac{2\pi}{\hbar} |M(q)|^2 \left( N_{\nu,q} + \frac{1}{2} \pm \frac{1}{2} \right) \delta_{\nu\nu'} \delta_{qq'}
\]

\[
\times \delta_{k_x k'_x} \delta_{k_y k'_y} \left[ E(\vec{k}) - E(\vec{k}') + \hbar \omega_{\nu,q} \right] F(\vec{k}) \left[ 1 - F(\vec{k}') \right],
\]

where \( M(q) \) is the matrix element:

\[
M(q) \equiv b \left( \int_{-\infty}^{\infty} \nabla \vec{u}_{\nu,q}(z) \chi_\nu^d(z)dz \right) / \kappa(\nu,q),
\]

\( N_{\nu,q} \) is the phonon number of the mode, and \( F(\vec{k}) = F[k_x, k_y] \) is the electron distribution function. In Eq. 4 the upper signs correspond to emission, and the lower ones correspond to absorption processes. We take into account the effect of electron screening of the DP by introducing the electron permittivity \( \kappa(\nu,q) = 1 + 2\pi e^2 dA(q) B(qd) / \kappa \). Here, \( A(q) \) is the polarization operator of two-dimensional electrons:

\[
A(q) = -2 \sum_{k} \frac{F(\vec{k}) - F(\vec{k} - \vec{q})}{E(\vec{k}) - E(\vec{k} - \vec{q})},
\]

where \( \kappa \) is the dielectric constant and the factor \( B(s) \) is

\[
B(s) = \frac{1}{s} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\zeta d\zeta' \chi^2(\zeta) \chi^2(\zeta') e^{-s |\zeta - \zeta'|},
\]

Now, we introduce the kinetic equation for the phonon number of the mode \( \nu, q \):

\[
\frac{dN_{\nu,q}}{dt} = \gamma^{(+)}_{\nu,q} (1 + N_{\nu,q}) - \gamma^{(-)}_{\nu,q} N_{\nu,q} - \beta_{\nu,q} N_{\nu,q},
\]

where \( \gamma^{(\pm)}_{\nu,q} \) are parameters which determine evolution of \( N_{\nu,q} \) in time due to the interaction with electrons. Both parameters can be found easily by calculating the \textit{total rates} of emission and absorption of phonons of a given mode by the summation of Eq. (4) over all initial and final electron states. The parameter \( \beta_{\nu,q} \) describes phonon losses. They can include phonon scattering or phonon absorption due to non-electronic mechanisms, phonon decay due to anharmonicity of the lattice, etc. In Eq. (4) the terms which correspond to stimulated processes can be represented by \( \gamma^{(+)}_{\nu,q} - \gamma^{(-)}_{\nu,q} \). From Eq. (4) the phonon increment (decrement) equal to

\[
\gamma_{\nu,q} = \frac{m^*}{\pi \hbar^2 q} |M(q)|^2 \left( \mathcal{I}^{(+)}(q) - \mathcal{I}^{(-)}(q) \right),
\]

\[
\mathcal{I}^{(\pm)}(q) = \int_{-\infty}^{\infty} dk_y F \left[ \text{sign}(q) \frac{m^* \omega_{\nu,q}}{\hbar |q|} + \frac{1}{2} \right],
\]

Here, \( m^* \) is the effective mass.

Depending on the shape of the electron distribution function, \( F[k_x, k_y] \), the value \( \gamma_{\nu,q} \) can be either positive, or negative. If the phonon increment caused by the electron-phonon interaction is positive and, in addition, it exceeds phonon losses, \( \gamma_{\nu,q} > \beta_{\nu,q} \), the population of corresponding mode(s) should increase in time, i.e., we obtain the effect of phonon generation.

One can introduce the amplification (absorption) coefficient for the confined acoustic modes which describes the rate of increase in the acoustic wave intensity per unit length. We obtain the amplification coefficient via the phonon increment: \( \alpha_{\nu,q} = \gamma_{\nu,q} / V_g \), where \( V_g = d\omega_{\nu,q} / dq \) is the group velocity of the wave. The signs of \( \gamma_{\nu,q} \) and \( \alpha_{\nu,q} \) are determined by the factor \( (\mathcal{I}^{(+)} - \mathcal{I}^{(-)}) \), which is to be calculated from the distribution function. This factor can be interpreted as the difference in the populations of the electron states, which are involved in the processes of emission and absorption. If this factor is positive, one obtains a kind of "population inversion".

We suppose that the electrons drift in an applied electric field along the QW layer. Under the realistic assumption of strong electron-electron scattering, the distribution function can be thought as the shifted Fermi distribution:

\[
F[k_x, k_y] = F_F \left[ k_x - \frac{m^*}{\hbar} V_{dr}, k_y \right],
\]

where \( F_F(\vec{k}) \) is the Fermi function, \( V_{dr} \) and \( T \) are the electron drift velocity and temperature, respectively. From Eq. (4) for phonons propagating along the electron flux (\( q > 0 \)), we immediately find that \( \gamma_{\nu,q}, \alpha_{\nu,q} > 0 \) if the electron drift velocity exceeds the confined phonon phase velocity: \( V_{dr} > \omega_{\nu,q} / |q| \). This criterion is, in fact, the well-known condition of the Cerenkov generation effect. If \( q < 0 \) we always have \( \gamma_{\nu,q}, \alpha_{\nu,q} < 0 \).
Typically, both velocities, $V_{dr}$ and $\omega_{\alpha q}/|q|$, are much less than the average electron velocity. This implies that there is a relatively small disturbance of the Fermi function. Thus, to estimate $\gamma_{\alpha q}$ and $\alpha_{\alpha q}$ we will take into account the shift in $F(k_x, k_y)$. While calculating the screening effect [see Eq. (3)] we can neglect this shift and use just the Fermi function $F_F(k)$. The latter approximation finalizes the description of amplification of the confined phonons by the drifting electrons.

Now we shall apply these results to confined phonons of different symmetry. It is easy to see that the functions $w_x(z)$ and $w_z(z)$ always have different symmetry. We define the symmetric shear-vertical (SSV) modes as those with $w_x(z) = w_x(-z)$, $w_z(z) = -w_z(-z)$ and the antisymmetric ones with $w_x(z) = -w_x(-z)$, $w_z(z) = w_z(-z)$. For a symmetric QW, the electrons are coupled with the SSV phonons. The displacement field distribution for one of the confined SSV modes is presented in Fig. 2 (b).

**FIG. 2.** The amplification coefficient versus frequency for two lowest SSV phonon branches. $T=50$, 100, 200 and 300 K. Increasing of $T$ leads to decreasing of $\alpha$ at maximum. The values of $\omega$ and $q$ which correspond to the maxima of $\alpha$ at $T=50$ K are indicated on the dispersion relations shown in the Insert.

We have performed numerical calculations of the amplification coefficient for different heterostructures. We have found that two effects contribute critically to amplification: the phonon confinement effect through the matrix element of Eq. (2) and the nonequilibrium population of electron states through the factor $(I^+ - I^-)$. For $\Pi - V$ and SiGe heterostructures, the acoustic mismatch is typically small and the lowest mode is an antisymmetric SV mode. Consequently, all SSV modes have finite frequency onsets. This determines two important features: a low-frequency cut-off of the amplification and a nonmonotonous dependence of the matrix element $M(q/\omega)$. The population factor of Eq. (1) limits phonon amplification at high frequencies. As a result, amplification band for each SSV phonon branch is relatively narrow. Two typical amplification bands are illustrated in Fig. 2. These results are obtained for a p-doped Si/Si$_{0.5}$Ge$_{0.5}$/Si structure. The heavy hole subband is the lowest one in the strained SiGe layer. We set $d = 5 \text{nm}$, the hole density is taken as $10^{12} \text{cm}^{-2}$ and the drift velocity is $V_{dr} = 2.5 V_{TA}$ with $V_{TA} = 3.4 \times 10^5 \text{cm/s}$ for the SiGe layer. One can see that amplification coefficient of the order of tens to hundreds $\text{cm}^{-1}$ can be achieved for confined modes in the sub-THz frequency range.

These values of $\alpha$ are well above unavoidable phonon losses due to the effects of anharmonicity and scattering on isotopes. The condition of phonon generation in a single passage device, $\alpha L_x \gg 1$, can be realized for reasonable extensions of the structure, $L_x$. At the maximum of amplification, the phonon wavelength equals 160 $\text{Å}$ and the generated phonon flux is confined to a layer of thickness of about 200 $\text{Å}$. Thus, a short-wavelength and highly-collimated beam of the coherent phonons can be amplified and generated in perfect QW heterostructures.

In conclusion, we have found that the drift of two-dimensional electrons can result in Cerenkov instability of the phonon subsystem: the phonon modes confined near the QW layer and propagating along the electron flux are amplified. The amplification coefficient for these modes has a sharp maximum in the sub-THz frequency range. The amplification coefficient can exceed hundreds of $\text{cm}^{-1}$ for the mode almost confined within the QW layer. Our results suggest that a simple electrical method for generation of high-frequency coherent phonons can be developed on the basis of the Cerenkov effect.

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