What is the right form of the probability distribution of the conductance at the mobility edge?

In a recent letter, Slevin and Ohtsuki \cite{1} reported finite size scaling results for the Anderson metal to insulator transition for the orthogonal and unitarity classes of the single electron tight-binding(TB) model. The average value of the conductance \( G=(e^2/h)g \) at the mobility edge, as well as the distribution of the conductance at the critical point, \( p_c(g) \), were calculated. Their studies showed that \( p_c(g) \) is independent of the system size. It also does not show any dip around \( g=0 \), as the \( \epsilon \) expansion results \cite{2} suggest. These conclusions were based on numerical results of system sizes of \( N \times N \times N \), with \( N=6,8 \) and 10. We will present new numerical data that indeed shows that \( p_c(g) \) has a dip for small \( g \).

We have systematically studied the conductance \( G \) of the 3d TB model by using the transfer matrix technique \cite{3}, which relates the conductance \( G \) with the transmission matrix \( t \), i.e. \( G=(e^2/h)g \), with \( g=2\text{Tr}(tt^*) \). The \( g \) defined here is for both spins. In Fig. 1 we present the results of \( p_c(g) \) for three different sizes of \( N=5, 10, \) and 20.

The mobility edge \cite{1} is at \( W=16.5 \) and \( E=0.0 \). Notice that as the size of the system increases a dip is developed at \( g=0 \), which is not present in the results presented in Fig. 2 of Ref. 1. We therefore have a size dependent \( p_c(g) \), which has a dip at small \( g \). It is well known that \( p(g) \) for extended states is gaussian, while for localized states is log-normal. However, it is not well known either experimentally \cite{4} or theoretically what is the correct form of the probability distribution at the mobility edge. \( p_c(g) \) obtained \cite{2} in the \( \epsilon \) expansion in the field theory has a hole at small \( g \) in agreement with the numerical results presented here. Recent results \cite{3} for a 2d TB model in the presence of a strong magnetic field show that \( p_c(g) \) is very broad with a dip at small \( g \). The 2d \( p_c(g) \) is very different from the 3d \( p_c(g) \) presented here. We have also calculated the average value of the conductance at the critical point (\( E=0.0 \) and \( W=16.5 \)) for \( N=5, 10 \) and 20 for 20000, 10000 and 8000 random configurations respectively. The results are summarized in Table I.

\begin{table}[h]
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\( N \) & \(< g >\) & \(\sigma_g\) & \(< \ln g >\) & \(\sigma_{\ln g}\) \\
\hline
5 & 0.72 & 0.64 & -0.857 & 1.19 \\
10 & 0.78 & 0.66 & -0.727 & 1.11 \\
20 & 0.86 & 0.68 & -0.587 & 1.09 \\
\hline
\end{tabular}
\end{table}

Notice that both \( g \) and \( \ln g \) have very large standard deviations, as big as their average values. Our results for both \( < g > \) and \( < g >_g= e^{<\ln g >} \) for the \( N=10 \) case (0.78, 0.48) are larger than the results presented (0.58, 0.30) in table III of Ref. 1 for the same model. This difference might be due \cite{6} to the different boundary conditions used by Ref. 1 (fixed) and ourselves (periodic). For the 2d case \cite{6} it is shown that \( < g >=1.00 \) and \( < g >_g=0.88 \) for the infinite size system. If we extrapolate our finite-size results to infinite sizes we obtain that \( < g >=1.00 \) and \( < g >_g=0.70 \). Remember that \( \sigma_g \) is comparable to \( < g > \).

In summary, we have numerically calculated the full probability distribution of the conductance, \( p_c(g) \), at the Anderson critical point. We find that \( p_c(g) \) has a dip at small \( g \) in agreement with the \( \epsilon \) expansion results \cite{2}. The \( p_c(g) \) for the 3d system is quite different from that of the 2d quantum critical point \cite{3}. The universality or not of these distributions is of central importance to the field of disordered systems.

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