Promoting understanding ability of semigroup concept through learning model of concept attainment

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Abstract. This study aimed to improve the ability of the understanding of semigroup concept through Concept Attainment Model. The method of this study is quasi experiment. The design of this study is using one group pre-test & post-test design. Sample of the study is 34 students of mathematics pre-service teacher at Abstract Algebra course. Instrument used in this study is a test about semigroup concept which contain 4 items. Data were analysed qualitatively and quantitatively. This study found that the improvement of students’ ability of the understanding of semigroup concept was 0.68, (medium category) and the achievement of of the concept understanding ability of semigroup was 75.44 (high category). Thus, concept attainment learning model was effective in developing the ability of the understanding of the students about semigroup concept.

1. Introduction

The weaknesses of students in understanding the mathematical concepts in Abstract Algebra or Algebraic Structure are: 1. the weakness in the compilation of proof lines [1]. The isomorphic concept and other concepts in Abstract Algebra are difficult for some students to understand [2]. The weakness of other students is the weakness in understanding the concept of derivative functions, especially in relating logical relations between various variables [3]. The concept of function becomes a prerequisite concept in studying semigroup concept because semigroup are grupoid that have associative properties [4]. On the other hand, grupoid is built based on the concept of function. Failure in algebra learning is caused by the low interaction of creativity of students [2].

Various efforts made by researchers to correct the weaknesses of students in understanding abstract algebraic concepts, especially semigroup concepts have been widely carried out, starting from aspects of curriculum, generalization to learning strategies. The improvement of the ability of students towards abstract algebraic material through curriculum improvement emphasizes on the ability to generalize concepts from a context [5]. In addition, the improvement of the students’ understanding of algebraic concepts is carried out through the improvement of teaching strategies [6]. Other improvements are the improvement of group concepts and isomorphism through informal strategies [7]. Inductive strategies are used to improve the understanding of algebraic materials [6]. The learning objectives of model of concept attainment builds and develops their understanding of concepts and critical thinking [8] can improve learning outcomes [9].

Concept attainment models are based on constructivist theory [10]. Students construct their own knowledge. The Concept Attainment learning model is effective in training activities for chemical workers [11]. This shows that learning model of concept attainment can be conducted for all topics of learning activities. In addition, it can be conducted at all levels of education, starting from elementary

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school to college [12]. Learning the Concept Attainment model is effective in teaching in secondary school [13].

This study will use the concept attainment learning model to improve and achieve the ability of the understanding of semigroup concepts.

The problem statements in this study were as follows:
a. How big is the achievement of the ability of the understanding of semigroup concepts, the students who were taught through the concept attainment learning model?
b. How big is the improvement of the ability of the understanding of semigroup concepts, the students who were taught through the concept attainment learning model?

2. Method
The type of this study was a quasi-experimental research with one group pre-test post-test design. The design of this study was described as follows [14].

\[
O_1 \times O_2
\]

- \( O_1 \) = Pretest
- \( O_2 \) = Posttest
- \( x \) = Treatment of the concept attainment learning model

Population of this study was 60 students who took Abstract Algebra of Even Semester Academic Year 2018-2019 at the Study Program of Mathematics Education, Teacher Training and Education Faculty, Universitas Bengkulu. The sample of this study was 34 students. The sampling was conducted purposively by adjusting the lecture schedule. The instrument about semigroup contained 4 items in the form of essay. Those four items were valid with a range of 0.5-1, while the device was reliable at 0.82 (high reliability). The data was analyzed qualitatively and quantitatively.

3. Result and Discussion
There were two measurements to find out about scores of semigroup after obtaining the concept attainment learning model, namely the achievement and improvement of the ability of the understanding of semigroup concept. The achievement of the scores of the ability of the understanding of semigroup concept can be seen from the scores of post-test, while the improvement of the scores of the ability of the understanding of semigroup concept can be seen from the scores of \( N \)-gain or the normalized gain. \( N \)-gain is an absolute gain divided by the possible maximum gain (ideal), namely:

\[
N - gain = \frac{\text{posttest score} - \text{pretest score}}{\text{ideal maximum score} - \text{pretest score}}
\]

Then, the data of \( N \)-gain is categorized with interpretation criteria [15] as seen in Table 1.

| \( N \)-gain Criteria | \( N \)-gain Interval |
|------------------------|----------------------|
| High                   | \( N \)-gain > 0,7    |
| Medium                 | \( 0,3 < N \)-gain \leq 0,7 |
| Low                    | \( N \)-gain \leq 0,3 |

On the other hand, the criteria of the score achievement of the ability of the understanding of semigroup concept used the criteria of Benchmark Reference Assessment as seen on Table 2.
Table 2. Criteria of the Achievement of the Ability of the Understanding of Semigroup Concept

| Criteria   | Score          | Score           |
|------------|----------------|-----------------|
| High       | Score ≥ X + S  | Score ≥ 66.67   |
| Medium     | X − S ≤ Score < X + S | 33.33 ≤ Score < 66.67 |
| Low        | Score < X − S  | Score < 33.33   |

Notes:

\[ X = \frac{1}{2} \text{ideal maximum score} = 50 \]
\[ S = \frac{1}{3} X = 16.67 \]

A grupoid \((G, \ast)\) is called semigroup, if \((x \ast y) \ast z = x \ast (y \ast z)\) for every \(x, y, z \in G\) [16]. Mathematical concepts contained in semigroup concepts include associative concepts, closure, grupoid, variable, and function. These concepts are prerequisite concepts for learning semigroup concepts. Based on the results of the semigroup material prerequisite ability test, the average score was 68. This means that the ability of the prerequisite concept of the students was sufficient to take semigroup lectures. The students have taken lectures that study the prerequisite concepts, namely Calculus, Fundamental Mathematics, and Algebra.

The ability of the understanding of the students towards the semigroup concepts is explained in Table 1.

Table 3. Descriptive Data of the Score of the Ability of the Understanding of Semigroup Concept

| Data          | Pre-test | Post-test | N-gain |
|---------------|----------|-----------|--------|
| N             | 34       | 34        | 34     |
| Minimum Score | 0        | 35        | 0      |
| Maximum Score | 70       | 100       | 1      |
| Average       | 23.68    | 75.44     | 0.68   |
| Standard Deviation | 18.10    | 18.44     | 0.26   |

Pre-test about semigroups provided to students before being provided the concept attainment learning model resulted in the initial abilities of the students of 23.68. This shows that the students did not understand the semigroup concepts. The students did not know that semigroups are related to grupoid, associative properties, operation, and function. An example of the result of the student’s work in solving the following pre-test questions is described below.

Item number 3: Show that the set of real numbers of \(R\) with binary operations.
\[ x \ast y = x + y - xy \text{ with } x, y \in R \text{ as a semigroup.} \]

One of the student’s answers (known as A) is described in Figure 1.

![Figure 1. The Pre-test Answer of Student A for the Item Number 3](image-url)
Based on the student’s answer in Figure 1, the student did not understand closeness and associative properties.

The treatment of the concept attainment learning model can be conducted with four stages of learning, such as: (1) Introduction; (2) Sample and Formulating a Hypothesis; (3) Cycle of Analysis; and (4) Closing and Implementation [9]. The first stage was that the lecturer introduced the definition of semigroup concept. The second stage was that the lecturer provided the exemplars of semigroup concept, formulated a hypothesis, and prove it. The third stage was that the lecturer demonstrated the flow of problem solving. The fourth stage was that the lecturer explained the implementation of semigroup concept at other situations.

The concept attainment learning model has been able to provide the semigroup materials properly to the students. The students were easier to understand semigroup concept more meaningfully and comprehensively. The significance of the concept was shown by the ability of the students to explain the stages of the emergence of semigroup definitions. In addition, the students could explain the mathematical concepts which formed semigroup concept. Furthermore, the students could implement semigroup concept comprehensively at other situations like implementing semigroup concept at the group concept. The students could explain that a group is a set $G$ together with an operation ($*$) such that the following four properties hold: (1) Closure; (2) Identity; (3) Inverse; and (4) Associative low [17].

After being provided the concept attainment learning model, the results of the understanding of semigroup concept showed that the achievement of the understanding of semigroup concept was much better with the average score of 75.44. Based on the criteria like Table 2, the achievement of the ability of semigroup concept is included in the high category. This showed that the concept attainment learning model could develop the ability of the understanding of semigroup concept properly.

The improvement (N-gain) of the understanding of semigroup concept was 0.68, including in the medium category. The improvement showed that several students experienced the change of knowledge about semigroup. At first, the students did not know about semigroup definition. Then, they knew about it. The students understood semigroup concept correctly. Semigrup is grupoid which has associative properties. They also understood grupoid as an algebraic structure with one binary composition [16]. Besides, the students could show that a mathematical statement is a semigroup. This is like the completion of the item number 3 of student A in Figure 2.

Figure 2. The Post-test Answer of Student A for the Item Number 3
The number of the students for each category in the posttest (achievement) and N-gain (improvement) of the score of the ability of the understanding of semigroup concept is presented in Figure 3.

![Figure 3. The Number of Students based on the Category of Posttest and N-gain](image)

Based on Figure 3, the percentage of the students with the achievement and improvement of the scores in the high category is more than the students with the medium and low categories.

The achievement and improvement of the ability of the understanding of semigroup concept contain the understanding of the forming process of semigroup definition, starting from set, function, binary composition (operation), algebraic structure, closure properties, associative properties, grupoid, and semigroup. The students started to understand that operation is a function. They could understand that ‘operation’ is related to ordered pair and function. They could explain that an operation \(*\) on \(A\) is a rule which assigns to each ordered pair \((a, b)\) of elements of \(A\) exactly one element \(a * b\) is \(A\) [18]. Besides, there was an improvement of the understanding of semigroup definition which is a grupoid that has associative properties. The students explained that if \((G, o)\) is a grupoid and \(\forall a, b, c \in G\) applies \((a o b) o c = a o (b o c)\) (associative properties), then \((G, o)\) is called as semigroup [19].

Based on the results of the study, there were 23 of 34 students that experienced the change of the knowledge about semigroup. In addition, the concept attainment learning model could build semigroup concept for the students by 68%. Through the concept attainment learning model, the students could explain about: 1) the definition of semigroup concept; 2) exemplars and non-exemplars of semigroups; and 3) Showing a mathematical statement that is a semigroup or not a semigroup.

The advantages of the concept attainment learning model to be able to build the ability of the understanding of the students towards semigroup concept are: 1) Being able to explain a concept in order; 2) Presenting exemplars and non-exemplars of the concept to be learned [10]; and 3) Having a systematic flow in proving a hypothesis, our approach to concept-attainment experiments emphasizes logical structure. A logical model is presented that can determine from any conceivable combination of positive and negative instances what concept or concepts are instantiated [20].

The descriptive results were strengthened with the result of \(t\)-test to find out the differences of the ability of the understanding of semigroup concept between pretest and posttest. The results can be shown in Table 4.
Table 4. The Statistical Test of the Test Results between the Pretest and Posttest Scores with the Paired Samples Test

| Paired Differences | Mean | Std. Dev. | Std. Error Mean | 95% Confidence Interval of the Difference | t | df | Sig. (2-tailed) |
|--------------------|------|-----------|-----------------|------------------------------------------|---|----|----------------|
| Pre-test – Post-test | -51.7647 | 21.3158 | 3.6556 | -59.2021 -44.3272 | -14.160 | 33 | .000 |

Based on Table 4, the difference of the pretest and posttest scores of the ability of the understanding of semigroup concept was very significant with a significance of 0.00. Thus, it can be concluded that the score improvement of the ability of the understanding of semigroup concept was very significant. This means that the concept attainment learning model was effective in developing the ability of the understanding of the students about semigroup concept.

4. Conclusion

The concept attainment learning model could improve the ability of the understanding of the students about semigroup concept of 0.68 (the medium category) and achieve the ability of the understanding of semigroup concept of 75.44 (the high category). The score improvement of concept understanding ability of semigroup concept was very significant, which means that the concept attainment learning model is effective in developing the ability of the understanding of the students about semigroup concept.

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