Development of a mathematical model for determining the EMF of the secondary winding of a measuring current transformer based on the Rogowski coil

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Abstract. This article is devoted to the development of a mathematical model for determining the electromotive force (EMF) of the secondary winding of a measuring current transformer based on a Rogowski coil. The study focuses on determining the EMF in the secondary winding of the current transformer at different positions of the primary bus, its forms, as well as the overall dimensions of the whole measuring equipment. The study was conducted on the basis of finite element modeling and elements of the experimental design techniques. Finite-element models have been developed for various primary bus shapes in the Femm program. Regression models were obtained for determining the magnetic flux passing through the Rogowski coil in order to determine the EMF in the secondary winding using the Statistica program. The research results showed the importance of taking into account the position of the primary bus relative to the center of the Rogowski coil, as well as the influence of the shape of the primary bus on the measurement result of EMF in the secondary winding of the current transformer.

1. Introduction
Currently, the constitution of an intelligent energy system with an active-adaptive network is actively developing in the power industry. The key component of this network is the “digital” substation. Currently, active work is underway to study this technology [1-4], as well as its implementation [5-7]. Such substations are equipped with modern control systems, digital devices, including measuring devices, and technologies of conversion of information. However, these technologies are relatively new and do not fully meet the requirements for reliability and efficiency of work. This statement is also true for technologies for diagnosing the state of digital substations and its elements. According to [8], on the example of a current transformer, the most common innovative measuring equipment at the substations of the new generation is the current transformer based on Rogowski coil. This is due to a number of its advantages, for example, lack of saturation of core, low weight and size, meeting the requirements in a wide range of flowing currents, etc. Rogowski coils are becoming increasingly used as primary converters for relay protection [9-11]. Practical interest for the implementation of measuring current transformers based on Rogowski coil are open coils. This raises the question of taking into account the dimensions of the Rogowski coil and the shape of the primary winding of the...
current transformer for the optimal ratio of dimensions in order to achieve optimal measurement accuracy. Thus, this article proposes the development of a mathematical model of a current transformer based on a Rogowski coil, which allows to determine the EMF in the secondary winding, taking into account the shape of the primary winding, as well as the dimensions of the measuring equipment.

2. Object of research

In a current transformer, the Rogowski coil is used as a current sensor, which is a hollow coil of special design (Figure 1). The Rogowski coil is wound uniformly around the annular cross section of the nonmagnetic frame [12].

![Figure 1. The principle of operation of the Rogowski coil](image)

The output voltage induced in the coil is determined by the equation:

\[ v(t) = -\frac{d\psi}{dt} \]  

where \( \psi \) - total magnetic flux in the coil.

The total magnetic flux \( \psi \) in the calculation of the Rogowski coil is determined by the equation:

\[ \Psi = \mu_0 \mu_r n \oint dl \int H dS \]  

where

- \( l \) - core length;
- \( n \) - winding density (windings per unit length);
- \( S \) - core cross-section area;
- \( H \) - magnetic field intensity;
- \( \mu_0 \) - permeability of air;
- \( \mu_r \) - relative permeability of the core material.

3. Materials and methods

As can be seen from formulas (1,2), the determination of the output voltage in the secondary winding is made difficult by the complexity of the calculation formulas. To simplify the procedure for determining the EMF in the secondary winding and at the same time obtaining mathematical model, it is proposed to use physical modeling using elements of the experimental design techniques.

To develop a mathematical model of a current transformer based on a Rogowski coil, preliminary studies were conducted to identify the influence of the shape of the primary bus and its centering on the accuracy of measuring the magnetic flux in the secondary winding of the current transformer. According to [12], the Rogowski coil can be made round, oval, rectangular, teardrop-shaped, and also in the form of an open-ended fork, which must be considered when designing, but the most common are round or rectangular shape, due to the fact that other cross-sectional shapes are less practical. According to [13], the shape of the primary winding can also be different - round, rectangular, box-shaped and four-strip. Preliminary studies include an experiment with a round primary bus, in which the bus diameter, the distance between the bus and the Rogowski coil — R, the X-axis and Y-axis
offset, and the diameter of the turn (Figure 2, a) were considered as factors. To solve the problem of determining the magnetic flux passing through the Rogowski coils, the numerical method is used - the finite element method [14]. For the finite element simulation, the Femm program was used. Based on the fact that the core of the secondary winding of the Rogowski coil is built on a non-magnetic material, air is chosen as the core material.

![Figure 2. Rogowski Coil:](image)

a) Rogowski coil with a round primary winding; b) Rogowski coil with a rectangular primary winding

Specifying the parameters \(d = 51.687 \text{ mm}, R = 78.329 \text{ mm}, r = 11.867 \text{ mm}\) and zero displacement of the primary winding relative to the center, finite-element modeling was performed to determine the magnetic flux in the Femm program:

\[
\Phi = 3.771 \times 10^{-5} \text{ Wb}.
\]

It was found that a statistically significant factor affecting the accuracy of determining the magnetic flux is the displacement of the primary winding relative to the center. The relative error at offset \(S1 = 20 \text{ mm}, S2 = 20 \text{ mm}\) is 2.56%, other factors are statistically insignificant, which indicate the importance of centering the primary winding.

To identify the influence of the shape of the primary bus on the result of determining the magnetic flux passing through the Rogowski coil, an additional experiment was conducted with a rectangular bus with dimensions \(a = 98.527 \text{ mm}; b = 8.319 \text{ mm}; r = 11.867 \text{ mm}; R = 78.329 \text{ mm}\) (Figure 2, b). The following result was obtained:

\[
\Phi_{\Sigma} = \frac{\sum_{n=1}^{2} \Phi_n}{2} = 4.015 \times 10^{-5} \text{ Wb}.
\]

Preliminary studies indicate the importance of taking into account the shape of the primary winding (for example, round and rectangular), since the difference between the measurement results is 6.5%. Thus, based on the obtained simulation results, it is required to develop a universal model of current transformer based on the Rogowski coil, which will take into account, in addition to the dimensions of the Rogowski coil, the shape of the primary winding of the current transformer.

With this consideration in mind, the EMF can be determined from the following system of equations:

\[
\begin{align*}
\psi &= f(i, G) \\
e &= f(\omega, n, t, \psi)
\end{align*}
\]

(3)

where \(n\) - number of turns of the secondary winding of the Rogowski coil, \(\psi\) - magnetic flux, \(i\) - current passing through the primary winding, \(G\) - size of the entire measuring equipment, \(t\) - time, \(\omega\) - cyclic frequency.

Rewrite the first equation using the basic formula of the transformer EMF:

\[
\begin{align*}
e_a &= \omega \cdot n \cdot \psi_a \\
\psi_a &= f(i, G)
\end{align*}
\]

(4)

According to (4), to determine the potential, it is necessary to determine the magnetic flux. The method of evaluating the magnetic flux depends on the shape of the primary bus. With the rectangular shape of the primary bus, the measurement is carried out at positions 1 and 2, after which it is summed.
up and subsequently averaged; with a round form, measurement is carried out at position 1 (Figure 2, a, b).

4. Development of mathematical models of measuring current transformer based on Rogowski coil

4.1. Mathematical model of measuring current transformer on the basis of a Rogowski coil with a round bus

To obtain a regression model by applying the experimental design techniques, the following intervals of variation of the factors R, r, I with d = 10 mm were chosen: R = 101.5 ± 20 mm, r = 10 ± 3.5 mm, I = 100 ± 10 A. The planning matrix of experiment is presented in table 1.

| №  | X1  | X2  | X3  | Ф, 10^-6 Wb |
|----|-----|-----|-----|--------------|
| 1  | +   | +   | +  | 110          | 2.801        |
| 2  | –   | +   | +  | 110          | 4.129        |
| 3  | +   | –   | +  | 110          | 2.248        |
| 4  | –   | –   | +  | 110          | 3.311        |
| 5  | +   | +   | –  | 90           | 2.292        |
| 6  | –   | +   | –  | 90           | 3.379        |
| 7  | +   | –   | –  | 90           | 1.839        |
| 8  | –   | +   | –  | 90           | 2.709        |

The results of the experiment were processed using the Statistica program (Figure 3):

Figure 3. Regression model of the first order of current transformer based on Rogowski coil.

The adequacy of the model is tested using the Fisher criterion. For this, it is necessary to compare the critical Fcr and the calculated Fcalc values of Fisher criterion. The critical value is selected from the Fisher distribution tables using the numbers of degrees of freedom f1 and f2, in this case Fcr = 3.01. The estimated value of the Fisher criterion, according to Figure 3, is 2.136. As a result of the fulfillment of the condition Fcr > Fcalc, the obtained model is adequate:

Fcr > Fcalc; 3.01 > 2.136.

\[ f_1 = N - L, \]
\[ f_2 = N (n - 1), \]

where N = 2^3 = 8 – number of experiments;
L - the number of significant coefficients of the regression equation – 4;
n – number of repetitions experiments – 3.

\[ f_1 = 8 - 4 = 4. \]
\[ f_2 = 8(3-1) = 16. \]

However, the adequacy of this model was obtained with narrow ranges of variation of factors. This may indicate the nonlinear nature of the factors under study. As a result of additional studies, it was detected that the dependence \( I = f(\Phi) \) is linear (Figure 4), and the dependence \( R = f(\Phi) \) is nonlinear, which indicates the need to develop a second-order model for an adequate description of the process (Figure 5).

The factors taken are \( d, R, r, I \). Interval of variation \( d = 31.104 \pm 20.583 \) mm; \( r = 15 \pm 3.133 \) mm; \( R = 90.529 \pm 12.2 \) mm; \( I = 80 \pm 28.288 \) A. The intervals of variation are taken in such a way that the smallest value of \( R \) is the largest radius of the round bus \( d \). As a result of the experiment, an adequate model was obtained with 3% deviations between repeated experiments:

\[ F_{cr} > F_{calc} \quad 1.85 > 0.45. \]

Regression model in coded values:

\[ \Phi = 2.97592 - 0.40608 \cdot X_2 + 0.0577 \cdot R^2 + 0.56926 \cdot X_3 + 
+ 1.06825 \cdot X_4 - 0.07719 \cdot X_2 \cdot X_3 - 0.14259 \cdot X_2 \cdot X_4 + 0.20221 \cdot X_3 \cdot X_4. \]
Regression model in named values:

\[ \Phi = 0.42127 - 0.0401 \cdot R + 0.00039 \cdot R^2 + 0.182 \cdot r + 0.04094 \cdot I - 0.00202 \cdot R \cdot r - 0.00041 \cdot R \cdot I + 0.00228 \cdot r \cdot I. \]

Rewrite (4) for a round bus:

\[ e_a = \omega \cdot n \cdot \psi_a \]
\[ \psi_a = 0.42127 - 0.0401 \cdot R + 0.00039 \cdot R^2 + 0.182 \cdot r + 0.04094 \cdot I - 0.00202 \cdot R \cdot r - 0.00041 \cdot R \cdot I + 0.00228 \cdot r \cdot I. \]

4.2. Mathematical model of measuring current transformer based on a Rogowski coil with a rectangular bus

For a rectangular bus (Figure 2, b) a similar experiment was conducted. The factors taken are the length and width of the bus, instead of the diameter. Considering the previous study, a second-order model is used with the following intervals of variation of factors: \( a = 62.5 \pm 36.027 \text{ mm} \); \( b = 5.5 \pm 2.819 \text{ mm} \); \( r = 15 \pm 3.133 \text{ mm} \); \( R = 95.529 \pm 15.333 \text{ mm} \); \( I = 80 \pm 31.328 \text{ A} \), with \( N = 43 \). The intervals of variation are chosen in the same way as in the previous experiment. The number of repeated experiments - 3, with a deviation between them equal to 3%.

As a result of the experiment, the resulting model is adequate:

\[ F_{cr} > F_{calc}; 1.58 > 0.8. \]

Rewrite (4) for a rectangular bus:

\[ e_a = \omega \cdot n \cdot \psi_a \]
\[ \psi_a = -0.1527 + 0.01007 \cdot a - 0.0348 \cdot R + 0.00036 \cdot R^2 + 0.18111 \cdot r + 0.04068 \cdot I - 0.00191 \cdot R \cdot r - 0.00039 \cdot R \cdot I + 0.00221 \cdot r \cdot I - 0.00009 \cdot R \cdot a. \]

The number of turns \( n \) is determined from the formula [15]:

\[ n = \frac{1}{d_w}k, \]

where \( l \) – coil length, \( d_w \) – winding wire diameter, \( k \) – fill factor.

5. Conclusion

As a result of the conducted research, the influence of the displacement of the position of the primary bus relative to the center of the Rogowski coil on the measurement of the magnetic flux, and as a result, the EMF in the secondary winding, was revealed. The relative error at offset \( S_1 = 20 \text{ mm} \), \( S_2 = 20 \text{ mm} \) is 2.56%, which indicate the importance of centering the primary winding. At the same time, the importance of taking into account the shape of the primary winding of the measuring current transformer on the basis of the Rogowski coil was revealed - the difference between the measurement results (using an example of a round and rectangular bus) is 6.5%. Two mathematical models were obtained that take into account both the shape of the primary bus and the dimensions of the measuring equipment for determining the EMF in the secondary winding of the current transformer based on the Rogowski coil.

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