Proton and electron acceleration through magnetic turbulence in relativistic outflows

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Abstract. Low frequency electromagnetic turbulence is generated in relativistically outflowing plasma that sweeps up particles from the surrounding environment. Electrons are energized by stochastic gyroresonant acceleration with the turbulence produced by the isotropization of captured protons and charged dust. Protons are accelerated stochastically by the dust induced turbulence. Analytical solutions for the proton and electron energy distributions are obtained and used to calculate broadband synchrotron emission. The solutions are compared with generic spectral behavior of blazars and gamma-ray bursts. Dust captured by a blast wave can generate turbulence that could accelerate protons to very high energies.

Key words: acceleration of particles – shock waves – turbulence – cosmic rays – dust – gamma rays: bursts

1. Introduction

Measurements of radio and optical polarization of blazar jet sources indicate that nonthermal lepton synchrotron emission is the dominant radiation mechanism at these frequencies (Begelman et al. 1984). The radio through X-ray afterglow and the prompt soft-gamma ray emission observed (Costa et al. 1997, van Paradijs et al. 1997, Djorgovski et al. 1997, Frail et al. 1998) from gamma-ray bursts (GRBs) is attributed to the same process (Tavani 1996, Paczyński & Rhoads 1993, Meszáros & Rees 1993). In the standard blast-wave physics that has been developed (Mészáros & Rees 1992, Vietri 1997, Waxman 1997) to explain the long wavelength afterglow emission from GRBs, particles captured from the external medium energize the blast wave and cause blast wave deceleration. When a blast wave sweeps up material from the surrounding medium, the free energy of the captured particles initially resides in the more massive protons, nucleons, or charged grains and dust, if the latter can survive capture by a relativistic wind, as assumed here. The less massive nonthermal electrons and positrons emit, however, most of the radiant energy. In order that high radiative efficiencies are possible, it is therefore required that there is efficient transfer of energy from the swept-up massive particles to electrons, or to very high-energy protons emitting synchrotron radiation or photons of short wavelengths. Major uncertainties in modeling relativistic outflows involve magnetic field generation and the mechanism for dissipating and transferring the free energy of protons, ions and dust particles to lighter particles (Meszáros, Rees & Papathanassiou 1993, Chiang & Dermer 1999). Non-thermal electrons and protons may both undergo first-order Fermi shock acceleration in relativistic blast waves to form power-law distributions reaching to very high energies, and the origin of ultra-high energy cosmic rays (UHECRs) has been attributed to shock acceleration of protons in the GRB blast waves (Vietri 1997, Waxman 1997). This mechanism has recently been called into question (Gallant & Achterberg 1999). Following the first shock crossing by a particle, subsequent cycles do not permit large gains of energy because the particle is captured by the shock before it has been scattered through a large angle. A stochastic mechanism for particle energization in relativistic shocks avoids these difficulties, but requires strong magnetic fields to retain UHECRs within the shell (Waxman 1997). However, the detailed physics of stochastic gyroresonant acceleration was not considered.

In this paper, we show that the magnetic turbulence generated by the capture of heavy charged particles can accelerate lighter charged particles to high energies through stochastic gyroresonant acceleration. This could account for the hard spectra observed from GRBs and blazars during their flaring states. The acceleration of particles to high energies could produce UHECRs through gyroresonant acceleration in relativistic outflows. The question of the survivability of dust against sublimation by the intense radiation fields of the GRB is addressed in the Appendix.
2. Particle energization in relativistic outflows

2.1. The relativistic pick-up model

A two-stream instability is formed when a blast wave with an entrained magnetic field encounters a medium of density $n_i^*$. According to the recent work of Pohl & Schlickeiser (2000), this causes the captured protons and electrons to rapidly isotropize in the blast wave plasma. The isotropization process operates on the comoving time scale (eq. (74) of PS2000) $t_{iso}(s) \equiv 0.7n_{b,s}/\Gamma_{300}n_i^*$, where $\Gamma = 300\Gamma_{300}$ is the bulk Lorentz factor of the blast wave and $n_{b,s}$ denotes the comoving blastwave density in units of $10^8$ protons cm$^{-3}$. The blast wave particle density in the comoving frame is given by $n_b(cm^{-3}) \sim E/(\Gamma_0m_pc^24\pi\Delta r\Delta r) \sim 10^7E_{54}/r_{10}^3$, and is implied by depositing an initial energy $10^{54}E_{54}$ ergs/(4$\pi$ sr) in a blast wave shell with comoving radius $\Delta r \sim r/\Gamma_0$, where $\Gamma_0$ is the initial blastwave Lorentz factor. Alfvén magnetic turbulence is generated by the incoming protons and electrons with an energy density (eq. (53) of PS2000) $U_A \equiv \beta_\Lambda n_i^*c^2\Gamma^2(1-1/\Gamma^2)$, where $\beta_\Lambda c$ is the Alfvén speed. The total energy density of the swept-up medium is $\sim 10^8\rho_{10}$ ergs/cm$^3$.

PS2000 calculated the emitted radiation in the thick-target limit when the captured, isotropized protons undergo nuclear inelastic collisions with thermal particles in the blastwave. Although this process involves the fraction $\sim (1-\beta_\Lambda)$ of the incoming energy retained in the protons, it operates on the comoving pion production time scale $t_{pp}(s) \sim 1.4 \times 10^7/n_{b,s}$, where $n_{b,s}$ denotes the comoving blastwave density in units of $10^8$ protons cm$^{-3}$. If the acceleration rate is sufficiently rapid, pion production could produce low-level prompt and extended emission in GRBs or blazars if $n_b$ remains at the estimated level. Moreover, luminous radiation from pion-decay electrons and positrons is only produced if $t_{pp}$ is much shorter than the diffusive proton escape time scale from the blast wave, given by $t_{esc}(\delta\rho/\kappa) \equiv r^2/(\Gamma_0ct_{iso})^2$, noting that the spatial diffusion coefficient $\kappa \equiv c^2t_{iso}$. This requires that $n_{b,s}n_i^*r_{10}^2 \gg 15\Gamma_{300}$, which holds well in jets but is only marginally satisfied in GRB blast waves.

2.2. The role of captured charged dust

Here we extend the work of PS2000 by considering the process of channeling the plasma turbulence energy into the energy of the swept-up primary electrons through gyroresonant interactions. As demonstrated in that paper, incoming charged particles with mass $m_i$, charge $Q_i$, and Lorentz factor $\Gamma$ generate Alfvénic turbulence with parallel wavenumbers $|k| > R_j^{-1}$, where the Larmor radius

$$R_j = m_jc^2\sqrt{1-1/(Q_j|e|B_0)}$$

and $B_0$ is the entrained magnetic field in the frame of the blastwave. For simplicity, we assume that the magnetic field is parallel to the blast wave velocity, and generalize the calculation of PS2000 by including the effects of negatively charged dust. According to eqs. (48)-(50) of PS2000, we obtain the power spectrum of backward (−) and forward (+) propagating Alfvén waves given by $I_{-}(k) \propto |Z(k)|$ and $I_{+}(k) \propto I_0^2(k)/|Z(k)|$, where $I_0(k)$ is the initial turbulence spectrum before isotropization, and

$$Z(k) = -\frac{1}{2} \frac{\omega_{p,i}}{c} \frac{n_p B^2}{n_b} \left(1 - \frac{(R_j/k)^{-1}}{H[|k| - R_j^{-1}]} \right).$$

The term $\omega_{p,i}(s^{-1}) = 1.3 \times 10^3\sqrt{n_b}$ is the proton plasma frequency and $H[x]$ denotes the Heaviside step function. This expression assumes $I_0(k) \ll |Z(k)|$ and makes use of the charge-neutrality condition $Q_{dnd} + n_e = n_0 = n_i^*$, where we assume that all dust particles have the same charge. A number of processes influence the charging of dust grains in a proton-electron-plasma. In the absence of any other process due to the higher electron mobility compared to the ions, dust particles will absorb more electrons than protons and thus attain a negative potential of order $-2.5k_B T_e/e$ (Spitzer 1968). However, a number of competing processes like neutral-atom collisions and ion impacts on the grain surface lead to electron loss from the dust grains. Also, a significant ultraviolet photon flux can charge the grain by the photoelectric effect to a positive potential. All these processes, together with poorly known grain properties, make an exact determination of the grain charge impossible (see e.g. McKee et al. (1977) for the variety of grain properties resulting from model calculations under various assumptions). Here we follow the argumentation of Ellison, Drury & Meyer (1997).

If the grain potential is $\phi$, then the charge on a spherical (of radius $a$) grain is of order $q = eQ_d \approx 4\pi\epsilon_0\omega_0\phi$, so that $Q_d \approx 700a/(10^{-7}m)(e/10^5)$, which is well in jets but is only marginally satisfied in GRB blast waves.

It is important to note that the densities of the incoming protons, electrons and dust particles are much smaller than the density of the blast wave plasma $n_b$, so that the properties of the low-frequency Alfvén waves, carried by the blast wave plasma, are not modified by the captured particles. Mathematically, this corresponds to the statement that the real part of the Alfvénic dispersion relation is solely determined by the blast wave electrons and protons. However, the free energy of beams of incoming protons, electrons and dust particles gives rise to a non-zero
imaginary part of the Alfvénic dispersion relation leading to a positive growth rate of backward moving (in the blast wave plasma) Alfvén waves. Moreover, for relativistic beams (Miller [1982] the growth rate of longitudinal electrostatic waves is much smaller than the growth rate of transverse Alfvénic waves, so that the free energy is dissipated by isotropising the incoming beam (i.e. capture) rather than heating the blast wave plasma by plateauing.

Two comments on the equilibrium wave spectrum (1) are appropriate:

(1) According to the quasilinear relaxation theory the equilibrium wave spectrum given in Eq. (1) is achieved formally after infinitely long time $t_{ql} \to \infty$ by transferring the free energy in the initial particle-beam-distributions into plasma waves. Numerical simulations of the electrostatic beam instability (see e.g. Grognard 1975) indicate that this asymptotic equilibrium distribution is established after $t_{ql} \simeq 100t_{iso}$.

(2) The wave spectrum in Eq. (1) is calculated from quasilinear wave kinetic equations that only take into account the wave growth from the unstable particle beams but neglect wave-wave interactions. Since the latter scale with the total magnetic field fluctuation energy density $(\delta B)^2$, this is justified as long as the total wave energy density is small to the initial energy density of the beam particles. Because according to Eqs. (53) and (64) of PS2000 $(\delta B)^2/(4\pi n_0) = \beta_A < 1$ this is indeed fulfilled.

The turbulence spectrum (1) leads to the quasilinear isotropization time scale $t_{iso}$ given above. This time scale and $t_{ql} = 100t_{iso}$ are much shorter than the blastwave crossing time $t_{bc}(s) \sim \Delta r/c \sim 1000r_{16}T_{300}$. Fig. 1 shows the resulting turbulence spectrum of backward propagating, left-handed circularly polarized Alfvén waves for a charge-weighted dust/proton density ratio $\epsilon = Q_d n_d/n_p = 0.01$, and a charge-weighted dust/proton mass ratio $Y_d = m_d/(Q_d m_p) = 10^6$. The relative contributions of dust to the turbulence spectrum (1) is determined primarily by the parameters $\epsilon$ and $Y_d$. Most of the magnetic turbulence energy density resides at low wave numbers and is provided by the charged dust for this value of $\epsilon$.

The total dust mass $M_d = m_d n_d = Y_d \epsilon M_p$, where the total proton mass $M_p = m_p n_p$. Thus the total mass in dust for the calculation in Fig. 1 is $\sim 10^4$ times the mass in the ionized gas, as might be found in the weakly ionized, low temperature star-forming environments that harbor GRB sources. By contrast, the relative mass in dust that in protons would be much less in the environments of active galactic nuclei, where the intense UV and X-ray radiation field would photo-disintegrate the dust.

2.3. Stochastic gyroresonant acceleration of electrons and protons

The primary electrons and protons may increase their energy by stochastic gyroresonant acceleration with the turbulence spectrum given by Eq.(1). Since a fraction $\beta_A$ of the incoming free energy is converted into transverse plasma wave turbulence, these particles can tap at most a fraction $\beta_A$ of the swept-up energy via this process. Efficient energy transfer therefore requires that $\beta_A \gg 0.01$. The e-folding time scale for stochastic gyroresonant acceleration is $t_{acc} \sim \beta_A^{-2} t_{iso}$, whereas the diffusive escape time $t_{esc} = R^2/\kappa = t_{iso}^2/\kappa_{1}$ (Barbosa 1979, Schlickeiser 1989). Because the turbulence spectrum in Eq.(1) is $\propto k^{-2}$, the acceleration, escape, and isotropization time scales are independent of particle energy. The comoving deceleration time scale $t_{dec}(s) = 1.3 \times 10^4(E_{54} n_{i}^4 T_{300}^{5/3} )^{2/3}$ (Mészáros & Rees 1993). Provided that $\beta_A \geq 0.03 n_{i}^{1/3} (T_{300}^{5/3} E_{54} n_{i}^{2})^{1/3}$, the acceleration time scale is much shorter than the time scale for evolutionary changes of the blast wave due to bulk deceleration. In this case, a steady-state calculation of the equilibrium particle energy spectrum is justified, which results from the balance of gyroresonant acceleration, diffusive escape, deceleration and radiative losses.

The steady-state kinetic equation for the phase-space density $f_e$ of relativistic electrons is (Schlickeiser 1984)

$$\frac{1}{\gamma} \frac{d}{d\tau} \left[ \frac{\gamma^4}{t_a} \frac{df_e}{d\gamma} + \left( \frac{\gamma^3}{t_{dec}} + \frac{\gamma^4}{\delta_{syn}} \right) f_e \right] - \frac{f_e}{t_{EE}} = -S_c \delta(\gamma - \Gamma) \ (2)$$

where $S_c = \sqrt{\Gamma^2 - 1} n_e c / (4\pi \Gamma r)$ and $\Gamma$ is the area of the blast wave that is effective in sweeping up material.

If no dust is present, then the electrons can only be accelerated to $\gamma < \Gamma R_p / R_e = \Gamma m_p / m_e$. In this range,
of the function is obtained by taking the asymptotic expansion $f(\gamma) = \gamma^{\lambda} / \Gamma[1 + 2\mu]$.

Likewise, the steady-state kinetic equation for the phase-space density $f_p$ of relativistic protons is

$$
\frac{1}{\gamma^2} \frac{d}{d\gamma} \left( \gamma^2 \frac{df_p}{d\gamma} \right) + \mu \frac{1}{t_{dec}} f_p = -f_p \delta(\gamma - \Gamma),
$$

where $S_p = \sqrt{\gamma^2 - 1} n_e^* c / (4\pi \Gamma \tau)$. Eq. (5) holds in the presence of dust when $\gamma < \gamma_{p,\max} = \Gamma R_d / R_p = \Gamma n_d / (m_p Q_d) = \Gamma Y_d$, and assumes that effects of photopion production do not limit the acceleration of protons to the highest energies. Evidently, the parameter $Y_d$ and the bulk Lorentz factor $\Gamma$ determine the maximum proton Lorentz factor $\gamma_{p,\max}$. If no dust is present, then protons are not significantly accelerated by gyroresonant processes unless long wavelength MHD turbulence is generated through processes not treated here.

The exact solution to Eq. (2) for electrons accelerated by turbulence with a spectrum given by Eq. (1) is

$$
f_e(\Gamma < \gamma < \gamma_{e,\max}) = t_{\text{acc}} S_e \frac{\Gamma[\mu - (3 - a)/2]}{\Gamma[1 + 2\mu]} \times \frac{\Gamma^{\mu + (a+1)/2} \gamma^{a - (a+3)/2 \mu}}{\Gamma[a - (a+3)/2]} \exp(-\gamma/\gamma_s) \times M(\mu - 3 - a, 1 + 2\mu, \frac{\Gamma}{\gamma_s} \gamma, 1 + 2\mu, \frac{\Gamma}{\gamma_s} \gamma)
$$

(Schlickeiser, 1984), where $\mu = \sqrt{\lambda + (3 - a)^2 / 4}$, $\lambda = t_{\text{acc}} / t_E$, $a = t_{\text{acc}} / t_{\text{dec}}$, $\Gamma(\alpha_0)$ denotes the gamma function, and $M(a_1, a_2, a_3)$ and $U(a_1, a_2, a_3)$ are the Kummer functions of the first and second kind, respectively. The value $\gamma_s = t_{\text{syn}} / t_{\text{acc}} \approx 370 \Gamma 300 n_e^* / n_{b,8}$ is the electron Lorentz factor where the electron synchrotron loss time scale equals the acceleration time scale. In the regime $\gamma \gg \Gamma$, an accurate approximation is obtained by taking the asymptotic expansion of the function $U$ for small arguments, resulting in

$$
f_e(\Gamma < \gamma < \gamma_{e,\max}) \approx \frac{t_{\text{acc}} S_e}{2\mu} \frac{\Gamma^{\mu + (1 + a)/2} \gamma^{-(3 + a)/2 - \mu}}{\Gamma[a - (a+3)/2]} \exp(-\gamma/\gamma_s).
$$

The exact solution to Eq. (3) for protons accelerated by turbulence with a spectrum given by Eq. (1) is

$$
f_p(\Gamma < \gamma < \gamma_{p,\max}) = \frac{t_{\text{acc}} S_p}{2 \mu_p} \frac{\Gamma^{\mu_p + (a_p + 1)/2} \gamma^{-(a_p + 3)/2 - \mu_p}}{\Gamma[a_p - (a_p + 3)/2 - \mu_p]},
$$

where $\mu_p = \sqrt{\lambda_p + (3 - a_p)^2 / 4}$, $\lambda_p = t_{\text{acc}} / \epsilon t_{\text{esc}}$, and $a_p = t_{\text{acc}} / \epsilon t_{\text{dec}} / \epsilon$.

In Fig. 2, we show the resulting proton and electron spectra, choosing $\Gamma_{300} = 1$, $\beta_A = 0.03$, $r_{16} = 0.1$, $n_{b,8} = 0.1$, $\epsilon = 1$, $Y_d = 10^8$, and $n_e^* = 1 \text{ cm}^{-3}$. As can be seen, the electron spectrum $dN_e/d\gamma \propto \gamma^{-5} \exp(-\gamma/11700)$ is very flat below the synchrotron cutoff, whereas the proton spectrum displays a spectrum $dN_p/d\gamma \propto \gamma^{-1.36}$ below the maximum Lorentz factor $\gamma_{p,\max} = 3 \times 10^8$. For these parameters, we find comoving acceleration times of $\sim 330 (\beta_A/0.03)^{-2} \text{s}$ for electrons, and $\sim 10$ times larger for the protons. The choosen value for $\beta_A$ implies a magnetic field strength of 14 Gauss. Larger values of $\beta_A$ will shorten the acceleration time scales and improve the accelerated particle efficiency. Requiring that the gyroradii of the swept-up dust particles $R_d = Y_d R_p$ in a magnetic field of strength $14 B_{14}$ Gauss is smaller than the comoving shell radius $10^{14} r_{16} \Gamma_{100}^{-1}$ cm limits the charge-weighted dust/proton mass ratio to values smaller than $Y_d < 5 \cdot 10^6 r_{16} B_{14} \Gamma_{100}^{-2}$, and consequently the maximum proton Lorentz factor in the comoving blast wave frame is $\gamma_{p,\max} = \Gamma Y_d < 5 \cdot 10^8 r_{16} B_{14} \Gamma_{100}^{-1}$.

Fig. 3 shows the resulting synchrotron emission spectrum produced by the electron distribution function (5). For the parameters used here, the synchrotron spectrum is very hard. Particle spectral indices $dN/d\gamma \propto \gamma^{-\alpha}$ with $\alpha > 1$ can be produced through this process, leading to synchrotron spectra $F_\nu \propto \nu^{-\alpha}$ with $\alpha = -1/3$, corresponding to the index of the elementary synchrotron emis-
sivity formula for monoenergetic electrons. In standard shock acceleration theory (Blandford & Eichler 1987), particles are accelerated with \( p \geq 2 \), leading to values of \( \alpha \geq 1/2 \). Harder synchrotron spectra can however be produced through nonlinear effects in first-order Fermi acceleration, or by introducing a low-energy cutoff in the electron distribution function. Synchrotron spectra from flaring blazar sources (Catanese et al. 1997, Cohen et al. 1997, Pian et al. 1998) and GRBs display very hard spectra requiring nonthermal lepton spectra with \( p < 2 \), consistent with the stochastic Fermi acceleration mechanism proposed here.

The hardening above the ankle of the cosmic ray spectrum and beyond the ZGK cutoff could be related to a hard UHECR component formed through stochastic acceleration in GRB blast waves (Waxman 1995, Takeda et al. 1998). Because turbulence is generated in the process of capturing and isotropizing particles from the external environment, a wave spectrum that can yield very high energy electrons and protons through gyroresonant acceleration must be present in the relativistic blast waves. If dust survives capture by the magnetized plasma wind, then long wavelength turbulence can be generated that could accelerate protons to ultra-high energies on time scales shorter than the duration over which the blast wave decelerates to nonrelativistic speeds. Gyroresonant acceleration of particles by MHD turbulence provides a mechanism to transfer energy from heavier to lighter species, and overcomes difficulties associated with accelerating UHECRs in relativistic blast waves through the first-order Fermi mechanism.

3. Summary and conclusions

We have shown that efficient and rapid stochastic gyroresonant acceleration of electrons and protons occurs when interstellar protons and dust are captured by a relativistic outflow. The MHD plasma wave turbulence, created within the blast wave plasma by the penetrating protons and charged dust particles, is transferred from the heavier to the lighter particles through the gyroresonant acceleration process. This mechanism is particularly attractive for the energization of nonthermal electrons in GRB sources, as radiation modelling often requires approximate energy equipartition between electrons and protons (e.g., Katz 1994, Beloborodov & Demianski 1995, Smolsky & Usov 1996, Chiang & Dermer 1999). Capture by and isotropization within a blast wave, as discussed by Pohl & Schlickeiser (2001), provides primary electrons with an energy 1836 times smaller than that of the protons. As shown here, the subsequent acceleration of electrons in the proton-induced turbulence leads to approximate energy equipartition of these nonthermal electrons. Moreover, if in addition charged dust particles penetrate the blast wave plasma, we find that further acceleration of the particles is possible to an energy determined by the radiative loss processes of the electrons and protons. For some parameter regimes, this could accelerate particles to \( \sim 10^{20} \) eV and account for UHECR production.

By solving the appropriate kinetic equations, we calculated the resulting particle energy spectra of the accelerated protons and electrons, and the observable synchrotron radiation spectrum of the accelerated electrons. The latter peaks at hard X-ray energies and is very flat, in agreement with observed photon spectra of GRBs and some blazars during their flaring states.

A crucial issue, particularly for the stochastic acceleration of protons, is the penetration of dust. While large amounts of dust are present in the environments of star-forming regions in galaxies, it is also necessary that the dust not be destroyed during capture by the blast wave or by the intense radiation associated with captured nonthermal particles in the blast wave. We address some aspects of this issue in Appendix A.

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4. Appendix A: Survivability of dust in GRB radiation field

A major uncertainty in this study is whether the dust can survive the intense GRB radiation field by sublimation before the blast wave arrives and sweeps up the dust particle. In the inhomogeneous environment surrounding a massive star/GRB progenitor (Dermer & Böttcher 2000), it is probable that the blast wave will impact a dense dusty cloud after passing through a nearly evacuated region. In that case, there will be little radiation in advance of the blast wave, because the blast wave will only start to be energized and radiate after it encounters the cloud. Under such circumstances, the survivability of a large fraction of the dust to sublimation is assured.

In the ideal case where a GRB blast wave passes through a uniform density medium before intercepting dust particles, we outline a calculation of dust sublimation and identify parameter sets that are compatible with dust survivability. We follow the treatment of Waxman & Draine (1999) for dust sublimation. Although reverse shock emission could be important for dust sublimation, we neglect (1999) for dust sublimation. Although reverse shock emission could be important for dust sublimation, we neglect it here. It has only been detected from GRB 990123 and can be strongly suppressed if, for example, the blast wave shell is thin and the reverse shock traverses the shell before becoming relativistic (Sari et al. 1999).

In this case the optical-UV emission radiating the dust, which is the photon energy range most important for dust sublimation (Waxman & Draine 1999), can be described by the $L_{\nu} = L_{\nu,\text{max}}(\nu/\nu_0)^{1/3}$ portion of the synchrotron emissivity spectrum where the quantity $L_{\nu,\text{max}} = 4\pi n_e^2 x^3 m_e c^2 \sigma_T B_0/9e$ (Sari et al. 1998), where $e$ is the proton charge. Using standard blast-wave physics, $B_0(G) = 0.388\Gamma\sqrt{n_e^*e_B}$, where $e_B < 1$ is the magnetic-field parameter. To give good fits to the prompt phase of GRBs, it is necessary that the electrons are in the weakly cooled regime (Chiang & Dermer 1999, Dermer et al. 2000). For a weakly cooled spectrum, $\nu_0 = \nu_m = \Gamma^2 \gamma m_e c^2 / 2\pi n_e e_c$, where $\gamma_m = e\Gamma(p_e - 2)m_p / m_e(p_e - 1)$ and $e_c > \sim 0.1$. Taking the electron injection index $p_e \approx 2.2 - 3.0 = 2.5$, as implied by burst spectroscopy in the prompt and afterglow phase, we find $\nu_m(\text{Hz}) = 4.1 \times 10^{11} c^4 \Gamma^4 \sqrt{n_e^*e_B}$ (see Sari et al. 1998, Dermer et al. 2000 for more details). The luminosity (ergs s$^{-1}$) between the 1 eV $= 2.41 \times 10^{14}$ Hz and 7.5 eV band is

$$L_{1-7.5} \approx 1.28 \times 10^{-5} n_e^{4/3} (e_B/e_c)^{1/3} x^{2/3} \Gamma^{2/3} \approx$$

$$2.0 \times 10^{46} \left( \frac{n_e^*}{100} \right)^{4/3} \left( \frac{e_B}{0.5} \right)^{1/3} \left( \frac{e_c}{0.5} \right)^{-3/2} \frac{x}{10^{16} \text{cm}} \frac{\Gamma(300)^{2/3}}{300},$$

which also defines our standard parameter set.

The relationship of $\Gamma$ and $x$ to observer time $t$ for an adiabatic blast wave decelerating in a uniform surrounding medium is

$$\frac{\Gamma}{\Gamma_0} = \frac{1}{\sqrt{1 + (4\tau)^{3/4}}}, \text{ and } \frac{x}{x_d} \approx \frac{\tau}{1 + 4^{-1/4} \tau^{3/4}},$$

where the deceleration radius $x_d = (3E_0 / 8\pi \Gamma_0^3 m_e e^2 n_e^*)^{1/3} \approx 2.1 \times 10^{16} (E_{54}/\Gamma_0^{3/2} m_2 n_e^*)^{1/3} \text{ cm}$, the dimensionless time $\tau = t/t_d = c\Gamma_0^2 e/xd$, and the apparent isotropic explosion energy is $10^{52} E_{54}$ ergs.

For the sublimation of dust, we consider the total energy change due to (1) heating by the GRB in the 1 - 7.5 eV band, (2) thermal reprocessing of dust, and (3) the energy carried away by the sublimated particles. In this approximation, the heating rate $dE/dt$ due to dust energization is given through

$$[4\pi a^3 k_B (\rho/m)]^{-1} \frac{dT}{dt} = \frac{dE}{dt} = \frac{L_{1-7.5}(t) a^2(t)}{4\pi r^2} - 4\pi a^2(t) \sigma_{SB} T^4 - 12\pi k_B T (\rho/m) a^2(t) \frac{d\alpha}{dt}. \quad (9)$$

The rate of shrinkage of dust grain radius $a(t)$ with time $t$, is given by

$$\frac{d\alpha}{dt} = -\left( \frac{m}{\rho} \right)^{1/3} \nu_0 \exp(-b/k_B T), \quad (10)$$

(Waxman & Draine 1999), where $\rho/m = 10^{23} \text{ cm}^{-3}$, $\nu_0 = 1 \times 10^{15} \text{ s}^{-1}$, and $b/k_B = 7 \times 10^4 \text{ K}$ for refractory grains.

We look for solutions of Eqs. (9) - (10) where the dust grain is closer than the blast wave to the GRB source at the moment the dust becomes completely sublimated. There are no interesting solutions for our standard parameters; in this case, dust at $10^{14} \text{ cm}$ becomes sublimated when the blast wave has only reached $x = 1.6 \times 10^{15} \text{ cm}$. It is easy, however, to find solutions for slightly less energetic GRBs in more tenuous environments. For example, the blast wave will reach dust particles before they sublime if the dust resides within $x \approx 9 \times 10^{15} \text{ cm}$ when $E_0 = 10^{52} \text{ ergs}$ and $n_e^* = 1 \text{ cm}^{-3}$. These values of the energy and density are not too different than the values found when modeling the afterglow of GRB 980425 (Wijers & Galama 1999). Thus we find that dust will survive destruction by sublimation in some, though not all, GRB environments if the surrounding medium is uniform. A clumpy surrounding medium improves the chances of dust survival.

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