Performance Analysis of Bit-Error-Rate and Channel Capacity of MIMO Communication Systems over Multipath Fading Channels

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ABSTRACT
In wireless communication systems (WCS), channel characteristics are random and unpredictable. The resultant effect of the randomness is the multipath effect that leads to multipath fading. Multipath fading is a major contributor to the unreliability of wireless communication links. In this paper, Multiple-Input, Multiple-Output (MIMO) schemes were investigated in mitigating the effect of signal fading. Space-time coded MIMO-OFDM was developed to enhance the performance of the WCS. The performance of the developed space-time coded MIMO-OFDM over both flat and frequency selective channels were compared with that of conventional WCS. The simulation results show that the space-time coded MIMO-OFDM considerably reduced the effect of multipath fading in WCS with improved bit error rate (BER) value.

Keyword: MIMO, Space-Time Code, OFDM, Bit-Error-Rate, Channel Capacity

1. INTRODUCTION
The rate of acquisition and usage of mobile telephony, internet, and multimedia services has put a lot of pressure on spectrum demand particularly in wireless communication systems (WCS). The allocated radio spectrum to wireless systems is limited making it scarce and expensive. As a result, the development of WCS has to focus on increasing data rate as well as improving performance without increase in bandwidth and transmission power [1-3].

The communication channels’ environment that WCS operates in determines their performance. These channels are characterized by time varying propagation medium, which influences the quality of signals being transmitted causing them to vary rapidly. The received signals may be impaired by multipath effects: such as lack of line-of-sight during transmission, multiple reflection of radiated energy from man-made objects, scattering, as well as mobility—also known as Doppler [4]. By the time-varying nature of the received signals, the resultant effects of the impairments could be constructive, and occasionally destructive, forming a periodically fading signal at the receiver. Fading, not only leads to degradation in the quality of the propagated signal but also, restricts the speed and reliability of the system [5]. Fading-effect is more pronounced in systems operating in the Unlicensed National Information Infrastructure (UNII) operating frequency range like the WLAN systems. A plethora of techniques has been proposed in the literature to reducing the effects of fading, with application to WCS particularly: (a) the combination of MIMO (multiple-input, multiple-output) and OFDM (orthogonal frequency division multiplexing) is potentially robust to channel frequency selectivity, as well as in combating fading effects [6-10]; (b) the forward error correction technique e.g. space-time coding in MIMO channels [11- 15]. These approaches have attempted to increase data rate by improving the bit error rate (BER) performance of the system.
In this paper, MIMO (multiple-input, multiple-output) schemes were investigated in mitigating the effect of signal fading as they affect wireless communication systems. A particular emphasis is placed on space-time coded MIMO-OFDM to enhance the performance of the WCS. The performance of the developed space-time coded MIMO-OFDM over both flat and frequency selective channels were compared with that of conventional WCS.

In Section 2, the mathematical description of the space-time coded OFDM is discussed. Section 3 discusses the MIMO system model using a 2x2 scheme as a case study. Section 4 contains the mathematical descriptions of capacity of an ergodic channel. Section 5 presents the performance curves generated based on simulation models developed in MATLAB. Conclusions are drawn in Section 6.

2. SPACE-TIME CODE AND ORTHOGONAL FREQUENCY-DIVISION MULTIPLEXING

The need to enhance the quality of wireless communication systems in the range of their wired counterparts leads to development of space-time processing for MIMO wireless communications. Space-time coding (STC) technique, a MIMO transmit strategy, exploits transmit diversity and gives high system reliability, and leads to various coding methods: e.g. space-time trellis codes (STTC); space-time block codes (STBC); space-time turbo trellis codes and layered space-time (LST) codes [2, 5, 15, 16]. STTC and LST have an advantage over STBC by offering coding gain although they are very difficult to design and require complex encoders and decoders [4, 17].

Though, the design of space-time codes in frequency-selective-fading channel is intricate because of the existence of Inter Symbol Interference (ISI) [18], the OFDM technique attempts to combat the ISI problem [9, 10, 18-20]. OFDM converts a frequency selective MIMO channel into a set of parallel frequency flat MIMO channels and randomize the burst errors caused by a wideband-fading channel [13, 21-24].

In an OFDM system, the entire channel is partitioned into sub-channels and a block of data is modulated to a set of subcarriers [20, 21]. Suppose there are N numbers of sub-channels in an OFDM symbol, and the pair of OFDM symbols in an STBC block as

\[ X = \begin{bmatrix} x_{(n)} & x_{(n+1)} \end{bmatrix} \]  

(1)

where \( x_{(n)} \) and \( x_{(n+1)} \) are the modulated symbols and \([.]^T\) denotes the transpose operation. The ST encoder maps \( X \) into

\[ X_{(n)} = \begin{bmatrix} x_{(n)} & -x_{(n+1)}^* \\ x_{(n+1)} & x_{(n)}^* \end{bmatrix} \]  

(2)

where \( X_{(n)} \) is the output of ST encoder, symbols \(-x_{(n+1)}^*\) and \(x_{(n)}^*\) are orthogonal copies of the original symbols.

Then, at the first time \( t \), the symbols \( x_{(n)} \) and \( x_{(n+1)} \) are transmitted simultaneously from the two transmit antennas. Assuming that each symbol has duration \( T \), at the next time slot \( t+T \), symbols \(-x_{(n+1)}^*\) and \(x_{(n)}^*\) are transmitted from the two antennas respectively. In this scheme, at the first timeslot, the original sequence is transmitted unaltered, while at the second timeslot, space-time coded version is transmitted. Upon application of Alamouti’s [5] space-time block coding, symbol can be transmitted both in space and time.

3. MIMO SYSTEM MODEL

Before delving into MIMO modeling, the SISO channel is considered. The input and output relationship of a SISO channel is given by:

\[ y = hx + n \]  

(3)

where \( y \) is the received signal, \( x \) is the transmitted signal and, \( h \) and \( n \) are respectively the impulse response of the channel and the channel noise. For a generalized MIMO scheme with \( N_t \) transmit and \( N_r \) receive antennas, the received signal vector \( y \) is modeled as [25]:

\[ y = Hx + n \]  

(4)

where \( x \) is the transmitted signal vector, \( H \) is the channel matrix—also known as channel state information—between transmitter and receiver, and \( n \) is the complex additive white Gaussian noise vector. The channel matrix is expressed thus:

\[ H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \]
where elements $h_{i,j}$ are channel coefficients from the $j$th transmit to $i$th receive antennas. The channel coefficients between transmitter and receiver are assumed constant during the transmission of two consecutive symbols. So, channel coefficients are constant across two successive symbol periods enabling the coefficients to be expressed by

$$h_{i,j}(t) = h_{i,j}(t + T) = h_{i,j} e^{j\theta_{i,j}}$$

where respectively, $h_{i,j}$ and $\theta_{i,j}$ are the amplitude gain and phase shift for the path from each $j$th transmit antenna to the $i$th receive antenna.

With proper cyclic extension, the vector of the received symbols at the first and second time slots after FFT process at the receive antennas, expressed in terms of the transmitted symbols and channel coefficients, is given by [22]:

$$
\begin{bmatrix}
    y_{1,1} \\
    y_{1,2} \\
    y_{2,1} \\
    y_{2,2}
\end{bmatrix} =
\begin{bmatrix}
    h_{1,1} & h_{1,2} & & & \\
    h_{2,1} & h_{2,2} & & & \\
    & & & & \\
    & & & & \\
\end{bmatrix}
\begin{bmatrix}
    x_{(n)} \\
    x_{(n)} \\
    x_{(n+1)} \\
    x_{(n+1)}
\end{bmatrix}
+ 
\begin{bmatrix}
    n_{1,1} \\
    n_{1,2} \\
    n_{2,1} \\
    n_{2,2}
\end{bmatrix}
$$

(7)

Basically in a multipath-fading channel with an additive white Gaussian noise (AWGN), the MIMO receiver is designed to exploit time and space diversity so as to maximize the diversity reception of the symbols. With proper cyclic extension, the received symbols at the first and second time slots after FFT process at the receive antennas are given by [2]:

$$
y_{1,1} = h_{1,1}x_{(n)} + h_{1,2}x_{(n+1)} + n_{1,1}$$

$$y_{1,2} = -h_{1,1}x_{(n+1)} + h_{1,2}x_{(n)} + n_{1,2}$$

$$y_{2,1} = h_{2,1}x_{(n)} + h_{2,2}x_{(n+1)} + n_{2,1}$$

$$y_{2,2} = -h_{2,1}x_{(n+1)} + h_{2,2}x_{(n)} + n_{2,2}$$

The AWGN is assumed as having zero mean but variance $\sigma^2_n$.

In general, the received signals at $i$th receive antenna during the two time instances satisfy the equations:

$$y_{i,1} = h_{i,1}x_{(n)} + h_{i,2}x_{(n+1)} + n_{i,1}$$

$$y_{i,2} = -h_{i,1}x_{(n+1)} + h_{i,2}x_{(n)} + n_{i,2}$$

(9)

To decode $x_{(n)}$ and $x_{(n+1)}$, the combiner is designed to give:

$$\hat{x}_{(n)} = h_{1,1}^* y_{1,1} + h_{1,2}^* y_{1,2} + h_{2,1}^* y_{2,1} + h_{2,2}^* y_{2,2}$$

$$\hat{x}_{(n+1)} = h_{1,1}^* y_{1,1} - h_{1,2}^* y_{1,2} + h_{2,1}^* y_{2,1} - h_{2,2}^* y_{2,2}$$

(10)

Detection of $x_{(n)}$ and $x_{(n+1)}$ is greatly simplified since there is no interference between $x_{(n)}$ and $x_{(n+1)}$; implying that,
\[ x_{(n)} = (\gamma_{1,1}^2 + \gamma_{1,2}^2 + \gamma_{2,1}^2 + \gamma_{2,2}^2) x_{(n)} + h_{1,1}^* n_{1,1} + h_{1,2}^* n_{1,2} + h_{2,1}^* n_{2,1} + h_{2,2}^* n_{2,2} \]
\[ x_{(n+1)} = (\gamma_{1,1}^2 + \gamma_{1,2}^2 + \gamma_{2,1}^2 + \gamma_{2,2}^2) x_{(n+1)} - h_{1,1} n_{1,1}^* - h_{1,2} n_{1,2}^* + h_{2,1} n_{2,1}^* + h_{2,2} n_{2,2}^* \]  
(11)

where \( x_{(n)} \) and \( x_{(n+1)} \) are the estimated received signals in both time slots of symbols transmissions, \( \gamma_{i,j}^2 \) and \( h_{i,j}^* \) are the squared magnitude and complex conjugate of the channel transfer function \( h_{i,j} \) respectively, and \( n_{i,j}^* \) is the complex conjugate of the noise \( n_{i,j} \) for both time slots. Alternatively,

\[ \hat{x}_{(n)} = \sum_{j=1}^{N} \sum_{i=1}^{N} \gamma_{i,j}^2 x_{(n)} + \sum_{j=1}^{N} \left\{ h_{1,1}^* n_{1,1} + h_{1,2}^* n_{1,2} \right\} \]  
(12)

\[ \hat{x}_{(n+1)} = \sum_{j=1}^{N} \sum_{i=1}^{N} \gamma_{i,j}^2 x_{(n+1)} + \sum_{j=1}^{N} \left\{ h_{1,1} n_{1,1}^* - h_{1,2} n_{1,2}^* \right\} \]  
(13)

The combined signals are sent to a maximum likelihood, or Zero Forcing decoder [23] to estimate the transmitted symbols.

4. **CHANNEL CAPACITY**

The capacity of an ergodic channel is given in terms of bits/sec or by normalizing with bandwidth by bits/sec/Hz. The SISO system channel capacity for ergodic channels is defined as:

\[ C = \log_2 (1 + SNR) \]  
(14)

for a constant data rate and relatively stable SNR, and it is independent of channel state information.

The capacity of MIMO channel can be derived as [7]:

\[ C = \log_2 \det \left( I_{N_t} + \frac{p}{\sigma_n^2} H H^\dagger \right) \]  
(15)

Upon expansion, and generalization, we write expression for channel capacity without channel state information at the transmitter:

\[ C = \sum_{k=1}^{K} \log_2 \left( 1 + \lambda_k^2 \frac{p_k}{N_t \sigma_n^2} \right) \]  
(16)

where \( p_k \) is the transmit power and \( \lambda_k^2 \) is the channel gain on the \( k \)th sub-channel. Alternatively,

\[ C = \log_2 \prod_{k=1}^{K} \left( 1 + \frac{P_{\text{eff},k}}{N_t \sigma_n^2} \right) \]  
(17)

where \( P_{\text{eff},k} \) is the effective power.

5. **SIMULATION RESULT AND DISCUSSION**

The simulation was run for over 10^7 transmitted blocks of data with varying signal-noise ratio values ranging from 0 – 30 dB with BPSK modulation. The ST coded OFDM system using BPSK is simulated and bit error performances are compared with the conventional system without STC. Performance comparison between these schemes is shown in Fig. 1. For instance, to achieve BER of 10^{-2} for an STC system, 21 dB SNR is needed. However, a 28 dB SNR is required to achieve the same BER for a conventional system. This implies that 7 dB more increase in signal power is required for conventional system to achieve the same BER of 10^{-2} as the ST coded OFDM employed in this work. From the results, it is observed that performance of STC-OFDM system performs significantly better than the system without ST code. This implies that, to achieve certain BER, power consumption by ST coded OFDM is comparatively reduced.
For the performance comparison of different MIMO schemes and in compliance to IEEE 802.11n requirements, we restrict our investigation to four antennas at both sides of the link; i.e. $N_t = N_r = 4$. The performance of different schemes is shown in Fig. 2. As observed, there is an increase in performance as the numbers of both transmit and receive antennas increase. And comparatively, as shown in Fig. 2, at 10 dB, 1×1 scheme has 3 bits/sec/Hz; 2×2 scheme has 6 bits/sec/Hz, 3×2 and 2×3 schemes have 7 bits/sec/Hz; 4×3 and 3×4 schemes have 10 bits/sec/Hz; and 4×4 scheme has 12 bits/sec/Hz channel capacities. It is also observed in Fig. 2 for OFDM-MIMO schemes with variable $N_t$ and $N_r$ but with equal total number of antennas overlapping show equal capacity of the systems. Furthermore, when complexity is considered, small number of antennas is adequate for suitable performance.
6. CONCLUSION

In this paper, MIMO scheme has been investigated for the wireless communication systems (WCS) over multipath fading channel with AWGN channels. Compared to a conventional communication systems; MIMO scheme provides improved data capacity at the given bandwidths leading to bandwidth efficiency of the scheme. Furthermore, performance of WCS was significantly improved by the implementation of space-time coding technique, which substantially reduced erroneous data transmission to give reliable communication systems. Simulation results show that the space-time coded OFDM-MIMO systems has less bit error rate and better performance with respect to conventional communication systems.

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Performance Analysis of Bit-Error-Rate and Channel Capacity of MIMO (Isiaka Ajewale)

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