The $\Sigma - D$ analysis of recently detected radio planetary nebulae in the Magellanic Clouds

(Research Note)

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ABSTRACT

Aims. We investigate and analyze the radio surface brightness to diameter ($\Sigma - D$) relation for recently detected, bright radio-continuum planetary nebulae (PNe) in the Magellanic Clouds (MC).

Methods. We applied a Monte Carlo analysis in order to account for sensitivity selection effects on measured $\Sigma - D$ relation slopes for bright radio PNe in the MCs.

Results. In the $\Sigma - D$ plane, these radio MCs PNe are positioned among the brightest of the nearby Galactic PNe and are close to the $D^{-2}$ sensitivity line of the MCs radio maps. The fitted Large Magellanic Cloud (LMC) data slope appears to be influenced by survey sensitivity. This suggests the MCs radio PN sample represents just the “tip of the iceberg” of the actual luminosity function. Specifically, our results imply that sensitivity selection tends to flatten the slope of the $\Sigma - D$ relation. Although MCs PNe appear to share the evolution properties of Galactic PNe, the small number of data points prevented us from further constraining their evolution properties.

Key words. methods: statistical – ISM: planetary nebulae: general – galaxies: Magellanic Clouds – radio continuum: ISM

1. Introduction

Filipović et al. (2009) has recently reported 15 radio-continuum PNe detected at the 3$\sigma$ level and above from various radio Magellanic Clouds (MCs) surveys. Their detections are mainly based on positional coincidences with optically detected planetary nebulae (PNe) and optical spectroscopy (Payne et al. 2008a,b). Included are Large Magellanic Cloud (LMC) mosaics (Dickel et al. 2005) with sensitivities of $\sim 0.5$ mJy beam$^{-1}$ at both 4.8 and 8.64 GHz with resolutions of 33 and 20", respectively. The Small Magellanic Cloud (SMC) mosaics have 0.5 mJy beam$^{-1}$ sensitivities at 4.8 and 8.64 GHz with resolutions of 30 and 15" (Dickel 2009). To compare these sources with Galactic PNe, we use radio surface brightness ($\Sigma$) since this quantity is distance independent:

$$\Sigma [\text{Wm}^{-2}\text{Hz}^{-1}\text{sr}^{-1}] = 1.505 \times 10^{-19} \frac{S[\text{Jy}]}{\theta^2[\text{''}]}.$$  (1)

where $S$ is the flux density and $\theta$, the angular diameter of the source. Theory predicts the existence of a linear relation between $\log \Sigma$ and $\log D$, with $D$ representing the diameter of the object (see Urošević et al. 2007; Urošević et al. 2009, and references therein).

All of the detected radio PNe are unresolved, despite five of them being observed in “snap-shot” mode with the resolution of up to 1" and sensitivity of $\sim 0.1$ mJy beam$^{-1}$ (for details see Filipović et al. 2009). The optical diameters are available for some 10 out of 15 (see Table 1 in Filipović et al. 2009). The MCs PNe data at 4.8 GHz are plotted in Fig. 1 with respective LMC map $3\sigma$ sensitivity line at 1.5 mJy beam$^{-1}$ (hereafter referred to as sensitivity line). Five LMC PNe are above the sensitivity line and one is below (“snap-shot” mode). Five PNe with estimated upper flux density limit of 1.5 mJy beam$^{-1}$ are plotted on the sensitivity line and the remaining one without optical diameter is not plotted. Four SMC PNe are plotted on the SMC map resolution limit of 30". We use the PNe sample of Galactic PNe with reliable individual distances from Stanghellini et al. (2008; hereafter SSV), and divide them into two subsamples (nearby PNe with distances smaller than 1 kpc and those with distances greater than 1 kpc). Urošević et al. (2009), in an extensive analysis of the empirical $\Sigma - D$ relation for PNe, argued that the SSV sample of nearby PNe is least influenced by selection effects. For comparison, we plot two radio PNe from the Sagittarius dwarf galaxy (Dudziak et al. 2000), also representative of extragalactic radio PNe. An arrow in Fig. 1 represents the $D^{-2}$ direction in which unresolved objects should move if their diameters were known.

Inspection of Fig. 1 reveal that the detected radio-continuum MCs PNe are very close to their sensitivity lines and they are as bright as the brightest Galactic PNe. When compared with Galactic PNe it appears that current data set is only the very peak of the actual MCs PNe luminosity function. In this paper, using $\chi^2$ analysis and Monte Carlo generated data samples, we...
investigate the sensitivity related selection effects on the $\Sigma - D$ properties of these objects and compare them with the properties of Galactic PNe.

As a convenient platform for distance determination, the $\Sigma - D$ relation has a ~50 year long history of exploitation for supernova remnants (SNRs). The study of the relation for these very luminous radio synchrotron sources began with the work of Shklovskii (1960), while the first empirical $\Sigma - D$ relation was presented by Poveda & Woltjer (1968). Decades of analysis resulted in a good understanding of the $\Sigma - D$ relation for SNRs and various data sample selection effects that influence that relation.

The previous analysis of PNe radio data samples (e.g. Phillips 2002, and references therein) were mainly focused on radius vs radio brightness temperature ($R - T_b$) empirical relations. These were sparse in the consideration of data selection effects. Recent works by Urošević et al. (2007) and Urošević et al. (2009) use a SNR formalism in the analysis of PNe data along with a discussion of these selection effects. Although they note that all PN samples suffer from selection effects caused by limitations in survey sensitivity and resolution, the nearby SSV sample, having a $\Sigma - D$ slope of $\beta = -2.61 \pm 0.21$, is least influenced by these effects.

2. Analysis

Samples for Monte Carlo simulations are made using sources from Table 1 in Filipović et al. (2009) that have a known optical diameter (LMC sources only) and are above the map sensitivity line for a given frequency. This includes 5 PNe at both frequencies (8.64 and 4.8 GHz) though not necessarily the same ones. Using adopted distances to the LMC of 50 kpc and to the SMC of 60 kpc (Alves 2004; Hilditch et al. 2005), the smallest sources in the selected samples have a mean geometrical diameter of 0.09 pc at both frequencies. The largest source has a mean geometrical diameter of 0.12 pc at 4.8 GHz and 0.15 pc at 8.64 GHz. Surface brightness ranges from $2.67 \times 10^{-18}$ to $8.65 \times 10^{-18}$ W m$^{-2}$ Hz$^{-1}$ sr$^{-1}$ at 8.64 GHz and from $4.70 \times 10^{-18}$ to $7.90 \times 10^{-18}$ W m$^{-2}$ Hz$^{-1}$ sr$^{-1}$ at 4.8 GHz. For the LMC maps, the 4.8 GHz sensitivity line is at $\Sigma = 7.46 \times 10^{-22}$ W m$^{-2}$ Hz$^{-1}$ sr$^{-1}$ with a $D^{-2}$ break at $D = 8.00$ pc and a 8.64 GHz sensitivity line at $\Sigma = 2.03 \times 10^{-21}$ and $D = 4.85$ pc, respectively.

The relative flux density errors at both frequencies of the LMC maps are <10% (Filipović et al. 2009) and they are approximately the size of the symbols shown in Fig. 1. If we assume that the optical diameters have no significant errors and that the absolute flux density error equals the flux density standard deviation $\sigma_r$, according to error propagation theory it follows from Eq. (1), that the standard deviation for log $\Sigma$ ($\sigma_{\log \Sigma}$) is $0.434 \Delta \Sigma / \langle \Sigma \rangle$. This gives $\sigma_{\log \Sigma} \approx 0.05$.

The resulting parameters of the log $\Sigma = a + \beta \cdot \log D$ fits are given in Table 1 for each frequency. The parameters $a$ and $\beta$, their standard uncertainties $\Delta a$ and $\Delta \beta$, the linear correlation coefficient $r$, the fit quality $r^2$, the probability $Q$ of obtaining larger WSSR (weighted sum of square residuals) and a ratio of WSSR to the number of degrees of freedom (ndof) are given in this table. A value of 0.1 $\geq 0 \geq 0.001$ is expected for a statistically acceptable fit (the case when errors of the dependent variable have a non-Gaussian distribution), if the data is were well approximated by the model. This should apply for fits presented in this paper since we have transformed flux density errors to log $\Sigma$ errors. When introduced in the least squares fitting procedure, our values of $Q$ implies a statistically acceptable fit.

Although the fits are statistically believable the resulting $\beta$ at 4.8 GHz substantially differs from $\beta$ at 8.64 GHz, likely in debt to a small number of data points. Sampling only the peak of the luminosity function and with the majority of the sample likely hidden below the sensitivity line, the resulting values for $\beta$ should not be taken seriously. Comparing the LMC radio PN sample and the sample of a nearby Galactic PNe, we made an
Table 1. Fit parameters of the LMC data sample.

| Frequency | $a$   | $\Delta a$ | $\beta$ | $\Delta \beta$ | $r$   | $r^2[\%]$ | $Q$  | $WSSR$ |
|-----------|-------|------------|---------|---------------|------|-----------|------|--------|
| 8.46 GHz  | -19.7 | 0.3        | -2.6    | 0.4           | -0.94| 88.36     | 5.02 $\times 10^{-2}$ | 2.60 |
| 4.80 GHz  | -18.1 | 0.5        | -0.9    | 0.5           | 0.64 | 49.86     | 9.96 $\times 10^{-2}$ | 2.08 |

Table 2. Monte Carlo simulation results at 4.8 GHz.

| The slope that is fitted | Mean simulated slope | Standard deviation of mean simulated slope | Mean slope after selection | Standard deviation of slope after selection | No. of generated samples that was fitted |
|-------------------------|----------------------|------------------------------------------|---------------------------|-------------------------------------------|------------------------------------------|
| scatter = 1.0           |                      |                                          |                           |                                           |                                          |
| -1.500000              | -1.492251            | 0.068066                                 | -1.514832                 | 0.064286                                  | 100                                     |
| -1.600000              | -1.598043            | 0.068590                                 | -1.623227                 | 0.062950                                  | 100                                     |
| -1.700000              | -1.699330            | 0.065987                                 | -1.728391                 | 0.064682                                  | 100                                     |
| -1.800000              | -1.810370            | 0.061776                                 | -1.827670                 | 0.062875                                  | 100                                     |
| -1.900000              | -1.904920            | 0.065998                                 | -1.939695                 | 0.063711                                  | 100                                     |
| -2.000000              | -2.003822            | 0.057447                                 | -2.001172                 | 0.059317                                  | 100                                     |
| -2.100000              | -2.099951            | 0.065622                                 | -2.043189                 | 0.062283                                  | 100                                     |
| -2.200000              | -2.195110            | 0.063386                                 | -2.070956                 | 0.068676                                  | 99                                      |
| -2.300000              | -2.297590            | 0.064135                                 | -2.074784                 | 0.130676                                  | 82                                      |
| -2.400000              | -2.398137            | 0.070116                                 | -2.110495                 | 0.211172                                  | 59                                      |
| -2.500000              | -2.501291            | 0.065666                                 | -2.180770                 | 0.426777                                  | 40                                      |
| -2.600000              | -2.594927            | 0.068783                                 | -2.223866                 | 0.527765                                  | 23                                      |
| -2.700000              | -2.700259            | 0.065226                                 | -2.347649                 | 0.527067                                  | 11                                      |
| -2.800000              | -2.801124            | 0.071673                                 | -2.339542                 | 0.345267                                  | 11                                      |
| -2.900000              | -2.892171            | 0.061980                                 | -2.325770                 | 0.384151                                  | 9                                       |
| -3.000000              | -2.995751            | 0.067793                                 | -2.348757                 | 0.608717                                  | 4                                       |
| -3.100000              | -3.093523            | 0.066956                                 | -2.789218                 | 0.225177                                  | 73                                      |
| -3.200000              | -3.093523            | 0.131417                                 | –                          | –                                         | 0                                       |
| -3.300000              | -3.304876            | 0.071690                                 | -2.841456                 | 0.077769                                  | 3                                       |
| -3.400000              | -3.304876            | 0.126657                                 | -2.841456                 | 0.077769                                  | 1                                       |
| -3.500000              | -3.502017            | 0.064626                                 | -2.471982                 | 0.471662                                  | 3                                       |

3. Discussion and conclusions

From Tables 2 and 3, it is evident that the sensitivity cutoff tends to flatten the intrinsic (simulated) slopes less than $-2$ (the most likely case for the real data). At 4.8 GHz, a slope of $-0.9 \pm 0.5$ (shallower than the sensitivity line) is not influenced by sensitivity related selection effects. From the other side, this slope does not have any physical interpretation and is only the consequence of scatter in the $\Sigma - D$ plane. The 8.64 GHz slope of $-2.6 \pm 0.4$ appears to correspond to somewhat steeper selection free slopes but due to a large error could have any values smaller than $-2.3$. This is within the range of the SSV slope at 4.8 GHz. With current LMC data samples it is not possible to constrain the lower limit, due to a smaller number of selected points for steeper slopes. The value of a mean slope after selection at 8.64 GHz starts to oscillate for the slopes $\leq -2.4$ (the standard deviation of slope after selection exceeds the interval between two successive simulated slopes). Table 2 is given here rather for completeness, but nevertheless shows that for the samples with small number of data points and a slope greater than $-2$ it is not possible to extract any meaningful information with this kind of simulations.

With these recently confirmed MCs radio PNe being amongst brightest Galactic PNe in the $\Sigma - D$ plane and given their proximity to the sensitivity line, it is likely they represent only the peak of the actual MCs PNe luminosity function, which is probably the “bright-end” extension of the Galactic PNe luminosity function (for a more detailed discussion of the nature of these MCs PNe see Filipović et al. 2009). The results of our LMC Monte Carlo simulations suggest that current sparse data samples cannot give meaningful $\Sigma - D$ slope. However, they
confirmed that the corresponding apparent $\Sigma - D$ slope flattens toward $\beta \approx -2$ because of the sensitivity related selection effects.

This should be kept in mind as a serious measure of caution in forthcoming studies of extragalactic PN samples. Future high sensitivity images of the MCs will surely provide even better and more complete radio samples of PN population and enable more robust constraining of PN evolution parameters.

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Table 3. Monte Carlo simulation results at 8.64 GHz.

| The slope that is simulated | Mean slope | Standard deviation of mean slope | Mean slope after selection | Standard deviation of slope after selection | No. of samples that was fitted |
|-----------------------------|------------|---------------------------------|---------------------------|-------------------------------------------|-------------------------------|
| scatter = 1.0               |            |                                 |                           |                                           |                               |
| $-1.500000$                 | $-1.503405$ | 0.052604                        | $-1.503405$               | 0.052604                                  | 100                           |
| $-1.600000$                 |            |                                 |                           |                                           |                               |
| $-1.700000$                 |            |                                 |                           |                                           |                               |
| $-1.800000$                 |            |                                 |                           |                                           |                               |
| $-1.900000$                 |            |                                 |                           |                                           |                               |
| $-2.000000$                 |            |                                 |                           |                                           |                               |
| $-2.100000$                 |            |                                 |                           |                                           |                               |
| $-2.200000$                 |            |                                 |                           |                                           |                               |
| $-2.300000$                 |            |                                 |                           |                                           |                               |
| $-2.400000$                 |            |                                 |                           |                                           |                               |
| $-2.500000$                 | $-2.501493$ | 0.043743                        | $-2.318480$               | 0.162147                                  | 98                            |
| $-2.600000$                 | $-2.601538$ | 0.043349                        | $-2.425642$               | 0.195200                                  | 98                            |
| $-2.700000$                 | $-2.709181$ | 0.091694                        | $-2.609181$               | 0.091654                                  | 100                           |
| $-2.800000$                 | $-2.801306$ | 0.077013                        | $-1.810825$               | 0.079255                                  | 100                           |
| $-2.900000$                 | $-2.901775$ | 0.080564                        | $-1.903992$               | 0.083989                                  | 100                           |
| $-3.000000$                 | $-3.002529$ | 0.093860                        | $-2.007141$               | 0.092602                                  | 100                           |
| $-3.100000$                 | $-3.100702$ | 0.041717                        | $-2.905552$               | 0.079765                                  | 100                           |
| $-3.200000$                 | $-3.200655$ | 0.097675                        | $-2.090592$               | 0.089702                                  | 100                           |
| $-3.300000$                 | $-3.302362$ | 0.042204                        | $-2.155513$               | 0.085328                                  | 100                           |
| $-3.400000$                 | $-3.404428$ | 0.049995                        | $-2.257452$               | 0.325416                                  | 15                            |
| $-3.500000$                 | $-3.501310$ | 0.043181                        | $-2.387478$               | 0.43336                                   | 14                            |

scatter = 2.0

| $-3.500000$                 | $-3.499001$ | 0.073477                        | $-1.506813$               | 0.071878                                  | 100                           |
| $-1.600000$                 | $-1.595066$ | 0.091924                        | $-1.609181$               | 0.091654                                  | 100                           |
| $-1.700000$                 | $-1.706313$ | 0.079195                        | $-1.720427$               | 0.077824                                  | 100                           |
| $-1.800000$                 | $-1.801306$ | 0.077013                        | $-1.810825$               | 0.079255                                  | 100                           |
| $-1.900000$                 | $-1.892254$ | 0.092443                        | $-1.903992$               | 0.083989                                  | 100                           |
| $-2.000000$                 | $-2.002529$ | 0.093860                        | $-2.007141$               | 0.092602                                  | 100                           |
| $-2.100000$                 | $-2.100702$ | 0.041717                        | $-2.905552$               | 0.079765                                  | 100                           |
| $-2.200000$                 | $-2.196983$ | 0.097675                        | $-2.090592$               | 0.089702                                  | 100                           |
| $-2.300000$                 | $-2.278865$ | 0.094011                        | $-2.155513$               | 0.085328                                  | 100                           |
| $-2.400000$                 | $-2.403615$ | 0.089715                        | $-2.257452$               | 0.325416                                  | 15                            |
| $-2.500000$                 | $-2.478826$ | 0.080478                        | $-2.194225$               | 0.191552                                  | 98                            |
| $-2.600000$                 | $-2.593142$ | 0.095809                        | $-2.264253$               | 0.276645                                  | 89                            |
| $-2.700000$                 | $-2.709181$ | 0.091667                        | $-2.402356$               | 0.329761                                  | 85                            |
| $-2.800000$                 | $-2.792243$ | 0.076143                        | $-2.297942$               | 0.374669                                  | 67                            |
| $-2.900000$                 | $-2.901775$ | 0.080564                        | $-2.406221$               | 0.411391                                  | 61                            |
| $-3.000000$                 | $-2.999507$ | 0.086134                        | $-2.591900$               | 0.528155                                  | 41                            |
| $-3.100000$                 | $-3.083982$ | 0.009140                        | $-2.527112$               | 0.497942                                  | 31                            |
| $-3.200000$                 | $-3.187769$ | 0.081336                        | $-2.453453$               | 0.617842                                  | 29                            |
| $-3.300000$                 | $-3.303730$ | 0.089221                        | $-2.681570$               | 1.138125                                  | 32                            |
| $-3.400000$                 | $-3.392252$ | 0.084091                        | $-2.712006$               | 0.649541                                  | 20                            |
| $-3.500000$                 | $-3.501420$ | 0.078870                        | $-2.584415$               | 0.885438                                  | 21                            |

\[
\Sigma = \frac{10^{-22}}{D[pc]} \frac{\nu}{W}[mHz]\]