Joint parameter determination using FRF decoupling method for connected solid plates

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Abstract. Use of the FRF decoupling method can be used to determine joint parameters for connected systems. The method has been verified to determine stiffness parameter for lumped mass systems and beam elements by the author in his earlier publication. Modal coupling method used to verify generated FRF in a coupled system for simple mass damper system. Sensitivity modes can be used for identifying the most accurate values for stiffness. In this paper, the method is extended to determine the stiffness parameter for a solid element. Though solid elements have only translational degree of freedoms (DOFs), this method can be used to determine rotational stiffness apart from translational stiffness for joint parameter.

1. Introduction

Considering mechanical joints as rigid will be inappropriate while analysing joint mechanical components. However, FEA modelling can predict results satisfactorily. Analysing mechanical joints without considering the stiffness of joints may lead to inaccurate results. So there is a need to identify stiffness and other joint parameters such as damping. Domain decomposition can be considered as one of method for decoupling. To be precise, dynamic substructuring can be considered as second-level domain decomposition. In the earlier time, component mode synthesis (Modal based identification) method was used, later on FRF decoupling method also introduced due to ease use of FRFs, damping parameters handling etc. The methods for decoupling can be modal based or FRF based. The advantages of FRF based methods over modal based are such as ease and simplicity in using FRF, more information in the frequency domain.

1.1 Derivation of FRF decoupling equation:

Receptance FRF has been considered in this methodology. Let, \( r \) = coordinates of subsystem of A only, \( j \) = connection coordinates of subsystem A, \( k \) = connection coordinates of subsystem of B, \( s \) = coordinates of subsystem of B only.

Figure 1. Two substructures connected with elastic coupling

Receptance FRF has been considered in this methodology. Let, \( r \) = coordinates of subsystem of A only, \( j \) = connection coordinates of subsystem A, \( k \) = connection coordinates of subsystem of B, \( s \) = coordinates of subsystem of B only.
Using compatibility and equilibrium conditions, below 4 equations for $K^*$ can be derived.

\[
[K^*] = [H_{rr}^{-1} - H_{sk}^{-1} - H_{kk}^{-1}]^{-1} \quad \text{--- (1)}
\]

\[
[K^*] = [H_{ss}^{-1} - H_{kk}^{-1}]^{-1} \quad \text{--- (2)}
\]

\[
[K^*] = [H_{sk}^{-1} - H_{kk}^{-1}]^{-1} \quad \text{--- (3)}
\]

\[
[K^*] = [H_{ss}^{-1} - H_{ss}^{-1}]^{-1} \quad \text{--- (4)}
\]

It can be observed that equation 2 and 3 involve coupled system transfer FRF, while equation 1 and 4 involve coupled system point FRF. In this paper, equation 1 and 4 has been used for joint identification.

In this study, equation 4 has been used. In general for experimental FRF decoupling method, for substructure A, one end can be fixed and other end can be connected to another substructure. FRF measurement can be considered for one or both substructures. Analytical methods can be used to generate FRFs from modal data using modal synthesis method.

1.2 Modal coupling method for simple lumped mass system:

In this study, FRF have been evaluated using Modal synthesis method. And to get closer to experimental FRFs, polluted FRFs have been used (In Matlab, we can use functions "normrnd" for randomly introducing errors in FRF.) Receptance (Displacement/Force) FRF has been considered in this study.

For viscous damped system with "N" DOF (Degree of freedom), Receptance FRF $H_{pq}(\omega)$ can be stated as,

\[
H_{pq}(\omega) = \frac{\sum_{r=1}^{n} \frac{\phi_{pr} \phi_{qr}}{(\omega_{r}^{2} - \omega^{2} + 2 j \gamma_{r} \omega \omega_{r})}} \quad \text{--- (5)}
\]

where $\phi_{pr}$ and $\phi_{qr}$ are $p$th and $q$th elements of mass normalized eigen vector of $r$th mode. $\omega_{r}$ and $\gamma_{r}$ are the $r$th mode natural frequency and viscous damping ratio. For structurally damped system with "N" DOF (Degree of freedom), Receptance FRF $H_{pq}(\omega)$ can be stated as,

\[
H_{pq}(\omega) = \frac{\sum_{r=1}^{n} \frac{\phi_{pr} \phi_{qr}}{(\omega_{r}^{2} - \omega^{2} + j \gamma_{r} \omega \omega_{r})}} \quad \text{--- (6)}
\]

Based on equation 6, $\omega_{r}$ and $\gamma_{r}$ are the $r$th mode natural frequency and $r$th mode loss factor respectively. Consider below 2 substructures which are lumped mass system.

![Figure 2. Lumped mass system](image-url)
Let the connecting stiffness parameter of 2000 N/mm used to connect two substructures.

![Coupled lumped mass system](image)

**Figure 3.** Coupled lumped mass system

$H^s_s$ (Point FRF for coupled system for end coordinate’s’) generated from modal synthesis method is shown in figure 4.

![Hcss point FRF and evaluated stiffness](image)

**Figure 4.** $H^s_s$ point FRF and evaluated stiffness

Modal coupling method is used to derive coupled system’s FRF, using modal quantities (mass normalized mode shape and modal frequencies) and stiffness matrix. Modal Coupling techniques, also referred as Time Domain or Component Mode Synthesis methods. These methods use a reduction performed on the number of modes used to describe each component model while still accounting for all the physical DOF.

The general equation for modal coupling method is as below.

$$
\begin{bmatrix}
\eta_A \\
\eta_B
\end{bmatrix}
+ 
\begin{bmatrix}
\phi_A^T & [0] & [0] & [0]
\end{bmatrix}
\begin{bmatrix}
K_C & [0] & [W_A^T] & [0]
\end{bmatrix}
\begin{bmatrix}
[0] & [K_C] & [0] & [0]
\end{bmatrix}
= \begin{bmatrix}
[0] & [0]
\end{bmatrix}

--- (7)

Here $\eta_A$ and $\eta_B$ are principal coordinates. The equivalent $K$ from above equation can be evaluated as

$$
\text{Equivalent \textquoteleft} K \text{\textquoteleft} = \begin{bmatrix}
\phi_A^T & [0] & [0] & [0]
\end{bmatrix}
\begin{bmatrix}
[K_C] & [0] & [W_A^T] & [0]
\end{bmatrix}
\begin{bmatrix}
[0] & [K_C] & [0] & [0]
\end{bmatrix}
\begin{bmatrix}
[0] & [0] & [W_A^T] & [0]
\end{bmatrix}

--- (8)
Using mass normalized modal matrix for substructures and stiffness matrix at connection point between substructures, coupled system’s modal parameters (modal frequencies and mass normalized mode shapes) can be found out.

As in case of previous example for lumped mass system, stiffness specified is 2000 N/mm and output parameter for stiffness using decoupling method is same value. Let us assume some deviation of stiffness value due to numerical error in decoupling method. When stiffness is changed from 2000 N/mm to 2500 N/mm in modal coupling method, the difference in coupled system’s FRF can be observed as below.

**Table 1** Natural frequency difference observed in modal coupling method due to variation in stiffness

| Natural frequency | Natural frequency (rad/sec) from Ansys for coupled system | Natural frequency (rad/sec) from Modal coupling method for stiffness of 2500 N/mm (correct stiffness = 2000 N/mm) |
|-------------------|----------------------------------------------------------|------------------------------------------------------------------------------------------|
| 1st               | 4.40                                                     | 4.43                                                                                     |
| 2nd               | 13.81                                                    | 14.02                                                                                   |
| 3rd               | 22.80                                                    | 22.84                                                                                   |
| 4th               | 31.25                                                    | 31.85                                                                                   |
| 5th               | 38.92                                                    | 38.94                                                                                   |
| 6th               | 45.65                                                    | 46.32                                                                                   |
| 7th               | 51.33                                                    | 51.81                                                                                   |
| 8th               | 55.95                                                    | 56.02                                                                                   |
| 9th               | 60.16                                                    | 62.19                                                                                   |
| 10th              | 65.23                                                    | 65.87                                                                                   |

\[ K^* = \left[ K_{ss} \right] \left[ H_{ss} \right]^{-1} \left[ H_{sk} \right] \left[ H_{jk} \right]^{-1} \quad (9) \]
The size of $K^*$ depends on number of connecting elements between substructures. As discussed earlier, in this study equation (4) has been used due to its practical usability, where one end of substructure can be fixed. Below are few examples for substructures which can be beams, lumped masses or solids.

![Figure 6. Substructures connected by $K^*$](image)

1. Lumped mass: Below two substructures which are lumped masses connected using stiffness $k_3$ and damping $c_3$.

![Figure 7. $K^*$ matrix dimension for lumped mass system](image)

$$[K^*] = \begin{bmatrix} [H_{ks}] & [H_{ss} - H_{ks}^*]^{-1} [H_{sk}] - [H_{jk}] - [H_{kk}] \end{bmatrix}^{-1}$$

**Figure 7. $K^*$ matrix dimension for lumped mass system**

The $K^*$ is complex number with real part as stiffness parameter while imaginary part as damping parameter.

2. $K^*$ with 2 DOF:

![Figure 8. $K^*$ matrix dimension for beam element with joint as 1 translational and 1 rotational stiffness](image)

$$[K^*] = \begin{bmatrix} [H_{ks}] & [H_{ss} - H_{ks}^*]^{-1} [H_{sk}] - [H_{jj}] - [H_{kk}] \end{bmatrix}^{-1}$$

**Figure 8. $K^*$ matrix dimension for beam element with joint as 1 translational and 1 rotational stiffness**

The $K^*$ matrix is 2*2 dimension, with (1,1) element representing translational parameter with corresponding real part as stiffness and imaginary part as damping value. The (2,2) element of $K^*$
represents rotational parameter (stiffness and damping). While elements (1,2) and (2,1) represents cross coupling stiffness and damping values.

3. \( K^* \) with 3 translational directional DOF:

\[
[K^*] = \begin{bmatrix}
[Hks] & [Hss] & [Hsk] \\
[Hsk] & [Hij] & [Hkk] \\
\end{bmatrix}
\]

\[
[K^*] = \begin{bmatrix}
[Hks] & [Hss] & [Hsk] \\
[Hsk] & [Hij] & [Hkk] \\
\end{bmatrix}^{-1}
\]

**Figure 9.** \( K^* \) matrix dimension for beam element with 3 translational stiffness

The \( K^* \) matrix is 3*3 dimension in above case. The element (1,1) represents stiffness in X direction (may be Y or Z, depending on matrix formulation), the element (2,2) represents stiffness in Y direction, the element (3,3) represents stiffness in Z direction.

4. \( K^* \) for extended interface approach method for substructuring

\[
[K^*] = \begin{bmatrix}
[Hks] & [Hss] & [Hsk] \\
[Hsk] & [Hij] & [Hkk] \\
\end{bmatrix}
\]

\[
[K^*] = \begin{bmatrix}
[Hks] & [Hss] & [Hsk] \\
[Hsk] & [Hij] & [Hkk] \\
\end{bmatrix}^{-1}
\]

**Figure 10.** \( K^* \) matrix dimension for extended interface approach method

The method can be used, when there is no information related to rotational FRF is available and rotational stiffness can be determined using translational DOF information as in case of solid element.

**1.4 Finding sensitive modes**

As for polluted FRF, stiffness evaluated at certain frequencies is more accurate compared to other natural frequencies. The modal frequencies which shows better accuracy are called as sensitive modes. Below 2 are methods to find most sensitive modes to evaluate stiffness parameter.

**Method 1:** Sensitivity analysis can be found by changing joint stiffness to see its effect on combined system’s FRF. Those modes which shows maximum change in shift of natural frequencies, will be the ones should be considered for stiffness measurements.

**Method 2:** Another method to find such sensitive modes / natural frequencies will be to find the Modal deformation difference (d) for connection coordinates, calculating ratio of those deformations with maximum absolute deformation (D) for given mode. The mode with maximum ratio (d/D) will be the most sensitive mode.
Consider 2 beam substrcutures connected by 3 translational stiffness as shown below.

In first case, stiffness in Y direction is modified by using 2 times of original value and plotting coupled system Y directional FRF. The difference in changed value for modal frequency gives information about the sensitive modes. In second case as, ratio of (d/D) is evaluated.
Table 2. Natural frequency difference observed in modal coupling method in beam system

| Direction | Mode number | Frequency in rad/sec | Shift in natural frequency | Most sensitive mode in ascending order |
|-----------|-------------|----------------------|----------------------------|-------------------------------------|
| Y         | 1           | 0                    | 0                          | 1                                   |
| Y         | 2           | 260                  | 0.075                      | 2                                   |
| Y         | 3           | 1817                 | 14.09                      | 3                                   |
| Y         | 4           | 3532                 | 47.49                      | 4                                   |
| Y         | 5           | 6262                 | 148.55                     | 5                                   |
| Y         | 6           | 10040                | 1328.26                    | 7                                   |
| Y         | 7           | 15410                | 416.37                     | 6                                   |
| Y         | 8           | 18171                | 2640.52                    | 8                                   |

Table 3. Method 2 parameters for determining most sensitive mode

| Direction | Mode number | Frequency in rad/sec | d  | D   | R = d/D | Most sensitive mode in ascending order |
|-----------|-------------|----------------------|----|-----|---------|-------------------------------------|
| Y         | 1           | 0                    | 0  | 183 | 0.0027  | 1                                   |
| Y         | 2           | 260                  | 0.36| 138.6| 0.0027  | 2                                   |
| Y         | 3           | 1817                 | 13.45| 125.8| 0.1069  | 3                                   |
| Y         | 4           | 3532                 | 34.61| 182.11| 0.1901  | 4                                   |
| Y         | 5           | 6262                 | 83.94| 146.39| 0.5735  | 5                                   |
| Y         | 6           | 10040                | 280.19| 195.75| 1.4313  | 7                                   |
| Y         | 7           | 15410                | 197.19| 232.97| 0.8468  | 6                                   |
| Y         | 8           | 18171                | 373.62| 231.07| 1.6169  | 8                                   |

2. Solid elements in evaluating stiffness parameter for joints

As method is already implemented for 2by2 (one rotation and one translation), same approach can be extended to 6by6 (3 translational and 3 rotational) case. The internal DOFs for residual subsystem B will be having nine DOF (s coordinates).

\[
[K^*] = \left[ \begin{bmatrix} H_{ks} \end{bmatrix}, \left[ \begin{bmatrix} H_{ss} \end{bmatrix} - \left[ \begin{bmatrix} H_{ss} \end{bmatrix} \right]^T \right]^{-1} \cdot \left[ H_{sk} \right] - \left[ H_{jj} \right] - \left[ H_{kk} \right] \right]^{-1}
\]

--- (9)

As it can be observed from equation, decoupling is done at nine interfaces DOF, while coupling DOF is six in number. This is extended interface approach with non-collocated case. As far as controllability between equilibrium and compatibility DOFs is ensured, interface DOFs need not necessary be same as compatibility DOFs.
2.1 FEA model details

The blue plate represents substructure A, while yellow plate represents substructure B. The DOF required at j and k coordinate should be six (Translational X, Y, Z and Rotational Rx, Ry, Rz). As there are only translational DOF available for SOLID 185 elements, we can use RBE2 element with mass 21 elements to get the six DOF at j and k coordinates. The nodes on one face end for substructure A fixed. Connecting coordinates ‘j’ and ‘k’ are chosen at master node of RBE2 element. The ‘s’ coordinate is based on previous extended interface approach, 9 nodes selected as s1,s2,s3,s4,s5,s6,s7,s8,s9.

Substructure A:

In Ansys FEA package, mass21 element defined with K3 option as "3-D with Rot inertia". Solid185 element is defined for meshing solid plates.
Figure 16. 6 stiffness defined using combin14 element between j and k coordinate

Three stiffness in translational direction and 3 stiffness in rotational direction have been defined. Six combin14 elements defined with stiffness value defined in corresponding real constants.

Stiffness in X translational direction, \( K_X = 500 \, \text{N/mm} \)
Stiffness in Y translational direction, \( K_Y = 700 \, \text{N/mm} \)
Stiffness in Z translational direction, \( K_Z = 900 \, \text{N/mm} \)
Stiffness in X rotational direction, \( K_{Rx} = 1200 \, \text{N/m} \)
Stiffness in Y rotational direction, \( K_{Ry} = 1500 \, \text{N/m} \)
Stiffness in Z rotational direction, \( K_{Rz} = 1800 \, \text{N/m} \)

Above values are considered for validation of FRF decoupling method for solid plates.

Figure 17. ‘s’ coordinates used to get information for rotational DOF

The Dofs evaluated for S1, S2, and S3 is Y direction translation, which can give information about rotation Z-axis at S2 (DOF Rz). The DOFs evaluated for S4, S5, S6 is X direction translation, which can give information about rotation Y-axis at S5 (DOF Ry). The DOFs evaluated for S7, S8, S9 is Z direction translation, which can give information about rotation X-axis at S8 (DOF Rx).
2.2 Matrix formulations for $K^*$ and FRF

\[
[K^*] = \left[ [H_{ks}] \left[ [H_{ss}]-\left[H^{'ss}\right] \right]^{-1} \left[ H_{sk} \right] - \left[ H_{jj} \right] \right]^{-1} \quad \text{--- (10)}
\]

\[
[K^*] = \begin{bmatrix}
Y & Y_{yx} & Y_{yx} & Y_{yx} & Y_{yx} & Y_{yx} \\
0 & 0 & 0 & 0 & 0 & 0 \\
X & X_{xy} & X_{xy} & X_{xy} & X_{xy} & X_{xy} \\
0 & 0 & 0 & 0 & 0 & 0 \\
Z & Z_{xz} & X_{xz} & Z_{xz} & Z_{xz} & Z_{xz} \\
0 & 0 & 0 & 0 & 0 & 0 \\
6 \times 6 & 6 \times 9 & 9 \times 9 & 9 \times 9 & 9 \times 6 & 6 \times 6 \\
6 \times 6 & 6 \times 9 & 9 \times 9 & 9 \times 6 & 6 \times 6 & 6 \times 6 \\
\end{bmatrix}
\]

[FRF matrix formulation details]

Figure 18.
3. Results

Figure 19. FEM Model for coupled system

![Figure 19](image1.png)

Figure 20. 6 direction stiffness curve

Results shows, when all modes (2004) are considered for calculating FRF, stiffness values for all 6 directions is constant over frequency. When for calculating FRFs from modes, first 1000 modes are accounted. This will introduce modal truncation error. Below figure shows the stiffness curve when all modes are not considered for calculating FRFs.
Figure 21. Stiffness curve when 1000 modes are used for calculating FRFs from available 2004 modes.

As can be seen, translational stiffness shows correct values for stiffness between 8000 to 14000 rad/sec. Rotational stiffness are having less sensitivity compared to translational stiffness.

The point FRF for coupled system which is HFSS is polluted 0.5% in Matlab code and result shows below stiffness curve.

Figure 22. Stiffness curve with polluted effect
Figure 23. Stiffness curve for X and Y translational direction

Stiffness in X direction for polluted FRF case at sensitive mode, values are near to specified stiffness of 500 N/mm. Stiffness in Y direction for polluted FRF case at sensitive mode, values are near to 700 N/mm.

Figure 24. Stiffness curve for Z and Rx direction

Stiffness in Z direction for polluted FRF case at sensitive mode, values are near to specified stiffness of 900 N/mm. Stiffness in Rx direction for polluted FRF. As modes are not so sensitive, different location needs to be excited to have good correlation for FRFs.

Figure 25. Stiffness values at sensitive modes for Ry and Rz directions.
Stiffness in Ry direction for polluted FRF, different measurement points may have evaluated correct values at sensitive modes Stiffness in Rz direction for polluted FRF. Near sensitive modes, average over frequency can be option for getting closer to specified stiffness values.

4. Conclusion
The use of FRF decoupling method for solid elements has been verified to determine joint parameters. Though solid elements do not have rotational DOFs, still using extended interface approach, it is possible to determine rotational stiffness along with translational directional stiffness for joints. Sensitivity of modes can be effectively used to determine most sensitive modes to evaluate stiffness.

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