Particle-like property of vacuum states

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Abstract

The wave-particle duality of the vacuum states of quantum fields is considered and the particle-like property of the vacuum state of a quantum field is proposed as a vacuum-particle which carries the vacuum-energy and the vacuum-momentum of this field. The vacuum-particles can be introduced into the quantum field theory (QFT) naturally without disturbing its mathematical structures and calculation results, but makes the QFT more self-consistent. For instance, the interactions between charged particles and vacuum state of electromagnetic field appears automatically in the Feynman diagrams of the quantum electrodynamics (QED), with which the atomic spontaneous emission and the Casimir effect can be interpreted directly by the QED perturbation theory. Besides, the relation between vacuum-particles and spontaneous symmetry breaking is also discussed.

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I. INTRODUCTION

Wave-particle duality is the central concept of quantum mechanics. It originated from the study of both wave-like and particle-like nature of lights, and was generalized to matters by Louis de Broglie in 1924 [1]. Later the de Broglie hypothesis, which argues that all matter and energy exhibits both wave-like and particle-like properties, was left to Erwin Schrodinger to discover the wave equations of quantum mechanics in 1926. Up to the present time many experiments have proved that all known particles have the wave-like property. However, the vacuum states of quantized fields, which carry non-zero energy, has not been treat by wave-particle duality yet.

An example is the Klein-Gordon field, whose momentum and energy after canonical quantization are showed in Eq.\[1\]

\[
\begin{align*}
P &= \sum_k (a_k^+ a_k + \frac{1}{2}) p_k \\
H &= \sum_k (a_k^+ a_k + \frac{1}{2}) \omega_k
\end{align*}
\]

Here \(\omega_k = \sqrt{p^2 + m^2}\) is the frequency of the mode with wave-vector \(k\), where \(k\) is the four-momentum. We adopt the natural unit \(\hbar = c = 1\) in the whole paper. In Eq.(1), when the eigenvalues of the particle-number operators in every mode \((a_k^+, a_k)\) are equal to zero, the Klein-Gordon field has the vacuum momentum (the sum of all modes is zero) \(\sum_k \frac{p_k}{2}\) and the vacuum energy \(\sum_k \frac{\omega_k}{2}\).

The vacuum state of the Klein-Gordon field are treated as the plane waves of different modes \((\frac{p_k}{2}, \frac{\omega_k}{2})\). An interesting question is what may occur if the vacuum state has the same wave-particle duality to all particles? It means that the vacuum states may also have the particle-like property like the excited states of fields (particles). For the vacuum state of the Klein-Gordon field, a plane wave with \((\frac{p_k}{2}, \frac{\omega_k}{2})\) is mathematically equivalent to a vacuum-particle who has the momentum \(\frac{p_k}{2}\) and the energy \(\frac{\omega_k}{2}\). The vacuum-particle can be excited to an one-particle state with \((\frac{3p_k}{2}, \frac{3\omega_k}{2})\) by absorbing momentum \(\frac{p_k}{2}\) and energy \(\frac{\omega_k}{2}\) (a particle of the Klein-Gordon field).

So we are interested in study such possibility. The paper is arranged as follows: In section 2, a vacuum particle hypothesis (VPH) is presented and the new lines which denote to vacuum-particles are introduced into Feynman diagrams automatically. In section 3, we
apply the VPH to the QED perturbation theory without disturbing the calculation of all S-matrix elements, but exhibit the role of vacuum-particles of dirac field and vacuum-photons in QED, with which the Casimir effect and the atomic spontaneous emission is studied by the QED perturbation theory more reasonably. Section 4 contents the discussion of the relationship between vacuum-particles and the spontaneous symmetry breaking, as well as the conclusion.

II. VACUUM-PARTICLE HYPOTHESIS (VPH)

A field is always defined and quantized at every point of the space, and the vacuum energy or the momentum of a field is the integrating value all over the space. We have proposed that the wave-particle duality of vacuum states is valid in the introduction, then the vacuum energy and momentum of a single-mode field are carried by a particle-like object named "vacuum-particle". We present a Vacuum-Particle Hypothesis (VPH) with two postulates:

1. Our space is filled with vacuum particles, each of which is the vacuum state of a single-mode quantum field and it carries the vacuum-momentum and vacuum-energy of this field.

2. The \( N \)-particle state of a single-mode field is that the vacuum-particle of this field overlapped by \( N \) field quanta (particles) of this field.

The two postulates are easy to understand by analogizing with a quantum harmonic oscillator. Its lowest energy state \( \frac{1}{2} \hbar \omega \) is analogized to the energy a vacuum-particle, its \( N \) quanta excited state \( (N + \frac{1}{2})\hbar \omega \) is analogized to the \( N \)-particle state of a single-mode field, and its quantum \( \hbar \omega \) is analogized to a field quantum (particle). Through the whole paper, the word "particle" always denotes a field quantum and the word "\( N \)-particle state" always denotes the state that a vacuum state overlapped by \( N \) field quanta.

According to postulate 1, all vacuum-particles need to be on-shell because the vacuum state is one of the eigenstate of a quantum field and the vacuum energy and momentum are always on the mass shell to compose the four-momentum of a vacuum-particle. According to postulate 2, the particle and the vacuum-particle of a single-mode field must have the same velocity so they can overlapped each other all the time. For a massive field, it means the momentum difference between the vacuum-particle and the particle comes from the differ-
FIG. 1: Simplified Feynman diagrams of the creation and annihilation processes of a Klein-Gordon particle. Solid lines with arrow denote to one-particle state of Klein-Gordon field, Solid lines with a circle denote to a vacuum-particle of Klein-Gordon field. The dot lines with arrows show the transformation of Feynman diagrams when the lines of vacuum particles are introduced.

ence of their proper mass. For a massless field, the same-velocity condition is automatically obeyed.

A vacuum particle can be naturally introduced into the physical processes of creation and annihilation of a particle in the quantum field theory (QFT). Figure 1 shows the difference between the Feynman diagrams of creating/annihilating a particle of Klein-Gordon field before and after the vacuum particle of the Klein-Gordon field is introduced. One is for the creation process

\[ a^+_k | \cdots, 0_k, \cdots \rangle = | \cdots, 1_k, \cdots \rangle, \]  

and the other is for the annihilation process

\[ a_k | \cdots, 1_k, \cdots \rangle = | \cdots, 0_k, \cdots \rangle. \]  

Here both the creation and the annihilation operator have the new physical meanings. For the Klein-Gordon field, the creation operator \( a^+_k \) in the mode \( k \) means exciting a vacuum-particle with four-momentum \( k/2 \) into the one-particle state with four-momentum \( 3k/2 \), while the four-momentum increase \((3k/2 - k/2)\) equals to the four-momentum of the field quantum \( k \). The annihilation operator \( a_k \) in the \( k \) mode means transferring an one-particle state with four-momentum \( 3k/2 \) into the vacuum-particle with four-momentum \( k/2 \), while the four-momentum loss is \((3k/2 - k/2 = k)\). The field operator is

\[ \phi(x) = \frac{1}{(2\pi)^{3/2}} \sum_k \sqrt{\frac{1}{2\omega_k}} (a^+_k e^{-ikx} + a_k e^{ikx}). \]  

Here the four momentum of the field quantum \( k \) has the new physical meaning of \( k = 3k/2 - k/2 \).

With the discussion above, we see after the VPH is introduced, only the energy of every eigenstate of a quantum field is redefined. The field quantum, field operator as well as
the Lagrangian are all unchanged. Therefore the introduction of vacuum-particles does not disturb any calculation process of quantum field theory (QFT). Instead, the vacuum states of quantum fields acquire more clear physical meanings and recover their momentum and energy by the VPH in the physical processes of QFT. It makes the QFT more self-consistent than removing the vacuum-momentum and vacuum-energy of a field by hand (the trick which is always used in the QFT). In next section we will focus on the role that the vacuum-particles of the Dirac field and the electromagnetic field (vacuum-photons) plays in the basic processes of QED perturbation theory.

III. QUANTUM ELECTRODYNAMICS WITH THE VPH

The interaction between charged particles and photons is well described by the quantum electrodynamics (QED). The calculation results of QED has showed great agreements with the experimental data. However, there are some phenomena that caused by the vacuum states of electromagnetic fields are widely studied, but has been not treated by QED yet. One is the Casimir effect [2], which has been observed and measured in the past few years. [3, 4, 5]. Another is the atomic spontaneous emission, which has been explained by the interactions between the excited electrons of atoms and the vacuum states of the electromagnetic fields. It can be controlled by cavities [6, 7, 8]. The two phenomena imply that the vacuum states of the electromagnetic fields can interact with the electric charged particles, but they are not described directly by the QED since the vacuum energy and momentum of the electromagnetic fields are replaced by hand in the QED.

In this section, we will merge the VPH into the QED perturbation theory naturally by redefining the momentum and the energy of every eigenstate, and the physical meaning of the creation and annihilation operators of Dirac and electromagnetic field (as we did to Klein-Gordon field in section 2). This process does not disturb the mathematical form of QED but can describe the coupling of electric charged particles, photons and vacuum particles of the two fields, with which the atomic spontaneous emission and the Casimir effect can be interpreted and studied directly by the QED perturbation theory.
Vacuum energy and momentum of the Dirac field are showed in Eq [5]

\[ \mathbf{P}_{\text{Dirac}} = \sum_{p} \sum_{\sigma=1}^{2} \left( c_{\sigma p}^{+} c_{\sigma p} + d_{\sigma p}^{+} d_{\sigma p} - 1 \right) \mathbf{p} \]
\[ H_{\text{Dirac}} = \sum_{p} \sum_{\sigma=1}^{2} \left( c_{\sigma p}^{+} c_{\sigma p} + d_{\sigma p}^{+} d_{\sigma p} - 1 \right) E_p \]

Here \( \mathbf{P} \) is the momentum vector and \( p \) is the four-momentum. When the eigenvalues of the particle-number operator of electrons \( (c_{\sigma p}^{+} c_{\sigma p}) \) and positrons \( (d_{\sigma p}^{+} d_{\sigma p}) \) are all equal to zero in every mode, according to the VPH, the vacuum state of a Dirac field is a vacuum-particle which can be excited to either an one-electron state or an one-positron state. Every vacuum-particle of the Dirac field has a negative momentum eigenvalue \(-p\) and a negative energy eigenvalue \(-E_p = -\sqrt{p^2 + m^2}\), which are just the additive inverses of the momentum and energy of a field quantum (either an electron or a positron) with four-momentum \( p \). It is still a problem that the vacuum energy of Dirac field is negative. Although some works have tried to solve it, this problem is beyond the aim of this paper so we treat them as negative values in the whole paper.

Similar as Eq [5], vacuum energy and momentum of the electromagnetic field can be found in Eq [6] when the eigenvalue of the photon-number operator \( a_{\lambda k}^{+} a_{\lambda k} \) equals to zero in every mode. Eq [6] are the results of the quantization of electromagnetic field in the Lorentz gauge, where \( k \) is the momentum vector and \( \mathbf{k} \) is the four-momentum.

\[ \mathbf{P}_{\text{EM}} = \sum_{k} \sum_{\lambda=1}^{4} \left( a_{\lambda k}^{+} a_{\lambda k} + \frac{1}{2} \right) \mathbf{k} \]
\[ H_{\text{EM}} = \sum_{k} \sum_{\lambda=1}^{4} \left( a_{\lambda k}^{+} a_{\lambda k} + \frac{1}{2} \right) \omega_k, \]

Here \( a_{\lambda k}^{+} = a_{\lambda k}^{+} \) when \( \lambda = 1, 2, 3 \) and \( a_{\lambda k}^{+} = -a_{\lambda k}^{+} \) when \( \lambda = 4 \). \( \lambda = 1, 2 \) means the two transverse polarizations, \( \lambda = 3 \) means the longitudinal polarization, and \( \lambda = 4 \) means the time-like polarization. According to the VPH, a vacuum-particle of the electromagnetic field, which is a vacuum-photon, has half energy \( \frac{\omega_k}{2} \) and half momentum \( \frac{\mathbf{k}}{2} \) of a quantum of electromagnetic field (a photon) with four-momentum \( \mathbf{k} \), while the speed of all vacuum-photons are always \( c \).

After the VPH is applied to the vacuum states of the Dirac field and the electromagnetic field, we can give the new physical meanings of their creation and annihilation operators as
we did to the Klein-Gordon field in section 2. For the Dirac field, $c_p^+ / d_p^+$ means exciting a vacuum-particle with four-momentum $-p$ into the one-electron/positron state with four-momentum 0, while the annihilation operator $c_p / d_p$ means the opposite process. The four-momentum increase or loss is $0 - (-p) = p$, which equals to a Dirac field quanta $p$. For the electromagnetic field, the creation operator $a_k^+$ means exciting a vacuum-photon with four-momentum $k$ (or a N-photon state with four-momentum $(N + \frac{1}{2})k$) into the one-photon state with four-momentum $3k/2$ or a $(N+1)$-photon state with four-momentum $(N + \frac{3}{2})k$, and the annihilation operator $a_k$ means the opposite process. The four-momentum increase or loss $\frac{3k}{2} - \frac{k}{2} = k$ equals to the field quanta (a photon) with four-momentum $k$.

Now the VPH can be mounted to QED perturbation theory. The field quanta and field operators of the Dirac field and the electromagnetic field are unchanged. The Lagrangian of QED is also unchanged. Only the vacuum state of the Dirac field is redefined as vacuum-particles with four-momentum $\sum_p - p$, and the vacuum state of the electromagnetic field is redefined as vacuum-photons with four-momentum $\sum_k \frac{k}{2}$. With the new physical meanings of the creation and annihilation operators of the Dirac field and the electromagnetic field, the Feynman diagrams of QED need to change into the new forms, where all three-line vertexes need to be replaced by six-line vertexes. It is just like we did to the Klein-Gordon field in figure 1. In the QED perturbation theory, all physical process according to the first-order S-matrix elements can not obey the energy and momentum conservation at same time, so the most basic physical process are described by second-order S-matrix elements, which are

$$S_{ji}^{(2)} = \langle f | - ie \left[ \bar{\varphi}(x_1) A_\mu(x_1) \varphi(x_1) \bar{\varphi}(x_2) A_\mu(x_2) \varphi(x_2) \right. $$

$$+ \bar{\varphi}(x_1) A_\mu(x_1) \varphi(x_1) \bar{\varphi}(x_2) A_\mu(x_2) \varphi(x_2) $$

$$+ \bar{\varphi}(x_1) A_\mu(x_1) \varphi(x_1) \bar{\varphi}(x_2) A_\mu(x_2) \varphi(x_2) $$

$$+ \bar{\varphi}(x_1) A_\mu(x_1) \varphi(x_1) \bar{\varphi}(x_2) A_\mu(x_2) \varphi(x_2) $$

$$+ \bar{\varphi}(x_1) A_\mu(x_1) \varphi(x_1) \bar{\varphi}(x_2) A_\mu(x_2) \varphi(x_2) $$

$$+ \bar{\varphi}(x_1) A_\mu(x_1) \varphi(x_1) \bar{\varphi}(x_2) A_\mu(x_2) \varphi(x_2) \left| i \right> \right.$$

where $\varphi(x)$ is the field operator of Dirac field and $A_\mu(x)$ is the field operator of electromagnetic field.

Figure 2 shows the new Feynman diagrams corresponding the Second-order S-matrix ele-
FIG. 2: Feynman diagrams of every second-order S-matrix element of the QED perturbation theory. Lines with up-arrow denote to electrons overlapped by their vacuum-particles and lines with down-arrow denote to positrons overlapped by their vacuum-particles. Lines with a circle denote to vacuum-particles of Dirac fields. Wave lines denote to photons overlapped by their vacuum-photons and wave lines with circles denote to vacuum-photons.

The lines of vacuum-photons emerges in the Feynman diagrams of QED perturbation theory. Phenomena cause by the interaction between vacuum-states of electromagnetic fields (vacuum-photons) and charged particles can be studied directly by the QED perturbation theory now. We see the vacuum-photons participle every physical process in figure 2 as the external-lines, whose four-momentum $\frac{1}{2}$ is invariant from initial state $|i\rangle$ to final state $\langle f|$. However, in some physical process corresponding to higher S-matrix elements, the four-momentum of a vacuum-photon may be changed from the initial state to the final state at a small space-time region, which can be considered as an interpret of Casimir effect from
FIG. 3: Feynman diagram of a fourth-order S-matrix element of QED perturbation theory which can be considered as an interpret of Casimir effect. Lines with arrow denote to electric charged particles overlapped by their vacuum-particles and lines with a circle denote to their vacuum-particles. Wave lines denote to photons overlapped by their vacuum-photons and wave lines with circles denote to vacuum-photons.

the QED view. For example, figure 3 is a case corresponding to a fourth-order S-matrix element. If the distances between each two of the space-time points \(x_1, x_2, x_3, \) and \(x_4\) fit the condition

\[
|x_1 - x_3| \approx |x_2 - x_4| \gg |x_1 - x_2| \approx |x_3 - x_4|,
\]

the process can be considered as a vacuum-photon with four-momentum \(k_0\) is absorbed at \(x_1\) and is emitted with four-momentum \(k'_0\) at \(x_2\), which is just a scattering process of a vacuum-photon near the space-time points \(x_1\) and \(x_2\). The Casimir effect \([2]\) requires such scattering processes to describe how a vacuum-photon can be reflected by the surface of a conducting plate, then the radiation force from the vacuum-photon can act on the surface of the conducting plate. According to the wave-particle duality, this process is equivalent to it that the plate reflects the plane wave of the vacuum-state of the electromagnetic field, which is the original interpret of the Casimir force \([2]\).

Figure 4 is the Feynman diagram of a third-order S-matrix element which can be described as the basic QED process of atomic spontaneous emission. An electron is perturbed by the vacuum states of electromagnetic field (vacuum-photons) at the space-time point \(x_2\) and excites a vacuum-photon with four-momentum \(\frac{k}{2}\) into a photon with four-momentum \(\frac{3k}{2}\) at \(x_2\). Then the electron becomes a off-shell electron and propagates to \(x_1\) to transfer the Coulomb interaction with a quark in the nuclear. After that process the off-shell electron recovers to be on-shell at \(x_1\). Through the whole process, the electron transits from a higher
FIG. 4: Feynman diagram of a third-order S-matrix element of QED perturbation theory which describes the basic process of atomic spontaneous emission. Thick line with arrow denote to a quark overlapped by its vacuum-particle, and thin line denote to an electron overlapped by its vacuum-particle. Wave lines denote to overlapped by their vacuum-photons and wave lines with circles denote to vacuum-photons.

energy level at $x_2$ to a lower energy level at $x_1$ with one photon emitted at $x_2$. The four-momentum of the emitted one-photon state $\frac{3k}{2}$ is determine by the four-momentum of the vacuum-photon $\frac{k}{2}$ and the photon $k$ at $x_2$.

The above two physical phenomena has been widely studied in the subject of quantum optics, where the vacuum state of electromagnetic field is treated as plane waves which carries the vacuum energy of the electromagnetic field. Because the possible particle-like property of these plane waves is not concerned before, the above two physical phenomena can not be studied by QED. After we considered the particle-like property of these plane waves by introduce the VPH, the interprets of the above two phenomena emerge from the QED perturbation theory automatically. This result shows the existence of the particle-like property of vacuum states is reasonable.

IV. DISCUSSIONS

We have introduced the VPH to the QED perturbation theory. Because the VPH is postulated for all kinds of quantum fields in the QFT, there are vacuum-particles that correspond to quarks, leptons besides electrons and positrons, and bosons besides photons. All of them can be defined from the vacuum-momentum and vacuum-energy of their fields like Eq(5) and Eq(6). So the VPH can be introduced into all non-abelian quantum gauge
theories such as electro-weak theory and quantum chromodynamics (QCD) without disturb their calculation process just like in QED. This topic will be interesting for the future study.

Besides, there is another important background theory of QFT in the standard model which may be highly related to the vacuum-particles. It is the spontaneous symmetry breaking. Since the Goldstone’s bosons emerges from the spontaneous continuous symmetry breaking, vacuum-particles of them can be easily defined. However, Goldstone’s bosons can not be observed in the nature because the spontaneous breaking of gauge symmetry do not generate them but generate Higgs bosons instead via Higgs mechanism. It is necessary and interesting to discuss the Higgs mechanism from the point of the vacuum-particle view. As we know, before spontaneous symmetry breaking, the Higgs field is a complex scalar field $\phi(x)$ with a non-zero vacuum expectation value, which means its vacuum-particle does not have the lowest possible energy and momentum. After the spontaneous symmetry breaking, a real vector field $A_\mu(x)$ obtain the proper mass from the Higgs field and the Higgs field becomes a Klein-Gorden field $\sigma(x)$. This process can be interpreted as the vacuum-particles of the vector field $A_\mu(x)$ obtain mass via coupling with the vacuum-particles of the Higgs field $\phi(x)$, and at the mean time the vacuum-particle of $\phi(x)$ becomes the vacuum-particle of field $\sigma(x)$, whose one-particle excited state is a one Higgs boson state. Thus in the VPH’s point of view, the Higgs mechanism is a process that happens due to the interaction between the vacuum-particles of different fields.

In conclusion, the vacuum-particle hypothesis (VPH) can be obtained naturally from the proposed particle-like property of vacuum state of quantum fields by the wave-particle duality. It recovers the energy and momentum of vacuum states, which benefit the self-consistency of the quantum field theory (QFT) without disturbing its mathematical form and calculation process. With the VPH, the Casimir effect and atomic spontaneous emission which are caused by the interaction between the matter and vacuum-state of the electromagnetic field can be interpreted and calculated within the frame of the QED perturbation theory directly. The Higgs mechanism can also be interpreted in a more clearly physical picture by the VPH.
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