On the Possibility of a Significant Increase in the Storage Time of Ultracold Neutrons in Traps Coated with a Liquid Helium Film

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It is shown that rough inner walls of a trap of ultracold neutrons can be coated with a superfluid helium film much thicker than the depth of penetration of ultracold neutrons into helium. This coating should reduce the rate of loss of ultracold neutrons caused by absorption in the walls of the trap by orders of magnitude. It is demonstrated that triangular roughness is more efficient than rectangular for the reduction of the rate of loss of ultracold neutrons. Triangular roughness is more easily implemented technically and such diffraction gratings are fabricated industrially. Other methods are proposed to increase the thickness of the protective helium film.

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1. INTRODUCTION

The study of the properties of a free neutron and its interaction with known or hypothetical fields provides valuable information on elementary particles and their interactions [1–5]. Accurate measurements of the lifetime of the neutron \( \tau_n \) make it possible to determine weak coupling constants, which is important for elementary particle physics, astrophysics, and cosmology [1–5]. Measurements of the asymmetry of the \( \beta \) decay of the neutron provide information on the ratio of the vector and axial vector weak coupling constants [6–8]. The search for a nonzero electric dipole moment of the neutron [9–11] limits the \( CP \) violation. Resonant transitions between discrete quantum energy levels of neutrons in the gravitational field of the Earth [12, 13] allow the study of the gravitational field at the microscale and impose constraints on dark matter. Neutron diffraction on crystals can be used to search for new internucleon interactions [14, 15].

The highest accuracy of measurement of the lifetime of the neutron is currently achieved with special traps for ultracold neutrons (UCNs) [16–19]. Ultracold neutrons in such traps are confined from above by the gravitational field of the Earth and from below and sides by a material that weakly absorbs neutrons and produces a potential barrier with the height \( V_0 \lesssim 300 \text{ neV} \) [7, 8, 10–12, 16–23]. The latest measurements with such traps give [19] \( \tau_n = (881.5 \pm 0.7 \, \text{[stat]} \pm 0.6 \, \text{[syst]} \) s. Since the neutron has a magnetic moment of 60 neV/T, the magnetogravitational capture of UCNs is possible [24–28]. The most accurate measurement of the lifetime of the neutron yields \( \tau_n = (877.7 \pm 0.7 \, \text{[stat]} + 0.4/-0.2 \, \text{[syst]} \) s [28]. According to the estimates in [19, 28], the errors of these methods are no more than 1 s, but the corresponding \( \tau_n \) values differ by almost 4 s. Such a large difference is apparently due to ignored or underestimated loss of UCNs in magnetic traps. The measurement of the lifetime of the neutron with a beam of cold neutrons [29–31], which is the main alternative to the measurement with UCNs, gives \( \tau_n = (887.7 \pm 1.2 \, \text{[stat]} \pm 1.9 \, \text{[syst]} \) s. The difference of this value from the measurements by other methods is even larger, which is a known unsolved enigma. Possible reasons for this difference are actively sought [32] from ignored errors in experiments with the neutron beams [33] to new decay channels of the neutron [31] or even dark matter [34].

The error of current experiments with traps of UCNs can also be larger than the estimate \( \leq 1 \) s because the procedure of determination of \( \tau_n \) from experimental data involves their extrapolation by more than 15 s to the limit of “zero loss” of neutrons in traps. One of the many possible reasons for incorrect extrapolation is the approximation of an isotropic (i.e., uniformly distributed in directions) impact of neutrons on the surface of a trap to calculate the loss rate. The real angular distribution of the velocities of UCNs is not isotropic and depends on the height: the larger the height from the bottom of the trap, the smaller the vertical component of the velocity of UCNs because of the gravitational field. This can affect both the geo-
metric and temperature extrapolations. This effect can
in principle be taken into account in Monte Carlo cal-
culations if the normal component of the velocity of
the neutron is determined for each impact with the
wall. The roughness of the surface also affects the loss
rate [20, 21], but it is more difficult to calculate this
effect. Although an extrapolation time of about 15 s is
the great achievement of last 15 years [16, 19], it is
rather long and prevents a further significant increase
in the accuracy of measurement of \( \tau_n \). To reduce the
extrapolation interval, it is necessary to reduce the loss
rate of UCNs. The main reason for this loss is the
interaction with the walls of traps, which slightly
absorb neutrons.

One of the fundamental solutions to the problem of
absorption of neutrons by the walls of the trap is their
coating with liquid \(^4\text{He}\), which does not absorb neu-
trons. Superfluid \(^4\text{He}\) coats all the walls of a vessel with
a thin film because of the van der Waals attraction.
However, the thickness of this film \( d_{\text{He}} \) is too small.
Since neutrons with energies below the potential bar-
rier \( V_0^{\text{He}} = 18.5 \text{ neV} \) rise to the maximum height
\( h_{\text{max}} = V_0^{\text{He}}/(m_\text{He}g) = 18 \text{ cm} \), we are interested in the
height above the helium level that is much larger than
the capillary length \( \kappa_{\text{He}} = \sqrt{\sigma_{\text{He}}/(g \rho_{\text{He}})} = 0.5 \text{ mm} \),
where \( \sigma_{\text{He}} = 0.354 \text{ dyn/cm} \) is the surface tension of
\(^4\text{He}\), \( g = 9.8 \text{ m/s}^2 \), and \( \rho_{\text{He}} = 0.145 \text{ g/cm}^3 \) is the
density of liquid \(^4\text{He}\). The film thickness at this height is
\( d_{\text{He}} \approx 10 \text{ nm} \), which is much smaller than the penetra-
tion depth \( \kappa_{\text{He}} = h/\sqrt{2m_\text{He}V_0^{\text{He}}} = 33.5 \text{ nm} \) of
neutrons into \(^4\text{He}\). Hence, such a film hardly protects neutrons
from absorption inside the wall of the trap.

The problem of an increase in the thickness of the
helium film is important only for the vertical (side)
walls of traps of UCNs. The bottom of the trap can be
easily coated with the helium film with the required
thickness \( d_{\text{He}} \gg \kappa_{\text{He}} = 33.5 \text{ nm} \); neutrons are con-
fined from above by the gravitational field of the
Earth. Only the lower part of the side walls to the
height \( h < a_{\text{He}} \sqrt{2} \ll h_{\text{max}} \) are coated with the meniscus
with the thickness \( d_{\text{He}} \sim a_{\text{He}} \gg \kappa_{\text{He}} \). To increase the
thickness of the helium film on the side walls of the
trap above the capillary length, it was proposed to store
neutrons in a rotating vessel with helium [35, 36].
However, the reflection of neutrons from the moving
surface gradually increases their kinetic energy, which
finally exceeds the potential barrier \( V_0^{\text{He}} = 18.5 \text{ neV} \),
and a neutron leaves the trap. Furthermore, the rotat-
ing liquid generates additional bulk and surface exci-
tations, which increases the inelastic scattering rate of
neutrons. Consequently, time-independent coating of
the walls of the trap with liquid \(^4\text{He}\) is necessary.

We recently assumed [37] that the thickness of the
helium film on the rough wall of the trap of UCNs
increases because of capillary effects. The roughness
of the wall usually increases the loss of neutrons by a
factor of 2–3 through absorption inside the walls of
the trap because this roughness makes the average repulsive potential of the walls smoother, so that the
wavefunction of the neutron penetrates deeper into the
wall [20, 21]. However, the average thickness of the
helium film deposited on such rough wall can be
strongly increased compared to the smooth surface
because of capillary effects, thus reducing the loss of
neutrons. Indeed, the roughness of the wall increases
its area and, correspondingly, the role of capillary
effects. If the scale of the surface roughness is \( l_R \ll a_{\text{He}} \),
to minimize the surface tension energy, the helium
film even on the rough wall should have an almost flat
interface with vacuum. Therefore, superfluid helium
fills all small cavities with the size \( l_R \ll d_{\text{He}} \) in the wall.

In this work, we develop this idea and propose
implementable variants of the side walls of the trap, for
which the thickness of the helium film is large and the
loss rate of UCNs through absorption in the wall of the
trap decreases by orders of magnitude.

2. HELIUM FILM ON THE ROUGH SURFACE:
GENERAL FORMULAS

To describe the profile of the helium film on the
rough surface, it is necessary to minimize the energy
functional of this film

\[
E_{\text{tot}} = V_g + E_s + V_w.
\]

Here, \( V_g \) is the gravity term given by the expression

\[
V_g = \rho_{\text{He}} g \int d_\text{He}(\mathbf{r}_\parallel) d^2 \mathbf{r}_\parallel,
\]

where \( \mathbf{r}_\parallel = \{x, z\} \) is the two-dimensional vector of the
horizontal, \( x \), and vertical, \( z \), coordinates on the wall,

\[
d_\text{He}(\mathbf{r}_\parallel) = \xi(\mathbf{r}_\parallel) - \xi_w(\mathbf{r}_\parallel)
\]

is the thickness of the helium film depending on the
coordinates, and \( \xi(\mathbf{r}_\parallel) \) and \( \xi_w(\mathbf{r}_\parallel) \) are the functions that
describe the profiles of the He surface and the wall of
the trap. The gravity term always reduces the thickness of
the helium film. Below, we consider the roughness
of the wall with the characteristic length scale
\( a_{\text{He}} \ll h_{\text{max}} \). The variation of the \( z \) coordinate at this
small length scale can be neglected compared to its
average value \( \langle z \rangle \) equal to the height \( h \) of the roughness
above the helium level in the trap. Therefore, the
\( z \) coordinate in Eq. (2) can be replaced by the height \( h \).

The second term \( E_s \) in Eq. (1) describes the surface
tension energy and is given by the formula

\[
E_s = \sigma_{\text{He}} \int \sqrt{1 + \left[\nabla \xi(\mathbf{r}_\parallel)\right]^2} d^2 \mathbf{r}_\parallel.
\]

Its root dependence complicates the problem of deter-
mination of the exact surface profile \( \xi(\mathbf{r}_\parallel) \). As a rule, this
problem can be solved analytically only in the limit of
small curvature of the surface $|\nabla \xi(x)| \ll 1$, when the approximation $\sqrt{1 + [\nabla \xi(x)]^2} = 1 + [\nabla \xi(x)]^2/2$ is valid. In our case, the condition $|\nabla \xi(x)| \ll 1$ is not necessarily fulfilled and we do not use this approximation for qualitative estimates. We instead search for the minimum of the initial functional specified by Eqs. (1)–(4) in the class of trial functions.

The gravity and surface tension terms in Eq. (1) are important at the macroscopic length scale $a_{He}$. The van der Waals term $V_W$ describing the attraction of helium to the material of the wall is noticeable only at much smaller distances $d_{He}^\text{min} = 10 \text{ nm} \ll a_{He}$. Since the range of van der Waals forces $d_{He}^\text{min}$ is five orders of magnitude smaller than the capillary length $a_{He}$, the effect of the gravitational force and surface tension of the helium surface on $V_W$ can be neglected and theoretical analysis is simplified. The van der Waals potential $V_W$ for the smooth wall depends only on the material of the wall and the film thickness: $V_W = V_W(d_{He})$. In the absence of surface tension, van der Waals forces would result in the coating of the rough surface with the helium film of the thickness $d_{He} \sim d_{He}^\text{min}$, which almost repeats the wall profile if the roughness scale is $l_R \gg d_{He}^\text{min}$. Thus, only the first two terms can be retained in the functional $E_{\text{tot}}(\xi) = \int \nabla \xi \cdot \nabla \xi \, dx$ given by Eq. (1), and the effect of the van der Waals term $V_W$ is reduced to the “boundary conditions” of the minimum thickness of the helium film $d_{He} \geq d_{He}^\text{min} = 10 \text{ nm}$.

Such a minimum helium film with the thickness $d_{He} \sim d_{He}^\text{min}$ caused by the van der Waals attraction provides an additional surface tension energy $\Delta E_s$, which can be even larger than the addition $\Delta V_W$ to the gravity term appearing because of an additional helium amount necessary for the helium surface to be flat: $\xi(x) = \text{const} = \max(\xi_{\text{He}}(x)) + d_{He}^\text{min}$. This additional helium amount depends on the wall roughness profile and can strongly increase the average thickness of helium films. This increase in the effective thickness of the helium film caused by capillary effects can apparently explain the difference in its experimental values determined by different methods [38–43]. Indeed, the total weight of helium is measured by the microweighing method [40]. This amount of helium includes filled cavities on the surface, which were not detected by optical methods in [39, 44]. Consequently, the microweighing method gives a thicker helium film on the rough surface [40]. Since the typical thickness of the helium film caused by the van der Waals forces is small, $d_{He} \sim 10 \text{ nm}$, even miniature surface roughness with a height of about 10 nm can strongly affect the measured values of the thickness of the helium film.

3. HELIUM FILM ON THE WALL WITH MODULATION IN THE FORM OF A TRIANGULAR WAVE

We consider the helium film on the rough wall in the form of a one-dimensional triangular wave with the period $l_R$ and depth $h_R$, as shown in Fig. 1. To approximately estimate the necessary roughness parameters of such a wall, we compare the energies of a very thin film with a thickness of $-d_{He}^\text{min}$ repeating the surface relief and the helium film with the flat surface shown by the blue dashed line in Fig. 1. Since the thickness of the film is larger than $d_{He}^\text{min}$, these two configurations have approximately the same van der Waals energy. Their gravitational energies (2) per unit area of the wall is $\Delta V_g = -g\rho_{He}h_R h/2$. The difference of surface tension energies (4) of these two configurations is equal to the product of the difference of their surface areas and the surface tension of liquid helium $\sigma_{He}$; this difference per unit surface area is $\Delta E_s = \sigma_{He}\left(\sqrt{1 + (2h_R/l_R)^2} - 1\right)$. The sum $\Delta V_g + \Delta E_s$ is positive; i.e., the flat free surface of the helium film is more favorable than that repeating the wall relief if two conditions are satisfied: (i) the modulation period is bounded from above:

$$l_R < l_R^\text{max} = 4\sigma_{He}/(g\rho_{He}) = 4a_{He}^2/h,$$

and (ii) the modulation depth is bounded from below:

$$h_R > h_R^\text{min} = \frac{4a_{He}^2/h}{(4a_{He}^2/(l_R h))^2 - 1} = \frac{l_R \eta}{\eta^2 - 1},$$

where

$$\eta = l_R h/(4a_{He}).$$

According to Eq. (5), $\eta < 1$.

The substitution of the value $a_{He} = 0.5 \text{ mm}$ and the maximum height $h_{max} = 18 \text{ cm}$ to which neutrons with energies lower than $V_{He}'$ can rise into Eq. (5) yields the maximum roughness period $l_R^\text{max}(h_{max}) \approx 5.6 \mu m$. Since $l_R^\text{max} \propto 1/h$, the modulation period at a smaller height can be larger. A part of the kinetic energy of the neutron that is determined only by the vertical compo-
ponent of its velocity $v_z$ is on average one-third of its total energy and one-fifth of the maximum energy $V_{0\text{He}}$. Consequently, the neutron usually collides with the vertical wall at a height below $h_{\text{max}}/5$, where the roughness period can be made larger by a factor of 5:

$$l_R < l_R^{\text{max}} (h_{\text{max}}/5) = 28 \mu \text{m} = 820\text{cm}^{-1}.$$  

According to Eq. (6), the minimum roughness amplitude of the wall at $\eta \ll 1$ is $l_R^{\text{min}} \propto l_R^2$. Therefore, at a smaller modulation period $l_R$, a less sharp roughness can be taken and, correspondingly, its brittleness is smaller. If the value $l_R = l_R^{\text{max}}/2$ is taken for reliability (in the case of strong fluctuations of $l_R$), which certainly satisfies Eq. (5), Eq. (6) takes the form $h_R > 2l_R/3$; i.e., modulation should be quite deep. If $l_R = l_R^{\text{max}}/4 = \alpha^2_{\text{He}}/h$, which satisfies Eq. (5) even with a larger margin, Eq. (6) has the form $h_R > 4\alpha h/15$; i.e., a smaller modulation depth can be taken and the roughness will be less brittle.

The two profiles of the helium film considered above are certainly not optimal, i.e., not corresponding to the minimum of the total energy (1). When the curvature of the surface is small, $|\nabla \xi |_{\eta} \ll 1$, the gravity term (2) is linear in $\xi$, whereas the capillary term (4) is quadratic in $\xi$. Consequently, the gravity term prevails at small $\xi$ values and, therefore, at least small curvature of the surface always exists and the flat surface $\eta = 0$ is impossible. At the same time, the repetition of the triangular wall profile by the free helium surface is also not profitable because it has corners. The real surface of this relief wall is described by a smooth periodic function $\xi(x)$ with the period $l_R$, which is schematically shown by the blue solid line in Fig. 1. Therefore, it is possible to take the approximate trial function

$$\xi(x) = \xi_0 \cos(2\pi x/l_R)$$  \hspace{1cm} (8)

and to determine the amplitude $\xi_0$ at which the total energy (1) has a minimum. From Eqs. (8) and (2), the gain in the gravitational energy because of such sinusoidal curvature of the surface per unit area of the wall is determined in the form

$$\Delta V_g = \rho_{\text{He}} g \rho_{\text{He}}^2 / l_R^{l_R/2} \int_{-l_R/2}^{l_R/2} [\cos(2\pi x/l_R) - 1] dx$$  \hspace{1cm} (9)

However, the substitution of Eq. (8) into Eq. (4) gives the following expression for curvature-induced loss in the surface tension energy per unit area of the wall:

$$\Delta E_c = \frac{\sigma_{\text{He}}}{l_R} \int_{-l_R/2}^{l_R/2} \left(1 + \left[\xi(x)\right]^2\right) - 1) dx$$  \hspace{1cm} (10)

where $E[x]$ is the complete elliptic integral of the second kind. The sum of Eqs. (9) and (11) gives the change in the total energy caused by sinusoidal curvature of the surface

$$\Delta E_{\text{tot}} = \rho_{\text{He}} g a_{\text{He}}^2 \left(2E[2\pi \xi_0^2]/\pi - 1 - 4\eta \xi_0^2\right),$$  \hspace{1cm} (12)

and $\xi_0^* = \xi_0/l_R$ is the normalized wave amplitude, which is determined by minimizing the total energy (12). It is seen that the position of the minimum $\xi^{\text{min}}_{\text{min}}$ depends only on the parameter $\eta = l_R h/(4a_{\text{He}}^2)$ given by Eq. (7). This parameter continuously appears in our problem. The dependence $\xi^{\text{min}}_{\text{min}}(\eta)$ obtained by minimizing Eq. (12) is shown by the blue solid line in Fig. 2. It diverges at $\eta \to 1$. Thus, we again arrive at Eq. (5) determining the maximum modulation period of the wall roughness. The function $\xi^{\text{min}}_{\text{min}}(\eta)$ at $\eta < 0.8$ is satisfactorily approximated by the formula (see Fig. 2)

$$\xi^{\text{min}}_{\text{min}}(\eta) = (2/\pi^2)\eta/(1 - 0.7\eta^2).$$  \hspace{1cm} (13)

The wave $\xi(x)$ specified by Eq. (8) does not touch the solid wall with the triangular profile, which is shown in Fig. 1, at

$$h_R > h_R^{\text{min}} = 2.3\xi^{\text{min}}_{\text{min}} \equiv 2.3 l_R \xi^{\text{min}}_{\text{min}}(\eta).$$  \hspace{1cm} (14)

Formula (14) and Fig. 2 determine the minimum depth of the triangular relief of the wall at which it does not touch the free helium surface anywhere except for bulges at integer values of $x/l_R$. This condition on $h_R$ is weaker by approximately a factor of 2 than the condition (6) because the curved surface of the helium film described by Eq. (8) with a small amplitude $\xi_0$ insignificant for the film to touch the solid wall is more favorable than the absolute flat surface. This weaken-
ing of the condition (6) on the roughness depth $h_{R}$ facilitates the practical implementation of such a wall. Nevertheless, the condition (5) on the roughness period does not change.

If the condition $\left| \nabla \xi \left( x_{i} \right) \right| \ll 1$ corresponding to $\xi_{0} \ll l_{R}$ were used from the beginning, we would obtain the linear function $\xi_{\text{min}}(\eta) = 2\eta/\pi^{2}$ shown by the orange dashed line in Fig. 2. According to Fig. 2, this linear approximation is applicable at $\eta < 0.4$, and it is completely inapplicable at $\eta \to 1$.

The triangular roughness of the wall shown in Fig. 1 has some significant advantages compared to the rectangular profile shown and considered in [37, Fig. 6].

First, the rectangular wall with the required parameters will be much less brittle than the rectangular one. It touches the free helium surface only at bulges at $x = x_{n} = h_{R}n$, where $n$ is an integer. The thickness of the helium film at these points decreases to $d_{\text{He}}^{\text{min}} \sim 10$ nm. However, since the derivative at these points is zero, $\xi_{\text{He}}(x_{n}) = 0$, the thickness of the film increases linearly with the distance $\Delta x = x - x_{n}$ from these points because of the linear dependence $\xi_{\text{He}}(\Delta x)$:

$$d_{\text{He}}(x) = d_{\text{He}}^{\text{min}} + |x - x_{n}|(2h_{R}/l_{R}).$$

According to the approximate semiclassical formula [37] of the exponential decrease in the wavefunction of the neutron inside the helium film, the helium film reduces the absorption of the neutrons by the wall by a factor of $10^{3}$.

Second, the proposed triangular profile corresponds to standard diffraction gratings, whose production technology was developed long ago and is used industrially. Consequently, such an artificial roughness can be easily created. Electron beam lithography can provide a much smaller period of the diffraction grating $l_{R} \leq 100$ nm [47]. Thus, the triangular surface roughness profile makes our idea of the coating of walls of traps of UCNs with a sufficiently thick film of superfluid $^{4}$He practically implementable.

4. OTHER METHODS

Well-developed powder metallurgy methods can be used to obtain the required rough/porous surface of the trap for the purpose of its further coating with the helium film. The simple pressing of a pure fine-grained beryllium powder at pressures of about 1.0–1.5 GPa and temperatures of about $1000^{\circ}$C will already give the surface profile applicable to hold the required amount of liquid helium on the vertical wall of the trap.

Moreover, efficient methods of obtaining pure porous beryllium were developed already in 1998 in [48] to coat the walls of nuclear fusion reactors. These methods can provide a large amount of beryllium with a controlled uniform porosity ranging from 10 to 70% and 100% open porosity [48].

The rough surface of traps of UCNs can also be obtained by one of the standard methods of sintering of small particles (dust) of a material weakly absorbing neutrons. In this case, an important requirement is a low concentration of impurity particles.

The coating of the walls of the trap of UCNs with a diamond nanopowder often used in experiments with UCNs [49] can also provide the required roughness of the surface to make the helium film sufficiently thick.

The superfluid $^{4}$He film protecting neutrons from absorption in the walls of the trap can be obtained using a thin metallic wire wound (entangled) to create a sufficiently large surface area $S$ per unit volume $V$ but with a small volume fraction $\phi_{\text{wire}} = V_{\text{wire}}/V$. Such a configuration of entangled wires can be suspended from above to hang loose under the action of the gravitational force. Such a wire should be made of a material.
rial weakly absorbing neutrons, e.g., beryllium. In this case, an insignificant fraction of the wire $\phi_{\text{wire}} \sim 0.1$ completely coated with $^4$He does not strongly absorb neutrons.

The surface and volume of the wire with the diameter $d$ and length $L$ per unit volume $V$ are $S_{\text{wire}} = L\pi d$ and $V_{\text{wire}} = L\pi d^2/4$, respectively. Owing to capillary effects, superfluid $^4$He will rise through such randomly entangled wires to the height $h$ at which the gravitational and surface energies of liquid helium are equal to each other: $hg\rho_{\text{He}} = S\sigma_{\text{He}}$. This imposes a constraint on the length $L$ and volume fraction $\phi_{\text{wire}}$ of the wire:

$$S = L\pi d > hg\rho_{\text{He}}/\sigma_{\text{He}} = hV/a_{\text{He}}^2,$$

which corresponds to the relation

$$\phi_{\text{wire}} > hd/(4a_{\text{He}}^2).$$

The substitution of $h = h_{\text{max}} = 18$ cm into Eq. (19) gives an upper bound for the thickness of the wire

$$d < d_{\text{max}} \approx 4\phi_{\text{wire}}a_{\text{He}}^2/h_{\text{max}}.$$

The condition $\phi_{\text{wire}} \leq 0.1$ means that the wire should be thinner than $d_{\text{max}} = 0.56 \mu$m. Under a more stringent condition $\phi_{\text{wire}} \leq 0.01$, $d_{\text{max}} = 14$ nm. It is likely more difficult technically to fabricate such a thin wire with a sufficient total length than to use the triangular roughness of the surface (diffraction grating) considered above. Entangled segments of the wire with a much smaller thickness of about 5 nm can be fabricated by the agglomeration of nanoparticles in $^4$He vortices [50, 51], but it is difficult to control the length and concentration of such wire segments by this method.

5. CONCLUSIONS

The main channel of loss of UCNs in material traps is the absorption of neutrons by the walls of traps. Neutrons are not absorbed by $^4$He. Therefore, the storage time of UCNs in a trap whose walls are coated with a helium film can be increased by several orders of magnitude because the inelastic scattering rate of UCNs on surface and bulk excitations of liquid helium is sufficiently low at $T < 0.4$ K [52]. However, because of van der Waals forces, the thickness of the superfluid helium film coating the vertical surfaces is $d_{\text{He}} = 10$ nm, which is much smaller than the penetration depth $\kappa^{-1}_{\text{He}} = h/\sqrt{2m_nV^0_{\text{He}}} = 33.5$ nm of neutrons in $^4$He. Consequently, such a film does not protect neutrons from absorption inside the wall of the trap. In this work, we have proposed a technically simple method of increasing the thickness of the helium film coating the inner surface of the trap of UCNs. It is necessary to make the walls of the trap rough, e.g., in the form of a standard diffraction grating with a triangular profile. Estimates of the necessary parameters of such roughness indicate that such a rough surface can be technically obtained by various inexpensive methods.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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