Magnonic Band Structure of Domain Wall Magnonic Crystals

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Magnonic crystals (MCs) are prototype magnetic metamaterials designed for the control of spin wave propagation. Conventional MCs are composed of single domain elements. If magnetization textures, such as domain walls (DWs), vortices, and skyrmions, are included in the building blocks of MCs, additional degrees of freedom over the control of the magnonic band structure can be achieved. We theoretically investigate the influence of DWs on the spin wave propagation and the corresponding magnonic band structure. It is found that the rotation of magnetization inside a DW introduces a geometric vector potential for the spin wave excitation. The corresponding Berry phase has quantized value $4n_w\pi$, where $n_w$ is the winding number of the DW. Due to the topological vector potential, the magnonic band structure of MCs with DWs as comprising elements differs significantly from an identical MC composed of only magnetic domains. This difference can be utilized to realize dynamic reconfiguration of magnonic band structure by a sole nucleation or annihilation of DWs in MCs.

Index Terms—Domain walls (DWs), magnonic bands, spin waves (SWs).

I. INTRODUCTION

Spin waves (SWs) are fundamental elementary excitations in magnetically ordered solid systems. Originally, the concept of SW was proposed to explain the famous $T^{3/2}$ law of the temperature dependence of saturation magnetization. Thereafter, the existence of SWs was confirmed by many experiments, and SWs became a basic ingredient in the field of magnetism and magnetic materials. Thanks to the rapid development of microstructuring technology, even the manipulation of the propagation of SW itself in periodic magnetic structures, dubbed magnonic crystals (MCs) [1], [2], becomes feasible nowadays. In practice, this degree of freedom in the manipulation of SW dynamics paves the route to information processing employing SWs. However, most of the contemporary studies involve only patterned magnetic domains and antiferromagnetically coupled nanowires [3], the role of magnetization texture still awaits for investigation [4], [5]. The inclusion of topological magnetization textures into the building elements of MCs would not only enlarge the horizon of the quest for new types of MCs, but also benefit from the additional tunability coming along with the response of magnetization textures to externally applied magnetic field [6] or electric current [7]. Actually, the interplay between topology and SW has already attracted theoretical interest, both in MCs [8] and in topological magnon insulators [9].

Despite the superficial different behavior exhibited by domain walls (DWs) and SWs, they are closely related to each other: SWs are the propagating excitation of a ferromagnet, while the wave function of the zero mode excitation of a ferromagnet is just the derivative of the DW profile [10], [11]. Here, we will consider the 1-D band structure where there is a $2\pi$ DW, which is the simplest topological entity in magnetic materials [10], in the unit cell of an MC. By analytically solving the SW eigenequation, we demonstrate that the topological nature of the underlying DW is transferred to the Berry phase experienced by the SW. Due to this topological nature of the Berry phase, the vector potential giving rise to the Berry phase determines a different magnonic band structure, compared with the band structure of an identical conventional MC composed of only uniform domains. We believe that the same topological effect is also effective in the skyrmion-based dynamic MCs [7], given that the homotopy group of both $2\pi$ DWs and skyrmions is identical [10].

The fundamental unit cell of a DWMC is shown schematically in Fig. 1. It is composed of two regions, regions 1 and 2. In our coordinate system, region 1 extends from $y = -d_1$...
to \( y = 0 \), and region 2 is bounded by \( y = 0 \) and \( y = d_2 \) planes. The magnetic materials in the two regions could be identical or different. For a usual MC, in order to induce an energy gap for SWs, the magnetic parameters of the two regions have to be different, which can be realized by using different materials or local modification of the magnetic properties of the same material through ion implantation [12], [13]. In our simplified 1-D treatment of DWMCs, what matters are the magnetic parameters in the two regions, which are differentiated by subscripts 1 and 2 for regions 1 and 2, respectively. For concreteness of the problem, we assume both regions have uniaxial perpendicular magnetic anisotropy along the \( z \)-axis and the MC structure is patterned from a continuous film lying in the \( xy \) plane.

The organization of our discussion of DWMCs is as follows. In Section II, we give the SW eigenfunctions by solving the linearized magnetization dynamics. With those magnonic eigenfunctions, the band structure of MCs composed of both single domains and DWs inside the unit cell is studied in Section III, by linearly superposing the eigenfunctions in each region. In Section IV, the dependence of the magnonic bandgaps and widths on magnetic material parameters will be investigated. Using the method of transfer matrix, the evolution from free propagating SWs without bandgaps to SWs characterized by band structure is described in Section V. Finally, Section VI discusses the possibility of creating DWMCs in experiments, and the conclusion is given in Section VII.

II. Spin Wave Eigenmodes

Before proceeding to the actual band structure of the MC, we first need to know the SW eigenmodes in each material. This requires the solution of the corresponding Landau–Lifshitz–Gilbert (LLG) equation [14]. Without an externally applied magnetic field and neglecting the damping term, the LLG equation for the normalized magnetization vector \( \mathbf{m} = M/\mathbf{M}_s \) reduces to

\[
\frac{d\mathbf{m}}{dt} = \omega_c \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_K + \mathbf{\nabla}^2 \mathbf{m})
\]  

with \( \omega_c = \gamma H_K \) the cutoff frequency for SWs and \( \delta = \sqrt{\mu/\mathbf{M}_s} \) the DW width constant. \( A \) is the exchange stiffness constant, and \( K \) is the uniaxial anisotropy constant. \( H_K \) is the anisotropy field, \( H_K = 2K/\mathbf{M}_s \), with \( \mathbf{M}_s \) the saturation magnetization. In (1), we consider only a perpendicular anisotropy field. In our simplified 1-D treatment of the SW dynamics, the demagnetization energy simplifies to two uniaxial anisotropies, one in plane and the other perpendicular to the film plane [15]. The latter can be incorporated into the interface perpendicular anisotropy field, forming an effective perpendicular anisotropy field. Inclusion of the former, in-plane demagnetization anisotropy in the form of a hard-axis anisotropy will only change the dispersion relation, making no difference to the understanding of the physics responsible for the formation of magnonic bands. Non-uniform dynamic demagnetization field [16], [17] can cause localization of SWs, and affect the SW modes in nanomagnets. Since we are interested in the influence on the magnonic dispersion relation of a periodic structure, we consider only the lowest lying mode with parabolic-like dispersion curves. Inclusion of higher modes will introduce more complex structures to the bands obtained with only the lowest mode, which is out of the scope of this paper. For the 1-D geometry considered here (Fig. 1), the gradient operator reduces to a differentiation on the \( y \) variable, since the periodic structure of the MC is along the \( y \)-axis and we consider only magnetization variation along this direction.

In the ground state, which is derived from the static LLG equation, \( \mathbf{m} \) is uniformly magnetized along the easy (\( z \)) axis in region 1, \( \mathbf{m} = \hat{z} \). To obtain the SW eigenfunction, a single harmonic deviation from the uniform solution is considered, \( \mathbf{m} \propto \hat{z} + \rho \exp(-i\omega t) \), where \( \rho \) is a vector in the film plane, thus perpendicular to the ground state magnetization direction \( \hat{z} \). Substitute this form of \( \mathbf{m} \) back into the LLG equation and retain only the first-order terms of \( \rho \), we can get the SW eigenfunction

\[
i\frac{\partial \rho}{\partial t} = \hat{z} \times (\delta^2 \rho - \rho)
\]  

where the abbreviation \( \delta^2 \rho = \partial^2 \rho/\partial y^2 \) is employed to make the equation compact. Let \( \phi = \rho_x + i\rho_y \), then the eigenfunction for \( \phi \) has a scalar form

\[
i\frac{\partial \phi}{\partial t} = (\delta^2 \phi - \phi).
\]  

The SW dispersion relation \( \omega/\omega_c = 1 + \delta^2 k^2_1 \) and the eigenfunction \( \phi = \exp(ik_1y) \) follows immediately. It is interesting to note that, except for the existence of an excitation gap, whose direct cause is the finite anisotropy field, the SW dispersion relation is identical to that of an electron traveling freely, whose effective mass is determined by the exchange constant and saturation magnetization only.

In region 2, the procedure to obtain the SW eigenfunction is essentially identical to that for region 1. The main difference is that, instead of a single domain state, there is a 2\( \pi \) DW present. A 2\( \pi \) DW can also be considered. We choose a 2\( \pi \) DW simply because the corresponding unit cell is simpler. Since a detailed derivation of the magnetization profile for a confined DW and the corresponding SW eigenfunction is given in Ref. [18], we will only describe the outline here. For brevity, in the following discussion of the DW magnetization profile and SW eigenfunction, the origin of the coordinate will be temporarily shifted to \( y = d_2/4 \). The ground state DW profile is described by a cosine Jacobian elliptic function [19]

\[
\cos \phi = -\sin(\sqrt{\mu/2} \theta),
\]

where \( \theta \) is the magnetization tilt angle in the \( xy \) plane, measured from the \( z \) axis, and \( K(\mu) \) is the first kind complete elliptic integral. The modulus \( \mu \) is determined by the length of region 2, \( 4\delta_2\sqrt{\pi K(\mu)} = d_2/4 \). Due to the continuous rotation of the magnetization vector, the SW excitation is oscillating in a plane perpendicular to the local magnetization vector. Viewed in this perpendicular plane, the SW oscillation is identical to that of a single domain, which is discussed earlier, except that the oscillation now is elliptical. Hence, although the SW oscillation is vectorial globally, it is locally oscillating in a plane. The global 3-D characteristics of the SW oscillation are restored if the rotation of the oscillating plane is considered. This feature of the SW
excitation guarantees that we can use again a scalar function to describe SWs inside DWs. By rotating the z-axis to the local magnetization direction, and taking into account of the elliptical characteristics of the SW oscillation, the scalar SW eigenequation becomes

\[- \left( 1 + \frac{d n^2(y_0, \mu)}{\mu} \right) \phi = \left( \frac{\gamma \phi^2}{\sqrt{\mu}} - 2 \phi s \left( \frac{y}{\sqrt{\mu} d}, \mu \right) \right) \]  

\( y_0 \) is an auxiliary constant related to \( \omega \) through \( \sqrt{\mu} \omega = cn(y_0, \mu)dn(y_0, \mu) \), and \( sn \) is the sine Jacobian elliptic function [19]. This equation has the form of a Schrödinger equation with an elliptic potential, known mathematically as the Lamé equation [20]. The same equation was obtained in the discussion of the excitation spectrum of the sine-Gordon equation [21]. The period of the potential in (4) is only half of the period of the magnetization distribution, which is \( d_2 \) for our case of a 2\( \pi \) DW. This halving of the real space periodicity indicates a doubling of the \( k \)-space periodicity for SWs, which is \( 4\pi/d_2 \). This difference in periodicity of the SW and the static magnetization is caused by the insensitivity of SWs to DW chirality and polarity.

The propagating solution of (4) is given by

\[ \phi = \frac{H(y/\sqrt{\mu} d + y_0)}{\Theta(y/\sqrt{\mu} d)} e^{-y Z(y_0)/\sqrt{\mu} d_2} \]  

\( y_0 \) is a periodic function with the period of the potential. To cast \( \phi \) into this form, the crystal wave vector has to be defined as \( k_2 = i Z(y_0)/\sqrt{\mu} d_2 + 2\pi/d_2 \). The constant \( 2\pi/d_2 \) corresponds to the first Brillouin zone boundaries. Due to this displacement of the dispersion relation, there are two energy minima in the first Brillouin zone for SWs in the DW, and they are located at the zone boundaries. Interestingly, there is no bandgaps opening in the whole Brillouin zone, although the potential function is periodic. The modulation function \( u_k \) is given by

\[ u_k = \exp(-2i\pi y/d_2) H(y/\sqrt{\mu} d + y_0)/\Theta(y/\sqrt{\mu} d_2) \],

which now satisfies \( u_k(y + d_2/2) = u_k(y) \). The periodicity of \( u_k \) can be seen from Fig. 2, where the real and imaginary parts of the modulation function \( u_k \) at the frequency of 15 GHz are shown. It is interesting to note that, although \( u_k \) is periodic with the same periodicity of the potential, its phase changes \( 2\pi \) after a displacement of the period of the potential, which corresponds to a \( \pi \) rotation of the magnetization vector. A \( 4\pi \) phase is achieved when another period of the potential is displaced, and the magnetization rotation is correspondingly \( 2\pi \). From this observation, the phase shift of \( u_k \) and the magnetization rotation of the DW is proportional to each other. The phase of \( u_k, \phi \), is a Berry phase [22], [23], in contrast to the dynamical phase related to the crystal momentum. The magnetization rotation divided by \( 2\pi \) is defined as the winding number [10] \( n_\omega \) in the configuration space. Therefore, the Berry phase of SWs in the DW is proportional to the winding number, which is a topological quantity. The relation between \( \phi \) and the winding number, \( \varphi = 4n_\omega \pi \), confirms that the SW Berry phase inherits the topological character of the underlying DW.

The potential in the Schrödinger equation [see (4)] and hence the whole Hamiltonian is invariant under spatial inversion, which implies that the eigenfunction can be chosen to have definite parity. The function \( \phi \), however, does not have a definite parity. Under the operation of spatial inversion \( \phi \) is transformed to \( -\phi(-y) = -\phi^* \), with \( \phi^* \) the complex conjugate of \( \phi \). Eigenfunctions with definite parity can be constructed from \( \phi \) and \( \phi^* \). Due to the CPT theorem [24], consecutive action of time reversal and spatial inversion will leave \( \phi \) intact, given that magnons are charge neutral particles. Hence, the Hamiltonian for SWs in the DW is invariant under time-reversal operation, and \( \phi \) and \( -\phi^* \) form time-reversal and space-inversion pairs. Due to the time-reversal symmetry, or equivalently, the inversion symmetry, the Berry phase of SWs with opposite \( k \) will have the opposite signs, signifying the different topological charge associated with positive or negative \( k \). As far as time-reversal symmetry is conserved, the back scattering of magnons at the Brillouin boundaries will be prohibited, thus forbidding the corresponding bandgap opening. This is consistent with the calculated SW energy spectrum. At this point, it should be emphasized that the dependence of the Berry phase on the crystal momentum is the consequence of the conservation of the time-reversal symmetry, in contrast to the non-reciprocity of SW propagation [25], [26], which requires the violation of the time-reversal symmetry.

The SW Berry phase \( \varphi \) can be expressed as the line integral of a field \( \beta(y) \), \( \varphi = -\int dy \beta \). From \( \beta \), a vector potential is derivable. The SW eigenfunction can be rewritten as \( \phi = \exp(i(\varphi + k_2 y)) \). With the definition of \( \beta \), it can be
easily seen that $\phi$ satisfies a simple equation $(\partial/\partial y + i\beta)\phi = ik_2\phi$, which enables us to define a covariant derivative operator [27] $D = \partial/\partial y + i\beta$. The explicit form of the vector potential is

$$\beta = 2\pi/d_2 + i\left(\frac{Z(y)}{\sqrt{\mu_0}d_2} + Z(y)/\sqrt{\mu_0}d_2 + \frac{cs(y)/\sqrt{\mu_0}d_2 + y_0}{\sqrt{\mu_0}d_2 + y_0}/\sqrt{\mu_0}d_2\right).$$

Potential-like field $\beta$ is generally not a real-valued field. Only when the magnetization is pointing along the easy axis, $\beta$ is a pure real number. $\phi$ is the eigenfunction of the covariant derivative operator with the eigenvalue $ik$, although $\phi$ is not an eigenfunction of the real derivative operator, $\partial/\partial y$. It is interesting to note that, instead of the real space, if we consider the configuration space of the magnetization $(m_1, m_2)$, the Berry phase can be expressed as the contour integral of a vector potential-like field in the configuration space, $\varphi = -\oint d\theta \beta_0$. The topological character of the Berry phase is easier to observe in the 2-D $(m_1, m_2)$ configuration space. As we will see in Section III, the difference between the crystal momentum and the vector potential-like field $\beta$ can be observed in the magnonic band structure reconfiguration induced by the presence of the DW in the unit cell.

III. BAND STRUCTURE OF DOMAIN-WALL MAGNONIC CRYSTALS

With those SW eigenmodes in regions 1 and 2 known, we are now ready to discuss the band structure of the whole MC. We use the same method employed previously for the discussion of the SW spectrum [28]–[32] in periodic magnetic structures. Micromagnetic simulations can also be used to deal with the same problem, similar to the treatment of magnonic band structure in a width-modulated MC [33]. While micromagnetic simulations can be more realistic, including non-uniform dipolar field and pinning effects, the analytic method employed here can provide more insight into the physical mechanism responsible for the magnonic band formation. For the MC considered here, the Bloch theorem requires that the eigenfunctions in both regions have the form

$$\psi_i = e^{ik_i y} u_i(y), \quad i = 1 \text{ or } 2 \quad (6)$$

where $k$ is the Bloch wave vector, confined to the first Brillouin zone $[-\pi/d, \pi/d]$. $d$ is the MC’s period and $u_i$ is arbitrary function with the same period of the MC, $u_i(y + nd) = u_i(y)$, where $n$ is an integer. $\psi_i$ here can be chosen as a linear combination of the SW eigenfunctions, $\psi_1 = a_1 \exp(ik_1y) + b_1 \exp(-ik_1y)$ in region 1 and $\psi_2 = a_2 \phi + b_2 \phi^*$ in region 2. The periodicity of $u_i$ will be guaranteed by a suitable choice of the coefficients $a_i$ and $b_i$. Since the eigenequation for $\psi_i$ is identical in form to a 1-D Schrödinger equation, coefficients $a_i$ and $b_i$ can be obtained by imposing the periodic boundary conditions for a Schrödinger equation. Specifically, this means that $u_i(0) = u_i(0)$ and $\bar{u}_i(0) = \bar{u}_i(0)$ at the origin (which is the central interface). Periodicity is guaranteed by the boundary conditions at the outer interfaces, $u_1(-d_1) = u_2(d_2)$ and $\bar{u}_1(-d_1) = \bar{u}_2(d_2)$. As discussed in Section II, the SW oscillation inside a DW is actually 3-D, and the scalar eigenfunction represents the oscillation in a plane perpendicular to the local magnetization vector. The boundary conditions employed here for the scalar functions in both regions 1 and 2 ensure that the spin current is conserved inside the unit cell. The continuity of the magnetization across the boundaries provides the applicability of the boundary conditions given earlier, which require that the SW oscillation is in the same plane. Correspondingly, the secular equation gives an implicit equation to determine the band structure

$$\cos kd = \cos k_1 d_1 \cos k_2 d_2 - \frac{k_1^2 + k_2^2}{2k_1 k_2} \sin k_1 d_1 \sin k_2 d_2. \quad (7)$$

As derived in Section II, $k_1$ is related to the frequency through $\omega/\omega_1 = 1 + \alpha^2 k_1^2$, which is the well-known dispersion relation for SWs in a single domain state. In the DW, however, the SW dispersion relation is not that simple any more, $\omega/\omega_2 = dn(\alpha, \mu_1)/\sqrt{\mu n c^2(\alpha, \mu_1)}$. $\alpha$ is a real parameter related to $y_0$ through $y_0 = i \alpha$, and $\mu_1$ is the complementary modulus, $\mu + \mu_1 = 1$. The crystal momentum in region 2 is given by $k_2 d_2 \sqrt{\mu} = Z(\alpha, \mu_1) + i \pi/2 K(\mu) - s c(\alpha, \mu_1) d n(\alpha, \mu_1)$. The constant $2\pi/d_2$ is omitted, because it makes no contribution to (7). $K(\mu) = K(\mu_1)$ is the complementary first kind complete elliptic integral and $q d_2 \sqrt{\mu} = -s c(\alpha, \mu_1) d n(\alpha, \mu_1)$. For comparison, the corresponding band structure for the MC structure without the 2-D DW is determined by the following equation:

$$\cos kd = \cos k_1 d_1 \cos k_2 d_2 - \frac{k_1^2 + k_2^2}{2k_1 k_2} \sin k_1 d_1 \sin k_2 d_2 \quad (8)$$

which is similar to the equation derived in the classical Kronig–Penney model [34]. This similarity is self-evident. In both cases, we use the same Schrödinger equation and Bloch theorem. Wave vectors $k_i$ are related to the frequency through $\omega/\omega_i = 1 + \alpha^2 k_i^2$ for this domain MC. If the two regions have identical material parameters, then the two wave vectors are equal to each other, $k_1 = k_2$, and the crystal wave vector $k$ reduces to the real wave vector, which means that there is no bandgaps for a continuous film.

Compared with (8), (7) shows that the main effect of the DW is to modify the momentum factor appearing in the implicit equation for the determination of the band structure. This modification can be understood on the fact that, due to the presence of the DW, the SW’s crystal momentum and the linear momentum are not the same quantity anymore. The crystal momentum of SWs in the DW is $k_2$, which corresponds to the eigenvalue of the covariant derivative operator. When the SW eigenfunction is displaced by $d_2$, its phase change is $k_2 d_2$, in analogy to the real linear momentum in continuous space. A similar conclusion was reached using micromagnetic simulation [35]. The linear momentum operator is proportional to the gradient $-i \nabla$, which reduces to $-ie/\beta y$ in our 1-D geometry. If there is no presence of the DW, a differentiation on the wave function gives a constant linear momentum. In the presence of the DW, the same differentiation on the corresponding SW wave function gives again the linear momentum. But the linear momentum is not a constant anymore. The linear momentum at the outer boundaries is given by $q = k_2 - \beta$, as compared with the crystal momentum $k_2$. This linear momentum will enter the momentum factor in the secular equation for the band structure. The disparity between the linear and crystal momenta derives from the
difference between the ordinary and covalent differentiation operators [27] acting on the SW wave function, as discussed in Section II.

To get numerical values, we need to specify the magnetic parameters. The magnetic parameters in the two regions should be different to get sizable energy gaps, which can be realized by ion implantation [12], [13] or using artificial superlattices [36], [37]. However, it should be noted that, in contrast to conventional 1-D MCs, DWMCs have energy gaps even if regions 1 and 2 have the same set of magnetic parameters [see Fig. 1 (dashed-dotted lines)], due to the different SW dispersion relation caused by the DW. The disappearance of magnonic bandgaps in conventional MCs is due to the restoration of the translation invariance, if the two regions have identical magnetic parameters. In the presence of the DW, the translation invariance is not restored even if the two regions are identical, which explains why there are still finite bandgaps in this case. It should be noted that this explanation is based on the unit cell structure shown in Fig. 1, where the DW is confined to region 2. In the case of identical magnetic parameters for the two regions, the DW will expand to occupy the whole unit cell, and the corresponding bandgaps will vanish if this effect is considered. This effect is ignored in all of our calculations, and we stick strictly to the unit cell shown in Fig. 1. As only \( \omega_c \) and \( \delta \) enter (7), we do not need to specify all three parameters, \( A \), \( K \), and \( M_z \). For region 1, we choose \( \omega_c, 1 = 10 \text{ GHz} \) and \( \delta_1 = 20 \text{ nm} \). We assume that, either due to ion implantation or material combination, in the second material, the anisotropy constant is reduced by a value of 10%. Other parameters remain the same in region 2. With those parameters specified, the band structure can be computed from (7) directly. An example band structure is shown in Fig. 3. As can be expected, the appearance of energy bands and bandgaps is obvious. Only the first two bands with a significant bandgap between them are shown. All the other bands with higher band index have negligible bandgaps.

From Fig. 3, we can see that the inclusion of the DW in the unit cell has significant effects. To facilitate a direct comparison, we use the same set of magnetic parameters to get the band structure for a conventional MC with the same unit cell, which is also shown in Fig. 3. Without the DW, the first band is very flat, meaning the group velocity there is very small. If the DW is present, the group velocity is increased. For higher energy bands, this effect is relatively less important. This transition from slow to fast propagation of SWs in the first band is actually caused by the increased cutoff frequency in the DW. For a DW characterized by the modulus \( \mu \), the cutoff frequency is \( \omega_c / \sqrt{\mu} \), scaled up by a factor of \( 1 / \sqrt{\mu} \), compared with the single domain case. In the first band, the SW is evanescent in region 2 for the domain MC, while it is propagating for the case of a DWMC. In addition, bandgap and gap position can both be tuned by the sole presence of the DW, as shown in Fig. 3. This signifies the main advantage of employing magnetization textures, whose representative is a DW, in the unit cell of an MC. Application of external field, either electric or magnetic, can tune the band structure. For the case considered in Fig. 3, the application of a magnetic field parallel to the \( z \)-direction can annihilate the DW, hence collapsing the band structure to that of a domain MC. After this transition, some forbidden states in the bandgap are allowed to propagate, realizing reconfigurable control over SWs’ propagation. Application of an external magnetic field parallel to the \( -z \) direction will modify the magnetization profile, thus affecting the SW characteristics and band structure, which will be investigated in the future.

To further illustrate the versatility of the DWMC, we consider unit cells with unequal widths for the comprising pieces, but fixed total cell size. In Fig. 4, the band structure of two asymmetric unit cells is given. It follows immediately that an expansion in the size of region 2, and hence a shrink in the size of region 1, has only quantitative significance. In contrast, a shrink in the size of region 2 changes the band structure qualitatively: there are only two bands with sizable bandgaps for \( d_2 = 0.3 \text{ \mu m} \), but that number increases to three with \( d_2 \) decreasing to 0.1 \( \text{ \mu m} \). This variation with \( d_2 \) is easily understood from the SW eigenequation. In a uniform domain, it is a Schrödinger equation with a constant potential. In the presence of the DW, the potential varies with position. A decrease in \( d_2 \) tightens the variation of the potential. As is well known from electronic band theory [34], the energy gap is related to the Fourier components of the potential. Hence, a change in the potential will definitely affect the band structure. This observation lends further support to the claim that the main advantage of DWMCs is to offer, besides the conventional modulation of magnetic parameters, an additional degree of tunability due to the adjustable magnetization profile in the unit cell.
Fig. 4. Magnonic energy bands in the positive half of the first Brillouin zone with asymmetric unit cell. The solid (red) and dashed (blue) lines are for the DWMCs and corresponding reference single domain MCs, respectively. The cell size is shown. (diamond). Filled (red) and open (blue) symbols denote results for the DWMCs and reference single domain MCs, respectively.

Fig. 5. Magnonic bandwidths of the first (top) and second (bottom) bands as a function of the reduction in the perpendicular magnetic anisotropy, $\delta K$, for fixed length of the unit cell $d = 0.4 \, \mu m$, but different ratios between the two regions inside the unit cell: $d_2 = 3d_1$ (square), $d_2 = d_1$ (circle), and $d_1 = 3d_2$ (diamond). Filled (red) and open (blue) symbols denote results for the DWMCs and reference single domain MCs, respectively.

### IV. Bandgaps and Bandwidths

For applications, the bandwidth and bandgap are important parameters. Fig. 5 shows the variation of the bandwidth for the first and second bands, as a function of the change in anisotropy, $\delta K = 1 - K_2/K_1$. For domain MC, it is obvious that the bandwidth of both the first and the second bands decreases with $\delta K$. At the value $\delta K = 0$, the bandwidth is given by the relation $\delta \omega = (2n - 1)\pi^2/d^2$, with $n$ the band index, derived from the dispersion relation. Since the unit cell length $d$ is the same in Fig. 5, the three curves should converge to the same point at $\delta K = 0$. At this point, the bandgap is zero, since there is no modulation of anisotropy and the translational invariance is restored. In contrast, the bandwidth for DWMC is not a monotonous function of $\delta K$. The effect of the nucleation of the DW can be observed conspicuously when the DW is more tightly confined: the bandwidth increases first, and then decreases with the increase of $\delta K$. For the bandwidth of the second band, its behavior is even more complex. There is a transition region, connecting the increasing and decreasing parts. If the length of region 2 is large, the effect of the DW is not that significant, and the bandwidth decreases with $\delta K$, similar to the case of domain MCs. The bandwidth of a band is determined by both the SW dispersion relations and the modulation in anisotropy in the unit cell. There are no simple rules to determine the behavior of the bandwidths before carrying out a numerical calculation. The qualitative behavior, however, is predictable. For example, when $\delta K$ is large, both the bandwidths of the first and second bands become small, since both bands are derived from evanescent waves in region 1. The resulting bands are narrow and have little dispersion, inherent of the localized characteristic of evanescent waves.

As a function of the modulation in anisotropy, the bandgaps for the first and second bands are shown in Fig. 6. The bandgaps of MC increases monotonously with $\delta K$, with inflection points in the plotted range for $\delta K$. In the presence of the DW in region 2, the bandgaps are no longer zero when the modulation in $K$ is absent, which is obviously derived from the violation of the translational invariance, due to the mere presence of the DW. The finite bandgap with no change of material parameters of DWMCs is in stark contrast to conventional domain MCs. For small $d_2$, there is a striking feature emerging, which is common to both types of MCs: the bandgap closes once or twice in the considered range of modulation in anisotropy, according to the geometrical structure of the unit cell, indicating the disappearance of bandgaps correspondingly. The sufficient and necessary condition to obtain a zero bandgap is easily obtained from (7) and (8), which is

$$k_1d_1 = n\pi, \quad k_2d_2 = m\pi$$

for both types of MCs. The same integer $m = n$ gives a zero bandgap at the zone center, and the zero bandgap will appear at the Brillouin zone boundaries if $m \neq n$. The condition can be interpreted as the condition for the wave function to interfere constructively, after propagating forward and reflected backward in each region, regions 1 and 2.
Or equivalently, the length of each region in the unit cell is an integer multiple of half of the wave length, which is similar in form to the condition for the formation of confined, discrete energy levels in a potential well. If the condition is fulfilled, the reflectivity of each individual part of the unit cell is zero, consistent with the constructive interference condition.

A similar behavior of bandwidth (Fig. 7) and bandgap (Fig. 8) can be observed as the unit cell size \( d \) is changed, while holding the ratio of the lengths of the two comprising parts in the unit cell constant. In contrast to the variation in anisotropy, a modification in \( d \) will not change the dispersion relation for each part in the unit cell of a domain MC. The only effect of the variation in \( d \) is to change the periodicity of the MC, and correspondingly the Brillouin zone boundaries.

Therefore, the Fourier components involved in the determination of the band structure will change correspondingly, making the observed variations. For the DWMC, in addition to the modified periodicity, the dispersion relation of the DW part in the unit cell is affected by a change in \( d \). This can be easily seen from the SW eigenequation (4), since the modulus \( \mu \) is determined by \( d_2 \), which is proportional to \( d \), and the dispersion relation is determined by \( \mu \).

V. DEVELOPMENT OF THE BAND STRUCTURE

Partial bandgap formation at the energy of an incident SW can be observed as an increased reflectivity. For a complete bandgap, the reflectivity should be unity. Hence, the SW reflectivity can serve as an indicator of bandgap formation. The SW reflectivity of the DWMC, or any 1-D structures composed
of single domains or \(2\pi\) DWs, can be calculated using the method of transfer matrix [38]. For this purpose, we need to know only two kinds of matrices: one is the propagation matrices describing the propagation of SW fields inside a medium, and the other is the interface matrices correlating the fields on both sides of an interface separating two media. For the case considered here, the propagation matrix in the single domain region is

\[
P = \begin{pmatrix}
e^{ik_1d_1} & 0 \\
0 & e^{-ik_1d_1}
\end{pmatrix}
\]

for the propagation of the field \(\psi = (a_1\exp(i k_1 y), b_1\exp(-i k_1 y))^T\) from \(y = -d_1\) to \(y = 0\). A similar expression holds for the propagation matrix in the presence of the \(2\pi\) DW

\[
\tilde{P} = \begin{pmatrix}
e^{ik_2d_2} & 0 \\
0 & e^{-ik_2d_2}
\end{pmatrix}
\]

describing the phase accumulated when the field \(\tilde{\psi} = (a_2\phi, b_2\phi^*)^T\) moves from \(y = 0\) to \(y = d_2\). The interface matrix at \(y = -d_1\) is defined through the continuity equation

\[
A \psi(-d_1) = \tilde{A} \psi(-d_1),
\]

which gives

\[
A = \begin{pmatrix}1 & 1 \\
 i k_1 & -i k_1
\end{pmatrix}, \quad \tilde{A} = \begin{pmatrix}1 & 1 \\
 i q & -i q
\end{pmatrix}.
\]

With those interface and propagation matrices, the outgoing wave function \(\psi_R\) can be related to the incoming wave function \(\psi_L\) by the transfer matrix \(T = A^{-1} \tilde{M}(MM)^NA\) through the relation \(\psi_R = T \psi_L\). Matrices \(M = APA^{-1}\) and \(\tilde{M} = A\tilde{P}A^{-1}\) are related to the interface and propagation matrices, and \(N\) is the number of the repeats of the unit cell. Since we inject and detect SWs in regions with the same parameters as region 1, \(N\) can actually be enumerated by the repeats of the DW. Fig. 9 shows the reflectivity for three values of the number of unit cells, \(N = 1, 5, \) and 10. It can be seen that with only \(N = 5\) unit cells, the lowest two bandgaps are already well developed. An increase to \(N = 10\) only improves the bandgap reflectivity slightly. Note that in Fig. 9, the bandgap development appears to be faster for the DWMC, as compared with the same conventional MC without the DW in the unit cell. However, this does not mean that DWMCs are superior to conventional MCs on this respect. The difference between those two MCs is caused by the different SW cutoff frequencies. When there is the DW in the unit cell, the cutoff frequency is increased significantly, to cover almost all the four bandgaps shown in Fig. 9, due to the small \(d_2\). The only observation is that the bandgaps close to the cutoff frequency develops faster, simply because of their evanescent characteristics. For experimental realization of an MC, our calculation demonstrates that bandgaps will develop in the presence of the order of 10 unit cells. This conclusion is valid only for ideal interfaces. If interface roughness is inevitably present, either due to ion implantation or imperfect growth of materials, more unit cells may be needed to observe significant reflection of SWs within bandgaps.

**VI. Discussion**

For the implementation of DWMCs, the nucleation or injection of DWs into the MC is the main obstacle. Although \(\pi\) DWs are extensively studied due to their potential use as information carriers, \(2\pi\) DWs receive little attention, although they frequently appear during the demagnetization process of a magnet [39]. During the demagnetization process of a magnetic tunnel junction, \(2\pi\) DWs were observed using Lorentz transmission electron microscopy [40]. In magnetic nanorings, meta-stable \(2\pi\) DWs were observed by magnetic force microscopy [41]. Later, the existence and stability of \(2\pi\) DWs was theoretically proven [42]. However, those randomly nucleated \(2\pi\) DWs are not amenable to be used in DWMCs, due to the difficulty in manipulating them. On this respect, multiple \(2\pi\) DWs can be injected into nanowires by cycling the polarity of field [43], or into a wedge shaped stripe by rotating field method [44]. In addition, Ar ion implantation can be employed to facilitate the formation of multiple \(2\pi\) DW state [45]. Making use of the Oersted field generated by a current-carrying wire, multiple \(2\pi\) DWs can also be nucleated [46], [47]. For the manipulation of DW chirality, recent micromagnetic studies showed that gold shunt pads could be employed to select the chirality of \(2\pi\) DWs [48]. For additional control of the DW position besides of the ion-implantation method, triangular antinotches can be used to pin the location of \(2\pi\) DWs [49], similar to the case of \(\pi\) DWs.
To create periodic DW structures, we can use ion-implantation to reduce the anisotropy in region 2 to create a potential well for DWs to settle in, and through cycling field polarity or rotating field method to nucleate and inject 2\(\pi\) DWs. With this method, it is easier to inject \(\pi\) DWs in ion-implanted magnetic structures to fabricate \(\pi\) DWMCs. Laser local heating can be used to enhance the probability of locally nucleating and trapping DWs [50]. In superlattices with anti-ferromagnetic or ferromagnetic coupling between adjacent magnetic layers, layers with lower DW energy can also accommodate DWs, following proper sequences of field preparation. But this would require a very large thickness of the soft layers. Periodic 2\(\pi\) DW structure was observed in double-layer Fe nanowires on W(110) during the demagnetization process [51], which would be an ideal model system for the study of DWMCs. Finally, by applying a large hard-axis field, and then reduce it to zero, 2\(\pi\) stripe DWs can appear [52]. This could be the simplest method to create DWMCs.

VII. CONCLUSION

In summary, the SW dispersion relation of a 2\(\pi\) DW and the corresponding magnonic band structure of a 1-D DWMC have been analytically calculated. Due to the continuous magnetization rotation of the DW, the effective potential in the eigenequation for SWs is periodic, with only half of the period of the magnetization profile. The consequence of the periodic potential is to shift the energy minima of the SW spectrum to the Brillouin zone boundaries. Despite the periodic potential experienced by SWs, there is no bandgap opening in the whole Brillouin zone, which is caused by the absence of back scattering of SWs provided by time-reversal symmetry. For the SW eigenfunction, the 2\(\pi\) DW induces a Berry phase, in addition to the dynamical phase related to the crystal momentum. Given those unique features of SWs propagating in the 2\(\pi\) DW, the DWMC exhibits a different magnonic band structure as compared with a reference MC containing only uniformly magnetized domains, which is made possible by the continuous magnetization rotation of the DW. We investigate systematically the band structure evolution as a function of the anisotropy modulation and unit cell size. It is found that the magnonic bandgaps can close, when magnons form quantum phases, the dynamic phase and the Berry phase, one advantage of including DWs in the unit cell is to obtain an additional control over the band structure of DWMCs. If realizable, reconfigurable switching between band structures of DWMCs and domain MCs can be achieved through the nucleation and annihilation of DWs by application of a magnetic field.

REFERENCES

[1] V. V. Kruglyak, S. O. Demokritov, and D. Grundler, “Magnonics,” *J. Phys. D, Appl. Phys.*, vol. 43, no. 26, p. 264001, 2010.

[2] B. Lenk, H. Ulrichs, F. Garbs, and M. Münzenberg, “The building blocks of magnonics,” *Phys. Rep.*, vol. 507, p. 107, Oct. 2011.

[3] J. Topp, D. Heitmann, M. P. Kostylev, and D. Grundler, “Making a reconfigurable artificial crystal by ordering bistable magnetic nanowires,” *Phys. Rev. Lett.*, vol. 104, p. 207705, May 2010.

[4] G. Duerr, R. Huber, and D. Grundler, “Enhanced functionality in magnonics by domain walls and inhomogeneous spin configurations,” *J. Phys., Condens. Matter*, vol. 24, no. 2, p. 024218, 2012.

[5] Z. Li, X. Wang, D. Wang, Y. Nie, W. Tang, and G. Guo, “Reconfigurable magnonic crystal consisting of periodically distributed domain walls in a nanostrip,” *J. Magn. Magn. Mater.*, vol. 388, p. 10, Aug. 2015.

[6] M. Krawczyk and D. Grundler, “Review and prospects of magnonic crystals and devices with reprogrammable band structure,” *J. Phys., Condens. Matter*, vol. 26, no. 12, p. 123202, 2014.

[7] F. Ma, Y. Zhou, H. B. Braun, and W. S. Lew, “Skyrmion-based dynamic magnonic crystal,” *Nano Lett.*, vol. 15, no. 6, pp. 4029–4036, 2015.

[8] R. Shindou, R. Matsumoto, S. Murakami, and J. Ohe, “Topological chiral magnonic edge mode in a magnonic crystal,” *Phys. Rev. B*, vol. 87, p. 174427, May 2013.

[9] A. Mook, J. Henk, and I. Mertig, “Edge states in topological magnon insulators,” *Phys. Rev. B*, vol. 90, p. 024412, Jul. 2014.

[10] H.-B. Braun, “Topological effects in nanomagnetism: From superparamagnetism to chiral quantum solitons,” *Adv. Phys.*, vol. 61, no. 1, pp. 1–116, 2012.

[11] G. Tatare, H. Kohno, and J. Shibata, “Microscopic approach to current-driven domain wall dynamics,” *Phys. Rep.*, vol. 468, no. 6, pp. 213–301, Nov. 2008.

[12] J. H. Franken, M. Hoeijmakers, R. Lavrijsen, and H. J. M. Swagten, “Domain-wall pinning by local control of anisotropy in Pt/Co/Pt strips,” *J. Phys., Condens. Matter*, vol. 24, no. 2, p. 024216, 2012.

[13] A. R. Buckingham *et al.*, “Switching the in-plane easy axis by ion implantation in rare earth based magnetic films,” *J. Phys., Condens. Matter*, vol. 25, no. 8, p. 086602, 2012.

[14] L. I. Landau, E. M. Lifshitz, and L. P. Pitaevski, *Statistical Physics*, 3rd ed. Oxford, U.K.: Pergamon, 1980.

[15] H.-B. Braun, “Fluctuations and instabilities of ferromagnetic domain-wall pairs in an external magnetic field,” *Phys. Rev. B*, vol. 50, p. 16485, 1994.

[16] V. V. Kruglyak, P. S. Keatley, R. J. Hicken, J. R. Childress, and J. A. Katine, “Dynamic configurational anisotropy in nanomagnets,” *Phys. Rev. B*, vol. 75, p. 024407, Jan. 2007.

[17] G. S. Gies, J. Podbielski, and D. Grundler, “Mode localization transition in ferromagnetic microscopic rings,” *Phys. Rev. B*, vol. 76, p. 014431, Jul. 2007.

[18] D. Wang, X.-G. Wang, and G.-H. Guo, “Magnetic momentum transfer force on domain walls confined in space,” *Europhys. Lett.*, vol. 101, no. 2, p. 27007, 2013.

[19] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions*. Gaithersburg, MD, USA: National Bureau of Standards, 1964.

[20] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis*, 4th ed. Cambridge, U.K.: Cambridge Univ. Press, 1927.

[21] B. Sutherland, “Some exact results for one-dimensional models of solids,” *Phys. Rev. A*, vol. 8, p. 2514, Nov. 1973.

[22] P. Bruno, V. K. Dugaev, and M. Taillefumier, “Topological Hall effect and berry phase in magnetic nanostructures,” *Phys. Rev. Lett.*, vol. 93, p. 096806, Aug. 2004.

[23] K. Y. Guslienko, G. R. Aranda, and J. M. Gonzalez, “Topological gauge field in nanomagnets: Spin-wave excitations over a slowly moving magnetization background,” *Phys. Rev. B*, vol. 81, p. 014414, Jan. 2010.

[24] W. Greiner and J. Reinhardt, *Field Quantization*. Berlin, Germany: Springer-Verlag, 1996.

[25] R. Verba, V. Tiberkevich, E. Bankowski, T. Meitzler, G. Melkov, and A. Slavin, “Conditions for the spin wave nonreciprocity in an array of dipolarly coupled magnetic nanopillars,” *Appl. Phys. Lett.*, vol. 103, no. 8, p. 082407, 2013.

[26] I. Lisenkov *et al.*, “Spin-wave edge modes in finite arrays of dipolarly coupled magnetic nanopillars,” *Phys. Rev. B*, vol. 90, p. 024417, Sep. 2014.

[27] W. Greiner and A. Schäfer, *Quantum Chromodynamics*. Heidelberg, Germany: Springer-Verlag, 1994.

[28] E. L. Albuquerque, P. Fulco, E. F. Sarmento, and D. R. Tilley, “Spin waves in a magnetic superlattice,” *Solid State Commun.*, vol. 58, no. 1, pp. 41–44, 1986.

[29] E. L. Albuquerque and M. G. Cottam, “Theory of spin waves in ferromagnetic superlattices with nonuniaxial single-ion anisotropy,” *Phys. Rev. B*, vol. 46, p. 14543, Dec. 1992.

[30] J. Barnaś, “Exchange modes in ferromagnetic superlattices,” *Phys. Rev. B*, vol. 45, p. 10427, May 1992.

[31] J. O. Vasseur, L. Dobrzynski, B. Djafari-Rouhani, and H. Puszkarski, “Magnon band structure of periodic composites,” *Phys. Rev. B*, vol. 54, p. 10443, Jul. 1996.

[32] X.-G. Wang, G.-H. Guo, Z.-X. Li, D.-W. Wang, Y.-Z. Nie, and W. Tang, “Spin-wave propagation in domain wall magnonic crystal,” *Europhys. Lett.*, vol. 109, no. 3, p. 37008, 2015.
[33] K.-S. Lee, D.-S. Han, and S.-K. Kim, “Physical origin and generic control of magnonic band gaps of dipole-exchange spin waves in width-modulated nanostrip waveguides,” *Phys. Rev. Lett.*, vol. 102, p. 127202, Mar. 2009.

[34] C. Kittel, *Introduction to Solid State Physics*, 8th ed. Hoboken, NJ, USA: Wiley, 2005.

[35] R. Hertel, W. Wulfhekel, and J. Kirchner, “Domain-wall induced phase shifts in spin waves,” *Phys. Rev. Lett.*, vol. 93, p. 257202, Dec. 2004.

[36] D. Wang, C. G. Morrison, A. R. Buckingham, G. J. Bowden, R. C. Ward, and P. A. J. de Groot, “Room temperature magneto optic exchange springs in DyFe$_2$/YFe$_2$DyFe$_2$/YFe$_2$ superlattices,” *J. Magn. Magn. Mater.*, vol. 321, pp. 586–589, Mar. 2009.

[37] R. Qiu, T. Huang, and Z. Zhang, “Magnon band structure and magnon density in one-dimensional magnonic crystals,” *J. Magn. Magn. Mater.*, vol. 368, p. 180, Nov. 2014.

[38] Z. Q. Qiu and S. D. Bader, “Surface magneto-optic Kerr effect,” *Rev. Sci. Instrum.*, vol. 71, no. 3, p. 1243, 2000.

[39] A. Hubert and R. Schafer, *Magnetic Domains: The Analysis of Magnetic Microstructures*. Berlin, Germany: Springer, 1998.

[40] X. Portier and A. Perfoid-Long, “The formation of 360° domain walls in magnetic tunnel junction elements,” *Appl. Phys. Lett.*, vol. 76, no. 6, p. 754, 2000.

[41] F. J. Castaño et al., “Metastable states in magnetic nanorings,” *Phys. Rev. B*, vol. 67, p. 184425, May 2003.

[42] C. B. Muratov and V. V. Osipov, “Theory of 360° domain walls in thin ferromagnetic films,” *J. Appl. Phys.*, vol. 104, no. 5, p. 053908, 2008.

[43] Y. Jang, S. R. Bowden, M. Mascaro, J. Unguris, and C. Ross, “Formation and structure of 360 and 540 degree domain walls in thin magnetic stripes,” *Appl. Phys. Lett.*, vol. 100, no. 6, p. 062407, 2012.

[44] M. Diegel, R. Mattheis, and E. Halder, “360° domain wall investigation for sensor applications,” *IEEE Trans. Magn.*, vol. 40, no. 4, pp. 2655–2657, Jul. 2004.

[45] M. O. Liedke et al., “Domain structure during magnetization reversal of PtMn/CoFe exchange bias micropatterned lines,” *J. Appl. Phys.*, vol. 100, no. 4, p. 043918, 2006.

[46] L. Thomas, M. Hayashi, R. Moriya, C. Rettnar, and S. Parkin, “Topological repulsion between domain walls in magnetic nanowires leading to the formation of bound states,” *Nature Commun.*, vol. 3, p. 810, May 2012.

[47] A. L. Gonzalez Oyarce, J. Llandro, and C. H. W. Barnes, “360° domain wall injection into magnetic thin films,” *Appl. Phys. Lett.*, vol. 103, no. 22, p. 222404, 2013.

[48] J. Zhang and C. A. Ross, “Gold shunt pads as a chirality filter for current-driven 360° domain wall motion in a ferromagnetic wire,” *Appl. Phys. Lett.*, vol. 103, no. 16, p. 162411, 2013.

[49] A. L. G. Oyarce, Y. Nakatani, and C. H. W. Barnes, “Static and dynamic behavior of 360° domain walls in patterned thin films,” *Phys. Rev. B*, vol. 87, p. 020401, Jun. 2013.

[50] J.-P. Tetienne et al., “Nanoscale imaging and control of domain-wall hopping with a nitrogen-vacancy center microscope,” *Science*, vol. 344, no. 6190, p. 1366, 2014.

[51] A. Kubetzka, O. Pietzsch, M. Bode, and R. Wiesendanger, “Spin-polarized scanning tunneling microscopy study of 360° walls in an external magnetic field,” *Phys. Rev. B*, vol. 67, p. 020401, Jan. 2003.

[52] G. Leaf et al., “Dynamic origin of stripe domains,” *Phys. Rev. Lett.*, vol. 96, p. 017201, Jan. 2006.