Micromachined Rate-Integrating Gyroscopes: Concept, Asymmetry Error Sources and Phenomena

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Abstract: Rate-integrating gyroscope (RIG) operation is an attractive operation scheme for micro gyroscopes, because it can provide direct angle measurement and a high dynamic range. However, its zero rate output (ZRO) has a severe drift issue. This work compared the difference between rate and rate-integrating gyroscope operations, and further modeled the gyroscope with asymmetric error terms. Then the influences of these non-ideal factors were analyzed to abridge the ZRO drift phenomena and the sources. The numerical verification proved the theory that such an oscillation pattern is caused by the stiffness mismatch, and decaying to X/Y axis manner resulted from the damping errors. The conclusions can contribute to aiding engineers to rapidly locate the bottlenecks of RIG operations.

Keywords: micro gyro; rate-integrating gyroscopes; whole angle mode gyroscopes; asymmetric micro-machined gyro

1. Introduction

Conventional micro-electro-mechanical systems (MEMS) rate-resonant gyroscopes have successfully been applied to a variety of applications as an attitude sensor, such as in automotive electronics [1], consumer electronics [2] and tactical weapons [3]. It is expected that MEMS gyro will be put into inertial navigation systems by fully taking the advantage of their low costs, power consumption and miniaturized size [4]. However, MEMS gyro’s relatively low performance prevents them being used in high-end applications. The bottle neck of rate gyroscopes is that they measure angular rate and are then integrated into angel for desired attitude estimation [5]. The error caused by bias drift will accumulate with the integration process which limits the reliable operating time.

In the past two decades, researchers have put lots of effort into improving the performance of MEMS gyro. Initially, the solutions were focused on novel materials, fabrication techniques and optimized structural design. Ayazi provided a poly silicon based ring gyro with nano gaps to increase the signal to noise ratio [6]. Alexander and Shkel investigated and improved the tuning fork type design of MEMS gyro [7]. Qiang Xu and Xuezhong Wu designed a tuning Fork gyroscope with a polygon-shaped vibration beam structure with high reliability. Qingsong Li and Dingbang Xiao have proposed a lumped mass micro gyro with a bias instability of navigation grade performance [8]. Apart from focusing on the mechanical systems, the electrically side of the gyro has also proven its potential to improve the performance. Kline developed a continuous mode reversal scheme to enhance...
the bias instability performance [9]. Haoyu Gu and Baolin Zhao developed a noise-suppressed mode reversal method for micro gyroscopes [10]. The advanced model predictive control method with a 1 bit quantizer was applied to gyro control to improve the noise performance [11]. Min Cui and Huiliang Cao proposed a novel characteristic compensation solution based on a temperature variable resistor [12]. A dual mode operation structure was developed to eliminate the rate random walk that is causing by the common mode errors [13]. Frequency modulated gyro architectures are also effective methods with which to reduce the influence of environmental variations on MEMS gyros. The artificial intelligence assisted techniques are also of interest to scholars for reducing errors of MEMS gyro [14,15].

The rate-integrating gyroscope (RIG), or namely, whole-angle mode gyros (WAMG), have raised interests both in the industry and academia in the last few years. Scientists and engineers consider this concept an alternative approach with which to conduct advanced micro angular sensors. It is claimed that RIG can directly measure the angle under this scheme to avoid the integration error issue, and provides unlimited dynamic range as a great bonus [16].

Shkel first demonstrated the RIG concept in MEMS gyroscopes in 2011 [17]. However, the RIG’s zero rate output (ZRO) was not stabilized as expected. Academia has put efforts into solving this major concern: Gregory proposed an in-run compensation system for RIG in 2012 with improved performance [18,19]. Shkel designed a 30 minutes of free vibration micro gyro for RIG operation without control electronics [20]. Gando proposed a dual gyroscope with catch-release mechanism to solve the ZRO drift and energy dissipation issue [21]. Zhongxu Hu utilized an off-line mode matching technique to reduce the ZRO drift issue [22]. At this point, the error mechanisms and calibration technique are still open, and call for further investigation for the sake of conducting reliable RIG.

In real practice, the ZRO of RIG systems will perform a random drift/oscillation pattern that confuses the engineers trying to locate the error sources. In this work, we comprehensively study the concept of the RIG and investigate the error mechanisms behind the phenomena.

The contributions of this manuscript are summarized as follows:

1. We applied a separation principle method to deeply characterize the error sources of RIG operation. A real MEMS gyro has a variety of non-ideal factors, and it is difficult to quantify their influences. Separated error study can simplify the analysis and better characterize the consequences.

2. We abridged the ZRO drift phenomena and scientific principles. The error factors of frequency split, damping and quality factor mismatch are added to the ideal gyro model and analyzed. Their errors, as time domain phenomena, are also explained to help engineers locate the major drifting source in practice.

3. We verified the theoretical study through a case to case comparison. A series of tests were utilized to demonstrate the influences of the major error sources with visualized trajectories.

The remainder of this manuscript gives a detailed theoretical analysis, verification and discussions.

2. Theoretical Analysis of RIG Errors and Phenomena

2.1. Ideal Symmetric RIG Principle

An ideal symmetric MEMS gyroscope can be modeled by a set of differential equations:

\[
\begin{align*}
mx'' + cx' + kx &= Fx - 2m\lambda\Omega_z y' \\
m\dot{y} + cy + ky &= Fy + 2m\lambda\Omega_z x,
\end{align*}
\]

where \(x\) and \(y\) are the displacements of the two modes (\(X\) and \(Y\)) of a gyro; \(m\) is the mass; \(k_x\) and \(k_y\) are the stiffness of \(X\) and \(Y\); \(c_x\) and \(c_y\) are the damping coefficients; \(F_x\) and \(F_y\) are the actuation forces; \(\lambda\) is the Coriolis angular gain depending on the mechanical structure design and modal orders; and \(\Omega_z\) is
the physical rotation rate that needs to be measured with the Z axis. For an ideal and symmetric gyro, it is notable that cross coupling terms are ignored, and assuming that $k_x = k_y$ to make sure the two modes are balanced.

Another set of useful definitions in practice is:

\[
\begin{align*}
\omega_x &= \sqrt{k_x/m} \\
\omega_y &= \sqrt{k_y/m} \\
c_x/m &= \omega_x/Q_x \\
c_y/m &= \omega_x/Q_y
\end{align*}
\]

where $\omega_x$ and $\omega_y$ are the resonant frequencies, and $Q_x$ and $Q_y$ are the quality factors (Q) of the two gyroscopic modes. The quality factor is a straightforward tool used to tailor the energy dissipation; typically, a higher Q is desired to increase the input signal sensitivity for gyros.

For a conventional rate gyro, the operation principle involves nulling the actuation force $F_y$ and letting $F_x = A_x \sin(\omega_xt)$, where $A_x$ is the drive force amplitude. The solution of a conventional gyro can be solved by averaging method with these assumptions: (a) ignoring the free response part of the system that is brought by initial conditions and (b) assuming the input force $A_x \sin(\omega_xt) >> 2\lambda \Omega_z y$:

\[
\begin{align*}
x(t) &= -\frac{A_x Q_x}{m\omega_x^2} \cos(\omega_xt) \\
y(t) &= -2\lambda \Omega_z \frac{A_x Q_x Q_y}{m\omega_x \omega_y} \cos(\omega_y t).
\end{align*}
\]

The detailed derivation of (3) is given in the Appendix A. This solution indicates that the X mode does not include any information about the rotation and Y is the sensitive axis of the angular rate to be extracted. The trajectory of an ideal rate gyro with a 45° step input is demonstrated in Figure 1a. The angular rate is as amplitude modulated (AM) to the carrier signal $\sin(\omega_y t)$ which can be demodulated through in-phase/quadrature (IQ) demodulation process.

The essence of the RIG operation is utilizing the free response of (1) when $F_x = F_y = 0$ and the initial conditions $x(0), y(0)$ are not all equal to zero. The difference can be also illustrated in Figure 1. Thus, (1) can be solved as:

\[
\begin{align*}
x(t) &= a \cos(\theta_0 + \int \Omega_z \ dt) \cos(\omega_xt) \\
y(t) &= a \sin(\theta_0 + \int \Omega_z \ dt) \cos(\omega_y t),
\end{align*}
\]

where $a$ is the amplitude of the vibration that depends on the initial conditions $x(0)$ and $y(0)$.

The AM signals $a \cos(\theta_0 + \int \Omega_z)$ and $a \sin(\theta_0 + \int \Omega_z)$ can be also demodulated via IQ demodulation to compute the rotation angle instead of rate:

\[
\theta_0 + \int \Omega_z \ dt = \arctan\left(\frac{a \cos(\theta_0 + \int \Omega_z \ dt)}{a \sin(\theta_0 + \int \Omega_z \ dt)}\right).
\]

The integral of $\Omega_z$ can be also defined as:

\[
\theta = \int \Omega_z \ dt,
\]
where θ is the orbit angle and is demonstrated in Figure 2. θ reflects the angle rotation in the Z axis instead of rate and can be calculated by measuring the amplitude x(t) and y(t) according to (5). It is feasible to extract \( \|x(t)\| \) and \( \|y(t)\| \) through the in-phase quadrature (IQ) modulation process.

\[
\begin{align*}
\ddot{x} + c_s \dot{x} + k_s x + c_{xy} \dot{y} + c_{yx} y &= F_x - 2m\lambda \Omega_z \dot{y} \\
\ddot{y} + c_y \dot{y} + k_y y + c_{yx} \dot{x} + k_{yx} x &= F_y + 2m\lambda \Omega_x \dot{x},
\end{align*}
\]  

(7)

Figure 1. Solution traces of MEMS gyros with different actuation conditions.

Figure 2. Mass center orbit of the rate-integrating gyroscope (RIG) gyro.

The trajectory of a ideal RIG gyro also with a 45\(^\circ\) angle rotation is illustrated in Figure 1b. Unlike the rate gyro in Figure 1a such that the energy dissipates along with time in the “sensitive axis” and the orbit angle goes back to 0\(^\circ\) eventually, the RIG gyro holds the orbit angle for users to measure the external rotation.

However, this promising RIG architecture has not been applied to engineering practice yet, due to the asymmetry and the existence of cross coupling terms which can be described by:
where \( k_{xy} = k_{yx} \) are the stiffness coupling coefficients, and \( c_{xy} = c_{yx} \) are the damping coupling coefficients. Moreover, the spring stiffness \( k_x, k_y \) and \( Q_x, Q_y \) are not balanced with each other in real practice because of the imperfections of the materials and the fabrication process, which is illustrated in Figure 3. The trajectories of the non-ideal rate/rig gyros are shown in Figure 1c,d. It was demonstrated that these error factors have induced distortions to both rate and RIG gyros, but the overall trends of the rate gyro converge to the ground truth. On the contrary, the behavior of RIG is very difficult to be quantitatively analyzed—the orbit angle oscillates and drifts away from the expected 45\(^\circ\) input reference. From the users’ point of view, the ZRO behaves as unpredictable rule-less/random drifts to keep RIG away from a commercial product.

![Figure 3. Model Comparison of (a) the ideal RIG model and (b) the RIG model with non-idealities.](image-url)

These complex RIG error behaviors have built-in barriers for engineers to locate the origins of the error sources and develop essential compensation/control methods to reduce these errors. The upcoming section will analyze the phenomena in a deep way and suggest practical ways to reduce the errors.

2.2. RIG with Anisoelasticity

Anisoelasticity is the most apparent error that hurts the ideal RIG operation. More specifically, the equivalent stiffnesses of the two modes, \( k_x \) and \( k_y \), are not balanced, which means there exists a frequency split of the two modes. This is mainly caused by multiple factors:

1. The anisotropic properties of the material. MEMS gyros are typically made by single crystal silicon, whose mechanical properties are different along different orientations because of its crystal structures. Even assuming that the mechanical structures of each vibration mode are fabricated as perfectly symmetric, the equivalent stiffness would be different.

2. Fabrication imperfection. The micro gyros are fabricated through a series of processes, including photolithography, etching and deposition. Though these processes are considered mature enough to withhold the mechanical tolerance, it is very common to yield a stiffness difference level of 5% between the two modes.

3. Thermal mechanical deformations. As the temperature changes, the dimensions of the MEMS gyro also expand/shrink to result in equivalent stiffness variation. The variations of the two modes will not be uniform which will lead to further splitting.

The resonant frequencies of each mode are defined by:

\[
\omega_x = \sqrt{\frac{k_x}{m}} \\
\omega_y = \sqrt{\frac{k_y}{m}} \\
\omega = \frac{(\omega_x + \omega_y)}{2}.
\]
The stiffness coupling term, \( k_{xy} \), is another anisoelasticity component in addition to the stiffness mismatch because the two modes can not be truly orthogonal. It results from the “electrode mis-alignment” of the gyroscope. As shown in Figure 4(a), the driving forces \( F_x \) and \( F_y \) are implemented through micro capacitive structures, which are not orthogonal to the axis of the modes. The force \( F_x \) applied to the X mode will equivalently induce a branch, namely, \( F_{x1} \), that will influence the Y mode. An alternative way to describe this non-ideality would be create cross coupling terms: the stiffness coupling \( k_{xy} \) and damping coupling \( c_{xy} \). The impact of \( c_{xy} \) will be discussed in the later subsection.

![Figure 4. Model comparison of (a) symmetric RIG model and (b) RIG model with non-idealities.](image)

The stiffness cross coupling term \( k_{xy} \) can be redefined similarly to the resonant frequency:

\[
\omega_{xy} = \sqrt{\frac{k_{xy}}{m}}.
\]  

(9)

Thus, the system equation for the behavioral analysis without considering the damping effects would be:

\[
\ddot{x} + \omega^2_x x + \omega^2_y y = -2\lambda_2 \dot{y}
\]

\[
\ddot{y} + \omega^2_y y + \omega^2_{xy} x = 2\lambda_2 \dot{x},
\]  

(10)

which is an equation that is difficult to be solved directly.

The alternative approach to evaluate the influence of the anisoelasticity would be examining the energy and angular momentum characteristics:

\[
E = \| x(t) \|^2 + \| y(t) \|^2
\]

\[
H = x \dot{y} - y \dot{x},
\]  

(11)

where \( E \) is the total kinetic energy and \( H \) is the angular momentum.

With the existence of frequency split and stiffness cross coupling, \( E \) is maintained because there is no damping effect and \( H \) can be obtained as [23]:

\[
H = H_m \cos(\frac{(\omega_x^2 - \omega_y^2)^2 - \omega_{xy}^2}{\omega} t),
\]  

(12)

where \( H_m \) is the maximum momentum that depends on the initial condition. This solution indicates that the orbit angle of the RIG with anisoelasticity is showing a periodically oscillation pattern, which is similar to the Lissajous pattern [24]. It can also be observed that the oscillation is more sensitive to a lower resonant frequency \( \omega \).
2.3. RIG with Damping Error Factors

Unlike the anisoelasticity components that cause the orbit of the gyro to be significantly changed, the consequences of asymmetry of the damping terms are more unspectacular and can be concluded to be part of stiffness mismatch. In fact, the damping errors can lead to another totally different phenomenon.

Similarly to anisoelasticity, the damping mismatch results from the isotropic nature of the silicon material and the fabrication deviation. The nonuniformity of the material leads to different thermal-elastic damping factors of the X and Y modes. The anchor losses of these two gyroscopic modes are also unbalanced due to the asymmetry of the mechanical structures. The major source of the damping cross coupling term $c_{xy}$ is already given in Figure 4.

The model is reformulated without anisoelasticity but damping errors for sake of technical analysis:

$$m\ddot{x} + c_{x}\dot{x} + k_{x}x + c_{yx}\dot{y} = -2m\lambda\Omega_{z}\dot{y}$$
$$m\ddot{y} + c_{y}\dot{y} + k_{y}y + c_{xy}\dot{x} = 2m\lambda\Omega_{z}\dot{x}.$$  (13)

Assuming that a step angle rotation $\theta$ is more steady than the state variables of (13) and the gyro dynamics can be considered as ideal during the rotation, it is applied. When the rotation is finished, the total energy of the gyro is $E_{0}$ and the instantaneous energy distributions of the two modes are:

$$E_{x}(0) = E_{0}\cos(\theta)$$
$$E_{y}(0) = E_{0}\sin(\theta),$$  (14)

where $E_{x}$ and $E_{y}$ are the energies of X and Y mode, respectively.

At this moment, the orbit angle $\theta$ is equal to the angle rotation and can be calculated through the energy observation of the two modes:

$$\theta_{0} = \arctan\left(\frac{E_{x}(0)}{E_{y}(0)}\right) = \arctan\left(\frac{E_{0}\cos(\theta)}{E_{0}\sin(\theta)}\right).$$  (15)

Even without energy control, if the $E_{x}$ and $E_{y}$ share the same decay rate, the calculation of $\theta$ is still reliable along with time. But in fact, the observed orbit angle according to the calculation would be:

$$\theta = \arctan\left(\frac{E_{x}(t)}{E_{y}(t)}\right) = \arctan\left(\frac{E_{0}\cos(\theta)e^{(-\tau_{x})}}{E_{0}\sin(\theta)e^{(-\tau_{y})}}\right),$$  (16)

where $\tau_{x}$ and $\tau_{y}$ are the time coefficients of the energy dissipation [25].

If the gyro has damping mismatch or damping cross coupling components, the dissipation rates are different from each other:

$$\tau_{x} = 4\frac{1}{c_{x} + c_{xy}\omega}$$
$$\tau_{y} = 4\frac{1}{c_{y} - c_{xy}\omega},$$  (17)

which indicates that the energy in one mode always decays faster than in the other and will result in the orbit angle drifting to the axis that has a longer decay time. The convergence time is proportional to the value of quality factor mismatch. It can be also found that a smaller resonant frequency $\omega$ and larger quality factor $Q$ can reduce the drift rate. However, this drift will always exist unless the gyro is perfectly symmetric. This process is illustrated in Figure 5. When the external rotation is applied, the
orbit angle will start to drift to the axis that has the smaller damping coefficient/higher quality factor, because the sub value of the other mode exhausts faster.

Figure 5. Unequalized energy dissipation-induced orbit angle drift.

3. Validation and Discussions

3.1. Simulation Configuration

A comprehensive simulation study was performed to verify the effectiveness of the theoretical analysis because real gyros cannot be configured into the desired situations; e.g., totally symmetric, no anisoelasticity, etc.

The simulated nominal gyro had an angular gain $\lambda = 1$, a resonant frequency of 10 kHz and a Q of 10 k for the convenience of comparison which are also close to real typical values. In the initial configuration, the two modes are perfectly matched ($k_x = k_y$, $c_x = c_y$) without any mode cross coupling ($c_x y$, $k_x y$). An input of a 45° angle rotation at 1 ms was applied for the following tests as a standard reference. The simulation tests were run in MATLAB SIMULINK 2019b.

3.2. Stiffness Mismatch and Coupling Results

The evaluation results of frequency split of X and Y are given in Figures 6 and 7a–d. A frequency split $\Delta f = 10$ Hz was first performed and is shown in Figure 6 to fully verify the predicted phenomena with a 40 ms simulation time. After the external rotation was applied, the orbit angle was the same as 45° at first, but the orbit started oscillating right after the rotation stopped, as the theory expected.

Comparison tests were also performed to evaluate the influences of varying $\Delta f$ and $f_n$. To make a clear argument, the simulation was shortened to 5 ms from 40 ms for sake of illustration. In Figure 7a, the parameters remained the same, and the final minor axis of the induced elliptical trajectory was about 0.1 at 5 ms. Then the $\Delta f$ was increased to 50 Hz to assess the consequences, and the fact is that the minor axis was about 5 times larger than in the previous case.

Figure 6. Trajectory of the gyro with frequency split for 40 ms.
Figure 7c,d changed the $f_n$ to 50 kHz, but the selections of $\Delta f$ were still 10 Hz and 50 Hz. It turned out that the oscillation decreased with a higher $f_n$ increase.

The stiffness cross coupling is potentially another error factor for RIG gyros, and the results are shown in Figure 8. The gyro was initialized back to the original parameters, and the coupling levels $k_{xy}$ were defined by the stiffness percentages of the modes. The values of $k_{xy}$ were chosen from 5% to 20% with a step size of 5%, which is based on the fact that the isolation levels for commercial gyros are around $-30$ dB. It can be observed that the stiffness cross coupling term can introduce a similar effect to frequency splitting and cause the orbit angle to oscillate.
3.3. Damping Coupling and Quality Factor Mismatch Results

Damping mismatch and cross coupling are actually severe issues for RIG operations and can lead the measured angle to drift back to one mode. The gyro with Q mismatch and damping coupling were verified to support the theoretical analysis.

Figure 9 shows the results regarding to the Q mismatch issue with a simulation time of 50 ms. Figure 9a,b demonstrates the trends of the gyro with 5 k of Q mismatch for X and Y, respectively. Though the orbit angles were not oscillating anymore, the angles drifted to the axis that had the lower damping factor. Then the quality factors were reduced to 5 k and 4.5 k for the evaluations, which are shown in Figure 9c,d. The drifted angles were more than 10 times higher than the \(Q = 50\) k case and proved that higher Q is desired for MEMS RIG. The last two sub-figures were used to verify that higher resonant frequencies can hurt the performance of bias stability. With the configuration of \(f_n = 20/40\) kHz and \(Q = 5/4.5\) k, the orbit drift was increasingly about two to three times worse than the one with \(f_n = 10\) kHz.

The damping coupling is also an important drifting source which tends to be overlooked. The verification started with the suggested nominal gyro with a 10% damping coupling level and then increased the resonant frequency \(f_n\) to 20 kHz to evaluate the dependency between the drift rate and \(f_n\), whose results are shown in Figure 10. According to these trajectories, the damping coupling term will also lead the orbit angle to one mode and is more sensitive to a higher resonant frequency.

![Figure 9. Trajectory of the gyro with damping mismatch and different \(f_n\).](image-url)
3.4. Discussions and Suggestions

Based on the theoretical study and validation presented above, it can be concluded that all of these error terms will fail the operation of RIG. The relationships between the phenomena and error factors can be summarized as:

1. The periodic oscillation motions at the angle output are caused by the frequency split or stiffness coupling.
2. The exponential drift toward one major axis/mode is induced by the quality factor mismatch and damping coupling term.

These two points can assist engineers in locating the major error sources for the RIG operations, and then improve the design/fabrication process to conduct improved gyros in the next generation. Though the zero error source gyro would never happen, here are some suggestions for MEMS designers that target for RIG operation:

1. When giving the symmetry the highest priority, e.g., designing a gyro in a converse way to reduce the frequency split/damping mismatch/mode coupling, even conducting a lower quality factor is acceptable. Developing a gyro using isotropic materials (e.g., poly silicon) is another solution.
2. Evaluating the RIG behavior, locating the major error source and modifying the design according to the analysis in this work. If the symmetry is already reaching the limits, it can be further optimized. If the ZRO pattern is dominated by drifting behavior, a higher Q and lower resonant frequency can improve the ZRO stability; if the ZRO behaves in an oscillation manner, higher resonant frequencies will reduce this error.

Though the optimized structure design, isotropic materials and advanced processes could improve the RIG ZRO stability, the RIG is much more sensitive to the rate scheme. The off-line auto tuning mechanisms may initially reduce the errors, but these undesired components are time variant. Despite the contribution to the noise power, a dynamic control system is still an ultimate solution to eliminate all the sources. Unlike the anisoeelasticity terms that can be minimized through the spring softening effect and tuning electrodes on the gyro, the damping mismatch/coupling terms are more difficult to be canceled. That is a major concern for the control system implementation.

4. Conclusions

The operation principles of micro RIG sensors with error sources were studied. The asymmetric factors will lead the orbit of the proof mass center to oscillate. The damping mismatch and coupling terms are the reasons to let the orbit angle drift to X/Y axis. A higher resonant frequency can reduce the oscillation with a cost of increasing the influence of damping errors. A dynamic control system study is recommended to reduce these error terms for long time stability RIG in the future.
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**Abbreviations**

The following abbreviations are used in this manuscript:

- ZRO Zero rate output
- MEMS Micro-electro-mechanical systems
- Q Quality factor
- RIG Rate-integrating gyroscopes
- WA Whole-angle

**Appendix A. Detailed Derivation of Conventional Rate Gyro System**

For a micro vibrating gyroscope, its full dynamics can be described by:

\[
\begin{align*}
\ddot{m}x + c_x \dot{x} + k_xx &= F_x - 2m \lambda \Omega_z \dot{y} \\
\ddot{m}y + c_y \dot{y} + k_yy &= F_y + 2m \lambda \Omega_z \dot{x},
\end{align*}
\] (A1)

where all the coefficients are explained in Section 2.

For the convenience of engineering practice, the following definitions are used:

\[
\begin{align*}
\omega_x &= \sqrt{\frac{k_x}{m}} \\
\omega_y &= \sqrt{\frac{k_y}{m}} \\
c_x/m &= \omega_x/Q_x \\
c_y/m &= \omega_x/Q_y.
\end{align*}
\] (A2)

For the conventional operation, the artificial input is designed with the following form:

\[
\begin{align*}
 f_x &= A_x \sin(\omega_x t) \\
 f_y &= 0.
\end{align*}
\] (A3)

Substituting (A2) and (A3) into the gyro system Equation (A1) and considering that the input force \( f_x \gg 2m \lambda \Omega_z \dot{y} \), the \( x \) mode would be:

\[
\ddot{x} + \frac{\omega_x}{Q_x} \dot{x} + \omega_x^2 x = \frac{A_x}{m} \sin(\omega_x t)
\] (A4)

and the \( Y \) mode would be:
\[ \ddot{y} + \frac{\omega_y}{Q_y} y + \omega_y^2 y = 2\lambda \Omega_z \dot{x}. \quad (A5) \]

Then the system can be solved from the X mode starting from (A4) by assuming the solution of \( x \) has the form of:

\[
\begin{align*}
  x(t) &= A_1 \cos(\omega_x t) \\
  \dot{x}(t) &= -\omega_x A_1 \sin(\omega_x t) \\
  \ddot{x}(t) &= -\omega_x^2 A_1 \cos(\omega_x t),
\end{align*}
\]

(A6)

where \( A_1 \) is the coefficient to be determined. Substituting (A6) into (A4), the following equation can be obtained:

\[- \omega_x^2 A_1 \cos(\omega_x t) - \frac{\omega_x}{Q_x} \omega_x A_1 \sin(\omega_x t) + \omega_x^2 x A_1 \cos(\omega_x t) = \frac{A_x}{m} \sin(\omega_x t). \quad (A7)\]

And this equation can be further processed as:

\[- \omega_x^2 A_1 \cos(\omega_x t) - \frac{\omega_x}{Q_x} \omega_x A_1 \sin(\omega_x t) - \frac{A_x}{m} \sin(\omega_x t) = 0. \quad (A8)\]

Then the coefficient \( A_1 \) can be solved as:

\[ A_1 = -\frac{A_x Q_x}{m \omega_x^2} \quad (A9) \]

So the solution of \( x \) can be obtained as:

\[
\begin{align*}
  x(t) &= -\frac{A_x Q_x}{m \omega_x^2} \cos(\omega_x t) \\
  \dot{x}(t) &= \frac{A_x Q_x}{m \omega_x} \sin(\omega_x t) \\
  \ddot{x}(t) &= \frac{A_x Q_x}{m} \cos(\omega_x t),
\end{align*}
\]

(A10)

where \( x(t) \) is the same as the one in (3), and \( \dot{x}(t) \) is used to solve \( y \) that is given in (A5):

\[ \ddot{y} + \frac{\omega_y}{Q_y} y + \omega_y^2 y = 2\lambda \Omega_z \frac{A_x Q_x}{m \omega_x} \sin(\omega_x t). \quad (A11) \]

Similarly to before, the assumed solution of \( y \) has the following form:

\[
\begin{align*}
  y(t) &= A_2 \cos(\omega_y t) \\
  \dot{y}(t) &= -\omega_y A_2 \sin(\omega_y t) \\
  \ddot{y}(t) &= -\omega_y^2 A_2 \cos(\omega_y t),
\end{align*}
\]

(A12)
where $A_2$ is the unknown coefficient to be solved.

(A12) can be combing with (A11) to obtain:

$$-\omega_y^2 A_2 \cos(\omega_y t) - \frac{\omega_y^2}{Q_y} \omega_y A_2 \sin(\omega_y t) + \omega_y^2 A_2 \cos(\omega_y t) = 2\lambda \Omega_2 \frac{A_x Q_x}{m \omega_x} \sin(\omega_x t).$$  \hspace{1cm} (A13)

Analyzing the equation and combing the similar terms:

$$-\omega_y^2 A_2 \cos(\omega_y t) + \omega_y^2 A_2 \cos(\omega_y t) = 0$$

$$-\omega_y^2 \omega_y A_2 \sin(\omega_y t) = 2\lambda \Omega_2 \frac{A_x Q_x}{m \omega_x} \sin(\omega_x t),$$

(A14)

The first cosine terms cancels out, and given that the gyro can be well matched, $\omega_x \approx \omega_y$ and $A_2$ will be eventually solved:

$$A_2 = -2\lambda \Omega_2 \frac{A_x Q_x}{m \omega_x \omega_y^2}. \hspace{1cm} (A15)$$

Since $A_2$ is the only unknown term in the assumed solution, $y$ can be solved:

$$y(t) = -2\lambda \Omega_2 \frac{A_x Q_x}{m \omega_x \omega_y^2} \cos(\omega_y t), \hspace{1cm} (A16)$$

which is also the same in (3).

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