The first excited $1/2^+$ state in $^9$Be and $^9$B

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Nuclear states observed around threshold energies provide us with interesting problems associated with the nuclear cluster structure [1, 2, 3, 4]. The first excited $J^\pi = 1/2^+$ state of $^9$Be [5], which is an $\alpha + \alpha + \textit{n}$ Borromean nucleus, is one of the typical examples in light nuclei. This state of $^9$Be has been observed as a sharp peak above the $^8$Be + n threshold energy in the photo-disintegration cross section of $\gamma + ^9$Be $\rightarrow \alpha + \alpha + \textit{n}$ [6, 7]. The strength of the peak has a strong influence on the reaction rate of the $^9$Be synthesis. We performed the calculations using an $\alpha + \alpha + \textit{n}$ three-body model [8, 9] and the complex scaling method (CSM), which well reproduces the observed photo-disintegration cross section. However, the result indicates that the $1/2^+$ state shows the $s$-wave virtual-state character of $^8$Be+$\textit{n}$. In addition to this problem, we discuss a mirror state problem of the first excite $1/2^+$ state in $^9$B.

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1 Introduction

Nuclear states observed around threshold energies provide us with interesting problems associated with the nuclear cluster structure [1, 2, 3, 4]. Most of them are also interesting astrophysically from the viewpoint of nucleosynthesis. The first excited $J^\pi = 1/2^+$ state in $^9$Be [5], which is an $\alpha + \alpha + \textit{n}$ Borromean nucleus, is one of the typical examples in light nuclei.

The reaction rate of the $^4$He($\alpha$, $\gamma$)$^9$Be reaction is crucial to understand the productions of heavy elements. In the $\alpha$(n, $\gamma$)$^9$Be reaction, a sequential process, $^4$He($\alpha$, $\gamma$) $^8$Be(n, $\gamma$) $^9$Be, has been considered as a dominant one. However, owing to the short life-time of the $^8$Be ground state ($\sim 10^{-16}$ s), a direct measurement of the $^8$Be(n, $\gamma$)$^9$Be reaction is impossible. For an alternative way, the cross section of its inverse reaction, $^9$Be($\gamma$, n)$^8$Be, has been measured to deduce the cross section of $^8$Be(n, $\gamma$)$^9$Be.

The low-lying $1/2^+$ state have a impact on the reaction rate of $^8$Be(n, $\gamma$)$^9$Be in stellar environments. This state of $^9$Be has been observed as a sharp peak above the $^8$Be + n threshold energy in the photo-disintegration cross section of $\gamma + ^9$Be $\rightarrow \alpha + \alpha + \textit{n}$ [6, 7]. The strength of the peak has a strong influence on the reaction rate of the $^9$Be synthesis. From a theoretical side, it is interesting to answer how the low-lying $1/2^+$ state of $^9$Be contributes to the $^8$Be(n, $\gamma$)$^9$Be reaction.

We perform the calculations using an $\alpha + \alpha + \textit{n}$ three-body model [8, 9] and the complex scaling method (CSM) [10, 11]. Applying the three-cluster potential, we show that the observed photo-disintegration cross section [6, 7] is well reproduced. And, the result indicates that the $1/2^+$ state shows the $s$-wave virtual-state character of $^8$Be + n.

In this report, we explain our results of the first excite $1/2^+$ state in $^9$Be in comparison with those of other previous studies [12 - 17], because it has been a long-standing problem whether the $1/2^+$ state is a resonant or virtual state. In addition to this problem, we discuss a mirror state problem of the first excite $1/2^+$ state in $^9$B.
In the next section, we will briefly explain the $\alpha + \alpha + n$ three-body model [8, 9], and show the results of the photo-disintegration cross section. In Sec. 3, the result of the complex scaling method for the $1/2^+$ state is discussed to show no resonance solutions for $^9\text{Be}$. In Section 4, the $1/2^+$ state in $^9\text{B}$ is shown to be obtained as a resonant state, and the comparison of energy levels for $^9\text{Be}$ and $^9\text{B}$ is discussed. Finally, summary is given in Section 5.

2 Photo-disintegration of $^9\text{Be}$

To understand the origin of a low-energy peak in the photo-disintegration cross section just above the $^8\text{Be} + n$ breakup threshold energy in $^9\text{Be}$, we investigate the $E1$-transition strength using an $\alpha + \alpha + n$ three-body model [8, 9]. The Hamiltonian for the relative motion of the $\alpha + \alpha + n$ three-body system for $^9\text{Be}$ is given as

$$H = \sum_{i=1}^{3} t_i - T_{cm} + \sum_{i=1}^{2} V_{an}(\xi_i) + V_{\alpha \alpha} + V_{\text{PF}} + V_3,$$

where $t_i$ and $T_{cm}$ are kinetic energy operators for each particle and the center of mass of the total system, respectively. The interactions between the neutron and the $\alpha$ particle is given as $V_{an}(\xi_i)$, where $\xi_i$ is the relative distance between them. We here employ the KKNN potential [18] for $V_{an}$. For the $\alpha-\alpha$ interaction $V_{\alpha \alpha}$, we employ a folding potential of the effective NN interaction [19] and the Coulomb interaction:

$$V_{\alpha \alpha}(r) = v_0 \exp(-ar^2) + \frac{4e^2}{r} \text{erf}(Br),$$

where $v_0 = -106.09$ MeV, $a = 0.2009$ fm$^{-2}$, and $\beta = 0.5972$ fm$^{-1}$. The pseudo-potential $V_{\text{PF}} = \lambda |\Phi_{\text{PF}}\rangle \langle \Phi_{\text{PF}}|$ with $\lambda = 10^6$ MeV is expressed by the projection operator to remove the Pauli forbidden states $\Phi_{\text{PF}}$ from the relative motion of $\alpha-\alpha$ and $\alpha-n$.

In the Hamiltonian of Equation (1), two-cluster potentials $V_{an}$ and $V_{\alpha \alpha}$ are fixed so as to reproduce the observed scattering data of $\alpha$ and $\alpha-\alpha$, respectively. Since the antisymmetrization effects are taken into account by the Pauli-potential $V_{\text{PF}}$ but a three-cluster exchange effect is not included explicitly in this calculation, we introduce the phenomenological three-cluster potential $V_3$ to investigate the photo disintegration of $^9\text{Be}$ by reproducing the breakup threshold energy into $\alpha + \alpha + n$. The explicit form of $V_3$ is given by

$$V_3 = v_3 \exp(-\mu r^2),$$

where $\rho$ is the hyper-radius of the $\alpha + \alpha + n$ system. The hyper-radius is defined as

$$\rho^2 = 2r^2 + \frac{8}{9}R^2,$$

where $r$ is the distance between two $\alpha$-particles and $R$ is that between the neutron and the center of mass of the $\alpha + \alpha$ subsystem. In Figure 1, calculated photo-disintegration cross sections are shown. The dashed and dotted lines are results with and without the three-body potential of $v_3 = 1.10$ MeV and $\mu = 0.02$ fm. The black solid line represents the cross section calculated by using an attractive three-body potential with $v_3 = -1.02$ MeV. The experimental data below $E_{\gamma} = 2.2$ MeV are taken from References [6, 7]. The arrow indicates the threshold energy of the $^8\text{Be}(0^+) + n$ channel.

The result calculated with an appropriate strength $v_3$ of the three-cluster interaction well reproduces the cross section peak observed just above the $^8\text{Be}(0^+) + n$ threshold.
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3 The virtual-state property of ⁹Be (1/2⁺)

For the problem that the first excited 1/2+ state in ⁹Be is resonant or virtual state, we have many studies so far [12 - 17]. To see whether the peak of the photo-disintegration cross section is due to resonances or not, we apply the complex scaling method to the α+ α + n model and search for the 1/2+ resonant states. The complex-scaled Schrodinger equation is given as

\[ H^\theta \Psi_J(\theta) = E^\theta_J \Psi_J(\theta), \]

where \( J \) is the total spin of the α+α+n system. The complex-scaled Hamiltonian and wave function are

\[ H^\theta = U(\theta) H U^{-1}(\theta), \]
\[ \Psi_J(\theta) = U(\theta) \Psi_J, \]

respectively. The complex scaling \( U(\theta) \) with a real parameter \( 0 \leq \theta \leq 45^\circ \) transforms the relative coordinates as

\[ U(\theta); \quad r \to r e^{i\theta}, \quad R \to R e^{i\theta}. \]

The calculated eigenvalue distribution of the 1/2⁺ states is shown in Figure 2. The result indicates no resonance solutions for \( \theta = 15^\circ \). Although there may exist a resonance solution with a large width, which cannot be solved with \( \theta = 15^\circ \), it is not consistent with observed data of the width \( \Gamma = 217\pm10 \text{ KeV} \) [5]. And we could not find such a resonant state by the analytical continuation for the three-cluster potential strength [8].

On the other hand, we obtain the resonant solution for the 1/2⁺ state in the mirror nucleus ⁹B, where the same Hamiltonian (Equation (1)) for the α + α + p model with the
Figure 2 - Energy eigenvalue distribution of 1/2+ states of 9Be measured from the \( \alpha + \alpha + n \) threshold with scaling angle \( \theta = 15^\circ \). The solid, dashed, and dotted lines represent the branch cuts for \( \alpha + \alpha + n \), \( ^8\text{Be}(0^+) + n \), and \( ^5\text{He}(3/2^-) + \alpha \) continua, respectively.

Coulomb interaction for the proton \( p \) are used. In Figure 3, the 1/2+ resonant state is shown with a circle. This resonance solution is understood to be reproduced by the Coulomb interaction between the valence proton and two \( \alpha \) clusters, which does not exist in the \( \alpha + \alpha + n \) system.

Figure 3 – Energy eigenvalue distribution of 1/2+ states of 9B measured from the \( \alpha + \alpha + p \) threshold with scaling angle \( \theta = 15^\circ \). The solid, dashed, and dotted lines represent the branch cuts for \( \alpha + \alpha + p \), \( ^8\text{Be}(0^+) + p \), and \( ^4\text{Li}(3/2^-) + \alpha \) continua, respectively.

The virtual state property of the 1/2+ state in 9Be was studied in detail by using the \(^8\text{Be}+n\) model [20-30]. It is confirmed that the virtual state of the neutron s-wave is embedded in the continuum without a barrier potential. Furthermore, it is shown that we cannot distinguish virtual state from resonant state in the shape of the cross section peak, when the resonance appears at a very small energy from the threshold.
4 Mirror States in $^9$Be and $^9$B

In addition to the $1/2^+$ state, low-lying states of $^9$Be are calculated within the $\alpha + \alpha + n$ model. The observed photo-disintegration cross sections [6, 7] are shown to be well explained over a wide energy region [9]. The energy levels of $^9$Be are presented in Figure 4 together with experimental results [5]. The first excited $1/2^+$ state does not have correspondence in the present calculation, and the $3/2^-_1$ state is predicted to be about 1 MeV lower than the experiment. However, other states are well reproduced.

![Figure 4](image)

Figure 4 – Energy levels of $^9$Be. The present calculation is compared with the experimental data taken from Ref. [5]

In Figure 5, we show the present result of energy levels for $^9$B in comparison with observed data [5]. The low-lying states, which are all resonant states, are well reproduced except for the first excited $1/2^+$ state. The calculated state is rather higher than the experimental one.

![Figure 5](image)

Figure 5 – Energy levels of $^9$B. The present calculation is compared with the experimental data taken from Ref. [5]
This state of $^9\text{B}$ is the mirror of the virtual state in $^9\text{Be}$, which is understood to have the $s$-wave configuration of the neutron around the $^8\text{Be}$ ($=\alpha+\alpha$) core. The Thomas-Ehrman effect [23] suggests that the $s$-wave proton of the mirror nucleus has a weak effect from the Coulomb interaction. Then the energy shift of the $s$-wave proton configuration is expected to be smaller than those of other states. Thus, the present result shows an inverse tendency of the energy relation between the $1/2^+$ states in $^9\text{Be}$ and $^9\text{B}$.

5 Summary

It has been a long standing problem that the peak of the photo-disintegration cross section observed just above the $^9\text{Be}+n$ threshold in $^9\text{Be}$ causes from the $1/2^+$ resonant state or a neutron $s$-wave virtual. The complex scaled $\alpha+\alpha+n$ model shows to reproduces the observed peak of the $1/2^+$ state due to a neutron $s$-wave virtual state of $^9\text{Be}(0^+) + n$. We discussed a mirror state problem of the first excite $1/2^+$ state in $^9\text{B}$.

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