Specification of Force Influencing a Particle in a Magnetic Field
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Abstract. We elaborate a special case of solving the problem of the calculating expression and obtaining the corresponding data for the ferro-particles magnetic capture force: in the paraxial zone between two opposing and likewise distanced magnetic elements. To the effect, we try to bring out the full potential of the traditionally used classical expression for a magnetic force (which is fair if applicable to the point weakly susceptible ferro-particle) with its possible adjustment for studying a real ferro-object (a ball).

Introduction
Magnetic separation is being widely employed in various industrial processes. For instance, it is still in high demand in biology and medicine.

Behaviour of a ferro-particle exposed to a magnetic field in a particular medium mainly depends on the force this ferro-particle experiences from the magnetic field.

Multiple discussions and numerous attempts to solve the fundamental issue regarding determination of magnetic force $F$, influencing a ferroparticle in the magnetic field (for example by magnetic separation) usually employ quite a famous classical model according to which the magnetic force is expressed as follows:

$$
\overrightarrow{F} = \mu_0 V \chi \nabla H \nabla H = \nabla \chi B \nabla B / \mu_0,
$$

where $\mu_0$ is the magnetic constant, $V$ denotes particle volume, $\chi$ is the particle magnetic susceptibility, $H$ or $B = \mu_0 H$ denote field intensity or density where the particle is located (self-evidently, in the medium magnetic permeability of which is close to 1).

Methods
Many researchers, who often mention and use the expression (1), totally overlook the fact that it is a corollary of an obviously idealized model, viz. the model of magnetic impact on a weakly susceptible particle of a small volume (a ‘point’ particle) which conventionally does not perturb the field it is exposed to.

Meanwhile, in practice (e.g. during magnetic separation) we virtually always talk about real particles having increased magnetic susceptibility and not being the point ones. Thus, the issue connected with the dogmatic usage of the expression (1), for instance, in comparative evaluation of major competing forces affecting such ferro-particles, also for the analysis of their magnetic capture conditions, in particular on the assumption of the concretely defined dynamic equation, calls for a mandatory additional discussion. We believe that the key position on the matter comes down to the question whether this approach, which is practically acknowledged as simplified and universal, can be valid. Frequent and seemingly self-explanatory use of the expression (1) speaks in favour of such recognition. Unfortunately, so far this intrinsically key issue concerning the magnetic capture mechanism itself has not been solved or even raised.

To address the issue, it is surely necessary to obtain a wide array of data (experimentally and/or theoretically) on the parameters constituting the expression (1), such as $H (B)$, $\nabla H (\nabla B)$, $\chi$; it is
also desirable to get them in a functional view as applied to specific cases of performing magnetic capture. We should mention that many studies neither feature such an array of data at all nor give some fragmentary information which disables the adequate use of this debatable (in its designated purpose) expression (1) and therefore denies obtaining valid final and interim results. In the absence of such an array of data, frequent reference to the expression (1) seems to be informative only in the most general sense.

Moreover, alongside with obtaining an expanded view of the model expression (1) (naturally, for particular cases of some operating zones with the magnet located there) to calculate specific values of the magnetic force, it is essential to have an adequate control, preferably direct, experimental data on the values of this force at our disposal. We may employ the data received with the help of various uniform operating zones and balls of different diameter [1, 2] inter alia.

Results

As for obtaining an expanded view of the model expression (1), this task can have an original solution, e.g. for a special case of a trial ferro-particle in an initially ‘unidirectional’ field with straightened magnetic flux lines (a linear problem). Let us consider, for example, that such a field is created in the central (paraxial) zone of the magnetic pole as well as between opposing, mutually distanced opposite magnetic poles which intensify the resultant field. In such cases the expression for the force (decreasing with \( x \) distancing from the pole) may be simplified to the following view:

\[
F = \mu_0 V \chi H \left| \frac{dB}{dx} \right| = V \chi B \frac{dH}{dx} / \mu_0,
\]

which is often considered quite acceptable for the practical application including magnetic separation problems.

Regarding functional versions of integral parameters constituting the expression (2), such as magnetic induction \( B \) or field intensity \( H \), one can say that the curves of their modulation (reduction) as \( x \) distances from the pole surface often have an exponential view. Thus, for an operating zone of a module consisting of two opposing (mutually distanced at \( b=35mm \)) magnetic elements Nd-Fe-B, these parameters up to a comparatively narrow zone in the vicinity of module symmetry plane (which does not exceed a quarter of the module operating zone) [3] are:

\[
B = B_0 \exp(-k_x \cdot x), \quad H = H_0 \exp(-k_x \cdot x),
\]

With the values of induction and/or intensity at the surface of the magnetic element (\( x=0 \)) amounting to \( B_0=0.4T \) and \( H_0=B_0/\mu_0=3,18 \cdot 10^5A/m \), whereas the value of a multiplier in the exponent is \( k_x=120 \, m^{-1} \) [3]. Therefore, the force factor entering in the expression (2) \( B |dB/dx| \) or \( H |dH/dx| \) taking into account parameters (3) and analytically easily defined characteristics of field non-homogeneity \( |dB/dx| \) or \( |dH/dx| \) can be functionally put down as:

\[
B |dB/dx| = B_0^2 k_x \exp(-2k_x \cdot x), \quad H |dB/dx| = H_0^2 k_x \exp(-2k_x \cdot x)
\]

Concerning functional version of such no less important parameter as magnetic susceptibility \( \chi \) of the ferro-particle exposed to a magnetizing force in the expression (2), we should note the two typical and rather tangible flaws which regretfully occur when the fundamental expressions (1) and (2) are commonly used.

Firstly, this is by no means always taken into consideration (even though a number of papers pinpoint it [3-8]) that magnetic susceptibility \( \chi \) of a certain body (a ferro-particle) greatly depends on its shape; besides, the connection of the body matter with its \( \chi \) susceptibility is [3, 5, 6, 8]: \( 1/\chi - 1/\chi_0 = N \), where \( N \) is the body demagnetizing coefficient. E.g. for a ball-shaped magnet when \( N=1/3 \) [5, 9, 10], this connection will result in:
\[ \chi = \frac{3 \chi_n}{(3 + \chi_n)} \quad (5) \]

Secondly, it is almost completely disregarded that magnetic susceptibility \( \chi_n \) of the magnet matter present in the expressions (4) and (5) and its permeability \( \mu_n = \chi_n + 1 \) is clearly expressed by a well-known distinctive extreme dependence on the magnetic field intensity \( H \); in the case considered, it naturally varies with the field intensity at a certain point located at \( x \) distance from the polar surface. Hence, the expression (5) can by no means (in defiance to the evolving opinion) be considered sufficient to be introduced in the expressions (1) and (2). There has to be information on functional dependence of \( \chi_n \) on \( H \) obtained, as nobody has yet stated and thus accounted for assuming, probably wrongly, that \( \chi_n \) is close to a constant.

Paper [8] provides a dependence of \( \mu_n \) on \( H \) for unannealed ball-bearing steel (matter) decreasing in the post-extreme domain; it may be easily transformed into the dependence of magnetic susceptibility \( \chi_n \) on \( H \) (Fig.1). Being depicted in logarithmical coordinates in the same

![Figure 1](image1.png)

Figure 1. Case study (by data [8]) of post-extreme domain dependence of magnetic susceptibility of unannealed ball-bearing steel.

![Figure 2](image2.png)

Figure 2. Post-extreme domain dependence of the unannealed ball-bearing steel ball magnetic susceptibility, obtained from (5) and Fig.1 data; linearization of the dependence in semi-logarithmic coordinates signifies its exponential nature (in the \( H \) range studied).

Area to the accuracy of invariable \( a \), this dependence may be easily ascertained to be shown as an inverse degree (with index \( n \), close to one [11]), i.e. \( \chi_n = (a/H)^n \). Then the expression (5) takes the following form:

\[ \chi = \frac{3(a/H)^n}{3 + (a/H)^n} \quad (6) \]
Yet, it is cumbersome to use this expression featuring multiple repetitions of parameter $H$. To obtain a simpler alternative expression, we may use dependence (5) and data of Fig.1 to initially find a graphical view of field dependence of the ball magnetic susceptibility (Fig.2). Then, noting that in the same post-extreme domain it is close to the exponential (Fig.2), expressions for $\chi$ including the allowances for characteristics (3) may be given as follows:

$$\chi = 2.9 \exp(-k_H H) = 2.9 \exp[-k_H H_0 \exp(-k_x x)],$$  
(7)

$$\chi = 2.9 \exp(-k_B B) = 2.9 \exp[-k_B B_0 \exp(-k_x x)],$$  
(8)

with dimensional parameters values of $k_H = 0.16 \cdot 10^{-5} \text{m/A}$ and $k_B = 1.3 \text{T}^{-1}$.

Then the expression (2), as a special case of the fundamental expression (1) in the fully expanded form (for the option considered), will be defined as:

$$F/V = 2.9 \mu_0 H_0^2 k_x \exp[-k_H H_0 \exp(-k_x x)]$$

$$- 2k_x x]$$  
(9)

$$F/V = 2.9 B_0^2 k_x \exp[-k_B B_0 \exp(-k_x x)] - 2k_x x] / \mu_0$$  
(10)

Objective testing of these expressions and a simultaneous proof of the input fundamental expressions (1, 2) to ascertain the validity of their application for ‘non-point’ magnets may naturally be carried out by employing the corresponding data of direct measurements. In particular, papers [1, 2] contain experimental results on immediate determination of magnetic force of attracting quite sizeable bodies, viz. balls of $d=6-10\text{mm}$ diameter in the aforementioned module. The fundamental result of these experiments has been the calculating formula of magnetic impact [1, 2] denoted as:

$$F/V = 1.9 A_b / x^2.$$  
(11)

We should emphasize that it is similar to the fundamental laws of gravitational and electric interaction; this formula holds true with the dimensional parameter value $A_b = 0.61 \text{N/mm}$ for special cases of the experiment [1, 2].

Fig.3 a,b feature the analytic data of the ball magnetic attraction specific force obtained by

![Graph](image)

Figure 3. Trial ball magnetic attraction specific force dependence on the distance to the polar surface of the magnet element in common (a) and logarithmic (b) coordinates; line 1 is calculation by (11), line 2 is calculation by (9, 10), dashed line 2/ defines the possibility of approximation of the tail section of dependence 2 by exponential function close to inverse quadratic one (similar to line 1)

The formula (11) and presented in both common and logarithmic coordinates for further functional validation. Please, keep in mind that the formula is applicable to the ferro-bodies (balls)
of real sizes which certainly perturb the field they are exposed to. Here we also demonstrate similar data by the formulae (9, 10), resulting from the fundamental expressions (1, 2) applicable, as mentioned above, to the point ferro-bodies which formally do not induce any perturbations in the field they are exposed to.

**Conclusions**

Based on the data obtained (Fig.3 a,b), we may conclude the following concerning the sheer possibility of the practical application of the expressions (1, 2).

Firstly, the data received by the formula (11) and depicted by line 1 in Fig.3 a,b considerably exceed the data obtained by the formulae (9, 10) and shown by line 2 *ibid.* Clearly it happens due to the really exhibited magnetic field perturbation factor, consequent strengthening of non-homogeneity of the field, and hence, the subsequent increase of the magnetic attractive force.

Secondly, in the distance range from the polar surface $x>6\text{mm}$ (i.e. for the distances no less than the magnetic body’s own size) the types of dependences $F$ on $x$, obtained both by the formulae (9, 10) and the formula (11), are close, namely $F \sim 1/x^2$ (Fig.3b, lines 1 and 2'). It means that for such $x$ area (for ‘non-point’ ferro-particles) there may still be applicable traditionally employed formulae.

![Figure 4. Case study of the partial mutual correspondence of dependences 1 and 2 shown in Fig.3a under condition that data of line 2 (here 2//) are increased with the allowance for the adjusting coefficient.](image)

(1) and (2), provided there is an adjusting coefficient being introduced (here the coefficient is 3.6). Then, for the aforementioned area (we iterate, the area is no less than the magnetic body’s proper size), the compared data and corresponding dependences practically coincide indeed (Fig.4, lines 1 and 2').

The discrepancies revealed in the defined data afford ground for a fruitful debate on the dogmatic usage (which has virtually become almost universally common) of the classical expression, especially by solving tasks of magnetic separation in biology and medicine [12, 13]. Thus, this expression might be applied to non-point magnetic bodies only in the range of distances from the polar surface no lesser than the own size of the magnetic body with imperative accounting for the essential adjusting coefficient.
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