Spin observables of the reaction \(pd \rightarrow ^3He\eta\) and quasi-bound \(^3He - \eta\) pole

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Abstract

A formalism for spin observables of the reaction \(pd \rightarrow ^3He\eta\) is derived in a model independent way. The general case with a full set of six independent spin amplitudes is studied. Furthermore, approximations by five and four spin amplitudes are investigated in the near threshold region. This region is of great interest to search for a quasi-bound \(^3He - \eta\) state, in particular, by measurement of energy dependence of relative phases of s- and p-wave amplitudes. Complete polarization experiments, allowing determination of spin amplitudes, are analyzed. It is shown that measurement of only analyzing powers and spin correlation coefficients hardly allows one to separate the s- and p-wave amplitudes, but additional measurement of polarization transfer coefficients simplifies this problem. Specific observables, given by products of one s- and one p-wave amplitudes, are found. Measurement of these observables will provide new independent information on the \(^3He - \eta\) pole position.

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1 Introduction

The reaction \(pd \rightarrow ^3He\eta\) has been arousing interest since the first measurement performed at SATURNE [1]. Strong enhancement of the cross section of this reaction was found near the threshold and confirmed later on in Ref. [2]. This enhancement was interpreted as a strong final state interaction effect arising due to presence of a quasi-bound \(^3He - \eta\) state [3]. This idea was supported by the calculation of this reaction within a two-step model [4, 5] and parameters of the quasi-bound state were estimated [4] from fit to the data. An independent experimental indication of existence of the quasi-bound \(^3He - \eta\) state was found in measurement of the photo-production of the \(\eta\)-meson on \(^3He\) in the reaction \(\gamma ^3He \rightarrow ^3He\eta\) [6], although this interpretation was questioned in Ref. [7]. Microscopic four-body calculations support quasi-bound [8] or anti-bound (virtual) [9] states in the \(^3He - \eta\) system within the existing uncertainties for the elementary \(\eta N\) interaction. A special type of experiments is required to decide whether the \(^3He - \eta\) system, if it really exists, is in a quasi-bound (for strong enough \(\eta N\) interaction) or anti-bound (for rather weak \(\eta N\) interaction) state [10].
Recently the near threshold cross section of the \(pd \rightarrow ^3He\eta\) reaction was measured at COSY \([11, 12]\) with a high precision. According to Ref.\([11]\), the pole corresponding to the quasi-bound (or anti-bound) state is located on the excitation energy plane at the point \(Q_0 = \left((-0.30\pm0.15_{\text{stat}}\pm0.04_{\text{syst}})\pm(0.21\pm0.29_{\text{stat}}\pm0.06_{\text{syst}})\right)\) MeV, that is very close to the threshold. As was noted in \([13]\), the presence of the pole in the s-wave amplitude of this reaction must lead to fast variation not only in its magnitude, but also in the phase. The latter provides a new criterion for identification of the pole. The specific phase behaviour comes from the following analytical form of the s-wave amplitude near the pole:

\[
f(p_\eta) = \frac{\xi}{p_\eta - ip_0},
\]

where \(p_\eta\) is the (real) c.m.s. momentum of the \(\eta\)-meson, \(p_0\) is the (complex) pole point directly related to the energy \(Q_0\), and \(\xi\) is a smooth function of \(p_\eta\). On the other hand, p-waves are expected to exhibit non-pole behaviour. Using the available unpolarized data \([11]\), the authors of Ref.\([13]\) found some non-direct indications of this specific phase behaviour of the s-wave amplitudes near the threshold. It is important to validate this interpretation by direct measurement of energy dependence of the s-wave amplitudes, which requires polarization experiments. Some of them were discussed in Ref.\([13]\) and it was assumed that the tensor analyzing power \(t_{20}\) and the spin correlation parameter \(C_{y,y}\) even in collinear kinematics could give necessary information about the s-wave amplitudes.

In this paper polarization measurements are discussed in detail. We consider a full set of spin observables including those which require measurement of polarizations of final particles. Moreover, in addition to the approximation by four spin amplitudes, used in Ref.\([13]\), we also investigate the case with five amplitudes and the general case, which includes a full set of six independent spin amplitudes of this reaction. We show that in collinear kinematics the complete polarization experiment is rather simple, but one cannot separate s- and p-wave amplitudes in this regime. Therefore, to determine the desirable phase dependence of the s-wave amplitudes in respect of p-wave ones one has to perform experiments beyond collinear kinematics. For the cases of five and four independent amplitudes we consider complete polarization experiments, which allow one to determine all these amplitudes. We find that one has to measure spin-transfer coefficients in addition to analyzing powers and spin-correlation parameters to complete the experiment. In the general case with six spin amplitudes the complete polarization experiment is too complicated to be really performed. Instead of performing it, we suggest to measure few optimal spin observables which could provide new independent information on the position of the quasi-bound pole.

In the next Section the spin structure of the transition matrix element is discussed. In section 3 all spin observables are derived in the general form, assuming conservation of P-parity and angular momentum. In Section 4 several versions of complete polarization experiments are discussed. Section 5 is devoted to the minimal set of spin observables, measurements of which should justify the position of the pole. A summary is given in the Conclusion. Some formulae for spin observables are given in the Appendix.
2 The transition matrix element

In terms of z-projections of spins of participating particles, there exist twelve different spin transitions in the reaction \( pd \rightarrow ^3He\eta \). Due to P-parity and angular momentum conservation only six of them are independent. This can be seen, for example, in the following way. Let us denote the orbital momentum in the initial (final) state as \( L_i (L_f) \) and the total angular momentum as \( J_i \) (\( J_f \)). Then the P-parity of the system is \( \pi = (-1)^{L_i} = (-1)^{L_f+1} \). The spin of the initial channel \( S \), which is defined as a vector sum of the deuteron and proton spins, takes two values \( S = \frac{1}{2} \) and \( S = \frac{3}{2} \). The sole relation \(|L_i - L_f| = 1\) is allowed for these values of \( S \) due to angular momentum and parity conservation. For the s-wave final state \( (L_f = 0) \) only two transitions are allowed with \( L_i = 1, J_i = J_f = \frac{1}{2} \) and \( S = \frac{1}{2} \) and \( S = \frac{3}{2} \). For the final p-wave state \( (L_f = 1) \) there are five transitions, two for \( L_i = 0 \) and three for \( L_f = 2 \). For all higher partial waves \( (L_f > 1) \) there are six independent transitions, three for \( L_i = L_f + 1 \) and three for \( L_i = L_f - 1 \). In a similar way one can find that for the given total angular momentum \( J_i = J_f \) there exist other six independent transitions.

In the non-orthogonal basis used in Ref.[13] and in notations of that paper, the transition operator of the reaction \( pd \rightarrow ^3He\eta \) can be written as

\[
\hat{F} = \epsilon \cdot T = A\epsilon \cdot \hat{p}_p + iB[\epsilon \times \sigma] \cdot \hat{p}_p + C\epsilon \cdot \hat{p}_\eta + iD[\epsilon \times \sigma] \cdot \hat{p}_\eta + iE(\epsilon \cdot n)(\sigma \cdot \hat{p}_p) + iF(\epsilon \cdot n)(\sigma \cdot \hat{p}_\eta),
\]

(2)

where \( \sigma \) is the Pauli matrix, \( \epsilon \) is the polarization vector of the deuteron, \( \hat{p}_p \) is the unit vector along the proton beam direction, \( \hat{p}_p \) and \( \hat{p}_\eta \) are the cms momenta of the proton and the \( \eta \)-meson, respectively, and \( n = [\hat{p}_\eta \times \hat{p}_p] \). We choose the coordinate system with the axes \( OZ \uparrow \uparrow \hat{p}_p, OY \uparrow \uparrow n, OX \uparrow \uparrow [n \times \hat{p}_p] \). Six independent terms in Eq.(2) correspond to six transitions discussed above and completely describe this reaction. As compared to Ref.[13], we also consider two additional terms, \( E \) and \( F \), both of the non-s-wave type. The s-wave amplitudes are contained in the terms \( A \) and \( B \) only. These are the only terms which do not disappear in the limit \( p_\eta \rightarrow 0 \) in Eq. (2). In the near threshold region, the terms \( A \) and \( B \) could contain mainly an admixture of p-waves, like \( A'(\hat{p}_p \cdot \hat{p}_\eta)(\epsilon \cdot \hat{p}_p) \) and \( iB'(\hat{p}_p \cdot \hat{p}_\eta)[\epsilon \times \sigma] \cdot \hat{p}_p \), respectively. These two p-wave terms and another p-wave amplitude \( E \) correspond to the initial d-wave state, whereas the p-wave terms \( C \) and \( D \) are related to the initial s-wave state. This is in agreement with the discussion given in the beginning of this section. The last term \( F \) in Eq. (2) corresponds to the d-wave (and higher partial waves) in the final state.

The unpolarized cms cross section takes the form (see also below Eqs. (8) and (9))

\[
d\sigma_0 = \frac{p_\eta}{3p_p} I,
\]

(3)

here the factor \( I \) is given as

\[
I = |A|^2 + 2|B|^2 + (|C|^2 + 2|D|^2)p_\eta^2 + 2Re(AC^* + 2BD^*)p_\eta \cos \theta_\eta + (|F|^2p_\eta^2 + |E|^2)p_\eta^2 \sin^2 \theta_\eta + 2Re(DE^* - BF^*)p_\eta \cos \theta_\eta)p_\eta^2 \sin^2 \theta_\eta
\]

(4)
with $\theta_\eta$ being the angle between the vectors $\mathbf{p}_p$ and $\mathbf{p}_\eta$. If one puts $F = 0$ and $E = 0$, Eq. (4) coincides with Eq.(4) from Ref.[13] and gives linear dependence in $\cos \theta_\eta$ for the differential cross section, observed in the existing data at $Q < 11$ MeV [11, 12]. In view of this observation, one may conclude the following. Firstly, the contribution of two last terms in Eq.(4), which are proportional to $\sin^2 \theta_\eta$, can be considered as negligible in the unpolarized cross section at $Q$ less than $\approx 10$ MeV. The latter gives grounds for using only four independent amplitudes and neglecting $E$ and $F$. This approximation, adopted in Ref.[13], will also be studied in this paper from the point of view of performing a complete polarization experiment. Secondly, the amplitudes $A$ and $B$ do not contain a sizeable contribution of p-waves at $Q < 11$ MeV [13]. Thus, near the threshold the amplitudes $A$ and $B$ are of the s-wave type and, therefore, are expected to contain the quasi-bound pole. On the other hand, all other amplitudes ($C$, $D$, $E$, and $F$) are expected to have smooth $p_\eta$ dependence near the pole. These assumptions will be essential in section 5, where the position of the quasi-bound pole on the momentum plane is considered. One should note that the d-wave term $F$ is most likely negligible as compared to the s- and p-wave terms in the near threshold region due to the centrifugal barrier. However, the term $E$ could be non-negligible in spin observables. Therefore, below we will also consider the case with five spin amplitudes ($A$, $B$, $C$, $D$, $E$) and the general case with the full set of six spin amplitudes.

3 Observables

Our strategy is the following. First, we derive a full set of spin observables for the general case of six independent spin amplitudes. After that the formalism for a particular case with five or four amplitudes can be obtained straightforwardly from the general formalism. Furthermore, having the full set of spin observables, one can find a solution for complete polarization experiments.

3.1 General case

Using Eq. (2), one can express any spin observables in terms of invariant spin amplitudes. With this aim, we present the Cartesian components of the vector-operator $\mathbf{T}$, defined by Eq. (2), as follows

$$T_x = M_1 + M_2 \sigma_y, \quad T_y = M_3 \sigma_x + M_4 \sigma_z, \quad T_z = M_5 + M_6 \sigma_y,$$

(5)

where

$$M_1 = C q_x, \quad M_2 = i(B + D q_z), \quad M_3 = -i(B + D q_z - F q_{z}^{2}), \quad M_4 = i(D + E + F q_{z}) q_x, \quad M_5 = A + C q_z, \quad M_6 = -i D q_z.$$  

(6)

\footnote{I am thankful to C. Wilkin for explanation of this feature.}
Here \( q_x \) and \( q_z \) are determined as
\[
q_x = -p_\eta \sin \theta_\eta, \quad q_z = p_\eta \cos \theta_\eta.
\] (7)

The unpolarized cross section can be written as
\[
d\sigma_0 = \frac{1}{6} \frac{p_\eta}{p_p} \sum_\alpha Tr T_\alpha T_\alpha^+ = \frac{1}{3} \frac{p_\eta}{p_p} I,
\] (8)
where \( \alpha = x, y, z \) and \( I \) has the following form:
\[
I = \frac{1}{2} \sum_\alpha Tr T_\alpha T_\alpha^+ = \sum_{i=1}^{6} |M_i|^2.
\] (9)

We use below definitions and notations of Ref. [14] for spin observables. The tensor analyzing powers of the deuteron take the form
\[
A_{yy} = 1 - \frac{3}{I} \left\{ |M_3|^2 + |M_4|^2 \right\},
\] (10)
\[
A_{xx} = 1 - \frac{3}{I} \left\{ |M_1|^2 + |M_2|^2 \right\},
\] (11)
\[
A_{zz} = 1 - \frac{3}{I} \left\{ |M_5|^2 + |M_6|^2 \right\},
\] (12)
\[
-\frac{I}{3} A_{xz} = Re(M_1 M_5^* + M_2 M_6^*).
\] (13)

The vector analyzing powers are
\[
\frac{I}{2} A^d_y = Im \{ M_5 M_1^* + M_6 M_2^* \},
\] (14)
\[
\frac{I}{2} A^d_y = Re \{ M_1 M_2^* + M_5 M_6^* \} - Im M_3 M_4^*.
\] (15)

The spin transfer coefficients \( K^{y'}_i(p) \), describing polarization transfer from the initial proton to the \(^3He\) nucleus, can be written as
\[
IK^{y'}_y(p) = |M_1|^2 + |M_2|^2 - |M_3|^2 - |M_4|^2 - |M_5|^2 + |M_6|^2, \quad (16)
\]
\[
IK^{x'}_x(p) = |M_1|^2 - |M_2|^2 + |M_3|^2 - |M_4|^2 + |M_5|^2 - |M_6|^2, \quad (17)
\]
\[
IK^{z'}_z(p) = |M_1|^2 - |M_2|^2 - |M_3|^2 + |M_4|^2 + |M_5|^2 - |M_6|^2, \quad (18)
\]
\[
-\frac{I}{2} K^{z'}_x(p) = Im \{ M_1 M_2^* + M_5 M_6^* \} - Re M_3 M_4^*, \quad (19)
\]
\[
-\frac{I}{2} K^{z'}_z(p) = Im \{ M_2 M_1^* + M_6 M_5^* \} - Re M_3 M_4^*. \quad (20)
\]

The spin transfer coefficients \( K^{y'}_j(d) \), describing polarization transfer from the initial deuteron to the \(^3He\) nucleus, are
\[
\frac{I}{2} K^{y'}_y(d) = Im(M_6 M_1^* + M_5 M_2^*),
\]

\[
\begin{align*}
\frac{I}{2} K^z_x(d) &= \text{Im}M_3 M_5^* - \text{Re}M_4 M_6^*, \\
\frac{I}{2} K^z_x(d) &= \text{Im}M_1 M_4^* - \text{Re}M_2 M_3^*, \\
\frac{I}{2} K^z_x(d) &= \text{Im}M_1 M_3^* + \text{Re}M_2 M_4^*, \\
\frac{I}{2} K^z_x(d) &= \text{Im}M_4 M_5^* + \text{Re}M_3 M_6^*, \\
\end{align*}
\]

The proton-deuteron spin correlation parameters are

\[
\begin{align*}
-\frac{I}{2} C_{y,y} &= \text{Im}M_2 M_5^* + \text{Im}M_1 M_6^*, \\
\frac{I}{2} C_{z,z} &= \text{Im}M_1 M_4^* + \text{Re}M_2 M_3^*, \\
\frac{I}{2} C_{x,x} &= \text{Im}M_3 M_5^* + \text{Re}M_4 M_6^*, \\
\frac{I}{2} C_{x,z} &= \text{Im}M_1 M_3^* - \text{Re}M_2 M_4^*, \\
\frac{I}{2} C_{z,x} &= \text{Im}M_4 M_5^* - \text{Re}M_3 M_6^*, \\
\end{align*}
\]

for the vector polarized deuteron and

\[
\begin{align*}
C_{y,yy} &= A_y^p + \frac{6}{I} \text{Im}M_3 M_4^*, \\
\frac{I}{2} C_{y,yy} &= \text{Re}(M_1 M_2^* + M_5 M_6^*) + 2\text{Im}M_3 M_4^*, \\
C_{y,xx} &= A_y^p - \frac{6}{I} \text{Re}M_2 M_1^*, \\
\frac{I}{2} C_{y,xx} &= \text{Re}(M_5 M_6^* - 2M_2 M_1^*) - \text{Im}M_3 M_4^*, \\
C_{y,zz} &= A_y^p - \frac{6}{I} \text{Re}M_5 M_6^*, \\
\frac{I}{2} C_{y,zz} &= \text{Re}(M_1 M_2^* - 2M_5 M_6^*) - \text{Im}M_3 M_4^*, \\
\frac{I}{3} C_{y,xz} &= \text{Re}(M_2 M_5^* + M_1 M_6^*), \\
\frac{I}{3} C_{x,yz} &= \text{Re}M_3 M_5^* - \text{Im}M_4 M_6^*, \\
\frac{I}{3} C_{z,yz} &= \text{Re}M_4 M_5^* - \text{Im}M_6 M_3^*, \\
\frac{I}{3} C_{x,xy} &= \text{Re}M_1 M_3^* - \text{Im}M_4 M_2^*, \\
\frac{I}{3} C_{z,xy} &= \text{Re}M_1 M_4^* - \text{Im}M_2 M_3^*, \\
C_{x,xx} &= C_{z,xx} = C_{x,zz} = C_{y,xy} = C_{y,yz} = C_{x,xz} = C_{x,zz} = C_{z,xx} = 0 \\
\end{align*}
\]
for the tensor polarized deuteron. Polarization transfer from the tensor polarized deuteron to the $^3He$ nucleus is described by the following observables:

\[ K_{yy}' = P_y^h + \frac{6}{I} Im M_1 M_3^*, \]

\[ \frac{I}{2} K_{yy}' = Re(M_1 M_2^* + M_5 M_6^*) - 2 Im M_4^* M_3, \]

\[ K_{xx}' = P_y^h - \frac{6}{I} Re M_2 M_1^*, \]

\[ \frac{I}{2} K_{xx}' = Re(M_5 M_6^* - 2 M_1 M_2^*) + Im M_3 M_4^*, \]

\[ K_{zz}' = P_y^h - \frac{6}{I} Re M_6 M_5^*, \]

\[ \frac{I}{2} K_{zz}' = Re(M_1 M_2^* - 2 M_5 M_6^*) + Im M_4 M_3^*, \]

\[ -\frac{I}{3} K_{yz}' = Re M_3 M_5^* + Im M_4 M_6^*, \]

\[ -\frac{I}{3} K_{xy}' = Re M_1 M_4^* + Im M_2 M_3^*, \]

\[ -\frac{I}{3} K_{zz}' = Re(M_2 M_5^* + M_1 M_6^*), \]

\[ -\frac{I}{3} K_{xz}' = Re M_4 M_5^* + Im M_6 M_3^*, \]

\[ -\frac{I}{3} K_{xy}' = Re M_1 M_3^* + Im M_5 M_4^*, \]

\[ K_{xx}' = K_{xx}' = K_{zz}' = K_{zz}' = K_{xy}' = K_{yz}' = K_{zz}' = K_{xx}' = 0, \quad (24) \]

where $P_y^r$ is the polarization of the final $^3He$ nucleus for the unpolarized beam and target, which can be written as

\[ \frac{I}{2} P_y^r = Re(M_1 M_2^* + M_5 M_6^*) + Im M_3 M_4^*, \quad P_x^r = P_z^r = 0. \quad (25) \]

One can also find the following relations:

\[ 3K_y^y(p) = 2A_{yy} + 1, \quad (26) \]

\[ K_y^y(d) = C_{y,y}. \quad (27) \]

### 3.2 Five independent amplitudes

Assuming $F = 0$ in Eq. (2), one has $M_2 = -M_3$. In this case the independent amplitudes in Eqs. (8)-(25) are

\[ M_1 = C q_x, \quad M_2 = i(B + D q_z), \quad M_3 = -i(B + D q_z), \]

\[ M_4 = i(D + E) q_x, \quad M_5 = A + C q_z, \quad M_6 = -i D q_x. \quad (28) \]
3.3 Four independent amplitudes

If one puts $E = F = 0$ in Eq. (2), then one has $M_2 = -M_3$ and $M_4 = -M_6$. In this case the spin structure of Eq. (2) coincides with that used in Ref. [13]. Thus, the following replacements in Eqs. (8)-(25)

\[ M_1 = C q_x, \quad M_2 = i(B + D q_z), \quad M_3 = -M_2, \quad M_4 = iD q_x, \quad M_5 = A + C q_z, \quad M_6 = -M_4 \]

reduce the general case to the approximation by four spin amplitudes. The spin observables in terms of the amplitudes $A$, $B$, $C$ and $D$ for this case are given in the Appendix.

4 Complete polarization experiment

4.1 Collinear kinematics

In collinear kinematics ($q_x = 0$) one has $M_1 = M_4 = M_6 = 0$ and $M_3 = -M_2$, as follows from Eq. (6). Therefore, only two spin amplitudes, $M_2 = M_2^{\text{coll}}$ and $M_5 = M_5^{\text{coll}}$, completely describe the reaction. These amplitudes can be determined from the measurement of the following four observables:

\[
A_{zz}^{\text{coll}} = 1 - \frac{3}{I^{\text{coll}}} |M_5^{\text{coll}}|^2, \tag{31}
\]

\[
-\frac{I^{\text{coll}}}{2} C_{y,y}^{\text{coll}} = \text{Im} M_2^{\text{coll}} M_5^{\text{coll}*}, \tag{32}
\]

\[
-\frac{I^{\text{coll}}}{2} C_{y,xz}^{\text{coll}} = \text{Re} M_2^{\text{coll}} M_5^{\text{coll}*}. \tag{33}
\]

From Eqs. (30) and (31) one can find moduli of these amplitudes, whereas the relative phase $\phi_{25}$ can be found from $\sin \phi_{25}$ and $\cos \phi_{25}$ given by Eqs. (32) and (33), respectively. Here the relative phase $\phi_{ij}$ of the amplitudes $M_i$ and $M_j$ is defined as $\text{Re} M_i M_j^* = |M_i||M_j| \cos \phi_{ij}$, $\text{Im} M_i M_j^* = |M_i||M_j| \sin \phi_{ij}$. In the general case, $M_2^{\text{coll}}$ and $M_5^{\text{coll}}$ are the following combinations of the $s$- and $p$-wave amplitudes: $M_5^{\text{coll}} = A \pm C p_\eta$ and $M_2^{\text{coll}} = B \pm D p_\eta$, where the $\pm$ sign refers to forward and backward production of the $\eta$ meson, respectively. Obviously, in collinear kinematics one cannot separate $s$- and $p$-wave amplitudes. For example, one can measure $t_{20}$ for forward and backward production, as suggested in Ref. [13], which gives $\text{Re} A^* C/(|A|^2 + p_\eta^2 |C|^2)$ and $\text{Re} B^* D/(|B|^2 + p_\eta^2 |D|^2)$ (see Eq.(11) in Ref.[13]). But the desirable values of $\cos \phi_{AC}$ and $\cos \phi_{BD}$ cannot be extracted from these measurements. Measurement of $C_{y,y}$ in collinear kinematics gives $\text{Re} A B^*$ with an admixture of $\text{Re}(A^* D + BC^*) p_\eta + \text{Re} C^* D p_\eta^2$ and, therefore, cannot help to disentangle the moduli and phases of the amplitudes. In order to separate the amplitudes $A$ and $B$ (or $C$ and $D$), measurements beyond collinear kinematics are required.

\[ \text{At the limiting point } p_\eta = 0, \text{ where the } p\text{-wave amplitudes vanish, the separation of two } s\text{-wave amplitudes can be performed exactly.} \]
4.2 Four spin amplitudes

4.2.1 Involving only analyzing powers and spin correlations

When neglecting the terms $E$ and $F$ in Eq. (2), one can find from Eqs. (A.19), (A.20) and (A.21) the only relation which contains moduli of the amplitudes and does not involve their phases:

$$I\frac{2}{3}[C_{y,y} - C_{x,x} + C_{z,z}] = I\frac{2}{3}(1 - A_{yy}) = |D|^2 q_x^2 + |B + Dq_z|^2.$$  (34)

However, this relation does not provide new information as compared to $A_{yy}$. Furthermore, taking into account the relation

$$A_{xx} + A_{yy} + A_{zz} = 0, \quad (35)$$

one can find from Eqs.(8), (9), (A.1)-(A.4), (A.19)-(A.21), and (34) that the unpolarized cross section $d\sigma_0$, tensor analyzing powers $A_{ij}$ and spin correlation coefficients $C_{i,j}$ and $C_{i,jk}$ allow one to derive only three independent linear equations containing the squared moduli of the following four amplitudes:

$$M_a = A + Cq_z, \quad M_b = B + Dq_z, \quad C \quad \text{and} \quad D.$$  

Therefore, one cannot determine the moduli independently of the phases of these amplitudes and has to find all of them simultaneously. For this aim, one can use nine observables $d\sigma_0, A_{xx}, A_{yy}, A_{zz}, C_{y,y}, C_{x,x}, C_{z,z},$ and $C_{z,z}$, which give nine (nonlinear) equations (A.1), (A.2), (A.3), (A.5), (A.19) – (A.23), connecting four moduli and six cosines of phases $\cos \phi_{CD}$, $\cos \phi_{ab}$, $\cos \phi_{Ca}$, $\cos \phi_{Cb}$, $\cos \phi_{Da}$, $\cos \phi_{Db}$. Due to Eq. (34), only eight equations are independent. Furthermore, since only three relative phases are independent, one should add to these system the following three linear equations connecting six relative phases:

$$\phi_{Cb} - \phi_{Ca} = \phi_{ab}, \quad \phi_{Db} - \phi_{Da} = \phi_{ab}, \quad \phi_{Cb} - \phi_{Db} = \phi_{CD}.$$  

However, in order to really use these relations one also has to employ equations containing sines of the phases. Therefore, one should take, for example, six observables $A_{y'_y}, A_{p'_y}, C_{y,y}, C_{y,y}, C_{z,z}, C_{z,z}$, and $C_{z,z}$ given by Eqs. (A.6), (A.7), (A.9), (A.11), (A.14) and (A.17), respectively. In total, one has 17 equations, which contain 14 spin observables and 16 unknown variables (four moduli, six cosines and six sines of the phases). Most likely, a solution to this system of equations can be found numerically, but this version of complete polarization experiment is too cumbersome.

4.2.2 Involving spin-transfer coefficients

The problem becomes much simpler if one can measure spin transfer coefficients. A possible solution is the following. The moduli of the amplitudes $M_a = A + Cq_z, \quad M_b = B + Dq_z, \quad D$ and $C$ can be found from the measurement of $d\sigma_0, A_{yy}, A_{xx}$ and $K'_z(p)$ as

$$|C|^2 q_x^2 = I\frac{2}{3}[2A_{yy} - A_{xx} - 1] + I\frac{2}{3}(1 - K'_z(p)), \quad (36)$$

$$|B + Dq_z|^2 = \frac{2I}{3}(1 - A_{yy}) - I\frac{2}{3}(1 - K'_z(p)), \quad (37)$$
In order to completely determine four complex amplitudes three relative phases have to be measured (note that the common phase is non-measurable and one can put it equal to zero). A possible solution for the relative phases of the amplitudes $C$, $M_b$, $D$, and $M_a$, whose moduli are given in Eqs.(36), (37), (38) and (39) is to measure $K^{x'}_x(p)$, $K^{x'}_z(p)$ and $K^{z'}_z(d)$. It gives $\cos \phi_{D_b}$, $\cos \phi_{C_b}$ and $\cos \phi_{D_a}$, as it follows from the following relations:

\[ I[K^{x'}_x(p) + K^{x'}_z(p)] = -2ReD^*(B + D q_z)q_x, \]
\[ I[K^{x'}_z(p) - \frac{1}{2}K^{z'}_z(d)] = q_x Re(B + D q_z)C^*, \]
\[ I[K^{z'}_x(p) - K^{z'}_z(p)] = -2q_x \{ Re(B + D q_z)C^* - (A + C q_z)D^* \}. \]  

In order to find $\sin \phi_{D_a}$, $\sin \phi_{D_b}$ and $\sin \phi_{C_b}$, one has to measure $C_{y,yy}$, $C_{y,xx}$ and $A_y^p$, as follows from Eqs. (A.8), (A.10), (A.12) and the following relation: $C_{y,yy} + C_{y,xx} + C_{y,zz} = 0$. Thus, in this version one has to perform ten accurate measurements.

The first equation in Eq.(34) can be considered as a necessary criterion for applicability of the approximation with four spin amplitudes in the matrix element.

### 4.3 Five spin amplitudes

Neglecting the d-wave amplitude $F$ in Eq.(2), one has the relation $M_2 = -M_3$. For this case moduli of five amplitudes can be found by measurement of $d\sigma_0$, $A_{xx}$, $A_{yy}$, $K^{x'}_x(p)$ and $K^{z'}_z(p)$:

\[ |M_1|^2 = \frac{I}{4} \left\{ K^{x'}_z(p) - K^{x'}_x(p) \right\} + \frac{I}{6} (1 + A_{yy} - 2A_{xx}), \]
\[ |M_2|^2 = -\frac{I}{4} \left\{ K^{x'}_z(p) - K^{x'}_x(p) \right\} + \frac{I}{6} (1 - A_{yy}), \]
\[ |M_4|^2 = \frac{I}{4} \left\{ K^{x'}_z(p) - K^{x'}_x(p) \right\} + \frac{I}{6} (1 - A_{yy}), \]
\[ |M_5|^2 = \frac{I}{2} \left\{ \frac{1}{3} + \frac{2}{3} A_{xx} + K^{x'}_z(p) \right\}, \]
\[ |M_6|^2 = \frac{I}{2} \left\{ \frac{1}{3} + \frac{2}{3} A_{yy} - K^{x'}_x(p) \right\}. \]

Among the available five phases of these amplitudes only four are independent. In order to determine two independent phases, one can measure, for example, the following four observables: $K^{x'}_x(d)$, $C_{z,x}$, $C_{z,yy}$ and $K^{y'}_y$. Indeed, one can see from Eqs. (21), (22), (23) and (24) that the above observables completely determine the phases $\phi_{45}$ and $\phi_{63}$. The other two phases $\phi_{13}$ and $\phi_{42}$ can be determined by measurement of $C_{x,z}$, $K^{x'}_z(d)$, $C_{x,xy}$ and $K^{y'}_x$. Thus, one needs 13 accurate measurements in this variant.
4.3.1 Reduction to four spin amplitudes

The solution obtained for the case of five spin amplitudes can be reduced to four amplitudes. This gives a new solution in addition to that found in section 4.2.2. Indeed, when neglecting the p-wave amplitude $E$ and d-wave amplitude $F$, one has $M_2 = -M_3$ and $M_4 = -M_6$. In this case one finds from Eqs. (43) and (45) the following relation:

$$K_x'(p) + K_z'(p) = 2A_{yy}.$$  \hspace{1cm} (46)

This relation can be used as a necessary condition for validity of the approximation by four spin amplitudes. Using Eq. (46) one can find from Eqs. (41)-(45) that four observables $d\sigma_0, A_{xx}, A_{yy}$ and $K_x'(p)$ determine four moduli $|M_1|, |M_2|, |M_4|$ and $|M_5|$. Taking into account Eqs. (29), one can see that these formulae coincide with that given by Eqs. (36)-(39). Three phases $\phi_{14}, \phi_{34}$ and $\phi_{56}$ can be determined by the observables $C_{z,z}, C_{y,yy}, C_{z,xy}, A_{y}', K_x'(p)$, and $K_z'(p)$ as it follows from Eqs. (22), (23), (15), (19) and (20). Finally, using the relation $\phi_{56} = \phi_{45} + \pi$, one can see that four amplitudes $(M_1, M_2, M_4, M_5)$ are completely determined by ten observables, $d\sigma_0, A_{xx}, A_{yy}, K_x'(p), C_{z,z}, C_{z,xy}, C_{y,yy}, A_{y}', K_x'(p)$ and $K_z'(p)$. Three of these observables (e.g., $C_{z,xy}, K_x'(p)$ and $K_z'(p)$) can be measured roughly.

4.4 Full set of spin amplitudes

In the general case, when all six spin amplitudes $M_i (i = 1, \ldots, 6)$ are included in the consideration, there is no simple way to determine moduli of these amplitudes. Indeed, there are seven linear equations for six squared moduli of these amplitudes given by Eqs. (9), (10), (11), (12), (16), (17), (18), which do not involve any phases. However, only five of them are linearly independent due to Eqs. (26) and (35). Therefore, in the general case in contrast to the case with four and five spin amplitudes, considered in sections 4.2.2 and 4.3, respectively, one cannot determine the moduli of the amplitudes $|M_i| (i = 1, \ldots, 6)$ independently of their phases. It means that the complete polarization experiment is too complicated in this case. Theoretically it suggests simultaneous determination of the moduli and phases of all amplitudes. Therefore, one has to deal with a large number of (nonlinear) equations, as in subsection 4.2.1. Furthermore, here one has six amplitudes instead of four and, therefore, the problem becomes even more complicated. Thus, we will no longer consider the complete polarization experiment for this general case.

5 Specific polarization observables

The complete polarization experiment provides full information about the reaction, including possible presence and position of the quasi-bound pole. However, in order to find solely the specific phase behaviour of the s-wave amplitudes caused by the $^3He - \eta$ pole, one should select only those observables which are most informative in this respect. In this connection one should note that at $q_z = 0$ the amplitudes $M_2, M_3$ and $M_5$ are of the s-wave type and, therefore, are expected to contain the $^3He - \eta$ pole. On the other
hand, the p- and d-wave amplitudes $M_1$, $M_4$ and $M_6$ are of the non-pole type with smooth $p_\eta$ dependence. We show below that one can get a product of s-wave and p-wave amplitudes without performing a complete polarization experiment. Indeed, as follows from Eqs. (21), (22), (23) and (24), the observables $K^{r'}_x(d)$, $C_{x,z}$, $C_{x,yz}$ and $K^{r'}_{y}$ determine the following complex numbers $M_1^* M_3$ and $M_4^* M_2$: $Re M_2 M_4^* = \frac{i}{4} [K^{r'}_x(d) - C_{x,z}]$, $Im M_2 M_4^* = \frac{i}{4} [K^{r'}_y - C_{x,yz}]$, which can be written as

$$Z_1 = M_2 M_4^* = q_x \left\{ B(D^* + E^*) + [D(E^* + D^*) + (B + D)F^*]q_x \right\},$$

$$Z_2 = M_3 M_1^* = -iq_x \left\{ BC^* + DC^* q_z - FC^* q_z^2 \right\}.$$  

(47)

At $q_z = 0$, both of them are products of one s-wave and one p-wave amplitude and, therefore, are expected to have the pole form of Eq. (1).

Likewise, one can find from Eqs.(21), (22), (23) and (24) that four observables $K^{r'}_x(d)$, $C_{x,z}$, $C_{x,yz}$ and $K^{r'}_y$ completely determine the following complex numbers:

$$Z_3 = M_5 M_4^* = -iq_x \left\{ A(D^* + E^*) + (AF^* + CD^* + CE^*) q_z + CF^* q_z^2 \right\},$$

$$Z_4 = M_5 M_6^* = \left\{ BD^* + |D|^2 q_z - FD^* q_z^2 \right\} q_x.$$  

(49)

which are also expected to have the same pole form at $q_z \to 0$. Thus, even without knowing the moduli of the amplitudes $|M_i|$ ($i = 1, \ldots, 6$), one can single out the pole dependence (1) of the measured complex numbers $Z_k$ ($k = 1, \ldots, 4$).

Furthermore, using Eq. (1), where $f(p_\eta) = Z_k(p_\eta)$, and taking real and imaginary parts of $(p_\eta - ip_0)Z_k$, one can find near the pole

$$(p_\eta + Imp_0)ReZ_k + Rep_0 ImZ_k = Re\xi \approx const,$$  

(51)

$$Rep_0 ReZ_k - (p_\eta + Imp_0)ImZ_k = -Im\xi \approx const;$$  

(52)

here $k = 1, \ldots, 4$ and const means independence of $p_\eta$. The magnitudes of $Imp_0$ and $Rep_0$ are restricted by the quasi-bound state energy $Q_0 = -p_0^2/2\mu$, determined from the existing unpolarized data [11]:

$$2\mu Re Q_0 = (Imp_0)^2 - (Rep_0)^2, \mu |Im Q_0| = |Rep_0||Imp_0|,$$  

(53)

where $\mu$ is the reduced mass of the $^3He - \eta$ system. Eqs. (51), (52) and Eq. (53) can be used for more precise determination of the magnitudes of $Imp_0$ and $Rep_0$ and can give their signs. Knowledge of the sign of the real part of $p_0$ allows one to conclude whether the $^3He - \eta$ state is quasi-bound ($Rep_0 > 0$) or anti-bound ($Rep_0 < 0$).

Measurement of spin transfer coefficients is a complicated experimental problem. In this connection one should note that measurement of only two observables $A_y^d$ and $A_{xz}$ completely determines the following complex number

$$Z_5 = M_1 M_5^* + M_2 M_6^* = \left\{ CA^* - BD^* + (|C|^2 - |D|^2) q_z \right\} q_x.$$  

(54)

---

4The contribution of the d-wave term $Fq_2^2$ to Eq. (48) is negligible in the near threshold region.

5At the pole $p_\eta = ip_0$ defined by Eq. (1), the radial part of the s-wave function of the $^3He - \eta$ relative motion has the asymptotic form of $\sim \exp(ip_\eta r)/r = \exp[\{-Rep_0 - iImp_0\}r]/r$. This function vanishes at $r \to \infty$ (bound) for $Rep_0 > 0$ and increases infinitely (anti-bound) for $Rep_0 < 0$.  

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This can be seen from the relations $ReZ_5 = -\frac{I}{3}A_{zz}$ and $ImZ_5 = -\frac{I}{2}A_{y}^*$, following from Eqs.(13) and (14), respectively. This result was found in Ref. [13], but within the approximation by four spin amplitudes. We should stress that Eq. (54) is also valid in the general case of six independent amplitudes. In the pole region, $Z_5$ is a linear combination of the $(p_\eta - ip_0)^{-1}$ and $(p_\eta + ip_0)^{-1}$ terms and, therefore, Eqs. (51) and (52) are not valid for $Z_5$. Nevertheless, this observable can be used as a new crucial test for the existing models of the $pd \rightarrow ^3He\eta$ and $^3He\eta \rightarrow ^3He\eta$ reactions. As follows from the $pd \rightarrow ^3He\eta$ data on $t_{20}$ [1], $|A| \approx |B|$. One can see from Eq. (54), that studying the $q_z$ dependence of $Z_5$ could allow one to check whether $|C| \approx |D|$. At $q_z = 0$, $Z_5$ gives a certain relation between the s-p phases $\phi_{CA}$ and $\phi_{BD}$.

Finally, there are other two complex numbers, each of them can be determined by measurement of two spin-correlation parameters $C_{ij}$ and $C_{ijk}$:

$$Z_6 = M_1 M_3^* - iM_2 M_4^*, \quad ReZ_6 = -\frac{I}{3}C_{x,xy}, \quad ImZ_6 = \frac{I}{2}C_{x,z},$$

$$Z_7 = M_4 M_5^* - iM_3 M_6^*, \quad ReZ_7 = -\frac{I}{3}C_{z,yz}, \quad ImZ_7 = \frac{I}{2}C_{z,x}.$$  \hfill (55)  \hfill (56)

The variables $Z_6$ and $Z_7$ are also combinations of the $(p_\eta - ip_0)^{-1}$ and $(p_\eta + ip_0)^{-1}$ terms. Note that all $Z_i$ ($i = 1, \ldots, 7$) vanish in the collinear regime ($q_z = 0$).

6 Conclusion

A possibility of performing a complete polarization experiment for the reaction $pd \rightarrow ^3He\eta$ is studied. Such measurement could give, in particular, the energy dependence of phases of s-wave transition amplitudes, providing a crucial test for existence of the quasi-bound state in the $^3He - \eta$ system near the threshold. Knowledge of energy dependence of the s-wave amplitudes near the threshold could allow one to determine the signs of the real and imaginary parts of the pole point in the complex momentum plane. In its turn, this allows one to determine whether the $^3He - \eta$ state is quasi-bound or anti-bound. We derived a full set of non-zero spin observables for this reaction. We show that in collinear kinematics the complete polarization experiment suggests only four measurements. However, one cannot separate s- and p-wave amplitudes in collinear regime. We considered two different cases beyond collinear kinematics. (i) In the case of four spin amplitudes, four observables $d\sigma_0$, $A_{yy}$, $A_{xx}$ and $K_x^z(p)$ completely determine moduli of the amplitudes. In order to get all four amplitudes with their relative phases, one needs ten observables (including spin transfer coefficients) and three of them can be measured roughly. On the contrary, knowledge of only analyzing powers and spin-correlation coefficients in the initial state does not allow one to determine moduli of the amplitudes independently of their phases and suggests performing 14 accurate measurements to get all four amplitudes. (ii) In the case of five independent amplitudes, measurement of spin transfer coefficients also simplifies the complete polarization experiment since allows one to determine five moduli of the amplitudes in terms of five observables $d\sigma_0$, $A_{yy}$, $A_{xx}$, $K_x^z(p)$ and $K_z^z(p)$. 

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Furthermore, we show that for the general case of six spin amplitudes the complete polarization experiment is too cumbersome and practically unrealizable. In view of this complexity, we suggest to measure those spin observables which allow one to single out the pole dependence of the s-wave amplitudes. There are two sets of such observables: $C_{z,x}, K_{z}^{x'}(d), C_{z,yz}, K_{y}^{z'}(d), C_{z,y}, K_{xy}^{z'}$. In general case of six independent amplitudes, we show also that measurement of two analyzing powers $A_{xz}$ and $A_{dy}$ near the threshold could provide new valuable information on the $^3He - \eta$ pole.

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7 Appendix

Here we present formulae for spin observables obtained in approximation by two s-wave $(A, B)$ and two p-wave $(C, D)$ amplitudes. The factor $I$ in Eqs. (8) and (9) is

$$I = |A + Cq_z|^2 + |B + Dq_z|^2 + |C|^2 q_z^2 + 2|D|q_z^2.$$  \hfill (A.1)

For analyzing powers one has (here we preserve all six spin amplitudes)

$$A_{xx} = 1 - \frac{3}{I} |C|^2 q_z^2 + |B + Dq_z|^2,$$  \hfill (A.2)

$$A_{yy} = 1 - \frac{3}{I} [|D + E + Fq_z|^2 q_z^2 + |B + Dq_z - Fq_z|^2],$$  \hfill (A.3)

$$A_{zz} = 1 - \frac{3}{I} |D|^2 q_z^2 + |A + Cq_z|^2,$$  \hfill (A.4)

$$-\frac{I}{3} A_{xz} = q_x \{ Re(A + Cq_z)C^* - Re(B^* + D^*q_z)D \},$$  \hfill (A.5)

$$\frac{I}{2} A_{dy} = Im(A + Cq_z)C^* q_x - Im(B^* + D^*q_z)Dq_z,$$  \hfill (A.6)

$$-\frac{I}{2} A_{yp} = \left[ Im(B + Dq_z)C^* - Im(B + Dq_z)D^* + E^* + F^* q_z \right] q_x.$$  \hfill (A.7)

Eqs. (A.2) –(A.6) are equivalent to the spherical tensors found in [13]: $t_{22} = \frac{1}{2\sqrt{3}} (A_{xx} - A_{yy})$, $t_{21} = -\frac{1}{\sqrt{3}} A_{xz}$, $t_{20} = A_{zz}/\sqrt{2}$, and $it_{11} = \frac{\sqrt{3}}{2} A_{dy}$.

The proton-deuteron spin-tensor correlation parameters are the following:

$$C_{y,yy} = A_{yp} - \frac{6}{I} Im(B + Dq_z) D^* q_x,$$  \hfill (A.8)
\[ -\frac{I}{2} C_{y,yy} = q_x Im \{(B + D q_z)C^* + \\
+ 2(B + D q_z - F q_z^2)(D^* + E^* + F q_z) + (A + C q_z)D^*\}, \]  
(A.9)
\[ C_{y,zz} = A_y^p + \frac{6}{I} Im D^*(A + C q_z)q_z, \]  
(A.10)
\[ -\frac{I}{2} C_{y,zz} = q_x Im \{(B + D q_z)C^* - (B + D q_z)D^* - 2(A + C q_z)D^*\}, \]  
(A.11)
\[ C_{y,xx} = A_y^p + \frac{6}{I} Im C^*(B + D q_z)q_z, \]  
(A.12)
\[ -\frac{I}{2} C_{y,xx} = q_x Im \{(A + C q_z)D^* - (B + D q_z)D^* - 2(B + D q_z)C^*\}, \]  
(A.13)
\[ -\frac{I}{3} C_{x,yz} = Im(B + D q_z)(A^* + C^* q_z), \]  
(A.14)
\[ -\frac{I}{3} C_{z,yz} = \{Im(A + C q_z)D^* + Im(B + D q_z)D^*\} q_z, \]  
(A.15)
\[ -\frac{I}{3} C_{x,xy} = \{Im(B + D q_z)C^* + Im(B + D q_z)D^*\} q_x \]  
(A.16)
\[ -\frac{I}{3} C_{z,xy} = Im CD^*q_z^2, \]  
(A.17)
\[ \frac{I}{3} C_{y,xz} = Im CD^*q_z^2 + Im(B + D q_z)(A^* + C^* q_z), \]  
(A.18)

The spin correlation parameters of the polarized proton and the vector polarized deuteron are the following:
\[ -\frac{I}{2} C_{y,y} = Re DC^* q_z^2 + Re(A + C q_z)(B^* + D^* q_z), \]  
(A.19)
\[ -\frac{I}{2} C_{x,x} = |D|^2 q_z^2 + Re(A + C q_z)(B^* + D^* q_z), \]  
(A.20)
\[ -\frac{I}{2} C_{z,z} = Re DC^* q_z^2 + |B + D q_z|^2, \]  
(A.21)
\[ \frac{I}{2} C_{x,z} = q_x Re(C - D)(B^* + D^* q_z), \]  
(A.22)
\[ \frac{I}{2} C_{z,x} = q_x \{Re(A + C q_z)D^* - Re(B + D q_z)D^*\}. \]  
(A.23)

The coefficients describing polarization transfer from the deuteron to the \(^3\)He nucleus can be written as
\[ -\frac{I}{2} K_y^y(d) = Re \{DC^* q_z^2 + (A + C q_z)(B^* + D^* q_z)\}, \]
\[ \frac{I}{2} K_x^x(d) = \{|D|^2 q_z^2 - Re(A + C q_z)(B^* + D^* q_z)\}, \]
\[ -\frac{I}{2} K_z^z(d) = Re DC^* q_z^2 - |B + D q_z|^2; \]
\[ \frac{I}{2} K_z^x(d) = Re \{C^*(B + D q_z) + D^*(B + D q_z)\}, \]
\[
\frac{I}{2} K_x'(d) = Re \{ D^*(A + Cq_z) + D^*(B + Dq_z) \}. \tag{A.24}
\]

The coefficients of polarization transfer from the initial proton to the $^3\text{He}$ nucleus are

\[
I K_y'(p) = |C|^2 q_x^2 + |A + Cq_z|^2,
\]
\[
I K_x'(p) = |C|^2 q_x^2 + |A + Cq_z|^2 - 2|D|^2 q_x^2,
\]
\[
I K_z'(p) = |C|^2 q_x^2 + |A + Cq_z|^2 - 2|B + Dq_z|^2 q_x^2,
\]
\[
\frac{I}{2} K_x'(p) = q_x Re \{(B + Dq_z)C^* - (A + Cq_z)D^* - (B + Dq_z)D^* \},
\]
\[
\frac{I}{2} K_z'(p) = q_x Re \{(A + Cq_z)D^* - (C^* + D^*) (B + Dq_z) \}. \tag{A.25}
\]

Eqs. (A.2) – (A.7), (A.9), (A.19) are valid in the general case of six spin amplitudes.

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