Scrutinizing Various Phenomenological Interactions In
The Context Of Holographic Ricci Dark Energy Models

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Abstract

In this paper, we examine two types of interacting holographic dark energy model using Pantheon supernova data, BAO BOSS DR12, CMB Planck 2015, fgas (gas mass fraction) and SZ/Xray (Sunyaev-Zeldovich effect and X-ray emission) data from galaxy clusters (GC). In particular, we considered the Holographic Ricci dark energy and Extended holographic Ricci dark energy models. During this analysis we considered seven type of phenomenological interaction terms (three linear and four non-linear) \( Q_1 = 3Hb\rho_D, Q_2 = 3Hb\rho_m, Q_3 = 3Hb(\rho_D + \rho_m), Q_4 = 3Hb(\rho_D + \frac{\rho_D^2}{\rho_D + \rho_m}), Q_5 = 3Hb(\rho_m + \frac{\rho_m^2}{\rho_D + \rho_m}), Q_6 = 3Hb(\rho_D + \rho_m + \frac{\rho_D^2}{\rho_D + \rho_m}), Q_7 = 3Hb(\rho_D + \rho_m + \frac{\rho_m^2}{\rho_D + \rho_m}) \) respectively. To find the best model we apply Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) and use the ΛCDM as the referring model for comparison. Using AIC and BIC models selection method we note that the \( Q_1 \) and \( Q_4 \) interaction terms are favored by observational data within the context of the holographic Ricci dark energy models. The obtained results also demonstrated that the considered types of holographic Ricci dark energy model are not favored by observational data since the ΛCDM is considered as the reference model. We also observed that the values of the deceleration parameter and the transition redshift for all models are compatible with the latest observational data and Planck 2015. In addition, we studied the jerk parameter for all models. Using our modified CAMB code, we observed that the interacting models suppress the CMB spectrum at low multipoles and enhances the acoustic peaks.

Keywords: Interacting holographic dark energy models, accelerated expanding Universe, observational constrains

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I INTRODUCTION

Raised in 1998 [1], dark energy has become one of the substantial cases in modern cosmology and many models have been proposed as a candidate to investigate it through the timeline of the Universe. Despite these proposed models, the dark energy is still a riddle in cosmology [2–10](to mention a few). The cosmological constant $\Lambda$ because of its proper explanation of the Universe’s expansion is the good candidate for study of the dark energy[11–14]. In spite of this coordination, the cosmological constant suffers from some drawbacks. Lack of ability to clarify, why densities of dark energy and dark matter are of the same order while they evolve in distinct way is of these drawbacks[9, 15–20].

Hence the holographic dark energy (HDE) as an alternative has been proposed and drawn many attentions in recent years [21–28]. This model is originated from the holographic principle to which all of the information in a particular region of space can be drawn out from its boundary area and considered by an IR cutoff [29, 30]. The energy density of HDE can be expressed by $\rho = 3c^2M_p^2/L^2$ [31–33]. In this equation, $c^2$ is a numerical constant, $M_p$ denotes the reduced Planck mass and $L$ can be taken as the size of the current Universe such as the Hubble scale[34, 35]. In addition, the HDE has some problems. The holographic dark energy with event horizon leads to causality and choosing other cutoffs such as Hubble Horizon and particle horizon could not satisfy the accelerated expansion of the Universe[36, 37]. Inspired by these problems from HDE, a model has been proposed which its length scale is the average radius of Ricci scalar curvature $|R|^{-1/2}$. This leads the dark energy density to be proportional to $R$. This is so-called the Holographic Ricci Dark Energy model (HRDE) [38]. Furthermore, the HRDE models has been extended to another model known as the Extended HRDE[39]. The HRDE can remove the fine tuning problem and also this model avoids causality and the coincidence problem[30, 31][40–43].

An attitude toward avoiding the coincidence problem also is the usage of interaction term as a non-gravitational component between dark sectors [44–49](to mention a few). In addition, because of the degeneracy between dark sectors in the Einstein’s gravity, it could be assumed that there is a non-gravitational coupling/interaction between them which can be non-linear [50–56]. These kinds of interaction term can be used for probing the dark energy related problems. Hence, the cosmologists have different options for selection and comparison of linear and non-linear models. In this case the phenomenological interactions have been studied in some works with holographic dark energy models. To be particular, Fu and et. al used three types of interactions ( $Q = 3Hb\rho_D$, $Q = 3Hb\rho_c$, $Q = 3Hb(\rho_D + \rho_c)$) in the context of holographic Ricci dark energy model (HRDE)[57]. Using SNIa, BAO and CMB as the latest observational data they found that HRDE models are not favored by these observational data and the BIC evidence is strongly against the model. These phenomenological interactions also have been used by Li and et. al along with $Q = 3Hb\sqrt{\rho_D\rho_c}$ and $Q = 3Hb\left(\rho_m + \frac{\rho_D\rho_c}{\rho_D+\rho_c}\right)$ [58]. Unanimously, they found that $Q = 3Hb\rho_D$ is better than the other interactions in their studies. The mentioned interactions also has been studied in[59] and using SNIa, BAO, CMB and $H_0$ as the latest observational data. It was observed that $Q = 3Hb\rho_D$ and $Q = 3Hb\left(\frac{\rho_D\rho_c}{\rho_D+\rho_c}\right)$ are the best models according to the results of AIC and BIC evidences.

Recently, some developments considering new forms of non-gravitational and non-linear interaction has been proposed[60]. We would like to use several of these interactions and investigate if disfavoring of the HRDE model can be alleviated by observational data using AIC and BIC [57, 61, 62]. We also check if these new non-linear interactions are better than the linear ones in this regards.

According to the discussion above, in this paper we compare two models of holographic dark energy with Ricci scalar curvature namely, Holographic Ricci Dark Energy model (HRDE) and Extended Holographic Ricci Dark Energy model (EHRDE) along with seven types of interaction ( $Q_1 = 3Hb(\rho_D + \rho_m)$, $Q_2 = 3Hb\rho_D$, $Q_3 = 3Hb\rho_m$, $Q_4 = 3Hb\left(\rho_D + \frac{\rho_D}{\rho_D+\rho_m}\right)$, $Q_5 = 3Hb\left(\rho_m + \frac{\rho_D^2}{\rho_D+\rho_m}\right)$, $Q_6 = 3Hb\left(\rho_D + \rho_m + \frac{\rho_D^2}{\rho_D+\rho_m}\right)$, $Q_7 = 3Hb\left(\rho_D + \rho_m + \frac{\rho_D^2}{\rho_D+\rho_m}\right)$ listed in Table1. Using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) as models selection tools.
we choose the most appropriate models among the other models. We also investigate the cosmographical aspects of the models. The scale factor is a component responsible for the dependency of spatial separation in the criterion of cosmology and can be considered as the key point of studying the kinematics of the Universe [63–65]. Hence, expanding the scale factor using Taylor series in the vicinity of the present time we have

\[ a(t) = \sum_{i=1}^{\infty} \frac{d^i a}{k! \, dt^i} (t - t_0)^i + 1, \quad (1) \]

and using this definition of scale factor we may write three terms of cosmography series

\[ H(t) = \frac{1}{a(t)} \frac{da}{dt}, \quad (2) \]
\[ q(t) = -\frac{1}{aH^2} \frac{d^2 a}{dt^2} = -1 - \frac{\dot{H}}{H^2}, \quad (3) \]
\[ j(t) = -\frac{1}{aH^3} \frac{d^3 a}{dt^3} = q + 2q^2 + \frac{\ddot{q}}{H}. \quad (4) \]

Extending this derivatives to the higher orders, for instance one can obtain snap parameter \( s \) for \( i = 4 \) to check how the evolution of the Universe deviates from the ΛCDM [66]. In the present work we restrict to \( i = 1, 2, 3 \) called as the Hubble parameter, the deceleration parameter and the jerk parameter respectively. By the use of the deceleration parameter it is possible to check the behavior of expansion of the Universe and also its transition from the decelerated \( (q > 0) \) to accelerated era \( (q < 0) \). In addition we may mark the transition redshift \( z_t \) (when \( q = 0 \)), the turning point redshift between two accelerated and decelerated era. The cosmic jerk parameter \( j \) as a dimensionless third derivative of the scale factor in terms of the cosmic time can compare the studied models with ΛCDM where \( j_0 = 1 \). Furthermore, an Universe with an accelerating rate of expansion has a positive value of jerk parameter.

The results of this paper for the models discussed above are based on the constraints from latest various observational datasets, namely the Pantheon Supernova type Ia, BAO from BOSS DR12, CMB from Planck 2015, and two categories of data originated from X-ray emitted from the galaxy clusters which are \( f_{\text{gas}} \) (gas mass fraction) and \( S/Z/X\)-Ray (Sunyaev-Zeldovich effect and X-ray emission) data. According to data analysis using these categories of data, the HRDE and EHRDE models remain disfavor by observational data. Also, we will see that the interaction \( Q_2 = 3Hb\rho_D \) will be the best model among the other ones similar to the results of Ref.[57–59] along with \( Q_4 = 3Hb \left( \rho_D + \frac{\rho_5}{\rho_D + \rho_m} \right) \) as the second best model.

The structure of this paper is as follows. In the next section (section 2) we briefly review the background equations of the models and introduce the interaction terms used in the current work. In section 3, we derive the differential equations of HRDE and EHRDE models and obtain the cosmological parameters of each model according to the chosen interaction terms. In section 4, the cosmographical behavior of the models has been studied. In the section 5, we provide the obtained results and discuss the aspects of the models. In section 6, we study the behavior of the present models in the CMB angular power spectrum. The last section is allocated to some concluding remarks.

II Background evolution

It is well-known that in a spatially flat FRW Universe, the Friedmann equation reads

\[ 3M_P^2 H^2 = \rho_D + \rho_m, \quad (5) \]
where $3M_P^2H^2$ is the critical density and $\rho_D$ and $\rho_m$ are density of dark energy and dark matter respectively. We may also write the dark energy and dark matter density with respect to the critical density as

$$\Omega_D = \frac{\rho_D}{3M_P^2H^2}, \quad \Omega_m = \frac{\rho_m}{3M_P^2H^2},$$  \hspace{1cm} (6)$$

and they obey the following relation

$$\Omega_D + \Omega_m = 1.$$  \hspace{1cm} (7)$$

The consideration of interaction between dark sectors makes the energy densities of the dark energy and the dark matter to be unable to satisfy the conservation laws. Hence, this leads to the following continuity equations

$$\dot{\rho}_m + 3H\rho_m = Q,$$  \hspace{1cm} (8)$$

$$\dot{\rho}_D + 3H(\rho_D + P_D) = -Q,$$  \hspace{1cm} (9)$$

in which $Q$ conveys the interaction term indicating energy flow between the components. Let us consider an explicit, non-gravitational form of interaction which phenomenologically originates from the energy transfer between the dark energy and the dark matter as$[67]$

$$Q = 3Hbq^n \left( \rho + \frac{\rho_i \rho_j}{\rho} \right).$$  \hspace{1cm} (10)$$

Where $n$ is a positive constant, $q$ is the deceleration parameter with $-1 - \frac{\ddot{H}}{H^2}$ defined in Eq.3, $H$ is the Hubble parameter and $\rho$ would be the summation of the dark energy density and dark matter density ($\rho_D + \rho_m$). The study of Ref.$[67]$ (which used different interacting Chaplygin gas models) shows that by choosing the sing changeable interaction originated from Eq.10, the stable critical points and the late time attractors cannot be found, while by fixing the sign of interaction the new late time attractors appears and describes, for instance, a Chaplygin gas dominated Universe. The most important achievement of Ref.$[67]$ is the new forms of scaling attractors demonstrating new solutions of the cosmological coincidence problem. For more details we refer the readers to Ref.$[67]$. In our study we choose the fixed sign of the interactions in the Eq.10 during the whole evolution of the Universe by consideration of $n = 0$. These kinds of interaction are very common types of non-linear interaction (as we see, for instance, in Ref.$[68–71]$). In this paper we will consider four non-linear terms of the interaction along with 3 linear ones which are listed in the Table 1.

In what follows, we implement the above interaction term in the context of HRDE and EHRDE and using observational data to obtain the best values of each model’s parameters. We also survey the cosmographical aspects of the models and then find the best models among the other ones using AIC and BIC evidences. Finally, we compare the models with $\Lambda$CDM as the reference model by the means of modified CAMB code package, as well.

### III Holographic Ricci Dark Energy Models

In this section we study the behavior of two most used types of holographic dark energy model namely, interacting Holographic Ricci Dark Energy (HRDE) and interacting Extended Holographic Ricci Dark Energy (EHRDE). First we produce two coupled differential equations to be solved numerically. This coupled differential equation shows the behavior of dark energy and Hubble parameter, suitable for both interacting and non-interacting models rather than the using an analytical solution for them. Secondly, we find the cosmological parameters of each case and provide the results of analysis in relevant tables.
Table 1: List of linear interactions \((Q_{1,3})\) and non-linear interactions \((Q_{4,7})\) considered in this paper.

| Mark | Interaction | Reference          |
|------|-------------|--------------------|
| \(Q_1\) | \(3Hb(\rho_D + \rho_m)\) | \([21, 44, 72–75]\) |
| \(Q_2\) | \(3Hbp_D\) | \([21, 44, 72–75]\) |
| \(Q_3\) | \(3Hbp_m\) | \([21, 44, 72–75]\) |
| \(Q_4\) | \(3Hb\left(\frac{\rho_D}{\rho_D + \rho_m}\right)\) | \([67]\) |
| \(Q_5\) | \(3Hb\left(\frac{\rho_m}{\rho_D + \rho_m}\right)\) | \([67]\) |
| \(Q_6\) | \(3Hb\left(\rho_D + \rho_m + \frac{\rho_D^2}{\rho_D + \rho_m}\right)\) | \([67]\) |
| \(Q_7\) | \(3Hb\left(\rho_D + \rho_m + \frac{\rho_m^2}{\rho_D + \rho_m}\right)\) | \([67]\) |

III.I Interacting Holographic Ricci Dark Energy Model

In a spatially flat Universe, the holographic dark energy is proportional to Ricci scalar curvature [38]

\[
R = -6 \left( \dot{H} + 2H^2 \right), \tag{11}
\]

and it is well-known that the density of dark energy can be written as [38]

\[
\rho_D = 3\alpha M_P^2 \left( \dot{H} + 2H^2 \right), \tag{12}
\]

in which \(H = \frac{\dot{a}}{a}\) is the Hubble parameter denoting the expansion rate of the Universe, the dot denotes the derivative in terms of \(t\), \(\alpha\) is a dimensionless parameter should be constrained as a free parameter and \(M_P = 1/\sqrt{8\pi G}\) is the reduced Planck mass and \(G\) is the Newton constant. Taking time derivative of Eq.5 and using Eqs.5, 9, 11 and 12 one can obtain the following coupled differential equations

\[
\dot{\Omega}_D = \left(2(1 - \Omega_D) \left( \frac{\Omega_D}{\alpha} - 2 \right) + 3(1 - \Omega_D - b\Omega_i) \right), \tag{13}
\]

\[
\frac{\dot{H}}{H^2} = \left( \frac{\Omega_D}{\alpha} - 2 \right), \tag{14}
\]

and

\[
\Omega_i = \frac{Q}{3M_P^2 H^3}. \tag{15}
\]

in which \(\dot{\Omega}_D = \Omega_{D}'H\) and \(\dot{H} = H'H\) where the prime denotes derivative with respect to \(x = \ln a\) and \(a = (1+z)^{-1}\). Then, the evolution of the density of dark energy and the Hubble parameter for HRDE in terms of redshift, after some algebra can be written as

\[
\frac{d\Omega_D}{dz} = - \left( \frac{1}{1+z} \right) \left(2(1 - \Omega_D) \left( \frac{\Omega_D}{\alpha} - 2 \right) + 3(1 - \Omega_D - b\Omega_i) \right), \tag{16}
\]

\[
\frac{dH}{dz} = - \left( \frac{H}{1+z} \right) \left( \frac{\Omega_D}{\alpha} - 2 \right). \tag{17}
\]

The results of the numerical calculation of these two coupled equations according to the combined observational data (see Appendix A for more details) can be seen in the Tables 2 and 3.
III.II Interacting Extended Holographic Ricci Dark Energy Model

A flexible form of HRDE has been proposed as the Extended HRDE with the following form of the dark energy density:

$$\rho_D = 3M_P^2 \left( \beta \dot{H} + \alpha H^2 \right), \quad (18)$$

where $\alpha$ and $\beta$ are constant parameters to be constrained by observational data, $M_P = 1/\sqrt{8\pi G}$ is the reduced Planck mass and $G$ is the Newton constant. It is clear that by assumption of $\alpha = 2\beta$ the EHRDE reduces to HRDE which has $\alpha_{ERDE} = \alpha_{RDE}$. Again, taking time derivative of Eq.5 and using Eqs.5, 9, 18 and $\Omega' = \frac{\Omega}{H}$ it is possible to reach the following coupled differential equations

$$\dot{\Omega}_D = \left( 2 (1 - \Omega_D) \left( \frac{9}{2} \left( \frac{\alpha - \frac{2}{3} \beta}{\alpha} - 1 \right) \frac{\Omega_D}{\alpha} \right) + \frac{2\beta}{\alpha} + \left( 3\alpha - 2\beta \right) - 3 (\Omega_D + b\Omega_i) \right), \quad (19)$$

$$\frac{\dot{H}}{H^2} = \left( \frac{9}{2} \left( \frac{\alpha - \frac{2}{3} \beta}{\alpha} - 1 \right) \frac{\Omega_D}{\alpha} \right), \quad (20)$$

and

$$\Omega_i = \frac{Q}{3M_P^2 H^3}. \quad (21)$$

in which $\dot{\Omega}_D = \Omega''_D H$ and $\dot{H} = H'H$ where the prime denotes derivative with respect to $x = \ln a$ and $a = (1 + z)^{-1}$. Then, the evolution of the density of dark energy and the Hubble parameter for EHRDE in terms of redshift can be written as

$$\frac{d\Omega}{dz} = \left( \frac{1}{1 + z} \right) \left( 2 (1 - \Omega_D) \left( \frac{9}{2} \left( \frac{\alpha - \frac{2}{3} \beta}{\alpha} - 1 \right) \frac{\Omega_D}{\alpha} \right) + \frac{2\beta}{\alpha} + \left( 3\alpha - 2\beta \right) - 3 (\Omega_D + b\Omega_i) \right), \quad (22)$$

$$\frac{dH}{dz} = \left( \frac{H}{1 + z} \right) \left( \frac{9}{2} \left( \frac{\alpha - \frac{2}{3} \beta}{\alpha} - 1 \right) \frac{\Omega_D}{\alpha} \right). \quad (23)$$

The results of the numerical calculation of these two coupled equations according to the combined observational data (see Appendix A for more details) can be seen in the Tables 4 and 5.

IV Cosmography

For studying cosmographical behavior of the model, we extend the Eq.1 to the third order which means we can study the behavior of the Hubble parameter Eq.2, the deceleration parameter Eq.3 and the jerk parameter Eq.4. In addition of calculation of the deceleration parameter, we obtain the transition redshift. For this, we employ the well-known Brent’s method which uses the combination of some methods with inverse quadratic interpolation as a secured version of the secant algorithm. This method by using three prior points can estimate the zero crossing. A description of this method can be found in "Numerical recipes in C" hand book [76].

The deceleration parameter of two interacting models of Ricci dark energy by substitution of the Eqs.17, 23 into Eq.3 can be written as

$$q_{HRDE} = -1 - \left( -2 + \frac{\Omega_D}{\alpha} \right), \quad (24)$$
respectively. Moreover, the value of \( q_0 \) for all models according to the best values of fitted parameters presented in Tables 2, 3, 4, 5 is approximately around \( q_0 \approx -0.63 \) for the present time which has good agreement with the value of the deceleration parameter by Planck \( (q_0 = -0.55)^{[73]} \) and shows an accelerated expansion of the Universe. The behavior of the deceleration parameter versus redshift for all models is plotted in Figs. 1 and 2. According to the plotted results in Figs 1 and 2, one can see that in region of 1σ interval level both lower and upper bounds of interacting and non-interacting HRDE and EHRDE models show the accelerating expansion within the redshift range \( z = (0.4, 0.8) \) which compatible with the recent observational works\[60, 77–80\](to mention few). The obtained results in the Tables 2, 3, 4 and 5 by the use of observational data show that the values of the transition redshift \( (z_t) \) of all models is in range of recent obtained values for transition redshift \( z_t = [0.4, 1]^{[60, 77–80]} \)(to mention few).

For the jerk parameter by substitution of Eqs. 24 and 25 into Eq.4 and with help of Eqs. 12 and 18, after some algebra, we may obtain

\[ j_{HRDE} = q_{HRDE} + 2q_{HRDE}^2 + 2 \left( 1 - \Omega_D \right) \left( -2 + \frac{\Omega_D}{\alpha} \right) + 3 - 3\Omega_D - 3\beta \Omega_i \]  

\[ (26) \]

\[ j_{EHRDE} = q_{EHRDE} + 2q_{EHRDE}^2 + \left( 2 \left( 1 - \Omega_D \right) \left( -\frac{3\alpha - 2\beta}{\alpha} - 3 \right) \frac{\Omega_D}{\alpha} \right) + \frac{2\beta}{\alpha} - \frac{2\beta - 3\alpha}{\alpha} - 3\beta \Omega_i - 3\Omega_D. \]  

\[ (27) \]

According to the jerk parameter we can explain the behavior of models in comparison to ΛCDM\[66, 81, 82\]. Compared to the deceleration parameter, the positive value for the jerk parameter demonstrates an accelerated expansion of the Universe. For ΛCDM in a flat Universe the value of jerk parameter...
parameter has a constant tendency to $j = 1$ \cite{66, 81, 82}. The observational constraints on the value of the cosmic jerk parameter in comparison with the deceleration parameter are weak $-5 < j_0 < 10$ \cite{83–86}. In this work we obtained the value of jerk parameters in range of $1 < j_0 < 2$ and its behavior for the interacting HRDE and interacting EHRDE is plotted in Figs. 3 and 4, respectively. The value of jerk parameter for interacting HRDE model remains positive and close to 1 between the redshift $z = [0.2, 0.6]$ and it crosses this line within the range of $1\sigma$. The EHRDE model has a tendency to reach the 1 at the early time and totally the values of its trajectory embrace the value of 1 in all redshifts within the range of $1\sigma$.

V Observational analysis

In this section we summarize the method used to analyze the models. In order to analyze the models we used SNIa, BAO, CMB, SZ/Xray (Sunyaev-Zeldovich effect and X-ray emission) and fgas (gas mass fraction) data introduced in Appendix A. For this purpose we employed the public codes EMCEE \cite{87} and GetDist Python package \footnote{https://getdist.readthedocs.io} for implementing the MCMC method and plotting the contours respectively.

By minimizing the $\chi^2$ we may obtain the best values of cosmological parameters

$$\chi^2_{total} = \chi^2_{Pantheon} + \chi^2_{BAO} + \chi^2_{CMB} + \chi^2_{SZ/Xray} + \chi^2_{Fgas}. \tag{28}$$

According to the obtained results listed in the Tables 2, 3, 4 and 5 we compare their compatibility with the very latest obtained cosmological parameters.

The Hubble constant $H_0$ is an important quantity in cosmology for calculation of age and size of the universe.
Figure 3: The evolution of the jerk parameter in terms of redshift for HRDE model (Eqs. 16 and 17) with the corresponding 1σ interval level according to the best fitted value listed in Tables 2 and 3 and using Eq.26 presenting the jerk parameter with interactions listed in Table 1. The straight line denotes the ΛCDM.

Figure 4: The evolution of the jerk parameter in terms of redshift for EHRDE model (Eqs. 22 and 23) with the corresponding 1σ interval level according to the best fitted value listed in Tables 4 and 5 and using Eq.27 presenting the jerk parameter with interactions listed in Table 1. The straight line denotes the ΛCDM.
Table 2: The fitted values of cosmological parameters for the holographic Ricci dark energy model (Eqs. 16 and 17) using linear and non-linear interactions listed in Table 1. The Pantheon supernova data, BAO BOSS DR12, CMB Planck 2015, fgas (gas mass fraction) and SZ/Xray (Sunyaev-Zeldovich effect and X-ray emission) data from galaxy clusters (GC) data has been used (See Appendix A).

| Params | Linear Interactions | Non-linear Interactions |
|--------|---------------------|-------------------------|
|        | $3H_0\rho_D$ | $3H_0\rho_m$ | $3H_0(\rho_D + \rho_m)$ |
| $H_0$ | $68.8878^{+0.5712}_{-0.5686}$ | $68.972^{+0.5961}_{-0.5987}$ | $68.8885^{+0.5845}_{-0.5821}$ | $68.9558^{+0.6072}_{-0.6066}$ |
| $\Omega_D$ | $0.7220^{+0.0178}_{-0.0155}$ | $0.6965^{+0.0131}_{-0.0162}$ | $0.7053^{+0.0158}_{-0.0151}$ | $0.6813^{+0.0131}_{-0.0200}$ |
| $\alpha$ | $0.4399^{+0.0148}_{-0.0165}$ | $0.4240^{+0.0155}_{-0.0200}$ | $0.4296^{+0.0150}_{-0.0160}$ | $0.4150^{+0.0162}_{-0.0166}$ |
| $b$ | $-0.0100^{+0.0045}_{-0.0057}$ | $0.0378^{+0.0029}_{-0.0020}$ | $0.0376^{+0.0052}_{-0.0053}$ | $0.0343^{+0.0046}_{-0.0046}$ |
| $M$ | $-19.3867^{+0.0206}_{-0.0209}$ | $-19.3846^{+0.0205}_{-0.0209}$ | $-19.3864^{+0.0877}_{-0.0857}$ | $-19.3841^{+0.0021}_{-0.0021}$ |
| $b_{fgas}$ | $0.7685^{+0.0754}_{-0.0988}$ | $0.844^{+0.1153}_{-0.0155}$ | $0.8169^{+0.0154}_{-0.0155}$ | $0.8186^{+0.0142}_{-0.0162}$ |
| $Age$ | $13.6126^{+0.4623}_{-0.4523}$ | $13.6740^{+0.4715}_{-0.4120}$ | $13.8396^{+0.3232}_{-0.4112}$ | $13.860^{+0.3441}_{-0.5925}$ |
| $\chi^2$ | $118.6778$ | $117.3842$ | $117.6833$ | $117.7193$ |
| $\chi_{dof}$ | $1.0231$ | $1.0231$ | $1.0145$ | $1.0148$ |

Table 3: The fitted values of cosmological parameters for the holographic Ricci dark energy model (Eqs. 16 and 17) using non-linear interactions listed in Table 1. The Pantheon supernova data, BAO BOSS DR12, CMB Planck 2015, fgas (gas mass fraction) and SZ/Xray (Sunyaev-Zeldovich effect and X-ray emission) data from galaxy clusters (GC) data has been used (See Appendix A).

| Non-linear Interactions |
|-------------------------|
| $p_D + \rho_D^{\rho_D} + \rho_m^{\rho_m}$ | $p_D + \rho_D^{\rho_D} + \rho_m^{\rho_m}$ | $p_D + \rho_D^{p_D} + \rho_m^{p_m}$ | $p_D + \rho_D^{p_D} + \rho_m^{p_m}$ |
| $H_0$ | $68.8035^{+0.5878}_{-0.5879}$ | $68.8097^{+0.5875}_{-0.5829}$ | $68.907^{+0.5712}_{-0.5761}$ | $68.7895^{+0.5976}_{-0.5978}$ |
| $\Omega_D$ | $0.6911^{+0.0121}_{-0.0161}$ | $0.7039^{+0.0142}_{-0.0161}$ | $0.6777^{+0.0122}_{-0.0193}$ | $0.6826^{+0.0122}_{-0.0191}$ |
| $\alpha$ | $0.4257^{+0.0339}_{-0.0587}$ | $0.4302^{+0.0371}_{-0.0597}$ | $0.4157^{+0.0180}_{-0.0392}$ | $0.4182^{+0.0231}_{-0.0501}$ |
| $b$ | $0.0306^{+0.0047}_{-0.0097}$ | $0.0342^{+0.0051}_{-0.0097}$ | $0.0303^{+0.0052}_{-0.0052}$ | $0.0312^{+0.0052}_{-0.0052}$ |
| $M$ | $-19.386{\pm}0.0211$ | $-19.388{\pm}0.0220$ | $-19.383{\pm}0.0205$ | $-19.386{\pm}0.0221$ |
| $b_{fgas}$ | $0.8561^{+0.0573}_{-0.0651}$ | $0.8186^{+0.0587}_{-0.0515}$ | $0.8971^{+0.0142}_{-0.0142}$ | $0.8810^{+0.0154}_{-0.0155}$ |
| $Age$ | $13.69078^{+0.3020}_{-0.5872}$ | $14.0109^{+0.3211}_{-0.3991}$ | $13.8764^{+0.3885}_{-0.7655}$ | $14.0221^{+0.3002}_{-0.5967}$ |
| $\chi^2$ | $117.4945$ | $117.7134$ | $117.8415$ | $117.7102$ |
| $\chi_{dof}$ | $1.0129$ | $1.0148$ | $1.0159$ | $1.0147$ |

Universe and also is a key factor for measuring the brightness and the mass of stars. This quantity corresponds to the Hubble parameter at the time of observation. Using the observational data in this work we obtained the value of the Hubble parameter for all models and we found a good consistency with the latest observational data, $H_0 = 67.78^{+0.91}_{-0.87}$ [88], $H_0 = 68^{+4.2}_{-4.1}$ [89], $H_0 = 66.76^{+0.42}_{-0.42}$ [73] and $H_0 = 70^{+12}_{-8}$ [90]. The value of dark energy density $\Omega_D$ also for all models has a suitable compatibility with latest obtained value $\Omega_D = 0.692^{+0.012}_{-0.012}$ [73]. However, the Q4 has the closest value between the studied models with $\Omega_D = 0.6911^{+0.0121}_{-0.0161}$ and $\Omega_D = 0.6914^{+0.0112}_{-0.0091}$ for HRDE and EHRDE respectively. For HRDE model, in spite of employing the latest observational data the value of $\alpha$ has not faced with remarkable change compared to the previous works [57, 61, 62]. For further information, in the case of $\alpha_{HRDE} = \alpha_{EHRDE}$ and $\beta = 2\alpha_{EHRDE}$ the EHRDE model reduces to HRDE. This ratio for all models stays in the range of $\beta/\alpha_{EHRDE} = 2^{+0.01}_{-0.01}$. According to this definition and considering the best fit values listed in Tables 2, 3, 4 and 5 we can see that the EHRDE model has a strong tendency
Table 4: The fitted values of cosmological parameters for the extended holographic Ricci dark energy model (Eqs.22 and 23) using linear and non-linear interactions listed in Table 1. The Pantheon supernova data, BAO BOSS DR12, CMB Planck 2015, fgas(gas mass fraction) and SZ/Xray(Sunyaev-Zeldovich effect and X-ray emission) data from galaxy clusters (GC) data has been used (See Appendix A).

| Parameters | Linear Interactions | Non-linear Interactions |
|------------|---------------------|-------------------------|
|            | 3HbρD | 3Hbρm | 3Hb(ρD + ρm) |
| H0         | 68.9841±0.8535 | 68.8995±0.7921 | 68.9532±0.7921 |
| | 68.9821±1.2332 | 68.9841±0.8535 | 68.9532±0.7921 |
| ΩD         | 0.6972±0.0007 | 0.7054±0.0008 | 0.6818±0.0012 |
| | 0.6968±0.0142 | 0.7054±0.0008 | 0.6818±0.0012 |
| α          | 0.4232±0.0044 | 0.4266±0.0377 | 0.4072±0.0551 |
| | 0.4432±0.0889 | 0.4266±0.0377 | 0.4072±0.0551 |
| β          | 0.8477±0.1222 | 0.8565±0.1102 | 0.8226±0.0612 |
| | 0.8497±0.0247 | 0.8565±0.1102 | 0.8226±0.0612 |
| b          | 0.0378±0.0050 | 0.0369±0.0022 | 0.0350±0.0028 |
| M          | −19.3853±0.1097 | −19.3846±0.0291 | −19.3868±0.0299 |
| | −19.3853±0.1097 | −19.3846±0.0291 | −19.3851±0.0311 |
| bgas       | 0.8429±0.0115 | 0.8442±0.0120 | 0.8884±0.0411 |
| | 0.8429±0.0175 | 0.8442±0.0120 | 0.8884±0.0411 |
| Age        | 13.7471±0.6564 | 13.6867±0.6721 | 13.8497±0.7211 |
| | 13.4711±0.6564 | 13.6867±0.6721 | 13.8497±0.7211 |
| zt         | 0.5411±0.1342 | 0.5473±0.0177 | 0.5412±0.0571 |
| | 0.5411±0.0401 | 0.5473±0.0491 | 0.5412±0.0571 |
| χ²         | 118.0536 | 117.3597 | 117.6568 |
| | 118.0536 | 117.3597 | 117.6568 |

Table 5: The fitted values of cosmological parameters for the extended holographic Ricci dark energy model (Eqs.22 and 23) using linear and non-linear interactions listed in Table 1. The Pantheon supernova data, BAO BOSS DR12, CMB Planck 2015, fgas(gas mass fraction) and SZ/Xray(Sunyaev-Zeldovich effect and X-ray emission) data from galaxy clusters (GC) data has been used (See Appendix A).

| Parameters | 3Hb(ρD + ρ_m) | 3Hbρm | 3Hb(ρD + ρ_m) |
|------------|---------------|-------|---------------|
| H0         | 68.8289±0.8431 | 68.8289±0.8431 | 68.9547±0.7005 |
| | 68.8156±0.7270 | 68.8289±0.8431 | 68.9547±0.7005 |
| ΩD         | 0.7043±0.0111 | 0.6803±0.0013 | 0.6833±0.0099 |
| | 0.6914±0.0112 | 0.6803±0.0013 | 0.6833±0.0099 |
| α          | 0.4316±0.0247 | 0.4024±0.0197 | 0.4171±0.0233 |
| | 0.4238±0.0364 | 0.4024±0.0197 | 0.4171±0.0233 |
| β          | 0.8605±0.0925 | 0.8246±0.1444 | 0.8352±0.0891 |
| | 0.8494±0.1001 | 0.8246±0.1444 | 0.8352±0.0891 |
| b          | 0.0325±0.0005 | 0.0325±0.0005 | 0.0325±0.0005 |
| | 0.0341±0.0003 | 0.0325±0.0005 | 0.0325±0.0005 |
| M          | −19.3880±0.1168 | −19.3851±0.0380 | −19.3865±0.0410 |
| | −19.3874±0.0301 | −19.3851±0.0380 | −19.3865±0.0410 |
| bgas       | 0.819±0.0091 | 0.8892±0.1602 | 0.8803±0.5254 |
| | 0.8556±0.1915 | 0.8892±0.1602 | 0.8803±0.5254 |
| Age        | 14.0112±0.9010 | 13.9164±0.8440 | 14.0429±0.7072 |
| | 13.7062±0.6012 | 13.9164±0.8440 | 14.0429±0.7072 |
| zt         | 0.5632±0.0541 | 0.5701±0.0371 | 0.5743±0.0523 |
| | 0.5563±0.0452 | 0.5701±0.0371 | 0.5743±0.0523 |
| χ²         | 117.7134 | 117.7102 | 117.7102 |
| | 117.4945 | 117.7102 | 117.7102 |

Towards HRDE model.
The value of the depletion component or the bias factor related to the gas dynamical simulation from fgas(gas mass fraction) data has been obtained bgas = 0.824±0.033[91]. This value has been used in some works as a fixed value[61, 92, 93]. We found that fixing of this parameter strongly affects the values of other parameters and the value of χ² as well. For example by using bgas = 0.824 we obtained ΩD = 0.66 and b = 0.08. Taking the bgas as a free parameter, we reached the bigger value for this quantity compared to Ref [91] except for Q3 and Q5 having smaller number. It should be mentioned that fixing this value makes the Universe older and out of the acceptable range of age. After fitting this value, the Age of the Universe for all models except for Q5 and Q7 is also in good agreement with the recent observational data (AgePlanck = 13.79Gyr)[73].


To compare the success of the models on fitting data we calculate $\chi_{dof}$ with $N = 116$ represents the entire data points used in this work. We notice that all the models are successful with reasonable value of the goodness of freedom (dof). The dof value for $\Lambda$CDM model with $\chi^2 = 116.0528$ is $\chi_{dof} = 1.0004$ which is slightly better than the other models. The non-interacting HRDE and EHRDE models have the biggest values of the degree of freedom and $Q_2$ for both HRDE and EHRDE shows the highest success on the fitting data.

These results show the consistency of HRDE and EHRDE models with latest observational data and also considering of interaction between dark sectors (all types of interaction in this work) does not impose any problem to this issue. In addition, these results show that the $f_{\text{gas}}$(gas mass fraction) and SZ/X-Ray data can play a rational role in the determination of free parameters for the cosmological models.

On the other hand, to determine the best cosmological models among several studied models we cannot rely on the fitted values of the relevant parameters. Despite the fact that minimizing $\chi^2$ is the most simple way to get the best fitting of free parameters, it is usually unreasonable to distinguish the best model between variety of studied models. Hence, for this issue Akaike Information Criterion (AIC)[94] and Bayesian Information Criterion (BIC) [95] have been proposed. For additional information see [96–99]. The AIC model selection function can be expressed as

$$AIC = -2 \ln L_{max} + 2k,$$

where $-2 \ln L_{max} = \chi^2_{\text{min}}$ is the highest likelihood, $k$ is the number of free parameters and $N$ is the number of data points used in the analysis. The BIC is similar to AIC with different second term

$$BIC = -2 \ln L_{max} + k \ln N.$$

It is obvious that a model favored by observations should give a small AIC and a small BIC.

The level of support for each model from AIC is

- **Less than 2**: This indicates there is substantial evidence to support the model (i.e., the model can be considered almost as good as the best model).
- **Between 4 and 7**: This indicates that the model has considerably less support.
- **Between 8 and 10 or bigger**: This indicates that there is essentially no support for the model (i.e., it is unlikely to be the best model).

The level of the evidence against models if the tool of selection is BIC:

- **Less than 2**: It is not worth more than a bare mention (i.e., the model can be considered almost as good as the best model).
- **Between 2 and 6**: The evidence against the model is positive.
- **Between 6 and 10**: The evidence against the candidate model is strong,(i.e., it can be merely the best model).
- **Bigger than 10**: The evidence is very strong (i.e., it is unlikely to be the best model).

Clearly, for both interacting and non-interacting Ricci dark energy model the values of $\chi^2$ in case of existence of interaction are smaller than the non-interacting models which is due to the additional parameter $b$. The interacting and non-interacting models have the bigger value of $\chi^2$ compared to the $\Lambda$CDM. From these analysis also it can be shown that the linear interaction terms lead to bigger value of the decoupling constant ($b$) in comparison with phenomenological interactions ( see Tables 2, 3, 4 and 5).

According to the AIC and BIC evidences shown in Table6 and graphical representation of the model comparison result in Fig 5, one can decide about choosing the appropriate interaction term. We take the $\Lambda$CDM model ($H_0 = 68.5846^{+0.7970}_{-0.8015}$, $\Omega_D = 0.6968^{+0.0174}_{-0.0173}$, $M = -19.3868^{+0.0202}_{-0.0207}$ and $bfg =$
According to the analysis above for two Ricci models namely, holographic Ricci dark energy model (HRDE) and extended holographic Ricci dark energy model (EHRDE) the best interaction models are $Q_2 = 3Hb\rho_D$ in the linear interaction’s category and $Q_4 = 3Hb\left(\rho_D + \frac{\rho^2_m}{\rho_D + \rho_m}\right)$ in the non-linear interaction’s category. The constraints on free parameters are summarized in Figs 8, 9, 10, 11 and Tables 2, 3, 4 and 5. By adding the gas (mass fraction) and SZ/X-Ray (Sunyaev-Zeldovich effect and X-ray emission) data compared to the previous works [57–59], we found that the best interaction still is the linear one $Q_2 = 3Hb\rho_D$. The HRDE models also similar to the previous works [57, 61, 62] remain disfavor by observational data in case of taking the $\Lambda$CDM as the reference model of comparison.
VI CMB Power Spectrums

In this section by the use of modified version of the Boltzmann code CAMB\(^\dagger\) [100, 101], we compare the power spectrums of the cosmic microwave anisotropy in all interacting and non-interacting HRDE and EHRDE models. Our results of the temperature power spectrum (TT) according to the fitted results are depicted in Figs. 6 and 7. From the figures, we see that both the HRDE (Eqs.16, 17) and the EHRDE (Eqs.22, 23) models for all types of interactions show the trends of squeezing power spectrum of the cosmic microwave anisotropy to small \(\ell\) or large angle scales. This squeezing can also be seen from the power spectrum of matter distribution in the Universe. Embodying on the large scale structure of matter distributions, all these models exhibit an as high as 20% peak power spectrum’s suppressing which occur in small \(k\) or large scale region. The origin of this suppression is mainly due to the relative lower baryon \(\Omega_b = 0.0464\) and neutrino \(\Omega_\nu = 0.00134\) occupation fraction in the cosmic contents partition scheme relative to \(\Lambda\)CDM model which are \(\Omega_b = 0.0468\) and neutrino \(\Omega_\nu = 0.00136\). Using \(\Lambda\)CDM as the reference model, we may observe that, \(Q_2 = 3Hb\rho_D, Q_3 = 3Hb\rho_m\) and \(Q_4 = 3Hb\left(\rho_D + \frac{\rho_m}{\rho_D + \rho_m}\right)\) are the three interaction models which are most close to \(\Lambda\)CDM. On the other hand, except for some very special case, the EHRDE model as a whole does not exhibit manifest advantage over the simple HRDE model.

VII Conclusion

In this work, we compared the behavior of seven types of interaction case \((Q_1 = 3Hb(\rho_D + \rho_m), Q_2 = 3Hb\rho_D, Q_3 = 3Hb\rho_m, Q_4 = 3Hb\left(\rho_D + \frac{\rho_m}{\rho_D + \rho_m}\right), Q_5 = 3Hb\left(\rho_m + \frac{\rho_D}{\rho_D + \rho_m}\right), Q_6 = 3Hb\left(\rho_D + \rho_m + \frac{\rho_m^2}{\rho_D + \rho_m}\right)\) and \(Q_7 = 3Hb\left(\rho_D + \rho_m + \frac{\rho_m^2}{\rho_D + \rho_m}\right)\) into the context of the holographic Ricci dark energy model (HRDE) defined by Eqs.16 and 17 and extended holographic Ricci dark energy model (EHRDE) defined by Eqs.22 and 23. We used SNIa compressed Pantheon data, Baryon Acoustic Oscillations (BAO) from BOSS DR12, Cosmic Microwave Background (CMB) of Planck 2015, fgas(gas mass fraction) and SZ/Xray(Sunyaev-Zeldovich effect and X-ray emission) as the observational data for constraining the

\(^\dagger\)https://camb.info
Figure 6: The CMB temperature spectra $C_\ell^{TT}$ of the HRDE (see Eqs.16, 17) with interaction terms listed in Table 1. All models’ $\Omega_D$ and $H_0 \equiv 100h[\text{km/s Mpc}]$ parameter are set as their best fitting values from Table 2 and 3. All models have equal $\Omega_b h^2 = 0.022$, $\Omega_\nu h^2 = 0.00064$ and manually tuned $\Omega_{cdm}$ so that $\Omega_b + \Omega_\nu + \Omega_{cdm} = 1 - \Omega_D$. Relative to $\Lambda$CDM model ($H_0 = 68.5846^{+0.7970}_{-0.8192}$, $\Omega_D = 0.6968^{+0.0174}_{-0.0173}$), the interacting and non-interacting HRDE models have the trend of yielding equal degree of anisotropies at larger angle scale or small $\ell$-poles.

potential free parameters of the models. For obtaining the results we employed and modified the Cosmo Hammer (EMCEE) Python package. We found that the deceleration parameter for all considered types of interaction, both linear and non-linear (see Table 1), shows the corresponding Universe is expanding with accelerating rate and is in good agreement with Planck 2015 data. In addition, according to the Figs.1 and 2 related to the deceleration parameter, it has been observed that the lower and upper bounds of $1\sigma$ confidence level for both interacting and non-interacting HRDE and EHRDE models enter the accelerating era within the redshift range $z = (0.4, 0.8)$. Using the Brent’s method we also obtained the transition redshift with good compatibility with recent studies in this
Figure 7: The CMB temperature spectra $c^T_T$ of the EHRDE (see Eqs. 22, 23) with interaction terms listed in Table 1. All models' $\Omega_D$ and $H_0 \equiv 100h[\text{km/s/Mpc}]$ parameter are set as their best fitting values from Table 4 and 5. All models have equal $\Omega_b h^2 = 0.022$, $\Omega_c h^2 = 0.00064$ and manually tuned $\Omega_{cdm}$ so that $\Omega_b + \Omega_c + \Omega_{cdm} = 1 - \Omega_D$. Relative to $\Lambda$CDM model ($H_0 = 68.5846^{+0.7970}_{-0.8015}$, $\Omega_D = 0.6968^{+0.0174}_{-0.0173}$), the interacting and non-interacting HERDE models have the trend of yielding equal degree of anisotropies at larger angle scale or small $\ell$-poles.

case $0.4 < z_t < 1$. Studying the jerk parameter, we observed that both models cross the $\Lambda$CDM line ($j_0 = 1$) within the range of $1\sigma$ confidence level. It has been observed that the EHRDE model is closer to $j_0 = 1$ in comparison to HRDE model. By employing two model selection tools (AIC and BIC) and obtaining the best value of parameters for the $\Lambda$CDM as the reference ($H_0 = 68.5846^{+0.7970}_{-0.8015}$, $\Omega_D = 0.6968^{+0.0174}_{-0.0173}$) we found that

1. The different types of HRDE whether interacting or non-interacting are not supported by observational data and are ruled out. This result is due to the opting of $\Lambda$CDM as the reference model. It can be mentioned that by changing the reference model from $\Lambda$CDM to a HDE model (such as
the interacting HRDE (AIC < 2) while BIC shows the positive evidences against the models (BIC < 4). Indeed with this situation it can be also mentioned that the HDE models have been proposed because of the fundamental problems of ΛCDM model mentioned in our discussion concerning to the motivation having alternative dark energy models.

2. Within the context of AIC and BIC, among the seven type of interactions we can pinpoint two of them \( Q_1 = 3Hb\rho_D \) and \( Q_4 = 3Hb(\rho_D + \frac{\rho_m}{\rho_D+\rho_m}) \) to be the best models.

Using modified version of CAMB package we observed the tendency of all models to small ℓ or large angle scale in power spectrum of the cosmic microwave background anisotropy and also show an as high as 20% degree of the matter power spectrum’s suppressing. Furthermore, we found that \( Q_2, Q_3 \) and \( Q_4 \) for both HRDE and HERDE are the closest models to ΛCDM.

Finally, using the combination of observational data we fitted the free parameters of the models. We observed that the cosmological parameters of the HRDE and HERDE for all linear and non-linear interactions have good agreement with latest obtained values of the cosmological parameters. Our results demonstrated that the Hubble constant value is in range of \( H_0 = [0.67, 71] \) having good consistency with the recent works on observational data. The obtained value of dark energy density for all models is in good agreement with latest Planck data. However, the model \( Q_4 \) with \( \Omega_D = 0.6911^{+0.0121}_{-0.0161} \) for HRDE and \( \Omega_D = 0.6914^{+0.0112}_{-0.0091} \) for EHRDE showed more compatibility. It worth to mention that adding two categories of galaxy clusters data namely, fgas(gas mass fraction) and SZ/X-ray (Sunyaev-Zeldovich effect and X-ray emission) did not change the results compared to the previous works (mentioned in the discussion) on phenomenological interactions and also the HRDE model. The HRDE models remain unsupported by observational data and the best interaction model still is the linear interaction (\( Q = 3Hb\rho_D \)). We also found that the depletion factor of fgas data \( b_{fgas} \) should be constrained. The results of the models are very sensitive to this parameter and assumption of \( b_{fgas} \) as a fixed parameter could result to having different value for \( \Omega_D \) and even age of the Universe.

In conclusion, we can note that the new non-linear interactions (studied in this work) are reliable for further study and compatible with the latest observational data. The results showed that the galaxy clusters data namely, fgas(gas mass fraction) and SZ/X-ray (Sunyaev-Zeldovich effect and X-ray emission) can play rational role in constraining the cosmological parameters.

It can be mentioned that for the deep understanding of phenomenological interactions, particularly the non-linear ones, more investigations should be done. Thus, for the future works, we would like to study the dynamical system methods for understanding the behavior of the non-linear interactions in the late time. We also are going to check how much these types of interactions are successful to alleviate the coincidence peroblem. In addition the perturbation analysis compare to the gravitational lenses and the Large Scale Structure can be performed.

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Appendix A

A.I Compressed Pantheon Supernovae Data

For the supernova type Ia (SNIa), we use 40 binned data points of the recent proposed Pantheon data with the range of redshift $z = [0.014, 1.62]$[102]. We use the systematic covariance $C_{sys}$ for a vector of binned distances

$$C_{ij,sys} = \sum_{n=1}^{i} \left( \frac{\partial \mu_i}{\partial S_n} \right) \left( \frac{\partial \mu_j}{\partial S_n} \right) (\sigma_{S_n})$$

in which the summation is over the $n$ systematics with $S_n$ and its magnitude of its error $\sigma_{S_n}$. According to $\Delta \mu = \mu_{data} - M - \mu_{obs}$ in which $M$ is a nuisance parameter we can write the $\chi^2$ relation for Pantheon SNIa data as

$$\chi^2_{Pantheon} = \Delta \mu^T \cdot C_{Pantheon}^{-1} \cdot \Delta \mu$$

Note that the $C_{Pantheon}$ is the summation of the systematic covariance and statistical matrix $D_{stat}$ having a diagonal component. The complete version of full and binned Pantheon supernova data can be found in the online source

A.II Baryon Acoustic Oscillations Data

We use the BOSS DR12 including six measured data points as the latest observational data for BAO [103]. The $\chi^2_{BAO}$ can be explained as

$$\chi^2_{BAO} = X^T \cdot C_{BAO}^{-1} \cdot X,$$

where $X$ for six data points is

$$X = \begin{pmatrix}
D_M(0.38) r_{s,fid} & -1512.39 \\
\frac{r_s(z_d)}{H(0.38) r_s(z_d)} & -81.208 \\
\frac{D_M(0.51) r_s(z_d)}{r_s(z_d)} & -1975.22 \\
\frac{H(0.51) r_s(z_d)}{r_s(z_d)} & -90.9 \\
\frac{D_M(0.61) r_s(z_d)}{r_s(z_d)} & -2306.68 \\
\frac{H(0.51) r_s(z_d)}{r_s(z_d)} & -98.964
\end{pmatrix},$$

and $r_{s,fid} = 147.78$ Mpc is the sound horizon of fiducial model, $D_M(z) = (1 + z) D_A(z)$ is the comoving angular diameter distance. The covariance matrices can be found at the MontePython online files **.

A.III Cosmic Microwave Background Data

Discovering the expansion history of the Universe, we check Cosmic Microwave Background (CMB). For this, we use the data of Planck 2015 [73]. The $\chi^2_{CMB}$ function may be explained as

$$\chi^2_{CMB} = q_1 - q_{data}^1 Cov_{CMB}^{-1} (q_1, q_3),$$

where $q_1 = R(z_*)$ is the shift parameter, $q_2 = l_A(z_*)$ in the acoustic scale, $q_3 = \omega_b$ is the density of baryonic matter and $Cov_{CMB}$ is the covariance matrix [73]. The CMB data of Planck 2015 are

$$q_{data}^1 = 1.7382,$$
$$q_{data}^2 = 301.63,$$
$$q_{data}^3 = 0.02262.$$
A.IV  Galaxy Clusters’ Data

This method has an explicit dependency to the diameter angular distance $d_A$ of the gas mass fraction data $f_{gas}$ from the galaxy clusters. In this technique we may consider that the baryonic fraction of the galaxy clusters proportionate to the global fraction of baryonic and dark matter. The gas mass fraction can be defined as

$$f_{gas} = \frac{M_{gas}}{M_t} \tag{39}$$

in which $M_{gas}$ is the gas mass of X-ray and $M_t$ is the total gravitational mass of the galaxy clusters. It is possible to explain the equation above according to $d_A$ \[104\]

$$f_{gas}^{\Lambda CDM} = \frac{b \Omega_b}{1 + 0.19 \sqrt{h \Omega_m}} \left( \frac{d_A^{\Lambda CDM}}{d_A} \right)^{1.5} \tag{40}$$

in which $f_{gas}$ is observational gas mass fraction data \[105\], $f_{gas}^{\Lambda CDM}$ is the gas mass fraction of the cosmology models (Here HRDE Models) compared to $\Lambda CDM$ as the reference model and $b$ is the depletion component which is the key factor of relation between the baryonic fraction in the galaxy clusters and the mean cosmic value\[106\]. We use 42 measured data points in range of $z = \{0.05, 1.1\}$ \[105\] and we may write the $\chi^2_{fgas}$ as

$$\chi^2_{fgas} = \sum_{n=1}^{42} \left( \frac{f_{gas}^{\Lambda CDM} - f_{th}^{fgas}}{\sigma_n} \right)^2 + \left( \frac{\Omega_b h^2 - 0.0214}{0.002} \right)^2 + \left( \frac{h^2 - 0.072}{0.08} \right)^2 + \left( \frac{b - 0.824}{0.089} \right)^2 \tag{41}$$

For SZ/Xray data, we use 25 measured data points of angular diameter distance $(d_{A,c})$ from galaxy clusters \[107\].This method can be related to the observing the galaxy clusters. The processes of sudden turbulence and compaction in Intra-clusters Medium causes the temperature to rise and by the Sunyaev-Zeldovich (SZ) effect and X-ray emission the galaxy clusters can be observed\[108\]. Using the SZ effect and X-ray emission of galaxy clusters it can be possible to measure the diameter angular distance $(d_A)$ of the clusters \[109\]. An error $\sigma_{de}$ is considered to each measurement which is derived by the combination of the uncertainties in the galaxy clusters and the statistical along with systematic errors. The usage of the statistical errors stems from galaxy clusters’ asphericity which is among the SZ point sources and the kinetic SZ effect\[110, 111\]. The $\chi^2$ for this procedure compared to the diameter angular distance can be written as

$$\chi^2_{SZ/Xray} = \sum_{n=1}^{25} \left( \frac{d_A - d_{A,c}}{\sigma_{de}} \right)^2 \tag{42}$$
Figure 8: The contour maps of the HRDE (see Eqs. 16 and 17) with three types of linear interaction $Q_1$, $Q_2$ and $Q_3$ listed in Table 1. In this figure $H_0$ is the Hubble parameter, $\Omega_D$ is the dark energy density, $\alpha = c^2$ is the dimensionless parameter, $b$ is the coupling constant, $M$ is the nuisance parameter of SNIa data, $b_{\text{fgas}}$ is the nuisance parameter of fgas mass fraction data, $z_t$ is the transition redshift and $Age$ is the age of the Universe for the HRDE model. The best fitted values of these parameter are listed in the Table 2.
Figure 9: The contour maps of the HRDE (see Eqs. 16 and 17) with four types of non-linear interaction \( Q_4, Q_5, Q_6 \) and \( Q_7 \) listed in Table 1. In this figure \( H_0 \) is the Hubble parameter, \( \Omega_D \) is the dark energy density, \( \alpha = c^2 \) is the dimensionless parameter, \( b \) is the coupling constant, \( M \) is the nuisance parameter of SNIa data, \( b_{\text{fgas}} \) is the nuisance parameter of fgas mass fraction data, \( z_t \) is the transition redshift and \( \text{Age} \) is the age of the Universe for the HRDE model. The best fitted values of these parameter are listed in the Table 3.
Figure 10: The contour maps of the EHRDE (see Eqs. 22 and 23) with three types of linear interaction $Q_1$, $Q_2$, and $Q_3$ listed in Table 1. In this figure $H_0$ is the Hubble parameter, $\Omega_D$ is the dark energy density, $\alpha$ and $\beta$ are the dimensionless parameter, $b$ is the coupling constant, $M$ is the nuisance parameter of SNIa data, $b_{fgas}$ is the nuisance parameter of fgas mass fraction data, $z_t$ is the transition redshift and $Age$ is the age of the Universe for the EHRDE model. The best fitted values of these parameter are listed in the Table 4.
Figure 11: The contour maps of the EHRDE (see Eqs. 22 and 23) with four types of non-linear interaction $Q_4$, $Q_5$, $Q_6$ and $Q_7$ listed in Table 1. In this figure $H_0$ is the Hubble parameter, $\Omega_D$ is the dark energy density, $\alpha$ and $\beta$ are the dimensionless parameter, $b$ is the coupling constant, $M$ is the nuisance parameter of SNIa data, $b_{fgas}$ is the nuisance parameter of fgas mass fraction data, $z_t$ is the transition redshift and $Age$ is the age of the Universe for the EHRDE model. The best fitted values of these parameter are listed in the Table 5.