Natural Characteristics of Multi-Stage Planetary Drives for Space Manipulator Joints Based on Modular Dynamic Model

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Abstract. In this paper, a modular dynamic model of multi-stage planetary gear drive for space manipulator joints is presented so as to overcome the shortcomings of the traditional overall modeling approach. Based on the modular idea, the system mass and stiffness matrices are respectively divided into four sub-modular matrices, then the modular program for the eigenvalue and eigenvector calculation of multi-stage planetary system is developed. The presented modular calculation program of natural characteristics can be used for the computational analysis of natural frequencies and modal vectors of the planetary gear drive with arbitrary drive stages and arbitrary drive parameters. Meanwhile, taking the four-stage planetary gear drive system for space manipulator joints for example, through the call of the modular calculation program, its natural frequencies and modal vectors can be quickly and easily acquired.

Keywords: Space Manipulator Joints, Multi-Stage Planetary Gears, Natural Characteristic, Modular.

1. Introduction
Since the drive joints offer larger capacity, higher reliability and longer service life in comparison with the harmonic gear drive joint, the multi-stage planetary gear transmission is commonly used in the transmission system for high-power space manipulator joints.

At present, many researches have been carried out on natural characteristic analysis for planetary gear drives [1-4]. However, in these above literatures, the dynamic model is set up based on the traditional integrated modeling. It is clear that the corresponding dynamic model need be re-established if the system transmission form is changed. Thus, the traditional modeling method is quite inconvenient.

In this paper, the modular modeling approach and the modular calculation program of natural characteristics for multi-stage planetary drives are presented. The modular modeling approach can overcome the shortcomings of the traditional overall modeling approach and set up a generalized modular dynamic model of the space manipulator joint drive. The modular calculation program of natural characteristics for multi-stage planetary drives can be used for the computational analysis of the natural frequencies and modal vectors of the planetary gear drive with arbitrary drive stages and arbitrary drive parameters.
2. Modular Dynamic model of Multi-Stage Planetary Gears for Space Manipulator Joints

In this paper, as is displayed in Fig. 1, the multi-stage 2K-H planetary drives connected in series are utilized. Each stage of planetary drives is comprised of a ring gear \( r_i \), \( N_i \) planet-gear \( p_i \), a sun-gear \( s_i \) and a carrier \( c_i \).

The pure torsional lumped-parameter dynamic model of the \( i \)-th stage 2K-H planetary drive for the multi-stage planetary gear drive system is exhibited in Fig. 2. According to Ren's study, the overall model of the drive system is divided into several level modules, and a generalized dynamic model of planetary gear drive system is able to be set up through calling different levels of modules. Based on the above modular approach, the nonlinear dynamic differential equations for the multi-stage planetary gear drive system is capable of being established as following:

![Lumped-parameter dynamic model of single-stage planetary drive](image)
\[
[M] \cdot \ddot{q} + ([K_m] + [K_b]) \cdot \{q\} = \{T\} + \{F\}
\]  
(1)

Where \(M\) denotes the mass matrix; \(K_m\) refers to the mesh stiffness matrix; \(K_b\) denotes the support stiffness matrix; \(T\) represents the external exciting force vector; \(F\) is the internal exciting force vector, \(q\) refers to the displacement vector.

3. Natural Characteristic Analysis Based on Modular Idea

The undamped free vibration equations can be obtained by rewriting the modular dynamic equation (1) of the multi-stage planetary gear drive system as

\[
[M] \cdot \ddot{q} + ([K_m] + [K_b]) \cdot \{q\} = 0
\]  
(2)

The eigenvalue problem corresponding to Equation 2 is as following:

\[
\omega_i^2 \cdot [M] \cdot \{\varphi_i\} = ([K_m] + [K_b]) \cdot \{\varphi_i\}
\]  
(3)

Where \(\omega_i\) is the \(i\)-th natural frequency of the system; \(\varphi_i\) is the mode vector corresponding to the \(i\)-th natural frequency.

It can be known from equation (3) that to obtain the natural frequency and mode vector of the drive system, only the specific values of the mass matrix and the stiffness matrix need to be known. In this paper, the modularization idea is adopted to compile the universal natural characteristic calculation program of the multi-stage planetary transmission system for space manipulator joint. Fig. 3 shows the schematic diagram of calculating the natural characteristics of the drive system. As shown in equation (4), the system mass matrix \(M\) is divided into four modules: a ring gear mass module \(M_r\), a sun gear mass module \(M_s\), a carrier mass module \(M_c\), and a planet gear mass module \(M_p\). These three mass modules are versatile. When the number of transmission stages and parameter values change, the integration of the overall mass matrix can be achieved via simple numerical modification. Likewise, the stiffness matrix \(K\) is divided into four general modules: a ring gear stiffness module \(K_r\), a sun gear stiffness module \(K_s\), a carrier stiffness module \(K_c\), and a planet gear stiffness module \(K_p\), as depicted in equation (5).

**Fig. 3** Schematic diagram of natural characteristic modularization calculation process

\[
M = \text{diag}[M_r, M_s, M_c, M_p]
\]  
(4)
\[ K = \text{diag}[K_r, K_s, K_c, K_p] \]  

(5)

Table 1 Simulation basic parameters of four-stage planetary drive system

| Item                     | Sun (1-4 Stage) | Planet (1-4 Stage) | Ring (1-4 Stage) | Carrier (1-4 Stage) |
|--------------------------|-----------------|--------------------|------------------|---------------------|
| Tooth number             | 21              | 39                 | 99               | —                   |
| Pressure angle (°)       | 20              | 20                 | 20               | —                   |
| Mass (kg)                | 0.72            | 2.45               | 4.65             | 1.16                |
| \( I/r^2 \) (kg)         | 0.35            | 1.08               | 2.15             | 0.58                |

Table 2. Stiffness parameters of four-stage planetary drive system

| Item                              | Symbol and value |
|-----------------------------------|------------------|
| Mesh stiffness/(N \cdot m^{-1})   | \( k_{sp} = k_{rp} = 5 \times 10^8 (i = 1, 2, 3, 4) \) |
| Support stiffness/(N \cdot m^{-1})| \( k_s = 1 \times 10^9 (i = 1, 2, 3, 4) \) |
| Torsional stiffness of central members/(N \cdot m^{-1}) | \( k_{cw} = k_{cw} = 0, k_{cw} = 1 \times 10^9 (i = 1, 2, 3, 4) \) |
| Torsional stiffness of coupling shaft/(N \cdot m/rad) | \( k_{csw} = 5 \times 10^8 (i = 1, 2, 3, 4) \) |

Tables 1 and 2 exhibit the simulation parameters of four-stage 2K-H planetary drive system for space manipulator joints. Natural characteristics of four-stage 2K-H planetary drive system are calculated using the proposed modular program for the eigenvalue calculation of planetary drive system to obtain its natural frequencies and mode vectors. Because of space limitations, only the calculation results of natural frequencies are given. Table 3 displays the calculated natural frequencies of four stage 2K-H planetary transmission system. The total number of natural frequencies is 4N+12, of which N is the number of planet gears per stage.

Table 3. Natural frequencies of four-stage 2K-H planetary drive when the planet number N is three

| multiplicity | Natural frequencies(HZ) |
|--------------|-------------------------|
| 1            | 63.4365                 |
| 1            | 70.6225                 |
| 1            | 96.1657                 |
| 1            | 120.7602                |
| 1            | 230.1777                |
| 1            | 249.2928                |
| 1            | 268.8623                |
| 8            | 279.5934                |
| 1            | 306.9394                |
| 1            | 643.9169                |
| 1            | 652.9207                |
| 1            | 663.1436                |
| 1            | 670.6278                |
| 1            | 1087.9852               |
| 1            | 1097.1306               |
| 1            | 1101.5808               |
4. Summary
In this paper, the natural characteristic modularization calculation of multi-stage planetary drive system for space manipulator joints is presented. Based on the modular idea, the mass and stiffness matrix is respectively divided into four sub-modular matrices. The modularization program for the eigenvalue and eigenvector calculation of multi-stage planetary drive system is developed. Based on the proposed modularization program, natural characteristics of four-stage planetary drive system for space manipulator joints are obtained.

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