Grand Unification and the Principle of Minimal Flavor Violation

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Abstract

Minimal Flavor Violation is an attractive approach to suppress unacceptably large flavor changing neutral currents from beyond the standard model physics at the TeV scale. It can be used in theories with low energy supersymmetry, multi Higgs doublet theories and other extensions of the minimal standard model. We show how minimal flavor violation can be implemented in Grand Unified theories.
1 Introduction

The minimal Standard Model (SM) contains three generations of quarks in left-handed $SU(2)_L$ doublets, $Q_{iL}$, and right-handed $SU(2)_L$ singlets, $u_{iR}$ and $d_{iR}$, and three generations of leptons in left-handed $SU(2)_L$ doublets, $L_{iL}$ and right-handed $SU(2)_L$ singlets $e_{iR}$. The only terms that break the $SU(3)_Q \times SU(3)_U \times SU(3)_D \times SU(3)_L \times SU(3)_E$ flavor symmetry are the quark and lepton Yukawa couplings to the Higgs doublet. Extensions of the standard model with new particles at the TeV scale that couple to the quarks are severely constrained by experimental limits on flavor changing neutral currents. An attractive way to prevent the new physics from generating unacceptably large flavor changing neutral currents is the principle of Minimal Flavor Violation (MFV) which attributes all the breaking of the $SU(3)_Q \times SU(3)_U \times SU(3)_D$ part of the flavor group to the quark Yukawas \[1, 2, 3\]. This principle has been implemented in specific models, such as strongly-interacting theories \[1\], low-energy supersymmetry \[2, 3\], and multi-Higgs models \[3, 4\], but it can also be formulated in terms of a generic non-renormalizable effective field theory valid around and below the TeV scale \[3\].

Neutrino masses can be incorporated into the SM by adding three generations of right-handed singlet neutrinos, $\nu_{iR}$, to the model. The flavor symmetry of the model is now extended by an additional $SU(3)_\nu$ factor. If the Majorana mass of these right handed neutrinos is large, when integrated out they generate dimension five operators that give Majorana masses to the left handed, non-singlet neutrinos. If, on the other hand, the right-handed neutrinos are light, then they combine with the left handed neutrinos into quasi-Dirac particles. Since neutrino masses break the leptonic flavor symmetry of the SM, one can consider extending the notion of MFV to the neutrino sector. The extension is trivial in the case of quasi-Dirac neutrinos, but not so for the case of heavy right handed neutrinos. The latter has the added feature that the small neutrino masses are generated by the see-saw mechanism. The (non-trivial) extension of the principle of minimal flavor violation to the lepton sector has recently been discussed in \[5\] (see also \[6\]).

In Grand Unified Theories (GUT) \[7\] the quarks and leptons appear in the same representations and so implementing the principle of minimal flavor violation requires some modifications. Consider the case where the three generations of fermions fall into the $5$, $10$ and $1$ representations of $SU(5)$. The $5$ representations $\psi_i$ contain the $d_{iR}$ quark fields and the lepton doublet fields $L_{iL}$, the $10$ representations $\chi_i$ contain the quark doublet fields $Q_{iL}$, $u_{iR}$ and the lepton fields $e_{iR}$. Finally the singlet $1$ representations $N_i$ contain the right-handed neutrinos $\nu_{iR}$. Evidently in such a unified theory the $SU(3)_Q \times SU(3)_U \times SU(3)_D \times SU(3)_L \times SU(3)_E \times SU(3)_\nu$ symmetry is reduced to $SU(3)_5 \times SU(3)_{10} \times SU(3)_1$ via the identifications $SU(3)_Q \sim SU(3)_5$ and $SU(3)_U \sim SU(3)_{10}$ and $SU(3)_D \sim SU(3)_1$. In this paper we show how to implement the principle of minimal flavor violation in grand unified theories based on the above particle content of the unifying gauge group $SU(5)$.
2 Flavor group and irreducible symmetry-breaking terms

The starting point to define the irreducible sources of flavor-symmetry breaking in a $SU(5)$ GUT framework are the following renormalizable Yukawa interactions

$$\mathcal{L}_{Y-GUT}^{(0)} = \lambda_5^{ij} \psi_i^T \chi_j H_5^* + \lambda_{10}^{ij} \chi_i^T \chi_j H_5 + \text{h.c.} \ ,$$

where $H_5$ is a Higgs field in the 5 of $SU(5)$ and we have explicitly indicated the $3 \times 3$ flavor indexes (and omitted the $SU(5)$ indexes). Imposing the invariance of $\mathcal{L}_{Y-GUT}^{(0)}$ under the transformations

$$\psi \rightarrow V_5^* \psi \ , \ \ \chi \rightarrow V_{10}^* \chi \ ,$$

which define the $SU(3)_c \times SU(3)_{10}$ flavor-symmetry group, allows us to identify the following spurion transformation properties:

$$\lambda_5 \rightarrow V_5^* \lambda_5 V_{10}^\dagger \ , \ \ \lambda_{10} \rightarrow V_{10}^* \lambda_{10} V_{10}^\dagger \ .$$

Projecting $\mathcal{L}_{Y-GUT}^{(0)}$ into the basis of $SU(3)_c \times SU(2)_L \times U(1)_Y$ light fields, or integrating out the heavy components of $H_5$, leads to the standard model couplings

$$\mathcal{L}_{Y-GUT}^{(0)} \supset \mathcal{L}_{Y-SM} = \lambda_5^{ij} \bar{u}_R Q_L^i h^* + \lambda_{10}^{ij} \bar{d}_R Q_L^i h + \lambda_5^{ij} \bar{e}_R L_L^i h + \text{h.c.} \ ,$$

where $h$ is the SM Higgs field and the Yukawa couplings satisfy the minimal GUT relations

$$\lambda_5 \propto \lambda_{10} \ , \ \ \lambda_d \propto \lambda_e^T \propto \lambda_5 \ .$$

A natural extension of this minimal set up is provided by the introduction of two Higgs fields, a 5 and a $\bar{5}$, in the $SU(5)$ Yukawa interaction, with appropriate assignments of $U(1)$ charges in order to avoid tree-level FCNCs:

$$\mathcal{L}_{Y-GUT}^{(2H)} = \lambda_5^{ij} \psi_i^T \chi_j H_5^* + \lambda_{10}^{ij} \chi_i^T \chi_j H_5 + \text{h.c.} \ ,$$

In this case the low-energy Yukawa interaction becomes a two-Higgs doublet model of type-II. This choice allows us to change the relative normalization of the spurions $\lambda_5$ and $\lambda_{10}$, but does not modify the proportionality between $\lambda_d$ and $\lambda_e^T$ in Eq. (5).

The proportionality between $\lambda_d$ and $\lambda_e^T$ implies the following relations (evaluated at the GUT scale):

$$m_\tau = m_b \ , \ \ \frac{m_\mu}{m_\tau} = \frac{m_s}{m_b} \ , \ \ \frac{m_e}{m_\mu} = \frac{m_d}{m_s} \ .$$

The last of these relations is badly broken by the experimental values of fermion masses and, as we will discuss in the next section, is very stable with respect to radiative corrections. As a result, we are forced to introduce new $SU(3)_5 \times SU(3)_{10}$ spurions which break the relation $\lambda_e^T \propto \lambda_d$. A simple way to achieve this goal is provided by the introduction of the dimension-five operator

$$\frac{1}{M} (\lambda_5')^{ij} \psi_i^T \Sigma \chi_j H_5 + \text{h.c.} \ ,$$

(see also [10])
where $M$ is a heavy scale ($M \gg M_{\text{GUT}}$), $\Sigma$ is a Higgs field in the adjoint representation of $SU(5)$, and $\lambda'_5$ is a new spurion (transforming as $\lambda_5$ under the flavor group). The high-scale vev of $\Sigma$ breaks $SU(5)$ preserving $SU(2)_L \times U(1)_Y$,

$$
\langle \Sigma \rangle = M_{\text{GUT}} \text{diag}(1, 1, 1, -3/2, -3/2),
$$

such that the GUT relations (5) are modified, thus:

$$
\lambda_u \propto \lambda_{10}, \quad \lambda_d \propto (\lambda_5 + \lambda'_5), \quad \lambda_{e}^T \propto \left(\lambda_5 - \frac{3}{2} \lambda'_5\right),
$$

where we have redefined the spurion $\lambda'_5$ incorporating the $M_{\text{GUT}}/M$ suppression factor. Note that the Higgs combination $\Sigma H_5$ appearing in Eq. (8) contains a $45$ representation of $SU(5)$. Thus a completely equivalent result can be obtained with a single $45$ Higgs field—with appropriate electroweak-scale vev breaking $SU(2)_L \times U(1)_Y$—and a renormalizable Yukawa interaction. The higher-dimensional operator in Eq. (8) has the advantage of providing a natural explanation for the smallness of $\lambda'_5$ which is supported by experimental data.

Whether we start from $\mathcal{L}_{Y^{-\text{GUT}}}^{(0)}$ or $\mathcal{L}_{Y^{-\text{GUT}}}^{(2H)}$, and whether we add the non-renormalizable term in Eq. (8) or a $45$ Higgs Yukawa term, a successful description of quark and charged-lepton masses requires the introduction of three independent symmetry-breaking spurions. From this point of view, the situation is very similar to the non-GUT MFV case analyzed in [3]. However, the flavor structure of the theory is quite different with respect to the non-GUT case: the symmetry group is substantially smaller and this allows the symmetry-breaking sources to appear in more ways in the low-energy effective operators.

There are also non-renormalizable operators that contribute to the up-type quark mass matrix. For example,

$$
\frac{1}{M} \langle \lambda_{10} \rangle^{ij} \lambda_i^T \Sigma \chi_j H_5.
$$

Dimensional analysis suggests such operators could contribute significantly to the up-quark mass. Since there are no mass relations for the up-type quarks we neglect such contributions in this paper. This convenient assumption has the advantage of leaving the up-type quark mass matrix symmetric, which simplifies our analysis of structures that give rise to flavor violation in the low energy effective theory. As above, we could replace the $\Sigma H_5/M$ combination by a $45$ Higgs which leads to a potentially large shift in the up-type Yukawa by an anti-symmetric matrix. We again neglect this contribution for simplicity, leaving the up-type quark mass matrix symmetric.

As far as the neutrino sector is concerned, the most natural choice in this framework is the introduction of three $SU(5)$-singlet fields corresponding to the right-handed neutrinos. In general, this implies the enlargement of the flavor group to $SU(3)_5 \times SU(3)_{10} \times SU(3)_1$ and the inclusion of two new spurions:

$$
\mathcal{L}_{Y^{-\text{GUT}}}^{(\nu)} = \lambda_i^{ij} N_i^T \psi_j H_5 + M_R^{ij} N_i^T N_j + \text{h.c.}
$$

Note that the spurion $M_R$ breaks both the flavor-symmetry subgroup $SU(3)_1$ and the $U(1)_{\text{LN}}$ associated to total lepton number. In addition, the right-handed neutrino mass
term breaks a discrete symmetry under which all spinor fields $f$ transform according to $f \to if$ and the Higgs field(s) transform as $H \to -H$. Hence it is technically natural to take $M_R$ to be significantly smaller than $M_{GUT}$. A second consequence of this discrete symmetry is that $M_R$ cannot be radiatively generated, even though the spurion combination $\lambda_1 \lambda_5^* \lambda_{10} (\lambda_1 \lambda_5^*)^T$ transform precisely like the spurion $M_R$ under the flavor symmetries. The third and most interesting consequence of this discrete symmetry is that it constrains the scale of the dimension five operator that gives rise to the left-handed neutrino masses in the low energy effective field theory. To see this, focus on the case where the eigenvalues of $M_R$ are all much larger than the scale of new physics $\Lambda$. In the low energy effective theory the left-handed neutrino mass arises from a dimension-five operator $h L^T_L L_L h$, where $h$ stands for the Higgs doublet and flavor and gauge indexes have been suppressed. The spurion analysis alone allows as coefficient to this operator the combination $\lambda_5 \lambda_{10}^* \lambda_5^T / \Lambda$, but this is forbidden by the discrete symmetry $L_L \to i L_L$, $h \to -h$. On the other hand, since the spurion $M_R$ transforms as $M_R \to -M_R$, the coefficient $\lambda_1^T M_R^{-1} \lambda_1$ is allowed by both flavor and discrete symmetries. Hence the dimension-five operator is necessarily suppressed by the large mass scale $M_R$.

If neutrinos are quasi-Dirac particles then $\lambda_1$ and $M_R$ are very small, $\lambda_1 \sim M_R/v \lesssim 10^{-11}$, where $v$ is the SM Higgs doublet expectation value. Hence, in the case that neutrinos are quasi-Dirac particles the effect of these matrices on flavor physics is unobservably small, and all new effects are codified in $\lambda_{10}$ and $\lambda_5^{(q)}$. This is not the case if neutrinos are Majorana particles, in which instance the right-handed neutrinos may be heavy. In fact, very heavy right-handed neutrinos, with mass below $M_{GUT}$ but well above the TeV scale give rise automatically to light Majorana neutrinos, of mass $\sim |\lambda_1 v|^2 / M_R$, through the see-saw mechanism. In this situation $\lambda_1$ may be large enough to have sizable effects on flavor physics. This is explored below in the following two sections where we largely focus on the heavy right-handed neutrino case.

Summarizing, the transformation properties of irreducible spurions and low-energy fields are:

\[
\begin{align*}
Q_L &\rightarrow V_{10} Q_L & \lambda_{10} &\rightarrow V_{10}^* \lambda_{10} V_{10}^T \\
 u_R &\rightarrow V_{10}^* u_R & \lambda_5 &\rightarrow V_{5}^* \lambda_5 V_{10}^T \\
d_R &\rightarrow V_{5}^* d_R & \lambda_5' &\rightarrow V_{5}^* \lambda_5' V_{10}^T \\
 L_L &\rightarrow V_{5} L_L & \lambda_1 &\rightarrow V_{1}^* \lambda_1 V_{5}^T \\
e_R &\rightarrow V_{10}^* e_R & M_R &\rightarrow V_{1}^* M_R V_{1}^T
\end{align*}
\]
3 Quark and Lepton Masses

Taking into account radiative corrections, the effective low-energy (electroweak-scale) Yukawa couplings responsible for quark and charged-lepton masses assume the following form:

\[ \lambda_u = a_u \left[ \lambda_{10} + \epsilon_{1u} \lambda_{10} \lambda_{10}^\dagger \lambda_{10}^\dagger + \epsilon_{2u} \lambda_{10} \lambda_{10}^\dagger \lambda_{5}^\dagger + \ldots \right], \]
\[ \lambda_d = a_d \left[ \left( \lambda_5 + \lambda_5' \right) + \epsilon_{d1} \lambda_5^\dagger + \epsilon_{d2} \lambda_5^\dagger \lambda_{10} \lambda_{10}^\dagger + \epsilon_{d3} \lambda_5^\dagger \lambda_5^\dagger \right] \lambda_{10}, \]
\[ \lambda_e^T = a_e \left[ \left( \lambda_5 - \frac{3}{2} \lambda_5' \right) + \epsilon_{e1} \lambda_5^\dagger + \epsilon_{e2} \lambda_5^\dagger \lambda_{10} \lambda_{10}^\dagger + \epsilon_{e3} \lambda_5^\dagger \lambda_5^\dagger \right], \quad (14) \]

where \( \lambda_5^{(i)} \) denotes either \( \lambda_5 \) or \( \lambda_5' \). The leading \( a_i \) coefficients encode potentially large QCD logarithms (in the case of the quark Yukawa couplings). Since we ignore the dynamical details of the underlying theory, we do not make specific assumptions about the values of these coefficients. We only impose (to simplify the analysis) the natural hierarchy

\[ a_i = O(1), \quad \epsilon_i \ll 1, \quad (15) \]

which holds in most scenarios. The small coefficients, \( \epsilon_i \), are induced by radiative corrections and are naturally of \( O(0.1) \). In this limit the series in Eq. (14) are dominated by the terms not suppressed by the \( \epsilon_i \). As we will discuss below, relaxing the assumption about the smallness of the \( \epsilon_i \) complicates the reconstruction of the GUT Yukawa couplings in terms of the measured fermion masses, but it does not modify in a significant way their hierarchical structure.

The two matrices \( \lambda_5 \) and \( \lambda_5' \) have eigenvalues that are definitely not proportional,\(^1\) but they have a similar hierarchical structure. A strong hierarchy is also exhibited by the eigenvalues of \( \lambda_{10} \). For this reason the higher-order combinations of \( \lambda_5^{(i)} \) and \( \lambda_{10} \) appearing in Eq. (14) have a very limited impact on the first two generations (compared to the linear terms): in absence of \( \lambda_5 \) the GUT relation \( m_e/m_\mu = m_d/m_s \) is almost unchanged even for \( \epsilon_i = O(1) \). In principle, one could hope to modify the \( m_e/m_\mu = m_d/m_s \) relation with a suitable choice of the \( \lambda_5^T \lambda_5^T \lambda_5 \) terms (\( \lambda_1 \) contains large mixing angles and is not very hierarchical). However, these terms have a non-trivial impact in Eqs. (14) only if the normalization of \( \lambda_1 \) is sufficiently high. As we will show in section 4 in this case one would induce too large FCNCs in the down-quark sector. As a result, the most natural solution to break the \( m_e/m_\mu = m_d/m_s \) relation is the introduction of the additional \( 5 \times 10 \) spurion \( \lambda_5' \). Motivated by the phenomenological success of the first GUT relation in Eq. (7), throughout this paper we will assume that the maximal eigenvalue of \( \lambda_5 \) is much larger than the maximal eigenvalue of \( \lambda_5' \).

The effective Majorana mass matrix of low-energy neutrinos, obtained by integrating out the \( N_i \) fields and other sources of \( U(1)_{LN} \) breaking, has the following general structure:

\[ m_\nu = \frac{\epsilon^2}{M_\nu} \left[ \lambda_1^T \frac{M_\nu}{M_R} \lambda_1 + \epsilon_{1\nu} \lambda_5^T \lambda_5^\dagger \lambda_{10} \lambda_{10}^\dagger + \ldots \right]. \quad (16) \]

\(^1\) Neglecting the higher-order terms, these matrices are diagonalized by bi-unitary transformations to go to the quark and lepton mass eigenstate basis.
Making contact with our previous papers \cite{5,12,13}, we have indicated with $M_\nu$ the average right-handed neutrino mass, which we assume to be the dominant (lighter) source of total lepton-number breaking. The $\epsilon_i$ in Eq. (16) take into account the possible effects of additional $U(1)_{L \nu}$ breaking terms (and additional breaking of the discrete symmetry $L_L \to i L_L$, $h \to -h$), normalized to $1/M_\nu$. Also in this case we assume $\epsilon_i \ll 1$, such that the standard see-saw mechanism dominates $m_\nu$.

To display explicitly the strength of the flavor changing neutral transitions, it is convenient to transform to the quark and lepton fields mass-eigenstate basis. To this end we make unitary transformations on the spinor fields as follows:

\begin{align}
U & \rightarrow V_{uL} u_L , \quad u_R \rightarrow V_{uR} u_R , \quad d_L \rightarrow V_{dL} d_L , \quad d_R \rightarrow V_{dR} d_R , \quad \text{(17)} \\
E & \rightarrow V_{eL} e_L , \quad e_R \rightarrow V_{eR} e_R , \quad \nu_L \rightarrow V_{\nuL} \nu_L , \quad \text{(18)}
\end{align}

These transformations are chosen to diagonalize the mass matrices,

\begin{align}
\lambda_u = V^T_{uR} \lambda_u V_{uL} , \quad \lambda_d = V^T_{dR} \lambda_d V_{dL} , \quad \lambda_e = V^T_{eR} \lambda_e V_{eL} , \quad \tilde{m}_\nu = V^T_{\nuL} m_\nu V_{\nuL} , \quad \text{(19)}
\end{align}

where $\lambda_u, \lambda_d, \lambda_e$ and $\tilde{m}_\nu$ are diagonal. Note that $V^*_{uR} = V_{uL}$, which follows from $\lambda^T_u = \lambda_u$, a feature in GUT models which does not generally hold in non-unified theories. Flavor changing interactions occur when the operators in the effective Hamiltonian fail to remain flavor diagonal after these unitary transformations are performed. The well known CKM matrix appears in the transformation of a charged current, $\bar{u}_L \gamma^\mu d_L \rightarrow \bar{u}_L V^+_{uL} d_L \gamma^\mu d_L$, so $V_{\text{CKM}} = V^+_{uL} V_{dL}$. Similarly, $U_{\text{PMNS}} = V^+_{eL} V_{\nuL}$.

Note that without loss of generality we can choose a $SU(5)$ invariant basis (or appropriate flavor rotations of the $\bar{5}$ and $10$ fermion fields) that have $\lambda_d$ diagonal. In this basis\textsuperscript{2},

\begin{align}
V_{dL} = V_{dR} = I , \quad V_{uL} = V^T_{\text{CKM}} , \quad V_{\nuL} = V_{eL} U_{\text{PMNS}} \quad \text{(20)}
\end{align}

so that the Yukawa matrices read

\begin{align}
\lambda_u = V^T_{\text{CKM}} \lambda_u V_{\text{CKM}} , \quad \text{(21)} \\
\lambda_e = V_{eR} \lambda_e V^T_{eL} , \quad \text{(22)} \\
\lambda_1 = \frac{M_R^{1/2}}{v} R \left(\tilde{m}_\nu\right)^{1/2} U^T_{\text{PMNS}} V^T_{eL} , \quad \text{(23)}
\end{align}

where $R$ is a complex orthogonal matrix \cite{14}. If we impose $M_R = M_\nu I$ then the last equation takes the form $\lambda_1 = (M_\nu^{1/2}/v) H (\tilde{m}_\nu)^{1/2} U^T_{\text{PMNS}} V^T_{eL}$, where $H$ is orthogonal and hermitian. If we further impose that CP is conserved in the neutrino Yukawas, then we can set $H \rightarrow I$. In this limit, the mixing matrices necessary to describe all possible flavor-changing processes are the well-known $V_{\text{CKM}}$ and $U_{\text{PMNS}}$, and the two additional matrices $V_{eL}$ and $V_{eR}$ which control the diagonalization of $\lambda_e$ in the basis where $\lambda_d$ is diagonal.

\textsuperscript{2} Here $I$ denotes the $3 \times 3$ identity matrix.
4 FCNC transitions

4.1 Old Mixing Structures

The low energy effects of new physics that results from integrating out fields at a scale $\Lambda$ is described by an effective Hamiltonian constructed from SM fields. It contains an infinite series of operators of ever increasing dimension, starting from dimension five, with inverse powers of $\Lambda$ included to give correct engineering dimensions. Hence the low energy effects are suppressed by powers of the low energy scale over $\Lambda$, and the higher the dimension of the operators the higher the power of the suppression. We will concentrate on operators of dimension six at most. The basic building blocks of operators that may produce flavor changing neutral effects are fermion bilinears. In the non-GUT framework analyzed in Ref. [3, 5], all the relevant FCNC amplitudes are constructed in terms of the following bilinear combinations of fermion fields:

- **quarks**: $\bar{Q}_L\lambda_u^\dagger\lambda_uQ_L$, $\bar{d}_R\lambda_d^\dagger\lambda_dQ_L$, (24)
- **leptons**: $\bar{L}_L\lambda_1^\dagger\lambda_1L_L$, $\bar{e}_R\lambda_e^\dagger\lambda_1L_L$. (25)

In these bilinears the Dirac indices are implicit and free, that is, they are not contracted. We have only retained combinations up to cubic order in the Yukawa couplings. Some combinations, like $\bar{Q}_L\lambda_u^\dagger\lambda_d^\dagger\lambda_dQ_L$, have not been listed because they produce smaller effects than the one listed, but they could become relevant in a two Higgs doublet model at large $\tan\beta$.

Consider the first operator in Eq. (24). It contains the down-type quark term which when transformed to the quark mass eigenstate basis becomes,

$$\bar{d}_L\lambda_u^\dagger\lambda_u d_L \rightarrow \bar{d}_L V_{dL}^\dagger \lambda_u^\dagger \lambda_u V_{dL} d_L = \bar{d}_L \Delta^{(q)} d_L$$

(26)

where $\Delta^{(q)}$ is completely specified in terms of the up-type quark masses and the CKM mixing angles:

$$\Delta^{(q)}_{ij} = V_{CKM}^\dagger \lambda_2^\dagger U_{PMNS} \bar{m}_u^2 \left[ (V_{CKM})_{3i}^* (V_{CKM})_{3j} + O(m^2_{c,u}/m_t^2) \right].$$

(27)

Similarly the first operator in Eq. (25) contains the charged lepton transition term

$$\bar{e}_L\lambda_1^\dagger\lambda_1 e_L \rightarrow \bar{e}_L V_{eL}^\dagger \lambda_1^\dagger \lambda_1 V_{eL} e_L = \bar{e}_L \Delta^{(l)} e_L,$$

(28)

with

$$\Delta^{(l)} \equiv U_{PMNS} (\bar{m}_\nu)^{1/2} R_i \frac{M_R}{v^2} R (\bar{m}_\nu)^{1/2} U_{PMNS}^\dagger.$$

(29)

Here the matrix $\Delta^{(l)}$ can be expressed in terms of neutrino masses and mixings if one makes the simplifying assumption of degenerate heavy neutrinos and approximate $CP$ invariance [5]. In this case

$$\Delta^{(l)}_{ij} = \frac{M_\nu}{v^2} \left[ m_{\nu_1} \delta_{ij} + (U_{PMNS})_{i2} (U_{PMNS}^*)_{j2} (m_{\nu_2} - m_{\nu_1})
+ (U_{PMNS})_{i3} (U_{PMNS}^*)_{j3} (m_{\nu_3} - m_{\nu_1}) \right] = \frac{M_\nu \sqrt{\Delta m^2_{\text{atm}}}}{v^2} \delta^{(l)}_{ij},$$

(30)
where, for later convenience we have introduced the ‘reduced’ couplings \( \delta_{ij}^{(l)} \) which are free of the (unknown) overall normalization. More explicitly, using the PDG notation of the PMNS matrix (with the convention \( s_{13} \geq 0 \)), assuming maximal mixing for the atmospheric neutrinos and denoting with \( s \) and \( c \) sine and cosine of the solar mixing angle, we find

\[
\delta_{21}^{(l)} = \frac{1}{\sqrt{2} \Delta m_{\text{atm}}^2} \left[ s_c (m_{\nu_2} - m_{\nu_1}) \pm s_{13} (m_{\nu_3} - m_{\nu_1}) \right], \tag{31}
\]

\[
\delta_{31}^{(l)} = \frac{1}{\sqrt{2} \Delta m_{\text{atm}}^2} \left[ -s_c (m_{\nu_2} - m_{\nu_1}) \pm s_{13} (m_{\nu_3} - m_{\nu_1}) \right], \tag{32}
\]

\[
\delta_{32}^{(l)} = \frac{1}{2 \sqrt{\Delta m_{\text{atm}}^2}} \left[ -c^2 (m_{\nu_2} - m_{\nu_1}) + (m_{\nu_3} - m_{\nu_1}) \right], \tag{33}
\]

where the + and − signs correspond to \( \delta = 0 \) and \( \pi \), respectively. In the normal hierarchy case (\( \nu_1 \) is the lightest neutrino), one has:

\[
m_{\nu_2} - m_{\nu_1} \xrightarrow{m_{\nu_1} \to 0} \sqrt{\Delta m_{\text{sol}}^2}, \quad m_{\nu_3} - m_{\nu_1} \xrightarrow{m_{\nu_1} \to 0} \sqrt{\Delta m_{\text{atm}}^2}, \tag{34}
\]

while in the inverted hierarchy case (\( \nu_3 \) is the lightest neutrino)

\[
m_{\nu_2} - m_{\nu_1} \xrightarrow{m_{\nu_1} \to 0} \frac{\Delta m_{\text{sol}}^2}{2 \sqrt{\Delta m_{\text{atm}}^2}}, \quad m_{\nu_3} - m_{\nu_1} \xrightarrow{m_{\nu_1} \to 0} - \sqrt{\Delta m_{\text{atm}}^2}. \tag{35}
\]

The key point which emerges by comparing Eqs. (31)–(33) and Eq. (27) is the fact that \( \delta_{ij}^{(l)} \) is substantially less hierarchical than \( \Delta_{ij}^{(q)} \). In the 2–3 case, whose result is insensitive to the value of \( s_{13} \) and is also very stable with respect to possible \( CP \)-violating parameters in \( H \), we have

\[
|\delta_{32}^{(l)}| \approx \frac{1}{2} \quad \text{vs.} \quad |\Delta_{32}^{(q)}| \approx 0.04. \tag{36}
\]

The difference is even more pronounced in the 1–2 case, where

\[
|\delta_{12}^{(l)}| \approx \max \left[ \frac{s_{13}}{\sqrt{2}}, \frac{s_c \sqrt{\Delta m_{\text{sol}}^2}}{\sqrt{2} \Delta m_{\text{atm}}^2} \right] \approx 0.1 \quad \text{vs.} \quad |\Delta_{12}^{(q)}| \approx 3 \times 10^{-4}. \tag{37}
\]

In principle, there are fine-tuned scenarios where the two components in Eq. (31) tend to cancel each other yielding smaller values of \( |\delta_{12}^{(l)}| \). However, if we allow non-vanishing \( CP \)-violating parameters in the neutrino Yukawa coupling, then \( \delta_{12}^{(l)} \) receives additional contributions proportional to \( \sqrt{\Delta m_{\text{atm}}^2} \) not suppressed by \( s_{13} \) [13]: these new terms naturally increase the size of \( |\delta_{12}^{(l)}| \), making these fine-tuned scenarios even more unlikely. Thus the estimate in (37) can be considered as a fairly general lower bound on \( |\delta_{12}^{(l)}| \).
4.2 New Mixing Structures

As anticipated, the restricted flavor group of the GUT framework allows more independent spurion combinations. In particular, potentially interesting effects arise from:

\begin{align}
\text{quarks :} & \quad \bar{Q}_L (\lambda_e \lambda_e^T)^T Q_L , \quad (38) \\
& \quad d_R \lambda_e^T (\lambda_e \lambda_e^T)^T Q_L , \quad (39) \\
& \quad \bar{d}_R (\lambda_e^T \lambda_e )^T d_R , \quad (40) \\
\text{leptons :} & \quad L_L (\lambda_d \lambda_d^T)^T L_L , \quad (41) \\
& \quad \bar{\ell}_R (\lambda_d \lambda_d^T)^T L_L , \quad (42) \\
& \quad \bar{\ell}_R \lambda_d^T \lambda_d^T e_R , \quad (43)
\end{align}

and terms obtained by the exchange \( \lambda_d \leftrightarrow \lambda_e^T \) in any of the bilinears in (24)–(25) and (38)–(43). Note the unusual structure, with right-handed down-type and charged-lepton fields not necessarily suppressed by the corresponding Yukawa couplings. To display explicitly the strength of the flavor changing neutral transitions we go to the quark and lepton mass eigenstate basis. While the structures in (24) and (25) can be expressed solely in terms of the diagonal Yukawas and the matrices \( \Delta^{(q)} \) and \( \Delta^{(l)} \), that is not the case with the bilinears allowed by the restricted flavor group of the GUT framework in (38)–(43).

Consider, for example, the bilinear (38). In the mass-eigenstate basis it gives the flavor neutral bilinears

\[ u_L \left( V^*_u \tilde{V} u_e \tilde{V}^T u_L \right) u_L , \quad \text{and} \quad d_L \left( V^*_d \tilde{V} d_e \tilde{V}^T d_L \right) d_L . \]

(44)

There is a new mixing matrix, \( C \equiv V^T_{eR} V_{dL} \), in terms of which these bilinears take the form

\[ u_L \left( V_{\text{CKM}} C^\dagger \tilde{\lambda}_e^2 C V_{\text{CKM}}^T \right) u_L , \quad \text{and} \quad d_L \left( C^\dagger \tilde{\lambda}_e^2 C \right) d_L . \]

(45)

The complete list of new, independent mixing matrices is

\[ C = V^T_{eR} V_{dL} \quad (46) \]
\[ G = V^T_{eL} V_{dR} \quad (47) \]

As shown in section 3, these two matrices diagonalize \( \lambda_e \) in the basis where \( \lambda_d \) is diagonal. Indeed \( C \to I \) and \( G \to I \) in the limit \( \lambda_e^T \to \lambda_d \). The flavor changing neutral bilinears that follow from (38)–(43) can be readily expressed in terms of \( C \) and \( G \). For example, the spurions in \( \bar{d}_L \otimes d_L \) of (38), including those that follow by replacing \( \lambda_e \to \lambda_e^T \) are

\[ C^\dagger \tilde{\lambda}_e^2 C , \quad C^\dagger \tilde{\lambda}_e G \lambda_d \], and \( \tilde{\lambda}_d G^\dagger \tilde{\lambda}_e C \).

(48)

Note that no new independent mixing matrices arise involving the transformations on \( u \)-quarks, because \( V^*_u \) is not independent of \( V_{uL} \) and the latter can be traded for \( V_{dL} \tilde{V}^T_{\text{CKM}} \). Had we kept the non-symmetric contribution to \( \lambda_u \) proportional to \( \lambda_{10}^T \) (see Eq. (11)) one more structure, the matrix \( V_{uL}^T V_{uR} \) would arise.

The two new mixing structures arise from different alignment between the mass matrices of charge \(-1/3\) quarks and charged leptons. As seen in Eq. (14), the misalignment,
due to $\lambda'_3$ and the higher order terms, is small compared to the largest eigenvalue. This allows one to determine the texture of the matrices $C$ and $G$ by proceeding in two steps: (i) first diagonalize the upper $2 \times 2$ block of $\lambda_d$ and $\lambda'_T$ via bi-unitary block-diagonal transformations; (ii) then diagonalize the remaining structure using perturbation theory, through unitary matrices of the form $U_\epsilon = I + \epsilon$, with $\epsilon^\dagger = -\epsilon$ and $\epsilon \sim O(e^2)$. It follows that the matrices $C$ and $G$ have a hierarchical structure,

$$
C = \left( \begin{array}{c|c} U_C & U_C \epsilon_C \\ \hline -\epsilon_C^\dagger & 1 \end{array} \right) + O(\epsilon^2),
$$

(49)

where $U_C$ is a unitary $2 \times 2$ matrix and $\epsilon_C$ is a $2 \times 1$ matrix with small components, of order $m_\mu/m_\tau$. The structure of the matrix $G$ is analogous, obtained by replacing $U_G$ for $U_C$ and $\epsilon_C$ for $\epsilon_G$. To understand the consequences of this structure consider, for example, the first combination in (48). A $b \to s$ or $b \to d$ transition may involve the largest eigenvalue $\bar{\lambda}'_2$ but is accompanied by one power of $\epsilon_C \sim 5 \times 10^{-2}$. Had we been ignorant about the structure of $C$ we would have been forced to conclude that any one off diagonal component of $C^\dagger \bar{\lambda}'_2 C$ could be in principle as large as $\bar{\lambda}'_2$.

### 4.3 Phenomenological constraints from FCNC processes

A substantial simplification on the possible new structures arises in the limit of a single light Higgs boson, or two Higgs bosons with similar vevs, where $(\bar{\lambda}_{e,d})_{33}/(\bar{\lambda}_u)_{33} \sim m_{b,\tau}/m_t \ll 1$. This strong suppression factor, combined with the hierarchy of $C$ and $G$, allows us to neglect all terms with at least two powers of $\lambda_{e,d}$. Note that this is not a good approximation in two Higgs doublet models at large $\tan \beta$. As a result, the complete list of phenomenologically relevant bilinear structures to be added to ‘old’ terms in (24) reduces to

$$
\begin{align*}
\bar{d}_R (\lambda'_{3} \lambda_1) \lambda_{1}^T \bar{d}_R & \quad \bar{d}_R G^T (\Delta^{(q)})^T G \bar{d}_R \\
\bar{d}_R (\lambda_\epsilon \lambda'_3 \lambda_1) \lambda_{1}^T \bar{d}_R & \quad \bar{d}_R G^T (\Delta^{(q)})^T \bar{\lambda}_e \bar{d}_R \\
\bar{d}_R (\lambda'_1 \lambda_1)^T \lambda_{d} \bar{d}_R & \quad \bar{d}_R G^T (\Delta^{(q)})^T \bar{\lambda}_d \bar{d}_R \\
\bar{\epsilon}_R \lambda_u \lambda^\dagger_1 e_R & \quad \bar{\epsilon}_R [C \Delta^{(q)} C^\dagger]^* e_R \\
\bar{\epsilon}_R \lambda_{u} \lambda^\dagger_{d} \lambda_{d} e_R & \quad \bar{\epsilon}_R [C \Delta^{(q)} \bar{\lambda}_d G^T]^* e_R \\
\bar{\epsilon}_R \lambda_u \lambda^\dagger_{e} \lambda_{e} \bar{L}_R & \quad \bar{\epsilon}_R [C \Delta^{(q)} C^\dagger]^* \bar{\lambda}_e e_R
\end{align*}
$$

(50)

where the arrow denotes the expressions relevant to down-type quarks and charged leptons (in the corresponding mass-eigenstate basis). In the following we will analyze some phenomenological consequences of these new structures derived from $\Delta F = 2$ quark processes and radiative FCNC decays.

First of all, it is clear that if the overall normalization of $\lambda_1$ is sufficiently small, we have no practical deviations from the non-GUT MFV scenario in the quark sector. As remarked above, this conclusion does not hold with two-Higgs doublets and large $\tan \beta$: given that $|C_{13}|$ is parametrically larger than $|(V_{\text{CKM}})_{13}|$, if $(\bar{\lambda}_e)_{33} \approx m_\tau \tan \beta/v$ is sufficiently large the bilinear in (45), which has two factors of $\lambda_{e}$, can generate sizable non-standard FCNC contributions in $b \to d$ and $s \to d$ transitions.
In order to quantify how small the normalization of $\lambda_1$ should be not to affect quark FCNC transitions, we analyze $\Delta F = 2$ processes, on which several precise experimental data are available [$\epsilon_K$ for $2 \leftrightarrow 1$ mixing, $\Delta M_{B_d}$ and $\mathcal{A}_{CP}(B_d \rightarrow \psi K^0)$ for $3 \leftrightarrow 1$, $\Delta M_{B_s}$ for $3 \leftrightarrow 2$]. Here the relevant dimension-six effective Hamiltonian describing new-physics effects (renormalized around the electroweak scale) contains

$$\mathcal{H}_{\Delta F=2} = \frac{1}{\Lambda^2} \left[ c_1 (\bar{Q}_L \lambda^T \lambda Q_L)^2 + c_2 (\bar{d}_R (\lambda^T \lambda_1)^T d_R)^2 \right], \quad (51)$$

where $\Lambda$ denotes the effective scale of new physics. The main virtue of the MFV hypothesis is to allow new physics close to the electroweak scale, as expected by a natural stabilization of the electroweak symmetry-breaking sector. Given the normalization of $\mathcal{H}_{\Delta F=2}$, the natural value of $\Lambda$ for $c_i = \mathcal{O}(1)$ is $\Lambda \lesssim 10$ TeV, if the new physics that generates these operators is related to the solution of the hierarchy problem.\(^3\) In the case of the first operator in $\mathcal{H}_{\Delta F=2}$, which survives also in the non-GUT case, present data implies the 95 %CL bound $\Lambda > 5.7$ TeV (for $c_1 = 1$)\(^1\), which is consistent with this naturalness assumption. We estimate that the second term in $\mathcal{H}_{\Delta F=2}$ is consistent with present experimental limits provided it does not exceed the largest allowed value of the first one (reached for $\Lambda = 5.7$ TeV with $c_1 = 1$). Therefore, with $c_2 = 1$, the new mixing term must satisfy the condition

$$\left| \left( G^\dagger (\Delta^{(i)})^T G \right)_{i \neq j} \right| = \frac{M_\nu \Delta m_{\text{ atm}}^2}{v^2} \left| \left( G^\dagger (\delta^{(i)})^T G \right)_{i \neq j} \right| \lesssim \Delta_{(q)}^{(i)} \quad (52)$$

which constraints the overall normalization of $\lambda_1$. The unknown structure of $G$ combined with the unknown hierarchy of the neutrino mass spectrum does not allow a precise evaluation of the l.h.s. of (52). However, the variability range is quite limited thanks to the non-hierarchical structure of $\Delta^{(i)}$ (see section 4.1). Barring accidental cancellations, we find

$$0.1 \lesssim \left| \left( G^\dagger (\delta^{(i)})^T G \right)_{i \neq j} \right| \lesssim 1. \quad (53)$$

The most conservative scenario (as far as the extraction of lower bounds on $M_\nu$ is concerned) is obtained for the normal hierarchy of the neutrino spectrum and $m_{\nu_1} \rightarrow 0$, where we obtain\(^4\)

$$M_\nu \lesssim \begin{cases} 
6 \times 10^{13} \text{ GeV} & \text{from } 2 - 3 \text{ mixing}, \\
2 \times 10^{12} \text{ GeV} & \text{from } 1 - 2 \text{ mixing}.
\end{cases} \quad (54)$$

Interestingly, the most stringent upper limit on $M_\nu$ in (54) is very close but not incompatible with the lower limit $M_\nu \gtrsim 10^{12}$ GeV derived in [13] as the condition for successful leptogenesis (with MFV and degenerate right-handed neutrinos). Since the most stringent constraint in (54) arises from 1–2 mixing, if $M_\nu \sim 10^{12}$ GeV we could

\(^3\) The effective FCNC operators generated within the SM by integrating out the top-quark and the heavy gauge-boson fields, correspond to an effective scale $\Lambda_0 = \sin \theta_W M_W / \alpha_\text{em} \approx 2.4$ TeV.\(^3\)

\(^4\) In the case of normal hierarchy and $m_{\nu_1} \rightarrow 0$, the 1–2 mixing term in (54) is $\mathcal{O}((\delta^{(i)})_{12})$. For inverted hierarchy and large 1–2 entries in $G$, the mixing can become larger resulting in more stringent constrains on $M_\nu$.\(^4\)
expect deviations from the non-GUT MFV relations of Ref. [3] in rare $K$ decays, while we should not see appreciable effects in $B$ physics. This is in contrast with the expectation of non-MFV GUT models such as the one analyzed in [17], where the large 2–3 mixing angle in the neutrino sector is used to advocate large FCNC contributions in $b \to s \gamma$. 

Once we impose that the normalization of $\lambda_1$ is low enough to pass the constraints from $\Delta F = 2$ processes, the two LR quark bilinears in (50) do not play a significant role in $b \to s \gamma$. The situation is much more interesting for the radiative lepton decays $l_i \to l_j \gamma$, whose relevant effective Hamiltonian (written below the scale of $SU(2)_L \times U(1)_Y$ breaking) contains

$$\Delta H^{\Delta F=1}_{\text{eff}} = \frac{\bar{v}}{\Lambda^2} e_R \left[ c'_1 \lambda_e \lambda_1 + c'_2 \lambda_u \lambda_e + c'_3 \lambda_u \lambda_3 \right] \sigma^{\mu \nu} e_L F_{\mu \nu}. \quad (55)$$

Here the new mixing terms induced by the quark Yukawa couplings could dominate over the non-GUT FCNC structures analyzed in [3].

We start recalling the upper bounds on $M_\nu$ following from the non-observation of the $l_i \to l_j \gamma$ processes in the non-GUT case ($c'_2 = c'_3 = 0$). In this case the parametric pattern of the leading FCNC couplings is

$$\left| (\tilde{\lambda}_e \Delta^{(l)})_{ij} \right| \sim \frac{M_\nu}{v^2} \sqrt{m^{2}_{\text{atm}}} \times \begin{cases} \mathcal{O}(1) \tilde{\lambda}_\tau \quad (\tau_R \to \mu_L) \\
\mathcal{O}(0.1) \tilde{\lambda}_\tau \quad (\tau_R \to e_L) \\
\mathcal{O}(0.1) \tilde{\lambda}_\mu \quad (\mu_R \to e_L) \end{cases} \quad (56)$$

The dominant constraint comes from the experimental limit $\mathcal{B}(\mu \to e\gamma) < 1.2 \times 10^{-11}$. Imposing the same conditions adopted in the quark sector ($\Lambda \lesssim 10$ TeV with $c'_1 = 1$), we find $M_\nu \lesssim 2 \times 10^{12}$ GeV [5]. Surprisingly enough, this limit is very similar to the one imposed by $\Delta F = 2$ quark mixing. This implies that $\mu \to e\gamma$ should be close to its present exclusion limit: if $M_\nu \sim 10^{12}$ GeV all the three terms in $\Delta H^{\Delta F=1}_{\text{eff}}$ can yield comparable contributions to the $\mu \to e\gamma$ rate, while if $M_\nu < 10^{12}$ GeV the two quark-induced terms start to dominate. The only way to suppress $\mathcal{B}(\mu \to e\gamma)$ below $10^{-12}$ is to push the new-physics scale to high values above 10 TeV.

The scenario where we can neglect the first term in $\Delta H^{\Delta F=1}_{\text{eff}}$ is quite similar, but not identical, to the one considered in Ref. [15] in the context of supersymmetry. We fully recover the flavor-mixing pattern of Ref. [15] in the limit where we neglect $\lambda'_5$ and $C, G \to I$. In the more general case, taking into account the hierarchical structure of $C, G$ and $\Delta^{(q)}$ the leading mixing terms in $\Delta H^{\Delta F=1}_{\text{eff}}$ have the following parametric structure:

$$\left| (C \Delta^{(q)} \tilde{\lambda}_d G^l)_{ij} \right| \sim \left| (C \Delta^{(q)} C^l \tilde{\lambda}_e)_{ij} \right| \sim \frac{m^2_{\ell}}{v^2} \times \begin{cases} \epsilon \tilde{\lambda}_\tau \quad (\tau_L \to \mu_R) \\
\epsilon \tilde{\lambda}_\tau \quad (\tau_L \to e_R) \\
\epsilon^2 \tilde{\lambda}_\mu \quad (\mu_L \to e_R) \end{cases} \quad (57)$$

where $\lambda \sim (V_{\text{CKM}})_{12} \sim 0.2$ and $\epsilon \sim \lambda^2$ denotes the parametric size of both $(V_{\text{CKM}})_{23}$ and $C_{ij}$ and $G_{ij}$ for $i = 1, 2$. The more pronounced hierarchy of the couplings relevant to $\tau$ FCNC decays in [17] vs. [56] implies that the present limit on $\mathcal{B}(\tau \to \mu, e\gamma)$ does not forbid values of $\mathcal{B}(\tau \to \mu, e\gamma)$ above $10^{-9}$ (as concluded in [5] looking only at the non-GUT terms).
5 Conclusions

In this work we have implemented the principle of the Minimal Flavor Violation in Grand Unified theories, focusing our attention on the $SU(5)$ gauge group.

Since quarks and leptons of a given family (including $\nu_R$) are grouped in three irreducible representations of $SU(5)$ ($\bar{5}$, $10$, and $1$), the flavor symmetry group $G_F = SU(3)_\bar{5} \times SU(3)_{10} \times SU(3)_1$ of the gauge Lagrangian is smaller compared to the standard case. We identified the irreducible sources of $SU(3)_\bar{5} \times SU(3)_{10} \times SU(3)_1$-breaking with the minimal set of couplings that leads to a consistent fermion mass spectrum at low energy. This can be done both with minimal and non-minimal Higgs content, e.g. introducing new Higgs fields transforming as a $45$.

One noteworthy consequence of the smaller flavor group is that the effective FCNC couplings are not determined anymore in terms of the diagonal fermion mass matrices and the CKM and PMNS mixing matrices. Due to the different alignment of the down-quarks and charged leptons mass matrices, two new mixing structures appear, of which only the hierarchical texture is known. The presence of new mixing structures precludes the possibility of deriving precise relations among the rates for different family transitions. Only an order-of-magnitude pattern can be identified. Despite this, a number of reasonably firm phenomenological consequences can be deduced:

- There is a well defined limit in which the standard MFV scenario for the quark sector is fully recovered: small $\tan \beta$ and $M_\nu \ll 10^{12}$ GeV. For $M_\nu \sim 10^{12}$ GeV and small $\tan \beta$, deviations from the standard MFV pattern can be expected in rare Kaon decays but not in $B$ physics. Ignoring fine-tuned scenarios, $M_\nu \gg 10^{12}$ GeV is excluded by the present constraints on quark FCNC transitions. Independently from the value of $M_\nu$, deviations from the standard MFV pattern can appear both in $K$ and in $B$ physics for $\tan \beta \gtrsim m_t/m_b$.

- Contrary to the non-GUT MFV framework, the rate for $\mu \to e\gamma$ (and other leptonic FCNC) cannot be arbitrarily suppressed by lowering the average mass $M_\nu$ of the heavy $\nu_R$. For $M_\nu \lesssim 10^{12}$ GeV, the GUT-induced contribution (controlled mainly by the top-quark Yukawa coupling and the CKM matrix) sets in. This implies that for values of the new physics scale $\Lambda \lesssim 10$ TeV the $\mu \to e\gamma$ rate is within reach of the experimental searches at MEG [18].

- Improved experimental information on $\tau \to \mu\gamma$ and $\tau \to e\gamma$ would be a powerful tool in discriminating the relative size of the standard MFV contributions (see Eqs. 56) versus the characteristic GUT-MFV contributions (see Eqs. 57), due to the different hierarchy pattern among $\tau \to \mu$, $\tau \to e$, and $\mu \to e$ transitions.

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