Torsional Vibration Analysis of Reciprocating Compressor Trains driven by Induction Motors

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Abstract. The dynamic study of electric motor driven compressors, for Oil&Gas (O&G) applications, are traditionally performed in two steps separating the mechanical and the electrical systems. The packager conducts a Torsional Vibration Analysis (TVA) modeling the mechanical system with a lumped parameter scheme, without taking into account the electrical part. The electric motor supplier later performs a source current pulsation analysis on the electric motor system, based on the TVA results. The mechanical and the electrical systems are actually linked by the electromagnetic effect. The effect of the motor air-gap on TVA has only recently been taken into account by adding a spring and a damper between motor and ground in the model. This model is more accurate than the traditional one, but is applicable only to the steady-state condition and still fails to consider the reciprocal effects between the two parts of the system. In this paper the torsional natural frequencies calculated using both the traditional and the new model have been compared. Furthermore, simulation of the complete system has been achieved through the use of LMS AMESim, multi-physics, one-dimensional simulation software that simultaneously solves the shafts rotation and electric motor voltage equation. Finally, the transient phenomena that occur during start-up have been studied.

1. Introduction

Stresses acting upon mechanical parts can cause premature failure of both the driving electrical machine shafts and other mechanical parts due to fatigue phenomena, especially in the case of resonance, when the exciting frequency matches a torsional natural frequency. It is for this reason that the standard codes for reciprocating compressors recommend that the “torsional natural frequencies of the complete driver-compressor system (including couplings and any gear unit) shall not be within 10% of any operating shaft speed and within 5% of any multiple of operating shaft speed in the rotating system up to and including the tenth multiple” [8]. In fact, reciprocating compressor systems are more subject to these phenomena due to the great variations in torque, consequently the preventive analysis of torsional vibrations in reciprocating compressors is essential for meeting the American Petroleum Institute (API) requirements, thus preventing the failure of the crankshaft, coupling, engine dampers, and compressor oil pumps.

In order to predict the vibration behavior of any torsional system, the inertia and stiffness properties of all the components are required. The steady-state torque and power output of a polyphase induction
motor are the result of electromagnetic fields which act across the air-gap between the stator and rotor. If the rotor has torsional vibration superimposed over a steady rotation, the same electromagnetic fields across the air-gap can produce an additional torque. Traditionally, the electromechanical interaction in the air-gap is ignored in standard torsional vibration analysis, but it can be important when improved prediction accuracy of natural frequencies or damping is required. In steady-state conditions, this effect can be taken into account by adding to the model a torsional spring and damper between the motor and the ground, obtained through a linearization of the electric motor model.

In this paper, the influence of motor dynamics in reciprocating compressor trains has been studied by means of different simulation profiles and the effect of induction motor electromagnetic stiffness and damping on torsional vibration has been evaluated considering the influence of air-gap torque. When transient phenomena have to be simulated, the above linearization is no longer applicable; furthermore, the analytical expressions of transient torque, which are usually provided by electric motor manufacturers, do not represent the real behavior of the system. For this reason, the transient analysis has to be performed by means of multi-physics simulation software, which makes it possible to simultaneously solve the electrical and mechanical equations of the system.

The paper is structured as follows: Section 2 outlines the torque effort paying particular attention to the resistant torque produced by reciprocating compressors. Section 3 describes the torque calculation in induction motors. Section 4 presents the modeling of the induction motor air-gap that can be added to the equivalent shaft to develop a more accurate simulation. This model can be used in TVA and gives more realistic results. Section 5 presents the results of simulations in two case studies. In sections 6 and 7, the study of the steady-state and transient forced responses is discussed. To conclude, further possible developments in this field are indicated.

2. Considerations on Torque Effort
Reciprocating machinery and especially reciprocating compressors show high torque variations. Absorbed torque can be represented by a variable and a constant term, which is the mean absorbed torque $C$,

$$ C = C_m + C(t) $$

where $C(t)$ represents a time variable function with a null mean value.

Reciprocating compressors are often driven by electric motors, which may be either synchronous or asynchronous. Electric motors react to these torque fluctuations with their own air-gap torque variations. These variations have an important effect on system torsional dynamics in terms of the natural frequencies of the system. This phenomenon can be intuitively understood by observing the induction motor torque-speed characteristic curve.
Figure 1 shows that the motor speed variation (mainly due to variable compressor torque) causes an adjustment of the developed motor torque (air-gap torque). As a result of this variation, there will also be an additional oscillation of the shaft speed.

The motion equation for an asynchronous (induction) motor (to which this study applies) is:

\[
\frac{J}{p} \frac{d^2 e}{dt^2} + D \frac{de}{dt} = C(t)
\]

where \( J \) is the motor inertia, \( p \) the number of pole pairs, \( D \) the damping, \( C(t) \) the torque fluctuation due to the compressor and \( \epsilon \) the flux angle between the applied voltage and a generic rotor winding.

3. Air-gap Torque Estimation

For a better understanding of the considerations that follow, it helps to consider the steady-state operation of the motor. The simplest mathematical expressions of motor equations are obtained if the frame of reference is chosen so as to rotate with the voltage vector of the supply, because in that frame the voltage and current appear as DC quantities.

Let us consider the following electrical and mechanical equations of an induction machine:

\[
u_s = i_s R_s + \frac{d\Psi_s}{dt} + j\omega \Psi_s
\]

\[
u_r = i_r R_r + \frac{d\Psi_r}{dt} + j(\omega_k - \omega) \Psi_r
\]

\[
J \frac{d\omega}{dt} + T_m = \frac{3}{2} p I_m (\Psi_s^* \Psi_r^*)
\]

where \( \Psi_s \), \( \Psi_r \), \( i_s \) and \( i_r \) are vectors of voltage, current and flux, of the stator (suffix s) and rotor (suffix r), \( \omega \) is the angular velocity of the chosen frame of reference, \( \omega_k \) is the mechanical angular velocity of the rotor, \( p \) is the number of pole pairs, \( J \) is the total moment of inertia, \( T_m \) is the air-gap torque, \( I_m \) represents the imaginary part of the quantity taken into account and \( \ast \) is the complex conjugate. \( R_s \) and \( R_r \) are the stator and the rotor resistance, respectively. At steady-state the flux vectors are of constant length, therefore in the chosen frame reference:

\[
\frac{d\Psi_s}{dt} = \frac{d\Psi_r}{dt} = 0.
\]

Thus we obtain:

\[
u_s = i_s \Psi_s
\]

\[
u_r = i_r \Psi_r + j(\omega_k - \omega) \Psi_r
\]

\[
T_m = \frac{3}{2} p I_m (\Psi_s^* \Psi_r^*)
\]

By considering the ratio of the magnetizing reactance and the stator reactance:

\[
k = \frac{X_m}{X_s}
\]

we obtain the motor torque curve as a function of the slip:

\[
T_m = \frac{3}{2} p I_m \left[ \frac{1}{X_s} + k^2 \frac{j R_s}{R_s + j X_s} \right] = \frac{3}{2} p k^2 I_m \left[ \frac{js}{R_s + js X_s} \right] = \frac{3}{2} \frac{p k^2}{R_r} \frac{s R_s}{R_r + s^2 + X_r^2}
\]

The torque curve has a maximum torque known as pull-out torque \( (T_p) \) at \( s_p \) (pull-out slip):

\[
T_p = k^2 \frac{1}{2 X_s}; \quad s_p = \frac{R_s}{X_s}
\]

As can be seen in Figure 2, the slope for low slips can be approximated by \( 2 T_p/s_p \). As a result the air-gap torque can be calculated by the following equation:

\[
T_m = \frac{2 T_p}{s_p} s
\]
By introducing the pull-out slip, it is possible to rewrite the stator and rotor current equations as:

$$i_s = \frac{1}{jX_s} + \frac{k^2}{X_p}s + js = \frac{1}{jX_s} + 2T_p \frac{s}{s_p + js}$$

(14)

$$i_r = 2T_p \frac{s}{s_j + js}$$

(15)

4. Motor Air-gap Influence on Torsional Vibration Analysis

Many references are available in technical literature on this subject, explaining the procedure for this kind of analysis [6]. The TVA is based on a lumped parameter model, approximating the behavior of the complete system, in which all the elements (compressor, flywheel, reducer, electric motors etc.) are represented by inertia disks linked by shaft pieces, modeled with springs and viscous dampers. API norms establish the rules and the simplification that can be done when making the lumped parameter model. Using the above mentioned model, the system is analyzed in a three-step procedure, calculating:

- Torsional Natural Frequencies and relevant Mode Shapes
- Steady-state Forced Response.
- Transient Forced Response.

During the last two stages, the excitation torques are applied to the system to evaluate the resulting torques on the shaft components and angular oscillations on the inertia disks.

According to API 618, the effect of the electric motor on the compression train is taken into account solely by ensuring that: “torsional natural frequencies are separated from the first and second multiples of the electrical power frequency by more than 10% and 5% respectively”. In fact, it is known that an electric motor produces an excitation couple at the frequency line and twice the frequency line. Recently, lumped parameter models of systems including induction motors, have been improved, taking into account the electric motor air-gap effect, in order to more accurately the system torsional natural behaviour [2][3].

This effect can be represented by an external stiffness and damping, like those shown in Figure 3, still provided that the induction motor model has been appropriately linearized, where $J_C$ is the Compressor Inertia [$kgm^2$], $J_M$ is the Electric Motor Rotor Inertia [$kgm^2$], $k$ is the Coupling Stiffness [Nm/rad], $d$ is the Coupling Damping [Nm·s/rad], $k_M$ is the Motor Air-gap Stiffness [Nm/rad], $d_M$ is the Motor Air-gap Damping [Nm·s/rad] and $m_C$ is the Compressor Resistant Torque [Nm]. $k_M$ and $d_M$ depend only on electric motor parameters (like breakdown torque, rated torque, number of poles, rated slip, etc.) as shown in the next relations:

$$k_M = n_p T_B \frac{x^2}{1 + x^2}; \quad d_M = \frac{k_M}{\omega T_L}; \quad x = \omega T_L; \quad T_L = \frac{1}{\omega} \frac{1}{2S_R} T_B;$$

(16)

where $T_B$ is the Electric Motor Time Constant [s], $T_R$ is the Rated Torque [Nm], $T_B$ is the Breakdown Torque [Nm] and $S_R$ is the Rated Slip.
Having considered the air-gap effect on the equivalent shaft has modified the original system. This has two major consequences on the system torsional natural frequencies, as shown in Figure 4, where letter a represent the fundamental natural frequency calculated ignoring the air-gap effect, while the new natural frequencies are represented with letter b.

- The previous first one, equal to zero (corresponding to the rigid mode of vibration), disappears and a new first one appears. The new frequency is very low effect, therefore, not considering it generally has no consequence.
The previous fundamental natural frequency is increased by a certain percentage that depends on the electro-mechanical interaction of the system, i.e. on the inertia ratio between the motor and compressor ($J_m/J_c$) and on the stiffness ratio between the motor shaft and the coupling ($k_m/k_c$). The shift of the fundamental natural frequency can be quickly estimated by using the following graph, based on analytical expressions.

As can be seen from the graph, the error in the fundamental natural frequency estimation, due to neglecting the air-gap effect, can be very significant when the system includes soft coupling and/or low motor inertia.

Therefore, not taking into account the air-gap effect could lead to unacceptable errors in calculation of torsional natural frequencies and consequently in forced response analysis.

5. Simulation Results

The theoretical results obtained in the previous paragraph have been verified in two case studies, named case A and case B. This has been done using traditional mathematical methods and with the multi-physics simulation software LMS AMESim. The natural frequencies obtained through the models, that neglect and include the air-gap effect, have been compared. In the following figure, a lumped parameters model (case A) realized with LMS AMESim is shown. In this model, the air-gap effect is not included.

### Table 1. System Parameters (Case A)

| Item | Description | Moment of Inertia $J$ [kgm$^2$] | Shaft Interval |
|------|-------------|---------------------------------|----------------|
| 1    | Compressor  | 0.059                           | [1-2] compressor |
| 2    | Compressor throw | 1.181                        | [2-3] compressor |
| 3    | Compressor throw | 1.171                        | [3-4] compressor |
| 4    | Compr. + fly. + coupling | 57.175                      | [4-5] coupling |
| 5    | Half coupling | 0.940                          | [5-6] induction motor |
| 6    | Induction motor | 52.570                        |                |

### Table 2. Undamped System Natural Frequencies Ignoring Air-Gap Effect (Case A)

| TNF | Hz  |
|-----|-----|
| 1ST | 7.7 |
| 2ND | 240.4 |
| 3TH | 279.5 |
| 4TH | 746.9 |
| 5TH | 2253.1 |

### Table 3. Motor Air-Gap Stiffness and Damping (Case A)

| Description | k$_m$ [Nm/rad] | d$_m$ [Nm/rad] | Stiffness Ratio $k_m/k_c$ | Inertia Ratio $J_m/J_c$ |
|-------------|---------------|---------------|--------------------------|------------------------|
| External stiffness | 5.68E+04     | 3.56E+01      |                          |                        |
| Stiffness Ratio $k_m/k_c$ | /            |               | 0.85                     |                        |
| Inertia Ratio $J_m/J_c$ | /            |               | 0.88                     |                        |

With the values of Table 1, the significant torsional natural frequencies (TNF) of the system are listed in Table 2.

Later, the air-gap effect was modeled with an external stiffness and damping obtained with equations (3) and (4), as explained in Paragraph 2 (see Table 3).

In this situation, the previous fundamental natural frequency results increased by 13% and, as explained in section 4, the natural frequency due to the rigid mode of vibration is no longer zero (see...
The results obtained with the scheme shown in fig. 7 have been confirmed also by using the induction motor model provided by LMS AMESim (fig. 8).

Adopting the same procedure used until now, the expected results have been obtained also for case B and the natural frequencies calculated by respectively including and ignoring the air-gap effect are summarized in table 6. Similar to case A, the results obtained with the scheme shown in figure 7 have been confirmed also by using the induction motor model provided by LMS AMESim (fig.8).

### Table 4. Torsional Natural Frequencies including Motor Air-Gap Effect

|   | Torsional Natural Frequency | Hz  |
|---|------------------------------|-----|
| 1st TNF | 3.1 (*new frequency) | 8.7 (+13%) |
| 2nd TNF | 240.4 | 279.5 |
| 3rd TNF | 746.9 | 2253.1 |

### Table 5. Motor Air-Gap Stiffness and Damping (Case B)

|   | External stiffness k_m Nm/rad | 9.66E+04 |
|---|-------------------------------|----------|
| External damping d_m Nm/rad | 7.90E+01 |
| Stiffness Ratio k_m/k_c | 0.92 |
| Inertia Ratio J_m/J_c | 1.30 |

### Table 6. Undamped System Natural Frequencies ignoring Air-Gap Effect and Torsional Natural Frequencies including Motor Air-Gap Effect (Case B)

|   | Torsional Natural Frequency | Hz  |
|---|------------------------------|-----|
| 1st TNF | 8.4 | 3.6 (*new frequency) |
| 2nd TNF | 240.4 | 9.2 (+10%) |
| 3rd TNF | 245.5 | 240.4 |
| 4th TNF | 746 | 245.5 |
| 5th TNF | 2747.2 | 746 |
| 6th TNF | 2747.2 |

**6. Steady-state Response Analysis**

According to API 618 the steady-state forced response is not required when the following conditions are verified:

- there are no torsional natural frequencies within 10% of operating shaft speed and within 5% of any multiple of operating shaft speed;
- torsional natural frequencies are separated from the first and second multiples of the electrical power frequency by more than 10% and 5% respectively.

![Figure 9. System A - Campbell Diagram](image-url)
To evaluate whether or not these conditions are verified, a Campbell diagram is commonly used. In this diagram, the resonances are identified with the intersection between torsional natural frequencies (horizontal line) and excitations torque (oblique lines). The Campbell Diagram for case study A is shown in figure 9. It should be noted that, in this case, ignoring the electric motor air-gap effect, leads to a lower evaluation of the fundamental torsional natural frequency, consequently reducing the separation margin.

As can be seen from the Campbell diagram shown above, for case study A, according to API 618, a steady-state forced response is not required.

However it has been carried out to evaluate the resulting torque acting on the shaft intervals, using the model shown in figure 8.

Figure 10 shows the coupling torque calculated for case A, when the nominal load torque (4000 Nm) is applied: the torque oscillation has an amplitude that remains within acceptable values and, as expected, it has frequency components at about 16 Hz and 32 Hz, corresponding to the shaft nominal speed (994 rpm) and its second multiple.

7. Transient Response Analysis
API 618 establishes that transient analysis is not mandatory when induction motors without Variable Frequency Drive (VFD) are used, as in the examples presented in this work. However, when a soft coupling is used, also in these cases the start-up could be a matter of concern, since, starting from stand still, the system will certainly pass through one or more natural frequencies, and, as a consequence, the torque fluctuation can be amplified.

During the start-up phase, also the electric motor oscillating torque represents an excitation source that must be included in the analysis. Motor manufacturers usually provide analytical expressions for the torque as a function of the time for the main transient phenomena (start up, bi-phase and tri-phase short circuit, auto-reclosing). But these expressions are valid only with a locked rotor and cannot be used to simulate what happens in the transient periods. For example, in figure 11, the torque vs. time curve for the start-up of the motor used in case A, is shown. It can be seen that the torque oscillation has a time constant of about 2.5 s, therefore the machine would have appreciable oscillations for at least 10 seconds (4x time constant) without speeding up. This is in contrast with what common practice suggests, i.e. the starting of the whole system lasts much less time (usually 2 to 5 seconds, depending on the load torque). To obtain more realistic results for the analysis of the system start-up, the authors used the multi–physical software LMS AMESim that simultaneously solves the voltage equations of the motor and the mechanical motion equations of the shaft.

Different possibilities are offered to model an induction machine with LMS AMESim. In order to achieve a representative model to analyze the transient start-up of the machine, a dynamic double cage induction machine model has been set. This model has the advantage of being simple while ensuring an accurate torque-speed characteristic in all machine operating states (locked rotor, breakdown torque and rated speed).

![Figure 10. System A – Steady-state coupling torque](image1)

![Figure 11. Induction motor Start-Up torque (provided by the manufacturer).](image2)
Figure 12. System A startup simulation performed by LMS AMESim

The voltage equations of the model expressed in the rotating reference frame linked to the electrical position of the rotor are the following:

\[ U_{sd} = -\psi_{qs} \omega_{es} + I_{ds} R_s + \frac{d\psi_{ds}}{dt} \]  
\( (17) \)

\[ U_{sq} = -\psi_{ds} \omega_{es} + I_{qs} R_s + \frac{d\psi_{qs}}{dt} \]  
\( (18) \)

As the rotor squirrel cage is short circuited:

\[ U_{rd} = U_{rq} = 0 \]  
\( (19) \)

\[ 0 = I_{dr1}' R_{r1}' + I_{dr2}' R_{r2}' + \frac{d\psi_{dr1}'}{dt} + \frac{d\psi_{dr2}'}{dt} \]  
\( (20) \)

\[ 0 = I_{qr1}' R_{r1}' + I_{qr2}' R_{r2}' + \frac{d\psi_{qr1}'}{dt} + \frac{d\psi_{qr2}'}{dt} \]  
\( (21) \)

The subscripts \( s \) and \( r \) refer respectively to the stator and to the rotor component of the quantity taken into account. The subscripts \( d \) and \( q \) refer to the direct and quadrature component of the Park’s transformation reference frame. The prime subscript stands for rotor variables referred to stator windings. The 1 and 2 subscript refer to the cage 1 or cage 2 parameters. \( \psi \) is the magnetic flux, \( U \) is the voltage, \( I \) is the current, \( R \) is the resistance, \( L \) is the inductance and \( p \) the pole pair number.

The fluxes of the model are expressed as follows:

\[ \psi_{ds} = L_s \cdot I_{ds} + L_m \cdot \left( I_{dr1}' + I_{dr2}' \right) \]  
\( (22) \)

\[ \psi_{qs} = L_s \cdot I_{qs} + L_m \cdot \left( I_{qr1}' + I_{qr2}' \right) \]  
\( (23) \)

\[ \psi_{dr} = L_m \cdot I_{dr} + L_{r1}' \cdot I_{dr1}' + L_{r2}' \cdot I_{dr2}' \]  
\( (24) \)

\[ \psi_{qr} = L_m \cdot I_{qr} + L_{r1}' \cdot I_{qr1}' + L_{r2}' \cdot I_{qr2}' \]  
\( (25) \)

where \( L_m \) is the magnetizing inductance common to \( L_s \) and \( L_{r1}', L_{r2}' \) are the stator, rotor cage 1 and rotor cage 2 inductances. The dynamic torque expression derives from the energy balance consideration. It is evaluated according to the following expression:

\[ T_e = p \cdot L_m \cdot \left[ \left( I_{qs} \cdot \left( I_{dr1}' + I_{dr2}' \right) \right) - \left( I_{ds} \cdot \left( I_{qr1}' + I_{qr2}' \right) \right) \right] \]  
\( (26) \)

The parameters of this model are given by the manufacturer in the framework of a single cage model. Common model parameters have been set according to this manufacturer’s datasheet information. We estimate the double cage parameters \( R_{r1}', L_{r1}', R_{r2}' \) and \( L_{r2}' \) thanks to an optimization process according to the torque speed characteristics provided by the manufacturer.

In figure 12, the results of the system A start-up simulation are shown. The simulation has been performed with a load torque of about 1/3 of the nominal torque, a value that approximates
effective resistant torque of a compressor started with valve unloaders on. The red line indicates the induction motor torque, the blue one the torque acting on the coupling, and the green one the rotor speed (starting from 0 rpm and reaching 994 rpm).

From the figure shown above some considerations can be made:

- Observing the torque oscillation of the electric motor, it can be deduced that the system electrical transient lasts only 1.2 s during the start-up (while the electric motor locked rotor transient lasts much more). However, the size of this oscillation (2.5 times the nominal torque) justifies a transient analysis although not required by the norms.
- The induction motor has oscillations that occur at line frequency (50 Hz) as expected. This justifies the API requirement to separate torsional natural frequencies from the first and second multiples of the electrical power supply frequency by more than 10% and 5% respectively.
- The coupling torque oscillations are not amplified when passing through resonance (516 rpm, 3.0 s). This is due to the presence of damping in the system and to the short time in which the compressor passes through the resonance region.
- The maximum torque acting on the coupling occurs at the electric motor breakdown torque.

8. Conclusions

In this work, the dynamic system of compression trains for O&G applications driven by electric motors has been studied. The study has been carried out considering the whole system, with the reciprocal effect that the mechanical and electrical parts have on each other. TVA has been performed in two case studies, comparing the results produced by a traditional lumped parameters model and a more recent model that represents the electric motor air-gap through a spring and a damper connected to earth. By using simulation software and traditional mathematical methods, it has been confirmed that the new model provides more accurate results in terms of system torsional natural frequencies.

However, the new model is obtained through a linearization of the non-linear induction motor model, therefore, it cannot be used to study the transient phenomena that occur, for example, during the motor start-up.

Furthermore, the use of the analytical expressions (usually provided by electric motor manufacturers) would lead to inaccurate results. In fact, these expressions are valid only in particular conditions (for example, with a locked rotor) and are obtained considering the electric motor separate from the mechanical part of the system, and consequently do not describe the real system behavior.

This limitation has been overcome by using the multi-physics simulation software LMS AMESim. This software makes it possible to consider the reciprocal effect of the electrical and mechanical part of the system by simultaneously solving the voltage equations of the electric motor and the mechanical equations of the shaft.

In this way, it has been possible to carry out the transient forced response of the system obtaining consistent results.

In further works, we will be extending our studies on the implications that the dynamic system of compression trains has on the supply line. To do this, we will be focusing more on the current pulsations that are generated on the line, due to the torque oscillations caused by the reciprocating compressors.

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