Instanton Content of the SU(3) Vacuum

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We study the topological content of the SU(3) vacuum using a method based on RG mapping developed for SU(2) gauge theory earlier. RG mapping, in which a series of APE-smearing steps is done while tracking the observables, reduces the short range fluctuations in the gauge fields while preserving the long structure. This allows us to study the instanton size distribution and topological susceptibility for SU(3) gauge theory. We arrive at a value for the topological susceptibility $\chi^{1/4}$, of 203(5) MeV. The size distribution peaks at $\rho = 0.3\text{fm}$, and is in good agreement with the prediction of the instanton liquid models.

Instantons play an essential role in the QCD vacuum. They explain the U(1) problem \cite{1} and there is growing evidence that they are responsible for chiral symmetry breaking and the low energy hadron spectrum \cite{2,3}. Phenomenological instanton liquid models describe the propagation of quarks as hopping from instanton to instanton. This requires the instantons and anti-instantons to overlap to provide continuous paths for this propagation. To understand if these paths are formed one has to determine the location and size distribution of the instantons in the vacuum.

Because instantons carry only a few percent of the action, they are hidden by vacuum fluctuations. To identify the instantons some method to reduce the short-range quantum fluctuations in the gauge fields while preserving the topological content of the vacuum is needed. The method of RG cycling used in Ref.\cite{4} to study SU(2) gauge theory is one of the best theoretically supported smoothing algorithms, but it is too expensive in terms of both processor time and memory to be of any use for SU(3) gauge theory. In Ref.\cite{5} RG cycling was fitted to a series of APE smearing steps. It was found that two APE steps with a staple weight of 0.45 were equivalent to one RG cycling step for SU(2). This method of smoothing the vacuum fluctuations will be referred to as RG mapping. RG mapping eliminates the expensive minimization step from RG cycling by fitting one RG cycling step to a series of APE-smearing steps.

Since APE-smearing slowly distorts the topological content of the lattice an extrapolation back to zero smearing steps is required. This means that the topological charge density must be measured at regular intervals as one APE smears the lattice. We found that the exact parameters for APE smearing are not important as long as one monitors the topological content over several steps and extrapolates back to zero steps.

Generalizing the RG mapping method described above, we study the instanton content of the SU(3) vacuum. We use the same parameters for the APE-smearing that were used for SU(2), a staple weight of 0.45 and measurements of the topological density every two smearing steps. Information before 12 smearing steps is discarded since the vacuum fluctuations are still too high to reliably identify the instantons. We ran on pure gauge configurations with the Wilson action at couplings $\beta = 5.85, 6.0, 6.1$. We used two different lattice sizes, $12^4$ for $\beta = 5.85$ & 6.0, and $16^4$ for $\beta = 6.0$ & 6.1. A detailed discussion of the method and results can found in Ref.\cite{6}.

The topological charge density was measured with a fixed point operator every two smearing steps between approximately 15 and 30 steps depending on the value of the coupling. The total charge, $Q_{FP}$ gave an integer value with 2-3 percent after 12 smearing steps. The topological susceptibility was calculated for the range of smearing steps for each coupling value. The susceptibility was very stable over smearing and
Figure 1. The susceptibility $\langle Q^2 \rangle$ vs number of $c = 0.45$ APE steps. Symbols are diamonds for $\beta = 5.85$, crosses for $\beta = 6.0$ on $12^4$ lattices, squares for $\beta = 6.0$ on $16^4$ lattices, and plusses for $\beta = 6.1$.

therefore there was no need to extrapolate back to zero smearing steps. In Fig. 1 $\langle Q^2 \rangle$ is plotted against the number of smearing steps done. We arrive at a final value for the topological susceptibility of $\chi^{1/4} = 203(5)\, MeV$. Before one compares this number with other works, we should note that we used a string tension that is about 5\% higher than the standard value. If we had used the more customary value of $\sqrt{\sigma} = 440\, MeV$ we would obtain a value of $\chi^{1/4} = 192\, MeV$, which is in complete agreement with the results from Refs. 2, 3.

The instantons must be monitored since they are slowly distorted as the lattice is smeared. In Fig. 2 we show the stable objects for a typical lattice configuration. The configuration has topological charge $Q_{FP}$ that is within five percent of the integer value of 4 from six smearing steps to 40 smearing steps. Five stable objects are found on the configuration, one anti-instanton and four instantons. The solid lines are the extrapolation to zero smearing based on smearing steps 16 to 24. The slopes for all the extrapolations, except the anti-instanton are less than 0.03. Since that anti-instanton size changes so rapidly and the extrapolation does not match the evolution out to large smearing steps, we interpret this as a misidentified vacuum fluctuation. That conclusion is also supported by the value of the total charge, $Q = 4$, on this configuration. On a typical configuration there are many “lumps” in the topological charge density that could be naively identified as instantons. Most of them are not stable but disappear after one or a few smearing steps. Further support for this can be seen in the instanton density at various levels of smearing. After 12 smearing steps the density is $4.1 fm^{-4}$ and after 24 smearing steps the density is $2.9 fm^{-4}$. This is still much higher than the phenomenologically expected value. It is important to separate the spurious objects from the true topological objects. Our criterion is to keep only stable objects. An object is stable if it can be identified at every analyzed smearing step and its size changes slowly. Since the size of instantons change almost linearly with the smoothing steps, we introduce a cut-off for the maximum acceptable change. This cut-off is chosen such that the topological susceptibility calculated from the reliably measured total charge $Q_{FP}$ agrees with that calculated from the charge $Q = I - A$ where I and A are the number of stable instantons and anti-instantons on a given configuration. When applying this cut-off, the resulting instanton density turns out to be $1 fm^{-4}$, in good agreement with the phenomenological expected value.

In Fig. 3 we show our instanton size distribution. The solid line in Fig. 3 is the instanton size distribution from the instanton liquid model for $\Lambda_{MS} = 200\, MeV$, provided by Shuryak. Our distribution along with our values for density and the average instanton size put us in good agreement with the various phenomenologically successful instanton liquid models.

We can now give a final description of the RG mapping method. RG mapping is a series of APE-smearing steps where the topological content is monitored at regular intervals over a range of smearing steps. The original content of the lattice is then found by making a linear extrap-
Figure 2. Radius versus APE-smearing steps of instantons (for clarity, two symbols, diamonds and squares both denote instantons) and anti-instantons (crosses) on a $16^4 \beta = 6.0$ configuration.

Figure 3. The size distribution of the instantons. The diamonds correspond to $\beta = 5.85$, octagons to $\beta = 6.0$, and squares to $\beta = 6.1$. The first bin of each distribution is contaminated by the cut-off.

olution to zero smearing steps while only accepting objects whose extrapolation has slope below a certain cut off. This cut-off is tuned so that $<Q_{FP}^2> = <(I-A)^2>$ where Q is the topological charge measured with the fixed point operator and I and A are the number instantons and anti-instantons found.

Using this method we calculate the relevant parameters of the instanton vacuum for SU(3). We find an instanton density of $1 fm^{-4}$ and an average instanton size of approximately $0.3 fm$. This is considerably smaller than the results given in Ref.\cite{7,8}. We attribute this difference to our extrapolation to zero smearing steps.

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