Defending Adversarial Attacks without Adversarial Attacks in Deep Reinforcement Learning

Xinghua Qu
School of Computer Science & Engineering
Nanyang Technological University
xinghua001e.ntu.edu.sg

Yew-Soon Ong
School of Computer Science & Engineering
Nanyang Technological University
asysong@ntu.edu.sg

Abhishek Gupta
Singapore Institute of Manufacturing Technology
Agency for Science, Technology and Research
abhishek_gupta@simtech.a-star.edu.sg

Zhu Sun
Department of Computing
Macquarie University
sunzhuntu@gmail.com

Abstract

Many recent studies in deep reinforcement learning (DRL) have proposed to boost adversarial robustness through policy distillation utilizing adversarial training, where additional adversarial examples are added in the training process of the student policy; this makes the robustness improvement less flexible and more computationally expensive. In contrast, we propose an efficient policy distillation paradigm called robust policy distillation that is capable of achieving an adversarially robust student policy without relying on any adversarial example during student policy training. To this end, we devise a new policy distillation loss that consists of two terms: 1) a prescription gap maximization loss aiming at simultaneously maximizing the likelihood of the action selected by the teacher policy and the entropy over the remaining actions; 2) a Jacobian regularization loss that minimizes the magnitude of Jacobian with respect to the input state. The theoretical analysis proves that our distillation loss guarantees to increase the prescription gap and the adversarial robustness. Meanwhile, experiments on five Atari games firmly verifies the superiority of our policy distillation on boosting adversarial robustness compared to other state-of-the-arts.

1 Introduction

The advancements in deep reinforcement learning (DRL) have demonstrated that deep neural networks (DNNs) as powerful function approximators can be trained to prescribe near-optimal actions on many complex tasks (e.g., Atari games [1] and robotics control [2]). Although remarkable achievements have been documented, the vulnerabilities of DNNs [3] and many successful attacks on DRL [4–7] have inspired us to improve the adversarial robustness of DRL policies, so as to defend against adversarial attacks in real-world deployments.

To achieve adversarial defense, many studies have investigated using policy distillation [8] combined with adversarial training [9] to obtain an accurate and robust student policy. In policy distillation, a robust student policy indicates that the action prescription by student policy should be consistent with the prescription by its teacher policy. With adversarial training, this consistency is learned through optimizing the student policy with adversarial attacks in training data. To generate such attacks, fast gradient sign method and projected gradient descent are respectively adopted by [10] and [11]. In a nutshell, such adversarial robustness is achieved by additional procedures of generating adversarial attacks in the training of student policy, which, however, makes improvements on robustness less...
flexible (e.g., adversarial robustness depending on one particular attack may fail when another attack happens) and more computationally expensive.

Instead of learning this consistency using adversarial training, we prove in Section 3.2 that such consistency can also be achieved by maximizing the student policy’s prescription gap between the teacher selected action and the remaining actions under attack. Most importantly, we derive that maximizing the prescription gap under attack can be transformed to simultaneously maximizing the prescription \textit{without attack} and minimizing the Jacobian with respect to input states; this provides us the possibility of achieving adversarial robustness without generating any attack during training. Therefore, we design a new policy distillation loss function that includes two parts: 1) prescription gap maximization (PGM) loss, and 2) Jacobian regularization (JR) loss. The PGM loss is different from most previous distillation loss functions (e.g., cross-entropy) that merely maximizes the probability of the action selected by teacher policy. More importantly, we also maximize the entropy of those actions not selected by the teacher policy, which enforces the student policy to have a larger prescription gap in order to resist attacks. The entropy term is weighted by the probability of the selected action; this allows the training to focus on accuracy at the beginning and pursue entropy maximization in the end. In order to further improve the adversarial robustness, we minimize the magnitude of Jacobian with respect to the input state, which is calculated based on PGM loss.

\textbf{Our main contributions can be summarized as:}

\begin{itemize}
  \item We propose a novel policy distillation paradigm to achieve a robust student policy that can defend adversarial attacks without relying on attack generation during training. In doing so, we design a novel policy distillation loss function that contains: 1) a PGM loss for simultaneously maximizing the probability of the action prescribed by teacher policy as well as the entropy of unwanted actions; 2) a JR loss that minimizes the norm of Jacobian with respect to the input state.
  \item The theoretical analysis proves that our distillation loss guarantees to increase the prescription gap and the adversarial robustness. Meanwhile, experiments on five Atari games show that the student policies trained via our policy distillation loss are not only as accurate as teacher policies but also more robust under adversarial attacks.
\end{itemize}

2 Related Work

In the context of DRL, Huang et al. [4] were among the first to analyze the vulnerability of DNN policies, where they utilized the fast gradient sign method (FGSM) [3] to generate adversarial perturbations. Lin et al. [5] explored a more complicated scenario by partially perturbing only selected frames, and they also investigated a designated targeted attack using a generative model. Qu et al. [6] studied a minimalistic attack to showcase that merely perturbing a single pixel in a few selected frames can significantly degrade the reward of state-of-the-art policies. Besides, Xiao et al. [7] provided a survey that refers many other attacks on RL with different settings.

To resist adversarial attacks in DRL, there have been several works that study the adversarial defense by using adversarial training. Mandlekar et al. [12] applied adversarial training on policy gradient algorithm by leveraging a simple FGSM to generate adversarial examples, but they just tested on some simple RL tasks. Pattanaik et al. [13] introduced much stronger attacks that are achieved by projected gradient descent (PGD) in adversarial training on Atari games. However, the results showcase that the robustness increase causes significant performance drop. To obtain better defense, Mirman et al. [10] and Fischer et al. [11] proposed adversarial training based policy distillation to build a more robust student policy, where FGSM and PGD are utilized respectively to generate adversarial attacks. Recently, Zhang et al. [14] proposed a DNN verification based adversarial training that utilizes the interval bound propagation (i.e., CROWN-IBP [15]) to boost robustness.

However, those adversarial training based approaches require additional procedures to generate adversarial attacks; this makes improvements on adversarial robustness less flexible and more computationally expensive. In contrast, our policy distillation approach is able to learn a robust student policy that can defend against adversarial attacks without replying on any type of attack generation during training.
3 Methodology

In this section, we first provide preliminaries on deep reinforcement learning and policy distillation, and then formulate our investigated problem. Accordingly, we devise a novel policy distillation loss $L_{PD}$, consisting of a prescription gap maximization loss and a Jacobian regularization loss. The proposed $L_{PD}$ is able to help achieve adversarial robustness of the student policy without depending on any specific type of attack. Finally, we theoretically prove that our distillation can increase the prescription gap and adversarial robustness.

3.1 Preliminaries

Deep Reinforcement Learning (DRL). In this paper, we consider a finite-horizon Markov decision process (MDP) that consists of a 4-tuple $(\mathcal{S}, \mathcal{A}, r, p)$, where $\mathcal{S}$ denotes the state space; $\mathcal{A}$ means the action space with size $|\mathcal{A}|$; $r(s_t)$ is the reward function when state $s_t$ transits to $s_{t+1}$ given action $a_t$; and $p$ represents the state transition function, e.g., $p(s_{t+1} | s_t, a_t)$, that is controlled by the environment. The aim of DRL algorithm (e.g., DQN \cite{11}) is to maximize the expected discounted reward $R(\pi_\theta) = \mathbb{E}[\sum_{t=0}^T \gamma^t r(s_t) | \pi_\theta]$ following a policy $\pi_\theta$, where $\pi$ is parameterized by $\theta$. However, $R(\pi_\theta)$ can be significantly degraded when an adversarial example $\delta_t : \mathcal{S} \rightarrow \mathcal{S}$ exists in state $s_t$. Note that in this paper $\pi_\theta(s_t)$ represents a prescribed distribution in action space of the policy $\pi_\theta$ on state $s_t$; $\pi_\theta(s_t, a_t)$ is the prescription on action $a_t$ given policy $\pi_\theta$ and state $s_t$. This $\delta_t$ is added on the original state $s_t$ in order to perturb the prescribed action distribution $\pi_\theta(s_t + \delta_t)$. Therefore, the perturbed action $a_t = \arg \max_a \pi_\theta(s_t + \delta_t, a)$ may be sub-optimal, thus reducing the reward of $\pi_\theta$. The expected perturbed accumulated reward is denoted as $R(\pi_\theta) = \mathbb{E}[\sum_{t=0}^T \gamma^t r(s_t, \delta_t) | \pi_\theta]$. To defend against $\delta_t$, adversarial training based policy distillations \cite{11} have been used.

Policy Distillation (PD). We follow the problem setting of PD \cite{8}, where a teacher policy $\pi_{\theta^T}$ (e.g., $Q$ value approximator) is first learned by RL algorithms. The aim of PD is to learn a student policy $\pi_{\theta^S}$ that can mimic the behavior of its teacher policy $\pi_{\theta^T}$. Most importantly, the student policy can even outperform its teacher policy with respect to: 1) higher accumulated reward value and 2) smaller size of policy network. Generally, PD is formulated to minimize the loss function $L(\theta^S)$ between the prescription from student policy $\pi_{\theta^S}(s_t)$ and that from the pre-trained teacher policy $\pi_{\theta^T}(s_t)$.

$$L(\theta^S) = \mathbb{E}_{s_t \sim \mathcal{S}} \left[ D(\pi_{\theta^S}(s_t), \pi_{\theta^T}(s_t)) \right],$$

where $D$ is a distance measurement and it usually adopts the Kullback–Leibler (KL) divergence \cite{8} or mean square error (MSE) \cite{11}. Although PD has documented many success stories on reward improvement \cite{8} and policy network compression \cite{16}, the adversarial robustness of the student policy $\pi_{\theta^S}$ has been less investigated so far. Some recent advancements have studied the adversarial defense by involving adversarial training in PD. In doing so, the loss function for a robust student policy $L_R(\theta^S)$ is reformulated as

$$L_R(\theta^S) = \mathbb{E}_{s_t \sim \mathcal{S}} \left[ \max_{\delta_t} D(\pi_{\theta^S}(s_t + \delta_t), \pi_{\theta^T}(s_t)) \right], \quad \|\delta_t\| \leq \epsilon,$$

where the norm value of $\delta_t$ is bounded by $\epsilon$. In order to generate adversarial examples $\delta_t$, many attack models (e.g., FGSM \cite{10} and PGD \cite{11}) have been applied. In particular, Zhang et al. \cite{14} proposed a certified defense utilizing an interval bound propagation technique (i.e., CROWN-IBP), but the certified bound is still iteratively obtained via forward and backward computation through the network. In summary, these adversarial training based policy distillations require additional procedures to generate adversarial examples; this makes adversarial robustness improvement less flexible. For instance, the robustness obtained depending on one particular attack may fail when the agent faces another attack. Furthermore, the generation of adversarial examples leads to more computational cost. Hence, a natural question to ask is: can we build a distillation paradigm that is capable of defending adversarial attacks without adversarial attacks in the training of student policy?

3.2 Problem Formulation

The aim of our policy distillation is to find a student policy $\pi_{\theta^S}$ that can maximize the accumulated reward $R(\pi_{\theta^S})$ even with adversarial perturbation $\delta_t$ on state $s_t$, while the distillation training is independent on adversarial attacks. In an MDP, the expected reward starting from $s_t$ is denoted by state value $V(s_t)$:

$$V(s_t) = \mathbb{E} \left[ \sum_{k=1}^T \gamma^k r(s_{t+k}) | \pi_{\theta^S} \right],$$

where the norm value of $\delta_t$ is bounded by $\epsilon$. In order to generate adversarial examples $\delta_t$, many attack models (e.g., FGSM \cite{10} and PGD \cite{11}) have been applied. In particular, Zhang et al. \cite{14} proposed a certified defense utilizing an interval bound propagation technique (i.e., CROWN-IBP), but the certified bound is still iteratively obtained via forward and backward computation through the network. In summary, these adversarial training based policy distillations require additional procedures to generate adversarial examples; this makes adversarial robustness improvement less flexible. For instance, the robustness obtained depending on one particular attack may fail when the agent faces another attack. Furthermore, the generation of adversarial examples leads to more computational cost. Hence, a natural question to ask is: can we build a distillation paradigm that is capable of defending adversarial attacks without adversarial attacks in the training of student policy?
Then, a robust distilled student policy is defined as,\[ \pi_{\theta \delta}(s_t + \delta_t, a_t) = \pi_{\theta \delta}(s_t, a_t) + \delta_t \nabla_{s_t} \pi_{\theta \delta}(s_t, a_t) + \omega_2, \] where \( \omega_1 \) and \( \omega_2 \) are truncation errors. For ease of analysis we assume \( \omega_1 - \omega_2 = 0 \). Thereby, Eq. (7) can be transformed as,
\[
\mathcal{G}_{\theta \delta}(s_t + \delta_t, a_t^T) = \min_{\delta_t, ||\delta_t|| \leq \epsilon} \left[ \pi_{\theta \delta}(s_t, a_t^T) + \delta_t \nabla_{s_t} \pi_{\theta \delta}(s_t, a_t^T) - \pi_{\theta \delta}(s_t + \delta_t, a_t^T) \right].
\]

Thus, in order to maximize the prescription gap \( \mathcal{G}_{\theta \delta}(s_t + \delta_t, a_t^T) \), the first term \( \mathcal{G}_{\theta \delta}(s_t, a_t^T) \) in Eq. (9) should be maximized. Note that \( \delta_t \) is optimized by an attacker to impact \( \mathcal{G}_{\theta \delta}(s_t, a_t^T) \) negatively. Although \( \delta_t \) cannot be controlled by \( \pi_{\theta \delta} \), we can control the Jacobian \( \nabla_{s_t} \mathcal{G}_{\theta \delta}(s_t, a_t^T) \) in policy

Note that in policy distillation, the student policy is trained to be consistent with its teacher policy \( \pi_{\theta \delta} \) by satisfying \( a^T = \arg \max_a \pi_{\theta \delta}(s_t, a) \). Therefore, to minimize the distance in Eq. (6), \( a^T = \arg \max_a \pi_{\theta \delta}(s_t + \delta_t, a) \) is to be ensured. In other words, we need to encourage the student policy \( \pi_{\theta \delta} \) to choose the action \( a^T \) selected by the teacher policy \( \pi_{\theta \delta} \), even under the condition that the adversarial attack \( \delta_t \) exists on state \( s_t \). Accordingly, the following proposition is put forth for a robust student policy based on a pre-trained teacher policy \( \pi_{\theta \delta} \).

**Proposition 1 (Robust student policy)** We assume that \( \pi_{\theta \delta} \) and \( \pi_{\theta \delta} \) are deterministic policies. The optimal action chosen by the teacher policy \( \pi_{\theta \delta} \) is \( a^T = \arg \max_a \pi_{\theta \delta}(s_t, a) \). Given bounded adversarial perturbations \( \delta_t, ||\delta_t|| \leq \epsilon \) on state \( s_t \), we design the prescription gap of student policy \( \pi_{\theta \delta} \) as,
\[
\mathcal{G}_{\theta \delta}(s_t + \delta_t, a^T) = \min_{\delta_t, ||\delta_t|| \leq \epsilon} \left[ \pi_{\theta \delta}(s_t + \delta_t, a^T) - \pi_{\theta \delta}(s_t + \delta_t, a_t) \right], \forall a \in A \cap a \neq a^T.
\]

Then, a robust distilled student policy \( \pi_{\theta \delta} \) must guarantee \( \mathcal{G}_{\theta \delta}(s_t + \delta_t, a^T) > 0 \).
distillation. Therefore, in maximizing robustness, the influence of the second term in Eq. (9) can be reduced by minimizing the magnitude of the Jacobian $\|\nabla_{s_t} g_{\theta^S}(s_t, a^T)\|$.

In sum, to maximize the prescription gap $G_{\theta^S}(s_t + \delta_t, a^T)$ with attack, we can alternatively maximize $g_{\theta^S}(s_t, a^T)$ without attack and simultaneously minimize the magnitude of Jacobian $\|\nabla_{s_t} g_{\theta^S}(s_t, a^T)\|$. Guided by this, we devise our distillation loss function in what follows.

### 3.3 Loss Function

Our policy distillation loss function $L_{PD}(\theta^S)$ is proposed as,

$$L_{PD}(\theta^S) = L_{pgm}(\theta^S) + \beta L_{jr}(\theta^S),$$

(10)

where $L_{pgm}(\theta^S)$ is the prescription gap maximization loss that not only maximizes the likelihood on action $a^T$ selected by teacher policy $\pi_{\theta^T}$, but also maximizes the entropy on the remaining actions; $L_{jr}(\theta^S)$ is the Jacobian regularization loss that aims to boost the robustness via minimizing the magnitude of Jacobian for $s_t$. The weight $\beta$ controls the strength of $L_{jr}(\theta^S)$. We illustrate the details of $L_{pgm}(\theta^S)$ and $L_{jr}(\theta^S)$ as follows.

**Prescription Gap Maximization (PGM).** With the goal of maximizing the prescription gap between action $a^T$ and the remaining actions in mind, we devise the PGM loss $L_{pgm}(\theta^S)$ as,

$$L_{pgm}(\theta^S) = -\pi_{\theta^S}(s_t, a^T) \eta \left[ -\sum_{a=1, a \neq a^T}^{|A|} \left( \frac{\pi_{\theta^S}(s_t, a)}{1 - \pi_{\theta^S}(s_t, a^T)} \right) \log \left( \frac{\pi_{\theta^S}(s_t, a)}{1 - \pi_{\theta^S}(s_t, a^T)} \right) \right],$$

(11)

where $\pi_{\theta^S}(s_t, a)$ is the prescription on action $a$; $\eta \in (0, 1)$ is a constant. The rationale behind Eq. (11) is that minimizing $L_{pgm}(\theta^S)$ enables to simultaneously maximize the likelihood of the action $a^T$ selected by the teacher policy $\pi_{\theta^T}(s_t, a^T)$, and the entropy over the remaining actions $-\sum_{a=1, a \neq a^T}^{|A|} \left( \frac{\pi_{\theta^S}(s_t, a)}{1 - \pi_{\theta^S}(s_t, a^T)} \right) \log \left( \frac{\pi_{\theta^S}(s_t, a)}{1 - \pi_{\theta^S}(s_t, a^T)} \right)$. The entropy maximization results in a smaller maximum over action $a$, $a \in A, a \neq a^T$. Hence, by maximizing $\pi_{\theta^S}(s_t, a^T)$ at the same time, we can facilitate a larger prescription gap $g_{\theta^S}(s_t, a^T)$. Note that, the entropy calculation is weighted by $\frac{1}{1 - \pi_{\theta^S}(s_t, a^T)}$, this makes the distillation training focus on maximizing $\pi_{\theta^S}(s_t, a^T)$ at the beginning when $\pi_{\theta^S}(s_t, a^T)$ is small. As $\pi_{\theta^S}(s_t, a^T)$ increases during training, $L_{pgm}(\theta^S)$ gradually shifts attention to the entropy maximization. In addition, $\eta$ balances the maximization on $\pi_{\theta^S}(s_t, a^T)$ and entropy regularization.

**Jacobian Regularization (JR).** As derived in Eq. (9), a robust policy distillation requires minimizing the Jacobian on the input $s_t$ as additional regularization. The concept of JR was introduced by Drucker and Le Cun [19] in double backpropagation to enhance generalization performance, where they trained neural networks not only by minimizing the gradient on weights but the gradient with respect to the input features. Hoffman et al. [20] utilized JR to increase the stability of image classifiers. However, how to effectively exploit JR in RL adversarial defence, especially in the policy distillation process, has so far remained under-explored. With that in mind, we thus propose the JR loss,

$$L_{jr}(\theta^S) = \left\| \frac{\partial L_{pgm}(\theta^S)}{\partial s_t} \right\|_F,$$

(12)

where $\frac{\partial L_{pgm}(\theta^S)}{\partial s_t}$ indicates the Jacobian on state $s_t$ w.r.t. the loss function $L_{pgm}(\theta^S)$; $F$ represents the Frobenius norm. It is worth noting that most start-of-the-art attack algorithms are on the basis of utilizing the Jacobian, thus minimizing the magnitude of Jocobian intuitively provides weaker gradient information; this makes a harder generation of $\delta_t$ for an attacker. In addition, according to the analysis in [21], if we maximize the prescription gap, it is able to alleviate the issue of gradient masking. A more detailed analysis on the improvement of adversarial robustness via minimizing the magnitude of Jacobian is provided in Theorem 2.

### 3.4 Theoretical Analysis

To support the design of our loss function for robust policy distillation, we analyze the policy prescription gap and the resultant improvement on adversarial robustness.
Theorem 1 (Policy prescription gap maximization) Given a particular prescription \( \pi_{\theta^s}(s_t, a^T) \) by student policy \( \pi_{\theta^s} \) on the action \( a^T \), if the PGM loss \( L_{pgm}(\theta^S) \) is minimized, we can ensure that the prescription gap \( G_{\theta^s}(s_t, a^T) \) in Eq. (9) is maximized. Moreover, if \( \pi_{\theta^s}(s_t, a^T) > \frac{1}{|A|} \) where \(|A|\) is the size of action space, \( G_{\theta^s}(s_t, a^T) \) is guaranteed to be positive.

The proof is deferred to Appendix A, which follows from the fact that maximum entropy is attained when the distribution over actions is uniform. This results in a minimized \( \pi_{\theta^s}(s_t, a) \), in turn maximizing the prescription gap \( G_{\theta^s}(s_t, a^T) = [\pi_{\theta^s}(s_t, a^T) - \pi_{\theta^s}(s_t, a)] \). Given \( \pi_{\theta^s}(s_t, a) > \frac{1}{|A|} \), we can derive that \( G_{\theta^s}(s_t, a^T) > 0 \).

Theorem 2 (Adversarial robustness) Given a student policy \( \pi_{\theta^s} \) and a minimized PGM loss \( L_{pgm}(\theta^S) \), we can guarantee an improvement on adversarial robustness if the JR loss \( L_{jr}(\theta^S) \) is minimized.

The proof is deferred to Appendix B, where the basic idea is that the minimized \( \| \frac{\partial L_{pgm}(\theta^S)}{\partial s} \| \) minimizes the impact of adversarial attack \( \delta_t \) on the PGM loss. Due to Theorem 1 has proved that minimized PGM loss ensures a maximized prescription gap. Thus the prescription gap with attacks is still maximized; this means an improvement on adversarial robustness.

4 Experiments
4.1 Experimental Setup
To provide a fair comparison, we test our approach on five Atari games (i.e., Freeway, Bank Heist, Pong, Boxing and Road Runner) that are utilized in the state-of-the-arts [11, 14]. For each game, the state is 4-stack of consecutive frames, and each frame is pre-processed to size 84 \times 84, where the pre-processing applies the environment wrapper based on the Arcade learning environment in Rainbow. After preprocessing, the pixel value from [0, 255] is normalized to [0, 1].

Teacher Policy Training. In our evaluation, the teacher policy is trained by DQN, where the network structure is based on Dueling-DQN and Noisy-Net following Rainbow [22]. Each teacher policy is trained with 4 million frames on a particular game (see source code [7]), which costs 12-40 hours on Nvidia 2080Ti. The other parameter settings for teacher policy training are provided in Appendix C.

Student Policy Distillation Training. In our implementation, the network structure of student policy \( \pi_{\theta^s} \) uses the Nature-DQN structure [11]. To train the student policy, we collect \( 1 \times 10^5 \) state-prescription pairs \( [s_t, \pi_{\theta^T}(s_t)] \) from the teacher policy \( \theta^T \), where 90% are treated as training data and the remaining 10% as testing data. We use Adam as the optimizer, and the implementation is based on Keras. The rest hyperparameter settings for policy distillation are provided in Appendix D.

Adversarial Attacks. To be align with the compared state-of-the-arts [11, 14], we use the untargeted Projected Gradient Decent (PGD) attack that performs \( K \) iteration updates of adversary \( \delta_t \), given by:

\[
s^{k+1} = s^k + \frac{\epsilon}{K} \mathcal{P} \left( \frac{\partial \mathcal{H}(\pi_{\theta^s}(s^k), a^T)}{\partial s^k} \right), \quad s^0 = s_t, \quad k = 0, 1, \ldots, K - 1, \tag{13}
\]

where \( s^{k+1} \) is the attacked input state with adversarial perturbation inside; \( \mathcal{H}(\pi_{\theta^s}(s^k), a^T) \) is the cross-entropy loss between student policy prescription \( \pi_{\theta^s}(s^k) \) and the one-hot vector encoded based on action \( a^T \) selected by teacher policy. \( \mathcal{P} \) is an operator projecting the input gradient into a constrained norm ball. \( \epsilon \) and \( \frac{\epsilon}{K} \) are the total norm constraint and the norm constraint for each iteration step, respectively. We explore three different values \{4, 10, 20\} of \( K \), where \( K = 4 \) and \( K = 10 \) are investigated in [11] and [14], respectively. We further explore the PGD attack with \( K = 20 \), that corresponds to a stronger attack than previous studies. The implementation of PGD attack is based on the adversarial robustness toolbox [23].

Evaluation Metric. In previous studies of adversarial robustness on DRL, the accumulated rewards under attack (i.e., usually PGD attack) are treated as the evaluation metric. However, in policy distillation, the performance of teacher policy (as a baseline model) also has a significant impact on

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1. Our code will be released upon acceptance.
2. https://github.com/Kaixhin/Rainbow
### Table 1: Averaged accumulated rewards over 15 episodes with and without PGD attack. The comparison with previous studies under PGD attack with different PGD iteration step $K$. Since the results on Boxing from Zhang et al. [14] has been not reported, we mark them as NA.

| Method | Evaluation Case | Freeway | Bank Heist | Pong | Boxing | Road Runner |
|--------|-----------------|---------|------------|------|--------|-------------|
| Ours   | No-attack       | 32.93 ± 0.25 | 1614.67 ± 12.58 | 21 ± 0 | 71.60 ± 9.26 | 16013.3 ± 1831.89 |
|        | PGD ($K = 4$)  | 32.61 ± 2.18 | 1606.67 ± 23.57 | 20.3 ± 0.79 | 70.20 ± 8.52 | 16120.3 ± 4655.93 |
|        | PGD ($K = 10$) | 30.63 ± 1.53 | 1614.0 ± 16.65 | 20.3 ± 0.67 | 72.40 ± 8.43 | 15953.3 ± 2834.28 |
|        | PGD ($K = 20$) | 31.53 ± 1.45 | 1614.67 ± 9.57 | 20.1 ± 0.94 | 70.20 ± 8.52 | 14160.0 ± 3091.67 |
| RS-DQN | No-attack       | 32.93 | 238.66 | 19.73 | 80.67 | 12106.67 |
|        | PGD ($K = 4$)  | 32.53 | 190.67 | 18.13 | 50.87 | 5753.33 |
| SA-DQN | No-attack       | 30.78 ± 0.5 | 1041.4 ± 12.3 | 21.0 ± 0 | NA | 15172.0 ± 791.7 |
|        | PGD ($K = 10$) | 30.36 ± 0.7 | 1043.6 ± 9.5 | 20.1 ± 0.0 | NA | 15280.0 ± 827.7 |

### Table 2: Comparison of relative robustness measured by $\mathcal{M}(\cdot)$ in Eq. (14) based on the results in Table 1. The comparison with previous studies under PGD attack with different PGD iteration $K$.

| Method | Evaluation Case | Freeway | Bank Heist | Pong | Boxing | Road Runner |
|--------|-----------------|---------|------------|------|--------|-------------|
| Ours   | $\pi_{T}$, PGD ($K = 4$), $\pi_{S}$ | $-0.58\%$ | $0.25\%$ | $-3.33\%$ | $-26.05\%$ | $-61.12\%$ |
|        | $\pi_{T}$, PGD ($K = 10$), $\pi_{S}$ | $-6.62\%$ | $0.71\%$ | $-3.33\%$ | $-23.74\%$ | $-61.52\%$ |
|        | $\pi_{T}$, PGD ($K = 20$), $\pi_{S}$ | $-3.87\%$ | $0.74\%$ | $-4.29\%$ | $-26.05\%$ | $-61.85\%$ |
|        | $\pi_{T}$, PGD ($K = 4$), $\pi_{S}$ | $-4.25\%$ | $0.69\%$ | $-4.29\%$ | $-1.96\%$ | $-11.57\%$ |
| RS-DQN | $\pi_{T}$, PGD ($K = 4$), $\pi_{S}$ | $-1.42\%$ | $-14.11\%$ | $-10.24\%$ | $-46.93\%$ | $-38.83\%$ |
|        | $\pi_{T}$, PGD ($K = 10$), $\pi_{S}$ | $-1.21\%$ | $-19.87\%$ | $-8.11\%$ | $-36.94\%$ | $-52.48\%$ |
| SA-DQN | $\pi_{T}$, PGD ($K = 10$), $\pi_{S}$ | $-7.72\%$ | $-20.24\%$ | $-2.90\%$ | NA | $-58.64\%$ |
|        | $\pi_{T}$, PGD ($K = 10$), $\pi_{S}$ | $-1.65\%$ | $0.21\%$ | $-4.29\%$ | NA | $0.71\%$ |

the reward of the distilled student policy; this inspires us to further design a new evaluation metric named as relative robustness $\mathcal{M}(R_{\delta}, R_{\beta})$.

$$\mathcal{M}(R_{\delta}, R_{\beta}) = \frac{R_{\delta} - R_{\beta}}{R_{\beta}} \cdot 100\% \quad (14)$$

where $R_{\delta}$ is the accumulated reward achieved by the student policy under attack, and $R_{\beta}$ is the accumulated reward achieved by the baseline policy (either teacher policy or student policy) without attack. This metric provides a percentage variation of accumulated rewards, which can measure the relative robustness when comparing different policy distillations with different baselines. Hence, we contend that it should be treated as a complement metric for evaluating adversarial robustness.

### 4.2 Experimental Results

**Evaluation of Distilled Student Policy $\pi_{S}$.** We evaluate the robustness the student policy trained by our robust policy distillation, and compare our results with state-of-the-arts, including (1) Fischer et al. [11] where DQN is the baseline policy; RS-DQN is the distilled student policy and (2) Zhang et al. [14] where DQN is the baseline policy; SA-DQN is the adversarially trained policy. The results are presented in Table 1.

In general, the accumulated rewards of our distilled policies $\pi_{S}$ under PGD attack are larger than those of both state-of-the-arts; this indicates that our defensive distillation loss can achieve a significant robustness. Specifically, several interesting findings are noted. (1) Under PGD ($K = 4$) attack, our $\pi_{S}$ achieves a much higher reward than that of [11]. Especially on Bank Heist and Road Runner, our $\pi_{S}$ achieves rewards of 1606.67 and 16120.3 respectively, whilst in [11] they only get 190.67 and 5753.33. Another noteworthy finding is on Boxing, where although the performance of our teacher policy $\pi_{T}$ is worse than the DQN in [11], our $\pi_{S}$ still outperforms RS-DQN in [11]. (2) The similar superiority of our approach can also be found in PGD ($K = 10$) attack versus Zhang et al. [14]. The performance of our $\pi_{T}$ is comparable with that of DQN in [14]. However, the performance of our $\pi_{S}$ is better than that of SA-DQN [14] with and without PGD ($K = 10$) attack. For instance, on Bank Heist, our $\pi_{S}$ achieves almost the
Analysis of Prescription Gap. To examine our claim on improving adversarial robustness via maximizing the prescription gap between optimal action and other sub-optimal actions, we compare the prescription gap between the teacher policy \( \pi_{\theta^T} \) and student policy \( \pi_{\theta^S} \) as shown in Figure 2. Assuming a prescription distribution \( P = [p_0, p_1, \ldots, p_N] \), the prescription gap is calculated as \( p_{i^*} - \max_{j \neq i^*} p_j \), where \( i^* = \arg \max_i p_i \). From Figure 2, on all the five games tested, the prescription gap values of student policies are far larger than those of teacher policies. Together with the above robustness analysis, it empirically demonstrates that the robustness of DRL policies are indeed improved via maximizing the prescription gap based on our policy distillation loss in Eq. (10).

Analysis of Hyper-parameter \( \beta \). We analyze the impact of \( \beta \) in Eq. (10) which controls the weight of JR loss. From the experimental results shown in Appendix E, we observe that as \( \beta \) increases, the accumulated reward of the distilled student policy under attack generally first goes up and then drops. Such observation is consistent with our analysis in Theorem 2, where the adversarial robustness is
based on a minimized $\mathcal{L}_{pgm}(\theta^S)$. Therefore, a too large $\beta$ will overemphasize the contribution of the JR loss, thus hurting the accumulated rewards of student policy even without attacks.

5 Conclusion

This paper proposes an efficient policy distillation paradigm to achieve a robust student policy that is capable of defending adversarial attacks without adversarial attacks during training. To this end, we introduce a novel distillation loss that consists of PGM loss and JR loss. Through theoretical analysis, we prove that our policy distillation ensures to increase the prescription gap between the optimal and sub-optimal actions as well as the adversarial robustness. Experiments on five Atari games show that our distilled student policy significantly outperforms the state-of-the-arts under strong attacks.

Broader Impact

In recent years, many success stories from DRL have been documented. However the vulnerabilities of DRL policies set a severe constraint on their real-world deployments, especially in those safety critical tasks (e.g., self-driving cars). Our approach can provide a realistic solution for improving the adversarial robustness of a deployed DRL policy. Through our robust policy distillation, we can train an accurate and robust student policy just according to the behaviour of the deployed DRL policy. It is noteworthy that our robust policy distillation is independent on any adversarial example; thus making our approach more realistic compared to adversarial training based methods. Taking self-driving car as an example, to improve adversarial robustness, the adversarial training requires the car to witness a huge mount of accidents as adversarial example. In contrast, such dangerous and un-affordable adversarial example is not required anymore in our robust policy distillation.

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**Appendix A: Proof of Theorem 1**

Given the condition that \( \pi_{\theta^*}(s_t, a^T) \) is a particular prescription, we define \( \pi_{\theta^*}(s_t, a^T) = C \) as a constant. Then the PGM loss in Eq. (11) can be rewritten as,

\[
L_{pgm}(\theta^S) = -C^n \cdot \left[ - \sum_{a=1, a \neq a_T}^{\left| A \right|} \left( \frac{\pi_{\theta^*}(s_t, a)}{1-C} \right) \log \left( \frac{\pi_{\theta^*}(s_t, a)}{1-C} \right) \right],
\]

As \( C \in (0, 1) \), we can get \( 1 - C \in (0, 1) \) and \( C^n \in (0, 1) \) where \( \eta \) is a positive constant. Therefore, minimizing \( L_{pgm}(\theta^S) \) is equal to maximizing the entropy \( h(\pi_{\theta^*}(s_t, a)) \),

\[
h(\pi_{\theta^*}(s_t, a)) = - \sum_{a=1, a \neq a_T}^{\left| A \right|} \pi_{\theta^*}(s_t, a) \log(\pi_{\theta^*}(s_t, a)).
\]

Given \( \pi_{\theta^*}(s_t, a^T) = C \), we have \( \sum_{a=1, a \neq a_T}^{\left| A \right|} \pi_{\theta^*}(s_t, a) = 1 - C \). According to the information theorem \[24\], the maximum of \( h(\pi_{\theta^*}(s_t, a)) \) is obtained when the distribution of \( \pi_{\theta^*}(s_t, a) \) is uniform; this results in a minimum \( \pi_{\theta^*}(s_t, a) = \frac{1}{\left| A \right| - 1} \). Therefore, we can get the maximized prescription gap

\[
G_{\theta^*}(s_t, a^T) = C - \frac{1 - C}{\left| A \right| - 1}
\]

\[
= \frac{\left| A \right|}{\left| A \right| - 1} C - \frac{1}{\left| A \right| - 1}.
\]

If \( \pi_{\theta^*}(s_t, a^T) > \frac{1}{\left| A \right|} \), then we can get \( G_{\theta^*}(s_t, a^T) > 0 \).

**Appendix B: Proof of Theorem 2**

In our policy distillation, the PGM loss \( L_{pgm}(\theta^S) \) is optimized to simultaneously maximize the probability on action \( a^T \) and the prescription gap \( G_{\theta^*}(s_t, a^T) \) as shown in the proof of Theorem 1. Therefore, in order to keep the policy \( \pi_{\theta^*} \) robust to adversarial attack \( \delta_t, \| \delta_t \| \leq \epsilon \), we have to ensure the PGM loss is still minimized with adversarial attack \( \delta_t \). For ease of analysis, we rewrite the PGM loss \( L_{pgm}(\theta^S) \) as a function of \( s_t \),

\[
L_{pgm}(s_t) = -\pi_{\theta^*}(s_t, a^T)^n \cdot \left[ - \sum_{a=1, a \neq a_T}^{\left| A \right|} \left( \frac{\pi_{\theta^*}(s_t, a)}{1 - \pi_{\theta^*}(s_t, a^T)} \right) \log \left( \frac{\pi_{\theta^*}(s_t, a)}{1 - \pi_{\theta^*}(s_t, a^T)} \right) \right].
\]

The PGM loss under adversarial attack \( \delta_t \) is \( L_{pgm}(s_t + \delta_t) \). Using Taylor expansion, we have

\[
L_{pgm}(s_t + \delta_t) = L_{pgm}(s_t) + \delta_t \frac{\partial L_{pgm}(s_t)}{s_t} + \omega_{pgm}, \forall \delta_t, \| \delta_t \| \leq \epsilon.
\]

We assume \( \omega_{pgm} \) is a small value that can be ignored. Note that \( \delta_t \) is generated by the attacker to negatively impact our distilled policy, which makes \( \delta_t \) not controllable from the distillation training perspective. Therefore, to minimize the impact of \( \delta_t \), \( \frac{\partial L_{pgm}(s_t)}{s_t} \), we can alternatively minimize the magnitude of Jacobian \( \| \frac{\partial L_{pgm}(s_t)}{s_t} \| \).

In summary, given that the \( L_{pgm}(s_t) \) without attack is minimized, if we minimize the norm of Jacobian \( \| \frac{\partial L_{pgm}(s_t)}{s_t} \| \), we can still achieve an minimized PGM loss \( L_{pgm}(s_t + \delta_t) \) with adversarial attack \( \delta_t \). Therefore, even with adversarial attack \( \delta_t \), the loss \( L_{pgm}(s_t + \delta_t) \) can still be minimized, which leads to a maximized \( G_{\theta^*}(s_t, a^T) > 0 \) as proved in Theorem 1. Accordingly, we can conclude that the adversarial robustness is improved.
Appendix C: Parameter Settings for Training Teacher Policy

Table 3 provides the hyper-parameter settings for training the teacher policy. In this paper, we use Rainbow DQN [22] to train the teacher policy, and explanations of each parameter can refer to [22].

| Parameters          | Settings      | Descriptions                      |
|---------------------|---------------|-----------------------------------|
| Optimizer           | Adam          | –                                 |
| Batch size          | 32            | –                                 |
| Learning rate       | 0.000625      | Adam learning rate                |
| φ₁                  | 0.9           | Adam decay rate 1                 |
| φ₂                  | 0.999         | Adam decay rate 2                 |
| Adam-eps            | $1.5 \times 10^{-4}$ | Adam epsilon                  |
| Start steps         | $2 \times 10^{4}$ | Number of steps before starting training |
| Environment ID      | 123           | The random seed in Arcade environment |
| T-max               | $1 \times 10^{7}$         | Number of training steps         |
| Max-episode-len     | $108 \times 10^{3}$ | Maximum episode length in game frames |
| h                   | 4             | Number of consecutive states processed |
| Hidden-size         | 512           | Network hidden size               |
| σ                   | 0.1           | Initial standard deviation of noisy linear layers |
| Atoms               | 51            | Discretised size of value distribution |
| V-min               | -10           | Minimum of value distribution support |
| V-max               | 10            | Maximum of value distribution support |
| Memory-length       | $1 \times 10^{6}$ | The length of replay buffer       |
| Target-update       | $1 \times 10^{4}$ | The frequency of updating target network |

Appendix D: Parameter Settings for Our Proposed Policy Distillation of $\pi_{\theta^S}$

Table 4 shows all the settings of parameters used to train the robust student policy with the loss function proposed in Section 3.3.

| Parameters          | Settings      | Descriptions                                  |
|---------------------|---------------|-----------------------------------------------|
| Batch size          | 32            | –                                             |
| Learning rate       | 0.00004       | Adam learning rate                            |
| φ₁                  | 0.9           | Adam decay rate 1                             |
| φ₂                  | 0.999         | Adam decay rate 2                             |
| Adam-eps            | $1 \times 10^{-7}$ | Adam epsilon                  |
| Max-epoch           | 1000          | The maximum number of epochs for policy distillation training |
| Patience            | 60            | Number of epochs that have no training improvement |
| β                   | 0.1           | To control the weight of $L_{j, r}(\theta^S)$ in Eq. 10 |
| η                   | 1/3           | Discount factor on $\pi_{\theta^S}(s_t, a_t^T)$ in Eq. 11 |
| φ                   | 0.9           | Weight balances Jacobian scale and Jacobian consistency in Eq. 12 |
| τ                   | 0.95          | Discount factor on Jacobian consistency with frame skip in Eq. 12 |
Appendix E: Analysis of Hyper-parameter $\beta$

![Figure 3: The comparison of distilled student policies with different value of $\beta$.](image)

To analyze the impact of $\beta$ in Eq. (10) which controls the weight of JR loss, we test 8 different settings on $\beta$ that are shown in Figure 3. Note that $\beta = 0$ represents the loss function does not consider the JR loss. From Figure 3, we observe that as long as $\beta \neq 0$, the distilled policies always behave robustly. In addition, as $\beta$ increases, the accumulated reward of the distilled policy with no attack gradually decreases; this indicates that overemphasizing of JR loss can hurt the accumulated reward without attack. This observation is consistent with our analysis in Theorem 2 where the adversarial robustness is based on a minimized $L_{pgm}(\theta^2)$. Therefore, a too large $\beta$ will overemphasize the contribution of the JR loss, thus hurting the accumulated rewards of student policy even without attacks. Moreover, the accumulated rewards under attack show that if we set $\beta$ as a suitable value (e.g., $\beta = 0.001$, $\beta = 0.01$ or $\beta = 0.1$), the distilled policies can behave robustly without sacrificing the reward.