Achieving Vanishing Rate Loss in Decentralized Network MIMO

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Abstract

In this paper, we analyze a Network MIMO channel with 2 Transmitters (TXs) jointly serving 2 users, where each TX has a different multi-user Channel State Information (CSI), potentially with a different accuracy. Recently it was shown the surprising result that this decentralized setting can attain the same Degrees-of-Freedom (DoF) as its genie-aided centralized counterpart in which both TXs share the best-quality CSI. However, the DoF derivation alone does not characterize the actual rate and the question was left open as to how big the rate gap between the centralized and the decentralized settings was going to be. In this paper, we considerably strengthen the previous intriguing DoF result by showing that it is possible to achieve asymptotically the same sum rate as that attained by Zero-Forcing (ZF) precoding in a centralized setting endowed with the best-quality CSI. This result involves a novel precoding scheme which is tailored to the decentralized case. The key intuition behind this scheme lies in the striking of an asymptotically optimal compromise between i) realizing high enough precision ZF precoding while ii) maintaining consistent-enough precoding decisions across the non-communicating cooperating TXs.

I. INTRODUCTION

Joint transmission in wireless networks is known to bring multiplicative improvements in network rates only under the assumption of perfect CSI [1]. The study of how imperfect or quantized CSI at the TXs (CSIT) affects the performance has focused on the assumption that the imperfect information is perfectly shared between the non-colocated transmitting antennas [1, 2]. However, this assumption may not be adapted to many applications within the upcoming wireless networks use cases, such as Ultra-Reliable Low-Latency Communication (URLLC) or heterogeneous backhaul deployments. As a result, there is a clear interest in looking at the scenario in which each TX may have a different information about the channel, denoted as Distributed CSIT setting [3].

We focus in this paper on a particular sub-case of the Distributed CSIT setting, so-called Distributed Network MIMO, wherein the TXs have access to all the information symbols of the users (RXs), yet do not share the same CSIT [4]. This model arises in presence of caching [5] and Cloud-RAN with high mobility [6], in which latency constraints impede efficient CSIT sharing within the channel coherence time. The DoF of this scenario has been

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studied in previous works. Specifically, it was shown that conventional ZF performs very poorly and several schemes were proposed to improve the robustness of the transmission with respect to CSIT inconsistencies [4], [7], [8]. One of the main successes has been obtained for the 2-user setting where the optimal DoF was shown to be equal to the DoF of the centralized setting [7] through an asymmetric precoding scheme where some TX deliberately throws away instantaneous CSIT.

Yet, these works suffer from the limitations of the metric used, as DoF only provides the asymptotic rate slope with respect to the SNR. Since it does not provide any information about the beamforming gain or the efficient power use at the TXs, schemes resulting in the same DoF may need a considerably different power to achieve the same rate [9]. Hence, the natural next step towards capacity characterization is to study the rate offset, which is the constant term in the linear approximation of the sum rate at high SNR, i.e.,

\[ R(P) = \text{DoF} \left( \log_2(\text{SNR}) - L_\infty \right) + o(1) \]  

where \( L_\infty \) represents the rate offset (vertical offset). Our main contributions read as follows:

- We provide a novel precoding scheme that achieves accurate ZF of the interference and, at the same time, a high beamforming gain through consistent transmission at the TXs.
- Through a new lower bound, we show that the proposed scheme achieves a vanishing rate loss at high SNR when compared to the centralized configuration with perfect CSIT sharing.

**Notations:** We use the Landau notation, i.e., \( f(x) = o(x) \) implies \( \lim_{x \to \infty} \frac{f(x)}{x} = 0 \), and \( f(x) = O(x) \) implies \( \lim_{x \to \infty} \frac{f(x)}{x} = M \), with \( 0 < |M| < \infty \). \( \mathbb{R}^+ \) stands for \( \{ x \in \mathbb{R} : x > 0 \} \), and \( \mathbb{E}_{|A} \) denotes the conditional expectation given an event \( A \). \( \|v\| \) stands for the Euclidean norm of the vector \( v \).

## II. Problem Formulation

### A. Transmission Model

We consider a setting with 2 single-antenna TXs jointly serving 2 single-antenna RXs over a Network MIMO setting –also known as Distributed Broadcast Channel (BC)–. The extension to multiple-antennas nodes follows naturally. Yet, it comes at the cost of heavier and less intuitive notations such that we focus here on the single antenna case. The extension to multiple-antennas TXs but single antenna receiver is more challenging and is relegated to the journal version of this work. The signal received at RX \( i \) is

\[ y_i = h_i^H x + z_i, \]  

where \( h_i^H \in \mathbb{C}^{1 \times 2} \) is the channel coefficients vector towards RX \( i \), \( x \in \mathbb{C}^{2 \times 1} \) is the transmitted multi-user signal, and \( z_i \in \mathbb{C} \) is the Additive White Gaussian Noise (AWGN) at RX \( i \), being independent of the channel and the transmitted signal, and drawn from a circularly symmetric complex Gaussian distribution \( \mathcal{N}_\mathbb{C}(0,1) \). We further define the channel matrix \( H \in \mathbb{C}^{2 \times 2} \) as

\[ H \triangleq \begin{bmatrix} h_1^H \\ h_2^H \end{bmatrix} \]  

(3)
with its \((i, k)\)-th element representing the channel coefficient from TX \(k\) to RX \(i\) and being denoted as \(h_{ik}\). The channel coefficients are assumed to be i.i.d. as \(N_C(0, 1)\) such that all the channel sub-matrices are full rank with probability one.

The transmitted multi-user signal \(x \in \mathbb{C}^{2 \times 1}\), is obtained from the precoding of the symbol vector \(s \triangleq [s_1 \ s_2]^T\), where the symbols \(s_i\) are i.i.d. as \(N_C(0, 1)\) and \(s_i\) denotes the symbol intended by RX \(i\) such that

\[
x \triangleq \bar{P} \begin{bmatrix} t_1 \\
t_2 \end{bmatrix} \begin{bmatrix} s_1 \\
s_2 \end{bmatrix},
\]

where \(\bar{P} \triangleq \sqrt{P}\) and \(P\) is the maximum instantaneous transmit power per TX. The vector \(t_i \in \mathbb{C}^{2 \times 1}\) denotes the normalized precoding vector towards RX \(i\). For further reference, we also introduce the multi-user precoder \(T \in \mathbb{C}^{2 \times 2}\) as \(T \triangleq \begin{bmatrix} t_1 \\
t_2 \end{bmatrix}\), and the TX \(j\)'s precoder as \(t_{TX,j} \triangleq \begin{bmatrix} \{t_1\}_j \\
\{t_1\}_j \end{bmatrix}^T\). We assume a per-TX instantaneous power constraint such that \(\|t_{TX,j}\| \leq 1, \forall j \in \{1, 2\}\).

**B. Grassmanian Random Vector Quantization**

We consider in this work that the RXs have perfectly estimated the channel coefficient to focus on the challenges of CSI feedback to the transmitter side. As we analyze the high SNR performance, we follow the same approach of the reference work from Jindal \cite{2} and study the performance of Grassmanian Random Vector Quantization (RVQ). For the sake of completeness, we will recall in the following some properties that will be needed in the proof of our main results. For more details about RVQ, see \cite{2}, \cite{10}.

In RVQ, a unit-norm channel vector \(\tilde{h} \in \mathbb{C}^M\) is quantized using \(B\) bits to a codebook \(C\) containing \(2^B\) unit-norm vectors isotropically distributed on the \(M\)-dimensional unit sphere. We consider a Grassmanian quantization scheme such that the quantized estimate denoted by \(\hat{h} \in \mathbb{C}^M\) is obtained to minimize the angle with the true channel, i.e.,

\[
\hat{h} = \arg\max_{w \in C} |\tilde{h}^H w|^2
\]

\[
= \arg\min_{w \in C} \sin^2(\angle(\tilde{h}, w)),
\]

where we have introduced the angle for unit-norm vectors in \(\mathbb{C}^M\) from \(\angle(x, y) \triangleq \arccos|x^H y|\). We define the quantization error as

\[
Z \triangleq \sin^2(\hat{h}, \tilde{h}).
\]

Since the elements of the codebook \(C\) are independent of \(\tilde{h}\) and isotropically distributed, the quantization error \(Z\) is obtained as the minimum of \(2^B\) independent beta \((M - 1, 1)\) random variables. Upon defining \(z = \sqrt{Z}\), and \(\bar{z} \triangleq \sqrt{1 - Z}\), we can write the true channel as a function of its quantized version as

\[
\tilde{h} = \bar{z}\hat{h} + z\delta,
\]

where \(\delta\) is a unit-norm vector isotropically distributed in the null space of \(\hat{h}\), and \(\bar{z}\) and \(Z\) are mutually independent. In our setting, since the vectors have \(M = 2\) elements, the quantization error \(Z\) is distributed as the minimum of \(2^B\) standard uniform random variables \(\text{\cite{2}}\).
C. Distributed CSIT Model

As previously mentioned, we consider here a Distributed CSIT configuration in which each TX receives a different imperfect estimate of the multi-user channel \[4\]. For sake of exposition, we consider that the CSI accuracy available at TX \( j \) is homogeneous across RXs.

Specifically, we consider that RX \( i \) feeds back to TX \( j \) a quantized version of the normalized vector \( \hat{h}_i \equiv \frac{h_i}{\|h_i\|} \in \mathbb{C}^2 \) using \( B^{(j)} \) bits, denoted as \( \hat{h}_i^{(j)} \). We assume that RX \( i \) uses random vector quantization codebooks \[2\], such that the codewords \( \hat{h}_i^{(j)} \) are unit-norm vectors uniformly distributed on the 2-dimensional complex unit sphere. After receiving the feedback from both RXs, TX \( j \) obtains a multi-user channel estimate \( \hat{H}^{(j)} = [\hat{h}_1^{(j)}, \hat{h}_2^{(j)}]^H \in \mathbb{C}^{2 \times 2} \). In order to avoid degenerated conditions, we assume that the codebooks of different RXs do not share any codeword. Moreover, we let the number of quantization bits grow linearly with \( \log_2(P) \) as \( B^{(j)} = \alpha^{(j)} \log_2(P) \).

This implies that \( P^{\alpha^{(j)}} = 2^{B^{(j)}} \). The scalar \( \alpha^{(j)} \) is called the CSIT scaling coefficient at TX \( j \). Under such feedback condition, it is known that the multiplexing gain (DoF) of our setting is equal to \( 1 + \min(\max(\alpha^{(1)}, \alpha^{(2)}), 1) \) \[4\], whereas this multiplexing gain collapses if the number of bits does not scale linearly with \( \log_2(P) \) \[1\], \[2\]. We assume that both \( \alpha^{(j)} \) are strictly positive. Given that one TX has the same CSIT quality (\( \alpha^{(j)} \)) for all the links, we can order them w.l.o.g. such that

\[ 1 \geq \alpha^{(1)} \geq \alpha^{(2)} > 0. \] (9)

The multi-user distributed CSIT configuration is represented through the multi-TX CSIT scaling vector \( \alpha \in \mathbb{R}^2 \) defined as

\[ \alpha \equiv \begin{bmatrix} \alpha^{(1)} \\ \alpha^{(2)} \end{bmatrix}. \] (10)

Importantly, we consider that due to delay constraints no additional communications are allowed between the TXs, such that the transmit coefficients at TX \( j \) are designed exclusively on the basis of its corresponding \( \hat{H}^{(j)} \) and the channel statistics, without any additional communication to the other TX.

D. Figure-of-Merit

Our figure-of-merit is the expected sum rate over both the fading realizations and the random codebooks. Let us define the expected rate of RX \( i \) as \( R_i \equiv \mathbb{E}[r_i] \), where \( r_i \) is the instantaneous rate of RX \( i \). In our setting, \( r_i \) writes as

\[ r_i \equiv \log_2 \left( 1 + \frac{P|\hat{h}_i^H t_i|^2}{1 + P|\hat{h}_i^H t_i|^2} \right), \] (11)

where we have introduced the notation \( \bar{i} \equiv i \mod 2 \) + 1. Then, the expected sum rate is given by \( R \equiv R_1 + R_2 \).

E. Centralized Zero-Forcing Precoding

We restrict in this work to ZF precoding schemes, which are known to achieve the optimal DoF in the centralized CSIT setting \[1\], \[2\] and that allow for analytical tractability. In this centralized setting, all the TXs own the same
channel estimate $\hat{H}$. Similarly to the distributed CSIT case, we define $\hat{H}_i$ as the shared estimate of the normalized vector channel towards RX $i$, obtained with a feedback rate of $B = \alpha \log_2(P)$ bits. Hence, in the centralized case $\hat{H}_i = \hat{h}_i(j) \; \forall j \in \{1, 2\}$. Let $v_i^*$ denote a unit-norm ZF precoder for RX $i$, computed on the basis of the estimate $\hat{H}_i$.

We can then write the centralized ZF precoding matrix as $T_{ZF} \triangleq \begin{bmatrix} \mu_1 v_1^* & \mu_2 v_2^* \end{bmatrix}$, where $\mu_i \in \mathbb{R}$ is a parameter that ensures that the instantaneous power constraint is fulfilled, and which will be explained later. From the ZF precoding definition, $v_i^*$ is a vector satisfying that $\hat{H}_i^\dagger v_i^* = 0$. (12)

Given that multiplying the beamformer $v_i^*$ by a phase-shift $e^{j\phi_i}$ does not impact the rate [11], we can select w.l.o.g., among all the possible $v_i^*$, the vector $v_i = e^{-j\phi_i} [\hat{h}_{i2}, -\hat{h}_{i1}]^T$, where $\phi_i$ is the phase of the second coefficient ($\hat{h}_{i1}$). Thus,

$$T_{ZF} \triangleq \begin{bmatrix} \hat{h}_{i1} & \hat{h}_{i2} & \lambda_1^* & 0 \\ \lambda_1^* & \lambda_2^* \end{bmatrix},$$

where we have introduced the notation $\lambda_i^* \triangleq \mu_i |v_{i,2}|$. From the unitary power constraint it holds that $0 \leq \lambda_i^* \leq 1$. Expression in (13) is just a rewriting of the conventional ZF matrix used in the literature [2], introduced to make the analogy with the distributed approach more explicit, such that we detach the interference-nulling part ($V^*$) and the power control ($\Lambda^*$).

F. Instantaneous Power Control

The power normalization strategy is performed by $\mu_i$ and follows any algorithm that belongs to a broad family of functions satisfying the per-TX instantaneous power constraint $\|t_{TX,j}\| \leq 1, \forall j \in \{1, 2\}$. Specifically, let $\lambda_i$ be the power-control value for RX $i$’s symbols, such that $\lambda_i \triangleq \mu_i |v_{i,2}|$. We model the power control as a function $\Lambda$ such that $\forall i \in \{1, 2\}$,

$$\lambda_i = \Lambda \left( \hat{H}, \alpha, P, i \right).$$

(14)

where $\lambda_i \in \mathbb{R}$. We assume that $\Lambda$ is $C^1$, i.e., all its partial derivatives exist and are continuous, and that its Jacobian Matrix $J_\Lambda$ satisfies $\|J_\Lambda\| \leq M_\Lambda < \infty$. Moreover, the probability density function of $\Lambda_i$, denoted as $f_{\Lambda_i}$ is bounded away from infinity such that

$$\max_x f_{\Lambda_i}(x) \leq f_{\Lambda_i}^{\text{max}} < \infty.$$  (15)

From the RVQ feedback assumption, $\hat{H}$ is distributed as $\hat{H}$ and hence the marginal pdf $f_{\Lambda_i}(x)$ is the same for perfect, imperfect centralized and distributed CSIT. To conclude, since the power control acts on the normalized precoder, the instantaneous power constraint per TX implies that

$$0 \leq \lambda_i \leq 1.$$  (16)
III. MAIN RESULTS

A. Proposed Hybrid Active-Passive ZF Precoding

Let us introduce the proposed distributed precoding scheme, coined Hybrid Active-Passive ZF Precoding (HAP), before presenting the main findings. The HAP precoder, denoted by $\mathbf{T}_{\text{HAP}} \in \mathbb{C}^{2 \times 2}$, is given by

$$
\mathbf{T}_{\text{HAP}} \triangleq \begin{bmatrix}
(h_{21}^{(1)})^{-1} h_{22}^{(1)} & (h_{11}^{(1)})^{-1} h_{12}^{(1)} \\
-1 & -1
\end{bmatrix} \odot \begin{bmatrix}
Q(\lambda_1^{(1)}) & Q(\lambda_2^{(1)}) \\
Q(\lambda_1^{(2)}) & Q(\lambda_2^{(2)})
\end{bmatrix}
$$

(17)

where $\odot$ denotes the Hadamard (element-wise) product and $\lambda^{(j)}_i$ is the distributed counterpart of $\lambda^*_i$. We can observe that the first matrix is equal to the interference-nulling matrix $\mathbf{V}^*$ in (13) based on the imperfect CSIT knowledge $\hat{\mathbf{H}}^{(1)}$ while the second matrix replaces the power normalization matrix $\mathbf{\Lambda}^*$.

The notation $Q(\cdot)$ represents the output of an arbitrary quantizer $Q$ satisfying that $Q(x) \leq x$ and that it exists a constant $M_Q < \infty$ such that

$$
\left| \mathbb{E}[Q(x)] \right| \leq M_Q,
$$

(P0)

which is a technical assumption that is satisfied by any non-degenerate quantizer. The role of $Q$ is to trade-off the accuracy of the power control with the consistency of the decision at the TXs, as the ZF orthogonality of (12) is preserved only if both TXs obtain the same quantization value. In order to emphasize the relevance of the quantizer, we define $\Omega$ as the set of estimates $(\hat{\mathbf{H}}^{(1)}, \hat{\mathbf{H}}^{(2)})$ that ensure that the ZF orthogonality is not violated, excluding degenerate cases, i.e.,

$$
\Omega \triangleq \left\{ (\hat{\mathbf{H}}^{(1)}, \hat{\mathbf{H}}^{(2)}) \mid \forall i \in \{1, 2\}, \ Q(\lambda_i^{(1)}) = Q(\lambda_i^{(2)}) \in \mathbb{R}^+ \right\}.
$$

(18)

In simple words, $\Omega$ encloses the cases when the TXs agree on the power normalization coefficients for both RXs and they are strictly positive. We further denote the complementary event of $\Omega$ as $\Omega^c$. We proceed by introducing two important properties for the quantizers.

**Definition 1 (Asymptotically Accurate Quantizers):** A quantizer $Q$ is said to be asymptotically accurate if

$$
\lim_{P \to \infty} Q(\lambda_i^{(j)}) = \lambda_i^{(j)} \quad \text{a.s.} \quad \forall i, j \in \{1, 2\},
$$

(P1)

where a.s. stands for almost surely.

**Definition 2 (Asymptotically Consistent Quantizers):** A quantizer $Q$ is said to be asymptotically consistent if

$$
\Pr(\Omega^c) = o\left( \frac{1}{\log_2(P)} \right).
$$

(P2)

In this work, we exhibit one particular quantizer satisfying properties (P1) and (P2), which is introduced in the following lemma. Optimizing further this quantizer is crucial to good performance at finite SNR and its optimization is an ongoing research topic.

**Lemma 1:** Let $Q_u$ be a uniform quantizer in the interval $[0, 1]$ with a step size of $\bar{P} - \alpha(2)$, such that

$$
Q_u(x) \triangleq \bar{P} - \alpha(2) \left\lfloor \frac{\bar{P} - \alpha(2)}{x} \right\rfloor.
$$

(19)

Then, $Q_u$ satisfies properties (P0), (P1) and (P2).

**Proof:** The proof is relegated to Appendix I. \hfill \blacksquare
B. Main Results

Let us denote by $R_{\text{HAP}}^{\text{ZF}}(\alpha^{(1)}, \alpha^{(2)})$ the expected sum rate achieved using HAP precoding in the Distributed CSIT setting with CSIT scaling quality $\alpha^{(1)}, \alpha^{(2)}$. Similarly, we denote as $R_{\text{ZF}}^{\text{ZF}}(\alpha^{(1)})$ the expected sum rate attained by the centralized ZF precoder of Section II-E on the basis of the estimate $\hat{H}^{(1)}$. Accordingly, the rate gap is defined as

$$\Delta R \triangleq R_{\text{ZF}}^{\text{ZF}}(\alpha^{(1)}) - R_{\text{HAP}}^{\text{ZF}}(\alpha^{(1)}, \alpha^{(2)})$$

(20)

and its instantaneous counterpart denoted as $\Delta r$, such that $\Delta r \triangleq r_{\text{ZF}}^{\text{ZF}}(\alpha^{(1)}) - r_{\text{HAP}}^{\text{ZF}}(\alpha^{(1)}, \alpha^{(2)})$ and $\Delta R = \mathbb{E}[\Delta r]$. We can now state our main results.

**Theorem 1:** The rate gap of ZF precoding with Distributed CSIT and instantaneous power constraint is upper bounded by

$$\Delta R \leq 2\mathbb{E}_{[\log_2 (\Gamma_1)]} + \text{Pr}(\Omega^c) R_{\text{ZF}}^{\text{ZF}}(\alpha^{(1)}),$$

(21)

where $\Omega$ is defined in (18), $\Gamma_1$ is defined as

$$\Gamma_1 \triangleq \left| \lambda_1^{(1)} \right|^2,$$

(22)

and it holds that $R_{\text{ZF}}^{\text{ZF}}(\alpha^{(1)}) \leq \log_2 (1 + 2P)$.

This bound depends on the set $\Omega$ and thus on the quantizer selected. Intuitively, a good quantizer has to ensure a high probability of agreement, so as to make $\text{Pr}(\Omega^c)$ small. This can be done by enlarging the quantization step, what will make the first term bigger, as $Q(\lambda_1^{(1)})$ needs to be as close to $\lambda_1^{(1)}$ as possible. This shows why finding the optimal quantizer is a challenging research topic. Nevertheless, there exists a family of quantizers that behaves asymptotically optimal, as stated in the following theorem.

**Theorem 2:** Let $Q$ be an arbitrary quantizer satisfying (P0), (P1) and (P2). Then, taking the limit in Theorem 1 yields

$$\lim_{P \to \infty} \Delta R \leq 0.$$

(23)

**Proof:** The proof follows from Theorem 1. First, note that the rate $R_{\text{ZF}}^{\text{ZF}}(\alpha^{(1)})$ is trivially bounded by the interference-free single-user rate to obtain that

$$R_{\text{ZF}}^{\text{ZF}}(\alpha^{(1)}) \leq \log_2 \left(1 + P\mathbb{E}\left[\|h_1\|^2\right]\right)$$

(24)

$$= \log_2 (1 + 2P),$$

(25)

what together with property (P2) implies that

$$\text{Pr}(\Omega^c) R_{\text{ZF}}^{\text{ZF}}(\alpha^{(1)}) = o(1).$$

(26)

Consequently, to conclude the proof it only remains to show that $\lim_{P \to \infty} \mathbb{E}_{[\log_2 (\Gamma_1)]} = 0$. From the definition of $\Gamma_1$, it holds that

$$\mathbb{E}_{[\log_2 (\Gamma_1)]} = \mathbb{E}_{[\log_2 (\lambda_1^{(1)})]} - \mathbb{E}_{[\log_2 (Q(\lambda_1^{(1)}))]}.$$
Note that, for any variable $x$ such that $0 \leq x \leq 1$, and for any two events $A, B$, such that $0 < \Pr(B \mid A) < 1$, it holds that

$$
\mathbb{E}[\log_2(x)] = \Pr(B \mid A)\mathbb{E}[\log_2(x)] + \Pr(B^c \mid A)\mathbb{E}[\log_2(x)],
$$

(28)

Since $0 \leq x \leq 1$, $\mathbb{E}[A \cap B \mid \log_2(x)] \leq 0$ and hence

$$
\mathbb{E}[A \cap B] \log_2(x) \geq \frac{1}{\Pr(B \mid A)} \mathbb{E}[A \mid \log_2(x)],
$$

(29)

Therefore, if $\mathbb{E}[A \mid \log_2(x)]$ exists, also $\mathbb{E}[A \cap B \mid \log_2(x)]$ exists and it is bounded below by (29) and above by 0. Let $A$ and $B$ be $A = \{Q(\lambda^{(1)}_i) > 0, \forall i\}$ and $B = \{Q(\lambda^{(1)}_i) = Q(\lambda^{(2)}_i), \forall i\}$. Thus, $\Omega = A \cap B$. It follows from (29) and (P0) that

$$
\mathbb{E}_{\Omega}[\log_2(Q(\lambda^{(1)}_i))] \geq \frac{\Pr(Q(\lambda^{(1)}_i) > 0, \forall i)}{\Pr(\Omega)} M_Q,
$$

(30)

where we have applied the fact that $\Pr(B \mid A) = \frac{\Pr(A \cap B)}{\Pr(A)}$. Hence, $\mathbb{E}_{\Omega}[\log_2(Q(\lambda^{(1)}_i))]$ is bounded. The same result follows for $\mathbb{E}_{\Omega}[\log_2(\lambda^{(1)}_i)]$ from the bounded density assumption of (15). Moreover, from the continuity of the log function and (P1), $\log_2(Q(\lambda^{(1)}_i))$ converges a.s. to $\log_2(\lambda^{(1)}_i)$. From all these facts, we can apply Lebesgue’s Dominated Convergence Theorem (12, Theorem 16.4) to interchange expectation and limit and show that

$$
\lim_{P \to \infty} \mathbb{E}_{\Omega}[\log_2(Q(\lambda^{(1)}_i))] = \mathbb{E}_{\Omega}[\log_2(\lambda^{(1)}_i)],
$$

(31)

and thus $\lim_{P \to \infty} \mathbb{E}_{\Omega}[\log_2(\Gamma_1)] = 0$, which concludes the proof.

Rate Offset with HAP precoder: It follows from Theorem 2 that the rate offset $L_\infty$ —defined in (1)— of ZF in the distributed CSIT setting is the same as for the genie-aided centralized setting, whose rate offset was shown in [2] to be constant with respect to Perfect CSIT ZF (and thus with respect to the capacity-achieving Dirty Paper Coding [13]) for $\alpha = 1$. Specifically, for a constant $b$, if $B^{(1)} = \log_2(P) - \log_2(b)$, then the rate offset with respect to Perfect CSIT ZF is given by $\log_2(b)$ [2].

The key for attaining such performance is the quantization of the power control, that allows the TXs to agree on the power normalization. Indeed, without quantization of the power values $\lambda^{(j)}_i$, it is easy to see from [17] that the orthogonality is lost. Interestingly, Lemma 1 illustrates that simple quantizers—as the uniform one—satisfy the sufficient conditions of convergence if we select the correct number of quantization levels. Moreover, since this quantizer is applied locally and no information exchange is done, the granularity of the quantizer does not increase the complexity of the scheme.

Let us consider that there is agreement between the TXs, i.e., that $Q(\lambda^{(1)}_i) = Q(\lambda^{(2)}_i), \forall i \in \{1, 2\}$, such that we can define

$$
\lambda^{Q}_i \triangleq Q(\lambda^{(j)}_i), \quad \forall i,j \in \{1,2\},
$$

(32)

what (P2) ensures that occurs with a probability high enough such that the disagreement is asymptotically negligible. In this case we can rewrite (17) with a conventional matrix multiplication to get

$$
T^{\text{HAP}} \triangleq \begin{bmatrix}
(\hat{h}^{(1)}_{21})^{-1} & (\hat{h}^{(1)}_{22})^{-1} & \lambda^{Q}_1 & 0 \\
-1 & -1 & 0 & \lambda^{Q}_2
\end{bmatrix}.
$$

(33)
It becomes then clear that the orthogonality (i.e., the interference attenuation) is ensured by the first matrix in (33) while the second diagonal matrix is only used to satisfy the power constraint. It is this division between the orthogonality and the power control that is key to the proof of the theorem, as the orthogonality has to be very precise — in the order of $P^{-\alpha(1)}$ — whereas the requirements for the power accuracy is much weaker. Regarding the quantizer $Q$, note that letting $Q$ having a single quantization point leads to a statistical power control, whereas letting $Q$ have infinite points leads to the unquantized version. In both cases, part of the DoF is lost.

The assumption of instantaneous power constraint stands in contrast with the previous works on the DoF of the Distributed Network MIMO, where a less practical average power constraint was considered [4], [7], [8]. Remarkably, considering an instantaneous power constraint, all the schemes presented in [4], [7], [8] do not achieve the optimal DoF. The only scheme achieving the optimal DoF is obtained from [4] where the transmit power scales in $P/\log(P)$ for a maximum instantaneous power of $P$. This leads to a very inefficient power normalization, and hence to a very poor rate offset.

IV. Proof of Theorem

We consider w.l.o.g. the rate difference at RX 1, denoted as $\Delta R_1$, since the proof for RX 2 is obtained after switching the RX’s indices. Its instantaneous counterpart $\Delta r_1$ is defined similarly, such that $\Delta R_1 = \mathbb{E}[\Delta r_1]$. Then, we can write

$$\Delta R_1 = \operatorname{Pr}(\Omega) \mathbb{E}_{\Omega}[\Delta r_1] + \operatorname{Pr}(\Omega^c) \mathbb{E}_{\Omega^c}[\Delta r_1].$$  (34)

First, we focus on the expectation of the first term $\mathbb{E}_{\Omega}[\Delta r_1]$ which encloses the consistent precoding cases. Conditioned on $\Omega$ it holds that $Q(\lambda_1^{(1)}) = Q(\lambda_2^{(2)})$, $\forall i \in \{1, 2\}$, and hence we can use the notation $\lambda_i^Q$ introduced in (32). Moreover, it can be observed from (13) and (33) that, conditioned on $\Omega$, the HAP precoder satisfies

$$t_i^{\text{HAP}} = \frac{\lambda_i^Q}{\lambda_i^*} t_i^{\text{ZF}}, \quad \forall i \in \{1, 2\}.  \quad \tag{35}$$

Since we assume that in the centralized ZF setting both TXs share the channel estimate of TX 1 ($\hat{H}^{(1)}$), we have that $\lambda_i^* = \lambda_i^{(1)}$. Given that $Q(x) \leq x$, it follows that $\lambda_i^Q/\lambda_i^* \leq 1$, $\forall i \in \{1, 2\}$. Let us recall that $\Gamma_i$ is defined as

$$\Gamma_i \triangleq \left| \frac{\lambda_i^{(1)}}{\lambda_i^*} \right|^2,$$

which satisfies then that $\Gamma_i \geq 1 \quad \forall i \in \{1, 2\}$. Thus, conditioned on $\Omega$ we can write that the SINR obtained through HAP precoding satisfies

$$1 + \frac{P \mathbb{E}[|h_1^H t_1^{\text{HAP}}|^2]}{1 + P \mathbb{E}[|h_2^H t_2^{\text{HAP}}|^2]} = 1 + \frac{1}{\Gamma_1} P \mathbb{E}[|h_1^H t_1^{\text{ZF}}|^2] \geq \frac{1}{\Gamma_1} \left( 1 + \frac{P \mathbb{E}[|h_1^H t_1^{\text{ZF}}|^2]}{1 + P \mathbb{E}[|h_1^H t_1^{\text{ZF}}|^2]} \right),$$

(37)

where (37) follows from (35)–(36) whereas (38) comes from the fact that $1/\Gamma_i \leq 1 \quad \forall i$. We can recognize in (38) the SINR at RX 1 for the centralized ZF scheme such that it holds:

$$R_{1|\Omega}^{\text{HAP}}(\alpha^{(1)}, \alpha^{(2)}) = \mathbb{E}_{\Omega} \left[ \log_2 \left( 1 + \frac{P |h_1^H t_1^{\text{HAP}}|^2}{1 + P |h_1^H t_1^{\text{HAP}}|^2} \right) \right]$$

$$\geq -\mathbb{E}_{\Omega} [\log_2 (\Gamma_i)] + R_{1|\Omega}^{\text{ZF}}(\alpha^{(1)}).$$

(39)
Since \( E[\Delta r_1] = R_{1|\Omega}^{ZF}(\alpha^{(1)}) - R_{1|\Omega}^{HAP}(\alpha^{(1)}, \alpha^{(2)}) \), it follows that
\[
E[\Delta r_1] \leq E[\log_2 (\Gamma_1)].
\] (41)

Focusing now on the inconsistent precoding cases, i.e., \( E[\Delta r_1] \), since \( R_{1|\Omega}^{HAP}(\alpha^{(1)}, \alpha^{(2)}) \geq 0 \) we can write that
\[
E[\Delta r_1] \leq R_{1|\Omega}^{ZF}(\alpha^{(1)}).
\] (42)

Putting these results together yields
\[
\Delta R_1 = R_{1|\Omega}^{ZF}(\alpha^{(1)}) - R_{1|\Omega}^{HAP}(\alpha^{(1)}, \alpha^{(2)}) \leq E[\log_2 (\lambda_i^{(1)}) + \Pr (\Omega) R_{1|\Omega}^{ZF}(\alpha^{(1)})],
\] (44)

where we have applied the fact that \( \Pr (\Omega) \leq 1 \). Thus, since \( \Gamma_1 \) and \( \Gamma_2 \) are identically distributed, it holds that
\[
\Delta R \leq 2\Delta R_1,
\] (45)

which concludes the proof.

V. NUMERICAL RESULTS

We illustrate in the following the performance for the uniform quantizer \( Q_u \) introduced in Lemma 1. For sake of exposition, we assume a simple power normalization that ensures the per-TX instantaneous constraint. Let us introduce the precoding vector of TX \( j \) before normalization as \( \mathbf{v}_{TX} = [v_{1,j}, v_{2,j}]^T \), such that the final precoder of TX \( j \) is \( \mathbf{t}_{TX} = [\mu_i v_{1,j}, \mu_i v_{2,j}]^T \). Then, \( \mu_i \) is chosen as
\[
\mu_i = \frac{1}{\max(\|v_{TX1}\|, \|v_{TX2}\|)} \quad \forall i \in \{1, 2\}.
\] (46)

In Fig. 1 we simulate the expected sum rate of the proposed scheme using Monte-Carlo runs and averaging over 1000 random codebooks and 1000 channel realizations, for the CSIT configuration \( \alpha^{(1)} = 1 \) and \( \alpha^{(2)} = 0.6 \).

We can see that the proposed scheme leads to a vanishing loss with respect to the centralized case (where both TXs are provided with the best CSIT), and that the lower-bound of Theorem 1 is considerably close to the actual rate. Furthermore, the scheme given in [4] using a scaled power normalization of \( P/\log_2(P) \) –so as to guarantee a full DoF and an instantaneous power constraint– can be seen to achieve also the optimal DoF although at the cost of a strong loss in rate offset. Finally, we can see how using an unquantized coefficient at TX2 leads to a loss in terms of DoF. This occurs because, as aforementioned, the mismatches between the precoding coefficients of each TX break the orthogonality needed for the interference nulling. Thus, this scheme only achieves a DoF proportional to \( \alpha^{(2)} \) instead of \( \alpha^{(1)} \). At intermediate SNR, this unquantized scheme outperforms the proposed HAP precoding scheme. Yet, this is a consequence of our focus in this work towards analytical tractability and asymptotic analysis. Optimizing the precoder for finite SNR performance will allow to bridge the gap between the two schemes to obtain a scheme outperforming both of them.


VI. CONCLUSION

Considering a decentralized scenario where each TX has a CSI with different SNR scaling accuracy, we have shown that there exists a linear precoding scheme that asymptotically recovers the rate of ZF precoding in the ideal centralized setting in which the best estimate is shared. Going beyond the setting considered, we have shown how using a low rate quantization of some parameters (here the power normalization) in combination with a higher-accuracy distributed decision allows to reach coordination without losing precision. The extension of the results to more antennas and more users, as well as the optimization at finite SNR, are interesting and challenging research problems currently under investigation.

APPENDIX I

PROOF OF LEMMA 1

We prove Lemma 1 by means of showing that it holds for a more general case. Specifically, we prove that \( \forall k > 1 \), the quantizer

\[
Q_u(x) \triangleq P - \alpha^{(2)} k \lfloor P \alpha^{(2)} x \rfloor,
\]

satisfies properties (P0), (P1) and (P2). We first prove property (P1). Afterwards, we demonstrate (P2) and finally (P0).

A. Proof of (P1): Convergence

In order to prove that \( Q_u \) satisfies (P1), i.e., that

\[
\lim_{P \to \infty} Q_u(\lambda_i^{(j)}) - \lambda_i^{(1)} = 0 \quad \text{a.s.} \quad \forall i, j \in \{1, 2\},
\]

Fig. 1: Expected sum rate of the proposed scheme for the setting with CSIT scaling parameters \( \alpha^{(1)} = 1, \alpha^{(2)} = 0.6 \), using the uniform quantizer of Lemma 1.
we demonstrate (48) for \( j = 2 \), as the case with \( j = 1 \) is straightforwardly proved following the same derivation. Let \( \mathbf{v}_H^{(j)} \in \mathbb{R}^{8 \times 1} \) be the column vector obtained by stacking the real and imaginary parts of the elements of \( \hat{H}^{(j)} \) one on top of another, such that

\[
\mathbf{v}_H^{(j)} = \begin{bmatrix}
\Re \left( \hat{h}_{11}^{(j)} \right) \\
\Im \left( \hat{h}_{11}^{(j)} \right) \\
\vdots \\
\Im \left( \hat{h}_{22}^{(j)} \right)
\end{bmatrix},
\]

where \( \Re(x) \) (resp. \( \Im(x) \)) denotes the real (resp. imaginary) part of \( x \in \mathbb{C} \). Using the Taylor’s expansion of \( \lambda_i^{(2)} \) centered in \( \lambda_i^{(1)} \), and introducing the notation

\[
\vartheta \triangleq (\mathbf{v}_H^{(2)} - \mathbf{v}_H^{(1)})^T \nabla \Lambda_i \left( \mathbf{v}_H^{(1)} \right) + o\left( \| \mathbf{v}_H^{(2)} - \mathbf{v}_H^{(1)} \| \right),
\]

where \( \left( \nabla \Lambda_i (\cdot) \right)^T \) is the \( i \)-th row of the Jacobian Matrix \( J_{\Lambda_i} \), we have that

\[
\lambda_i^{(2)} - \lambda_i^{(1)} = \vartheta.
\]

From (51) and \( Q_u \)’s definition in (47), it follows that

\[
Q_u(\lambda_i^{(2)}) - \lambda_i^{(1)} = \tilde{P}^{u(2)} - \tilde{P}^{u(1)} (\lambda_i^{(1)} + \vartheta) - \lambda_i^{(1)}.
\]

Then, since \( c \left[ \frac{1}{c} (x + y) \right] - x \leq y \), we obtain that

\[
Q_u(\lambda_i^{(2)}) - \lambda_i^{(1)} \leq \vartheta.
\]

Similarly, since \( c \left[ \frac{1}{c} (x + y) \right] - x \geq c \left[ \frac{y}{c} \right] \geq y - c \), we can bound (52) from below as

\[
Q_u(\lambda_i^{(2)}) - \lambda_i^{(1)} \geq \vartheta - \tilde{P}^{u(2)}.
\]

From (53) and (54), it holds that it is sufficient to prove that

\[
\lim_{P \to \infty} \vartheta = 0 \quad \text{a.s.}
\]

to demonstrate that \( \lim_{P \to \infty} Q_u(\lambda_i^{(2)}) = \lambda_i^{(1)} \) almost surely. To do so, we make use of the following lemma, whose proof is relegated to Appendix II.

Lemma 2: Let \( \hat{H}^{(j)}, \forall j \in \{1, 2\} \), be a quantized version of the matrix \( \hat{H} \), such that each row vector \( \hat{h}_{i}^{(j)} \), \( \forall i \in \{1, 2\} \), is quantized with \( B^{(j)} = \alpha^{(j)} \log_2(P) \) bits. Then, it holds that

\[
\lim_{P \to \infty} \| \mathbf{v}_H^{(2)} - \mathbf{v}_H^{(1)} \| = 0 \quad \text{a.s.}
\]

Since \( \| \nabla \Lambda_i \| \leq \| J_{\Lambda_i} \| \leq M_{\Lambda} \), it holds that

\[
|\vartheta| \leq \| \mathbf{v}_H^{(2)} - \mathbf{v}_H^{(1)} \| M_{\Lambda} + o(\| \mathbf{v}_H^{(2)} - \mathbf{v}_H^{(1)} \|)
\]

and thus we obtain from Lemma 2 that

\[
\lim_{P \to \infty} \vartheta = 0 \quad \text{a.s.}
\]

Consequently, \( Q_u \) satisfies (P1).
B. Proof of (P2): Probability of agreement

We want to prove that \( Q_u \) satisfies that

\[
\Pr(\Omega^c) = o\left(\frac{1}{\log_2(P)}\right),
\]

where \( \Pr(\Omega^c) = 1 - \Pr\left(\forall i \in \{1, 2\} \; Q(\lambda_i^{(1)}) = Q(\lambda_i^{(2)}) \in \mathbb{R}^+\right) \). Note that, for any two events \( A, B \), it holds that

\[
1 - \Pr(A \land B) \leq 1 - \Pr(A) + 1 - \Pr(B).
\]

Since the probability of agreement for \( \lambda_1 \) is the same as for \( \lambda_2 \), we can write

\[
\Pr(\Omega^c) \leq 2 \left(1 - \Pr\left(Q_u(\lambda_1^{(1)}) = Q_u(\lambda_1^{(2)}) \in \mathbb{R}^+\right)\right). \tag{61}
\]

Moreover, it holds that

\[
1 - \Pr\left(Q_u(\lambda_1^{(1)}) = Q_u(\lambda_1^{(2)}) \in \mathbb{R}^+\right) \leq \Pr\left(Q_u(\lambda_1^{(1)}) \neq Q_u(\lambda_1^{(2)})\right) + \Pr\left(Q_u(\lambda_1^{(1)}) = 0\right). \tag{62}
\]

Focusing on the last term of (62) it follows that

\[
\Pr\left(Q_u(\lambda_1^{(1)}) = 0\right) = \Pr\left(\lambda_1^{(1)} \leq \bar{P}_{u_n(2)}\right) \leq f^\text{max}_{\lambda_1^{(1)}} \bar{P}_{u_n(2)} \tag{63} \leq \frac{1}{\log_2(P)}, \tag{64}
\]

where (63) follows from the step-size of \( Q_u \), and (64) follows from the bounded density assumption of (15).

Let us focus on the probability of disagreement \( \Pr\left(Q_u(\lambda_1^{(1)}) \neq Q_u(\lambda_1^{(2)})\right) \). Let \( \ell_n \) be the \( n \)-th reconstruction level of \( Q_u \), \( n \in \mathbb{N}_N \triangleq \{1, \ldots, N\} \) with \( N = \left[\bar{P}_{u_n(2)}\right] \). We assume that \( \bar{P}_{u_n(2)} \in \mathbb{N} \) in order to ease the notation, although the result holds for any \( \bar{P}_{u_n(2)} \in \mathbb{R} \). Let us define \( L_n \) as the input interval that outputs \( \ell_n \), i.e.,

\[
L_n = \{x \mid Q_u(x) = \ell_n\}. \tag{66}
\]

\( L_n \) has a range \([\ell_n, \ell_{n+1}]\) such that \( \ell_{n+1} - \ell_n = \bar{P}_{u_n(2)} \) (\( \ell_{N+1} = 1 \)). We split \( L_n \) in two areas, \( B_n \) and \( C_n \), depicted in Fig. 2. The area \( B_n \) is the border area, i.e.,

\[
B_n = \left\{x \in L_n \mid x - \ell_{n}^{\text{min}} < \bar{P}_{u_n(2)} - \ell_n^{\text{max}} - x < \bar{P}_{u_n(2)}\right\}, \tag{67}
\]

whereas the area \( C_n \) is the central area, i.e.,

\[
C_n = \{x \in L_n \setminus B_n\}. \tag{68}
\]

Intuitively, the probability of disagreement is very high if \( \lambda_i^{(1)} \) lies in the border area \( B_n \), whereas this probability vanishes in the central area \( C_n \). Mathematically, we have that

\[
\Pr\left(Q_u(\lambda_1^{(1)}) \neq Q_u(\lambda_1^{(2)})\right) \leq \Pr\left(\lambda_1^{(1)} \in \bigcup_{n \in N_N} B_n\right) + \Pr\left(Q_u(\lambda_1^{(1)}) \neq Q_u(\lambda_1^{(2)}) \mid \lambda_1^{(1)} \in \bigcup_{n \in N_N} C_n\right). \tag{69}
\]

From the bounded density assumption of (15), the probability that a computed value \( \lambda_1^{(1)} \) is in \( B_n \) is
Fig. 2: Illustration of a reconstruction level $L_n$ of the quantizer and the two sub-areas in which we divide it: The central area $C_n$ and the edge area $B_n$.

\[
\Pr \left( \lambda_1^{(1)} \in B_n \right) \leq f_{\lambda_1}^{\max} |B_n| = f_{\lambda_1}^{\max} 2 \bar{P}^{-\alpha(2)} ,
\]

where $|B_n|$ denotes the length of $B_n$. Since there are $N = \bar{P}^{\alpha(2)}$ cells, the probability of being in the border of any cell is

\[
\Pr \left( \lambda_1^{(1)} \in \bigcup_{n \in \mathbb{N}_N} B_n \right) \leq \bar{P}^{\alpha(2)} f_{\lambda_1}^{\max} 2 \bar{P}^{-\alpha(2)}
\]

\[
= 2 f_{\lambda_1}^{\max} \bar{P}^{-\alpha(2)}
\]

\[
= o \left( \frac{1}{\log_2 (P)} \right).
\]

Focusing on $C_n$, since the minimum distance from any point of $C_n$ to the border of $L_n$ is $\bar{P}^{-\alpha(2)}$, it holds that

\[
\Pr \left( Q_n(\lambda_1^{(1)}) \neq Q_n(\lambda_1^{(2)}) \mid \lambda_1^{(1)} \in \bigcup_{n \in \mathbb{N}_N} C_n \right) \leq \Pr \left( |\lambda_1^{(1)} - \lambda_1^{(2)}| \geq \bar{P}^{-\alpha(2)} \mid \lambda_1^{(1)} \in \bigcup_{n \in \mathbb{N}_N} C_n \right).
\]

Given that, for two events $A, C$, $\Pr(A \mid C) \leq \Pr(A) / \Pr(C)$, it follows that

\[
\Pr \left( |\lambda_1^{(1)} - \lambda_1^{(2)}| \geq \bar{P}^{-\alpha(2)} \mid \lambda_1^{(1)} \in \bigcup_{n \in \mathbb{N}_N} C_n \right) \leq \frac{1}{\Pr \left( \lambda_1^{(1)} \in \bigcup_{n \in \mathbb{N}_N} C_n \right)} \Pr \left( |\lambda_1^{(1)} - \lambda_1^{(2)}| \geq \bar{P}^{-\alpha(2)} \right)
\]

\[
\leq \frac{1}{1 - 2 f_{\lambda_1}^{\max} \bar{P}^{-\alpha(2)}} \frac{\Pr \left( |\lambda_1^{(1)} - \lambda_1^{(2)}| \geq \bar{P}^{-\alpha(2)} \right)}{\bar{P}^{-\alpha(2)}}
\]

where (77) follows from (73) and (78) from Chebyshev’s Inequality. In the following, we obtain the expectation $\mathbb{E} \left[ |\lambda_1^{(1)} - \lambda_1^{(2)}|^2 \right]$. From Taylor’s Theorem it follows that

\[
\mathbb{E} \left[ |\lambda_1^{(1)} - \lambda_1^{(2)}|^2 \right] \leq \mathbb{E} \left[ \left( v_h^{(2)} - v_h^{(1)} \right)^T \nabla \Lambda_1 \left( v_h^{(1)} \right) \right]^2 + \mathbb{E} \left[ o \left( \|v_h^{(2)} - v_h^{(1)}\| \right)^2 \right]
\]

\[
\leq M_f^2 \mathbb{E} \left[ \|v_h^{(2)} - v_h^{(1)}\|^2 \right] + \mathbb{E} \left[ o \left( \|v_h^{(2)} - v_h^{(1)}\| \right) \right],
\]

where (80) comes from the fact that $\|\nabla \Lambda_i\| \leq \|J_\Lambda\| \leq M_f$. We present in the following a useful lemma whose proof is relegated to Appendix III.
Lemma 3: Let \( \tilde{H}^{(i)} \), \( i \in \{1, 2\} \), be a quantized version of the matrix \( \tilde{H} \), such that each row vector \( \tilde{h}_i^{(i)} \), \( \forall i \in \{1, 2\} \), is quantized with \( B^{(i)} = \alpha^{(i)} \log_2(P) \) bits. Let \( \kappa \) be a positive constant and \( \alpha^{(1)} \geq \alpha^{(2)} \). Then, it holds that

\[
E \left[ \| v^{(2)}_{\tilde{H}} - v^{(1)}_{\tilde{H}} \|^2 \right] = \kappa P^{-\alpha^{(2)}}. \tag{81}
\]

It follows from Lemma 2 and Lemma 3 than

\[
E \left[ o \left( \| v^{(2)}_{\tilde{H}} - v^{(1)}_{\tilde{H}} \| \right) \right] = o \left( P^{-\alpha^{(2)}} \right). \tag{82}
\]

Including Lemma 3 and \( P_0 \) in (80) yields

\[
E \left[ \lambda^{(1)}_{1} - \lambda^{(2)}_{1} \right] \leq \kappa M^2 P^{-\alpha^{(2)}} + o \left( P^{-\alpha^{(2)}} \right). \tag{83}
\]

Since \( \bar{P} = \sqrt{P} \), substituting (83) in (78) we obtain that

\[
\Pr \left( Q_u(\lambda^{(1)}_{1}) \neq Q_u(\lambda^{(2)}_{1}) \mid \lambda^{(1)}_{1} \in \bigcup_{n \in \mathbb{N}_N} C_n \right) \leq \frac{1}{1 - 2f_{\Lambda_1}^{\max} \bar{P}^{-\alpha^{(2)}}} \frac{\kappa M^2 P^{-\alpha^{(2)}} + o \left( P^{-\alpha^{(2)}} \right)}{P^{-\alpha^{(2)}}}, \tag{84}
\]

\[
= O \left( \frac{1}{P^{-\alpha^{(2)}}} \right), \tag{85}
\]

\[
= o \left( \frac{1}{\log_2(P)} \right) \tag{86}
\]

for any \( k > 1 \). From (65), (74) and (86) it follows that

\[
\Pr (\Omega^c) \leq 2 \left( \Pr \left( Q_u(\lambda^{(1)}_{1}) \neq Q_u(\lambda^{(2)}_{1}) \right) + \Pr \left( Q_u(\lambda^{(1)}_{1}) = 0 \right) \right) \tag{87}
\]

\[
= o \left( \frac{1}{\log_2(P)} \right), \tag{88}
\]

what concludes the proof for property \( P_2 \).

C. Proof of \( P_0 \): Bounded Expectation

We show in the following that \( \exists M < \infty \) such that \( \forall P \) it holds that

\[
\left| E_{Q_u(\lambda^{(i)}) > 0} \left[ \log_2 \left( Q_u(\lambda^{(j)}) \right) \right] \right| \leq M. \tag{89}
\]

Let us denote \( x \triangleq \lambda^{(j)}_i \), \( i, j \in \{1, 2\} \) and the quantization step size as \( q \triangleq \bar{P}^{-\alpha^{(2)}} / k \). First, we easily upper bound it as \( 0 \leq \lambda^{(j)}_i \leq 1 \) implies that

\[
E_{Q_u(x) > 0} \left[ \log_2 \left( Q_u(x) \right) \right] \leq 0. \tag{90}
\]

In order to lower bound it, note that

\[
E_{Q_u(x) > 0} \left[ \log_2 \left( Q_u(x) \right) \right] \triangleq \sum_{i=1}^{M} \log_2(iq) p_{i | Q_u(x) > 0}(iq), \tag{91}
\]

where \( M \triangleq \left\lfloor \frac{1}{2} \right\rfloor - 1 \) because the quantization level \( Q_u(x) = 0 \) \( (i = 0) \) is excluded from \( Q_u(x) > 0 \). Besides this, the term \( p_{i | Q_u(x) > 0}(iq) \) stands for

\[
p_{i | Q_u(x) > 0}(iq) \triangleq \Pr \left( Q_u(x) = iq \mid Q_u(x) > 0 \right), \tag{92}
\]
where \( \Pr(Q_u(x) = iq) = \Pr(iq \leq x \leq (i + 1)q) \). The expectation in (91) is bounded for a given finite \( P \) because \( q = \bar{P}^{-\alpha(2)/k} > 0 \). In the following we prove that it is bounded also when \( P \to \infty \). We can write that

\[
\Pr(Q_u(x) = iq \mid Q_u(x) > 0) = \frac{\Pr(Q_u(x) = iq \land Q_u(x) > 0)}{\Pr(Q_u(x) > 0)}
\]

(93)

\[
= \frac{\Pr(Q_u(x) = iq)}{\Pr(Q_u(x) > 0)}
\]

(94)

\[
\leq \frac{f_{\Lambda}^{\max}}{\max(0, 1 - f_{\Lambda}^{\max} \bar{P}^{-\alpha(2)}/\bar{x})} q,
\]

(95)

where (93) comes from (64) as \( 1 - \Pr(Q_u(x) > 0) \leq f_{\Lambda}^{\max} \bar{P}^{-\alpha(2)}/\bar{x} \) and from the fact that \( \Pr(Q_u(x) = iq) \leq f_{\Lambda}^{\max} q \).

Note that \( \exists P_{\min} \) such that \( \forall P > P_{\min}, 1 - f_{\Lambda}^{\max} \bar{P}^{-\alpha(2)}/\bar{x} > 0 \). As we focus on the limit as \( P \to \infty \), we assume hereinafter that \( 1 - f_{\Lambda}^{\max} \bar{P}^{-\alpha(2)}/\bar{x} > 0 \). We introduce the notation

\[
p'_{\max} = \frac{f_{\Lambda}^{\max}}{1 - f_{\Lambda}^{\max} \bar{P}^{-\alpha(2)}/\bar{x}}.
\]

Hence, since \( M \leq \frac{1}{q} \) (and thus \( q \leq \frac{1}{M} \)) and \( \forall i \leq \frac{1}{q} \) it holds that \( \log_2(iq) \leq 0 \), it follows that

\[
\mathbb{E}_{Q_u(x) > 0}[\log_2(Q_u(x))] \geq \sum_{i=1}^{M} \log_2(iq) p'_{\max} q
\]

(97)

\[
\geq \sum_{i=1}^{M} \log_2 \left( \frac{i}{M} \right) \frac{p'_{\max}}{M} \sum_{i=1}^{M} \log_2(iq) - \sum_{i=1}^{M} \log_2(M)
\]

(98)

\[
= p'_{\max} \left( \frac{\log_2(M!)}{M} - \log_2(M) \right).
\]

(99)

We have that

\[
\lim_{M \to \infty} \left( \frac{\log_2(M!)}{M} - \log_2(M) \right) = \frac{-1}{\ln(2)},
\]

(100)

what together with the fact that \( \lim_{P \to \infty} p'_{\max} = f_{\Lambda}^{\max} \) implies that

\[
\lim_{P \to \infty} \mathbb{E}_{Q_u(x) > 0}[\log_2(Q_u(x))] \geq \frac{-f_{\Lambda}^{\max}}{\ln(2)},
\]

(101)

what concludes the proof.

**APPENDIX II**

**PROOF OF LEMMA2**

Let us denote the first element of the vector \( v^{(j)}_H \in \mathbb{R}^{8 \times 1} \) as \( \hat{h}_{\Re}^{(j)} \), i.e., \( \hat{h}_{\Re}^{(j)} = \Re(h_{11}^{(j)}) \). Similarly, \( \hat{h}_{\Re} \) denotes the real part of the normalized channel coefficient, \( \hat{h}_{\Re} = \Re(h_{11}) \). Therefore, since the elements of \( v^{(j)} \) are i.i.d.,

\[
\|v^{(2)}_H - v^{(1)}_H\| \xrightarrow{a.s.} 0 \iff |\hat{h}_{\Re}^{(2)} - \hat{h}_{\Re}^{(1)}| \xrightarrow{a.s.} 0.
\]

(103)

Furthermore, from the feedback model it holds that

\[
|\hat{h}_{\Re}^{(2)} - \hat{h}_{\Re}^{(1)}| \xrightarrow{a.s.} 0 \iff \hat{h}_{\Re}^{(j)} - \hat{h}_{\Re} \xrightarrow{a.s.} 0 \forall j \in \{1, 2\}.
\]

(104)
Let $A_n = \{ |X_n - X| > \varepsilon \}$. Then,
\[ X_n \xrightarrow{a.s.} X \iff \Pr(A_n \ i.o.) = 0 \ \forall \varepsilon > 0, \tag{105} \]
where
\[ A_n \ i.o. \triangleq \{ w : w \in A_n \text{ for infinitely many } n \} \tag{106} \]

\[ = \limsup_n A_n. \tag{107} \]

Let $X_n = \hat{h}_{\mathbb{R}}^{(j)} - \tilde{h}_{\mathbb{R}}$ and $X = 0$. We obtain in the following $\Pr(A_n) = \Pr(|\hat{h}_{\mathbb{R}}^{(j)} - \tilde{h}_{\mathbb{R}}| > \varepsilon)$. The absolute value of the difference can be bounded as
\[ |\hat{h}_{\mathbb{R}}^{(j)} - \tilde{h}_{\mathbb{R}}| = |(1 - z_1^{(j)})\hat{h}_{\mathbb{R}}^{(j)} - z_1^{(j)}\delta_{\mathbb{R}}^{(j)}| \leq (1 - z_1^{(j)}) + z_1^{(j)} \tag{108} \]

where (108) comes from the estimate model in (7) and (109) because $|\hat{h}_{\mathbb{R}}^{(j)}| \leq 1$ and $|\delta_{\mathbb{R}}^{(j)}| \leq 1$. The absolute value is omitted in (109) because $0 \leq z_1^{(j)} \leq 1$. Let us remind that
\[ z_1^{(j)} = \sqrt{1 - (z_1^{(j)})^2}. \tag{110} \]

Since $1 - \sqrt{1 - x^2} \leq x$ for $0 \leq x \leq 1$, it holds that
\[ |\hat{h}_{\mathbb{R}}^{(j)} - \tilde{h}_{\mathbb{R}}| \leq 2z_1^{(j)}. \tag{111} \]

Hence,
\[ \Pr(|\hat{h}_{\mathbb{R}}^{(j)} - \tilde{h}_{\mathbb{R}}| > \varepsilon) \leq \Pr(2z_1^{(j)} > \varepsilon) = \Pr(Z_1^{(j)} > \varepsilon^2/4), \tag{112} \]

since $Z_1^{(j)} = (z_1^{(j)})^2$. The quantization error $Z_1^{(j)}$ is distributed as the minimum of $n = 2^{B^{(j)}}$ (and $2^{B^{(j)}} = P^{(j)}$) standard uniform random variables $[2]$, $[10]$. Hence, upon denoting $\varepsilon' = \varepsilon^2/4$, it holds that
\[ \Pr(Z_1^{(j)} > \varepsilon') = (1 - \varepsilon')^n. \tag{114} \]

By definition –see (107)–, $\Pr(A_n \ i.o.)$ satisfies
\[ \Pr(A_n \ i.o.) \leq \lim_{n \to \infty} \sum_{m=n}^{\infty} \Pr(A_n). \tag{115} \]

Introducing (114) in (115) yields
\[ \Pr(A_n \ i.o.) \leq \lim_{n \to \infty} \sum_{m=n}^{\infty} (1 - \varepsilon')^n = \lim_{n \to \infty} \frac{(1 - \varepsilon')^{n-1}}{\varepsilon'} = 0. \tag{118} \]

where (117) comes from the application of the geometric series' formula. This implies that $\Pr(A_n \ i.o.) = 0 \ \forall \varepsilon > 0$, and thus Lemma [2] is proven.
APPENDIX III
PROOF OF LEMMA 3

In this section we prove Lemma 3, i.e., that

$$E \left[ \| \mathbf{v}_H^{(2)} - \mathbf{v}_H^{(1)} \|^2 \right] \leq \kappa P^{-\alpha(2)}. \quad (119)$$

As defined in the previous appendix, let $\hat{h}_R^{(j)} = \Re(\hat{h}^{(j)}_{11})$ and $\tilde{h}_R = \Re(\hat{h}_{11})$. We start by noting that, since the elements of $\mathbf{v}_H^{(2)} \in \mathbb{R}^{8 \times 1}$ are i.i.d., it holds that

$$E \left[ \| \mathbf{v}_H^{(2)} - \mathbf{v}_H^{(1)} \|^2 \right] = 8 E \left[ | \hat{h}_R^{(2)} - \hat{h}_R^{(1)} |^2 \right]. \quad (120)$$

The absolute value of the difference can be bounded as

$$\left| \hat{h}_R^{(2)} - \hat{h}_R^{(1)} \right| \leq \left| \hat{h}_R^{(2)} - \tilde{h}_R \right| + \left| \tilde{h}_R - \hat{h}_R^{(1)} \right| \leq 2z_1^{(2)} + 2z_1^{(1)}, \quad (121)$$

what follows from (111). Since $z_1^{(2)}$ is drawn from the same distribution as $z_1^{(1)}$ but with higher variance, it holds that

$$E \left[ (2z_1^{(2)} + 2z_1^{(1)})^2 \right] \leq E \left[ (4z_1^{(2)})^2 \right] \quad (123)$$

and consequently

$$E \left[ \left| \hat{h}_R^{(2)} - \hat{h}_R^{(1)} \right|^2 \right] \leq 16 E \left[ (z_1^{(2)})^2 \right] \quad (124)$$

$$\leq 16 P^{-\alpha(2)}, \quad (125)$$

where (125) is obtained from [2, Lemma 1]. This concludes the proof of Lemma 3.

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