Chain reconfiguration in active noise

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Abstract
In a typical single molecule experiment, the dynamics of an unfolded protein is studied by determining the reconfiguration time using long-range Förster resonance energy transfer, where the reconfiguration time is the characteristic decay time of the position correlation between two residues of the protein. In this paper we theoretically calculate the reconfiguration time for a single flexible polymer in the presence of active noise. The study suggests that though the mean square displacement grows faster, the chain reconfiguration is always slower in the presence of long-lived active noise with exponential temporal correlation. Similar behavior is observed for a worm-like semiflexible chain and a Zimm chain. However it is primarily the characteristic correlation time of the active noise and not the strength that controls the increase in the reconfiguration time. In brief, such active noise makes the polymer move faster but the correlation loss between the monomers becomes slow.

Keywords: reconfiguration time, active noise, long temporal correlation, polymer dynamics

(Some figures may appear in colour only in the online journal)

1. Introduction

Active processes giving rise to non-equilibrium fluctuations are ubiquitous in biological systems. This is notably distinct from the incessant motion exhibited by particles in any fluid, known as Brownian motion, which results from the constant collision of the particles with their surrounding solvent molecules [1]. However, in biological systems active motion is driven by the chemical energy produced from the hydrolysis of adenosine triphosphate (ATP). For instance the motion of the cytoskeleton inside cells is controlled by motor proteins which involves ATP hydrolysis [2]. Other examples would be cell membranes which are
perpetually out of equilibrium through active processes [3] and swimming bacteria which control the active transport of nutrients in aqueous media [4]. In a recent study it has been shown that the dynamics of DNA is also influenced by the processes dependent on the energy derived from ATP hydrolysis [5]. Drawing motivations from these active processes there have been attempts to build models for polymers in active baths [6, 7] as well as in active and viscoelastic baths [8]. A series of simulation studies have also been performed to investigate the looping dynamics in biological systems in a viscoelastic crowded environment [9]. It is shown that the presence of crowders influences the chain dynamics to a great extent depending on their size and the stiffness of the polymer [10, 11]. Shin et al have recently shown that in the presence of self-propelled particles, the loop formation in polymers becomes faster due to increased diffusion [12]. In another study it has been found that looping is also faster when the polymer itself is active, having a catalytic monomer. This catalytic monomer generates a concentration gradient prompting faster diffusion of the non-catalytic monomer resulting in rapid ring-closure [13]. Such studies are extremely important as loop formation in biopolymers is an essential process in protein folding, DNA replication, etc [14].

Experimentally there have been many attempts to study the dynamics of unfolded proteins mainly involving long-range Förster resonance energy transfer (FRET) [15, 16]. In this particular technique two residues of a protein are labeled with a donor and an acceptor using fluorescence probes to study the fluctuation of the distance between them (figure 1). This distance is temporally correlated with a characteristic decay time, referred to as the reconfiguration time \( \tau_{\text{NO}} \) which is determined by fitting the long time decay of the second order intensity correlation function [17, 18]. This reconfiguration time provides a general notion of the rate of change of the configuration of an unfolded protein and one would also expect the looping time to follow a similar trend, especially when the calculations are carried out in the Wilemski–Fixman framework [19]. To the best of our knowledge no such experimental study has been performed to date which would provide insight into the reconfiguration dynamics of a chain in an active medium. In this paper we theoretically analyze the dynamics of a single chain polymer in the presence of active noise. By active noise we refer to a long-lived temporal noise where the time correlation is independent of the ambient temperature. In contrast to the recent simulation studies we find such long temporally correlated noise to result in a slower reconfiguration of a polymer chain, be it flexible or semi-flexible. Even in

![Figure 1. Schematic showing the end-to-end monomers of a protein labeled with the donor and the acceptor. The arrow depicts the distance \( R_{\text{NO}}(t) \) between the donor and acceptor monitored in the experiment.](image-url)
the presence of non-local hydrodynamic interactions in addition to the active noise, reconfiguration of the chain is slower.

The paper is arranged as follows. In section 2 we introduce the model for active noise and in section 3 the calculation methods are discussed. The results are presented in section 4 and the paper is concluded in section 5.

2. Model

For a one-dimensional Brownian particle, moving in a harmonic trap, the dynamics in the over-damped regime is described by the Langevin equation [20, 21]

\[
\frac{dx(t)}{dt} = -kx(t) + f(t)
\]

where, \(k\) is the force constant and \(f(t)\) is the Gaussian random force with first and second moments

\[
\langle f(t) \rangle = 0, \quad \langle f(t') f(t'') \rangle = 2\xi k_B T \delta(t' - t'').
\]

Here the strength of the correlation depends on the ambient temperature \(T\). Now when the system is subjected to an additional active noise of strength \(f_A\) the equation of motion becomes [22],

\[
\frac{dx(t)}{dt} = -kx(t) + f(t) + f_A(t)
\]

Here, \(f_A\) is considered to be exponentially correlated with a characteristic decay time \(\tau_A\) and a Gaussian distribution with moments

\[
\langle f_A(t) \rangle = 0, \quad \langle f_A(t') f_A(t'') \rangle = Ce^{-|t' - t''|/\tau_A}.
\]

This kind of exponential time correlation of noise is a manifestation of Ornstein–Uhlenbeck processes, where the noise itself undergoes an over-damped motion in a harmonic trap. In the ‘Kubo–Anderson process’, a similar exponential temporal correlation of the noise is observed where the noise consists of discontinuous Markovian processes. This process is commonly known as ‘telegraphic noise’ and ‘dichotomous noise’ [23]. Importantly, \(C\), the strength of the active noise, is independent of \(T\) and can be related to the probability of active force \(P(f_A)\) and the force \(f_A\) acting on the particle as \(f C \sim P(f_A) f_A^2\). Being independent of ambient temperature \(T\) the active noise drives the system out of equilibrium and only in the infinite time limit can a stationary state be realized. In between, the system remains in a nonequilibrium state. Such a choice of noise correlation comes from earlier simulation studies on red-blood-cell membrane fluctuations, where the force \(f_A(t)\) originates from the non-equilibrium fluctuations of the motor proteins [24].

The position correlation function \(\langle x(t) x(0) \rangle\) with active noise is analytically trackable and has the following expression. Readers are referred to the appendix for the detailed derivation.

\[
\phi(t) = \langle x(t) x(0) \rangle = \frac{k_B T}{k} e^{-\frac{t}{\tau}} + Ce^{-\frac{t}{\tau_A}} \left( \frac{1}{1 + \frac{1}{\tau_A}} + \frac{e^{\frac{t}{\tau_A}} - 1}{\frac{1}{\tau^2} - \frac{1}{\tau_A^2}} \right).
\]
In the absence of active noise, \( C \rightarrow 0 \), the above expression is reduced to

\[
\lim_{C \to 0} \phi(t) = \frac{k_B T}{k} e^{-\frac{k}{C}}.
\]

This is the time-correlation function for an over-damped Brownian particle in a harmonic potential in the presence of only thermal noise or the Ornstein–Uhlenbeck process and \( \tau = \frac{\xi}{k} \) is the corresponding relaxation time [25]. It is obvious from equation (5) that although the correlation function is translationally invariant, it is not single exponential. However, this can be approximated as a single exponential with an effective relaxation time \( \tau_{\text{eff}} \).

\[
\phi_{\text{eff}}(t) \approx \frac{k_B T_0}{k} e^{-\frac{k}{C} t},
\]

with

\[
\tau_{\text{eff}} = \int_0^\infty \frac{d\phi(t)}{\phi(0)} = \tau \left[ \frac{(k_B T_0^2 + C \tau \tau_A)(\tau + \tau_A)}{C \tau^2 \tau_A k + k_B T_0^2 (\tau + \tau_A)} \right] \tag{7}
\]

where, \( \tau_{\text{eff}} \) approximately gives the relaxation time of the Brownian particle in the presence of the active noise and is bound from above and below by \( \tau \) and \( \tau + \tau_A \), respectively. In the limit \( T \to \infty, \tau_{\text{eff}} = \tau \), at the other extreme \( T \to 0, \tau_{\text{eff}} = \tau + \tau_A \). Similarly, as \( C \to \infty \), the correlation decay also becomes slower with \( \tau_{\text{eff}} = \tau + \tau_A \) and in the absence of noise when \( C \to 0, \tau_{\text{eff}} = \tau \). Other than \( \tau_{\text{eff}} \), another parameter is \( T_0 \), which defines the effective correlation function \( \phi_{\text{eff}}(t) \). \( T_0 \) is related to the ambient temperature as follows, \( k_B T_0 = k_B \left( T + \frac{C k}{\xi k_B \left( \frac{1}{\tau} + \frac{1}{\tau_A} \right)} \right) \).

Thus, \( k_B T_0 \) defines a renormalized thermal energy, but only in the limit \( t \to \infty \). This directly follows from the mean square displacement (MSD) of the particle.

\[
\langle (x(t) - x(0))^2 \rangle = 2(\phi(0) - \phi(t)) = \frac{2k_B T}{k} (1 - e^{-\frac{k}{C} t}) + \frac{2C}{\xi^2 \left( \frac{1}{\tau} + \frac{1}{\tau_A} \right)} (1 - e^{-\frac{k}{C} t}) - \frac{2C (e^{-\frac{k}{C} t} - e^{-\frac{2k}{C} t})}{\xi^2 \left( \frac{1}{\tau} + \frac{1}{\tau_A} \right)} \tag{8}
\]

In the limit \( t \to \infty \)

\[
\lim_{t \to \infty} \langle (x(t) - x(0))^2 \rangle = \frac{2k_B}{k} T + \frac{C k}{\xi k_B \left( \frac{1}{\tau} + \frac{1}{\tau_A} \right)}.
\]

From equation (8) it is evident that the MSD grows with time \( t \) and saturates as expected due to confinement. However, the initial growth of the MSD is faster in the presence of active noise \( (C \neq 0, \tau_A \neq 0) \). A detailed derivation of the MSD is presented in the appendix.

This model can be further extended to describe a many-body system such as a flexible or a semi-flexible polymer as long as the chain is assumed to have a Gaussian distribution.
2.1. Rouse chain

The Rouse model is the simplest and most widely used model to describe a polymer with Gaussian statistics devoid of any hydrodynamics and excluded volume interaction. The equation of motion of the $n$th monomer is given by

$$\frac{\xi}{k_B T} \frac{\partial^2 R_n(t)}{\partial t^2} = k \frac{\partial R_n(t)}{\partial n^2} + f(n, t), \tag{9}$$

where $R_n(t)$ is the position of the $n$th monomer at time $t$, and $n$ can vary from 0 to $N$ for a polymer with $N + 1$ monomers. The friction coefficient is denoted by $\xi$, which is proportional to solvent viscosity and $k$ is the spring constant which is related to the Kuhn length,$b$, as $k = \frac{\eta b^2}{k_B T}$ with $f(n, t)$ being the random force acting on the $n$th monomer at time $t$ which accounts for the collision between the monomer with its surrounding solvent molecules,

$$\langle f(n, t) \rangle = 0, \quad \langle f_m(n, t') f_{m'}(m, t'') \rangle = 2\xi k_B T \delta_{m, m'} \delta(n - m) \delta(t' - t''). \tag{10}$$

As a simple extension of the above model one can consider a Rouse chain in the presence of active noise $f_A(t)$.

$$\frac{\xi}{k_B T} \frac{\partial^2 R_n(t)}{\partial t^2} = k \frac{\partial R_n(t)}{\partial n^2} + f(n, t) + f_A(n, t). \tag{11}$$

It is a very standard procedure to decouple the equation of motion of the monomers using normal modes with independent motions as follows,

$$\frac{\xi}{k_B T} \frac{dX_p(t)}{dr} = -k_p X_p(t) + f_p(t) + f_{A,p}(t), \tag{12}$$

where $k_p = \frac{5\xi k_B T \pi^2}{6N}$ and $\xi_p = 2N\xi$. The relaxation time for the $p$th normal mode in the absence of any active noise is $\tau_p = \frac{\xi_p}{k_p} = \frac{\tau_p}{\tau^R_p}$, where $\tau_p = \frac{5\xi k_B T \pi^2}{6N \xi}$ is known as the Rouse time. $f_p(t)$ and $f_{A,p}(t)$ are random and active forces, respectively, which follow

$$\langle f_{p,\alpha} (t) \rangle = 0, \quad \langle f_{p,\alpha}(t') f_{p,\beta}(t'') \rangle = 2\xi k_B T \delta_{\alpha, \beta} \delta(t' - t'') \tag{13}$$

$$\langle f_{A,p,\alpha} (t) \rangle = 0, \quad \langle f_{A,p,\alpha}(t') f_{A,q,\beta}(t'') \rangle = 2N \xi \delta_{\alpha, \beta} \delta(t' - t''). \tag{14}$$

The above equation (12) is structurally the same as that of equation (3), the only difference being that it is for the $p$th mode of a chain. It is obvious that each mode of the chain behaves as an over-damped Brownian particle in the presence of the active noise trapped in a harmonic well.

2.2. Zimm chain

When pre-averaged hydrodynamic interactions are considered under a $\theta$ condition, the normal modes of the polymer behave very similarly to that of a Rouse chain and have the same structure [20]

$$\xi_p \frac{dX_p(t)}{dr} = -k_p Z X_p(t) + f_p(t) + f_{A,p}(t), \tag{15}$$

with $\xi_p = \xi \frac{5N\xi k_B T \pi^2}{3}$, where $k_p = \frac{k_p Z}{k_p}$, $\tau^Z_p = \frac{\tau^R_p}{k_p}$ and $\tau^Z = \frac{5\xi k_B T \pi^2}{64\xi^2 k_B T}$. [26].
2.3. Worm-like chain

The semi-flexible polymer is modeled as a Kratky–Porod worm-like chain which is unstretchable and includes the effect of bending energy [20, 27]. The equation of motion for a semi-flexible chain without incorporating the effects from hydrodynamic interactions is given by

\[\xi \frac{\partial R_n(t)}{\partial t} = k \frac{\partial^2 R_n(t)}{\partial n^2} - \kappa \frac{\partial^4 R_n(t)}{\partial n^4} + f(n, t) + f_A(n, t).\]  

(16)

In a normal mode description the semi-flexible chain is similar to a flexible chain except for \(k_p\) which has a fourth order dependence on the mode number \(p\) unlike the flexible chain,

\[\xi_p \frac{dX_p(t)}{dt} = -k_p^S X_p(t) + f_p(t) + f_{A,p}(t),\]  

(17)

where \(k_p^S = \frac{6k_BT^p}{N^p} + \frac{2\kappa p_x^p}{N^p}\) and \(\kappa\), bending rigidity, is related to the persistence length \(l_p\) of the polymer as follows \(\kappa = k_BTl_p\). However, \(\xi_p^S = \xi_p = 2N\xi\) and \(\tau_p^S = \frac{\xi_p^S}{\kappa}\).

3. Calculation methods

The time-correlation function for the normal modes has a very general structure for the flexible as well as the semi-flexible chain and the structure remains the same even when the hydrodynamic interactions are incorporated. The form of the expression is identical to that of a single over-damped Brownian particle moving in a harmonic well in the presence of active noise

\[\langle X_{pq}(t)X_{q'q}(0) \rangle = \frac{k_N T}{k_p^S} \delta_{pq} \delta_{pq'} e^{-\tau_p^S t} + \frac{2NC}{\xi_p^S} \delta_{pq} \delta_{pq'} e^{-\frac{1}{\tau_p^S} - \frac{1}{\tau_A^S}} \left( \frac{1}{\tau_p} + \frac{1}{\tau_A} \right) + \frac{\varepsilon_{pq}^S}{\tau_p^2} \right).\]  

(18)

To find the exact expression for the flexible, semi-flexible, or Zimm chain one just needs to select the exact forms of \(k_p, \xi_p\) and \(\tau_p\) for a Rouse chain, \(k_p^Z, \xi_p^Z, \tau_p^Z\) for a Zimm chain and \(k_p^S, \xi_p^S, \tau_p^S\) for a semi-flexible chain.

The time-correlation function for the vector \((R_{0N})\) connecting the \(N\)th and the \(0\)th monomer can easily be calculated from the above expression, which is the summation over the correlation functions of all odd normal modes describing the polymer

\[\Phi^o_{N0}(t) = \langle R_{0N}(t) \cdot R_{0N}(0) \rangle = 16 \sum_{p=odd}^\infty 3\langle X_p(t)X_p(0) \rangle.\]  

(19)

The reconfiguration time \((\tau_{N0})\) corresponding to the fluctuation of the distance between the end-to-end monomers is theoretically calculated by taking a time integration of the corresponding normalized correlation function \((\Phi_{N0}(t))\) [28, 29]

\[\tau_{N0} = \int_0^\infty dt \Phi_{N0}(t),\]  

(20)

where, \(\Phi_{N0}(t) = \frac{\Phi_{N0}(t)}{\Phi_{N0}(0)}\).

Similarly, the expression of the MSD of the vector \((R_{0N})\) can also be derived from the MSD of the normal modes [28, 30]. This is again similar to that of a single particle. In a recent
study, Ghosh et al [22] demonstrated how the MSD of a semi-flexible chain grows in the presence of similar active noise. The higher the strength of the active noise is, the faster the growth,

$$
\langle (R_{N0}(t) - R_{N0}(0))^2 \rangle = 16 \sum_{p=\text{odd}}^{\infty} 3 \left( \frac{2k_B T}{k_p} (1 - e^{-\frac{t}{\tau}}) + \frac{2C}{\xi_p^2 \left( \frac{1}{\tau_p} + \frac{1}{\tau_a} \right)} \right) \\
\times (1 - e^{-\frac{t}{\tau}}) \frac{2C(e^{-\frac{t}{\tau}} - e^{-\frac{t}{\tau_a}})}{\xi_p^2 \left( \frac{1}{\tau_p} - \frac{1}{\tau_a} \right)}.
$$

(21)

4. Results and discussion

In figure 2 we show the normalized correlation function for the Rouse chain in the presence and absence of active noise which is calculated using the generalized expression given in equation (18). The parameters are chosen inconsistently with the real values such as $N = 100$, $b = 3.8 \times 10^{-10}$ m, $T = 300$ K and $\xi = 9.42 \times 10^{-12}$ kgs$^{-1}$, which corresponds to the viscosity of water. As mentioned earlier, $C$ is the strength of the active noise and $C = \frac{P(f_a \sigma_a^2)}{b}$. It has been experimentally observed that in biological systems motor proteins such as myosin and kinesin exert a force in the $\sim 5 - 10$ pN range [31]. For our calculations we have considered $f_a = 10 \times 10^{-12}$ N and $P(f_a) = 1$. For a fixed value of $C$ we have chosen two different values of $\tau_A$, 0.2$\tau_1$ and 5.0$\tau_1$, which are in the same order of magnitude of $\tau_1$. From the plot it can be seen that correlation decay is always slower in the presence of active noise even when the characteristic decay time of the active noise $\tau_A$ is very small, and as $\tau_A$
increases the decay of $\Phi_{N0}$ becomes even slower. However, this correlation loss has a very weak dependence on the strength of the active noise $C$. Changing the strength causes practically no difference in the correlation function. The log–log plot of reconfiguration time against chain length ($N$) is shown in figure 3, where the reconfiguration time is calculated using equation (20) and, as expected, the reconfiguration time increases as the temporal correlation loss of the active noise becomes slower. For a 100 monomer chain $\tau_{N0}$ increases $\sim 1.4$ times in the presence of active noise when $\tau_A = 0.2\tau_1$, whereas it becomes $\sim 7$ times higher when the decay time of the active noise $\tau_A = 5\tau_1$. But, surprisingly, the chain-length dependence remains unchanged even in the presence of active noise. In all three cases $\alpha = 2$ where, $\tau_{N0} \sim N^\alpha$. It is well known that the reconfiguration time is a summation of the relaxation times of each mode i.e. $\tau_{N0} = 16\sum_{p=\text{odd}}^{\infty} 3\tau_p \langle X_p(t)X_p(0) \rangle$, which has the analytically exact expression

$$\tau_{N0} = 16 \sum_{p=\text{odd}}^{\infty} 3\tau_p \left[ \frac{(k_BT\xi_p^2 + C\tau_p\tau_A k_p)(\tau_p + \tau_A)}{C\tau_p^2\tau_A k_p + k_BT\xi_p^2(\tau_p + \tau_A)} \right],$$

where, the $N$ dependence comes through $\tau_p$, $\xi_p$ and $k_p$. A careful analysis of the preceding expression shows that if the active noise strength $C$ is very small, i.e. $C \rightarrow 0$, the above expression reduces to $\tau_{N0} \approx 16\sum_{p=\text{odd}}^{\infty} 3\tau_p$ and since, $\tau_p \sim N^2$, the dependence of the reconfiguration time is also identical. Now, what happens if $C$ becomes very large, i.e. $C \rightarrow \infty$, $\tau_{N0} \approx 16\sum_{p=\text{odd}}^{\infty} 3\tau_p + \bar{\tau}$, where $\bar{\tau}$ is a constant, even in this case the $N$ dependence comes only from $\tau_p$ and $\tau_{N0} \sim N^2$. Between these two extreme cases the active noise causes a very small change in the $N$ dependence of the reconfiguration time which is reflected in figure 3.

The same set of calculations have been performed for a flexible polymer including the pre-averaged hydrodynamic interaction under the $\theta$ condition. The plot of the normalized time-correlation function against time is shown in figure 4 which shows a similar trend as that
of the Rouse chain, i.e. the correlation loss becomes slower whenever active noise is introduced into the system. Next, the chain-length dependence of the reconfiguration time is determined for the Zimm chain from Figure 5, and it is found to be \( \sim N^{1.5} \) which is in agreement with the previous work done by Chakrabarti [26]. In this case the chain-length dependence of the reconfiguration time also does not differ in the presence of active noise.

Figure 4. Plot of \( \Phi_{N0}(t) \) versus \( t \) for the Zimm chain. The values of the parameters used in the calculations are \( N = 100, \ b = 3.8 \times 10^{-10} \text{ m}, \ T = 300 \text{ K} \) and \( \xi = 9.42 \times 10^{-12} \text{ kgs}^{-1} \).

Figure 5. Log–log plot of reconfiguration time \( (\tau_{N0}) \) versus chain length \( (N) \) for the Zimm chain. The values of the parameters used in the calculations are \( b = 3.8 \times 10^{-10} \text{ m}, \ T = 300 \text{ K} \) and \( \xi = 9.42 \times 10^{-12} \text{ kgs}^{-1} \).
Figure 6. Plot of $\Phi_{N_0}(t)$ versus $t$ for semi-flexible chain. The values of the parameters used in the calculations are $N = 100$, $b = 3.8 \times 10^{-10}$ m, $T = 300$ K and $\xi = 9.42 \times 10^{-12}$ kgs$^{-1}$.

Figure 7. Log–log plot of reconfiguration time ($\tau_{N_0}$) versus chain length ($N$) for the semi-flexible chain. The values of the parameters used in the calculations are $b = 3.8 \times 10^{-10}$ m, $T = 300$ K and $\xi = 9.42 \times 10^{-12}$ kgs$^{-1}$.

Figure 6 shows the normalized time-correlation function of the end-to-end vector for a semi-flexible chain. Here also the behavior of the correlation loss in the presence of active noise is identical to the Rouse and Zimm chain. The Kuhn length for the semi-flexible chain has been considered to be $b = 50 \times 10^{-9}$ m. This is roughly the Kuhn length of DNA which has a series of different Kuhn lengths depending upon the solvent condition [32]. The persistence length of the semi-flexible chain has been considered to be half of the Kuhn length.
during the calculations. When the chain-length dependence of the semi-flexible chain is determined from figure 7, it is found to be \( \sim N^2 \). This might seem surprising since \( \tau_{\rho}^\text{s} \) has a fourth order dependence on \( N \) in addition to the usual second order dependence, for which the dependence on chain length should be higher than that of the Rouse model. However, if we take a look at the expression for \( \tau_{\rho}^\text{s} \), since the value of \( \kappa \) considered in the calculations is very small, the contribution from \( N^4 \) is negligibly small and that is the reason for the semi-flexible chain, \( \tau_{N0} \sim N^2 \).

5. Conclusion

In this work we analyzed the effect of active noise in chain reconfiguration for a flexible polymer where the active noise is modeled with a long temporally correlated non-equilibrium force. It can be clearly seen that in the presence of active noise the chain reconfiguration become slower. However, the slowing down of the reconfiguration dynamics seems to be controlled by the correlation time \( \tau_\alpha \) rather than the strength of the correlation. Thus in a typical FRET-like experiment, the measurement of the reconfiguration time of a protein should show slowing down in an environment with such active noise. Similar behaviors are also observed when pre-averaged hydrodynamic interaction is considered. For a worm-like semi-flexible chain the trend remains the same. However, the dependence on the chain length of the reconfiguration time does not change in the presence of active noise. Keeping a long story short, our study suggests that the presence of active noise cannot always guarantee faster reconfiguration of a polymer chain. In an environment where a long temporal noise acts on the chain, a FRET type measurement would show the chain retaining the correlation for a longer time than in the absence of such noise.
Another important aspect of the problem we are currently investigating is the combined
effect of excluded volume and active noise in the chain dynamics [33]. In other words, how
the solvent quality along with the active noise influence the reconfiguration dynamics of a
chain. This seems analytically trackable at the mean field level, as recently implemented in
analyzing the effect of solvent quality on chain dynamics by introducing a Flory type
exponent in the time-correlation function [34, 35] between the chain segments.

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Appendix

A.1. Correlation function

The equation of motion for a single over-damped Brownian particle trapped in a harmonic
potential in the presence of active noise is

\[ \frac{dx(t)}{dt} + \frac{k}{\xi}x(t) = \frac{1}{\xi}(f(t) + f_A(t)). \]

Multiplying the integrating factor \( e^{\frac{kt}{\xi}} \) on both sides we obtain

\[ \left( \frac{dx(t)}{dt} + \frac{k}{\xi}x(t) \right)e^{\frac{kt}{\xi}} = \frac{1}{\xi}(f(t)e^{\frac{kt}{\xi}} + f_A(t)e^{\frac{kt}{\xi}}). \]

Integrating both side from \(-\infty\) to \(t\) (which means we assume that the system starts evolving
at an infinite past),

\[ \int_{-\infty}^{t} \frac{dx(t')}{dt} e^{\frac{kt'}{\xi}} dt' = \frac{1}{\xi} \int_{-\infty}^{t} \left( f(t')e^{\frac{kt'}{\xi}} + f_A(t')e^{\frac{kt'}{\xi}} \right) dt' \]

or \( x(t) = \frac{e^{\frac{kt}{\xi}}}{\xi} \int_{-\infty}^{t} f(t')e^{\frac{kt'}{\xi}} dt' + f_A(t')e^{\frac{kt'}{\xi}} \) as, \( \frac{\xi}{k} = \tau \).

Since the thermal and the active noise are uncorrelated, they come separately as a
summation in the correlation function, and the position correlation function for the thermal
noise has a standard solution which is not shown here,

\[ \langle x(t)x(0) \rangle = \frac{e^{\frac{kt}{\xi}}}{\xi} \int_{-\infty}^{t} dt' \int_{-\infty}^{0} dt'' \left( \langle f(t')f(t'') \rangle + \langle f_A(t')f_A(t'') \rangle \right) e^{\frac{kt'}{\xi} + \frac{kt''}{\xi}} \]

\[ = \frac{k_B T}{k} e^{-\frac{kt}{\xi}} + \frac{C}{\xi^2} e^{-\frac{kt}{\xi}} \int_{-\infty}^{t} dt' \int_{-\infty}^{0} dt'' e^{\frac{kt'}{\xi}} e^{-\frac{kt''}{\xi}}. \]

The position correlation function for the time-correlation function of the active noise involves
a modulus of time, therefore the integration is split into two parts, one where \( t' > t'' \) and
another considering \( t' < t'' \), as shown in figure 8.
\[ \langle x(t)x(0) \rangle = \frac{k_B T}{k} e^{-\frac{\xi}{k}} + \frac{C}{\xi^2} e^{-\frac{\xi}{k}} \left( \int_{r=-\infty}^{0} dr'' \int_{r'=0}^{0} dr' e^{\left( -\frac{\xi}{2k} \right) g\left( r + r' \right)} + \int_{r'=0}^{0} dr' e^{\left( -\frac{\xi}{2k} \right) g\left( r + r' \right)} \right) \\
= \frac{k_B T}{k} e^{-\frac{\xi}{k}} + \frac{C}{\xi^2} e^{-\frac{\xi}{k}} \left( 2 \int_{r=-\infty}^{0} dr'' \int_{r'=0}^{0} dr' e^{\left( -\frac{\xi}{2k} \right) g\left( r + r' \right)} \right) \\
= \frac{k_B T}{k} e^{-\frac{\xi}{k}} + \frac{C}{\xi^2} e^{-\frac{\xi}{k}} \left( 2 \int_{r=-\infty}^{0} dr'' \int_{r'=0}^{0} dr' e^{\left( -\frac{\xi}{2k} \right) g\left( r + r' \right)} \right) \\
+ \frac{1}{\left( \frac{1}{\xi} + \frac{1}{\gamma} \right) \left( \frac{1}{\xi} - \frac{1}{\gamma} \right)} e^{\left( -\frac{1}{\xi} + \frac{1}{\gamma} \right) - 1} \right) \\
= \frac{k_B T}{k} e^{-\frac{\xi}{k}} + \frac{C}{\xi^2} e^{-\frac{\xi}{k}} \left( \frac{1}{\left( \frac{1}{\xi} + \frac{1}{\gamma} \right)} + e^{-\frac{1}{\xi} + \frac{1}{\gamma} - 1} \right). \tag{23} \]

A.2. MSD

The MSD of a single particle in one dimension is as follows,

\[ \langle (x(t) - x(0))^2 \rangle = \frac{e^{-\frac{\xi}{k}}}{\xi^2} \int_{-\infty}^{0} dr' \left( f'(t') e^{\xi} + f_A(t') e^{\xi} \right) \]

\[ x(t) - x(0) = \frac{e^{-\frac{\xi}{k}}}{\xi^2} \int_{-\infty}^{0} dr' \left( f'(t') e^{\xi} + f_A(t') e^{\xi} \right) - \frac{1}{\xi} \int_{-\infty}^{0} dr' \left( f'(t') e^{\xi} + f_A(t') e^{\xi} \right) \]

\[ \langle (x(t) - x(0))^2 \rangle = \frac{e^{-\frac{\xi}{k}}}{\xi^2} \int_{-\infty}^{0} dr' \int_{-\infty}^{0} dr'' \left( \langle f(t') f(t'') \rangle + \langle f_A(t') f_A(t'') \rangle \right) e^{\left( -\frac{\xi}{2k} \right) g\left( r + r' \right)} \]

\[ + \frac{1}{\xi^2} \int_{-\infty}^{0} dr' \int_{-\infty}^{0} dr'' \left( \langle f(t') f(t'') \rangle + \langle f_A(t') f_A(t'') \rangle \right) e^{\left( -\frac{\xi}{2k} \right) g\left( r + r' \right)} \]

\[ - \frac{2}{\xi^2} \int_{-\infty}^{0} dr' \int_{-\infty}^{0} dr'' \left( \langle f(t') f(t'') \rangle + \langle f_A(t') f_A(t'') \rangle \right) e^{\left( -\frac{\xi}{2k} \right) g\left( r + r' \right)} \]

Again the MSD for an over-damped Brownian particle in a harmonic potential is well known and the derivation is not presented here [20],
\[
\frac{e^{-\frac{\xi}{\xi}}}{L^2} \int_{-\infty}^{\infty} dt' \int_{-\infty}^{t'} dt'' \left( f(t') f(t'') + \langle f_A(t') f_A(t'') \rangle \right) e^{i \xi (t' - t'' - t)}
\]
\[
= \frac{k_B T}{k} + C \left( e^{-\frac{\xi}{\xi}} \left( \frac{1}{\frac{\xi}{\xi} + \frac{1}{\gamma_0}} + \frac{2}{\xi} \right) - \frac{2e^{-\frac{\xi}{\xi}}}{\xi} \left( \frac{1}{\frac{\xi}{\xi} + \frac{1}{\gamma_0}} \right) \right)
\]
\[
+ \frac{1}{\frac{\xi}{\xi} + \frac{1}{\gamma_0}} \left( 1 - e^{-\frac{\xi}{\xi}} \right) - \frac{2e^{-\frac{\xi}{\xi}}}{\xi} \left( \frac{1}{\frac{\xi}{\xi} + \frac{1}{\gamma_0}} \right) \left( 1 - e^{-\frac{\xi}{\xi}} \right)
\]
\[
= \frac{k_B T}{k} + C \left( \frac{1}{\frac{\xi}{\xi} + \frac{1}{\gamma_0}} \right)
\]
\[
\frac{1}{\xi^2} \int_{-\infty}^{t'} dt' \int_{-\infty}^{t'} dt'' \left( f(t') f(t'') + \langle f_A(t') f_A(t'') \rangle \right) e^{i \xi (t' - t'' - t)}
\]
\[
= \frac{k_B T}{k} e^{-\frac{\xi}{\xi}} + \frac{2C}{\xi} e^{-\frac{\xi}{\xi}} \left( \frac{1}{\xi} \frac{1}{\xi} + \frac{1}{\gamma_0} \right) + \frac{e^{-\frac{\xi}{\xi}} - \frac{1}{\xi}}{\frac{1}{\xi} \frac{1}{\xi} + \frac{1}{\gamma_0}}
\]
\[
\langle (x(t) - x(0))^2 \rangle = \frac{2k_B T}{k} (1 - e^{-\frac{\xi}{\xi}}) + \frac{2C}{\xi^2} \frac{1}{\frac{\xi}{\xi} + \frac{1}{\gamma_0}} (1 - e^{-\frac{\xi}{\xi}}) - \frac{2C (e^{-\frac{\xi}{\xi}} - e^{-\frac{\xi}{\xi}})}{\xi^2} \left( \frac{1}{\xi} \frac{1}{\xi} + \frac{1}{\gamma_0} \right).
\]

(24)

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