Thick slab punching with symmetry reductions

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Abstract
The contribution presents a test series of four geometrically similar specimens of different size designed to investigate the punching behavior of thick reinforced concrete slabs. The thickness ranges from 300 to 650 mm and the slab radius varies between 1.12 and 2.45 m. For experimental execution, an innovative test method is employed and updated towards thick slabs. It uses the symmetry of specimens and reduces them corresponding to the symmetry grade to quarters. By applying this method, geometrical dimensions and associated test loads are reduced by 75%. The tests demonstrate on the one hand that the updated method is suitable for punching tests of thick specimens. And, on the other hand, the results indicate a less pronounced size effect for multilayered, highly reinforced slabs than predicted in theory.

KEYWORDS
large-scale tests, punching shear, size effect, size-dependent punching behavior, slab quarter, symmetry

1 | INTRODUCTION

The sizes of reinforced concrete structures like high-rise buildings or power plants rapidly grow since more efficient planning, construction and material technologies become available. Increasingly larger members are employed to bridge rising spans and carry higher loads. Typical examples are flat slabs supported locally on columns. In building practice, they reach thicknesses of 1.2 m and more. A key topic of discussion with increasing size is the matter of safe and economic designs.

Punching failure is typically the governing failure mode of flat slabs in the ultimate limit state. It occurs when slab resistances in the vicinity of a column are exceeded. Different empirical, semiempirical and mechanically based models are available for design. Because of the complexity of the failure type, safety and economical evaluations are mainly conducted on experimental data.

Numerous experimental studies are available that investigate the influence of various parameters on the punching resistance including the reinforcement ratio, the slab thickness, concrete properties, and shear reinforcement. However, there is no published data on experiments of shear-slender slabs with thickness above 500 mm. Moreover, different data collections reveal that about 90% of the results are derived from tests with effective depths of less than 200 mm. The main obstacles are huge financial efforts and hardly manageable dimensions of specimens, associated self-weights and appropriate test loads.

The lack of results on thick slabs is particularly critical, since punching is affected by a size effect. This means, that the normalized resistance decreases non-

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linearily with increasing thickness. In general, two different types of size effects are distinguished. First, the statistical size effect that describes the failure of a structure by the weakest-link model. This behavior can be captured by Weibull’s theory of random strength and is primarily known from metallic structures which fail at initiation of microscopic fractures. And second, the energetic (deterministic) size effect, which applies for quasi-brittle failures characterized by stable crack growth like punching shear failures in concrete structures. This type of size effect can be described theoretically by fracture mechanics (size-effect law) or is estimated fully empirically by data fitting. However, regardless of the approach, large-scale experiments are needed to provide benchmarks and determine the actual level of safety.

The purpose of the present study is to investigate the punching behavior of thick slabs. To overcome the size limitation of conventional test setups, an innovative test method for reinforced concrete structures is applied. It reduces the testing effort by exploiting the principle of symmetry. The paper describes the experimental implementation with respect to the major challenges of thick specimens and demonstrates its abilities on a test series of four large-scale specimens with thicknesses between 300 and 650 mm.

2 | EXPERIMENTAL PROGRAM

2.1 | Test method

The test series is conducted with a specific test method. The basic idea is to exploit symmetry in experimental investigations. Similar to numerical simulations, symmetrical parts of the specimens are substituted by a modular bearing construction. As a result, test loads, geometrical dimensions and self-weights are reduced corresponding to the symmetry grade. Primary objective of the approach is to provide access to entirely new test ranges for large-scale specimens while maintaining the existing lab infrastructure.

On the left, Figure 1 shows the method’s principle applied to an isolated slab-column connection that is typically used for punching shear investigations. The support conditions in these tests are rotationally symmetric with a column stub at center and circumferentially distributed loads at a defined perimeter. Due to the orthogonal reinforcement mesh, the specimen exhibits two axes of symmetry and can therefore be divided into four identical segments with the same structural behavior. Both, the self-weight and the test load of these segments are reduced by a factor of 4.

To respect the symmetry, additional boundary conditions must be fulfilled to replace the removed parts and achieve identical results compared to full-size members. On the right, Figure 1 illustrates these conditions. At the symmetry planes, the bending moments are clamped with no axial rotations \( \varphi = 0 \), while vertical and horizontal motions can occur without restrictions. Moreover, constraints that result from the slab design at the interconnections between concrete and steel have to be avoided.

Table 1 lists the theoretical symmetry conditions and their experimental implementation, namely rigid steel bearings, a back-anchoring system to fix the bending reinforcement, sliding planes along the steel surfaces and subdivided sliding plates to allow for curvature. Their design poses a major challenge because they must neither strengthen nor weaken the specimens and thus influence the test results.
Figure 2 presents the experimental implementations at the symmetry planes in detail. The connection between specimen and bearing acts like a shear hinge, so it transfers only bending moments \((M)\) split into tensile \((F_S)\) and compressive \((F_C)\) forces. As shown in Figure 2a, tensile forces are induced by screwed-in reinforcement and transmitted to the rear side of the bearing’s front plate via the back-anchoring system. By contrast, compressive forces are directly applied to the front side. In order to prevent any form of gapping or axial rotation \(\phi\) between specimen and bearing, all threaded rods of the back-anchoring system are prestressed before testing. The employed connectors are decoupled with silicone layers \((t=2\text{ mm})\) to avoid unintended strengthening of the concrete near the symmetry planes.\(^{25}\)

On both sides of the front plate, sliding planes are arranged to ensure almost free vertical and horizontal movement and prevent a transfer of shear forces. The planes consist of chambered, greased polytetrafluoroethylene (PTFE) layers with very low thickness \((t=0.5\text{ mm})\) to avoid unintended rotations \(\phi\) due to their low Young’s modulus. This construction exhibits a very low friction coefficient even for high contact pressures (Section 2.5).

To prevent constraints from radial bending, the steel sliding plates are divided (Figure 2b). Gaps of \(t=4\text{ mm}\) are provided between two adjacent elements to allow free lateral elongation or compaction. All components have been initially developed, improved, and verified in separate performance tests. A more detailed description of the method, its development and verification including the testing of slab quarters of 300 mm, is published elsewhere.\(^{15,22,25}\)

### Table 1

| Boundary conditions | Theory                        | Experiment                                      | References |
|---------------------|-------------------------------|-------------------------------------------------|------------|
| a \(\phi_y(x=0)=0\) \(\phi_x(y=0)=0\) | Full clamping                 | Rigid steel bearings                             | 15, 16     |
| b \(u_y(y=0)=0\) \(u_x(x=0)=0\) | No horizontal movements       | Back-anchoring system and noncompressible sliding planes | 16, 23     |
| c \(u_y(x=0)\neq0\) \(u_x(y=0)\neq0\) \(u_z(x=0,y=0)\neq0\) | Free sliding                  | Low friction sliding planes of greased PTFE      | 23, 24     |
| d \(\Delta u \neq 0\) \(\Delta e \neq 0\) | No constraints                | Divided steel sliding plates to ensure free elongation and compaction of the slab | 16, 25     |

Abbreviation: PTFE, polytetrafluoroethylene.

**Figure 2** Experimental realization of the symmetry planes (a) section A-A and (b) top view

2.2 | Test specimens

The experimental program consists of four specimens, one full-size reference slab \((s30)\) and three slab quarters \((sq30\text{–}sq65)\). The abbreviations denote the type of slab \((s \triangleq \text{full size}, \text{sq} \triangleq \text{quarter})\) as well as its thickness in \((\text{cm})\). The series is designed to investigate the size-
dependent punching shear behavior. Therefore, all slabs are geometrically similar but different in size. The thickness ranges from 300 to 650 mm and the slab radius \( r_s \) varies between 1.12 and 2.45 m. To keep the shear-slenderness ratio \( a_v/d \) and the perimeter-thickness ratio \( u_0/d \) constant, the column radius \( r_c \) and the shear distance \( a_v \) are increased linearly with the thickness. The loading is applied at the radius \( r_q \). All parameters of the geometry can be read from Table 2 whereas Figure 3 shows the typical geometry of a slab quarter.

All specimens are designed to fail in punching not in flexure. Moreover, they do not contain any shear reinforcement. In order to ensure punching failure, the calculative flexural strength \( V_{\text{flex}} \) was computed using the yield line theory. As a demand from size-dependency, the geometrical reinforcement ratio \( \rho_l \) is kept constant for all thicknesses. This causes that punching theoretically occurs on different levels of flexural strength. Birkel and Dilger, for example, allowed \( \rho_l \) to vary in their punching investigations to control cracking. On the contrary, their ratio of punching to flexural strength remains constant. Since the reinforcement ratio influences both, the punching and the deformation capacities, this approach was not adopted here.

The reinforcement layout of sq65 is shown in Figure 3. Its crosswise pattern is typical for all specimens. The upper flexural reinforcement consists of two or four layers. s30 and sq30 possess two layers Ø 20/100, while sq50 (Ø 16/100—first + second layer and Ø 20/100—third + fourth layer) and sq65 (Ø 16/100—first + second layer and Ø 25/100—third + fourth layer) require four.

To anchor the flexural reinforcement \( u \)-shaped stirrups of Ø 10 (s30 and sq30) or Ø 14 (sq50 and sq65) are arranged. The bottom reinforcement within the compression zone consists of Ø 8 bars (s30 and sq30) or Ø 10 bars (sq50 and sq65) with a spacing of 100 mm in both directions. Additionally, edge reinforcement (Ø 10) was placed at the free lateral surfaces of all slabs. Both were not screwed to the sliding plates and just serve as surface reinforcement.

The symmetry planes of the slab quarters are subject to very high precision demands, which must already be taken into account during manufacture. Due to the statically indeterminate support conditions and high back-anchoring forces of up to 5 MN/m, even low geometric uncertainties in the submillimeter range cause unwanted eigenstresses. For this reason, sq30 was cast in a precise CNC-milled steel formwork and a strict procedure was followed during installation to align the bearings to the specimen. However, this approach is not suitable for the large-scale specimens. The main reasons are their less manageable and varying dimensions (e.g., self-weight increases 10 times from sq30 to sq65) and a significantly larger and much more complex test setup. Therefore, a

**Table 2** Parameters of geometry

| Specimen | \( h \) (m) | \( d \) (m) | \( r_c \) (m) | \( r_q \) (m) | \( r_s \) (m) | \( a_v/d \) (−) | \( u_0/d \) (−) |
|----------|-------------|-------------|-------------|-------------|-------------|----------------|----------------|
| s30      | 0.30        | 0.24        | 0.20        | 0.97        | 1.12        | 3.21           | 5.24           |
| sq30     | 0.30        | 0.24        | 0.20        | 0.97        | 1.12        | 3.21           | 5.24           |
| sq50     | 0.50        | 0.40        | 0.33        | 1.62        | 1.87        | 3.21           | 5.24           |
| sq65     | 0.65        | 0.55        | 0.46        | 2.20        | 2.45        | 3.21           | 5.24           |

**Figure 3** Test specimen—general geometry and reinforcement layout of sq65
different approach is followed. As can be seen from Figure 4, sq50 and sq65 are directly concreted against the assembled setup using match casting. In contrast to sq30, all sliding plates are tensioned individually before casting. Thus, the development of eigenstresses is prohibited.

2.3 | Material properties

The same ready-mixed concrete with a maximum aggregate size of \(d_g = 16\) mm is used for all specimens. s30 and sq30 are concreted simultaneously from one batch, whereas sq50 and sq65 are cast separately.

All specimens contain hot-rolled steel bars with a pronounced yielding plateau. While s30 is made from ordinary bars, all slab quarters are equipped with bars with threaded ends. Table 3 summarizes averaged material properties.

2.4 | Test setups

The experimental program was carried out on three different but comparable test setups. Initially, verification tests between s30 and sq30 were conducted on two tailor-made small-scale setups demonstrating the general feasibility of symmetry reduction in punching tests. After successful verification, the concept was upgraded towards large scales. In the following, only that large-scale version will be focused on. Both small-scale setups are detailed in Reference 25.

Figure 5 shows the large-scale test setup equipped with specimen sq65 and the steel girders of load application. On the right and on the backside, three steel bearings \((h = 1.2\) m, \(b \approx 1\) m) form a symmetry plane. The shear force is induced at the loading perimeter by two layers of single-span beams and transferred to a quartered circular column stub located at the intersection of the symmetry planes below the specimen. The setup is designed for slabs up to 700 mm thickness. The geometrical dimensions are about \(4 \times 4\) (m) in ground view. The total weight of the steel elements without specimen and portal frame (hydraulic cylinders and connecting cross beam) amounts to about 25 t. To clamp the bending moments from testing, the bearings are prestressed to the strong floor with eight bar tendons and a total force of about 13 MN.

A specific assembly concept is elaborated using 3D-CAD-Software to ensure the same conditions in all tests.\(^{28,29}\) Figure 6 presents this concept with four major steps:

a. Positioning and alignment of the bearing construction.

b. Prestressing of the bearings against the strong floor.

c. Assembly and casting of the specimen.

d. Installation of the loading girders and the load frame.

Figure 6a illustrates the positioning and the alignment of the bearings on the strong floor. Three solid steel base plates \((t = 96\) mm) with integrated slots \((t = 10\) mm) are placed on the floor. They are first leveled with screws and then grouted with mortar to achieve a direct load

### Table 3 Material properties of concrete and reinforcement

| Specimen | Concrete | Reinforcement |
|----------|----------|---------------|
|          | \(f_{cm} (\text{N/mm}^2)\) | \(E_{cm} (\text{N/mm}^2)\) | \(f_{ym} (\text{N/mm}^2)\) | \(\rho_l (%)\) | \(E_{sm} (\text{N/mm}^2)\) | \(\mathcal{O}_l (\text{mm})\) |
| s30      | 22.8     | 21,700        | 552  | 1.31  | 206,200        | 20         |
| sq30     | 22.6     | 22,300        | 522  | 1.31  | 207,800        | 20         |
| sq50     | 21.5     | 25,825        | 556  | 1.29  | 207,900        | 16         |
| sq65     | 20.2     | 23,688        | 575  | 1.27  | 209,300        | 16         |
|          |          |               | 538  | 1.27  | 184,700        | 25         |

Note: \(f_{cm}\) average concrete compressive strength; \(E_{cm}\) Young’s modulus of concrete; \(f_{ym}\) yield strength of flexural reinforcement; \(\rho_l\) geometrical reinforcement ratio; \(E_{sm}\) Young’s modulus of flexural reinforcement; \(\mathcal{O}_l\) diameter of flexural reinforcement.
transfers. The bearing elements are precisely aligned face to face to the slots. They are also equipped with soft layers for stiffness control (Section 2.6).

Two different types of bearings are used. Both exhibit a comb-like back structure and an 80 mm front plate with openings for the back anchoring. Bearing Type 1 is equipped with six back plates \( t = 40 \text{ mm} \) and Type 2 with four \( t = 50 \text{ mm} \). Their overall bending stiffness is kept constant not to affect the highly stiffness-dependent load transfer.

Figure 6b displays the prestressing of the bearings by bar tendons (65 WR; DYWIDAG-Systems International). The bars induce forces up to 2.1 MN. To ensure a well-defined distribution of the prestressing force, steel
girders, and elastomeric pads \((t = 21 \text{ mm}, \text{CR-2000, Calenberg Ingenieure GmbH [CI GmbH]})\) are applied. That way the bearings do not lift off up to a bending moment of about 1,700 kNm/m.

Figure 6c demonstrates the assembly and casting preparations for the large-scale slabs. As mentioned, the slabs are concreted directly within the test setup (match casting) to avoid eigenstresses. The assembly process starts with the positioning of the bottom formwork along with the column stub. Both serve as a supporting surface for the specimen. On this surface up to 24 sliding plates are arranged step-by-step. Before installation, the plates are already fully equipped with the sliding components (tape, PTFE, and grease) and the bending reinforcement. On assembly, additional PVC and PTFE fillers ensure small gaps in between two adjacent plates and in the corner region. Both fillers are removed before testing to prevent restraints as a result from radial bending (Table 1, d). Then the remaining reinforcement (e.g., bottom and stirrups) is inserted. All open sides are shuttered for concrete. Finally, the sliding plates are prestressed against the bearings.

Figure 6d shows the load application. The specimen is loaded from top through two hydraulic cylinders \((1 \times 2.5 \text{ MN and } 1 \times 3.0 \text{ MN})\) which are connected by a cross beam \((h = 1 \text{ m}; b = 0.4 \text{ m}; l = 6.20 \text{ m})\). Inside the mounting supports elastomeric pads \((t = 20 \text{ mm}, \text{Compact Bearing S70, CI GmbH})\) are provided to prevent rotational restraints and to ensure a statically determined load transfer. Both cylinders are coupled and steered in deformation control. The load ratio results from their distance to the load application. This kind of loading structure proved to be very flexible. Steplessly, it can be used for all specimen sizes.

The load is applied at the free edges onto the slab. Four loading points simulate a line-like loading. To divide the load equally and independent from the local stiffness of a cracked concrete slab, two layers of centrically loaded single-span beams are placed below the cross beam. Additionally, calottes, PTFE, and reinforced elastomeric pads \((t = 30 \text{ mm}, \text{Sandwich Bearing Q, CI GmbH})\) are used to prevent restraints. Both the load application radius \(r_q\) and the area of the four loading points are increased linearly with size.

### 2.5 | Sliding planes

Figure 7a shows an exploded drawing of the back-anchoring system of sq65. It consists of two greased sliding planes at the front and at the backside of the front slab of the bearings, a sliding and an anchorage plate as well as a screwing system for fixing. Due to prestressing, friction forces \(F_\mu\) at the sliding planes must be taken into account, although the friction coefficients are very small. \(F_\mu\) reduces the reaction forces and thus the shear stresses at the column. However, a direct measurement with load cells in the punching tests is not feasible. For this reason, extensive performance tests with isolated sliding components and tests with coupled sliding systems have been conducted before.

In Figure 7b, the friction coefficients over the relevant sliding distances are shown. The red curves represent preliminary small-scale sliding tests with two isolated anchorage plates which are prestressed against a bearing by two threaded rods with a force of approximately 650 kN. A detailed description of these tests and their results can be found elsewhere. In contrast, the blue curve is extracted from a large-scale test on a halved beam using a sliding configuration similar to sq65 (Figure 7a). The beam is back-anchored at the symmetry plane with three rods applying a pre-

### TABLE 4  | Bearing stiffnesses with or without soft layers

| Bearing | Bearing stiffness \(c\) \((\text{kNm/} \text{rad/m})\) | Bearing stiffness \(c\) \((\text{kNm/} \text{rad/m})\) | Bearing stiffness \(c\) \((\text{kNm/} \text{rad/m})\) |
|---------|----------------------------------|----------------------------------|----------------------------------|
| Theory  | Infinite                         | Infinite                         | Infinite                         |
| Without stiffness control | 1,100,000                      | 1,100,000                       | 1,100,000                       |
| With stiffness control    | 1,100,000                       | 391,000                          | 203,000                          |

### TABLE 5  | Computed tangential moments acting on the bearing elements compared to stiffnesses and soft layers for control

| Bearing | \(m_t\) \((\text{kNm/m})\) | Bearing stiffness \(c\) \((\text{kNm/} \text{rad/m})\) | Soft layer | Type | Young's modulus |
|---------|------------------------|----------------------------------|------------|------|-----------------|
| Bearing 1 | 1,352                  | 1,100,000                        | None       | —    | —               |
| Bearing 2 | 488                    | 391,000                          | Pad\(^a\)  | 5 mm | 80 MPa          |
| Bearing 3 | 242                    | 203,000                          | Pad\(^a\)  | 8 mm | 55 MPa          |

\(^a\)Caoutchouc.
stressing force of approximately 975 kN. Both test arrangements proved that the sliding construction exhibits a friction coefficient of about \( \mu = 0.4\% \) up to sliding paths of 30 mm. Applied to the test series, it leads to a reduction of the ultimate punching load of 3 kN per sliding plate for slab quarters up to 50 cm (two rods) and 4 kN per sliding plate for sq65 (three rods). Table 6 summarizes the friction forces \( F_\mu \) for all specimens.

### Table 6

| Specimen | \( V_{R,test} \) (kN) | \( F_\mu \) (kN) | \( V_{R,test,red} \) (kN) | \( V_{\text{Flex}} \) (kN) | \( V_{Rm,EC2} \) (kN) | \( V_{R,CSCT} \) (kN) | \( V_{R,test,red}/V_{Rm,EC2} \) (−) | \( V_{R,test,red}/V_{R,CSCT} \) (−) |
|----------|------------------|-----------------|----------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| s30\(^b\) | 323              | 0               | 323                  | 680             | 334             | 329             | 0.97            | 0.98            |
| sq30     | 364              | 30              | 334                  | 650             | 333             | 328             | 1               | 1.02            |
| sq50     | 1,041            | 54              | 987                  | 1,851           | 802             | 757             | 1.18            | 1.3             |
| sq50-2   | 936              | 54              | 891                  | 1,851           | 843             | 790             | 1.06            | 1.13            |
| sq65     | 1,593            | 96              | 1,497                | 3,431           | 1,385           | 1,243           | 1.08            | 1.2             |
| Average  |                  |                 |                      |                 |                 |                 | 1.06            | 1.13            |

\(^a\)Mean level \( (C_{Rm,c} = 0.234) \).

\(^b\)Recalculated to slab quarter.

**Figure 7** Back-anchoring system: (a) configuration for sq65 and (b) measured friction coefficients displayed over the distance of sliding.

### 2.6 Control of bearing stiffness

Different from the demanded symmetry conditions, the bearing elements are not infinitely rigid. In case of statically indeterminate members, such as slabs, internal moments unintentionally redistributed indirectly proportional to the stiffness. To maintain the intended distribution of sectional forces, a control mechanism was established that systematically reduces the stiffness of the
individual bearing elements and counteracts the balancing of moments.

Figure 8 demonstrates this approach. It shows the tangential moment response of sq65 along the slab radius \( r \) for three different stiffness distributions at ultimate punching load. The theoretically ideal moment curve with infinitely rigid supports is shown in red. This curve represents the intended distribution and is identical to the simplified analytical approach of Markus.\(^3^0\) Taking into account the actual steel structure, the green curve indicates the moment distribution that arises without any stiffness modifications. It is estimated from a linear-elastic FE calculation. Moments are redistributed from the highly loaded column towards the outer edges of the slab. Finally, the blue curve shows the response with additive softness for Bearings 2 and 3. For softness, elastomeric pads\(^3^1\) are placed between the base plates and the front plate of the bearings (Figure 6a). Softness is controlled by the thickness and corresponds to the decrease of bending from bearing to bearing (Table 5). Young’s moduli are linearly approximated from the product specifications.\(^3^1\) All stiffnesses are summarized in Table 4 whereas Table 5 provides details on the pads and the alignment of stiffness-ratios to moment-ratios from bearing to bearing.

### 3 | RESULTS

#### 3.1 | Load–deflection behavior

Figure 9 presents the load–deflection response of all specimens. The vertical test load \( V_{R,test} \) includes the applied piston force and the self-weight of the load application. The deflection \( w \) is measured between the perimeter of load application \( r_q \) and the center of the column stub in a distance of approximately 2 cm from the symmetry plane. Obviously, all slabs exhibit a brittle failure after reaching the ultimate load, characterized by a sharp drop of forces. Both the vertical test loads and the test loads reduced by friction forces \( V_{R,test,red} \) can be read from Table 6.

For the thick specimens, the slab deflections seem slightly increased. Despite a rising slab stiffness, the gradients in the ascending branches of sq50 and sq65 are almost identical.\(^8,1^4\) This is attributed to two aspects, namely to elastic bearing deformations and to the growing distance between the measuring devices with increasing size. A lift-off of bearings was not observed.

#### 3.2 | Saw cuts and crack pattern

Saw cuts were performed to examine the inner shear cracks after testing (Figure 10). While s30 is sliced at the symmetry planes, all slab quarters are first separated from the lateral steel elements and then cut into two equal parts. Consequently, only one layer of top reinforcement can be seen for s30, whereas the quarters disclose all layers, so two for sq30 and four for sq50 and sq65. In the vicinity of the symmetry planes, approximately 10 cm of the concrete is lost due to the embedded back-anchoring system and the screwed-in reinforcement (marked with dashed lines).

All specimens indicate a typical punching failure of slabs with no shear reinforcement that is characterized by a dominantly inclined shear crack running from the edge of the column to the upper reinforcement. The mean inclination complies for all slabs and is about 30°. Near the symmetry planes, no branching of cracks occurs.

Figure 11a shows the crack pattern of sq50 disassembled from the test setup. Radial cracks arise in equal distances running from the inner corner to the outer edges. Tangential cracks concentrate near the
support up to about $2.0d$ away from the column at the radius of final failure. It is worth to note that cracks uniformly distribute even at the single sliding plates confirming an almost neutral influence of the symmetry planes.

3.3 Optical measurement

sq65 was additionally investigated with the optical measurement system ARAMIS. The measurement consists of 252 intervals, with four images per interval. Figure 11b
**FIGURE 12** Distribution of vertical deflections from optical measurements of sq65: (a) vertical displacements of the surface after failure and (b) displacements along the y-axis for different load stages.

**FIGURE 13** Development of radial strains along the first and second reinforcement layer near the symmetry axes for (a) s30 and sq30, (b) sq50, and (c) sq65.
illustrates the measured crack pattern after failure. Due to the size of the concrete surface, only the region up to the load application \((r \approx 2.0 \text{ m})\) could be recorded by the cameras. The oval dot near the column’s center is caused by a transport anchor.

Cracks are visualized by localized deformations recalculated to artificial strains. The crack development starts with a first tangential crack over the column stub. Subsequently, radial cracks nucleate from the column region to the free edge of the slab. They evenly distribute and divide the slab into segments. Then, new tangential cracks form starting near the column and propagate to the free edges. At failure, an outermost tangential crack suddenly arises accompanied by a sharp drop in the load. This critical crack develops in a distance of 0.90 m \((-1.6d)\) from the column’s perimeter.

Figure 12a shows the vertical displacements \(w\) on the upper surface after failure. As intended, they are distributed almost axially symmetric and reach up to 18 mm. Figure 12b plots the development of the displacements at different load stages along the y-axis. Pronounced curvature occurs only in the first 460 mm from the column’s perimeter. Outside, the slab shows almost rigid body rotations characterized by a constantly increasing deflection. At failure, a superproportional rise of deflections in the outer region is observed.

### 3.4 Reinforcement strains

Figure 13 presents the development of radial strains for all specimens. It shows the strains measured near both symmetry axes (left: y and right: x) in the first and second reinforcement layer for different load stages along with the normalized radius \(\rho\). For all slabs, the strains seem axially symmetric and qualitatively comply well with the theoretical radial moment of a slab. As intended, yielding \((\varepsilon_y \approx 2.7\%)\) does not occur until punching failure, so a bending failure is successfully prohibited. Quantitatively, the strains agree well to a cracked slab under bending, especially for higher loads (green + orange).32

### 4 DISCUSSION OF RESULTS

#### 4.1 General

A first analysis of sq50 revealed restraints at the support and at the regions of load application. They were caused by unintended deformations of the test setup. To overcome the restraints, a sliding plane similar to that of the back-anchoring system was installed below the slab on the column stub. Furthermore, additional PTFE layers were added to the load application. After modification, the same specimen was reloaded. In this second test, sq50 reached an ultimate load of \(V_{R,\text{test}} = 936 \text{ kN}\) with \(f_{cm} = 24.4 \text{ MPa}\) and showed a rather ductile type of failure. For the sake of clarity, all results from reloading are marked as sq50-2. They are not considered as a true punching capacity, but are used for discussion, as they represent the lower limit of the punching shear resistance. For sq65, the described updates were installed before testing; hence, no distinction must be made here.

Another key aspect from experimental implementation related to the test setup is the confinement action in the compression zone. This effect ensures that a conical shell is formed and a multi-axial stress state evolves. Without confinement the strength and stiffness in the compressive zone and thus the punching resistance decreases. Strain measurements on the bottom reinforcement in tangential direction near the column perimeter in case of s30 and sq30 show good agreement. It indicates equal confinement for small-scale specimens. However, for sq50 and sq65 with much higher load levels, such measurements are not available. This is an important issue for future experiments.

#### 4.2 Comparison of ultimate loads

The ultimate loads \(V_{R,\text{test,red}}\) are compared with the database of Vocke33 in order to analyze the size-dependent punching behavior. The database consists of axially symmetrical tests on slabs without shear reinforcement. In Figure 14, the normalized shear stresses at failure \(\nu_u\) according to Equation (1) at a distance of 1.5d are plotted along with the average effective depth \(d\). To establish
comparability, differences in concrete strength $f_{c,\text{test}}$ and reinforcement ratios $\rho_{l,\text{test}}$ are normalized with a power law to the basic values of $\rho_{l,\text{norm}} = 1.0\%$ and $f_{c,\text{norm}} = 25\text{ MPa}$. The length of the critical control perimeter $u_{\text{crit}}$ is computed according to Equations (2) and (3) depending on the geometrical type of the column being either a rectangle or a circle.

\[
v_u = \frac{V_{R,\text{test,red}}}{u_{\text{crit,l}} \cdot d} \left( \frac{\rho_{l,\text{norm}} f_{c,\text{norm}}}{\rho_{l,\text{test}} f_{c,\text{test}}} \right)^{1/3} \quad (1)
\]

with:

\[
u_{\text{crit,circ}} = (r_c + 1.5 \cdot d) \cdot \pi / 2 \quad (2)
\]

\[
u_{\text{crit,rect}} = 1.5 \cdot d \cdot \pi / 2 + r_c \quad (3)
\]

In Figure 14, the gray triangles represent tests taken from the database. They clearly demonstrate the lack of results for effective depths greater than 200 mm. The average shear stress at failure yields $v_{\text{sum}} = 1.44\text{ MPa}$ (dashed red line).

The presented series is depicted in green dots. s30 ($v_u = 1.44\text{ MPa}$) and sq30 ($v_u = 1.49\text{ MPa}$) correspond very well to the average value. In contrast, sq50 exceeds it ($v_u = 1.63\text{ MPa}$) by approximately 13%. As mentioned, this is attributed to restraints. The second attempt sq50-2 with the updated setup yields a substantially lower capacity ($v_u = 1.41\text{ MPa}$). As expected, sq65 exhibits the lowest normalized shear resistance ($v_u = 1.34\text{ MPa}$). Compared to sq30, it decreases by around 10%.

To extend the graph, tests of Winkler et al.$^{15}$ and Guandalini et al.$^8$ are added. In the first prototypical slab quarter test by Winkler et al., stiffening effects from shear studs and membrane actions led to an overestimation of the capacity ($v_u = 2.02\text{ MPa}$). These unintended effects have been identified and eliminated for all quarters of the present study.

Guandalini et al. provide results of full-size slabs with very similar properties (PG-1 and PG-6, both $v_u = 1.36\text{ MPa}$) and the largest, axially symmetric slab test (PG-3) known to the authors (blue triangles). The latter exhibits a significant lower shear stress ($v_u = 0.98\text{ MPa}$) but also possesses a considerably lower reinforcement ratio ($\rho_l = 0.33\%$).

Summing up, the shear stresses $v_u$ of the present study are close to the mean value of the database. Moreover, they decrease with increasing $d$. However, this decrease is considerably less pronounced than PG-3 suggests. This is attributed to the high longitudinal reinforcement quantities in combination with a multilayered reinforcement type. Already Leonhardt and Walther concluded from beam tests that the size effect depends not only on the effective depth, but also on reinforcement amounts and bond properties.$^{34}$ Especially, the latter are improved in the presented tests by using multilayered bars of smaller diameters rather than one layer with increased diameters but reduced surface. This reduces cracking and thus maintains the load transfer via aggregate interlock to a greater extent.

### 4.3 Punching resistance predictions

The punching resistances are estimated according to Eurocode 2 (EC2)$^3$ and the critical shear crack theory (CSCT).$^6$ Both approaches specify resistances from a nominal shear strength along a specific control perimeter. For the sake of comparability, all punching strengths are computed in the ultimate state with mean values from Tables 2 and 3. Moreover, no partial safety factors are considered.

The ultimate resistance $V_{Rm,EC2}$ according to EC2 is calculated with Equation (4) as a function of the reinforcement ratio $\rho_l$, an empirical factor $C_{Rm,c}$, the concrete compressive strength $f_{ck}$, the column geometry with a control perimeter $u_{2.0}$ located at a distance of $2.0d$ and the average effective depth $d$. Size dependency is taken into account by the factor $k$ that reduces the resistances for effective depths greater than 200 mm.

\[
V_{Rm,EC2} = C_{Rm,c} \cdot k \cdot (100 \cdot \rho_l f_{ck})^{1/5} \cdot u_{2.0} \cdot d \quad (4)
\]

\[
k = 1 + \frac{\sqrt{200}}{d \text{[mm]}} \leq 2.0 \quad (5)
\]

On a characteristic level, the empirical factor $C_{Rk,c}$ is based on a 5% quantile. Thus, calculated shear resistances would underestimate 95% of all tests.$^{19,35}$ Mean values are around 1.2–1.4 times higher than the computed characteristic ones.$^{35,36}$ To compare the measured ultimate loads with mean ones, $C_{Rm,c}$ is set to $C_{Rk,c} \times 1.3 = 0.234$. At the same time, $f_{cm}$ is considered with $f_{ck} = f_{cm} – 4\text{ MPa}$ (laboratory conditions).

For the calculation of the punching resistance $V_{R,\text{CSCT}}$, Equation (6) is employed. It is derived analytically from the CSCT.$^6$ This theory is e.g. used in the current draft of punching shear provisions for the new generation of Eurocode 2 (prEN1992-1-1)$^{37}$ The basic idea is, that the critical shear crack (CSC) governs the ability of a slab to transfer shear forces. Hence, the resistance depends on the crack opening and the roughness of the interfaces. Doing so, the CSCT accounts for strain and size effects.$^{20}$
where \( k_\beta = \sqrt{8 \cdot a \cdot d / b_{0.5}} \geq 1.0 \) is the shear enhancement coefficient with \( a = 8 \) for inner columns, \( \rho_l \) is the geometrical reinforcement ratio, \( f_c \) is the concrete compressive strength, \( d_{dg} \) is the reference value of the roughness of the CSC, \( r_{sc} \) is the distance from the column axis to the line of contraflexure, \( d \) is the effective depth and \( b_{0.5} = (2 \cdot \pi \cdot (r_c + d/2)) \) is the control perimeter in a distance of 0.5d. The results of both approaches are summarized in Table 6.

Both modeling approaches are in good agreement with the test results. The predictions of CSCT are slightly more conservative with an average ratio of measured to calculated values of 1.13. EC2 (1.06) comes closer to the mean. As a trend, the differences between predictions and measurements increase with the slab thickness for both approaches (EC2 = 8%, CSCT = 18%) what indicates a less pronounced size effect than theoretically assumed. This is in line with the findings in Section 4.1.

\[ V_{R,CSCT} = k_b \left( 100 \rho_l \cdot f_c \cdot \frac{d_{dg}}{r_{sc}} \right)^{3/5} \cdot b_{0.5} \cdot d \leq 0.55 \cdot b_{0.5} \cdot d \cdot \sqrt{f_c} \]  \hfill (6)

5 | CONCLUSIONS

The paper demonstrates that the principle of symmetry can be exploited to overcome the current size limitations in punching shear investigations. Double symmetric, full-size slabs are reduced to quarters. As a result, the test capacity is increased by almost 400% giving rise to first punching tests of slabs with 650 mm thickness. The symmetry conditions are satisfied by an elaborated test concept. The main components are a rigid modular bearing construction, a back-anchoring system, almost frictionless sliding planes and a slip-free vertical anchoring system for a high load transfer.

From the test series comprising four large-scale specimens, the following conclusions can be drawn:

- The test results indicate that high reinforcement ratios in combination with multilayered reinforcements lead to a less pronounced size effect than predicted by theory.
- The punching resistances correspond well to the mean values of the database of Vocke and EC2. The predictions of CSCT are slightly more conservative.
- Due to physical limitations, the test setup is not infinitely rigid as theoretically demanded by the symmetry conditions. In order to prevent unwanted moment redistribution, a stiffness control mechanism is used.
- Elastomeric pads as soft layers have proven to be suitable for this purpose.
- The optical measurements, the strain measurements, and the crack patterns prove that the slab quarters perform the intended axially symmetric punching behavior.
- Match casting provides several advantages for large slab thicknesses and helps to avoid eigenstresses from the back-anchoring process.
- Since this is the first series of tests on large-scale specimens, more research (including true replicates) is needed in order to generalize findings and exclude potential influences from the complex technical implementation.

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