Pressure fluctuation analysis of axial-flow pump based on local mean decomposition

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Abstract. Pressure fluctuation is inevitable in the operation of pumps. With an axial-flow pump as example, a new method for time-frequency analysis based on local mean decomposition (LMD) method was proposed to make a research on the characteristics of pressure fluctuation. LMD method is a self-adaptive method, by which any complicated multi-component signal could be decomposed into a number of product functions (PFs) whose instantaneous frequencies have physical significance in theory. Firstly, LMD method was introduced. Secondly, LMD method was compared with empirical mode decomposition (EMD) method and the results showed the superiority of LMD method. Finally, LMD method was applied to the pressure fluctuation signal of axial-flow pump and the PFs were extracted. Compared with EMD method, the results showed that LMD method can improve the time-frequency resolution and extract main vibration components accurately and effectively.

1. Introduction
Pressure fluctuation is inevitable in the operation of pumps. Monitoring and analysis of pressure fluctuation are essential for ensuring the stable and safety operation [1]. In order to make a further analysis, some conventional signal processing methods have been introduced to extract the characteristics of pressure fluctuation signals such as Fast Fourier Transform (FFT) and Wavelet Transform (WT) [2][3]. However, the above methods are not self-adaptive signal processing methods and based on the assumptions of linearity and stationary of the signals. In recent years, a new signal processing method, namely empirical mode decomposition (EMD) was proposed to analyze non-stationary signals [4][5]. EMD method can decompose the original signal into a number of intrinsic mode functions (IMFs) which represent the natural oscillatory mode embedded in the signal. By performing Hilbert transform to each IMF, the corresponding instantaneous amplitude (IA) and instantaneous frequency (IF) could be calculated. EMD is a self-adaptive method. However, the use of cubic spline and Hilbert transform in the EMD process induces a loss of amplitude and frequency information [6][7]. In order to solve this problem, local mean decomposition (LMD) was put forward [8].

LMD uses smoothed local mean to determine a more credible and reliable IF directly from the natural oscillatory mode embedded in the signal. LMD can decompose the original signal into a number of product functions (PFs), each of which is the product of an envelope signal from which IA can be got and a purely frequency modulated signal could be calculated. Thus, both the end effect and the unexplainable negative instantaneous frequency produced in the process of EMD can be avoided effectively. Furthermore, the complete time frequency distribution of the original signal could be
obtained by assembling IA and IF of all PFs. Recently, LMD has been widely used in the analysis of non-stationary and nonlinear signals [9].

In this paper, with an axial-flow pump as example, LMD method is applied to extract the time-frequency features of pressure fluctuation. And the results are compared with EMD method, the results show that LMD method can improve the time-frequency resolution and extract main vibration components from pressure fluctuation signals accurately and effectively.

The rest of the paper is organized as follows. In Section 2, LMD method is briefly reviewed. In section 3, LMD method is compared with empirical mode decomposition (EMD) method and the results show the superiority of LMD method. In section 4, LMD method will be applied to analyze the pressure fluctuation of axial-flow pump, and the comparison of LMD and EMD is demonstrated to show the advantages of LMD in detecting the pressure fluctuation. Finally, the experiments results will be summarized.

2. LMD algorithm

LMD can decompose the original signal into a number of PFs which is the product of an envelope signal and a purely frequency modulated signal. The envelope signal represent the IA, and the IF can be directly calculated from the purely frequency modulated signal without Hilbert transform. The IA and IF calculated by LMD are more precise than those obtained by EMD because LMD uses smoothed local means which facilitate a more natural decomposition than that using the cubic spline approach of EMD.

Given the original signal \( x(t) \), it can be decomposed as follows:

Identify all the local extremes \( n_i \), then the local mean value \( m_i \) can be calculated by calculating the mean of successive maximum and minimum.

\[
m_i = 0.5 \times (n_i + n_{i+1})
\]  

(1)

And the local envelope \( a_i \) is defined as

\[
a_i = 0.5 \times (n_i - n_{i+1})
\]  

(2)

Smooth the local mean \( m_i \) and the local envelope \( a_i \) with moving average filter, then local mean function \( m_{11}(t) \) and the local envelope function \( a_{11}(t) \) are formed. Subtract the local mean function \( m_{11}(t) \) from the original signal \( x(t) \) and the resulting signal \( h_{11}(t) \) is defined as in equation (3).

\[
h_{11}(t) = x(t) - m_{11}(t)
\]  

(3)

Get the frequency modulated signal \( s_{11}(t) \), then

\[
s_{11}(t) = h_{11}(t)[a_{11}(t)]^{-1}
\]  

(4)

\( s_{11}(t) \) is a purely frequency modulated signal when the local envelope function \( a_{12}(t) \) of \( s_{11}(t) \) satisfy \( a_{12}(t)=1 \). Otherwise, \( s_{11}(t) \) is regarded as the original signal \( x(t) \) and the above procedures need to be repeated until \(-1 \leq s_{1n}(t) \leq 1 \). \( s_{1n}(t) \) can be calculated as equation (5) and equation (6) mentioned.

\[
h_{1n}(t) = s_{1n-1}(t) - m_{1n}(t)
\]  

(5)

\[
s_{1n}(t) = h_{1n}(t)[a_{1n}(t)]^{-1}
\]  

(6)

Envelope signal \( \hat{a}_{n}(t) \) can be derived by multiplying all local envelope functions.
\[
\tilde{a}_i(t) = a_{i_1}(t)a_{i_2}(t)\cdots a_{i_n}(t) = \prod_{q=1}^{n} a_{i_q}(t)
\]  

Get the first product function PF1 by multiplying envelope signal \( \tilde{a}_i(t) \) and the purely frequency modulated signal \( s_{i_n}(t) \), that is

\[
PF_1(t) = \tilde{a}_i(t)s_{i_n}(t)
\]

PF1 contains the highest frequency oscillations of the original signal. The IA is the envelope signal \( \tilde{a}_i(t) \) and the IF is defined as

\[
f_1(t) = \frac{1}{2\pi} \frac{d[\arccos(s_{i_n}(t))]}{dt}
\]

Subtract PF1 from the original signal \( x(t) \), the result is named as \( u_1(t) \), which is treated as the new original signal. Repeat all the above procedure until \( u_k(t) \) becomes monotonic function, where \( k \) is the time of the iteration.

\[
u_k(t) = u_{k-1}(t) - PF_k(t)
\]

Thus, the original signal \( x(t) \) was decomposed into \( k \)-product and a monotonic function,

\[
x(t) = \sum_{p=1}^{k} PF_p(t) + u_k(t)
\]

3. Application to simulation signals

To compare the effectiveness of LMD and EMD method, a simulation signal provided as follows is analyzed by LMD and EMD respectively.

\[
x(t) = \left[ 1 + 0.5\cos(0.01t)\right] \cos(0.5t + 2\cos(0.02t)) + 4\sin(0.004t)\sin(0.12t)
\]

**Figure 1.** The waveform of the PFs.

**Figure 2.** Instantaneous frequency of the PFs.
The signal can be decomposed in two amplitude modulation and frequency modulation components. The first one is consisted of 0.25Hz basic signal and 0.01Hz of frequency modulation amplitude modulation. The other is consisted of 0.06Hz. The simulation signal is decomposed by LMD method and two PFs are obtained which is shown in figure 1. The instantaneous frequency of PFs are shown in figure 2. PF1 and PF2 correspond to the two components of the original signal respectively.

The simulation signal is decomposed by EMD method next. 7 IMFs are obtained. Calculating the correlation coefficients of IMFs, the results are listed in table 1. According to table 1, IMF1 and IMF2 are the most sensitive IMFs which is shown in figure 3. Therefore, the analysis is only adapted to IMF1 and IMF2. The instantaneous frequency of IMFs are shown in figure 4.

| IMF1  | IMF2  | IMF3  | IMF4  | IMF5  | IMF6  | IMF7  |
|-------|-------|-------|-------|-------|-------|-------|
| 0.5471| 0.8392| 0.0113| 0.0203| 0.0232| 0.0214| 0.0168|

IMF1 and IMF2 are correspond to the two components of the original signal also. But compare with figure 2 and figure 4, it is clear that notable distortion has been appeared in instantaneous frequency of IMFs, especially in the two end of the signal. While the end effect and frequency fluctuation are not obvious in the decomposition results of LMD.

**Figure 3.** The waveform of the IMFs.

**Figure 4.** Instantaneous frequency of the IMFs.

4. **Application to the pressure fluctuation analysis**

To verity the effectiveness of LMD in the feature extraction, LMD method is applied to the experimental pressure fluctuation signals analysis. In this paper, all the experimental data are obtained from an axial-flow pump experiment and the experiment system is given in figure 5.

**Figure 5.** The experiment system of the axial-flow pump.
The pressure fluctuations are measured by pressure transducers which are installed in the vicinity of the impeller inlet, the impeller outlet and outlet conduit. The number of blade is 3. The rated speed of the electric motor is 1250 r/min. The advanced data acquisition and analysis system is EN900 supported by ENVADA Company in Beijing. The sampling frequency is 256 times of the rotation frequency and 1024 points are collected. The pressure fluctuation of outlet conduit (head 3.4m, angle 4°) is shown in figure 6.

**Figure 6.** The pressure fluctuation signal of outlet conduit.

The pressure fluctuation signal was decomposed by EMD first. The results are shown in figure 7. There are 7 IMFs obtained in the process of EMD.

**Figure 7.** Decomposition results of pressure fluctuation signal by EMD method.

For choosing the most sensitive IMFs, the correlation coefficients of IMFs are calculated which are listed in table 2. According to the results of table 2, IMF 5, IMF 6 and IMF7 are the most sensitive IMFs.

**Table 2.** The correlation coefficients of IMFs (pressure fluctuation).

|       | IMF1   | IMF2   | IMF3   | IMF4   | IMF5   | IMF6   | IMF7   |
|-------|--------|--------|--------|--------|--------|--------|--------|
| EMD   | 0.0553 | 0.0919 | 0.0692 | 0.1003 | 0.8029 | 0.2634 | 0.3846 |
To compare the effectiveness of LMD and EMD, the pressure fluctuation signal was decomposed by LMD and the results are shown in figure 8.

![Figure 8. Decomposition results of pressure fluctuation signal by LMD method.](image)

Spectrum analysis is applied to IMFs and PFs, and the results are shown in figure 9 and figure 10. By LMD and EMD method, 62.5Hz, 20.83Hz and 10.4 Hz are extracted clearly. But 5.2Hz only can be found from LMD results. As is mentioned before, the rotation is 1250r/min, so the rotation frequency is 20.83HZ \( (f_0) \). The characteristic frequency of EMD are 3\( f_0 \), \( f_0 \) and 0.5 \( f_0 \). While the characteristic frequency of LMD are 3 \( f_0 \), \( f_0 \), 0.5 \( f_0 \) and 0.25 \( f_0 \), which is more consistent with the actual situation. As is well-known, the frequency 3 \( f_0 \) results from the influence of impeller on the water flow, the rotation frequency \( f_0 \) results from the speed of pump. The low frequency 0.25 \( f_0 \) and 0.5 \( f_0 \) is the results of the irregular movements of turbulence.

![Figure 9. Spectrum analysis of the IMFs.](image)

From the experiment, it can be found that LMD method can achieve the more precise decomposition results than EMD method. Compared with the EMD method, there is no false component of PFS and more effect features can be extracted by LMD. But, there are some problems need to be resolved such as mode mixing problem and end effect in the operation of LMD. For the wide application of LMD, there are a lot of problems need to be studied further.
5. Conclusions
In this paper, LMD method is introduced to analyze the characteristics frequency of pressure fluctuation. The approach avoids the interference of false components. Furthermore, LMD gives the better decomposition performance for the lower frequency components. The experiment results indicate that LMD method is efficient for the multi-component signals. Finally, there are still some problems to improve the stability and validity of LMD method in the next research.

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Figure 10. Spectrum analysis of the PFs.