LETTER

Observation of Dirac monopoles in a synthetic magnetic field

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Magnetic monopoles—particles that behave as isolated north or south magnetic poles—have been the subject of speculation since the first detailed observations of magnetism several hundred years ago. Numerous theoretical investigations and hitherto unsuccessful experimental searches have followed Dirac’s 1931 development of a theory of monopoles consistent with both quantum mechanics and the gauge invariance of the electromagnetic field. The existence of even a single Dirac magnetic monopole would have far-reaching physical consequences, most famously explaining the quantization of electric charge. Although analogues of magnetic monopoles have been found in exotic spin ices and other systems, there has been no direct experimental observation of Dirac monopoles within a medium described by a quantum field, such as superfluid helium-3 (refs 10–13). Here we demonstrate the controlled creation of Dirac monopoles in the synthetic magnetic field produced by a spinor Bose–Einstein condensate. Monopoles are identified, in both experiments and matching numerical simulations, at the termini of vortex lines within the condensate. By directly imaging such a vortex line, the presence of a monopole may be discerned from the experimental data alone. These real-space images provide conclusive and long-awaited experimental evidence of the existence of Dirac monopoles. Our result provides an unprecedented opportunity to observe and manipulate these quantum mechanical entities in a controlled environment.

Maxwell’s equations refer neither to magnetic monopoles nor to the magnetic currents that arise from their motion. Although a simple symmetrization with respect to the electric and magnetic fields, respectively E and B, leads to equations that involve these magnetic charges, it also seemingly prevents their description in terms of the familiar scalar and vector potentials, respectively V and A, alone. Because the quantum mechanical Hamiltonian is expressed in terms of potentials, rather than electromagnetic fields, this modification immediately leads to serious theoretical challenges.

In a celebrated paper that combined arguments from quantum mechanics and classical electrodynamics, Dirac identified electromagnetic potentials consistent with the existence of magnetic monopoles. His derivation relies on the observation that in quantum mechanics the potentials V and A influence charged-particle dynamics either through the Hamiltonian or, equivalently, through modifications of the complex phase of the particle wavefunction. Armed with these equivalent perspectives, Dirac then considered the phase properties of a wavefunction pierced by a semi-infinite nodal line with non-zero phase winding. He discovered that the corresponding electromagnetic potentials yield the magnetic field of a monopole located at the endpoint of the nodal line. The vector potential in this case also exhibits a nonphysical line singularity, or ‘Dirac string’, that terminates at the monopole.

We experimentally create Dirac monopoles in the synthetic electromagnetic field that arises in the context of a ferromagnetic spin-1 $^{87}$Rb Bose–Einstein condensate (BEC) in a tailored excited state. The BEC is described by a quantum mechanical order parameter that satisfies a nonlinear Schrödinger equation, and the synthetic gauge potentials describing a north magnetic pole (Fig. 1) are generated by the spin texture. This experiment builds on studies of synthetic electric and magnetic fields, respectively $B_\ast$ and $A_\ast$, in atomic BECs, which is an emerging topic of intense interest in the simulation of condensed-matter systems with ultracold atoms. Unlike monopole experiments in spin ices, liquid crystals, skyrmion lattices and metallic ferromagnets, our experiments demonstrate the essential quantum features of the monopole envisioned by Dirac.

Physically, the vector potential, $A_\ast$, and synthetic magnetic field, $B_\ast = hV \times A_\ast$, are related to the superfluid velocity, $v_s$, and vorticity, $\Omega = \nabla \times v_s$, respectively. (Here $h$ denotes Planck’s constant divided by $2\pi$.) Our primary evidence for the existence of the monopole comes from images of the condensate density taken after the creation of these fields (Figs 2 and 3), which reveal a nodal vortex line with $4\pi$ phase winding terminating within the condensate. The images also display a three-dimensional spin structure that agrees well with the results of

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Figure 1 | Schematic representations of the monopole creation process and experimental apparatus. a–c, Theoretical spin orientation (red arrows) within the condensate when the magnetic field zero (black dot) is above (a), entering (b) and in the middle of (c) the condensate. The helix represents the singularity in the vorticity. d, Azimuthal superfluid velocity, $v_\phi$ (colour scale and red arrow), scaled by equatorial velocity, $v_\phi$. Black arrows depict the synthetic magnetic field, $B_\ast$. e, Experimental set-up showing magnetic quadrupole (Q) and bias field (BX, BY and BZ) coils. Red arrows (OT) show beam paths of the optical dipole trap, and blue arrows indicate horizontal (H) and vertical (V) imaging axes. Gravity points in the $–z$ direction.
time-varying magnetic field, condensate. The spin rotation occurs as following the method introduced in ref. 14, a magnetic field zero is initially located on the horizontal axis for the three spin states \(|1\rangle, |0\rangle, |\bar{1}\rangle\); the colour map is given in f. Yellow arrows indicate the location of the nodal lines. The rightmost three columns show images taken along the vertical axis. The scale is 285 µm \(\times\) 285 µm (horizontal) and 220 µm \(\times\) 220 µm (vertical), and the peak column density is \(\bar{n}_p = 1.0 \times 10^9\) cm\(^{-2}\).

Numerical simulations (Fig. 4). We analyse these findings and discuss their implications below.

The spinor order parameter corresponding to the Dirac monopole\(^{13,17}\) is generated by an adiabatic spin rotation in response to a time-varying magnetic field, \(B(r, t)\). Similar spin rotations have been used to create multiply quantized vortices\(^8\) and skyrmion spin textures\(^{19}\). The order parameter \(\Psi(r, t) = \psi(r, t)z(r, t)\) is the product of a scalar order parameter, \(\psi\), and a spinor, \(z = (z_1, z_2, z_3)\). The condensate is initially spin-polarized along the z axis, that is, \(z = (1, 0, 0)^T\). Following the method introduced in ref. 14, a magnetic field \(B(r, t) = b_y(x\hat{x} + y\hat{y} - 2z\hat{z}) + B_z(t)\hat{z}\) is applied, where \(b_y > 0\) is the strength of a quadrupole field gradient and \(B_z(t)\) is a uniform bias field. The magnetic field zero is initially located on the z axis at \(z = B_z(t)/(2b_y)\). The condensate spin rotation occurs as \(B_z\) is reduced, drawing the magnetic field zero into the region occupied by the superfluid.

Ideally, the condensate spin adiabatically follows the local direction of the field (Fig. 1a–c). Our numerical analysis indicates, and both simulations and experiment confirm, that the fraction of atoms undergoing non-adiabatic spin-flip transitions is of order 1% for our experimental parameters. The spin texture in the adiabatic case is conveniently expressed in a scaled and shifted coordinate system with \(x' = x, y' = y, z' = 2z - B_z\bar{n}_p\), corresponding derivatives \(\nabla'\), and spherical coordinates \((r', \theta', \phi')\). This transformation scales the z axis by a factor of two and shifts the origin of coordinates to coincide with the zero of the magnetic field. The applied magnetic field is then \(B = b_y(x'\hat{x} + y'\hat{y} - z'\hat{z})\). As \(B_z\) is reduced, each spin rotates by an angle \(\pi - \theta'\) about an axis \(\hat{n}(r', \theta', \phi') = -\hat{x}'\sin\phi' + \hat{y}'\cos\phi'\). This spatially dependent rotation leads to a superfluid velocity

\[\nu = \frac{\hbar}{M r^2} \sin\theta' \cos\phi'\]

and vorticity

\[\Omega = -\frac{\hbar}{M r^2} r' + \frac{4\pi\hbar}{M} \delta(x')\delta(\theta')\theta(z')r'\]

where \(M\) is the atomic mass, \(\delta\) is the Dirac delta function and \(\Theta\) is the Heaviside step function. The vorticity is that of a monopole attached to a semi-infinite vortex line singularity, of phase winding \(4\pi\), extending along the \(+z'\) axis.

The synthetic vector potential arising from the spin rotation can be written as \(A^* = -\nu \hat{n}/\hbar\), with the line singularity in \(A^*\) coincident with the nodal line in \(\Psi\). However, this singularity is nonphysical, because it depends on the choice of gauge and can even be made to vanish\(^{20}\) (Supplementary Information). The synthetic magnetic field of the monopole is therefore simply

\[B^* = \frac{\hbar}{r'^2} r'\]

The fields \(\nu\) and \(B^*\) are depicted in Fig. 1d.
Figure 4 | Quantitative comparison between experiment and simulation. 

a. Experimental (solid lines) and simulated (dashed and dotted lines) column densities $n$ of the condensate from the vertical images in Fig. 3, with cross-sections taken as shown in the insets. Dotted lines show the approximate effect of three-body losses (see text). The origin $x = 0$ coincides with the hole in state $|0\rangle$. b. Fractions in each spin state for different positions of the centre of mass of the $|0\rangle$ state $(z_c)$ relative to that of the condensate $(z_s)$, in units of the axial Thomas–Fermi radius $(Z)$. Solid lines are simulated values and points marked with letters and numbers correspond to panels a–e of Figs 2 and 3, respectively. Typical error bars that reflect uncertainties in the calibration of the imaging system are shown for several points.

The experimental set-up is shown schematically in Fig. 1e. The optically trapped $^{87}$Rb BEC consists of $N \approx 1.8 \times 10^5$ atoms in the $|F = 1, m = 1\rangle = |1\rangle$ spin state, where the uncertainty reflects shot-to-shot variations and the calibration of the detection system. The calculated radial and axial Thomas–Fermi radii are $R = 6.5 \mu m$ and $Z = 4.6 \mu m$, respectively, and the corresponding optical trap frequencies are respectively $\omega_r \approx 2 \pi \times 160$ Hz and $\omega_z \approx 2 \pi \times 220$ Hz. Four sets of coils are used to produce $B_y$, $B_z$ and the transverse magnetic field components $B_x$ and $B_p$, which are used to guide the applied magnetic field zero into the condensate. At the beginning of the monopole creation process, the bias field is $B_z = 10$ mG. The quadrupole field gradient is then linearly ramped from zero to $B_y = 3.7$ G cm$^{-1}$, placing the magnetic field zero approximately 30 $\mu m$ above the condensate. The field zero is then brought down into the condensate by decreasing $B_z$ linearly to $B_{zf}$ at the rate $\dot{B}_z = -0.25$ G s$^{-1}$. We call this the ‘creation ramp’.

The atomic density of each spinor component $|m\rangle$ is imaged as established by the local spin rotation during the creation ramp (Methods). As the field zero passes through the condensate (Fig. 2a–f), the distribution of particles in the three spin states changes in a manner indicative of the expected spin rotation shown in Fig. 1. The nodal line appears in the images taken along the vertical axis as holes in the $|1\rangle$ and $|0\rangle$ components, and in the side images as regions of reduced density extending vertically from the top of the condensate towards, but not through, the $|1\rangle$ component. This nodal line extends more deeply into the condensate as $B_{zf}$ is reduced. Ultimately it splits into two vortex lines (Fig. 2f; see also Extended Data Fig. 1)—the characteristic signature of the decay of a doubly quantized vortex—illustrating its 4$\pi$ phase winding.

We compare the experimental images of the vertically (Fig. 3a) and horizontally (Fig. 3c) imaged density profiles with those given by numerical simulations (Fig. 3b, d) in which the monopole is near the centre of the condensate. The simulation data are obtained by solving the full three-dimensional dynamics of the spinor order parameter (Methods). The locations of the doubly quantized and singly quantized vortices in spinor components $|\pm 1\rangle$ and $|0\rangle$ are visible in the experimentally acquired density profiles, as are other structures discernible in the images obtained from the numerical simulations. The observed vertical spatial separation of the spinor components (Fig. 3c) confirms that the vortex line terminates within the bulk of the condensate.

The quantitative agreement between experiment and simulation is apparent in Fig. 4, which shows cross-sections of the density profiles taken through the centre of the condensate. The differences observed in the peak densities (Fig. 4a) of the experimental (solid lines) and simulated (dashed lines) data are due to effects not taken into account in the simulation, such as three-body losses that were observed to be $\sim 10\%$ in the experiment. To show their effect, we have scaled the simulated data accordingly (dotted lines). Noting the absence of free parameters, the experimental data are in very good agreement with the numerical simulation.

We also show the fraction of the condensate in each spinor component for different vertical monopole locations within the condensate (Fig. 4b), including data from images in which the nodal line of the order parameter does not necessarily coincide with the $z$ axis.

The physical observable is the position of the centre mass of the $|0\rangle$ component, $z_0$, relative to the centre mass of the whole condensate, $z_c$. Again, we find that the experiments and simulations are in very good quantitative agreement without any free parameters.

An alternative description of the origins of the velocity and vorticity profiles (equations (1) and (2)) can be presented in terms of the motion of the monopole (Supplementary Information). As the monopole approaches the condensate, it is a source not only of the synthetic magnetic field, $B^s$ (equation (3)), but also of an azimuthal synthetic electric field, $E^a$, described by Faraday’s law, $\nabla \times E^a = -\partial B^s/\partial t$. Each mass element of the superfluid is given a corresponding azimuthal acceleration by $E^a$. The monopole motion thereby induces the appropriate superfluid velocity and vorticity profiles within the condensate, in a manner similar to the induction of electric current in a superconducting loop by the motion of a (natural) magnetic monopole.

In our case, the condensate itself is the monopole detector, analogous to the superconducting loop. Being three-dimensional, however, it is sensitive to the entire $4\pi$ solid angle surrounding the monopole.

The creation and manipulation of a Dirac monopole in a controlled environment opens up a wide range of experimental and theoretical investigations. The time evolution and decay of the monopole are of particular interest because it is not created in the ground state.

Interactions between the monopole and other topological excitations, such as vortices, present another fundamental research avenue with a variety of unexplored phenomena. There exists also the possibility of identifying and studying condensate spin textures that correspond to other exotic synthetic electromagnetic fields, such as that of the non-Abelian monopole.

Finally, the experimental methods developed in this work can also be directly used in the realization of a vortex pump, which paves the way for the study of peculiar many-body quantum states, such as those related to the quantum Hall effect.

Note added in proof: The effects of the Lorentz force arising from an inhomogeneous synthetic magnetic field have recently been observed in condensate dynamics.

METHODS SUMMARY

Imaging. After the creation ramp, we non-adiabatically change $B_y$ from $B_{zf}$ to a large value (typically several hundred milligauss) to project the condensate spinor components $|m\rangle$ into the approximate eigenstates of the Zeeman Hamiltonian while preserving the monopole spin texture. We call this the ‘projection ramp’. The condensate is then released from the trap and allowed to expand for 22.9 ms. The three spin states are separated along the $x$ axis during the expansion by a 3.5-ms pulse of the magnetic field gradient with the magnetic bias field pointing in the $x$ direction. We take images simultaneously along the horizontal and vertical axes.

Data. The images shown in Figs 2 and 3 are selected from among several dozen similar images taken under identical conditions, and hundreds of similar images taken under similar conditions (see also Extended Data Fig. 2 for representative examples). Not every experimental run yields an image of a monopole, because drifts in the magnetic field and location of the optical trap cause the magnetic field zero to pass outside the BEC. Under optimal conditions, five to ten consecutive images may be taken before drifts require adjustment of the bias fields.

Simulation. We solve the full three-dimensional Gross–Pitaevskii equation with simulation parameters chosen to match those of the experiment, excepting the effects of three-body losses and the magnetic forces arising from the gradient...
during the spin state separation just before imaging. To show the effects of the expansion, we present integrated particle densities of the condensate from the numerical simulation immediately after the creation ramp, and while the magnetic field zero is still in the condensate, in Extended Data Fig. 3. The volume considered varies from $20 \times 20 \times 20 a_r^3$ to $320 \times 320 \times 320 a_r^3$, where $a_r = \sqrt{\hbar / M_0} \approx 0.9 \mu m$ is the radial harmonic oscillator length. The size of the computational grid changes from $180 \times 180 \times 180$ to $1,024 \times 1,024 \times 1,024$ points.

Online Content Any additional Methods, Extended Data display items and Source Data are available in the online version of the paper; references unique to these sections appear only in the online paper.

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Author Contributions M.W.R., S.K. and D.S.H. developed and conducted the experiments, after which M.W.R. and D.S.H. analysed the data. E.R. performed the numerical simulations under the guidance of M.M., who also developed the gauge transformations presented in Supplementary Information. Interactive feedback between the experiments and simulations carried out by M.W.R., D.S.H., E.R. and M.M. was essential to achieving the reported results. All authors discussed both experimental and theoretical results and commented on the manuscript.

Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to D.S.H. (dshall@amherst.edu).
Decreasing $B_\text{r}$, on the other hand, results in lengthier exposures of the BEC to magnetic field noise that can possibly induce undesirable spin transitions. The noise associated with the power mains (at frequencies that are odd-integer ($n$) multiples of 60 Hz) is the most serious, being resonant at a field of $n \times 85 \mu$G. The choice $B_r = 0.25 \, \text{G} \text{ s}^{-1}$ ensures that the resonance condition is passed in less time than a single oscillation period of the noise, at least up to $n = 7$. The effect of this noise is merely to distort slightly the path traced by the field zero during the creation ramp.

With the experimental parameters described above, we find that a negligible number of atoms are excited out of the SFSS after the field zero is moved fully through the condensate. Only when we increase the ramp rates by an order of magnitude do we find a discernible fraction of the atoms in the weak-field seeking state. This is consistent with the simulations and the calculation of $P_{\text{ad}}$. We conclude that non-adiabatic spin flips are not important in the monopole creation process with the parameters used in the experiments, and that the Landau–Zener model describes this phenomenon well.

Imaging. At the end of the creation ramp, $B_r$ is rapidly decreased (in 0.040 ms) until $|B_r/B_\text{rI}| > 1$, a stage we call ‘the projection ramp’. This non-adiabatic field ramp keeps the order parameter essentially unchanged but takes the spin states $|m\rangle$ to be the approximate eigenstates of the Zeeman Hamiltonian. As described below, we image the particle density in each of these new eigenstates, accessing the detailed structure of the monopole established by the creation ramp.

Immediately after the projection ramp, the magnetic field gradient is turned off in 0.350 ms. The optical trapping beams are then extinguished, releasing the condensate from the trap and permitting it to expand freely for 4 ms. The field is then increased adiabatically (in 1 ms) to 13.7 G in the $z$ direction as $B_r$ is simultaneously reduced to zero. After a 1.5-ms delay, the magnetic gradient coils are pulsed on for 3.5 ms to 20.1 G cm$^{-1}$ (radial) to separate the spin states horizontally.

The total time of flight of the atoms is 22.9 ms, counted from the moment of release from the optical trap. After expansion, the condensates are imaged absorbptively along both the vertical ($z$) and horizontal ($y$) axes simultaneously (to within 14 μG) in the presence of a 0.1-G imaging field directed along $z$. In the absorption images, we correct for neither the slightly different sensitivities of the different spin states to the probe beam nor the slightly different expansions that result from the applied magnetic field gradient.

Although we describe in this paper the creation of the monopole with the initial parameters $b_z > 0$, $B_r(t = 0) > 0$, and $B_\text{rI} < 0$, the procedure yields essentially identical results experimentally when $b_z > 0$, $B_r(t = 0) < 0$, and $B_\text{rI} > 0$ in the creation ramp, except that the field zero enters the condensate along the negative $z$ axis and thereby changes the sense of the spin rotation. Similarly, $B_r$ can be rapidly increased in either the $+z$ or the $-z$ direction in the projection ramp with the same outcomes (Extended Data Fig. 2).

The images shown in Figs 2 and 3 are selected from among dozens of similar images taken under identical conditions, and hundreds of images taken under different (Extended Data Fig. 2). Not every image demonstrates the signature presence of a monopole, because drifts in the magnetic fields and the location of the optical trap eventually cause the magnetic field zero to miss the condensate. Under optimal conditions, we find that we can take five to ten sequential images in which the field zero passes through the condensate, after which we must adjust the magnetic bias fields to re-centre the magnetic field zero on the condensate.

The first images of pairs of singly quantized vortices that indicate the passage of the monopole through the condensate were taken on 6 February 2013. Consistent images of the condensate density distributions associated with the monopole were first obtained on 1 March 2013.

Numerical simulation. The experimental set-up is simulated by solving the full three-dimensional Gross–Pitaevskii equation. The simulation parameters are chosen to match those of the experiment, but we include neither the effects of three-body losses nor the magnetic forces arising from the gradient during the spin state separation just before imaging. The particle number is held fixed at $N = 1.8 \times 10^5$, corresponding to the initial number of atoms in the experiment. We can roughly account for the three-body losses by scaling the obtained particle density by the fraction of atoms that remain at the end of the experiment. Otherwise, the simulations are performed with the time-dependent parameters identical to those used in the experiment.

The volume considered varies from $20 \times 20 \times 20a_0^3$ to $320 \times 320 \times 320a_0^3$, where $a_0 = \sqrt{\hbar/M_\text{rI}}=0.9 \, \mu\text{m}$ is the radial harmonic oscillator length. The size of the computational grid changes from $180 \times 180 \times 180$ to $1,024 \times 1,024 \times 1,024$ points. The initial spin-polarized state is obtained with a relaxation method and the temporal evolution is computed using a split-operator technique employing Fourier transforms for the kinetic energy part. The time required for the computation is reduced with the help of graphics processing units, coordinate transformations and an adaptive computational grid.
Effects of free expansion. The condensate must be allowed to expand freely to image its spin structure and determine the presence of the monopole. The condensate is therefore not imaged while the magnetic field zero is within the condensate. To demonstrate the effects of the free expansion on the spin structure, we show the simulated particle densities of the condensate just after the creation ramp in Extended Data Fig. 3. The images are created from an intermediate step in the complete simulation that is used to produce Fig. 2. The principal effects of the release of the condensate are its expansion with different speeds in different directions, the increase in the relative vortex core sizes, the partial filling of the vortex cores with other spinor components, and the slight separation of the different spinor components. The last three effects are due exclusively to the repulsive interactions between the atoms during the first few milliseconds of expansion. Because there is excellent agreement between the simulated and experimentally observed results in Fig. 3, we conclude that Extended Data Fig. 3 is a suitable representation of the condensate just after the creation ramp, while the field zero is still within the superfluid.

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Extended Data Figure 1 | Decay of the doubly quantized vortex. Images of the condensate time evolution after moving the magnetic field zero completely through the condensate. The evolution time is shown at the bottom right of each panel. The maximum pixel intensity corresponds to a peak column density $n_p = 1.0 \times 10^9$ cm$^{-2}$, and the field of view is 246 μm × 246 μm. Each image represents a separate condensate, and $B_z = 3$ G s$^{-1}$. After roughly 10 ms the vortex splits in two, demonstrating the initial $4\pi$ phase winding of the nodal line.
Extended Data Figure 2 | Additional representative images of Dirac monopoles. Each row contains images of the same condensate. The maximum pixel intensity corresponds to $\bar{n}_p = 8.2 \times 10^8$ cm$^{-2}$, and the field of view is 220 μm $\times$ 220 μm in the vertical images and 285 μm $\times$ 285 μm in the horizontal images. The arrow points to the density depletion that is identified as the nodal line. In a–c, we use the same protocol outlined in the paper: an off-centre monopole (a); an angled nodal line that is visible in the side image but not in the vertically directed image (b); and a nodal line that appears to be splitting into two vortices in the $|m = -1|$ component (c). d, An example of a monopole spin structure in which the creation ramp is as described in the text but the projection ramp is reversed (that is, $B_z$ is rapidly increased until $|B_z/B_{z,d}| \approx 1$). e, Monopole spin structure created by moving the field zero into the condensate from below with $B_z > 0$. The projection ramp is performed as described in d.
Extended Data Figure 3 | Numerical simulation of integrated particle densities before expansion. Vertically (a) and horizontally (b) integrated particle densities of a condensate just before the projection ramp, with $B_z$, chosen such that the monopole is in the centre of the condensate. The fields of view are 17.2 $\mu$m $\times$ 17.2 $\mu$m (a) and 17.2 $\mu$m $\times$ 11.4 $\mu$m (b); in b, it is reduced in the $z$ direction for a more convenient comparison with the simulations shown in Fig. 3. The maximum pixel intensity corresponds to $\bar{n}_p = 2.98 \times 10^{11}$ cm$^{-2}$. 

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