Measuring the Virial Factor in SDSS DR5 Quasars with Redshifted Hβ and Fe II Broad Emission Lines

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Abstract

Under the hypothesis of gravitational redshift induced by a central supermassive black hole, and based on line widths and shifts of redward-shifted Hβ and Fe II broad emission lines for a sample of 1973 z < 0.8 Sloan Digital Sky Survey DR5 quasars, we measured the virial factor in determining supermassive black hole masses, usually estimated by the reverberation mapping method or the relevant secondary methods. The virial factor had been believed to be from the geometric effect of the broad-line region. The measured virial factor of Fe II is larger than that of Hβ for 98% of these quasars. The virial factor is very different from object to object and for different emission lines. For most of these quasars, the virial factor of Hβ is larger than these averages that were usually used in determining the masses of black holes. There are three positive correlations among the measured virial factor of Hβ, dimensionless accretion rate, and Fe II/Hβ line ratio. A positive three-dimensional correlation is found among these three quantities, and this correlation indicates that the virial factor is likely dominated by the dimensionless accretion rate and metallicity. A negative correlation is found between the redward shift of Hβ and the scaled size of the broad-line region radius in units of the gravitational radius of the black hole. This negative correlation will be expected naturally if the redward shift of Hβ is mainly from the gravity of the black hole. Radiation pressure from the accretion disk may be a significant contributor to the virial factor.

Unified Astronomy Thesaurus concepts: Black hole physics (159); Active galactic nuclei (16); Emission line galaxies (459); Quasars (1319); Supermassive black holes (1663)

Supporting material: machine-readable table

1. Introduction

Mass, M∗, is an important fundamental parameter of black holes, and reliable measurement of M∗ always will be a key issue of black-hole-related research, such as the research and understanding of the formation, growth, and evolution of the black holes in the universe and the coevolution debates of supermassive black holes (SMBHs) and host galaxies (e.g., Komendy & Ho 2013). Assuming virialized motion of clouds in the broad-line region (BLR), the reverberation mapping (RM) method or the relevant secondary methods based on single-epoch spectra were widely used to measure M∗ by an RM black hole mass M_{RM} = f_{FWHM}^{2} n_{BLR} / G for active galactic nuclei (AGNs), where f is the virial factor, v_{FWHM} is the FWHM of the broad emission line, r_{BLR} is the radius of the BLR, and G is the gravitational constant (e.g., Peterson et al. 2004). f is commonly considered the main source of uncertainty in M_{RM}. If the line width v_{FWHM} is replaced with the second moment of emission line, σ_{line} f becomes f. The virial factor had been believed to be induced by the geometric effect of the BLR. Based on the photoionization assumption (e.g., Blandford & McKee 1982; Peterson 1993), r_{BLR} = τ_{ob} c / (1 + z), where c is the speed of light, z is the redshift of the source, and τ_{ob} is the time lag observed between the broad-line and continuum variations. For non-RM AGNs with single-epoch spectra, r_{BLR} can be estimated with the radius–luminosity relation, i.e., the empirical r_{BLR} = L(5100 Å) relation for the Hβ emission line, established on the basis of the RM AGNs, where L(5100 Å) is the AGN continuum luminosity at rest-frame wavelength 5100 Å (e.g., Kaspi et al. 2000; Bentz et al. 2013; Du et al. 2018b; Du & Wang 2019; Yu et al. 2020).

RM observation research has been undertaken for more than 100 AGNs over the past several decades (e.g., Kaspi & Netzer 1999; Kaspi et al. 2000; Peterson et al. 2005; Bentz et al. 2006; Kaspi et al. 2007; Bentz et al. 2010; Denney et al. 2010; Barth et al. 2011; Haas et al. 2011; Pozo Nuñez et al. 2012; Du et al. 2014; Pei et al. 2014; Wang et al. 2014; Barth et al. 2015; Du et al. 2015, 2016; Lu et al. 2016; Pei et al. 2017; Du et al. 2018a, 2018b; Xiao et al. 2018a, 2018b; Zhang et al. 2019; Feng et al. 2021a, 2021b). RM surveys have been run, such as the OzDES AGN spectroscopic RM project (King et al. 2015; Hoormann et al. 2019) and the Sloan Digital Sky Survey (SDSS) spectroscopic RM project (Shen et al. 2015a, 2015b, 2016; Grier et al. 2017; Shen et al. 2019). The single-epoch spectrum was widely used to estimate M_{RM} for the SDSS quasars (e.g., Hu et al. 2008; Liu et al. 2019) and the high-z quasars (e.g., Willott et al. 2010; Wu et al. 2015; Wang et al. 2019). However, the virial factor is very uncertain owing to the unclear kinematics and geometry of the BLR (e.g., Peterson et al. 2004; Woo et al. 2015). Radiation pressure of the accretion disk has significant influence on the BLR clouds and the dynamics of clouds (e.g., Marconi et al. 2008; Netzer & Marziani 2010; Krause et al. 2011, 2012; Naddaf et al. 2021). The dynamics of clouds can determine the three-dimensional
Thus, measuring the geometry of the BLR (Naddaf et al. 2021). Thus, radiation pressure may be a contributor to the virial factor. Radiation pressure was not considered in estimating $M_{\text{RM}}$. Averages of $f \approx 1$ and/or $f_s \approx 5$ were derived based on the $M_\text{vir} - \sigma_*$ relation for the low-$z$ inactive and quiescent galaxies, where $\sigma_*$ is the stellar velocity dispersion of the galaxy bulge (e.g., Tremaine et al. 2002; Onken et al. 2004; Piotrovich et al. 2015; Woo et al. 2015). $f \approx 1$ and/or $f_s \approx 5$ were usually used to estimate $M_{\text{RM}}$ by the RM and/or the single-epoch spectra of AGNs. Thus, measuring $f$ and/or $f_s$ independently by a new method for individual AGNs is necessary and important to understand the physics of the BLR and the issues related to black hole masses.

Liu et al. (2017) proposed a new method to measure $f$ based on the widths and shifts of redward-shifted broad emission lines for the RM AGNs. The Fe III $\lambda\lambda2039$–2113 UV line blend arises from an inner region of the BLR (Mediavilla et al. 2018), and a lot of evidence shows that the UV blend originates close to the SMBH (e.g., Mediavilla & Jiménez-Vicente 2021). Large values of $f$ are obtained from the widths and redward shifts of these UV blends, and an average of $\langle f_{\text{Fe III}} \rangle = 14.3 \pm 2.4$ is derived for 10 lensed quasars with a mean Eddington ratio of $\sim 0.8$ (Mediavilla et al. 2020). This average value is much larger than the widely accepted one of $f \approx 1$. However, the origin of redward shifts of broad emission lines is unclear, because the origin of broad emission lines in AGNs is not yet clear (e.g., Wang et al. 2017). A tight correlation between broadening and redward shift of the Fe III $\lambda\lambda2039$–2113 blend for quasars in the BOSS survey supports the gravitational interpretation of its redward shift (Mediavilla et al. 2018).

Alternative explanations, such as inflow, will need additional physics to explain the observed trend between broadening and redshift (Mediavilla et al. 2018). The redward shifts of the rms profiles of broad emission lines with respect to narrow emission lines and the BLR radii in Mrk 110 follow the gravitational redshift prediction (see Figure 3 in Kollatschny 2003). The velocity-resolved time lags of H$\beta$ broad emission lines for Mrk 50 and SBS 1518+593 show characteristics of a Keplerian disk or virialized motion (Barth et al. 2011; Du et al. 2018a). A sign of the gravitational redshift $z_g$ was found in a statistical sense for broad H$\beta$ in the single-epoch spectra of SDSS DR7 quasars (Tremaine et al. 2014).

Hu et al. (2008) suggested that inflow may generate the redward shifts of broad emission lines. Absorption lines that are redshifted with respect to the quasar’s systemic velocity are an unambiguous signature of inflow (Rubin 2017, p. 95). Inflow generates the redward shifts of broad absorption lines relative to the quasar’s systemic velocity determined from narrow emission lines, and the broad absorption and emission lines may be from different gas regions owing to their distinct velocities (Zhou et al. 2019). Redward shifts of broad emission lines of H$_\gamma$, H$\beta$, and He I lines for Mrk 817 seemingly have an origin of outflow that is denoted by the redward asymmetric velocity-resolved lag profiles of these lines (Lu et al. 2021). However, the redward shifts of broad emission lines are commonly believed to be from inflow that will lead to the blueward asymmetric velocity-resolved lag profiles. This discrepancy implies that the redward shifts of broad emission lines do not originate from inflow. For each RM observation cycle of NGC 5548, Lu et al. (2016) found that the variations of average 5100 Å luminosity lead the changes of $t_\text{BR}$ by $\tau_{r-L} = 2.35$ yr, which is consistent with a dynamical timescale of $t_{\text{BLR}} \approx 2.10$ yr for the BLR, and they obtained that the BLR could be jointly controlled by the radiation pressure of the accretion disk and the central black hole gravity. Krause et al. (2011) found that stable orbits of clouds in the BLR exist for very sub-Keplerian rotation, for which the radiation pressure force contributes substantially to the force budget. Thus, the radiation pressure force might result in significant influence on the virial factor. In this work, SDSS DR5 quasars with redward-shifted H$\beta$ and Fe II broad emission lines (see Table 2 in Hu et al. 2008) are used to investigate the virial factor, relations between the virial factor and other physical quantities for these quasars, and the origin of the redward shift of the H$\beta$ broad emission line.

The structure is as follows. Section 2 presents the method. Section 3 describes the sample selection. Section 4 presents a discussion and conclusions. Throughout this paper, we assume a standard cosmology with $H_0 = 70$ km s$^{-1}$Mpc$^{-1}$, $\Omega_M = 0.3$, and $\Omega_\Lambda = 0.7$ (Spergel et al. 2007).

2. Method

A BLR cloud is subject to the gravity of a black hole, $F_g$, and radiation pressure force, due to central continuum radiation. Under the resultant force of $F = F_g + F_r$, the total mechanical energy and angular momentum are conserved for the BLR clouds because $F_g$ and $F_r$ are central forces. Under various assumptions, $F_r$ can be calculated for more than hundreds of thousands of lines, with detailed photoionization, radiative transfer, and energy balance calculations (e.g., Dannen et al. 2019). In principle, $M$ could be estimated by the BLR cloud motions, as the numerical calculation methods give $F_r$. However, the various assumptions may significantly influence the reliability of $F_r$. Especially, many unknown physical parameters are likely various for different AGNs. Thus, a new method was proposed to measure $f$ when avoiding the numerical calculation of $F_r$ (Liu et al. 2017).

The virial factor formula in Liu et al. (2017) was derived from the Schwarzschild metric for a static cloud. In fact, the BLR clouds are not static, e.g., in the virialized motion. The gravitational redshift in the Schwarzschild spacetime for the BLR clouds can be expressed as (see Equation (12) in Chakraborty & Bhattacharyya 2018)

$$z_g = \left(1 - \frac{3GM}{c^2 r_{\text{BLR}}}\right)^{-1/2} - 1,$$

where the gravitational and transverse Doppler shifts are taken into account. $M$ is estimated as

$$M = \frac{1}{3} G^{-1} c^2 r_{\text{BLR}} [1 - (1 + z_g)^{-2}],$$

and the first-order approximation is

$$M = \frac{2}{3} G^{-1} c^2 z_g r_{\text{BLR}}.$$  

as $z_g \ll 1$ or $r_g/r_{\text{BLR}} \ll 1$ for optical broad emission lines (the gravitational radius $r_g = GM/c^2$).

Here Equation (3) is the same as Equation (3) of Mediavilla et al. (2018), in which the weak-field limit of the Schwarzschild metric was assumed. At the optical BLR scales, the Schwarzschild metric is valid and matches the weak-field limit. Equation (1) is valid for a disklike BLR (see Chakraborty & Bhattacharyya 2018). The disklike BLR is preferred by some
RM observations of AGNs, e.g., NGC 3516 (e.g., Denney et al. 2010; Feng et al. 2021a), and the VLTI instrument GRAVITY observations of quasar 3C 273 (Sturm et al. 2018). For rapidly rotating BLR clouds, the relativistic beaming effect can give rise to a profile asymmetry with an enhanced blue side in broad emission lines, i.e., blueshifts of broad emission lines (Mediavilla & Insertis 1989). Thus, the relativistic beaming effect should be neglected for the redward-shifted broad emission lines, which should be dominated by the gravitational redshift and transverse Doppler effects. The factor of 2/3 in Equation (3) results from correcting the transverse Doppler shift, which is essentially that the moving clock becomes slower. The factor of 2/3 does not appear in the formulae used to estimate $M_\ast$, in Kollatschny (2003) and Liu et al. (2017) because they did not consider the transverse Doppler shift. If $M_\ast$ estimated with Equation (2) is equal to $M_{\text{RM}}$, we have the virial factor

$$f = \frac{1}{3} \frac{c^2}{v_{\text{FWHM}}^2} \left[1 - (1 + z_r)^{-2}\right]. \quad (4)$$

If $v_{\text{FWHM}}$ is replaced with $\sigma_{\text{line}}$, $f$ becomes $f_r$.

Because $r_{\text{BLR}} \gg r_g$ for optical broad emission lines, Equation (1) can give the multi-broad-line approach of measuring $M_\ast$ as

$$M_\ast \approx \frac{2}{3} G^{-1} c^2 \Delta z \left(\frac{1}{r_{\text{BLR},i}^{-1} - 1} - \frac{1}{r_{\text{BLR},j}^{-1}}\right)^{-1}, \quad (5)$$

where $\Delta z_{ij} = z_i - z_j = z_{g,i} - z_{g,j}$ is the redshift difference between the broad lines $i$ and $j$ with the relevant BLR radius $r_{\text{BLR},i}$ and $r_{\text{BLR},j}$. Here Equations (2), (3), (4), and (5) have a factor of 2/3 more than Equations (4), (5), (9), and (7) derived in Liu et al. (2017), respectively. The reliability of the redward shift method was confirmed by the consistent masses estimated from their Equations (4) and (7) based on four broad emission lines for Mrk 110 (Liu et al. 2017). Thus, Equations (2) and (5) in this work can also give consistent black hole masses for Mrk 110. The RM observations of multiple broad emission lines for AGNs might further test the reliability of this method, based on Equations (2) and (5). Hereafter, $M_{\text{RM}}$ denotes $M_\ast$ measured with the RM method and/or the relevant secondary methods, and $f_{g,i}$ denotes the virial factor that comes from the geometric effect of the BLR.

### 3. Sample Selection

Hu et al. (2008) reported a systematical investigation of optical Fe II emission in a large sample of 4037 $z < 0.8$ quasars selected from the SDSS DR5, for which they had developed and tested a detailed line-fitting technique, taking into account the complex continuum and narrow and broad emission line spectra. The line widths and redward velocity shifts of the Fe II and H$\beta$ spectra are given in Table 2 of Hu et al. (2008). On the basis of $\Delta v - \sigma(\Delta v) > 0$, where $\Delta v$ is the redward velocity shift for the H$\beta$ and Fe II broad emission lines (i.e., $\Delta v > 0$) and $\sigma(\Delta v)$ is the error of $\Delta v$, 1973 quasars are selected out of these 4037 quasars as our sample. This selection condition makes sure that the velocity shift is larger than zero within 1σ uncertainties. If $\Delta v - \sigma(\Delta v) \leqslant 0$, it is possible that the velocity shift is redshift, blueshift, or no shift. Thus, the redward velocity shift seems much less reliable if $\Delta v - \sigma(\Delta v) \leqslant 0$, and this selection condition of $\Delta v - \sigma(\Delta v) > 0$ seems reasonable.

Because the empirical $r_{\text{BLR}}-L_{5100}$ relation is established for broad emission line H$\beta$, the relevant research of the virial factor is made mainly with the broad H$\beta$ line. Some physical quantities are taken from Table 2 in Hu et al. (2008), including the cosmological redshift of the source ($z$), $v_{\text{FWHM}}(H\beta)$, $\sigma_{\text{line}}(H\beta)$, the redward velocity shift of broad H$\beta$ ($\Delta v(H\beta)$), $v_{\text{FWHM}}($Fe II$)$, $\Delta v($Fe II$)$, $L_{5100}$, the black hole mass, the Eddington ratio, and the line ratio of Fe II to H$\beta$. The bolometric luminosity in the Eddington ratio was estimated in Hu et al. (2008) using $L_{\text{bol}} = 9L_{5100}$ (Kaspi et al. 2000). The details of sample are listed in Table 1. The virial factor is estimated by Equation (4), and the relevant values for the H$\beta$ and Fe II broad emission lines are listed in Table 1. The dimensionless accretion rate $\dot{M}_{\ast} = L_{\text{bol}}/L_{\text{Edd}}(f_{g,i})/\eta$, where $\eta$ is the efficiency of converting rest-mass energy to radiation, $L_{\text{bol}}$ is the bolometric luminosity, $L_{\text{Edd}}$ is the Eddington luminosity, $f_{g,i} = 1$ for $v_{\text{FWHM}}$, and $f_g = 5.5$ for $\sigma_{\text{line}}$. Here we adopt $\eta = 0.038$ (Du et al. 2015).

### 4. Analysis and Results

The Spearman rank correlation test shows that the virial factor is positively correlated with the dimensionless accretion rate $\dot{M}_{\ast} = 5.5$ for these 1973 quasars (see Figure 1 and Table 2). The virial factor and $\dot{M}_{\ast} = 5.5$ are related to the line width, and this line width dependency may result in a false correlation between them. The partial correlation analysis gives a confidence level of 99.99% for the positive correlation of log$f_{g,i}$–log$\dot{M}_{\ast} = 5.5$ when excluding the dependence on the line width $\sigma_{\text{line}}$. Since the virial factor may be affected by $F_\beta$, it is possible that the virial factor is correlated with $L_{5100}$. Hence, we analyze the virial factor and $L_{5100}$ and find no correlation between them (see Figure 2). Thus, the positive correlation exists between the virial factor and $\dot{M}_{\ast} = 5.5$. This positive correlation is qualitatively consistent with the logical expectation when the overall effect of $F_\beta$ on the BLR clouds is taken into account to estimate $M_{\text{RM}}$. In addition, $f_{g,i} > f_g = 5.5$ and $f_{g,i} > f_g = 1$ for H$\beta$ in most of the quasars (see Figure 1).

In order to test the gravitational origin of the redward velocity shift of the broad emission line, we compare $\Delta v(H\beta)$ to $r_{\text{BLR}}/r_g(f_g = 5.5)$, the BLR radius in units of the gravitational radius of the black hole. The Spearman rank correlation test shows negative correlation between the velocity shift and $r_{\text{BLR}}/r_g(f_g = 5.5)$ (see Figure 3(a) and Table 2). This negative correlation is qualitatively consistent with the expectation when $\Delta v(H\beta)$ is mainly from the gravity of the central black hole. The values of $r_{\text{BLR}}/r_g(f_g = 5.5)$ in Figure 3(a) are estimated based on the uncorrected $M_{\text{RM}}(f_g = 5.5)$. However, $M_{\text{RM}}(f_g = 5.5)$ could not be corrected individually for each quasar owing to the absence of the different individual virial factor that is independent of $\Delta v(H\beta)$. The overall correlation of $M_{\text{RM}}(f_g = 5.5)$ for these 1973 quasars can be made by a factor of 3.4 derived from $f_g = 5.5$ and an average of $f_g = 18.5$ presented in Figure 1(a) (see Figure 3(b)). This overall correlation is equivalent to the overall parallel shift of the data in Figure 3(a). Figure 3(b) shows that the negative correlation expectation is basically consistent with the trend between $\Delta v(H\beta)$ and the corrected $r_{\text{BLR}}/r_g(f_g = 5.5)$. This indicates that $\Delta v(H\beta)$ is dominated by the gravity of the central black hole. In addition, $r_g/r_{\text{BLR}} \lesssim 0.01 \leqslant 1$ for the $x$-axis values in
Table 1
The Relevant Parameters for 1973 Quasars in SDSS DR5

| Designation | z   | $v_{\text{FWHM}}(\text{H}\beta)$ | $\Delta v(\text{H}\beta)$ | $v_{\text{FWHM}}(\text{Fe II})$ | $\Delta v(\text{Fe II})$ | $\log L$ | $M_\text{BH}$ | $L_\text{edd}$ | $R_\text{Fe}$ | $f(\text{H}\beta)$ | $f(\text{Fe II})$ | $f(\text{Fe II})/f(\text{H}\beta)$ | $\log \dot{M}/\dot{M}_{\text{Edd}}$ | $r_{\text{BLR}}$ |
|-------------|-----|-------------------------------|--------------------------|--------------------------------|--------------------------|----------|-------------|----------------|-------------|----------------|----------------|-----------------------------------|-------------------|-------------|
| 000011.96-000225.3 | 0.4784 | 3135.8 ± 62.6 | 1893.5 | 542.3 ± 35.4 | 1898.6 ± 74.1 | 387.6 ± 38.4 | 44.74 | 28.1 | 0.140 | 1.318 | 11.0 ± 0.8 | 30.2 ± 3.7 | 21.5 ± 2.7 | 0.57 | 4535.0 |
| 001111.9-002011.5 | 0.5173 | 3666.5 ± 125.3 | 2267.1 | 486.9 ± 66.3 | 2496.5 ± 406.2 | 1082.1 ± 156.9 | 44.60 | 32.2 | 0.088 | 0.609 | 7.2 ± 1.1 | 18.9 ± 3.2 | 34.5 ± 15.9 | 0.37 | 3159.8 |
| 000131.42+144610.6 | 0.5309 | 5054.5 ± 306.4 | 2299.3 | 481.9 ± 103.9 | 3359.2 ± 658.1 | 1595.2 ± 214.5 | 44.44 | 25.8 | 0.077 | 0.640 | 3.8 ± 0.9 | 18.2 ± 4.4 | 28.0 ± 17.1 | 0.31 | 3069.8 |

Note. Column (1): object name. Column (2): redshift. Column (3): $v_{\text{FWHM}}$ of $\text{H}\beta$ broad emission line. Column (4): $\sigma_{\text{line}}$ of $\text{H}\beta$ broad emission line. Column (5): the redward velocity shift of $\text{H}\beta$. Column (6): $v_{\text{FWHM}}$ of $\text{Fe II}$ broad emission line. Column (7): the redward velocity shift of $\text{Fe II}$. Column (8): $\log L(5100 \text{ Å})$ in units of erg s$^{-1}$. Column (9): the black hole mass. Column (10): the Eddington ratio. Column (11): the $\text{Fe II}/\text{H}\beta$ line ratio. Column (12): the virial factor estimated from $v_{\text{FWHM}}$ of $\text{H}\beta$. Column (13): the virial factor estimated from $\sigma_{\text{line}}$ of $\text{H}\beta$. Column (14): the virial factor estimated from $v_{\text{FWHM}}$ of $\text{Fe II}$. Column (15): logarithm of $\dot{M}/\dot{M}_{\text{Edd}}$. Column (16): $r_{\text{BLR}}$ in units of $r_g$, where $r_{\text{BLR}} = 22.3L_\odot^{0.49}$ light-days with $L_\odot = L(5100 \text{ Å})/(10^{44} \text{ erg s}^{-1})$. Columns (2)-(11) are taken from Table 2 of Hu et al. (2008) or converted from the relevant quantities in Table 2 of Hu et al. (2008). (This table is available in its entirety in machine-readable form.)
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Figure 1. (a) Hβ-σmea-based fσ vs. fσ. The Spearman test shows a positive correlation between these two physical quantities, which have averages corresponding to the red filled circle. The dashed line denotes fσ = 5.5 for σmea. (b) Hβ-σFWHM-based fσ vs. fσ. The Spearman test shows a positive correlation between these two physical quantities, which have averages corresponding to the red filled circle. The dashed line denotes fσ = 1 for σFWHM.

Table 2
Spearman’s Rank Analysis Results

| X                  | Y                  | r_s | P_s  |
|--------------------|--------------------|-----|------|
| F(Hβ) or Lbol/Ledd | fσ                | 0.54| <10^-6 |
| F(Hβ) or Lbol/Ledd | fσ                | 0.61| <10^-6 |
| L(5100 Å)          | fσ                | 0.02| 0.41 |
| L(5100 Å)          | fσ                | 0.03| 0.14 |
| rBLR/rβfσ = 5.5    | Δν/σ               | 0.34| <10^-6 |
| Δν/σ               | (fmea/σ)²          |     |      |
| R(Fe II)/Hβ        | fσ                | 0.43|      |
| F(Hβ) or Lbol/Ledd | R(Fe II)/Hβ        | 0.54| <10^-6 |

Note. X and Y are the relevant quantities presented in Figures 1–6. The SPEAR (Press et al. 1992) gives r_s and P_s (the Spearman rank correlation coefficient and the p-value of the hypothesis test, respectively).

Figures 3(a) and (b) and the values of rσ/rBLR corrected by fσ in Table 1. At these optical BLR scales of quasars, the Schwarzschild metric is valid and still matches the weak-field limit. The RM research of AGNs may shed light on the origin of Δν(Hβ), and the relevant discussion is presented in the next section.

The virial factor of Fe II is larger than that of Hβ for 98% of these quasars (see Figure 4). In addition, Figure 4 shows that the virial factor is very different from object to object and for different emission lines. If the stratified photoionization found for broad emission lines is prevalent in AGNs, the optimized photoionization zones in BLRs will be different for different lines. For clouds at a given radius, Fσ is the resultant force from the different ions with negligible drift velocities between the gas constituents within these clouds, which generate a prominent broad emission line component. In addition, the typical size of clouds of the BLR is much less than the extent of the BLR. Hence, Fσ on clouds at different radii is different, thus potentially resulting in different virial factors. Fe II may be from a region outside the BLR of Hβ, i.e., rBLR(Fe II) > rBLR(Hβ) (e.g., Hu et al. 2008, and references therein). The RM of quasar 3C 273 showed rBLR(Fe II) > rBLR(Hβ) (Zhao et al. 2019). For broad emission lines with different rBLR, there will be f ∝ rBLR (α > 0), as Fσ is considered in the virialized motion for a given AGN (Liu et al. 2017). If rBLR(Fe II) > rBLR(Hβ) and f ∝ rBLR, it will be expected that the measured virial factor of Fe II is larger than that of Hβ for most of the quasars in our sample.

Netzer & Trakhtenbrot (2007) have suggested that RBLR is a BLR metallicity indicator for SDSS type 1 AGNs. Panda et al. (2018, 2019) have suggested that RBLR is associated with the BLR metallicity. RBLR increases with increasing metallicity. The BLR metallicity can influence Fσ owing to the line-driven force dominated by the metallic elements (e.g., Ferland et al. 2009; Dannen et al. 2019). Thus, RBLR may influence the virial factor. Three positive correlations exist among fσ, RBLR, and F(Hβ) (see Table 2 and Figure 5). Since three correlations exist among them, there should be a correlation in the form of fσ (F(Hβ), RBLR). In fact, there is a positive correlation at the confidence level of >99.99%, log fσ = −0.41 + 0.11 log F(Hβ) + 0.28 log RBLR. Thus, Fσ is dominated by RBLR and F(Hβ) (or the Eddington ratio). This should be understood that Fσ exerted on the BLR clouds will be larger as the BLR metallicity is higher and/or the radiation of the accretion disk is stronger. Thus, the observed envelope delineating the data should be a consequence of physical effects, such as the Doppler effects, the gravitational redshift, and the line-driven force, which depend on the black hole mass, the bolometric luminosity of the black hole, and the BLR metallicity.

5. Discussion and Conclusions

As z_s ≪ 1, Equation (4) can give for σmea, fσ, and z_s = Δν/c

$$\log\left(\frac{\sigma_{\text{line}}}{c}\right)^2 = -\log\left(\frac{3}{2}f_\sigma\right) + \log\left(\frac{\Delta\nu}{c}\right).$$

(6)

which is similar to Equation (6) in Mediavilla et al. (2018), where a tight correlation was found between the widths and redward shifts of the Fe III λ2039–2113 blend for their quasars, and this correlation supports the gravitational interpretation of the Fe III λ2039–2113 redward shifts. The Spearman rank correlation test shows a positive correlation between the line width and velocity shift of the Hβ line for these 1973 quasars (see Table 2). A series of lines based on Equation (6) with different fσ are compared to the observational data points (see Figure 6). From top to bottom, the corresponding fσ increases. Because of the codependence between the Eddington ratio, dimensionless accretion rate, and σmea, the large ranges of the former two quantities may lead to
the large span in the direction roughly perpendicular to these lines (see Figure 6). In addition, the metallicity difference of the BLR might decrease correlations in Figure 1. Microturbulence within the BLR clouds can act as an apparent metallicity controller for the Fe II, and the reduction in the value of the metallicity can be up to a factor of 10 for the Fe II/Hβ line ratio.
redward asymmetric lag maps. However, the asymmetric lag maps and shifts of broad emission lines for AGNs usually differ from the expectations of inflow, such as 3C 273 (e.g., Zhang et al. 2019), PG 0026+129 (Hu et al. 2020), NGC 3516 (e.g., Denney et al. 2010; Feng et al. 2021a), and NGC 2617 (e.g., Feng et al. 2021b). The redward-shifted broad emission lines with the blueward asymmetric lag maps might be generated by an elliptical disklike BLR or a circular disklike BLR plus a spiral-arm-like BLR (Feng et al. 2021a). Eccentricities and orientations of cloud orbits significantly influence the full two-dimensional transfer function (2DTF) of a single disklike BLR (see Figure 3 in Kovačević et al. 2020), and the redward-shifted broad emission lines with various lag maps may originate from the clouds in viral motion with various asymmetric responses in 2DTF. Virialized BLRs are suggested by the symmetric lag maps of redward-shifted broad emission lines for SBS 1116 + 583A (Bentz et al. 2009), Mrk 50 (Barth et al. 2011), and SBS 1518 + 593 (Du et al. 2018a). Therefore, the redward-shifted broad emission lines in AGNs do not necessarily originate from inflow.

These 2485 quasars with $\Delta v(H\beta) > 0$ in Table 2 of Hu et al. (2008) have a median of $\Delta v(H\beta) = 190$ km s$^{-1}$ with a typical error of 55 km s$^{-1}$ and an average and standard deviation of $\langle \Delta v(H\beta) \rangle = 238 \pm 67$ km s$^{-1}$. A total of 1973 quasars in our sample have $\Delta v(H\beta) = 242 \pm 52$ km s$^{-1}$ and $\langle \Delta v \rangle (H\beta) = 281 \pm 63$ km s$^{-1}$. Considering uncertainties, these distributions of $\Delta v(H\beta)$ do not seem obviously different. The relevant distributions of $\Delta v(H\beta)$ are presented in Figure 7(a).

Sources with smaller $\Delta v(H\beta)$ are more likely excluded by $\Delta v - \sigma(\Delta v) > 0$, which means that the relative error of $\Delta v$ is smaller than 1. The relative error distributions in Figure 7(b) for the 1973 and 2485 quasar samples show that the relative error is more likely larger for the smaller $\Delta v$. Correlation analyses for 2485 quasars show, at the confidence level of $>99.99\%$, three positive correlations among $f_\beta$, $R_{\text{Edd}}$, and $A_{f_\beta=5.5}$, a positive correlation in the form of $f_\beta(A_{f_\beta=5.5}, R_{\text{Edd}})$, and a negative correlation between $\Delta v(H\beta)$ and $r_{\text{BLR}}/r_{(f_\beta=5.5)}$. These results indicate that correlations found for 1973 quasars do not originate from the selection effect, i.e., $\Delta v - \sigma(\Delta v) > 0$ will not result in illusory correlations, though this condition will make $\Delta v$ larger for the selected quasars. The fraction of blueshifted broad-line H$\beta$ with $\Delta v + 3\sigma(\Delta v) < 0$ is about 14% for quasars in Hu et al. (2008), and these blueshifted quasars may be explained by additional blueshift of a kinematic origin arising from radial motion, e.g., outflows. Outflows seem to exist even if AGNs are in their low-flux states (e.g., Mehdiour et al. 2022). A larger quasar sample, e.g., SDSS DR7 quasars in Liu et al. (2019), who gave the detailed parameters of spectra, will be used in the next work.

The radiative efficiency is closely related to a black hole spin, but it is difficult to measure the spin of the black hole in the AGN. Usually, the Eddington ratio is regarded as a proxy of the accretion rate of a black hole. Even though these correlations of the dimensionless accretion rate with the other physical quantities are likely influenced by the unknown real individual value of radiative efficiency, there are still correlations of the Eddington ratio with these physical quantities, because only a difference of 0.038 exists between $A_{f_\beta=5.5}$ and $f_{\text{sed}}/L_{\text{Edd}}$ in Table 1. Based on $L_{\text{Edd}}$ and the mass accretion rate, Davis & Laor (2011) determined $\eta$ for a sample of 80 Palomar-Green quasars and found a strong correlation of $\eta = 0.089M_\bullet^{0.52}$, where $M_\bullet$ is the black hole mass in units of $10^8 M_\odot$. In order to test the influence of a fixed radiative
Figure 6. log(\(\sigma_{\text{line}}/c\))^2 vs. \(\Delta v/c\) for the H\(\beta\) line. The values labeled on the dashed lines represent \(f_\sigma\) in Equation (6).

Figure 7. Distributions of \(\Delta v(\text{H}\beta)\) and its relative error. (a) The solid lines are the distributions for 2485 quasars with \(\Delta v(\text{H}\beta) > 0\) in Hu et al. (2008). The dashed lines are the distributions for 1973 quasars in our sample. (b) Distributions of the relative error of \(\Delta v(\text{H}\beta)\).

Figure 8. (a) H\(\beta\)--\(\sigma_{\text{line}}\)-based \(f_\sigma\) vs. \(\mathcal{M}_{f=5.5}\). The Spearman test shows a positive correlation between these two physical quantities. (b) H\(\beta\)--\(v_{\text{FWHM}}\)-based \(f\) vs. \(\mathcal{M}_{f=5.5}\). The Spearman test shows a positive correlation between these two physical quantities. \(\mathcal{M}_{f=5.5}\) is the value estimated by \(\eta = 0.089 \mathcal{M}_{\odot}\) rather than \(\eta = 0.038\).
efficiency $\eta = 0.038$, this empirical relation is used to estimate $\eta$. Correlation analyses are made for those quantities in Figures 1 and 5 with $H_f = 5.5$ to be reestimated by $L_{\text{red}}/L_{\text{edd}}$ in Table 1 and the estimated $\eta$. There are still correlations very different from object to object and for different emission lines $f$, basically reproduce the distribution of $\sigma_{\text{line}}$, $\Delta v$ for the Hβ line (see Figure 4). A series of lines, based on Equation (6) with different $f_r$, basically reproduce the distribution of $(\sigma_{\text{line}}, \Delta v)$ for the Hβ line (see Figure 6), supporting the gravitational interpretation of $\Delta v$ for the Hβ line. There are three positive correlations among $f_r, \Delta v$, $H_f = 5.5$, and $R_{\text{edd}}$. A correlation, $log f_r = -0.41 + 0.11 \log H_f = 5.5 + 0.28 \log R_{\text{edd}}$, indicates that the virial factor is dominated by $H_f = 5.5$ and metallicity, which will influence $F_r$ on the BLR clouds. $\Delta v(\text{H}\beta)$ is anticorrelated with $r_{\text{BLR}}/r_{\text{edd}} = 5.5$, supporting the gravitational origin of the redward shift of Hβ. Our results indicate that $F_r$ may be an important contributor to the virial factor, and the redward-shifted broad emission lines show the potential of measuring $\eta$. We are very grateful to the anonymous referees for constructive comments leading to significant improvement of this paper. We thank the helpful discussions of Prof. J. R. Mao. We thank the financial support of the National Key R&D Program of China with grant No. 2021YFA1600404, and the National Natural Science Foundation of China (NSFC; grant No. 11991051). We acknowledge the science research grants from the China Manned Space Project with grant No. CMS-CSST-2021-A06.

References

Barth, A. J., Bennert, V. N., Canalizo, G., et al. 2015, ApJS, 217, 26
Barth, A. J., Pancoast, A., Thorman, S. J., et al. 2011, ApJL, 743, L4
Bentz, M. C., Denney, K. D., Cackett, E. M., et al. 2006, ApJ, 651, 775
Bentz, M. C., Denney, K. D., Grier, C. J., et al. 2013, ApJ, 767, 149
Bentz, M. C., Walsh, J. L., Barth, A. J., et al. 2009, ApJ, 705, 199
Bentz, M. C., Walsh, J. L., Barth, A. J., et al. 2010, ApJ, 716, 993
Blandford, R. D., & McKee, C. F. 1982, ApJ, 255, 419
Chakraborty, C., & Bhattacharyya, S. 2018, PrRvD, 98, 3021
Condon, J. J., Hutchings, J. B., & Gower, A. C. 1985, AJ, 90, 1642
Dannen, R. C., Proga, D., Kallman, T. R., & Waters, T. 2019, ApJ, 882, 99
Davis, S. W., & Laor, A. 2011, ApJ, 728, 10
Denney, K. D., Peterson, B. M., Pogge, R. W., et al. 2010, ApJ, 721, 715
Du, P., Brotherton, M. S., Wang, K., et al. 2018a, ApJL, 869, 142
Du, P., Hu, C., Lu, K. X., et al. 2014, ApJ, 782, 45
Du, P., Hu, C., Lu, K. X., et al. 2015, ApJ, 806, 22
Du, P., Lu, K. X., Hu, C., et al. 2016, ApJ, 820, 27
Du, P., Zhang, Z. X., Wang, K., et al. 2018b, ApJ, 856, 6
Du, P., & Wang, J. M. 2019, ApJ, 886, 42
Dyda, S., & Proga, D. 2018, MNRAS, 481, 5263
Feng, H. C., Hu, C., Li, S. S., et al. 2021a, ApJ, 909, 18
Feng, H. C., Liu, H. T., Bai, J. M., et al. 2021b, ApJ, 912, 92
Ferland, G. J., Hu, C., Wang, J. M., et al. 2009, ApJL, 707, L82
Grier, C. J., Trump, J. R., Shen, Y., et al. 2017, ApJ, 851, 21
Haas, M., Chini, R., Ramolla, M., et al. 2011, A&A, 535, A73
Hoormann, J. K., Martini, P., Davis, T. M., et al. 2019, MNRAS, 487, 3650
Hu, C., Du, P., Lu, K. X., et al. 2015, ApJ, 804, 138
Hu, C., Li, S. S., Guo, W. J., et al. 2020, ApJ, 905, 75
Hu, C., Wang, J. M., Ho, L. C., et al. 2008, ApJ, 687, 78
Kang, D., & Woo, J. H. 2018, ApJ, 864, 124
Kaspi, S., Brandt, W. N., Maoz, D., et al. 2007, ApJ, 659, 997
Kaspi, S., & Netzer, H. 1999, ApJ, 524, 71
Kaspi, S., Smith, P. S., Netzer, H., et al. 2000, ApJ, 533, 631
King, A. L., Martini, P., Davis, T. M., et al. 2015, MNRAS, 453, 1701
Kollatschny, W. 2003, A&A, 422, L61
Kormendy, J., & Ho, L. C. 2013, ARARA, 51, 511
Kovačević, A. B., Wang, J.-M., & Popović, L. Ć. 2020, A&A, 635, A1
Krause, M., Burkert, A., & Schartmann, M. 2011, MNRAS, 411, 550
Krause, M., Schartmann, M., & Burkert, A. 2012, MNRAS, 425, 3172
Li, H. T., Feng, H. C., & Bai, J. M. 2017, MNRAS, 466, 3323
Li, H. Y., Liu, W. J., Dong, X. B., et al. 2019, ApJS, 243, 21
Liu, H. Y., Liu, W. J., Dong, X. B., et al. 2019, ApJS, 243, 21
Liu, X. D., Du, P., Hu, C., et al. 2016, ApJ, 821, 118
Liu, K. X., Wang, J. G., Zhang, Z. X., et al. 2021, ApJ, 918, 50
Marconi, A., Axon, D. J., Maiolino, R., et al. 2008, ApJ, 678, 693
Mas-Ribas, L., & Mauclard, R. 2019, ApJ, 886, 151
Mediavilla, E., & Insertis, F. M. 1989, A&A, 214, 79
Mediavilla, E., & Jiménez-Vicente, J. 2021, ApJ, 914, 112
Mediavilla, E., Jiménez-Vicente, J., Fian, C., et al. 2018, ApJ, 862, 104
Mediavilla, E., Jiménez-Vicente, J., Mejía-Restrepo, J., et al. 2020, ApJ, 895, 111
Meena, B., Crenshaw, D. M., Schmitt, H. R., et al. 2021, ApJ, 916, 31
Meidipour, M., Kriss, G. A., Brenneman, L. W., et al. 2022, ApJ, 925, 84
Nadaf, M. H., Czerny, B., & Szczepka, R. 2021, ApJ, 920, 30
Netzer, H., & Marziani, P. 2010, ApJ, 728, 318
Netzer, H., & Trakhtenbrot, B. 2007, ApJ, 654, 754
Nomura, M., Ohuga, K., & Done, C. 2020, MNRAS, 494, 3616
Onken, C. A., Ferrarese, L., Merritt, D., et al. 2004, ApJ, 615, 645
Panda, S. 2021, A&A, 650, A154
Panda, S., Czerny, B., Adhikari, T. P., et al. 2018, ApJ, 866, 115
Panda, S., Czerny, B., Done, C., & Kubota, A. 2019, ApJ, 875, 133
Pei, L., Barth, A. J., Aldering, G. S., et al. 2014, ApJ, 795, 38
