Inflation models after WMAP year three

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Abstract. The survey of models in astro-ph/0510441 is updated. For the first
time, a large fraction of the models is ruled out at more than 3σ.

Keywords: CMBR experiments, inflation

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1. Introduction

In a recent paper [1] we discussed a range of models for the origin of the curvature perturbation and the tensor perturbation, including constraints on the spectral index $n$ coming from WMAP year one data. In this note we update the discussion to include WMAP year three data [2]. The models assume that the curvature perturbation is generated from the vacuum fluctuation of the inflaton field, so that it is directly related to the inflationary potential\(^1\). Some of them work with the type of field theory that is usually invoked when considering extensions of the Standard Model, while others work within a framework derived more or less directly from string theory.

The prediction for $n$ typically depends on $N$, the number of $e$-folds of slow-roll inflation occurring after the observable Universe leaves the horizon. With a high inflation scale, and radiation and/or matter domination between the end of inflation and nucleosynthesis,

$$N = 54 \pm 7.\tag{1.1}$$

More generally the range has to be

$$14 < N < 75,\tag{1.2}$$

the lower bound coming from the requirement to form early objects weighing a million solar masses, and the upper bound from imposing $P/\rho < 1$ on the pressure and energy density [4].

Following [1], we call a model small-field if the change $\Delta\phi_N$ of the inflaton field during the $N$ $e$-folds is $\lesssim M_P$, and large-field if it is $\gg M_P$.

\(^1\) Models where instead the curvature perturbation is generated from the vacuum fluctuation of some curvaton-type field, and their status after WMAP year three, are considered elsewhere [3].
2. Small-field models

Small-field models predict a negligible tensor fraction $r$. With that constraint, the WMAP constraint is

$$n = 0.948^{+0.015}_{-0.018}.$$  (2.1)

This is actually the constraint obtained by combining WMAP data with the SDSS galaxy survey, but it hardly changes if some other data set is used, including WMAP alone.

Essentially the same constraint was obtained using WMAP year one data, in conjunction with the 2dF galaxy survey alone [5] or with 2dF and other data sets [6], but a higher result compatible with $n = 1$ was obtained using WMAP year one data alone or WMAP with [7] SDSS. The crucial point now is that even in the last two cases the scale-invariant value $n = 1$ is excluded at around the 3σ level.

The value (2.1) assumes negligible running, $n' \equiv d n / d \ln k = 0$. If $n'$ is allowed to float the value of $n$ does not change much, and $n' = 0$ is allowed at the 2-σ level. A fit including Lyman-α data [8] gives

$$n' = -(2.0 \pm 1.2) \times 10^{-2}.$$  (2.2)

2.1. A class of allowed models

For small-field models with a concave-downward potential, $\epsilon \lesssim 0.0002$ [9]. Then the prediction [10] $n = 1 + 2\eta - 6\epsilon$ becomes just $n = 1 + 2\eta$. For a wide class of concave-downward models this becomes [11]

$$n = 1 - \frac{p - 1}{2 - 2N},$$  (2.3)

with $p \gtrsim 3$ or $p \leq 0$. (Here $N$ is actually $N(k) \equiv N - \ln(H_0/k)$, but the variation presumably is negligible over the range $\Delta N \sim 10$ over which the observational constraint applies.) For these models, the observed normalization of the spectrum requires a high inflation scale, so that equation (1.1) will be appropriate for a standard post-inflationary cosmology, but still the tensor fraction is negligible.

The case $p \leq 0$ is realized in some hybrid inflation models in which the potential necessarily steepens significantly towards the end of inflation. The case $p < 0$ corresponds to a potential

$$V \simeq V_0 \left[ 1 - \left( \frac{\phi}{\mu} \right)^p \right],$$  (2.4)

with $V_0$ dominating so as to permit inflation. This can come from mutated hybrid inflation [12,13], with integer values of $p$ favoured but not mandatory. With integral $p$ it can also come from $N = 2$ supergravity [14] or D-brane cosmology [15]. The limit $p \to 0$ corresponds to a logarithmic potential achieved in the simplest and perhaps unrealistic version of $F$-term [16]–[18] or $D$-term [18,19] hybrid inflation. The limit $p \to -\infty$ corresponds to an exponential potential, which may be generated by a kinetic term passing through zero [18] or by appropriate non-Einstein gravity (non-hybrid) inflation [11] (see also [20]).

2 We adopt the definitions [10] $2\epsilon = (M_P V' / V)^2$ and $\eta = M_P^2 V'' / V$, with $V(\phi)$ the inflationary potential.
Figure 1. The prediction (2.3) for different $p$. The bold full line is the limit $|p| \to \infty$. Above it from the top down are the lines $p = 0, -2$ and $-4$, and below it from the bottom up are the lines $p = 3, 4$ and 5. The observational bounds from [2] are indicated.

The case $p \gtrsim 3$ also corresponds to equation (2.4). This case is attractive because it gives the potential a maximum, at which eternal inflation can take place providing the initial condition for the subsequent slow roll [9, 21, 22]. As (2.4) is only supposed to be an approximation lasting for a sufficient number of e-folds, $p$ need not be an integer, but it has to be well above 2 for the prediction (2.3) to hold. It could correspond to non-hybrid inflation with $\Delta \phi \ll M_P$ (New Inflation [23, 24]) or else with $\Delta \phi \sim M_P$ (Modular Inflation which has a long history [25]–[27] and is currently under intense investigation in the context of string theory [28]–[31]). It could also correspond to mutated hybrid inflation [13], or else [9] to one of the $p \leq 0$ models, modified by the addition of a non-renormalizable term.

An attractive proposal which can give equation (2.3) is to make the inflaton a pseudo-Nambu–Goldstone boson so that the flatness of its potential is protected by a symmetry. Realizations of this proposal include a two-component model giving $p = 3$ [32] and a hybrid model giving $p = 0$ [33]. (A different proposal for ensuring the flatness is described in [34] based on earlier work [16, 18], but it has not been carried through to the point where a definite form for the potential is proposed.)

In figure 1, the prediction (2.3) is shown against $N$ for a few values of $p$. Very low values of $N$ are forbidden, as is seen clearly in figure 2. With $N$ in the reasonable range (1.1), the prediction $n(p)$ is shown in figure 3, where we see that all values of $p$ are just about allowed at the $2\sigma$ level.

These cases give a spectral index more or less in agreement with the observed one because their prediction is $n - 1 \simeq -1/N$. The case of equation (2.4) with $p = 2$ is quite different [9, 25, 35]. The tilt is now $n - 1 = -2\mu^2 M_P^4/V$, which might have had any value in the range $-1 \ll n - 1 < 0$. It depends on the parameters of the potential, not just on its functional form as in the previous case.

Now that observation requires such a small tilt, the case $p = 2$ requires a rather abrupt steepening of the potential after cosmological scales leave the horizon. This significantly
restricts the parameter space for non-hybrid models [33, 35]. It can be achieved in an inverted hybrid model [13] by choice of parameters, but this typically involves fine-tuning [36] because of the negative coupling of the inflaton to the waterfall field.

For \( p = 2 \) there is no running of the spectral index, while the prediction (2.3) gives \( n' = (n - 1)/N \simeq 10^{-3} \). The latter is far below the uncertainty of the current constraint, but it might be detected with very accurate cmb polarization measurements [38] combined with other types of data.

### 2.2. Running mass inflation

The idea of running mass inflation [46, 49] is to use a loop correction, to flatten a tree-level potential which would otherwise be too steep for inflation. Supersymmetry is assumed, with a soft mass term.
The original model [46] uses a tree-level quadratic potential (equation (2.5) with \( p = 2 \)) modified by a one-loop correction. In the slow-roll approximation the prediction [47] is 
\[ n - 1 = 2(s - c) \text{ and } n' \equiv dn/d\ln k = 2sc, \]
where \( c \) is a measure of the coupling between the chiral superfield containing the inflaton and the superfield in the loop and \( s \) is related to the field value at the end of inflation. If the inflaton has a gauge coupling one probably needs [48] \( c \sim 10^{-1} \); then the observed \( n \) requires \( s \approx c \) and \( n' \sim 10^{-2} \). Such a value is not ruled out by the current bound (2.2), but it will be either ruled out or detected by a modest improvement in the accuracy. If instead the inflaton has only Yukawa couplings one can have \( c \sim 10^{-2} \) which would make the running undetectable so that the model can agree with observation.

An alternative model [49] makes the inflaton a two-component modulus. It typically gives either negligible tilt or tilt with rather strong running, but further investigation is needed to see whether it is ruled out.

### 2.3. Models ruled out

If the observation of negative tilt holds up it will represent a very significant development. Speaking generally, it avoids the criticism that \( n = 1 \) might have had some simple explanation, overlooked so far, which has nothing to do with field theory or inflation. Within the context of slow-roll inflation, negative tilt excludes several possibilities for the inflationary potential in a small-field model.

(a) Concave-upward potentials. A concave-upward potential gives positive tilt if \( 2\eta > 6\epsilon \). That is generally the case for small-field models. In particular it is true for small-field models with
\[
V = V_0 \left[ 1 + \left( \frac{\phi}{\mu} \right)^p \right].
\]
Indeed, \( V_0 \) must dominate to achieve small-field inflation, but then \( \epsilon \sim (\phi/M_P)^2 \eta^2 \ll \eta \). An attractive realization of this potential is the original hybrid model with \( p = 2 \) [39]. An integer \( p \geq 3 \) also corresponds to tree-level hybrid inflation [13, 40, 41], while the case \( p \leq -1 \) corresponds to dynamical supersymmetry breaking [42]. These are less attractive because small-field inflation occurs only over a limited range of \( \phi \) and it is not clear how the field is supposed to arrive within this range [11].

(b) Very flat potentials. If the potential is very flat, \( \epsilon \) and \( \eta \) will be negligible and so will the tilt \( n - 1 \). This happens in some models which seek to explain the inflationary scale \( V^{1/4} \) by identifying it with the relatively low scale of supersymmetry breaking in the vacuum [43]–[45]. Low-scale supersymmetric inflation may be viable within a more pragmatic framework [35].

(c) Generic modular inflation and supergravity. A string theory modulus is expected generically to have a potential of the form \( V = V_0 f(\phi/M_P) \), with \( f(x) \) and its low derivatives of order 1 at a generic point in the range \( \phi \lesssim M_P \). Near a maximum this gives \( \eta \sim -1 \) which only marginally allows inflation and gives \( n - 1 \sim -1 \). A similar result, \( |\eta| \gtrsim 1 \), is expected in a generic supergravity theory for any field. One of the most
Figure 4. The curved lines are the Natural Inflation predictions for $N = 40$ and 75, and the horizontal lines are the corresponding multi-component Chaotic Inflation predictions. The junction of each pair of lines corresponds to single-component Chaotic Inflation. The darker blue region corresponds to the allowed region at the 68% confidence level and the lighter blue (inner) region corresponds to the 95% confidence level. The regions allowed by observation with various assumptions are taken from [2] with the modified contours [37].

3. Large-field models

Large-field models allow an observable tensor fraction $r$. The single-component models are chaotic inflation [50], the multi-component version of that [51, 52], and Natural Inflation [53]. The situation for these models is illustrated in figure 4. (It shows the WMAP/SDSS constraint, but it hardly changes if WMAP is combined with other data sets.) There is no dramatic change from the situation with earlier constraints derived from WMAP year-one.

The generic prediction for a chaotic inflation potential $V \propto \phi^\alpha$ is

$$r = \frac{4\alpha}{N},$$

$$n - 1 = 2\eta - 6\epsilon = -\frac{2 + \alpha}{2N}. \quad (3.2)$$

As pointed out already [2], the year-three WMAP data rule out rather firmly $\alpha \geq 4$. Interestingly enough, the allowed case $\alpha = 2$ is also the best-motivated one in the context
of received ideas about field theory [31, 33, 54, 55], because it is reproduced by a Natural Inflation potential with a large period.

4. Summary and outlook

Over the last twenty-five years many field-theory models of slow-roll inflation have been proposed. We have seen that the WMAP year three results for $n$ and $r$ rule out a large fraction of these models. The remainder seem to be in two broad classes; large field models giving measurable tensor fraction $r$ and small-field models giving negligible $r$.

Within a few years, the PLANCK result for $n$ and the Clover result for $r$ will decide between these two classes. If $r$ is not detected, further discrimination between small-field models will come from measurements of the spectral index and its running.

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