Physical approach to analysis of induction motor braking under machinery load

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Abstract. The mechanical braking process of induction motors during short interruptions, and full switching off is studied in the paper. The approximate approach to solving a differential equation is used for the theoretical analysis of the braking process. The comparison of this approach with the exact solution shows its good accuracy. The used procedure is simple for a deep understanding of physical processes. It is shown that good results could be obtained for both full-stopping and braking due to sags if the right collocation point is chosen. This approach could be used for the study of different transients in electric motors and drives. It helps beginner students to understand clearly the main features of transients. However, it is also useful for experienced engineers who can easily obtain design formulas.

1. Introduction

Short circuits in electrical networks lead to short-term power supply interruptions [1, 2]. Induction motors (IM) slow down during voltage sags and short interruptions in electric networks [2–4]. Therefore, in order to maintain continuation and quality of technological processes, industrial plants should have re-acceleration schemes to allow a noninterfering restart of production processes.

Sometimes the full stopping and delayed restarts of the IMs for noncritical loads are acceptable for industry plants [2, 5]. However, some technological process can be disrupted by a temporary interruption interval from 3 to 60 s [3]. The induction motors are kept connected to the supply during voltage sags and short interruptions instead of disconnecting and re-starting them [3, 6]. This kind of induction motors connection helps to reduce their influence on the technological processes. When a voltage interruption takes place, the motors slow down until the voltage is restored, and then motors speed becomes increasing. This speed decreasing could be insignificant for many industrial technological processes [7–9].

In spite of occurrence of software for accurate transient calculation [10–12], simple analytical methods for calculating key self-starting parameters are also important for practice [5, 13]. The motor speed decrease is the most important and critical parameter for its restarting. The possibility of uninterrupted technological process is determined by the lowest speed and duration of speed decreasing [7, 13]. Other processes also play an important role in IM stopping and re-starting [14]. However, the speed decrease during free braking causes changing the back EMF and the residual voltage on the substation buses and re-starting current [14].
A mechanical characteristic of different technological units is a complex nonlinear function of the speed. Currently, there are well-known expressions for the load torque which are used in the analysis of electric motor starting and braking [7, 15]. However, their form is quite difficult for understanding by students, practitioners and specialists in the electrical engineering [15].

Therefore, when considering the IM starting, braking and restarting, one needs to pay great attention and efforts to perform purely mathematical issues, or just accept the final solution. In both cases, the student does not get the opportunity to deeply understand the processes inside the IM, since their attention is concentrated on the analysis of formal mathematical relationships [15, 16].

The proposed paper describes the approximate solution of the equations of the IM braking. In other words, there is an attempt to obtain mathematical relationships by the approach [16]. At the same time, the mathematical operations themselves are extremely simple and understandable.

The depth of voltage drop is the only difference between short interruptions and voltage sags [1]. The interruption has the strongest impact on the technological process quality and stability. Therefore, the only short interruption will be considered further as it is required for finding the critical speed decrease. The following solution is also valid for braking during the full stopping of disconnected IMs.

2. Problem formulation

The angular speed change in time is found from the motion equation:

\[ T - T_L = J \frac{d\omega}{dt}, \]

where \( T \) and \( T_L \) are the electromagnetic and load torques, N·m, \( J \) is the full moment of inertia of the motor and mechanical load, which is equal to the net moment of the motor inertia and the load inertia reduced to the motor shaft, kg·m², \( \omega \) is the angular speed, rad/s.

When the short supply interruption takes place, the IM electromagnetic torque is zero, and the motion equation is:

\[ -T_L = J \frac{d\omega}{dt}. \]

Dividing both equation sides by the rated torque \( T_r \) and taking into account that the rated power \( P_r = \omega_r T_r \), where \( \omega_r \) is the rated angular speed, gives

\[ -m_L = \tau_j \frac{d\omega}{dt}, \quad (1) \]

where \( m_L = \frac{T_L}{T_r} \) is the mechanical load torque in per units, p.u., \( \tau_j = \frac{J \omega}{P_r} \) is the mechanical time constant, \( \omega_r \) is the angular speed in per units, p.u.

The per unit mechanical characteristics of various mechanical loads are represented by the expression in the generalized form [15]:

\[ m_L = m_{L0} + (m_{L1} - m_{L0}) \omega^\gamma, \]

where \( m_{L0} \) is the load torque at zero angular rotational speed (\( \omega = 0 \)), determined by friction forces, p.u., \( m_{L1} \) is the load torque, \( \gamma \) is the coefficient depending on the type of the mechanical load, p.u. If angular speed is rated (\( \omega = 1 \)), p.u., the load torque is equal to the motor load factor \( k_L \), p.u., i.e. \( m_{L1} |_{\omega=1} = k_L \). Therefore, the mechanical load characteristic has the form:

\[ m_L = m_{L0} + (k_L - m_{L0}) \omega^\gamma. \quad (2) \]

The coefficient \( \gamma \) takes the following values:

- \( \gamma = 0 \) for mechanisms with a constant load torque;
- \( \gamma = 1 \) for mechanisms with a linear dependence of the load torque;
\[ \gamma = 2 \text{ for centrifugal mechanisms (fans, centrifugal pumps) in modes without backpressure;} \]
\[ \gamma = 3 \text{ for main pumps of oil transportation in modes with backpressure [15].} \]

Substituting (2) in (1) gives the generalized motion equation for various load mechanisms:
\[ -m_{t,0} - (k_L - m_{t,0}) \omega'' = \tau_j \frac{d\omega}{dt}. \] (3)

The numerical solution of (3) for mechanisms with different mechanical characteristics can be obtained. However, the simplest case \( m_{t,0} = 0 \) illustrating the method is considered:
\[ k_t \omega'' = \tau_j \frac{d\omega}{dt}. \] (4)

3. Motion equation analysis and solution

The terms of (4) are analyzed below. Since the motor initially works in the steady-state mode, its initial speed is equal to the rated one \( \omega_1 = \omega \).

Further, due to the load torque, the speed decreases to a new steady-state value \( \omega = 0 \), if the time interval is enough and the acceleration \( \frac{d\omega}{dt} \) is equal to zero:
\[ \frac{d\omega}{dt} \bigg|_{t \to \infty} = -k_t \omega'' = \frac{-k_t 0''}{\tau_j} = 0. \]

It also follows from (4) with \( \gamma \geq 1 \) that the rate of change of the angular speed or acceleration \( \frac{d\omega}{dt} \), has the maximum at the initial time moment and further decreases at \( t \to \infty \). This behavior is similar to the exponential function (Figure 1) [17,18], and the best approximation to this curve is the decaying function:
\[ \omega = e^{-t/T}, \] (5)
where \( T \) is an unknown time constant [16].

When the time constant value is determined, it is assumed that the exact and the approximate solutions have coincided at a certain point. Since the exact and the approximate solutions have coincided at the beginning and end of the process, it is necessary to take the point in the middle of the process. This approach is called collocation. It recommends taking this point at the exponential function decrease equal to 2/3 [16], then:
\[ k_L \left( \frac{2}{3} \right)^\gamma = \frac{\tau_j}{T} \frac{2}{3}. \]

The design formula for the time constant is obtained from the equation above:
\[ T = \frac{\tau_j}{k_L} \left( \frac{3}{2} \right)^{\gamma^{-1}}. \] (6)

When choosing a point at the exponential function decrease equal to 1/3 [16] the time constant is:
\[ T = \frac{\tau_j}{k_L} 3^{\gamma^{-1}}. \] (7)

Figure 1 shows the exact solution of the motion equation and its approximate solutions according to (6), (7) with the collocation time constant values 2/3, 1/3. As it follows from Figure 1, the first solution (6) better approximates the transition process start and the second solution (7) better approximates the transition process end. However, the approximate solution for the collocation time constant 2/3 found from (6) is the closest to the exact solution.
Thus, it is advisable to use the design formula (7) for consideration of the braking and stopping of IMs. If the IM braking is considered during the short interruption of power supply, then it is preferable to use (6).

It follows from the design formulas (6) and (7) that the duration of the braking process is determined not only by the motor inertia constant but also by the load factor and the coefficient $\gamma$. It follows directly from these expressions that the transition process slows down when the load factor is decreased, and the gamma index is increased.

**Conclusion**

A simplified approach to the theoretical analysis of the braking process of an induction motor under load has been proposed. An approximate solution has been obtained for the motor braking process taking into account the load torque on its shaft. This solution reflects the physical sense of the braking process in the understandable form. In contrast to existing approaches, the simplified solution allows understanding more easily the main essence of the braking process and does not require complex mathematical calculations.

This approach compared with the exact solution, has shown its high accuracy. It follows directly from the obtained design formulas that braking duration is proportional to the motor inertia constant, and the transition process slows down when the load factor is decreased, and the gamma index is increased.

This approach could be used for the study of different transients in electric motors and drives. Beginner students gain a clear understanding of solution physical meaning while experienced engineers can easily obtain design formulas.

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