The anomalous Hall effect in ferromagnetic Fe: 
Skew scattering or side jump?

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Abstract

The question is investigated whether the anomalous Hall effect (AHE) in Fe films is due to skew scattering or side jump. For this purpose sandwiches of FeIn are investigated in which the conduction electrons carry their drift velocity across the interface. This yields an additional AHE conductance $\Delta G_{xy}$ whose dependence on the In mean free path is used to determine the mechanism of the AHE in the Fe film. The structure of the Fe film is kept constant.
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In ferromagnetic metals and metals with magnetic impurities one observes two contributions to the Hall effect, (i) the normal Hall effect and (ii) the anomalous Hall effect (AHE). The AHE is caused by spin-orbit scattering through the interaction of the conduction electron spin with the magnetic moments of the sample. The anomalous Hall effect was already observed by Hall a century ago [1]. However, theoretically it is a rather complicated problem. There are two main mechanisms discussed in the literature, (a) skew scattering and (b) side jump. Both require a scattering mechanism for the conduction electrons and vanish in pure samples. The first models of skew scattering were developed by Karpulus and Luttinger [2] and Smit [3], while the side jump was proposed by Berger [4]. Due to its importance in spintronics, the anomalous Hall effect has been intensively studied in recent years [5], [6], [7], [8] (for further references see [9]). Recently an additional
mechanism has been under discussion which is connected with the Berry phase and believed to occur even in the absence of any scattering (see for example [10]). Here we restrict the discussion to the skew scattering and the side jump.

It is often stated in theoretical papers that for skew scattering the anomalous Hall resistivity $\rho_{yx}$ is proportional to the resistivity $\rho_{xx}$ while for the side jump $\rho_{yx}$ is proportional to the square of the resistivity $\rho_{xx}^2$. A number of experimental investigation tried to identify the mechanism of the AHE by changing the resistivity of their sample and analyzing the dependence of $\rho_{yx}$ on $\rho_{xx}$.

In this paper we will first show that the power law $\rho_{yx} \propto \rho_{xx}^p$ with $p = 1$ for skew scattering and $p = 2$ for the side jump is rather poorly justified. In the second part of the paper we use a very different experimental approach to identify the origin of the AHE in thin amorphous Fe films.

We briefly review the AHE resistivity for skew scattering and the sidessump in a ferromagnetic sample where the magnetic moments are aligned in the $z$-direction. The sample is disordered and has a finite $\rho_{xx}$. Part of the scattering will be potential scattering, i.e. spin-independent, and another part will be magnetic or spin-dependent.

We begin with **skew scattering**. First we consider conduction electrons with spin $\sigma$. In Fig.1 an electron propagates in the x-direction. A part of the wave is skew-scattered by a magnetic moment. We describe the potential scatterers by their concentration $n_i$ and their total scattering cross section $a_i$ (The index $i$ for impurity). Similarly the magnetic scatterers have the concentration $n_m$ with the scattering cross section $a_{m\sigma}$, i.e. the strength of the scattered wave is given by $a_{m\sigma}$ ($a_{m\sigma}$ depends on the spin of the conduction electron). The integrated momentum of the skew scattered wave possesses an electron momentum in the y-direction with the weight $a_{AH,\sigma}$.Here $a_{AH,\sigma}$ is the AHE cross section. It is defined so that $\hbar k_F a_{AH,\sigma}$ is equal to the $y$-component of the integrated momentum of the scattered wave. (A possible forward scattering should be incorporated into the scattering cross section $a_{m\sigma}$.) With the definition of a relaxation time $\tau_\sigma$ and the corresponding mean free path $l_\sigma$

\[
\frac{1}{\tau_\sigma} = \frac{v_{F,\sigma}}{l_\sigma} = v_{F,\sigma} (n_i a_i + n_m a_{m\sigma})
\]

one obtains for the longitudinal and transverse resistivities $\rho_{xx,\sigma}$ and $\rho_{yx,\sigma}$ for
each spin direction $\sigma$

$$\rho_{xx,\sigma} = \frac{m}{n_\sigma e^2 \tau}\sigma$$
$$\rho_{yx,\sigma} = \rho_{xx,\sigma} n_m a_{AH,\sigma} = \rho_{xx,\sigma} \frac{n_m a_{AH,\sigma}}{n_i a_i + n_m a_{m\sigma}}$$

The ratio $\rho_{yx,\sigma}/\rho_{xx,\sigma}$ (and therefore $\rho_{yx,\sigma}$) is rather intuitive. According to Fig.1 the electrons propagate the distance of the mean free path $l_\sigma$ (MFP) before they lose their drift velocity. After the skew scattering the fraction $n_m a_{AH,\sigma}/(n_i a_i + n_m a_{m\sigma})$ of the electron propagates the same MFP $l_\sigma$ in $y$-direction. This yields the ratio $l_{y,\sigma}/l_{x,\sigma} = \rho_{yx,\sigma}/\rho_{xx,\sigma} = n_m a_{AH,\sigma}/(n_i a_i + n_m a_{m\sigma})$.

![Diagram](image1.png)

Fig.1a: An (spin up) electron wave with momentum $k = (k_x,0,0)$ propagates in the x-direction. A part $a_{m\sigma}$ of the wave is skew-scattered by a magnetic moment and carries a momentum in the y-direction.

b) This time the side jump displaces the scattered electrons wave by $\Delta y_\sigma$ in the y-direction.

Equation (1) yields the well known statement that the AHE resistivity $\rho_{yx,\sigma}$ is proportional to the resistivity $\rho_{xx,\sigma}$, for a single spin direction!
Since the electron has two spins we have to (i) invert the resistivity tensors $(\rho_{ij})^{↑↑}$ for each spin to obtain the conductivity tensors $(\sigma_{ij})^{↑↓}$, (ii) add the two conductivity tensors and (iii) invert the resulting tensor.

$$(\rho) = \left( (\rho)^{-1} + (\rho)^{-1}_↓ \right)^{-1}$$

(2)

Since the MFPs of spin up and down electrons in the ferromagnet are generally quite different the original linearity between $\rho_{yx,\sigma}$ and $\rho_{xx,\sigma}$ for the individual spin is replaced by a complicated dependence.

For the side jump the electron does not propagate in the y-direction after the scattering but the whole scattered electron is displaced by the distance $\Delta y_{\sigma}$ in the y-direction. For the event sketched in Fig.1 the propagation in the x-direction is again $l_{x,\sigma} = l_{\sigma}$ while the "propagation" in the y-direction is equal to $l_{y,\sigma} = \Delta y_{\sigma} n_m a_{m\sigma} / (n_i a_i + n_m a_{m\sigma})$. This yields for $\rho_{yx,\sigma}$ (for each spin) the following value for the side jump

$$\rho_{yx,\sigma} = \rho_{xx,\sigma} \frac{l_{y,\sigma}}{l_{\sigma}} = \rho_{xx,\sigma} \Delta y_{\sigma} n_m a_{m\sigma}$$

(3)

If $\rho_{xx,\sigma}$ is proportional to density of magnetic scattering centers $n_m$ then $\rho_{yx,\sigma}$ is proportional to the square of $\rho_{xx,\sigma}$ if the scattering cross section $a_{m\sigma}$ and the side jump are independent of the resistivity, for a single spin direction! Using eq. (2) for the total AHE resistivity destroys the quadratic relation. Furthermore the parameters $a_i, a_{m\sigma}, a_{AH,\sigma}, \Delta y_{\sigma}$ are not independent of the disorder. The scattering potential is generally not the atomic potential but the deviation from the periodic potential. This potential is generally not spherically symmetric but is rather the gradient of a spherical potential. All of the scattering parameters $a_i, a_{m\sigma}, a_{AH,\sigma}$ and $\Delta y_{\sigma}$ will change in complicated ways with the disorder or alloying of the ferromagnet. In particular the behavior of $\Delta y_{\sigma}$ as a function of disorder is very critical. As Berger showed the side jump, which is caused by the spin-orbit interaction, only becomes significant because the spin-orbit interaction in the magnetic atoms can be enhanced by a factor of $3 \times 10^4$. Even the smallest change in the local environment of the magnetic atom could change this enhancement factor.

We summarize: The simple power law dependence of $\rho_{yx}$ on the resistivity $\rho_{xx}$ for skew scattering and side jump may not be reliable for the following reasons:

- The contribution of two kinds of electrical carriers in ferromagnets, spin
up and down electrons, destroys the simple relation between $\rho_{yx}$ and $\rho_{xx}$.

- The scattering parameters $a_i, a_{m\sigma}, a_{AH,\sigma}$ and $\Delta y_\sigma$ will change in complicated ways with the disorder or alloying of the ferromagnet.

- The enhancement of the spin-orbit interaction (which determines the side-jump parameter) by a factor of $3 \times 10^4$ will be very sensitive to the disorder.

In the present investigation we use a new approach to investigate the AHE of a ferromagnetic film. We use a thin ferromagnetic film as the target of a scattering experiment by exposing it to incident electrons. The momentum of the incident electrons is varied and the electrons are scattered by the target. Their (integrated) angular scattering is measured. This appears to be a conventional scattering experiment but there is an important difference. The probing electrons are the conduction electrons of a normal metal film which is condensed on top of the ferromagnetic film. The sandwich is shown in Fig. 2. In the experiment we use amorphous Fe with a very short MFP for the ferromagnet and In with a much larger MFP for the normal metal. This simplifies the underlying physics and the evaluation of the experiment.

![Fig. 2: A sandwich consisting of ferromagnetic amorphous Fe and the normal metal In is quench condensed. In the presence of an electric field the conduction electrons in the In carry their larger drift velocity into the upper layer of the Fe and create a large anomalous Hall effect (AHE) in the Fe. Its dependence on the mean free path of the conduction electrons in the In identifies the origin of the AHE.](image)

In the presence of an electric field $E$ (in the x-direction) the electrons accumulate finite drift velocities: $v_n$ in the normal metal and $v_{f\uparrow}, v_{f\downarrow}$ in the
amorphous ferromagnet. The electrons of both metals cross the interface. The electrons which cross from the normal metal into the ferromagnet increase the current density in the upper layers of the Fe dramatically because they carry a much larger drift velocity. This injected high current density in the ferromagnet is proportional to the MFP in the normal metal. It creates an additional AHE in the Fe. If the AHE is due to the side-jump mechanism then the injected current yields an AHE conductance which is proportional to the MFP $l_n$ in the normal metal. If the AHE is due to skew scattering then a large fraction of the scattered electrons returns into the normal metal and propagates there the distance $l_n$. Therefore their contribution to the AHE conductance is proportional to the square of the MFP in the normal metal. By changing the MFP in the normal metal we can analyse the origin of the AHE in the ferromagnet without changing the structure of the ferro-magnet.

Our FeIn sandwiches are prepared at liquid helium temperatures. To obtain very flat and homogeneous Fe films we first condense 10 atomic layers of insulating amorphous Sb. On this fresh substrate the Fe film shows conductance already for one mono layer. The thickness of the Fe films lies in the range of 5 to 10 atomic layers. On top of the Fe film the In is condensed in several steps up to a thickness of 25nm. The MFP of the In lies in the range of $5-20$nm while the MFP of the Fe is of the order of a few Angstroms. Fig.3 shows the anomalous Hall curves for a sandwich of 5 atomic layers of amorphous Fe covered with increasing layers of In. The normal Hall conductance is subtracted. From these curves we obtain the AHE conductance $L_{xy}^{AHE}$ by back extrapolation of the high field part of the curve to zero magnetic field. We observe an increase of the AHE conductance with the In thickness. We denote the additional AHE conductance as interface AHE conductance.
Fig. 3: The anomalous Hall conductance curves for a thin Fe film covered with In of increasing thickness.

In Fig. 4 we have plotted the interface AHE conductance $\Delta L_{xy}$ as a function of the In thickness for two different amorphous Fe film thicknesses of 5 and 8.6 atomic layers. The fact that the two curves lie very close to each other demonstrates that the interface AHE conductance does not depend on the thickness of the ferromagnet (as long as the thickness is larger than the
MFP).

Fig. 4: The interface anomalous Hall conductance of two FeIn sandwiches as a function of the In thickness. The thicknesses of the amorphous Fe films are 5 and 8.6 atomic layers.

For the evaluation and interpretation of the experiment we calculated the AHE conductance of an FN (ferromagnet/non-magnetic metal) sandwich. (The details will be published elsewhere). We applied the (linearized) Boltzmann equation using Chamber’s method of the vector mean free path [11]. The fact that the Fermi energies differ in the two metals complicates the calculation considerably. Therefore we follow here the examples in the theory of giant magneto-resistance and the superconducting proximity effect where the first theoretical approaches simplified the problem by assuming identical electronic properties in both metals. Furthermore we take the densities of spin up and down electrons in the ferromagnet as identical and equal to \( n/2 \).

The thickness and MFPs of the Fe film are denoted as \( d_f, l_{f\uparrow}, l_{f\downarrow} \) for spin up and down and \( d_n, l_n \) for the In. Because the Fe and In film are in parallel their conductances would simply add if there would be no interface crossing between the films. Without the crossing the In would not contribute to the AHE.
The conduction electrons in the In with $k_z < 0$ cross through the interface into the Fe. In the following we call them the injected electrons. In the Fe they carry an injected current $I_{in}$ for each spin which for $l_n >> l_{f\uparrow}, l_{f\downarrow}$ is

$$I_{in} = \frac{1}{16} e^2 N_0 v_F l_n (l_{f\uparrow} + l_{f\downarrow}) E$$

where $N_0$ is the density of electron states per spin. This current flows in a thin layer of the Fe whose thickness is half the MFP, i.e., $l_{f\uparrow}/2$ or $l_{f\downarrow}/2$ for spin up and down electrons and is proportional to the MFP in the normal metal. The result for the longitudinal part of the conductance $G_{xx} = I_{in}/E$ is similar to Fuchs [12] and Sondheimer [13] for thin films but extended to sandwiches. The injected current yields an additional large AHE. The resulting contribution to the anomalous Hall conductance depends on the mechanism of the AHE.

**side jump:** The electrons which cross from the normal metal into the ferromagnet contribute to the side jump. They yield an additional AHE conductance

$$\Delta G_{xy}^{(sj)} = \frac{1}{16} e^2 N_0 v_F l_n (l_{f\uparrow} \Delta y_{\uparrow} n_m a_{0\uparrow,m} + l_{f\downarrow} \Delta y_{\downarrow} n_m a_{0\downarrow,m})$$

The electrons which cross from the ferromagnet to the normal metal do not contribute to the AHE.

**skew scattering:** In contrast to the side jump, here part of the important physics happens after the scattering because half of the skew scattered electrons propagate back towards the normal film. Therefore one obtains two additional contributions due to the normal metal film: (i) The conduction electrons which are accelerated in the ferromagnet and cross into the normal metal after the scattering and (ii) the conduction electrons which are accelerated in the normal metal, cross the interface into the ferromagnet, experience skew scattering and then cross back into the normal metal. The second effect is proportional to $l_n^2$ and is dominant. It yields an additional anomalous Hall conductance

$$\Delta G_{xy}^{(ss)} = \frac{\beta}{32} e^2 N_0 v_F l_n^2 n_m (a_{ah\uparrow} l_{f\uparrow} + a_{ah\downarrow} l_{f\downarrow})$$

The additional factor $\beta/2$ (with respect to the current) is due to the fact that only half of the scattered electrons move back towards the normal metal. Since they are roughly the distance $l_{f\uparrow}/2, l_{f\downarrow}/2$ from the interface only the
fraction $\beta$ reaches the normal metal without being scattered in the ferromagnet. The factor $\beta$ is less than one and of the order of $1/2$. (If the scattering in the ferromagnet would be isotropic then $\beta$ would have the value $1/2$)

For skew scattering the additional anomalous Hall conductance is proportional to the square of the MFP in the normal metal. This result is quite physical. The interface AHE conductance is (i) proportional to the drift velocity, i.e. $l_n$, and (ii) proportional to the distance the electrons travel after the scattering, which yields another factor of $l_n$.

For the analysis of the experiment we plot in Fig.5 the total anomalous Hall conductance of the two sandwiches versus the MFP of the conduction electrons in the In film. Together with the experimental points is shown a linear fit. Obviously the interface AHE conductance is linear in the MFP $l_n$ of the In. This proves clearly that the anomalous Hall effect in the amorphous Fe film is due to the side jump. Originally Berger’s argument for observing the side jump in metals with small MFP was that there the small MFP reduces the magnitude of the skew scattering so that one could observe the side jump. Our conclusion goes beyond that suggestion. Since the propagation in the normal metal yields a contribution which is proportional to the square of the MFP in the normal metal our experiment should detect a contribution of the skew scattering even if its contribution in the amorphous Fe would be very small. So the conclusion of our experiment is that there is practically
no skew scattering in the amorphous Fe.

![Graph](image)

Fig.5: The total anomalous Hall conductance of the two FeIn sandwiches shown in Fig.5 as a function of the In mean free path.

In this paper we have investigated the mechanism of the AHE in amorphous Fe. The standard approach assumes that $\rho_{yx}$ depends on $\rho_{xx}$ linearly for skew scattering and quadratically for side jump. First we pointed out that this assumption is not well justified. Instead we introduced a method for which the structure of the ferromagnet is kept constant. By preparing a sandwich of amorphous Fe with the non-magnetic metal In we observed an increased AHE conductance because conduction electrons with a larger drift velocity cross from the In into the Fe and cause an additional ”interface” AHE within the MFP of the Fe. For the side jump this interface AHE conductance $\Delta G_{xy}$ is proportional the MFP $l_{In}$ of the In. If the AHE is due to skew scattering then about $1/4$ of the skew-scattered electrons cross back into the In and propagate the distance $l_{In}$. This yields a quadratic dependence of $\Delta G_{xy}$ on $l_{In}$ for skew scattering. The great advantage of the interface AHE is that the structure and scattering potentials of the ferromagnet are kept constant.

Our experimental results yield a linear dependence of $\Delta G_{xy}$ on the In MFP $l_{In}$. This not only shows that the side jump is the dominant mechanism
for the AHE in amorphous Fe, but the experiment did not detect any skew
scattering in the amorphous Fe film.

Abbreviations: AHE=anomalous Hall effect, MFP=mean free path.

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