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Sensitivity analysis of COVID-19 with quarantine and vaccination: A fractal-fractional model

Abdul Malik a,b,* , Musaed Alkholief b , Fahad M. Aldakheel c , Azmat Ali Khan d , Zubair Ahmad e,f , Warda Kamal g , Mansour Khalil Gatasheh h , Aws Alshamsan a,b

a Department of Pharmaceutics, College of Pharmacy, King Saud University, Riyadh, Saudi Arabia
b Nanobiotechnology Unit, Department of Pharmaceutics, College of Pharmacy, King Saud University, Riyadh, Saudi Arabia
c Department of Clinical Laboratory Sciences, College of Applied Medical Sciences, King Saud University, Riyadh 11564, Saudi Arabia
d Pharmaceutical Biotechnology Laboratory, Department of Pharmaceutical Chemistry, College of Pharmacy, King Saud University, Riyadh, Saudi Arabia
e Dipartimento di Matematica e Fisica, Università degli Studi della Campania “Luigi Vanvitelli”, Caserta, 81100, Italy
f Novel Global Community Educational Foundation, Australia
g Biomediotronics, Enzymics, 7 Peterlee Place, Hebersham, NSW 2770, Australia
h Department of Biochemistry, College of Science, King Saud University, Riyadh, Saudi Arabia

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Abstract  To eradicate most infectious diseases, mathematical modelling of contagious diseases has revealed that a combination of quarantine, vaccination, and cure is frequently required. However, eradicating the disease will remain a difficult task if they aren’t provided at the appropriate time and in the right quantity. Control analysis has been shown to be an effective way for discovering the best approaches to preventing the spread of contagious diseases through the development of disease preventive interventions. The method comprises reducing the cost of infection, implementing control measures, or both. In order to gain a better understanding of COVID-19’s future dynamics, this study presents a compartmental mathematical model. The problem is modelled as a highly nonlinear coupled system of classical order ODEs, which is then generalised using the Mittag-Leffler kernel’s fractal-fractional derivative. The uniqueness of the fractional model under discussion has also been demonstrated. The boundedness and non-negativity of the considered model are also established. The next generation technique is used to examine basic reproduction, and disease free and endemic equilibrium. We used reported cases from Australia in this investigation due to the high risk of infection. The reported cases are considered between 1st July 2021 and 20th August 2021. On the basis of previous data, the spread of infection is predicted for the next 600 days which is shown through different graphs. The graphical solution of the considered nonlinear model is

* Corresponding author at: Department of Pharmaceutics, College of Pharmacy, King Saud University, Riyadh, Saudi Arabia.
E-mail address: amoinuddin@ksu.edu.sa (A. Malik).
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obtained via numerical scheme by implementing the MATLAB software. Based on the fitted values of parameters, the basic reproduction number \( R_0 \) is calculated as \( R_0 \approx 1.58276 \). Furthermore, the impact of fractional and fractal parameter on the disease spread among different classes is demonstrated. In addition, the impact of quarantine and vaccination on infected people is dramatically depicted. It’s been argued that public awareness of the quarantine and effective vaccination can drastically reduce infection rates in the population.

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1. Introduction

The severe acute respiratory syndrome coronavirus 2 that causes severe acute respiratory illness causes Coronavirus disease 2019 (COVID-19) (SARS-CoV-2) [1]. Because of its rapid geographic spread, COVID-19 has emerged as a worldwide health hazard. Preventive measures, coupled with effective illness isolation and community containment, play a vital role in limiting viral public transmission. The improvement of a vaccination to remove the virus from the host is still a problem [2]. COVID-19 has infected over 208,652,652 persons worldwide, resulting in 4,382,962 fatalities and 187,037,365 recoveries (as of August 17, 2021, 01:11 GMT) [3]. Current research is examining the repurposing of US FDA-approved medications in conjunction with vaccine production, and some are currently in clinical trials [4] (see Table 1).

SARS-CoV-2, which was discovered in Chinese (Wuhan city) first time, is infectious in individuals and has spread fast throughout the world through adjacent human contact or the split of cough and sneeze of infected persons. As a result of the increasing infection rate outside of China, WHO designated the COVID-19 epidemic a “pandemic” on March 12th, 2020. COVID-19 is classified as a coronavirus and belongs to the Sarbecovirus subgenus, which includes many other species that cause moderate to severe human illnesses. COVID is the seventh coronavirus to infect humans, following 229E, NL63, OC43, HKU1, MERS-CoV, and the preceding SARS-CoV [5]. The envelope (E), membrane (M), nucleocapsid (N), and spike (S) are the four primary structural proteins found in the corona virus genome (S). S, E, M, and N proteins are virulence proteins that facilitate DNA replication. These proteins bind to the human angiotensin-converting enzyme 2 (ACE2) receptor as soon as they enter a human cell. As a result, these viral proteins might be exploited as a therapeutic target to halt the reproduction of the virus [6].

In early 2020, Australia, like many other nations, had an increase in coronavirus disease (COVID-19) cases. From late June to early August, COVID-19 infections were concentrated in Victoria. Between June 14th and August 10th, 13,078 incidents were registered, with daily case counts reaching at 686 on August 5th in Australia’s second most populated region. The majority of the incidents (95%) took place in Melbourne, the state capital with a population of 4.93 million. Following its implementation, the mandatory mask usage policy significantly increased public mask use, which was linked to a large decrease in reported COVID-19 incidents [7]. COVID-19 was initially detected in Australia in late January 2020, and as of May 18th, there have been 7,060 confirmed cases, 99 fatalities, and 6,389 cases recovered. Over a million tests were performed, with a 0.7 percent overall positive rate [8]. WHO collected 39,096 confirmed cases of COVID-19 between 3 January 2020, 6:41 pm CEST, and 16 August 2021, including 958 fatalities. 13,723,146 vaccine doses had been given as of August 7, 2021 [9].

Differential equations have been utilised to model dynamic processes in a variety of areas, including fluid mechanics, epidemiology, biochemistry, and engineering [10–15]. In epidemiology, the application of mathematics and computer approaches is gaining popularity. It has been a hot topic for scholars for decades, and many techniques are employed to describe or explain the dynamics of various infectious illnesses in terms of mathematical equations [16–21]. In addition, the notion of optimal control is used to these mathematical models, which has been shown to be effective in the prevention of many pandemic outbreaks in various parts of the world at various times [22,23]. The theoretical studies for the dynamics of different infectious diseases through mathematical approaches can be seen Refs [24–28]. Peter et al. [24] analyzed the dynamics of COVID-19 in Pakistan using real data and calculated the basic reproduction number for the estimated parameters values using next generation technique. Abdullah et al. [25] investigated the numerical solutions for the COVID-19 model and compared their results with the real data from Wuhan, China. Similarly, Al-Ajlan [26] investigated the role of an e-learning management system so called blackboard system during the COVID-19 pandemic in Saudi Arabia. Their study revealed that the use of blackboard system developed the skills of students who are newly admitted to the Taif University. Alnaser et al. [27] studied the dynamics of COVID-19 in Bahrain.

### Table 1 List of Parameters with Description:

| Parameter | Description | Dimension/Unit |
|-----------|-------------|----------------|
| \( \sigma \) | Interaction rate between infected and susceptible individuals | Time\(^{-1}\) |
| \( \chi \) | Incubation period | Time\(^{-1}\) |
| \( \phi \) | Quarantined rate of infected individuals | Time\(^{-1}\) |
| \( \gamma \) | Vaccination rate of susceptible individuals | Time\(^{-1}\) |
| \( \rho \) | Recovery or removal rate of infected individuals | Time\(^{-1}\) |
| \( \zeta \) | Recovery or removal rate of quarantined individuals | Time\(^{-1}\) |
| \( \theta \) | Recruitment rate | Population/Time |
| \( \Theta \) | Natural mortality rate | Population/Time |

Differential equations have been utilised to model dynamic processes in a variety of areas, including fluid mechanics, epidemiology, biochemistry, and engineering [10–15]. In epidemiology, the application of mathematics and computer approaches is gaining popularity. It has been a hot topic for scholars for decades, and many techniques are employed to describe or explain the dynamics of various infectious illnesses in terms of mathematical equations [16–21]. In addition, the notion of optimal control is used to these mathematical models, which has been shown to be effective in the prevention of many pandemic outbreaks in various parts of the world at various times [22,23]. The theoretical studies for the dynamics of different infectious diseases through mathematical approaches can be seen Refs [24–28]. Peter et al. [24] analyzed the dynamics of COVID-19 in Pakistan using real data and calculated the basic reproduction number for the estimated parameters values using next generation technique. Abdullah et al. [25] investigated the numerical solutions for the COVID-19 model and compared their results with the real data from Wuhan, China. Similarly, Al-Ajlan [26] investigated the role of an e-learning management system so called blackboard system during the COVID-19 pandemic in Saudi Arabia. Their study revealed that the use of blackboard system developed the skills of students who are newly admitted to the Taif University. Alnaser et al. [27] studied the dynamics of COVID-19 in Bahrain.
Saudi Arabia and Egypt using mathematical tools. They related their results with the surrounding temperature, population density, life style and nutrition. Qureshi et al. [28] investigated the transmission of measles among the human population using the fractional mathematical model. They analysed and compared their results with the real data taken from WHO.

Memory has been presented as a technique using fractional-order models [29,30]. Most of these works aim to demonstrate how nonlinear equations may make even basic dynamical models complicated. In nature, certain physical problems may be governed by the power law, whereas others may be governed by the Mittag–Leffler and exponential decay laws. Riemann-Liouville, Caputo-Fabrizio, and Atangana-Baleanu differential operators are only a few of the many fractional derivative definitions. The implementation of these fractional differential operators having these kernels can be seen in science and engineering such as mass spring damper systems [31], logistic population growth model [32] and dynamics of infectious diseases [28]. Fractional models has shown its importance in the last few decades but no one has coupled fractal derivative with fractional derivative until Atangana [33] introduced novel operators of differentiation in his work in 2017, such as a convolution of a power law, exponential decay law, and extended Mittag-Leffler law with fractal derivative. Fractal-fractional derivative were supplied to the many fractional operators. The fractal-fractional (FF) operator, which combines fractal and fractional differentiation, is a novel idea whose characteristics and features are being researched right now. Fractal-fractional differentiation may be found in chaotic attractors, chemical processes, complex dynamical systems, and electrical circuits [33–37].

The goal of this study is to provide a compartmental model for COVID-19’s spread among people. The model is then applied to real-world data from Australia for the period of July 1, 2021, to August 20, 2021. The examined model’s boundedness and non-negativity have been established, and equilibria and the basic reproduction number have been obtained. Furthermore, the classical model is extended using the Mittag-Leffler kernel’s fractal-fractional differential operator. The solution fractal-fractional model’s uniqueness is also proved. The considered model’s graphical solutions are obtained using a numerical technique and computational software MATLAB. The future scenario for the propagation of COVID-19 is assessed over the next 600 days using available data. The effect of the fractional parameter and the fractal dimension is graphically depicted. Some plots are also shown to demonstrate the influence of quarantine and vaccination on the spread of infection among various classes. Finally, some conclusions are drawn, as well as some suggestions for reducing the risk of infection.

2. Mathematical preliminaries

This section introduces some key concepts and prepositions that will help the remainder of the study [33,36,38].

**Definition 2.1.** For a continuous function \( p(t) \in C^n[0, T] \), the fractal-fractional operator with Mittag-Leffler type kernel having fractional order \( \delta \) and fractal order \( \lambda \) can be written as:

\[
 aFFM^\delta_\lambda \ p(t) = \frac{Q(\delta)}{1 - \delta} \frac{d}{dt} \int_0^t \! E_\delta \left( \frac{-\delta(t - \zeta)\lambda}{1 - \delta} \right) p'(\zeta) d\zeta,
\]

where

\[
 \frac{dp(t)}{dt} = \lim_{\lambda \to \delta} \frac{p(t) - p(t_\lambda)}{t^\lambda - t_\lambda^\lambda}(2 - \lambda)
\]

and \( 0 < \delta, \lambda \leq m \in \mathbb{N} \). \( Q(\delta) = 1 - \delta + \frac{1}{(1 - \delta)} \) and \( E_\delta(\cdot) \) is the normalization and Mittag-Leffler functions respectively.

**Definition 2.2.** The integral operator for the fractal-fractional operator with Mittag-Leffler type kernel having fractional order \( \delta \) and fractal order \( \lambda \) can be written as:

\[
 aFFM^\delta_\lambda \ p(t) = \frac{\lambda(1 - \delta)^{\lambda - 1}}{Q(\delta)} \ p(t) + \frac{\delta\lambda}{Q(\delta)} \int_0^t \! \zeta^{\lambda - 1} (t - \zeta)^{\delta - 1} p'(\zeta) d\zeta.
\]

**Definition 2.3.** Let us suppose a fractional ODE:

\[
 ^{ABC}_\beta \partial^\lambda \ K(t) = y(t, K(t)) \quad \text{with} \quad K(0) = K_0
\]

The numerical structure for equation (4) can be written as:

\[
 K_{n+1}^\lambda = K_n + \frac{\beta}{\phi_{n+1}^\lambda} y(t, K_n(t)) + \frac{\beta}{\phi_{n+1}^\lambda} \sum_{n} \left[ \frac{\phi_{n+1}^\lambda}{\phi_{n+1}^\lambda} \left( (w + 1 - q)^2 (w + 2 - q + 3\delta) - (w - q)^2 (w + 2 - q + 3\delta) \right) \right]
\]

3. Mathematical modeling

The present model is carried out for the transmission of COVID-19 among the human population. The overall human population is symbolized with \( N(t) \) which is further divided in to various subclasses i.e., susceptible population is represented by \( S(t) \), infected individuals are denoted by \( I(t) \). Similarly, \( Q(t) \) are the quarantined and \( R(t) \) are the recovered or removed/deceased individuals. The vaccinated individuals are denoted by \( V(t) \). The flow diagram for the considered problem is demonstrated in Fig. 1, where susceptible people can interact with \( I(t) \) having a route \( \sigma SI \) and become exposed. Some of susceptible individuals are vaccinated with a vaccination rate of \( \gamma \) and enters to the vaccination class \( V(t) \). We have considered the vaccination of only susceptible individuals because susceptible individuals are at high risk of infection. The immunity of exposed and infected individuals is already familiar with the virus behavior and structure and their antibodies are more active as compared to the susceptible individuals. Therefore, we ignored the vaccination of these classes for the time being. Similarly, the exposed individuals complete its incubation period and enters to the infected class \( I(t) \) with incubation rate \( \chi \). Similarly, some of the infected individuals will recover or die out due to the infection with the positive rate \( \varepsilon \) while some of them are quarantined with a positive rate \( \phi \). At the last, these quarantined individuals are recovered or removed due to the disease with the rate \( \xi \). The natural death rate of all the classes is symbolized with \( \theta \) while the recruitment or birth rate is denoted by \( \Theta \). All the parameters
are defined in Table 1. The flow diagram of the considered model for the spread of COVID-19 is given as:

The system of equations for the considered problem with non-negative IC’s is given by:

$$
\begin{align*}
\frac{dS}{dt} &= \Theta - \sigma SI - (\gamma + \theta)S, \\
\frac{dE}{dt} &= \sigma SI - (\chi + \theta)E, \\
\frac{dQ}{dt} &= \chi E - (\phi + \sigma + \theta)I, \\
\frac{dI}{dt} &= \phi I - (\zeta + \theta)Q, \\
\frac{dV}{dt} &= \zeta Q - \delta R, \\
\frac{dR}{dt} &= \gamma S - \theta V,
\end{align*}
$$

(6)

With

$$
S(0) = S^* \geq 0, \quad E(0) = E^* \geq 0, \quad R(0) = I^* \geq 0, \\
Q(0) = Q^* \geq 0, \quad R(0) = R^* \geq 0, \quad V(0) = V^* \geq 0.
$$

(7)

4. Positivity and boundedness of the COVID-19 model

This section shows that the solution of the considered problem is bounded as well as non-negative. For this, we consider the lemma given as [11,12]:

**Lemma 4.1.** Consider \( \Phi \subset \mathbb{R} \times \mathbb{C}^n \) is open, \( g_q \in \mathbb{C}(\Phi, \mathbb{R}), \)

\( q = 1, 2, \ldots, n. \) If \( g_q \big|_{Y_q=0,Y_q \in \mathbb{C}^n} \geq 0, \quad Y_i = (Y_{1i}, Y_{2i}, \ldots, Y_{ni})^T, \)

\( q = 1, 2, \ldots, n, \) then \( \mathbb{C}^n_{\geq 0}(\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n) : \varphi \in \mathbb{C}^n_{\geq 0}) \) is the invariant domain of the subsequent equations.

$$
\begin{align*}
dv_q(t) &= g_q(t, Y_q), t \geq \rho, q = 1, 2, \ldots, n. \\
\mathbb{R}^n_{\geq 0}(Y_1, Y_2, \ldots, Y_n) : Y_q \geq 0, \quad q = 1, 2, \ldots, n.
\end{align*}
$$

(8)

**Proposition 4.1.** The system (6) is invariant in \( \mathbb{R}^n_+ \).

**Proof:** As a result of restating the system (6):

$$
\frac{dW}{dt} = M(W(t)), W(0) = W_0 \geq 0.
$$

(9)

We noted that

$$
\begin{align*}
\frac{dS}{dt} |_{S=0} &= \Theta \geq 0, \\
\frac{dE}{dt} |_{E=0} &= \sigma SI \geq 0, \\
\frac{dI}{dt} |_{I=0} &= \chi E \geq 0, \\
\frac{dQ}{dt} |_{Q=0} &= \phi I \geq 0, \\
\frac{dV}{dt} |_{V=0} &= \zeta Q \geq 0, \\
\frac{dR}{dt} |_{R=0} &= \gamma S \geq 0.
\end{align*}
$$

(10)

Hence, \( \mathbb{R}^n_+ \) is invariant set.

**Proposition 4.2.** Model (6) is bounded in:

\( \Phi = \left\{ (S(t), E(t), I(t), Q(t), R(t), V(t)) \in \mathbb{R}^6 : N(t) = \frac{\Theta}{\Theta} \right\}. \)

**Proof:** By adding all the Eqs. of model (6), we get:

$$
\frac{dN(t)}{dt} = \Theta - \theta N, \quad \text{with} \quad N(0) = N_0 \geq 0.
$$

(11)

Which leads us:

$$
N(t) \leq N_0 e^{-\theta t} + \frac{\Theta}{\theta} (1 - e^{-\theta t}).
$$

(12)

Equation (14) clearly shows that if \( t \to \infty \) then \( N(t) \leq \frac{\Theta}{\theta} \), indicating that the feasible region is:

$$
\Phi = \left\{ (S(t), E(t), I(t), Q(t), R(t), V(t)) \in \mathbb{R}^6 : N(t) \leq \frac{\Theta}{\theta} \right\}.
$$

(13)

Hence the solution of the considered COVID-19 model is bounded.

5. Model Equilibria, basic reproduction number and stability analysis

The possible equilibria of system (6) can be obtained via the steady state by assuming:
\[
\frac{dS}{dt} = \frac{dE}{dt} = \frac{dV}{dt} = \frac{dI}{dt} = \frac{dQ}{dt} = \frac{dR}{dt} = 0 \tag{16}
\]

Using equation (16), model (6) becomes:

\[
\begin{align*}
0 &= \Theta - \sigma SI - (\gamma + \theta)S, \\
0 &= \sigma SI - (\chi + \theta)E, \\
0 &= \chi E - (\phi + \sigma + \theta)I, \\
0 &= \phi I - (\xi + \theta)Q, \\
0 &= \mu I + \xi Q - \theta R, \\
0 &= \gamma S - \theta V,
\end{align*}
\tag{17}
\]

The following is the possible unique disease-free equilibrium (DFE) of system (17):

\[
\Psi_{DFE} = (S^0, E^0, I^0, Q^0, R^0, V^0) = \left(\Theta, 0, 0, 0, 0, \frac{\gamma \Theta}{\theta(\gamma + \theta)}\right). \tag{18}
\]

Now have to compute the basic reproduction number \(R_0\), we introduce the two matrices \(F\) and \(V\) at \(\Psi_{DFE}\) by applying the procedure of Next Generation Technique [39]:

\[
F = \begin{bmatrix}
0 & \frac{\sigma I}{\gamma + \theta} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \tag{19}
\]

and

\[
V = \begin{bmatrix}
\chi + \theta & 0 & 0 \\
-\chi & \phi + \sigma + \theta & 0 \\
0 & -\phi & \xi + \theta
\end{bmatrix} \tag{20}
\]

According to the next generation technique given in [39], the spectral radius \(\rho(FV^{-1})\) is the essential \(R_0\) which is given below:

\[
R_0 = \frac{\Theta \sigma \gamma}{(\theta + \chi)(\gamma + \theta)(\theta + \phi + \sigma)} \tag{21}
\]

Next, we have to find the endemic equilibrium (EE) of the considered problem which is symbolized by \(\Psi_{EE} = (S^*, E^*, I^*, Q^*, R^*, V^*)\) and is given as:

\[
\begin{align*}
S^* &= \frac{1}{\sigma(\chi + \sigma + \theta + \phi + \gamma + \theta)}(\Theta \chi - (\theta^2 + \theta(\phi + \chi + \sigma + \chi(\phi + \sigma)))I^*), \\
E^* &= \frac{1}{\sigma(\chi + \sigma + \theta + \phi)}(\Theta \sigma + \sigma(\chi + \sigma)(\chi + \sigma + \phi + \gamma + \theta)), \\
I^* &= \frac{R_0^* I^*}{\gamma / (\gamma + \theta)} - \frac{(\phi + \gamma + \phi + \gamma + \theta + \phi + \gamma + \phi + \gamma + \phi + \gamma + \theta)}{\gamma / (\gamma + \theta)}, \\
Q^* &= \frac{\phi \gamma}{\gamma / (\gamma + \theta)}R^*, \\
R^* &= \frac{1}{\gamma / (\gamma + \theta)}(\xi \phi + \mu + \theta(\theta + \xi))I^*, \\
V^* &= \frac{\gamma / (\gamma + \theta)}{\sigma(\chi + \sigma + \theta + \phi + \gamma + \theta)}(\Theta \chi - (\theta^2 + \theta(\phi + \chi + \sigma + \chi(\phi + \sigma)))I^*).
\end{align*}
\tag{22}
\]

\textbf{Theorem 5.1} ([40]). The DFE \(\Psi_{DFE}\) of the system (6) is locally asymptotically stable if \(R_0 < 1\).
6. Fractal-Fractional COVID-19 model

To generalize the classical model of COVID-19 given in equation (6), we apply the Fractal-Fractional (FF) differential operator given in definition 2.1. Thus, equation (6) becomes:

\[
\begin{align*}
\text{ABR} & \phi^{\delta}_{\lambda}\left(S(t)\right) = \Theta - \sigma SI - (\gamma + \theta)S, \\
\text{ABR} & \phi^{\delta}_{\lambda}\left(E(t)\right) = \sigma SI - (\chi + \theta)E, \\
\text{ABR} & \phi^{\delta}_{\lambda}\left(I(t)\right) = \chi E - (\phi + \sigma + \theta)I, \\
\text{ABR} & \phi^{\delta}_{\lambda}\left(Q(t)\right) = \phi I - (\zeta + \theta)Q, \\
\text{ABR} & \phi^{\delta}_{\lambda}\left(R(t)\right) = \phi I + \zeta Q - \theta R, \\
\text{ABR} & \phi^{\delta}_{\lambda}\left(V(t)\right) = \gamma S - \theta V,
\end{align*}
\]

where \(\text{ABR} \phi^{\delta}_{\lambda}(\cdot)\) is the definition of fractal-fractional derivative having fractional order \(\delta\) and fractal dimension \(\lambda\).

6.1. Existence and uniqueness of the Fractal-Fractional model

The existence of unique solution to the FF model is demonstrated in this subsection of the article, which is provided in Eq. (29). Let us deliberate the system in the way given below:

\[
\begin{align*}
0\text{ABR} & \phi^{\delta}_{\lambda}\left(S(t)\right) = \lambda t^{\delta\eta-1}(R(t, S)), \\
0\text{ABR} & \phi^{\delta}_{\lambda}\left(E(t)\right) = \lambda t^{\delta\eta-1}(R(t, E)), \\
0\text{ABR} & \phi^{\delta}_{\lambda}\left(I(t)\right) = \lambda t^{\delta\eta-1}(R(t, I)), \\
0\text{ABR} & \phi^{\delta}_{\lambda}\left(Q(t)\right) = \lambda t^{\delta\eta-1}(R(t, Q)), \\
0\text{ABR} & \phi^{\delta}_{\lambda}\left(R(t)\right) = \lambda t^{\delta\eta-1}(R(t, R)), \\
0\text{ABR} & \phi^{\delta}_{\lambda}\left(V(t)\right) = \lambda t^{\delta\eta-1}(R(t, V)),
\end{align*}
\]

where,

\[
\begin{align*}
R(t, S) &= \Theta - \sigma SI - (\gamma + \theta)S, \\
R(t, E) &= \sigma SI - (\chi + \theta)E, \\
R(t, I) &= \chi E - (\phi + \sigma + \theta)I, \\
R(t, Q) &= \phi I - (\zeta + \theta)Q, \\
R(t, R) &= \phi I + \zeta Q - \theta R, \\
R(t, V) &= \gamma S - \theta V.
\end{align*}
\]

We can rewrite system (30) as:

\[
0\text{ABR} \phi_{\lambda}\left(A(t)\right) = \lambda t^{\delta\eta-1}X(t, A(t)), \\
A(0) = A^*;
\]

where

\[
A(t) = \begin{bmatrix}
S(t) \\
E(t) \\
I(t) \\
Q(t) \\
R(t) \\
V(t)
\end{bmatrix}, \\
0\text{ABR} \phi_{\lambda}\left(A(t)\right) = \begin{bmatrix}
S(0) \\
E(0) \\
I(0) \\
Q(0) \\
R(0) \\
V(0)
\end{bmatrix}, \\
X(t, A(t)) = \begin{bmatrix}
R(t, S) \\
R(t, E) \\
R(t, I) \\
R(t, Q) \\
R(t, R) \\
R(t, V)
\end{bmatrix}
\]

Let \(\text{B}(w)\) be a Banach space of the real-valued continuous functions with supremum norm defined on the interval \(w = [0, T]\) and \(T = \text{B}(w) \times \text{B}(w) \times \text{B}(w) \times \text{B}(w) \times \text{B}(w)\) with the norm \(\| X \| = \sup \{|X(t)| : t \in w\}\).

Define an operator \(Y : T \to T\) as:

\[
Y(X(t)) = X(0) + \frac{2(1-\delta)}{Q(\delta)} t^{\delta\eta-1}X(t, A(t)) + \frac{\delta\lambda}{Q(\delta)} \times \int_0^{T} t^{\lambda\delta-1}(t-\zeta)^{\frac{\delta}{\eta}}X(\zeta, A(\zeta))d\zeta.
\]

Now, by imposing growth and Lipschitz condition on \(X(t, A(t))\), in the form:

- For each \(A \in T\), \(\exists\) constants \(\vartheta_X\) and \(k_X\) such that

\[
|X(t, A(t))| \leq \vartheta_X |A(t)| + k_X.
\]

- For each \(X, \bar{X} \in T\), \(\exists\) a constant \(Y_X > 0\) such that

\[
\left|X(t, A(t)) - X\left(t, A(t)\right)\right| \leq Y_X |A(t) - \bar{A}(t)|.
\]

Theorem 6.1. The model (29) has at least one solution if \(X : [0, T] \times T \to \mathbb{R}\) is a continuous function and holds condition (36).

Proof: It is important to show that \(Y\) is continuous. As, \(X\) is continuous which means that \(Y\) is continuous.

Suppose \(Q = \{A \in B : \| A \| \leq \| \cdot \|, \| \cdot \| > 0\}\). Now for any \(X \in T\), we have:

\[
\| Y(X(t)) \| = \sup \left| X(0) + \frac{2(1-\delta)}{Q(\delta)} t^{\delta\eta-1}X(t, A(t)) \right|
\]

\[
\leq X(0) + \frac{2(1-\delta)}{Q(\delta)} \int_0^{T} t^{\lambda\delta-1}(t-\zeta)^{\frac{\delta}{\eta}}X(\zeta, A(\zeta))d\zeta
\]

\[
\leq X(0) + \frac{2(1-\delta)}{Q(\delta)} \left( \delta_X \| A \| + k_X \right)
\]

\[
\leq X(0) + \frac{2(1-\delta)}{Q(\delta)} \left( \delta_X \| A \| + k_X \right) \ell(\delta, \lambda) \leq \| \delta_X \| A \| + k_X \ell(\delta, \lambda).
\]

Here, \(\ell(\delta, \lambda)\) is the beta function.

Hence the operator \(Y\) is uniformly bounded.

For equicontinuity of \(Y\), let us take \(t_1 < t_2 \leq T\). Now, consider:

\[
\| Y(X(t_1)) - Y(X(t_2)) \| = \left| \frac{2(1-\delta)}{Q(\delta)} \int_0^{T} t^{\lambda\delta-1}(t-\zeta)^{\frac{\delta}{\eta}}X(\zeta, A(\zeta))d\zeta \right|
\]

\[
\leq \left( \frac{2(1-\delta)}{Q(\delta)} \delta_X \| A(t_1) \| + k_X \right) \ell(\delta, \lambda)
\]

\[
\leq \left( \frac{2(1-\delta)}{Q(\delta)} \delta_X \| A(t_2) \| + k_X \right) \ell(\delta, \lambda)
\]

\[
\leq \left( \frac{2(1-\delta)}{Q(\delta)} \delta_X \| A(t_2) \| + k_X \right) \ell(\delta, \lambda).
\]
Here, it is clearly noticed that when $t_1 \to t_2$ then $Y(X(t_2)) - Y(X(t_1)) \to 0$. Consequently, we can say that $\| Y(X(t_2)) - Y(X(t_1)) \| \to 0$ as $t_1 \to t_2$.

As a result, $T$ is a contraction. Hence, the provided model has a unique solution according to the Banach contraction principle.

6.2. Numerical scheme for the Fractal-Fractional model

Consider the system (29)

\[
\begin{align*}
0 & A \left( t \right) B R S (t) = I(t, S), \\
0 & A \left( t \right) B R S (E(t)) = \dot{I}(t, E), \\
0 & A \left( t \right) B R S (I(t)) = \dot{I}(t, I), \\
0 & A \left( t \right) B R S (Q(t)) = \dot{I}(t, Q), \\
0 & A \left( t \right) B R S (R(t)) = \dot{I}(t, R), \\
0 & A \left( t \right) B R S (V(t)) = \dot{I}(t, V).
\end{align*}
\]

We can also write:

\[
\begin{align*}
0 & A \left( t \right) B R S (S(t)) = I(t, S), \\
0 & A \left( t \right) B R S (E(t)) = I(t, E), \\
0 & A \left( t \right) B R S (I(t)) = I(t, I), \\
0 & A \left( t \right) B R S (Q(t)) = I(t, Q), \\
0 & A \left( t \right) B R S (R(t)) = I(t, R), \\
0 & A \left( t \right) B R S (V(t)) = I(t, V).
\end{align*}
\]

Now by replacing $0 \ A \ B \ C \ \varphi_\theta^\alpha$ with $0 \ A \ B \ C \ \varphi_\theta^\alpha$ and applying the procedure given in equation (5), we obtain the numerical algorithm in the following form:

\[
S_{n+1} = S(t_n) + \frac{\mu}{\Delta t} \dot{I}(t_n, S(t_n)) + \frac{\mu}{\Delta t} \sum_{i=0}^{n} \frac{\mu}{\Delta t} \dot{I}(t_i, S(t_i)) \left[ \frac{\varphi_\theta^\alpha(\dot{I}(t_i, S(t_i)))}{\varphi_\theta^\alpha(1)} \right] \left\{ (w+1-q)^{(w+2-q+\delta)} - (w-q)^{(w+2-q+2\delta)} \right\},
\]

7. Estimation of parameters

In this section, the estimation of parameters for the spread of COVID-19 among people in Australia for the time interval between 1st July 2021 and 20th August 2021. The cumulative reported infection cases for the given time interval are displayed through the bar chart given in Fig. 2. Some parameters are taken from the literature while some are fitted or estimated. The total population of Australia which is symbolized by $N(0)$ and recorded as $N(0) = 2587301$ for the year 2021 [41]. The number of cumulative infected individuals on July 1st 2021 was recorded as 30610, so we consider $I(0) = 30610$ [42]. Similarly, the number of Cumulative vaccinated individuals with only single dose and fully vacinated (double dose) on 1st July 2021 was given as 6,243,880 and 1,726,273 [43]. But here, we will consider only fully vacinated individuals, so $V(0) = 1726273$. The other initial conditions are arbitrary assumed. The average life span in Australia is 83.94 [41]. The birth rate is calculated as $\theta = N(0) \times \theta$ and Table 2 lists the other parameter’s values that were fitted. The dynamics of the investigated model are fitted for the values in Table 2, and the comparison is shown in Fig. 3. The parameter estimates with the confidence interval are given in Table 4. As seen in the figure, our model dynamics are in excellent agreement with the real data. The basic reproduction number $R_0$ is approximated as $R_0 \approx 1.58276$ based on these parameter values.
8. Global sensitivity analysis

A sensitivity analysis for an epidemic model’s objective is to find the most critical parameters associated with a certain intervention that has a substantial influence on disease dynamics. The characteristics that produce a substantial variation in the value of the fundamental reproduction number are of interest to us. The sensitivity indices can be used to determine the corresponding variation in the state variable caused by a parameter change. These indices have been calculated using the definition given in [44]. The sensitivity index is calculated using partial derivatives, as shown below:

\[ \psi_{\ell} = \frac{\partial R_0}{\partial \ell} \times \frac{\ell}{R_0} \]  

(43a)

The analytical formulae for the \( R_0 \) sensitivity indices may be computed using the definition presented above. The positive indices parameters in Table 3 indicate that COVID infection would rise in the population, whereas the negative indices parameters in Table 3 suggest that COVID infection will fall in the population. From the Table 3 and Fig. 4, it can be observed that the parameters having high negative indices are vaccination rate \( c \) and quarantine rate \( \theta \). It means that vaccination rate \( \gamma \) and quarantine rate \( \phi \) are the most sensitive parameters having negative indices which can control the value \( R_0 \) so that the COVID-19 transmission can be controlled by producing the awareness of quarantine and vaccination in the population. Other parameters having negative indices are \( \theta \) and \( \sigma \). \( \theta \) is the natural mortality rate which cannot be changed and is fixed for every specific region while \( \sigma \) is the recovery rate of infected individuals from the disease and this parameter can be increased by implementing proper medication and treatment of the infected individuals which will definitely effect the spread of infection. The sensitivity of other parameters \( \Theta, \chi \times \sigma \) are also portrayed in Fig. 4 and the values of sensitivity indices of all the parameters are given in Table 3.

| Parameter | Value | Confidence Interval |
|-----------|-------|---------------------|
| \( \sigma \) | \( 2.25 \times 10^{-9} \) | \( (0,5.87 \times 10^{-9}) \) |
| \( \chi \) | 0.0067 | \( (0.001,0.009) \) |
| \( \phi \) | 0.003 | \( (0.0014,0.0057) \) |
| \( \gamma \) | 0.00031 | \( (0,0.00052) \) |
| \( \sigma \) | 0.00045 | \( (0.00232,0.00077) \) |
| \( \xi \) | 0.001 | \( (0.00034,0.002) \) |

Table 3  Sensitivity Indices of \( R_0 \) versus different parameters.

| Parameter | Sensitivity Indices |
|-----------|--------------------|
| \( \sigma \) | 1 |
| \( \chi \) | 0.04455 |
| \( \phi \) | \( -0.861416 \) |
| \( \gamma \) | \( -0.904742 \) |
| \( \sigma \) | \( -0.129212 \) |
| \( \Theta \) | 1 |
| \( \theta \) | \( -0.14918 \) |

Table 2  Parameters values for the considered model:

| Parameter | Value | Source |
|-----------|-------|--------|
| \( \sigma \) | \( 2.25 \times 10^{-9} \) | Fitted |
| \( \chi \) | 0.0067 | Fitted |
| \( \phi \) | 0.003 | Fitted |
| \( \gamma \) | 0.00031 | Fitted |
| \( \sigma \) | 0.00045 | Fitted |
| \( \xi \) | 0.001 | Fitted |
| \( \Theta \) | \( N(0) \times \theta \) | Estimated |
| \( \theta \) | \( \frac{1}{2} \) | [41] |

Fig. 2  Cumulative Cases of COVID-19 between 1st July 2021 and 20th August 2021.

Fig. 3  Real Data versus Curve Fitting.
9. Results and discussions

This section provides the graphical results of the considered COVID-19 model. The influence of fractional parameter $\delta$ and fractal dimension $\lambda$ is portrayed in Figs. 5 and 6 respectively. By varying $\delta$ as well as $\lambda$ gives us interesting results and a variety of solutions rather than one such as integer order of classical models. On the basis of chosen interval of time that is between 1st July 2021 and 20th August 202, the spread of infection is predicted for the next 600 days. It is worth mentioning that by taking $\lambda \rightarrow 1$, the considered fractal-fractional model of COVID-19 reduces to the simple fractional model of Atangana-Baleanu (AB) fractional model. Similarly, by keeping $\lambda \rightarrow 1$ and $\delta \rightarrow 1$, the model as well as the solution reduce to the classical sense which is given in model (1). Hence, it proves that the considered fractal-fractional COVID-19 model is more general than the simple fractional and classical one and it can lead us to best fit the real data with the solution of considered problem by adjusting both the parameters $\delta$ and $\lambda$.

The influence of vaccination rate $\gamma$ is depicted in Fig. 7. It can be observed from the graph that by increasing values of $\gamma$ increases the population in vaccination class $V(t)$ and decreases the susceptible population $S(t)$ more quickly. This is because, by increasing $\gamma$, it means that we are increasing the vaccination awareness in the people and more people are going to be vaccinated. These people leave the susceptible class $S(t)$ and join the vaccinated class $V(t)$ which have a strong immunity as comparatively to the susceptible people and by this way, $V(t)$ increases while $S(t)$ decreases over time. Similarly, increase in $\gamma$ also decreases the exposed $E(t)$, infected $I(t)$, quarantined $Q(t)$ as well as recovered $R(t)$. This is due of decrease in susceptible individuals because if the number of susceptible class $S(t)$ decreases and more people join the vaccinated class $V(t)$, so there will be less chances of infection and less individuals will be infected which leads us to decrease in $E(t)$ and $I(t)$. In the same way, if there minimum of people got infected then it will affect the quarantined $Q(t)$ as well as recovered class $R(t)$ because the route of these classes are dependent on $I(t)$. If $I(t)$ decreases, then $Q(t)$ and $R(t)$ decreases because there will be not more population left in $I(t)$ to be recovered or quarantined.

The influence of quarantine rate $\phi$ on the COVID-19 dynamics is portrayed in Fig. 8. It may be observed in the figure that by increasing $\phi$ decreases the exposed $E(t)$ as well as infected class $I(t)$. This shows that the awareness of quarantine among infected individuals can drastically decreases the infection in the population because it will decrease the interaction among infected and susceptible individuals. Similarly, increase in $\phi$ increases $S(t)$, $V(t)$ and $Q(t)$. By increasing $\phi$, $S(t)$ will take more time to vanish. This is due to decrease in $E(t)$ and $I(t)$ because it will decrease the interaction between healthy people and infected ones which will decrease the spread of infection in the susceptible class and we will have more time to vaccinate the susceptible individuals which also increases the number of vaccinated people $V(t)$ over time. In the same way, increase in $\phi$, it increases the awareness among infected individuals to be quarantined which increases the number of quarantined individuals $Q(t)$.

Figs. 9 and 10 depicts the influence of sensitive and control parameters on basic reproduction number $R_0$. In Fig. 7, the influence of contact rate $\sigma$ and vaccination rate $\gamma$ versus $R_0$ through 3D and contour plots. It can be seen clearly that $\sigma$ increases $R_0$ while $\gamma$ reduces $R_0$. It means that by increasing awareness of vaccination among people and reducing contact rate $\sigma$ through awareness can dramatically decreases the value of $R_0$ which results to decrease the infection among the population. Similarly, Fig. 10 shows the impact of incubation period $\chi$ and quarantine rate $\phi$ versus $R_0$. It is noticed that $\chi$ rises while $\phi$ decreases the value of $R_0$. Hence, it is concluded that by producing awareness of being quarantine among the infected individuals can drastically control the infection among the population.

10. Concluding remarks

The dynamics of COVID-19 in the human population is examined in this paper. This is performed by the formulation of a mathematical model based on a highly nonlinear coupled system of ODEs with ICs. Using the fractal-fractional (FF) differential operator, the classical system of ODEs is then transformed into a fractal-fractional model. The model’s equilibrium and positivity are also demonstrated. Furthermore, the next generation approach is used to estimate the basic reproduction number. The fractal-fractional model’s solution has also been proven to be unique. In order to find the model’s most sensitive parameters, a sensitivity analysis of the current model is also done. For the considered fractal-fractional COVID-19 model, a numerical method is built, and simulation is done using MATLAB. The model is calibrated using real COVID-19 data collected in Australia between July 1 and August 20, 2021. Some of the parameters for data fitting are obtained from the literature while some are fitted and based on these fitted parameters values, basic reproduction number is calculated as $R_0 \approx 1.58276$. Additionally, The effect of fractional parameter and fractal dimension is shown graphically. Furthermore, the effect of various sensitive and control parameters on the COVID-19 dynamics is graphically depicted, as well as the impact of these parameters on the basic reproduction number, using 3D graphs and contours. It has been established that being conscious of being quarantined and getting
adequate vaccination can help us reduce the virus risk. We argue that if the government restricts individuals to their homes only, avoids shaking hands, follows appropriate hand washing procedures, limits travels to endemic regions, limits locations where illness may spread further, and closely monitors physical and social distances, the number of infected cases would decrease more quickly. Due to high rate of spreading and occurrence of second and third wave throughout the world, scientist and researcher across the globe are doing hard work and paying more attention towards the development of effective vaccines and screening and designing of natural and synthetic compounds along with the repurposing study on FDA approved several drugs. It may help to combat the COVID-19 in Australia along with the globe.

Fig. 5 Influence of fractional parameter $\delta$ on the COVID-19 dynamics among various classes.
Fig. 6  Influence of fractal dimension $\lambda$ on the COVID-19 dynamics among various classes.
Fig. 7 Impact of vaccination rate $\gamma$ on the COVID-19 dynamics in different compartments.
Fig. 8  Impact of quarantine rate $\phi$ on COVID-19 dynamics in different compartments.
Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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