Application of differential evolution algorithm in the problems of gliding descent optimization

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Abstract. Despite the closure of the Space Shuttle program, spaceplane remain a promising vehicle, which is being developed in many countries, such as USA, European Union and China. Interest in spaceplane is due to the possibility of their reusability and wider maneuverability in comparison with reentry capsule. However, it’s reusability is directly related to satisfying the temperature limit while descending from orbit. Also, the spaceplane has ricochets in its trajectory due to a high lift-to-drag ratio that decreases landing accuracy. The paper aim is the synthesis of an optimal control program in the problems of gliding descent to the maximizing longitudinal range and lateral range with temperature limitation and the absence of ricochets. The synthesis of the optimal trajectory can be obtained by the control channels of attack angle and bank angle. The approach is to represent the control channels of attack angle and bank angle in the form of Fourier series, and the search for the optimal solution is carried out using the differential evolution algorithm. The paper presents the results of solving problems using a different number of the Fourier series terms.

1. Introduction
Currently the only active spaceplane is a Boeing-X37B. Despite this, many countries are developing spaceplane, for example, American Dream Chaser, European Space RIDER and Chinese Shenlong.

One of the classic problems inherent in a spaceplane is its gliding descent. In this regard, various optimization methods are currently being developed to solve this problem. For example, in paper [1] a hybrid heuristic optimization method Fractional-Order Particle Swarm Gravitational Search algorithm (FPSOGSA) is proposed that has better accuracy and convergence compared to another considered optimization method. FPSOGSA was used for gliding descent taking into account limitations on the dynamic pressure and heating rate. In another article [2] guidance method based on analytical prediction with a self-adaptive capability during missions was proposed. Zhou et al [3] proposed using the altitude-range phase portrait to optimize the gliding descent and to track the fulfillment of the restriction on phase variables instead of attack angle. A similar technique was also presented in paper [4]. However, in addition to the altitude-range phase portrait, the altitude-velocity is also used and it does not rely on any pre-planned attack angle scheme. The paper [5] proposed a method in which the dynamic pressure is chosen as the design parameter, since it reduces the complexity of problem. According to the assurances of the authors, using this approach and the application of the particle swarm method allows to obtain an optimal trajectory with lower computational costs compared to pseudospectral methods.

This paper considers the problems of gliding descent to the maximum longitudinal range and lateral range taking into account limitations on the temperature and the absence of ricochets. Optimization problems are solved by the differential evolution algorithm that was firstly proposed by Storn and Price.
Due to its efficiency, the differential evolution algorithm is actively used in aerospace engineering. One of the examples of the application of the differential evolution algorithm is the determination of the spacecraft attitude and moments of inertia due to the current collection from solar panels [7]. On the basis of this algorithm, authors [8] developed constraint differential evolution, which made it possible to solve the problem of unmanned aerial vehicle (UAV) path planning in the disaster scenario. The developed method was compared with other variations of differential evolution and showed the best result when solving a problem with restrictions on flight altitude, angle of the UAV and limited UAV slope. Melton [9] compared the particle swarm method and differential evolution in the time-optimal slew-maneuver problem and found that differential evolution is more efficient. He also found that the combination of these optimization methods can reduce computation time by 40%.

2. Mathematics model

2.1. Flight dynamics

It is more convenient to solve problems in the trajectory coordinate system. In this coordinate system, the following axes are taken: X-axis is directed to the spaceplane velocity vector, and Y-axis perpendicular to X-axis and is directed upward from the Earth surface [10]. The system of differential equations in the trajectory coordinate system that describes mass center motion relative to ellipsoidal Earth has the following view [11]:

\[
\begin{align*}
\dot{V} &= -\sigma_x \rho V^2 + g_e \sin \theta + g_{\phi} \sin \chi \cos \theta + \frac{P_x}{m} + R \omega_E^2 \cos \varphi \left(\sin \theta \cos \varphi - \cos \theta \sin \varphi \sin \chi\right), \\
\dot{\theta} &= \sigma_x \rho V \cos \gamma_a + \left(\frac{V}{R} - \frac{g_e}{V}\right) \cos \theta - \frac{g_{\phi}}{V} \sin \chi \sin \theta + \frac{P}{Vm} \sin \chi \sin \theta + 2 \omega_E \cos \varphi \cos \chi + \\
&\quad + \frac{R \omega_E^2}{V} \cos \varphi \left(\cos \theta \cos \varphi + \sin \theta \sin \varphi \sin \chi\right), \\
\dot{\chi} &= -\sigma_x \rho V \sin \gamma_a - \frac{V \cos \theta}{R} \tan \varphi \cos \chi + g_{\phi} \frac{R \cos \chi}{V \cos \theta} - \frac{P_x}{Vm \cos \theta} - 2 \omega_E \left(\sin \varphi - \cos \varphi \sin \chi \tan \theta\right) - \frac{R \omega_E^2}{V \cos \theta} \sin \varphi \cos \varphi \cos \chi, \\
\dot{R} &= V \sin \theta, \\
\dot{\varphi} &= \frac{V \cos \theta}{R} \sin \chi, \\
\dot{\lambda} &= \frac{V \cos \theta \cos \chi}{R \cos \varphi}, \\
\dot{L} &= \dot{\lambda} R_s, \\
\dot{D} &= \dot{\varphi} R_s,
\end{align*}
\]

where \(V\) is spaceplane velocity, \(\theta\) is trajectory angle, \(\chi\) is path angle, \(R\) is radius-vector of mass center, \(\varphi\) is geocentric latitude, \(\lambda\) is geocentric longitude, \(m\) is spaceplane mass, \(\sigma_x, \sigma_{\phi}\) are ballistic coefficients, \(\rho\) is air density, \(g_e, g_{\phi}\) are radial and meridional components of the gravitational acceleration respectively, \(P_x, P_{\phi}, P\) are projections of thrust on the corresponding axis, \(\omega_E\) is angular velocity of Earth rotation, \(\gamma_a\) is bank angle, \(L\) is longitudinal range, \(D\) is lateral range, \(R_s\) is Earth’s surface radius.

The air density \(\rho\) at flight altitude \(H\) is calculated by the exponential form [12]:
\[ \rho = \rho_{45} \exp \left( -\frac{H - H_{45}}{R_T M} \right). \]  

where \( \rho_{45} \) is air density at flight altitude of 45 km, \( R_g \) is universal gas constant, \( T_M \) is molar temperature. The flight altitude \( H \) for ellipsoidal Earth is obtained as:

\[ H = R - R_e. \]

Here \( R_e \) is Earth’s surface radius at a given latitude \( \varphi \) defined as

\[ R_e = R_e \left( 1 - \alpha_{cmp} \sin^2 \varphi \right), \]

where \( R_e \) is Earth’s equator radius, \( \alpha_{cmp} = 1/298.25 \) is Earth polar compression.

The radial and meridional components of the gravitational acceleration have the following view:

\[ g_r = \frac{\mu}{R^2} \left( 1 + \left( \alpha_{cmp} - \frac{q}{2} \right) \left( \frac{R}{R_e} \right)^2 \sin^2 \varphi \right), \]

\[ g_\varphi = -\frac{\mu}{R^2} \left( \alpha_{cmp} - \frac{q}{2} \right) \left( \frac{R}{R_e} \right)^2 \sin 2\varphi, \]

here \( \mu \) is standard gravitational parameter, \( q \) is ratio of centripetal acceleration to the gravitational acceleration on the equator:

\[ q = \frac{\omega_e^2 R_e^3}{\mu}. \]

Projections of thrust \( P \) on the corresponding axes are obtained by the following relations:

\[ P_x = P \cos \alpha, \]

\[ P_y = P \sin \alpha \cos \gamma_a, \]

\[ P_z = P \sin \alpha \sin \gamma_a, \]

where \( \alpha \) is attack angle.

Ballistic coefficients are calculated by the following formulas:

\[ \sigma_x = \frac{C_D S}{2m}, \]

\[ \sigma_y = \frac{C_L S}{2m}, \]

where \( C_D, C_L \) are drag coefficient and lift coefficient, respectively, and \( S \) is wing area.

Since drag coefficient \( C_D \) and lift coefficient \( C_L \) are functions of attack angle \( \alpha \), flight altitude \( H \) and Mach number \( M \), the three-cubic polynomials are used to describe them accurately:

\[ C_D(\alpha, H, M) = \sum_{i=0}^{3} \sum_{j=0}^{3} \sum_{k=0}^{3} a_{i,j,k} \alpha^i H^j M^k, \]

\[ C_L(\alpha, H, M) = \sum_{i=0}^{3} \sum_{j=0}^{3} \sum_{k=0}^{3} b_{i,j,k} \alpha^i H^j M^k, \]

where \( a_{i,j,k}, b_{i,j,k} \) are polynomial coefficients obtained by solving the system of linear equations.
Calculation of heat flux $q_T$ and temperature at the stagnation point can be performed by the following relations:

$$q_T = 1.27 \cdot 10^{-3} \left( \frac{\rho}{r_{crv}} \right)^{1/2} V^{3.05},$$  \hspace{0.5cm} (15)

$$T = \left( \frac{q_T}{\nu \sigma} \right)^{1/4},$$  \hspace{0.5cm} (16)

where $r_{crv}$ is radius of nose, $\varepsilon$ is emissivity, $\sigma$ is Stefan-Boltzmann constant.

2.2. Problem formulation

The gliding descent problem to the maximum longitudinal range can be formulated as follows: it is necessary to synthetize a control by attack angle $\alpha(t)$ that maximizes longitude at the finite flight altitude:

$$F(H_f = 20) = \lambda \rightarrow \max.$$  \hspace{0.5cm} (17)

Similarly, gliding descent problem to the maximum lateral range can be formulated: it is necessary to synthetize a control by attack angle $\alpha(t)$ and bank angle $\gamma(t)$ that maximizes latitude at the finite flight altitude:

$$F(H_f = 20) = \varphi \rightarrow \max.$$  \hspace{0.5cm} (18)

The following penalty function is used to take into account the limitations on the temperature at the stagnation point $T_{lim}$ and the absence of ricochets:

$$F(t_f) = \lambda - Q \frac{T_{max}}{T_{lim}} - Q_1 \theta_{max} \rightarrow \max.$$  \hspace{0.5cm} (19)

$$F(t_f) = \varphi - Q \frac{T_{max}}{T_{lim}} - Q_1 \theta_{max} \rightarrow \max.$$  \hspace{0.5cm} (20)

where $Q, Q_1$ are positive integers.

3. Numerical approach

3.1. Technique of optimal solution searching

As it is shown in [11] control program can have quite a complex view. Thus, control channels of attack angle and bank angle can be presented as Fourier series, that can describe a complex dependence and be easily scaled in time:

$$\alpha(t) = \frac{a_0}{2} + \sum_{n=1}^{k} A_n \cos \left( \frac{2\pi k}{2\tau} t + \theta_k \right),$$  \hspace{0.5cm} (21)

$$\gamma(t) = \frac{g_k}{2} + \sum_{n=1}^{k} G_n \cos \left( \frac{2\pi k}{2\tau} t + \theta_k \right),$$  \hspace{0.5cm} (22)

where $\tau$ is integration time.

Thus, the differential evolution algorithm will search for the value of $a_0, g_0, A_k, G_k, \theta_k$ and $\theta_k$ corresponding to the optimum.

The following technique is proposed to find the solution of optimization problem on the basis of the Fourier series and the differential evolution algorithm:

- In the first approximation, the control program has the form of a Fourier series with $k = 3$.  

• After obtaining an approximate solution using the Fourier series with $k = 3$, the value of $k$
increases until the difference in the functional between the current and previous iterations does
not exceed 1%.
• A value increase $k$ in the Fourier series can lead to a significant decrease in the solution
convergence by using parameter $F = 0.5$. To increase the convergence solution, parameter $F$
can be taken as a random variable at each iteration, that is chosen according to a uniform law.

3.2. Initial data

The solution of the tasks is carried out for a hypothetical spaceplane shown in Figure 1. The
spaceplane aerodynamic characteristics were obtained in paper [13].

![Geometrics model of hypothetical spaceplane](image)

Figure 1. Geometrics model of hypothetical spaceplane

The spaceplane has design parameters that are presented in Table 1.

| Parameter          | Value |
|--------------------|-------|
| Wing area (m²)     | 18.44 |
| Radius of nose (m) | 0.5   |
| Mass (kg)          | 5000  |

Table 1. Spaceplane design parameters

Limitations on temperature at the stagnation point, trajectory angle and control are shown in Table
2.

| Parameter                          | Min. Value | Max. Value |
|------------------------------------|------------|------------|
| The temperature at the stagnation point (°C) | -          | 1600       |
| Trajectory angle (degree)          | -          | 0          |
| Attack angle (degree)              | 0          | 40         |
| Bank angle (degree)                | 0          | 180        |

Table 2. Limitations
Initial conditions of spaceplane motion are presented in Table 3.

Table 3. Initial conditions

| Parameter                | Value |
|--------------------------|-------|
| Flight altitude (km)     | 100   |
| Velocity (km/s)          | 7.367 |
| Trajectory angle (degree)| -1    |
| Path angle (degree)      | 0     |
| Longitude (degree)       | 0     |
| Latitude (degree)        | 0     |

4. Results

4.1. The gliding descent to maximum to the maximum longitudinal range

It is necessary to assess spaceplane maximum maneuverability before starting to solve the gliding descent problem to the maximum longitudinal range. To carry out this assessment, it is necessary to solve the posed problem without taking into account the restrictions on the phase variables. The differential evolution algorithm with $k = 2$ in Fourier series formed control of the attack angle in such a way, that the lift-to-drag ratio was close to the its maximum value as it is shown in Figure 2. The maximum longitudinal range is 15915 km. An increase in $k$ to 4 increases the value of the longitudinal range to 15920 km, which indicates that optimal control was obtained at $k = 2$. However, using this control of the attack angle leads to multiple ricochets in the trajectory and the temperature at the stagnation point exceeds 2000 °C (Figure 3). Thus, it is necessary to solve this problem taking into account the restrictions on the temperature at the stagnation point and the absence of ricochets.

Solving the problem taking into account limitation of temperature at the stagnation point lead to a significant change in the control program of the attack angle (Figure 4). It is expressed in the spaceplane motion for most of the trajectory at values of attack angle, close to the maximum allowable. The spaceplane motion with this control program led to a noticeable decrease in the intensity of ricochets, and therefore it several times “touches” the limiting temperature value (Figure 5). The longitudinal range decreased to 13018 km compared to the longitudinal range obtained without taking into account restrictions.
Increasing of parameter $k$ to 4 led to a noticeable decrease in the spaceplane motion at values of attack angle, that are close to the maximum permissible (Figure 6). Such changes led to a significant decrease in the height of the first ricochet, as a result of which the flight duration increased and, consequently, the longitudinal range increased to 14060 km (Figure 7).

A further increase of parameter $k$ to 6 in Fourier series resulted in a noticeable change in the control program of attack angle (Figure 8). The initial value of attack angle decreased to 35 degrees and subsequently its value is close to the maximum lift-to-drag ratio. This resulted in an increase in the time of the spaceplane motion and, accordingly, the longitudinal range, which increased to 15420 km (Figure 9). The increasing of parameter $k$ to 8 in the Fourier series resulted in a slight increase in longitudinal range. Thus, the control obtained for $k = 6$ in the Fourier series can be considered as optimal. Despite the obtained optimal control, there are ricochets in the spaceplane trajectory and therefore it is necessary to introduce an additional restriction on the trajectory angle $\theta$. 
The solution of the problem, taking into account the limitations on two phase variables with a constant parameter $F = 0.5$ has weak convergence. Consequently, to resolve this problem, the parameter $F$ was taken as a random variable at each iteration of optimization procedure. The presence of the second restriction resulted in a slight change in the control program of attack angle (Figure 10). At the initial time moment, the spaceplane flies with the maximum allowable value of attack angle to smooth out the first ricochet. Subsequently, the attack angle assumes a value close to the maximum lift-to-drag ratio. The use of this control program led the spaceplane trajectory to a smooth view, in which it flies close to the temperature limit for a larger motion section (Figure 11). The presence of the second restriction decreased the longitudinal range to 14916 km.

4.2. The gliding descent to maximum to the maximum lateral range
As in the problem of gliding descent to the maximum longitudinal range, the solution of this problem to the maximum lateral range firstly performed without taking into account the restrictions to assess the limiting maneuverability of spaceplane. The control program of attack angle was formed in such a way that its value corresponded to the maximum lift-to-drag ratio (Figure 12). To achieve maximum lateral range, the initial value of bank angle is 97 degrees and its value decreases almost monotonically. The lateral range value is 2563 km. An increase of parameter $k$ to 4 increases the value of the lateral range
to 2567 km, that indicates that optimal control was obtained at $k = 2$. However, the use of this control program is unacceptable due to the presence of ricochets in the spaceplane trajectory and exceeding the temperature limit, the value of which was 2137 °C.

The presence of a temperature limitation led to a noticeable change in the spaceplane control program (Figure 14). This is expressed in its movement at the maximum admissible values of the attack angle for $\approx 600$ seconds. Also, the initial value of the bank angle significantly decreased to 55 degrees. Such changes resulted in decrease in the immersion depth of the spaceplane from 61 km to 75 km at the first ricochet due to the fulfillment of the temperature limit. The lateral range value is 2033 km when using this control program.

The use of Fourier series with $k = 4$ resulted in a slight decrease in the spaceplane motion time at the maximum permissible values of attack angle (Figure 16). On the other hand, the bank angle control program has changed significantly, the form of which has a more complex view. Such changes in control programs led to a noticeable smoothing of the trajectory, which has only one ricochet (Figure 17). The lateral range increased to 2074 km.
The presentation of control programs for the attack angle and bank angle in the form of a Fourier series with \( k = 6 \) resulted in a significant reduction in the spaceplane motion time at the maximum permissible values of attack angle (Figure 18). The bank angle control program has also undergone significant changes. At the initial moment of time, the spaceplane flies with a zero bank angle and then with larger values in comparison with the program obtained with \( k = 4 \). The use of formed control programs has led to a complete smoothing of the spaceplane trajectory in the presence of only temperature limitation (Figure 19). With this control program, the lateral range value increased to 2281 km.

Finally, a further increase of parameter \( k \) to 8 in the Fourier series resulted in a minor change in the control programs (Figure 20). The time of spaceplane motion at the maximum admissible values of attack angle slightly increased, and at the zero value of bank angle decreased. Despite the minor changes in the control programs, the trajectory was formed in such a way, that the spaceplane moves along the temperature limit (Figure 21). Wherein, the trajectory also has smooth view. The lateral range increased to 2369 km. The synthesis of optimal control with \( k = 10 \) in the Fourier series resulted in slight increase in lateral range to 2389 km. Therefore, the control obtained for \( k = 8 \) in Fourier series can be considered as optimal.
5. Conclusion
The optimization problem of spaceplane gliding descent was considered. The optimal control program was obtained by the represent of control channels of attack angle and bank angle, and the differential evolution algorithm. In conclusion, the next points can be claimed:

- The problem of gliding descent optimization with maximizing longitudinal and lateral ranges without limitations can be solved with \( k = 3 \) in the Fourier series. The maximum values of longitudinal and lateral ranges are 15915 km, and 2563 km, respectively.
- The parameter \( k = 7 \) in the Fourier series was used for obtaining optimal control of gliding descent to the maximum longitudinal range, taking into account both one and two limitations on the phase variables. However, at each iteration the random parameter \( F \) was used to obtain optimal control with two limitations due to the low solution convergence at its constant value. The maximum value of longitudinal range taking into account one and two limitations on phase variables are 15420 km and 14916 km, respectively.
- The optimal problem of gliding descent solution to the maximum lateral range taking into account limitations can be solved by using parameter \( k = 9 \) in the Fourier series. Similarly, with problem of gliding descent to the maximum longitudinal range with two restrictions, calculation carried out with the random parameter \( F \) at each iteration. The optimal control program obtained with temperature limitation also satisfied restriction of the absence of ricochets. The maximum value of lateral range is 2369 km.

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