Programmable View Update Strategies on Relations

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ABSTRACT

View update is an important mechanism that allows updates on a view by translating them into the corresponding updates on the base relations. The existing literature has shown the ambiguity of translating view updates. To address this ambiguity, we propose a robust language-based approach for making view update strategies programmable and evaluable. Specifically, we introduce a novel approach to use Datalog to describe these update strategies. We propose a validation algorithm to check the well-behavedness of the written Datalog programs. We present a fragment of the Datalog language for which our validation is both sound and complete. This fragment not only has good properties in theory but is also useful for solving practical view updates. Furthermore, we develop an algorithm for optimizing user-written programs to efficiently implement updatable views in relational database management systems. We have implemented our proposed approach. The experimental results show that our framework is feasible and efficient in practice.

1. INTRODUCTION

View update [11, 20, 21, 22, 33] is an important mechanism in relational databases. This mechanism allows updates on a view by translating them into the corresponding updates on the base relations [21]. Consider a view V defined by a query get over the database S, as shown in Figure 1a. An update translator T maps each update u on V to an update T(u) on S such that it is well-behaved in the sense that after the view update is propagated to the source, we will obtain the same view from the updated source, i.e., u(V) = get(T(u)(S)). Given a view definition get, the known view update problem [21] is to derive such an update translator T.

However, there is an ambiguity issue here. Because the query get is generally not injective, there may be many update translations on the source database that can be used to reflect view update [20, 21]. This ambiguity makes view update an open challenging problem that has a long history in database research [22, 20, 21, 11, 34, 33, 30, 36, 45, 42, 41]. The existing approaches either impose too many syntactic restrictions on the view definition get that allow for limited unambiguous update propagation [21, 15, 11, 35, 43, 41, 44, 45, 46] or provide dialogue mechanisms for users to manually choose update translations with users’ interaction [34, 42]. In practice, commercial database systems such as PostgreSQL [4] provide very limited support for updatable views such that even a simple union view cannot be updated.

In this paper, we propose a new approach for solving the view updating problem practically and correctly. The key idea is to provide a formal language for people to directly program their view update strategies. On the one hand, this language can be considered a formal treatment of Keller’s dialogue [34], but on the other hand, it is unique in that it can fully determine the behavior of bidirectional update propagation between the source and the view.

This idea is inspired by bidirectional programming [25, 19] in the programming language community, where update propagation from the view to the source is formulated as a put, a so-called putback transformation, which maps the updated view and the original source to an updated source, as shown in Figure 1b. This put not only captures the view update strategy but also fully describes the view update behavior. First, it is clear that if we have such a put, the translation T is obtained for free:

$$T(u)(S) = \text{put}(S, u(\text{get}(S)))$$

Second, and more interestingly, while there may be many puts for a given get, there is at most one get for a given put for a well-behaved view update [32, 24, 23, 38, 37]. Thus, the corresponding get can be deterministically derived from a put in general. Although several languages have been proposed for writing put for updatable views over tree-like data structures [56, 38, 37], whether we can design such a language for solving the traditional view update problem on relations remains unclear.

There are several challenges in designing a formal language for programming put, a view update strategy, on relations.

- The language is desired to be expressive in practice to cover users’ update strategies.

Figure 1: The view update problem (a) and bidirectional transformation (b).
To make every view update consistent with the source database, an update strategy put must satisfy some certain properties, as formalized in previous work [25, 23, 24]. Therefore, there is a need for a validation algorithm to statically check the well-behavedness of user-written strategies and whether they respect the view definition if the view is defined beforehand.

To be useful in practice rather than just a theoretical framework, the language must be efficiently implemented when running in relational database management systems (RDBMSs).

In contrast to the existing approaches [56, 38, 37] where new domain-specific languages (DSLs) are designed, we argue that Datalog, a well-known query language, can be used as a formal language for describing view update strategies in relational databases. Our contributions are summarized as follows.

We introduce a novel way to use nonrecursive Datalog with negation and built-in predicates for describing view update strategies. We propose a validation algorithm for statically checking the well-behavedness of the described update strategies.

We identify a fragment of Datalog, called linear-view guarded negation Datalog (LVGN-Datalog), in which our validation algorithm is both sound and complete. Furthermore, the algorithm can automatically derive from view update strategies the corresponding view definition to confirm the view expected beforehand.

We develop an incrementalization algorithm to optimize view update strategy program. This algorithm integrates the standard incrementalization method for Datalog with the well-behavedness in view update.

We have implemented all the algorithms in our framework, called BIRDS. The experiments on a benchmarks collected in practice show that our framework is feasible for checking most of the view update strategies. Interestingly, LVGN-Datalog is expressive enough for solving many types of views in practice and can be efficiently implemented by incrementalization in existing RDBMSs.

The remainder of this paper is organized as follows. After presenting some basic notions in Section 2, we present our proposed method for specifying view update strategies in Datalog in Section 3. The validation and incrementalization algorithms for these update strategies are described in Section 4 and Section 5, respectively. Section 6 shows the experimental results of our implementation. Section 7 summarizes related works. Section 8 concludes this paper.

2. PRELIMINARIES

In this section, we briefly review the basic concepts and notations that will be used throughout this paper.

2.1 Datalog and Relational Databases

Relational databases. A database schema $D$ is a finite sequence of relation names (or predicate symbols, or simply predicates) $(r_1, \ldots, r_n)$. Each predicate $r_i$ has an associated arity $n_i > 0$ or an associated sequence of attribute names $A_1, \ldots, A_{n_i}$. A database (instance) $D$ of $D$ assigns to each predicate $r_i$ in $D$ a finite $n_i$-ary relation $R_i$. $D(r_i) = R_i$.

An atom (or atomic formula) is of the form $r(t_1, \ldots, t_k)$ (or written as $r(\vec{t})$) such that $r$ is a $k$-ary predicate and each $t_i$ is a term, which is either a constant or a variable. When $t_1, \ldots, t_k$ are all constants, $r(t_1, \ldots, t_k)$ is called a ground atom.

A database $D$ can be represented as a set of ground atoms [18, 17], where each ground atom $r(t_1, \ldots, t_k)$ corresponds to the tuple $\langle t_1, \ldots, t_k \rangle$ of relation $R_i$ in $D$. As an example of a relational database, consider a database $D$ that consists of two relations with respective schemas $r_1(A, B)$ and $r_2(C)$. Let the actual instances of these two relations be $R_1 = \{(1, 2), (2, 3)\}$ and $R_2 = \{(3), (4)\}$, respectively. The set of ground atoms of the database is $D = \{r_1(1, 2), r_1(2, 3), r_2(3), r_2(4)\}$.

Datalog. A Datalog program $P$ is a nonempty finite set of rules, and each rule is an expression of the form [18]:

$$H := L_1, \ldots, L_n,$$

where $H, L_1, \ldots, L_n$ are atoms. $H$ is called the rule head, and $L_1, \ldots, L_n$ is called the rule body. Predicates in $P$ are divided into two categories: extensional database (EDB) and intensional database (IDB) predicates. Each EDB predicate corresponds to a relation called the EDB relation, which is physically stored in a database $D$. Meanwhile, IDB predicates occur in the head of some rules in the Datalog program. Following the convention used in [18], throughout this paper, we use lowercase characters for predicate symbols and uppercase characters for variables in Datalog programs. In a Datalog rule, variables that occur exactly once can be replaced by an anonymous variable, denoted as “.”.

A Datalog program takes as input a database $D$ of EDB relations to derive all relations, called IDB relations, corresponding to IDB predicates in $P$. If we restrict the output of a Datalog program $P$ to a single IDB relation $R$ corresponding to IDB predicate $r$, then we have a Datalog query, denoted as $(P, R)$. We say that an IDB predicate $r$ is satisfiable (or the query $(P, R)$ is satisfiable) if there exists a database $D$ such that the result of $P$ over $D$ restricted to $R$ is nonempty [10].

We can extend Datalog by allowing built-in predicates, such as equality (=) or comparison (>), in the Datalog rule bodies but in a safe way in which all the variables in the built-in predicate must appear in some positive atoms [18].

2.2 Bidirectional Transformations

A bidirectional transformation (BX) [25] is a pair of a forward transformation get and a backward (putback) transformation put, as shown in Figure 1b. The forward transformation get is a query over a source database $S$ that results in a view relation $V$. The putback transformation put takes as input the original database $S$ and an updated view $V'$ to produce a new database $S'$. To ensure consistency between the source database and the view, a BX must satisfy the following round-tripping properties, called GetPut and

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1 A prototype implementation is available at https://dangtv.github.io/BIRDS/.
3. THE LANGUAGE FOR VIEW UPDATE STRATEGIES

As mentioned in the introduction, it may be surprising that the base language that we are using for view update strategies is nonrecursive Datalog with negation and built-in predicates (e.g., =, ≠, <, >) [18]. One might wonder how the pure query language Datalog can be used to describe updates. In this section, we show that delta relations enable Datalog to describe view update strategies. We will define a fragment of Datalog, called LVGN-Datalog, which is not only powerful for describing various view update strategies but also important for our later validation.

3.1 Formulating Update Strategies as Queries Producing Delta Relations

Recall that a view update strategy is a putback transformation put that takes as input the original source database and an updated view to produce an updated source. Our idea of specifying the transformation put in Datalog is to write a Datalog query that takes as input the original source database and an updated view to yield updates on the source; thus, the new source can be obtained.

We use delta relations to represent updates to the source database. The concept of delta relations is new and is used in the study on the incrementalization of Datalog programs [28]. Unlike the use of delta relations to describe incrementalization algorithms at the meta level, we let users consider both relations and their corresponding delta relations at the programming level.

Let $R$ be a relation and $r$ be the predicate corresponding to $R$. Following [27, 39, 53], we use two delta predicates $+r$ and $-r$ and write $+r(t)$ and $-r(t)$ to denote the insertion and deletion of the tuple $t$ into/from relation $R$, respectively. An update that replaces tuple $t'$ with a new one $t^\prime$ is a combination of a deletion $-r(t')$ and an insertion $+r(t^\prime)$. We use a delta relation, denoted as $\Delta R$, to capture both these deletions and insertions. For example, consider a binary relation $R = \{(1, 2), (1, 3)\}$; applying a delta relation $\Delta R = \{-(1, 2), +r(1, 1)\}$ to $R$ results in $R' = \{(1, 1), (1, 3)\}$. Let $\Delta_R^+$ be the set of insertions and $\Delta_R^-$ be the set of deletions in $\Delta R$. Applying $\Delta R$ to the relation $R$ is to delete tuples in $\Delta_R^-$ from $R$ and insert tuples in $\Delta_R^+$ into $R$. Considering set semantics, the delta application is the following:

$$R' = R \oplus \Delta R = (R \setminus \Delta_R^-) \cup \Delta_R^+$$

An update strategy for a view can now be specified by a set of Datalog rules that define delta relations of the source database from the updated view.

Example 3.1. Consider a source database $S$, which consists of two base relations, $R_1$ and $R_2$, with respective schemas $r_1(A)$ and $r_2(A)$, and a view relation $V$ defined by a union over $R_1$ and $R_2$: $V = get(S) = R_1 \cup R_2$. To illustrate the ambiguity of updates to $V$, consider an attempt to insert a tuple $(3)$ into the view $V$. There are three simple ways to update the source database: (i) insert tuple $(3)$ into $R_1$, (ii) insert tuple $(3)$ into $R_2$, and (iii) insert tuple $(3)$ into both $R_1$ and $R_2$. Therefore, the update strategy for the view needs to be explicitly specified to resolve the ambiguity of view updates. Given original source relations $R_1$ and $R_2$ and an updated view relation $V$, the following Datalog program is one strategy for propagating data in the updated view to the source:

$$-r_1(X) :- r_1(X), \neg v(X),$$

$$-r_2(X) :- r_2(X), \neg v(X),$$

$$+r_1(X) :- v(X), \neg r_1(X), \neg r_2(X).$$

The first two rules state that if a tuple $(X)$ is in $R_1$ or $R_2$ but not in $V$, it will be deleted from $R_1$ or $R_2$, respectively. The last rule states that if a tuple $(X)$ is in $V$ but not in either $R_1$ nor $R_2$, it will be inserted into $R_1$. Let the actual instances of the source and the updated view be $S = \{r_1(1), r_2(2), r_2(4)\}$ and $V = \{v(1), v(3), v(4)\}$, respectively. The input for the Datalog program is a database of both the source and the view $(S, V) = \{(r_1(1), r_2(2), r_2(4)), (v(1), v(3), v(4))\}$. Thus, the result is delta relations $\Delta R_1 = \{+r_1(3)\}$ and $\Delta R_2 = \{-r_2(2)\}$. By applying these delta relations to $S$, we obtain a new source database $S' = \{(r_1(1), r_1(3), r_2(4))\}$. \hfill \Box

Formally, consider a database schema $S = \{r_1, \ldots, r_n\}$ and a single view $v$. Let $S$ be a source database and $V$ be an updated view relation. We use $\Delta S$ to denote all insertions and deletions of all relations in $S$. For example, the $\Delta S$ in Example 3.1 is $\Delta S = \{+r_1(3), -r_2(2)\}$. We say that $\Delta S$ is non-contradictory if it has no insertion/deletion of the same tuple into/from the same relation. Applying a non-contradictory $\Delta S$ to a database $S$, denoted as $S \oplus \Delta S$, is to apply each delta relation in $\Delta S$ to the corresponding relation.
in \( S \). We use the pair \((S, V)\) to denote the database instance \( I \) over the schema \((r_1, \ldots, r_n, v)\) such that \( I(r_i) = S(r_i) \) for each \( i \in [1, n] \) and \( I(v) = V \). A view update strategy \( \text{put} \) is formulated by a Datalog query \( \text{putdelta} \) over the database \((S, V)\) that results in a \( \Delta S \) (shown in Figure 2) as follows:

\[
\text{put}(S, V) = S \oplus \text{putdelta}(S, V)
\]  

(1)

The Datalog program \( \text{putdelta} \) is called a Datalog putback program (or putback program for short). The result of \( \text{putdelta} \), \( \Delta S \), should be non-contradictory to be applicable to the original source database \( S \).

**Definition 3.1 (Well-definedness).** A putback program is well defined if, for every source database \( S \) and view relation \( V \), the program results in a non-contradictory \( \Delta S \).

### 3.2 LVGN-Datalog

We have seen that nonrecursive Datalog with extensions including negation and built-in predicates can be used for specifying view update strategies. We now focus on the extensions of Datalog in which the satisfiability of queries is decidable. This property plays an important role in guaranteeing that the validity of putback programs is decidable. Specifically, we define a fragment of Datalog, LVGN-Datalog, which is an extension of nonrecursive guarded negation Datalog (GN-Datalog [13]) with equalities, constants, comparisons [18] and linear view predicate. This Datalog fragment allows not only for writing many practical view update strategies but also for decidable checking of validity later.

#### 3.2.1 Nonrecursive GN-Datalog with Equalities, Constants, and Comparisons

We consider a restricted form of negation in Datalog, called GN-Datalog [12, 13], in which we can decide the satisfiability of any queries. In this way, we define LVGN-Datalog as an extension of this GN-Datalog fragment without recursion as follows:

- Equality is of the form \( t_1 = t_2 \), where \( t_1/t_2 \) is either a variable or a constant.
- Comparison predicates \( < (> \, \text{on totally ordered domains in the form of } X < c \quad (X > c) \), where \( X \) is a variable and \( c \) is a constant.
- Constants may freely be used in Datalog rule bodies or rule heads without restriction.
- Every rule is negation guarded [13] such that for every atom \( L \) (or equality, or comparison) occurring either in the rule head or negated in the rule body, the body must have a positive atom or equality, called a guard, containing all variables occurring in \( L \).

**Example 3.2.** The following rule is negation guarded:

\[
b(X, Y, Z) := \exists_{\text{guard}}(X, Y, Z), \neg Z \lor \exists_{\text{equality}}(X, Y, Z).
\]

because the negated atom \( \exists_{\text{guard}}(X, Y, Z) \), negated equality \( \neg Z = 1 \) and the head atom \( b(X, Y, Z) \) are all guarded since all variables \( X, Y, \) and \( Z \) are in the positive atom \( \exists_{\text{guard}}(X, Y, Z) \).

#### 3.2.2 Linear View

As formally proven in [24], the putback transformation \( \text{put} \) must be lossless (i.e., injective) with respect to the view relation. This means that all information in the view must be embedded in the updated source. To enable tracking this behavior of putback programs in LVGN-Datalog, we introduce a restriction called linear view, which controls the usage of the view in the programs. By linear view, we mean that the view is linearly used such that there is no self-join and projection on the view. Every program in LVGN-Datalog conforms to the linear view as follows:

**Definition 3.2 (Linear View).** A Datalog putback program conforms to a linear view if the view relation occurs only in the rules defining delta relations, and in each of these delta rules, there is at most one (positive or negative) view atom, which contains no anonymous variable (\( \_ \)).

**Example 3.3.** Consider a source relation \( R \) of arity 3 and a view relation \( V \) of arity 2. The following rule defines delta relation \( \Delta R \), where the predicate \( +r/\text{¯r} \) corresponds to the insertion/deletion set:

\[
\begin{align*}
- \text{r}(X, Y, Z) &:= - \text{r}(X, Y, Z), \neg v(X, Y) \quad \text{(rule 1)} \\
- \text{r}(X, Y, Z) &:= - \text{r}(X, Y, Z), \neg v(X, Z) \quad \text{(rule 2)} \\
+ \text{r}(X, Y, Z) &:= v(X, Y), v(Y, Z), \neg \text{r}(X, Y, Z). \quad \text{(rule 3)}
\end{align*}
\]

The rule (rule 1) conforms to a linear view because the atom \( v(X, Y) \) occurs once in the rule body, whereas the rules (rule 2) and (rule 3) do not because there is an anonymous variable (\( \_ \)) in the atom of \( v \) in (rule 2) and there is a self-join of \( v \) in (rule 3).

#### 3.2.3 Integrity Constraints

Since an updateable view can be treated as a base table, it is natural to create constraints on the view. Similar to the idea of negative constraints introduced in [17], we extend the rules in LVGN-Datalog by allowing a truth constant \( \text{false} \) (denoted as \( \bot \)) in the rule head for expressing integrity constraints. In this way, a constraint, called the guarded negation constraint, is of the form \( \forall \vec{X}, \Phi(\vec{X}) \to \bot \), where \( \Phi(\vec{X}) \) is the conjunction of all atoms and negated atoms in the rule body and \( \Phi(\vec{X}) \) is a guarded negation formula. The universal quantifiers \( \forall \vec{X} \) are omitted in Datalog rules.

**Example 3.4.** Consider a view relation \( v(X, Y, Z) \). To prevent any tuples having \( Z > 2 \) in the view \( v \), we can use the following constraint:

\[ \bot := \text{r}(X, Y, Z), \neg Z > 2. \]

#### 3.2.4 Properties

We say that a query \( Q \) is satisfiable if there is an input database \( D \) such that the result of \( Q \) over \( D \) is nonempty. The problem of determining whether a query in nonrecursive GN-Datalog is satisfiable is known to be decidable [13]. It is not surprising that allowing equalities, constants and comparisons in nonrecursive GN-Datalog does not make the satisfiability problem undecidable since the same already holds for guarded negation in SQL [13]. The idea is that we can transform such a GN-Datalog program into an equivalent guarded negation first-order (GNFO) formula whose satisfiability is decidable [12].
The view \texttt{ced} contains information about the current departments of each employee. We express the following update strategy for propagating updated data in this view to the base tables \texttt{ed} and \texttt{eed}. If a person is in a department according to \texttt{ed} but he/she is currently no longer in this department according to \texttt{ced}, this department becomes his/her previous department and thus needs to be added to \texttt{eed}. If a person used to be in a department according to \texttt{eed} but he/she returned to this department according to \texttt{ced}, then this department of him/her needs to be removed from \texttt{eed}. The view \texttt{residents1962} is defined from the view \texttt{residents} such that \texttt{residents1962} contains all residents that have a birth date in 1962. Interestingly, because the view \texttt{residents} is now updatable, \texttt{residents} can be considered as the source relation of \texttt{residents1962}. Therefore, we can write an update strategy on \texttt{residents1962} for updating \texttt{residents} instead of updating the base tables \texttt{male}, \texttt{female} and \texttt{others} as follows:

\textbf{Constraints:}
\begin{itemize}
  \item $\bot := \texttt{residents1962}(E,B,G), B \geq 1962-12-31$.
  \item $\bot := \texttt{residents1962}(E,B,G), B \leq 1962-01-01$.
\end{itemize}

\textbf{Update rules:}
\begin{itemize}
  \item $\texttt{residents1962}(E,B,G) := \texttt{residents}(E,B,G), \neg \texttt{residents}(E,B,G)$.
  \item $\texttt{residents1962}(E,B,G) := \texttt{residents}(E,B,G), B < 1962-01-01$.
  \item $\texttt{residents1962}(E,B,G) := \texttt{residents}(E,B,G), B > 1962-12-31$.
\end{itemize}

We define the constraints to guarantee that in the updated view \texttt{residents1962}, there is no tuple having a value of the attribute \texttt{birth date} not in 1962. Any view updates that violate these constraints are rejected. In this way, our update strategy is to insert into the source table \texttt{residents} any new tuples appearing in \texttt{residents1962} but not yet in \texttt{residents}. On the other hand, we delete only tuples in \texttt{residents} having \texttt{birth date} in 1962 if they no longer appear in \texttt{residents1962}.

The view \texttt{employees} contains residents who are employed, whereas \texttt{retired} contains residents who retired. Since \texttt{employees} and \texttt{retired} are defined from two updatable views \texttt{residents} and \texttt{ced}, we can use \texttt{residents} and \texttt{ced} as the source relations to write an update strategy of \texttt{employees}:

\textbf{Constraints:}
\begin{itemize}
  \item $\bot := \texttt{employees}(E,B,G), \neg \texttt{ced}(E,\_)$.
\end{itemize}

\textbf{Update rules:}
\begin{itemize}
  \item $\texttt{employees}(E,B,G) := \texttt{employees}(E,B,G), \neg \texttt{residents}(E,B,G)$.
  \item $\texttt{employees}(E,B,G) := \texttt{employees}(E,B,G), \neg \texttt{ced}(E,\_), \neg \texttt{employees}(E,B,G)$.
\end{itemize}

Interestingly, in this strategy, we use a constraint to specify more complicated restrictions of updates on \texttt{employees}. The constraint implies that there must be no tuple \texttt{(E, B, G)} in the updated view \texttt{employees} having the value \texttt{E} of the attribute \texttt{emp name}, which cannot be found in any tuples of \texttt{ced}. In other words, the constraint does not allow insertion into \texttt{employees} an actual new employee who is not mentioned in the source relation \texttt{ced}. The update strategy then reflects...
updates on the view employees to updates on the source residents.

For retired, we describe an update strategy to update the current employment status of residents as follows:

- $\text{ced}(E, D) \leftarrow \text{ced}(E, D), \text{retired}(E).$
- $\text{ced}(E, D) \leftarrow \text{residents}(E, \_), \neg \text{retired}(E), \neg \text{ced}(E, D), D = \text{unknown}'.

$\text{residents}(E, B, G) \leftarrow \text{retired}(E), G = \text{unknown}', \neg \text{residents}(E, \_), B = '00-00-00'.$

We have presented the formal way to describe the view update strategy using Datalog. In the next section, we will present our proposed validation algorithm (see Algorithm 1) for checking view update strategies. In fact, if an update strategy specified in LVGN-Datalog is valid, the corresponding view definition can be automatically derived and expressed in GN-Datalog. For all the update strategies in our case study, the corresponding view definitions derived by our validation algorithm are the same as the expected ones shown in Figure 3.

4. VALIDATION ALGORITHM

In this section, we present an algorithm for checking the validity of view update strategies.

4.1 Overview

Checking the validity of a view update strategy based on Definition 2.1 is challenging since it requires constructing a view definition satisfying both the GetPUT and PutGET properties. Instead, we shall propose another way for the validity check based on the following important fact:

**Lemma 4.1.** Given a valid view update strategy $put$, if a view definition $get$ satisfies GetPUT, then $get$ must also satisfy PutGET with $put$.

Lemma 4.1 implies that if $put$ is valid, we can construct a view definition $get$ that satisfies both GetPUT and PutGET by choosing any $get$ satisfying GetPUT. In other words, if $put$ is valid, we can consider only GetPUT for the $get$ derivation.

By Lemma 4.1, the idea of our validation algorithm is detecting contradictions for the assumption that the given view update strategy $put$ is valid. Assuming that $put$ is valid, we first check the existence of a view definition $get$ satisfying GetPUT with $put$. We consider the expected view definition $expected\_get$ if available as a candidate for the $get$ definition and construct the $get$ definition if $expected\_get$ does not satisfy GetPUT. Clearly, if $get$ does not exist, we can conclude that $put$ is invalid. Otherwise, we continue to check whether $get$ also satisfies PutGET with $put$ (Lemma 4.1). If this check passed, we actually complete the validation and it is sufficient to conclude that $put$ is valid because the $get$ found satisfies both GetPUT and PutGET. Furthermore, the constructed $get$ is useful to confirm the initially expected view definition especially when they are not the same. For the case in which the expected view definition is not explicitly specified, the view definition is automatically derived.

In particular, we are given a putback program $putdelta$, which is written in nonrecursive Datalog with negation and built-in predicates, and maybe an expected view definition ($expected\_get$) if it is explicitly described. The validation algorithm consists of three passes (see Figure 4): (1) checking the well-definedness of the putback program, (2) checking the existence of a view definition $get$ satisfying GetPUT with the view update strategy $put$ specified by the putback program and deriving $get$, and (3) checking whether $get$ and $put$ satisfy PutGET. If one of the passes fails, we can conclude that $put$ is invalid. Otherwise, $put$ is valid because the derived $get$ satisfies GetPUT and PutGET with $put$.

4.2 Well-definedness

Consider a database schema $S = \langle r_1, \ldots, r_n \rangle$ and a view $v$. Given a putback program $putdelta$, the goal is to check whether the delta $\Delta S$ resulting from $putdelta$ is non-contradictory for any source database $S$ and any view relation $V$. In other words, we check whether in $\Delta S$, there is no pair of insertion and deletion, $+r_i(\bar{t})$ and $-r_i(\bar{t})$, of the same tuple $\bar{t}$ on the same relation $R_i$. To check this property, we add the following new rules to $putdelta$:

$$d_i(\bar{X}_i) := +r_i(\bar{X}_i), -r_i(\bar{X}_i). \quad (i \in [1, n])$$

(2)

The problem of checking whether $\Delta S$ is non-contradictory is reduced to the problem of checking whether each IDB predicate $d_i$ in the Datalog program is unsatisfiable. When $putdelta$ is in LVGN-Datalog, because each rule (2) is trivially negation guarded, according to Theorem 3.2, the satisfiability of $d_i$ is decidable.

4.3 Existence of A View Definition Satisfying GetPUT

Consider a view update strategy $put$ specified by a putback program $putdelta$ and a set of constraints $\Sigma$. Assume that $put$ is valid. If an expected view definition $expected\_get$ is explicitly written by users, we check whether $expected\_get$ satisfies GetPUT with $put$. With the view defined by $expected\_get$, the GetPUT property means that $put$ makes no change to the source. Therefore, checking the GetPUT property is reduced to checking the unsatisfiability of each delta relation in the Datalog program $putdelta$. This check is decidable if $putdelta$ and $expected\_get$ is in LVGN-Datalog.

If $expected\_get$ is not explicitly written or if it does not satisfy GetPUT, we construct a view definition $get$ satisfying GetPUT as follows. For each source database $S$, we find a steady-state view $V$ such that the putback transformation $put$ makes no change to the source database $S$. In other words, $V$ must satisfy the constraints in $\Sigma$ and $put(S, V) = S$. We define $get$ as the mapping that maps each $S$ to the $V$. If there exists an $S$ such that we cannot find any steady-state view, then there is no view definition satisfying GetPUT, and we conclude that $put$ is invalid. Otherwise, the constructed
get satisfies GetPUT with put. Moreover, the view relation \( V \) resulting from \( get \) over \( S \) always satisfies \( \Sigma \).

**Example 4.1 (Intuition).** Consider the update strategy put in Example 3.1. For an arbitrary source database instance \( S \), the goal is to find a steady-state view \( V \) such that put\((S,V) = S\), i.e., both of the source relations \( R_1 \) and \( R_2 \) are unchanged. Recall that the putback transformation put is described by Datalog rules that compute delta relations of each source relation \( R_1 \) and \( R_2 \). For \( R_1 \), we compute \( \Delta_{R_1}^+ \) and \( \Delta_{R_1}^- \), which are the set of insertions and the set of deletions on \( R_1 \), respectively. \( R_1 \) is unchanged if all inserted tuples are already in \( R_1 \) and all deleted tuples are actually not in \( R_1 \). Similarly, for \( R_2 \), all tuples in \( \Delta_{R_2}^- \) must not be in \( R_2 \) (we do not have \( \Delta_{R_2}^+ \)). This leads to the following:

\[
\begin{align*}
\Delta_{R_1}^+ \cap R_1 &= \emptyset \\
\Delta_{R_2}^- \cap R_2 &= \emptyset \\
\Delta_{R_1}^- \setminus R_1 &= \emptyset
\end{align*}
\]

Let us transform each delta predicate \( \neg r_1 \), \( \neg r_2 \), and \( + r_1 \) in the Datalog program putdelta to the form of relational calculus query [10]: \( \varphi_{r_1} = r_1(X) \land \neg v(X), \varphi_{r_2} = r_2(X) \land \neg v(X), \varphi_{r_3} = v(X) \land \neg r_1(X) \land \neg r_2(X). \) The constraint \( (3) \) is equivalent to the constraint that all the relational calculus queries \( \varphi_{r_1}(X) \land r_1(X), \varphi_{r_2}(X) \land r_2(X) \) and \( \varphi_{r_3}(X) \land \neg r_1(X) \) result in an empty set over the database \((S,V)\) of both the source and view relations. In other words, \((S,V)\) does not satisfy the following first-order sentences:

\[
\begin{align*}
(S,V) \not\models \exists X, \varphi_{r_1}(X) \land r_1(X) \\
(S,V) \not\models \exists X, \varphi_{r_2}(X) \land r_2(X) \\
(S,V) \not\models \exists X, \varphi_{r_3}(X) \land \neg r_1(X)
\end{align*}
\]

By applying \( \neg \exists X, \xi(X) \equiv \forall X, \xi(X) \rightarrow \bot \), we have

\[
\begin{align*}
(S,V) \models \forall X, \varphi_{r_1}(X) \land r_1(X) \rightarrow \bot \\
(S,V) \models \forall X, \varphi_{r_2}(X) \land r_2(X) \rightarrow \bot \\
(S,V) \models \forall X, \varphi_{r_3}(X) \land \neg r_1(X) \rightarrow \bot
\end{align*}
\]

The idea for checking whether a view relation \( V \) satisfying the above logical sentences exists is that we swap the atom \( v(X) \) appearing in these sentences to either the right-hand side or the left-hand side of the implication formula. For this purpose, we apply \( p \land \neg q \rightarrow \bot \equiv p \rightarrow q \) and obtain:

\[
\begin{align*}
(S,V) \models \forall X, r_1(X) \rightarrow v(X) \\
(S,V) \models \forall X, r_2(X) \rightarrow v(X) \\
(S,V) \models v(X) \rightarrow (\neg r_1(X) \land \neg r_2(X))
\end{align*}
\]

By combining all sentences that have \( v(X) \) on the right-hand side and combining all sentences that have \( v(X) \) on the left-hand side, we obtain:

\[
(S,V) \models \begin{cases} 
\forall X, r_1(X) \lor r_2(X) \rightarrow v(X), \\
\forall X, v(X) \rightarrow (\neg r_1(X) \land \neg r_2(X))
\end{cases}
\]

Note that \( S \) is an instance over \((r_1,r_2)\) and \( V \) is the view relation corresponding to predicate \( v \). The first sentence provides us the lower bound \( V_{\min} \) of \( V \), which is the result of a first-order (FO) query \( \psi_1 = r_1(X) \lor r_2(X) \) over \( S \). The second sentence provides us the upper bound \( V_{\max} \) of \( V \), which is the result of the first-order query \( \psi_2 = \neg (r_1(X) \land r_2(X)) \) over \( S \). In fact, for each \( S \), all the \( V \) such that \( V_{\min} \subseteq V \subseteq V_{\max} \) satisfy \((4)\), i.e., are steady-state instances of the view. Thus, a steady-state instance \( V \) exists if \( V_{\min} \subseteq V_{\max} \). Indeed, by applying equivalence \( \neg (p \lor q) \equiv \neg p \land \neg q \) to \( \psi_2 \), we obtain the same formula as \( \psi_1 \), thus \( \forall X, \psi_1(X) \rightarrow \psi_2(X) \) holds, leading to that \( V_{\min} \subseteq V_{\max} \) holds. Now by choosing \( V_{\min} \) as a steady-state view instance, we can construct a get as the mapping that maps each \( S \) to \( V_{\min} \). In other words, get is a query equivalent to the FO query \( \psi_1 \) over the source \( S \). Since \( \psi_1 \) is a safe-range formula\(^3\), we transform \( \psi_1 \) to an equivalent Datalog query\(^4\) as follows:

\[
\begin{align*}
v(X) &:= r_1(X). \\
v(X) &:= r_2(X).
\end{align*}
\]

This is the view definition get that satisfies GetPUT with the given put.

**4.3.1 Checking the existence of a steady-state view**

In general, similar to the idea shown in Example 4.1, for an arbitrary putback program putdelta and a set of constraints \( \Sigma \) in LVGN-Datalog, we can always construct a guarded negation first-order (GNFO) sentence to check whether a steady-state view \( V \) satisfying \( \Sigma \) and \( put(S,V) = S \) (i.e., \( S \models putdelta(S,V) = S \)) exists.

**Lemma 4.2.** Given a LVGN-Datalog putback program putdelta and a set of guarded negation constraints \( \Sigma \), there exist first-order formulas \( \phi_1, \phi_2, \phi_3 \) such that for a given database instance \( S \), a view relation \( V \) satisfies \( \Sigma \) and \( S \models putdelta(S,V) = S \) iff

\[
\begin{align*}
(S,V) \models \forall X, \xi(X) \equiv \forall X, \xi(X) \rightarrow \bot \\
(S,V) \models \forall X, \xi(X) \rightarrow \bot \\
(S,V) \models \forall X, \xi(X) \rightarrow \bot
\end{align*}
\]

where \( v \) is the predicate corresponding to the view relation \( V \) and \( \phi_1, \phi_2, \phi_3 \) have no occurrence of the view predicate \( v \). Both \( \phi_2(Y) \) and \( \phi_3 \) are safe-range GNFO formulas, and \( v(Y) \land \phi_1(Y) \) is equivalent to a GNFO formula.

The third constraint \( S \not\models \phi_3 \) in (7) is simplified to \( S \not\models \phi_3 \) because the FO sentence \( \phi_3 \) has no atom of \( v \) as a subformula. This means that \( \phi_3 \) must be unsatisfiable over any database \( S \). Since \( \phi_3 \) is a GNFO sentence, we can check whether \( \phi_3 \) is satisfiable. If it is satisfiable, we conclude that the view relation \( V \) does not exist; thus, put is invalid.

For the two other constraints in (7), by applying the logical equivalence \( p \land \neg q \rightarrow \bot \equiv p \rightarrow q \), we have:

\[
\begin{align*}
(S,V) \models \forall X, r_1(X) \rightarrow v(X) \\
(S,V) \models \forall X, r_2(X) \rightarrow v(X) \\
(S,V) \models v(X) \rightarrow (\neg r_1(X) \land \neg r_2(X))
\end{align*}
\]

Because \( \phi_1 \) and \( \phi_2 \) do not contain an atom of \( v \) as a subformula, there exists an instance \( V \) if

\[
\begin{align*}
S \models \forall Y, \phi_1(Y) \land \phi_2(Y) \rightarrow \neg \phi_1(Y) \\
S \models \forall Y, \phi_1(Y) \land \phi_2(Y) \rightarrow \bot
\end{align*}
\]

This means that the sentence \( \exists Y, \phi_1(Y) \land \phi_2(Y) \) is not satisfiable. In this way, checking the existence of a \( V \) is now

\(^3\)A FO query \( \psi \) over \( D \) results in all tuples \( t \) s.t. \( D \models \psi(t) \).

\(^4\)Due to the equivalence between nonrecursive Datalog queries and safe-range FO formulas [10].
reduced to checking the satisfiability of $\exists \vec{Y}, \phi_1(\vec{Y}) \land \phi_2(\vec{Y})$.
The idea of checking the satisfiability of $\exists \vec{Y}, \phi_1(\vec{Y}) \land \phi_2(\vec{Y})$ is to reduce this problem to that of a GNFO sentence. For this purpose, we introduce a fresh relation $r$ of an appropriate arity. We have the fact that $\exists \vec{Y}, \phi_1(\vec{Y}) \land \phi_2(\vec{Y})$ is satisfiable if and only if $\exists \vec{Y}, r(\vec{Y}) \land \phi_1(\vec{Y}) \land \phi_2(\vec{Y})$ is satisfiable. Because $v(\vec{Y}) \land \phi_1(\vec{Y})$ is equivalent to a GNFO formula, $r(\vec{Y}) \land \phi_1(\vec{Y})$ is also equivalent to a GNFO formula. On the other hand, $\phi_2(\vec{Y})$ is equivalent to a GNFO formula; hence, we can transform $\exists \vec{Y}, r(\vec{Y}) \land \phi_1(\vec{Y}) \land \phi_2(\vec{Y})$ into an equivalent GNFO sentence whose satisfiability is decidable.

4.3.2 Constructing a view definition

If both $\phi_1$ and $\exists \vec{Y}, \phi_1(\vec{Y}) \land \phi_2(\vec{Y})$ are unsatisfiable, there exists a steady-state view $V$ satisfying $\Sigma$ such that $S \oplus \text{putdelta}(S,V) = S$ for each database $S$. One steady-state view $V$ is the one resulting from the safe-range formula $\phi_2$ over $S$. Indeed, such a $V$ satisfies (8); hence, it satisfies $\Sigma$ and $S \oplus \text{putdelta}(S,V) = S$. By choosing this steady-state view, we construct a view definition $\text{get}$ as the Datalog query equivalent to the safe-range formula $\phi_2$. We present the detailed transformation from safe-range formula to Datalog query in Ex B. Since $\phi_2$ is a GNFO formula, $\text{get}$ is in nonrecursive GN-Datalog with equalities, constants and comparisons.

4.4 The PutGet Property

To check the PutGet property that $\text{get}(\text{put}(S,V)) = V$ for any $S$ and $V$, we first construct a Datalog query over database $(S,V)$ equivalent to the composition $\text{get}(\text{put}(S,V))$. Recall that $\text{put}(S,V) = S \oplus \text{putdelta}(S,V)$. The result of $\text{put}(S,V)$ is a new source $S'$ obtained by applying $\Delta S$ computed from $\text{putdelta}$ to the original source $S$. Let us use predicate $r_i^{new}$ for the new relation of predicate $r_i$ in $S$ after the update. The result of applying a delta $\Delta S$ to the database $S$ is equivalent to the result of the following Datalog rules ($i \in [1,n]$):

$r_i^{new}(X_i) :- r_i(X_i), \neg r_i(X_i).

r_i^{new}(X_i) :- r_i(X_i), \neg r_i(X_i).

By adding these rules to the Datalog putback program $\text{putdelta}$, we derive a new Datalog program, denoted as $\text{newsource}$, that results in a new source database. The result of $\text{get}(\text{put}(S,V))$ is the same as the result of the Datalog query $\text{get}$ over the new source database computed by the program $\text{newsource}$. Therefore, we can substitute each EDB predicate $r_i$ in the $\text{get}$ program with the new program $r_i^{new}$ and then merge the obtained program with the program $\text{newsource}$ to obtain a Datalog program, denoted as $\text{putget}$.

The result of $\text{putget}$ over $(S,V)$ is exactly the same as the result of $\text{get}(\text{put}(S,V))$. For example, the Datalog program $\text{putget}$ for the view update strategy in Example 4.1 is:

$\neg r_1(X) :- r_1(X), \neg v(X).

\neg r_2(X) :- r_2(X), \neg v(X).

r_1(X) :- v(X), \neg r_1(X), \neg r_2(X).

r_2^{new}(X) :- r_1(X), \neg r_2(X).

r_2^{new}(X) :- r_2(X), \neg r_2(X).

v^{new}(X) :- r_1^{new}(X).

v^{new}(X) :- r_2^{new}(X).

Checking the PutGet property is now reduced to checking whether the result of Datalog query $\text{putget}$ over database $(S,V)$ is the same as the view relation $V$. By transforming $\text{putget}$ to the FO formula $\phi_{\text{putget}}(\vec{Y})$, we reduce checking the PutGet property to checking the satisfiability of the two following sentences:

$\Phi_1 = \exists \vec{Y}, \phi_{\text{putget}}(\vec{Y}) \land \neg v(\vec{Y})$ (9)

$\Phi_2 = \exists \vec{Y}, v(\vec{Y}) \land \neg \phi_{\text{putget}}(\vec{Y})$ (10)

The PutGet property holds if and only if $\Phi_1$ and $\Phi_2$ are not satisfiable. Because all the rules in the programs $\text{newsource}$ and $\text{get}$ are negation guarded, $\text{get}$ is in guarded negation Datalog. Therefore, $\Phi_1$ is a guarded negation first-order sentence; hence, its satisfiability is decidable. $\Phi_1$ is satisfiable if and only if $\Phi_1' = \exists \vec{Y}, \phi_{\text{putget}}(\vec{Y}) \land r(\vec{Y}) \land v(\vec{Y})$ is satisfiable, where $r$ is a fresh relation of an appropriate arity. Since $\Phi_1'$ is a guarded negation first-order sentence, its satisfiability is decidable by Theorem 3.2.

4.5 Soundness and Completeness

Algorithm 1 summarizes the validation of Datalog putback programs $\text{putdelta}$. After all the checks have passed, the corresponding view definition is returned and $\text{putdelta}$ is valid. For LVGN-Datalog in which the query satisfiability is decidable (Theorem 3.2), Algorithm 1 is sound and complete:

**Theorem 4.3 (Soundness and Completeness).**

- If a LVGN-Datalog putback program $\text{putdelta}$ passes all the checks in Algorithm 1, $\text{putdelta}$ is valid.

- Every valid LVGN-Datalog putback program $\text{putdelta}$ passes all the checks in Algorithm 1.

It is remarkable that if $\text{putdelta}$ is not in LVGN-Datalog, but in nonrecursive Datalog with unrestricted negation and built-in predicates, we can still perform the checks in the validation algorithm by feeding them to an automated theorem prover. Though, Algorithm 1 may not terminate and not successfully construct the view definition $\text{get}$ because of the undecidability problem [10, 54]. Therefore, Algorithm 1

**Algorithm 1: Validate(expected_get, putdelta, $\Sigma$)**

get ← null;

// Checking the well-definedness of putdelta
check if all predicates $d_i$ ($i \in [1,n]$) in (2) are unsatisfiable under $\Sigma;

if expected_get is not null then

// Checking if expected_get satisfies GetPut
if all delta relations of putdelta are unsatisfiable under $\Sigma$ with the view defined by expected_get then

get ← expected_get;

if (expected_get is null) or (get is null) then

// Constructing a $\text{get}$ satisfying GetPut
check if $\phi_3$ in (7) is unsatisfiable under $\Sigma;

check if $\exists \vec{Y}, \phi_1(\vec{Y}) \land \phi_2(\vec{Y})$ ($\phi_1$ and $\phi_2$ in (8)) is unsatisfiable under $\Sigma;

// Constructing a $\text{get}$
get ← Translating FO formula $\phi_2$ in (8) to an equivalent Datalog query;

// Checking the PutGet property
check if $\Phi_1$ and $\Phi_2$ in (9) and (10) are unsatisfiable under $\Sigma;

return get;
is sound for validating the pair of putdelta and expected_get that once it terminates, we can conclude putdelta is valid.

5. INCREMENTALIZATION

We have shown that an updatable view is defined by a valid put, which makes changes to the source to reflect view updates. However, when there is only a small update on the view, repeating the put computation is not efficient. In this section, we further optimize the computation of the putback program by exploiting its well-behavedness and integrating it with the standard incrementalization method for Datalog.

5.1 Incrementalizing Putback Program

Consider the steady state before a view update in which both the source and the view are unchanged; due to the GetPut property, a valid putdelta results in a ∆S having no effect on the original source S: S ⊕ ∆S = S. This means that ∆S can be either an empty set or a nonempty set in which all deletions in ∆S are not yet in the original source S and all insertions in ∆S are already in S. If the view is updated by a delta ∆V, there will be some changes to ∆S, denoted as ∆2S, that have effects on the original source S.

Example 5.1. Consider the database in Example 3.1: S = {r1(1), r2(2), r3(4)}. Let ∆S = {+r1(1), +r2(2), −r3(3)} be a delta of S. Clearly, S ⊕ ∆S = S. Now, we change ∆S by a delta of ∆S, denoted as ∆2S, which includes a set of deletions to ∆S, ∆2S = {+r1(1), −r2(3)}, and a set of insertions to ∆S, ∆2S = {+r1(3), −r2(4)}. We obtain a new delta of S:

\[ ∆S' = (ΔS \setminus Δ2S) \cup Δ2S = \{+r1(3), +r2(2), −r2(4)\} \]

and the new database S' = S ⊕ ∆S' = {r1(1), r1(3), r2(2)}. In fact, we can also obtain the same S' by applying only ∆2S directly to S: S' = S ⊕ ∆2S.

Intuitively, for each base relation Ri in the source database S, we obtain the new Ri by applying to Ri the delta relations ∆Ri and ∆Ri from ∆S. Because all the tuples in ∆Ri are not in Ri and all the tuples in ∆Ri are in Ri, if we remove some tuples from ∆Ri or ∆Ri, then the result Ri has no change. Only the tuples inserted into ∆Ri or ∆Ri make some changes in Ri. Therefore, S' can be obtained by applying to the original S the part ∆2S of ∆2S, i.e., ∆S' and ∆2S are interchangeable.

Proposition 5.1. Let S be a database and ∆S be a non-contradictory delta of the database S such that S ⊕ ∆S = S. Let ∆2S be a delta of ∆S, and the following equation holds:

\[ S' = S \oplus \Delta S' = S \oplus \Delta 2S \]

where ∆S' = ∆S ⊕ ∆2S and ∆2S is the set of new tuples inserted into ∆S by applying ∆2S.

Proposition 5.1 is the key observation for deriving from putdelta an incremental Datalog program ∂put that computes ∆S more efficiently (Figure 5). To derive ∂put, we first incrementalize the Datalog program putdelta to obtain Datalog rules that compute ∆2S from the change ∆V on the view V. This step can be performed using classical incrementalization methods for Datalog [28]. We then use ∆2S in ∆2S as an instance of ∆S for applying to the source S.

5.2 Incremental View Maintenance

Example 5.2 (Intuition). Given a source relation R of arity 2 and a view relation V defined by a selection on R: v(X,Y) := r(X,Y), Y > 2. Consider the following update strategy with a constraint that updates on V must satisfy the selection condition Y > 2:

\[ +(r(X,Y)) := +v(X,Y), ¬r(X,Y). \]
\[ −r(X,Y) := ¬v(X,Y), ¬v(X,Y). \]

Let \( \Delta^+ / \Delta^- \) be the set of insertions/deletions into/from the view V. We use two predicates +v and −v for \( \Delta^+ \) and \( \Delta^- \), respectively. To generate delta rules for computing changes of ∆V when the view is changed by \( \Delta^+ \) and \( \Delta^- \), we adopt the incremental view maintenance techniques introduced in [28] but in a way that derives rules for computing the insert and deletion set for ∆V separately. When \( \Delta^+ \) and \( \Delta^- \) are disjoint, by applying distribution laws for the first Datalog rule, we derive two rules that define the changes to \( \Delta^+ \) and a set of deletions \( \Delta^- \), as follows:

\[ +(r(X,Y)) := +v(X,Y), ¬r(X,Y). \]
\[ −(r(X,Y)) := +v(X,Y), ¬r(X,Y). \]

where predicates +(r) and -(r) correspond to \( \Delta^+ \) and \( \Delta^- \), respectively. Similarly, we derive rules defining changes to \( \Delta^+ \) and \( \Delta^- \), as follows:

\[ +(r(X,Y)) := ¬m(X,Y), ¬v(X,Y). \]
\[ −(r(X,Y)) := ¬m(X,Y), +v(X,Y). \]

Finally, as stated in Proposition 5.1, \( \Delta^2S \) and \( \Delta^2S \) are interchangeable. Since \( \Delta^2S \) contains \( \Delta^+ (\Delta^2S) \) and \( \Delta^- (\Delta^2S) \), we can substitute \( ¬r + r \) for the predicates +(r) and +(r), respectively, to derive the program ∂put as follows:

\[ m(X,Y) := r(X,Y), Y > 2. \]
\[ +r(X,Y) := +v(X,Y), ¬r(X,Y). \]
\[ −r(X,Y) := ¬m(X,Y), ¬v(X,Y). \]

Because \( \Delta^+ \) and \( \Delta^- \) are generally much smaller than the view V, the computation of \( \Delta^+ (\Delta^2S) \) in the derived rules is more efficient than the computation of \( \Delta^2S \) in putdelta.

The incrementalization algorithm that transforms a putback program putdelta in nonrecursive Datalog with negation and built-in predicates into an equivalent program ∂put is as follows:

- Step 1: We first stratify the Datalog program putdelta. Let \( v_1, l_1, \ldots, l_m, r_1, \ldots, r_n \) be a stratification [18] of the Datalog program putdelta, which is an order for the evaluation of IDB relations of putdelta.

- Step 2: To derive rules for computing changes of each IDB relation \( l_1, \ldots, l_m \) when the view v is changed, we adopt the incremental view maintenance techniques
introduced in [28] but in a way that derives rules for computing each insertion set (+\(i\)) and deletion set (−\(i\)) on IDB relation \(l_i\) separately (see the details in Ex C).

- **Step 2:** Similar to Step 1, we perform the following steps: (1) performing update requests to obtain a larger modification request on the view. When combining multiple DML statements into one transaction to perform the following steps: (1) handling update requests to obtain the incremental program \(\text{putdelta}\). This is because \(\Delta^S\) can be used as a substitute for positive predicate \(v\) for positive predicates of the view, \(+v\) for positive predicates of the view, and \(−v\) for negative predicate \(−v\).

**Lemma 5.2.** Every valid LVGN-Datalog putback program \(\text{putdelta}\) for a view relation \(V\) is equivalent to an incremental program that is derived from \(\text{putdelta}\) by substituting delta predicates of the view, \(+v\) and \(−v\), for positive and negative predicates of the view, \(v\) and \(−v\), respectively.

### 6. Implementation and Evaluation

#### 6.1 Implementation

We have implemented a prototype for our proposed validation and incrementalization algorithms in Ocaml (The full source code is available at https://github.com/dangtv/birds). For the case in which the view update strategy is not in LVGN-Datalog, our framework feeds each check in our validation algorithm to the Z3 automated theorem prover. As mentioned in Section 4, the validation algorithm may not terminate, though it is sound for checking the pair of views defined in Section 4.

As shown in Example 5.2, for a LVGN-Datalog program in which the view predicate \(v\) occurs at most once in each delta rule, the transformation from a putback program \(\text{putdelta}\) to an incremental one \(\text{putdelta}\) is simplified to substituting \(+v\) for positive predicate \(v\) and \(−v\) for negative predicate \(−v\).

Our translation is conducted because nonrecursive Datalog queries can be expressed in SQL [10]. We use a similar approach to the translation from Datalog to SQL queries used in [29]. The SQL view definition is of the form CREATE VIEW \(<\text{view-name}>\) AS \(<\text{sql-defining-query}>\). Meanwhile, the implementation for the update strategy is achieved by generating a SQL program that defines triggers [52] and associated trigger procedures on the view. These trigger procedures are automatically invoked in response to view update requests, which can be any SQL DML statements of INSERT/DELETE/UPDATE. Our framework also supports combining multiple DML statements into one transaction to obtain a larger modification request on the view. When there are view update requests, the triggers on the view perform the following steps: (1) handling update requests to the view to derive deltas of the view (see Ex D), (2) checking the constraints if applying the deltas from step (1) to the view, and (3) computing each delta relation and applying them to the source. The main trigger has the following form:

CREATE TRIGGER <update-strategy>

**INSTEAD OF** INSERT OR UPDATE OR DELETE ON <view V>

BEGIN
-- Deriving changes on the view
Derive from view update requests \(\Delta^+_V\) and \(\Delta^-_V\)
-- Checking constraints
FOR EACH \(<\text{constraint}> \forall \vec{X}, \Phi_l(\vec{X}) \rightarrow \perp\) DO
IF EXISTS (\(<\text{SQL-query-of}> \Phi_l(\vec{X})\>) THEN
RAISE "Invalid view updates";
END IF;
END FOR;
-- Calculate and apply delta relations
FOR EACH <source relation \(R_i\)> DO
CREATE TEMP TABLE \(\Delta^+_R_i\) AS <sql-query-of> \(+r_i>;
CREATE TEMP TABLE \(\Delta^-_R_i\) AS <sql-query-of> \(−r_i>;
DELETE FROM \(R_i\) WHERE \text{ROW} (\(R_i\)) \in \(\Delta^-_R_i\);
INSERT INTO SELECT * FROM \(R_i\) \(\Delta^+_R_i\);
END FOR;
END;

#### 6.2 Evaluation

To evaluate our approach, we conduct two experiments. The goal of the first experiment is to investigate the practical relevance of our proposed method in describing view update strategies and to evaluate the performance of our framework in checking these described update strategies. In the second experiment, we study the efficiency of our incrementalization algorithm when implementing updatable views in a commercial RDBMS.

**6.2.1 Benchmarks**

To perform the evaluation, we collect benchmarks of views and update strategies from two different sources:

- View update examples and exercises collected from the literature: textbooks [52, 26], online tutorials [2, 3, 6, 5, 8] (triggers, sharded tables, and so forth), papers [15, 33] and our case study in Section 3.
- View update issues asked on online question & answer sites: Database Administrators Stack Exchange [1] and Stack Overflow Public Q&A [7].

All experiments on these benchmarks are run using Ubuntu server LTS 16.04 and PostgreSQL 9.6 on a computer with 2 CPUs and 4 GB RAM.

**6.2.2 Results**

As mentioned previously, we perform the first experiment to investigate which users’ update strategies are expressible and verifiable by our approach. In our benchmarks, the view update strategies collected are either implemented in SQL triggers or naturally described by users/systems. We manually use nonrecursive Datalog with negation and built-in predicates (NR-Datalog) to specify these update strategies as \(\text{putdelta}\) programs and input them with the expected view definition to our framework. Table 1 shows the validation results. In terms of expressiveness, NR-Datalog can be used to formalize most of the update strategies except the one for aggregation views. This is because we have not considered aggregation in Datalog. Interestingly, LVGN-Datalog can also express many update strategies for many views defined by selection, projection, union, set difference and semi

\(^5\)For the update strategies implemented in SQL triggers, rewriting them into \(\text{putdelta}\) programs can be automated.
Table 1: Validation results. S, P, SJ, IJ, LJ, RJ, FJ, U, D and A stand for selection, projection, semi join, inner join, left join, right join, full join, union, set difference and aggregation, respectively. PK, FK, ID, and C stand for primary key, foreign key, inclusion dependency, and domain constraint, respectively.

| ID  | View                  | Operator in view definition | Program size (LOC) | Constraint | LVGN-Datalog | NR-Datalog | Validation Time (s) | Compiled SQL (Byte) |
|-----|-----------------------|----------------------------|-------------------|------------|--------------|------------|---------------------|---------------------|
| 1   | car_master            | P                          | 4                 | ✓          | ✓            | ✓          | 1.74                | 8417                |
| 2   | goodstudents          | P S                        | 5                 | C          | ✓            | ✓          | 1.86                | 9182                |
| 3   | luxuryitems           | S                          | 5                 | C          | ✓            | ✓          | 1.77                | 8938                |
| 4   | usa_city              | P S                        | 5                 | C          | ✓            | ✓          | 1.77                | 9059                |
| 5   | coll                  | LJ                         | 6                 | ✓          | ✓            | ✓          | 1.72                | 8847                |
| 6   | residents1962         | S                          | 6                 | C          | ✓            | ✓          | 1.73                | 9099                |
| 7   | employees             | SJ P                       | 6                 | ID         | ✓            | ✓          | 1.76                | 9538                |
| 8   | researchers           | SJ P S                    | 6                 | ✓          | ✓            | ✓          | 1.79                | 9058                |
| 9   | retired               | SJ P D                    | 6                 | ✓          | ✓            | ✓          | 1.76                | 9048                |
| 10  | paramountmovies      | P S                        | 7                 | ✓          | ✓            | ✓          | 1.81                | 9721                |
| 11  | officeinfo            | P                          | 8                 | ✓          | ✓            | ✓          | 1.8               | 9966                |
| 12  | vw_brands             | U P                        | 8                 | C          | ✓            | ✓          | 1.78                | 10934               |
| 13  | tracks2               | P                          | 8                 | ✓          | ✓            | ✓          | 1.81                | 9824                |
| 14  | residents             | U                          | 10                | ✓          | ✓            | ✓          | 1.77                | 14504               |
| 15  | tracks3               | S                          | 11                | C          | ✓            | ✓          | 1.88                | 14430               |
| 16  | tracks1               | LJ                         | 12                | PK         | ✓            | ✓          | 1.92                | 95606               |
| 17  | bstudents             | LJ P S                    | 15                | PK         | ✓            | ✓          | 2.13                | 22431               |
| 18  | all_cars              | LJ                         | 13                | PK, FK     | ✓            | ✓          | 1.89                | 23013               |
| 19  | measurement           | U                          | 13                | C, ID      | ✓            | ✓          | 1.78                | 12624               |
| 20  | newpc                 | LJ P S                    | 15                | JD         | ✓            | ✓          | 2.06                | 44665               |
| 21  | activestudents        | LJ P S                    | 19                | PK, JD     | ✓            | ✓          | 2.19                | 31766               |
| 22  | vw_customers          | LJ P                       | 19                | PK, FK, JD | ✓            | ✓          | 2.92                | 26286               |
| 23  | emp_view              | LJ P A                    | -                 | ✓          | ✓            | ✓          | -                   | -                   |
| 24  | ukaz_lok              | S                          | 6                 | C          | ✓            | ✓          | 1.79                | 10104               |
| 25  | message               | U                          | 8                 | C          | ✓            | ✓          | 1.8                | 15749               |
| 26  | outstanding_task      | P SJ                       | 10                | ID, C      | ✓            | ✓          | 10.07               | 18253               |
| 27  | pol_view              | P LJ                       | 12                | PK         | ✓            | ✓          | 2.1                | 24741               |
| 28  | phonelist             | U                          | 14                | ✓          | ✓            | ✓          | 1.94                | 16553               |
| 29  | products              | LJ                         | 16                | PK, FK, C  | ✓            | ✓          | 3.6                 | 58394               |
| 30  | koncerty              | LJ                         | 17                | PK         | ✓            | ✓          | 1.93                | 29147               |
| 31  | purchaseview          | P LJ                       | 19                | PK, FK, JD | ✓            | ✓          | 1.89                | 27462               |
| 32  | vehicle_view          | P LJ                       | 20                | PK, FK, JD | ✓            | ✓          | 2.04                | 29226               |

Join. Inner join view is not expressible in LVGN-Datalog because the definition of inner join is not in guarded negation Datalog\(^a\). Many common constraints are also expressible in NR-Datalog\(^b\). LVGN-Datalog is limited in expressing primary key (functional dependency) or join dependency because these dependencies are not negation guarded\(^b\). Even for the cases that LVGN-Datalog cannot express, thus far, all the well-behavedness checks in our experiment terminate after an acceptable time (approximately a few seconds). The checking time almost increases with the number of Datalog rules, but this time also depends on the number of attributes of source and view relations. For example, view #25 has the longest checking time because this view and its source relations have many more attributes than other views. Similarly, the size of the generated SQL implementations is larger for the more complex Datalog update strategies.

We perform the second experiment to evaluate the efficiency of the incrementalization algorithm in optimizing view update strategies. Specifically, we compare the performance of the incrementalized update strategy with the original one when they are translated into SQL trigger programs and run in PostgreSQL database. For this experiment, we select some typical views in our benchmarks including: luxuryitems (Selection), officeinfo (Projection), outstanding_task (Join) and vw_brands (Union). For each view, we randomly generate data for the base tables and measure the running time of the view update strategy against the base table size (number of tuples) when there is an SQL DML statement that attempts to modify the view. Figure 6 shows the comparison between the original view update strategies (black lines) and the incrementalized ones (blue lines). It is clear that as the size of the base tables increases, our incrementalization significantly reduces the running time to a constant value, thereby improving the performance of the view update strategies.

7. RELATED WORK

The view update problem is a classical problem that has a long history in database research [22, 20, 21, 11, 34, 48, 33, 40, 29, 16, 36, 44, 45, 46, 42, 41]. It was realized very early that a database update that reflects a view update may not always exist, and even if it does exist, it may not be unique [20, 21]. To solve the ambiguity of translating view updates to updates on base relations, the concept of view complement is proposed to determine the unique update translation of a view [11, 35, 43, 41]. Keller [34] enumerates all view update translations and chooses the one through interaction with database administrators, thereby solving the ambiguity.

\(^a\)An example of inner join is \(v(X, Y, Z) := s_1(X, Y), s_2(Y, Z)\), which is not a guarded negation Datalog rule.

\(^b\)Primary key \(A\) on relation \(r(A, B)\) is expressed by the rule \(\neg r(A, B_1), r(A, B_2), \neg B_1 = B_2\), where the equality \(B_1 = B_2\) is not guarded.
problem. Some other researchers allow users to choose the one through an interaction with the user at view definition time [34, 42]. Some other approaches restrict the syntax for defining views [21] that allow for unambiguous update propagation. Recently, intention-based approaches have been proposed to find relevant update policies for several types of views [44, 45, 46]. In another aspect, because some updates on views are not translatable, some works permit side effects of the view update translator [48] or restrict the kind of updates that can be performed on a view [33]. Some other works use auxiliary tables to store the updates, which cannot be applied to the underlying database [40, 29]. The authors of [16, 36] studied approximation algorithms to minimize the side effects for propagating deletion from the view to the source database. However, these existing approaches can only solve a very restricted class of view updates.

By generalizing view update as a synchronization problem between two data structures, considerable research effort has been devoted to bidirectional programming [19] for this problem not only in relational databases [15, 31] but also for other data types, such tree [25, 47], graph [30] or string data [14]. The prior work by Bohannon et al. [15] employs bidirectional transformation for view update in relational databases. The authors propose a bidirectional language, called relational lenses, by enriching the SQL expression for defining views of projection, selection, and join. The language guarantees that every expression can be interpreted forwardly as a view definition and backwardly as an update strategy such that these backward and forward transformations are well-behaved. A recent work [31] has shown that incrementalization is necessary for relational lenses to make this language practical in RDBMSs. However, this language is less expressive than general relational algebra; hence, not every updatable view can be written. Moreover, relational lenses still limit programmers from control over the update strategy.

Melnik et al. [49] propose a novel declarative mapping language for specifying the relationship between application entity views and relational databases, which is compiled into bidirectional views for the view update translation. The user-specified mappings are validated to guarantee the generated bidirectional views to roundtrip. Furthermore, the authors introduce the concept of merge views that together with the bidirectional views contribute to determining complete update strategies, thereby solving the ambiguity of view updates. Though, merge views are exclusively used and validating the behavior of this operation with respect to the roundtripping criterion is not explicitly considered. In comparison to [49], where the proposed mapping language is restricted to selection-projection views (no joins), our approach focuses on a specification language, which is in lower level but more expressive that more view update strategies can be expressed. Moreover, the full behaviour of the specified view update strategies are validated by our approach.

Our work was greatly inspired by the putback-based approach in bidirectional programming [32, 50, 51, 24, 38, 37]. The key observation in this approach is that thanks to well-behavedness, putback transformation uniquely determines the get one. In contrast to the other approaches, the putback-based approach provides languages that allow programmers to write their intended update strategies more freely and derive the get behavior from their putback program. A typical language of this putback-based approach is BiGUL [38, 37], which supports programming putback functions declaratively while automatically deriving the corresponding unique forward transformation. Based on BiGUL, Zan et al. [55] design a putback-based Haskell library for bidirectional transformations on relations. However, this language is designed for Haskell data structures; hence, it cannot run directly in database environments. The transformation from tables in relational databases to data structures in Haskell would reduce the performance of view updates. In contrast, we propose adopting the Datalog language for implementing view update strategies at the logical level, which will be optimized and translated to SQL statements to run efficiently inside a SQL database system.

8. CONCLUSIONS

In this paper, we have introduced a novel approach for relational view update in which programmers are given full control over deciding and implementing their view update strategies. By using nonrecursive Datalog with extensions as the language for describing view update strategies, we propose algorithms for validating user-written update strategies and optimizing update strategies before compiling them into SQL scripts to run effectively in RDBMSs. The experimental results show the performance of our framework in terms of both validation time and running time.

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A. PROOFS

A.1 Proof of Theorem 2.1

Proof. By contradiction. If there are two view definitions \( \text{get}_1 \) and \( \text{get}_2 \) that satisfy the condition, then by applying the \( \text{GETPUT} \) and \( \text{PUTGET} \) properties to the expression \( E = \text{get}_1(\text{put}(S, \text{get}_2(S))) \), we have \( E = \text{get}_1(S) \) and \( E = \text{get}_2(S) \), respectively. This means that \( \text{get}_1(S) = \text{get}_2(S) \) for any database \( S \), i.e., \( \text{get}_1 \) and \( \text{get}_2 \) are equivalent.

A.2 Proof of Lemma 3.1

Proof. We shall transform Datalog query into an equivalent guarded negation first-order (GNFO) formula [12]. Let \( P \) be a Datalog program in nonrecursive GN-Datalog with equalities, constants and comparisons. Without loss of generality, we assume that in \( P \), for every pair of head atoms \( h_1(X_1), h_2(X_2) \) in \( P \), \( h_1 = h_2 \) implies \( X_1 = X_2 \) (this can be achieved by variable renaming).

Since there are constants that can occur in both atoms and equalities. We first remove all constants appearing in atoms by transforming them to constants appearing in equalities. This can be done by introducing a fresh variable \( X \) for each constant \( c \) in the atoms of the Datalog rule (head or body), then adding the equality \( X = c \) to the Datalog body and substitute \( X \) for the constant \( c \). By this transformation, we consider equalities of the form \( X = c \) and a positive atom as a guard for negative predicates or head atom of Datalog rules. In other words, for each head atom or negative predicate \( \beta \), there is a positive atom \( p(Y) \) such that all the free variables in \( \beta \) must appear in \( p(Y) \) or in an equality of the form \( X = c \). For example, the following rule

\[
    h(Y, 1) : = p(Z, W, 3), \neg r(W, 4)
\]

is transformed into

\[
    h(Y, X_1) : = p(Z, W, X_2), \neg r(W, X_3), X_1 = 1, X_2 = 3, X_3 = 4.
\]

in which the negated atom \( r(w, X_3) \) is guarded by the positive atom \( p(Z, W, X_2) \) and the equality \( X_3 = 4 \). The head atom \( h(Y, X_1) \) is guarded by \( p(Z, w, X_2) \) and \( X_1 = 1 \).

We inductively define a FO formula \( \varphi_r \) equivalent to the Datalog query \( P \) restricted to a predicate \( r \). In other word, for every database \( D \), the output of \( P \) over \( D \) restricted to IDB relation \( R \) corresponding to IDB predicate \( r \) \( (P(D))_R \) is the same as the set of tuple \( t \) satisfying \( \varphi_r (\{ t \mid \varphi(t) \}) \).

The construction of \( \varphi_r \) is inductively defined as the following:

- (Base case) \( r \) is an EDB relation, i.e., \( r \in S \cup \{v\} \):
  \[
  \varphi_r = r(X_r), \text{ where } X_r \text{ denotes } (X_1, \ldots, X_{\text{arity}(r)}).
  \]
• (Inductive case) \( r \) is an IDB relation, i.e., \( r \) occurs in the head of some rules. Suppose that there are \( m \) rules:

\[
\begin{align*}
    r(X_r) := & \alpha_1,1, \ldots, \alpha_{1,n_1} \\
    \vdots \\
    r(X_r) := & \alpha_{m,1}, \ldots, \alpha_{m,n_m}
\end{align*}
\]

Let \( \varphi_{r,i}(X_r) \) be the FO formula for \( r \) when considering only the \( i \)-th rule:

\[
\varphi_{r,i}(X_r) = \exists \bar{E}_i \left( \bigwedge_{j=1}^{n_i} \beta_{i,j} \right)
\]

where \( \bar{E}_i \) contains the bound variables of the \( i \)-th rule (variables not in the rule head),

\[
\beta_{i,j} = \begin{cases} 
    \varphi_w(\bar{Z}), & \text{if } \alpha_{i,j} \text{ is an atom } w(\bar{Z}) \\
    \neg \varphi_w(\bar{Z}), & \text{if } \alpha_{i,j} \text{ is an negated atom } \neg w(\bar{Z}) \\
    \alpha_{i,j} & \text{if } \alpha_{i,j} \text{ is an equality predicate or a negated equality predicate} \\
    C_{<c}(X) & \text{if } \alpha_{i,j} \text{ is a comparison predicate } X < c \\
    C_{>c}(X) & \text{if } \alpha_{i,j} \text{ is a comparison predicate } X > c
\end{cases}
\]

Here we introduce fresh predicates \( C_{<c}(X) \) and \( C_{>c}(X) \) for the comparisons. We have:

\[
\varphi_r(X_r) = \bigvee_{i=1}^{m} \varphi_{r,i}(X_r) = \bigvee_{i=1}^{m} \left( \exists \bar{E}_i \left( \bigwedge_{j=1}^{n_i} \beta_{i,j} \right) \right)
\]

In each conjuction \( \varphi_{r,i}(X_r) = \exists \bar{E}_i \left( \bigwedge_{j=1}^{n_i} \beta_{i,j} \right) \), each negative predicate \( \beta_{i,j} \) is guarded by a positive atom \( \alpha_{i,j}(\bar{Y}) \) and many equalities. Moreover, there exists an positive atom \( \alpha_{i,j}(\bar{Y}) \) containing all the free variables of \( X_r \).

Let us briefly recall the syntax of GNFO formulas with constants proposed by Bárány et al. [12]. GNFO formulas with constants are generated by the recursive definition

\[
\varphi := r(t_1, \ldots, t_n) | t_1 = t_2 | \varphi_1 \land \varphi_2 | \varphi_1 \lor \varphi_2 | \exists \bar{x} \varphi | \alpha \land \neg \varphi
\]

where each \( t_i \) is either a variable or a constant symbol, and, in \( \alpha \land \neg \varphi, \alpha \) is an atomic formula of EDB predicate containing all free variables of \( \varphi \).

We shall transform \( \varphi_r(X_r) \) into a GNFO formula by structural induction on \( \varphi_r(X_r) \). Since GNFO is close under conjunction and existential quantifier, \( \varphi_r(X_r) \) is transformed into a GNFO formula. We first group each negative predicate \( \beta_{i,j} \) with its guard atom \( \alpha_{i,j}(\bar{Y}) \). If a free variables \( X \) appearing in \( \beta_{i,j} \) but not in \( \alpha_{i,j}(\bar{Y}) \), \( X \) must appear in an equality \( X = c \), we then substitute \( c \) for \( X \) in \( \beta_{i,j} \) and obtain \( \varphi_{w_{i,j}}(\bar{Y}) \land \beta_{i,j} \), where \( \bar{Y} \) contains all the free variable of \( \beta_{i,j} \). If two negare predicates share the same guard atom then the guard atom can be used twice.

\[
\varphi_{r,i}(X_r) = \exists \bar{E}_i \left( \bigwedge_{k} \beta_{i,k} \right) \land \left( \bigvee_{j} \left( \varphi_{w_{i,j}}(\bar{Y}) \land \beta_{i,j} \right) \right)
\]

Because each \( \beta_{i,k} \) in \( \bigwedge_{k} \beta_{i,k} \) is a positive predicate, we inductively transform each to GNFO. Now consider each formula \( \psi = (\varphi_{w_{i,j}}(\bar{Y}) \land \beta_{i,j}) \).

• If \( w_{i,j} \) is an EDB predicate, \( \varphi_{w_{i,j}}(\bar{Y}) = w_{i,j}(\bar{Y}) \), thus \( \psi \) is a GNFO formula.

• If \( w_{i,j} \) is an IDB predicate, by the construction of \( \varphi_{w_{i,j}}(\bar{Y}) \), we have \( \varphi_{w_{i,j}}(\bar{Y}) = \bigvee \varphi_{w_{i,j}}(\bar{Y}) \). As mentioned before in each \( \varphi_{w_{i,j}}(\bar{Y}) \) there is an IDB atom \( u_l(\bar{Z}) \) containing all variables of \( \bar{Y} \). Therefore,

\[
\psi = \bigvee_{l} \varphi_{w_{i,j}}(\bar{Y}) \land \beta_{i,j}
\]

We continue to inductively transform each \( \varphi_{w_{i,j}}(\bar{Y}) \) and \( \varphi_{u_l}(\bar{Z}) \land \beta_{i,j} \).
D and use all (finite) the suitable values of the active domain of D to construct a relation corresponding to each predicate $C_{<0}(X)/C_{=0}(X)$. Obviously, $D'$ satisfies $\Phi$ and $\varphi_r(X_i)$. Conversely, if there is a signature $D'$ that satisfies $\Phi$ and $\varphi_r(X_i)$ we can construct a database $D$ by an isomorphic copy of all relations from $D'$ except the relations corresponding to predicates $C_{<0}(X)$ and $C_{=0}(X)$. It is known that for GNFO formulas, satisfiability over finite structure coincides with satisfiability over unrestricted structures. In other words, any structures satisfying the GNFO formula are finite. Therefore $D'$ is a finite structure, i.e. a database. Since the satisfiability of a GNFO sentence is decidable, the satisfiability of the Datalog query $(P, R)$ is also decidable.

A.3 Proof of Theorem 3.2

Proof. As in Lemma 3.1, we first transform a query $Q$ in nonrecursive GN-Datalog with equalities, constants and comparisons into an equivalent guarded negation first-order formula $\varphi_r(\vec{Y})$. The result of $Q$ over a database $D$ is not empty iff $D$ satisfies the sentence $\exists \vec{Y}, \varphi_r(\vec{Y})$. Let $\Sigma$ be a set of guarded negation constraints and $\sigma_i = (X_i, \Phi_i(X_i)) \rightarrow \bot$ (i ∈ [1, m]) be a constraint in $\Sigma$, where $\Phi_i(X_i)$ is a conjunction of (negative) atoms. Clearly, each $\Phi_i(X_i)$ is a guarded negation formula since there is a guard atom in the rule body $\Phi_i(X_i)$. We rewrite $\sigma_i$ as an equivalent sentence $\sigma_i \equiv -\exists X_i, \Phi_i(X_i)$. Now, the query $Q$ is satisfiable under $\Sigma$ iff there exists a database $D$ satisfying all $\sigma_i$ such that $D$ satisfies $\exists \vec{Y}, \varphi_r(\vec{Y})$. This means that we need to check whether there exists a database $D$ such that $D$ satisfies all $\sigma_i$ and $\exists \vec{Y}, \varphi_r(\vec{Y})$: $D |= \left( \bigwedge_{i=1}^{m} -\exists X_i, \Phi_i(X_i) \right) \land \left( \exists \vec{Y}, \varphi_r(\vec{Y}) \right)$. Note that there is no free variable in $\exists \vec{Y}, \varphi_r(\vec{Y})$ and all $\Phi_1, \ldots, \Phi_m$ and $\varphi_r(\vec{Y})$ are GNFO formulas, the conjunction $\left( \bigwedge_{i=1}^{m} -\exists X_i, \Phi_i(X_i) \right) \land \left( \exists \vec{Y}, \varphi_r(\vec{Y}) \right)$ is a GNFO formula. Thus, the problem now is reduced to the satisfiability of a GNFO formula, which is decidable.

A.4 Proof of Lemma 4.1

Proof. From Definition 2.1, we know that there exists a view definition $get^d$ that satisfies both $GetPut$ and $PutGet$ with the given valid put. Let get be an arbitrary view definition satisfying $GetPut$ with put, i.e., $put(S, get(S)) = S$ for any $S$. By applying the query $get^d$ to both the right-hand side and left-hand side of this equation, and using the $PutGet$ property of $get^d$ and $put$, we obtain:

$\text{get}^d(\text{put}(S, \text{get}(S))) = \text{get}^d(S) \Rightarrow \text{get}(S) = \text{get}^d(S)$

This means that $\text{get}(S) = \text{get}^d(S)$ for any $S$, i.e., get and $get^d$ are the same. Thus, get satisfies $PutGet$ with put.

A.5 Proof of Lemma 4.2

Let $(r_1, \ldots, r_n)$ be a source database schema and $S$ be a database instance of this schema, i.e., $S$ contains all relations $R_1, \ldots, R_n$ corresponding to the schema $r_1, \ldots, r_n$. Let $B$ be a view over the source database. Let $\Sigma$ be a set of $m$ guarded negation constraints over the view and the source database; each constraint is of the form $\sigma_i = (X_i, \Phi_{\sigma_i}(X_i)) \rightarrow \bot$.

Let us consider a LVGN-Datalog putback program $putdelta$ for the view $v$. $putdelta$ takes as an updated view instance $V$ and the original source database $S$ to result in a delta $\Delta S$ of the source. $V$ is a steady state of the view if $\Delta S$ has no effect on the original $S$, i.e., $S \oplus \Delta S = S$. Recall that $\Delta S$ contains all the tuples that need to be inserted/deleted into/from each source relation $R_i$ (i ∈ [1, n]), represented by two sets $\Delta^+_{R_i}$ and $\Delta^-_{R_i}$ for these insertions and deletions, respectively. $S \oplus \Delta S = S$ iff

$\Delta^-_{R_i} \cap R_i = \Delta^+_{R_i} \setminus R_i = \emptyset, \forall i \in [1, n]$ (11)

Note that each $\Delta^+_{R_i}/\Delta^-_{R_i}$ is the result of the Datalog program $putdelta$ restricted to the delta predicate $+r_i/-r_i$ and the source database $(S, V)$. Since $putdelta$ is nonrecursive, we have an equivalent relational calculus query $\varphi_{+r_i}(X_i)/\varphi_{-r_i}(X_i)$ for each $\Delta^+_{R_i}/\Delta^-_{R_i}$. Equation (11) is equivalent to the condition that two relational calculus queries $\varphi_{-r_i}(X_i) \land r_i(X_i)$ and $\varphi_{+r_i}(X_i) \land -r_i(X_i)$ must be empty over the view and source database $(S, V)$. In other words, the first-order sentences $\exists X_i, \varphi_{-r_i}(X_i) \land r_i(X_i)$ and $\exists X_i, \varphi_{+r_i}(X_i) \land -r_i(X_i)$ are not satisfiable over the view and source database $(S, V)$. Combined with the constraint set $\Sigma$, a steady-state view $V$ satisfies $S$ and $S \oplus putdelta(S, V) = S$ iff:

$\left\{ \begin{array}{ll}
(S, V) \not\models \exists X_i, \varphi_{-r_i}(X_i) \land r_i(X_i), i \in [1, n]
\end{array} \right.$ (12)

$\left\{ \begin{array}{ll}
(S, V) \not\models \exists X_i, \varphi_{+r_i}(X_i) \land -r_i(X_i), i \in [1, n]
\end{array} \right.$ (13)

We now find such a $V$ satisfying (13).

Claim 1. Given a putback program $putdelta$ written in LVGN-Datalog for a view $v$ and a source schema $(r_1, \ldots, r_n)$, each relational calculus formula $\varphi_r(X_i)$ of the query $putdelta$ restricted to a predicate $r$ can be rewritten in the following linear-view form:

$\left( \bigvee_{k=1}^{p} \exists E_{1k}, \psi_{1k} \land \psi_{1k} \right) \lor \left( \left( \bigvee_{k=1}^{q} \exists E_{2k}, -\psi_{2k} \land \psi_{3} \right) \lor \psi_{3} \right)$

where view atom $v$ does not appear in $\psi_{1k}$, $\psi_{2k}$ or $\psi_{3}$. Each of formulas $\exists E_{1k}, \psi_{1k} \land \psi_{1k}, \exists E_{2k}, -\psi_{2k} \land \psi_{3}$ is a safe-range GNFO formula and has the same free variables $X_i$.

Proof. The proof is conducted inductively on the transformation (presented in Subsection A.2 - the proof of Lemma 3.1) between the Datalog query $putdelta$ restricted to a predicate $r$ and an equivalent GNFO formula $\varphi_r(X_i)$. Note that in this transformation, each $\varphi_{+r_i}$ is a safe-range formula, i.e., is a relational calculus [10].

We inductively prove that every $\varphi_r$ can be transformed into the linear-view form. The base case is trivial. For the inductive case, due to the linear-view restriction, if $r$ is a normal predicate (not a delta predicate), then there is no view atom $v$ in all the rules defining $r$; thus, $\varphi_r =$
The first case is that there is no query \( b \) in the transformation, if \( p = q = 0 \). The other hand, if \( r \) is a delta predicate, in each \( i \)-th rule \( r(X_r) : - \alpha_i, \ldots, \alpha_i, \eta_i \), there are two cases. The first case is that there is no \( \alpha_{i,j,0} \) of a view atom \( v \), \( \varphi_{r,i} = \Box_{t,x}^{n_{i+1}} \beta_{k,j} \) is in linear-view form, where \( \psi_3 = \varphi_{r,i} \) and \( p = q = 0 \). In the second case, there is only one \( \alpha_{i,j,0} \), which is an atom \( \nu(Y_i) \) or a negated atom \( \neg v(Y_i) \). Thus, \( \varphi_{r,i} = \Box_{t,x}^{n_{i+1}} \beta_{k,j} \) or \( \varphi_{r,i} = \Box_{t,x}^{n_{i+1}} \nu(Y_i) \land \Box_{t,x}^{n_{i+1}} \neg \beta_{k,j} \). Therefore, \( \varphi_{r,i} \) is rewritten in linear-view form. Note that if two formulas are in linear-view form, then the disjunction of them can be transformed into the linear-view form. Indeed,
\[
\left( \bigvee_{k=1}^{p_1} \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_1} \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3
\]
\[
\left( \bigvee_{k=1}^{p_2} \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_2} \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3
\]
\[
\equiv \left( \bigvee_{k=1}^{p_2} \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_2} \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3
\]
In this way, \( \varphi_r = \bigvee_{i=1}^{p_1} \varphi_{r,i} \) is rewritten in linear-view form.

As proven in [13], we can continue to transform each safe-range formula \( \varphi_{r,i} \) into a GNFO formula. In other words, in the linear-view form of \( \varphi_r \), each \( \Box_{t,x}^{k} \psi_1 \lor \Box_{t,x}^{k} \neg \psi_2 \lor \psi_3 \) can be transformed into a safe-range GNFO formula. In this transformation, if \( \Box_{t,x}^{k} \psi_1 \lor \Box_{t,x}^{k} \neg \psi_2 \lor \psi_3 \) is transformed into \( \Box_{t,x}^{k} \psi_1 \lor \Box_{t,x}^{k} \neg \psi_2 \lor \psi_3 \), we will transform it into \( \Box_{t,x}^{k} (\psi_1 \lor \neg \psi_2) \lor (\psi_1 \lor \neg \psi_2) \). In this way, we finally obtain a safe-range GNFO formula of \( \varphi_r \), which is also in linear-view form.

We now know that the relational calculus formula \( \varphi_{x,r} \) of each delta predicate \( \pm \) is rewritten in linear-view form. For each constraint \( \sigma_i \) of the form \( \forall \bar{X}_i, \Phi_i(\bar{X}_i) \rightarrow \bot \), we can consider a Datalog rule \( b_i(\bar{X}_i) : = \Phi_i(\bar{X}_i) \) in putdelta. The conjunction \( \Phi_i(\bar{X}_i) \) is equivalent to the relational calculus query \( \varphi_i(\bar{X}_i) \) of relation \( b_i \). Therefore, \( \Phi_i(\bar{X}_i) \) can also be transformed to the linear-view form.

Since \( \varphi_{x,r}(\bar{X}_r) \) can be rewritten in linear-view form, the conjunction \( \varphi_{x,r}(\bar{X}_r) \land r(\bar{X}_r) \) can be rewritten in linear-view form by applying the distribution of existential quantifier over disjunction:
\[
\varphi_{x,r}(\bar{X}_r) \land r(\bar{X}_r) \equiv \left( \bigvee_{k=1}^{p_1} \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_1} \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3
\]
\[
\equiv \left( \bigvee_{k=1}^{p_2} \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_2} \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3
\]

\( \bar{X}_r \) is the free variable of \( \varphi_r(\bar{X}_r) \); hence, no existential variable in \( \bar{E}_{1k} \) or \( \bar{E}_{2k} \) is in \( \bar{X}_r \). We can push \( r(\bar{X}_r) \) into the existential quantifier \( \exists \bar{E}_{1k} / \exists \bar{E}_{2k} \) and obtain:
\[
\left( \bigvee_{k=1}^{p_1} \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_1} \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3
\]
\[
\equiv \left( \bigvee_{k=1}^{p_2} \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_2} \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3
\]

This is in linear-view form. Therefore, the disjunction \( (\varphi_{x,r}(\bar{X}_r) \land r(\bar{X}_r)) \lor (\varphi_{x,r}(\bar{X}_r) \land r(\bar{X}_r)) \) can be rewritten in linear-view form. The constraint (13) is now rewritten as:
\[
(S,V) \not\models \exists \bar{X}_r, \left( \bigvee_{k=1}^{p_1} \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_1} \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3, i \in [1, n]
\]
\[
(S,V) \not\models \exists \bar{X}_r, \left( \bigvee_{k=1}^{p_1} \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_1} \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3, i \in [n + 1, n + m]
\]

By applying the distribution of existential quantifier over disjunction:
\[
\exists \bar{X}_r, \xi_1(\bar{X}_r) \lor \xi_2(\bar{X}_r) \equiv (\exists \bar{X}_r, \xi_1(\bar{X}_r)) \lor (\exists \bar{X}_r, \xi_2(\bar{X}_r))
\]
we have:
\[
(S,V) \not\models \left( \bigvee_{k=1}^{p_1} \Box_{t,x}^{k} \exists \bar{E}_{1k}, \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_1} \Box_{t,x}^{k} \exists \bar{E}_{2k}, \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3, i \in [1, n]
\]
\[
(S,V) \not\models \left( \bigvee_{k=1}^{p_1} \Box_{t,x}^{k} \exists \bar{E}_{1k}, \Box_{t,x}^{k} \psi_1 \right) \lor \left( \bigvee_{k=1}^{q_1} \Box_{t,x}^{k} \exists \bar{E}_{2k}, \Box_{t,x}^{k} \neg \psi_2 \right) \lor \psi_3, i \in [n + 1, n + m]
\]

Here, we have disjunction of many formulas on the right-hand side, and we can apply the equivalence between \((S,V) \not\models \xi_1 \lor \xi_2 \) and \((S,V) \not\models \xi_1 \land (S,V) \not\models \xi_2 \) to separate the disjunction on the right-hand side and obtain \( n_3 \) sentences as follows:
\[
(S,V) \not\models \exists \bar{E}_{1k}, \Box_{t,x}^{k} \psi_3, k \in [1, n_1]
\]
\[
(S,V) \not\models \exists \bar{E}_{1k}, \Box_{t,x}^{k} \psi_3, k \in [n_1 + 1, n_2]
\]
\[
(S,V) \not\models \exists \bar{E}_{1k}, \Box_{t,x}^{k} \psi_3, k \in [n_2 + 1, n_3]
\]

where \( n_1 = \sum_{i=1}^{p_1} p_1, n_2 = n_1 + \sum_{i=1}^{q_1} q_1 \) and \( n_3 = n_2 + n + m \).

All variables in \( \bar{Y}_k \) are in \( \bar{E}_k \) for any \( k \).

Note that \( \exists W_i(v(Y_i) \lor \psi_k) \equiv (v(Y_i)) \exists W_i \psi_k \) if \( W_i \) is not a free variable in \( v(Y_i) \). In this way, we push existential variables in \( \bar{E}_k \) but not in \( \bar{Y}_k \), denoted by \( \bar{Z}_k, \) into the subformula \( \psi_k \).

In the case that there is a variable \( X \) appearing more than once in \( \bar{Y}_k \), we can introduce a new fresh variable \( X' \) and
add the equality $X = X'$ to the formulas after the quantifier $\exists Y$. For example,

$$\exists Y_1 Y_2, v(Y_1, Y_1, Y_2) \equiv \exists Y_1 Y'_1 Y_2, v(Y_1, Y'_1, Y_2) \land Y_1 = Y'_1$$

We then substitute the variables in each $Y_i$ to obtain the same $\bar{Y} = Y_1, \ldots, Y_n$. Then, we have $n_3$ FO sentences that $(S, V)$ must not satisfy:

$$\begin{align*}
(S, V) &\not\models \exists Y, v(\bar{Y}) \land \exists Z_k, \psi_k(\bar{E}_k), k \in [1, n_1] \\
(S, V) &\not\models \exists Y, v(\bar{Y}) \land \exists Z_k, \psi_k(\bar{E}_k), k \in [n_1 + 1, n_2] \\
(S, V) &\not\models \exists E_k, \psi_k(\bar{E}_k), k \in [n_2 + 1, n_3]
\end{align*}$$

Because $((S, V) \not\models \xi_1) \land ((S, V) \not\models \xi_2)$ is equivalent to $(S, V) \not\models \xi_1 \lor \xi_2$, we have:

$$\begin{align*}
(S, V) &\not\models \bigvee_{k=1}^{n_1} (\exists Y, v(\bar{Y}) \land \exists Z_k, \psi_k(\bar{E}_k)) \\
(S, V) &\not\models \bigvee_{k=n_1+1}^{n_2} (\exists Y, \neg v(\bar{Y}) \land \exists Z_k, \psi_k(\bar{E}_k)) \\
(S, V) &\not\models \bigvee_{k=n_2+1}^{n_3} (\exists E_k, \psi_k(\bar{E}_k))
\end{align*}$$

By applying the distribution of existential quantifier over disjunction $(\exists Y, \xi_1(\bar{Y})) \lor (\exists Y, \xi_2(\bar{Y})) \equiv \exists Y, \xi_1(\bar{Y}) \lor \xi_2(\bar{Y})$, we have:

$$\begin{align*}
(S, V) &\not\models \exists Y, \bigvee_{k=1}^{n_1} (v(\bar{Y}) \land \exists Z_k, \psi_k(\bar{E}_k)) \\
(S, V) &\not\models \exists Y, \bigvee_{k=n_1+1}^{n_2} (\neg v(\bar{Y}) \land \exists Z_k, \psi_k(\bar{E}_k)) \\
(S, V) &\not\models \bigvee_{k=n_2+1}^{n_3} (\exists E_k, \psi_k(\bar{E}_k))
\end{align*}$$

By applying the distribution of conjunction over disjunction $(p \land q) \lor (p \land r) \equiv (p \land (q \lor r))$, we have:

$$\begin{align*}
(S, V) &\not\models \exists Y, \bigvee_{k=1}^{n_1} (v(\bar{Y}) \land \exists Z_k, \psi_k(\bar{E}_k)) \\
(S, V) &\not\models \exists Y, \bigvee_{k=n_1+1}^{n_2} (\neg v(\bar{Y}) \land \exists Z_k, \psi_k(\bar{E}_k)) \\
(S, V) &\not\models \bigvee_{k=n_2+1}^{n_3} (\exists E_k, \psi_k(\bar{E}_k))
\end{align*}$$

Note that in (14), each $\exists E_k, \psi_k(\bar{E}_k)$ is a safe-range GNFO formula; hence, $\exists Y, \psi(\bar{Y})$ is a safe-range GNFO formula. Each $\exists Z_k, \psi_k(\bar{E}_k)$ is a safe-range GNFO formula; hence, $\exists Y, \psi(\bar{Y})$ is a safe-range GNFO formula. Each $\exists E_k, v(\bar{E}_k) \land \psi_k, k \in [1, n_1]$ is a safe-range GNFO formula; hence, $\exists Y, \psi(\bar{Y}) \equiv \bigvee_{k=1}^{n_1} (\exists Y, v(\bar{Y}) \land \exists Z_k, \psi_k(\bar{E}_k)) \equiv \bigvee_{k=1}^{n_2} (\exists E_k, \psi_k(\bar{E}_k))$, which is a safe-range GNFO formula.

### A.6 Proof of Proposition 5.1

**Proof.** Consider a database $S$ over schema $(r_1, \ldots, r_n)$. $S \cup S' = S$ means that $\Delta^\pm R_i \cap R_i = \emptyset$ and $\Delta^\pm R_i \setminus R_i = \emptyset$ ($i \in [1, n]$). Let $\Delta^2 S$ be the change on $\Delta S$, i.e., $\Delta^2 S$ contains insertions and deletions into/from each $\Delta^\pm R_i$ and $\Delta^\pm R_i$. we use $\Delta^\pm R_i$ as an abbreviation for $\Delta^\pm R_i$ and $\Delta^\pm R_i$. Let $\Delta^+ (\Delta^- R_i)$ and $\Delta^- (\Delta^+ R_i)$ be the set of insertions and the set of deletions for $\Delta^\pm R_i$, respectively. The new instance $\Delta^\pm R_i$ of each $\Delta^\pm R_i$ in $\Delta S$ is obtained by:

$$\Delta^\pm R_i = (\Delta^- R_i \setminus (\Delta^+ R_i)) \cup (\Delta^+ R_i)$$

We finally obtain a new source database $S'$ by applying each $\Delta^\pm R_i$ to the corresponding relation $R_i$ in database $S$:

$$\begin{align*}R'_i &= (R_i \setminus (\Delta^- R_i) \cup (\Delta^+ R_i)) \\
&= (R_i \setminus ((\Delta^- R_i) \cup (\Delta^+ R_i))) \cup (\Delta^+ R_i) \cup (\Delta^+ R_i)
\end{align*}$$

Because $\Delta^- R_i$ and $\Delta^+ R_i$ are disjoint, and because $\Delta^- R_i \cap R_i = \emptyset$ and $\Delta^+ R_i \setminus R_i = \emptyset$, we can simplify the above equation to:

$$R'_i = R_i \setminus \Delta^+ R_i \cup \Delta^+ R_i \cup \Delta^+ R_i$$

(17)

Note that $\Delta^+ (\Delta^- R_i)$ and $\Delta^- (\Delta^+ R_i)$ contain all the tuples inserted into $\Delta^\pm R_i$ and $\Delta^\pm R_i$, respectively. In other words, $\Delta^+ (\Delta^- R_i)$ and $\Delta^- (\Delta^+ R_i)$ are delta relations in $\Delta^\pm S$. This means that the source database $S'$ is obtained by applying $\Delta^\pm S$ to $S$: $S' = S \oplus \Delta^\pm S$.

### A.7 Proof of Lemma 5.2

**Proof.** Consider a valid LVGN-Datalog putback program putdelta for a view $v$ and source database schema $(r_1, \ldots, r_n)$. Since putdelta is in LVGN-Datalog, the view predicate occurs only in the rules defining delta relations of the source $(\pm r_1, \ldots, \pm r_n)$, and at most once in each rule. When the view relation is changed, only delta relations, $\pm r_1, \ldots, \pm r_n$, are changed, all other relations (intermediate relations in putdelta) are unchanged. Therefore, to incrementize putdelta, we use only rules defining delta relations (having a predicate $\pm r_i$ as the head), to derive the rules computing changes to the delta relations.

A Datalog rule having a delta predicate $\pm r_i$ in the head and a view predicate $v$ in the body is in one of the following forms:

$$\begin{align*}
\pm r_i(\bar{X}) &\leftarrow v(\bar{Y}), Q(\bar{Z}) \\
\pm r_i(\bar{X}) &\leftarrow v(\bar{Y}), Q(\bar{Z})
\end{align*}$$

(positive view)

$$\begin{align*}
\pm r_i(\bar{X}) &\leftarrow v(\bar{Y}), Q(\bar{Z}) \\
\pm r_i(\bar{X}) &\leftarrow v(\bar{Y}), Q(\bar{Z})
\end{align*}$$

(negative view)

where $Q(\bar{Z})$ is the conjunction of the rest of the rule body, $Q(\bar{Z})$ is unchanged, whereas the view relation $v$ is changed to $v' = (v \setminus v + v')$, where $+v$ and $-v$ corresponds to the insertions set of deletions set, respectively. Similar to the the incrementalization technique in [28], by distributing joins over set minus and union we obtain:

$$\begin{align*}
{+}(\pm r_i)(\bar{X}) &\leftarrow v(\bar{Y}), Q(\bar{Z}) \\
{-}(\pm r_i)(\bar{X}) &\leftarrow v(\bar{Y}), Q(\bar{Z})
\end{align*}$$

for the case of positive view and:

$$\begin{align*}
{+}(\pm r_i)(\bar{X}) &\leftarrow v(\bar{Y}), Q(\bar{Z}) \\
{-}(\pm r_i)(\bar{X}) &\leftarrow v(\bar{Y}), Q(\bar{Z})
\end{align*}$$

for the case of negative view, where new delta relations is obtained by $\pm r'_i = (\pm r_i \setminus (-\pm r_i)) \cup (+\pm r_i)$. Proposition 5.1 implies that the set of insertions to the delta relation, $+(\pm r_i)$, can be used as $\pm r'_i$ to apply to the
source relation $r_i$ to obtain the same new source. Therefore, the rule computing $-v(\pm r_i)$ is redundant, $\pm r_i'$ can be computed by the rules of $+(\pm r_i)$:

$$\pm r_i'(X) := \pm v(Y), Q(Z).$$

for the case of positive view and

$$\pm r_i'(X) := -v(Y), Q(Z).$$

for the case of negative view. This shows that the transformation from origin $putdelta$ to an incremental one is substituting delta predicates of the view, $+v$ and $-v$, for positive and negative predicates of the view, $v$ and $\neg v$, respectively. $\square$

B. TRANSFORMATION FROM SAFE-RANGE FO FORMULA TO DATALOG

In this section, we present the transformation from a safe-range FO formula $\varphi$ to an equivalent Datalog query.

We briefly the algorithm that transforms a safe-range FO formula $\varphi$ into relational algebra normal form (RANF) described in [10]. Let’s assume that $\varphi$ is in safe-range normal form (SRNF) in which there is no universal quantifiers, no implications, and there is no conjunction or disjunction sign that occurs directly below a negation sign (every FO formula can be transformed into an SRNF formula). Recall that the set of range-restricted variables of the FO formula $\varphi$ ($rr(\varphi)$) is inductively defined as follows [10]:

- if $\varphi = R(e_1, \ldots, e_n)$, $rr(\varphi)$ is the set of variables in $\{e_1, \ldots, e_n\}$
- if $\varphi = (x = a)$ or $\varphi = (a = x)$, $rr(\varphi) = x$
- if $\varphi = \varphi_1 \land \varphi_2$, $rr(\varphi) = rr(\varphi_1) \cup rr(\varphi_2)$
- if $\varphi = \varphi_1 \land x = y$, $rr(\varphi) = rr(\varphi_1)$ if $(x, y) \cap rr(\varphi_1) = \emptyset$ and $rr(\varphi) = rr(\varphi_1) \cup \{x, y\}$ otherwise
- if $\varphi = \varphi_1 \lor \varphi_2$, $rr(\varphi) = rr(\varphi_1) \cap rr(\varphi_2)$
- if $\varphi = \neg \varphi_1$, $rr(\varphi) = \emptyset$
- if $\varphi = \exists \xi \varphi_1$, $rr(\varphi) = rr(\varphi_1) - \{x\}$ if $\{x\} \subseteq rr(\varphi_1)$ and $rr(\varphi) = \bot$ otherwise

where for each $Z$, $\bot \cup Z = \bot \cap Z = \bot - Z = Z - \bot = \bot$, $\bot$ indicates that some quantified variables are not range restricted. Let $free(\varphi)$ be the set of free variables of $\varphi$. $\varphi$ is a safe-range FO formula iff $rr(\varphi) = free(\varphi)$.

**Definition B.1.** An occurrence of a subformula $\psi$ in $\varphi$ is self-contained if its root is $\land$ or $\lor$.

- $\psi = \psi_1 \lor \ldots \lor \psi_n$ and $rr(\psi) = rr(\psi_1) = \ldots = rr(\psi_n) = free(\psi)$;
- $\psi = \exists \xi \psi_1$ and $rr(\psi_1) = free(\psi_1)$.

A safe-range SRNF formula $\varphi$ is in relational algebra normal form (RANF) if each subformula of $\varphi$ is self-contained.

The algorithm that transforms a safe-range SRNF formula $\varphi$ into an equivalent RANF formula is based on the following rewrite rules for each subformula $\psi$ in $\varphi$:

- **Push-into-or:** Consider the subformula
  $$\psi = \psi_1 \land \ldots \land \psi_n \land \xi$$
  where $\xi = \xi_1 \lor \ldots \lor \xi_m$. If $rr(\psi) = free(\psi)$ but $rr(\xi \lor \ldots \lor \xi_m) \neq free(\xi \lor \ldots \lor \xi_m)$, we nondeterministically choose a subset $\{i_1, \ldots, i_k\}$ of $\{1, \ldots, n\}$ such that $\xi = (\xi_1 \land \psi_{i_1} \land \ldots \land \psi_{i_k}) \lor \ldots \lor (\xi_m \land \psi_{i_1} \land \ldots \land \psi_{i_k})$ satisfies $rr(\xi') = free(\xi')$. Let $\{j_1, \ldots, j_l\} = \{1, \ldots, n\} \setminus \{i_1, \ldots, i_k\}$, we rewrite $\psi$ into $\psi'$:
  $$\psi' = (\psi_{j_1} \land \ldots \land \psi_{j_l} \land \xi')$$

- **Push-into-quantifier:** Consider the subformula
  $$\psi = \psi_1 \land \ldots \land \psi_n \land \exists \xi$$
  If $rr(\psi) = free(\psi)$, but $rr(\xi) \neq free(\xi)$, we nondeterministically choose a subset $\{i_1, \ldots, i_k\}$ of $\{1, \ldots, n\}$ such that $\xi' = \psi_{i_1} \land \ldots \land \psi_{i_k} \land \xi$ satisfies $rr(\xi') = free(\xi')$. We replace $\psi$ by $\psi' = \psi_{i_1} \land \ldots \land \psi_{i_k} \land \exists \xi'$
  where $\{j_1, \ldots, j_l\} = \{1, \ldots, n\} \setminus \{i_1, \ldots, i_k\}$.

- **Push-into-negated-quantifier:** Consider the subformula
  $$\psi = \psi_1 \land \ldots \land \psi_n \land \neg \exists \xi$$
  If $rr(\psi) = free(\psi)$ but $rr(\xi) \neq free(\xi)$, we replace $\psi$ by $\psi' = \psi_{1} \land \ldots \land \psi_{n} \land \neg \exists \xi'$
  where $\xi' = \psi_{i_1} \land \ldots \land \psi_{i_k} \land \xi$
  and where $rr(\xi') = free(\xi')$ and $\{i_1, \ldots, i_k\} \subseteq \{1, \ldots, n\}$.

$\psi'$ is equivalent to $\psi$ because the propositional formulas $p \land q \land \neg \tau$ and $p \land q \land \neg (p \land \tau)$ are equivalent.

$$\psi' = \psi_{1} \land \ldots \land \psi_{n} \land \neg (\psi_{1} \land \ldots \land \psi_{n} \land \exists \xi)$$

And we continue to apply the Push-into-quantifier procedure.

Now we transform the RANF formula $\varphi$ into an equivalent Datalog query $(P_\varphi, G_\varphi)$. Suppose $\{x_1, \ldots, x_k\} = free(\varphi)$, $(P_\varphi, G_\varphi)$ is inductively constructed as follows:

- $\varphi = R(e_1, \ldots, e_n)$, where $\{x_1, \ldots, x_k\}$ is the set of free variables in $\{e_1, \ldots, e_n\}$:
  $P_\varphi = \{G_\varphi(x_1, \ldots, x_k) := R(e_1, \ldots, e_n)\}$
  and the datalog query is $(P_\varphi, G_\varphi)$.

- $\varphi = x = a$ or $a = x$:
  $P_\varphi = \{G_\varphi(x) := x = a\}$

- $\varphi = \psi_1 \land \ldots \land \psi_n$, we divide $\{\psi_1, \ldots, \psi_n\}$ into a set of positive subformulas $\{\psi_{i_1}, \ldots, \psi_{i_m}\}$ and a set of equalities $\{x = a \lor a = x \lor x = x \lor x = y\}$ $\{\psi_{m+1}, \ldots, \psi_{m+n}\}$, and a set of negative subformulas $\{\neg \psi_{m+1}, \ldots, \neg \psi_m\}$. Let $\{x_{i_1}, \ldots, x_{i_k}\} = free(\psi_i)$, we construct $(P_{\psi_i}, G_{\psi_i})$ for each $\psi_i$ in $\{\psi_1, \ldots, \psi_m\}$, and for each $\psi_i \in \{\psi_{m+1}, \ldots, \psi_m\}$. The result is as follows:

$$P_\varphi = \bigcup_{i=1}^{m} P_{\psi_i} \cup \bigcup_{i=m+1}^{m} P_{\psi_i} \cup \left\{\begin{array}{l}
G_{\psi_i}(x_1, \ldots, x_k) := G_{\psi_{i_1}}(x_{i_1}, \ldots, x_{i_k}) \ldots \ldots \\
G_{\psi_{m+1}}(x_{m+1}, \ldots, x_{m+n}) \ldots \ldots \\
G_{\psi_m}(x_{m+1}, \ldots, x_{m+n}) \ldots \ldots \\
G_{\psi_{m+1}}(x_{m+1}, \ldots, x_{m+n}) \ldots \ldots \\
G_{\psi_m}(x_{m+1}, \ldots, x_{m+n}) \ldots \ldots \\
G_{\psi_{m+1}}(x_{m+1}, \ldots, x_{m+n}) \ldots \ldots \\
G_{\psi_m}(x_{m+1}, \ldots, x_{m+n}) \ldots \ldots \\
G_{\psi_{m+1}}(x_{m+1}, \ldots, x_{m+n}) \ldots \ldots \\
G_{\psi_m}(x_{m+1}, \ldots, x_{m+n}) \ldots \ldots \end{array}\right\}$$
Join and Selection

| Joint | Negation |
|-------|----------|
| \[ h(X) \leftarrow r_1(Y), r_2(Z). \]
| \( \text{vars}(X) = \text{vars}(Y) \cup \text{vars}(Z) \) | \[ h(X) \leftarrow r_1(X), \neg r_2(Y). \]
| \( \text{vars}(X) \supseteq \text{vars}(Y) \) | \( \text{vars}(X) \subseteq \text{vars}(Y) \) |

- \[ -h(X) \leftarrow \neg r_1(Y), r_2(Z). \]
- \[ h(X) \leftarrow r_1(Y), \neg r_2(Z). \]
- \[ +h(X) \leftarrow +r_1(Y), r_2(Z). \]
- \[ +h(X) \leftarrow r_1(Y), +r_2(Z). \]
- \[ h^+(X) \leftarrow r_1^+(Y), r_2^+(Z). \]

Projection

| Union | Negation |
|-------|----------|
| \[ h(X) \leftarrow r_1(X,Y). \] | \[ h(X) \leftarrow r_1(X,Y). \] |
| \[ \#h(X) \leftarrow +r_1(X,Y), \neg h(X). \] | \[ -h(X) \leftarrow -r_1(X,Y), \neg r_2(Y). \] |
| \[ -h(X) \leftarrow -r_1(X,Y), \neg r_2(Y). \] | \[ -h(X) \leftarrow -r_1(X), +r_2(Y). \] |
| \[ +h(X) \leftarrow +r_1(X), +r_2(Y). \] | \[ +h(X) \leftarrow +r_1(X), \neg r_2^+(Y). \] |
| \[ h^+(X) \leftarrow r_1^+(X,Y). \] | \[ h^+(X) \leftarrow r_1^+(X), \neg r_2^+(Y). \] |

Figure 7: Rules for incrementalizing Datalog putback program. \( X \) denotes a tuple of variables, \( \text{vars}(X) \) denotes the set of all variables in \( X \).

- \( \varphi = \psi_1 \lor \ldots \lor \psi_n \), where \( \text{free}(\psi_1) = \ldots = \text{free}(\psi_n) = \{x_1, \ldots, x_k\} \). We construct \((P_\varphi, G)\) (with the same goal predicate \( G \)) for each \( \psi_i \) in \( \{\psi_1, \ldots, \psi_n\} \) and obtain:

\[ P_\varphi = \bigcup_{i=1}^{n} P_{\psi_i} \]

\[ G_\varphi = G \]

- \( \varphi = \exists y_1, \ldots, y_m. \psi(z_1, \ldots, z_n) \), where \( \{x_1, \ldots, x_k\} = \{z_1, \ldots, z_n\} \setminus \{y_1, \ldots, y_m\} \):

\[ P_\varphi = P_0 \cup \{G_\varphi(x_1, \ldots, x_k) \leftarrow G_0(z_1, \ldots, z_n)\} \]

C. RULES FOR INCREMENTALIZING PUTBACK PROGRAMS

Considering a putback program \( \text{putdelta} \) in nonrecursive Datalog with negation, NR-Datalog\(^*\), we shall derive Datalog rules to compute changes to delta relations of the source database when the view relation is changed. The derived Datalog rules form an incrementalized program of \( \text{putdelta} \).

Our idea is that we first transform \( \text{putdelta} \) into an equivalent Datalog program, in which every IDB relation is defined from at most 2 other relations. We then inductively apply the incrementalization rules in Figure 7 to derive Datalog rules for computing changes to each IDB relation.

**Lemma C.1.** For every NR-Datalog\(^*\) program \( P \) with the goal is restricted to a set of IDB relations \( G \), there is a NR-Datalog\(^*\) \( P’ \), in which each IDB relation is defined from at most two other relations, that the queries \((P, G)\) and \((P’, G)\) are equivalent.

**Proof.** (Sketch) There exists such a transformation between these two Datalog programs because non-recursive Datalog program with negation is equivalent to relational algebra, in which each binary relational operator can be simulated by Datalog rules with two relations in the rule bodies.

For a relation \( h \) defined from two relations \( r_1 \) and \( r_2 \), we have four incrementalization rules corresponding to four cases, shown in Figure 7. In each case, we derive Datalog rules for computing separately insertions (\( \Delta^+_{h} \)) and deletions (\( \Delta^-_{h} \)) to \( h \) when there are changes to relations \( r_1 \) and \( r_2 \). Note that in these derived Datalog rules, if \( \Delta^+_{h} \) and \( \Delta^-_{h} \) are disjoint, then the obtained \( \Delta^+_{h} \) and \( \Delta^-_{h} \) are disjoint. Therefore, we can inductively apply the four incrementalization rules when \( h \) is used to define other IDB relations. We have formally proven the correctness of these incrementalization rules by using an assistant theorem prover.

**Lemma C.2.** For each case in Figure 7, the new relation \( h^+ \) computed from its defining rules is the same as the result obtained by applying delta relations \( +h \) and \( -h \) computed by the derived Datalog rules to the old relation \( h \).

Our incrementalization rules can be easily extended for built-in predicates (e.g., \( =, <, > \)) in the Datalog program by considering these predicates as unchanged relations in our incrementalization rules.

D. DERIVING VIEW DELTAS

**Algorithm 2:** View-Delta\((u_1, \ldots, u_n)\)

\[ \Delta^+_{V} \leftarrow \emptyset; \Delta^-_{V} \leftarrow \emptyset; \]

for each DML statement \( u \) in \( u_1, \ldots, u_n \) do

Derive the set \( \delta^+/-\delta^- \) of inserted/deleted tuples;

\[ \Delta^+_{V} \leftarrow (\Delta^+_{V} \cup \delta^+); \]

\[ \Delta^-_{V} \leftarrow (\Delta^-_{V} \cup \delta^-); \]

Our incrementalization on putback transformation requires deriving a delta relation \( \Delta^\top \) of the view \( V \) in the form of insertions and deletions when there are any view update requests. In RDBMSs, these update requests are declarative DML (data manipulation language) statements of the forms [52] \text{INSERT INTO} \ V \ \text{VALUES} (\ldots), \text{DELETE FROM} \ V \ \text{WHERE} <\text{condition}> \), and \text{UPDATE} \ V \ \text{SET} \ \text{expr} = \ldots \ \text{WHERE} <\text{condition}> \). Fortunately, it is trivial to obtain from the \text{INSERT/DELETE} statement the tuples that need to be inserted or deleted. Meanwhile, an \text{UPDATE} \ V \ \text{statement} on the view can be represented as deletions followed by insertions; hence, we can also derive the deleted/inserted tuples.

A view update request can be a sequence of DML statements rather than a single one. This sequence is combined into one transaction by using the SQL command \text{BEGIN} before the sequence and the command \text{END} after the sequence. To address this case, we propose a procedure for calculating \( \Delta^\top \) and \( \Delta^\bot \) of the whole view update transaction, as shown in Algorithm 2. Concretely, for each DML statement in the sequence, we derive the insertion set \( \delta^+\) and the deletion set \( \delta^-\), and we merge these changes to \( \Delta^\top \) and \( \Delta^\bot \). In this way, later statements have stronger effects than earlier statements. For example, if the sequence is inserting a tuple \( \bar{t} \) and then deleting this tuple \( \bar{t} \), the tuple \( \bar{t} \) is no longer inserted, i.e., we remove \( \bar{t} \) from \( \Delta^\top \).