Light Quark Mass Determinations from the Lattice.

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This paper is a review of recent lattice determinations of the light quark masses. It describes the method employed to calculate quark masses in the lattice formulation, and the extrapolations required to reach the physical regime. This review is designed to be accessible to a general audience, not specifically lattice theorists.

1. INTRODUCTION

The masses of the light quarks are fundamental parameters of the Standard Model, and yet are still surprisingly poorly determined. The Particle Data Book gives their values as

\begin{align}
m_u(1\text{GeV}) & = 2 \rightarrow 8 \text{ MeV} \\
m_d(1\text{GeV}) & = 5 \rightarrow 15 \text{ MeV} \\
m_s(1\text{GeV}) & = 100 \rightarrow 300 \text{ MeV}
\end{align}

The bulk of lattice calculations have studied the isospin averaged quark mass \( m_l = \frac{1}{2}(m_u + m_d) \) and I will restrict my attention to these calculations. I note though that there has been a preliminary lattice investigation of the isospin mass splitting \( m_d - m_u \) by using electromagnetic as well as gluonic fields. They obtain a value for \( m_u/m_d \) of \( \approx 0.5 \) in excellent agreement with other methods showing that isospin splitting effects are also technically accessible on the lattice.

In the next section I review the principles involved in lattice calculations of light quark masses. I then discuss the various extrapolations that are required in order to make contact with the real world in section 3. This section will be the main focus of the review. In section 4, I review the recent lattice results, attempting to explain some of the inconsistencies that occur. In this section I give rough lattice values of the light quark masses, based on the data and the issues raised in section 3. In the last section I compare these values with estimates using other methods.

For other reviews on this topic, including summaries of lattice results prior to 1994 see \[\text{[8,9]}\]. I note also that \[\text{[10] and [11]}\] contain very recent reviews of this topic.

2. HOW THE LATTICE CALCULATES THE LIGHT QUARK MASSES

The generic lattice approach to spectrum calculations studies two-point correlation functions, \( G_2(t) \), of hadronic interpolating operators in a background sea of glue (and sea quarks in the case of “unquenched” calculations). These background configurations are weighted by the appropriate Boltzmann factor (which means that the approach is identical to calculation of correlation functions in statistical mechanics). Lattice calculations are fully non-perturbative, and, in principle, only involve approximations that can be systematically improved. In practice the inclusion of dynamical sea quarks (i.e. virtual quark - anti-
quark pairs) is a technical headache because it involves a huge increase in computational requirements. However, more and more lattice results are being published with dynamical sea quarks (also called “unquenching”) and this trend is certain to gain momentum.

In the calculation of $G_2(t)$, if one were fortunate enough to have The Interpolating Operator of the hadron in question, then $G_2(t)$ would be The Two-point Function. However, when using a non-perfect operator, $O$, $G_2(t)$ receives contributions from all hadronic states which have non-zero overlap with $O$. It is straightforward to show that $G_2(t)$ has the following form

$$G_2(t) = \sum_i Z_i e^{-M_i t}$$

where the sum is over the hadronic states, and $M_i$ and $Z_i$ are the hadronic mass and overlap of the $i$-th state. Note that since the calculation is performed in Euclidean space-time, the excited states are exponentially suppressed with respect to the fundamental state (i.e. the exponentials have real arguments). This means that $G_2(t)$ becomes The Two-point Function only for sufficiently large values of $t$. Obviously the parameters of the ground state, in particular the mass $M_0 = M$, can be extracted by fitting $G_2(t)$ for $t$ sufficiently large, i.e.

$$G_2(t) \to Z e^{-Mt} \quad \text{as} \quad t \to \infty.$$  

Here comes the crucial point. This mass, $M$, is not the mass of the physical hadron. It is the mass of a hadron in a universe where the quark masses are the same as those input into the lattice calculation, i.e. the lattice is simulating a form of QCD, but not the QCD of the real world.

In fact it is a little more complicated than this. The mass $M$ (and all lattice predictions) are functions of all the input parameters of the lattice calculation. Specifically these parameters are:

- quark masses, $m$;
- lattice volume, $V$;
- lattice spacing, $a$;
- treatment of sea quark effects (i.e. quenched or unquenched);
- type of lattice action used.

In this section I consider only the dependence on the first quantity above; I leave the other effects to the next section.

Obviously we do not know the correct values of the quark masses to use in the calculation - these are, of course, the parameters that we wish to calculate. So the next step is to adjust these light quarks masses in the Lagrangian until the hadronic mass, $M$, agrees with the experimental values. Thus, for instance, we equate:

$$M_V(m_s, m_l) = M_{K^*} = 770\text{MeV},$$  \hspace{1cm} (2)

where $M_V(m_s, m_l)$ is the lattice value of the vector meson composed of quark with masses $m_s$ and $m_l$. We are naturally free to use any hadron containing a strange quark in order to determine the strange quark mass, e.g.

$$M_V(m_s, m_s) = M_\phi = 1020\text{MeV},$$

or

$$M_{PS}(m_s, m_l) = M_K = 494\text{MeV},$$  \hspace{1cm} (3)

where $PS$ stands for pseudoscalar. The differences in these various determinations of $m_s$ are systematic effects and should be included in the error determination. Note also that the determination of $m_s$ from the $K^*$ and $\phi$ are generally in excellent agreement. This is due to the facts that (i) the experimental values of these hadronic masses are linear in their constituent quark masses, and (ii) to a very good approximation, the lattice values of $M_V(m_1, m_2)$ are also linear in $m_1$ and $m_2$.

Normally, due to the computational expense, the lattice calculation is performed using only a small number (say 3 - 5) of quark masses. The adjustment of the quark masses mentioned above is done by means of an interpolation/extrapolation of these data points.

Note that there is an alternative approach to the calculation of light quark masses on the lattice using the axial Ward identity (see [2] & [12]).
There has also been a very recent lattice recent calculation of the “renormalisation group invariant mass” in [13].

Once the values of the lattice quark masses have been found, they are then expressed in the \( \overline{MS} \) scheme, traditionally at the scale 2 GeV. (For convenience, I note the conversion factor between 2 GeV & 1 GeV is \( \frac{\overline{m}(1 GeV)}{\overline{m}(2 GeV)} \approx 1.4 \).) The matching procedure uses the relationship between the two schemes,

\[
\overline{m}(\mu) = Z_m(\mu a)m(a),
\]

where \( \overline{m}(\mu) \) is the quark mass in the \( \overline{MS} \) scheme at the scale \( \mu \). \( Z_m \) has been calculated in perturbation theory [14] and its renormalisation group-improved form at next-to-leading order has been given in [2]. Note that the expansion parameter \( g^2 \) relevant for the matching coefficient \( Z_m \) is a short-distance (i.e. small) coupling constant, so perturbation theory should be fairly well behaved for this quantity (barring “tadpole” contributions [3]). This contrasts with the large distance scales governing hadronic quantities like \( M \) (which require a non-perturbative tool such as the lattice to calculate correctly).

There are two families of the lattice actions currently used, Wilson and Staggered. While the staggered action has features that are particularly endearing for chiral physics, it has the big disadvantage that its \( \mathcal{O}(g^2) \) coefficient in \( Z_m \) is very large; typically this term is 50 - 100% of the zero order term [3]. Therefore connecting any lattice quark mass value with continuum schemes introduces unknown higher order effects which are uncontrolled. For this reason I do not consider results from the staggered approach in this review.

In the Wilson action, the quark mass is additively renormalised. The connection between the quark mass parameter in the lattice Lagrangian, \( K^{-1} \), and the quark mass is

\[
m(a) = a^{-1} \frac{1}{2} \left( \frac{1}{K} - \frac{1}{K_c} \right),
\]

where \( K_c \) corresponds to a massless pion, i.e.

\[
M^2_{PS}(K_c, K_c) \equiv 0.
\]

All fields and couplings in the lattice action have had their dimensions removed by multiplication of appropriate powers of \( a \). These powers of \( a \) must be put back in all dimensionful predictions (see eq.(4)). The lattice spacing \( a \) can be obtained by fixing the lattice prediction of any dimensionful physical quantity with its experimental value. Thus one of the physical ‘predictions’ from the lattice is used to set \( a \) (this corresponds to setting the scale \( \Lambda_{QCD} \)), and one lattice ‘prediction’ is used for each quark mass determination (see above). The spacing \( a \) can be set from any dimensionful physical quantity (that has been measured by experiment!). Different physical quantities can lead to slightly different values of \( a \), and this systematic effect should again be reflected in the error estimate.

In this section I have described, in principle, how lattice calculations of light quark masses are performed. In the next section, I review the details of this approach, in particular the effect that the extrapolations in the quark masses, volume, lattice spacing etc. have on the final lattice prediction.

3. EXTRAPOLATIONS

3.1. Basic Ideas

In the previous section I stressed that there are several input parameters in lattice calculations of strong interaction physics: the quark masses, volume, lattice spacing, type of lattice action used and the number of quark flavours included in the sea, \( N_f \).\(^4\) Assuming that all of these parameters were set to their physical value, then the lattice would give absolute predictions of QCD. However, for technical reasons, these input parameters are almost never set equal to their physical values, and so the following extrapolations are required (note that in lattice calculations, one is free to use sea quark masses, \( m_{sea} \), different to the “valence” quark masses, \( m_V \)):

\[
\{m_{sea}\} \rightarrow \{m_u, m_d, \ldots\}
\]

\(^3\)Recently the lattice component of \( Z_m \) has been calculated non-perturbatively [3] which means that the only perturbative component remaining in \( Z_m \) is in the continuum matching.

\(^4\)The quenched approximation corresponds to \( N_f = 0 \).
Thus the lattice simulates QCD, but with an unusual (i.e. unphysical) set of parameters - the light quark masses are larger than their physical values (typically they are $\gtrsim 50$ MeV), the volume is finite, space-time is discretised, and the number of dynamical quark flavours does not always correspond with the real world. It is essential that all of the extrapolations eq.(6) are under control in order for the lattice to make accurate determinations of physical quantities.

A reliable extrapolation requires data points over a range of values of the parameter in question. Thus, for example, an extrapolation in $a$ requires data at several values of $a$. Due to the present limitations in computing power, it is not possible to cover a large amount of the (multi-dimensional) parameter space in order to perform all of the extrapolations. Therefore most lattice projects concentrate on the effects of varying only one or two of the above parameters. Different groups often concentrate their efforts in studying and extrapolating different parameters and this fact can sometimes lead to different final predictions for the quark mass. The wide variation in lattice predictions for the quark masses referred to in the introduction is primarily due to this effect. Data at the same parameter values from different groups are generally compatible.

Unfortunately, all of the extrapolations in eq.(\ref{eq:6}) are required in order to obtain The Lattice Prediction of the quark mass (or, in fact, of any physical quantity). Thus, while the lattice calculations performed so far are very encouraging, more work will be needed to reach this ultimate goal. Fortunately, many physical quantities aren’t nearly as sensitive to these input parameters as the quark mass, and therefore don’t suffer from these extrapolation effects to the same extent.

In the following subsections I will discuss the effects of extrapolations in the lattice spacing, $a$, the sea quark mass, $m^{sea}$, as well as the related systematics due to unquenching effects. I do not consider volume effects, since they are relatively small. The extrapolations in $m^{V}$ are basically those discussed in the previous section.

3.2. Lattice Spacing Effects

In \cite{3}, a collection of world data for $\langle m_l(2GeV) \rangle$ and $\langle m_s(2GeV) \rangle$ was extrapolated to the $a = 0$ limit. They found that the quenched Wilson data had a strong dependence on $a$. Assuming a linear dependence $m_l(a) \sim m_l(0) \times (1 + xa)$, the continuum extrapolant, $m_l(0)$, was as much as 40\% lower than the simulated data. (This applied to both $\langle m_l \rangle$ and $\langle m_s \rangle$.) An analysis of unquenched Wilson and staggered data was presented as “preliminary” since these simulations were in their early stages.

In \cite{4}, some evidence for an $O(a)$ effect was found and this was included their final quark mass estimate.

Another group, \cite{5}, analysed quenched Wilson and improved Wilson $m_s$ data from the APE collaboration over a range of fairly fine lattice spacing. They found that any $O(a)$ effects were at least as small as the statistical errors in the calculation and therefore no extrapolation in $a$ was
performed.

It is my belief that more data of better quality is required to unambiguously determine the quark mass dependency on \( a \).

### 3.3. Sea Quark and Unquenching Effects

A very interesting study of the dependency of \( m_l \) and \( m_s \) on the sea quark mass was presented in [6]. The calculation was at a fixed value of the strong coupling constant (naively corresponding to a fixed value of \( a \)) and used three values of the sea quark mass, the heaviest being at around \( \approx 0.6 \) (see e.g. [17]). Important also they have data for \( m_V \neq m_s \) effects. However, in this relatively early stage of unquenched calculations, one is forced to ignore such effects in order to make progress.

The SESAM collaboration also has data at non-degenerate “valence” quark masses \( m_V \), \( m_s \). This means that they are able to apply eqs. (6 & 3) directly to obtain \( m_s \) rather than relying on SU(3) flavour symmetry (see, e.g. [17]). Importantly also they present arguments about the correct procedure for extrapolating in \( m_V \) and \( m_s \) pointing out the perils of following an incorrect procedure [6]. I reproduce their arguments here.

Their explanation can be understood by studying Fig. 2 which is taken from Fig. 4 in the first reference in [6] (and slightly generalised). In this plot, the sea and valence lattice quark mass parameters \( 1/K^V \) and \( 1/K^s \) are plotted against each other. The solid diagonal line corresponding to \( K^V = K^s \) corresponds to the degenerate case where much of the earlier simulations have been performed. The three diagonal dashed lines correspond to the constraints \( M_{PS}(K^V, K^s) = 0 \), \( M_V(K^V, K^s) = M_{PS} \), and \( M_V(K^V, K^s) = M_\phi \) (labelled [3], [2] & [4] respectively). Note that, as constructed in [6], these lines are parallel, and empirically their slopes are \( \approx -1 \). The points labelled “\( \rho \)” and “\( \phi \)” are the physical points; \( \rho \) consisting of two light quarks in a sea of light quarks, and the \( \phi \) consisting of two strange quarks in a sea of light quarks. The point marked “0” corresponds to \( m_V = m_s = 0 \).

The two displacements on the y-axis denote the correctly determined quark masses \( m_l \) and \( m_s \) (see eqs. [3]) - i.e. they are calculated in a sea consisting of physical “\( l \)” quarks. The vertical displacements in the body of the figure show the incorrectly determined values of \( m_l \) and \( m_s \). The incorrect \( m_l \) value, marked “\( m_l^Q \)”, is obtained if one performed a “quenched-like” analysis - i.e. an extrapolation in \( m_V \) at fixed \( m_s \). Note that this

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6The strong dependence on \( m_s \) on the static quark potential and hadron masses has also recently been observed by the UKQCD collaboration [4].
is around twice the correct value of $m_l$. The $m_s^Q$ value is the corresponding “quenched-like” value of $m_s$. Note that this is surprisingly close to the correct $m_s$ value! Finally, $m_s^{\text{deg}}$ is the $m_s$ value obtained if one worked with degenerate sea and valence quarks. This can be seen to be roughly half of the correct $m_s$ value.

### 3.4. Summary of Extrapolation Effects

In concluding this section, it is clear that further work is required to fully understand and compensate for the effects of the extrapolations of the input parameters to their physical values in a lattice simulation. As detailed above, both the continuum extrapolation ($a \to 0$, see Sect. 3.2) and the sea quark effects (Sect. 3.3) introduce potentially large systematics that can be as much as a factor of two. It is these extrapolations that are the main source of the large spread of lattice values for the light quark masses presented in contemporary literature.

Despite these uncertainty, lattice calculations performed by different groups at common parameter values are in generally excellent agreement leading to great optimism for the future.

### 4. RECENT LATTICE RESULTS

For the reasons outlined in Sect. 2, I will restrict myself to discussing recent lattice results from the Wilson family of actions. There have been many calculations of light quark masses over the years (see [8,9] and references therein), but I will concentrate on the more recent results. These come from 4 groups [2–6] and are shown in Table 4. (Note that the APE/Rome group has new data in [5] which are compatible with those from [2–4].) Alongside the values for $m_l(2GeV)$ and $m_s(2GeV)$ are indicated whether (a) the simulations were performed in the quenched approximation, (b) the correct $m_{sea}$ extrapolation was performed (see Sect. 3.3) (c) an attempt was made to allow for $O(a)$ errors (see Sect. 3.2), and, (d) whether an “improved action” was used.

The quotation marks around the unquenched results for the LANL and FNAL groups signify that these are considered as preliminary, or are based on early unquenched results and plausibility arguments.

It is interesting to note that the small value for the unquenched $m_l(2GeV)$ in [4] and [5] are not reproduced in the fuller unquenched analysis of [6]. This is due to two extrapolation systematics: (i) the sea quark effects are correctly taken account of in [6], but not in [4] (see the discussion surrounding “$m_s^{\text{deg}}$” in Sect. 3.3); and (ii) the $O(a)$ effects are treated differently - [6] does no $a \to 0$ extrapolation since only one value of the lattice spacing was used (furthermore an un-improved action was used), whereas both [3] and [7] these are lattice actions with reduced discretisation systematics.
Table 1
Recent lattice results for the light quark masses.

| Group | $\bar{m}_l$(2GeV) [MeV] | $\bar{m}_s$(2GeV) [MeV] | Unquenched? | Correct $m^{sea}$ extrapolation? | $a \to 0$ taken? | "Improved Action" used? |
|-------|-----------------|-----------------|-------------|-------------------------------|-----------------|------------------|
| APE   | 128(18)         |                 | No          | -                             | -               | Yes              |
| APE   | 122(20)         |                 | No          | -                             | -               | Yes & No         |
| LANL  | 3.4(4)(3)       | 100(21)(10)     | No          | -                             | Yes             | Yes & No         |
| FNAL  | 3.6(6)          | 95(16)          | No          | -                             | Yes             | Yes & No         |
| SESAM | 5.5(5;2)        | 166(15)         | No          | -                             | No              | No               |

Table 2
Recent non-lattice results for $m_l$ and $m_s$.

| QCD Sum Rules | $\bar{m}_l$(2GeV) = 4.9(1.1)MeV |
|---------------|----------------------------------|
| 18            | 3.4MeV $\leq \bar{m}_l$(2GeV) $\leq$ 5.6MeV |
| 19            | $\bar{m}_s$(2GeV) $\approx$ 5MeV |
| 20            | $\bar{m}_l$(2GeV) = 114(21)MeV |
| 21            | $\bar{m}_s$(2GeV) = 141(21)MeV |
| 22            | $\bar{m}_s$(2GeV) = 146(14)MeV |
| 23            | $\bar{m}_s$(2GeV) = 102(13)MeV |
| Chiral Perturbation Theory | $m_s/m_l = 24.4(1.5)MeV$ |
| Experiment    | $\bar{m}_s$(2GeV) $\approx$ 160MeV |

I incorporate a linear $a \to 0$ extrapolation procedure (see Sect. [2.3]).

The other interesting feature to note is that the relative quenching effects in $\bar{m}_l$(2GeV) appear small, whereas they appear significant in the case of $\bar{m}_s$(2GeV) [3]. This can again be understood in terms of the discussion in Sect. 3.3 surrounding Fig.2.

By considering the extrapolation effects discussed in Sect. 3, I give the following rough estimates for the values of the (unquenched !) quark masses based on the results in Table 1:

$\bar{m}_l$(2GeV) = $2 \to 5$ GeV, \hspace{1cm} (7)

$\bar{m}_s$(2GeV) = $80 \to 140$ GeV. \hspace{1cm} (8)

The $\bar{m}_l$(2GeV) range is chosen so that it covers the values in Table 1. The $\bar{m}_s$(2GeV) estimate is obtained by considering the fact that the LANL (unquenched) predictions are likely to be too low due to their sea quark treatment, whereas the SESAM value is possibly too high due to $O(a)$ effects. The values in eqs.(7,8) are obviously only meant as a guide, and until the parameter space of lattice calculations is more fully explored, it is not possible to perform all the extrapolations required to determine The Lattice Prediction of $\bar{m}_l$(2GeV) and $\bar{m}_s$(2GeV).

5. COMPARISONS WITH OTHER METHODS

I give in Table 2 some determinations of light quark masses from non-lattice methods. (These have all been run to the scale 2 GeV.) Note the preliminary result from the Aleph Collaboration for $m_s$, [25]. The lattice estimates in eqs.(7,8) are in fairly good agreement with these results, although they seem generally on the low side. This point has been investigated in [24] where lower bounds on $m_l$ and $m_s$ are derived from fairly fundamental principles. The lower limit for $\bar{m}_s$(2GeV) in eq.(8) is in some conflict with the bounds in [20]. (See also [27].)

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DISCUSSIONS

A.P. Contogouris, University of Athens
Some time ago, P. Lepage of Cornell Univ. proposed a method for lattice calculations and claimed that it could be used in a small computer, even a laptop(!). Did you use his method? And if not, why? More generally, would you like to comment about the present stage of Lepage’s approach?

C.R. Allton
No, the calculations that I have described do not use Lepage’s “coarse-lattice” method. This is a proposal that adds extra terms to the lattice action in order to reduce the effects of discretisation - i.e. it is a so-called “improvement” scheme. The important feature of Lepage’s approach is that the coefficients of these additional terms are set by a re-arranged perturbation series (contrasting with other improvement methods which use a fully non-perturbative technique). Lepage’s approach has had success in reducing the rotational non-invariance of the static quark potential, and in quantities heavily dependent on that potential (such as physics of the heavy sector). However, progress in the light quark sector has been limited so far and this is the reason why the method has not been used for the determination of light quark masses.

K.F. Liu, University of Kentucky
I want to add to what Chris has said. While the improved action program has been working remarkably well for the pure gauge sector and heavy quarks for NRQCD, its progress for light quarks is still limited at this stage.