Viscosity of neutron star matter and r-modes in rotating pulsars

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Age-period diagram for pulsars

Young pulsars’ periods are much larger than the Kepler limit.

\[ \nu_K \approx 1.2 \text{ kHz}(M/M_\odot)^{1/2}(10\text{ km}/R)^{3/2} \]

\[ \tau_{s.d.} = \frac{P}{2(\dot{P})} \]

The ATNF pulsar catalogue

recycled pulsars

stability of old pulsars?
R-modes in rotating neutron star

Consider a neutron star with a radius \( R \) rotating with a rotation frequency \( \Omega \) and a perturbation in the form

\[
\delta v(r, t) = a(t) R \Omega \left( \frac{r}{R} \right)^l r \times \nabla Y_l e^{i(\omega t - t/\tau)}
\]

and the oscillation amplitude

\[
\omega = -\Omega \frac{(l-1)(l+2)}{(l+1)}
\]

analogous to Rossby waves in oceans and atmosphere

1998 Andersson and Friedman, Morsink showed that in NS these modes are unstable

for \( a \ll 1 \) amplitude changes

\[
\dot{a}(t) \approx -\frac{a}{\tau}
\]

\[
\frac{1}{\tau} = -\frac{1}{\tau_G} < 0
\]

gravitational radiation drives r-mode unstable

(most unstable for \( l=m=2 \))

talks by Kostas Kokkotas week 2 and Nils Anderson week 3
If the r-modes are undamped, the star would lose its angular moment on the time scale of $\tau_G$, because of an enhanced emission of gravitation waves.

\[
\frac{1}{\tau_G} = \frac{1}{15.6 \text{ s}} R_6^7 \Omega_4^6 \frac{\rho_c}{\rho_0} \]

[Lindblom, Owen, Morsnik, PRL80 (98) 4843]

\[
\frac{1}{\tau} = -\frac{1}{\tau_G} + \frac{1}{\tau_V} \quad \text{viscous damping}
\]

**Viscosity**

**Euler equation:**

\[
\rho \partial_t \mathbf{v}_i + \nabla_i p = -\nabla_i \rho + \partial_k \sigma_{ki}\]

\[
\sigma_{ik} = \eta \left( \nabla_k \mathbf{v}_i + \nabla_i \mathbf{v}_k - \frac{2}{3} \delta_{ik} \nabla \mathbf{v} \right) + \zeta \delta_{ik} \nabla \mathbf{v}
\]

**shear viscosity**

\[
\text{Dissipation when there is a velocity gradient}
\]

**bulk viscosity**

\[
\text{Dissipation after uniform volume change}
\]

Maxwell (1860) kinetic theory calculations

\[
\eta \sim \rho v_{\text{rms}} l \quad \text{Units:} \quad \frac{g}{\text{cm} \cdot \text{s}} = \text{Poise}
\]

\[\eta_{\text{Air}} \approx 1.8 \cdot 10^{-4} \text{Poise} \quad \eta_{\text{water}} \approx 1.0 \cdot 10^{-2} \text{Poise}\]
**R-modes stability**

R-mode is unstable if the time \( \tau > 0 \)

\[
\frac{1}{\tau} = -\frac{1}{\tau_G} + \frac{1}{\tau_\eta} + \frac{1}{\tau_\zeta}
\]

gravitational time scale

\[
\tau_G^{-1} = \frac{4096}{164025} G \Omega^6 R^7 \langle \rho \rangle_6 \approx \frac{6.43 \cdot 10^{-2}}{s} \frac{R_6^7 \Omega_4^6 \rho_{\text{cen}}}{\rho_0}
\]

damping rate due to the shear viscosity

\[
\tau_\eta^{-1} = \frac{5}{R^2} \frac{\langle \eta \rangle_4}{\langle \rho \rangle_6} \approx \frac{5.98 \cdot 10^{-5}}{s} \frac{\langle \eta_{20} \rangle_4}{R_6^2 \rho_{\text{cen}}/\rho_0}
\]

damping rate due to the bulk viscosity

\[
R_6 = R/10^6 \text{ cm} \\
\Omega_4 = \Omega/10^4 \text{ s} \\
\eta_{20} = \eta/(10^{20} \frac{g}{\text{cm s}}) \\
\zeta_{20} = \zeta/(10^{20} \frac{g}{\text{cm s}})
\]

\[
\tau_\zeta^{-1} = \frac{4\pi}{690} \left( \frac{\Omega^2}{\pi G \rho} \right)^2 \frac{\langle \zeta (1 + 0.86(r/R)^2) \rangle_8}{R^2 \langle \rho \rangle_6} \approx \frac{2.20 \cdot 10^{-7}}{s} \frac{R_6^4 \Omega_4^4 \langle \zeta_{20} [1 + 0.86(r/R)^2] \rangle_8}{(M/M_\odot)^2 (\rho_{\text{cen}}/\rho_0)}
\]

\[
\langle \ldots \rangle_n = \frac{1}{R^{n+1}} \int_0^R (\ldots) r^n \, dr
\]

profile averages

[Lindblom, Owen, Morsnik PRL 80 (98) 4843]
Owen, Lindblom, Cutler, Schutz, Vecchio, Andersen PRD58 (98) 084020]
**Shear viscosity**: collisional viscosity:

- lepton, nucleon, phonon, neutrino contributions

**Bulk viscosity**: collisional viscosity

- “soft-mode” viscosity (Leontovich-Mandelstam)

**Inputs**: EoS, density profiles in NS, pairing gaps, NN-interaction

- **neutron star structure**

  - mass and radius

  - density profiles

\[
\frac{\rho(r)}{\rho(0)} \approx \frac{n(\rho)}{n(0)} \approx f\left(\frac{r}{R}\right) = 1 - \left(\frac{r}{R}\right)^2
\]

\[
\frac{X_p(r)}{X_p(0)} \approx \chi\left(\frac{r}{R}\right) = \left[1 - \left(\frac{r}{R}\right)^2\right]^{3/5}
\]

for \(n > 0.6n_0\) HDD EoS

for \(n \leq 0.6n_0\) Friedman-Pandharipande-Skyrme EoS
Profiles of critical temperatures for the singlet neutron and proton pairing

We ignore $^3P_2$ neutron pairing

The heavier a NS is the smaller is the size of the region with nucleon pairing
shear viscosity
\[ \eta = \frac{1}{15 T} \sum_a \int \frac{d^3p}{(2\pi)^3} \tau_a \left( v^2, p^2 \right) n_a(p) \left( 1 \pm n_a(p) \right) \]

degenerated fermions \( T \ll T_F \)
\[ \eta = \frac{1}{5} \sum_a n_a \rho_F, a \tau_a \]

collision time of the particle \( a \)

Fermi liquid result
\[ \tau_a \propto \frac{1}{T^2} \]

bulk viscosity
\[ \zeta = \frac{1}{9 T} \int \frac{d^3p}{(2\pi)^3} \tau \left( v^2 - 3\nu_s^2 \right)^2 m_n^2 n_p \left( 1 \pm n_p \right) \]
Lepton shear viscosity

Lepton shear viscosity = electron + muon contribution

\[ \eta_{e/\mu} = \eta_e + \eta_\mu \]

low T, Fermi liquid results:

\[ \eta_l = \frac{1}{5} n_l p_{F,l} \tau_l \]

Lepton collision time \( \tau_l \) is determined by lepton-lepton and lepton-proton collisions.

Flowers and Itoh:

\[ \eta^{(FI)}_{e/\mu} \approx \eta^{(FI)}_e = 4.2 \cdot 10^{17} \left[ \frac{g}{\text{cm} \cdot \text{s}} \right] \left( \frac{\rho}{\rho_0} \right)^2 T^{-2}_9 \]

Important role of the phonon modification (plasmon exchange)

for QCD plasma [Heiselberg, Pethick Phys. Rev. D 48 (1993) 2916]

[Shternin, Yakovlev, Phys. Rev. D 78 (2008) 063006]

leading terms for small \( T \)

\[ \eta_e = 1.82 \cdot 10^{19} \left[ \frac{g}{\text{cm} \cdot \text{s}} \right] \left( \frac{n_p}{n_0} \right)^{\frac{14}{9}} \left( \frac{n_e}{n_p} \right)^2 \frac{T_{9}^{-\frac{5}{3}}}{(1 + r)^{\frac{2}{3}}} \]

\[ r = \left( p_{F,e}^2 + p_{F,\mu}^2 \right) / p_{F,p}^2 \]

\[ \eta_\mu = \left( \frac{n_\mu}{n_e} \right)^{\frac{5}{3}} \eta_e \]

muon contributions are important
Lepton shear viscosity vs. neutron star mass

Effects of proton pairing on lepton shear viscosity

[Shternin, Yakovlev, Phys. Rev. D 78 (2008) 063006]
Nucleon shear viscosity

\[ \eta_n = \frac{3n_n p_{F,n}^2 m_N^2}{80 m_n^* 4 T^2 S_{nn}} \]

Fermi liquid result

\[ Q_{nn} = \frac{1}{4} \sum_{\text{spin}} |M_{nn}|^2 \]

**Effective NN cross section**

\[ S_{nn} = \frac{m_N^2}{16 \pi^2} \int_0^1 dx' \int_0^{\sqrt{1-x'^2}} dx \frac{12 x^2 x'^2 Q_{nn}(q, q')}{\sqrt{1 - x^2 - x'^2}} \]

**FOPE:**

\[ S_{nn}^{\text{FOPE}} \approx \frac{3 m_n^2 f_{\pi NN}^4}{40 \pi} \approx \frac{1.1}{m_{\pi}^2} \]

**MOPE:**

\[ S_{nn}^{\text{MOPE}} = K S_{nn}^{\text{FOPE}} \]

\[ K = \frac{30 \pi \Gamma^4 p_{F,n}^3}{128 \gamma \bar{\omega}^3} \left[ 1 + \frac{2}{3 \gamma p_{F,n}} \bar{\omega} \right] \]

**Modification factor**

\[ K(n = n_0) \approx 0.3 \quad \rightarrow \quad [\text{Bacca et al., PRC80}] \]

\[ K(n = 2.6 n_0) \approx 1 \]

\[ K(n \geq 3n_0) \approx 2 \quad \eta_n \ll \eta_l \]
Phonon shear viscosity

We consider the interaction of the phonon (Anderson-Bogoliubov) mode with neutrons

\[
\eta_{\text{ph}} = \int \frac{d^3 q}{(2\pi)^3} \frac{\tau_{\text{ph}}}{15 T} \frac{(sv_{F,n})^2 q^2}{(e^{sv_{F,n}q/T} - 1)(1 - e^{-sv_{F,n}q/T})} = \frac{2\pi^2}{25} \frac{T^4}{v_{F,n}^3} \bar{\tau}_{\text{ph}} \quad s = 1/\sqrt{3}
\]

From the Larkin-Migdal equation for anomalous vertex \[\text{[E.K. Voskresensky, PRC77, PRC81]}\]

\[
\tilde{\tau}_{V,0} = \frac{-2 \Delta \omega}{\omega^2 - \frac{1}{3} v_{F,n}^2 q^2 - i\omega \gamma_{\text{ph}}(\omega, q)} \tau_{V,0}
\]

bare p.-h. vertex

\[
\gamma_{\text{ph}}(q) = 1/\tau_{\text{ph}} \approx \frac{2\pi}{3} v_{F,n} q e^{-\frac{\sqrt{3} \Delta}{T}}
\]

The phonon lifetime \(\bar{\tau}_{\text{ph}} \approx 5.9 \cdot 10^{-22} \frac{e^{\sqrt{3} \Delta}}{T_0} \) s must be smaller than the balistic time

\[
\tau_{\text{bal}} \sim \frac{1}{s v_F} \approx 1.6 \cdot 10^{-5} \left(\frac{n_0}{n}\right)^{\frac{1}{3}} \left\frac{m^*}{m_N}\right\text{ size of the region of the neutron pairing}
\]
\[ \eta_{ph} \simeq 2.1 \cdot 10^{23} \left( \frac{g}{\text{cm} \cdot \text{s}} \right) \left( \frac{n_0}{n} \right) \left( \frac{m_n^*}{m_N} \right)^3 T_9^4 \frac{\min\{\tau_{ph}, \tau_{bal}\}}{s} \]

\[ \tau_{ph} \simeq 5.9 \cdot 10^{-22} \frac{e^{\sqrt{\frac{3}{2}} \frac{\Delta}{T}}}{T_9} \text{ s} \]

\[ \tau_{bal} \simeq 1.6 \cdot 10^{-5} \text{ s} \left( \frac{n_0}{n} \right)^{\frac{1}{3}} \frac{m_n^*}{m_N} \]

\[ \eta_{ph} \] strongly depends on the pairing gap, contributes at temperatures slightly below \( T_c \).

1. pairing gaps reduced in medium
2. vacuum pairing gaps

[Hebeler, Schwenk, Friman]
**Neutrino shear viscosity**

With the temperature increase the neutrino mean free path decreases and for sufficiently high temperatures neutrinos become trapped inside the neutron star interior.

\[
\eta_\nu = 2 \int \frac{2d^3q}{(2\pi)^3} \frac{\tau_\nu}{15T} \frac{v_\nu^2 q^2}{\left(e^{v_\nu q/T} + 1\right)\left(1 + e^{-v_\nu q/T}\right)} = \frac{7\pi^2}{225 v_\nu^3} T^4 \tau_\nu
\]

\[
\mu_\nu \approx 0 \quad \quad v_\nu = c
\]

Neutrino mean free path is determined by inverse MMU and PU processes

\[
\bar{\tau}_\nu \simeq \frac{8.7}{T_9^4} \frac{s}{F_{\text{MMU}}(n)} \left(\frac{m_N}{m_N^*}\right)^4 \frac{(n_0/n_e)^{\frac{1}{3}}}{1 + \chi_{\text{PU}}(n, T)}
\]

\[
\eta_\nu \simeq \frac{3.08 \cdot 10^{22}}{1 + \chi_{\text{PU}}(n, T)} \left[\frac{g}{\text{cm}\cdot\text{s}}\right] \left(\frac{n_0}{n_p}\right)^{\frac{1}{3}} \left(\frac{m_N}{m_N^*}\right)^4 \frac{F_{\text{MMU}}(n)}{1 + \chi_{\text{PU}}(n, T)}
\]

**weak T dependence**

contributes only in regions where neutrinos are trapped

\[
\eta_\nu^{(\text{opac})}(r, T) = \eta_\nu(n(r)) \theta(r_{\text{opac}} - r)
\]

**opacity radius** is determined as

\[
v_\nu \bar{\tau}_\nu(n(r_{\text{opac}}), T) = R - r_{\text{opac}}
\]
Lepton shear viscosity

\[ \langle \eta \rangle_4 \text{[g/(cm s)]} \]

\[ \log_{10} \left( \frac{M}{M_\odot} \right) \]

- \[ \langle \eta \rangle_4 \] vs. \[ T \text{[K]} \]
- Lines represent different mass-to-solar-mass ratios: 1.0 (solid blue), 1.5 (dashed blue), 2.05 (dotted blue)
- Regions labeled: leptons, neutrinos, MMU, MMU+PU

[Graph showing the relationship between lepton shear viscosity, mass-to-solar-mass ratio, and temperature.]
Bulk viscosity

\[ \zeta_{\text{coll}} = \frac{m_N^*}{162\pi^2} \frac{m_N^*}{n_0} \tau T^4 \left[ \frac{n_0}{n} \right]^{1/3} F_0^2 \]

[Sykes, Brooker, Ann. Phys. 56(1970) 1]

\( F_0 \) is the zeroth harmonics of the dimensionless scalar Landau-Migdal parameter, \( F_0 \sim 1 \)

\[ \tau \sim \frac{m_\pi^2}{(m_N^* T^2)} \] nucleon relaxation time;

\[ \zeta_{\text{coll}} \sim 90 \left[ \frac{g}{\text{cm}\cdot\text{s}} \right] T^2 g \left[ \frac{n_0}{n} \right]^{1/3} F_0^2 \]

small contribution
Energy dissipation of the mode: \[ \dot{E}_{\text{mode}} = P \dot{V} - \epsilon_\nu \] \( \star \) is neutrino emissivity

Energy of the mode decreases if the pressure depends on an order parameter, which variation is delayed with respect to the variation of the density [Mandelstam, Leontovich, ZhETF 7 (1937) 438]

**in neutron stars**

order parameter is \( X_l = n_l/n \) lepton concentration \( \delta \mu_l = \mu_n - \mu_p - \mu_l \neq 0 \)

\[ \delta \dot{X}_l = -\frac{\delta X_l}{\tau_{X,l}} + n \frac{\partial \delta \mu_l}{\partial n} \delta n(t) \] relaxation time

\[ \zeta_{\text{s.m.}} \approx -\frac{\partial P}{\partial X_l} \frac{dX_l}{dn} \left\{ \frac{n \tau_{X,l}}{1 + \omega^2 \tau_{X,l}^2} \right\} \]

**soft mode contribution**

[ Sawar PRD39, Haensel, Levenfish, Yakovlev A&A357, A&A372 ]
\[ \zeta_{\text{s.m.}} \approx - \frac{\partial P}{\partial X_l} \frac{dX_l}{dn} \frac{n \tau_{X,l}}{1 + \omega^2 \tau_{X,l}^2} \approx - \frac{\partial P}{\partial X_l} \frac{dX_l}{dn} \frac{n}{\omega^2 \tau_{X,l}} \quad \text{for} \quad \omega \tau_{X,l} \gg 1 \]

\[ \frac{1}{\tau_{X,l}} = \sum_r \mathcal{R}^{[r]} = \sum_r \frac{1}{\tau_{X,l}^{[r]}} \]

\[ \zeta_{\text{s.m.}} \approx \sum_r \zeta_{[r]} \]

**Direct Urca (DU);** \[ \zeta_{[\text{DU}]} \propto T^5 \]

**Modified Urca (MU);** \[ \zeta_{[\text{MU}]} \propto T^7 \]

[calculated with FOPE]

**Medium MU (MMU);** \[ \zeta_{[\text{MMU}]} \propto F_{\text{MMU}}(n)T^7 \]

[calculated with MOPE]

**Pion condensate Urca (PU);** \[ \zeta_{[\text{PU}]} \propto T^5 \]

\[ \langle \zeta_{\text{s.m.}} (1 + 0.86r^2/R^2) \rangle_{\text{s}} \]

\[ T = 10^9 \text{K,} \quad \omega = 4/3 \times 10^4 \text{Hz} \]
Shear and bulk viscosities. Results

![Graph showing the relationship between shear and bulk viscosities and temperature, with various lines representing different mass-to-mass ratios.](image)
R-mode stability window

\[ \tau^{-1}_G(\nu_c) = \tau^{-1}_\eta(\nu_c) + \tau^{-1}_\zeta(\nu_c) \quad \rightarrow \quad \nu_c = \nu_c(T) \]

\[ \nu = \Omega/2\pi \]

![Graph showing R-mode stability window with different mass ranges and shear bulk conditions]
1. star is born hot and rapidly rotating (point A)

2. cooling time (heat transport!) >> spin-down time
   \[ t_{\text{spin-down}} \sim \frac{100s}{a_{\text{max}}^2 \nu_3^6} \]
   for max. r-mode amplitude \( a_{\text{max}} \sim 1 \)

3. star moves along line AB because of r-modes

4. line BC, cooling and magnetic breaking

Minimum point B must be above 62 Hz (PSRJ0537-6910)
Minimum of the stability window

Mass is too close to the limiting one
Rotation of LMXB pulsars cannot be explained. Shear viscosity is too small.

Alternative mechanisms

- differential drift in magnetic field
  [Rezzolla, Lamb, Shapiro]
- weak reactions with hyperons + hyperon pairing
  [Jones; Nayyar Owen]
- core-crust coupling
  [Bildsten, Ushomirsky, Levin]
- saturation of r-mode amplitude at small values
  [Arras, Bondaresku, Wasserman]
- non-linear decay of r-modes
  [Kastaun]
- coupling to more stable modes
  [Gusakov, Chugunov, Kantor]
- vortex flux-tube interactions
  [Haskell, Glampedakis, Andersson]
Conclusions

In-medium modifications of modes is important in the NS core:
- moderate increase nucleon shear viscosity for small NS masses
- partial trapping of neutrinos in the star core at $T \sim 2 \times 10^9 - 10^{10}$ K
- large neutrino shear viscosity for $T > 2 \times 10^9$
- strong increase of the bulk viscosity

Phonon shear viscosity (due to the phonon-neutron interaction)
- could be important if the gap is sufficiently large

Rotation periods of young pulsars can be understood
- if the pion softenning is taken into account

Shear viscosity at $T \sim 10^8$ K is too small to explain recycled pulsars in LMXB

- differential rotation of the star $\rightarrow$ increase of the critical frequency
Outer part of the core does not rotates. Crust rotates \[
\frac{\Omega_{\text{fin}}(r)}{\Omega_c} \approx \theta(r_c - r)
\]

Minimal \( r_c \) is determined by the size of the proton paring zone.
Soft bosonic modes in rapidly rotating systems

dispersion laws of the low-lying bosonic excitations

What happens if the medium flows with a velocity \( v > v_c \)?

\[ v_c = \frac{\epsilon(k_0)}{k_0} \]

There appears a Bose condensate of excitations, which will carry a part of the momentum and the fluid will move with smaller velocity.

\[ \delta E = - (v k - \epsilon(k)) |\varphi_0|^2 + \frac{1}{2} \Lambda |\varphi_0|^4 \]

differential rotation of the star