Understanding the Charged Meson Z(4430)

Gui-Jun Ding

Department of Modern Physics,
University of Science and Technology of China, Hefei, Anhui 230026, China

Abstract

The difference between Z(4430) as a $D^*D_1$ molecule and a tetraquark state and how to distinguish between them are discussed. We construct an effective Lagrangian with $D^*D_1$ contact interactions constrained by the heavy quark symmetry and chiral symmetry to study Z(4430). We find that if Z(4430) is a $D^*D_1$ molecule state, there should be a $B^*B_1$ bound state as well, and its mass is about 11048.6 MeV.

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Recently the Belle Collaboration has reported a new state $Z(4430)$ in the $\pi^+\psi'$ invariant mass spectrum in $B \to K\pi^+\psi'$ with statistical significance greater than $7\sigma$. The Breit Wigner fit for this resonance yields the peak mass $M_Z = 4433 \pm 4\text{(stat)} \pm 1\text{(syst)}\text{MeV}$ and the width $\Gamma = 44_{-13}^{+17}\text{(stat)}_{-11}^{+30}\text{(syst)}\text{MeV}$. The product branching fraction is determined to be $\mathcal{B}(B \to KZ(4430)) \times \mathcal{B}(Z(4430) \to \pi^+\psi') = (4.1 \pm 1.0\text{(stat)} \pm 1.3\text{(syst)}) \times 10^{-5}$. Differing from other hidden charmonium-like states such as $X(3872)$ and $Y(4260)$ etc, $Z(4430)$ is a positively charged state, therefore it must not be a conventional $c\bar{c}$ state. It would be an exotic state beyond the naive quenched quark model, if it is confirmed by the further experiments.

Some theoretical studies have been carried out to understand the structure and properties of this interesting state. Because its mass is so close to the threshold of $D^*D_1(2420)$, Rosner suggested that $Z(4430)$ is a S-wave threshold effect. Meng and Chao proposed that $Z(4430)$ is a S-wave $D^*D_1$ resonance, re-scattering mechanism has been suggested to explain the absence of the signal in $\pi^+J/\psi$ for properly chosen parameters. The mass of $Z(4430)$ as a $J^P = 0^-D^*D_1$ molecule was calculated from the QCD sum rule. A dynamical study of whether $Z(4430)$ could be a S-wave molecular state of $D^*D_1$ has been performed, where the authors assumed that the long distance one pion exchange dominates. They found that the attraction from the one pion exchange potential alone is not strong enough to form a bound $D^*D_1$ molecular state. Short range force maybe plays an important role in the dynamics of $Z(4430)$.

Maiani et al suggested that $Z(4430)$ is a diquark-antidiquark state with flavor $[cu][\bar{c}d]$, it is the radial excitations of $X_{ud}^+(1^{+-};1S)$ with mass about 3880 MeV, which mainly decays into $J/\psi\pi^+$ and $\eta_c(1S)\rho^+$. Tetraquark interpretation is also suggested based on the QCD-string model. Other theoretical interpretations such as baryonium and threshold cusp effect are put forward. The mass and the production of the bottom analog of $Z(4430)$ have been studied as well.

Just as $X(3872)$ may be a weakly bound state of $DD^*\psi[12, 13, 14, 15]$, the closeness of $Z(4430)$ to the $D^*D_1(2420)$ threshold strongly suggests that $Z(4430)$ could be a weakly bound $D^*D_1(2420)$ molecular state. This is a old and very interesting idea which has been applied to a variety of mesons with unusual characteristics such as the $\psi(4040)[16, 17]$ and $f_0(980)$. 
Although the mass of $Z(4430)$ is also close to the threshold of $D^*(2010)D'_1(2430)$, $D'_1(2430)$ is very broad, therefore it decays so quickly that it isn’t possible to form a $D^*D'_1$ molecular state. The component of $D^*(2010)D'_1(2430)$ (or $D^*(2010)D'_1(2430)$) could be neglected in the molecular state interpretation for $Z(4430)$.

If $Z(4430)$ is a weakly bound molecular state, it plays the role of deuteron in the meson antimeson interactions, which is sometimes called deuson [20]. So we can apply the methods developed for the description of deuteron to $Z(4430)$ [21, 22, 23, 24]. We will use an effective field theory to describe $Z(4430)$, which is similar to the pionless effective theory of shallow nuclear bound state [21, 22, 23]. Since the binding energy of $Z(4430)$ is small, the size of this bound state is quite large. Consequently the particular details of the interactions between the heavy mesons and antimesons are irrelevant to the description of the molecule state, and we can use the effective lagrangian with four-meson interactions consistent with both the heavy quark symmetry and chiral symmetry to study this system.

The paper is organized as follows. We discuss the crucial signals which can distinguish between the molecule and tetraquark interpretation in Sec. II. In Sec. III we construct the effective Lagrangian consistent with heavy quark symmetry and chiral symmetry. The binding of $Z(4430)$ is studied by considering the transition amplitude for $Z(4430) \rightarrow Z(4430)$. Taking into account the scaling of the effective coupling constant, we predict the mass of the bottom analog of $Z(4430)$. A summary of our results is given in Sec. IV.

II. $Z(4430)$: MOLECULE OR TETRAQUARK?

Since the S-wave inter-hadron forces are strongest, it is natural to expect that $D^*D'_1$ is in relative S-wave, then the quantum number $J^P$ of $Z(4430)$ can be $0^−$, $1^−$ and $2^−$. For the $2^−$ assignment, its production in $B \rightarrow Z(4430)K$ is strongly suppressed by the small phase space. the $1^−$ state has a larger mass and $0^−$ state should be more stable as suggested by the authors in [4]. So we assume $Z(4430)$ as a $D^*D'_1$ molecule with $J^P = 0^−$ in this work. Since $Z(4430)$ was reconstructed in the $\pi^+\psi'$ final state, from isospin and $G$-parity conservation, we learn $Z(4430)$ is a isovector state with positive $G$-parity. Under $G$−parity transformation, $D^{*0}(D'_1) \rightarrow D^{*+}(D'_1)$ and $D^{*+}(D'_1) \rightarrow -D^{-0}(-D'_1)$, therefore the flavor wavefunction of $Z(4430)$ is

$$|Z(4430)⟩ = \frac{1}{\sqrt{2}}(|D^{*+}D'_1⟩ - |D'_1D^*⟩)$$ (1)
In [19], Maiani et al. predicted two $1^{++}$ states, and their masses are approximately 3754 MeV and 3882 MeV respectively. They identified Z(4430) with the first radial excitation of the higher $1^{++}$ state, then the radial excitation of the lower $1^{++}$ state with mass about 4344 MeV should be observed in the $\psi'\pi^+$ final state as well. Searching this state at Belle or Babar is an important test of the structure of Z(4430).

The difference between the molecular state and the tetraquark interpretation is obvious. In the tetraquark picture, $J^P$ of Z(4430) is $1^+$ which is different from $0^-, 1^-, 2^-$ in the S-wave $D^*D_1$ molecular state case. For the molecule interpretation, the leading source of decay is dissociation, to good approximation dissociation will proceed via the free space decay of the constituent mesons. Since $D_1$ dominantly decays into $D^*\pi$, $D^*D^*\pi$ should be the main decay mode for Z(4430) as a $D^*D_1$ molecule. While the decay of Z(4430) could proceed through the "fall apart" decay mechanism in the tetraquark picture, it can decay into $DD^*$ and $D^*D^*$ in both S-wave and D-wave besides the $J/\psi\pi$, $J/\psi\rho$, $\eta_c(1S)\rho$ and $\psi(2S)\pi$ final states, however, it can not decay into $DD$ due to its unnatural spin-parity. So whether the three body mode $D^*D^*\pi$ has considerable branch ratios or the two body decay modes $DD^*$, $D^*D^*$, $J/\psi\pi$, $J/\psi\rho$, $\eta_c(1S)\rho$ and $\psi(2S)\pi$ is another important test of the nature of Z(4430).

III. Z(4430) AS A $D^*D_1$ MOLECULE FROM THE EFFECTIVE FIELD THEORY

The general effective Lagrangian required to describe the $D^*D_1$ molecule is constrained by both the heavy quark symmetry and chiral symmetry, it consists of the one-body interaction terms and the two-body interaction terms

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$$

The one-body effective lagrangian $\mathcal{L}_1$ which describes the strong interaction of heavy meson with one heavy quark(antiquark) is given by the heavy-hadron chiral perturbation theory [23, 26, 27, 28]

$$\mathcal{L}_1 = -i \text{Tr}[\overline{H}_a(Q)(\bar{\psi} \cdot D_{ba} + \frac{D^2_{ba}}{2m_P})H_b(Q)] + \frac{i}{2} g \text{Tr}[\overline{H}_a(Q)H_b(Q)\gamma_\mu\gamma_5(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)_{ba}]$$

$$+ \frac{\lambda_2}{m_Q} \text{Tr}[\overline{H}_a(Q)\sigma^{\mu\nu}H_b(Q)\sigma_{\mu\nu}] + \text{Tr}[\overline{T}_a(Q)^\mu(i\bar{\psi} \cdot D_{ba} - \delta m_T\delta_{ba} + \frac{D^2_{ba}}{2m_T})T^\mu_{ba}]$$

$$+ \frac{i}{2} g'' \text{Tr}[\overline{T}_a(Q)^\mu T^\mu_{ba}\gamma_\mu\gamma_5(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)_{ba}] + (H_a(Q) \rightarrow \overline{H}_a(Q), \overline{H}_a(Q) \rightarrow H_a(Q),$$

$$T_a(Q)^\mu \rightarrow \overline{T}_a(Q)^\mu, T_a(Q)^\mu \rightarrow T_a(Q)^\mu + \ldots$$

(3)
where \( H_a^{(Q)} \) and \( T_a^{(Q)\mu} \) are the matrix representations of the heavy mesons, \( H_a^{(Q)} \) and \( T_a^{(Q)\mu} \) are the matrix representations of the heavy antimesons, and the ellipsis denotes higher order terms in the chiral expansion. The covariant derivative \( D^\mu_{ab} = \partial^\mu \delta_{ab} - \frac{1}{2}(\xi^a \partial^\mu \xi + \xi \partial^\mu \xi^a) \), \( \delta m_T = M_{P_1} - M_P \), and the mass difference between \( P^* \) and \( P \) is \( \Delta \equiv M_{P^*} - M_P = -\frac{8\sqrt{2}}{m_Q^2} \).

The superfield multiplets \( H_a^{(Q)} \), \( T_a^{(Q)\mu} \), \( H_a^{(\overline{Q})} \) and \( T_a^{(\overline{Q})\mu} \) are as follows

\[
H_a^{(Q)} = \frac{1}{2} \left[ P_a^{(Q)\gamma\mu} \gamma_\mu - P_a^{(Q)\gamma_5} \right],
\]

\[
T_a^{(Q)\mu} = \frac{1}{2} \left\{ P_{2a}^{(Q)\gamma\mu} \gamma_\mu - \sqrt{\frac{3}{2}} \right\} P_{1a}^{(Q)\gamma_5} [g_\nu^\mu - \frac{1}{3} \gamma_\nu (\gamma^\mu - \nu^\mu)]\right) \right\} \frac{1-\beta^\mu}{2}.
\]

\[
H_a^{(\overline{Q})} = \left[ P_a^{(\overline{Q})\gamma\mu} \gamma_\mu - P_a^{(\overline{Q})\gamma_5} \right] \frac{1-\beta^\mu}{2},
\]

\[
T_a^{(\overline{Q})\mu} = \left\{ P_{2a}^{(\overline{Q})\gamma\mu} \gamma_\mu - \sqrt{\frac{3}{2}} P_{1a}^{(\overline{Q})\gamma_5} [g_\nu^\mu - \frac{1}{3} (\gamma^\mu - \nu^\mu) \gamma_\nu]\right\} \frac{1-\beta^\mu}{2}.
\]

The pseudogoldstone boson octet is introduced via the exponential representation

\[
\xi = \exp(i \mathcal{M}/f_\pi), \quad \Sigma = \xi^2
\]

with \( f_\pi = 132 \text{ MeV} \) and

\[
\mathcal{M} = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- \\
K^- \\
\pi^+ \\
\pi^0 + \frac{1}{\sqrt{6}} \eta \\
K^0 \\
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\frac{1}{\sqrt{2}} \eta
\end{pmatrix}
\]

The two-body effective Lagrangian \( \mathcal{L}_2 \) which describes the interactions between the heavy mesons and antimesons is the local four-boson contact interactions as follows

\[
\mathcal{L}_2 = \frac{1}{4} h_1 \{ \text{Tr}[H_a^{(Q)} H_a^{(Q)} \gamma_\mu] \text{Tr}[T_b^{(Q)\alpha} T_b^{(Q)\mu} \gamma_\mu] + \text{Tr}[T_a^{(Q)\alpha} T_a^{(Q)\mu} \gamma_\mu] \text{Tr}[H_b^{(\overline{Q})} H_b^{(\overline{Q})} \gamma_\mu] \}
\]

\[
+ \frac{3}{10} h_2 \{ \text{Tr}[H_a^{(Q)} H_a^{(Q)} \gamma_\mu \gamma_5] \text{Tr}[T_b^{(Q)\alpha} T_b^{(Q)\mu} \gamma_\mu \gamma_5] + \text{Tr}[T_a^{(Q)\alpha} T_a^{(Q)\mu} \gamma_\mu \gamma_5] \text{Tr}[H_b^{(\overline{Q})} H_b^{(\overline{Q})} \gamma_\mu \gamma_5] \}.
\]

If \( Z(4430) \) is indeed a \( D^* D_1 \) molecule, heavy quark symmetry requires the existence of a \( B^* B_1 \) molecular state. To predict the properties of \( B^* B_1 \) molecule from \( Z(4430) \), we need to determine how the coupling constants \( h_1 \) and \( h_2 \) scale with the heavy meson mass \( M \). We rescale all energy \( q^0 \rightarrow q^0/M \) and the coordinate \( t \rightarrow M \bar{t} \) so that the dimensional quantities have the same size (i.e., are measured in unit of the momentum \( p \)). If we demand that the action is independent of \( M \), then since the measure \( d^4x \sim M \), the Lagrangian density \( \mathcal{L} \sim 1/M \). The kinetic term determines that the heavy meson field \( H_a^{(Q)} (H_a^{(\overline{Q})}) \) and \( T_a^{(Q)\mu} \)
FIG. 1: The Feynman diagrams for the transition $Z(4430) \rightarrow Z(4430)$

$(T^{(Q)}_a)$ scale as $M^0$, so the couplings

$$h_1 \sim h_2 \sim 1/M$$

The two-body interaction terms relevant to the $P^* P_1$ part is

$$L_{2, P^* P_1} = h_1 [P_a^{(Q)*\alpha\dagger} P_{ba}^{(Q)*\beta\dagger} P_{1b\beta}^{(Q)*} + P_{1a\alpha}^{(Q)*\alpha\dagger} P_b^{(Q)*\beta\dagger} P_{b\beta}^{(Q)*}]$$

$$+ h_2 [P_a^{(Q)*\alpha\dagger} P_{1b\alpha}^{(Q)*\beta\dagger} P_a^{(Q)*\beta\dagger} P_{1b\beta}^{(Q)*} - P_a^{(Q)*\alpha\dagger} P_{1b\alpha}^{(Q)*\beta\dagger} P_{b\beta}^{(Q)*} + P_{1a\alpha}^{(Q)*\beta\dagger} P_{1b\beta}^{(Q)*} - P_{1a\alpha}^{(Q)*\alpha\dagger} P_{b\beta}^{(Q)*\beta\dagger} P_{1b\beta}^{(Q)*}]$$

(9)

Here we concentrate on $Z(4430)$, which we assume to be a $D^* D_1$ bound state. Setting $a = 2$ and $b = 1$ in Eq.(9), we obtain the effective interactions relevant to $Z(4430)$

$$L_{2, Z} = h_1 [D^{*+\alpha\dagger} D_1^{\alpha\dagger} D_1^{\beta\dagger} D_1^{\beta\dagger} D_1^{0\beta\dagger} D_1^{0\beta\dagger} D_1^{0\beta\dagger}] + h_2 [D^{*+\alpha\dagger} D_1^{\alpha\dagger} D_1^{\beta\dagger} D_1^{0\beta\dagger} D_1^{0\beta\dagger} D_1^{0\beta\dagger} - D^{*+\alpha\dagger} D_1^{\alpha\dagger} D_1^{\beta\dagger} D_1^{0\beta\dagger} D_1^{0\beta\dagger} D_1^{0\beta\dagger} - D^{*+\alpha\dagger} D_1^{\alpha\dagger} D_1^{\beta\dagger} D_1^{0\beta\dagger} D_1^{0\beta\dagger} D_1^{0\beta\dagger} - D^{*+\alpha\dagger} D_1^{\alpha\dagger} D_1^{\beta\dagger} D_1^{0\beta\dagger} D_1^{0\beta\dagger} D_1^{0\beta\dagger}]$$

(10)

A superscript $\dagger$ on a field represents its complex conjugate. If the above interactions in Eq.(10) is treated nonperturbatively, there should be S-wave bound state which can be identified with the $Z(4430)$.

We denote the transition amplitude for $Z(4430) \rightarrow Z(4430)$ by $i\mathcal{A}(E)$, it depends only on the total energy $E$ in the center-of-mass frame, and $Z(4430)$ corresponds to a pole of $i\mathcal{A}(E)$. $i\mathcal{A}(E)$ can be calculated nonperturbatively by summing the loop graphs in Fig.1

$$i\mathcal{A}(E) = \frac{4ih}{1 - ihL(E)}$$

(11)

where $h = h_1 - 3h_2$, and $L(E)$ is the amplitude for the propagation of $D^* D_1$ between successive interaction vertex, it is given by

$$L(E) = \int \frac{d^4 q}{(2\pi)^4} \frac{i}{E_2 - q_0 - \Delta - \frac{q^2}{2MD^*} + i\epsilon} \frac{i}{E_2 - q_0 - \delta m_T - \frac{q^2}{2MD_1} + i\epsilon}$$
\[
-2M_{D^*D_1} i \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2 - 2M_{D^*D_1}(E - \Delta - \delta m_T) - i\epsilon} \tag{12}
\]

where \(M_{D^*D_1} = \frac{M_{D^*}M_{D_1}}{M_{D^*} + M_{D_1}}\) is the reduced mass of the \(D^*D_1\) system. From Eq. (12), we see that \(L(E)\) has a linear ultraviolet divergence, which can be removed by the renormalization of the coupling constant \(h_1\) and \(h_2\). \(L(E)\) is finite in dimensional regularization, which corresponds to an explicit subtraction of the linear divergence, and we denote the renormalized \(h\) as \(h_R\). \(L(E)\) in dimensional regularization is given by

\[
L(E) = \frac{iM_{D^*D_1}}{2\pi} \sqrt{-2M_{D^*D_1}(E - \Delta - \delta m_T)} \tag{13}
\]

Inserting the above expression for \(L(E)\) into the amplitude \(iA(E)\) in Eq. (11), then the amplitude reduces to

\[
iA(E) = \frac{ih_R}{1 + h_RM_{D^*D_1}/(2\pi)\sqrt{-2M_{D^*D_1}(E - \Delta - \delta m_T)}} \tag{14}
\]

If \(h_R < 0\), \(A(E)\) has a pole corresponding to \(Z(4430)\)

\[
E_{\text{pole}} = \Delta + \delta m_T - \frac{2\pi^2}{h_R^2M_{D^*D_1}^3} \tag{15}
\]

The above energy is measured relative to twice the pseudoscalar mass \(M_D\), therefore the binding energy of \(Z(4430)\) is

\[
E_{Z,b} = M_{D^*} + M_{D_1} - (2M_D + \Delta + \delta m_T - \frac{2\pi^2}{h_R^2M_{D^*D_1}^3}) = \frac{2\pi^2}{h_R^2M_{D^*D_1}^3} \tag{16}
\]

If future experiments confirm the molecular state nature of \(Z(4430)\), the binding energy of the bottom analog of \(Z(4430)\) can be predicted. From the scaling of the coupling constants \(h_1\) and \(h_2\) with \(M\) in Eq. (8), we obtain \(h_R(D)M_{D^*D_1} \sim h_R(B)M_{B^*B_1}\). We denote the the bottom analog of \(Z(4430)\) and its binding energy as \(Z'\) and \(E_{Z',b}\) respectively, then

\[
E_{Z',b} \sim E_{Z,b} \frac{M_{D^*D_1}}{M_{B^*B_1}} \tag{17}
\]

Since the \(D^*D_1\) threshold is \(M_{D^{*+}} + M_{D^0_1} = 4432.3 \pm 1.7\) MeV \([29]\), and the \(Z(4430)\) mass is \(4433 \pm 4(\text{stat}) \pm 1(\text{syst})\) MeV, now we can not determine the binding energy of \(Z(4430)\) because of the large uncertainty. As an illustration, we assume \(E_{Z,b} = 1\) MeV for the moment, then from Eq. (17) we predict that the binding energy of the bottom analog of \(Z(4430)\) is about 0.4 MeV and the mass approximately is 11048.6 MeV. Our result for the
mass of the bottom analog of \( Z(4430) \) is larger than the quark model prediction (10730±100) MeV\(^{10} \) and the QCD sum rule prediction (10.74±0.12) GeV\(^{11} \). We note that the effective theory prediction for the mass of \( 1^{++} \ BB^{*} \) molecule state is larger than the quark model predictions as well\(^{30}, 31, 32 \).

For the \( PP_1 \) sector, from Eq.\(^{7} \) we can obtain the relevant contact interactions

\[
\mathcal{L}_{2,DD_1} = -h_1 [P_a^{(Q)\dagger} P_{\bar{a}}^{(Q)} P_{b}^{(\bar{Q})\dagger} P_{\bar{b}}^{(\bar{Q})} + P_{1a}^{(Q)\dagger} P_{1\bar{b}}^{(\bar{Q})\dagger} P_{b}^{(Q)} P_{\bar{b}}^{(\bar{Q})}] \quad (18)
\]

This Lagrangian involves only one coupling constant \( h_1 \). Performing similar calculations as for \( Z(4430) \), we find that only if \( h_{1R} > 0 \) there is a charged \( DD_1 \) molecule, and the binding energy is \( 2\pi^2/(h_{1R}^2 M_{DD_1}^3) \), where \( M_{DD_1} \) is the reduced mass of \( DD_1 \) system. Therefore the existence of \( D^*D_1 \) molecule doesn’t in general implies the existence of \( DD_1 \) bound molecule.

The above discussions can be straightforwardly generalized to the flavor SU(3) symmetry, and a nonet(Octet and Singlet) which \( Z(4430) \) belongs to is predicted\(^{5}, 11 \). For the positive charged \( D_s^*D_1 \) (or \( D_{s1}D_s^* \)) molecule \( Z_s^+ \), its flavor wavefunction generally is

\[
|Z_s^+\rangle = c_1 |D_s^{+}\bar{D}_1^0\rangle + c_2 |D_{s1}^{+}\bar{D}_0^0\rangle \quad \text{with} \quad |c_1|^2 + |c_2|^2 = 1.
\]

The relevant four-boson contact interactions are the \( a = 3, b = 1 \) terms in Eq.\(^{19} \), which is as follows

\[
\mathcal{L}_{2,Z_s^+} = h_1 [D_s^{+}\bar{D}_1^0 |D_s^{+}\bar{D}_1^0\rangle + D_{s1}^{+}\bar{D}_0^0 |D_{s1}^{+}\bar{D}_0^0\rangle + h_2 |D_s^{+}\bar{D}_1^0 |D_s^{+}\bar{D}_1^0\rangle + D_{s1}^{+}\bar{D}_0^0 |D_{s1}^{+}\bar{D}_0^0\rangle ] \quad (19)
\]

Therefore the binding energy of \( Z_s^+ \) is \( 2\pi^2/(h_{1R}^2 M_{D^*D_1}^3) \) under the SU(3) flavor symmetry. It could decay into \( D^*D_s^{+}\pi \), \( D^*D_s^{+}K \) and \( K^{+}\psi' \), so the experimental search for these final states is very interesting.

The assumption of \( Z(4430) \) as a weakly bound \( D^*D_1 \) molecule is particularly predictive. The small binding energy, which is much smaller other QCD scales such as \( \Lambda_{QCD} \) and the pion mass \( m_{\pi} \) etc, implies that the molecule has universal properties\(^{33} \). This can be further exploited through factorization formulae to predict the production and decay properties of \( Z(4430) \) just similar to the X(3872) study\(^{34} \). The relevant work is in progress\(^{35} \).

\section*{IV. SUMMARY}

The proximity of \( Z(4430) \) mass to the \( D^*D_1 \) threshold favors a \( D^*D_1 \) molecule state interpretation of \( Z(4430) \). In this work we first discuss how to distinguish the molecule interpretation from the tetraquark picture, and we find that experimentally measuring whether
Z(4430) dominantly decays into $D^*D^*\pi$ or the two body modes $DD^*$, $D^*D^*$, $J/\psi\pi$, $J/\psi\rho$, $\eta_c(1S)\rho$ and $\psi(2S)\pi$ is very important for understanding the structure of Z(4430). If Z(4430) is a tetraquark state, another state with mass about 4344 MeV should be found in the $\psi'\pi^+$ final state in addition.

We have studied Z(4430) as a $D^*D_1$ molecule from the effective field theory, and an effective Lagrangian with four-boson contact interactions is constructed to describe this system. We find that if Z(4430) is a $D^*D_1$ molecule state, there should be a $B^*B_1$ molecule with mass about 11048.6 MeV. The existence of the $D^*D_1$ bound state doesn’t in general implies a bound state in the $DD_1$ system.

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