Warping the young stellar disc in the Galactic Centre

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Abstract. We examine influence of the circum-nuclear disc (CND) upon the orbital evolution of young stars in the Galactic Centre. We show that gravity of the CND causes precession of the orbits which is highly sensitive upon the semi-major axis and inclination. We consider such a differential precession within the context of an ongoing discussion about the origin of the young stars and suggest a possibility that all of them have originated in a thin disc which was partially destroyed due to the influence of the CND during the period of $\sim 6\text{Myr}$.

1. Introduction
Recent observations of the central stellar cusp of the Milky Way have revealed a numerous population of massive young stars within the (projected) distance of $\lesssim 0.5\text{pc}$ from the supermassive black hole (e.g. Genzel et al. 1996, Paumard et al. 2006). The apparent youth of these stars is considered to be in contradiction with their vicinity to the black hole which is assumed to prevent stellar formation due to strong tidal forces acting on the parent gaseous clouds. Further analyses have shown that subset of stars above the projected radius $r \approx 0.04\text{pc}$ forms a coherently rotating ‘clockwise’ disc (CWS). It has been proposed that these stars may have been born in a massive gaseous disc fragmenting due to it own gravity (Levin & Beloborodov 2003, Nayakshin 2006). In spite of its attractiveness, this model, cannot explain origin of all young stars in the central parsec as more than one half of them have large inclinations with respect to the CWS. Even if we admit existence of another ‘counter-clockwise’ disc of $\lesssim 20$ young stars (Genzel et al. 2003, Paumard et al. 2006) which could also have been born via fragmentation of a gaseous disc, we will still be facing a problem of the origin of several tens of massive young stars that apparently do not belong to any disc-like structure in the Galactic Centre.

The aim of this contribution is to discuss dynamical processes that should be taken into considerations about the relation of the initial and current kinematical states of the young stars. In the following section, we present notes on the dynamics of stars in the idealised model of the Galactic Centre. In Section 4 we discuss consequences of the orbital evolution of the young stars with regard to the stability of a disc-like stellar system. We summarise our results in Section 5.

2. Stellar dynamics in the central parsec
Gravitational field within the central parsec is dominated by the supermassive black hole. Relativistic effects can be safely ignored at the radii of $r \gtrsim 0.04\text{pc}$ and, therefore, we will treat the black hole as a source of purely Keplerian potential. In spite of the extreme stellar
density in this region, characteristic time-scale of two-body relaxational processes are longer than the lifetime of the young stars (Hopman & Alexander 2006). Therefore, we will model the gravitational field of the stellar cusp with a smooth spherically symmetric potential

\[ V_c(r) = \frac{4GM_c}{R_h} \left( \frac{r}{R_h} \right)^{1/4} \]  

(1)

where \( M_c \) is the mass of the star cluster within the radius of the influence of the black hole, \( R_h \approx 1.5 \text{pc} \). The effect of the stellar cusp upon the dynamics of the individual stars lies in a (negative) advance of the pericentre of their orbits. Finally, we assume that a non-spherical component of the gravitational field is determined by the massive molecular torus. For the sake of simplicity we will treat it as an infinitesimally narrow ring of mass \( M_{\text{CND}} \) and radius \( R_{\text{CND}} \). Dynamics of the stellar orbit is then equivalent to the dynamics in a reduced hierarchical triple system. Hence, we may describe secular evolution of the orbital elements with equations (Kozai 1962, Lidov 1962):

\[ T_K \sqrt{1 - e^2} \frac{de}{dt} = \frac{15}{8} e (1 - e^2) \sin 2\omega \sin^2 i, \]  

(2)

\[ T_K \sqrt{1 - e^2} \frac{di}{dt} = -\frac{15}{8} e^2 \sin 2\omega \sin i \cos i, \]  

(3)

\[ T_K \sqrt{1 - e^2} \frac{d\omega}{dt} = \frac{3}{4} \{2 - 2e^2 + 5 \sin^2 \omega [e^2 - \sin^2 i]\}, \]  

(4)

\[ T_K \sqrt{1 - e^2} \frac{d\Omega}{dt} = -\frac{3}{4} \cos i \{1 + 4e^2 - 5e^2 \cos^2 \omega\}, \]  

(5)

where

\[ T_K = \frac{M_\star}{M_{\text{CND}}} \frac{R_{\text{CND}}^3}{a \sqrt{GM_\star a}}. \]  

(6)

is the characteristic time-scale of the orbital evolution, \( a \) is the semi-major axis of the orbit, \( e \) is its eccentricity, \( i \) is inclination with respect to the CND, \( \omega \) is the argument of the pericentre and \( \Omega \) stands for the longitude of the ascending node (all angles are measured in the reference frame in which CND lies in the equatorial plane). Due to the presence of two integrals of equations (2) – (5) and periodic nature of the angles \( \omega \) and \( \Omega \), inclination and eccentricity of the orbit periodically oscillate. Example of these so called ‘Kozai oscillations’ is presented in Fig. 1 (solid line).

Gravity of the stellar cusp can be included by means of an additional term to equation (4) which describes the additional pericentre shift (Ivanov, Polnarev & Saha 2005). In spite of that its explicit form is not known for the arbitrary profile of \( V_c(r) \), we may safely state that, in the absolute value, it overwhelms the pericentre advance due to the CND already for \( M_c \gtrsim 0.1 M_{\text{CND}} \). The periodic nature of the changes of eccentricity and inclination is preserved. Nevertheless, its period is shortened and the amplitude is substantially damped (Karas & Šubr 2007). As in the case of the Galactic Centre the mass of the star cluster is well above the threshold value, we may omit the discussion of the detailed profile of the stellar cusp. We may also consider secular evolution of the eccentricity and inclination to be negligible. Formula (5) for the precession rate can be then simplified by averaging over one period of \( \omega \) which yields:

\[ \Delta \Omega = -\frac{3}{4} \cos i \left( \frac{a}{R_{\text{CND}}} \right)^{3/2} \frac{\sqrt{GM_\star}}{R_{\text{CND}}^3} \frac{M_{\text{CND}}}{M_\star} \frac{1 + \frac{3}{2} e^2}{\sqrt{1 - e^2}} \Delta t \]  

\[ = -5417^o \cos i \left( \frac{a}{R_{\text{CND}}} \right)^{3/2} \left( \frac{M_\star}{3.5 \times 10^6 M_\odot} \right)^{1/2} \left( \frac{R_{\text{CND}}}{1 \text{pc}} \right)^{-3/2} \frac{1 + \frac{3}{2} e^2}{\sqrt{1 - e^2}} \frac{M_{\text{CND}}}{M_\star} \Delta t \text{1Myr} \]  

(7)
Figure 1. Evolution of the orbital elements of two example orbits. The solid line represents a trajectory in the gravitational field of the central mass $M_\bullet = 3.5 \times 10^6 M_\odot$ and a ring of radius $R_{\text{CND}} = 1.5\,\text{pc}$ and mass $M_{\text{CND}} = M_\bullet$. The dotted line shows an orbit integrated in the field including in addition a spherical cusp of mass $M_c = 0.1M_\bullet$. In both cases the initial values of the orbital elements are: $a = 0.1R_{\text{CND}}$, $e = 0.1$, $i = 80^\circ$, $\omega = 0$ and $\Omega = 0$.

We have performed several numerical tests of the validity of formula (7). Figure 2 shows iso-contours of $\Delta\Omega$ as a function of $M_{\text{CND}}$ and $\cos i$ obtained by means of numerical integration of the test particle trajectory. The slope of the curves follows the theoretically predicted one. The systematic shift of the iso-contours with respect to the values given by eq. (7) as shown for $\Delta\Omega = 30^\circ$ is mainly due to an inconsistency between the mean and osculating values of the orbital elements. Eqs. (2) – (7) are valid for orbital elements averaged over one revolution; on the other hand, $a = 0.15\,\text{pc}$ stands as an initial value of the osculating semi-major axis in the numerical integration, while the mean value of semi-major axis is slightly smaller due to the mass of the stellar cusp. We haven’t made correction to this discrepancy as it is not straightforward and the introduced error of $\Delta\Omega$ is smaller than 20%.

3. Results

An important consequence of formula (7) is a strong dependence of the precession rate upon the semi-major axis and inclination. In the latter case, the dependence is pronounced for $i \approx 90^\circ$. For example, the absolute value of the change of $\Omega$ for $a = 0.4\,\text{pc}$, $e = 0.5$, $i = 85^\circ$, $R_{\text{CND}} = 1.5\,\text{pc}$, $M_{\text{CND}} = 10^6 M_\odot$ and $\Delta t = 6\,\text{Myr}$ is $|\Delta\Omega| \approx 100^\circ$. On the other hand, for $a = 0.04\,\text{pc}$ and $i = 89^\circ$ and keeping the other parameters unchanged, we obtain $|\Delta\Omega| \approx 1^\circ$. Note that, if both orbits start with identical value of $\Omega$, they would be initially corotating. These considerations indicate that a popular model of the origin of the young stars in the Galactic Centre which assumes their formation in a fragmenting gaseous disc needs to include gravitational influence of the CND in order to follow further evolution of the initially flat stellar disc.
Figure 2. $\Delta \Omega$ as a function of the mass of the CND and the mutual inclination of the CND and a reference stellar orbit which evolves for a period of 6Myr. Fixed is the radius of the CND, $R_{\text{CND}} = 1.5\text{pc}$, semi-major axis of the orbit, $a = 0.15\text{pc}$, and its eccentricity, $e = 0.5$. Solid lines represent iso-contours of $\Delta \Omega$ as obtained by means of the numerical integration of the orbit. Dashed line shows iso-contour of $\Delta \Omega = 30^\circ$ determined by formula (7).

3.1. Orientation and mass of the CND

Properties of the molecular torus are determined from radio observations of emission of various molecules, both neutral and ionised. The maximum of the observed emission is at a distance of $\sim 1.5\text{pc}$ from SgrA*+. Its total mass is rather uncertain — values ranging from $2 \times 10^5 M_\odot$ to $2 \times 10^6 M_\odot$ can be found in the literature. The most recent works prefer higher masses (e.g. $M_{\text{CND}} \approx 10^6 M_\odot$, Christopher et al. 2005). Inclination with respect to the plane of the sky is $i_{\text{CND}}' \approx 70^\circ$ and the position of the ascending (receding) node measured from the North is $\Omega_{\text{CND}}' \approx 25^\circ$ (Jackson et al. 1993). (Note that angles $i$ and $\Omega$, i.e. without primes, used in the previous section are measured in the coordinate system in which the CND lies in the $z = 0$ plane.) Uncertainty in the values of both angles determining orientation of the CND is larger than $10^\circ$.

Presence of the coherently rotating disc of young stars poses some constraints upon the parameters of the CND. Assumption that the stellar disc was stable for the period of $\sim 6\text{Myr}$ determines an upper limit of the rate of the precession of its members — the change of $\Omega$ must not exceed the current value of the opening angle of the CWS which is $\approx 20^\circ$ (Paumard et al. 2006). Figure 2 shows $\Delta \Omega$ as a function of $M_{\text{CND}}$ and $i$ for an orbit at the outer edge of the stellar disc. A particular diagonal line which represents the iso-contour of $\Delta \Omega = 30^\circ$ in 6Myr can be treated as an boundary of the (shaded) region of the values of $M_{\text{CND}}$ and $i$ that would not lead to destruction of the CWS during its lifetime. Thin dash-dotted line which indicates the estimated mass of the CND crosses the boundary at $i \approx 85^\circ$. This limiting mutual inclination of the stellar disc and the CND also determines an upper limit for the initial opening angle of the stellar disc to $|90^\circ - i_0| \approx 5^\circ$. 
Figure 3. Model of the warped disc in terms of sinusoidal projection of normal vectors (small crosses) of individual orbits in the observer’s reference frame. Latitude represents angular distance from the line of sight, longitude is measured counterclockwise from the North. Large crosses denote position of the vectors normal to the CND; dashed curve is a set of all vectors perpendicular to the axis of the CND. The orbits have evolved under the influence of the CND for a period of 6Myr, all of them originating from a small region around the point \((\varphi_0', \theta_0') = (270^\circ, 60^\circ)\) the boundary of which is represented by solid line.

3.2. Single warped disc

The values of parameters that lead to non-destructive precession represent rather small subset of their whole definition range. Hence, it is likely that some stars were out of this region initially and, subsequently, they were detached from the initial planar structure due to precession larger than 20°. Such a scenario could give an answer to the intriguing question about the origin of those young stars in the Galactic Centre which apparently do not belong to any of the recognised stellar discs.

The high sensitivity of the precession upon undetermined orbital parameters of the observed stars as well as the lack of robust determination of the mass and orientation of the CND prohibits us to track the stars back in time and to seek for their parent disc(s). We assume that forthcoming observations will enable us to measure accelerations of the young stars in the Galactic Centre and, consequently, to determine their orbital elements. Then, sets of stars that were corotating in the past should be distinguished due to specific pattern formed by their normal vectors. We show an example of the warped disc on simulated data in Figure 3. We plot the configuration of normal vectors of 100 orbits that were detached from a 5° neighbourhood of the vector \(\mathbf{n}_0 = (\varphi_0', \theta_0') = (270^\circ, 60^\circ)\) which corresponds to \((i_0', \Omega_0') = (120^\circ, 90^\circ)\). The space occupied initially by the normal vectors is denoted with a small circle (deformed due to the sinusoidal projection), while their positions after 6Myr of the orbital evolution are denoted with crosses. Distribution of the semi-major axes is \(n(a) \propto a^{-1}\) which transforms to the surface density \(\propto r^{-2}\); eccentricities are drawn randomly with equal probability from an interval \((0, 1)\). Parameters of the CND were set to \(M_{\text{CND}} = M_\star\), \(R_{\text{CND}} = 1.5\)pc and...
Most of the orbits’ normal vectors remained relatively close to their original positions, forming a structure that can be still recognised as a stellar disc. Those stars that have undergone more rapid precession form a specific pattern in the space $(\varphi', \theta')$ — a tail that is parallel with the circumference perpendicular to the CND.

In Šubr, Schovancová & Kroupa (2009) we have performed a test of compatibility of the hypothesis of a common origin of the young stars in the GC in a single thin disc with the observational data published by Paumard et al. (2006, Table 2). We have examined a $1\sigma$ neighbourhood of the velocities of 72 stars (excluding S-stars and stars with undetermined velocity component along the line of sight) searching for such a state, initial orientation of which is close to a given normal vector $\mathbf{n}_0$. We assumed the orbital evolution to be solely due to the precession around the axis of the CND, orientation and mass of which stand as free parameters. We have shown that there exists a class of the parameters of the CND for which all stars might have a common origin in a thin disc. Not surprisingly, the best fits gave initial orientation of the disc around $(\varphi'_0, \Omega'_0) = (120^\circ, 90^\circ)$, which is close to the orientation of the clockwise stellar disc (Levin & Beloborodov 2003, Paumard et al. 2006). Hence, the hypothesis of the common origin of all young stars in the Galactic Centre (except for S-stars) can be considered viable.

4. Conclusions

Within our idealised model, the orbits of stars in the central parsec of the Galaxy precess around the symmetry axis of the massive molecular torus. We have verified by means of numerical integration of the equations of motion that this process is robust and it is not substantially modified when the axial symmetry is broken and CND is represented by a finite number of discrete particles. We estimate that relaxation processes act on the young stars on time-scales considerably longer than their presumed lifetime. Therefore, we suggest that the precession due to the gravity of the CND played a dominant role in the orbital evolution of these stars during the past $\sim 6$Myr.

The rate of precession is sensitive upon the semi-major axis of the orbit and its inclination with respect to the CND. This process is of particular importance for the dynamics of a stellar disc – the differential precession tends to destroy a disc-like system which consists of orbits within some range of semi-major axis an inclination. As an example, we have shown that for a system with $a \in (0.04\text{pc}, 0.4\text{pc})$ and an opening angle of $\approx 5^\circ$ which is nearly perpendicular to the CND, the precession will drag normal vectors of a considerable fraction of stars by more than $20^\circ$ away from their original positions.

We point out that, due to the differential precession, there is a nontrivial mapping between initial and current orientations of orbits of the young stars in the Galactic Centre. We suggest a partial destruction (warping) due to the gravity of the massive gaseous torus of an initially coherently rotating disc as an explanation of the origin of those young stars in the Galactic Centre that do not belong to the recently identified clockwise stellar disc. Our hypothesis of a warped disc predicts that normal vectors of all of its member stars should lie close to the circumference perpendicular to the axis of the CND. We have verified that this hypothesis is compatible with the observational data published by Paumard et al. 2006. Nevertheless, the lack of robust determination of the current values of the orbital elements form the observational data prevents a convincing identification of the warped stellar disc. Finally, let us remark that the hypothesis of the single warped stellar disc also cannot explain apparently random orientations of the orbits of the S-stars, as these are the least affected by the gravity of the CND.

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