Study on the stability of waterpower-speed control system for hydropower station with upstream and downstream surge chambers based on regulation modes

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Abstract. In allusion to the hydropower station with upstream and downstream surge chambers, a complete mathematical model of waterpower-speed control system that includes pipeline system and turbine regulation system is established under the premise of the breakthrough of Thoma assumption in this paper. The comprehensive transfer functions and free movement equations that characterize the dynamic characteristics of system are derived when the mode of governor is respectively frequency regulation and power regulation. Then according to Routh-Hurwitz theorem, the stability domain that describes the good or bad of stability is drawn in the coordinate system with the relative areas of upstream and downstream surge chambers as abscissa and ordinate respectively. Finally, the effects of Thoma assumption, flow inertia, regulation modes, and governor parameters on the stability of waterpower-speed control system are analyzed by means of stability domain. The following conclusions have been come to: Thoma assumption made the stability worse. The flow inertia $T_w$ has unfavorable effect on the stability of the two regulation modes. The stability of power regulation mode is obviously superior to frequency regulation mode under the same condition, but the parametric variation sensibility of the former is inferior to the latter. For the governor parameters, the stability continually gets better with the increase of temporary droop $b_t$ and damping device time constant $T_d$, while the stability of frequency regulation would get worse with the increase of temporary droop $b_t$ when the damping device time constant $T_d$ takes small value. As the increase of permanent droop $b_p$, the stability of power regulation mode gets worse.

1. Introduction
With the booming career of the hydroelectric development, more and more upstream and downstream surge chambers are utilized for hydropower stations and pumped-storage power stations of complicated arrangement form of water delivery and power generation system. Only the arrangement place, sectional areas combination, regulation modes and governor parameters are selected reasonably can we ensure the safe and stable operation of the power station.

There are some researches on the stability of the regulating system with upstream and downstream surge chambers in power station. Evangelisti and Gubin [1] proposed two computational formulas of the stable sectional area. Jaeger [2] gave three groups of stability domain curves. Jiandong Yang [3] presented the design criteria and the computing method for stable sectional areas of upstream and downstream surge chambers. The research results above are all based on Thoma assumption and...
ignore the effects of penstock, generator, turbine and governor. Xu Lai [4] introduced the governor and the unit equations into the mathematical model and then analysed the effect of governor parameters on the stability of upstream and downstream surge chambers hydropower station. However, he only used frequency regulation mode for the governor.

Under the premise of the breakthrough of Thoma assumption, a complete mathematical model of waterpower-speed control system that includes pipeline system and turbine regulation system is established in this paper. The comprehensive transfer functions and free movement equations that characterize the dynamic characteristics of system are derived when the mode of governor is selected as frequency regulation and power regulation, respectively. According to Routh-Hurwitz theorem, the stability domain is drawn. Then the effects of Thoma assumption, flow inertia, regulation modes and governor parameters on the stability of waterpower-speed control system are analysed by using stability domain.

2. Mathematical Model
The diversion power system of hydropower station with upstream and downstream surge chambers is illustrated in Figure 1.

Figure 1. Diversion power system of hydropower station with upstream and downstream surge chambers.

(1) Pipe network equations [5]:
Momentum equation of diversion tunnel:
- upstream: \[ z_1 - \frac{2h_{10}}{H_0} q_{11} = T_{w11} \frac{dq_{11}}{dt} \] (1)
- downstream: \[ z_2 - \frac{2h_{20}}{H_0} q_{21} = T_{w21} \frac{dq_{21}}{dt} \] (2)

Continuity equation of surge chamber:
- upstream: \[ q_{11} = q_{12} - T_{F1} \frac{dz_1}{dt} \] (3)
- downstream: \[ q_{21} = q_{22} - T_{F2} \frac{dz_2}{dt} \] (4)

Momentum equation of penstock:
\[ h = -z_1 - z_2 - T_{w12} \frac{dq_{12}}{dt} - \frac{2h_{10}}{H_0} q_{12} - T_{w22} \frac{dq_{22}}{dt} - \frac{2h_{20}}{H_0} q_{22} \] (5)

(2) Hydraulic turbine regulating system equations [5, 6]
Moment equation and flow equation of hydraulic turbine:
- moment equation: \[ m_e = e_x h + e_y x + e_z y \] (6)
- flow equation: \[ q_e = e_{qx} h + e_{qx} x + e_{qy} y \] (7)

in which: \[ q_e = q_{12} = q_{22} \] (8)
The first derivative differential equation of generator:
Governor equations:

The PID governor is adopted by frequency regulation, and its block diagram is shown in Figure 2. When a hydropower station takes responsibility for the task of frequency modulation, it operates in the manner of no deviation of frequency, i.e. \( b_p = 0 \). Hence, the differential equation of this governor is:

\[
\frac{dx}{dt} = m_t - \left( m_e + e_x \right)
\]  

(9)

3. Solution of comprehensive transfer function

3.1. Frequency regulation mode

By using the method of Laplace transform to the equations (1)-(10), the comprehensive transfer function of waterpower-speed control system can be derived:

\[
G(s) = \frac{X(s)}{M_s(s)} = \frac{d_1 d_2 \left( b_T T_s s^2 + b_T s \right)}{a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}
\]  

(12)

Through the comprehensive transfer function (12), the ninth order free movement equation of frequency regulation mode is obtained:

\[
a_0 \frac{d^9 x}{dt^9} + a_6 \frac{d^9 x}{dt^9} + a_5 \frac{d^8 x}{dt^8} + a_4 \frac{d^7 x}{dt^7} + a_3 \frac{d^6 x}{dt^6} + a_2 \frac{d^5 x}{dt^5} + a_1 \frac{d^4 x}{dt^4} + a_0 \frac{d^3 x}{dt^3} + a_1 \frac{d^2 x}{dt^2} + a_6 \frac{dx}{dt} + a_0 = 0
\]  

(13)

3.2. Power regulation mode

The equation of governor for power regulation mode could be transferred into the form similar to the equation of frequency regulation mode:

\[
b_T T_s \frac{d^2 y}{dt^2} + b_T T_s \frac{dy}{dt} = -b_p \left[ T_e T_s \frac{d^2 x}{dt^2} + T_e \frac{dx}{dt} \right] + (e_x + 1) x
\]  

(14)

By the method of Laplace transform to the equations (1)-(9) and (14), the comprehensive transfer function of waterpower-speed control system for power regulation mode can be derived:
Using the comprehensive transfer function (15), we can derive the ninth order free movement equation of power regulation mode:

$$a_0 \frac{d^9x}{dn^9} + a_1 \frac{d^8x}{dn^8} + a_2 \frac{d^7x}{dn^7} + a_3 \frac{d^6x}{dn^6} + a_4 \frac{d^5x}{dn^5} + a_5 \frac{d^4x}{dn^4} + a_6 \frac{d^3x}{dn^3} + a_7 \frac{d^2x}{dn^2} + a_8 \frac{dx}{dn} + a_9 = 0$$

(16)

The expressions of coefficients in equations (13) and (16) are presented in Appendix B.

4. Stability conditions of the waterpower-speed control system

Applying Routh-Hurwitz criterion [2], the stability conditions of waterpower-speed control system can be gotten as follows (taking frequency regulation mode for example, $a_i$ should be changed into $a'_i$ in the case of power regulation mode):

1. $a_i > 0 \quad (i = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$

2. $\begin{vmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ 0 & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ 0 & 0 & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & 0 & 0 & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \\ 0 & 0 & 0 & 0 & a_0 & a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & 0 & 0 & 0 & a_0 & a_1 & a_2 & a_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_0 & a_1 & a_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_0 & a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_0 \end{vmatrix} > 0$

3. $\begin{vmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ 0 & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ 0 & 0 & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & 0 & 0 & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \\ 0 & 0 & 0 & 0 & a_0 & a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & 0 & 0 & 0 & a_0 & a_1 & a_2 & a_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_0 & a_1 & a_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_0 & a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_0 \end{vmatrix} > 0$

5. Analyses of stability for the waterpower-speed control system

Thoma assumption assumes that when a hydropower station operates isolated and ignores the variation of turbine efficiency, the governor can keep the output constant strictly. Let the amplification coefficient of surge chamber sectional area $n_i$ be $F_{i}/F_{i0} \quad (i=1, 2)$, where $F_{i0}$ is the Thoma stable sectional area with each surge chamber working individually. In the coordinate system with $n_j$ as abscissa and $n_z$ as ordinate, the region that meets the stability conditions is called stability domain [5], which is shown in Figure 4. The stability domain can be obtained by substituting the characteristic parameters of regulation system in different conditions into the stability conditions. According to the reference [3], we can know that the horizontal dashed L1 is the horizontal asymptote, the vertical dashed L2 is vertical asymptote, and the closed region surrounded by two asymptotes and stable boundary is resonance region. Points A and B are a couple of interference points, and point C is the resonance point.

The effects of regulation modes, flow inertia of tunnel and penstock and governor parameters on the stability of waterpower-speed control system are analyzed by the control variable method. Taking a power station for example, the basic information are as follows: The turbine rated output is 306.12 MW, the rated head is 419m, the rated discharge is 81.56 m³/s and the rated rotation speed is 81.56 r/min. $T_{w1}/T_0=0.176$, $T_{w2}/T_0=0.227$, $T_{w3}/T_0=0.026$, $T_{w4}/T_0=0.117$, $h_{110}/H_0=0.006$, $h_{120}/H_0=0.014$, $h_{220}/H_0=0.003$, $h_{230}/H_0=0.004$. The ideal values of turbine transfer coefficients are as follows: $c_8=1.5$, $c_9=0.6$, $c_{16}=1.0$, $c_{17}=0.6$.
\( e_x = -1, e_y = 1, e_{th} = 0.5, e_{aq} = 0, e_{qy} = 1. \) The default variable values are as follows: \( e_g = 0, T_n = 0.6s, T_y = 0.02s, b_p = 0.04, b_t = 0.15, T_d = 1.5s. \)

5.1. Effect of flow inertia of penstock

The stability domains, considering the function of the flow inertia of penstock and Thoma assumption, are calculated and drawn (shown in Figure 5), respectively. \( T_{w_1}/T_a \) is taken as a variable and its value is assigned as 0.1, 0.2 and 0.3 in turn.

Figure 5 shows the effects of \( T_{w_i}/T_a \) \((i=1, 2)\) of diversion penstock and tailrace penstock on the stability domains:

1. Under the same condition, the stability domain becomes smaller with the increase of \( T_{w_i}/T_a \). In power regulation mode, the rangeability of stable boundary is well-distributed, while in frequency regulation mode, it is considerably small.
2. Considering penstock, the stability domain of frequency regulation mode is smaller than that of Thoma assumption, but both horizontal asymptotes are coincide generally. The stability domain of power regulation mode is possibly greater or smaller than that of Thoma assumption, and their horizontal asymptotes are far apart while the vertical asymptotes are close.

5.2. Effect of flow inertia of diversion tunnel and tailrace tunnel

The variables \( T_{w_1}/T_a \) and \( T_{w_2}/T_a \) are selected to analyze their effects on stability domain. The stability domains are shown in Figure 6.

Figure 6 shows the effects of \( T_{w_i}/T_a \) \((i=1, 2)\) of diversion tunnel and tailrace tunnel on the stability domains:

1. The abscissa \( n_1 \) of resonance point becomes smaller and the ordinate \( n_2 \) becomes bigger when the value of \( T_{w_1}/T_a \) increases in both two regulation modes. However, the increase of \( T_{w_2}/T_a \) has an opposite effect. Hence, the changes of upstream and downstream are symmetrical.
(2) The coordinate values of interference points are also affected by the value of $T_{w1}/T_a$ in power regulation mode. When the value of $T_{w1}/T_a$ increases individually, the location of point B changes a little but point A moves upper right, their coordinate values both become larger. On the contrary, when the value of $T_{w2}/T_a$ increases individually, the location of point B changes and its coordinate values become larger. Therefore, we can know that either $T_{w1}/T_a$ or $T_{w2}/T_a$ increases, $n_1$ and $n_2$ should enlarge synchronously instead of individually, so that the stability of the system can be guaranteed. This phenomenon is more sensitive in power regulation than that in frequency regulation.

5.3. Effect of governor parameters

The effects of temporary droop $b_t$, damping device time constant $T_d$ and the permanent droop $b_p$ on the stability domain are analysed. The results are shown in Figure 7.

Through the analysis of Figure 7 we get:

(1) Figure 7-f shows that if $b_t$ and $T_d$ take small value, the stability domain of those two regulation modes are less than that of Thoma assumption, but the position of horizontal asymptote in power regulation is still lower than that of Thoma assumption. On the contrary, if $b_t$ and $T_d$ take large value, the stability domain of those two regulation modes are greater than that of Thoma assumption. But the
stability domain, where close to point B of Thoma assumption, is still less than that of Thoma assumption.

(2) Figure 7-a and 7-c show that the stability domain of power regulation is bigger than that of frequency regulation in the same conditions. If $b_t$ or $T_d$ changes, the range ability of the stability domain of power regulation changes much more obviously than that of frequency regulation. That indicates the sensibility of parametric variation in power regulation is weaker than that in frequency regulation.

(3) As $b_t$ increases, $T_d$ is constant, the stability domain of power regulation becomes bigger. But in frequency regulation, if $T_d$ takes small value, the stability domain would be smaller, and when $T_d$ takes big value, the stability domain turns bigger. Therefore, there is a critical value about $T_d$ in frequency regulation. When $T_d$ is smaller than the critical value, it goes against to the stability with the increase of $b_t$. However, when $T_d$ is bigger than the critical value, it benefits to the stability with the increase of $b_t$.

(4) As $T_d$ increases, $b_t$ is constant, the stability domain of power regulation becomes bigger. However, in frequency regulation, if $b_t$ takes small value, the stability domain decreasing first and then increasing. Obviously, there is a critical value about $T_d$ in frequency regulation. When $T_d$ increases in the range of less than the critical value, it goes against to the stability. But if $b_t$ takes big value, it is conducive to the stability.

(5) With the increase of $b_p$, the stability of power regulation gets worse.

6. Conclusions
Some conclusions are formed by studying the stability of waterpower-speed control system for hydropower station with upstream and downstream surge chambers based on regulation modes:

(1) When the function of the flow inertia of penstock is taken into consideration, the stability of frequency regulation is worse than that of Thoma assumption, while the stability of power regulation is likely to be better or worse than that of Thoma assumption.

(2) The effects of the flow inertia of diversion tunnel and tailrace tunnel on the stability of system are symmetrical. Either $T_{w11}/T_a$ or $T_{w21}/T_a$ increases, the sectional areas of upstream and downstream surge chambers should enlarge synchronously instead of individually, so that the stability of the system can be guaranteed.

(3) If governor is taken into consideration, the relationship of stability between frequency or power regulation and Thoma assumption is related to the evaluation of governor parameters. Generally speaking, the smaller the governor parameters are, the worse the stability becomes after considering the governor, the better the stability becomes after considering the Thoma assumption.

(4) The stability domain is larger for the power regulation under the same conditions. However, no matter in worst or good characteristic of system, the sensibility of parametric variation is weaker for power regulation. The stable boundary of both two regulation modes have intersect areas, which make the combination point coordinates only in that areas satisfy the stability requirements of frequency regulation.

(5) The governor parameters $b_t$, $T_d$ and $b_p$ have great influence on the stability. There is a critical value about $T_d$ in frequency regulation, if $T_d$ increases in the range of less than the critical value, it goes against to the stability as $b_t$ increases, while it benefits to the stability in power regulation as $b_t$ increases or $b_p$ decreases.

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Appendix A
The subscript 0 represents the value of initial moment. \( i = 1 \) or 2, in which 1 represents downstream, \( j = 1 \) or 2, in which 1 represents diversion tunnel or tailrace tunnel and 2 represents penstock.

\[
Q_{ij} \quad \text{flow of the number \( ij \) pipeline} \\
H_i \quad \text{working head of spiral case inlet (\( i = 1 \)) or draft tube outlet (\( i = 2 \))} \\
h_{i0} \quad \text{head loss of the number \( ij \) pipeline} \\
Y \quad \text{guide vane opening} \\
P_g \quad \text{given power at any time} \\
F_i \quad \text{area of surge chamber} \\
T_{aij} \quad \text{time constant of fluid inertia in the number \( ij \) pipeline} \\
T_a \quad \text{time constant of unit inertia} \\
\Delta Z_i \quad \text{change of surge chamber water level (positive downward for upstream and positive upward for downstream)} \\
e_n, e_r, e_q \quad \text{transfer coefficients of hydraulic turbine torque} \\
e_{\alpha n}, e_{\alpha r}, e_{\alpha q} \quad \text{transfer coefficients of hydraulic turbine flow} \\
b_1, T_s, T_n, T_e, e_g, b_p \quad \text{governor parameters} \\
z_i = \Delta Z_i / H_0 \quad \text{relative change of surge chamber water level} \\
q_i = (Q_{ij} - Q_{i0}) / Q_0 \quad \text{relative change of the flow of the number \( ij \) pipeline} \\
h_i = (H_i - H_{i0}) / H_0 \quad \text{relative working head of spiral case inlet (\( i = 1 \)) or draft tube outlet (\( i = 2 \))} \\
h = (H - H_{i0}) / H_0 \quad \text{relative working head of the turbine} \\
m_g = (M_g - M_{g0}) / M_{g0} \quad \text{relative dynamic torque} \\
x = (n - n_0) / n \quad \text{relative rotational speed} \\
y = (Y - Y_0) / Y_0 \quad \text{relative guide vane opening} \\
m_e = (M_e - M_{e0}) / M_{e0} \quad \text{relative resistance torque} \\
p_c = (P_c - P_{c0}) / P_{c0} \quad \text{relative actual power} \\
p_g = (P_g - P_{g0}) / P_{g0} \quad \text{relative given power} \\
T_{ni} = F_i H_0 / Q_0 \quad \text{time constant of surge chamber}
\]

**Appendix B**

\[
a_0 = f_1 f_2 b_T a_T \quad \quad \quad \quad \quad \quad \quad a_i = f_1 f_2 b_T a_T + (f_1 f_3 + f_2 f_3) b_T T_2 + f_1 f_{10} T_T a_T \\
a_2 = (f_2 f_3 + f_2 f_1) b_T T_2 + (f_1 f_0 + f_2 f_8 + f_3 f_7) b_T T_2 T_e + f_1 f_{10} T_T a_T + (f_1 f_1 + f_2 f_10) T_T a_T \\
a_3 = (f_1 f_0 + f_2 f_8 + f_3 f_7) b_T T_2 + (f_2 f_9 + f_3 f_8 + f_4 f_7) b_T T_2 T_e + f_1 f_{10} + (f_1 f_{11} + f_2 f_{10}) T_T + (f_2 f_{11} + f_3 f_{10}) T_T a_T \\
a_4 = (f_1 f_0 + f_2 f_8 + f_3 f_7) b_T T_2 + (f_2 f_9 + f_3 f_8 + f_4 f_7 + k_1) b_T T_2 T_e + (f_1 f_{11} + f_2 f_{10}) + (f_2 f_{11} + f_3 f_{10}) T_T \\
+ (f_3 f_{11} + f_4 f_{10} - k_3) T_T a_T \\
a_5 = (f_3 f_{11} + f_4 f_{10} + f_5 f_{21} + k_4) b_T T_2 + (f_5 f_{21} + f_6 f_{21} + f_7 f_{21} + k_5) b_T T_2 T_e + (f_2 f_{11} + f_3 f_{10}) \\
+ (f_3 f_{11} + f_4 f_{10} - k_3) T_T a_T \\
a_6 = (f_3 f_{11} + f_4 f_{10} + f_5 f_{21} + k_4) b_T T_2 + (f_5 f_{21} + f_6 f_{21} + f_7 f_{21} + k_5) b_T T_2 T_e + (f_2 f_{11} + f_3 f_{10} - k_3) \\
+ (f_3 f_{11} + f_4 f_{10} - k_3) T_T a_T \\
a_7 = (f_3 f_{11} + f_4 f_{10} + k_3) b_T T_2 + (f_6 f_{21} + k_4) b_T T_2 T_e + (f_4 f_{11} + f_5 f_{10} - k_3) + (f_2 f_{11} + f_3 f_{10} - k_3) T_T a_T \\
+ (f_3 f_{11} + f_4 f_{10} - k_3) T_T a_T
\]
\[ a_b = (f_a f_b + k_1) b T_d + (f_a f_{1-1} - k_2) T_d + (f_3 f_{1-1} + f_6 f_{10} - k_3) \quad a_b = f_6 f_{1-1} - k_8 \]

\[ a_b = f_1 f_2 T_1 \quad a_b = f_1 f_f b T_a + (f_6 f_7 + f_1 f_8) b_T T_y + f_1 f_{10} b_p \left(T_a e_y + T_d + T_u\right) \]

\[ a_2' = (f_1 f_2 + f_1 f_3) b T_d + (f_6 f_7 + f_1 f_8 + f_5 f_7 + k_1) b T_2 T_a + (f_1 f_2 + f_6 f_{10} b_p (e_1 + 1) \]

\[ a_4' = (f_1 f_2 + f_6 f_8 + f_6 f_9 + f_1 f_8) b T_d + (f_6 f_9 + f_6 f_8 + f_6 f_9 + k_2) b T_2 T_a + (f_1 f_2 + f_6 f_{10} b_p (e_1 + 1) \]

\[ a_6' = (f_1 f_2 + f_6 f_8 + f_6 f_9 + f_1 f_8 + f_1 f_9 + k_3) b T_d + (f_6 f_9 + f_6 f_{10} - k_3) b T_2 T_a \]

\[ a_8' = (f_1 f_2 + f_6 f_8 + f_6 f_9 + k_4) b T_d + (f_6 f_9 + f_1 f_9 + k_5) b T_2 T_a + (f_6 f_9 + f_1 f_{10} - k_5) b T_2 T_a \]

\[ k_1 = e_{q_4} (T_{f_1} + T_{f_2}) T_{w_{11}} T_{w_{21}} \quad k_2 = e_{q_4} (T_{f_1} + T_{f_2}) \left(T_{w_{11}} + 2h_{210} + T_{w_{21}} + 2h_{110} - T_{w_{21}} \right) \]

\[ k_3 = e_{q_4} \left(T_{w_{11}} + T_{w_{21}} + 4h_{110} h_{210} H_0 T_{f_1} + T_{f_2} \right) \quad k_4 = e_{q_4} e_{q_4} \left(2h_{110} + 2h_{210} H_0 \right) \quad k_5 = e_{q_4} (T_{f_1} + T_{f_2}) T_{w_{11} + w_{21}} \]

\[ k_6 = e_{q_4} (T_{f_1} + T_{f_2}) \left(T_{w_{11}} + 2h_{210} + T_{w_{21}} + 2h_{110} - T_{w_{21}} \right) \quad k_7 = e_{q_4} \left(T_{w_{11}} + T_{w_{21}} + 4h_{110} h_{210} H_0 T_{f_1} + T_{f_2} \right) \]

\[ k_8 = e_{q_4} \left(2h_{110} + 2h_{210} - T_{w_{21}} \right) \quad f_1 = e_{q_1} T_{f_1} T_{f_2} \left(T_{w_{11}} + T_{w_{22}} \right) T_{w_{11} + w_{21}} \]

\[ f_2 = e_{q_1} T_{f_1} T_{f_2} \left(T_{w_{11}} + T_{w_{22}} \right) \left(T_{w_{11}} + 2h_{210} + T_{w_{21}} + 2h_{110} - T_{w_{21}} \right) + T_{f_1} T_{f_2} \left(T_{w_{11}} + T_{w_{21}} \right) \left(1 + e_{q_1} \left(2h_{210} + 2h_{210} \right) \right) \]

\[ f_3 = T_{f_1} T_{f_2} \left(T_{w_{11}} + 2h_{210} H_0 + T_{w_{21}} + 2h_{110} - T_{w_{21}} \right) + e_{q_1} \left(T_{w_{11}} + T_{w_{22}} \right) \left(T_{f_1} T_{f_2} + 4h_{110} h_{210} H_0 T_{f_1} + T_{f_2} + T_{w_{21}} \right) + (T_{f_1} + T_{f_2}) T_{w_{11} + w_{21}} \]

\[ f_4 = 4h_{110} h_{210} H_0 + T_{f_1} T_{w_{11}} + T_{f_2} T_{w_{21}} \left(1 + e_{q_1} \left(2h_{210} + 2h_{210} \right) \right) \]

\[ + e_{q_1} \left(T_{w_{11}} + T_{w_{22}} \right) \left(T_{f_1} + T_{f_2} \right) \left(2h_{210} H_0 + T_{f_1} + \frac{2h_{110} H_0}{T_{f_2}} + T_{w_{21}} \right) + e_{q_1} \left(T_{w_{11}} + T_{w_{22}} \right) \left(T_{f_1} + T_{f_2} \right) \left(2h_{210} H_0 + T_{f_1} + \frac{2h_{110} H_0}{T_{f_2}} \right) \]
\[ f_s = \left( T_{f_1} \frac{2h_{10}}{H_0} + T_{f_2} \frac{2h_{210}}{H_0} \right) \left( 1 + e_{\phi} \frac{2h_{210} + 2h_{220}}{H_0} \right) + e_{\phi} \left( T_{f_1} + T_{f_2} \right) \frac{4h_{10}h_{210}}{H_0^2} + T_{u_{11}} + T_{u_{21}} + T_{u_{12}} + T_{u_{22}} \]  
\[ f_s = 1 + e_{\phi} \frac{2h_{10} + 2h_{210} + 2h_{220} + 2h_{220}}{H_0} \]  
\[ f_s = T_s \left( 1 + e_{\phi} \frac{2h_{10} + 2h_{210}}{H_0} \right) + \left[ e_{\phi} \left( e_x - e_y \right) + e_x e_{\phi} \right] \left( T_{u_{12}} + T_{u_{22}} \right) \]  
\[ f_s = \left( e_x - e_y \right) \left( 1 + e_{\phi} \frac{2h_{210} + 2h_{220}}{H_0} \right) + e_x e_{\phi} \frac{2h_{220} + 2h_{210}}{H_0} \]  
\[ f_{11} = \left( e_x e_{\phi} - e_y e_{\phi} \right) \frac{2h_{220} + 2h_{210}}{H_0} + e_y \]  
\[ d_1 = 1 + \frac{e_{\phi}}{1 + \left( q_1 + q_2 \right) e_{\phi}} \left( \frac{p_1}{1 + T_{f_1} s p_1} + \frac{p_2}{1 + T_{f_2} s p_2} \right) \]  
\[ d_2 = \left[ 1 + \left( q_1 + q_2 \right) e_{\phi} \right]^2 \left( 1 + T_{f_1} s p_1 \right) \left( 1 + T_{f_2} s p_2 \right) \]  
\[ p_1 = T_{u_{11}} s + \frac{2h_{10}}{H_0} \]  
\[ p_2 = T_{u_{21}} s + \frac{2h_{210}}{H_0} \]  
\[ q_1 = T_{u_{12}} s + 2h_{210} / H_0 \]  
\[ q_2 = T_{u_{22}} s + 2h_{220} / H_0 \]  

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