Decoupling Solution to SUSY Flavor Problem via Extra Dimensions

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Abstract

We discuss the decoupling solution to SUSY flavor problem in the fat brane scenario. We present a simple model to yield the decoupling sfermion spectrum in a five dimensional theory. Sfermion masses are generated by the overlap between the wave functions of the matter fields and the chiral superfields on the SUSY breaking brane. Two explicit examples of the spectrum are given.
In building models with supersymmetry (SUSY), we must take into account that the sfermion mass spectrum of the first and the second generations is severely constrained from the flavor changing neutral current (FCNC) processes such as $K^0 - \bar{K}^0$ mixing etc. Mainly, three approaches to address this problem (SUSY flavor problem) have been discussed in the literature; (1) the degeneracy \cite{1}, (2) the alignment \cite{2} and (3) the decoupling \cite{3}.

In this letter, we consider the decoupling solution to SUSY flavor problem in the context of higher dimensional theories. The basic idea is very simple. In extra dimensions, it is well known that if the matter wave functions are localized at different points in extra dimensions, Yukawa hierarchy can be obtained by the suppression factor of the overlap of wave functions \cite{4}. Since the fermion mass hierarchy is $m_1 < m_2 < m_3$ where $m_i$ is a fermion mass of the i-th generation, the matter of the third generation is localized close to the Higgs fields and the first generation is localized most distant from the Higgs fields. Introducing SUSY breaking brane in which the chiral superfield with nonvanishing F-term is localized correlates the sfermion masses with the fermion masses. If the SUSY breaking brane is put close to the first generation matter fields, the sfermion mass hierarchy is inverted, $\tilde{m}_1 > \tilde{m}_2 > \tilde{m}_3$ where $\tilde{m}_i$ is the sfermion mass of the i-th generation. Therefore, we expect that the decoupling solution can be a natural solution. Namely, the sfermion masses of the first and the second generation is the order of 10 TeV and the sfermion mass of the third generation is the order of 100 GeV for naturalness.

Let us discuss the model in detail. We consider an $\mathcal{N} = 1$ supersymmetric theory in five dimensions. We introduce two 3-branes at $y = 0$ and $y = L$, where $y$ denotes the fifth coordinate in five dimensional space-time. The gauge supermultiplets of the Standard Model (SM) gauge groups lives in the bulk and its zero mode wave functions are flat in the fifth dimension. The matter fields also lives in the bulk and its zero mode wave functions are assumed to be Gaussian. Higgs doublets are assumed to be localized on the brane at $y = 0$, we refer to this brane as “H-brane”. Further, extra chiral superfields $X, \Phi'$ and $\bar{\Phi}'$ localized on the brane at $y = L$ are introduced. $X$ is a chiral superfield with nonvanishing F-term (i.e. $X = \theta^2 F$). $\Phi'$ and $\bar{\Phi}'$ are vector-like superfields with a mass $M$. A pair of vector-like superfields are introduced for each matter chiral superfields, namely $Q', \bar{Q}'$ for $Q$, and $U', \bar{U}'$ for $U$ and $L', \bar{L}'$ for $L$ etc. We refer to the brane at $y = L$ as “SUSY breaking brane”.

\footnote{For readers interested in the localization mechanism of the chiral superfields, see Appendix of Ref. \cite{5}.}
Naive expectation is that the gaugino mass is generated from the term

$$
\delta(y - L) \int d^2 \theta \frac{X(x)}{M_s^2} W^\alpha(x, y) W_\alpha(x, y) \rightarrow M_\lambda = \frac{|F|}{M^2 L_c},
$$

(1)

where \(x\) is a coordinate of the four dimensional space-time, \(W_\alpha\) is the field strength tensor superfield living in the bulk, \(L_c\) is the width of the thick wall which should be considered as the compactification length in our framework and \(M_s\) is the Planck scale in five dimensions. Note that the gaugino masses receive only the volume suppression factor.

The soft breaking mass of the \(i\)-th generation chiral superfield \(\Phi_i\) is naively generated from the term

$$
\delta(y - L) \int d^4 \theta \frac{X^\dagger(x) X(x)}{M^3_s} \Phi^\dagger_i(x, y) \Phi_i(x, y)
$$

$$
\rightarrow \tilde{m}_{ij}^2 \equiv \frac{|F|^2}{M^2 L_c} \exp[-M^2(L - y_i)^2 - M^2(L - y_j)^2],
$$

(2)

where the form of the zero mode wave function of the matter fields is assumed as \(\Phi_i^{(0)} \sim \exp[-M^2(y - y_i)^2]\). The sfermion masses receives not only the volume suppression but also the exponential suppression, therefore are negligibly small compared to the gaugino masses. This spectrum is similar to the gaugino mediation scenario \[3\]. Although the spectrum of the gaugino mediation is phenomenologically interesting, this is not the subject in this paper.

As is clear from the above argument, the modification is needed to obtain the spectrum of the decoupling solution. The wayout is to replace \(M_s\) with some scale \(M < M_s\) such that the enhancement by \(M\) compensates for the exponential suppression. We consider here the following superpotential on the SUSY breaking brane

$$
W = \delta(y - L) \int dy \lambda X(x) \Phi_i(x, y) \Phi^\dagger_i(x) + M \Phi'(x) \Phi'(x),
$$

(3)

$$
= \lambda \epsilon_i X(x) \Phi_i(x) \Phi^\dagger_i(x) + M \Phi'(x) \Phi'(x),
$$

(4)

where \(\lambda\) is a dimensionless constant of order unity. The second expression is obtained by integrating out the fifth dimensional degree of freedom. \(\epsilon_i\) is a suppression factor coming from the zero mode wave function of \(i\)-th generation. At the scale below \(M\), we can integrate out the massive fields \(\Phi', \Phi\). Then, the effective superpotential of Eq.(4) vanishes, and the effective Kähler potential receives the correction at tree level such as

$$
\delta K = \frac{\epsilon_i^2}{M^2} X^\dagger X \Phi^\dagger_i \Phi_i.
$$

(5)
As mentioned above, it is necessary to introduce a pair of vector-like fields for each matter chiral superfield to obtain the above Kähler potential. Otherwise, the Kähler potential with the flavor mixing will arise in general. We do not consider this possibility. Sfermion masses coming from Eq. (5) becomes
\[
\tilde{m}_i^2 = \epsilon_i^2 \frac{|F|^2}{M^2}.
\]  
This result has the desirable features that the suppression factor is not only replaced with \( M < M^* \), but also proportional to \( \epsilon \simeq (\text{Yukawa})^{-1} \). Note that this argument holds in the case \( F < M^2 \). Also, we assumed here that the overall sign of the Kähler potential is positive.

In our scenario, since the information of the location where the matter fields are localized is necessary to derive the sfermion mass spectrum, we briefly discuss Yukawa hierarchy. We consider the up-type Yukawa coupling for example
\[
W = \delta(y) \int dyQ_i(x, y)\bar{U}_j(x, y)H(x),
\]
where the order one coefficient is implicit. Integrating out the fifth dimensional degree of freedom, we obtain the effective Yukawa coupling in four dimensions at the compactification scale \( L_c^{-1} \) as
\[
y_{\text{eff}} \simeq \exp[-M^2(y_{Q_i}^2 - y_{\bar{U}_j}^2)].
\]
In order to realize the Yukawa hierarchy
\[
y_t \sim \mathcal{O}(1), \ y_c \sim \mathcal{O}(10^{-2}), \ y_u \sim \mathcal{O}(10^{-5}),
\]
\[
y_b \sim \mathcal{O}(10^{-2}), \ y_s \sim \mathcal{O}(10^{-4}), \ y_d \sim \mathcal{O}(10^{-5}),
\]
the location of the matter fields are determined\footnote{To be correct, the effective Yukawa couplings (5) have to be evolved down to the weak scale by the renormalization group equation (RGE) and matched to Eqs. (9), (10) after diagonalizing Yukawa matrix. We simply neglect this RGE effects and the mixing angles.} for instance,
\[
y_H \simeq y_{\bar{H}} \simeq y_{Q_3} \simeq y_{U_3} \simeq 0, \ |y_{Q_2}| \simeq |y_{U_2}| \simeq \sqrt{\ln 10 M_s^{-1}}, \ |y_{D_3}| \simeq \sqrt{2 \ln 10 M_s^{-1}},
\]
\[
|y_{Q_1}| \simeq |y_{U_1}| \simeq |y_{D_1}| \simeq \sqrt{\frac{5}{2} \ln 10 M_s^{-1}}, \ y_{D_2} \simeq \sqrt{3 \ln 10 M_s^{-1}}.
\]
Now, we turn to the sfermion mass spectrum. The decoupling solution requires that the masses of the first and the second generations should be heavier than \( \mathcal{O}(10) \) TeV.
and that of the third generation be around $\mathcal{O}(100)$ GeV for naturalness. We consider two cases of the decoupling constraints according to Eqs. (11) and (12). The first case is $\tilde{m}_{Q_2} = \tilde{m}_{U_2} \simeq 10$ TeV (solution 1), and the second one is $\tilde{m}_{D_2} \simeq 10$ TeV (solution 2).

Let us consider the solution 1 at first. This case imposes

$$10^2 \simeq \epsilon_2/\epsilon_3 \simeq \exp[(M_sL)^2 - (M_sL - \sqrt{\ln10})^2],$$

which leads to

$$M_sL \simeq \frac{3}{2} \sqrt{\ln10}.$$

This implies that SUSY breaking brane is located between the first and the second generations. This is the interesting feature of our model. In the conventional SUSY breaking models in extra dimensions, the visible brane and the SUSY breaking brane are separated in the extra dimensional spaces and the dangerous flavor violating sfermion masses are suppressed by the locality. In our model, SUSY breaking brane are not separate from the visible brane (more correctly, the visible wall) but is predicted to be located within the visible wall to yield the decoupling sfermion mass spectrum.

Requiring that the gaugino mass should be around 100 GeV,

$$M_\lambda \simeq \frac{F}{M_s M_s L_c} \simeq 100 \text{ GeV},$$

we obtain

$$F \simeq (M_s L_c) \times 10^2 M_s.$$  

Since $\tilde{m}_{Q_2} = \tilde{m}_{U_2} \simeq 10$ TeV,

$$\tilde{m}_{Q_2} = \tilde{m}_{U_2} \simeq \frac{F}{M} \, \exp[-(M_s L - \sqrt{\ln10})^2],$$

$$\simeq \frac{10^2 (M_s L_c) (M_s L)}{M L} \, \exp[-(\frac{3}{2} \sqrt{\ln10} - \sqrt{\ln10})^2] \simeq 10 \text{ TeV},$$

$$\Leftrightarrow M L \simeq (M_s L_c) \times 10^{-2} \times \frac{3}{2} \sqrt{\ln10} \, \exp[-\ln10/4].$$

Using Eqs. (14, 16, 19), we obtain the sfermion masses for other generations

$$\tilde{m}_{Q_1} = \tilde{m}_{U_1} = \tilde{m}_{D_1} \simeq \frac{F}{M} \, \exp[-(M_s L - \sqrt{5\ln10/2})^2] \simeq 18 \text{ TeV},$$

$$\tilde{m}_{Q_3} = \tilde{m}_{U_3} \simeq \frac{F}{M} \, \exp[-9\ln10/4] \simeq 100 \text{ GeV},$$

$$\tilde{m}_{D_2} \simeq \frac{F}{M} \, \exp[-(3\sqrt{\ln10/2} - \sqrt{3\ln10})^2] \simeq 15.7 \text{ TeV},$$

$$\tilde{m}_{D_3} \simeq \frac{F}{M} \, \exp[-(3\sqrt{\ln10/2} - \sqrt{2\ln10})^2] \simeq 17.5 \text{ TeV}.$$
We note that these sfermion masses are generated at the compactification scale $L^{-1}_c$.

Let us comment on scales in our model. Assuming $M_* \simeq 10^{18}\text{GeV}$ and $M_* L_c \simeq 10$,

$$F \simeq 10^{21}\text{GeV}^2, \quad M \simeq 5.6 \times 10^{16}\text{GeV}, \quad L^{-1}_c \simeq 10^{17}\text{GeV} \quad (24)$$

are obtained. As mentioned above, $F < M^2$ is satisfied. It is also interesting that the mass of the additional vector-like superfields is close to the compactification scale.

Next we will show the case of the solution 2. Instead of the condition (13), we can impose

$$10^2 \simeq \varepsilon_2/\varepsilon_3 \simeq \exp[(M_* L)^2 - (M_* (L - \sqrt{3\ln 10})^2)], \quad (25)$$

which leads to

$$M_* L \simeq \frac{5}{6} \sqrt{3\ln 10}. \quad (26)$$

From the gaugino mass constraint, we obtain

$$F \simeq (M_* L_c) \times 10^2 M_* \quad (27)$$

On the other hand, $\tilde{m}_{D_2} \simeq 10$ TeV leads to

$$ML \simeq (M_* L_c) (ML) \times 10^{-2}\exp[-\ln 10/12]. \quad (28)$$

Other sfermion masses at the compactification scale can be estimated as the same analyses of the previous case as,

$$\tilde{m}_{Q_3} = \tilde{m}_{U_3} \simeq 100 \text{ GeV}, \quad \tilde{m}_{D_3} \simeq 12.1 \text{ TeV}, \quad (29)$$

$$\tilde{m}_{Q_2} = \tilde{m}_{U_2} \simeq 7.7 \text{ TeV}, \quad \tilde{m}_{Q_1} = \tilde{m}_{U_1} = \tilde{m}_{D_1} \simeq 11.6 \text{ TeV}. \quad (30)$$

As for scales, it is almost the same as the solution 1,

$$F \simeq 10^{21}\text{GeV}^2, \quad M \simeq 8.25 \times 10^{16}\text{GeV}, \quad L^{-1}_c \simeq 10^{17}\text{GeV} \quad (31)$$

We give some comments here. First, in our scenario, $\tilde{m}_{D_3}$ is the order of $O(10)$ TeV, which is somewhat large from the viewpoint of the decoupling solution. This means that the large $\tan \beta$ case is preferable. Second, it is known that the decoupling scenario generically suffers from a problem that the third generation sfermion mass squareds are driven

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Since the mass of the vector-like fields $M$ is a parameter in four dimensional theory, it has nothing to do with the fundamental Planck scale $M_*$. 

to be negative through the two-loop RGE effects of the heavy first-two generation sfermion masses \[7\]. Since the sfermion masses we obtained are generated at the compactification scale, the third generation sfermion mass squareds might be negative at low energy. To avoid this, we have to add the extra fields with negative SUSY breaking mass squareds as discussed in Ref. \[8\]. We do not discuss this point in detail in this paper.

In summary, we have proposed the decoupling scenario in the fat brane approach. In this approach, Yukawa hierarchy is determined by the overlap of wave functions of the matter fields localized at the different points in extra dimensions. The lighter matter fields are localized closer to the point where Higgs fields are localized. We introduced the SUSY breaking brane in which the chiral superfield with nonzero F-term is localized and discussed whether the spectrum of the decoupling solution is possible or not, namely, the first-two generation sfermion masses are the order of 10 TeV, and the third generation sfermion masses are of order 100 GeV. Naively, sfermion masses generated from the Kähler potential suppressed by the fundamental scale are negligibly small compared to the gaugino masses since sfermion masses receive the additional exponential suppression by the overlap between the wave functions of the matter fields and of the chiral superfield on the SUSY breaking brane. In order to obtain the spectrum of the decoupling scenario, we have introduced the extra vector-like superfields and Yukawa interaction between the vector-like superfields, MSSM superfields and the chiral superfield with nonzero F-term. Integrating out the vector-like fields leads to the Kähler potential proportional to the suppression factor with the inverse of Yukawa hierarchy and suppressed by the mass of the vector-like fields, which is smaller than the fundamental scale. This becomes the dominant source of the sfermion masses. Two explicit examples of the spectrum have been shown in this paper. It has turned out that the large tan$\beta$ case is preferable because $\tilde{m}_{D_3}$ is somewhat large. It is interesting that SUSY breaking brane is predicted to be localized within the thick visible wall unlike the conventional SUSY breaking scenario in extra dimensions.

Finally, we have touched on the negative sfermion mass squareds problem. Since the sfermion masses we have obtained is generated at the compactification scale, a detailed RGE analysis is necessary to see whether the third generation sfermion mass squareds are indeed positive at low energy. We leave this subject for future work.
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