Higgs Bosons Production with Photons at Lepton-Antilepton Colliders

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Abstract

By model independent way scalar and pseudoscalar neutral Higgs boson production with photon in the tree process $\mu^+\mu^- \rightarrow H^0\gamma$ are considered. For the Standard Model and Minimal Supersymmetric Standard Model cases numerical estimates are obtained. The model independent flavour changing Higgs bosons production in the tree processes $e^+e^-, \mu^+e^- \rightarrow H_f^0\gamma$ is also considered.
1. Introduction

As known, Higgs bosons interactions with fermions are proportional to their masses. This property of Higgs bosons make possible their production at $\mu^+ \mu^-$-colliders in resonance \cite{1}, \cite{2} (for references on $\mu^+ \mu^-$-colliders see ref. \cite{3} and references therein):

$$\mu^+ \mu^- \rightarrow H^0.$$ (1)

Analogous process in $e^+ e^-$-collisions is suppressed by the smallness of electron mass.

However, mass of the Higgs bosons is the free parameter of the theory and, thus, we don’t know which energies are necessary for $H^0$-bosons production in resonance (i.e. at $s = m_H^2$).

Here we study scalar and pseudoscalar Higgs boson production by model independent way in the processes\cite{2}

$$\mu^+ \mu^- \rightarrow H^0 \gamma,$$ (2)

as described by the diagrams of Fig.1, where $H^0$-are scalar or pseudoscalar Higgs bosons (see Appendix A for model independent interaction of Higgs bosons with fermions as well as Higgs bosons interactions with fermions in the framework of the Minimal Supersymmetric Standard Model (MSSM), see \cite{8, 9} and references therein).

Whereas at $e^+ e^-$-colliders the tree contribution is suppressed by the smallness of electron mass in comparison with loop contribution \cite{5}, \cite{6}, \cite{7}, at $\mu^+ \mu^-$-colliders, as we will see below under some conditions the tree contribution may exceed loop effects.

We also consider similar processes (see Fig.1)\cite{2}

$$\mu^+ e^- \rightarrow H^0_{fe} \gamma,$$ (3)

$$\mu^+ \mu^-, e^+ e^- \rightarrow H^0_{fe} \gamma,$$ (4)

\footnote{Scalar and pseudoscalar Higgs bosons production in the process (2) has been considered previously in \cite{4} however the author of these references do not consider the effect of muon mass (formula (6) below) and in numerical results for the total cross section cut angles near colliding beams direction, where the cross section is divergent.}

\footnote{for references on $\mu^+ e^-$-colliders see \cite{11} and references therein}
where \( H^0_{Jc} \)-are flavour changing scalars or pseudoscalars \([10]\). Such particles are predicted by many extensions of the Standard Model, for example, they are contained into SUSY theories with \( R \)-parity violation \([12]-[18]\) (the role of \( H^0_{Jc} \)-bosons in such theories plays scalar neutrino).

In Appendix A the most general interaction of \( H^0_{Jc} \)-bosons with fermions is presented. It must be noted that in some models (in particular in models with \( R \)-parity violation) Yukawa couplings are not necessarily proportional to the fermion masses and consequently the processes (3),(4) may also be measurable as we see below at sufficiently large couplings.

Flavour changing Higgs bosons exchanges may contribute to the processes with lepton flavour violation. In \([19]\) has been considered their contribution to the decay \( \mu \to e\gamma \), in \([20, 21, 11]\), their contribution to the muonium-antimuonium conversion (for details of muonium-antimuonium experiments see \([22]\)).

Flavour changing scalars and pseudoscalars may also be produced virtually or in resonance in \( \mu^+\mu^- \)-collisions \([11]\), \( \mu^+\mu^- \)-collisions, \( e^+e^- \)-collisions \([23]-[26]\).

Although the cross section of the processes (2),(3) is essentially smaller than the cross section of the resonant process (1) and \( \mu^+\mu^- \to H^0_{Jc} \), the processes (2),(3) are allowed under more soft restriction \( \sqrt{s} > m_H \) where \( m_H \) is the mass of the \( H^0 \) or \( H^0_{Jc} \)-bosons.

Produced in reactions (3),(4) \( H^0_{Jc} \)-bosons decay unambiguously into leptons (\( H^0_{Jc} \to \bar{l}_i l_j \)) and consequently there are no backgrounds to the process (3) with subsequent decays of \( H^0_{Jc} \)-bosons in contrast to the process (2), where backgrounds like \( \mu^+\mu^- \to \bar{b}b\gamma \) imitating appropriated Higgs boson decays exist. The backgrounds are absent for the process (3) and resonant production of \( H^0_{Jc} \)-bosons even if produced flavour changing scalar or pseudoscalar decay into fermions of the same flavours.

2. The \( \mu^+\mu^- \to H^0\gamma \) process

For the differential cross section for both scalar and pseudoscalar Higgs boson cases we obtain the following result \([1]\).

\( ^3 \)The cross sections for the scalar and pseudoscalar cases are not equal to each other if we do not neglect terms proportional to \( m_\mu^4 \) and of higher order in \( m_\mu \).
\[ \frac{d\sigma(\mu^+\mu^- \to H^0\gamma)}{dt} = \frac{\pi\alpha^2}{4\sin^2\theta_W} \frac{m_{\mu}^2}{m_W^2} F_{S_P}^2 \frac{1}{s^2} \left( \frac{m_H^4 + s^2}{(t - m^2_\mu)(u - m^2_\mu)} - 2m_\mu^2 m_H^2 \left( \frac{1}{(t - m^2_\mu)^2 + (u - m^2_\mu)^2} \right) \right). \]

Here we use the following notations: \( s = (k_1 + k_2)^2, t = (k_1 - k_3)^2, u = (k_2 - k_3)^2, m_\mu \) - mass of the muon.

If we neglect muon mass we obtain:

\[ \frac{d\sigma(\mu^+\mu^- \to H^0\gamma)}{d\cos\theta} = \frac{\pi\alpha^2}{2\sin^2\theta_W} \frac{m_{\mu}^2}{m_W^2} F_{S_P}^2 \frac{1}{s - m_H^2} \frac{s^2 + m_H^4}{s^2} \frac{1}{\sin^2\theta}. \] (6)

Here \( \theta \) is the angle between the photon momentum \( \vec{k}_3 \) and muon momentum \( \vec{k}_1 \).

As we see, our result contains a collinear singularity at \( \theta = 0, \pi \), however, photons with \( \vec{k}_3 \) nearly parallel to the beam direction can not be detected and we cut some cone near this direction as has been done for the \( e^+e^- \to Z^0\gamma \) process (see ref. \[27\] and references therein).

It must be noted however that even if Higgs boson momentum lies in these cones it is possible that momentums of the Higgs boson decay products (or part of them) will be placed beyond these cones and if we do not exclude such events (i.e. heavy fermions from Higgs boson decays with missing energy from undetected photons) we mustn’t in principle cut these cones.

For the total cross sections where we take into account mass of \( \mu \)-meson we obtain the following result \[4]\:

\[ \sigma(\mu^+\mu^- \to H^0\gamma) = \frac{\pi\alpha^2}{2\sin^2\theta_W} \frac{m_{\mu}^2}{m_W^2} F_{S_P}^2 \frac{1}{s - m_H^2} \left( 1 + \frac{m_H^4}{s^2} \log\left( \frac{s}{m_\mu^2} \right) - 2 \frac{m_H^2}{s} \right). \] (7)

It must be noted that our calculations are valid only at \( \sqrt{s} - m_H \gg \Gamma_H \) where \( \Gamma_H \) is the total width of the \( H^0 \)-bosons.

\[4\]Despite \( \sqrt{s}, m_H \gg m_\mu \) we do not neglect the terms proportinal \( m_\mu^2 \) because the terms \( (t - m_\mu^2)^{-2},(u - m_\mu^2)^{-2} \), in formulas (5),(8) contain after integration over \( \cos\theta \) singularity \( m_\mu^{-2} \). Terms of the higher degrees of \( m_\mu \) in numerators may be neglected. The author expresses his sincere gratitude to F.Cuypers for this note.
In general, near threshold it is necessary to consider virtual $H^0$-bosons with finite width in propagator and also take into account radiative corrections which cancell infrared singularity in the process (3).

Analogous situation takes place also for $Z^0$-boson production (see e.g. ref. [28] and references therein) in the vicinity of peak where it is also necessary to consider the finite width of $Z^0$-boson and take into account QED-correction which cancell infrared singularities of the process $e^+e^- \rightarrow Z^{0*}\gamma$.

Besides the contribution to the processes (2),(3) from the Fig.1 there is also a contribution from loops [5, 6] with virtual $W^\pm$-and $t$-quarks and with other heavy particles in various extensions of the Standard Model such as contributions from squarks, charged Higgs bosons, chargino [7].

It is of interest to compare both tree and loop contributions. For example, within the Standard Model essentially below $m_Z$ as seen from the Fig.2 and from Fig.8 of Ref.[5] the tree contribution exceeds the loop contribution. At energies above $m_Z$ loop contribution dominate over the tree. For instance, at $\sqrt{s} = 200GeV \gg m_H$ as seen from Fig.2 and from figures of Ref.[5] standard loop contribution exceeds the tree contribution.

On the other hand, at $\sqrt{s} \gg 2m_t$ loop contribution also decreases faster than tree contribution, because loop integrals contain additional degree of $s^{-1}$ at $\sqrt{s} \gg 2m_t$.

Analogous situation takes place within the MSSM, at $\sqrt{s} < m_Z$ the tree contribution exceeds the loop contribution.

In the MSSM loop contribution to the process (2) depends on many unknown parameters such as SUSY particle masses, $\tan \beta$, various SUSY mass parameters containing in the couplings of Higgs and $Z^0$-bosons with SUSY particles and the cross section of the loop contribution to the process (2) within the MSSM may be increased or decreased up to two orders in comparision with the loop contribution in the Standard model (see Fig.3-11 in the ref. [7]), whereas the tree contribution is enhanced by factors $(\tan \beta)^2, (\cos \alpha \cos \beta)^2, (\sin \alpha \cos \beta)^2$, (see Appendix A), where $\tan \beta$ may be as high as $\sim \frac{m_t}{m_b} \approx 35$ [29, 30] (which enhanced the number of events $H_i^0\gamma$ by 1225 times in comparision with result of Fig.2). As seen from Fig.3-11 of the ref. [7] at $\sqrt{s} = 200GeV$ at the wide range of
SUSY parameters considered in [7] the ratio \( \sigma(e^+e^- \to H^0\gamma)/\sigma(e^+e^- \to \mu^+\mu^-)/ < 10^{-4} \), whereas the same ratio for tree contribution at large \( \tan \beta \) and far from threshold is about \( \frac{m_\mu^2}{m_W^2} \tan^2 \beta \log(s/m_\mu^2) \). For example, at \( \tan \beta = 20 \) this ratio for tree contribution is about 0.01 and consequently exceeds loop contribution even at \( \sqrt{s} \gg m_H \).

It must be noted also, that near threshold (i.e. at \( \sqrt{s} \sim m_H \)) tree contribution as seen from formula (7) enhanced as \( \sigma_{\text{tree}} \sim 1/(s - m_H^2) \) whereas loop contribution is suppresed as \( \sigma_{\text{loop}} \sim (s - m_H^2)^3 \) (see also Fig.8,10 of the Ref. [7]).

At \( \sqrt{s} \) much larger than masses of particles ( \( W^{\pm} \)-boson, \( t \)-quark, charged Higgs boson and their superpartners) in loop contributions as well as in the Standard Model are suppresed by additional degree of \( s^{-1} \) from loop integrals.

The number of Higgs bosons produced in the process (2) is shown in Fig.2 for case of yearly luminosity \( L = 1000 fb^{-1} \) and at \( F = 1 \) (the muon colliders with luminosities of order \( 1000 fb^{-1} \) and \( \sqrt{s} = 4 TeV \) has been considered e.g. in ref. [3]).

3. The \( \mu^+e^- \to H^0_{f,c}\gamma, e^+e^- \to H^0_{f,c}\gamma \) processes

For the differential cross section for both scalar and pseudoscalar flavour changing Higgs boson cases we obtain the following result:

\[
\frac{d\sigma(\mu^+e^- \to H^0_{f,c}\gamma)}{dt} = \frac{1}{4} \alpha(h_{S,P}^{\mu e})^2 \frac{1}{s^2} \frac{(m_H^4 + s^2)}{(t - m_H^2)(u - m_e^2)} - 2m_H^2 \frac{m_\mu^2}{(t - m_H^2)^2} + \frac{m_e^2}{(u - m_e^2)^2},
\]

where \( m_e \) is electron mass.

For the total cross sections we obtain the following result:

\[
\sigma(\mu^+\mu^- \to H^0_{f,c}\gamma) = 2\alpha(h_{S,P}^{\mu e})^2 \frac{1}{s - m_H^2} \frac{(1 + m_H^4)}{(s^2)} \log(\frac{s}{m_\mu m_e}) - 2\frac{m_H^2}{s}.
\]

It must be noted, that all the above mentioned remarks concerning collinear and infrared singularities, missing photons near colliding beam direction and details of calculation connected with nonzero masses of the electron and muon remain true also in cases of the processes (3),(4). The cross section of the processes \( \mu^+\mu^- , e^+e^- \to H^0_{f,c}\gamma \) may be obtained by replacements \( m_\mu \to m_e, m_\mu \to m_e \) respectively.
It must be noted that although in (A7) are considered the interaction of flavour changing pure scalar and pseudoscalars whereas in (A8) \( \tilde{\nu} \) is the mixing of scalar and pseudoscalar, the cross sections of the \( \tilde{\nu} + \gamma \) and \( H^0_{fc}(P^0_{fc}) + \gamma \) production at the same Yucawa couplings (i.e. at \( \lambda = h \)) are the same if we neglect terms \( O(m^4_{\mu}) \) and higher degrees of \( m_{\mu} \).

The number of flavour changing Higgs bosons produced in the process (3) is shown in Fig.3 at \( h_{S,P}^{le} = 10^{-3} \).

As well as in the case of reaction (2) the processes (3),(4) are most effective at low \( \sqrt{s} \sim m_H \).

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Note Added

When this paper has been completed the author received the information that neutral scalar Higgs boson production with photon within the Standard Model \( \mu^+\mu^- \rightarrow H^0\gamma \) has been also studied in [36],[37]. It must be noted, we disagree with formulas (3),(4) of ref. [36]. The nonlogarithmic terms in our result (formula (7)) and in formula (4) of the ref.[36] are different even after neglecting terms of order \( O(m^2_{\mu}) \) and higher in the last formula.
Appendix A

We parametrized the model independent lagrangian of scalar and pseudoscalar Higgs boson \( (H^0 = S^0, P^0) \) interaction with fermions by the following way:

\[
\mathcal{L} = \frac{g m_f}{2 m_W} F_S \bar{f} f S^0 + \frac{g m_f}{2 m_W} F_P \bar{f} \gamma_5 f P^0 \quad (A.1)
\]

In the Standard Model there is only one physical scalar \( (F_S = 1) \) and pseudoscalars are absent \( (F_P = 0) \).

In the MSSM, the Higgs sector contains two doublets of Higgs bosons with opposite hypercharge \( (Y = \pm 1) \).

After spontaneous symmetry breaking the following physical states appear: charged Higgs bosons \( H^\pm \), and three neutral ones, \( H_1^0, H_2^0, H_3^0 \).

At tree level the masses of scalars \( H_1^0, H_2^0 \) and an angle \( \alpha \) (which described the mixing of scalar states) are being expressed through the mass of pseudoscalar \( H_3^0 \) and \( \tan \beta = \frac{v_2}{v_1} \), where \( v_2, v_1 \) are both doublets vacuum expectations by following relationships:

\[
m_{H_1, H_2}^2 = \frac{1}{2} \left[ m_{H_3}^2 + m_Z^2 \pm \left( (m_{H_3}^2 + m_Z^2)^2 - 4m_Z^2 m_{H_3}^2 \cos^2 2\beta \right)^{1/2} \right] \quad (A.2)
\]

\[
\tan 2\alpha = \frac{m_{H_2}^2 + m_Z^2}{m_{H_3}^2 - m_Z^2} \tan 2\beta. \quad (A.3)
\]

It follows from (A1) that MSSM guaranties the existence of, at least, one light Higgs boson with \( m_{H_2} < m_Z \).

Interactions of the \( H_1^0 \)-bosons with fermions are described by lagrangian:

\[
\mathcal{L} = \frac{g m_d \cos \alpha}{2 m_W} d d H_1^0 + \frac{g m_d \sin \alpha}{2 m_W} s d H_2^0 + \frac{g m_d}{2 m_W} \tan \beta \bar{d} \gamma_5 d H_3^0 \quad (A.4)
\]

At \( \tan \beta \gg 1 \) the mass relation (A2),(A3) and formula (A4) are strongly reduced:

\[
m_{H_2} = m_{H_3}, m_{H_1} = m_Z, F_{H_2} = \tan \beta \gg F_{H_1} \text{at } m_{H_3} > m_Z, \quad (A.5)
\]
\[ m_{H_2} = m_{H_3}, m_{H_1} = m_{H_3}, F_{H_2} = \tan \beta \gg F_{H_1} \text{ at } m_{H_3} > m_Z. \] \hspace{1cm} (A.6)

It must be noted, that radiative corrections \([31]-[33]\) can strongly change relations (A1),(A2) however in the large \(\tan \beta\) limit and at \(m_{H_3} < m_Z\) or at \(m_{H_3} \gg m_Z\) formulas (A2),(A3),(A5) hold approximately true even after taking into account the radiative corrections.

We parametrized the model independent lagrangian of flavour changing scalar and pseudoscalar Higgs bosons \(H^0_{f \bar{c}} = S^0_{f \bar{c}}, P^0_{f \bar{c}}\) interaction with fermions by following way (see e.g. \([27]\)):

\[
\mathcal{L} = ih^{ij}_{S} \bar{f}_i f_j S^0_{f \bar{c}} + ih^{ij}_{P} \bar{f}_i \gamma_5 f_j P^0_{f \bar{c}} + H.c. \hspace{1cm} (A.7)
\]

Analogous interactions may also derive from \(R\)-parity braking part of the superpotential in the above mentioned theories with \(R\)-parity violation. In four-component Dirac notations \(R\)-parity violating interactions has the following form :

\[
\mathcal{L} = \lambda_{ijk}(\bar{l}^{\nu}_{i} j^{\nu}_{j} l^{R}_{k} + \text{neutrino interactions}) + H.c. \hspace{1cm} (A.8)
\]

where \(l^{i}, \bar{\nu}^{j}\) are charged lepton and scalar neutrino respectively.

sections (8).
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Figure 1: Diagrams corresponding to the processes (2),(3).

Figure 2: Number of Higgs bosons produced in the process (2) as a function of square root $s$ at fixed $m_H$. Curves 1,2,3 correspond to the $m_H = 70, 200, 500 GeV$ respectively.
Figure 3: Number of Higgs bosons produced in the process (3) as a function of square root $s$ at fixed $m_H$ at $h_{S,P}^{\mu e} = 10^{-3}$. Curves 1,2 correspond to the $m_H = 100,200 GeV$ respectively.
Figure 4: Number of Higgs bosons produced in the process (3) as a function of square root $s$ at fixed $m_H$ at $h_{S,P}^{\mu e} = 10^{-3}$. Curves 1,2 correspond to the $m_H = 100, 200 GeV$ respectively.