Adaptive Active Disturbance Rejection Control for Rendezvous of a Swarm of Drones

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ABSTRACT This paper addresses the time-varying rendezvous problem of a swarm of drones with the leader-follower consensus hierarchy. Each drone in the swarm is under the influence of external disturbances in the form of wind gusts. To control the swarm of perturbed drones, this paper proposes a fully distributed adaptive leader-following time-varying disturbance rejection pinning control for the rendezvous of drones using the framework of the extended state observer that depends on the state estimates of the neighboring agents rather than the actual states. The communication topology within the swarm is modelled using algebraic graph theory. Unlike most of the literature in which the leader agent act as a virtual reference generator, an active leader agent with non-zero control input is considered. The states of the leader drone are only available to a subset of the graph. The communication topology between leader-follower is directed while that between follower-follower is undirected. Based on the relative output information between the neighboring drones, the extended state observer estimates the local states and disturbances of each drone and the adaptive control actively compensates for the external disturbances simultaneously. Finally, it is shown analytically as well as in simulations that the time-varying rendezvous can be achieved by the proposed algorithm effectively.

INDEX TERMS Consensus, drone, extended state observer, rendezvous, wind gust.

I. INTRODUCTION

Aerial vehicles have attained substantial attention in recent years due to their enormous usage in industry and academia. The aerial vehicles can be used to perform search and inspection [1], surveillance and rescue [2], navigation and mapping [3], fire fighting [4], smart irrigation [5], state-of-the-art aerial photography [6], parcel delivery services [7], obstacle avoidance and path planning in urban environment [8], smart transportation [9] and military [10]. The extensive applications of aerial vehicles result from the maneuverability in three-dimensional space and the capability of vertical take-off and landing with almost zero carbon emission in most cases, which is environment friendly. Swarms of these vehicles, especially drones (quadrotor) are highly flexible, re-configurable, scalable and easy to maintain [11].

In the pioneering works [12], [13], the foundation of consensus control theory was laid and the effect of the control gains on the efficiency of multi-agent systems
follows

The main contributions of this work are summarized as state observer design, a pinning control-based adaptive wind gusts. Thus, inspired by the distributed extended affected by the uncertainties and disturbances such as the real flight, the flight operation of drones are adversely linearities. These assumptions and simplifications are far

integrator type [26] with simplified Lipschitz type non-

systems (MASs), but in most of the cases, the leader

is represented as a node (vertex), whereas, the adjacency

matrix represents the interconnections between the con-

nected neighboring drones. Some essential and necessary pre-

liminaries to understand the inter-drone connectivity using

graph topology for nonlinear multi-agent systems, the form-

ation control protocol for obstacle avoidance is presented in [21]. By considering the directed communication graph topology for nonlinear multi-agent systems, the for-

mation control protocol for obstacle avoidance is presented in [22]. For nonlinear multi-agent systems having Lipschitz type nonlinearities, a directed communication-based leader-

follower distributed tracking algorithm is presented in [23]. A leader-follower consensus tracking controller for linear time-invariant multi-agent systems with undirected communication between the agents having simplified disturbances is proposed in [24]. These control protocols are designed using a passive leader agent with zero control input or the dynamics of the leader are chosen as oscillator type, hence, the leader agent works as a virtual reference generator.

Although there is a lot of literature available on the consensus tracking control of leader-follower multi-agent systems (MASs), but in most of the cases, the leader is chosen as virtual reference generator and the intrinsic dynamics of the system are taken as sinusoidal [25] or integrator type [26] with simplified Lipschitz type nonlinearities. These assumptions and simplifications are far from real scenarios. It is worth mentioning that, during the real flight, the flight operation of drones are adversely affected by the uncertainties and disturbances such as wind gusts. Thus, inspired by the distributed extended state observer design, a pinning control-based adaptive leader-following time-varying disturbance rejection algorithm for the rendezvous of drones is proposed in this paper. The main contributions of this work are summarized as follows

- A pinning control-based adaptive leader-following time-varying disturbance rejection rendezvous protocol using distributed extended state observer framework is proposed that depends on the state estimates of the neighboring agents rather than the actual states. The designed algorithm is independent of graph topology and global information exchange between the drones. Any drone can leave or join the communication topology without affecting the overall response of the system. Thus, the designed control methodology can be classified as a fully distributed.
- By using the multiple Lyapunov function approach, the stability analysis is performed and algebraic Riccati inequalities (AREs) are obtained by the subsequent analysis. The controller and observer gains are obtained by solving the AREs according to the Schur’s complement.

The remainder of this paper is organized as follows. Section II presents problem formulation for the multi-drone system using the notions of algebraic graph theory. Section III presents an adaptive leader-following active disturbance rejection time-varying rendezvous protocol using the architecture of extended state observer. Section IV comprises simulation results as well as comparative analysis and finally, in section V, conclusions for the current work and future aspects of the designed algorithm are discussed.

II. PRELIMINARIES

Consensus requires some states of the system to follow a desired trajectory in space and time simultaneously. Usually, the consensus is achieved based on the output trajectories of the system and the connectivity between connected neighboring drones. The physical connectivity between the drones can be modelled by using the matrices and graph theory. In accordance with the algebraic graph theory [27], the dynamical multi-drone system can be represented as a graph (tree diagram) in which each drone is represented as a node (vertex), whereas, the adjacency matrix represents the interconnections between the connected neighboring drones. Some essential and necessary preliminaries to understand the inter-drone connectivity using algebraic graph theory are summarized in the following subsection.

A. NOTATIONS

Let the set of real numbers, the n dimensional Euclidean space and the set of n × m real matrices are denoted by \( \mathbb{R} \), \( \mathbb{R}^n \) and \( \mathbb{R}^{n \times m} \), respectively. Furthermore, \( \mathbf{0}_n \) and \( \mathbf{I}_n \) represent an n dimensional column vector having all entries as zero and one, respectively, whereas \( \mathbf{I}_n \) represent n × n identity matrix. \( \lambda_i \), \( \lambda_{\min} \) and \( \lambda_{\max} \) show the \( i^{th} \) eigenvalue, minimum eigenvalue and maximum eigenvalue, respectively. An n × n positive definite matrix is represented by \( M^{n \times n} > 0 \) and \( M^T \) indicate the transpose of matrix, whereas the Euclidean
norm is expressed as \( \| \cdot \|_k, \forall k \in 1, 2, \ldots, \infty \). Furthermore, the matrix \( M = [m_{ij}] \) is non-singular and \( \Re(\lambda_i(M)) > 0 \) if \( m_{ij} < 0, \forall i \neq j \).

### B. ALGEBRAIC GRAPH THEORY

This subsection recalls some essential results and basic definitions from the algebraic graph theory [28] that will be utilized to express the inter-drone connectivity in the form of matrices and graphs.

In a multi-drone system, each node (vertex) is represented by a nonempty finite set of \( N \) nodes i.e., \( \mathcal{V} = \{v_0, v_1, v_2, \ldots, v_N\} \) having the node index set as \( i = (0, 1, 2, \ldots, N) \). A set of edges show the connection between any two nodes represented by a tuple of vertices as \( E \subset \{(v_i, v_j) ; v_i, v_j \in \mathcal{V}\} \). The associated adjacency matrix of a graph lists all the edges and represented as \( \mathcal{A}_{ij} = [a_{ij}] \in \mathbb{R}^{N \times N} \). The neighborhood (i.e., connected neighboring nodes) of any node ‘i’ is represented by \( \mathcal{N}_i = \{j|(v_i, v_j) \in E\} \). Thus, the graph representing the inter-drone connectivity is represented as \( \mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A}) \). Each element of the adjacency matrix represent the weighted edge between any two vertices. The off-diagonal element of the adjacency matrix \( a_{ij} > 0 \) represent an inter-connection between two nodes and the connection (i.e., flow of information) start from node ‘j’ and terminate at node ‘i’, whereas, \( a_{ij} = 0 \) represent that the corresponding nodes are not connected. The diagonal elements of the adjacency matrix \( a_{ii} = 0, \forall (i, j) \in N \) represent that there exist no self-loops and repeated edges.

If the information flow is unidirectional from one node to the rest of the neighboring nodes, then the graph is classified as directed, whereas, if the information flow is bidirectional, then the graph is said to be undirected. For an undirected graph, the out-degree and in-degree matrices represent the number of outgoing and incoming edges from a node, respectively. The number of outgoing and incoming edges from a node are equal in the case of undirected graph topology, hence, the degree matrix for undirected graph is given as, \( \mathcal{D} = \text{diag}(d_i) \in \mathbb{R}^{N \times N} \), where \( d_i = \sum_{j \in N_i} a_{ij} \). The algebraic sum of the adjacency and degree matrices is mathematically expressed as \( \tilde{\mathcal{L}} = \mathcal{D} - \mathcal{A} \Rightarrow \tilde{\mathcal{L}} = [l_{ij}]^{N \times N} \), where \( \tilde{\mathcal{L}} \) denote the Laplacian matrix of the graph and \( l_{ii} = \sum_{j=1}^{N} a_{ij} \), \( l_{ij} = -a_{ij}, \forall i \neq j, \forall i = 1, 2, \ldots, N \). A graph is categorized as connected if there exist a connection between all the nodes of the graph, whereas, a graph is classified as strongly connected, if any node can be traversed from all of the remaining nodes. If there exist a directed connection between the nodes \( v_i \) and \( v_j \) and the directed connection start from node \( v_i \) and terminate at node \( v_j \), then the root of the graph tree is at the node \( v_i \) and this node corresponds to the leader drone. For the considered leader-following graph topology, the communication topology between leader and connected followers is directed which is represented by the subgraph \( \mathcal{G}_f \), whereas, the communication topology between the followers is undirected which is represented by the subgraph \( \mathcal{G}_f \). Thus, the overall communication graph containing the directed and undirected communication topologies can be written as \( \mathcal{G} = \mathcal{G}_f \cup \mathcal{G}_r \). The matrix \( \tilde{\mathcal{L}} \) associated to the subgraph \( \mathcal{G}_f \) representing the undirected communication can be written as

\[
\tilde{\mathcal{L}} = \begin{bmatrix}
a^*_{11} & -a_{12} & \cdots & -a_{1N} \\
-a_{21} & a^*_{22} & \cdots & -a_{2N} \\
\vdots & \ddots & \ddots & \vdots \\
-a_{N1} & \cdots & -a_{NN}
\end{bmatrix}
\]

(1)

where, \( a^*_{ij} = a_{12} + a_{13} + \cdots + a_{1N}, a^*_{ii} = a_{21} + a_{23} + \cdots + a_{2N} \) and \( a^*_{ij} = a_{N1} + a_{N2} + \cdots + a_{N,N-1} \). Furthermore, the matrix \( \mathcal{J} \) associated to the subgraph \( \mathcal{G}_f \) describing the directed connectivity between the leader and followers is given as

\[
\mathcal{J} = \begin{bmatrix}
f_1 & 0 & \cdots & 0 \\
0 & f_2 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & f_N
\end{bmatrix}
\]

(2)

where, \( f_j \) represent that there is a directed link between leader and the \( j^{th} \) drone. One may now write the overall communication topology as

\[
\mathcal{L} = \tilde{\mathcal{L}} + \mathcal{J}.
\]

(3)

Some assumptions and lemmas which are required to accomplish the stability analysis are given below.

**Assumption 1:** In the considered graph topology \( \mathcal{G} \), the leader drone is only connected to a few followers as defined by the graph subset \( \mathcal{G}_f \) and has directed paths to all the connected follower drones. There is no flow of information from the follower drones to the leader drone. The connection between the nodes \( v_l \) and \( v_l \) originates from node \( v_l \) and terminate at node \( v_l \) represent that the graph tree contain a root at node \( v_l \). The node \( v_l \) is the leader node. Only the partial states of the leader as governed by the output matrix of the leader are accessible to the followers present in the graph subset \( \mathcal{G}_f \), whereas, none of the followers receives the control input of the leader. The Laplacian matrix \( \mathcal{L} \) describe the overall graph topology in which \( \tilde{\mathcal{L}} \) represent the communication topology between followers and \( \mathcal{J} \) represent the directed connection between the leader and followers. Furthermore, the minimum eigenvalue of \( \mathcal{L} \) is defined as \( \gamma = \lambda_{\min}(\mathcal{L}) > 0 \) i.e., the smallest non-zero eigenvalue.

**Lemma 1:** Consider a dynamical system represented by the matrices \( \mathcal{A} \in \mathbb{R}^{n \times n}, \mathcal{A} \in \mathbb{R}^{(n+m) \times (n+m)}, \mathcal{B} \in \mathbb{R}^{n \times m}, \mathcal{B} \in \mathbb{R}^{(n+q) \times m}, C \in \mathbb{R}^{q \times n}, \mathcal{C} \in \mathbb{R}^{q \times (n+q)} \). According to the Popov-Belevitch-Hautus (PBH) lemma [29]

1. The dynamical systems defined by the pairs \( (\mathcal{A}, \mathcal{B}) \) and \( (\mathcal{A}, \mathcal{B}) \) are controllable if the controllability matrices \( C(\mathcal{A}, \mathcal{B}) \) and \( C(\mathcal{A}, \mathcal{B}) \) have full row rank i.e. \( n \) and \( n + m \), respectively. Moreover, the pairs \( (\mathcal{A}, \mathcal{B}) \) and \( (\mathcal{A}, \mathcal{B}) \) are stabilizable, if \( \Re(\lambda_i(\mathcal{A} - \lambda I_m)) \geq 0 \) and \( \Re(\lambda_i(\mathcal{A} - \lambda I_{(n+m)})) \geq 0 \).

2. Similarly, for the pairs \( (\mathcal{A}, \mathcal{C}) \) and \( (\mathcal{A}, \mathcal{C}) \) to be observable, the observability matrices \( O(\mathcal{A}, \mathcal{C}) \) and \( O(\mathcal{A}, \mathcal{C}) \) have full column rank i.e. \( n \) and \( n + m \), respectively.
Moreover, the pairs \((A, C)\) and \((\tilde{A}, \tilde{C})\) are detectable if,
\[
\text{Re}(A - \lambda J_i) \geq 0 \quad \text{and} \quad \text{Re}(\tilde{A} - \lambda I_{n+m}) \geq 0.
\]

**Lemma 2:** Consider a scalar \(\alpha\) and the matrices, \(A, B, C\) and \(D\) having the appropriate dimensions. Kronecker product of two matrices \(A^{mn}\) and \(B^{pq}\) is defined as
\[
A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1q}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mq}B \end{bmatrix}.
\]

Some of the properties of Kronecker product that are used in the subsequent analysis are given below
\begin{enumerate}
\item \(\alpha \otimes A = A \otimes \alpha = \alpha A\).
\item \(A^{mn} \otimes B^{pq} = C^{mp \times nq}\).
\item \((A \otimes B)^T = A^T \otimes B^T\).
\item \((A \otimes B)^{-1} = A^{-1} \otimes B^{-1}\).
\item \((A + B) \otimes C = A \otimes C + B \otimes C\).
\item \((AC) \otimes (BD) = (A \otimes B) \times (C \otimes D)\).
\end{enumerate}

**Lemma 3:** Consider the graph Laplacian matrix \(L\) as defined in (3) and define a vector such that \(\xi(t) = [\xi_1^T(t), \xi_2^T(t), \ldots, \xi_N^T(t)]^T\). Then, one may obtain \(L\xi(t) = \sum_{j=1}^N a_{ij} [\xi_j(t) - \xi_i(t)] + f_i(t), \forall i \in 1, 2, \ldots, N\). Proof: Since, \(L\) is obtained from undirected communication topology, one can easily verify that \(L\) is symmetric and \(L = LT^T\). Furthermore, from (2), one can see that \(\mathcal{J}\) is a diagonal matrix, hence, \(\mathcal{J}^T = \mathcal{J}\). Thus, \(\hat{L} + \mathcal{J} = [\hat{L} + \mathcal{J}]^T\). Since, \((i, j) \in 1, 2, \ldots, N,\) using (3), one may write
\[
L\xi(t) = \begin{bmatrix} a_{11} & -a_{12} & \ldots & -a_{1N} \\ -a_{21} & a_{22} & \ldots & \cdots \\ \vdots & \ddots & \ddots & \vdots \\ -a_{N1} & \cdots & -a_{NN} \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_N(t) \end{bmatrix}
+ \begin{bmatrix} f_1 & 0 & \cdots & 0 \\ 0 & f_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & f_N \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_N(t) \end{bmatrix}
\]
\[
\therefore L\xi(t) = \sum_{j=1}^N a_{ij} [\xi_j(t) - \xi_i(t)] + f_i(t)
\]

Thus,
\[
L\xi(t) = \sum_{j=1}^N a_{ij} [\xi_j(t) - \xi_i(t)] + f_i(t)
\]

Moreover, since \(L\) is symmetric \(\Rightarrow [L\xi(t)]^T = \xi^T(t)L\).

**Assumption 2:** The control inputs associated with each drone are bounded i.e. \(\sum_{i=0}^N \|u_i(t)\|_\infty \leq \varepsilon_1\). The external disturbances acting on each drone and the coupling gains between each drone are bounded and differentiable i.e. \(\sum_{i=0}^N \|z_i(t) + \tilde{z}_i(t)\|_\infty \leq \varepsilon_2 \Rightarrow \lim_{t \to \infty} \tilde{z}_i(t) \to 0 \Rightarrow \lim_{t \to \infty} \tilde{z}_i(t) \to 0\) and \(\sum_{i=0}^N \|\Gamma_i(t) + \tilde{\Gamma}_i(t)\|_\infty \leq \varepsilon_3 \Rightarrow \lim_{t \to \infty} \tilde{\Gamma}_i(t) \to 0 \Rightarrow \lim_{t \to \infty} \tilde{\Gamma}_i(t) \to 0\).

**III. PROBLEM FORMULATION**

The multi-agent system considered in this paper assumes a leader-follower hierarchy with \(N\) follower drones and one active leader drone. Each drone is represented as a node in the graph and the communication topology as well as the weighted connections between the neighboring drones are listed in the adjacency matrix. Each drone is subjected to external disturbances in terms of wind gusts. From [30], [31] and [32], one can obtain the dynamical model of the \(i^{th}\) drone in the standard state-space representation as
\[
\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + z_i(t)
\]
\[
y_i(t) = Cx_i(t) + Du_i(t)
\]

where, \(\forall i \in (0, \ldots, N), A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{q \times n}, D \in \mathbb{R}^{q \times m}\) are the system, input, output and feedback matrices, respectively. \(u_i(t) \in \mathbb{R}^{n \times 1}\), \(x_i(t) \in \mathbb{R}^{n \times 1}\) and \(y_i(t) \in \mathbb{R}^{q \times 1}\) are the control, state and output vectors, respectively. \(z_i(t) \in \mathbb{R}^{m \times 1}\) is the vector of time-varying external disturbances exerted on the system i.e., wind gusts. It is worthy to mention that the system represented by (5) is a general linear system, which usually may consist of any first order, second order and higher order integrator type dynamics. Unlike most of the existing literature in which the leader is taken only as a virtual reference generator with zero control input, in this paper an active leader under non-zero control input is considered. Let the index \(i = 0\) represent the leader drone, then one can write (5) as follows
\[
\dot{x}_0(t) = Ax_0(t) + Bu_0(t) + z_0(t)
\]
\[
y_0(t) = Cx_0(t) + Du_0(t)
\]

where the leader control input \(u_0(t)\) and the applied external disturbances \(z_0(t)\) are bounded and differentiable according to Assumption 2. Let the overall state vector, control vector and disturbances vector for all the follower drones is defined as \(x(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T\), \(u(t) = [u_1^T(t), u_2^T(t), \ldots, u_N^T(t)]^T\) and \(z(t) = [z_1^T(t), z_2^T(t), \ldots, z_N^T(t)]^T\), respectively. Now, for \(N\) follower drones, using lemma 2, (5) can be expressed in concise equation as
\[
\dot{x}(t) = (IN \otimes A)x(t) + (IN \otimes B)(u(t) + z(t))
\]

By incorporating the estimate of external disturbances, the observer dynamics for the \(i^{th}\) follower drone are defined as
\[
\dot{\hat{x}_i}(t) = A\hat{x}_i(t) + Bu_i(t) + \hat{z}_i(t)
\]

where \(\hat{z}_i(t)\) is a vector of the estimate of external disturbances. Now define the overall state estimation vector and disturbance estimation vector as \(\hat{x}(t) = [\hat{x}_1^T(t), \hat{x}_2^T(t), \ldots, \hat{x}_N^T(t)]^T\) and \(\hat{z}(t) = [\hat{z}_1^T(t), \hat{z}_2^T(t), \ldots, \hat{z}_N^T(t)]^T\), respectively. Then using lemma 2, one can compactly write (8) as follows
\[
\dot{\hat{x}}(t) = (IN \otimes A)\hat{x}(t) + (IN \otimes B)(u(t) + \hat{z}(t))
\]

Equations (7) and (9) are the overall representation of the system and observer dynamics, respectively.

**IV. EXTENDED STATE OBSERVER-BASED DISTRIBUTED ADAPTIVE CONTROLLER WITH ACTIVE DISTURBANCE REJECTION**

In this section, inspired by the methodology of extended state observer design, an adaptive control for disturbance estimation and rejection is proposed. A disturbance observer estimates the disturbances acting on the system based on the
relative state information sharing between followers and the limits on the external disturbances [33]. Moreover, the disturbance estimates are integrated into the controller such that the disturbances are estimated and rejected simultaneously [34].

In order to proceed with the proposed distributed extended state observer design, the disturbance vector \( \zeta(t) \) is incorporated in the state vector of the system (7) as

\[
\dot{x}(t) = \begin{bmatrix} \dot{x}(t) \\ \zeta(t) \end{bmatrix} = I_N \otimes \begin{bmatrix} A & B \\ 0_{m \times n} & 0_m \end{bmatrix} \begin{bmatrix} x(t) \\ \zeta(t) \end{bmatrix} + I_N \otimes \begin{bmatrix} B \\ 0_m \end{bmatrix} u(t) + I_N \otimes \begin{bmatrix} 0_{m \times n} \\ I_m \end{bmatrix} \delta(t)
\]

where, \( \zeta(t) = \delta_i(t) \). Thus, one may obtain the extended state-space description of compact system (7) as

\[
\dot{x}(t) = I_N \otimes (\tilde{A} x(t) + \tilde{B} u(t) + E \delta(t))
\]

where, \( \tilde{A} \in \mathbb{R}^{(n+m) \times (n+m)}, \tilde{B} \in \mathbb{R}^{(n+q) \times (m)}, \tilde{C} \in \mathbb{R}^{(q) \times (n+q)} \) are the system, input and output matrices of the extended state-space, and \( E \in \mathbb{R}^{(n+m) \times (m)} \). The dynamics of the perturbed system are represented in (7) and the observer in (9).

These both equations contain the disturbance terms and the difference between the state vector and the observer vector is due to the appearance of disturbances, hence, the observer dynamics in the extended state-space can be obtained by incorporating the disturbance estimation vector \( \hat{\zeta}(t) \) in (9).

The dynamics of the disturbance estimate for the \( i \)-th drone dependent on its own states and the states of the connected neighboring drones are given as

\[
\dot{\hat{\zeta}}_i(t) = HC \left( \sum_{j=1}^{N} a_{ij} [x_i(t) - x_j(t)] - \left[ \hat{x}_i(t) - \hat{x}_j(t) \right] + \hat{\chi}_i(t) \right)
\]

where, \( HC \in \mathbb{R}^{(n+m) \times (q)} \) represent the observer gain matrix. For \( i \in 1, 2, \ldots, N \), one may expand (11) as

\[
\dot{\hat{\zeta}}_i(t) = \begin{bmatrix} \hat{\zeta}_1(t) \\ \hat{\zeta}_2(t) \\ \vdots \\ \hat{\zeta}_N(t) \end{bmatrix} = HC 0 \ldots 0 = \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \\ \vdots \\ \hat{x}_N(t) \end{bmatrix} - \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix}
\]

where, \( \hat{\chi}_i(t) = a_{i1}^* \chi_1 + a_{i2}^* \chi_2 + \ldots + a_{in}^* \chi_n \) and \( a_{i1}, a_{i2}, \ldots, a_{in} \) are defined in (1). Let the state estimation error is described as \( \xi_i(t) = \hat{x}(t) - x(t) \). Using lemma 2, it is possible to compactly write (12) as

\[
\dot{\hat{\zeta}}_i(t) = (L \otimes HC) \xi_i(t) \tag{13}
\]

Thus, using (13), the observer dynamics (9) in the extended state-space form can be formulated as

\[
\dot{x}(t) = I_N \otimes \begin{bmatrix} A & B \\ 0_{m \times n} & 0_m \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{\zeta}(t) \end{bmatrix} + I_N \otimes \begin{bmatrix} B \\ 0_m \end{bmatrix} u(t) + L \otimes \begin{bmatrix} 0_n \\ 0_m \end{bmatrix} \xi_i(t)
\]

where, \( L \in \mathbb{R}^{(m+q) \times (m+q)} \). Now a distributed adaptive control law \( u_i(t) \) is proposed for each follower drone that depends upon the states of the observer of the connected neighboring drones, the leader states, the coupling gains between drones and the disturbance estimate. The control algorithm actively estimate and reject the external disturbances and simultaneously update the coupling gains between the drones as follow

\[
u_i(t) = \Gamma_i K \left( \sum_{j=1}^{N} a_{ij} \left[ \hat{x}_i(t) - \hat{x}_j(t) \right] + \hat{\chi}_i(t) \right) - x_0(t) + \Gamma_i \text{sgn} \left( K \sum_{j=1}^{N} a_{ij} \hat{x}_i(t) - \hat{x}_j(t) \right) - \hat{\chi}_i(t) \tag{15}
\]

where, \( K \in \mathbb{R}^{m \times n} \) represent the controller gain matrix and the dynamics of adaptive coupling gains are given as

\[
\dot{\Gamma}_i(t) = \left[ \sum_{j=1}^{N} a_{ij} [\hat{x}_i(t) - \hat{x}_j(t)] + \hat{\chi}_i(t) - x_0(t) \right] + \left[ \sum_{j=1}^{N} a_{ij} [\hat{x}_i(t) - \hat{x}_j(t)] + \hat{\chi}_i(t) - x_0(t) \right]
\]

where, \( \Gamma_i(t) \in \mathbb{R}^{m \times n} \) represent the adaptive coupling gain of the respective follower, \( \varphi \in \mathbb{R}^{n \times n} \) is defined in theorem 1 and \( a_{ij} \) are the elements of adjacency matrix. The following theorem assists in the closed-loop analysis of the multi-drone system and finding the controller and observer parameters.

**Theorem 1:** Consider the drone swarm system (5) under the designed extended state observer (14). If the Assumptions 1 and 2 are satisfied then the leader-following time-varying rendezvous for position consensus can be achieved by the adaptive control (15) with active disturbance estimation and rejection controller, simultaneously. The controller and observer design parameters are defined as \( K = -\rho B^T Q^{-1} \),
The Lyapunov function defined in (23) is quadratic, hence, $V_o(t) \geq 0, \forall t \geq 0$. Taking the time derivative along the trajectories of the system (21), one may have

$$
\dot{V}_o(t) = 2\xi_\sigma^T(t)(L \otimes P^{-1})(I_N \otimes \tilde{A} + L \otimes H\tilde{C})\xi_\sigma(t) - (I_N \otimes E)\delta(t)
$$

(24)

Let there be a change of variables in error-space defined as

$$
\tilde{\xi}_\sigma(t) = (I_N \otimes P^{-1})\xi_\sigma(t).
$$

(25)

Substituting $H = -P^{-1}\tilde{C}$ and using the assumption 2, one may write

$$
V_o(t) = \tilde{\xi}_\sigma^T(t)(L \otimes (P\tilde{A} + \tilde{A}^T P) - 2L^2 \otimes \tilde{C}\tilde{C})\tilde{\xi}_\sigma(t)
$$

(26)

+ 2\|\tilde{\xi}_\sigma(t)(L \otimes E)\|_1\|\delta(t)\|_\infty

using the assumption 1 and lemma 2, one may obtain

$$
\dot{V}_o(t) \leq \gamma \tilde{\xi}_\sigma^T(t)(P\tilde{A} + \tilde{A}^T P - 2\gamma \tilde{C}\tilde{C})\tilde{\xi}_\sigma(t)
$$

(27)

Thus, solving (27) for $P > 0$ such that $P\tilde{A} + \tilde{A}^T P - 2\gamma \tilde{C}\tilde{C} < 0$, the Lyapunov function $\dot{V}_o(t)$ can be rendered as negative definite. Now define the state tracking error between leader and follower drones as $\xi_i(t) = x_i(t) - x_0(t)$. Taking the time derivative and substituting (5), (6) and (15), one may have

$$
\dot{\xi}_i(t) = Ax_i(t) + B\Gamma_i(k_i \sum_{j=1}^N a_{ij}[\hat{x}_j(t) - \hat{\xi}_j(t)])
$$

(29)

$$
+ J_i[\hat{x}_i(t) - x_0(t)]
$$

Similarly, $\dot{\xi}_i(t) - x_0(t) + x_i(t) = [\hat{x}_i(t) - x_i(t)] + [x_i(t) - x_0(t)]$, one may obtain

$$
\dot{\xi}_i(t) - x_0(t) = \xi_{\epsilon i}(t) + \xi_\sigma(t)
$$

(30)

Thus, using (29) and (30), one can write (28) as

$$
\dot{\xi}_{\epsilon i}(t) = A\xi_{\epsilon i}(t) - B\xi_{\epsilon i}(t) - B[u_0(t) + \xi_\sigma(t)]
$$

+ $B\Gamma_i(k_i \sum_{j=1}^N a_{ij}[\xi_{\epsilon j}(t) - \xi_{\epsilon j}(t) + \xi_{\epsilon j}(t) - \hat{\xi}_j(t)])$

$$
+ J_i[\xi_{\epsilon i}(t) - \xi_{\epsilon i}(t)]
$$

+ $B\xi_{\epsilon i}(t) - B[\xi_\sigma(t) - \xi_\sigma(t)]$

(31)

Let the vector of joint estimation errors dynamics is defined as $\xi_\sigma(t) = [\xi_\sigma^T(t), \xi_\sigma^T(t)]^T$ and augmenting (19) and (20), one may obtain

$$
\dot{\xi}_\sigma(t) = \begin{bmatrix}
\dot{\xi}_\eta(t) \\
\dot{\xi}_\xi(t)
\end{bmatrix} = \begin{bmatrix} A \\ 0_{m \times n} \end{bmatrix} \begin{bmatrix}
\xi_\eta(t) \\
\xi_\xi(t)
\end{bmatrix}
$$

$$
+ \begin{bmatrix} L \otimes H \begin{bmatrix} C & 0_{n \times m} \end{bmatrix} \\ 0_{n \times n} \end{bmatrix} \begin{bmatrix}
\xi_\eta(t) \\
\xi_\xi(t)
\end{bmatrix} - I_N \otimes \begin{bmatrix} 0_{n \times m} \\ I_m \end{bmatrix} \delta(t)
\end{bmatrix}
$$

$$
\dot{\xi}_\sigma(t) = (I_N \otimes \tilde{A} + L \otimes H\tilde{C})\xi_\sigma(t) - (I_N \otimes E)\delta(t)
$$

(21)

Now consider the following composite quadratic Lyapunov function dependent on the joint estimation errors and tracking error as

$$
V(t) = V_o(t) + V_{\epsilon}(t)
$$

(22)

The Lyapunov function $V_o(t)$ dependent on the joint estimation errors dynamics is designed as

$$
V_o(t) = \xi_\sigma^T(t)(L \otimes P^{-1})\xi_\sigma(t)
$$

(23)
For $i \in 1, 2, \ldots, N$ follower drones, one may write (31) in augmented matrix form as below
\[
\begin{bmatrix}
\ddot{\xi}_1(t) \\
\ddot{\xi}_2(t) \\
\vdots \\
\ddot{\xi}_N(t)
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
\xi_{1N}(t) \\
\xi_{2N}(t) \\
\vdots \\
\xi_{NN}(t)
\end{bmatrix}
+ \begin{bmatrix}
A \\
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
\xi_{1N}(t) \\
\xi_{2N}(t) \\
\vdots \\
\xi_{NN}(t)
\end{bmatrix}
\] 

where $\tilde{a}_1 = a_1^a + J_1, \tilde{a}_2 = a_2^a + J_2, \ldots, \tilde{a}_N = a_N^a + J_N$ and $a_1^a, a_2^a, \ldots, a_N^a$ are the diagonal elements of the matrix defined in (1). Thus, using lemma 2, one may express the augmented system in compact representation as follow
\[
\begin{bmatrix}
\ddot{\xi}_1(t) \\
\ddot{\xi}_2(t) \\
\vdots \\
\ddot{\xi}_N(t)
\end{bmatrix}
= \begin{bmatrix}
\xi_{1N}(t) \\
\xi_{2N}(t) \\
\vdots \\
\xi_{NN}(t)
\end{bmatrix}
= \begin{bmatrix}
\begin{bmatrix}
A \\
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
\xi_{1N}(t) \\
\xi_{2N}(t) \\
\vdots \\
\xi_{NN}(t)
\end{bmatrix}
\] 

Furthermore, using (29) and (30), one may write (16) as
\[
\dot{\Gamma}_i(t) = \sum_{j=1}^{N} a_{ij} [\xi_{ij}(t) - \xi_{ij}(t) - \xi_{ij}(t) + \xi_{ij}(t)]
+ \eta_i [\xi_{ij}(t) + \xi_{ij}(t)]
\] 

Now another composite Lyapunov function $V_\tau(t)$ is defined that is dependent on the adaptive coupling gain associated to each follower drone and the tracking error between the leader and follower drones and as below
\[
V_\tau(t) = \xi_\tau^T(t)(L \otimes Q^{-1})\xi_\tau(t) + \sum_{i=1}^{N} (\Gamma_i(t) - \epsilon)^2
\] 

It is elementary to verify intuitively that the Lyapunov function $V_\tau(t)$ is quadratic and always positive semi-definite. Hence, $V_\tau(t) \geq 0, \forall t \geq 0$. Now taking the derivative of (34) along the trajectories of the system (32) and (33), one may have
\[
\dot{V}_\tau(t) = 2 \xi_\tau^T(t)(L \otimes Q^{-1}A + L\Gamma(t)L \otimes Q^{-1}B)K\xi_\tau(t)
+ 2 \xi_\tau^T(t)L\Gamma(t) + Q^{-1}B \text{sgn}(L \otimes K)\xi_\tau(t)
+ (L \otimes K)\xi_\tau(t) - 2 \xi_\tau^T(t)L \otimes Q^{-1}B(\eta_0(t) + \xi_\tau(t))
+ 2 \xi_\tau^T(t)L \otimes Q^{-1}B(\eta_0(t) + \xi_\tau(t)) + 2 \xi_\tau^T(t)
\] 

Let there be a transformation in error dynamics defined as
\[
\tilde{\xi}_\tau(t) = (I_N \otimes Q^{-1})\xi_\tau(t)
\] 

Substituting $K = -B^TQ^{-1}$ and $\varphi = Q^{-1}BB^TQ^{-1}$. Moreover, for a vector $z \in \mathbb{R}^{n \times 1}$, one may write, $z = \text{sgn}(z)|z| \Rightarrow \|z\|_1 = |z|$, hence, $z^T \text{sgn}(z) = \|z\|_1$ [35]. Thus, using lemma 3 and the transformation (36), one may write (35) as
\[
\tilde{V}_\tau(t) = \tilde{\xi}_\tau^T(t)(L \otimes (Q \Gamma + A \Omega))\tilde{\xi}_\tau(t) + 2 \eta_0(t) + \xi_\tau(t)\|_{\infty}
\]
In compact form (38) can be written as

\[
\begin{align*}
-2\epsilon & \left\| (L \otimes B^T) \tilde{\zeta}_1(t) \right\|_1 \nonumber + \tilde{\zeta}_1^T(t)(L \otimes B) \tilde{\zeta}_1(t) \\
& + \tilde{\zeta}_1^T(t) \left( L \otimes B^T \right) \tilde{\zeta}_1(t)
\end{align*}
\] (37)

Since, \(|u_0(t)|_\infty \leq \epsilon_1 \), \(|\xi_0(t)|_\infty \leq \epsilon_2 \) and \(|\Gamma|_\infty \leq \epsilon_3 \Rightarrow \|\epsilon_1 + \epsilon_2 + \epsilon_3\|_\infty \leq \epsilon \). Thus, by selecting \( \epsilon > (\epsilon_1 + \epsilon_2 + \epsilon_3) \) and using the lemma 2 and assumptions 1 and 2, one may obtain

\[
\dot{V}_r(t) \leq \gamma \tilde{\zeta}_1^T(t)(QA^T + AQ - 2\epsilon \gamma BB^T)\tilde{\zeta}_1(t) \\
- \tilde{\zeta}_1^T(t)(\gamma \epsilon_3 BB^T)\tilde{\zeta}_1(t) - \tilde{\zeta}_1^T(t)(\gamma \epsilon_3 BB^T)\tilde{\zeta}_1(t) \\
- \tilde{\zeta}_1^T(t)(2(\epsilon - \epsilon_3)\gamma^2 BB^T)\tilde{\zeta}_1(t)
\] (38)

In compact form (38) can be written as

\[
\dot{V}_r(t) \leq \left[ \begin{array}{c} \tilde{\zeta}_1(t) \\ \tilde{\zeta}_2(t) \\ \tilde{\zeta}_3(t) \end{array} \right]^T \Theta \left[ \begin{array}{c} \tilde{\zeta}_1(t) \\ \tilde{\zeta}_2(t) \\ \tilde{\zeta}_3(t) \end{array} \right]
\] (39)

where

\[
\Theta = \begin{bmatrix} 0 & 0 & \gamma B \\ 0 & -2\gamma^2(\epsilon - \epsilon_3)BB^T - \gamma \epsilon_3 BB^T & \gamma \Theta \\ \gamma BB^T & -\gamma \epsilon_3 BB^T & \gamma \Theta \end{bmatrix}
\]

and

\[
\tilde{\Theta} = QA^T + AQ - 2\epsilon \gamma BB^T < 0
\] (40)

Solving (40) for \( Q > 0 \), such that \( \tilde{\Theta} \leq 0 \) \( \Rightarrow \Theta \leq 0 \) \( \Rightarrow \dot{V}_r(t) \leq 0 \). Thus, from (27) and (40), it can be concluded that the time derivative of (22) is negative semi-definite. This completes the proof.

Remark 1: The condition for the observability and controllability for the tuple \((A, C)\) and \((A, B)\) is necessary to compute the observer and controller gains. Moreover, for the solvability of the LMI (17), the necessary condition requires the tuple \((A, C)\) to be detectable and for the LMI (18), the tuple \((A, B)\) to be stabilizable. Thus, the Schur complement lemma [36], solving the LMIs to find the distributed adaptive extended state disturbance rejection observer based controller parameters, the closed-loop trajectories of the multi-drone system are convergent and asymptotically stable, i.e. the error trajectories

\[
\lim_{t \to \infty} (|\tilde{\xi}_1(t)|, |\tilde{\xi}_2(t)|, |\tilde{\xi}_3(t)|) \to 0.
\]
A static position reference signal for the figures 3–8 is given by a sigmoid function with initial values represented in Cartesian coordinates as $(x_i(0), y_i(0)) = [(0, 0), (0.15, 0.15), (-0.15, -0.15), (0.3, 0.3), (-0.3, -0.3), (0.45, 0.45), (-0.45, -0.45), (0.6, 0.6), (-0.6, -0.6), (0.75, 0.75), (-0.75, -0.75), (0.9, 0.9)], \forall i \in 0, 1, \ldots, 11$ and final values as $(\psi(6), x(1), y(3), z(0.1))^T = \left[\frac{\pi}{4}, 1, 1, 3\right]^T$. A block diagram to represent the overall communication hierarchy of the multi-drone system architecture is shown in Fig. 2. Each drone is subjected to external disturbances $\zeta_i(t)$ in terms of wind gusts. The time-varying reference trajectory $X_d(t)$ is given to the leader drone only and all the drones transmit their states $(X, \dot{X})_i(t)$ to the communication network and receive the relative states of the neighbors $(X, \dot{X})_N(t)$ that are decided by (1). Since, $\tilde{L}$ is symmetric, thus, $a_{ij} = a_{ji}$. For the communication topology shown in Fig. 1, the matrix $\tilde{L} \in \mathbb{R}^{12 \times 12}$, hence, only the non-zero elements are mentioned here i.e., $a_{12}, a_{14}, a_{15}, a_{23}, a_{24}, a_{34}, a_{45}, a_{47}, a_{56}, a_{67}, a_{610}, a_{78}, a_{79}, a_{710}, a_{89}, a_{911}, a_{1011} = 1$. Moreover, the pinning matrix defining the directed connection between leader and followers is defined as $\mathcal{J} = \text{diag}(1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)$. 

**FIGURE 3.** Linear and angular positions for a static reference input.

**FIGURE 4.** Linear and angular velocities for a static reference input.

**FIGURE 5.** RMSE of disturbance estimation errors for a static reference input.

**FIGURE 6.** RMSE of state estimation errors for a static reference input.

**FIGURE 7.** RMSE of tracking errors between leader and followers for a static reference input.

**FIGURE 8.** Adaptive coupling gains associated to each drone for a static reference input.
Each drone has three subsystems, in which the (10) is implemented in the system block and (14) is implemented in the observer block, whereas (15), (16) are implemented in the controller subsystem. The use of $\rho$ provides an extra bit of leverage to improve the transient performance and in this case it is set to 1. Solving the linear matrix inequalities (17) and (18) according to algorithm 1, and setting $\gamma = \lambda_{\min}(L) = 0.5$ and $\epsilon \geq \|\epsilon_1 + \epsilon_2 + \epsilon_3\|_{\infty} = 10$, one may obtain the controller gain matrix $K \in \mathbb{R}^{4 \times 12}$ and the extended observer gain matrix $H \in \mathbb{R}^{16 \times 4}$ as below:

$$K = \begin{bmatrix}
0 & -1.0216 & 0.0062 & 0 \\
0 & 0.0050 & -1.0401 & 0 \\
0 & 0 & 0 & -1.0178 \\
0 & 7.2278 & 9.6069 & 0 \\
0 & -9.2660 & 2.1265 & 0 \\
-4.1186 & 0 & 0 & 0 \\
0 & -15.876 & 1.5677 & 0 \\
0 & 1.4977 & -17.938 & 0 \\
0 & 0 & 0 & -5.2643 \\
0 & 7.6703 & 12.676 & 0 \\
0 & -12.188 & 4.7140 & 0 \\
-7.5358 & 0 & 0 & 0
\end{bmatrix}^T$$

$$H = \begin{bmatrix}
0 & -217.47 & 449.39 & 0 \\
0 & -446.35 & -273.84 & 0 \\
528.86 & 0 & 0 & 0 \\
0 & 216.38 & 57.634 & 0 \\
0 & 53.865 & 275.03 & 0 \\
0 & 0 & 0 & 13.114 \\
0 & -95.460 & 152.46 & 0 \\
0 & -153.59 & -66.024 & 0 \\
55.557 & 0 & 0 & 0 \\
0 & 26.375 & 3.2228 & 0 \\
0 & 3.2228 & 30.099 & 0 \\
0 & 0 & 0 & 10.327 \\
0 & 0 & 0 & 4.9725 \\
0 & -0.6042 & 1.6274 & 0 \\
0 & -1.4895 & -1.0488 & 0 \\
12.097 & 0 & 0 & 0
\end{bmatrix}$$

In this work, the external disturbance model is chosen as the wind gust model that is an integration of mean wind velocity, shear effect model, continuous Dryden turbulence model and horizontal wind model [38]. Thus, the wind gust model is more realistic in order to mimic the actual world scenario since it is not sinusoidal and undeterministic. The linear and angular position trajectories of 12 drones are shown in Fig. 3, whereas, Fig. 4 show the linear and angular velocities. The root mean squared errors (RMSEs) of the disturbance estimation error, state estimation error and tracking error are calculated as, $RMSE_{\xi(t)} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\hat{\xi}(t) - \xi(t))^2}$,
RMSE_{ξ_i}(t) = \sqrt{\frac{1}{\Delta} \sum_{i=1}^{N} (\hat{x}_i(t) - x_i(t))^2}$ and $RMSE_{η_i}(t) = \sqrt{\frac{1}{\Delta} \sum_{i=1}^{N} (\hat{\eta}_i(t) - \eta_i(t))^2}$, respectively, where $\Delta$ is the number of time samples. The RMSE of these error trajectories are shown in Fig. 5, Fig. 6 and Fig. 7, respectively. Fig. 8 show the time-varying adaptive coupling gains associated with each follower drone.

In addition to static reference signal, a more realistic, practical and dynamic 3D trajectory is designed in which the desired time-varying position reference input is given as $X_d = [0, 10, 10\cos(0.25t), 10\sin(0.5t), 3]^T$. Each drone is set at different initial positions represented in Cartesian coordinates as $(x_i(0), y_i(0)) = [(10, 5), (8, 3), (12, 0), (13, -1), (12, 1), (5, 4), (5, 0), (0, 0), (4, -4), (6, -6), (3, -6), (5, -8)], \forall i \in 0, 1, \ldots, 11$ as shown by the ground level markers $X_i$ at altitude $z_i = 0$. Furthermore, the final position of each drone is set 1 meter apart from its neighboring drone to keep the snake/chain formation topology as shown in Fig. 13. The trajectory is $(\infty)$ shaped with center at $(0, 0)$ and $(x_{min}, y_{min}, z_{min}) = (-10, -10, 0)$ and $(x_{max}, y_{max}, z_{max}) = (10, 10, 3)$. The leader and the $n^{th}$ follower drone is connected to only 1 drone and all other drones are connected to 2 immediate neighbors to maintain this formation as shown by the markers at $z_i = 3$. Moreover, the root mean squared errors (RMSEs) of the disturbance estimation error, state estimation error and tracking error are shown in Fig. 9, Fig. 10 and Fig. 11, respectively, whereas, Fig. 12 show the time-varying adaptive coupling gains associated with every follower drone. The considered wind gust model is state dependent and hence the effect of wind disturbances is larger for a dynamic reference trajectory. It can be seen from the 3D output trajectories that even for the dynamic input reference, the transient and steady state performance of the system is smooth without large overshoots under the adaptive protocol.

In addition, a comparative analysis is provided using non-adaptive coupling gains associated with every follower. For the figures 14–17, the coupling gains are chosen as $\Gamma = [1, 3, 5, 10]$. The root mean squared errors (RMSEs) of the disturbance estimation error, state estimation error and tracking error are shown in Fig. 14, Fig. 15 and Fig. 16,
respectively. It can be seen that the magnitude of these errors is larger as compared to the adaptive case. Finally, Fig. 17 show the 3D-tracking for different values of fixed coupling gains. It can be noted that there remain significant errors in the case of fixed coupling gains, whereas, the tracking results using adaptive control protocol even under the effect of external disturbances are quite efficient.

Remark 2: Typically miniature drones are not equipped with processing modules that can compute high-end vision and learning-based computations and requires a base station for these computations. Moreover, the commonly available miniature drones have altitude sensors such as flow deck, sonar etc to estimate the height and one visual sensor i.e., camera mounted on the front end for navigation and obstacle avoidance. To estimate the distance between connected miniature drones, a Lidar cannot be used due to its bulky size and power requirement. Thus, each miniature drone must be equipped with at-least four sensors to estimate the lateral distance between the connected neighbors. Our proposed algorithm can be applied in the real world scenario using parallel ultra-low power shield (PULP [39]) in combination with flow-deck (altitude sensor [40]) and multi-ranger deck (front, back, right, left and upside distance sensor [41]). By combining these modules, the miniature drone becomes a fully-autonomous platform and enables us to perform sensor fusion and track the reference trajectory while maintaining the desired formation. Furthermore, integrating the PULP shield and MCUNET [42] module with the miniature drones, it is possible to implement the vision and learning-based computationally complex algorithms on-board. Thus, the miniature drones can operate in highly cluttered urban environment full-autonomously.

VI. CONCLUSION AND FUTURE WORKS

In this paper, the leader-follower time-varying rendezvous problem of a swarm of drones under the influence of external disturbances is investigated, where a more realistic external disturbances model in terms of wind gusts that is closer to the actual scenario is applied to each drone. The wind model is chosen as an integration of mean wind velocity, shear effect model, continuous Dryden turbulence model and horizontal wind model. The proposed distributed adaptive active disturbance rejection controller achieves the underlying problem by simultaneously estimating and rejecting the external disturbances based on an ESO. The controller and observer gains are calculated based on the sufficient conditions that are formed as LMI and then are tractable. Furthermore, depending on the relative state information between corresponding neighboring drones, the coupling gains associated with each follower drone are designed to be adaptive to improve the transient and steady-state response of the trajectory tracking.

An interesting future aspect is to investigate a leader-follower hierarchy having multiple leaders and followers with heterogeneous dynamics. For the space missions, the algorithm can also be tested for a swarm of CubeSat. Furthermore, in order to make the communication between the drones more secure, blockchain technology can be utilized.

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