Lower Order Approximation of Bounded-Parameter Uncertain Systems using Amplitude Matching and Whale Optimization Algorithm with Constriction Factor

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Abstract. Lower order modelling of uncertain bounded systems is presented in this paper using amplitude matching and Whale Optimization Algorithm (WOA) with Constriction Factor (CF). The amplitude matching technique ensures to retain the stability and dynamics of the original higher-order systems. The proposed optimization algorithm for minimum Integral Square Error (ISE) approximates the model so that the responses match closely. The results are validated by comparing the ISE values of existing models. The presented technique is applicable for both Linear Time Invariant Continuous Systems (LTICS) and Linear Time Invariant Discrete Systems (LTIDS) without any need for tedious calculation of domain transformation.

Keywords: Uncertain- Interval System – Order Reduction -LTICS- LTIDS – Amplitude Matching- Whale Optimization Algorithm (WOA) – Constriction Factor (CF)

1. Introduction

Many engineering applications are uncertain or imprecision systems in nature. Based on the causes and characterization of the uncertainty, they can be demonstrated as probabilistic, fuzzy or interval models. The real time situations like ambiguous measurement, external instabilities and aging which ensue in parametric uncertainties are dealt with examples in [1] [2]. Such uncertain structural systems can be conducted and denoted using interval analysis in which the imprecision parameters are represented by a simple range [3].

The analysis of interval systems is established by using classical approaches such as Routh criterion, Nyquist and Root locus methods. The model order reduction of such systems is realized in many practical applications in the engineering fields such as heat transfer, flow control, circuit analysis, aerodynamics and so on. For the lower order modelling of interval systems, Routh Pade approximation [4], \(\gamma-\delta\) Routh Approximations [5], Dominant pole retention methods [6], differentiation methods [7] and other mixed methods [8], [9] and [10] have been developed.

In this paper, model reduction of interval systems is formulated by employing the amplitude matching technique which has been developed for fixed parameter systems [11] [12] and the Whale Optimization Algorithm with Constriction Factor [13].
2. Proposed Method - LTIDS

If $R$ is a real number field, then the closed interval represented by $[a, \bar{a}]$ is the set of real numbers bounded between upper and lower limits as $[a, \bar{a}] = \{a \in \mathbb{R}; a \leq a \leq \bar{a}\}$. The stable original nth order uncertain interval LTIDS is represented by

$$\left[ G(z), \overline{G}(z) \right] = \left[ \frac{N(z)}{D(z)}, \frac{\overline{N}(z)}{\overline{D}(z)} \right]$$

where $m < n$,$\sum_{i=0}^{m} \frac{a_i}{a_i}$ (0 $\leq i \leq m$) and $\sum_{i=0}^{n} \frac{\overline{a}_i}{\overline{a}_i}$ (0 $\leq i \leq n$) are scalar values; $N(z)$ is the numerator polynomial; $D(z)$ is the denominator polynomial. Assuming the reduced second-order uncertain but bounded LTIDS to be derived as,

$$\left[ \frac{R(z)}{R(z)}, \frac{\overline{R}(z)}{\overline{R}(z)} \right] = \left[ \frac{k_1 + \bar{k}_1}{\overline{l}_1 + \overline{l}_1}, \frac{k_0 + \bar{k}_0}{\overline{l}_0 + \overline{l}_0} \right]$$

(2)

where $\left[ k_1, \bar{k}_1 \right]$, $\left[ k_0, \bar{k}_0 \right]$, $\left[ l_1, \bar{l}_1 \right]$, and $\left[ l_0, \bar{l}_0 \right]$ are scalar constants. Let the reduced order lower order models are designed for each bound individually. Hence, (2) can also be written as,

$$\left[ R(z), \overline{R}(z) \right] = \left[ \frac{\frac{k_1 z + k_0}{z^2 + \frac{\bar{k}_1}{\bar{l}_1}}, \frac{k_1}{z^2 + \frac{\bar{k}_0}{\bar{l}_0}}}{z^2 + \frac{l_1}{l_1}, \frac{l_0}{l_0}} \right]$$

(3)

The amplitude matching and WOA with CF technique is proposed to obtain the lower order model of the uncertain but bounded system.

2.1. Amplitude Matching

In the proposed technique, the unknown parameters in (2) are obtained by the combination of amplitude matching and the Whale Optimization Algorithm (WOA) with Constriction Factor (CF). An effective lower order system response should hold the equal amplitudes as that of its original higher order model response in all the time-indexes. This can be closely realized by equating the amplitude at the first time-index and steady state value of original step response with the corresponding reduced order system response amplitudes [12] and [14]. In general, the amplitudes at desired time indexes in the sequence are obtained from the Laurent series expression of the model.

A Laurent series is a series with both positive and negative power of $(z-z_0)$. It gives the analytic expression of a function on an annular region $\{ r_1 < |z-z_0| < r_2 \}$. If $f(z)$ is analytic in the annular region between and on assumed two concentric circles $n_1$ and $n_2$ centered at $z=z_0$ and of radii $r_1$ and $r_2$<r1 respectively, then there exists a unique series called Laurent series. When related to the definition of the z transform, the Laurent series of the step response gives the step response magnitudes. The amplitudes corresponding to first time indexes that are extracted from the Laurent series and the steady state values of higher order and reduced order systems are matched to retain the dynamics of the responses. Let the first time-index amplitude is $\alpha_1$ and the steady state value is $\alpha_s$. Steady state value of the original and reduced order system $G(z)$ and $R(z)$, from (1) and (3) can be obtained as

$$\left[ G(z), \overline{G}(z) \right]_{z=1} = \left[ \alpha_{ss}, \alpha_{ss} \right] = \left[ R(z), \overline{R}(z) \right]_{z=1}.$$  

Considering the steady state values of the higher order system as critical, the upper and lower limit models are taken separately and matched with the corresponding reduced order models and hence the steady state values are strictly retained.

$$\left[ G(z), \overline{G}(z) \right]_{z=1} = \left[ \alpha_{ss}, \alpha_{ss} \right] = \left[ \frac{a_m + a_{m-1} + \ldots + a_1 + a_0}{b_n + b_{n-1} + \ldots + b_1 + b_0}, \frac{\overline{a}_m + \overline{a}_{m-1} + \ldots + \overline{a}_1 + \overline{a}_0}{\overline{b}_n + \overline{b}_{n-1} + \ldots + \overline{b}_1 + \overline{b}_0} \right]$$

(4)

$$\left[ R(z), \overline{R}(z) \right]_{z=1} = \left[ \alpha_{ss}, \alpha_{ss} \right] = \left[ \frac{k_1 + k_0}{l_1 + l_0}, \frac{\overline{k}_1 + \overline{k}_0}{\overline{l}_1 + \overline{l}_0} \right]$$

(5)
Matching (4) and (5),
\[
\left( k_1 + k_0 \right) \left( 1 + \frac{1}{t_1 + t_0} \right) = \frac{\Sigma_{i=0}^{m} a_i}{\Sigma_{i=0}^{n} b_i} = \alpha_{ss} \quad \text{and} \quad \left( k_1 + \bar{k}_0 \right) \left( 1 + \frac{1}{t_1 + \bar{t}_0} \right) = \frac{\Sigma_{i=0}^{m} a_i^\prime}{\Sigma_{i=0}^{n} b_i} = \bar{\alpha}_{ss}
\] (6)

To obtain the amplitude at first time-index, it is assumed that \( m = (n-1) \) and the interval is upheld by proceeding the formulation without extracting the individual models. Unit step response and it’s Laurent series of the original interval system is
\[
U\left( G(z), \bar{G}(z) \right) = \left( \frac{z}{z-1} \right) \left[ \begin{array}{c} a_{n-1}, \bar{a}_{n-1} \\ a_{n-2}, \bar{a}_{n-2} \\ \vdots \\ a_1, \bar{a}_1 \\ a_0, \bar{a}_0 \end{array} \right] z^{n-1} + \left( \frac{b_n}{b_{n-1}} \right) \left( \begin{array}{c} a_{n-1}, \bar{a}_{n-1} \\ a_{n-2}, \bar{a}_{n-2} \\ \vdots \\ a_1, \bar{a}_1 \\ a_0, \bar{a}_0 \end{array} \right) z^{n-2} + \ldots
\] (7)

Unit step response and Laurent series of the reduced order interval system is
\[
U\left( R(z), \bar{R}(z) \right) = \left( \frac{z}{z-1} \right) \left[ \begin{array}{c} k_1, \bar{k}_1 \\ k_0, \bar{k}_0 \end{array} \right] z^{n-1} + \left( \frac{1}{t_1 + l_0} \right) \left( \begin{array}{c} k_1, \bar{k}_1 \\ k_0, \bar{k}_0 \end{array} \right) z^{n-2} + \ldots
\] (8)

From (7), the amplitude corresponding to first time index of the higher order system
\[
\left[ \alpha_1, \bar{\alpha}_1 \right] = \left[ \frac{a_{n-1}}{b_n}, \frac{\bar{a}_{n-1}}{\bar{b}_n} \right] = \left[ \frac{a_{n-1}, \bar{a}_{n-1}}{b_n, \bar{b}_n} \right] = \left[ \frac{1}{b_n}, \bar{\alpha}_1 \right]
\] (9)

From (8), for the reduced order system,
\[
\text{The amplitude corresponding to first time index} \left[ \alpha_1, \bar{\alpha}_1 \right] = \left[ k_1, \bar{k}_1 \right]
\] (10)

Matching the amplitudes from (9) and (10), we get
\[
\alpha_1 = k_1 = \min \left( \frac{a_{n-1}}{b_n}, \frac{a_{n-1}}{b_n}, \frac{a_{n-1}}{b_n}, \frac{a_{n-1}}{b_n} \right)
\] (11)

\[
\bar{\alpha}_1 = \bar{k}_1 = \max \left( \frac{a_{n-1}}{b_n}, \frac{a_{n-1}}{b_n}, \frac{a_{n-1}}{b_n}, \frac{a_{n-1}}{b_n} \right)
\] (12)

Substituting amplitudes from (6), (11) and (12), \( R(z) = \frac{k_1 z + k_0}{z^2 + t_1 z + l_0} \) and \( \bar{R}(z) = \frac{k_1 z + \bar{k}_0}{z^2 + l_1 z + l_0} \) become,
\[
R(z) = \frac{\alpha_1 z + \alpha_{ss} \left( 1 + \frac{1}{t_1 + l_0} \right) - \alpha_1}{z^2 + \frac{1}{t_1} z + \frac{l_0}{t_1}} \quad \text{and} \quad \bar{R}(z) = \frac{\bar{\alpha}_1 z + \bar{\alpha}_{ss} \left( 1 + l_1 + \bar{l}_0 \right) - \bar{\alpha}_1}{z^2 + \frac{1}{l_1} z + \frac{l_0}{l_1}}
\] (13)

The two unknown values of each reduced model from (13) are estimated by using WOA with CF for the minimum Integral Square Error (ISE). In theory, any control scheme performance is measured by the integral of error in the response for a fixed period. In general ISE of a system can be given as,
\[
\text{ISE} = \sum_{n=0}^{N} \left[ y(t_n) - y_r(t_n) \right]^2
\] (14)

Where the \( y(t_n) \) and \( y_r(t_n) \) are the outputs of the original and lower order models at \( n^{th} \) time index respectively. \( N \) is the number of time indexes considered to calculate the ISE.
2.2. WOA with CF

The Whale Optimization Algorithm which is developed by Seyedali Mirjalili and Andrew Lewis in 2016 has been employed in this paper with the introduction of Constriction Factor to tune the unknown parameters of reduced order models. As the WOA encounters the random exploration of solution in the search trajectory, the CF is included to restrict the explosion. The WOA with CF shows improved results in terms of better solution and convergence speed when experimented with multi-dimensional benchmark functions [13]. WOA involves a swarm of agents searching for the best solution and the position by employing ‘Bubble-net’ attacking technique of humpback whales, which includes shrinking encircling, spiral updating and random exploration for prey. The leader/best-solution agent’s position is defined, and it is assumed as the closest position to the optimum-value/prey. Whales always encircle the prey once spotted. A mathematical model of encircling mechanism for other search agents to update their positions towards best solution agent is given below. The constriction factor developed for the best convergence by Clerc and Kennedy for Particle Swarm Optimization technique is

\[ CF = \frac{2 \kappa}{2 - \phi - \sqrt{\phi^2 - 4 \phi}} \]  

Where 0 < \kappa ≤ 1. Kappa (\kappa) mostly gets the value 1. And \phi ≥ 4. Now for the WOA with CF, improved distance and position equations of encircling, spiral strategy and random exploration are given below. The position update,

\[ \tilde{X}(t + 1) = \tilde{X}^*(t) - \tilde{A} \cdot \tilde{D}_{\text{CF}}' \] (16)

\[ \tilde{D}_{\text{CF}}' = CF \cdot (|\tilde{X}^*(t) - \tilde{X}(t)|) \] (17)

where, \( t \) – current iteration; \( \tilde{A} \), \( \tilde{C} \) - coefficient vectors; \( \tilde{X}(t) \) – the position of the search agent; \( \tilde{X}^*(t) \) – the position of the best-solution agent; \( \tilde{A} = 2 \tilde{a} . \tilde{r} - \tilde{a} \); \( \tilde{C} = 2. \tilde{r} \); \( \tilde{a} \) -distance control parameter, linearly decreases from 2 to 0; \( \tilde{r} \) – random number [0,1]. Spiral equation for position update, \( \tilde{X}(t + 1) = \tilde{D}_{\text{CF}}' . e^{bt} \cos(2\pi t) + \tilde{X}^*(t) \)

where \( b \) – constant defines the shape of a logarithmic spiral; \( b \) – random number in [-1, 1].

The random position overhaul model

\[ \tilde{X}(t + 1) = \tilde{X}_{\text{rand}} + \tilde{X}(t) \] (18)

The general step by step procedure for the WOA is shown in the flowchart Figure 1.

3. Proposed Method - LTICS

The stable original nth order uncertain interval LTICS is represented by

\[ [\bar{G}(s), \tilde{G}(s)] = \left[ \bar{\mathbf{A}}(s), \tilde{\mathbf{A}}(s) \right] = \sum_{l=0}^{n} \left[ A_l, \tilde{A}_l \right] s^l \]
\[
\left( A_m, \overline{A}_m \right) s^{m+1} + \left( B_m, \overline{B}_m \right) s^{m+1} + \cdots + \left( A_1, \overline{A}_1 \right) s + \left( B_0, \overline{B}_0 \right)
\]

(21)

where \( m < n \); \( A_i, \overline{A}_i \) \((0 \leq i \leq m)\) and \( B_i, \overline{B}_i \) \((0 \leq i \leq n)\) are scalar values; \( A_i, B_i \) and \( \overline{A}_i, \overline{B}_i \) are lower and upper bounds. \( N(s) \) is the numerator polynomial; \( D(s) \) is the denominator polynomial; \( m=n-1 \). Let the reduced second-order uncertain but bounded LTICS to be derived as

\[
\left[ R(s), \overline{R}(s) \right] = \left[ \frac{K_1, \overline{K}_1}{s^2 + L_1}, \frac{K_0, \overline{K}_0}{s^2 + L_0} \right] = \left[ \frac{K_1 s + K_0}{s^2 + L_1 s + L_0}, \frac{\overline{K}_1 s + \overline{K}_0}{s^2 + \overline{L}_1 s + \overline{L}_0} \right]
\]

(22)

where \( K_1, \overline{K}_1 \), \( K_0, \overline{K}_0 \), \( L_1, \overline{L}_1 \), and \( L_0, \overline{L}_0 \) are scalar constants.

The Shamash method is considered for \( s \) to \( z \) domain transformation. Referring [15], [11] and [13] extending the amplitude matching technique to continuous systems is direct and simple which doesn’t need actual tedious domain transformation procedure.

- The proposed technique involves only the first amplitude terms of numerator and denominator polynomials. The first parameters \( \left[ A_m, \overline{A}_m \right] \) and \( \left[ B_n, \overline{B}_n \right] \) of numerator and denominator polynomials remain unaltered even after the direct domain transformation by Shamash method, the unit step response calculation and inverse domain transformation by Shamash.

- The steady state values of \( s \) domain and \( z \) domain transfer function models are always same.

The amplitude corresponding to first time index of the higher order system

\[
\left[ \alpha_1, \alpha_\overline{1} \right] = \left[ \frac{A_n-1, \overline{A}_{n-1}}{B_n, \overline{B}_n} \right] = \left[ \frac{A_{n-1}, \overline{A}_{n-1}}{B_n, \overline{B}_n} \right] \left[ \frac{1}{B_n, \overline{B}_n} \right] = \left[ \min \left( \frac{A_{n-1}, \overline{A}_{n-1}}{B_n, \overline{B}_n}, \frac{A_{n-1}, \overline{A}_{n-1}}{B_n, \overline{B}_n} \right), \max \left( \frac{A_{n-1}, \overline{A}_{n-1}}{B_n, \overline{B}_n}, \frac{A_{n-1}, \overline{A}_{n-1}}{B_n, \overline{B}_n} \right) \right]
\]

(23)

Similar to (8), for the continuous reduced order system, the amplitude corresponding to first time index

\[
\left[ \alpha_1, \alpha_\overline{1} \right] = \left[ K_1, \overline{K}_1 \right]
\]

(24)

Matching the amplitudes from (23),(25) and (26), we get,

\[
\alpha_1 = K_1 = \min \left( \frac{A_{n-1}, \overline{A}_{n-1}}{B_n, \overline{B}_n}, \frac{A_{n-1}, \overline{A}_{n-1}}{B_n, \overline{B}_n} \right)
\]

(25)

\[
\overline{\alpha_1} = \overline{K}_1 = \max \left( \frac{A_{n-1}, \overline{A}_{n-1}}{B_n, \overline{B}_n}, \frac{A_{n-1}, \overline{A}_{n-1}}{B_n, \overline{B}_n} \right)
\]

(26)

From (21), Steady state value from higher order system transfer function

\[
\left[ G(s), \overline{G}(s) \right]_{s=0} = \left[ \alpha_{ss}, \overline{\alpha}_{ss} \right] = \left[ \frac{A_0}{B_0}, \overline{A}_0, \overline{B}_0 \right]
\]

(27)

From (22), Steady state value from reduced order system transfer function

\[
\left[ R(s), \overline{R}(s) \right]_{s=0} = \left[ \alpha_{ss}, \overline{\alpha}_{ss} \right] = \left[ \frac{K_0}{L_0}, \overline{K}_0, \overline{L}_0 \right]
\]

(28)

When (27) and (28) are matched

\[
K_0 = \frac{A_0}{B_0} = \alpha_{ss} \quad ; \quad \overline{K}_0 = \frac{\overline{A}_0}{\overline{B}_0} = \overline{\alpha}_{ss}
\]

(29)

Substituting amplitudes from (25), (26) and (29),

\[
R(s) = \frac{\alpha_{ss} s + \overline{\alpha}_{ss} (L_0)}{s^2 + \overline{L}_1 s + \overline{L}_0} \quad \text{and} \quad \overline{R}(s) = \frac{\overline{\alpha}_{ss} s + \alpha_{ss} (L_0)}{s^2 + \overline{L}_1 s + \overline{L}_0}
\]

(30)

The unknown values are obtained by employing WOA with CF as explained in Section 3.

4. Illustrations

4.1 Example 1 – LTICS

Consider the LTICS interval system given by Bandopadhyay et al. in 1997 [5]
From the amplitude matching procedure (Section 4), the amplitudes obtained for the given example are \([\alpha_1, \alpha_2] = [1.8094, 2.2105]\) and \([\alpha_{SS}, \alpha_{SS}] = [3.26152, 3.26152]\). Hence from (30) the reduced models become \(R(s) = \frac{1.8094 s + 3.26152 (L_0)}{s^2 + L_1 s + L_0}\) and \(\bar{R}(s) = \frac{2.2105 s + 3.26152 (L_0)}{s^2 + L_1 s + L_0}\). Invoking the WOA with CF for finding the unknown values results in, \(\bar{R}(s) = \frac{1.8095 s + 0.2721 + 0.0834}{s^2 + 0.7309 s + 0.0834}\) and \(\bar{R}(s) = \frac{2.2105 s + 0.5737 + 0.1759}{s^2 + 0.7309, 1.0334 s + [0.0834, 0.1759]}\). The reduced order model formulated by proposed method is

![Figure 2. Lower Bound Responses - Example 1](image1)

![Figure 3. Upper Bound Responses - Example 1](image2)

| Reduced order Models | Lower | Upper |
|----------------------|-------|-------|
| Bandyopadhyay Model (1997) [5] | 299.52 | 108.35 |
| \([1.16,1.14] s^2 + 0.27,0.53\) | \([2.92,2.83]\) |
| Selvaganesan Model (2007) [19] | 1228.7 | 1196.2 |
| \([260.955,861.331] s^2 + 175.232,218.581\) | \([364.72,366.62]\) |
| \([562.4,555.6] s^2 + [181.6, 205.4]\) | \([319.49,406.63]\) |
| Devender PSO Model (2010) [20] | 133.97 | 134.23 |
| \([562.4,555.6] s^2 + [181.6, 205.4]\) | \([319.49,406.63]\) |
| Devender GA Model (2010) [18] | 129.38 | 129.72 |
| \([553.9,560.3] s^2 + [181.9, 205.4]\) | \([319.49,406.63]\) |
| Priya Model (2013) [21] | 692.77 | 476.44 |
| \([4.4052,5.6923] s^2 + 2.7494, 2.7892\) | \([319.49,406.63]\) |
| **Proposed Method** | 73.12 | 47.136 |
| \([1.8095,2.2105] s^2 + [0.2721, 0.5737]\) | \([73.12,47.136]\) |

Table 1. ISE Comparison of Reduced Interval Models - Example 1

The lower bound and upper bound reduce order model responses are compared with existing models in Table 2 for 70 seconds with 0.01 seconds interval. From the Table 1, Figure 2 and Figure 3, it can be observed that the proposed reduced model preserve the stability and closely match with the original higher order system. Also the proposed method gives minimum error when compared with existing methods.
4.2 Example 2 – LTIDS

The real-time digital control system transfer function given by Choudhary & Nagar in [22] is

\[ G(z) = \frac{1.6484, 1.7156 \cdot z^7 + [1.0937, 1.1383] \cdot z^6 + [-0.2142, -0.2058] \cdot z^5 + [0.1490, 0.1550] \cdot z^4 + [-0.5263, -0.5057] \cdot z^3 + [-0.2672, -0.2568] \cdot z^2 + [0.0431, 0.0449] \cdot z + [-0.0061, -0.0059]}{[23.52, 24.48] \cdot z^6 + [-1.7156, -1.6484] \cdot z^5 + [-1.1383, -1.0937] \cdot z^4 + [0.2058, 0.2142] \cdot z^3 + [-0.1550, -0.1490] \cdot z^2 + [0.5057, 0.5263] \cdot z^1 + [0.2568, 0.2672] \cdot z + [-0.0449, -0.0431] \cdot z + [0.0059, 0.0061]} \]

The given higher order model is reduced using above explained proposed methods and then compared to the existing models. Let the reduced model to be found is as in (3). Employing the amplitude matching technique as explained in Section 2 the reduced models become ,

\[ R(z) = \frac{\alpha_s \cdot z + \alpha_{ss} \cdot (1 + \bar{l}_1 + \bar{l}_0)}{z^2 + \bar{l}_1 \cdot z + \bar{l}_0} \quad \text{and} \quad \bar{R}(z) = \frac{\bar{\alpha}_1 \cdot z + \bar{\alpha}_{ss} \cdot (1 + \bar{l}_1 + \bar{l}_0)}{z^2 + \bar{l}_1 \cdot z + \bar{l}_0}. \]

Where \( \alpha_1, \bar{\alpha}_1 \) = [0.0673, 0.0729] from (11) and (12) and \( \alpha_{ss}, \bar{\alpha}_{ss} \) = [0.0896, 0.0922] from (6). Now the \( R(z) \) and \( \bar{R}(z) \) are subjected to WOA with CF to estimate the unknown values. Hence, \( [R(z), \bar{R}(z)] = [0.0673, 0.0729] \cdot z + [-0.0367, -0.0424] \)

\[ z^2 + [-1.1763, -1.1411] \cdot z + [0.5181, 0.4721] \]

Figure 4 Lower Bound Responses - Example 2

Figure 5 Upper Bound Responses - Example 2
Table 2. ISE Comparison of Reduced Interval Models - Example 2.

| Reduced order Models     | Lower       | Upper       |
|--------------------------|-------------|-------------|
| Amit Model [22]          | 1.7991      | 0.4104      |
| [0.02,0.05]z²+[0.007,0.01]z+[-0.04,–0.01] |            |             |
| [1.38,1.45]z²+[-1.91,–1.89]z+[0.64,0.71] |            |             |
| Proposed Method          | 3.2364×10⁻⁴ | 3.5743×10⁻⁴ |
| [0.0673,0.0729]z+[-0.0367,–0.0398] |            |             |
| z²+[-1.1763,–1.1692]z+[0.5181,0.5282] |            |             |

The reduced order model derived is compared with the existing model in Table 2. From Figure 4 and Figure 5 it can be concluded that the proposed method works well for interval system with good results.

5. Conclusion
The procured uncertain approximated models using presented technique are strictly able to maintain the stability. The adoption of proposed method to both LTICS and LTIDS is accomplished without any complex computation. From the illustrations it is exhibited that the lower model responses closely trail their large-scale systems. When compared to the existing method, the proposed method is the simplest one and also giving the lowest ISE.

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