Effect of Fermi surface evolution on superconducting gap in superconducting topological insulator

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Abstract

We study the bulk electronic states of a superconducting topological insulator, which is a promising candidate topological superconductors. Recent experiments suggest that the three-dimensional Fermi surface evolves into two-dimensional one. We show that the superconducting energy gap structure on the Fermi surface systematically changes with this evolution. It is clarified that the bulk electronic properties, such as spin–lattice relaxation rate and specific heat, depend on the shape of the Fermi surface and the type of the energy gap function. These results serve as a guide to determine the pairing symmetry of Cu$_x$Bi$_2$Se$_3$.

Keywords: topological superconductor, topological insulator, superconducting topological insulator, pairing symmetry, bulk electronic properties

(Some figures may appear in colour only in the online journal)

1. Introduction

Recently, topological insulators (TIs) and topological superconductors (TSCs) have become star materials, drawing intense research interest because of their novel physical properties and potential applications in electronic devices [1–4]. The remarkable feature of a TI is the presence of surface Dirac cone that is protected by the topological invariant $Z_2$, which is defined in a bulk time-reversal invariant insulating state [2, 4]. Thanks to the existence of the surface Dirac cone, we can expect new exotic transport phenomena and future applications for spintronics devices in TIs. The superconducting analog of TIs is dubbed as the TSC, which also has a gapless surface state called surface Andreev bound state (SABS), which is protected by the bulk topological invariant [3, 5–7]. One of the important features of TSC is that the SABS can be Majorana fermion [8]. There are several proposals that Majorana fermion is applicable for fault tolerant quantum computation [9].

A typical example of the three-dimensional (3D) TI is Bi$_2$Se$_3$ [10, 11], which is widely studied because it has a simple Dirac-cone at $\Gamma$-point and large bulk band gap comparable to the room temperature [4]. Recent research studies of Bi$_2$Se$_3$ reveal that Cu-doped Bi$_2$Se$_3$ shows superconductivity with transition temperature $T_c = 3.8K$ [12]. Soon after the observation of superconductivity, an angle resolved photoemission spectroscopy measurement in the normal state clarified that the topological surface state remains even after the Cu doping [13]. In this sense, Cu$_x$Bi$_2$Se$_3$ can be called a superconducting topological insulator (STI) and considered as the best platform for studying the relationship between TI and superconductivity. Moreover, owing to its peculiar band structure from the strong spin–orbit interaction, we can naturally imagine that TSC can be realized.

Tunneling spectroscopy measurement is one of the most promising tools for detecting the SABS, which can help reveal the pairing symmetry. Actually, the SABSs in the cuprates and Sr$_2$RuO$_4$ manifest as a zero bias conductance peak (ZBCP) [14–19], and the presence of a ZBCP becomes evidence for an unconventional pairing, such as a $d$-wave or $p$-wave. Some experimental results indicate the realization of topological superconductivity in Cu$_x$Bi$_2$Se$_3$. The ZBCP has been reported in the point contact spectroscopy by Sasaki et al [20]. Using theoretical calculations for possible pair potentials [20–24], it has been revealed that the resulting ZBCP originates from the...
There are some conflicting experimental results that indicate the nontopological superconductivity without the SABS in Cu$_x$Bi$_2$Se$_3$. In the scanning tunneling spectroscopy, the ZBCP has not been observed, but a U-shaped fully gapped spectrum has appeared [30,31]. Nevertheless, more recently, Mizushima et al. have suggested spin– triplet pairing [42]. In this model, we consider the cubic lattice structure, whereas the crystal structure of Cu$_x$Bi$_2$Se$_3$ is rhombohedral [43]. This simplification does not affect the low energy electronic structures. The basis of the orbitals in equation (2) consists of two effective $p_z$ orbitals of Se and Bi on the upper and lower sides in a quintuple layer, respectively. Hereafter, we call this the orbital basis and refer to the labels following the diagonalization of $H_0(k)$ as the band basis.

$$U^+(k)H_0(k)U(k) = c(k)\tilde{\sigma}_0\tilde{s}_0 + \eta(k)\tilde{\sigma}_0,$$

where $\eta(k) = \sqrt{m^2(k) + v^2(k)(\sin^2k_x a + \sin^2k_y a) + v^2(k)\sin^2k_z c}$ and $\tilde{\sigma}_0$ is the Pauli matrix that denotes the band index, i.e., $\tilde{\sigma}_1 = 1$ for the conduction band and $\tilde{\sigma}_2 = -1$ for the valence band.

Before we describe the pair potential types, we discuss the shape of the Fermi surface. If we adopt the parameters of the normal-state Hamiltonian, as reported in [20], it gives a 3D Fermi surface around $\Gamma$-point. These parameters describe the electronic structure of Bi$_2$Se$_3$ well. However, in Cu$_x$Bi$_2$Se$_3$, because Cu is intercalated between the quintuple layers, the values of the hopping terms along the $c$-direction ($c_1, m_1, v_c$) and the chemical potential $\mu$ can be modulated because of the change of the $c$-axis length and the electron doping, respectively. In this paper, we use the parameter reported in [20] for $\mu = 0.40$. For $\mu = 0.65$, we use the half-value of $c_1, m_1$, and $v_c$. For other $\mu$, we interpolate the values of $c_1, m_1$, and $v_c$ linearly. For larger $\mu$ or smaller values of hopping terms along the $c$-axis ($c_1, m_1, v_c$), the shape of the Fermi surface becomes cylindrical.

Next, we explain the possible types of the pair potential for these two orbital models [33]. Because Cu$_x$Bi$_2$Se$_3$ is not a strongly correlated system, we assume that the Cooper pairs within a unit cell are dominant and therefore, the pair potential does not depend on $k$. In this case, Fermi statistics allows six types of pair potential: $\Delta_{\sigma_0\tilde{s}_0}, \Delta_{\sigma_1\tilde{s}_0}, \Delta_{\sigma_2\tilde{s}_0}, \Delta_{\sigma_0\tilde{s}_1}, \Delta_{\sigma_1\tilde{s}_1}, \Delta_{\sigma_2\tilde{s}_1}$. They are classified into four types of irreducible representation of the $D_{3d}$ point group: $A_1g$ ($\Delta_{\sigma_0\tilde{s}_0}$ and $\Delta_{\sigma_1\tilde{s}_0}$), $A_{1u}$ ($\Delta_{\sigma_2\tilde{s}_0}$), $A_{2u}$ ($\Delta_{\sigma_0\tilde{s}_1}$), and $A_{2u}$ ($\Delta_{\sigma_1\tilde{s}_1}$). In this paper, we consider the cases of $\Delta_1 = \Delta_{\sigma_0\tilde{s}_0}, \Delta_2 = \Delta_{\sigma_1\tilde{s}_1}, \Delta_3 = \Delta_{\sigma_0\tilde{s}_1}$, and $\Delta_4 = \Delta_{\sigma_1\tilde{s}_0}$. We do not consider $\Delta_{\sigma_2\tilde{s}_0}$ because the spin–orbit interaction in equation (2) does not favor the $\Delta_{\sigma_2\tilde{s}_0}$ pairing. Note that the result for $\Delta_{\sigma_2\tilde{s}_0}$ is obtained using the $\Delta_4$ by four-fold rotation around the $z$-axis.
In the preceding pairings, $\Delta_1$ is an even-parity Bardeen–Cooper–Schrieffer (BCS) pairing and $\Delta_2$, $\Delta_3$, and $\Delta_4$ are odd-parity pairings.

Now we consider the pair potential in the band basis, as introduced in equation (5), by the unitary transformation,

$$\Delta_{\sigma_i s_j} \rightarrow U^\dagger(k)\Delta_{\sigma_i s_j}U(k) = \Delta_{\bar{\sigma}_i s_j}. \quad (6)$$

Here, we consider the electron-doped TI, as realized in $\text{Cu}_x\text{Bi}_2\text{Se}_3$, where the Fermi surface consists of only the conduction band. Because the band gap is much larger than the superconducting gap [13], we can ignore the interband and intraband pairing within the valence band. Next, we obtain the superconducting gap in the tight-binding model from the (1, 1) component of the Puli matrix $\bar{\sigma}$ in the band basis.

$$|\Delta_1(k)| = \Delta \quad (7)$$

$$|\Delta_2(k)| \leq \sqrt{\frac{m(k)}{m(k)^2 + v_F^2 \sin^2 k_c c}} \frac{v^2(\sin^2 k_c a + \sin^2 k_y a)}{\eta(k)} + \frac{v_F^2 \sin^2 k_c c}{m^2(k) + v_F^2 \sin^2 k_c c} \quad (8)$$

$$|\Delta_3(k)| = \Delta \sqrt{\frac{\sin^2 k_c a + \sin^2 k_y a}{\eta^2(k)}} \quad (9)$$

$$|\Delta_4(k)| = \Delta \sqrt{\frac{v_F^2 \sin^2 k_c a + v_F^2 \sin^2 k_y c}{\eta^2(k)}} \quad (10)$$

Detailed derivations of the pair potentials in the tight-binding model from the (1, 1) component of the Puli matrix $\bar{\sigma}$ in the band basis have been reported in [34, 35].

### 3. Energy gap structure with evolution of Fermi surface

In the case of 3D Fermi surface, the energy gap structures for the possible pair potentials have been clarified in the previous studies. In this case, $\Delta_1$ and $\Delta_2$ show fully gapped structures, whereas $\Delta_3$ and $\Delta_4$ have point nodes along the $k_c$- and $k_y$-axis, respectively [20, 23, 33]. In the following, we show the change of the energy gap structures on the Fermi surface through $\text{Cu}$ intercalation. We also show the DOS, which is useful for understanding the calculated results of the spin–lattice relaxation rate and the specific heat shown in the later sections.

In the calculation of the DOS, we adopt the quasi-classical approximation $\Delta \ll v_F k_F$, where $v_F$ is the group velocity along the normal vector to the Fermi surface. Then, the DOS in the superconducting state $N_s(E)$, normalized by its value in the normal state $N_n(E = 0)$, is given by

$$\frac{N_s(E)}{N_n(0)} = \int_{FS} \text{Re} \left[ \frac{|E|}{v_F \sqrt{E^2 - \Delta^2(k)}} \right] dS / \int_{FS} 1 dS \quad (11)$$

where $\int_{FS} dS$ denotes the integral over the Fermi surface. In the second line of equation (11), we assume that $v_F$ is constant on the Fermi surface for simplicity. Because the $k$-dependence of the energy gap and the DOS are trivial for $\Delta_1$, we show the DOS for $\Delta_2$, $\Delta_3$, and $\Delta_4$.

#### 3.1. Energy gap structure and DOS for $\Delta_2$

In figure 2, we show the evolution of the Fermi surface, the superconducting gap (a)–(c) and the DOS (f)–(j) for the case of $\Delta_2$. The color on the Fermi surface indicates the magnitude of the energy gap at the Fermi momenta. In the case of $\mu = 0.40$ and 0.45, the Fermi surface is 3D (figure 1(a) and (b)). With increasing $\mu$, the volume of the Fermi surface becomes larger and the top and the bottom of the Fermi surface reach the boundary of the Brillouin zone at $\mu = 0.50$, as seen from figure 1(c). With increasing $\mu$, the Fermi surface becomes cylindrical and the two-dimensionality of the Fermi surface is much stronger for larger $\mu$ (figure 1(d) and (e)). As seen from figure 1, the energy gap other than the case of $\mu = 0.50$ have fully gapped structures. On the other hand, it is closed at $(k_x, k_y, k_z) = (0, 0, \pm \pi/c)$ in the case of $\mu = 0.50$. It is noted that the topological phase changes between the spherical and the cylindrical Fermi surfaces. In the fully gapped odd–parity superconductors with time-reversal symmetry, the 3D winding number is odd (even) when the number of the time-reversal-invariant momenta enclosed by the Fermi surface is odd (even) [6]. In the present cases, the number of the time-reversal-invariant momenta enclosed by the Fermi surface is one for $\mu < 0.50$ and two for $\mu > 0.50$, and therefore, the topological nature of the STI for $\Delta_2$ changes [41]. Because the topological phase does not change without an energy gap closing, a nodal point appears between the two different topological phases at $\mu = 0.50$. Owing to this gap closing, the energy gap near $(k_x, k_y, k_z) = (0, 0, \pm \pi/c)$ is suppressed. Thus, $\Delta_2(k)$ is highly anisotropic for $\mu = 0.45$ and 0.65, where the ratio of the maximum to the minimum of the energy gap is almost 2. On the other hand, anisotropy is small for $\mu = 0.40$ and 0.80. Further, the DOS for $\mu = 0.40$ and 0.80 has a large peak at $E/\Delta = 1$ and rapidly decreases to zero for $E/\Delta < 1$. This is similar to that for an isotropic-gap superconductor, such as $\Delta_1$.

#### 3.2. Energy gap structure and DOS for $\Delta_3$

In figure 2, we show the evolution of the Fermi surface, the superconducting gap (a)–(c), and the DOS (f)–(j) for the case of $\Delta_3$. As seen from equation (9), $\Delta_3(k) = 0$ along the $k_c$-axis ($k_z = k_y = 0$). Thus, in the case of the 3D Fermi
surface ($\mu < 0.50$), the point nodes appear at the north and south poles on the Fermi surface, as seen from figures 2(a) and (b). Because of these point nodes, the DOS near $E = 0$ is proportional to $E^2$, as shown in figures 2(f) and (g). Because the Fermi surface is vertically long, the curvature of the Fermi surface has a maximum at the nodal points. This curvature diverges at $\mu = 0.50$, where the Fermi surface reaches the zone boundary $k_z = \pi$. Then, the coefficient of $E^2$ in the DOS diverges and $E$-linear behavior appears as shown in figure 2(h). In contrast, for the case of a 2D Fermi surface $\mu > 0.50$, the DOS shows fully gapped structures because the nodal points disappear. With increasing $\mu$, the anisotropy of the energy gap becomes small because the two-dimensionality of the Fermi surface increases.

### 3.3. Energy gap structure and DOS for $\Delta_4$

In figure 3, we show the evolution of the Fermi surface, the superconducting gap (a)–(e), and the DOS (f)–(j) in the case of $\Delta_4$. In the case of the 3D Fermi surface, point nodes exist along the $k_x$-axis ($k_x = k_z = 0$). Thus, the DOS near $E = 0$ is proportional to $E^2$, as shown in figures 2(f) and (g). As compared to the case of $\Delta_3$, the coefficient of $E^2$ in the DOS is smaller because the curvature of the Fermi surface at the point nodes is smaller than that for $\Delta_3$. When the Fermi surface
In this paper, we set the normalized magnitude of the quasi-particle damping.

The normalized spin–lattice relaxation rate $T_1^{-1}$ is given by

$$
\frac{T_{1N}}{T_1} = \frac{2}{N_c(0)^2} \int_0^\infty dE N_s^2(E) \left( 1 + \frac{<\Delta_{i1}>}{E} \frac{<\Delta_{i1}^*>}{E} \right) \left( -\frac{dJ(E)}{dE} \right).
$$

(12)

where $N_s(E)$ is the DOS in the superconducting state given by equation (11), $f(E)$ is the Fermi distribution function, and $<\Delta_{i1}>$ ($i = 1, 2, 3, 4$) denotes the average of the pair potential on the Fermi surface [45]. This average part is equal to zero for $\Delta_2$, $\Delta_3$, and $\Delta_4$ because these pair potentials are odd parity. For the temperature dependence of pair potential, we consider a phenomenological form obtained from the BCS theory [46],

$$
\Delta(T) = a k_B T \tanh \left( \frac{T_c}{T} \right).
$$

(13)

where $a$ is a coupling constant. In the conventional BCS superconductor, $a$ is known to be 1.76. However, $a$ often deviates from the BCS value [47]. The estimated $a$ for Cu$_x$Bi$_2$Se$_3$ is 2.3 from the measurements of upper critical magnetic field $H_c2$. Thus, we use $a = 2.3$ in this paper. To consider the quasi-particle damping effect, we add the imaginary part $E$ by replacing $E$ with $E + i\Delta$ in equation (11), where $\Delta$ denotes the magnitude of the quasi-particle damping. In this paper, we set the normalized magnitude of the quasi-particle damping as $\Delta/\Delta = 10^{-3}, 10^{-2}$, and $10^{-1}$.

In figure 4, we show the temperature dependence of the spin–lattice relaxation rate for the isotropic-gap superconductor $\Delta_1$ with $\delta/\Delta = 10^{-3}, 10^{-2}$, and $10^{-1}$. We can clearly see a prominent Hebel–Slichter peak. The height of the Hebel–Slichter peak decreases with increasing $\delta/\Delta$. However, this peak remains for the case with large quasi-

Figure 3. (a)–(c) Evolution of the Fermi surface and superconducting gap, (f)–(i) the DOS, as a function of the chemical potential $\mu$ in the case of $\Delta_v$. The color of the Fermi surface indicates the magnitude of the superconducting gap at the Fermi momenta.
particle damping $\delta/\Delta = 10^{-3}$. Because the energy gap structure for $\Delta_1$ is unchanged by the concentration of Cu, the same results are obtained for both 2D and 3D Fermi surfaces.

Next, we show the temperature dependence of the spin–lattice relaxation rate $T_1^{-1}$ in the case of isotropic gap pairing $\Delta_1$. The normalized magnitude of the quasi-particle damping is set to $\delta/\Delta = 10^{-3}$ (blue solid line), $10^{-2}$ (green dashed line), and $10^{-1}$ (red dotted line).

In the case of $\Delta_3$, the Hebel–Slichter peak for 3D Fermi surface ($\mu < 0.50$) is suppressed, even in the case of small quasi-particle damping $\delta/\Delta = 10^{-3}$, as shown in figure 5(b). This is because the DOS at $E/\Delta = 1$ is smaller than that of the other pairings because of the formation of point nodes. By contrast, with increasing $\mu$, the energy spectra become fully gapped. As a result, the Hebel–Slichter peak appears for 2D Fermi surface ($\mu < 0.50$) with small quasi-particle damping ($\delta/\Delta = 10^{-3}$ and $10^{-2}$). Because of the disappearance of point nodes in a 2D Fermi surface, the power-law behavior of $T_1(T_c)/T_1(T)$ at a low temperature for a 3D Fermi surface changes to exponential one. In the case of $\Delta_4$, the height of the Hebel–Slichter peak decreases with the evolution of the Fermi surface. This is consistent with the suppression of the peak in the DOS at $E/\Delta = 1$. In other words, the suppression of the Hebel–Slichter peak is because of the emergence of the additional point nodes at the zone boundary. Note that the Hebel–Slichter peak does not appear for large magnitudes of the quasi-particle damping in both cases with a 2D and 3D Fermi surface.

Here, we summarize the results of $T_1$. In the weak quasi-particle damping cases ($\delta/\Delta = 10^{-3}$ and $10^{-2}$), the temperature dependence of $T_1(T_c)/T_1(T)$ is different for each type of pairing. The Hebel–Slichter peak appears for (i) $\Delta_1$, (ii) $\Delta_2$, and (iii) $\Delta_3$ with a 2D Fermi surface or (iv) $\Delta_4$ with a 3D Fermi surface. The height of the Hebel–Slichter peak for $\Delta_4$ is much larger than that for other pairings. Thus, with strong quasi-particle damping ($\delta/\Delta = 10^{-1}$), the Hebel–Slichter peaks for all odd–parity pairings ($\Delta_2$, $\Delta_3$, and $\Delta_4$) disappear, whereas that for even–parity pairings ($\Delta_1$) remains. In usual cases, because the magnitude of the pair potential is much smaller than the chemical potential, the quasi-particle damping effect is strong. Therefore, it is concluded that the absence of the Hebel–Slichter peak can support the odd–parity superconductivity.

5. Specific heat

In this section, we calculate the temperature dependence of the specific heat of the STI for the possible pair potentials. The specific heat is given by

$$C_s = \frac{2\beta}{N} \sum_k \left( E_i^2(k) + \beta E_i(k) \frac{\partial \Delta}{\partial \beta} \frac{\partial E_i(k)}{\partial \Delta} \right) \times \left( -\frac{\partial f(E_i(k))}{\partial E_i(k)} \right),$$

(14)

where $N$ is the number of unit cells, $\beta = 1/k_B T$, and $E_i(k)$ ($i = 2, 3, 4$) is the energy dispersion for each possible pair potential. The detailed forms of $E_i(k)$ are given in [34]. For the temperature dependence of the pair potential, we use equation (13) with $\alpha = 2.3$, which is the same as that used in the calculation of the spin–lattice relaxation rate. In general, the temperature dependence of $C_s(T)/T$ for a full gap superconductor has an exponential behavior at a low temperature. In the case of superconductors with point nodes and line nodes, $C_s(T)/T$ is proportional to $T^2$ and $T$ near $T = 0$, respectively. On the other hand, the magnitude of a jump of $C_s(T)/T$ at $T = T_c$ increases with the coupling constant $\alpha$. In addition, because the following entropy valance relation must be satisfied,

$$\int_0^{T_c} dT \frac{C_s(T)}{T} = C_n(T)/T = 0,$$

(15)

the specific heat at low and high temperatures more or less influence each other.

In figure 6, we show the specific heat for the STI as a function of the temperature. The first row shows the temperature dependence of $C_s(T)/T$ for $\Delta_2$ with $\mu = 0.40$, $0.45$, $0.50$, $0.65$, and $0.80$. The second and third row shows it for $\Delta_3$ and $\Delta_4$, respectively. In the case of $\Delta_2$, with the evolution of the Fermi surface, the energy gap closes at $\mu = 0.50$. Near the transition point, the magnitude of the specific heat jump is the smallest among the calculated results for $\Delta_2$ because the anisotropy of the energy gap becomes the largest. The magnitude of the specific heat jump at $T = T_c$ becomes larger as $\mu$ is farther from $\mu = 0.50$. At a low temperature, $C_s(T)/T$ has an exponential behavior except for when $\mu = 0.50$, as shown figure 6(a). For $\mu = 0.50$, it is proportional to $T$, which is consistent with $E$-linear behavior in the DOS.
Figure 5. Temperature dependence of $\frac{C_v(T)/C_v(T_c)}{\Delta}$ for (a) $\Delta_2$, (b) $\Delta_3$, and (c) $\Delta_4$ with $\mu = 0.40, 0.45, 0.50, 0.65$, and $0.80$. The normalized magnitude of the quasi-particle damping is set to $\delta \Delta = 10^{-3}$ (blue solid line), $10^{-3}$ (green dashed line) and $10^{-1}$ (red dotted line).

Figure 6. Temperature dependence of $C_v(T)/T$ for (a) $\Delta_2$, (b) $\Delta_3$ and (c) $\Delta_4$ with $\mu = 0.40, 0.45, 0.50, 0.65$, and $0.80$. The black and blue lines show the specific heat in the superconducting and normal states, respectively.
As we can see in figure 6(b), the temperature dependence of $C_s(T)/T$ for $\Delta_1$ changes with the evolution of the Fermi surface. This is because the energy gap structure changes from a point nodal one to a fully gapped one with the increase of $\mu$. For the same reason, $C_s(T)/T$ near $T = 0$ shows a $T^3$ behavior and an exponential behavior for 3D and 2D Fermi surfaces, respectively. The magnitude of the specific heat normalized by its value in the normal state just below $T_c$ is almost two for a 3D Fermi surface. For $\mu > 0.50$, this value increase with $\mu$ because of the suppression of anisotropy of the energy gap.

According to figure 6(c), the temperature dependence of $C_s(T)/T$ for $\Delta_4$ also changes with the evolution of the Fermi surface. In the case of $\Delta_4$, because the energy spectra have point nodes both for a 2D and 3D Fermi surface, $C_s(T)/T$ is proportional to $T^2$. However, the magnitude of the specific heat jump to a 3D Fermi surface is larger than that for a 2D one. This is because the number of point nodes increases on a heat jump to a 3D Fermi surface. This is because the energy gap structure changes with the evolution of the Fermi surface.

In conclusion, we have shown that the wide variation of the temperature dependence of the specific heat appears depending on the shape of the Fermi surface. Especially, the magnitude of the specific heat jump changes systematically with the evolution of the Fermi surface. In the case of $\Delta_4$, the magnitude of the specific heat jump decreases once, down to $\mu = 0.50$, and increases with increasing $\mu$. On the other hand, the magnitude of the specific heat jump for $\Delta_3$ and $\Delta_4$ is enhanced and reduced, respectively, almost monotonically with increasing $\mu$. Namely, the systematical changes of the specific heat jump are different in each pair potential. For this reason, it is possible to identify the pairing symmetry from the measurements of specific heat by changing the amount of Cu intercalation.

6. Discussion and summary

In this paper, we studied the bulk electronic states of the STI, Cu$_x$Bi$_2$Se$_3$. We focused on the evolution of the Fermi surface changing from a 3D to a 2D shape and its influence on bulk physical quantities. It has been shown that the superconducting energy gap structure on the Fermi surface systematically changes with this evolution. We clarified that the spin–lattice relaxation rate and the specific heat depend on the shape of the Fermi surface and type of energy gap function. These obtained results are useful for identifying the pairing symmetry of Cu$_x$Bi$_2$Se$_3$.

Here, we comment on the effect of the Fermi surface evolution on tunneling spectra. For a 3D Fermi surface, the tunneling spectra can have the ZBCP in the case of $\Delta_2$ or $\Delta_4$ [20–22, 32]. For a 2D Fermi surface, the tunneling spectra have been calculated for $\Delta_1$ and $\Delta_2$ [32]. In both cases, the tunneling spectra have not shown the ZBCP [32]. In the case of $\Delta_3$, because the SABSs are absent on (001) surface, which corresponds to the (111) surface in Cu$_x$Bi$_2$Se$_3$, it is expected that the tunneling spectra in the low transmissivity junction will have similar behaviors to the bulk DOS shown in figure 2. On the other hand, in the case of $\Delta_4$, there are two possibilities; the ZBCP becomes broader than that for a 3D Fermi surface or the zero bias dip appears. In either case, it is expected that the value of the conductance at zero bias voltage decreases with the evolution of the Fermi surface. A detailed analysis on tunneling spectra with the Fermi surface evolution is a problem for future studies. Our work could also be extended to anisotropic pairing realized by correlation effect [50, 51].

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