Determining Dynamic Properties of Elastomer-Dampers by Means of Impact Testing

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Abstract
Background Damping elements made of elastomer materials are used in almost every mechanical system to prevent damage to components caused by impact-like excitations and the resulting high-frequency, large-amplitude oscillations. The dynamics of these operating conditions exceed the performance limits of conventional experimental testing methods, impeding validated predictions of the damper’s transmission behaviour.

Objective A method is proposed to directly investigate the influence of impacts on the transmission behaviour of elastomeric dampers by impact testing.

Methods Torsional-loaded elastomer dampers were experimentally investigated using a drop tower. During the experiment, a mass is brought into impact contact with a lever arm connected to the tested coupling. Measurements on resulting torsional oscillations and a comparison of the measurement results with a simple analytical model of the system allow for determining the coupling parameters stiffness and damping ratio.

Results The characteristic parameters stiffness and damping ratio of the elastomer damper were mapped as a function of excitation-amplitude and frequency. A comparison of drop-tower test results with servo-hydraulic measurements validated the determined parameters.

Conclusions Determining the transmission behaviour of elastomeric dampers from highly dynamic and impact-induced oscillation states proved to be a good approach to supplement established testing methods.

Keywords Drop tower · Torsional impact · Elastomer coupling · Viscoelasticity · Payne effect

Introduction

Elastomer dampers decisively contribute to the functioning of our globalized world. Used in applications like marine propulsion, electric drives, conveyor belts, and rolling mills [1], these dampers can be found wherever the occurrence of impact loads is unavoidable. The reason lies in their ability to isolate vibration and mitigate impact loads [2]. Until a few decades ago, it was sufficient to solve vibration problems by integrating any elastomer damper into the respective system when undesirable vibrations occurred. The only condition was that the damper withstood the mechanical loads. Since the noise-vibration and harshness (NVH) behaviour of drivetrains is increasingly perceived as a safety and quality feature [3], a more differentiated approach in solving vibration problems is needed today. NVH models of many commonly used drive components are constantly being developed [4–6]. This development enables model-based prediction and optimisation of systems in the development process. However, the modelling quality of elastomer components is below that of other drive components [7] and thus renders system optimisation less effective.

To predict the transmission behaviour of elastomer components it is necessary to correctly predict the behaviour of the material elastomer itself. This prediction is impeded by a multitude of material effects [8]: The correlation of material behaviour and temperature is strongly non-linear [9]. The so-called Mullins effect softens the material and only occurs during the first few loading cycles [10]. As seen in Fig. 1, three further material effects, namely hyperelasticity, viscoelasticity, and Payne effect, are decisive for the material behaviour.

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Hyperelasticity causes non-linear stress–strain ratios even under quasi-static conditions (Fig. 1(a) [12, 13]. Viscoelasticity results in time–frequency dependence of the material (Fig. 1(b) [14, 15]. The Payne effect describes changes in stiffness and damping depending on the load amplitude (Fig. 1(c) [16, 17]. The most commonly used approaches to model the resulting behaviour of these effects are constitutive-analytical approaches and finite-element simulation. However, all approaches need corresponding experimental data of the to-be-modelled use-conditions either for validation or parameter identification purposes.

To obtain these experimental data, dynamic experiments need to be conducted. These experiments are usually based on subjecting the investigated test specimen or component to sinusoidal loads in a dynamic mechanical analysis (DMA). During the experiment, information on the material behaviour is gathered by measuring the response deformation [18]. By changing frequency and amplitude separately, the viscoelasticity and the Payne effect can be quantified. A commonly applied extension of the DMA is the dynamic mechanical thermal analysis (DMTA) [19]. The DMTA applies a method developed by Williams et al. to infer frequency changes through temperature changes [20]. On that basis, the investigated frequency range can be extended, by performing DMA at different temperatures. Validating a modelling approach, however, cannot be based on DMTA, validating a model based on another model contradicts the purpose of validation. Therefore, a testing device is required to reproduce the entire in-use load spectrum of the investigated component. If this requirement can be met depends on both the test device and the to-be-investigated load spectrum.

Regarding the load spectrum, elastomer dampers usually exhibit low stiffness and are subjected to large load amplitudes caused by impact loads. The combination of both causes oscillations at large deformation amplitudes above 5% strain and frequencies above 50 Hz, as impacts can excite high natural frequencies.

The most commonly deployed test devices are servo-hydraulic test benches, which can investigate high frequencies up to 200 Hz due to fast switching valves. Proper test setups can also be used for large amplitudes. However, the combination of both, large amplitudes and high frequencies, may not be achieved due to test bench limitations. The limitations are a capped oil supply through hydraulic valves and resonances of the test setup, which increasingly superimpose test results at higher frequencies. The application of servo-hydraulic test methods therefore does not allow impact testing. The other commonly used dynamic test device, electrodynamic test benches, exhibit the same limitation: They can be used to test elastomer components at frequencies up to 2 kHz, but only at deformation amplitudes less than 1% [21, 22].

Since commonly used test devices are insufficient for testing impact subjected elastomer dampers, alternative testing devices and corresponding testing procedures are increasingly sought in research. Millet and Bourne, for example investigated the impact reaction behaviour of polychloroprene, which they placed between steel plates and bombarded with a gas gun. Thereby they were able to demonstrate changes in the properties of the material compared to slower hydraulic test methods [23]. Kren and Naumov used electromagnets to cause an indenter to strike an elastomer sample at a precisely defined velocity. Measurements of material relaxation provided information on time-dependent material behaviour [24]. Both Freidenberg et al. [25] and MeraM [26] used drop towers to subject elastomer samples to impact loads. By varying the impact velocity, viscoelastic material parameters were obtained for FE simulation.

Especially impact tests on drop towers proved to be a sufficient alternative for dynamic investigations of elastomer material specimens. Still, there is no test setup available today that allows for testing different elastomer parts other than test samples. Especially rotationally loaded parts to be characterized in the rotational degree of freedom cannot be investigated by means of impact testing yet. Additionally, required variation of test parameters, in particular of the loading speed, is not yet investigated to a sufficient extent.

The aim of this work is to introduce a low-cost experimental procedure to characterize the rotational transmission behaviour of elastomer dampers under typical in-use working conditions, including impact loads. Using a drop tower for lateral impact generation on a lever arm was found to be a sufficient approach. A test configuration was prototyped and used for testing an elastomer coupling.
Compared to tests on servo-hydraulic test rigs, the method proved to produce qualitatively and quantitatively correct results. It was possible to investigate parameter combinations on the prototype that cannot or only with great difficulty be achieved on servo-hydraulic test benches.

**Material and Methods**

**Theory of Impact Investigation for Measuring Rotational Transmission Behaviour**

To predict the behaviour of elastomer couplings, their mechanical properties can be easily modelled as a one degree of freedom spring-damper system [27]. Figure 2 shows the corresponding system in which the investigated elastomer damper is represented by a spring and a damper. In this work, the mass reflects the system subjected to impact loads. The accuracy of this simplified model can be amplified by implementing the dependence of the elastomer’s stiffness and damping on frequency and amplitude. For an experimental method to potentially replace established methods, it has to offer the ability to determine the system’s parameters stiffness and damping at different frequencies, amplitudes and preloads.

To understand which measurement data needs to be obtained, the system’s equation of motion can be derived from equality of forces (equation (1)). The equation can be equally applied to a rotational system by replacing mass \( m \) by inertia \( J \) and longitudinal position \( x \) by rotational angle \( \varphi \), equation (2).

\[
m\ddot{x} + d\dot{x} + cx = 0 \quad (1)
\]

\[
J\ddot{\varphi} + d\dot{\varphi} + c\varphi = 0 \quad (2)
\]

Considering the system’s undamped rotational eigenfrequency \( \omega_0 = \sqrt{c/J} \), the damped frequency \( \omega = \omega_0 \cdot \sqrt{1 - D^2} \), Lehr’s damping measure \( D = d / (2J\omega_0) \) and decay coefficient \( \delta = d / 2J \), the approach \( \varphi(t) = \Phi e^{i\omega t} \) can be used to find the solution in equation (3).

\[
\varphi(t) = e^{-\delta t} (\Phi_1 e^{i\omega t} + \Phi_2 e^{-i\omega t}) = \Phi \cdot e^{-\delta t} \cos(\omega t - \Phi) \quad (3)
\]

In this work, it is assumed that an impact induces energy \( E_{\text{imp}} \) into the one-mass spring-damper system, which beforehand was neither strained nor in movement. The impact duration is assumed to be very short compared to the cycle time and is therefore assumed to end at \( t = 0 \). Since the impact happens very fast, the system is still in a non-deformed state (i.e., \( \varphi(t = 0) = 0 \)) after the impact, which allows determining \( \Phi = \pi / 2 \) as the only non-trivial solution. Directly after impact, the system’s whole energy is stored as kinetic energy \( E_{\text{kin}} \) according to equation (4).

\[
E_{\text{imp}} = E_{\text{kin}} = \frac{1}{2} \cdot J \cdot \varphi(t = 0)^2 \quad (4)
\]

The fact that the potential Energy of the spring \( E_{\text{pot}} = 1/2 \cdot c \cdot \Phi^2 \) in an undamped system \( \delta = 0 \) has to carry the system’s whole energy allows for calculating the angular amplitude \( \Phi \) in equation (5).

\[
\Phi = \sqrt{2E_{\text{imp}} / c} \quad (5)
\]

One example of the resulting movement is shown in Fig. 3, which represents an approximation of the expected measurement data. Note that this simplified system neglects the fact that stiffness and damping of elastomer components change their properties during the experiment based on current deformation and load.

Figure 3 shows a damped, thus decaying, sinusoidal oscillation of both torsional angle and transmitted torque. The decomposition of the torque into a viscous component and an elastic component illustrates the phase shift between the total torque and the torsional angle. Also shown in Fig. 3 is the energy of the overall system, which is composed of the kinetic energy of the moving mass and the potential energy of the spring and decreases logarithmically. From the shown course of measured torque and angle, it is possible to determine the sought-after parameters frequency, angular amplitude, preload, stiffness and damping.

It is necessary to ensure that the measured system performs an oscillation which is not influenced from the outside. The torsional angle and the torque transmitted by the coupling must be recorded during this oscillation. The data acquisition must cover at least three quarters of an oscillation period to be able to deduce damping from the decrease of the amplitude. To make it possible to derive characteristic diagrams of the coupling’s properties, varying the test parameters is obligatory: The frequency can be influenced by varying the inertia and the amplitude by varying the energy input.

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Fig. 2 One mass spring damper system: \( c \) represents the spring’s stiffness, \( d \) is the damping coefficient, \( m \) stands for the mass in movement and \( x \) is the time varying position of the mass
Theory of Impact Design

According to Theory of Impact Investigation for Measuring Rotational Transmission Behaviour, the basic test principle consists of transferring energy by impact contact into a system capable of oscillating freely. Since elastomer couplings are loaded in torsional degree of freedom, the resulting oscillation has to be a rotation. This makes it necessary to either have a translational impact force act on a lever arm or to bring two rotational systems into impact contact. Since the latter is more difficult to implement, the translational approach was applied in this work: An impact mass brought two rotational systems into impact contact. Since elastomer couplings are loaded in torsional degree of freedom, the resulting system capable of oscillating freely. Since elastomer couplings are loaded in torsional degree of freedom, the resulting oscillation has to be a rotation. This makes it necessary to either have a translational impact force act on a lever arm or to bring two rotational systems into impact contact. Since the latter is more difficult to implement, the translational approach was applied in this work: An impact mass.

If masses that move relatively to each other are brought into contact, the faster moving mass transfers energy and momentum to the other mass until the contact ends—a process described by the impact law. Considering a translational motion of mass \( m_1 \) with velocity \( v_1 \), stationary mass \( m_2 \) and coefficient of restitution \( k \), the resulting velocities \( v'_1 \) and \( v'_2 \) of both masses can be calculated according to equations (6) and (7) [28].

\[
\begin{align*}
v'_1 &= \frac{m_1 \cdot v_1 + m_2 \cdot v_1 \cdot k}{m_1 + m_2} \\
v'_2 &= \frac{m_1 \cdot v_1 - m_1 \cdot v_1 \cdot k}{m_1 + m_2}
\end{align*}
\]

Equations (6) and (7) can be applied equivalently to two rotational systems or to systems consisting of a combination of both. For this purpose, rotational and translational systems can be converted into equivalent systems by calculating substitute masses and velocities according to equations (8) and inserting the result in equations (6) and (7). In this context, \( L \) stands for the lever arm length, i.e. the orthogonal distance from rotational axis to impact point. \( J \) is the inertia and \( \omega_i \) represents the angular velocity.

\[
m_i = \frac{J_i}{L^2} \quad \text{and} \quad v_i = \omega_i \cdot L
\]

Based on the calculated velocities, the equations of motion of the lever arm and the drop mass after impact can be determined. The velocity \( v_1 \) of the drop mass before impact is only dependant on drop height \( h \) and gravity constant \( g \) and equals \( v_1 = \sqrt{2 \cdot h \cdot g} \), which leads to velocity \( v'_1 \) of the drop mass and velocity \( v'_2 \) of the tip of the lever arm after impact in equation (9).

\[
v'_1 = \frac{\sqrt{2 \cdot h \cdot g} \cdot (m_1 + \frac{J_2}{L^2} \cdot k)}{m_1 + \frac{J_2}{L^2}} \quad v'_2 = \frac{\sqrt{2 \cdot h \cdot g} \cdot (m_1 - \frac{J_2}{L^2} \cdot k)}{m_1 + \frac{J_2}{L^2}}
\]

As the drop mass continues to move freely after impact, the only influencing factor on the motion of the mass is gravity, with results in the equation of motion in equation (10).

\[
x_1(t) = v'_1 \cdot t + 0.5 \cdot g \cdot r^2
\]

To obtain the translational motion of the tip of the lever arm, the rotational equation of motion (3) is transformed into a translational movement, which requires for the translational amplitude \( \hat{x} \). Taking into account equation (5) while considering \( E_{\text{imp}} = 0.5 \cdot m_2 \cdot v^2_2 = 0.5 \cdot \frac{J_2}{L^2} \cdot v^2_2 \) and the translational stiffness \( c_{\text{trans}} = c/L^2 \), results in equation (11) which finally delivers the equation of motion in equation (12).

\[
\hat{x} = \sqrt{\frac{2 \cdot E_{\text{imp}}}{c_{\text{trans}}}} = \sqrt{\frac{J_2 \cdot v_2^2}{c}}
\]
Figure 4 visualises the movements and by that the expected test sequence after impact for different inertias of the lever arm \( J \). It can be seen that the drop mass is accelerated towards the lever arm until impact at \( t = 0 \). The impact leads to either a reversal of direction of motion \( (J = 0.1) \) or only to a slowing down of the motion \( (J = 0.15 \) and \( J = 0.5 \)). If no reversal is achieved or if the escape velocity of mass 1 after the impact is too low \( (J = 0.15) \), the masses undergo a second impact within the first oscillation period of the lever arm. As measurement data are expected to decrease in quality by a second impact, mass ratios must be chosen in a way that no second impact is evident. To verify whether specific combinations of parameters result in a second impact, equations (10) and (12) can be equated and solved numerically. If the calculated impact-times is greater than the lever arm’s cycle time, the parameter combination does not cause a double impact.

**Experimental Setup**

**Drop Tower**

To implement the test concept described in *Theory of Impact Investigation for Measuring Rotational Transmission Behaviour* and *Theory of Impact Design*, the test rig shown in Fig. 5 was constructed. In the drop tower, a drop mass can be positioned at various heights by a rope hoist while a rope sensor measures the height. The mass is held in place by a permanent magnet which can be rendered ineffective by application of an electrical current to initiate the experiment. The impact structure is positioned below the drop tower. The tested elastomer coupling is connected to a lever arm on the one side and to a flange on the other side, which are both mounted on ball-bearings. The test setup is positioned in a way, that the drop mass comes into contact with a piezo sensor (PCB: 200C50) at the end of the lever arm, which measures the impact force. The coupling flange is connected to a force sensor (HBM: K-U10M) whose measurements allow for calculating torques transmitted by the coupling. A laser sensor (Keyence: LK-H152) detects the coupling’s torsional angle indirectly via a distance measurement. Data acquisition is performed with a measurement amplifier (HBM: Quantum X) and the downstream software Catman (HBM) attaining a maximum sampling rate of 38 kHz. The total cost of the configuration is below 15,000 €.

Parameter variation is achieved by various drop heights, different drop masses and different additional masses on the lever arm. By applying a force on the lever arm using a
screw, the coupling can be pre-deformed prior to the experiment. Available parameters are listed in Table 1, including catalogue parameters of the investigated coupling’s damping ratio $d_{ratio} = E_{diss}/E_{el}$ as quotient of the dissipated energy per full load cycle $E_{diss}$ and the stored elastic energy $E_{el}$. Also given are information on the coupling’s material. Detailed information on the elastomer compound was not provided by the manufacturer.

The tested coupling is a full volume elastomer disc coupling, in which the torque is transmitted in axial direction through an elastomer ring with a thickness of approximately 50 mm, an inner diameter of 200 mm and an outer diameter of 270 mm. Due to the ring-shaped design, the elastomer undergoes almost pure shear, which results in geometrically linear and backlash-free transmission behaviour.

### Servo-Hydraulic Test Setup

The servo-hydraulic test setup shown in Fig. 6 is used to validate the measurement results of the drop tower. In the servohydraulic test setup, elastomer couplings can be subjected to dynamic torsional loads by a hydraulic actuator (Schenk hydropuls rotary actuator 8kNm). The torsional angle of the coupling as well as the torque transmitted by the coupling are measured during the test. The investigable angle is $\pm 20^\circ$, the maximum test frequency is 200Hz and the maximum torque is $\pm 8kNm$. Measurement data can be recorded with a maximum sampling rate of 2048Hz with an uncertainty of $\pm 5Nm$ for the torque and $\pm 0.02^\circ$ for the angle.

### Design of Experiment

The drop tower experiments were planned to investigate the entire parameter space in order to show that both, the amplitude and frequency dependence of the elastomer damper, can be correctly captured. Table 2 lists all parameters which can be changed independently. In the first column, the available inertias of the lever arm and the drop masses are given, as they cannot be changed independently of each other: Following Theory of Impact Design, a combination of 2kg drop mass and rotational inertia 1 would result in a double impact. For this reason, the experiments with inertia 1 were carried out with the small 1kg drop mass (subsequently referenced as case 1). For the experiments with inertia 2 (subsequently referenced as case 2) and inertia 3 (subsequently referenced as case 3) the 2kg drop mass was deployed. Every possible combination of the three different inertias, ten different drop heights, and three different preloads shown in Table 2 was investigated separately. For statistical insurance, each test was carried out five times. This results in a total of $3 \times 10 \times 3 \times 5 = 450$ recorded impact tests.

Prior to the experiments, the coupling was pre-strained to avoid an influence of the Mullins effect. The applied pre-straining procedure is visualized in Fig. 7 and includes

| Table 1 Test parameters of the drop tower and properties of the investigated elastomer coupling |
|-----------------------------------------------------------------------------------------------|
| Drop height $h$ [mm] | 5 – 6000 |
| Length lever arm [mm] | 200 |
| Coupling catalogue stiffness [Nm/rad] | 916 |
| Coupling catalogue damping ratio [-] | 1.6 |
| Rotating Inertia $J_{rot}$ [kg m$^2$] | 1: 0.1557 Material: Natural rubber |
| | 2: 0.359 Shore A hardness: 60 |
| | 3: 0.5233 |
| Drop mass $m_{drop}$ [kg] | 1 kg, 2 kg Quasistatic shear modulus [N/mm$^2$] | 0.59 |

| Table 2 Parameters in drop tower test. Each possible combination of the parameters was tested five times |
|---------------------------------------------------------------|
| Inertia [kg m$^2$] / drop mass [kg] / drop height [m] / Preload [kNm] |
| 0.1557 / 1 (case 1) | 0.05 | 0 |
| 0.359 / 2 (case 2) | 0.2 | 0.5 |
| 0.5233 / 2 (case 3) | 0.4 | 1 |
| | 0.55 | |
| | 0.7 | |
| | 1 | |
| | 1.4 | |
| | 1.8 | |
| | 3.5 | |
| | 6 | |

![Fig. 6 Servohydraulic test bench with mounted elastomer coupling](image-url)
subjecting the coupling to three very slow oscillations of the torsional angle. The constant amplitude of 28° is the highest angular amplitude, which the tested coupling can withstand without damage. After these large deformations, the coupling’s material exhibits inner tensions by the Payne effect. Therefore, the amplitude is slowly reduced to zero at a higher frequency (∼100 oscillation cycles). After this procedure, the coupling is left to rest for 30 min for inner stresses caused by viscoelastic material behaviour to dissipate. The Mullins effect can be seen in a decrease of the torque amplitude during the first two oscillation cycles. The pre-straining procedure was considered sufficient as no further decrease can be observed in the third cycle, while the angular amplitude is kept constant.

The tests on the servo-hydraulic test rig are intended to provide comparative values for validation. To map the influence of the frequency, three frequency sweeps were performed in which the load frequency was varied at constant angular amplitude. The amplitude dependency was tested by applying amplitude sweeps, in which the load amplitude was varied at constant frequency.

Results and Discussion

Experimental Data in Time Domain

In every drop tower experiment, measurement data regarding the impact force, torsional angle and transmitted torque are obtained in time domain. Figure 8 shows examples of test results obtained for three different combinations of test parameters without preload. By a detailed investigation of the measured impact torque, it can be seen that the duration of the force application due to impact is approx.0.3ms. The impact duration decreases slightly by reducing the drop mass. The maximum applied torque increases with increasing drop height, increasing drop mass and increasing inertia of the rotating mass and amounts to 26.2kNm. The course of the rotational angle immediately after the impact shows the expected decaying sinusoidal oscillation described in Theory of Impact Investigation for Measuring Rotational Transmission Behaviour. The frequency of the oscillation increases with reduced inertia, the amplitude increases with increased drop height. It becomes clear that the assumption of the system as a one-mass spring-damper is justified. With a minimum cycle time greater than 20ms, the assumption of a very short impact duration (0.3ms) also seems appropriate Fig. 8 shows that along with the decaying sinusoidal oscillation on the torque trend over time, additional superimposed oscillations are present. Since these oscillations are not reflected in the measured values for the torsion angle, they are presumable due to resonances in the test setup on the force measurement side and higher natural frequencies of the elastomer coupling.

Evaluation of Experimental Results and Parameter Identification

The observed superimposed torque-oscillations impede determining the sought coupling parameters. If, for example,
the coupling’s stiffness was calculated from the quotient of the maximum angle and the torque measured at the same time, the oscillations would result in a random variance of ±55%. For this reason, the approach derived in Theory of Impact Investigation for Measuring Rotational Transmission Behaviour is used to improve the quality of analysis. Equation (3) is adapted to the course of the measured angle by means of quadratic optimisation. By taking into account the variable parameters \( \hat{\phi}_{var}, \delta_{var}, \omega_{var} \) and \( \Phi_{var} \) the approach function in equation (13), the error function with the square weighted error \( \text{erf} \) and the measured angle \( \phi_{meas} \) in equation (14) are obtained. The quality of the fit can be optimized by varying the length of the time window \( t_{win} \):

\[
\phi(t) = \hat{\phi}_{var} \cdot e^{-\delta_{var} \cos(\omega_{var} t)}
\]

\[
\text{erf} \left( \hat{\phi}_{var}, \delta_{var}, \omega_{var}, \Phi_{var} \right) = \frac{1}{\sum_{(t)} \sqrt{(\phi(t))^2 - \phi_{max}^2(t)}}
\]

The upper graph in Fig. 9 shows the curve fit for the angle course also shown in Fig. 8. Only the first complete cycle time is considered for the fitting. It is clear that the selected fitting function reproduces the measurement data of the first oscillation period with high accuracy. However, the longer the test lasts, the less accurate the prediction of the transfer behaviour becomes. The reason for that is the change in material properties during the test: The PAYNE effect causes an increase in stiffness when the amplitude decreases due to damping, which leads to a higher frequency. This effect is neglected in the single-mass spring-damper model.

From the identified parameters, the sought-after parameters stiffness and damping can be determined directly according to equations (15) and (16).

\[
c_{dyn,M1} = J \cdot (\omega_{var}^2 + (\delta_{var} \cdot 2 \cdot J)^2)
\]

\[
d_{ratio} = \frac{E_{diss}}{E_{ef}} = \frac{1}{2 \cdot \left( e^{-\frac{t}{\tau_{meas}}} \right)^2 - 1}
\]

To verify whether this calculation is valid, the measured torque signal was used for a comparative evaluation and for that was also analysed via curve fit. It was considered that the measured angle is proportional to the deformation of the coupling material. Since this deformation causes the reaction torque, the frequency of the measured torque curves must correspond to the frequency of the measured angle.

The same applies to the time-dependent amplitude decay, so that the variable parameters \( \hat{\phi}_{var,T} \) and \( \Phi_{var,T} \) can be used according to the angle function. The function in equation (17) with the variable parameters \( \hat{\phi}_{var,T} \) and \( \Phi_{var,T} \) was fitted to the torque measurement data. The result of the fit is shown in the lower diagram in Fig. 9.

\[
T(t) = \hat{\phi}_{var,T} \cdot e^{-\delta_{var} \cos(\omega_{var} t - \Phi_{var,T})}
\]

Table 3 shows the identified parameters. The table also includes a column showing a comparison of coupling parameters determined from curve fitting to angle measurement (equations (15) and (16)) to coupling stiffness \( c_{dyn,M2} = T(\varphi = \varphi_{max}) / \varphi_{max} \) calculated with respect to the measured torque. It is clear to see that both methods provide similar results. Discrepancies between the stiffnesses remain less than 5%. These probably originate from inaccuracies in the determination of the phase shift between the angle and torque curves due to the superimposed oscillations in the torque signal. From this point, only parameters, which are calculated based on the course of the angle, are considered.

Since the dependency of stiffness and damping on frequency and angular amplitude is to be determined in the drop tower test, it is necessary to calculate a reference frequency and a reference angular amplitude which are characteristic for the respective test. The frequency can be read

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**Table 3** Function parameters and resulting stiffness and damping

| No | \( \hat{\phi}_{var} \) | \( \delta_{var} \) | \( \omega_{var} \) | \( \hat{\phi}_{var,T} \) | \( \Phi_{var,T} \) | \( c_{dyn,M1} \) | \( c_{dyn,M2} \) | \( d_{ratio} \) |
|----|-----------------|----------------|---------------|-----------------|----------------|----------------|----------------|---------|
| 1  | 1.6473          | 33.031         | 293.64        | 0.4487          | 282.92         | 254.341        | 267.490        | 1.5552  |
| 2  | 2.8063          | 18.373         | 164.81        | 0.5672          | 176.10         | 205.306        | 194.464        | 1.5295  |
| 3  | 0.6261          | 16.426         | 154.59        | 0.2002          | 157.24         | 313.555        | 315.711        | 1.4005  |
directly from Table 3 as relative to angular velocity $\omega_{\text{var}}$. However, the angular amplitude decreases in the course of the experiment due to the damping of the elastomer coupling. For this reason, the amplitude in the middle of the evaluated time period according to equation (18) is assumed as the reference angular amplitude $\hat{\varphi}_{\text{var,ref}}$.

\[
\hat{\varphi}_{\text{var,ref}} = \hat{\varphi}_{\text{var}} \cdot e^{-\delta_{\text{var}} t_{\text{max}}/2}
\]  

Parameter Variation

Since the influence of different load conditions on the coupling behaviour is to be investigated at the drop tower, it is necessary to vary test parameters. As described in Drop Tower, the influence of frequency and angular amplitude on the coupling behaviour needs to be examined. The angular amplitude can be increased easily by introducing additional kinetic energy into the oscillating system. For this purpose, either the drop mass or the drop height can be increased. Since a larger drop mass potentially leads to multiple contacts between the lever arm and the drop mass, as described in Theory of Impact Design, the drop height was changed first and foremost. Figure 10(a) shows the influence of the ten adjustable drop heights on the angular amplitude. For each drop height five impact tests were performed. It becomes clear that the variation of the drop height has the desired effect on the angular amplitude.

To investigate different frequencies, it is possible to attach additional mass to the lever arm. Thus, the natural frequency of the system is decreased. Figure 10(b) shows the influence of different inertias on the frequency. As expected, the frequency decreases with increasing inertia. However, the frequency is also subject to change without changes of the inertia. The observed decrease in frequency with increasing angular amplitude is a consequence of the Payne effect described in Theory of Impact Investigation for Measuring Rotational Transmission Behaviour. Larger amplitudes can lead to a lower stiffness of the elastomer, resulting in a lower frequency. Figure 10(c) illustrates this effect and furthermore the viscoelastic behaviour of the elastomer, which leads to an increase in stiffness with higher frequency. Table 4 gives an overview of the determined coupling parameters for different combinations of drop height and inertia.

Preloading

As an additional crucial parameter that may influence the coupling behaviour, the preload has the potential to cause significant changes in the transmission behaviour as well. The mechanisms of this influence are the hyperelastic material behaviour (cf. chapter 1) and non-linear geometries. A non-linear geometry is particularly present if the topology of a component under load deviates significantly from the topology in the unloaded state.

According to Drop Tower, the preload can be applied to the elastomer coupling before the experiment is carried out. Figure 11 shows that no influence of this preload on the recorded coupling behaviour can be observed, although the measurement results are consistent even under preload. The result suggests that the drop tower might also be used to detect the influence of preload on geometrically non-linear couplings. Unlike that, the proof can only be provided in future work, since the coupling under investigation shows geometrically linear as well as non-hyperelastic behaviour.

Constitutive Modelling

As described in Parameter Variation, the parameters frequency and angular amplitude cannot be controlled independently of each other. If, for example, the drop height is changed in order to influence the angular amplitude, the resulting frequency is also subject to change. Thus, it is particularly difficult in the drop tower to specifically set certain parameter combinations. A model description of the measured behaviour is required, which allows for conclusions to be drawn from the investigated conditions to other conditions.

In this work, a descriptive model was developed based on an approach developed by Austrell [29] to consider the viscoelastic material behaviour and the Payne effect independent of each other. By that, the total stiffness $c_{\text{total}}$ is the sum of viscoelastic stiffness $c_{\text{vis}}$ and elastoplastic stiffness $c_{\text{EP}}$. The elastoplastic stiffness is modeled as a two-member Prony series due to the observed degressive relationship between stiffness and angular amplitude in analogy to the commonly used modeling of elasto-plasticity as a spring-friction element [30]. It is known from literature that viscoelastic stiffness follows an exponential relationship with
frequency [31]. Following this, the viscoelastic stiffness is mapped proportionally to $\sqrt{f}$ and an additional Prony term. The total stiffness $c_{\text{total}}(\hat{\phi}, f) = c_{\text{EP}}(\hat{\phi}) + c_{\text{vis}}(f)$ is thus obtained as a function of the four elastoplastic Prony parameters $k_1$ to $k_4$ and the viscoelastic model parameter $k_5$ to $k_7$ in equation (19).

$$c(\hat{\phi}, f) = k_1 \cdot 10^{-k_2 \hat{\phi}} + k_3 \cdot 10^{-k_4 \hat{\phi}} + k_5 \cdot \sqrt{f} + k_6 \cdot 10^{-k_7 \hat{\phi}}$$ (19)

Equation (19) represents a continuous plane equation in three-dimensional space of stiffness, amplitude and frequency and can thus be adapted to the measurement results by means of quadratic optimisation. To achieve this optimisation, the Matlab Curve Fitting Toolbox was applied. Figure 12 shows the calculated model parameters and compares experimentally determined coupling parameters with model predictions. It can be clearly seen that the model, visualised in the right diagram in Fig. 12, is able to accurately reproduce the experimental results.

To simplify the model of the coupling’s damping, it is considered that, in accordance with the double logarithmic representation in Fig. 1(b), stiffness and damping run as straight lines over a wide frequency range. Their relationship can thus be described as a frequency-independent factor. If the dependence of this factor on the load amplitude is modelled as a two-part Prony series, in the same way as in equation (19), equation (20) is obtained. If the stiffness in equation (19) is applied to equation (20), the amplitude- and frequency-dependent damping can be determined.

$$\frac{c(\hat{\phi}, f)}{d(\hat{\phi})} = k_8 \cdot 10^{-k_9 \hat{\phi}} + k_{10} \cdot 10^{-k_{11} \hat{\phi}} + k_{12} \cdot 10^{-k_{13} \hat{\phi}}$$ (20)

Since equation (20) describes only a two-dimensional correlation, it can easily be adapted to the existing measured data by quadratic optimisation. Figure 13 shows the determined model parameters $k_8$ to $k_{13}$ as well as a comparison of measurement data and model prediction. The measured data for damping are subject to slightly higher dispersion than the measured data for stiffness in Fig. 12, but the results of the test cases clearly follow the same relationship, which is reproduced with moderate deviation from the model.

### Measurements on Servo-Hydraulic Test Bench

The experimental setup described in Servo-Hydraulic Test Setup was used to generate reference data to validate drop tower measurements. Three frequency sweeps were performed at a respective constant amplitude of 0.5°, 1.5° and 2.5°. In these

| $\hat{\phi}$ (deg) | $f$ (Hz) | $c$ (Nm/deg) | $d$ | $\hat{\phi}$ (deg) | $f$ (Hz) | $c$ (Nm/deg) | $d$ |
|------------------|---------|---------------|-----|------------------|---------|---------------|-----|
| 0.172            | 60.18   | 390.0         | 0.620 | 0.982           | 28.38   | 201.6         | 1.484 |
| 0.305            | 55.26   | 330.0         | 0.991 | 0.836           | 29.21   | 213.6         | 1.468 |
| 0.481            | 50.65   | 278.4         | 1.543 | 1.559           | 26.37   | 174.2         | 1.522 |
| 0.570            | 49.13   | 262.6         | 1.759 | 1.775           | 25.80   | 166.6         | 1.447 |
| 0.660            | 47.82   | 249.1         | 1.900 | 2.721           | 24.46   | 149.6         | 1.351 |
| 0.889            | 45.32   | 223.9         | 1.957 | 3.201           | 23.92   | 142.9         | 1.252 |
| 0.652            | 47.98   | 250.8         | 1.883 | 0.144           | 30.71   | 341.2         | 0.505 |
| 1.121            | 43.64   | 207.4         | 1.871 | 0.334           | 27.90   | 282.4         | 0.814 |
| 0.996            | 44.50   | 215.9         | 2.010 | 0.539           | 25.87   | 243.1         | 0.957 |
| 1.765            | 40.78   | 181.0         | 1.735 | 0.621           | 25.09   | 228.9         | 1.076 |
| 2.295            | 39.20   | 167.0         | 1.587 | 0.740           | 24.32   | 215.2         | 1.140 |
| 0.242            | 36.13   | 325.0         | 0.862 | 0.867           | 23.51   | 201.7         | 1.442 |
| 0.458            | 32.77   | 267.9         | 1.094 | 1.046           | 22.91   | 191.4         | 1.419 |
| 0.693            | 30.60   | 234.0         | 1.275 | 1.503           | 21.79   | 173.3         | 1.467 |
| 0.814            | 29.54   | 218.3         | 1.399 | 1.944           | 21.04   | 161.4         | 1.363 |

### Table 4 Characteristic diagram of coupling’s stiffness and damping relative to amplitude and frequency

![Fig. 11 Influence of preload on the coupling’s transmission behaviour](image-url)
sweeps the maximum frequency was determined by the test bench’s maximum oil flow rate to 100 Hz, 60 Hz and 20 Hz, respectively. Furthermore, three amplitude sweeps were carried out at constant frequencies of 1 Hz, 10 Hz and 30 Hz, with respective maximum angular amplitudes of 10°, 6° and 2°.

The evaluation of the experiments is carried out according to [32], whereby the Savitzky-Golay filter with a width of an eighth of a cycle time is used to smooth the test data. The test parameters determined in the test evaluation are always considered and not the target specifications for the test rig.

Validation of Drop Tower Results

To validate the developed model (cf. Constitutive Modelling) and by that to also the proposed drop tower testing method, parameters predicted by the model were compared to parameters derived from measurements on the servo-hydraulic test bench (cf. Servo-Hydraulic Test Setup and Measurements on Servo-Hydraulic Test Bench). The model was not adapted to the values determined on the servo-hydraulic test bench.

Figure 14(a) visualizes stiffness values from servohydraulic experiments in three amplitude sweeps each at a constant frequency of 1 Hz, 10 Hz and 30 Hz, respectively. Further, the corresponding stiffness predictions by the drop tower model are shown. It is clear to see that both, measured stiffness and predicted stiffness, follow the same qualitative course. The stiffness decreases with increasing amplitudes (by a factor of 3 within the observed parameter space) and is generally higher at higher frequencies. Measurement and prediction are in good quantitative agreement with deviations below 2%, even at amplitudes above 3.5° and below 0.3° which are beyond the measuring range of the drop tower.

Looking at the measured and predicted damping ratios in Fig. 14(b) for the same amplitude sweeps, qualitatively identical courses can be found. At small angles, the damping ratio initially increases with increasing angle, reaches a plateau and then decreases. Both, the existence of the plateau and the angular amplitude at which the plateau occurs, are correctly predicted by the model. Furthermore, an increase in the relative damping with increasing frequency can be observed, which is also predicted correctly. Regarding the quantitative quality of the prediction a significant decrease can be observed above and below the measuring range of the drop tower. However, within the measuring range the error stays below 6.5%.

A comparison of predicted and measured stiffness in three frequency sweeps each at a constant angular amplitude of 0.5°, 1.5° and 2.5°, respectively, is given in Fig. 14(c). Again an excellent qualitative agreement can be seen. Model and measurement exhibit increasing stiffness with increasing frequency and generally higher stiffness at smaller angular amplitudes. Within the measuring range of the drop tower, above 20 Hz, almost no deviation between measured and predicted values (< 1%) is to be observed. At smaller frequencies, the deviation amounts up to 6%.

Damping ratio predictions and measurements in the same frequency sweeps are compared in Fig. 14(d). An increase
in damping ratio with increasing frequency can be observed in measurement and prediction. Within the measuring range of the drop tower quantitative deviations are small (<2%) and amount up to 12% at lower frequencies.

All in all, the change of the investigated parameters stiffness and damping within the tested parameter space amounts to 300% and 200%, respectively. The observed discrepancy of model and measurements stays below 6% for stiffness and about 7% for damping. That means that most of the respective changes are predicted correctly. Increasing deviations can be observed especially at amplitudes and frequencies beyond the measuring range of the drop tower, which indicates that most of the error evolves from extrapolating the drop tower measurements to the servo-hydraulic measuring range. Nevertheless, it is clear to see that measured values and predictions are quantitatively close to each other and show the exact same qualitative progression.

Conclusions

An experimental set-up was designed and developed based on the theory of impact law to investigate the dynamic transmission behaviour of torsional-impact subjected elastomer dampers. The setup consists of a drop tower in which a drop mass is released from a precisely adjustable height. The drop mass impacts a lever arm connected to the test damper which results in a torsional oscillation. It was found that this oscillation corresponds to the expected motion of a single-mass spring-damper for at least one cycle of motion. An optimisation-based adaptation of the corresponding equation of motion to experimental results allows for determining stiffness and damping parameters of the coupling. Test parameters can be adjusted by varying the drop height and the mass of the lever arm. In 400 impact tests, a characteristic diagram of the coupling’s parameters was made. Based on the overlay model, plane equations were developed in parameter space of frequency, load amplitude, and stiffness resp. damping and adapted to measurement results by optimisation-based parameter identification. The equations allow for predicting the damper’s behaviour for conditions which are not considered in experiments. The entire evaluation process was validated by comparing the calibrated model to experiments on a servo-hydraulic test bench. It was shown that the drop tower, even in its prototypical implementation, delivers measurement results that quantitatively correspond to servo-hydraulic measurements. A significantly lower scatter of the results can be achieved with investment costs many times lower (<10%).
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Declarations

Conflict of Interest The authors declare that they have no conflict of interest.

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