Maximizing time and energy resolution for photons detected by transition-edge sensors

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(Dated: December 16, 2019)

Background: X-ray telescopes are powerful tools for the study of neutron stars and black holes. In order to probe such fascinating astrophysical objects future X-ray telescopes will carry superconducting transition-edge sensors. The analysis of the signals produced by X-rays absorption onto transitional-edge sensor is already mature. Several methods have already been developed (e.g., principal component analysis, nonlinear optimal filtering or high-rate processing method) to analyse the pulses that result from X-rays absorbed in these sensors.

Purpose: Our goal is to develop a lightweight, linear filter that will maximize energy and time resolution when X-ray photons are detected by transition-edge sensors. Such a method could be implemented in the new generation of X-ray space telescopes. Furthermore, we find the minimal sampling rate that will not degrade the energy and time-resolution of these techniques.

Method: Our method is designed for the widest range of photon energies (from 0.1 keV to 30 keV). Transition-edge sensors exhibit a non-linear response that becomes more pronounced with increasing photon energy; therefore, we need to treat high-energy photons differently from low-energy photons. In general, the switching-point energy depends on the properties of the detector and corresponds to the energy of the photon that begins to saturate the superconductor to normal transition of the TES (at about 4.6 keV here). In order to retrieve the energy and the arrival time of the photon, we fit simulations of the evolution of the current including the typical noise sources in a sensor with simulated theoretical models. The curve-fitting parameters are interpolated to extract the energy and time resolution.

Results: For energies from 0.1 keV to 30 keV and with a sampling rate of 195 kHz, we obtain a 2$\sigma$-energy resolution between 1.67 eV and 6.43 eV. Those results hold if the sampling rate decreases by a factor two. About time resolution, with a sampling rate of 195 kHz we get a 2$\sigma$-time resolution between 94 ns and 0.55 ns for a sensor with the physical parameters as those used in the HOLMES experiment.

Conclusions: We have successfully developed a new method that enables to maximize the energy and the arrival time of photons detected by a TES, using a very simple implementation. In order to make this method useful on a larger scale, it will be essential to get a more general description of the noise in a TES, and it will be necessary to develop a robust way to identify pile-up events.

I. INTRODUCTION

X-ray telescopes enable us to study fascinating compact objects, such as neutron stars and black holes. In the future, several missions, e.g., Athena [1], Colibrì [2] or Lynx [3], are planned to carry arrays of superconducting transition-edge sensors (TESs). Consequently, maximizing energy resolution and timing for photons detected with such sensors is a crucial goal in X-ray astronomy [4].

Recently, several techniques to analyze X-ray data from transition-edge sensors have been developed, such as principal component analysis [5], optimal filtering of the resistance signal [6], nonlinear optimal filtering [7] and optimal fitting [8–10]. Those techniques achieve an energy resolution between 0.7 keV and 3.4 keV full width at half maximum (FWHM), for low energies (below 6 keV). However, those techniques may be difficult to implement easily on an X-ray space telescope.

Therefore, we propose a new and lighter method, in order to get the energy and the arrival time of photons detected by a TES. We aim to use this technique to predict the behavior of the new generation of non-focusing X-ray telescopes. Indeed, such X-ray telescopes will be studying variable X-ray emissions coming from neutron stars and black holes, and probe the region very close to them, where the dynamical timescales of those region are of the order of microseconds. Since accreting neutron stars and black holes shine bright in the 0.5 – 10 keV range, we developed a technique that maximizes time and energy resolution over the widest range of photon energies (here, from 0.1 keV to 30 keV).

This paper first describes our detector model and how events were simulated. We next address the issue of noise in TESs and present our method that enables to maximize energy resolution and timing. Finally, we discuss the outcomes.

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II. TRANSITION-EDGE SENSORS TO DETECT X-RAYS

A. Detector model

A transition-edge sensor is made of a superconducting metal film functioning near its transition temperature (typically 0.1 K). While electrons move freely in a superconducting metal, they encounter some significant resistance when the metal switches to its normal phase. The transition from superconductor to normal metal occurs within about a narrow 1 mK change in the temperature, but results in a large change in resistance. Thus after an X-ray photon deposits energy in the sensor, the superconductor heats up, the resistance increases, the electric current drops and an X-ray photon is detected.

Indeed, when an X-ray photon hits a TES, the provided energy provokes the needed rise in temperature for the transition to happen; therefore measuring how much the intensity of the current diminishes and how long the sensor takes to recover enables us to determine the energy of that photon. Finally, after the absorption of a photon, a TES needs to be cooled down to its initial temperature by using a cooling bath, at temperature \( T_{\text{bath}} \), in order to be able to detect the next photon. In this work, we simulated current pulses in a TES, using differential equations from the Irwin-Hilton model [11]. The temperature \( T \) and the current \( I \) of the detector evolve as follows

\[
\frac{dI}{dt} = -\frac{R(T,I) + R_L I}{L} + \frac{V}{L}, \tag{2.1}
\]

\[
\frac{dT}{dt} = \frac{R(T,I)}{C_V} I^2 - \frac{k}{C_V} (T^n - T_{\text{bath}}^n), \tag{2.2}
\]

where the detector resistance is given by [7]

\[
R(T,I) = \frac{R_N}{2} \left\{ 1 + \tanh \left[ \frac{T - T_C + (I/A)^{2/3}}{2 \ln(2) T_W} \right] \right\}. \tag{2.3}
\]

We assume that the physical parameters in Eq. (2.1), Eq. (2.2) and Eq. (2.3) are the same as for detectors being developed at NIST (National Institute of Standards and Technology) for the HOLMES experiment. Those parameters are summarized in [12] and are reported in Table I.

In order to test those parameters, we ran our TES simulation without any photon arrivals. We replaced \( R(T,I) \) by \( R_0 \) in Eq. (2.1) and Eq. (2.2), and we used the initial temperature \( T_0 \) and current \( I_0 \) given in Table I. We obtained a slightly different current at quiescence, \( I_0 = 63.87 \mu A \). This new current at quiescence is the one that will be used throughout this work.

B. Simulations of events

In order to develop a method that gives the energy and the arrival time of incoming photons, while maximizing the resolution for those two parameters, we simulated single events. To simulate each photon arrival, we solved Eq. (2.1) and Eq. (2.2) letting the initial temperature and current of the TES be:

\[
T_{0,\gamma} = T_0 + \frac{E_\gamma}{C_V}, \tag{2.4}
\]

\[
I_{0,\gamma} = I_0, \tag{2.5}
\]

where \( E_\gamma \) is the energy of each simulated incoming photon. The current evolution is shown in Fig. 1 for a 7 keV photon. We observe that current in a TES drops with a relaxation time of about 0.5 ms, which limits how much the acquisition time can be reduced.

We generated single events at energies in the 0.1 – 30 keV range, for the three different values of inductance coming from Table I. We observed that the total drop in the current in the TES increases with the energy of the incoming photon, until it clearly saturates for \( E_\gamma \geq 10 \) keV. Once the current drop saturates, the relaxation time increases significantly with the energy of the incoming photon. Those two simple observations, are the basis of the method we have developed.

In order to illustrate those two observations, we plot the maximum current drop in the TES as a function of \( E_\gamma \) (see Fig. 2), and the FWHM of the current drop as a function of \( E_\gamma \) (see Fig. 3). The maximum current drop has

| TABLE I. Physical parameters used for detectors being developed at NIST for the HOLMES experiment [12]. |

| Physical parameters                        | Assumed | Derived |
|--------------------------------------------|---------|---------|
| \( n = 3.25 \)                             |         |         |
| \( k = 23.3 \) nW.K\(^{-1}\)               |         |         |
| \( T_c = 0.1 \) K                          |         |         |
| \( C_V = 0.5 \) pJ.K\(^{-1}\)              |         |         |
| \( R_L = 0.3 \) m\( \Omega \)              |         |         |
| \( T_{\text{bath}} = 0.07 \) K             |         |         |
| \( R_N = 10 \) m\( \Omega \)               |         |         |
| \( R_0 = 2 \) m\( \Omega \)                |         |         |
| \( L \in \{ 12, 24, 48 \} \) n\( \Omega \) |         |         |

* We use this value of inductance throughout this work, except mentioned otherwise.

| TABLE II. Sampling parameters used for simulations of events in a TES. |

| Sampling parameters | Acquisition time: \( T = 1.28 \) ms |
|---------------------|---------------------------------------|
| Number of points:   | Sampling interval: \( \Delta t \) (\( \mu s \)) | Sampling rate: \( f_s \) (kHz) |
| \( N \)             | \( \Delta t \)                         | \( f_s \)                      |
| 250                 | 5.12                                  | 195*                          |
| 125                 | 10.2                                  | 97.7                          |
| 62                  | 20.6                                  | 48.4                          |
| 31                  | 41.3                                  | 24.2                          |
| 13                  | 98.5                                  | 10.2                          |

* All plots are at this sampling rate, except where mentioned otherwise.
FIG. 1. Change in current for a single event in a TES. The incoming photon has an energy of 7 keV.

a linear behavior for $E_{\gamma} \leq 4.6$ keV, whereas the FWHM of the current drop becomes linear for $E_{\gamma} \geq 4.6$ keV. Although the magnitude of X-ray pulses as a function of energy has already been used to retrieve the energy of an incoming photon [13], the energy resolution obtained at high-energy is poor because TESs have a very non-linear response. Consequently, to maximize time and energy resolution for X-ray photons, we need to treat high-energy and low-energy photons with two different techniques. In general, the switching-point energy depends on the properties of the detector and corresponds to the energy of the photon that begins to saturate the superconductor to normal transition of the TES.

FIG. 2. Maximum current drop in a TES as a function of the energy of the incoming photon.

III. NOISE IN A TRANSITION-EDGE SENSOR

Two irreducible sources of noise in a TES are thermal fluctuation noise (also known as phonon noise), and Johnson–Nyquist noise [14]. Thermal fluctuation (TF) noise is the statistical fluctuations that arise from energy exchange between the detector and the heat sink. Johnson–Nyquist (JN) noise is produced by the thermal agitation of electrons in a TES, in other words, it is the electronic noise at equilibrium.

In order to implement our method, we first need to add noise to our simulations. Following the power spectrum of the ARMA-generated noise used in [12], we generate a power spectral density (PSD) for the thermal fluctuation noise and for the Johnson-Nyquist noise, at equilibrium:

$$PSD_{TF}^{eq}(f) = \frac{9.4 \times 10^{-21}}{1 + \left(\frac{f}{4 \times 10^3 \text{Hz}}\right)^2} \text{A}^2\text{Hz}^{-1}$$

$$PSD_{JN}^{eq}(f) = 0.6 \times 10^{-21} \text{A}^2\text{Hz}^{-1},$$

where the variables with indices “eq” are taken at equilibrium. The PSD of the total noise is simply the sum of the two PSDs. Then, we convert these power spectral densities into real noise signals, for each type of noise by performing an inverse Fourier transform using each power spectral density with randomly chosen phases; that is, we assume that the two noise sources are uncorrelated and add them incoherently.

In order to account for non-stationary effects in the noise [14], we scaled both these noises sources at equilibrium, using current, temperature and resistance in a TES at a given moment, $t_j$:

$$N_{TF}^{\text{non-eq}}(t_j) = \frac{T R_{eq}}{R T_{eq} dT} \left(\frac{dR}{dT}\right)^{-1} N_{TF}^{eq}(t_j)$$

$$N_{JN}^{\text{non-eq}}(t_j) = \sqrt{\frac{T R_{eq}}{R T_{eq}}} N_{JN}^{eq}(t_j).$$

The total noise is simply the sum of the two noise signals. In order to obtain the resulting equilibrium and
non-equilibrium PSDs, we perform a Fourier transform of the noise signals, yielding resulting depicted in Fig. 4. We compare the PSDs of the total non-equilibrium noise (averaged over 1,000 iterations) to the one at equilibrium. They slightly differ for photons with high energies because of the scaling effects in non-stationary noise [11]. As the sensor cools from absorbing a 9.5 keV photon, the thermal fluctuation noise is slightly larger than at equilibrium and conversely the Johnson-Nyquist noise is slightly smaller. Simulating these non-stationary noise signals enables us to test our methods in close-to-real conditions.

![Graph comparing non-equilibrium and equilibrium PSDs](image)

**FIG. 4.** Comparison between the non-equilibrium PSDs (dashed lines) and the ones at equilibrium (solid lines). The Johnson-Nyquist noise has a constant PSD whereas the thermal fluctuation noise has a PSD which drops off at high frequencies. The incoming photon has an energy $E_\gamma = 9.5$ keV.

**IV. MEASURING THE ENERGY AND THE ARRIVAL TIME OF AN INCOMING X-RAY PHOTON**

We now present our method to maximize energy and time resolution for photons hitting a TES. We use the physical parameters outlined in Tab. I with $L = 48$ nH and a sampling interval of 5.12 μs.

**A. High-energy photons**

As explained in Sec. II B, we need to treat photons differently according to their energy. In this section, we develop a technique to measure energy and arrival time for high-energy photons. We first describe our method through an example. We simulate a single event at 9.5 keV, with $L = 48$ nH. We work with that inductance so that the onset of the pulse can be resolve even with sampling rates less than 100 kHz. We then use a theoretical model at 7 keV to fit the noisy pulse. This model was obtained by interpolating current-drop simulations, described in Sec. II B, at a given energy. We split this theoretical model in two parts (see Fig. 5), the “onset” part, and the “decay” part:

$$I_{\text{onset}} = \frac{3}{4}I_{\text{TES}}(t < t_{\text{max}})$$

$$I_{\text{decay}} = \frac{3}{4}I_{\text{TES}}(t > t_{\text{max}}),$$

where $3/4$ is a numerical factor that enables to have enough points for curve fitting while avoiding effects from the shape of the current-drop maximum, and $t_{\text{max}}$ is the time at which the current drop is maximum. The “onset” part enables us to obtain the arrival time, whereas the decay part enables us to get the energy of the incoming photon. The theoretical model was chosen for an event at 7 keV; we however do not expect the energy of the theoretical model to affect the resolution, as long as it has a large enough energy to saturate the transition, that is, $E > 4.6$ keV.

![Graph showing single event and theoretical model](image)

**FIG. 5.** Single event at $E_\gamma = 9.5$ keV, and theoretical model at $E_\gamma = 7$ keV. We zoom in (box) to attest the presence of noise in the simulation.

The curve-fitting model is the following:

$$t_{\text{onset}} = k_{\text{onset}}I_{\text{onset}}(t - t_{\text{onset}}),$$

$$t_{\text{decay}} = I_{\text{decay}}(t - t_{\text{decay}}).$$

We then ran simulations for energies between 0.1 keV and 30 keV, and retrieved the parameter values for each energy. The response of $k_{\text{onset}}$ is linear at low energies, which gives a first glimpse of the method used for low-energy photons. On the other hand, the response of $(t_{\text{decay}} - t_{\text{onset}})$ is linear at high energies, therefore this parameter is used to obtain the energy of the incoming photon. The parameter $t_{\text{onset}}$ is used to get the arrival time. In our simulations, the actual arrival time is zero, so the value of $t_{\text{onset}}$ in the simulations yields the uncertainty in the measurement arrival times.
Finally, for each photon energy, we run 1,000 simulations and retrieve the energy and arrival time uncertainty. Going back to our example where we fit a photon of energy 9.5 keV, we obtained the energy of the incoming photon with an uncertainty of \{+2.35 eV, −2.23 eV\} within the 68-percent confidence region. Moreover, the resolution on the arrival time is \{+0.45533 ns, −0.48716 ns\} with 68-percent confidence.

B. Low-energy photons

In this section, we develop a technique to measure energy and arrival time for low-energy photons. We simulated a single event at 0.5 keV. We then used a theoretical model at 1 keV to fit the whole noisy pulse [8–10]. This is showed in Fig. 6.

The curve-fitting model is the following:

\[
I_{\text{shape}}^\text{fit} = k_{\text{shape}} I_{\text{shape}}(t - t_{\text{shape}}),
\]

We then ran the simulations for energies between 0.1 keV and 30 keV, and retrieved the parameters for each energy. The response of \(I_{\text{shape}}\) is linear at low energies, this is therefore the parameter that we use to obtain the energy of the incoming photon. The parameter \(t_{\text{shape}}\) is used to determine the arrival time.

Finally, for each energy, we run 1,000 simulations and retrieve the energy and arrival time uncertainty. Going back to our example, we obtained the energy of the incoming photon with a resolution of \{+0.87 eV, −0.83 eV\}. In addition, the resolution on the arrival time is \{+11.69120 ns, −10.84495 ns\}. Both of these results are 68-percent confidence intervals.

C. Energy and arrival time resolution

We now summarize the energy resolutions for all photon energies in Fig. 7. As one would expect, \(k_{\text{shape}}\) gives the best energy resolution at low energies and \(t_{\text{decay}} - t_{\text{onset}}\) gives it at high energies. One can see that the switching point energy is at about 4.6 keV. Our method enables us to reach an energy resolution at 2\(\sigma\) between 1.67 eV and 6.43 eV, for 0.1 keV < \(E_{\gamma}\) < 30 keV.

Regarding the arrival time, one can notice that the resolution saturates at high energies. Our method enables us to reach a resolution at 2\(\sigma\) between 94 ns and 0.55 ns, for 0.1 keV < \(E_{\gamma}\) < 30 keV (see Fig. 8).

These resolutions were obtained with a very simple method, which is promising for future analysis of X-ray-telescope data. However, pile-up events are known to alter the obtained resolution [15]. Therefore, whenever a consistent photon energy cannot be retrieved with our method, the event can be treated separately as a pile-up event.
V. ENERGY AND TIME RESOLUTION FOR DIFFERENT PARAMETERS OF A TES

The resolution obtained depend on the physical parameters of a detector, such as the sampling rate and the heat capacity. Consequently, we need to change those parameters to see how the resolution is impacted. We also digitized the signal with 16 bits to see how the resolution would be affected by signal processing.

For simulations at 195 kHz, we use the better energy resolution given by either \( t_{\text{decay}} - t_{\text{onset}} \) or \( k_{\text{shape}} \), and the best time resolution given by either \( t_{\text{onset}} \) or \( t_{\text{shape}} \). However, for lower sampling rates not enough points are present in the onset part of the curve fitting. Therefore, for those lower sampling rates, we use the better energy resolution given by either \( t_{\text{decay}} - t_{\text{shape}} \) or \( k_{\text{shape}} \), and the time resolution is given by \( t_{\text{shape}} \).

Energy resolutions for different TES parameters are showed in Fig. 9. One notices that digitizing the signal with 16 bits has no effect on energy resolution and time resolution, which is a promising result for a future use of this method on a real telescope. Increasing the heat capacity however diminishes our energy resolution at low energies. Reducing the sampling rate to 97.7 kHz doesn’t influence our energy resolution; this enables us to use lower sampling rates than in previous methods [5]. One can see that if the sampling rate goes as low as 48.4 kHz, then the energy resolution decreases by a factor two. To account for the decrease in resolution with lower sampling rates, one could be tempted to increase the heat capacity and the acquisition time. However this would increase the FWHM of the signal and would not let enough time to the TES to cool down for the next incoming photon. Consequently, the sampling rate is the only parameter that can be modified without losing significant resolution.

![Energy uncertainty range (within 2\( \sigma \)) for different parameters of the TES.](image)

VI. CONCLUSION

TESs are key elements for future X-ray astrophysics telescopes [16]. While retrieving the energy and the arrival time of a photon detected by a TES is not new; we have successfully developed a new method that enables to optimize the measurements of the energy and the arrival time of photons detected by a TES, using a very simple linear-filter implementation. Such a simple method is promising for future X-ray analysis. Our method treats high-energy photons and low-energy photons separately. Indeed, since TESs have a very non-linear response, scaling effects in non-stationary noise are more important at high energies. We retrieve the energy and arrival time of low-energy photons by fitting the entire current drop in a TES and by interpolating the magnitude of the drop as a function of the energy. This technique is not new, but gives poor results for high-energy photons. Consequently, for the latter, we developed a new technique; we use the width of the current drop instead of its size, by splitting the curve fitting in two parts. In order to work in closest to real conditions, we generated non-stationary noise in a TES. We noticed that using stationary noise does not change the energy resolution at low-energies, but does for high-energies (where the scaling effects are more important).

We successfully retrieved energy and arrival time for an incoming photon, with resolutions similar to the ones obtained with previous methods at low energies. However, thanks to our new technique, we improve the energy resolution for high-energy photons.

In order to make this method useful on a larger scale, it will be essential to get a more general description of the noise in a TES, and it will be necessary to develop a robust way to identify pile-up events. Eventually, we will determine how to efficiently deal with real-time processing, and therefore enable its implementation in future X-ray telescopes.

ACKNOWLEDGMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada, the Canada Foundation for Innovation, the British Columbia Knowledge Development Fund. This research has made use of NASA’s Astrophysics Data System Bibliographic Services.

Changing the TES parameters affects the time resolution as well, but in any case it remains far below 1 \( \mu \)s.
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