Developing a direct gravity trip distribution model for air passenger demand

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Abstract. All Airport network and Commercial Flight Service networks need to be planned and assessed periodically. The planning must be developed based on the Air Passenger Demand prediction. Direct Trip Distribution Model seems the most appropriate for this prediction. Direct Gravity Trip Distribution model was tried to be developed, calculated by using the iterative method. The research gives an indication that the Direct Gravity Trip Distribution can be developed and used. The general formula has been developed. The appropriate formula variables, incorporating the attraction coefficient, the attraction mass, the impedance and its powering value still need to investigate. The calculation method has been developed but needs to be refined, including the optimum and the accuracy indicator. The Direct Gravity Trip Distribution Model can be used either for Air Transport and Sea Transport.

1. Introduction

Airport Network and Commercial Flight Service Network need to be assessed and planned periodically. The planning must be based on the prediction of Passenger Demand. In Transport Demand, the Trip Distribution is the base for Predicting the Future Condition. Therefore, in general we know The Base Year Trip Distribution which is in general developed by using Gravity Model, and Predicted Year Trip Distribution which is in general developed by using the Growth Factor Method [1]. In general, Transport Modeling knows three groups of transportation modeling: the Direct Model, the Conventional Model (a full 4 steps model) and the Unconventional Model (modelling based on Traffic Volume Data). The Four Steps Transport Modelling has been done a lot. Trip Generation model for person trip has been done, for freight has been done also, special conventional transport model for Public Transport Line, trip production modelling method for ensuring good quality model, and investigating the correlation between the coefficient of determination and accuracy. Research on Trip Distribution has been done a lot and deeply also, i.e stratified trip distribution, evaluating different trip distribution formulas, evaluating trip length distribution patterns, and finding the correct formula for deterrence function [2], [3]. Examples on Research on Modal Split can be mentioned as follows: developing modal split for Chennai, finding the efficient modal split, the correlation between modal travel delay and modal split, and different approach on modal split calculation [4], [5]. An example of research on Traffic Assignment is developing a method to validate and calibrate the Traffic Assignment Model. Direct Trip Distribution Model is developed based on Origin Destination (OD) Data [6].

Gravity Model principal is also used in Economic Interaction among regions, countries and others. Its theoretical foundation has been researched, developed and written [7], [8]. An analysis has been
done on trade and direct investment in Central and European countries by using the Gravity Model [9]. Later on, this Gravity Model has been used and developed to model special cases of trade, in linearizing the Gravity Formula by using Logit Formula, in investigating the variable characteristics [10-12]. Even, World Trade Organization (WTO) produce Advance Guide for Trade Policy Analysis based on Structural Gravity Model.

A full Four Steps Transport Model is not appropriate to model Air Passenger Demand, since it deals with Trip Generation, Modal Split, Trip Distribution and Traffic Assignment. A Direct Model is common for modelling special case of urban transport, such as freight transport, since the Trip Generation is not obvious. The Gravity Model is popular in Trip Distribution Modelling and in Economic Interaction modelling. The Direct Gravity Trip Distribution Model for air passenger demand seems very and the most appropriate, since modal split and traffic assignment is not needed. Thus, trying to develop a Direct Gravity Trip Distribution Model for Air Passenger Demand is very important.

This paper presents the result of an attempt to develop a Direct Gravity Trip Distribution Model for Air Passenger Demand. Blangkajeren Air Passenger Demand is taken as a study case.

2. Research method
The main question is the possibility of developing a gravity model for modelling Trip Distribution directly for Air Transportation. These include defining the model formula, the data needed, the calculation method, and the accuracy calculation. Therefore, the research followed these steps: research objective definition, gravity formula development, calculation method development, case study designation, model development, conclusions reflection.

3. Model development

3.1. Blangkajeren commercial flight service
Blangkajeren is a Kabupaten in Aceh Province. It has an airport, the Senubung Airport. The airport is connected by Air Transportation to Banda Aceh and Medan. One flight a week for both directions. The flight map is presented in Figure 1 below.

![](image1.png)

Figure 1. Flight Map: Blanglkajeren – Banda Aceh – Medan.

3.2. Development of the direct gravity trip distribution formula
Basic Gravity follows the dictum that the Gravity Force is a multiplication of the gravity acceleration with two masses divided by the square of its distance. The Basic Gravity formula is presented in Formula 1 below.
\[ F_{ij} = g \frac{m_i m_j}{d_{ij}^2} \]  

(1)

Where:
- \( F_{ij} \): gravity force between two masses
- \( g \): gravity acceleration
- \( m_i \): mass of \( i \)
- \( d_{ij} \): the distance between mass \( i \) and mass \( j \)

As can be seen, the Gravity Principle can be used to express the Attraction Force between two points or among points. The Attraction Force, between two areas, can be in the form of a number of trips, trade volume, investment, number of courier, etc. Three main components play in this attraction force phenomena, i.e. attraction coefficient, attraction masses, and impedance between points. In order to be used for the general case, this Basic Gravity formula must be modified into an expression containing: attraction coefficient, attraction mass, and impedance. Therefore, Formula 1 must be modified into a formula of the General Attraction Force as shown in Formula 2 below.

\[ Z_{ij} = K \times M_i^a \times M_j^b \times I_{i,j}^c \]  

(2)

Where:
- \( Z_{ij} \): attraction force
- \( K \): attraction coefficient
- \( M_i \): attraction mass of \( i \)
- \( I_{i,j} \): impedance between \( i \) and \( j \)
- \( K, a, b, c \): constants

For the purposes to express Trip Distribution among different points, the Formula 2 must be modified into General Direct Gravity Trip Distribution formula expressed as Formula 3 below.

\[ T_{ij} \propto K \times M_i^a \times M_j^b \times I_{i,j}^c \]  

(3)

Where:
- \( T_{ij} \): number of trip from \( i \) to \( j \)
- \( K \): attraction coefficient
- \( M_i \): mass of \( i \)
- \( I_{i,j} \): impedance from \( i \) to \( j \)
- \( K, a, b, c \): constants

Certain variables of the above General Direct Gravity Trip Distribution formula must be defined. For this study, Attraction Mass is defined as Population and Economic Mass (Regional Gross Domestic Product – RGDP), while the Impedance is defined as Distance and Ticket Tarriff. Therefore the Formula 3 is modified into Formula 4 below.

\[ T_{ij} \propto K \times P_i^a \times E_i^b \times D_{ij}^c \times T_{a_{ij}}^d \]  

(4)

Where:
- \( T_{ij} \): number of passenger trip from \( i \) to \( j \)
- \( P_i \): population of \( i \)
- \( E_i \): economic condition (RGDP) of zone \( i \)
- \( D_{ij} \): distance between zone \( i \) to zone \( j \)
- \( T_{a_{ij}} \): ticket tarriff from zone \( i \) to zone \( j \)
- \( K, a, b, c, d \): constants

3.3. Calculation method

The modelling calculation deals with finding several coefficient values. This can be calculated by using successive iterative calculation for each constant. For each calculation, the objective is finding the coefficient value to minimize the Sum of Square Error (SSE). The iterations must be repeated until a solution can be gotten. For each step of the iteration, each constant is optimized separately while keeping the other constant unchanged. SSE is an addition of the square error of \( T_{ij} \) produced by each
step of the iteration. Afterward, the accuracy of the model is measured by the percentage of absolute error of calculated $T_{ij}$. Error is the difference between the real $T_{ij}$ value to the modelled $T_{ij}$ value.

3.4. Research data

Data needed for this modelling are all secondary data. Those data are flight related data and attraction force related data. The flight related data consists of the number of passengers, distance and airfare tariff. The attraction force data consist of population and RGDP data. Those data are presented in Table 1 and Table 2 below.

Table 1. Flight data.

| No | Flight   | Passenger  | Distance | Tarriff |
|----|----------|------------|----------|---------|
| 1  | GYO - BJT| 28,75      | 288,47   | 372     |
| 2  | GYO - KNO| 29,5       | 154,61   | 342     |

source: interview survey

Table 2. Attraction masses

| No | Zone       | Population | RGDP       |
|----|------------|------------|------------|
| 1  | Blangkajeren| 95.379     | 2.438.949 |
| 2  | Banda Aceh | 356.983    | 16.808.137|
| 3  | Medan      | 2.210.624  | 204.299.850|

source: BPS Blangkajeren 2019, BPS Banda Aceh 2019, BPS Medan 2019

4. Direct gravity trip distribution model development

4.1. General direct gravity trip distribution model and its variations

The General Direct Gravity Trip Distribution Model follows the models used commonly, either for transportation modelling and trade distribution modelling. The population and the GRDP are used for the representation of attraction force, while distance and ticket tariff are used for representation of the impedance or friction among the origin and destination. The General Direct Gravity Trip Distribution Model is presented in the following Formula 5.

Four different models as variations of the General Model are then developed. The difference among those four is on the powering grouping. In the first model, only the Impedances are powered in once. In the second model, the Impedance are powered separately. The third model is an extension of the second model, by powering both populations and RGDP’s into two groups. The fourth model is the direct variation of the third model by powering the attraction forces separately between the i’s and the j’s.

$$T_{ij} = K \times P_i^a \times P_j^b \times E_i^c \times E_j^d \times D_{ij}^e \times T_{aij}^f$$

(5)

Notation for the next four models are the same as the above notation, thus the notations are not repeated for the next four formulae.

4.2. General direct gravity trip distribution model and its variations

Model 1 uses the powered impedance with an unpowered attraction force. The power is done once for the two Impedance Components. The Attraction Forces are not powered. The iteration is stopped at SSE value equal to 815.319. The accuracy for Blangkajeren – Medan prediction is very bad, by having an error of 99.3%. While passenger demand prediction for Blangkajeren – Banda Aceh is accurate enough, at 3.6% of error. The Model is presented in Formula 6 below, while the accuracy calculation result is presented in Table 3 as follows.
\[ T_{ij} = 1.05(E - 17) \times(P_iP_jE_iE_j) \times(D_{ij}T_{aij})^{-0.980} \] (6)

### Table 3. Accuracy calculation for model 1.

| SSE        | Flight         | \( T_{ij} \) pax/month | \( T_{ij}' \) pax/month | \(|D|\) | D%   |
|------------|----------------|-------------------------|--------------------------|-------|------|
| 815,309    | GYO-BJT        | 28,750                  | 29,773                   | 1,023 | 3.6  |
|            | GYO-KNO        | 29,500                  | 0,198                    | 29,302| 99.3 |

4.3. Impedance variables are powered separately

Model 2 uses differently powered impedances with unpowered attraction mass. The powers are different for each Impedance Components. The Attraction Forces are not powered. The SSE value obtained is 815.155. The calculation gives a relatively accurate prediction for Blangkajeren – Banda Aceh passenger demand, with an error of 4.8%. While the prediction for Blangkajeren – Medan is totally inaccurate, with an error of 99.3%. The Model is presented in Formula 7 below, while the accuracy calculation result is presented in Table 4.

\[ T_{ij} = 1.00(E - 17) \times(P_iP_jE_iE_j) \times(D_{ij}T_{aij})^{-0.995} \] (7)

### Table 4. Accuracy Calculation.

| SSE       | Flight         | \( T_{ij} \) pax/month | \( T_{ij}' \) pax/month | \(|D|\) | D%   |
|-----------|----------------|-------------------------|--------------------------|-------|------|
| 815,155   | GYO-BJT        | 28,750                  | 30,140                   | 1,390 | 4.8  |
|           | GYO-KNO        | 29,500                  | 0,210                    | 29,290| 99.3 |

4.4. Separate powering for impedance and attraction forces

Model 3 uses separately powered attraction forces and impedances. The power for attraction forces is done separately between for the Populations and for the RGDPs. The power is done separately also for the two Impedance Components. This calculation gives an SSE value of 814.770. The model give an accurate prediction for Blangkajeren – Banda Aceh with an error of 2.1%, but very inaccurate for Blangka – Medan with an error of 99.3%. The Model is presented in Formula 8, while the accuracy calculation result is presented in Table 5.

\[ T_{ij} = 1.00(E - 17) \times(P_iP_jE_iE_j) \times(D_{ij}T_{aij})^{-0.999} \times T_{aij}^{-1.000} \] (8)

### Table 5. Accuracy Calculation.

| SSE       | Flight         | \( T_{ij} \) pax/month | \( T_{ij}' \) pax/month | \(|D|\) | D%   |
|-----------|----------------|-------------------------|--------------------------|-------|------|
| 814,770   | GYO-BJT        | 28,750                  | 29,360                   | 0,610 | 2.1  |
|           | GYO-KNO        | 29,500                  | 0,210                    | 29,290| 99.3 |

4.5. Separate powering for impedance and attraction force based on zone

Model 4 is the extension of Model 3. The Attraction Forces are powered separately based on zone, therefore data of Population and RGDP are powered at once for each zone. The Impedance Factor is powered separately. The calculation stopped at SSE value of 815.155. The modeling gave a pretty accurate prediction for Blangkajeren – Banda Aceh passenger demand, with an error of 3.2%. On the other hand, the modeling gives a very inaccurate prediction for Blangkajeren – Medan, with an error of 99.3%. The Model is presented in Formula 9 below, while the accuracy calculation result is presented in Table 6 as follows.

\[ T_{ij} = 1.00(E - 17) \times(P_iP_jE_iE_j)^{0.928} \times(D_{ij}T_{aij})^{-0.999} \times T_{aij}^{-1.000} \] (9)
4.6. Comparisons of the 4 models

The four models are very similar to each other; the model gives errors of 2.1% - 4.3% for Blangkajeren – Banda Aceh passenger demand, and give errors of 99.3% for Blangkajeren _ Medan, with four similar SSE values. It is interesting to assess the correlation between the SSE value and the Error Value. The modelling shows that, in general, the less SSE value gives less error. In other words, Better SSE give better accuracy. The most accurate model is gotten by Model 3, in which the Attraction Forces are grouped into Population and Economic Condition. The accuracy comparisons are presented in Table 7 and Graph 1 below.

| Flight | SSE  | Tij  | Tij' | | |
|--------|------|------|------|------|------|
| GYO-BJT | 28,750 | 29,660 | 0,910 | 3.2 |
| GYO-KNO | 29,500 | 0,200 | 29,300 | 99.3 |

Table 6. Accuracy Calculation.

Table 7. Accuracy Comparison.

| Model | SSE  | Error 1 | Error 2 |
|-------|------|---------|---------|
| 1     | 815,309 | 3.6     | 99.3    |
| 2     | 815,155 | 4.3     | 99.3    |
| 3     | 814,770 | 2.1     | 99.3    |
| 4     | 815,155 | 3.2     | 99.3    |

Figure 2. The relation between accuracy and the SSE for Flight GYO-BJT.

5. Closure

The research has been finally finished. Several main conclusions can be presented as follows.

- The most accurate model is the Model 3, in which the Population values and the RGDP values are powered separately by the group.
- Direct Gravity Trip Distribution Model can be developed.
- Population and Economic Conditions can be used as Attraction Forces.
- Distance and Ticket Tariff can be used as Impedances.
- Different combinations of attraction force and impedance factors must be investigated.
- Different usage of minimizing measures must be investigated.
- Accuracy measures must be formulated.
It can be stated here, that the Direct Gravity Trip Distribution Model can be used for cases in which calculation of Trip Generation and Traffic Assignment is not an obligation, such for Air Transport and Sea Transport.

Further researches work is needed for investigating using a different measure for optimality, for investigating using different variables of attraction force and impedance factor, for developing the calculation method systematically, and for investigating the usage of gravity model for different air passenger demand cases.

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