The Modeling of Time-Structured Multiturn Injection
into Fermilab Main Injector

(Microbunch Injection with Parasitic Longitudinal Painting)

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Abstract

This paper presents the modeling of time-structured multturn injection for an upgraded Main Injector with the 8-GeV Superconducting RF proton driver, or an ILC-style linac, or a Project-X linac. The Radio-Frequency mismatch between a linac and the upgraded Main Injector will induce parasitic longitudinal painting in RF-phase direction. Several different scenarios with a choice of different RF parameters for single RF system and double RF system in the presence of longitudinal space charge have been investigated. From the studies of microbunch injection with the aid of ESME (2003) numerical simulations, it is found that the dual RF system with a choice of appropriate RF parameters allows us to overcome the space-charge limitation set by beam intensity during the multturn-injection process. A double RF system with a harmonic ratio \( R_H = H_2/H_1 \) of 2.0 and a voltage ratio \( R_V = V_2/V_1 \) of 0.5 are most favored to reduce both longitudinal and transverse effects of space charge in the Main Injector.
Microbunch Injection into the Main Injector

from a Superconducting RF linac into the Main Injector

After studying the method of time-structured multiturn injection from the present 400-MeV linac to Fermilab’s Booster [1], we have further explored the method of microbunch injection for applications to the Main Injector: from a future Superconducting RF (SRF) linac to an upgraded Main Injector (FMI-2) including parasitic longitudinal painting. The future SC linac referred herein can be the 8-GeV SC xsRF linac proton driver [2–4], or an ILC-style linac, or the Project-X linac [5].

Overview of the Main Injector

The Main Injector (MI) is a ring with a circumference of about 3.3 (km). The central roles of the MI is to connect to the Tevatron, the Booster, the Antiproton source, switchyard, and the Recycler ring via a number of beam transport lines in the Fermilab accelerator complex. The MI can accelerate and decelerate particles between 8 GeV and 150 GeV, depending on the mode of operation. The harmonic number of the MI is 588 and harmonic RF at injection is 52.8114 (MHz).

Overview of the 8-GeV Superconducting RF Linac

An 8-GeV SC linac has been proposed as a single-stage $H^-$ injector into the Main Injector as a replacement for the aging 400-MeV Linac and 8-GeV Booster. This would be the highest energy $H^-$ multiturn injection system in the world. Fermilab has been carrying out design studies [4, 6] of the SC linac and injection systems [7] for the last several years. The linac design [8] utilizes a warm-temperature 325-MHz RFQ and rebuncher cavities to bunch the beam at 325 MHz. At $\beta = 0.89$ (about $E_{kin} = 1.1 GeV$), the RF of the SC cavities

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1 We will use Main Injector (FMI) and upgraded Main Injector (FMI-2) interchangeably herein.
2 For the sake of brevity and convenience, MI RF will be referred to as 53 MHz hereafter.
is 1.3 GHz. The ultimate bunch structure required for injection into the MI will be formed by a 325-MHz fast chopper system [9]. The fast chopper system will be required to remove individual 325-MHz bunches or groups of bunches to be matched to the MI RF structure and provide a beam abort notch: two out of every six microbunches are to be removed.

**Time Structure of the Main Injector**

In the injection modeling for the MI with the aid of ESME [10], a train of four microbunches, produced by the fast chopper system located at the front end of the 8-GeV SC RF linac, are injected into the upgraded Main Injector. Figure 1 is a schematic illustrating one MI RF bucket that is populated with an initial train of four microbunches. The two chopped microbunches are represented by two consecutive empty 325-MHz RF buckets.

![Figure 1: Three consecutive MI RF buckets of 53 MHz. A train of four microbunches of 325 MHz are synchronously injected into each standing 53-MHz bucket. Two chopped microbunches are equivalent to 6 ns.](image)

More details of the time structure of the MI RF bucket after the first synchronous injection from a SC linac are illustrated in Figure 2.

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3 The *microbunch* is referred to as a 325-MHz bunch hereafter.
Figure 2: The time structure of a single RF system of an upgraded Main Injector: principal RF harmonic of 52.8 (MHz), sub-harmonic of 325 (MHz), and a beam abort notch of 6 ns.

Figure 3: Single RF system: the injection of 4 microbunches at the 1st turn with RF waveform drawn in the background.
A total beam notch per MI RF bucket is about 6 ns, which corresponds to two 325-MHz RF buckets. Figure 3 is a phase-space ($\Delta E, \theta$) plot of the very first train of four microbunches with a RF-voltage waveform drawn in the background. The following is a list of longitudinal parameters that can be found in the header of each ESME phase-space plot in this memo.

$iter$ (number of turns), $H_B$ (bucket height), $S_B$ (bucket area), $S_b$ (bunch area), $V$ (RF voltage), $E_s$ (synchronous energy), $\nu_s$ (synchrotron tune), $p$ ($dp/dt$), $\eta$ (slip factor), $\tau_s$ (revolution period), $h$ (harmonic number), $\Psi$ (synchronous phase), and $N$ (number of macroparticles)

The horizontal axis is $\Delta \theta (= \theta - \theta_s)$ in units of degrees with one full revolution representing 360 (deg), the vertical axis on the left is $\Delta E (= E - E_s)$ in units of MeV, and the vertical axis on the right is $V_{rf}$ in units of kV for the RF waveform.

The azimuthal-density and energy-density distribution of one of four microbunches are plotted in Figures 4 and 5, respectively. The root-mean-square (RMS) width of one microbunch is $1.96 \times 10^{-4}$ (deg), which corresponds to 6.05 (ps). The red area indicates the tail portion of a microbunch. The fractional tail portion is about 1.34 %. To make the tail portion of a microbunch stand out, the density distribution is also plotted on a logarithmic scale. The RMS value of initial energy spread is about 0.26 (MeV) per each microbunch.

Regarding the MI to be a 11.1338-$\mu$s, or 360-degree ring, we used the following conversion factor:

$$0.032335 \ (deg/\text{ns})$$

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4 The **azimuthal density** is referred to as the charge density, or the azimuthal profile of macroparticles.
5 The **energy-density distribution** is referred to as the profile of macroparticles in energy ($\Delta E$) direction.
Figure 4: Zero-centered charge density with tail portion; (left) on a linear scale (right) on a logarithmic scale

Figure 5: Zero-centered energy density; (left) on a linear scale (right) on a logarithmic scale
The principal MI RF is 53 (MHz) and the SC-linac bunching frequency is 325 (MHz). From the MI RF, we can obtain an integer harmonic number of 588. The width of a MI RF bucket is 18.935 (ns), into which trains of microbunches are repeatedly injected over 270 turns, filling in a MI RF bucket in the region of ± 6 (ns) around the center of a MI RF bucket. The beam notch that is kept free of beams is 3.3 (ns) long, following an earlier injection study with a long bunch [11]. In the following sections, we will explore and discuss several microbunch-injection scenarios, based upon the RF parameters used in the present MI operation.

**Microbunch-Injection Scenarios for the Main Injector**

The goals of the modeling of the time-structured multiturn injection into FMI-2 with a SC linac are threefold:

1. To find the optimized Fermilab Main Injector RF system with the following aspects:
   1. Efficient RF capture methods with minimum particle losses
   2. Main Injector RF system with adequate RF parameters
   3. Minimization of space-charge effects in all degrees of freedom
   4. Optimization of total capture time and total injection time

2. To design a fast beam-chopper system to make nearly loss-free injection attainable

3. To investigate any other limitations on the intensity upgrades from the present Main Injector

With the above goals in mind, we modeled four different scenarios of synchronous injection over 270 injection turns, but with different RF harmonic systems and RF parameters. The following is the parameters of the Main Injector that are consistently employed in ESME simulations for all scenarios presented herein.
Table 1: Main Injector Parameters for ESME Simulations

| Parameter                                      | Value                  |
|------------------------------------------------|------------------------|
| Mean Radius                                    | 528.297 (m)            |
| Beam Momentum (at injection)                   | 8.889 (GeV/c)          |
| $\gamma^2_{tr}$ (transition gamma)             | 466.53572              |
| Principal Harmonic No.                         | 588                    |
| $\Phi_s$ (synchronous phase)                   | 0.0 (deg)              |
| effective beam radius (at injection)           | 0.0050 (m)             |
| effective beam-pipe radius (at injection)      | 0.051 (m)              |
| No. of Space-Charge Bins                       | 64                     |
| No. of FFT Bins                                | 64                     |
| Microbunch Intensity                           | $2.65 \times 10^8$    |
| Total Intensity (after injection is complete)  | $1.54 \times 10^{14}$ |

SCENARIO (I)

- synchronous injection of a train of 4 microbunches into a standing RF bucket
- single RF Harmonic (53 MHz)
- $V_{rf} = 800$ (kV) (fixed RF voltage with no ramping)
- 270 injection turns ($\sim 3$ ms) $\rightarrow$ 2,700 turns ($\sim 30$ ms)
- longitudinal space charge included
- longitudinal painting in RF-phase ($\theta$) direction

We begin with a single RF system of 53 (MHz) and the fixed RF voltage ($V_{rf}$) of 800 (kV). Referring to Figure 3, a series of four blobs captured in a RF bucket represent the first train of microbunches that are synchronously injected into a standing RF bucket. The dotted sinusoidal curve for RF voltage waveform in the background of the phase-space figure indicates the single RF harmonic and the amplitude of RF voltage.
The single turn in the MI in terms of the average machine circumference and the revolution period can be expressed as follows:

\[ 360 \text{ (deg)} = 11.1338 \text{ (\mu s)} = 3319.388 \text{ (m)} \]  

(2)

For one MI RF bucket, the following Eqn. (3) is obtained from Eqn. (2) as illustrated in Figure 2:

\[ 52.8114 \text{ (MHz)} = 18.9353 \text{ (ns)} = 0.6122 \text{ (deg)} = 5.645 \text{ (m)} \]  

(3)

Due to the longitudinal mismatch, or RF mismatch between the SC linac and the FMI-2, an inherent phase slip between 325-MHz microbunch structure and the MI stationary buckets could be parasitically induced in the form of longitudinal painting in RF-phase direction. With \( f \) and \( \lambda \) being radio frequency and RF wavelength, respectively, we computed the RF ratio (\( R_{rf} \)).

\[
R_{rf} = \frac{f_{PD}}{f_{MI}} = \frac{325 \text{ MHz}}{52.8114 \text{ MHz}} = \frac{\lambda_{MI}}{\lambda_{PD}} = \frac{5.6453 \text{ (m)}}{0.9173 \text{ (m)}} = 6.154
\]

(4)

Referring to Figure 2, Eqn. (4) implies that a total of 6 linac RF buckets of 325 MHz can fit in one MI RF bucket. Since the RF ratio is a non-integer number, a modulus of the RF ratio is computed as for the case of the Booster injection modeling [1].

\[
R_{rf} \bigg|_{\text{mod}} = 0.154
\]

(5)

One can then arrive at the range of phase slip within one MI RF bucket.

\[ 0.155 \times 0.0995 \text{ (deg)} = 0.0154 \text{ (deg)} \]  

(6)
Figure 6 shows trains of microbunches and discrete charge distribution after 3 injection turns with phase slips included. For each successive MI RF bucket, the 325-MHz bunches advance by 0.0154 (deg). Hence, it is estimated that every 6 turns the phase of injected microbunches slips through one 325-MHz bucket.

\[
\delta \theta_{rf} = \frac{0.0995 \text{ (deg)}}{0.0154 \text{ (deg)}} = 6.46
\]

Having this amount of phase slip included, we simulated the SC linac-to-MI injection with ESME over 270 injection turns. After the 270 turns, the longitudinal phase space is painted in RF-phase space as illustrated by Figure 7. As in Figure 8, one-peaked but continuous distribution of charge density\(^6\) at the 270-th turn is shown. The evolution of longitudinal emittance growth for the case of a single RF harmonic is shown in Figure 9. As the number of macroparticles increases, so does the longitudinal emittance up to about 0.08 (eV-s). Then, the emittance flattens out until the end of the simulation run. The induced voltage (\(V_{sc}\)) due to space-charge fields (\(E(z)\)) is proportional to the \(g\)-factor, which is usually defined as \(1 + 2\ln(R_w/R_b)\). Hence, from the charge distribution, space-charge-induced voltage per turn is computed at a specified turn. In particular, we looked at the induced voltage at the last injection turn of 270-th turn as shown in Figure 11.

\[
V_{sc}' \propto \frac{1}{\beta\gamma^2} \left[ 1 + 2\ln\left(\frac{R_w}{R_b}\right) \right] \frac{d\lambda}{dz}
\]

where \(R_w\), and \(R_b\) denote effective beam pipe radius, and effective beam radius, respectively. In the case of the Main Injector at injection energy of 8.0 GeV, the \(g\)-factor is about 5.64. In Figure 12, the peak collective voltage in frequency domain (\(\hat{V}_{FD}\)) is drawn at each turn. The collective voltage goes up to about 50.0 (keV) at the end of the injection.

\(^6\) Charge density, or azimuthal density is referred to as charge line density, for convenience.
Figure 6: **Scenario (I)** Phase space and discrete charge density after 3 injection turns
Figure 7: [Scenario (l)] Synchronous injection of microbunches with a single RF system (a) after 270 turns (b) after 2,700 turns; Note that LSC in the figure title stands for Longitudinal Space Charge.
Figure 8: [Scenario (I)] Distribution of charge density with single RF harmonic at the 270-th turn and at the 2,700-th turn.
Figure 9: [Scenario (I)] Evolution of longitudinal emittance with a single RF harmonic over 270 turns

Figure 10: [Scenario (I)] Evolution of the number of injected macroparticles with a single RF harmonic over 270 turns
Figure 11: [Scenario (I)] Induced $\Delta E$ per turn due to space charge ($\Delta E/\text{turn}$) with a single RF harmonic at $270^{th}$ turn

Figure 12: [Scenario (I)] Time evolution of collective voltage in frequency domain over 270 turns
Figure 13: [Scenario (I)] Time evolution of bunching factor

As the bunching factor is usually defined as average current ($\langle I \rangle$) over peak current ($\hat{I}$) for convenience, the value is upper-bounded at unity. Figure 13 shows that the bunching factor converges to about 0.3 for the case of Scenario (I).
**SCENARIO (II)**

In Scenario (II), the injection process described in Scenario (I) is followed by ramping up the RF voltage linearly up to 150% of the initial RF voltage. Thus, the RF voltage ramping lasts for additional 27 (ms), and the total injection time elapses about 30 (ms).

- \( V_{rf, i} = 800 \text{(kV)}, V_{rf, f} = 1,200 \text{(kV)} \)
- ramping RF voltage after the injection is complete

At the end of RF voltage ramping, the macroparticles are well captured in a RF bucket as shown in Figure [14]. Figure [15] shows the variation of RF voltage from 800 (kV) to 1,200 (kV) starting from the 270-th turn through the 2,700-th turn. In comparison to Figure [8] from Scenario (I), the gradient of charge distribution \( (dλ/dz) \) are reduced and spread more out with ramping RF voltage over extended 2,700 turns (cf. Figure [16]). However, it still shows a peaked distribution centered around the origin. The longitudinal emittance \( (ε_l) \) grows up to 0.085 (eV-s) with a fixed RF voltage at 800. (kV). Once the RF voltage starts ramping linearly, the emittance continues to grow gradually up to 0.09 (eV-s) as shown in Figure [17].

In an attempt to reduce further the space-charge-induced voltage and to produce a more uniform charge distribution, a dual RF harmonic system is explored in Scenarios (III) and (IV) in the following subsections.
Figure 14: [Scenario (II)] Synchronous injection of microbunches with a single RF harmonic and ramping RF voltage

Figure 15: [Scenario (II)] RF voltage curve over 2,700 turns
**Figure 16:** [Scenario (II)] Distribution of charge density with a single RF harmonic and ramping RF voltage after 2,700 turns

**Figure 17:** [Scenario (II)] Time evolution of longitudinal emittance ($\varepsilon_l$) with a single RF system
SCENARIO (III)

- Dual RF Harmonics:
  \[ f_{rf, 1} = 53 \text{ MHz and } f_{rf, 2} = 106 \text{ MHz} \]
- \( H_1 = 588 \) and \( H_2 = 1176 \)
  \[ R_H = \frac{H_2}{H_1} = 2.0 \]
- \( V_{rf, 1} = 400 \text{ (kV)} \) and \( V_{rf, 2} = 300 \text{ (kV)} \) (fixed RF voltages)
  \[ R_H = \frac{V_{rf, 2}}{V_{rf, 1}} = 0.75 \]

In scenario (III), a dual RF harmonic system of 400 (kV) on 53 (MHz) and 300 (kV) on 106 (MHz) is explored. The higher-order RF harmonic \((H_2 = 1176)\) is twice of the principal RF harmonic \((H_1 = 588)\). The secondary RF voltage \((V_{rf, 2})\) is 75% of the principal RF voltage \((V_{rf, 1})\). The RF voltage waveform of the higher harmonic for a dual RF system can be

\[ V = V'_{rf, 1} \sin(H_1 \phi_1 + \psi_1) + V'_{rf, 2} \sin(H_2 \phi_2 + \psi_2), \quad (9) \]

where \( V'_{rf}, H, \phi, \) and \( \psi \) are RF voltage, harmonic number, phase angle of each macroparticle, and phase of RF cavity, respectively. As can be seen in Figure [18], the waveform has a negative slope around the stable phase of 0 (deg). By adding a higher secondary harmonic RF voltage to a principal harmonic RF voltage, we can create a flat-bottom potential energy. As a result, the RF-bucket contour with a dual RF system is also flattened at the top and bottom on a phase-space plot. Hence, the charge distribution follows the flat contour shape. Also, the principal RF voltage can be half as high as the RF voltage used for the case of single RF harmonic. Since the flat-bottom potential energy irons out the peaked charge distribution, the utilization of a dual RF system gives a great advantage to the single RF system. Eventually, the dual RF system will help us lower the beam-current limitations caused by space-charge effects. After 2,700 turns, the formation of localized macroparticle distributions are observed around three local bumps of the dual-harmonic
voltage waveform. Figure 20 shows the time evolution of injected macroparticles inside a dual RF bucket. From turn 1 through turn 270, a train of four microbunches are injected with parasitic phase offsets. After the injection process is complete, in order to reach an equilibrium state, injected macroparticles are circulated up to 2,700 turns with no further injection. Note that as time elapses after the completion of injection process, sporadic white gaps in between lumps of macroparticles are gradually disappeared. In Figures from 22 through 24 three plots of longitudinal phase space, charge density, and energy density comprise each row corresponding to turn number under a phase-space plot. With the choice of RF voltages $R_V = 0.75$, a bi-modal charge distribution is created due to a pair of potential wells around the stable phase of 0 (deg). However, with the help of longitudinal painting, the contour of charge distribution becomes smoother as the turn number increases, but the dual peaks remain in the charge distribution.

Figure 18: [Scenario (III)] RF bucket and dual RF waveform in the background
Figure 19: [Scenario (III)] Distribution of macroparticles with longitudinal painting
Figure 20: [Scenario (II)] Distribution of charge density with longitudinal painting included
Figure 21: [Scenario (III)] Time evolution of phase space with longitudinal painting included, starting from the 1st injection turn through 2700 turns.
Figure 22: **Scenario (III)** Time evolution of phase space with longitudinal painting starting from $100^{th}$ turn through $300^{th}$ turns
Figure 23: [Scenario (III)] Time evolution of phase space with longitudinal painting starting from 600th turn through 1500th turns
Figure 24: [Scenario (III)] Time evolution of phase space with longitudinal painting starting from 1800th turn through 2700th turns

(a) 1800th turn  (b) charge density  (c) energy density

(d) 2100th turn  (e) charge density  (f) energy density

(g) 2400th turn  (h) charge density  (i) energy density

(j) 2700th turn  (k) charge density  (l) energy density
SCENARIO (IV)

• Dual RF Harmonics:
  
  \[ f_{r_f, 1} = 53 \text{ MHz and } f_{r_f, 2} = 106 \text{ MHz} \]
  
  \[ H_2 = 1176 \text{ and } H_1 = 588 \]
  
  \[ R_H = \frac{f_{r_f, 2}}{f_{r_f, 1}} = 2.0 \]

• \( V_{r_f, 1} = 400 \text{ (kV) and } V_{r_f, 2} = 200 \text{ (kV) (fixed RF voltages)} \)

Scenario (IV) is under the same conditions as in scenario (III), except that the secondary RF voltage is 50% of the principal RF voltage. It is more advantageous in that the RF waveform has a nice plateau around the stable phase of 0 (deg) as shown in Figure 25. Through the longitudinal painting at each turn, a total of 1080 (270 \times 4) microbunches that are injected over 270 turns and further circulated up to 30 (ms) gradually transform into a continuous macrobunch, or long bunch spanning in between -0.2 (deg) and +0.2 (deg). Unlike in Figure 20(a), localization of macroparticles is not observed in Figure 27(a) after the injection of 270 turns is complete. After further circulation of beams up to 2,700 turns with no further injection, the beam charge distribution becomes smoother in the region of -0.2 (deg) and +0.2 (deg). (see Figures 1.19 and 1.20). As a consequence, the bi-modal distribution of charge density observed from the outcome of Scenario (III) is not observed as shown in Figures 22 through 24. As the number of injected macroparticles increases, longitudinal painting progresses, and time elapses, the fine structure of charge distribution gradually disappears. Eventually, after the 2,700 turns, the contour of charge distribution becomes smoother. Including phase offsets without energy jitter, this effect stands out in charge distribution, rather than energy distribution as observed in Scenario (III).
Figure 25: [Scenario (IV)] RF waveform for a dual RF system with the RF voltage ratio of 0.5
Figure 26: **Scenario (IV)**  Synchronous injection of microbunches with a dual RF harmonic (a) at 270\textsuperscript{th} turns and (b) at 2700\textsuperscript{th} turns
Figure 27: [Scenario (IV)] Distribution of charge density with a dual RF harmonic (a) at 270\textsuperscript{th} turns and (b) at 2700\textsuperscript{th} turns
Figure 28: [Scenario (IV)] Time evolution of phase space with longitudinal painting included, starting from the 1st injection turn through 2700 turns
Figure 29: [Scenario (IV)] Time evolution of phase space with longitudinal painting starting from $100^{th}$ turn through $300^{th}$ turns
Figure 30: [Scenario (IV)] Time evolution of phase space with longitudinal painting starting from 600<sup>th</sup> turn through 1500<sup>th</sup> turns
Figure 31: [Scenario (IV)] Time evolution of phase space with longitudinal painting starting from 1,800\textsuperscript{th} turn through 2,700\textsuperscript{th} turns
As Figure 32 shows, the encouraging result is obtained with the dual RF harmonic system. The longitudinal emittance grows about 50% less than in the cases of a single RF harmonic system. The peak induced voltage ($\hat{V}_{sc}$) due to space charge is around 40 (kV) per turn as in Figure 33. Figure 34 shows the evolution of peak voltage induced by space charge. By using the phase modulation in a controlled fashion, which will allow us to maneuver charge distribution, we will be able to lower space-charge voltage further. The bunching factor calculated at each turn for Scenario (IV) turns out to be close to that of Scenario (I) with a single RF harmonic as in Figure 35. It should be noted that the bunching factor calculations are important in that it serves as an indicator of how large tune spreads will be prior to 3-D space-charge calculations.

**Figure 32:** [Scenario (IV)] The growth of longitudinal emittance with a dual RF system over 2,700 turns
Figure 33: [Scenario (IV)] Additional $\Delta E$ induced by longitudinal space charge with a dual RF harmonics after 2,700 turns

Figure 34: [Scenario (IV)] Evolution of peak voltage in frequency domain with a dual RF harmonics over 2,700 turns
Concluding Remarks

We have investigated different scenarios of microbunch injection methods between a superconducting linac and the MI ring under the influence of longitudinal space charge: from the 8-GeV linac proton driver to the Main Injector.

The RF mismatch between a linac and a ring can induce phase shifts with trains of microbunches, which serve as longitudinal painting in a parasitic fashion. Hence, it would be rather advantageous to use harmonics of non-integer ratio between a linac and a ring in order to induce longitudinal painting. Besides, subsequent charge redistribution in longitudinal direction can be achieved through additional beam circulation with no further beam injection. Since the roaming of charge in longitudinal phase space can reduce the gradient of charge distribution, induced voltage due to space charge can be reduced accordingly. In addition to painting in phase, future simulations are planned to include both
phase and energy jitters due to errors in the SC RF linac. Because of the short bunch length of the linac beam, it is anticipated that the impact of the broad-band impedance may play an important role in the longitudinal dynamics [13]. An optimized dual RF system and longitudinal painting can overcome beam-intensity limitations induced by space charge in high-intensity machines. Four injection scenarios manifest that a double RF system with the harmonic ratio ($R_{hh} = 1176/588$) of 2.0 and the voltage ratio ($R_v = 200kV/400kV$) of 0.5 are most favored. All of the scenarios for the time-structured multiturn injection including animated simulation results are available on a Fermilab web site [14].

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