Interference through quantum dots

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\textit{New Journal of Physics} 9 (2007) 113
Received 31 October 2006
Published 9 May 2007
Online at \url{http://www.njp.org/}
doi:10.1088/1367-2630/9/5/113

\textbf{Abstract.} We discuss the effect of quantum interference on transport through a quantum dot (QD) system. We introduce an indirect coherent coupling parameter $\alpha$, which provides constructive/destructive interference in the transport current depending on its phase and the magnetic flux. We estimate the current through the QD system using the non-equilibrium Green’s function method as well as the Bloch equation method under a large bias voltage condition. The visibility of the Aharonov–Bohm oscillation is evaluated. For a large inter-dot Coulomb interaction, the current is strongly suppressed by the quantum interference effect, while the current is restored by applying an oscillating resonance field with the frequency of twice the inter-dot tunnelling energy.
1. Introduction

Quantum phase coherence in mesoscopic systems is strikingly demonstrated with the principle of superposition, or interference experiments. Aharonov–Bohm (AB) interference is the most fundamental type and has been experimentally confirmed in metallic and semiconductor rings. Recently, in interference experiments with an AB ring containing a quantum dot (QD) in one of the arms, quasi-periodic modulation of the tunnelling current has been demonstrated as a function of the magnetic flux through the ring [1]–[3]. This confirms that phase coherence is maintained during the tunnelling process through a QD. The Fano effect is another type of interference in mesoscopic physics, which occurs in a system in which discrete and continuum energy states coexist [4, 5].

More recently the AB oscillations of a tunnelling current passing through a laterally coupled double QD (DQD) system were observed [6, 7]. These experimental results have motivated theoretical investigations of electron transport through such a system [8]–[11]. DQD has been attracting attention as an important device structure for entangled spin qubit operations [12]–[14]. There is also an interesting theoretical prediction that cotunnelling currents passing through spin-singlet and triplet states have different AB oscillation phases [15].

In this paper, we consider the transport through an AB interferometer containing a laterally coupled DQD. We introduce the indirect coupling parameter $\alpha$, which characterizes the strength of coherent coupling via the reservoirs between two QDs [16]. A system with the maximum coupling $|\alpha| = 1$ has already been widely studied theoretically [8]–[10]. In actual systems, however, such a case is very special and most experimental situations correspond to $|\alpha| < 1$. The situation where $\alpha = 0$ has also been explored in the context of the orbital Kondo problem [17, 18]. We calculate the tunnelling current through the DQD systems in terms of Green’s function techniques [19, 20] as well as the Bloch equation method. Although electron spin is crucial in the previous theoretical proposals, here we disregard it and focus on the quantum interference properties of spinless electrons with/without inter-dot Coulomb interaction.

This paper is organized as follows. In section 2, a standard tunnelling Hamiltonian is employed to describe an AB interferometer containing a laterally coupled DQD. We introduce the indirect coupling parameter $\alpha$. The current formula for the non-interacting case is provided in section 3 and the visibility of the AB oscillation is discussed in the large bias limit. In section 4,
Figure 1. (a) Schematic diagram of an AB interferometer containing a laterally coupled DQD. The magnetic fluxes threading the left and right sub-circuits are $\Phi_L$ and $\Phi_R$, respectively, and cause the AB effect. (b) Equivalent model of a single dot containing two energy levels, which are in the bias window.

we provide the current expression in the limit of a strong inter-dot Coulomb interaction and a large bias. In some situations, the current is completely suppressed because of the system being trapped to a ‘dark state’ and a finite current is restored by applying oscillating field to the system. Our results are summarized in section 5. Three sections in the appendices provide the detailed solutions of the Bloch equation.

2. Model and formulation

We studied a laterally coupled DQD both of which are tunnel-coupled to left (L) and right (R) reservoirs as shown in figure 1(a). The Hamiltonian is $\mathcal{H} = \mathcal{H}_R + \mathcal{H}_{DQD} + \mathcal{H}_T$ with

$$\mathcal{H}_R = \sum_{\nu \in \{L, R\}} \sum_k \epsilon_{\nu k} c_{\nu k}^\dagger c_{\nu k},$$

$$\mathcal{H}_{DQD} = \sum_{\zeta \in \{A, B\}} \epsilon_\zeta d_\zeta^\dagger d_\zeta - t_\zeta (d_A^\dagger d_B + h.c.) + Ud_A^\dagger d_B,$$

$$\mathcal{H}_T = \sum_{\nu \in \{L, R\}} \sum_k \sum_{\zeta \in \{A, B\}} [\tau_{\nu k}^{(\zeta)} (\phi_\zeta) c_{\nu k}^\dagger d_\zeta + h.c.],$$

where $c_{\nu k}^\dagger (c_{\nu k})$ and $d_\zeta^\dagger (d_\zeta)$ represent creation (annihilation) operators of the reservoir $\nu = L/R$ and the QD $\zeta = A/B$, respectively. $\mathcal{H}_R$ represents the reservoirs $\nu = L/R$ with their modes $k$. $\mathcal{H}_{DQD}$ is the Hamiltonian for the DQD with level energies $\epsilon_A$ and $\epsilon_B$. We disregarded the spin degree of freedom and we adopt a large limit for the intra-dot Coulomb interaction, hence only one level is relevant in each dot. $U$ and $t_\zeta$ characterize the inter-dot Coulomb interaction and
We considered that each reservoir have the following relations
\[ G_{\nu}(\epsilon) = \int \frac{d\phi_0}{\pi} \frac{f_{\nu}(\epsilon - \phi_0)}{\epsilon - \phi_0}, \]
where the flux \( \Phi_0 \) is defined by the mode \( \nu = L(R) \). The magnetic flux dependence of the tunnelling amplitude is \( t_{\nu k}(\phi_0)/t_{\nu k}^0(\phi_0) \propto \exp(\pm i\phi_0) \), where the upper (lower) sign is for \( \nu = L(R) \), and the effective magnetic flux \( \phi_0 = 2\pi\Phi_0/\Phi_0 \), which is defined by the flux \( \Phi_0 \) threading through the area formed by DQD and the reservoir \( \nu \) and the magnetic flux quantum \( \Phi_0 = \hbar/e \). This Hamiltonian also describes the system of a single dot with two relevant energy levels as shown in figure 1(b) when \( t_\nu = 0 \) and \( \phi_0 = 0 \), where \( U \) is now interpreted as an intra-dot Coulomb interaction.

In general, the tunnelling current is obtained with the non-equilibrium Green’s function (NEGF) formalism by
\[ I = \frac{ie}{2\hbar} \int d\epsilon \text{Tr} \{ (\Gamma^L - \Gamma^R) G^< (\epsilon) + [f_L(\epsilon)\Gamma^L - f_R(\epsilon)\Gamma^R] [G^\prime (\epsilon) - G^\ast (\epsilon)] \}, \]
where \( G^\prime (\epsilon) \) and \( G^\ast (\epsilon) \) are the retarded and advanced Green’s function of the DQD, and \( G^< (\epsilon) \) is the lesser Green’s function [19, 20]. The boldface denotes the 2 \times 2 matrix and \( f_\nu(\epsilon) \equiv 1/[1 + e^{(\epsilon - \mu_\nu)/k_B T}] \) is the Fermi distribution function where \( \mu_\nu, k_B \) and \( T \) are the chemical potential of the reservoir \( \nu \), the Bolzmann constant, and the absolute temperature, respectively. We considered that each reservoir \( \nu \) is in local thermal equilibrium. The line-width functions are defined as
\[ \Gamma_\nu^{\nu, \nu'}(\epsilon) = 2\pi \sum_k \frac{t_{\nu k}^{(\nu)^*}(\phi_0) t_{\nu k}^{(\nu')}(\phi_0) \delta(\epsilon - \epsilon_{\nu k})}{}, \]
and the off-diagonal component has the following property \( \Gamma_\nu^{\nu, \nu'} = \Gamma_\nu^\ast \) for \( \nu = L(R) \). The wide-band limit approximation disregards the energy dependence of \( \Gamma_\nu^{\nu, \nu'} \). The Green’s functions have the following relations
\[ G^\ast (\epsilon) = [G^\prime (\epsilon)]^\ast, \quad G^< (\epsilon) = G^\prime (\epsilon) \Sigma^< (\epsilon) G^\ast (\epsilon), \]
where \( \Sigma^< (\epsilon) = i[f_L(\epsilon)\Gamma^L + f_R(\epsilon)\Gamma^R] \) is the self-energy. We have previously discussed the linear conductance for a zero offset \( \Delta \equiv \epsilon_B - \epsilon_A = 0 \) in [21] for non-interacting case \( (U = 0) \), where the current formula is simpler as follows
\[ I = \frac{e}{h} \int d\epsilon \text{Tr} \{ f_L(\epsilon) - f_R(\epsilon) \} \text{Tr} \{ G^\prime (\epsilon) \Gamma^L G^\ast (\epsilon) \Gamma^R \}. \]
However, if the interaction \( U \) is finite, we have to use equation (4) as demonstrated in the following section.

By contrast, we also derived the Bloch equation using the method proposed by Gurvitz et al [22, 23], which is appropriate for the large applied bias condition \( \mu_L - \mu_R \gg \Delta \), \( |\Gamma_\nu^{\nu, \nu'}|, t_\nu, k_B T \) and allows the interaction effect to be taken into account relatively easily. This approach only works if all the electrons tunnel from the left to the right reservoir since it disregards a possible interference of the electron transferred in the opposite directions. We specify the state of a DQD by the occupation number in these dots \( (N_A, N_B) \). The states \( (0, 0), (1, 0), (0, 1), (1, 1) \) are abbreviated as \( l = \{0', 'A', 'B', 'D', '\} \), respectively. \( \rho_{l,r} \) is the reduced density matrix element of a DQD after integrating out the reservoir modes in the total system density matrix. The dynamics
of the electrons passing through a DQD is characterized by the following set of differential equations with an appropriate initial condition

$$\frac{d\rho_{00}}{dt} = -(\Gamma^L_{AA} + \Gamma^L_{BB})\rho_{00} + \Gamma^R_{AA}\rho_{AA} + \Gamma^R_{BB}\rho_{BB} + \Gamma^R_{AB}\rho_{AB} + \Gamma^R_{BA}\rho_{BA}, \quad (8)$$

$$\frac{d\rho_{AA}}{dt} = \Gamma^L_{AA}\rho_{00} - (\tilde{\Gamma}^L_{BB} + \Gamma^R_{AA})\rho_{AA} + \Gamma^R_{BB}\rho_{DD} + \left(\frac{\tilde{\Gamma}^L_{AB} - \Gamma^R_{AB}}{2} + i\xi\right)\rho_{AB} + \left(\frac{\tilde{\Gamma}^L_{BA} - \Gamma^R_{BA}}{2} - i\xi\right)\rho_{BA}, \quad (9)$$

$$\frac{d\rho_{BB}}{dt} = \Gamma^L_{BB}\rho_{00} - (\tilde{\Gamma}^L_{AA} + \Gamma^R_{BB})\rho_{BB} + \tilde{\Gamma}^R_{AA}\rho_{DD} + \left(\frac{\tilde{\Gamma}^L_{AB} - \Gamma^R_{AB}}{2} - i\xi\right)\rho_{AB} + \left(\frac{\tilde{\Gamma}^L_{BA} - \Gamma^R_{BA}}{2} + i\xi\right)\rho_{BA}, \quad (10)$$

$$\frac{d\rho_{DD}}{dt} = \tilde{\Gamma}^L_{BB}\rho_{AA} + \tilde{\Gamma}^L_{AA}\rho_{BB} - (\tilde{\Gamma}^R_{AA} + \tilde{\Gamma}^R_{BB})\rho_{DD} - \tilde{\Gamma}^L_{AB}\rho_{AB} - \tilde{\Gamma}^L_{BA}\rho_{BA}, \quad (11)$$

$$\frac{d\rho_{AB}}{dt} = \Gamma^L_{BA}\rho_{00} + \frac{\tilde{\Gamma}^L_{BA} - \Gamma^R_{BA}}{2}(\rho_{AA} + \rho_{BB}) + i\xi(\rho_{AA} - \rho_{BB}) - \tilde{\Gamma}^R_{BA}\rho_{DD} + \left(i\Delta - \frac{\tilde{\Gamma}^L_{AA} + \tilde{\Gamma}^L_{BB} + \Gamma^R_{AA} + \Gamma^R_{BB}}{2}\right)\rho_{AB}, \quad (12)$$

$$\frac{d\rho_{BA}}{dt} = \Gamma^L_{AB}\rho_{00} + \frac{\tilde{\Gamma}^L_{AB} - \Gamma^R_{AB}}{2}(\rho_{AA} + \rho_{BB}) - i\xi(\rho_{AA} - \rho_{BB}) - \tilde{\Gamma}^R_{AB}\rho_{DD} - \left(i\Delta + \frac{\tilde{\Gamma}^L_{AA} + \tilde{\Gamma}^L_{BB} + \Gamma^R_{AA} + \Gamma^R_{BB}}{2}\right)\rho_{BA}. \quad (13)$$

The functions \(\tilde{\Gamma}^{\nu}_{\zeta\zeta'}(\epsilon)\), which describe the tunneling rate of electrons into (out of) dot(s) \(\zeta, \zeta'\), when an electron (two electrons) already occupying the DQD, are obtained by replacing \(\epsilon_{\nu k}\) in equation (5) with \(\epsilon_{\nu k} - U\). In the large limit for the interaction, \(U \gg \mu_L - \mu_R\), the tunneling-in process \(\tilde{\Gamma}^L_{\zeta\zeta'}\) is absent and we can set \(\rho_{DD} = 0\).

The physical meaning of the rate equations for the diagonal components of the density matrix is rather transparent. They are interpreted in terms of loss and gain processes [23]. For example, the first term of equation (8) is the loss of the probability \(\rho_{00}\) by the tunnelling-in processes from the left reservoir to the dot A or dot B. The second and the third terms are the gain by the tunnelling-out processes from the dot A and dot B to the right reservoir, respectively. The last two terms are the gain by the coherent tunneling-out processes from the superposed states described by \(\rho_{AB}\) and \(\rho_{BA}\) with parameters \(\Gamma^R_{AB}\) and \(\Gamma^R_{BA}\) to the right reservoir. The plus sign for \(\tilde{\Gamma}^L_{\zeta\zeta'}\) in the last two terms in equations (9) and (10) is from the anti-commutation rule of the Fermi statistics. The physical meaning of the rate equations for the off-diagonal components...
Deserve attention. The first (fourth) term is a gain process where the coherent tunnelling from left reservoir into the empty DQD (to right reservoir from doubly occupied DQD, with the minus sign from the Fermi statistics). The second term is a gain made up of the two processes with intermediate states of ‘D’ by $\tilde{\Gamma}_{BA}^L$ and ‘0’ by $\tilde{\Gamma}_{BA}^R$. The third term describes the coherent inter-dot tunnelling process. The last term is made up of coherent Rabi oscillation and loss processes to the left and right reservoirs. Recently, coherent effects in magnetotransport though Zeeman-split levels have been discussed in the similar method [24]. By interpreting the Zeeman-split spin sublevels as our two (spinless) levels, the comparison of the Bloch equation of these two models might be interesting. The reservoir studied in [24] is highly non-equilibrium, namely the chemical potential of the left reservoir and a given polarization is much larger than the QD level energies as in our model, and there are continuum of empty states available in the left reservoir whose polarization is opposite. Therefore, counting the right reservoir with two polarizations, there is one reservoir for tunnelling-in and three reservoirs for tunnelling-out processes, and there is no direct relation between the two Bloch equations. Rather, the model considered in [25], where the reservoirs are ferromagnetic semi-metals and no density of states for minority spins, seems strongly related to our model.

The above two approaches (NEGF and Bloch equation) are sufficiently general for us to discuss the effect of interaction and interference in the transport through a DQD. However, since the off-diagonal line-width function elements $\Gamma_{\zeta\zeta'}^{v} (\zeta \neq \zeta')$, which controls the strength of the coherence of the transport, is strongly dependent on the microscopic model, and we need further simplification to grasp the fundamental physics of this system. Here, we define the indirect coherent coupling parameter $\alpha_v$, which was first introduced in [16]. Using $\alpha_v$, the off-diagonal part of the line-width function becomes

$$\Gamma_{AB}^{v} = \alpha_v \sqrt{\Gamma_{AA}^{v} \Gamma_{BB}^{v}} e^{\mp i \phi_v},$$

(14)

where the upper (lower) sign is for $v = L(R)$. All the parameters $\Gamma_{\zeta\zeta'}^{v}$, $\alpha_v$ are independent of energy in the wide-band limit. We also disregarded the energy dependence of the effective flux induced by changes in the electron trajectory. The parameter $\alpha_L (\alpha_R)$ characterizes the coherent injection into the DQD from the reservoir L (the coherent emission from the DQD to the reservoir R). When the time-reversal symmetry is broken by a magnetic field, $\Gamma_{AB}^{v}$ is a complex parameter. Here we factorized the magnetic flux dependence in the phase exp ($\mp i \phi_v$) as shown in equation (14) and the parameter $\alpha_v$ is real. $|\alpha_v| = 1$ corresponds to full coherence and $\alpha_v = 0$ denotes zero coherence, corresponding to a situation where the two QDs are independently coupled to the reservoir. The explicit derivation of $\alpha_v$ for a highly localized QD and flat tunnelling barriers to two-dimensional (2D) or 3D reservoirs is described in detail in [21]. When the tunnelling barrier height is much higher than the chemical potential $\mu_v$, we obtained $\alpha_v = \frac{3}{k_{F}^s} J_1 (k_{F}^s s) (\alpha_v = \frac{2}{k_{F}^s} J_1 (k_{F}^s s))$ for 3D (2D) reservoir $v$. Here, $s$ is the distance between the two dots and $k_{F}^s$ is the Fermi wavenumber of the reservoir $v$. $J_1 (j_1)$ is the first order (spherical) Bessel function. When the barrier is not so high, we had an expression of more smooth dependence on the distance $s$, namely $\alpha_v = (2y_0^v / \sqrt{s^2 + (2y_0^v)^2})^D$, where $y_0^v$ is the distance between the DQD to the reservoir $v$ of dimension $D = 2$ or 3. In general, for the reservoir of higher dimension, the parameter $|\alpha_v|$ becomes small faster with the inter-dot distance $s$ since there are more modes selectively coupled to one of the two dots. There has been a detailed analysis of the coherence in a metallic reservoir in [26].

Equations (8)–(13) differ from that obtained with a similar method [27] and from that obtained with the gradient expansion method [28]. Both differ from ours as regards the sign.
of \( \tilde{\Gamma}_{\zeta'} \) with \( \zeta \neq \zeta' \) and the former is missing the first term in equations (12) and (13) which represents coherent injection of an electron into DQD from the left reservoir. We can check that our formula provides a reasonable result in a limiting situation as follows and in the next section. Let us consider a symmetric system, namely, \( \Gamma^v_{AA} = \Gamma^v_{BB} \equiv \gamma_v \), zero flux \( \phi_v = 0 \), i.e., \( \Gamma^v_{AB} = \gamma_v \), and non-interacting \( U = 0 \). We transform from the dot local \( A/B \) basis to the symmetric/antisymmetric (s/a) state basis \( |\Psi_{s/a}\rangle = \frac{1}{\sqrt{2}} (|A\rangle \pm |B\rangle) \) under the condition of zero offset \( \Delta = 0 \). The density matrix is then transformed to a new basis as

\[
\rho_{AA} = \frac{1}{2} (\rho_{ss} + \rho_{aa} + \rho_{sa} + \rho_{as}),
\]

\[
\rho_{BB} = \frac{1}{2} (\rho_{ss} + \rho_{aa} - \rho_{sa} - \rho_{as}),
\]

\[
\rho_{AB} = \frac{1}{2} (\rho_{ss} - \rho_{aa} - \rho_{sa} + \rho_{as}),
\]

\[
\rho_{BA} = \frac{1}{2} (\rho_{ss} - \rho_{aa} + \rho_{sa} - \rho_{as}),
\]

with invariant \( \rho_{00} \) and \( \rho_{DD} \). Then we define state dependent line-width functions \( \gamma^v_{s/a} = (1 \pm \alpha_v) \gamma_v \) with \((+)\) for a symmetric (antisymmetric) state. The Bloch equation for the new basis is

\[
\frac{d\rho_{00}}{dt} = -(\gamma^L_s + \gamma^L_a) \rho_{00} + \gamma^R_s \rho_{ss} + \gamma^R_a \rho_{aa},
\]

\[
\frac{d\rho_{ss}}{dt} = -(\gamma^R_s + \gamma^L_a) \rho_{ss} + \gamma^L_s \rho_{00} + \gamma^R_s \rho_{DD},
\]

\[
\frac{d\rho_{aa}}{dt} = -(\gamma^L_s + \gamma^R_a) \rho_{aa} + \gamma^L_a \rho_{00} + \gamma^R_a \rho_{DD},
\]

\[
\frac{d\rho_{DD}}{dt} = -(\gamma^R_s + \gamma^R_a) \rho_{DD} + \gamma^L_s \rho_{ss} + \gamma^L_a \rho_{aa},
\]

\[
\frac{d\rho_{sa}}{dt} = -[1/2(\gamma^L_s + \gamma^L_a + \gamma^R_s + \gamma^R_a) + 2it_c] \rho_{sa},
\]

and there is a similar equation with \( 2it_c \to -2it_c \) for \( \rho_{as} \). Because of the relaxation term \(-(\gamma^L_s + \gamma^L_a + \gamma^R_s + \gamma^R_a)/2 \), the quantum coherence term \( \rho_{sa} \), which is independent of other elements, simply disappears from any initial condition for the steady state limit. Therefore, equations (15)–(19) correctly describe the independent dynamics of symmetric and antisymmetric channels with state dependent line-width functions. It should be noted that when \( \alpha_v = 1 \) (\( \alpha_v = -1 \)), the antisymmetric (symmetric) state is decoupled from the reservoir \( v \), \( \gamma^v_s = 0 \) (\( \gamma^v_a = 0 \)). This is because of the perfect destructive interference.

From equations (8)–(13), we obtain the steady state density matrix at \( t \to \infty \) by employing the auxiliary relation \( \rho_{00} + \rho_{AA} + \rho_{BB} + \rho_{DD} = 1 \). Using the result, the steady current is obtained as follows

\[
I = e \Gamma^R_{AA} \rho_{AA} + \Gamma^R_{BB} \rho_{BB} + (\tilde{\Gamma}^R_{AA} + \tilde{\Gamma}^R_{BB}) \rho_{DD} + \Gamma^R_{AB} \rho_{AB} + \Gamma^R_{BA} \rho_{BA}.
\]

The detail of the derivation of the current formula is summarized in appendices.
3. Non-interacting system

First, we discuss the system without interaction \((U = 0)\). For simplicity, we restrict to highly symmetric coupling to the reservoirs, \(\Gamma_{\mu\nu} \equiv \gamma\) and symmetric fluxes \(\phi_L = \phi_R \equiv \phi/2\). The retarded Green’s function is straightforwardly obtained by solving the equation of motion (EOM). Its Fourier transform is

\[
G'(\epsilon) = \begin{pmatrix}
\epsilon - \epsilon_A + i\gamma & t_c + \frac{1}{2}i\gamma \left(\alpha_L e^{-i\phi/2} + \alpha_R e^{i\phi/2}\right) \\
t_c + \frac{1}{2}i\gamma \left(\alpha_L e^{i\phi/2} + \alpha_R e^{-i\phi/2}\right) & \epsilon - \epsilon_B + i\gamma
\end{pmatrix}^{-1}.
\] (21)

For a non-interacting conductor, the transmission probability of the electron with energy \(\epsilon\) is defined as

\[
\mathcal{T}(\epsilon) = \text{Tr}\{G'(\epsilon)\Gamma^L G^\alpha(\epsilon)\Gamma^R\},
\] (22)

which appears in equation (7). The linear conductance \(G\) at zero temperature is obtained in the Landauer formula

\[
G = \frac{e^2}{h} \mathcal{T}(0),
\] (23)

where the energy is measured from the (average) chemical potential of the reservoirs. The explicit formula for zero-offset, \(\Delta = 0\), is shown in equation (18) of [21]. The function \(\mathcal{T}(0)\) for \(\alpha = 0\) and \(|\alpha| = 1\) at zero flux \(\phi = 0\) has following simple physical meaning

\[
\mathcal{T}(0) = \begin{cases}
\frac{\gamma^2}{(\epsilon_0 - t_c)^2 + \gamma^2} + \frac{\gamma^2}{(\epsilon_0 + t_c)^2 + \gamma^2} & \text{for } \alpha = 0, \\
\frac{(2\gamma)^2}{(\epsilon_0 - t_c)^2 + (2\gamma)^2} & \text{for } |\alpha| = 1,
\end{cases}
\] (24)

where \(\alpha = 0\) corresponds to two independent Breit–Wigner resonances through the symmetric and antisymmetric states with line-width \(\gamma\), while \(\alpha = 1\) \((\alpha = -1)\) represents Breit–Wigner resonance through only the symmetric (antisymmetric) state with doubled line-width \(2\gamma\). It has been shown that the period of AB oscillation of the linear conductance is \(2\pi\) when \(t_c = 0\) and \(4\pi\) when \(t_c \neq 0\). For a flux with non-integer multiple of \(2\pi\), the \(\epsilon_0\) dependence of the conductance shows Fano line shape when \(|\alpha| = 1\) [4]. However, this Fano effect is quickly suppressed if \(|\alpha|\) becomes less than 1.

The current for a finite bias \(eV \equiv \mu_L - \mu_R\) at \(T = 0\) is

\[
I = \frac{e}{h} \int_{\mu_R}^{\mu_L} d\epsilon \mathcal{T}(\epsilon).
\] (25)

In the limit of a large bias, this can be evaluated by the contour integral and the result for \(\alpha_L = \alpha_R \equiv \alpha\) and \(\Delta = 0\) is

\[
I = e\gamma \frac{t_c^2 + \gamma^2(1 - \alpha^2 \sin^2 \phi/2)}{t_c^2 + \gamma^2}.
\] (26)

4 The definition of the sign of \(t_c\) is reversed and \(\epsilon_A = \epsilon_B \equiv \epsilon_0, \alpha_L = \alpha_R \equiv \alpha\).
Now the period of the current oscillation with the flux is $2\pi$ independent of $t_c$. At zero flux, $\phi = 0$, the current is $e\gamma$ independent of $\alpha$, which is explicitly checked from equation (24) by replacing $\epsilon_0$ with $\epsilon_0 - \epsilon$ and by integrating with $\epsilon$ for $(-\infty, \infty)$. The current is the sum of $e\gamma/2$ from symmetric and antisymmetric states for $\alpha = 0$, and the current is $e(2\gamma)/2$ from symmetric or antisymmetric state for $|\alpha| = 1$. The energy offset $\Delta$ dependence of the current is shown for various values of $\alpha$ in figure 2. It should be noted that for sufficiently large offset $\Delta \gg \gamma$, the current is independent of $\alpha$.

The current for the large bias limit is also derived by the Bloch equation with $\tilde{\Gamma}_{\nu}^{\nu} = \Gamma_{\nu}^{\nu}$. The result is shown in equation (A.3) in appendix A. For $\alpha_R = \alpha_L \equiv \alpha$ and $\phi_L = \phi_R = \phi/2$, we have

$$I = e\gamma \frac{\Delta^2 + 4(1 - \alpha^2 \cos^2 \phi/2)(t_c^2 + \gamma^2(1 - \alpha^2 \sin^2 \phi/2))}{\Delta^2 + 4(1 - \alpha^2 \cos^2 \phi/2)(t_c^2 + \gamma^2)},$$

(27)

which provides the same result for $\Delta = 0$ as that obtained by the NEGF method, equation (26) and for $\Delta \neq 0$ shown in figure 2 (left panel). The current is the maximum, $I = e\gamma$, at $\phi = 2n\pi$ and the minimum at $\phi = (2n + 1)\pi$ with an integer $n$. The visibility of the AB oscillation is

$$v \equiv \frac{I(\phi = 0) - I(\phi = \pi)}{I(\phi = 0)} = \frac{4\gamma^2 \alpha^2}{\Delta^2 + 4(t_c^2 + \gamma^2)},$$

(28)

As shown in section 2, the Hamiltonian also describes the system of a single dot with two levels. Usually, a coherent injection process to the multiple levels in a QD is not considered, except for [29, 30]. Here we clarify the condition when this is justified. By putting $t_c = 0$ and $\phi_{\nu} = 0$ in equation (A.3), we have the current formula

$$I = e\gamma \frac{\Delta^2 + \gamma^2(4 - 2(\alpha_L^2 + \alpha_R^2))}{\Delta^2 + \gamma^2(4 - (\alpha_L + \alpha_R)^2)}.\tag{29}$$

The total current deviates from $e\gamma$, just the sum of the current via each level, $\frac{1}{2}e\gamma$, by the effect of quantum interference when $\alpha_L \neq \alpha_R$. This effect is maximum if $\Delta = 0$. When one of the $\alpha$'s is one and the other is zero, the current becomes $\frac{2}{3}e\gamma$. Moreover, when $\alpha_L/\alpha_R = -1$ which is
expected for a 1D QD [30], we have \( I = e\gamma(1 - \alpha_R^2) \) and we expect complete suppression of current when \( \alpha_L = -\alpha_R = \pm 1 \), since the left (right) reservoir selectively couples to symmetric (antisymmetric) states, or vice versa. This effect of interference vanishes for large offset \(|\Delta| \gg \gamma\) and the current is \( e\gamma \). This behavior is shown in figure 2 (left panel).

4. Strong interaction limit

Here, we consider the case of \( U \to \infty \) and a large bias. The general form of the steady current is obtained in appendix B. For simplicity, we restricted ourselves to the symmetrical coupling \( \Gamma_{AA}^{\nu} = \Gamma_{BB}^{\nu} \equiv \gamma_\nu \). In the special case where zero flux \( \phi_\nu = 0 \), we obtain from equation (B.1)

\[
I = e \frac{2\gamma_R \gamma_L}{2\gamma_L + \gamma_R} \frac{(\Delta^2/(\gamma_R^2 + 4t_c^2)) + 1 - \alpha_R^2}{(\Delta^2/(\gamma_L^2 + 4t_c^2)) + (2\gamma_L(1 - \alpha_L \alpha_R) + \gamma_R(1 - \alpha_R^2))/(2\gamma_L + \gamma_R)}. \tag{30}
\]

When the electron tunnelling-out process to the reservoir R is incoherent, \( \alpha_R = 0 \), the current value becomes the classical limit

\[
I_{\text{incoherent}} = e \frac{2\gamma_R \gamma_L}{2\gamma_L + \gamma_R}. \tag{31}
\]

Interestingly, if \( \alpha_L = \alpha_R \), the current has the same value \( I_{\text{incoherent}} \) as if the coherent transport is absent. In both cases, the current value is independent of \( \Delta \) and \( t_c \). Under general \( \alpha_\nu \) conditions, the current value approaches \( I_{\text{incoherent}} \) if the condition \( \Delta^2 \gg \gamma_R^2 + 4t_c^2 \) is satisfied. This behaviour is derived for a more general situation (asymmetric couplings) in equation (B.3) in appendix B.

When \( |\alpha_R| \to 1 \) and \( \Delta \to 0 \), the current is completely suppressed although we are supplying the system with a large bias.\(^5\) This is evident from the plot of overall dependence on positive \( \alpha_R \) and \( \alpha_L \) in figure 3(a). This can be understood as the system being trapped in the ‘dark-state’,

\(^5\) We need to keep \( \alpha_L \neq \text{sign}(\alpha_R) \), since \( \alpha_R = \alpha_L = \pm 1 \) provides finite current \( I_{\text{incoherent}} \).
which in this context (for \( \alpha_R \to 1 \)) means the antisymmetric state that cannot couple to the R reservoir as discussed in the previous section. The steady state density matrix in this limit is \( \rho_{AA} = \rho_{BB} = \frac{1}{2}, \quad \rho_{AB} = \rho_{BA} = -\frac{1}{2}, \quad \rho_{00} = 0, \) which is the density matrix of the pure state: \( \rho = |\Psi_a\rangle\langle\Psi_a| \) with the antisymmetric state \( |\Psi_a\rangle = \frac{1}{\sqrt{2}}(|A - |B\rangle). \) The limit of \( \alpha_R \to -1 \) is explained by exchanging the role of symmetric and antisymmetric states. This mechanism has been discussed in a triple dot system as a coherent population trapping (CPT) mechanism [31]. The current in such a triple dot system is estimated in appendix C and we found the \( \Delta \) dependence of the current is similar to equation (30).

We demonstrate the collapse of current suppression by applying an oscillating electric field [28, 32, 33]. We evaluated the effect of a weak oscillating field \( \Delta(t) = \delta \cos \omega t \) for \( \alpha_R = 1 \) in the perturbation theory in \( \delta \). The lowest order contribution of the leakage steady current is the second order of \( \delta \)

\[
I|_{\alpha_R=1} = \frac{e\gamma_R \delta^2 (\gamma_R^2 + 4t_c^2 + \omega^2)}{2(1 - \alpha_L)(\gamma_R^4 + (4t_c^2 - \omega^2)^2 + 2\gamma_R^2(4t_c^2 + \omega^2))},
\]

which is plotted in figure 3(b). The current peaks with a value \( e\gamma_R \delta^2 \gamma_R^2 \gamma_L^2 \gamma_A^2 \) at a frequency \( \omega \sim \pm 2t_c \), which corresponds to the emission of one photon and the system transits from the antisymmetric state to the symmetric state, that allows the electron to leak into the right reservoir.

The flux dependence of the current in the large bias limit is following. We only consider the symmetric configuration: \( \gamma_R = \gamma_L = \gamma \) and \( \phi_L = \phi_R = \phi/2 \) and \( \alpha_R = 1 \) and use equation (B.1).

When the offset \( \Delta \) is zero

\[
I = e\gamma \frac{2t_c^2(1 - \cos \phi)}{\gamma^2 + (5 - 2\alpha_L)t_c^2 - (\alpha_L^2\gamma^2 + (1 + 2\alpha_L)t_c^2) \cos \phi},
\]

therefore the visibility of AB oscillation of the current is 1. When the inter-dot tunnelling \( t_c \) is zero

\[
I = e\gamma \frac{2\Delta^2}{3\Delta^2 + 2\alpha_L \Delta \gamma \sin \phi + 2\gamma^2(1 - \alpha_L \cos \phi)},
\]

By defining \( \theta = \arctan \Delta/\gamma \) with \( A = \sqrt{\gamma^2 + \Delta^2} \), the denominator reduces to \( 3\Delta^2 + 2\gamma^2 - 2\alpha_L \gamma A \cos (\phi + \theta) \) and the maximum and minimum current condition is determined by the factor \( \alpha_L \cos (\phi + \theta) \). Therefore its visibility is

\[
v = \frac{4|\alpha_L| \gamma \sqrt{\gamma^2 + \Delta^2}}{3\Delta^2 + 2\gamma^2 + 2|\alpha_L| \gamma \sqrt{\gamma^2 + \Delta^2}},
\]

which is a monotonic decreasing function of \( |\Delta|/\gamma \) and \( |\alpha_L| \). The linear (two-terminal) conductance should be symmetric with respect to the reversal of the flux in accordance with Onsager’s relation, hence is an even function of \( \phi \). However, the current shown in equation (34) is obviously not an even function. This is not the problem since we are discussing the current in a non-linear regime, that is out of the boundary of Onsager’s argument about a linear-response regime [34, 35].

We also studied the current expression using the NEGF method for the special condition \( \Delta = 0 \). For simplicity, all tunnelling amplitudes are the same and we use the single parameter \( \gamma \).
We transform the basis to the symmetric/antisymmetric basis with \( \epsilon_s = \epsilon_0 - t_c \) and \( \epsilon_a \equiv \epsilon_0 + t_c \) and \( \epsilon_0 = (\epsilon_A + \epsilon_B)/2 \). The inter-dot interaction Hamiltonian becomes \( U n_n n_{\bar{n}} \) where \( n_n \equiv d_n^\dagger d_n \) with \( n = s/a \). On this basis, the line-width function matrix is

\[
\Gamma^v = \gamma \begin{pmatrix} 1 + \alpha_v & 0 \\ 0 & 1 - \alpha_v \end{pmatrix}.
\] (36)

We define the total line-width \( \Gamma \equiv \Gamma^L + \Gamma^R \). We calculated the retarded Green’s function \( G_{nn}^\prime(\epsilon) = \frac{\delta_{nm} \chi(\epsilon)}{\epsilon - \epsilon_n + \frac{i}{2} \Gamma_{nn} \chi(\epsilon)} \),\( G_{nn}^\prime(\epsilon) = \frac{1 - \langle n_n \rangle}{\epsilon - \epsilon_n + \frac{i}{2} \Gamma_{nn}(1 - \langle n_{\bar{n}} \rangle)} \),\( \chi(\epsilon) \equiv 1 + \frac{U/\langle n_{\bar{n}} \rangle}{\epsilon - \epsilon_n - U} \).

Then we have in the limit of \( U \to \infty \),

\[
\langle n_n \rangle = \frac{1}{2\pi i} \int \frac{d\epsilon}{\epsilon} G_{nn} \lesssim (\epsilon), \tag{39}
\]

which should be evaluated self-consistently. The linear conductance at zero temperature for \( \alpha_L = \alpha_R = \alpha \) is obtained by

\[
G = \frac{e^2}{h} \left\{ \tilde{\gamma}_s^2 + \tilde{\gamma}_a^2 + \frac{\tilde{\gamma}_s^2}{\epsilon_s^2 + \tilde{\gamma}_s^2/2} - 2 \left( \alpha \sin \left( \frac{\phi}{2} \right) \right)^2 (1 - \langle n_s \rangle)(1 - \langle n_a \rangle) \right\} \left( \frac{\epsilon_s \epsilon_a + \tilde{\gamma}_s \tilde{\gamma}_a}{(\epsilon_s^2 + \tilde{\gamma}_s^2)(\epsilon_a^2 + \tilde{\gamma}_a^2)} \right), \tag{40}
\]

where \( \tilde{\gamma}_{s/a} \equiv \gamma \left( 1 \pm \alpha \cos \left( \frac{\phi}{2} \right) \right) (1 - \langle n_{s/a} \rangle) \).

In the limit of a large bias, equation (39) reduces to the following self-consistent equation

\[
\langle n_{s/a} \rangle = \frac{(1 \pm \alpha_L)(1 \mp \alpha_R)}{2 \pm \alpha_L \pm \alpha_R}, \tag{41}
\]

and the solutions are

\[
\langle n_{s/a} \rangle = \frac{(1 \pm \alpha_L)(1 \mp \alpha_R)}{3 - 2\alpha_L \alpha_R - (\alpha_R)^2}. \tag{42}
\]

Putting these in the formula of the current,

\[
I = \sum_{n \in \{s, a\}} I_n, \tag{43}
\]

\[
I_{s/a} = e\gamma(1 \pm \alpha_L)(1 \mp \alpha_R) \frac{1 - \langle n_{s/a} \rangle}{2 \pm \alpha_L \pm \alpha_R}, \tag{44}
\]

we finally obtain the same result as that obtained by the Bloch equation, equation (30) with \( \Delta = 0 \)

\[
I = 2e\gamma \frac{1 - \alpha_R^2}{3 - 2\alpha_L \alpha_R - \alpha_R^2}. \tag{45}
\]
5. Conclusions

We discussed the effect of quantum interference on the transport through a QD system. We stressed the role of the indirect coherent coupling parameter $\alpha$, which provides constructive/destructive interference in the transport current depending on its phase. We derived the current using the NEGF method as well as the Bloch equation method under a large bias condition. For a large inter-dot Coulomb interaction, the current is strongly suppressed by the quantum interference effect, where the current is restored by applying an oscillating resonant field.

Acknowledgments

We thank A Aharony, O Entin-Wohlman, J Tobiska, M Pioro-Ladri`ere, T Hatano and S Tarucha for valuable discussions and useful comments. YT is partly supported by SORST-JST.

Appendix A. Derivation of current for a non-interacting system

Steady state density matrix elements are derived from the Bloch equation by setting $\frac{d\rho}{dt} = 0$. In matrix notation, we have

$$\frac{d\bar{\rho}}{dt} = \hat{M} \cdot \bar{\rho},$$

where $\bar{\rho} = (\rho_{00}, \rho_{AA}, \rho_{BB}, \rho_{DD}, \rho_{AB}, \rho_{BA})^T$ and the complex matrix $\hat{M}$ is defined from equations (8)–(13). The steady state condition is $\hat{M} \cdot \bar{\rho} = 0$. Since the six algebraic equations are not independent, we need the equation for conservation of probability, $\rho_{00} + \rho_{AA} + \rho_{BB} + \rho_{DD} = 1$. To proceed further, we define new matrix $\hat{M}_1$ by replacing the first row of $\hat{M}$ by $(1, 1, 1, 1, 0, 0)$ then the equation becomes

$$\hat{M}_1 \cdot \rho = \bar{u},$$

where $\bar{u} = (1, 0, 0, 0, 0, 0)^T$. Then the density matrix at the steady state condition is obtained by $\bar{\rho}_{\text{steady}} = \hat{M}_1^{-1} \bar{u}$. The steady state current is evaluated using equation (20).

When the interaction is absent, $\hat{\Gamma}_{\zeta\zeta}^v = \hat{\Gamma}_{\zeta\zeta}^\gamma$. Evaluating the current for symmetric coupling $\Gamma_{AA}^v = \Gamma_{BB}^v = \gamma$, we obtain

$$I = e\gamma \frac{N_I}{D_I},$$

$$N_I = \Delta^2 + \gamma^2[4 - 2(\alpha_L^2 + \alpha_R^2) + \alpha_L^2\alpha_R^2 \sin^2(\phi_L + \phi_R)] + 2t_c^2[2 - \alpha_L^2 \cos^2 \phi_L - \alpha_R^2 \cos^2 \phi_R],$$

$$D_I = \Delta^2 + \gamma^2[4 - \alpha_L^2 - \alpha_R^2 - 2\alpha_L\alpha_R \cos(\phi_L + \phi_R)] + t_c^2[4 - (\alpha_L \cos \phi_L + \alpha_R \cos \phi_R)^2].$$

Appendix B. Derivation of current in the strong inter-dot interaction limit

We solve the Bloch equation equations (8)–(13) for $U \to \infty$, namely, neglecting the term $\rho_{DD}$ and $\hat{\Gamma}_{\zeta\zeta}^v$ in the steady state condition. First, we restrict ourselves to the symmetrical coupling.
\[ I = \frac{2 \gamma_L \gamma_R \Delta^2}{D} \{ \Delta^2 - 2 \alpha_R^2 \gamma_R^2 \cos 2 \phi_R + (1 - \alpha_R^2) \gamma_R^2 + 2(2 - \alpha_R^2) \}, \]  

where \[ D = \Delta^2 (2 \gamma_L + \gamma_R) + 2 \alpha_L \alpha_R \Delta \gamma_L \gamma_R \sin (\phi_L + \phi_R) + 4 \gamma_L^2 \gamma_R (1 - \alpha_R^2) \cos^2 \phi_R \] 

and \[ 2 \gamma_L \gamma_R^2 (1 - \alpha_L \alpha_R \cos (\phi_L + \phi_R)). \]  

In the model of a single QD with two levels (\( \gamma_L = 0, \phi_L = 0 \)) as shown in figure 1(b), current is evaluated for the most general choices of \( \gamma_{\xi \epsilon}^\nu \), 

\[ I = I_{\text{incoherent}} K(\Delta), \]  

where \( K(\Delta) = \frac{4 \Delta^2 + (1 - \alpha_R^2) (\Gamma_{AA}^R + \Gamma_{BB}^R)^2}{4 \Delta^2 + (1 - \alpha_R^2) \Gamma_{AA}^R \Gamma_{BB}^R + \Gamma_{AA}^R \Gamma_{BB}^L + \Gamma_{BB}^R \Gamma_{AA}^L - 2 \alpha_L \alpha_R K (\Gamma_{AA}^R + \Gamma_{BB}^R)^2}. \]  

\[ \kappa \equiv \sqrt{\Gamma_{AA}^R \Gamma_{BB}^R \Gamma_{AA}^L \Gamma_{BB}^L}. \]  

In this model, the incoherent current is suppressed if one of the couplings to the right reservoir, \( \Gamma_{AA}^R \) or \( \Gamma_{BB}^R \), is very small [36]. The corresponding local state in the QD is now the ‘dark state’ to suppress the current. The function \( K(\Delta) \) is one when the coherent coupling in the right reservoir is absent, \( \alpha_R = 0 \) irrespective of the values of \( \Delta, \alpha_L \), and so forth. When \( \alpha_R \to 1 \), \( K(\Delta) \) is suppressed for small \( \Delta \), while for large offset, \( \Delta, K(\Delta) \to 1. \)

\section*{Appendix C. Derivation of current through the triple dot system}  

We consider the triple-dot model used in [31] with two lateral dots, A and B, with energies \( \epsilon_A \) and \( \epsilon_B \) coupled to left reservoirs independently and one dot, C, with energy \( \epsilon_c \) coupled to the right reservoir. The dots A and B are tunnel coupled to C with amplitude \( t \). Because of the charging effect, the total number of electrons is zero or one and the applied bias is very large. Setting up the Bloch equation as done in the main text, we obtain the formula of steady current 

\[ I = \frac{2 \gamma_L \gamma_R \Delta^2}{(2 \gamma_L + \gamma_R + \frac{\alpha_R^2}{2} (2 (\epsilon_0 - \epsilon_c)^2 + (\Delta^2 + \gamma_R^2)/2)) \Delta^2 + 8 \gamma_L t^2}, \]  

where \( \epsilon_0 = \frac{\epsilon_A + \epsilon_B}{2} \) and \( \Delta = \epsilon_B - \epsilon_A \). This formula resembles the result of equation (30) with \( \alpha_R = 1 \), where the current is strongly suppressed near \( \Delta = 0 \), while the behaviour for large \( |\Delta| \) is different.
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