A study of the decays of $S$–wave $\bar{D}^*K^+$ hadronic molecules: the scalar $X_0(2900)$ and its spin partners $X_{J(J=1,2)}$

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In this work, we investigated the decays of the fully open-flavor tetraquark state $X_0(2900)$ which was observed by the LHCb Collaboration very recently. Here, the $X_0(2900)$ was assigned as a $S$–wave $\bar{D}^*K^+$ hadronic molecule with $I = 0$, and the effective lagrangian approach was applied to estimate the partial decay widths. Moreover, we also predicted the decay behaviors of the other unobserved $X_{J(J=1,2)}$, which were the spin partners of the $X_0(2900)$ in the $S$–wave $\bar{D}^*K^+$ picture. It was pointed out that the $X_1$ state with $I = 0$ was a broad state with the width more than one hundred MeV, while another $X_2$ state with $I = 0$ was a narrow state with the width approaching half of that for the $X_0(2900)$. In addition, our results also showed that the $\bar{D}^*K$ mode was expected to be the dominant decay mode for both $X_1$ and $X_2$. Searching for those unobserved $X_{J(J=1,2)}$ in the future experiments might be helpful to understand the nature of $X_0(2900)$.

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I. INTRODUCTION

Until now, the exotic family is no longer thin due to the great efforts from the experimental side. Traces of their existence have been found in $B = 0$ meson sector, baryon sector as well as the $B = 2$ dibaryon sector, namely, the tetraquark states, pentaquark states, and hexaquark states. The $X(3872)$, $D_{sJ}^*(2317)$, $Z_c$, $P_c$ are the typical examples of the remarkable exotic states (more information can be found in the review papers [1–12]). Concerned to the constituent quarks, most of the exotic states contain a pair of quark-antiquark, $c\bar{c}$ or $u\bar{u}$ for instance, which makes them hidden-flavor. Besides of the hidden-flavor structure, the exotic states can be composed of fully open-flavor quarks. The first fully open-flavor exotic state, as well as the only one before September 2020, was observed in the 2016 named $X(5568)$[13]. It was observed by the D0 Collaboration and was expected to be consist of $\bar{b}su\bar{d}$, which made it obviously exotic[13]. The $X(5568)$ was interesting and attracted a great attention for both experimentalists and theorists [2, 5, 6, 8, 9, 11, 12]. However, the later negative results for the $X(5568)$ from other collaborations bogged down the interests of the study of fully open-flavor states[14, 15].

The situation dramatically changed very recently, since the LHCb Collaboration reported their first amplitude analysis of the $B^+ \rightarrow \bar{D}^*D^*K^+$ process[16, 17] and where they have to introduce one spin-0 $X_0(2900)$ state and another spin-1 $X_1(2900)$ in their model in order to describe the data. Their obtained resonance parameters were,

\begin{align}
X_0(2900) : \quad M &= 2866 \pm 7 \pm 2\text{MeV}, \\
&\Gamma = 57 \pm 12 \pm 4\text{MeV}, 
\end{align}

and

\begin{align}
X_1(2900) : \quad M &= 2904 \pm 5 \pm 1\text{MeV}, \\
&\Gamma = 110 \pm 11 \pm 4\text{MeV}.
\end{align}

Their $P$ parities were determined to be positive for the spin–$0$ state, and negative for the spin–$1$ state based on the $D^*K^+$ decay channel. Besides, the isospin $I$ was still unknown, while there were two possible assignments $I = 0$ and $I = 1$. Therefore, the $I(J^P)$ quantum numbers of the $X_0(2900)$ and $X_1(2900)$ were $0/1(0^+)$ and $0/1(1^-)$, respectively.

It should be stressed that the $D^*K^+$ final state indicated the exotic structure of the observed $X_0(2900)$ and $X_1(2900)$, e.g., $cd\bar{s}u$ quark flavors. Therefore, the two resonances were fully open-flavor states similar to the $X(5568)$, and unambiguously differed from the conventional hadrons. Those exotic states have been explained as the tetraquark states. The hadronic molecules and compact tetraquarks are two types of tetraquark states. In the former case, the four quarks form two hadrons, which are bounded via the strong interaction. In the later case, the quarks form a compact structure. For the particular $cd\bar{s}u$ structure here, Ref. [18] calculated its anti-particle in 2010, a bound $D^*K^+$ decaying to $D\bar{K}$. The predicted mass, width and quantum numbers were 2848 MeV, 59 MeV and $I(J^P) = 0(0^+)$, respectively. Moreover, the authors of Ref. [19] also predicted a $cs\bar{u}\bar{d}$ state with the mass 2850 MeV. Besides, the charmed partners of the $X(5568)$, whose structure were $su\bar{d}\bar{c}$, were predicted [20, 21], however, the mass $M = 2550\text{MeV}$ did not fit the present observation.
Stimulated by the observation of the \( X_0(2900) \) and \( X_1(2900) \), many theoretical analyses of the two resonances have been carried out by employing various approaches\cite{22–41}. Ref. \cite{22} and \cite{25, 31} interpreted the \( X_0(2900) \) as the compact tetraquark based on the constituent quark model and QCD sum rules, respectively. Moreover, the \( X_1(2900) \) was explained as the compact tetraquark state in Refs. \cite{27, 31, 33, 34, 36, 39}. Applying the chromomagnetic interactions diquark configuration model, the \( J^P = 0^+ \) resonance was also considered as a radial excited tetraquark, while the \( J^P = 1^- \) one was assigned as an orbitally excited tetraquark\cite{24}. However, a calculation based on the extended relativized quark model disfavored the tetraquark interpretation\cite{28}.

It should be mentioned that the hadronic molecules assignments were proposed\cite{26, 27, 29, 30, 32, 34, 36, 39}. By considering the \( J^P \) quantum numbers and mass threshold, the \( X_0(2900) \) was explained as the \( S^- \)-wave \( D^* K^+ \) hadronic molecule, while the \( X_1(2900) \) was explained as the \( D_1(2460) \) and \( P^- \)-wave \( D^* K^+ \) hadronic molecules\cite{29}. There was also a negative result for the \( D_1K \) molecule interpretation for the \( X_1(2900) \), where the author found that the potential between \( D_1K \) was too weak to form any bound state\cite{41}. To explore the nature of the \( X_0(2900) \) and \( X_1(2900) \), the production mechanism was also analysed\cite{38}. In additions, Ref. \cite{23} considered the triangle singularity to be the origin of \( X_0(2900) \) and \( X_1(2900) \).

Whether the \( X_0(2900) \) and \( X_1(2900) \) were compact tetraquarks, hadronic molecules or due to kinetic effects was unclear so far. In the present work, we followed the \( S^- \)-wave \( D^* K^+ \) interpretation for the \( X_0(2900) \) with isospin \( I = 0 \) proposed in Ref. \cite{26, 30, 32, 34} to investigate its decay behaviors via the effective lagrangian approach. In particular, in the \( S^- \)-wave \( D^* K^+ \) hadronic molecule scenario, two spin partners of \( X_0(2900) \) were predicted with \( J = 1 \) and \( J = 2 \)\cite{30, 32, 34}. Here we would refer \( X_1 \) and \( X_2 \) to the \( J = 1 \) and \( J = 2 \) states, respectively. One should note that the \( X_1 \) hereafter was not the \( X_1(2900) \) in Eq. (2), while the \( X_0 \) corresponding to the \( X_0(2900) \) in Eq. (1). Within the same molecule scenario, we also investigated the decay behaviors of \( X_1 \) and \( X_2 \).

The present paper is assigned as follows. The effective lagrangians and decays are given in the next section. Sec. III shows our numerical results and discussion. The summary is presented in the last section.

### II. EFFECTIVE LAGRANGIANS AND DECAYS

The effective lagrangian approach was applied to estimate the decays of experimental observed \( X_0(2900) \) in the present work, where it was considered as the \( S^- \)-wave \( D^* K^+ \) hadronic molecules with the isospin \( I(X_0) = 0 \). Besides, the decays of the predicted \( X_{j(J=1,2)} \), being the spin partners of the \( X_0(2900) \) in the \( S^- \)-wave \( D^* K^+ \) picture, were also investigated, where two possible isospins \( I(X_{j(J=1,2)}) = 0 \) and \( I(X_{j(J=1,2)}) = 1 \) were adopted for our analyses.

We firstly constructed the effective lagrangians describing the interaction between the molecular state and its components,

\[
L_{X_0}(x) = g_{X_0}X_0(x) \int dy\Phi(y^2) \left[ D_{p_\tau}^-(x + \omega_{K\cdot D\cdot y})K_{\mu}^+(x - \omega_{D\cdot K\cdot y}) - \tilde{D}^{0p}(x + \omega_{K\cdot D\cdot y})K_{\mu}^0(x - \omega_{D\cdot K\cdot y}) + \text{H.C.} \right], \\
L_{X_1}(x) = ig_{X_1}\epsilon_{\mu\nu\rho\sigma}\partial^\rho X_1^\nu(x) \int dy\Phi(y^2) \left[ D_{p_\tau}^-(x + \omega_{K\cdot D\cdot y})K_{\mu\nu}^{++}(x - \omega_{D\cdot K\cdot y}) \pm \tilde{D}^{0p}(x + \omega_{K\cdot D\cdot y})K_{\mu\nu}^{0+}(x - \omega_{D\cdot K\cdot y}) + \text{H.C.} \right], \\
L_{X_2}(x) = g_{X_2}X_2^\mu(x) \int dy\Phi(y^2) \left[ D_{p_\tau}^-(x + \omega_{K\cdot D\cdot y})K_{\mu\nu}^{++}(x - \omega_{D\cdot K\cdot y}) \pm \tilde{D}^{0p}(x + \omega_{K\cdot D\cdot y})K_{\mu\nu}^{0+}(x - \omega_{D\cdot K\cdot y}) + \text{H.C.} \right],
\]

where the \( \pm \) corresponding to \( X_{j(J=1,2)} \) states with \( I = 1 \) and \( I = 0 \), respectively. The coupling constant \( g_{X_0, X_1, X_2} \) can be determined by the compositeness condition\cite{42–44}. The \( \omega_{AB} = m_A/(m_A + m_B) \), the correlation function \( \Phi(y^2) \) carries the distribution information of the components in the hadronic molecule. Within the Fourier transformation, \( \Phi(y^2) = \int d^4p/(2\pi)^4e^{-ipx}\Phi(-p^2) \). It should be mentioned that the Gaussian form \( \Phi(p_{\mu}^2) = \exp(-p_{\mu}^2/\Lambda^2) \) was widely used to estimate the decays of hadronic molecules\cite{42–47}. In Eqs. (3)-(5), the \( \Lambda \) is the model parameter related to the size of the hadronic molecule.

Considering the two-body decays, the \( X_0 \) can decay to \( \tilde{D}K \), the \( X_1 \) can decay to \( D^* K \) and \( D^* \bar{K} \), and the \( X_2 \) can decay to \( D \bar{K} \), \( D^* K \) and \( D \bar{K} \). These transitions occurred via the triangle diagrams (presented in Fig. 1), where the hadronic molecule and the final state are connected through the \( D^* \) and \( K^* \) by exchanging a proper hadrons. Here, the exchanged hadrons can be either pseudoscalar meson and vector meson, including

\[
P: \pi, \eta, \eta', \quad V: \rho, \omega.
\]

As we can see from the Fig. 1, the effective lagrangians describing the interaction between the charmed (strange)
The $P$ stands for $\pi$, $\eta$ and $\eta'$, where
\begin{align}
\pi &= \left( \begin{array}{c} \pi^0 \\ \sqrt{2}\pi^+ \\ -\pi^0 \end{array} \right), \\
\eta &= \left( \begin{array}{c} \rho^0 \\ \sqrt{2}\rho^+ \\ -\rho^0 \end{array} \right),
\end{align}
and the vector meson $V$ can be $\rho$, $\omega$, or $\omega'$, where
\begin{align}
\pi &= \left( \begin{array}{c} \pi^0 \\ \sqrt{2}\pi^+ \\ -\pi^0 \end{array} \right), \\
\eta &= \left( \begin{array}{c} \rho^0 \\ \sqrt{2}\rho^+ \\ -\rho^0 \end{array} \right),
\end{align}

In our numerical calculations, we simple employ the coupling constants $g_{D^{*}\pi} = 12.2$, which was estimated via the experimental measured decay width of process $D^* \to D\pi$ [50]. The $g_{D^{*}\rho} = 11.9$ was from the Ref. [49]. Applying the VMD method to the process $D^* \to D\pi$, one can obtain the $g_{D^{*}\pi} = 2.82$ [49]. The $g_{D^{*}\rho} = 2.52$ was determined by the same VMD method [48, 49]. In addition, the coupling constants $g_{K^0\pi}$ and $g_{K^+\rho}$ are related via a gauge coupling $g$.

\begin{align}
\frac{g_{K^0\pi}}{g_{K^+\rho}} &= \frac{1}{4},
\end{align}

The $P$ and $V$ stand for the exchanged pseudoscalar mesons and vector mesons, respectively, including the $\pi^0$, $\eta$, $\eta'$, $\rho^0$ and $\omega$. Besides, three additional diagrams with the intermediate $D^{*0}$ were not presented here, which also contributed to the process $X_{H,(0,1.2)} \to D^{*0}K^{*0}$ and were considered in the calculation.
Other two diagrams with the $\bar{D}^{(*)}K^{(*)}$ intermediate states, in which the exchanged states are $\pi^-$ and $\rho^-$, also contribute to the process $X_0 \rightarrow D^+K^+$, we can obtain the corresponding Feynman amplitude via the isospin symmetry,

$$\mathcal{M}_{X_0 \rightarrow D^- K^+} = -2M_{X_0 \rightarrow D^- K^+}^0,$$

$$\mathcal{M}_{X_0 \rightarrow D^- K^+} = -2M_{X_0 \rightarrow D^- K^+}^1.$$

Similarly, we can write out the Feynman amplitudes for the processes $X_1 \rightarrow D^{*-}K^+, X_1 \rightarrow D^- K^+, X_2 \rightarrow D^- K^+, X_2 \rightarrow D^- K^{*+}$, and $X_2 \rightarrow D^- K^{*+}$. The detailed expressions were presented in the Appendix.

Now, the total contributions of the processes $X_{I(J=0,1,2)} \rightarrow D^{(*)}K^{(*)+}$ were,

$$M_{X_{I,J=0,1,2} \rightarrow D^{(*)}K^{(*)+}} = M^0 + M^I + M^{0} + M^{0},$$

$$+M^I \pm (\mathcal{R}^{\pi} + \mathcal{R}^{\rho}),$$

where in the right side the lower index $X_{I,J=0,1,2} \rightarrow D^{(*)}K^{(*)+}$ of $M$ was ignored, the $+$ corresponding to $I = 0$ and $I = 1$ cases, respectively. Finally, we can derive the partial decay widths of the processes $X_{I,J=0,1,2} \rightarrow D^{(*)}K^{(*)+}$,

$$\Gamma(X_{I,J=0,1,2} \rightarrow D^{(*)}K^{(*)+}) = \frac{1}{2J + 1} \frac{1}{8\pi M^2} \times |\mathcal{M}_{X_{I,J=0,1,2} \rightarrow D^{(*)}K^{(*)+}}|^2,$$

where the $J$ and $M$ are the angular momentum and mass of the initial state, respectively. $|\beta|$ is the three-momentum of the final state in the rest frame of the initial state, the overline represents the sum of the polarization for the initial and final states.

In terms of the isospin symmetry, the partial decay width of the $\bar{D}^{(*)}K^{(*)0}$ is the same as the $D^{(*)}K^{(*)+}$ mode. Therefore,

$$\Gamma(X_{I,J=0,1,2} \rightarrow \bar{D}^{(*)}K^{(*)}) = \Gamma(X_{I,J=0,1,2} \rightarrow D^{(*)}K^{(*)+}) + \Gamma(X_{I,J=0,1,2} \rightarrow \bar{D}^{(*)}K^{(*)0})$$

$$= 2\Gamma(X_{I,J=0,1,2} \rightarrow D^{(*)}K^{(*)+}).$$

III. NUMERICAL RESULTS AND DISCUSSION

In Fig. 2, the numerical results of partial decay width for $X_0 \rightarrow \bar{D}K$ process were presented, where the $\Lambda$ and $\Lambda_1$ were the two parameters in our present approach. Since they cannot be determined by the first principle, the experimental data is usually applied to constrain them. Assuming that the partial decay width of $\bar{D}K$ mode was the experimental measured total decay width of $X_0$ resonance, then, the parameters can be constrained via the experimental measured data. On the other hand, we attempted to constrain the parameters within the range $0.5 - 1.5$ GeV, while other regions for the cut-off parameters seem unreasonable. The solid line in Fig. 2 corresponding to the center value of the experimental measured $X_0$ decay width, which is $\Gamma(X_0) = 57$ MeV. Based on this line, a series sets of parameters can be determined, here, we gave several typical values of the constrained parameter. For $\Lambda = 0.8, 0.9, 1.0, 1.1, 1.2$ and $1.3$ GeV, the corresponding $\Lambda_1$ are $1.41, 1.31, 1.24, 1.18, 1.14, 1.11$ and $1.08$ GeV, respectively.

The above typical values of parameters were applied to predict the decay properties of the $X_1$ and $X_2$, the two spin partners of $X_0(2900)$. Here, both the masses of the $X_1$ and $X_2$ were assigned to be 2866 MeV, which were predicted in Ref. [32]. In Fig. 3, the numerical results of partial decay widths for the $X_1 \rightarrow \bar{D}K$ and $X_1 \rightarrow \bar{D}K^*$ processes were presented. For the $I(X_1) = 0$ case, we found that $\Gamma(X_1 \rightarrow \bar{D}K^*)$ varied from $123.6$ MeV to $101.0$ MeV within the constrained parameters, which weakly depended to the parameters. The partial decay width for another $\bar{D}K^*$ mode varied from $16.4$ MeV to $14.7$ MeV. The numerical results for $I(X_1) = 1$ case were much smaller compared to those for $I(X_1) = 0$ case, where the partial decay widths for the $\bar{D}K$
and $\bar{D}K^*$ were 19.2 – 15.9 MeV and 4.01 – 2.75 MeV, respectively. It could be concluded that for the both two cases, the $D^*K$ mode was the dominant decay mode. Besides, we also found that the $X_1$ with $I = 0$ was a broad state since the corresponding estimated width was more than 100 MeV.

As for another $X_2$ state, the numerical results of its partial decay widths were presented in the Fig. 4. One can find that the $D^*K$ mode was the dominant decay mode both for the $I(X_2) = 0$ and $I(X_2) = 1$ cases, the corresponding partial decay widths in the constrained parameter range were $19.7 – 10.4$ MeV for the $I(X_2) = 0$ case and $3.56 – 1.54$ MeV for the $I(X_2) = 1$ case. Compared to the $D^*K$ mode, the partial decay width for the $\bar{D}K^*$ mode was expected to be, at least, one order of magnitude smaller. In particular, the $\Gamma(X_2 \rightarrow \bar{D}K^*)$ was $0.649 – 0.385$ MeV for the $I(X_2) = 0$ case and $0.0921 – 0.0507$ MeV for $I(X_2) = 1$ case. Besides of the $D^*K$ and $\bar{D}K^*$ mode, the $X_2$ can also decay to the $\bar{D}K$, which was the channel that $X_0$ observed in. The corresponding partial width was $4.06 – 2.09$ MeV for the $I(X_2) = 0$ case, and $0.692 – 0.296$ MeV for the $I(X_2) = 1$ case. Similar to the $X_1$ case, the partial decay widths for the $X_2$ state with $I = 0$ was much larger than that with $I = 1$.

Besides, we got that the partial decay width for the $I = 1$ states were almost one-seventh of that for $I = 0$. We concluded that the $X_1$ state with $I = 0$ was a broad state with the width more than 100 MeV, while others were narrow state. Both for the $X_1$ and $X_2$ state, the $D^*K$ mode was the dominant decay mode.

Finally, the observation of the $X_0(2900)$ opened a new area for the fully open multi-quark states. The inner structure of the $X_0(2900)$ is still controversial. It is valuable to determine the isospin number of $X_0(2900)$ experimentally. Meanwhile, searching for its spin partners and the flavor partners can also help us to understand the nature of $X_0(2900)$. We hoped that more progress can be carried out in the near future.

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**FIG. 4:** The partial decay widths of the transitions from $X_1$ to $\bar{D}K$, $D^*K$ and $\bar{D}K^*$ with the constrained parameters. The left column corresponding to the results with $I(X_2) = 0$ and the right column was the results with $I(X_2) = 1$.

**TABLE I:** Predicted partial decay widths for the $X_1$ and $X_2$. The results were based on the typical values of parameters $\Lambda = 1.0$ GeV, $\Lambda_1 = 1.24$ GeV.

| Partial Decay Width (MeV) | $X_1$ | $X_2$ |
|---------------------------|--------|--------|
| $\Gamma(\bar{D}K)$        | -      | 3.15   |
| $\Gamma(D^*K)$            | 11.5   | 15.4   |
| $\Gamma(\bar{D}K^*)$      | 16.3   | 3.58   |

Based on the above analyses, it was found that the predicted results depended weakly to the parameters. Therefore, in Tab. I, we also summarized our predictions for the partial decay widths of $X_1$ and $X_2$ with the typical parameters $\Lambda = 1.0$ GeV and $\Lambda_1 = 1.24$ GeV.

### IV. SUMMARY

In the present work, we investigated the decay behaviors of $X_0(2900)$ in the $S$-wave $D^*K^*$ scenario with the isospin $I = 0$. With the help of the effective lagrangian approach, the contributions from the triangle diagrams were estimated. Moreover, in order to represent the off-shell effect of the coupling constants, a phenomenological form factor was considered. The obtained partial decay width for the $X_0 \rightarrow \bar{D}K$ process was in agreement with the experimental data with the model parameters $\Lambda$ and $\Lambda_1$ that were selected to be around 1 GeV.

Within the constrained model parameters, we further calculated the decay behaviors of another two $S$-wave $D^*K^*$ hadronic molecules $X_1$ and $X_2$, where both the $I = 0$ and $I = 1$ cases were taken into account. The $X_1$ can decay to $D^*K$ and $\bar{D}K^*$, and the $X_2$ can decay to $\bar{D}K$, $D^*K$ and $\bar{D}K^*$. In the constrained parameter ranges, the partial decay widths for the $X_1$ state with $I = 0$ were,

\[
\Gamma(X_1 \rightarrow D^*K) = 124 - 101 \text{ MeV}, \quad (28)
\]

\[
\Gamma(X_1 \rightarrow \bar{D}K^*) = 16.4 - 14.7 \text{ MeV}. \quad (29)
\]

and for the $X_2$ state with $I = 0$,

\[
\Gamma(X_2 \rightarrow D^*K) = 4.06 - 2.09 \text{ MeV}, \quad (30)
\]

\[
\Gamma(X_2 \rightarrow \bar{D}K^*) = 19.8 - 10.4 \text{ MeV}, \quad (31)
\]

\[
\Gamma(X_2 \rightarrow \bar{D}K) = 0.649 - 0.385 \text{ MeV}. \quad (32)
\]
Appendix A: The amplitudes of the transition from the \( X_{J=1,2} \) to \( D^{(*)}K^{(*)} \)

The diagrams contributing to the process \( X_{J=1,2} \) to \( D^{(*)}K^{(*)} \) were presented in Fig. 1, we can write out the corresponding Feynman amplitudes. For the \( X_1 \rightarrow D^- K^+ \) process,

\[
\mathcal{M}^{p}_{X_1 \rightarrow D^- K^+} = \int \frac{d^4 q}{(2\pi)^4} \Phi \left( p_1 - w_{12} p^2 \right) \left\{ \frac{1}{\sqrt{2}} g_{X_1 \epsilon J y} (ip^p) \times \epsilon^\dagger (p_1) \left[ \frac{g_{D^* D^* p^V \epsilon_{\rho\sigma\nu\tau}} (ip^p) (-ip^p) e^\dagger (p_3) \right] \times \left[ -ig_{K^* K p} (-iq_\rho - ip_\delta) \right] - g_\rho^\eta + p_1^p p_1^1/m_1 \right\} \frac{-g_\rho^\eta + p_1^p p_1^1/m_1}{p_1^2 - m_1^2} \times \frac{-g_\rho^\eta + p_1^p p_1^1/m_1}{p_1^2 - m_1^2} \times \frac{1}{q^2 - m_q^2} \frac{1}{\sqrt{p_1^2}} \right\} \times \mathcal{F}^2 (m_q, \Lambda_1), \quad (A1)
\]

For the \( X_1 \rightarrow D^- K^{*+} \) process,

\[
\mathcal{M}^{p}_{X_1 \rightarrow D^- K^{*+}} = \int \frac{d^4 q}{(2\pi)^4} \Phi \left( p_1 - w_{12} p^2 \right) \left\{ \frac{1}{\sqrt{2}} g_{X_1 \epsilon J y} (ip^p) \times \epsilon^\dagger (p_1) \left[ \frac{g_{D^* D^* p^V \epsilon_{\rho\sigma\nu\tau}} (ip^p) (-ip^p) e^\dagger (p_3) \right] \times \left[ -ig_{K^* K p} (-iq_\rho - ip_\delta) \right] - g_\rho^\eta + p_1^p p_1^1/m_1 \right\} \frac{-g_\rho^\eta + p_1^p p_1^1/m_1}{p_1^2 - m_1^2} \times \frac{-g_\rho^\eta + p_1^p p_1^1/m_1}{p_1^2 - m_1^2} \times \frac{1}{q^2 - m_q^2} \frac{1}{\sqrt{p_1^2}} \right\} \times \mathcal{F}^2 (m_q, \Lambda_1), \quad (A2)
\]

For the \( X_2 \rightarrow D^- K^+ \) process,

\[
\mathcal{M}^{p}_{X_2 \rightarrow D^- K^+} = \int \frac{d^4 q}{(2\pi)^4} \Phi \left( p_1 - w_{12} p^2 \right) \left\{ \frac{1}{\sqrt{2}} g_{X_1 \epsilon J y} (ip^p) \times \epsilon^\dagger (p_1) \left[ \frac{g_{D^* D^* p^V \epsilon_{\rho\sigma\nu\tau}} (ip^p) (-ip^p) e^\dagger (p_3) \right] \times \left[ -ig_{K^* K p} (-iq_\rho - ip_\delta) \right] - g_\rho^\eta + p_1^p p_1^1/m_1 \right\} \frac{-g_\rho^\eta + p_1^p p_1^1/m_1}{p_1^2 - m_1^2} \times \frac{-g_\rho^\eta + p_1^p p_1^1/m_1}{p_1^2 - m_1^2} \times \frac{1}{q^2 - m_q^2} \frac{1}{\sqrt{p_1^2}} \right\} \times \mathcal{F}^2 (m_q, \Lambda_1), \quad (A5)
\]

For the \( X_2 \rightarrow D^- K^{*+} \) process,

\[
\mathcal{M}^{p}_{X_2 \rightarrow D^- K^{*+}} = \int \frac{d^4 q}{(2\pi)^4} \Phi \left( p_1 - w_{12} p^2 \right) \left\{ \frac{1}{\sqrt{2}} g_{X_1 \epsilon J y} (ip^p) \times \epsilon^\dagger (p_1) \left[ \frac{g_{D^* D^* p^V \epsilon_{\rho\sigma\nu\tau}} (ip^p) (-ip^p) e^\dagger (p_3) \right] \times \left[ -ig_{K^* K p} (-iq_\rho - ip_\delta) \right] - g_\rho^\eta + p_1^p p_1^1/m_1 \right\} \frac{-g_\rho^\eta + p_1^p p_1^1/m_1}{p_1^2 - m_1^2} \times \frac{-g_\rho^\eta + p_1^p p_1^1/m_1}{p_1^2 - m_1^2} \times \frac{1}{q^2 - m_q^2} \frac{1}{\sqrt{p_1^2}} \right\} \times \mathcal{F}^2 (m_q, \Lambda_1), \quad (A6)
\]

For the \( X_2 \rightarrow D^- K^{*+} \) process,
For the process $X_2 \rightarrow D^- K^{++}$

\[
M_{X_2 \rightarrow D^- K^{++}}^\mu = \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12} p)^2] \left[ \frac{1}{\sqrt{2}} g_{X_2} \epsilon_{\xi \lambda}(p) \right] \times [igD^\alpha_D p_{\alpha}][ - g_{K^- K^+} \epsilon_{\eta \rho \sigma \tau} ] \times (-ip_2^\mu)(ip_4^\nu) \epsilon^\nu(p_4) \frac{-g^{\nu \mu} + p_1^\nu p_1^\mu / m_1^2}{p_1^2 - m_1^2} 
\times -g^{\nu \tau} + p_2^\nu p_2^\tau / m_2^2 \frac{1}{p_2^2 - m_2^2} 
\times -g^{\nu \rho} + q_\nu q_\rho / m_q^2 \frac{1}{q^2 - m_q^2} 
\times F^2(m_q, \Lambda_1), \quad (A9)
\]

\[
M_{X_2 \rightarrow D^- K^{++}}^\nu = \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12} p)^2] \left[ \frac{1}{\sqrt{2}} g_{X_2} \epsilon_{\xi \lambda}(p) \right] \times [g_D^\alpha_D p_\alpha][ - i g_{K^- K^+} ] \times (-ip_2^\mu)(ip_4^\nu) \epsilon^\nu(p_4) \times (iq_\rho) g^{\tau \rho} - (iq_\rho) g^{\mu \tau} + (-ip_2^\nu) g^{\mu \rho} 
\times -g^{\alpha \beta} + p_1^\nu p_1^\mu / m_1^2 \frac{-g_\alpha g_\beta + p_1^\nu p_2^\beta / m_2^2}{p_1^2 - m_1^2} 
\times -g^{\alpha \tau} + q_\alpha q_\tau / m_q^2 \frac{1}{q^2 - m_q^2} 
\times -g^{\alpha \rho} + q_\alpha q_\rho / m_q^2 \frac{1}{q^2 - m_q^2} F^2(m_q, \Lambda_1). \quad (A10)
\]
