Weyl Symmetry Inspired Inflation and Dark Matter

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Abstract

Motivated by the original Weyl scaling gauge symmetry, we present a theoretical framework to explain cosmic inflation and dark matter simultaneously. This symmetry has been resurrected in recent attempts to formulate the gauge theory of gravity. We show the inspired inflation can be well consistent with observations and probed in future. Furthermore, Weyl gauge boson can be a dark matter candidate and decay due to its novel couplings to standard model Higgs field, which can lead to testable signatures in dark matter indirect searches, Higgs invisible decay and production rate at colliders.
Introduction: The accumulated compelling evidence for dark matter (DM) has been challenging the standard model (SM) of fundamental physics for decades. The supporting observations, such as cosmic microwave background (CMB), large-scale structure, rotation curves, scope from cosmological to galactic scales [1, 2]. For the intrinsic nature of DM, however, we are still lacking sufficient information since the robust evidence is only able to suggest that DM must have gravitational interaction. Nevertheless, explanations of DM would require extensions of SM, either in the sector of particle physics or gravity.

Gauge symmetry has played a guiding principle for constructing fundamental laws of nature since last century when Weyl first proposed [3] the scaling symmetry and tried to unify the electromagnetic interaction with Einstein’s general relativity. The original scale factor has to be modified as a phase to account for the gauge $U(1)$ theory for electromagnetic interaction [4]. $U(1)$ is later generalized to non-abelian theory by Yang and Mills [5], which describes the interactions of all known fundamental particles in SM by incorporating the Higgs mechanism [6–8]. Variants of Weyl/scaling symmetry, however, still stimulate explorations of theoretical and phenomenological studies, see Refs. [9–35] for examples in cosmology and particle physics. Recently, Refs. [36, 37] has shown the original Weyl symmetry can play a crucial role in formulating the gauge theory of gravity.

In this paper we propose that the original Weyl symmetry can provide a framework to explain the cosmic inflation [38–41] and DM simultaneously [42]. The starting inflationary Lagrangian can be Weyl invariant and responsible for the generation of Planck scale. Theoretical predictions of observables in this scenario are consistent with current experiments and testable in future. The gauge boson associated with local Weyl symmetry may be identified as a DM candidate if its mass and coupling are in the right region. Thanks to the novel connection with SM Higgs field, Higgs’s invisible decay and single- or double-production at colliders can be used to probe this scenario. Additionally, the intrinsic decay of Weyl boson into SM particles would induce signatures at DM indirect searches in cosmic-ray, gamma-ray and neutrino spectra.

Framework: To illustrate the main physical points, we start with the following general bosonic Lagrangian with two scalars $\varphi$ and $\phi$,

$$\mathcal{L} \supset \sqrt{-g} \left[ \alpha \left( \varphi^2 R - 6 \partial_{\mu} \varphi \partial^{\mu} \varphi \right) + \beta \left( \phi^2 R - 6 \partial_{\mu} \phi \partial^{\mu} \phi \right) + \zeta_1 \frac{1}{2} D_{\mu} \varphi D^{\mu} \varphi + \zeta_2 \frac{1}{2} D_{\mu} \phi D^{\mu} \phi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - V(\phi, \varphi) \right],$$

(1)
where $R$ is the Ricci scalar, $W_\mu$ the Weyl field, $F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ and the covariant derivative $D_\mu = \partial_\mu - g_{\mu\nu} W_\nu$ for scalars. More complete Lagrangian can be found in Refs. [36, 37] where gravity is formulated as a gauge theory of the fundamental field $\chi^a_\mu$ with its connection to metric, $\chi^a_\mu \chi^b_\nu \eta_{ab} = g_{\mu\nu}$. Throughout our paper, we use the sign convention, $\eta_{ab} = (1, -1, -1, -1)$. The potential $V$ has a general form of $\sum_{i=0}^{4} c_i \phi^i \varphi^{4-i}$. The parameters in the front of scalar kinetic terms can have two possible signs, $\zeta_i = \pm 1$. The negative sign can appear in theories with compactified Kaluza-Klein extra-dimension and is not necessary associated with theoretical issues, as long as the total energy of the system is positive [43]. We shall take $\zeta_i = 1$ unless otherwise stated. Note that $W_\mu$ does not couple to fermions directly, which can be understood from the Lagrangian for a massless fermion $\psi$, $\sqrt{-g} \left( \psi \gamma^\mu D_\mu \psi - \bar{D}_\mu \psi \gamma^\mu \psi \right)$. Since there is no $i$ in the covariant derivative for $W_\mu$, $W_\mu$-dependent terms will cancel in the parentheses.

The above general Lagrangian has a local symmetry and is invariant under the following local Weyl or scaling transformation

$$\begin{align*}
g_{\mu\nu}(x) &\rightarrow g'_{\mu\nu}(x) = \lambda^2(x) g_{\mu\nu}(x), \\
\varphi(x) &\rightarrow \varphi'(x) = \lambda^{-1}(x) \varphi(x), \\
\phi(x) &\rightarrow \phi'(x) = \lambda^{-1}(x) \phi(x), \\
W_\mu(x) &\rightarrow W'_\mu(x) = W_\mu(x) - \frac{\partial_\mu \lambda(x)}{g_W \lambda(x)},
\end{align*}$$

where the scale factor $\lambda(x)$ acts as a gauge parameter that may be taken in the domain $\lambda > 0$ without losing generality. When we fix $\phi^2 = v^2$, Einstein-Hilbert term $R$ is recovered. In such an Einstein basis, Weyl boson $W_\mu$ gets a mass $M_W = g_W v$ due to the kinetic term of $\phi$. Afterwards, the theory describes Einstein’s gravity with a non-minimally coupled scalar $\varphi$ and the massive gauge boson $W_\mu$. We shall show that $\varphi$ can be responsible for cosmic inflation and $W_\mu$ can be a dark matter candidate if its mass light and coupling constant are in the proper range.

Inflation: To demonstrate how the above framework can provide a mechanism for cosmic inflation, we shall illustrate with a concrete example with $\beta = 0$ and $V = c(\varphi^2 - \phi^2)^2$. Detailed analysis with general $\beta$ and $V$ will be presented elsewhere. After fixing $\phi^2 = v^2$, for the scalar and gravity part we have

$$\mathcal{L} \supset \sqrt{-g} \left[ \alpha \left( \varphi^2 R - 6 \partial_\mu \varphi \partial^\mu \varphi \right) + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - c(\varphi^2 - v^2)^2 \right].$$

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Now the potential has a minimum at $\varphi = v$, resulting in an effective Planck scale $M_p^2 = 2\alpha v^2$.

To make the formalism more familiar, we can change to Einstein frame [44] by redefining the metric $\bar{g}_{\mu\nu} = 2\alpha \varphi^2 / M_p^2 \cdot g_{\mu\nu}$ and scalar field $\sigma = M_p / \sqrt{2\alpha} \cdot \ln(\varphi / M_p)$, and obtain

$$\mathcal{L} / \sqrt{-\bar{g}} \supset \frac{M_p^2}{2} \bar{R} + \frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - c M_p^4 \left( 1 - \frac{1}{2\alpha} \exp \left( - \frac{2\sqrt{2\alpha}}{M_p} \sigma \right) \right)^2.$$  \hspace{1cm} (7)

Now the formalism describes the Einstein’s gravity with a minimally coupled scalar $\sigma$. Using $\varphi = M_p \exp (\sqrt{2\alpha} \sigma / M_p)$, the standard inflationary slow-roll parameters [45] can be calculated and expressed concisely as

$$\epsilon = \frac{16\alpha M_p^4}{(2\alpha \varphi^2 - M_p^2)^2}, \quad \eta = \frac{32\alpha M_p^2 (M_p^2 - \alpha \varphi^2)}{(2\alpha \varphi^2 - M_p^2)^2},$$  \hspace{1cm} (8)

which are related with the observables, spectral index $n_s = 1 - 6\epsilon + 2\eta$ and tensor-to-scalar ratio $r = 16\epsilon$. To solve the flatness and horizon problem, the Universe has to inflate at least by $e^N$ (the typical e-folding number $N \simeq 50 \sim 60$) before inflation ends at $\varphi^2 = M_p^2 (1/2 + 2\sqrt{\alpha}) / \alpha$. To give the correct amplitude of scalar power spectrum, parameter $c$ should be around $5\alpha \times 10^{-11}$, a typical value in large-field inflation models. Thus, only $\alpha$ is an effectively free parameter. This minimal model gives an one-parameter inflationary scenario, in which the Hubble scale $\mathcal{H}$ is at order of $\sim 10^{13}$GeV.

We numerically solve the inflationary dynamics and present in Fig. 1 the calculated values of $(n_s, r)$ for e-folding number $N = 50$(square) and 60(circle), in comparison with the shaded regions allowed by the latest *Planck* [46] results with 1-$\sigma$ (blue) and 2-$\sigma$ (purple). From top to bottom, each of four red dotted segments between the square and circle indicates the continuous change of $N$ from 50 to 60 for the four choices of $\alpha$, $\alpha = 0.001, 0.01, 0.1, 0.5$. Future CMB experiments [47, 48] that can probe $r \sim 10^{-3}$ would reach the parameter domain $\alpha \sim \mathcal{O}(0.1)$.

**Weyl Boson as Dark Matter:** To check whether Weyl boson can be a viable DM candidate, we need to investigate various constraints from different searches [49]. The important connection between Weyl boson and the SM of particle physics is through the Higgs field. To be Weyl-symmetric, the SM Higgs doublet $H$ can minimally couple to Weyl boson as

$$\mathcal{L} \supset \sqrt{-\bar{g}} \left[ (\mathcal{D}_\mu H)^\dagger \mathcal{D}^\mu H - \lambda_H \left( H^\dagger H - v_H^2 / 2 \right)^2 \right],$$  \hspace{1cm} (9)

where $\mathcal{D}_\mu$ includes both Weyl boson and electroweak gauge bosons. The vacuum expectation value $v_H$ can originate from the coupling between $H$ and $\phi$. For example, we can replace $v_H^2$
FIG. 1. Illustration of \((n_s, r)\) for \(\alpha = 0.001, 0.01, 0.1, 0.5\) (from top to bottom). The theoretical values of \((n_s, r)\) are shown for e-folding number \(N = 50\) (squares) and \(60\) (circles), in comparison with the shaded regions allowed by Planck [46] with 1-\(\sigma\) (blue) and 2-\(\sigma\) (purple). Dashed red lines indicate the gradual change from \(N = 50\) to \(N = 60\).

with \(\lambda_{\phi H} \phi^2\) and impose the coupling constant \(\lambda_{\phi H} = v_H^2/v^2\). Within the unitary gauge for electroweak symmetry, we can substitute \(H\) with \((0, v_H + h)^T/\sqrt{2}\). After some algebra and without showing the electroweak part, we finally obtain

\[
\mathcal{L} \supset \sqrt{-g} \left[ \frac{1}{2} \partial^\mu \tilde{h} \partial_\mu \tilde{h} - \frac{1}{2} m_H^2 \tilde{h}^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} m_W^2 \overline{W}^\mu W_\mu + L_{\text{int}} \right],
\]

where the interaction part is given by

\[
L_{\text{int}} = \frac{1}{2} g_W^2 \left( \frac{2v_H \tilde{h}}{C_H} + \frac{\tilde{h}^2}{C_H^2} \right) \left( \overline{W}^\mu W_\mu + \frac{2g_W v_H}{m_W^2 C_H} \overline{W}_\mu \partial^\mu \tilde{h} + \frac{g_W^2 v_H^2}{m_W^4 C_H^2} \partial^\mu \tilde{h} \partial^\nu \tilde{h} \right)
- \frac{g_W}{C_H^2} \left( \overline{W}^\mu \partial_\mu \tilde{h} + \frac{g_W v_H}{m_W^2 C_H} \partial^\mu \tilde{h} \partial^\nu \tilde{h} \right) - \frac{\lambda_H \tilde{h}^3}{C_H^3} \left( v_H + \tilde{h} \right). \tag{11}
\]

Here \(\overline{W} = W_\mu - \partial_\mu h \cdot g_W v_H/m_W^2\) is the redefined Weyl boson in order to diagonalize the kinetic terms due to the mixing between \(W_\mu\) and \(h\). Other parameters are defined as

\[
m_W^2 = M_W^2 + g_W^2 v_H^2, \quad m_\tilde{h} = v_H \sqrt{2 \lambda_H/C_H}, \quad \tilde{h} = C_H h, \quad C_H \equiv \sqrt{1 - \frac{g_W^2 v_H^2}{m_W^2}}. \tag{12}
\]
Here $M_W$ is the mass of Weyl boson before electroweak symmetry breaking, which can be different from its value at high energy scale due to the renormalization-group running or some other Higgs-like field’s contributions. Note that the redefinition of Weyl boson induces an additional kinetic term for Higgs field, resulting in $C_H$ for normalization. The physical mass for $\bar{h}$ in SM will be identified as $m_{\bar{h}} = 125\text{GeV}$.

From the above Lagrangian, besides the novel interactions with derivative couplings, we observe immediately that there is no symmetry to guarantee the stability of Weyl boson, though the interaction $\bar{W}^\mu \partial_\mu \bar{h}$ alone can not directly cause the decay $W_\mu \rightarrow 2\bar{h}$ due to the property for on-shell Weyl boson, $\partial_\mu \bar{W}^\mu = 0$. However, if this interaction appears for an off-shell $W_\mu$ in a decay process, channels like $W_\mu \rightarrow \bar{h} + W_\mu^* \rightarrow 3\bar{h}$ at the tree-level can happen thanks to $\bar{h}\bar{W}^{\mu\nu}W_\mu$ interaction. For $m_W \gg m_{\bar{h}}$, $W_\mu \rightarrow 2\bar{h} + W_\mu^* \rightarrow 4\bar{h}$ would be the dominant process, which has a decay width as

$$\Gamma_W \sim \frac{g_W^6 m_W}{32\pi^5} \simeq 1.8 \times 10^{-25}\text{s}^{-1} \times \left(\frac{g_W}{9 \times 10^{-10}}\right)^6 \times \left(\frac{m_W}{2.2 \times 10^9\text{GeV}}\right)^6.$$  (13)

Note that in general $m_W$ and $g_W$ can be independent. In the minimal model presented above we have the simple relation, $m_W^2 \sim g_W^2 M_P^2/\alpha$ since $v \gg v_H$. However, in extended models with more Higgs-like fields that can have negative $\zeta$ for the kinetic term, $m_W$ and $g_W$ would be connected more complicatedly. For phenomenological studies, we shall treat $m_W$ and $g_W$ as independent from now on.

Gravitational decay is also possible, but usually subdominant unless the mass is very large. Evidently, to be a possible DM candidate, $W_\mu$ would need to have a very tiny $g_W$ to be long-lived enough, namely, at least longer than the age of our Universe, $t_U \simeq 4 \times 10^{17}\text{s}$. As a matter of fact, indirect DM searches in cosmic-ray, gamma-ray and neutrino spectra have already given stronger limits, $\Gamma_W \lesssim 10^{-26}\text{s}^{-1}$. For lighter $W_\mu$, the dominant decay channels are those with three fermion-antifermion pairs, such as $W_\mu \rightarrow 3b + 3\bar{b}$ (bottom quark) or $W_\mu \rightarrow 3\tau^+ + 3\tau^-$ (tau lepton). We show the approximate constraints in Fig. 2 as the color-shaded regions, orange for the tau lepton channel, red for bottom quark and brown for Higgs. It should be kept in mind that these constraints are subjected to $O(1)$ uncertainties due to approximate estimations of the decay widths. Weyl boson can also scatter with nucleons through Higgs exchange, giving signals in DM direct searches, such as PandaX [51] and XENON1T [50]. However, the relevant high-mass region has already been excluded by indirect searches, see the red dotted curve in the right-top of Fig. 2 by XENON1T.
FIG. 2. Various constraints for Weyl boson on the plane $m_W$ vs. $g_W$. Colored regions above the individual curves are excluded by the corresponding physics discussed in the context, the red dashed curve from XENON1T [50], the blue dot-dashed line from the constraint on signal strength of Higgs at LHC, the purple long-dashed line from invisible decay of Higgs particle, the dashed orange line from $3\tau^+3\tau^-$ decay, the red line from $3b\bar{b}$ decay and the long-dashed brown line from $4h$ decay. The white area is still allowed, except that it is subjected to the relic abundance requirement that depends on cosmology. For instance, the three vertical dashed lines indicate the contributions from vacuum fluctuation with Hubble scale $H = 10^{13}$GeV. Blue and black bands give the correct DM abundance from thermal production by Higgs annihilation for $T_h = 10^3$GeV and $10^{15}$GeV, respectively. See text for details.

Due to the normalization of Higgs field, all the couplings in operators like $h^n\mathcal{O}$ would get rescaled by a factor of $1/C_H^n$. The decay width would be enhanced by $1/C_H^2$, compared with SM value $\Gamma_{\text{SM}} \simeq 4.3\text{MeV}$. The current upper bound from LHC measurement is $\Gamma_{\text{SM}} < 13\text{MeV}$ at 95% confidence level [52], which can give $C_H^2 \gtrsim 1/3$, $m_W/g_W \gtrsim 300\text{GeV}$. The rate for single Higgs production at the LHC will also be enhanced by $1/C_H^2$, which would modify the signal strength in the observed decay channel. The combined analysis of ATLAS and CMS [53] gives the favored signal strength $\mu = 1.09 \pm 0.11$, constraining...
$C_H^2 \gtrsim 1/1.31$, $m_W/g_W \gtrsim 505\text{GeV}$ at $2\sigma$ level and excludes the blue-shaded region with dot-dashed boundary in Fig. 2. Higgs-pair production at the moment is not as sensitive as the above two. Future colliders with more precise measurements of Higgs observables would probe this scenario further.

If $m_{\bar{h}} > 2m_W$, there is a new decay model for Higgs, $\bar{h} \to W_\mu + W_\mu$. The decay width is calculated as

$$\Gamma (\bar{h} \to W_\mu + W_\mu) = \frac{g_W^4 v_H^2 m_h^3}{32\pi} \frac{m_W^4}{m_h^4} \sqrt{1 - x_W} \left(1 - x_W + \frac{3}{4} x_W^2\right),$$  \hspace{1cm} (14)

where $x_W = 4m_W^2/m_h^2$. Since Weyl boson has a lifetime at least comparable to the age of Universe, the above channel would show itself as an invisible decay for Higgs at colliders. Current data from ATLAS [54] and CMS [55] give a limit on the invisible branching ratio, $Br_{inv} \lesssim 26\%$, which is shown as the purple long-dashed line for $m_W < 62.5\text{GeV}$ in Fig. 2 and provides the strongest bound in the low mass region.

The white area in Fig. 2 is not constrained by any terrestrial experiment searches yet, therefore is still allowed, except that it is subjected to the relic abundance requirement that depends on the details of cosmological history. We consider the production of Weyl boson in the early universe from two sources, vacuum fluctuation and thermal production. Vacuum fluctuation [56, 57] gives the relic abundance $\Omega_W$,

$$\Omega_W \simeq \Omega_{DM} \times \sqrt{\frac{m_W}{6 \times 10^{-11}\text{GeV}}} \times \left(\frac{\mathcal{H}}{10^{13}\text{GeV}}\right)^2,$$  \hspace{1cm} (15)

where $\Omega_{DM} \simeq 0.25$. We show three vertical dashed lines which from left to right correspond to $\Omega_W = (10^{-2}, 10^{-1}, 1)\Omega_{DM}$ for $\mathcal{H} = 10^{13}\text{GeV}$. Thermal annihilation from Higgs particles contributes to

$$\Omega_W \simeq \Omega_{DM} \times \left(\frac{g_W}{3 \times 10^{-7}}\right)^4 \left(\frac{10^2\text{GeV}}{m_W}\right)^3 \left(\frac{T_h}{10^3\text{GeV}}\right)^3,$$  \hspace{1cm} (16)

where $T_h$ is the highest temperature of Higgs particles in the thermal bath after inflation. Note that $T_h$ is not necessarily equal to the reheating temperature, because Higgs particle may not be in thermal equilibrium with the reheated sector just after inflation. As a demonstration, the blue and black bands give the correct DM abundance for $T_h = 10^{3}\text{GeV}$ and $10^{15}\text{GeV}$, respectively.

Finally, the possible kinetic mixing between Weyl boson and photon $\gamma$ through $\kappa F_{\mu\nu} \gamma^{\mu\nu}$ would open up an additional portal to probe this scenario. Search strategies for general dark photons [58] due to the mixing also apply to the case of Weyl boson.
Conclusion: We have presented a theoretical observation that the original Weyl scaling symmetry can provide a unified framework to explain the cosmic inflation and dark matter simultaneously. The inspired inflationary scenario is well consistent with current observations and can be tested in future CMB experiments. Moreover, thanks to the coupling to Higgs boson, Weyl gauge boson can induce testable signatures in Higgs sector, such as invisible decay, single- and double-Higgs production. The Weyl boson as dark matter is intrinsically unstable and decays into standard model particles, which can be probed complementarily at indirect searches in cosmic-ray, gamma-ray and neutrino spectra.

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