Probing the anomalous triple gauge boson couplings in $e^+e^- \rightarrow W^+W^-$ using $W$ polarizations with polarized beams

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ABSTRACT: We study the anomalous $W^+W^-V$ ($V = \gamma, Z$) couplings in $e^+e^- \rightarrow W^+W^-$ using the complete set of polarization observables of $W$ boson with longitudinally polarized electron ($e^-$) and positron ($e^+$) beams. For the effective $W^+W^-V$ couplings, we use the most general Lorentz invariant form factor parametrization as well as $SU(2) \times U(1)$ invariant dimension-6 effective operators. We estimate simultaneous limits on the anomalous couplings in both the parametrizations using Markov-Chain–Monte-Carlo (MCMC) method for an $e^+e^-$ collider running at centre of mass energy of $\sqrt{s} = 500$ GeV and integrated luminosity of $L = 100$ fb$^{-1}$. The best limits on the anomalous couplings are obtained for $e^-$ and $e^+$ polarization being $(\pm 0.8, \mp 0.6)$ and they are better than the best available limit from various experiments.

KEYWORDS: Polarization of $W$ boson, anomalous triple gauge boson couplings (aTGC), MCMC, beam polarizations
1 Introduction

The non-abelian gauge symmetry $SU(2) \times U(1)$ of the Standard Model (SM) allows the $WWV$ ($V = \gamma, Z$) couplings after the Electro-Weak Symmetry Breaking (EWSB) by Higgs field, discovered at the LHC [1]. To test the EWSB, the $WWV$ couplings have to be measured precisely, which is still lacking. We intend to study the measurement of these couplings using polarization observables of spin-1 boson [2–8]. To test the SM $WWV$ couplings one has to hypothesize beyond the SM (BSM) couplings and make sure they do not appear at all or are severely constrained. One approach is to consider $SU(2) \times U(1)$ invariant higher dimension effective operators which provide the $WWV$ form factors after EWSB [9]. The effective Lagrangian considering the higher dimension operators can be written as

$$L_{\text{eff}} = L_{\text{SM}} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_i \frac{c_i^{(8)}}{\Lambda^4} \mathcal{O}_i^{(8)} + \ldots , \quad (1.1)$$

where $c_i^{(6,8)}$ are the couplings of the higher dimension operators $\mathcal{O}_i^{(6,8)}$ and $\Lambda$ is the energy scale below which the theory is valid. To the lowest order (upto dimension-6) the operators contributing to $WWV$ couplings are [10, 11]

$$\mathcal{O}_{WWW} = \text{Tr}[W_{\mu\nu}W^{\nu\rho}W_{\rho}] \ ,$$
$$\mathcal{O}_W = (D_\mu \Phi)^\dagger W^{\mu\nu}(D_\nu \Phi) \ ,$$
$$\mathcal{O}_B = (D_\mu \Phi)^\dagger B^{\mu\nu}(D_\nu \Phi) \ .$$
\[ \mathcal{O}_{WWW} = \text{Tr}[\bar{W}_{\mu\nu}W^{\rho\mu}W^{\rho\nu}] , \]
\[ \delta \tilde{W} = (D_\mu \Phi)\bar{W}^{\mu\nu} (D_\nu \Phi) , \]  \hspace{1cm} (1.2)

where \( \Phi \) is the Higgs doublet field and

\[
D_\mu = \partial_\mu + \frac{i}{2} g \tau^I W_\mu^I + \frac{i}{2} g' B_\mu ,
\]
\[
W_{\mu\nu} = \frac{i}{2} g \tau^I (\partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g\epsilon_{IJK} W_\mu^J W_\nu^K) ,
\]
\[
B_{\mu\nu} = \frac{i}{2} g' (\partial_\mu B_\nu - \partial_\nu B_\mu) .
\]  \hspace{1cm} (1.3)

Here \( g \) and \( g' \) are the \( SU(2) \) and \( U(1) \) couplings, respectively. Among these operators \( \mathcal{O}_{WWW} \), \( \mathcal{O}_W \) and \( \mathcal{O}_B \) are CP-even, while \( \mathcal{O}_{\tilde{W}} \) and \( \delta \tilde{W} \) are CP-odd. These effective operators, after EWSB, also provides \( ZZV \), \( HZV \) couplings which can be examined in various processes, e.g. \( ZV \) production, \( WZ \) production, \( HV \) production processes. The couplings in these processes may contain some other effective operator as well.

The other alternative to step beyond the SM \( WWV \) structure is to consider the most general Lorentz invariant effective form factors in a model independent way. A Lagrangian for the above parametrization is given by [12]

\[
\mathcal{L}_{WWV} = ig_{WWW} (g_1^V W_{\mu\nu}^{+} W^{-\mu} - W^{+\mu} W_{\mu\nu}^-) V^\nu + ig_4^V W_{\mu}^{+} W_{\nu}^- (\partial^\mu V^\nu + \partial^\nu V^\mu)
\]
\[
- ig_5^V \epsilon^{\mu\rho\sigma} (W_{\mu}^{+} \partial_\rho W_{\nu}^- - W_{\rho}^{+\mu} W_{\nu}^-) V_\sigma + \frac{\lambda^V}{M_W^2} W_{\mu}^{+\nu} W_{\nu}^{-\rho} V_\rho
\]
\[
+ \frac{\lambda\tilde{V}}{M_W^2} W_{\mu}^{+\nu} W_{\nu}^{-\rho} \tilde{V}_\rho + \kappa^V W_{\mu}^{+\nu} W_{\nu}^- V_{\mu\nu} + \kappa\tilde{V} W_{\mu}^{+\nu} W_{\nu}^- \tilde{V}_{\mu\nu} \right) .
\]  \hspace{1cm} (1.4)

Here \( W_{\mu\nu} = \partial_\mu W_{\nu}^\pm - \partial_\nu W_{\mu}^\pm \), \( V_{\mu\nu} = \partial_\mu V_{\nu} - \partial_\nu V_{\mu} \), \( \tilde{V}_{\mu\nu} = 1/2 \epsilon^{\mu\rho\sigma} V_{\rho\sigma} \), and the overall coupling constants are defined as \( g_{WWW} = - g \sin \theta_W \) and \( g_{WZW} = - g \cos \theta_W \), \( \theta_W \) being the weak mixing angle. In the SM \( g_1^V = 1 \), \( \kappa^V = 1 \) and other couplings are zero. The anomalous part in \( g_1^V \), \( \kappa^V \) would be \( \Delta g_1^V = g_1^V - 1 \), \( \Delta \kappa^V = \kappa^V - 1 \), respectively. The couplings \( g_1^V, \kappa^V \) and \( \lambda^V \) are CP-even (both \( C \) and \( P \)-even), while \( g_4^V, \kappa\tilde{V} \) and \( \lambda\tilde{V} \) are CP-odd. On the other hand \( g_5^V \) is both \( C \) and \( P \)-odd making it \( CP \)-even. We label these set of 14 anomalous couplings to be \( c_i^V \) as given in Eq. (A.2) in appendix A for later uses.

On restricting to the \( SU(2) \times U(1) \) gauge, the coupling \( (c_i^Z) \) of the Lagrangian in Eq. (1.4) can be written in terms of the couplings of the operators in Eq. (1.2) as [10, 11, 13]

\[
\Delta g_1^Z = c_WM_Z^2 ,
\]
\[
g_4^Z = g_5^Z = \Delta g_1^\gamma = 0 ,
\]
\[
\lambda^\gamma = \lambda^Z = \lambda^V = c_{WWW} \frac{3g^2M_W^2}{2\Lambda^2} ,
\]
\[
\lambda\tilde{V} = \lambda\tilde{Z} = \lambda\tilde{V} = c_{WWW} \frac{3g^2M_V^2}{2\Lambda^2} ,
\]
\[
\Delta \kappa^\gamma = (c_W + c_B) \frac{M_W^2}{2\Lambda^2},
\]
\[
\Delta \kappa^Z = (c_W - c_B \tan^2 \theta_W) \frac{M_W^2}{2\Lambda^2},
\]
\[
\kappa^\gamma = c_W \frac{M_W^2}{2\Lambda^2},
\]
\[
\kappa^Z = -c_W \tan^2 \theta_W \frac{M_W^2}{2\Lambda^2}.
\]
\[\text{(1.5)}\]

It is clear from above that some of the vertex factor couplings are dependent on each others and they are

\[
\Delta g^Z_4 = \Delta \kappa^Z + \tan^2 \theta_W \Delta \kappa^\gamma,
\]
\[
\kappa^Z + \tan^2 \theta_W \kappa^\gamma = 0.
\]
\[\text{(1.6)}\]

We label the non-vanishing 9 couplings in $SU(2) \times U(1)$ gauge as $c_i^{Z\gamma}$ given in Eq. (A.3) in appendix A for later uses.

The anomalous $WWV$ couplings have been studied in the effective operator approach as well as in the effective vertex formalism subjected to $SU(2) \times U(1)$ invariance for $e^+e^-$ collider [12, 14–24], Large Hadron electron collider (LHeC) [25–27], $e^-\gamma$ collider [28] and hadron collider (LHC) [21, 22, 29–37]. Some $CP$-odd $WWV$ couplings have been studied in Refs. [24, 35].

On the experimental side, the anomalous $WWV$ couplings have been explored and stringent limits on them have been obtained at the LEP [3, 38–40], the Tevatron [41, 42], the LHC [43–58] and Tevatron-LHC [59]. The tightest one parameter limit obtained on the anomalous couplings from experiments are given in Table 1. The tightest limits on operator couplings ($c_i^{\alpha}$) are obtained in Ref. [56] for $CP$-even ones and in Ref. [45] for $CP$-odd ones. These limit translated to $c_i^{Z\gamma}$ using Eq. (1.5) are also given in Table 1. The tightest limits on the couplings $g_4^Z$ and $g_5^Z$ are obtained in Ref. [38, 39] considering the Lagrangian in Eq. (1.4).

The $W^+W^-$ production is one of the important processes to be studied at the future International Linear Collider (ILC) [60–62] for precision test [63] as well as for BSM physics. This process has been studied earlier for SM phenomenology as well as for various BSM physics with and without beam polarization [12, 64–68]. Here we intend to study $WWV$ anomalous couplings in $e^+e^- \rightarrow W^+W^-$ at $\sqrt{s} = 500$ GeV and integrated luminosity of $\mathcal{L} = 100 \text{ fb}^{-1}$ using the cross section, forward-backward asymmetry and 8 polarizations asymmetries of $W^-$ for a set of choices of longitudinally polarized $e^+$ and $e^-$ beams in the channel $W^- \rightarrow l^-\bar{\nu}_l$ ($l = e, \mu$)\(^1\) and $W^+ \rightarrow \text{hadrons}$. The polarization of $Z$ and $W$ are being used widely recently for various BSM studies [69–75] along with studies with anomalous gauge boson couplings [3, 7, 76, 77]. Recently the polarizations of $W/Z$ has been measured in $WZ$ production at the LHC [78]. Besides the final state polarizations, the initial state beam polarizations at the ILC can be used to enhance the relevant signal

\(^1\)For simplicity we do not include tau decay mode as the tau decays to neutrino within the beam pipe giving extra missing momenta affecting the reconstruction of the events.
Table 1: The list of tightest limits obtained on the anomalous couplings of dimension-6 operators in Eq (1.2) and effective vertices in Eq. (1.4) in $SU(2) \times U(1)$ gauge (except $g_Z^4$ and $g_Z^5$) at 95% C.L. from experiments.

| $c_i^\nu$ | Limits (TeV$^{-2}$) | Remark |
|-----------|---------------------|--------|
| $c_{WWW}^\nu$ | $[-1.58, +1.59]$ | CMS $\sqrt{s} = 13$ TeV, $\mathcal{L} = 35.9$ fb$^{-1}$, $SU(2) \times U(1)$ [56] |
| $c_W^\nu$ | $[-2.00, +2.65]$ | CMS [56] |
| $c_B^\nu$ | $[-8.78, +8.54]$ | CMS [56] |
| $c_{WWW}^\nu$ | $[-11, +11]$ | ATLAS $\sqrt{s} = 7(8)$ TeV, $\mathcal{L} = 4.7(20.2)$ fb$^{-1}$ [45] |
| $c_W^\nu$ | $[-580, 580]$ | ATLAS [45] |

| $\lambda^\nu$ | Limits (×10$^{-2}$) | Remark |
|---------------|---------------------|--------|
| $\lambda^V$ | $[-0.65, +0.66]$ | CMS [56] |
| $\Delta \kappa^\gamma$ | $[-4.4, +6.3]$ | CMS $\sqrt{s} = 8$ TeV, $\mathcal{L} = 19$ fb$^{-1}$, $SU(2) \times U(1)$ [44] |
| $\Delta g_Z^4$ | $[-0.61, +0.74]$ | CMS [56] |
| $\Delta \kappa_Z^4$ | $[-0.79, +0.82]$ | CMS [56] |
| $\tilde{\lambda}^V$ | $[-4.7, +4.6]$ | ATLAS [45] |
| $\tilde{\kappa}_Z^4$ | $[-14, -1]$ | DELPHI (LEP2), $\sqrt{s} = 189$-209 GeV, $\mathcal{L} = 520$ pb$^{-1}$ [39] |

| $c_i^Z$ | Limits (×10$^{-2}$) | Remark |
|-----------|---------------------|--------|
| $g_Z^4$ | $[-59, -20]$ | DELPHI [39] |
| $g_Z^5$ | $[-16, +9.0]$ | OPAL (LEP), $\sqrt{s} = 183$-209 GeV, $\mathcal{L} = 680$ pb$^{-1}$ [38] |

to background ratio [63, 66, 68, 79, 80]. It also has the ability to distinguish between $CP$-even and $CP$-odd couplings [63, 81–90]. We note that an $e^+e^-$ machine will run with longitudinal beam polarizations switching between $(\eta_3, \xi_3)$ and $(-\eta_3, -\xi_3)$ [63], where $\eta_3(\xi_3)$ is the longitudinal polarization of $e^-$ ($e^+$). For integrated luminosity of 100 fb$^{-1}$, one will have half the luminosity available for each polarization configurations. The most common observables, the cross section for example, studied in literature with beam polarizations are the total cross section

$$\sigma_T(\eta_3, \xi_3) = \sigma(+\eta_3, +\xi_3) + \sigma(-\eta_3, -\xi_3)$$

(1.7)

and the difference

$$\sigma_A(\eta_3, \xi_3) = \sigma(+\eta_3, +\xi_3) - \sigma(-\eta_3, -\xi_3)$$

(1.8)

We find that combining the two opposite beam polarizations at the level of $\chi^2$ rather than combining them as in Eq. (1.7) & (1.8), we can limit the anomalous couplings better in this
analysis, see appendix C for explanation.

We note that there exist 64 polarization correlations [12] apart from \(8+8\) polarizations for \(W^+\) and \(W^-\). The measurement of these correlations requires identification of light quark flavours in the above channel, which is not possible, hence we are not including polarization correlations in our analysis. In the case of both the \(W\)'s decaying leptonically, there are two missing neutrinos and reconstruction of polarization observables suffers combinatorial ambiguity. Here we aim to work with a set of observables that can be reconstructed uniquely and test their ability to probe the anomalous couplings including partial contribution up to \(\mathcal{O}(\Lambda^{-4})^2\).

The rest of the paper is arranged in the following way. In Sect. 2 we introduce the complete set polarization observables of a spin-1 particle along with the forward-backward asymmetry and study the effect of beam polarizations on the observables. In Sect. 3 we use the vertex form factors for the Lagrangian in Eq. (1.4) and obtained expressions for all the observables. In this section, we cross-validate analytical results against the numerical result from MadGraph5 [91] for sanity checking. We also study the \(\cos \theta\) (of \(W\)) dependences of the observables and study their sensitivity on the anomalous couplings. In this section, we also estimate simultaneous limits on \(c_L^\alpha\), \(c_O^\alpha\) and the translated limits on \(c_L^g\). We give an insight into the choice of beam polarizations in this process in Sect. 3.3 and conclude in Sect. 4.

2 Observables and effect of beam polarizations

We study \(W^+W^-\) production at ILC running at \(\sqrt{s} = 500\) GeV and integrated luminosity \(\mathcal{L} = 100\) fb\(^{-1}\) using longitudinal polarization of \(e^-\) and \(e^+\) beams giving 50 fb\(^{-1}\) to each choice of beam polarization. The Feynman diagram for the process is shown in Fig. 1 where Fig. 1(a) corresponds to the \(\nu_e\) mediated \(t\)-channel diagram and the Fig. 1(b) corresponds to the \(V (Z, \gamma)\) mediated \(s\)-channel diagram containing the anomalous triple gauge boson couplings (aTGC) contributions represented by the shaded blob. The decay mode is chosen to be

\[
W^+ \to q_u \bar{q}_d , \quad W^- \to l^- \bar{\nu}_l , \quad l = e, \mu ,
\]

Figure 1: Feynman diagram of \(e^+e^- \to W^+W^-\), (a) \(t\)-channel and (b) \(s\)-channel with anomalous \(W^+W^-V\) \((V = \gamma, Z)\) vertex contribution shown as blob.

\(\text{We calculate cross section upto } \mathcal{O}(\Lambda^{-4}), \text{i.e., quadratic in dimension-6 (as linear approximation is not valid, see appendix B) and linear in dimension-8 couplings choosing dimension-8 couplings to be zero to compare our result with current LHC constraints on dimension-6 parameters [45, 56].}\)
where $q_u$ and $q_d$ are up-type and down-type quarks, respectively. We use complete set of eight spin-1 observables of $W^-$ boson [6, 7].

The $W$ boson being a spin-1 particle, its normalised production density matrix in the spin basis can be written as [2, 5]

$$
\rho(\lambda, \lambda') = \frac{1}{3} \left[ I_{3\times3} + \frac{3}{2} \vec{p} \vec{S} + \sqrt{\frac{3}{2}} T_{ij} (S_i S_j + S_j S_i) \right],
$$  \hspace{1cm} (2.2)

where $\vec{p} = \{p_x, p_y, p_z\}$ is the vector polarization of a spin-1 particle, $\vec{S} = \{S_x, S_y, S_z\}$ is the spin basis and $T_{ij}$ ($i, j = x, y, z$) is the 2nd-rank symmetric traceless tensor, $\lambda$ and $\lambda'$ are helicities of the particle. The tensor $T_{ij}$ has 5 independent elements, which are $T_{xy}$, $T_{xz}$, $T_{yz}$, $T_{xx} - T_{yy}$ and $T_{zz}$. Combining the $\rho(\lambda, \lambda')$ with normalised decay density matrix of the particle to a pair of fermion $f$, the differential cross section would be [5]

$$
\frac{1}{\sigma} \frac{d\sigma}{d\Omega_f} = \frac{3}{8\pi} \left[ \frac{2}{3} - (1 - 3\delta) \frac{T_{zz}}{\sqrt{6}} + \alpha \ p_z \cos \theta_f + \sqrt{\frac{3}{2}} (1 - 3\delta) \ T_{zz} \cos^2 \theta_f 
\right.
\left. + \left( \alpha \ p_x + 2 \sqrt{\frac{2}{3}} (1 - 3\delta) \ T_{xz} \cos \theta_f \right) \sin \theta_f \ \cos \phi_f 
\right.
\left. + \left( \alpha \ p_y + 2 \sqrt{\frac{2}{3}} (1 - 3\delta) \ T_{yz} \cos \theta_f \right) \sin \theta_f \ \sin \phi_f 
\right.
\left. + (1 - 3\delta) \left( \frac{T_{xx} - T_{yy}}{\sqrt{6}} \right) \sin^2 \theta_f \cos(2\phi_f) 
\right.
\left. + \sqrt{\frac{2}{3}} (1 - 3\delta) \ T_{xy} \sin^2 \theta_f \sin(2\phi_f) \right].
$$  \hspace{1cm} (2.3)

Here $\theta_f$, $\phi_f$ are the polar and the azimuthal orientation of the fermion $f$, in the rest frame of the particle ($W$) with its would be momentum along the $z$-direction. In this case $\alpha = -1$ and $\delta = 0$. The vector polarizations $\vec{p}$ and independent tensor polarizations $T_{ij}$ are calculable from the asymmetries constructed from the decay angular distribution of lepton (here $l^-$). For example $p_x$ can be calculated from the asymptmetry $A_x$ as

$$
A_x = \frac{\sigma(\cos \phi_f > 0) - \sigma(\cos \phi_f < 0)}{\sigma(\cos \phi_f > 0) + \sigma(\cos \phi_f < 0)} = \frac{3\alpha p_x}{4}.
$$  \hspace{1cm} (2.4)

The asymmetries corresponding to all other polarizations, vector polarizations $p_y$, $p_z$ and independent tensor polarizations $T_{ij}$ are $A_y$, $A_z$, $A_{xy}$, $A_{xz}$, $A_{yz}$, $A_{x^2-y^2}$, $A_{zz}$, see Ref. [7] for details.

Owing to the $t$-channel process (Fig. 1a) and absence of a $u$-channel process, like in $ZV$ production [7, 76], the $W^\pm$ produced are not forward-backward symmetric. We include forward-backward asymmetry, defined as

$$
A_{fb} = \frac{1}{\sigma_{W^+W^-}} \left[ \int_0^1 \frac{d\sigma_{W^+W^-}}{d\cos \theta_{W^-}} - \int_{-1}^0 \frac{d\sigma_{W^+W^-}}{d\cos \theta_{W^-}} \right],
$$  \hspace{1cm} (2.5)

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of the $W^-$ to the set of observables making total of ten observables including the cross section as well. Here $\theta_{W^-}$ is the production angle of the $W^-$ w.r.t. the $e^-$ beam direction and $\sigma_{W^+W^-}$ is the production cross section.

These asymmetries can be measured in a real collider from the final state lepton $l^-$. One has to calculate the asymmetries in the rest frame of $W^-$ which require the missing $\bar{\nu}_l$ momenta to be reconstructed. At an $e^+e^-$ collider, as studied here, reconstructing the missing $\bar{\nu}_l$ is possible because only one missing particle is involved and no Parton distribution function (PDF) is involved, i.e., initial momentas are known. But for a collider where PDF is involved, reconstructing the actual missing momenta may not be possible.

![Figure 2](image-url)

**Figure 2:** Production cross section $\sigma_{W^+W^-}$ in pb (left-panel) and polarization asymmetry $A_x$ (right-panel) in the SM as a function of longitudinal beam polarization $\eta_3$ (for $e^-$) and $\xi_3$ (for $e^+$) at $\sqrt{s} = 500$ GeV. The asterisk represents the unpolarized point and the number near it corresponds to the SM values for corresponding observables for unpolarized beams.

We explore the dependence of the cross section and asymmetries on longitudinal polarization $\eta_3$ of $e^-$ and $\xi_3$ of $e^+$. In Fig. 2 we show the production cross section $\sigma_{W^+W^-}$ and $A_x$ as a function of beam polarization as an example. The cross section decreases along $\eta_3 = -\xi_3$ path from 20 pb on the left-top corner to 7.2 pb at the unpolarized point and further to 1 pb in the right-bottom corner. This is because of the $W^\pm$ couples to left chiral $e^-$ i.e., it requires $e^-$ to be negatively polarized and $e^+$ to be positively polarized for the higher cross section. The variation of $A_{fb}$ (not shown) with beam polarization is the same as cross section but very slow above the line $\eta_3 = \xi_3$. From this, we can expect that a positive $\eta_3$ and a negative $\xi_3$ will reduce the SM contributions to observables increasing the $S/\sqrt{B}$ ratio ($S =$ signal, $B =$ background). Some other asymmetries like $A_x$ have opposite dependence on the beam polarizations compared to the cross section, its modulus reduces for negative $\eta_3$ and positive $\xi_3$. 


\[ V^\star \mu W^− \alpha W^+ \beta P q \bar{q} = i g_{WWV} \Gamma^{\alpha\beta}_V(P, q, \bar{q}) \]

\[ \Gamma^{\alpha\beta}_V = f^V_1(q - \bar{q})^\mu g^{\alpha\beta} - \frac{f^V_2}{M_W^2} (q - \bar{q})^\mu P^\alpha P^\beta + \frac{f^V_3}{M_W^2} (P^\alpha g^{\mu\beta} - P^\beta g^{\mu\alpha}) + i f^V_4 (P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}) \]

\[ + i f^V_5 q^\alpha e^{\mu\alpha\beta\rho} (q - \bar{q})_\rho - f^V_6 e^{\mu\alpha\beta\rho} P_\rho + \frac{\tilde{f}^V_7}{M_W^2} q^\alpha e^{\mu\beta\rho\sigma} + q^\beta e^{\mu\alpha\rho\sigma} \bar{q}_\rho \bar{q}_\sigma \]

where \( P, q, \bar{q} \) are the four-momenta of \( V, W^-, W^+ \), respectively. The momentum conventions are shown in Fig. 3. The form factor \( f_i \)s has been obtained from the Lagrangian in Eq. (1.4) using \textit{FeynRules} \{92\} to be

\[ f^V_1 = g^V_1, \quad f^V_2 = \lambda^V, \quad f^V_3 = g^V_1 + \kappa^V + \lambda^V, \]

\[ f^V_4 = g^V_4, \quad f^V_5 = g^V_5, \quad f^V_6 = \tilde{\lambda}^V + \left(1 - \frac{s}{2M_W^2}\right) \tilde{\lambda}^V, \quad \tilde{f}^V_7 = \tilde{\lambda}^V \].

\[ (3.2) \]

We use the vertex factors in Eq. (3.1) for the analytical calculation of our observables and cross validate them numerically with \textit{MadGraph5} \{91\} implementation of Eq. (1.4). As an example, we present two observables \( \sigma_{W^+W^-} \) and \( A_{zz} \) for the SM (\( c^V_1 = 0.0 \)) and for a chosen couplings point \( c^V_1 = 0.05 \), in Fig. 4. The agreement between the analytical and the numerical calculations over a range of \( \sqrt{s} \) indicates the validity of relations in Eq. (3.2), specially the \( s \) dependence of \( f^V_1 \) and \( f^V_6 \).

Analytical expressions of all the observables have been obtained and their dependence on the anomalous couplings \( c^V_1 \) are given in Table 5 in appendix A. The \( CP \)-even couplings in \( CP \)-even observables \( \sigma, A_x, A_z, A_{xz}, A_{x^2-y^2}, A_{zz} \) appear in linear as well as in quadratic form but do not appear in the \( CP \)-odd observables \( A_y, A_{xy}, A_{yz} \). On the other hand \( CP \)-odd couplings appears linearly in \( CP \)-odd observables and quadratically in \( CP \)-even observables. Thus the \( CP \)-even couplings may have double patch in their confidence interval leading to asymmetric limits which will be discussed in Sect. 3.1. On the other hand the \( CP \)-odd
couplings will have a single patch in their confidence interval and will poses symmetric limits.

3.1 Sensitivity of observables on anomalous couplings and their binning

The sensitivity of an observables $O$ depending on anomalous couplings $\vec{f}$ with beam polarization $\eta_3, \xi_3$ is given by

$$S_O(\vec{f}, \eta_3, \xi_3) = \frac{|O(\vec{f}, \eta_3, \xi_3) - O(\vec{0}, \eta_3, \xi_3)|}{|\delta O(\eta_3, \xi_3)|} ,$$

(3.3)

where $\delta O = \sqrt{(\delta O_{stat.})^2 + (\delta O_{sys.})^2}$ is the estimated error in $O$. The error for the cross section would be,

$$\delta \sigma(\eta_3, \xi_3) = \sqrt{\frac{\sigma(\eta_3, \xi_3)}{L} + \epsilon_\sigma^2 \sigma(\eta_3, \xi_3)^2}$$

(3.4)

whereas the estimated error in the asymmetries would be,

$$\delta A(\eta_3, \xi_3) = \sqrt{\frac{1 - A(\eta_3, \xi_3)^2}{L \sigma(\eta_3, \xi_3)} + \epsilon_A^2} .$$

(3.5)

Here $L$ is the luminosity of the data set, $\epsilon_\sigma$ and $\epsilon_A$ are the systematic fractional error in the cross section and asymmetries, respectively. We take $L = 50$ fb$^{-1}$ for each choice of beam polarizations, $\epsilon_\sigma = 2\%$ and $\epsilon_A = 1\%$ as a benchmark scenario for the present analyses. The sensitivity of all 10 observables have been studied on all the 14 couplings of the Lagrangian in Eq. (1.4) with the chosen $\sqrt{s}$, $L$ and systematic uncertainties. The sensitivity of all observables on $g_4^Z$ and $\Delta \kappa^3$ are shown in Fig. 5 as representative. Being $CP$-odd (either only linear or only quadratic terms present) $g_4^Z$ has single patch in the confidence interval,
while the $\Delta\kappa^\gamma$ being CP-even (linear as well as quadratic terms present), it has two patches in the sensitivity curve, as noted earlier. The CP-odd observable $A_y$ provides the tightest one parameter limit on $g^\gamma_1$. The tightest 1σ limit on $\Delta\kappa^\gamma$ is obtained using $A_{fb}$, while at 2σ level, a combination of $A_{fb}$ and $A_z$ provide the tightest limit.

Here, we have a total of 14 different anomalous couplings to be measured, while we only have 10 observables. A certain combination of large couplings may mimic the SM within the statistical errors. To avoid these we need more number of observables to be included in the analysis. We achieve this by dividing $\cos\theta_W$ into eight bins and calculate the cross section and polarization asymmetries in all of them. In Fig. 6 the cross section and the polarization asymmetries $A_z$, $A_x$, and $A_y$ are shown as a function of $\cos\theta_W$ for the SM and some aTGC couplings for both polarized and unpolarized beams. The SM values for unpolarized case is shown in dotted (blue) lines, SM with polarization of $(\eta_3, \xi_3) = (+0.6, -0.6)$ is shown in dashed (black) lines. The solid (red) lines correspond to unpolarized aTGC values while dashed-dotted (green) lines represent polarized aTGC values of observables. For the cross section (left-top-panel) we take $\Delta g^\gamma_1$ to be 0.1 and all other couplings to zero in the case of both polarized and unpolarized beam. We see that the fractional deviation from the SM value is larger in the most backward bin ($\cos\theta_W \in (-1.0, -0.75)$) and gradually reduces in the forward direction. The deviation is even larger in case of beam polarization. The sensitivity of the cross section on $\Delta g^\gamma_1$ is thus expected to be high in the most backward bin. In the case of asymmetries $A_z$ (right-top-panel), $A_{zz}$ (left-bottom-panel) and $A_y$ (right-bottom-panel) the aTGC is assumed to be $\Delta\kappa^Z = 0.05$, $\lambda^Z = 0.05$ and $g^\gamma_2 = 0.05$, respectively, while others are kept at zero. The change in the asymmetries due to aTGC is larger in the backward bin for both polarized and unpolarized case. We note that the asymmetries may not have the highest sensitivity in the most backward bin but some other bin. We consider the cross section and eight polarization asymmetries in all 8 bins, i.e., we have 72 observables in our analysis.

One parameter sensitivity of the set of 9 observables in all 8 bin has been studied. We show sensitivity of $A_y$ on $g^\gamma_2$ and of $A_z$ on $\Delta\kappa^\gamma$ in the 8 bin in Fig. 7 as representative. The tightest limits based on sensitivity (coming from one bin) is roughly twice as tight.
as compared to the unbin case in Fig. 5. Thus we expect simultaneous limits on all the couplings to be tighter when using binned observables.

We perform a set of MCMC analyses with a different set of observables for different kinematical cuts with unpolarized beams to understand their roles in providing limits on the anomalous couplings. These analyses are listed in Table 2. The corresponding 14-dimensional rectangular volume\(^3\) made out of 95\% Bayesian confidence interval (BCI) on the anomalous couplings are also listed in Table 2 in the last column. The simplest analysis would be to consider only the cross section in the full \(\cos \theta\) domain and perform MCMC analysis which is named as \(\sigma\)-ubinned. The typical 95\% limits on the parameters range

\(^3\)This volume of limit is the the volume of a 14-dimensional rectangular box bounding by the 95\% BCI projection of simultaneous limits in each coupling, which can be a measure of goodness of the benchmark beam polarization. We computed the cross section and other asymmetries keeping term up to quadratic in couplings. In this case, even a single observable can give a finite volume of limit and constrain all 14 couplings, which would not be possible if only terms linear in couplings were present.
\[ c \theta \in [-1.00, -0.75) \]
\[ c \theta \in [-0.75, -0.50) \]
\[ c \theta \in [-0.50, -0.25) \]
\[ c \theta \in [-0.25, +0.00) \]
\[ c \theta \in [+0.00, +0.25) \]
\[ c \theta \in [+0.25, +0.50) \]
\[ c \theta \in [+0.50, +0.75) \]
\[ c \theta \in [+0.75, +1.00) \]

\[ \Delta \kappa^\gamma \]  

\[ S(A_y) \]

\[ \frac{g_4}{Z} \]

\[ S(A_z) \]

Figure 7: The one parameter sensitivity of \( A_x \) on \( g_4^Z \) (left-panel) and of \( A_z \) on \( \Delta \kappa^\gamma \) (right-panel) in 8 bin at \( \sqrt{s} = 500 \text{ GeV} \), \( \mathcal{L} = 100 \text{ fb}^{-1} \) with \( c_\theta = \cos \theta_{W^-} \) for unpolarized beams.

Table 2: The list of analyses performed in the present work and set of observables used with a different kinematical cut to obtain simultaneous limits on anomalous couplings at \( \sqrt{s} = 500 \text{ GeV} \), \( \mathcal{L} = 100 \text{ fb}^{-1} \) with unpolarized beams. The rectangular volume of couplings at 95% BCI is shown in the last column for each analyses (see text for details).

| Analysis name  | Set of observables | Kinematical cut on \( \cos \theta_{W^-} \) | Volume of Limits |
|----------------|--------------------|---------------------------------------------|------------------|
| \( \sigma \)-ubinned | \( \sigma \) | \( \cos \theta_{W^-} \in [-1.0, 1.0] \) | \( 4.4 \times 10^{-11} \) |
| Unbinned       | \( \sigma, A_{fb}, A_i \) | \( \cos \theta_{W^-} \in [-1.0, 1.0] \) | \( 3.1 \times 10^{-12} \) |
| \( \sigma \)-binned | \( \sigma \) | \( \cos \theta_{W^-} \in \left[ \frac{m-5}{4}, \frac{m-4}{4} \right], m = 1, 2, \ldots, 8 \) | \( 3.7 \times 10^{-12} \) |
| Pol.-binned    | \( A_i \) | \( \text{``} \) | \( 1.6 \times 10^{-15} \) |
| Binned         | \( \sigma, A_i \) | \( \text{``} \) | \( 5.2 \times 10^{-17} \) |

from \( \sim \pm 0.04 \) to \( \pm 0.25 \) giving the volume of limits to be \( 4.4 \times 10^{-11} \). As we have polarizations asymmetries, the straight forward analysis would be to consider all observables for the full domain of \( \cos \theta_{W^-} \). This analysis is named \textbf{Unbinned} where limits on anomalous couplings get constrained better reducing the volume of limits by a factor of 10 compared to the \( \sigma \)-ubinned. To see how binning improve the limits we perform an analysis named \textbf{\( \sigma \)-binned} using only the cross section in 8 bin. We see the analysis \( \sigma \)-binned is better than \( \sigma \)-ubinned and comparable to the analysis \textbf{Unbinned}. To see the strength of the polarization asymmetries, we perform an analysis named \textbf{Pol.-binned} using just the polarization asymmetries in 8 bin. We see that this analysis is much better than the \( \sigma \)-binned. The most natural and complete analysis would be to consider all the observables after binning. The analysis is named as \textbf{Binned} which has limits much better than any analysis. The comparison between the analysis \( \sigma \)-binned, Pol.-binned and Binned is shown in Fig. 8 in the panel \( \lambda^\gamma-\lambda^Z \) in two-parameter (left-panel) as well as in multi-parameter (right-panel).
We perform MCMC analysis to estimate simultaneous limits on the anomalous couplings using the binned observables in both effective vertex formalism with 14 independent couplings and effective operator approach with 5 independent couplings for a set of chosen beam polarizations \((\eta_3, \xi_3)\) to be \((0, 0), (+0.2, -0.2), (+0.4, -0.4), (+0.6, -0.6), (+0.8, -0.6), (+0.8, -0.8)\) along with their opposite values. The beam polarization \((+\eta_3, +\xi_3)\) and its opposite \((-\eta_3, -\xi_3)\) are combined at the level of \(\chi^2\) as

\[
\chi^2_{\text{tot}}(\pm \eta_3, \pm \xi_3) = \sum_{\text{bin}} \sum_{N} \left( \chi^2 [\mathcal{O}_N(+\eta_3, +\xi_3)] + \chi^2 [\mathcal{O}_N(-\eta_3, -\xi_3)] \right),
\]

where \(N\) runs over all the observables. The 95% simultaneous limits for the chosen set of beam polarizations combined according to Eq. (3.6) are shown in Table 3 for effective vertex formalism \((c_L^\eta)\) and in Table 4 for effective operator approach \((c_O^\eta)\). The corresponding

**Figure 8:** The \(\chi^2 = 4\) contours in the left-panel and 95 % C.L. contours from simultaneous analysis in the right-panel in the \(\lambda^\gamma - \chi^2\) plane using the binned cross section \((\sigma)\) alone in dotted (black) line, just binned polarizations asymmetries \((\text{Pol.})\) in dashed (blue) line and the bin cross section together with binned polarization asymmetries \((\sigma + \text{Pol.})\) in solid (green) line for \(\sqrt{s} = 500\) GeV, \(L = 100\) fb\(^{-1}\).

We also calculate one parameter limit on all the couplings at 95 % C.L. considering all the binned observables with unpolarized beams in the effective vertex formalism as well as in the effective operator approach and list them in the last column of Tables 3 & 4, respectively for comparison. In the next subsection, we study the effect of beam polarization on the limit of the anomalous couplings.

### 3.2 Effect of beam polarizations to the limits on the anomalous couplings

The right-panel reflects the Table 2 and even in the two parameter analysis (left-panel) by keeping all other parameter to zero the behaviour is same, i.e, the bounded region for \(\chi^2 = 4\) is smaller in Pol.-binned (Pol.) than \(\sigma\)-binned (\(\sigma\)) and smallest for Binned (\(\sigma+\text{Pol.}\)).
translated limit to the vertex factor couplings $c_L^g$ are also shown in the Table 4 using relation from Eq. (1.5). While presenting limits the following notation is used

$$\text{low} \leq [\text{low}, \text{high}]$$

with low being lower limit and high being upper limit. A pictorial visualization of the limits shown in Table 3 & 4 is given in Fig. 9 for the easy comparisons. The limits on the couplings get tighter as the beam polarization is increased along $\eta_3 = -\xi_3$ path and become tightest at the extreme beam polarization $(\pm 0.8, \mp 0.8)$. However, the choice $(\pm 0.8, \mp 0.6)$ is best to put constraints on the couplings within the technological reach [93, 94].

To show the effect of beam polarization the marginalised 1D projection for the couplings $\lambda^\gamma$, $\Delta g_1^Z$ and $\Delta \kappa^Z$ as well as 2D projection at 95% C.L. on $\lambda^\gamma - \lambda^Z$, $\Delta g_1^Z - \kappa^Z$ and $\Delta \kappa^\gamma - \Delta \kappa^Z$ planes are shown in Fig. 10 for the effective vertex formalism ($c_L^{g}$) as representative. We observe that as the beam polarization is increased from $(0,0)$ to $(\pm 0.8, \mp 0.8)$ the contours

Figure 9: The pictorial visualisation of 95% BCI limits (a) : on the anomalous couplings $c_L^g$ in left-panel, (b) : on $c_O^C$ in right-top-panel and (c) : on $c_L^g$ in right-bottom-panel for $\sqrt{s} = 500$ GeV, $\mathcal{L} = 100$ fb$^{-1}$ using the binned observables. The numerical values of the limits can be read of in Tables 3 & 4.
get smaller centred around the SM values in the 2D projection which is reflected in the 1D projection as well. In the \( \Delta \kappa^\gamma-\Delta \kappa^Z \) panel the contour gets divided into two part at \((\pm 0.4, \mp 0.4)\) and become one single contour later centred around the SM values. In the case of effective operator approach \((c_O^p)\) all the 1D and 2D (95 % C.L.) projections after marginalization are shown in Fig. 11. In this case the couplings \(c_W\) and \(c_B\) has two patches up-to beam polarization \((\pm 0.2, \mp 0.2)\) and become one single patch starting at beam polarization \((\pm 0.3, \mp 0.3)\) centred around SM values. As the beam polarization is increased along the \(\eta_3 = -\xi_3\) line the measurement of the anomalous couplings gets improved. The set of beam polarization chosen here are mostly along \(\eta_3 = -\xi_3\) line, but some choice off to the line might provide the same results. A discussion on the choice of beam polarization is given in the next section.

### 3.3 On the choice of beam polarizations

In the previous section, we found that \((\pm \eta_3, \pm \xi_3) = (\pm 0.8, \pm 0.6)\) is the best choice of beam polarization to provide simultaneous limits on the anomalous couplings obtained by MCMC analysis. Here, we discuss the average likelihood or the weighted volume of the parameter

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**Figure 10**: The marginalised 1D projection for the couplings \(\lambda^\gamma\), \(\Delta g_1^Z\) and \(\Delta \kappa^Z\) in top-panel and 2D projection at 95 % C.L. on \(\lambda^\gamma-\lambda^Z\), \(\Delta g_1^Z-\kappa^Z\) and \(\Delta \kappa^\gamma-\Delta \kappa^Z\) planes in bottom-panel from MCMC for a set of choice of beam polarizations as shown for \(\sqrt{s} = 500\) GeV, \(L = 100\) fb\(^{-1}\) using the binned observables in the effective vertex formalism. The legend labels are same as in Figs. 9 \& 11.
\[ \sqrt{s} = 500 \text{ GeV}, \mathcal{L} = 100 \text{ fb}^{-1} \]

95% C.L.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure11.png}
\caption{All the marginalised 1D projection and 2D projections at 95% C.L. from MCMC in triangular array for the effective operators (TeV$^{-2}$) for a set of choice of beam polarizations for $\sqrt{s} = 500$ GeV, $\mathcal{L} = 100$ fb$^{-1}$ using the binned observables.}
\end{figure}

space defined as [76]

\[ L(V_f : \eta_3, \xi_3) = \int_{V_f} \exp \left[ -\frac{1}{2} \chi^2_{\text{tot}}(\vec{f}, \eta_3, \xi_3) \right] d\vec{f} \]  \hspace{1cm} (3.7)\]

to cross-examine the beam polarization choices made in the previous section. Here $\vec{f}$ is the coupling vector and $V_f$ is the volume of parameters space over which the average is done and $L(V_f : \eta_3, \xi_3)$ corresponds to the volume of the parameter space that is statistically consistent with the SM. One naively expects the limits to be tightest when $L(V_f : \eta_3, \xi_3)$ is minimum. We calculate the above quantity as a function of $(\pm \eta_3, \pm \xi_3)$ for Binned
Figure 12: The averaged likelihood $L_{Av} = L(V_f; \eta_3, \xi_3)$ in log scale as a function of $(\pm \eta_3, \pm \xi_3)$ in the effective vertex formalism for $\sqrt{s} = 500$ GeV, $\mathcal{L} = 100$ fb$^{-1}$. In this case in the effective vertex formalism given in Lagrangian in Eq. (1.4) and presented in Fig. 12. As the opposite beam polarizations are combined, only the half-portion are shown in the $\eta_3 - \xi_3$ plane. The dot (*) points along the $\eta_3 = -\xi_3$ are the chosen choice of beam polarizations for the MCMC analysis. We see that the average likelihood decreases along $\eta_3 = -\xi_3$ line while it increases along $\eta_3 = \xi_3$ line. The constant lines or contours of average likelihood in the figure imply that any beam polarization along the lines will provide the similar shape of $1D$ and $2D$ projections of couplings and their limits. For example, the point $(\pm 0.8, \mp 0.6)$ is equivalent to the point $(\pm 0.7, \mp 0.7)$ as well as $(\pm 0.6, \pm 0.8)$ roughly in providing simultaneous limits which are verified from the limits obtained by MCMC analysis. From the figure, it is confirmed that the polarization $(\pm 0.8, \mp 0.6)$ is indeed the best choice to provide simultaneous limits on the anomalous couplings within the achievable range.

4 Conclusion

In conclusion, we studied anomalous triple gauge boson couplings in $e^+e^- \rightarrow W^+W^-$ with longitudinally polarized beams using W boson polarization observables together with the total cross section and the forward-backward asymmetry for $\sqrt{s} = 500$ GeV and luminosity of $\mathcal{L} = 100$ fb$^{-1}$. We have 14 anomalous couplings, whereas we have only 10 observables to measure them. So we binned all the observables ($A_{fb}$ excluded) in 8 regions of the $\cos \theta_{W^-}$ to increase the number of observables to measure the couplings. We estimated simultaneous limit on all the couplings for several chosen set of beam polarization in both effective vertex formalism and effective operator approach. The limits on couplings are tighter when $SU(2) \times U(1)$ symmetry is assumed. We show the consistency between the best choice of beam polarizations and minimum likelihood averaged over the anomalous couplings. We
find that the polarization ($\pm 0.8, \mp 0.6$) to be the best to provide the tightest constraint on the anomalous couplings in both approaches at the ILC within the technological reach. Our one parameter limits with unpolarized beams and simultaneous limits for best polarization choice are much better than the one parameter limits from experiment, see Table 4. Our analysis consider certain simplifying assumptions, such as the absence of initial-state/final-state radiation and detector effects. While the former might dilute the limits by little amount the later is expected to have no effects on the results as only the leptonic channel is assumed and no flavour tagging or reconstruction is required.

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A The dependences of observables on anomalous couplings

The anomalous gauge boson couplings $c^O_i$ of effective operator in Eq. (1.2) and the couplings $c^\varphi_i$ of the Lagrangian in Eq. (1.4) and the couplings $c^\varphi_{gi}$ of Lagrangian in $SU(2) \times U(1)$ gauge (given in Eq. (1.5)) are labelled as

$$c^O_i = \{ c_{WWW}, c_W, c_B, c_{W^\prime W}, c_{W^\prime} \} \ , \quad (A.1)$$

$$c^\varphi_i = \{ \Delta g_V^Y, g_4^V, g_5^V, \lambda^V, \lambda^V, \Delta \kappa^V, \kappa^V \} , \quad V = \gamma, Z \ , \quad (A.2)$$

$$c^\varphi_{gi} = \{ \lambda^V, \lambda^V, \Delta \kappa^V, \kappa^V, \Delta g_1^Z, \Delta \kappa^Z, \kappa^Z \} \ . \quad (A.3)$$

B Note on linear approximation

If the cross section $\sigma$ is express as a function of couplings $c_i$ as,

$$\sigma = \sigma_0 + \sum_i \sigma_i \times c_i + \sum_{i,j} \sigma_{ij} \times c_i c_j \ , \quad (B.1)$$

linear approximation for the BSM operator will be possible if the quadratic contributions are much smaller than the linear contribution, i.e.,

$$|\sigma_i \times c_i| \gg |\sigma_{ii} \times c_i^2| , \quad \text{or} \quad |c_i| \ll \frac{\sigma_i}{\sigma_{ii}} \ . \quad (B.2)$$

As an example, consider the $\lambda^Z$ dependent unpolarized cross section given by

$$\sigma(0.0, 0.0) = 1037. + 57. \times \lambda^Z + 12241. \times (\lambda^Z)^2 \ . \quad (B.3)$$

The linear approximation is valid for $|\lambda^Z| \ll 0.004$. However, the limit on $\lambda^Z$ is $\pm 0.36$ at $1\sigma$ level at 100 fb$^{-1}$ (2% systematic is used) assuming linear approximation of Eq. (B.3), which is much beyond the validity of the linear approximation. To derive a sensible limit one needs to include the quadratic term which appear at $\mathcal{O}(\Lambda^{-4})$. However, at $\mathcal{O}(\Lambda^{-4})$ one also has contribution from dimension-8 operators at linear order. Our present analysis include quadratic contributions in dimension-6 operators and does not include dimension-8 contributions to compare our result with current LHC constrain, Table 1.
Combining beam polarization with its opposite values

\[ \chi^2 = 4 \] contours of the unbinned cross section for \( \sigma = \sigma(+\eta_3, +\xi_3) \) in solid/green line, \( \tilde{\sigma} = \sigma(-\eta_3, -\xi_3) \) in big-dashed/black line, \( \sigma_T = \sigma(+\eta_3, +\xi_3) + \sigma(-\eta_3, -\xi_3) \) in dotted/blue line, \( \sigma_A = \sigma(+\eta_3, +\xi_3) - \sigma(-\eta_3, -\xi_3) \) in dash-dotted/red line and the combined \( \chi^2 \) of \( \sigma \) and \( \tilde{\sigma} \) in dashed/magenta for polarization \((\eta_3, \xi_3) = (+0.6, -0.6)\) on \( \lambda^\gamma - \lambda^Z \) plane are shown in the left-panel. The 95 % C.L. contours from simultaneous analysis in \( \lambda^\gamma - \lambda^Z \) plane for the beam polarization \((+0.6, -0.6)\), \((-0.6, +0.6)\) and their combined one \((\pm 0.6, \mp 0.6)\) are shown in the right-panel using all the binned observables, i.e., in Binned case. The analyses are done for \( \sqrt{\mathcal{s}} = 500 \text{ GeV} \) and \( \mathcal{L} = 50 \text{ fb}^{-1} \) luminosity to each polarization set.

To reduce the systematic errors in analysis due to luminosity the beam polarization is flipped between two opposite choices frequently giving half the total luminosity to both the polarization choices in an \( e^+ - e^- \) collider. One can, in principle, use the observables, e.g., the total cross section \( \sigma_T \) or their difference \( \sigma_A \) as in Eqs. (1.7) & (1.8), respectively or for the two opposite polarization choices \( \sigma \) & \( \tilde{\sigma} \) separately for a suitable analysis. In this work, we have combined the opposite beam polarization at the level of \( \chi^2 \) as given in Eq. (3.6) not at the level of observables as the former constraints the couplings better than any combinations and of-course the individuals. To depict this, we present the \( \chi^2 = 4 \) contours of the unbinned cross sections in Fig. 13 (left-panel) for beam polarization \((+0.6, -0.6)\) (\( \sigma \)) and \((-0.6, +0.6)\) (\( \tilde{\sigma} \)) and the combinations \( \sigma_T \) and \( \sigma_A \) along with the combined \( \chi^2 \) in the \( \lambda^\gamma - \lambda^Z \) plane for \( \mathcal{L} = 50 \text{ fb}^{-1} \) luminosity to each polarization choice as representative. A systematic error of 2% is used as a benchmark in the cross section. The nature of the contours can be explain as follows: In the \( WW \) production, the aTGC contributions appear only in the s-channel (see Fig. 1) where initial state \( e^+e^- \) couples through \( \gamma/Z \) boson and both left and right chiral electrons contribute almost equally. The t-channel diagram, however, is pure background.
and receives contribution only from left chiral electrons. As a result the \( \bar{\sigma} \) (big-dashed/black) contains more background than \( \sigma \) (solid/green) leading to a weaker limit on the couplings. Further, inclusion of \( \bar{\sigma} \) into \( \sigma_T \) (dotted/blue) and \( \sigma_A \) (dashed-dotted/red) reduces the signal to background ratio and hence they are less sensitive to the couplings. The total \( \chi^2 \) for the combined beam polarization shown in dashed (magenta) is, of course, the best to constrain the couplings. This behaviour is reverified with the simultaneous analysis using the binned cross section and polarization asymmetries (72 observables in the Binned case) and depicted in Fig. 13 (right-panel) in the same \( \lambda^+ - \lambda^Z \) plane showing the 95% C.L. contours for beam polarizations \((+0.6, -0.6), (-0.6, +0.6), \) and their combinations \((\pm 0.6, \pm 0.6)\). Thus we choose to combine the opposite beam polarization choices at the level of \( \chi^2 \) rather than combining them at the level of observables.

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Table 3: List of Posterior 95% BCI of anomalous couplings $c_i^\mathcal{L}$ ($10^{-2}$) of the Lagrangian in Eq. (1.4) at $\sqrt{s} = 500$ GeV, $\mathcal{L} = 100$ fb$^{-1}$ for a chosen set of longitudinal beam polarizations $\eta_3$ and $\xi_3$ from MCMC in \textit{Binned} case. The limits for the best choice of beam polarization within technological reach, i.e., ($\pm0.8, \mp0.6$) is marked in \textbf{bold}. The pictorial visualisation for these 95% BCI of $c_i^\mathcal{L}$ is shown in Fig. 9 in the left-panel. The one parameter ($1P$) limits ($10^{-2}$) at 95% BCI with unpolarized beams are given in the last column for comparison. The notation used here is $^\text{high}_{\text{low}} \equiv [\text{low}, \text{high}]$ with low being lower limit and high being upper limit.

| param | (0,0) | ($\pm0.2, \mp0.2$) | ($\pm0.4, \mp0.4$) | ($\pm0.6, \mp0.6$) | ($\pm0.8, +=0.8$) | ($\pm0.8, =-0.8$) | 1P(0,0) |
|-------|-------|-------------------|-------------------|-------------------|-------------------|-------------------|---------|
| $\Delta g_1^\gamma$ | $\pm5.5$ | $\pm3.3$ | $\pm2.7$ | $\pm2.1$ | $\pm1.7$ | $\pm1.6$ | $\pm1.3$ |
| $g_1^\gamma$ | $\pm6.0$ | $\pm5.3$ | $\pm4.0$ | $\pm3.0$ | $\pm2.5$ | $\pm2.2$ | $\pm1.9$ |
| $g_5^\gamma$ | $\pm6.1$ | $\pm5.1$ | $\pm2.6$ | $\pm1.4$ | $\pm1.1$ | $\pm1.0$ | $\pm1.9$ |
| $\lambda^\gamma$ | $\pm1.4$ | $\pm1.2$ | $\pm1.2$ | $\pm1.0$ | $\pm0.89$ | $\pm0.81$ | $\pm0.77$ |
| $\tilde{\lambda}^\gamma$ | $\pm1.6$ | $\pm1.4$ | $\pm1.1$ | $\pm0.88$ | $\pm0.82$ | $\pm0.77$ | $\pm1.0$ |
| $\Delta \kappa^\gamma$ | $\pm6.1$ | $\pm5.2$ | $\pm4.0$ | $\pm2.9$ | $\pm2.6$ | $\pm2.3$ | $\pm2.3$ |
| $\kappa^\gamma$ | $\pm6.0$ | $\pm5.2$ | $\pm3.9$ | $\pm3.0$ | $\pm2.6$ | $\pm2.3$ | $\pm2.3$ |
| $\Delta g_1^\mathcal{Z}$ | $\pm7.2$ | $\pm5.6$ | $\pm4.5$ | $\pm2.1$ | $\pm1.8$ | $\pm1.6$ | $\pm1.3$ |
| $g_1^\mathcal{Z}$ | $\pm4.8$ | $\pm4.3$ | $\pm3.3$ | $\pm2.5$ | $\pm2.2$ | $\pm2.0$ | $\pm1.4$ |
| $g_5^\mathcal{Z}$ | $\pm4.7$ | $\pm4.0$ | $\pm2.1$ | $\pm1.3$ | $\pm1.0$ | $\pm0.86$ | $\pm1.2$ |
| $\lambda^\mathcal{Z}$ | $\pm1.1$ | $\pm1.0$ | $\pm0.80$ | $\pm0.49$ | $\pm0.47$ | $\pm0.44$ | $\pm0.56$ |
| $\tilde{\lambda}^\mathcal{Z}$ | $\pm1.3$ | $\pm1.1$ | $\pm0.90$ | $\pm0.77$ | $\pm0.73$ | $\pm0.68$ | $\pm0.57$ |
| $\Delta \kappa^\mathcal{Z}$ | $\pm3.6$ | $\pm3.2$ | $\pm3.1$ | $\pm0.56$ | $\pm0.43$ | $\pm0.36$ | $\pm0.43$ |
| $\kappa^\mathcal{Z}$ | $\pm4.7$ | $\pm4.2$ | $\pm3.3$ | $\pm2.5$ | $\pm2.2$ | $\pm2.1$ | $\pm1.5$ |
Table 4: The list of posterior 95% BCI of anomalous couplings \( c_i^\gamma \) (TeV\(^{-2}\)) of effective operators in Eq. (1.2) and their translated limits on the couplings \( c_i^{Z_0} \) (10\(^{-2}\)) for \( \sqrt{s} = 500 \) GeV, \( \mathcal{L} = 100 \) fb\(^{-1}\) in Binned case for a chosen set of longitudinal beam polarizations \( \eta_3 \) and \( \xi_3 \) from MCMC. The pictorial visualisation for these 95% BCI of \( c_i^\gamma \) and \( c_i^{Z_0} \) are shown in Fig. 9 in right-top and right-bottom panel, respectively. Rest details are same as in Table 3.

| param \( \frac{c_{W\gamma W}}{\Lambda^2} \) | (0, 0) | \( \pm 0.2, \mp 0.2 \) | \( \pm 0.4, \mp 0.4 \) | \( \pm 0.6, \mp 0.6 \) | \( \pm 0.8, \mp 0.8 \) | \( 1\sigma(0,0) \) |
|---|---|---|---|---|---|---|
| \( \frac{c_{W\gamma W}}{\Lambda^2} \) | +1.3 | +1.2 | +1.2 | +1.1 | +1.1 | +1.0 | +0.84 |
| \( \frac{c_{W\gamma W}}{\Lambda^2} \) | −1.9 | −1.4 | −1.1 | −0.96 | −1.0 | −0.94 | −0.97 |
| \( \frac{c_{W\gamma W}}{\Lambda^2} \) | +5.0 | +4.6 | +0.83 | +0.58 | +0.60 | +0.55 | +0.55 |
| \( \frac{c_{W\gamma W}}{\Lambda^2} \) | −1.4 | −1.1 | −0.86 | −0.72 | −0.73 | −0.63 | −0.58 |
| \( \frac{c_{W\gamma W}}{\Lambda^2} \) | +2.7 | +1.9 | +0.98 | +0.62 | +0.56 | +0.47 | +1.2 |
| \( \frac{c_{W\gamma W}}{\Lambda^2} \) | −23.7 | −20.2 | −1.3 | −0.75 | −0.64 | −0.53 | −1.3 |
| \( \frac{c_{W\gamma W}}{\Lambda^2} \) | +1.4 | +1.1 | +0.97 | +0.94 | +0.91 | +0.87 | +0.97 |
| \( \frac{c_{W\gamma W}}{\Lambda^2} \) | −1.4 | −1.1 | −0.97 | −0.93 | −0.90 | −0.87 | −0.98 |
| \( \Lambda^V \) | +0.52 | +0.50 | +0.49 | +0.46 | +0.45 | +0.42 | +0.35 |
| \( \Lambda^V \) | −0.79 | −0.58 | −0.46 | −0.40 | −0.41 | −0.39 | −0.40 |
| \( \frac{\Delta\kappa^V}{\Lambda^V} \) | +0.52 | +0.44 | +0.40 | +0.39 | +0.37 | +0.36 | +0.40 |
| \( \frac{\Delta\kappa^V}{\Lambda^V} \) | −0.60 | −0.45 | −0.40 | −0.38 | −0.37 | −0.36 | −0.41 |
| \( \frac{\Delta\kappa^\eta}{\Lambda^V} \) | +0.52 | +0.44 | +0.28 | +0.24 | +0.25 | +0.23 | +0.56 |
| \( \frac{\Delta\kappa^\eta}{\Lambda^V} \) | −6.4 | −5.1 | −0.38 | −0.32 | −0.32 | −0.28 | −0.61 |
| \( \frac{\Delta\kappa^Z}{\Lambda^V} \) | +3.9 | +3.2 | +2.1 | +1.3 | +1.0 | +0.84 | +3.2 |
| \( \frac{\Delta\kappa^Z}{\Lambda^V} \) | −3.9 | −3.2 | −2.1 | −1.3 | −1.0 | −0.84 | −3.2 |
| \( \Delta\kappa^\eta \) | +0.52 | +0.44 | +0.28 | +0.24 | +0.25 | +0.23 | +0.56 |
| \( \Delta\kappa^Z \) | +3.9 | +3.2 | +2.1 | +1.3 | +1.0 | +0.84 | +3.2 |
| \( \Delta\kappa^\eta \) | −3.9 | −3.2 | −2.1 | −1.3 | −1.0 | −0.84 | −3.2 |
| \( \Delta\kappa^Z \) | +3.9 | +3.2 | +2.1 | +1.3 | +1.0 | +0.84 | +3.2 |
| \( \kappa^\eta \) | +1.1 | +0.92 | +0.62 | +0.38 | +0.29 | +0.24 | +0.92 |
| \( \kappa^Z \) | −1.1 | −0.91 | −0.61 | −0.38 | −0.30 | −0.24 | −0.93 |
Table 5: Dependences of observables (numerator) on anomalous couplings in the form of $c_L^2$ (linear), $(c_L^2)^2$ (quadratic) and $c_L^2 c_J^2$, $i \neq j$ (interference) in the process $e^+ e^- \rightarrow W^+ W^-$. Here $V \in \{\gamma, Z\}$. The “✓” (checkmark) represents the presence and “—” (big-dash) corresponds to absence.

| Parameters | $\sigma$ | $\sigma \times A_x$ | $\sigma \times A_y$ | $\sigma \times A_z$ | $\sigma \times A_{xy}$ | $\sigma \times A_{xz}$ | $\sigma \times A_{yz}$ | $\sigma \times A_{x2-z2}$ | $\sigma \times A_{z2}$ | $\sigma \times A_{fb}$ |
|------------|---------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $\Delta g_1^V$ | ✓ | ✓ | — | ✓ | — | ✓ | ✓ | ✓ | ✓ | ✓ |
| $g_4^V$ | — | — | ✓ | — | ✓ | — | ✓ | — | — | — |
| $g_5^V$ | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓ | ✓ | ✓ |
| $\lambda^V$ | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓ | ✓ | ✓ |
| $\lambda^V$ | — | — | ✓ | ✓ | — | ✓ | — | ✓ | — | — |
| $\Delta \kappa^V$ | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓ | ✓ | ✓ |
| $\Delta g_1^V g_4^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $\Delta g_1^V g_5^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $\Delta g_1^V \lambda^V$ | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓ | ✓ | ✓ |
| $\Delta g_1^V \lambda^V$ | — | — | ✓ | ✓ | — | ✓ | — | ✓ | — | — |
| $\Delta g_1^V \Delta \kappa^V$ | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓ | ✓ | ✓ |
| $\Delta g_1^V \Delta \kappa^V$ | — | — | ✓ | ✓ | — | ✓ | — | ✓ | — | — |
| $g_4^V g_5^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $g_4^V \lambda^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $g_4^V \lambda^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $g_4^V \Delta \kappa^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $g_4^V \Delta \kappa^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $\lambda^V \lambda^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $\lambda^V \lambda^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $\lambda^V \Delta \kappa^V$ | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓ | ✓ | ✓ |
| $\lambda^V \Delta \kappa^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $\lambda^V \Delta \kappa^V$ | — | — | — | — | — | — | — | ✓ | — | — |
| $\lambda^V \Delta \kappa^V$ | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓ | ✓ | ✓ |