A microscopic study of the proton-neutron symmetry and phonon structure of the low-lying states in $^{92}\text{Zr}$

N. Lo Iudice$^1$ and Ch. Stoyanov$^2$

$^1$Dipartimento di Scienze Fisiche, Università di Napoli ”Federico II”
and Istituto Nazionale di Fisica Nucleare, Monte S Angelo, Via Cinthia I-80126 Napoli, Italy
$^2$ Institute for Nuclear Research and Nuclear Energy, 1784 Sofia, Bulgaria

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Abstract

We studied in a microscopic multiphonon approach the proton-neutron symmetry and phonon structure of some low-lying states recently discovered in $^{92}\text{Zr}$. We confirm the breaking of F-spin symmetry, but argue that the breaking mechanism is more complex than the one suggested in the original shell model analysis of the data. We found other new intriguing features of the spectrum, like a pronounced multiphonon fragmentation of the states and a tentative evidence of a three-phonon mixed symmetry state.

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*Electronic address: loiudice@na.infn.it
†Electronic address: stoyanov@inrne.bas.bg
I. INTRODUCTION

The experiments on $^{94}$Mo $^{1, 2, 3, 4}$, $^{96}$Ru $^{5, 6}$ and other nuclei $^{7}$ have provided conclusive evidence in favor of the existence of F-spin mixed symmetry states in nearly spherical nuclei, thereby confirming a major prediction of the proton-neutron interacting boson model (IBM2) $^{8}$. The many levels and transition probabilities produced by the experiments fit extraordinarily well in the IBM2 scheme according to the F-spin quantum number and the number of quadrupole bosons. Indeed, states with the same F-spin symmetry differing by one-phonon were found to be strongly coupled by isoscalar E2 transitions, while $F = F_{\text{max}}$ symmetric and $F = F_{\text{max}} - 1$ mixed symmetry states with the same number of phonons were connected through strong M1 transitions, a feature that qualifies the mixed-symmetry states as scissors-like excitations $^{9, 10}$ built on quadrupole vibrational states.

The phonon structure as well as the proton-neutron (p-n) symmetry were confirmed by a microscopic calculation using a multiphonon basis, whose phonons were generated in random-phase approximation (RPA) $^{11, 12}$. This approach, known as quasiparticle-phonon model (QPM) $^{13}$, accounted with good accuracy for the energies, transition probabilities and selection rules. A good description was also given by a truncated shell model (SM) calculation $^{14}$.

In the attempt of establishing how far the F-spin symmetry holds valid, the same experimental group performed a photon scattering experiment $^{15}$ on $^{92}$Zr, which is supposed to be at the border line of the F-spin symmetry domain, and analyzed the data in the framework of the SM adopted in Ref. $^{14}$ for studying the $^{94}$Mo. The most meaningful outcome of their analysis was that the two lowest $2^+$ states are one-phonon excitations, at variance with the other nuclei of the region where the second $2^+_2$ is a two-phonon state. More remarkably, the first $2^+_1$ is basically a pure neutron excitation, while the second $2^+_2$ is purely isovector.

This latter point, implying a severe breaking of F-spin, gave us the main motivation for the present QPM study. This approach, in fact, using explicitly a RPA phonon basis and taking into account a very large number of configurations excluded in SM, provides a more stringent microscopic test for the the validity of the F-spin symmetry in $^{92}$Zr.
II. CALCULATION AND RESULTS

We used the same Woods-Saxon potential and the same separable two-body Hamiltonian adopted in [11, 12]. We used also the same single particle basis, which encompasses all bound states from the bottom of the well up to the quasi-bound states embedded into the continuum. Following the same strategy, we fit the strength $\kappa_2$ of the quadrupole-quadrupole (Q-Q) interaction on the energy and $E2$ decay strength of the $2^+_1$ and the coupling constant $G_2$ of the quadrupole pairing on the overall properties of the low-lying $2^+$ isovector state. As we shall see, this came out to be $G_2 \simeq \kappa_2$, slightly larger than in the case of $^{94}$Mo ($G_2 \simeq 0.8\kappa_2$). The other Hamiltonian parameters remained unchanged. Because of the large model space, we used effective charges very close to the bare values, namely $e_p = 1.1$ for protons and $e_n = 0.1$ for neutrons. We also used the spin-gyromagnetic quenching factor $g_s = 0.7$.

A. A very brief outline of the procedure

Following the QPM procedure, we transformed the Hamiltonian into the phonon form

$$H_{QPM} = \sum_{i\mu} \omega_{i\lambda} Q_{i\lambda\mu}^\dagger Q_{i\lambda\mu} + H_{vq},$$

where the first term is the unperturbed phonon Hamiltonian and $H_{vq}$ is a phonon-coupling piece whose exact expression can be found in Ref. [13]. Both terms are expressed in terms of the RPA phonon operators

$$Q_{i\lambda\mu}^\dagger = \frac{1}{2} \sum_{jj'} \left\{ \psi_{jj'}^{\dagger \lambda} [\alpha_j^{\dagger \mu} \alpha_{j'}]_{\lambda\mu} - (-1)^{\lambda-\mu} \varphi_{jj'}^{\dagger \lambda} [\alpha_{j'}^{\dagger \mu} \alpha_{j}]_{\lambda-\mu} \right\}$$

of multipolarity $\lambda\mu$ and energy $\omega_{i\lambda}$, where $\alpha_{jm}^{\dagger}(\alpha_{jm})$ are quasiparticle operators obtained from the corresponding particle operators through a Bogoliubov transformation. The phonon operators fulfil the normalization conditions

$$\frac{1}{2} \sum_{jj'} \left[ \psi_{jj'}^{\dagger \lambda} \psi_{jj'}^{\dagger \lambda'} - \varphi_{jj'}^{\dagger \lambda} \varphi_{jj'}^{\dagger \lambda'} \right] = \delta_{i'i'} \delta_{\lambda\lambda'}.$$  

It is worth to point out that, among the RPA phonons, only few are collective, composed of a coherent linear combination of two-quasiparticle configurations. The Boson Hamiltonian is then accordingly diagonalized in a space spanned by RPA multiphonon states. As in Ref. [11, 12], we included up to three-phonon states.
TABLE I: Neutron and proton quadrupole transition amplitudes and IV/IS ratios for the lowest two $[2^+]_{RPA}$ states

| g.s. $\rightarrow [2^+]_{RPA}$ | $G_2$ | $M^{(n)}_i (fm^2)$ | $M^{(p)}_i (fm^2)$ | $R_i(IV/IS)$ | $R^2_i(IV/IS)$ |
|-----------------------------|--------|-------------------|-------------------|-------------|-------------|
| g.s. $\rightarrow [2^+]_{RPA}$ | 0.0    | 67.67             | 41.98             | 0.234       | 0.055       |
|                             | 1.0    | 72.2              | 49.6              | 0.185       | 0.034       |
| g.s. $\rightarrow [2^+]_{RPA}$ | 0.0    | 52.67             | 46.40             | 0.063       | 0.004       |
|                             | 1.0    | 14.4              | 28.4              | -0.327      | 0.107       |

B. RPA analysis

The preliminary condition for QPM being able to test the symmetry properties and phonon composition of the low-lying states as described in IBM2, is the occurrence at low energy of at least two RPA quadrupole phonons, which we denote $[2^+]_{RPA}$ and $[2^+]_{RPA}$. In the specific case of $^{92}$Zr, even in QPM the two lowest lying states are to be one-phonon states, the first dominated by $[2^+]_{RPA}$, the second by $[2^+]_{RPA}$. To test the proton-neutron symmetry of these RPA states we compute the ratios

\[ R_{i=1,2}(IV/IS) = \frac{M_i^{(n)} - M_i^{(p)}}{M_i^{(n)} + M_i^{(p)}}, \quad R^2_{i}(IV/IS) = |R_i(IV/IS)|^2, \tag{4} \]

where

\[ M_i^{(n/p)} = \langle 2^+_i \big| \sum_k \frac{(p/n)}{r_k^2} Y_2(\Omega_k) \big| g.s. \rangle. \tag{5} \]

If F-spin is preserved to a good extent, we must have $R^2_{1}(IV/IS) < 1$ and $R^2_{2}(IV/IS) > 1$.

In order to meet all the above requirements we have only one parameter at our disposal, the quadrupole pairing constant. This, while affecting little the lowest $[2^+]_{RPA}$, has an appreciable impact on the second $[2^+]_{RPA}$. For $G_2 = 0$, the ratios shown in Table I qualify not only the first but also the second $[2^+]_{RPA}$ as p-n symmetric. The second is, actually, even more symmetric and collective than the first. Moreover, the corresponding one-phonon QPM state is pushed above a two-phonon state and becomes the third excited state, contrary to the experimental data. As we increase $G_2$, the transition amplitudes, specially the neutron one, decrease, but not enough. Even for $G_2/k = 1$, in fact, the neutron amplitude is small but positive, so that the corresponding IV/IS ratio is much larger than in the case of vanishing $G_2$, but, still, appreciably smaller than one. Only for values of $G_2$ considerably larger than
κ, the $[2^+_2]_{RPA}$ becomes p-n non symmetric ($R(IV/IS) > 1$), but looses completely its collectivity. The corresponding E2 strength is negligible, at variance with experiments. We therefore chose $G_2 = \kappa$ which allows to fulfills more closely the experimental requirements.

For such a value, the lowest RPA $2^+_1$ is collective, though to a less extent than in other nuclei of the same region. Its RPA E2 decay strength (Table II) is smaller than the corresponding one in $^{94}$Mo [12] by more than a factor two. Such a quenching reflects the diminished role of the proton with respect to the neutron component in $^{92}$Zr. The neutron dominance, however, is far less pronounced than in SM and does not alter dramatically the symmetry of the state. In fact, not only all proton and neutron components are in phase, but also the ratio of the isovector to the isoscalar quadrupole transition amplitudes is small, though not negligible. Such a test qualifies the $2^+_1$ as a $\Delta T = 0$ p-n symmetric state with a small, though non negligible, admixture of non symmetric pieces.

The F-spin breaking is more substantial in the second $[2^+_2]_{RPA}$. Indeed, its isoscalar to isovector ratio $R_2(IV/IS)$ is considerably smaller than unity, indicating that the transition is promoted with comparable strengths by both isoscalar and isovector quadrupole operators. This conclusion seems to be contradicted by the structure of the wave function (Table II), whose largest proton and neutron components are in opposition of phase.

In order to solve this puzzle, we have analyzed more closely the neutron contributions to the $IV/IS$ ratio. As shown in Table III, the contribution of the largest neutron component to the matrix element of the quadrupole field, $M_2^{(n)}$, is negative and opposite to the proton contribution. This neutron component would, therefore yield $R_2(IV/IS) > 1$, as expected. On the other hand, the remaining neutron configurations are in phase with the protons (Table II) and, because of the collective nature of the $2^+_1$, are in large number. Moreover, for most of these configurations, the two-quasiparticle matrix elements of the quadrupole field are quite sizeable (Table III) and compensate for their small amplitude coefficients in the wave function. Their integrated contribution overcomes the one due to the dominant component, yielding a total $IV/IS$ ratio smaller than one. The breaking of the p-n symmetry is therefore the result of a competition between a dominant configuration, with relatively small quadrupole matrix elements, and a large number of small components yielding large quadrupole transition amplitudes and acting coherently. We may, therefore, infer from this RPA analysis that F-spin is more severely broken in the second rather than the lowest $2^+$. Our findings are quite different from the SM results. These discrepancies can be explained
TABLE II: Structure of the lowest RPA phonons (only the largest components are given) and corresponding E2 reduced transition strengths in $^{92}$Zr. The states are normalized according to Eq. (3).

| $\lambda_\pi^\pi$ | $\omega_\lambda^\pi$ (MeV) | $B(E2 \downarrow)$ (w.u.) | Structure |
|-----------------|-----------------|------------------|------------|
| $2^+_1$         | 1.18            | 7.6              | 0.99($2d_{5/2} \otimes 2d_{5/2}$)$_n$ 48.4% |
|                 |                 |                  | +0.23($2d_{5/2} \otimes 3s_{1/2}$)$_n$ 4.8% |
|                 |                 |                  | +0.17($2d_{5/2} \otimes 1g_{9/2}$)$_n$ 2.5% |
|                 |                 |                  | total 60% |
|                 |                 |                  | +0.64($1g_{9/2} \otimes 1g_{9/2}$)$_p$ 20.5% |
|                 |                 |                  | +0.23($1f_{5/2} \otimes 2p_{1/2}$)$_p$ 5.5% |
|                 |                 |                  | +0.23($2p_{3/2} \otimes 2p_{1/2}$)$_p$ 5.2% |
|                 |                 |                  | +0.15($1f_{5/2} \otimes 1f_{5/2}$)$_p$ 1.1% |
|                 |                 |                  | +0.14($1g_{9/2} \otimes 2d_{5/2}$)$_p$ 2.1% |
|                 |                 |                  | total 40% |
| $2^+_2$         | 2.07            | 2.2              | $-0.99(2d_{5/2} \otimes 2d_{5/2})_n$ 48.8% |
|                 |                 |                  | +0.09($2d_{5/2} \otimes 3s_{1/2})_n$ 0.72% |
|                 |                 |                  | +0.11($2d_{5/2} \otimes 1g_{9/2})_n$ 0.82% |
|                 |                 |                  | total 51% |
|                 |                 |                  | +0.79($1g_{9/2} \otimes 1g_{9/2})_p$ 30.7% |
|                 |                 |                  | +0.25($1f_{5/2} \otimes 2p_{1/2})_p$ 5.7% |
|                 |                 |                  | +0.25($2p_{3/2} \otimes 2p_{1/2})_p$ 5.7% |
|                 |                 |                  | +0.18($1f_{5/2} \otimes 1f_{5/2})_p$ 1.3% |
|                 |                 |                  | +0.15($1g_{9/2} \otimes 2d_{5/2})_p$ 1.9% |
|                 |                 |                  | total 49% |

with the severely truncated space used in the SM, which is spanned only by two neutrons and two protons external to the inert cores $N_c = 50$ and $Z_c = 38$, respectively. As shown in tables II and III, this truncation excludes many configurations whose contribution change dramatically the nature of the second $[2^+_2]_{RPA}$ state.
TABLE III: Contribution of selected neutron configurations of the second RPA phonon \([2_2^+]_{RPA}\) to the matrix element \(\mathcal{M}_2^{(n)}(RPA)\) of the quadrupole field in \(^{92}\text{Zr}\). \(\mathcal{M}_2^{(n)}(q_1q_2)\) gives the pure two-quasiparticle matrix elements.

| \(q_1 \otimes q_2\) | \(\mathcal{M}_2^{(n)}(q_1q_2)\) (\(fm^2\)) | \(\psi\)  | \(\phi\)  | \(\%\)  | \(\mathcal{M}_2^{(n)}(RPA)\) (\(fm^2\)) |
|-----------------|-----------------|----------|----------|--------|-----------------|
| \(2d_{5/2} \otimes 2d_{5/2}\) | 17.86            | -0.99    | 0.09     | 48.8%  | -16.07          |
| \(2d_{5/2} \otimes 3s_{1/2}\) | 25.33            | 0.10     | 0.04     | 0.72%  | 3.51            |
| \(1g_{9/2} \otimes 2d_{5/2}\) | 34.85            | 0.11     | 0.07     | 0.82%  | 6.19            |
| \(2p_{3/2} \otimes 2f_{7/2}\) | 33.17            | 0.02     | 0.01     | 0.01%  | 1.12            |
| \(1f_{7/2} \otimes 1h_{11/2}\) | 57.68            | 0.05     | 0.04     | 0.07%  | 4.79            |
| \(1g_{9/2} \otimes 1i_{13/2}\) | 72.83            | 0.05     | 0.04     | 0.09%  | 7.01            |
| \(1d_{3/2} \otimes 1g_{7/2}\) | 33.83            | 0.02     | 0.02     | 0.01%  | 1.22            |
| \(\cdots\)          | \(\cdots\)      | \(\cdots\) | \(\cdots\) | \(\cdots\) | \(\cdots\)      |

C. QPM results

Let us now investigate the QPM states (Table IV) and how their phonon composition affects the E2 (Tables V) as well as the E1 and M1 transitions (Table VI).

The first \(2^+_1\) is mostly accounted for by the lowest RPA one-phonon component. The appreciable, but not overwhelming, neutron dominance is therefore confirmed and is consistent with the magnetic properties of the state. Indeed, the QPM yields for the gyromagnetic factor \(g(2^+_1) = -0.20\), very close to the experimental value \(g_{\text{exp}}(2^+_1) = -0.18(1)\). Apparently, the experimental negative g-factor claims only a moderately large neutron weight.

The second \(2^+_2\) is a one-phonon state, dominated by the second \([2_2^+]_{RPA}\). As pointed out already, this is peculiar of \(^{92}\text{Zr}\), since the \(2^+_2\) in the nearby nuclei \(^{94}\text{Mo}\) and \(^{136}\text{Ba}\) was found to be a two-phonon symmetric state \(^{12}\). This one-phonon \(2^+_2\) undergoes an E2 decay to the ground state (Table V) and a M1 transition to the symmetric \(2^+_1\) (Table VI). The computed M1 strength, though fairly large, is smaller than in \(^{94}\text{Mo}\), while the E2 strength is unusually large. These two correlated facts, already pointed out in the experimental analysis \(^{15}\), indicate that the \(2^+_2\) is not a pure ”mixed symmetry” state but has an appreciable F-spin symmetric component. The latter piece, in fact, is responsible for the enhancement of the E2 and quenching of the M1 strengths, respectively, with respect to \(^{94}\text{Mo}\). The breaking of
TABLE IV: Energy and phonon structure of selected low-lying excited states in $^{92}$Zr. Only the dominant components are shown.

| State | E (keV) | Structure,% |
|-------|---------|-------------|
|       | EXP | QPM |        |
| $2_1^+$ | 934 | 1017 | 92.6%$[2_1^+]_{RPA}$ |
| $2_2^+$ | 1847 | 1945 | 90%$[2_2^+]_{RPA}$ |
| $2_3^+$ | 2067 | 2008 | 19%$[2_3^+]_{RPA} + 65%[2_1^+ \otimes 2_1^+]_{RPA}$ |
| $2_4^+$ | 3386 |       | 32%$[2_4^+]_{RPA} + 51%[2_1^+ \otimes 2_1^+]_{RPA}$ |
| $4_1^+$ | 1495 | 1598 | 78%$[4_1^+]_{RPA} + 17%[2_1^+ \otimes 2_1^+]_{RPA}$ |
| $4_2^+$ | 2398 | 2188 | 18%$[4_2^+]_{RPA} + 59%[2_1^+ \otimes 2_1^+]_{RPA}$ |
| $4_3^+$ | 2863 | 2906 | 27%$[4_3^+]_{RPA} + 32%[2_1^+ \otimes 4_1^+]_{RPA} + 28%[2_1^+ \otimes 2_2^+]_{RPA}$ |
| $1_1^+$ | 3472 | 3235 | 93%$[2_1^+ \otimes 2_2^+]_{RPA} + 6%[1_1^+]_{RPA}$ |
| $1_2^+$ | 3638 | 3781 | 91%$[1_1^+]_{RPA}$ |
| $3_2^+$ | 3180 |       | 74%$[2_1^+ \otimes 2_2^+]_{RPA}$ |
| $1_1^-$ | 3370 | 3398 | 99.3%$[2_1^+ \otimes 3_1^-]_{RPA}$ |
| $3_1^-$ | 2339 | 2387 | 81%$[3_1^-]_{RPA} + 14%[2_1^+ \otimes 3_1^-]_{RPA}$ |

F-spin is even more pronounced than what predicted by the present QPM calculation. This, in fact, overestimates the experimental M1 strength and underestimates the E2 transition probability roughly by the same factor $\sim 1.5$. Our QPM result is therefore at variance with the SM findings. This discrepancy emerges clearly from the analysis of the magnetic moments. The QPM g-factor for the $2_1^+$ state is $g(2_1^+) = -0.31$, quite different in sign and magnitude from the SM value $g_{SM}(2_1^+) = 0.9$. Clearly, a measure of this quantity would discriminate between the two descriptions and, therefore, would shed light on the p-n symmetry of this state.

Another distinguishing feature of $^{92}$Zr with respect to the nearby nuclei is the fragmentation of the QPM states into several multiphonon components. The symmetric two-phonon $[2_1^+ \otimes 2_1^+]_{RPA}$ accounts only for 65% of the $2_3^+$. For the sake of comparison, the two-phonon counterpart in $^{94}$Mo represents the 82% of the $2_2^+$ state. The non symmetric $[2_1^+ \otimes 2_2^+]_{RPA}$ is spread over several $2^+$ states. It is dominant in the $2_7^+$ and sizeable in $2_8^+$. These two states are therefore predicted to have strong E2 decays to the p-n non symmetric $2_7^+$ state.
TABLE V:  $E2$ transitions connecting some excited states in $^{92}$Zr calculated in QPM. The experimental data are taken from Ref. [15].

| $B(E2; J_i \rightarrow J_f)(w.u.)$ | EXP   | QPM |
|----------------------------------|-------|-----|
| $B(E2; 2_1^+ \rightarrow g.s.)$  | 6.4(6) | 6.5 |
| $B(E2; 2_2^+ \rightarrow g.s.)$  | 3.7(8) | 2.1 |
| $B(E2; 2_2^+ \rightarrow 2_1^+)$ | 0.3(1) | 0.39|
| $B(E2; 2_3^+ \rightarrow 2_1^+)$ | 6.8   |     |
| $B(E2; 4_1^+ \rightarrow 2_1^+)$ | 4.04(12)| 1.0 |
| $B(E2; 4_2^+ \rightarrow 2_1^+)$ | 8.4   |     |
| $B(E2; 4_2^+ \rightarrow 2_1^+)$ | 2.4   |     |
| $B(E2; 4_2^+ \rightarrow 2_1^+)$ | 2.6   |     |
| $B(E2; 1_1^+ \rightarrow 2_1^+)$ | <8.0(6) | 2.0 |
| $B(E2; 3_2^+ \rightarrow 2_1^+)$ |       | 1.6 |
| $B(E2; 3_2^+ \rightarrow 2_2^+)$ |       | 4.4 |

The few available data are closely reproduced by the calculation. Unfortunately, most of the predicted strong transitions have not been detected yet.

The $[2_1^+ \otimes 2_1^+]_{RPA}$ is spread among several $4^+$ QPM states, thereby generating several $4^+$ excitations with appreciable, if not strong, $E2$ decay. The experiments have detected only one $4^+$ and measured the strength of its $E2$ transition to the $2_1^+$ as well as its g-factor, which resulted to be $g_{exp}(4^+) = -0.50$. It is unlikely that this excitation corresponds to the first QPM $4_1^+$ dominated by the RPA one-phonon $[4_1^+]_{RPA}$ with an appreciable admixture (17%) of the two-phonon component $[2_1^+ \otimes 2_1^+]_{RPA}$. Its $E2$ decay strength to the $2_1^+$ is a factor four smaller than the measured value and its g-factor is $g(4_1^+) = 0.57$, opposite in sign to the experimental value. It is more natural to associate the observed excitation to the second $4_2^+$, composed of a dominant two-phonon component with a small one-phonon admixture. Its $E2$ strength is only a factor two larger than the measured value and its g-factor is $g(4_2^+) = -0.32$, reasonably close to the experimental value. This comparative analysis claims also a more pronounced fragmentation leading to a further quenching of the $E2$ decay strength of the $4_2^+$ and to an enhancement of its g-factor. On the other hand, a further fragmentation would
enhance the amplitude of the two-phonon component $\left[ 2^+_1 \otimes 2^+_1 \right]_{RPA}$ and, therefore, the E2 decay strength of the first $4^+_1$. The observation of additional $4^+$ states and the measurement of their E2 decays would represent a precious test for testing the multiphonon fragmentation of the states predicted in QPM.

The $1^+$ and $3^+$ states are also of importance for testing the p-n symmetry. As shown in Table IV, only the first $1^+_1$ is predominantly a two-phonon non symmetric state and, therefore, would be the analogue of the IBM mixed-symmetry state, if F-spin were conserved. The other has a dominant spin excitation component. Spin indeed contributes mainly to the strength of the $M1$ decay of the second $1^+_2$. It gives also a small but non negligible contribution to the decay of the $1^+_1$. Such a contribution is crucial for attaining a good agreement with experiments. The strong E2 decay of the $1^+_1$ to the symmetric $2^+_1$ is also consistent with the experiments and mirrors the unusually strong E2 decay of the $2^+_2$ to the ground state. It represents, therefore, an additional signature of F-spin breaking. A further confirm may be provided by the E2 decay of the $3^+_2$. This contains a very large $\left[ 2^+_1 \otimes 2^+_1 \right]_{RPA}$ component and is predicted to decay to the $2^+_1$ with a strong E2 transition. An experimental test would be desirable.

Very intriguing is the case of the $2^+_{3.263}$ level observed at $E = 3.263$ MeV. This state decays to the first $2^+_1$ with an appreciable M1 strength and an E2 strength of the order of one single particle unit. This level is close to the energy $E \simeq 3.7$ MeV of the three phonon state $\left| 2^+_3 \right> = \left[ 2^+_1 \otimes 2^+_1 \otimes 2^+_2 \right]_{2^+}$. Moreover, the strength of the M1 transition of this three phonon state to the first $2^+_1$ has the same structure of and is comparable in magnitude to the strength of the M1 decay of the non symmetric $1^+_1$ to the ground state. For a pure, properly antisymmetrized, three-phonon state, we get $B(M1, 2^+_3 \rightarrow 2^+_1) = 0.06 \mu^2_N$, close to $B(M1, 1^+_1 \rightarrow 0_{gr}) = 0.07 \mu^2_N$ and smaller than the measured strength by a factor two. Thus, it is tempting to consider this $2^+_{3.263}$ as a good candidate for being a three-phonon excitation with small admixture of two-phonon components. If confirmed by more complete calculations, this level would provide the first evidence of a three-phonon non symmetric $2^+$ state.

Valuable pieces of information come from the study of the low-lying negative parity states. Consistently with previous QPM calculations, the first $1^-_1$ is a pure $\left[ 2^+_1 \otimes 3^-_1 \right]_{RPA}$ two-phonon state. The computed energy is higher than the measured value and close to the sum of the $2^+_1$ and $3^-_1$ energies. The strength of the E1 decay to the ground state is in good
TABLE VI: QPM versus experimental $M_1$, $E_1$, and $E_3$ transitions between some excited states in $^{92}$Zr. The experimental data are taken from Ref. [15].

| $B(M_1; J_i \rightarrow J_f)(\mu_2^N)$ | EXP | QPM | QPM ($g_s = 0$) |
|--------------------------------------|-----|-----|-----------------|
| $B(M_1; 2^+_1 \rightarrow 2^+_1)$    | 0.46(15) | 0.68 | 0.22 |
| $B(M_1; 2^+_2 \rightarrow 2^+_1)$    | 0.16(2) | 0.06$^a$ | |
| $B(M_1; 1^+_1 \rightarrow g.s.)$     | 0.094(4) | 0.069 | 0.031 |
| $B(M_1; 1^+_2 \rightarrow g.s.)$     | | 0.081 | 0.018 |
| $B(M_1; 1^+_1 \rightarrow 2^+_1)$    | $< 0.089(6)$ | $1 \times 10^{-4}$ | $5 \times 10^{-5}$ |
| $B(E_1; J_i \rightarrow J_f)$         |       |       |     |
| $B(E_1; 1^-_1 \rightarrow g.s.)$     | $0.037(4) \times 10^{-3}(e^2 fm^2)$ | $0.045 \times 10^{-3}(e^2 fm^2)$ |     |
| $B(E_1; 3^-_1 \rightarrow 2^+_1)$    | | $0.43 \times 10^{-3}$ ( w.u.) |     |
| $B(E_1; 3^-_1 \rightarrow 2^+_2)$    | | $1.9 \times 10^{-3}$ ( w.u.) |     |
| $B(E3; 3^-_1 \rightarrow g.s.)$      | | 24 ( w.u.) |     |

$^a$Computed under the assumption that the state is a pure, properly antisymmetrized, three-phonon state.

agreement with experiments and results from a destructive interference between the surface quadrupole-octupole mode and the IVGDR. The $3^-_1$ is predominantly an octupole mode and is predicted to undergo a strong $E_3$ decay to the ground state.

Because of the isovector nature of the electric dipole operator, the $E_1$ transitions of the $3^-_1$ to the two lowest $2^+$ states may be used as a test of their p-n symmetry. The ratio $R_{E_1} = B(E_1; 3^-_1 \rightarrow 2^+_1)/B(E_1; 3^-_1 \rightarrow 2^+_2)$ was measured recently [17] and found to be $R_{E_1} = 2.7(2)$, larger than 1 but an order of magnitude smaller than in the nearby nuclei like $^{94}$Mo. The authors of Ref. [17], driven by the truncated SM calculation, ascribed this suppression to the neutron character of the first $2^+_1$. Our QPM calculation yields for this ratio $R_{E_1} = 4.4$, larger than the experimental value by only a factor $\sim 1.5$. This rather satisfactory agreement suggests that the suppression might be due to F-spin breaking in both $2^+_1$ and, specially, $2^+_2$ states. Moreover, the overestimation of the experimental ratio suggests a more pronounced p-n symmetry breaking than in our QPM scheme.
III. CONCLUDING REMARKS

On the ground of the present study, we may draw the conclusion that, consistently with the experimental analysis [15], the lowest two $2^+$ are RPA one-phonon states. At variance with the conclusion drawn in [15], based on a calculation carried out within a too severely truncated SM space, we found that the $2^+_1$ state has appreciable but not huge neutron dominance which does not destroy its p-n symmetric character. The F-spin, instead, is broken more substantially in the second $2^+$ state. A selective comparison of the QPM with the experimental transition strengths strongly suggests that this breaking is even more pronounced than what predicted by the QPM calculation. Such a F-spin admixture provides the key for a consistent description of all experimental levels and transitions. The measure of selected additional transitions, as pointed out in the text, would greatly contribute to a conclusive clarification of this issue.

Another remarkable result is the fragmentation of QPM states among several multiphonon components. Because of such a spread, the corresponding IBM states should also be linear combinations of several multi-bosonic basis states.

Last but not least, the present study offers arguments in favor of the first experimental evidence of a three-phonon non symmetric $2^+$ state.

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