An Ignored Assumption of $\Lambda$CDM Cosmology
and An Old Question:
Do We Live On The “Center” of The Universe?

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We point out that $\Lambda$CDM cosmology has an ignored assumption. That is, the $\Lambda$ component of the universe moves synchronously with ordinary matters on Hubble scales. Usually, this assumption is made when we write down the Einstein equations either. For details of the argument in this paragraph, please refer to [1], sections. 11.8-9, [2] and [3].

If $\Lambda$ component is vacuum energy, then we have no reason to say that it moves synchronously with ordinary matters, even on the Hubble scales. On the contrary, the conception that it acts as an absolute background with negative pressures may be more favored by our intuitions. However, if $\Lambda$ component is vacuum energy and does not move synchronously with ordinary matters, then a serious problem must be notified. That is, our current explanation of the accelerating expansion of the universe should be re-considered because it is based on the assumption that $\Lambda$ component moves synchronously with ordinary matters. As we state in the previous paragraph, when the $\Lambda$ current is included in the energy momentum tensor, the scale factor of the universe cannot be defined globally, i.e.,we can define scale factors $a_1$ and $a_2$ through relations $U(t, r_1) = a_1(t) \cdot f(r_1)$ and $U(t, r_2) = a_2(t) \cdot f(r_2)$ respectively, but we cannot assure that $a_1(t) = a_2(t)$. In this case, the explanation of the accelerating expansion of the universe must be very different from that in the usual $\Lambda$CDM cosmology where the scale factor of the universe is defined globally.

If the vacuum energy of quantum field must be considered in cosmology and it is required to move synchronously with ordinary matters, we guess but are not sure some kinds of regularization is necessary. As we know, in the studying of Casimir effects between two parallel conducting plates, the vacuum energy is regularized, with negative pressures and is of course strictly co-moving with our conducting plates. But we do not know when the regularization is applied on our universe, it is possible or not a vacuum energy density of order $G^{-1}H_0^2$ which is required by astro-physical observations can be obtained. Whatever order the regularized vacuum energy density is of, we think when the regularization is performed, the cosmological constant problem should not be so serious as we usually think.

From the following Gendanken experiment, we can also see that the vacuum energy which does not co-move with ordinary matters cannot contribute to the accelerating expansion of the universe. First imagine that our universe only consists of two test particles and an omnipresent vacuum energy. For each of the test particles, the repulsion force comes from the different direction of the background vacuum energy cancels each other as long as the background space time is infinitely large. So our two test particles can only move towards each other at the inter-gravitations. Then imagine that our universe consists only of a galaxy clusters in which our Milky Way galaxy lies in and an omnipresent vacuum energy. In this case, the repulsion exerted on the galaxies inside the clusters comes from the different direction of the background vacuum energy also cancels each other, and the galaxies only move towards the center of the clusters at the galaxy-galaxy-between gravitations. Depends on the relative value of the initial kinetic energy and the inter-gravitating potential of the galaxies, our galaxy cluster as a whole can be at accelerating contraction phase accelerating expansion phase. But its contraction or expansion...
phase has nothing to do with the omnipresent vacuum energy. It is the pressures originated from the random moving of the galaxies that determines the phase our galaxy cluster lies on. Finally let us imagine that, our universe consists of many many galaxy clusters and galaxies and the omnipresent vacuum energy, the only difference of this case from that of the one galaxy-cluster universe is that, when we are considering the cluster-cluster, cluster-galaxy and galaxy-galaxy interactions, we only need to count contributions from those clusters and galaxies lie in the particle horizon of the universe.

Further analyzing our Gedanken experiment of one galaxy cluster universe, we doubt we may forget something of very importance when we extrapolate the concepts of pressures obtained from the ideal gas in laboratories to the expanding universe. To make our doubt more clearly expressed, let us consider three kinds of gases, (i) a bottle of closed gas in our laboratories, (ii) a galaxy cluster as a gas whose basic molecules are galaxies such as our Milky Way and (iii) the total observable universe as gas whose basic molecules are galaxy clusters and galaxies. In the first case, the pressure of the gases originates from the random moving of the basic molecules and the system is kept in bounding state because the bottle has walls which cannot be penetrated by the basic molecules. In the second case, the pressure also originates from the random moving of the basic molecules - the galaxies, the system is kept in balance at the self-gravitation and the pressures. In the third case, we are usually told that if dark energy were not introduced, the cosmic fluid is a zero-pressure gas. Is it reasonable to neglect the pressures originating the random moving of galaxy clusters and galaxies?

We must note that it is just the same pressures that kept the galaxy clusters in a stable state in the second example. In the second example, if initially, the composite galaxies were given too much kinetic energy, then the total system may not be able to be kept in balance, some of its composite molecules may escape, just as man-made satellites escape from our planet. Some people may say that what we state here is just the same thing occurs in a totally matter dominated open universe. We will say to these peoples that, partly this is the case, but it cannot be totally think so. What we would like to emphasize here is that, even in a totally matter dominated universe, the pressures originating from the random moving of the composite molecules of it cannot be neglected as we are usually taught. What’s more, in a homogeneous and isotropic universe, if the Hubble recession of galaxies are also count as random movings and also contribute to pressures, we will expect that the energy momentum tensor describing our cosmic fluid should have such a property that, its pressure part (the abstract value) should increase as a function of the co-moving distance. In the next section, we will provide a strict solution of Einstein equation, whose energy momentum tensor indeed has this property, please see eqs (1) and (2).

II. A NEW MECHANISM EXPLAINING THE ACCELERATING RECESSION OF SUPER-NOVAES

At this moment, we are not sure that the existence of particle horizon guarantees the regularization of vacuum energy and the regularized vacuum energy will affect the expansion of our observable universe. So, rather than integrating the vacuum energy of quantum field and regularizing it to get the cosmological constant, we prefer to propose another mechanism which can explain the accelerating expansion of our universe and which is of no necessary to introduce the Λ term in the Einstein equation. That is, the pressures originating from the random moving of galaxy clusters and galaxies cannot be neglected.

We find that the following metric ansatz solve Einstein equation $G_{\mu \nu} = -8\pi G T_{\mu \nu}$,

$$ds^2 = -dt^2 + e^{r_t/A}(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2)$$

(1)

with the energy momentum tensor given by

$$8\pi G T_{\mu \nu} =$$

$$\begin{bmatrix}
3e^{r_t/A} - \frac{2e^{r_t/A}}{A} & -A^{-1} & 0 & 0 \\
-A^{-1} & \frac{r_t}{A} - \frac{3e^{r_t/A}r_t}{4A^2} & 0 & 0 \\
0 & 0 & T_{22} & 0 \\
0 & 0 & 0 & T_{33}
\end{bmatrix}$$

(2)

Where $A$ is a constant with dimension of $[\text{length}]^2$. Comparing eq (1) with the usual Friedman-Robertson-Walker metric $ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2)$, we can see that the most remarkable feature of the metric (1) is that, the scale factor is $r$ dependent and at a given position $r$, the scale factor increase exponentially as time passes by. So such a metric can describe an accelerating universe. In fact it is just using being able to describe an accelerating universe as a criteria that we construct the metric ansatz eq (1), and then using Einstein equation we get the energy momentum tensor eq (2).

So at this moment, we are not very familiar with the physical meaning of each term in the energy momentum tensor eq (2). But we know it describes some kinds of fluid whose pressures cannot be neglected, and (the abstract value) is an increasing function of the co-moving distance increases. This point coincides with our expectation expressed at the end of previous section. So we guess it may have relevance with our realistic cosmological fluid, when the pressures originating from the random moving (including the Hubble recession) of galaxy clusters and galaxies must be considered. Its non-diagonal terms only appear on the $t - r$ and $r - t$ position, this may indicate the fact that the pressures in cosmology has differences from that of the gas in our laboratories. In the studying of the gas pressures in laboratories, we need not to consider the time for a gas molecule to run from the container’s walls to another molecule after collision with the container walls. But in cosmologies, to
consider the effects of pressures originating from the random moving or Hubble recessions of galaxy clusters and galaxies, the time for a basic composite galaxy to run from the Horizon edge to us must be considered. In this case we expect that the energy momentum tensor must have non-zero $t - r$ and $r - t$ components.

Of course, since the metric (1) deviates from the standard Friedmann-Robertson-Walker metric so much that, we are very not sure that it describes our real universe indeed. But we are sure that, if the pressures originating from the random moving (including the Hubble recession) of galaxy clusters and galaxies in our universe must be considered, then the metric of our universe must have the same features as eq(1). We are also sure that, in this case the energy-momentum tensor is non-diagonal, it has non-zero components and its energy density and pressures are both time and position dependent. For example we can check that the following metric ansatz also solve Einstein equation:

$$ds^2 = -dt^2 + e^{t/r}(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2)$$ (3)

but with the energy momentum tensor written as

$$8\pi G T_{\mu\nu} = \begin{bmatrix}
\frac{3-r^2}{r^2} & 0 & 0 \\
0 & -\frac{3e^{t/r} - 4e^{-t/r} - r^2}{4r^2} & 0 \\
0 & 0 & T_{22} \\
0 & 0 & 0 & T_{33}
\end{bmatrix}$$

$$T_{22} = -\frac{3e^{t/r} - 2e^{-t/r} - r^2}{4}, T_{33} = T_{22}\sin^2\theta$$ (4)

We are not sure if this metric describe an accelerating expanding universe or not. Because, naively looking, the recession velocity of an object in such a space-time reads: $\frac{dr}{dt} \propto \frac{1}{r}$, the farer is an object lies away from the origin, the smaller its recession velocity will be. But, in this space time, since the scale factor of universe is $r$ dependent, the definition Hubble recession’s velocity may not be the same as that in the usual Friedmann-Robertson-Walker space-time.

We must emphasize again that, the metrics eq(1) and (3) is obtained by an inverse method. We are not sure it can describe a realistic universe. We provide it here only to illustrate that if the pressures originating from the random moving of galaxy clusters and galaxies in the universe cannot be neglected, the metric of the space time and the energy momentum tensor should have the same features as those of eqs(1), (2) and eqs(3), (4).

Originally, we think that to get a solution of Einstein equation in which the scale factor of metric is both time and position dependent and if the dependence cannot be factorized, so that we can have a position dependent Hubble recession, our observable universe must be lying on the center of some very big super-super-clusters, in this super-super-cluster, the matter distribution has spherical symmetry, so in the general solution of Einstein equation $ds^2 = -dt^2 + U(t,r)dr^2 + V(t,r)d\Omega^2$, the function $U(t,r)$ cannot be factorized as $a^2(t)f(r)$ as we are usually taught in the text-book such as [1], we test the ansatz eq(3) and get the energy momentum tensor eq(4). Although we find that, the energy density and pressures are decreasing function of co-moving distances, but the metric may not describe an accelerating universe as indicated by the super-novae observations. Then we test metric ansatz eq(1) and get the energy momentum tensor eq(4), in this case we find the metric describe an accelerating expansion universe, but the energy-density and pressures is an increasing function of co-moving distances. In this case, we propose that in the usually Friedmann-Robert-Walker universe, the pressures originating from the random moving of galaxy clusters and galaxies may be neglected un-appropriately. If our original imagination were correct, we may really live on the center of the “universe”, so we name our this paper as “An Ignored Assumption of ΛCDM Cosmology and An Old Question: Do We Live On The Center of The "Universe"?”

Although we have realized that, as long as we consider the pressures originating from the random moving (including the Hubble recession) of galaxy clusters and galaxies, obtaining an non-factorized scale factor of the universe is possible even discarding the assumption that we are living on the center of the “universe”, we still would like to name this paper as “An Ignored Assumption of ΛCDM Cosmology and An Old Question: Do We Live On The "Center" of The Universe?”

but now with the position of quotation mark changed! Because from some aspects, we are living on a minimum pressure point in the universe, it may be thought as the “center” of the universe!

### III. CONCLUSIONS

In the first part of this paper, we point out that ΛCDM cosmology has an ignored assumption. That is, the Λ component of the universe moves synchronously on Hubble scales. We think this is very difficult to understand if it is vacuum energy of quantum field. But, considering that in the studying of the Casimir effects of two parallel conducting plates, after regularization, the vacuum energy obtained is not only co-moving with the conducting plates, but also with negative pressures, we think that to get the correct cosmological constant by integrating the vacuum energy, some kind of regularization is necessary.

In the second part of the paper, we propose a new mechanism which can explain the accelerating recession of super-novae but with no necessary to introduce Λ term in the Einstein equation. In this mechanism, the pressures of the cosmological fluid originating from the random moving of galaxy clusters and galaxies is considered. We provide an analytical solution of Einstein equation which may describe some kinds of cosmological
fluid whose pressures is considered. We are not very sure that our metric describes our realistic universe indeed. But from the metric we can see that, if the pressures originating from the random moving of galaxy-clusters and galaxies must be considered, the scale factor of the universe metric should depend on the time and positions at the same time, and the dependence on the two variables cannot be factorized. At the same time, the energy-momentum tensor of the universe should not be diagonal, it should have non-zero component on the $t - r$ and $r - t$ position to indicate the fact that, for a test particle to move from the edge of the horizon to us, time is needed.

[1] Steven Weinberg, 1972, Gravitations and Cosmology, John Wiley press.
[2] D.F. Zeng and Y. H. Gao, Spherical Collapse Model and Dark Energy(I), [astro-ph/0505163](http://arxiv.org/abs/astro-ph/0505163).
[3] D.F. Zeng and Y. H. Gao, Spherical Collapse Model and Dark Energy(II), [astro-ph/0505164](http://arxiv.org/abs/astro-ph/0505164).