1. Transversity

The transversity distribution function \( h_1 \) (or \( \delta q \)) is a measure of how much of the transverse spin of a polarized proton is transferred to its quarks, i.e. the density of transversely polarized quarks inside a transversely polarized proton.

The transversity distribution function is a chiral-odd or helicity flip amplitude. Observables involving transversity should therefore be (helicity flip)\(^2\). This is the reason \( h_1 \) cannot be measured in inclusive Deep Inelastic Scattering (DIS) (\( ep \rightarrow e'X \)); it enters the cross section suppressed by a factor of \( m_a/Q \), where \( m_a \) is the mass of a quark of flavor \( a \) and \( Q \) is the invariant mass of the virtual photon that probes the proton. A further complication is that in charged current experiments the virtual photon that probes the proton. A further complication is that in charged current exchange processes chiral-odd functions like \( h_1 \) cannot be accessed. This is a drawback regarding a future flavor separation for such functions.

To measure transversity there are essentially two options left: single or double transverse spin asymmetries in (semi-inclusive, neutral current) \( ep \) or \( pp \) processes. Few such experiments have been performed to date and this is the reason that no experimental data on \( h_1 \) is available thus far. Hints of nonzero \( h_1 \) come only from the HERMES and E704 experiments (the latter is a measurement of \( D_{NN} \), see below). But a number of future experiments (e.g. HERMES,COMPASS,RHIC) are expected to provide detailed information on transversity functions.

2. Transversity asymmetries

The Drell-Yan process of two colliding transversely polarized hadrons producing a lepton pair was originally thought to be the best way of accessing the transversity distribution, for instance at RHIC. This double transverse spin asymmetry \( A_{TT}^{DY} \) is proportional to \( h_1(x_1) T_1^a(x_2) \). The problem is that \( h_1 \) for antiquarks (\( h_1^a = \bar{T}_1^a \)) inside a proton is presumably much smaller than for quarks and the asymmetry is not expected to be large. In fact, by using Soffer’s inequality \( |h_1(x)| \leq \frac{1}{2} [g_1(x) + g_1(x)] \), \( A_{TT}^{DY} \) has been shown \(^3\) to be small at RHIC, just beyond the experimental reach (but a future upgrade would be very promising).

Also the double transverse spin asymmetry in jet production (directly proportional to \( (h_1)^2 \) at high \( p_t \)) poses experimental problems, because of the tiny cross sections one is dealing with. In Ref. \(^3\) it is shown that for RHIC the statistical error is not a problem, but due to the small values of the asymmetry, the systematic errors need to be of extremely good control.

Another possible way to access the transversity distribution function via a double transverse spin asymmetry, involves the transversity fragmentation function \( H_1 \). It measures the probability of \( q(s_f) \rightarrow h(S_f) + X \), where \( h \) is a spin-1/2 hadron, for instance a \( \Lambda \). The double transverse spin asymmetry \( D_{NN} \) –the transverse polarization transfer– involving both \( h_1 \) and \( H_1 \) occurs in the processes \( ep \rightarrow e' \Lambda^+ X \) and \( p p \rightarrow \Lambda^+ X \). The latter observable has been measured by E704 \(^3\) and found to be sizeable, but a solid conclu-
sion about $h_1$ cannot be drawn due to the rather low $p_T$ range, which casts doubt on the use of a factorized expression for the cross section. Furthermore, $H_1$ is also unknown and although it could be extracted from the double transverse spin asymmetry in $e^+e^- \rightarrow \Lambda^+ \Lambda^0 X$, this will also pose quite a challenge.

In short, double transverse spin asymmetries do not seem promising to extract the transversity distribution in the near future. This leaves the option of single spin asymmetries (SSA), which all exploit fragmentation functions of some sort. There are three options: 1) measuring the transverse momentum of the final state hadron compared to the jet axis or, equivalently, the quark momentum, see Fig. 1 of Ref. [11]. The Collins effect will be denoted by a fragmentation function $H_1^{\perp}(z,k_{\perp})$ and if nonzero, it can lead to SSA in $e^+p_T \rightarrow e^+\pi^0 X$ and $p^+_T \rightarrow \pi X$. There are some experimental indications that the Collins effect is indeed nonzero, e.g., SSA measured by HERMES [2,7] and SMC [8] at relatively low energies. Also, SSA in $p^+_T \rightarrow \pi X$ as measured by E704 and at the AGS (BNL) can be (at least partially) explained as arising from a nonzero Collins function [9].

3. Collins effect asymmetries

The Collins effect refers to a nonzero correlation between the transverse spin $s_T$ of a fragmenting quark and the distribution of produced hadrons. More specifically, a transversely polarized quark can in principle fragment into particles (with nonzero transverse momentum $k_{\perp}$) having a $k_{\perp} \times s_T$ angular distribution around the jet axis or, equivalently, the quark momentum, see Fig. 1 of Ref. [11]. The Collins effect will be denoted by a fragmentation function $H_1^{\perp}(z,k_{\perp})$ and if nonzero, it can lead to SSA in $e^+p_T \rightarrow e^+\pi^0 X$ and $p^+_T \rightarrow \pi X$. There are some experimental indications that the Collins effect is indeed nonzero, e.g., SSA measured by HERMES [2,7] and SMC [8] at relatively low energies. Also, SSA in $p^+_T \rightarrow \pi X$ as measured by E704 and at the AGS (BNL) can be (at least partially) explained as arising from a nonzero Collins function [9].

3.1. Collins effect in semi-inclusive DIS

Collins [10] considered semi-inclusive DIS (SIDIS) $e^+p_T \rightarrow e^+\pi^0 X$, where the spin of the proton is orthogonal to the direction of the virtual photon $\gamma^*$. and one observes the pion transverse momentum $P^{\perp}_\gamma$, which has an angle $\phi^\gamma$ compared to the lepton scattering plane. Collins has shown that the cross section for this process has an asymmetry that is proportional to the transversity function: $A_T \propto \sin(\phi^\gamma + \phi^s) |S_T| h_1^{\perp}$. To discuss this SSA further, we will first project it out from the cross section (cf. Ref. [11]). Consider the cross sections integrated, but weighted with a function $W = W(|P^{\perp}_\gamma|, \phi^\gamma)$:

$$\langle W \rangle \equiv \int d^2P^{\perp}_\gamma \ W \frac{d\sigma_{e^+e^-\rightarrow e^+\pi^0 X}}{dxdydzd\phi^\gamma |d^2P^{\perp}_\gamma|}, \quad (1)$$

where we restrict to the case of $|P^{\perp}_\gamma|^2 \ll Q^2$. We will focus on

$$O \equiv \langle \sin(\phi^\gamma) |P^{\perp}_\gamma| \rangle \frac{4\pi \alpha^2 s/Q^4}{|M_\pi|^2} = |S_T|(1-y) \sum_{a,a^\prime} c_a^2 x h^a_1(x) z H_1^{\perp}(1)(\gamma)(z), \quad (2)$$

where $\phi^\gamma = \phi^\gamma + \phi^s$ and $H_1^{\perp}(z) \equiv \int d^2k_{\perp} \frac{k_{\perp}^2}{2z^2M_\pi^2} H_1^{\perp}(z,k_{\perp}^2)$. (3)

At present all phenomenological studies of the Collins effect are performed using such tree level expressions. But the leading order (LO) evolution equations are known for $h_1$ (NLO even) and $H_1^{\perp}(z)$ (at least in the large $N_c$ limit [12]), showing that both functions evolve autonomously (and vanish asymptotically). This provides one with the LO $Q^2$ behavior of the observable $O$, which arises solely from the LO evolution of $h_1$ and $H_1^{\perp}(z)$. This is however a nontrivial result, since this semi-inclusive process is not a case where collinear factorization applies. In the differential cross section $d\sigma/d^2P^{\perp}_\gamma$ itself, beyond tree level soft gluon corrections do not cancel, such that Sudakov factors need to be taken into account and a more complicated factorization theorem applies [13,14]. In fact, the observable $O$ (Eq. (2)) is the only $|P^{\perp}_\gamma|$-moment of the Collins asymmetry in the cross section, that is not sensitive to the Sudakov factors. This observable would therefore be better suited for a LO analysis than the full $|P^{\perp}_\gamma|$-dependent asymmetry.

Now we will look at the latter in the explicit example of the Collins effect asymmetry in SIDIS;
more specifically, $e^+ d \rightarrow e^+ \gamma^*(q_T) p \rightarrow e^+ \pi X (q_T = -2P_T^\perp$ and $q_T^2 = Q_T^2 \ll Q^2)$

\[
\frac{d\sigma(e^+ p \rightarrow e^+ \pi X)}{dxdzdyd\phi_c d^2q_T} \propto \{1 + |S_T| \sin(\phi_c)A(q_T)\}.
\]

To get an idea about the effect of Sudakov factors, we have assumed Gaussian transverse momentum dependence for $H_1^\perp (z, k_T)$, such that the asymmetry’s analyzing power $A(q_T)$ is given by

\[
A(q_T) = \frac{\sum_a e_a^2 b(y) h_1^a(x) H_1^{\perp(1)a}(z)}{\sum_b e_b^2 a(y) f_1^b(x) D_1^b(z)} A(Q_T),
\]

where $a(y) = (1 - y + \frac{2}{z} y^2), b(y) = (1 - y)$. In Ref. [14] we studied $A(Q_T)$ for a generic nonperturbative Sudakov factor, because of lack of experimental input for this quantity. It was found that $A(Q_T)$ at $Q = M_Z$ becomes considerably smaller and broader than the tree level expectation. We also observed that $\max[A(Q_T)] \sim Q^{-0.5} - Q^{-0.6}$. Thus, tree level estimates tend to overestimate transverse momentum dependent azimuthal spin asymmetries and Sudakov factors cannot be ignored at present-day collider energies.

3.2. Collins effect in $e^+ e^- \rightarrow \pi^+ \pi^- X$

In order to obtain the Collins function itself, one can measure a $\cos(2\phi)$ asymmetry in $e^+ e^- \rightarrow \pi^+ \pi^- X$, that has a contribution proportional to the Collins function squared [15] (at equal momentum fractions). A first indication of such a nonzero (but small) asymmetry comes from a preliminary analysis [16] of the 91-95 LEP1 data (DELPHI). A similar study using the off-resonance data from the B-factory BELLE at KEK, is planned [17].

Also for this Collins effect observable, the tree level asymmetry expression is not sufficient if results from different experiments are to be compared. Beyond tree level Sudakov factors need to be included. If the differential cross section is written as

\[
\frac{d\sigma(e^+ e^- \rightarrow \pi^+ \pi^- X)}{d^2q_T} \propto \{1 + \cos(2\phi)A(q_T)\},
\]

with $q_T^2 \ll Q^2$, then assuming again Gaussian transverse momentum dependence for the Collins function, we find

\[
A(q_T) \propto \frac{H_1^{\perp(1)}(z_1)H_1^{\perp(1)}(z_2)}{D_1(z_1)D_1(z_2)} A(Q_T).
\]

Again, a generic example [14] shows that Sudakov factors now produce an order of magnitude suppression at $Q = M_Z$ compared to a typical tree level result (in addition, now $\max[A(Q_T)] \sim Q^{-0.9} - Q^{-1.0}$). Therefore, this Collins effect observable is best studied with two jet events at $\sqrt{s} \ll M_Z$ (a requirement satisfied by BELLE, which operates on and just below the $\Upsilon(4S)$).

Nevertheless, the extraction of the Collins function from this asymmetry is not straightforward, since there is asymmetric background from hard gluon radiation (when $Q_T \sim Q$) and from weak decays. The former enters the $Q_T$ dependent asymmetry proportional to $\alpha_s Q_T^2/Q^2$, which at lower values of $Q^2$ need not be small. This contribution could be neglected at LEP energies [15].

Luckily it is calculable and so is the background from weak decays, e.g. $e^+ e^- \rightarrow \pi^+ \pi^- \rightarrow \pi^+ \pi^- X$.

As in the case of the Collins asymmetry in SIDIS, there is one particular $Q_T$ moment of the asymmetry that is not sensitive to Sudakov factors, namely the first $Q_T^2$ moment: $\int dQ_T^2 Q_T^2 d\sigma/dQ_T$. Unfortunately, it is mostly sensitive to the high $Q_T^2 (\sim Q^2)$ hard gluon radiation. This contribution could in principle be cut off by imposing a maximum $Q_T$ cut, but this introduces a further source of uncertainty.

4. Interference fragmentation functions

Jaffe, Jin and Tang [18] pointed out the possibility that the Collins effect averages to zero in the sum over final states $X$. Instead, they proposed to measure two pions in the final state ($\pi^+ \pi^-$) out of $X$ ($\pi^+, \pi^-$ belong to the same jet), which presumably depends on the strong phase shifts of the $(\pi^+ \pi^-)$ system. The interference between different partial waves could give rise to a nonzero chiral-odd fragmentation function called the interference fragmentation function (IFF). The IFF would lead to single spin asymmetries in $e p^+ \rightarrow e' (\pi^+ \pi^-) X$ and $p p^+ \rightarrow (\pi^+ \pi^-) X$, both proportional to the transversity function.
4.1. IFF in semi-inclusive DIS

The SSA expression for $e p^+ \rightarrow e^+ (\pi^+ \pi^-) X$ in terms of the IFF $\delta q_1(z)$ is [[13]]

$$\langle \cos(\phi_{S_T}^\ell + \phi_{R_T}^\ell) \rangle \propto F|S_T||R_T|h_1(x)\delta q_1(z),$$  (8)

where $z = z^+ + z^-$; $R_T = (z^+ k^- - z^- k^+)/z$; $F = F(m^2) = \sin\delta_0 \sin\delta_1 \sin(\delta_0 - \delta_1)$, where $\delta_0, \delta_1$ are the $\ell = 0, 1$ phase shifts and $m^2$ is the $\pi^+ \pi^-$ invariant mass. Note the implicit assumption of factorization of $z$ and $m^2$ dependence, which leads to the prediction that on the $p$ resonance the asymmetry is zero (according to the experimentally determined phase shifts). More general $z, m^2$ dependences have been considered [14] and this should be tested.

Like-sign $(\pi^+ \pi^\pm)$ asymmetries are expected to be tiny, which provides another useful test.

The asymmetry expression ([5]) is based on a collinear factorization theorem (soft gluon contributions cancel, no Sudakov factors appear). Thus an analysis beyond tree level is conceptually straightforward. The evolution of $\delta q_1(z)$ equals that of $H_1(z)$ and is known to NLO Ref. [20]. A NLO analysis is thus feasible (cf. Ref. [21]).

4.2. IFF in $e^+e^-$ annihilation

For the extraction of the interference fragmentation functions themselves one can study a $\cos(\phi_{R_T}^\ell + \phi_{R_T}^\ell)$ asymmetry [[12]] in $e^+ e^- \rightarrow (\pi^+ \pi^-)_{\text{jet1}} (\pi^+ \pi^-)_{\text{jet2}} X$ which is proportional to $(\delta q_1)^2$ and which is again possible at BELLE.

There is no expected asymmetric background. Combining such a result with for instance the single spin asymmetry in $p p^+ \rightarrow \pi^+ \pi^- X$ to be measured at RHIC, seems –at present– to be one of the most realistic ways of obtaining information on the transversity function.

Finally, a cross-check observable that is interesting to study is a $\sin(\phi_{S_T}^\ell + \phi_{R_T}^\ell)$ asymmetry in $e^+ e^- \rightarrow (\pi^+)_{\text{jet1}} (\pi^+)_{\text{jet2}} X$, which is proportional to a product of the Collins and the interference fragmentation function.

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