σ meson mass and width at finite density

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Abstract

The σ meson mass and width are studied at finite baryonic density in the framework of a chiral unitary approach which successfully reproduces the meson meson phase shifts and generates the $f_0$ and σ resonances in vacuum.

1 Introduction

There is mounting evidence showing the existence of a light scalar isoscalar meson [1, 2, 3, 4]. However, its nature as a genuine meson state or as a resonance dynamically generated in the \( \pi \pi \) scattering is still under debate. A review of the current status of the discussion can be found in ref. [5].

In any case, it is such a broad resonance that its effects are hardly visible in any phase shifts or decay plots. The situation could be quite different at finite densities and/or temperatures where the σ could become lighter and narrow [6, 7]. Apart from its implications related to chiral symmetry restoration, this topic is of special interest due to the relevance of the \( I = J = 0 \) channel in the nucleon-nucleon interaction. Any substantial change of this meson properties could alter the in medium nucleon-nucleon interaction and therefore our current understanding of the nuclear matter in heavy ion collision, neutron stars or even in normal nuclei if the changes occur at low densities.

The density dependence of the σ properties has been studied in several models. Bernard et al. [8, 9, 10] have found a decrease of the mass as a function of the density using a generalized Nambu Jona-Lasinio model, until it converges to the mass of the pion, its chiral partner. It is noticeable, that in the same model the \( \rho, \omega \) and \( \pi \) mesons have almost constant masses.

In ref. [11], Hatsuda et al. studied the σ propagator in the linear σ model and found an enhanced and narrow spectral function near the \( 2\pi \) threshold.
caused by the partial restoration of the chiral symmetry, where the $m_\sigma$ mass would approach $m_\pi$. The same conclusions were reached using the nonlinear chiral lagrangians in ref. [12].

Similar results, with large enhancements in the $\pi\pi$ amplitude around the $2\pi$ threshold, have been found in a quite different approach by studying the $s-wave$, $I = 0$ $\pi\pi$ correlations in nuclear matter [13, 14, 13, 16]. In these cases the modifications of the $\sigma$ channel are induced by the strong $p-wave$ coupling of the pions to the particle-hole ($ph$) and $\Delta$-hole ($\Delta h$) nuclear excitations. It was pointed out in [17, 18] that this attractive $\sigma$ selfenergy induced by the $\pi$ renormalization in the nuclear medium could be complementary to additional $s-wave$ renormalizations of the kind discussed in [11, 12] calling for even larger effects.

On the experimental side, there are also several results showing strong medium effects in the $\sigma$ channel at low invariant masses in the $A(\pi, 2\pi)$ [19, 20, 21, 22, 23] and $A(\gamma, 2\pi)$ [24] reactions. At the moment, the cleanest signal probably corresponds to the $A(\gamma, 2\pi^0)$ reaction, which shows large density effects that had been predicted in both shape and size in ref. [25], using a model for the $\pi\pi$ final state interaction along the lines of the present work.

Our aim in this paper is to study the $\sigma$ mass and width at finite densities in the context of the model developed in [26, 27, 28, 29, 30, 31]. These works, which provide a very economical and successful description of a wide range of hadronic phenomenology, use as input the lowest orders lagrangian of chiral perturbation theory [32] and calculate meson-meson scattering in a coupled channels unitary way. The $\sigma$ meson is not a basic ingredient on this approach, it is dynamically generated and appears as a pole of the $\pi\pi$ scattering amplitude in the second Riemann sheet.

The nuclear medium effects on the scalar isoscalar channel were implemented in this framework in refs. [16, 33]. As in other approaches, large medium effects were found. Namely, the imaginary part of the $\pi\pi$ scattering amplitude showed a clear shift of strength towards low energies as the density increases.

In the next section we present, for the sake of completeness, a brief description of the model used for the $\pi\pi$ interaction in both vacuum and dense medium, which is already published elsewhere. The following section describes the method used to calculate the $\pi\pi$ scattering amplitude in the complex energy plane and to search for the $\sigma$ pole position at finite densities.
2 $\pi\pi$ interaction

In this work we consider only the scalar isoscalar ($\sigma$) channel and follow the simple method of ref. [31] for $\pi\pi$ interaction in vacuum and refs. [16, 33] for the nuclear medium effects. Additional information on this and related approaches for different spin isospin channels can be found in refs. [31, 28, 29, 34].

2.1 Vacuum

The basic idea is to solve a Bethe Salpeter (BS) equation, which guarantees unitarity, matching the low energy results to the chiral perturbation theory ($\chi PT$) predictions. We consider two coupled channels, $\pi\pi$ and $KK$ and neglect the $\eta\eta$ channel which is not relevant at the low energies we are interested in.

The BS equation is given by

$$T = V + VGT.$$  \hspace{1cm} (1)

Eq. (1) is a matrix integral equation which involves the two mesons one loop divergent integral, (see Fig. 1), where $V$ and $T$ appear off shell. However, for this channel both functions can be factorized on shell out of the integral. The remaining off shell part can be absorbed by a renormalization of the coupling constants as it was shown in refs. [31, 35]. Thus, the BS equation becomes purely algebraic and the $VGT$ originally inside the loop integral becomes then the product of $V$, $G$ and $T$, with $V$ and $T$ the on shell amplitudes independent of the integration variables, and $G$ given by the expression

$$G_{ii}(P) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m^2_{fi} + i\epsilon} \frac{1}{(P - q)^2 - m^2_{fi} + i\epsilon}$$  \hspace{1cm} (2)

where $P$ is the momentum of the meson-meson system. This integral is regularized with a cut-off adjusted to optimize the fit to the $\pi - \pi$ phase shifts ($\Lambda = 1.03$ GeV). The potential $V$ appearing in the BS equation is taken from the lowest order chiral Lagrangian

$$L_2 = \frac{1}{12 f^2} < (\partial_\mu \Phi \Phi - \Phi \partial_\mu \Phi)^2 + M\Phi^4 >.$$  \hspace{1cm} (3)
where the symbol \(<>\) indicates the trace in flavour space, \(f\) is the pion decay constant and \(\Phi, M\) are the pseudoscalar meson and mass \(SU(3)\) matrices. This model reproduces well phase shifts and inelasticities up to about 1.2 GeV. The \(\sigma\) and \(f_0(980)\) resonances appear as poles of the scattering amplitude. The coupling of channels is essential to produce the \(f_0(980)\) resonance, while the \(\sigma\) pole is little affected by the coupling of the pions to \(K\bar{K}\) \[31\].

2.2 The nuclear medium.

As we are mainly interested in the low energy region, which is not very sensitive to the kaon channels, we will only consider the nuclear medium effects on the pions. The main changes of the pion propagation in the nuclear medium come from the p-wave selfenergy, produced basically by the the coupling of pions to particle-hole (\(ph\)) and Delta-hole (\(\Delta h\)) excitations. For a pion of momentum \(q\) it is given by

\[
\Pi(q) = \frac{\left(\frac{D+F}{2f}\right)^2 \bar{q}^2 U(q)}{1 - \left(\frac{D+F}{2f}\right)^2 g' U(q)}
\]  

with \(g' = 0.7\) the Landau-Migdal parameter, \(U(q)\) the Lindhard function and \((D + F) = 1.257\). The expressions for the Lindhard functions are taken from ref. \[36\].

Thus, the in medium BS equation will include the diagrams of Fig. 2 where the solid line bubbles represent the \(ph\) and \(\Delta h\) excitations.

![Diagram](image)

Figure 2: Terms of the meson-meson scattering amplitude accounting for \(ph\) and \(\Delta h\) excitation.

In fact, as it was shown in \[37\], the contact terms with the \(ph\) (\(\Delta h\)) excitations of diagrams (b)(c)(d) cancel the off shell contribution from the meson meson vertices in the term of Fig. 2(a). Hence, we just need to calculate the diagrams of the free type (Fig. 1) and those of Fig. 2(a) with the amplitudes factorized on shell. Therefore, at first order in baryon density, we are left with simple meson propagator corrections which can be readily
incorporated by changing the meson vacuum propagators by the in medium ones.

There are some other medium corrections, like the one depicted in Fig. 3 which has been considered in several papers like [11, 38, 12] using the linear $\sigma$ selfenergy diagram.

In our approach, we start with only pseudoscalar meson fields. As it was discussed previously, the sigma is generated dynamically through the rescattering of the pions. A possible analog to the diagram of Fig. 3 is given by the diagrams of Fig. 4.

![Figure 3: Tadpole $\sigma$ selfenergy diagram.](image)

Figure 4: Some tadpole diagrams contributing to the meson-meson scattering amplitude

We evaluate the contribution of these diagrams by using the chiral Lagrangians [39, 40, 41, 42] involving the octet of baryons and the octet of pseudoscalar mesons. We find these terms to be proportional to $\bar{p}\gamma^\mu p - \bar{n}\gamma^\mu n$ and thus they vanish in symmetric nuclear matter.

Also we would need to consider diagrams of the type shown in Fig. 5. We get a negligible contribution to the the $\pi\pi$ scattering in the nuclear medium from this diagram [33] and for simplicity we do not include it in this work.

The $\pi\pi$ scattering amplitude obtained using this model exhibits a strong shift towards low energies. In Fig. 6 we show the imaginary part of this amplitude for several densities. Quite similar results have been found using different models [15] and it has been suggested that this accumulation of strength, close to the pion threshold, could reflect a shift of the $\sigma$ pole which would approach the mass of the pion. Other pion selfenergy contributions
related to $2ph$ excitations, and thus proportional to $\rho^2$, can be incorporated in the pion propagator. As we are most interested in the region of low energies we can take as estimation the corresponding piece of the optical potentials obtained from pionic atoms data, following the procedure of ref. [16] and substituting in Eq. 4

$$ \left( \frac{D + F}{2f} \right)^2 U(q) \rightarrow \left( \frac{D + F}{2f} \right)^2 U(q) - 4\pi C_0^* \rho^2 $$

with $\rho$ the nuclear density and $C_0^* = (0.105 + i0.096)m^{-6}_\pi$. Its effects are small except at large densities as can be appreciated by comparing Fig. 6 with Fig. 7 of ref. [16] where this piece is included.

### 3 $\sigma$ meson mass and width as a function of the baryon density

In this section we will consider the lightest pole position of the $\pi\pi$ scattering amplitude in the $S=I=0$ channel in the complex energy plane, which corresponds to the $\sigma$ resonance. In our model, the vacuum pole position occurs at a mass around 500 MeV and the width is around 400 MeV. A compilation of values for these magnitudes obtained in several modern analyses can be found in ref. [43]. See also Fig. 1 of ref. [2] and [44].

The analytical structure of the meson meson scattering amplitude is driven by the meson loops, which are given by the formula

$$ G_{\pi\pi} = i \int_0^\infty \frac{d^4q}{(2\pi)^4} D_\pi(q^0, \vec{q})D_\pi(\sqrt{s} - q^0, -\vec{q}) $$

where $D_\pi$ is the $\pi$ propagator which, in the nuclei, incorporates the p-wave coupling of the pions to particle-hole and $\Delta$-hole excitations. We can simplify the calculation by using the Lehmann representation for the meson
propagators

\[
D_\pi(q^0, \vec{q}) = -\frac{2}{\pi} \int_0^\infty dx \frac{Im D_\pi(x, \vec{q})}{(q^0)^2 - x^2 + i\epsilon}
\]  

(7)

After some algebraic manipulation we obtain

\[
G_{\pi\pi} = \int_0^\infty dE \frac{1}{2\pi} \left( \frac{1}{P^0 - E + i\epsilon} - \frac{1}{P^0 + E - i\epsilon} \right) F(E)
\]  

(8)

where \( F(E) \) is a real function given by

\[
F(E) = \int \frac{d^3q}{(2\pi)^3} \int_{-E}^E \frac{dx}{\pi} \text{Im} D_\pi((E + x)/2, \vec{q}) \text{Im} D_\pi((E - x)/2, \vec{q})
\]  

(9)

which in vacuum takes the simple form

\[
F_{\text{vac}}(E) = \frac{p_\pi}{4\pi E} = \frac{1}{4\pi} \sqrt{\frac{1}{4} - \frac{m_\pi^2}{E^2}}
\]  

(10)

As we are interested in the positive energy region, we can concentrate our attention on the first term of Eq. 8 which is the only one that has a cut. We can rewrite that term, neglecting the \( \epsilon \), like

\[
\frac{F(E)}{P^0 - E} \rightarrow \frac{F(\text{Real}(P^0))}{P^0 - E} + \frac{F(E) - F(\text{Real}(P^0))}{P^0 - E}
\]  

(11)
The $E$ integration of the first term it is the only piece that depends on the Riemann sheet and can be readily evaluated. In the second sheet the integral takes the value

$$
\int_{0,(II)}^{\infty} dE \left( \frac{F(\text{Real}(P^0))}{P^0 - E} \right) = \int_{0,(II)}^{\infty} dE \left( \frac{F(\text{Real}(P^0))}{P^0 - E} \right) + 2\pi i F(\text{Real}(P^0)) \quad (12)
$$

Substituting this result in Eq. 8 we get the following expression for the two meson propagator in the second sheet

$$
G^{(II)}_{\pi\pi} = i F(\text{Real}(P^0)) + \int_{0}^{\infty} \frac{dE}{2\pi} \left( \frac{1}{P^0 - E} - \frac{1}{P^0 + E} \right) F(E) \quad (13)
$$

The rest of the pieces entering the BS equation are analytical single valued functions and therefore we can proceed to look for poles in the scattering amplitude.

4 Results

The function $F(E)$ includes the phenomenological information on the pion selfenergy in the nuclear medium. The results for several densities are shown in Fig. 7.

The $F(E)$ strength is related to the imaginary part of the $\sigma$ propagator and therefore reflects the energy and density dependence of the different $\sigma$ decay channels. We can start classifying them according to their density dependence (See. Fig.).

At low densities, the $\sigma$ meson can only decay into two pions, $\sigma \to \pi + \pi$. Therefore we have a threshold at $E = 2m_{\pi}$. As the density grows the $\sigma \to \pi + (Nh)$ decay channel becomes relevant and we have a new threshold at $E = m_{\pi}$, which is clearly visible in the curve corresponding to $\rho = \rho_0 / 8$. Finally, at larger densities, mechanisms such as $\sigma \to 2(Nh)$ which go like $\rho^2$ become important. They are possible even at very low energies.

Using the two meson propagator of Eq. 8 we can solve the BS equation, obtain the meson-meson scattering amplitude and look for the poles in the complex energy plane. The results for the $\sigma$ pole position are shown in Fig. 9 for densities up to 1.5 $\rho_0$. Note, however, that the calculation is more reliable at low densities because some contributions of order $\rho^2$ or higher are missing. We find that both mass and width decrease as the density increases, reaching a mass around 250 MeV and a similar width at 1.5 times the nuclear density. At large densities the $2ph$ pieces of the pion selfenergy become relevant decreasing further both mass and width. These results are little sensitive to the kaon channel. Its omission affects less than 1 percent
the mass and increases the width around 5 percent. The results of Fig. 9 can be cast in terms of an effective potential which can be approximated by

$$V_{\rho} = a \frac{\rho}{\rho_0} + b \left( \frac{\rho}{\rho_0} \right)^2$$

(14)

with $a = -358 - i 108$ MeV and $b = 140 + i 23.6$ MeV.

Qualitatively similar results for the mass are found in other models. See for instance Fig. 10 of ref. [45]. However, we should stress that there the changes respond to a reduction of the pion decay constant ($f$) value which we have kept constant. The basic ingredient that drives the mass decrease in our calculation is the p-wave interaction of the pion with the baryons in the medium.

Although the $\sigma$ mass drops significantly, the width stays relatively large, even when the mass is close or below the $2\pi$ threshold. This width comes mostly from medium decay channels, namely, the decay into a single pion and a $\rho\hbar$ excitation or $2\rho\hbar$ excitations. Therefore we should not expect any signal of narrow $\sigma$ mesons in the medium as expected in [7] or in models that include only purely mesonic decays. Nonetheless the low mass could modify
5 Conclusions

In this work, we have searched for the lightest pole of the $\pi\pi$ scattering amplitude, the $\sigma$ meson, in the complex energy plane as a function of the baryon density.

The growing consensus that the $\sigma$ is rather a $\pi\pi$ resonance of dynamical origin than a genuine QCD state built from $q\bar{q}$ pairs, finds its support in recent chiral unitary formulations of the $\pi\pi$ interaction which we have followed in the present work. This approach has been extended to include the interaction of the pions with a nuclear medium, which allows us to trace the density dependence of the $\sigma$ pole.

A quite general agreement has been reached about these medium modifications of the $\pi\pi$ interaction between different groups, either working in the present approach or using other $\pi\pi$ scattering formulations, as long as they satisfy some minimal chiral constraints. All these models are, however, involved numerically to the point that extrapolating analytically the results to the complex plane is a difficult task, particularly when some pieces of the $\pi$ selfenergy, like the $2p2h$ contributions, are taken from a comparison with pionic atoms data and not from a theoretical model.

In this work, the analytical extrapolation was made possible by the use of the Lehmann representation for the $\pi$ propagators. This allowed us to express the scattering matrix in terms of the pion propagators evaluated on the real energy axis.

The results obtained show a dropping of the $\sigma$ mass as a function of the density, down to values close to two pion masses at normal nuclear density. The width decreases moderately and reaches values around 300 MeV at $\rho = \rho_0$. Yet, this decrease is surprising in view of the fact that in the nuclear matter.
medium there are more channels for the $\sigma$ decay, like pion-$ph$ or $2ph$. The presence of these channels partly makes up for the strong reduction of the $2\pi$ channel due to the smaller phase space available when the $\sigma$ mass drops.

Altogether, the changes in the mass and width of the $\sigma$ are quite large, compared to other typical meson or baryon properties in a dense medium. Therefore, we expect drastic signals in the invariant mass distributions of two pions (or photons) in $2\pi$ ($2\gamma$) production experiments if those pions (photons) can couple strongly to the scalar isoscalar channel. Some present data seem to support these claims, although further experiments and calculations should be performed to test these findings.

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References

[1] D. E. Groom et al. [Particle Data Group Collaboration], Eur. Phys. J. C 15 (2000) 1.

[2] N. A. Tornqvist, arXiv:hep-ph/0008135.

[3] E. M. Aitala et al. [E791 Collaboration], Phys. Rev. Lett. 86 (2001) 770 arXiv:hep-ex/0007028.

[4] G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B 603 (2001) 125 arXiv:hep-ph/0103088.

[5] S. Ishida et al., “Possible existence of the sigma-meson and its implications to hadron physics. Proceedings, Workshop, Sigma-Meson 2000, Kyoto, Japan, June 12-14, 2000,” KEK-PROCEEDINGS-2001-04. http://amaterasu.kek.jp/YITPws/

[6] G. E. Brown and M. Rho, Phys. Rept. 269 (1996) 333 arXiv:hep-ph/9504250.

[7] T. Hatsuda, Nucl. Phys. A 698 (2002) 243 arXiv:hep-ph/0104139.

[8] V. Bernard, U. G. Meissner and I. Zahed, Phys. Rev. Lett. 59 (1987) 966.

[9] V. Bernard and U. G. Meissner, Phys. Rev. D 38 (1988) 1551.

[10] V. Bernard and U. G. Meissner, Nucl. Phys. A 489 (1988) 647.

[11] T. Hatsuda, T. Kunihiro and H. Shimizu, Phys. Rev. Lett. 82, 2840 (1999).

[12] D. Jido, T. Hatsuda and T. Kunihiro, Phys. Rev. D 63 (2001) 011901 arXiv:hep-ph/0008076.

[13] P. Schuck, W. Norenberg and G. Chanfray, Z. Phys. A 330 (1988) 119.

[14] R. Rapp, J. W. Durso and J. Wambach, Nucl. Phys. A 596 (1996) 436 arXiv:nucl-th/9508026.

[15] Z. Aouissat, R. Rapp, G. Chanfray, P. Schuck and J. Wambach, Nucl. Phys. A 581 (1995) 471 arXiv:nucl-th/9406010.

[16] H. C. Chiang, E. Oset and M. J. Vicente-Vacas, Nucl. Phys. A 644 (1998) 77 arXiv:nucl-th/9712047.
[17] Z. Aouissat, G. Chanfray, P. Schuck and J. Wambach, Phys. Rev. C 61 (2000) 012202.

[18] D. Davesne, Y. J. Zhang and G. Chanfray, Phys. Rev. C 62 (2000) 024604 [arXiv:nucl-th/9909032].

[19] F. Bonutti et al. [CHAOS Collaboration], Phys. Rev. Lett. 77 (1996) 603.

[20] F. Bonutti et al. [CHAOS Collaboration], Nucl. Phys. A 638 (1998) 729.

[21] P. Camerini, N. Grion, R. Rui and D. Vetterli, Nucl. Phys. A 552 (1993) 451 [Erratum-ibid. A 572 (1993) 791].

[22] F. Bonutti et al. [CHAOS Collaboration], Phys. Rev. C 60 (1999) 018201.

[23] A. Starostin et al. [Crystal Ball Collaboration], Phys. Rev. Lett. 85 (2000) 5539.

[24] V. Metag, talk given at the International Workshop on Chiral Fluctuations, Orsay, 2001.

[25] L. Roca, E. Oset and M. J. Vicente Vacas, [arXiv:nucl-th/0201054].

[26] A. Dobado, M. J. Herrero and T. N. Truong, Phys. Lett. B 235 (1990) 134.

[27] A. Dobado and J. R. Pelaez, Phys. Rev. D 47 (1993) 4883 [arXiv:hep-ph/9301276].

[28] J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. Lett. 80 (1998) 3452 [arXiv:hep-ph/9803242].

[29] J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. D 59 (1999) 074001 [Erratum-ibid. D 60 (1999) 099906] [arXiv:hep-ph/9804209].

[30] J. A. Oller and E. Oset, Phys. Rev. D 60 (1999) 074023 [arXiv:hep-ph/9809337].

[31] J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438 [Erratum-ibid. A 652 (1997) 407] [arXiv:hep-ph/9702314].

[32] J. Gasser and H. Leutwyler, Nucl. Phys. B 250 (1985) 517.
[33] E. Oset and M. J. Vicente Vacas, Nucl. Phys. A 678 (2000) 424 [arXiv:nucl-th/0004030].

[34] J. Nieves and E. Ruiz Arriola, Nucl. Phys. A 679 (2000) 57 [arXiv:hep-ph/9907469].

[35] J. Nieves and E. Ruiz Arriola, Phys. Lett. B 455 (1999) 30 [arXiv:nucl-th/9807035].

[36] E. Oset, P. Fernandez de Cordoba, L. L. Salcedo and R. Brockmann, Phys. Rept. 188 (1990) 79.

[37] G. Chanfray and D. Davesne, Nucl. Phys. A 646 (1999) 125.

[38] T. Kunihiro, [arXiv:hep-ph/9905262]

[39] A. Pich, Rept. Prog. Phys. 58 (1995) 563 [arXiv:hep-ph/9502360].

[40] U. G. Meissner, Rept. Prog. Phys. 56 (1993) 903 [arXiv:hep-ph/9302247].

[41] V. Bernard, N. Kaiser and U. G. Meissner, Int. J. Mod. Phys. E 4 (1995) 193 [arXiv:hep-ph/9501384].

[42] G. Ecker, Prog. Part. Nucl. Phys. 35 (1995) 1 [arXiv:hep-ph/9501357].

[43] V. E. Markushin, Z. Xiao and H. Q. Zheng, Nucl. Phys. A 695 (2001) 273 [arXiv:hep-ph/0011260].

[44] E. van Beveren and G. Rupp, [arXiv:hep-ph/0201006]

[45] T. Hatsuda and T. Kunihiro, [arXiv:nucl-th/0112027]