Fast Track Communications

Super Kähler oscillator from $SU(2|1)$ superspace

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Abstract
We construct a new version of the worldline $SU(2|1)$ superspace as a deformation of the standard $\mathbb{N} = 4, d = 1$ superspace, and show that it naturally provides off-shell and on-shell descriptions of a general supersymmetric Kähler oscillator model considered earlier, at the classical level, within the Hamiltonian approach. The basic object is a generalized chiral $SU(2|1), d = 1$ superfield with the off-shell field content $(2, 4, 2)$. The frequency of the oscillator and the strength of the external magnetic field are defined in terms of two parameters: the contraction parameter $m$ and the new parameter $\lambda$ which reflects the freedom in defining the chiral $SU(2|1), d = 1$ superspace. We treat both classical and quantum cases.

Keywords: supersymmetry, superfields, deformation

1. Introduction

Recently, we proposed a new type of supersymmetric quantum mechanics (SQM) based on the worldline realization of the supergroup $SU(2|1)$ in the appropriate $\mathbb{N} = 4, d = 1$ superspace [1]. The corresponding SQM models are deformations of the standard $\mathbb{N} = 4$ SQM models with the intrinsic mass parameter $m$, such that the $\mathbb{N} = 4$ models are reproduced in the limit $m = 0$. The $SU(2|1)$ supersymmetry acts on the worldline multiplets of the same off-shell dimension as in the standard $\mathbb{N} = 4, d = 1$ supersymmetry. In [1] we considered models associated with the off-shell $d = 1$ multiplets $(1, 4, 3)$ and $(2, 4, 2)$. In models of the first type the $SU(2|1)$ supersymmetry is recognized on the shell as the ‘weak supersymmetry’ of [4], while the models of the second type provide some interesting deformations of the standard $\mathbb{N} = 4, d = 1$ Kähler sigma models [5], such that they contain proper couplings to the...
external magnetic field and intrinsic potential terms related to the Kähler potential and vanishing in the \( m = 0 \) limit. Another peculiarity of these models is the very restrictive form of the extra superpotential terms. No such restrictions appear in the \((2, 4, 2)\) SQM models with the standard \( \mathcal{N} = 4, d = 1 \) supersymmetry.

On the other hand, there is a different class of mechanical systems with \( SU(2|1) \) as the underlying supersymmetry, known in the literature as supersymmetric Kähler oscillators \([6, 7]\). These models deal with the same number of physical bosonic and fermionic variables (multiples of \( 2 + 4 \)) as our \((2, 4, 2)\) \( SU(2|1) \) models, but differ essentially in the structure of the relevant supercharges and Hamiltonian. They involve two deformation parameters: the strength of a constant magnetic field \( B \) and the frequency of the oscillator \( \omega \). While the first parameter can be identified with the contraction parameter \( m \), it was unclear how to interpret the second parameter within our \( SU(2|1), d = 1 \) superspace approach.

In the present note we explain how to incorporate the models of \([6, 7]\) into the \( SU(2|1), d = 1 \) superspace formalism. To this end, one needs to define the \( SU(2|1), d = 1 \) superspace in a way different from that employed in \([1]\). The basic difference is that the Hamiltonian of the system should be identified with the whole intrinsic \( U(1) \) charge of the \( SU(2|1) \) superalgebra, but not with the central charge as in \([1]\).

The new version of the \( SU(2|1), d = 1 \) superspace still contains the chiral subspace having half as many Grassmann coordinates, but the definition of this subspace now reveals a freedom parameterized by a new parameter \( \lambda \). It is just this freedom that is responsible for the appearance of the correct Kähler potential term in the Hamiltonian with the frequency \( \omega \sim \sin 2\lambda \). The extra superpotentials also exhibit more freedom compared to those appearing in the ‘old’ \((2, 4, 2)\) \( SU(2|1) \) models: they are parameterized by an arbitrary holomorphic function of the complex bosonic variables \( z^i \).

The paper is organized as follows. In sections 2 and 3 we define the new version of the \( SU(2|1), d = 1 \) superspace and explain its relation to the superspace of \([1]\), as well as the origin of the \( \lambda \) freedom mentioned above. In section 4 we introduce the generalized chiral \( SU(2|1) \) superfield which involves the parameter \( \lambda \) appearing in the off-shell transformations of the component fields. The general off-shell and on-shell Lagrangians of these chiral superfields, including the extra superpotentials, are constructed in section 5, starting from the superfield approach. We also compute the relevant Hamiltonian, supercharges and other \( SU(2|1) \) generators, in both the classical and the quantum cases, and check that they do indeed form the algebra \( su(2|1) \). Concluding remarks are collected in section 6.

2. The new \( SU(2|1) \) superspace

2.1. The \( su(2|1) \) superalgebra and its \( u(1) \) extension

The standard form of the superalgebra \( su(2|1) \) is

\[
\begin{align*}
\{ Q^i, Q^j \} &= 2 m t^i_j + 2\delta^i_j \tilde{H}, \\
[ t^i_j, t^k_l ] &= \delta^i_k t^j_l - \delta^i_l t^j_k, \\
[ t^i_j, \tilde{Q}_i ] &= \frac{1}{2} \delta^i_j \tilde{Q}_i - \delta^i_i \tilde{Q}_j, \\
[ t^i_j, Q^k ] &= \delta^i_k Q^j - \frac{1}{2} \delta^i_j Q^k, \\
[ \tilde{H}, Q^i ] &= \frac{m}{2} \tilde{Q}_i, \\
[ \tilde{H}, \tilde{Q}_i ] &= -\frac{m}{2} Q^i,
\end{align*}
\]

(2.1)

all other (anti)commutators vanishing. The generators \( t^i_j \) generate \( SU(2) \) symmetry, while the mass-dimension generator \( \tilde{H} \) is a \( U(1) \) symmetry generator. The arbitrary mass parameter \( m \) is treated as a contraction parameter: sending \( m \to 0 \) leads to the \( \mathcal{N} = 4, d = 1 \) Poincaré
superalgebra. In the limit $m = 0$, $\tilde{H}$ becomes the canonical Hamiltonian and the generators $I_i^j$ become the outer $SU(2)$ automorphism generators.

To see the difference between the $SU(2|1)$, $d = 1$ superspace introduced in [1] and its modification that we are going to deal with here, it is instructive to extend (2.1) with the external $U(1)$ automorphism symmetry generator $F$ which has non-zero commutation relations only with the supercharges [2]:

$$ \left[ F, \bar{Q}_i \right] = -\frac{1}{2} \bar{Q}_i, \quad \left[ F, Q^i \right] = \frac{1}{2} Q^i. \quad (2.2) $$

The redefinition $\tilde{H} \equiv H - mF$ brings the extended superalgebra $su(2|1) \oplus u(1)_{ext}$ to the form in which it looks like a centrally extended superalgebra $\hat{su}(2|1)$:

$$ \{ Q^i, \bar{Q}_i \} = 2m \left( I_i^{\prime} - \delta_i^j F \right) + 2\delta_i^j H, \quad \left[ I_i^{\prime}, H \right] = \delta_i^j I_j^{\prime} - \delta_i^j I_i^{\prime}, $$

$$ \left[ I_i^{\prime}, \bar{Q}_i \right] = \frac{1}{2} \delta_i^j \bar{Q}_j - \delta_i^j \bar{Q}_i, \quad \left[ I_i^{\prime}, Q^j \right] = \delta_i^j Q^j - \frac{1}{2} \delta_i^j Q^j, $$

$$ \left[ F, \bar{Q}_i \right] = \frac{1}{2} \bar{Q}_i, \quad \left[ F, Q^j \right] = \frac{1}{2} Q^j. \quad (2.3) $$

In the new basis $(F, H)$ the generator $F$ becomes the internal $U(1)$ generator, while $H$ commutes with all generators and so can be treated as a central charge.

2.2. Two $d = 1$ supercosets of $SU(2|1)$

The $SU(2|1)$, $d = 1$ superspace introduced in [1] can be identified with the following coset of the extended superalgebra $su(2|1) \oplus u(1)_{ext}$ in the basis $H, F$:

$$ \frac{SU(2|1) \times U(1)_{ext}}{SU(2) \times U(1)_{ext}} \sim \left\{ Q^i, \bar{Q}_i, H, F, I_i^{\prime} \right\}. \quad (2.4) $$

Its main property is that the time coordinate $t$ is associated as the coset parameter with the central charge generator $H$. This generator commutes with all other ones and, in the corresponding SQM models, can naturally be identified with the Hamiltonian.

In the present paper we will deal with another $SU(2|1)$ coset:

$$ \frac{SU(2|1) \times U(1)_{ext}}{SU(2) \times U(1)_{ext}} = \frac{SU(2|1)}{SU(2)} \sim \left\{ Q^i, \bar{Q}_i, \tilde{H}, I_i^{\prime} \right\}. \quad (2.5) $$

The role of the Hamiltonian is now played by the full internal $U(1)$ generator $\tilde{H}$. Though it does not commute with the supercharges, the Noether expressions for the latter are still conserved due to the presence of an explicit dependence on $t$ in them. This situation is reminiscent of what happens, e.g., in the conformal and superconformal mechanics [8].

2.3. The second $SU(2)$ group

For what follows, it will be useful to have a different notation for the superalgebra (2.3). Introducing supercharges with the doublet $SU(2)_{ext} \times SU(2)$ indices as$^2$

$^2$ The doublet indices are raised and lowered in the standard way by the $\epsilon$ symbols, e.g., $Q^i = \epsilon^i_\alpha Q^\alpha$, $\bar{Q}^{\alpha} = \epsilon^{\alpha}_i \bar{Q}_i$, $\epsilon_{12} = -\epsilon^{12} = 1$. 

$$ \varphi = \epsilon^i_\alpha \varphi^\alpha, \quad \epsilon_{12} = -\epsilon^{12} = 1. $$
the relations (2.3) can be rewritten in the ‘quasi’-\(SU(2)_{\text{ext}}\times SU(2)\) covariant form as

\[
(Q_m, Q_n) = 2imc_{ab} I_{ij} + 2e_{ij} c_{ab} \tilde{H},
\]

\[
[I_{ij}, Q_{ab}] = \frac{1}{2}(e_{ij} Q_{ab} + e_{ab} Q_{ij}),
\]

\[
[H, Q_{ab}] = \frac{i m}{2} c^b_{ab} Q_{ab}.
\]

Here \(c_{ab}\) is a constant real triplet of the \(SU(2)_{\text{ext}}\) symmetry which breaks the latter down to \(U(1)_{\text{ext}} \subset SU(2)_{\text{ext}}\):

\[
c_{ab} = c_{a0}, \quad (c_{ab}) = \epsilon^{ac} \epsilon^{bd} c_{cd}, \quad c^{ab} c_{ab} = 2.
\]

Choosing, e.g., the \(SU(2)_{\text{ext}}\) frame in such a way that \(c_{12} = c_{21} = -i, \quad c_{11} = c_{22} = 0\), we reproduce (2.1), with the external automorphism generator \(F\) as the only remnant of this second \(SU(2)\). Another frame corresponds to the choice \(c_{12} = 0, \quad c_{11} = c_{22} = 1\), which yields the equivalent form of the \(SU(2|1)\) superalgebra

\[
\{Q_i^+, Q_j^\}$\} = \{Q_i^+, Q_j^\}$\} = 2imI_{ij}, \quad \{Q_i^-, Q_j^\}$\} = 2e_{ij} \tilde{H}.
\]

In this frame, the residual \(U(1)_{\text{ext}}\) automorphism acts as \(O(2)\) rotations of the 2-vector \((Q_i^+, Q_j^\})\) (the same holds as regards the action of the internal \(U(1)\) charge generator \(\tilde{H}\)).

Thus the full \(SU(2)_{\text{ext}}\) symmetry is not the automorphism group of the \(su(2|1)\) superalgebra, only some \(U(1)_{\text{ext}} \sim O(2)_{\text{ext}}\) subgroup of it is. On the other hand, in the limit \(m \to 0\), this group becomes one of the \(SU(2)\) factors of the full automorphism group \(SU(2)\times SU(2)\) of the \(N = 4, d = 1\) Poincaré supersymmetry. The generalized \(SU(2|1)\) chirality that we will consider in section 3 is directly related to the existence of this \(SU(2)_{\text{ext}}\) freedom. The new parameter \(\lambda\) associated with this chirality can be interpreted as a parameter of some \(U(1)_{\text{ext}} \subset SU(2)_{\text{ext}}\). Just because \(SU(2)_{\text{ext}}\) is not an automorphism of \(SU(2|1)\), \(\lambda\) is a physical parameter at \(m \neq 0\). It becomes removable and hence unphysical only in the limit \(m = 0\), when \(SU(2)_{\text{ext}}\) turns into the automorphism symmetry of the underlying supersymmetry.

3. Superspace technicalities

3.1. Transformations

We parameterize the supercoset (2.5) with the coordinates \((t, \theta, \bar{\theta})\). The time coordinate is associated with the generator \(\tilde{H}\), which plays, in what follows, the role of the Hamiltonian. An element of the supercoset is defined as

\[
g = \exp \left( i\bar{\theta} Q^\prime - i\theta \bar{Q}^\prime \right) \exp \left( i\tilde{H} \right).
\]

where

\[
\bar{\theta} = \left[ 1 - \frac{2m}{3} (\theta \cdot \theta) \right] \theta, \quad (\bar{\theta}) = \bar{\theta}.
\]

One can check that, with the order of factors as in (3.1), the superspace coordinates \((t, \theta, \bar{\theta})\) transform in the same manner as those in [1]:

\[3\] We use the following conventions: \(\tilde{x} \cdot \zeta = \tilde{x}^\dagger \zeta, \quad \tilde{x} \cdot \zeta = \tilde{x}^\dagger \zeta, \quad \zeta = x^\dagger \zeta, \quad (\zeta)^\dagger = x \cdot \zeta, \quad (\tilde{x}^\dagger \zeta)^\dagger = \tilde{x} \cdot \zeta.\]
Then the invariant measure of integration over the new $SU(2|1)$ superspace is defined by the same formula as in [1]:

$$d\mu := d\theta^i d\bar{\theta}^i \left[ 1 + 2m(\bar{\theta} \cdot \theta) \right].$$

(3.4)

Though the transformations (3.3) coincide with those for the coordinates of the superspace (2.4), the transformation properties of the relevant superfields (and covariant derivatives) are essentially different because the superfields defined on (2.5) cannot carry any internal $U(1)$ charge (the latter is absent in the stability subgroup). Besides, the time coordinate in (2.4) undergoes independent translations under the central charge generator $H$, while the $\tilde{H}$ translations of $t$ in (3.1) necessarily affect the Grassmann coordinates as well.

### 3.2. Covariant derivatives

Applying the same general method of Cartan 1-forms as in [1], it is easy to find the corresponding covariant derivatives:

$$\mathcal{D} = e^{-i\omega_2} \left[ \left( 1 + m(\bar{\theta} \cdot \theta) - \frac{3m^2}{4}(\bar{\theta} \cdot \theta)^2 \right) \frac{\partial}{\partial \theta_j} - m \bar{\theta} \theta_j \frac{\partial}{\partial \bar{\theta}} - i\bar{\theta} \frac{\partial}{\partial t} \right]$$

$$- \left( m \bar{\theta}^i \bar{\theta}_j - \frac{m^2}{2}(\bar{\theta}^i \bar{\theta}_j)^2 \theta^i \theta_j \right),$$

$$\tilde{\mathcal{D}} = e^{i\omega_2} \left[ \left( 1 + m(\bar{\theta} \cdot \theta) - \frac{3m^2}{4}(\bar{\theta} \cdot \theta)^2 \right) \frac{\partial}{\partial \bar{\theta}_j} + m \bar{\theta} \theta_j \frac{\partial}{\partial \theta} + i\bar{\theta} \frac{\partial}{\partial t} \right]$$

$$+ \left( m \theta^i \bar{\theta}_j - \frac{m^2}{2}(\theta^i \bar{\theta}_j)^2 \bar{\theta}_j \bar{\theta}_j \right),$$

$$\mathcal{D}_{\psi} = \partial_i.$$  

(3.5)

The objects within the square brackets coincide with the covariant derivatives on the superspace (2.4), modulo the absence of the matrix parts with the internal $U(1)$ charge generator which is now out of the stability subgroup. Non-trivial $U(1)_{int}$ transformations of the ‘old’ covariant derivatives are now compensated by the transformations of the time-dependent factors in (3.5), so the new covariant derivatives are $U(1)_{int}$-inert. In contrast to the case for [1], the covariant derivatives (3.5) undergo only induced $SU(2)$ transformations (forming the stability subgroup), while the subgroup $U(1)_{int}$ is realized solely as transformations of the supercoset coordinates. The superalgebra of the covariant derivatives mimics the superalgebra $su(2|1)$:

$$\{\mathcal{D}^i, \tilde{\mathcal{D}}_j\} = 2m \delta^i_j + 2i\delta^i_j \mathcal{D}_{\psi},$$

$$\left[ \mathcal{D}^i, \tilde{F}_j \right] = \delta^i_j \tilde{F}_j - \frac{1}{2} \delta^i_j \mathcal{D}_j,$$

$$\left[ \tilde{D}_j, \mathcal{D}^i \right] = \frac{1}{2} \delta^i_j \mathcal{D}^j - \delta^i_j \tilde{F},$$

$$\left[ \mathcal{D}_{\psi}, \tilde{D}_j \right] = \frac{i}{2} \mathcal{D}_j, \quad \left[ \mathcal{D}_{\psi}, \tilde{D}^j \right] = -\frac{i}{2} \tilde{D}^j.$$  

(3.6)

Note that one can ascribe to $\mathcal{D}^i$ and $\tilde{\mathcal{D}}_j$ (as well as to $\theta_i$ and $\bar{\theta}_j$) the opposite charges with respect to the $U(1)_{int}$ generator $F$ which can be formally kept in the stability subgroup in (2.5).
However, in contrast to the internal generator $F$ in (2.4), the automorphism generator $F$ in (2.5) never appears in the r.h.s. of the anticommutators of the supercharges. In the first case, the internal $U(1)$ invariance is the necessary consequence of supersymmetry and therefore should be respected by any corresponding SQM model. In the second case, the automorphism $U(1)$ symmetry is not automatically implied by supersymmetry. So this $U(1)$ invariance is merely an additional possible restriction on the SQM models: one may impose it, or not.

4. Generalized chiral $SU(2;1)$ superfields

The standard form of the chiral and antichiral conditions is

\[(a) \ \hat{D} \Phi = 0, \quad (b) \ \hat{D} \tilde{\Phi} = 0. \tag{4.1}\]

In the framework of the superspace approach of [1], this was the only option for describing the multiplet $(2,4,2)$. It is uniquely specified by the covariance with respect to the stability subgroup $U(2) = SU(2) \times U(1)_{\text{int}}$. In the case that we consider here, the stability subgroup is actually $SU(2)$ (modulo the unessential automorphism $U(1)_{\text{ext}}$ group). Capitalizing on that, we can generalize the chiral condition (4.1) as

\[(a) \ \hat{D} \Phi = 0, \quad (b) \ \hat{D} \tilde{\Phi} = 0, \tag{4.2}\]

where

\[\hat{D} = \cos \lambda \ D - \sin \lambda \ \hat{D}, \quad \hat{D}' = \cos \lambda \ D' + \sin \lambda \ \hat{D}'. \tag{4.3}\]

Clearly, in the approach based on the superspace (2.4), the constraints (4.2) are not covariant under $U(1)_{\text{int}}$ which multiplies $D$ and $\hat{D}$ by the mutually conjugated phase factors. In the case under consideration, $D$ and $\hat{D}$ undergo no supersymmetry-induced $U(1)$ phase transformations, and so the conditions (4.2) are $SU(2\|1)$ covariant at any $\lambda$. The linear combinations (4.3) can be interpreted as a result of the rotation of the $SU(2)_{\text{ext}}$ doublet $D_\alpha = (D, \hat{D})$ by some one-parameter subgroup of $SU(2)_{\text{ext}}$ acting on the doublet index $a$ (this $SU(2)$ group is the same as in section 2.3). Since this subgroup is not an automorphism of (2.3), the $\lambda$ dependence cannot be removed from (4.2), (4.3) by any redefinition of the Grassmann coordinates $\theta, \hat{\theta}$. This is possible only in the limit $m = 0$, when $SU(2)_{\text{ext}}$ becomes the automorphism group of the $N = 4, \ d = 1$ superalgebra.

The conditions (4.2) amount to the existence of the left and right chiral subspaces:

\[(t_L, \hat{\theta}), \quad (t_R, \hat{\theta}) \tag{4.4}\]

where

\[t_L = t + \cos 2\lambda(\hat{\theta} \cdot \theta) + \frac{i}{2} \sin 2\lambda \left[ (\hat{\theta})^2 e^{-im} + (\theta)^2 e^{im} \right] \quad \text{and} \quad \hat{t}_R = \left[ \cos \lambda \ \theta e^{\frac{i}{m}} + \sin \lambda \ \hat{\theta} e^{-\frac{i}{m}} \right] \left[ 1 - \frac{m}{2} (\hat{\theta} \cdot \theta) \right]. \tag{4.5}\]

The right subspace coordinates are obtained via complex conjugation. Indeed, in the basis $(t_L, \hat{\theta}, \hat{\theta}^\dagger)$ the constraint (4.2a) is reduced to the form $\partial_{\theta} \Phi = 0 \Rightarrow \Phi = \varphi(t_L, \hat{\theta})$. As it should be, the coordinate set (4.5) is closed under the $SU(2\|1)$ transformations.
\[
\delta \theta = \cos \lambda \left[ e^{\frac{1}{2} \theta m} + m \left( \bar{e} \cdot \theta \right) \bar{\theta} e^{-\frac{1}{2} \theta m} \right] + \sin \lambda \left[ e^{\frac{1}{2} \theta m} - m \left( e \cdot \bar{\theta} \right) \bar{\theta} e^{-\frac{1}{2} \theta m} \right],
\]
\[
\delta \lambda = 2 \cos \lambda \left( e \cdot \bar{\theta} \right) e^{-\frac{1}{2} \theta m} + \sin \lambda \left( e \cdot \bar{\theta} \right) e^{\frac{1}{2} \theta m}.
\]
Equations (4.5), (4.6) suggest that in the SU (2(1)) case there exists a family of non-equivalent chiral subspaces parameterized by \( \lambda \). It is straightforward to show that in the \( m = 0 \) case the \( \lambda \) dependence can be entirely absorbed into the proper redefinition of the Grassmann coordinates and the supersymmetry transformation parameters. This is not possible at \( m \neq 0 \), i.e. in the SU (2(1)) case.

The rotated covariant derivatives (4.3) satisfy the relations
\[
\left\{ \hat{D}_I, \hat{D}_J \right\} = -2 m \sin 2 \lambda \hat{I}_{ij} \quad \text{and c.c.},
\] which means that the chiral superfields \( \varphi \) cannot carry any external SU(2) index, i.e. they must be SU(2) singlets. The left chiral superfield \( \varphi (t_I, \bar{\theta}) \) as a solution of (4.2) is given by the standard expansion
\[
\varphi (t_I, \bar{\theta}) = z + \sqrt{2} \bar{\theta} \xi^i + \left( \bar{\theta} \right)^2 B, \quad \left( \xi^i \right) = \xi_i.
\]
The odd transformations (4.6) induce the following off-shell transformations of the component fields in (4.8):
\[
\delta z = - \sqrt{2} \cos \lambda \left( e \cdot \xi \right) e^{\frac{1}{2} \theta m} + \sqrt{2} \sin \lambda \left( \bar{e} \cdot \xi \right) e^{-\frac{1}{2} \theta m},
\]
\[
\delta \xi^i = \sqrt{2} e^i \left[ \cos \lambda \left( \bar{z} - \sin \lambda \bar{B} \right) e^{-\frac{1}{2} \theta m} - \sqrt{2} e^i \left[ \sin \lambda \bar{z} + \cos \lambda \bar{B} \right] e^{\frac{1}{2} \theta m} \right],
\]
\[
\delta B = \sqrt{2} \cos \lambda \left[ i \left( \bar{e} \cdot \xi \right) + \frac{m}{2} \left( \bar{e} \cdot \xi \right) \right] e^{-\frac{1}{2} \theta m} + \sqrt{2} \sin \lambda \left[ i \left( e \cdot \xi \right) - \frac{m}{2} \left( e \cdot \xi \right) \right] e^{\frac{1}{2} \theta m}.
\]
In the special case with \( \sin \lambda = 0 \), the \( \theta \) expansion of \( \varphi \) takes the form
\[
\varphi (t_I, \bar{\theta}) = z + \sqrt{2} \bar{\theta} \xi^i + \left( \bar{\theta} \right)^2 B = z + \sqrt{2} e^{\frac{1}{2} \theta m} \theta^i \xi_i + e^{\theta m} \left( \theta \right)^2 B.
\]
After making the field redefinitions
\[
\xi_i \rightarrow \xi_i, \quad e^{\frac{1}{2} \theta m} B \rightarrow B,
\]
this particular \( \varphi (t_I, \bar{\theta}) \) is recognized as the chiral superfield of [1], with zero \( U(1)_{int} \) charge. The analogous equivalence holds for \( \cos \lambda = 0 \). No such redefinition is possible for general \( \lambda \), so the time dependence in the transformations (4.9) is unremovable in this case.

Finally, note that the one-parameter set of the chirality conditions (4.2), (4.3) is in fact the most general linear set. One could start in (4.2a) with the general linear combination of \( \hat{D}_I \) and \( \hat{D}_J \) involving two complex coefficients. Then, using the freedom of multiplying such a constraint by an arbitrary non-zero factor and carrying out the proper phase rotation of the Grassmann coordinates, one can reduce it just to the form (4.2a).

5. The supersymmetric Kähler oscillator

5.1. The general kinetic Lagrangian

The most general sigma-model part of the SU (2(1)) invariant action of the generalized chiral superfields \( \varphi^a (t_I, \bar{\theta}), a = 1, \ldots N, \) is specified by an arbitrary Kähler potential \( f (\varphi^a, \bar{\varphi}^a) \).
\[ S_{\text{kin}} = \int dt \mathcal{L}_{\text{kin}} = \frac{1}{4} \int dt f(p^\a, \dot{q}^\a). \] (5.1)

In the generic case we will consider only the corresponding bosonic Lagrangian, since working out the fermionic terms is straightforward. This Lagrangian reads

\[ \mathcal{L}_{\text{kin}} = g_{ab} \bar{z}^a \dot{z}^b - \frac{m}{2} \cos 2\lambda \left( \bar{z}^a f_a - \bar{z}^a f_a \right) - \frac{m}{2} \sin 2\lambda \left( \bar{B}^a f_a + s^a B^a \right) + g_{ab} \bar{B}^a B^b, \] (5.2)

where \( g_{ab} = \partial \bar{z} \partial f (z, \bar{z}), \) \( f_a = \partial \bar{f} (z, \bar{z}). \) After elimination of the auxiliary fields \( \bar{B}^a \) and \( B^b, \) the bosonic Lagrangian becomes

\[ \mathcal{L}^\text{on} = g_{ab} \bar{z}^a \dot{z}^b - \frac{m}{2} \cos 2\lambda \left( \bar{z}^a f_a - \bar{z}^a f_a \right) - \frac{m^2}{4} g_{ab} \sin^2 2\lambda f_a f_b, \quad g_{\bar{a}b} := (g_{ab})^{-1}. \] (5.3)

It is recognized as the Lagrangian of the Kähler oscillator (the sum of the first and the third terms) extended by a coupling to an external magnetic field (the WZ term). All three terms arise from the single superfield term (5.1). Both the strength of the magnetic field and the oscillator frequency prove to be expressed through the same parameter \( \lambda. \) One more interesting feature of the Lagrangian (5.3) is that either the WZ term or the Kähler oscillator potential \( \sim g_{ab} f_a f_b \) can be eliminated through the proper choice of \( \lambda (\cos 2\lambda = 0 \text{ or } \sin 2\lambda = 0), \) for an arbitrary \( f(z, \bar{z}). \) In the \( (2, 4, 2) \) Lagrangian of [1] the WZ term vanishes only for some very special choices of the Kähler potential and non-zero intrinsic \( U(1) \) charge of the chiral superfields.

Performing a Legendre transformation, we find the bosonic Hamiltonian

\[ \mathcal{H}_{\text{bos}} = g_{ab} \left[ \left( p_b - \frac{m}{2} \cos 2\lambda f_b \right) \left( p_a + \frac{m}{2} \cos 2\lambda f_a \right) + \frac{m^2}{4} \sin^2 2\lambda f_a f_b \right]. \] (5.4)

It is just the Hamiltonian of [7], with the relevant magnetic field \( B = m \cos 2\lambda \) and the frequency \( \omega = (m/2) \sin 2\lambda. \) In the limit \( m = 0 \) all the \( \lambda \)-dependent terms in (5.3) and (5.4) drop out, in accord with the discussion in section 4.

For simplicity, from now on we will limit our attention to one complex chiral superfield. The corresponding full Lagrangian reads

\[ \mathcal{L}_{\text{kin}} = g_{\bar{a}b} \left( \bar{z}^{\bar{a}} \dot{z}^b + \frac{m}{2} \cos 2\lambda \left( \bar{z}^a f_a - \bar{z}^a f_a \right) + \frac{i}{2} \left( \bar{z} \cdot \xi \right) \left[ \bar{z} g_{\bar{a}} - \bar{z} g_{\bar{a}} \right] - \frac{1}{2} \left( \xi \right)^2 B_{\bar{a}} - \frac{1}{2} \left( \bar{\xi} \right)^2 B_{\bar{a}} \right. \\
+ \left. g_{\bar{B}B} + \frac{1}{4} \left( \xi \right)^2 \left( \bar{\xi} \right)^2 g_{\bar{a}b} - \frac{m}{2} \cos 2\lambda \left( \bar{z} f_a - \bar{z} f_a \right) - \frac{m}{2} \sin 2\lambda \left( \bar{B} f_a + f_a B \right) \right] \\
+ \frac{m}{4} \sin 2\lambda \left( \left( \xi \right)^2 f_{\bar{a}} + \left( \bar{\xi} \right)^2 f_{\bar{a}} \right) - \frac{1}{2} mg \sin 2\lambda \left( \bar{\xi} \cdot \xi \right). \] (5.5)

Eliminating the auxiliary fields, we obtain the on-shell Lagrangian

\[ \mathcal{L}_{\text{kin}}^\text{on} = g_{\bar{a}b} \left( \bar{z}^{\bar{a}} \dot{z}^b + \frac{m}{2} \cos 2\lambda \left( \bar{z}^a f_a - \bar{z}^a f_a \right) + \frac{i}{2} \left( \bar{z} \cdot \xi \right) \left[ \bar{z} g_{\bar{a}} - \bar{z} g_{\bar{a}} \right] + \frac{1}{4} \left( \xi \right)^2 \left( \bar{\xi} \right)^2 R \right. \\
- \left. \frac{1}{2} m \cos 2\lambda \left( \bar{z} f_a - \bar{z} f_a \right) - \frac{m^2}{4} g_{\bar{a}b} \sin^2 2\lambda f_a f_b - \frac{1}{2} mg \sin 2\lambda \left( \bar{\xi} \cdot \xi \right) \right] \\
+ \frac{m}{4} \sin 2\lambda \left( \left( \xi \right)^2 f_{\bar{a}} + \left( \bar{\xi} \right)^2 f_{\bar{a}} \right) + \frac{m}{4} \sin 2\lambda \left( \bar{\xi} \cdot \xi \right) \left( f_{\bar{a}} - g f_{\bar{a}} \right). \] (5.6)
where

\[ R = g_{i\bar{j}} - \frac{g_i g_{\bar{j}}}{g}, \quad (5.7) \]

and the corresponding on-shell supersymmetry transformations

\[
\delta \xi = -\sqrt{2} \cos \lambda \left( \epsilon \cdot \xi \right) e^{\frac{i}{2} \omega} + \sqrt{2} \sin \lambda \left( \bar{\epsilon} \cdot \bar{\xi} \right) e^{-\frac{i}{2} \omega}, \\
\delta \xi^i = \sqrt{2} \bar{e}^i \left[ I_{\cos \lambda \bar{\zeta}} - \frac{1}{2} g^{-1} \sin \lambda \left( m \sin 2\lambda f_{\bar{\xi}} + (\bar{\xi})^2 g_{\bar{i}} \right) \right] e^{\frac{i}{2} \omega} \\
- \sqrt{2} e^i \left[ I_{\sin \lambda \bar{\zeta}} + \frac{1}{2} g^{-1} \cos \lambda \left( m \sin 2\lambda f_{\xi} + (\bar{\xi})^2 g_i \right) \right] e^{\frac{i}{2} \omega}. \quad (5.8)
\]

5.2. The superpotential

The chiral subspace integration measure \( d\mu_L \) is invariant under the transformations (4.6):

\[ d\mu_L = d\ell \, d\hat{\theta}, \quad \delta d\mu_L = \left( \partial_{\epsilon} \delta L - \partial_{\bar{\epsilon}} \delta \bar{L} \right) d\mu_L = 0. \quad (5.9) \]

Due to this property, it is possible to define the general external superpotential

\[ S_{\text{pot}} = \int dt \, \mathcal{L}_{\text{pot}} = \frac{\bar{m}}{4} \left[ \int d\mu_L \, U(\varphi) + \int d\bar{\mu}_{\bar{L}} \, \bar{U}(\bar{\varphi}) \right]. \quad (5.10) \]

where \( U(\varphi) \) is an arbitrary holomorphic function. In components, (5.10) yields

\[ \mathcal{L}_{\text{pot}} = \frac{\bar{m}}{4} \left[ 2\bar{B}\bar{\bar{U}}_{\bar{\xi}}(\bar{\xi}) + 2BU_{\xi}(z) - (\xi)^2 U_{\xi}(z) - (\bar{\xi})^2 \bar{U}_{\bar{\xi}}(\bar{\xi}) \right]. \quad (5.11) \]

In the limit \( m = 0 \), this expression goes over to the potential term of the standard \( \mathcal{N} = 4, d = 1 \) multiplet (2, 4, 2). The total off-shell Lagrangian \( \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} \) thus includes the following bosonic potential term:

\[ -\frac{1}{2} B \left( m \sin 2\lambda f_{\bar{\xi}} - \bar{m} U_{\bar{\xi}} \right) - \frac{1}{2} B \left( m \sin 2\lambda f_\xi - m U_{\xi} \right) + g\bar{B}B. \quad (5.12) \]

The intrinsic bosonic potential in the on-shell Lagrangian (5.6) is then modified as

\[ -\frac{1}{4} g^{-1} m^2 \sin^2 2\lambda f_{\bar{\xi}} \rightarrow -\frac{1}{4} g^{-1} \left( m \sin 2\lambda f_{\bar{\xi}} - \bar{m} \bar{U}_{\bar{\xi}} \right) \left( m \sin 2\lambda f_\xi - m U_{\xi} \right). \quad (5.13) \]

A similar modification comes about in terms \( (\xi)^2 \) and \( (\bar{\xi})^2 \). For simplicity, in what follows we will not consider the superpotential contributions.

In the approach based on the superspace (2.4) the chiral integration measure is multiplied by some induced \( U(1)_{\text{int}} \) phase factor under the \( SU(2|1) \) supersymmetry. This non-invariance of the measure imposes severe restrictions on the form of the admissible superpotentials [1]. No such restrictions arise in the approach that we deal with here.
5.3. The Hamiltonian formalism

The canonical Hamiltonian corresponding to the Lagrangian (5.6) reads

\[
\hat{H} = g^{-1} \left( p_\xi - \frac{i}{2} m \cos 2\lambda f_\xi + \frac{i}{2} g \xi \xi \right) \left( p_\tilde{\xi} + \frac{i}{2} \tilde{m} \cos 2\lambda f_{\tilde{\xi}} - \frac{i}{2} \tilde{g} \xi \tilde{\xi} \right) \\
- \frac{1}{4} \left( \xi \right)^2 \left( \tilde{\xi} \right)^2 R + \frac{m^2}{4} \sin^2 2\lambda f_\xi f_{\tilde{\xi}} + \frac{m}{2} \cos 2\lambda \xi \xi \\
- \frac{m}{4} \sin 2\lambda \left( \xi \right)^2 \left( f_\xi - \frac{f_{\tilde{\xi}}}{g} \right) - \frac{m}{4} \sin 2\lambda \left( \tilde{\xi} \right)^2 \left( f_{\tilde{\xi}} - \frac{g f_\xi}{g} \right).
\]

(5.14)

Taking into account that we deal with one complex chiral superfield, it can be checked that this Hamiltonian coincides with \( \mathcal{H}_{\text{SUSY}}^{(N = 1)} \) of [7]. The notation used there is related to ours as follows:

\[
m \sin 2\lambda = 2\omega, \quad m \cos 2\lambda = B, \quad m = 2\Lambda.
\]

(5.15)

In the particular case \( \omega = 0 \) (\( \lambda = 0, \pi \)), the Hamiltonian \( \hat{H} \) reproduces the expression for the difference of generators \( H - mF \) defined in [1].

The remaining conserved Noether charges are found to be

\[
Q' = \sqrt{2} e^{im} \left[ \cos \lambda \xi \left( p_\xi - \frac{i}{2} mf_\xi + \frac{i}{2} g \xi \xi \right) \\
- \sin \lambda \tilde{\xi} \left( p_{\tilde{\xi}} - \frac{i}{2} \tilde{m} f_{\tilde{\xi}} - \frac{i}{2} \tilde{g} \xi \tilde{\xi} \right) \right],
\]

\[
Q = \sqrt{2} e^{im} \left[ \cos \lambda \tilde{\xi} \left( p_{\tilde{\xi}} + \frac{i}{2} \tilde{m} f_\xi - \frac{i}{2} \tilde{g} \xi \xi \right) \\
+ \sin \lambda \xi \left( p_\xi + \frac{i}{2} mf_{\tilde{\xi}} + \frac{i}{2} g \xi \tilde{\xi} \right) \right],
\]

\[
I' = g \left( \xi \xi_{\tilde{\xi}} - \frac{1}{2} \xi \xi \xi \xi \right).
\]

(5.16)

The Poisson (Dirac) brackets are imposed as

\[
\{ z, p_\xi \} = 1, \quad \{ \xi, \tilde{\xi} \} = -i \delta^i_j g^{-1}, \quad \{ p_j, \tilde{\xi}^i \} = \frac{1}{2} g_{ij} \xi \xi_{\tilde{\xi}}.
\]

(5.17)

We see that the supercharges explicitly depend on time, and this dependence is such that they satisfy the generalized conservation laws:

\[
\frac{d}{dt} Q' = \partial_\xi Q' + \{ Q', \hat{H} \} = 0, \quad \frac{d}{dt} \tilde{Q} = \partial_{\tilde{\xi}} \tilde{Q} + \{ \tilde{Q}, \hat{H} \} = 0.
\]

(5.18)

This can be easily checked, using the Poisson brackets

\[
\{ \hat{H}, Q' \} = \frac{im}{2} Q', \quad \{ \hat{H}, \tilde{Q} \} = -\frac{im}{2} \tilde{Q}.
\]

(5.19)

Note that for the special cases \( \sin \lambda = 0 \) or \( \cos \lambda = 0 \) the time dependence of the supercharges in (5.16) can be absorbed into a redefinition of the fermionic variables \( \xi^i \) and \( \xi_{\tilde{\xi}}^i \); after that the supercharges start to satisfy the standard form of the conservation laws. This is in line with the remark after (4.9): for these values of \( \lambda \) one recovers the chiral \( (2, 4, 2) \) multiplet defined on the ‘old’ \( SU(2|1) \) superspace (2.4).
To prepare the system for quantization, it is useful to make the substitution
\[ (z, \xi) \rightarrow (z, \eta), \quad \eta := g^\frac{1}{2} \xi. \] (5.20)

In the new variables, the brackets become
\[ \{z, p_i\} = 1, \quad \{\eta^i, \bar{\eta}_j\} = -i \delta^i_j, \quad \{p_i, \eta^i\} = \{p_i, \bar{\eta}_j\} = 0. \] (5.21)

The Noether charges (5.16) and the Hamiltonian (5.14) are then rewritten as
\[ Q^\prime = \sqrt{2} g^{-\frac{1}{2}} \cos \lambda \eta^i \pi (m) - \sin \lambda \eta^i \pi (m) \]
\[ \hat{Q}_i = \sqrt{2} g^{-\frac{1}{2}} \cos \lambda \bar{\eta}_i \pi (m) + \sin \lambda \eta^i \pi (m) \]
\[ l_i^\prime = \eta^i \bar{\eta}_i - \frac{1}{2} \delta^i_j \eta^j \bar{\eta}_i, \]
\[ \hat{H} = g^{-\frac{1}{2}} \left[ \bar{\pi} (m \cos 2\lambda) \pi (m \cos 2\lambda) - \frac{1}{4} g^{-1} (\eta^i \bar{\eta}_i)^2 R + \frac{m^2}{4} \sin^2 2\lambda \bar{f}_i f_i \right. \]
\[ - \frac{m}{4} \sin 2\lambda (\eta^i \bar{\eta}_i)^2 \left( f_{\bar{\pi}}^i - \frac{f_i g_{\bar{\pi}}}{g} \right) \]
\[ - \frac{m}{2} g \cos 2\lambda \bar{\eta}_i \eta^i \right], \] (5.22)

where we defined
\[ \pi (m) = p_{\xi} - \frac{i}{2} m f_{\xi} + \frac{i}{2} g^{-1} g_{\xi} \eta^i \bar{\eta}_i, \quad \bar{\pi} (m) = p_{\xi} + \frac{i}{2} m f_{\xi} - \frac{i}{2} g^{-1} g_{\xi} \eta^i \bar{\eta}_i. \] (5.23)

### 5.4. Quantization

The brackets (5.21) are quantized in the standard way:
\[ \left[ \hat{\xi}, \hat{\rho}_j \right] = i, \quad \left\{ \eta^i, \bar{\eta}_j \right\} = \delta^i_j, \quad \left[ \hat{\rho}_i, \bar{\eta}_j \right] = \left[ \hat{\rho}_i, \eta^j \right] = 0, \quad \hat{\rho}_i = -i \partial_{\eta^i}, \quad \hat{\eta}_i = \frac{\partial}{\partial \eta^i}. \] (5.24)

It will be helpful to use the relation
\[ [\pi (m), \bar{\pi} (m)] = mg - \frac{1}{2} g^{-1} R \left[ \eta^i, \bar{\eta}_i \right]. \] (5.25)

where
\[ \pi (m) = -i \partial_{\eta^i} - \frac{i}{2} m f_{\xi}, \quad \bar{\pi} (m) = -i \partial_{\eta^i} + \frac{i}{2} m f_{\xi} \]
\[ \bar{\pi} (m) = -i \partial_{\eta^i} - \frac{i}{2} m f_{\xi} \] (5.26)

The general quantization scheme for SQM models was developed in [9]. Following this procedure, we obtain the quantum operators\(^4\)

\(^4\) We pass to the picture in which the supercharges bear no explicit \( t \) dependence: \( \psi \rightarrow U \psi, \)
\( \left( Q_{\text{con}}, \bar{Q}_{\text{con}} \right) \rightarrow U \left( Q_{\text{con}}, \bar{Q}_{\text{con}} \right) U^{-1}, \quad U = e^{i \theta}. \)
They satisfy the $su(2|1)$ superalgebra (2.1) with the quantum Hamiltonian
\begin{align}
\hat{H} &= g^{-1} \left[ \pi (m \cos 2\lambda \eta \pi (m) - \sin \lambda \eta^{\dagger}\pi (-m)) \right. \\
&\quad - \frac{m}{4} \sin 2\lambda (\eta)^{\dagger} \left( f_{\varepsilon} - \frac{f_{\varepsilon}g_{\varepsilon}}{g} \right) - \frac{m}{4} \sin 2\lambda (\eta^{\dagger})^{\dagger} \left( f_{\varepsilon} - \frac{g f_{\varepsilon}}{g} \right) \\
&\quad + \frac{m}{2} g \cos 2\lambda \eta^{\dagger}\eta \right].
\end{align}
(5.28)

6. Concluding remarks

In this paper, which is a natural continuation of our previous work [1], we constructed and studied a new class of the $SU(2|1)$ SQM models as a deformation of the standard $N = 4$, $d = 1$ SQM based on the chiral supermultiplet (2, 4, 2). We found that, as distinct from the standard $N = 4$ case, in the $SU(2|1)$ case two different definitions of the worldline superspace are possible. In one of them, the time coset coordinate is associated with the central charge generator, while in the other the corresponding coset generator is the full internal $U(1)$ generator. The first option was used in [1], and the second one was elaborated on in the present paper. In accordance with the existence of two different $SU(2|1)$ superspaces, there exist two different versions of the chiral $SU(2|1)$ multiplet (2, 4, 2) which are both reduced to the same standard linear chiral $N = 4$, $d = 1$ multiplet in the contraction limit $m = 0$. The general $SU(2|1)$ SQM models based on the first version were considered in [1], while here we presented the characteristic features of the alternative class of the $SU(2|1)$ SQM models which are built on the second version of the multiplet (2, 4, 2). One of these peculiarities is the presence of new dimensionless parameter $\lambda$ which reflects the non-uniqueness of embedding of the relevant chiral superspace into the full $SU(2|1)$ superspace. Since the Hamiltonian in the second version of the $SU(2|1)$ superspace formalism does not commute with the supercharges, the corresponding transformations of the component fields contain an explicit dependence on the time variable which is removable only in some special cases, when the chiral superfields are reduced to those defined on the ‘old’ $SU(2|1)$ superspace. Note that the (1, 4, 3) models considered in [1] have the same description within the new $SU(2|1)$ superspace, as the constraints constructed with the help of the spinor derivatives (3.5) coincide with those used in [1].

The SQM models that we have deduced from our $SU(2|1)$ superspace approach were earlier constructed in the Hamiltonian on-shell component formalism in [6, 7] and were called there ‘supersymmetric Kähler oscillators’. New features provided by the superfield approach are the off-shell formulation of this class of models: the relevant component Lagrangians with auxiliary fields (including the general external superpotential) and the new off-shell $\lambda$-dependent worldline realization of $SU(2|1)$ supersymmetry. We also presented the full set of the quantum $SU(2|1)$ generators in these models. All generators can now be systematically derived from the invariant Lagrangians by the standard Noether procedure.
It would be interesting to explicitly solve the corresponding quantum mechanical problems (the spectrum of $\tilde{H}$, etc) for a few simple choices of the underlying Kähler potential $f$. We expect a splitting of the spectrum into two towers of $N = 2$ supersymmetry multiplets, like in the models considered in [1, 4]. The relation to the $N$-fold supersymmetries of [10] also deserves further clarification. It is of obvious interest to look whether the $N = 4$ SQM models based on the nonlinear version of the multiplet $(2, 4, 2)$ [11] admit a generalization to the $SU(2|1)$ case, and which of the two different $SU(2|1)$ superspace approaches is preferable for such a generalization. The study of other $SU(2|1)$ counterparts of the basic $N = 4$ off-shell multiplets and the associated SQM models by exploiting the $SU(2|1)$ superspaces of both types is also a problem for future analysis.

An interesting issue concerns possible implications of the dimensionless parameter $\lambda$ in the higher-dimensional supersymmetric theories on curved manifolds [2, 3]. The supergroup $SU(2|1)$ extended by an external $R$ symmetry generator was already utilized in [2] for the space $S^3 \times S^1$, and the chiral superfields constructed in [1] can be alternatively recovered through a dimensional reduction of those defined in [2]. It remains to see whether the generalized chiral $SU(2|1)$ superfields considered here also have some higher-dimensional counterparts.

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