Angular asymmetries in the reactions $\bar{p}p \rightarrow d\pi^+\eta$ and $\bar{p}n \rightarrow d\pi^0\eta$
and $a_0 - f_0$ mixing

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(November 13, 2018)

Abstract

The reactions $pp \rightarrow d\pi^+\eta$ and $pn \rightarrow d\pi^0\eta$ are of special interest for investigating the $a_0(980)$ ($J^P = 0^+$) resonance in the process $NN \rightarrow da_0 \rightarrow d\pi\eta$.
We study some aspects of those reactions within a general formalism and also in a concrete phenomenological model. In particular, it is shown that the presence of nonresonant (i.e. without excitation of the $a_0$ resonance) contributions to these reactions yields nonvanishing values for specific polarization observables, i.e. to effects like those generated by $a_0^0 - f_0$ mixing. An experimental determination of these observables for the reaction $pp \rightarrow d\pi^+\eta$
would provide concrete information on the magnitude of those nonresonant contributions to $\pi\eta$ production. We discuss also the possibility of extracting information about $a_0^0 - f_0$ mixing from the reaction $\bar{p}n \rightarrow d\pi^0\eta$ with polarized proton beam.

PACS: 13.60.Le, 13.75.-n, 14.40.Cs
I. INTRODUCTION

The reactions \( pp \to dK^+K^0 \) and \( pp \to d\pi^+\eta \) are presently the subject of experimental investigations by the ANKE collaboration at the COSY accelerator in Jülich [1–3]. The main issue of this study is to obtain further information about the scalar \( a_0^+ \) (980) resonance, which decays dominantly into the \( K^+K^0 \) and \( \pi^+\eta \) channels [4]. Also a measurement of the production of the neutral \( a_0^0 \) meson in the reaction \( pn \to da_0^0 \) with the ANKE spectrometer is planned [5–7]. The \( a_0^0 \) production is closely related to the problem of \( a_0^0 \to f_0 \) mixing [8]. A nonzero value for the transition amplitude \( a_0^0 \leftrightarrow f_0 \) provides a forward-backward asymmetry for the reaction \( pn \to da_0^0 \). As was shown in ref. [9], near the threshold this forward-backward asymmetry is large, of the order of 10% – 15%. Thus, it is evident that the study of this asymmetry can provide useful information on the process of \( a_0^0 \to f_0 \) mixing.

In a recent paper [10] some aspects of the reaction \( \vec{p}n \to da_0^0 \) were discussed for the case of a perpendicular polarized proton beam. Specifically, it was shown that for energies close to threshold the angular-asymmetry parameter defined by

\[
A(\theta, \varphi) = \frac{\sigma(\theta, \varphi) - \sigma(\pi - \theta, \varphi + \pi)}{\sigma(\theta, \varphi) + \sigma(\pi - \theta, \varphi + \pi)},
\]

(1)

with

\[
\sigma(\theta, \varphi) \equiv \frac{d\sigma}{d\Omega}(\theta, \varphi),
\]

(2)

(\( \theta \) and \( \varphi \) are the polar and azimuthal angles of the outgoing \( \pi\eta \) system in the CMS of the reaction) is proportional to the \( a_0^0 \to f_0 \) mixing amplitude. I.e.

\[
A(\theta, \varphi) \sim \xi \cdot k,
\]

(3)

where \( k \) is the relative momentum of the \( a_0^0 \) meson with respect to the deuteron and \( \xi \) is the \( a_0^0 \to f_0 \) mixing parameter. This result is valid in lowest order with respect to the momentum \( k \), i.e. keeping only contributions that are at most linear in \( k \). It was shown in ref. [10] that corrections to \( A(\theta, \varphi) \) from isospin-conserving terms are of order \( k^3 \) and, therefore, they are of relevance at higher energies only. Thus, the study of the angular asymmetry (Eq. 1) near the \( a_0^0 \) threshold in the reaction \( \vec{p}n \to da_0^0 \) gives information on various invariant amplitudes for this reaction [10] but, in particular, on the \( a_0^0 \to f_0 \) mixing parameter \( \xi \).

The present paper focusses on the reaction \( NN \to d\pi\eta \). We take into consideration that the \( \pi\eta \) system can not only be produced via the formation of the \( a_0(980) \) resonance, i.e. in an \( S \)-wave state, but also via other reaction mechanisms and then can be in a \( P \) wave or in even higher partial waves. For brevity, in the following we will refer to the latter contributions as nonresonant or as background of the \( NN \to da_0 \to d\pi\eta \) amplitude. (Note that there can be also a background in the \( S \)-wave state, as discussed in Refs. [11,12] – but this is not the issue we are concerned with here.) Consequences of this nonresonant background contribution for polarization observables of the reactions \( \vec{p}p \to d\pi^+\eta \) and \( \vec{p}n \to d\pi^0\eta \) will be discussed. Specifically, it will be shown that, because of the presence of the nonresonant background, in these reactions the angular-asymmetry parameter \( A(\theta, \varphi) \) becomes nonzero even without isospin-violation. Indeed, like the effect induced by the \( a_0^0 \to f_0 \) mixing discussed
above, the contribution of the nonresonant background is also linear in \( k \) and therefore is difficult to separate from effects of the \( a_0^0 - f_0 \) mixing in the reaction \( \vec{p}n \rightarrow d\pi^0\eta \). However, a measurement of this asymmetry in the reaction \( \vec{p}p \rightarrow d\pi^+\eta \) can provide us with information on the contribution of the nonresonant background to the reaction \( pp \rightarrow da_0^+ \rightarrow d\pi^+\eta \).

Recently the reactions \( pp \rightarrow dK^+\bar{K}^0 \) and \( pp \rightarrow d\pi^+\eta \) were studied theoretically in a chiral unitary approach taking into account the coupling between the \( K^+\bar{K}^0 \) and \( \pi^+\eta \) channels [13]. The elementary \( \pi\eta \) production amplitude was assumed to be given by the diagram shown in Fig. 1a where the \( \pi \) and \( \eta \) mesons emerge from different nucleons and rescatter with each other before being emitted. Since chiral dynamics suppressed the coupling of the \( \pi\eta \) system to \( P \)-waves [14], it follows within this approach that the \( \pi^+\eta \) system is preferably produced in \( S \)-waves and that \( P \) waves, i.e. the nonresonant background discussed above, should be practically negligible \(^1\). Their result motivated us to consider in the present paper a different production mechanism that does not involve the \( \pi\eta \) amplitude directly and therefore is not constrained by chiral symmetry. We assume that the reaction \( pp \rightarrow d\pi\eta \) proceeds via pion exchange between the nucleons followed by the exitation of the \( \Delta(1232) \) and the \( N^*(1535) \) resonances [12] which then produce the \( \pi \) and \( \eta \) mesons in their respective decay, cf. Fig. 1b. Such a reaction mechanism is certainly suppressed at energies near the \( \pi\eta \) threshold where chiral dynamics should be dominating. However, in the region of the \( a_0 \) resonance the excess energy for the \( \pi\eta \) system is already around 300 MeV. Therefore this mechanism could be already of relevance and, specifically, it will introduce \( P \)-wave contributions. We will present corresponding results for the angular symmetry \( A(\theta, \varphi) \) and also for differential cross sections. Clearly, in this context an experimental study of this asymmetry parameter is very interesting because it may shed light on the validity of the chiral unitary approach [13] for the calculation of amplitudes for the reactions \( pp \rightarrow dK^+\bar{K}^0 \) and \( pp \rightarrow d\pi^+\eta \) near the \( a_0 \) threshold.

Finally, we will show that a systematic study of the angular-asymmetry parameters for both reactions, \( \vec{p}p \rightarrow d\pi^+\eta \) and \( \vec{p}n \rightarrow d\pi^0\eta \), may allow to obtain quantitative information on the \( a_0^0 - f_0 \) mixing parameter.

The paper is organized as follows. In Sect. II we provide the general form of the reaction amplitude for the process \( NN \rightarrow d\pi\eta \) near threshold where we allow the \( \pi\eta \) system to be in an \( S \) or a \( P \) wave. Furthermore we derive expressions for the corresponding differential cross sections. In Sect. III we consider a phenomenological model for the nonresonant (\( P \) wave) contributions to the reaction \( pp \rightarrow d\pi^+\eta \) and present concrete estimations for the angular symmetry \( A(\theta, \varphi) \) and also for differential cross sections. Implications of a possibly non-zero background contribution to the reaction \( NN \rightarrow d\pi\eta \) on the issue of \( a_0^0 - f_0 \) is discussed in Sect. IV. The paper ends with a short summary.

\(^1\)Note, however, that \( P \)-waves arise naturally in the effective Lagrangian approach of Achasov et al. [15] based on the anomalous Wess-Zumino action.
II. GENERAL FORM OF REACTION AMPLITUDE

Let us consider first the reaction $pp \rightarrow d\pi^+\eta$. If the $\pi^+\eta$ system is produced by the decay of the $a_0^+$ meson it is in an $S$-wave. However, if we produce the $\pi^+\eta$ state with an invariant mass of $m \approx m_{a_0} \approx 980$ MeV but not via the $a_0^+$ resonance then the orbital angular momentum of the $\pi\eta$ system may be large because, as mentioned, the energy available in the $\pi\eta$ system is already around 300 MeV.

We start with the most general form of the amplitude $M$ for the reaction $NN \rightarrow d\pi\eta$. It is given by

$$M = \phi_1^T \sigma_y [F + G \cdot \sigma] \phi_2,$$

where $\phi_1^T$ and $\phi_2$ are the spinors of the nucleons ($T$ indicates the transposed state vector). The amplitude in Eq. (4) involves two terms, $F$ and $G \cdot \sigma$, corresponding to the initial total spin of the nucleons of $S_{NN} = 0$ and $S_{NN} = 1$, respectively.

Note, that both functions $F$ and $G$ are to be linear functions of the polarization vector $\epsilon^*$ of the outgoing deuteron. In the following we limit ourselves to the consideration of $S$- and $P$-wave states for the produced $\pi\eta$ system and, accordingly, we write both functions $F$ and $G$ as

$$F_S = a_0 \epsilon^* \cdot \mathbf{k} + b_0 \epsilon^* \cdot \mathbf{p} + c_0 \mathbf{p} \cdot \epsilon^* \cdot \mathbf{k} + d_0 \mathbf{p} \cdot \epsilon^* \cdot \mathbf{p} \cdot \mathbf{k}$$

for the case of the $\pi\eta$ system being in an $S$-wave (e.g. production via the $a_0$ resonance) and

$$F_P = a_1 \epsilon^* \cdot \mathbf{p} \times \mathbf{q} + b_1 \epsilon^* \cdot \mathbf{p} \times \mathbf{q} + c_1 \mathbf{p} \cdot \epsilon^* \cdot \mathbf{p} \times \mathbf{q} + d_1 \mathbf{p} \cdot \epsilon^* \cdot \mathbf{p} \times \mathbf{q} \cdot \mathbf{p} \cdot \mathbf{q}$$

for the case of the $\pi\eta$ system being produced in a $P$-wave. Here $a_0, a_1, b_0, b_1, c_0, c_1, d_0,$ and $d_1$ are independent scalar amplitudes, which may be considered as being basically constants for the near-threshold production ($k$ is small) of the $\pi\eta$ system with an invariant mass $m = m_{a_0}$ of the $a_0$ meson. In Eqs. (5)-(7) we used the following notations:

$\mathbf{p}$ is the initial relative momentum in the center-of-mass system (CMS) of the reaction;

$\mathbf{k}$ is the final relative momentum of the deuteron with respect to the $\pi\eta$ system in the CMS of the reaction;

$\mathbf{q}$ is the relative momentum between the pion and the $\eta$ in the $\pi\eta$ CM frame.

The matrix element $\mathcal{M}$ (Eq. (4)) squared and summed over the polarizations of the initial neutron is given by

$$|\mathcal{M}|^2 = \frac{1}{2} \left[ |F|^2 + 2 \text{Re}(F^* G \cdot \zeta) + (G^* \cdot G) + i (\zeta \cdot [G \times G^*]) \right],$$

where $\zeta$ is the polarization vector of the deuteron.
where $\zeta$ is the polarization vector of the initial proton, i.e. $\zeta = \phi_1^+ \sigma \phi_1$. In what follows we shall consider the vector $p$ to be aligned in the direction of the $z$-axis and the vector $\zeta$ to be aligned in $x$-direction, so that $\zeta \perp p$.

Since we are interested in the behaviour of the differential cross section $d^2\sigma/dm\,d\Omega_{\mathbf{k}}$ and in the angular-asymmetry parameter $A(\theta, \varphi)$, we are to integrate the expression (8) over the direction of the momentum $\mathbf{q}$:

$$
\frac{d^2\sigma}{dm\,d\Omega_{\mathbf{k}}} = N \int \frac{d\Omega_{\mathbf{q}}}{4\pi} \left| \mathcal{M} \right|^2 := \sigma(m; \theta, \varphi).
$$

Here $m$ is the invariant mass of the $\pi\eta$ system, $N = kq/(4\pi)^4 ps$ and $s$ is the square of the total energy in the CMS of the reaction. After the integration over $d\Omega_{\mathbf{q}}$ the $S - P$ interference term in $\sigma(m; \theta, \varphi)$ disappears. The contribution of the $S$-wave part of $\mathcal{M}$ (4) to $\sigma(m; \theta, \varphi)$ can be obtained from Eqs. (22) and (23) of ref. [10] and reads

$$
\left( \frac{d^2\sigma}{dm\,d\Omega_{\mathbf{k}}} \right)_S = N \left\{ p^2 k^2 \left[ \frac{1}{2} (|a_S|^2 + |b_S|^2) + \left[ |b_S|^2 + \frac{1}{2} |b_S + p^2 d_S|^2 + \text{Re} \left( a_S^* c_S + (a_S + c_S)^*(b_S + p^2 d_S) \right) \right] \cos^2 \theta + \zeta pk \text{Im} \left( a_S^* b_S + a_S^* c_S + b_S^* c_S + p^2 d_S^* c_S \right) \sin \theta \cos \theta \sin \varphi \right\}.
$$

The contribution from the $P$-wave part can be deduced from Eqs. (6)–(9) and amounts to

$$
\left( \frac{d^2\sigma}{dm\,d\Omega_{\mathbf{k}}} \right)_P = N \left\{ \frac{k^2 q^2}{6} \left[ 2|a_0|^2 + 2|b_0|^2 p^4 \cos^2 \theta + |c_0|^2 p^4 \sin^2 \theta + 4\text{Re} \left( a_0^* b_0 \right) p^2 \cos^2 \theta \right.ight.
\left. - \frac{q^2}{3} \text{Re} \left( a_0^* a_1 - a_0^* c_1 - c_0^* a_1 p^2 \right) (\zeta \cdot [\mathbf{k} \times \mathbf{p}]) \right.
\left. + \frac{p^4 q^2}{6} \left[ (3|a_1|^2 + |b_1|^2 + |c_1|^2) + p^4 |d_1|^2 \right] + 2\text{Re} \left( a_1^* (b_1 + c_1 + p^2 d_1) + b_1^* c_1 + (b_1^* + c_1^*) p^2 d_1 \right) \right\}.
$$

It can be easily seen from Eq. (11) that the only contribution to the angular-asymmetry parameter $A(\theta, \varphi)$, defined in Eq. (1), comes from the term proportional to $\zeta \cdot [\mathbf{k} \times \mathbf{p}] = \zeta pk \sin \theta \sin \varphi$, where $\theta$ and $\varphi$ are the polar and azimuthal angles of the outgoing $\pi\eta$ system in the CMS of the reaction. Hence, because of the $P$-wave (nonresonant background) contribution, we get a nonvanishing angular asymmetry and this asymmetry is linear in $k$. Note that $A(\theta, \varphi) = 0$ in the case where the $\pi\eta$ system is produced in an $S$-wave, as follows from Eq. (10).

Therefore, it is clear that the angular asymmetry $A(\theta, \varphi)$ in the reaction $\bar{p}p \to d\pi^+\eta$ can provide information on the magnitude of contributions from partial waves with $l \geq 1$ in the $\pi\eta$ system.
III. Estimation of the Angular Asymmetry for Nonresonant $\pi^+\eta$ Production

Let us now come to a concrete estimation for the angular-asymmetry parameter. For that purpose we consider a simple model for the reaction $pp \to d\pi^+\eta$ in which the $\pi^+\eta$ system can be produced with nonzero internal angular momentum ($l \geq 1$). Specifically, we adopt the one-pion-exchange diagram in Fig. 1b. We assume that the dominant contribution arises from the intermediate $\Delta(1232)$ state in the subprocess $\pi N \to \pi N$ and from the $S$-wave amplitude of the subprocess $\pi N \to \eta N$. The latter is basically given by the contribution of the $N^*(1535)$ resonance state. This diagram with the same subprocesses was already used in ref. [12] in the context of the $a_0$ production process in $pN \to d\pi\eta$. However, in that work the authors were primarily interested in the $S$-wave part of the one-pion-exchange $\pi\eta$ production amplitude. Therefore, the angular distribution of the outgoing pion was not calculated in [12].

The evaluation of this diagram, treating the intermediate nucleons and the final deuteron nonrelativistically, leads to the following expression for the corresponding $\pi\eta$ production amplitude $M$:

$$M = M_1 + M_2, \quad M_{1,2} = \pm A_{1,2} \phi^T_{1,2} \sigma_y (e^* \cdot \sigma) \left(2n_{1,2} \cdot n_2 + i \sigma \cdot [n_{1,2} \times n_2]\right) \phi_{2,1},$$  \hspace{1cm} (12)

Note that the amplitude $M$ is antisymmetric with respect to the initial nucleons, i.e. $M_1 \leftrightarrow -M_2$ when the nucleons are interchanged. The quantities $A_{1,2}$ are given by

$$A_{1,2} = T \frac{E + m_N}{2m_N \sqrt{2m_N}} M_{\pi^0 N \to \eta N} M_{\pi N \to \pi N} \int \frac{dq}{(2\pi)^3} u(q) G\pi(t_{1,2}).$$  \hspace{1cm} (13)

Further, $2n_{1,2} \cdot n_2 + i \sigma \cdot [n_{1,2} \times n_2]$ is the spin operator of the $\Delta$ state in the $\pi N$ scattering amplitude and $n_{1,2}' = r_{1,2}/r_{1,2}$ and $n_2 = k_2/k_2$ are unit vectors. The corresponding momenta, $r_{1,2} = k_2 + q_2 - p_{2,1}$ and $k_2$, are the 3-momenta of the intermediate (virtual) and final pion in the CMS of the reaction, in the notation used in Fig. 1b. $T = 4 \sqrt{2}/3$ is an isospin factor and $m_N$ and $E$ are the mass and total CMS energy of the initial proton. The $S$-wave amplitude for $\pi^0 N \to \eta N$ is obtained from the relation (see also ref. [16])

$$|M_{\pi^0 N \to \eta N}(s_1)|^2 = 8\pi s_1 \frac{p_{cm}}{p_{cm}} \sigma_{\pi^0 p \to \eta N} \left(s_1 = m^2_{\pi N},\right)$$  \hspace{1cm} (14)

where the cross section $\sigma_{\pi^0 p \to \eta N} = (21.2 \pm 1.8) p^0_{cm} [\mu b]$ ($p^0_{cm}$ in MeV/$c$) is taken from the experiment [17]. The scalar amplitude for $\pi N \to \pi N$ resulting from the $\Delta$ resonance reads

$$M_{\pi N \to \pi N}(m_{\pi N}) = \frac{g^2_{\pi N \Delta}}{m_{\pi N} - M_\Delta + i\Gamma_\Delta / 2} k_{cm}(\pi'), \quad g^2_{\pi N \Delta} = \frac{4\pi M_\Delta \Gamma_{\Delta \rightarrow N\pi}}{k^3_R},$$  \hspace{1cm} (15)

where $k_{cm}(\pi')$, $k_{cm}(\pi)$ are the relative momenta at the $\Delta N\pi$ vertices with the virtual and final pion, respectively. $k_R$ is the relative momentum in the decay $\Delta \rightarrow N\pi$ at the nominal mass $m_{\pi N} = M_\Delta$.

In the following all the values which are taken out of the loop integral (13) are calculated at fixed values $q_1 = q_2 = p_d/2$ of the intermediate on-mass-shell nucleons, where $p_d$ is the
momentum of final deuton in the reaction CM. The loop integral in Eq. (13) contains only the wave function \( u(q) \) of the deuteron and the pion propagator \( G_\pi(t) \). We use the S-wave part of the deuteron wave function of the full Bonn potential [18]. The propagator of the virtual pion, including a form factor \( F_\pi(t) \) of monopole type for each \( \Delta N\pi \) vertex, reads

\[
G_\pi(t) = \frac{F_\pi^2(t)}{t - \mu^2 + i0}, \quad F_\pi(t) = \frac{\mu^2 - \Lambda^2}{t - \Lambda^2},
\]

where \( \mu \) is the pion mass. For the cutoff parameter \( \Lambda \) we consider the values \( \Lambda = 1 \div 1.3 \text{ GeV} \) [12]. The four-momentum transfer squared \( t \) for nonrelativistic intermediate nucleons is given by the relations \( t = t_{1,2} \) for the corresponding part \( \mathcal{M}_{1,2} \) of the total antisymmetric amplitude \( \mathcal{M} \) according to Eq. (12)

\[
t_{1,2} - \mu^2 + i0 = -x \left[ (q - \Delta_{1,2})^2 - a_{1,2}^2 - i0 \right], \quad x = (E - \omega_2)/m,
\]

\[
a_{1,2}^2 = \frac{1}{x} \left[ (T_N - \omega_2)^2 + (p_{2,1} - k_2)^2 \left( \frac{1}{x} - 1 \right) - \mu^2 \right], \quad \Delta_{1,2} = \frac{p_{2,1} - k_2}{x} - \frac{p_\pi}{2},
\]

where \( T_N = E - m_N \) and \( \omega_2 \) is the total energy of the final pion in the CMS of the reaction.

Let us rewrite the total amplitude \( \mathcal{M} \) (Eq. (12)) in the form of Eq. (4). Then we get

\[
\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 = \phi_1^T \sigma_y [F + G \cdot \sigma] \phi_2, \quad \mathcal{M}_{1,2} = \pm A_{1,2} \phi_{1,2}^T \sigma_y [F_{1,2} + G_{1,2} \cdot \sigma] \phi_{2,1},
\]

where

\[
F = A_1 F_1 + A_2 F_2, \quad G = A_1 G_1 - A_2 G_2.
\]

\[
F_{1,2} = i(\epsilon \cdot [n_{1,2} \times n_2]), \quad G_1 = 2(n_{1,2} \cdot \epsilon) n_2 - (n_2 \cdot \epsilon) n_{1,2}.
\]

Using Eqs. (8), (9) and (12)-(19), we evaluated the angular asymmetry \( A(\theta, \varphi) \), defined in Eq. (1), at the angle \( \theta = 90^0 \), where the term \( 2\text{Re}(F^* G \cdot \zeta) \sim \zeta \cdot [k \times p] = \zeta pk \sin \theta \sin \varphi \) should produce the maximal effect. The calculations were performed at \( m = 980 \text{ MeV} \) (i.e. in the \( a_0 \)-meson region). It turned out that the resulting angular asymmetry parameter \( A(\theta, \varphi) \) is very small, i.e. \( A(90^0, \varphi) \leq 1\% \). In order to understand this we need to go back to the equations above. Indeed one can immediately see, that in the case of zero relative phase between the quantities \( A_1 \) and \( A_2 \) given by Eq. (13), one should obtain exactly \( A(\theta, \varphi) = 0 \). This is due to the relative phase of \( 90^0 \) between the functions \( F \) and \( G \) in the term \( 2\text{Re}(F^* G \cdot \zeta) \) of Eq. (8) because of the factor \( i \) in \( F \) (see Eq. (19) for \( F_1 \)). The latter results from the spin structure of the \( \pi N \) scattering amplitude, which in our case is given by the excitation of the \( \Delta \) resonance alone, and is connected with the Hermitian form of the interaction (cf. the factor \( i \) in front of the \( \sigma \) matrix in Eq. (12)). In fact, the loop integral in Eq. (13) generates an imaginary part from the on-mass-shell contribution of the exchanged pion. Therefore, due to permutation of the initial protons, the quantities \( A_1 \) and \( A_2 \) acquire some nonzero relative phase. But still there is only a small effect on the angular asymmetry.

In this context let us mention that the interaction in the initial \( NN \) system, which is neglected in the simple model calculation presented here, should also introduce a relative phase between those two amplitudes and therefore might lead to an enhancement in the predictions for the angular-asymmetry parameter.
In any case, the smallness of this asymmetry effect does not mean that the $P$-wave fraction in the $\pi\eta$ system is also small. To illustrate this, we present here some results for the reaction with unpolarized proton beam, namely the distribution $d^2\sigma/dmdz_1$ at selected values of the invariant mass $m$ of the $\pi\eta$ system around the $a_0$ mass and for two values of the cutoff parameter $\Lambda$ ($\Lambda = 1$ and 1.3 GeV). Here $z_1 = \cos\theta_1$, where $\theta_1$ is the polar angle of the outgoing $\pi^+$ meson with respect to the direction of the proton-beam momentum in the $\pi^+\eta$ rest frame. The calculations were done at the proton beam energy $T_{lab} = 2.65$ GeV. The resulting angular spectra are shown in the Figs. 2 and 3. Evidently, they are not isotropic but exhibit a strong angular dependence and, therefore, demonstrate that there are significant contributions from higher partial waves ($l \geq 1$). Thus, a measurement of the angular distribution for the produced $\pi^+\eta$ system would be rather instructive. In particular it should allow to examine the validity of the chiral unitary approach used in ref. [13], which implies a completely isotropic angular distribution, in the energy region of the $a_0(980)$ resonance.

IV. THE REACTION $\vec{p}n \to d\pi^0\eta$ AND THE $a_0-f_0$ MIXING AMPLITUDE

Let us now consider the reaction $pp \to d\pi^0\eta$. As discussed in ref. [10], if the $\pi^0\eta$ system is produced in an $S$-wave then the only nonzero contribution to the angular-asymmetry parameter comes from the $a_0^0-f_0$ mixing amplitude. Indeed, we have shown there that – in lowest order in $k$ – the isospin-violating contribution to $A(\theta, \varphi)$ is proportional to $\xi \cdot k$. Thus, an extraction of the $a_0^0-f_0$ mixing parameter $\xi$ would, in principle, be feasible from experimental information on the angular asymmetry.

However, as should be clear after the discussion in Sect. II, there could be also contributions to $A(\theta, \varphi)$ from isospin-conserving terms, because of the possible presence of $P$ waves in the $\pi\eta$ system. Such contributions arise from the term in the differential cross section proportional to $\zeta \cdot [k \times p]$. This term is maximal at $\theta = 90^\circ$ and vanishes at $\theta = 0^\circ$. As a consequence, a separation of both contributions to $A(\theta, \varphi)$ is rather complicated.

Still, there is a possibility to separate the contribution from the isospin-violating part, namely by carrying out a combined study of the polarized differential cross sections for both reactions $\vec{p}p \to d\pi^+\eta$ and $\vec{p}n \to d\pi^0\eta$. In order to illustrate how this can be achieved, let us first remind the reader that the amplitudes of those reactions are related by

\[ M(pn \to d\pi^0\eta) = \frac{1}{\sqrt{2}} M(pp \to d\pi^+\eta), \]

if isospin is conserved. This relation suggests that one should consider a “subtracted” differential cross section,

\[ \sigma_\Delta(m; \theta, \varphi) := \frac{d^2\sigma_0}{dm \Omega_k} - \frac{1}{2} \frac{d^2\sigma_+}{dm \Omega_k}, \]

where $\sigma_0$ and $\sigma_+$ are the cross sections of the processes $\vec{p}n \to d\pi^0\eta$ and $\vec{p}p \to d\pi^+\eta$, respectively. Evidently, the angular asymmetry $A(\theta, \varphi)$, evaluated according to Eq. (1) for this “subtracted” differential cross section $\sigma_\Delta(m; \theta, \varphi)$ does not contain terms induced by
isospin-conserving processes and hence provides only information on isospin violating effects. Note that in the simple model considered in Sect. III the angular asymmetry in the reaction \( \vec{pp} \rightarrow d\pi^+\eta \) is small in any case and, therefore, one might expect that any effects seen in the experiment should come mainly from isospin-violating \( a_0^0 - f_0 \) mixing.

V. SUMMARY

We presented a discussion on effects of background contributions to differential cross sections and the angular asymmetry \( A(\theta, \varphi) \) for the reactions \( pp \rightarrow d\pi^+\eta \) and \( pp \rightarrow d\pi^+\eta \). Specifically, we pointed out that already in lowest order in the relative momentum between the deuteron and the \( \pi\eta \) system the angular asymmetry \( A(\theta, \varphi) \) can be nonzero – even without isospin mixing effects – because of the presence of higher partial waves in the \( \pi\eta \) system. As a consequence, a measurement of the angular asymmetry \( A(\theta, \varphi) \) with polarized proton beams for the reaction \( \vec{pp} \rightarrow d\pi^+\eta \) should provide useful information on the role of higher partial waves in this reaction. It would also allow to examine the validity of model calculations of the reaction \( pp \rightarrow d\pi^+\eta \) based on chiral constraints [13] for energies around the \( a_0 \) threshold.

We also argued that a combined analysis of differential cross sections for both reactions, \( \vec{pp} \rightarrow d\pi^+\eta \) and \( \vec{pn} \rightarrow d\pi^0\eta \), may facilitate the extraction of isospin-violating effects induced by \( a_0^0 - f_0 \) mixing and allow to shed light on the nature of these scalar mesons.

ACKNOWLEDGEMENTS

The authors are thankful to V.V. Baru and M. Büscher for useful discussions. This work was partly supported by the DFG-RFBI Grant No. 02-02-04001 (436 RUS 113/652/1-1) and by the RFBR Grant No. 00-15-96562.
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FIGURES

FIG. 1. Nonresonant mechanisms for the reaction $pp \rightarrow d\pi^+\eta$. (a) $\pi\eta$ rescattering. (b) Pion-exchange diagram involving the $\pi N \rightarrow \Delta \rightarrow \pi N$ and $\pi N \rightarrow \eta N$ reaction amplitudes.

FIG. 2. Angular distributions of the outgoing $\pi^+$ meson in the $\pi^+\eta$ rest frame for several values $m$ of the $\pi^+\eta$ invariant mass in the $a_0$-mass region. The dashed, solid and dotted curves correspond to $m = 950, 980$ and $1020$ MeV/$c^2$, respectively. The calculations are performed at $T_{lab} = 2.65$ GeV and for a cutoff parameter (16) $\Lambda = 1$ GeV/$c$.

FIG. 3. The same distributions as in Fig. 2 but for $\Lambda = 1.3$ GeV/$c$. 
Fig. 1a

Fig. 1b
\[ T_{\mu\nu} = 2.65 \text{ GeV}, \quad \Lambda = 1 \text{ GeV} \]

\[ \frac{d^2\sigma}{dm^2dz}, \text{ (\mu b/GeV)} \]

- \( m = 950 \text{ MeV} \)
- \( m = 980 \text{ MeV} \)
- \( m = 1020 \text{ MeV} \)

Fig. 2
Fig. 3