Effects of Mutual Transits by Extrasolar Planet–Companion Systems on Light Curves

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Abstract

We consider the effects of mutual transits by extrasolar planet–companion systems (in a true binary or a planet–satellite system) on light curves. We show that induced changes in light curves depend strongly on a ratio between a planet–companion’s orbital velocity around their host star and a planet–companion’s spin speed around their common center of mass. In both the slow and fast-spin cases (corresponding to long and short distances between them, respectively), a certain asymmetry appears in light curves. We show that, especially in the case of short distances, occultation of one faint object by the other, while the transit of the planet–companion system occurs in front of its parent star, causes an apparent increase in light curves, and characteristic fluctuations appear as important evidence of mutual transits. We show also that extrasolar mutual transits provide a complementary method for measuring the radii of two transiting objects, their separation and masses, and consequently for identifying the system as a true binary, a planet–satellite system, or others. Monitoring $10^5$ stars for 3 yr with Kepler may lead to the discovery of a second Earth–Moon-like system if the fraction of such systems for an averaged star is larger than 0.05, or we may put an upper limit on the fraction as $f < 0.05$.

Key words: eclipses — occultations — planets and satellites: general — stars: planetary systems — techniques: photometric

1. Introduction

It is of general interest to discover a second Earth–Moon system. Detections of extrasolar planet–satellite or binary-planet systems will bring important information to planet (and satellite) formation theory (e.g., Jewitt & Sheppard 2005; Canup & Ward 2006; Jewitt & Haghighipour 2007).

It is not clear whether the IAU definition for planets in the solar system can be applied to extrasolar planets as it is. The IAU definition in 2006 is as follows. A planet is a celestial body that (a) is in orbit around the Sun, (b) has sufficient mass so that it assumes a hydrostatic equilibrium (nearly round) shape, and (c) has cleared the neighborhood around its orbit.

We call a gravitationally bound system of two extrasolar planet-size objects simply as extrasolar binary planets. They constitute a true binary if the following conditions are satisfied instead of (c) in addition to the two criteria: (a) with replacing the Sun by a host star, and (b). (c1) Their total mass is dominant in the neighborhood around their orbits. (c2) Their common center of mass is above their surfaces. If it is below a surface of one object, we may call them an extrasolar planet–satellite.

There have been theoretical studies on the existence of planets with satellites. The solar system’s outer gaseous planets have multiple satellites, each of which notably has a similar fraction ($\sim 10^{-4}$) of their respective planet’s mass. For instance, Canup and Ward (2006) found that the mass fraction is regulated to be $\sim 10^{-4}$ by a balance between two competing processes of the material inflow to the satellites and the satellite loss through orbital decay driven by the gas. They suggested that similar processes could limit the largest satellite of extrasolar giant planets. Such theoretical predictions await future observational tests. There still remains a possibility of detecting Jupiter-size binary planets with comparable masses. Furthermore, we should note that their model does not hold for solid planets. It may be possible to detect binary solid planets (perhaps Earth-size ones). Therefore, future detection of extrasolar planet–companion systems, or a larger mass fraction ($> 10^{-4}$) of satellites around gaseous exoplanets will give definite information on the planet and satellite formation theory. In any case, unexpected findings will open up the possibilities of new configurations, such as binary planets.

Recent direct imaging of a planetary mass of $\sim 8 M_J$ with an apparent separation of 330 AU from the parent star (Lafrenière et al. 2008) indicates the likely existence of long-period exoplanets ($> 1000$ yr). In this paper, we consider such exoplanets as well as close ones.

Since the first detection of a transiting extrasolar planet (Charbonneau et al. 2000), photometric techniques have been successful (e.g., Deming et al. 2005 for probing atmosphere; Ohta et al. 2005; Winn et al. 2005; Gaudi & Winn 2007; Narita et al. 2007, 2008 for measuring stellar spins). In addition to COROT, 1 Kepler 2 has been very recently launched. It will monitor about $10^5$ stars with an expected 10 ppm ($= 10^{-5}$) photometric differential sensitivity. This enables the detection of Moon-size objects.

Sartoretti and Schneider (1999) first suggested a photometric detection of extrasolar satellites. Cabrera and Schneider (2007) developed a method based on the imaging of a planet–companion as an unresolved system (but resolved from its host star) using planet–companion mutual transits and mutual shadows. As an alternative method, timing offsets for a single

1. [http://www.esa.int/SPECIALS/COROT/]
2. [http://kepler.nasa.gov/]

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2. Mutual Transits of Extrasolar Planet–Companion Systems

2.1. Approximations and Notation

For the sake of simplicity, we assume that the orbital plane of a planet–companion system orbiting around its common center of mass (COM) is the same as the orbital plane of the COM in orbit around the host star with radius $R$. This coplanar assumption is a reasonable starting point, because it seems that planets are born from fragmentations of a single protostellar disk, and thus their spins and the orbital angular momentum are nearly parallel to the spin axis of the disk.

The inclination angles of the orbital plane with respect to the line of sight are chosen to be 90°, because transiting planets can be observed only for (nearly) the edge-on case. In order to clearly show our idea, the binaries are in circular motion as an approximation by (c1). It is straightforward to extend the circular motion to elliptic motions.

For investigating transits, we need the transverse position, $x$, and velocity, $v$. We denote those of the COM for planet–companion systems as $x_{\text{CM}}$ and $v_{\text{CM}}$, where the origin of $x$ is chosen as the center of the star. The positions and velocities of individual planets with mass $m_1$ and $m_2$ in the binary system with separation $a$ are denoted as $x_i$ and $v_j$ ($i = 1, 2$), respectively. We express the positions of individual planets as

$$x_1 = x_{\text{CM}} + a_1 \cos\omega(t - t_0),$$

$$x_2 = x_{\text{CM}} - a_2 \cos\omega(t - t_0),$$

where the orbital radii of individual planets around their COM are denoted by $a_i$, the angular velocity of the binary motion is denoted by $\omega$, and $t_0$ means the time when the binary separation becomes perpendicular to the line of sight (see table 1 for a list of the parameters and their definitions). For the sake of simplicity, we set $t_0 = 0$ below.

One can approximate $v_{\text{CM}}$ as being constant during the transit, because the duration is much shorter than the orbital period for the binary around the host star.

2.2. Transits in Light Curves

The decrease in the apparent luminosity due to mutual transits is expressed as

$$L = \frac{S - \Delta S}{S},$$

where $S = \pi R^2$, $S_1 = \pi r_1^2$, $S_2 = \pi r_2^2$, and $\Delta S = S_1 + S_2 - S_{12}$. Here, $r_1$ and $r_2$ denote the radii of planets 1 and 2, and $S_{12}$ denotes the area of the apparent overlap between them, which is seen from the observer. Without any loss of generality, we assume $r_1 \geq r_2$.

2.3. Effects on Light Curves

We now consider light curves by mutual transits of planet–companion systems. The time derivative of equation (2) becomes $v_2 = v_{\text{CM}} + a_{2\omega} \sin \omega (t - t_0)$. Hence, the apparent retrograde motion is observed if $v_{\text{CM}} < a_{2\omega}$, which we call the fast planet–companion’s spin. If $v_{\text{CM}} > a_{2\omega}$, we call it slow spin. The Earth–Moon and Jupiter–Ganymede systems represent slow and marginal cases, respectively.

Figure 1 shows light curves during mutual transits for two cases. One is the zero-spin limit, with $\omega \rightarrow 0$ as a reference. In this case, the motion of the two objects is nothing but a translation. Because of the time lag between the first and second transits, a certain plateau appears in the light curves. The other is a slow-spin case, such as $W = 1$, where $W$ denotes the dimensionless spin ratio, defined as $a_{2\omega} / v_{\text{CM}}$. Its basic feature is the same as that for the zero-spin case, except for certain changes that are due to the relative motion between the two
Table 1. List of quantities characterizing a system in this paper.

| Symbol | Definition |
|--------|------------|
| \( P_{\text{CM}} \) | Orbital period around a host star |
| \( P \) | Spin period of a planet–companion system |
| \( \omega \) | Angular velocity of a planet–companion system (\( = 2\pi / P \)) |
| \( a_{\text{CM}} \) | Distance of planet–companion’s center of mass from their host star |
| \( a \) | Separation of a planet–companion system |
| \( a_{\perp} \) | Apparent separation of a planet–companion system |
| \( R \) | Host star’s radius |
| \( r_1 \) | Planet’s radius |
| \( r_2 \) | Companion’s radius |
| \( m_1 \) | Planet’s mass |
| \( m_2 \) | Companion’s mass |
| \( m_{\text{tot}} \) | \( m_1 + m_2 \) |
| \( x_{\text{CM}} \) | Transverse position of a planet–companion’s center of mass |
| \( x_1 \) | Planet’s transverse position |
| \( x_2 \) | Companion’s transverse position |
| \( t_0 \) | Time at the maximum apparent separation of planet–companion |
| \( T_{\text{top}} \) | Time duration: width of a hill’s top in light curves |
| \( T_{\text{bottom}} \) | Time duration: width of a hill’s bottom in light curves |
| \( T_{12} \) | Time lag between the first and second transits |
| \( p \) | Detection probability for a given set of parameters |
| \( f \) | Fraction of Earth–Moon-like systems for an averaged star |

Fig. 1. Light curves: Solid red one denotes the zero binary’s spin limit as a reference (\( W \equiv a\omega / v_{\text{CM}} = 0 \)). Dashed green one is a slow-spin case (long distance) for \( W = 1 \). The vertical axis denotes the apparent luminosity (in percentages). The horizontal one is time in units of the half-crossing time of the star by the COM of the binary, defined as \( R / v_{\text{CM}} \). For the sake of simplicity, we assume a binary with a common mass density, a radius ratio of \( R : r_1 : r_2 = 20:2:1 \), and \( a/R = 0.9 \). planets. In the slow case, the averaged inclination of each slope in the light curve is dependent on the time, especially at the start and end of the mutual transit. Here, we assume a planet–companion system with a common mass density, a radius ratio of \( R : r_1 : r_2 = 20:2:1 \), and \( a/R = 0.9 \). Fast-spin cases are shown by figures 2 and 3 (\( W = 3 \) and 6, respectively), where the apparent retrograde motion produces characteristic fluctuations. Here, we assume the same configuration as that in figure 1, except for the shorter distance from the host star. These figures also show the transverse positions of the planets with time, which would help us to understand the chronological changes in the light curves. In particular, it can be understood that such characteristic patterns appear only when two faint objects are in front of the host star and one of them transits (or occults) the other.

2.4. Parameter Determinations through Mutual Transits

In all of the above cases, the amount of decrease in the light curves or the magnitude of fluctuations gives the ratios among the radii of the star and two faint objects, \((R, r_1, r_2)\). Through the behaviors of the apparent light curves in both the slow and fast cases, \( a\omega \) (as its ratio to \( v_{\text{CM}} \)) can be obtained as shown by figures 1–3. Here, \( v_{\text{CM}} \) is determined as \( v_{\text{CM}} = 2R/T_E \) by measuring the duration of the whole transit time, \( T_E \), because \( R \gg r_1, r_2 \), if the stellar radius, \( R \) (and mass \( m_3 \)), is known for instance by its spectral type. Therefore, \( a\omega \) is determined separately. The planet–companion’s spin velocity, \( a\omega \), determines the gravity between the objects.

The spin period, \( P \) (and thus \( \omega \)), can be determined, especially for the fast rotation that produces multiple “hills”, because an interval between neighboring “hills” is nothing but half of the binary period. As a result, the binary separation, \( a \), is obtained separately. Hence, one can determine the total mass of the binary as being \( Gm_{\text{tot}} = a^3 \omega^2 \) from Kepler’s third law, where \( G \) denotes the gravitational constant.

If we also assume that the mass density is common for two objects constituting the binary (this may be reasonable, especially for similar size objects, as \( r_1 \sim r_2 \)), each mass is determined as \( m_1 = r_1^3(r_1^3 + r_2^3)^{-1}m_{\text{tot}} \) and \( m_2 = r_2^3(r_1^3 + r_2^3)^{-1}m_{\text{tot}} \). Therefore, the orbital radius of each body around the COM is obtained as \( a_1 = r_2^3(r_1^3 + r_2^3)^{-1}a \) and \( a_2 = r_1^3(r_1^3 + r_2^3)^{-1}a \). At this point, importantly, the two objects can be
Fig. 2. Top panel: a light curve for a fast-spin case (short distance). The radius and mass ratios are the same as those in figure 1. We assume $W = 3$. Brightness fluctuations appear with a width of $T_{\text{top}} = 0.033$ and $T_{\text{bottom}} = 0.1$. These values satisfy equations (5) and (6). Bottom panel: the motion of each body in the direction of $x$ normalized by $R$ (solid red for the primary and dotted green for the secondary). When one faint object transits or occults the other in front of the host star, mutual transits occur, and “hills” appear in the light curve.

Fig. 3. Top panel: a light curve for a faster-spin case (shorter distance). Bottom panel: the motion of each body. The parameters are the same as those in figure 1, except for $W = 6$. Compared with figure 2, the number of “hills” increases, and the shape of the light curve becomes more complicated, especially at the bottom. A plateau at around $t = 0.5$ is due to a single transit of one faint object, since the other has passed across a host star.

In the slow-spin case, on the other hand, the apparent separation, $a_\perp$ (normal to the line of sight), is determined as $a_\perp = T_{12}v_{\text{CM}}$ from measuring the time lag, $T_{12}$, between the first and second transits, because $v_{\text{CM}}$ is known (see above).

Before closing this subsection, we briefly touch on the time scale of the brightness fluctuation. The full width of a “hill”, $T_{\text{hill}}$, corresponds to the crossing time of two planets, $2r/a\omega$. We thus obtain

$$a\omega = \frac{2r}{T_{\text{hill}}}.$$  

By measuring the width, therefore, $a\omega$ can be determined directly and independently only for the fast spin that produces spiky patterns. To be more precise, the full widths at the top and bottom of a “hill” are expressed as (also see figure 4)

$$T_{\text{top}} = \frac{2(r_1 - r_2)}{a\omega},$$

$$T_{\text{bottom}} = \frac{2(r_1 + r_2)}{a\omega}.$$  

Only for symmetric binaries ($r_1 = r_2$), we have $T_{\text{top}} = 0$, and thus true spikes. Otherwise, truncated spikes (or “hills”) appear. With $r_1$ and $r_2$ determined from brightness changes, measuring either $T_{\text{top}}$ or $T_{\text{bottom}}$ provides $a\omega$. This can be
verifed in figure 2. Figure 5 shows a flow chart of the parameter determinations that are discussed above.

The half width for giant planets is about

$$\frac{r}{a\omega} \sim 5 \times 10^3 \left( \frac{r}{5 \times 10^4 \text{km}} \frac{10^4 \text{km} \text{s}^{-1}}{a \omega} \right) \text{s.} \quad (7)$$

Therefore, the detections of such fluctuations due to mutual transits of extrasolar binary planets require frequent observations, say every hour. Furthermore, more frequency (e.g., every ten minutes) is necessary for parameter estimations of the binary.

Let us mention the connection of the present result with current space telescopes. A decrease in the apparent luminosity due to the secondary planet is $O(r_2^2/R^3)$. Besides the time resolution (or observation frequency) and mission lifetimes, the detection limits by COROT with the achieved accuracy of photometric measurements (700 ppm in one hour) could make $r_2/R \sim 2 \times 10^{-2}$. The nominal integration time is 32 s but coadded over 8.5 min except for 1000 selected targets for which the nominal sampling is preserved. By the Kepler mission with an expected 10 ppm differential sensitivity for solar-like stars with $n_T = 12$, the lower limit will be reduced to $r_2/R \sim 3 \times 10^{-3}$. An analogy of the Earth–Moon $(r_2/R \sim 2.5 \times 10^{-3}, W \sim 0.03)$ and Jupiter–Ganymede $(r_2/R \sim 4 \times 10^{-3}, W \sim 0.8)$ will be marginally detectable. Figure 6 shows a light curve due to an analogy of the Earth–Moon system. Observations both with high frequency (at least during the time of transits) and with good photometric sensitivity are desired for future detections of mutual transits. COROT satisfies these requirements, and thus has a chance to find mutual transits. Kepler (with CCDs readout every 3 s) is one of the most suitable missions to date for this goal.

2.5. Event Rate and Possible Bounds on Earth–Moon–like Systems

The probability of detecting mutual transits is expressed as $p = p_1 p_2 p_3$, where the probability for an object (with orbital period $P_{CM}$) transiting its host star during the observed time, $T_{\text{obs}}$, is denoted as $p_1 = T_{\text{obs}}/P_{CM}$, for that one component transiting (or occulting) the other during the eclipse with duration $T_E$ is denoted as $p_2 = T_E/P$ for slow cases ($p_2 = 1$ for $P < T_E$), and that for the condition that an observer is located in directions where the eclipse can be seen is denoted as $p_3 = \theta_{\text{max}}/(\pi/2)$. Here, the maximum angle from the orbital plane becomes $\theta_{\text{max}} \equiv R/a_{CM}$. Hence, we obtain

$$p = \frac{2RT_E T_{\text{obs}}}{a_{CM} P P_{CM}}, \quad (8)$$

which becomes $p \sim 6 \times 10^{-5} T_{\text{obs}} \text{yr}^{-1}$ for an Earth–Moon–like system. We thus need to monitor a number of stars ($N_S > 10^4$). Let $f$ denote the fraction of such systems for an averaged star. We have the expected events, $n$, for observing $N_S$ stars during $T_{\text{obs}}$ as $n = f p N_S$. Therefore, monitoring $10^7$ stars for 3 yr with Kepler may lead to the discovery of a second Earth–Moon–like system if the fraction is larger than 0.05, or it may put upper limits on the fraction as $f < (p N_S)^{-1} \sim 0.05$ (3 yr/$T_{\text{obs}}$) $10^7/N_S$.

We should note that there exist constraints due to some physical mechanisms on our parameters, especially the orbital separation. For instance, the companion’s orbital radius must be larger than the Roche limit and smaller than the Hill radius (e.g., Danby 1988; Murray & Dermott 2000). These stability conditions are satisfied by the Earth–Moon system. Therefore, we can use equation (8) for Earth–Moon analogies. For general cases, such as “exoearth–exomoon” systems that have much smaller separations, or are located at a much shorter distance from their parent star, however, we should take account of corrections due to certain physical constraints (e.g., Sartoretti
& Schneider 1999 for estimates of such conditional probabili-
ties with incorporating the Roche and Hill radii).

3. Conclusion

We have shown that light curves by mutual transits of
extrasolar planets strongly depend on a planet–companion’s
spin velocity, and especially for small-separation cases where
occultation of one faint object by the other transiting a parent
star causes an apparent increase in the light curves, and char-
acteristic fluctuations appear. We have also shown that extra-
solar mutual transits provide a complementary method for
measuring the radii of two transiting objects, their distance, and
masses, and consequently for identifying them as a true binary,
a planet–satellite system, or others. Event rates and possible
bounds on the fraction of Earth–Moon-like systems have been
presented. Up to this point, we have considered only the obscu-
ration effect in a simplistic manner. When actual light curves
are analyzed, we should incorporate (1) a small deviation of
the inclination angle from 90°, (2) elliptical motions of the
binary, and (3) perturbations as three- (or more-) body inter-
actions (e.g., Danby 1988; Murray & Dermott 2000). Limb
darkenings also should be taken into account.

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