The longitudinal structure function $F_L$: perturbative QCD and $k_T$-factorization versus experimental data at fixed $W$

A.V. Kotikov

Institut für Theoretische Teilchenphysik
Universität Karlsruhe
D-76128 Karlsruhe, Germany

A.V. Lipatov

Department of Physics
Lomonosov Moscow State University
119899 Moscow, Russia

N.P. Zotov

Skobeltsyn Institute of Nuclear Physics
Lomonosov Moscow State University
119992 Moscow, Russia

Abstract

We use results for the structure function $F_L$ for a gluon target having nonzero transverse momentum square at order $\alpha_s$, obtained in our previous paper, to compare with recent H1 experimental data for $F_L$ at fixed $W$ values and with collinear GRV predictions at LO and NLO approximation.

PACS number(s): 13.60.Hb, 12.38.Bx, 13.15.Dk
The longitudinal structure function (SF) $F_L(x, Q^2)$ is a very sensitive QCD characteristic and is directly connected to the gluon content of the proton. It is equal to zero in the parton model with spin $-\frac{1}{2}$ partons and has got nonzero values in the framework of perturbative Quantum Chromodynamics. The perturbative QCD, however, leads to a quite controversial results. At the leading order (LO) approximation $F_L$ amounts to about $10 \div 20\%$ of the corresponding $F_2$ values at large $Q^2$ range and, thus, it has got quite large contributions at low $x$ range. The next-to-leading order (NLO) corrections to the longitudinal coefficient function are large and negative at small $x$ [1]-[5] and can lead to negative $F_L$ values at low $x$ and low $Q^2$ values (see [5, 6]). Negative $F_L$ values demonstrate a limitations of the applicability of perturbation theory and the necessity of a resummation procedure, that leads to coupling constant scale higher than $Q^2$ (see [5], [7]-[9]).

The experimental extraction of $F_L$ data requires a rather cumbersome procedure, especially at small values of $x$ (see [10], for example). Recently, however, there have been presented new precise preliminary H1 data [11] on the longitudinal SF $F_L$, which have probed the small-$x$ region $10^{-5} \leq x \leq 10^{-2}$.

In this paper the standard perturbative QCD formulas and also the so called $k_T$-factorization approach [12] based on Balitsky-Fadin-Kuraev-Lipatov (BFKL) dynamics [13] (see also recent review [14] and references therein) is used for the analysis of the above data. The perturbative QCD approach is called hereafter as collinear approximation and applied at LO and NLO levels using GRV parameterizations for parton densities (see [15]). The corresponding coefficient functions are taken from the papers [3, 1].

In the framework of the $k_T$-factorization approach, which is of primary consideration in our paper, a study of the longitudinal SF $F_L$ has been done firstly in Ref. [16], where the small $x$ asymptotics of $F_L$ has been obtained analytically using the BFKL results for the Mellin transform of the unintegrated gluon distribution and the longitudinal Wilson coefficient functions for the full perturbative series has been calculated at asymptotically small $x$ values. In this note we follow a more phenomenological approach [18] where we analyzed $F_L$ data in a broader range at small $x$ and, thus, we use parameterizations of the unintegrated gluon distribution function $\Phi_g(x, k_T^2)$ (see Ref. [14]).

A similar study has been already done $^2$ in our paper [18] using previous H1 data [21]. The recent H1 preliminary experimental data [11] is essentially more precise, that stimulates the present additional study.

1. The unintegrated gluon distribution $\Phi_g(x, k_T^2)$ ($f_g$ is the (integrated) gluon distribution in the proton multiplied by $x$ and $k_\perp$ is the transverse part of the gluon 4-momentum $k^\mu$)

$$f_g(x, Q^2) = \int_{Q^2}^{\infty} dk_\perp^2 \Phi_g(x, k_\perp^2) \quad \text{(hereafter} \quad k^2 = -k_\perp^2) \quad (1)$$

is the basic dynamical quantity in the $k_T$-factorization approach $^3$. It satisfies the BFKL equation [13].

$^2$Note that the studies of the $F_L$ structure function in the framework of the $k_T$-factorization have been done also in [19, 20].

$^3$In our previous analysis [17] we have shown that the property $k^2 = -k_\perp^2$ leads to the equality of the Bjorken $x$ value in the standard renormalization-group approach and in the Sudakov one.
Then, in the $k_T$-factorization the SF $F_{2,L}(x, Q^2)$ are driven at small $x$ primarily by gluons and are related in the following way to the unintegrated distribution $\Phi_g(x, k_T^2)$:

$$F_{2,L}(x, Q^2) = \int_x^1 \frac{dz}{z} \int Q^2 dk_T^2 \sum_{i=u,d,s,c} e_i^2 \cdot \tilde{C}^g_{2,L}(x/z, Q^2, m_i^2, k_T^2) \Phi_g(z, k_T^2),$$

(2)

where $e_i^2$ are charge squares of active quarks.

The functions $\tilde{C}^g_{2,L}(x, Q^2, m_i^2, k_T^2)$ can be regarded as SF of the off-shell gluons with virtuality $k_T^2$ (hereafter we call them hard structure functions by analogy with similar relations between cross-sections and hard cross-sections). They are described by the sum of the quark box (and crossed box) diagram contribution to the photon-gluon interaction (see, for example, Fig. 1 in [17] and [18]).

2. Notice that the $k_T^2$-integral in Eqs. (1) and (2) can be divergent at lower limit, at least for some parameterizations of $\Phi_g(x, k_T^2)$. To overcome the problem we change the low $Q^2$ asymptotics of the QCD coupling constant within hard structure functions. We apply here two models: the “freezing” procedure and Shirkov-Solovtsov analytization.

The “freezing” of the strong coupling constant is very popular phenomenological model for infrared behavior of $\alpha_s(Q^2)$. The “freezing” can be done in the hard way and in the soft way.

In the hard case (see [22], for example), the strong coupling constant itself is modified: it is taken to be constant at all $Q^2$ values less than some $Q_0^2$, i.e. $\alpha_s(Q^2) = \alpha_s(Q_0^2)$, if $Q^2 \leq Q_0^2$.

In the soft case (see [20], for example), the subject of the modification is the argument of the strong coupling constant. It contains the shift $Q^2 \rightarrow Q^2 + M^2$, where $M$ is an additional scale, which strongly modifies the infrared $\alpha_s$ properties. For massless produced quarks, $\rho$-meson mass $m_\rho$ is usually taken as the $M$ value, i.e. $M = m_\rho$. In the case of massive quarks with mass $m_i$, the $M = 2m_i$ value is usually used. Below we will use the soft version of “freezing” procedure.

Shirkov and Solovtsov proposed [23] a procedure of analytization of the strong coupling constant $\alpha_s(Q^2)$, which leads to a new strong analytical coupling constant $a_{an}(Q^2)$ having nonstandard infrared properties. We are not in position to discuss here theoretical aspects of the procedure and use only the final formulae for the analytical coupling constant $a_{an}(Q^2)$. They have the following form

$$\frac{a_{an}(Q^2)}{4\pi} = \frac{1}{\beta_0} \left[ \frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} \right],$$

(3)

in the LO approximation and

$$\frac{a_{an}(Q^2)}{4\pi} = \frac{1}{\beta_0} \left[ \frac{1}{\ln(Q^2/\Lambda^2) + b_1 \ln[1 + \ln(Q^2/\Lambda^2)/b_1]} + \frac{1}{2} \frac{\Lambda^2}{\Lambda^2 - Q^2} - \frac{\Lambda^2}{Q^2} C_1 \right],$$

(4)

in the NLO approximation, where $\beta_0$ and $\beta_1$ are the two first terms in the $\alpha_s$-expansion of $\beta$-function and $b_1 = \beta_1/\beta_0^2$. The constant $C_1 = 0.0354$ is very small.
The first terms in the r.h.s. of Eqs. (3) and (4) are the standard LO and NLO representations for $\alpha_s(Q^2)$. The additional terms modify its infrared properties.

Note that numerically both infrared transformations, the “freezing” procedure and Shirkov-Solovtsov analytization, lead to very close results (see below Fig. 1 and also Ref. [24] and discussion therein).

Figure 1: $Q^2$ dependence of $F_L(x, Q^2)$ (at fixed $W = 276$ GeV). The H1 preliminary, $e^+p$ and $e^-p$ experimental data are shown as the black points, black and white squares, respectively (see [11]). Theoretical curves obtained in the $k_T$–factorization approach with the JB unintegrated gluon distribution: solid curve corresponds to ”frozen” coupling constant, dashed curve - analytical coupling constant, dash-dotted - ”frozen” argument of the unintegrated gluon distribution function.

3. As it was already noted above, the purpose of the paper is to describe new preliminary H1 experimental data for the longitudinal SF $F_L(x, Q^2)$ using our calculations of the hard SF $\hat{C}_{2tL}^g(x, Q^2, m^2, k_{\perp}^2)$ given in our previous study [17] and infrared modifications of $\alpha_s(Q^2)$, explained above. For the unintegrated gluon distribution $\Phi(x, k_{\perp}^2, Q_0^2)$ we use the so called Blumlein’s parametrization (JB) [25]. Note that there are also several other popu-
lar parameterizations, which give quite similar results excepting, perhaps, the contributions from the small $k_\perp^2$-range: $k_\perp^2 \leq 1 \text{ GeV}^2$ (see Ref. [14] and references therein).

The JB form depends strongly on the Pomeron intercept value. In different models the Pomeron intercept has different values (see [26]). So, in our calculations we apply the H1 parameterization [27] based on the corresponding H1 data, which are in good agreement with perturbative QCD (see Refs. [27, 28]).

We calculate the SF $F_L$ as the sum of two types of contributions: the charm quark one $F_L^c$ and the light quark one $F_L^l$:

$$F_L = F_L^l + F_L^c$$  \hspace{1cm} (5)

For the $F_L^l$ part we use the massless limit of hard SF (see [17, 18]). We always use $f = 4$ in our fits, because our results depend very weakly on the exact $f$ value (for similar results see fits of experimental data in [29] and discussions therein). The weak dependence comes from two basic properties. Firstly, the charm part of $F_L$, $F_L^c$, is quite small at the considered $Q^2$ values (see Ref. [18] for the $F_L^c$ study). Secondly, the strong coupling constant very weakly depends on $f$ because of the corresponding relations between $\Lambda$ values at different $f$ (see [30]).

In Fig. 1 we show the SF $F_L$ with “frozen” and analytical coupling constants, respectively, as a function of $Q^2$ for fixed $W$ in comparison with H1 experimental data sets (see [11]). The results are mostly coincide with each other. They are presented as bold and dashed curves, which cannot be really resolved in the figure.

The dash-dotted curve shows the results obtained with “frozen” argument of the unintegrated gluon density. The difference between the bold and dash-dotted lines is not so big, that demonstrates the unimportance of the infrared modifications of the density argument. Below we only restrict ourselves only to the modification of the argument in the strong coupling constant entering the hard structure function.

Fig. 2 contains the same bold curve as Fig. 1 and shows also the collinear results for $F_L$ values. We use the popular GRV parameterizations [15] at LO and NLO approximations. The $k_T$-factorization results lie between the collinear ones, that demonstrates clearly the particular resummation of high-order collinear contributions at small $x$ values in the $k_T$-factorization approach.

We also see excellent agreement between the experimental data and collinear approach with GRV parton densities at NLO approximation. The NLO corrections are large and negative and decrease the $F_L$ value by an approximate factor of 2 at $Q^2 < 10 \text{ GeV}^2$.

In Figs. 1 and 2, our $k_T$-factorization results are in good agreement with the data for large and small parts of the $Q^2$ range. We have, however, some disagreement between the data and theoretical predictions at $Q^2 \sim 3 \text{ GeV}^2$. The disagreement exists in both cases: for collinear QCD approach at the LO approximation and for $k_T$-factorization.

Comparing these results with Fig. 4 of Lobodzinska’s talk in Ref. [11] we conclude that the disagreement comes from the usage of the LO approximation. Unfortunately, at the moment in the $k_T$-factorization approach only the LO terms are available. The calculation of the NLO corrections is a very complicated problem (see [31] and discussion therein).

A rough estimation of the NLO corrections in the $k_T$-factorization approach can be done in the following way. Consider first the BFKL approach. A popular resummation of the NLO corrections is done in [8] at some approximation. Ref. [8] demonstrates, that is the
Figure 2: $Q^2$ dependence of $F_L(x, Q^2)$ (at fixed $W = 276$ GeV). The experimental points are as in Fig. 1. Solid curve is the result of the $k_T$–factorization approach with the JB unintegrated gluon distribution and “frozen” coupling constant, dashed curve - the GRV LO calculations, dash-dotted curve - the GRV NLO calculations, dotted curve - the result of the GRV LO calculations with $\mu^2 = 127Q^2$.

The basic effect of the NLO corrections, that is the strong rise of the $\alpha_s$ argument from $Q^2$ to $Q_{eff}^2 = K \cdot Q^2$, where $K = 127$, i.e. $K \gg 1$, which is in agreement with [5], [7] and [9]. The use of the effective argument $Q_{eff}^2$ in the DGLAP approach at LO approximation leads to results which are very close to the ones obtained in the case of NLO approximation: see the dot-dashed and dotted curves in Fig. 2. Thus, we hope that the effective argument represents the basic effect of the NLO corrections in the framework of the $k_T$–factorization, which in some sense lies between the DGLAP and BFKL approaches as it was noted above already.

The necessity of large effective arguments is also demonstrated in Fig. 3, where we show the $k_T$–factorization and collinear results for nonrunning coupling constant. Its argument is fixed at $Q^2 = M_Z^2$ giving $\alpha_s \approx 0.118$ (see [32]), i.e. the considered argument is larger than...
Figure 3: $Q^2$ dependence of $F_L(x, Q^2)$ (at fixed $W = 276$ GeV). The experimental points are as in Fig. 1. Solid curve is the result of the $k_T$-factorization approach with the JB unintegrated gluon distribution and $\mu^2 = M_Z^2$, dashed curve - the GRV LO calculations at $\mu^2 = M_Z^2$.

the most part of the $Q^2$-values of the considered experimental data. 

The results obtained in the $k_T$-factorization and collinear approaches based on $Q^2_{eff}$ argument are presented in Fig. 4. In comparison with the ones shown in Fig. 1, they are close to each other because the effective argument is essentially larger than the $Q^2$ value. There is very good agreement between the experimental data and both theoretical approaches.

Moreover, we also present in Fig.4 the $F_L$ results based on the $R_{world}$-parameterization for the $R = \sigma_L/\sigma_T$ ratio (see [33]) (because $F_L = F_2R/(1 + R)$), improved in [34, 35] for low $Q^2$ values and the parameterization of $F_2$ data used in the our previous paper [18]. The results are in good agreement with other theoretical predictions as well as with experimental data.

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4The study is also initiated by conversation with L.Lönnblad, we thank him.
Figure 4: $Q^2$ dependence of $F_L(x, Q^2)$ (at fixed $W = 276 \text{ GeV}$). The experimental points are as in Fig. 1. Solid curve is the result of the $k_T$–factorization approach with the JB unintegrated gluon distribution and at $\mu^2 = 127Q^2$; dashed curve - the GRV LO calculations at $\mu^2 = 127Q^2$, dash-dotted curve - from the $R_{\text{world}}$-parametrization.

4. Resume. In the framework of $k_T$–factorization we have applied the results of the calculation of the perturbative parts for the structure functions $F_L$ and $F_L^c$ for a gluon target, having nonzero momentum square, in the process of photon-gluon fusion [17, 18] to the analysis of recent H1 preliminary data. The perturbative QCD predictions are presented also at LO and NLO approximations.

We have found very good agreement between the experimental data and collinear results based on GRV parameterization at NLO approximation. The LO collinear and $k_T$–factorization results show disagreement with the data at some $Q^2$ values. We argued that the disagreement comes from the absence of the NLO corrections in the framework of the $k_T$–factorization. We modeled these NLO corrections by choosing large effective argument of the strong coupling constant and argued for our choice. The effective corrections signifi-
cantly improve the agreement with the H1 data under consideration.

Acknowledgements

We thank S.P. Baranov for careful reading of manuscript and useful remarks. The our study is supported in part by the RFBR grant. One of the authors (A.V.K.) is supported in part by Alexander von Humboldt fellowship. A.V.L. is supported in part by INTAS YSF-2002 grant N° 399 and ”Dinastiya” Fundation. N.P.Z. also acknowledge L. Jönsson for discussion of the H1 data [11] and the support of Crafoord Fundation (Sweden).

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