Bid Prediction in Repeated Auctions with Learning

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Abstract

We consider the problem of bid prediction in repeated auctions and evaluate the performance of econometric methods for learning agents using a dataset from a mainstream sponsored search auction marketplace. Sponsored search auctions is a billion dollar industry and the main source of revenue of several tech giants. A critical problem in optimizing such marketplaces is understanding how bidders will react to changes in the auction design. We propose the use of no-regret based econometrics for bid prediction, modelling players as no-regret learners with respect to a utility function, unknown to the analyst. We apply these methods in a real-world dataset from the BingAds sponsored search auction marketplace and show that no-regret econometric methods perform comparable to state-of-the-art time-series machine learning methods when there is no co-variate shift, but significantly out-perform machine learning methods when there is a co-variate shift between the training and test periods. This portrays the importance of using structural econometric approaches in predicting how players will respond to changes in the market. Moreover, we show that among structural econometric methods, approaches based on no-regret learning out-perform more traditional, equilibrium-based, econometric methods that assume that players continuously best-respond to competition.

1 Introduction

Sponsored search auctions are one of the most prominent revenue sources of modern tech giants and among the most profitable electronic marketplaces. Understanding how to design and optimize mechanisms for online ad auctions has been the focus of a long line of work at the intersection of economics and computation, with several streams of research analyzing the design of approximately optimal simple auctions (Edelman et al., 2007; Caragiannis et al., 2015; Lucier et al., 2012), optimizing reserve prices (Mohri and Medina, 2016), estimation of returns-on-investment (Lewis and Rao, 2015) and analyzing structural parameters (Athey and Nekipelov, 2010; Syrgkanis et al., 2015).

An important step in optimizing sponsored search marketplaces is the ability to understand how bidders will respond to market changes or market shocks. One can take a fully unstructured approach to the bid prediction problem by treating it as a time-series forecasting problem. However, such approaches can potentially overfit to the current market design or market setting and will not be able to extrapolate well, when co-variate shifts arise in the data.

Economic and econometric theory can potentially be beneficial for performing such extrapolation tasks and improve the ability to predict counterfactual behavior. In this paper we perform an empirical evaluation of this statement. The environment that the bidders are facing can be best thought of as a repeated game-theoretic strategic interaction, where the bids of one player affect the reward of another. Thereby, learning models of repeated strategic interactions are most appropriate. One can potentially think of the task as a multi-agent inverse reinforcement learning problem (IRL) (Russell, 1998; Ng and Russell, 2000; Yu et al., 2019). However, the players in a sponsored search auction are facing a very complex auction design and a dynamically changing strategic environment.
Therefore classical approaches to multi-agent IRL that typically assume that players form consistent beliefs can be problematic. Similar problems arise if one uses more classical economic approaches that assume that the system is at equilibrium (Athey and Nekipelov [2010], Paarsch et al. [2006]), or variations of equilibrium notions that incorporate bounded rationality (Rong et al., 2016).

An alternative that has received recent attention is modelling players as invoking no-regret learning algorithms [Syrgkanis et al. (2015), Nisan and Noti (2017a,b), Braverman et al. (2018), Alaei et al. (2019)]. However, all prior work focused primarily at uncovering structural parameters of the utility of the bidders, such as the value per click, and did not address the empirical performance of these methods in terms of their predictive power. In this work we test whether such models of no-regret learning behavior can have predictive power and out-perform baselines. We note that even though this theory is making several strong behavioral assumptions on the bidders, we mostly use these assumptions to impose structure on our estimation approach so as to regularize and extrapolate better. The ultimate judge is how well these methods perform in terms of prediction. Thus, in the spirit of George Box’s writings (Box [1976]), even if these theories could be wrong, our goal is to test whether they could potentially be useful.

Using a large auction dataset from Microsoft’s BingAds sponsored search auction marketplace, we show that regret-based bid prediction methods perform comparable to machine learning baselines when there is no co-variate shift, but outperform these baselines in a statistically significant manner in the presence of a co-variate shift.

2 Bid Prediction in Repeated Ad Auctions

We consider a repeated sponsored search auction setting. At each period $t$ a set of $n_t$ bidders participate in an auction for advertisement slots that will appear alongside the search results triggered by user queries. We assume that there are $n_t$ bidders and $m$ slots and each slot corresponds to a different probability of receiving a click. Each player $i$ submits a bid $b_{i,t}$ at each time $t$ and based on the vector of bids, the auction decides an allocation of the slots and a price that each bidder needs to pay. For the goal of our analysis, the actual mechanics of the auction are not important and moreover are too complex to describe, even if proprietary constraints where not at play.

What is important are the following fundamentals. At every period $t$, we associate two functions that are sufficient statistics for the strategic reasoning of player $i$: a probability of click curve $x_{i,t}^c : \mathbb{R}_+ \rightarrow [0, 1]$ and a cost-per-click (CPC) curve $p_{i,t} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, which for every bid $b$ return the average probability of receiving a click and the average cost-per-click for the auctions that occurred during that period had the player submitted a bid $b$. In practice, such curves are reported to the bidders at periodic intervals through revenue optimization feedback tools provided by sponsored search auction marketplaces. Given that from now on we will be mostly focusing on the perspective of a single bidder, we will drop the index $i$. The competition stemming from other bidders is summarized in the sufficient statistic of the cost and click curves. Our main question is the following:

**Given a sequence of bids $b_{1:t}$ for an advertiser, can we forecast the future time series $b_{t+1:T}$? Moreover, can we forecast this series when there is a change in the market at time $t$ (a co-variate shift)?**

For each of these variants of the question we will consider two forecasting tasks. In the first, we are asked to produce a future time-series $b_{t+1:T}$, where the prediction of bid $b_{\tau}$, for $\tau > t$, is performed solely with knowledge of the bids $b_{1:t}$ and the past cost and click curves $x_{1:t}, p_{1:t}$, and the cost and click curves up till time $\tau - 1$, i.e. $x_{t+1:\tau-1}$, $p_{t+1:\tau-1}$. In the second, we will consider a one-step-ahead prediction, where we forecast $b_{\tau}$ from bids $b_{1:\tau-1}$ and curves $x_{1:\tau-1}, p_{1:\tau-1}$.

3 No-Regret Learning and Structural Econometrics

We will consider a structural econometric approach to the bid prediction task, invoking techniques from classical auction theory and econometrics in auctions (see e.g. (Paarsch et al. [2006])). Since most classical econometric theory in auctions tackles static auction settings or imposes very strong dynamic equilibrium conditions, we will primarily focus on the recent line of work at the intersection

\[\text{Typically forecasting cost and click curves is an easier task, since they only depend on aggregate market statistics, hence for our structural econometric methods we will assume that such forecasts are available.}\]
of economics and computer science, that models players as no-regret learners and performs econometrics under such a behavioral assumption (Nekipelov et al. 2015; Nisan and Noti 2017a,b). In this section we describe the assumptions that these structural methods are making and how to transfer these assumptions to an estimation and prediction strategy.

In order to understand how bidders will behave in the future in a model-based manner, one needs to model the objective that the bidders are optimizing and the approach that they use to optimize over time and handle uncertainty. A standard assumption in auction theory is that players have utility from each auction that takes the form: \( u_t(b; v) = (v - p_t(b))x_t(b) \), i.e. the utility is the expected number of clicks times the value-per-click (VPC) \( v \) minus the expected payment. Thus, the only parameter that we need to estimate from the data for each player is the value per-click \( v \).

A classical framework in machine learning on repeated decision making in the face of uncertainty is that of no-regret learning. The no-regret learning framework posits that bidders will choose a bid \( b_t \) at each period, such that their regret against submitting any fixed bid in hindsight vanishes to zero as they play for more and more periods, i.e.:

\[
\text{Regret}(u_{1:T}, b_{1:T}; v) = \sup_b \frac{1}{T} \sum_{t=1}^{T} (u_t(b; v) - u_t(b_t; v)) = o_p(1)
\]

We will adopt the regret framework and assume that bidders use some form of no-regret algorithm to optimize their bid. We note that contrary to the no-regret framework, traditional econometrics in auctions typically assumes that players best respond to the competition (i.e. \( b^* = \arg \max_b u_t(b; v) \)) and use this property to identify the value \( v \) (see e.g. (Athey and Nekipelov 2010) for such an econometric approach applied to sponsored search auction data). However, we will see in the empirical part that such best-response algorithms are out-performed by no-regret based algorithms in terms of their predictive power.

Since we are not only interested in uncovering structural parameters of the setting, but also in predicting future behavior, we will consider several classes of no-regret algorithms and fit their parameters to data. We will make the assumption that the utility of the player is a concave function of the bid, which renders the problem that the bidder is facing an online convex optimization problem (Zinkevich 2003; Shalev-Shwartz 2012) in one dimension. Thus we will consider several widely studied algorithms for this setting. In each of these algorithms we will describe the update rule \( h(\cdot) \) (the next bid of a player as a function of past bids, clicks and cost curves) and provide some context on where this update rule stems from.

**Online Gradient Descent (OGD) (Zinkevich 2003):**

\[
b_{t+1} = h_{\text{OGD},\eta,v}(b_t, u_t) := b_t + \eta \nabla_b u_t(b_t; v)
\]

OGD can be thought of as regularized best response with momentum, with respect to the last-period “linearized” utility \( \tilde{u}_t(b; v) = \nabla_b u_t(b_t; v) \cdot (b - b_t) \), i.e.: \( b_{t+1} = \arg \max_b \tilde{u}_t(b; v) - \frac{\eta}{2} \| b - b_t \|^2 \). Moreover, it can also be thought of as regularized best response to the past average of linearized utilities, with shrinkage bias: \( b_{t+1} = \arg \max_b \sum_{\tau=1}^{t} \tilde{u}_\tau(b; v) - \frac{\eta}{2} \| b \|^2 \).

**Implicit OGD (BR-Reg):** \( b_{t+1} \) is defined as the solution to the equation: \( b_{t+1} - b_t = \eta \nabla_b u_t(b_{t+1}; v) \). This is also referred to as the implicit gradient descent (Toulis and Airoldi 2017). This algorithm has the same interpretation as OGD of a regularized best-response with momentum, but without the linearization of the utility component, i.e. it is equivalent to:

\[
b_{t+1} = h_{\text{BR-Reg},\eta,v}(b_t, u_t) := \arg \max_b u_t(b; v) - \frac{1}{2\eta} \| b - b_t \|^2
\]

**FTRL without linearization and with recency bias:**

\[
b_{t+1} = h_{\text{FTRL},\eta,v}(u_{1:t}) := \arg \max_b \sum_{\tau=1}^{t} \beta^{t-\tau} u_\tau(b; v) - \frac{1}{2\eta} \| b \|^2
\]

This implies that \( b_{t+1} \) is defined as the solution to the equation: \( b_{t+1} = \eta \sum_{\tau=1}^{t} \beta^{t-\tau} \sum_b u_\tau(b_{t+1}; v) \). Recency bias has been analyzed in the context of no-regret algorithms (Fudenberg and Levine 2014) and relates to learning in changing environments (Hazan and Seshadhri 2009; Adamskiy et al.)
and fast convergence in games (Syrgkanis et al., 2015). We will also refer to the special case where $\beta = 1$ and $\eta = \infty$ as the Follow-the-Leader algorithm (FTL), which is also a no-regret algorithm when the utility functions are strongly concave (see e.g. [Hazan et al., 2007]).

Algorithm-independent VPC estimation: We will estimate the VPC $v$ of the player solely based on the no-regret condition and irrespective of the update algorithm. We consider the value estimation algorithms proposed in (Nekipelov et al., 2015; Nisan and Noti, 2017b). The basic approach one could take (as described in (Nekipelov et al., 2015)) is to choose the parameter $v$ that achieves the smallest possible regret level for the player, referred to as the min-regret estimate, i.e.: $v_{\text{mr}} = \arg\min_{v \in \mathcal{V}} \text{Regret}(u_{1:T}, b_{1:T}; v)$. A more stable alternative was provided in [Nisan and Noti, 2017b] that propose the use of a soft-min version of min regret, referred to as the quantal-regret estimate:

$$v_{qr} = \frac{\sum_{v \in \mathcal{V}} v \exp\{-\lambda \text{Regret}(u_{1:T}, b_{1:T}; v)\}}{\sum_{v \in \mathcal{V}} \exp\{-\lambda \text{Regret}(u_{1:T}, b_{1:T}; v)\}}$$

where $\mathcal{V}$ is a set of candidate valuations. The authors also provide a Bayesian justification of this choice as imposing a prior on the space of valuations.

We found empirically that the quantal-regret value estimate is more stable and less sensitive to estimation errors than the min-regret estimate. Moreover, Figure 1 justifies the use of $v_{qr}$ over $v_{mr}$ on our data, as it leads to more reasonable predictions on how much players shade their bid (i.e. what fraction of their value is their bid), and how their valuation varies across days of the week, if we were to learn a separate value on solely the dataset of each day. Moreover, we find that the bid difference is positively correlated with the recent gradient of the utility evaluated at the quantal-regret estimate, as predicted by the OGD algorithm, providing further justification for the use of the quantal-regret value (see Appendix A).

![Figure 1: Comparison of min regret (a & b right) vs. quantal regret (a & b left) value estimates.](Image)

Algorithm-specific step-size estimation: All the algorithms that we consider contain a step-size parameter $\eta$ that intuitively controls how aggressively the algorithm responds to new evidence. We will estimate $\hat{\eta}$ of each algorithm from the data by minimizing the mean squared prediction error on the training set: $\arg\min_{\eta} \frac{1}{T} \sum_{t=1}^{T} (\hat{b}_{t} - h(b_{t-1, u_{t-1}}))^2$, where $h$ is the algorithm update rule. Since this is a scalar parameter, in the worst-case the optimization requires a grid search. Observe that for OGD, finding $\eta$ that minimizes the mean squared prediction error on the training set:

$$\hat{\eta}_\text{ODG} := \arg\min_{\eta \in \mathbb{R}_+} \frac{1}{T} \sum_{i=1}^{T} (b_{i+1} - b_i - \eta \nabla_b u_t(b_i))^2$$

is equivalent to finding the $\eta$ from the linear regression of $b_{i+1} - b_i$ on $\nabla_b u_t(b_i)$. For the other algorithms that we consider we perform a grid search to solve the optimization problems.

2It is possible to view the econ counterfactual-curve based algorithms, in which players respond to their estimates on the other players’ average behavior, as mean-field algorithms Iyer et al. (2014) with no additional constraints. The most basic example would be the simple best response to the last observed click and cost curves. We have also evaluated FTL with recency bias, i.e., the special case of FTRL with $\eta = \infty$ and $\beta = 0.9$ that can be viewed as mean field where the beliefs of the players are estimated by the running averages of recent curves. However, in the paper we present the basic variant of FTL with $\beta = 1$ (i.e., with no recency bias) that achieves better results on our empirical dataset.

3In practice we take $\mathcal{V}$ to be a grid of values ranging from 1% of the average bid to 6 times the average bid.

4In practice we enforce positivity of $\eta$ by returning the absolute value of the unconstrained optimal solution.
4 Data Description

We analyze sponsored-search auction data from the BingAds sponsored search auction marketplace. The dataset consists of bidding data for 13 high-volume keywords collected in a period of two weeks. For each bidder, we analyze data that include the bids the player made, the bidder’s cost-per-click (CPC), the slot and the clicks that the bidder won in each auction, as well as counterfactual data of CPC and click rates that the bidder could have obtained for different counterfactual bids according to the competition in the auction. These counterfactual information was generated via the Genie system (Bayir et al., 2019). For each bidder and auction, the counterfactual data consist of CPC and probability of click points for counterfactual bids according to 12 different multipliers of the actual bid the bidder made: \((0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 5.0)\).

Typically, bidders participate in multiple auctions every hour, and we aggregate the data by hour for each bidder. The hourly aggregated bid of a player is the average of bids the player placed during that hour. The hourly counterfactual data for each bidder is a collection of the 12-point discrete curves of each of the auctions the player participated during the hour. We aggregate these hourly curves for each player by fitting the click data points to a concave function of the form \(ax^b + x\), which gave the best out-of-sample MSE in a 5-fold cross validation on a validation keyword among the functional forms we tested (including a linear function, a sigmoid \(\frac{a}{1+e^{-bx+c}}\), and a convex function \(ax^2 + bx^4\)).

The CPC data points are fitted to a linear function \(ax\) to ensure concave utilities. This is of course a modeling assumption that is put to the test in the prediction performance of the economic models.

The dataset that we analyze consists of data of players who participated in auctions in at least 100 hours and won the top position at least once during the whole two weeks of data. In addition, we require that the players place only positive bids and have non-zero variance in both their train and test data (see details on data splitting in Section 5.1).

The full dataset, after the filtering described above, includes data of multiple thousands of bidders with an average of \(202.4\) active hours per bidder, which aggregate in total a number of auctions of the order of multiple millions. Figure 2a shows the normalized distribution of hourly-aggregated bids across all players in the dataset. The bidding levels are diverse and span several orders of magnitudes, with, roughly, a power-law distribution, as seen in the logarithmic scale inset. Figure 2b shows the distribution of sequence lengths across bidders. Figure 2c shows the distribution of coefficient of variation of bids across bidders. The most frequent bidding behavior is of moderate variability of around \(5\%\) deviation from the average. However, as seen in the median at \(12.8\%\), the majority of players have high variability bids. The average is as high as \(21.2\%\), due to a non-negligible number of bidders with extremely high variability, as seen by the power-law tail distribution; see the inset in logarithmic scale of players with coefficient of variation of above \(0.15\).

5 Predictive Performance

In this section we evaluate the predictive performance of the prediction methods in a setting in which the train and test data have similar distributions, which we refer to as the in-sample setting. We show that in this setting, ML methods do well and that the structured econ OGD method manages to achieve comparable results. We also observe large differences between BR and the no-regret
OGD method. In Section 5.4, we evaluate the performance of the methods discussed in Section 3 that interpolate between BR and OGD. The results show that the no-regret interpolating methods outperform those that are not regret minimizing.

5.1 The In-Sample Prediction Setting

We analyze the dataset described in Section 4. We use one keyword for development and validation and exclude this keyword from the prediction analysis. The dataset without the validation keyword consists of 96.2% of the players in the full dataset. For each player we divide the bid sequence and use the first 90% as training data and the last 10% for test. Training sequence lengths range between 90 and 303 hours with an average of 182.2, and test sequence lengths range between 10 and 33 hours, with an average of 19.7. We evaluate the predictive performance of the methods both in series prediction, where each model is trained on the training sequence of a player and then remains fixed for the prediction phase on the entire test sequence, and in a stepahead prediction task, where the models are re-fitted on the true data at every step and predict only a single step at a time.

5.2 Benchmark Machine-Learning (ML) Methods

We implemented the following ML benchmarks: A linear model which receives input of the two recent bids (“lag-2 input”); we refer to this model as AR2; As a non-linear benchmark we use a random forest model with lag-2 input (RF2); As a deep-learning benchmark we use multi-layered perceptron models with lag-2 input (MLP2); Facebook’s Prophet model Taylor and Letham (2018): an additive regression model that is designed to produce smooth forecasts of scalar data across time and to capture long and short term trends, as well as periodic signals, and is a natural state-of-the-art benchmark for the series bid prediction task. Prophet is trained for each player on the full sequence of training bids and the corresponding date and hour in day of each bid. See more details on these ML algorithms in Appendix B.

5.3 ML vs. Structured Econ Methods in the In-Sample Prediction Setting

Here we compare the predictive performance of the structured econ methods and the ML benchmarks. We evaluate the prediction success by the Mean Absolute Percentage Error (MAPE) across bidders. The MAPE is defined for each bidder \(i\) by

\[
\text{MAPE}_i = \frac{1}{|\text{Test}_i|} \sum_{t \in \text{Test}_i} \left| \frac{b_i^t - \hat{b}_i^t}{b_i^t} \right|
\]

Percentage error naturally allows aggregation of errors across bidders with bids of different scale (see Figure 3a).

Figures 3a and 3b show the distributions of performance in the in-sample setting. Overall, the results show that in this setting, where the train and test sequences come from similar distributions, the bid-based ML methods indeed do well. OGD manages to achieve comparable results to the ML algorithms in terms of the main mass of the distributions in both the series and the stepahead tasks, but has higher error in series prediction in terms of the mean error; this difference is statistically significant from RF2 and AR2, see Table 1a in Appendix C.1 for mean errors and confidence intervals. We also see that BR is significantly inferior to the other methods. Note that BR is depicted only in the series prediction results; the predictions of BR are the same for the series and the stepahead prediction settings as they are not a function of the previous bids but of the economic feedback.

Among the ML methods, the top performing methods in the series prediction task (Figure 3a and Table C.1a), are the non-linear RF2 model and the linear AR2 model. They outperform the state-of-the-art Prophet, probably due to the relatively small training data, on which the they manage to
train more effectively. In the in-sample stepahead prediction task (Figure 3b), all methods except Prophet perform similarly well, with a median absolute percentage error of less than 5%. Prophet is less suited for receiving step-by-step input and has the worst performance in this stepahead task. This disadvantage is statistically significant in terms of the means (see Table C.1b).

In the Appendix we show examples of bid curves and predictions of the OGD and the ML methods in the series and the stepahead prediction tasks. The series prediction is a hard task since errors may be accumulated with time. The examples show how Prophet manages to capture the bid dynamics well even in non-trivial dynamics, and that also OGD usually manages to capture the correct direction of bid change. The predictions of RF2, MLP2 and AR2 are qualitatively similar, all usually “cut” the bid curves somewhere close to their average. See Appendix D.1 for more details.

All in all, OGD achieves comparable results to the ML benchmarks, showing that the players’ actions are consistent with the economic feedback, as is captured by the utility functions estimated from the data (see Section 4). In Section 6 we show that the structured econ methods that rely on the economic feedback are particularly useful in a setting where there is a change in the bid distributions.

5.4 BR vs. OGD in the In-Sample Prediction Setting

We have seen that OGD is significantly better than BR. Figure 3c shows the performance of the methods that interpolate between BR and OGD presented in Section 3. The figure also shows the Momentum-BR method, which is a direct interpolation between BR and OGD that sets the next bid to the average between the current bid and the best-reply bid. It can be clearly seen that the no-regret methods predict closer to the actual bidding data than the methods that do not minimize regret. FTL, which is a no-regret algorithm for strongly concave utility functions, has higher error than the classic no-regret learning algorithms. The figure also shows the effect of the two key features that distinguish between BR and OGD – best reply to the history and regularization – in the performance of the interpolating methods. The comparison of BR and FTL shows that replying to the entire history improves the performance compared to the memory-less BR. However the more substantial improvement is obtained by adding a regularization term, as can be seen in the lower MAPE score of FTRL and BR-Reg. Among the no-regret methods, the BR-Reg and the computationally-efficient OGD have the best performance, with an advantage that is statistically significant compared to the other methods (see Table C.1a).

6 Predictive Performance with a Co-variate Shift

In this section we evaluate the methods in a setting where there is a change in the bid distribution. Our results show that in this more challenging co-variate shift prediction setting, the structured econ methods outperform the bid-based ML benchmarks that fail to adapt to the different data.

6.1 The Co-variate Shift Dataset

We wish to evaluate the prediction performance in a setting in which the test bids are significantly different than the training bids. For this purpose, we subselect days from the data described in Section 4 (excluding the validation keyword), for which the distribution of bids during the day
(10am to 9pm) is significantly different than the distribution of bids during the night (10pm to 9am), and use the day bids as the test set. Since the ML benchmarks usually benefit from larger training data than the 12 data points of a single night, we allow all methods to use all night data of a player as training. Specifically, to create the co-variate shift dataset, we consider full days with sufficiently non-trivial activity, i.e. days in which bidders participated in auctions throughout all 24 hours with a coefficient of variation of bids of at least 0.1. We subselect days where the day bids are different from all night bids for a player according to a Kolmogorov-Smirnov test (with \( p < 0.001 \)) and apply a two-sided t-test to confirm that the day bid distribution is not only distinguishable from the train data but also has a statistically significant higher average (with \( p < 0.05 \)).

In total, the co-variate shift dataset consists of 762 days for prediction for 260 bidders. The average number of training hours across bidders is 141.2 with a standard deviation of 29.8. The number of test hours is 12 for each test day (10am to 9pm), to a total of \( 12 \cdot 762 = 9,144 \) steps for prediction. Figure 4a shows the distributions of train bids (including all night bids for each player in the dataset) and test bids, both normalized by the average of the training bids for each player. The figure illustrates that indeed the distribution of test data is different than the training data. Figure 4b shows the average bid by the hour in day, across all test days in the co-variate shift dataset. The plot shows the average of test sequences on the right of the vertical line, and their preceding nights on the left, where each 24-hour sequence is normalized by its average. The shaded area shows the 25 to 75 percentiles for every hour. As can be seen, night bids are on average as low as 85% of the average bid, and the test bids are on average as high as 115% of the average bid.

6.2 ML vs. Structured Econ Methods with a Co-variate Shift

Figures 5a and 5b show the MAPE distributions of the bid-based ML methods and the structured econ methods BR and OGD in the co-variate shift setting. Appendix C.2 presents the mean MAPE across players as well as the confidence bounds. Clearly, the simple and computationally-efficient OGD achieves the best performance both in the series and in the stepahead prediction tasks. The bid-based ML methods have a higher error, and even the BR method, that had the worst performance in the in-sample setting (see Section 5) is now comparable to the ML methods. These results show that when there is a change in the data, it is better to react to the economic feedback than to the bid history that is no longer relevant.

In the series prediction task (Figures 5a and C.2a), among the ML methods, Prophet has the best median error, but still makes large errors for some of the players, as seen by the distribution width. RF2 has the second-best median, followed by AR2 and MLP2. Note that in this setting the length of the predicted series is 12, while in the in-sample setting (Section 5) the average predicted series length is 20, and therefore the error levels are not comparable. In contrast, it is possible to compare error levels across settings in the stepahead prediction. Figure 5b shows that as could be expected, in the challenging co-variate prediction setting all methods have higher median errors than in the in-sample setting. The OGD has the best performance with a median absolute error of 6.4% of the true bid.

To portray the qualitative difference of the different prediction methods, we present in the Appendix the prediction curves of the OGD and the ML methods alongside the actual bids, for a sample of bidders. The plots show how OGD typically matches the new bid level of the test data. In contrast, the ML methods fail to adapt to the new level of bids and their predictions typically remain close to the lower bids on which they were trained. See Appendix D.2 for more details.
Figure 5c shows the performance of the econ methods interpolating between BR and OGD that were discussed in Section 3. Unlike the results in the in-sample setting (see Section 5.3), in the co-variate shift setting the separation between no-regret methods and the methods that do not minimize regret is less clear. However, the results still show that the BR, which does not incorporate any element of learning, is worse than the other methods that do combine some form of no-regret learning.

7 ML Models with Economic Features

Figure 6 shows the MAPE results in the in-sample and the co-variate shift prediction settings. In the in-sample setting (Figure 6a), it can be seen that the models with the additional economic features lead to similar or slightly worse performance than the purely bid-based models. Thus, in this prediction setting, the bid information was a sufficient predictor of future bids and the extra econ features introduced noise. This is similar to previous results in repeated normal-form games [Kolumbus and Noti (2019)]. In contrast, Figure 6b shows that in the co-variate shift prediction setting in which the bids in the training data are different than the test data, the econ features can be useful in augmenting the ML methods. The most significant utilization of the economic features is achieved by the MLP2 method; MLP2 with economic features (“MLP2Econ”) achieves the best performance among all ML methods, while the bid-based MLP2 has the worst performance in this setting. Still, the simple econometric-based OGD that models regret-minimizing players has the best performance. These results further highlight the importance of structured-economic feedback when there are changes in the market that lead to changes in bid distributions, and the advantage of simple structured econ models in this setting.

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APPENDICES

A  Hypothesis Testing for OGD

Figure 7: Hypothesis testing related to the OGD algorithm’s plausibility. Distribution across players of the negative log of p-value of the correlation between $b_{t+1} - b_t$ and $\nabla_b u_t (b_t; v_{qr})$, multiplied by the sign of correlation. The distribution is shown for the subset of players with average $b_{t+1} - b_t$ of at least 1 cent across their bid series. We observe that for almost all such players OGD with value $v_{qr}$ is a plausible model (as most signed p-values are strictly positive).

B  Time-Series ML Methods

We implemented the following machine learning (ML) benchmarks:

**AR2**: A linear model that is implemented using LinearRegression in the python scikit-learn package [Pedregosa et al. (2011)]. The input to this model at each time step is the two recent bids. In the series prediction setting the two recent bids are the last two predictions made by the model and in the stepahead setting the input is the two recent true bids. We call this type of input “lag-2 input.”

**RF2**: as a non-linear machine learning benchmark we use a random forest model with lag-2 input. The random forest predictor has 100 trees with a maximum depth of 2, and bootstrap sub-sampling was used to build each tree. The model is implemented using RandomForestRegressor in the scikit-learn python package.

**MLP2**: As a deep-learning benchmark we use multi-layered perceptron models (fully connected feed-forward neural networks) with lag-2 inputs. The networks have two hidden layers with 128 units in each layer, relu activation function and are optimized using the ADAM optimizer with the legacy parameters of [Kingma and Ba (2014)] for 100 epochs, with a batch size of 10 and a learning rate of 0.0001. The networks are implemented using scikit-learn MLPRegressor. We fitted the number of hidden layers, the size of each layer, the learning rate, and the number of epochs on validation data of a single keyword that is separate from the test data analyzed in the paper.

**Prophet**: Facebook’s Prophet model [Taylor and Letham (2018)] is a modern additive regression model for time series forecasting. Prophet is trained for each player on the full sequence of training bids and the corresponding date and hour in day of each bid. Prophet is especially designed to produce smooth forecasts of scalar data across time and to capture long and short term trends, as well as periodic signals, and is a natural state-of-the-art benchmark for the series bid prediction task. For the stepahead prediction task we re-train Prophet before each prediction with the sequence of true bids up to the previous timestep and produce a new prediction at each step with the newly updated model. We implemented Prophet in python using the original source code available by the authors of [Taylor and Letham (2018)].

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7See also: [https://research.fb.com/blog/2017/02/prophet-forecasting-at-scale/](https://research.fb.com/blog/2017/02/prophet-forecasting-at-scale/)
## C MAPE Results: Means and Confidence Bounds

### C.1 The In-Sample Prediction Setting

| Method  | Mean  | StdErr | Lower Bound | Upper Bound |
|---------|-------|--------|-------------|-------------|
| RF2     | 0.104 | 0.002  | 0.101       | 0.108       |
| AR2     | 0.107 | 0.002  | 0.104       | 0.111       |
| Prophet | 0.114 | 0.002  | 0.111       | 0.118       |
| MLP2    | 0.117 | 0.002  | 0.114       | 0.121       |
| BR-Reg  | 0.118 | 0.002  | 0.115       | 0.122       |
| OGD     | 0.120 | 0.002  | 0.117       | 0.124       |
| FTRL    | 0.122 | 0.002  | 0.119       | 0.156       |
| FTL     | 0.238 | 0.002  | 0.234       | 0.243       |
| Momentum-BR | 0.253 | 0.002 | 0.249     | 0.257     |
| BR      | 0.274 | 0.002  | 0.270       | 0.279       |

(a) Series prediction

| Method  | Mean  | StdErr | Lower Bound | Upper Bound |
|---------|-------|--------|-------------|-------------|
| MLP2    | 0.080 | 0.002  | 0.077       | 0.083       |
| AR2     | 0.080 | 0.002  | 0.077       | 0.083       |
| OGD     | 0.080 | 0.002  | 0.077       | 0.083       |
| RF2     | 0.085 | 0.002  | 0.081       | 0.088       |
| Prophet | 0.101 | 0.002  | 0.098       | 0.104       |

(b) Stepahead prediction

Table 1: In-Sample Setting. Mean MAPE score across players, standard error of the mean and 95% confidence interval, excluding outliers according to the standard method of McGill et al. (1978).

### C.2 The Co-Variate Shift Prediction Setting

| Method  | Mean  | StdErr | Lower Bound | Upper Bound |
|---------|-------|--------|-------------|-------------|
| OGD     | 0.165 | 0.006  | 0.154       | 0.177       |
| FTL     | 0.168 | 0.007  | 0.155       | 0.181       |
| FTRL    | 0.172 | 0.006  | 0.159       | 0.184       |
| BR-Reg  | 0.173 | 0.006  | 0.161       | 0.184       |
| Momentum-BR | 0.177 | 0.006 | 0.164     | 0.190     |
| Prophet | 0.191 | 0.007  | 0.178       | 0.204       |
| BR      | 0.197 | 0.006  | 0.185       | 0.210       |
| RF2     | 0.203 | 0.006  | 0.190       | 0.215       |
| AR2     | 0.221 | 0.006  | 0.209       | 0.233       |
| MLP2    | 0.226 | 0.007  | 0.212       | 0.241       |

(a) Series prediction

| Method  | Mean  | StdErr | Lower Bound | Upper Bound |
|---------|-------|--------|-------------|-------------|
| OGD     | 0.086 | 0.005  | 0.077       | 0.095       |
| MLP2    | 0.092 | 0.004  | 0.083       | 0.100       |
| AR2     | 0.104 | 0.005  | 0.094       | 0.113       |
| RF2     | 0.106 | 0.005  | 0.097       | 0.114       |
| Prophet | 0.133 | 0.006  | 0.122       | 0.144       |

(b) Stepahead prediction

Table 2: The Co-Variate Shift Setting. Mean MAPE score across players, standard error of the mean and 95% confidence interval, excluding outliers according to the standard method of McGill et al. (1978).
D Prediction Examples

D.1 Prediction Examples in the In-Sample Prediction Setting

Figures 8 and 9 show examples of bid curves and predictions of the OGD and the ML methods in the in-sample prediction setting. The series prediction is a hard task since errors of a model are served as inputs to the next prediction steps and the errors may be accumulated. We see that Prophet manages to capture the bid dynamics well in most cases. E.g., Figure 8d shows an impressive projection of Prophet 30 hours to the future in a non-trivial behavior. Also OGD usually manages to capture the correct direction of bid change. In 8a it is interesting to see that both Prophet and OGD outline similar curves, not far from the true bids, although the models are very different and are relying on very different inputs. All models predict relatively smooth curves compared to the large hourly fluctuations observed in the bidding data. The predictions of the non-linear RF2 and MLP2 and the linear AR2 are qualitatively similar, all usually “cut” the bid curves somewhere close to their average. The better MAPE score of RF2 shows that it is usually closer to the average than the other two methods. Figure 9 shows the stepahead prediction for the same sample of 6 bidders. As can be seen, in the stepahead prediction task where the models are re-trained in each step with the true recent bid, the predictions of all models remain closer to the actual bid curve than in the series prediction task. Also the Prophet model, which is less suited for receiving step-by-step input, seems to benefit from this input and its predictions only get closer to the true bids with time; e.g. compare Prophet’s predictions for example f in stepahead (Figure 9) and in series (Figure 8) prediction tasks.

![Figure 8: Series predictions in the in-sample setting: Example curves (y-axis removed).](image)

![Figure 9: Stepahead predictions in the in-sample setting: Example curves (y-axis removed).](image)
D.2 Prediction Examples in the Co-variate Shift Prediction Setting

Figures 10 and 11 show examples of bid curves and predictions of the OGD and the ML methods in the co-variate shift prediction setting. In the series prediction task (Figure 10), the plots show how OGD typically matches the new bid level of the test data even though it did not see bids from that bid distribution in the training data. In contrast, the ML methods fail to adapt to the new level of bids and their predictions typically remain close to the lower bids on which they were trained. The Prophet model tends to predict periodicity even when it is absent in the true data, possibly due to Fourier analysis on a relatively small training dataset for each player that may produce noise artifacts. For example, in 10c, Prophet catches the increasing trend in bids, but still forces a periodic change. RF2, MLP2, and AR2, that all receive the same input of the 2 previous bids, predict similar smooth dynamics, though RF2 and MLP2 seem somewhat closer to the higher bid levels of the test data than the linear AR2 model. Figure 11 shows prediction examples for the same bidders in the stepahead prediction task. In this more simple prediction task, where the models are re-trained at each step with the true recent bid, the predictions of all models are closer to the actual bid curve than in the series prediction task, though still the OGD predictions seem to match the new bid level better than the ML methods which usually predict lower than the true bid.

Figure 10: Series predictions in the co-variate shift setting: Example curves (y-axis removed).

Figure 11: Steapahead predictions in the co-variate shift setting: Example curves (y-axis removed).