Long Range Correlations in the Disordered Phase of a Simple Three State Lattice Gas

G. Korniss, B. Schmittmann and R. K. P. Zia

Center for Stochastic Processes in Science and Engineering
and
Department of Physics, Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061-0435, USA
(June 10, 1996)

We investigate the dynamics of a three-state stochastic lattice gas, consisting of holes and two oppositely “charged” species of particles, under the influence of an “electric” field, at zero total charge. Interacting only through an excluded volume constraint, particles can hop to nearest neighbour empty sites. With increasing density and drive, the system orders into a charge-segregated state. Using a combination of Langevin equations and Monte Carlo simulations, we study the steady-state structure factors in the disordered phase where homogeneous configurations are stable against small harmonic perturbations. They show a discontinuity singularity at the origin which in real space leads to an intricate crossover between power laws of different kinds.

PACS: 64.60Cn, 66.30Hs, 82.20Mj

Keywords: Driven lattice gas; Monte Carlo simulations; Langevin equations; Long-range correlations.

For a system with short range microscopics, placed in thermal equilibrium, long range spatial correlations are absent in general. Their presence is a typical signal that the system is at a singular point. On the other hand, for systems in non-equilibrium steady states, long range correlations are often observed [4]. With conserved dynamics they generically arise if the system has strong spatial anisotropies and lacks detailed balance. A good example is the driven Ising lattice gas [1]. The role of the drive (“external field”) is to bias hopping rates along a particular direction on the lattice. In addition to many other unexpected features such as non-Hamiltonian fixed points controlling the critical behavior [2], the system exhibits long range spatial correlations at all temperatures above criticality, as a result of the breakdown of the fluctuation-dissipation theorem (FDT). In momentum space, this appears as a discontinuity singularity of the structure factor at the origin [3].

One can generalize the model of ref [3] in a fashion similar to the one leading to the Ising model to spin-1 [4] or Potts [5] models, by considering more than one species of particles. To start with, we have two species of particles (referred as +’s and −’s) driven in opposite directions and holes (empty sites). To keep the model simple, however, we may neglect the usual Ising nearest neighbor interaction and keep “only” the excluded volume constraint and the bias. This multi-species model, in both one and two dimensions, has been thoroughly investigated [6–13]. In the simplest scenario charge (+−) exchange is not allowed. Monte Carlo simulations [7] in two dimensions and mean-field studies [8] show that there is a transition, controlled by particle density and drive, from a spatially homogeneous (disordered) phase to a charge segregated one, where the excluded volume constraint leads to the mutual blocking of particles. If we soften the excluded volume constraint – by allowing exchange between nearest neighbor, oppositely charged particles on a much slower time scale than the particle hole exchange – the above transition still survives [11]. In the disordered phase, these two versions behave in much the same way. Thus, for simplicity, we will study only the case without charge exchange in this paper and focus on the spatial correlations and structure factors.

We consider a two dimensional fully periodic lattice with \( L_\perp \times L_\parallel \) sites, each of which can be empty or occupied by a single particle. Since we have two species we need two occupation variables \( n_1^x \) and \( n_2^x \), with \( n \) being 0 or 1, depending on whether a positive or negative particle is present at site \( x \). Although we refer to these particles as “charged”, they do not interact via the Coulomb potential. Instead, they respond to an external, uniform electric field \( E \), directed along the \(+x_\parallel\) axis. We restrict ourselves to zero total charge, i.e. \( \sum_x [n_1^x - n_2^x] = 0 \). In the absence of the drive the dynamics does not distinguish between the different species: both types hop randomly to nearest neighbor empty sites, with a rate \( \Gamma \). The electric field introduces a bias into these rates in such a way that jumps against the force will be exponentially suppressed. During one Monte Carlo step 2\( L_\perp L_\parallel \) nearest neighbor bonds are picked randomly. If a particle-hole pair is encountered, an exchange takes place with probability \( W = \Gamma \min\{1, \exp(qE \delta x_\parallel)\} \), where \( q = \pm 1 \) is the charge of the particle and \( \delta x_\parallel = \pm 1 \) is the change of the \( x_\parallel \) coordinate of the particle due to the jump.

For our simulations, we set \( \Gamma = 1 \). Using 30 × 30 and 60 × 60 lattices, we initialized the system with random configurations of various particle densities and carried out runs ranging from 2.5 to 5 × 10^5 MCS. After allowing 62500 MCS for the system to settle into a steady state, we measured the Fourier transform of \( n_1^\pm \) every 125 MCS. The structure factors are then constructed from averages of these measurements, according to...
\begin{equation}
S^{\alpha\beta}(k) = \frac{1}{L_z L_y} \sum_{x,y} e^{-i k \cdot (x-y)} \langle n_x^\alpha n_y^\beta \rangle ,
\end{equation}

where \( \alpha, \beta = +, - \) and \( k = \frac{2 \pi m_j}{L_z} \). Note that \( S^{\alpha\beta} \) is the Fourier transform of the usual equal-time correlation function \( G^{\alpha\beta}(x) \equiv \langle n_x^\alpha n_y^\beta \rangle - \langle n_x^\alpha \rangle \langle n_y^\beta \rangle \). Thus, if \( G \) is even in \( x \), \( S \) will be real. Equivalently, an imaginary part of \( S \) corresponds to the part of \( G \) which is odd in \( x \). Due to charge symmetry, we expect \( G^{+-} = G^{-+} \). Clearly, both must be even in \( x \), so that the associated \( S \)'s are real. On other hand, due to the drive, we have

\begin{equation}
G^{+-}(x,E) = G^{+-}(x_\perp, -x_\parallel, -E)
\end{equation}

so that \( S^{+-} \) may have an imaginary part (which must be odd in \( E \)). Finally, \( G^{-+}(x) = G^{+-}(-x) \) is just a mathematical identity.

In Fig. 1, we present the results for the three independent \( S \)'s found in the larger system. Before discussing the data in detail, we will first present the theoretical framework within which they can be understood. In particular, we will illustrate the emergence of discontinuity singularities in the structure factors at \( k = 0 \), and their consequences for long-range correlations in real space. This will then be followed by a comparison between our theoretical predictions and the simulation results.

Since our interest lies in the behavior at large distances (or small \( k \)), the simplest approach relies on continuum field theory. Starting from the dynamics at the microscopic level, specified by the above rates and the associated Master equation, there are several ways to arrive at a coarse-grained description, in terms of equations of motions

\[ \frac{\partial}{\partial t} \rho(\vec{x}, t) = \nabla \cdot \left( \nabla \chi(\vec{x}, t) \right) \]

\[ \chi(\vec{x}, t) \]

\[ \chi(\vec{x}, t) = \chi^\perp(\vec{x}, t) + \chi^\parallel(\vec{x}, t) \]

For generality, we consider the \( d \)-dimensional case when \( x_\parallel \) is directed along the electric field and \( x_\perp \) is in the \( d-1 \) dimensional subspace, perpendicular to the field. After taking the continuum limit the result for the macroscopic part is a set of mean-field equations of motions for the slowly varying local densities:

\begin{equation}
\partial_t \bar{g}^\pm = -\nabla \Gamma \left( \begin{array}{c} \bar{g}^+ \\ \bar{g}^- \end{array} \right) \nabla \left( 1 - \bar{g}^+ - \bar{g}^- \right) \pm \varepsilon x_\parallel \bar{g}^\pm (1 - \bar{g}^+ - \bar{g}^-) ;
\end{equation}

where

\[ \Gamma = \begin{pmatrix} \Gamma_\perp & 0 \\ 0 & \Gamma_\parallel \end{pmatrix} \]

is the diffusion-matrix. \( \Gamma_\perp \) is diagonal and isotropic in the \( d-1 \) dimensional subspace, thus characterized by a number \( \Gamma_\perp \). \( \nabla \) is the asymmetric gradient operator, namely for any two functions \( f \) and \( g \),

\[ \nabla f \cdot \nabla g = f \nabla g - g \nabla f . \]

\( \varepsilon = 2 \tanh(E/2) \) is the coarse-grained bias and \( \varepsilon x_\parallel \) is the unit vector along the \( x_\parallel \) direction.

Since \( \Gamma \) is a continuity equation, it trivially admits \( t \)-independent solutions which are homogeneous in space:

\[ \bar{g}^\pm(\vec{x}, t) = \bar{g}^\pm . \]

For simplicity, and to ease comparison with simulation data, we have chosen equal densities for both species. At the mean-field level, this solution describes the steady state. The correlations of interest are then associated with \( \chi^\pm \), the scaled deviations \( \bar{g} \) from the mean-field steady state.

Returning to the \( \Omega \)-expansion, the result for the fluctuating part, at lowest order, is a Fokker-Planck equation. For our purposes, the equivalent Langevin equation is more transparent. At this order, its deterministic part is linear and the (conserved) noise is Gaussian. Focusing on the fluctuations about the homogeneous phase, the result is:

\begin{equation}
\partial_t \left( \begin{array}{c} \chi^+(\vec{x}, t) \\ \chi^-(\vec{x}, t) \end{array} \right) = \mathcal{L}(\nabla) \left( \begin{array}{c} \chi^+(\vec{x}, t) \\ \chi^-(\vec{x}, t) \end{array} \right) - \left( \begin{array}{c} \nabla n^+(\vec{x}, t) \\ \nabla n^-(\vec{x}, t) \end{array} \right) ,
\end{equation}

where

\[ \mathcal{L}(\nabla) = \begin{pmatrix} \mathcal{L}^{++}(\nabla) & \mathcal{L}^{+-}(\nabla) \\ \mathcal{L}^{-+}(\nabla) & \mathcal{L}^{--}(\nabla) \end{pmatrix} = \begin{pmatrix} (1 - \bar{g}) \nabla \Gamma \nabla - (1 - 3\bar{g})\varepsilon \Gamma_\parallel \partial_\parallel \\ \bar{g} \nabla \Gamma \nabla - \bar{g} \varepsilon \Gamma_\parallel \partial_\parallel \end{pmatrix} \left( \begin{array}{c} \bar{g} \nabla \Gamma \nabla + \bar{g} \varepsilon \Gamma_\parallel \partial_\parallel \\ (1 - \bar{g}) \nabla \Gamma \nabla + (1 - 3\bar{g})\varepsilon \Gamma_\parallel \partial_\parallel \end{array} \right) .
\]
Here, $\eta^\pm(x, t)$ are Gaussian, white noise terms, satisfying:

$$
\langle \eta_i^\alpha(x, t) \rangle = 0, \quad \langle \eta_i^\alpha(x, t) \eta_j^\beta(x', t') \rangle = 2\delta^{\alpha\beta} \sigma_{ij} \delta(x - x') \delta(t - t')
$$

(8)

$$
\alpha, \beta = +, -; \quad i, j = 1, 2, \ldots d.
$$

$(\sigma_{ij}) = \sigma$ is the noise matrix:

$$
\sigma = \begin{pmatrix} \sigma_\perp & 0 \\ 0 & \sigma_\parallel \end{pmatrix}. \tag{9}
$$

Similar to $\Gamma_\perp$, $\sigma_\perp$ is diagonal and isotropic in the d-1 dimensional subspace, characterized by a number $\sigma_\perp$. In equilibrium systems, the FDT guarantees $\sigma \propto \Gamma$. In particular, in the absence of the drive, we would have $\sigma = \bar{\rho}(1 - 2\bar{\rho})\Gamma$ here. However, when driven, the proportionality is not expected to hold, in that the diffusion and noise matrices would be renormalized differently by the drive $\varepsilon$. This is certainly the situation in the driven single species case [3]. Finally, we point out that $\eta^+$ and $\eta^-$ are uncorrelated, due to the fact that charge exchange is not allowed.

To find the correlations and structure factors from eqns. (6-8), we introduce the Fourier components for the fluctuations:

$$
\chi^\pm(k, \omega) = \int dt dx \chi^\pm(x, t) e^{-i(\omega t + kx)}, \tag{10}
$$

and similar ones for the noise, so that

$$
\langle \eta_i^\alpha(k, \omega) \rangle = 0, \quad \langle \eta_i^\alpha(k, \omega) \eta_j^\beta(k', \omega') \rangle = 2\delta^{\alpha\beta} \sigma_{ij} \left[(2\pi)^{d+1} \delta(k + k') \delta(\omega + \omega')\right]
$$

(11)

$$
\alpha, \beta = +, -; \quad i, j = 1, 2, \ldots d.
$$

Then the solution to (3) is trivial:

$$
\begin{pmatrix} \chi^+(k, \omega) \\ \chi^-(k, \omega) \end{pmatrix} = (\mathcal{L}(ik) - i\omega)^{-1} \begin{pmatrix} ik\eta^+(k, \omega) \\ ik\eta^-(k, \omega) \end{pmatrix}. \tag{12}
$$

Note that, in $k$ space, $(\mathcal{L}^+, \mathcal{L}^-)$ and $(\mathcal{L}^+, \mathcal{L}^-)$ are complex conjugate pairs.

As expected, $\langle \chi^\pm(k, \omega) \rangle = 0$, since these are the fluctuations about the conserved average densities. Their correlations are just the dynamic structure factors, easily obtained from (11) and (12). From the definition:

$$
\langle \chi^\alpha(k, \omega) \chi^\beta(k', \omega') \rangle \equiv S^{\alpha\beta}(k, \omega) \left[(2\pi)^{d+1} \delta(k + k') \delta(\omega + \omega')\right]
$$

(13)

we find the two independent $S$’s:

$$
S^{++}(k, \omega) = \frac{2k\sigma k}{|\Lambda(k, \omega)|^2} \left| i\omega - \mathcal{L}^-(ik) \right|^2 + \left| \mathcal{L}^{++}(ik) \right|^2
$$

$$
S^{+-}(k, \omega) = \frac{2k\sigma k}{|\Lambda(k, \omega)|^2} \left(-2\mathcal{L}^+(ik)\mathcal{L}^-(ik)\right) \tag{14}
$$

where

$$
\Lambda(k, \omega) = \det(\mathcal{L}(ik) - i\omega). \tag{15}
$$

To find the equal-time correlations, we need the steady state structure factors, which can be obtained from (14) by an integration over $\omega$. The results are:

$$
S^{++}(k) = \frac{k\sigma k}{\frac{1}{2} \text{Tr} \mathcal{L}(ik)} \frac{|\mathcal{L}^{++}(ik)|^2}{\det \mathcal{L}(ik)}
$$

$$
S^{+-}(k) = \frac{k\sigma k}{\frac{1}{2} \text{Tr} \mathcal{L}(ik)} \frac{-2\mathcal{L}^+(ik)\mathcal{L}^-(ik)}{\det \mathcal{L}(ik)}. \tag{16}
$$

Note that $-\frac{1}{2} \text{Tr} \mathcal{L}(ik) = (1 - \bar{\rho})k\Gamma k$ is positive definite, while we need to require $\det \mathcal{L}(ik) > 0$ for all $k \neq 0$ to assure that the system is within the linear stability boundary. Finally, using (2), eqs. (14) take the explicit form:
\begin{equation}
S^{++}(k) = \frac{(1 - \tilde{\varrho}) k\sigma k}{(1 - 2\tilde{\varrho}) k\Gamma k} \frac{(k\Gamma k)^2 + (1 - 3\tilde{\varrho})^2 r^2_{\parallel} k^2_{\parallel}}{(1 - \tilde{\varrho})^2 k\Gamma k + (1 - 4\tilde{\varrho}) \Gamma^2_{\parallel} e^2 k^2_{\parallel}}
\end{equation}

\begin{equation}
\text{Re}\{S^{+}(k)\} = - \frac{\tilde{\varrho} k\sigma k}{(1 - 2\tilde{\varrho}) k\Gamma k} \frac{(k\Gamma k)^2 - (1 - 3\tilde{\varrho})^2 r^2_{\parallel} k^2_{\parallel}}{(1 - \tilde{\varrho})^2 k\Gamma k + (1 - 4\tilde{\varrho}) \Gamma^2_{\parallel} e^2 k^2_{\parallel}}
\end{equation}

\begin{equation}
\text{Im}\{S^{+}(k)\} = \frac{2\tilde{\varrho} k\Gamma k}{(1 - \tilde{\varrho}) (k\Gamma k)^2 + (1 - 4\tilde{\varrho}) \Gamma^2_{\parallel} e^2 k^2_{\parallel}}.
\end{equation}

A key feature of these structure factors is that, unlike in equilibrium cases, all are singular at the origin. For both \(S^{++}\) and \(\text{Re}\{S^{+-}\}\), this singularity is exhibited as a discontinuity, i.e., \(\lim_{k_\parallel \to 0} S(0, k_\parallel) \neq \lim_{k_\perp \to 0} S(k_\perp, 0)\). In particular,

\begin{equation}
\lim_{k_\perp \to 0} S^{++}(0, k_\parallel) = \left[ \frac{\sigma_{\parallel} \Gamma_{\perp}}{\Gamma_{\parallel} \sigma_{\perp}} \right] \frac{(1 - 3\tilde{\varrho})^2}{(1 - \tilde{\varrho})^2 (1 - 4\tilde{\varrho})}
\end{equation}

and

\begin{equation}
\lim_{k_\perp \to 0} \text{Re}\{S^{+-}(0, k_\parallel)\} = - \left[ \frac{\sigma_{\parallel} \Gamma_{\perp}}{\Gamma_{\parallel} \sigma_{\perp}} \right] \frac{(1 - 3\tilde{\varrho})}{(1 - \tilde{\varrho})^2 (1 - 4\tilde{\varrho})}.
\end{equation}

In general these ratios are not unity. Note that these singularities come not only from the generic FDT-breaking property, namely \(\frac{\Gamma_{\perp}}{\Gamma_{\parallel}} \neq \frac{\sigma_{\parallel}}{\sigma_{\perp}}\), but also from the specifics of this particular driven system. This second factor is a monotonically increasing function of \(\tilde{\varrho}\) reaching \(\infty\) at \(\tilde{\varrho} = 1/4\). This divergence is related to the instability of the homogeneous phase, as we will see below.

On the other hand, though \(\text{Im}\{S^{+-}(k)\}\) vanishes for \(k \to 0\) in any direction, discontinuities are present in higher derivatives. In configuration space, these singularities translate into power law decays of the equal-time correlation functions \(\langle \chi^\alpha(x' + x, t)\chi^\beta(x', t) \rangle\), which is, thanks to translational invariance, independent of \(x'\). More precisely, it is just

\begin{equation}
G^{\alpha\beta}(x) = \int \frac{d^d k}{(2\pi)^d} S^{\alpha\beta}(k) e^{i k \cdot x}.
\end{equation}

To carry out the transform, it is convenient define a “mass” \(m\) via

\begin{equation}
4m^2 = (1 - 4\tilde{\varrho}) \Gamma^2_{\parallel}
\end{equation}

To keep the system in the homogeneous phase, it is sufficient to impose \(m^2 > 0\). We should note that, in the limit \(\varepsilon L_{\parallel} \to \infty\), the mean-field phase boundary is given precisely by \(m^2 = 0\). Otherwise, for \(finite\ \varepsilon L_{\parallel}\), the system does not reach the stability limit until \(\tilde{\varrho}\) equals \(\frac{1}{4} \left(1 + [2\pi / \varepsilon L_{\parallel}]^2\right)\). Further, it is convenient to rescale the lengths

\begin{equation}
x_\perp \to \frac{x_\perp}{\Gamma_{\perp}}, \quad x_{\parallel} \to \frac{x_{\parallel}}{\Gamma_{\parallel}}
\end{equation}

so that \(\Gamma\) becomes the unit matrix. In terms of these rescaled \(x\), let us define \(r \equiv |x|\). Also, we should rescale the elements of the noise matrix, to keep the notation simple:

\begin{equation}
\sigma_{\perp} \to \frac{\sigma_{\perp}}{\Gamma_{\perp}}, \quad \sigma_{\parallel} \to \frac{\sigma_{\parallel}}{\Gamma_{\parallel}}.
\end{equation}

Now, the transform can be carried out exactly. The results are:

\begin{equation}
G^{++}(x) = \nabla \sigma \nabla \left\{-A^{++} \frac{1}{r^{d-2}} + B^{++} \left( \cosh(mx_{\parallel}) \left( \frac{m}{r} \right)^{\frac{d-2}{2}} K_{\frac{d-2}{2}}(mr) \right) \right\}
\end{equation}

\begin{equation}
G^{+-}_c(x) = \nabla \sigma \nabla \left\{-A^{+-}_c \frac{1}{r^{d-2}} + B^{+-}_c \left( \cosh(mx_{\parallel}) \left( \frac{m}{r} \right)^{\frac{d-2}{2}} K_{\frac{d-2}{2}}(mr) \right) \right\}
\end{equation}

\begin{equation}
G^{+-}_o(x) = \nabla \sigma \nabla \left\{ B^{+-}_o \text{sgn}(\varepsilon) \sinh(mx_{\parallel}) \left( \frac{m}{r} \right)^{\frac{d-2}{2}} K_{\frac{d-2}{2}}(mr) \right\},
\end{equation}
where $G^{\pm \pm}_{\pm \pm}$ are the parts of $G^{\pm \pm}$ even or odd in $x_\parallel$, being the transforms of the real and imaginary parts of $S^{\pm \pm}$. The full correlation is, of course,

$$G^{\pm}(x) = G^{\pm \pm}_{e}(x) + G^{\pm \pm}_{o}(x).$$

(27)

This decomposition simply reflects the symmetries of the system in the presence of the field (22). In eqns. (24-26), $K_\nu(z)$ is the modified Bessel function and, provided $\varepsilon \neq 0$, the $A$’s and $B$’s are positive constants depending on $\Gamma_\perp$, $\Gamma_\parallel$, and $\rho$.

The first terms in (24) and (25), being proportional to $r^{-d}$, are the well known power law decays [15] due to “FDT-violation” [3]. The $B$-terms in (24-26) produce an exponential decay for large $r$, except along the field. This can be easily deduced, since, for large $|x_\parallel|$, the exponentials of the hyperbolic and Bessel functions cancel. Without presenting the detailed asymptotic expansions of (24-26), we illustrate the main results as $mr \to \infty$.

For $x_\perp \neq 0$:

$$G^{++}(x), \quad G^{--}(x) \propto \frac{\sigma_\parallel - \sigma_\perp}{r^d} \left[ \frac{x_\parallel^2 - (d-1)x_\perp^2}{r^2} \right] + \cdots ,$$

(28)

where the $\cdots$ represent exponential, short-ranged tails. In this form, we emphasize the three key ingredients of the “FDT-violating” power law decays, namely, the dependence on $\sigma_\parallel \neq \sigma_\perp$ (or $\sigma_\parallel/\Gamma_\parallel \neq \sigma_\perp/\Gamma_\perp$), the dipole amplitude and, of course, the $r^{-d}$. Also, note that the part of $G^{\pm \pm}$ odd in $x_\parallel$ is seen to be only short-ranged.

The more interesting limit is for $x_\perp = 0$. Then, in addition to the above power laws, we have another power: $|x_\parallel|^{-(d+1)/2}$. For all $d > 1$, this power will dominate over the “FDT-violating” component. In fact, if we study $d > 3$ systems, even the next leading term in the asymptotic expansion of $K_\nu$ will be more important than $r^{-d}$. For typical simulations in $d = 2$, we would write:

$$G^{++}(0, x_\parallel), \quad G^{--}(0, x_\parallel) \propto -|x_\parallel|^{-\frac{3}{2}} + O\left(|x_\parallel|^{-2}\right),$$

$$G^{++}(0, x_\parallel) \propto -\text{sgn}(\varepsilon x_\parallel) |x_\parallel|^{-\frac{3}{2}} + O\left(|x_\parallel|^{-\frac{5}{2}}\right).$$

(29)

Note that, to emphasize the sign of the amplitude of the leading power, we included explicit factors of $(-1)$, so that the proportionality constants in (28) are positive. In conclusion, the spatial correlations are dominated by the expected $r^{-d}$ power law, except along the field, where a novel $|x_\parallel|^{-(d+1)/2}$ decay takes over. This new power law comes from the coupling between the two species as a result of the excluded volume constraint and the opposite bias.

Finally, let us turn to comparisons with simulation results. Typically, we find that power law tails are difficult to observe, in relatively small systems such as ours. Thus, we focus on the structure factors. Deferring details to a later publication, we point out some general characteristics. On the whole, the agreement between data (Fig. 11 and 17) is quite impressive, considering that the latter is only the “first approximation”. For example,

- in all three cases, $S(k_\perp, 0)$ is independent of $k_\perp$;
- the value where Re{$S^+(0, k_\parallel)$} vanishes agrees well with the theoretically predicted $k_\parallel = \sqrt{(1-3\bar{\rho})/(1-\bar{\rho})} \varepsilon$;
- similarly, the ratio Re{$S^+(k_\perp, 0)$}/$S^{++}(k_\perp, 0)$ is found to be quite close to the expected value of $-\bar{\rho}/(1-\bar{\rho})$.

On the other hand, the data and [17] do not agree so well for, e.g., Re{$S^+(0, \frac{2\pi}{L_\parallel})$}/$S^{++}(0, \frac{2\pi}{L_\parallel})$. We suspect the origin of this discrepancy to be the following. Given that our data were collected quite close to a second order transition (so as to observe relatively large fluctuations), we should expect the longitudinal component of the two-point function to suffer considerable renormalization. Certainly, this is the case in the proto model [11,14]. It would be very interesting to take into account finite size corrections, to compute these effects in field theory, and to carry out extensive runs for detailed comparison.

In summary, we have examined the spatial correlations and the structure factors in a simple model of biased diffusion of two species. Using both simulation and analytic techniques, we find the expected power law decay, $r^{-d}$, typical of non-equilibrium steady states of a system with anisotropy and subjected to a conservation law. In addition, a novel power, $r^{-(d+1)/2}$, is found for correlations along the bias. The general agreement between simulations and a simple mean-field theory is surprisingly good, while we await a better theory in order to make detailed quantitative comparisons.
ACKNOWLEDGEMENTS

We thank R. Bausch and Z. Toroczkai for many stimulating discussions. This research is supported in part by grants from the National Science Foundation through the Division of Materials Research.

[1] D. Beysens and M. Gbadamassi, Phys. Rev. A22 (1980) 2250; H. Kiefte, M.J. Clouter and R. Penney, Phys. Rev. B30 (1984) 4017; B.M. Law, P.N. Segrè, R.W. Gammon and J.V. Sengers, Phys. Rev. A41 (1990) 816; P.N. Segrè, R.W. Gammon, J.V. Sengers and B.M. Law, Phys. Rev. A45 (1992) 714; P.N Segrè, R. Schmitz and J.V Sengers, Physica A195 (1993) 31.

[2] B. Schmittmann and R.K.P. Zia, in: Phase Transitions and Critical Phenomena Vol. 17, eds. C. Domb and J.L. Lebowitz, (Academic Press, N.Y., 1995).

[3] S. Katz, J.L. Lebowitz and H. Spohn, Phys. Rev. B28 (1983) 1655; J. Stat. Phys. 34 (1984) 497.

[4] H.K. Janssen and B. Schmittmann, Z. Phys. B64 (1986) 503; K.-t Leung and J.L.Cardy, J. Stat. Phys. 44 (1986) 567 and (1986) 1087.

[5] R.K.P. Zia, K. Hwang, B. Schmittmann and K.-t Leung, Physica A194 (1993) 183.

[6] M. Blume, V.J. Emery and R.B. Griffiths, Phys. Rev. A4 (1971) 1071.

[7] R.B. Potts, Proc. Camb. Phil. Soc. 48 (1952) 106; F.Y. Wu, Rev. Mod. Phys. 54 (1982) 235.

[8] B. Schmittmann, K. Hwang and R.K.P. Zia, Europhys. Lett. 19 (1992) 19.

[9] I. Vilfan, R.K.P. Zia and B. Schmittmann, Phys. Rev. Lett. 73 (1994) 2071.

[10] D.P. Foster, C. Godrèche, J. Stat. Phys. 76 (1994) 1129.

[11] G. Korniss, B. Schmittmann and R.K.P. Zia, Europhys. Lett. 32 (1995) 49 and to be published in J. Stat. Phys.

[12] O. Biham, A.A. Middleton, and D. Levine, Phys. Rev. A46 (1992) R6128; K.-t. Leung, Phys. Rev. Lett. 73 (1994) 2386

[13] T.M. Ligett Interacting Particle Systems (Springer, N.Y., 1985); L.H. Gwa and H. Spohn Phys. Rev. Lett. 68 (1992) 725; L.H. Gwa and Spohn H. Phys. Rev. A46 (1992) 844; D. Dhar, Phase Transitions 9 (1987) 51; J. Krug Phys. Rev. Lett. 67 (1991) 1882; B. Derrida, E. Domany and D. Mukamel, J. Stat. Phys. 69 (1992) 667; B. Derrida and M.R. Evans J. Phys. I. France 3 (1993) 311; M.R. Evans, D.P. Foster, C. Godrèche and D. Mukamel, Phys. Rev. Lett. 78 (1995) 208 and J. Stat. Phys. 80 (1995) 69.

[14] N. van Kampen, Adv. Chem. Phys. 34 (1976) 245

[15] M.Q. Zhang, J.-S. Wang, J.L Lebowitz and J.L. Vallés, J. Stat. Phys. 52 (1988) 1461.
FIG. 1. Steady state structure factors (a) $S^+(k)$, (b) $\text{Re}\{S^+(k)\}$, (c) $\text{Im}\{S^+(k)\}$ for a $60 \times 60$ system at $E = 0.471$ and $\bar{\rho} = 0.175$. Structure factors are plotted against the integer $m_{||} = \frac{k_{||} L_{||}}{2\pi}$, while $m_{\perp} = \frac{k_{\perp} L_{\perp}}{2\pi}$ is taken as a parameter.