Feasible Region of Transmission Sections: Weak Factors Identification and Expansion

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ABSTRACT Feasible region limitation of transmission sections hinders the resource complementarity among regional power networks and the accommodation of renewable energies. To expand the feasible region of transmission sections, this paper proposes an optimization planning method with weak factors identification. In the method, the feasible region of transmission sections is characterized based on multi-parametric programming considering the operational constraints of power systems. Then, based on the characterized feasible region, a weak factors identification method is proposed to capture the critical factors limiting the feasible region of transmission sections, such as specific transmission capacity constraints of lines and minimum output constraints of generators. Furthermore, an optimization planning model for upgrading the identified transmission lines and generators is constructed. With weak factors identification, the planning model can avoid traversing all transmission lines and generators and significantly reduce the computational burden. The effectiveness of the proposed method is demonstrated on IEEE 30-bus and 118-bus systems.

INDEX TERMS Power system planning, feasible region, transmission section, weak factors identification.

NOMENCLATURE

SETS AND INDICES

\(a\) Set of parameters or variables in the active constraints
\(ia\) Set of parameters or variables in the inactive constraints
\(S_L\) Set of transmission lines meeting the active constraints
\(S_G\) Set of generators meeting the active constraints
\(\Omega_L\) Set of identified weak factors of transmission lines
\(\Omega_G\) Set of identified weak factors of generators
\(t\) Index of dispatch time interval
\(s\) Index of operational scenario
\(l\) Index of transmission line
\(g\) Index of thermal power generator
\(w\) Index of wind power generator
\(f\) Index of line in transmission section
\(\Omega_m^L\) Set of thermal power generators connected to bus \(m\)
\(\Omega_m^W\) Set of wind power generators connected to bus \(m\)

PARAMETERS

\(C\) Vector of the production cost function coefficient
\(P_D\) Vector of load demand
\(P_m^G\) All-one vector associated with \(P_G\)
\(P_m\) All-one vector associated with \(P_D\)
\(\bar{A}_{PTDF}\) Power Transfer Distribution Factor (PTDF) matrix
\(I_G\) Connection matrix of generators
\(\bar{P}_L\) Vector of capacity of transmission lines
\(\bar{P}_G\) Vector of maximum power output of generators
\(\bar{P}_m\) Vector of minimum power output of generators
\(T\) Number of dispatch time intervals
\(N_c\) Number of all combinations of active and inactive constraints
\(N_l\) Number of transmission lines
\(N_g\) Number of thermal power generators
I. INTRODUCTION

Renewable power has been significantly installed in these years. In China, the installed capacity reached 184 GW for wind power generation and 174 GW for solar power generation at the end of 2018 [1]. With the rapid development of renewable energies and demands, the feasible region of the power system is decreased. As a result, a large amount of renewable electricity is curtailed [2]. One distinct obstacle for the accommodation of large-scale renewable energy is the feasible region limitation of transmission sections (e.g. the transmission capacity constraint of lines can restrict the feasible region of transmission sections). Expanding the feasible region of transmission sections can increase the accommodation of large-scale renewable energy and promote the balance of renewable energy and power demand among the regional power networks [3].

Many studies have investigated methods for expanding the feasible region of transmission sections. For power system operation, flexibilities of the power system (e.g. the deep peak regulation of generators [4], emergency transmission rates of lines [5] and transmission switching [6]) are beneficial for expanding the feasible region of transmission sections. For power system planning, energy storage [7], flexible AC transmission systems (FACTS) [8], new transmission lines [9] are generally utilized to expand the feasible region of transmission sections and deliver more renewable energy. In this paper, we focus on expanding the feasible region of transmission sections through the power system planning.

In the power system planning, to get the optimal placement and capacity of lines and generators, a complicated optimization planning model needs to be solved. Most studies formulate the planning model as a Mixed Integer Linear Programming (MILP) problem, and some advanced mathematic optimization algorithms (e.g. Branch-and-bound [10], Benders decomposition [11]) are used to solve the MILP problem. However, with a large number of scenarios and candidate planning objects that need to be considered, the integer variables of the MILP problem will increase exponentially, leading to the heavy computational burden, or even difficult to solve the problem.

To alleviate the computational burden of the planning model, some optimization methods are investigated by researchers, divided into the following two types:

1) Scenario reduction techniques: The methods that model the uncertain scenarios of renewable energetic and demands include analytical formulations, uncertainty sets and scenario representations [12], [13]. The analytical formulations use the probability density function to model the uncertainties [14]. The uncertainty set method provides the generation...
interval of renewable energy, the optimization model for planning is to search for optimal investment decisions under the worst-case scenario [15]. The main drawback of analytical formulations and uncertainty sets is difficult to solve. Scenario representation method uses massive scenarios to represent the uncertainties of renewable energy and load [16]. To reduce the computational burden, scenario reduction techniques are widely used to obtain a small number of representative scenarios [17], [18].

2) Searching space reduction: The search space of planning problem means the number of integer variables. To reduce the integer variables of the optimization planning model, a list of candidate lines is suggested in transmission expansion planning [19]. In [20]–[22], candidate lines are obtained according to the times of line congestion. In [23], [24], authors relax the line capacity constraints when solving the optimal power flow. After calculation, the lines, whose constraints are violated, that have no negative effect on other congested lines are selected. In [19] and [25], the candidate lines are selected based on the nodal price difference between two buses, and a higher price difference has more potential benefit of installing a new line. These methods are easy and intuitive. However, since the selected congested lines or lines with a higher price difference are used for reducing the generation cost, it may be not suitable for expanding the feasible region of transmission sections.

This paper focuses on searching space reduction. To avoid these mentioned defects, this paper proposes a new weak factors identification method for the feasible region of transmission sections, with a special focus on its application in the power system planning. The contributions are twofold:

1) A weak factors identification method is proposed to capture the critical factors that restrict the feasible region of transmission sections. Firstly, based on multi-parametric programming, the feasible region of transmission sections is characterized when the operational constraints of the power system are preserved, and vertices of the feasible region can be obtained. Then, based on the obtained vertices, the weak factors which restricting the feasible region can be identified through the active part of operational constraints related to these vertices.

2) An optimization planning model for expanding the feasible region of transmission sections is constructed. By strengthening the identified weak factors of transmission lines and generators, the feasible region of transmission sections can be expanded effectively. Besides, with the weak factors identification method, the planning model can avoid traversing all the transmission lines and generators and significantly reduce the computational burden, verified by case studies.

The rest of the paper is organized as follows. The weak factors identification method is proposed in Section II. The optimization planning model is constructed in Section III. Case studies are presented in Section IV, followed by the conclusions in Section V.

II. WEAK FACTORS IDENTIFICATION METHOD

A. OVERVIEW OF WEAK FACTORS IDENTIFICATION METHOD

In essence, the feasible region of transmission sections depends on the operational constraints of the power system, including the power balance equations, generation output constraints, and transmission capacity limits. The purpose of the proposed weak factors identification method is to find out the critical constraints from these operational constraints. The main procedure is shown in Fig. 1.

1) Characterize the feasible region of transmission sections based on multi-parametric programming, as introduced in subsection B.

2) Identify the weak factors based on the active part of operational constraints related to the vertices of the feasible region, as introduced in subsection C.

B. FEASIBLE REGION OF TRANSMISSION SECTIONS

For the power system operation, the operational point is generally obtained by DC optimal power flow (DC OPF) model. In this paper, the focus is to improve the transmission capacity of transmission sections to achieve resource complementarity among regional power networks and improve the accommodation of large-scale renewable energies. The DC OPF model aims to find the minimum power production costs and the power generation deployment satisfying the operational constraints. Thus, the feasible region of transmission sections should also satisfy the operational constraints of the power system. The feasible region can be obtained by using multi-parametric programming.

A DC OPF model can be expressed as equations (1)-(5), including objective function and operational constraints [26].

\[
\begin{align*}
\text{min} \quad z &= CP_G \\
\text{s.t.} \quad e_G P_G &= e_D P_D \\
P_L &= A_{PTDF}(I_G P_G - P_D) \\
-\bar{P}_L &\leq P_L \leq \bar{P}_L \\
P_G &\leq P_G \leq \bar{P}_G
\end{align*}
\]
Equation (1) is the objective function minimizing production costs of the power system. Equation (2) is the power balance equation ensuring that the total generation output is equal to the total load demand. Equation (3) is the power flow equation, and the DC power flow in the form of Power Transfer Distribution Factor (PTDF) matrix is used to calculate line flows, the detailed principle of the PTDF matrix can be seen in [27]. Equation (4) is the capacity constraint of transmission lines, and equation (5) is the power output constraint of generators.

In general, the transmission section is composed of some specific lines in transmission lines, it usually plays an important role in connecting regional power networks and stabilizing the power networks operation [28]. As shown in equation (6), vector $P_F$ is the sub vector of $P_L$ (i.e., $P_F \in P_L$), representing the flow of transmission section, and $P_{L,F}$ represents the flow of other lines except transmission section.

$$P_L = [P_F, P_{L,F}]^T$$

(6)

Therefore, equations (3)-(4) can be rewritten as follows:

$$[P_F, P_{L,F}]^T = A_{PTDF}(G_{PG} - P_D)$$

(7)

$$-[P_F, P_{L,F}]^T \leq [P_F, P_{L,F}]^T \leq [P_F, P_{L,F}]^T$$

(8)

For the brevity, the optimization model (1)-(2), (5) and (7)-(8) can be represented as a compact formulation (9)-(10) with variables $P_G$ and $P_F$. The general constraint (10) is used representing operational constraints (2), (5) and (7)-(8), among that $G(P_G, P_F)$ is the linear function of $P_G$ and $P_F$, and $m, n$ and $b$ are coefficient matrices of the general constraint, derived from equations (2), (5) and (7)-(8).

$$\min z = CP_G$$

$$G(P_G, P_F) = mP_G - nP_F - b \leq 0$$

(9)

(10)

To characterize the feasible region of the transmission section $P_F$, the detailed procedure is shown in the following:

Suppose $P^*_G$ and $P^*_F$ are the optimal solution of the optimization model (9)-(10). By submitting the optimal solution into (10), the active constraint $G_0(P_G, P_F) = 0$ and inactive constraint $G_{ia}(P_G, P_F) < 0$ can be determined. Furthermore, the Karush-Kuhn-Tucker (KKT) optimality conditions of the model (9)-(10) can be represented as (11)-(15) according to the determined active and inactive constraints [29].

$$m^T\lambda_a + m^T_{ia}\lambda_{ia} = 0$$

(11)

$$m_a P^*_G - n_a P^*_F - b_a = 0$$

(12)

$$\lambda_a \geq 0$$

(13)

$$m_{ia} P^*_G - n_{ia} P^*_F - b_{ia} < 0$$

(14)

$$\lambda_{ia} = 0$$

(15)

Solving equations (11)-(12) and (15), the variables $P^*_G$ and $\lambda_a$ can be represented as linear functions associated with variable $P^*_F$, as functions $f_{P_G}$ and $f_{\lambda_a}$ shown in equation (16).

By submitting (16) into (13)-(14), the partial feasible region $CR_i$ can be obtained as (17).

$$\begin{bmatrix}
P^*_G \\
\lambda_a
\end{bmatrix} = \begin{bmatrix}
0 & m^T_a \\
m_a & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
0 \\
n_a P^*_F + b_a
\end{bmatrix}$$

(16)

$$CR_i = \{P^*_F | f_{\lambda_a}(P^*_F) \geq 0, m_{ia} P^*_G - n_{ia} P^*_F - b_{ia} < 0\}$$

(17)

If active constraints $G_{0a}(P_G, P_F) = 0$ and inactive constraints $G_{ia}(P_G, P_F) < 0$ remain unchanged, we can obtain a specific partial region $CR_i$, and the linear relationships in (17) always hold in the partial region $CR_i$. Thus, the partial feasible region depends on the combination of active constraints and inactive constraints. Exploring all partial feasible regions considering all combinations of active and inactive constraints is a typical multi-parametric programming process. By using the algorithm in [30], the feasible region of transmission section can be obtained as $CR_{in}$ in (18), where $\alpha_i$ and $\beta_i$ are coefficient matrices, derived from equations (16)-(17).

$$CR_{in} = \bigcup_{i=1}^{N_c} CR_i \{ \bigcup_{i=1}^{N_c} \alpha_i P_F + \beta_i \leq 0\}$$

(18)

1) WEAK FACTORS IDENTIFICATION

After obtaining the feasible region of transmission section, vertices on boundaries of the feasible region can be obtained, as the red solid dots $\circ$ shown in Fig. 1. Through (17), we can know that the boundaries of the feasible region are determined by the combination of active constraints and inactive constraints, and the information of the active constraints limiting the feasible region is contained in the vertices on the boundaries. Thus, these active constraints can be obtained based on the vertices, and the weak factors can be further identified through the active constraints, the process is as follows:

Selecting the points of interest $P^*_F$ from the vertices, and the selection criterion is to select the points related to the expansion direction we focus on of the feasible region. The relationship between the selected points and the expansion direction is that the feasible region will be expanded if the selected points move on the expansion direction. For example, as shown in Fig. 1, if the expansion direction we focus on is $D_1$, then points 1, 2 and 5 can be selected, and the points 3 and 4 are ignored, because the feasible region will be expanded if points 1, 2 and 5 move on the direction $D_1$, but points 3 and 4 will not. Based on the selected points $P^*_F$, the flow of transmission section can be determined as (19).

$$P_F = P^*_F$$

(19)

Submitting (19) into the DC OPF model (1)-(5), transmission lines and generators which meet the active constraints can be determined as (20), including capacity constraints of transmission lines and minimum power output constraints of generators. It should be noted that, for the upgrading planning generally does not consider to improve the maximum
power output of generators, so the maximum power output constraints of generators are not considered into the weak factors in this paper.

\[
S_L = \{ P_l | |P_l| = \bar{P}_l, P_l \in P_L, \bar{P}_l \in \bar{P}_L, \forall l = 1, 2, \ldots, N_l \} \\
S_G = \{ P_g | P_g \in P_G, \forall g = 1, 2, \ldots, N_g \}
\]  

(20)

In (20), the identified transmission lines and generators are the weak factors that limiting the feasible region of transmission sections under a dispatch time interval \( t \). After traversing all the time intervals \( T \), we can get the complete sets of weak factors, as \( \Omega_L \) and \( \Omega_G \) shown in (21). Moreover, the importance of weak factors can be ranking by the number of times they are repeatedly identified, i.e., the more times a weak factor is repeatedly identified, the more important it is.

\[
\Omega_L = \bigcup_{t=1}^{T} S_L, \quad \Omega_G = \bigcup_{t=1}^{T} S_G
\]  

(21)

Thus, formulations (1)-(21) is the proposed weak factors identification method. Based on the proposed method, the weak factors limiting the feasible region of transmission sections can be identified, and can be further used to reduce search space for the following optimization planning model.

III. OPTIMIZATION PLANNING MODEL

In this section, the optimization planning model for upgrading transmission lines and generators is constructed based on the identified weak factors of transmission lines \( \Omega_L \) and generators \( \Omega_G \).

A. OBJECTIVE FUNCTION

Planning model generally needs to consider the uncertainty of renewable energies and load demands, and the method of typical operational scenarios are widely used [16]. Based on the typical operational scenarios of renewable energies and load demands, the objective function (22) is established to expand the feasible region of transmission sections by maximizing all the combinations of power flow on the transmission section.

\[
\max \sum_{i=1}^{N_L} \sigma_i \sum_{t=1}^{T} \left( \epsilon_{ij}^{(1)} f_{ij}^{(1)} + \epsilon_{ij}^{(2)} f_{ij}^{(2)} + \ldots + \epsilon_{all}^{(N_f)} f_{all}^{(N_f)} \right)
\]

\[
\begin{cases}
  f_{ij}^{(1)} &= P_{f,i,s,t} \quad \forall i = 1, 2, \ldots, N_f \\
  f_{ij}^{(2)} &= P_{f,i,s,t} + P_{f,j,s,t} \quad \forall i, j = 1, 2, \ldots, N_f; i < j \\
  f_{all}^{(N_f)} &= \sum_{i} P_{f,i,s,t}
\end{cases}
\]  

(22)

In the objective function (22), \( f_{ij}^{(1)} \), \( f_{ij}^{(2)} \), \ldots, \( f_{all}^{(N_f)} \) are subfunctions, and \( \epsilon_{ij}^{(1)}, \epsilon_{ij}^{(2)}, \ldots, \epsilon_{all}^{(N_f)} \) are the corresponding weight coefficients of these subfunctions. Each subfunction \( f \) is a combination of power flow on the transmission section, representing an expansion direction of the feasible region of transmission section. For example, subfunction \( f_{ij}^{(2)} \) represents the combination of power flow on two lines (line \( i \) and line \( j \)) in transmission section, it also represents the expansion direction of the combination of line \( i \) and line \( j \) axis, and the subfunction \( f_{ij}^{(2)} \) is used to expand the feasible region by increasing the combination of power flow on line \( i \) and line \( j \). Meanwhile, the value of weight coefficient (e.g. \( \epsilon_{ij}^{(2)} \)) reflects the importance of the corresponding expansion direction, and the larger the weight coefficient is, the more we want to expand the feasible region of transmission section in this direction.

B. UPGRADING MODELS OF TRANSMISSION LINE AND GENERATOR

1) TRANSMISSION LINE

A part of lines become transmission bottlenecks due to its capacity shortage, and also restrict the transmission capacity of transmission sections greatly. Constructing new lines is one of the most effective approach to solve this problem, but the process is time consuming and expensive, and also faces with significant political and social challenges, such as the obstacle of land acquisition [31]. Upgrading existing power transmission infrastructure can avoid constructing new corridors and save the investment. A widely used method is to replace the low-level transmission lines with large section conductors (LSC) to improve the transmission capacity. If line \( l \) is replaced, its corresponding value of impedance \( x_l \) and transmission capacity \( \bar{P}_l \) also change, as shown in (23)-(24).

\[
\bar{P}_l = \begin{cases}
  \bar{P}_l, & \text{if } u_l = 0 \\
  k_l \bar{P}_l, & \text{if } u_l = 1
\end{cases} \quad (l \in \Omega_L)
\]

(23)

\[
x_l = \begin{cases}
  x_l, & \text{if } u_l = 0 \\
  r_l x_l, & \text{if } u_l = 1
\end{cases} \quad (l \in \Omega_L)
\]

(24)

Since there are many types of LSC with different capacity and investment cost, but only one can be selected for application, so the relationship between capacity scale factor \( k_l \) and the corresponding investment cost \( c_l \) of different LSC is demonstrated as the discrete points [32], as shown in Fig. 2. Each point Type-i represents a type of LSC with the corresponding capacity scale factor \( k_{l,i} \) and investment cost \( c_{l,i,j} \).

Based on the relationship in Fig. 2, the model (25)-(27) can be used to calculate the investment cost of transmission
line $c_l$. Among them, the value of $z_i$ indicates whether the type $i$ of LSC is selected ($z_i = 1$) or not ($z_i = 0$).

$$c_l = \sum_{i=1}^{N_L} z_i c_{l,i}$$  \hspace{1cm} (25)

$$\sum_{i=1}^{N_L} z_i = 1$$  \hspace{1cm} (26)

$$z_i \in \{0, 1\}$$  \hspace{1cm} (27)

2) GENERATOR

The minimum output constraint of generators can also limit the feasible region of transmission sections. In China, to maintain the stable combustion of the boiler for the old thermal unit, the minimum output of thermal generator is approximately 60% of the maximum output. The minimum output of generator can be decreased by applying some flexibility retrofit (optimize burners, coal pulverizing system, and so on) [33]. These methods of flexibility retrofit can stable combustion of the boiler at lower-level of generation, and the minimum output of generators can be reduced to 30%-45% of the maximum output.

Considering the flexibility retrofit of thermal power units, the minimum power output $P_{G_{min}}$ can be rewritten as (28). The coefficient of retrofit depth factor $k_g$ represents the ratio of minimum output after and before taking flexibility retrofit.

$$P_g = \begin{cases} P_g, & \text{if } u_g = 0 \\ \frac{k_g}{k_m} P_g, & \text{if } u_g = 1 \end{cases} \quad (g \in \Omega_G)$$  \hspace{1cm} (28)

The relationship between the retrofit depth factor $k_g$ and the corresponding investment cost $c_g$ can be expressed as Fig. 3 [33].

Based on the relationship in Fig. 3, the calculation model of the investment cost $c_g$ is expressed as (29)-(30).

$$c_g = w_g (1 - k_g) P_g$$  \hspace{1cm} (29)

$$k_{g_{min}} \leq k_g \leq 1$$  \hspace{1cm} (30)

Thus, the above equations (23)-(27) and (28)-(30) are the upgrading models of transmission line and generator respectively.

C. SYSTEM CONSTRAINTS

The system constraints include the investment cost constraints (31)-(32) and the operational constraints (33)-(39). Besides, for calculation convenience, the DC power flow in the form of phase angle is usually applied to power system planning. So, in equation (34), the phase angle is used to calculate the line flows instead of the PTDF matrix.

$$\sum_{g \in \Omega_G} c_g + \sum_{l \in \Omega_L} c_l \leq C_{budget}$$  \hspace{1cm} (31)

$$\sum_{g \in \Omega_G} u_g + \sum_{l \in \Omega_L} u_l \leq U_{\text{max}}$$  \hspace{1cm} (32)

$$\sum_{i \in \Omega_{G_i}} P_{g,i,t} + \sum_{j \in \Omega_{W,j,t}} P_{w,j,t} + \sum_{b \in \Omega_{L_b}} \bar{P}_{l,b,s,t} = P_{d,n,s,t}$$  \hspace{1cm} \forall s \in N_S, \forall t \in T, \forall m \in N_m$$  \hspace{1cm} (33)

$$P_{L,s,t} = \frac{1}{x_l} \Delta \theta_{s,t}$$  \hspace{1cm} \forall s \in N_S, \forall t \in T$$  \hspace{1cm} (34)

$$-\pi \leq \theta_{s,t} \leq \pi$$  \hspace{1cm} \forall s \in N_S, \forall t \in T$$  \hspace{1cm} (35)

$$0 \leq P_{W,s,t} \leq \bar{P}_{W,s,t}$$  \hspace{1cm} \forall s \in N_S, \forall t \in T$$  \hspace{1cm} (36)

$$P_{G} \leq P_{G,s,t} \leq \bar{P}_{G}$$  \hspace{1cm} \forall s \in N_S, \forall t \in T$$  \hspace{1cm} (37)

$$-\bar{P}_L \leq P_{L,s,t} \leq \bar{P}_L$$  \hspace{1cm} \forall s \in N_S, \forall t \in T$$  \hspace{1cm} (38)

$$R_{down} \Delta t \leq P_{G,s,t} - P_{G,s,t-1} \leq R_{up} \Delta t$$  \hspace{1cm} \forall s \in N_S, \forall t, t-1 \in T$$  \hspace{1cm} (39)

Equations (33)-(34) represent the nodal power balance constraint and power flow balance constraint, respectively. Constraints (35)-(38) represent the upper and lower limits of nodal phase angle ($\theta$), power output of wind power generators ($P_W$), power output of thermal power generators ($P_G$), and power flow on transmission lines ($P_{L}$) respectively in turn. Constraint (39) is the ramp up/down rate limit of thermal power generators.

D. CALCULATION PROCEDURE

The optimization planning method with weak factors identification is composed of equations (1)-(39). The flow chart of the calculation procedure is shown in Fig. 4. The proposed model is a MILP problem, and can be solved by commercial solvers such as Gurobi [34].

Through the process in Fig. 4, for expanding the feasible region of transmission sections, the optimal planning strategy of upgrading transmission lines and generators can be determined.

IV. CASE STUDIES

The proposed method is tested via numerical experiments with IEEE 30-bus system and IEEE 118-bus system.

A. SYSTEM INFORMATION

In the IEEE 30-bus system, as shown in Fig. 5, the power system network is divided into three areas, numbered Area 1, Area 2 and Area 3 respectively. In Area 1, nodes 1, 2 and 5 are connected with three wind power generators (1#, 2# and 3# generator) respectively. In Area 2, nodes 11, 13 and
15 are connected with three thermal power generators (4#, 5# and 6# generator) respectively. Besides, nodes 4 and 6 of Area 1 are connected with nodes 12 and 10 of Area 2 by the transmission section ($P_{f1}$ and $P_{f2}$). The positive direction of power transmission is from Area 1 to Area 2. In the modified IEEE-30 bus system, the installed capacity of wind power generators is 300 MW, the installed capacity of thermal power generators is 300 MW. Typical operation scenarios are used to consider the uncertainty of load demand and wind power.

In the IEEE 118-bus system, nodes 23 and 30 are connected with nodes 24 and 38 respectively by the transmission section. The positive direction of power transmission is from nodes 23 and 24 to nodes 30 and 38. There are 10 wind power generators connected in nodes 10, 12, ..., 59, and 12 thermal power generators connected in nodes 61, 65, ..., 116 respectively. The total installed capacity of wind generators is 2700MW, and that of thermal power generators is 3900 MW.

The detailed information, such as load curves, wind power prediction curves, bus/generator/branch data, operational constraints, etc., is given to [35].

### B. ANALYSIS OF RESULTS

#### 1) RESULTS OF WEAK FACTORS IDENTIFICATION

Based on the proposed method, the feasible regions of the transmission section, in each time interval of dispatching under typical scenarios, are characterized. The feasible regions of the transmission section in the minimum load time interval ($t = 5$) and the maximum load time interval ($t = 19$) under scenario 1 of IEEE 30-bus system are shown in Fig. 6. The feasible regions of the transmission section are limited by the operational constraints of power system, including the capacity constraints of transmission lines and output constraints of generators.

After characterizing the feasible region of the transmission section, the vertices of the feasible region are obtained (the red bold dots shown in Fig. 6). The obtained vertices of transmission section under $t = 5$ and $t = 19$ are listed in Table 1.

| Dispatching interval | Extreme feasible points ($P_{f1}, P_{f2}$) (MW) |
|----------------------|-----------------------------------------------|
| $t = 5$              | (-10.5, -43.6), (-10.4, -44.3), (8.3, 2.8), (-7.1, -28.6), (13.7, -4.6), (10.5, -14.8), (9.5, -17.1), (2.4, -4.0), (-8.2, -32.3), (18.8, 12.2), (17.9, 13.7) |
| $t = 19$             | (32.5, 46.0), (34.1, 41.3), (25.8, 30.4), (32.7, 29.5), (34.0, 31.9), (34.1, 39.6), (29.5, 23.9), (23.1, 18.1), (23.4, 15.1) |

As shown in Table 2, the identified weak factors are 30# branch, 4#, 5# and 6# generators in $t = 5$, and 6#, 30# and 35# branches, 5# and 6# generators in $t = 19$. It indicates that the feasible region of the transmission section is limited by 30# branch capacity constraint, and 4#, 5# and 6# generator minimum output constraints in $t = 5$. In $t = 19$, the feasible region is limited by 6#, 30# and 35# branch capacity constraints, and 5# and 6# generator minimum output constraints.
TABLE 2. Identified weak factors in $T = 5$ and $T = 19$ under scenario 1 of IEEE 30-Bus system.

| Dispatching interval | Weak factors                        | Generator |
|----------------------|-------------------------------------|-----------|
| $t=5$                | 30# branch (15-23)                  | 4# generator |
|                      |                                     | 5# generator |
|                      |                                     | 6# generator |
| $t=19$               | 6# branch (2-6)                     | 5# generator |
|                      | 30# branch (15-23)                  | 6# generator |
|                      | 35# branch (23-27)                  |            |

FIGURE 7. Feasible regions of the transmission section in $t = 5$ (a) before and after upgrading with 5# generator and in $t = 19$ (b) before and after upgrading with 35# branch.

TABLE 3. Identified weak factors in IEEE 30-Bus system and IEEE 118-Bus system under all typical operation scenarios.

| Test system       | Identified weak factors |
|-------------------|-------------------------|
|                   | Line                    | Generator |
| IEEE 30-bus system| 6# branch (2-6)         | 4# generator |
|                   | 30# branch (15-23)      | 5# generator |
|                   | 35# branch (23-27)      | 6# generator |
| IEEE 118-bus system| 12# branch (8-9)       | 11# generator |
|                   | 15# branch (9-10)       | 12# generator |
|                   | 42# branch (26-25)      | 13# generator |
|                   | 43# branch (26-30)      | 14# generator |
|                   | 63# branch (38-65)      | 15# generator |
|                   | 112# branch (65-68)     | 16# generator |
|                   | 115# branch (68-81)     | 17# generator |
|                   | 140# branch (81-80)     | 18# generator |
|                   | 149# branch (86-87)     | 19# generator |
|                   | 2# generator            | 20# generator |
|                   |                         | 21# generator |
|                   |                         | 22# generator |

FIGURE 8. Feasible regions of the transmission section before and after taking the planning results in $t = 19$ under scenario 1 of IEEE 30-bus system (a) and IEEE 118-bus system (b).

To verify the limitation effect of the identified weak factors on the feasible region of the transmission section, the feasible regions of the transmission section before and after upgrading 5# generator (reduce its minimum output by 50%) in $t = 5$, and upgrading 35# branch (double its transmission capacity) in $t = 19$ are compared. The corresponding feasible regions of the transmission section are shown in Fig. 7.

It can be seen from Fig. 7 that the feasible region has been obviously expanded after upgrading 5# generator and 35# branch. The total transfer capacity of the transmission section has been increased from 31.6 MW to 58.1 MW in $t = 5$ and increased from 78.5 MW to 91.1 MW in $t = 19$. The above results verify the limitation effectiveness of the identified weak factors on power transmission capacity of the transmission section.

Similar to the process of above examples, weak factors limiting feasible regions of transmission sections in IEEE 30-bus system and IEEE 118-bus system under all typical operation scenarios are obtained, as listed in Table 3. In IEEE 30-bus system, the identified weak factors consist of three lines (6#, 30# and 35# branch) and three generators (4#, 5# and 6# generator). In IEEE 118-bus system, the identified weak factors consist of nine lines (12#, 15# ...and 149# branch) and twelve generators (11#, 12# ...and 22# generator).

2) RESULTS OF OPTIMIZATION PLANNING

In the proposed upgrade planning method for expanding the feasible region of the transmission section, identified weak factors in Table 3 can be used as candidate lines and generators for the optimization planning model. To verify the validity of proposed upgrade planning method, the following methods M0-M2 are compared in the following:

M0: The power system has not been upgraded.

M1: The upgrade planning method without weak factors identification (candidates for upgrading include all transmission lines and thermal power generators).

M2: The proposed upgrade planning method with weak factors identification (candidates for upgrading include all transmission lines and thermal power generators).

In Table 4, in the IEEE 30-bus system, the planning results solved by M1 and M2 are the same, i.e., increasing the capacity of branches 30# and 35# by 24.0MW and 7.2MW respectively, and reducing the minimum output of generators 5# and 6# by 30.0MW both. Compared with M0, after taking the planning results, the daily average transfer capacity of the transmission section is increased by 30.7%, from 1395.4MWh to 1823.8MWh, while the daily average consumption of wind power is increased by 11.6%, from 4299.2 MWh to 4798.4MWh. Since the candidates for upgrading include all transmission lines and thermal power
TABLE 4. Upgrade planning results of IEEE 30-Bus system and IEEE 118-Bus system with M0-M2 under typical operation scenarios.

| Test system       | Method | Planning result                                                                 | Investment cost ($) | Computational time (s) | Daily average transfer capacity of transmission section (MWh) | Daily average consumption of wind power (MWh) |
|-------------------|--------|---------------------------------------------------------------------------------|---------------------|------------------------|--------------------------------------------------------------|-----------------------------------------------|
| IEEE 30-bus system| M0     | /                                                                              |                     |                        |                                                              |                                               |
|                   | M1     | 30# (2.0p.u. / 24.0MW) 35# (1.6p.u. / 7.2MW) 5# (0.5p.u. / 30.0MW) 6# (0.5p.u. / 30.0MW) | 8,000,000           | 87360.9                | 1823.8                                                        | 4798.4                                        |
|                   | M2     | 30# (2.0p.u. / 24.0MW) 35# (1.6p.u. / 7.2MW) 5# (0.5p.u. / 30.0MW) 6# (0.5p.u. / 30.0MW) |                     | 499.8                  | 1823.8                                                        | 4798.4                                        |
| IEEE 118-bus system| M0     | /                                                                              |                     |                        |                                                              |                                               |
|                   | M1     | The value of convergence gap still exceeds 20%                                 | 16,000,000          | >48h                   | ---                                                          | ---                                           |
|                   | M2     | 43# (2.0p.u. / 198MW) 115# (1.3p.u. / 59.4MW) 140# (1.3p.u. / 59.4MW) 17# (0.5p.u. / 30MW) |                     | 3693.3                 | 9947.1                                                        | 49767.7                                       |

generators in M1, the planning result of M1 is absolute optimal solution, and the same planning results of M1 and M2 verify the effectiveness of the proposed method M2. Furthermore, the computational time of M2 (499.8 seconds) is up to over 170 times faster than M1 (87360.9 seconds), indicating the computational burden is reduced greatly by using the weak factors identification method.

In IEEE 118-bus system, it cannot obtain the planning result with M1 due to its heavy computational burden, and the value of convergence gap still exceeds 20% with the calculation time more than 48 hours. However, the optimal planning result can be obtained using the proposed method M2, within 3694 seconds approximately, and the planning result is to increase the capacity of branches 43#, 115# and 140# by 198MW, 59.4MW and 59.4MW respectively, and reduce the minimum output of generators 17# by 30.0MW. This demonstrates the superiority of the proposed method again. Similarly, after taking the planning result, the daily average total transfer capacity of the transmission section has increased by 36.4%, from 7290.9MWh to 9947.1MWh, and the daily average consumption of wind power has increased by 4.3%, from 47717.9 MWh to 49767.7MWh.

In order to show the effect of upgrading planning intuitively, feasible regions of the transmission section before and after taking the planning results in $t = 19$ under scenario 1 of IEEE 30-bus system and IEEE 118-bus system are compared respectively, as shown in Fig. 8. It is obvious that the feasible region of the transmission section has been expanded effectively after taking the upgrading planning measures, verifying the effectiveness of the proposed planning method.

V. CONCLUSIONS

In this paper, an optimization planning method with weak factors identification is proposed to expand the feasible region of transmission sections. On the one hand, a new weak factors identification method is proposed. In the method, the feasible region of transmission sections is characterized firstly, and then the weak factors that limiting the feasible region are identified through the operational constraints of the power system. On the other hand, based on the weak factors identification method, an optimization planning model is proposed to expand the feasible region of transmission sections, through strengthening the identified weak factors of transmission lines and generators.

Numerical experiment results verify the effectiveness of the proposed method. The feasible region of transmission sections can be expanded significantly by strengthening the identified weak factors, and the consumption of renewable energy is improved greatly. Furthermore, based on the weak factors identification method, the computational burden of the planning model can be reduced significantly, which is beneficial to efficiency improvement.

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