CONTROL OF STAR FORMATION IN GALAXIES BY GRAVITATIONAL INSTABILITY

YUEXING LI,1 MORDECAI-MARK MAC LOW,2 AND RALF S. KLESSEN3

Received 2004 July 12; accepted 2005 January 3; published 2005 January 21

ABSTRACT

We study gravitational instability and consequent star formation in a wide range of isolated disk galaxies, using three-dimensional smoothed particle hydrodynamics simulations at a resolution sufficient to fully resolve gravitational collapse. Stellar feedback is represented by an isothermal equation of state. Absorbing sink particles are inserted in dynamically bound, converging regions with number density \( n > 10^3 \text{ cm}^{-3} \) to directly measure the mass of gravitationally collapsing gas available for star formation. Our models quantitatively reproduce not only the observed Schmidt law, but also the observed star formation threshold in disk galaxies. Our results suggest that the dominant physical mechanism determining the star formation rate is just the strength of gravitational instability, with feedback primarily functioning to maintain a roughly constant effective sound speed.

Subject headings: galaxies: ISM — galaxies: spiral — galaxies: star clusters — Galaxy: evolution — Galaxy: kinematics and dynamics — stars: formation

1. INTRODUCTION

Stars form in galaxies at hugely varying rates (Kennicutt 1998a). The mechanisms that control the star formation rate from interstellar gas are widely debated (Shu et al. 1987; Elmegreen 2002; Larson 2003; Mac Low & Klessen 2004). Gravitational collapse is opposed by gas pressure, supersonic turbulence, magnetic fields, and rotational shear. Gas pressure is in turn regulated by radiative cooling and stellar and turbulent heating. Despite this complexity, star-forming spiral galaxies follow two empirical laws. First, stars only form above a critical gas surface density (Martin & Kennicutt 2001) that appears to be determined by the Toomre (1964) criterion for gravitational instability. Second, the rate of star formation is proportional to a power of the total gas surface density (Schmidt 1959; Kennicutt 1998b).

A number of groups have simulated disk galaxies in isolation or in mergers, or in cosmological contexts (e.g., Mihos & Hernquist 1994; Friedli & Benz 1995; Sommer-Larsen et al. 1999; Springel 2000; Barnes 2002; Governato et al. 2004). Robertson et al. (2004) review this work. However, in these simulations, star formation is generally set up with empirical recipes a priori. The origin of the observed Schmidt law remains unclear.

Recent cosmological simulations with moderate-mass resolution by Kravtsov (2003) show that the Schmidt law is a manifestation of the overall density distribution of the interstellar medium and that feedback does not strongly influence it. However, the strength of gravitational instability was not directly measured in his work, so a direct connection could not be made between instability and the Schmidt Law, as we do here. The importance of gravitational instability in controlling large-scale star formation was emphasized by Friedli & Benz (1995) and Elmegreen (2002). A similar conclusion comes from the observation that thin dust lanes in galaxies only form in gravitationally unstable regions (Dalcanton et al. 2004).

We simulate a large set of isolated galaxies to investigate gravitational instability and consequent star formation. In this Letter, we examine star formation as a function of gravitational instability and compare the global Schmidt law and star formation thresholds derived from our simulations to the observations.

2. COMPUTATIONAL METHOD

We use the smoothed particle hydrodynamics code GADGET (Springel et al. 2001), modified to include absorbing sink particles (Bate et al. 1995), to directly measure the mass of gravitationally collapsing gas. Sink particles, representing star clusters (SCs), replace gravitationally bound regions of converging flow that reach number densities \( n > 10^3 \text{ cm}^{-3} \). (These regions have pressures \( P/lk \sim 10^7 \text{ K cm}^{-3} \), typical of star-forming regions.)

Our galaxy model consists of a dark matter halo and a disk of stars and isothermal gas. The galaxy structure is based on the analytical work by Mo et al. (1998), as implemented numerically by Springel & White (1999) and Springel (2000). The isothermal sound speed is chosen to be either \( c_s = 6 \text{ km s}^{-1} \) in models with low temperature (LT) or \( c_s = 15 \text{ km s}^{-1} \) in models with high temperature (HT). Table 1 lists the most important model parameters. The Toomre criterion for gravitational instability that couples stars and gas, \( Q_{sg} \), is calculated following Rafikov (2001), and the minimum value is derived using the wavenumber \( k \) of greatest instability and lowest \( Q_{sg} \) at each radius.

The gas, halo, and disk particles are distributed with the number ratio \( N_h : N_b : N_g = 5 : 3 : 2 \). For the halo and disk, the gravitational softening lengths are \( \epsilon_b = 0.4 \text{ kpc} \) and \( \epsilon_g = 0.1 \text{ kpc} \), respectively, while that of the gas \( \epsilon_g \) is given in Table 1 for each model. The minimum spatial and mass resolutions in the gas are given by \( \epsilon_g \) as twice the kernel mass (\( \sim 80m_\odot \)). We adopt typical values for the halo concentration parameter \( c = 5 \), spin parameter \( \lambda = 0.05 \), and Hubble constant \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (Springel 2000). The spin parameter used is a typical one for galaxies subject to the tidal forces of the cosmological background. Reed et al. (2003) suggest a wide range of \( c \) for galaxy-size halos. However, this parameter is based on a simple model of the halo formation time (Navarro et al. 1997), with poorly known distribution (Mo et al. 1998). Springel & White (1999) suggest that \( c = 5 \) is theoretically expected for flat, low-density universes.

Models of gravitational collapse must satisfy three numerical criteria: the Jeans resolution criterion (Bate & Burkert 1997, hereafter BB97; Whitworth 1998), the gravity-hydro balance criterion for gravitational softening (BB97), and the equipartition criterion.
The local SFE of dense molecular clouds is around 30%. We therein starburst galaxies dominated by molecular gas. This suggests roughly constant. Kennicutt (1998b) shows a median SFE of 30% star formation efficiency (SFE) in molecular clouds remains constant. Kennicutt (1998b) shows a median SFE of 30% for particle masses (Steinmetz & White 1997). Truelove et al. (1998) suggest that a Jeans mass must be resolved with far more than the $N_e = 2$ smoothing kernels proposed by BB97. Therefore, we performed a resolution study of model G100-1 (LT), with $N_e = 10^3$, 8 $\times$ 10$^3$, and 6.4 $\times$ 10$^4$, corresponding to $N_e \approx 0.4$, 3.0, and 23.9, respectively. We find convergence at within 10% of the global amount of mass accreted by sink particles between the two highest resolutions, suggesting that the BB97 criterion is sufficient for the problem considered here.

We performed 24 simulations satisfying all three criteria, including six models of low-mass galaxies with low temperature ($T$) to study the effect of changing the effective sound speed. We also set a minimum value of $N_e \geq 10^4$ particles for lower mass galaxies resolved with fewer particles.

### 3. GLOBAL SCHMIDT LAW

To derive the Schmidt law, we average $\Sigma_{\text{SFR}}$ and $\Sigma_{\text{gas}}$ over the star-forming region, following Kennicutt (1989), with radius $a$ chosen to encircle 80% of the mass in sinks. To estimate the star formation rate, we make the assumption that individual sinks represent dense molecular clouds that form stars at some efficiency. Observations by Rownd & Young (1999) suggest that the local star formation efficiency (SFE) in molecular clouds remains roughly constant. Kennicutt (1999b) shows a median SFE of 30% in starburst galaxies dominated by molecular gas. This suggests the local SFE of dense molecular clouds is around 30%. We therefore adopt a fixed local SFE of $\epsilon = 30\%$ to convert the mass of sinks to stars. Note that this local efficiency is different from the global star formation efficiency in galaxies, which measures the fraction of the total gas turned into stars. The global SFE can be quantitatively measured from the radial profile, as indicated in the middle panel, which shows a sharp drop at virial radius.

Figure 1 shows the Schmidt law derived from our simulations. The best fit to the observations by Kennicutt (1998b) gives a fit to the observations by Rownd & Young (1999) suggests that the Schmidt law $\Sigma_{\text{SFR}} = A \Sigma_{\text{gas}}^\alpha$, with global efficiency $A = (2.5 \pm 0.7) \times 10^{-2}$ and a power law $\alpha = 1.4 \pm 0.15$, where $\Sigma_{\text{SFR}}$ is given in units of $M_\odot$ kpc$^{-2}$ yr$^{-1}$ and $\Sigma_{\text{gas}}$ is given in units of $M_\odot$ pc$^{-2}$. A least-squares fit to the models listed in Table 1 (both low $T$ and high $T$) gives $A = (1.4 \pm 0.4) \times 10^{-4}$ and $\alpha = 1.45 \pm 0.07$, agreeing with the observations to within the errors. Note that LT models tend to have slightly higher SF rates than equivalent HT models. Thus, observations may be able to directly measure the effective sound speed (roughly equivalent to velocity dispersion) of the star-forming gas in galactic disks and nuclei. More simulations will be needed to demonstrate this quantitatively.

Our chosen models do not populate the lowest and highest star formation rates observed. Interacting galaxies can produce very unstable disks and trigger vigorous starbursts (e.g., Li et al. 2004). Quiescent normal galaxies form stars at a rate below our mass resolution limit. Our most stable models indeed show no star formation in the first few billion years.

### 4. STAR FORMATION THRESHOLD

A threshold is clearly visible in the spatial distribution of gas and stars in our galaxy models, as illustrated in Figure 2. The critical value of the instability parameter at the threshold can be quantitatively measured from the radial profile, as indicated in the middle panel, which shows a sharp drop of $\Sigma_{\text{SFR}}$ at $R \sim 2R_e$. The critical values of $Q_a$ and $Q_h$ at the threshold $R_{th}$ are shown in the bottom panel of Figure 2 for all the models.

### Table 1: Galaxy Models and Numerical Parameters

| Model | $f_e$ | $Q_a$(LT) | $Q_a$(HT) | $N_e$ | $\epsilon$ | $m_p$ |
|-------|-------|-----------|-----------|-------|-----------|-------|
| G50-1 | 1     | 1.22      | 1.45      | 1.0   | 10        | 0.08  |
| G50-2 | 2.5   | 0.94      | 1.53      | 1.0   | 10        | 0.21  |
| G50-3 | 4.5   | 0.65      | 1.52      | 1.0   | 10        | 0.37  |
| G50-4 | 9     | 0.33      | 0.82      | 1.0   | 10        | 0.75  |
| G100-1| 1     | 1.08      | ...       | 6.4   | 7         | 0.10  |
| G100-2| 2.5   | 1.07      | 1.07      | 1.0   | 10        | 1.65  |
| G100-3| 4.5   | 0.82      | 1.0       | 10    | 2.97      |
| G100-4| 9     | 0.42      | 1.0       | 20    | 5.94      |
| G120-1| 1     | 1.34      | ...       | 20    | 2.72      |
| G120-2| 2.5   | 0.89      | 1.0       | 20    | 6.80      |
| G120-3| 4.5   | 0.52      | 1.0       | 30    | 12.2      |
| G120-4| 9     | 0.26      | 1.5       | 40    | 16.3      |
| G220-1| 1     | 0.65      | ...       | 6.4   | 15        |
| G220-2| 1     | 1.11      | ...       | 20    | 7.07      |
| G220-3| 2.5   | 0.66      | 1.2       | 30    | 14.8      |
| G220-4| 4.5   | 0.38      | 2.0       | 40    | 15.9      |
| G220-4| 9     | 0.19      | 4.0       | 40    | 16.0      |

* The first number is the rotational velocity in units of kilometers per second at virial radius.
* Percentage of total halo mass in gas.
* Minimum initial $Q_a$ for low-T model. Missing data indicate models not run at full resolution.
* Million particles in high-resolution runs.
* Gravitational softening length of gas in units of parsecs.
* Gas particle mass in units of $10^4 M_\odot$.

---

Fig. 1.—Schmidt law from fully resolved low-T (open symbols) and high-T (filled symbols) models listed in Table 1 that showed gravitational collapse. The colors indicate the galaxy rotational velocities, while the symbol shapes indicate the gas fractions, as specified in the legend. The black line is the best fit to the observations from Kennicutt (1998b), while the red line is the best fit to the simulations.
Fig. 2.—Top: Star formation threshold illustrated by the low-\(T\) model G220-1 with \(N_g = 6.4 \times 10^6\). Log of gas surface density is shown, with values given by the color bar. Yellow circles indicate SCs, while the red circle shows \(R_{\text{sc}}\). Middle: Radial profiles of star formation rate (yellow circles) and Toomre \(Q\) parameters for stars \(Q_s\) (asterisks), gas \(Q_g\) (circles), and stars and gas combined \(Q_{sg}\) (diamonds). The red line shows \(R_{\text{sc}}\). Bottom: Critical values of \(Q_s\) (filled symbols) and \(Q_{sg}\) (open symbols) at \(R_{\text{sc}}\) for both low-\(T\) (red) and high-\(T\) (black) models.

Fig. 3.—Star formation timescale \(\tau_{\text{SF}}\) as a function of initial \(Q_{\text{eq}}\)(min) for both low-\(T\) (open symbols) and high-\(T\) (filled symbols) models. The solid line is the least-squares fit.

fully resolved models listed in Table 1. The critical values of \(Q_s\) appear to be generally higher than \(Q_g\) in the same galaxy, and both have lower values (<1) in more unstable models.

Most galaxies not classified as starbursts have gas fractions comparable to or less than our most stable models, so the observation of a threshold value of \(Q_s \sim 1.4\) may reflect the stability of the galaxies in the sample (Martin & Kennicutt 2001). Observed variations in the threshold also appear to occur naturally. If we only use the Toomre criterion for the gas \(Q_g\), we get slightly larger scatter than if we include the stars and use the combined criterion \(Q_{sg}\), but the effect is small.

5. DISCUSSION AND SUMMARY

What controls star formation in different galaxies? Our models suggest the answer is the nonlinear development of gravitational instability. Figure 3 shows the correlation between the star formation timescale \(\tau_{\text{SF}}\) and the initial minimum \(Q_{\text{eq}}\)(min) for fully resolved models listed in Table 1. The best fit is \(\tau_{\text{SF}} = (34 \pm 7 \text{ Myr}) \exp \left[ (4.2 \pm 0.3)Q_{\text{eq}}\text{(min)} \right]\). Quiescent star formation occurs where \(Q_{\text{eq}}\) is large, while vigorous starbursts occur where \(Q_{\text{eq}}\) is small. This differs from the emphasis on supersonic turbulence by Kravtsov (2003). The maximum strength of instability \(Q_{\text{eq}}\)(min) depends on the mass of the galaxy and the gas fraction. The larger the halo mass, or the larger the gas fraction, the smaller \(Q_{\text{eq}}\)(min), and thus the shorter \(\tau_{\text{SF}}\).

Typical observed starburst times of \(10^8\) yr are consistent with our fit for \(\tau_{\text{SF}}\) (Kennicutt 1998b). This also agrees with the observations by MacArthur et al. (2004) that the star formation rate depends on the galaxy potential. McGaugh et al. (2000) show a break in the Tully-Fisher relation for galaxies with \(V \leq 90\) km s\(^{-1}\), suggesting a transition at this scale. Indeed, our models with \(V \leq 100\) km s\(^{-1}\) and gas fraction \(\leq 50\%\) of the disk mass appear to be rather stable \((Q_{\text{eq}} > 1.0)\), with no star formation in the first 3 Gyr, while models with \(V \geq 120\) km s\(^{-1}\) become less stable, forming stars easily. This is also
consistent with the rotational velocity above which dust lanes are observed to form (Dalcanton et al. 2004).

We have deferred inclusion of explicit feedback, magnetic fields, and gas recycling to future work. However, we believe each will have minor effects on the questions considered here. The assumption of an isothermal equation of state for the gas implies substantial feedback to maintain the effective temperature of the gas against radiative cooling and turbulent dissipation. Real interstellar gas has a wide range of temperatures. However, the rms velocity dispersion generally falls within the range 6–12 km s^{-1} (e.g., Scale & Elmegreen 2004). Direct feedback from starbursts may play only a minor role in quenching subsequent star formation (e.g., Kravtsov 2003; Monaco 2004), perhaps because most energy is deposited not in the disk, but above it as superbubbles blow out (e.g., Fujita et al. 2003; Avillez & Breitschwerdt 2004). Kim & Ostriker (2001) demonstrate that swing and magneto-Jeans instabilities operating in a gaseous disk occur at Q_{\text{\rho}} \sim 1.4, suggesting that magnetostatic support is unimportant. The lack of gas recycling both from disrupted molecular clouds and from massive stars will change the detailed patterns of star formation, but probably not the overall results.

Simulations of isolated isothermal disks by Robertson et al. (2004) show large-scale collapse in their centers, leading to disks far smaller than observed, which they argued was caused by an isothermal equation of state. This behavior does not occur in our model, which has physical parameters close to theirs but a resolution sufficient to resolve the Jeans length. Similarly, Governato et al. (2004) argue that several long-standing problems in galaxy simulations, such as the angular momentum catastrophe, may well be caused by inadequate resolution or a violation of the other numerical criteria. We will present more resolution studies in future work.

In summary, our models quantitatively reproduce not only the Schmidt law, but also the star formation threshold in disk galaxies. We find a direct correlation between the star formation rate and the strength of gravitational instability. This suggests that gravitational instability in effectively isothermal gas may be the dominant physical mechanism that controls the rate and location of star formation in galaxies. Unstable galaxies were more common at early cosmic times, so our results, together with merger-induced starbursts (Li et al. 2004), may account for the Butcher-Oemler effect (Butcher & Oemler 1984) of increasing blueness of galaxies with redshift. Massive galaxies form stars quickly, which may account for the downsizing effect: star formation first occurs in big galaxies at high redshift, while modern starburst galaxies are small (Cowie et al. 1996; Poggianti et al. 2004; Ferreras et al. 2004). The slow evolution of star formation in our low-mass models resembles that observed in low surface brightness galaxies (van den Hoek et al. 2000).

We thank V. Springel for making both GADGET and his galaxy initial condition generator available, and for useful discussions; A.-K. Jappsen for participating in the implementation of sink particles in GADGET; and F. Adams, J. Dalcanton, R. Kennicutt, J. Lee, C. Martin, D. McCray, T. Quinn, M. Shara, and J. van Gorkom for useful discussions. The referee, F. Governato, also gave valuable comments. This work was supported by NSF grants AST99-85392 and AST03-07854, NASA grant NAGS-13028, and DFG Emmy Noether grant KL1358/1. Computations were performed at the Pittsburgh Supercomputer Center, supported by the NSF, on the Parallel Computing Facility of the AMNH, and on an Ultraspire III cluster generously donated by Sun Microsystems.

REFERENCES

Avillez, M. A., & Breitschwerdt, D. 2004, A&A, 425, 899
Barnes, J. E. 2002, MNRAS, 333, 481
Bate, M. R., Bonnell, I. A., & Price, N. M. 1995, MNRAS, 277, 362
Bate, M. R., & Burkert, A. 1997, MNRAS, 288, 1060 (BB97)
Butcher, H., & Oemler, A., Jr. 1984, ApJ, 285, 426
Cowie, L. L., Songaila, A., Hu, E. M., & Cohen, J. G. 1996, AJ, 112, 839
Dalcanton, J., Yoachim, P., & Bernstein, R. A. 2004, ApJ, 608, 189
Elmegreen, B. G. 2002, ApJ, 577, 206
Ferreras, I., Silk, J., Böhm, A., & Ziegler, B. 2004, MNRAS, 355, 64
Friedli, D., & Benz, W. 1995, A&A, 301, 649
Fujita, A., Martin, C. L., Mac Low, M.-M., & Abel, T. 2003, ApJ, 599, 50
Governato, F., et al. 2004, ApJ, 607, 688
Kennicutt, R. C., Jr. 1989, ApJ, 344, 685
———. 1998a, ARA&A, 36, 189
———. 1998b, ApJ, 498, 541
Kim, W. T., & Ostriker, E. C. 2001, ApJ, 559, 70
Kravtsov, A. V. 2003, ApJ, 590, L1
Larson, R. B. 2003, Rep. Prog. Phys., 66, 1651
Li, Y., Mac Low, M.-M., & Klessen, R. S. 2004, ApJ, 614, L29
MacArthur, L. A., Courteau, S, Bell, E., & Holtzman, J. A. 2004, ApJS, 152, 175
Mac Low, M.-M., & Klessen, R. S. 2004, Rev. Mod. Phys., 76, 125
Martin, C. L., & Kennicutt, R. C., Jr. 2001, ApJ, 555, 301
McGaugh, S. S., Schombert, J. M., Bothun, G. D., & de Blok, W. J. G. 2000, ApJ, 533, L99
Mihos, C. J., & Hernquist, L. 1991, ApJ, 431, L9
Mo, H. J., Mao, S., & White, S. D. M. 1998, MNRAS, 295, 319
Monaco, P. 2004, MNRAS, 352, 181
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Poggianti, B. M., Bridges, T. J., Komiyama, Y., Yagi, M., Carter, D., Mobasher, B., Okamura, S., & Kashikawa, N. 2004, ApJ, 601, 197
Ratnikov, R. R. 2001, MNRAS, 323, 445
Redd, D., Governato, F., Verde, L., Gardner, J., Quinn, T., Stadel, J., Merritt, D., & Lage, G. 2003, preprint (astro-ph/0312544)
Robertson, B., Yoshida, N., Springel, V., & Hernquist, L. 2004, ApJ, 606, 32
Rownd, B. K., & Young, J. S. 1999, AJ, 118, 670
Scalo, J., & Elmegreen, B. G. 2004, ARA&A, 42, 275
Schmidt, M. 1959, ApJ, 129, 243
Shu, F., Adams, F. C., & Lizano, S. 1987, ARA&A, 25, 23
Somer-Larsen, J., Gelato, S., & Vedel, H. 1999, ApJ, 519, 501
Springel, V. 2000, MNRAS, 312, 859
Springel, V., & White, S. M. D. 1999, MNRAS, 307, 162
Springel, V., Yoshida, N., & White, S. D. M., 2001, NewA, 6, 79
Steinmetz, M., & White, S. D. M. 1997, MNRAS, 288, 545
Toomre, A. 1964, ApJ, 139, 1217
Trueblood J. K., Klein, R. I., McKee, C. F., Holliman, J. H., II, Howell, L. H., Greenough, J. A., & Woods, D. T. 1998, ApJ, 495, 821
van den Hoek, L. B., de Blok, W. J. G., van der Hulst, J. M., & de Jong, T. 2000, A&A, 357, 397
Whitworth, A. P. 1998, MNRAS, 296, 442