Duane - Hunt Relation Improved

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Abstract

In present paper the Duane-Hunt relation for direct measurement of the Planck constant is improved by including relativistic corrections. New relation to determine the Planck constant, suggested in this paper contains Duane-Hunt relation as first term and can be applied in a wide range of energies.

Keywords: Special relativity, Old quantum theory, Bremsstrahlung, Spacetime Physics.

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1 Introduction

The origin of quantization and nature of the Planck constant remain the most intriguing problems of physics, from the very moment of the creation of the Quantum Theory (QT) at the beginning of the last century. From the beginning a great number of attempts to explain the nature of the Planck constant were made. One interesting way to do this was that suggested by Boyer in 1978 [1] (see also [2] -[4] and references therein), within the framework of “Zero-Point Radiation”. Another idea that should be mentioned here was recently implemented in the framework of the electrodynamics model of the atom proposed by M. Percovac [5]. Within this approach (as it was shown by author) the Planck constant value could probably depends on the energy (see also experimental papers cited below in which such a dependence was discussed).

In our papers [6], [7], [8] we suggest a more natural manner to explain the origin of the Planck constant due to the geometrical quantization of action. This way not only allows us to calculate the correct value of the Planck constant from the first principles (i.e. from cosmological parameters, or from the geometry of our Universe), but it also allows us to unify the QT and the relativistic physics. Moreover, from the observational fact that geometry of our universe is changed adiabatically on time, it immediately follows that the Planck constant should change its value with time. It is important also to mention here the work of V. Garcia Morales [9] in which, starting from the thermodynamics, he argued that neither time nor space needs to be discrete but it is just action what is quantized. All this facts suggest the importance of accurate measurement of
Planck’s constant in all energy ranges for the possible detection of more subtle effects and, as a consequence, for experimental justification of the theory.

Recently there appears a compilation of experimental measurements of the Planck’s constant for wide range of energies [10, 11-25], where the probable experimentally measurable dependence of the Planck constant on energy was supposed. So, more careful experiments over all energy ranges are clearly needed.

At the moment there are a lot of measurements of the Planck constant at small energies from 0.001 to 1eV [10, 11-18]. Also the measurements for 1MeV are available [19, 20-23], but in the range of energies from 1eV to 1MeV there are no precise data reported [10, 11-23]. In this case it is of great importance not only elaborate new experimental technique, but also increase precision of actually available methods.

One such method of direct measurement of the Planck constant should be mentioned here is the technique based on the Duane-Hunt (D-H) relation. This relation was written from classical point of view, without relativistic corrections. For this reason it shows dramatic discrepancy for the measured Planck constant in respect to the CODATA value [19, 20-22] which appears in second - third digit. For this reason the D-H relation in its classical form cannot be used for precise measurements of the Planck constant [24, 25-27].

In present paper we have revised D-H relation by considering relativistic corrections. An exact relativistic relation between the incident electron energy and energy of produced X-ray photon is obtained, and 4 terms of its expansion are suggested. This relation is precise if compared with the Duane – Hunt expression, which actually corresponds to the first term of the expansion on parameter v/c.

2 Bremsstrahlung

Let’s consider in details the geometry and physics of the process of scattering of a relativistic electron on a tungsten atom with the emission of a photon. In the Minkowski spacetime we represent it in fig.1.
Fig1. Schematic representation of the momentum four-vectors of each of the entities involved, where the subscript \((e)\) represents the electron and the subscript \((w)\) corresponds to Tungsten, the prima superscript indicates the quantities after collision, and \(hk\) is the momentum of the emitted photon.

In this case the conservation equations for temporal and spatial parts of the 4-momentum \([28]\) are

\[
E_e + E_w = E'_e + E'_w + E_\gamma ,
\]

(1)

\[
p_e + p_w = p'_e + p'_w + p_\gamma ,
\]

(2)

were the subscript \(\gamma\) denotes the emitted photon. We can rewrite Eq(2) in the corresponding projections as follows:

\[
p_e - p_w \cos \theta = p'_e \cos \psi - p'_w \cos \varphi ,
\]

(3)
\[ p_w \sin \theta = p'_w \sin \psi - p'_w \sin \varphi + \hbar k \quad , \] (4)

where \( h \) is the reduced Planck’s constant, \( k \) is the wavenumber of the emitted photon, and \( p, p' \) are the 3- moments of each constituent of the system before \((p)\) and after \((p')\) interaction.

In consequence with experimental evidence and D-H relation, we also assume that the initial electron transfers all of its energy and momentum to the photon (this is the case in the framework of a Duane – Hunt relation). i.e. \( p'_e \ll p_e \) and \( p'_e \approx 0 \). Thus Eq.(3) and (4) take the form

\[ p_e - p_w \cos \theta = -p'_w \cos \varphi \quad , \] (5)

\[ p_w \sin \theta = \hbar k - p'_w \sin \varphi \quad . \] (6)

By squaring Eq.(5), we get

\[ p'_w \cos^2 \varphi = (p_w \cos \theta - p_e)^2 \quad , \] (7)

and rewrite Eq.(6) to have

\[ \hbar k = p_w \sin \theta + p'_w \sin \varphi \quad . \] (8)

From Eq.(7) through a trigonometric identity, we obtain

\[ p'_w \cos^2 \varphi = p'_w - (\hbar k - p_w \sin \theta)^2 \quad , \] (9)

and from Eqs.(7) and (9), we have

\[ (p_w \cos \theta - p_e)^2 = p'_w - (\hbar k - p_w \sin \theta)^2 \quad . \] (10)

Besides that, the relativistic equations for energy also should be applied. They are \( E^2 = m^2 c^4 + p^2 c^2 \) and \( E_w = K_w + m c^2 \), where \( E \) is the total energy, \( K \) kinetic energy, and \( p \) is the 3-momentum. From this relation and Eq.(1) we have the following expression for energy:

\[ eU + m_e c^2 + K_w + M c^2 = K'_e + m_e c^2 + K'_w + M c^2 + \hbar \nu \quad , \] (11)

where by assumption \( V'_c \ll V_e \) (it implies that \( K'_e \approx 0 \)). Therefore

\[ eU + K_w = K'_e + K'_w + \hbar \nu \quad . \] (12)

When the electron reaches the anode, it has a total energy \( E_e \), whereas \( K_w, K'_w \) are nonrelativistic, namely they can be substituted by classical expression \( \frac{p^2}{2m} \), then

\[ eU + \frac{p^2_{w}}{2M} = \hbar \nu + \frac{p^2_{w}}{2M} \quad . \] (13)

Hence, the square of momentum of the Tungsten nucleus after the interaction is
\[ p_w^2 = 2M \left( eU + \frac{p_w^2}{2M} - h\nu \right) \] . \hspace{1cm} (14)

The momentum of the electron \( p_e \) in terms of its total energy is

\[ p_e c = \sqrt{(eU)^2 + 2m_e c^2 eU} \] . \hspace{1cm} (15)

Substituting Eqs. (14) and (15) into Eq. (10) and solving it for \( h\nu \), one can obtain the following expression,

\[ h\nu = \left( Mc^2 - p_w c \sin \theta \right) \left[ -1 + \sqrt{1 + \frac{2Mc^2 eU + 2p_w c \cos \theta \sqrt{(eU)^2 + 2m_e c^2 eU} - (eU)^2 - 2m_e c^2 eU}{(Mc^2 - p_w c \sin \theta)^2}} \right] \] \hspace{1cm} (16)

This expression was obtained in the approximation of the smallness of the relativistic corrections to the classical energy of the tungsten atom. Let’s evaluate how accurate this expression is. For the electron, we used exact relativistic expressions for energy and momentum, while for tungsten we limited ourselves to the second term of the expansion (the kinetic energy of the atom). The expansion for the energy of the tungsten atom up to the third term is:

\[ E_w = Mc^2 + \frac{(Mc^2/2)(V/c)^2}{2} + \frac{(3/8)(Mc^2/2)(V/c)^4}{4} \].

For thermal velocities \( V = 10^5 \) that correspond to a temperature of 2000 K, we obtain the estimate for this correction: \( \delta E/E = \delta h/h = (3/8)(V/c)^4 = 3 \cdot 10^{-23} \). As one can see, this is quite sufficient for accurate measurements of the Planck constant.

Finally, to compare our result with the original D-H relation \( h\nu = eU \), we should expand this expression in respect to the small parameter \( V/c \). By leaving the first order terms in the expansion, we immediately obtain

\[ h\nu = eU + \frac{p_w \cos \theta \sqrt{(eU)^2 + 2m_e c^2 eU}}{Mc} - \frac{(eU)^2}{2Mc^2} - \frac{m_e}{M} (eU) \] \hspace{1cm} (17)

So, as one can see the first term coincides exactly with the D-H relation, while the subsequent three terms are first-order relativistic corrections to the relation.

3 Conclusion

In present paper the improved expression for the D-H relation is obtained. This expression takes into account the fact that velocities of atoms are differ from zero and relativistic corrections for the electron energy-momentum are significant. We suggest exact formula (16), which generalizes the D-H relation by taking into account relativistic effects. We also suggest expansion of the relation obtained, in order to compare our result with the original D-H relation. As one can see,
the first term of this expansion coincide precisely with the Duane-Hunt law $h\nu = eU$, and the next terms correspond to the relativistic corrections.

Obtained expression allows us to determine the Planck’s constant (with appropriate experimental data) with greater precision and in great range of energies (from 1eV to 1MeV) which was unavailable for other experiments. For this reason one can hope that obtained expression will be useful to close this great gap in experimental measurements of the Planck constant.

To conclude it should be mentioned again that there exist a large number of physical theories, particularly based on the geometric approach [6], [7], [8] and [30, 31], which not only allow us to calculate the Planck constant from the first principles, but also predict its variation. Thus, the problem of accurate experimental determination of Planck constant is of crucial importance, since such measurements will make it possible to discriminate a large number of theories by choosing those that correspond to the experiment.

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