Elastic, Inelastic, and Path Length Fluctuations in Jet Tomography

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Jet quenching theory using perturbative QCD is extended to include (1) elastic as well as (2) inelastic parton energy losses and (3) jet path length fluctuations. The extended theory is applied to non-photonic single electron production in central Au+Au collisions at $\sqrt{s} = 200$ AGeV. The three effects combine to significantly reduce the discrepancy between theory and current data without violating the global entropy bounds from multiplicity and elliptic flow data. We also check for consistency with the pion suppression data out to 20 GeV. Fluctuations of the jet path lengths in realistic geometry and the difference between the widths of fluctuations of elastic and inelastic energy loss are essential to take into account.

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Possibility (3) is of course the most radical and would imply the persistence of non-perturbative physics in the sQGP down to extremely short wavelengths. Processes can be postulated to improve the fit to the data\cite{15}, but at the price of losing theoretical control of the tomographic information from jet quenching data. DGVW\cite{11} showed that by arbitrarily increasing the initial sQGP densities to unphysical values, the non-photonic electrons from heavy quarks can be artificially suppressed to $R_{AA} \sim 0.5 \pm 0.1$. Thus, to approach the electron data, conventional radiative energy loss requires either a violation of bulk entropy bounds or nonperturbatively large $\alpha_s$ extrapolations of the theory. Even by ignoring the bottom contribution, Ref. 18 found that a similarly excessive transport coefficient $\hat{q}_{eff} \sim 14 \text{GeV}^2/\text{fm}$, was necessary to approach the level of suppression of electrons in the data.

Bottom quark jets are very weakly quenched by radiative energy loss. Using the FONLL production cross-sections, their contribution significantly reduces the single electron suppression\cite{11} compared to that of the charm jets alone. The ratio $R_{AA}$ is not sensitive to the scaling of all cross-sections by a constant. However, it is sensitive to any uncertainty in the relative contribution of charm and bottom jets to the electrons\cite{10}. Recent data from STAR on electrons from p+p collisions\cite{7} may indicate an even larger uncertainty in the production than expected from FONLL. However, PHENIX p+p to electron data are compatible with the upper limit of FONLL predictions\cite{21, 22}, similar to the comparison between FONLL and Tevatron data.

The discrepancy between the ‘DGLV Rad only’ predictions and the data in Fig. 1 and recent work\cite{22, 23, 24} motivated us to revisit the assumption that pQCD elastic energy loss\cite{26} is negligible compared to radiative. In earlier studies, the elastic energy loss\cite{26, 27, 28, 29, 30, 31} was found to be $\Delta E/E \sim 0.3 - 0.5 \text{ GeV/fm}$, which was erroneously considered to be small compared to the several GeV/fm expected from radiative energy loss. The apparent weakness of conventional pQCD collisional energy loss mechanisms was also supported by parton transport theory results\cite{32, 33}, which showed that the typical thermal pQCD elastic cross section, $\sigma_{el} \sim 3 \text{mb}$, is too small to explain the differential elliptic flow at high $p_T > 2 \text{ GeV}$ and also underestimates the high $p_T$ quenching of pions.

In contrast, Mustafa\cite{23} found that radiative and elastic average energy losses for heavy quarks are in fact comparable over a very wide kinematic range accessible at RHIC. In Fig. 2 we confirm Mustafa’s finding and extend it to the light quark sector as well. The fractional energy loss, $\Delta E/E$, from DGLV radiative for $u, c, b$ quarks (solid curves; see also App. IB) is compared to TG\cite{21} and BT\cite{23} estimates of elastic (dashed curves; see also App. IA). For light quarks, the elastic energy loss decreases more rapidly with energy than radiative energy loss, but even at 20 GeV the elastic is only 50% smaller than the radiative.

From Fig. 2 we see that for $E > 10 \text{ GeV}$ light and charm quark jets have elastic energy losses smaller but of the same order of magnitude as the inelastic losses. But due to the large mass effect\cite{34, 35, 36}, both radiative and elastic energy losses remain significantly smaller for bottom quarks than for light and charm quarks, but the elastic energy loss can now be greater than inelastic up to $\sim 15 \text{ GeV}$. We present both TG and BT as a measure of the theoretical uncertainties of the Coulomb log (see App IC for benchmark numerical examples). These are largest for the heaviest $b$ quark. As they are not ultrarelativistic, the leading log approximation\cite{27, 28} breaks down in the kinematic range accessible at RHIC. More rigorous computations of elastic energy loss\cite{37} and numerical covariant transport techniques\cite{32} can be used to reduce the theoretical uncertainties in the elastic energy loss effects.

Theoretical Framework.

The quenched spectra of partons, hadrons, and leptons are calculated as in\cite{11} from the generic pQCD convolution

$$\frac{E d^3\sigma(e)}{dp^3} = \frac{E_i d^3\sigma(Q)}{dp_i^3} \otimes P(E_i \rightarrow E_f) \otimes D(Q \rightarrow H_Q) \otimes f(H_Q \rightarrow e),$$

where $Q$ denotes quarks and gluons. For charm and bottom, the initial quark spectrum, $E d^3\sigma(Q)/dp^3$, is com-
is the locally determined effective path length of the jet distribution of initial hard jet production points, \( \vec{x} \) with inelastic cross section \( P_{el}(\vec{x}, \phi) \) are computed at leading order as in \[5\]. \( P(E_i \rightarrow E_f) \) is the energy loss probability, \( D(Q \rightarrow H_Q) \) is the fragmentation function of quark \( Q \) to hadron \( H_Q \), and \( f(H_Q \rightarrow e) \) is the decay function of hadron \( H_Q \) into the observed single electron. We use the same mass and factorization scales as in \[40\] and employ the CTEQ5M parton densities \[42\] with no intrinsic \( k_T \). As in \[40\] we neglect shadowing of the nuclear parton distribution in this application.

We assume that the final quenched energy, \( E_f \), is large enough that the Eikonal approximation can be employed. We also assume that in Au+Au collisions, the jet fragmentation function into hadrons is the same as in \( e^+e^- \) collisions. This is expected to be valid in the deconfined medium case, where hadronization of \( Q \rightarrow H_Q \) cannot occur until the quark emerges from the sQGP.

The main difference between our previous calculation \[11\] and the present one is the inclusion of two new physics components in the energy loss probability \( P(E_i \rightarrow E_f) \). First, \( P(E_i \rightarrow E_f) \) is generalized to include for the first time both elastic and inelastic energy loss and their fluctuations. We note that Vitev \[43\] was the first to generalize the GLV formalism to include initial state elastic energy loss effects in d+Au. In this work, \( P_{el}(\Delta_{rad}, L(\vec{x}_{\perp}, \phi)) \) extends the formalism to include final state elastic energy loss effects in \( A+A \).

The second major change relative to our previous applications is that we now take into account geometric path length fluctuations as follows:

\[
P(\Delta_{rad}, L(\vec{x}_{\perp}, \phi)) = \frac{d\phi}{2\pi} \int \frac{d^2\vec{x}}{N_{bin}(\vec{b})} T_{AA}(\vec{x}_{\perp}, \vec{b}) \otimes P_{el}(\Delta_{el}, L(\vec{x}_{\perp}, \phi)).
\]

Here

\[
L(\vec{x}_{\perp}, \phi) = \int d\tau \rho_p(\vec{x}_{\perp} + \tau \hat{n}(\phi)) / \langle \rho_p \rangle
\]

is the locally determined effective path length of the jet given its initial production point \( \vec{x}_{\perp} \) and its initial azimuthal direction \( \phi \) relative to the impact parameter plane \((x, y)\). The geometric path averaging used here is similar to that used in \[11\] and by Eskola et al. \[43\]. However, the inclusion of elastic energy losses together with path fluctuations in more realistic geometries was not considered before.

We consider a diffuse Woods-Saxon nuclear density profile \[46\], which creates a participant transverse density, \( \rho_p(\vec{x}_{\perp}) \), computed using the Glauber profiles, \( T_A(\vec{x}) \), with inelastic cross section \( \sigma_{NX} = 42 \text{ mb} \). The bulk sQGP transverse density is assumed to be proportional to this participant density, and its form is shown (for the \( y = 0 \) slice) in Fig. 3 by the curve labeled \( \rho_{QGP} \). The distribution of initial hard jet production points, \( \vec{x}_{\perp} \), is assumed on the other hand to be proportional to the binary collision density, \( T_{AA}(\vec{x} + \vec{b}/2)T_A(\vec{x} - \vec{b}/2) \). This is illustrated in Fig. 3 by the narrower curve labeled \( \rho_{i\text{et}} \).

The combination of fluctuating DGLV radiative \[32\] with the new elastic energy losses and fluctuating path lengths (via the extra \( d^2\vec{x}, d\phi \) integrations) adds a high computational cost to the extended theory specified by Eqs. \[12\]. In this first study with the extended theory, we explore the relative order of magnitude of the competing effects by combining two simpler approaches.

In approach I, we parameterize the heavy quark pQCD spectra by a simpler power law, \( E d^3Q/d^3k \propto 1/k_T^{n+2} \), with a slowly varying logarithmic index \( n \approx n(p_T) \). For the pure power law case, the partonic modification factor, \( R_Q = d\sigma_Q^{\text{final}} / d\sigma_Q^{\text{initial}} \), (prior to fragmentation) is greatly simplified. This enables us to perform the path length fluctuations numerically via

\[
R_Q = \int \frac{d\phi}{2\pi} \int \frac{d^2\vec{x}_{\perp}}{N_{bin}(\vec{b})} T_{AA}(\vec{x}_{\perp}, \vec{b}) \int d\epsilon (1 - \epsilon)^nP_Q(\epsilon; L(\vec{x}_{\perp}, \phi)),
\]

where

\[
P_Q(\epsilon; L) = \int dx P_Q(\epsilon; x)P_{Q,el}(\epsilon - x; L).
\]

Both the mean and width of those fractional energy losses depend on the local path length. (See App ID for numer-
To illustrate the difference in approach II, consider the case of power law initial $Q$ distributions as in Eq. (4). In this case

$$R^I_Q(p_T, L_Q) = ((1 - \epsilon_Q(L_Q))^{n}(1 - \epsilon_Q(L_Q))^{n})_{\Delta E}. \quad (7)$$

The branching implementation is seen via the product of two distinct factors in contrast to the one quenching factor in Eq. (4). For small $\langle \epsilon_Q \rangle$ both approaches obviously give rise to the same $R_Q = 1 - n(\epsilon_Q)$.

Due to the high computational cost in approach I, only the TG elastic is used for the heavy quarks and only BT for light quarks. The Coulomb log uncertainties are estimated only in approach II.

In both approaches, fluctuations of the radiative energy loss due to gluon number fluctuations are computed as discussed in detail in Ref. [11, 38]. This involves using the DGLV generalization of the GLV opacity expansion to heavy quarks. Bjorken longitudinal expansion is taken into account by evaluating the bulk density at an average time $\tau = L/2$. For elastic energy loss, the full fluctuation spectrum is approximated here by a Gaussian centered at the average energy loss with variance $\sigma^2_{el} = 2T(\Delta E_{el}(p_T, L))$. In approach I the correct, numerically intensive integration through the Bjorken expanding medium provides $\Delta E_{el}(p_T, L)$. In approach II the $\tau = L/2$ approximation is again used; numerical comparisons show that for $L \sim 2 - 7$ fm this reproduces the full calculation well. Finally, we note that we use the additional numerical simplification of keeping the strong coupling constant $\alpha_s$ fixed at 0.3.

**Numerical Results: Parton Level**

In Fig. 5 we show the quenching pattern of $Q$ from the second approach for a “typical” path length scale $L = 5$ fm, similar to that used in previous calculations. The curves show $R_Q(p_T)$, prior to hadronization, for $Q = g, u, c, b$. The dashed curves show the quenching arising from only the DGLV radiative energy loss. The solid curves show the full results after including TG elastic as well as DGLV radiative energy loss. Adding elastic energy loss is seen to increase the quenching of all flavors for fixed path length. Note especially the strong nonlinear increase of the gluon suppression and the factor $\sim 2$ increase of the bottom suppression. The curious switch of the $u$ and the $c$ quenching reflects the extra valence (smaller index $n_u$) contribution to light quarks.

Fig. 5 emphasizes the unavoidable result of using a fixed, “typical” path length scale, $L$, in jet tomography: the pion and single electron quenching can never be similar. If pions were produced only by charm, then we would expect comparable quenching for both. However, contributions from highly quenched gluons decrease the pion $R_{AA}$ while weakly quenched bottom quarks increase the electron $R_{AA}$. Therefore, in the fixed length scenario, we expect a noticeable difference between pion and single electron suppression patterns.
Fig. 2 shows that the energy loss of c quarks is somewhat less than for u quarks; however, the higher $p_T$ power index, $n_c$, of c relative to $u$ – as predicted by pQCD and due to the valence component of $u$ – compensates by amplifying its quenching.

However, none of the distributions can be categorized as surface emission. The characteristic widths of these distributions range from $\Delta x \approx 3 - 6$ fm. We show below that such a large dynamic range of path length fluctuations is essential for consistent reproduction of both electron and pion data.

We turn next to Figs. 6 and 7, which show the interplay between the dynamical geometry seen in Fig. 5 and the elastic-enhanced quenching of partons. In Figs. 6 and 7 the solid green curves labeled “DGLV+TG/BT: Full Geometry” are the results using approach I based on Eq. (4). The curves labeled TG and BT are from approach II based on Eq. (7). The effective fixed $L_Q$ in II were taken to match approximately the green curves in which full path length fluctuations are taken into account. This procedure is not exact because of the different numerical approximations involved, but the trends are well reproduced. The $L_Q$ are determined only to $\sim 0.5$ fm accuracy, as this suffices for our purposes here. We show the comparison between approaches I and II for heavy quarks in Fig. 6 using $L_c = 4.5$ and $L_b = 5.5$ fm and for gluons and light quarks in Fig. 7 using $L_g = 3.5$ and $L_u = 5.0$ fm; see Fig. 4 for a visual comparison of the input length distributions used. This hierarchy of $Q$-dependent length scales is in accord with that expected from Fig. 5.

Note that in comparison to the fixed $L = 5$ fm case in Fig. 5 geometric fluctuations reduce gluon jet quenching in Fig. 7 by a factor $\sim 2$. Nevertheless, even with path length fluctuations the gluons are still quenched by a factor of 10 when elastic energy loss is included in addition to radiative.

The amplified role of elastic energy loss is due to its...
smaller width for fluctuations relative to radiative fluctuations. Even in moderately opaque media with $L/\lambda \sim 10$, inelastic energy loss fluctuations are large because only a few, 2-3, extra gluons are radiated. Thus, gluon number fluctuations, $O(1/\sqrt{N_g})$ lead to a substantial reduction in the effect of radiative energy loss. On the other hand, elastic energy loss fluctuations are controlled by collision number fluctuations, $O(\sqrt{\lambda/L})$, which are relatively small in comparison for a significant proportion of the length scales probed. Therefore, fluctuations of the elastic energy loss do not dilute the suppression of the nuclear modification factor as much as $N_g$ fluctuations. The increase in the sensitivity of the final quenching level to the opacity is a novel and useful byproduct of including elastic energy loss fluctuations, which are relatively small in comparison for a significant proportion of the length scales probed. Therefore, fluctuations of the elastic energy loss do not dilute the suppression of the nuclear modification factor as much as $N_g$ fluctuations. The inclusion of elastic energy loss significantly reduces the fragility of pure radiative quenching and therefore increases the sensitivity of jet quenching to the opacity of the bulk medium.

**Numerical Results: Pions and Electrons**

We now return to Fig. 1 to discuss the consequence of including elastic energy loss of $c$ and $b$ quarks on the electron spectrum. The inclusion of the collisional energy loss significantly improves the comparison between theory and the single electron data. That is, the lower yellow band can reach below $R_{AA} < 0.5$ in spite of keeping $dN_g/dy = 1000$, consistent with measured multiplicity, and using a conservative $\alpha_s = 0.3$. A large source of the uncertainty represented by the lower yellow band is the modest but poorly determined elastic energy loss, $\Delta E/E \approx 0.0 - 0.1$, of bottom quarks (see Fig. 2). There is additional uncertainty from the relative contributions to electrons from charm and bottom jets. The dashed lines show an extreme version of this in which charm jets are the only source of electrons. If the charm to bottom ratio given by FONLL calculations is accurate, the current data suggests that even the combined radiative+elastic pQCD mechanism is not sufficient to explain the single electron suppression.

As emphasized in [11], any proposed energy loss mechanisms must also be checked for consistency with the extensive pion quenching data [1], for which preliminary data now extend out to $p_T \sim 20$ GeV. This challenge is seen clearly in Fig. 3 where for fixed $L = 5$ fm, the addition of elastic energy loss would overpredict the quenching of pions. However, the simultaneous inclusion of path fluctuations leads to a decrease of the mean $g$ and $u,d$ path lengths that partially offsets the increased energy loss. Therefore, the combined three effects considered here makes it possible to satisfy $R_{AA} < 0.5 \pm 0.1$ without violating the bulk $dN_g/dy = 1000$ entropy constraint and without violating the pion quenching constraint $R_{AA}^\pi < 0.2 \pm 0.1$ now observed out to 20 GeV; see Fig. 8. We note that the slow rise of $R_{AA}^\pi$ with $p_T$ in the present calculation is due in part to the neglect of initial $k_T$ smearing that raises the low $p_T$ region and the EMC effect that lowers the high $p_T$ region (see [5]).

**Conclusions**

The elastic component of the energy loss cannot be neglected when considering pQCD jet quenching. While the results presented in this paper are encouraging, further improvements of the jet quenching theory will be required before stronger conclusions can be drawn.

From an experimental perspective, there is at present significant disagreement between measured p+p to electron baselines [1, 3]. In addition, direct measurement of $D$ spectra will be essential to deconvolute the different bottom and charm jet quark dynamics.

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**Fig. 7**: As in Fig. 6 but for light $u,d$ quarks and gluons. The yellow bands are computed in this case with effective $g, u$ path lengths $L_g = 3.5$ and $L_u = 5.0$ fm based on Eq. (7). Note that charm and light quark quenching are similar in this $p_T$ range.

**Fig. 8**: The consistency of the extended jet quenching theory is tested by comparing its prediction to the nuclear modification of the $\pi^0$ spectra observed by PHENIX [1].
On the theoretical side, more work is needed to sort out coherence and correlation effects between elastic and inelastic processes that occur in a finite time and with multiple collisions. Classical electrodynamics calculations presented in [48] suggested that radiative and elastic processes could destructively interfere over lengths far longer than previously thought. As described in [49], a proper accounting of the current shows finite size effects persist out only to the expected lengths of order the screening scale, $1/\mu_D \lesssim 1$ fm. Additionally, work on the quantum mechanical treatment of elastic energy loss in a finite medium [50, 51] also concluded that finite size effects on $R_{AA}$ remain small except in peripheral collisions.

There are several other open problems that require further study. The inclusion of all the initial state effects from [8] will be needed to fully check the consistency of the pion predictions with the data. Only an approximate fluctuation spectrum for elastic energy loss has been included here; still needed is an examination of the effect of the full fluctuation spectrum.

The radiative and elastic energy losses depend sensitively on the coupling, $\Delta E^{\text{rad}} \propto \alpha_s^3$ and $\Delta E^{\text{el}} \propto \alpha_s^2$. Future calculations will have to relax the current fixed $\alpha_s$ approximation. In [52], the running of the coupling is seen to increase the magnitude of the elastic energy loss and alter the energy dependence. More complete calculations of both radiative and elastic energy losses will involve integrals that probe momentum scales that are certainly nonperturbative. Therefore it will be important to study the irreducible uncertainty associated with the different maximum $\alpha_s$ cutoff prescriptions commonly used.

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I. APPENDIX

A. Collisional Energy Loss

The leading logarithmic expression for the elastic energy loss of a jet with color Casimir $C_R$ in an ideal quark-gluon plasma with $n_f$ active quark flavors and temperature, $T$, is given by [26]

$$ \frac{dE^{\text{el}}}{dx} = C_R \pi \alpha_s^2 T^2 (1 + \frac{n_f}{6}) f(v) \log(B_c) $$

where the Coulomb log is controlled by the ratio $B_c$ that involves relevant minimum and maximum momentum transfers or impact parameters. For scattering in an assumed ultrarelativistic ($m = 0$) gas of partons in the jet velocity dependence is

$$ f(v) = \frac{1}{v^2} \left( v + \frac{1}{2} (v^2 - 1) \log(\frac{1 + v}{1 - v}) \right) \xrightarrow{v \to 1} 1 $$

Estimates for $B_c$ differ below asymptotic ($E \gg T$) energies and are given in [24, 27], and [28] that we denote by Bj, TG, and BT respectively:

$$ B_{\text{Bj}} = \frac{(4E_p T)}{(\mu^2)} $$

$$ B_{\text{TG}} = \left( \frac{4pT}{(E_p - p + 4T)} \right) / (\mu) $$

$$ B_{\text{BT}} = \left\{ \begin{array}{ll}
2^{\pi^2/\gamma} & 0.85 E_p T / (\frac{\mu^2}{\lambda}) \quad E_p \gg \frac{M^2}{T} \\
\frac{\ln(\frac{E_p T}{\mu})}{(\frac{\mu^2}{\lambda})} & E_p \ll \frac{M^2}{T}
\end{array} \right. $$

with the crossover between $E_p \ll \frac{M^2}{T}$ and $E_p \gg \frac{M^2}{T}$ being taken at $E_p = \frac{2M^2}{T}$ for numerical computation.

B. DGLV Radiative Energy Loss

For completeness we also record the DGLV formula for radiative energy loss used in our calculations. We neglect finite kinematic limits on the momentum transfer $q$ integral, and perform the finite $0 < k_\perp \leq k_{\text{max}} = 2px(1-x)$ and azimuthal $0 \leq \phi \leq 2\pi$ integrals analytically. The mean fractional radiative energy loss can be then evaluated numerically from the expression

$$ \frac{\Delta E^{(1)}_{\text{rad}}}{E} = \frac{C_F \alpha_s}{\pi} \frac{L}{\lambda_g} \int_{0}^{\infty} \int_{0}^{\infty} \frac{4 \mu^2 q^4 dq}{(\frac{E_p}{L})^2 + (q^2 + \beta^2)^2} \times (A \log B + C) $$

where

$$ \beta^2 = m_g^2 (1 - x) + M^2 x^2 $$

$$ \lambda_g^{-1} = \rho_g \sigma_{gg} + \rho_g \sigma_{gg} $$

$$ \sigma_{gg} = \frac{9}{2} \frac{\alpha_s}{2 \mu^2} $$

We employ the asymptotic 1-loop transverse gluon mass $m_g = \mu / \sqrt{2}$. The A,B,C functions denote

$$ A = \frac{2 \beta^2}{f^2_f b} $$

$$ B = \frac{(\beta^2 + K)(\beta^2 Q_{\mu}^\pm + Q_{\mu}^+ Q_{\mu}^- + Q_{\mu}^+ f_\beta)}{\beta^2 (Q_{\mu}^\pm - K) - Q_{\mu}^\pm K + Q_{\mu}^+ Q_{\mu}^- + f_\beta} $$

$$ C = \frac{1}{2 q^2 f^2_f b} \left[ \beta^2 \mu^2 (2q^2 - \mu^2) + \beta^2 (\beta^2 - \mu^2) K + Q_{\mu}^\pm (\beta^4 - 4q^2 Q_{\mu}^\pm) + f_\beta (\beta^2 - 3q^2 + \mu^2) + 2q^2 Q_{\mu}^\pm + 3\beta^2 q^2 Q_K \right] $$
the distribution of fluctuating Debye mass squared is

cylinder with density is computed from

\[ \rho(\tau) = \frac{1}{\pi R^2 \tau} \frac{dN}{dy} \] (24)

We assume \( R = 6 \text{ fm} \). The temperature evolves as

\[ T(\tau) = \left( \frac{\pi^2}{1.202} \frac{\rho(\tau)}{9n_f + 16} \right)^{\frac{1}{\beta}} \]

where \( n_f \) is the number of active quark flavors. The effective static approximation simulates the effect of Bjorken expansion by evaluating \( T \) at \( \tau = L/2 \), where \( L \) is the path length to the cylinder surface. The gluon density is computed from \( \rho_g = \frac{1.202}{\pi^2} \times 16T^3 \), and the density of quarks plus antiquarks is \( \rho_q = \frac{1.202}{\pi^2} \times 9n_f T^3 \). The Debye mass squared is \( \mu^2 = 4\pi\alpha_s^2 T^2(1 + \frac{4}{\alpha_s}) \). In Table I the results for \( n_f = 0 \) are given for a charm jet of energy \( 10 \leq E \leq 15 \).

### D. Energy Loss Fluctuation Spectrum

This section illustrates the fluctuation spectra of induced gluon number and the distribution of fluctuating energy loss for a specific case of a 15 GeV up quark jet with path length 5 fm. Fig. 9 shows the first order induced gluon number distribution \( dN_g/\text{dy} \) for this case. Fig. 11 shows the fractional radiative energy loss distribution taking into account Poisson fluctuations of the gluon number computed as in [4]. The finite probability, \( P(n_g = 0) = 0.2377 \), of radiating zero gluons contributes a \( \delta(\epsilon) \) that is not shown. There is also a finite probability, 0.0213, of complete stopping with \( \epsilon > 1 \).

The width of the elastic energy fluctuations seen in Fig. 11 is significantly smaller than the radiative width. The narrowing of the convoluted elastic plus radiative distributions significantly reduces the distortion effects due to fluctuations. Because of the steep \( p_T \) fall off of the initial unquenched parton spectra, the smaller width of the elastic energy fluctuations considerably amplifies the quenching effect due to collisional energy loss in comparison to the larger but much broader radiative contribution. In terms of an effective renormalization \( \langle \epsilon \rangle \rightarrow Z_{eff}(\epsilon) \) as discussed in [4], \( Z_{eff} \) is closer to unity than the renormalization \( Z_{rad} \sim 0.5 \) characteristic of pure radiative energy loss distributions.

### TABLE I: \( \Delta E \) benchmark test cases for a charm jet (\( m = 1.2 \) GeV, \( C_R = \frac{3}{2} \)) with fixed \( \alpha_s = 0.3 \). The path length is \( L = 5 \) fm; \( hc = 0.197 \) GeV fm. The density is \( \frac{dN}{dy} = 1000 \) \((n_f = 0)\), giving \( T = 0.2403 \) GeV, \( \mu = 0.4666 \) GeV, and \( \lambda_g = 1.2465 \) fm. For radiative, the \( q \) integration limits are taken to be 0.0001 and 50.

| \( E_{jet} \) (GeV) | Radiative \( \frac{\Delta E}{E} \) | Collisional \( \frac{\Delta E}{E} \) |
|-----------------|----------------|----------------|
| 10              | 0.2111         | 0.1594         |
| 11              | 0.2126         | 0.1566         |
| 12              | 0.2129         | 0.1538         |
| 13              | 0.2123         | 0.1450         |
| 14              | 0.2110         | 0.1400         |
| 15              | 0.2093         | 0.1379         |

where the abbreviated symbols denote

\[ K = k_{\text{max}}^2 = 2px(1 - x) \] (19)

\[ Q_\beta^\pm = q^2 \pm \mu^2 \] (20)

\[ Q_k^\pm = q^2 \pm k_{\text{max}}^2 \] (21)

\[ f_\beta = f(\beta, Q_\beta^+, Q_\beta^-) \] (22)

\[ f_\mu = f(\mu, Q_k^+, Q_k^-) \] (23)

with \( f(x, y, z) = \sqrt{x^4 + 2x^2y + z^2} \).
The narrow (green) curve corresponds to BT elastic energy loss fluctuations; based on the input from Fig. 9, the lower, broader (red) curve corresponds to inelastic energy loss due to gluon number fluctuations. The part of the radiation spectrum with $\epsilon > 1$ is replaced with 0.0213$(1-x)$, and there is a contribution from zero gluon number fluctuations, 0.2377$\delta(x)$, not shown. The continuous part of the radiative distribution has integrated norm 0.7374 with mean 0.2026.

The top (black) curve corresponds to the convolution of elastic and inelastic energy fraction fluctuations.

FIG. 11: The ratio of rms width, $\sigma(L)$, to the mean fraction energy loss $\langle \epsilon \rangle$ for radiative, elastic and convolved energy loss distributions is shown as a function of the path length, $L$, for the Bjorken expanding plasma with $dN_g/dy = 1000$. The case of an up quark jet with $E = 15$ GeV is shown. Notice that the elastic distribution is significantly narrower than the radiative one. This amplifies the effect of elastic energy loss on $R_{AA}$ relative to radiative.

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