Chiral Symmetry and Low Energy Pion-Nucleon Scattering

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In these lectures, I examine the effect of the meson factory $\pi N$ data on the current algebra/PCAC program which describes chiral symmetry breaking in this system. After historical remarks on the current algebra/PCAC versus chiral Lagrangians approaches to chiral symmetry, and description of the need for $\pi N$ amplitudes with virtual (off-mass-shell) pions in nuclear force models and other nuclear physics problems, I begin with kinematics and isospin aspects of the invariant amplitudes. A detailed introduction to the hadronic vector and axial-vector currents and the hypothesis of partially conserved axial-vector currents (PCAC) follows. I review and test against contemporary data the PCAC predictions of the Goldberger-Treiman relation, and the Adler consistency condition for a $\pi N$ amplitude. Then comes a detailed description of the current algebra Ward-Takahashi identities in the chiral limit and a brief account of the on-shell current algebra Ward-Takahashi identities. The latter identities form the basis of so-called current algebra models of $\pi N$ scattering. I then test these models against the contemporary empirical $\pi N$ amplitudes extrapolated into the subthreshold region via dispersion relations. The scale and the $t$ dependence of the “sigma term” is determined by the recent data.

1 Introduction

The implementation of chiral symmetry in hadronic physics began around 1960. Its consequences were examined with two basic approaches. One is based on the concept of a partially conserved axial-vector current (PCAC) coupled with the algebra of vector and axial-vector hadronic currents. This current algebra is expressed as equal time commutation relations (the analogue of angular momentum commutation relations in quantum mechanics). The other (Lagrangian form) is based on chiral Lagrangians with small explicit chiral symmetry breaking terms. A famous example of the latter is the linear sigma model of Gell-Mann and Levy which explicitly exhibits both PCAC and the current algebra. To adapt J. B. S. Haldane’s famous remark about the Deity and beetles, chiral symmetry seems to be inordinately fond of pions. Single pion exchange accounts for about 70% of the binding of light nuclei (and perhaps all nuclei) and pions make up the most prominent non-nucleon degree of freedom in nuclear physics. Thus it should not be surprising that chiral symmetry was applied to nuclear physics as soon as 1967. It is the aim of these lectures, motivated by the nuclear physics problems briefly mentioned below, to discuss the PCAC-current algebra approach with the aid of contemporary experimental knowledge of the low energy pion nucleon interaction. Relationships between the two approaches to chiral symmetry breaking will be mentioned when useful. This introductory lecture is primarily for motivation and will freely employ undefined concepts that will be defined and derived in detail in the following lectures.
One of the earliest and boldest uses, in nuclear physics, of the PCAC form of chiral symmetry was the relationship obtained, by Blin-Stoyle and Tint, between the $\beta$-decay pion-exchange operator and a phenomenological two-body (nucleons) pion production operator. With this relation, they attempted to analyze the process $p+p \rightarrow d+\pi^+$ using two-body terms obtained from a comparison of $\beta$-decay of the tritium nucleus and $\beta$-decay of the neutron. Neither the pion production data, the three-body wavefunction, nor the $f_t$ values of the two $\beta$-decays were known in the 60’s well enough to obtain a quantitative conclusion. Nearly the same technique was used 30 years later to obtain a rather reliable calculation of the process $p+p \rightarrow d+e^++\nu_e$, so important for stellar nucleosynthesis. It may seem reasonable to anyone that the latter process of weak capture of protons by protons might be related to weak $\beta$-decay. But it is the introduction of an isovector axial-vector hadronic current to play a role in both strong and weak interactions which lead to the perhaps more startling recognition of a relation between a strong (pion production) and a weak ($\beta$-decay) process. We shall see how this comes about later.

Another explicit use of PCAC alone (in the form of Adler’s consistency condition) was in an envisioned re-scattering of a virtual pion from one nucleon of a three-nucleon system. This process establishes a three-nucleon interaction due to two-pion exchange. Brown et al. showed that the three-nucleon force contribution to the binding energy of nuclear matter could be obtained from the isospin symmetric pion-nucleon forward scattering amplitude extrapolated off the pion mass shell, and was quite small. This analysis knowingly neglected the pion-nucleon sigma term, a measure of chiral symmetry breaking (the sigma term is proportional to the non-conserved axial-vector current). Somewhat later the full panoply of current algebra and PCAC constraints (labelled current algebra/PCAC) was brought to bear on the off-shell pion-nucleon amplitude. These current algebra/PCAC “soft pion theorems” led to a scenario in which the chiral symmetry breaking sigma term could not be neglected, but instead was quite prominent in the three-body interaction. However, the original insight of Refs. [5, 6, 7, 8], based on PCAC and later on current algebra/PCAC, that pion exchange based three-nucleon interactions are small compared to two-nucleon interactions remains true in the Lagrangian form of chiral symmetric theories. A currently employed three-nucleon interaction according to a chiral Lagrangian is the Brazil three-body force (TBF). The first version of this TBF had a sigma term contribution which did not come from a Lagrangian and in a later version the sigma term contribution was altered to conform to the current algebra/PCAC constraints which had previously guided the Tucson-Melbourne two-pion exchange TBF.

A technical trick in Refs. [6, 8], which directly relates pion-nucleon scattering to a TBF contribution in nuclear matter, leads me to my third (and final) illustrative example of chiral symmetry in nuclear physics: pion condensation in nuclear matter. An approximate evaluation of a three-body diagram in an translationally invariant system like nuclear matter can be made by summing and averaging the active nucleon over the Fermi sea. Then one obtains a modified one-pion-exchange-potential between the other two nucleons, which can easily be evaluated in a many-body
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That is, the (now) single exchanged pion has an effective mass \( m^* \) which is proportional to the isospin even, forward \( \pi N \) amplitude multiplied by the density (from the summation). A useful way to think about pion condensation is to extend the idea of a virtual pion rescattering from the active nucleon of a three-nucleon cluster to the picture of a pion rescattering again and again from the nucleons of nuclear matter. The criterion for pion condensation can be expressed in terms of \( m^* \) (or the self energy in the pion propagator) which again is directly related to forward \( \pi N \) amplitudes. Pion condensation in neutron matter was examined with such amplitudes subject to the current algebra/PCAC constraints \[12\]. Actually, this third example of chiral symmetry had been discussed earlier with the aid of chiral Lagrangians \[13\]. This last problem has a contemporary reverberation which is somewhat amusing in that, of the three problems so far, it surely is the least constrained by experiment. Yet the relationships between the two forms of chiral symmetry have been clarified by a small debate on, of all things, kaon condensation in dense nuclear matter. This debate was between a group \[14\] who, fifteen years after the Tucson group \[12\], re-examined the current algebra/PCAC program of pion condensation, and practitioners \[15\] of the contemporary effective Lagrangian form of chiral symmetry known as chiral perturbation theory.

These three examples share the idea of a virtual pion rescattering from a nucleon (pion production in \( NN \) collisions and two-pion exchange TBFs) or from the many nucleons of nuclear matter (pion condensation). Another example which I will not discuss much is the two-pion exchange part of the \( NN \) interaction itself. As a Feynman diagram, this process has pion loops and the other three problems need only tree diagrams. As with the three-nucleon interaction, the first use of chiral symmetry in the two-nucleon interaction was again by Gerry Brown who applied the current algebra/PCAC constraints on the \( \pi N \) amplitudes (and dropped the sigma term) in a series of articles titled “Isn’t it time to calculate the nucleon-nucleon force?” and “Soft pioneering determination of the intermediate range nucleon-nucleon interaction” \[16\]. The chiral symmetry aspect of two-pion exchange NN diagram can (as expected) and has been treated with chiral Lagrangian techniques recently \[17, 18\]. I now cut off this introductory and historical survey and turn to the concept of “soft pions”.

Each of these nuclear physics problems can be thought of as dependent on a \( \pi N \) scattering amplitude with at least one of the exchanged pions off its mass shell: \( q^2 = q_0^2 - \vec{q}^2 \neq m_\pi^2 \). For example, short range correlations between two nucleons of nuclear matter suggests that the virtual pions of a TBF are spacelike with \( q_0 \approx 0 \) and \( \vec{q}^2 \leq 10m_\pi^2 \) \[13\], and the calculation of Ref. \[12\] found that, at the condensation density, \( q_c^2 \leq -2m_\pi^2 \). The “soft pion theorems” strictly apply to pions with \( q \to 0 \) which means that every component of the four-vector goes to zero. In particular, since \( q_0 \to 0 \) then \( q_0^2 = m_\pi^2 \to 0 \) and a soft pion is a massless pion. In the language of QCD, this means that the quark mass goes to zero and the chiral symmetry of the QCD Lagrangian is restored. The axial-vector current would be conserved if the pion was massless. The mass of the pion is small on the scale of the other hadrons \( (m_\pi^2/m_N^2 \approx 1/45) \) so one of the ideas of PCAC is that the non-conservation of the axial vector current is small. Another formulation of PCAC suggests that
one can make a smooth extrapolation from the exact amplitudes with soft pions to obtain either theoretical amplitudes with on-mass-shell pions (“hard” pions in the old jargon) or the off-shell amplitudes of the nuclear physics problems. Certainly the soft pion constraints of current algebra/PCAC are within the assumed off-shell extrapolations used in these problems.

In the 1960’s the current algebra/PCAC approach and the chiral Lagrangian approach to chiral symmetry (and how it is broken in the non-chiral world we do experiments in) developed in parallel and each approach paid close attention to the other. The 1970 lectures by Treiman on current algebra and by Jackiw on field theory provide a useful (and pedagogical!) summary of this development[19]. For example, the linear $\sigma$ model was an early chiral Lagrangian motivated by the current algebra/PCAC program. This model reflects the feeling in the 1960’s that the ultimate justification of the results obtained from a chiral Lagrangian rests on the foundation of current algebra. On the other hand, an early puzzle was the current algebra demonstration that the (observed) decay $\pi \rightarrow \gamma\gamma$ should be zero. In his lectures, Jackiw used the linear $\sigma$ model to demonstrate that the “conventional current algebra” techniques were inadequate. He went on from this demonstration of a violation of the axial-vector Ward identity with the nucleon level linear $\sigma$ model to introduce a study of anomalies which is documented in Ref. [19]. (The quark level linear $\sigma$ model, however, does appear to describe the decay $\pi \rightarrow \gamma\gamma$ and 22 other radiative meson decays [20], so the final denouement of this dialogue may be still to come.) In any event, anomalies play no role in the nuclear physics problems of these lectures, and will not be discussed here. A very useful pedagogical paper, specifically aimed at the nuclear physicist, commented on the relation between the two approaches to chiral symmetry. In it, David Campbell showed that, in a given chiral model field theory with a specific choice of canonical pion fields, certain of the theorems expected from current algebra/PCAC (in particular the Adler consistency condition) will not be true [21]. This is one of the excellent papers which I hope the present lectures will prepare the student to appreciate.

In 1979, Weinberg [22] introduced a “most general chiral Lagrangian” constructed from powers of a chiral-covariant derivative of a dimensionless pion field. This Lagrangian was aimed at calculating purely pionic processes with low energy pions. The most general such phenomenological Lagrangian, unlike earlier closed form models, is an infinite series of such operators of higher and higher dimensionality. He was easily able to show that the lowest order Feynman diagrams constructed from the Lagrangian are tree graphs. These tree graphs reproduce his earlier [23] analysis of low energy $\pi\pi$ scattering obtained from i) the Ward identities of current algebra and ii) PCAC in the form of a smooth extrapolation from soft to physical pions. The importance of the 1979 paper lies in its analysis of the more complicated Feynman diagrams of the infinite perturbation expansion of the chiral Lagrangian. The phenomenological Lagrangian would produce amplitudes of the form: $T \sim E^\nu$, where $E$ is the energy. This fact was obtained using dimensional analysis and $\nu$ is an integer determined by the structure of the Feynman diagram. The QCD picture of chiral symmetry breaking (for example in a world with only light $u$ and $d$ quark fields) imposes a further constraint upon $\nu$: that more complicated diagrams
necessarily have larger values of \( \nu \). Thus, provided that \( E \) is smaller than some intrinsic energy scale, \( \Lambda \), the perturbation series is a decreasing series in \( E/\Lambda \). The derivative structure of the Lagrangian guarantees that amplitudes from loops and other products of higher order perturbation theory produce only larger values of \( \nu \). The Lagrangian cannot be renormalized because this is an effective field theory where all possible terms consistent with the symmetries assumed must be included. The non-renormalizability means that more and more unknown constants appear at higher (arranged in powers of \( \nu \)) and higher orders of perturbation theory but their effect is suppressed by factors of \( E/\Lambda \). Since it is a phenomenological Lagrangian the unknown constants must be determined by experiment, and one hopes that meaningful results can be obtained at a low enough energy such that the number of terms needed remains tractable. That is the disadvantage of this approach. An advantage is the systematic nature of the scheme with respect to the breaking of chiral symmetry. I quote from the seminal paper: “the soft \( \pi \) and soft \( K \) results of current algebra, which would be precise theorems in the limit of exact chiral symmetry, become somewhat fuzzy, depending for their interpretation on a good deal of unsystematic guesswork about the smoothness of extrapolations off the mass shell. ... phenomenological Lagrangians can serve as the basis of an approach to chiral symmetry breaking, which has at least the virtue of being entirely systematic” [22]

The introduction of the nucleon into this scheme (now called chiral perturbation theory or ChPT) led to a major industry in particle physics and to a reversal of the old idea that a symmetry imposed on an effective Lagrangian can only be legitimized by an underlying theory such as current algebra [24]. The new effective field theory program does not attempt to find really fundamental laws of nature, but does exploit systematically the symmetries encoded in the phenomenological Lagrangian. The belief in the power of this program leads to astounding remarks in chiral perturbation theory papers. Consider:

“Although the purpose of this comment is not to discuss the experimental situation, it may be one of nature’s follies that experiments seem to favour the original LEG [Low Energy Guesses of pion photoproduction from the nucleon] over the correct LET [Low Energy Theorems from ChPT]. One plausible explanation for the seeming failure of the LET is the very slow convergence of the expansion in \( m_\pi \).” [25]

Or:

“We can compare the situation with that of the decay \( \eta \to \pi^0\gamma\gamma \) where the one loop ChPT prediction is approximately 170 times smaller than the experimental result. The \( \mathcal{O}(p^6) \) contributions [the next order in the expansion] then bring the ChPT result into satisfactory accord with experiment.” [26]

On a more cautious note:

“The one-loop calculation [of \( \gamma\gamma \to \pi^0\pi^0 \)] in ChPT disagrees with the data even near threshold.” ... “In conclusion, a self-consistent, quantitative description of \( \gamma\gamma \to \pi^0\pi^0 \) and \( \eta \to \pi^0\gamma\gamma \) data at \( \mathcal{O}(p^6) \) is still
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problematic. A good description of the $\gamma\gamma \rightarrow \pi^0\pi^0$ cross section has been achieved whereas a satisfactory, quantitative prediction of the decay width seems to be beyond the reach of an ordinary calculation at $\mathcal{O}(p^6)$ [such a calculation involves tree-level, one- and two-loop Feynman diagrams].” 

Finally, on elastic $\pi N$ scattering, the subject of these lectures:

“the chiral expansion converges to the experimental values, but the convergence seems to be rather slow, in a sense that contributions to different orders are comparable. This fact seems to show that despite of the relative success in describing elastic $\pi N$ scattering at threshold, the third order is definitely not the whole story. A complete one-loop calculation, which will include the fourth order of the chiral expansion, is probably needed for sufficiently reliable description of this process” 

Although nature seems to have pulled up its socks since the first comment was made (more recent measurements of pion photoproduction seem to favor ChPT results near threshold), one is still left with a not fully satisfied feeling by these comments.

More recently Weinberg applied this procedure to systems with more than one nucleon \[30\] so that effective field methods could be extended to nuclear forces and nuclei \[30\]. This program is being continued vigorously by van Kolck and others, thereby generating another minor industry in nuclear physics a decade or so after ChPT hit particle physics.

In the following lectures, I will review the current algebra and PCAC program and its applications to the three nuclear physics problems of this introduction. These problems have also been attacked by the effective field theory program. The former approach to chiral symmetry can be closely tied to the experimental program in pion-nucleon scattering and the latter approach takes some of its undetermined constants from pion-nucleon scattering. Before proceeding, I recommend the following review articles on this field. Pion-nucleon scattering is treated, more extensively than I will, in Field Theory, Chiral Symmetry, and Pion-Nucleon Interactions by D. K. Campbell \[31\]. The mathematical aspects of global symmetries in Lagrangian forms of field theory is discussed cogently in lectures at an earlier Indian-Summer School: Elements of Chiral Symmetry by M. Kirchbach \[32, 33\]. A very useful account (which I shall freely borrow from) of the original approach to chiral symmetry is Current algebra, PCAC, and the quark model by M. D. Scadron \[34\]. The nuclear physics aspects of effective field theories are well described in Effective Field Theory of Nuclear Forces by U. van Kolck \[35\] and Dimensional Power Counting in Nuclei by J. L. Friar \[36\]. The titles of these review articles should suggest to the student where to go for further studies.

2 Kinematics

We begin with the scattering amplitude for $\pi^i(q) + N(p) \rightarrow \pi^i(q') + N(p')$ where $p, q, p', q'$ are nucleon and pion initial and final momenta. We ignore for the time
being spin and isospin aspects of the problem ($i$ and $j$ are pion (Cartesian) isospin indices). For elastic scattering $q + p = q' + p'$ and the scalar product of these four-vectors is $ab \equiv a_0b_0 - \vec{a} \cdot \vec{b}$. Define the “$s$-channel” Mandelstam invariants

$$
\begin{align*}
    s & \equiv (p + q)^2 = (p' + q')^2 \\
    t & \equiv (q - q')^2 = (p' - p)^2 \\
    u & \equiv (p - q')^2 = (p' - q)^2.
\end{align*}
$$

The invariant $s$ in this $s$-channel corresponds to the square of the total energy for the process. Since four-momentum conservation is but one constraint upon the four momenta, there are three independent combinations of these momenta (and energies), but only two independent combinations of Lorentz scalar products. So $s$, $t$, and $u$ are not independent and it can quickly be shown with the aid of $q + p = q' + p'$ that

$$
s + t + u = p^2 + p'^2 + q^2 + q'^2.
$$

If the nucleons are on-mass-shell ($p^2 = p_0^2 - \vec{p}^2 = m^2$) and the pions are on-mass-shell ($q^2 = q_0^2 - \vec{q}^2 = \mu^2$), this relation becomes $s + t + u = 2m^2 + 2\mu^2$.

These Lorentz invariants $s$, $t$, and $u$ can be visualized in different coordinate systems. For example, in the $s$-channel center of mass frame the incoming (on-mass-shell) momenta are pion $q = (q_0, \vec{q}_{cm})$ and nucleon $p = (E, -\vec{q}_{cm})$, the final (on-mass-shell) momenta are $q' = (q_0, \vec{q}_{cm}')$ and $p' = (E, -\vec{q}_{cm}')$, where $|\vec{q}_{cm}| = |\vec{q}_{cm}'|$ and the three-vector $\vec{q}_{cm}$ is simply rotated by the angle $\theta_{cm}$.

In this frame one can evaluate

$$
\begin{align*}
    s & \equiv (p + q)^2 = m^2 + \mu^2 + 2[(m^2 + q_{cm}^2)^{\frac{1}{2}}(\mu^2 + q_{cm}'^2)^{\frac{1}{2}} + q_{cm}'^2] \\
    t & \equiv (q - q')^2 = -2q_{cm}^2(1 - \cos \theta_{cm}) \\
    u & \equiv (p - q')^2 = m^2 + \mu^2 - 2[(m^2 + q_{cm}^2)^{\frac{1}{2}}(\mu^2 + q_{cm}'^2)^{\frac{1}{2}} + q_{cm}^2 \cos \theta_{cm}] .
\end{align*}
$$

Note that $s \geq (m + \mu)^2$ and $t \leq 0$ for physical $\pi N$ scattering where $q_{cm}^2 \geq 0$ and $-1 \leq \cos \theta_{cm} \leq 1$. The $cm$ energy of the on-mass-shell nucleon is $E = (s - m^2 -$
Partial wave phase shifts are naturally expressed in terms of $q_{cm}^2$ and its associated Mandelstam variable $s$.

Now consider the $s$-channel laboratory frame in which the target nucleon is at rest and the kinetic energy of the incoming pion is defined by $T_\pi \equiv \omega - \mu = \sqrt{k^2 + \mu^2} - \mu$, where $\omega$ is the lab energy of the incoming pion:

\[ q = (\omega, \vec{k}) \quad p = (m, 0) \]

\[ q' = (\omega', \vec{k}') \quad p' = (E, \vec{p}) \]

In this laboratory frame the Mandelstam invariants $s$ and $u$ take the form

\[ s \equiv (p + q)^2 = m^2 + \mu^2 + 2m\omega = (m + \mu)^2 + 2mT_\pi \]

\[ u \equiv (p - q')^2 = m^2 + \mu^2 - 2m\omega' . \]

In either frame, it is clear that the threshold for physical $\pi N$ scattering is $s_{th} = (m + \mu)^2$ from $q_{cm}^2 = 0$ or $T_\pi = 0$, $t_{th} = 0$ from $q_{cm}^2 = 0$, and $u_{th} = (m - \mu)^2$ from $q_{cm}^2 = 0$ or $\omega' = \mu$.

Now introduce the variable

\[ \nu = \frac{s - u}{4m} \]

which has the threshold value $\nu_{th} = \mu$ in the $s$-channel. For on-shell nucleons and pions $\nu = \omega + t/(4m)$ so that in the forward direction, $t = 0$, the variable $\nu$ represents the lab energy of the incoming pion. Pion-nucleon scattering amplitudes are often given in terms of the pair of variables $(\nu, t)$ rather than $(s, t)$ which would be appropriate for a partial wave representation, for example. A reason for this is that the variable $\nu$ has a definite symmetry under crossing, a concept to which we now turn. Crossing is the interchange of a particle with its antiparticle with opposite four-momentum. I can “cross” the pions by adding nothing to the $s$-channel relation $\pi^j(q) + N(p) \to \pi^i(q') + N(p')$ as follows:

\[ \pi^j(q) \quad +N(p) \quad \rightarrow \quad \pi^i(q') \quad +N(p') \]

Since I have changed nothing this $s$-channel process is equivalent to $\pi^i(-q') + N(p) \to \pi^j(-q) + N(p')$.
which is called the $u$-channel because $u = (p - q')^2$ is now the sum of the incoming momenta and in this channel $u$ is the square of the total energy. Because the antiparticle of a pion is still a pion ($\pi^+ = \pi^-$ and $\pi^0 = \pi^0$) both the $s$-channel and the $u$-channel describe $\pi N$ scattering.

Carrying on with “crossing” one can convince oneself that

$s$-channel In this channel $s$ is the total energy squared and for physical scattering $s \geq s_{th} = (m + \mu)^2$ and $t \leq 0$ (and $u \leq 0$).

$u$-channel In this channel $u$ is the total energy squared and for physical scattering $u \geq u_{th} = (m + \mu)^2$ and $t \leq 0$ (and $s \leq 0$).

$t$-channel In this channel the incoming particles are a pion and an anti-pion and the outgoing particles are $N$ and $\bar{N}$. For physical scattering the total energy squared must be larger than the rest mass of the heaviest particle, so that $t \geq t_{th} = (m + m)^2$, $s \leq 0$, and $u \leq 0$.

The scattering amplitude $T(s, t, u)$ is a function of the three (not all independent) variables. The physical regions of the variables of the three channels are disjunct. We have determined the threshold values “by inspection”. It is slightly more complicated to work out the boundaries of the physical regions in the Mandelstam plane. They are given by the zeros of the Kibble function $\Phi$.

$$\Phi = t[su - (m^2 - \mu^2)].$$

The physical regions correspond to the regions where $\Phi \geq 0$. This criterion essentially characterizes the need for the scattering angle to satisfy $-1 \leq \cos \theta_{cm} \leq 1$. Clearly $t = 0$ or $\cos \theta_{cm} = 1$ is a boundary of the physical region no matter what the values of $s$ and $u$ are. The other zero of $\Phi$ then shows the dependence of a lower limit to $t$ for $s$-channel $\pi N$ scattering (for example) which depends upon the values of $s \geq s_{th} = (m + \mu)^2$ and $u \leq 0$. Elastic scattering depends upon two independent variables: some sort of energy and some sort of scattering angle. As mentioned before, if you wanted to end up with a partial wave representation the natural variables are the pair $(s, t)$ because $t$ has a simple interpretation in terms of $q_{cm}^2$ and $\theta_{cm}$, for example. In the following discussion, the pair $(\nu, t)$ is more natural because $\nu \rightarrow -\nu$ under the interchange of $s$ and $u$: $s \Leftrightarrow u$. Then the physical regions of the Mandelstam plane are bounded by a hyperbola in the $(\nu, t)$ plane.
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and the straight line \( t = 0 \) (See Fig 1). The boundaries of the physical regions for \( \pi + N \rightarrow \pi + N \) and for \( \pi + \pi \rightarrow N + \bar{N} \) form branches of the same hyperbola. Note that the asymptotes of the boundary hyperbola are the lines \( s = 0 \) and \( u = 0 \).

Let us turn from the relativistic invariants in \( T(\nu, t) \) to isospin considerations in the three channels. The incoming particles in the \( s \) and \( u \) channels have isospin \( \frac{1}{2} \) (nucleon) and 1 (pion). The total isospin \( I_s \) is then either \( \frac{1}{2} \) or \( \frac{3}{2} \). In the \( t \)-channel \( (\pi \bar{\pi} \rightarrow N\bar{N}) \) \( I_t = 0 \), so there are also two amplitudes in isospin: \( T(+) \) with \( I_t = 0 \) and \( T(-) \) which has \( I_t = 1 \). The \( t \)-channel isospin is especially convenient because the pions obey Bose symmetry when crossed: ie \( I_t^0 \rightarrow I_t^0 \) and \( I_t^1 \rightarrow -I_t^1 \) when \( s \leftarrow u \). To make contact with the \( s \)-channel amplitudes (and the charge states) we need the \( s \leftarrow t \) crossing relations:

**Figure 1:** Mandelstam diagram of pion-nucleon scattering
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\[ T^{(+)} = \frac{1}{3}(T(\frac{1}{2}) + 2T(\frac{3}{2})) \]
\[ T^{(-)} = \frac{1}{3}(T(\frac{1}{2}) - T(\frac{3}{2})) \]
\[ T^{(2)} = T^{(+)} + 2T^{(-)} \]
\[ T^{(\frac{3}{2})} = T^{(+)} - T^{(-)} . \]

The pion field operators transform as components of a vector in isospin space with Cartesian components defined as:

\[ \pi^+ = +\frac{(\pi^1 + i\pi^2)}{\sqrt{2}} \]
\[ \pi^0 = \pi^3 \]
\[ \pi^- = +\frac{(\pi^1 - i\pi^2)}{\sqrt{2}} , \]

and

\[ T(\pi^+ p \rightarrow \pi^+ p) = T(\pi^0 n \rightarrow \pi^0 n) = \frac{2}{\sqrt{3}}(T(\frac{3}{2}) - T(\frac{1}{2})) \]
\[ T(\pi^- p \rightarrow \pi^- p) = T(\pi^+ p \rightarrow \pi^+ p) \]
\[ = T(\pi^- p \rightarrow \pi^- p) \]
\[ = \frac{2}{\sqrt{3}}(T(\frac{3}{2}) - T(\frac{1}{2})) \]
\[ = -\sqrt{2}T(-) , \]

for example.

Now we are in a position to examine the isospin structure of the T-matrix elements which describe the scattering \( \pi^j(q) + N(p) \rightarrow \pi^j(q') + N(p') \). They are defined as:

\[ \langle q' p' | S - 1 | q p \rangle = +i(2\pi)^4\delta^4(p' + q' - p - q)T^{ij}(\nu, t; p^2 = m^2, p'^2 = m'^2, q^2, q'^2) , \]

where we have displayed the (assumed) on-mass-shell nucleons and left the four-momentum of the pions as a variable. The isospin structure of \( T^{ij} \) is perhaps most easily visualized from the Feynman diagram with an intermediate nucleon pole state and the isospin “scalar” vertex \( \bar{N}\tau N \cdot \vec{\pi} \):

\[ T^{ij}\tau^i\tau^j = T^{(+)}\frac{1}{2}(\tau^i\tau^j + \tau^j\tau^i) + T^{(-)}\frac{1}{2}(\tau^i\tau^j - \tau^j\tau^i) \]
\[ = T^{(+)}\delta^{ij} + T^{(-)}i\epsilon^{ijk}\tau^k . \]

The second equality is easily proved from the properties of the \( SU(2) \) \( \tau \)-matrices: \( \{\tau^i, \tau^j\} = 2\delta^{ij} \) and \( [\tau^i, \tau^j] = 2i\epsilon^{ijk}\tau^k \). With this representation, it is clear that \( T^{(+)} \) (\( T^{(-)} \)) must be even (odd) under the interchange of the pions \( i \leftrightarrow j \) and \( s \leftrightarrow u \). We also note that \( \pi^i\pi^j\delta^{ij} = \vec{\pi} \cdot \vec{\pi} = \pi^2 \) could be realized by the t-channel exchange of an isoscalar scalar \( \pi\pi \) resonance— the putative sigma meson. In a similar manner, the t-channel odd exchange, \( \pi^i\pi^j i\epsilon^{ijk}\tau^k = i\vec{\pi} \times \vec{\pi} \cdot \vec{\pi} \) could be realized by the isovector \( \rho \) meson. We will discuss these models in a later lecture.

Finally, we let the Dirac spinors \( u(p) \) carry the spin of the nucleons and write the Lorentz invariant \( T = +\bar{u}(p')\{M(\nu, t)\}u(p) \) where \( M \) could be made up of scalars, vectors, and higher order tensors constructed of the vectors \( p, p', q, q' \) and the gamma matrices \( 1, \gamma^\mu, \gamma^\mu\gamma^5, \gamma_5, \gamma^\mu\gamma_5 \). That is, one could form:

\[ M = A + B^\mu \gamma_\mu + C^\mu\nu[\gamma_\mu, \gamma_\nu] + D^\mu\gamma_\mu\gamma_5 + E\gamma_5 \]

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but conservation of parity eliminates $D^\mu$ and $E$ as candidates. With the aid of the Dirac equation for free (on-mass-shell) nucleons,

$$(p^\mu \gamma_\mu - m)u(p) = 0 = \bar{u}(p')(p'^\mu \gamma_\mu - m),$$

all the combinations one can write down for $C^\mu_\nu$ reduce to $A + B^\mu \gamma_\mu$, where $A$ is a scalar and $B$ is a four-vector formed of those available: $p, p', q, q'$. $B^\mu$ cannot be $p$ or $p'$ because the Dirac equation would make $T \sim \bar{m}u(p')u(p)$ already included in $A$. So $B^\mu$ must be linear in $q$ and $q'$, but $B^\mu$ cannot be $(q - q')^\mu = (p' - p)^\mu$ for the same reason. We conclude that

$$T^\pm = \bar{u}(p')\{A^\pm(\nu, t) + \frac{1}{2}(\bar{q}' + \bar{q})B^\pm(\nu, t)\}u(p)$$

where $\bar{q'} \equiv q'^\mu \gamma_\mu$ and the factor $\frac{1}{2}$ is inserted to make the expressions for $s$ and $u$-channel nucleon poles in $B$ simple. With the aid of $\nu = (s - u)/4m = (q' + q) \cdot (p' + p)/4m$ and the free particle Dirac equation, one can rewrite this as

$$T^\pm = \bar{u}(p')\{A^\pm(\nu, t) + \nu B^\pm(\nu, t) - \frac{1}{4m}(\bar{q}' \bar{q})B^\pm(\nu, t)\}u(p). \quad (3)$$

Define the combination $A + \nu B = F$, which is called $D$ in Höhlers book [39] and in much of the literature. It can be shown that this combination of invariant amplitudes corresponds to the non-relativistic forward $(p = p')$ scattering of a nucleon from a pion in which the spin of the nucleon remains unchanged (non-spin flip); for example, see Ref. [40], pp 612. For this reason the invariant amplitude $F$ is sometimes called the “forward amplitude” but obviously we can study the combination $A + \nu B$ for any value of $\nu$ and $t$.

Expressions of chiral symmetry in the form of soft pion theorems and their on-mass-shell analogues are most naturally expressed as conditions on the four amplitudes $F^\pm(\nu, t)$ and $B^\pm(\nu, t)$, rather than the set $A^\pm(\nu, t)$ and $B^\pm(\nu, t)$. As we shall see in the following, the predictions of chiral symmetry breaking are all in the subthreshold crescent of the Mandelstam representation (Figure 2). This crescent is below the $s$-channel threshold $s_{th} = (m + \mu)^2$ for $\pi N \rightarrow \pi N$, below the $u$-channel threshold $u_{th} = (m + \mu)^2$ for $\pi N \rightarrow \pi N$, and below the $t$-channel threshold $t_{th} = (m + m)^2$ for $\pi \pi \rightarrow NN$. Therefore the invariant amplitudes in this subthreshold crescent are real functions of the real variables $\nu$ and $t$.

### 3 Current Algebra and PCAC

The formalism in this lecture follows very closely the exposition of Scadron in his review article [41] and textbook [42]. It is included here to make the lectures somewhat self-contained and to enable me to compare the current algebra predictions for pion-nucleon scattering with the current experimental results, a comparison which has not been emphasized in most contemporary discussions of the meson factory data.

We begin with the basic ideas of the current algebra-Partially Conserved Axial-vector Current (PCAC) implementation of chiral symmetry in hadronic physics.
Recall that in non-relativistic quantum mechanics the charge operator $Q(t)$ obeys the Heisenberg equation of motion:

$$\frac{dQ(t)}{dt} = \frac{\partial Q(t)}{\partial t} - i\{Q(t), H(t)\}.$$  

(4)

Then, if $Q$ is explicitly independent of time, conservation of charge is equivalent to $Q$ commuting with the Hamiltonian. In relativistic quantum mechanics we can define current densities and Hamilton densities as

$$Q(t) \equiv \int d^3x J_0(t, \vec{x})$$

(5)

$$H(t) \equiv \int d^3x \mathcal{H}(t, \vec{x}).$$

(6)

The equation of continuity for charge

$$\frac{dQ(t)}{dt} = \int d^3x \left( \frac{\partial J_0(t, \vec{x})}{\partial t} \right) + \vec{\nabla} \cdot \vec{J}(t, \vec{x}) \equiv \int d^3x \partial J(x)$$

(7)

allows one to rewrite (4) in the local density form

$$i\partial J(x) = \{Q, \mathcal{H}(x)\}$$

(8)

if $Q$ does not depend explicitly on time.
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Commutators such as this one form the underlying dynamics in current algebra. We now know that at the hadronic level QCD is spontaneously broken down into a vector $SU(2)$ algebra and an axial-vector $SU(2)$ algebra. Current algebra is based on the $SU(2)$ equal-time commutation relations of isotopic vector charges

$$[Q^i, Q^j] = i\epsilon^{ijk}Q^k$$

which was extended in the 1960’s by Gell-Man’s suggestion of adding axial charge ($Q^5$) commutators

$$[Q^i, Q^5] = i\epsilon^{ijk}Q^k, \quad [Q^5, Q^5] = i\epsilon^{ijk}Q^k$$

to complete the chiral algebra. Models of $H$ for strong, electromagnetic, and weak transitions as products of currents then predict observable hadron current divergences according to (8) with the aid of the charge algebra and its current algebra generalizations. We defer discussion of the current algebra per se until after this introductory material is discussed.

The $SU(2)$ notation is the same as before with states $|\pi_i\rangle$, and (to be defined) vector currents $J^i_\mu$ and axial-vector currents $A^i_\mu$, where $i = 1, 2, 3$. Define isotopic charges from current densities as

$$Q^i = \int d^3x J^i_\mu(t, \vec{x})$$

The $SU(2)$ hadron states transform irreducibly as $Q^i |P_j\rangle = f_{ijk} |P_k\rangle$, where $f_{ijk} = \epsilon_{ijk}$. In the generalization to $SU(3)$ the anti-symmetric structure constant $f_{ijk}$ is related to the Gell-Man $\lambda_i$ matrices, $i = 1, \cdots, 8$ (for a tabulation, see Ref. [34]). Now consider the $SU(2)$ and $SU(3)$ structure of the electromagnetic current

$$J^\gamma_\mu = J^S_\mu + J^V_\mu = \frac{1}{\sqrt{3}} J^8_\mu + J^3_\mu,$$

where $J^V_\mu = J^3_\mu$ is the isovector current and $J^S_\mu = \frac{1}{\sqrt{3}} J^8_\mu$ corresponds to $2J^Y_\mu$ the hypercharge current. The corresponding charges are

$$Q = \int d^3x J^S_0(x), \quad \frac{1}{2} Y = \int d^3x J^V_0(x), \quad I_3 = \int d^3x J^3_0(x).$$

The equation of continuity (7), coupled with the fact that the electromagnetic charge $Q$ is conserved in the strong interaction, implies $\partial J^\gamma(x) = 0$. The $SU(3)$ structure of the photon is consistent with the separate conservation of isospin and hypercharge in the strong interactions and suggests the Gell-Mann-Nishijima relation $Q = \frac{1}{2} Y + I_3$.

### 3.1 Conserved $SU(2)$ Vector Currents

We want to treat $J^S_\mu$ and $J^{V,i}_\mu$ (where $J^V_\mu = J^{V,i}_i$) as conserved hadronic currents for the strong interactions, $\partial J^{V,i} = 0$ and $\partial J^S = 0$. To illustrate this, define the isovector part of the $SU(2)$ strong vector current by its nucleon matrix elements:

$$\langle N_p | J^{V,i}_\mu(x) | N_p \rangle = \bar{N}_p \frac{1}{2} [F^1_1(q^2)\gamma_\mu + F^2_1(q^2)i\sigma_{\mu\nu}q^\nu / 2m] N_p e^{iq\cdot x},$$

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where \((q = p' - p)\), \(F^V_1(q^2)\) is the nucleon isovector charge form factor and \(F^V_2(q^2)\) the nucleon isovector magnetic moment form factor. The isoscalar and isovector decomposition is defined as \(F^S_{1,2}(q^2) = F^p_{1,2}(q^2) + F^n_{1,2}(q^2)\) and \(F^V_{1,2}(q^2) = F^p_{1,2}(q^2) - F^n_{1,2}(q^2)\). This definition is made because we identify the \(i = 3\) component of \(J^V_\mu\) plus \(J^S_\mu\) (with a similar definition) as the electromagnetic current for the proton or the neutron. For the complete electromagnetic current, \(J^V_\mu = J^S_\mu + J^V_\mu\), charge is conserved and \(F^V_1(0) = 1\). In terms of isospin this conservation law becomes \(F^V_1(0) = F^p_1(0) + F^n_1(0) = 1\), where \(F^p_1(0) = 1\) and \(F^n_1(0) = 0\). With the aid of the free Dirac equation one can show \(\bar{u}_p q^\mu \gamma_\mu u_p = 0\) and \(\sigma^\mu \sigma_\mu q^\nu = 0\), thus demonstrating that the divergence of the isovector current (13) (and the analogue isoscalar current) is indeed zero.

In a similar manner, we can extend the electromagnetic charged pion current to the isovector-vector hadron current:

\[
\langle \pi'_\mu | J^V_{\mu j}(x) | \pi^\nu_p \rangle = e^{ijk} F^\nu_\mu (q^2) \delta^{ij}(q' + p)_\mu e^{iqx},
\]

where the charge form factor of the pion is normalized to \(F_\pi(0) = 1\). In our isospin convention \(J^V_\mu = J^{V,3}_\mu\), and I note that \(J^S_\mu\) does not couple to pions; this would violate \(G\)-parity. The current of (14) is conserved for \(p'^2 = p^2\) up to a term proportional to \((q' - p)^\mu\) which disappears in \(\langle \pi | \partial J^V_{\mu j} | \pi \rangle = 0\). But the general \(SU(2)\) vector current is conserved as an operator

\[
\partial J^I(x) = 0 \quad i = 1, 2, 3
\]

such that the nucleon and pion matrix elements of (13) vanish, consistent with the vanishing divergences of (14) and (15) for on-shell equal-mass hadrons.

One can continue to demonstrate the vanishing divergence of other matrix elements of the hadronic vector current. For example, the existence of the vector mesons \(\rho\) and \(\omega\) suggests a direct \(\rho - \gamma\) and \(\omega - \gamma\) transition. We write the \(\rho\)-to-vacuum matrix elements of the hadronic isovector vector current as

\[
\langle 0 | J^{V,i}_\mu(x) | \rho^\mu(q) \rangle = \frac{m_\rho^2}{g_\rho} \epsilon_\mu(q) \delta^{ij} e^{-iqx},
\]

and the hadronic isoscalar vector current as

\[
\langle 0 | J^{S,i}_\mu(x) | \omega(q) \rangle = \frac{m_\omega^2}{g_\omega} \epsilon_\mu(q) e^{-iqx}.
\]

These currents are conserved as well because \(\partial J^{V,S} \propto q \cdot \epsilon(q) = 0\) for on-shell spin-1 polarization vectors \(\epsilon_\mu(q)\).

3.2 \(SU(2)\) Axial-vector Current \(A^i_\mu\)

We introduce this current with the simplest matrix element (and the analogue of the \(\rho\)-to-vacuum matrix element of the vector current) \(\pi\)-to-vacuum:

\[
\langle 0 | A^i_\mu(x) | \pi^\lambda(q) \rangle = i f_\pi g_\mu \delta^{ij} e^{-iqx},
\]
where $f_\pi \approx 93$ MeV is called the pion decay constant and its value is measured in the weak decay $\pi^+ \to \mu^+ \nu_\mu$. The divergence of (18) is

$$\langle 0 | \partial A^i(0) | \pi^j \rangle = \delta^{ij} f_\pi \mu^2$$

(19)

for $i, j = 1, 2, 3$ and an on-shell pion $q^2 = \mu^2 \equiv m_\pi^2$. From this exercise we learn that *axial-currents are not conserved, even if $SU(2)$ is an exact symmetry.* But the pion mass is small relative to all other hadrons: $\mu^2 / m^2 \approx 1/45$. In 1960 Nambu suggested that $\langle 0 | \partial A^i(0) | \pi^j \rangle \approx 0$ and even $\partial A^i \approx 0$ in an operator sense [41]. Next we define the nucleon matrix elements of $A^i_\mu$:

$$\langle N_p' | A^i_\mu(x) | N_p \rangle = \bar{N}_p' \tau^i 2 \left[ g_A(q^2)(t) i\gamma_\mu \gamma_5 + h_A(q^2) i\gamma_\mu \gamma_5 \right] \gamma_5 e^{-iq \cdot x},$$

(20)

where $q = (p' - p)$ as usual and $\bar{\gamma}_5 = \gamma_5$ as in Refs. [34, 40]. Finally we present a diagrammatic representation of these matrix elements:

![Diagram](image)

**Figure 3:** Matrix elements of the axial-vector current

which will be useful in the discussion of PCAC and later on of current algebra.

### 3.3 PCAC

We now review three ways of looking at the partial conservation of the axial vector current (PCAC) and establish a *soft-pion theorem* which will be used and tested against data in the following. The first (Nambu) statement of PCAC is simply that $\partial A^i \approx 0$ in an operator sense. We now consider the general emission of a very low energy pion $A \to B + \pi^i$ so that the emitted pion is soft ($m_\pi^2 \approx 0$). Replace the pion by an axial-vector and then remove the pion pole in this diagrammatic way:

This diagrammatic equation relates the axial-vector $M$-function $M_\mu$ and the pion pole contribution $M_\pi$ as

$$M^i_\mu = (-i) (-if_\pi q_\mu) \left( \frac{i}{q^2 - m_\pi^2 + i\epsilon} \right) M^i_\pi(q) + M^i_\mu.$$

(21)

To establish this (second) $S$-matrix form of PCAC, let i) $m_\pi^2 \approx 0$ in the pion propagator, ii) take the divergence of both sides of (21) and iii) use the Nambu version in the form of $q^\mu M^i_\mu \approx 0$ to arrive at

$$if_\pi M^i_\pi(q) = q^\mu M^i_\mu$$

(22)
Chiral Symmetry…

Figure 4: Pion-pole dominance of axial-vector current matrix elements.

Relation (22) can also be derived ([34], pp 221) for \( m_\pi^2 \neq 0 \) with the aid of the field theoretic statement \( \partial A = f_\pi m_\pi^2 \phi^I_\pi(x) \) where \( \langle 0 | \phi^I_\pi(\pi) | \pi \rangle = \delta^{I J} \) and \( \phi^I_\pi(x) \) is some pseudoscalar field operator with the quantum numbers of the pion. It can be shown [34] that equation (22) holds for either \( m_\pi^2 \rightarrow 0 \) or \( q^2 \rightarrow 0 \), provided that the pion pole is first removed from \( q^\mu M^I_\mu \).

The third and most useful form of PCAC (for our study of \( \pi N \) scattering) is obtained from the soft-pion limit \( (q \rightarrow 0) \) of (22) rewritten as \( M^I_\mu(q) = -i f_\pi q^\mu \overline{M}_\mu \). The right-hand side of this relation has contribution \( M^I_\mu(\text{poles}) \sim \mathcal{O}(1) \) which vanish as \( q^\mu \rightarrow 0 \). However, the \( \mathcal{O}(1/q) \) poles from “tagging on” the axial-vector to external nucleon lines will not vanish, giving the soft pion theorem:

\[
M^I_\mu(q) \xrightarrow{q \rightarrow 0} = -i f_\pi q^\mu \overline{M}_\mu(\text{poles}) + \mathcal{O}(q) ,
\]

and the soft pion version of PCAC: after removal of the pion poles and \( \mathcal{O}(1/q) \) poles from the axial-vector amplitude the (truly) background amplitude is a smoothly varying function of \( q^2 \) such that

\[
q^\mu \overline{M}_\mu(\text{non-pole}) \approx 0 ,
\]

and

\[
M^I_\mu(q^2) \approx M^I_\mu(0) .
\]

This soft pion version of PCAC (25) is now a statement about pion amplitudes and can be used as such. It is a sharp statement, comparable to other characterizations of PCAC, such as “What is special is that the pion mass is small, compared to the characteristic masses of strong interaction physics; thus extrapolation over a distance of \( m_\pi^2 \) introduces only small errors” [42], pp 43.

3.4 The Goldberger-Treiman Relation

The Goldberger-Treiman (GT) relation between strong and weak interaction parameters was displayed already in 1958 [43] and explained by Nambu a short time later [41]. Here we show that the GT relation can be regarded as a single soft pion prediction of PCAC and pion pole dominance of axial-vector, hadronic transitions. First let us notice that the divergence of (21) coupled with the (Nambu)
PCAC statement that $\langle N_p' | \partial A^i_\mu (x) | N_p \rangle \approx 0$ implies that the axial form factors obey

$$2m g_A(q^2) + q^2 h_A(q^2) \approx 0 .$$

where we have used $\gamma^\mu q_\mu \gamma_5 \to 2m \gamma_5$, when sandwiched between the spinors of on-mass-shell nucleons. To go farther, we dominate the axial-vector matrix element with the pion pole exactly as in Fig. 4, but this time we have an effective $\pi NN$ coupling $H_{\pi NN} = g_{\pi NN} \bar{N} \vec{\tau} \cdot \pi \gamma_5 N$ which gives an explicit form to the pion pole $M_i^\pi(q)$ of (21). Carrying this out we find

$$\langle N_p' | A^i_\mu (x) | N_p \rangle \approx g_{\pi NN} \bar{u}_{p'} \tau^i \gamma_5 u_p \frac{i}{q^2 - m_\pi^2 + i\epsilon} (-i)(-i f_\pi q_\mu) .$$

Neglecting $m_\pi^2$ and comparing with (20) shows that it is the form factor $h_A(q^2) i q_\mu \gamma_5$ which has the pion pole:

$$h_A(q^2) \approx -\frac{2 f_\pi g_{\pi NN}}{q^2} .$$

Now let $q^2 \to 0$ to suppress the non-pion-pole terms, and the $\approx$ in (28) becomes an equality, turning (26) into the exact relation

$$2m g_A(q^2 = 0) - 2 f_\pi g_{\pi NN}(q^2 = 0) = 0 ,$$

which takes the familiar Goldberger-Treiman form

$$m g_A(0) = f_\pi g_{\pi NN} ,$$

our first soft pion prediction.

To test the GT relation empirically in the chirally broken real world, convert it to a Goldberger-Treiman discrepancy

$$\Delta = 1 - \frac{m_N g_A(0)}{f_\pi g} .$$

The experimental values are [14]

$$m_N = \frac{1}{2}(m_p + m_n) = 938.91897 \pm 0.00028 \text{ MeV},$$

and [15]

$$f_\pi = 92.6 \pm 0.2 \text{ MeV},$$

We use the current best value of $g_A(t = 0)$ as determined by two consortia at the Institute for Nuclear Theory [3, 16]. They find, by averaging modern results for the neutron lifetime and decay asymmetries,

$$g_A(0) = 1.2654 \pm 0.0042 .$$

The least well known, and somewhat controversial, strong interaction parameter is
Chiral Symmetry

\[ g_{\pi NN}(q^2 = m_{\pi}^2) \approx 13.12 \]  [47]
\[ g_{\pi NN}(q^2 = m_{\pi}^2) \approx 13.02 \]  [48]

down about 2% from the pre-meson factory value of \( g_{\pi NN} \approx 13.40 \)  [49, 50]. The GT discrepancy then becomes

\[ \Delta \approx 0.023 \]  [47]  \hspace{1cm} (31)
\[ \Delta \approx 0.015 \]  [48]  \hspace{1cm} (32)

or a discrepancy of only 2%! This numerical fact is an, better than usual, example of the soft pion form of PCAC: \( M_\pi(q^2) \approx M_\pi(0) \).

The Goldberger-Treiman relation is exact in the chiral limit \( m_{\pi}^2 \to 0 \) (\( \partial A = 0 \)). In our derivation we neglected \( m_{\pi}^2 \) and then took the limit \( q^2 \to 0 \). Both limits are necessary to make the relationship exact. This distinction becomes important as one attempts to use the \( q^2 \to 0 \) limit to guide the low \( q^2 \) variation of the \( \pi NN \) vertex function for an off-mass shell pion in models of the \( NN \) and \( NNN \) forces  [49].

The value of \( \Delta \) indicates a 2% decrease in the coupling from the on-shell coupling \( q^2 = m_{\pi}^2 \) to \( q^2 = 0 \). One should parameterize the \( \pi NN \) “form factor” to have this “GT slope” which reflects chiral symmetry breaking. The usual \( \pi NN \) form factor of the Tucson-Melbourne \( NNN \) force  [11] has been parameterized to have about a 3% GT slope. That is, if \( F_{\pi NN}(q^2) = (\Lambda^2 - m_{\pi}^2)/(\Lambda^2 - q^2) \) then \( \Lambda \approx 800 \) MeV.

3.5 The Adler Consistency Condition

Another soft pion result which is independent of current algebra follows from an Adler-Dothan version of the soft pion theorem. We start with \( M_\pi^2(q) = \frac{-i}{q} q^\mu T^\mu_\pi \) for the general hadronic amplitude \( A \to B + \pi^i \), and examine the origin of the \( O(q^{-1}) \) nucleon poles which survive in the limit \( q \to 0 \). Then only the axial vector (designated by * “tagging” onto external nucleon lines of the general hadronic ingoing line \( A \) and outgoing line \( B \) in the diagram below) will generate nucleon propagators \( O(q^{-1}) \). The * represents a \( g_A(q^2) \)-type coupling of the axial vector, since \( h_A(q^2) \) is already included in \( M_\pi^2(q^2) \) from the pion pole in \( h_A(q^2) \), see (28).

\[ \begin{array}{c}
| \text{ } \text{ } \text{ } | \\
| \text{ } \text{ } \text{ } | \\
A \xrightarrow{*} \xrightarrow{\mu} + B \\
A \xrightarrow{*} \xrightarrow{\mu} + B
\end{array} \]

Now we apply the GT relation \( mg_A(0) = f_\pi g_{\pi NN} \) to identify the nucleon pole parts of the axial background (i.e., pion-pole removed) amplitude \( q^\mu T^\mu_\pi \) with the
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pseudoscalar $\pi N$ interaction, so that the diagrammatic representation of the background becomes:

\[
\begin{align*}
A & + B \\
A & + (M_0\gamma_5\tau^i + \gamma_5\tau^iM_0)
\end{align*}
\]

The hadronic amplitude $A \rightarrow B$ labeled $M_0$ contains nucleons but has had, as we have seen, the soft pion removed and the axial vector removed. Now let $q^\mu \rightarrow 0$ in $M^i_\pi(q) = \frac{g_{\pi NN}}{f_\pi}q^\mu M^i_\mu$, to suppress all further background parts in $M^i_\mu$ of $O(q^0)$. Only the nucleon poles $O(q^{-1})$ with pseudoscalar pion-nucleon coupling are left and we have the soft pion theorem proved by Adler and Dothan [51]:

\[
M^i_\pi(q) \xrightarrow{q\rightarrow 0} M^i_{\pi N\text{poles}}(q) + M^i_\pi(q \rightarrow 0)
\]

(33)

where

\[
\overline{M}^i_\pi(q \rightarrow 0) = \frac{g_{\pi NN}}{2m}(M_0\gamma_5\tau^i \gamma_5\tau^iM_0).
\]

(34)

This version of the soft pion theorem, valid for either an incoming or an outgoing soft pion, is the analog of the soft photon theorem of Low [52]. It allows us to turn the Adler zero [53], $M^i_\pi(q \rightarrow 0) = 0$ provided that $\overline{M}^i_\mu$ in (22) has no poles, into the Adler PCAC consistency condition for $\pi N$ scattering.

To begin this demonstration, let us display explicitly the $s$-channel and $u$-channel nucleon poles in $\pi^i(q) + N(p) \rightarrow \pi^i(q') + N(p')$:

\[
\begin{align*}
\begin{array}{c}
q' \downarrow i \\
p' \leftarrow \\
q \downarrow j \\
p \rightarrow i
\end{array} & =
\begin{array}{c}
q' \downarrow i \\
p' \leftarrow \\
q \downarrow j \\
p \rightarrow i
\end{array} +
\begin{array}{c}
q' \downarrow i \\
p' \leftarrow \\
q \downarrow j \\
p \rightarrow i
\end{array} +
\begin{array}{c}
q' \downarrow i \\
p' \leftarrow \\
q \downarrow j \\
p \rightarrow i
\end{array} +
\begin{array}{c}
q' \downarrow i \\
p' \leftarrow \\
q \downarrow j \\
p \rightarrow i
\end{array}
\end{align*}
\]

Figure 5: Nucleon pole terms in $\pi N$ scattering

Both nucleon poles are present and are added together by the Feynman rules because the crossed pions are bosons. Now let the final pion become soft ($q' \rightarrow 0$) and the other three particle be on-mass-shell. As we have separated out the pseudoscalar nucleon poles, we can apply (34) with $M_0 = -g_{\pi NN}\tau^i\gamma_5$ from $N \rightarrow N + \pi$. Then

\[
\overline{M}^i_\pi(q \rightarrow 0) = -\frac{g_{\pi NN}}{2m}(g_{\pi NN}\tau^j\gamma_5\gamma_5\tau^i + \gamma_5\tau^i g_{\pi NN}\tau^j\gamma_5) \quad (35)
\]
where we have used \( \{ \tau^i, \tau^j \} = 2 \delta^{ij} \) and remind the reader that \( \gamma^5 \) = −1 in this (Schweber) convention.

Now we restate the Adler consistency condition (36) as a condition on the invariant amplitude \( F^+ = A^+ + \nu B^+ \) (since the isospin condition is \( t \)-channel even). The kinematic variables for \( q^2 = m^2 \) and \( q' \to 0 \) are \( t = (q^2 - q'^2) = \mu^2 \) and \( \nu = 0 \) because \( s = u = m^2 \). Then the Adler consistency condition becomes

\[
F^+(\nu = 0, t = \mu^2; q^2 = \mu^2, q'^2 = 0) = A^+(\nu = 0, t = \mu^2; q^2 = \mu^2, q'^2 = 0) = \frac{g^2}{m},
\]

and we note the pseudoscalar nucleon poles do not contribute to \( A^{(\pm)} \) but only to \( B^{(\pm)} \). Specifically,

\[
A^{(\pm)}_{ps} = 0 \quad B^+_{ps} = \frac{g^2}{m} \frac{\nu}{\nu_B^2 - \nu^2} \quad B^-_{ps} = \frac{g^2}{m} \frac{\nu B}{\nu_B^2 - \nu^2}.
\]

To make contact with \( \pi N \) data analyses obtained from dispersion relations, it is natural to evaluate pseudoscalar nucleon poles, not as field theory Feynman diagrams but in the sense of dispersion theory so that the residue in \( \nu^2 \) of \( F^+ \) is evaluated at the value of \( \nu \) at the \( s \)-channel nucleon pole, \( 2m(\nu_B - \nu) = (m^2 - s) \) or \( \nu_B = -q \cdot q'/2m \):

\[
F^+ = \frac{g^2}{m} \frac{\nu_B^2}{\nu_B^2 - \nu^2} \quad F^- = \frac{g^2}{m} \frac{\nu B}{\nu_B^2 - \nu^2},
\]

(see Ref. [10], pp 340-343). The difference between the two prescriptions lies only in \( F^+ \):

\[
F^+_p(\nu, t) = F^+_{ps}(\nu, t) + \frac{g^2}{m}.
\]

Now restate the Adler consistency condition in the form of a condition on the background \( \pi N \) amplitude defined as

\[
F^{(\pm)} = F^{(\pm)}_p + \bar{F}^{(\pm)},
\]

so that

\[
\bar{F}^+ = \bar{F}^+_p - \frac{g^2}{m}.
\]

In the single soft (Adler) limit (37) becomes

\[
F^+_p \to 0, \quad \bar{F}^+ \to \frac{g^2}{m} - \frac{g^2}{m} = 0.
\]

The knowledgeable reader may have noticed that for \( F^+ \) the dispersion-theoretic nucleon pole with pseudoscalar coupling is the same as the field theoretic nucleon pole.
pole of $F^+$ with pseudovector coupling, so one can think, if one wishes, think of the background $\bar{F}^+$ as the full amplitude minus the pseudovector poles. It is often said that the Adler PCAC consistency condition of chiral symmetry forces the use of pseudovector coupling, but it is obvious from the above that this soft pion theorem makes no such demand. In the future, the phrase “nucleon poles” refer to dispersion theory poles with pseudoscalar coupling.

Invoking PCAC in the form of (25), we can expect that putting the final pion back on-mass-shell (and holding fixed $t$ and $\nu$) should not change the Adler consistency condition much:

$$
\bar{F}^+(0, \mu^2) \equiv \bar{F}^+(\nu = 0, t = \mu^2; q^2 = \mu^2, q'^2 = \mu^2) \approx 0.
$$

This “Adler Low Energy Theorem” (LET) point is in the subthreshold crescent region of the Mandelstam plane (see Fig. 2) and the value of $\bar{F}^+$ can be reliably determined from $\pi N$ scattering data with the aid of dispersion relations. It is $\bar{F}^+ \approx -0.03 \mu^{-1}$ or $\bar{F}^+ \approx -0.08 \mu^{-1}$, extrapolations obtained from the most recent phase shift analysis called SM98 \[54\]. As the amplitude, unlike the Goldberger-Treiman discrepancy, has dimensions we must compare this result to the overall scale $-1.3 \mu^{-1} \leq \bar{F}^+(\nu, t) \leq 6 \mu^{-1}$ within the subthreshold crescent. Then we see that this PCAC low energy theorem \[14\] is also rather impressively confirmed by the data \[57\]. Indeed one can go a step further and notice that this background amplitude has a zero in the subthreshold crescent which, beginning at the Adler LET, passes very near the threshold point $(\nu = \mu, t = 0)$ \[55\]. The nucleon pole contribution is quite small ($\approx -0.13 \mu^{-1}$) at threshold, so the overall unbarred isoscalar scattering length $a_0 \approx F^+(\mu, 0)/4\pi \approx 0.01 \mu^{-1} \approx 0$ is a threshold consequence of the PCAC Adler consistency condition and has nothing to do with current algebra.

### 3.6 Current Algebra and $\pi N$ Scattering

The current algebra representation of low-energy $\pi N$ scattering not only utilises on-mass-shell ($q^2 = q'^2 = \mu^2$) axial Ward-Takahashi identities (analogous to the conditions gauge invariance imposes on photon-target scattering, but incorporating a current algebra commutation relation) but also make a specific prediction for the amplitude $\pi^0(q) + N(p) \rightarrow \pi^0(q') + N(p')$ when both pions are soft \[23\]. This latter prediction can be compared to the data by invoking PCAC in the form of (25) to bring each pion back to the mass-shell. But first we must establish this current algebra representation \[58\] \[54\].
Begin with \( SU(2) \) axial currents “scattering” off target nucleons and write the covariant amplitude (to be sandwiched between on-shell nucleon spinors) as

\[
M_{ij}^{\mu\nu} = i \int d^4 x e^{iq' \cdot x} T[A_i^\mu(x), A_j^\nu(0)] \theta(x_0),
\]
(45)

where \( \Delta = q - q' = p' - p \) and the momentum transfer is \( t = \Delta^2 \). Contract (45) with \( q' \) (i.e. take a divergence in coordinate space), integrate the right hand side by parts, and drop the surface term at infinity. Using the identity \( \partial^\mu T(A_\mu(x)\ldots) = T(\partial A\ldots) + \delta(x_0)A_0\ldots \), we find

\[
q'^\mu M_{ij}^{\mu\nu} = i \int d^4 x e^{iq' \cdot x} T[\partial^\mu A_i^\mu(x), A_j^\nu(0)] \theta(x_0) - i\epsilon^{ijk} \Gamma^k_{\nu} (\Delta),
\]
(46)

where we have used the Equal Time Commutation relationship (47) to bring in the three-point vertex function \( \Gamma^k_{\nu}(\Delta) \) which depends only on the momentum transfer \( \Delta = (p' - p) \).

Before going on, let us pause to examine the extension to currents of the charge algebra of (4) and (5):

\[
\begin{align*}
[Q^i, J^k_{\nu}(x)]_{ETC} &= i\epsilon^{ijk} J^k_{\nu}(x) \\
[Q^i, A^j_{\nu}(x)]_{ETC} &= i\epsilon^{ijk} A^j_{\nu}(x) \\
[Q^5, J^j_{\nu}(x)]_{ETC} &= i\epsilon^{ijk} A^j_{\nu}(x) \\
[Q^5, A^j_{\nu}(x)]_{ETC} &= i\epsilon^{ijk} J^k_{\nu}(x).
\end{align*}
\]
(47)
The axial charge is, of course, defined as $Q_A^i = \int d^3x A_i^A(t, \vec{x})$, by analogy to $Q_V^3 = \int d^3x J_3^A(t, \vec{x})$, and the $SU(3)$ structure constants $f^{ijk}$ reduce to $\epsilon^{ijk}$ for $i = 1, 2, 3$ of $SU(2)$ pion-nucleon scattering. Then one can recover the charge algebra (10) and (11) from the current algebra relations (17) by setting $\nu = 0$ and integrating over all space. Notice that if the currents in (11) were the conserved isovector-current $J_{iu}^a(k)$ and $J_{iu}^a(k')$ then (16) would become simply $k^{\mu} M_{\mu}^{ij} = -i\epsilon^{ijk} \Gamma_0^k(\Delta)$. The latter is the isovector-Ward-Takahashi identity for virtual isovector photons which replaces the gauge invariance equation for real photons.

Now we go on, by contracting (16) by $q^\nu$ and converting the $x$ dependence of the currents from $\partial A^i$ to $A^i_{\mu}$ so that we can integrate by parts once again. The result is the “double” Ward-Takahashi identity

$$q^\nu M_{\mu}^{ij} q^\nu =$$

$$\int d^4x e^{-iq.x} T[\bar{\chi}A^i_{\mu}(0), A^j_{\mu}(x)]\theta(x_0) = -i\epsilon^{ijk} \Gamma_\nu^k(\Delta) q^\nu + [i\partial^\mu A^i_{\mu}(0), Q_A^j]_{ETC}$$

We can symmetrise the current algebra term $i\epsilon^{ijk} \Gamma^k_\nu(\Delta) q^\nu = i\epsilon^{ijk} \Gamma^k_\nu(\Delta) Q^\nu$ by utilising $\Gamma^k_\nu(\Delta) \delta^\nu = 0$ and the definition $Q = \frac{1}{2}(q + q')$. This term can be identified with the measured electromagnetic form factors of nucleons [58].

$$\Gamma^k_\nu(\Delta) Q^\nu = \frac{5}{2} [F^Y_1(t)\gamma_{\nu} Q^\nu - \frac{1}{4m} F^Y_2(t)[\gamma^\nu q_{\nu}, \gamma^\nu q_{\nu}]]$$,

(49)

where we have put back in the suppressed nucleon spinors and used the defining equation (13). The second commutator term on the RHS of (16), reinstating the nucleon spinors, is the pion-nucleon “sigma” term

$$(N_{\nu'}[i\partial^\mu A^i_{\mu}(0), Q_A^j]_{ETC}|N_p) = \delta^{ij} \sigma_{N}(t) \bar{N}_{\nu'} N_p ,$$

(50)

which like the current algebra term can be only a function of $t$. The amount of the $t$ dependence of the sigma term cannot be determined by theory, but like the $t$ dependence of the the current algebra term is obtained by comparing with measurement. The sigma term is isospin symmetric, as can can be established by reversing the order of momentum contractions of $M_{\mu}^{ij}$. The sigma term is a measure of chiral symmetry breaking since it is proportional to the non-conserved current $\partial A^i = 0$.

Having discussed the two $t$ dependent terms on the RHS of (16), we now relate the LHS to pion-nucleon scattering by dominating the LHS by pion poles according to Figure 8.

The $S$-matrix version of PCAC $if_{\pi} M_{\pi A}^i(q) = q^\mu \bar{M}_{\mu}^i$ holds no matter if we let $\mu^2 \to 0$ or $q^2 \to 0$, or keep both non-zero. So we can take the double divergence of Fig. 8 as in (19), first setting $\partial A = 0$, which gets rid of the integral on the RHS of (19), and equate the result to the RHS of (16), giving

$$-f_{\pi}^2 M_{\pi A}^{ij} + q^\nu M_{\mu}^{ij} q^\nu = -i\epsilon^{ijk} \frac{\tau^k}{2} [F^Y_1(t)\gamma_{\nu} Q^\nu - \frac{1}{4m} F^Y_2(t)[\gamma^\nu q_{\nu}, \gamma^\nu q_{\nu}]] + \delta^{ij} \sigma_{N}(t) .$$

(51)
The minus sign of $M_{\pi\pi}$ is the reverse of the sign associated with the third (the one we really want) diagram on the RHS of the figure because of the first two diagrams.

This relationship between $\pi N$ scattering and a double divergence has been obtained from the covariant amplitude (22) with the aid of pion-pole dominance and the S-Matrix version of PCAC (22). In order to reproduce two noteworthy current algebra/PCAC theorems, the Weinberg double soft-pion theorem (23) and the Adler-Weisberger double soft-pion relation (59) we now use the soft pion theorem (25). The latter can be applied only if all poles are removed from both amplitudes: $M_{\pi\pi}$ and the non-pion pole axial-Compton amplitude $\bar{M}_{\pi\pi}^{\mu
u}$. The nucleon poles are removed from $M_{\pi\pi}$ as in Fig. 5 and in an analogous figure for nucleon poles in $\bar{M}_{\pi\pi}^{\mu
u}$, with the important distinction that the nucleon-axial coupling is not pseudoscalar $g_{\pi NN}\gamma_5 \equiv g_{\pi NN}$ but instead is defined by (20). This distinction introduces the Adler contact term back into the isospin-even $\pi N$ amplitude. With the removal of the nucleon poles the resulting $q'' \mu \bar{M}_{\pi\pi}^{\mu
u} q''$ vanishes as $q \to 0, q' \to 0$.

The upshot is the generic double soft-pion result:

$$M_{\pi\pi}(q, q') \approx M_{\pi\pi}^N (q, q') + \bar{M}_{\pi\pi}(q \to 0, q' \to 0),$$

which has the isospin decomposition (4)

$$\bar{M}_{\pi\pi}^+(q \to 0, q' \to 0) = \frac{g^2}{m} - \frac{\sigma_N(0)}{f^2_\pi},$$

derived by Weinberg (23), and

$$\nu^{-1}M_{\pi\pi}^-(q \to 0, q' \to 0) = \frac{1}{f^2_\pi} - \frac{g^2}{2m^2} = \frac{1}{f^2_\pi}(1 - g_A^2),$$

the Adler-Weisberger relation (59). To obtain the crossing symmetric relation (53), we divide $F_1^V(t) \gamma_5 Q''$ of (21) by $\nu = (p + p') \cdot (q + q')/4m$, before taking the $q \to 0, q' \to 0$ limit, to get $\frac{1}{4} F_1^V(t) \to \frac{1}{2}$ because $F_1^V(0) = 1$. We then use the GT relation to obtain the far RHS of (53).
3.7 On-pion-mass-shell current algebra Ward-Takahashi identities

We have derived the Ward identities in the chiral limit such that $q^\mu \bar{M}^{ij}_{\mu \nu} q^{\nu}$ vanishes as $q \to 0, q' \to 0$, where $\bar{M}^{ij}_{\mu \nu}$ is the background axial-Compton amplitude with pion and nucleon poles removed. A derivation similar to the above, but keeping both pions on-mass-shell at all stages, yields on-shell Ward identities which impose current algebra constraints on pion-nucleon scattering [60]. These identities are most conveniently expressed by writing the (not necessarily zero) double divergence in the same manner as the $M$ function for $\pi N$ scattering (3)

$$M^{(\pm)}_{\pi\pi} = F^\pm(\nu, t) - \frac{1}{4m} \{ g, g' \} B^\pm(\nu, t)$$

$$q^\mu \bar{M}^{\pm \nu}_{\mu \nu} q^{\nu} = C^{\pm}(\nu, t) - \frac{1}{4m} \{ g, g' \} D^{\pm}(\nu, t)$$

Now define the background $\pi N$ amplitude as $\bar{M} = M - M_P$ where $M_P$ is the dispersion-theoretic pole of (39), see Figs. (2) and (5), for pseudoscalar $\pi NN$ coupling. The on-pion-mass-shell Ward-Takahashi identities take the form

$$\bar{F}^+(\nu, t) = \frac{\sigma_N(t)}{f_\pi^2} + C^+(\nu, t)$$

$$\nu^{-1}\bar{F}^-(\nu, t) = \frac{F^Y(\nu, t)}{2f_\pi^2} - \frac{g^2}{2m^2} + \nu^{-1}C^-(\nu, t)$$

$$\nu^{-1}\bar{B}^+(\nu, t) = \nu^{-1}D^+(\nu, t)$$

$$\bar{B}^-(\nu, t) = \frac{F^Y(\nu, t) + F_2^Y(\nu, t)}{2f_\pi^2} - \frac{g^2}{2m^2} + D^-(\nu, t)$$

where we notice that removal of the dispersion theory pole removes the contact term $g^2/m$ from (57). The on-shell analogue [53] of the Adler-Weisberger double soft pion point (52) goes to (53) because $C^-(\nu, t)$ is defined as a double divergence in coordinate space which vanishes as $(q \to 0, q' \to 0)$. Comparing with the double-soft pion Weinberg limit (54), which can be written as

$$\bar{F}^+(0, q' \to 0) = -\frac{\sigma_N(0)}{f_\pi^2}$$

we note that the sign change is due to an analytic power series expansion in $q$ and $q'$ (scaled to a typical hadron mass such as $m$) which obeys the Adler zero [13] [61]. In fact, Brown, Pardee, and Peccei [60] suggest the sigma-term structure $\sigma_N(t)((q^2 + q'^2)/\mu^2 - 1)$ which manifests the sign change and the Adler zero. They also confirm the on-shell Cheng-Dashen (CD) low energy theorem of 1971 [54] :

$$\bar{F}^+(0, 2\mu^2) = \frac{\sigma_N(2\mu^2)}{f_\pi^2} + \mathcal{O}(\mu^4)$$

With the on-shell Ward identities we are now in a position to test the current algebra representation with the data of $\pi N$ scattering extrapolated to the sub-threshold crescent of Fig. 2. These tests can establish the magnitude of the sigma
term in (57), suggest (with the aid of the PCAC hypothesis) the \( t \) dependence of the sigma term, and confirm or deny the \( t \) dependence of the current algebra terms (from (47)) in (58) and (60). We turn to these tests in the next section.

4 Tests of Current Algebra and Soft Pion Theorems

In this lecture we use contemporary analyses of on-mass-shell \( \pi N \) scattering to i) test the structure of current algebra in this context, and to ii) examine the validity of the PCAC hypothesis (already validated for the Adler LET (44)) that the exact double soft-pion theorems of Weinberg (54) and of Adler and Weisberger (55) should be evident in the \( \pi N \) data. The former tests of current algebra were initiated by the data analysis of the Karlsruhe group [62] which use fixed-\( t \) dispersion relations to extrapolate from the(\( s \)) \( s \)-channel experimental phase shifts into the subthreshold region around \( (\nu = 0, t = 0) \) (see Fig. 1). In fact, the experimental information in this region, once the nucleon pole contributions (see Fig. 2) have been removed, can be expressed in terms of the expansion coefficients of the four background \( \pi N \) invariant amplitudes about this point. The values of these H"ohler expansion coefficients obtained from data taken in the 1970’s, before the meson factories were built, are summarized in the encyclopedic Ref. [39]. The two current algebra models [63, 64] of \( \pi N \) amplitudes, which have been adapted to the construction of two-pion exchange three-nucleon forces [8-11], were calibrated against these pre-meson factory H"ohler coefficients.

Recently two of the twenty-eight subthreshold subthreshold coefficients have been re-evaluated [54] via fixed-\( t \) dispersion relations from the latest partial wave analysis [57] of meson factory data. We will use the more comprehensive determination [57] of the amplitudes \( \tilde{F}^+(\nu, t) \) and \( \nu^{-1} \tilde{F}^-(\nu, t) \) in the subthreshold crescent to fully carry out the tests i) and ii) of chiral symmetry. This determination also is from the VPI phase-shift analysis SM98 [56], but the analysis used interior dispersion relations (IDR), pioneered by Hite, et al. [65], and advocated by H"ohler for the purpose of testing chiral symmetry [66]. Interior dispersion relations are evaluated along hyperbolas in the Mandelstam plane which correspond to a fixed angle in the \( s \)-channel laboratory frame. For \( t < 0 \) the path of fixed lab angle lies entirely within the \( s \)-channel physical region and passes through the \( s \)-channel threshold point. (The IDR paths are similar to the lines of fixed center of mass three-momentum \( q^2 \) and \( x = \cos \theta_{\text{cm}} \) of Fig. 9, from Ref. [67] and illustrating a construction of invariant amplitudes from a simple summation of partial-wave amplitudes continued into the subthreshold crescent). With the IDR paths, one can reliably extrapolate the invariant amplitudes to any point in the subthreshold crescent and in particular evaluate the amplitudes along the vertical axis \( (\nu = 0, 0 \leq t \leq 4\mu^2) \) to test the on-shell analogues of the soft pion theorems.
Fig. 9. A portion of the (all four particles on-mass-shell) Mandelstam $(\nu, t)$ plane, which includes the $s$-channel physical region and the subthreshold crescent.

But first we must attempt to calibrate the IDR invariant amplitudes from SM98 phase shifts against independent measurements. The IDR value at the $s$-channel threshold point ($\nu = \mu, t = 0$) the scattering lengths

$$a^{(+)} = \frac{1}{4\pi(1 + \mu/m)} F^+(\mu, 0) \approx -0.005 \mu^{-1}$$

$$a^{(-)} = \frac{1}{4\pi(1 + \mu/m)} F^-(\mu, 0) \approx +0.087 \mu^{-1}$$

in good agreement with the preliminary values $a^{(+)} = .0016 \pm .0013 \mu^{-1}$ and $a^{(-)} = 0.0868 \pm 0.0014 \mu^{-1}$ from the 1s level shifts and widths in pionic hydrogen and deuterium [68]. In addition, at the pseudothreshold point $\nu = 0$, $t = 4\mu^2$ (a focus of the boundary hyperbola; see Fig. 1) the $I = 0$ $\pi\pi$ scattering length, $a_{00}$, can be evaluated with the method of Ref. [69]. The IDR value $a_{00} \approx 0.20\mu^{-1}$ again agrees well with the recent determination of $a_{00} = 0.204 \pm 0.014 \pm 0.008 \mu^{-1}$ from the totally independent reaction $\pi N \rightarrow \pi\pi N$ [70]. That the IDR give reasonable values of $a_{00}$ and $a^{(+)}$ at opposite points on the boundary of the subthreshold crescent adds confidence to the values in the central region.
Our first test of a combined current algebra-PCAC prediction uses the IDR determination of the isospin odd amplitude $\nu^{-1}\bar{F}^-(\nu, t)$ at the point $(\nu = 0, t = 0)$. The exact current algebra Adler-Weisberger result is

$$\nu^{-1}\bar{F}^-(q \rightarrow 0, q' \rightarrow 0) = \nu^{-1}\bar{F}^-(0, 0; 0, 0) = \frac{1}{f^2}(1 - g_A^2) = -0.62 \mu^{-2}, \quad (63)$$

where we have used the values of section 3.4. The empirical IDR $\nu^{-1}\bar{F}^-(0, 0) \approx -0.44 \mu^{-2}$, indicating that if $\nu$ and $t$ are kept fixed, the PCAC extrapolation from the chiral symmetric Adler-Weisberger limit to the chirally broken real world is minimal. Perhaps not as small as the 2% single soft pion Goldberger-Treiman result of section 3.4 nor the ($\approx 5\%$) single soft pion Adler consistency condition result of section 3.5, but still the PCAC hypothesis appears to work well in this more stringent test. Since the $\pi N$ amplitude can be written with the choice of variables $\nu, t; q^2, q'^2$ or $\nu, q \cdot q'; q^2, q'^2$ or indeed some other combination, the magnitude of the PCAC corrections will depend on which pair $(\nu, t)$ or $(\nu, \nu_B = q \cdot q'/2m)$ is held fixed during the extrapolation from $q \rightarrow 0$ to $q^2 = \mu^2 [71]$. From our experience with (64) in Section 3.5, we argue that holding fixed $(\nu, t)$ (see Fig. 10) is the correct way to apply PCAC.

Given these empirical values of the on-shell amplitude $\bar{F}^+(\nu, t)$ and $\nu^{-1}\bar{F}^-(\nu, t)$, one can now visualize in Fig. 10 the proposed tests of the (isospin-even) PCAC low energy theorems (LET) labeled Adler LET (44) and Weinberg LET, the latter one can now visualize in Fig. 10 the proposed tests of the (isospin-even) PCAC amplitude. The extrapolations shown are from the soft pion points A and A’ where $\bar{F}^+(q \rightarrow 0) = \bar{F}^+(q' \rightarrow 0) = 0$ and from the double soft Weinberg point $\bar{F}^+(q \rightarrow 0, q' \rightarrow 0) = \bar{F}^+(0, 0; 0, 0) = -\frac{\sigma_N(0)}{f^2}$ to the on-pion-mass-shell line, holding fixed $\nu$ and $t$. The scale of the various points on the figure and of the isospin-even tests is given by the Cheng-Dashen LET [12] and the (expected by PCAC) “anti-Cheng-Dashen” value at the Weinberg point (54).

The IDR amplitude takes the following values on the on-mass-shell line:

$$\bar{F}^+(0, t = 2\mu^2) = \frac{\sigma_N(2\mu^2)}{f^2} + \mathcal{O}(\mu^4) \approx +1.35 \mu^{-1}$$
$$\bar{F}^+(0, t = \mu^2) = 0 + \mathcal{O}(\mu^2) \approx -0.08 \mu^{-1} \quad (64)$$
$$\bar{F}^+(0, t = 0) = -\frac{\sigma_N(0)}{f^2} + \mathcal{O}(\mu^2) \approx -1.34 \mu^{-1}$$

Let us make a careful distinction between the expected corrections indicated in (64). The $\mathcal{O}(\mu^4)$ corrections of the top line are the result of a rigorous on-shell derivation of a Ward identity [60, 61]. The putative $\mathcal{O}(\mu^2)$ corrections of the lower two lines are what one might expect from the already discussed corrections to the Goldberger-Treiman relation, the Adler zero, and the Adler-Weisberger relation as one goes from the chiral symmetric world to the world of $\pi N$ scattering. It is these latter presumed PCAC corrections which we are trying to test. If one extends the observed pattern of small PCAC corrections to the bottom line of (64), one can interpret $\bar{F}^+(0, t = 0) \approx -\bar{F}^+(0, t = 2\mu^2)$ as indicating that the $t$
dependence of $\sigma(t)$ \cite{50}, as determined from the $\pi N$ scattering data, is quite small indeed. The alternative picture of $\frac{\sigma_N(2\mu^2)}{f_\pi^2} - \frac{\sigma_N(0)}{f_\pi^2} \approx 0.25 \, \mu^{-1}$ \cite{73} would demand a quite large PCAC correction along the lower plane of Fig. 10 to get back to the empirical amplitude; much larger than any other PCAC correction evaluated in the $\pi N$ system or elsewhere (Ref. \cite{34}). Furthermore, we will see in Fig. 11 that the amplitude $\bar{F}^+(0, t)$ is nearly linear in $t$ in the interval between $t = 0$ and $t = 2\mu^2$ (with increasing curvature at $t$ approaches the $\pi\pi \to N\bar{N}$ pseudo-threshold at $4\mu^2$). We will return to this issue after investigating the uniqueness of the IDR amplitudes from which these conclusions are drawn.

Fig. 10. The geometry of the off-mass-shell $\pi N$ amplitude $\bar{F}^+(\nu, t, q^2, q'^2)$ for $\nu = 0$.

The value of the IDR amplitude $\bar{F}^+(\nu = 0, t = 2\mu^2) \approx 1.35 \, \mu^{-1}$ at the Cheng-Dashen point leads to a value of the sigma term from \cite{62} of

$$\sigma_N(2\mu^2) = \bar{F}^+(0, 2\mu^2)f_\pi^2 \approx 83 \, \text{MeV}$$
Fig. 11. The values of $\bar{F}^+(0, t)$ at the on-mass-shell line of Fig. 10. The solid line and the filled circles are from the pre-meson factory data analysed in Ref. [39]. The double line corresponds to the IDR analyses of meson factory phaseshifts SM98 (Ref. [55]) and the short dashed line is constructed with the subthreshold coefficients determined from SM98 with forward dispersion relations (Ref. [54]). The star includes the “curvature corrections” to estimate the latter amplitude at the Cheng-Dashen point.

This value is at the high end of a range of 40 MeV to 80 MeV presented in 1997 at the MENU97 conference and reviewed there by Wagner [72]. An independent application of forward dispersion relations to the SM98 partial wave analysis (itself heavily influenced by various sets of dispersion relation constraints [54]) gives

$$\sigma_N(2\mu^2) = \bar{F}^+(0, 2\mu^2)f_\pi^2 \approx (f_1^+ + 2\mu^2 f_2^+)f_\pi^2 \approx 77 \text{ MeV}$$

where we use the notation of Appendix A of Ref. [11] for the Höhler subthreshold coefficients. To this the authors of Ref. [54] estimate in various ways a “curvature correction” to arrive at a final value of 82 to 92 MeV for $\sigma_N(2\mu^2)$. They also obtain at the Weinberg LET $\bar{F}^+(0, t = 0) \equiv f_1^+ = -1.30 \mu^{-1}$ which agrees well with the IDR value. We have already noted in section 3.5 that the Adler zero is closely emulated both by the IDR value $\bar{F}^+(0, t = \mu^2) \approx -0.08 \mu^{-1}$ and by the forward dispersion relation value $\bar{F}^+(0, t = \mu^2) = f_1^+ + \mu^2 f_2^+ = (-1.30 + 1.27) \mu^{-1} = -0.03 \mu^{-1}$. These two different dispersion relation analyses of the same set of phase shifts agree well with each other on the subthreshold (but on-shell) line ($\nu = 0, 0 \leq t \leq 2\mu^2$) important for tests of PCAC (See Fig. 11). Both analyses
include the statements that if the authors replace the SM98 phase shifts by the old Karlsruhe phase shifts (from the pre-meson factory data) they reproduce the Karlsruhe dispersion relation results. It would seem that this value of the sigma term follows from the SM98 phase shifts and is not an artifact of a particular type of dispersion theory analysis. Be warned, however, that extrapolations to the unphysical Cheng-Dashen point with potential model amplitudes fit to (perhaps) different data sets give sigma terms at the low end of the MENU97 range and the reader is encouraged to continue monitoring the situation, especially the CNI-experiment with CHAOS at TRIUMF [68].

We have established the scale set by the size of the sigma term, the near linearity and change of sign of the empirical amplitude $\bar{F}^+(\nu, t)$ in the range $0 \leq t \leq 2\mu^2$, and the validity of the PCAC hypothesis for the off-shell amplitudes $\bar{F}^+(0, t; q^2, q'^2)$ and $\bar{F}^-(0, t; q^2, q'^2)$. It remains to discuss the $t$ dependence of the first terms in the on-shell current algebra Ward identities of Eqs. (57), (58), and (60). The current algebra terms (49) are simply given by the measured electromagnetic form factors. Given that the intrinsic $t$ dependence of $\sigma_N(t)$ is quite small, one can set

$$
\bar{F}^+(\nu, t) = \left( \frac{\sigma_N(2\mu^2)}{f^2_\pi} \right) \left[ 1 + \beta \left( \frac{t}{\mu^2} - 2 \right) \right] + C^+(\nu, t),
$$

where the background amplitude $C^+(\nu, t)$ of (56) is modeled by the overwhelmingly dominant $\Delta(1232)$ isobar. The two approaches to this background amplitude have used dispersion theory for the (over 20) invariant amplitudes of the axial-vector nucleon amplitude $M_{1\mu}^{ij}$ [63] or a $\Delta$-propagator field theory model [64]. Both models of the background amplitude give quite similar results for $C^+(0, t)$ in the low $t$ regime [57]. The dispersion theoretic $C^+(\nu, t; q^2, q'^2)$ contains an unknown subtraction constant in the $g_{\mu\nu}$ term of the axial-nucleon amplitude $\bar{M}_{\mu\nu}^+$, which is moved into the unknown $\beta$ of (refeq:pnamp) and ultimately determined by the data.

This particular $t$ dependence of the multiplier of $\sigma_N(t)$ is suggested by a low energy expansion similar to the Weinberg amplitude for low-energy $\pi\pi$ scattering. This amplitude in the linear approximation satisfying all current algebra/PCAC and quark model $((3, 3) + (3, 3))$ constraints is [23]

$$
T_{\pi\pi} = \frac{1}{f^2_\pi} \left( (s - \mu^2) \delta^{ab} \delta^{cd} + (t - \mu^2) \delta^{ac} \delta^{bd} + (u - \mu^2) \delta^{ad} \delta^{bc} \right) + O(\mu^4),
$$

along with the quark model $\pi\pi$ $\sigma$ term $\sigma_{\pi\pi} = \mu^2$. Generalizations of (70a) to $SU(3)$ pseudoscalar meson-meson scattering were worked out by Osborn [74] and by Li and Pagels [75]. In particular, for $\pi P \rightarrow \pi P$ scattering, the off-shell low-energy generalization of (66) in the linear approximation includes a $(t$-channel) isospin-even part

$$
T_{PP}^{t\text{-even}} = \frac{\sigma_{PP}}{f^2_\pi} \left[ (1 - \beta_P) \left( \frac{q^2 + q'^2}{\mu^2} - 1 \right) + \beta_P \left( \frac{t}{\mu^2} - 1 \right) \right] + O(\mu^4)
$$

where the background amplitude $\sigma_{PP}$ of (56) is modeled by the overwhelmingly dominant $\Delta(1232)$ isobar.
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for

$$\beta_\pi, \beta_K, \beta_{N\sigma} = 1, \frac{1}{2}, 0$$

and

$$\sigma_{\pi\pi}, \sigma_{KK}, \sigma_{N\sigma} = \left(1, \frac{1}{2}, \frac{1}{3}\right) \mu^2.$$ (68)

In fact the $t$ dependent structure of (65) follows from (67) (with constant sigma terms) for scattering of on-shell pions from a meson target. Moreover, this linear (in $t$) structure of both (65) and (67) manifests the Adler and Weinberg soft pion theorems.

For $\pi N \to \pi N$ scattering, however, the fact that the nucleon four-momentum cannot become soft means that $\beta_P$ in (65) cannot be a priori predicted as it is in (68) for meson targets. Instead $\beta$ in (65) for on-mass-shell $\pi N \to \pi N$ scattering is fitted to the the IDR curve in Fig. 11 to find $\beta \approx 0.45$, quite near to $\beta_K = 1/2$ in (68), perhaps reflecting the same isospin structure of K and N. The above current algebra/PCAC analysis in (65) and in (66-68) does not mean that the $\sigma$ term occurring in four-point function $\pi N$ scattering has an intrinsic $t$ dependence. Rather, the linear $t$ dependent factor in (65) is a PCAC realization of the unknown subtraction constant $\beta q' \cdot q \sigma_N$, with the $\beta$ determined by the Adler and Weinberg LEIs.

An alternative ansatz for the $t$ dependence of the sigma term stems from the SU(2) linear $\sigma$ model (LoM) with $N, \pi, \sigma$ as elementary fields [1]. (For a recent review of the resurgence of interest in the $\sigma$ meson, see the references in [77].) Using a pseudoscalar rather than pseudovector $\pi NN$ coupling means that the t-channel $\sigma$ pole has a background $F^+\sigma$ amplitude [76] proportional to $(m^2\sigma - \mu^2)(m^2\sigma - t)^{-1} - 1$. This structure automatically complies with the 3 low-energy theorems; e. g. it vanishes at $t=\mu^2$ as does the Adler zero. Thus the isospin-even background $\pi N$ amplitude can be expressed in the LoM at $\nu = 0$ as [76]

$$\bar{F}^+_{\text{LoM}}(0, t) = \frac{g^2_{\pi NN}}{m} \left[\frac{m^2\sigma - \mu^2}{m^2\sigma - t} - 1\right].$$ (69)

To obtain a quantitative fit to the empirical amplitude this must be supplemented by the $\Delta$ contribution. In the dispersion relation model the amplitude becomes

$$\bar{F}^+(0, t) = \frac{g^2_{\pi NN}}{m} \left[\frac{m^2\sigma - \mu^2}{m^2\sigma - t} - 1\right] + \beta' q \cdot q' + C^+(\nu, t)$$ (70)

where again the parameter $\beta'$ shifts the unknown subtraction constant in the $\Delta$ contribution to the PCAC realization $\beta' q' \cdot q \sigma_N$. A fit which is within the two lines of the double line of Fig. 11 for $0 \leq t \leq 2 \mu^2$ can be made with $\beta' = 1.44 \mu^{-3}$ and $m_\sigma = 4.68 \mu \approx 653$ MeV [78], the latter a quite reasonable value when compared with the current $\sigma$ meson phenomenology [77].

With these determinations of the $t$ dependence of the sigma term in (57), we can finally finish the tests suggested in the last sentence of Section 3. That is, how well do the current algebra models describe the $\pi N$ data extrapolated to the subthreshold crescent of Fig. 2? Away from the $\nu = 0$ line in the amplitude $F^+$, these tests are given by expanding the isobar modeled $C^\pm(\nu, t)$ and $D^\pm(\nu, t)$ and
comparing the theoretical expansion coefficients with the empirical Höhler coefficients. This is an old story and the general trends are summarized in Appendix A of Ref. [1]. The 28 subthreshold coefficients are matched very well indeed by the current algebra amplitudes (57-60), once the $t$ dependence of the sigma term is fixed empirically (as above). The $t$ dependence of the two current algebra terms in (58) and (60) is indeed given quite well by the isovector vector current of (13). The preliminary determinations of the subthreshold Höhler coefficients from the meson factory data [7] does not change the qualitative picture given in [11, 13]. Only the scale of $F^+(\nu, t)$, set by the size of the sigma term, has changed with the advent of increasingly more precise $\pi N$ data.

The $t$ dependence of the sigma term was suggested by PCAC off-shell constraints (67) or by the linear sigma model (69). The success of the on-shell tests and of the examples of the PCAC hypothesis suggests a reliable PCAC off-pion-mass shell extrapolation for the not-so far extrapolations of the $\pi N$ amplitude discussed in the Introduction. However, the shorter range parts of the two-pion exchange three-body force will be quite different for the $t$ dependence of the sigma term from (67) or (69). The shorter range parts which follow from (67) have been discussed and compared with those of chiral perturbation theory in Ref. [80] which used the techniques of Ref. [15]. The implications of (69) for three-body forces, threshold pion production, and pion condensation remain to be worked out.

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