Leveraging Cross Feedback of User and Item Embeddings for Variational Autoencoder based Collaborative Filtering

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Abstract

Matrix factorization (MF) has been widely applied to collaborative filtering in recommendation systems. Its Bayesian variants can derive posterior distributions of user and item embeddings, and are more robust to sparse ratings. However, the Bayesian methods are restricted by their update rules for the posterior parameters due to the conjugacy of the priors and the likelihood. Neural networks can potentially address this issue by capturing complex mappings between the posterior parameters and the data. In this paper, we propose a variational auto-encoder based Bayesian MF framework. It leverages not only the data but also the information from the embeddings to approximate their joint posterior distribution. The approximation is an iterative procedure with cross feedback of user and item embeddings to the others’ encoders. More specifically, user embeddings sampled in the previous iteration, alongside their ratings, are fed back into the item-side encoders to compute the posterior parameters for the item embeddings in the current iteration, and vice versa. The decoder network then reconstructs the data using the MF with the currently re-sampled user and item embeddings. We show the effectiveness of our framework in terms of reconstruction errors across five real-world datasets. We also perform ablation studies to illustrate the importance of the cross feedback component of our framework in lowering the reconstruction errors and accelerating the convergence.

1 Introduction

In this paper, we focus on improving the performance of Bayesian matrix factorization (BMF) in collaborative filtering (CF). BMF models impose appropriate prior distributions over user and item embeddings. These embeddings then construct a likelihood term over the rating data. Training BMF models amounts to infer the joint posterior distribution of the embeddings analytically or by approximation. In most cases, distributions involving multiple variables cannot be derived analytically, and thus approximation inference techniques are used instead.

BMF models adopt two types of approximation inference frameworks: Gibbs sampling and variational inference. Both of them rely on the conjugacy of the priors and the likelihood over the embeddings. The BMF models have mostly exploited Normal-Normal and Poisson-Gamma conjugacy for their likelihood and priors.

Despite its algebraic convenience, the conjugate updates on the posterior parameters of each embedding are independent and thus fail to utilize the information from each other. This limits the performance of the BMF models. In this case, we want the data information for different embeddings to be shared to refine the inference of their posterior distributions.

To enable the sharing of the data information, it is common to model the posterior parameters as regression over the data. The regression weights are learned to map similar data patterns into values in close proximity in the latent space. Naturally, the mapping is non-linear. This motivates us to use artificial neural networks, which can fit arbitrarily complex mapping functions, to predict the posterior parameters for the embeddings. In particular, variational auto-encoders (VAEs) provide the foundation to achieve all of the above. It connects variational inference of posterior distributions with neural networks.

In this paper, we propose a novel BMF framework based
on variational autoencoders. It conducts mean-field variational inference for user and item embeddings by predicting their posterior parameters using multi-layer perceptron (MLP) encoders. These encoders take in not only the data but also the embeddings. This input arrangement is motivated by the dependence in statistical inference. In the context of collaborative filtering, the inference of a user’s embeddings is dependent on both his/her ratings and the embeddings of the rated items, and vice versa.

Our framework leverages not only the data information but also the embedding information in an iterative manner. In each iteration, user and item embeddings are sampled from their respective encoders. The inputs to the user-side encoders include the ratings and the item embedding samples fed back from the last iteration, and vice versa. A decoder then reconstructs the ratings using matrix factorization with the currently sampled user and item embeddings.

Our framework achieves overall lower reconstruction errors compared to seven state-of-the-art neural network based CF methods across five real-world rating datasets. It also yields semantically coherent embeddings that represent different types of users and items. The ablation study also reveals that the cross feedback of user and item embeddings to the others’ encoders is useful for further lowering the errors.

2 Related Work

Matrix factorization [7] (MF) is the most widely applied approach in collaborative filtering (CF). It assumes that a matrix can be well approximated by the multiplication of two low rank matrices. In the context of CF, the target matrix stores the user-item ratings, while the two low-rank matrices comprise the latent representations, called the embeddings, of users and items. Among various MF models, the probabilistic matrix factorization (PMF) by Salakhutdinov et al. [10] proposed to assign Normal priors to the user and item embeddings. These embeddings further form a Normal likelihood over the ratings. Each rating is modelled to follow a Normal distribution centered on a user-item embedding dot product. Optimization of the maximum a posteriori (MAP) yields point estimates for the embeddings.

2.1 Bayesian Matrix Factorization

Extending from the PMF model are the Bayesian matrix factorization (BMF) models [2][3][11]. They impose various priors over the user and item embeddings. These priors are usually conjugate to the likelihood over the rating data. In [11], a fully Bayesian treatment was applied to the BMF in which Normal-Wishart priors were imposed over the embeddings. Due to the conjugacy of such priors and a Normal likelihood, the conditional posterior distributions of the embeddings can be sampled in a closed form via the Gibbs sampling. Gopalan et al. [2][3] proposed to impose Gamma distributions as the priors over the embeddings. In this case, the likelihood is assumed to follow a Poisson distribution which is conjugate to the Gamma priors. These work leverages the variational inference to estimate the posterior Gamma shapes and rates for the embeddings. The conjugate priors employed enable the inference but the update rules they exert on the posterior parameters are very often restricted in capturing useful data information.

2.2 Variational Autoencoder Based Bayesian Matrix Factorization

Autoencoders [1] (AE) are a family of unsupervised generative neural networks that reconstruct their input data and meanwhile generate embeddings of the input data. Among them, variational autoencoders [6] (VAE) are known to be able to approximate intractable posterior distributions of latent variables with different types of neural networks. This is achieved via computing the posterior parameters of the latent variables using particular neural networks (e.g. the MLP). As a result, VAEs can capture arbitrarily complex mapping relationships between the posterior parameters and the data, which failed to be captured by the conjugate updates of the BMF models. VAEs can also share the posterior update information while the updates in BMF are independent.

In recent years, research has started to emerge which integrates VAEs with the BMF in collaborative filtering. Li and She [8] proposed to leverage VAEs for estimating the posterior means and variances of item embeddings from the item content. Meanwhile, the generated item embeddings are involved in the traditional BMF modeling. In [9], only the posterior means and variances of the user embeddings are estimated by the VAEs from the rating data. Item-specific weights of a MLP network are learned to map the user embeddings non-linearly into the multinomial predictions for the implicit data. To our best knowledge, there has not been any work which estimates the posterior parameters for both the user and the item embedding using the VAEs.

2.3 Other Neural Network Based Matrix Factorization Models

Apart from the VAEs, there have been many other types of neural networks (NN) applied to the matrix factorization for collaborative filtering. In [9], the authors also proposed a denoising autoencoder (DAE) based user-oriented MF model. It randomly adds noise to the in-
put data and then learns user embeddings to recover the original data. This model is very similar to another major DAE based user-oriented model proposed in [13]. The main difference is that the former model adopts the multinomial log-likelihood loss which is more robust than the logistic loss employed by the latter. Earlier work has mostly used vanilla AEs to replace the MF approaches. One of the pioneering work [12] proposed two variants of the vanilla AEs for separately encoding users and items from the user-item and the item-user rating matrices respectively. Their decoders then reconstruct the above rating matrices from the resulting user and item embeddings respectively.

Unlike AE based approaches which take in the rating data, there are other NN based approaches that take in the one-hot encodings of the user and item IDs. In [5], non-linear interactions between the user and the item embeddings (mapped from their encoded IDs) are captured via a MLP network. The two embeddings are concatenated before passed through the MLP. Different from the concatenation, He et al. [4] proposed to perform outer product between the user and the item embeddings to better capture their interactive patterns. The resulting latent interaction maps are passed through a convolutional neural network (CNN) which learns high-order correlations among the embedding dimensions.

3 Proposed Framework

In this paper, we use the symbols $I$ and $J$ to denote the number of users and the number of items involved in collaborative filtering. We denote the embedding of the $i$-th user as $u_i$ and that of the $j$-th item as $v_j$ where $i = 1, ..., I$ and $j = 1, ..., J$. The number of dimensions of $u_i$ and $v_j$ are $K$. The user and the item embedding matrices are expressed as $U \in \mathbb{R}^{I \times K}$ and $V \in \mathbb{R}^{J \times K}$ respectively. The ratings of all the users on all the items are formulated as a matrix $R$ of size $I \times J$. Its $(i,j)$-th entry stores the rating $r_{ij}$ given by the $i$-th user to the $j$-th item. If the rating is missing, we set $r_{ij} = 0$ for convenience of mathematical expression.

Our framework focuses on inferring the posterior distributions for the user and item embeddings. It leverages the mean-field variational inference method. This method assumes that the embeddings are independent and estimates their posterior parameters by minimizing the negative evidence lower bound (ELBO) loss. In collaborative filtering, the embeddings are usually assumed to follow Normal distributions. In this case, the negative ELBO loss, denoted by $Q$, for the Normal likelihood over the ratings $R$ can be expressed as follows:

$$Q = \| (R - UV^T) \odot M \|_2 - \lambda \sum_{k=1}^{K} \sum_{i=1}^{I} \sum_{j=1}^{J} KL(N(\mu_{ik}, \sigma^2_{ik}), N(\mu_{ik}, \sigma^2_{ik}))$$

In Equation (1) $M \in \mathbb{R}^{I \times J}$ is a masking matrix. Its $(i,j)$-th entry is either zero if $r_{ij} = 0$ or one otherwise. The symbol $\odot$ denotes element-wise multiplication between matrices. The symbols $\mu_{ik}$ and $\sigma^2_{ik}$ denote the posterior mean and variance for the $k$-th component of user $i$’s embedding $u_i$. Correspondingly, $\mu$ and $\sigma^2$ are the prior mean and variance (for both user and item embedding components). The parameter $\lambda > 0$ controls the magnitudes of the KL divergence terms in the loss function.

For the mean-field variational inference, the conjugacy of the Normal prior and likelihood over the embeddings allows closed-form posterior updates on their means and variances. However, the update rules are limited in their expressive power. Moreover, the updates for the embeddings are independent to one another. As a result, the posterior parameter estimation and the model performance can be susceptible to the possible sparsity in the rating data. Combining BMF with VAEs can solve both problems. VAEs gain their expressive power via activation functions and their depths. Meanwhile, the weights between the layers can map similar inputs to similar outputs. This exerts a smoothing effect on the noisy sparse data information.

3.1 Framework Structure

Figure 1 shows the architecture of the proposed framework. Note that for simplicity, the posterior means and variances are shown to be computed by the same set of MLP networks. For the experiment, we implement separate MLP networks to compute these two parameters.

Our framework inherits the encoder-decoder structure from the VAEs. In terms of the decoder, it reconstructs the rating matrix $R$ as the multiplication of the user and the item embedding matrices $UV^T$. As for the encoder part, there are user-side and item-side encoders which respectively generate the matrices $U$ and $V$. Both encoders possess two types of inputs: the rating input and the embedding input. The user-side rating input (layer) takes in the rating matrix $R$, while the item-side input takes in its transpose $R^T$.

As for the embedding input (layer) of the user-side encoder, it is constructed based on the item embed-
Figure 1: The architecture of our framework.

Positioning in the document:

1. The architecture of our framework.
2. More specifically, it is constructed as a matrix $Z^{(U)} \in \mathbb{R}^{I \times (JK)}$. Its $i$-th row is a concatenation of the embeddings of all the items after the embeddings of those unrated by user $i$ were masked into zeros. As an example, suppose user $i$ has rated the second of three items and the embedding dimension $K = 3$. Then, the input row corresponding to this user is $z_i = [0, 0, 0, v_{21}, v_{22}, v_{23}, 0, 0, 0]$. Likewise, the embedding input at the item-side takes in a matrix $Z^{(V)} \in \mathbb{R}^{J \times (IK)}$ which is constructed in the same manner as $Z^{(U)}$.

Both the rating and the embedding inputs at each side are passed through and transformed by their own MLP networks to compute the corresponding posterior parameters. To make the framework description succinct, we only describe the MLP networks of the user-side encoder that compute the posterior means for the user embeddings. Initially, the rating input of the user-side encoder is transformed by an MLP network as follows:

$$
F_1^{(U)} = q(R\Theta_0^{(U)} + 1^{(U)}\alpha_0^{(U)})
$$

$$
F_2^{(U)} = q(F_1^{(U)}\Theta_1^{(U)} + 1^{(U)}\alpha_1^{(U)})
$$

$$
\vdots
$$

$$
F_{L+1}^{(U)} = q(F_L^{(U)}\Theta_L^{(U)} + 1^{(U)}\alpha_L^{(U)})
$$

In Equation 2 $\Theta_0^{(U)} \in \mathbb{R}^{J \times M^{(1)}}$ and $\alpha_0^{(U)} \in \mathbb{R}^{1 \times M^{(1)}}$ are respectively the weight matrix and the bias vector specific to the rating input layer. Here, $M^{(1)}$ is the number of neurons for the first hidden layer subsequent to the rating inputs. The term $1^{(U)}$ denotes a column vector of size $I \times 1$ with its entries being all ones. The matrix output of this input layer is then transformed by the activation function $q$. The result $F_1^{(U)} \in \mathbb{R}^{I \times M^{(1)}}$ serves as the input to the first hidden layer.

There are in total $L'$ hidden layers in the user-side encoder. The $l'$-th hidden layer, where $l' = 1, \ldots, L'$, possesses a weight matrix $\Theta_{l'}^{(U)} \in \mathbb{R}^{M^{(l')}} \times M^{(l'+1)}$ and a bias vector $\alpha_{l'}^{(U)} \in \mathbb{R}^{1 \times M^{(l'+1)}}$. Its corresponding input matrix is $F_{l'}^{(U)} \in \mathbb{R}^{I \times M^{(l')}}$. Finally, the output of the $L$-th hidden layer, $F_{L+1}^{(U)} \in \mathbb{R}^{I \times M^{(L'+1)}}$, is the intermediate user embedding matrices derived from the rating input $R$. For convenience, we use $K' = M^{(L'+1)}$ to denote the dimension of each intermediate user embedding.

Similar to the rating input $R$, the item embedding input $Z^{(U)}$ is passed through an MLP network and trans-
formed as follows:

\[
G_1^{(U)} = q(Z^{(U)}Φ_0^{(U)} + 1^{(U)}β_0^{(U)}) \\
G_2^{(U)} = q(G_1^{(U)}Φ_1^{(U)} + 1^{(U)}β_1^{(U)}) \\
\ldots \\
G_{L+1}^{(U)} = q(G_L^{(U)}Φ_L^{(U)} + 1^{(U)}β_L^{(U)})
\]

The symbols and terms in Equation 3 correspond to those in Equation 2. The only difference is that the weight matrix of the embedding input layer \( Φ_0^{(U)} ∈ \mathbb{R}^{JK \times M^{(1)}} \) is much larger than its counterpart \( Θ_0^{(U)} \). Other than this, we let the two MLP networks have the same number of hidden layers \( L \), the same number of hidden neurons \( M^{(l)} \) at each layer \( l \) and the same dimension \( K' \) for the intermediate embeddings.

After generating the intermediate embedding matrices \( F_{L+1}^{(U)} \) and \( G_{L+1}^{(U)} \), we concatenate them in a column-wise manner. This creates the input \( S^{(U)} = [F_{L+1}^{(U)} G_{L+1}^{(U)}] ∈ \mathbb{R}^{I \times 2K'} \) to the final MLP network. It computes the posterior means for the user embeddings as follows:

\[
H_1^{(U)} = q(SΨ_0^{(U)} + 1^{(U)}γ_0^{(U)}) \\
H_2^{(U)} = q(H_1^{(U)}Ψ_1^{(U)} + 1^{(U)}γ_1^{(U)}) \\
\ldots \\
μ_U = H_{L+1}^{(U)} = h(H_L^{(U)}Ψ_L^{(U)} + 1^{(U)}γ_L^{(U)})
\]

From Equation 4, the input layer of the final MLP has a weight matrix \( Ψ_1^{(U)} ∈ \mathbb{R}^{2K' \times N^{(1)}} \) and a bias vector \( γ_0^{(U)} ∈ \mathbb{R}^{1 \times N^{(1)}} \). The number of hidden neurons in this case is \( N^{(1)} \). The other network parameters include the weight matrices \( Ψ_l^{(U)} ∈ \mathbb{R}^{N^{(l)} \times N^{(l+1)}} \) and the bias vectors \( γ_l^{(U)} ∈ \mathbb{R}^{1 \times N^{(l+1)}} \) at each hidden layer \( l = 1, \ldots, L \). Here, \( L \) is the number of hidden layers in the network. The last hidden layer possesses a different activation function \( h \). It outputs the posterior mean matrix \( μ_U ∈ \mathbb{R}^{I \times K} \) for all the \( I \) users (with \( K = N^{(L+1)} \)).

The posterior variance matrix \( \text{diag}\{σ_U\} ∈ \mathbb{R}^{I \times K} \) is computed using the same Equations 2, 3, and 4 but with separate sets of weights and biases. Finally, the user embedding matrix \( U \) is generated based on \( μ_U \) and \( \text{diag}\{σ_U\} \) using the reparametrization trick.

3.2 Cross Feedback Loops of User & Item Embeddings

There are two feedback loops existing in our framework. As shown by Figure 1, one feeds the user-side (encoder) output back into the item-side embedding input layer. The other feeds the item-side output back into the user-side embedding input layer. According to Algorithm 1, this cross feedback mechanism is conducted iteratively alongside the variational inference for the embeddings \( U \) and \( V \). In our algorithm, steps 6 and 15 specify the cross feedback loops between the embedding inputs at one side and the encoder outputs at the other side. More specifically, the embeddings \( U, V \) sampled at step 15 will be fed across and back into each other’s embedding input layers at step 6. This cross feedback mechanism prevents our framework from being trapped in an infinite loop of gradient computation.

In Algorithm 1, users and items are batched according to the predefined user and item batch sizes \( B_U, B_V \). As a result, the rating and the embedding inputs of the encoders at both sides need to be sliced by the corresponding batches before passed through the networks. In addition, our algorithm employs nested iteration across the item and the user batches rather than sequential iteration. The latter alternates the update of the user-side and the item-side weights and biases while the nested iteration updates these parameters all at once. We found in the experiment that the nested iteration allows our framework to achieve higher prediction accuracy at the cost of more running time.

Algorithm 1: VAE based BMF with Cross Feedback of User and Item Embeddings

1. **Input:** rating matrices \( R \), user batch size \( B_U \), item batch size \( B_V \)
2. Sample entries of \( U, V \) from \( \mathcal{N}(μ, σ^2) \)
3. Sample \( U \) user batches and \( V \) item batches
4. Randomly initialize user-side and item-side encoder weights and biases
5. For not converged or not max iterations do
   6. Build user-side inputs \( Z^{(U)} \) from \( V \) and item-side inputs \( Z^{(V)} \) from \( U \)
   7. For each user batch do
      8. Slice \( R, Z^{(U)} \) by the user batch
      9. For each item batch do
         10. Slice \( R^T, Z^{(V)} \) by the item batch
         11. Compute gradients \( \nabla Q \) w.r.t. all the weights and biases over the slices
         12. Update the weights and biases with the computed gradients
      End
   End
   14. Resample \( U, V \) using reparametrization trick
End
4 Experiments and Results

In this section, we compare the rating prediction performance of our framework with various state-of-the-art CF models across five real-world datasets. We further study the importance of the embedding cross feedback mechanism in accelerating the model convergence and improving the model performance. This includes an ablation study in which we conduct performance comparison with the framework itself by taking its components away.

4.1 Datasets

We investigate five medium to large real-world datasets. The major information of these datasets is summarized in Table 2.

MovieLens-1M, 10M, 20M (ML-1M, ML-10M, ML-20M): These are medium to large sets of user-movie ratings collected from MovieLens\(^2\) a movie recommendation service, over different periods of time.

Amazon pet supplies and Kindle stores (Amazon-Pet, Amazon-Kindle): These datasets contain ratings from Amazon on different types of products.\(^3\) For these two datasets, we only preserve users and items that have at least 10 ratings.

4.2 Experimental setup

To evaluate the performance of our framework, we adopted the training-validation-testing evaluation routine. We randomly split the observed rating data into the training, validation and testing data subsets with the 70%-15%-15% ratio. The evaluation metric employed in the experiment is the root mean squared error (RMSE) over the testing datasets.

We perform hyper-parameter optimization for our framework based on its RMSE scores on the validation datasets. More specifically, a grid search is conducted on the embedding dimensions \(K\) and \(K'\) among \{10, 15, 25, 50\}, the number of encoder hidden layers \(L'\) and \(L\) among \{0, 1, 2\}, the number of corresponding hidden neurons \(M'(l)\) and \(N(l)\) at each layer among \{10, 25, 50\} and the controlling parameter \(\beta\) for the KL terms among \{0.00001, 0.0001, 0.001, 0.01, 0.1, 1\}. We restrict the grid search such that \(K < K'\) and \(K, K' \leq M(l), N(l)\) at any hidden layer.

As for the user and the item batch sizes \(B_U, B_V\), we enlarge them when the numbers of users and items in the datasets are larger. This accelerates the framework training at the cost of more memory consumption and the degraded framework performance in RMSE. In the experiment, we carefully tuned the batch sizes considering this trade-off. For the ML-1M dataset, we set \(B_U, B_V\) to be 100 given its relatively small numbers of users and items, while for the other datasets, \(B_U, B_V\) are set to be 1,000.

4.3 Baselines

We compare our framework with the following state-of-the-art CF models in terms of the testing RMSE. The optimization of the hyper-parameters for each of these models is based on the validation RMSE. Table 1 summarizes the ranges of the hyper-parameter values of each model for the optimization.

Neural Collaborative Filtering (NCF): This model uses an MLP network to replace the traditional matrix factorization over the user and item embeddings. We adhere to the choice of ReLU as the activation function from the original paper.

Convolutional Neural Network based Collaborative Filtering (CNN-CF): This model applies a CNN network to the outer product of the user and the item embeddings to capture the user-item interactive patterns. We adopt the default kernel size \(2 \times 2\) and the ReLU activation function from the original paper. Furthermore, we change the original Bayesian personalized ranking loss on the implicit data to the MSE loss on our explicit rating data.

Autoencoder based User-side & Item-side Collaborative Filtering (U-AutoRec, I-AutoRec): U-AutoRec encodes and decodes the rating matrix \(R\) with a standard autoencoder. The encoder in this case can only generate user embeddings. Likewise, I-AutoRec encodes and decodes \(R^T\) with the encoder only generating item embeddings. We adopt the ReLU function for the hidden layers and the sigmoid function for the output layer as in the paper.

Variational & Denoising Autoencoder based Collaborative Filtering (VAE-CF, DAE-CF): These models are essentially user-side autoencoders that only generate embeddings for the users. We change their multinomial likelihood loss functions on the implicit data to the MSE loss over our rating data. The other parts of the models remain unchanged. These include the controlling parameter \(\beta\) for the KL term and its decay strategy in VAE-CF, the weight decay strategies and the dropout probabilities in both models.

4.4 Predictive Analysis

In this section, we evaluate the performance of all the models on predicting the ratings in the testing datasets.
that our framework clearly defeats all the baselines with testing RMSE scores of different models across different datasets) and testing. Figure 2 shows the changes of the minimizing the model hyper-parameters (with the validation percentages we have taken are 1% users and the items, to make it sparse. The sub-sampling ML-1M dataset, which has abundant ratings across the users and the items, which is the case in the Movielens data. However, we can see from Table 2 that the ratings in the Amazon datasets are much sparser across the users and the items. It performs better when ratings are abundant across the items which is the case in the Movielens data. The sparsity at both sides cause the I-AutoRec and the U-AutoRec to degrade in their performance. In comparison, our framework appears to be more robust when the sparsity occurs in the Amazon datasets.

To further verify our conjecture, we sub-sampled the ML-1M dataset, which has abundant ratings across the users and the items, to make it sparse. The sub-sampling percentages we have taken are 1%, 2%, 3%, 5% and 10%. The sub-sampled datasets are used to train each model. The rest of the data is split equally for optimizing the model hyper-parameters (with the validation datasets) and testing. Figure 2 shows the changes of the testing RMSE scores of different models across different sub-sampling percentages. It can be seen from the figure that our framework clearly defeats all the baselines with significant performance margins across all the percentages. When the percentage is 1% and 10%, the margin between our framework and the second-best model is respectively the largest (i.e. by 0.047) and the smallest (i.e. by 0.018).

In this case, the NCF model still ranks at the second place in terms of the testing RMSE. The best explanation to the robustness of the NCF towards the user and item rating sparsity is that the MLP network takes in dense embeddings as the input rather than take in the sparse ratings directly (as by the I-AutoRec and U-AutoRec).

The VAE-CF, DAE-CF and U-AutoRec models are generally under-performing for both the full and the sub-sampled datasets. We conjecture that this is because all of them are user-oriented. Since the rating sparsity is overall more severe at the user side for all the datasets, the user-oriented approaches tend to be affected more in terms of their performance.

### 4.5 Ablation Study

In this section, we conduct an ablation study to illustrate the importance of the two main components of our framework: the rating data inputs and the embedding

| Models | $K$ | # hidden layers | # neurons | $L_2$ |
|--------|-----|-----------------|-----------|-------|
| NCF    | {5, 10, 25, 50, 100} | 0 to 2 | {10, 25, 50, 100} | {0, 0.01, 0.1, 1} |
| CNN-CF | {8, 16, 32, 64, 128} | 1 to 6 | {64 × 64, 32 × 32, 16 × 16, ...} | {0, 0.01, 0.1, 1} |
| U/I-AutoRec | {5, 10, 25, 50, 100} | 0 to 2 | {10, 25, 50, 100} | {0, 0.01, 0.1, 1} |
| VAE-CF | {10, 25, 50, 100, 200} | 0 to 2 | {50, 100, 200, 400, 600} | N/A |
| DAE-CF | {10, 25, 50, 100, 200} | 0 to 2 | {50, 100, 200, 400, 600} | {0, 0.01, 0.1, 1} |

Table 1: The hyper-parameter value ranges of each baseline model for the hyper-parameter optimization on the validation RMSE.

| Datasets | # users | # items | # ratings |
|----------|---------|---------|-----------|
| ML-1M    | 6,040   | 3,900   | 1,000,209 |
| ML-10M   | 69,878  | 10,631  | 9,999,989 |
| ML-20M   | 135,543 | 21,651  | 19,972,967 |
| Amazon-Pet | 46,403  | 61,673  | 1,008,336 |
| Amazon-Kindle | 92,351  | 55,924  | 1,619,155 |

Table 2: The summary of the key attributes of the datasets used in the experiment.

Table 3 shows the testing RMSE scores of all the models across the five datasets. It can be observed that our proposed framework has achieved lower testing RMSE than all the baselines on each dataset. It manages to outperform the second-best model by at least 0.007 in the testing RMSE (at the ML-1M dataset). It yields the largest advantage against the second-place model, which is 0.016, at the Amazon-Kindle data. It is also interesting to see that the I-AutoRec model holds the second place across all the Movielens datasets while it is the NCF model for the Amazon datasets. We conjecture that this has to do with how the I-AutoRec model works. It performs better when ratings are abundant across the items which is the case in the Movielens data. However, we can see from Table 2 that the ratings in the Amazon datasets are much sparser across the users and the items. The sparsity at both sides cause the I-AutoRec and the U-AutoRec to degrade in their performance. In comparison, our framework appears to be more robust when the sparsity occurs in the Amazon datasets.
Table 3: The RMSE results of each model across the five rating datasets under the 70%-15%-15% training-validation-testing split ratio.

| Model      | ML-1M | ML-10M | ML-20M | Amazon-Pet | Amazon-Kindle |
|------------|-------|--------|--------|------------|---------------|
| NCF        | 0.870 | 0.831  | 0.822  | 1.098      | 0.734         |
| CNN-CF     | 0.875 | 0.844  | 0.839  | 1.119      | 0.770         |
| U-AutoRec  | 0.883 | 0.893  | 0.833  | 1.136      | 0.772         |
| I-AutoRec  | 0.859 | 0.823  | 0.818  | 1.213      | 0.787         |
| DAE-CF     | 0.911 | 0.852  | 0.841  | 1.247      | 0.762         |
| VAE-CF     | 0.903 | 0.834  | 0.836  | 1.211      | 0.747         |
| Ours       | 0.852 | 0.812  | 0.809  | 1.087      | 0.718         |

Table 4: The RMSE results of our framework with each of its main components ablated away. The study shows that both components contribute significantly to the framework performance. Especially, the implicit information leveraged by the embedding cross feedback component shows its role of further improving the performance anchored by the explicit data information.

| Ablated components | ML-1M | ML-10M | ML-20M | Amazon-Pet | Amazon-Kindle |
|--------------------|-------|--------|--------|------------|---------------|
| No data inputs     | 0.917 | 0.902  | 0.878  | 1.241      | 0.813         |
| No embedding cross feedback | 0.864 | 0.829  | 0.824  | 1.110      | 0.739         |
| Full Model         | 0.852 | 0.812  | 0.809  | 1.087      | 0.718         |

Cross feedback mechanism. The rating data inputs include the rating matrix $R$ and its transpose $R^T$ that are fed to compute the intermediate embeddings $F_{L+1}^{(U)}$ and $F_{L+1}^{(V)}$ respectively. Removing them and the entire MLP networks that generate the corresponding intermediate embeddings from the framework means that the explicit data information is no longer exploited. The embedding cross feedback component involves the two embedding inputs $Z^{(U)}$ and $Z^{(V)}$ that are fed to compute the intermediate embeddings $G_{L+1}^{(U)}$ and $G_{L+1}^{(V)}$ respectively. Removing this component from our framework means that it no longer leverages the implicit data information for the variational inference.

With each of its components taken away, we train and optimize the hyper-parameters of our framework with the same training and validation datasets. Afterwards, we evaluate it on the same testing datasets in terms of the testing RMSE. Table 4 shows the study results. It can be observed that both components are important to our framework. This is because when each component is removed, the performance of our framework degrades significantly. Especially, the degradation caused by removing the data inputs is much more greater than that caused by removing the cross feedback mechanism. This is expected as the explicit data information is more directly useful in improving the model performance compared to the implicit information. Nevertheless, the ablation study shows the efficacy of the embedding cross feedback mechanism in improving the performance of our framework.

### 4.6 Convergence analysis

To further investigate the importance of the embedding cross feedback mechanism, we inspect how the framework performance (i.e. the testing RMSE scores) converges with and without the mechanism. More specifically, we inspect the performance of the following four variants of our framework:

**Variant A**: our framework without the cross feedback mechanism and the number of hidden layers $L' = 0$ for the MLP networks over the rating data.

**Variant B**: our framework with the cross feedback mechanism and the number of hidden layers (before “concat”) $L' = 0$ for the MLP networks over the rating data and the embedding feedback. The number of hidden layers (after “concat”) $L = 0$ in this case.

**Variant C**: our framework without the cross feedback mechanism and the number of hidden layers $L' = 1$ for the MLP networks over the rating data.

**Variant D**: our framework with the cross feedback mechanism and the number of hidden layers (before “concat”) $L' = 1$ for the MLP networks over the rating data and the embedding feedback. The number of hidden layers (after “concat”) $L = 0$.

Here, the only difference between variants A and B, and
between variants C and D is the presence of the cross feedback mechanism. We would like to see whether variants B and D which possess the mechanism will not only perform better but also converge faster than their counterparts without the mechanism.

Figure 3 shows how the testing RMSE scores of the four variants change during the first 100 training iterations over the ML-1M data. It can be observed that the two variants with the cross feedback mechanism constantly outperform their counterparts. Especially, the performance of variant C rises much more quickly than variant D at the early stage of the training. This indicates that the cross feedback mechanism can quickly improve the framework performance even when the posterior inference for the embeddings has just begun.

We also note that the line of variant B is much smoother than that of variant A which still fluctuates after 50 iterations. Considering the two variants have the same optimization configuration, this clearly shows that the cross feedback mechanism smooths and accelerates the convergence of our framework.

Finally, there is a huge performance gap between variants A and B. This gap reconfirms that the cross feedback mechanism can significantly boost the framework performance when the explicit data information is not sufficiently exploited (due to no hidden layers in the corresponding MLPs).

5 Conclusion

In this paper, we propose a variational auto-encoder based Bayesian matrix factorization framework for collaborative filtering. Compared to the previous work, this framework leverages not only the explicit data information from the ratings but also the implicit information from the user and item embeddings for improving the variational approximation of their posterior parameters.

Our framework is characterized by the iterative cross feedback of the user and item embeddings to each other's encoder input layers. The cross feedback inputs provide useful implicit information regarding the counterparts. Both the explicit and implicit information is learned by dedicated MLP networks whose final outputs are concatenated. Then, the result is fed into another MLP network which fuses and map the two types of information into the posterior parameters of the embeddings.

Experimental results show that our framework outperforms six state-of-the-art NN based collaborative filtering approaches in terms of the reconstruction error over both the full and sub-sampled data. It is also found that the cross feedback mechanism not only improves the framework performance but also accelerates its convergence.

As the future work, we will investigate the possibility of incorporating the attention mechanisms and the transformers into the encoder structure to refine the posterior parameter estimation for the embeddings. We will also adapt our framework to be able to model the implicit data in recommendation systems.

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