Research on Fractal Characteristics of Building Energy Consumption Time Series

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Abstract. The problem of nonlinear building energy consumption characteristics is studied. The fractal energy theory and the rescaled range analysis method are used to construct the building energy consumption analysis model. Through the analysis of the building energy consumption curve, it is concluded that the building energy consumption changes have a certain periodic law and self-similarity in terms of geometry. In terms of geometry, building energy consumption has self-similarity. By calculating the fractal dimension, it is found that the building energy consumption has self-similarity at the time scale. At the same time, the analysis of different types of buildings energy consumption characteristics reveals that there is no difference in the fractal characteristics. It provides a new perspective for the study of building energy consumption characteristics, building energy consumption predictions and troubleshooting.

1. Introduction
Architecture, industry and transportation are the three main aspects of energy consumption. Building energy consumption accounts for 25% to 30% of total social energy consumption [1]. Determining reasonable energy-saving measures for building energy consumption are an important part of achieving energy conservation and consumption reduction. The basis for building energy conservation is the analysis and modeling of building energy consumption. Domestic and foreign scholars have also done a lot of research on building energy consumption. These studies can be divided into two categories: the first type of research [2-3] is on construction structures, and this kind of research is mainly applied in the architectural design stage. The building energy consumption time-by-time simulation software is used to predict building energy consumption. The second type of research is targeted at building energy consumption data. This kind of research is mainly used in the construction use stage, using linear regression algorithm [4], artificial intelligence algorithm [5], data mining algorithm [6] for analysis, finding the law of energy consumption data, and finding the relationship between it and its influencing factors to predict building energy consumption and guide building energy efficiency. However, there are few studies on the characteristics of the energy consumption data itself. The historical energy consumption data are used to analyze the characteristics of energy consumption in order to predict the building energy consumption. By observing the time series curve of the building energy consumption, it is found that this is a very typical nonlinear curve.
The concept of fractal has been proposed for more than a hundred years. At that time, fractals only exist in geometric figures, and later some scientists proposed some theory of fractals. Fractal properties exist not only in geometry but also in time series. In recent years, fractal theory is widely used in power load forecasting [7], traffic flow forecasting [8], and mineralization forecasting [9]. Literature [10-11] analyzes the fractal characteristics of power load time series. Natural things generally have fractal characteristics. Through the preliminary observation of the geometric characteristics of the building energy consumption curve, it is found that the building energy consumption has fractal characteristics. Therefore, based on the theory of fractal theory, the fractal characteristics of building energy consumption are studied by analyzing the characteristics of building energy consumption, which provides a certain theory for applying fractal theory to building energy consumption prediction and fault diagnosis.

2. Fractal theory

2.1 Fractal characteristics

2.1.1 Self-similarity
Self-similarity means that certain structural or process characteristics are similar from different time scales or spatial scales. There is a self-similarity between the whole and the whole, the parts and the parts, and the parts and the whole. Self-similarity can be divided into three categories: exact self-similarity, which is usually only found in regular fractal graphs produced by mathematical equations; approximate self-similarity, which exists only within a certain scale; statistical self-similarity, the characteristics are not visually obvious, but the fractal dimension remains basically the same as that the curve is enlarged.

2.1.2 Scale-invariance
Scale-invariance means that an area is arbitrarily selected on the fractal pattern, and the shape obtained by enlarging or reducing it is consistent with the characteristics of the original form. The scale-free interval of exact self-similarity is infinite, and their approximation intervals are limited for approximate self-similarity or statistical self-similarity.

2.2 Method for judging fractal characteristics
The method of distinguishing fractal characteristics can be divided into two categories: qualitative discrimination and quantitative discrimination. Qualitative discrimination is to draw the curve of the research object at a certain time scale (spatial scale) to observe the trend and variation of the curve. If the trend of the curve is almost the same on this time scale (spatial scale) for the same subject, it can be considered to have fractal characteristics. The quantitative discrimination is judged against the fractal dimension.

2.3 Fractal dimension
Fractal dimension is the main tool for studying fractals. In the fractal system, there is always a constant amount - fractal dimension. Therefore, quantitative analysis of whether there is a fractal feature is to calculate its fractal dimension. For the same research object, if the fractal dimension is almost the same under a certain time scale (spatial scale), it can be considered that it has fractal characteristics. Common methods for calculating the fractal dimension include the repetitive range analysis method, the correlation dimension method, and the partition function method.

2.4 Rescaled Range (R/S) Analysis
The R/S analysis was first proposed by the Hydrologist Hurst in 1907 when studying the water storage of the Nile Dam. The steps of this analysis are given below.
A sequence $S$ of length $N$ is divided into the consecutive subintervals of length $L$, $L = 1, 2, 3...n$. The number of subintervals is $N$, $N=1, 2, 3...N$. Each subinterval can be represented as $X_N$. For each subinterval of length $L$, the mean values can be expressed as:

$$\bar{X}_N = \frac{1}{n} \sum_{i=1}^{n} X_{N,i}$$  \hspace{1cm} (1)

For a single subinterval, its cumulative mean dispersion is calculated as follows:

$$D_{N,k} = \sum_{i=1}^{k} (X_{N,i} - \bar{X}_N), k = 1, 2, \cdots n$$  \hspace{1cm} (2)

According to the above formula, the range of the individual subintervals is defined as:

$$R_N = \max(D_{N,k}) - \min(D_{N,k}), k = 1, 2, \cdots n$$  \hspace{1cm} (3)

Then the standard deviation of each subinterval should be calculated as below:

$$S_N = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (X_{N,i} - \bar{X}_N)}$$  \hspace{1cm} (4)

According to the above, for the division length $n$, the rescale degree of $N$ sub-intervals can be calculated as below:

$$(R/S)_c = \frac{1}{n} \sum_{i=1}^{N} \frac{R_N}{S_N}$$  \hspace{1cm} (5)

It is known that $\log(R/S)$ and $\log(n)$ have the following linear relationship:

$$\log(R/S) = \log c + H \times \log(n)$$  \hspace{1cm} (6)

Where $H$ is the Hurst exponent. According to the above formula, the value of $H$ is solved by fitting the least squares method. After calculating the $H$ value, the $D_f$ can be calculated based on Equation (7):

$$D_f = 2 - H$$  \hspace{1cm} (7)

3. Building energy consumption fractal characteristics

3.1 Fractal characteristics of campus building energy consumption

Take the energy consumption record of a campus building on December 4, 2017, from 0:00 to 23:59, once every hour, and get the energy consumption time series of the building. The energy consumption time series is analyzed by R/S analysis method. Figure 1 shows the energy consumption curve for the day. It can be seen from figure 1 that the energy consumption curve is non-linear. When the fractal dimension of the time series is obtained, the curve has a distinct linear relationship, which explains the time series is scale invariant.

![Figure 1. The energy consumption curve of the campus on December 4, 2017](image1)

![Figure 2. Find the curve of the ln(N)–ln(R/S) fractal dimension](image2)
Next, analyze the energy consumption time series curve for seven days in a week. As can be seen from figure 3, the energy consumption per day is not the same, especially, the value on Saturday and Sunday is significantly lower than the work days, but it is very similar in geometry. By calculating their fractal dimensions, it can be concluded that they are self-similar, and the results are shown in table 1.

![Figure 3. Energy consumption curve of campus buildings for one week in a row](image1)

![Figure 4. Weekly campus building energy consumption curve for one month in a row](image2)

Table 1. Fractal dimension of campus building energy consumption for one week in a row

|       | Dimensions  |       | Dimensions  |
|-------|-------------|-------|-------------|
| Sun   | 1.0052      | Thu  | 1.0238      |
| Mon   | 1.1035      | Fri  | 1.0583      |
| Tue   | 1.0598      | Sat  | 1.0034      |
| Wed   | 1.1067      |      |             |

Then analyze the energy consumption curve on the weekly time scale. As can be seen from figure 4, the energy consumption per week is slightly different in value. But there is a high degree of similarity in the trend of geometrical figures. By calculating the weekly fractal dimension, it can be concluded that they are self-similar. The calculation results are shown in table 2.

Table 2. Fractional dimension of campus building energy consumption for one month in a row

| Weeks | First | Second | Third | Forth |
|-------|-------|--------|-------|-------|
|       | 1.1630| 1.1532 | 1.1345| 1.2869|

Next, the fractal dimensions of building energy consumption are analyzed at the same spatial scale and different time scales, and calculate the energy fractal dimension of one day, one week and one month. The results are shown in table 3.

Table 3. Fractal Dimensions of Campus Building Energy Consumption at Different Time Scales

| Length of time | Day | Week | Month |
|----------------|-----|------|-------|
| Dimensions     | 1.1035| 1.1630| 1.4536|

It can be seen from the results that for the energy consumption of campus buildings in the same period of time (for example, one week or one month in a row), the fractal dimension of energy consumption remains basically the same. But the fractal dimension increases with time. If the building energy consumption is regarded as a set, the fractal dimension increases with time, which is approximately in line with the nature of the fractal set.

Different dates of campus buildings and different weeks of energy consumption can be considered as different parts of the overall energy consumption in time. So on the time scale, there is self-similarity between the part and the part of campus building energy consumption. This self-similarity is approximate self-similarity.
3.2 Fractal characteristics of commercial building energy consumption

In the same way, the energy consumption record of a commercial building on July 1, 2018 is obtained, and the energy consumption time series of the commercial building is obtained. The energy consumption time series is analyzed by using the R/S analysis method. Figure 5 shows the energy consumption curve for the day. It can be clearly seen that the energy consumption curve is also non-linear. Similarly, by obtaining the fractal dimension, it is proved that the energy consumption time series has scale invariance.

Then the daily energy consumption curve for one week is analyzed. From figure 6, it can be seen that the daily energy consumption is different in value. Especially the energy consumption on Friday is significantly greater than other time, but the shape of the geometry is very similar. The fractal dimensions were also calculated and found to have self-similarity between them. The calculation results are shown in table 4.

| Table 4. Fractal dimension of commercial building energy consumption for one week in a row |
|-----------------------------------------------|
| Sun          | 1.3828 | Thu          | 1.3757 |
| Mon          | 1.3733 | Fri          | 1.3822 |
| Tue          | 1.3826 | Sat          | 1.3798 |
| Wed          | 1.3841 |              |        |

Then analyze the energy consumption curve on a weekly time scale. As can be seen from figure 7, the energy consumption per week is slightly different in value. But geometrically, their geometry is still similar. And their fractal dimension is calculated. It can be found that they have self-similarity. The calculation results are shown in table 5.

| Table 5. Weekly commercial building energy consumption fractal dimension for one month in a row |
|-----------------------------------------------|
| Weeks | First  | Second | Third  | Forth |
|-------|--------|--------|--------|-------|
| Dimensions | 1.4324 | 1.4415 | 1.4484 | 1.4346 |
Like the campus building, calculate the fractal dimension of energy consumption for one day, one week, and one month. The results are shown in table 6.

Table 6. Fractal Dimensions of Commercial Building Energy Consumption at Different Time Scales

| Length of time | Day   | Week  | Month |
|----------------|-------|-------|-------|
| Dimensions     | 1.3828| 1.4324| 1.4936|

It can be seen from the results that, like campus buildings, the fractal dimension of energy consumption remains basically the same for the energy consumption of commercial buildings in the same period of time, and its fractal dimension increases with time. Different dates of commercial buildings and different weeks of energy consumption can be considered as different parts of the overall energy consumption in time, so it can be concluded that there is self-similarity between the part and the part of the building energy consumption. This self-similarity is also approximate self-similarity.

3.3 Fractal characteristics of medical building energy consumption

In the same way, take the energy consumption record of a medical building on April 1, 2016, the energy consumption time series of the medical building is obtained, and then analyze the energy consumption time series by R/S analysis method. It can be seen from figure 8 that the building energy consumption curve is non-linear, and the same fractal dimension is proved to prove that the time series has scale invariance.

Then analyze the daily energy consumption curve for one week. From figure 9, it can be seen that the daily energy consumption is different in value. Especially the energy consumption on Saturday and Sunday is significantly less than other time, but the shape of the geometry is very similar. The same calculation of their fractal dimension proves that there is self-similarity between them, and the calculation results are shown in table 7.

Table 7. Fractal dimension of medical building energy consumption for one week in a week

| Dimensions | Sun 1.0541 | Tue 1.0643 | Wed 1.0527 |
|------------|------------|------------|------------|
|            | Mon 1.0451 |           |            |
|            | Fri 1.0036 |           |            |
|            | Sat 1.0024 |           |            |
|            |            |           |            |

Then analyze the energy consumption curve on a weekly time scale. As can be seen from figure 10, the energy consumption per week is slightly different in value, but the shapes of the geometric figures are similar, and their fractal dimensions are calculated to prove their self-similarity, and the calculation results are shown in table 8.

Table 8. Fractal dimension of medical building energy consumption for one week in a week

| Dimensions | Sun 1.0541 | Thu 1.005 |
|------------|------------|-----------|
|            | Mon 1.0451 | Fri 1.0036|
|            | Tue 1.0643 | Sat 1.0024|
|            |            |           |

Figure 9. Medical building energy consumption curve for one week in a row

Figure 10. Weekly medical building energy consumption curve for one month in a row
Table 8. Weekly medical building energy consumption fractal dimension for one month in a row

| Weeks    | First  | Second | Third  | Forth  |
|----------|--------|--------|--------|--------|
| Dimensions | 1.1865 | 1.193  | 1.1822 | 1.1852 |

Like the previous two buildings, calculate the fractal dimension of energy consumption for one day, one week, and one month. The results are shown in table 9.

Table 9. Fractal Dimensions of Medical Building Energy Consumption at Different Time Scales

| Length of time | Day  | Week | Month |
|----------------|------|------|-------|
| Dimensions     | 1.005| 1.1865| 1.3957|

It can be seen from the results that, with the previous two buildings, the fractal dimension of the building energy consumption remains basically the same for the same period of time, and the numerical value increases with time. Different dates of medical buildings and different weeks of energy consumption can be considered as different parts of the overall energy consumption in time, so there is also self-similarity between the part and the part of the building energy consumption. This self-similarity is also approximate self-similarity.

4. Conclusions
In summary, the building energy consumption time series has fractal characteristics, and it also has self-similarity on the time scale. For different building types, the building energy consumption time series have the same self-similarity. The correct analysis of the characteristics of building energy consumption is the basis for building energy consumption prediction. The fractal theory is used to describe the characteristics of building energy consumption, which can introduce the fractal theory basis for building energy consumption prediction. At the same time, it provides a new method and new ideas for analyzing the building energy consumption in the future, which will be of great significance for building energy consumption prediction and fault diagnosis.

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