A Fractal Prediction Model for Evaluating Normal Contact Stiffness of Micro-pitting Gear

Xiaopeng Wang and Shijun Liu
Zhengzhou Research Institute of Mechanical Engineering, China Academy of Machinery Science and Technology, Zhengzhou, Henan, China, 450052.
Email: wangxiaopeng4109@hotmail.com

Abstract. Estimating normal contact stiffness of micro-pitting is one important subject to study the dynamic fault characteristics of gear system. Combined with the fractal parameters of surface morphology and the distribution area of contact points on the microscopic scale, a fractal model of joint surface with normal contact stiffness was established by investigated the elastic-plastic contact mechanical properties of micro-convex. The estimated fractal parameters for micro-pitting characteristic was established with the macroscopic physical dimension of involute gear and microscopic physical cause of micro-pitting, to investigate normal contact stiffness of involute gear under different machining technology of tooth surface and different levels of micro-pitting. The result shows that the surface microscopic elements have different effects on normal load and normal contact stiffness. The surface contact coefficients are different along with the engagement point of the tooth profile, and improving the accuracy of the tooth surface (reducing the roughness) can improve the normal contact stiffness of meshing tooth surface. The normal contact stiffness of tooth is lower significantly with micro-pitting. The simulation results are consistent with the actual engineering. The proposed mathematical model of micro-pitting can be used as a basis for an analysis of dynamic response and expansion of micro-pitting.

1. Introduction
When cyclical contact stress of the tooth surface exceeds the fatigue limit of materials, micro-cracks are generated at the surface or subsurface of the tooth with the cyclical contact stress, and the pits are formed which the minute metal piece falls off from the tooth surface with developing of crack. This phenomenon is called fatigue pitting [1]. The position of pitting is more common on the lower side of the pitch line, as shown in Figure 1.

![Figure 1. Schematics of fatigue pitting failure of tooth surface](image)

Pitting, as one important feature of gear fatigue failure, not only affects contact strength of the tooth surface but also affects contact stiffness of the contact interface. The formation mechanism of pitting features and the strength properties and dynamic response characteristics after pitting have...
been the subject of continuous research by scholars [2-4]. Micro-pitting is an initial failure feature of the tooth surface in the fatigue pitting. But relative research is still lacking with the different effects of spatial scale. With the research on contact characteristics of contact surface micro-morphology in the micro spatial scale, contact characteristics of contact interface are closely related to an external load, surface micro-morphology and material properties [5-7]. Tian [8, 9] investigated normal contact stiffness of the fixed interface by a mathematical and mechanical model of asperity surface with microscopic geometry. Goerke [10] established a mathematical model and experiment system of the normal fractal contact surface to analyze the influence of different parameters (morphology, hardness, etc.) on the load capacity of the contact surface. The effect of fractal properties of asperity surface on the normal and tangential load capacity of the contact surface and normal contact stiffness were investigated in the references [11, 12]. Yuan [13] proposed an improved M-B model for the asperity contact surface by combination with elastic-plastic theory. Contact strength and contact stiffness of the tooth surface were analyzed qualitatively with fractal theory by Zhao [14] and Chen [15].

Based on the above research theory and engineering application of the fractal model for contact surface, the load capacity and normal contact stiffness of the tooth surface is future investigated combine with the material parameters and load effect of operating surface in a real meshing process of gear pair. A mathematical fractal prediction model of surface morphology of micro-pitting is established to investigate quantitatively the influence of micro-pitting feature on normal contact stiffness combine with the forming physical phenomenon of micro-pitting. The calculation mathematical/mechanical model of contact stiffness for gear system dynamics and response characteristics of tooth surface fatigue pitting are further improved and perfected.

2. Micro-pitting Formation

Under the action of cyclical contact stress, cracks always appear along the direction of frictional force by the sliding of tooth surface in the meshing process of a pair of involute spur gears, as shown in Figure 2. The pitch line is as a boundary, the friction direction and cracks direction are alternately presented on the top and down tooth surface. The contact model of any engagement position on tooth surface can be approximately simulated as the contact of two cylindrical rollers. The contact model of two cylindrical rollers (driving and driven) is shown in Figure 3, the physical symbols of $v_1$ and $v_2$ are the speed of driving and driven roller, and $v_1$ is greater than $v_2$. The driving roller is defined as an overtaking face, and the driven roller is an overlooking face. While the clearance between two rollers is filled with lubricating oil, the oil also flows into the cracks on the surface. The friction direction and crack direction are generated according to the movement direction which is shown in Figure 3a. And the surface makes contact from right to left. In the sub contact model of cracks on the overlooking face (left of Figure3b), the crack notch is first contacted and sealed subsequently with the contact points moves. And the lubricating oil pressure is increased in the crack, stress concentration exists in the crack tip in the meantime so that causing the crack growth. But in the sub contact model of cracks on the overtaking face (right of Figure3b), the crack flank is contacted and the lubricating oil in the crack

![Figure 2. Crack initiation direction of tooth surface](image)

![Figure 3. The formation mechanism of pitting](image)
is squeezed out. And the lubricating oil pressure is decreased in the crack which slows down the crack growth. According to the contact characteristic of tooth surface in the meshing process, the overtaking face is along with tooth profile from pitch line to the tooth root, and the overlooking face is along with tooth profile from pitch line to the tooth tip. Therefore, cracks are expanded for pitting easily below the pitch line, which is consistent with the schematics of pitting failure feature in Figure 1.

3. Fractal Contact Model of Asperity Surface

3.1. Contact Model of Micro-convex

Based on the fractal function of W-M and dimension independence of fractal parameters, the elastic-plastic contact is equivalently simplified to the contact between asperity surface and rigid ideal surface in the fractal contact M-B model [16] of asperity surface, as shown in Figure 4. In the schematic diagram, \( z \) is the height of a given asperity measured from the mean of original asperity heights. \( d \) is the separation between the rigid flat surface and mean of original asperity heights. \( \delta \) is the total deformation of asperity surface. Therefore, the deformation of the given asperity surface under the load is written as \( \delta = z - d \). And from the Weierstrass-Mandelbrot function, the roughness is a cosine wave for which the micro-convex before deformation is given as equation (1), the schematic diagram is given as Figure 5.

\[
z(x) = G^{D-1} l^{2-D} \cos \left( \frac{\pi x}{l} \right), \quad -l/2 < x < l/2
\]

(1)

Which \( D \) is the fractal dimension, \( G \) is the fractal roughness parameter of surface and \( l \) denotes the length scale. Then combine with the cosine function of micro-convex and calculation method of any curvature radius, the curvature radius of micro-convex and max deformation of the peak can be derived as equation (2) and equation (3).

\[
R = \lim_{l \to 0} \left( 1 + \frac{dz(x)}{dx} \right) \frac{d^2z(x)}{dx^2} = \frac{l^D}{\pi^2 G^{D-1}} = \frac{a_{0}^{D/2}}{\pi^2 G^{D-1}}
\]

(2)

\[
\delta_{\text{max}} = G^{D-1} a_{0}^{(2-D)/2}
\]

(3)

Substitute curvature radius \( R \) into equation (4), the critical deformation \( \delta_c \) which exchanges from elastic to plastic is then derived. The critical contact area \( a_c \) of elastic-plastic transformation is derived by equation (3) and equation (4), which is shown in equation (5). The mathematical relationship of elastic contact area, curvature radius, and elastic deformation can be obtained as equation (6) by the contact geometric deformation of micro-convex.

\[
\delta_c = \left[ \pi K \sigma_y / (2E) \right] R = \left( K \phi / 2 \right)^2 \cdot a_{0}^{D/2} / G^{D-1}
\]

(4)

\[
a_{c} = \left( K \phi / 2 \right)^{2(1-D)} \cdot G^{2}
\]

(5)

\[
a = \pi R \delta
\]

(6)
Where, $a$ is the elastic contact area, $H$ is the surface hardness, $\sigma_y$ is the material yield strength, and $E$ is the synthetic Yang’s modulus. $K$ is a ratio of surface hardness to the material yield strength, which is expressed as $K=H/\sigma_y$, and the material parameter $\phi$ is expressed as $\phi=\sigma_y/E$.

$$
\begin{align*}
\begin{cases}
P_e &= \frac{4}{3}ER^{1/2}\delta^{3/2} \\
P_p &= Ha
\end{cases}
\Rightarrow
\begin{cases}
P_e(a) &= \frac{4}{3}\sqrt{\pi}EG^{D-1}a^{(3-D)/2} & \text{Elastic} \\
P_p(a) &= K\sigma_ya & \text{Plastic}
\end{cases}
\end{align*}
$$

(7)

From the Hertz elastic contact theory, the normal elastic load of a micro-convex is expressed in equation (7), and when the plastic deformation of micro-convex is produced, the plastic load related to hardness and contact area can be expressed in equation (7).

### 3.2. Model of Normal Load and Normal Contact Stiffness

To defined $a_l$ as the max contact area of a micro-convex, any contact point of the tooth surface is simplified as two cylindrical contacts. The contact coefficient of cylindrical surface $\lambda_c$ is introduced since the number of micro-convex is different between two cylindrical contacts and one cylinder and one flat contact. So that the area distribution function of tooth surface contact points $n(a)$ is given as equation (8). Which $B$ is the tooth width, $R_1$ is the curvature radius of contact point for pinion and $R_2$ is the curvature radius for gear.

$$
n(a) = \left|\frac{dN(A>a)}{da}\right| = \frac{1}{2}\lambda_cD\frac{a^{D-2}}{a^{2-D}} \quad \text{and} \quad \lambda_c = \left[\frac{4B}{\pi E}(\frac{R_1}{R_1+R_2})^{3/2}/(\pi R_1+\pi R_2)\right]^{1/2}
$$

(8)

When the $a_l$ is greater than the $a_c$, the contact state of asperity surface includes two contact formation, one is elastic contact, and the other is plastic. The total normal load can be derived as equation (9) by combine with elastic load and plastic load.

$$
P = \int_{a_c}^{a_l} p_e(a)n(a)da + \int_{a_c}^{a_l} p_p(a)n(a)da = \frac{4}{3}\sqrt{\pi}EG^{D-1}\frac{D}{3-2D}a_l^{D/2}\left(a_l^{(3-D)/2}-a_c^{(3-D)/2}\right) + \frac{D}{2-D}\lambda_cK\sigma_ya_l^{D/2}a_l^{(3-D)/2}
$$

(9)

When the $a_l$ is smaller than the $a_c$, the asperity surface exhibits a plastic contact state, and its total normal load only contains a plastic load, which is derived as equation (10).

$$
P = \int_{a_c}^{a_l} p_p(a)n(a)da = D\lambda_cK\sigma_ya_l/(2-D)
$$

(10)

According to the relation between elastic load and deformation in equation (7), the normal contact stiffness is obtained by differential calculation of the contact deformation amount by the elastic load, as shown in equation (11).

$$
k_n = dp_e/d\delta = 2ER^{1/2}\delta^{1/2}
$$

(11)

Substitute equation (6) into equation (11), the total normal contact stiffness related to the contact area is derived as equation (12) by combine with the area distribution function of contact points $n(a)$ (equation (8)).

$$
K_n = \sum_{a_c}^{a_l} k_n = \int_{a_c}^{a_l} k_n n(a)da = \frac{2E\lambda_c}{\sqrt{\pi}} D a_l^{D/2}(a_l^{(3-D)/2}-a_c^{(3-D)/2})
$$

(12)

### 3.3. Nondimensionalization of the Proposed Model

While the $a_l$ is greater than the $a_c$, the tooth contact is in the form of incomplete plasticity. In order to further investigate the relationship between total load and contact area and reduce the input of parameters in numerical simulation, a fractal contact model considered the asperity surface in the dimensionless form is given as equation (13), and the dimensionless parameters and simplified expressions in equation (13) satisfy the following formulas in equation (14).
established by the above relationship, which is given as equation (16). The dimensionless normal load, the dimensionless real contact area, the dimensionless critical contact area, the dimensionless nominal contact area, and the dimensionless critical contact area are the expressions of fractal dimension and contact coefficient of cylindrical surface. The normal contact stiffness $K_n$ (equation (12)) considered the asperity surface in a dimensionless form is given as equation (15). $g_1(D)$ is the expression of fractal dimension and contact coefficient of cylindrical surface.

$$K_n^* = \frac{2\lambda_s}{\sqrt{\pi}} g_1(D) A_c^{*0.2} \left[ \frac{2-D}{DD_\lambda_c} A_c^{*(1-D)/2} - a_c^{*(1-D)/2} \right]$$ and $g_1(D) = \frac{D}{2-D} \left( \frac{2-D}{DD_\lambda_c} \right)^{D/2}$

From equation (13) and equation (15), the dimensionless normal load $P^*$ and dimensionless normal contact stiffness $K_n^*$ have a certain mathematical expression correlation with the dimensionless real contact area $A_c^*$ and the dimensionless critical contact area $a_c^*$ respectively. The mathematical relationship of the dimensionless normal load $P^*$ and dimensionless normal contact stiffness $K_n^*$ is established by the above relationship, which is given as equation (16).

$$P^* \sim (A_c^*, a_c^*), K_n^* \sim (A_c^*, a_c^*) \Rightarrow (K_n^* \sim P^*)$$

4. Simulation and Discussion of the Proposed Model

In order to visualize the simulation data display, the drawing method with a single logarithmic coordinate is used to draw the simulation results. Figure 6 ($\lambda_s=0.7$, $G^*=10^{10}$) shows that the normal load increases with the increase of real contact area, and decreases first and increases subsequently with the increase of fractal dimension. It means that the normal load has a minimum extreme value about the fractal dimension $D$. Therefore, fractal dimension $D$ from 1.50 to 1.65 is selected for the detailed simulation analysis of normal load, the result is shown in Figure 7. The three-dimensional topological diagram of normal load present concave with the variable of fractal dimension and real contact area in Figure 7a. And the curved surface concave is more obvious with the increase of the real contact area. It can be seen in the contour diagram (Figure 7b) that the minimum extreme value of normal load approximately appears when the fractal dimension is equal to 1.57. Figure 8 shows that the normal contact stiffness increases with the increase of fractal dimension, and increases with the increase of real contact area.
The influence of roughness parameter on normal load and normal contact stiffness are analyzed with the fractal dimension and contact coefficient of cylindrical surface ($D=1.6$, $\lambda_c=0.7$). In Figure 9, the normal load increases with the increase of roughness parameter, but the normal contact stiffness decreases with the increase of roughness parameter. It can be obtained from equation (5) that critical contact area which exchanges from elastic to plastic decreases with the decrease of roughness parameter under the same conditions of hardness coefficient, material parameter, and nominal contact area, then the surface smoothness and normal contact stiffness are improved. Considering the result of Figure 6, Figure 8, and Figure 9, there is a certain coordination relation between the fractal dimension and the roughness parameter, which makes the optimal effect of normal load and normal contact stiffness.

Figure 9. Effect of roughness parameter $G^*$ on normal load $P^*$ and normal contact stiffness $K_n^*$

Figure 10. Effect of cylindrical surface contact coefficient $\lambda_c$ on normal load $P^*$ and normal contact stiffness $K_n^*$

Figure 10 shows that the normal load increases with the increase of contact coefficient of cylindrical surface, and the increased curve presents nonlinearly, the load difference decrease gradually in the same real contact area. The effect of real contact area and contact coefficient of cylindrical surface on normal contact stiffness has the same variational tendency as the effect on normal load.

Figure 11. Effect of surface hardness on normal load $P^*$ and normal contact stiffness $K_n^*$
Figure 12. Effect of material parameter on normal load $P^*$ and normal contact stiffness $K^*_n$.

The effect of surface hardness and material parameter on normal load and normal contact stiffness is shown in Figure 11 and Figure 12, the normal load and normal contact stiffness both increase with the increase of the variable $H$ and $\phi$. And the effect of two variable on the above on the critical contact area are further analyzed. Figure 13 shows that the critical contact area decreases nonlinearly with the increase of surface hardness and material parameter, but increases with the increase of roughness parameter. It means that under the same cold machining technology, the critical contact surface which exchanges from elastic to plastic is improved by the application of different material and heat treatment technology, so as to improve its load capacity and deformation resistance.

Figure 13. Effect of material parameter and surface hardness $H$ on critical contact surface $a_c^*$.

Figure 14. Scanning electron microscope diagrams of a micro-pitting area [17]

Through the discussion of the influence of various parameters on the above, the parameters include fractal dimension, roughness parameter, contact coefficient of cylindrical surface, etc. do not affect the normal load and normal contact stiffness alone, but act together and to determine the surface micro-topography feature. Therefore, the influence of asperity surface factors on normal load and normal contact stiffness should be considered comprehensively for the engineering application of fractal theory.

5. Normal Contact Stiffness Model of Micro-pitting of Tooth Surface

The formation mechanism of micro-pitting and the relevant research in reference [18-20] shows that micro-pitting is produced firstly on the tooth surface of pinion, and the micro-pitting area mainly occurs from the pitch line to the dedendum (Figure 1 and Figure 14). The feature size of pitting ranges from 0.01mm (micro-pitting) to 0.8mm (extended pitting), and the failure feature of spalling occurs on the tooth surface in which the pitting expands under the load further. The cracks include surface crack and sub-surface crack are produced on the tooth surface as shown in Figure 14a. Under the joint action
of engagement force and lubricating oil press, the maximum feature size of a micro-pitting caused by surface crack is about 10 \( \mu \text{m} \), while the feature size of micro-pitting caused by sub-surface crack ranges from 20 \( \mu \text{m} \) to 100 \( \mu \text{m} \). Because the dimension feature of micro-pitting is very small, the mathematical analysis model of micro-pitting is established according to the simulation method of the roughness of asperity surface \[21, 22\], and to simulate the different size of micro-pitting by the change of tooth surface roughness \[23\].

According to the measured data and theoretical calculation of the asperity surface with different machining technology in the references \[24, 25\], the approximate relationship between roughness, fractal dimension and roughness parameter is obtained which presents power-exponential relation and is given as equation (17).

\[
Ra^{0.042} = 1.528 / D = -5.26 / \log G
\]  

(17)

![Figure 15. Meshing process of a spur gear pair](image)

**Table 1. Calculation parameters of involute spur gear pair**

| Physical parameters               | Value               |
|----------------------------------|---------------------|
| Number of teeth                  | 25, 33              |
| Module /\text{mm}                | 4                   |
| Face width /\text{mm}            | 28                  |
| Pressure angle /\text{deg}       | 20                  |
| Addendum coefficient             | 1                   |
| Tip clearance coefficient        | 0.25                |
| Yang's modulus /\text{N/mm}²     | 2.06×10⁴            |
| Poisson's ratio                  | 0.3                 |
| Yield strength of material /\text{N/mm}² | 835          |
| Power /\text{kW}                 | 30                  |
| Speed of revolution /\text{r/min} | 1000               |

Figure 15a shows that a meshing process of a spur gear pair in which the contact ratio ranges from 1 to 2, and the tooth surface along the meshing line is divided for single meshing region and double meshing region in Figure 15b. The segment \( CP \) in the single meshing region, which the main region of micro-pitting occurs on the tooth surface, is selected to investigate the effect of micro-pitting on normal contact stiffness. The main calculation parameters of spur gear pair are shown in Table 1.

With the drawn method of Bobillier and the basic parameter of tooth profile of involute gear, the relationship between curvature radius and engagement angle at any engagement point in segment \( CP \) of the single meshing region is derived as equation (18). And the contact half-width related to the nominal contact area and engagement angle is derived as equation (19) according to the Hertz elastic contact theory of two-cylinder.

\[
\rho_{mn} = mz_p \cos \alpha \tan \alpha_{mn} / 2 \quad \text{and} \quad \rho_{qm} = mz_q \cos \alpha \tan \alpha_{qm} / 2
\]  

(18)
The derived parameter relation with engagement angle is obtained by bringing the basic parameter of involute spur gear pair into the equation (18), equation (19) and equation (8). Figure 16 shows that the comprehensive curvature radius and surface contact coefficient increase slightly from engagement point C to pitch point P. The engagement force and nominal contact area are also increased in Figure 17.

The seven different roughness are selected to calculate the normal contact stiffness of tooth surface, which simulates the existed machining technology of gear (hobbling, grinding, finishing, etc.). The four first kinds are to simulate engineering application of roughness of tooth surface accuracy. And the latter three kinds are to simulate the different levels of micro-pitting of tooth surface topography. The fractal parameter of the proposed model is obtained by substitute the simulated value to equation (17). The result in Table 2 shows that the fractal dimension decreases with the increase of roughness, which means the fractal is rougher, but the roughness parameter increases with a roughness, which means the surface smoothness decrease. The variational tendency is consistent with the real surface topography of asperity surface.

| Roughness                  | Fractal dimension D | Roughness parameter G |
|----------------------------|---------------------|-----------------------|
| Different machining technology |                     |                       |
| 0.1                        | 1.68                | \(10^{5.79}\)         |
| 0.8                        | 1.54                | \(10^{5.31}\)         |
| 1.6                        | 1.50                | \(10^{5.16}\)         |
| 3.2                        | 1.46                | \(10^{5.01}\)         |
| 12.5                       | 1.37                | \(10^{4.73}\)         |
| Simulated micro-pitting features |                 |                       |
| 20                         | 1.35                | \(10^{4.64}\)         |
| 50                         | 1.30                | \(10^{4.46}\)         |

The design and calculation parameters are brought into the proposed micro-pitting model of normal contact stiffness. Figure 18 shows that normal contact stiffness increases slightly along the segment CP with the different machining technology, which is different from the linear model of Yang-Sun [26]. The decrease of roughness, which means the accuracy of tooth surface is higher, gives rise to an increase of the normal contact stiffness. The level of normal contact stiffness at the Ra=0.1 is obviously higher than others, which has the same effect as improving the surface accuracy to enhance the contact fatigue strength of gear [27]. Comparison with the Yang-Sun model and semi-empirical formula which is proposed in the NASA technical report [28], the level of normal contact stiffness is \(10^8\) N/m under the ordinary machining technology between the proposed model and the two models. The curve of Yang-Sun model is between the resulting curve of Ra=0.8 and Ra=1.6. Since the
Yang-Sun model is independent of load and deformation, it means that the contact stiffness is a constant when the geometric parameters are determined, which is contrary to the actual condition. The semi-empirical formula is between the resulting curve of $Ra=1.6$ and $Ra=3.2$. The different result in the Figure 19 is produced as the morphology of asperity surface is not taken into account in the two models. The normal contact stiffness has a different response with the different machining technology in the results. When high quality, high parameter and high performance of gear are needed for engineering, in addition to considering the macroscopic geometric size, the contact feature of gear needs to be analyzed qualitatively and quantitatively under different machining technology. And the Yang-Sun model and semi-empirical formula can be used as the initial stiffness prediction of gears with low-quality requirements or the initial stiffness prediction of new product design.

When the micro-pitting is produced on the tooth surface, the result of normal contact stiffness in Figure 20 decreases obviously. Comparison with the contact stiffness of the non-pitting surface, the two results differ 1 or 2 order of magnitude. The load capacity of tooth is also reduced. The simulated result is consistent with the engineering application, and the proposed mathematical model of micro-pitting can be used as a basis for an analysis of dynamic response and expansion of micro-pitting.

6. Results

Based on the fractal theory of asperity surface, the mathematical fractal prediction model of surface morphology of micro-pitting is established to investigate quantitatively the influence of micro-pitting feature on normal contact stiffness combine with the forming physical phenomenon of micro-pitting. The results are shown in the below:

1) The normal load decreases first and increase subsequently with the increase of fractal dimension. It means that the normal load has a minimum extreme value about the fractal dimension. And the normal load also increases with the increase of roughness parameter, surface contact coefficient, surface hardness, and material parameter. the normal contact stiffness increases with the increase of the fractal parameters except the roughness parameter.

2) The normal contact stiffness of tooth surface is improved by improving the surface accuracy, which has the same effect as improving the surface accuracy to enhance the contact fatigue strength of gear. Comparison with the Yang-Sun model and semi-empirical formula, the proposed fractal model of normal contact stiffness can roundly predict the normal contact stiffness under different machining technology.

3) When the micro-pitting is produced on the tooth surface, the result of normal contact stiffness decreases obviously. The load capacity of tooth is also reduced. The simulated result is consistent with the engineering application, and the proposed mathematical model of micro-pitting can be used as a basis for an analysis of dynamic response and expansion of micro-pitting.

7. Acknowledgement

The authors gratefully acknowledge the support by the National Key Research and Development Plan of China through Grants No. 2018YFB2001700.
8. References

[1] Alban L E. *Systematic analysis of gear failures* (Ohio: American Society for Metals).
[2] Luo Y, Baddour N and Liang M *Mechanical Systems and Signal Processing* **119** 155-81.
[3] Liang XH, Zhang HS, Liu LB and Zuo MJ *Mechanism and Machine Theory* **106** 1-15.
[4] Sanchez MB, Pleguezuelos M and Pedrero JI *Mechanism and Machine Theory* **109** 231-49.
[5] Ge SR, Chen G. *Wear* **231** 249-55.
[6] Huang K, Zhao H, Chen Q *Tribology* **28** 529-33.
[7] Wang R, Zhu L and Zhu C *International Journal of Mechanical Sciences* **134** 357-69.
[8] Tian HL, Zhu DL and Qin HL *Journal of China Three Gorges University* **24** 83-88.
[9] Tian HL, Zhong XY, Qin HL, Zhao CH, Fang ZF, Zhu DL, Chen BJ and Zhang FJ. *Journal of Mechanical Engineering* **49** 108-22.
[10] Goerke D, Willner K. *Wear* **264** 589-98.
[11] Kucharski S, Starzyński G. *Wear* **440-441** 1-14.
[12] Pan W, Li X, Wang LL, Guo N and Mu JX. *European Journal of Mechanics - A/Solids* **66** 94-102.
[13] Yuan Y, Cheng Y, Liu K and Gan L. *Applied Surface Science* **425** 1138-57.
[14] Zhao LL, Huang XP and Zhou ZW. *Journal of Chongqing University of Technology* **08** 78-83.
[15] Chen Q, Zhao H, Huang K and Xu S. *China Mechanical Engineering* **21** 1014-17.
[16] Majumdar A, Bhushan B. *ASME Journal of Tribology* **113** 1-11.
[17] Weibring M, Gondecki L and Tenberge P. *Tribology International* **131** 299-307.
[18] Li S, Kahraman A. *International Journal of Fatigue* **59** 224-33.
[19] Morales-Espejel G, Rycerz P and Kadiric A, *Wear* **38** 99-115.
[20] Shi Pley EE, Machine Design **39** 152–162.
[21] Chen H, Hu YZ, Wang H and Wang WZ. *Lubrication Engineering* **10** 52-55.
[22] Jiang YJ, Huang WQ, Sun ZY and Sun QC. *Machinery Design and Manufacture* **08** 8-10.
[23] Miao X, Huang X. *Wear* **309** 146-51.
[24] Chen Q. *Hefei University of Technology*. 2010.
[25] Ge SR, Zhu H *Fractal tribology* (China Machine Press).
[26] Yang DCH, Sun Z. *Journal of Mechanisms, Transmissions & Automation in Design* **107** 529-35.
[27] Zhao GH, Liu SJ, Li JQ and Wang XP. *Journal of Mechanical Strength* **395** 40-44.
[28] Lin HH, Townsend DP and Oswald, FB. *Profile Modification to Minimize Spur Gear Dynamic Loading* (NASA Technical Memorandum 89901) pp 1-22.