1 Introduction

The Higgs boson is one of the key components of the Standard Model (SM). And it is, as yet, undiscovered. Over the years, several experiments have looked for the Higgs and ruled out its existence over certain mass ranges. The first important step in this direction was taken by the LEP experiments which ruled out a Higgs mass less than 114.4 GeV [1]. The latest in this line are the CMS and ATLAS experiments at the LHC. The ATLAS collaboration has ruled out [2] the mass ranges 112.9–115.5 GeV, 131–238 GeV and 251–466 GeV for the Standard Model Higgs while the CMS collaboration has ruled out [3] the entire range of 127–600 GeV. The Tevatron too has ruled out a subset of this range [4]. However, that is not all. Lately, both the CMS and the ATLAS [2, 3] have observed the first traces of what may be a signal for the SM Higgs. This excess is seen the region 124–126 GeV and, although the statistics is insufficient for a discovery to be claimed immediately, particle physicists across the globe are enthused [5, 6, 7] by the fact that the excess is seen in multiple channels and in the region favoured by electroweak precision tests [8].

If and when the Higgs is discovered, questions regarding its origin and properties will have to be addressed. Is the Higgs only a single, neutral CP-even scalar or does it have charged/CP-odd
partners? Is the Higgs really a fundamental particle or is it a composite with a dynamic origin? The answers to some of these questions may involve physics beyond the Standard Model and will be revealed by studying the properties of the particle that is discovered. Two issues need to be borne in mind. Given that the SM, necessarily, can only be an effective theory, what prevents the SM Higgs from acquiring a large radiative correction to its mass? In other words, the stabilization of the Higgs mass (or, equivalently, the solution of the hierarchy problem) requires some mechanism beyond the SM. In a related vein, a SM Higgs mass of $\sim 125$ GeV runs afoul of the constraints from vacuum stability which, in turn, demands that there be some new physics at not too large a mass scale.

Further, with the Higgs being responsible for the generation of masses in the Standard Model and the top quark being the most massive particle in the model, it is very likely that the top and the Higgs sectors are closely intertwined. Probing one sector could reveal any new physics in the other. This is explicitly borne out by a large class of models that go beyond the SM in trying to explain electroweak symmetry breaking [9]. Of particular interest in this context are mechanisms for dynamic breaking of electroweak symmetry through the formation of $t\bar{t}$ condensates [10]. Apart from these, many other models could lead to large anomalous couplings of the top. A partial list of examples wherein heavy fermions may play a role would include Little Higgs models [11] or models with extra space-time dimensions [12, 13, 14]. Similarly, if the SM is augmented by colour-triplet or colour-sextet scalars that have Yukawa couplings with the top-quark [15], integrating out the former could also result in such eventualities.

The exact effects of such an extended sector on low-energy observables would, understandably, depend on the details of the model. However, on very general grounds, the act of integrating out the heavy fields would result in higher-dimensional operators in the low-energy effective theory [16]. The form and magnitude of such operators would depend on the nature and the sizes of the couplings that the SM fields under consideration have with those that have been integrated out. As we have argued above, the larger coupling of the top with the electroweak symmetry breaking sector is expected to play a defining role in such situations. Further, from a phenomenological point of view, the high threshold for top production has meant that its couplings are still not well measured and can yet accommodate significant deviations from the SM.

In this paper, we consider Higgs production and decay as probes of anomalous couplings of the top quark. At the LHC, the primary mode for Higgs production is through gluon fusion. As the latter is dominated by the top loop, only those anomalous couplings of the top that lead to any deviations in the $ttH$ and/or the $ttg$ vertices are expected to modify the production rate. A modification of the $ttH$ vertex occurs even at the tree-level in theories with extra Higgs fields and has already been explored extensively, for example, in the context of supersymmetric theories. Here, we concentrate, instead, on the top-gluon vertex, arguing that the limits already obtained in the literature [17, 18, 19, 20, 21, 22, 23] still allow for substantial deviation of the Higgs production cross-section from its value within the SM.

Similarly, the Higgs decay width into a pair of photons can be modified by invoking an anomalous $HWW$ vertex on the one hand, and anomalous $WW\gamma$ or $tt\gamma$ vertices on the other. Electroweak symmetry, though, relates the first of these to the $HZZ$ vertex, and the absence of any deviation [2]
in either the already measured $ZZ^* \rightarrow 4\ell$ channel or in the $WW^*$ channel strongly constrains any large modification of this vertex. Enhancement in this vertex is also constrained by the non-observation of a Higgs signal at the Tevatron \[\text{4]\]. Further, there are strong constraints from the LEP experiments on the modification of the $WW\gamma$ vertex. We shall, thus, limit ourselves to a discussion of the $tt\gamma$ vertex in the context of the Higgs decay.

## 2 Analytic and Numerical Results

### 2.1 Anomalous $ttg$ couplings

The simplest gauge-invariant modification to the top-gluon vertex can be wrought by augmenting the Standard Model Lagrangian by an effective operator of the form

$$\mathcal{L} \ni g_s A (\bar{Q}_L \sigma_{\mu\nu} T^a t_R) F^{\mu\nu} \bar{\phi} + \text{h.c.}$$

where $Q_L$ (containing $t_L$) and the scalar field $\phi$ are the usual SM doublets and $\bar{\phi} = -i\sigma_2 \phi^*$. The constant $A \sim \mathcal{O}(\Lambda^{-2})$ where $\Lambda$ represents the cut-off scale for the effective Lagrangian. Indeed, the above is the only new Lorentz structure available as long as one restricts to dim-6 operators. Any other modification can only be in terms of a momentum (form-factor) dependence of either $A$ or even the canonical vector current. As this entails further suppression in $\Lambda^{-1}$, we shall neglect such behaviour.

After the breaking of electroweak symmetry, eq.(1) gives

$$\mathcal{L} \ni g_s A (\bar{t}_L \sigma_{\mu\nu} T^a t_R) F^{\mu\nu} \frac{(H + v)}{\sqrt{2}} + \text{h.c.}$$

With the inclusion of this term, the $ttg$ gluon vertex gets modified. In addition, interaction terms involving $ttgg$, $ttgH$ and $ttggH$ are generated. The corresponding Feynman rules are given in Fig.1.

\[\text{Figure 1: New Feynman rules. } p^a \text{ is the momentum of the incoming gluon. } B = Re(\mathcal{A}) + i\text{Im}(\mathcal{A}) \gamma_5.\]
The terms proportional to $v$ correspond to anomalous chromomagnetic and chromoelectric dipole moments for the top and are often parameterized as

$$\frac{g_s}{\Lambda} i \sigma_{\mu \nu} T^a (\rho + i \rho' \gamma_5) t F_{a \mu \nu}, \quad (3)$$

where, $\rho, \rho' = +1, -1, 0$ and $\Lambda$ represents the energy scale of the new physics that may lead to such operators. Equations (2) and (3) are, thus, related by

$$\frac{\rho}{\Lambda} = \frac{v}{\sqrt{2}} \text{Re}(A), \quad \frac{\rho'}{\Lambda} = \frac{v}{\sqrt{2}} \text{Im}(A). \quad (4)$$

Clearly, the terms of eq. (3) would modify top production rates from processes such as $pp, p\bar{p} \to t\bar{t}$ and $e^+e^- \to t\bar{t}g$, and these have been used in the past to impose constraints on the couplings [19, 22].

A few comments are necessary at this stage. It might seem, at first sight, that we could have started with eq. (3) rather than invoking eq. (1) for the former is invariant under $SU(3)_C \times U(1)_{em}$. However, the lack of invariance under the full electroweak symmetry has consequences that we discuss below. Also note that, although $\Lambda$ appears to be an arbitrary parameter, for the effective field theory formalism to be applicable, $\Lambda$ must be greater than any other mass scale in the theory. For example, Ref. [24] suggests $\Lambda > 2m_t$. Following this, we may parameterize $\Lambda \equiv \zeta m_t$ where $\zeta > 2$. In other words, sensitivity to $\Lambda$ may be translated to sensitivity to the dimensionless parameter $\zeta$. We shall return to this discussion once again in Sec. 3.

With the addition of the operator of eq. (2) to the Lagrangian, the lowest order (LO) amplitudes that contribute to $gg \to H$ are those shown in Fig. 2. Note that two of the diagrams, viz. Fig. 2(c, d), would not arise if we had started with the operator of eq. (3) instead.

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1In the Standard Model, $\rho/\Lambda \sim O(\alpha_s/\pi m_t)$ at the one-loop level. The coefficient of the $CP$-violating term, $\rho'/\Lambda$, is non-zero only at the three-loop level.
Two additional diagrams (see Fig.3) also arise, but each of these can be seen to vanish identically being proportional to the trace of a single Gell-Mann matrix. In other words, whereas the vertex structure stipulates that the two gluons need to be in a colour-antisymmetric state, for the process under consideration, they must be in a singlet state. Indeed, both these vertices can contribute only to the NLO processes, e.g., $gg \rightarrow H + g$.

\begin{align*}
\text{Figure 3: Additional LO contributions to } gg \rightarrow H \text{ due to eq.}(1). \text{ These, however, are identically zero.}
\end{align*}

The Standard Model amplitude for $gg \rightarrow H$ is finite. Indeed, this has to be so as such a term does not exist at the tree-level and hence no counterterms can be written. The introduction of the anomalous term changes matters considerably. Owing to its non-renormalizable nature, it could beget divergent quantum corrections to terms that were absent at the tree-level. The $ggH$ vertex is one example of a term that could receive a divergent correction, but it is not the only one. Additional divergences could be generated at every higher order of perturbation theory. At the one-loop level though, the situation is under control. Consider, for example, Fig.2(a, b). With the higher dimensional nature of the anomalous coupling manifesting itself only in terms of the external momenta, the naive divergence of the loop remains unaltered from its SM counterpart. The situation would change for the worse if the anomalous coupling appeared at an internal vertex. This, however, can happen only at higher loops. Presently, we ignore this aspect as the theory under consideration is an effective one and such questions are meaningful only in the context of an ultraviolet completion.

Formally, the amplitudes corresponding to Fig.2(a, b) appear to be linearly divergent, and, hence, a naive use of Feynman parameterization and subsequent shifting of variables is fraught with danger, as it might introduce non-trivial boundary terms. However, once these two amplitudes are added, the linearly divergent piece cancels exactly, leaving behind only a logarithmic divergence. If we were to start with the Lagrangian of eq. (3), the resultant amplitude would, typically, be proportional to $(\rho/\Lambda) \ln(\Lambda/M)$, where $M$ denotes some combination of the mass scales present in the theory, viz $m_t, m_H$. Such a structure is only to be expected in a non-renormalizable effective theory with a finite cut-off scale. Note that the term formally vanishes as the cutoff $\Lambda \rightarrow \infty$. For the diagrams of Fig.2(c, d), once again, the apparent quadratic divergence gets cancelled leaving behind only a logarithmic divergence. Once the amplitudes from the two sets of diagrams (i.e. Fig.2(a, b) and Fig.2(c, d)) are added, the divergences cancel exactly, leaving behind only a finite residue. A particular consequence of the inclusion of Fig.2(c, d) is that, for a given $A$, the amplitude now is

\footnote{Note, though, that the calculation of the finite residue has to be done with a gauge-invariant prescription. In other words, the individual logarithmic divergences need to be regularized in a gauge invariant manner, such as dimensional regularization.}

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smaller (on account of the large logarithm vanishing) and, hence, the limits obtained on $\mathcal{A}$ are more conservative.

Figure 4: Limits on $\Lambda$ (with $\rho = \pm 1$) obtained by imposing the restriction that the $\sigma(gg \rightarrow H)$ remain within $m\%$ of its SM value. Results are plotted for $m = 5, 10, 20, 30$. In each case, the region below the curve is ruled out.

Using this amplitude (the full expressions are given in the Appendix), one may then obtain the Higgs production cross-section as a function of $\mathcal{A}$. It is interesting to note that, as long as $\text{Im}(\mathcal{A}) \neq 0$, a non-zero $CP$-violating coupling of the gluons to the Higgs is generated as well. This, of course, is not unexpected given the structure of the couplings. However, in keeping with the spirit of effective field theories, we restrict ourselves to $\mathcal{O}(\mathcal{A})$ in our computation of the cross-section. Taking into account the $\mathcal{O}(\mathcal{A}^2)$ terms would have necessitated the inclusion of $\mathcal{O}(\mathcal{A}^2)$ terms in the Lagrangian (eq.(1)) itself. Due to this, $\text{Im}(\mathcal{A})$ does not contribute and consequently, constraints can be placed only on $\text{Re}(\mathcal{A})$, or equivalently, on $\rho/\Lambda$. The allowed range for $\Lambda/\rho$ for different $m_H$ is shown in Fig.4. Allowing for a 30% deviation in the cross-section, one finds that the region below the yellow (dot-dashed) line is ruled out. If one reduces the allowed deviation, a larger range of $\Lambda$ gets ruled out. This is the region below the blue (dotted) line for a deviation of 20%, the green (dashed) line for 10% and the red (solid) line for 5%. As expected, greater accuracy in the measurement of the Higgs production cross-section leads to more stringent limits on $\Lambda$. We display the results for Higgs masses in the range 115–130 GeV. In this range, the limits have only a very mild dependence on $m_H$. 
2.2 Anomalous $tt\gamma$ couplings

For a low mass Higgs that is favoured by current data, the most promising discovery mode is $H \to \gamma\gamma$, and we now turn to this channel. It could be argued that the signal strength for $pp \to H \to \gamma\gamma$ may receive anomalous contributions at both the production and the decay vertices and that unravelling the two is impossible. Fortunately, though, excesses have been reported in $H \to ZZ^* \to 4\ell$ channel as well and even the $H \to WW^*$ channel is being pursued assiduously. Thus, observables such as $\Gamma(H \to \gamma\gamma)/\Gamma(H \to ZZ^*)$ are likely to be well-measured. With the $HZZ$ and $HWW$ vertices being driven by tree level couplings, the partial width $\Gamma(H \to \gamma\gamma)$ would be measurable and, thus, would constitute a very good probe for the effective $H\gamma\gamma$ vertex.

Analogous to eq. (2), an effective operator for the $tt\gamma$ vertex can be written as

$$L \ni eQ_t A' (\bar{t}_L \sigma_{\mu\nu} t_R) F_{\mu\nu} (H + v) \sqrt{2} + h.c.,$$

where $Q_t$ is the charge of the top quark in units of $e$. Once again, this may be written in terms of anomalous magnetic and electric dipole moments of the top quark.

$$\frac{\eta}{\Lambda'} = \frac{v}{\sqrt{2}} \text{Re}(A'), \quad \frac{\eta'}{\Lambda'} = \frac{v}{\sqrt{2}} \text{Im}(A')$$

with $\eta, \eta' = +1, -1, 0$ and $\Lambda'$ being the relevant new physics scale. Note that the arguments in Sec. 2.1 pertaining to the lower limit on $\Lambda$, are also applicable for $\Lambda'$.

The calculation of the top-loop contribution to $\Gamma(H \to \gamma\gamma)$ is akin to that for $\Gamma(H \to gg)$, except that there is no analogue of Fig. 3 (which, in any case, vanished identically). Once again, restricting to $O(A')$ results in limits being obtained only on $\eta/\Lambda'$. These are shown in Fig. 5. $\Lambda'$ values below the yellow (dot-dashed) lead to a deviation of 30% or more in $\Gamma(H \to \gamma\gamma)$. Similarly, the blue (dotted), green (dashed) and red (solid) lines represent, respectively, the 20%, 10% and 5% limits. It is interesting to see that even the weakest limits displayed here are comparable to those that may be obtained from the measurement of $tt\gamma$ production with 30 fb$^{-1}$ of data from the LHC operating at a centre-of-mass energy of 14 TeV [25].

Note that, in contrast to the case for the gluon coupling, the constraints on the anomalous magnetic moment correspond to a much lower energy scale, well within the energy reach of the LHC. Hence it would be possible to draw definitive conclusions about the existence of such anomalous moments, once sufficient data has been accumulated.

3 These couplings are also experimentally constrained by the Tevatron experiments [4].

4 Note that the analysis of Ref. [25] did not take into consideration all the experimental effects and, thus, the sensitivity projected therein is likely to suffer further degradation.
3 Discussions and Summary

Higgs production at the LHC is dominated by gluon fusion through a top loop. A precise measurement of the Higgs production cross-section can, thus, be used to probe possible non-SM contributions to the $t\bar{t}g$ vertex. Similarly, a measurement of the decay width of the Higgs into a $\gamma\gamma$ final state can constrain anomalous $t\bar{t}\gamma$ couplings. This assumes particular significance in the light of a possible sighting of the Higgs by both the ATLAS and the CMS collaborations, and the fact that the reported excess, while somewhat larger than that expected for a SM Higgs, is not inconsistent with such a hypothesis.

Parameterizing deviations of the said vertices in terms of higher-order operators in an effective field theory, we have performed such a study. We find that if the Higgs cross-section can be measured even to an accuracy of 20%, new physics giving rise to non-standard $ttg$ couplings can be ruled out up to an energy scale of nearly 20 TeV for a Higgs boson of mass 115–130 GeV. This would constitute an improvement over direct constraints from $tt$ production at the LHC [19] or even a polarized linear collider [20, 22]. In fact, these limits would be better than those obtained from similar (loop-induced) indirect measurements such as $b \to s\gamma$ or the ratio $\Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ [23]. Indeed, the accuracy reach seems to be comparable to the sizes of the anomalous chromomagnetic moment expected in a large class of models [20].
In the spirit of effective theories, we have retained terms only up to $O(A)$ or, equivalently, up to $O(\Lambda^{-2})$ in our calculation of the cross-section. At this point, let us consider the inclusion of the $O(A^2)$ terms. As in the case of the $O(A)$ terms, equal and opposite logarithmic divergences are encountered for each of the two subsets Fig.2(a, b) and Fig.2(c, d). In other words, once again, although the chromomagnetic term, on its own, leads to a divergent contribution, the inclusion of the full $SU(2)_L \times U(1)_Y$ invariant term leads to a cancellation of the divergences. Thus, at the end of the day, the $O(A^2)$ contribution seems a meaningful one. Indeed, the inclusion of this contribution would lead to a stronger constraint on $A$. We, nonetheless, omit this term while extracting limits, as we have not included terms higher order in $\Lambda^{-1}$ in our effective Lagrangian.

At this stage, we must consider possible higher-order corrections to the Higgs production process, both within the SM and in the extended theory. At first sight, it seems that, with a non-renormalizable term in the Lagrangian, this is fraught with danger. However, experience with higher order calculations of the $gg \rightarrow H$ production in the SM shows that it need not be so. NLO corrections to $\sigma(gg \rightarrow H)$ have been calculated in the Standard Model \cite{27} using the effective field theory approach by introducing a term of the form $\lambda_{\text{eff}} F^\mu\nu F^\mu\nu H$ in the Lagrangian, with the coupling strength $\lambda_{\text{eff}}$ being determined in terms of the one-loop calculation of this vertex within the SM. The difference between the infinite-$m_t$ approximation that such a treatment implies and the full calculation \cite{28} is found to be relatively small. The reason is not far to seek. The functional dependence on $m_t$ is a slow one and, thus, the numerical difference caused by the $m_t \rightarrow \infty$ approximation is a small one. For the additional contribution due to the top chromomagnetic moment, the story is similar, though not exactly the same. As eq. (17) shows, the $O(A)$ term can be expressed as $A m_t f(m_t/m_H)$ where $f(x)$ is a very slowly varying function. Naively, the presence of the $A m_t$-factor seems to indicate that the infinite-$m_t$ limit is inapplicable here. However, as $A \sim 1/v\Lambda = 1/(v\zeta m_t)$, this apparent dependence on $m_t$ is only a superficial one.\cite{3} With the remaining dependence on $m_t/m_H$ being a very slow one, the anomalous contribution can be subsumed in a suitably rescaled $\lambda_{\text{eff}}$. It is, thus, quite apparent that the NLO K-factor in this theory would be very similar to that within the SM. In other words, the $A$-induced scaling of the $gg \rightarrow H$ cross-section is expected to be largely insensitive to higher order corrections.\cite{4}

Higher-order QCD corrections are not the only uncertainties plaguing the cross-section calculation. The choice of the parton distributions as well as the factorization scale, together, have significant uncertainties associated with them. The estimates vary, ranging from $\sim 10\%$ \cite{29} to $\sim 20\%$ \cite{30}. This is almost irreducible in the present context and cannot be entirely circumvented in the absence of still higher order calculations. Indeed, this uncertainty is applicable to any effort to establish this resonance as the SM Higgs. Comparison of the production cross-sections across modes is expected to reduce this uncertainty to some extent but not entirely. Note, though, that bounds obtained from $t\bar{t}$ production etc. would also be plagued by similar uncertainties. At an $e^+e^-$ collider, ratios such as $\sigma(t\bar{t} + \text{jet})/\sigma(t\bar{t})$ could be expected to give additional information. However, the efficacy of this observable is not clear in the context of the LHC both on account of the reduced statistics that the observation of an additional jet entails, and also due to the uncertainties in the very definition of such semi-inclusive cross-sections. In view of this, it is quite apparent that the bounds advocated here should be treated as complementary to direct ones.

\footnote{With $\Lambda$ being the cut-off scale, all momentum integrals (and masses) have to be limited to $\Lambda$ or below. In other words, the infinite-$m_t$ limit can only be taken with $m_t/\Lambda$ being finite.}

\footnote{While we have argued this for the $O(A)$ term, clearly it holds as well for the $O(A^2)$ term.
As far as top-photon interactions are concerned, the comparison of the Higgs signal in the diphoton mode with that in the four-lepton channel (with signals in both having been reported) would, for the same range of Higgs masses, rule out new physics contributions to the $tt\gamma$ vertex upto more than 5 TeV. This energy regime is also within reach of the LHC, and hence, it should be possible to detect direct signals of any new physics that is involved as well. Once again, for reasons exactly analogous to those expressed earlier, the QCD corrections are almost identical for the SM decay and the decay in this extended model. Furthermore, this comparison of the modes almost entirely frees one from the aforementioned uncertainties due to the choice of the parton distributions and the factorization scale.

It should be appreciated that the bounds were obtained starting with effective operators invariant under the full SM gauge symmetry. Were we to consider only (chromo-)magnetic dipole couplings in isolation—i.e. admit terms that respect only $SU(3)_C \times U(1)_{em}$—the corresponding contributions to the Higgs partial widths would have received logarithmic enhancements and the consequent bounds would have been even stronger.

Understandably, (chromo-)electric dipole moments cannot be constrained well using these observables. However, once sufficient data is accumulated to permit determination of the CP properties of the putative resonance, even this would be possible. Indeed, were it to be established to be a pseudoscalar, it would be a challenging task to establish such large cross-sections in any given model, and effective operators such as those we have considered could provide a guideline for this task.

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## Appendix

The diagrams of Fig.2(a, b) give us, for the effective $ggH$ vertex,

$$\mathcal{M}_1^{\mu \nu} = \left( g_\ast^2 m_t \right) Tr[T_a T_b] \int \frac{d^4k}{(2\pi)^4} \frac{Tr[(\bar{\psi}_1 + m_t) \Gamma^\mu (\bar{k} + m_t) \Gamma^\nu (\bar{k} - \psi_2 + m_t)]}{[(k + p_1)^2 - m_t^2][k^2 - m_t^2][(k - p_2)^2 - m_t^2]}$$

(7)

$$\mathcal{M}_2^{\mu \nu} = -\left( g_\ast^2 m_t \right) Tr[T_b T_a] \int \frac{d^4k}{(2\pi)^4} \frac{Tr[(\bar{k} - \psi_2 - m_t) \Gamma^\nu (\bar{k} - m_t) \Gamma^\mu (\bar{k} + \psi_1 - m_t)]}{[(k - p_2)^2 - m_t^2][k^2 - m_t^2][(k + p_1)^2 - m_t^2]}.$$  (8)
Similarly, those in Fig. 2(c, d) lead to

\[ \mathcal{M}^{\mu\nu}_3 = -g_s^2 \text{Tr}[T_a T_b] \int \frac{d^4k}{(2\pi)^4} \frac{\text{Tr}[\Gamma^\mu_2 (k + m_t) \Gamma_2^\nu (k - p_2 + m_t)]}{[k^2 - m_t^2][(k - p_2)^2 - m_t^2]} \] (9)

\[ \mathcal{M}^{\mu\nu}_4 = -g_s^2 \text{Tr}[T_b T_a] \int \frac{d^4k}{(2\pi)^4} \frac{\text{Tr}[\Gamma^\nu_2 (k + m_t) \Gamma_1^\mu (k - p_1 + m_t)]}{[k^2 - m_t^2][(k - p_1)^2 - m_t^2]} , \] (10)

where

\[ \Gamma_1^\mu = \gamma^\mu + \sqrt{2} i B v \sigma^{\mu\alpha} p_1^\alpha , \quad \tilde{\Gamma}_1^\mu = \sqrt{2} i B \sigma^{\mu\alpha} p_1^\alpha \] (11)

\[ \Gamma_2^\nu = \gamma^\nu + \sqrt{2} i B v \sigma^{\nu\beta} p_2^\beta , \quad \tilde{\Gamma}_2^\nu = \sqrt{2} i B \sigma^{\nu\beta} p_2^\beta . \] (12)

with \( B = \text{Re}(A) + i \text{Im}(A) \gamma_5 \).

Whereas \( \mathcal{M}^{\mu\nu}_{1,2} \) have apparent linear divergences, for \( \mathcal{M}^{\mu\nu}_{3,4} \) the naive degree of divergence is quadratic. For on-shell gluons, terms proportional to \( p_1^2 \) and \( p_2^2 \) vanish identically. Since the gluon couples to a conserved current, terms proportional to \( p_1^\mu \) or \( p_2^\nu \) vanish as well. Consequently, the quadratic and linear divergences disappear, leaving behind terms that are either finite or, at worst, logarithmically divergent. The divergences can be regularized using any gauge-invariant prescription such as dimensional regularization. On summing all the contributing amplitudes and performing dimensional regularization, one obtains a finite and gauge-invariant form for the vertex function as a series in \( A \), namely

\[ \mathcal{M}^{\mu\nu} = i \left( \frac{g_s^2}{4\pi^2} \right) \delta_{ab} \left[ C_0 I + C_1 J + C_2 \left( J - \frac{1}{2} \right) \right] \] (13)

\[ C_0 = \frac{1}{v} \left( \frac{m_H^2}{2} g^{\mu\nu} - p_1^\nu p_2^\mu \right) \] (14)

\[ C_1 = 4m_t \left[ \frac{\text{Re}(A)}{\sqrt{2}} \left( \frac{m_H^2}{2} g^{\mu\nu} - p_1^\nu p_2^\mu \right) + \frac{\text{Im}(A)}{\sqrt{2}} \left( \epsilon^{\mu\alpha\beta\nu} p_{1\alpha} p_{2\beta} \right) \right] \] (15)

\[ C_2 = 4m_t^2 v \left[ \frac{\text{Re}(A^2)}{2} \left( \frac{m_H^2}{2} g^{\mu\nu} - p_1^\nu p_2^\mu \right) + \frac{\text{Im}(A^2)}{2} \epsilon^{\mu\alpha\beta\nu} p_{1\alpha} p_{2\beta} \right] \] (16)
Defining \( w = \frac{m_H^2}{m_t^2} \), we have for the integrals

\[
I(w) = \int_0^1 dx \int_0^{1-x} dy \frac{1 - 4xy}{1 - wxy}
\]

\[
= \frac{2}{w} - \left( \frac{4 - w}{w^2} \right) \left[ \text{Li}_2 \left( \frac{2w}{w + \sqrt{w^2 - 4w}} \right) + \text{Li}_2 \left( \frac{2w}{w - \sqrt{w^2 - 4w}} \right) \right]
\]

\[
J(w) = \int_0^1 dx \int_0^{1-x} dy \left[ \log \left( 1 - wxy \right) + 2 \right]
\]

\[
= -\frac{1}{2} + \sqrt{\frac{4 - w}{w}} \tan^{-1} \sqrt{\frac{w}{4 - w}} + \frac{1}{w} \left[ \text{Li}_2 \left( \frac{2w}{w + \sqrt{w^2 - 4w}} \right) + \text{Li}_2 \left( \frac{2w}{w - \sqrt{w^2 - 4w}} \right) \right]
\]

where \( \text{Li}_2(x) \) is the dilogarithm or Spence function.
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