Interference of an array of atom lasers

Giovanni Cennini, Carsten Geckeler, Gunnar Ritt, and Martin Weitz

Physikalisches Institut der Universität Tübingen,
Auf der Morgenstelle 14, 72076 Tübingen, Germany

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Abstract

We report on the observation of interference of a series of atom lasers. A comb-like array of coherent atomic beams is generated by outcoupling atoms from distinct Bose-Einstein condensates confined in the independent sites of a mesoscopic optical lattice. The observed interference signal arises from the spatial beating of the overlapped atom laser beams, which is sampled over a vertical region corresponding to 2 ms of free fall time. The average relative de Broglie frequency of the atom lasers was measured.

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Soon after the realization of optical lasers, the temporal interference signal among two such sources was observed [1]. This enabled investigations of the difference frequency and the frequency stability of the optical sources. To date, advances in the field of synthesizing and controlling optical frequencies allow for the measurement and comparison of optical frequencies with radiofrequency precision [2, 3]. Atom lasers are coherent atom sources whose outcoupled wave can be seen as a matter wave analog to the radiation emitted by optical lasers [4, 5, 6, 7, 8].

We report on the measurement of the relative frequency of several distinct atom laser beams. In the sites of a mesoscopic optical lattice potential, rubidium atoms are initially cooled to quantum degeneracy to form independent microcondensates. By smoothly ramping down the confining lattice potential, the condensates are coupled out into a comb-like array of independent, coherent atomic beams directed downwards along the earth’s gravitational acceleration. Due to their divergence, the atom laser beams soon overlap. The resulting interference pattern, as observed by absorption imaging, gives a streak-camera like image of the beating of the coherent matter wave beams. The temporal evolution of the atom lasers relative phase was monitored over a vertical range corresponding to 2 ms of free fall time. The average difference frequency between adjacent atom laser sources was in a proof of principle experiment measured to an accuracy of 46 Hz.

Bose-Einstein condensates are coherent ensembles of particles, all populating a single one-particle state [9]. The coherence of such quantum degenerate samples has first been verified in interference experiments with two atomic Bose-Einstein condensates, leading to the conclusion that each condensate upon measurement can be described by a single, macroscopic quantum phase [10]. Recently, the relative phase of two Bose-Einstein condensates was continuously sampled by light scattering off the internal atomic structure [11]. On the other hand, light scattering can also imprint an atom phase [12]. Bose-Einstein condensated atoms can be outcoupled from their confining potentials to form atom laser beams [4, 5, 6, 7, 8]. By splitting up and recombining a single atom laser beam, its spectral line width was determined [13].

Our experiment is based on the technique of direct generation of Bose-Einstein condensates in optical dipole traps [8, 14, 15, 16]. Initially, cold thermal rubidium atoms \(^{87}\text{Rb}\) are confined in the antinodes of a one-dimensional optical standing wave generated using mid-infrared radiation derived from a CO\(_2\)-laser. Neighbouring lattice sites are spaced by
\[ d = \frac{\lambda_{\text{CO}_2}}{2} \approx 5.3 \mu m. \] For our experimental parameters, atom tunnelling between adjacent sites is negligible, which guarantees that the atom lasers operate truly independently. By applying a magnetic field gradient of typically 10 G/cm, we remove atoms in magnetic field sensitive spin projections, yielding an ensemble of atoms all populating the \( m_F = 0 \) component of the lowest hyperfine ground state \((F = 1)\). The thermal atoms are cooled evaporatively to quantum degeneracy by lowering the trapping laser beam power to produce an array of independent disk-shaped microcondensates, as described more in detail in an earlier work [17]. At the end of the evaporation stage, the CO\(_2\)-laser beam power is 40 mW on a 30 \( \mu \)m beam radius. We generate an array of in average seven disk-shaped \( m_F = 0 \) microcondensates. The total number of atoms in this array is 2000. The atom laser beams are generated by subsequently smoothly ramping the CO\(_2\)-laser beam intensity towards zero in a 30 ms long ramp. The outcoupling occurs once the dipole trapping force does not anymore sustain the atoms against gravity. In this way, a comb-like structure of parallel atom laser beams directed downwards is produced. Due to the use of \( m_F = 0 \) condensates in our all-optical technique, fluctuations of the chemical potential due to stray magnetic fields are suppressed to within the 14 fK/(mG)\(^2\) quadratic Zeeman shift of this clock-state [8].

We note that the spatial phase evolution of each atom laser is determined by the de Broglie wavelength \( \lambda = h/p \) with \( p^2/2m = \hbar \omega - mgz \). One expects \( \lambda \) to decrease during the free fall as the momentum \( p \) increases. Despite the strong chirping of the spatial phase, monochromaticity of the matter waves is clearly defined [18, 19, 20]. In contrast, a time-dependent, fluctuating value of the chemical potential of the condensates would lead to deviations from merely gravitationally chirped matter waves, and relative variations can be measured in interference experiments with other atom laser beams. More generally, such a comparison can also reveal other sources of differences in the independent beams frequency, such as variations in the vertical trap position, which will lead to a gravitational phase difference, or possibly even tiny differences in the atoms mass.

A scheme of the potential experienced by atoms during the onset of the atom laser beams operation is shown in Fig. 1. Before the condensates outcoupling, the trap vibrational frequencies are 1.9 kHz and 150 Hz along the longitudinal and the radial trapping directions of the lattice respectively. The atom laser beams from adjacent sites are estimated to start overlapping at a time of 500 \( \mu s \) after outcoupling, and after 4 ms a complete overlap of all beams occurs.
We experimentally observe the far-field interference pattern of the atom lasers using absorption imaging. Fig. 2a shows a typical interference pattern for an average of 7 populated sites in the lattice, which equals the number of interfering atom laser beams. The image gives a streak-camera like recording of the interference pattern, in which the vertical position \( z \) relates to the fall time via the ballistic free fall formula. Fig. 2b shows a transverse profile of the interference pattern averaged over a vertical length of 44 \( \mu \)m.

Let us next discuss the expected interference signal for such an array of overlapping atom laser beams. We expect that the macroscopic wavefunction of each atom laser beam is given by an eigensolution of the Gross-Pitaevskii equation in the gravitational field. As the total number of atoms emitted from each source is fixed, we do not expect the atom lasers to have a well-defined phase. However, upon measurement of the interference pattern of the overlapping beams a projection upon a state with well-defined relative phase is performed.

The optical lattice is in our experiment aligned orthogonally to the axis of gravity. Let us denote the macroscopic condensate wavefunction at the \( n \)-th site \((n = 1, 2, \ldots, N)\) centered at position \( \mathbf{r}_n \) at time \( t = 0 \) as \( a_n e^{i\theta_n} \), where \( a_n \) gives the amplitude and \( \theta_n \) the phase, the latter being random in each realization of the experiment. After outcoupling, the total wavefunction of the overlapping beams at position \( \mathbf{r} \) and time \( t \) (with \( t > 0 \)) can be formulated using path integrals evaluated along classical space-time trajectories \[21\]:

\[
\psi(\mathbf{r}, t) = \sum_{n=1}^{N} a_n e^{i\theta_n} C(\mathbf{p}, t_{\text{out}}) \exp \left[ \frac{i}{\hbar} \left( \int_{\mathbf{r}_{\text{trap}, n}}^{\mathbf{r}} \mathbf{p} \, d\mathbf{r}' \right) - \int_0^t \hbar \omega_n \, dt' \right] \tag{1}
\]

where \( \omega_n \) is the temporal de Broglie frequency of each atom laser defined as \( \hbar \omega_n = mgz_{\text{trap}, n} + \mu_n(t') \). This frequency is the sum of the initial atomic kinetic energy and \( n \)-th condensate’s chemical potential \( \mu_n \), where \( m \) denotes the atom mass and \( g \) the gravitational acceleration. We stress that the temporal phase fluctuations of both the initial condensates and the outcoupling process are accounted for by assuming time-dependent chemical potentials \( \mu_n(t') \). After outcoupling, due to energy conservation \( \hbar \omega_n \) remains constant, i.e. \( \hbar \omega_n = mgz + \mu_n(t_{\text{out}}) \) for \( t' > t_{\text{out}} \) (with \( 0 \leq t_{\text{out}} \leq t \)). We are interested in the total wavefunction outcoupled from the \( N \) sources all emitting at the same vertical position \( z_{\text{trap}, n} = z_{\text{trap}}, y_{\text{trap}, n} = 0 \), and equally spaced along the \( x \) axis with \( x_{\text{trap}, n} = d \cdot n \), where \( d = \lambda_{\text{CO}_2}/2 \) is the
lattice spacing. In the far field one finds 
\[ p_{z,n} = \sqrt{2m(mg(z_{\text{trap}} - z) - \mu_n(t_{\text{out}})) - (p_x^2 + p_y^2)} \]
and \( p_x = mx/t_{\text{exp}} \), where \( t_{\text{exp}} = t - t_{\text{out}} \) denotes the free expansion time. After carrying out the above integrals, we find that the probability to detect a particle at fixed time \( t \) and position \( r \) can be written as:

\[
|\psi(r, t)|^2 = |C(p, t_{\text{out}})|^2 \sum_{n=1}^{N} A_n \cos \left[ \frac{n \hbar}{ht_{\text{exp}}} x + \delta_n + \beta_n(z, t) \right], \tag{2}
\]

where \( A_0 = \sum_{m=1}^{N} a_m^2 \), \( \delta_0 = 0 \) and for \( 1 \leq n \leq N - 1 \):

\[ A_n e^{i\delta_n} = 2 \sum_{m=1}^{N-n} a_m a_{m+n} e^{i\Delta \theta_m}. \]

Both \( A_n \) and the phase angles \( \delta_n \) shall here be real numbers. These coefficients can be evaluated in a random walk model with \( N - 1 \) steps assuming that the phase angles \( \Delta \theta_m = \theta_m - \theta_{m+n} \) are randomly distributed \[20\]. If all amplitudes \( a_m \) are equal, for \( N \gg 1 \) the average fringe visibility \( V \equiv A_1/A_0 \) is given by \( \sqrt{\pi/N} \). The \( z \)-dependent phase in eq. 2 is

\[
\beta_n(z, t) = \frac{1}{\sum_{m'=1}^{N-n} e^{i\Delta \theta_m}} \sum_{m=1}^{N-n} \left[ \varphi_m(z, t) - \varphi_{m+n}(z, t) \right] e^{i\Delta \theta_m}, \tag{3}
\]

which again can be evaluated with a random walk model. In this formula \( \varphi_n(z, t) = (1/\hbar) \int_0^{t_{\text{out}}(z)} \mu_n(t') dt' + \mu_n(t_{\text{out}}(z))(t - t_{\text{out}}(z)). \)

We expect that the mean square value of this \( z \)-dependent phase is approximately given by \( \sqrt{\langle \Delta \beta_n^2 \rangle} \approx \sqrt{\langle \Delta \varphi^2 \rangle} \), which is independent of the number of condensates. Small statistical fluctuations of the chemical potential, which would lead to a line broadening, should affect the multiple beams interference pattern in a similar amount than a two atom lasers interference experiment. In case that the lattice axis is tilted against the axis of gravity by an angle \( \alpha \) (with \( \alpha \ll 1 \)), the cosine factor in eq. 2 for \( |\psi|^2 \) contains an additional factor 
\[ \frac{\hbar}{n} \sin(\alpha) \sqrt{2m^2 g(z_{\text{trap}} - z)} \], which in turn will lead to a tilting of the fringe pattern.

The experimental interference pattern shown in Fig. 2 has high contrast (near 50%). With our present imaging resolution of \( 6 \mu m \), we spatially resolve only the interference pattern between adjacent lattice sites. Let us begin an analysis of the atom lasers interference pattern by restricting ourselves to signal profiles for a given fall distance \( |z - z_{\text{trap}}| \), which corresponds to a fixed outcoupling time \( t_{\text{out}} \). In this case, the amplitude and phase of the interference pattern are evaluated analogous to earlier work on the interference of independent Bose-Einstein condensates \[22, 23\]. The average fringe contrast of a set of eight observed interference patterns with its corresponding standard deviation was \( (24 \pm 5) \% \). Within the quoted uncertainties, this value agrees very well with the results of our previous works
studying the interference of a variable number $N$ of Bose-Einstein condensates \[17\], from which we interpolate a value of 25\% for $N = 7$. From this we conclude that the atom laser output coupling mechanism, within our experimental measurement uncertainties, does not introduce high frequency phase noise and is reproducible from site to site.

The present work involves atom lasers with (quasi-)continuous output coupling, and we can study the variation of the phase of the interference pattern with vertical position $z$, which directly is related to the duration of free fall. This allows us to monitor the atom lasers relative frequency. The vertical, stripe-like nature of the fringes shown in Fig. 2 clearly shows that the temporal evolution of the atom laser phase over the observed 2 ms time window is well controlled. Fluctuations of the relative de Broglie frequencies here would lead to jittering or blurred fringes.

From Eq. 2 we expect that the fringe spacing increases linearly with the expansion time $t_{\text{exp}}$ of atoms and is given by $ht_{\text{exp}}/(md)$, where $d$ is the spatial separation of sources. As the atoms during the outcoupling process already transversally expand before being completely removed from the trap potential, we set $t_{\text{exp}} = t_{\text{fall}} + t_{\text{delay}}$ with $z = z_{\text{trap}} - gt_{\text{fall}}^2/2$ describing the ballistic free fall and a constant term $t_{\text{delay}}$ to account for the time lag during outcoupling in which atoms are not yet exposed to the earth’s gravitational field alone. A fit to the experimental data for the pattern of Fig. 2 gives $t_{\text{delay}} \approx 3$ ms, which is consistent with a simple numerical simulation. Fig. 3a shows the experimental fringe spacing along with the fit (solid line) as a function of the expansion time.

Of particular interest is an analysis of the variation of the phase of the fringe pattern on the expansion time. The corresponding data is shown in Fig. 3b. Over the observed 2 ms time interval, corresponding here to roughly 250 $\mu$m of fall distance, the phase remains relatively constant with in average showing variations of order of $2\pi/10$. The experimental data can both be fitted with a constant (dashed line) and linear (solid line) function of fall time, with the latter fit yielding a smaller sum of residuals. The non-zero slope of the latter fit translates to a finite difference frequency between adjacent atom lasers, as is e.g. expected for a CO$_2$-laser lattice beam non-perfectly aligned orthogonal to the axis of gravity. Table I gives the fitted average atom laser difference frequency for this measurement (data set #1) along with the result for three other high contrast fringe patterns. The average difference frequency between adjacent atom laser beams for the four data sets equals $(-51 \pm 46)$ Hz. Notably, this result corresponds to the difference frequency of independently generated matter wave...
beams.

To conclude, we have observed the interference of an array of atom laser sources. The average relative phase fluctuations and difference frequencies between adjacent matter wave sources were monitored. For the future, we expect that the relative measurement of atom laser frequencies can allow for novel matter wave metrology techniques. An intriguing question is if ultimately frequency measurements of small atomic mass differences can be carried out.

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FIG. 1: Scheme of the periodic trapping potential during the atom lasers emission. The emission occurs at the downwards directed surfaces of the microscopic traps, as indicated in the figure.

FIG. 2: Far field interference pattern of independent all-optical atom lasers. (a) Absorption image of the interference pattern of seven overlapped atom lasers beams, recorded at an expansion time of 15 ms after the onset of outcoupling of atoms from the sites of the optical lattice. The field of view is $60 \mu m \times 220 \mu m$. (b) Horizontal density profile of image (a) averaged over the marked vertical region with $44 \mu m$ height. The experimental data is represented by connected solid dots and the fitted fringe pattern by the dashed line.

FIG. 3: Fringe spacing and phase for the interference pattern of Fig. 2, as function of the expansion time and of the corresponding free fall distance. (a) Experimental data for the fringe spacing evolution along with a linear fit (solid line). (b) Evolution of the fringe phase. The data has been fitted both with a constant (dashed line) and a linear function (solid line). The linear fit gives a finite slope which corresponds to an (average) difference frequency between adjacent atom lasers of $-77$ Hz.

| Data set number | $\delta \omega_{n,n-1}$ (Hz) |
|-----------------|-------------------------------|
| 1               | $-77$                         |
| 2               | $-75$                         |
| 3               | $-133$                        |
| 4               | $82$                          |
| Average         | $-51 \pm 46$                  |

TABLE I: Measured (averaged) difference frequency of neighbouring atom laser beams. From a total of eight recorded interference patterns, with, due to the intrinsically random nature of the individual condensates phases, statistically distributed fringe contrast, these four sets had high enough contrast to allow for a reliable extraction of the average relative de Broglie frequency. We have fitted the measured differential phase with a linear function of the expansion time, as shown in Fig. 3b for data set #1. In the bottom row, the mean value for the matter wave frequency along with its standard deviation is given.
