Price and lead time differentiation, capacity strategy and market competition

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We study a duopoly market in which customers are heterogeneous, and can be segmented as price or time sensitive. Each firm tailors (differentiates) its products/services for the two customer classes solely based on guaranteed lead time and the corresponding price. Our objective is to understand how competition affects price and lead time differentiation of the firms in the presence of different operations strategy (shared versus dedicated capacity), product substitution and asymmetry between the competing firms. Our results suggest that when firms use dedicated resources to serve the two market segments, pure price competition always tends to decrease individual prices as well as price differentiation, irrespective of the market behaviour. Further, the effect of competition is more pronounced when customers are allowed to self-select, thereby introducing substitutability between the two product options. On the other hand, when firms compete in time, in addition to price, the effect of competition on product differentiation depends crucially on the behaviour of the market. Our results further suggest that the firm with a larger market base should always maintain a larger price and lead time differentiation between the two market segments. Similarly, the firm with a capacity cost advantage should also maintain a larger lead time differentiation.

Keywords: operations strategy; competition; price; lead time; product differentiation

1. Introduction

Many firms today use lead time guarantee in their promotion campaigns. Some firms even exploit customers’ sensitivity to time to extract a price premium for the same product by promising them a shorter lead time. Amazon.com, for example, charges more than double for shipment to guarantee a delivery in two days against its normal delivery time of around a week (Ray and Jewkes 2004). Such firms exploit customers’ heterogeneity in their preferences for time and price to charge different prices from different market segments (and promise them different lead times). Such price discrimination strategies are known to help firms extract a greater consumer surplus (Tirole 1998).

Different firms in an industry may compete with each other by offering better deals, either in the form of lower prices, better lead time guarantees or both to their customers. Attractive price and lead time can generate demand. Failure to meet the guaranteed lead times may, however, result in penalties, either in the form of a discount or partial refund, denting a firm’s margin. A striking example in this context is the case of seven online retailers, including Macys.com, Toystrus.com and CDNOW, that paid fines to the tune of $1.5 million to settle a Federal Trade Commission lawsuit over late deliveries made in 1999 (Pekgun 2007). Firms, therefore, target to meet their guaranteed lead times with at least a given level of reliability. In make-to-order or service industries, this usually translates into a better (server) capacity management, an operations-related decision. Marketing and operations managers of such firms, therefore, need to jointly decide on the optimal prices and lead times for the different market segments, and the required capacity levels.

Keeping the above discussion in mind, we study a market with two competing firms. Each firm sells a menu of products/services, differentiated only in their prices and lead time guarantees, to exploit customers’ heterogeneity in their preferences for time and price. The main issue that a firm faces in such a market is how to optimally differentiate its products, based on their prices and guaranteed lead times, for customers with different preferences, and accordingly decide its optimal capacity levels. In a competitive market, it also needs to take into account the reaction from other firms, and its impact on its own demand. While it is well known that competition, in general, drives prices down, its effect on price differentiation (price discrimination, as is popularly termed in Economics literature) is not clear, all the more so when the price discrimination is based on some endogenous category such as the lead time guarantee.
The traditional theory on price discrimination, which predicts that market competition decreases a firm’s ability to use price discrimination, has often been challenged by very contrasting empirical results (Gerardi and Shapiro 2007). Moreover, the effect of competition on lead time differentiation between different customer segments itself has not been studied.

Competing firms may either use separate (dedicated) resources (capacities) for the different market segments or may share them across the market segments. Industry presents evidences in favour of both: (i) dedicated capacities; (ii) shared capacities. For example, web hosting and content delivery firms maintain dedicated servers for customers like news sites whose content for online delivery is time sensitive. This makes possible real-time update to their data. Other customers whose data do not require frequent updates are served using a different set of servers. In contrast, a third-party logistics service provider shares its fleet of vehicles to serve multiple firms with different delivery time guarantees. There are also evidences of two competing firms in the same industry using very different capacity strategies. For example, FedEx uses separate facilities for its express and ground services. UPS, in contrast, delivers express and ground services using one integrated network (Jayaswal 2009).

In light of the above discussion, we try to address the following research questions: (1) How does competition affect the price and lead time differentiation decisions of a firm relative to a monopolistic setting? Traditional theory suggests competition should result in lower prices; should it result in lower price discrimination as well? How does the result change if firms compete in time, in addition to price? (2) How does the operations strategy, specifically the capacity strategy (dedicated versus shared capacities), used by competing firms’ affect the equilibrium price and lead time decisions? (3) How does asymmetry in firms’ operating conditions (in terms of capacity cost or market penetration) affect the equilibrium price and lead time differentiation decisions? Answers to these questions are summarised as propositions (Propositions 4 and 6 for Question 1) and observations (Observation 1 for Question 2, and Observations 2 and 3 for Question 3).

Rest of the paper is organised as follows. In Section 2, we provide a review of the related literature. We present our mathematical model and the underlying assumptions in Section 3. In Section 4, we describe the best response of a firm, both for dedicated and shared capacity settings, given its competitor’s price and lead time decisions. Section 5 discusses the Equilibrium solution for the duopoly problem, followed by analysis of important results. The paper concludes with a summary of our main results in Section 6.

2. Literature review

The work in this paper relates closely to price discrimination, which has been studied extensively in the Economics (Industrial Organisation) literature, and to price and/or time competition in Operations Management (OM) field. We briefly review each of these two areas of research separately.

The textbook theory argues that competitive firms cannot price discriminate since they are price-takers, while monopolists can price discriminate to the extent that there exists both heterogeneity in consumers’ demand elasticities and a useful sorting mechanism to distinguish between consumer types (Gerardi and Shapiro 2007). The textbook theory, therefore, predicts that competition should decrease price discrimination. This is further corroborated by the theoretical model of Rochet and Stole (1999). However, the theoretical models of Gale (1993), Stole (1995) produce exactly the opposite results. Further, different empirical studies have again produced very contrasting results. Borenstein (1989), Borenstein and Rose (1994) found evidence of increasing price dispersion with competition in airline industry, thereby suggesting that competition decreases price discrimination. However, a more detailed empirical study by Gerardi and Shapiro (2007) found a negative relation between market competition and price dispersion, thereby suggesting that competition decreases price discrimination.

Literature in OM on price and/or time competition model a firm’s operations in a greater detail. These papers can mostly be classified into (Cachon and Harker 2002) (i) papers on inventory games, and (ii) those on queueing games. Papers on inventory games are relevant in a make-to-stock setting where firms use inventory as their strategic tool to compete in the market. Papers on queueing games are pertinent to make-to-order or service industries, where firms use better (server) capacity/queue management to adjust their price and lead time decisions, and thus compete in the market. Since our model is relevant to make-to-order or service industries, our focus is on the latter category.

Literature on queueing games can further be categorised into (i) papers that aggregate price and waiting time into a single measure called ‘full price’, and (ii) those that model price and delivery/waiting time as independent explanatory variables. Levhari and Luski (1978), Loch (1991), Lederer and Li (1997), Chen and Wan (2003), Armony and Haviv (2003) belong to the former subcategory. All these papers assume that customers associate a specific cost rate with their waiting time, and that they make their selection of a firm based only on their ‘full price’, which is the sum of the actual price charged and the expected delay cost, disregarding any other service attributes. Further, they assume that all customers are in a position to assess the equilibrium steady state waiting times they will experience.

The second subcategory of papers on queueing games, which model price and lead time as independent explanatory variables, include So (2000), Pekgun, Griffin, and Keskinocak (2008), Allon and Federgruen (2007, 2009). These papers
model customers’ aggregate demand for a firm’s service as a function of its price, lead time and/or other service attributes, each of which is explicitly advertised by the firm. We position our work in this category since we treat price and lead times as independent variables announced by a firm. Although our demand model bears some similarity with those used by Pekgun, Griffin, and Keskinocak (2008), Allon and Federgruen (2007), it is fundamentally different from them in that these models consider only a single customer class, and thus there is no market segmentation. To our knowledge, Loch (1991), Lederer and Li (1997), Armony and Haviv (2003), Allon and Federgruen (2009) are the only works in this subcategory to have addressed the phenomenon of market segmentation. As noted earlier, these papers, except for Allon and Federgruen (2009), assume that customers aggregate the price and lead time into a full price, and that they select the service provider on the basis of this full price only. In doing so, they assume that all customers are in a position to assess the equilibrium steady state waiting times they will experience, while in our model, waiting time standards are advertised to the different classes. Moreover, they consider the firms’ capacity levels as exogenously given, and not a decision variable.

Thus, Allon and Federgruen (2009) appears to be the closest to our model, although the effect of competition on price discrimination is not their focus. However, they study completely segmented markets, which means that each customer is strictly assigned to a specific class, and she cannot switch between different classes. This is tantamount to saying that the specific service package (price and lead time combination) offered to a given customer class is not available to any other class, and hence is non-substitutable. In marketing/economics parlance, their work is pertinent to third-degree price discrimination (see Talluri and Ryzin 2004, for a definition). Our demand model is more generalised, which also captures product substitution. Our model, therefore, allows us to study both second- and third-degree price discrimination. Moreover, Allon and Federgruen (2009) use a service level that is based on expected lead times. It is, therefore, quite possible that a large portion of the demands are actually not served within their promised lead times, even if the promised lead times are met on average. We, therefore, assume that firms select their capacity levels so as to fulfil their promised lead times with a high level of reliability (generally 99%). This makes the lead time guarantees more attractive, although it makes the problem a lot more challenging to solve.

3. Decision model

We consider a service or a make-to-order manufacturing industry with two firms, indexed by \( i \in \{1, 2\} \) and \( j = 3 - i \), competing in a market that is segmented into two customer classes, indexed by \( c \in \{h, l\} \). Class \( h \) customers are high-priority/express customers who are more time sensitive and are willing to pay a price premium for a shorter lead time. Class \( l \) customers are low-priority/regular customers who are more price sensitive and are willing to accept a longer lead time for a price discount. Firm \( i \) competes for its market share by selecting its prices \( p_i \) and guaranteed lead times \( L_i \) offered to market segment \( c \in \{h, l\} \). Firm \( i \) faces a demand from class \( c \), generated according to a Poisson process with rate \( \lambda_i^c \left( p_i, L_i \right) \), \( c \in \{h, l\}, i \in \{1, 2\} \), which depends on the decisions of both the firms in the following way: (i) each firm’s expected demand from a given market segment is (i) decreasing in its price and lead time offered to that segment, (ii) increasing in its price and lead time offered to the other market segment, and (iii) increasing in the price and lead time offered by the other firm to the same segment. We model this using the following system of linear equations:

\[
\begin{align*}
\lambda_h^i &= d^i - \beta_p p_i + \theta_p (p_i - p_h^i) - \beta_L L_i^h + \theta_L (L_i^h - L_h^i) + \gamma_p (p_h^i - p_h^i) + \gamma_L (L_h^i - L_h^i) \\
\lambda_l^i &= d^i - \beta_p p_i + \theta_p (p_i - p_l^i) - \beta_L L_i^l + \theta_L (L_i^l - L_l^i) + \gamma_p (p_l^i - p_l^i) + \gamma_L (L_l^i - L_l^i)
\end{align*}
\]

where,

- \( 2a^i \) : market base of firm \( i \)
- \( \beta_p^c \) : sensitivity of class \( c \) demand to its own price
- \( \beta_L^c \) : sensitivity of class \( c \) demand to its own guaranteed lead time
- \( \theta_p \) : sensitivity of demand to inter-class price difference
- \( \theta_L \) : sensitivity of demand to inter-class lead time difference
- \( \gamma_p \) : sensitivity of demand to inter-firm price difference
- \( \gamma_L \) : sensitivity of demand to inter-firm lead time difference

\( 2a^i \) parameterises firm \( i \)’s market base. Mathematically, it is the demand faced by firm \( i \) when price and lead time offered by each firm to each customer class is zero. It captures the aggregate effect of all the factors such as a firm’s brand image, service quality, etc., other than price and lead time on demand. Hence, the firm offering the lowest price and the shortest lead time to a market segment need not capture its entire demand. The relative values of \( d^i \) and \( a^i \) can be loosely used to describe comparative advantage in terms of a firm’s market penetration. This may reflect the underlying preferences of customers for one firm over the other, which may be due to customers’ appeal for a brand.
The above demand model generalises the demand model used by Jayaswal, Jewkes, and Ray (2011) to a competitive setting. It also generalises the demand models used by Tsay and Agarwal (2000), Pekgun, Griffin, and Keskinocak (2008) to segmented markets. Further, it generalises the demand model used by Allon and Federgruen (2009) by taking into account product substitution ($\theta_p$ and $\theta_L$). The total market size ($\sum_{i \in \{1, 2\}} \sum_{c \in \{h, l\}} \lambda^i_c$) in our model is invariant to any changes in inter-firm or inter-class sensitivities, which only affects the distribution of the total demand among the firms and the customer classes. However, the pricing and lead time decisions of the two firms affect both the total market size as well as the resulting demand for each firm and from each market segment.

By definition, $\beta^i_p > 0$, $\beta^i_L > 0$, $\theta_p \geq 0$, $\theta_L \geq 0$, $\gamma_p \geq 0$, $\gamma_L \geq 0$, $\beta^h_p < \beta^l_p$, and $\beta^h_L > \beta^l_L$. $\theta_p > 0$, $\theta_L > 0$ signifies product substitution, while $\gamma_p > 0$, $\gamma_L > 0$ signify the presence of price competition and lead time competition in the market. $\gamma_p = \gamma_L = 0$ makes the demand of the two firms independent, and hence decouples their decision-making, resulting in a monopolistic setting.

We use a reliability level $\alpha^i$ with which firm $i$ tries to meet its lead time guarantee. In our model, we restrict our discussions only to cases where the service reliability for each firm is the same, i.e., $\alpha^i = \alpha$. This is applicable in situations where there exists some industry standard and published reports (like those published by the Aviation Consumer Protection Division of the U.S. Department of Transportation, and Expedia for airline industry) such that the delivery performance of each firm is readily available to customers. In this way, firms are discouraged from performing below the standard such that the market share is then mainly affected by their promised times and prices, as depicted by our demand model. Of course, our model and analysis also allows for different service reliabilities for different firms.

We assume that the time firm $i$ takes to serve a customer from class $c$ is exponentially distributed with a rate $\mu^i_c$. Firm $i$, therefore, behaves like an $\text{M}/\mu^i_c$/1 queueing system. Customers within each class are served on a first-come-first-serve (FCFS) basis. The firm can invest in its installed capacity to increase its processing rate $\mu^i_c$. We assume there are no economies of scale in investing in capacity. This is a common assumption made in the literature, primarily for tractability of the resulting model (Boyaci and Ray 2003, 2006; Jayaswal, Jewkes, and Ray 2011). So a unit increment in $\mu^i_c$ always costs $SA^i$. We also assume that firm $i$ incurs the same operating cost of $\text{S}m^i$ in serving a customer of either class.

The industry is assumed to have established a standard lead time for regular customers, and hence $L^i_l = L^i_h = L^i$. Firm $i$ selects and announces its express lead time and the two prices ($L^i_h$, $p^i_h$, $p^i_l$) so as to maximise its profit. Firm $i$ does so taking into account the lead time and prices ($L^j_h$, $p^j_h$, $p^j_l$) selected by firm $j = 3 - i$ since they have an impact on firm $i$’s demands, and hence on its profit. It also needs to simultaneously select its optimal service rates (i.e. installed capacities) $\mu^i_c$ in order to meet the guaranteed lead times with at least a minimum level of reliability $\alpha^i$.

4. A firm’s best response

Given the price, lead time and capacity decisions $(p^i_h, p^i_l, L^i_h, \mu^i_h, \mu^i_l)$ of firm $j = 3 - i$, firm $i \in \{1, 2\}$ tries to select its own corresponding decisions $(p^i_h, p^i_l, L^i_h, \mu^i_h, \mu^i_l)$ that maximise its profit and also ensure that its lead time commitments are met reliably. As clear from the demand model (1)–(2), the demands for firm $i \in \{1, 2\}$, and its decisions in turn, depend on the price and lead time decisions made by firm $j = 3 - i$. Firm $i$’s demand and its decisions, however, do not depend directly on the capacity level ($\mu^j_h, \mu^j_l$) selected by firm $j$. While competing with the other firm, each firm, therefore, possesses only two types of essential strategic instruments: prices and the lead times. Firm $i$’s strategy can be defined as a vector of its strategic decision variables $s^i := (p^i_h, p^i_l, L^i_h)$, which it uses to compete against the other firm $j$. The best response of firm $i$ to firm $j$’s strategy, $s^j := (p^j_h, p^j_l, L^j_h)$, is thus a strategy $s^{i*} := (p^{i*}_h, p^{i*}_l, L^{i*}_h)$ such that $\pi^i(s^{i*}, s^j) = \max_{i \neq j} \pi^i(s^i, s^j)$, $i \in \{1, 2\}$ and $j = 3 - i$. Firm $i$’s best response is the solution to the following optimisation problem (called the pricing and lead time decision problem $\{\text{PLDP}^i\}$):

\[
\text{max} \quad \pi^i = (p^i_h - m^i)\lambda^i_h + (p^i_l - m^i)\lambda^i_l - A^i(\mu^i_h + \mu^i_l)
\]

s.t.

\[
L^i_h < L^j_l
\]

\[
p^i_h, p^i_l, \lambda^i_h, \lambda^i_l, \mu^i_h, \mu^i_l \geq 0
\]

Stability condition

\[
S^i_h(L^i_h) = P(W^i_h \leq L^i_h) \geq \alpha
\]

\[
S^i_l(L^i_l) = P(W^i_l \leq L^i_l) \geq \alpha
\]

where $\lambda^i_h$ and $\lambda^i_l$ are given by (1) and (2), respectively. The objective function (3) is the profit per unit time. Constraint (4) requires that the guaranteed lead time for high-priority customers be shorter than that for the other class. Constraint set (5) is
needed to define a realistic problem setting. Constraint (6) is the stability condition for the queuing system, which models the service facility at the firm. Constraints (7) and (8) are lead time reliability constraints (also called service-level constraints), which require that the steady-state actual lead time $W^i_h$ (resp., $W^j_l$) of a customer should not exceed its guaranteed lead time $L^i_h$ (resp., $L^j_l$) with a probability of at least $\alpha$.

Note that a firm’s best response problem has a form very much similar to a firm’s optimal decision in a monopolistic setting, discussed by Jayaswal, Jewkes, and Ray (2011). Difference still arises between the two due to the strategic interaction between the competing firms, which is captured in the demand model (1)–(2). Therefore, the best response of a firm can also be obtained by adapting the solution method developed by Jayaswal, Jewkes, and Ray (2011) for the monopolistic setting.

In what follows, we adapt $[PLDP^j]$ for a firm using dedicated or shared capacity strategy by specifying the form of constraints (6)–(8).

### 4.1 Dedicated capacity setting

For a dedicated capacity setting, where each customer class is served by a separate M/M/1 server, the sojourn time distribution for either class of customers is known to be exponential (Ross 2010). In this case, there is a separate stability condition for each of the queues. Hence, constraints (6)–(8) can be expressed as:

\[
\lambda^i_c - \mu^i_c < 0, \quad c \in \{h, l\}
\]  

\[
S^i_h(L^i_h) = P(W^i_h \leq L^i_h) = 1 - e^{(\lambda^i_h - \mu^i_h)L^i_h} \geq \alpha
\]  

\[
S^i_l(L^i_l) = P(W^i_l \leq L^i_l) = 1 - e^{(\lambda^i_l - \mu^i_l)L^i_l} \geq \alpha
\]

**Proposition 1** For a given strategy $s^j := (p^j_h, p^j_l, L^j_h)$ by firm $j = 3 - i$, the best response express lead time $L^{i^*}_i$ of firm $i \in \{1, 2\}$ using dedicated capacities is given by the unique root of (9) in the interval $[0, L^i_l)$

\[
\frac{\partial \pi^i(L^i_l)}{\partial L^i_l} = -\left(\beta^i_h + \theta_L + \gamma_L\right)\left(p^i_{h^*}(L^i_l) - m^i - A^i\right) + \theta_L\left(p^i_{l^*}(L^i_l) - m^i - A^i\right) - \frac{A \ln(1 - \alpha)}{(L^i_h)^2}
\]

where $p^i_{h^*}(L^i_l)$ and $p^i_{l^*}(L^i_l)$ are the optimal prices for a fixed express lead time of firm $i$, given by:

\[
p^i_{h^*}(L^i_l) = \frac{(\beta^i_p + 2\theta_p + \gamma_p)a^i - (\beta^i_p(\theta_L + \gamma_L) + \beta^i_h(\beta^i_p + \theta_p + \gamma_p) + \theta_p\gamma_L + \gamma_L\gamma_p + \theta_L\gamma_p)L^i_h}{2D} + \frac{[(\beta^i_p + \gamma_p)\theta_L - (\beta^i_L + \gamma_L)\theta_p]L^i_l + (\beta^i_p\gamma_p + \gamma_p\theta_p + \gamma_p^2)\beta^i_h + (\theta_p\gamma_p)\beta^i_l}{2D} + \frac{A^i + m^i}{2}
\]

\[
p^i_{l^*}(L^i_l) = \frac{(\beta^i_p + 2\theta_p + \gamma_p)a^i - (\beta^i_p(\theta_L + \gamma_L) + \beta^i_l(\beta^i_p + \theta_p + \gamma_p) + \theta_p\gamma_L + \gamma_L\gamma_p + \theta_L\gamma_p)L^i_l}{2D} + \frac{[(\beta^i_p + \gamma_p)\theta_L - (\beta^i_L + \gamma_L)\theta_p]L^i_l + (\theta_p\gamma_p)\beta^i_h + (\beta^i_p\gamma_p + \gamma_p\theta_p + \gamma_p^2)\beta^i_l}{2D} + \frac{A^i + m^i}{2}
\]

and $D = \beta^i_p\beta^i_l + \beta^i_h\theta_p + \beta^i_l\theta_p + \beta^i_p\gamma_p + \beta^i_l\gamma_p + 2\theta_p\gamma_p + \gamma_p^2$.

The corresponding optimal price differentiation is then:

\[
\frac{p^i_{h^*}(L^i_l) - p^i_{l^*}(L^i_l)}{2D} = \frac{((\beta^i_l - \beta^i_h)a^i + (\beta^i_h + \beta^i_l)\theta_L + (\gamma_L + 2\theta_L)\gamma_p)(L^i_l - L^i_h)}{2D} - \frac{-(\beta^i_p\beta^i_l + \beta^i_h\gamma_p + \beta^i_l\gamma_p)L^i_h + (\beta^i_p\beta^i_l + \beta^i_h\gamma_p + \beta^i_p\gamma_p)L^i_l + (\beta^i_p + \gamma_p)\gamma_p\beta^i_h}{2D} - \frac{-(\beta^i_p + \gamma_p)\gamma_p\beta^i_l + (\beta^i_p + \gamma_p)\gamma_pL^i_h - (\beta^i_p + \gamma_p)\gamma_pL^i_l}{2D}
\]
**Proof**  See Appendix A.

**Proposition 2**  Given a strategy \( s^j := (p^j, \mu^j, L^j) \) of firm \( j = 3 - i \), a decrease in the express lead time \( L^j_h \) by firm \( i \in \{1, 2\} \) using dedicated capacities results in: (a) an increase in \( p^i_s \); (b) a decrease in \( p^i_s \) if \( \theta_L/(\beta^i_L + \gamma_L) > \theta_p/(\beta^i_p + \gamma_p) \); an increase in \( p^i_s \) if \( \theta_L/(\beta^i_L + \gamma_L) < \theta_p/(\beta^i_p + \gamma_p) \); no change in \( p^i_s \) otherwise.

**Proof**  Follows directly from Proposition 1.

Proposition 2 suggests that given the decisions of the other firm, a firm’s best response express price always increases with a decrease in its own express lead time, which is intuitive. The effect of any change in its express lead time on the regular price, however, depends on the behaviour of the market. Specifically, the regular price decreases with any decrease in its express lead time if the relative sensitivity of customers to the inter-class lead time difference (with respect to their own lead time and inter-firm lead time difference) is greater than their relative sensitivity to the inter-class price difference (with respect to their own price and inter-firm price difference), such that \( \theta_L/(\beta^i_L + \gamma_L) > \theta_p/(\beta^i_p + \gamma_p) \), \( c \in \{h, l\} \). We call such a market as Time Difference Sensitive (TDS). On the other hand, a decrease in its express lead time increases its regular prices if the relative sensitivity of customers to the inter-class lead time difference (with respect to their own price and inter-firm lead time difference) is smaller than their relative sensitivity to the inter-class price difference (with respect to their own price and inter-firm price difference), such that \( \theta_L/(\beta^i_L + \gamma_L) < \theta_p/(\beta^i_p + \gamma_p) \), \( c \in \{h, l\} \). We call such a market as Price Difference Sensitive (PDS).

### 4.2 Shared capacity setting

The firm’s choice of shared capacity is modelled using a single server, which serves both customer classes employing a simple fixed priority scheme that always gives priority to time-sensitive customers. Customers within each class are served to a non-preemptive priority discipline.

For a shared capacity setting, the sojourn time distribution \( S^i_h(\cdot) \) for high-priority customers in a preemptive priority queue is known to be exponential (Chang 1965). Hence, the lead time reliability constraint (7) has an analytical closed-form representation, similar to that for the dedicated capacity setting. We assume the single server serves customers of either class at the same rate \( \mu^i = \mu^j = \mu^l \). Constraints (6) and (7) in a shared capacity setting can then be expressed as:

\[
\lambda^j + \lambda^j - \mu^j < 0 \tag{6SC}
\]

\[
S^i_h(L^j_h) = P(W^i_h \leq L^j_h) = 1 - e^{(\lambda^j - \mu^j)L^j_h} \geq \alpha \quad \tag{7SC}
\]

However, a closed-form expression for the sojourn time distribution \( S^i_l(\cdot) \) for low-priority customers, appearing in constraint (8), is not known (see Jayaswal, Jewkes, and Ray 2011, for a detailed discussion). Further, even an analytical characterisation of the sojourn time distribution or a good approximation will not produce an analytical solution similar to that for the dedicated capacity setting since it cannot be guaranteed at the outset which of the constraints will be binding at optimality. So, the model in a shared capacity setting does not lend itself to an easy solution using conventional optimisation methods. We resolve this difficulty by solving it in two stages. We first solve the problem for a fixed \( L^j_h \) (we term it as pricing decision problem \( [PDP^j_{SC}] \)) numerically using the matrix geometric method in a cutting plane framework (see Appendices B, C and D). Solution to \( [PDP^j_{SC}] \) is then used to solve the pricing and lead time decision problem \( [PDP^i_{SC}] \) using the golden section search method. In the following, we discuss the solution method very briefly, and refer our readers to Jayaswal (2009), Jayaswal, Jewkes, and Ray (2011) for the details.

#### 4.2.1 The pricing decision problem \( [PDP^j_{SC}] \)

On substituting (1) and (2) into (3), the objective function for \( [PDP] \) is quadratic. All constraints are linear, except for (8), which does not have a closed-form expression. Although the exact form of \( S^i_l(\cdot) \) in constraint (8) is unknown, we exploit its special structure, determined numerically using the matrix geometric method (see Appendix A). Plots of \( S^i_l(\cdot) \) vs. \((p^i_h, p^i_l)\), and \( S^i_l(\cdot) \) vs. \( \mu^j \) are shown in Figure 1. These plots suggest that \( S^i_l(\cdot) \) is concave in \((p^i_h, p^i_l)\) and separately in \( \mu^j \). However, this does not necessarily show the joint concavity of \( S^i_l(\cdot) \) in \((p^i_h, p^i_l, \mu^j)\). We will, therefore, integrate into our solution method a mechanism to ensure that the concavity assumption is not violated (see Appendix D).
Assuming $S_i^i(\cdot)$ is concave, it can be approximated by a set of tangent hyperplanes at various points $(p_{ik}^i, p_{ik}^i, \mu_{ik})$, $\forall k \in K$, that is:

$$ S_i^i(\cdot) = \min_{k \in K} \left\{ S_i^i(\cdot) + (p_k^i - p_{ik}^i) \left( \frac{\partial S_i^i(\cdot)}{\partial p_h^i} \right) + (p_k^i - p_{ik}^i) \left( \frac{\partial S_i^i(\cdot)}{\partial p_l^i} \right) + (\mu - \mu_{ik}) \left( \frac{\partial S_i^i(\cdot)}{\partial \mu^i} \right) \right\} $$

where $S_i^i(\cdot)$ denotes the value of $S_i^i(\cdot)$ at a fixed point $(p_{ik}^i, p_{ik}^i, \mu_{ik})$, $\frac{\partial S_i^i(\cdot)}{\partial p_h^i}$, $\frac{\partial S_i^i(\cdot)}{\partial p_l^i}$, and $\frac{\partial S_i^i(\cdot)}{\partial \mu^i}$ are the partial gradients of $S_i^i(\cdot)$ at $(p_{ik}^i, p_{ik}^i, \mu_{ik})$, which can be obtained using the finite difference method, described in Appendix C. Constraint (8) in a shared capacity setting can thus be replaced by the following set of linear constraints:

$$ S_i^{ik}(\cdot) + (p_k^i - p_{ik}^i) \left( \frac{\partial S_i^{ik}(\cdot)}{\partial p_h^i} \right) + (p_k^i - p_{ik}^i) \left( \frac{\partial S_i^{ik}(\cdot)}{\partial p_l^i} \right) + (\mu - \mu_{ik}) \left( \frac{\partial S_i^{ik}(\cdot)}{\partial \mu^i} \right) \geq \alpha \quad \forall k \in K $$

Substituting the above set of constraints in place of (8), and the expressions (1) and (2) for $\lambda_i^j$ and $\lambda_i^j$ results in the following quadratic programming problem (QPP) with a finite but a large number of constraints, which makes it suitable for the cutting plane method (Kelley 1960).

$$ \text{[PDP]}: \max_{p_h^i, p_l^i, \mu^i} \pi^i = -(\beta_h^i + \theta_p + \gamma_p)(p_h^i)^2 - (\beta_p^i + \theta_p + \gamma_p)(p_l^i)^2 + 2\theta_p p_h^i p_l^i $$

subject to:

$$ p_h^i, p_l^i, \mu^i \geq 0 $$

$$ - (\beta_h^i + \theta_p + \gamma_p)p_h^i + \theta_p p_l^i \geq (\beta_p^i + \theta_p + \gamma_p)L_h^i - \theta_L L_h^i - \gamma_p L_l^i - a^i $$

$$ - (\beta_p^i + \theta_p + \gamma_p)p_l^i \geq -\theta_L L_h^i + (\beta_p^i + \theta_p + \gamma_p)L_l^i - p_l^i - a^i $$

$$ - (\beta_h^i + \gamma_p)p_h^i - (\beta_p^i + \gamma_p)p_l^i - \mu^i $$

$$ < (\beta_h^i + \gamma_p)L_h^i + (\beta_p^i + \gamma_p)L_l^i - \gamma_p(p_h^i + p_l^i) - \gamma_p L_h^i - a^i $$

Figure 1. Service level vs. prices and capacity.
Appendix D).

To investigate the effect of capacity strategy on equilibrium price and lead time decisions, we study the following three scenarios: (1) DD: both firms using dedicated capacities; (ii) SS: both firms using shared capacities; (iii) DS: firm 1 using dedicated capacities while firm 2 using shared capacities. However, we study the DD scenario in a greater detail as it allows us to obtain analytical results.

4.2.2 The pricing and lead time decision problem \[ P L D P_{i}^{SC} \]

The pricing and lead time decision problem \[ P L D P_{i}^{SC} \] adds an additional dimension to \[ P D P_{i}^{SC} \] by treating \( L_{h} \) as a decision variable, which the firm tries to jointly optimise along with \( p_{h}^{i} \), \( p_{L}^{i} \) and \( \mu^{i} \) for a given strategy of firm \( j \neq i \). This makes constraint \( (7^{SC}) \) non-linear, and the model substantially more challenging to solve. We use the solution to \[ P D P_{i}^{SC} \] and the golden section search method (Luenberger 1984) to solve \[ P L D P_{i}^{SC} \], which can be rewritten as:

\[
\max_{L_{h}^{i} \in [0, L_{i}]} f(L_{h}^{i})
\]

where \( f(L_{h}^{i}) \) is a \[ P D P_{i}^{SC} \] for a fixed \( L_{h}^{i} \). We have shown in a dedicated capacity setting that \( f(L_{h}^{i}) \) has a unique maximum when \( a^{i} \) is high (see Appendix A). Our extensive numerical experiments with \( L_{h}^{i} \) suggest that a sufficiently large \( a^{i} \) guarantees that \( f(L_{h}^{i}) \) has a unique maximum in a shared capacity setting as well, and hence \[ P L D P_{i}^{SC} \] can be solved efficiently using the golden section search method. At each step, the algorithm solves a \[ P D P_{i}^{SC} \] to evaluate \( f(L_{h}^{i}) \) for a given value of \( L_{h}^{i} \).

5. Duopoly problem

We now study the price and lead time decisions for a duopoly problem. One basic question is to investigate whether an equilibrium exists, and if so, how does the equilibrium change under different operational settings and market characteristics.

To investigate the effect of capacity strategy on equilibrium price and lead time decisions, we study the following three scenarios: (1) DD: both firms using dedicated capacities; (ii) SS: both firms using shared capacities; (iii) DS: firm 1 using dedicated capacities while firm 2 using shared capacities. However, we study the DD scenario in a greater detail as it allows us to obtain analytical results.

Under competition, both firms simultaneously announce their price and lead time decisions. We assume that firm \( i \in \{1, 2\} \) has full knowledge of the operational setting of firm \( j = 3 - i \), including its capacity strategy and also its parameters \( A, m \) and \( a \). Firm \( i \) can thus correctly anticipate the best response of firm \( j \) to its own moves, and can hence strategically plan its own strategy. A Nash equilibrium is a vector of strategies \((s^{i*}, s^{j*})\) such that for each firm \( i, \pi^{i}(s^{i*}, s^{j*}) = \max_{s^{i}} \pi^{i}(s^{i}, s^{j*}), \) \( i \in \{1, 2\} \) and \( j = 3 - i \). In other words, the strategy used by either firm is the best response to the strategy chosen by the other.

We first study the equilibrium results under pure price competition in a DD scenario.
5.1 Pure price competition in a DD setting

Proposition 1 gives the best response prices of a firm. Thus, when the lead time decisions are fixed, such that firms compete purely using prices, equilibrium prices can be obtained in closed form by the simultaneous solution of the four linear equations given by (10) and (11) (two equations corresponding to each $i \in \{1, 2\}$).

**Proposition 3** Pure price competition in a DD setting has a Nash equilibrium. Further, if the firms are identical, then the equilibrium prices are symmetric. The corresponding price differentiation for a given $L_h$ is then:

$$p^*_h(L_h) - p^*_l(L_h) = \frac{2(\beta^l_p - \beta^h_p)a + 2(\beta^l_p + \beta^h_p + \gamma_p)\theta_L(L - L_h) + \beta^l_L(2\beta^h_p + \gamma_p)L}{D_1} - \left(2\beta^l_p + \gamma_p\right)\beta^h_L L_h + \left(\beta^l_p - \beta^h_p\right)\gamma_p(A + m)$$

(21)

**Proof** See Appendix E.

5.1.1 Effect of pure price competition in a DD setting

We now study the effect of price competition on a firm’s price and lead time decisions in a dedicated capacity setting. We know competition generally drives prices down. But how does competition affect price differentiation/discrimination? To answer this, we compare the optimal prices of a monopolist with its equilibrium prices when it faces price competition from an identical firm. A monopolist setting can be represented as a special case of our competitive model with two identical firms, each with a market base $2a$, but with $\gamma_p = \gamma_L = 0$. This represents two identical firms operating in (geographically or otherwise) different markets such that they do not poach each other’s market share. In contrast, a competitive setting represents a situation in which two firms operate in the same market, each with a market base $2a$, such that each firm’s demand is affected by the relative prices of the two firms. Mathematically, this corresponds to $\gamma_p > 0, \gamma_L > 0$ in our competitive model.

**Proposition 4** Pure price competition in a dedicated capacity setting always results in: (a) a lower express price $p^*_h$, (a) a lower regular price $p^*_l$, and (c) a lower price differentiation $(p^*_h - p^*_l)$. Further, the effects are more pronounced in presence of product substitution.

**Proof** See Appendix F

5.2 Price and lead time competition in a DD setting

We now consider a more general situation wherein firms have flexibility in quoting the lead times to their express customers. We still assume there is a standard lead time for regular customers established by the industry (Section 3 discusses the situations in which a standard lead time for regular customers is justified). The equilibrium express lead times are given by the simultaneous solution of the system of two non-linear equations, given by (9) = 0 for $i = 1, 2$. In the absence of a closed-form solution for this system of non-linear equations, we design an iterative procedure, described in Algorithm 1, that always converges to the equilibrium solution. We solve for an equilibrium solution assuming the game is played dynamically, starting at an initial solution, until none of the firms has an incentive to deviate from its decision unilaterally.

**Proposition 5** The iterative procedure given in Algorithm 1 converges to a Nash Equilibrium for a DD setting.

**Proof** See Appendix G.
Algorithm 1. Iterative Algorithm for Nash Equilibrium

1: Initialisation: For each firm $i$, set $p^h_i = p^l_i = m^l, L^h_i = 0$ or $L^l_i = L_i$.
2: Iterative step: Start with $i = 1$. Use the best response obtained for Firm $i$ problem. Repeat this for $i = 2$.
3: Convergence criteria: Repeat step 2 until each firm’s decision values differ from their previous values by less than some predetermined tolerance level $\epsilon$.

Table 1. Market parameters used in numerical examples.

| Market Type | $\beta^h_p$ | $\beta^l_p$ | $\theta_p$ | $\beta^h_L$ | $\beta^l_L$ | $\theta_L$ | $\gamma_p$ | $\gamma_L$ |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TDS         | 0.55        | 0.75        | 0.15        | 0.9         | 0.7         | 0.5         | 0.4         | 0.4         |
| PDS         | 0.5         | 0.7         | 0.4         | 0.9         | 0.7         | 0.1         | 0.4         | 0.4         |

5.2.1 Effect of price and lead time competition

When firms use lead time, in addition to price, as a strategic tool to attract demand and compete in the market, this leads to another question of interest: how does competition affect both price and lead time differentiation? To answer this, we compare the equilibrium prices and lead time decisions in a competitive setting with that under a monopolistic setting. The effect of competition on price and lead time differentiation, in general, depends on the relative intensities of price competition ($\gamma_p$) and lead time competition ($\gamma_L$), as well as other demand parameters. The following proposition summarises the effect of competition for special cases.

**Proposition 6** Price and lead time competition in a dedicated capacity setting: (a) decreases both lead time differentiation and price differentiation when $\gamma_L = 0$; (b) increases both lead time differentiation and price differentiation when $\gamma_p = 0$.

**Proof** See Appendix H

The above proposition suggests that price and lead time competition may increase or decrease price and lead time differentiation, depending on customers’ behaviour. This is intuitive. When $\gamma_L = 0$, customers’ choice of a firm is not influenced by the relative lead times but by the relative prices offered by the two firms. In such a situation, firms tend to cut prices to attract customers. At the same time, they increase their express lead time (and hence decrease their lead time differentiation) in order to cut their capacity cost and maintain their profit. It further follows from (21) that a smaller lead time differentiation (and hence larger express lead time) also results in a smaller price differentiation. On the other hand, when $\gamma_p = 0$, customers’ choice of a firm is not influenced by the relative prices but by the relative lead times offered by the two firms. In such a situation, firms try to cut their lead times to attract customers. This results in a smaller express lead time, and hence a larger lead time differentiation. Again, it follows from (21) that a larger lead time differentiation also allows the firms to maintain a larger price differentiation.

5.3 Effect of capacity strategy

As discussed in Section 1, different firms, even in the same industry, use different capacity strategies (for example, FedEx versus UPS). A natural question then is: how does firms’ capacity strategy affect equilibrium price and lead time decisions? To answer this, we compare the equilibrium decisions of otherwise symmetric firms under the three scenarios (DD, SS and DS). The best response of a firm in a shared capacity setting, however, lacks an analytical characterisation. Therefore, it is not possible to derive analytical results for equilibrium decisions when at least one of the competing firms uses shared capacities for its different market segments. We, therefore, test our models numerically under different combinations of parameter values. Our numerical results suggest that the price and lead time competition always has a unique equilibrium, obtained using Algorithm 1. Generalisations based on observable patterns that emerge from these numerical experiments are reported as observations.

We present a small sample from our extensive numerical experiments to illustrate the effect of capacity strategy on equilibrium decisions. We use the parameter setting described in Tables 1 and 2 for two levels of capacity cost: (i) $A = 0.01$ (for small capacity cost) and (ii) $A = 0.25$ (for large capacity cost). A comparison of the equilibrium prices and lead times in an SS versus a DD setting is shown in Table 3, and for a DS setting is shown in Table 4. Results from the numerical experiments are summarised in the following Observation.
Table 2. Firm-specific parameters used in numerical examples

|       | Firm 1 |       | Firm 2 |
|-------|--------|-------|--------|
| $a^1$ | 10     | $m^1$ | 3      | $a^2$ | 10 | $m^2$ | 3 |
| $\alpha^1$ | 0.99 | $L^1_l$ | 1      | $\alpha^2$ | 0.99 | $L^2_l$ | 1 |

Figure 2. Effects of capacity cost asymmetry on product differentiation decisions in a DD setting.

(a) Delivery time differentiation versus marginal capacity cost when capacity cost is low
(b) Price differentiation versus marginal capacity cost when capacity cost is low
(c) Delivery time differentiation versus marginal capacity cost when capacity cost is high
(d) Price differentiation versus marginal capacity cost when capacity cost is high

Observation 1:

- Price and lead time competition under SS, compared to DD, results in: (a) a larger price differentiation at equilibrium, and (b) a larger lead time differentiation at equilibrium if capacity cost is high, but a smaller lead time differentiation when capacity cost is small.
- Price and lead time competition under DS results in: (a) a larger price differentiation at equilibrium for the firm using shared capacities, and (b) a larger lead time differentiation at equilibrium for the firm using shared capacities if capacity cost is high, but a smaller lead time differentiation for the firm using shared capacities when capacity cost is small.
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Figure 3. Effects of capacity cost asymmetry on product differentiation decisions in an SS setting.

(a) Delivery time differentiation versus marginal capacity cost when capacity cost is low

(b) Price differentiation versus marginal capacity cost when capacity cost is low

(c) Delivery time differentiation versus marginal capacity cost when capacity cost is high

(d) Price differentiation versus marginal capacity cost when capacity cost is high

Table 3. Numerical results for DD and SS settings.

|       | $A = 0.01$ |       | $A = 0.25$ |
|-------|------------|-------|------------|
|       | $PDS$     | $TDS$ | $PDS$     | $TDS$     |
|       | DD         | SS    | DD         | SS         |
| $L_h^*$ | 0.079835   | 0.07984 | 0.075916   | 0.075927   |
| $L_l - L_h^*$ | 0.920165   | 0.92016 | 0.924084   | 0.924073   |
| $p_h^*$ | 8.48642    | 8.4861  | 8.487575   | 8.487409   |
| $p_l^*$ | 7.4262     | 7.42033 | 6.748059   | 6.742129   |
| $p_h^* - p_l^*$ | 1.06022    | 1.06577 | 1.739516   | 1.74528    |

|       | $DD$       | SS     | $DD$       | SS         |
|-------|------------|--------|------------|------------|
| $L_h^*$ | 0.40887    | 0.375944 | 0.393785   | 0.382392   |
| $L_l - L_h^*$ | 0.59113    | 0.624056 | 0.606215   | 0.617608   |
| $p_h^*$ | 8.475324   | 8.496719 | 8.397922   | 8.407862   |
| $p_l^*$ | 7.536797   | 7.410724 | 6.933531   | 6.790338   |
| $p_h^* - p_l^*$ | 0.938345   | 1.085995 | 1.464391   | 1.617524   |

5.4 Effect of asymmetry between firms

When the competing firms are asymmetric, they will try to exploit their competitive advantage of a lower capacity cost $A$, or a higher market base $a$. We study the effects of such asymmetries on their equilibrium decisions.
5.4.1 Asymmetry in capacity cost

**Observation 2:** If one of the firms, which are otherwise identical, has a higher capacity cost, then compared to the other firm at equilibrium:

Table 4. Numerical results for DS setting

|       | PDS | TDS |
|-------|-----|-----|
|       | D   | S   | D   | S   |
| $L_h^*$ | 0.079836 | 0.079838 | 0.075915 | 0.075928 |
| $L_i - L_h^*$ | 0.920164 | 0.920162 | 0.924085 | 0.924072 |
| $p_h^*$ | 8.486179 | 8.486345 | 8.48746 | 8.487527 |
| $p_i$ | 7.425453 | 7.42108 | 6.747257 | 6.742933 |
| $p_h - p_i^*$ | 1.060726 | 1.065265 | 1.740203 | 1.744595 |

|       | PDS | TDS |
|-------|-----|-----|
|       | D   | S   | D   | S   |
| $L_h^*$ | 0.409076 | 0.375427 | 0.393641 | 0.381862 |
| $L_i - L_h^*$ | 0.590924 | 0.624573 | 0.606359 | 0.618138 |
| $p_h^*$ | 8.469203 | 8.502327 | 8.395587 | 8.410847 |
| $p_i$ | 7.520666 | 7.426818 | 6.914207 | 6.809853 |
| $p_h - p_i^*$ | 0.948537 | 1.075509 | 1.481381 | 1.600994 |
• in a DD setting, it has (a) a smaller lead time differentiation, and (b) a smaller price differentiation (Refer to Figure 2).
• in an SS setting, it has (a) a smaller lead time differentiation, and (b) a smaller price differentiation if the absolute capacity costs are very small, but a larger price differentiation if the absolute capacity costs are high (Refer to 3).

Figure 2 shows the equilibrium price and lead time differentiations of the two firms in a DD setting that differ in their capacity costs but are otherwise identical. Figure 3 shows similar plots for an SS setting. Although we present these plots for market parameter values shown in Table 1 corresponding to a PDS-type market, the qualitative results are independent of the specific market parameters. Firm-specific parameters are as shown in Table 2. In one set of experiments, we fix the capacity cost of firm 1, \( A^1 \), at 0.01 and vary that for firm 2, \( A^2 \), from 0.01 to 0.10. In another set of experiments, we fix \( A^1 \) at 0.25 and vary \( A^2 \) from 0.25 to 1.0. This helps us capture the effect of a larger capacity cost incurred by firm 2 on the decisions of the two firms at equilibrium. As evident from the plots, when the firms are symmetric (\( A^2 = A^1 \)), the lead time and price differentiations of both firms coincide. Any increase in firm 2’s capacity cost (\( A^2 \)) always decreases its lead time differentiation at equilibrium, irrespective of the capacity strategy used by the two firms. An increase in \( A^2 \) also decreases firm 2’s price differentiation at equilibrium in a DD setting. In an SS setting, an increase in \( A^2 \) decreases firm 2’s price differentiation only when \( A^1 \) and \( A^2 \) are still small (\( \leq 0.1 \)); when \( A^1 \) and \( A^2 \) are high (\( \geq 0.25 \)), an increase in \( A^2 \) increases firm 2’s price differentiation. However, the effect of an increase in firm 2’s capacity cost \( A^2 \) may have a similar or a contrasting effect on firm 1, depending on the market parameters and the level of the capacity cost. Whatever the effects on individual firms, when \( A^2 > A^1 \), firm 2 always has a smaller lead time differentiation and a smaller price differentiation in a DD setting, but a higher price differentiation for larger absolute capacity costs in an SS setting.

5.4.2 Asymmetry in market base

**Observation 3:** If one of the firms, which are otherwise identical, has a larger market base, then compared to the other firm at equilibrium:
- it always has (a) a larger lead time differentiation, and (b) a larger price differentiation, irrespective of the capacity strategy of either firm (Refer to Figures 4 and 5).

We illustrate this result using a sample from our numerical experiments. We consider two firms that have different market bases (\( a^1 \neq a^2 \)), but are otherwise identical. Difference in the market bases of the two firms means that one firm always has a higher mean demand even if they both offer the same lead times at the same prices. This may be the result of a difference in their brand appeal to the customers or due to a more convenient locations or a better customer experience at one of the firms. We assume the market is PDS type (parameter values shown in Table 1), although the generalisations drawn are independent of the specific market parameters. Firm-specific parameters are as shown in Table 2. The market base \( a^1 \) for firm 1 is now fixed at 10, while that for firm 2 (\( a^2 \)) is varied. Figures 4 and 5 show the equilibrium price and lead time differentiations of the two firms in a DD and an SS setting, respectively. This helps us capture the effect of a larger market base of firm 2 on the decisions of the two firms at equilibrium. As evident from the plots, when the firms are symmetric (\( a^2 = a^1 \)), the lead time and price differentiations of both the firms coincide. Any increase in firm 2’s market base (\( a^2 \)) increases its lead time differentiation as well as the price differentiation at equilibrium. Although firm 1’s price and lead time differentiation decisions also increase with \( a^2 \) in this case, this is specific only to this set of market parameters. In general, the behaviour of firm 1’s decisions depends on the market parameters. Whatever be the effects on individual firms, when \( a^2 > a^1 \), firm 2 always has a larger lead time differentiation and a larger price differentiation, irrespective of the capacity strategy and market parameters.

6. Conclusion

In this paper, we studied the product differentiation strategies of competing firms, which may use different capacity strategies. Our primary objective was to understand the effect of competition on a firm’s product differentiation strategy in the presence of different capacity strategies (dedicated versus shared) used by competing firms. For this, we developed a general mathematical model, special cases of which captures a firm’s best response to its competitor’s decisions, depending on whether it uses dedicated or shared capacities to serve different market segments.

Our study provides insights into the effect of competition on price differentiation/discrimination. We showed that when firms use dedicated capacities, pure price competition always reduces individual prices as well as price differentiation (Proposition 4). However, when firms use lead times, in addition to prices, as strategic variables to compete in the market, the effect of competition on product differentiation further depends on customers’ behaviour (Proposition 6). Our study also brings out the effect of firms’ capacity strategy on their price and lead time differentiation decisions. Specifically, when the
processing capacities are expensive, then the firm with shared capacities should offer more differentiated products (both in prices as well as lead times) to the two market segments, compared to the firm with dedicated capacities. However, when the capacity cost is small, then the firm using shared capacities should offer less differentiated lead times (Observation 1). When asymmetry in capacity costs exists between firms, the way a firm should exploit its lower capacity cost depends on its own capacity strategy as well as that of its competitor. Specifically, the firm with cheaper capacities should make its products more differentiated if both firms use dedicated capacities. If both firms use shared capacities, then the firm with cheaper capacities should again make its lead times more differentiated, but whether it should offer more homogeneous or more differentiated prices depends further on the capacity cost levels (high or low) for the two firms (Observation 2). Our results further suggest that the firm with a larger market base should always offer more differentiated products, irrespective of the capacity strategy of either firm (Observation 3).

**Disclosure statement**

No potential conflict of interest was reported by the authors.

**Supplemental data**

Supplemental material containing Appendices A to H is available online [http://dx.doi.org/10.1080/00207543.2016.1145816](http://dx.doi.org/10.1080/00207543.2016.1145816).

**Notes**

1. $A^i$ may be different for different customer classes if they are served by different service capacities. Using the same marginal capacity cost for the two customer classes, however, allows a meaningful comparison between the dedicated and the shared capacity settings.

2. Although we make this assumption mainly for the tractability of the model, this is still a reasonably realistic representation of certain business settings. For example, in most of the online retail web hosting services, any updating of content, if not done in express fashion, must be done within one day (Boyaci and Ray 2003).

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