APPLICATION OF CURRENT ALGEBRA OR CHIRAL SYMMETRY TO TAU HADRONIC DECAYS\textsuperscript{*}

L. BELDJoudi  
Centre de Physique Théorique, Ecole Polytechnique  
91128 Palaiseau, France  
E-mail: beljoudi@orphee.polytechnique.fr

and

T. N. TRUONG  
Centre de Physique Théorique, Ecole Polytechnique  
E-mail: truong@orphee.polytechnique.fr

Abstract  

$\tau \to \pi\pi\nu$, $\tau \to \pi K\nu$, $\tau \to K\eta\nu$, $\tau \to 3\pi\nu$ and $\tau \to \pi\pi K\nu$ have been investigated using chiral symmetry with dispersion relation in agreement with the unitarity condition.

1. Introduction

Current Algebra was invented a long time ago to study phenomena involving emission of soft pions (and kaons) and also the chiral symmetry breaking effects. It was soon discovered that the current algebra low energy theorems (LET) can be obtained by the effective lagrangian method at the tree graph order. Using it, we can calculate matrix elements of physical processes by the Feynman perturbation method in a straightforward way. Because of the chiral properties of the Nambu-Goldstone bosons, the effective lagrangian involves the derivatives of the pion fields which make the theory non-renormalisable. Chiral Perturbation Theory ($\chi$PT) is a well defined perturbation procedure which can take into account in a systematic way of higher orders (higher loops) and of the chiral symmetry breaking effects. At any order, these relations form low energy theorems of QCD. Despite the beauty of the method, it has limitations in phenomenological applications. We distinguish three main problems with this theory:

1) Because of the non-renormalisability of the theory, the number of parameters increases as we calculate higher loops.

2) $\chi$PT is effectively a power series expansion in terms of pion momenta, hence it cannot take into account of the resonance effects (Breit

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Wigner form cannot be expanded in a convergent power series of momenta. It can only explain the low energy tail of the resonance.

3) Because $\chi$PT is a perturbation theory, it only satisfies the unitarity relation order by order. This approximation is not good enough for strong interaction or for resonance physics.

Because of these problems, at first sight, $\chi$PT cannot be used to study Tau physics where most of the hadronic modes are dominated by resonances and where the pions emitted are not soft. The remedy for these problems were given a long time ago even before $\chi$PT became popular. One method is to use the Current Algebra soft pion theorems and use dispersion relation to take into account of the unitarity. The second method is to extend the validity of the $\chi$PT by resumming the perturbation series in order to take into account of the unitarity relation. The third method is to assume the vector meson dominance in the electroweak processes and requiring at low energy or at zero momentum transfer the low energy theorem is recovered. In fact, the three methods are equivalent as they all satisfy the LET and the unitarity relation to a good approximation.

Using the basic Equal Time Commutator Relations (ETCR) as postulated by Gell-Mann,

$$\left[ Q^a_V V^b_{\mu}(x) \right] = i f^{abc} V^c_{\mu}(x) \quad (1)$$
$$\left[ Q^a_A A^b_{\mu}(x) \right] = i f^{abc} V^c_{\mu}(x) \quad (2)$$
$$\left[ Q^a_V A^b_{\mu}(x) \right] = i f^{abc} A^c_{\mu}(x) \quad (3)$$

$V^a_{\mu}$ and $A^a_{\mu}$ are the vector and axial currents generated by the approximate symmetry $SU(3)_L \times SU(3)_R$ of QCD, $Q^a_V$ and $Q^a_A$ are the corresponding charges.

Together with the basic Current Algebra formula for a soft pion emission:

$$\lim_{k_\mu \to 0} \langle \pi^a(k) B | O(0) | A \rangle = - \frac{i}{f_\pi} \langle B | [Q^a_A, O(0)] | A \rangle \quad (4)$$

Eq(4) sets the scale for the relevant matrix element. In the dispersion relation language it can be used as a substraction constant for a substracted dispersion relations.

More explicitly, let us denote the Current Algebra result as $A_{LET}(0)$, thus the substracted dispersion relation can be written as follows:

$$A(s) = A_{LET}(0) + \frac{s}{\pi} \int_{s_0}^{\infty} dz \frac{\text{Im} A(z)}{z(z - s - i\epsilon)} \quad (5)$$

Because of the substracted dispersion relation, the high energy contribution in the dispersion integral, which is difficult to calculate reliably, is
suppressed. This result is in contrast with the physics done in the 60’s (e.g. Bootstrap) which involves unsubtracted dispersion relation with uncontrollable approximation (because the high energy contribution is not suppressed).

In the physical world, where the pion are not soft, corrections must be made. First the chiral symmetry breaking effect can be taken into account approximately by hand. The second correction to be made is to impose unitarity relation which is crucial for hadronic processes. This can be done either by strong interaction dynamics models e.g. Vector Meson Dominance or some unitarisation schemes.

Some of these relations were calculated in 1978\[1\]. It is desirable to update these calculations with a more sophisticated technique in order to compare with more recent experimental data and also to the $\chi$PT technique. As an illustration we present applications to $\tau \to \pi\pi\nu$, $\tau \to \pi K\nu$, $\tau \to \pi^*\nu$, $\tau \to \rho\nu$, and $\tau \to \rho\nu$ decays.

2. $\tau \to \pi\pi\nu$ decay

Using the CVC hypothesis and Lorentz invariance, one can write straightforwardly the following matrix elements:

$$\langle \pi^a(p_1)\pi^b(p_2) | V^c_\mu(0)|0 \rangle = i\epsilon^{abc} f(s)(p_2 - p_1)_\mu$$  \hspace{1cm} (6)
$$\langle \pi^a(p_1)\pi^b(p_2) | \hat{m}(\bar{u}u + \bar{d}d)|0 \rangle = \delta^{ab} \Gamma(s)$$  \hspace{1cm} (7)

At zero momentum transfer we have the following normalization: $f(0) = 1$ and $\Gamma(0) = m^2_\pi$. Assuming the elastic unitarity condition, we deduce the following relations:

$$\text{Im} f(s) = f(s) \exp(-i\delta^1_1 \sin \delta^1_1)$$
$$\text{Im} \Gamma(s) = \Gamma(s) \exp(-i\delta^0_0 \sin \delta^0_0)$$  \hspace{1cm} (8)

Hence $f(s)$ must have the phase of the P wave $\pi\pi$ phase shift, and $\Gamma(s)$ the S wave phase $\pi\pi$ shift.

The general solutions to this equation are well known, they are of the Muskhelishvili Omnès type\[2\]:

$$f(s) = P_f(s)\Omega_1(s)$$  \hspace{1cm} (9)
$$\Gamma(s) = \Gamma(0)P_T(s)\Omega_0(s)$$  \hspace{1cm} (10)

where

$$\Omega_1(s) = \exp\left(\frac{s}{\pi} \int_{4m^2_\pi}^{\infty} \frac{\delta^1_1 dz}{z(z - s - i\epsilon)}\right)$$  \hspace{1cm} (11)
\[ \Omega_0(s) = \exp \left( \frac{s}{\pi} \int_0^\infty \frac{\delta_0^2 dz}{z(z - s - i\epsilon)} \right) \] (12)

\( P_f \) and \( P_\Gamma \) are polynomials normalized to unity at \( s = 0 \) which determine the high energy behavior of the form factors. They could also represent the low energy contribution of the higher mass intermediate states to the form factors. In the following we assume the dominance of the elastic unitarity relation and hence we set \( P_f(s) = P_\Gamma(s) = 1 \).

At one loop chiral perturbation theory we have:

\[
\begin{align*}
    f(s) &= 1 + \frac{r^2}{6} s - \left(96\pi f_\pi^2\right)^{-1} \left[ (s - 4m_\pi^2)(h(s) - h(0)) + 4m_\pi^2 h'(0)s \right] \\
    \Gamma(s) &= \frac{\Gamma(0)}{1 - \frac{r^2}{6} s - \left(16\pi f_\pi^2\right)^{-1} \left[ (s - m_\pi^2)(h(s) - h(0)) + \frac{m_\pi^2}{2} s h'(0) \right]} \tag{13}
\end{align*}
\]

\[
\begin{align*}
    f(s) &= \frac{1}{1 - \frac{r^2}{6} s + \left(96\pi f_\pi^2\right)^{-1} \left[ (s - 4m_\pi^2)(h(s) - h(0)) + 4m_\pi^2 h'(0)s \right]} \\
    \Gamma(s) &= \frac{\Gamma(0)}{1 - \frac{r^2}{6} s + \left(16\pi f_\pi^2\right)^{-1} \left[ (s - m_\pi^2)(h(s) - h(0)) + \frac{m_\pi^2}{2} s h'(0) \right]} \tag{14}
\end{align*}
\]

If we make a bubble summation consistently with the elastic unitarity condition, we obtain:

\[
\begin{align*}
    f(s) &= 1 + \frac{r^2}{6} s - \left(96\pi f_\pi^2\right)^{-1} \left[ (s - 4m_\pi^2)(h(s) - h(0)) + 4m_\pi^2 h'(0)s \right] \\
    \Gamma(s) &= \frac{\Gamma(0)}{1 - \frac{r^2}{6} s - \left(16\pi f_\pi^2\right)^{-1} \left[ (s - m_\pi^2)(h(s) - h(0)) + \frac{m_\pi^2}{2} s h'(0) \right]} \tag{15}
\end{align*}
\]

where

\[
h(s) = \frac{1}{\pi} \rho(s) \ln \left( \frac{1 + \rho(s)}{-1 + \rho(s)} \right) \tag{16}
\]

\( \rho(s) \) is the phase space factor.

Eq.(13) can also be obtained by the diagonal Padé approximant method by using Eq.(11) and (12). This approximation is identical to the Vector Meson Dominance approach of Gell-Mann-Sharp and Wagner where the self energy correction is taken into account to make the vector meson unstable and with \( \rho\pi\pi \) coupling given by KSRF relation.

An alternative method to calculate the scalar and vector form factors is to use the S and P wave \( \pi\pi \) phase shifts and the Omnès representation (see Eq.(9) and Eq.(10)). For more details see Ref. The P wave \( \pi\pi \) phase shift is given in Fig.(1).

Because there are no experimental information on the scalar form factor, we only compare the vector form factor with the experimental data. It is seen that the agreement between theory and experimental data is satisfactory although the peak values of the experimental form factor squared at the \( \rho \) mass is about 40 as compared with the theoretical calculation value 32 or an error of the order of 20% as it can be seen in Fig.(2).
Figure 1: The solid line $I = 1, l = 1$ pion scattering phase shift calculated from unitarized ChPT Dashed line corresponds to the similar phase shift where the left hand cut is neglected.

Figure 2: The solid line is the vector pion form factor squared calculated from the Omnès representation. The dashed line represents the same quantity calculated by the Padé Method
This discrepancy is probably due to the inelastic effect of the $\omega\pi$ channel as was previously pointed out.

We can also calculate the vector and scalar pion rms radii using the following formula:

$$\langle r^2_V \rangle = \frac{6}{\pi} \int_{4m^2_\pi}^{\infty} \frac{\delta_1^1 dz}{z^2}$$

$$\langle r^2_S \rangle = \frac{6}{\pi} \int_{4m^2_\pi}^{\infty} \frac{\delta_0^0 dz}{z^2}$$

Numerical integration gives $\langle r^2_V \rangle = 0.40 \text{ fm}^2$, and $\langle r^2_S \rangle = 0.47 \text{ fm}^2$ compared to the experimental value $\langle r^2_V \rangle = 0.439 \pm 0.03 \text{ fm}^2$, and the $\langle r^2_S \rangle = 0.5 \text{ fm}^2$ which is obtained from the experimental $\pi K$ scalar radius and from SU(3) symmetry. The agreement between experimental data and theoretical calculations is satisfactory.

3. $\tau \rightarrow \pi K\nu$ decay

The most general $\tau \rightarrow \pi K\nu$ decay amplitudes are given in terms of two form factors:

$$\langle \pi^0 K^- | V_{\mu}^{4-i5} (0) | 0 \rangle = f_1(s)(p_2 - p_1)_{\mu} + f_2(s)(p_1 + p_2)_{\mu}$$

where $p_1$, and $p_2$ are, respectively, the pion and kaon momenta, and $s = (p_1 + p_2)^2$ is the time-like momentum transfer and $V_{\mu}^{4-i5}$ is the vector current operator with the superscript indices referring to the SU(3) octet currents. $f_1(s)$ is the P wave $\pi K$ form factor, $f_2(s)$ is a linear combination of S and P states as can be seen by taking the divergence of Eq (15):

$$g(s) = -i \langle \pi^0 K^- | \partial^\mu V_{\mu}^{4-i5} | 0 \rangle = (m^2_K - m^2_\pi) f_1(s) + s f_2(s)$$

$g(s)$ is therefore a pure scalar which describes the S wave $\pi K$ form factor. $g(s)$ measures the $SU(3)$ violating effect because, in the exact $SU(3)$ limit, the vector current is conserved. We expect therefore in the $\tau \rightarrow \pi K\nu$ decay, the P wave form factor $f_1(s)$ dominates.

Because of the octet current hypothesis the two channels $\pi^0 K^-$ and $\pi^- K^0$ matrix elements are related by the Clebsh Gordon coefficient

$$\langle \pi^- K^0 | V_{\mu}^{4-i5} | 0 \rangle = \sqrt{2} \langle \pi^0 K^- | V_{\mu}^{4-i5} | 0 \rangle$$

Using the Ademollo-Gatto theorem $f_1(0) = 1/\sqrt{2}$ and hence $g(0) = (m^2_K - m^2_\pi)/\sqrt{2}$ for $\pi^0 K^-$ system.
Using the standard current algebra technique and the $SU(2)_L \times SU(2)_R$ commutation relation by taking the pion momentum $p_1$ soft we have the well known Callan-Treiman relation:\footnote{5}

$$f_1(m_K^2) + f_2(m_K^2) = \frac{f_K}{f_\pi \sqrt{2}} \tag{20}$$

where $f_K$ and $f_\pi$ are, respectively, the K and $\pi$ decay constants $f_K/f_\pi = 1.22$.

By evaluating Eq(16) at $t = m_K^2$ and noting that $f_2(m_K^2)$ is proportional to $m_\pi^2/m_K^2$ we have: $g(m_K^2) \approx g(0)f_K/f_\pi$

The elastic unitarity condition, which should be valid in the physical region of the $\tau \to \pi K\nu$ decay gives:

$$Im f_1(s) = f_1(s) \exp -i\frac{\delta_{s}^{1/2}}{2} \sin \frac{\delta_{p}^{1/2}}{2} \tag{21}$$
$$Img(s) = g(s) \exp -i\frac{\delta_{s}^{1/2}}{2} \sin \frac{\delta_{p}^{1/2}}{2} \tag{22}$$

where $\delta_{s}^{1/2}$ and $\delta_{p}^{1/2}$ are respectively the phase of S and P wave $I=1/2$ $\pi K$ scattering amplitude. The general solutions are given by the Muskhelishvili Omnès representation:\footnote{6}

$$f(s) = f(0) \exp \left(\frac{s}{4m_\pi^2} \int \frac{\delta_{p}^{1/2} dz}{z(z - s - i\epsilon)}\right) \tag{23}$$
$$g(s) = g(0) \exp \left(\frac{s}{4m_\pi^2} \int \frac{\delta_{s}^{1/2} dz}{z(z - s - i\epsilon)}\right) \tag{24}$$

At one loop chiral perturbation theory we have:

$$f(s) = f(0) + f^{\text{loop}}(s) \tag{25}$$
$$g(s) = g(0) + g^{\text{loop}}(s) \tag{26}$$

$f^{\text{loop}}(s)$ and $g^{\text{loop}}(s)$ are given elsewhere\footnote{7}. If we make the usual bubble summation, consistent with the elastic unitarity condition, we obtain:

$$f(s) = \frac{f(0)}{1 - f^{\text{loop}}(s)/f(0)}$$
$$g(s) = \frac{g(0)}{1 - g^{\text{loop}}(s)/g(0)} \tag{27}$$

From the expression for $f_1(s)$, the phase of the form factor which is identical to the P wave phase shifts of $\pi K$ scattering amplitude, can be calculated using the experimental value of $\langle r_v^2 \rangle = 0.34 \pm 0.03 fm^2$. Using
this value we have \( m_{K^*} = 810 \pm 30 \text{ MeV} \) which agrees with the experimental data \( m_{K^*} = 892 \text{ MeV} \). Its width satisfies the following modified KSRF relation:

\[
\Gamma_{K^*} = \frac{\lambda^{3/2}(m_{K^*}^2, m_\pi^2, m_K^2)}{128\pi m_{K^*}^3 f_{\pi}^2}
\]  

Using the experimental value \( m_{K^*} = 892 \text{ MeV} \) the numerical result of the right hand side of Eq(26) is 55 MeV, compared to the experimental value of \( 49.8 \pm 0.8 \text{ MeV} \). In Fig.(3), we give a graphic representation of \( f_1(s) \) squared.

The branching ratio \( B.R = \frac{\Gamma(\tau \rightarrow \pi K \nu)}{\Gamma(\tau \rightarrow all)} \) is 1.0\% and is in agreement with the experimental result of \( B.R_{\text{exp.}} = (1.4 \pm 0.2)\% \).

Because the S wave \( \pi K \) scattering length does not vanish, \( g(s) \) has a square root threshold singularity at the threshold (the derivative of \( g(s) \) is discontinuous at this point) as it can be seen in Fig.(4).

The scalar form factor contributes very little to the \( \pi K \) spectrum owing to the fact that it appears as a square of the amplitude. The forward backward asymmetry, being proportional to the amplitude, is reasonably large. It is about 10\% in the \( K^* \) resonance region where the number of events is maximum as it can be seen in Fig.(5).
The forward-backward asymmetry could be a useful quantity for studying the relative phases of the S and P waves.

The Omnès representation with the S and P wave $I = 1/2$ $\pi K$ scattering amplitude studied in Ref. 8 (and given in Fig. 6) yields a branching ratio of $1.15\%$ for $\tau \to \pi K \nu$ decay, in a better agreement with the experimental value $1.4 \pm 0.2\%$.

4. $\tau \to K \eta \nu$

In this process elastic unitarity condition does not apply. One should consider the two channels $\pi K$ and $K \eta$ in the final state interaction. Chiral Perturbation Theory cannot describe the $K \eta$ scattering even at threshold since the $K^*$ resonance occurs below $K \eta$ threshold. According to the equivalence between the Padé Method and the VMD approach (as it was pointed out in the pion form factor calculation), we will use in the following the $K^*$ dominance for studying the $\tau \to K \eta \nu$ decay. VMD method introduces the inelastic effects through the $K^*$ self energy. Using the experimental value of $\tau \to K^* \nu$ branching ratio and SU(3) symmetry, we predict a branching ratio of $1.6 \times 10^{-4}$. The experimental value of
Figure 5: Prediction for the forward-backward asymmetry $A_{FB}$ defined in eq(5) as a function of the $\pi K$ invariant mass squared. The dashed/solid curves correspond respectively to the calculations with/without the left hand cut of $\pi K$ scattering amplitude.

Figure 6: The solid line represents the $I=1/2$, $l=1$ $\pi K$ scattering phase shift calculated from the unitarized CPTh. The dashed line corresponds to the similar phase shift when the left hand cut is neglected. The dot-dashed line is the CPTh prediction phase (which is not the same as the phase shift due to the violation of the full elastic unitarity relation in this method).
$\tau \to K\eta\nu$ branching ratio is $(2.5 \pm 0.5) \times 10^{-4}$.

5. $\tau \to \pi\rho\nu$, $\tau \to \pi K^\star\nu$ and $\tau \to K\rho\nu$

The most general matrix element can be written as:

$$\langle \pi^-(k)\rho^0(p)|A_{\mu}^{1-\iota2}(0)|0 \rangle = f_1(Q^2)\epsilon_\mu + \epsilon.k \left((k+p)_{\mu}f_2(Q^2) + (k-p)_{\mu}f_3(Q^2)\right)$$

(29)

where $Q^2 = (k+p)^2$ and $\epsilon$ is the polarisation vector of $\rho$. $f_1$, $f_2$, and $f_3$ are complex form factors and are only functions of $Q^2$. Current algebra soft pion theorem, which is obtained by taking the limit $k_\mu \to 0$, gives only information on $f_1$ but not on the other two form factors. In an explicit model, it was shown that they contribute little to the $\tau \to \pi\rho\nu$. Interested readers are referred to the original article. (We assume here that the decay constant of $\pi'$ is sufficiently small and hence can be neglected). Using the standard low energy current algebra theorem and taking the limit $k_\mu \to 0$ we have:

$$\lim_{k_\mu \to 0} \langle \pi^-(k)\rho^0(p)|A_{\mu}^{1-\iota2}(0)|0 \rangle = -\sqrt{2}f_\rho f_\pi \epsilon_\mu(p)$$

(30)

where $f_\pi = 93 MeV$, and $f_\rho$ is defined by the rate of $\rho \to e^+e^-$. Using the experimental data we obtain, $f_\rho = 0.118 GeV^2$. This value of $f_\rho$ is equivalent to writing approximately the pion form factor as $F_\pi(s) = m_\pi^2(1 + \delta s/m_\pi^2)/\left(m_\pi^2 - s - i m_\rho \Gamma_\rho(s)\right)$. A good fit to the experimental data is obtained with $\delta = 0.2$. In fact, the more general form of Eq(3) reads

$$\lim_{k_\mu \to 0} \langle \pi^-(k)\rho^+(q_1)\pi^-(q_2)|A_{\mu}^{1-\iota2}(0)|0 \rangle = -\sqrt{\frac{2}{f_\pi}} F_\pi(s)(q_1 - q_2)_{\mu}$$

(31)

For convenience we shall first use Eq(3). The 3$\pi$ matrix element below the $\rho\pi$ threshold can be straightforwardly obtained from Eq(28). Using Eq(26) in (27) we have:

$$f_1(m_\rho^2) = -\sqrt{2} \frac{f_\rho}{f_\pi}$$

(32)

Let us start with the narrow width approximation for the $A_1$ propagator. Using $A_1$ dominance for the form factor we have:

$$f_1(Q^2) = -\sqrt{2} f_\rho \frac{(m_A^2 - m_\rho^2)}{m_A^2 - Q^2}$$

(33)

The generalisation of Eq[30] to take into account of the unstable nature of $A_1$ can be straightforwardly made. Using the $A_1$ dominance hypothesis
for the axial current, the general expression for $f_1(Q^2)$ is:

$$f_1(Q^2) = -\sqrt{2} \frac{f_\rho m_A^2 - m_\rho^2 - \pi(m_\rho^2)}{f_\pi m_A^2 - Q^2 - \pi(Q^2)}$$

(34)

where we use the standard prescription for describing an unstable particle, with $\pi(Q^2)$ being the self energy operator of the $A_1$ resonance and is obtained by the bubble summation of the $\pi\rho$ intermediate states, similar to the treatment of the W and Z propagators in the standard model. In order to have the usual Breit Wigner description of a resonance, we must make a twice substracted dispersion relation with $Re[\pi(m_A^2)] = Re[\pi'(m_A^2)] = 0$

where $m_A$ is the $A_1$ mass:

$$Re[\pi(Q^2)] = \frac{(Q^2 - m_A^2)^2}{\pi} \frac{\int dz \frac{Im[\pi(z)] - Im[\pi(m_A^2)] - (z - m_A^2)Im[\pi'(m_A^2)]}{(z - m_A^2)^2(z - Q^2)}}\frac{g_{A\rho\pi}^2}{8\pi} \sqrt{\lambda(Q^2, m_\rho^2, m_\pi^2)} \left(1 + \frac{\lambda(Q^2, m_\rho^2, m_\pi^2)}{12m_\rho^2Q^2}\right)$$

(35)

where we define the $\pi^0\rho^+ A_1^-$ vertex as $g_{A\rho\pi} = (A, \epsilon(\rho), \lambda(Q^2, m_\rho^2, m_\pi^2) = (Q^2 - (m_\rho + m_\pi)^2)/(Q^2 - (m_\rho - m_\pi)^2)$, and $P$ stands for the principal part integration.

If we assume that the acceptance correction to the ALEPH data was negligible, our best fit to the $3\pi$ spectrum gives the following ranges of $m_A$ and $\Gamma_A$: $m_A = 1.24 \pm 0.02 GeV$, $\Gamma_A = 0.43 \pm 0.02 GeV$.

Using these results, the calculated branching ratio for $\tau \to 3\pi\nu$ is $19 \pm 3\%$. The central value corresponds to our best fit which is shown in Fig.(7). This value agrees with the ALEPH branching ratio $19.14 \pm 0.48 \pm 0.44\%$.

A similar treatment for $\tau \to \pi K^*\nu$ and $\tau \to K\rho\nu$ decays can be done using the axial vector dominance of $Q_1(1270)$ and $Q_2(1400)$ (for more details see Ref.8). Numerical calculation gives:

$$BR(\tau \to \rho^0 K^- \nu) = 0.1\%$$

$$BR(\tau \to \pi^- K^0\nu) = 0.4\%$$

(36)

These results are in good agreement with the values of TPC/Two-Gamma collaboration: $B(\tau \to K^{*0}\pi^-\nu, neutral) = 0.51 \pm 0.2 \pm 0.13$ and $B(\tau \to K^-\pi^+\pi^-\nu) = 0.7 \pm 0.2$. The $K^-\rho^0\nu$ mode is therefore consistent with zero.

6. Conclusions

To conclude we would like to outline some main points:
Figure 7: Our best fit for the $\tau \rightarrow 3\pi\nu$ spectrum corresponding to $m_A = 1.24 \text{ GeV}$, $\Gamma_A = 0.43 \text{ GeV}$

(i) Perturbation theory satisfies unitarity order by order and that is known as perturbative unitarity.

This method is sufficient to describe physical phenomena if the coupling constant is small like in QED. But it fails if there is a bound state or a resonance state: in QED the positronium bound state is solved by Bethe-Salpeter equations (Ladder type summation).

(ii) Notice that in the Standard Model, the unitarity is preserved by the bubble summation of the $Z^0$ propagator.

– In the Nambu-Jona-Lasinio Model the bubble summation is done to satisfy the unitarity. The Higgs and Nambu Goldstones are generated as bound states due to four fermion interaction.

(iii) Low energy Nucleon-Nucleon interaction has resonance in the singlet S state and a bound state (Deuteron) in the triplet state. This problem was solved by H. Bethe in the 50’s using effective range expansion which preserves the unitarity of the S matrix. More explicitly, instead of expanding the amplitude like in Chiral Perturbation Theory, the Effective Range Theory makes an expansion of $k \cot(\delta)$

$$k \cot(\delta) = \frac{1}{a} + \frac{1}{2} r_0 k^2 + ... \quad (37)$$

with $a$ and $r_0$ being respectively the scattering length and the effective range.
This expansion, unlike $\chi$PT, preserves unitarity.

(iiii) In this paper we have shown that Chiral Perturbation Theory should be modified to describe the main problems of $\tau$ physics. We have extended the range of validity of Chiral Perturbation Theory using the Padé Approximant Method or VMD hypothesis. Both methods satisfy unitarity. A good agreement with the experimental data is obtained for the exclusive hadronic modes of $\tau$ decay.

1. T.N Pham, C.Roiesnel and T.N Truong, Phy. Rev. Lett.41(1978)371
   T.N Pham, C.Roiesnel and T.N Truong, Phy.Lett.78B(1978)623
2. N.I.Muskhelishvili, Singular integral equations (Noordhoff, Groningen,1953);
   R. Omnes, Nuovo Cimento 8(1958)316.
3. B. Costa de Beauregard, T.N.Pham, B.Pire and T.N. Truong, Phy. Lett.B67 (1977)213.
4. K.Kawarabayashi and M. Suzuki, Phy. Rev. Lett. 16(1966)255.
   Riazuddin and Fayyazuddin, Phy. Rev.147(1966)1071.
5. C.Callan and S.Treiman,Phys.Rev.Lett.,16,153(1966).
6. L.Beldjoudi and T.N Truong, Ecole Polytechnique preprint (CPTH A292.0294, hep-ph/9403348)
7. L.Beldjoudi and T.N Truong, Phy. Lett. B351(1995)357.
8. L.Beldjoudi and T.N. Truong, Phy. Lett. B344(1995)419.
9. P. Estabrooks and A.D. Martin, Nucl.Phys. B79 (1974)301.
10. P. Estabrooks et al., Nucl.Phys. B133(1978) 490.