Transverse single spin asymmetry in $e + p^+ \rightarrow e + J/\psi + X$ and transverse momentum dependent evolution of the Sivers function

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(Dated: May 11, 2014)

We extend our analysis of transverse single spin asymmetry (SSA) in electroproduction of $J/\psi$ to include the effect of the scale evolution of the transverse momentum dependent (TMD) parton distribution functions and gluon Sivers function. We estimate SSA for JLab, HERMES, COMPASS, and eRHIC energies using the color evaporation model of charmonium production, using an analytically obtained approximate solution of TMD evolution equations discussed in the literature. We find that there is a reduction in the asymmetry compared with our predictions for the earlier case considered by us, wherein the $Q^2$ dependence came only from DGLAP evolution of the unpolarized gluon densities and a different parametrization of the TMD Sivers function was used.

PACS numbers: 13.88.+e, 13.60.-r, 14.40.Lb, 29.25.Pj
I. INTRODUCTION

The factorization theorems of quantum chromodynamics (QCD), which enable us to calculate the cross sections of hadronic processes as a convolution of partonic cross section with parton distribution functions (PDFs) and fragmentation functions (FFs), are based on a collinear approximation. In this approximation, the intrinsic transverse momentum is integrated over in the definition of PDFs and FFs. However, this collinear factorization at leading twist is unable to account for the single spin asymmetries (SSAs) that have been observed in processes involving scattering off polarized hadrons \[ 1, 2 \]. One of the approaches that has been used to explain these SSAs is based on the transverse momentum dependent TMD factorization formalism wherein the PDFs and FFs depend on intrinsic transverse momentum \( k_T \) also in addition to the momentum fraction variable \( x \). TMD factorization is particularly useful for describing the processes that are sensitive to the parton’s intrinsic transverse momentum. A formalism for TMD factorization has been developed by Collins and Soper \[ 3 \] and has been used to study Drell-Yan (DY), semi-inclusive deep inelastic scattering (SIDIS) and back-to-back hadron production in \( e^+e^- \) annihilation at small transverse momentum \[ 4 \]. There has been huge interest amongst both theoreticians and experimentalists in understanding to what extent the above factorization holds, and in the study of the TMD PDFs, TMD FFs, and the resulting SSA’s with the aim of probing the transverse momentum distribution of partons inside hadrons and the spin structure of hadrons. Henceforth, we will refer to TMD PDFs and TMD FFs collectively as “TMDs.”

One of the important TMDs is the Sivers function which describes the transverse momentum distribution of an unpolarized parton inside a transversely polarized hadron. Estimates of Sivers asymmetry have been given in the Drell Yan (DY), SIDIS and D-meson production based on a generalized factorization formula involving TMD PDFs and TMD FFs. In a previous work \[ 5 \], we proposed \( J/\psi \) production as a probe of gluon Sivers function and gave estimates for SSAs in low virtuality electroproduction of \( J/\psi \) at JLab, HERMES, COMPASS, and eRHIC energies. The use of heavy quark and quarkonium production to get information about the gluon densities, as well as about the underlying QCD dynamics involved in the formation of the \( QQ \) bound states, has been explored for a variety of beams, polarized and unpolarized, as well as a variety of targets \[ 6 \]. At leading order (LO), the charmonium production receives a contribution only from a single partonic subprocess \( \gamma g \rightarrow c \bar{c} \). Hence, SSA in \( e + p_T \rightarrow e + J/\psi + X \) can be used as a clean probe of gluon Sivers function. There exist three different models for charmonium production. In the color singlet model \[ 7 \] the cross section for charmonium production is factorized into a short distance part for \( c \bar{c} \) pair production calculable in perturbation theory and a nonperturbative matrix element for the formation of a bound state, which is produced in a color singlet state. In the color evaporation model, first proposed by Halzen and Matsuda \[ 8 \] and Fritsch \[ 9 \], a statistical treatment of color is made and the probability of finding a specific quarkonium state is assumed to be independent of the color of the heavy quark pair. In later versions of this model, it has been found that the data are better fitted if a phenomenological factor is included in the differential cross section formula, which depends on a Gaussian distribution of the transverse momentum of the charmonium \[ 10 \]. A more recent model of charmonium production is the color octet model \[ 11 \]. This is based on a factorization approach in nonrelativistic QCD (NRQCD), and it allows \( c \bar{c} \) pairs to be produced in color octet states. Here again, one requires knowledge of the nonperturbative color octet matrix elements, which are determined through fits to the data on charmonium production. The investigation of SSAs in charmonium production as a possible tool towards resolution of the charmonium production mechanism puzzle has been discussed within the NRQCD framework some time back \[ 12 \]. It was shown in Ref. \[ 13 \], using general arguments, that SSA in heavy quarkonium production is sensitive to the color configuration of the \( q \bar{q} \) pair produced at a short distance. Based on an analysis of initial and final state interactions, the authors argued that in the NRQCD framework, asymmetry is nonzero in \( e-p \) collisions only in color octet model and in \( p-p \) collisions it is non-zero only in the color singlet model.

In our earlier work in Ref. \[ 11 \] we had performed a phenomenological study of the SSA in charmonium production. We had calculated the asymmetry in the process \( e + p_T \rightarrow e + J/\psi + X \) using the color evaporation model of charmonium production \[ 11 \]. As discussed therein, in this exploratory study of SSA as a probe of the gluon Sivers function, we had chosen to work with CEM due to its simplicity. In the older work, we had used parameters of Sivers function fitted to SIDIS data in Ref. \[ 14 \]. Earlier parametrizations of TMDs like the ones we had used \[ 11 \] had been obtained by assuming a parton model picture of TMD factorization and directly fitting \[ 15, 16 \] the calculated cross sections to experimental data from HERMES and COMPASS. However, the \( Q^2 \) - evolution of TMDs had not been taken into account in obtaining these parametrizations. Our earlier work that used these thus could not include the effect of evolution of TMDs.

In the present work, we improve our previous estimates by taking into account the \( Q^2 \) evolution of TMD PDFs and the Sivers function, and energy dependence of the Sivers asymmetry has attracted a lot of attention in the recent past \[ 17–20 \]. A TMD factorization framework taking into account the \( Q^2 \) evolution of TMDs has been proposed \[ 21 \]. This framework has been used to obtain evolved TMDs from fixed scale fits. A simple strategy for extracting the TMD evolved Sivers function has been proposed, and a best fit of the SIDIS asymmetries has been performed taking into account the \( Q^2 \) dependence of TMDs \[ 22 \] using data from Ref. \[ 23 \]. The authors have also compared these fits
with those obtained earlier \[20\]. In fact, the TMDs evolve much faster than the integrated PDFs, for which the scale evolution is given by the DGLAP equation. In this work, we apply the formalism of Ref. \[28\] to \(J/\psi\) electroproduction to improve our earlier estimates, taking into account the effect of \(Q^2\) evolved TMDs. In the current investigation, we restrict ourselves to the use of analytical formulas for the evolved TMDs, given in Ref. \[28\].

In Sec. II, we review the formalism for \(J/\psi\) production in CEM and also summarize the TMD evolution formalism of Ref. \[28\]. In Sec. III, we present our estimates for asymmetry obtained by taking into account the TMD evolution and scale dependence of the Gaussian width of PDFs. Section IV contains the summary and discussion of our results.

II. FORMALISM

A. Asymmetry in \(J/\psi\) production

We consider the LO parton model cross section for low virtuality electroproduction (photoproduction) of \(J/\psi\) within the color evaporation model. According to CEM, the cross section for charmonium production is proportional to the rate of production of the \(c\bar{c}\) pair integrated over the mass range \(2m_c\) to \(2m_D\) \[19\]

\[
\sigma = \frac{1}{9} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{d\hat{\sigma}_{c\bar{c}}}{dM_{c\bar{c}}}
\]

(1)

where \(m_c\) is the charm quark mass and \(2m_D\) is the \(D\bar{D}\) threshold. The partonic cross section \(\frac{d\hat{\sigma}}{dM_{c\bar{c}}}\) is calculable perturbatively, \(M_{c\bar{c}}\) being the invariant mass of the \(c\bar{c}\) pair. The differential cross section for \(\gamma + p \to J/\psi + X\) is given by

\[
\frac{d\sigma_{\gamma p \to c\bar{c}}}{dM^2_{c\bar{c}}} = \int dx_f f_{g/p}(x_f) \frac{d\hat{\sigma}_{\gamma p \to c\bar{c}}}{dM^2_{c\bar{c}}}
\]

(2)

where \(f_{g/p}(x)\) is the gluon distribution in the proton.

Using the Weizsacker-Williams approximation \[30\], one can convolute the cross section given by Eq. (2) with a photon flux factor to obtain the electroproduction cross section for \(e + p \to e + J/\psi + X\)

\[
\frac{d\sigma_{e p \to e + J/\psi + X}}{dM^2_{c\bar{c}}} = \int dx_\gamma f_{\gamma/e}(x_\gamma) \frac{d\hat{\sigma}_{\gamma p \to c\bar{c}}}{dM^2_{c\bar{c}}}.
\]

(3)

In the above, \(f_{\gamma/e}(x_\gamma)\) is the distribution function of the photon in an electron with \(x_\gamma\) denoting the energy fraction of the electron that the photon carries. We use the expression for \(f_{\gamma/e}(x_\gamma)\) from Refs. \[31, 32\] given by

\[
f_{\gamma/e}(x_\gamma, E) = \alpha \left(\frac{1 + (1 - x_\gamma)^2}{x_\gamma} \left(\ln \frac{E}{m} - \frac{1}{2}\right) + \frac{x_\gamma}{2} \left[\ln \left(\frac{2 - x_\gamma}{2 - 2x_\gamma}\right) + 1\right]\right)
\]

\[
\quad + \frac{(2 - x_\gamma)^2}{2x_\gamma} \ln \left(\frac{2 - 2x_\gamma}{2 - x_\gamma}\right),
\]

(4)

where \(x_\gamma\) is the energy fraction of the electron carried by the photon, \(m\) is the mass of the electron, and \(E\) is the energy of the electron.

Thus, the cross section for electroproduction of \(J/\psi\) using WW approximation is given by

\[
\sigma_{e p \to e + J/\psi + X} = \int_{4m_c^2}^{4m_D^2} dM^2_{c\bar{c}} \int dx_\gamma dx_f f_{\gamma/e}(x_\gamma) f_{g/p}(x_f) \frac{d\hat{\sigma}_{\gamma p \to c\bar{c}}}{dM^2_{c\bar{c}}}.
\]

(5)

To calculate SSA in the scattering of electrons off a polarized proton target, we assume a generalization of the CEM expression by taking into account the transverse momentum dependence of the Weizsacker-Williams function and gluon distribution function \[11\]

\[
\frac{d\sigma_{e p \to e + J/\psi + X}}{dM^2} = \int dx_\gamma dx_g \left[d^2 k_{\perp\gamma} d^2 k_{\parallel g}\right] f_{g/p}(x_g, k_{\perp g}) f_{\gamma/e}(x_\gamma, k_{\parallel\gamma}) \frac{d\hat{\sigma}_{\gamma p \to c\bar{c}}}{dM^2}
\]

(6)

where \(M^2 \equiv M_{c\bar{c}}^2\). The difference in \(d\sigma^+\) and \(d\sigma^-\) is parametrized in terms of the gluon Sivers function

\[
d\sigma^+ - d\sigma^- = \int dx_\gamma dx_g d^2 k_{\perp\gamma} d^2 k_{\parallel g} \Delta N f_{g/p}(x_g, k_{\parallel g}) f_{\gamma/e}(x_\gamma, k_{\parallel\gamma}) \frac{d\hat{\sigma}_{\gamma p \to c\bar{c}}}{dM^2}
\]

(7)
where \( d\sigma \) is the elementary cross section for the process \( \gamma g \rightarrow c\bar{c} \) given by

\[
d\sigma = \frac{1}{2s} \frac{d^3p_c}{2E_c} \frac{d^3p_{\bar{c}}}{2E_{\bar{c}}} \frac{1}{(2\pi)^2} \delta^4(p_\gamma + p_g - p_c - p_{\bar{c}}) |M_{\gamma g \rightarrow c\bar{c}}|^2.
\]  

Following the procedure in Ref. [11], we obtain

\[
\frac{d^4\sigma^\uparrow}{dydM^2d^2q_T} - \frac{d^4\sigma^\downarrow}{dydM^2d^2q_T} = \frac{1}{s} \int [d^2k_{\perp g} \Delta^N f_{g/p}(x_g, k_{\perp g}) f_{\gamma/c}(x_\gamma, k_{\perp \gamma}) \times \delta^2(k_{\perp \gamma} + k_{\perp g} - q_T) \delta_0^\gamma g \rightarrow c\bar{c}(M^2)]
\]

and

\[
\frac{d^4\sigma^\uparrow}{dydM^2d^2q_T} + \frac{d^4\sigma^\downarrow}{dydM^2d^2q_T} = \frac{2}{s} \int [d^2k_{\perp g} \Delta^N f_{g/p}(x_g, k_{\perp g}) f_{\gamma/c}(x_\gamma, k_{\perp \gamma}) \times \delta^2(k_{\perp \gamma} + k_{\perp g} - q_T) \delta_0^\gamma g \rightarrow c\bar{c}(M^2)]
\]

where

\[
x_{g,\gamma} = \frac{M}{\sqrt{s}} e^{\pm y}
\]

and the partonic cross section is given by [32]

\[
\sigma_0^{\gamma g \rightarrow c\bar{c}}(M^2) = \frac{1}{2} e^{2\pi \alpha_s \alpha} [(1 + v - \frac{1}{2} |v|^2) \ln \frac{1 + \sqrt{1 - v}}{1 - \sqrt{1 - v}} - (1 + v) \sqrt{1 - v}].
\]

Here, \( v = \frac{4m^2}{M^2} \) and \( M^2 \equiv s \).

Integrating Eqs. (9) and (10) over \( M^2 \), we obtain the difference and sum of \( \frac{d^4\sigma^\uparrow}{dyd^2q_T} \) and \( \frac{d^4\sigma^\downarrow}{dyd^2q_T} \) for \( J/\psi \) production.

The Sivers asymmetry is defined as

\[
A_N^{\sin(\phi_T - \phi_S)} = \frac{\int d\phi_T [d\sigma^\uparrow - d\sigma^\downarrow] \sin(\phi_T - \phi_S)}{\int d\phi_T [d\sigma^\uparrow + d\sigma^\downarrow]}
\]

where \( d\sigma^\uparrow \) is the differential cross section in \( q_T \) or the \( y \) variable and \( \phi_T \) and \( \phi_S \) are the azimuthal angles of the \( J/\psi \) and proton spin respectively. The weight factor in the numerator projects out the Sivers asymmetry. To evaluate asymmetry in \( y \) distribution, we substitute

\[
d\sigma^\uparrow - d\sigma^\downarrow = \int d\phi_T \int q_T dq_T \int [d^2k_{\perp g}] \int [d^2k_{\perp \gamma}] \Delta^N f_{g/p}(x_g, k_{\perp g}) f_{\gamma/c}(x_\gamma, q_T - k_{\perp \gamma}) \delta_0(M^2)
\]

and

\[
d\sigma^\uparrow + d\sigma^\downarrow = 2 \int d\phi_T \int q_T dq_T \int [d^2k_{\perp g}] \int [d^2k_{\perp \gamma}] f_{g/p}(x_g, k_{\perp g}) f_{\gamma/c}(x_\gamma, q_T - k_{\perp \gamma}) \delta_0(M^2).
\]

Thus at LO, the SSA depends on the Weizsacker-Williams function, gluon distribution function and gluon Sivers function. Let us recapitulate some of the details of our choices, discussed in more detail in [11]. For \( k_{\perp g} \) dependence of the unpolarized PDFs we use a simple factorized and Gaussian form [34]:

\[
f_{g/p}(x_g, k_{\perp g}) = f_{g/p}(x_g) \frac{1}{\pi(k_{\perp g}^2)} e^{-k_{\perp g}^2/(k_{\perp g}^2)}.
\]
In addition to this, we need to specify the transverse momentum dependence of the WW functions. In principle, it would be interesting to see whether one can derive an expression for this starting from the first principle. But at present we use a simple Gaussian form for the WW function following the corresponding one for the unpolarized PDFs. The form we use is given by

\[ f_{\gamma/e}(x_\gamma, k_{\perp \gamma}) = f_{\gamma/e}(x_\gamma) \frac{1}{\pi\langle k_{\perp \gamma}^2 \rangle} e^{-k_{\perp \gamma}^2/\langle k_{\perp \gamma}^2 \rangle}. \]  

(17)

In fact, in our earlier work, we had used a dipole form as well. However, finding our results to be rather insensitive to the assumed form, in this work we have used only the Gaussian form and further use comparable values of \(\langle k_{\perp \gamma}^2 \rangle\) and \(\langle k_{\perp g}^2 \rangle\).

In Ref.\[11\], we have used the Sivers function given by\[35\]

\[ \Delta^N f_{g/p^\uparrow}(x_g, k_{\perp g}) = \Delta^N f_{g/p^\uparrow}(x_g) \frac{1}{\pi\langle k_{\perp g}^2 \rangle} h(k_{\perp g}) e^{-k_{\perp g}^2/\langle k_{\perp g}^2 \rangle} \cos(\phi_{k_{\perp}}) \]  

(18)

where the gluon Sivers function, \(\Delta^N f_{g/p^\uparrow}(x_g)\) is defined as

\[ \Delta^N f_{g/p^\uparrow}(x_g) = 2 N_g(x_g) f_{g/p}(x_g). \]  

(19)

Here, \(N_g(x_g)\) is an \(x\)-dependent normalization for gluon and

\[ h(k_{\perp g}) = \sqrt{2} e^{k_{\perp g}/M_1} k_{\perp g}^{-k_{\perp g}/M_1^2} \]  

(20)

\(M_1\) is the parameter obtained by fitting the recent experimental data corresponding to pion and kaon production at HERMES and COMPASS.

The parametrizations for the quark Sivers function \(N_u(x)\) and \(N_d(x)\) have been fitted from SIDIS data and are given by \[21\]

\[ N_f(x) = N_f x^{a_f} (1 - x)^{b_f} \frac{(a_f + b_f)(a_f + b_f)}{a_f a_f + b_f b_f} \]  

(21)

Here \(a_f, b_f, N_f\) for \(u\) and \(d\) quarks are free parameters obtained by fitting the data. However, there is no information available on \(N_g(x)\). In our analysis, we have used two parametrizations \[30\]

(a) \(N_g(x) = \langle N_u(x) + N_d(x) \rangle / 2\),

(b) \(N_g(x) = N_d(x)\).

The first choice assumes that the gluon Sivers function is the average of the up and down quark Sivers function while the second choice is motivated by the fact that the gluon distribution function is close to the \(d\) quark distribution function. There can be other more general parametrizations also; however, in this work we use only these two choices. It may be noted that out of the remaining two choices proposed in Ref.\[30\], one is irrelevant for us and the other choice differs from choice (b) only in terms of Gaussian width; as per the results of Ref.\[30\] this is not expected to have too much impact, and hence we do not include it.

\[\text{B. Scale evolution of TMDs}\]

In our previous analysis, we had chosen the scale of PDFs and Sivers function to be \(\hat{s}\). The TMD PDFs and Sivers function were obtained through DGLAP evolution by considering the evolution of the factorized collinear part. The issue of scale dependence of TMDs has attracted a lot of attention in the recent past and a new formalism for the \(Q^2\) evolution of TMDs has been developed\[22, 23, 37\]. Based on this TMD evolution, best fits of the SIDIS Sivers asymmetry have been performed and compared with earlier estimates extracted without using TMD evolution\[28\]. In this work, we employ the strategy used in Ref.\[28\] to take into account the TMD evolution of PDFs and the Sivers function and compare the resulting asymmetries in \(J/\psi\) production with our earlier estimates which were obtained using DGLAP evolution. Since we are following the formalism of Ref.\[28\], we will be brief in our description of the same. The details can be found in Ref.\[28\].
In this formalism, the \( Q^2 \) evolution of the \( k_\perp \) dependent distribution function is given by \(^2\)

\[
\hat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x, Q_0) R(Q, Q_0) \frac{e^{-k_\perp^2/w^2}}{\pi w^2},
\]

(22)

where \( f_{q/p}(x, Q_0) \) is the usual integrated PDF evaluated at the initial scale \( Q_0 \) and \( w^2 \equiv w^2(Q, Q_0) \) is the “evolving” Gaussian width, defined as

\[
w^2(Q, Q_0) = \langle k_{\perp}^2 \rangle + 2g_2 \ln \frac{Q}{Q_0}.
\]

(23)

Here, the evolution factor \( R(Q, Q_0) \) is the limiting value of a function \( R(Q, Q_0, b_T) \) that drives the \( Q^2 \) evolution of TMDs in coordinate space and is given by \(^2\) \(^2\)

\[
R(Q, Q_0, b_T) \equiv \exp \left\{ \ln \frac{Q}{Q_0} \int_{Q_0}^{Q} \frac{d\mu'}{\mu'} \gamma_F(\mu') + \int_{Q_0}^{Q} \frac{d\mu}{\mu} \gamma_F \left( \frac{\mu}{\mu^2} Q^2 \right) \right\}
\]

(24)

where \( b_T \) is the parton impact parameter and

\[
\mu_0 = \frac{C_1}{b_*(b_T)}, \quad b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}.
\]

(25)

with \( C_1 = 2e^{-\gamma_E} \) and \( \gamma_E = 0.577 \) \(^3\).

In the limit \( b_T \to \infty \), \( R(Q, Q_0, b_T) \to R(Q, Q_0) \) and \( b_* \to b_{max} \). \( \gamma_F \) and \( \gamma_K \) are anomalous dimensions that are given, at \( O(\alpha_s) \), by

\[
\gamma_F(\mu; Q^2 \mu^2) = \alpha_s(\mu) \frac{C_F}{\pi} \left( \frac{3}{2} - \ln \frac{Q^2}{\mu^2} \right)
\]

(26)

\[
\gamma_K(\mu) = \alpha_s(\mu) \frac{2C_F}{\pi}.
\]

(27)

The TMD evolved Sivers function is given by\(^2\)

\[
\Delta N \hat{f}_{q/p^\prime}(x, k_\perp; Q) = \frac{k_\perp}{M_1} \sqrt{2e} \left( \frac{k_{\perp}^2}{k_{\perp}^2} \right)^2 \Delta N f_{q/p^\prime}(x, Q_0) R(Q, Q_0) \frac{e^{-k_\perp^2/w_0^2}}{\pi w_0^2},
\]

(28)

with

\[
w_0^2(Q, Q_0) = \langle k_{\perp}^2 \rangle + 2g_2 \ln \frac{Q}{Q_0}
\]

(29)

where

\[
\frac{1}{\langle k_{\perp}^2 \rangle} = \frac{1}{M_1^2} + \frac{1}{\langle k_{\perp}^2 \rangle}.
\]

(30)

\[
\langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2, \quad g_2 = 0.68, \quad b_{max} = 0.5 \text{ GeV}^{-1}.
\]

(31)

Here, \( M_1 \) is a best fit parameter \(^2\), \( Q^2 = \hat{s} \), and \( Q_0^2 = 1.0 \text{ GeV}^2 \) \(^3\).

\section{Asymmetry in J/ψ production using evolved TMDs}

Using the TMD evolved PDF and Sivers function given in Eqs. \(^2\) \(^2\) and \(^2\), and following the procedure in Ref. \(^1\), we obtain the expression for the numerator of asymmetry as

\[
\frac{d^3\sigma^+}{dy dq^T} - \frac{d^3\sigma^-}{dy dq^T} = \frac{1}{s} \int_{4M_1^2}^{4m_0^2} dM^2 \Delta N f_{q/p^\prime}(x_g, Q_0) f_{\gamma/\epsilon}(x_\gamma) \sqrt{2e} \frac{q_T}{M_1} \times R(Q, Q_0) \frac{((k_S^2))^2 \exp[-q_T^2/(w_0^2 + (k_{\perp}^2))] \cos(\phi_{q_T}) \delta^g_{\perp}^{\gamma \rightarrow e\bar{e}}(M^2)}{\pi[w_0^2 + (k_{\perp}^2)]^2(k_{\perp}^2)}
\]

(32)
photon distribution [32]. Obtained using GRV98LO for unpolarized gluon distribution functions [42] and the Weizsacker-Williams function for $k_\perp$ fact, also performed the effective range of integration to be less than the value at which the above-mentioned tail starts. We have, in as the finite width of the Gaussian dependence of the WW function; in fact, it provides additional damping, making the effective range of integration to be less than the value at which the above-mentioned tail starts. We have, in fact, also performed the $k_{1g}$ integration in Eqs. (14) and (15) numerically over a finite range allowed by Gaussian approximation and verified that the result is the same as obtained by analytical integration.

III. NUMERICAL ESTIMATES FOR THE ASYMMETRY IN $J/\psi$ PRODUCTION USING TMD EVOLVED SIVERS FUNCTION

We will now estimate the magnitude of asymmetry using the TMD evolved PDFs and Sivers function and compare the results with our previous estimates of asymmetry using DGLAP evolution [11]. We will also compare our results with asymmetry calculated using DGLAP evolution with another set of parameters extracted from DGLAP fits at $Q_0 = 1$ GeV.

For the DGLAP evolution, we have estimated asymmetry using two different sets of parameters. The first set, which we call DGLAP1, consists of the values of the best fit parameters that we have used in Ref. [11]:

$$ N_u = 0.40, \ a_u = 0.35, \ b_u = 2.6, $$
$$ N_d = -0.97, \ a_d = 0.44, \ b_d = 0.90, $$
$$ M_1^2 = 0.19 \ GeV^2. $$

(34)

These parameters are from new HERMES and COMPASS data [39, 40] fitted at $Q^2 = 2.4 \ GeV^2$ [20].

The second set of parameters, which we call DGLAP2, has been extracted from DGLAP fits at $Q_0 = 1$ GeV [28]:

$$ N_u = 0.45, \ a_u = 1.08, \ b_u = 6.9, $$
$$ N_d = -1.00, \ a_d = 1.7, \ b_d = 6.9, $$
$$ M_1^2 = 0.19 \ GeV^2. $$

(35)

In both cases, the value of $\langle k_{1g}^2 \rangle$ is chosen to be the same as $\langle k_{1e}^2 \rangle$ for quarks obtained in Ref. [41] by analysis of the Cahn effect in unpolarized SIDIS from data collected in different energy and $Q^2$ ranges assuming a constant Gaussian width. The value of $\langle k_{1g}^2 \rangle$ has been chosen to be $0.25 \ GeV^2$ as in our earlier work.

For TMD evolved Sivers function, we have used the parameter set fitted at $Q_0 = 1$ GeV given in Ref. [28].

$$ N_u = 0.75, \ N_d = -1.00, \ b = 4.0, $$
$$ a_u = 0.82, \ a_d = 1.36, \ M_1^2 = 0.34 \ GeV^2. $$

(36)

We have estimated the asymmetry with both kinds of parametrizations [labeled (a) and (b)]. The estimates are obtained using GRV98LO for unpolarized gluon distribution functions [12] and the Weizsacker-Williams function for photon distribution [32].

In Figs. 1-6 we have performed a comparative study of Sivers asymmetry $A_N^{\sin(\phi_{qT} - \phi_S)}$ for DGLAP evolution and TMD evolution as a function of rapidity $y$ and $q_T$, the transverse momentum of $J/\psi$, respectively for JLab ($\sqrt{s} = 4.7$ GeV), HERMES ($\sqrt{s} = 7.2$ GeV), COMPASS ($\sqrt{s} = 17.33$ GeV) and eRHIC ($\sqrt{s} = 31.6$ GeV and $\sqrt{s} = 158.1$ GeV) energies. Before we start with a detailed discussion, we make one observation. In all the cases the asymmetries are smaller than our earlier estimates. However, they still remain sizable, except for the lowest energy at JLab, where it is a difficult measurement due to the low event rate.

In Figs. 1-5, SSA with DGLAP evolution with parameter sets DGLAP1 and DGLAP2 and TMD evolution is compared using parametrization (b). To get the $q_T$ distribution of asymmetry we have integrated over all possible values of $y$, and to get the $y$ distribution of asymmetry we have integrated over $q_T$ from 0 to 1.0 GeV. The masses
of the $c$ quark and the $D$ meson are taken to be $1.275$ and $1.864$ GeV, respectively. In the TMD evolved PDFs and Sivers function, the initial scale $Q_0^2$ has been chosen to be $1.0$ GeV as we are using Sivers function parameters of Ref. [28] fitted at this scale. Since we are using the color evaporation model, evolved TMDs are evaluated at $Q^2 = \hat{s}$, which varies between $4m_c^2$ and $4m_D^2$, and is the only relevant scale for $J/\psi$ production in the CEM.

We note that all the asymmetries are suppressed with respect to our earlier predictions using the older model and parameter set DGLAP1[11]. To be specific, we see that for parametrization (b) of the gluon Sivers function, with TMD evolution, the maximum asymmetry in the $y$ distribution is reduced from approximately 15% to 5% at different rapidity values, for energies of JLab, HERMES, COMPASS, eRHIC-1, and eRHIC-2 experiments. For the $q_T$ distribution of asymmetry with parametrization (b), the maximum asymmetry is reduced from approximately 15% to 5% at $q_T=0.75$ GeV for energies of JLab, HERMES, COMPASS and eRHIC-1 experiments. For higher eRHIC energy, the asymmetry in $q_T$ distribution is reduced from approximately 5% to 1%.

We have also compared our estimates of asymmetry using TMD evolved PDFs with the asymmetry calculated using DGLAP evolved PDFs with the parameter set DGLAP2. The parameters in these two cases have been extracted from TMD fits at $Q_0 = 1.0$ GeV and DGLAP fits at $Q_0 = 1.0$ GeV respectively, assuming a Gaussian width of 0.5 GeV at this initial scale. In this case also, we find appreciable suppression but the difference between DGLAP and TMD estimates is smaller now. In particular, the asymmetry is not changing much for the TMD evolution and DGLAP evolution in case of the $y$ distribution of JLab experiments. For the $q_T$ distribution of asymmetry with parametrisation (b), maximum asymmetry is reduced from approximately 8% to 5% for eRHIC-1 experiment. For the $q_T$ distribution of asymmetry in the eRHIC-2 experiment, the asymmetry is reduced from approximately 1.5% to 1%. In general, we find that the estimates using this parameter set are closer to the estimates using TMD evolved PDFs as compared to our earlier estimates using a parameter set fitted at $Q^2 = 2.4$ GeV$^2$. This seems to suggest the importance of the $Q^2$ dependence of the Gaussian width as well the earlier DGLAP fits of the Sivers function have been obtaining assuming a constant, scale-independent Gaussian width.

In Fig. 6, we have compared the SSA for parametrizations (a) and (b) using TMD evolution. As in case of DGLAP evolution, we find that the asymmetry is higher for parametrization (b) than that for parametrization (a) in the case of TMD evolution as well.

IV. DISCUSSION AND SUMMARY

In this work, we have given numerical estimates for $J/\psi$ production in low virtuality electroproduction at JLAB, COMPASS, HERMES, and eRHIC energies, using the TMD evolved parton distribution functions and Sivers function. At leading order, this asymmetry is a clean probe of the gluon Sivers function. We use here the analytic formulation for incorporating the scale evolution of the TMDs and calculate the asymmetries in the color evaporation model for $J/\psi$ production. We compared these estimates with those obtained by us previously using a different parametrization wherein the $Q^2$ dependence of Gaussian width was assumed. The present estimates, while reduced substantially, when compared to our earlier ones, are still sizable for most experiments.

Noting that the average $Q^2$ for our process is $\sim 10$ GeV$^2$, the suppression we find is seen consistent with the suppression in going from HERMES to COMPASS energies, obtained in Ref. [25] in the context of SIDIS. It should also be remembered that in the case of $J/\psi$ production, the transverse momentum dependence of the WW function, can also have non trivial effects.

In the model under consideration, the amount of asymmetry is affected by the evolution factor $R(Q, Q_0)$ as well as by the $Q^2$ dependence of the Gaussian width. In the color evaporation model, $Q^2$ varies between $4m_c^2$ and $4m_D^2$ and $R(Q, Q_0)$ does not vary much over this range. Hence, this factor approximately gets canceled between the numerator and denominator in the evaluation of asymmetry. Thus, the noticeable suppression comes from the logarithmic dependence on $Q^2$ of the Gaussian width. This is consistent with the observation in [28] that the substantial decrease in asymmetry is mainly due to the $Q^2$ dependence of the Gaussian width of TMD PDFs. Hence, it is important to understand this $Q^2$ dependence of Gaussian width in order to be able to make predictions that can be compared with future experiments. In the case of $J/\psi$ production, there is yet another convolution with a transverse momentum dependent WW function. In principle, one needs to model its scale dependence as well. In the end it should be noted that, since the hard scale is the same for all beam energies, the Sivers function is probed in the same $Q^2$ range. This explains the similar amounts of reduction of asymmetries seen at all energies.

Our estimates in the present work are based on the approximate form of Sivers function in Eq. (28) which was obtained in Ref. [28] using TMD evolution equations of the Refs. [22, 24]. This form, called the analytic form in Ref. [28], is obtained by making an approximation $b_T \rightarrow \infty$ in Eq. (24). An exact form can be obtained from Eq. (24) by first evaluating the TMD evolved PDFs in coordinate space $\tilde{F}(x, b_T; Q)$ and then taking their Fourier transform. However, the approximate form we have used is valid for $b_T \geq 1.0\, GeV^{-1}$ if the typical $k_t$ involved is of the order of 1 GeV. Using the exact form may lead to slight differences from the present estimates, and we plan to address this
issue in future work.

V. ACKNOWLEDGEMENTS

R.M.G. wishes to acknowledge support from the Department of Science and Technology, India, under Grant No. SR/S2/JCB-64/2007 under the J.C. Bose Fellowship scheme. A. Misra and V.S.R. would like to thank the Department of Science and Technology, India, for financial support under Grant No. SR/S2/HEP-17/2006 and to the Department of Atomic Energy-BRNS, India, under the grant No. 2010/37P/47/BRNS.

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FIG. 1. The Sivers asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for $e+p^+ \to e+J/\psi + X$ at JLab energy ($\sqrt{s} = 4.7$ GeV) as a function of $y$ (left panel) and $q_T$ (right panel) for parametrization (b). The solid (blue) line corresponds to results obtained using TMD evolution. The dashed (red) and dotted (black) line corresponds to DGLAP evolution with DGLAP fit parameters at $Q_0 = \sqrt{2.4}$ GeV and $Q_0 = 1$ GeV respectively. The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-0.25 \leq y \leq 0.25)$.

FIG. 2. The Sivers asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for $e+p^+ \to e+J/\psi + X$ at HEMRES energy ($\sqrt{s} = 7.2$ GeV) as a function of $y$ (left panel) and $q_T$ (right panel) for parametrization (b). The solid (blue) line corresponds to results obtained using TMD evolution. The dashed (red) and dotted (black) line corresponds to DGLAP evolution with DGLAP fit parameters at $Q_0 = \sqrt{2.4}$ GeV and $Q_0 = 1$ GeV respectively. The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-0.6 \leq y \leq 0.6)$.

FIG. 3. The Sivers asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for $e+p^+ \to e+J/\psi + X$ at COMPASS energy ($\sqrt{s} = 17.33$ GeV) as a function of $y$ (left panel) and $q_T$ (right panel) for parametrization (b). The solid (blue) line corresponds to results obtained using TMD evolution. The dashed (red) and dotted (black) line corresponds to DGLAP evolution with DGLAP fit parameters at $Q_0 = \sqrt{2.4}$ GeV and $Q_0 = 1$ GeV respectively. The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-1.5 \leq y \leq 1.5)$. 
FIG. 4. The Sivers asymmetry $A_N^{\sin(\Phi_{qT} - \Phi_S)}$ for $e + p^+ \rightarrow e + J/\psi + X$ at eRHIC-1 energy ($\sqrt{s} = 31.6$ GeV) as a function of $y$ (left panel) and $q_T$ (right panel) for parametrization (b). The solid (blue) line corresponds to results obtained using TMD evolution. The dashed (red) and dotted (black) line corresponds to DGLAP evolution with DGLAP fit parameters at $Q_0 = \sqrt{2.4}$ GeV and $Q_0 = 1$ GeV respectively. The integration ranges are ($0 \leq q_T \leq 1$) GeV and ($-2.1 \leq y \leq 2.1$).

FIG. 5. The Sivers asymmetry $A_N^{\sin(\Phi_{qT} - \Phi_S)}$ for $e + p^+ \rightarrow e + J/\psi + X$ at eRHIC-2 energy ($\sqrt{s} = 158.1$ GeV) as a function of $y$ (left panel) and $q_T$ (right panel) for parametrization (b). The solid (blue) line corresponds to results obtained using TMD evolution. The dashed (red) and dotted (black) line corresponds to DGLAP evolution with DGLAP fit parameters at $Q_0 = \sqrt{2.4}$ GeV and $Q_0 = 1$ GeV respectively. The integration ranges are ($0 \leq q_T \leq 1$) GeV and ($-3.7 \leq y \leq 3.7$).

FIG. 6. The Sivers asymmetry $A_N^{\sin(\Phi_{qT} - \Phi_S)}$ for $e + p^+ \rightarrow e + J/\psi + X$ at COMPASS energy ($\sqrt{s} = 17.33$ GeV) as a function of $y$ (left panel) and $q_T$ (right panel) using TMD evolution. The solid (red) line corresponds to results obtained using parametrization (a) and the dashed (blue) line correspond to parametrization (b). The integration ranges are ($0 \leq q_T \leq 1$) GeV and ($-1.5 \leq y \leq 1.5$).