The plasma inside a dust free void: hotter, denser, or both?

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\textbf{Abstract.} The existence of a dust-free void, as often observed in dusty plasmas with small particles, or in dusty plasmas under micro-gravity conditions, requires a maximum of the ionization inside the void. Enhanced optical emission inside the void has indeed been observed. The extra losses of plasma on the dust has to be compensated for by extra ionization, which means that the electron temperature must rise. Inside the void there is no depletion of electrons, so a rise in electron temperature is not immediately obvious. It was therefore proposed that the relatively high electron density in the void with respect to the surrounding dusty region, where the electrons are depleted, causes the higher ionization inside the void. Different observations and models have until now not given a decisive answer, however. Using a global model, we predict that a homogeneous dusty plasma without a void should have an increased electron temperature, but at a reduced electron density. A dusty plasma with a fully formed void should have both an increased electron temperature and density in the void. A fully self-consistent two-dimensional model agrees with these conclusions and also shows that the void is a complex system, which is heated by the dust on the outside, but has most of the ionization on the inside.

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1. Motivation

Dusty plasma is plasma containing small solid particles as well as the usual plasma components. A good overview of the physics of dusty plasmas is given in [1, 2]. In a plasma where small sub-micron particles form through chemical reactions [3], or in dusty plasma experiments done under micro-gravity conditions [4], where fabricated dust particles are introduced, a dust-free void is usually observed. This void results from flowing ions being scattered and collected by the dust particles, thereby transferring momentum and pushing them in the direction of the dominant ion flow. Consequently, this void requires a maximum of the ionization inside the dust-free void. The corresponding maximum in optical emission inside the void has indeed been observed in many cases, for instance in [5, 6], and more recently in [7, 8]. These cases include both capacitively coupled, as well as inductively coupled plasmas, both at high input powers of several tens of Watts, and at low input powers of a few Watts. These low input powers are closer to the input power we have in our modelled discharges in this paper, which is roughly 0.1 W.

Dust particles in a plasma collect ions and electrons. In equilibrium, the electron current to the dust equals the ion current and the potential of the dust particles reaches the floating potential. Thus, plasma continuously recombines on the dust and the dust acts as an extra sink for plasma. This loss of plasma has to be compensated by ionization. In the region of depletion, this requires an increase in the average electron energy (i.e. electron temperature), because every single electron must have a higher ionization probability. One can then wonder why the ionization maximum occurs inside the void, since there is no depletion of plasma there.

It was first proposed that the increased ionization is due to an increased electron temperature in the void, caused by the electron depletion in the dust cloud. In a way, this depletion increases the resistivity of the plasma, increasing the electric field [3, 5]. This increased electron temperature was also shown in several simulations [9]–[11].

In experiments however, no increase of the electron temperature was observed [12]. It was then supposed that the increased ionization was due to the enhanced electron density in the void, with respect to the electron depleted plasma in the surrounding dusty region [6, 13]. In a recent experiment in an inductively coupled dusty plasma [8], it was shown that both the electron temperature and the electron density increase in the void. This was also the result of a
simulation [14]. In this one-dimensional simulation the dust and plasma components were not coupled self-consistently, however.

Finally, a recent paper [15] shows that the void is also present when very few particles are used. This means that also in a dust-free discharge the same principles must apply, namely high ionization in the centre of the discharge, resulting in the diffusion of ions from the centre outwards. Apparently, this is still so when a large amount of dust is present.

In this paper, we show how this complex structure of the void comes about. We first demonstrate by using a simple global model, similar to the approach followed in [16, 17], that we always expect an increase in the electron temperature when dust is added to a plasma, but that the behaviour of the electron density in the plasma strongly depends on whether the dust causes a volume loss process or a surface like process, which happens when a void forms. We then continue by describing a fully self-consistent two-dimensional model [18] with which we simulate a dusty discharge under micro-gravity conditions. We show that the plasma in the void is both hotter as well as denser, and we show why this is so.

2. Plasma response to dust: global model

The electron temperature in a discharge is determined by the balance between plasma creation (ionization) and plasma losses (fluxes to the walls). In a global model [16, 17], we can write this in terms of the volume- and surface-averaged quantities as

\[ n_{\text{gas}} n_e \langle \sigma v \rangle V = 0.6 n_e v_{\text{Bohm}} S, \]

where \( V \) is the volume in which ionization takes place, \( \langle \sigma v \rangle \) is the (electron energy dependent) ionization rate, \( v_{\text{Bohm}} \) is the Bohm velocity, \( v_{\text{Bohm}} = \sqrt{k_B T_e / m_e} \), with \( T_e \) the electron temperature and \( m_e \) the ion mass. \( S \) is the surface area of the walls surrounding the plasma. The input power then determines the plasma density, since

\[ P_{\text{input}} = n_{\text{gas}} n_e \langle \sigma v \rangle V \epsilon_{\text{ion}}, \]

with \( \epsilon_{\text{ion}} \) the effective energy required to produce one ion. This effective energy is used to take other mechanisms which remove electron energy into account, such as the electron losses to the walls and other inelastic collisions, i.e. electron impact excitation. It effectively requires more energy to produce one ion than simply the ionization energy.

Suppose now that through chemical reactions small particles have formed homogeneously throughout the discharge. They are well below 30 nm, so that no void forms [19]. We can then assume that the loss of plasma on the dust results in a volume loss process. We therefore rewrite equation (1) into

\[ n_{\text{gas}} n_e \langle \sigma v \rangle V = 0.6 n_e v_{\text{Bohm}} S + n_D n_e k_{\text{rec}} V, \]

where \( n_D \) is the dust particle density and \( k_{\text{rec}} \) is the recombination rate of ions and electrons on the dust, which follows from orbital motion limited (OML) charging theories, as explained in section 3.2. Rewriting this gives

\[ \left( n_{\text{gas}} \langle \sigma v \rangle - n_D k_{\text{rec}} \right) \frac{V}{S} = 0.6 v_{\text{Bohm}}. \]

Since no void has formed, we assume that \( V/S \) does not change. Figure 1 shows the volume production term and the Bohm loss, as well as the production term corrected for the volume losses on the dust found from equation (4). This plot is for a radius of the plasma volume of
Figure 1. This figure shows the losses to the walls, represented by the black line and labelled ‘A’, the volume production without extra losses, represented by the red line labelled ‘B’, and the production corrected for the volume losses, represented by the green line, labelled ‘C’. The points where the last two lines cross the first correspond to the equilibrium points. We see that with volume losses, the equilibrium shifts to a higher temperature resulting in higher plasma production.

2.3 cm, such that \( V/S \approx 0.0075 \). This corresponds to a plasma with the size of the void shown in section 5, homogeneously filled with dust. \( \langle \sigma v \rangle \) is calculated for argon and is the same as used in our fluid model. In this plot, the volume losses to the dust correspond to 50% of the volume production in the plasma.

We see that an extra volume loss term shifts the balance of production and loss to a higher temperature. This of course corresponds to a higher plasma production rate. Assuming that the input power is constant, and that the effective energy required to form an ion also does not change, it follows from equation (2) that this increase in production requires a decrease in the plasma density, \( n_e \). The conclusion then is that volume loss processes increase the electron temperature, but decrease the plasma density.

Assume now that the dust particles have grown further, and a void forms. This way the losses on the dust particles make a transition from a homogeneous volume loss process to a more localized surface loss process at the interface between the dust free void and the dust cloud. We consider this interface more or less as a wall, assuming a rather short penetration depth [20]. We therefore rewrite the particle balance as

\[
 n_{\text{gas}} n_e \langle \sigma v \rangle \tilde{V} = 0.6 n_e v_{\text{Bohm}} \tilde{S},
\]

where \( \tilde{S} \) is the effective absorption surface of the void-dust cloud interface. \( \tilde{V} \) is the new volume in which plasma is formed, which is the volume inside the void. Even though a true sheath might not form near the void-dust interface, so that the Bohm velocity might be an overestimate, see for instance [21], double space charge layers do build up and the void-dust interface acts like a wall [20]. Furthermore, the plasma density is higher in the void than in the sheath regions, so that the total flux arriving at the void-dust interface might still be close to the Bohm flux.

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We rewrite this again as:

$$n_{\text{gas}} \langle \sigma v \rangle \frac{\vec{V}}{S} = 0.6v_{\text{Bohm}}.$$  \hspace{2cm} (6)

We see that now the volume to surface ratio becomes important. Both the dust density and size (which determine $S$) as well as the discharge settings (which play a role in the volume of the void, $V$) determine the losses in the plasma. Figure 2 shows the production and losses again, for two ratios of the volume over the surface. We see that a smaller void (which has a smaller volume over surface ratio) results in a higher electron temperature and more production.

For an eight times smaller volume of the void (going from the blue to the red line in figure 2), we see that the production doubles. This way, we find from equation (2), that the plasma density must be four times higher. So, when the dust forms a void and acts like a wall, with dominant surface loss processes, the electron temperature goes up together with the plasma density. We now turn to the self-consistent fluid model, and show in more detail what happens to the plasma properties in a dusty plasma when a void forms.

3. Description of the 2D model

3.1. Fluid model for the plasma species

The model for the plasma species is based on the drift-diffusion approximation for the fluxes of ions and electrons, which requires the solution of the electric field found from the Poisson equation including the dust charge. The ion transport is governed by an effective electric field, since they are too heavy to follow the high frequency harmonic potential applied to the electrodes.
The electron energy density (i.e. the product of the electron density and the average electron energy \( w_e = n_e \epsilon = n_e \frac{3}{2} k_e T_e \)) is calculated self-consistently from the energy balance. The losses of electron energy include excitation, ionization and recombination of electrons on the dust. The source is Ohmic heating. To compute the sources and sinks as a function of the average electron energy, \( \epsilon \), tables are generated using a two-term Boltzmann solver for the electron energy distribution function. Thus, we account for deviations from a Maxwellian distribution function. The power dissipated by the ions is locally transferred to the neutral gas.

The boundary conditions are a vanishing density for all species with a negative charge and a vanishing density gradient for species with a positive charge. All the details of the plasma model can be found elsewhere [11, 18, 22].

### 3.2. Fluid model for the dust

#### 3.2.1. Charging of the dust

Dust particles are added in the model by means of a source term for the dust fluid. So, we model dust which is introduced into the plasma, and not the formation of dust particles through chemical pathways. Dust particles in a plasma act as small probes and collect ions and electrons. When the electron and ion currents are equal, the ‘floating potential’ is reached with respect to the surrounding plasma. The time averaged currents to the dust are calculated in our model using OML theory [23]. From the condition \( I_i + I_e = 0 \), the floating potential \( V_d \) is found with a Newton method; the dust charge for a spherical particle is then computed as \( e Q_d = -4\pi\epsilon_0 r_d V_d \), with \( \epsilon_0 \) the permittivity of vacuum and \( r_d \) the dust particle radius. Near the walls the dust charge can become positive, due to the large value of \( n_i/n_e \) in the sheath. From the equilibrium current, the recombination rate is calculated.

Even though recent work has shown the importance of ion–neutral collisions for the charging of dust [24, 25], we do not take them into account here. In a previous paper [18], we show that the mean free path in and near the void is much larger than the screening length, even though this does not directly exclude the importance of ion–neutral collisions. Furthermore, in PIC/MC calculations [26] of a similar discharge, which includes all the (in-)elastic collisions between plasma particles and the background neutrals and the dust, we have shown that the OML value for the dust charge is reasonable.

#### 3.2.2. Forces acting on the dust

Different forces act on the charged dust particles in the plasma. The gravitational force is neglected here, thus simulating micro-gravity conditions. Since the dust particles are charged, they are accelerated by the electric field. Due to their high mass, the dust particles only follow the time averaged electric field. Charged dust particles also collect and deflect ions, which results in the ion drag force. Our model takes the effect of moderate nonlinear scattering, the effect of ion drift, as well as the effect of ion-neutral collisions into account [27]–[29]. Due to the heating of the background gas caused by the dissipation of power by the ions and by the elevated surface temperature of the dust, temperature gradients arise in the background gas, whereas the temperature of the surrounding walls is kept at room temperature. The thermophoretic force therefore originates from the heating by processes in the plasma interior, and not from externally heated walls or electrodes. Finally, the neutral drag is calculated. We ignore any advection of the background gas.

By assuming that this neutral drag always balances the other forces, a drift–diffusion-like equation can be found for the transport of the dust. The diffusion part in this drift–diffusion equation comes from the dust fluid pressure gradient and depends on the incompressibility of
the dust fluid. The diffusion coefficient is derived from the equation of state of the dust fluid, and we follow the method described in [30]. A complete description of the calculation of the dust charging and transport can be found in [18].

4. Modelled geometry

We model the geometry of the PKE–Nefedov experiment, which was used aboard the International Space Station [4]. This is a RF device with two cylindrical electrodes, 3 cm apart, with a radius of 2.1 cm, both powered by a 13.56 MHz harmonic potential in push–pull mode. The electrodes are shielded by a so called dark-shield with a radius of 2.4 cm to prevent the generation of a discharge between the sides of the electrodes and the outer wall.

The simulation is started without dust at a constant potential of 100 V peak-to-peak. This corresponds to a total power of $W_{\text{tot}} = 6 \times 10^{-2}$ W. The background pressure is set at $P_{\text{gas}} = 204$ mTorr (27 Pa) at 273 K. In this plasma, the total power equally divides between the ions ($W_+ = 2.8 \times 10^{-2}$ W) and the electrons ($W_e = 3.2 \times 10^{-2}$ W).

When the dust-free plasma reaches a periodic solution, the dust is added. In the experiment dust is introduced with two shakers in the electrodes, which we model by adding source terms for the dust just above the lower and below the upper electrode. The dust particles are mono-disperse and have a radius of 6.8 µm. During the run the void evolves self-consistently. With the dust present, we keep the total power constant (at $6 \times 10^{-2}$ W), which in the final equilibrium state requires a lower driving potential of only $V = 78.8$ V peak-to-peak. We add 500 000 dust particles in total and run the simulation until the equilibrium dust structures are formed. With this amount of dust, the power is distributed over the plasma as $W_e = 4 \times 10^{-2}$ W and $W_+ = 2 \times 10^{-2}$ W.

Even though we do not present them here, the results for a dusty discharge at a constant driving potential of 100 V (instead of constant applied power) are similar. It is interesting to note that the total power then increases to $8 \times 10^{-2}$ W.

5. Results

When we introduce dust in the discharge, the electron density and temperature change. This is shown in figure 3. The general conclusions of the global model are indeed confirmed. When a small number of dust particles is added, the loss terms are best described by a volume loss, which means an increase in temperature, with a decrease in density. When more and more dust particles are introduced, and a void forms, the dust acts more and more as a wall. The temperature still increases, but the density also starts to rise. Similar behaviour is also observed in [31].

The exact details, such as the intermediate rise and fall in electron density, do not follow from the global model. These details depend also on the exact way we model the sources of the dust particles. Most important is another effect, which will be the focus of this paper from now on. It has to do with the spatial separation of the source of electron heating, which drives the ionization, and on the position where this heat is used for ionization.

We start with a dust-free discharge. The left panel of figure 4 shows the power taken up by the electrons. Most of the power is taken up in the pre-sheaths. The average electron temperature ($\epsilon = \frac{3}{2}k_bT_e$) is homogeneous, however, as shown in figure 4 on the right.
Figure 3. The central electron density, represented by the solid black line labelled ‘A’, and the average electron energy (eV), represented by the dotted red line labelled ‘B’, for different times after the dust sources are turned on. It is shown against the total amount of dust particles added to the discharge.

Figure 4. Left: the energy taken up by the electrons in kW m$^{-3}$. The electrons mainly pick up energy in the pre-sheaths. Right: the average electron energy (eV), which is reasonably homogeneous for a large part of the discharge.

The ionization profile peaks in the centre of the discharge, despite the power dissipation in the pre-sheaths. The electron density is also highest there, as shown in figure 5. Already in the dust-free case, we see a clear separation between the position where the electrons gain energy and are heated, and in the position where they deposit this heat and ionize the background gas.

We now turn our attention to the discharge with added dust particles. The final dust density is shown in figure 6 on the left side. The dust-free void is clearly visible. The electron density on the right shows how indeed the plasma is confined inside the void and has a higher density of $4 \times 10^{15}$ m$^{-3}$ instead of $2 \times 10^{15}$ m$^{-3}$.

The heating of the electrons is shown in figure 7 on the left-hand side. We see that most of the power is taken up inside the dust clouds. This is expected since the electron depletion is high there. The average electron energy is again reasonably homogeneous in the bulk of the plasma, but is higher than in the dust-free case, about 6 eV instead of 5 eV.
Figure 5. Left: the ionization rate in $10^{20} \text{ m}^{-3} \text{ s}^{-1}$, which is highest in the centre. Right: the electron density in $10^{15} \text{ m}^{-3}$.

Figure 6. Left: the dust density in $10^{10} \text{ m}^{-3}$. The dust-free void in the centre is clearly visible. Right: The electron density in $10^{15} \text{ m}^{-3}$. The plasma is confined within the volume of the void.

The ionization profile peaks inside the void, as seen in figure 7 in the bottom panel, and reaches much higher values of $9 \times 10^{20} \text{ m}^{-3} \text{ s}^{-1}$, instead of $2 \times 10^{20} \text{ m}^{-3} \text{ s}^{-1}$. This high ionization rate agrees with the high amount of optical emission observed in dusty discharges under micro-gravity conditions.

Obviously, the heating of the electrons and the ionization are again spatially separated, but the effect is clearly enhanced by the presence of the dust. The time averaged energy balance for the electrons can be written as

$$\nabla \cdot \mathbf{\Gamma}_{\text{w_e}} = \mathbf{J}_e \cdot \mathbf{E} - \mathbf{S}_{\text{w_e}}. \quad (7)$$

The bar indicates averaging in time over one RF cycle. The term on the left-hand side is the divergence of the heat flux. The first term on the right-hand side is the source of heating, the Ohmic heating, and the second term contains all the sinks, with the main loss term due to electron impact ionization. Regions of negative divergence of the energy flux thus correspond to regions with more ionization than heating, whereas regions of positive divergence correspond to regions with a net Ohmic heating of the electrons.
Figure 7. Left: the power dissipation in the dusty discharge at constant input power in kW m$^{-3}$. Right: the average electron energy (eV), which is again homogeneous in the plasma bulk, but has a higher value than in the dust free case. Bottom: the ionization rate in 10$^{20}$ m$^{-3}$ s$^{-1}$ in the dusty discharge. The ionization clearly peaks inside the void, and is much more pronounced in the presence of dust, which means much higher optical emission from the plasma in comparison with a dust-free discharge.

Figure 8 shows the time averaged axial energy flux over the central axis in the dusty discharge. We clearly see that the region with positive divergence ($\partial_z \Gamma_{w_e z} > 0$) lies inside the dust clouds. The central void corresponds with the region of negative divergence ($\partial_z \Gamma_{w_e z} < 0$). The energy flux is such that the heat produced in the dusty region is transported to the central void, where it is lost in ionizations.

In the dust-free discharge, the situation is similar, where the heat production in the pre-sheaths is also transported inwards, where it is lost to ionization, but in case of the dusty discharge it is much more pronounced.

Writing out the electron energy density flux and assuming that the diffusion of electron energy density is negligible to the contribution of the drift in the electric field, we find,

$$\Gamma_{w_e} = -\frac{5}{3} \mu_e \left( w_e \overline{E} + \overline{w_e} \overline{E} \right),$$

where the tilde indicates the time dependent oscillations of the quantities around the average value. Our computations show that the second term on the right-hand side is much smaller than the first term on the right-hand side, in most of the discharge, except in the sheaths. Therefore, it can be neglected here.

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Figure 8. The axial electron energy flux (W m$^{-2}$) along the central axis of the discharge, between the electrodes, for the case without dust, presented by the red dotted line labelled ‘A’, and for the case with dust presented by the black solid line labelled ‘B’. The heating in the cloud and the ionization in the void, together with the transport of the heat from the dust to the void is clearly shown.

Using this to write out the left-hand side of equation (7), we find

$$\nabla \cdot \Gamma_w \approx -5\mu_e\epsilon \mathbf{E} \cdot \nabla n_e - 5\mu_e n_e \mathbf{E} \cdot \nabla \epsilon. \quad (9)$$

We have dropped the bars since all quantities are time-averaged. In the above equation we have assumed that quasi-neutrality holds, $\nabla \cdot \mathbf{E} = 0$. The time averaged electric field points from the centre to the walls. The gradient of the time averaged electron density points in the opposite direction. This means that the first term on the right-hand side of equation (9) is always positive. Therefore, the negative sign of the divergence of the heat flux from the dust clouds to the void is due to the gradient in the electron temperature acting together with the time averaged electric field.

6. Conclusions

A global model predicts an increase of the electron temperature when dust is added to a plasma. When the dust is homogeneous and no void forms, for instance during early stages of dust formation in chemically active plasmas, a decrease of the electron density is predicted. When more dust of larger size is added and a void forms, the dust starts acting as a wall and a rise of the electron density is predicted. These predictions are confirmed by a self-consistent two-dimensional fluid model, but an important effect is caused by the separation of the electron heating in the dust cloud and the ionization inside the void. Therefore, the electron density is not only influenced by a transition from volume recombination on the dust to a surface-like recombination on the dust, but also by a strong divergence of the heat flux, which grows for an increasing amount of dust in the discharge.

Our fluid model shows how in the dust-free discharge the electrons primarily pick up energy in the pre-sheaths. The ionization is highest in the centre of the discharge, which is
confirmed by the observations in [15]. When dust is added, the electrons are heated inside the dust clouds, due to the depletion there, but the ionization has a maximum inside the void. We have shown that the heat produced inside the dust clouds is transported to the void and results in plasma production there. The heat is transported by a growing gradient in the electron temperature, acting together with the electric field, and not by the gradient in the electron density in this field, since this acts in the opposite direction.

The final equilibrium is much like a kettle on a stove, which is heated on the outside, but has water boiling inside. However, the kettle keeps the boiling water inside, whereas here the ‘boiling’ plasma keeps the ‘kettle’ out by making a void. This makes the void such a fascinating and complex structure.

Comparing our results with the observations mentioned in the motivation, we conclude that indeed the electron temperature inside the void is lower than in the surrounding dusty area, but it is still higher than in the dust-free case. This elevated temperature is due to the transport of heat from the electron depleted regions, into the void. This, together with the increased electron density, causes the enhanced ionization, which is also observed in terms of enhanced optical emission during and after void formation.

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