Alternative way to understand the unexpected results of the JLab polarization experiments to measure the Sachs form factors ratio

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In the one-photon exchange approximation we discuss questions related to the interpretation of unexpected results of the JLab polarization experiments to measure the Sachs form factors ratio $G_E/G_M$ in the region $1.0 \leq Q^2 \leq 8.5 \text{ GeV}^2$. For this purpose, we developed an approach which essentially is a generalization of the constituent-counting rules of the perturbative QCD (pQCD) for the case of massive quarks. We assume that at the lower boundary of the considered region the hard-scattering mechanism of pQCD is realized. Within the framework of the developed approach we calculated the hard kernel of the proton current matrix elements $J_{p}^{0,\beta,\delta}$ for the full set of spin combinations corresponding to the number of the spin-flipped quarks, which contribute to the proton transition without spin-flip ($J_{p}^{0,\beta,\delta}$) and with the spin-flip ($J_{p}^{\beta,\delta}$). This allows us to state that (i) around the lower boundary of the considered region, the leading scaling behavior of the Sachs form factors has the form $G_E/G_M \sim 1/Q^6$, (ii) the dipole dependence ($G_E/G_M \sim 1/Q^4$) is realized in the asymptotic regime of pQCD when $\tau \gg 1$ ($\tau = Q^2/(4M^2)$) in the case when the quark transitions with spin-flip dominate, (iii) the asymptotic regime of pQCD in the JLab experiments has not yet been achieved, and (iv) the linear decrease of the ratio $G_E/G_M$ at $\tau < 1$ is due to additional contributions to $J_{p}^{\beta,\delta}$ by spin-flip transitions of two quarks and an additional contribution to $J_{p}^{0,\beta,\delta}$ by spin-flip transitions of three quarks.

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I. INTRODUCTION

Experiments aimed at studying the proton form factors (FFs), the electric ($G_E$) and magnetic ($G_M$) ones, which are frequently referred to as the Sachs FFs, have been performed since the mid 1950 s [1, 2] by using elastic electron-proton scattering. In the case of unpolarized electrons and protons, all experimental data on the behavior of the proton FFs were obtained by using the Rosenbluth formula [1] for the differential cross section for the reaction $ep \rightarrow ep$; that is,

$$\frac{d\sigma}{d\Omega_e} = \frac{\alpha^2 E^2 \cos^2(\theta_e/2)}{4E_1^4 \sin^4(\theta_e/2)} \frac{1}{1 + \tau} \left( \frac{G_E^2}{2} + \frac{\tau G_M^2}{\varepsilon} \right).$$

(1)

Here, $\tau = Q^2/4M^2$, $Q^2 = -q^2 = 4E_1E_2 \sin^2(\theta_e/2)$ is the square of the momentum transfer to the proton and $M$ is the proton mass; $E_1$, $E_2$, and $\theta_e$ are, respectively, the initial-electron energy, the final-electron energy, and the electron scattering angle in the rest frame of the initial proton; $\varepsilon$ is the degree of the linear polarization of the virtual photon [3, 4], $\varepsilon^{-1} = 1 + 2(1 + \tau) \tan^2(\theta_e/2)$; and $\alpha = 1/137$ is the fine-structure constant. Expression (1) was obtained in the one-photon exchange approximation and the electron mass was set to zero.

With the aid of Rosenbluth’s technique, it was found that the experimental dependences of $G_E$ and $G_M$ on $Q^2$ are well described up to 10 GeV$^2$ by the dipole-approximation expression

$$G_E = G_M/\mu = G_D(Q^2) \equiv (1 + Q^2/0.71)^{-2},$$

(2)

where $\mu$ is the proton magnetic moment ($\mu = 2.79$).

In [3], Akhiezer and Rekalo proposed a method for measuring the ratio of the Sachs FFs. Their method relies on the phenomenon of polarization transfer from the longitudinally polarized initial electron to the final proton. Precision experiments based on the method [4] were performed at JLab [6, 7]. They showed that in the range of $0.5 < Q^2 < 5.6$ GeV$^2$, there was a linear decrease in the ratio $R = \mu G_E/G_M$ with increasing $Q^2$,

$$R = 1 - 0.13(Q^2 - 0.04),$$

(3)

which indicates that $G_E$ falls faster than $G_M$. This is in contradiction with data obtained with the aid of Rosenbluth’s technique; according to those, the approximate equality $R \approx 1$ must hold. Repeated, more precise, measurements of the ratio $R$ using the polarization transfer method [9, 10] and by Rosenbluth’s method [11] only confirmed this contradiction. In order to resolve this contradiction, it was assumed that the discrepancy in question may be caused by disregarding, in the respective analysis, the contribution of two-photon exchange (TPE) (see work [12], the reviews [13, 14], and references therein). At the present time, three experiments aimed at studying the contribution of TPE are known. It is an experiment

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\[1\] An analogous prediction was made by D.V. Volkov in 1965 based on SU(6) symmetry for baryon octet.
at the VEPP-3 storage ring in Novosibirsk, the OLYMPUS experiment at the DORIS accelerator at DESY, and the EG5 CLAS experiment at JLab.

In 15, we proposed a new alternative to the method 4 for determining the Sachs FFs in the process e⃗p → e⃗p on the basis of measuring cross sections for spin-flip and non-spin-flip transitions for protons.

The aims of this paper are (i) the interpretation in the one-photon exchange approximation unexpected results of the JLab polarization experiments to measure the Sachs FFs ratio as well as the explanation of the reason for the linear dependence in 3, and (ii) the determination of the conditions for the realization of the Sachs FFs dipole dependence based on the use of the hard-scattering mechanism (HSM) of perturbative QCD (pQCD) under the assumption that the onset of pQCD starts around the lower boundary of the considered region.

It is, in general, admitted that the onset of the asymptotic regime of pQCD starts around the J/Ψ mass squared. It was first observed in work 16 that the proton magnetic FF $G_M$ follows the asymptotic pQCD predictions of $17, 18$ and $Q^2G_M$ becomes nearly constant (with the logarithmic accuracy, modulo log($Q^2$) factors) starting at $Q^2 \approx 9$ GeV$^2$. The answer to the question what is in general admitted at present on the onset of pQCD can be found in 19–21. In Refs. 19, 20, based on using completely different approaches, it is shown that the point of transition from non-perturbative QCD to pQCD correspond to a momentum scale $Q_0 \sim 1$ GeV. For this reason we will below assume that HSM of pQCD starts at the lower boundary of the considered region, i.e. around $Q_0 \sim 1$ GeV. In 21 within the analytic perturbation theory (APT) approach using the rules of the Gerasimov-Drell-Hearn, it is shown that the point of "crosslinking" of the perturbative and nonperturbative regimes in APT is significantly lower than that obtained in the framework of the standard pQCD, where $Q_0 \sim 1$ GeV. The main reason for such a significant forwarding down of $Q$ within the APT approach is the disappearance of the nonphysical singularities of the perturbation theory series. It should be noted that in the known work of Belitsky et al. 22 the authors have performed numerical calculations in the framework of pQCD in the region of $0.5 \leq Q^2 \leq 5.5$ GeV$^2$; therefore, they proceeded from the assumption that the onset of pQCD starts already at $Q^2 = 0.5$ GeV$^2$. It is very likely that the results of Ref. 22 are an indirect proof of the correctness results of Ref. 21 obtained in the framework of the APT.

In order to achieve goals, we will use the formalism of the method for calculating the matrix elements of QED processes in the diagonal spin basis (DSB) 23, 24.

II. PHYSICAL MEANING OF THE SACHS FFS

It is well known that in the Breit frame of the initial and the final proton, the Sachs FFs $G_E$ and $G_M$ describe the distributions of the proton charge and magnetic moment, respectively, and their advantage is due to the simplification of expression (1). The question of whether there is any physical meaning behind the decomposition of $G_E^2$ and $G_M^2$ in Rosenbluth’s cross section was not raised and not discussed either in textbooks or in scientific literature. Nevertheless, it was shown many years ago in the work of Sikach 23 that the FFs $G_E$ and $G_M$ factorize in the DSB even at the level of amplitudes in calculating (in an arbitrary reference frame) the proton current matrix elements in the cases of non-spin-flip and spin-flip transitions for the proton.

A. Diagonal spin basis

In the DSB, the spin four-vectors $s_1$ and $s_2$ of fermions with four-momenta $q_1$ (before the interaction) and $q_2$ (after it) have the form 23

$$s_1 = -\frac{(v_1v_2)v_1 - v_2}{\sqrt{(v_1v_2)^2 - 1}}, \quad s_2 = \frac{(v_1v_2)v_2 - v_1}{\sqrt{(v_1v_2)^2 - 1}},$$

(4)

where $v_1 = q_1/M$ and $v_2 = q_2/M$. They satisfy ordinary conditions – that is, $s_1s_1 = s_2s_2 = 0$ and $s_1^2 = s_2^2 = -1$ – and are invariant under the transformations of a little group of Lorentz group $L_{q_1,q_2}$ common to particles with 4-momenta $q_1$ and $q_2$: $L_{q_1,q_2}q_1 = q_1$ and $L_{q_1,q_2}q_2 = q_2$. This group is isomorphic to the one-parameter subgroup of the rotational group $SO(3)$ with an axis whose direction is determined by the three-dimensional vector 26

$$a = q_1/q_{10} - q_2/q_{20}.$$  

(5)

For the two particles in question, the spin projections onto the direction specified by the vector $a$ in Eq. 5 simultaneously have specific values 26, 2.

Let us consider the realization of the DSB in the initial proton rest frame, where $q_1 = (q_{01}, q_1) = (M, 0)$. In this case for the vector $a$ in Eq. 5 we have $a = n_2 = q_3/[q_3^2]$; that is, the direction of the final proton motion is a common direction onto which one projects the spins in question. Therefore, in the rest frame of the initial proton the polarization state of the final proton is a helicity state, while the spin four-vectors $s_1$ and $s_2$ in the DSB 41 have the form

$$s_1 = (0, n_2), \quad s_2 = (|v_2|, v_{20}n_2).$$

(6)

Note in the DSB the particles with the 4-momenta $q_1$ (before interaction) and $q_2$ (after interaction) have common spin operators 24, 27. This makes it possible to

2 The vector $a$ in Eq. 5 is the difference of two three-dimensional vectors, and the geometric image of the difference of two three-vectors is a diagonal of the parallelogram. This is the reason why the term “DSB” was introduced by academician F.I. Fedorov. Note the Breit frame, where $q_2 = -q_1$ is a particular case of the DSB.
separate the interactions with and without change in the spin states of the particles involved in the reaction and, thus, to trace the dynamics of the spin interaction. 3

B. Amplitudes of the proton current in DSB

The matrix elements of the proton current in the one-photon exchange approximation has the form

$$ (J^\mu_{\delta,\delta})_\mu = \tau^\delta(q_2, s_2) \Gamma_\mu(q^2) u^\delta(q_1, s_1), \quad (7) $$

$$ \Gamma_\mu(q^2) = F_1 \gamma_\mu + \frac{F_2}{4M} (\hat{q} \gamma_\mu - \gamma_\mu \hat{q}), \quad (8) $$

where $u(q_1, s_1)$ and $u(q_2, s_2)$ are the bispinors of the protons with four-momenta $q_1$ and $q_2$ and spin four-vectors $s_1$ and $s_2$; accordingly, we have $q_1^2 = M^2$, and $\tau(q_1) u(q_1) = 2M (i = 1, 2)$; $q = q_2 - q_1$ is the four-momentum transfer to the proton; $\gamma_\mu$ and $\hat{q}$ are the Dirac operators, $\hat{q} = \gamma^\mu q_\mu$; $F_1$ and $F_2$ are, respectively, the Dirac and Pauli FFs.

The matrix elements of the proton current corresponding to the proton transitions without and with spin-flip calculated in the DSB have the form [23, 25]

$$ (J^\delta_{\delta,\delta})_\mu = 2M G_E(b_0)_\mu, $$

$$ (J^\delta_{-\delta})_\mu = -2 M \delta \sqrt{G_M(b_\delta)_\mu}, \quad (10) $$

where $G_E$ and $G_M$ are the Sachs FFs

$$ G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2. \quad (11) $$

In expressions (9), (10) we used an orthonormalized basis (tetrad) of four-vectors $b_A (A = 0, 1, 2, 3)$; that is,

$$ b_0 = q_+ / \sqrt{q^2}, \quad b_3 = q_- / \sqrt{-q^2}, $$

$$ (b_1)_\mu = \varepsilon_{\mu\nu\rho\sigma} b^\nu b^\rho b^\sigma / 2, \quad (b_2)_\mu = \varepsilon_{\mu\nu\rho\sigma} q^1_\sigma / \rho. \quad (12) $$

Here, $q_+ = q_2 + q_1$, $q_- = q = q_2 - q_1$, $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita tensor ($\varepsilon_{0123} = -1$), $p_1$ is the four-momentum of the initial electron, and $\rho$ is determined from the normalization conditions $b_1^2 = b_3^2 = b_2^2 = -b_0^2 = -1$, where $b_{\pm,\delta} = b_1 \pm i \delta b_2$, $b_3^2 = b_{-\delta}^2 = -2$, $\delta = \pm 1$.

Note that the matrix elements of the proton current in the DSB that correspond to the proton transitions without and with spin-flip given by Eqs. (9), (10) are expressed only in terms of the electric, $G_E$, and magnetic, $G_M$, FFs, respectively. It is precisely because of this factorization of $G_E$ and $G_M$ that Rosenbluth’s formula is decomposed for the sum of two terms containing only $G_E$ and $G_M$, which are responsible for the contributions of the transitions without and with spin-flip of the proton, respectively.

In the case of pointlike particles having a mass $m_q$, their current amplitudes have the form

$$ (J^\delta_{\delta,\delta})_\mu = 2m_q (b_0)_\mu, $$

$$ (J^\delta_{-\delta})_\mu = -2 m_q \delta \sqrt{G_M(b_\delta)_\mu}, \quad \tau_q = Q_q^2 / 4m_q^2. \quad (14) $$

In the ultrarelativistic (massless) case, only spin-flip transitions contribute to the cross section for the process being considered, since the amplitudes without spin-flip in Eq. (9) and Eq. (13) vanish. At first glance, this conclusion contradicts the well-known fact that in the massless limit, only amplitudes of the processes corresponding to helicity-conserving transitions do not vanish. Such processes are frequently referred to as non-spin-flip processes. However, this terminology is highly conditional since the particles involved have different directions of motion before and after the interaction event. Moreover, it is erroneous since in helicity-conserving processes at high energies the spins of the particles are in fact flipped. There is no contradiction here since in the DSB the initial state for ultrarelativistic particles is a helicity state, while the final state has a negative helicity, with the result

$$ M^{\delta,\delta} = M^{-(\delta),\lambda} = M^{\lambda,\lambda}, \quad M^{\delta,\delta} = M^{-(\delta),\lambda} = 0. \quad (15) $$

Along with the representation (8) for $\Gamma_\mu(q^2)$, another equivalent representation is often used,

$$ \Gamma_\mu(q^2) = G_M \gamma_\mu - \frac{(q_1 + q_2)_\mu}{2M} F_2. \quad (16) $$

On the basis of the explicit form (8) and (16) for $\Gamma_\mu(q^2)$, it is often stated (see e.g. [6, 7, 28]) that the proton Dirac FF $F_1$ (proton Pauli FF $F_2$) corresponds to helicity-conserving (helicity-flip) transitions of the proton, respectively. In fact, it is $G_M (G_E)$ rather than $F_1$ ($F_2$) that is responsible for helicity-conserving (helicity-flip) transitions at high $q_1$ and $q_2$ [see Eqs. (9), (10), (13)].

We note that in the literature sometimes there is no clear understanding of the physical meaning of the quantity $\varepsilon$ in formula (11). So in [6, 7, 11, 14, 25] it is written that the quantity $\varepsilon$ is a degree of the longitudinal polarization of the virtual photon. In fact $\varepsilon$ is the degree of the linear polarization of the virtual photon (see [23, 24]).

III. THE $Q^2$ DEPENDENCE OF THE SACHS FFs $G_E$ AND $G_M$

Let us consider the $Q^2$ dependence of the absolute values of the matrix elements of the proton currents (9), (10) and pointlike-particle ones (12), (13). We note that the factorization of $2M$ and $2m_q$ in expressions (9), (10), and (13), (14) is due to normalizing the particle bispinors by the condition $\bar{u}_i u_i = 2m_i$. In performing further calculations, it is more convenient to employ the normalization conditions $\bar{u}_i u_i = 1$. Since $|b_0| = 1$ and $|b_\delta| = \sqrt{2}$ for

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3 The spin states of massless particles in the DSB coincide up to sign with helical states [23]; in this case, the DSB formalism is equivalent to the CALKUL group method [24].
the absolute values of the matrix elements of the proton currents $J_p^{\pm,\delta}$ and pointlike-particle ones $J_q^{\pm,\delta}$, we get the following expressions

$$J_p^{\pm,\delta} = G_E, J_p^{-\delta,\delta} = \sqrt{\tau} G_M,$$  
$$J_q^{\pm,\delta} = 1, J_q^{-\delta,\delta} = \sqrt{\tau_q}.$$ \(17\)

(18)

In these expressions due to $|b_2| = \sqrt{2}$ it was necessary to write correctly not $\tau$ and $\tau_q$ but $\tau' = 2\tau$ and $\tau_q' = 2\tau_q$, but below we shall omit the primes.

Let us consider the HSM of pQCD \(17\) in the process $e p \rightarrow e p$ that is realized as we believe at $Q^2 \geq 1$ GeV$^2$. In this case the leading contribution to the proton current can be presented as a sum of the hard gluon exchange processes, where the proton is replaced by a set of three almost on mass shell quarks as illustrated in Fig. 1.

![Fig. 1: Typical Born diagrams for the proton FFs.](image)

Below we will suppose the masses of quarks $m_q$ to be equal to $1/3$ of the proton mass $M$ and the fraction of their transfer momenta to be equal. So we have

$$\tau_q = \tau.$$ \(19\)

Under such simplifying assumptions it can easily be verified that the matrix element corresponding to the sum of two gauge-invariant diagrams, shown in Fig. 1, has the form

$$(J_{p_1,2}^{\pm,\delta})^\mu = (J_q^{\pm,\delta})^\nu (J_q^{\pm,\delta})_\nu (J_q^{\pm,\delta})^\mu / Q^6,$$ \(20\)

where $Q^6$ in the denominator corresponds to the product of two gluon propagators, of an order of magnitude $1/Q^2$ and two quark propagators of an order $1/Q$. Therefore, the absolute magnitudes of the proton current matrix elements $J_p^{\pm,\delta}$ that correspond to the contribution of the full set of possible Feynman diagrams can be written as the product of three point-quark current amplitudes $J_q^{\pm,\delta}$ divided by $Q^6$,

$$J_p^{\pm,\delta} \sim J_q^{\pm,\delta} J_q^{\pm,\delta} J_q^{\pm,\delta} / Q^6.$$ \(21\)

Relations \(17\), \(18\), \(19\), and \(21\) make it possible to show how there arises the $Q^2$ dependence of $G_E$ and $G_M$ in the HSM of pQCD and explain the results of polarization experiments at JLab.

There are two possibilities for a proton non-spin-flip transition: (i) none of the three quarks undergoes a spin-flip transition and (ii) two quarks undergo a spin-flip transition, while the third does not. We denote the number of such ways as $n_{qE}^{\delta,\delta} = [0, 2]$. Proton spin-flip can also proceed in two ways: (i) one quark undergoes a spin-flip transition, while the other two do not, and (ii) all three quarks undergo a spin-flip transition. We denote the number of such ways by $n_{qM}^{\delta,\delta} = [1, 3]$. Thus, there are in all four combinations to be considered:

$$n_{qE}^{\delta,\delta} \times n_{qM}^{\delta,\delta} = (0, 1) \oplus (0, 3) \oplus (2, 1) \oplus (2, 3).$$ \(22\)

Note due to Eqs. \(18\), \(19\) at $\tau \ll 1$ ($\tau \gg 1$) the quark transition without (with) spin-flip dominates. Therefore, the sets (0,1) and (2,3) with the minimal and maximal number of spin-flip quarks are realized at $\tau \ll 1$ and $\tau \gg 1$, respectively.

### A. The set (0,1), $G_E, G_M \sim 1/Q^6$, $G_E/G_M \sim 1$

Let us consider the first (0,1) set corresponding to a proton non-spin-flip transition $J_p^{\delta,\delta}$ for the case where there is no spin-flip for any of the three quarks and corresponding to the proton transition $J_p^{-\delta,\delta}$ where spin-flip occurs only for one quark. According to Eqs. \(17\), the matrix elements of the proton current $J_p^{\delta,\delta}$ and $J_p^{-\delta,\delta}$ must be proportional to $G_E$ and $G_M$, respectively; as a result, we have

$$J_p^{\delta,\delta} = G_E \sim 1 \times 1 \times 1 / Q^6,$$ \(23\)

$$J_p^{-\delta,\delta} = \sqrt{\tau} G_M \sim \sqrt{\tau} \times 1 \times 1 / Q^6,$$ \(24\)

where the factors of unity and $\sqrt{\tau}$ on the right-hand side of Eqs. \(23\) and \(24\) correspond to non-spin-flip transitions for three pointlike quarks and to the spin-flip transition for one quark. As a result, we have

$$G_E \sim 1 / Q^6, G_M \sim 1 / Q^6, G_E / G_M \sim 1.$$ \(25\)

Therefore, for the set (0,1) the FFs ratio $G_E/G_M$ behaves in just the same way as in the dipole case. However, the dependencies $G_E, G_M \sim 1/Q^6$ are not dipole ones.

### B. The set (0,3), $G_E \sim 1/Q^6$, $G_M \sim 1/Q^4$

Let us consider the (0,3) set. For this purpose we write equalities similar to \(23\) and \(24\); that is,

$$J_p^{\delta,\delta} = G_E \sim 1 \times 1 \times 1 / Q^6,$$ \(26\)

$$J_p^{-\delta,\delta} = \sqrt{\tau} G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau} / Q^6.$$ \(27\)

From here, we obtain

$$G_E \sim 1 / Q^6, G_M \sim \tau / Q^6, G_E / G_M \sim 1 / \tau \sim 4 M^2 / Q^2,$$ \(28\)

$$Q^2 G_E / G_M \sim 4 M^2 = \text{const.}$$ \(29\)

Relation \(29\) is sometimes called in the literature the Brodsky saturation law; it really corresponds to a maximal possible number of the quark spin-flip transitions.
C. The set (2,1), \( G_E \sim 1/Q^4, G_M \sim 1/Q^6 \)

Let us consider the set (2,1) in Eq. (22). Following the same line of reasoning as above, we have

\[
G_E \sim \frac{\tau}{Q^6}, \quad G_M \sim \frac{1}{Q^6}, \quad \frac{G_E}{G_M} \sim \tau \sim \frac{Q^2}{4M^2}, \quad (30)
\]

\[
Q^2 \frac{G_M}{G_E} \sim 4M^2 = \text{const}. \quad (31)
\]

D. The set (2,3), \( G_E, G_M \sim 1/Q^4, G_E/G_M \sim 1 \)

The set (2,3) is generated by spin-flip transitions for two quarks in the case of the contribution to \( J_p^{-\delta,\delta} \) and by spin-flip transitions for all three quarks in the case of the contribution to \( J_p^{\delta,\delta} \). For this case we have

\[
J_p^{\delta,\delta} = G_E \sim \sqrt{\tau} \times \sqrt{\tau} \times 1/ Q^6, \quad (32)
\]

\[
J_p^{-\delta,\delta} = \sqrt{\tau} G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau} / Q^6. \quad (33)
\]

Hence, we obtain

\[
G_E \sim \frac{1}{Q^4}, \quad G_M \sim \frac{1}{Q^4}, \quad \frac{G_E}{G_M} \sim 1. \quad (34)
\]

Therefore, the dipole dependence in the behavior of the FFs \( G_E \) and \( G_M \) on \( Q^2 \) occurs in the set (2,3) at \( \tau \gg 1 \) in the case when a number of quark transitions with spin-flip saturation takes place.

Thus, our approach is in fact a generalization of constituent-counting rules for the massive quarks. Note, in Ref. [29] to estimate the leading contribution of the HSM in the proton magnetic FF within the standard pQCD with massless quarks, a method similar to our approach was used. At the same time, formulas (16), (17) in Ref. [29] and our formula (27) are the same and reproduce the well-known result obtained in the works of Brodsky [30] within the framework of the constituent-counting rules before the development of QCD.

IV. SPIN PARAMETRIZATION FOR \( G_E/G_M \)

The non-spin-flip \( (J_p^{\delta,\delta}) \) and spin-flip \( (J_p^{-\delta,\delta}) \) proton-current amplitudes can be represented as the linear combinations

\[
J_p^{\delta,\delta} = \alpha_0 J_q^{\delta,\delta} J_q^{\delta,\delta} J_q^{\delta,\delta} + \alpha_2 J_q^{-\delta,\delta} J_q^{-\delta,\delta} J_q^{-\delta,\delta}, \quad (35)
\]

\[
J_p^{-\delta,\delta} = \beta_1 J_q^{\delta,\delta} J_q^{\delta,\delta} J_q^{\delta,\delta} + \beta_3 J_q^{-\delta,\delta} J_q^{-\delta,\delta} J_q^{-\delta,\delta}, \quad (36)
\]

where the coefficients \( \alpha_0, \alpha_2, \beta_1, \) and \( \beta_3 \) have a clear physical meaning that is determined by their indices. With the aid of Eqs. (35) and (36), one can readily obtain a general expression for the ratio \( G_E/G_M \). The result is

\[
\frac{G_E}{G_M} = \frac{\alpha_0 + \alpha_2 \tau}{\beta_1 + \beta_3 \tau}. \quad (37)
\]

This expression may serve as a basis for constructing spin parametrization and fits experimental data obtained by measuring the ratio \( G_E/G_M \).

We showed above that at \( \tau \ll 1 \) the quark transition without spin-flip dominates; the set (0,1) with the minimal number of spin-flip quarks, where \( G_E/G_M \sim 1 \), must occur. In this case the coefficients \( \alpha_0 \) and \( \beta_1 \) in Eq. (37) must have the values close to unity. With allowance for this comment, we expand the right-hand side of (37) in a power series for \( \tau \). As a result, we get the law of a linear decrease in the ratio \( R = G_E/G_M \) as \( Q^2 \) increases,

\[
R \approx 1 - (\beta_3 - \alpha_2) \tau. \quad (38)
\]

V. CONCLUSION

We have discussed in the one-photon exchange approximation the questions related to the interpretation of the JLab polarization experiment’s unexpected results to measure the Sachs FFs ratio \( G_E/G_M \) in the region \( 1.0 \leq Q^2 \leq 8.5 \text{GeV}^2 \). For this purpose, in the case of the HSM of the pQCD, we calculated the hard kernel of the proton current matrix elements \( J_p^{\delta,\delta} \) for the full set of spin combinations corresponding to a number of the spin-flipped quarks, which contribute to the proton transition without spin-flip \( (J_p^{\delta,\delta}) \) and with the spin-flip \( (J_p^{-\delta,\delta}) \). This allows us to state that (i) around the lower boundary of the considered region the leading scaling behavior of the Sachs FFs has the form \( G_E/G_M \sim 1/Q^6 \), (ii) the dipole dependence \( (G_E, G_M \sim 1/Q^4) \) is realized in the asymptotic regime of pQCD when \( \tau \gg 1 \) in the case when the quark transitions with spin-flip dominate, (iii) the asymptotic regime of pQCD in the JLab experiments has not yet been achieved, and it is likely that the asymptotic regime for \( G_E \) occurs at higher values \( Q^2 \) than for \( G_M \), (iv) and the linear decrease of the ratio \( G_E/G_M \sim 1/Q^4 \) at \( \tau < 1 \) is due to additional contributions to \( J_p^{\delta,\delta} \) by spin-flip transitions of two quarks and an additional contribution to \( J_p^{-\delta,\delta} \) by spin-flip transitions of three quarks.

Thus, abandoning the massless quarks, we were able to explain in the one-photon exchange approximation the unexpected results of measurements of the proton Sachs FFs ratio and analytically derive the experimentally established formula of the linear decrease law for this ratio at \( \tau < 1 \). We believe that the interpretation presented above can be considered as a possible way to solve the \( G_E/G_M \) problem.

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