ERBL and DGLAP kernels for transversity distributions.

Two-loop calculations in covariant gauge

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Abstract

The results of a two-loop calculation in the Feynman gauge of both the DGLAP and the ERBL evolution kernels for transversely polarized distributions are presented. The structure of these evolution kernels is discussed in detail. In addition, the effect of the two-loop evolution on the distribution amplitude of a twist-2 transversely polarized meson is explored.

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1. Introduction

Evolution kernels are the main ingredients of the well-known evolution equations for the parton distribution in DIS processes [1] and for the parton distribution amplitudes [2] in hard exclusive reactions. These equations describe the dependence of the parton distributions on the renormalization scale $\mu^2$. Previously, two-loop calculations were performed for the unpolarized forward DGLAP evolution kernel $P(z)$ in [1, 3, 4] and, what is more cumbersome, for the nonforward ERBL kernel $V(x, y)$ in [2, 5, 6] that was challenging and complicated technical tasks. Here, we present the results of a direct calculation of evolution kernels for the transversity distributions in next-to-leading-order (NLO) performed in the $\overline{\text{MS}}$ scheme. These calculations are carried out in the Feynman gauge within a single mold for both the forward kernels and the nonforward ones, i.e.,

$$P_T(x) = a_s \frac{P_T^0(x)}{2} + a_s^2 \frac{P_T^1(x)}{2} + \ldots,$$

$$V_T(x, y) = a_s \frac{V_T^0(x, y)}{2} + a_s^2 \frac{V_T^1(x, y)}{2} + \ldots,$$

where $a_s = \alpha_s(\mu^2)/(4\pi)$.

Note that the kernel $P_T^1$ was first obtained in [7] within a light-cone gauge calculation and shortly thereafter the corresponding anomalous dimensions $\gamma^T_1(n)$ were presented in [8, 9]. The kernel $V_T^1$ was reconstructed in [10] on the basis of the knowledge of the structure of symmetry-breaking terms for the kernel, which first appeared at the two-loop level. For the reader’s convenience, let us explain these issues in more detail. Those terms of $V_T^1$ that are responsible for the conformal-symmetry breaking can be fixed and expressed via some special convolutions of the known [10–12] one-loop kernel elements. At the same time, the remaining part or, in other words, the symmetrical part (in terms of a conformal-group representation) of this kernel can eventually be restored from a certain part of the forward kernel $P_T^1$ [10]. This possibility of “guessing” will not be pursued here.

The calculation of $V_T$ or $P_T$ can be performed following the standard procedure to find the renormalization-group generators in the $\overline{\text{MS}}$ scheme. Expressions for them in terms of the renormalization constant $Z_T$ for every diagram $\Gamma$ in (see, e.g., [14] and [6]) are given by

$$Z_T = 1 - \hat{K} R'(\Gamma), \quad V(P) = -a_s \partial_{a_s} \left( Z_T^{(1)} \right) = a_s \partial_{a_s} \left( \hat{K} R'(\Gamma) \right) \frac{\text{NLO}}{2\hat{K} R'(\Gamma)}.$$ 

Here, (i) $R'$ is the incomplete BPHZ $R$-operation; $D = 4 - 2\varepsilon$ is the space-time dimension, (ii) $\hat{K}$ separates out poles in $\varepsilon$, whereas $\hat{K}_1$ picks out a simple pole, and (iii) $Z^{(1)}$ is the coefficient of the simple pole in the expansion of $Z_T$. To introduce an appropriate notation for the analysis of the two-loop results, let us start with the leading order $V_T^0 (P_T^0)$ results obtained in a covariant $\xi$-gauge$^1$,

$$P_T^0(x) = C_F \left[ p_0(x) + \delta(1-x) \right],$$

$$V_T^0(x, y) = C_F \left[ 2F_T(x, y) + (x \to \bar{x}, \ y \to \bar{y}) \right] - \delta(y-x).$$

Here, $p_0(x) \equiv \frac{4x}{1-x}$; $F_T(x, y) \equiv \frac{x}{y(x-y)}$; $\bar{x} = 1-x$, $\bar{y} = 1-y$; symbol $(\ldots)_+$ denotes different distributions like $p(x)_+ = p(x) - \delta(1-x) \int_0^1 p(z) \ dz$ and $V(x, y)_+ = V(x, y) - \delta(y-x) \int_0^1 V(z, y) \ dz$.

$^1$ The gauge parameter $\xi$ is defined via the gluon propagator in lowest-order perturbation theory which reads

$$iD_{\mu\nu}^a(k^2) = \frac{-i\delta_{ab}}{k^2 + i\epsilon} \left( g_{\mu\nu} - \xi^{\mu\nu}_k \right).$$
The diagrammatic expansion of the kernels is presented in the Table below, where $\xi$-dependent terms appear in the partial diagrams $a,c$ canceling out each other in the complete results in Eqs. (1.4–1.5), as expected. The slash on the line of each of these diagrams denotes the delta function $\delta(x - nk/nP)$, where $k$ is the momentum on this line, while $n$ is a light-cone vector ($n^2 = 0$). These diagrammatic calculation rules can be traced to the momentum representation of the composite operator $\bar{\psi}(0)\sigma_{\mu\nu}\psi(\lambda n)$, denoted here by $\otimes$, and were elaborated in detail in [6]. The abbreviation $MC$ in the figures denotes the mirror–conjugate diagrams, while the symbol $C$ denotes the corresponding mirror conjugation of arguments, $\mathcal{C}\theta(y > x)f(x,y) \equiv \theta(y > x)f(x,y) + \theta(y < x)f(\bar{x},\bar{y})$. The local current, corresponding to the operator $\otimes$, is not conserved and, therefore, there is no “plus” prescription imposed on Eqs. (1.4), (1.5). Therefore, the separate $\delta$-functions survive.

Noting that the product $(y\bar{y})V_{a}^{T}(x,y)$ is symmetric under the exchange $x \leftrightarrow y$, one realizes that the corresponding anomalous dimension matrix can be diagonalized in the Gegenbauer basis $\{\psi_n(x) = (x\bar{x})C_n^{3/2}(2x - 1)\}$. The deeper reason for this is that conformal symmetry survives at the LO level [15]. On the other hand, at NLO the conformal symmetry does not hold true (in the MS scheme) owing to renormalization effects. These generate specific terms in $V_1^{T}$ that break this $x \leftrightarrow y$ symmetry as well as the diagonalization property mentioned above. For brevity, the corresponding terms will be referred to as “nondiagonal (diagonal)” ones.

In the next section, the contributions to $P_1^{T}$ and $V_1^{T}$ for each of the 2-loop diagrams will be demonstrated explicitly. In Sec. 3, we analyze the structure of both calculated kernels which are in accord with the expected manifestation of these symmetry breaking terms. Finally, we confirm the results for $P_1^{T}$, calculated in [7], as well as the result for $V_1^{T}$ found in [10]. The kernel $V_1^{T}$ provides the key ingredient, necessary for any complete NLO analysis of exclusive processes involving transversely polarized vector mesons via a QCD evolution of their distribution amplitudes. For an illustration of this NLO evolution, we analyze in Sec. 4 how it affects the transversely polarized $\rho$-meson distribution amplitude (DA) at the characteristic scale $\mu_B^2$ applicable to the $B$-meson semileptonic decay [13]. Our main findings are summarized in Sec. 5 together with our conclusions.

Table 1: Diagrammatic expansion of the one-loop kernels with $MC$ denoting the mirror–conjugated diagrams

| Diagram                                      | Equation                                                                 |
|----------------------------------------------|--------------------------------------------------------------------------|
| $P_a(x)$                                     | $-C_F\xi\delta(1-x)$                                                   |
| $V_a(x,y)$                                   | $-C_F\xi\delta(y-x)$                                                   |
| $P_b(x)$                                     | $C_F\left(p_0\right) = \frac{4x}{1-x}$                                  |
| $V_b(x,y)$                                   | $C_F\left(\mathcal{C}\theta(y > x)\ 2F_T\right)$                      |
| $P_c(x)$                                     | $-C_F(1-\xi)\delta(1-x)$                                               |
| $V_c(x,y)$                                   | $-C_F(1-\xi)\delta(y-x)$                                               |
2. Diagram-by-diagram presentation for $P^T_1$ and $V^T_1$

Here, we present the diagram-by-diagram results of the calculation of the DGLAP, $P^T_1$, and ERBL, $V^T_1$, kernels at the two-loop level for $\xi = 0$. In all there are 19 diagrams in the list below where we also display the diagrams with a zero contribution. The full list of them can be found in [4]. Note that superscripts \( \star \) mark the obtained new result for each diagram. The results for the other diagrams can be restored from those obeying the DGLAP [4] or ERBL [6] evolution kernels. Diagrams \( f^\star \) and \( h \) with gluon-loop insertions include also the corresponding ghost loops. Let us remark that there are only four basic scalar topologies of integrals, the latter being presented \(^2\) in [6].

\[
\begin{align*}
\text{d} & \\
\text{P}(x) &= -C_F \left[p_0 \left(1 + \ln \bar{x} \right) \right]_+ \\
\text{V}(x,y) &= -2C_F^2 \left[\mathcal{C} \theta(y > x) F^T \left(1 + \ln \left(1 - \frac{x}{y} \right) \right) \right]_+ \\
\text{e}^\star & \\
\text{P}(x) &= C_F T_r N_f \frac{8}{9}\delta(1 - x) \\
\text{V}(x,y) &= C_F T_r N_f \frac{8}{9}\delta(y - x) \\
\text{f}^\star & \\
\text{P}(x) &= -C_F C_A \frac{16}{9}\delta(1 - x) \\
\text{V}(x,y) &= -C_F C_A \frac{16}{9}\delta(y - x) \\
\text{g} & \\
\text{P}(x) &= -C_F T_r N_f \left[p_0 \left(\frac{20}{9} + \frac{4}{3} \ln x \right) \right]_+ \\
\text{V}(x,y) &= -2C_F T_r N_f \left[\mathcal{C} \theta(y > x) F^T \left(\frac{20}{9} + \frac{4}{3} \ln \frac{x}{y} \right) \right]_+ \\
\text{h} & \\
\text{P}(x) &= C_F C_A \left[p_0 \left(\frac{31}{9} + \frac{5}{3} \ln x \right) \right]_+ \\
\text{V}(x,y) &= 2C_F C_A \left[\mathcal{C} \theta(y > x) F^T \left(\frac{31}{9} + \frac{5}{3} \ln \frac{x}{y} \right) \right]_+ 
\end{align*}
\]

\(^2\) see also corrections to these results in Appendix B in [16]
| i* | $P(x) = -4C_F \left( C_F - \frac{C_A}{2} \right) \ln \bar{x}$ |
|---|---|
| | $V(x,y) = -2C_F \left( C_F - \frac{C_A}{2} \right) C \theta(y > x) \frac{1}{y} \ln \left( 1 - \frac{x}{y} \right)$ |
| k* | $P(x) = 4C_F \left( C_F - \frac{C_A}{2} \right) \left( \bar{x} + \eta \bar{x} \right)$ |
| | $V(x,y) = 4C_F \left( C_F - \frac{C_A}{2} \right) C \left[ \theta(y > x) \frac{x}{y} + \theta(y > \bar{x}) \frac{\bar{x}}{y} \right]$ |
| l | $P(x) = C_F \left( C_F - \frac{C_A}{2} \right) \left[ p_0 \left( 1 - 3 \ln x - \ln^2 x + \ln \bar{x} \right) + 12 \ln \bar{x} \right]_+$ |
| | $V(x,y) = 2C_F \left( C_F - \frac{C_A}{2} \right) C \left[ \theta(y > x) \frac{x}{y} + \theta(y > \bar{x}) \frac{\bar{x}}{y} \right]$ |
| m | $P(x) = C_F C_A \left[ p_0 \left( \frac{3}{2} - \frac{1}{2} \ln x - \frac{1}{4} \ln^2 \bar{x} \right) + \ln^2 \bar{x} \right]_+$ |
| | $V(x,y) = C_F C_A \left[ \theta(y > x) \left[ F^T \left( 3 - \ln \frac{x}{y} - \ln^2 \left( 1 - \frac{x}{y} \right) \right) + \frac{1}{2y} \ln^2 \left( 1 - \frac{x}{y} \right) \right] \right]_+$ |
| n* | $P(x) = C_F C_A \left[ \ln^2 \bar{x} + 4 \ln \bar{x} + p_0 \left( \frac{1}{4} \ln^2 x - \ln \bar{x} \ln x \right) \right]_+$ |
| | $V(x,y) = C_F C_A \left[ \theta(y > x) \left[ \frac{1}{2y} \ln^2 \left( 1 - \frac{x}{y} \right) + \frac{2}{y} \ln \left( 1 - \frac{x}{y} \right) + \frac{2}{yy} \ln x \ln \bar{x} + F^T \left( \frac{1}{2} \ln^2 \frac{x}{y} - 2 \ln \left( 1 - \frac{x}{y} \right) \right) + 2 F^T \ln \frac{x}{y} \ln \left( 1 - \frac{x}{y} \right) \right] \right]_+$ |
\[ P(x) = C_F \left( C_F - \frac{C_A}{2} \right) \left[ p_0 \left( 4 \text{Li}_2(1-x) - \ln^2 x \right) + 8 \ln \bar{x} ight. \\
+ \eta \ 2p_0 \left( \text{Li}_2 \left( \frac{|x|}{1+|x|} \right) - \text{Li}_2 \left( \frac{1}{1+|x|} \right) + \frac{1}{2} \ln^2 |x| \right) \\
- \ln |x| \ln(1+|x|) \right] \]

\[ V(x,y) = 2C_F \left( C_F - \frac{C_A}{2} \right) \left\{ C_\theta(y > x) \left[ \frac{2}{y} \ln \left( 1 - \frac{x}{y} \right) + \frac{1}{yy} \ln x \ln \bar{x} - \ln \left( 1 - \frac{x}{y} \right) \ln \left( \frac{1}{y} \right) \right] + G^T(x, y) \right\} \]

\[ P(x) = C_F C_A \left[ p_0 \left( 1 + \ln x + S(x) + \frac{1}{4} \ln^2 x - \frac{1}{2} \ln \bar{x} + \frac{1}{4} \ln^2 \bar{x} \right) \\
- 4 \ln \bar{x} - 2 \ln^2 \bar{x} \right]_+ \]

\[ V(x,y) = C_F C_A \left\{ C_\theta(y > x) \left[ F^T \left( 2 + 2 \ln \frac{x}{y} + \frac{1}{2} \ln^2 \frac{x}{y} + \frac{4}{2} \right) \\
- \ln \left( 1 - \frac{x}{y} \right) - \frac{3}{2} \ln^2 \left( 1 - \frac{x}{y} \right) \right) \\
- \frac{1}{y} \ln \left( 1 - \frac{x}{y} \right) - \frac{2}{y} \ln \left( 1 - \frac{x}{y} \right) \right\} \]

\[ P(x) = C_F \left( C_F - \frac{C_A}{2} \right) \left[ p_0 \left( 2 \ln^2 x + 4S(x) \right) - 16 \ln \bar{x} \right]_+ + C_F C_A \left[ p_0 \left( 2 + 2S(\bar{x}) + \frac{1}{2} \ln^2 x + \ln \bar{x} + \frac{1}{2} \ln^2 \bar{x} \right) \right]_+ \]

\[ V(x,y) = 2C_F \left( C_F - \frac{C_A}{2} \right) \cdot \left\{ C_\theta(y > x) \left[ F^T \left( 2 \ln^2 \frac{x}{y} + 4S(\frac{x}{y}) \right) - \frac{4}{y} \ln \left( 1 - \frac{x}{y} \right) \right] \right\} \]

\[ + C_F C_A \cdot \left[ C_\theta(y > x) F^T \left( 4 + 4S \left( 1 - \frac{x}{y} \right) + \ln^2 \frac{x}{y} + 2 \ln \left( 1 - \frac{x}{y} \right) \right. \right. \]

\[ + \ln^2 \left( 1 - \frac{x}{y} \right) \right\]
Here $S(x) \equiv \text{Li}_2(x) - \text{Li}_2(1)$ and the special notation $G^T(x)$ will be clarified in the next section (Eq. (3.13)).

There is, in general, a mixing of quark and antiquark densities in higher-loop calculations. At NLO, the diagrams $k^*$, $o^*$ contribute to the kernel $P_{1qq}$ expressing the probability to find a quark inside a quark (at $\eta = 0$), they also contribute to the kernel $P_{1q\bar{q}}$ giving the probability to find an antiquark inside a quark. Actually, in the latter case, one should consider two kernels, viz., $P_{\pm} = P_{1qq} \pm P_{1q\bar{q}}$ for $\eta = \pm 1$, [7]. We shall separate these contributions and focus on the results for $P_{1qq}$ and $V_1^T$ in the next section.

3. The structure of the evolution kernels in NLO

In this section, we discuss the total results of the two-loop calculation and also the general structure of the evolution kernels at NLO. We commence with those elements that appear in the renormalization procedure at NLO of both the kernels $P$ and $V$.

3.1. DGLAP kernel

Collecting the “quark-quark” contributions to the NLO DGLAP kernel, presented above, leads for $P_{1qq}^T$ to the final expression

$$P_{1qq}^T(x) = C_F^2 \cdot P_F^T(x) + C_F C_A \cdot P_G^T(x) + C_F N_f T_r \cdot P_N^T(x), \quad (3.1)$$

where

$$P_F^T(x) = 4\bar{x} - \left[ p_0(x) \left( 3 \ln(x) + 4 \ln(x) \ln(\bar{x}) \right) \right]_+ + \delta(\bar{x}) \left( \frac{43}{2} + 8\zeta(3) - \frac{8\pi^2}{3} \right), \quad (3.2)$$

$$P_G^T(x) = -2\bar{x} + \left[ p_0(x) \left( \ln^2(x) + \frac{11}{3} \ln(x) + \frac{67}{9} - \frac{\pi^2}{3} \right) \right]_+ - \delta(\bar{x}) \left( \frac{365}{18} - \frac{4\pi^2}{3} + 4\zeta(3) \right), \quad (3.3)$$

$$P_N^T(x) = - \frac{4}{3} \left[ p_0(x) \left( \ln(x) + \frac{5}{3} \right) \right]_+ + \frac{26}{9} \delta(\bar{x}). \quad (3.4)$$
This expression together with the expression for \( P_{1q}^T \) can be reduced, after some simple algebraic manipulations, to those found in [7].

Let us rewrite the expression for \( P_{1q}^T \) in Eqs. (3.1)–(3.4) in such a form that corresponds to the structure of \( \tilde{K}_1 R' \) at the two-loop level—see Eq. (1.3). Following [12], we consider the renormalization of the diagram \( \Gamma \) and its contracted one-loop subgraph \( E \) that can be written symbolically as \( \Gamma = E \cdot W \), where \( W \) is the one-loop remainder. As the result, the pole part of \( E \) should be multiplied by the finite part of the remainder \( W \) and vice versa. All those subgraphs \( E_i \) that are related to the charge renormalization of the intrinsic vertex in the various diagrams (see, e.g., diagrams \( d, g, h, l, m \) in the list) contribute to the coefficient of the QCD \( \beta \)-function \( b_0 = \frac{11}{3} C_A - \frac{4}{3} T_r N_f \). After contracting each of these \( E_i \) terms, the remainder reduces to one of the one–loop diagrams \( a, b, c \) from Table 1. The appropriate finite part of each of these diagrams in dimensional regularization can be obtained from the differentiation of the auxiliary kernel (cf. similar kernels in [12]) \( P(x; \varepsilon) = 4x^{1+\varepsilon} \bar{x}^{-1} \) with respect to the parameter \( \varepsilon \):

\[
\dot{p}_0(x) = \frac{d}{d\varepsilon} P(x; \varepsilon) \bigg|_{\varepsilon=0} = p_0(x) \ln(x) .
\] (3.5)

On the other hand, the composite operator illustrated, e.g., in diagrams \( a^*, b^*, c^*, o^*, q, r \) calls for a different sort of renormalization. Notably, the contracted subgraph \( E \) should include the composite operator that coincides with that in the one-loop diagrams in Table 1. The latter generates the kernel \( P_0^T \) (or \( V_0^T \)), while the finite part of the remainder is formed from the finite part of \( \frac{1}{\varepsilon} P(x; \varepsilon) \), i.e., by \( \dot{p}_0 \) that finally leads to the contribution

\[
(\dot{p}_0 \ast (p_0)_+) (x) = p_0(x) \left( 4 \ln(x) + 4 \ln(x) \ln(\bar{x}) - 2 \ln^2(x) \right) .
\] (3.6)

Here the symbol \( \ast \) denotes the Mellin convolution, \((f \ast g)(x) = \int_0^1 dy \frac{dy}{y} f(y) g(x/y)\). Collecting all these terms together, one recasts \( P_1^T \) in the form given by the first term in the curly brackets below:

\[
P_{1qq}^T(x) = \left\{ C_F \dot{p}_0 \ast \left[ b_0 1 - P_0^T \right] + p_0(x) C_F \left[ C_A \left( \frac{67}{9} - \frac{\pi^2}{3} \right) - \frac{20}{9} N_f T_r \right] \right\} + \\
+ C_F \left( C_F - \frac{C_A}{2} \right) \left[ 4\bar{x} - 2 \left( p_0(x) \ln^2(x) \right)_+ \right] \\
+ \delta(\bar{x}) C_F \left[ C_F \frac{43}{9} - C_A \frac{365}{18} + N_f T_r \frac{26}{9} + \left( C_F - \frac{C_A}{2} \right) 8 \left( \zeta(3) - \frac{\pi^2}{3} \right) \right] .
\] (3.7a–c)

The second term in the curly brackets in (3.7a) originates from the product of the finite parts of the contracted subgraphs \( E_i \), or, more specifically, from the finite part of the charge renormalization (diagrams \( g, h, l, m \)) and another finite and specific (see diagrams \( n^*, o^*, q, r \), part of the composite operator, as well as from the pole parts of the remainder that are proportional to \( p_0 \). In this respect, the coefficient of \( p_0 \) appears to be proportional [17] to the two-loop cusp anomalous dimension [18],

\[
\frac{1}{4} \Gamma_{cusp}^{(1)} = C_F \left[ C_A \left( \frac{67}{9} - \frac{\pi^2}{3} \right) - \frac{20}{9} N_f T_r \right] .
\] (3.8)

The terms in (3.7b) are formed by the diagrams \( k^* \) and \( o^* \) with nonplanar elements that also contribute to the “quark-antiquark” part \( P_{1qq}^T \) of the kernel. Finally, the \( \delta \)-function in (3.7c) manifests the fact that the corresponding local current is not conserved. Let us emphasize at this point that the expressions in the r.h.s. of (3.7a), (3.7c) together with Eq. (3.8) has the general structure of any nonsinglet NLO DGLAP kernel that follows from the renormalization procedure.
3.2. ERBL kernel

Collecting the partial contributions from the diagram-expansion list we arrive at

\[ V^T_1(x, y) = C_F^2 \cdot V^T_F(x, y) + C_F C_A \cdot V^T_G(x, y) + C_F N_f T_r \cdot V^T_N(x, y), \]

\[ V^T_F(x, y) = 4C \left[ \theta(y > x) \frac{x}{y} + \theta(y > \bar{x}) \frac{\bar{x}}{y} \right] + 2G^T(x, y) + 4 \left\{ C\theta(y > x) \left[ F^T \ln^2 \frac{x}{y} \right. \right. \]
\[ + \frac{1}{y \bar{y}} \ln x \ln \bar{x} - \frac{3}{2} F^T \ln \frac{x}{y} - (F^T - \bar{F}^T) \ln \frac{x}{y} \ln \left(1 - \frac{x}{y}\right) \left. \right\}_{+} \]
\[ + 4 \left[ \frac{11}{8} + 6\zeta(3) - 2\frac{\pi^2}{3} \right] \delta(x - y), \] (3.10)

\[ V^T_G(x, y) = -2C \left[ \theta(y > x) \frac{x}{y} + \theta(y > \bar{x}) \frac{\bar{x}}{y} \right] - G^T(x, y) \]
\[ + \left[ C\theta(y > x) 2F^T \left( \frac{11}{3} \ln \frac{x}{y} + \frac{67}{9} - \frac{\pi^2}{3} \right) \right]_{+} \]
\[ + \left[ - \frac{221}{18} - 12\zeta(3) + 4\frac{\pi^2}{3} \right] \delta(y - x), \] (3.11)

\[ V^T_N(x, y) = -\frac{4}{3} \left[ C\theta(y > x) 2F^T \left( \ln \frac{x}{y} + \frac{5}{3} \right) \right]_{+} + \frac{26}{9} \delta(y - x). \] (3.12)

Here and below

\[ G^T(x, y) = -4C \left[ \theta(y > x) \left( \bar{F}^T \ln \bar{x} \ln y - F^T [\text{Li}_2(x) + \text{Li}_2(\bar{y})] + \frac{\pi^2}{6} F^T \right) + \theta(\bar{y} < x) \]
\[ \times \left( (F^T - \bar{F}^T) [\text{Li}_2(1 - \frac{x}{y}) + \frac{1}{2} \ln^2 x] + F^T [\text{Li}_2(\bar{y}) - \ln x \ln y + \bar{F}^T \text{Li}_2(\bar{x})] \right). \] (3.13)

The term \( G^T \) is “diagonal”, \( \text{i.e. } y\bar{y}G^T(x, y) = x\bar{x}G^T(y, x) \), \( G^T \) coincides with the similar term \( G \) in the unpolarized kernel \( V_1 \) [6] by performing there \([10]\) the replacement \( F^T \rightarrow \bar{F} \) and excluding the third term \( \frac{x^2}{\bar{y}}F^T \) in the first line of (3.13). This term is tied to \( G^T \) in order to preserve \( \Gamma_{\text{cusp}} \) in the general structure of \( V^T_1 \).

The origin of the structure of the ERBL kernel can be considered by analogy with the DGLAP case, as explained in [12]. Taking into account that the ERBL auxiliary kernel is \( V(x, y; \varepsilon) = 1/2C (\theta(y > x) P(x/y; z/y) \varepsilon) / y \) leads to the following finite part of the corresponding remainder \( V^T_0 = 1/2C (\theta(y > x) p_0(x/y)/y) \), which, after introducing an appropriate convolution for the one-loop elements [12], \( (f \otimes g)(x, y) = \int_{-1}^1 dz f(x, z) g(z, y), \) leads to the expression

\[ V^T_1(x, y) = \left\{ C_F V^T_0 \otimes \left( b_0 \mathbb{1} - V^T_0 \right) + C\theta(y > x) 2F^T \Gamma_{\text{cusp}}^{(1)} \left( \frac{I}{4} + C_F [g_+, \otimes V^T_0] \right) \right\}_{+} \]
(3.14a)

\[ + C_F \left( C_F - \frac{C_A}{2} \right) \left\{ 4C \left[ \theta(y > x) \frac{x}{y} + \theta(y > \bar{x}) \frac{\bar{x}}{y} \right] + 2G^T(x, y) \right\} \] (3.14b)

\[ + \delta(y - x) C_F \left[ C_F \frac{27}{2} - C_A \frac{221}{18} + N_f T_r \frac{26}{9} \right]. \] (3.14c)
The structure of the elements of $V_1^T$ in (3.14a)-(3.14c) resembles that of $P_1^T$ in (3.7a)-(3.7c) with a natural replacement of notation for the convolution and the symbols $\otimes \to *$, $V_0^T \to \bar{p}_0$, $V_0^T \to P_0^T$, $2F_T \to p_0$ with the exception of the important third term in the curly bracket in (3.14a). In addition to the “nondiagonal” term proportional to $b_0$ and to the proper operator renormalization (see the first convolution in Eq. (3.14a)), there appears an additional “nondiagonal” term which is represented by the commutator $[g_+ , \otimes V_0^T] \equiv g_+ \otimes V_0^T - V_0^T \otimes g_+$. This term is induced by the leading-order anomaly in special conformal transformations of conformal operators,

$$
g(x, y) = -2C \frac{\theta(y > x)}{y - x} \ln \left(1 - \frac{x}{y} \right),
$$

(3.15)

an interesting issue explained in [10, 11]. All the other terms in $V_1^T$ are “diagonal”. Concluding these considerations let us mention that the expressions in the r.h.s. of (3.14a), (3.14c) give us the elements of the general structure of any NLO ERBL kernel.

4. Effects of two-loop evolution for the meson DA

The subject of this section concerns the effects of the two-loop QCD evolution in an appropriate example inspired by calculations of the $B \to \rho \nu e$ decay [13, 19]. For the leading-twist DA of the transversely polarized $\rho$-meson expanded in a Gegenbauer series

$$
\varphi(x, \mu_0^2) = \sum_{n=0} c_n(\mu_0^2)\psi_n(x)
$$

(4.1)

the two-loop evolution of each harmonic $\psi_n$ from $\mu_0^2$ to $\mu^2$, $\psi_n(x) \to \Phi_n(x, \mu^2)$, can be approximately represented\(^3\) as [12]

$$
\Phi_n(x, \mu^2) = \exp \left( - \int_{a_s(\mu_0^2)}^{a_s(\mu^2)} \frac{d\alpha}{\beta(\alpha)} \left( \gamma(n, \alpha) \right) \right) \left[ \psi_n(x) + a_s \sum_{m>n} \frac{d_{mn}}{N_m} \psi_m(x) \right].
$$

(4.2)

Here we have

$$
\gamma(n, a_s) = a_s \gamma_0^T(n) + a_s^2 \gamma_1^T(n),
$$

(4.3)

$$
\beta(a_s) = -a_s^2 b_0 - a_s^3 b_1,
$$

(4.4)

$$
d_{mn} = \frac{Z_{mn}}{\gamma_0(n) - \gamma_0(m) - b_0} \left[ 1 - \left( \frac{a_s(\mu^2)}{a_s(\mu_0^2)} \right)^{\gamma_0(n) - \gamma_0(m) - b_0} \right],
$$

$$
Z_{mn} = C_n^{3/2} \otimes V_1^T \otimes \psi_m, \quad N_m \delta_{nm} = C_n^{3/2} \otimes \psi_m = \frac{(n + 1)(n + 2)}{4(2n + 3)} \delta_{nm}
$$

and $\gamma(n)$ is the anomalous dimension with $\gamma_1^T(n) = Z_{nn}/N_n$. The coefficients $d_{mn}/N_m$ can be calculated analytically (in the form of lengthy sums) by virtue of the knowledge of the structure of the “nondiagonal” and “diagonal” terms in expressions (3.14a)-(3.14b). Their evaluation for the values of the input parameters $\mu_0^2 = 1$ GeV\(^2\), $\mu_B^2 = 36$ GeV\(^2\) (the latter being the characteristic scale of the $B \to \rho \nu e$ decay) for $N_f = 4$ is presented in Table 2.

---

\(^3\) The NLO evolution that preserves the renormalization-group property for the “nondiagonal” elements was worked out in [20].
\[ m \mid n \]

\begin{tabular}{c|cccc}
  & 0 & 2 & 4 & 6 \\
2 & 0.398 & 0 & 0 & 0 \\
4 & 0.013 & 1.08 & 0 & 0 \\
6 & 0.024 & 0.297 & 1.269 & 0 \\
8 & 0.024 & 0.094 & 0.485 & 1.288 \\
10 & 0.02 & 0.026 & 0.216 & 0.585 \\
12 & 0.015 & 0.001 & -0.103 & 0.303 \\
14 & 0.012 & 0.008 & -0.049 & 0.168 \\
16 & 0.01 & 0.011 & -0.022 & 0.097 \\
18 & 0.008 & 0.012 & -0.007 & 0.057 \\
20 & 0.006 & 0.012 & 0.000 & -0.033 \\
\end{tabular}

Table 2. Numerical values of the \( d_{nm}/N_n \) coefficients for the first 6 harmonics \( \psi_n \).

These coefficients \( d_{nm}/N_n \) decrease not so fast as those for the unpolarized case [12]. The numerical calculation shows that truncating the sum in (4.2) after the 10th term provides us with a 0.03% accuracy at the scale 36 GeV².

To get an estimate for the difference between the two-loop and the one-loop result, let us compare the corresponding first inverse moments of the DA, \( \langle x^{-1} \rangle \) = \( \int_0^1 \varphi(x)/xdx \). The ratios \( (\langle x^{-1}_{2\text{-loop}} \rangle /\langle x^{-1}_{1\text{-loop}} \rangle - 1) \% \) at this scale are -4.1% for \( \psi_0(x) \), -1.4% for \( \psi_2(x) \), -0.3% for \( \psi_4(x) \), and even less for higher harmonics. As regards the model distribution amplitude \( \varphi^T_\rho \) normalized at \( \mu_0^2 \approx 1 \text{ GeV}^2 \),

\[ \varphi^T_\rho(x; \mu_0^2) = \psi_0(x) + 0.29\psi_2(x) + 0.41\psi_4(x) - 0.32\psi_6(x) , \]

which was obtained obtained for a transversely polarized \( \rho \)-meson in [13], the ratio of the first inverse moments takes the value 3.6%. The evolution effect on the meson distribution amplitude \( \varphi^T_\rho(x; \mu_0^2) \) is illustrated in Fig. 1. The dashed black line shows the unevolved expression, while the result of the two-loop evolution to the scale \( \mu_B^2 = 36 \text{ GeV}^2 \) is represented by a solid red line, and the one-loop result is shown as a blue dashed-dotted line.

5. Conclusions

Let us summarize our findings. In Sec. 2, we presented the diagram-by-diagram results of a direct two-loop calculation of the DGLAP, \( P^T \), and the ERBL, \( V^T \), evolution kernels for transversity distributions, employing the Feynman gauge. The mutual correspondence between the \( V \) and \( P \) results, for each of the diagrams, was checked making use of the relation \( P(z) = \lim_{\eta \rightarrow 0} \frac{1}{|\eta|} V \left( \frac{z}{\eta} , \frac{1}{\eta} \right) \).

It was found that the total result for \( P^T_1 \) coincided with the one in [7] (obtained within a light-cone gauge calculation), whereas the total result for \( V^T_1 \) turned out to agree with the prediction obtained in [10].

We worked out the general structure of any nonsinglet NLO DGLAP kernel, Eqs. (3.7),(3.8), and any NLO ERBL kernel, Eqs. (3.14), respectively, subject to the renormalization procedure.

The NLO evolution of the DA of twist 2 (for transversely polarized \( \rho \)-meson) was considered and its relative effect was estimated for the inverse moment of the corresponding DA. This effect amounts to a few per cents (4% for the zero Gegenbauer harmonic) after evolving from the low scale \( \mu_0^2 \approx 1 \text{ GeV}^2 \) to the characteristic scale \( \mu_B^2 = 36 \text{ GeV}^2 \) of the B-decay process.
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