Mathematical analysis of hybrid mediated blood flow in stenosis narrow arteries

Azad Hussain, Lubna Sarwar, Aysha Rehman, Qasem Al Mdallal, Abdulrazak H. Almaliki & A. S. El-Shafay

In this paper the behavior of flow of blood under stenosis suppositions is studied. Nanoparticles of Ag and Cu are being used with blood as base fluid. The problem governing equations are modeled into PDE’s, which are transformed into set of ODE’s with the help of useful similarity transformation. We investigated the solution numerically for various parameters on temperature and velocity distribution and shown in the form of tables and graphs. It is found that the velocity of blood increases while the temperature curve goes down by increasing the concentration of nanoparticles and also temperature curve decreases by increasing the values of gamma and Prandtl number. Furthermore, the calculated results shows that increment in flow parameter gamma caused an increase in velocity values. In the field of biomedicine, the important approach of nanotechnology is the use of nanoparticles in chemotherapy.

List of symbols

- $R$: Radius of constricted region
- $R_o$: Width of unobstructed area
- $\lambda$: Maximum height of stenosis
- $L_o/2$: Length of stenosis
- $(u, v)$: Velocity components
- $(r, x)$: Coordinate components
- $\mu_{hf}$: Dynamic viscosity of hybrid nano-fluid
- $\rho_{hf}$: Density of hybrid nano-fluid
- $k_{hf}$: Thermal conductivity for hybrid nano-fluid
- $(\rho C_p)_{hf}$: Heat capacitance of hybrid nano-fluid
- $\rho_f$: Blood density
- $C_f$: Skin friction
- $Nu$: Nusselt number
- $Re_x^{1/2}$: Reynolds number
- $T$: Dimensional temperature
- $\mu_f$: Dynamic viscosity of base fluid
- $k_f$: Thermal conductivity for base fluid
- $\phi_1, \phi_2$: Nanoparticles volume fraction
- $(\rho C_p)_f$: Heat capacitance of base fluid
- $(\rho C_p)_1, (\rho C_p)_2$: Heat capacitance of nanoparticles
- $k_{s1}, k_{s2}$: Thermal conductivity for nanoparticles
- $\rho_{s1}, \rho_{s2}$: Densities of solid nanoparticles
- $\psi$: Stream function
- $\nu_f$: Kinematic viscosity of base fluid
- $\gamma$: Flow parameter
- $\eta$: Transformed coordinate

1Department of Mathematics, University of Gujrat, Gujrat 50700, Pakistan. 2Department of Mathematical Sciences, UAE University, Alain 15551, United Arab Emirates. 3Department of Civil Engineering, College of Engineering, Taif University, P. O. Box 11099, Taif 21944, Saudi Arabia. 4Department of Mechanical Engineering, College of Engineering, Prince Sattam Bin Abdulaziz University, Alkhairj 16273, Saudi Arabia. 5Mechanical Power Engineering Department, Faculty of Engineering, Mansoura University, Mansoura 35516, Egypt. *email: q.almdallal@uaeu.ac.ae
The mechanical investigation of blood flow through arterial stenosis allows some important aspects because of its significant medical and engineering applications. It is very normal to find the narrowing’s or plaques in the arterial system of humans, known as arterial stenosis. The normal pattern of flow of blood disturb through the stenosed artery. To filter waste produces our kidneys require particular amount of flow of blood and that normal amount of rich-oxygen blood can't reach our kidneys due to tapered arteries and this may cause high blood pressure and many injuries. Stenosis in arteries can cause death due to worsen over time. Ellahi et al.1 investigated the flow of blood through permeable walls stenosis and use perturbation method to find the solution of their problem. Haghighi et al.2 analyzed the blood flow behavior mathematically through constrict arteries and solved the problem by using finite difference method. Hussain et al.3 described the blood flow behavior through tapered arteries by assuming blood as non-Newtonian fluid and investigated the flow problem numerically. Sankar et al.4 analyzed the narrow surface of arteries and did the comparative study by assuming it as two fluid model and consequences shows that resistance to flow increases with the increase of stenosis width and height. Shah et al.5 discussed some new properties in the inquiry of nanoscience’s that highlight the nanoparticles concentration in blood flow and studied the consequences of different parameters like source, sink, clot size and stenosis height etc. Pokhrel et al.6 studied abnormal flow of blood through diseased artery by using Navier stoke equations and investigated pressure drop, the ratio of minimum, maximum shear stress. Many investigations7–10 have been considered to understand the effects of stenosed arteries on blood flow. Tanveer et al.11 considered non-Newtonian fluid behavior of blood and presented theoretical analysis in a microchannel. Some other arterial diseases also discussed by many researchers12–15.

In biological systems the nanoparticles characteristics such as surface shape, chemistry, size can be controlled to increase their function. These properties vary for every nanofluid depending upon base fluid and the tiny particles. Nanoparticles have more ability to conduct heat as compared to base fluid and also have more impact on the heat transfer enhancement. In biomedical applications a large number of nanoparticles have been improved and in treatment some of them have shown great potential, to activate the growth of blood vessels and imaging of diseases. They are also treated as drug carrying vehicles. Cu-Ag nanoparticles are the most impactful materials in nanoscience and nanomedicine. Ardahaie et al.15 studied the consequences of nanoparticles through stenosis and also investigated the magnetic effects in a porous blood artery. Nadeem and Ijaz16 presented the characteristics of flow of blood through curved channel stenosis and also discussed the behavior of various parameters like slip parameter, Darcy number, Prandtl number and nanoparticles volume fraction. Zaman17 analyzed the behavior of unsteady blood flow in a stenosed channel in the presence of copper and silver nanoparticles. By using curvilinear coordinates he developed the equation of momentum and energy. Tripathi et al.18 investigated the characteristics of unsteady flow of blood through stenosed artery with the addition of (copper, silver) nanoparticles by considering porous arterial wall. They computed the various results for wall shear stress, temperature, velocity, flow rate for particular height of stenosis. Many researchers19–32 studied the applications of nanoparticles.

In order to indicate that how addition of nanoparticles can be helpful in blood flow through arteries, a mathematical model is investigated. In our problem, the base fluid blood is Newtonian fluid and hybrid nanoparticles are added to it. The results of various parameters including blood flow parameter, Prandtl number and nanoparticles volume fraction are studied and their consequences on flow are presented in the form of tables and graphs. In our paper firstly the flow governing equations are analyzed and solution is attained numerically by using MATLAB bvp4c technique. Next, the consequences of different parameters have been shown by plotting the graphs. At the last the important results are given. Present study is applicable in different biomedical fields.

**Flow geometry and coordinate system**

The conditions of abnormal blood flow due to stenosis are developed and become the reason for disease in artery. In under discussion problem we assumed that flow of blood reacts like steady, incompressible, two dimensional, viscous fluid through constrict artery of length $L_0/2$, where blood flow along $x$-axis and $r$-axis is perpendicular to the flow.

The sketch of the problem as presented in Fig. 1, is described as Verma33, where blood flow through cosine shape constriction of artery having width $2R_0$ of unblocked area, radius of the blocked region of artery is $R(x)$ and the maximum height of constrict area is $\lambda$. The red dots shows the blockage in the artery and these blockage effect the blood flow. Stenosed region profile is selected as

$$R(x) = R_0 - \frac{1}{2} \left( 1 + \cos \left( \frac{4x \pi}{L_0} \right) \right), \quad -\frac{L_0}{4} < x < \frac{L_0}{4}$$

$$R(x) = R_0, \quad \text{Otherwise.}$$

**Governing equation and boundary conditions**

The flow governing steady boundary layer equations for Newtonian hybrid nanofluid are defined as34,35:

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0,$$
Thermophysical properties of nanofluids are defined as follows:

\[ \rho_{nf} = \rho_f \left( 1 - \phi \right) + \phi \frac{\rho_s}{\phi_f}, \]

\[ \mu_{nf} = \mu_f \left( 1 - \phi \right)^{2.5} \cdot \frac{\mu_s}{\phi_f}, \]

\[ (\rho C_p)_{nf} = (\rho C_p)_f \left( 1 - \phi \right) + \phi \frac{(\rho C_p)_s}{\phi_f}, \]

\[ k_{nf} = \frac{k_f + 2k_f - 2\phi (k_f - k_s)}{k_f + 2k_f + \phi (k_f - k_s)}. \]

where \( \rho_{nf} \) is the density, \( \mu_{nf} \) is viscosity, \( k_f \) and \( k_{nf} \) represents thermal conductivity of Cu-Ag nanoparticles and blood, \( (\rho C_p)_{nf} \) represents the heat capacity of fluid, which are described in Table 1. By introducing stream function \( \psi \) The continuity Eq. (2) is satisfied for \( u \) and \( v \) such that

\[ u \left( \frac{\partial}{\partial x} + \frac{v}{\partial r} \right) u = \frac{k_{nf}}{\rho_{nf}} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right), \]

\[ \left( u \frac{\partial}{\partial x} + \frac{v}{\partial r} \right) T = \frac{k_{nf}}{(\rho C_p)_{nf}} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right), \]

The continuity and energy equations are satisfied for \( u, v \) and \( T \) such that

\[ \frac{\partial v}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0 \text{ at } r = 0. \]

**Table 1.** Base fluid (blood) and Silver (Ag), Copper (Cu) nanoparticles experimental values.

| Material  | Symbol | \( \rho \) (kg/m\(^3\)) | \( C_p \) (Jkg\(^{-1}\)K\(^{-1}\)) | \( k \) (Wm\(^{-1}\)K\(^{-1}\)) |
|-----------|--------|----------------|----------------|----------------|
| Blood     | –      | 1050           | 3617           | 0.52           |
| Silver    | Ag     | 10,500         | 235            | 429            |
| Copper    | Cu     | 8933           | 385            | 400            |

Figure 1. Geometrical structure of arterial stenosis.
\[ u = r^{-1} \frac{\partial \psi}{\partial r}, \quad v = -r^{-1} \frac{\partial \psi}{\partial x}. \]  

Now Eqs. (3–4) becomes

\[
\begin{align*}
\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi}{\partial r} \right) - \frac{1}{r} \frac{\partial}{\partial x} \left( \frac{1}{r} \frac{\partial \psi}{\partial x} \right) &= \frac{\mu_{h\psi}}{\rho_{h\psi}} \frac{\partial}{\partial r} \left( \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} \right), \\
\left( \frac{1}{r} \frac{\partial}{\partial r} \right) \frac{\partial T}{\partial x} - \frac{1}{r} \frac{\partial}{\partial x} \left( \frac{1}{r} \frac{\partial T}{\partial x} \right) &= \frac{k_{h\psi}}{\rho C_p} \frac{\partial}{\partial r} \left( \frac{\partial T}{\partial r} \right).
\end{align*}
\]

We can find expression for shear stress

\[
\tau_w = \mu_{h\psi} \left. \frac{\partial u}{\partial r} \right|_{r=R},
\]

and heat flux is

\[
q_w = -k_{h\psi} \left. \frac{\partial T}{\partial r} \right|_{r=R}.
\]

The non-dimensional parameters in above equations are Prandtl number \( Pr = k_f / (\mu C_p) \), flow parameter is \( \gamma = \sqrt{\nu_l L_0 / u_0 R^2} \) and concentration of Cu-Ag nanoparticles are presented by \( \phi_1 \) and \( \phi_2 \) respectively. The physical quantities i.e., heat transfer coefficient \( N_u \) and Skin friction coefficient \( C_f \) of flow field.

\[
C_f = \frac{\tau_w}{\frac{1}{2} \rho_f u_0^2},
\]

\[
N_u = \frac{xq_w}{k_f (T_1 - T_0)}.
\]

The non-dimensional form of Eqs. (17–18) becomes
where \( \text{Re}^{-1/2} \) shows the Reynolds number.

**Graphical results and explanation**

The effects of stenosed artery on the blood flow pattern and the consequences of various parameters are investigated. The graphical results for temperature and velocity are presented. The aim of this study is to obtain the best possible combination of physical and chemical properties in a unique fluid by using different materials. Figure 1 shows the geometry of constrict artery. The behavior of fluid flow parameter \( \gamma \) on blood temperature is presented in Fig. 2. Temperature of blood goes by rising \( \gamma \) values. Figure 3 shows the temperature results against \( \text{Pr} \). The curve of temperature decreases by increasing \( \text{Pr} \). The behavior of nanoparticles on temperature curve is shown in Fig. 4. Curve of temperature decreases due to increase in \( \phi \). Basically the results in Figs. 2, 3, 4 reveals that temperature decreases with the addition of hybrid nanoparticles (copper, silver) for increasing values of \( \text{Pr} \).
Concluding remarks

In current article, the model studies the behavior of Prandtl number, flow parameter and nanoparticle volume fraction on temperature and velocity of blood. The hybrid nanofluid is considered as a mixture of blood and Cu-Ag NPs. Some main lists of the results can be summarized as follows:
Figure 6. Effects of nanoparticles volume fraction on velocity.

Figure 7. Consequences of Pr and γ on $-\theta'(0)$.

Figure 8. Variations of nanoparticle volume fraction and γ on $\frac{1}{2}C_f Re$. 
| $\gamma$ | Pr | $\frac{k_f}{\gamma} \theta'(0)$ |
|-----|-----|------------------|
| 0.1 | 2.0 | -0.24123 |
| 0.12 | | -0.20365 |
| 0.14 | | -0.16682 |
| 0.1 | 4.0 | -0.24123 |
| 6.0 | | -1.17916 |
| 6.0 | | -2.19785 |

Table 2. Nusselt number $-\frac{k_f}{\gamma} \theta'(0)$ with respect to $\gamma$ and Pr.

| $\gamma$ | $\phi$ | $\frac{1}{2} C_f Re$ |
|-----|-----|------------------|
| 0.1 | 0.0 | 0.6970 |
| 0.12 | | 0.6455 |
| 0.14 | | 0.5959 |
| 0.1 | | 0.6970 |
| 0.1 | 1.0 | 1.0834 |
| 0.2 | | 2.0290 |

Table 3. Values of $\frac{1}{2} C_f Re$ with respect to $\phi$ and $\gamma$.

1. Rising the values of $\gamma$ and the nanoparticle size intensifies the velocity and flow of blood.
2. Temperature of blood decreases due to increase in Pr, $\gamma$ and nanoparticle volume fraction values.
3. Heat transfer curve diminishes with the increase in $\gamma$ and Pr values.
4. Skin friction curve declines by rising the values of nanoparticle volume fraction and $\gamma$.
5. The results indicate that addition of nanoparticles can help in improving flow of blood through arterial stenosis.

Received: 4 October 2021; Accepted: 14 March 2022
Published online: 26 July 2022

References
1. Ellahi, R., Rahman, S. U. & Nadeem, S. Blood flow of Jeffrey fluid in a catheterized tapered artery with the suspension of nanoparticles. Phys. Lett. A 378, 2973–2980 (2014).
2. Haghihighi, A. R. & As, M. S. Mathematical modeling of micropolar fluid flow through an overlapping arterial stenosis. Int. J. Biomath. 8(4), 1530056 (2015).
3. Hussain, A., Sarwar, L., Akbar, S. & Nadeem, S. Mathematical model for blood flow through the stenosed channel. Phys. Scr. 95, 2 (2019).
4. Sankar, D. S. & Yatim, Y. Comparative analysis of mathematical models for blood flow in tapered constricted arteries. Abstr. Appl. Anal. 34, 235960 (2012).
5. Shah, S. R. & Kumar, R. Mathematical modeling of blood flow with the suspension of nanoparticles through a tapered artery with a blood clot. Front. Nanotechnol. https://doi.org/10.3389/fnano.2020.596475 (2020).
6. Pokhrel, P. R., Kafle, J., Kattel, P. & Gaire, H. P. Analysis of blood flow through artery with mild stenosis. J. Inst. Sci. Technol. 25(2), 33–38 (2020).
7. Ijaz, S. & Nadeem, S. Examination of nanoparticles as a drug carrier on blood flow through catheterized composite stenosed artery with permeable walls. Comput. Methods Prog. Biomed. 1339, 83–94 (2016).
8. Ahmed, A. & Nadeem, S. Biomathematical study of time-dependent flow of a Carreau nanofluid through inclined catheterized arteries with overlapping stenosis. J. Cent. South Univ. 24, 2725–2744, https://doi.org/10.1007/s11771-017-3685-4 (2017).
9. Nadeem, S., Ijaz, S. & Akbar, N. S. Nanoparticle analysis for blood flow of Prandtl fluid model with permeable walls. Int. Nano Lett. 3, 35 (2013).
10. Ellahi, R., Rahman, S. U., Nadeem, S. & Vafai, K. The blood flow of Prandtl fluid through a tapered stenosed arteries in permeable Walls with magnetic field. Commun. Theor. Phys. 63(3), 353, https://doi.org/10.1088/0253-6102/63/3/353 (2015).
11. Tanveer, A., Salahuddin, T., Khan, M., Malik, M. Y. & Alqarni, M. Theoretical analysis of non-Newtonian blood flow in a microchannel. Comput. Methods Prog. Biomed. 191, 105280 (2019).
12. Teviranus, G. R. S. Psychoses by attacks from subverted mast cells: A role for arterial intramural flow badly steered by the nasal ganglia?. Psychiatr. Danub. 32(Suppl 1), 93–104 (2020).
13. Yang, Y. et al. Postnatal exposure to DNP was associated with greater alterations of lipidomic markers for hepatic steatosis than DEHP in postweaning mice. Sci. Total Environ. 758, 143631, https://doi.org/10.1016/j.scitotenv.2020.143631 (2021).
14. Tang, W., Wan, S., Yang, Z., Tischendorff, A. E. & Zou, Q. Tumor origin detection with tissue-specific mRNA and DNA methylation markers. Bioinformatics 34(3), 398–406, https://doi.org/10.1093/bioinformatics/btx622 (2018).
15. Ardahae, S. S., Amiri, A. J., Amouei, A., Hosseinzadeh, K. & Ganji, D. D. Investigating the effect of adding nanoparticles to the blood flow in presence of magnetic field in a porous blood arterial. Inform. Med. Unlocked 10, 71–81 (2018).
16. Nadeem, S. & Ijaz, S. Theoretical analysis of metallic nanoparticles on blood flow through stenosed artery with permeable walls. Phys. Lett. A 379, 542–554 (2015).
17. Zaman, A. Effects of nanoparticles (Cu (Copper), Silver (Ag)) and slip on unsteady blood flow through a curved stenosed channel with aneurysm. Therm. Sci. Eng. Prog. 5, 482–491, https://doi.org/10.1016/j.tsep.2018.02.004 (2018).
18. Tripathi, J., Vasu, B., Anwar Bég, O., Reddy Gorla, R. S. & Kameswaran, P. K. Computational simulation of rheological blood flow containing hybrid nanoparticles in an inclined catheterized artery with stenotic, aneurysmal and slip effects. Comput. Biol. Med. 139, 105009 (2021).
19. Toghraie, D. et al. Two-phase investigation of water-Al2O3 nanofluid in a micro concentric annulus under non-uniform heat flux boundary conditions. Int. J. Numer. Meth. Heat Fluid Flow 30(4), 1795–1814. https://doi.org/10.1108/HFF-11-2018-0628 (2020).
20. Chamkha, A. J., Dogonchi, A. S. & Ganji, D. D. Magneto-hydrodynamic flow and heat transfer of a hybrid nanofluid in a rotating system among two surfaces in the presence of thermal radiation and Joule heating. AIP Adv. 9, 025103. https://doi.org/10.1063/1.5086247 (2019).
21. Raza, J., Mebarek-Oudina, F. & Chamkha, A. J. Magnetohydrodynamic flow of molybdenum disulfide nanofluid in a channel with shape effects. Multidiscip. Model. Mater. Struct. 15(4), 1573–6105 (2019).
22. Shashikumar, N. S., Gireesha, B. J., Mahanthesh, B., Prasannakumara, B. C. & Chamkha, A. J. Entropy generation analysis of magneto-nanoliquids embedded with aluminium and titanium alloy nanoparticles in microchannel with partial slips and convective conditions. Int. J. Numer. Meth. Heat Fluid Flow 29(10), 0961–5539 (2019).
23. Selimefendigil, F., Ismael, M. A. & Chamkha, A. J. Mixed convection in superposed nanofluid and porous layers in square enclosure with inner rotating cylinder. Int. J. Mech. Sci. 124–125, 95–108 (2017).
24. Chamkha, A. J. Non-Darcy fully developed mixed convection in a porous medium channel with heat generation/absorption and hydromagnetic effects. Int. J. Comput. Methodol. 32, 6. https://doi.org/10.1080/10407789708913991 (1997).
25. Chamkha, A. I. On laminar hydromagnetic mixed convection flow in a vertical channel with symmetric and asymmetric wall heating conditions. Int. J. Heat Mass Transf. 45, 2509–2525 (2002).
26. Chamkha, A. J. Unsteady laminar hydromagnetic fluid-particle flow and heat transfer in channels and circular pipes. Int. J. Heat Fluid Flow 21(6), 740–746. https://doi.org/10.1016/S0142-727X(00)00031-X (2000).
27. Rasool, G. et al. Entropy generation and consequences of binary chemical reaction on MHD Darcy–Forchheimer Williamson nanofluid flow over non-linearly stretching surface. Entropy 22(1), 18. https://doi.org/10.3390/e22010180 (2020).
28. Hussain, A., Rehman, A., Nadeem, S., Riaz Khan, M. & Issakhov, A. A computational model for the radiated kinetic molecular postulate of fluid-originated nanomaterial liquid flow in the induced magnetic flux regime. Math. Probl. Eng. 17, 690366. https://doi.org/10.1155/2021/690366 (2021).
29. Hussain, A. et al. A thermal conductivity model for hybrid heat and mass transfer investigation of single and multi-wall carbon nano-tubes flow induced by a spinning body. Case Stud. Therm. Eng. 28, 101449. https://doi.org/10.1016/j.csteme.2021.101449 (2021).
30. Hussain, A. et al. Heat transmission of engine-oil-based rotating nanofluids flow with influence of partial slip condition: a computational model. Energies 14(13), 3859 (2021).
31. Rehman, A., Hussain, A. & Nadeem, S. Assisting and opposing stagnation point pseudoplastic nano liquid flow towards a flexible Riga sheet: A computational approach. Math. Probl. Eng. 2021, 1–14 (2021).
32. Sarwar, L. & Hussain, A. Flow characteristics of Au blood nanofluid in stenotic artery. Int. Commun. Heat Mass Transf. 127, 105486. https://doi.org/10.1016/j.icheatmasstransfer.2021.105486 (2021).
33. Verma, S. R. Study of blood flow through modelled vascular stenosis. Int. Commun. Heat Mass Transf. 127, 105486. https://doi.org/10.1016/j.icheatmasstransfer.2021.105486 (2021).
34. Vajravelu, K., Prasad, K. V. & Santhi, S. R. Axisymmetric magneto-hydrodynamic (MHD) flow and heat transfer. Comput. Fluids 104, 740–746. https://doi.org/10.1016/j.compfluid.2012.06.008 (2012).
35. Elgazery, N. S. Flow of non-Newtonian magneto-fluid with gold and alumina nanoparticles through a non-Darcian porous medium at a non-isothermal stretching cylinder. J. Egypt. Math. Soc. 27, 39. https://doi.org/10.1016/j.joems.2019.01-0017-x (2019).
36. Tayebi, T., Chamkha, A. J., Melabari, A. A. & Raouache, E. Effect of internal heat generation or absorption on conjugate thermal-free convection of a suspension of hybrid nanofluid in a partitioned circular annulus. Int. Commun. Heat Mass Transf. 126, 105397. https://doi.org/10.1016/j.icheatmasstransfer.2021.105397 (2021).
37. Ahmed, A. & Nadeem, S. Effects of magnetohydrodynamics and hybrid nanoparticles on a micropolar fluid with 6-types of stenosis. Results Math. 7, 4130–4139 (2017).

Acknowledgements
The authors would like to acknowledge and express their gratitude to the United Arab Emirates University, Al Ain, UAE for providing financial support with Grant No. 125086.

Author contributions
A.H.; conceptualization, supervision. L.S.; data curation, formal analysis, investigation, writing-original draft. A.R.; revised as well as analyzed the abstract section. Q.A.M.; improved the introduction part and also corrected the relations for thermal conductivity. A.H.A.; revised the flow geometry and explain the limitations and applications of nanoparticles to blood base fluid as asked by both reviewers. A.S.E.S.; explained the numerical method used in the simulation properly and also revised graphical results and discussion section critically.

Funding
UAE University, Post–Doctoral Grant 12S086.

Competing interests
The authors declare no competing interests.

Additional information
Correspondence and requests for materials should be addressed to Q.A.M.
Reprints and permissions information is available at www.nature.com/reprints.
Publisher’s note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.
