Effect of engine location on flutter speed and frequency of a tapered viscoelastic wing

Y S Matter\textsuperscript{1}, T T Darabseh\textsuperscript{1} and A-H I Mourad\textsuperscript{1}

\textsuperscript{1} Mechanical Engineering Department, United Arab Emirates University, P.O. Box 15551, Al Ain, UAE

E-mail: 201570005@uaeu.ac.ae

Abstract. The flutter of a tapered viscoelastic wing carrying an engine and subjected to a follower thrust force is investigated. The wing is considered as a cantilever tapered Euler-Bernoulli beam, made of a linear viscoelastic material where Kelvin-Voigt model is assumed to represent the viscoelastic behaviour of the material. In addition, quasi-steady and unsteady aerodynamic forces models are introduced along with the follower thrust force. The mass and inertia of the engine are modelled in order to achieve more realistic behaviour of the engine upon flutter characteristics of the system. Moreover, the governing equations of motion are derived through the Extended Hamilton’s Principle. The generalized function theory is used to more accurately consider the contribution of the mass and its follower force in the governing equations. The resulting partial differential equations are solved by Galerkin method along with the classical flutter investigation approach. Parametric studies highlighting the sensitivity of the chord-wise engine location, the span-wise engine location, and the vertical engine location on the flutter speed and flutter frequency are reported. It is found that the location of the engine in the three directions play an important role in the dynamic stability of the wing.

1. Introduction

Aeroelasticity is the term used to denote the field of study concerned with the interaction among aerodynamics, elasticity, inertia forces, and the phenomena that can result. Aeroelastic phenomena are usually classified as being either static (as the divergence) or dynamic (as the flutter). Divergence is a phenomenon that occurs when the moments resulting from aerodynamic forces overcome the elastic restoring forces due to structural stiffness. The flutter is defined as a dynamic lack of stability that occurs in a flexible structure subjected to aerodynamic loads at certain speed and frequency - called the flutter speed and the flutter frequency - which cause the structure to undergo divergent oscillations.

Bending-torsion aeroelastic instabilities have been investigated by many authors. References [1] and [2] studied the flutter phenomenon of a uniform wing by analysing a set of partial differential equations governing the motion of the wing. The use of quasi-steady aerodynamic theory for aeroelastic analysis of the lifting surfaces is a good approximation in the field of aeroelasticity as found in [3-6], and others. In Reference [7], a systematic approach based on Galerkin method was developed to investigate the flutter speed and frequency for a wing subjected to quasi-steady aerodynamic forces. The quasi-steady aerodynamic model can be used for low frequencies with acceptable results as investigated in [8]. However, Reference [9] observed that the quasi-steady
aerodynamic theory give inaccurate flutter results for a lifting surface under incompressible flow compared to the unsteady aerodynamic model.

Viscoelastic materials are typically used for enhancing damping and reduce structural vibration. Reference [10] analysed the dynamic response of Kelvin-Voigt viscoelastic simply supported beam. It was shown that the flutter speed associated with the viscoelastic wing might be greater or lower than that associated with the elastic wing. In addition, it was pointed out that the use of viscoelastic materials has either stabilization or destabilization contributions on the response of the structure, as in [11-15].

The engine’s thrust, which acts as a non-conservative follower force on a mass attached to the wing, may affect the behaviour of the wing vibration. Many authors have studied bending-torsional flutter of elastic systems exposed to such non-conservative follower forces. For elastic systems, two books have been presented discussing comprehensively the dynamic stability of conservative and non-conservative elastic systems [16, 17]. By applying a method of expansion of fractional power of parameters, Reference [18] used a cantilever viscoelastic bar subjected to a tip follower force to show that, for realistic damping behaviour, there is a part of quasi-stability region should be added to the instability region. References [19] and [20] examined the effects of a transverse follower force on stability regions of HALE wing; however, they did not account for the inertia properties of the engine and considered only one location along the wing. The dynamic stability of wings carrying external stores and subjected to a lateral follower force was examined by [21]. The study observed that the engine mass, thrust, and location are of great influence on the dynamic stability of the aircraft wing. The dynamic instability of a cantilever composite wing with an attached mass subjected to follower force representing the thrust of the engine was studied in [22]. Therein, it was reported that the ply angle, engine location, the magnitude of engine mass and thrust have significant effects on the aeroelastic stability of the composite wings.

The objective of this work is to examine the dynamic response and determine regions of stability and flutter speeds and frequencies of a tapered viscoelastic wing carrying an engine and subjected to a follower thrust force modelled as a cantilever tapered Euler-Bernoulli beam. Furthermore, a non-dimensional parametric study is conducted to investigate the effect of the engine location on the flutter speed and frequency. The governing equations of motion are developed using Hamilton’s principle with the quasi-steady and unsteady models for the aerodynamic forces. The equations of motion are solved using Galerkin method and classical flutter investigation procedure.

2. Formulation

2.1. Wing Model

The cantilever viscoelastic tapered wing carrying a concentrated engine mass and subjected to follower thrust force and aerodynamic loading is shown in Figure 1(a) with \( x_e \) denotes the engine location along the span. Furthermore, the deformed typical section of the wing is modelled in Figure 1(b) with \( y_a \) is the chord-wise distance between the wing centre of gravity (\( C_g_a \)) and the elastic axis and \( y_0 \) is the chord-wise distance from the wing’s leading edge to the elastic axis. The chord-wise distance between the engine centre of gravity (\( C_g_e \)) and the elastic axis of the wing is denoted by \( y_e \) and the vertical distance from the engine centre of gravity to the wing chord line is denoted by \( z_e \). The points \( AE \) and \( AC \) refer to the wing elastic axis and the aerodynamic centre of the wing, respectively.

The coordinate axes \( x, y, z \) are fixed on the wing root in which the \( x \)-axis lies exactly on the elastic axis and directed along the length of the wing in the span-wise direction. The orthogonal axes \( x', y', z' \) are attached to represent the wing after deformation. The external loads cause the wing to be deformed such that the elastic axis of the wing is moved by an amount of \( h \) in the \( z \)-direction. Moreover, the wing rotates about its elastic axis by an angle of \( \theta \). Therefore, the system has two degrees of freedom.

It is important to note that the engine thrust (denoted here by \( P \)) is exerted exactly on the engine \( C_g \) and directed along the chord-wise direction of the wing. This gives the ability to recognize the thrust of the engine as a transverse follower force.
2.2. Equations of Motion

The wing is made of a linear viscoelastic material, where the stress is linearly proportional to the strain history. Various models were constructed to represent the viscoelastic behaviour of the material. It was predicted that the spring–dashpot models are useful to conceive how viscoelastic behaviour can arise. Kelvin-Voigt model (as shown in Figure 2) is used to describe the behaviour of the viscoelastic material. The Kelvin–Voigt model consists of a spring and dashpot connected in parallel. Because the two elements are subjected to the same strain, the total stress is the sum of stress in each element, so

$$\frac{\sigma_n}{\varepsilon} = \{ E + \eta_E \partial_t \} = E^{*} \tag{1}$$

Similarly

$$\frac{\sigma_s}{\gamma} = \{ G + \eta_G \partial_t \} = G^{*} \tag{2}$$

where $\partial_t = \frac{\partial}{\partial t}$, $\sigma_n$, and $\sigma_s$ are the normal and shear stresses, respectively; $E$ and $G$ are Young’s and the shear moduli, respectively; $\eta_E$ and $\eta_G$ are the coefficients of viscous damping forces in bending and torsion, respectively; and $\varepsilon$ and $\gamma$ are the normal and the shear strains, respectively.

The equations of motion are derived via the variational Hamilton’s principle that can be expressed as

$$\int_{t_1}^{t_2} \left[ \delta U - \delta T_w - \delta T_e - \delta W_A - \delta W_F \right] = 0 \tag{3}$$

where $U$ here is the potential energy (not the air speed), $T_w$ is the kinetic energy of the wing, $T_e$ is the kinetic energy of the engine, $W_A$ is the virtual work of the distributed aerodynamic loads, $W_F$ is the virtual work of the concentrated engine thrust, and $\delta$ is the variational operator.
The first variation of the potential energy of the wing is given by
\[
\delta U = \frac{1}{2} \int \left[ 2G^* J \dot{\theta} \delta \dot{\theta} + 2E^* I h^\prime h^\prime \delta h^\prime + 2P(x_c - x) H(x_c - x) \delta \dot{h}^\prime \right] dx
\]
where \( E^* \) and \( G^* \) are the wing bending and torsional rigidities, respectively. \( H(x_c - x) \) is the Heaviside function, which is used in order to account for the location of the engine thrust force.

The first variation of the wing kinetic energy is given by
\[
\delta T_w = \int_0^l \frac{1}{2} \left[ 2m h \dot{h} + 2m y \dot{\theta} \dot{h} + 2m y \dot{\theta} \dot{h} + 2I_{EA} \dot{\theta} \dot{\theta} \right] dx
\]
where \( m \) is the wing mass per unit span, \( I_{EA} \) is the wing moment of inertia about the elastic axis.

The first variation of the engine kinetic energy is given by
\[
\delta T_e = \int_0^l \left[ \frac{1}{2} \left( M_c (z^2 \dot{h}^\prime + z^2 \dot{h}^\prime + \dot{h} \dot{h} + y_c \dot{h} \dot{\theta} + y_c \dot{h} \dot{\theta} + y_c \dot{h} \dot{\theta} - \dot{h} \dot{\theta} \dot{\theta} \right) \right] \delta h + f \delta \dot{\theta} \right] dx
\]
where \( M_c \) is the engine mass, \( I_{me} \) is the engine moment of inertia, and \( \delta h(x_c - x) \) is the Dirac-Delta function, which is used in order to precisely account for the location of the engine mass along the wing span.

The variation of the virtual work of the distributed aerodynamic loads is given by
\[
\delta W_A = \int_0^l (L \delta h + M \delta \theta) \, dx
\]
where \( L \) and \( M \) are the aerodynamic lift force and twisting moment per unit span, respectively.

The variation of the virtual work of the concentrated engine thrust is given by
\[
\delta W_P = \int_0^l \left[ P \delta h + \left( (P y_c \theta - P z_c) \delta h + f \delta \dot{\theta} \right) \right] dx
\]
Using the previous expressions for the variation of the potential and kinetic energies and the variation of the virtual work as well as using Kelvin-Voigt model to represent the viscoelastic behaviour of the material, the equations of motion are obtained as
\[
\begin{align*}
m \ddot{h} + my \ddot{\theta} + (E \dot{h})'' + (J \dot{\theta})'' + P(x_c - x)H(x_c - x) \theta'' - 2PH(x_c - x) \theta' \\
+ [M_c \ddot{h} + M_c y \ddot{\theta} - M_c z^2 \dot{h}^\prime - P \theta \delta h(x_c - x)] = -L \\
I_{EA} \ddot{\theta} + my \ddot{h} - (G \dot{h})' - (J \dot{\theta})' + P(x_c - x)H(x_c - x) h'^\prime \\
+ [M_c y \ddot{h} + I_{me} (z_c^2 + y_c^2) \ddot{\theta} + Pz_c - P y_c \dot{\theta} \delta h(x_c - x) \dot{\theta}] = M
\end{align*}
\]

2.3. Aerodynamic Models

In this study, the quasi-steady and unsteady models for subsonic 2-dimensional flow are considered to represent the aerodynamic forces about the elastic axis. The lift and moment equations based on the quasi-steady model are [3]
\[
L_{qs} = \frac{\rho U^2}{2} C_{Lq} \left[ \theta + \frac{\dot{h}}{U} + \frac{c}{U} \left( \frac{3 - \gamma_e}{4} \right) \dot{\theta} \right]
\]
where $c$ is the chord length and $\rho_a$ and $U$ are the density and speed of air, respectively. The term $dC_L/d\theta$ is considered to be constant, with a value of $2\pi$ obtained theoretically for incompressible flow.

In the unsteady aerodynamic model, the lift and moment equations are [23, 24]

$$
L = \frac{\pi \rho \alpha c^2}{4} \left[ \dot{h} + U \dot{\theta} \left( y_0 - \frac{c}{2} \right) \right] + \frac{\rho \alpha c^2}{2} \frac{dC_L}{d\theta} C(k) \left[ \theta + \frac{\dot{h}}{U} + c \left( 3 - \frac{y_0}{c} \right) \right]
$$

(12.a)

$$
M = \frac{\pi \rho \alpha c^2}{4} \left[ \left( y_0 - \frac{c}{2} \right) \dot{h} - U \left( \frac{3c}{4} - y_0 \right) \dot{\theta} - c^2 \left( \frac{9}{32} + \frac{y_0}{c} - 1 \right) \right]
$$

+ \frac{\rho \alpha c^2}{2} \frac{dC_L}{d\theta} C(k) \left( \frac{y_0}{c} \right) \left[ \theta + \frac{\dot{h}}{U} + c \left( 3 - \frac{y_0}{c} \right) \right]

(12.b)

where $C(k)$ is the Theodorsen’s function, which can be approximated by [23]

$$
C(k) = 1 - \frac{0.165}{1 - \frac{0.045}{k}} - \frac{0.335}{1 - \frac{0.30}{k}}, \quad k \leq 0.5
$$

(13.a)

$$
C(k) = 1 - \frac{0.165}{1 - \frac{0.041}{k}} - \frac{0.335}{1 - \frac{0.32}{k}}, \quad k > 0.5
$$

(13.b)

where $i$ is the imaginary unit and $k$ is the reduced frequency, given by

$$
k = \frac{\alpha c}{U}
$$

(14)

The final governing equations of motion, for the tapered viscoelastic cantilever wing subjected to bending and torsion loading governed by the aerodynamic strip theory, can be obtained for both aerodynamic models.

For the Quasi-Steady model, substitute equation (11.a) in (9) and equation (11.b) in (10). For the Unsteady model, substitute equation (12.a) in (9) and equation (12.b) in (10).

3. Solution procedure

In this study, a beam model that tapers in one plane, namely the xy-plane, is considered. Therefore, the general equations for the cord length $c(x)$, the wing mass per unit length $m(x)$, the elastic axis location $y_0(x)$, the offset between the elastic axis and the wing centre of gravity $y_0(x)$, the moment of inertia $I(x)$, the polar moment of inertia $J(x)$, the bending rigidity $EI(x)$, the torsional rigidity $GJ(x)$, and the mass moment of inertia per unit length $I_{Es}(x)$ of the wing are given by

$$
\beta(x) = \beta_r \left( 1 - c_t \frac{x}{l} \right)
$$

(15)

where $\beta$ represents the parameter, the subscript $r$ refers to the value of that parameter at the wing root and $c_t$ represents the taper ratio.

Throughout the numerical analysis, the following non-dimensional parameters are introduced
Due to the complexity of the partial differential equations of motion, a closed form solution cannot be found. However, the solution can be investigated via Galerkin approximate solution technique. At the flutter boundary, the equations of $h$ and $\theta$ are

\[
\begin{align*}
\dot{h}(\xi,t) &= \bar{h} f(\xi)e^{i\omega t} \\
\dot{\theta}(\xi,t) &= \bar{\theta} \phi(\xi)e^{i\omega t}
\end{align*}
\]

where $\bar{h}$ and $\bar{\theta}$ are the amplitudes of motion, and are dimensionally the same as $h$ and $\theta$ respectively. $\omega$ represents the frequency of harmonic oscillations occurring at the flutter boundary. $f(\xi)$ and $\phi(\xi)$ (given below) represent the uncoupled bending and torsion mode shapes of a cantilever beam, respectively.

\[
f(\xi) = [\cosh(1.875\xi) - \cos(1.875\xi)] - 0.734[\sinh(1.875\xi) - \sin(1.875\xi)]
\]

\[
\phi(\xi) = \sin\left(\frac{\pi\xi}{2}\right)
\]

Substituting equations (16) and (17) into equations (9) and (10), respectively and multiplying equation (9) by $f$ and equation (10) by $\phi$ and integrating over the wing span, a set of two algebraic equations are obtained; which can be written in matrix form as

\[
\begin{bmatrix}
(A_1) & (B_1) \\
(A_2) & (B_2)
\end{bmatrix}
\begin{bmatrix}
\bar{h} \\
\bar{\theta}
\end{bmatrix}
= 0
\]

The coefficients $A_i$ and $B_i$ depend on which aerodynamic model is used. Therefore, there are two sets of the coefficients $A_i$ and $B_i$ as given below. The determinant of the coefficient matrix has to vanish for nontrivial solution. With the determinant being complex in general, both its real and imaginary parts must vanish. This leads to two equations with two unknowns $U_{cr}$ and $\omega$, which are the flutter speed and flutter frequency, respectively.

- For the quasi-steady aerodynamic model:

  \[
  A_1 = a_{11} + i\omega a_{12} - \omega^2 a_{13} - \omega^2 a_{14} + \omega^2 a_{15} + i\omega U_{cr} a_{16}
  \]

  \[
  B_1 = -\omega^2 b_{11} + b_{12} - b_{13} - \omega^2 b_{14} - b_{15} + U_{cr}^2 b_{16} + i\omega U_{cr} b_{17}
  \]

  \[
  A_2 = \omega^2 a_{21} - a_{22} + \omega^2 a_{23} + i\omega U_{cr} a_{24}
  \]

  \[
  B_2 = b_{21} + i\omega b_{22} + \omega^2 b_{23} + \omega^2 b_{24} + b_{25} + U_{cr}^2 b_{26} + i\omega U_{cr} b_{27}
  \]

- For the unsteady aerodynamic model:
where the coefficients \( a_{ij} \) and \( b_{ij} \) are given in the appendix.

Galerkin method can be effectively applied to aeroelastic analysis because of its versatility. The implementation of Galerkin method-based aeroelastic analysis is developed entirely within a numerical code. The numerical values of the dimensions and material mechanical properties of the model used in this study are listed in Table 1 as found in [1].

Table 1. Properties of Goland’s wing at root section.

| Parameter                  | Value       |
|----------------------------|-------------|
| Wing span                  | 20 ft.      |
| Chord                      | 6 ft.       |
| Bending rigidity           | \( 23.6 \times 10^6 \) lbf ft\(^2\) |
| Torsional rigidity         | \( 2.39 \times 10^6 \) lbf ft\(^2\) |
| Mass of the wing per unit length | 0.746 slugs/ft. |
| Mass moment of inertia     | 1.943 slugs ft\(^2\)/ft. |
| Elastic axis position      | 33% of the chord |
| Inertial axis position     | 43% of the chord |
| Air density                | 0.00237 slugs/ft\(^3\) |

4. Results and discussion

4.1. Validation
To verify the accuracy of the model, the results for a clean elastic straight wing (in the absence of the engine, taper ratio, and viscoelastic damping of the material) are validated against those in Reference [9] for both aerodynamic models, and good agreement is observed as seen in Table 2. The slight difference is due to the fact that they used ‘p’ and ‘p-k’ methods in their study whereas the determinant method is utilized in this work.

Table 2. Validation of flutter speed and frequency for a clean straight wing.

| Aerodynamic Model | Velocity (ft/sec) | Frequency (rad/sec) |
|-------------------|-------------------|---------------------|
| Ref. [9] Quasi-Steady | 110             | 93                  |
| Unsteady          | 449              | 70                  |
| Present work Quasi-Steady | 110.36      | 94                  |
| Unsteady          | 464.9            | 68.6                |
| Exact [1] Unsteady | 451             | 70.7                |

4.2. Effect of span-wise engine location
The influence of the span-wise engine location \((x_e)\) on the non-dimensional flutter speed and frequency for a straight elastic wing \((c_t = \eta_E = \eta_G = 0)\) is shown in Figure 3. The engine’s Cg is located exactly on the elastic axis of the wing \((y_e = 0)\) with \(z_e = 0, P = 1,\) and \(M_e = 1.\) Figure 3 reveals that, for the unsteady aerodynamic model, moving the engine from the wing root to almost 40% of the wing span \((x_e = 0.4)\) decreases the flutter speed. As the engine slides further towards the wing tip, the flutter speed dramatically increases. For the quasi-steady model, the flutter speed slightly decreases as the engine moves from the wing root to around 25% of the span \((x_e = 0.25)\), further sliding towards the wing tip increases the flutter speed. These results show agreement with those in [22]. The flutter frequency decreases by moving the engine towards the wing tip for the two aerodynamic models. It is also observed that the quasi-steady aerodynamic model provide more conservative results than the unsteady model although the two models give almost the same behaviour.

Figure 3. Effect of span-wise engine location on (a) non-dimensional flutter speed and (b) non-dimensional flutter frequency for straight elastic wing \((P = 1, M_e = 1)\).

4.3. Effect of chord-wise engine location

Figure 4 demonstrates the effect of the chord-wise location of the engine \((y_e)\) on the non-dimensional flutter speed and frequency for a straight elastic wing \((c_t = \eta_E = \eta_G = 0)\). The engine is located at quarter-span \((x_e = 0.25)\) with \(z_e = 0, P = 1,\) and \(M_e = 1.\) The plot shows that, for the unsteady aerodynamic model, the flutter speed slightly increases as the engine slides from the wing leading edge up to 10% of the chord before the wing elastic axis \((y_e = -0.1)\). Moving the engine further towards the wing trailing edge decreases the flutter speed. In addition, the quasi-steady aerodynamic model provides that the flutter speed decreases as the engine moves from the leading edge to the trailing edge. For both of the aerodynamic models, the flutter frequency increases as the engine slides from the leading edge to the trailing edge. Indeed, similar behaviour was obtained in [21] and [22], where in the former it was pointed out that moving the engine from trailing edge to the leading edge in chord-wise direction makes the wing more stable.

Figure 4. Effect of chord-wise engine location on (a) non-dimensional flutter speed and (b) non-dimensional flutter frequency for straight elastic wing \((P = 1, M_e = 1)\).
4.4. Effect of vertical engine location

The effect of the vertical location of the engine \( (z_e) \) on the non-dimensional flutter speed and frequency for a straight elastic wing \( (c_t = \eta_e = \eta_G = 0) \) is illustrated in Figure 5. The engine is located at quarter-span \( (x_e = 0.25) \) with \( y_e = 0, P = 1, \) and \( M_e = 1. \) The quasi-steady aerodynamic model provides that the wing becomes more stable as the engine goes further below the wing. However, different behaviour is observed when the unsteady aerodynamic model is considered, where the wing becomes less stable as the engine goes further below the wing.

![Figure 5. Effect of vertical engine location on (a) non-dimensional flutter speed and (b) non-dimensional flutter frequency for straight elastic wing \( (P = 1, M_e = 1). \)](image)

5. Conclusions

Flutter is one of the significant phenomena of Aeroelasticity. This dangerous aeroelastic phenomenon can occur to any flexible structure subjected to aerodynamic forces such as aircraft wings, bridges, buildings, etc. It is important to analyse the flutter in order to predict the speed and frequency at which it occurs so that structural damages and failures can be avoided. By incorporating the effects of the engine location in the three directions, this work is a growing out of the aeroelastic research pertains the quasi-steady and unsteady aerodynamic formulations.

The purpose of this work is modelling and analysing a tapered viscoelastic wing carrying an engine and subjected to a follower thrust force as well as to aerodynamic forces from an aeroelastic viewpoint. The effects of the engine location in the three directions on the wing flutter characteristics have been studied.

The unsteady aerodynamic model provides more reliable results than the quasi-steady model which offers more conservative predictions. In most cases, the two models yield the same behaviour or at least the same trend. In some other cases, however, they give diverse results.

Parametric studies show that the engine location along the span, the chord-wise engine location, and the vertical location of the engine have significant effects on the flutter speed and frequency of the aircraft wing. Therefore, all these parameters must be taken into consideration to get accurate stability investigation results.

According to this work, the wing will become more stable if the engine is located away from the fuselage towards the wing tip, the engine centre of gravity is moved towards the wing leading edge and/or the engine is placed right below the wing (i.e. the vertical distance between the engine centre of gravity and the wing chord-line is minimal).

References

[1] Goland M 1945 The flutter of a uniform cantilever wing J. Applied Mechanics 12 pp A197-A208
[2] Goland M and Luke Y L 1948 The flutter of a uniform wing with tip weights J. Applied Mechanics 15 pp 13-20
[3] Fung Y C 1955 An Introduction to the Theory of Aeroelasticity 2nd ed. (New York: Dover Publications)
[4] Dowell E H 1967 Nonlinear oscillations of a fluttering plate II *AIAA J.* 5 pp 1856-62
[5] Dowell E H and Voss H M 1965 Theoretical and experimental panel flutter studies in the Mach number range 1.0 to 5.0 *AIAA J.* 3 pp 2292-304
[6] Meirovitch L 1975 *Elements of Vibration Analysis* (New York: McGraw-Hill)
[7] Moosavi M R, Oskouei A N and Khelil A 2005 Flutter of subsonic wing *Thin-walled structures* 43 pp 617-27
[8] Acum W E A 1959 *The Comparison of Theory and Experiment of Oscillating Wings* vol 2 AGARD manual on aeroelasticity
[9] Haddadpour H and Firouz-Abadi R D 2006 Evaluation of quasi-steady aerodynamic modeling for flutter prediction of aircraft wings in incompressible flow *Thin-walled structures* 44 pp 931-6
[10] Jia-ju S and Ke-hwa J 1981 Dynamic response of viscoelastic beam *Applied Mathematics and Mechanics* 2 pp 255-64
[11] Hilton H H 1957 Pitching instability of rigid lifting surfaces on viscoelastic supports in subsonic or supersonic potential flow *Proc. 3rd Midwestern Conf. on Solid Mechanics* (Michigan) pp 1-19
[12] Hilton H H 1960 The divergence of supersonic, linear viscoelastic lifting surfaces, including chordwise bending *J. Aerospace Sciences* 27 pp 926-34
[13] Hilton H H 1991 Viscoelastic and structural damping analysis *Proc. on Damping’91* (Ohio) Air Force Technical Report WL-TR-91 3078 pp 1-15
[14] Ungar E E 1971 Chapter 14 in *Noise and Vibration Control* ed L L Beranek (New York: McGraw-Hill)
[15] Yi S Ahmad M F and Hilton H H 1996 Dynamic responses of plates with viscoelastic free layer damping treatment *J. vibration and acoustics* 118 pp 362-7
[16] Bolotin V V 1962 *The Dynamic Stability of Elastic Systems* vol 1 (California: Aerospace Corp El Segundo)
[17] Bolotin V V 1963 *Nonconservative Problems of the Theory of Elastic Stability* (Oxford: Pergamon Press)
[18] Bolotin V V and Zhinzher N I 1969 Effects of damping on stability of elastic systems subjected to nonconservative forces *Int. J. Solids and Structures* 5 pp 965-89
[19] Hodges D H 2001 Lateral-torsional flutter of a deep cantilever loaded by a lateral follower force at the tip *J. Sound and Vibration* 247 pp 175-83
[20] Hodges D H, Patil M J and Chae S 2002 Effect of thrust on bending-torsion flutter of wings *J. Aircraft* 39 pp 371-6
[21] Fazelzadeh S A, Mazidi A and Kalantari H 2009 Bending-torsional flutter of wings with an attached mass subjected to a follower force *J. Sound and Vibration* 323 pp 148-62
[22] Amoozgar M R, Irani S and Vio G A 2013 Aeroelastic instability of a composite wing with a powered-engine *J. Fluids and Structures* 36 pp 70-82
[23] Theodorsen T 1935 *General Theory of Aerodynamic Instability and the Mechanism of Flutter* NACA TR 496 (Washington: National Advisory Committee for Aeronautics)
[24] Hodges D H and Pierce G A 2011 *Introduction to Structural Dynamics and Aeroelasticity* 2nd ed. (New York: Cambridge University Press)