Effective $\Delta S = 1$ weak chiral Lagrangian
from the instanton-induced chiral quark model

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Abstract

We present the recent investigation of the $\Delta S = 1$ effective weak chiral Lagrangian within the framework of the instanton-induced chiral quark model. Starting from the effective four-quark operators, we derive the effective weak chiral action by integrating out the constituent quark fields. Employing the derivative expansion, we obtain the effective weak chiral Lagrangian to $O(p^4)$ order with low energy constants.

Chiral perturbation theory ($\chi$PT), known as an effective field theory in very low-energy regime, explains low-energy phenomena in strong interactions very well. Based on its success in describing strong interactions, $\chi$PT was also applied to the nonleptonic processes. However, the effective weak chiral Lagrangian of order $O(p^4)$ brings in too many low energy constants (LECs) to be determined by experimental data, as the chiral corrections are considered. In order to proceed to make numerical calculations, it is inevitable to rely on models which provide the LECs.

Recently, Antonelli et al. applied the $\chi$QM in order to obtain the effective weak chiral Lagrangian in the lowest order ($O(p^2)$) in the chiral expansion. The large $N_c$ expansion was also considered in orders $O(N_c^2)$, $O(N_c)$, and $O(\alpha_s N_c)$. In their study, the correction from order $O(\alpha_s N_c)$ plays an essential role in reaching the agreement with phenomenology. However, one should mention that treating order $O(\alpha_s N_c)$ requires a special care. As noted by Ref. [4], $O([\alpha_s N_c]^2)$ is in fact the same order as $O(\alpha_s N_c)$ and might be nonnegligible.

More recently, Ref. [5] extended the work of Ref. [3] to the next-to-leading order in the chiral expansion. However, they calculated directly the transition amplitudes of the decay $K \to \pi\pi$ to order $O(p^4)$, which is in a sense different from the original scheme of chiral perturbation theory.

In this work we rather want to furnish the LECs not only to order $O(p^2)$ but also to order $O(p^4)$ within the framework of the instanton-induced chiral quark model.

The effective chiral action with the $\Delta S = 1$ effective weak Hamiltonian can be written as follows:

$$\exp \left( - S_{\text{eff}}^{\Delta S=1} \right) = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \exp \left[ \int d^4x \left( \psi^\dagger D\psi - H_{\text{eff}}^{\Delta S=1} \right) \right], \tag{1}$$

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where $D$ is the Dirac operator with the pseudo-Goldstone boson $\bar{\psi}
abla \psi$.

$\quad D = i\bar{\psi} + iM\gamma^5. \quad (2)$

The constituent quark mass $M$ is a momentum-dependent mass which characterizes the instanton-induced chiral quark model. The effect of the momentum dependence of the $M$ will be considered in a later work $[7]$. $U^{\gamma_5}$ denotes the chiral meson field. The effective weak quark Hamiltonian $\mathcal{H}^{\Delta S=1}_{\text{eff}}$ consists of ten four-quark operators among which only seven operators are independent:

$\mathcal{H}^{\Delta S=1}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i c_i(\mu) \mathcal{Q}_i(\mu) + \text{h.c.} \quad (3)$

The Hamiltonian $\mathcal{H}^{\Delta S=1}_{\text{eff}}$ can be found elsewhere (for example, see Ref. $[8]$). Since the Fermi constant $G_F$ is very small, we can expand Eq.(1) with regard to it and obtain the following expression:

$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{1}{\mathcal{N}} \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{H}_{\text{eff}}^{\Delta S=1} \exp \left[ \int d^4x \psi^\dagger \mathcal{D}\psi \right] . \quad (4)$

By integrating out the quark fields and making the derivative expansion, we derive the effective chiral Lagrangian as follows:

$\mathcal{L}_{\text{eff}}^{\Delta S=1, \mathcal{O}(\mu^2)} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* f_\pi^4 g_8 \left[ \lambda_3^2 L^\mu L^\mu \right] + \text{h.c.} \quad (5)$

$\quad \mathcal{L}_{\Delta S=1, \mathcal{O}(\mu^2)}^{(1/2)} = \mathcal{L}_{\Delta S=1, \mathcal{O}(\mu^2)}^{(1/2)} + \frac{5}{9} \mathcal{L}_{\Delta S=1, \mathcal{O}(\mu^2)}^{(3/2)}, \quad (6)$

where

$\quad \mathcal{L}_{\Delta S=1, \mathcal{O}(\mu^2)}^{(1/2)} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* f_\pi^4 g_8 \left[ \lambda_3^2 L^\mu L^\mu \right] + \text{h.c.}, \quad (6)$

$\quad \mathcal{L}_{\Delta S=1, \mathcal{O}(\mu^2)}^{(3/2)} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* f_\pi^4 g_{27} \left( \lambda_3^3 L^\mu L^\mu - \lambda_2^3 L^\mu L^\mu - 5 \lambda_2^3 L^\mu L^\mu \right) + \text{h.c.}, \quad (6)$

$\quad \mathcal{L}_{\Delta S=1, \mathcal{O}(\mu^2)}^{(3/2)} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* f_\pi^4 g_{27} \left( \lambda_3^3 L^\mu L^\mu + 2 \lambda_2^3 L^\mu L^\mu + \lambda_2^3 L^\mu L^\mu \right) + \text{h.c.} \quad (6)$

Note that the isospin $T = 1/2$ part of the eikosiheptaplet is suppressed by the prefactor, so that one can treat the eikosiheptaplet part of the Lagrangian as a pure $T = 3/2$ one. $g_8$ and $g_{27}$ can be extracted from the $K \to \pi\pi$ decay rate. At the tree level, the $\Delta T = 1/2$ enhancement is reflected in these constants.

We first revisit the recent results of Ref. $[3]$ but we concentrate only on the low energy constants. To obtain the comparable results to the empirical data, Ref. $[3]$ considered the effect of the gluon condensates coming from the external background gluon fields $[4]$ in order $\mathcal{O}(\alpha_s N_c)$. With orders $\mathcal{O}(N_c^2), \mathcal{O}(N_c)$ and $\mathcal{O}(\alpha_s N_c)$ taken into account, one arrives at
following results in the leading order $\mathcal{O}(p^2)$:

\[
g_{27}^{\delta_{GG}} = \left(-\frac{2}{5} + \frac{1}{N_c} (1 - \delta_{GG}) \frac{3}{5}\right) c_1 + \left(\frac{3}{5} - \frac{1}{N_c} (1 - \delta_{GG}) \frac{2}{5}\right) c_2 + \frac{1}{N_c} (1 - \delta_{GG}) c_3 + c_4 + \left(\frac{\langle \bar{q}q \rangle}{N_c f_\pi^2 M} - \frac{\langle \bar{q}q \rangle M}{8 f_\pi^4 \pi^2}\right) c_5 + \left(\frac{\langle \bar{q}q \rangle}{f_\pi^2 M} - \frac{N_c \langle \bar{q}q \rangle M}{8 f_\pi^4 \pi^2}\right) c_6 + \left(\frac{3}{5} + \frac{1}{N_c} (1 - \delta_{GG}) \frac{2}{5}\right) c_9 + \left(\frac{2}{5} - \frac{1}{N_c} (1 - \delta_{GG}) \frac{3}{5}\right) c_{10}
\]

(7)

\[
g_{27}^{\delta_{GG}} = \left(1 + \frac{1}{N_c} (1 - \delta_{GG})\right) \left(\frac{3}{5} c_1 + \frac{3}{5} c_2 + \frac{9}{10} c_9 + \frac{9}{10} c_{10}\right)
\]

(8)

where

\[
\delta_{GG} = \frac{N_c \langle \alpha_s G G / \pi \rangle}{2 \frac{16}{\pi^2} f_\pi^4}.
\]

(9)

Figure draws the dependence of the $g_8/g_{27}$ on the gluon condensate.

![Graph showing the dependence of $g_8/g_{27}$ on the gluon condensate](image)

**Fig.3:** Dependence of the $g_8/g_{27}$ on the gluon condensate ($\alpha_s GG$). The solid curve denotes the LO renormalization scheme in Ref. [3], while the dashed curve and dot-dashed one stand for the NDR and the HV schemes, respectively. The value of the constituent quark mass $M = 300$ MeV is used and the quark condensate $\langle \bar{q}q \rangle / 2 = -(250$ MeV)$^3$ is employed.

As the gluon condensate increases, the ratio $g_8/g_{27}$ is improved remarkably, so that we are able to obtain the reasonable value of the ratio around $(380 \text{ MeV})^4$, which is acceptable with the counterterms being considered [4]. However, there is one important caveat. As was mentioned in Ref. [4], the $\mathcal{O}(\alpha N_c)^2$ corrections which are of order $\mathcal{O}(1)$ as well were neglected, based on the argument that they involve condensates of higher dimension requiring higher powers of the normalization scale. However, keeping in mind the fact that the $\mathcal{O}(\alpha_s N_c)$ corrections enhance the result of the ratio $g_8/g_{27}$ so radically as shown above, the higher-order terms might give some contribution. We would rather regard the above results as a mere indication of the importance of gluon effects in the nonleptonic processes.
An effective weak chiral Lagrangian to order $\mathcal{O}(p^4)$ in $\chi$PT with a minimal set of independent terms was given by Ecker et al. [10] and Esposito-Farèse [2] (presented in Minkowski space). A part of the corresponding low energy constants are obtained from the present study:

\[
L_{\text{eff}}^{\Delta S=1, \mathcal{O}(p^4)} = - \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* f^2 \left[ (N_1^{(8)} \langle \lambda_6 \lambda_\mu L^\mu L^\nu \rangle + N_2^{(8)} \cdot \langle \lambda_6 L_\mu L^\nu \rangle + N_3^{(8)} \langle \lambda_6 L_\mu L^\nu \rangle + N_4^{(8)} \langle \lambda_6 L_\mu L^\nu \rangle + N_5^{(8)} i \epsilon_{\mu \nu \rho \delta} \langle \lambda_6 L_\mu \rangle \langle L^\nu L^\rho L^\delta \rangle \right]
\]

As noted by G. Ecker et al. [10], the LECs $N_1^{(8)}$, $N_2^{(8)}$, $N_3^{(8)}$, and $N_4^{(8)}$ contribute to the process $K \to 3\pi$ while $N_5^{(8)}$ does to the radiative $K$-decays. In particular, the $N_5^{(8)}$ is related to the chiral anomaly [11]. Note that all LECs in the eikosheptaplet are degenerate.

Taking into account the $N_c$ corrections, the LECs are derived as follows:

\[
N_1^{(8)} = \left( -\frac{N_c^2 M^2}{128 \pi^4 f_\pi^2} + \frac{N_c}{8 \pi^2} - \frac{f_\pi^2}{2 M^2} \right) c_0 + \left( -\frac{N_c M^2}{128 \pi^4 f_\pi^2} + \frac{1}{8 \pi^2} - \frac{f_\pi^2}{2 N_c M^2} \right) c_5,
\]

\[
N_2^{(8)} = \frac{N_c}{60 \pi^2} \left( -2 + \frac{1}{N_c} \right) c_1 + \left( -3 + \frac{1}{N_c} \right) c_2 + \frac{1}{N_c} 5 c_3 + 5c_4
\]

\[
+ \left( -3 + \frac{1}{N_c} \right) c_9 + \left( 2 - \frac{1}{N_c} \right) c_{10},
\]

\[
N_3^{(8)} = 0,
\]

\[
N_4^{(8)} = \frac{N_c}{60 \pi^2} \left( -\frac{1}{2} - \frac{1}{N_c} \right) c_1 + \left( -1 + \frac{1}{N_c} \right) c_2 + \frac{5}{2} c_3 + \frac{1}{N_c} 5 c_4
\]

\[
- \frac{5}{2} c_5 - \frac{1}{N_c} 5 c_6 + \left( -\frac{3}{2} + \frac{1}{N_c} \right) c_9 + \left( -\frac{3}{2} + \frac{1}{N_c} \right) c_{10},
\]

\[
N_5^{(8)} = \frac{N_c}{60 \pi^2} \left( -\frac{1}{2} + \frac{1}{N_c} \right) c_1 + \left( 1 - \frac{3}{N_c} \right) c_2 - \frac{5}{2} c_3 - \frac{1}{N_c} 5 c_4
\]

\[
- \frac{5}{2} c_5 - \frac{1}{N_c} 5 c_6 + \left(-1 + \frac{3}{N_c} \right) c_9 + \left( \frac{3}{2} - \frac{1}{N_c} \right) c_{10},
\]

\[
N_6^{(27)} = N_7^{(27)} = N_8^{(27)} = N_9^{(27)} = 0,
\]

\[
N_2^{(27)} = -N_3^{(27)} = -N_4^{(27)} = N_5^{(27)} = N_6^{(27)} = \frac{N_c}{60 \pi^2} \left( 1 + \frac{1}{N_c} \right) \left(-3 c_1 - 3 c_2 - \frac{9}{2} c_3 - \frac{9}{2} c_4 \right).
\]

The corresponding numerical results and further discussions such as the contribution of the quark axial-vector constant $g_A$ and the momentum-dependence of the constituent quark mass will appear soon [4].
In summary, we have determined a part of the LECs for the $\mathcal{O}(p^4)$ and discussed the relevance of the gluon condensate to the LECs in the leading order.

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