Dependency of Statistical Correlation between Ply Elastic Properties of FRP

Dong Xu¹*, Jine Huang¹, Yang Zhang¹, Hongwei Cheng¹ and Shufeng Zhang²

¹ Naval Research Academy, Beijing 100161, China
² Science and Technology on Integrated Logistics Support Laboratory, National University of Defense Technology, China
Email: xudongcrazy@163.com

Abstract. Statistical correlation between ply (or lamina) mechanical properties is recently recognized as an important factor in composite structure reliability evaluation. This work provides a sensitivity analysis of stochastic variation of ply elastic properties to constitutional material (microscale) properties to reveal essential causes of the statistical correlation. Relationship between linear correlation coefficients of ply elastic properties and microscale uncertainty is derived. It is shown that if the microscale uncertainty is dominated by the uncertainty of FVR, there would be great statistical correlation between all ply in-plane elastic properties for both unidirectional and plain-weave fibre reinforced composite. Statistical correlation between ply elastic properties of plain-weave fibre reinforced composite additionally depends on the uncertainty level of unit-cell geometry.

1. Introduction

Fibre reinforced plastic (FRP) is extensively used as load bearing structures in aircraft, ships, vehicles, buildings, etc. Fibre reinforced composite commonly provides superior mechanical properties including high stiffness, high strength and good fatigue resistance, but a noteworthy limitation of current fibre reinforced composite is the large uncertainty on its mechanical properties, which is mainly caused by poor consistency in constituent material properties and manufacture process [1, 2]. It is experimentally shown that the coefficient of variation (CoV) of elastic properties of carbon fibre reinforced plastic (CFRP) and glass fibre reinforce plastic (GFRP) could be around 5%-10%, while the CoV of its strength properties could be around 10%-20% [3, 4]. Therefore, probabilistic analysis and structure reliability evaluation of composite structures is more essential than that for counterpart metal structures [5].

Roughly before 2010, most publically reported studies on reliability analysis of composite structures consider uncertainties such as ply & laminate mechanical properties, ply & laminate geometrical configurations, external loading, etc. [6-10]. These studies assume that ply mechanical properties are statistically independent with one another. By multiscale numerical investigation, Shaw et al. [11] for the first time shows that ply mechanical properties of unidirectional fibre reinforced composite could be statistically correlated with one another. Similar statistical correlation is later reported by Mustafa et al. [12]. By stochastic finite element analysis (SFEA), Zhang et al. [13] shows that mechanical properties of plain-weave composite could also be statistically correlated. Recently, Zhang et al. [14, 15] reported that the statistical correlation between ply elastic properties would provide large influence on composite structure reliability for certain loading configurations. Calvário et al. [16] takes account the statistical correlation to investigate the sensitivity of composite failure.
probability with different failure criterion. Overall, several studies have realized the significant influence of the statistical correlation between composite mechanical properties on structure reliability. However, current studies are still limited to the level of knowing the existence of statistical correlation between composite mechanical properties, but little work has been reported on comprehensive analysis about the dependency of the statistical correlation.

The present work aims to provide a comprehensive understanding on the dependency of the statistical correlation between ply (or lamina) elastic properties. By a combination of micromechanical models and Monte-carlo simulation, the sensitivity of ply elastic properties to constituent material properties is firstly investigated. Correlation coefficients between ply elastic properties are estimated by using different sets of combination of CoV on constituent material properties. The dependency of the statistical correlation is studied on both unidirectional and plain-weave fibre reinforced composites, and causes of large statistical correlation between ply elastic properties are discussed.

2. Micromechanical Models

For unidirectional fibre reinforced composite, Huang [17] micromechanical model was employed to derive ply effective elastic properties from constituent material (microscale) properties. The Huang micromechanical model is based on cylindrical RVE (represent volume element) and a bridging matrix which connects the stress state in fibre to the stress state in matrix. The Huang micromechanical model provides explicit analytical prediction on lamina elastic properties and agrees well with experimental observations [11]. By the Huang micromechanical model, in-plane elastic properties of unidirectional fibre reinforced lamina are written as

\[
E_{11} = V_f E_{f11} + V_m E_m
\]

\[
E_{22} = \frac{(V_f + V_m a_{11})(V_f + V_m a_{22})}{(V_f + V_m a_{11})(V_f S_{f22} + a_{12} V_m S_{m22}) + V_f V_m (S_{m12} - S_{f12}) a_{12}}
\]

\[
\nu_{12} = V_f \nu_{f11} + V_m \nu_m
\]

\[
G_{12} = \frac{(G_{f12} + G_m) + V_f (G_{f12} - G_m)}{(G_{f12} + G_m) - V_f (G_{f12} - G_m)}
\]

where \(a_{11}, a_{22}, a_{12}, S_{f11}, \ldots, S_{m12}\) are given in Huang [17]. In a comparison with Chamis micromechanical model [18], the prediction of \(E_{11}\) and \(\nu_{12}\) of the two micromechanical models is the same while that of \(E_{22}\) and \(G_{12}\) is different.

For plain-weave fibre reinforced composite, FE models on a representative unit-cell is employed to derive ply effective elastic properties. A schematic illustration of the unit-cell is shown in figure 1, while the unit-cell size is defined by yarn space (s), thickness (h) and width (w). The geometry of the yarn is represented by sinusoidal function as shown in equations 5-9. Since the unit cell shown in figure 1 is a quarter of a full unit cell, reflectional symmetry is set as boundary constraint for the unit-cell in the FE modeling. The effective in-plane elastic properties are obtained by homogenized strain/stress of the unit-cell. More details on the FE modeling are shown in our previous publication [13].

\[
y_1(x) = \frac{h}{2} \left[ \cos \frac{\pi x}{s} + 1 \right] \quad (0 < x < s)
\]

\[
y_2(x) = \frac{h}{2} \left[ \cos \frac{\pi x}{s} - 1 \right] \quad (0 < x < s)
\]

\[
y_3(x) = -h \cos \frac{\pi x}{\beta} \quad (0 < x < \frac{w}{2})
\]

\[
y_4(x) = -h \cos \left[ \frac{\pi (x - (s - \beta))}{\beta} \right] \quad (s - \frac{w}{2} < x < s)
\]
Figure 1. Plain-weave composite: (a) sketch of unit-cell; (b) definition of geometrical parameters [13].

\[
\beta = \frac{\pi w}{2 \arccos[\sin^2(\frac{\pi w}{4s})]}
\]  

(9)

3. Sensitivity of Ply Elastic Properties

Sensitivity of ply elastic properties to constituent material properties would provide clues for why ply elastic properties are statically correlated. In this work, the sensitivity is represented by linear correlation coefficient between stochastic samples of ply elastic properties and microscale parameters. The statistics of T300 carbon fibre, E-glass fibre and epoxy are shown in Table 1. As most CoV of constituent material properties have been experimentally shown to be around 5% [15], in this work the CoV of all constituent material properties are assumed to be 5% to provide fair sensitivity analysis over all microscale parameters. It is important to notice that carbon fibre is transversely isotropic while E-glass fibre is isotropic, and hence it is necessary to investigate whether this difference affects the statistical correlation of ply elastic properties. The statistics of unit-cell geometrical parameters of glass fibre plain-weave composite is shown in Table 2. For the sensitivity analysis of unidirectional GFRP and CFRP, 5×10^4 implementation of Monte-carlo simulation was conducted to derive the sensitivity. For the sensitivity analysis of plain-weave composite, Latin hypercube sampling is employed as the FE modeling is much time consuming and 500 implementation is shown to be good enough to provide convergent solution on the statistics of ply elastic properties.

| Table 1. Statistics of constituent material properties [13]. |
|---------------------------------------------------------------|
|                        | Mean value | CoV  | Distribution |
| E-glass fibre          |            |      |              |
| \(E_f\)               | 72 GPa     | 5%   | Normal       |
| \(\nu_f\)             | 0.2        | 5%   | Normal       |
| Carbon fibre           |            |      |              |
| \(E_{11}\)            | 213.7 GPa  | 5%   | Normal       |
| \(E_{22}\)            | 13.8 GPa   | 5%   | Normal       |
| \(\nu_{12}\)          | 0.2        | 5%   | Normal       |
| \(G_{12}\)            | 13.8 GPa   | 5%   | Normal       |
| \(\nu_{23}\)          | 0.3        | 5%   | Normal       |
| Epoxy                 |            |      |              |
| \(E_m\)               | 3.45       | 5%   | Normal       |
| \(\nu_m\)             | 0.35       | 5%   | Normal       |
| Fibre volume ratio     |            |      |              |
| \(V_f\)               | 0.66       | 5%   | Normal       |
Table 2. Statistics on geometrical parameters of plain-weave composite.

| E-glass fabric/epoxy | Mean value | CoV  | Distribution type |
|----------------------|------------|------|-------------------|
| S                    | 1.96 mm    | 5%   | Normal            |
| w [19]               | 1.70 mm    | 5%   | Normal            |
| h [19]               | 0.15 mm    | 5%   | Normal            |

Figure 2. Sensitivity of ply elastic properties of unidirectional E-glass/epoxy composite.

Figure 3. Sensitivity of ply elastic properties of unidirectional carbon/epoxy composite.

Figure 4. Sensitivity of ply elastic properties of E-glass plain-weave/epoxy composite.

The sensitivity of ply elastic properties of unidirectional GFRP, unidirectional CFRP and E-glass plain-weave composite is shown in figures 2–4, respectively. Here, sensitivity smaller than 5% (significance level) is not presented. In addition, the $V_f$ in figures 2 and 3 represents the FVR of composite while in figure 4 $V_f$ represents FVR of yarn. From figures 3 and 4, it is seen that the FVR significantly affects all ply elastic properties of unidirectional GFRP and CFRP, and $E_m$ affects both $E_{22}$ and $G_{12}$. Other microscale parameters affect only one ply elastic properties. Similarly, ply elastic properties of E-glass plain-weave composite all depend on the FVR of yarn, and $v_{12}$ and $G$ both depend on $E_m$. In addition, $E$ and $G$ also have great negative dependence on $s$ (yarn space) and positive dependence on $w$ (yarn width), and $v_{12}$ and $G$ both depend on $E_f$. Therefore, it is shown that stochastic variation of certain microscale parameters simultaneously affects two or more ply elastic properties, which essentially explains why ply mechanical properties could be statistically correlated.

4. Dependency of Statistical Correlation

Since all ply elastic properties of unidirectional fibre reinforced composite show great sensitivity on the FVR, a comparison is conducted with different combinations of values of CoV of FVR and other microscale parameters. Tables 3 and 4 show linear correlation coefficients of ply elastic properties at different status of microscale uncertainty, where cov2 represents the CoV of FVR and cov1 represents the CoV of other microscale parameters (see table 1). It is shown that the scenarios of cov1 = 0.01 & cov2 = 0.01 and cov1 = 0.05 & cov2 = 0.05 provide almost the same linear correlation coefficients between ply elastic properties. In the scenario of cov1 = 0.01 & cov2 = 0.05, all linear correlation coefficients are over 0.9, which indicates that all ply elastic properties are strongly correlated. On the contrary, in the scenario of cov1 = 0.05 & cov2 = 0.01, all linear correlation coefficients are very small.
(except for the linear correlation coefficient between $G_{12}$ and $E_{22}$), indicating the ply elastic properties are almost independent with one another.

**Table 3.** Linear correlation coefficients of elastic properties of GFRP.

| cov1=0.01 | $\rho$ | cov2=0.01 | $\rho$ | cov1=0.05 | $\rho$ | cov2=0.05 | $\rho$ |
|-----------|--------|-----------|--------|-----------|--------|-----------|--------|
| $E_{11}$  | 1.00   | $E_{11}$  | 1.00   | $E_{11}$  | 1.00   |
| $E_{22}$  | 0.71   | 1.00      | $E_{22}$ | 0.97      | 1.00   |
| $\nu_{12}$ | -0.30  | -0.33     | 1.00   | $\nu_{12}$ | -0.90 | -0.90     | 1.00   |
| $G_{12}$  | 0.69   | 0.98      | -0.46  | 1.00      | $G_{12}$ | 0.97      | 1.00   |

| cov1=0.05 | $\rho$ | cov2=0.01 | $\rho$ | cov1=0.05 | $\rho$ | cov2=0.05 | $\rho$ |
|-----------|--------|-----------|--------|-----------|--------|-----------|--------|
| $E_{11}$  | 1.00   | $E_{11}$  | 1.00   | $E_{11}$  | 1.00   |
| $E_{22}$  | 0.33   | 1.00      | $E_{22}$ | 0.71      | 1.00   |
| $\nu_{12}$ | -0.01  | 0.10      | 1.00   | $\nu_{12}$ | -0.27 | -0.32     | 1.00   |
| $G_{12}$  | 0.27   | 0.90      | -0.20  | 1.00      | $G_{12}$ | 0.69      | 0.98   |

**Table 4.** Linear correlation coefficients of elastic properties of CFRP.

| cov1 = 0.01 | $\rho$ | cov2 = 0.01 | $\rho$ | cov1 = 0.05 | $\rho$ | cov2 = 0.05 | $\rho$ |
|-------------|--------|-------------|--------|-------------|--------|-------------|--------|
| $E_{11}$    | 1      | $E_{11}$    | 1      | $E_{11}$    | 1      |
| $E_{22}$    | 0.53   | 1           | $E_{22}$ | 0.97      | 1      |
| $\nu_{12}$  | -0.32  | -0.28      | 1      | $\nu_{12}$ | -0.92 | -0.92      | 1      |
| $G_{12}$    | 0.61   | 0.83        | -0.48  | 1          | $G_{12}$ | 0.97      | 0.99   |

| cov1 = 0.05 | $\rho$ | cov2 = 0.01 | $\rho$ | cov1 = 0.05 | $\rho$ | cov2 = 0.05 | $\rho$ |
|-------------|--------|-------------|--------|-------------|--------|-------------|--------|
| $E_{11}$    | 1      | $E_{11}$    | 1      | $E_{11}$    | 1      |
| $E_{22}$    | 0.07   | 1           | $E_{22}$ | 0.53      | 1      |
| $\nu_{12}$  | 0.00   | 0.11        | 1      | $\nu_{12}$ | -0.34 | -0.30      | 1      |
| $G_{12}$    | 0.08   | 0.57        | -0.18  | 1          | $G_{12}$ | 0.62      | 0.83   |

A more detailed comparison of the linear correlation coefficient between $E_{11}$ and $E_{22}$ is shown in figure 5, which shows that the correlation coefficient increases with the value of cov2/cov1. Therefore, it is shown that although all ply elastic properties heavily depend on the FVR as shown in figures 2 and 3, a large CoV of FVR not necessarily results heavy correlation between ply elastic properties. If the CoV of FVR is much larger than that of other microscale parameters, i.e. the microscale uncertainty is dominated by the stochastic variation of FVR, significant correlation between ply elastic properties would be resulted. On the contrary, if the microscale uncertainty is dominated by the stochastic variation of other parameters such as $E_m$, small or even negligible statistical correlation between ply elastic properties is resulted. It is important to see that the correlation coefficient between $E_{22}$ and $G_{12}$ for all of the four scenarios is large. This is because of that $E_{22}$ and $G_{12}$ not only have great sensitivity to FVR but also $E_m$, and in any scenario the microscale uncertainty is dominated by the uncertainty of FVR or $E_m$. 


Figure 5. Linear correlation coefficient between $E_{11}$ and $E_{22}$ at different CoV of microscale parameters.

For the E-glass plain-weave composite, a comparison is conducted with two different combinations of values of CoV of FVR and other microscale parameters as shown in table 5, where cov2 represents the CoV of FVR and cov1 represents the CoV of other microscale parameters. From the scenario of cov1 = 0.05 & cov2 = 0.01, where the microscale uncertainty is not dominated by the uncertainty of FVR, a great linear correlation coefficient between $E$ and $G$ (0.69) is still observed, which is different from counterpart observation for unidirectional fibre reinforced composite. This is explained by the fact that $E$ and $G$ are not only sensitive to FVR but also to unit-cell geometrical parameters $s$ and $w$, as shown in figure 4. In the scenario of cov1 = 0.05 & cov2 = 0.01, uncertainty of $s$ and $w$ takes a primary part of the microscale uncertainty, therefore a large linear correlation coefficient between $E$ and $G$ is still resulted. This phenomenon is similar to the large correlation between $E_{22}$ and $G_{12}$ for the unidirectional CFRP and GFRP as discussed above. Hence, it can be deduced that if two (or more) ply elastic properties both greatly depend a certain microscale parameter, and the uncertainty of this microscale parameter takes a primary part of microscale uncertainty, there will be notable or significant statistical correlation between the two (or more) ply elastic properties.

Table 5. Linear correlation coefficients of elastic properties of E-glass plain-weave composite.

| cov1 = 0.05 | cov2 = 0.05 | cov1 = 0.05 | cov2 = 0.01 |
|-------------|-------------|-------------|-------------|
| $E$ | 1 | 1 | $E$ |
| $E$ | 0.9 | 0.69 | 1 |
| $G$ | 0.50 | 1 | 0.33 |
| $G$ | 0.25 | 0.33 |
| $v$ | -0.11 | 1 |

5. Conclusions

Recent studies on the reliability analysis of fibre reinforced composite structures have realized the existence of statistical correlation between ply (or lamina) mechanical properties and its large influence on structure reliability. This study probably provides a first detailed investigation on the dependency of the statistical correlation (to the authors’ knowledge). The sensitivity of ply elastic properties is analyzed in a stochastic approach, and correlation coefficients between ply elastic properties are compared at different scenarios of microscale uncertainty. Several conclusions can be drawn as follows:

1) Stochastic variation of all ply in-plane elastic properties shows great sensitivity to the stochastic variation of FVR, despite of unidirectional or plain-weave fibre reinforced composite. Stochastic variation of ply in-plane elastic properties of plain-weave composite also shows great sensitivity to randomness of unit-cell geometry.

2) If the microscale uncertainty is dominated by the uncertainty of FVR, correlation between all ply elastic properties would be great. On the contrary, correlation between ply elastic properties would
be little or even negligible, except that certain ply elastic properties both heavily depend on other microscale parameters possessing large uncertainty.

(3) More generally, if two (or more) ply elastic properties both greatly depends a certain microscale parameter, and the uncertainty of this microscale parameter is much larger than other microscale parameters, there will be great statistical correlation between the two (or more) ply elastic properties.

Recognizing the significance level of statistical correlation between ply mechanical properties is important for accurate reliability evaluation, simplified reliability calculation process and related experimental verification on composite structures. For example, if it has been known that there is little statistical correlation between $E_{11}$ and $E_{22}$, hence the microscale uncertainty could not be dominated by the uncertainty of FVR, and consequently the statistical correlation between $E_{11}$ and $G_{12}$ (or $E_{11}$ and $v_{12}$, $E_{22}$ and $v_{12}$, $G_{12}$ and $v_{12}$) should also be insignificant. This helps on simplifying structure reliability calculation by neglecting the statistical correlation and also gets rid of requirement for costively experimental assessment on the statistical correlation. The methodology and conclusions presented in this study would also provide helpful reference on statistical correlation between mechanical properties of short fibre or particle reinforced composite.

References

[1] Mesogitis T S, Skordas A A and Long A C 2014 Uncertainty in the manufacturing of fibrous thermosetting composites: A review Composites Part A: Applied Science and Manufacturing 57 67-75

[2] Sanei S H R and Fertig R S 2016 Length-scale dependence of variability in epoxy modulus extracted from composite prepreg Polym Test 50 297-300

[3] Sriramula S and Chryssanthopoulos M K 2019 Quantification of uncertainty modelling in stochastic analysis of FRP composites Composites Part A: Applied Science and Manufacturing 40 1673-84

[4] Li Z, Shufeng Z, Yuanxiang J, Junyong T and Xun C 2016 Compressive behaviour of fibre reinforced plastic with random fibre packing and a region of fibre waviness Journal Reinforced Plastic Composite 36 323-37

[5] Chiachio M, Chiachio J and Rus G 2012 Reliability in composites - A selective review and survey of current development Compos Part B-Enging 43 902-13

[6] Chen N Z, Sun H H and Soares C G 2003 Reliability analysis of a ship hull in composite material Composite Structures 62 59-66

[7] Di Sciuva M and Lomario D 2003 A comparison between Monte Carlo and FORMS in calculating the reliability of a composite structure Composite Structures 59 155-62

[8] Frangopol DM and Recek S 2003 Reliability of fiber-reinforced composite laminate plates Probability Engineering Mechanics 18 119-37

[9] Lekou D J and Philippidis T P 2008 Mechanical property variability in FRP laminates and its effect on failure prediction Composites Part B: Engineering 39 1247-56

[10] Antonio C C and Hoffbauer L N 2009 An approach for reliability-based robust design optimisation of angle-ply composites Composite Structures 90 53-9

[11] Shaw A, Sriramula S, Gosling P D and Chryssanthopoulos M K 2010 A critical reliability evaluation of fibre reinforced composite materials based on probabilistic micro and macro-mechanical analysis Composites Part B: Engineering 41 446-53

[12] Mustafa G, Suleman A and Crawford C 2015 Probabilistic micromechanical analysis of composite material stiffness properties for a wind turbine blade Composite Structures 131 905-16

[13] Zhang S, Wang H, Zhang L and Chen X 2018 Statistical correlation between elastic properties of plain-weave composite and its influence on structure reliability Composite Structures 200 939-945
[14] Zhang S, Zhang C and Chen X 2015 Effect of statistical correlation between ply mechanical properties on reliability of fibre reinforced plastic composite structures *Journal of Composite Material* **49** 2935-45

[15] Zhang S, Zhang L, Wang Y, Tao J and Chen X 2016 Effect of ply level thickness uncertainty on reliability of laminated composite panels *Journal of fibre Reinforced Plastic Composite* **35** 1387-1400

[16] Calvário M, Teixeira A P and Soares C G 2018 Uncertainty propagation and sensitivity analysis of a laminated composite beam *4th International Conference on Maritime Technology and Engineering* 395-402

[17] Huang Z 2001 Micromechanical prediction of ultimate strength of transversely isotropic fibrous composites *International Journal of Solids Structures* **38** 4147-72

[18] Chamis C 1989 Mechanics of Composite Materials: Past, Present and Future NASA *Technical Memorandum* 100793

[19] Wang H and Wang Z 2016 Quantification of effects of stochastic feature parameters of yarn on elastic properties of plain-weave composite – Part 2: Statistical predictions vs. mechanical experiments *Composites Part A: Applied Science and Manufacturing* **84** 147-57