Quantum black holes in the horizon quantum mechanics model at the Large Hadron Collider

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Abstract

Quantum black hole production at the Large Hadron Collider is investigated using the horizon quantum mechanics model. This model has novel implications for how black holes might be observed in collider experiments. Black hole production is predicted to be possible below the Planck scale, thus leading to the intriguing possibility that black holes could be produced even if the Planck scale is slightly above the collider centre of mass energy. In addition, the usual anticipated resonance in the black hole mass distribution is significantly widened in this model. For values of the Planck scale above the current lower limits, the shape of the black hole mass distribution is almost independent of the Planck scale and depends more on the number of extra dimensions. These model features suggest the need for alternative search strategies in collider experiments.

1 Introduction

Low-scale gravity provides an interesting possibility for gaining insight into the hierarchy problem. A wide variety of models based on different paradigms \cite{1, 2, 3} have been proposed. A speculative, but intriguing, possibility of most models is the production of quantum black holes in hadron colliders \cite{4, 5}.

The cross section for black hole production is typically chosen to be the classical geometric form $\hat{\sigma} \approx \pi r_g^2$, where $r_g$ is the gravitational radius which is a function of the black hole mass $M$ and depends on the fundamental parameters of the model. In the large extra dimensions paradigm proposed in Ref. \cite{1, 2}, the model parameters are the higher-dimensional Planck scale $M_D$ and total number of space-time dimensions $D$. We will consider the case of a tensionless non-rotating spherically symmetric solution for the gravitational radius \cite{6}.

In proton–proton collisions, only a fraction of the total centre of mass energy $\sqrt{s}$ is available in the hard-scatter process. We define $s x_a x_b \equiv s \tau \equiv \hat{s}$, where $x_a$ and $x_b$ are the fractional energies of the two colliding partons (assumed massless) relative to the proton energies. The full particle-level cross section $\sigma$ is obtained from the parton-level cross section $\hat{\sigma}$ by using \cite{7}

$$\sigma_{pp \to BH+X}(s) = \sum_{a,b} \int_{M_D^2/s}^1 d\tau \int_1^1 \frac{dx}{x} f_a \left( \frac{\tau}{x} \right) f_b(x) \Theta(M-M_{th}) \hat{\sigma}_{ab \to BH}(\hat{s} = M^2), \quad (1)$$

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where \(a\) and \(b\) are the parton types in the two protons, and \(f_a\) and \(f_b\) are parton distribution functions (PDFs) for the proton. The sum is over all possible quark and gluon pairings. While several pre-factors to the cross section have been suggested (see Ref. [7] for a summary) they are not important for this study and will not be considered.

The usual ansatz is that black holes can not be produced with \(M\) below some minimum mass threshold \(M_{\text{th}}\). This is emphasized by the use of the Heaviside step function \(\Theta\) in Eq. (1). The value of \(M_{\text{th}}\) is typically taken to be \(M_D\) for quantum black holes or a few times \(M_D\) for classical black holes. Unfortunately, results depend on the subjective choice of the \(M_{\text{th}}\) cutoff.

A modification to the typical model of black hole formation in hadron colliders is made by the horizon quantum mechanics (HQM) model [8, 9]. The HQM model makes a modification to black hole production by treating the source of the black hole and its horizon as individual quantum objects with their own wave functions. This serves to make the location of both the source and horizon fuzzy. The system exhibits properties of a black hole when the source is located within the quantized horizon, with the probability of the system being a black hole given by

\[
P_{\text{BH}} = \int_0^\infty P_S(r < r_H)P_H(r_H)dr_H,
\]

where \(P_H(r_H)\) is the probability density of horizon radii \(r_H\) and \(P_S(r < r_H)\) gives the probability that the source is within \(r_H\). Explicit expressions of these probabilities are giving in Ref. [8, 9]. Qualitatively, the use of the HQM probability in the calculation of the proton–proton cross section is akin to replacing the step function located at \(M_{\text{th}}\) with a sigmoid-like function that varies with \(M/M_D\) and depends on \(D\).

The purpose of the work presented here is to evaluate the impact of the HQM model on the production of quantum black holes with emphasis on the signatures for experiments at the Large Hadron Collider (LHC). We begin with a brief description of Monte Carlo (MC) event generation in the HQM model, with more details of the implementation described in Appendix A. Possible reasons for differences between our results and a previously published result [10] can be found in Appendix B. We discuss the effects of HQM on the total proton–proton cross section and the differential proton–proton cross section as a function \(M\). The possibility of quantum black hole detection in the HQM model in LHC experiments is discussed.

We make use of the following conventions. When comparing models, the QBH model refers to the quantum black hole model with Heaviside step function turn-on typically used by ATLAS and CMS searches at \(\sqrt{s}\) of 7 TeV [11, 12, 13], 8 TeV [14, 15, 16, 17, 18, 19], and 13 TeV [20, 21, 22, 23, 24, 25, 26, 27] that does not include any HQM effects. The HQM model will be the model with horizon quantum mechanics effects included. The only difference between these two models is their production turn-on behaviour in \(M/M_D\) for different \(D\). The total number of space-time dimensions \(D = n + 4\), where \(n\) is the number of extra dimensions, unlike in Ref. [10], where \(D = n + 3\) only represents the total number of spacial dimensions.

2 Black hole production probability

For the purpose of cross section calculations along with event generation, the QBH 3.00 MC quantum black hole event generator\(^1\) is used [28]. In this model [29, 30, 31], we consider tensionless non-rotating black holes. HQM effects are added to the proton–proton cross section by including the factor \(P_{\text{BH}}\) of Eq. (2) into Eq. (1):

\(^1\)We use QBH to refer to the quantum black hole model and QBH to refer to the quantum black hole generator of the same name.
Figure 1: Horizon quantum mechanics (HQM) probability curves $P_{BH}$ versus black hole mass $M$ relative to the Planck scale $M_D$ for selected total number of space-time dimensions $D$. The dashed black line represents the step function used in quantum black hole (QBH) models.

\[
\sigma_{pp\rightarrow BH+X}(s) = \sum_{a,b} \int_{M^2/s}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x} f_a \left( \frac{\tau}{x} \right) f_b(x) P_{BH}(M) \hat{\sigma}_{ab\rightarrow BH}(\hat{s} = M^2),
\]

where $P_{BH}$ requires another numerical integration. The cross section formula is now independent of $M_{th}$ and the model has one less free parameter.

In order to visualise how the HQM probability varies with $M$, $M_D$, and $D$, we have computed the integral in Eq. (2) explicitly, as shown in Fig. 1.

The probability curves suggest some interesting phenomena that are not seen in the QBH model. First, instead of a step function at $M = M_D$, the new curves are smooth. The most notable consequence is that there is a finite probability that a black hole can be formed with $M < M_D$. Second, we see that the probability for a black hole to be produced near $M_D$ is suppressed for high $D$. In other words, one generally expects more black holes to be produced for low $D$. This is at odds with the usual effect of dimensionality in the QBH model, where greater $D$ corresponds to a greater geometric cross section. A third observation is that most of the curve is significantly above the value of $M/M_D = 1$. And lastly, the slope in the curves at $P_{BH} = 0.5$ are not particularly steep.

We can roughly quantify the extent to which the $P_{BH}$ curves create a threshold in the $M$ distribution by considering the midpoint of each curve as the point where $P_{BH} = 0.5$. These values are shown in Table 1. For $D = 6$, the black hole mass threshold rises to slightly above the usual $M_D$ threshold in the QBH model. For $D = 10$, the threshold is more than twice $M_D$. 

Table 1
This means that more dimensions will cause heavy suppression of black hole production in the HQM model, unlike the QBH model in which more black holes will be produced at higher $D$.

Table 1: Ratio of black hole mass $M$ to Planck scale $M_D$ at $P_{BH} = 0.5$ for different total number of space-time dimensions $D$ in the horizon quantum mechanics model.

| $D$ | 6   | 7   | 8   | 9   | 10  | 11  |
|-----|-----|-----|-----|-----|-----|-----|
| $M/M_D$ | 1.38 | 1.61 | 1.80 | 1.97 | 2.11 | 2.25 |

3 Proton–proton total cross section

We start by analyzing how the inclusion of HQM impacts the proton–proton total cross section as a function of $M_D$ and $D$. There are two competing factors at play. On one hand, we are multiplying the parton-level cross section by a factor between 0 and 1, which in general decreases the cross section. On the other hand, we are considering a wider range of possible $M$ than in the QBH model. In addition, while it is unreasonable to think of producing events in the QBH model if $M_D > \sqrt{s}$, the smooth cutoff imposed by HQM allows for black holes when $M_D$ is above the collider energy. The phenomena are shown in Fig. 2.

The inclusion of HQM suppresses the total cross section for low $M_D$ but predicts a higher
cross section than the QBH model at high $M_D$. It is also interesting to note how the role of dimensionality is reversed in the two models. For a given $M_D$, higher cross sections occur at lower $D$ in the HQM model, except for a small region below about 2 TeV. Also, in the HQM model the cross section at a given $M_D$ is significantly different for different $D$ as $M_D$ increase. Thus over most of the $M_D$ range, dimensionality is significantly more important in the HQM model.

It is also useful to determine the $M_D$ value at which the HQM model cross section crosses over the QBH model cross section, and thus where the HQM model might become more significant. For $D = 6$, $D = 8$, and $D = 10$, the crossovers in $M_D$ occur at approximately 5.4 TeV, 8.2 TeV, and 9.7 TeV, respectively. To understand which region of $M_D$ is interesting, we consider the current lower-limits, at the 95% confidence level, on $M_D$ of 9.9 TeV, 6.3 TeV, and 5.3 TeV for $D = 6$, $D = 8$, and $D = 10$, respectively, set by the CMS [32] and ATLAS [33] experiments. At these $M_D$ limits, black hole production in the HQM model is still well below the QBH model except for $D = 6$ where the HQM model predicts a cross section of about three orders of magnitude higher than the QBH model.

The most glaring difference between models occur above the $M_D$ lower limits. While the QBH model cross sections falls sharply as $M_{th} = M_D$ is pushed toward $\sqrt{s}$, the HQM model cross sections exhibit a more gradual drop that becomes less steep at higher $M_D$. This results in some notable properties unique to the HQM model. First, black holes may be produced even if $M_D > \sqrt{s}$. Second, since the cross sections do not converge to zero at $M_D = \sqrt{s}$, dimensionality plays a greater role at high $M_D$.

Of particular importance for observing quantum black holes in experiments is the number of black hole events we are able to produce. Typically, a minimum of ten signal events is sought to form a reasonable claim of discovery. In Fig. 3 we plot the luminosity required to produce ten events in proton–proton collisions at $\sqrt{s} = 13$ TeV. Analysis performed by ATLAS and CMS using the full run-2 dataset typically quote a luminosity of about 139 fb$^{-1}$. Using this luminosity, more than ten events can be produced in the QBH model for $M_D$ less than about 8.7 TeV, 9.2 TeV, and 9.5 TeV for $D = 6$, $D = 8$, and $D = 10$, respectively. The lower limits on $M_D$ would exclude $D = 6$ black holes in the QBH model. The current best lower limit from a direct QBH search is $M_{th} = M_D > 9.4$ TeV for $D = 10$ [27]. Even with a luminosity of 1 ab at $\sqrt{s} = 13$ TeV, the limit on $M_{th}$ is unlikely to go above about 10.5 TeV. Thus, the QBH model is being significantly restricted even at current luminosities.

The LHC is able to produce black holes at much higher values of $M_D$ in the HQM model for most $D$. At a current luminosity of 139 fb$^{-1}$, values of $M_D$ in the HQM model are not constrained by the lower limits on $M_D$, and quantum black holes could exist in the LHC experiment’s current datasets. However, as we will see next it will be non-trivial to detect HQM black holes in current ATLAS and CMS datasets even if produced.

### 4 Proton–proton differential cross section

The inclusion of HQM in quantum black hole production has notable implications on the $M$ distribution of black holes. Since the cross sections of QBH and HQM models typically differ

\footnote{At this point, we are assuming a perfect search for black holes.}
Figure 3: Luminosity required to produce ten black hole events as a function of Planck scale $M_D$ at a centre of mass energy of 13 TeV. Curves for different models and total number of space-time dimensions $D$ are shown. Solid curves are used for the horizon quantum mechanics (HQM) model and dashed curves are used for the quantum black hole (QBH) model. The horizontal dotted line represents a luminosity of 139 fb$^{-1}$.

by over an order of magnitude (except at very low $M_D$ and near the crossing), it is illustrative to compare the normalized shapes of distributions for $M_D$ of interest. Figure 4 compares $M$ distributions for four selected values of $M_D$ and $D = 10$.

For a small $M_D$, the HQM model gives the peak structure of the QBH model, but this changes for higher $M_D$, and $M$ is distributed over a wide range: $2 \lesssim M \lesssim 10$ TeV. This difference in shape is a direct consequence of the shapes of the PDFs and the $P_{BH}$ curve from HQM. The PDFs fall rapidly as parton energies approach $\sqrt{s}/2$. For $M_D = 12$ TeV in the QBH model, a very small cross section is expected since $M$ is limited to the range $12 < M < 13$ TeV. In the $M_D = 12$ TeV HQM model, the lower mass for black holes is dictated by the $P_{BH}$ curve. Black hole masses below 2 TeV are suppressed since $P_{BH} \approx 0$, and likewise black holes with mass above about 10 TeV are suppressed by the PDFs. This interplay in the HQM model between the convolution of PDFs and $P_{BH}$ gives rise to the shape of the $M$ distributions.

The peak in the QBH $M$ distribution moves up with increasing $M_D$ since the model’s definition of $M_{th}$ is a strict cutoff in $M$. In contrast, the HQM model $M$ distribution does not appear to shift up much above $M_D \gtrsim 7$ TeV. This phenomena is explored further in Fig. 5. While the QBH model $M$ distribution moves up with increasing $M_D$ acting as a minimum mass threshold, the HQM model $M$ distributions are much more spread out and the shape of the distributions do not change significantly once $M_D$ exceeds a few TeV. We also observe that in the HQM model it is very difficult to produce black hole masses above $\sim 11$ TeV, even though $M_D$ is not limited.
Figure 4: Quantum black hole (QBH) model and horizon quantum mechanics (HQM) model mass $M$ distributions normalized to unity for a) $M_D = 1$ TeV, b) $M_D = 4$ TeV, c) $M_D = 8$ TeV, and d) $M_D = 12$ TeV. The centre of mass energy is 13 TeV and $D = 10$.

The progression of black hole $M$ distributions with $M_D$ in both models is shown in Fig. 6 which plots the mean $M$ as a function of $M_D$. The QBH model curve gives exactly what is expected, since most black holes are produced with mass $M_D$, a linear increase in the mean $M$ is observed for all $D$. This is in contrast to the HQM model which resembles a linear increase only for small $M_D$ and then levels off at a constant mean $M$ for $M_D \gtrsim 8$ TeV. The value of the mean $M$ to which the trend converges is dependent on $D$. The reason for this is an interplay between the $P_{BH}$ curves which approach zero as $M$ approaches zero and the PDFs which approach zero as $M$ approaches $\sqrt{s}$. The consequence is a “pinching off” that serves to create a mass distribution that does not change shape significantly between the two mass regions where the production of black holes is vanishingly small. The mean $M$ increases with $D$ due to the $P_{BH}$ curve being shifting higher in $M/M_D$ with increasing $D$, as previously shown in Fig. 1.

Finally, the shape of the HQM model $M$ distribution has implications on how black holes in this model may be detected in the ATLAS and CMS experiments. In the QBH model, black holes are expected to predominantly decay into two-body final states. The majority of these decay products would be quarks and gluons that would hadronize to produce jets. For
Figure 5: Quantum black hole (QBH) model a) and horizon quantum mechanics (HQM) model b) proton–proton differential cross sections $d\sigma/dM$ versus black hole mass $M$ for selected values of the Planck scale $M_D$. The centre of mass energy is 13 TeV and total number of space-time dimensions $D = 10$. 
Figure 6: Mean mass of black hole events as a function of Planck scale $M_D$ at a centre of mass energy of 13 TeV. Quantum black hole (QBH) and the horizon quantum mechanics (HQM) models are shown for two values of total number of space-time dimensions $D$.

This reason, ATLAS and CMS have searched for resonances in the mass distribution of dijet events. To investigate how black holes in the HQM model would appear in these searches, we use 139 fb$^{-1}$ of ATLAS data recorded during run-2 at $\sqrt{s} = 13$ TeV \cite{27}. Quantum black hole events are simulated using the same selection criteria, at the particle level, as in the ATLAS analysis. We understand that particle-level selection will only roughly emulate the geometrical acceptance of events in the ATLAS detector, but the signal yields should be indicative of a full experimental analysis\cite{27}.

Figure 7a) shows an example QBH resonance for $M_D = 9.5$ TeV and $D = 10$. This resonance is beyond the highest dijet mass event obtained by ATLAS. In addition, the decisive lack of such a resonance structure in the dijet mass spectrum has allowed ATLAS to limit black holes in the QBH model to $M_{th} > 9.4$ TeV for $D = 10$ at the 95% confidence level \cite{27}. Thus, the QBH model in its simplest form is close to being ruled out.

For the HQM model, dijet distributions are shown in Fig. 7b), Fig. 7c), and Fig. 7d) for $(M_D = 8$ TeV, $D = 10)$, $(M_D = 8.5$ TeV, $D = 8)$, and $(M_D = 9$ TeV, $D = 6)$, respectively. Although ATLAS and CMS have not set limits on the HQM model they have eliminated a wide variety of resonances in the dijet mass spectrum from trigger turn on to about 8 TeV. Thus HQM black hole production resulting in sizable deviations from the smoothly falling dijet mass distribution are not allowed. The values of $M_D$ in the figures have been chosen high enough

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\footnote{The data is taken from the HEpdata repository \url{https://www.hepdata.net/record/ins1759712}}

\footnote{The quantum black holes we consider are only effected by the rapidity requirements; the simulated events pass the transverse momentum, invariant mass, and azimuthal angle requirements.}
Figure 7: Black hole dijet mass distributions superimposed on the ATLAS dijet mass $m_{jj}$ spectrum measured at a centre of mass energy of 13 TeV and a luminosity of 139 fb$^{-1}$ [27]. The results are shown for the quantum black hole (QBH) model and horizon quantum mechanics (HQM) model for various values of the Planck scale $M_D$ and the total number of extra space-time dimensions $D$.

to not result in a clear enhancement in the dijet mass distribution that ATLAS and CMS have not seen. On the other hand, if the $M_D$ values are chosen higher the number of events becomes insignificant for masses above the ATLAS and CMS data points. It would thus be extremely difficult to observe black holes in the HQM model in the current dijet invariant mass spectrum.

5 Discovery potential in the dijet mass distribution

In order to predict the discovery potential for observing quantum black holes, we take into consideration both the number of events above background and the significance of the signal. For the significance, we use the gaussian approximation without uncertainty:

$$\text{significance} = \frac{\text{signal}}{\sqrt{\text{background}}} \sigma,$$

where “signal” is the number of signal events above background and “background” is the number
of background events excluding signal events. We understand that this formula will break
down with small number of events, and that we should really include background uncertainties.
However, such an analysis is beyond the scope of this work, and is unlikely to change the
qualitative findings.

We consider a significant observation to be greater than $5\sigma$. Using a cut-and-count method,
significance is calculated by counting events above $M_{4\ell}$. While this is natural for the QBH model,
it is perhaps not so meaningful for the HQM model since many of the events have $M < M_D$.
For the sake of comparison, we consider two approaches to calculating the significance for the
HQM model. The first is the usual definition, where we consider $M_D$ as a cutoff. In this method
$M_D$ values beyond $\sqrt{s}$ can not be probed. In the second method, we consider all black hole
events and count the background from the least massive signal event. We understand that the
latter method would be extremely difficult, and probably not even desirable, to realize in an
experiment’s analysis, but it might be more indicative of a shape-fit procedure that might likely
be used.

The event count and significance are presented in Fig. 8 and Fig. 9 respectively. While
counting HQM model events over the entire mass range gives the greater number of events, the
method of counting HQM model events only above $M_D$ give better significances. This could
have been anticipated given the large number of background events at low dijet masses. Using
either approach to calculating the significance, the discovery potential at allowed values $M_D$
is less for the HQM model than the QBH model. Since the ATLAS background that we are
using does not extend beyond 8.1 TeV, and because of simple significance formula Eq. (4), the
significance curves in Fig. 9 end at $M_D = 8$ TeV.

Using the $M > M_D$ counting method and by noting the minimum $M_D$ value given by the
ten event and $5\sigma$ criteria, we assess the possibility of detecting HQM black holes in ATLAS
and CMS. For $D = 10$, the number of signal events is greater than ten for $M_D \lesssim 7.5$ TeV.
The corresponding significance is greater than $5\sigma$ for $M_D \lesssim 7.4$ TeV, and this sets the upper
limit on $M_D$ to observe black holes in the HQM model. For the $D = 6$ case, greater than ten
events occurs when $M_D \lesssim 8.0$ TeV and the significance is greater than $5\sigma$ at $M_D \lesssim 8.0$ TeV.
However, with only one background event, the significance as defined in Eq. (4) does not have
have meaning. In any case, the lower limit on $M_D$ from the CMS experiment [32] for $D = 6$ is
9.9 TeV at the 95% confidence level, thus eliminating the HQM model for $D = 6$.

Given the increase in luminosity and $\sqrt{s}$ in subsequent LHC runs, these discovery potentials
stand to increase somewhat. With this thought in mind, we make some predictions at $\sqrt{s} =
13$ TeV on the luminosity required at a given $M_D$ for a meaningful discovery. We assume that the
number of background events, based on the data from Ref. 27, scales linearly with luminosity.
When calculating the significance using $M > M_D$ as a cutoff in the cut-and-count method, we
have made the additional assumption that event-count is the limiting factor for $M_D > 8$ TeV as
this is the highest dijet mass at which the ATLAS background estimate is given. The results are
shown in Fig. 10 where we only consider luminosities above 139 fb$^{-1}$. It is seen that the increase
in probing $M_D$ with a reasonable increase in luminosity is not very significant, indicating that
we are close to exhausting the search for black holes in both QBH and HQM models using the
dijet mass distribution at $\sqrt{s} = 13$ TeV. Although we have used a very simplistic approach
to estimating the discovery potential, this conclusion is unlikely to change with a more robust
estimate.

6 Angular search

Given the small potential for observation of HQM black holes in the dijet mass distribution,
a discovery in the invariant mass variable is unlikely. Alternatively, an angular search may be
Figure 8: Predicted number of black hole events versus Planck scale $M_D$ for a centre of mass energy of 13 TeV and luminosity of 139 fb$^{-1}$ when selecting events at the parton level according to the same criteria as the search in Ref. [27]. The solid curves are for total space-time dimension $D = 10$ and the dashed curves for $D = 6$.

Figure 9: Significance of a black hole observation above dijet background versus Planck scale $M_D$ for a centre of mass energy of 13 TeV and luminosity of 139 fb$^{-1}$. Event were selected at the parton level according to the same criteria as the search in Ref. [27]. The solid curves are for total space-time dimension $D = 10$ and the dashed curves for $D = 6$. 
performed to distinguish an enhancement of events due to black hole production above QCD background \[11, 12, 20, 22, 23\]. The HQM model does not yet predict any modification to the usual decays in the QBH model; there is no difference between the two models in terms of the shape of angular distributions.

An example angular search could be in the variable $\chi$ defined as

$$\chi = e^{\left|y_1 - y_2\right|},$$  \hspace{1cm} (5)

where $y_1$ and $y_2$ are the rapidities of the two jets. QCD $t$-channel scattering constituting the background is approximately constant in $\chi$, while $s$-channel resonances tend to be enhanced at low $\chi$. Because of this, an angular search could help uncover the wide $s$-channel mass enhancement that is predicted by HQM. Since the predictions of an angular search are highly dependent on the analysis and detector details, we leave it to the ATLAS and CMS collaborations to perform such a search.

7 Conclusions

Microscopic black hole formation as predicted by HQM was implemented in the QBH MC event generator to investigate the impact on possible black hole production at the LHC. The inclusion of the HQM model serves to decrease the total black hole cross section for small $M_D$, but the new model is not restricted by a threshold mass requirement. Therefore, HQM predicts black holes may be produced at $M_D \sim \sqrt{s}$. The HQM model is also highly dependent on dimensionality and predicts that a greater number of events may be produced with a smaller $D$. The $M$ distribution is also greatly affected by HQM with a much wider spread of black hole masses. In other words,
there is no resonance structure in the HQM model. This wide $M$ distribution converges to a constant shape for large $M_D$, which can be considered to be one of the defining features of the HQM model.

The predicted signal in the dijet mass distribution along with ATLAS run-2 data were used to estimate the number of signal events and significance. Observations of quantum black holes governed by HQM were predicted to be limited to $M_D \lesssim 8.0$ TeV for $D = 6$ and $M_D \lesssim 7.2$ TeV for $D = 10$. Most likely the mass spread of HQM black holes is too large to allow observation using a resonance search alone, and an angular search is an exciting possibility.

Some of the above results were first mentioned in Ref. [10]. Unfortunately, that paper could only make use of ATLAS and CMS results at a centre of mass energy of 8 TeV. We view our analysis as more comprehensive, close to experiment, and up to date.

Lastly, although the HQM model has been used, we do not believe the results presented here depend specifically on the formula presented in Ref. [9]; similar results would be obtained for any non-step-like threshold mass production of black holes.

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**A Monte Carlo event generation**

In order to visualise how the HQM probability $P_{BH}$ varies with $D$, $M_D$, and $M$, we computed the integral in Eq. (2) explicitly using numerical integration. As shown in Fig. 1, good accuracy was achieved with the use of Simpson’s method and an adequate large number of subdivisions.

A more elegant means of producing the appropriate $P_{BH}$ factor can be performed by MC integration. As a check, we have also produced the curves in Fig. 1 using MC sampling. By integrating and inverting the $P_H$ distribution, random values of the horizon radius $r_H$ can be sampled using a uniform distribution of random numbers. Since $P_H$ is a probability density function, using random $r_H$ values to calculate $P_S$ for a large number of samples effectively computes the expected value for $P_S$ (or equivalently, $P_{BH}$). For completeness, we present this calculation.

We begin from Eq 3.7 in Ref. [9]:

$$P_H(r_H) = a^\frac{d}{2} \frac{2(d-2)}{\Gamma(s,1)} \Theta(r_H - R_d) \exp \left( -a^2 r_H^{2(d-2)} \right) r_H^{d-1},$$

where in this appendix we use the notion of Ref. [9] except we take the total number of spatial dimensions to be $d$. We have define $a = (d-2)/(2m)$ and $s = d/[2(d-2)]$, and used $\Delta = m$ as in Ref [9]; $\Gamma(s,x) = \int_x^\infty t^{s-1} e^{-t} dt$ is the upper incomplete Gamma function. In addition, we are using Planck units since we are only interested in lengths and masses relative to $M_D$.

By taking the Heaviside step function to be one, the indefinite integral can be computed:

$$\int P_H(r_H) dr_H = -\frac{\Gamma \left( s, a^2 r_H^{2(d-2)} \right)}{\Gamma(s,1)}.$$

Substituting a lower limit of $R_d = [2m/(d-2)]^{1/(d-2)}$ and upper limit of $r_H$, to allow calculation of the cumulative distribution function, gives
\[ \text{CDF}[P_H(r_H)] = 1 - \frac{\Gamma(s, a r_H^{2(d-2)})}{\Gamma(s, 1)}. \] (8)

If we generate a uniform random real number \( u \) in the interval \((0, 1) \) or \( 1 - u \) in the interval \((1, 0) \) and set it equal to Eq. (8), we can solve for \( r_H \) by inverting the incomplete Gamma function with respect to its second parameter:

\[ r_H = R_d \left( Q^{-1}(s, Q(s, 1) u) \right)^{\frac{1}{2(d-2)}}. \] (9)

Note that \( Q^{-1}(s, Q(s, x)) = x \), where \( Q^{-1} \) is the inverse of the regularized upper incomplete Gamma function \( Q(s, x) = \Gamma(s, x)/\Gamma(s) \). There are numerical methods to optimise this inversion.

Upon randomly sampling the horizon radii from Eq. (9), we return values of \( P_S(r < r_H) \) as given by Eq. 3.5 in Ref. [9]:

\[ P_S(r < r_H) = \frac{\gamma\left(\frac{d}{2}, m^2 r_H^2\right)}{\Gamma\left(\frac{d}{2}\right)}, \] (10)

where \( \gamma(s, x) = \int_0^x t^{s-1}e^{-t}dt \) is the lower incomplete Gamma function.

The above random horizon generation can simply be looped over with an average of all \( P_S \) values giving an approximate value for \( P_{BH} \). We easily recreate the same probability curves as in Fig. 1 which used Simpson’s method.

Both the MC method and Simpson’s method for calculating \( P_{BH} \) have been implemented in QBH. Despite both methods producing the same results, there are technical pros and cons of each method. The MC HQM calculation just presented is the default method.

One additional technicality should be mentioned. Since black hole production in the HQM model allows for \( M \) less than \( M_D \) there is no lower-mass cutoff in the generator. Instead, the \( P_{BH} \) curve imposes its own smooth limit as it becomes arbitrarily small. To sample \( M \) via a power transformation of the cross section used to increase efficiency, we choose an arbitrary minimum of 100 GeV since in practise it is exceedingly rare to generate an event with \( M \) this low. For example, selecting a 200 GeV minimum has a negligible impact on the results.

We point out that our curves of \( P_{BH} \) are identical to the corresponding figure in Ref. [10] within our ability to read values from their figure. Equation (7) in Ref. [10] disagrees with Eq. (3.8) Ref. [9], although the later cites the former. We believe Eq. (7) in Ref. [10] has the inverse power of \((m_d/m)\) and a normalization difference of \((D-2)^2\). If the formula in the paper was actually used to generate the plot, the curves continue to increase above one with increasing mass and do not represent probability distributions.

### B Comparison of QBH and BLACKMAX event generators

A first HQM study was made in Ref. [10] and the discrepancy between their results and those presented here should be addressed. In Ref. [10], the black hole event generator BLACKMAX [34] was used. It is designed using the same cross section formula as QBH, with some additional prefactors, so we expect approximate agreement between the two results. However, it is consistently found that cross sections reported in Ref. [10] are approximately an order of magnitude larger than we obtain with QBH.

We present two plausible reasons for the discrepancy. The first is the event generator parameters. Results were produced using BLACKMAX 2.02.0 in the “standard configuration”. We interpret this to mean the set of default parameters included in the BLACKMAX 2.02.0 tarball, changing only the options absolutely required for the simulation at hand (i.e. \( \sqrt{s}, M_D, \) and \( D \)).
Additionally, no PDF set was specified in Ref. [10]. While we used the CTEQ6L1 [35] PDF set in this analysis, a different PDF set could give different results.

An alternative explanation is that some parameters are misreported. BLACKMAX accepts a parameter for the number of extra dimensions $n$, which is equivalent to $D = 4 + n$ in our convention. Erroneously setting $n = D$ (or $n = d$) as input to BLACKMAX reproduces the results reported in Ref. [9] closer, which suggest the difference may be superficial.

As a sanity check, we embarked on a more complete comparison between BLACKMAX and QBH to ensure our results are consistent both with and without the HQM modifications. In summary, we can get good agreement between the two generators after accounting for small differences between models. This suggest the results in Ref. [10] are misreported, or possibly even erroneous.

First, the CTEQ6L1 PDF set was chosen for both QBH and BLACKMAX. Both generators can be linked to LHAPDF 6.2.1 [36] as version 6 contains support for the legacy interfaces used by BLACKMAX. We do not build BLACKMAX with PYTHIA as it is not needed for our comparisons. BLACKMAX can be built using a modern version of gcc (we used 5.4) provided the necessary Fortran libraries are included in the system. Some depreciated compiler flags may need to be removed depending on the particular compiler version. The parameters that need to be changed (in the parameter.txt file of the BLACKMAX distribution) in order to bring QBH and BLACKMAX into good agreement are as follows:

1. The Choose_a_case option should be set to 1 to consider tensionless non-rotating black holes.

2. The choose_a_pdf_file option should be set to 10042 to use the CTEQ6L1 PDF set.

3. The other_definition_of_cross_section option should be set to 2. By default BLACKMAX uses an angular momentum form factor which QBH does not.

4. The Mass_loss_factor, momentum_loss_factor, and Angular_momentum_loss_factor should be set to 0.0 in BLACKMAX since these options are not considered in QBH.

5. The QBH member function qbh->setQscale(false) should be called in the user-defined main.cc file. This uses the black hole mass as the QCD scale. This scale is hardcoded in BLACKMAX.

6. Comment out line 456 in the default BLACKMAX.c source file. This line modifies the differential cross section by accounting for the position of the black hole with respect to the extra dimensions. QBH does not include this feature.

7. To include HQM modifications in BLACKMAX, we read in the $P_{BH}$ values from a file. As BLACKMAX is written in C, we do not have access to the Boost gamma function library that we use in QBH for calculating $P_{BH}$. Since we know that both presented methods of including $P_{BH}$ in QBH are identical, we simply read the nearest-mass value from a pre-calculated table into BLACKMAX in order to modify the parton-level cross section. Reading in values is done at the beginning of the main function into a global array. The cross section is modified within the “cross section by Monte Carlo integrals” section of BLACKMAX.c. The variables of interest are: Mb the randomly generated black hole mass; Mpl the Planck scale; and fact the parton-level cross section factor. The modification is made after the fact=-2*u*Mb*A1*Lx0 line in the BLACKMAX source code.

With these modifications there is good agreement between the generators. The only major discrepancy is between the non-HQM BLACKMAX and QBH cross sections when $M_D \gtrsim 11$ TeV. Here, the QBH cross section is about 2% higher than the BLACKMAX value.
It is important to note that this difference is not seen in the HQM calculation, suggesting that it is likely a result of the rising minimum mass value. The precise reason for this remains unknown, but we will outline what has been tried. First, QBH and BLACKMAX use different power transforms when sampling masses. Manipulating the exact transform does not produce a large change in the calculation. Second, BLACKMAX and QBH include the PDF samples in the proton-level cross section in slightly different ways. The consequence of manipulating the methodology was also found to be negligible. The only facet of the calculation which was not subject to intense scrutiny was the process of sampling of the PDFs. While both generators use the same CTEQ6L1 PDFs with LHAPDF 6.2.1, QBH uses the LHAPDF 6 interface while BLACKMAX uses the legacy Fortran interface. Furthermore, the control flow for evaluation of the PDFs is significantly different. This suggests that the remaining discrepancy may be a consequence of this. If further tweaking of the generators is required, we suggest the investigation resumes here.

In summary, aside from a difference in cross sections at higher $M_D$ in the non-HQM case, there is good agreement between the generators. Importantly, this difference is only a matter of a small percent uncertainty as opposed to an order of magnitude disagreement seen in Ref. [10]. This gives us confidence in the implementation of our modified QBH generator.

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, The Hierarchy problem and new dimensions at a millimeter, Phys. Lett. B429 (1998) 263 [hep-ph/9803315].

[2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, New dimensions at a millimeter to a Fermi and superstrings at a TeV, Phys. Lett. B436 (1998) 257 [hep-ph/9804398].

[3] L. Randall and R. Sundrum, A Large mass hierarchy from a small extra dimension, Phys. Rev. Lett. 83 (1999) 3370 [hep-ph/9905221].

[4] S. B. Giddings and S. D. Thomas, High-energy colliders as black hole factories: The end of short distance physics, Phys. Rev. D65 (2002) 056010 [hep-ph/0106219].

[5] S. Dimopoulos and G. L. Landsberg, Black holes at the LHC, Phys. Rev. Lett. 87 (2001) 161602 [hep-ph/0106295].

[6] R. C. Myers and M. J. Perry, Black Holes in Higher Dimensional Space-Times, Ann. Phys. (N.Y.) 172 (1986) 304.

[7] D. M. Gingrich, Black hole cross-section at the Large Hadron Collider, Int. J. Mod. Phys. A21 (2006) 6653 [hep-ph/0609055].

[8] R. Casadio, Horizons and non-local time evolution of quantum mechanical systems, Eur. Phys. J. C75 (2015) 160 [1411.5848].

[9] R. Casadio, R. T. Cavalcanti, A. Giugno and J. Mureika, Horizon of quantum black holes in various dimensions, Phys. Lett. B760 (2016) 36 [1509.09317].

[10] N. Arsene, R. Casadio and O. Micu, Quantum production of black holes at colliders, Eur. Phys. J. C76 (2016) 384 [1606.07323].
[11] ATLAS collaboration, *Search for New Physics in Dijet Mass and Angular Distributions in pp Collisions at $\sqrt{s} = 7$ TeV Measured with the ATLAS Detector*, New J. Phys. 13 (2011) 053044 [1103.3864].

[12] ATLAS collaboration, *ATLAS search for new phenomena in dijet mass and angular distributions using pp collisions at $\sqrt{s} = 7$ TeV*, JHEP 01 (2013) 029 [1210.1718].

[13] CMS collaboration, *Search for Narrow Resonances and Quantum Black Holes in Inclusive and b-Tagged Dijet Mass Spectra from pp Collisions at $\sqrt{s} = 7$ TeV*, JHEP 01 (2013) 013 [1210.2387].

[14] ATLAS collaboration, *Search for new phenomena in photon+jet events collected in proton–proton collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, Phys. Lett. B728 (2014) 562 [1309.3230].

[15] ATLAS collaboration, *Search for Quantum Black Hole Production in High-Invariant-Mass Lepton+Jet Final States Using pp Collisions at $\sqrt{s} = 8$ TeV and the ATLAS Detector*, Phys. Rev. Lett. 112 (2014) 091804 [1311.2006].

[16] ATLAS collaboration, *Search for high-mass dilepton resonances in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, Phys. Rev. D90 (2014) 052005 [1405.4123].

[17] ATLAS collaboration, *Search for new phenomena in the dijet mass distribution using p–p collision data at $\sqrt{s} = 8$ TeV with the ATLAS detector*, Phys. Rev. D91 (2015) 052007 [1407.1376].

[18] CMS collaboration, *Search for resonances and quantum black holes using dijet mass spectra in proton-proton collisions at $\sqrt{s} = 8$ TeV*, Phys. Rev. D91 (2015) 052009 [1501.04198].

[19] CMS collaboration, *Search for lepton flavour violating decays of heavy resonances and quantum black holes to an eµ pair in proton-proton collisions at $\sqrt{s} = 8$ TeV*, Eur. Phys. J. C76 (2016) 317 [1604.05239].

[20] ATLAS collaboration, *Search for new phenomena in dijet mass and angular distributions from pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, Phys. Lett. B754 (2016) 302 [1512.01530].

[21] ATLAS collaboration, *Search for new phenomena in different-flavour high-mass dilepton final states in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, Eur. Phys. J. C76 (2016) 541 [1607.08079].

[22] ATLAS collaboration, *Search for new phenomena in dijet events using 37 fb$^{-1}$ of pp collision data collected at $\sqrt{s} = 13$ TeV with the ATLAS detector*, Phys. Rev. D96 (2017) 052004 [1703.09127].

[23] CMS collaboration, *Search for new physics with dijet angular distributions in proton-proton collisions at $\sqrt{s} = 13$ TeV*, JHEP 07 (2017) 013 [1703.09986].

[24] ATLAS collaboration, *Search for new phenomena in high-mass final states with a photon and a jet from pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, Eur. Phys. J. C 78 (2018) 102 [1709.10440].
[25] CMS collaboration, *Search for lepton-flavor violating decays of heavy resonances and quantum black holes to eµ final states in proton-proton collisions at √s = 13 TeV*, *JHEP* **04** (2018) 073 [1802.01122].

[26] ATLAS collaboration, *Search for lepton-flavor violation in different-flavor, high-mass final states in pp collisions at √s = 13 TeV with the ATLAS detector*, *Phys. Rev. D* **98** (2018) 092008 [1807.06573].

[27] ATLAS collaboration, *Search for new resonances in mass distributions of jet pairs using 139 fb−1 of pp collisions at √s = 13 TeV with the ATLAS detector*, *JHEP* **03** (2020) 145 [1910.08447].

[28] D. M. Gingrich, *Monte Carlo event generator for black hole production and decay in proton-proton collisions*, *Comput. Phys. Commun.* **181** (2010) 1917 [0911.5370].

[29] P. Meade and L. Randall, *Black Holes and Quantum Gravity at the LHC*, *JHEP* **05** (2008) 003 [0708.3017].

[30] X. Calmet, W. Gong and S. D. H. Hsu, *Colorful quantum black holes at the LHC*, *Phys. Lett. B668* (2008) 20 [0806.4605].

[31] D. M. Gingrich, *Quantum black holes with charge, colour, and spin at the LHC*, *J. Phys. G37* (2010) 105008 [0912.0826].

[32] CMS collaboration, *Search for new physics in final states with an energetic jet or a hadronically decaying W or Z boson and transverse momentum imbalance at √s = 13 TeV*, *Phys. Rev. D97* (2018) 092005 [1712.02345].

[33] ATLAS collaboration, *Search for dark matter and other new phenomena in events with an energetic jet and large missing transverse momentum using the ATLAS detector*, *JHEP* **01** (2018) 126 [1711.03301].

[34] D.-C. Dai, C. Issever, E. Rizvi, G. Starkman, D. Stojkovic and J. Tseng, *Manual of BlackMax. A black-hole event generator with rotation, recoil, split branes, and brane tension. Version 2.02*, *Comput. Phys. Commun.* **236** (2019) 285 [0902.3577].

[35] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky and W. K. Tung, *New generation of parton distributions with uncertainties from global QCD analysis*, *JHEP* **07** (2002) 012 [hep-ph/0201195].

[36] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht et al., *LHAPDF6: parton density access in the LHC precision era*, *Eur. Phys. J. C75* (2015) 132 [1412.7420].