Conformal Prediction Interval Estimations with an Application to Day-Ahead and Intraday Power Markets

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Abstract

We discuss a concept denoted as Conformal Prediction (CP) in this paper. While initially stemming from the world of machine learning, it was never applied or analyzed in the context of short-term electricity price forecasting. Therefore, we elaborate the aspects that render Conformal Prediction worthwhile to know and explain why its simple yet very efficient idea has worked in other fields of application and why its characteristics are promising for short-term power applications as well. We compare its performance with different state-of-the-art electricity price forecasting models such as quantile regression averaging (QRA) in an empirical out-of-sample study for three short-term electricity time series. We combine Conformal Prediction with various underlying point forecast models to demonstrate its versatility and behavior under changing conditions. Our findings suggest that Conformal Prediction yields sharp and reliable prediction intervals in short-term power markets. We further inspect the effect each of Conformal Prediction’s model components has and provide a path-based guideline on how to find the best CP model for each market.

Keywords: Energy forecasting, Prediction intervals, Electricity price forecasting, Probability forecasting, Quantile regression, Linear models

1. Introduction

Our society is full of forecasts, whether it is economic data, weather or customer demand. Unsurprisingly, this general statement equally counts for the energy industry. Amjady & Hemmati (2006) enlighten the demand for accurate price predictions from two perspectives. A transaction based explanation requires a) the exchange bidding to be precise in order to be executed and b) bilateral deals to be realistically priced. If we consider the role of market participants as the other angle, the necessity for reliable price opinions for a) producers to maximize their profit in power plant dispatch and b) consumers to hedge and minimize their price uncertainty becomes evident. Thus, forecasting electricity prices is a vivid field of research. Overviews on the status quo and available approaches are supplied by Aggarwal et al. (2009); Weron (2014). Whilst a broad variety of several point forecasts, i.e., the determination of a concrete numerical estimate for the price, is already available and being constantly improved by academics, the factor uncertainty is only gaining more attention lately. Inevitably, all forecasts imply uncertainty about their level of preciseness, so why stopping at the estimated price and not quantifying the unknown deviation that comes along with it? This is where prediction intervals (PI) come into play. Based on the idea of embracing uncertainty, a prediction interval tries to identify a bandwidth that will most likely cover the true value. Unfortunately, extensive studies of density or interval prediction are just getting more attention lately. The most prominent technique is quantile regression averaging (QRA) in Maciejowska & Nowotarski (2016); Maciejowska et al. (2016); Nowotarski & Weron (2014, 2015); ?. It showed convincing results in various applications and marks the current status quo for energy markets. Other models are given by bootstrapping (see a GARCH model in Khosravi et al. (2013)) or quantile regression as in Bunn et al. (2013). For a more detailed discussion on probabilistic fore-
casting, the interested reader might refer to a comprehensive study in Nowotarski & Weron (2017).

We aim at contributing to the research community in two ways: 1) we introduce a relatively unknown concept called Conformal Prediction with applications to day-ahead and intraday power prices. It is designated to predict intervals based on errors and new information, features weak assumptions on data characteristics and is versatile with regards to the underlying point prediction model. It might be seen as an expansion to an existing point prediction estimator. But how does Conformal Prediction perform under changing market conditions and in comparison to other approaches? Can the approach deal with alternating point forecasts and the specialties of hourly prices? To find an answer to these questions, the remainder of this paper is structured into four parts. To start with, we will thoroughly introduce the barely known concept of Conformal Prediction to the world of forecasters in section 2. Before turning from a general Conformal Prediction toy example towards a more dedicated electricity price scheme, we need to discuss the characteristics of electricity prices based on three selected markets in section 3. Once the theoretical foundation and time series description are dealt with, we turn our attention towards the detailed models. We will discuss the general model setup in 4.1, our point forecasts in 4.2 and close the model description by elaborating our PI estimators in 4.3. Section 5 provides the results of our empirical study based on several performance measures such as the Winkler Score or pinball loss. In that context, we modify a very basic model step by step until it equals Conformal Prediction so that we can assess which specific aspect causes the highest impact on performance. Finally, we conclude our findings in section 6 and critically assess potential improvements for further research.

2. The concept of Conformal Prediction

Conformal Prediction (CP)\(^1\) describes an entire framework thoroughly analyzed for the first time in Gammerman et al. (1998) or latter in Shafer & Vovk (2008) and Vovk et al. (2005) for both regression and classification problems. Conformal Prediction was initially introduced in an online or transductive manner, such that different data realizations are iteratively presented to the learning algorithm. This is not only computational costly but also less practice-oriented. Many real world applications require batch processing meaning that there is one learning set of historical observations and a function that tries to derive a generic rule applicable to new data. Inductive Conformal Prediction translates the transductive approach into a batch or inductive setting. Please note that we will refer to the batch case for regression problems when mentioning CP. But what renders CP special and why should forecasters know about it? We will firstly address its specifics:

- CP yields valid prediction intervals that meet the designated confidence level \(1 - \alpha\). The user predefines the desired confidence level.
- Only the weak assumption of exchangeability is made. The common assumption of i.i.d. residuals fulfills this postulation.
- CP can be coupled with every singular prediction model as it solely uses the final outcome of a classification or regression model.
- The framework itself offers high versatility with its applications in regression, classification or an online or batch setting. It post-processes point or classification model estimates and is independent from the underlying point forecast model characteristics and assumptions.
- Conformal Prediction computes symmetric prediction intervals while other approaches focus on quantiles in a separate manner.

\(^1\)If we think in a broader sense, Conformal Prediction describes an entire framework with different sub-models. For reasons of clarity we will denote our sub-models as ’Conformal Prediction’ as well. Hence, the framework and model specific definition are used analogously in this paper.

The most crucial aspect from above is the pre-processing characteristic. Alike an additional layer, CP adds an interval estimate to an existing point forecasting model. A core principle of this second layer is the existence of a non-conformity score \(\lambda\). It determines how uncommon an observation is in comparison to the real value. Suppose, we have (according to Johansson et al. (2014))

- A dataset \(\mathcal{Z}_h = \{(x_{1,h}, y_{1,h}), \ldots, (x_{L,h}, y_{L,h})\}\) that we randomly split into:
A random forecast model that exploits $Z_{\text{train}}$ for training and yields estimate $\hat{y}_i$. Please note that we train on $Z_{\text{train}}$ and supply it with the data of $Z_{\text{calib}}$ to obtain unbiased out-of-sample alike estimates $\hat{y}_{M+1,h}, \ldots, \hat{y}_{L,h}$.

- The most simple non-conformity score $\lambda_i = |y_{i,h} - \hat{y}_{i,h}|$, only applied on the estimates in $Z_{\text{calib}}$.

The random division into training and calibration is essential since we explicitly fit a model on $Z_{\text{train}}$ and exploit $Z_{\text{calib}}$ in an out-of-sample context. Please note that we intentionally use sampling here to increase generalization. The point forecast model is trained to minimize the error made for $Z_{\text{train}}$. Only considering $Z_{\text{train}}$ results in a construction of intervals on the basis of explicitly minimized errors and causes an unrealistic estimation for unknown data. It does not reflect the model behavior in an out-of-sample environment and could overfit the prediction interval. Whereas the determination of the non-conformity...
scores $\lambda_{M+1,h},...,\lambda_{L,h}$ is trivial, the absolute error does not work for $i = L + 1$ anymore since there is no observed value $y_{L+1,h}$ present. However, the CP application in classification problems allows for a derivation of a solution. Assume every possible label is known and given by an approximate target value $\tilde{y}_{L+1,h}$. The non-conformity measure $A_{L+1,h}^{i\tilde{y}}$ is computed for every possible value $\tilde{y}_h$ and then compared to all other scores of $Z_{\text{calib}}$ to determine its level of uniqueness in (taken from Johansson et al. (2014))

$$p(\tilde{y}_h) = \frac{\{i = M + 1,..,L : \lambda_i \geq A_{L,h}^{i\tilde{y}}\}}{|Z_{\text{calib}}| + 1}. \quad (1)$$

Common CP literature denotes the result of this comparison as the p-value of each value $\tilde{y}$ or in more intuitive words the share of non-conformity scores that are larger or equal to $A_{L+1,h}^{i\tilde{y}}$ (see for instance Vovk et al. (2005)). Based on the assumption that each possible value $\tilde{y}_{L+1,h}$ is known, all values under a significance level $\alpha$ are excluded. This leaves an interval of all possible realizations of the true value determined by

$$\tilde{y}_h : p(\tilde{y}_h) > \alpha. \quad (2)$$

Unfortunately, it is impossible to compute all possible values $\tilde{y}_{L+1,h}$ in the regression example at hand since $y \in \mathbb{R}$. As an alternative, $A_{L+1,h}^{i\tilde{y}}$ provides a probabilistic threshold so that the non-conformity score for the true value $y_{L+1,h}$ will not exceed $A_{L+1,h}^{i\tilde{y}}$ with confidence $1 - \alpha$. The threshold value $A_{L+1,h}^{i\tilde{y}}$ is identified by the equation (based on Johansson et al. (2014))

$$\frac{\{|i = M + 1,..,L : \lambda_i < A_{L,h}^{i\tilde{y}}\}| + 1}{|Z_{\text{calib}}| + 1} \geq 1 - \alpha. \quad (3)$$

A little toy example might be helpful in understanding this concept. For the sake of simplicity, we ignore hourly effects and set $i = 1,..,8$ where 8 is the instance were we only face the given variables $x_8$ and need to forecast an interval for $y_8$. Figure 1 presents a solution minding Eq. (3) and a more implementation-oriented introduction to Inductive Conformal Prediction as it is easily implemented in any programming language. We can sort all non-conformity scores in a descending order and then compute the $\alpha - \text{th}$ percentile of the given sample. The results (a value of 3 in our toy example) are equal in both calculations and form the interval around the point forecast in

$$\tilde{y}_{L+1,h} \pm A_{L+1,h}^{i\tilde{y}}. \quad (4)$$

This symmetric interval comprises the true price with confidence $1 - \alpha$ under exchangeability in the underlying dataset. For a more technical derivation of such validity the interested reader might study Vovk et al. (2005). Whereas other models fail to meet this requirement, CP leaves in theory no concern about validity but the range of the interval itself. It might yield broader intervals than other approaches if the underlying point forecast model is not precise or specific time-series characteristics are not regarded. But why do we think that CP is suitable for electricity price forecasting? Firstly, we will discuss our time series in scope and their characteristics and then present an adjusted CP scheme together with a set of point forecasts and other PI expert learners.

### 3. Data and case study framework

We examine datasets that comprise electricity spot prices from three different power markets: Nord Pool spot day-ahead (year 2012 - 2013) prices, the German EPEX intraday market (year 2013-2016) and the price track of the Global Energy Forecasting Competition 2014 (GEFCom 2014, data for years 2011 - 2013). The choice of markets covers geographical and chronological differences and provides insight in the model performance under varying market conditions. At the same time, we
3.1 Considered power markets

have chosen markets that were at least partially regarded by other authors to have reproducible findings. Our findings are comparable to Nowotarski & Weron (2014) for the Nord Pool market. While there is no EPEX intraday study available when this paper was written, Nowotarski & Weron (2017) provide a benchmark for the GEFCom dataset. The case study employs a common rolling estimation framework which recalculates the model parameters on a daily basis and consequently shifts the entire training, calibration and forecast window by 24 hours as shown in Figure 3.

Figure 3: Out-of-sample rolling estimation scheme for our case study. The split into training and calibration applies to NCP models only. Please also note the random split depicted by changing areas of training and calibration.

The parameterization period (i.e., training and calibration phase) spans 330 days and yields 182 days of Nord Pool forecasts or 831 daily intraday intervals respectively. GEFCom models yield 8760 out-of-sample predictions. We deliberately expand the estimation window for intraday and GEFCom data to assess whether the models are capable of reaching stable coverage ratios over a longer time horizon. Conformal Prediction models and the naive benchmark are applied on the entire parameterization period while QRA is based on point forecasts and cuts of eight weeks of parameterization data to train the quantile regression model. For more information on reproducibility one might check the data files mentioned in the appendix.

3.1 Considered power markets

The first time series we regard is the Nord Pool Spot system price which is determined in a closed-form day-ahead auction at 12:00 CET. It describes the unconstrained day-ahead price for the entire Nordic bidding zone (e.g. Norway, Denmark, Sweden and Finland). It comprises hourly spot electricity prices reported in EUR/MWh from 8.8.2012 to 31.12.2013. The price series might be obtained from the Nord Pool Spot web page (http://www.nordpoolspot.com). Our case study refers to previous work of Nowotarski & Weron (2014) which is why we replicate their basic setup: We calibrate the models from 8.8.2012 - 3.7.2013 and report out-of-sample results for a 182 day period spanning from 4.7.2013 to 31.12.2013.

A different short-term price series is provided by hourly prices of German EPEX intraday trading reported in EUR/MWh. While the Nord Pool market allows entering a single round of bids establishing the prices in a day-ahead auction, the EPEX intraday market is a continuous one that is tradable up to 30 minutes\(^3\) prior to delivery. Please note that this lead time has changed per July 2015 from 45 to 30 minutes. We will consider the volume weighted average price (VWAP) of all transactions for the specific delivery hour. The data series might be obtained from the EEX historical data service and ranges from 21.7.2013 to 30.9.2016. The initial training and calibration window spans data from 21.7.2013 - 22.6.2014. We conducted the out-of-sample test over 831 days to have valid findings not being influenced by any annual or seasonal effects. In contrast to the Nord Pool data, we apply a set of external factors for the German intraday market. The model is enriched with the ENTSO-E total load forecast obtainable from https://transparency.entsoe.eu\(^4\) and estimated wind injection (freely available for download at https://www.eex-transparency.com\(^5\)). These determinants are not only assumed to improve accuracy but increase complexity of the forecast model. Hence, we can validate our model behavior under the usage of price information or multi-dimensional regressor matrices. Please note that we have decided to ignore photovoltaics production as this requires a more complex regression setup. Usually one would leave a photovoltaics variable out of the model during night times when there is no generation and add it in daylight hours. We have sacrificed the additional input for the sake of a similar regression setup in all three power markets.

The last dataset stems from the Global Energy Forecasting competition 2014 and is available for download in the appendix of Hong et al. (2016). It covers hourly zonal prices, zonal load forecasts and system load predictions. The original market or exchange has never been communicated by the authors but due to its usage in a large-scale price forecasting competition it serves as a transparent, reproducible benchmark dataset. We use all available data points which implies a time period from 1.1.2011 - 17.12.2013. We follow the study in Nowotarski & Weron (2017) and compute out-of-sample estimations from

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\(^3\)As of July 2017 EPEX allows to trade up to 30 minutes before delivery from one German control area to another.
3.2 Pre-Processing

18.12.12 to 17.12.13 to have comparable findings.

But why have we chosen these price series? Figure 2 depicts how different the markets are. The three price series suggest a mean-reverting tendency but the intraday (ID) time series features higher volatility and negative prices. GEFCom data equals the intraday data in volatility but shows an overall higher price level. Table 1 supports the assumption of divergent character-

istics. Both the standard deviation (SD) and the interquartile range (IQR) are much higher for intraday and GEFCom data. Interestingly, the spread between the 1st and 3rd quantile is much higher with GEFCom prices while the difference between minimum and maximum is lower than the other two markets. Hence, we do not only have entirely different price series in terms of geographical and timely characteristics but also the statistics support the impression of diversity.

3.2 Pre-Processing

Our time series exhibits hourly granularity which renders a slight transformation necessary. Daylight saving time causes one doubled hour and one missing value. We partly follow Weron (2007) and average the duplicate hours. The latter is computed using multiple imputations as mentioned in Buuren & Groothuis-Oudshoorn (2011). Figure 2 reveals another concern. Outliers are found in both datasets. Conformal Prediction exploits descending errors and could sacrifice preciseness to outliers. Hence, we firstly tried the IQR based Tukey method (see Hoaglin (2003) for a more detailed description). Outliers are defined by 1.5*IQR (like whiskers in common box-plot graphics) and are replaced by multiple imputations after removal. This process should usually ensure greater generalization abilities and less danger of wide intervals. It also marks an adjustment to the underlying time series and needs to be treated very carefully. Given our data and the parameterization window, forecast performance (based on coverage, PI width and the Winkler Score) was decreased by around 5 - 10 % which is why we have decided to leave outliers unchanged in our final time series.

Prices and input factors for regression are usually transformed since many models demand stable variance. We apply a Box-Cox based power transformation denoted as Yeo-Johnson transformation. The great benefit of that approach over the plain Box-Cox one is the capability to deal with negative prices or zero values. It is defined in $\psi$ as

$$\psi(\lambda, y_h, t) = \begin{cases} \left( (y_{h,t} + 1)^{\lambda_h} - 1 \right) / \lambda_h & \text{if } \lambda_h \neq 0, y_{h,t} \geq 0 \\ \log(y_{h,t} + 1) & \text{if } \lambda_h = 0, y_{h,t} \geq 0 \\ -(y_{h,t} + 1)^{2-\lambda_h} - 1) / (2 - \lambda_h) & \text{if } \lambda_h \neq 2, y_{h,t} < 0 \\ -\log(-y_{h,t} + 1) & \text{if } \lambda_h = 2, y_{h,t} < 0. \end{cases}$$

(5)

Please note that we optimize each $\lambda$ individually per model and market using R’s caret package.

4. Prediction Models

4.1 General forecasting approach

Our main target with regards to the regression model is standardization across the empirical study and its models. The input parameters are similar to the comparative studies in Nowotarski & Weron (2017) and Nowotarski & Weron (2014). We partially deviate due to the fact that we use a harmonized regression matrix in all of the markets. We vary the input factors only individually per model and market using R’s caret package.

| Nord Pool day-ahead | EPEX intraday | GEFCom 2014 |
|---------------------|--------------|-------------|
| mean                | 36.38        | 31.81       | 44.83       |
| SD                  | 8.64         | 14.52       | 15.26       |
| 1st quartile        | 32.55        | 24.04       | 33.42       |
| 3rd quartile        | 39.90        | 38.98       | 53.93       |
| min                 | 1.38         | -155.52     | 12.52       |
| max                 | 138.76       | 155.52      | 85.53       |

Table 1: Descriptive statistics of our price series.
Electricity price regression model itself is given by
\[ y_{h,t-1} \sim A_k F_{k,t}, \]  
where \( A_k \) are the load factors and \( F_{k,t} \) the principal components of yesterday’s prices. The components shall comprise all daily price information and are determined using all 24 hours. Please note that \( k = 1, \ldots, 24 \) because 24 hours yield 24 components. As with conventional PCA, the first few factors comprise sufficient information to be included. In our case, three components are utilized. For another application of PCA in the context of electricity price dimension reduction one might check ?.

4.2. Individual Point Forecast Models

A common basis for many PI estimators are point forecasts in the form of a simple regression where the actual price is a function of input factors and an error term. We apply a variety of different models starting from a naive benchmark over an advanced linear regression model to some machine learning algorithms.

4.2.1. Naive expert learner

A simple model is required to assess if more sophisticated approaches truly add any benefit. Therefore, we assume that the best guess for today’s price is the last available similar day price. Based on the scheme laid out in Nowotarski & Weron (2017) we use yesterday’s hourly price if the day to be predicted is a Tuesday, Wednesday, Thursday or Friday. If not, then the price of the hour of the previous week is assumed to be the forecast. This Naive benchmark does not require any computations nor transformations but regards weekly effects and the daily term-structure due to its multivariate approach.

4.2.2. Lasso regression

Our second expert learner combines point forecasting with feature selection. Introduced in Tibshirani (1996), the least absolute shrinkage and selection operator (Lasso) enhances the common ordinary least squares (OLS) scheme in a way that unnecessary variables are penalized or even removed. The Lasso estimator expands OLS by adding a linear penalty factor \( \lambda_h \geq 0 \) in
\[ \hat{\beta}_{\text{lasso}} = \arg \min_{\beta_h} \left( \sum_{t=1}^{T} (y_{h,t} - \sum_{j=1}^{p} \beta_{h,j} x_{h,t,j})^2 + \lambda_h \sum_{j=1}^{p} |\beta_{h,j}| \right). \]

In case of \( \lambda_h = 0 \) we obtain OLS results while \( \lambda_h \to \infty \) causes all variables to be removed from the model. We compute a solution for \( \hat{\beta}_{\text{lasso}} \) using the coordinate descent algorithm implemented in the R package glmnet of ?. The algorithm itself leaves the hyperparameter \( \lambda_{h,t} \) to be optimized. We use a two-fold cross-validation and identify the ideal tuning parameter each hour and day out of an equidistant grid between 0.1 and...
0.001 with step size 0.001. Although this results in more computational effort, a recent study in [?] highlights the importance of recursive Lasso hyperparameter tuning and its beneficial effect on performance.

4.2.3. K-nearest neighbor regression

The idea of the K-nearest neighbor (denoted as KNN) algorithm is based on the fact that patterns in data will repeat in the future. The model implies that a comparable set of input factors will most likely result in the same output as observed with analogous input factors. Therefore, KNN approaches use a similarity measure to identify observations with likewise patterns. The most similar values are then regarded as the prediction. The parameter $k$ defines how many similar observations are taken into account. If $k > 1$, the different realizations for the target variable are usually averaged in order to yield an estimate for the true value. We have explicitly incorporated this rather simple approach to have an alternative estimator based on a simple mapping rule. We use $k = 50$ for Nord Pool and GEFCOM data and $k = 200$ for EPEX intraday prices and determine similarity based on Euclidean distance. We use unprocessed prices for the KNN calculation. For another application on Spanish day-ahead prices the interested reader might refer to Lora et al. (2002).

4.2.4. Support vector machine regression

A support vector machine regression (SVM) maps the regression data to a high-dimensional space and tries to find simple linear decision rules in a new space. The solution obtained is a global one. A large variety of kernel functions renders SVM\(^4\) models to be very flexible which is why most of their practical applications present are in the context of hybrid models that combine several model layers together. Typical examples of such are to be found in Che & Wang (2010) or Zhang et al. (2012). In contrast to that, we apply a very simple stand-alone model based on the R package kernlab that uses a radial basis function kernel. We set $\text{sigma} = 0.005$ and apply a cost of constraint violation of $C = 1.25$. The algorithm itself is restricted to a maximum of 1,000 iterations and works on Yeo-Johnson transformed prices.

4.3 Prediction interval models

A proper point forecast model is only the first step to retrieve prediction intervals. A bit of attention must be paid to the intervals and its notation. A $(1 - \alpha)$ prediction interval implies that the interval contains the true label with probability $(1 - \alpha)$. Transferring this idea to the calculation of quantiles leads to $\tau = \frac{\alpha}{2}$ for the lower and $\tau = (1 - \frac{\alpha}{2})$ for the upper bound. For instance, we calculate the 5% and 95% quantile which yields a 90% prediction interval if the bandwidth between the two quantiles is regarded. A note must also be made on symmetry. Models can estimate quantiles or PIs in a symmetric fashion by adding or subtracting from one point forecast. Other models compute quantiles independently such that we construct the PIs from two quantiles without any point forecast in between.

4.3.1. Empirical error distribution approach

As a probabilistic benchmark, we introduce a simplistic, model-autarkic benchmark approach called empirical error distribution (the suffix _E will be used in the following). Assume any expert leaner from the previous sub-chapter. We simply compute their forecast in the calibration and training time window, calculate the forecast-individual residuals $\varepsilon_{h,t}$ (using the absolute error) and compute the sample quantile of errors $q_t(\varepsilon_{h,t})$. We expand the point forecast for the unknown data to $\hat{y}_{h,t} = y_{h,t-1} \pm q_t$ to retrieve the upper and lower bounds. This procedure does not demand any assumptions on time series characteristics nor requires any greater effort and marks the minimum to be reached for all other models. Please note that we do not use any sampling for our quantile calculation such that one could argue that this automatically leads to overfitting or intervals too narrow. This definitely holds true for very small samples. However, given our sample size we follow the asymptotic theory and assume that we do not conduct a large error. Besides, leaving out sampling -one of Conformal Prediction’s key factors- puts us in a position to specifically analyze its influence in a dedicated study in sub-chapter 5.4. Another possible point of criticism is the choice of the absolute error as the basis for the quantile computation. We want to compute a symmetric estimator but acknowledge that another residual definition could influence results, which is why we briefly touch asymmetric quantiles in sub-chapter 5.4 as well. We assume the effect to be rather minor as the residuals itself are nearly symmetric. In such a setting,
there is no substantial deviation if one calculates a quantile for absolute values or their unadjusted equivalents.

4.3.2. Quantile regression averaging

Recent studies as well as the GEFCOM (see Hong et al. (2016) for results) have shown how powerful the quantile regression averaging (QRA) model of Nowotarski & Weron (2015) is. It stems from the thought of combining forecasts to improve performance (e.g. in Bordignon et al. (2013); Nowotarski et al. (2014)). The approach uses a set of individual point forecasts as an input for a quantile regression. The output is a quantile of either forecast errors (see Maciejowska & Nowotarski (2016) for instance) or price levels (applied in Nowotarski & Weron (2015)). The underlying problem formulation is to be found in Nowotarski & Weron (2014) as

$$q_{\tau,h}(y_{h,t}) = \omega_{\tau,h} \hat{y}_{h,t} + e_{\tau,h},$$

where $\hat{y}_{h,t}$ is the vector of point forecasts and $\omega_{\tau,h}$ a vector of weights to multiply the model output with. The term $q_{\tau}(y_{h,t})$ denotes the conditional quantile of the electricity price distribution given the user specified nominal coverage in $\tau$. The weights are determined by an optimization in

$$\arg\ min_{\omega} \left[ \sum_{t=1}^{L} \rho_{\tau}(y_{h,t} - \omega' \hat{y}_{h,t}) \right],$$

where $1, \ldots, L$ describes the in-sample period and $\rho_{\tau}(z) = (\tau - |\hat{y}_{h,t} - z|)_{+}$. Equation (10) is the equivalent to a likelihood function of a linear regression with asymmetric Laplace-errors and yields numerical values for the upper and lower bound. Please also note that QRA does not explicitly account for heteroscedasticity. It necessitates point forecast estimates as input factors but if these models do not consider the different price realizations of weekdays and hours, the model might end up biased for electricity prices.

4.3.3. Normalized Conformal Prediction

The toy example was helpful in understanding the basic concept but equally important is to fine-tune the CP approach to electricity prices. They feature high volatility and a strong seasonality observable in weekly and daily patterns. Weekends tend to show lower price levels just like night hours where less electricity is needed. Therefore, the inductive CP model introduced in chapter 2 requires to take into account new data

| Point Forecast | 50% | 90% | Naive | LASSO | LASSO | KNN | KNN | SVM | SVM | QRA | Naive | LASSO | LASSO | KNN | KNN | SVM | SVM | QRA |
|----------------|-----|-----|-------|-------|-------|-----|-----|-----|-----|-----|-------|-------|-------|-----|-----|-----|-----|-----|-----|
| PI width (€/MWh) | 3.3  | 2.6  | 2.9   | 3.1   | 3.4   | 2.2 | 2.1 | 2.1 | 2.1 | 2.1 | 12.3  | 7.7   | 8.4  | 7.7 | 11.9 | 6.7  | 8.3  | 6.2  |
| daily coverage (%) | 60.2 | 51   | 53.2  | 48.7  | 56.2  | 50.5 | 50.3 | 44.9 | 44.9 | 44.9 | 95.6  | 89.5  | 93.1 | 90  | 95.9 | 90.3 | 93.3 | 80.9 |
| UC-Test passed (%) | 41.6 | 95.8 | 95.8  | 95.8  | 54.1  | 100  | 95.8 | 83.3 | 83.3 | 83.3 | 25   | 95.8  | 66.7 | 79.2 | 37.5 | 95.8 | 66.7 | 16.7 |
| CC-Test passed (%) | 29.1 | 75   | 66.7  | 33.3  | 0     | 79.1 | 70.8 | 50  | 50  | 50  | 16.6  | 75    | 54.1 | 16.6 | 75  | 54.1 | 16.7 |
| $\phi$ Pinball loss | 0.79 | 0.66 | 0.69  | 0.77  | 0.76  | 0.61 | 0.59 | 0.59 | 0.59 | 0.59 | 0.38  | 0.27  | 0.28 | 0.28 | 0.33 | 0.24 | 0.27 | 0.26 |
| $\phi$ Winkler score | 8.4  | 5.2  | 5.4   | 6.1   | 8.6   | 4.8  | 4.7 | 4.7 | 4.7 | 4.7 | 15   | 10.8  | 11.2 | 11.2 | 13.3 | 9.4  | 10.8 | 10.2 |
| PI width (€/MWh) | 12.6 | 6.9  | 6.7   | 12.6  | 10.8  | 7.4  | 6.3 | 6.3 | 6.3 | 6.3 | 37   | 17.7  | 17.4  | 32.7 | 27.9 | 20.2 | 17.9 | 17.5 |
| daily coverage (%) | 53.7 | 52.5 | 52.9  | 50.2  | 48.6  | 51.8  | 49.7 | 47.2 | 47.2 | 47.2 | 91.2  | 90.8  | 89.4  | 89.4 | 90.2 | 90.4 | 83.3 |
| UC-Test passed (%) | 75   | 91.6 | 75    | 100   | 83.3  | 91.6  | 34.5 | 50  | 50  | 50  | 91.6  | 100   | 100   | 95.9 | 100 | 95.9 | 90  | 94  |
| CC-Test passed (%) | 0    | 54.2 | 41.6  | 0     | 0     | 75   | 100  | 62.5 | 62.5 | 62.5 | 12.5  | 58.3  | 20.8  | 0   | 58.3 | 100 | 33.3 |
| $\phi$ Pinball loss | 3.34 | 1.71 | 1.7   | 3.62  | 3.18  | 1.88 | 1.78 | 1.81 | 1.81 | 1.81 | 1.72  | 0.66  | 0.66  | 1.59 | 1.93 | 0.75 | 0.71 | 0.78 |
| $\phi$ Winkler score | 26.7 | 13.7 | 13.6  | 26.1  | 22.6  | 15   | 14.3 | 14.5 | 14.5 | 14.5 | 54.8  | 26.5  | 26.7  | 49.6 | 41.3 | 30.1 | 28.7 | 31.5 |
| PI width (€/MWh) | 7    | 10.8 | 6.9   | 17.5  | 14.1  | 8.9  | 5.2 | 12.3 | 12.3 | 12.3 | 32.5  | 21.4  | 21.5  | 40.2 | 36.3 | 24.5 | 19.4 | 30.4 |
| daily coverage (%) | 43.3 | 48.6 | 46    | 60.1  | 58.6  | 46.1  | 40.7 | 48.5 | 48.5 | 48.5 | 83.4  | 87.7  | 87.3  | 85.4 | 85.4 | 86.2 | 82.1 | 87.4 |
| UC-Test passed (%) | 33.3 | 95.8 | 66.7  | 12.5  | 25    | 91.6  | 100  | 0   | 0   | 0   | 12.5  | 91.7  | 62.5  | 48.8 | 37.5 | 62.5 | 0   | 79.2 |
| CC-Test passed (%) | 0    | 50   | 50    | 0     | 0     | 0     | 0   | 0   | 0   | 0   | 0     | 37.5  | 0     | 0   | 0   | 0   | 0   | 12.5 |
| $\phi$ Pinball loss | 4.26 | 3.1  | 2.77  | 6.05  | 5.75  | 3.45 | 3.5  | 3.29 | 3.29 | 3.29 | 2.44  | 1.3   | 1.43  | 3.56 | 3.66 | 1.69 | 2.23 | 1.21 |
| $\phi$ Winkler score | 3.41 | 24.3 | 22.2  | 48.4  | 46    | 27.5 | 27.9 | 26.3 | 26.3 | 26.3 | 97.6  | 52    | 57.3  | 142.7 | 146.5 | 67.5 | 89.2 | 45.6 |

Table 2: Selected prediction interval sharpness and reliability results for empirical two-sided prediction intervals. Please note that the pinball loss is a metrics for each quantile which we have averaged for the respective PI, such that the 90% PI describes the average Pinball loss of the 5th and 95th quantile.
and information to address the issue of heteroscedasticity. We aim to minimize any bias by an extended Conformal Prediction scheme referred to as Normalized Conformal Prediction (NCP) in Papadopoulos & Haralambous (2010) that considers new data as well. Whereas the toy example only uses historical data for the determination of non-conformity scores, the expanded version also incorporates the information set applied for the regression model. But what is different to the calculus mentioned in section 2? It is mainly the non-conformity score.

A non-conformity score $\lambda_{h,i}$ exists for every pair of $(x_{i,h}, y_{i,h})$ in $i = M + 1, ..., L$. Please note that we deviate from the $h, t$ notation here and introduce $i$ to a) establish a connection to the

![Figure 4: Differences between the empirical coverage of the predictors and the nominal coverage per percentile. While we have left out the 50th percentile in the initial calculation, it was depicted here using interpolation. This ensures that we do not create any unwanted bias through a steeper step between the 55th and 45th percentile. The hatched gray area reflects the minimum possible coverage, i.e., the 5th percentile cannot have a higher negative deviation than 5 percent.](image-url)
examples of sub-chapter 2 and b) to highlight the different order due to sampled training and calibration that indeed is different from the chronological $h, t$ order. The non-conformity score is given by
\[ \lambda_{h,t} = \frac{|y_{h,t} - \hat{y}_{h,t}|}{\hat{\epsilon}_{h,t}} \tag{11} \]
with $\hat{\epsilon}_{h,t}$ being the estimated error predicted by a second, explicit error estimation model. This model predicts the estimated error for the out-of-sample data. The interval forecast is given by
\[ y_{a,h,t} = \hat{y}_{h,t} \pm (\lambda_{a,h,t} \hat{\epsilon}_{h,t}) \tag{12} \]
The NCP algorithm depends on two autarkic prediction models. One of them aims to deliver a point forecast. It might also be regarded as a stand-alone predictor if one disregards the Conformal Prediction framework. Based on such, the errors made in the training process are calculated. The second model uses these values as the response and forecasts the inaccuracy present in the actual prediction approach. Once both models are parameterized, they equally generate their prediction on the novel calibration dataset.

5. Empirical results

5.1. General performance metrics

Prediction intervals require to be reliable and sharp (Nowotarski & Weron 2017). The term reliability itself refers to the empirical coverage being close or equal to the designated coverage level. It is also noteworthy that reliability and sharpness share a close interdependency. The sharper an interval gets, the less it is near the true coverage. Moreover, we are facing a tradeoff between the two quality criteria. As a first approach to the topic, we compare the empirical coverage with the nominal values under consideration of the PI width in the upper rows of Table 2. From left to right, it depicts values for each model and interval. The first impression is that NCP models yield good coverage. Yet, this is not a uniform statement as the results differ per model and interval. There is no single best model for coverage nor for sharpness. If we take a closer look at the different markets, it appears as if the EPEX intraday and GEFCom markets are more difficult to predict in a probabilistic manner as their error measures are higher than Nord Pool ones on average. This intuitively makes sense as these markets are the more volatile ones. Higher volatility seems to widen the difference in predictions and observations. Our QRA model shows good performance but remains behind the Conformal Prediction models. Please note that we can validate our QRA results by means of the findings reported in Nowotarski & Weron (2014) for the Nord Pool market as they were very similar. QRA results obtained in Nowotarski & Weron (2017) for the GEFCom dataset were slightly better than our QRA model which might be due to the changed selection of point forecast models. We have chosen our predictors mostly out of the field of machine learning while the aforementioned authors have used a wider set of traditional time series approaches. However, since the results do not fundamentally differ we see that as further cross-literature validation of our models.

A downside of the previous analysis is the strict focus on both the 50% and 90% prediction interval and its associated 25/75 and 5/95 percentiles. It enforces symmetry and does not evaluate the upper and lower parts of the PI in a separate way which leaves room for netting effects in errors. In order to assess the prediction quality one needs to focus on all other percentiles as done in Figure 4. It depicts the deviation between empirical and nominal coverage computed for all percentiles in steps of 5 and shows the asymmetric estimation quality. The first striking fact is that contrary to Table 2, the Nord Pool and GEFCom markets appear to be harder to predict since the distance to the true coverage is higher than anticipated by Table 2. Most of the models seem to suffer around the 55 and 45 percentile which is usually a hard region to predict due to the high density of observations in that area. We did not compute the 50 percentile as this is typically estimated by median point forecasts and is not directly associated to the Conformal Prediction technique anymore. For reasons of a clear depiction, the 50 percentile area was only interpolated. There is no single best predictor but different markets with diverse performance. Support vector machines tend to show a constant level of differences in comparison with other estimation approaches. If we compare the empirical quantiles with their NCP equivalents it is not possible to favor one over the other. The choice of the best model seems to be heavily connected with the market to be predicted and the underlying point forecast. Our last finding is associated with the observation of differences between Table 2 and Figure 4. Obviously, singular percentiles are harder to foretell. But if one, for instance, considers the QRA performance in the EPEX intraday market, an interesting relationship becomes
5.2 Christofersten Test

Besides the nominal coverage, there is a commonly used test set provided by Christofersten (1998) which examines unconditional coverage (UC), independence, and conditional coverage (CC). The sooner evaluates true coverage while independence takes clustering effects into account. We stick to Weron & Misiorek (2008) and restrict on the first observation which renders conditional coverage to be the sum of independence and unconditional coverage. That being said, it is in this case sufficient to test for unconditional coverage and its conditional equivalent as the latter comprises the independence information. The tests are processed in the Likelihood-Ratio (LR) framework and use a hit series (1 if the interval is correct, 0 otherwise) as input. We also test hourly observations so that no daily effects can falsely create any signals of dependence across several hours. The detailed test statistics are displayed in Figure 5. We plot the test output in the form of LR test statistics against each hour of the

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Figure 5: Christofersten unconditional coverage (UC) and conditional coverage (CC) test results reported as hourly Likelihood ratio (LR) statistics. Please note that all LR values above 20 are set equal to 20 for the graphical depiction.

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evident. The deviation switches from positive just to become very negative. If we recap that the 50% PI should cover the range of the 25 and 75 quantiles we might assume that some of the models benefit from netting effects out of symmetry. This explains why the Nord Pool study reveals higher deviations in Figure 4. The 50% PI is near an empirical result of 50% but the two individual quantiles are less close to the nominal coverage.
day and do so per model and market. The dashed gray line symbolizes the 5% significance level of the test statistics and determines the acceptance criterion for both the UC and CC test. In more naive words: all statistics above the gray line point towards a lack of reliability under our test setup. The first striking observation in Figure 5 is that Conformal Prediction appears to have a positive effect on the LR statistics. Taking the Lasso, for instance, most of the NCP plots per market are below their empirical counterparts which speaks for the theoretical foundation that postulates true coverage for (N)CP. We can also observe such performance for SVM_NCP predictions. KNN leaves a mixed impression. It seems to have consistent problems in all markets with the stricter CC test in particular. QRA’s 50% PI values are mostly under the 5% significance level. The associated 90% PI test statistics create a different impression as they mostly do not meet our acceptance criterion. These findings are in line with Nowotarski & Weron (2017) in case of GEFCom but partially differ in the Nord Pool case. Nowotarski & Weron (2014) report a higher ratio of accepted hours for the 90% PI. Still, this might be caused by the different blend of point forecasts. Finally, Naive_E and its connected missing acceptance require a deeper look. Our naive benchmark yields insufficient reliability in all of the considered power markets which delivers evidence to the fact that a more advanced prediction interval determination approach brings additional benefit. If we compare the intra-market results, it becomes evident that GEFCom is the time series that is the hardest challenge in terms of reliability, even for the most performant models. In contrast to that, EPEX ID and Nord Pool seem to have roughly the same range of errors across all models. Unfortunately, we do not know enough about the GEFCom origin to establish any further connection between fundamental characteristics and the problems with reliability. Yet, we can acknowledge that out of our three time series the Christoffersen test confirm that GEFCom is the hardest to be estimated.

### 5.3 Winkler Score and pinball loss

All previous assessments have focused either on reliability or sharpness in a separate manner. A metrics known as the Winkler Score (see Winkler (1972) for the derivation) allows to jointly elicit both given in (cases representation adopted from Maciejowska et al. (2016))

\[
W_{h,t} = \begin{cases} 
B_{h,t} & \text{for } y_{h,t} \in B_{h,t} \\
B_{h,t} + \frac{2}{\alpha} (L_{h,t} - y_{h,t}) & \text{for } y_{h,t} < B_{h,t} \\
B_{h,t} + \frac{2}{\alpha} (y_{h,t} - U_{h,t}) & \text{for } y_{h,t} > B_{h,t},
\end{cases}
\]

where \( B_{h,t} \) represents the bandwidth of the two-sided prediction interval and \( L_{h,t}, U_{h,t} \) its lower and upper bounds. The Winkler Score penalizes deviating coverage and examines the bandwidth. All results are depicted in Figure 6. The upper part under section a) tries to contribute to the question of additional benefits of using Normalized Conformal Prediction in combination with different point forecasts. Which point forecast models gains the most from Normalized Conformal Prediction and consequently feature the lowest Winkler Score? Figure 6 shows the decrease in the latter if we use NCP instead of the error distribution approach (using an _NCP model instead of an _E one). For GEFCom and Nord Pool, one can observe a decrease in the Winkler Score of about 10% - 20%, happening mostly with the 90% PIs marked in gray. In EPEX intraday markets, NCP additions do not have an impact on the Winkler Score which underlines our diverse choice of markets and how different the results are. The KNN model even shows an increase in the German intraday market albeit for all other markets there is at least a bit of decrease in the error measure. A possible connection could be connected to Figure 4 where the EPEX ID market features low deviation from the true coverage. If we recap that the Winker Score takes into coverage we might assume that this market is overall less complex in its prediction characteristics and, therefore, does not benefit from further model complexity. Still, this is just a first, trivial explanation and requires more empirical analysis that goes beyond the scope of this paper. All in all, Normalized Conformal Prediction seems to have a positive impact on the Winkler Score in two out of three markets. On the other hand, the performance varies with the underlying point forecast models even in the same market.

Section a) is giving a good first impression but left the time structure of a short-term price forecast aside. We want to assess hourly differences and have plotted a corresponding curve of hourly Winkler Scores for each market in section b). In order to reduce the complexity of the graphical depiction, we have narrowed down the analysis and only compare the best Normalized Conformal Prediction model (LASSO_NCP for EPEX ID and GEFCom, SVM_NCP in case of Nord Pool data) based
on Table 2 with QRA and our naive benchmark. Not very surprisingly, the Winkler Score curve of the naive approach is much higher which implies less preciseness. This holds true for all of the three markets. QRA and NCP are very close: in our Nord Pool and EPEX intraday application, NCP features slightly lower curves while with GEFCom data, QRA and NCP are almost equal. If one takes a deeper look at the hourly shape of each individual curve it becomes evident that night hours appear to show lower Winkler Scores. There are spikes in the error measure during the off-peak/peak time block shifts (around hour 8 and 20) in the Nord Pool market. This effect is often observed in electricity spot markets or day-ahead markets in particular and might be explained by additional power plants ramped up or down to cover peak load during the day. While intraday markets are usually used to cover residual loads or renewables adjustments, day-ahead markets serve as a market place for the entire daily fleet generation or load. Therefore, we observe such a strong block shifting effect in the Nord Pool day-ahead data while there is less in the intraday equivalent. Taking the hourly shapes into account, we have to favor QRA or NCP over the naive benchmark with NCP showing a slightly lower Winkler Score in some instances.

Our second test statistic is a very popular one. The pinball loss (PB loss) was chosen to be the official scoring rule for the GEFCom 2014 probabilistic forecasting track in Hong et al. (2016) and gained the reputation of a common measure for probabilistic forecasts. Its representation is given by

\[
P_B(q_{y_{h,t}}(\tau), y_{h,t}, q) = \begin{cases} (1-\tau)(q_{y_{h,t}}(\tau) - y_{h,t}) & \text{for } y_{h,t} < Q_{y_{h,t}}(\tau) \\ q(y_{h,t} - q_{y_{h,t}}(\tau)) & \text{for } y_{h,t} \geq Q_{y_{h,t}}(\tau), \end{cases}
\]

where \(q_{y_{h,t}}(\tau)\) is the \(\tau\)-th estimated quantile of the the electricity price series \(y_{h,t}\). The pinball loss is a quantile specific measure but can simply be averaged across hours or quantiles in order to have a more comprehensive sharpness indicator. The analysis of the pinball loss goes into a different direction compared to the Winkler assessments since it focuses on percentiles in order to determine how an approach behaves under varying probabilistic assumptions. This modus operandi also shifts the focus towards asymmetric performance and sets each percentile in a performance relation. In contrast to that, Table 2 focused on prediction intervals which imply symmetry. All findings are
5.4 Path dependent evaluation of Conformal Prediction performance drivers

The previous sub-chapters have only taken a global view on estimation capabilities and compared the model performance with QRA and a naive benchmark. We have not discussed the question of why Conformal Prediction is performing in a decent manner. Leaving the technical concept aside, Conformal Prediction features three possible origins of which performance might stem from. Firstly, it forces the forecast to be symmetric. We sort the non-conformity measure \( \lambda_{h,t} \) and consider the respective value corresponding to the desired PI. The identified non-conformity score is added or subtracted from the forecast such that there is no designated differentiation between quantiles. For instance, we determine the 50% PI by subtracting and adding the same \( \lambda_{h,t} \) from our point forecast. In contrast to that, an asymmetric approach determines the 25th and 75th quantile in an independent manner. Combining these two quantiles yields the 50% PI in a second step. The second potential source of performance gains is the sampling technique described in chapter two. Conformal Prediction randomly splits the available set of information into training and calibration to ensure a maximum of generalization. But does this step really improve the models? The third aspect of Conformal Prediction is at least out of an intuitive guess a very important one. In the case of Normalized Conformal Prediction, we adjust the non-conformity score by estimated errors as mentioned in Eq. (11). When it comes to the forecast value \( \tilde{y}_{h,t} \), this small modification ensures that all new information is regarded in the prediction interval determination by firstly estimating the error for \( t+1 \) and then plugging it in in Eq. (12). Without any quantitative backing, one will surely assume that this is a reasonable operation with a positive impact on predictive performance, especially if we consider the strong daily effects of electricity price time series. Heteroscedasticity caused by weekly effects is taken into account since we include the daily dummy in the new information set.

We have run a simulation path of different combinations of the above three model expansions that jointly form the core of Conformal Prediction in the same out-of-sample fashion that was already applied in the empirical analysis in the previous sub-chapters. All data inputs, transformations, and the out-of-sample rolling window approach remain unchanged. At the same time, we only consider the 3 point forecast models that serve as a basis for the Conformal Prediction PI determination. Speaking of values, we utilize our two common error measures pinball loss and Winkler Score in order to identify the impact normalization, sampling and symmetry have. In addition, we compare the PI width. Although this is not a traditional error measure per se, it helps in understanding differences. Figure 8 and 9 illustrate both the different simulations paths as well as the connected error measures separated into the 50% and 90% prediction interval. We start the analysis with the most basic form of PI estimation by computing the quantiles of the empirical error distribution\(^5\), depicted in the very left (model with no colored dots). This model neither samples any of the data nor uses new information. Please also note that we receive an asymmetric estimation as we independently compute the quantile for the upper and lower part of the PI, i.e., do not use absolute errors for quantile determination. We have initially assumed that this PI predictor is by far the worst one but were proven wrong by our empirical study. In comparison to Normalized Conformal Prediction, the asymmetric empirical quantiles tend to perform

\(^5\)Please note that this differs from the \(E\) model used before due to the lack of symmetry. All \(E\) models are symmetric ones.
5.4 Path dependent evaluation of Conformal Prediction performance drivers

![Graphs showing Pinball loss score per percentile for Nord Pool, GEFCom, and EPEX ID.]

**Figure 7:** Pinball loss score per percentile as mentioned in Eq. (14). Please note that we have taken the average over all 24 hours to depict model performance under different percentiles and have plotted every 5th percentile except the 50th one.

very well. In the German intraday market, the results are nearly equal to NCP while NCP yields lower errors in Nord Pool and GEFCom. But as a first result, we can say that the most modest form of probabilistic forecasting is more accurate than expected.

Based on the asymmetric quantiles, we separately add all three extension stages to the basic model. The second estimator uses normalization via estimated errors (blue dots and lines), its other two equivalents add symmetry (gray dots and lines) and sampling of data depicted by red dots and lines. Please
note that the symmetric empirical PIs equal our empirical error distribution approach (previously denoted as \_E models) under sub-chapter 4.3.1. Performance-wise, one can observe a clear picture. Adding symmetry lowers the Winkler Score and pinball loss. Interestingly, the interval width is not widened at the same time which reflects that we yield more accurate intervals and that the previous one was not just too narrow. Such a clear indication came unanticipated as the technical model difference is rather small. Instead of stand-alone quantiles we compute absolute empirical errors and add or subtract them from the point forecast. While this is a very small change in terms of computation, its impact is impressive and speaks for symmetry in residuals.

Sampling to avoid overfitting appears to further increase accuracy which at least partially refutes our asymptotic argument of making no mistake without sampling. Still, the effect is very small. Normalization or the addition of new information is a bit problematic in a stand-alone application. The asymmetric empirical model computes negative and positive normalization values in the form of estimated errors to be subtracted or added to the probabilistic forecast. These values are then normalized by the expected error, which is either a positive or negative value. It might occur that the sign of some of the values changes the entire PI to unrealistic estimations which is why we adjust the normalization numbers to be always positive. All in all, the computation of normalized asymmetric quantiles does not really make sense. However, for reasons of completeness, we show the model results. In some cases, we observe a tremendous performance drop after normalization which further underlines the argument of a known misconstruction.

We could end our analysis at this point. But that would imply linear additivity of the specific model extensions. Is it intuitively possible to add for instance symmetry and normalization and yield the sum of each extension’s performance? We expand our models into three different paths to answer this question. Firstly, we add the three other extensions. The normalized empirical quantiles are changed to a symmetric version (blue and gray dots). This step shall further validate our findings with regards to the inefficiency of non-symmetric normalization. And indeed symmetry heals the issue of misconstructed PIs. Error measures are lower while the PI width is narrowed down as well which reflects an improvement of sharpness under reliability.

A different picture is painted if we extend the sampled asymmetric quantiles to a symmetric estimator, depicted by gray and red dots. Please note that this predictor equals Conformal Prediction mentioned in chapter two. The Winkler Score and pinball loss only slightly change in some instances which highlights that sampling helps on a case by case basis. Adding normalization to sampled quantiles (blue and red dots) causes the same bias as with the normalized quantiles. Due to the lack of symmetry, the sign of the output could change the entire prediction which causes the Winkler Score and pinball loss to be much higher. Hence, our empirical study suggests that the path from empirical quantiles to normalized and then normalized and sampled ones does not make sense to apply.

Last but not least, we focus on Normalized Conformal Prediction as our last layer. In some cases, such as the GEFCom predictions, it makes sense to utilize all three extensions jointly together. In other scenarios such as EPEX intraday the addition of sampling to norm-symmetric intervals was not beneficial with regards to performance. This finding perfectly matches the impression from the Winkler Score analysis. The less complex markets with regards to estimations, namely Nord Pool and EPEX ID, do not seem to benefit from model extension in a way the GEFCom data set does.

So after computing 144 models what does the path-dependent analysis suggest? We can assume that there is no singular model that outperforms all the others. Our choice of markets was indeed a very diverse one which causes results to be very different. The same counts for the point prediction models itself. There are some universal tendencies such as beneficial effects of symmetric estimations. That being said, the usage of normalization and sampling only adds value in some of the cases. We advise every forecaster to carefully test the probabilistic models in question, especially if the market to be predicted features statistical similarity to the GEFCom data. Conformal Prediction serves as a good framework but still requires fine-tuning with regards to the optimal blend of its key components.

6. Conclusion and outlook

The underlying research motivation of this paper was a thorough introduction of Conformal Prediction with a particular focus on short-term electricity prices. We have discussed the theoretical concept and demonstrated that Conformal Prediction works like a second layer to any given point forecast at hand. By exploiting errors made from these point forecasts, symmet-
We have averaged for the respective PI, such that the 50% PI describes the average pinball loss of the 25th and 75th quantile. To evaluate which part of Conformal Prediction accounts for most of the gains in preciseness. Please note that the pinball loss is a metric for each quantile which normalization and sampling for the 50% PI. We have applied the same empirical setup as described in the previous sub-chapters but changed the models bit by bit to identify the key performance drivers based on a path-dependent analysis of the three different model extensions symmetry, normalization, and sampling for the 50% PI. We have applied the same empirical setup as described in the previous sub-chapters but changed the models bit by bit to evaluate which part of Conformal Prediction accounts for most of the gains in preciseness. Please note that the pinball loss is a metric for each quantile which we have averaged for the respective PI, such that the 50% PI describes the average pinball loss of the 25th and 75th quantile.

**Figure 8:** Identification of Conformal Prediction’s key performance drivers based on a path dependent analysis of the three different model extensions symmetry, normalization, and sampling for the 50% PI. We have applied the same empirical setup as described in the previous sub-chapters but changed the models bit by bit to evaluate which part of Conformal Prediction accounts for most of the gains in preciseness. Please note that the pinball loss is a metric for each quantile which we have averaged for the respective PI, such that the 50% PI describes the average pinball loss of the 25th and 75th quantile.
Figure 9: Identification of Conformal Prediction’s key performance drivers based on a path dependent analysis of the three different model extensions symmetry, normalization and sampling for the 90 % PI. The plot is equivalent to Figure 8 besides the different PI.
ric prediction intervals are computed. The other two novelties in that sense are the sampling Conformal Prediction does in order to achieve a high level of generalization and the normalization by means of estimated errors. We explicitly consider new information when we adjust the PI with an estimated error for $t + 1$. This helps to account for electricity price characteristics like heteroscedasticity caused by contrasting load scenarios at different days since we include information about such in the probabilistic estimation process. Leaving the theory behind, we have tested multiple probabilistic forecasting concepts in three independent pricing regimes and establish a connection to the empirical results of Nowotarski & Weron (2014) and Nowotarski & Weron (2017) by adopting a comparable QRA model. We demonstrate that Conformal Prediction can live up to the expectations and yields valid prediction intervals even with changing point forecast inputs. In comparison to a naive benchmark, the well-known QRA and a simple error distribution approach applied to an alike set of point forecasts, NCP is equal or even better in terms of Winkler Score, Christoffersen Test or pinball loss. Connected to the decent performance is the question for key performance drivers. An additional evaluation that independently analyzes Conformal Prediction’s three key aspects symmetry, normalization and sampling of input data brings more clarity. We have simulated different paths leading to a total of 144 computations. The overall picture was rather unclear. Conformal Prediction and its different modifications show varying performance across markets. As a consequence, we advise energy companies to compute a path like done in sub-chapter 5.4 for their forecasting problems and only then to decide on one specific Conformal Prediction model.

At the same time, we have to acknowledge that these findings right now only account for short-term electricity prices. We have deliberately chosen to focus on these in order to yield maximum objectivity in our analysis. Future research might as well look at other possible applications such as wind forecasting or load prediction. Apart from that, we did not discuss any extension of the known Normalized Conformal Prediction framework. Following the idea of QRA, Conformal Prediction intervals might also be averaged to get even better results. However, current research has not yet found a solution that yields valid intervals although the first findings were promising. The interested reader might take a closer look at ? for a description of multiple aggregated Conformal Predictors or ? for the idea of combining various p-values or non-conformity scores.

It might be worth to expand these ideas to the world of energy-related forecasting. Last but not least, we want to encourage researchers to not only focus on a theoretical discussion but to take into account the economic effects arising out of forecasts. We did not touch this topic but it might be interesting to see what monetary benefits a probabilistic estimation can bring over a usual point forecast.

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Appendix A. Research data

Supplementary data to this article such as detailed model specification and code samples for reproduction can be found online at DOI: will be added later during the revision phase.

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