A spectroscopic look at the gravitationally lensed Type Ia supernova 2016geu at $z = 0.409$

Zach Cano,$^{1}$* Jonatan Selsing,$^{2}$ Jens Hjorth,$^{2}$ Antonio de Ugarte Postigo,$^{1,2}$ Lise Christensen,$^{2}$ Christa Gall$^{2}$ and D. A. Kann$^{1}$

$^{1}$Instituto de Astrofísica de Andalucía (IAA-CSIC), Glorieta de la Astronomía s/n, E-18008 Granada, Spain
$^{2}$Dark Cosmology Centre, Niels Bohr Institute, University of Copenhagen, Juliane Maries Vej 30, DK-2100 København Ø, Denmark

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ABSTRACT

The spectacular success of Type Ia supernovae (SNe Ia) in SN-cosmology is based on the assumption that their photometric and spectroscopic properties are invariant with redshift. However, this fundamental assumption needs to be tested with observations of high-$z$ SNe Ia. To date, the majority of SNe Ia observed at moderate to large redshifts ($0.4 \leq z \leq 1.0$) are faint, and the resultant analyses are based on observations with modest signal-to-noise ratios that impart a degree of ambiguity in their determined properties. In rare cases, however, the Universe offers a helping hand: To date a few SNe Ia have been observed that have had their luminosities magnified by intervening galaxies and galaxy clusters acting as gravitational lenses. In this paper, we present long-slit spectroscopy of the lensed SN Ia 2016geu, which occurred at a redshift of $z = 0.409$, and was magnified by a factor of $\approx 55$ by a galaxy located at $z = 0.216$. We compared our spectra, which were obtained a couple of weeks to a couple of months past peak light, with the spectroscopic properties of well-observed, nearby SNe Ia, finding that SN 2016geu's properties are commensurate with those of SNe Ia in the local Universe. Based primarily on the velocity and strength of the Si II $\lambda 6355$ absorption feature, we find that SN 2016geu can be classified as a high-velocity, high-velocity-gradient and 'core-normal’ SN Ia. The strength of various features (measured though their pseudo-equivalent widths) argue against SN 2016geu being a faint, broad-lined, cool or shallow-silicon SN Ia. We conclude that the spectroscopic properties of SN 2016geu imply that it is a normal SN Ia, and when taking previous results by other authors into consideration, there is very little, if any, evolution in the observational properties of SNe Ia up to $z \approx 0.4$.

Key words: gravitational lensing: strong – supernovae: general – supernovae: individual: SN 2016geu – cosmology: miscellaneous.

1 INTRODUCTION

The study of Type Ia supernovae (SNe Ia) encompasses many fields of astrophysical research: They are connected with the end-points of stellar evolution in binary systems, and elements nucleosynthesised during their explosions, along with their enormous energy input, enrich the environments of their host galaxies. They also play a central role in SN cosmology and provide an extremely useful tool for determining cosmological distances. These, in turn, provide constraints on fundamental cosmological properties, including the expansion rate of the Universe and the amount of matter and energy contained therein (Riess et al. 1998; Perlmutter et al. 1999).

The role of SNe Ia in cosmology cannot be overstated. The absolute luminosity and the width of their light-curves are correlated (Phillips 1993), which provides a means to reduce the amount of scatter in SN Ia Hubble diagrams and provide better constraints on cosmological models. Hence, two-parameter luminosity calibrations are currently used in SNe Ia research, which includes a colour term in the fitting process (e.g. Tripp 1998; Guy et al. 2007), while additional techniques aimed at reducing the Hubble-diagram scatter are constantly being sought after (e.g. Höflich et al. 2010). For example, in several SNe Ia cosmology analyses a third parameter related to their host galaxy properties (stellar mass and/or star formation rate) was sought and used when standardizing their SN Ia samples (e.g. Lampeitl et al. 2010; Conley et al. 2011; Betoule et al. 2014; Uddin et al. 2017).

Despite the success of SNe Ia as cosmological probes, fundamental understanding of why they are such good standardizable candles is generally lacking. Sub-types of SNe Ia were identified as early as the mid-1980s (e.g. SN 1984A Branch 1987; Phillips et al. 1987), and later in the early 1990s (e.g. SNe 1991bg and 1991T...
Filippenko et al. 1992a,b; Phillips et al. 1992). Indeed the overall spectroscopic diversity of SNe Ia (Filippenko 1997), followed by the classification of a fraction of SNe Ia into sub-types (see Section 7), prompts the question of just why do the bulk of observed SNe Ia, i.e. ‘normal SNe Ia’, have such ideal properties that facilitate their successful use in SN cosmology?

Further uncertainties fuel this question: what are the progenitor systems of SNe Ia? How do their environments affect their observed properties (Hamuy et al. 1995; Howell et al. 2007; Kelly et al. 2015a)? Do all SNe Ia, and their corresponding sub-classes, appear the same across all of cosmic time, or is there a redshift evolution? Certainly the physical and chemical properties (mass, age, metal and dust content) of galaxies change with time, and if the physical environments hosting SNe Ia directly affect their physical and observational properties, perhaps by influencing the amount of nickel synthesized during the explosion (e.g. Höflich, Wheeler per cent Thielemann 1998; Timmes, Brown & Truran 2003), this will have a direct influence on a given SN’s luminosity. Indeed, as current SN Ia cosmology is based on the assumption that the observed correlations between the luminosity, light-curve shape and colour of SNe Ia (and the properties of their host galaxies) hold with changing redshift, unambiguously addressing this assumption is of great importance to the field.

In this paper, we have attempted to address the question of redshift invariance of SNe Ia. Previous studies have tested this assumption using different approaches. The seminal works of Riess et al. (1998) and Perlmutter et al. (1999), who investigated SNe Ia up to $z = 0.5$, found no evolution in their spectra and light curves. However, Riess et al. (1999) compared a low-$z$ and high-$z$ sample of SNe Ia, and found the rise times of the two groups differed by more than $5\sigma$. Hook et al. (2005) studied a sample of $N = 14$ SNe Ia in the redshift range $0.17 \leq z \leq 0.83$, and found that both the spectral time sequences and the measured Ca ii H & K velocities were indistinguishable from low-$z$ SNe Ia. Conversely, using the assumption that SNe Ia arise from two populations (prompt and delayed), Howell et al. (2007) found that SNe Ia at redshifts up to $z = 1.5$ may be, on average, 12 per cent more luminous than their low-$z$ counterparts. Using 87 SNe Ia detected by the Supernova Legacy Survey (SNLS) during its first three years of operation, Bronger et al. (2008) found no evolution in the rest-frame equivalent widths and ejecta velocities of SNe Ia up to a redshift of unity. Additional, later data obtained by the SNLS further supported this conclusion (Walker et al. 2011). Interestingly, when analysing ESO/VLT spectra obtained for the SNLS (opposed to the Gemini spectra presented in Bronger et al. 2008), Balland et al. (2009) found that the spectra of SNe Ia at $z < 0.5$ had deeper intermediate mass element absorption compared with the $z > 0.5$ sample. Moreover, based on Hubble Space Telescope (HST) spectroscopic observations of low-$z$ and high-$z$ SNe Ia ($0.4 \leq z \leq 0.9$), Maguire et al. (2012) found a modest evolution in their NUV (near-ultraviolet) spectra ($\approx 3\sigma$ significance), where there appeared to be a UV-flux deficit in the more distant SNe Ia relative to the nearby sample. Collectively, these studies may suggest that there is a small, yet appreciable, difference in the photometric and spectroscopic properties of nearby and high-$z$ SNe Ia, though this is far from being certain.

Many of the aforementioned studies relied on spectroscopic observations of high-$z$ SNe Ia, which are inevitably faint and suffer from low signal-to-noise ratios (S/Ns) that limit the amount of unambiguous information that can be ascertained of these events. Fortunately, nature sometimes reveals glimpses of her secrets in beautiful and awe-inspiring ways. Massive galaxies and galaxy clusters have been observed to act as weak and strong gravitational lenses, where they magnify distant objects that intersect the line of sight with themselves and Earth. To date a few gravitationally lensed SNe have been observed, including the famous type II SN Refsdal (Kelly et al. 2015b), which as predicted by Refsdal (1964) had multiple images of the same SN, where the time delay between them is proportional to the value of the Hubble constant. Lensed SNe Ia have also been detected: PS1-10afx, at a redshift of $z = 1.39$, was shown by Quimby et al. (2013) to be more than 30 times brighter than expected for an unlensed SNe Ia at a comparable redshift. Petrushovska et al. (2017) recently demonstrated that the spectroscopic properties of PS1-10afx, in comparison to the median spectra of nearby SNe Ia, showed no evidence for spectral evolution between it and low-$z$ SNe Ia. SN HFF14Tom ($z = 1.3457$) was discovered behind the galaxy cluster Abell 2744 (0.308), where the authors demonstrated its use as a direct probe of galaxy lens models (Rodney et al. 2015).

Here, we report on the spectroscopic properties of another lensed SNe Ia, SN 2016geu, which occurred at a redshift of $z = 0.409$ and was $\approx 55$ times brighter than an unlensed SNe Ia at the same distance. Such an increase in the brightness of a SN Ia has provided us with an opportunity to investigate the spectroscopic properties of these objects at a redshift where such properties are poorly understood using the largest telescopes and the most advanced spectrographs currently available. Our goal therefore was to determine its spectral properties as a function of time (where its photometric properties were determined elsewhere by Goobar et al. 2017). This paper is organized as follows. In Section 2, we present relevant background information of SN 2016geu published by Goobar et al. 2017. In Section 3, we describe our data reduction procedures, while in Section 4 we describe our method to account for and remove contaminating flux from the lensing galaxy in our spectra. In Section 5, we compare our spectroscopic time series with nearby SNe Ia 1994D and 2011fe. In Section 6, we determine the pseudo-equivalent widths (pWs) and line velocities of several transitions in our spectra, and compare them with those presented in the literature. We sub-classify SN 2016geu in Section 7, and compare our observations with synthetic SYN++ spectra in Section 8. Finally, in Section 9, we discuss and summarize our conclusions. All times and phases are given in the rest frame unless specified otherwise, while $\langle \rangle$ indicates an average value.

2 SN 2016GEU – BACKGROUND INFORMATION

SN 2016geu (Goobar et al. 2017) was discovered by the intermediate Palomar Transient Factory (iPTF) on 2016 September 05 at RA = 21h04m15.86 and Dec. = $-06\deg20\arcmin24.5\arcsec$ (J2000) in SDSS galaxy J210415.89$-062024.7$. Spectroscopic classification of the optical transient was performed with the SED Machine (Ritter et al. 2014) on the Palomar 60-inch telescope (P60) on 2016 October 2, where it was found to be a spectroscopically normal SN Ia at $z \approx 0.4$. At this redshift, its apparent R-band magnitude ($R = 19.5$) was more than 3.5 mag brighter than a normal SN Ia at the same redshift, which prompted the realization that its increased brightness may be due to an intervening galaxy acting as a gravitational lens. From narrow Ca ii H & K and Na D absorption features, as well as weak H and [N ii] emission lines in the Palomar 200-inch telescope (P200) and the 2.2-m Nordic Optical Telescope (NOT) spectra, Goobar et al. (2017) found the redshift of the SN and the lensing system to be $z = 0.409$ and 0.216, respectively. In a Keck K-band adaptive-optics image obtained on 2016 October 13, multiple images of the SN were recorded, which consisted of a partial Einstein ring of
Table 1. SN 2016geu: vital statistics.

| SN 2016geu | Reference |
|------------|-----------|
| RA(J2000) = 21°04′15″86 | G17 |
| Decl.(J2000) = −06°20′24.″9 | G17 |
| zSN = 0.409 | G17 |
| zlens = 0.216 | G17 |
| V\text{max} = 4.37 ± 0.15 | G17 |
| E(B−V) = 0.31 ± 0.05 mag | G17 |
| \(t_0(JD) = 245 7629.9 ± 0.3\) | G17 and this work |
| \(m^\text{peak}_B = 18.42 ± 0.22\) | G17 |
| \(t_\text{max}^\text{peak}(JD) = 245 7654.6 ± 0.2\) | G17 |
| \(\Delta m_{15,B} = 1.08 ± 0.19\) | G17 and this work |
| \(v^\text{max}_{\text{SII}} = 11 950 ± 140\text{ km s}^{-1}\) | This work |
| \(v_{10} = 10 850 ± 250\text{ km s}^{-1}\) | This work |
| \(i = −110.3 ± 10.0\text{ km s}^{-1}\) | This work |

Note. G17: Goobar et al. (2017).

3 DATA REDUCTION

We obtained four epochs of spectroscopy of SN 2016geu with GTC/OSIRIS as part of the GTC DDT proposal GTC2016-058 (PI: Z. Cano) using grism R1000R. The GTC spectra were reduced using standard techniques with IRAF-based scripts. The reduced GTC spectra were flux calibrated with the r-band acquisition images obtained each night using aperture photometry, where the aperture size was chosen so that the entire SN+host+ lens system was contained within the aperture. The calibration was performed via a zero-point, which was determined using two dozen SDSS standard stars in the SN field of view, and the AB flux density was converted from units of erg s\(^{-1}\) cm\(^{-2}\) Hz\(^{-1}\) into units of erg s\(^{-1}\) cm\(^{-2}\) Å\(^{-1}\) using an effective wavelength for the GTC/OSIRIS r-band filter of \(\lambda_0 = 6122.3\text{ Å}\). A normalization factor between the flux density of the r-band acquisition image and the GTC/OSIRIS spectrum was manually determined and applied to the latter. Two spectra were obtained with the X-shooter instrument (Vernet et al. 2011) mounted on Unit Telescope 2 (UT2, Kueyen) of the Very Large Telescope (VLT) at the Paranal Observatory as part of ESO Programme ID 098.A-0648(A) (PI: J. Hjorth), which were reduced using the VLT/X-shooter pipeline version 2.8.4 (Modigliani et al. 2010) run in physical mode along with dedicated PYTHON scripts for the post-processing of the images [see Gall et al. (in preparation) for a full description of the X-shooter data reduction and calibration procedure]. All the absolute-flux-calibrated spectra were then corrected for line-of-sight extinction, i.e. foreground Milky Way extinction (Schlegel, Finkbeiner & Davis 1998; Schlafly & Finkbeiner 2011), and rest-frame extinction (Goobar et al. 2017). A summary of all spectra used in this current analysis is given in Table 2, while the complete spectroscopic time series is presented in Fig. 1.

4 LENS GALAXY SUBTRACTION

As the spectroscopic slit covered not only SN 2016geu, but also part of its host galaxy and the lensing galaxy (both of which are thought to be elliptical galaxies; Goobar et al. 2017), the obtained spectra consist of integrated light from three distinct components at two different redshifts. Removing this contaminating flux is non-trivial, especially as the spectra were obtained with the two instruments at different angles (the GTC spectra were obtained always at the parallactic angle, whereas the X-shooter were obtained at +86°), and different slit widths (1 arcsec for the GTC spectra, and 1.0 arcsec/0.9 arcsec/0.9 arcsec for the UVB/VIS/NIR X-shooter spectra). Undoubtedly these different instrument setups will result in different amounts of contaminating host and lens flux.

To the best of our knowledge there is no ideal method to remove this flux, nevertheless we have attempted to remove this flux using the following method, but are mindful of the caveats arising from the different instrument configurations. Therefore, in order to correctly investigate the pWs of SN 2016geu (Section 6), the spectra need to be cleaned of these contaminating sources. To accomplish this, the ‘colour matching’ technique outlined in Foley et al. (2012) was employed. As the relative flux contributions and spectral shapes of the lensing galaxy, host galaxy and the SN are unknown in the spectra, the optimal approach is to make the lens- and host-subtracted spectra match the SN colours, the latter of which are obtained through difference imaging. For consistency, the spectra were all scaled to the acquisition camera magnitudes using an aperture that covered the entire source. Additionally, we used a Mannucci elliptical template (Mannucci et al. 2001, 2005) to represent the host and lensing galaxies.

To do the colour matching, we determined the SN colours at each epoch by fitting the light curves published by Goobar et al. (2017) with the SALT2 light-curve fitting tool (Guy et al. 2007), which were executed using SNCosmo (Barbary et al. 2016) and the best-fitting light curves were obtained through the use of EMCEE (Foreman-Mackey et al. 2013). The best-fitting model parameters are \(t_0 = 57654.20^{+0.20}_{-0.20}, x_0 \times 10^3 = 3.96^{+0.10}_{-0.10}, x_1 = 0.15^{+0.20}_{-0.20}\) and \(C = 0.25^{+0.04}_{-0.04}\). We corrected for Milky Way dust extinction using the dust maps of Schlegel et al. (1998) and Schlafly & Finkbeiner (2011). These derived values are entirely consistent with those obtained by Goobar et al. (2017) (see e.g. Section 2).

Equipped with the SALT2 model light-curve fits, we calculated equivalent synthetic magnitudes from each spectroscopic
Table 2. SN 2016geu: spectroscopy log.

| UT date       | $t - t_0$ (d)$^a$ | $\Delta t_{\text{max}}^B$ (d)$^b$ | Wavelength range (Å) | Telescope and instrument | Grism   | Exposure (s) |
|---------------|-------------------|-------------------------------|----------------------|--------------------------|---------|--------------|
| 2016 October 15 | 33.7              | +16.2                         | 5100–10000           | GTC-OSIRIS               | R1000R  | 5 $\times$ 300 |
| 2016 October 18 | 35.2              | +17.7                         | 3200–22000           | VLT-X-shooter UVB+VIS+NIR| 4 $\times$ 1184 |
| 2016 October 30 | 43.8              | +26.2                         | 3200–22000           | VLT-X-shooter UVB+VIS+NIR| 4 $\times$ 1184 |
| 2016 November 18 | 57.8              | +40.3                         | 5100–10000           | GTC-OSIRIS               | R1000R  | 3 $\times$ 900  |
| 2016 November 30 | 66.3              | +48.8                         | 5100–10000           | GTC-OSIRIS               | R1000R  | 3 $\times$ 1200 |
| 2016 December 18 | 77.0              | +59.4                         | 5100–10000           | GTC-OSIRIS               | R1000R  | 4 $\times$ 1200 |

Notes. All times and phases are given in the rest frame of SN 2016geu ($z = 0.409$).

$^a$Phase relative to the estimated time of explosion ($t_0 = 2457629.9 \pm 0.3$).

$^b$Phase relative to the peak B-band light ($t_{\text{max}}^B = 245 7654.6 \pm 0.2$, see Goobar et al. 2017).

Figure 1. Spectroscopic time series of SN 2016geu obtained with GTC/OSIRIS (blue) and VLT/X-shooter (black), which have been dereddened for foreground and host extinction. Contributions from the lensing and host galaxies have not been removed. Observer-frame and rest-frame wavelengths are shown on the bottom and top x-axes, respectively. Times (rest frame) are given relative to peak B-band light ($t_{\text{max}}^B = 245 7654.6$). Regions of contamination from telluric lines are denoted as the shaded-grey regions. All spectra are in their native resolution, and some strong lines related to noise have been removed manually for presentation purposes.

observation using the SDSS ugriz filter transmission curves\(^1\) and the two X-shooter spectra (as they span the widest wavelength range and hence more photometric bands that can be used to constrain the relative contribution of the lens and host). For both X-shooter observations, the host contamination contribution was found to be negligible compared to the lens. However, as we see the Ca H&K absorption lines from the host galaxy, and host flux is clearly visible in the adaptive optics, K-band Keck/Osiris observations presented in Goobar et al. (2017) (their fig. 3), we know that there is some host contamination, but the colour matching is best achieved with a single template (i.e. lens galaxy only). Next, we used the colour-matching technique on the GTC spectra, but as the wavelength coverage of the GTC spectra is insufficient to constrain both the lens and host contributions, we were forced to assume no host contribution based on the results of the colour-match method from the X-shooter spectra. Hence, the relative SN and lens contributions were determined, and the latter removed, from the GTC spectra.

\(^1\) http://www.sdss.org/instruments/camera/#Filters
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Figure 2. Comparison of the rest-frame spectra of SN 2016geu (which include contributions from the underlying host and lensing galaxy) with two nearby SNe Ia: SN 1994D (green) and SN 2011fe (red). Left-hand panel: GTC spectrum obtained 16.2 d after peak B-band light. Right-hand panel: X-shooter spectrum obtained 26.2 d after peak B-band light. Note that the X-shooter spectrum spans a wider wavelength range than the GTC one. The main absorption features seen in the comparison spectra are presented, which consist of iron-peak elements (Fe II, Cr II and Co II) and intermediate-mass elements (Ca II, Mg II, Na I and Si I). Many of the features in the SN 2016geu spectra can also be attributed to these elements and ions. Both SN 2016geu spectra have been smoothed, and regions of telluric absorption (oxygen bands A & B, and water) removed, for presentation purposes.

Finally, the derived \( p_Ws \) (Section 6) and comparison with synthetic spectra (Section 8), have had the lensing galaxy contribution removed using the above-described method. However, there may be a small, but unknown, contribution from the underlying host galaxy that we have been unable to quantify and remove. As such, strictly speaking the \( p_Ws \) should be considered as lower limits, as any additional host-template spectra removal will change, and likely decrease, the continuum levels of the spectra.

5 COMPARISON WITH OTHER SNE Ia

To date, several nearby SNe Ia have been detected within half a dozen Mpc. Their close proximity has led to the acquisition of exquisite high-cadence and excellent S/N photometric and spectroscopic data sets. One such event, which is now widely considered to be the archetype SN Ia, is SN 2011fe, which exploded in the Pinwheel Galaxy (\( D = 6.4 \) Mpc, Paturel et al. 2003), and it was shown to suffer from very little, if any reddening in its host galaxy (Pereira et al. 2013). Even closer SNe Ia have been detected, including SN 2014J (which exploded in M82, at a distance of \( D = 3.5 \) Mpc), however, unlike SN 2011fe, SN 2014J suffered from a large amount of extinction (\( E(B - V) = 1.22 \pm 0.5 \) mag), and like other highly reddened SNe Ia (e.g. SN 2006X, Wang et al. 2008), had a reddening law of \( R_V = 1.4 \pm 0.1 \) (Amanullah et al. 2014; Goobar et al. 2014).

In this work, we compared our GTC and X-shooter spectroscopic observations of SN 2016geu with those of two nearby, well-observed SNe Ia: SN 1994D (Patat et al. 1996), which exploded in NGC 4526 (distance of \( 16.9 \pm 1.6 \) Mpc) and SN 2011fe (Pereira et al. 2013). These data sets provide us with a tool to determine the spectral phase and a first-order approximation of the chemical content of the ejecta of SN 2016geu via direct comparison. The spectra of the comparison SNe Ia were dereddened for foreground (Schlafly & Finkbeiner 2011) (assuming \( R_V = 3.1 \)) and rest-frame extinction using the values from the aforementioned papers. The rest-frame reddening was applied to rest-frame spectra (\( z = 0.001 494 \) and 0.000 80, respectively). We compared the spectra of SN 2016geu from two (rest-frame) epochs: the first GTC spectrum obtained at \( t_{\text{max}} = +16.2 \) d, and the second X-shooter spectrum obtained at \( t_{\text{max}} = +26.2 \) d, which are shown in Fig. 2.

Visual inspection of the first GTC spectrum reveals several typical transitions seen in SN Ia at a couple weeks past peak light (e.g. Parrent, Friesen & Parthasarathy 2014), including blended iron-peak elements (Fe II, Cr II and Co II), and several intermediate-mass elements (Ca II, Mg II, Na I and Si I). The strong absorption feature near 6150 Å, which is likely blueshifted Si II \( \lambda 6355 \), is clearly present in the first GTC spectrum (\( +16.2 \) d) and the two X-shooter spectra (\( +17.7 \) and \( +26.2 \) d), but absent in the latter three (later than \( +40.3 \) d), implying that the layers probed by the photosphere at these epochs are devoid of this element.

6 LINE VELOCITIES AND (PSEUDO-)EQUIVALENT WIDTHS

6.1 Line velocities

Under the assumption that the absorption feature near 6150 Å is unblended Si II \( \lambda 6355 \), we fit it with a single Gaussian profile to determine its minimum, and hence its blue-shifted velocity. Using a method employed in previous works (Can et al. 2014, 2015, 2017), we used PYTHON to do the fitting, and employed a bootstrap (i.e. a Monte Carlo sampling and replacement) routine to estimate the errors in the local minima by utilizing the error spectrum from each epoch. In this manner, both the statistical and flux errors were appropriately considered. Systematic errors will undoubtedly
be introduced from the selection of the pW end-points, and we have endeavored to be as consistent as possible with the spectra obtained with different instruments. Nevertheless, such errors are likely present and may be underrepresented in the quoted pW errors. The bootstrap-fitting procedure was performed on our GTC and X-shooter spectra, as well as those published in Goobar et al. (2017), which were retrieved from the Weizmann Interactive Supernova data REPository2 (WiseREP, Yaron & Gal-Yam 2012). All spectra were corrected for foreground and host extinction and converted into rest-frame wavelengths. A plot of the expansion velocity of Si ii λ6355 for SN 2016geu is presented in Fig. 3, and the values can be found in Table 3. Data of the comparison SNe Ia, which were chosen to represent the various sub-types of SNe [i.e. normal, faint, high velocity (HV), etc.; see Section 7], are from Phillips et al. (1992), Garavini et al. (2004), Wang et al. (2009), Marion et al. (2013), Marion et al. (2015) and Pan et al. (2015), and references therein.

Next, we determined the rate of change of the Si ii λ6355 expansion velocity by fitting a straight line to the data to determine its slope (\( \dot{v} \)) and y-intercept (i.e. the expansion velocity at peak B-band light). The errors were determined using an additional bootstrap (i.e. Monte Carlo sampling and replacement using the error spectra) analysis. The best-fitting values are \( \dot{v} = -110.3 \pm 10.0 \text{ km s}^{-1} \text{ d}^{-1} \) and \( v_{\text{y}} = 11.950 \pm 140 \text{ km s}^{-1} \). The velocity at \( v_{\text{y}} + 10 \) d is \( v(10) = 10.850 \pm 250 \text{ km s}^{-1} \). We note that the inferred velocity at peak B-band light is calculated from the assumption that the Si ii λ6355 expansion velocity evolves linearly for times from peak light

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**Table 3.** SN 2016geu: line velocities and (pseudo-)equivalent widths.

| Phase | \( v_{6355} \) (10^3 km s\(^{-1}\)) | \( pW1 \) (Å) | \( pW3 \) (Å) | \( pW4 \) (Å) | \( pW7 \) (Å) | Telescope | Spectra Reference |
|-------|-----------------|-------|-------|-------|-------|----------|-----------------|
| +7.9 | 11.04 ± 0.19    | 80.3 ± 12.8 | 118.8 ± 17.2 | 143.8 ± 13.1 | 65.4 ± 17.4 | P200     | SN+host+lens   |
| +7.9 | 11.15 ± 17.9    | 157.4 ± 24.1 | 197.3 ± 18.3 | 69.4 ± 24.5 | P200     | SN+host   |
| +9.3 | 10.93 ± 0.16    | 69.2 ± 11.2 | 121.5 ± 12.4 | 140.5 ± 11.6 | 70.4 ± 4.5 | P200     | SN+host+lens   |
| +9.3 | 12.75 ± 15.7    | 202.4 ± 17.4 | 242.5 ± 16.2 | 98.3 ± 12.0 | P200     | SN+host   |
| +11.9| 10.68 ± 0.20    | 80.0 ± 14.3 | 173.3 ± 11.5 | 182.0 ± 9.4  | 76.4 ± 14.2 | NOT      | SN+host+lens   |
| +11.9| 14.14 ± 20.0    | 314.6 ± 29.6 | 282.2 ± 15.1 | 64.4 ± 19.9 | NOT      | NOT      |
| +16.2| 10.08 ± 0.18    | 44.4 ± 17.8 | 174.8 ± 13.4 | 183.7 ± 16.2 | >39.6    | GTC      | SN+host+lens   |
| +16.2| 177.0 ± 32.0    | 483.3 ± 25.3 | 439.1 ± 18.5 | 115.2 ± 11.9 | GTC      | SN+host   |
| +17.7| 10.07 ± 0.30    | 52.4 ± 12.4 | 186.9 ± 16.6 | 187.0 ± 8.0  | 103.1 ± 13.2 | XS       | SN+host+lens   |
| +17.7| 124.9 ± 23.4    | 398.2 ± 50.9 | 376.9 ± 19.9 | 210.7 ± 15.6 | XS       | SN+host   |
| +26.2| 9.04 ± 0.30     | 36.8 ± 10.0 | 117.0 ± 18.7 | 188.1 ± 8.6  | 115.0 ± 7.3 | XS       | SN+host+lens   |
| +26.2| 108.3 ± 33.6    | 410.0 ± 73.1 | 334.0 ± 18.7 | 264.2 ± 17.5 | XS       | SN+host   |

**Note.** 
*Phase relative to the peak B-band light.*

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Figure 3. Blueshifted velocity of the Si ii λ6355 transition. Left-hand panel: A comparison with other well-observed SNe Ia. Right-hand panel: The velocity evolution with time for SN 2016geu is \( \dot{v} = 110.3 \pm 10.0 \text{ km s}^{-1} \text{ d}^{-1} \), which is steeper than the combined (all), post-maximum sample of Folatelli et al. (2013) by \( \approx 2\sigma \).

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2 https://wiserep2.weizmann.ac.il/
and later (i.e. we have simply extrapolated to $t_{\text{max}}$ using the linear fit). Inspection of the $\mathrm{Si\ II}$ λ6355 velocity evolution of the comparison SNe Ia in Fig. 3 reveals that while some do evolve linearly from peak $B$-band light onwards, for example, SN 2002bo, which is an HV SN Ia (see Section 7), and SNe 1994D and 2005cf, which are normal SNe Ia, others do not (e.g. SNe 1984A and 2006X, both HV SNe Ia).

### 6.2 Pseudo-equivalent widths

The expansion velocity, and its rate of change with time, can be used to sub-classify a given SN Ia (see Section 7). Another useful classification diagnostic is the strength of various transitions in the optical spectra of SNe Ia, which are inferred by the equivalent widths.

#### 7 CLASSIFYING SN 2016GEU

In order to better understand the diversity of SNe Ia, and hence facilitate their use as standardizable candles with ever-improving precision, several statistical studies have been performed. The results of these studies are sub-classifications of SNe Ia, based primarily on different spectral features, including blueshifted velocities and equivalent widths of several Ca, Si, Mg and Fe transitions.

#### 7.1 The Benetti et al. (2005) scheme

Benetti et al. (2005) investigated a sample of $N = 26$ SNe Ia, including SNe 1991T and 1991bg, and found three general sub-classes: FAINT, high velocity gradient (HVG) and low velocity gradient (LVG). In their classification scheme, the primary indicator was the $\mathrm{Si\ II}$ λ6355 transition velocity, as well as its rate of change with time ($\dot{\nu}$). FAINT SNe Ia had peak magnitudes commensurate with that of SN 1991bg ($(M_B) = -17.2, \sigma = 0.6 \text{ mag}$), low expansion velocities $(v_{10} = 9200 \text{ km s}^{-1}, \sigma = 600 \text{ km s}^{-1})$ and rapid velocity gradients $(\dot{\nu} = -87 \text{ km s}^{-1} \text{ d}^{-1}, \sigma = 20 \text{ km s}^{-1} \text{ d}^{-1})$. LVG SNe Ia, as their name suggests, have large expansion velocities $(v_{10} = 12200 \text{ km s}^{-1}, \sigma = 1100 \text{ km s}^{-1})$ and gradients $(\dot{\nu} = -97 \text{ km s}^{-1} \text{ d}^{-1}, \sigma = 16 \text{ km s}^{-1} \text{ d}^{-1})$, while those of the LVG group have much slower velocity gradients $(\dot{\nu} = -37 \text{ km s}^{-1} \text{ d}^{-1}, \sigma = 18 \text{ km s}^{-1} \text{ d}^{-1})$ and intermediate expansion velocities $(v_{10} = 10300 \text{ km s}^{-1}, \sigma = 300 \text{ km s}^{-1})$. The

Figure 4. Evolution of the $pW$s features with time: $pW1$ (Ca ii H&K, in blue), $pW3$ (Mg ii, in red), $pW4$ (Fe ii, in green), and $pW7$ (Si ii λ6355, in black). Values derived from the spectra containing flux from the SN + host + lens are shown as filled circles and solid lines, while those obtained from the lens-subtracted spectra are shown as filled triangles and dashed lines.

For SN 2016geu, the $pW1$ values of the SN + host + lens slightly decrease over the time of the observations from 80 Å to 40 Å, but with large error bars. For the lens-subtracted spectra, the initial and final values are around 110–120 Å, and reach a maximum of almost 180 Å in the GTC spectrum at $t_{\text{max}} + 16$ d.

The $pW3$ (Mg ii) values in the CSP sample also have a wide range of values at all epochs (relative to $B$-max), but most have a value near $pW3 \approx 100$ Å near peak $B$-band light, and a sharp increase to ~250–300 Å between 5 and 15 d post-peak. Similar behaviour is seen here for SN 2016geu, where the SN + host + lens $pW3$ values increase from ~120 Å to ~170 Å between 9.3 d and 11.9 d past $B$-max. For the lens-subtracted spectra, the $pW3$ values increase steadily from 160–480 Å from $t_{\text{max}} + 7.9$ to +16.2 d, and level out around 400 Å in the final two epochs. We note that the $pW3$ values from the GTC and X-shooter spectra taken two days apart differ by about 85 Å, where the GTC spectrum gives larger values. Interestingly, in the final X-shooter spectrum, the values for the lens-subtracted/unsubtracted spectra differ by a factor of almost four, meaning that the removal of the elliptical lens template was vital for properly charting the evolution of this particular $pW$s.

Next, the $pW4$ values also evolve in tandem with the $pW3$ values, for both the lens-subtracted and -unsubtracted spectra, and they do so at a rate similar to that seen in the CSP sample. Finally, the $\mathrm{Si\ II}$ λ6355 feature ($pW7$), is also seen to increase in the CSP sample in the 20 d following peak $B$-band light, increasing from ~100 Å to ~200 Å (with large scatter) during this time. For SN 2016geu, the value of $pW7$ also increases with time, though its value at all epochs appears to occur at the lower end of the $pW7$ distribution seen in the CSP sample for the unsubtracted spectra, while the lens-subtracted values are generally larger and more closely match the behaviour seen in the CSP sample. Early on, the $pW7$ values are similar to those of the SS sample, but then evolve around $t_{\text{max}} + 15$ to +20 d to those of the CN/CL/BL SNe Ia (bottom right-hand panel in Fig. 5).
The lens-subtracted $pW$ values of SN 2016geu (filled black stars and solid lines) relative to the CSP SN Ia sample in Folatelli et al. (2013) and Gall et al. (2017). The colours and filled symbols indicate the different sub-types: CN (core-normal, red circles), CL (cool, orange squares), BL (broad-lined, blue triangles) and SS (shallow silicon, green diamonds). In general, the $pW$ values of SN 2016geu are consistent with the comparison samples within their respective errorbars. Some differences are seen for the $pW_3$ values, where the maximum values from $t_{\text{max}}^B \approx +15$ d and onwards are larger than the comparison samples. Moreover, initially the $pW_7$ values of SN 2016geu appear similar to the SS (green) sample, but then increase to match those of the other sub-type samples after $t_{\text{max}}^B \approx +15$ d. The evolution of the $pW_4$ values also appears to be different to the comparison samples, where the decrease at late epochs is generally not seen in the latter samples.

The latter two groups have similar absolute, peak $B$-band magnitudes ($\langle M_B \rangle = -19.3$, $-19.3$, respectively) and similar light-curve widths ($\langle \Delta m_{15,B} \rangle = 1.2, 1.1$ mag, respectively).

In the Benetti et al. (2005) classification scheme, SN 2016geu has a light-curve width ($\langle \Delta m_{15,B} \rangle = 1.1 \pm 0.2$ mag) similar to the averages of the HVG and LVG groups, but is more than $8\sigma$ smaller than the average of the FAINT group. In terms of the $v_{10}$ diagnostic, the value measured for SN 2016geu is almost $3\sigma$ larger than the FAINT average, and within $1\sigma$ and $2\sigma$ of the HVG and LVG group averages, respectively. Finally, in terms of the velocity gradient, SN 2016geu is consistent with both the FAINT and HVG groups, but is more than $4\sigma$ larger than the average of the LVG group. Of all three groups, SN 2016geu is most consistent with the HVG group, and quite inconsistent with the FAINT group.

7.2 The Branch et al. (2006) scheme

Building upon the classification scheme devised by Folatelli (2004), and extended by several authors (Garavini et al. 2007; Blondin et al. 2012), and notably Branch et al. (2006), Folatelli et al. (2013) analysed a spectroscopic data set of 604 spectra of $N = 93$ SNe Ia that spanned a range of $-12$ to $+150$ d relative to peak $B$-band light. Based on the strengths of the $pW_6$ and $pW_7$ ($\lambda 5972$ and $\lambda 6355$, respectively; see Section 6), four sub-classifications were devised: (i) cool (CL): $pW_6 > 30$ Å; (ii) broad-lined (BL): $pW_6 < 30$ Å and $pW_7 > 105$ Å; (iii) shallow-silicon (SS): $pW_7 \leq 70$ Å; and (iv) core-normal (CN): $pW_6 \leq 30$ Å and $70 \leq pW_7 \leq 105$ Å.

Furthermore, the $\lambda 6355$ velocity evolution of the four sub-classes differ (near peak $B$-band light), as depicted in Fig. 3: (i) CL: $-177 \pm 20$ km s$^{-1}$ d$^{-1}$; (ii) BL: $-250 \pm 36$ km s$^{-1}$ d$^{-1}$; (iii) SS: $-42 \pm 16$ km s$^{-1}$ d$^{-1}$; and (iv) CN: $-86 \pm 14$ km s$^{-1}$ d$^{-1}$. The combined value for all sub-classes near peak $B$-band light was $-112 \pm 17$ km s$^{-1}$ d$^{-1}$ and $-77 \pm 16$ km s$^{-1}$ d$^{-1}$ for the phase range of $+7$ to $+25$ d from peak $B$-band light.

For SN Ia spectra near peak light, both $pW$ regions are visible; however, a couple of weeks later $\lambda 5972$ ($pW_6$) is no longer detectable, but $\lambda 6355$ is, as in the case of SN 2016geu. The classifications above are based on the strength of the $pW$ features at peak light, whereas the earliest epoch investigated here corresponds to $t_{\text{max}}^B = +7.9$ d (from Goobar et al. 2017). Inspection of fig. 11 from Folatelli et al. (2013) shows that, for a given SN Ia, the value of $pW_7$ is approximately the same for the $5$–$10$ d either side of $t_{\text{max}}^B$. 

Figure 5.
7.3 The Wang et al. (2009) scheme

Wang et al. (2009) investigated a sample of \( N = 158 \) SNe Ia near peak \( B \)-band light, and classified their SNe into two sub-classes: normal and HV. These two groups were based on the \( \text{Si II} \lambda 6355 \) velocity at peak \( B \)-band light, where the samples had averages of \( \langle v \rangle = 10600 \pm 400 \text{ km s}^{-1} \) and \( \langle v \rangle \geq 11800 \text{ km s}^{-1} \), respectively. Wang et al. (2009) find that the HV group has narrower \( M_B \) and \( \Delta m_{15,B} \) distributions than the normal group, and there is overlap in phenomenological properties of the HV group with the HVG of Benetti et al. (2005). These authors find that the normal group has a velocity evolution after peak \( B \)-band light of \( \langle i \rangle \approx -40 \text{ km s}^{-1} \text{ d}^{-1} \), which is similar to that measured for the LVG of Benetti et al. (2005).

The value of \( v_B^{\text{max}} = 11950 \pm 140 \text{ km s}^{-1} \) measured for SN 2016geu places it in the HV group, and is more than \( 3\sigma \) more rapid than the normal group average.

8 SPECTRAL MODELLING WITH SYN++

In Section 5, visual comparison of the GTC and X-shooter spectra with those of well observed, nearby SNe Ia 1994D and 2011fe revealed the presence of iron-peak elements and intermediate-mass elements. To gain a deeper understanding of which elements in the SN ejecta may be responsible for the various absorption features in the spectra of SN 2016geu, we created synthetic SN spectra using SYN++ (Thomas, Nugent & Meza 2011) for two of the spectra presented here: \( t_B^{\text{max}} + 16.2 \) and \( +26.2 \) d. The original and synthetic spectra are presented in Fig. 6, while the best-fitting parameters are given in Table 4.

In the left-hand panel of Fig. 6 are the GTC spectra from \( t_B^{\text{max}} + 16.2 \). When creating the synthetic SYN++ spectra, we assumed an excitation temperature of \( T_{\text{exc}} = 7000 \text{K} \) following the SYNOW modelling performed by Branch et al. (2008) for SNe Ia a similar post-maximum epoch. The photospheric velocity \( (v_p) \), the blackbody temperature \( (T_{\text{BB}}) \), the density of each element/ion \( (\log \tau) \) and its maximum velocity \( (v_{\text{max}}) \) were manually changed to produce a good visual fit. Following Parrent et al. (2012), we used a constant term of 1.40 and a quadratic warp term of \(-1.9\), to get the synthetic spectrum to match the luminosity of the GTC spectrum, and to obtain good agreement between the synthetic spectrum and the observations at the blue end. For the X-shooter spectrum, we used a constant term of 0.9 and a linear term of \(-1.2\).

As seen visually in Fig. 2, the elements/ions responsible for the main absorption features are (in increasing atomic number): \( \text{Na I, Mg II, Si II, Ca II, Ti II, Cr II, Fe II and Co II} \). \( \text{Si II} \) is, as expected, most prominent in the wavelength range \( 6000 \lambda \leq 6330 \text{Å} \), while also contributing some of the absorption seen between 3600 and 3900 Å. Much of the absorption seen between 4000 and 5300 Å can be attributed to \( \text{Fe II} \), with minor contributions from \( \text{Cr II} \) over the same wavelength interval. \( \text{Ti II} \) also contributes to absorption between 3500 and 4650 Å, in particular, the absorption line near 4190 Å. \( \text{Na I} \) can be attributed to the feature between 5560 and 5800 Å, while the absorption bluewards of 4000 Å is due to \( \text{Ca II} \) (H&K).

Modest absorption due to \( \text{Mg II} \) is seen between 4290 and 4450 Å and due to \( \text{Co II} \) between 3500 and 4650 Å, which is the most
prominent absorption near \( \approx 4000 \, \text{Å} \). The densities of each element/ion are similar \((\log \tau = -0.1\) to \(-0.5\)) however, that of Cr II is significantly larger \((\log \tau = 2.0)\). The maximum velocity of each element/ion ranges from 12,000 km s\(^{-1}\) for Co II to 22,000 km s\(^{-1}\) for Ca II. The maximum velocity of Fe II was found to be 16,000 km s\(^{-1}\); larger velocities resulted in too much line blending in the synthetic spectra. The X-shooter spectrum from \( t_{\text{max}}^{+} = 26.2 \) is shown in the right-hand panel of Fig. 6, which spans a wider range in wavelength than the earlier GTC spectrum. This time we are able to investigate both the blue and red Ca II absorption features. As before, we kept the excitation temperature at 7000 K. The BB temperature was 6200 K and the photospheric velocity was found to be 8000 km s\(^{-1}\). Given that only 10 d had passed since the GTC epoch, the elements and ions likely responsible for the various absorption features are similar to the previous epoch. Indeed the relative densities and maximum velocities are comparable between the two epochs, though the \(\log(\tau)\) values are generally smaller in this epoch. The strongest transitions and on the cusp between the LVG and HVG group classifications. Visual comparison with the nearby SNe Ia revealed line-transitions corresponding to several iron-peak and intermediate-mass elements, which are typical of post-maximum SNe Ia at \( +2 \) to \(+4\) weeks.

Next, we classified SN 2016geu using the schema of Benetti et al. (2005), Branch et al. (2006) and Wang et al. (2009):

(i) Benetti et al. (2005): SN 2016geu is most consistent with the HVG group and inconsistent with the FAINT group.
(ii) Branch et al. (2006): SN 2016geu is most consistent with the CN group and inconsistent with the BL, SS and CL classes.
(iii) Wang et al. (2009): SN 2016geu is most consistent with the HV group and more than 3\( \sigma \) more rapid than the normal sub-class.

It is interesting to note here that the CN SNe Ia in the CSP sample (Folatelli et al. 2013) are not HV objects – indeed there are very few cases where a SN Ia has been classified as both a CN and an HV (e.g. SNe 2006is and 2005ku, both in the CSP sample). Instead, most of the HV objects in their sample belong to the BL class.

When fitting the \( pWs \), we found that both the Fe II and Mg II \( pWs \) increased sharply around \( t_{\text{max}}^{+} = 15\) d. This behaviour was also observed in the sample investigated by Folatelli et al. (2013). The \( \lambda 6355\) \( pW \) increased steadily over the investigated time-period, while the Ca II H&K \( pW \) remained constant. The strength of a given \( pW \) can be used as a proxy for how much of a given element is present at or above the photosphere at the observation time. Over time, the SN ejecta expands and cools, and the observer is offered a deeper and deeper view of the SN ejecta. Thus, if the value of a given \( pW \) decreases as the velocity decreases (and hence time increases), this can imply a more central location of the element/ion. Thus, the measured increase in the \( pWs \) associated with Fe II and Mg II may imply a more central location for a sizable amount of these species. In contrast, the smoother \( pW \) evolution of Si II \( \lambda 6355 \) implies a more mixed and homogeneous distribution. We note, however, that this conclusion is rather simplified, and it does not consider the effects arising from temperature and the recombination rates of various elements in the ejecta. As such, while an increase of a given \( pW \) measurement of a given element/ion can imply its increased abundance, temperature can affect the recombination rate, which, in turn, also affects the strength of the measured \( pWs \). Moreover, as the Fe and Mg \( pWs \) are affected by blending, their measured changes can arise from changes in the blending of the various transitions.
giving rise to the specific $pW$, rather than an increase in its relative abundance. Thus, the inferred location of the Fe, Mg and Si in the SN ejecta needs to be interpreted with these caveats in mind.

Next, we created synthetic spectra using the 1D SYN++ code (Thomas et al. 2011) for the two aforementioned GTC and X-shooter spectra. As from the visual comparison with the nearby SNe Ia, the main elements and ions needed to reproduce the observations were iron-peak elements (Ti II, Cr II, Fe II and Co II) and Sn Ia, the main elements and ions needed to reproduce the X-shooter spectra. As from the visual comparison with the nearby (Thomas et al. 2011) for the two aforementioned GTC and Cr II and Fe II increased in the second epoch, implying their increased abundance. This increase also appears to correspond with the increase in their measured $pW$ values.

Based on the above classifications and modelling, we can conclude with some confidence that SN 2016geu is a normal SN Ia, and it possesses properties similar to SNe Ia in the local Universe. This result supports the conclusions of Hook et al. (2005) and Petrushevska et al. (2017), which suggest there is very little, if any, difference in the observational properties of SNe Ia up to $z \approx 0.4$, as shown here, and perhaps even up to $z \approx 1.4$, as shown by the latter authors. We note that the redshift of SN 2016geu is at the lower end of the redshift range of SNe Ia investigated by Maguire et al. (2012), and hence we cannot comment on the slight redshift-variance observed in their sample. Inevitably more moderate to high-$z$ SNe Ia are required to further test the assumption of the invariance of their observational properties, the conclusions of which will have severe implications on their continued use in SN-cosmology.

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