Testing Higgs Self-Couplings at High-Energy Linear Colliders

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Abstract. In order to verify the Higgs mechanism experimentally, the Higgs self-couplings have to be probed. These couplings allow the reconstruction of the characteristic Higgs potential responsible for the electroweak symmetry breaking. The couplings are accessible in a variety of multiple Higgs production processes. The theoretical analysis including the most relevant channels for the production of neutral Higgs boson pairs at high-energy and high-luminosity $e^+e^-$ linear colliders will be presented in this note.

1. Within the Higgs mechanism [1] the electroweak gauge bosons and fundamental matter particles acquire their masses through the interaction with a scalar field. The self-interaction of the scalar field induces, via a non-vanishing field strength $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV, the spontaneous breaking of the electroweak $SU(2)_L \times U(1)_Y$ symmetry down to the electromagnetic $U(1)_{EM}$ symmetry.

To establish the Higgs mechanism experimentally, the self-energy potential of the Standard Model [2], $V = \lambda (\Phi^\dagger\Phi - v^2/2)^2$, with a minimum at $\langle \Phi \rangle_0 = v/\sqrt{2}$ must be reconstructed. This task requires the measurement of the Higgs self-couplings of the physical Higgs boson $H$, which can be read off directly from the potential

$$V = \frac{M_H^2}{2} H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4$$

(1)

As evident from Eq. (1), in the SM the trilinear and quadrilinear vertices are uniquely determined by the mass of the Higgs boson, $M_H = \sqrt{2\lambda}v$.

The trilinear self-coupling $\lambda = 6\sqrt{2}\lambda$ in units of $v/\sqrt{2}$ is accessible directly in Higgs pair production at high-energy $e^+e^-$ linear colliders. For c.m. energies up to about 1 TeV, double Higgs-strahlung [3,4]

$$e^+e^- \rightarrow ZHH$$

(2)

is the most promising process [5]. The process includes the amplitude involving the trilinear Higgs self-coupling and two additional amplitudes due to the standard electroweak gauge interactions, cf. Fig. 1, so that it is a binomial in $\lambda_{HHH}$. 
FIGURE 1. Subprocesses contributing to double Higgs-strahlung, $e^+e^- \rightarrow ZHH$, in the Standard Model at $e^+e^-$ linear colliders.

FIGURE 2. (a) The cross section for double Higgs-strahlung, $e^+e^- \rightarrow ZHH$, in the Standard Model at two collider energies: $\sqrt{s} = 500$ GeV and 800 GeV. The electron/positron beams are taken oppositely polarized. The vertical bar corresponds to a variation of the trilinear Higgs coupling between 0.8 and 1.2 of the SM value. (b) The cross section for $WW$ double Higgs fusion, $e^+e^- \rightarrow \nu_e\bar{\nu}_e HH$, at $\sqrt{s} = 500$ GeV, 1 TeV and 1.6 TeV. The initial beams are polarized.

As evident from Fig. 2a the cross section is very sensitive to the trilinear Higgs self-coupling and amounts up to 0.35 fb for $M_H = 120$ GeV and a c.m. energy of 500 GeV. Scaling with the energy, it decreases to 0.3 fb at $\sqrt{s} = 800$ GeV. Experimental detector simulations of signal and background processes in the SM have demonstrated that the Higgs self-coupling can be extracted with an accuracy of $\sim 20\%$ for $M_H = 120$ GeV at high luminosity $\int \mathcal{L} = 2$ ab$^{-1}$ [6].

The $WW$ double Higgs fusion process [4,7]

$$ e^+e^- \rightarrow \nu_e\bar{\nu}_e HH \tag{3} $$

which increases with rising $\sqrt{s}$, can be exploited for larger energies, cf. Fig. 2b. [$\sigma = 0.37$ fb for $M_H = 120$ GeV and $\sqrt{s} = 1$ TeV, polarized $e^+e^-$ beams.]

Triple Higgs production is sensitive to the quadrilinear Higgs self-coupling. Due to the suppressed coupling and an additional particle in the final state, the cross section $\sigma(e^+e^- \rightarrow ZHHH)$ is only of $\mathcal{O}(ab)$ and therefore not measurable at typical linear collider energies and luminosities [5].

2. In the Minimal Supersymmetric Extension of the Standard Model (MSSM) with five physical Higgs particles $h, H, A$ and $H^\pm$ [8], a plethora of trilinear and quadrilinear Higgs self-couplings can be realized. The CP-invariant couplings
TABLE 1: The trilinear couplings between neutral CP-even and CP-odd MSSM Higgs bosons, which can generically be probed in double Higgs-strahlung and associated triple Higgs-production, are marked by a cross. [The matrix for WW fusion is isomorphic to the matrix for Higgs-strahlung.]

| $\lambda$ | double Higgs-strahlung | triple Higgs-production |
|-----|-----------------|-----------------|
| $h h h$ | $\times$ | $\times$ |
| $H h h$ | $\times \times$ | $\times \times$ |
| $H H h$ | $\times \times$ | $\times \times$ |
| $H H H$ | $\times$ | $\times$ |
| $h A A$ | $\times$ | $\times \times \times$ |
| $H A A$ | $\times$ | $\times \times \times$ |

among the neutral Higgs bosons, $\lambda_{h h h}, \lambda_{H h h}, \lambda_{H H h}, \lambda_{H H H}, \lambda_{h A A}, \lambda_{H A A}$, are involved in a large number of processes [5,9]. The double and triple Higgs production processes and the trilinear couplings, that can be probed in the respective process, are listed in Table 1. The system is solvable for all $\lambda$’s up to discrete ambiguities. However, in practice, not all processes are accessible experimentally so that one has to follow the reverse direction in this case: Comparing the theoretical predictions with the experimental results of the accessible channels, the trilinear Higgs self-couplings can be tested stringently.

The process $e^+ e^- \rightarrow Z h h$ is sensitive to the trilinear coupling of the light CP-even Higgs boson $h$,

$$\lambda_{h h h} = 3 \cos 2\alpha \sin(\beta + \alpha) + \mathcal{O}(G_F M_t^4/M_Z^2)$$

expressed in the mixing angles $\alpha$ and $\beta$, in a large range of the MSSM parameter space, as can be inferred from Fig. 3. It shows the $2\sigma$ sensitivity area in the [$M_A, \tan \beta$] plane for a non-zero coupling at an integrated luminosity of 2 ab$^{-1}$. The cross section is required to exceed 0.01 fb. The sensitivity areas are significantly smaller for processes involving heavy Higgs bosons $H$ and $A$ in the final state. Details can be found in Ref. [5].

### 3. In summary.

The measurement of the Higgs self-couplings is essential for the reconstruction of the characteristic self-energy potential. The large luminosities, which are available at future high-energy $e^+ e^-$ linear colliders, allow the measurement of the trilinear Higgs self-couplings via double Higgs-strahlung and WW double Higgs fusion.

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FIGURE 3. MSSM: Sensitivity to the coupling $\lambda_{hhh}$ of the light CP-even neutral Higgs boson $h$ in the process $e^+e^- \rightarrow Zhh$ for a collider energy of 500 GeV (no mixing).

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