QoS Aware Robot Trajectory Optimization With IRS-Assisted Millimeter-Wave Communications

CRISTIAN TATINO©1,2 (Member, IEEE), NIKOLAOS PAPPAS©3 (Senior Member, IEEE), AND DI YUAN©4 (Senior Member, IEEE)

1Department of Science and Technology (ITN), Linköping University, 581 83 Linköping, Sweden
2Department of Baseband Radio Resource Management, Ericsson AB, 16483 Stockholm, Sweden
3Department of Computer and Information Science, Linköping University, 581 83 Linköping, Sweden
4Department of Information Technology, Uppsala University, 752 36 Uppsala, Sweden

CORRESPONDING AUTHOR: C. TATINO (e-mail: cristian.tatino@ericsson.com)

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ABSTRACT This paper considers the motion energy minimization problem for a robot that uses millimeter-wave (mm-wave) communications assisted by an intelligent reflective surface (IRS). The robot must perform tasks within given deadlines, and it is subject to quality of service (QoS) constraints. This problem is crucial for fully automated factories governed by the binomial of autonomous robots and new generations of mobile communications, i.e., 5G and 6G. In this new context, robot energy efficiency and communication reliability remain fundamental problems that couple in optimizing robot trajectory and communication QoS. More precisely, to account for the mutual dependency between robot position and communication QoS, robot trajectory and beamforming at the IRS and access point all need to be optimized. We present a solution that can decouple the two problems by exploiting mm-wave channel characteristics. Then, a closed-form solution is obtained for the beamforming optimization problem, whereas the trajectory is optimized by a novel successive-convex optimization-based algorithm that can deal with abrupt line-of-sight (LOS) to non-line-of-sight (NLOS) transitions. Specifically, the algorithm uses a radio map to avoid obstacles and poorly covered areas. We prove that the algorithm can converge to a solution satisfying the Karush-Kuhn-Tucker conditions. The simulation results show a fast convergence rate of the algorithm and a dramatic reduction of the motion energy consumption with respect to methods that aim to find maximum-rate trajectories. Moreover, we show that passive IRSs represent a powerful solution to improve the radio coverage and motion energy efficiency of robots.

INDEX TERMS Energy efficient motion, intelligent reflective surface, millimeter-waves, robot path planning.

I. INTRODUCTION

Robotics and wireless technologies are driving the new industrial revolution, i.e., Industry 4.0, and playing a crucial role in the digital transition of manufacturing processes, warehousing, and logistics [2]. However, the massive exploitation of robots and the rising of new industrial applications, with stringent quality of service (QoS) requirements, will stress the performance of the next generation of mobile communications, i.e., 6G. Specifically, real-time industrial applications, such as augmented and virtual reality for assisted manufacturing or mining, may require Gbps for peak data rates [3], [4]. Moreover, swarms consisting of hundreds of sensing robots in future warehouses may need to operate with latency and reliability of 1 ms and up to 99.9999%, respectively [3], [5].

Millimeter-wave (mm-wave) spectrum has been identified as a possible solution for providing wireless communications in industrial scenarios [6]. However, high-band communications suffer from high blockage sensitivity [7], [8] that reduces communication reliability when a robot moves...
in environments with obstacles. In addition to avoiding obstacles, trajectory planning highly affects mm-wave performance as it determines whether the robot is in line-of-sight (LOS) or non-line-of-sight (NLOS). Moreover, robots are battery-limited and have tasks usually characterized by stringent deadlines. By optimizing the robot’s movement, it is possible to dramatically decrease its energy consumption with a significant reduction in the total electrical energy consumption for manufacturing processes. Consequently, in the last decades, robot trajectory planning has been one of the most relevant problems in robotics [9]–[14]. This problem has assumed increasing importance for wirelessly connected robots, where trajectories must be optimized according to the radio coverage [15]–[17].

Besides trajectory optimization, several solutions have been proposed to enhance the coverage in mm-waves scenarios, e.g., relays [18] and intelligent reflective surfaces (IRSs) [19], [20]. IRSs consist of arrays of reflective elements that can be electronically controlled to adjust the angle and the phase of the reflected signals to be either added coherently or destructively to the receiver [21]–[23]. Due to the short wavelength at mm-wave frequency ranges, IRSs with many reflective elements can be deployed to provide strong alternative signal paths when the LOS path is blocked, which improves robot communications’ throughput and reliability. Moreover, in comparison to active relays, the lower energy consumption and cost [24], [25] make passive IRSs an ideal candidate for increasing the energy efficiency of fully autonomous robots. The energy consumption can be further reduced when energy-harvesting IRSs can be deployed [26]. However, beamforming at the IRS must be set according to the channel that depends on the robot trajectory. Therefore, we consider a trajectory and beamforming co-optimization to minimize the motion energy consumption of a wirelessly connected robot in IRS-assisted mm-wave scenarios.

To solve this problem, we propose a modified successive convex optimization (SCO) algorithm that accounts for the knowledge of the environment and a radio map to avoid collisions and satisfy time and QoS constraints.

A. RELATED WORKS

Energy-aware trajectory optimization has been one of the most crucial problems in robotics [9]–[12]. In [9], the authors model the power consumption of a DC motor-equipped robot as a function of the speed. By optimally controlling the robot’s speed, it is possible to achieve up to 50% energy-saving. The work in [10] uses a graph-based method and A* algorithm to determine the robot’s minimum cost path, where the cost of an edge represents the corresponding motion energy consumption. A convex optimization approach is adopted in [11], which presents an alternating quadratic programming method to determine the path that minimizes the energy consumption in scenarios with obstacles. Multi-robot scenarios are studied in [12], where the authors propose a distributed algorithm for optimizing locations and times of robots’ rendezvous. This problem includes the battery level of the robots as a constraint. The corresponding energy consumption is derived from the optimal control problem that minimizes the energy consumption along the robot trajectory. In contrast to the previously mentioned studies, the work in [15] deals with wirelessly connected robots. The authors propose joint robot communication and motion energy minimization by controlling the transmit power and the robot’s speed along a fixed trajectory.

Controlling the robot motion introduces a new degree of freedom for resource allocation problems in wireless communications. In the past few years, several studies have explored this topic [15]–[17], [27]–[30]. Similar to [15], the work in [16] proposes an optimal control problem for motion and communication energy minimization subject to a certain amount of data to be transmitted and power limit. In this case, multiple robots transmit data to an access point (AP) in a scenario without obstacles. In [17], the authors define a convex optimization problem to minimize the energy consumption of a moving relay and multiple mobile sensing robots by controlling trajectories and transmit power. The sensing robots move in a scenario without obstacles along paths that depend on the assigned tasks. Joint task and trajectory optimization for multiple robots is explored in [27]–[30], where [30] deals with directional communication scenarios.

Directional communications are typical for mm-wave communications that has recently attracted the interest of industrial applications [4], [31]–[34]. Mm-wave connected robots are considered in [34], where the authors present several instances of an association and path planning problem in multi-AP mm-wave networks. A graph-based algorithm is used in [34] to minimize handovers and travel time of multiple robots, where a radio map is used to account for obstacles and communication blockages. However, the potentials of mm-waves transmissions for wirelessly connected robots need to be further explored. Several studies have been performed for unmanned aerial vehicle (UAV) scenarios, as shown in [35], where joint communication and trajectory planning problems are particularly relevant [36]–[41]. Specifically, similar to [34], the work in [41] uses a graph-based method and a radio map to optimize UAVs’ flying distance while ensuring a target QoS requirement.

In the last few years, researchers have considered the application of IRSs to enhance UAV-user communications [42]–[44]. Especially in [42] and [43], authors study UAV-users communications assisted by stationary IRSs. They study joint UAV trajectory and IRS beamforming optimization problems to maximize the users’ received power. A similar scenario is considered in [44], where IRSs mounted on UAVs are used to establish indirect LOS links to maximize the minimum rate among user clusters. The resulting joint UAV positioning and IRS beamforming optimization problem is solved by using a hybrid particle swarm optimization-based heuristic algorithm. IRSs for enhancing low-frequency robot-AP communications are studied in [45], where the authors use a graph-based method to minimize the robot traversal time while ensuring a
minimum data rate requirement at each location along the trajectory. However, as detailed in later sections, maximum rate trajectories and graph-based methods would provide high energy consumption or infeasible trajectories if applied to our problem.

B. CONTRIBUTIONS

The contributions of this paper are summarized as follows:

- We consider a novel robot trajectory optimization problem with IRS-assisted mm-wave communications. The problem aims to minimize the motion energy consumption while satisfying the minimum average data rate and deadline constraints. Moreover, the robot must avoid collisions with obstacles. To solve this problem, we account for the mutual dependence of the energy consumption and achieved data rate on the robot trajectory as well as beamforming (at the AP and IRS). To the best of our knowledge, energy-efficient trajectory planning problems have not been considered for wirelessly connected robots using mm-waves, which have peculiar signal propagation conditions.

- In mm-wave scenarios, when obstacles create abrupt LOS-NLOS transitions, the received data rate is not a convex function of the robot’s position. Therefore, to solve the problem mentioned above, we propose a novel method that combines SCO-based algorithms with radio map information. The former exploits a convex approximation of the data rate that is obtained by fitting the analytical expression of the signal-to-noise ratio (SNR) with the radio map itself. The latter is used to account for the abrupt SNR variations caused by LOS-NLOS transitions. Previous SCO-based algorithms for UAVs, e.g., in [42], [43], may lead to infeasible paths or higher energy consumption if applied to our problem. Moreover, UAV communications do not present the same characteristics as wirelessly connected robots of which the altitude cannot be adapted. Graph-based methods can also include radio maps [45], but they cannot account for the average data rate constraint and may poorly perform when applied to our problem.

- We prove that the proposed SCO algorithm converges and, under certain conditions, it converges to a point satisfying KKT conditions. The proposed algorithm can dramatically reduce the robot’s energy consumption with respect to trajectories that maximize the data rate. Moreover, we show that IRSs can enhance the motion energy efficiency for QoS-constrained wirelessly connected robots. Specifically, the algorithm converges to a solution corresponding to the minimum energy consumption trajectory by increasing the number of IRS’s reflective elements.

The rest of the paper is organized as follows. In Section II we describe the system model. In Section III, we formulate the problem; moreover, we decouple beamforming and trajectory optimizations. In Section IV, we solve the latter by using an SCO algorithm, and in Section V, we provide performance evaluation. Finally, Section VI concludes the paper.

II. SYSTEM MODEL AND ASSUMPTIONS

We consider an industrial scenario, e.g., an industrial plant, where a robot moves from a starting position \( q_s \) to its goal \( q_d \) within a time horizon of fixed duration. The robot moves on the horizontal plane of a 3D restricted area containing several 3D obstacles. These are represented by a set \( O \) of cylinders with elliptic bases and given heights.\(^1\) The area is covered by an AP using mm-waves to which the robot needs to transmit uplink data by maintaining a given communication QoS. This is expressed as a minimum average data rate requirement\(^2\) \( (r_{\text{min}}) \). The robot is equipped with a single antenna, whereas the AP is equipped with \( N \) antennas. The robot-AP communication is assisted by a passive IRS consisting of a uniform linear array (ULA)\(^3\) of \( M \) reflective elements, of which the phase shifters are adjusted by a controller (IRS controller in Fig. 1). The controller shares the channel state information (CSI) with the AP. A scheduler, which we assume to be co-located with the controller, optimizes the robot trajectory. The goal is to minimize the motion energy consumption accounting for both active and passive beamforming at the AP and IRS, respectively.

**Notations:** we denote scalars by italic letter, whereas vectors are denoted by bold-face letters. \((.)^T\) and \((.)^H\) represent the transpose and the conjugate transpose, respectively; \( \text{diag}(.) \) returns the diagonalization of a vector and \( \arg(\cdot) \)

\(^1\) Note that arbitrarily shaped obstacles can be often approximated by the intersection and the union of several convex shapes [46]. In this paper, we consider 3D cylinders with elliptic bases.

\(^2\) A minimum average data rate requirement can model applications where robots can store data in a buffer and transmit when channel conditions are favorable. This choice makes the problem more general, and the solution presented in this manuscript can be easily extended to instantaneous data rate requirements.

\(^3\) This study can be generalized to IRSs consisting of uniform planar arrays (UPA). Specifically, the problem formulation and algorithm in Sections III and IV, respectively, are still valid for the UPA case, for which we show the results in Section V-C.
TABLE 1. Summary of the notation.

| Symbol | Description |
|--------|-------------|
| \( Q \) | robot trajectory |
| \( q_s \) | robot’s starting position |
| \( q_a \) | AP’s position |
| \( z_a \) | AP’s height |
| \( d_{a,k} \) | robot-AP distance when the robot is at position \( q_k \) |
| \( \mathcal{O} \) | obstacle set |
| \( K \) | number of timeslots |
| \( D_{\text{max}} \) | maximum distance that a robot can travel in a timeslot |
| \( M \) | number of reflective elements at the IRS |
| \( h_{r,k} \) | robot-IRS channel when the robot is at position \( q_k \) |
| \( \theta_{m,k} \) | phase shift of reflective element \( m \) when the robot is at position \( q_k \) |
| \( \mathbf{w}_k \) | normalized beamforming vector when the robot is at position \( q_k \) |
| \( E \) | total motion energy consumption |
| \( \text{SNR}_k \) | received SNR when the robot is at position \( q_k \) |
| \( \tau \) | achieved data rate vector along the robot trajectory |
| \( \tilde{\text{SNR}}_k \) | estimated optimal SNR when the robot is at position \( q_k \) |
| \( r_{\text{opz}} \) | concave approximation of the average achieved data rate |
| \( T_k \) | trust region for \( q_k \) |
| \( \tilde{\nu} \) | estimated robot-IRS path loss exponent |
| \( Q_0 \) | initial solution for Algorithm 1 (RMAP) |
| \( q_k \) | \( k \)-th robot’s position along the trajectory |
| \( q_d \) | robot’s final position |
| \( q_e \) | IRS’s position |
| \( z_e \) | IRS’s height |
| \( d_{r,k} \) | robot-IRS distance when the robot is at position \( q_k \) |
| \( q_{c,o} \) | center’s position of obstacle \( o \) in \( \mathcal{O} \) |
| \( r_{\text{min}} \) | minimum average data rate requirement |
| \( N \) | number of antennas at the AP |
| \( G_k \) | AP-IRS channel when the robot is at position \( q_k \) |
| \( H_k \) | robot-AP channel when the robot is at position \( q_k \) |
| \( \Phi_k \) | phase shifts matrix when the robot is at position \( q_k \) |
| \( B_w \) | system bandwidth |
| \( E_k \) | robot motion energy consumption in timeslot \( k \) |
| \( r_k \) | achieved data when the robot is at position \( q_k \) |
| \( \tau^* \) | average achieved data rate |
| \( \tau_{\text{map}} \) | optimized data rate |
| \( \tilde{\tau} \) | average achieved data rate obtained by the radio map |
| \( \mu \) | trust region reduction parameter |
| \( \tilde{Q}_j \) | estimated robot-AP path loss exponent solution of Algorithm 1 (RMAP) at iteration \( j \) |

denotes the phase of a complex number. Finally, \( ||.||_n \) represents the \( n \)-norm. Moreover, a summary of the notation is available in Table 1.

A. ROBOT MOTION MODEL

The robot must avoid collisions with obstacles and reach the destination within a given deadline. We divide the time horizon, which is defined by the deadline, in \( K \) small slots of duration \( \Delta_t \). Within a timeslot, the robot can travel for a maximum distance of \( D_{\text{max}} = v_{\text{max}} \Delta_t \), where, \( v_{\text{max}} \) is the maximum speed. A trajectory of the robot is represented as a sequence of \( K+1 \) positions, i.e., \( Q = \{q_0, q_1, \ldots, q_K\} \), where \( q_0 = q_s \) and \( q_K = q_d \). The terms \( q_k = [x_k, y_k] \), \( k = 0, \ldots, K \), represent the Cartesian coordinates of the robot’s positions on the horizontal plane along the trajectory.

Let \( q_s = [x_s, y_s] \) and \( q_f = [x_f, y_f] \) be the fixed positions of the AP and the IRS, respectively. The altitude of the robot is fixed at its antenna height \( z_r \). The AP and the IRS are installed at heights \( z_a \) and \( z_e \), respectively. Let \( v_k \) be the speed of the robot at the \( k \)-th timeslot. Then, the motion energy consumption of the DC motor-equipped robot along the path can be written as follows [15]:

\[
E = \sum_{k=1}^{K-1} E_k = \sum_{k=1}^{K} c_1 \| v_k \|^2 \Delta_t + c_2 v_k \Delta_t + c_3 \Delta_t,
\]

where, \( c_1, c_2, \) and \( c_3 \) are positive constants depending on the characteristics of the robot and external load.

B. CHANNEL MODEL

As shown in Fig. 1, let \( \mathbf{h}_r \in \mathbb{C}^M \) be the channel vector between the robot and the IRS and \( \mathbf{G} \in \mathbb{C}^{M \times N} \) denote the channel matrix between the IRS and the AP. The direct channel between the robot and the AP is represented by vector \( \mathbf{h}_d \in \mathbb{C}^N \). Then, the received baseband signal at the AP when the robot is at position \( q_k \) can be written as follows:

\[
y_k = (\mathbf{h}_{r,k}^H \Phi_k \mathbf{G}_k + \mathbf{h}_{d,k}^H) \mathbf{w}_k \sqrt{p_t} s_k + \eta_k, \tag{2}
\]

where, \( s_k \) and \( p_t \) are the transmit signal and the transmit power in the uplink, respectively, \( \eta_k \sim \mathcal{C}(0, \sigma^2) \) denotes the additive white Gaussian noise (AWGN). The term \( \mathbf{w}_k \in \mathbb{C}^M \) is the normalized beamforming vector at the AP, and \( \Phi_k = \text{diag}(e^{j\theta_{1,k}}, \ldots, e^{j\theta_{M,k}}) \) is a diagonal matrix that accounts for the phase shifts \( \theta_{m,k} \in [0, 2\pi) \) associated with the reflective elements of the IRS. This IRS model can capture the dependency between the received QoS and the number of IRS’s reflective elements by keeping a good level of tractability and low energy consumption [47]. Due to the high path loss of mm-wave transmissions, signals that are reflected more than once are subject to severe attenuation and are not considered in (2). Thus, the received SNR for \( q_k \) can be written as follows:

\[
\text{SNR}_k = \frac{|(\mathbf{h}_{r,k}^H \Phi_k \mathbf{G}_k + \mathbf{h}_{d,k}^H) \mathbf{w}_k|^2}{\sigma^2 p_t}, \tag{3}
\]

where, the superscript \( H \) represents the hermitian. Moreover, let \( d_{r,k} = (z_r - z_i)^2 + ||q_k - q_{k-1}||_2^2 \) and \( d_{a,k} = (z_r - z_i)^2 + ||q_k - q_{k-1}||_2^2 \) be the robot-IRS and robot-AP distances, respectively. Then, channel vectors \( \mathbf{h}_{r,k} \) and \( \mathbf{h}_{d,k} \) can be modeled as follows:

\[
\mathbf{h}_{r,k} = \sqrt{p_d} \tilde{\mathbf{h}}_{r,k}, \tag{4}
\]

\[
\mathbf{h}_{d,k} = \sqrt{p_d} \tilde{\mathbf{h}}_{d,k}, \tag{5}
\]

where, \( \tilde{\mathbf{h}}_{r,k} \sim \mathcal{C}(0, \mathbf{I}) \) and \( \tilde{\mathbf{h}}_{d,k} \sim \mathcal{C}(0, \mathbf{I}) \) are complex Gaussian vectors whose elements are independent and identically distributed (i.i.d) with zero means and unit
Finally, for fixed $\Phi_k$, $w_k$, and position $q_k$, we obtain the data rate by using Shannon’s formula as follows:
\[
r_k = B_w \log_2 (1 + SNR_k) = B_w \log_2 \left( 1 + \frac{|\langle pd_{d,k}^{-\frac{1}{2}} h_{d,k}^H \Phi_k G_k + \langle pd_{a,k}^{-\frac{1}{2}} h_{a,k}^H \rangle w_k |^2}{\sigma^2} p_i \right). \tag{6}
\]

To obtain (6) we use (3), (4), and (5). The term $B_w$ represents the system bandwidth. Let $\mathbf{r} = [r_0, r_1, \ldots, r_K]$ be a vector, of which the elements represent the data rates along the robot trajectory $Q = [q_0, q_1, \ldots, q_K]$. Thus, the average data rate for a trajectory $Q$ is given by:
\[
\bar{r} = \frac{1}{K} \sum_{k=0}^{K} r_k. \tag{7}
\]

We can observe that $\bar{r}$ is a function of $\Phi_k$, $w_k$, and $q_k$. The latter is included in $d_{i,k}$ and $d_{a,k}$. Thus, the robot’s position and beamforming affect the data rate, which in turn affects the trajectory and QoS. To account for these dependencies, in the next section, we formulate the joint beamforming and trajectory optimization problem introduced in this section.

### III. PROBLEM FORMULATION

In this section, we formulate the problem introduced in Section II. Let $\Phi = [\Phi_0, \Phi_1, \ldots, \Phi_K]$ and $w = [w_0, w_1, \ldots, w_K]$, then the joint robot trajectory and beamforming problem can be formulated as follows:

**P1:**
\[
\min_{Q, \Phi, w} \quad E \tag{8a}
\]
\[
s.t. \quad \bar{r} \geq r_{\text{min}}, \quad (8b)
\]
\[
\|q_k - q_{k-1}\|_2 \leq D_{\text{max}}, \quad k = 1, \ldots, K, \quad (8c)
\]
\[
q_0 = q_s, \quad q_K = q_f, \quad (8d)
\]
\[
(q_k - q_{c,o})^T P_o^{-1} (q_k - q_{c,o}) \geq d_s, \quad \forall k, \forall o \in \mathcal{O}, \quad (8e)
\]
\[
\|w_k\|_2^2 \leq 1, \quad \forall k, \quad (8f)
\]
\[
\Phi_k = \text{diag}(e^{i\theta_{k,1}}, \ldots, e^{i\theta_{k,M}}), \quad \forall k, \quad (8g)
\]
\[
0 \leq \theta_{m,k} \leq 2\pi, \quad \forall m, \forall k, \quad (8h)
\]

where, the objective function (8a) is given by (1) and represents the total robot motion energy consumption along the trajectory. Note that the communication energy consumption of the robot is negligible with respect to the motion energy consumption; hence, the objective function does not include the former. The first constraint (8b) represents the QoS requirement to complete the task, where $\bar{r}$ is defined in (7) and $r_{\text{min}}$ is the minimum required average data rate. Constraints (8c) allow the robot to move in a timeslot for a maximum distance of $D_{\text{max}}$, whereas (8d) fix the starting and the goal positions. To avoid collisions with obstacles, we include (8e). More precisely, as described in the previous section, obstacles are approximated by the intersection and the union of several ellipsoids $o \in \mathcal{O}$ on the horizontal plane. Each of them is described by a center $q_{c,o}$ and a symmetric and positive definite matrix $P_o$. The latter defines the length and the rotation of the ellipse. The term $d_s \geq 1$ represents a safety distance between the robot and the obstacle. Finally, constraints (8f) and (8h) impose the norm of $w_k$ to be at most one and $\theta_{m,k}$ to be continuous, respectively.

Problem P1 is non-linear and non-convex. However, as shown in the following sections, it is possible to decouple the beamforming and the trajectory optimization problems. More precisely, we maximize the left-hand side (LHS) of (8b) in P1 by deriving closed-forms of $\Phi$ and $w$ that maximize the average data rate for each trajectory. Then, we can obtain an optimization problem equivalent to P1, of which the only optimization variable is $Q$. This resulting trajectory optimization problem is solved in Section IV by using an SCO-based algorithm.

### A. AVERAGE RATE MAXIMIZATION

In this section, we first find closed-form solutions of $\Phi$ and $w$ that maximize average data rate $\bar{r}$. Specifically, for a fixed trajectory we solve the following problem:

**P2:**
\[
\max_{\Phi, w} \quad \bar{r} \tag{9a}
\]
\[
s.t. \quad (8f), (8g), (8h), \quad (9a)
\]

where, $\bar{r}$ is given by (7).

**Proposition 1:** By replacing the LHS of (8b) in P1, with the optimum data rate resulting from solving P2, we obtain a trajectory optimization problem that is equivalent to P1.

**Proof:** We first note that in P1, the LHS of (8b) is the only expression that depends on $\Phi_k$ and $w_k$, which are not contributing to the cost (8a). Moreover, by solving P2, we obtain an optimum data rate expression ($\bar{r}^*\Phi^*$) that depends only on $Q$ such that $\bar{r}^* \geq \bar{r} \forall Q$. Thus, by replacing the LHS of (8b) in P1 with $\bar{r}^*$ we obtain an optimization problem that depends only on $Q$ with a feasible region that includes the feasible region of P1.

To solve P2, we can assume that the IRS and the AP are installed with a LOS link. In mm-wave communications, the LOS path presents a much higher gain than the sum of NLOS paths. Thus, the IRS-AP channel can be approximated by a rank-one matrix [20]:

\[
G_k = \sqrt{NMd_{\text{los}}^2} \tilde{G}_k = \sqrt{NM} \gamma \tilde{a}_i \tilde{b}_k^T, \quad \forall k, \quad (10)
\]

where, $\gamma = \sqrt{\rho_{\text{los}}^2}$. The term $d_{\text{los}}$ is the distance between the AP and the IRS that is fixed and does not depend on
\( q_k \). The path loss exponent of the LOS path between the AP and the IRS is two and \( \rho \) accounts for the path loss at the reference distance and antenna gain. The terms \( \tilde{a}_k \in \mathbb{C}^M \) and \( \tilde{b}_k \in \mathbb{C}^N \) are the normalized array response vectors at \( q_k \) associated with the IRS and the AP, respectively. These can be expressed as follows:

\[
\tilde{a}_k = \frac{1}{\sqrt{M}} \left[ 1, e^{-j \frac{2\pi}{\lambda} w a_k}, \ldots, e^{-j \frac{2\pi}{\lambda} w(M-1)a_k} \right],
\]

\[
\tilde{b}_k = \frac{1}{\sqrt{N}} \left[ 1, e^{-j \frac{2\pi}{\lambda} w b_k}, \ldots, e^{-j \frac{2\pi}{\lambda} w(N-1)b_k} \right],
\]

where, \( a_k \) is the cosine of angle-of-arrival (AoA) and \( b_k \) is the cosine of angle-of-departure (AoD). The term \( \lambda \) is the carrier wavelength, whereas \( w \) is the antenna separation.

Maximizing \( P_2 \) is equivalent to maximizing the received SNR at each robot’s position \( q_k \) (3). Assuming \( \bar{G}_k = \tilde{a}_k^* \tilde{b}_k^* \), and \( \Phi_k = e^{\psi_k} \tilde{b}_k^* \), this problem has a closed-form solution [20], which is given by:

\[
\psi_k^* = -\arg\left( \tilde{b}_k^* H \tilde{h}_{d,k} \right),
\]

\[
\Phi_k^* = \text{diag}(e^{-j \arg(g_{1,k})}, \ldots, e^{-j \arg(g_{M,k})}),
\]

\[
w_k^* = \frac{e^{\psi_k} \sqrt{\rho d_{l,k}^{-\nu} h_{l,k}^* \Phi_k^* G_k + \sqrt{\rho d_{r,k}^{-\mu} h_{r,k}^* \Phi_k^* G_k}}}{\|e^{\psi_k} \sqrt{\rho d_{l,k}^{-\nu} h_{l,k}^* \Phi_k^* G_k + \sqrt{\rho d_{r,k}^{-\mu} h_{r,k}^* \Phi_k^* G_k}}\|^2},
\]

where, \( g_k = \sqrt{\rho d_{l,k}^{-\nu} h_{l,k}^* \Phi_k^* G_k} \) and \( \circ \) denotes the element-wise product. By putting (13), (14), and (15) into (3), we obtain the following optimal SNR expression for \( q_k \):

\[
\text{SNR}_k^* = \left( N |\rho| |\gamma|^2 \| \tilde{h}_{d,k} \|_2 d_{l,k}^{-\nu} + 2\sqrt{2|\rho| |\gamma| \| \tilde{h}_{d,k} \|_2 d_{r,k}^{-\nu} d_{l,k}^{-\mu} + |\rho| \| \tilde{h}_{d,k} \|_2^2 d_{l,k}^{-\mu}} \right) \frac{P_t}{\sigma^2}.
\]

\[
= \left( A d_{l,k}^{-\nu} + B d_{r,k}^{-\nu} d_{l,k}^{-\mu} + C d_{l,k}^{-\mu} \right) \frac{P_t}{\sigma^2}.
\]

IV. TRAJECTORY OPTIMIZATION

In this section, we provide an algorithm to solve problem P3 that, as introduced in Section III-A, is a trajectory optimization problem. We first derive the following:

**Lemma 1:** Given \( c_1 \geq 0, c_2 \geq 0, \) and \( c_3 \geq 0, \) the objective function of P3 (19a) is a convex function of \( \bar{Q} \).

**Proof:** We prove Lemma 1 by induction. As in (1), let \( E|_{K=n} \) be the motion energy consumption of the robot when \( K = n \): \( E|_{K=n} = \sum_{k=1}^{n} c_1 \|q_k - q_{k-1}\|^2 + c_2 \|q_k - q_{k-1}\|_2 + c_3 \Delta t \). We first prove that \( E|_{K=1} \) is convex and then, by assuming that convexity holds for \( E|_{K=n-1} \) we prove that \( E|_{K=n} \) is a convex function of \( \bar{Q} = \{q_0, \ldots, q_n\} \). It is easy to show that \( E|_{K=1} = c_1 \|q_1 - q_0\|^2 + c_2 \|q_1 - q_0\|_2 + c_3 \Delta t \) is a convex function of \( q_0 \) and \( q_1 \) because it consists of the sum of two convex functions, i.e., \( c_1 \|q_1 - q_0\|^2 \) and \( c_2 \|q_1 - q_0\|_2 \), and a constant term. Now, assume that \( E|_{K=n-1} \) is convex, we consider \( E|_{K=n} = E|_{K=n-1} + c_1 \|q_n - q_{n-1}\|^2 + c_2 \|q_n - q_{n-1}\|_2 + c_3 \Delta t \). By following the same reasoning, we can observe that \( E|_{K=n} \) is the sum of three convex functions of \( \bar{Q} = \{q_0, \ldots, q_n\} \): \( E|_{K=n-1} \) that is convex by hypothesis, \( \|q_n - q_{n-1}\|^2 \), and \( c_2 \|q_n - q_{n-1}\|_2 \).

Thus, the objective function of P3 is a convex function of \( \bar{Q} \). However, P3 is non-convex because the LHS of (19b) and (8e) are not concave functions of \( q_k \). For this reason, we perform a convex local approximation of these...
two constraints and solve the problem iteratively by using an SCO algorithm. Starting from constraint (19b), we have the following lemma:

**Lemma 2:** Given \( \hat{A} \geq 0, \hat{B} \geq 0, \hat{C} \geq 0, \hat{\nu} \geq 0, \) and \( \hat{\mu} \geq 0, \) \( \hat{r}^* \) is a convex function of \( d_{a,k} \) and \( d_{i,k} \) with \( k = 0, \ldots, K. \)

**Proof:** See Appendix A.

Thus, since any convex function can be lower-bounded by its first-order Taylor expansion, we have the following:

\[
\hat{r}^* \geq \hat{r}_{apx}^* = \frac{B_w}{K} \sum_{k=0}^{K} \left[ \left(-\frac{\hat{d}_a d_{i,k}}{2\beta d_{a,k}^2 d_{i,k}^2 + \hat{d}_a d_{i,k}^2 + \hat{C} d_{a,k}^2 + \hat{C} d_{i,k}^2} \right) \right].
\]

We now consider the following lemma:

**Lemma 3:** Given non-negative parameters \( \hat{A}, \hat{B}, \hat{C}, \hat{\nu}, \) and \( \hat{\mu}, \) \( \hat{r}_{apx}^* \) is a concave function of \( q_k. \)

**Proof:** See Appendix B.

Thus, in a small neighborhood of \( d_{a,0,k} \) and \( d_{i,0,k}, \) we can derive \( \hat{r}_{apx}^* \) that is a concave function of \( q_k \) and a lower bound of \( \hat{r}^*. \) The same reasoning can be applied to constraints (8e) leading to the following inequality:

\[
\left(q_k - q_{c,o}\right)^T P_{o}^{-1} \left(q_k - q_{c,o}\right) \geq \left(q_{0,k} - q_{c,o}\right)^T P_{o}^{-1} \left(q_{0,k} - q_{c,o}\right) + \left(q_{0,k} - q_{c,o}\right)^T P_{o}^{-1} \left(q_k - q_{0,k}\right).
\]

We now consider an SCO algorithm. Starting from constraint (19b), we have

\[
\begin{align*}
\sum_{k=0}^{K} \log_2 \left( 1 + \left( \hat{d}_a d_{i,k} + \hat{d}_i d_{a,k} + \hat{C} d_{a,k} + \hat{C} d_{i,k} \right) \right) \\
\geq \log_2 \left( 1 + \left( \hat{d}_a d_{i,k} + \hat{d}_i d_{a,k} + \hat{C} d_{a,k} + \hat{C} d_{i,k} \right) \right)
\end{align*}
\]

We now consider the following lemma:

**Lemma 2:** Given non-negative parameters \( \hat{A}, \hat{B}, \hat{C}, \hat{\nu}, \) and \( \hat{\mu}, \) \( \hat{r}_{apx}^* \) is a concave function of \( q_k. \)

**Proof:** See Appendix B.

Thus, since any convex function can be lower-bounded by its first-order Taylor expansion, we have the following:

\[
\hat{r}^* \geq \hat{r}_{apx}^* = \frac{B_w}{K} \sum_{k=0}^{K} \left[ \left(-\frac{\hat{d}_a d_{i,k}}{2\beta d_{a,k}^2 d_{i,k}^2 + \hat{d}_a d_{i,k}^2 + \hat{C} d_{a,k}^2 + \hat{C} d_{i,k}^2} \right) \right].
\]

We now consider the following lemma:

**Lemma 3:** Given non-negative parameters \( \hat{A}, \hat{B}, \hat{C}, \hat{\nu}, \) and \( \hat{\mu}, \) \( \hat{r}_{apx}^* \) is a concave function of \( q_k. \)

**Proof:** See Appendix B.

Thus, since any convex function can be lower-bounded by its first-order Taylor expansion, we have the following:

\[
\hat{r}^* \geq \hat{r}_{apx}^* = \frac{B_w}{K} \sum_{k=0}^{K} \left[ \left(-\frac{\hat{d}_a d_{i,k}}{2\beta d_{a,k}^2 d_{i,k}^2 + \hat{d}_a d_{i,k}^2 + \hat{C} d_{a,k}^2 + \hat{C} d_{i,k}^2} \right) \right].
\]

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**Proof:** See Appendix B.

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\]

We now consider the following lemma:

**Lemma 3:** Given non-negative parameters \( \hat{A}, \hat{B}, \hat{C}, \hat{\nu}, \) and \( \hat{\mu}, \) \( \hat{r}_{apx}^* \) is a concave function of \( q_k. \)

**Proof:** See Appendix B.

Thus, since any convex function can be lower-bounded by its first-order Taylor expansion, we have the following:

\[
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\]

We now consider the following lemma:

**Lemma 3:** Given non-negative parameters \( \hat{A}, \hat{B}, \hat{C}, \hat{\nu}, \) and \( \hat{\mu}, \) \( \hat{r}_{apx}^* \) is a concave function of \( q_k. \)

**Proof:** See Appendix B.

Thus, since any convex function can be lower-bounded by its first-order Taylor expansion, we have the following:

\[
\hat{r}^* \geq \hat{r}_{apx}^* = \frac{B_w}{K} \sum_{k=0}^{K} \left[ \left(-\frac{\hat{d}_a d_{i,k}}{2\beta d_{a,k}^2 d_{i,k}^2 + \hat{d}_a d_{i,k}^2 + \hat{C} d_{a,k}^2 + \hat{C} d_{i,k}^2} \right) \right].
\]
the NLOS and LOS transitions created by the obstacles. The algorithm stops if the sequence of solutions converges or when a maximum number of iterations ($N_	ext{it}$) is reached. More precisely, we can prove that the algorithm converges and, under some conditions, it converges to a Karush-Kuhn-Tucker (KKT) point of $P3$.

**Theorem 1:** RMAP provides a non-increasing and convergent sequence of solutions. Moreover, if at each iteration $j$ we have that $\bar{r}_{\text{map},j} \geq r_{\text{min}}$, RMAP converges to a KKT point of $P3$.

**Proof:** The sequence of solutions provided by RMAP is non-increasing because, the solution of $P4$ at iteration $j - 1$, $\bar{Q}_{j-1}$, is a feasible solution of minimization problem $P4$ at iteration $j$. For the rest of the proof, see Appendix C and Appendix D.

**Proposition 2:** If the average measured data rate corresponding to the initial solution satisfies $\bar{r}_{\text{map},0} \geq r_{\text{min}}$, then the solution to which RMAP converges satisfies this constraint as well.

This is a direct consequence of Lines 7-9 of RMAP and Theorem 1. Namely, if $\bar{r}_{\text{map},j} < r_{\text{min}}$ then $\bar{Q}_j = \bar{Q}_{j-1}$. Moreover, assuming that the initial solution satisfies $\bar{r}_{\text{map},0} \geq r_{\text{min}}$ and RMAP converges to a solution, this solution must satisfy the above constraint.

**Proposition 2** highlights the importance of initial solutions, of which the quality affects the performance of SCO-based methods. The quality of a solution of SCO-based algorithms also depends on the initial solution. Specifically, different initial paths can lead to different local optima. In this work, we obtain $\bar{Q}_0$ by using a graph-based method. More precisely, we compute the shortest path on a time-expanded graph as done in [34]. The edges and vertices of the graph are defined on a discrete set of positions free from obstacles. In each timeslot, a robot may either stay at a vertex or move to an adjacent one. The distance between vertices is set according to the robot’s maximum speed. On this graph, the costs of the edges are set to generate two different initial solutions. The first one minimizes the motion energy consumption (ME), whereas the second solution maximizes the data rate (MR). The radio map is used to obtain the SNR and the data rate for the positions corresponding to vertices and edges. Then, RMAP uses initial solution ME if it is feasible and MR, otherwise. If the latter is not feasible, the algorithm declares infeasibility.

**V. NUMERICAL RESULTS**

In this section, we provide a numerical validation of RMAP for solving $P3$. Similar to [34], we consider a $50 \times 30$ m² rectangular-shaped indoor scenario. The robot’s starting position is [9.5, 15.5], whereas the destination is [40.5, 14.5]. There are an AP and an IRS placed at [25.30] and [25,0], respectively, operating in the 60 GHz band as in [33], with bandwidth $B_\nu = 200$ MHz. The height of the IRS is 2.5 m, whereas, as in [34], we set the heights of the AP to 5 m to account for the higher ceilings of industrial plants, and the robot’s antenna to 0.5 m. For the sake of clarity, we first present results for a scenario consisting of four ellipse obstacles that are placed as in Fig. 2, represented by the grey shaded areas (base scenario). This scenario includes several robot-AP and robot-IRS channel conditions, i.e., LOS and NLOS positions. Then, we present results that are averaged over ten instances in which 20 obstacles are randomly placed (random scenario). In both scenarios, the obstacles’ length, width, and height are 6 m, 4 m, and 2 m, respectively.

Similar to [33], the path loss at a reference distance of 1 m is 68 dB, and the path loss exponents of the robot-IRS and robot-AP channels in (4) and (5), respectively, are set to 2 for LOS, and 4.5 for NLOS. Without loss of generality, the antenna gain of the reflective elements is set to 0 dBi. Moreover, we set the transmit and the noise powers to 20 dBm, and $-80$ dBm, respectively. We show the results, for several values of $r_{\text{min}}$ and $M$. Specifically, given the area size and the considered $r_{\text{min}}$ values, as we can observe by the results in Section V-B, RMAP can provide solutions that approach a lower bound of $P3$ with $0 \leq M < 200$. Unless otherwise specified, we set the following parameters values: $K = 30$, $\tau = 0.5$, $\nu = 16$, $\Delta_t = 1$ s, $v_\text{max} = 3$ m/s, $d_1 = 1.35$, $N_\nu = 100$, $\epsilon = 0.01$, $T = 1$ m, $c_1 = 4.39$, $c_2 = 24.67$, and $c_3 = 14.77$ [15]. Finally, to derive (17), we estimate $\tilde{A} \geq 0$, $\tilde{B} \geq 0$, $\tilde{C} \geq 0$, $\tilde{c} \geq 0$, and $\tilde{p} \geq 0$. These parameters are obtained by fitting (16) on a radio map by solving a nonlinear least squares problem. The radio map is obtained from the averaging of 10,000 channel measurements on a grid of $500 \times 300$ points.

**A. BASE SCENARIO**

In Fig. 2, we first show ME and MR initial solutions for several values of $M$. The MR initial solution considers the trajectory that maximizes the data rate. Depending on the number of reflective elements ($M$), MR trajectories tend to avoid NLOS areas with respect to both the AP and the IRS.
In Fig. 3(a) we show robot trajectories resulting from RMAP for $K = 30$, $r_{min} = 2.0$ Gbps, and several values of $M$. For $M = 0$, we can observe that the robot avoids NLOS areas with respect to the AP. Specifically, RMAP uses initial solution MR. When $M$ increases, the IRS enhances the coverage such that the robot can find a trajectory with lower energy consumption ($E$) by using initial solution ME. The resulting trajectory crosses the NLOS area with respect to both the AP and the IRS. Note that, for $r_{min} = 2.0$ Gbps, values of $M$ that are higher than 64 do not provide further gain, thus the trajectories for $M = 64$ and $M = 128$ coincide. This is not true when $r_{min} = 2.5$ Gbps, for which the trajectories are shown in Fig. 3(b). More precisely, when $r_{min}$ increases, RMAP selects the MR initial solutions for all the values of $M$, and the resulting paths are either closer to the AP or the IRS to improve the coverage and increase the data rate. For $M = 0$ and $M = 64$, the robot trajectories completely avoid NLOS areas with respect to the AP, whereas, for $M = 128$, the paths closer to the IRS provide higher data rates. We can also observe that the robot decreases its speed when the data rate is higher. Specifically, in LOS positions closer to the AP and the IRS, the robot travels for a smaller distance in each timeslot to exploit better coverage.

In general, $E$ increases for higher values of $r_{min}$ and decreases when $M$ and $K$ increase. This is more evident in Fig. 4(a) and Fig. 4(b) where we show energy consumption $E_j$ corresponding to the sequence of solutions $Q_j$ provided by RMAP for several values of $M$, $r_{min}$, and $K$. First, we can observe that, by increasing $M$, RMAP converges to paths with lower $E$. The energy consumption also decreases when $K$ increases. Specifically, for a fixed value of $\Delta_t$, higher values of $K$ correspond to longer deadlines, and the robot can decrease the speed to reach the destination. Then, as described by (1), lower speeds correspond to lower values of $E$. Finally, we can observe that $E_j$ is non-increasing and RMAP converges in few iterations. However, as explained and shown better in the following section, the number of iterations within which RMAP converges depends on the value of $\tau$. 
B. RANDOM SCENARIO

In this section, we present results that are averaged over 10 instances. In each instance, \(N_o = 20\) obstacles are randomly placed. The area, the obstacles dimension, and the robot’s starting position and destination are the same that are used in Section V-A. In Fig. 5, we show the average energy consumption corresponding to the trajectory resulting from RMAP for several values of \(M\), \(r_{min}\), and \(K\). In addition, Fig. 5 depicts lower bounds to the average energy consumption that are computed by solving a relaxed version of problem P3, where obstacle avoidance (8e) and QoS (19b) constraints are relaxed. This relaxation results in a convex optimization problem that provides lower bounds to solutions of P3. Conversely, it is important to highlight that RMAP provides upper bounds to P3. Specifically, the gap between the solution provided by RMAP to the global optimum (optimality gap) is equal to or lower than the gap with respect to the lower bound that is shown in Fig. 5. Moreover, we show the average \(E\) corresponding to robot trajectories that maximize the data rate under time and collision avoidance constraints. Maximum data rate trajectories are obtained by solving a modified version of P3, where the LHS of (19b) represents the objective function of the problem. This problem is solved by using an SCO-based algorithm where, as done in Section IV for P3, the objective function and the LHS of (8e) are approximated by convex functions. Note that maximum-rate trajectories are feasible solutions of P3, whereas trajectories resulting from computing the lower bounds may not.

In Fig. 5, we can observe that, for both \(r_{min} = 2.0\) Gbps and \(r_{min} = 2.5\) Gbps, RMAP can reduce dramatically the average \(E\) with respect to the maximum data rate approach. Particularly, for \(r_{min} = 2.0\) Gbps and \(K = 40\) the gain is close to 100% and the optimality gap is less than 5%. Fig. 5 shows that the average \(E\) resulted from RMAP decreases and approaches the lower bound as \(M\) increases. Specifically, by increasing the number of reflective elements at the IRS, we enhance the coverage, and the robot can find a higher number of feasible trajectories. However, while for \(r_{min} = 2.5\) Gbps, increasing \(M\) results in a monotonic decrease of \(E\), for \(r_{min} = 2.0\) Gbps, we note that above a certain threshold (\(M \geq 32\)) increasing the value of \(M\) does not provide significant gains. Additional gain can be obtained by increasing the values of \(K\) as also explained in Section V-A. These observations do not hold for maximum data rate trajectories for which the corresponding energy consumption have not monotonic decreasing behaviors with respect to \(M\).

Finally, in Fig. 6, we show the effects of \(\tau\) on the performance of RMAP. As explained in Section IV, in each iteration of RMAP for which the solution does not satisfy \(r_{map,j} \geq r_{min}\), we multiply the trust region size (of the position where the data rate drops the most) by \(\tau\). Specifically, when \(\tau\) is smaller, the sizes of the trust regions may decrease faster, leading to a faster algorithm convergence. However, higher values of \(\tau\) may result in higher quality solutions. This phenomenon can be observed in Fig. 6, which shows the tradeoff between the number of iterations that RMAP needs to converge and the average energy consumption. More precisely, we show the average \(E\) corresponding to the solutions within which RMAP converges for several values of \(\tau\). These are normalized to the solutions that are obtained for \(\tau = 0.75\). As introduced above, we can observe that the number of iterations needed for the convergence monotonically increases when \(\tau\) assumes higher values, whereas average \(E\) has the opposite behavior. However, even when \(\tau = 0\), RMAP provides high-quality solutions in fewer iterations and negligible loss (\(\leq 1\%\)) with respect to the ones provided for \(\tau = 0.75\). Note that the number of iterations highly affects the computational complexity of SCO algorithms [43]. Specifically, at each iteration, RMAP solves convex optimization problem P4. By using the interior-point method, problem P4 can be solved in the order of few tens of ms with a laptop with 8 GB of RAM and a 7th generation Intel Core i7 processor.
C. UNIFORM RECTANGULAR ARRAY IRS

In this section, we consider an IRS consisting of a uniform rectangular array (URA) of $M = M_x M_z$ elements, where $M_x = M_z$. The terms $M_x$ and $M_z$ are the numbers of reflective elements on the horizontal and vertical axes of the IRS, respectively. For URA-modeled IRSs, the analysis and the resulting algorithm in Sections III and IV still hold. However, the normalized IRS array response vector $a_{URA} \in \mathbb{C}^M$ is defined as follows [49]:

$$a_{URA} = \frac{1}{\sqrt{M}} \left\{ e^{-j \frac{2\pi}{M} w[m_x \sin \vartheta_d \cos \vartheta_e + m_z \sin \vartheta_d \sin \vartheta_e]} ,
\ldots , e^{-j \frac{2\pi}{M} w[(M_x-1) \sin \vartheta_d \cos \vartheta_e + (M_z-1) \sin \vartheta_d \sin \vartheta_e]} \right\},$$

(24)

where, $\vartheta_d$ and $\vartheta_e$ are the azimuth and elevation AoA, respectively, whereas $w$ represents the spacing between the elements along both the y and z axes.

As done in Section V-B for the ULA-modeled IRS, in Fig. 7, we show the average $E$ of the solutions of RMAP, maximum data rate trajectories, and lower bounds for the URA-modeled IRS for several values of $M$, $r_{\text{min}}$, and $K$. We can observe that the higher number of reflective elements results in better solutions of RMAP. Specifically, in Fig. 7, RMAP can reach a lower optimality gap with respect to the results shown in Fig. 5. Moreover, the asymptotic behavior of RMAP solutions, already observed in Fig. 5, is better highlighted in Fig. 7, in which the value of $M$ reaches the number of $M_x^2 = 1024$ reflective elements.

D. SALEH-VALENZUELA CHANNEL MODEL

In this section, we show that RMAP can be efficiently used for other channel models. We now assume that the robot-AP and the robot-IRS channel vectors, $h_r$ and $h_d$, respectively, at the robot’s position $q_k$, follow the geometric Saleh-Valenzuela (SV) channel model for mm-wave communications [20]:

$$h_{r,k} = \sqrt{\frac{M}{L_{r,k}}} \left( \frac{\rho_{0,k} a(\alpha_{0,k}) + \sum_{l=1}^{L_{r,k}-1} \rho_{l,k} a(\alpha_{l,k})}{\rho_{0,k} b(\beta_{0,k}) + \sum_{l=1}^{L_{d,k}-1} \rho_{l,k} b(\beta_{l,k})} \right),$$

(25)

$$h_{d,k} = \sqrt{\frac{N}{L_{d,k}}} \left( \frac{\rho_{0,k} b(\beta_{0,k}) + \sum_{l=1}^{L_{d,k}-1} \rho_{l,k} b(\beta_{l,k})}{\rho_{0,k} a(\alpha_{0,k}) + \sum_{l=1}^{L_{r,k}-1} \rho_{l,k} a(\alpha_{l,k})} \right),$$

(26)

where, $L_{d,k}$ and $L_{r,k}$ are the numbers of paths for the robot-AP and robot-IRS links, respectively. The terms $a$ and $\alpha_{l,k}$ are the normalized array response vector and the cosine of AoA for the $l$-th path at the IRS, respectively. The corresponding entities for the AP are denoted by $b$ and $\beta_{l,k}$. Moreover, $\rho_{0,k}$ and $\rho_{l,k}$ are the complex gains associated with LOS and NLOS paths, respectively. The IRS-AP channel is approximated as in (10). Furthermore, we assume the same expressions of $\rho_{0,k}$ and $\rho_{l,k}$ as in [20]. However, we adjust the corresponding parameters (e.g., path loss at a reference distance, path loss exponents, and noise power) to the ones adopted in Section V to account for indoor industrial scenarios. Finally, unless otherwise specified, the rest of the parameters are set as in Section V.

In Fig. 8, we show the counterpart results of Fig. 5 for the geometric SV channel model. We can observe that although the higher amount of contributions of the reflected paths in LOS conditions of the geometric SV channel model may lead to a decrease in the energy consumption, the differences between Fig. 5 and Fig. 8 are not significant, and most of the conclusions derived from the former still hold for the latter. More precisely, the average $E$ resulted from RMAP decreases and approaches the lower bound as $M$ increases. Moreover, the optimality gap remains less than 5%. This can be explained by the following reason: The average rate optimization in Section III-A, the radio map, and the optimal $\text{SNR}$ estimation ($\text{SNR}^*$) in (17) make the trajectory optimization in Section IV independent of the assumed channel model. Additionally, the feasible region...
of P3 appears rather insensitive to the channel model. Therefore, RMAP may provide similar trajectories and corresponding energy consumption for different assumed channel models.

VI. CONCLUSION

In this work, we have proposed a novel robot trajectory optimization problem with QoS constrained communications to minimize motion energy consumption. The robot must avoid collisions with obstacles, reach the destination within a deadline, and transmit data to an AP operating at mm-wave frequency bands that is assisted by an IRS. The uplink transmission is subject to a minimum average data rate. To the best of our knowledge, energy-efficient trajectory planning problems for wirelessly connected robots have not been considered for mm-wave communications. We have proposed a solution that accounts for the challenging signal propagation conditions at such high frequencies and the mutual dependence between the channel conditions and the robot trajectory. Specifically, we have decoupled the beamforming and the trajectory optimization problems by exploiting the mm-wave propagation characteristics. The latter is solved by an SCO-based algorithm (RMAP) for which the convergence is proved. RMAP can deal with sudden data rate drops due to LOS-NLOS transitions by using the information that is stored in a radio map. Given this information, RMAP can find trajectories that avoid obstacles and poorly connected areas for satisfying data rate requirements.

We have shown trajectories and corresponding energy consumptions at which the algorithm converges for several scenarios and system parameters. The algorithm converges in few iterations to solutions that approach the lower bound to the energy consumption, which is dramatically reduced with respect to trajectories that maximize the data rate. Additionally, the proposed algorithm can be efficiently used for several channel and IRS models. Finally, we have shown that, by increasing the number of IRS’s reflective elements, we can improve the coverage and reduce the energy consumption of wirelessly connected robots. Thus, given the negligible power consumption of passive IRSs, they represent powerful solutions to enhance the energy efficiency of fully connected and autonomous factories.

APPENDIX A

To prove Lemma 2, we prove that \( r^*_k \), which is the estimated data rate at position \( q_k \), is a convex function of \( d_{i,k} \) and \( d_{a,k} \). Then, \( r^* \) is convex because it is a sum of convex functions. We first compute the partial derivatives of \( r^*_k \) with respect to \( d_{i,k} \) and \( d_{a,k} \). These are given by:

\[
\frac{\partial r^*_k}{\partial d_{i,k}} = \frac{-(\hat{\mu} \hat{\nu} \hat{A} d_{i,k} \hat{\nu} - 2 - \hat{\nu}/2 \hat{B} \hat{d}_{i,k} \hat{\nu} + 2 \hat{d}_{i,k} \hat{\nu}/2)}{F_k} \frac{p_k}{\sigma^2},
\]

(27)

\[
\frac{\partial r^*_k}{\partial d_{a,k}} = \frac{-(\hat{\mu} \hat{\nu} \hat{C} d_{a,k} \hat{\nu} - 2 - \hat{\nu}/2 \hat{B} \hat{d}_{a,k} \hat{\nu} + 2 \hat{d}_{a,k} \hat{\nu}/2)}{F_k} \frac{p_k}{\sigma^2},
\]

(28)

where, \( F_k = \ln(2)(1 + (\hat{\nu} \hat{A} d_{i,k} \hat{\nu} + \hat{\nu}/2 \hat{B} d_{i,k} \hat{\nu} + 2 \hat{d}_{i,k} \hat{\nu}/2) \frac{p_k}{\sigma}) > 0 \). Then, the second order partial derivatives are given by:

\[
\frac{\partial^2 r^*_k}{\partial d_{i,k}^2} = \frac{\partial^2 r^*_k}{\partial d_{a,k}^2} = \frac{F_k}{F_k^2} \frac{(\hat{\mu} \hat{\nu} \hat{A} d_{i,k} \hat{\nu} - 2 - \hat{\nu}/2 \hat{B} d_{i,k} \hat{\nu} + 2 \hat{d}_{i,k} \hat{\nu}/2)}{F_k^2} \frac{p_k}{\sigma^2},
\]

(29)

\[
\frac{\partial^2 r^*_k}{\partial d_{i,k} \partial d_{a,k}} = \frac{F_k}{F_k^2} \frac{(\hat{\mu} \hat{\nu} \hat{A} d_{i,k} \hat{\nu} - 2 - \hat{\nu}/2 \hat{B} d_{i,k} \hat{\nu} + 2 \hat{d}_{i,k} \hat{\nu}/2)}{F_k^2} \frac{p_k}{\sigma^2},
\]

(30)

\[
\frac{\partial^2 r^*_k}{\partial d_{i,k}^2} \frac{\partial^2 r^*_k}{\partial d_{a,k}^2} = \frac{F_k}{F_k^2} \frac{(\hat{\mu} \hat{\nu} \hat{A} d_{i,k} \hat{\nu} - 2 - \hat{\nu}/2 \hat{B} d_{i,k} \hat{\nu} + 2 \hat{d}_{i,k} \hat{\nu}/2)}{F_k^2} \frac{p_k}{\sigma^2},
\]

(31)

We observe that \( \frac{\partial^2 r^*_k}{\partial d_{i,k}^2} > 0 \), \( \frac{\partial^2 r^*_k}{\partial d_{a,k}^2} > 0 \) and \( \frac{\partial^2 r^*_k}{\partial d_{i,k} \partial d_{a,k}} = \frac{\partial^2 r^*_k}{\partial d_{a,k} \partial d_{i,k}} \leq 0 \). Therefore, the Hessian is positive definite and \( r^* \) is a convex function of \( d_{i,k} \) and \( d_{a,k} \).

APPENDIX B

We prove Lemma 3 by following the same reasoning of Appendix A. Note that \( r^*_{app,k} \) in (20) is the sum of \( K+1 \) functions each of them depending only on the robot’s position \( q_k \):

\[
r^*_{app,k} = \frac{1}{K} \sum_{k=0}^{K} r^*_{app,k}
\]

\[= \frac{1}{K} \sum_{k=0}^{K} \bigg( 1 + (\hat{\nu} \hat{A} d_{i,k} \hat{\nu} + \hat{\nu}/2 \hat{B} d_{i,k} \hat{\nu} + 2 \hat{d}_{i,k} \hat{\nu}/2) \frac{p_k}{\sigma} \bigg)
\]

\[+ \frac{\partial r^*_k}{\partial d_{a,k}} \left( d_{a,0,k} - d_{a,0,k} \right) \frac{\partial r^*_k}{\partial d_{a,k}} \left( d_{a,0,k} - d_{a,0,k} \right)
\]

\[+ \frac{\partial r^*_k}{\partial d_{i,k}} \left( d_{i,0,k} - d_{i,0,k} \right) \left( d_{i,0,k} - d_{i,0,k} \right)
\]

\[+ \frac{\partial r^*_k}{\partial d_{a,k}} \left( d_{a,0,k} - d_{a,0,k} \right) \left( d_{a,0,k} - d_{a,0,k} \right)
\]

\[+ \frac{\partial r^*_k}{\partial d_{i,k}} \left( d_{i,0,k} - d_{i,0,k} \right) \left( d_{i,0,k} - d_{i,0,k} \right)
\]

\[+ \frac{\partial r^*_k}{\partial d_{a,k}} \left( d_{a,0,k} - d_{a,0,k} \right) \left( d_{a,0,k} - d_{a,0,k} \right)
\]

\[+ \frac{\partial r^*_k}{\partial d_{i,k}} \left( d_{i,0,k} - d_{i,0,k} \right) \left( d_{i,0,k} - d_{i,0,k} \right)
\]

(32)

where, \( \frac{\partial r^*_k}{\partial d_{i,k}} < 0 \) and \( \frac{\partial r^*_k}{\partial d_{a,k}} < 0 \) are given by (27) and (28), respectively. Note that in (32) we have replaced \( d_k \) with \([x_k, y_k]\). Let us define \( D_k = \sqrt{(x_k - x_0)^2 + (x_k - x_0)^2 + (y_k - y_0)^2} \), then, we can compute the partial derivatives of \( r^*_{app,k} \) with respect to \( x_k \) and
Now, we prove that this condition holds, Lines 7-9 of RMAP is guaranteed for every feasible initial solution and every $Tk$.

Note that adding this affine constraint to P3 does not change the convexity of the latter. As done in Appendix C, we can prove that RMAP converges to a KKT point of $P^3$’ if we have that $\bar{r}_\text{map,j} \geq r_{\text{min}}$ holds $\forall j > n + 1$. Otherwise, if $\tilde{r}_\text{map,j} < r_{\text{min}}$ for a certain iteration, the proof follows Case 1 or Case 2.

**Case 2:** $Tk = 0$ for a certain position $q_k$ and iteration $n$. In this case, we have that constraint (23e) of P4 becomes:

$$||q_{j,k} - q_{j-1,k}|| \leq T_k = 0, \forall j > n.$$ 

This is equivalent to adding the following affine constraints to P3 and P4, respectively:

$$q_k = q_{n-1,k} \text{ and } q_{j,k} = q_{n-1,k}, \forall j > n.$$ 

The constraints, in fact, fix the position of the robot $q_k$. Given these constraints, we are free to set $T_k > 0$. Thus, from iteration $j > n$, RMAP solves a modified version of P3 ($P^3’$), starting from initial solution $q_{n-1}$ and $T_k > 0$, $\forall k$ by iteratively solving a modified version of P4 ($P^4’$). Note that adding this affine constraint to P3 and P4 does not change the convexity of the latter. As done in Appendix C, we can prove that RMAP converges to a KKT point of $P^3’$ if we have that $\bar{r}_\text{map,j} \geq r_{\text{min}}$ holds $\forall j > n + 1$. Otherwise, if $\tilde{r}_\text{map,j} < r_{\text{min}}$ for a certain iteration, the proof follows Case 1 or Case 2, whether $T_k > 0$, $\forall k$ or $T_k = 0$ for a certain position, respectively. If $T_k = 0$, $\forall k$, the proof follows Case 3.

**Case 3:** $T_k = 0$. In this case, the robot’s position for each $k = 0, \ldots, K$ is fixed, and RMAP has converged to a solution.

Thus, we have that RMAP either converges to a KKT point of P3 or to a KKT point of a modified problem where, for some or all $k = 0, \ldots, K$, the robot’s positions are fixed.

APPENDIX C

In this Appendix, we prove that the sequence of solutions provided by RMAP converges to a KKT point of P3 if, in each iteration $j$, $\bar{r}_\text{map,j} \geq r_{\text{min}}$. We first observe that, when this condition holds, Lines 7-9 of RMAP do not affect the solution. Then, solving P3 by RMAP is equivalent to solving P3 by using an SCO algorithm for which the convergence to a KKT point of P3 follows from [50]. The convergence is guaranteed for every feasible initial solution and every trust region size $T_k > 0$. This proves the second part of Theorem 1.

APPENDIX D

Now, we prove that RMAP converges even when condition $\bar{r}_\text{map,j} \geq r_{\text{min}}$ does not hold for each iteration. More precisely, for each iteration $n$ such that $\bar{r}_\text{map,n} < r_{\text{min}}$, RMAP sets $Q_n = Q_{n-1}$ and decreases $T_k$ for a certain position $q_k$. Then, we can have one of the following three cases:

- **Case 1:** $T_k > 0$, $\forall k$. RMAP continues solving problem P3 from iteration $j = n+1$ with $Q_{n-1}$ as the initial feasible solution and $T_k > 0$, $\forall k$. As proved in Appendix C, if for the successive iterations, i.e., $\forall j > n + 1$, we have that $\bar{r}_\text{map,j} \geq r_{\text{min}}$, RMAP still converges to a KKT point of P3. Otherwise, for each iteration $n$ such that $\bar{r}_\text{map,n} < r_{\text{min}}$, Lines 8 and 9 are repeated and this proof follows either Case 1 or Case 2 whether $T_k > 0$, $\forall k$ or $T_k = 0$ for a certain position, respectively. If $T_k = 0$, $\forall k$, the proof follows Case 3.

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