The factorisation of the hard amplitude for exclusive meson production in deep inelastic scattering is considered in the framework of a simple model. It is demonstrated explicitly how gauge invariance ensures the cancellation of non-factorising contributions.

1 Introduction

Recent HERA measurements of exclusive vector meson electroproduction at high photon energy and virtuality open new possibilities for the investigation of the interplay of hard and soft physics. The original crude calculations based on a simple model of two gluon exchange have since been refined by several authors. Recently, the mechanism of factorisation of the calculable hard amplitude and the nonperturbative component, given by the non-diagonal parton densities and the meson wave function, has been discussed in a rather general framework.

It is the purpose of this talk to demonstrate explicitly, in the framework of a simple model, how gauge invariance ensures the cancellations required for the factorisation of the hard part of the amplitude.

In the simplest diagrammatic description, the incoming photon splits into a quark antiquark pair which then interacts with the target via two gluon exchange and finally forms the outgoing vector meson. We focus on the problems associated with the vertex that couples the outgoing quarks to the vector meson. In the case of light flavours, this vertex is nonperturbative and so our knowledge of it is far from complete, though its analytic structure is known. If we neglect spin, it is a function $V(u, v)$ of the squared 4-momenta on the two quark legs, $u$ and $v$. Branch points in each of its variables, $u$ and $v$, are associated with “normal thresholds” and possibly also “anomalous thresholds”. We show that even in leading power this results in a breakdown of the desired diagrammatic factorisation.

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However, this does not necessarily imply that the factorisation theorem is invalid. For a generic nonperturbative vertex function $V$, the naive two gluon exchange calculation is not QCD gauge invariant. To achieve gauge invariance, graphs in which either or both of the exchanged gluons couples directly into the nonperturbative meson vertex function have to be included. The question we discuss is whether these additional diagrams restore the factorisation theorem.

We are not able to give a definitive answer to this question, because of the need to introduce two further nonperturbative vertex functions, and we know as little about these as we do about $V$. However, we report a calculation based on an explicit simple model in which we find that the factorisation theorem is indeed restored. This encourages the belief that its validity may be general.

Our model is a simple one in which all three vertex functions have the analytic structure that is expected on general grounds and in which they are related in such a way that the complete leading-power amplitude for the exclusive meson production is gauge invariant. We then find that the amplitude contains leading contributions from diagrams that cannot be separated into a hard production process and soft wave function corrections. However, gauge invariance leads to a cancellation of the unwanted contributions and leaves the factorisation theorem intact. The final formula can be obtained from the leading order diagrams by a redefinition of the rules for calculating them: basically, this amounts to ignoring the branch points in the vertex function.

In Sec. 2, we recall some aspects of the calculation in the case where the structure of the nonperturbative vertex function $V$ may be neglected. In Sec. 3 we explain the complications that arise when its structure is taken into account, introduce a simple model for this structure and analyse its consequences.

2. Structureless vertex function

In this section, we focus on the case of a structureless vertex function corresponding to a point-like coupling of the quarks to the outgoing meson (see Fig. 1). For simplicity, we take the meson to be spinless, and also pretend that the photon and the quarks have no spin. The coupling of the gluons to the scalar quarks is given by $-ig(r_\mu + r'_\mu)$, where $r$ and $r'$ are the momenta of the directed quark lines, and the coupling of the photon is $ie$, where $e$ has the dimensions of mass. We use light-cone co-ordinates and work in a frame where $q, q', P, P'$ have zero transverse components and large `$+$' and `$-$' components respectively. We concentrate on the forward production, so that the transverse component of the momentum transfer $\Delta = q' - q = \ell - \ell' = P - P'$ vanishes, $\Delta_\perp = 0$.

We study the limit $W^2 \gg Q^2 \gg m_\gamma^2$, where $W^2$ is the $\gamma^* p$ energy. The diagrams of Fig. 1 are the only ones that contribute in leading power when $V$ is constant. The lower bubble in these diagrams in principle has a complicated structure. However, we shall assume that the main contribution comes from values of its subenergy $\sigma$ that are not too large, $\sigma = (P - \ell)^2 \ll W^2$. Then $\ell_+ \ll q_+$, and the dependence of the lower bubble on $\sigma$ may be approximated by $\delta(\sigma)$. Further, when the upper parts of the diagrams are added together the main contribution arises from small values of $\ell_-$, $\ell_- \ll P_-$, so that also $\ell'_- \ll P_-$ and $\ell^2 \sim \ell'^2 \sim -\ell^2_\perp$. This may be seen most simply by calculating the imaginary part of the amplitude, where the left-most of the
two quarks to which $\ell'$ is attached is on shell. Hence the important part of the lower bubble effectively has the structure

$$F^{\mu\nu}(\ell, \ell', P) \approx \delta(P - \ell_+) F(\ell_+^2) P^\mu P^\nu,$$  \hspace{1cm} (1)

which is defined to include both gluon propagators and all colour factors. A similar expression was found by Cheng and Wu in a tree model for the lower bubble, though we do not need to restrict ourselves to such a simple model. We assume that $F$ restricts the gluon momentum to be soft, $\ell_+^2 \ll Q^2$. In the high energy limit it suffices to calculate

$$M = \int \frac{d^4\ell}{(2\pi)^4} T^{\mu\nu} F_{\mu\nu} \approx \int \frac{d^4\ell}{4(2\pi)^4} T_{++} F_{--},$$  \hspace{1cm} (2)

where

$$T^{\mu\nu} = T^{\mu\nu}(\ell, \ell', q) = T^{\mu\nu}_a + T^{\mu\nu}_b + T^{\mu\nu}_c$$  \hspace{1cm} (3)

is the sum of the upper parts of the diagrams in Fig. 1.

The lower amplitude $F_{\mu\nu}$ in the diagrams of Fig. 1 is symmetric with respect to the two gluon lines. This symmetry of the lower amplitude allows us to replace the properly-symmetrised upper amplitude $T^{\mu\nu}_{sym}$, with the unsymmetrised amplitude $T^{\mu\nu}$ corresponding to the sum of the diagrams in Fig. 1. The symmetrised amplitude is

$$T^{\mu\nu}_{sym}(\ell, \ell', q) = \frac{1}{2} [T^{\mu\nu}(\ell, \ell', q) + T^{\mu\nu}(\ell', \ell, q)].$$  \hspace{1cm} (4)

The two exchanged gluons together must form a colour singlet and so, at least for our calculation that takes account only of the lowest order in $\alpha_S(Q^2)$, the symmetrised amplitude $T^{\mu\nu}_{sym}$ satisfies the same Ward identity as for two photons:

$$T^{\mu\nu}_{sym}(\ell, \ell', q)\ell_\mu \ell_\nu = 0.$$  \hspace{1cm} (5)

Writing this equation in light-cone components and setting $\ell_\perp = \ell'_\perp$, we see that for the small values of $\ell_-, \ell'_-, \ell_+$ and $\ell'_+$ that we need,

$$T_{sym,++} \sim \ell_+^2$$  \hspace{1cm} (6)

for $\ell_+^2 \to 0$. Here we have used the fact that the tensor $T^{\mu\nu}_{sym}$, which is built from $\ell, \ell$ and $q$, has no large ‘-$’ components. The $\ell_-$ integration makes this equation hold also for the original, unsymmetrised amplitude:

$$\int d\ell_- T_{++} \sim \ell_+^2.$$  \hspace{1cm} (7)

This is the crucial feature of the two-gluon amplitude that will simplify the calculation and lead to the factorising result of the next section.

It is convenient to begin with the contribution from diagram a) of Fig. 1 to the $\ell_-$ integral of $T_{++}$, which is required in Eq. (2):

$$\int d\ell_- T_{a,++} = -4e g^2 q_+ \int \frac{d^4k}{(2\pi)^3} \frac{z(1-z)}{N^2 + (k_+ + \ell_+)^2} \frac{V(k^2, (q' - k)^2)}{k^2(q' - k)^2}.$$  \hspace{1cm} (8)

Here $N^2 = z(1-z)Q^2$, $z = k_+/q_+$ and the condition $\ell_+ = 0$, enforced by the $\delta$-function in Eq. (3), has been anticipated.

Now $\int d\ell_- T_{b,++}$ and $\int d\ell_- T_{c,++}$ each carry no $\ell_+$ dependence. So to ensure the validity of Eq. (6) the sum of the three diagrams must be

$$\int d\ell_- T_{++} = 4e g^2 q_+ \int \frac{d^4k}{(2\pi)^3} z(1-z)N \frac{V(k^2, (q' - k)^2)}{k^2(q' - k)^2},$$  \hspace{1cm} (9)
where
\[ N = \left[ \frac{1}{N^2 + k_{\perp}^2} - \frac{1}{N^2 + (k_{\perp} + \ell_{\perp})^2} \right] \sim \frac{\ell_{\perp}^2}{N^4}. \] (10)

We have used the softness of the wave function, which results in the dominant contribution to the integral arising from values of \( k_{\perp}^2 \ll Q^2 \), and the rotational symmetry, which makes \( k_{\perp} \cdot \ell_{\perp} \) integrate to 0. Note the \( 1/Q^4 \) behaviour obtained after a cancellation of \( 1/Q^2 \) contributions from the individual diagrams. This cancellation, which is closely related to the well-known effect of colour transparency\(^8\), has been discussed in\(^2\) in the framework of vector meson electroproduction.

Introduce the light-cone wave function of the meson
\[ \psi(z, k_{\perp}^2) = -i q' + \frac{2}{\pi^2} \int dk_{\perp}^+ dk_{\perp}^- \frac{V(k_{\perp}^2, (q' - k)^2)}{(2\pi)^4 k_{\perp}^2} \delta(k_{\perp}^+ - z q'). \] (11)

The final result following from Eqs. (2) and (9) is a convolution of the production amplitude of two on-shell quarks and the light-cone wave function:
\[ M = i e g^2 W^2 \left( \int \frac{d^2 \ell_{\perp}}{2(2\pi)^3} \ell_{\perp}^2 F(\ell_{\perp}^2) \right) \int dz \int d^2 k_{\perp}^\perp \frac{z(1 - z)}{N^4} \psi(z, k_{\perp}^2). \] (12)

This corresponds to the \( O(\ell_{\perp}^2) \) term in the Taylor expansion of the contribution Eq. (8) from Fig. 1a).

3. Factorisation for a simple model wave function

In the previous section we have supposed that the vertex function \( V \) has no structure. We used the gauge invariance of the amplitude to argue that there is a cancellation among the three diagrams of Fig. 1a). Of course, we may instead obtain the same result by explicit calculation of each of the three diagrams. This may be done most simply by calculating their imaginary parts and using the known fact that the complete amplitude must be pure-imaginary when its energy dependence is \( (W^2)^{1.0} \). Alternatively, the contributions to the amplitude itself may be calculated by doing the \( k_\perp \) integration, completing the integration contour with an infinite semicircle and taking appropriate pole residues.

Consider Fig. 1b) for example. In either method of calculation, if \( V \) has no structure the upper quark line gets put on shell. However, if we now take account of the known structure of \( V \), there is an additional contribution to the imaginary part corresponding to cutting the graph through the vertex function — the \( k_\perp \) integration would need to take account also of branch points of \( V \), not just the poles of the propagators. The gauge-invariance argument breaks down because the set of diagrams by itself is no longer gauge invariant: one must add to it diagrams where either or both of the gluons couples directly into the vertex function.

![Fig 2a)](image1.png) Model for the nonperturbative vertex function \( V \). b) Diagram with a gluon coupling into the vertex.

In order to study this, we use the simplest model for the quark-quark-meson vertex function \( V(u, v) \) that incorporates at least part of its known branch-point structure. It is the purely nonperturbative vertex function that we are modelling: it goes to zero suitably rapidly when
either of the squared 4-momenta \( u \) or \( v \) of the quarks becomes large. We do not consider its perturbative tail, which would be obtained by exchanging a perturbative gluon between the quarks. It is a familiar notion that the correct analytic properties of nonperturbative amplitudes are those corresponding to Feynman graphs, even though the numerical values of such graphs have no physical significance. In order to model the vertex function \( V \), therefore, we use the simple Feynman graph of Fig. 2a), where the line that joins the quarks is a scalar (like the quarks themselves in our simple calculations) which couples to them with strength \( \lambda' \) and where the right-hand internal vertex is taken to be a constant \( \lambda \). This model has branch points in each of the variables \( u \) and \( v \), and it has the appropriate softness.

![Fig. 3](https://example.com/fig3.png)

**Fig. 3** Diagram for meson production with the vertex modelled by scalar particle exchange.

When the vertex function of Fig. 2a) is used in Fig. 1a), we obtain Fig. 3. The expression for the upper part of the diagram is Eq. (8) with

\[
V(k^2, (q' - k)^2) = \int \frac{d^4 k'}{(2\pi)^4} \frac{i\lambda \lambda'^2}{k'^2 (q' - k')^2 (k - k')^2}.
\]

The diagram of Fig. 3 by itself gives no consistent description of meson production since it lacks gauge invariance. This problem is not cured by just adding the two diagrams 1b) and c) with the blob replaced by the vertex of Fig. 2a). It is necessary also to include diagrams where a gluon is coupled into the vertex (see, e.g., Fig. 2b)). Furthermore, diagrams where both gluons couple into the vertex have to be included. The complete set of these additional diagrams is shown in Fig. 4.

The same gauge invariance arguments that lead to Eq. (7) apply to the sum of all the diagrams in Figs. 3 and 4. Therefore, the complete result for \( T_{++} \), which is now defined by the sum of the upper parts of all these diagrams, can be obtained by extracting the \( \ell_\perp^2 \) term at leading order in \( W^2 \) and \( Q^2 \). Such a term, with a power behaviour \( \sim \ell_\perp^2 W^2/Q^4 \), is obtained from the diagram in Fig. 3 (see Eqs. (8) and (13)) by expanding around \( \ell_\perp = 0 \). It can be demonstrated that none of the other diagrams gives rise to such a leading-order \( \ell_\perp^2 \) contribution (see\footnote{5} for more details).

The complete answer is given by the \( \ell_\perp^2 \) term from the Taylor expansion of Eq. (8). The amplitude \( M \) is precisely the one of Eqs. (12) and (11), with \( V(k^2, (q' - k)^2) \) given by Eq. (13). We have also checked the correctness of this simple factorising result by explicitly calculating all diagrams of Fig. 4.

5. Conclusions

The mechanism of factorisation in exclusive meson production has been analysed in the framework of a simple scalar model. In this model the meson is formed by two scalar quarks interacting via the exchange of a scalar boson. From a calculation of all contributing diagrams within the restriction of two-gluon exchange the following picture emerges.

The complete result contains leading contributions from diagrams that cannot be factorised into quark-pair production and meson formation. Nevertheless, in Feynman gauge the answer to the calculation can be anticipated by looking only at one particular factorising diagram. The
Fig. 4 The remaining diagrams contributing to meson production within the above simple model for the meson wave function.

reason for this simplification is gauge invariance. In the dominant region where the transverse momentum $\ell_\perp$ of the two $t$-channel gluons is small, gauge invariance requires the complete quark part of the amplitude to be proportional to $\ell_\perp^2$. The leading $\ell_\perp^2$ dependence comes exclusively from one diagram. Thus, the complete answer can be obtained from this particular diagram, which has the property to factorise explicitly if the two quark lines are cut. The resulting amplitude can be written in a factorised form.

It was our intention to demonstrate the above mechanism of gauge invariance induced cancellations in as simple and as explicit a way as possible. Our analysis supplements the otherwise much more general and complete discussion of by making it more explicit and by handling the known structure of the nonperturbative meson vertex function.

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