Constraining weak annihilation using semileptonic $D$ decays

Zoltan Ligeti,1 Michael Luke,2 and Aneesh V. Manohar3
1Ernest Orlando Lawrence Berkeley National Laboratory, University of California, Berkeley, CA 94720
2Department of Physics, University of Toronto, 60 St. George Street, Toronto, Ontario, Canada M5S 1A7
3Department of Physics, University of California at San Diego, 9500 Gilman Drive, La Jolla, CA 92093

The recently measured semileptonic $D_s$ decay rate can be used to constrain weak annihilation (WA) effects in semileptonic $D$ and $B$ decays. We revisit the theoretical predictions for inclusive semileptonic $D_{(s)}$ decays using a variety of quark mass schemes. The most reliable results are obtained if the fits to $B$ decay distributions are used to eliminate the charm quark mass dependence, without using any specific charm mass scheme. Our fit to the available data shows that WA is smaller than commonly assumed. There is no indication that the WA octet contribution (which is better without using any specific charm mass scheme. Our fit to the available data shows that WA is smaller than commonly assumed. There is no indication that the WA octet contribution (which is better without using any specific charm mass scheme) is the dominant contribution in a phase space region which affects the so-called weak annihilation (WA) terms, give a significant contribution in a phase space region which affects the so-called weak annihilation (WA) terms. For this reason, the extraction of the WA contribution is necessary to determine whether there is, in fact, a conflict with the standard model predictions.

The WA contribution to the total $B \to X_u \ell \bar{v}$ decay rate [3] and to the charged lepton (or neutrino) energy spectrum [3] are calculable in terms of the matrix elements of local four-quark operators. However, there is so far no first-principles derivation of the WA contribution to the double or triple differential spectra. For this reason, the extraction of the WA contribution from the differential $B \to X_u \ell \bar{v}$ spectra is model dependent, and any model-independent bound on the magnitude of the WA matrix elements is important. It was pointed out by Voloshin [3] that the same matrix elements that enter $B \to X_u \ell \bar{v}$ decay can be constrained by the semileptonic rate difference between $D$ and $D_s$ mesons, since the $B$ and $D$ meson matrix elements are related by heavy quark symmetry. In this paper we revisit the theoretical calculations of semileptonic $D$ decays and extract bounds on the WA contribution to $D$ and $B$ decays.

At order $\Lambda_{QCD}/m_c$, there are dimension-6 four-quark operators in the OPE for the semileptonic $D$ decay rate, the WA operators,

$$O^{(q)}_{V-A} = (\bar{e}_u \gamma^\mu q_L) (\bar{q}_L \gamma_\mu c_v),$$
$$O^{(q)}_{S-P} = (\bar{e}_u q_L) (\bar{q}_L c_v),$$

where $q = s, d$ and $c_v$ is the heavy quark effective theory charm quark field. The matrix elements $B^{(q,i)}_{1,2}$ of these operators are defined by

$$\langle D_i | O^{(q)}_{V-A} | D_i \rangle = \frac{1}{8} f_D^2 m_D B^{(q,i)}_1,$$
$$\langle D_i | O^{(q)}_{S-P} | D_i \rangle = \frac{1}{8} f_D^2 m_D B^{(q,i)}_2,$$

where $i = u, d, s$ labels the flavor of the light quark in the $D$ meson. Compared to the dimension-3, 5, and other dimension-6 operators, the matrix elements of these operators are enhanced by $16\pi^2$, and contribute to the semileptonic decay widths of the three $D$ mesons as [3]

$$\Gamma_{WA}^{(D_i)} \sim \sum_{q=s,d} \frac{f_D^2 m_D |V_{cq}|^2}{m_c^3} 16\pi^2 \left( B^{(q,i)}_2 - B^{(q,i)}_1 \right),$$

relative to the semileptonic width at lowest order in the OPE and at tree-level in $\alpha_s$,

$$\Gamma_0(m_c) \equiv \frac{G_F^2 m_c^5}{192\pi^3}.$$

If one assumes factorization and the vacuum saturation approximation, then the WA contribution vanishes, since $B_1 = B_2 = 1$ or 0 depending on whether or not $q$ is the same as $i$. Deviations from the factorization ansatz are usually estimated at the 10% level [3].

The above analysis also holds for $B$ decays, with the replacement of $D$ meson quantities by the corresponding $B$ meson ones. The matrix elements in the $c$ and $b$ sectors are related by heavy quark symmetry. The same matrix element estimate for $B$ decays (i.e., $|B_1 - B_2| \sim 0.1$), along with $f_B \approx 200$ MeV [3], implies that the four-quark operators contribute $\sim 3\%$ to the total $B \to X_u \ell \bar{v}$ rate, making it difficult to accurately determine the WA contribution from $B$ decays. However, the WA contribution to $D$ decay is formally $(m_b/m_c)^3$ enhanced relative to $B$ decay, and is comparable to the leading order decay rate, $\Gamma_0$, due to the $16\pi^2$ enhancement. Thus studying WA effects in $D$ decay is a good way to constrain the matrix elements of the four-quark operators. Note that to determine the WA contribution to $B$ decays at the 1% level only requires the four-quark matrix elements to $\sim 30\%$ accuracy. Even if this $1/m_c^3$ contribution to $D$ decays is
comparable to the leading order rate, this does not necessarily mean that the $1/m_c$ expansion breaks down, since the WA contribution is the only $16\pi^2$ enhanced contribution at $\mathcal{O}(1/m_c^2)$.

It has long been known that the difference in the semileptonic branching ratios \[ B(D^0 \rightarrow X e^+\nu) = (6.49 \pm 0.09 \pm 0.11)\% \], \[ B(D^+ \rightarrow X e^+\nu) = (16.13 \pm 0.10 \pm 0.29)\% \], is mainly due to the lifetime difference, and that the semileptonic widths $\Gamma(D^0 \rightarrow X e^+\nu)$ \(\approx\) $\Gamma(D^+ \rightarrow X e^+\nu)$ are equal to within 3%. Recently CLEO-c measured \[ \Gamma(c \rightarrow e) \] the $D_s$ semileptonic branching ratio \[ B(D_s^+ \rightarrow X e^+\nu) = (6.52 \pm 0.39 \pm 0.15)\% \].

The expressions for semileptonic $D(s)$ \(\rightarrow X \ell \bar{\nu})\) decays are well known in the literature. Schematically, \[ \Gamma_{\text{SL}} = \Gamma_{\text{c\bar{c}}} + \Gamma_{\text{WA}} \] where $\Gamma_{\text{WA}}$ is defined in Eq. (1), and \[ \Gamma_{\text{c\bar{c}}} = \Gamma_0 \left[ |V_{c3}|^2 (1 - 8r + 8r^3 - r^4 - 12r^2 \ln r) + |V_{cd}|^2 \right. \] \[ + \left( \lambda_1 + \frac{T_1 + 3T_2}{m_c} \right) \frac{1}{2m_c^2} - \left( \lambda_2 + \frac{T_3 + 3T_4}{3m_c} \right) \frac{9}{2m_c^2} \] \[ + \frac{77\rho_1 + 27\rho_2}{6m_c^2} + \mathcal{O}(\epsilon^4, m_c^4) \], \[ \lambda_1, \lambda_2, \rho_1, \rho_2 \] are the matrix elements of dimension-5 and 6 two-quark operators, $T_{1,2,3,4}$ are the matrix elements of time-ordered products, and $r = m_2^2/m_c^2$. These may all be determined from fits to various $B$ decay spectra \[ \Gamma_0, T_1 \text{ - 13}. \] The complete expression including $\epsilon$ corrections is complicated. In our analysis, we include the radiative corrections to order $\epsilon^2$ \[ \Gamma_0, T_1 \text{ - 14}. \] and power corrections to order $\epsilon^4/m_c^4$ \[ \Gamma_0, T_1 \text{ - 15}. \]. For the leading term in the OPE, we include the effect of a nonzero strange quark mass, since it affects the semileptonic width by $\sim 6\%$, and set $m_s \rightarrow 0$ elsewhere. For nonzero $r$, the $\rho_1$ contribution to $\Gamma_{\text{c\bar{c}}}$ has a log $r$ divergence as $r \rightarrow 0$. In the OPE with $r = 0$, this divergence is effectively absorbed into the matrix elements of WA operators. Including the WA contribution converts the $q^3$ spectrum into a plus distribution \[ \Gamma_{\text{c\bar{c}}} \rightarrow \Gamma_{\text{c\bar{c}}}^\pm (6m_2^2) \] to contribution to the total semileptonic rate in Eq. (6).

The terms in $\Gamma_{\text{c\bar{c}}}$ are all independent of the flavor of the spectator quark in the heavy meson, and so give equal contributions to $\Gamma_{\text{SL}}$ for all three $D$ (or $B$) mesons. The leading term in $\Gamma_{\text{SL}}$ which depends on the flavor of the spectator quark, and thus produces a difference in the semileptonic partial widths, is $\Gamma_{\text{WA}}$. $\Gamma_{\text{WA}}$ depends on two independent matrix elements in the flavor $SU(3)$ limit, since the operators in Eq. (1) have an $SU(3)$ singlet and octet part, each of which yield one invariant with the two $D$ fields. We can write the decay rates of the three $D$ mesons in terms of these two parameters as

\[ \Gamma_{\text{SL}}^{(D^0)} = \Gamma_{\text{c\bar{c}}} + \Gamma_0 \left[ |V_{c3}|^2 \left( \frac{a_0 - a_8}{3} + |V_{cd}|^2 \frac{a_0 - a_8}{3} \right) \right], \] \(\text{(9)}\)

\[ \Gamma_{\text{SL}}^{(D^\pm)} = \Gamma_{\text{c\bar{c}}} + \Gamma_0 \left[ |V_{c3}|^2 \left( \frac{a_0 - a_8}{3} + |V_{cd}|^2 \frac{a_0 + 2a_8}{3} \right) \right], \] \(\text{(10)}\)

\[ \Gamma_{\text{SL}}^{(D_s)} = \Gamma_{\text{c\bar{c}}} + \Gamma_0 \left[ |V_{c3}|^2 \left( \frac{a_0 + 2a_8}{3} + |V_{cd}|^2 \frac{a_0 - a_8}{3} \right) \right]. \] \(\text{(11)}\)

Here we normalized the weak annihilation contribution to the observed semileptonic $D_{\pm,0}$ decay rate,

\[ G_0 \equiv 0.16 \text{ ps}^{-1} \equiv \Gamma_0(m_{\text{ref}}), \] \(\text{(12)}\)

where $m_{\text{ref}} = 1.357$ GeV and $a_0$ and $a_8$ are dimensionless numbers proportional to the singlet and octet matrix elements,

\[ \frac{G_F^2 m_c^2 f_D^2 m_D}{12\pi^3} B(\frac{q^2}{m_D^2}) (B^{(q,i)}_2 - B^{(q,i)}_1) \equiv \delta_{q,i} a_8 + \frac{1}{3} (a_0 - a_8). \] \(\text{(13)}\)

The size of $a_{0,8}$ is then (approximately) the fraction of the meson semileptonic width due to WA.

The difference $\Gamma_{\text{SL}}^{(D^0)} - \Gamma_{\text{SL}}^{(D^\pm)}$ is suppressed by $|V_{cd}/V_{c3}|^2$. Neglecting this correction, it is straightforward to extract $a_8$ from the measured difference of the semileptonic widths of the $D^0, \pm$ and the $D_s$, as proposed in Refs. \[ \text{12 - 13}. \] This difference combined with any of the individual semileptonic widths also allows $a_0$ to be extracted, but this requires a reliable computation of $\Gamma_{\text{c\bar{c}}}$. Since the charm mass is not particularly large compared with nonperturbative QCD scales, this computation suffers from both large perturbative and $1/m_c^2$ corrections, limiting the precision with which WA can be studied in charm decays.

The leading perturbative corrections to the semileptonic decay widths are given by a perturbation series multiplying the free-quark decay width $\Gamma_0$ given in Eq. (4). This perturbation series depends on the choice of scheme for $m_c$, and $m_c$ could be determined from the $B$ decay data using the method of Ref. \[ \text{20}. \] However, it is well-known that the perturbation series for this leading term in the OPE is badly behaved when the rate is expressed in terms of the charm quark pole or $\overline{\text{MS}}$ masses. Including the known results up to order $\epsilon^2$ \[ \text{14}. \] and using $\alpha_s = 0.35$ gives the series

\[ \frac{\Gamma_{\text{c\bar{c}}}}{\Gamma_0(m_{\text{pole}})} = 1 - 0.269 \epsilon - 0.360 \epsilon^2_{\text{BLM}} + 0.069 \epsilon^2 + \ldots, \] \(\text{(12)}\)

and

\[ \frac{\Gamma_{\text{c\bar{c}}}}{\Gamma_0(m_c)} = 1 + 0.474 \epsilon + 0.513 \epsilon^2_{\text{BLM}} - 0.142 \epsilon^2 + \ldots, \] \(\text{(13)}\)

respectively. Here $\epsilon \equiv 1$ counts the order in the perturbation series, and the BLM subscript refers to the $\epsilon^n \beta_0^{n-1}$ terms in the perturbation series. In Eqs. \[ \text{12 - 10}. \] we
set \(m_s \to 0\) for simplicity; this has no effect on our discussion.] These series are poorly behaved, and do not appear to converge.

The bad behavior of these perturbation series is understood theoretically from \(b\) decays, and arises from a poor choice for the heavy quark mass. A better behaved series is obtained by using a threshold mass scheme, such as the 1S [13–15], kinetic [16] or PS [17] mass schemes. As observed already in [13], the perturbation series relating \(\Gamma_{cc}\) to the 1S mass is reasonably well-behaved, and extracting \(m_c\) using the method of Ref. [20]

\[
\frac{\Gamma_{cc}}{\Gamma_0[m_c^{1S}(1\text{GeV})]} = 1 - 0.133 \epsilon - 0.006 \epsilon^2_{\text{BLM}} - 0.017 \epsilon^2. \tag{14}
\]

The series is less well-behaved in the PS or kinetic schemes (defining both with a 1 GeV factorization scale). For the PS scheme we find

\[
\frac{\Gamma_{cc}}{\Gamma_0[m_c^{1S}(1\text{GeV})]} = 1 + 0.262 \epsilon + 0.217 \epsilon^2_{\text{BLM}} - 0.079 \epsilon^2, \tag{15}
\]

while for the kinetic scheme, as previously noted [20], the series is considerably worse

\[
\frac{\Gamma_{cc}}{\Gamma_0[m_c^{\text{kin}}(1\text{GeV})]} = 1 + 0.628 \epsilon + 0.631 \epsilon^2_{\text{BLM}} - 0.126 \epsilon^2. \tag{16}
\]

In addition to the uncertainties in the above series, there will be additional uncertainties in extracting a charm quark threshold mass from other physical quantities, such as moments of \(B\) decay spectra [13, 14]. Since the charm quark mass is an intermediate quantity which is not required for our analysis, we can minimize this source of theoretical uncertainty by bypassing any choice of charm mass scheme, and instead directly relate the semileptonic \(D\) decay widths to the values of \(m_c^{1S}\) and \(\Delta = m_b - m_c\) extracted from a global fit to \(B\) decay spectra. From Eq. (12) and the relation between the \(b\) quark pole and 1S masses [22]

\[
\frac{m_b}{m_b^{1S}} = 1 + 0.011 \epsilon + 0.019 \epsilon^2_{\text{BLM}} - 0.003 \epsilon^2 + \ldots, \tag{17}
\]

we find the reasonably well-behaved perturbation series

\[
\frac{\Gamma_{cc}}{\Gamma_0[m_b^{1S} - \Delta]} = 1 - 0.075 \epsilon - 0.013 \epsilon^2_{\text{BLM}} - 0.021 \epsilon^2, \tag{18}
\]

using \(m_b^{1S} = 4.7\) GeV, \(\Delta = 3.4\) GeV, \(\alpha_s(m_b) = 0.22\), and, as in the previous expressions, we have continued to set \(m_s\) to zero. We will therefore use this method to determine the \(D\) semileptonic widths theoretically.

The masses \(m_b^{1S}\) and \(m_b - m_c\) and HQET parameters \(\lambda_{1,2}, \rho_{1,2}\) and \(\lambda_{2,3,4}\), as well as their correlated uncertainties, are obtained using a fit to the \(B\) decay spectra [11, 12]. The values for \(D\) decay are related to those for \(B\) decay by renormalization group evolution between \(m_b\) and \(m_c\). \(\lambda_1\) is not renormalized due to reparameterization invariance [22], while \(\lambda_2(m_c) = \kappa_c \lambda_2(m_b)\), with \(\kappa_c \approx 1.2\). Radiative corrections to the \(1/m_b^2\) terms are computed in Refs. [23–30]. Since they are small and were not included in the \(B\) decay fits, we neglect them here.

The errors from the fits include the experimental uncertainties, as well as additional theoretical uncertainties due to neglected higher order corrections, as given in Ref. [11, 12]. We treat the \(B\) and \(D\) decay calculations as independent. Thus the \(B\) decay fit results will be held fixed (at order \(\epsilon^2_{\text{BLM}}\)) while we vary the order of the \(D\) decay results between tree-level and \(\epsilon^2\). While this may be formally inconsistent, numerically, the \(\alpha_s\) and \(1/mQ\) corrections are significantly larger for \(D\) than for \(B\) decay.

Using the value of \(\Gamma_{cc}\) obtained as discussed above, and fitting to the experimentally measured rates in Eqs. (3) and (4) gives the WA annihilation parameters \(a_0\) and \(a_s\). Figure 1 shows the 90% CL contours at tree level, order \(\epsilon\), and order \(\epsilon^2\). The best fit parameters at order \(\epsilon^2\) are

\[
a_0 = 1.25 \pm 0.15, \tag{19}
\]

\[
a_s = -0.20 \pm 0.12, \tag{19}
\]

where the error is from the order \(\epsilon^2\) fit. The series of \(\alpha_s^a\) corrections to \(\Gamma_{cc}\) are flavor independent, and lead to a shift in \(a_0\) depending on the order in \(\epsilon\), but do not affect \(a_s\), which can be determined from \(\Gamma_s^{(\text{PS})} - \Gamma_s^{(\text{EF})}\). \(\Gamma_{cc}\) cancels in this difference, so \(a_s\) is not affected by the convergence of the \(\alpha_s\) expansion. The shift in \(a_0\) between \(\epsilon\) and \(\epsilon^2\) is 0.06, which is smaller than other uncertainties.

The expansion in \(1/m_c\) is also not as rapidly convergent as in the \(B\) meson system, so there are significant uncertainties which mainly affect \(a_0\). We find that the \(1/m_c^2\) and \(1/m_c^3\) terms in Eq. (3) contribute roughly \(-50\%\) and \(+35\%\) to the semileptonic widths. These corrections are much larger than the corresponding ones for the hadron masses, because of the larger coefficients of \(\lambda_2, \rho_1,\) and \(\rho_2\). One could estimate the uncertainty corresponding to these large corrections by including an additional error of 0.2 in \(a_0\), which is half the \(1/m_c^3\) term. In the \(N_c \to \infty\) limit, the meson sector of QCD has a \(U(3)_q \otimes U(3)_{\bar{q}}\) symmetry [24] and this implies that \(a_0 = a_s\) (see, e.g., Ref. [23]), which is shown as the black line in Fig. 1.

Neglecting Cabibbo-suppressed terms, the correspondence between our notation and that of Ref. [6] is

\[
a_s = \frac{m_s^2 m_D f_D^2}{m_{\text{ref}}} 16 \pi^2 (B_{2s}^D - B_{1s}^D), \tag{20}
\]

and the same equation with \(a_s \to a_0\) and the non-singlet \(B_{1,2}\) replaced by the singlet ones. Taking \(f_D \approx 200\) MeV [8] gives \((B_{1s}^D - B_{2s}^D) \approx -a_0/4\approx 0.05 \pm 0.03\), which is somewhat smaller than (although consistent with) the simple estimate \((B_{1s}^D - B_{2s}^D) \approx 0.1\) in [6, 8].

The linear combinations \((a_0 + 2a_s)/3\) and \((a_0 - a_s)/3\) that contribute to the decay rates in Eq. (3) are

\[
(a_0 + 2a_s)/3 = 0.29 \pm 0.10, \tag{21}
\]

\[
(a_0 - a_s)/3 = 0.48 \pm 0.06, \tag{21}
\]

where only the fit uncertainty is quoted, as discussed above. The 90% confidence level contours in these variables are shown in Fig. 1. While there are significant
uncertainties in the fit result for the WA contribution in Eqs. (19) and (21), it still has important implications for $B$ and $D$ decays and the determination of $|V_{ub}|$.

It has often been assumed that the WA term where the light quark in the operator matches that in the heavy meson is much larger than when the light quarks differ, i.e., $|a_0 + 2a_8| \gg |a_0 - a_8|$. Indeed, the central values of our results suggest that the WA contribution to $B^0$ decay is larger than that to $B^{\pm}$ decay. The WA matrix element in which the light quark field of the operator is contracted with the spectator quark in the heavy meson is helicity suppressed by $m_l^2/m_h^2$, where $m_l$ is the lepton mass, and gives a contribution of relative order $\Lambda_{QCD}^2 m_l^2/m_h^2$ to the decay width. Other diagrams, in which the spectator quark is not annihilated by the four-quark operator, are of relative order $\Lambda_{QCD}^2/m_h^2$. In a quark model, they would contain additional suppression factors from gluon exchange to connect the spectator light quark with the rest of the diagram, but nothing as small as $m_l^2/m_h^2$.

The $D$ meson lifetimes also depend on the WA matrix elements through both the semileptonic and non-leptonic decay rates. The non-leptonic rates depend on two additional color octet operators, and the behavior of the $\alpha_s$ perturbation series is even worse than for the semileptonic case. Neglecting the color octet matrix elements and $SU(3)$ violation (as before), one would predict

$$\frac{\Gamma(D^\pi) - \Gamma(D^\pi_{SL})}{\Gamma(D^\pi_{total}) - \Gamma(D^\pi_{total})} = \frac{3}{8 C_+ C_- \cos^2 \theta_C} \approx 0.3,$$

where $C_- = C_{\pi}^{-2} = [\alpha_s(m_c)/\alpha_s(m_W)]^{12/25}$, and we have used $C_+ = 1.6$ and $C_+ = 0.8$ for the numerical values.

The $D$ branching ratios Eqs. (18) and (19) and the lifetimes yield $0.07 \pm 0.02$. This shows that there must be some other large contribution to the nonleptonic decay rates, e.g., large color octet matrix elements, $\alpha_s$ corrections, or higher order $1/m_c$ terms, so the total widths do not provide a useful bound on $a_{0,8}$.

It is often stated that the difference between the $B^\pm$ and $B^0$ semileptonic rates can be used to constrain the impact of WA on the extraction of $|V_{ub}|$ from $B \to X_d \ell \bar{\nu}$ decays. However, $\Gamma(B^\pm) - \Gamma(B^0) \propto a_8$, while individually $\Gamma(B^\pm) - \Gamma(B^0)$, which determine $|V_{ub}|$, depend on both $a_0$ and $a_8$. We find no evidence that $|a_0| \ll |a_8|$, so the $\Gamma(B^\pm) - \Gamma(B^0)$ width difference will not strongly constrain the WA contribution to $|V_{ub}|$. While the uncertainties in our analysis are substantial, it gives strong indication that the WA contribution to the $B \to X_d \ell \bar{\nu}$ rate is less than the $\sim 3\%$ estimate often used. Our conclusions are unchanged if $SU(3)$ breaking or higher order $1/m_Q$ corrections are included, since these will only shift the estimate of the WA contribution by $\sim 20\%$ of its value. The 2$\sigma$ discrepancy in $V_{ub}$ mentioned in the introduction cannot be explained away using WA.

Our results imply that the WA contribution to $B$ decays, which is a factor $(m_c/m_b)^3 \sim 0.03$ smaller than the corresponding contribution to $D$ decays, is around 1%. If we use heavy quark symmetry for the bag parameters instead of the matrix elements, scaling with $(m_c^2/m_b^2)(f_B^2/f_D^2)$ gives 2.5%, still smaller than past estimates. A recent CDF measurement of the $B$ meson and $A_s$ baryon lifetimes also indicates that spectator effects in the $b$ hadron decays may be smaller than previously thought.

**ACKNOWLEDGMENTS**

We thank Frank Tackmann for helpful conversations, and C. S. Park and S. Stone for correspondence about Ref. [20]. The work of ZL was supported in part by the Director, Office of Science, Office of High Energy Physics of the U.S. Department of Energy under contract DE-
ML was supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1] J. Chay, H. Georgi, and B. Grinstein, Phys. Lett. B247, 399 (1990).
[2] I. I. Y. Bigi, N. G. Uraltsev, and A. I. Vainshtein, Phys. Lett. B293, 430 (1992), arXiv:hep-ph/9207214 [Erratum, ibid. B 297 (1993) 477].
[3] I. I. Y. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Phys. Rev. Lett. 71, 496 (1993), arXiv:hep-ph/9304222.
[4] A. V. Manohar and M. B. Wise, Phys. Rev. D49, 1310 (1994), arXiv:hep-ph/9308246.
[5] I. I. Y. Bigi and N. G. Uraltsev, Nucl. Phys. B423, 33 (1994), arXiv:hep-ph/9310285.
[6] M. B. Voloshin, Phys. Lett. B515, 74 (2001), arXiv:hep-ph/0106040.
[7] A. K. Leibovich, Z. Ligeti, and M. B. Wise, Phys. Rev. D59, 074017 (1999), arXiv:hep-ph/9811233.
[8] J. Laiho, E. Lunghi, and R. S. Van de Water, Phys. Rev. D81, 034503 (2010), arXiv:0910.2928 [Unknown].
[9] C. Amsler et al. (Particle Data Group), Phys. Lett. B677, 1 (2008).
[10] D. M. Asner et al. (The CLEO Collaboration)(2009), arXiv:0912.4322 [hep-ex].
[11] C. W. Bauer, Z. Ligeti, M. Luke, A. V. Manohar, and M. Trott, Phys. Rev. D70, 094017 (2004), arXiv:hep-ph/0408002.
[12] C. W. Bauer, Z. Ligeti, M. Luke, and A. V. Manohar, Phys. Rev. D67, 054012 (2003), arXiv:hep-ph/0210027.
[13] O. Buchmuller and H. Flacher, Phys. Rev. D73, 073008 (2006), arXiv:hep-ph/0507253.
[14] T. van Ritbergen, Phys. Lett. B454, 353 (1999), arXiv:hep-ph/9903228.
[15] M. Greyn and A. Kapustin, Phys. Rev. D55, 6924 (1997), arXiv:hep-ph/9603448.