Fractal structure of near-threshold quarkonium production off cold nuclear matter

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We investigate near-threshold production of quarkonium resonances in cold nuclear matter through a scaling theory with two exponents which are fixed by existing data on near-threshold $J/\psi$ production in proton-nucleus collisions. Interestingly, it seems possible to extend one of the multifractal dimensions to the production of other mesons in cold nuclear matter. The scaling theory can be tested and refined in experiments at the upcoming high-intensity FAIR accelerator complex in GSI.

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There are unexplored systematics for the production of quarkonia close to threshold. For the $J/\psi$, a few experimental studies were carried out in the days before the understanding of QCD was mature. Attention shifted to the high-energy frontier, since perturbative QCD turned out to be the tool appropriate to that region, and weak-coupling theory was applied successfully to higher-energy production of quarkonium in pA collisions. However, the planned FAIR in GSI presents a grand opportunity for the study of near-threshold particle production in cold nuclear matter, and test and refine a scaling theory which is developed here. This is interesting for several reasons. First, as we argue here, testing the limits of such a scaling theory shows where the crossover between hadron and quark descriptions of matter lies. Second, this measurement can be used to test factorization where it cannot be proved, and therefore has implications on the understanding of CP violations. Third, a detailed understanding of cold nuclear effects in quarkonium production in heavy-ion collisions. Finally, the discovery of scaling laws, which is our main result, is of fundamental interest since the exponents can define universality classes across very broad ranges of physical phenomena.

The basic toolkit which we bring to this study is the modern understanding that the renormalization group is expressed via scaling. We wish to ascertain whether there is a dynamical symmetry of cold nuclear matter so that a small change in the amount of nuclear matter can be compensated for by a corresponding change in the energy of the probe. Invariance under such scaling would manifest itself in the form of exponents which are eigenvalues of renormalization group transformations. These scaling exponents are also called fractal dimensions or anomalous dimensions. At high energies they can be computed in perturbation theory. Near the threshold of particle production, they have to be discovered in data and understood non-perturbatively. Discovery of scaling exponents, which is our main result, conversely implies the scaling symmetries.

We are interested in the production of a quarkonium state, $H$, in pA collisions near the threshold energy $\sqrt{S_0}$, which is the minimum energy in the center of mass required to produce $H$. We follow the convention of writing the CM energy, $\sqrt{S}$, in the equivalent pp system; for a fixed target configuration this means $S = 2M_p(E_b + M_p)$, where $E_b$ is the beam energy and $M_p$ the proton mass. The threshold energy, $\sqrt{S_0} = 2M_p + M_H$, where $M_H$ is the mass of $H$. The total inclusive cross section, $\sigma$, can be a function of $\sqrt{S}$, $M_H$, $M_p$, and the nuclear mass, $M_A$. Then, a dimensional argument allows us to write

$$S_0 \sigma = f(A, Y, h), \quad \text{where} \quad A = \frac{M_A}{M_p},$$

$$Y = \frac{1}{2} \log \left( \frac{S}{S_0} \right), \quad h = \frac{M_H}{M_p},$$

and $f$ is a dimensionless function. In this definition of $A$, we neglect the effects of nuclear binding, which are expected to be less than 1%, and isospin effects, which could be slightly larger. We take masses and branching ratios from $[3]$. In this paper we report a scaling analysis of $J/\psi$ cross sections in a dilepton channel in pp and pA collisions from the lowest up to ISR energies, but not beyond. Within this data corpus, corrections for kinematic acceptance limitations of each experiment needed for global analyses are discussed in $[3, 22]$. We decided to examine $B\sigma$ rather than $\sigma$, where $B$ is the branching ratio in the dielectron or dimuon channel. The reason is that over the years the value of $B$ has moved by more than its error bar. When the inclusive cross section in one of these dilepton channels is measured, this uncertainty does not affect the result. Some experiments correct their data for nuclear effects according to a formula $A^{\alpha}$, with $\alpha$ obtained from their data. We undid this correction, since this is part of our global analysis.
Near threshold the variable $Y$ is small and close to zero. In proton nucleus scattering, we expect some Fermi motion: even though the CM of the nucleus is at rest, individual nucleons may be moving. The typical energy of this movement is of the order of the binding energy per nucleon $[24]$, and hence comparable to other effects which we have neglected. Clearly, Fermi motion can be detected with experiments close to threshold since the cross section vanishes otherwise. However, for $Y > 0.1$, the effect can be neglected.

When $\sqrt{S}$ is large, then one expects to be able to compute $f(A,Y,h)$ in models inspired by perturbative QCD; this is true whether $h$ is large and $Y$ small $[25]$ or $h$ is small and $Y$ large $[1,4]$. An important pre-requisite for these computations is the factorization of the initial state into parton distribution functions. If these factorization theorems were valid then certain kinematic scalings could be expected to hold $[26]$ which are seen to fail for $Y \approx 1$ $[22]$. So, in the near-threshold region for $J/\psi$ perturbative QCD inspired models for quarkonium production is not viable.

When $Y$ is small, one expects the cross section to have a Taylor expansion in $Y$. In pp collisions, where $A = 1$, one may then expect the particularly simple parametrization $f_1(Y,h) = Y^\beta f_1(\tilde{h})$, where $\beta$ is a scaling dimension and $\tilde{h}$ is a scaled variable. Such a scaling law indeed describes the $A = 1$ part of the data corpus well with

$$Bf_1/S_0 = 2.0 \pm 0.4 \text{ nb}, \quad \beta = 3.20 \pm 0.26,$$  

with covariance of $-0.934$.

More generally, we can write $f(A,Y,h) = Y^\beta f(A,\tilde{h})$, where the “renormalized” variables are taken to have the form $A = A/(Y+y)^{\mu}$, where $\mu$ is an anomalous dimension and $y$ is an additive renormalization constant. Then a change from $Y$ to $Y' = YY$ can be compensated by a change of the target nucleus from $A$ to $A' = \xi A$ with $\xi^{1/\mu} = (Y/y)/(Y+y)$. One has a similar renormalized $\tilde{h}(h,Y)$. A power law $f(A,\tilde{h}) = A^\alpha f(\tilde{h})$ then gives a multifractal exponent $A^\alpha(Y)$. Generally, such a power law parametrization may be expected when $A \gg 1$. Using the extensive data taken by NA50 $[20]$ we find that this is indeed a very good description of the data for $A > 50$. Furthermore it is significantly less probable that this behaviour extends to all $A$.

Moreover, from the data corpus it is clear that a constant power, independent of $Y$, is inadequate. Instead we write

$$f(A,Y,h) = Y^\beta A^{\alpha(Y)} f(\tilde{h}),$$

and choose a particularly simple multifractal exponent with a linear dependence on $Y$. The data corpus supports such a scaling law with

$$\alpha = (0.76 \pm 0.02) + (0.10 \pm 0.01)Y,$$  

with covariance $-0.984$, as shown in Figure 1. The data from CERN-PS $[10]$ was not used in this fit, because of the large errors in this measurement; its inclusion does not change the central values of the fit significantly but increases the errors. It would be interesting to test in future experiments whether this exponent is modified near $Y = 0.1$ as the thresholds for $\psi(2S)$ and the $\chi$S are approached.

For a test of the scaling behaviour in eq. (3) we constructed the scaled cross section $Bf/A^\alpha$, using the largest $A$ from each experiment which had $A > 50$. Fitting by a power of $Y$ gives

$$Bf/S_0 = 3.2 \pm 0.5 \text{ nb}, \quad \beta = 3.0 \pm 0.3,$$  

with covariance of $-0.970$. In dimensionless units we have $f = (3.4 \pm 0.5) \times 10^{-3}$. The exponent is consistent with that obtained in eq. (4). In fact, we have plotted the pp data also in Figure 2 to show that the two sets of data are close, but not identical. This is consistent with our expectation that the multifractal exponent of $A$ is valid for large $A$. It is clear from Figure 2 that the data corpus does not provide a very stringent test of scaling in near-threshold production cross sections. Additional data would therefore be very useful.

It is interesting to extend the parametrization of eqs. (3,4) to other resonances. The exponent $\alpha$ for the ground state quarkonium is unlikely to have a close relationship to that for higher resonances, since they are more susceptible to decay due to medium effects. On the other hand, one may ask whether there is any universality in exponents of ground state quarkonia. To answer this, we examine the $\Upsilon$, for which E772 $[21]$ reported $\alpha = 0.962 \pm 0.006 \pm 0.008$ at $\sqrt{S} = 38.8$ GeV. Extrapolating the fit in eq. (4) gives $\alpha = 0.883 \pm 0.008$. 

![Figure 1: The multifractal exponent $\alpha$ for the production of $J/\psi$ in pA collisions. The band encloses the 68% confidence limits of the two parameter fit of eq. (4).](image-url)
To improve this we add a resonance mass dependence:
\[ \alpha = \alpha_0 + \alpha_1 Y + \alpha_2 h. \]
The fit of eq. 4 along with the above measurement gives \( \alpha_0 = 0.012 \pm 0.002 \), and \( \alpha_0 = 0.044 \pm 0.029 \). Interestingly, optical models of shadowing in very low-energy nucleon-nucleus scattering predict \( \alpha_0 = 2/3 \). This value is in agreement with results for low energy \( \pi \) production \([28]\). Coincidentally, the formula is also in agreement with the measured value of \( \alpha \) for \( K, \rho \), and \( \omega \) production at low energy \([29]\).

It is interesting to compare the parametrization of eq. 3 with other studies. The Glauber model treated in the eikonal approximation has also been used in cold nuclear matter with the parametrization
\[ f(A, Y, h) = A \exp \left(-\gamma A^{1/3}\right) f(Y, h). \]
In the model, the dimensionless number \( \gamma = \rho \sigma_{abs} \lambda \), with \( \rho \) being the nuclear density, \( \sigma_{abs} \) having the interpretation of a cross section for absorption of \( J/\psi \) in cold nuclear medium, and \( \lambda A^{1/3} \) being the path length of the \( J/\psi \) in the nucleus (see, for example, \([30]\)). Although the functional form \( F(A) \) seems to be very different from \( A^\alpha \), they can be numerically close. As a result, the Glauber model cannot be experimentally distinguished from the simpler model of eq. 3. In fact, the data corpus can be used to parametrize \( \gamma = (0.27 \pm 0.02) - (0.092 \pm 0.007)Y \). Then, using the values \( \rho = 0.16/\text{fm}^3 \) and \( \lambda = 1.1 \text{ fm} \), we can write
\[ \sigma_{abs} = (15 \pm 1) - (5.2 \pm 0.4)Y \text{ mb}, \quad \text{for } Y \leq 2.5. \]
This is consistent with the values extracted in \([22]\).

Different forms have also been used for the scaling function \( f(Y, h) = S_0 \sigma A^{-\alpha} \). A parametrization suggested in \([31]\) is compatible with the data for \( Y > 1 \), but differs from that of eq. 3 near threshold. A parametrization of the threshold effect as \( (1 - 1/\exp Y)^\nu \), as in \([16]\), is compatible with data for \( Y > 1 \), but differs from eq. 3 near the threshold, as shown in Figure 2. Fermi motion based explanations of the discrepancy with data for \( Y < 1 \) are ruled out because two pieces of the data come from pp experiments. We argue that the crossover between these forms is physical and interesting. Near threshold \( Y \approx 1 - 1/\exp Y \), so the powers \( \beta \) and \( \nu \) can be compared. The fit value \( \beta \approx 3 \) can be interpreted within the old “spectator counting rules” \([32]\), as indicating that there are 2 spectators. Since hidden charm production is an OZI-violating process, this is consistent with a purely hadronic origin of near threshold processes. On the other hand, the best fit value of \( \nu \approx 11 \) (Figure 2) implies that there are 6 spectators. This is consistent with counting the valence quarks from two participating hadrons in a process dominated by gluon fusion. So the crossover from one regime to another may give us a clue to the regime of validity of perturbative QCD.

![FIG. 2: Scaling plot for the experimental data on the total inclusive cross section for the production of \( J/\psi \) in \( \pi A \) collisions. The band enclosed by full lines is the 68% confidence limit of the two parameter fit of eq. 3 to \( \pi A \) data for \( A > 50 \), so excluding \( p \)-Be data from IHEP \([3]\) and E331 \([8]\) and \( p \)-Si data from E771 \([5]\). The band enclosed between the dashed curves is the 68% confidence limit of a two parameter fit to the form \( K[1 - e^{-Y}]^\nu \) to all \( \pi A \) data.](image)
scaling laws can be tested well. This would make the GSI an ideal test bed for exploring the near-threshold production process for $J/\psi$ as well as cold nuclear effects, including questions about factorization and $p_T$ and $x_F$ distributions. A wish list would contain measurements with pp and a variety of pA collisions to check the scaling of eq. [4]. The pp data could also be used to check the scaling exponent of eq. [2] and whether it is compatible with the pA result of eq. [5]. A range of $A$ can be used to test the region of validity of the power law $A^\alpha$. A systematic study of $p_T$ and $x_F$ distributions would also be extremely useful.

In summary, we have extracted a power law parametrization of the cross section for near-threshold $J/\psi$ production off cold nuclear matter. The results are given in eqs. [3]-[5]. Such power laws are more than just a parametrization, since they reveal certain dynamical symmetries of hadronic systems, which equate a physical system with one $Y$ and $A$ to another with different $Y$ and $A$. These identities constitute renormalization group transformations, and should eventually be computable from QCD. Interestingly, it seems that the multifractal exponent $\alpha$ of eq. [4] can be extended to the production of the $\Upsilon$ via

$$\alpha(Y, h) = (0.64 \pm 0.02) + (0.10 \pm 0.01)Y + (0.012 \pm 0.002)h. \quad (9)$$

Coincidentally, this form it also reproduces the exponent required for inclusive $\pi$, $K$, $\rho$ and $\omega$ production, but not for $\psi'$ or $\phi$ production. These scaling laws present fundamental restrictions on QCD, and therefore should be of priority in upcoming low-energy and high-intensity experiments at the SIS-100/300.

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