Testing the Standard Model and Schemes for Quark Mass Matrices with CP Asymmetries in B Decays

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Abstract

The values of \(\sin 2\alpha\) and \(\sin 2\beta\), where \(\alpha\) and \(\beta\) are angles of the unitarity triangle, will be readily measured in a B factory (and maybe also in hadron colliders). We study the standard model constraints in the \(\sin 2\alpha - \sin 2\beta\) plane. We use the results from recent analyses of \(f_B\) and \(\tau_B/V_{tb}\)\textsuperscript{2} which take into account heavy quark symmetry considerations. We find \(\sin 2\beta \geq 0.15\) and most likely \(\sin 2\beta \gtrsim 0.6\), and emphasize the strong correlations between \(\sin 2\alpha\) and \(\sin 2\beta\). Various schemes for quark mass matrices allow much smaller areas in the \(\sin 2\alpha - \sin 2\beta\) plane. We study the schemes of Fritzsch, of Dimopoulos, Hall and Raby, and of Giudice, as well as the “symmetric CKM” idea, and show how CP asymmetries in B decays will crucially test each of these schemes.

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CP asymmetries in neutral B decays will provide a unique way to measure the CKM parameters. In a high-luminosity $e^+e^-$ collider running at the energy of the $\Upsilon(4S)$ resonance (a “B factory”), two of the three angles of the unitarity triangle (see Fig. 1) will be readily measured \cite{1}: the CP asymmetry in e.g. $B \to \pi^+\pi^-$ will determine $\sin 2\alpha$, while that in e.g. $B \to \psi K_S$ will determine $\sin 2\beta$. It may also be possible to measure $\sin 2\beta$ in a hadron collider, but $\sin 2\alpha$ would be difficult due to the large background (see, e.g., \cite{2}). The experimental measurements are expected to be highly accurate and the theoretical calculations are, to a large extent, free of hadronic uncertainties. Furthermore, CP asymmetries in neutral B decays are a powerful probe into possible sources of CP violation beyond the standard model (SM). The richness of available B decay modes would allow one to determine detailed features of the new sources of CP violation if the SM predictions are not borne out. In this work, we refer to both aspects of CP asymmetries in B decays, namely the determination of the CKM parameters within the SM, and the testing of extensions of the SM, with a special emphasis on the information that can be extracted by measuring two angles of the unitarity triangle rather than, say, $\sin 2\beta$ alone.

In the first part of this work, we investigate in detail the SM predictions for $\sin 2\alpha$ and $\sin 2\beta$. In particular, we study the correlation between the two quantities and present our results in the $\sin 2\alpha - \sin 2\beta$ plane. We update previous analyses with emphasis on recent theoretical developments which involve the heavy quark symmetry.

In the second part of this work, we show how various schemes for quark mass matrices can be tested through their predictions for $\sin 2\alpha$ and $\sin 2\beta$. We analyze the Dimopoulos-Hall-Raby (DHR) scheme \cite{3}, the Giudice scheme \cite{4}, the Fritzsch scheme \cite{5}, and the idea that the CKM matrix is symmetric in the absolute values of its entries \cite{6} (including the two-angle parametrization of Kielanowski \cite{7}). Each of these schemes allows a range for the asymmetries which is much smaller than in the SM and thus may be clearly excluded when the asymmetries are measured.

Various bounds on the CKM parameters are usually presented as constraints on the form of the unitarity triangle (for a review see \cite{3,1,2} and references therein). However, the quantities directly measurable via CP violation in a B-factory are $\sin 2\alpha$ and $\sin 2\beta$, so we will present our constraints in terms of these observables. The time-dependent CP asymmetry in the decay of a B or $\bar{B}$ into some final CP-eigenstate $f$ is given by

$$\frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)} = -\text{Im}\lambda(f) \sin \Delta M t,$$

(1)

where $\Delta M \equiv M(B_{\text{Heavy}}) - M(B_{\text{Light}})$, $B^0(t)$ ($\bar{B}^0(t)$) is a state which starts out as the flavor eigenstate $B^0$ ($\bar{B}^0$) at a time $t = 0$, and $\lambda(f)$ is a complex number with (almost exactly) unit magnitude. Then, within the SM (and in all schemes considered in this
work),
\[ \text{Im} \lambda(\pi^+\pi^-) = \sin 2\alpha, \quad \text{Im} \lambda(\psi K_S) = \sin 2\beta \]  
where we took into account the fact that \( \psi K_S \) is a CP-odd state. Thus, our figures in the \( \sin 2\alpha - \sin 2\beta \) plane simply present the allowed range in the \( \text{Im} \lambda(\pi^+\pi^-) - \text{Im} \lambda(\psi K_S) \) plane. This gives an important advantage to our method: the presentation in the \( \text{Im} \lambda(\pi^+\pi^-) - \text{Im} \lambda(\psi K_S) \) plane allows a direct comparison of the SM predictions (or the experimental results) with models of new physics where the asymmetries are not related to angles of the unitarity triangle.

We use the following relations to transform from the \( (\rho, \eta) \) coordinates of the free vertex \( A \) of the unitarity triangle to \( (\sin 2\alpha, \sin 2\beta) \):
\[ \sin 2\alpha = \frac{2\eta[\eta^2 + \rho(\rho - 1)]}{[\eta^2 + (1 - \rho)^2][\eta^2 + \rho^2]}, \]
\[ \sin 2\beta = \frac{2\eta(1 - \rho)}{\eta^2 + (1 - \rho)^2}. \]  
Note that these coordinate transformations are highly nonlinear; hence the predictions in the \( \sin 2\alpha - \sin 2\beta \) plane will be very different from the more familiar constraints in the \( \rho - \eta \) plane. Furthermore, since (3) are not (uniquely) invertible, we may not simply map the regions in the \( \rho - \eta \) plane allowed by each of the various constraints into corresponding regions in the \( \sin 2\alpha - \sin 2\beta \) plane, and then assume that the overlap in the latter is allowed. To see this, note that a single point in the overlap region in the \( \sin 2\alpha - \sin 2\beta \) plane may correspond to two different points in the \( \rho - \eta \) plane. If each of these two points is allowed by one constraint but forbidden by the other, then the original point in the \( \sin 2\alpha - \sin 2\beta \) plane is in fact forbidden though it is in the overlap of two regions allowed by the individual constraints. We therefore form the overlap in the \( \rho - \eta \) plane first, and then map this overall-allowed region into \( \sin 2\alpha - \sin 2\beta \) coordinates. Finally, even in the \( \rho - \eta \) plane the overlap of two allowed regions may not all be allowed: a given point in the overlap may meet the various constraints only by using different values of some parameter which enters into both constraints. But this correlation is unimportant in practice, since the uncertainties in the parameters which enter into more than one constraint never dominate both constraints.

We now analyze the SM predictions for \( \sin 2\alpha \) and \( \sin 2\beta \), updating previous analyses of constraints on the CKM parameters. The most significant update is in the constraint from \( B-\bar{B} \) mixing, which determines the length of one side of the unitarity triangle:
\[ (1 - \rho)^2 + \eta^2 = \left( \frac{1.3 \times 10^7 \text{GeV}}{B_B y_f} \right)^2 x_d \]
\[ \times \left| y_2(y_i) \langle \tau_B | V_{cb} |^2 \rangle | V_{cd} |^2 \eta_B \right| \]  
\[ (4) \]
where \( \eta_B = 0.85 \) is a QCD correction, \( y_t = (m_t/M_W)^2 \) and \( f_2(x) = 1 - \frac{3}{4}x(1+x)(1-x)^{-2}[1 + 2x(1-x^2)^{-1}\ln(x)] \). Recently, both lattice and QCD sum-rule calculations of the \( f_B \) decay constant were made which rely on heavy quark symmetry considerations. Results from the two techniques now converge to a consistent range and, we believe, should be preferred over previous, more model-dependent, calculations. We use the result of ref. [10] from QCD sum-rules, which is consistent with lattice calculations (see [11] and references therein),

\[
f_B = 190 \pm 50 \text{ MeV.} \tag{5}
\]

Since the \( B_B \) factor is expected to be close to unity, we simply take \( B_B = 1 \) and neglect the uncertainty in \( B_B \) relative to that in \( f_B \) (or, equivalently, absorb it into the uncertainty in (5)). Heavy quark symmetry considerations have also been applied to find the combination \( |V_{cb}|^2 \tau_B \). We again believe that the new results, in which only the corrections to the heavy quark limit are model-dependent, should replace previous calculations which were completely model dependent. We take the analysis of ref. [12] with updated input data [13]:

\[
|V_{cb}| \left( \tau_b/1.3 \text{ ps} \right)^{1/2} = 0.040 \pm 0.005. \tag{6}
\]

For the mixing parameter \( x_d \), we use [14]

\[
x_d = 0.67 \pm 0.11. \tag{7}
\]

Finally, we use \( |V_{cd}| = |V_{us}| = 0.221 \pm 0.002 \).

Our second constraint comes from the endpoint of the lepton spectrum in charmless semileptonic B decays. We adopt the range quoted by the Particle Data Group [15]:

\[
|V_{ub}/V_{cb}| = 0.10 \pm 0.03. \tag{8}
\]

This determines the length of the other side of the unitarity triangle:

\[
\rho^2 + \eta^2 = \left| \frac{V_{ub}}{V_{cb}V_{cd}} \right|^2. \tag{9}
\]

The third constraint comes from the \( CP \)-violating \( \epsilon \) parameter in the \( K^0 \) system:

\[
\rho = \left[ 1 + \frac{(\eta_3 f_3(y_c, y_t) - \eta_1) y_c}{\eta_2 y_t f_2(y_t) |V_{cb}|^2} \right] - \frac{1}{\eta} \left[ \frac{2.5 \times 10^{-5} |\epsilon|}{\eta_2 y_t f_2(y_t)|V_{cb}|^4 B_K |V_{cd}|^2} \right] \tag{10}
\]

where \( \eta_1 = 0.7, \eta_2 = 0.6 \) and \( \eta_3 = 0.4 \) are QCD corrections [16], \( y_c = (m_c/M_W)^2 \) and \( f_3(x, y) = \ln(y/x) - \frac{3}{4}y(1-y)^{-1}[1 + y(1-y)^{-1}\ln(y)] \). The uncertainties here lie in the value of the \( B_K \) parameter, estimated to be

\[
B_K = 2/3 \pm 1/3, \tag{11}
\]
and in the range for $|V_{cb}|$. Using $\tau_B = 1.29 \pm 0.05$ ps, we deduce from (8):

$$|V_{cb}| = 0.040 \pm 0.007.$$  

(12)

We further use $|\epsilon| = (2.26 \pm 0.02) \times 10^{-3}$ and $m_c(m_c) = 1.27 \pm 0.05$ GeV.

Since the $x_d$ and $\epsilon$ constraints depend on $m_t$, we have carried out our analysis for various $m_t$ values within the range $90$ GeV $\leq m_t \leq 185$ GeV. We present our results in Fig. 2 in two ways. First, the thin black curves encompass all values of $(\sin 2\alpha, \sin 2\beta)$ which satisfy all three constraints using values of the input parameters within their $1-\sigma$ ranges (or within the theoretically favored ranges for the parameters $B_K$ and $f_B$). That is, the SM can accommodate a B-factory result anywhere within these curves without stretching any input parameter beyond its $1-\sigma$ range. We will refer to these regions as the “allowed” areas of the SM. (A somewhat similar plot of $\sin 2\alpha$ versus $\rho$ appears in [11]). Second (and similarly to [18]), in order to get a sense of the expected value of $(\sin 2\alpha, \sin 2\beta)$ given our current knowledge of the various input parameters, we generated numerous sample values for these parameters based on a Gaussian distribution for $|V_{cd}|$, $\tau_B|V_{cb}|^2$, $|V_{ub}/V_{cb}|$, $\tau_B$, $x_d$, $m_c$ and $|\epsilon|$, and a uniform distribution (= 0 outside of the “1-\sigma” range) for $f_B$. For each sample set we used the constraints (4) and (9) to determine $\rho$ and $\eta$, and then rejected those sets which did not meet the constraint (10) for $1/3 \leq B_K \leq 1$. We binned the sets which passed in the $\sin 2\alpha-\sin 2\beta$ plane, and thus obtained their probability distribution. We show in Fig. 2 the resulting 68% and 90% probability contours in dark gray and light gray, respectively. Since we do not know the true origin of the CKM parameters and thus do not know the true probability distribution from which the experimental inputs result, and since the theoretical restrictions on $f_B$ and $B_K$ cannot be posed statistically, we can only interpret these probability contours as an indication of likely outcomes for B-factory results based on the SM. For example, the “tail” of the allowed areas which extends towards small values of $(\sin 2\alpha, \sin 2\beta)$ requires many of the parameters to be stretched to their $1-\sigma$ bounds and so seems unlikely and lies outside both probability contours.

Similarly to previous analyses (see, e.g., [2, 11, 13, 21, 20, 24, 23]), we find that $\sin 2\alpha$ can have any value in the full range from $-1$ to $1$, while $\sin 2\beta$ is always positive and has a lower bound

$$\sin 2\beta \geq 0.15.$$  

(13)

Furthermore, $\sin 2\alpha$ is likely to be positive if the top mass is near its present lower bound, and most importantly the favored values for $\sin 2\beta$ are above 0.5. We also find that the bounds on the two quantities are correlated (as also noted in [21]). In particular, we note that:
• The magnitude of at least one of the two asymmetries is always larger than 0.2, and probably larger than 0.6.

• If $\sin 2\beta \leq 0.4$, then $\sin 2\alpha$ must be positive—in fact, above 0.2.

Once the top mass is measured firmer predictions will of course be possible, based on one of the graphs in Fig. 2.

Various estimates may be made of the allowed ranges for the input parameters. In particular, there is no single obvious way to evaluate theoretical uncertainties. Furthermore, future improvement in both experimental measurements and theoretical analyses would certainly strengthen the constraints. Thus, it is useful to understand the sensitivity of our analysis to the various uncertainties. To this end we have displayed in Fig. 3 how the allowed regions of the SM depend on the choice of input parameters, for a representative top mass of 130 GeV. For Figs. 3a, 3b and 3c we have allowed somewhat larger ranges for $0.05 \leq |V_{ub}/V_{cb}| \leq 0.15$ and $100 \text{ MeV} \leq f_B \leq 300 \text{ MeV}$. All other ranges are kept as before. The 5 solid lines of Fig. 3a correspond, from bottom to top, to the constraint (1) when $|V_{ub}/V_{cb}|$ increases from 0.05 to 0.15. The 12 solid lines of Fig. 3b correspond, from left to right, to the constraint (11) when the values of $f_B$ and $\tau_B |V_{cb}|^2$ decrease within their respective ranges. The 6 solid lines of Fig. 3c correspond, from left to right, to the constraint (11) when $|V_{cb}|^2$ and $B_K$ decrease within their respective ranges. (Note that each solid line in these figures must meet all three constraints. For Fig. 3b this disallows the lower end of the range for $f_B$ and $\tau_B |V_{cb}|^2$, while for Fig. 3c it is the lower end of the range for $|V_{cb}|^2$ and $B_K$ that is not allowed.) One can then read off the approximate allowed region for a more restricted choice of input parameter ranges. For completeness we have also plotted in Fig. 3d the allowed region obtained by accepting the range $0.15 \leq |V_{ub}/V_{cb}| \leq 0.20$ suggested by Isgur et al. [24], while keeping all other parameters as in the rest of Fig. 3. In this case it is likely that $\sin 2\beta$ is very close to unity, or else (and this is unlikely) $\sin 2\alpha \sim \sin 2\beta$ and they can both be as small as roughly 0.1 if $|V_{ub}/V_{cb}|$ and $B_K$ are as large as possible and $f_B$ is as small as possible.

We next turn to the testing of various schemes for quark mass matrices. We use the following ranges for quark masses at 1 GeV [17]:

$$m_c = 1.36 \pm 0.05 \text{ GeV}, \quad m_b = 5.6 \pm 0.4 \text{ GeV},$$  \(14\)

and for mass ratios:

$$\frac{m_d}{m_s} = 0.051 \pm 0.004, \quad \frac{m_u}{m_c} = 0.0038 \pm 0.0012, \quad \frac{m_s}{m_b} = 0.030 \pm 0.011.$$  \(15\)

In the remainder of our analysis we allow only $1-\sigma$ ranges for all inputs, since we believe that if any of these schemes need to be stretched beyond their $1-\sigma$ predictions
then their motivation is largely lost. These $1 - \sigma$ ranges should only be viewed as the favored values within the schemes; one should not rule out any scheme simply on the basis that the experimental results do not quite fall within the $1 - \sigma$ predictions we obtain. In Fig. 4 we display these predictions of the four schemes for the same sample values of $m_t$ as in Fig. 2. Only the symmetric CKM ansatz admits a sufficiently large range of $m_t$ to be included in more than one graph. For reference we have also indicated, in gray, the $1 - \sigma$ allowed areas of the SM.

We first discuss the Fritzsch scheme [5],

$$M_u = \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & a_d e^{i\phi_1} & 0 \\ a_d e^{-i\phi_1} & 0 & b_d e^{i\phi_2} \\ 0 & b_d e^{-i\phi_2} & c_d \end{pmatrix}. \quad (16)$$

It fits ten parameters (6 masses, 3 mixing angles and a CP-violating phase) with eight parameters and therefore makes two predictions. It is now nearly excluded [25]. The main difficulty lies in the relation

$$|V_{cb}| = \left| \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}} \right|, \quad (17)$$

which can only be fulfilled if the top quark is close to the experimental lower bound:

$$m_t \sim 90 \text{ GeV}. \quad (18)$$

If the top quark is indeed this light, then the next crucial test for the Fritzsch scheme would be its predictions for CP asymmetries in $B^0$ decays. The allowed range for $(\sin 2\alpha, \sin 2\beta)$ is shown as the black wedge in Fig. 4a. We find

$$0.10 \leq \sin 2\alpha \leq 0.67; \quad 0.56 \leq \sin 2\beta \leq 0.60. \quad (19)$$

We turn next to the scheme of Giudice [4], which requires the charged fermion mass matrices to have the following form at the GUT scale:

$$M_u = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & f e^{i\phi} & 0 \\ f e^{-i\phi} & d & 2d \\ 0 & 2d & c \end{pmatrix}, \quad M_\ell = \begin{pmatrix} 0 & f & 0 \\ f & -3d & 2d \\ 0 & 2d & c \end{pmatrix}. \quad (20)$$

This scheme fits the quark and lepton mass matrices with six parameters and therefore makes seven predictions. Among them we find

$$m_t \sim 125 - 155 \text{ GeV}, \quad |V_{cb}| \sim 0.048, \quad 0.07 \leq |V_{ub}/V_{cb}| \leq 0.084 \left( \frac{130 \text{ GeV}}{m_t} \right). \quad (21)$$
Note that our allowed range for \( m_t \) is smaller than in ref. \([4]\), due to our stronger bounds on \( |V_{ub}/V_{cb}| \). (This range is very sensitive to the bottom quark mass, and thus could be enlarged by adopting more conservative estimates of the uncertainty in \( m_b \).) It is not unlikely that this scheme would survive the various measurements until a B-factory starts running. Then it allows only a narrow band in the \( \sin 2\alpha - \sin 2\beta \) plane, as shown in Fig. 4b. The overall constraint is

\[
-0.98 \leq \sin 2\alpha \leq +1.0; \quad 0.2 \leq \sin 2\beta \leq 0.7. \tag{22}
\]

However, for low \( \sin 2\beta \) values, there is a strong correlation between the two asymmetries. In particular, if \( \sin 2\beta < \sim 0.45 \), then \( \sin 2\alpha \geq 0.65 \).

The scheme by DHR \([3]\) requires that, at the GUT scale, charged fermion mass matrices are of the following form:

\[
M_u = \begin{pmatrix}
0 & c & 0 \\
c & 0 & b \\
0 & b & a
\end{pmatrix}, \quad M_d = \begin{pmatrix}
0 & f & e^{i\phi} & 0 \\
e^{-i\phi} & e & 0 \\
0 & 0 & d
\end{pmatrix}, \quad M_\ell = \begin{pmatrix}
0 & f & 0 \\
f & -3e & 0 \\
0 & 0 & d
\end{pmatrix}. \tag{23}
\]

It has seven parameters and therefore six predictions, among which we find (c.f. \([26]\))

\[
m_t \sim 185 \text{ GeV}, \quad |V_{cb}| \sim 0.047, \quad |V_{ub}/V_{cb}| \sim 0.065. \tag{24}
\]

(Note that the latter prediction, which is at the top of the \( 1-\sigma \) range for this scheme, is just below our allowed range. We therefore predict a very narrow range of the DHR parameter \( \chi \) which accounts for much of the uncertainty in this scheme: \( \chi^2 \simeq 4/3. \)) Thus, future measurements of \( m_t \), or theoretical improvement in determining \( |V_{cb}| \) or \( |V_{ub}/V_{cb}| \), may easily exclude the DHR scheme. If it survives these tests, then it would provide very powerful predictions for \( CP \) asymmetries in \( B^0 \) decays. Only a very narrow range in the \( \sin 2\alpha - \sin 2\beta \) plane is allowed, as shown in Fig. 4d. The overall constraint is

\[
-0.58 \leq \sin 2\alpha \leq -0.33; \quad 0.51 \leq \sin 2\beta \leq 0.60. \tag{25}
\]

Once again the values of the two asymmetries are correlated, providing an even stronger test than implied by (25).

Our last example is the symmetric ansatz \([6]\) for the CKM matrix,

\[
|V_{ij}| = |V_{ji}|. \tag{26}
\]

The theoretical motivation for this ansatz is more obscure than for the previous ansätze. In particular, it is still to be demonstrated that the constraints (26) can result from some symmetry of the lagrangian \([27]\). This ansatz leads to (c.f. \([28, 29, 30, 31]\))

\[
m_t \gtrsim 160 \text{ GeV}, \quad |V_{ub}/V_{cb}| \geq |V_{cd}|/2 \simeq 0.11. \tag{27}
\]
This bound on $m_t$ is lower than in some previous analyses due to our higher allowed range of $f_B$, as already remarked in [18]). $CP$ asymmetries in $B^0$ decays would be extremely powerful in testing (20). The correlation between $\sin 2\alpha$ and $\sin 2\beta$ is strongest here, as (26) leads to

$$\rho = 1/2 \implies \sin 2\alpha = -2 \sin 2\beta \cos 2\beta. \quad (28)$$

For a fixed $m_t$ value, (28) leads to an allowed curve in the $\sin 2\alpha - \sin 2\beta$ plane, as shown in Figs. 4c and 4d. For the overall bounds we find

$$-1.0 \leq \sin 2\alpha \leq -0.76; \quad 0.68 \leq \sin 2\beta \leq 0.91. \quad (29)$$

The two-angle parametrization of the CKM matrix proposed by Kielanowski [7] is a special case of this ansatz, in which $\eta \simeq 1/(2\sqrt{3})$ (to within a few percent). Consequently (c.f. [29, 30]) $\sin 2\alpha = -\sqrt{3}/2 = -\sin 2\beta$, as indicated by the small filled circle in Figs. 4c and 4d.

Before concluding, let us mention a discussion of the structure of quark mass matrices by Bjorken [32]. His assumptions lead to a prediction for the angle $\gamma$ of the unitarity triangle, $\gamma \approx \pi/2$. For the asymmetries discussed here, this implies $\sin 2\alpha = \sin 2\beta$, which coincides with the predictions of the superweak scenario. A discussion of the experimental prospects of excluding such a relation can be found in refs. [33, 34].

To summarize, we have examined the predictions of the SM and of various quark mass matrix schemes for $\sin 2\alpha$ and $\sin 2\beta$ or, equivalently, for the $CP$ asymmetries in $B \to \pi\pi$ and $B \to \psi K_S$. Our main results are presented in Figs. 2 and 4. We have displayed them in the $\sin 2\alpha - \sin 2\beta$ plane to facilitate direct comparison with future experiments or non-standard models, and to show the importance of the correlation between the predictions for $\sin 2\alpha$ and for $\sin 2\beta$. (This correlation was also used in [21, 35]). The predictions are quite encouraging for experimenters:

- Recent improvements in theoretical calculations lead to a lower bound on the asymmetry in $B \to \psi K_S$ of order 0.15, somewhat higher than previous analyses.

- If the asymmetry in $B \to \psi K_S$ is close to its lower bound, then it is highly correlated with the asymmetry in $B \to \pi\pi$ and at least one of the two is larger than 0.2.

- For the asymmetries to both be small, many parameters have to assume values close to their $1 - \sigma$ bounds, which is improbable. It is more likely that at least one of the asymmetries is larger than 0.6.
Various schemes for quark mass matrices allow a much smaller range for the two asymmetries than does the SM. Therefore, they would be stringently tested when the asymmetries are measured.

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Figure Captions

Fig. 1: The unitarity condition $V^*_{ub}V_{ud} + V^*_{cb}V_{cd} + V^*_{tb}V_{td} = 0$ represented as a triangle in the complex plane. The sides have been divided by $|V_{cb}V_{cd}|$ so that the vertices may be placed at $(0,0)$, $(1,0)$ and $(\rho, \eta)$. The angles $\alpha$ and $\beta$ are measured counterclockwise as shown.

Fig. 2: The SM predictions in the $\sin 2\alpha$ (horizontal) $- \sin 2\beta$ (vertical) plane, for four different top quark masses: (a) 90 GeV, (b) 130 GeV, (c) 160 GeV and (d) 185 GeV. The regions allowed by the $1-\sigma$ ranges for all parameters described in the text are outlined by the thin black lines. The 68% probability contours generated as described in the text are shown as thick dark-gray lines, while the 90% contours are indicated by thinner light-gray lines.

Fig. 3: The dependence of the allowed regions in the $\sin 2\alpha - \sin 2\beta$ plane on the input parameter ranges, for a representative value of $m_t = 130$ GeV. The largest region allowed by all three constraints is outlined by the dashed lines. For this figure we have allowed the wider range $100$ MeV $\leq f_B \leq 300$ MeV but kept all other ranges as before, with the exception of $|V_{ub}/V_{cb}|$: in 3a, b and c we allow the wider range $0.05 \leq |V_{ub}/V_{cb}| \leq 0.15$, while in 3d we adopt the higher range of ref. [24], $0.15 \leq |V_{ub}/V_{cb}| \leq 0.20$. The 5 solid lines in 3a correspond, from bottom to top, to the constraint (I) when $|V_{ub}/V_{cb}|$ increases from 0.05 to 0.15. The 12 solid lines of 3b correspond, from left to right, to the constraint (II) when its right-hand side increases from 0.29 to 2.71. The 6 solid lines of 3c correspond, from left to right, to the constraint (III) when its first bracketed expression increases from 1.34 to 1.40 and the second increases from 0.24 to 0.96.

Fig. 4: The $1-\sigma$ allowed regions predicted by various mass matrix schemes in the $\sin 2\alpha - \sin 2\beta$ plane, for the same sample values of $m_t$ as in Fig. 2. The allowed regions within the SM are outlined for reference in light gray. In 4a the value of $m_t = 90$ GeV is consistent only with the Fritzsch ansatz, which predicts the values within the thin black wedge. A top mass of $m_t = 130$ GeV in 4b is compatible only with the scheme of Giudice, which allows the region within the band outlined in black. A symmetric CKM matrix is consistent with a top mass of 160 GeV (4c) and 185 GeV (4d); its predictions lie along the short black curve, while the special case of Kielanowski is shown as the small filled circle in each of these figures. The DHR scheme predicts the heavy top mass $m_t = 185$ GeV of 4d, and allows only the tiny region shown in black.