Modeling of competition between agents of the same level in a multi-level system with information asymmetry

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Abstract. We propose a game-theoretic model of resource-based competition between agents of the same level in a multi-level system with information asymmetry. It is shown that under conditions of uncertainty, the optimism of agents leads to non-optimal solutions and an increase in the loss of the total added value in the system due to resource waste. The mechanisms of optimistic shifts in agent estimates and agent’s Bayesian adjustment are described. Equilibrium conditions are obtained in the proposed model.

1. Introduction
Problems of estimating uncertainty are often named among the most significant factors that impede decision making in systems with partially independent agents, both technical and social. Among such systems, we could distinguish multilevel systems in which this problem is combined with the problem of agent opportunism [3, 11, 17]. For example, consider venture capital funds with some project filtering mechanisms, where reviewing experts, managers, decision makers, fund managers and shareholders are all agents of different levels; distributed hierarchical management systems, such as a regional management system; social network structures with agents of various levels of access and influence, including markets with imperfect competition, etc. In the models of such systems, the heterogeneity of individual agents is usually taken into account by means of various utility functions, and the differences in levels are taken into account by means of principal-agent models in modeling of relationships between agents [11, 15]. For such systems there is a need to model decision-making taking into account uncertainty, and the most common approach is to reduce uncertainty to risk where it is possible [7, 8]. The problem arises when such reduce is impossible without a significant loss of information.

A closer look allows us to distinguish two components of the problem of estimating uncertainty – rational decision-making mechanisms and management of risk attitude mechanisms. If a large number of works were the first to be devoted [2, 13, 16], then risk propensity management issues were addressed much less frequently [5], most often risk propensity in modern models is accepted by the immanent characterization of an agent that is not subject to changes [2, 6].

At the same time, the question of the optimal risk appetite or the optimal ratio of agents with different risk appetites seems to be important for the study of distributed decision-making systems (both technical and social) that are faced with the uncertainty of the future. Such systems are based on the aggregation of implicit knowledge, the carriers of which are various agents, for making decisions based on the forecast of the future state of the external environment. An example of such a system is...
the innovation investment system, when the evaluation of innovative projects is difficult due to their uniqueness, and the proportion of potentially successful projects is small enough so that a naive approach based on the initial screening of pseudo-innovators [9, 10] and the subsequent financing of all projects was irrational. In reality, the supply side in the innovation investment market is represented by many investors, differing in institutional characteristics and decision-making models. For some of them, risk appetite is normatively limited (investment funds), for others (business angels) - there are no such restrictions. This means that an effective mechanism for managing the system of innovative investment (and more broadly – systems with heterogeneous agents in an uncertain environment, both social and artificial) must take into account the existence of persistent deviations from risk neutrality in the behavior of agents. In this regard, it is of interest to study the effectiveness of various economic mechanisms in a multi-agent system with uncertainty at different risk appetite of agents.

While modeling social and economic systems the risk attitude for is usually represented as utility functions of risky gains. A convex utility function is characteristic for risk-prone agents, concave - for risk-avoiding agents. Arrow-Pratt's risk aversion measure, which characterizes the elasticity of the marginal utility of money in terms of consumption, is often used as the basis for assessing risk appetite [1, 14, 18]. For situations of symmetric information and risk (i.e., the known probabilistic distribution of uncertain future payments), this representation makes it possible to very well take into account the individual characteristics of agents in modeling interactions (insurance market, lotteries, stock market, etc.). In the case of information asymmetry or uncertainty, when the probability distribution parameters are unknown, this approach does not give the desired result, since in addition to the individual risk appetite, one must also take into account deviations of the estimates of the probability distribution of future incomes and expenses from the true values.

Thus, the question arises of how to model the risk appetite of an agent under uncertainty. Assume that agent wins are cashable. Then deviation from risk-neutral behavior is not optimal. The literature shows that this is true both for conditions of complete information and for risk conditions [4, 18]. Consider the uncertainty situation as a risk situation with true distribution parameters unknown to the agents (expectation, variance, asymmetry, shift, etc.). Let us imagine the risk appetite of the agent in the form of the combined effect of several irrational deviations of the estimate, which shift the subjective assessment of the probability of the occurrence of the event from some true probability. For competing innovators, this may be optimism about their performance, for business angels, optimism about their ability to assess the prospects of an innovative project. In the event of a dumping war, this may be the underestimation of the competitors' budget capabilities. For regional agents in a distributed hierarchical system, this is optimism regarding their ability to attract external and internal resources, etc.

2. The Model

Suppose that in a multi-level open system there are several agents with different risk appetite, competing for one long-term contract with the principal.

Consider a model of competition between two agents \( i_1 \) and \( i_2 \), each of which has a certain budget \( R_i(t) \) at time \( t\geq 0 \). For simplicity, we take the initial value \( R_i(0) \) per unit. \( R \) reflects the agent's ability to bear the costs at the bidding stage for the subject of the contract, for example, the cost of participation in exhibitions, the manufacture and presentation of prototypes, and the preparation of marketing materials.

The game begins with a preliminary period, followed by a depletion game, if the participant decides to take part in it. In the preliminary period, each participant chooses to participate in the fight or not, and if either of the two chooses to participate, then a war of attrition type game begins [12], otherwise the status quo is saved and the budgets of the agents are not wasted. If the war of attrition began, then in each time period \( t\geq 0 \) both agents simultaneously decide to continue to fight or to leave the game. Continuing the fight entails costs for both participants. These costs can be represented as a
decrease in the agent’s budget with a certain speed $v_i$, so that the costs of agent $i$ over time $t$ will be as follows:

$$\frac{dR_i}{dt} = -v_i R_i(t)$$

(1)

So, we can deduce that $R_i(t) = e^{-v_i t}$.

Suppose that $v_i$ is independently taken from the distribution $F_i(\hat{\theta}, \gamma^2)$, known to agents, where any $\gamma$ is in the range from 0 to 1. Agent $i$ knows its $v_i$ value (its type) exactly and has an estimate about the type of the other agent (assumption regarding the probability distribution).

Let us introduce the concept of optimistic bias in agents’ assessments of the types of competing agents. Suppose that the expected distribution of competitor type $F_i$ can be represented as $\hat{F}_{-i}(\hat{\theta} + K_i \frac{\gamma^2}{\sigma_i^2})$, where $\hat{\theta} + K_i$ is the estimate of mathematical expectation, and $\frac{\gamma^2}{\sigma_i^2}$ is the variance estimate. We now consider two types of optimistic biases in the assessment – “underestimating the opponent” ($K_i > 0$) and “re-evaluating the accuracy of the forecast” ($\sigma_i^2 > 1$). Underestimating an opponent is the average error in assessing the likelihood of an early exit of an opponent from the game. In the model, it takes the form of increasing the mathematical expectation of costs in each period of the game, which reduces the opponent’s incentives to continue the rivalry in assessing player $i$. The second type of optimistic bias is a reassessment of the accuracy of your forecast. The higher $\sigma_i$, the more confident player $i$ is in its assessment of opponent’s costs.

Two circumstances should be noted. Firstly, underestimating an opponent in our model is a deviation from optimal behavior, because it fully explains the discrepancy between the expected distribution $\hat{F}_{-i}$ with the true distribution $F_i$. Secondly, underestimating the opponent leads to asymmetric configurations, because in its absence, the expected distributions of the players coincide.

For illustration purposes, Fig. 1 shows all four possible types of the deviations $\hat{F}_{-i}$ from $F_i$ - a) underestimation of the opponent, b) overestimation of accuracy, c) both types of bias combined and d) no bias (zero deviation).

![Figure 1](image). Types of deviations caused by differences in subjective ratings $\hat{F}_{-i}$ (shown by a dotted line) from the true distribution of $F_i$ (shown by a solid line).

In each period of the game, agents choose whether to continue to compete or to leave the game. The game ends when at least one of them leaves the game. If one agent refused to continue the
competition, and the second did not, then the winner receives the resources of the loser. If both refused to continue in the same round, they remain with their sources of resources.

Thus, if agent \(i\) intends to leave the game at step \(T_i\), its expected gain depends on the actions of the opponent and can be described as follows:

\[
U_i(T_i, T_{-i}, v_i) = \begin{cases} 
-T_i, & T_i < T_{-i} \\
-T_i + V_i^S(T_i), & T_i = T_{-i} \\
-T_i + V_i^W(T_i), & T_i > T_{-i}
\end{cases}
\]

(2)

where \(V_i^S(T_i), V_i^W(T_i)\) – expected final payouts, respectively, in case of simultaneous completion of the game and victory in the game.

Denoting the distribution of expectations of agent \(i\) with respect to the future exit point of the opponent from the game by \(\tilde{G}_{-i}(\cdot)\) and the corresponding probability density function by \(\tilde{g}_{-i}(\cdot)\) we obtain the expected payout of the agent

\[
V_i(T_i, v_i) = -T_i[1 - \tilde{G}_{-i}(T_i)] + \int_0^{T_i} [V_i^W(x) - x] \tilde{g}_{-i}(x) dx
\]

(3)

The equilibrium in this game is found as the Bayesian Nash equilibrium [8]. The strategy of agent \(i\) is described by a pair \((s_i, T_i(v_i))\), where the second element is a function that determines the optimal predicted exit time for each possible value of \(v_i\).

3. Conclusion

Denoting the distribution of expectations of agent \(i\) with respect to the future exit point of the opponent from the game by \(\tilde{G}_{-i}(\cdot)\) and the corresponding probability density function by \(\tilde{g}_{-i}(\cdot)\) we obtain the expected payout of the agent

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Acknowledgements

The research was supported by Russian Foundation of Basic Research (RFBR), research projects No. 19-010-00376 A, 19-010-00577 A.

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