

1. Introduction

Mold oscillation plays an important role in continuous casting. It decides whether the continuous casting progress is smooth or not. The velocity rules of mold oscillation went through rectangular, trapezoid, sinusoidal, and nonsinusoidal oscillation. With the demand growing for the surface quality and casting speed of slab, nonsinusoidal oscillation has been one of the key technologies to realize highly efficient continuous casting [1–4].

Nowadays, mold oscillation technologies used in production mainly include sinusoidal and nonsinusoidal oscillations. Sinusoidal oscillation could reduce negative strip time and improve surface quality of slab when high frequency and small amplitude operation are adopted [5, 6]. Lin et al. [7] optimized the sinusoidal oscillation parameters for Nanjing steel, the lubrication between the mold and slab had been improved, the demoulding resistance was reduced, the hook-type oscillation mark was eliminated, and the slab quality was enhanced. Wang et al. [8] found that the hook-type oscillation marks of slabs could be reduced significantly by increasing frequency and reducing amplitude. Although sinusoidal oscillation can reduce negative strip time, the positive strip time also reduces. In contrast, at the time of reducing negative strip time for nonsinusoidal oscillation, positive strip time can be prolonged which is useful for mold powder consuming. Nonsinusoidal oscillation can obtain reasonable technological parameters and is an ideal oscillation mode for mold [9]. The research of nonsinusoidal oscillation is mainly focused on oscillation waveform, realization way, and parameter control. The waveform functions of nonsinusoidal oscillation mainly contain entire function and piecewise function. An oscillation waveform of entire function was developed by Suzuki et al. [10], which had been used in many iron and steel enterprises. The principle of nonsinusoidal oscillation was studied by means of the rotation vector method, and Li et al. proposed an oscillation waveform which could be realized by using a
couple of elliptical gears [11], which had been applied in many companies and achieved good results. Zhang et al. [12, 13] proposed a nonsinusoidal oscillation waveform composed of two sinusoidal functions, which could be realized by double eccentric shafts. Liu et al. presented the entire function of nonsinusoidal oscillation waveform which could be realized by noncircular gears [14]. Hong et al. [15] proposed an oscillation waveform function composed of Fourier series. Although the entire function is easy to be constructed, the values of basic parameters have a certain limit. An oscillation waveform function consisting of three sections of sine trigonometric functions was proposed. The displacement, velocity, and acceleration curves are continuous. However, the maximum acceleration is larger and it has bigger impact for the oscillator when the mold is at the uppermost point and lowest point. Then, an oscillation waveform function was proposed in this paper, which has lower maximum acceleration and inertia force. Based on nonsinusoidal oscillation waveform of seven section functions, the calculation methods of technological parameters were given, the multitechnological parameter curve was obtained, and the synchronous control model of casting speed and oscillation frequency was determined. The calculation results indicate that the oscillation waveform function proposed in this paper will be useful to further improve the quality of slab and highly efficient continuous casting.

2. Construction of Nonsinusoidal Oscillation Waveform

The lower acceleration for oscillator results in smaller inertia force. The velocity function of nonsinusoidal oscillation waveform is constructed, which is composed of seven sections, shown in Figure 1. From Figure 1, it can be seen that AB, CD, FG, and HK are straight lines and BC, DEF, and GH are parabolas. The functions of oscillation waveform are as follows.

The velocity function is

\[
v(t) = \begin{cases} 
  v_B, & 0 \leq t \leq t_B, \\
  -k(t - t_B)^2 + v_B, & t_B \leq t \leq t_C, \\
  -2k(t_C - t_B)(t - t_C), & t_C \leq t \leq t_D, \\
  k\left(t - \frac{1}{2f}\right)^2 - v_B - 2k(t_C - t_B)\left(\frac{1}{2f} + t_B - 2t_C\right), & t_D \leq t \leq t_F, \\
  2k(t_C - t_B)\left(t + t_C - \frac{1}{f}\right), & t_F \leq t \leq t_G, \\
  -k\left(t + t_B - \frac{1}{f}\right)^2 + v_B, & t_G \leq t \leq t_H, \\
  v_B, & t_H \leq t \leq t_K,
\end{cases}
\]

where \(v\) is the velocity of mold, mm/s, \(h\) is the oscillation amplitude, mm, \(f\) is the oscillation frequency, Hz, and \(\alpha\) is the waveform modification ratio of nonsinusoidal oscillation. \(t_C, t_D, t_F,\) and \(t_G\) are the time periods of different positions for mold, \(s, v_B\) is the velocity of point B, \(t_F = (1/f) - t_D, t_G = (1/f) - t_C,\) and \(t_H = (1/f) - t_B, \) \(v_B = k(t_C - t_B)^2, k = (3h)/[3t_C(t_C - t_B)^2 - (t_C - t_B)^3], t_B = \alpha/[f(1 + \alpha)], t_C = (1 + \alpha)/(4f),\) and \(t_D = 1/(2f) + t_B - t_C.\)
The displacement function is

\[ s = \begin{cases} 
 v_B t, & 0 \leq t \leq t_B, \\
 -\frac{k}{3} (t - t_B)^3 + v_B t, & t_B \leq t \leq t_C, \\
 -k(t_C - t_B)(t - t_C)^2 + c_1, & t_C \leq t \leq t_D, \\
 \frac{k}{3} \left[ (t - \frac{1}{2}f)^3 - v_B t - 2kt(t_C - t_B) \left( \frac{1}{2}f + t_B - 2t_C \right) + c_2 \right], & t_D \leq t \leq t_F, \\
 k(t_C - t_B) \left[ t - \left( \frac{1}{f} - t_C \right)^2 \right] + c_3, & t_F \leq t \leq t_G, \\
 -\frac{k}{3} \left[ t - \left( \frac{1}{f} - t_B \right) \right]^3 + v_B t + c_4, & t_G \leq t \leq t_H, \\
 v_B \left( t - \frac{1}{f} \right), & t_H \leq t \leq t_K, 
\end{cases} \]

where \( s \) is the displacement of mold, mm,
\[
c_1 = v_B t_C - \left( \frac{k}{3} \right) (t_C - t_B)^3 = \left( \frac{v_B^2}{3} \right) (t_B + 2t_C),
\]
\[
c_2 = \left( \frac{v_B^2}{2f} \right) + \left( \frac{k}{f} \right) (t_C - t_B) \left( \frac{1}{2f} \right) + t_B - 2t_C),
\]
\[
c_3 = -\left( \frac{v_B^3}{3} \right) (t_B + 2t_C), \text{ and } c_4 = -\left( \frac{v_B}{f} \right).
\]

The acceleration function is

\[ a = \begin{cases} 
 0, & 0 \leq t \leq t_B, \\
 -2k(t - t_B), & t_B \leq t \leq t_C, \\
 -2k(t_C - t_B), & t_C \leq t \leq t_D, \\
 2k(t_C - t_B), & t_D \leq t \leq t_F, \\
 2k \left( t + t_B - \frac{1}{f} \right), & t_F \leq t \leq t_G, \\
 -2k \left( t + t_B - \frac{1}{f} \right), & t_G \leq t \leq t_H, \\
 0, & t_H \leq t \leq t_K, 
\end{cases} \]  \quad (3)

with oscillation waveform of five sections with the same basic parameters, shown in Figure 3. Then, the inertia force is smaller. In brief, the oscillation waveform has good dynamic characteristics.

3. Technological Parameters and Synchronous Control Model

Technological parameters decide the production of continuous caster and the surface quality of slab, so it is necessary to analyze them.

3.1. Negative Strip Time Curves. Negative strip time is the period time that the velocity of mold moving downward is faster than casting speed shown in Figure 4. If the negative strip time is longer, the oscillation mark will be deeper:

\[ t_N = 2 \left( \frac{1}{2f} - t_1 \right). \]  \quad (4)

For processing,

\[ t_1 = \frac{1}{2} \left( \frac{1}{f} - t_N \right). \]  \quad (5)

The velocity of mold is \( v_c \) at time \( t_1 \). Submitting equation (5) into equation (1) (the third equation),

\[ t_N = \frac{1 - \alpha - \frac{v_c (1 - \alpha)^2 (a^2 + 4\alpha + 1)}{24hf^2 (\alpha + 1)^2}}{2f}. \]  \quad (6)

where \( t_N \) is the negative strip time, \( s \), and \( v_c \) is the casting speed, mm/s.

If \( Z = (2h/E) (\text{mm} \cdot \text{min}/\text{m}) \) and \( E = (v_c/1000) \times 60 \), the relationship between \( t_N \) and \( Z \) is obtained as
Figure 1: Sketch of velocity curve for nonsinusoidal oscillation.

Figure 2: Nonsinusoidal oscillation waveform of seven section functions. (a) Displacement curves. (b) Velocity curves. (c) Acceleration curves.
when $\alpha$ equals 20% and $Z$ equals different values; negative strip time curves are shown in Figure 5.

### 3.2 Negative Strip Ratio

Negative strip ratio is defined as

$$NS = 1 - \frac{4fh}{v_c(1 - \alpha)}.$$  \hspace{1cm} (8)

The larger negative strip ratio is helpful for removing the slab from the mold. Submitting (8) into equation (6), the relationship between $t_N$ and NS can be expressed as

$$t_N = \frac{1 - \alpha}{2f} - \frac{25(1 - \alpha)^2(\alpha^2 + 4\alpha + 1)}{18f^2(\alpha + 1)^2Z},$$  \hspace{1cm} (7)

when $\alpha$ equals 20% and $Z$ equals different values; negative strip time curves are shown in Figure 5.

### 3.3 Negative Strip Time Ratio

Negative strip time ratio is the ratio between negative strip time and half an oscillation period:

$$NSR = \frac{t_N}{0.5T} = 1 - \frac{v_c(\alpha - 1)^2(\alpha^2 + 4\alpha + 1)}{12hf(\alpha + 1)^2}.$$  \hspace{1cm} (9)

When $\alpha$ equals 20%, NS equals different values, and the change of $t_N$ with $f$ is shown in Figure 5.

$$t_N = \frac{1}{2f} - \frac{(1 - \alpha)(\alpha^2 + 4\alpha + 1)}{6f(\alpha + 1)^2(1 - NS)}.$$  \hspace{1cm} (10)

Where NSR is the negative strip time ratio, %.
3.4. Negative Strip Distance. Negative strip distance is the displacement of mold moving downward relative to the slab during the negative strip time of an oscillation period. Then, NSA (the area, shown in Figure 4) can be calculated as follows:

\[
\text{NSA} = \int_{t_1}^{t_2} \left[ v_m - v_c \right] \, dt = 2 \int_{(T/2)-(T_N/2)}^{T/2} \left[ v_m - v_c \right] \, dt
\]

\[
= 2 \left\{ \int_{(T/2)-(T_N/2)}^{T_D} \left[ 2k(t_C - t_B)(t - t_C) \right] \, dt + \int_{(T_D)-(T_N/2)}^{T/2} \left[ v_B + 2k(t_C - t_B) \left( \frac{1}{2f} + t_B - 2t_C \right) - k \left( \frac{1}{2f} \right)^2 \right] \, dt \right\} - v_c t_N
\]

\[
= 2k(t_C - t_B) \left( \frac{1}{2f} + t_B - 2t_C \right) - v_c \left( \frac{1}{2f} \right)^2 t_N + \frac{4h(1 - \alpha)^2}{\alpha^2 + 6\alpha + 1} + \frac{4\alpha}{\alpha^2 + 4\alpha + 1} + \frac{v_c(\alpha - 1)}{4f} - \frac{v_c t_N}{2}
\]

where NSA is the negative strip distance, mm.

3.5. Positive Strip Time. Positive strip time \( t_p \) is the time of the mold moving upward relative to the slab in an oscillating period. It can be determined as

\[
t_p = T - T_N = \frac{1}{f} - T_N = \frac{1 + \alpha}{2f} + \frac{v_c(1 - \alpha)^2(\alpha^2 + 4\alpha + 1)}{24hf^2(\alpha + 1)^2}
\]

where \( t_p \) is the positive strip time, s.

Table 1: Caster parameters.

| Item                        | Unit   | Value   |
|-----------------------------|--------|---------|
| Slab section size           | mm × mm| 180 × 1800 |
| Amplitude                   | mm     | ±4      |
| Frequency                   | min⁻¹  | 104     |
| Casting speed               | m/min  | 1.15    |

Table 2: Technological parameters of nonsinusoidal oscillation.

| \( v_c \) (m/min) | \( f \) (min⁻¹) | \( t_N \) (s) | NS (%) | NSA (mm) | NSR (%) | \( t_p \) (s) | \( \Delta v \) (m/min) |
|-------------------|-----------------|--------------|---------|----------|---------|------------|---------------------|
| 0.3               | 60              | 0.357        | -300    | 6.107    | 71.48   | 0.643      | 1.239               |
| 0.4               | 60              | 0.343        | -200    | 5.523    | 68.64   | 0.657      | 1.339               |
| 0.5               | 66              | 0.305        | -164    | 5.214    | 67.09   | 0.604      | 1.533               |
| 0.6               | 72              | 0.274        | -140    | 4.962    | 65.80   | 0.559      | 1.727               |
| 0.7               | 78              | 0.249        | -122.86 | 4.753    | 64.71   | 0.520      | 1.921               |
| 0.8               | 84              | 0.228        | -110    | 4.577    | 63.77   | 0.487      | 2.115               |
| 0.9               | 90              | 0.210        | -100    | 4.426    | 62.96   | 0.457      | 2.309               |
| 1.0               | 96              | 0.195        | -92     | 4.296    | 62.25   | 0.431      | 2.503               |
| 1.1               | 102             | 0.181        | -85.45  | 4.182    | 61.63   | 0.407      | 2.697               |
| 1.15              | 105             | 0.175        | -82.61  | 4.130    | 61.34   | 0.396      | 2.794               |
| 1.2               | 108             | 0.170        | -80     | 4.081    | 61.07   | 0.386      | 2.890               |
| 1.3               | 114             | 0.159        | -75.38  | 3.993    | 60.57   | 0.367      | 3.084               |
| 1.4               | 120             | 0.150        | -71.43  | 3.913    | 60.12   | 0.350      | 3.278               |
| 1.5               | 126             | 0.142        | -68     | 3.842    | 59.72   | 0.334      | 3.472               |

Note. h = 4 mm; \( \alpha = 20\% \).

3.6. Positive Strip Velocity. The positive strip velocity \( \Delta v \) is the maximum velocity of mold moving upward relative to the slab. That is,

\[
\Delta v = |v_{u,max}| + |v_c| = \frac{6fh(1 + \alpha)}{\alpha^2 + 4\alpha + 1} + |v_c|, \tag{13}
\]

where \( v_{u,max} \) is the maximum velocity of mold moving upward. A group of curves of positive strip velocity are illustrated in Figure 6 with different constants of \( \Delta v \).

3.7. Curves for Multiparameters. Nonsinusoidal oscillation technological parameters of \( t_N, NS, t_p, \) and \( \Delta v \) are illustrated in Figure 6. According to Figure 6, the values of these...
technological parameters can be taken into account simultaneously when the relationship between oscillation frequency and casting speed is determined. So by using multiparameter curves of oscillation shown in Figure 6, it is convenient to determine the synchrocontrol model of casting speed and oscillation frequency.

By using equations (6), (8), (12), and (13), a group of curves for negative strip time, negative strip ratio, positive strip time, and positive strip velocity are obtained with different values of $t_N$, NS, $t_p$, and $\Delta v$, which are shown in Figure 6. When NS equals 57.41%, $t_N$ equals zero. If NS is greater than or equal to 57.41%, there will be no negative strip time which is not permitted for steel continuous casting production. If the oscillation parameters of $\alpha$ and $h$ changed, the multiparameter curves should be calculated again. According to the multiparameter curves, the synchrocontrolling model between casting speed and oscillation frequency determined can be suitable for different kinds of steel continuous casting.

The parameters of sinusoidal oscillation used in a steel plant are shown in Table 1. To further enhance the slab surface quality, without changing oscillation amplitude of 4 mm and making modification ratio equal to 20%, the synchrocontrol model of nonsinusoidal oscillation between casting speed and oscillation frequency is determined according to Figure 6 as follows:

$$f = \begin{cases} 
60, & v_c < 0.4, \\
60 \cdot v_c + 36, & v_c \geq 0.4.
\end{cases}$$

(14)

Using the relationship of equation (14), the nonsinusoidal oscillation technological parameters are calculated and illustrated in Table 2.

The comparison of technological parameters between sinusoidal oscillation and nonsinusoidal oscillation is illustrated in Table 3. From Table 3, it can be found that under the same casting speed, the negative strip time and the positive strip velocity of the nonsinusoidal oscillation are smaller than those of sinusoidal oscillation, which would make the depth of the oscillation marks shallower and the tensile friction on the slab reduce. The negative strip distance of the nonsinusoidal oscillation is larger than that of sinusoidal oscillation, which is helpful to remove the slab from mold, weld the cracks in a solidified shell together, and increase casting speed. The positive strip time of nonsinusoidal oscillation is longer than that of sinusoidal oscillation, which will lead to a better lubrication condition with a larger quantity of casting powder consumption, reduce the tensile stress, and minimize cracks of solid shell. Thus, nonsinusoidal oscillation is superior to sinusoidal oscillation for improving surface quality of slab and decreasing the probability of steel breakout.

| Oscillation mode | $v_c$ (m/min) | $f$ (min$^{-1}$) | $t_N$ (s) | NSA (mm) | $t_p$ (s) | $\Delta v$ (m/min) |
|------------------|--------------|----------------|----------|----------|----------|------------------|
| Sinusoidal       | 1.15         | 104            | 0.205    | 3.259    | 0.372    | 3.764            |
| Nonsinusoidal    | 1.15         | 105            | 0.175    | 4.130    | 0.396    | 2.794            |

### 4. Conclusions

A nonsinusoidal oscillation waveform proposed could control the maximum acceleration, which has good dynamic characteristics due to the continuity of velocity and acceleration. By selecting reasonable basic oscillation parameters and the synchrocontrol model, it could satisfy the continuous casting production of different kinds of steel. The synchrocontrol model of oscillation frequency and casting speed proposed is reasonable and could gain better technological oscillation parameters. Compared with sinusoidal oscillation, the nonsinusoidal oscillation is beneficial to improve the surface quality of the slab and enhance casting speed. In the future, the nonsinusoidal oscillation waveform can be realized by different oscillators, which can reduce the inertia force and prolong the service life of the equipment.

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare that there are no conflicts of interest.

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### Table 3: Technological parameters of sinusoidal and nonsinusoidal oscillation.

| Oscillation mode | $v_c$ (m/min) | $f$ (min$^{-1}$) | $t_N$ (s) | NSA (mm) | $t_p$ (s) | $\Delta v$ (m/min) |
|------------------|--------------|----------------|----------|----------|----------|------------------|
| Sinusoidal       | 1.15         | 104            | 0.205    | 3.259    | 0.372    | 3.764            |
| Nonsinusoidal    | 1.15         | 105            | 0.175    | 4.130    | 0.396    | 2.794            |
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