Universal spin-1/2 fermion field localization on a 5D braneworld

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In this work we present a refined method for the localization of spin–1/2 fermions on the 5D braneworld paradigm. We begin by proposing a more natural ansatz for the Yukawa coupling in the 5D bulk fermionic action, that guarantees the localization of the ground states for the 4D fermions with right or left chirality. Furthermore, we show that the fermion ground states localization allow us to show the absence of tachyonic modes in the left– and right– chiral Kaluza-Klein mass spectrum. More precisely, we show that localization of gravity in the 5D braneworld implies the localization of the spin–1/2 fermions.

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I. INTRODUCTION

At the end of the nineties decade a new approach for the hierarchy problem in high energy physics came into light, it was a geometric point of view born from the generalization of the ideas of Kaluza and Klein. The main idea was to use metric that could in principle depend on the extra dimensions in order to solve the mass hierarchy problem. Some of the models that raised this idea were the thin braneworld models $[1]$–$[8]$, which prove to successfully address the mass hierarchy problem and the gravity localization in a 3 + 1-braneworld. This scenario of the universe located at a space-time submanifold was pointed out in string theory $[9]$–$[11]$. An important issue of this scenario was the mechanisms for the localization of matter fields in this submanifold called thin brane. Nowadays, a more general picture arises, the thick braneworld models $[12]$–$[20]$, they are a more realistic approach because the submanifold is not located at a singularity as in the thin brane models, and in some cases the effects of the thickness can be important. This fact can lead to a more phenomenological study of the physics on the brane because the matter is a non singular mass distribution localized along the fifth dimension. Furthermore the thick brane models may also have a more fundamental motivation originating from supergravity theories as shown in $[21]$. In this context any conjecture about the mechanism for the matter field confinement in the brane can be made $[22]$–$[42]$. However the assumptions can be very speculative and the effect is that the matter localization mechanisms then prefers some models to others, in fact, the method for localizing the matter appears to be a discriminant among the braneworld models, for example spin–0 the scalar field localization performed in $[22]$–$[27]$ is closely related to the localization of gravity; in other words, the localization of gravity guarantees the localization of the KK ground state for the spin–0 matter field. A drawback on the other hand is that the localization mechanism performed by $[22]$–$[27]$ was not able to localize gauge fields with spin–1 for some thick and thin branewords. However a recent work $[34]$ menaced to localize de zero mode of the gauge fields with spin–1 in the 5D braneworld models, where the localization had not been suitable. The localization mechanism employed for matter fields living in a 5D braneworld scenarios has been the subject of many studies. Moreover the most popular method for the localization of spin–1/2 fermions is made in a rather speculative way $[35]$–$[42]$: this is because of the freedom one has to propose the Yukawa coupling term. In the present letter we propose a more natural proposal for the Yukawa coupling term $F(z)\bar{\Psi}\Psi$ annexed in to the 5D bulk action, which avoids arbitrariness and guarantees the localization for the ground states of the KK spectrum for the right– and left– chiral spin–1/2 fermions, this choice also allows us the possibility of find more bound states since we can manipulate the deep of the 5D KK potentials, taking care of leaving the KK spectrum free of tachyonic states.

This work is presented in three sections; we first give a brief introduction to the mechanism of spin–1/2 fermions localization, in section two we address the choice for the $F(z)$ function with allows us to obtain a left– and right–chiral KK mass spectrum free on tachyonic instabilities, as well as the localization of the ground sate for the fermions with left– or right– chirality. Furthermore in section three we show that in our natural proposal the localization of
gravity on the braneworld implies the localization of spin–1/2 fermions as well; finally we present our conclusions in section four.

II. LOCALIZATION OF SPIN–1/2 FERMION FIELDS ON A BRANeworld

The method we use is the standard mechanism for the localization of spin–1/2 fermions, see for example \[35\]–\[42\], the procedure employed is showed in the following lines. The ansatz for the 5D geometry is

\[
dS^2_5 = e^{2A(z)} (\hat{g}_{\mu\nu}(x) dx^\mu dx^\nu + dz^2),
\]

where \(z\) is the 5D coordinate, \(\hat{g}_{\mu\nu}\) with \(\mu, \nu = 0, 1, 2, 3\) is the 4D induced metric on the brane and the function \(e^A\) is called the warp factor. In this models we can assume, as it is shown in \[13\]–\[16\], that gravity is localized in the brane, meaning that the zero mode of the KK spectrum associated with the 4D graviton \(\Psi_{\text{grav}}\) \(\sim e^{2A(z)}\) is a normalizable function on the fifth dimension. We suppose that the 5D fermionic fields have a weak interaction with gravity (we do not consider the backreaction effects) in such a way that the localization properties of the 5D graviton \(\Psi_{\text{grav}}\) does not change.

In 5D spacetime, fermions are four–component spinors and their Dirac structure can be described by \(\Gamma^M = e^M_M \Gamma^M\) with \(e^M_M\) being the vierbein and \(\{\Gamma^M, \Gamma^N\} = 2g^{MN}\). In this Subsection, \(\hat{M}, \hat{N}, \cdots = 0, 1, 2, 3, 5\) and \(\hat{\mu}, \hat{\nu}, \cdots = 0, 1, 2, 3\) denote the 5D and 4D local Lorentz indices, respectively, and \(\Gamma^\hat{M}\) are the gamma matrices in 5D flat spacetime.

For our set-up, the vierbein and the gamma matrices are defined through

\[
e^M_{\hat{M}} = \begin{pmatrix} e^A e^\hat{\mu} & 0 & 0 \\ 0 & -e^A \end{pmatrix}, \quad \Gamma^M = e^{-A}(\hat{\epsilon}^\mu_{\hat{\nu}} \gamma^\nu, -i\gamma^5) = e^{-A}(\gamma^\mu, -i\gamma^5),
\]

where \(\gamma^\mu = e^\mu_{\hat{\nu}} \gamma^\nu\), \(\gamma^\nu\) and \(\gamma^5\) are the usual flat gamma matrices in the 4D Dirac representation.

The Dirac action of a spin–1/2 fermion with a mass term can be expressed as \[24\]

\[
S_\Psi = \int d^5x \sqrt{-g} \left[ \bar{\Psi} \Gamma^M (\partial_M + \omega_M) \Psi - F(z) \bar{\Psi} \Psi \right].
\]

Here \(\omega_M\) is the spin connection defined as \(\omega_M = 4 \omega^\hat{M}\hat{N} \Gamma^\hat{M} \Gamma^\hat{N}\) while \(\omega^\hat{M}\hat{N}\) is defined as

\[
\omega^\hat{M}\hat{N} = \frac{1}{2} e^{N\hat{M}} \left( \partial_N e^\hat{\mu}_{\hat{S}} - \partial_{\hat{S}} e^\hat{\mu}_{\hat{N}} \right) - \frac{1}{2} e^{N\hat{N}} \left( \partial_N e^{\hat{\mu}}_{\hat{M}} - \partial_{\hat{M}} e^{\hat{\mu}}_{\hat{N}} \right) - \frac{1}{2} \epsilon^{P\hat{S}} e^{\hat{Q}} \left( \partial_P e_{QR} - \partial_Q e_{PR} \right) e^R_{\hat{M}},
\]

and \(F(z)\) is some general scalar function of the extra dimensional coordinate \(z\). We will discuss about the properties of the scalar function \(F(z)\) later in section two, in the context of the localization of KK fermion modes. The non–vanishing components of the spin connection \(\omega_M\) for the background metric \[11\] are

\[
\omega_M = \frac{1}{2} (\partial_z A) \gamma_5 + \hat{\omega}_\mu,
\]

where \(\hat{\omega}_\mu = \frac{1}{4} \hat{\omega}^{\hat{\mu}\hat{\nu}} \Gamma^\hat{\mu} \Gamma^\hat{\nu}\) is the spin connection derived from the metric \(\hat{g}_{\mu\nu}(x) = e^\mu_{\hat{\nu}}(x) e^\nu_{\hat{x}}(x) \eta_{\hat{\mu}\hat{\nu}}\). Thus, the equation of motion corresponding to the action \[2\] varying over \(\Psi\) can be written as

\[
[i \gamma^\mu(\partial_\mu + \hat{\omega}_\mu) + \gamma^5 (\partial_z + 2 \partial_\mu A) - e^A F(z)] \bar{\Psi} \Psi = 0,
\]

where \(\gamma^\mu(\partial_\mu + \hat{\omega}_\mu)\) is the Dirac operator on the brane.

We turn now to investigate the 5D Dirac equation \[5\], and write the spinor in terms of 4D effective fields. On account of the fifth gamma matrix \(\gamma^5\), we anticipate the left– and right–handed projections of the 4D part to behave differently. From Eq. \[5\], we propose the following KK decomposition for the 5D spinors \(\Psi\) and \(\bar{\Psi}\)

\[
\Psi = e^{-2A} \left( \sum_n \psi_{Ln}(x) L_n(z) + \sum_n \psi_{Rn}(x) R_n(z) \right),
\]
The KK decomposition proposal for the 5D Dirac spinors is inspired in the chirality projectors, this decomposition imposes chirality on the massive KK modes inherited from the 4D projectors

\[ \psi_{Ln} = \frac{1}{2} (I_4 - \gamma^5) \psi(x)_n, \]  
\[ \psi_{Rn} = \frac{1}{2} (I_4 + \gamma^5) \psi(x)_n, \]  

where \( \gamma^5 \psi_{Ln} = \frac{1}{2} (\gamma^5 - I_4) \psi(x)_n = -\psi_{Ln} \) and \( \gamma^5 \psi_{Rn} = \frac{1}{2} (\gamma^5 + I_4) \psi(x)_n = \psi_{Rn} \) are the left-handed and right-handed components of a 4D Dirac field, respectively. In order to decouple the Dirac equation (4) in its 4D and 5D part, we must assume that the right-handed eigenfunctions \( \psi_{Ln}(x) \) and \( \psi_{Rn}(x) \) satisfy the 4D Dirac equations. The consequences of this assumption allow us to end up with a system of two coupled differential equations for the eigenvalues of the KK modes \( L_n(z) \) and \( R_n(z) \) given by:

\[ (\partial_z + e^A F(z)) L_n(z) = m_n R_n(z), \]  
\[ (\partial_z - e^A F(z)) R_n(z) = -m_n L_n(z). \]  

The system above can be decoupled by mixing (9) and (10) equations and after some algebra we get a pair of Schrödinger type equations

\[ (-\partial^2 + V_L(z)) L_n(z) = m^2_n L_n(z), \]  
\[ (-\partial^2 + V_R(z)) R_n(z) = m^2_n R_n(z), \]  

where the corresponding left and right potentials read

\[ V_L(z) = \left( e^{A(z)F(z)} \right)^2 - \left( e^{A(z)F(z)} \right)', \]  
\[ V_R(w) = \left( e^{A(z)F(z)} \right)^2 + \left( e^{A(z)F(z)} \right)', \]  

in this notation a prime denotes \( \partial_z \).

III. THE PROPOSAL FOR \( F(z) \)

We can immediately see that the equations (11) and (12) equations are restricted by the behavior of the function \( F(z) \). In what follows we will find a way to choose this function by directly looking at the integrability conditions on \( L(z) \) or \( R(z) \) in the equations (11) y (12). We are mainly interested in the localization properties of the massless mode of the KK excitations. Let us thus consider the \( m_n = 0 \) case; if the identification \( y(z) = e^{A(z)}F(z) \) is made we can rewrite the potential \( V_L(z) \) as

\[ V_L(z) = y^2 - y'. \]  

The last equation is a Riccati differential equation and a particular solution can be obtained by performing the following change of variable \( y = -\frac{F'}{F} \), now we have a differential equation for the new variable that reads

\[ -Z''(z) + V_L(z)Z(z) = 0. \]  

By taking a closer look at equation (16) we see that if is a Schrödinger–like equation, the same equation that the massless version of (11). Thus, \( Z = L_0 \) is a natural choice for the solution of the equation (16). On the other hand, as we mentioned above, the wave function of the massless graviton is normalized and takes the form \( \Psi_{grav} \sim e^{\frac{1}{2}A(z)} \) (see [13–20]). This fact suggests that a simple choice for \( L_0 = Z \) can be a power law of the zero mode for the graviton, namely

\[ Z(z) = e^{MA(z)}, \]  

where \( M \) is a real and positive constant, that can be set in such a way that the \( Z(z) \) function is normalizable in the fifth dimension. By taking into account the above relation, the coupling \( F(z) \) can be written as

\[ F(z) = M \partial_z e^{-A(z)}. \]
The above relations imply that the Yukawa coupling is entirely determined by the bulk geometry. In refs. [17, 18], are studied braneworld models for a class of geometries where the warp factor has the following behaviour at infinity

\[ e^A \sim \frac{1}{|z|^\gamma} \quad \text{when} \quad z \to \infty. \]

In the cited references it is shown that if the 4D Planck mass is finite, massless graviton is localized and there is not space–time singularities, the parameter \( 1/3 < \gamma \leq 1 \). By considering this kind of metrics, the coupling \( F \) takes the form

\[ F \sim M \gamma \text{sgn}(z)|z|^{\gamma-1}, \quad \text{when} \quad z \to \infty. \]

Hence, the field \( F \) is finite along the extra dimension. This fact is important because it is guarantees the stability of the field \( F \).

Now we take the ansatz for \( F(z) \) and substitute it into the equations (19) and (20), allowing us to recast the left and right pair of equations as follows

\[
\begin{align*}
\left( \partial_z - MA' \right) L_n(z) &= m_n R_n(z), \\
\left( \partial_z + MA' \right) R_n(z) &= -m_n L_n(z).
\end{align*}
\]

In order to obtain the zero modes for the fermions with right– and left–chirality we must set \( m_n = 0 \) in (19) and (20) to get:

\[
\begin{align*}
L_0 &\propto e^{MA(z)}, \\
R_0 &\propto e^{-MA(z)}.
\end{align*}
\]

The normalization of the zero mode spin-2 graviton ensures that \( e^A \to 0 \) when \( z \to \pm \infty \). Then, for a given sign of \( M \) the above relations tell us that it is not possible to have both massless left– and right–chiral KK fermion modes localized on the brane at once, since when one is localized, the other one is not.

In general if we ask for \( e^{MA(z)} \) to be a normalizable function on the fifth dimension and combine the equations (19) and (20), we can obtain the Schrödinger–like equations for the left– and right–chiral KK modes of fermions:

\[
\begin{align*}
-\partial_z^2 L_n(z) + V_L(z) L_n(z) &= m_{L,n}^2 L_n(z), \\
-\partial_z^2 R_n(z) + V_R(z) R_n(z) &= m_{R,n}^2 R_n(z).
\end{align*}
\]

Then the ansatz for \( F(z) \) fixes the form of the potentials \( V_{L,R} \) that can be recast as follows

\[
\begin{align*}
V_L(z) &= M^2 A'^2 + MA'' , \\
V_R(z) &= M^2 A'^2 - MA''.
\end{align*}
\]

By analyzing the shape of the potentials (25) and (26), it is easy to see that the deep of the potential wells \( V_{L,R}(z) \) is determined by the size of the parameter \( M \). Furthermore if we demand that the fermions with right/left chirality are localized in the 3-brane and that they present a symmetric distribution around the brane origin, then the \( Z(z) \) must necessarily be an even function, implying that that \( F(z) = M \partial_z e^{-A(z)} \), which is an odd function. This statement can be explicitly demonstrated if we substitute \( F(z) \) in to equations (19) and (20), if we take into account that \( e^A \) is an even function, then the potentials \( V_{L,R} \) are also even functions around the center of the brane in the fifth dimension. We show that the KK mass spectrum for the fermions with right– and left–chirality is free of tachyonic modes (see [43] and references there in) by writing their quantum mechanic supersymmetric analogue form from the Schrödinger–like equations (23) and (24)

\[
\begin{align*}
(-\partial_z - MA') (\partial_z - MA') L_n(z) &= m_{L,n}^2 L_n(z), \\
(\partial_z - MA') (-\partial_z - MA') R_n(z) &= m_{R,n}^2 R_n(z).
\end{align*}
\]

#### IV. RELATION BETWEEN GRAVITY AND FERMION LOCALIZATION

A more direct relation between gravity localization and spin-1/2 fermionic fields localization can be made if we set \( M \geq \frac{3}{2} \). In particular if we chose \( M = 3/2 \) a direct relation between the localization of the left chiral ground state
and the zero mode of KK spectrum for the 5D graviton $\Psi_{\text{grav}} \sim e^{\frac{2}{3}A(z)}$ can be made. The relation is immediate if we see the expressions for the potentials (25) and (26) and the analogue supersymmetric quantum mechanic potential for the KK graviton ground state equation [13],

$$V_L(z) = \frac{9}{4} A'^2 + \frac{3}{2} A''$$

(29)

$$V_R(z) = \frac{9}{4} A'^2 - \frac{3}{2} A''$$

(30)

It is important to state that if we set $M = -M$ in the equations (23) and (24), a similar treatment can be performed. The results of this new scenario are the followings: the zero mode of the right chirality fermion $\psi_0^R(z)$ is localized, while its analogue left ground state $\psi_0^L(z)$ remains delocalized. The analog supersymmetric quantum mechanic form of the Schrödinger equations (27) and (28) can be written as

$$(-\partial_z - MA') (\partial_z - MA') R_n = m^2_{R_n} R_n,$$

(31)

$$(-\partial_z - MA') (-\partial_z - MA') L_n = m^2_{L_n} L_n.$$

(32)

Once we obtain the eigenfunctions $L_n, R_n$ associated to the bound states of the KK spectrum via the Schrödinger–like equation, and use them along with the determinant of the metric $\sqrt{-g}$ and the warp factor $e^A$ to perform a dimensional reduction over the 5D action in such a way that the corresponding field configuration in 4D is given by

$$S_{\frac{5}{2}} = \int d^5 x \sqrt{-g} \bar{\Psi} \left[ i \Gamma^M (\partial_M + \omega_M) - F(z) \right] \Psi,$$

$$= \sum_n \int d^4 x \sqrt{-\hat{g}} \bar{\psi}_n \left[ i \gamma^\mu (\partial_\mu + \hat{\omega}_\mu) - m_n \right] \psi_n,$$

(33)

where the eigenfunctions $L_n$ and $R_n$ form a closed set of linearly independent functions that satisfy the following orthogonality relations:

$$\int_{-\infty}^{+\infty} L_m L_n dz = \delta_{mn},$$

(34)

$$\int_{-\infty}^{+\infty} R_m R_n dz = \delta_{mn},$$

(35)

$$\int_{-\infty}^{+\infty} L_m R_n dz = 0.$$  

(36)

V. CONCLUSIONS

In this letter we have proposed a natural way of determine the Yukawa coupling $F(z) \bar{\Psi} \Psi$ for the 5D bulk action, the form of the $F(z)$ function allow us to localize the spin–$\frac{1}{2}$ fermionic fields in the paradigm of the 5D braneworld scenarios. The structure acquired by the $V_{L,R}$ potentials is induced by the ansatz for the $F(z)$ function and we show that this choice allow the localization of the left– or right–chiral KK zero modes $L_0(z), R_0(z)$ and excludes the tachyonic modes of the KK mass spectrum $m_{L_n}$ and $m_{R_n}$. The proposal for the function $F(z)$ leaves behind other speculative proposals. This can be seen more clearly if we set $M = \pm 3/2$, that is a more phenomenologically viable ansatz. This choice allows for a direct relation between the localization of the ground state for the left– or right–chiral KK mass spectrum and the localization of the zero mode for the 4D graviton in the brane. We can also infer the number of left and right bound states as well as the existence of a mass gap in the KK mass spectrum of the right and left fermion.

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