Probing SUSY with 10 TeV stop mass in rare decays and CP violation of kaon

Morimitsu Tanimoto\textsuperscript{1} and Kei Yamamoto\textsuperscript{2,}\textsuperscript{*}

\textsuperscript{1}Department of Physics, Niigata University, Niigata 950-2181, Japan
\textsuperscript{2}KEK Theory Center, IPNS, KEK, Tsukuba, Ibaraki 305-0801, Japan
\textsuperscript{*}E-mail: kei.yamamoto@kek.jp

Received August 29, 2016; Accepted October 5, 2016; Published December 11, 2016

We probe SUSY at the 10 TeV scale in the rare decays and CP violation of kaon. We focus on the processes of $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$, combined with the CP-violating parameters $\epsilon_K$ and $\epsilon_K'/\epsilon_K$. The Z-penguin mediated by the chargino loop cannot enhance $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ because the left–right mixing of the stop is constrained by the 125 GeV Higgs mass. On the other hand, the Z-penguin mediated by the gluino loop can enhance the branching ratios of both $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$. The former increases up to more than $1.0 \times 10^{-10}$, which is much larger than the SM prediction even if the constraint of $\epsilon_K$ is imposed. It is remarkable that the Z-penguin mediated by the gluino loop can simultaneously enhance $\epsilon_K'/\epsilon_K$ and the branching ratio of $K_L \rightarrow \pi^0\nu\bar{\nu}$, which increases up to $1.0 \times 10^{-10}$. We also study the decay rates of $K_L \rightarrow \mu^+\mu^-$, $B^0 \rightarrow \mu^+\mu^-$, and $B^+ \rightarrow \mu^+\mu^-$, which correlate with the $K_L \rightarrow \pi^0\nu\bar{\nu}$ decay through the Z-penguin. It is important to examine the $B^0 \rightarrow \mu^+\mu^-$ process since we expect enough sensitivity of this decay mode to SUSY at LHCb.

1. Introduction

The rare decays and CP violation of kaon have given us important constraints for new physics (NP) since the standard model (SM) contributions are suppressed due to the flavor structure of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1,2]. Typical examples are the rare decay processes $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$, which are clean theoretically [3,4]. These processes have been considered to be one of the powerful probes of NP [5–17]. In order to improve previous experimental measurements [18,19], new experiments are going on. One is the J-PARC KOTO experiment, which is to measure the decay rate of $K_L \rightarrow \pi^0\nu\bar{\nu}$ approaching the SM-predicted precision [20,21]. Another is the CERN NA62 experiment to observe the $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay [22].

In particular, the $K_L \rightarrow \pi^0\nu\bar{\nu}$ process is the CP-violating one and provides direct measurement of the CP-violating phase in the CKM matrix. On the other hand, the indirect CP-violating parameter $\epsilon_K$, which is induced by $K^0–\bar{K}^0$ mixing, has given us precise information about the CP-violating phase of the CKM matrix. Another CP-violating parameter, $\epsilon_K'/\epsilon_K$, was measured in $K \rightarrow \pi\pi$ decay. Therefore, the $K_L \rightarrow \pi^0\nu\bar{\nu}$ process is expected to open an NP window in CP violation by combining with $\epsilon_K$ and $\epsilon_K'/\epsilon_K$.

The $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decays are dominated by the Z-penguin process, which is the flavor-changing neutral current (FCNC) through loop diagrams. The Z-penguin process also gives a large contribution to $\epsilon_K'/\epsilon_K$ due to the enhancement of the $\Delta I = 1/2$ amplitude [23]. Actually,
it cancels the dominant QCD penguin contribution significantly in the SM since it has the opposite sign to the QCD penguin amplitude. On the other hand, \( \epsilon_K \) is given by the box diagram. We expect deviation from the SM prediction with correlation among \( K_L \to \pi^0\nu\bar{\nu}, K^+ \to \pi^+\nu\bar{\nu}, \epsilon_K \), and \( \epsilon'_K/\epsilon_K \) due to the NP effect. Furthermore, there may be other correlations of NP with the kaon rare decay \( K_L \to \mu^+\mu^- \) [24] and the B meson rare decays \( B^0 \to \mu^+\mu^-, B_s \to \mu^+\mu^- \), which have been observed in the LHCb and CMS experiments [25,26] since the Z-penguin process also contributes to these processes.

In this work, we discuss the minimal supersymmetric SM (MSSM) as the typical NP. Recent searches for SUSY particles at the LHC have given us important constraints. Since the lower bounds of masses of SUSY particles increase gradually, the gluino mass is supposed to be beyond the scale of 2 TeV [27–32]. SUSY models have also been seriously constrained by the Higgs discovery, in which the Higgs mass is 125 GeV [33,34]. These facts suggest a class of SUSY models with heavy sfermions. If the squark masses are expected to be \( \mathcal{O}(10) \) TeV, the lightest Higgs mass can be pushed up to 125 GeV [35], whereas all SUSY particles can be out of the reach of the LHC experiment. Therefore, the indirect search for SUSY particles becomes important in low-energy flavor physics [36–38]. We discuss CP-violation-related phenomena such as \( K_L \to \pi^0\nu\bar{\nu}, K^+ \to \pi^+\nu\bar{\nu}, \epsilon_K \), and \( \epsilon'_K/\epsilon_K \) in the framework of high-scale SUSY with \( \mathcal{O}(10) \) TeV.

We can also consider the SUSY model with a split family [39,40] in which the third family of squarks/sleptons is heavy, \( \mathcal{O}(10) \) TeV, while the first and second families of squarks/sleptons and gauginos have relatively low masses \( \mathcal{O}(1) \) TeV. This model is motivated by the Nambu–Goldstone hypothesis for quarks and leptons in the first two generations [41]. Although there are no signals of SUSY particles in the LHC experiment at present, this scenario is not in conflict with the present bound of SUSY particles. The split-family model is consistent with the 125 GeV Higgs mass [33,34] and the muon \( g - 2 \) [42]. The stop mass with \( \mathcal{O}(10) \) TeV pushes up the lightest Higgs mass to 125 GeV [35]. The deviation from the SM prediction of the muon \( g - 2 \) [43,44] is explained by the sleptons of the first and second families with mass less than 1 TeV [40]. Therefore, it is important to examine the split-family model in the rare decays and CP violation at low energy [36–38], as well as the direct search at the LHC.

For many years, the rare decays and CP violation in the \( K \) and \( B \) mesons have been successfully understood within the framework of the SM, where the source of CP violation is the Kobayashi–Maskawa (KM) phase [2]. On the other hand, there are new sources of CP violation if the SM is extended to the SUSY model. For example, the soft squark mass matrices contain CP-violating phases, which contribute to FCNC with CP violation [45]. Therefore, one expects to discover a SUSY contribution in CP-violating phenomena at low energy. Actually, we have found that the SUSY contribution could be up to 40% in the observed \( \epsilon_K \), but it is minor in CP violation of the \( B \) meson at a high scale of 10–50 TeV [38]. Moreover, we have also found a sizable contribution of high-scale SUSY to \( K_L \to \pi^0\nu\bar{\nu} \) and \( K^+ \to \pi^+\nu\bar{\nu} \) in the nonminimal-flavor-violation (non-MFV) scenario [46].

It is also important to take account of \( \epsilon'_K/\epsilon_K \), because the SM has potential difficulties in describing the data for \( \epsilon'_K/\epsilon_K \) [23]. Therefore, we study \( \epsilon'_K/\epsilon_K \) in the SUSY model with the non-MFV scenario. We discuss \( K_L \to \pi^0\nu\bar{\nu} \) and \( K^+ \to \pi^+\nu\bar{\nu} \) with CP violations, \( \epsilon_K \), and \( \epsilon'_K/\epsilon_K \) in the framework of SUSY at \( \mathcal{O}(10) \) TeV. In addition, we discuss the SUSY contribution to the decay processes \( K_L \to \mu^+\mu^- \), \( B^0 \to \mu^+\mu^- \), and \( B_s \to \mu^+\mu^- \).

We have already presented the numerical predictions of the branching ratios \( K_L \to \pi^0\nu\bar{\nu} \) and \( K^+ \to \pi^+\nu\bar{\nu} \) in Ref. [46], where all squarks/sleptons and gauginos are at \( \mathcal{O}(10) \) TeV. However,
those numerical results should be revised with those of this paper since the relevant constraints are not imposed enough there. In this paper, we also reexamine them comprehensively by taking account of the gluino contribution, as well as the chargino one, with a large left–right mixing angle of squarks.

Our paper is organized as follows. In Sect. 2, we discuss the formulation of the rare decays, \( K_L \to \pi^0 \nu \bar{\nu}, K^+ \to \pi^+ \nu \bar{\nu}, K_L \to \mu^+ \mu^-, B^0 \to \mu^+ \mu^-, \) and \( B_s \to \mu^+ \mu^-, \) and CP violations of \( \epsilon_K \) and \( \epsilon_K'/\epsilon_K. \) Section 3 gives our setup of SUSY with 10 TeV squark masses. In Sect. 4, we present our numerical results. Section 5 is devoted to discussions and a summary. The relevant formulae are presented in Appendices A, B, and C.

2. Observables

2.1. \( K_L \to \pi^0 \nu \bar{\nu} \) and \( K^+ \to \pi^+ \nu \bar{\nu} \)

Let us begin by discussing the kaon rare decays \( K_L \to \pi^0 \nu \bar{\nu} \) and \( K^+ \to \pi^+ \nu \bar{\nu}, \) which are dominated by the Z-penguin process in the SM. In the estimation of the branching ratios of \( K \to \pi \nu \bar{\nu}, \) the hadronic matrix elements can be extracted with the isospin symmetry relation \([47,48]\). These processes are theoretically clean because the long-distance contributions are small \([14]\), and then the theoretical uncertainty is estimated below several percent. Accurate measurements of these decay processes provide crucial tests of the SM. In particular, the \( K_L \to \pi^0 \nu \bar{\nu} \) process is a purely CP-violating one, which can reveal the source of the CP-violating phase. The basic formulae are presented in Appendix C.1. The SM predictions have been discussed Refs. \([4,49,50]\). They are given as

\[
\text{BR}(K_L \to \pi^0 \nu \bar{\nu})_{\text{SM}} = (3.36 \pm 0.05) \times 10^{-11} \cdot \left[ \frac{|V_{ub}|}{3.88 \times 10^{-3}} \right]^2 \left[ \frac{|V_{cb}|}{40.7 \times 10^{-3}} \right]^2 \left[ \frac{\sin(\gamma)}{\sin(73.2^\circ)} \right]^2.
\]

\[
\text{BR}(K^+ \to \pi^+ \nu \bar{\nu})_{\text{SM}} = (8.39 \pm 0.30) \times 10^{-11} \cdot \left[ \frac{|V_{cb}|}{40.7 \times 10^{-3}} \right]^{2.8} \left[ \frac{\gamma}{73.2^\circ} \right]^{0.74}.
\]

On the experimental side, the upper bound of the branching ratio of \( K_L \to \pi^0 \nu \bar{\nu} \) is given by the KEK E391a experiment \([18]\), and the branching ratio of \( K^+ \to \pi^+ \nu \bar{\nu} \) was measured by the BNL E787 and E949 experiments as \([19]\):

\[
\text{BR}(K_L \to \pi^0 \nu \bar{\nu})_{\text{exp}} < 2.6 \times 10^{-8} \quad (90\% \text{ C.L.}),
\]

\[
\text{BR}(K^+ \to \pi^+ \nu \bar{\nu})_{\text{exp}} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}.
\]

At present, the J-PARC KOTO experiment is an in-flight measurement of \( K_L \to \pi^0 \nu \bar{\nu} \) approaching the SM-predicted precision \([20,21]\), while the CERN NA62 experiment \([22]\) is expected for the precise measurement of the \( K^+ \to \pi^+ \nu \bar{\nu} \) decay.

The SUSY contribution has been studied in many works \([11,51–55]\). The sizable enhancement of these kaon decays was expected through large left–right mixing of the chargino interaction in \( sL\tilde{h}_LX^- \) and \( dL\tilde{h}_LX^- \) at the SUSY scale of \( \mathcal{O}(1) \) TeV \([51,54]\). We find that even at the \( \mathcal{O}(10) \) TeV scale, these decays are enhanced through the Z-penguin mediated by the gluino with large left–right mixing.

---

1 In our calculation, we use the CKM elements in the study of the so-called universal unitarity triangle including the data of the CP asymmetry \( S_{K^L\phi K_S} \) and the mass differences of \( B \) mesons without inputting \( \epsilon_K \) (strategy S1 in Ref. \([50]\)). In this case, the SM prediction for \( K \to \pi \nu \bar{\nu} \) shifts lower.
2.2. $\epsilon_K$
Let us discuss another CP-violating parameter $\epsilon_K$, which has been measured precisely. Its hadronic
matrix element $\hat{B}_K$ is reliably determined by the lattice calculations as \[56,57\]

$$\hat{B}_K = 0.766 \pm 0.010.$$  \hfill (4)

Another theoretical uncertainty in $\epsilon_K$ is also reduced by removing the QCD correction factor of the
two charm box diagram \[58\]. Thus, the accurate estimate of the SM contribution enables us to search
for NP such as SUSY. A nonnegligible SUSY contribution has been expected in $\epsilon_K$ even at a scale
of $\mathcal{O}(100)$ TeV \[36–38\]. Consequently, $\epsilon_K$ gives us one of the most important constraints to predict
the SUSY contribution in the $K \to \pi \nu \bar{\nu}$ decays. In our calculation of $\epsilon_K$, we investigate the SUSY
contributions for the box diagram, which is correlated with the $K_L \to \pi^0 \nu \bar{\nu}$ process directly.

2.3. $\epsilon'_K/\epsilon_K$
The direct CP violation $\epsilon'_K/\epsilon_K$ is also important to constrain the NP. The basic formula for $\epsilon'_K/\epsilon_K$ is
given as \[12,23,59\]

$$\frac{\epsilon'_K}{\epsilon_K} = \text{Im}(V_{td} V_{ts}^* \cdot F_{\epsilon'}),$$  \hfill (5)

where

$$F_{\epsilon'} = P_0 + P_X X + P_Y Y + P_Z Z + P_E E,$$  \hfill (6)

with

$$X = C - 4 B^{(u)}, \quad Y = C - B^{(d)}, \quad Z = C + \frac{1}{4} D.$$  \hfill (7)

Functions $B, C, D, E$ denote the loop functions including SM and SUSY effects, which come from
boxes with external $d \bar{d}(B^{(d)}), u \bar{u}(B^{(u)}), Z$-penguin $(C),$ photon-penguin $(D),$ and gluon-penguins $(E).$
The coefficients $P_i$ are given by

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8,$$  \hfill (8)

with the nonperturbative parameters $B^{(1/2)}_6$ and $B^{(3/2)}_8$ defined as

$$R_6 \equiv B^{(1/2)}_6 \left[ \frac{114.54 \text{ MeV}}{m_s(m_c) + m_d(m_c)} \right]^2, \quad R_8 \equiv B^{(3/2)}_8 \left[ \frac{114.54 \text{ MeV}}{m_s(m_c) + m_d(m_c)} \right]^2.$$  \hfill (9)

The numerical values of $r_i^{(0,8,6)}$ are presented in Ref. [23].

The most important parameters to predict $\epsilon'_K/\epsilon_K$ are the non-perturbative parameters $B^{(1/2)}_6$ and
$B^{(3/2)}_8$. Recently, the RBC-UKQCD lattice collaboration \[60,61\] gave

$$B^{(1/2)}_6 = 0.57 \pm 0.15, \quad B^{(3/2)}_8 = 0.76 \pm 0.05,$$  \hfill (10)

which predict $(\epsilon'_K/\epsilon_K)_{\text{SM}} = (1.9 \pm 4.5) \times 10^{-4}$ in the SM \[23\]. This SM prediction is much smaller
than the experimental result \[62\]

$$(\epsilon'_K/\epsilon_K)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}.$$  \hfill (11)
This disagreement between the SM prediction and the experimental value may suggest NP in the kaon system; however there are several open questions that have to be answered to conclude the issue [23]. We use these values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ with $3\sigma$ in our calculation.

The dominant contribution to the Z-penguin $C$ comes from a chargino-mediated one and a gluino-mediated one if the large left–right mixing of squarks is allowed. On the other hand, the effects of neutralinos are suppressed [51–53]. The chargino-mediated Z-penguin $C(\chi^{\pm})$ and the gluino-mediated Z-penguin $C(\tilde{g})$ are given as

$$V_{td}V_{ts}^* C(\chi^{\pm}) = \frac{1}{8} \left( \frac{4m_W^2}{3\cos^2\theta_W} \right) \left[ P_{ZL}^{sd}(\chi^{\pm})^* + \frac{c_w^2}{s_w^2} P_{ZR}^{sd}(\chi^{\pm})^* \right],$$

$$V_{td}V_{ts}^* C(\tilde{g}) = \frac{1}{8} \left( \frac{4m_W^2}{3\cos^2\theta_W} \right) \left[ P_{ZL}^{sd}(\tilde{g})^* + \frac{c_w^2}{s_w^2} P_{ZR}^{sd}(\tilde{g})^* \right],$$

where $c_w^2 = \cos^2\theta_W$ and $s_w^2 = \sin^2\theta_W$, with the Weinberg angle $\theta_W$, and the Z-penguin amplitudes $P_{ZL(R)}^{sd}(\chi^{\pm})$ and $P_{ZL(R)}^{sd}(\tilde{g})$ are given in Eqs. (B1) and (B4) in Appendix B.

The box diagram effect is suppressed compared with the penguin diagram if the SUSY-breaking scale $M_S$ satisfies $M_S \gg m_W$ [11]. Thus, the dominant SUSY contribution to $\epsilon'_K/\epsilon_K$ is given by the Z-penguin mediated by the chargino and gluino. Therefore, we should consider the correlation between $\epsilon'_K/\epsilon_K$ and the branching ratio of $K_L \to \pi^0\nu\bar{\nu}$.

Let us write $\epsilon'_K/\epsilon_K$ as

$$\left( \frac{\epsilon'_K}{\epsilon_K} \right) = \left( \frac{\epsilon'_K}{\epsilon_K} \right)_{SM} + \left( \frac{\epsilon'_K}{\epsilon_K} \right)_Z + \left( \frac{\epsilon'_K}{\epsilon_K} \right)_R,$$

where the second and third terms denote the Z-penguin induced by the left-handed and right-handed interactions of SUSY, respectively. The contributions are written as follows [24]

$$\left( \frac{\epsilon'_K}{\epsilon_K} \right)_Z + \left( \frac{\epsilon'_K}{\epsilon_K} \right)_R = -2.64 \times 10^3 B_8^{(3/2)} \left[ \text{Im} \Delta_L^{sd}(Z) + \frac{c_w^2}{s_w^2} \text{Im} \Delta_R^{sd}(Z) \right],$$

where

$$\Delta_{L(R)}^{sd}(Z) = \frac{g_2^2 m_W^2}{8\pi^2 c_w^2} P_{ZL(R)}^{sd}. $$

In order to see the correlation between $\epsilon'_K/\epsilon_K$ and the $K_L \to \pi^0\nu\bar{\nu}$ decay, it is helpful to write down the $K_L \to \pi^0\nu\bar{\nu}$ amplitude induced by the chargino- and gluino-mediated Z-penguin in terms of $\Delta_{L(R)}^{sd}(Z)$ as

$$A(K_L \to \pi^0\nu\bar{\nu})_Z \sim \left[ \text{Im} \Delta_L^{sd}(Z) + \text{Im} \Delta_R^{sd}(Z) \right],$$

as seen in Appendix C.1.

The Z-penguin amplitude mediated by the chargino dominates the left-handed coupling of the Z boson. Therefore, the chargino contribution to $\epsilon'_K/\epsilon_K$ is opposite to $K_L \to \pi^0\nu\bar{\nu}$. If the Z-penguin mediated by the chargino enhances $\epsilon'_K/\epsilon_K$, the $K_L \to \pi^0\nu\bar{\nu}$ decay is suppressed considerably. On the other hand, the Z-penguin amplitude mediated by the gluino gives equal left-handed and right-handed Z couplings. Then, the right-handed Z coupling of the Z-penguin amplitude is a factor of
$e_{\mu}^2/s_{\mu}^2 \simeq 3.3$ larger than the left-handed one. Therefore, we can obtain the SUSY contribution, which can enhance, simultaneously, $\epsilon'_K/\epsilon_K$ and the branching ratio for $K_L \to \pi^0 v \bar{v}$. Actually, by choosing $\text{Im} \Delta_{L}^{sd}(Z) > 0$ and $\text{Im} \Delta_{R}^{sd}(Z) < 0$, the region

$$|\text{Im} \Delta_{R}^{sd}(Z)| < |\text{Im} \Delta_{L}^{sd}(Z)| < 3.3 |\text{Im} \Delta_{R}^{sd}(Z)|$$  \hspace{1cm} (18)$$
can enhance both $\epsilon'_K/\epsilon_K$ and the branching ratio for $K_L \to \pi^0 v \bar{v}$. We discuss this case in our numerical results.

2.4. $K_L \to \mu^+\mu^-, B^0 \to \mu^+\mu^-, and B_s \to \mu^+\mu^-$ decays

The Z-penguin also contributes $K_L \to \mu^+\mu^-, B^0 \to \mu^+\mu^-$, and $B_s \to \mu^+\mu^-$ decays. These decay amplitudes are governed by the axial semileptonic operator $O_{10}$, which is created by the Z-penguin top loop and the W box diagram in the SM. Those general formulae are presented in Appendix C.2. The CMS and LHCb Collaborations have observed the branching ratio for $B_s \to \mu^+\mu^-$, and $B^0 \to \mu^+\mu^-$ has also been measured [26]:

$$\text{BR}(B_s \to \mu^+\mu^-)_{\text{exp}} = (2.8^{+0.7}_{-0.6}) \times 10^{-9},$$

$$\text{BR}(B^0 \to \mu^+\mu^-)_{\text{exp}} = (3.9^{+1.6}_{-1.4}) \times 10^{-10}. \hspace{1cm} (19)$$

The SM predictions have been given as [63]

$$\text{BR}(B_s \to \mu^+\mu^-)_{\text{SM}} = (3.65 \pm 0.23) \times 10^{-9},$$

$$\text{BR}(B^0 \to \mu^+\mu^-)_{\text{SM}} = (1.06 \pm 0.09) \times 10^{-10}. \hspace{1cm} (20)$$

On the other hand, the long-distance effect is expected to be large in the $K_L \to \mu^+\mu^-$ process [64]. Therefore, it may be difficult to extract the effect of the Z-penguin process. The SM prediction of the short-distance contribution was given as [24]

$$\text{BR}(K_L \to \mu^+\mu^-)_{\text{SM}} = (0.8 \pm 0.1) \times 10^{-9}. \hspace{1cm} (21)$$

The experimental data for $K_L \to \mu^+\mu^-$ is [62]

$$\text{BR}(K_L \to \mu^+\mu^-)_{\text{exp}} = (6.84 \pm 0.11) \times 10^{-9}, \hspace{1cm} (22)$$

from which the constraint on the short-distance contribution has been estimated as [64]

$$\text{BR}(K_L \to \mu^+\mu^-)_{\text{SD}} \leq 2.5 \times 10^{-9}. \hspace{1cm} (23)$$

Thus, the SUSY contribution through the Z-penguin is expected to be correlated among the rare decays of $K_L \to \pi^0 v \bar{v}$, $K^+ \to \pi^+ v \bar{v}$, $K_L \to \mu^+\mu^-$, $B^0 \to \mu^+\mu^-$, and $B_s \to \mu^+\mu^-$, as well as CP violations of $\epsilon_K$ and $\epsilon'_K/\epsilon_K$.

3. SUSY flavor mixing

Recent LHC results for the SUSY search may suggest high-scale SUSY, $\mathcal{O}(10–1000) \text{ TeV}$ [36–38] since the lower bounds of the gluino mass and squark masses are close to 2 TeV. Taking account of these recent results, we consider the possibility of high-scale SUSY at 10 TeV, in which the $K \to \pi v \bar{v}$ decays and $\epsilon'_K/\epsilon_K$ with the constraint of $\epsilon_K$ are discussed.

We also consider the split-family model, which has a specific spectrum of the SUSY particles [39,40]. This model is motivated by the Nambu–Goldstone hypothesis for quarks and leptons in the
first two generations [41]. Therefore, the third family of squarks/sleptons is heavy, e.g., $O(10)$ TeV, while the first and second family of squarks/sleptons have relatively low masses of $O(1)$ TeV. Close to the experimental lower bound, the masses of bino and wino are assumed to be small, less than 1 TeV. The model was first discussed in the $B_s - \bar{B}_s$ mixing [39]. It successfully explained both the 125 GeV Higgs mass and the muon $g-2$ simultaneously [40]. The stop mass with $O(10)$ TeV pushes up the Higgs mass to 125 GeV. The deviation of the muon $g-2$ is explained by the sleptons of the first and second families with mass less than 1 TeV. Since the squark masses of the first and second families are also relatively low, as well as the sleptons, we expect the SUSY contribution in the kaon system to become large.

The new flavor mixing and CP violation effect are induced through the quark–squark–gaugino and the lepton–slepton–gaugino couplings. The $6 \times 6$ squark mass matrix $M^2_q$ in the super-CKM basis is diagonalized to the mass eigenstate basis in terms of the rotation matrix $\Gamma(q)^\dagger$ as

$$m^2_q = \Gamma(q) M^2_q \Gamma(q)^\dagger,$$

where $\Gamma(q)$ is a $6 \times 6$ unitary matrix, and it is decomposed into $3 \times 6$ matrices as $\Gamma(q) = (\Gamma_L^{(q)}, \Gamma_R^{(q)})$. The explicit matrix is shown in Appendix A. We introduce twelve mixing parameters $s_{12}^{qLqR}, s_{23}^{qLqR}$, and $s_{13}^{qLqR}$, where $q = u, d$ for squark mixing. In addition, we also introduce left–right (LR) mixing angles $\theta_{LR}^{ij}$. In practice, we take $s_{12}^{qLqR} = 0$, which is motivated by the almost-degenerate squark masses of the first and the second families to protect the large contribution to the $K^0 - \bar{K}^0$ mass difference $\Delta M_K$. It is also known that the single mixing effect of $s_{12}^{qLqR}$ to $K \rightarrow \pi \nu \bar{\nu}$ is minor [51]. Actually, we have checked numerically that the contribution of $s_{12}^{qLqR} = 0 \sim 0.3$ is negligibly small. There also appear the phases $\phi_{ij}^{qL}$ and $\phi_{ij}^{qR}$ associated with the mixing angles, which bring new sources of CP violations. In our work, we treat those mixing parameters and phases as free parameters in the framework of the non-MFV scenario.

Since the $Z$-penguin processes give the dominant contribution for $K \rightarrow \pi \nu \bar{\nu}$ and $\epsilon'_K/\epsilon_K$, we calculate the $Z$-penguin mediated by the chargino and gluino. The interaction is presented in Appendix B. The relevant parameters are presented in the following section.

4. Numerical analysis

4.1. Setup of parameters

Let us discuss the decay rates of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ processes by choosing a sample of the mass spectrum in the high-scale SUSY model at $O(10)$ TeV. The enhancements of these kaon rare decays require large left–right mixing with large squark-flavor mixing. In order to show our results clearly, we take a simple setup for the high-scale SUSY model as follows:

- We fix the gluino, wino, and bino masses $M_i$ ($i = 3, 2, 1$) with $\mu$ and $\tan \beta$ as
  $$M_3 = 10 \text{ TeV}, \quad M_2 = 3.3 \text{ TeV}, \quad M_1 = 1.6 \text{ TeV}, \quad \mu = 10 \text{ TeV}, \quad \tan \beta = 3,$$

  for high-scale SUSY.

- We take the masses of stop $\tilde{t}_1, \tilde{t}_2$, and sbottom $\tilde{b}_1, \tilde{b}_2$ as a sample set
  $$m_{\tilde{t}_1} = 10 \text{ TeV}, \quad m_{\tilde{t}_2} = 15 \text{ TeV}, \quad m_{\tilde{b}_1} = 10 \text{ TeV}, \quad m_{\tilde{b}_2} = 15 \text{ TeV}.$$  

On the other hand, we take the masses of the first and second family up-type and down-type squarks around 15 TeV within 5–15% as relevant. This mass spectrum of the first and second
families does not change our numerical results much because the third family squarks dominate the Z-penguin induced by the chargino and gluino interactions in our model.

- We take the left–right mixing angles
  \[ \theta_{tLR}^t = 0.07 \quad \text{and} \quad \theta_{bLR}^b = 0.1\text{–}0.3, \]

  where \( \theta_{tLR}^t \) is estimated by input of the stop masses in Eq. (26) with the large \( A \) term, which is constrained by the 125 GeV Higgs mass due to the large radiative correction [35]. On the other hand, there is no strong constraint for the left–right mixing of the down-squarks from the \( B \) meson experiments in the region of \( O(10) \) TeV [2]. Therefore, we take rather large values to see the enhancement of the \( K_L \to \pi^0 \nu \bar{\nu} \) decay.

- The flavor mixing parameters \( s_{ij}^L \) and \( s_{ij}^R \) of the up and down sectors are free parameters, and are varied in
  \[ s_{i3}^{ul}, s_{i3}^{dl} = 0 \sim 0.3 \quad (i = 1, 2), \quad s_{i3}^{ur}, s_{i3}^{dr} = 0 \sim 0.3 \quad (i = 1, 2), \]

  where the upper bound 0.3 is given by the experimental constraint of the \( K^0 - \bar{K}^0 \) mass difference \( \Delta M_K \). As discussed in the previous section, we ignore mixing between the first and second family of squarks, \( s_{12}^{ul} \), and then can avoid the large contribution from \( s_{12}^{ul} \) to \( \Delta M_K \). This single mixing effect of \( s_{12}^{ul} \) to the Z-penguin mediated by the chargino is known to be minor compared with the double mixing effect [51,54]. Namely, the SUSY contributions of the \( K_L \to \pi^0 \nu \bar{\nu} \) and \( K^+ \to \pi^+ \nu \bar{\nu} \) processes are dominated by the double mixing of the stop and sbottom.

- The phase parameters \( \phi_{13}^{L(R)} \) and \( \phi_{23}^{L(R)} \) are also free parameters. We scan them in \( -\pi \sim \pi \) randomly.

- We neglect the minor contribution from the sleptons and sneutrinos. We also neglect the charged Higgs contribution, which is tiny due to the CKM mixing.

- For non-perturbative parameters \( B_6^{(1/2)} \) and \( B_8^{(3/2)} \), which are the key ones for estimating \( \epsilon'_K / \epsilon_K \), we use the RBC-UKQCD result \( B_6^{(1/2)} = 0.57 \pm 0.15 \) and \( B_8^{(3/2)} = 0.76 \pm 0.05 \) in Eq. (10). We scan them within the 3\( \sigma \) error bar.

We use the CKM elements \( |V_{cb}|, |V_{ub}|, |V_{td}| \) in Ref. [50] with 3\( \sigma \) error bars, which are obtained in the framework of the SM. If there is a large SUSY contribution to the kaon and the \( B \) meson systems, the values of the CKM elements may be changed. Actually, the SUSY contribution is comparable to the SM one for \( \epsilon_K \) in our following numerical analyses, although very small for CP violations and the mass differences of the \( B \) mesons at the \( O(10) \) TeV scale of squarks [38]. We use the CKM element in the study of the unitarity triangle, including the data of CP asymmetries and the mass differences of \( B \) mesons without inputting \( \epsilon_K \) (strategy S1 in Ref. [50]).

### Results in SUSY at 10 TeV

Let us discuss the case of high-scale SUSY, where all squarks/sleptons are at the 10 TeV scale.

First, we discuss the contribution of the Z-penguin induced by the chargino to the \( K_L \to \pi^0 \nu \bar{\nu} \) and \( K^+ \to \pi^+ \nu \bar{\nu} \) processes. In this case, the left–right mixing of the up-squark sector controls the magnitude of the Z-penguin amplitude. Since the \( A \) term is considerably constrained by the

\[ \text{The metastability of vacuum can also constrain the left–right mixing for the down-squark sector [65]. In order to justify our setup of the left–right mixing angle, a more precise analysis of the vacuum stability is important.} \]
125 GeV Higgs mass, the left–right mixing angle cannot be large in our mass spectrum, at most $\theta_{LR}^b = 0.07$ as presented in the above setup. Therefore, we cannot obtain the enhancement of those processes.\footnote{\textsuperscript{3}If we take the smaller mass for $m_{\tilde{t}_2}$ in Eq. (26), e.g., 12 TeV, the left–right mixing angle can be chosen to be larger than 0.1. However, the contribution of $m_{\tilde{t}_1}$ is canceled by that of $m_{\tilde{t}_2}$ due to the small mass difference.} Actually, the predicted branching ratios of $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ deviate from the prediction of the SM with order 10%. Thus, we conclude that the $Z$-penguin mediated by the chargino cannot bring a large enhancement for the $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ decays due to the constraint of the 125 GeV Higgs mass. This result is consistent with the recent work in Ref. \cite{66}, where the metastability of vacuum constrains the left–right mixing for the up-squark sector.

On the other hand, the $Z$-penguin induced by the gluino could be large due to the large down-type left–right mixing $\theta_{LR}^b = 0.1–0.3$. In our setup of parameters, we show the predicted branching ratios, $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \to \pi^+ \nu \bar{\nu})$ in Fig. 1, where the mixings $s_{13}^{dL,dR}$ and $s_{23}^{dL,dR}$ are scanned in 0–0.3 and the left–right mixing angle $\theta_{LR}^b$ is fixed at 0.3. Here the Grossman–Nir bound is shown by the slanted green line \cite{67}. In order to see the $\theta_{LR}^b$ dependence, we also present the $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \to \pi^+ \nu \bar{\nu})$ in Figs. 2 and 3, in which $\theta_{LR}^b$ is fixed at 0.2 and 0.1 respectively. As seen in Figs. 1–3, the branching ratio of $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ depends considerably on the left–right mixing angle $\theta_{LR}^b$. The enhancement of $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ requires the left–right mixing angle to be larger than 0.1.

\textbf{Fig. 1.} The predicted region for $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \to \pi^+ \nu \bar{\nu})$ without imposing $\epsilon_K$ where $\theta_{LR}^b = 0.3$. The green line corresponds to the Grossman–Nir bound. The dashed red lines denote the 1σ experimental bounds for $\text{BR}(K^+ \to \pi^+ \nu \bar{\nu})$. The pink area indicates the SM prediction.
Fig. 2. The predicted region for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, without imposing $\epsilon_K$, where $\theta_{LR}^b = 0.2$. Notation is the same as in Fig. 1.

Fig. 3. The predicted region for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, without imposing $\epsilon_K$, where $\theta_{LR}^b = 0.1$. Notation is the same as in Fig. 1.

Fig. 4. The predicted region for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, with imposing $\epsilon_K$, where $\theta_{LR}^b = 0.3$. Notation is the same as in Fig. 1.

We show the predicted region for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, imposing $\epsilon_K$ where $\theta_{LR}^b = 0.3$ is fixed in Fig. 4. There are two directions in the predicted plane of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. The direction of the enhancement of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ corresponds to $\phi_{13}^{dL} - \phi_{23}^{dL} \simeq -\pi/2$ and $\phi_{13}^{dR} - \phi_{23}^{dR} \simeq \pi/2$, and the enhancement of $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ to $\phi_{13}^{dL,dR} - \phi_{23}^{dL,dR} \simeq 0, \pi$.

As a result, it is found that $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ can be enhanced up to $4 \times 10^{-10}$, which is much larger than the SM enhancement, with the $\epsilon_K$ constraint satisfied.
Therefore, the SUSY contribution does not spoil the agreement between the real part of the SM. The addition of the imaginary part of the $K$ and $\text{Im} \Delta M_{K}$ contributes to another kaon rare decay $K_{L} \rightarrow \pi^{0} \nu \bar{\nu}$, and the $B$ meson rare decays, $B_{d}^{0} \rightarrow \mu^{+} \mu^{-}$, and $B_{s} \rightarrow \mu^{+} \mu^{-}$. Therefore, we expect them to correlate with the $K \rightarrow \pi \nu \bar{\nu}$ decays. In the $K_{L} \rightarrow \mu^{+} \mu^{-}$ process, the long-distance effect is estimated to be large in Ref. [64]. Therefore, we discuss only the short-distance effect, which is dominated by the $Z$-penguin. We show $\text{BR}(K_{L} \rightarrow \pi^{0} \nu \bar{\nu})$ versus $\text{BR}(K_{L} \rightarrow \mu^{+} \mu^{-})$ in Fig. 7, where the constraint from the $K^{0} - \bar{K}^{0}$ mass difference $\Delta M_{K}$. Our SUSY contribution of $\Delta M_{K}$(SUSY) is comparable with the SM contribution $\Delta M_{K}$(SM). It is possible to fit the following condition, keeping the enhancement of $\text{BR}(K_{L} \rightarrow \pi^{0} \nu \bar{\nu})$:

$$\frac{\Delta M_{K}}{\Delta M_{K}(\text{SM})} = 0.75 \sim 1.25,$$

which is the criterion of the allowed NP contribution in Ref. [68]. We also estimate the SUSY contributions to $\Delta M_{B_{0}}$ and $\Delta M_{B_{s}}$, which are at most 10% of the SM.

Let us discuss the correlation between $\text{BR}(K_{L} \rightarrow \pi^{0} \nu \bar{\nu})$ and $\epsilon_{K}' / \epsilon_{K}$, as discussed in Sect. 2.3, both processes come from the imaginary part of the same $Z$-penguin, and can be enhanced simultaneously once the condition of Eq. (18) is imposed. In Fig. 5, we show the correlation between $\text{BR}(K_{L} \rightarrow \pi^{0} \nu \bar{\nu})$ and $\epsilon_{K}' / \epsilon_{K}$, where $Zsd$ coupling satisfies the condition of Eq. (18). The constraint from $\epsilon_{K}$ is also imposed. It is remarkable that the $Z$-penguin mediated by the gluino enhances $\epsilon_{K}' / \epsilon_{K}$ and the branching ratio for $K_{L} \rightarrow \pi^{0} \nu \bar{\nu}$ simultaneously. While the estimated $\epsilon_{K}' / \epsilon_{K}$ fits the observed value, the branching ratio of $K_{L} \rightarrow \pi^{0} \nu \bar{\nu}$ increases up to $1.0 \times 10^{-10}$. In this region, the phase of $\text{Im} \Delta_{L}^{sd}$ and $\text{Im} \Delta_{R}^{sd}$ becomes opposite, so the enhanced region of $\text{BR}(K_{L} \rightarrow \pi^{0} \nu \bar{\nu})$ is somewhat reduced by the cancelation between the left-handed coupling of $Z$ and the right-handed one partially, compared with the result in Fig. 4.

The real parts of $\Delta_{L}^{sd}$ and $\Delta_{R}^{sd}$ are sufficiently small since $\phi_{13}^{dL,dR} - \phi_{23}^{dL,dR} \simeq \pm \pi / 2$ is taken. Therefore, the SUSY contribution does not spoil the agreement between the real part of the $K \rightarrow \pi \pi$ amplitude in the SM and the experimental data.

In Fig. 6, we show the correlation between $\text{BR}(K_{L} \rightarrow \pi^{0} \nu \bar{\nu})$ and $\text{BR}(K^{+} \rightarrow \pi^{+} \nu \bar{\nu})$. In the parameter region where $\text{BR}(K_{L} \rightarrow \pi^{0} \nu \bar{\nu})$ and $\epsilon_{K}' / \epsilon_{K}$ are enhanced, the branching ratio of $K^{+} \rightarrow \pi^{+} \nu \bar{\nu}$ does not deviate from the SM. It is understandable because $\phi_{13}^{dL,dR} - \phi_{23}^{dL,dR} \simeq \pm \pi / 2$ is taken in order to enhance $\text{BR}(K_{L} \rightarrow \pi^{0} \nu \bar{\nu})$ with the $\epsilon_{K}$ constraint. On the other hand, $\text{BR}(K^{+} \rightarrow \pi^{+} \nu \bar{\nu})$ is dominated by the considerably sizable real part of the SM. The addition of the imaginary part of the SUSY contribution does not change the SM prediction significantly.
Fig. 6. The predicted region for $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \to \pi^+ \nu \bar{\nu})$, where the $Zsd$ coupling satisfies the condition of Eq. (18). Notation is the same as in Fig. 1.

Fig. 7. The predicted $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K_L \to \mu^+ \mu^-)$. The pink area indicates the SM with $3\sigma$. The solid red line denotes the bound for the short-distance contribution.

$\epsilon_K$ is imposed. It is noticed that the predicted value almost satisfies the bound for the short-distance contribution in Eq. (23), presented as the red line.

The clear correlation between two branching ratios is understandable because $\text{BR}(K_L \to \mu^+ \mu^-)$ is sensitive only to the real part of $Z$-couplings. When the enhancement of $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ is found in the future, $\text{BR}(K_L \to \mu^+ \mu^-)$ will remain less than $10^{-9}$. On the other hand, when $\text{BR}(K_L \to \mu^+ \mu^-)$ is larger than $10^{-9}$, there is no enhancement of $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$. This relation is testable in future experiments.

We also show $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ versus $\text{BR}(B^0 \to \mu^+ \mu^-)$ in Fig. 8. We can expect the enhancement of $\text{BR}(B^0 \to \mu^+ \mu^-)$ in our setup even if $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ is comparable to the SM one. Since LHCb will observe the $\text{BR}(B^0 \to \mu^+ \mu^-)$ [69], this result is the attractive one in our model.

On the other hand, we do not see the correlation between $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ and $\text{BR}(B_s \to \mu^+ \mu^-)$ since the SM component of $\text{BR}(B_s \to \mu^+ \mu^-)$ is relatively large compared with $B^0 \to \mu^+ \mu^-$. The enhancement of the $K_L \to \pi^0 \nu \bar{\nu}$ decay rate is still consistent with the present experimental data of $\text{BR}(B_s \to \mu^+ \mu^-)$.

4.3. Results in the split-family model with 10 TeV stop and sbottom

Let us discuss the case of the split-family SUSY model with 10 TeV stop and sbottom, where first and second family squark masses are around 2 TeV. The constraint of $\epsilon_K$ is seriously tight for CP-violating phases associated with squark mixing in the split-family SUSY model. Moreover, the $|\Delta F| = 2$ processes receive overly large contributions from the the first and second squarks because
they are relatively light, at $\mathcal{O}(1)$ TeV. Actually, $\Delta M_K$, $\Delta M_{B^0}$, and $\Delta M_{B_s}$ are predicted as

$$\frac{\Delta M_K}{\Delta M_K (\text{SM})} \simeq 400, \quad \frac{\Delta M_{B^0}}{\Delta M_{B^0 (\text{SM})}} \simeq 50, \quad \frac{\Delta M_{B_s}}{\Delta M_{B_s (\text{SM})}} \simeq 3. \quad \text{(30)}$$

In addition, the large left–right mixing generates large contributions to the $b \to s \gamma$ decay, therefore, the left–right mixing angle is severely constrained by the experimental data for $b \to s \gamma$. Therefore, it is impossible to realize the enhancement of $\text{BR}(K_L \to \pi^0 \nu \bar{\nu})$ in the split-family model satisfying constraints $|\Delta F| = 1, 2$ transitions in the kaon and the B meson systems.

4.4. EDMs of neutron and mercury

Finally, we add a comment on the electric dipole moments (EDMs) of the neutron and mercury (Hg), $d_n$ and $d_{\text{Hg}}$, which arise through the chromo-EDM of the quarks, $d_q^C$ due to gluino–squark mixing [70–75]. If both left-handed and right-handed mixing angles are taken to be large, such as $s_{13}^{d L} = s_{13}^{d R} \simeq 0.3$ or $s_{23}^{d L} = s_{23}^{d R} \simeq 0.3$, with large left–right mixing, $d_n$ and $d_{\text{Hg}}$ are predicted to be one and two orders larger than the experimental upper bound [62], respectively, $|d_n| < 0.29 \times 10^{-25} \text{e} \cdot \text{cm}$ and $|d_{\text{Hg}}| < 3.1 \times 10^{-29} \text{e} \cdot \text{cm}$.

However, there still remains the freedom of phase parameters. For example, by tuning $\phi_{13}^{d L}$ and $\phi_{13}^{d R}$ ($i = 1, 2$) under the constraint from $\epsilon_K$, we can suppress the EDMs sufficiently. This tuning does not spoil our numerical results above.

5. Summary and discussions

In order to probe SUSY at the 10 TeV scale, we have studied the processes of $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ combined with the CP-violating parameters $\epsilon_K$ and $\epsilon_K' / \epsilon_K$. The Z-penguin mediated by the chargino loop cannot enhance $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ because the left–right mixing of the stop is constrained by the 125 GeV Higgs mass. On the other hand, the Z-penguin mediated by the gluino loop can enhance the branching ratios of both $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$, where the former increases more than $1.0 \times 10^{-10}$, much larger than the SM prediction even if the constraint of $\epsilon_K$ is imposed. Thus, the $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ decays provide us with very important information to probe the SUSY.

It is remarkable that the Z-penguin mediated by the gluino loop can simultaneously enhance $\epsilon_K' / \epsilon_K$ and the branching ratio for $K_L \to \pi^0 \nu \bar{\nu}$. While the estimated $\epsilon_K' / \epsilon_K$ fits the observed value, the branching ratio of $K_L \to \pi^0 \nu \bar{\nu}$ increases up to $1.0 \times 10^{-10}$. 

13/21
Appendix A. Squark-flavor mixing matrix

The flavor mixing and CP violation are induced through the quark–squark–gaugino and the lepton–slepton–gaugino couplings. The Lagrangian of the gaugino–quark–squark interaction is written as

$$\mathcal{L}_{\text{int}}(\tilde{G}q \tilde{q}) = -i\sqrt{2}g_{1,2,3} \sum_{q} \tilde{q}_i^q (T^a) \tilde{G}^a \left[ (\Gamma^{(q)}_L)_i^j L + (\Gamma^{(q)}_R)_i^j R \right] q_j + \text{H.c.}, \quad (A1)$$

where $\tilde{G}^a$ is the gaugino field, $T^a$ is the generator of the gauge group, and $L, R$ are projection operators. The left-handed and right-handed mixing matrixes $\Gamma^{(q)}_L$ and $\Gamma^{(q)}_R$ diagonalize the $6 \times 6$ squark mass matrix $M_{\tilde{q}}^2$ in the super-CKM basis to the mass eigenstate basis as

$$M_{\tilde{q}}^2 = \Gamma^{(q)\dagger} \text{diag}(m_{\tilde{q}}^2) \Gamma^{(q)} = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{RL}^2 & M_{RR}^2 \end{pmatrix}, \quad (A2)$$

where $\Gamma^{(q)}$ is the $6 \times 6$ unitary matrix, and it is decomposed into $3 \times 3$ matrices as $\Gamma^{(q)} = (\Gamma^{(q)}_L, \Gamma^{(q)}_R)$. The squark mass matrix $M_{\tilde{q}}^2$ in the super-CKM basis is the same as that in SLHA notation [76,77]. We write $\Gamma^{(q)}_{L,R}$ as

$$\Gamma^{(q)}_L = \begin{pmatrix} c_{13}^{qL} & 0 & s_{13}^{qL} e^{-i\phi_{13}^{qL}} & 0 & 0 & -s_{13}^{qL} e^{-i\phi_{13}^{qL}} e^{i\phi_{23}^{qL}} \\ -s_{23}^{qL} c_{13}^{qL} e^{i(\phi_{23}^{qL} - \phi_{13}^{qL})} & c_{23}^{qL} & s_{23}^{qL} e^{-i\phi_{23}^{qL}} & -s_{23}^{qL} c_{13}^{qL} & 0 & e^{i\phi_{23}^{qL}} \\ -s_{23}^{qL} c_{13}^{qL} e^{i(\phi_{23}^{qL} - \phi_{13}^{qL})} & -s_{23}^{qL} c_{13}^{qL} e^{i(\phi_{23}^{qL} - \phi_{13}^{qL})} & c_{13}^{qL} c_{23}^{qL} e^{i\phi_{23}^{qL}} & 0 & 0 & -c_{13}^{qL} c_{23}^{qL} e^{i\phi_{23}^{qL}} \\ 0 & 0 & 0 & c_{13}^{qL} c_{23}^{qL} e^{i\phi_{23}^{qL}} & -c_{13}^{qL} c_{23}^{qL} e^{i\phi_{23}^{qL}} & e^{i\phi_{23}^{qL}} \end{pmatrix}^T.$$
where we use abbreviations $c_{ij}^{qL,qR} = \cos \theta_{ij}^{qL,qR}$, $s_{ij}^{qL,qR} = \sin \theta_{ij}^{qL,qR}$, $c_{qR} = \cos \theta_{q}$, and $s_{qR} = \sin \theta_{q}$ with \(\theta_{ij}^{qL,qR}\) being the mixing angles between \(i\)th and \(j\)th families of squarks. In these mixing matrices, we take $s_{12}^{qL,qR} = 0$.

The $3 \times 3$ submatrix $M_{LR}^2$ is given as

$$M_{LR}^2 = (m_{q_{3,1}}^2 - m_{q_{3,2}}^2) \cos \theta_{LR}^q \sin \theta_{LR}^q e^{i \phi_{LR}^q} \times \left( \begin{array}{ccc} s_{13}^{qL,qR} & c_{13}^{qL,qR} & c_{13}^{qL,qR} \\ c_{13}^{qL,qR} & c_{13}^{qL,qR} & c_{13}^{qL,qR} \\ s_{13}^{qL,qR} & c_{13}^{qL,qR} & c_{13}^{qL,qR} \end{array} \right),$$

where the left–right mixing angles $\theta_{LR}^q$ are given approximately as

$$\theta_{LR}^b \simeq \frac{m_{b} (A_{33}^{u,\alpha} - \mu \tan \beta)}{m_{bR}^2 - m_{bL}^2}, \quad \theta_{LR}^t \simeq \frac{m_{t} (A_{33}^{u,\alpha} - \mu \cot \beta)}{m_{tL}^2 - m_{tR}^2}.$$

### Appendix B. Chargino- and gluino-interactions-induced Z-penguin

The $Z$-penguin amplitude mediated by the chargino, $P^{sd}_{ZL}(\chi^\pm)$ in our basis [78] is given as

$$P^{sd}_{ZL}(\chi^\pm) = \frac{g_s^2}{4m_W} \sum_{a,b,I,J} (1^{(d)}_{CL})^I_{aI} (\bar{1}^{(d)}_{CL})^J_{bJ} \left\{ \delta^I_J (U_+)^I_{aI} (U_+)^J_{bJ} \right\} \left\{ \log x_{qR}^I + f_2(x_{qL}^I, x_{qR}^I) \right\}$$

$$- 2\delta^I_J (U_-)^I_{aI} (U_-)^J_{bJ} \left\{ x_{qL}^I x_{qR}^I \right\} \left\{ \delta^I_J (\tilde{\Gamma}_L)^I_{aI} f_2(x_{qL}^I, x_{qR}^I) \right\},$$

where

$$(\Gamma^{(d)}_{CL})^I_{aI} = (\Gamma^{(d)}_{L})^I_{aI} (V_{CKM})^I_{aI} (U_+)^a_J + \frac{1}{g_2} (\Gamma^{(d)}_{R})^I_{aI} f_2(x_{qL}^I, x_{qR}^I).$$

and

$$(\tilde{\Gamma}_L)^I_{aI} = (\Gamma^{(d)}_{L})^I_{aI},$$

with $q =$ s, d, $I = 1 – 6$ for up-squarks, and $\alpha = 1, 2$ for charginos. Here, $(U_\pm)^{a}_{I}$ denotes the mixing parameters between the wino and the higgsino.

The right-handed $Z$-penguin $P^{sd}_{ZR}(\chi^\pm)$ is also given simply by replacements between L and R [78].

The $Z$-penguin amplitude mediated by the gluino, $P^{sd}_{ZL}(\tilde{g})$ [78] is written as

$$P^{sd}_{ZL}(\tilde{g}) = -\frac{2}{3} \frac{g_s^2}{m_W} \sum_{I,J} (1^{(d)}_{GL})^I_{aI} (\tilde{\Gamma}^{(d)}_{GL})^J_{aJ} f_2(x_{qL}^I, x_{qR}^I),$$

where

$$(\Gamma^{(d)}_{GL})^I_{aI} = (\Gamma^{(d)}_{L})^I_{aI} (V_{CKM})^I_{aI} (U_+)^a_J + \frac{1}{g_2} (\Gamma^{(d)}_{R})^I_{aI} f_2(x_{qL}^I, x_{qR}^I).$$

$$(\tilde{\Gamma}_L)^I_{aI} = (\Gamma^{(d)}_{L})^I_{aI},$$

and

$$(\tilde{\Gamma}_R)^I_{aI} = (\Gamma^{(d)}_{R})^I_{aI},$$

with $q =$ s, d, $I = 1 – 6$ for up-squarks, and $\alpha = 1, 2$ for charginos. Here, $(U_\pm)^{a}_{I}$ denotes the mixing parameters between the wino and the higgsino.
where
\[ (\Gamma_R^{(d)})^J_I \equiv (1_R^{(d)})^I_J. \] (B5)

The right-handed Z-penguin \( P_{ZP}^{dR}(\bar{g}) \) is also given simply by replacements between L and R.

### Appendix C. Basic formulae

#### C.1. \( K^+ \to \pi^+ \nu \bar{\nu} \) and \( K_L \to \pi^0 \nu \bar{\nu} \)

The effective Hamiltonian for \( K \to \pi \nu \bar{\nu} \) in the SM is given as [3]

\[ H_{\text{eff}}^{\text{SM}} = \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} \sum_{i=e,\mu,\tau} \left[ V_{ci}^* V_{cd} X_c + V_{ti}^* V_{td} X_t \right] (\bar{s}_L \gamma^\mu d_L) (\bar{\nu}_L^{\mu} \nu_L) + \text{H.c.,} \] (C1)

which is induced by the box and the Z-penguin mediated by the W boson. The loop function \( X_c \) denotes the charm-quark contribution of the Z-penguin, and \( X_t \) is the sum of the top-quark exchanges of the box diagram and the Z-penguin in Eq. (C1).

Let us define the function \( F \) as

\[ F = V_{cs}^* V_{cd} X_c + V_{ts}^* V_{td} X_t. \] (C2)

The branching ratio of \( K^+ \to \pi^+ \nu \bar{\nu} \) is given in terms of \( F \). Taking the ratio of it to the branching ratio of \( K^+ \to \pi^0 e^+ \nu \), which is the tree level transition, we obtain a simple form:

\[ \frac{\text{BR}(K^+ \to \pi^+ \nu \bar{\nu})}{\text{BR}(K^+ \to \pi^0 e^+ \nu)} = \frac{2}{|V_{ud}|^2} \left( \frac{\alpha}{2\pi \sin^2 \theta_W} \right)^2 \sum_{i=e,\mu,\tau} |F|^2. \] (C3)

Here the hadronic matrix element has been removed by using the fact that the hadronic matrix element of \( K^+ \to \pi^0 e^+ \nu \), which is well measured as \( \text{BR}(K^+ \to \pi^0 e^+ \nu)_{\text{exp}} = (5.07 \pm 0.04) \times 10^{-2} [62] \), is related to that of \( K^+ \to \pi^+ \nu \bar{\nu} \) with isospin symmetry:

\[ \langle \pi^0 | (\bar{d}_L \gamma^\mu s_L) | \bar{K}^0 \rangle = \langle \pi^0 | (\bar{s}_L \gamma^\mu u_L) | K^+ \rangle, \] (C4)

\[ \langle \pi^+ | (\bar{s}_L \gamma^\mu d_L) | K^+ \rangle = \sqrt{2} \langle \pi^0 | (\bar{s}_L \gamma^\mu u_L) | K^+ \rangle. \] (C5)

Finally, the branching ratio for \( K^+ \to \pi^+ \nu \bar{\nu} \) is expressed as

\[ \text{BR}(K^+ \to \pi^+ \nu \bar{\nu}) = 3\kappa |F|^2, \quad \kappa = \frac{2}{|V_{ud}|^2} r_{K^+} \left( \frac{\alpha}{2\pi \sin^2 \theta_W} \right)^2 \frac{\text{BR}(K^+ \to \pi^0 e^+ \nu)}{\text{BR}(K^+ \to \pi^0 e^+ \nu)}, \] (C6)

where \( r_{K^+} \) is the isospin breaking correction between \( K^+ \to \pi^+ \nu \bar{\nu} \) and \( K^+ \to \pi^0 e^+ \nu [47,48] \), and the factor 3 comes from the sum of three neutrino flavors. It is noticed that the branching ratio for \( K^+ \to \pi^+ \nu \bar{\nu} \) depends on both the real and imaginary parts of \( F \).

For the \( K_L \to \pi^0 \nu \bar{\nu} \) decay, the \( K^0 - \bar{K}^0 \) mixing should be taken into account, and one obtains

\[ A(K_L \to \pi^0 \nu \bar{\nu}) = \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} (\bar{v}_L^{\mu} \gamma^\mu \nu_L) \langle \pi^0 | [F(\bar{s}_L \gamma^\mu d_L) + F^*(\bar{d}_L \gamma^\mu s_L) | K_L \rangle \]

\[ = \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} (\bar{v}_L^{\mu} \gamma^\mu \nu_L) \left( \frac{1}{\sqrt{2}} [F(1 + \bar{\epsilon})(\pi^0 | (\bar{s}_L \gamma^\mu d_L) | K^0) + F^{*}(1 - \bar{\epsilon})(\pi^0 | (\bar{d}_L \gamma^\mu s_L) | \bar{K}^0)] \right) \]

\[ \simeq \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} (\bar{v}_L^{\mu} \gamma^\mu \nu_L) \left( \frac{1}{\sqrt{2}} 2i \text{Im} F(\pi^0 | (\bar{s}_L \gamma^\mu d_L) | K^0) \right), \] (C7)
where we use

\[ |K_L⟩ = \frac{1}{\sqrt{2}} \left[ (1 + \bar{\epsilon})|K^0⟩ + (1 - \bar{\epsilon})|\bar{K}^0⟩ \right], \quad (C8) \]

with

\[ CP|K^0⟩ = -|\bar{K}^0⟩, \quad ⟨π^0|(\bar{d}_Lγμs_L)|K^0⟩ = -⟨π^0|(\bar{s}_Lγμd_L)|K^0⟩. \quad (C9) \]

We neglect CP violation in \( K^0 - \bar{K}^0 \) mixing, \( \bar{\epsilon} \), due to its smallness, \( |\bar{\epsilon}| \sim 10^{-3} \). Taking the ratio between the branching ratios of \( K^+ \rightarrow π^0 e^+ ν \) and \( K_L \rightarrow π^0 ν \bar{ν} \), we have the simple form

\[ \frac{BR(K_L \rightarrow π^0 ν \bar{ν})}{BR(K^+ \rightarrow π^0 e^+ ν)} = \frac{2}{|V_{us}|^2} \left( \frac{α}{2π} \sin^2 θ_W \right)^2 \frac{τ(K_L)}{τ(K^+)} \sum_{i=e,μ,τ} (|Im F|)^2. \quad (C10) \]

Therefore, the branching ratio of \( K_L \rightarrow π^0 ν \bar{ν} \) is given as

\[ BR(K_L \rightarrow π^0 ν \bar{ν}) = 3K \cdot \frac{r_{K_L}}{r_{K^+}} \frac{τ(K_L)}{τ(K^+)} (|Im F|)^2, \quad (C11) \]

where \( r_{K_L} \) denotes the isospin breaking effect [47,48]. It is remarked that the branching ratio of \( K_L \rightarrow π^0 ν \bar{ν} \) depends on the imaginary part of \( F \).

The effective Hamiltonian in Eq. (C1) is modified due to new box diagrams and penguin diagrams induced by SUSY particles. Then, the effective Lagrangian is given as

\[ L_{eff} = \sum_{i,j=e,μ,τ} \left[ C_{ij}^{VLL}(\bar{s}_Lγμd_L) + C_{ij}^{VRL}(\bar{s}_Rγμd_R) \right] \left( \bar{ν}_j L γμν_i L \right) + \text{H.c.}, \quad (C12) \]

where \( i \) and \( j \) are the indices of the flavor of the neutrino final state. Here, \( C_{ij}^{VLL,VRL} \) is the sum of the box and Z-penguin contributions:

\[ C_{ij}^{VLL} = -B_{VLL}^{adj} Q_{ZL}^{(v)} P_{ZL}^{pd} δ_{ij}, \quad C_{ij}^{VRL} = -B_{VRL}^{adj} - \frac{α^2}{4π} Q_{ZL}^{(v)} P_{ZR}^{pd} δ_{ij}, \quad (C13) \]

where the weak neutral-current coupling \( Q_{ZL}^{(v)} = 1/2 \), and \( B_{V_{LL(R)L}}^{adj} \) and \( P_{ZL(R)}^{pd} \) denote the box contribution and the Z-penguin contribution, respectively, and \( V, L, \) and \( R \) denote the vector, left-handed, and right-handed couplings, respectively. In addition to the W boson contribution, there are the gluino- \( (\tilde{g}) \), the chargino- \( (\tilde{χ}^±) \), and the neutralino- \( (\tilde{χ}^0) \) mediated contributions.

The branching ratios of \( K^+ \rightarrow π^+ ν \bar{ν} \) and \( K_L \rightarrow π^0 ν \bar{ν} \) are obtained by replacing internal effect \( F \) in Eqs. (C6) and (C11) to \( C_{ij}^{VLL} + C_{ij}^{VRL} \):

\[ BR(K^+ \rightarrow π^+ ν \bar{ν}) = \kappa \sum_{i=e,μ,τ} |C_{ij}^{VLL} + C_{ij}^{VRL}|^2, \quad (C14) \]

\[ BR(K_L \rightarrow π^0 ν \bar{ν}) = κ \cdot \frac{r_{K_L}}{r_{K^+}} \frac{τ(K_L)}{τ(K^+)} \sum_{i=e,μ,τ} |Im(C_{ij}^{VLL} + C_{ij}^{VRL})|^2. \quad (C15) \]
C.2. $B_s \rightarrow \mu^+\mu^-$, $B^0 \rightarrow \mu^+\mu^-$, and $K_L \rightarrow \mu^+\mu^-$

The Z-penguin process appears in $B_s \rightarrow \mu^+\mu^-$ and $B^0 \rightarrow \mu^+\mu^-$ decays. We show the branching ratio for $B_s \rightarrow \mu^+\mu^-$, which includes the Z-penguin amplitude [78]:

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) = \frac{\tau_{B_s}}{14\pi} \frac{m_{B_s}^2}{m_{B_s}} \left( \frac{\alpha}{4\pi} \right)^2 \frac{m_\mu^2}{m_{B_s}^2} \left( 1 - \frac{4m_\mu^2}{m_{B_s}^2} \right)^{1/2} \left| C^{(\mu)}_{\text{VRA}} - C^{(\mu)}_{\text{VLA}} \right|^2, \quad (C16)$$

where

$$\frac{\alpha}{4\pi} C^{(\mu)}_{\text{VLA}} = -B^{(bs\mu\mu)}_{\text{VLL}}(\text{SM}) - \frac{\alpha_2}{4\pi} \frac{1}{4} \frac{g^2}{2m_W^2} V_{tb} V^*_{ts} B_0(x_t). \quad \text{(C17)}$$

We include the box diagram only for the SM, which is

$$B^{(bs\mu\mu)}_{\text{VLL}}(\text{SM}) = -\frac{\alpha_2}{4\pi} \frac{g^2}{2m_W^2} V_{tb} V^*_{ts} B_0(x_t). \quad \text{(C18)}$$

On the other hand, the SM component of the Z-penguin amplitude is

$$P^{bs}_{ZL} = \frac{g^2}{2m_W^2} V_{tb} V^*_{ts} \times 4 C_0(x_t), \quad \text{(C19)}$$

where $B_0(x_t)$ and $C_0(x_t)$ are well-known loop functions depending on $x_t = m_t^2/m_W^2$. We have neglected other amplitudes such as the Higgs-mediated scalar amplitude since we focus on NP in the Z-penguin process.

The branching ratio of $B^0 \rightarrow \mu^+\mu^-$ is given by a similar expression. For the $K_L \rightarrow \mu^+\mu^-$ decay, its branching ratio is given as [79]

$$\text{BR}(K_L \rightarrow \mu^+\mu^-)_{\text{SD}} = \kappa_\mu \left[ \frac{\text{Re}\lambda_i}{\lambda^5} Y(x_t) + \frac{\text{Re}\lambda_c}{\lambda} P_c \right]^2, \quad \text{(C20)}$$

$$\kappa_\mu = (2.009 \pm 0.017) \times 10^{-9} \left( \frac{\lambda}{0.225} \right)^8, \quad \text{(C21)}$$

where $\lambda$ is the Wolfenstein parameter, $\lambda_i = V^*_{is} V_{id}$, and the charm-quark contribution $P_c$ is calculated in NNLO as $P_c = 0.115 \pm 0.018$, and $Y$ is the same as in Eq. (7). We use its SM value as $Y(x_t) = 0.950 \pm 0.049$ ($x_t \equiv m_t^2/M_W^2$).

References

[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[3] A. J. Buras, arXiv:hep-ph/9806471 [Search INSPIRE].
[4] A. J. Buras, D. Buttazzo, J. Girrbach-Noe, and R. Kneijsens, J. High Energy Phys. 11, 033 (2015) [arXiv:1503.02693 [hep-ph]] [Search INSPIRE].
[5] S. Bertolini and A. Masiero, Phys. Lett. B 174, 343 (1986).
[6] I. I. Y. Bigi and F. Gabbiani, Nucl. Phys. B 367, 3 (1991).
[7] G. F. Giudice, Z. Phys. C 34, 57 (1987).
[8] B. Mukhopadhyaya and A. Raychaudhuri, Phys. Lett. B 189, 203 (1987).
[9] G. Couture and H. Konig, Z. Phys. C 69, 167 (1995) [arXiv:hep-ph/9503299] [Search INSPIRE].
[10] T. Goto, Y. Okada, and Y. Shimizu, Phys. Rev. D 58, 094006 (1998) [arXiv:hep-ph/9804294] [Search INSPIRE].
[11] A. J. Buras, G. Colangelo, G. Isidori, A. Romanino, and L. Silvestrini, Nucl. Phys. B 566, 3 (2000) [arXiv:hep-ph/990837] [Search INSPIRE].
[12] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager, and L. Silvestrini, Nucl. Phys. B 592, 55 (2001) [arXiv:hep-ph/0007133] [Search INSPIRE].
[13] A. J. Buras, arXiv:hep-ph/0505175 [Search INSPIRE].
[14] A. J. Buras, F. Schwab, and S. Uhlig, Rev. Mod. Phys. 80, 965 (2008) [arXiv:hep-ph/0405132] [Search INSPIRE].
[15] M. Blanke, Acta Phys. Polon. B 41, 127 (2010) [arXiv:0904.2528 [hep-ph]] [Search INSPIRE].
[16] C. Smith, arXiv:1409.6162 [hep-ph] [Search INSPIRE].
[17] W. S. Hou, M. Kohda, and F. Xu, Phys. Lett. B 751, 458 (2015) [arXiv:1411.1988 [hep-ph]] [Search INSPIRE].
[18] J. K. Ahn et al. [E391a Collaboration], Phys. Rev. D 81, 072004 (2010) [arXiv:0911.4789 [hep-ex]] [Search INSPIRE].
[19] A. V. Artamonov et al. [BNL-E949 Collaboration], Phys. Rev. D 79, 092004 (2009) [arXiv:0903.0030 [hep-ex]] [Search INSPIRE].
[20] M. Togawa, J. Phys. Conf. Ser. 556, 012046 (2014).
[21] K. Shiomi [for the KOTO Collaboration], arXiv:1411.4250 [hep-ex] [Search INSPIRE].
[22] V. Kozhuharov [NA62 Collaboration], EPJ Web Conf. 80, 00003 (2014) [arXiv:1412.0240 [hep-ex]] [Search INSPIRE].
[23] A. J. Buras, M. Gorbahn, S. Jager, and M. Jamin, J. High Energy Phys. 11, 202 (2015) [arXiv:1507.06345 [hep-ph]] [Search INSPIRE].
[24] A. J. Buras, J. High Energy Phys. 04, 071 (2016) [arXiv:1601.00005 [hep-ph]] [Search INSPIRE].
[25] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 110, 021801 (2013) [arXiv:1211.2674 [hep-ex]] [Search INSPIRE].
[26] V. Khachatryan et al. [CMS and LHCb Collaborations], Nature 522, 68 (2015) [arXiv:1411.4413 [hep-ex]] [Search INSPIRE].
[27] G. Aad et al. [ATLAS Collaboration], J. High Energy Phys. 09, 176 (2014) [arXiv:1405.7875 [hep-ex]] [Search INSPIRE].
[28] S. Chatrchyan et al. [CMS Collaboration], J. High Energy Phys. 06 055 (2014) [arXiv:1402.4770 [hep-ex]] [Search INSPIRE].
[29] G. Aad et al. [ATLAS Collaboration], J. High Energy Phys. 11, 118 (2014) [arXiv:1407.0583 [hep-ex]] [Search INSPIRE].
[30] ATLAS Collaboration, ATLAS-CONF-2016-078, [Search INSPIRE].
[31] CMS Collaboration, CMS-PAS-SUS-16-014, [Search INSPIRE].
[32] CMS Collaboration, CMS-PAS-SUS-16-019, [Search INSPIRE].
[33] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716, 1 (2012) [arXiv:1207.7214 [hep-ex]] [Search INSPIRE].
[34] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012) [arXiv:1207.7235 [hep-ex]] [Search INSPIRE].
[35] P. Draper, P. Meade, M. Reece, and D. Shih, Phys. Rev. D 85, 095007 (2012) [arXiv:1112.3068 [hep-ph]] [Search INSPIRE].
[36] W. Altmannshofer, R. Harnik, and J. Zupan, J. High Energy Phys. 11, 202 (2013) [arXiv:1308.3653 [hep-ph]] [Search INSPIRE].
[37] T. Moroi and M. Nagai, Phys. Lett. B 723, 107 (2013) [arXiv:1303.0668 [hep-ph]] [Search INSPIRE].
[38] M. Tanimoto and K. Yamamoto, Phys. Lett. B 735, 426 (2014) [arXiv:1404.0520 [hep-ph]] [Search INSPIRE].
[39] M. Endo, S. Shirai, and T. T. Yanagida, Prog. Theor. Phys. 125, 921 (2011) [arXiv:1009.3366 [hep-ph]] [Search INSPIRE].
[40] M. Ibe, T. T. Yanagida, and N. Yokozaki, J. High Energy Phys. 08, 067 (2013) [arXiv:1303.6995 [hep-ph]] [Search INSPIRE].
[41] S. K. Mandal, M. Nojiri, M. Sudano, and T. T. Yanagida, J. High Energy Phys. 01, 131 (2011) [arXiv:1004.4164 [hep-ph]] [Search INSPIRE].
[42] G. W. Bennett et al. [Muon g-2 Collaboration], Phys. Rev. D 73, 072003 (2006) [arXiv:hep-ex/0602035] [Search INSPIRE].

[43] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, and T. Teubner, J. Phys. G 38, 085003 (2011) [arXiv:1105.3149 [hep-ph]] [Search INSPIRE].

[44] M. Davier, A. Hoecker, B. Malaescu, and Z. Zhang, Eur. Phys. J. C 71, 1515 (2011); 72, 1874 (2012) [erratum] [arXiv:1010.4180 [hep-ph]] [Search INSPIRE].

[45] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, Nucl. Phys. B 477, 321 (1996) [arXiv:hep-ph/9604387] [Search INSPIRE].

[46] M. Tanimoto and K. Yamamoto, Prog. Theor. Exp. Phys. 2015, 053B07 (2015) [arXiv:1503.06270 [hep-ph]] [Search INSPIRE].

[47] W. J. Marciano and Z. Parsa, Phys. Rev. D 53, 1 (1996).

[48] F. Mescia and C. Smith, Phys. Rev. D 76, 034017 (2007) [arXiv:0705.2025 [hep-ph]] [Search INSPIRE].

[49] J. Brod, M. Gorbahn, and E. Stamou, Phys. Rev. D 83, 034030 (2011) [arXiv:1009.0947 [hep-ph]] [Search INSPIRE].

[50] M. Blanke and A. J. Buras, Eur. Phys. J. C 76, 197 (2016) [arXiv:1602.04020 [hep-ph]] [Search INSPIRE].

[51] G. Colangelo and G. Isidori, J. High Energy Phys. 09, 009 (1998) [arXiv:hep-ph/9808487] [Search INSPIRE].

[52] Y. Nir and M. P. Worah, Phys. Lett. B 423, 319 (1998) [arXiv:hep-ph/9711215] [Search INSPIRE].

[53] A. J. Buras, A. Romanino, and L. Silvestrini, Nucl. Phys. B 520, 3 (1998) [arXiv:hep-ph/9712398] [Search INSPIRE].

[54] A. J. Buras, T. Ewerth, S. Jager, and J. Rosiek, Nucl. Phys. B 565, 3 (2000) [arXiv:hep-ph/0408142] [Search INSPIRE].

[55] G. Isidori, F. Mescia, P. Paradisi, C. Smith, and S. Trine, J. High Energy Phys. 09, 009 (1998) [arXiv:hep-ph/9808487] [Search INSPIRE].

[56] T. Bae et al., PoS LATTICE 2013, 476 (2014) [arXiv:1310.7319 [hep-lat]] [Search INSPIRE].

[57] S. Aoki et al., Eur. Phys. J. C 74, 2890 (2014) [arXiv:1310.8555 [hep-lat]] [Search INSPIRE].

[58] Z. Ligeti and F. Sala, arXiv:1602.08494 [hep-ph] [Search INSPIRE].

[59] S. Bosch, A. J. Buras, M. Gorbahn, S. Jager, M. E. Lautenbacher, and L. Silvestrini, Nucl. Phys. B 565, 3 (2000) [arXiv:hep-ph/9904408] [Search INSPIRE].

[60] T. Blum et al., Phys. Rev. D 91, 074502 (2015) [arXiv:1502.00263 [hep-lat]] [Search INSPIRE].

[61] Z. Bai et al. [RBC and UKQCD Collaborations], Phys. Rev. Lett. 115, 212001 (2015) [arXiv:1505.07863 [hep-lat]] [Search INSPIRE].

[62] K. A. Olive et al. [Particle Data Group], Chin. Phys. C 38, 090001 (2014).

[63] C. Bobeth, M. Gorbahn, T. Herrmann, M. Misiak, E. Stamou, and M. Steinhauser, Phys. Rev. Lett. 112, 101801 (2014) [arXiv:1311.0903 [hep-ph]] [Search INSPIRE].

[64] G. Isidori and R. Unterdotterf, J. High Energy Phys. 01, 009 (2004) [arXiv:hep-ph/0311084] [Search INSPIRE].

[65] J. H. Park, Phys. Rev. D 83, 055015 (2011) [arXiv:1011.4939 [hep-ph]] [Search INSPIRE].

[66] M. Endo, S. Mishima, D. Ueda, and K. Yamamoto, arXiv:1608.01444 [hep-ph] [Search INSPIRE].

[67] Y. Grossman and Y. Nir, Phys. Lett. B 398, 163 (1997) [arXiv:hep-ph/9701313] [Search INSPIRE].

[68] A. J. Buras and J. G. Girrbach, Rept. Prog. Phys. 77, 086201 (2014) [arXiv:1306.3775 [hep-ph]] [Search INSPIRE].

[69] J. N. Butler et al. [Quark Flavor Physics Working Group Collaboration], arXiv:1311.1076 [hep-ex] [Search INSPIRE].

[70] M. Pospelov and A. Ritz, Phys. Rev. D 63, 073015 (2001) [arXiv:hep-ph/0010037] [Search INSPIRE].

[71] J. Hisano and Y. Shimizu, Phys. Lett. B 581, 224 (2004) [arXiv:hep-ph/0308255] [Search INSPIRE].

[72] J. Hisano and Y. Shimizu, Phys. Rev. D 70, 093001 (2004) [arXiv:hep-ph/0406091] [Search INSPIRE].

[73] J. Hisano, M. Nagai, and P. Paradisi, Phys. Rev. D 80, 095014 (2009) [arXiv:0812.4283 [hep-ph]] [Search INSPIRE].

[74] K. Fuyuto, J. Hisano, and N. Nagata, Phys. Rev. D 87, 054018 (2013) [arXiv:1211.5228 [hep-ph]] [Search INSPIRE].
[75] K. Fuyuto, J. Hisano, N. Nagata, and K. Tsumura, J. High Energy Phys. 1312, 010 (2013) [arXiv:1308.6493 [hep-ph]] [Search INSPIRE].

[76] B. C. Allanach et al., Comput. Phys. Commun. 180, 8 (2009) [arXiv:0801.0045 [hep-ph]] [Search INSPIRE].

[77] P. Z. Skands et al., J. High Energy Phys. 07, 036 (2004) [arXiv:hep-ph/0311123] [Search INSPIRE].

[78] T. Goto, http://research.kek.jp/people/tgoto/, date last accessed November 15, 2016.

[79] M. Gorbahn and U. Haisch, Phys. Rev. Lett. 97, 122002 (2006) [arXiv:hep-ph/0605203] [Search INSPIRE].