Singlet Couplings and (0,2) Models*

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We use the quantum symmetries present in string compactification on Landau-Ginzburg orbifolds to prove the existence of a large class of exactly marginal (0,2) deformations of (2,2) superconformal theories. Analogous methods apply to the more general (0,2) models introduced in [1], lending further credence to the fact that the corresponding Landau-Ginzburg models represent bona-fide (0,2) SCFTs. We also use the large symmetry groups which arise when the worldsheet superpotential is turned off to constrain the dependence of certain correlation functions on the untwisted moduli. This allows us to approach the problem of what happens when one tries to deform away from the Landau-Ginzburg point. In particular, we find that the masses and three-point couplings of the massless $E_6$ singlets related to $H^1(\text{End}(T))$ vanish at all points in the quintic Kähler moduli space. Putting these results together, and invoking some plausible dynamical assumptions about the corresponding linear $\sigma$-models, we show that one can deform these Landau-Ginzburg theories to arbitrary values of the Kähler moduli.

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1. Introduction

Conformal field theories with (0,2) worldsheet supersymmetry are of great interest because of their role in constructing string-based models of elementary particle physics with spacetime supersymmetry \([2]\). Calabi-Yau \(\sigma\)-models with the vacuum gauge connection identified with the spin connection actually give rise to (2,2) superconformal field theories \([3]\), and the moduli spaces of such solutions have been explored in great detail over the past several years \([4,5,6]\). Their (0,2) generalizations, which include Calabi-Yau \(\sigma\)-models with more general choices for the gauge field vacuum expectation value \([7]\) (at least to all orders in \(\sigma\)-model perturbation theory), have remained largely mysterious.

In some recent papers, techniques which allow one to study string compactifications on (0,2) supersymmetric Landau-Ginzburg orbifolds have been developed and exploited \([6,8,1]\). However, although it has been made plausible that the (0,2) Landau-Ginzburg models studied in \([6,8,1]\) do indeed represent classical solutions of string theory, no rigorous proof of their existence as conformal field theories has been supplied. Especially in view of the fact that generic (0,2) Calabi-Yau \(\sigma\)-models might be destabilized by worldsheet instantons \([9]\), one would like to have such a proof.

In \(\S\) 2, we prove that a large class of (0,2) deformations of (2,2) Landau-Ginzburg orbifolds have nontrivial infrared fixed points. This is accomplished by showing that they correspond to exactly flat directions in the spacetime superpotential of the (2,2) theory. Symmetry considerations analogous to those which were used in \([10]\) allow us to infer the existence of many such flat directions. The novelty is that we make use of the quantum R-symmetry which is characteristic of Landau-Ginzburg orbifolds \([11]\). As an example, we discuss the (0,2) moduli space of the quintic where, in addition to the 101 complex structure deformations, 200 extra \(E_6\) singlets are allowed to assume arbitrary expectation values. This confirms our intuition that the space of (2,2) Landau-Ginzburg orbifolds is but a small subspace of the space of (0,2) models.

In \(\S\) 4, we use considerations analogous to those of \(\S\) 2 in the case of (0,2) Landau-Ginzburg theories which are not obviously deformations of (2,2) theories. We cannot rigorously prove that such theories exist as conformal theories by the technique of \(\S\) 2, since we cannot choose an expansion point which we know to be conformal. However, by assuming that one (0,2) Landau-Ginzburg theory exists as a conformal theory, we will be able to prove that the neighboring (0,2) theories (obtained by changing the parameters in the Landau-Ginzburg superpotential) are also conformally invariant, i.e. that the parameters in the (0,2) superpotential do indeed correspond to flat directions in the spacetime
superpotential. This is all discussed in the context of a particular example studied in detail in [1], which at large radius corresponds to a (0,2) theory on a complete intersection Calabi-Yau manifold in $\mathbb{P}_{1,1,1,2,2}$.

Having found in §2 a large space of exactly marginal (0,2) deformations of the (2,2) Landau-Ginzburg theory, we would like to know to what extent it is possible to turn on the remaining $E_6$ singlets. In particular, we would like to know what, if anything, of this picture persists when we turn on the Kähler modulus. In §3, we use the approach of [12] to study the couplings of the twisted sector singlets (including the Kähler modulus) in the particular case of the quintic. We use the $SU(5) \times SU(5)$ symmetry which arises when one turns off the worldsheet superpotential to constrain the dependence of the superpotential for the twisted sector singlets on the untwisted moduli. This allows us to argue both that the 224 $E_6$ singlets related to $H^1(\text{End}(T))$ remain massless throughout the Kähler moduli space of the (2,2) quintic, and that the three-point couplings of these singlets also vanish. However, the four-point couplings of the 24 twisted sector singlets related to $H^1(\text{End}(T))$ are nonzero, indicating that they are not moduli. Our arguments are not powerful enough to prove the existence of new exactly flat directions at arbitrary radius, but we return to that question using other techniques in §5.

After providing strong evidence that the (0,2) Landau-Ginzburg models do indeed correspond to bona-fide CFTs in §2 and §4, we turn in §5 to the subject of (0,2) models at finite radius. Making some very plausible assumptions about the renormalization group flow of the (0,2) linear $\sigma$-models we have been studying, and restricting ourselves to models with 1 (complex) dimensional Kähler moduli spaces, we are able to prove that the large (0,2) moduli spaces we have found at the Landau-Ginzburg radius exist at all values of the Kähler modulus. For example, the full 102-dimensional (2,2) moduli space of the quintic is actually a submanifold of a 302 dimensional (0,2) moduli space. Our proof uses elementary ideas of Morse theory, applied to Zamolodchikov’s c-function [1].

In §6, we discuss some interesting questions which remain to be answered by future explorations of (0,2) moduli space.

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1 Morse theory and the c-function have also come together elsewhere in the string literature [13].
2. (0,2) Deformations

R-symmetries are particularly useful tools in establishing the existence of exactly flat directions in supersymmetric field theories. Consider a supersymmetric gauge theory with some collection of chiral fields $\Phi_1, \ldots, \Phi_M$ and suppose furthermore that this theory has an R-symmetry under which the superfields $\Phi_1, \ldots, \Phi_N$ ($N \leq M$) are invariant. Then since the spacetime superpotential $W$ is not invariant under the symmetry, no terms of the form $f(\Phi_1, \ldots, \Phi_N)$ can appear in the superpotential. If furthermore no terms of the form $f(\Phi_1, \ldots, \Phi_N)X$ can appear in the spacetime superpotential for $X = \Phi_{N+1}, \ldots, \Phi_M$, then the directions in field space corresponding to the scalar components of $\Phi_1, \ldots, \Phi_N$ are necessarily F-flat. Therefore, as long as the constraints of D-flatness are also satisfied, the VEVs of the scalar components of $\Phi_1, \ldots, \Phi_N$ parametrize a space of degenerate ground states for the supersymmetric theory in question.

In conformal perturbation theory, one needs to examine certain correlation functions of the zero-momentum vertex operators corresponding to the fields $\Phi_i$ in order to show that they correspond to exactly marginal deformation of the worldsheet superconformal field theory (i.e. that they preserve superconformal invariance). But this is exactly the same condition as demanding the spacetime equations of motion be satisfied. Here is where spacetime supersymmetry comes to our aid. We only need to check F- and D-flatness to assure ourselves that the spacetime equations of motion are satisfied, and this involves examining a much smaller and more tractable set of worldsheet correlation functions than would be the case without spacetime supersymmetry. In fact, if the fields $\Phi_i$ are gauge-singlets, D-flatness is automatic, so we only need to check F-flatness.

It is well known that many (2,2) Calabi-Yau compactifications possess, at special points in their complex structure moduli space $M_C$, extra “classical” discrete R-symmetries (which are essentially symmetries of the defining equations of the Calabi-Yau in some ambient weighted projective space) [14]. In the context of string compactification on the quintic, these R-symmetries have been used to prove the existence of exactly flat (0,2) directions at special points in $M_C$. In particular, since conformal perturbation theory about an interior point in (2,2) moduli space does not miss any non-perturbative $\sigma$-model effects, demonstrating the existence of a flat (0,2) direction by such macroscopic reasoning guarantees that the corresponding (0,2) theories are not destabilized by worldsheet instantons [10].

The Landau-Ginzburg orbifolds are distinguished by the fact that they all possess at least one discrete R-symmetry, namely the “quantum” symmetry which counts the twisted
sector $k$ of origin of the various physical states. The VEVs of the massless gauge-singlet fields which arise in the untwisted sector and are uncharged under the quantum symmetry are therefore guaranteed to be moduli of the spacetime supersymmetric field theory, and on the string worldsheet these fields will be represented by mutually integrable moduli of the conformal field theory. The massless singlets which arise in the untwisted sector of the (2,2) Landau-Ginzburg theories typically include many $E_6$ singlet fields which are related to neither complex structure nor Kähler structure deformations— at large radius, these modes are related to the cohomology group $H^1(\text{End}(T))$. Giving VEVs to these fields breaks the (2,2) worldsheet supersymmetry to (0,2) supersymmetry.

For concreteness, let us focus attention on the quintic hypersurface in $\mathbb{C}P^4$. The Landau-Ginzburg theory is a point of enhanced symmetry in the Kähler moduli space. One normally says that the Landau-Ginzburg orbifold has a $\mathbb{Z}_5$ quantum symmetry, but since one needs to include both NS and R sectors for the left-movers, there are actually 10 sectors in the Landau-Ginzburg orbifold. So one might better think of the quantum symmetry as $\mathbb{Z}_{10} = \mathbb{Z}_2 \times \mathbb{Z}_5$. Actually, this definition of the quantum symmetry is a little awkward because the different components, under the decomposition $E_6 \supset SO(10) \times U(1)$, of a given $E_6$ representation transform with different weight under this $\mathbb{Z}_{10}$ symmetry. To fix this, we can compose this symmetry with an element of the center of $E_6$, to obtain a $\mathbb{Z}_{30} = \mathbb{Z}_3 \times \mathbb{Z}_{10}$ symmetry which acts homogeneously on $E_6$ multiplets. In the language of $\mathbb{R}$, this $\mathbb{Z}_{30}$ is generated by

$$S_Q = e^{2\pi i (3k-2q)/30}$$

(2.1)

where $k = 0, \ldots, 9$ labels the sector number of the Landau-Ginzburg orbifold, and $q$ is the left-moving $U(1)$ charge. The charges of the various massless multiplets under $S_Q$ are listed in Table 1 as integers $\in \mathbb{Z}/30\mathbb{Z}$.

| $S_Q$ | 27 | \overline{27} | C, S | R | S' | $\mathcal{W}$ |
|------|----|-------------|-----|---|---|------|
|      | -2 | 8           | 0   | 6 | 6 | -6   |

**Table 1:** Charges ($\in \mathbb{Z}/30\mathbb{Z}$) of the spacetime matter multiplets, and of the spacetime superpotential, $\mathcal{W}$, under $S_Q$, the “quantum” R-symmetry present at the Landau-Ginzburg point.

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2 The states found in \cite{8} were the massless spacetime fermions; for a fermion which comes from the $(k+1)$st twisted sector, its scalar superpartner comes from the $k$th twisted sector.
On the world sheet, $S_Q$ simply enforces the fact that sector number is conserved mod 10 in correlation functions. It is easy to see that, in spacetime, $S_Q$ generates a discrete R-symmetry, under which the spacetime superpotential has charge $-6 \text{ mod } 30$. That is, one should add 3 to the entries in Table 1 to obtain the $S_Q$-charge of the corresponding right-handed fermions in these chiral multiplets. Clearly $27^3$ and $\overline{27}^3$ are couplings in the superpotential allowed by the discrete $R$-symmetry, whereas, say, $27^2 \overline{27}^2$ is not.

In Table 1, we have divided the 224 singlets corresponding to elements of $H^1(\text{End} (T))$ into the 200, denoted by $S$, which arise in the untwisted sector of the Landau-Ginzburg orbifold, and the 24, denoted by $S'$, which arise in the $k = 2$ twisted sector.

Let us recall how this distinction arises [8]. The $H^1(\text{End} (T))$ singlets can be identified with operators of the form

$$S = \lambda^i P_i(\phi)$$

where $P_i(\phi)$ are a set of five quartics satisfying

$$\phi^i P_i(\phi) = 0 .$$

There are $5 \times 70 - 126 = 224$ operators (2.2) satisfying (2.3). However, in the Landau-Ginzburg theory, precisely 24 of these are $\overline{Q} +$-trivial. Namely, we need to mod out the polynomials satisfying (2.3) by the equivalence relation ($W(\phi)$ is the (2,2) worldsheet superpotential)

$$P_i(\phi) \sim P_i(\phi) + A_{i}^{j} \partial_j W - \frac{1}{5} \delta_{i}^{j} A_{j}^{k} \partial_{k} W$$

for an arbitrary traceless matrix $A_{i}^{j}$.

The 24 singlets from the twisted sector which “replace” the missing singlets from the untwisted sector take the form

$$S'_{ij} = (\overline{\lambda}^i_{-3/10} \phi^j_{-1/5} - \frac{1}{5} \delta^{ij} \overline{\lambda}_{-3/10} \cdot \phi_{-1/5}) S_2$$

where $S_2$ is the field that creates the ground state of the twisted sector.

Let us now see what restrictions on the $S$-dependence of the spacetime superpotential are imposed by this quantum R-symmetry. First of all, since both the $C$s and the $S$s are neutral under $S_Q$, and a term in $W$ must have charge $-6 \text{ mod } 30$, we see that, at the Landau-Ginzburg point, no term in the superpotential of the form $W = f(C, S)X + \ldots$, where $X$ is any singlet, is allowed. Thus, at the Landau-Ginzburg point, for an arbitrary complex structure, all 200 $S$s correspond to flat directions in the superpotential.
In the conformal field theory, these are exactly marginal operators, which break \((2,2)\) superconformal symmetry to \((0,2)\), while preserving \(E_6\) as the spacetime gauge group.

We should note at the same time that \(\mathcal{W} = 27 \overline{27} S^n + \ldots\) is also forbidden, so that turning on \(S\) does not cause the \(\overline{27}\) and a \(27\) to pair up and get a mass.

At the Landau-Ginzburg point in its Kähler moduli space, we have found that the quintic has, at least, a 301 dimensional \((0,2)\) moduli space of \(E_6\) preserving \((0,2)\) deformations. Two questions naturally arise:

- Do these \((0,2)\) deformations remain exactly marginal when we turn on the Kähler modulus?
- Are any of the 24 singlets, \(S'\), which occur in the same twisted sector as the Kähler modulus, mutually integrable with the deformations we have found?

We will address these questions in the next section. It will turn out that the answer to the second question is no. The \(S'\) are charged under the quantum R-symmetry, so it is possible for the to have a nontrivial superpotential, spoiling their flatness. The quantum R-symmetry dictates that the lowest possible term is quartic, \(\mathcal{W} = S'^4 + \ldots\)

The answer to the first question is likely, yes, but we will not see that until §5. What would it mean if the answer to the first question turned out to be no? It would imply that the Landau-Ginzburg theory is a multicritical “point” in the \((0,2)\) moduli space \(\mathcal{M}_{(0,2)}\). \(\mathcal{M}_{(0,2)}\) would consist of two components – a 102-dimensional space of \((2,2)\) symmetric theories, and a 301-dimensional space of what are generically \((0,2)\) symmetric theories – which meet along the 101-dimensional locus of \((2,2)\) symmetric Landau-Ginzburg theories. This is schematically depicted in Fig. 1.

![Fig. 1: Schematic picture of the moduli space of the quintic, showing the intersection of \(\mathcal{M}_{(2,2)}\) and \(\mathcal{M}_{(0,2)}\) along the locus of \((2,2)\) Landau-Ginzburg theories.](image)
One might ask what this picture looks like under mirror symmetry. The Landau-Ginzburg locus in the Kähler moduli space is simply the locus $\psi = 0$ in the complex structure moduli space of the mirror quintic, where the polynomial is of Fermat form. The 200 singlets in question all arise in blowing up the singularities of the mirror, as do 100 of the 101 Kähler moduli. All of these are mutually-integrable marginal perturbations. So we see that, for the Fermat form of the quintic mirror, there is a 200 parameter family of $(0,2)$ deformations at arbitrary radius!

One intriguing possibility – not realized in this example – is that a generic $(0,2)$ compactification might freeze the radius at some Planckian value. This is what would seem to happen when the theory is formulated on the original quintic, if the answer to the first question was no: The theory would seem to be stuck at the Landau-Ginzburg radius. However, in the mirror picture, it is clear that there are still some directions in $\mathcal{M}_{(0,2)}$ which ought to be interpreted as “large radius”.

3. Twisted Sector Singlet Couplings

Consider a general point in the 301 dimensional moduli space of $(0,2)$ Landau-Ginzburg theories that we have found on the quintic. The worldsheet superpotential can be written in $(0,2)$ superspace as

$$\int d^2zd\theta \Lambda^a F_a(\Phi) = \int d^2 zd\theta F_{aijkl} \Lambda^a \Phi^i \Phi^j \Phi^k \Phi^l$$

If we neglect the superpotential, the theory possesses an $SU(5) \times SU(5)$ symmetry under which the $\Lambda$s and the $\Phi$s rotate independently.

Properly speaking, we should also include a $U(1) \times U(1)$ phase symmetry as well. One of these $U(1)$s is generated by our old friend $q$, and doesn’t teach us anything new. Unfortunately, the peculiar quantization [15] of the zero mode of the scalar field (which is noncompact when we turn off the $F$s) spoils the conservation of the remaining $U(1)$ (even when we neglect the explicit breaking by the $F$s). So, in the end, the only new symmetry we have to exploit is $SU(5) \times SU(5)$.

The coupling constants $F_{aijkl}$ break this symmetry explicitly, transforming as the $(\overline{5}, 70')$ representation. Since this is the only source of $SU(5) \times SU(5)$-breaking in the theory, we will be able to constrain the dependence on the $F$s of various correlation functions of the massless multiplets by demanding that they transform correctly under $SU(5) \times SU(5)$. 

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We will be particularly interested in the couplings of the 25 singlets from the twisted sector. We denote them collectively by

\[ S'^a \phi^i S \]

and clearly they transform as the \((\bar{5}, 5)\) representation.

We saw in the previous section that the quantum symmetry dictates that the lowest nonvanishing term in the superpotential for the \(S'\)s is at least quartic. So we will be interested in computing an \(\langle S'^4 \rangle\) coupling.

Before we launch in, however, there is clearly a subtlety we must deal with. The spacetime superpotential is a section of a line bundle over the moduli space (that is, over the space of \(\mathcal{F}\)s). We therefore need to supply some trivialization of that line bundle in order to specify it. There is no obvious candidate for such a trivialization, even locally on the space of the \(\mathcal{F}\)s.

To evade this ambiguity, we will simply note that ratios of superpotential couplings transform as sections of a trivial line bundle, and so are canonically-defined (up to scale) as functions on the moduli space. Since we are interested in exploiting the \(SU(5) \times SU(5)\) transformation properties, a natural candidate to normalize our correlation functions is the \(\langle \overline{27}^3 \rangle\) coupling, which is, after all, an \(SU(5) \times SU(5)\) singlet. So we will denote

\[ \langle\langle S'^4 \rangle\rangle = \frac{\langle S'^4 \rangle}{\langle \overline{27} \rangle} \]  

(3.1)

The somewhat attentive reader might wonder how the statements of this paragraph are to be reconciled with the oft-repeated statement that “the \(\overline{27}^3\) coupling is independent of the complex structure moduli” \[16\]. In the notation of \[17\], the spacetime superpotential is a section of the line bundle \(\tilde{L}^3\). The \(\overline{27}\)s are sections of \(T_{\mathcal{M}_{R}} \otimes L\), the tangent bundle to the Kähler moduli space, twisted by the line bundle \(L\). So the cubic coupling is a linear map from \(S^3(T_{\mathcal{M}_{R}} \otimes L) \rightarrow \tilde{L}^3\). In other words, it is a section of \(S^3(T_{\mathcal{M}_{R}}^* \otimes \mathcal{L}^{-1} \otimes \overline{\mathcal{L}}^3)\). But, restricted to the complex structure moduli space, \(T_{\mathcal{M}_{R}}\) and \(\mathcal{L}^{-1} \otimes \overline{\mathcal{L}}\) are trivial bundles. Hence it makes sense to say that the coupling, a section of a trivial bundle, is a constant.

A related point is that we need to pin down the ambiguity in the \(\mathcal{F}\)-dependence of the normalization of the operator \(S_2\) (and the corresponding fermion state \(|3\rangle\)) which go into defining the vertex operators for the \(S'\)s. But the \(\overline{27}^3\) coupling can be represented, for instance, as the matrix element \(\langle 3 \mid S_4 \mid 3 \rangle\). So if we define the relative normalization to be such that \(S_4\) appears with unit coefficient in the OPE of two \(S_2\)s then all ambiguity in the normalization of these operators disappears from the ratio (3.1).
Our task is to determine the dependence of this correlation function on the $F$s. Clearly, $SU(5) \times SU(5)$ symmetry is not going to be enough to determine the complete dependence for us. It is possible to write a polynomial in the $F$s (the lowest degree of such a polynomial is 10) which is an $SU(5) \times SU(5)$ singlet. Our determination of the correlation function is therefore ambiguous up to multiplication by an arbitrary $SU(5) \times SU(5)$ singlet function of the $F$s.

Modulo this ambiguity, we can still place some powerful constraints on the $SU(5) \times SU(5)$ nonsinglet part of the $F$ dependence.

1) Analyticity. The spacetime superpotential depends holomorphically on the moduli, and so is a function of the $F$s, but not the $\bar{F}$s.

2) Quintality. The correlation function (3.1) transforms as the 4th symmetric power of $(\bar{5}, 5)$. By quintality, a polynomial in the $F$s (which, recall, are in the $(\bar{5}, 70')$ representation) which transforms in this representation must have degree $4 + 5n$.

3) Flatness of the $(2,2)$ moduli. Recall that the Kähler modulus $R = \delta_{\bar{a}i} S'^{\bar{a}i}$. Under the diagonal $SU(5) \subset SU(5) \times SU(5)$, the $F_{aijkl}$ transform as $126' \oplus 224$. The $(2,2)$ theory is obtained by setting to zero the $224$ piece, and considering $F_{aijkl}$ which are totally symmetric on their five indices. Since $R$ is indeed a modulus of the $(2,2)$ theory, $\langle \langle R^4 \rangle \rangle = \delta_{\bar{a}_1i_1} \cdots \delta_{\bar{a}_4i_4} \langle \langle S'^{\bar{a}_1i_1} \cdots S'^{\bar{a}_4i_4} \rangle \rangle$ must vanish when we set the $F$ to their $(2,2)$ symmetric values.

4) Flatness of “twisted (2,2)” moduli. The above case corresponded to choosing polynomials $F_a(\phi) = \partial_a W(\phi)$.

However, the most general form for the $F$s which preserves a (2,2) supersymmetry is $F_a(\phi) = ((U^T)^{-1})^i_a \partial_i W(V^{-1} \phi)$

for some invertible matrices $U, V$. When $U = V$, this is just a harmless $GL(5)$ transform of (3.2), and the left-moving supersymmetry is the standard one. When $U^{-1} = V^5$, this is the global form of the deformations in $H^1(End(T))$ in the ideal (2.4). Expanding $U = 1 - A T$, $V = 1 + \frac{1}{5} A T$, we find that $F_a(\phi)$ is a deformation of $\partial_a W(\phi)$ by an element of the ideal (2.4). This is a $\bar{Q}_+$-trivial deformation, but it forces us to redefine the left-moving supersymmetries. Instead of $G^+ = \delta_{\bar{a}i} \left( -\frac{4}{5} \bar{\lambda}^a \partial \phi^i + \frac{1}{5} \partial \bar{\lambda}^a \phi^i \right)$ we have $G^+ = \delta_{\bar{a}a} (UV^{-1})^a_i \left( -\frac{4}{5} \bar{\lambda}^a \partial \phi^i + \frac{1}{5} \partial \bar{\lambda}^a \phi^i \right)$. (3.4)
Naturally, then, we should redefine the Kähler modulus to be
\[
R = \delta_{\bar{a}a} (UV^{-1})^{\bar{a}i} \bar{\lambda}^a \phi^i S_2
\]  
(3.5)

Contracting the $S^I$ correlation function with $\delta_{\bar{a}_1a_1} (UV^{-1})^{a_1 i_1} \ldots$, we must find that the redefined $R^4$ coupling vanishes for $F$s of the form (3.3).

5) In addition to the $R^4$ coupling, the $\langle \langle S^{\bar{a}_i} R^3 \rangle \rangle$ coupling must also vanish. This is clear, since we know that the deformed theory is conformal, so the 1-point functions (in this case, of $S'$) vanish.

6) The correlation function (3.1) should transform covariantly under the $G = (GL(5) \times GL(5))/\mathbb{C}^*$ action (3.3). It is convenient to fix the $\mathbb{C}^*$ symmetry by choosing the gauge
\[
\det(U^{-1} V) = 1
\]

Then, under the $G$ action,
\[
\langle \langle S^{|\bar{a}_1 i_1} \ldots S^{|\bar{a}_4 i_4} \rangle \rangle \to (\delta_{\bar{a}_1 a_1} (U^T)^{-1})^{a_1 b_1} \delta_{b_1 b_1} \ldots (\delta_{\bar{a}_4 a_4} (U^T)^{-1})^{a_4 b_4} \delta_{b_4 b_4} \times \\
\times (V)^{i_1 j_1} \ldots (V)^{i_4 j_4} \langle \langle S^{b_1 j_1} \ldots S^{b_4 j_4} \rangle \rangle
\]  
(3.6)

Together, these are quite stringent constraints. Up to an $SU(5) \times SU(5)$ singlet function of the $F$s, $f(F)$, we obtain
\[
\langle \langle S^{\bar{a}_1 i_1} S^{\bar{a}_2 i_2} S^{\bar{a}_3 i_3} S^{\bar{a}_4 i_4} \rangle \rangle = \delta_{\bar{a}_1 a_1} \delta_{\bar{a}_2 a_2} \delta_{\bar{a}_3 a_3} \delta_{\bar{a}_4 a_4} \times \\
\times \epsilon_{i_1 j_1 k_1 l_1 m_1} \epsilon_{i_2 j_2 k_2 l_2 m_2} \epsilon_{i_3 j_3 k_3 l_3 m_3} \epsilon_{i_4 j_4 k_4 l_4 m_4} \times \\
\times F_{a_1 j_1 k_2 l_1 m_1} F_{a_2 k_2 k_3 l_2 m_2} F_{a_3 l_3 l_4 m_3} F_{a_4 m_1 m_2 m_3 m_4} f(F)
\]  
(3.7)

From (3.3), we learn that, under the $G$ action, $f(F) \to \det(V)^4 f(F)$. An example of an invariant function which transforms with this weight is
\[
f(F) = (\epsilon^{a_1 \ldots a_5} \epsilon^{a_6 \ldots a_10} \epsilon^{i_2 \ldots i_6} \epsilon^{i_7 \ldots i_{10} i_1} \ldots \epsilon^{l_5 \ldots l_9} \epsilon^{l_{10} l_1 \ldots l_4} \times \\
\times F_{a_1 i_1 j_1 k_1 l_1} \ldots F_{a_{10} i_{10} j_{10} k_{10} l_{10}})^{-2/5}
\]

More generally, we can consider a $(4 + 5n)$-point function,
\[
\langle \langle S^{\bar{a}_1 i_1} S^{\bar{a}_2 i_2} S^{\bar{a}_3 i_3} S^{\bar{a}_4 i_4} R^{5n} \rangle \rangle
\]  
(3.8)

Note that the Kähler modulus $R$, being the tangent vector to the Kähler moduli space, can be normalized in a fashion independent of the complex structure. That is, the corresponding $(0)$-picture vertex operator can be defined to be independent of the $F$s. All of
the conditions 1)–6) generalize to (3.8). Thus, up to, perhaps, a different choice of the function \( f(F) \), (3.8) must have exactly the same form as the right hand side of (3.7).

Certainly, we haven’t proven that (3.7) is the only \( SU(5) \times SU(5) \) structure that can occur. However, we have not been able to find a structure at higher orders which satisfied conditions 1)-6) and could not be reduced to the form (3.7). Perhaps, at sufficiently high order in the \( F \)s, one exists, but we have not been able to find it.

In any case, the striking feature of (3.7) is that, not only does it vanish when three or four of the \( S' \)s are in fact Rs, but it vanishes when any of the \( S' \)s are Rs. Thus the zero, one, two and three point functions of the \( S' \)s vanish at arbitrary values of the Kähler modulus! Even if (3.7) turns out not to be unique and there exist higher order invariants not reducible to (3.7), it is very likely that those invariants share this property.

One can go further than this. Differentiating (3.7) with respect to the \( F \)s, and then setting them equal to their (2,2)-symmetric values has the effect of inserting untwisted sector singlets (\( S \)s, or \( C \)s) into the correlation function. Schematically,

\[
\partial_F \langle \langle S'^4 R^{5n} \rangle \rangle = \langle \langle SS'^4 R^{5n} \rangle \rangle - \langle \langle S'^4 R^{5n} \rangle \rangle \langle \langle S^2 T^{27} \rangle \rangle
\]

We can contract indices appropriately to turn some of these \( S' \)s into \( R \)s. Having shown that the zero, one, two and three point functions of the \( S' \)s vanish, we see, from explicitly differentiating the RHS of (3.7), that the zero, one, two and three point functions of any combination of \( S' \)s and \( S \)s also vanish. In particular, we have 224 massless \( H^1(\text{End}(T)) \) singlets at an arbitrary point in the Kähler moduli space.

The lowest nonvanishing singlet couplings in the (2,2) Landau-Ginzburg theory, consistent with both the quantum symmetry and (3.7) are: \( S'^4 R^{4+5n} \), \( S^3 S'^3 R^{3+5n} \), \( S'^2 S'^2 R^{2+5n} \), \( SS'^3 R^{1+5n} \), and \( S'^4 R^{5n} \). At finite \( R \), these are all quartic couplings among the \( H^1(\text{End}(T)) \) singlets, there being no invariant distinction between the \( S \)s and \( S' \)s at finite \( R \). So the general statement is that the superpotential for the \( H^1(\text{End}(T)) \) singlets starts at quartic order at an arbitrary point in the Kähler moduli space of the (2,2) theory.

We saw in the previous section that there is, at least, a 200-dimensional space of \( E_6 \)-preserving (0,2) deformations of the quintic Landau-Ginzburg theory. The four point function (3.7) is the obstruction to extending this further to include the 24 singlets in the twisted sector. It is not a terribly difficult calculation to compute this obstruction explicitly in, say, the Gepner model.
The correlation function one wants to evaluate is \( \langle S'_{15} S'_{25} S'_{35} S'_{45} \rangle \). The \((-1)\)-picture vertex operator for \( S'_{15} \) is given in the tensor product of minimal models by the operator \((0^{-2}_{0} 0^{-2}_{0}) (1^{-1}_{1} 0^{-1}_{0})^{3} (2^{0}_{0} 0^{0}_{0})\), and similarly for the other \( S'_{45} \). We need to shift two of these operators by \((0^{0}_{0} 0^{0}_{0})^{5}\) to produce the corresponding \((-1/2)\)-picture fermion vertex operators, and we need to shift one of the remaining vertex operators by \((0^{0}_{0} 0^{0}_{0})^{4} (0^{0}_{0} 0^{0}_{2})\) to produce the \((0)\)-picture vertex operator which is to be integrated over the worldsheet. Even without explicitly calculating the integrated 4-point function, we can readily see that the N=2 minimal model fusion rules are compatible with its being nonzero.

The hard question, which we have not been able to address, is how many \((0,2)\) directions are obstructed at finite \( R \)? At the Landau-Ginzburg point, it is the 24 \( S' \) that are obstructed, whereas the 200 \( S \)s are unobstructed. When we turn on the Kähler modulus, the distinction between the \( S \)s and \( S' \)s is effaced, and some combinations of these 224 singlets are obstructed. Perhaps all of them are, if not by this term, then by higher terms in the superpotential which we have not yet considered. We will see in §5 that this is very likely not the case, and that, in fact, 200 of them remain unobstructed. However, it is clear from the computations of this section that precisely \textit{which} combinations of singlets are unobstructed is a complicated function of both the Kähler and complex structure moduli.

4. More General \((0,2)\) Theories

We have seen that the quantum R-symmetry of \((2,2)\) Landau-Ginzburg models is enough to guarantee that, in many cases, the \((2,2)\) moduli space is a small part of a much larger \((0,2)\) moduli space. What can \( S_Q \) do for us in the context of \((0,2)\) Landau-Ginzburg theories that are not obviously obtained as deformations of \((2,2)\) theories \([1]\)?

The Landau-Ginzburg theories discussed in \([1]\) were described by a \((0,2)\) worldsheet superpotential of the form

\[
\int d^{2}z d\theta \sum W_{j}(\Phi) + \Lambda^{a} F_{a}(\Phi)
\]  

(4.1)

defined in such a way that the theory possesses both a U(1) symmetry with charge \( q \), and a U(1) R-symmetry with charge \( \overline{q} \), where the charges of the fields are given in Table 2.
Table 2: $U(1)$ and $U(1)_R$ Charges of the (0,2) Landau-Ginzburg superfields.

| $\Phi_i$ | $q_i$ | $\bar{q}_i$ |
|----------|-------|-------------|
| $\Lambda^a$ | $q_a - 1$ | $q_a$ |
| $\Sigma^j$ | $q_j - 1$ | $q_j$ |

The charges are constrained to satisfy

$$\sum_{j=1}^n (q_j - 1) = - \sum_{i=1}^{n+D+1} q_i$$

$$\sum_{a=1}^{r+1} q_a = 1$$

$$\sum (q_j - 1)^2 + \sum q_a^2 = 1 + \sum q_i^2$$

where $D = 3$ for Calabi-Yau threefolds. In the infrared, the $U(1)$ current $\bar{J}$, associated to $\Psi$, becomes the $U(1)$ current in the (0,2) superconformal algebra. We can read off the infrared central charge from the $\bar{J}$-$J$ anomaly. The constraints (4.2) ensure that this gives

$$\bar{c}/3 = \sum (q_i - 1)^2 - \sum q_a^2 - \sum q_j^2 = D .$$

Similarly, $J$, the $U(1)$ current associated to $q$, generates a left-moving $U(1)$ current algebra in the infrared, whose central extension can be read off from the $J$-$\bar{J}$ anomaly:

$$r = \sum (q_j - 1)^2 + \sum (q_a - 1)^2 - \sum q_i^2 .$$

And, of course, consistency requires that the $J$-$\bar{J}$ anomaly vanish:

$$0 = \sum (q_i - 1)q_i - \sum q_j(q_j - 1) - \sum q_a(q_a - 1)$$

which, again, is assured by (4.2).

In fact, the situation is even better than that. Even in the off-criticality theory, the operators

$$T'(z) = - \sum_i \left( \partial \phi_i \partial \tilde{\phi}_i + \frac{q_i}{2} \partial (\phi_i \partial \tilde{\phi}_i) \right) + \sum_a \left( \lambda_a \partial \bar{\lambda}_a - \frac{1 - q_a}{2} \partial (\lambda_a \bar{\lambda}_a) \right)$$

$$+ \sum_j \left( \sigma_j \partial \bar{\sigma}_j - \frac{1 - q_j}{2} \partial (\sigma_j \bar{\sigma}_j) \right),$$

$$J'(z) = - \sum_i q_i \phi_i \partial \tilde{\phi}_i + \sum_a (1 - q_a) \lambda_a \bar{\lambda}_a + \sum_j (1 - q_j) \sigma_j \bar{\sigma}_j$$

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commute with the right-moving supersymmetry generator $\bar{Q}_+$, and generate a Virasoro$\times\hat{U}(1)$ algebra on the $\bar{Q}_+$-cohomology. This algebra coincides with the left-moving chiral algebra in the infrared, and one can again compute, using the free field methods of [13], the Virasoro central charge ($c = 6 + r$), and the $\hat{U}(1)$ central charge ($r$), in agreement with above.

Finally, to embed this theory in a heterotic string theory, we need to orbifold the Landau-Ginzburg theory (4.1) by the $\mathbb{Z}_2$ group generated by $e^{-i\pi q} \times (-1)^{F_f}$, where $F_f$ is the fermion number for a set of $16 - 2r$ free left-moving Majorana-Weyl fermions which represent the gauge degrees of freedom [8]. Here $m$ is the least common denominator of all the charges in Table 2.

But all of these calculations assume that (0,2) supersymmetry is unbroken in the infrared limit. If (0,2) supersymmetry is spontaneously broken in the infrared, then all bets are off. Since we don’t have as firm a grasp of the dynamics of the theory (4.1) as we do of the more familiar (2,2) Landau-Ginzburg theories, any consistency checks that we can apply should bolster our confidence that (0,2) supersymmetry is indeed unbroken in the infrared, and that the Landau-Ginzburg orbifold is a bona-fide string vacuum.

One such consistency check is provided by the quantum R-symmetry. Assume that one point in the moduli space of the (0,2) Landau-Ginzburg theory does exist as a (0,2) conformal theory. As in the (2,2) case discussed in §2, this (0,2) Landau-Ginzburg orbifold will have a quantum R-symmetry generated on the worldsheet by

$$S_Q = e^{2\pi i(kr-2q)/2mr}$$

(4.4)

where $k = 0, 1, \ldots, 2m - 1$ is the sector number, and $r$ is the “rank of the vacuum gauge bundle” – $r = 3, 4, 5$ for spacetime gauge groups $E_6$, $SO(10)$, and $SU(5)$. One finds, as in the (2,2) case, that the R-symmetry guarantees that all of the singlets corresponding to $\bar{Q}_+$-nontrivial deformations of the (0,2) superpotential (4.1) are indeed flat directions. They all are neutral under the quantum R-symmetry because they all come from the untwisted sector of the Landau-Ginzburg orbifold. So assuming that one point in the moduli space of the (0,2) Landau-Ginzburg theory exists, we are able to prove that all of the $\bar{Q}_+$ non-trivial parameters in the worldsheet superpotential correspond to moduli of

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5 That this algebra is satisfied on the quantum level requires that the conditions (4.2) hold [18].
the theory. This is a non-trivial self-consistency check on the assumption that these (0,2) Landau-Ginzburg theories have infrared fixed points with the desired properties.

For example, consider the model discussed in detail in §4.1 of [1] (the $Y_{W;4,4}$ model listed in [19]). The Calabi-Yau $\sigma$-model description of this theory consists of a rank 4 vacuum gauge bundle over a complete intersection manifold in $W^{\mathbb{P}_5_{1,1,1,1,2,2}}$ defined by the vanishing loci of two degree four polynomials. Therefore, this theory yields an $SO(10)$ observable gauge group in spacetime. At the Landau-Ginzburg point in its Kähler moduli space, it has a $\mathbb{Z}_{10}$ quantum symmetry (like the quintic). However, this definition of the quantum symmetry is somewhat awkward because the different components of a given $SO(10)$ representation, under the decomposition $SO(10) \supset SO(8) \times U(1)$, transform with different weight under the $\mathbb{Z}_{10}$ symmetry. We can fix this as we did in the case of the quintic, by multiplying by an element of the center of $SO(10)$, to obtain a $\mathbb{Z}_{20}$ symmetry which acts homogeneously on $SO(10)$ multiplets. This $\mathbb{Z}_{20}$ is generated by

$$S_Q = e^{2\pi i \frac{2k-q}{20}}$$  \hspace{1cm} (4.5)

where $k = 0, \ldots, 9$ labels the sector number of the Landau-Ginzburg orbifold and $q$ is the left-moving $U(1)$ charge. The charges of the various multiplets under $S_Q$ are listed in Table 3 as integers $\in \mathbb{Z}/20\mathbb{Z}$.

| $S_Q$ | $16$ | $10$ | $10'$ | $S$ | $S'$ | $\mathcal{W}$ |
|-------|------|------|------|-----|------|---------|
|       | $-1$ | $-2$ | $6$  | $0$ | $4$  | $-4$    |

**Table 3:** Charges ($\in \mathbb{Z}/20\mathbb{Z}$) of the spacetime matter multiplets, and of the spacetime superpotential, $\mathcal{W}$, under $S_Q$, the “quantum” R-symmetry present at the Landau-Ginzburg point.

80 16s of $SO(10)$, 72 10s and 318 gauge singlets $S$ arise in the $k = 0$ sector of the Landau-Ginzburg theory (for generic choices of the defining data). There are also 21 singlets $S'$ which arise in the $k = 2$ twisted sector, and 2 10s of $SO(10)$ which arise in the $k = 4$ sector and are denoted as $10'$ in Table 3. The detailed forms of the corresponding states can be found in [1] §4.1, and will not be important in what follows.

One sees immediately that the quantum symmetry (4.3) guarantees that no terms of the form $f(S)$ or $f(S)S'$ (where $f$ is an arbitrary function of the 318 untwisted singlets) can appear in the spacetime superpotential $\mathcal{W}$. Therefore, as in §2, one is guaranteed that the corresponding 318 vertex operators are mutually integrable moduli of the (0,2) theory.
Of course, we had to assume that one point in the moduli space of this (0,2) Landau-Ginzburg theory existed as a conformal theory to run this argument. Making this assumption, we have proved that an entire 318 dimensional moduli space of (0,2) Landau-Ginzburg theories exists. The 318 singlets $S$ correspond to the $\bar{Q}_+$ nontrivial deformations of the Landau-Ginzburg superpotential.

We should go on at this point to analyse the spacetime superpotential for the twisted-sector singlets $S'$, to see if there is a flat direction which we can interpret as moving away from the Landau-Ginzburg point in Kähler moduli space. The analysis is, unfortunately, somewhat more complicated than the case of the quintic. The flavour symmetry group is $SU(7) \times SU(4) \times SU(2)$, and the polynomial coefficients lie in three different irreducible representations of this group. We hope to present this analysis elsewhere.

5. The Renormalization Group

The conclusions that we have been able to draw so far may seem a little anæmic. Using the quantum symmetry, we have been able to establish the self-consistency of the (0,2) Landau-Ginzburg theory, so we can be fairly confident that the Landau-Ginzburg theory, and the theory at infinite radius, are (0,2) superconformal in the infrared. Moreover, these theories are clearly distinct. The former has a discrete spectrum, whereas the latter has a continuous spectrum of states when quantized on the circle.

We have not, however, gotten very far in showing that these theories remain superconformal as we deform in the Kähler modulus. Nevertheless, with some plausible dynamical assumptions about the behaviour of the linear $\sigma$-models, we have enough information, for the simple case of models with a single Kähler parameter, to prove that the whole phase diagram of the linear $\sigma$-model is superconformal.

In the linear $\sigma$-model, the worldsheet superpotential is unrenormalized, even nonperturbatively. The issue which we need to grapple with is the renormalization of the coefficient of the Fayet-Iliopoulos D-term in the linear $\sigma$-model action. This coefficient, in the infrared limit, is nothing other than the Kähler parameter.

Demanding that the one loop divergence vanish imposes a condition on the sum of the scalar charges in the model, which is easily satisfied in the models of interest. Beyond

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6 The simplest way to see this is to note that the flavour symmetries exploited in §3 forbid any nontrivial renormalization of the $F$s.
one loop, there is a nonrenormalization theorem which says that the Fayet-Iliopoulos D-term is unrenormalized to all orders in perturbation theory. What we need to worry about is whether the D-term is renormalized nonperturbatively, say by gauge instantons. If this happens, we might find ourselves in a situation where the running coupling \( r(\mu) \) runs off to infinity as we flow to the infrared. In that case, even though the the linear \( \sigma \)-model \textit{seemed} to contain a continuously-variable Kähler parameter, the infrared limit consists of only one point, the infinite radius theory.

More generally, the infrared limit might consist of several points, or it might consist of the entire 2 real dimensional Kähler moduli space (what we hope to prove). Spacetime supersymmetry, which requires the moduli to form chiral multiplets, forbids the remaining possibility – a component of the Kähler moduli space of real dimension 1.

We now make some plausible assumptions about the behaviour of these theories.

1) If, along some RG trajectory, (0,2) supersymmetry is unbroken in the infrared, then the central charge in the infrared limit is accurately given by (4.3). That is, we assume that the \( U(1) \) R-symmetry that is present in these models becomes the \( U(1) \) current \( \bar{J} \) in the right-moving N=2 algebra in the infrared limit.

2) The only trajectories for which (0,2) supersymmetry is broken in the infrared are those which flow to \( r = \theta = 0 \), the “phase transition point” [6]. This is almost certainly true, because the Witten index is well-defined and \textit{nonzero} for all these theories \textit{except} at \( r = \theta = 0 \) where the vacuum manifold becomes noncompact [6].

We now examine the Zamolodchikov c-function [20] as a function of the Kähler parameter for these theories.\textsuperscript{8} Assume (counterfactually) that the critical points of the c-function are isolated. The (0,2) supersymmetric critical points must, in fact, be local minima, since spacetime supersymmetry implies the nonexistence of tachyons ( (0,2)-preserving relevant perturbations). Assumption 1) says that these minima are all degenerate. We have established the existence of at least two such minima: the infinite radius theory, and the Landau-Ginzburg theory. The only critical points which are \textit{not} minima must have (0,2) supersymmetry spontaneously broken. But, by assumption 2), the only candidate for such a critical point is \( r = \theta = 0 \). So we have a function with two (or more) isolated minima,

\textsuperscript{7} The existence of a 1-dimensional moduli space of \textit{non-supersymmetric} fixed points is precluded by assumption 2) below.

\textsuperscript{8} Equivalently, one could think of the renormalization group flow as defining a vector field on the Kähler parameter space, and apply the Lefschetz fixed-point theorem.
and one other critical point. This is impossible. The c-function is a Morse function on the Kähler parameter space (topologically a sphere). The alternating sum of its critical points, $(\# \text{minima}) - (\# \text{saddle-points}) + (\# \text{maxima})$, must give the Euler characteristic, and the number of critical points of index $i$ must be greater than or equal to the $i^{th}$ Betti number. Under the above assumptions, these Morse inequalities must be violated. Hence the hypothesis that the critical points are isolated must be false, and the whole Kähler moduli space is superconformal.

Note that this argument relied on the crucial fact that we had at least two minima. Had we not established that the Landau-Ginzburg theory was superconformal, then we could readily satisfy the Morse inequalities by giving the c-function one minimum (the infinite radius theory) and one maximum ($r = \theta = 0$).

This argument can be generalized to higher-dimensional Kähler moduli spaces, provided we have sufficient control over the locus in the parameter space on which supersymmetry may be broken. In the higher dimensional case, the critical point sets of the c-function are no longer points, but manifolds $M_j$, so we need to use a simple generalization of the Morse inequalities due to Bott [21],

$$b_i(M) \leq \sum_j b_{i-\text{ind}(M_j)}(M_j)$$

$$\chi(M) = \sum_j (-1)^{\text{ind}(M_j)} \chi(M_j).$$

6. Discussion

Let us review what we have seen in the previous sections:

1) There is strong evidence that the (0,2) Landau-Ginzburg orbifolds of [1] have nontrivial infrared fixed points. For certain (0,2) deformations of (2,2) theories, the discussion of §2 constitutes a proof of this, and demonstrates that many (2,2) Landau-Ginzburg theories are adjoined to much larger spaces of (0,2) Landau-Ginzburg theories.

2) The (2,2) and (0,2) Landau-Ginzburg theories often contain many massless $E_6$ singlets which a priori might become massive as one deforms away from the Landau-Ginzburg point. Symmetry arguments like those of §3 indicate that often, most of these $E_6$ singlets remain massless as one leaves the Landau-Ginzburg point. These symmetry arguments also constrain the singlet $n$-point functions for low $n$.

3) For the case of one-parameter Kähler moduli spaces, one can prove using simple topological arguments that the (0,2) moduli spaces of 1) continue to exist at finite
radius. For (2,2) theories like the quintic, this indicates that the full (2,2) moduli space is a submanifold of a much larger (0,2) moduli space. Similar topological arguments may allow one to prove analogous statements for theories with multi-dimensional Kähler moduli spaces, given sufficient knowledge of the “phase diagram.”

In the (0,2) context, having an extended chiral algebra with (2,2) superconformal supersymmetry is a very rare situation and requires highly nongeneric defining data. Still, it would not be surprising if the locus in (0,2) moduli space on which this occurred had several disjoint components. In this situation, a picture like Fig. 2 could arise. A single (0,2) moduli space could connect different (2,2) moduli spaces, whose large-radius limits correspond to different Calabi-Yau manifolds, which are perhaps not even birationally equivalent.

![Diagram](https://via.placeholder.com/150)

**Fig. 2:** Two different (2,2) moduli spaces characterized by the same number of generations and antigenerations arising as different enhanced symmetry points in a larger (0,2) moduli space.

As we have seen, the number of massless generations and antigenerations do not change as we move about in the particular (0,2) moduli space we have discovered here. Thus the two different Calabi-Yau manifolds in the above picture must have the same Hodge numbers.

Equally possible is that there are other “special points” in (2,2) moduli space which are multicritical, in that extra (0,2) flat directions appear there. In that case, the picture of the moduli space becomes even more complicated than Fig. 2 (exceeding our artistic abilities to depict it). One can pass from a (2,2) theory to a (0,2) theory to another.
(2,2) theory to yet another (0,2) theory to \ldots In this way, one might conjecture that all Calabi-Yau manifolds with the same Hodge numbers are continuously connected to each other.

Of course, we have only considered turning on VEVs for gauge-singlets. It is also possible (and, indeed, cases are known \textsuperscript{[10]} that there are F- and D-flat perturbations which break the $E_6$ gauge symmetry. In this case, the only invariant we expect to be preserved as one moves about in the (0,2) moduli space is the difference between the number of generations and antigenerations.\textsuperscript{9}

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\textsuperscript{9} Which is given by $\frac{1}{2} \chi$ for (2,2) Calabi-Yau theories and $\frac{1}{2} c_3(E)$ for (0,2) Calabi-Yau theories with vacuum gauge bundle $E$.
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