Numerical Simulation of Hydrofoil Cavitation in Pitching Motion

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Abstract—Based on CFD, the NACA4412 pitching motion hydrofoil is compared and analyzed for the lift/drag coefficient, the pressure and the gas volume fraction at different attack angles. The result is that the lift/drag coefficient curve of the dynamic hydrofoil is a closed curve. When the hydrofoil is pitching up, the lift/drag coefficient will be generally greater than that in the static state, which first increases and then decreases as attack angles increases. When the hydrofoil is down, the lift/drag coefficient will be generally smaller than that in the static state, which first decreases and then increases as attack angles decreases. Moreover, when the hydrofoil is pitching upwards, there will be a sudden change in pressure on the subsurface, resulting in an extreme value. In addition, there is more comprehensive cavitation on the upper surface of the dynamic hydrofoil, and local cavitation on the subsurface.

1. Introduction
The cavitation phenomenon occurs when the solid surface is in contact with water and has a high-speed relative motion. Generally, it will damage the solid surface\cite{1}, namely the phenomenon of cavitation erosion.

Li Zengliang et al. \cite{2} studied the cavitation law of different airfoil blades of vertical-axis tidal energy turbines. Lu Yuheng et al. \cite{3} compared and analyzed the turbulence models for numerical simulation of airfoil gap cavitation. Sun Zhaocheng et al. \cite{4} conducted optimization research on the lift-drag ratio and cavitation performance of tidal current turbine airfoil. Cheng Huaiyu et al. \cite{5} performed a large eddy simulation study on the effect of cavitation on the evolution characteristics and characteristic parameters of the tip clearance leakage vortex. Therefore, most of the studies mentioned above have focused on static hydrofoils. In this paper, a numerical simulation is carried out for the dynamic hydrofoil cavitation in pitching motion.

2. Numerical Model and Algorithm
In this paper, the Singhal cavitation model\cite{6} and the RNG $k$-$\epsilon$ turbulence model\cite{7} are selected, and the SIMPLEC algorithm\cite{8} is used to solve the N-S equation.

2.1. Governing Equation and Singhal Cavitation Model
The governing equations are:

\begin{equation}
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 \quad (1)
\end{equation}
Momentum equation:
\[
\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}[(\mu + \mu_t)(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})] + \rho g_i.
\]  

(2)

The void mass fraction in the Singhal cavitation model[6] is
\[
\frac{\partial (\rho f)}{\partial t} + \frac{\partial (\rho u_i f)}{\partial x_j} = \frac{1}{\rho} = f_{\omega} + f_{\omega} + \frac{1 - f_{\omega} - f_{\omega}}{\rho_l}.
\]  

(3)

2.2. RNG k-ε Turbulence Model
The turbulent kinetic energy and dissipation rate in the RNG k-ε turbulence model[7] are
\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_i k)}{\partial x_j} = \frac{\partial (\rho e)}{\partial t} + \frac{\partial (\rho e u_i)}{\partial x_j} =
\frac{\partial}{\partial x_j}[(\mu + \mu_t)(\frac{\partial k}{\partial x_j})] + G_k - \rho e,
\frac{\partial}{\partial x_j}[(\mu + \mu_t)(\frac{\partial e}{\partial x_j})] + c_1G_k \frac{e}{k} - c_2\rho \frac{e^2}{k}.
\]  

(4)

2.3. Calculation Parameters
NACA4412 hydrofoil is adopted, span \(c = 100\text{mm}\), and cavitation number \(\sigma = 0.5\). What is more, the motion law of the attack angle \(\alpha\) is[9] \(\alpha = \alpha_0 + \Delta \sin(\omega t)\), and the initial attack angle \(\alpha_0 = 10^\circ\). Amplitude \(\Delta \alpha = 10^\circ\), speed \(V = 15\text{m/s}\), and frequency \(K = 0.2\). Besides, angular velocity \(\omega = 2VK/c = 60\text{rad/s}\), and time step \(\Delta t = 0.005\text{s}\). Then, calculate 8 pitching cycles.

2.4. Computational Domain and Mesh
With the help of an O-shaped mesh, the calculation area is a circle surrounding the hydrofoil with a radius of about 20c. Moreover, draw boundary layer meshes according to the requirement that \(y^+\) is less than 1, and the number of mesh is about 30,000.

3. Analysis of Calculation Results

3.1. Lift/Drag Coefficient
Fig.1 shows how the dynamic hydrofoil lift coefficient \(C_l\) and drag coefficient \(C_d\) change with time \(t\). It can be seen from Fig.1 that the lift/drag coefficient of the hydrofoil oscillates periodically with time, and the oscillating law is unified with the oscillating law of attack angles. The data is basically stable after 8 pitching cycles. Moreover, from Fig.1(a), it can be seen that the lift/drag coefficient changes almost monotonously during the pitching motion. In other words, the lift/drag coefficient increases as attack angles increases, and decreases as attack angles decreases. What is more, when the hydrofoil is pitching up, the lift coefficient changes from increasing to decreasing before approaching the maximum attack angle, and the decrease lasts for a long time. When the hydrofoil is down, the lift coefficient changes from decreasing to increasing before approaching the minimum attack angle, and the increase lasts for a long time. When the hydrofoil is pitching up, the drag coefficient decreases with the increase of attack angle near the minimum attack angle, and then turns to increase with the increase of attack angle. Additionally, the drag coefficient changes from increasing to decreasing before approaching the maximum attack angle, and the decreasing duration is shorter. When the hydrofoil is down, the drag coefficient changes from decreasing to increasing before approaching the minimum attack angle, and the increase lasts for a short time. When \(t = 0.74\text{s}\), the lift coefficient...
reaches the extreme value of 0.086. When \( t = 0.79\)s, the lift coefficient reaches the extreme value of 0.002, and the maximum amplitude is 0.084. In addition, it can be seen from Fig.1(b) that when \( t = 0.75\)s, the drag coefficient reaches the extreme value of 0.027. When \( t = 0.79\)s, the drag coefficient reaches the extreme value of 0.004, and the maximum amplitude is 0.023. The amplitude of the lift coefficient is greater.

![Lift coefficient](image1.png)

**Fig.1 Variation curve of dynamic hydrofoil lift/drag coefficient with time**

**3.2. Pressure**

Fig.3 shows how the lift/drag coefficient of hydrofoil varies with the attack angle \( \alpha \). It can be seen from Fig.2 that the law that the lift/drag coefficient of static and dynamic hydrofoil varies with the attack angle is different. The lift/drag coefficient curve of a static hydrofoil is a monotonous curve without the extreme value, while the lift/drag coefficient curve of the dynamic hydrofoil is a closed curve with the maximum value. Moreover, when \( \alpha \) raises to 14.06°, the lift coefficient will reach the extreme value of 0.086. When \( \alpha \) descends to 7.27°, the lift coefficient will reach the extreme value of 0.002. What is more, when \( \alpha \) raises to 18.51°, the drag coefficient will reach the extreme value of 0.027. When \( \alpha \) descends to 7.27°, the drag coefficient will reach the extreme value of 0.004. In addition, when the hydrofoil is pitching up, the lift/drag coefficient will be generally greater than that in the static state, which generally increases first and then decreases with the increase of the attack angle. When the hydrofoil is down, the lift/drag coefficient will be generally smaller than that in the static state, which generally decreases first and then increases as the attack angle decreases. The lift/drag coefficient of the hydrofoil is generally greater when it is pitching up than that when it is pitching down.

![Pressure distribution](image2.png)

**Fig.3 Distribution of hydrofoil surface pressure along relative chord length \( x/c \) at different attack angle. ↑ refers to the upward process, and ↓ means the downward process. It can be seen from Fig.3 that the pressure difference of the hydrofoil is generally greater when it is pitching up than that when it is pitching down. The main reason is that the pressure on the subsurface of the hydrofoil is greater, which causes the lift and drag coefficient of the hydrofoil to be generally greater when pitching up than that when pitching down, and there is more comprehensive cavitation on the upper surface of the hydrofoil. Moreover, when the attack angle is small, there is local cavitation on the subsurface of the hydrofoil near the leading edge. When the hydrofoil is descending, and the attack angle is large, there is local cavitation on the subsurface of the hydrofoil near the trailing edge. When the hydrofoil is in the upward pitching process, and the attack angle is small, there will be a sudden change in pressure on the subsurface near the leading edge 0.2c-0.4c. Meanwhile, the pressure maxima will be produced, and the sudden change point moves to the leading edge as the attack angle increases. In addition, when \( \alpha \) raises to 14.06°, the pressure difference on the hydrofoil surface will reach its
extreme value. When $\alpha$ descends to $7.27^\circ$, the pressure difference on the hydrofoil surface will reach the minimum.

Fig. 2 Curve of hydrofoil lift/drag coefficient with the attack angle

(a) Lift coefficient  
(b) Drag coefficient

Fig. 3 Pressure

(a) Ascending process  
(b) Descending process

3.3. Gas Volume Fraction

Fig. 4 shows the distribution of the gas volume fraction on the surface of the hydrofoil at different attack angles. It can be seen from Fig. 4 that the cavitation bubbles continually fall off during the pitching motion of the hydrofoil. Moreover, the degree of cavitation on the upper surface of the hydrofoil is generally smaller when it pitches up than when it pitches down. As a result, the pressure difference when the hydrofoil pitches up is generally greater than that when it pitches down. What is more, the cavitation zone on the upper surface of the hydrofoil is longer, and there is not cavitation only in a small area near the trailing edge. When the attack angle is small, there will be a cavitation zone with a length of about $0.4c$ on the subsurface of the hydrofoil near the leading edge. Besides, when the hydrofoil is descending, and attack angle is large, there is local cavitation on the subsurface of the hydrofoil near the trailing edge, and as the attack angle decreases, it moves to the leading edge and gradually collapses and disappears.
4. Conclusion

Through analyzing the numerical simulation of NACA4412 pitching motion hydrofoil cavitation, it is concluded that:

1. The variation law of the lift and drag coefficient in the dynamic hydrofoil over time is consistent with the law of the attack angle movement, showing periodic oscillations;

2. The lift/drag coefficient curve of the dynamic hydrofoil is a closed curve. When the hydrofoil pitches up, the lift and drag coefficient is generally greater than that in the static state. Moreover, the lift and drag coefficient increases first and then decreases with the increase of the attack angle. When the hydrofoil is down, the lift and drag coefficient is generally smaller than in the static state, which first decreases and then increases as the attack angle decreases. In addition, the lift/drag coefficient of the hydrofoil is generally greater when it is pitching up than that when it is pitching down;

3. More comprehensive cavitation occurs on the upper surface of the hydrofoil. When the attack angle is small, local cavitation appears on the subsurface of the hydrofoil near the leading edge. In
addition, when the hydrofoil is descending, and the attack angle is large, local cavitation occurs on the subsurface of the hydrofoil near the trailing edge;

(4) When the hydrofoil is pitching up, and the attack angle is small, the pressure jump will occur at the subsurface near the leading edge 0.2c-0.4c. What is more, the pressure maxima will be generated, and the sudden change point moves to the leading edge as the attack angle increases;

(5) When the attack angle is small, there will be a cavitation zone with a length of about 0.4c on the subsurface of the hydrofoil near the leading edge. Moreover, when the hydrofoil is descending, and the attack angle is large, the local cavitation that occurs on the subsurface of the hydrofoil near the trailing edge moves toward the leading edge with the decrease of the attack angle and gradually collapses and disappears.

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