FREQUENCY AND STIFFNESS ANALYSIS OF CRACKED BOLTED COMPOSITE GEOMETRIES UNDER THE APPLICATION OF SHOCK LOAD

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Abstract—This work presents the development of an approximate analytical model to study the effect of application of shock loading on frequency response and stiffness of single lap multi bolted cracked composite laminates. These configurations are transformed into mass-spring systems and mathematical model is completed to determine natural frequency of bolted geometries. Analytical results are validated by comparing the results with Finite Element (FE) simulations run in ANSYS workbench. Conclusions reveal that changes in stiffness, because of increase in number of bolts and presence of cracks lead to reduction in fundamental frequency of the system. Therefore, this sensitive nature of frequency response can be used in modeling larger mechanical and aerospace structures.

Keywords—crack; shock loading; frequency response; stiffness; mass-spring system

I. INTRODUCTION

Composite materials play a vital role in mechanical and aerospace applications because of their high stiffness & strength to weight characteristics, good creep resistance and extended fatigue life. These composite structural components during their operational phase are subjected to and are influenced by different types of stresses and hazardous loadings that cause damages or failure and reduce the working life & strength of structures. Bolted / riveted joints are often used to assemble composite plates and panels within aerospace wing and tail plane structures. However, mechanically fastened joints are source of weakness, generate regions of high stress concentration and contribute to an excessive weight. Therefore, in order to utilize full potential of composite materials as structural members, their strengths, weaknesses, damages induced in them because of presence of bolted connections and hazardous loadings must be understood. Present investigation is to develop an analytical procedure to determine frequency response of a system under shock load and same is extended to incorporate the effect of cracks. The current background study is conducted considering literature related to spring based approach, the shock loading and crack formulation in parallel.

Scientists and researchers have always taken it as a challenge to develop accurate and efficient design tools, with a view to minimize analysis time and cost. Spring based methods have always been considered as highly efficient and low-cost methods of analysis for designing of multi-bolt joints [1-6]. This approach involves idealization of structure as series of spring and masses with a value of stiffness being assigned to each component. McCarthy [4] used this method to provide analytical procedures for shear loading in single and multi-bolt configurations. In [7] the author developed methods to describe fastener pull through mode of loading in the joints. In addition to this, time consuming detailed FE models are mostly used [5] to calibrate the results. These researchers either used tensile, shear or pull as forcing function in their mass-spring based analytical models. Moreover, the results were acquired after ignoring the mass of both bolt and composite laminates. To the best of author’s knowledge, spring based analytical model to determine composite lap configuration behavior in terms of frequency response under the influence of shock loading requires consideration.

Nonlinear dynamic frequency and displacement response of laminated composite plates and panels subjected to shock loads has been investigated by numerous researchers. Soykasap [8]
presented an experimental setup to determine the dynamic behavior of composite material cylindrical shells subjected to shock loads. The mathematical formulations for different types of shock loads have been developed, to acquire the displacement or deflection response of systems under consideration [9]. Mostly, shock load is analytically modeled as blast, step and triangular load. Vaziri et.al. [10] studied the fracture and deflection of sandwich plates subjected to intense and uniform impulsive pressure load i.e. the shock load. The shock load was applied in the form of uniform pressure over the surface of plate. The formulation for shock was given by \( P(t) = p_0 \exp(-t/t_0) \), where \( p_0 \) and \( t_0 \) denoted peak pressure and decay time of shock. Nemes and Randles [11] developed a methodology to determine the response of firm composites to shock loading. Composite plates under consideration were made of graphite / epoxy. The shock formulation in the study was given by \( P(t) = p_0 e^{-t/t_0} \). Here the arrival of wave was considered at \( t=0 \) and \( \theta_t \), the characteristic time was given in micro seconds. The developed shock formulations just contained information related to instantaneous rise with exponential decay. It is found that there exists a need for developing a shock load formula that may assess the natural frequency of carbon fiber epoxy composites. Composite structures during their operational life in civil, mechanical and aerospace applications are subjected to dynamic loading and frequency response analysis provides an efficient way to determine the system behavior under such conditions.

Defects in structural component because of presence of crack effect its stiffness and dynamic performance. Many factors contribute to creation and presence of cracks. Poor fatigue life generates fatigue cracks in components under working conditions. Cracks may also be caused by mechanical defects and some cracks occurring inside the material are generated during manufacturing process. In the available literature related to analysis of system with crack mostly experimental investigations are conducted to determine the displacement, deflection and frequency response [12-15]. Okamura et.al. [16] developed the formulation for stiffness of region around crack by keeping length of crack under consideration. Krawczuk and Ostachowicz [17] determined frequencies of graphite composites having transverse open crack. The formulation consisted of crack depth and location. Nayak [18] preformed vibration analysis and developed formula for cracked region by considering the crack depth and location. Guo and Sun [19] developed an approach to determine stiffness of crack by transforming it into rotational spring. The stiffness formulation of that spring consisted of its depth and location. In available crack formulation, the researchers have either considered crack length or crack depth. There exists a need to develop a crack formulation that may contain information related to both crack depth & length and attempt has been made in present study to fill this gap.

In current work an approximate analytical model is developed to study the frequency response of bolted composite plate made of carbon fiber reinforced epoxy matrix composite with quasi-isotropic layup having a crack at specified location subjected to shock load. Composite bolted, cracked and un-cracked, configurations are transformed into mass-spring systems where bolts, laminates, the clamped region, stiffness of crack and the region around the crack are represented by a series of linear springs and masses. Shock load, applied to top surface of geometry, is defined as a decaying function of time. Attempts are made to validate the analytical model by comparing the results with \( FE \) simulations. The proposed solution shows that the frequency response obtained by shock loading is sensitive to increasing number of bolts and crack in plate. This work mainly covers the advantages of using obtained frequency data as a means of detecting changes in the stiffness and frequency results. In the proceeding section formulation of bolted lap configuration of flat composite plates is discussed, frequency response of bolted lap geometry without and with crack effects under shock load are obtained next, and then validation of the approximate analytical model with \( FE \) simulations are presented followed by discussion on results and conclusion.

II. APPROXIMATE VIBRATION MODEL OF BOLTED LAP COMPOSITE PLATES

In the bird's eye view, two flat composite plates, positioned in the shape of lap, with number of bolts varying from one to three is used as a test specimen and frequency analysis of the structure is
performed when it is subjected to shock load. In the second step, crack of some uniform depth and different lengths is introduced at different locations of bolted lap structure. The crack stiffness formulations are then introduced into approximated analytical model to determine the effect of presence of crack on stiffness and frequency characteristics. A single bolt single lap composite plate Figure 1b is used for initial development of analytical model to predict vibration frequency response of structure. The material under consideration was carbon-fiber/epoxy, designated as HTA/6376 [5].

![Figure 1](image)

Figure 1. Geometry specifications of bolt and single lap composite plate (a) Titanium bolt (b) 1 - bolt lap geometry

The length of each plate is 500mm. These plates are positioned one above the other to produce single lap structure and the region of overlap is 90mm. The overall length of this geometry is 910mm and width is 500mm. The composite laminates have a quasi-isotropic layup with sequence of stacking as [45/0/-45/90]s. Each layer has a ply thickness of 0.13mm that led to total thickness of 4.16mm for each plate of composite material. The titanium M8 bolts Figure 1a are introduced into the geometry having bolt holes, with diameter, similar to bolt shank. The geometry is considered to be completely fixed from the left and right end. The unidirectional & homogenized lamina material properties and aerospace grade titanium alloy bolt material properties are obtained from McCarthy [20] and is referred here in Table 1. However, the current analytical model is carried out using homogenized laminate material properties obtained from classical laminate theory. In order to assess the model’s accuracy in terms of predicting frequency response of multi-bolt composite joints, the 2 and 3 - bolts single lap joint connections are also modeled and investigated.

| Material properties [20] |
|--------------------------|
| Unidirectional properties: HTA/6376 | Homogenized | laminate | Titanium properties |
| E₁₁ (GPa) | 140 | Eₓₓ (GPa) | 54.25° | E₁₁(GPa) | 110 |
| E₂₂ (GPa) | 10 | Eᵧᵧ (GPa) | 54.25° | ν | 0.29 |
| E₃₃ (GPa) | 10 | Eᵧᵧ (GPa) | 12.59 | G₁₂ (GPa) | 5.2 |
| G₁₂ (GPa) | 5.2 | Gₓₓ (GPa) | 20.72° | G₁₃ (GPa) | 5.2 |
| G₁₃ (GPa) | 5.2 | Gₓᵧ (GPa) | 4.55 | G₂₂ (GPa) | 3.9 |
| G₂₂ (GPa) | 3.9 | Gᵧᵧ (GPa) | 4.55 | 𝜈₁₂ | 0.3 |
| 𝜈₁₂ | 0.3 | 𝜈ₓᵧ | 0.309° | 𝜈ₓₓ | 0.332 |
| 𝜈₁₃ | 0.3 | 𝜈ᵧₓ | 0.332 | 𝜈ᵧᵧ | 0.5 |

a Verified by classical laminate theory

The frequency response obtained from the analytical model are only validated with 3D FE simulation as no experimental frequency response results are available for this type of joint configuration with shock load as forcing function.

Several researchers have used spring-mass system for analyzing the composite bolted lap configuration structures [4-5]. An effort has been made in this study to analyze the shock loading behavior of composite bolted lap geometries. The bolted cracked and un-cracked single lap joint (with number of bolts varying from 1 to 3) has been transformed into a system of spring and masses
This methodology is applicable to any type of bolted lap geometries. The shock load $F(t)$ is applied in the lateral direction to the top surface of the plate. This 2-DoF spring mass system consists of stiffness of bolt $k_B$, stiffness of clamped region $k_C$, stiffness of laminates $k_{L_1}$ & $k_{L_2}$, stiffness of cracked region $k_{L\text{ec}}$, mass of composite laminate plates $m_{L_1}$ & $m_{L_2}$, mass of bolt(s) $m_B$ and the geometrical parameters. Analysis of spring mass system is based on the following assumptions:

- The masses are free to move in lateral direction only
- The springs have stiffness only in lateral direction
- Turning moment of bolts is not considered in the analysis
- Gray and McCarthy [5] ignored the mass effects in their analysis, however, the current investigation incorporates both the mass of laminates and the bolts with system subjected to shock load applied in lateral direction. Each mass in the system is inspected with the help of free-body diagrams and equations of motion obtained are;

$$ m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = F(t) $$  \hspace{0.5cm} (1)

$$ m_2\ddot{x}_2 + k_2x_2 - k_1x_1 = 0 $$  \hspace{0.5cm} (2)

$x_1$ & $x_2$ are the displacement of mass 1 & 2 respectively. The system of linear equations for this mass - spring model subjected to shock load may be written in matrix form as;

$$ \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix} $$  \hspace{0.5cm} (3)

$m_1$ - mass of laminate 1 & 2, $m_2$ - mass of bolt, $k_1$ - stiffness of laminate 1& 2 (for un-cracked geometry) and it is added stiffness of laminates and cracked region (for cracked geometry), $k_2$ - added stiffness of bolt ($k_B$) and clamped region ($k_C$).

The stiffness of different parts and regions of mass-spring system are obtained as follows:

a. According to Gray and McCarthy [5] the stiffness of bolt can be represented by (4).

$$ k_B = \frac{E_s A_{\text{shank}}}{L_{\text{shank}}} $$  \hspace{0.5cm} (4)

$E_s$ - Young’s modulus of bolt material, $A_{\text{shank}}$ and $L_{\text{shank}}$ are cross-sectional area and length of bolt shank respectively.

b. Stiffness of laminate’s clamped region is dependent on the $\frac{a}{d_{b,0}}$ ratio i.e., if $\frac{a}{d_{b,0}} > 6$ [5 , 21] then,

$$ k_{\text{clamped}} = \frac{\pi Ed_{b,0} \tan \alpha}{2 \ln \left( \frac{\gamma + 3}{\gamma + \gamma LAM \times \tan \alpha + 3d_{b,0}} \right)} $$  \hspace{0.5cm} (5)

\begin{figure}[h]
\centering
\begin{subfigure}[b]{0.3\textwidth}
\includegraphics[width=\textwidth]{single_bolt}
\caption{Single bolt}
\end{subfigure} \hspace{0.5cm}
\begin{subfigure}[b]{0.3\textwidth}
\includegraphics[width=\textwidth]{double_bolt}
\caption{Double bolt}
\end{subfigure} \hspace{0.5cm}
\begin{subfigure}[b]{0.3\textwidth}
\includegraphics[width=\textwidth]{triple_bolt}
\caption{Triple Bolt}
\end{subfigure}
\caption{Mass-spring systems for bolted lap geometries}
\end{figure}

\begin{figure}[h]
\centering
\begin{subfigure}[b]{0.3\textwidth}
\includegraphics[width=\textwidth]{single_bolt_clamped}
\caption{Single bolt}
\end{subfigure} \hspace{0.5cm}
\begin{subfigure}[b]{0.3\textwidth}
\includegraphics[width=\textwidth]{double_bolt_clamped}
\caption{Double bolt}
\end{subfigure} \hspace{0.5cm}
\begin{subfigure}[b]{0.3\textwidth}
\includegraphics[width=\textwidth]{triple_bolt_clamped}
\caption{Triple Bolt}
\end{subfigure}
\caption{Mass-spring systems for bolt plate geometries having crack of length ‘b’}
\end{figure}
Otherwise, the stiffness of clamped region is:

\[
k_{\text{clamped}} = \frac{\pi E \tan \alpha}{2} \ln \left( \frac{\gamma + 3}{\gamma - 1} \right) \left( \frac{a - d_{\text{Bolt}}}{a + 3d_{\text{Bolt}}} \right) + 4 \left( \frac{th_{\text{LAM}} \times \tan \alpha - a + pd_{\text{Bolt}}}{(a + 3d_{\text{Bolt}})(a - d_{\text{Bolt}})} \right)
\]

(6)

Where; \(a\) - length of plate, \(d_{\text{Bolt}}\) - bolt shank diameter, \(th_{\text{LAM}}\) - thickness of laminate, \(\gamma = \frac{\sqrt{E_{\text{w}}/E_{\text{Bolt}}}}{d_{\text{w}}/d_{\text{Bolt}}}\), \(E\) - Young’s modulus of material (For composite materials \(E\) can be replaced by \(\sqrt{E_{\text{w}}/E_{\text{Bolt}}}\)), \(\alpha\) - pressure distribution angle = 36° [5]

c. The stiffness of laminate defined in [4] is modified by using the homogenized laminate modulus in global z-direction, and leads to following form;

\[
k_{\text{LAM}} = \left( \frac{E_{\text{w}} \times w \times th_{\text{LAM}}}{p - d} \right)
\]

(7)

where; \(k_{\text{LAM}}\) - stiffness of laminate, \(E_{\text{w}}\) - homogenized laminate modulus in global z-direction, \(w\) - width of laminate, \(th_{\text{LAM}}\) - thickness of laminate, \(p\) - pitch of bolt, \(d\) - diameter of bolt

d. A crack of varying length and constant depth is introduced in transverse direction in one of the laminate at some distance i.e. \(L_0\) from the fixed end. The width of composite plate is \(w\) and height is \(h\). First mode of vibrations is assumed here. The crack introduced in composite plate is modeled as a linear spring representing the stiffness of crack and the region around the crack in terms of crack length & depth. The stiffness of crack is then incorporated in the developed mass-spring systems to determine the effect of crack length, depth and location on the stiffness of laminate and frequency of the entire system. The crack introduced in the geometry and the same incorporated as equivalent crack stiffness in mass-spring system is elaborated for three cases as stated in Figure 3.

Okamura et. al. [16] obtained the stiffness of spring by considering the crack length, i.e.,

\[
k_{\text{crack}} = \frac{EhW^2}{72(1 - \vartheta^2)F\left(\frac{b}{W}\right)}
\]

(8)

where, \(k_{\text{crack}}\) - stiffness of crack, \(E\) - elastic modulus of plate in z-direction, \(h\) - plate thickness, \(w\) - plate width, \(\vartheta\) - poisson’s ratio, \(F\left(\frac{b}{W}\right)\) - function representing crack length to plate width ratio, \(b\) - length of crack

where; \(F\left(\frac{b}{W}\right)\) is crack length to plate width ratio and the function \(F\left(\frac{b}{W}\right)\) in (8) is stated in [16] as,

\[
F\left(\frac{b}{W}\right) = 1.98 \left(\frac{b}{W}\right)^2 - 3.277 \left(\frac{b}{W}\right)^5 + 14.43 \left(\frac{b}{W}\right)^4 - 31.26 \left(\frac{b}{W}\right)^6 + 63.56 \left(\frac{b}{W}\right)^8 - 103.36 \left(\frac{b}{W}\right)^9 + 147.52 \left(\frac{b}{W}\right)^{10} - 127.69 \left(\frac{b}{W}\right)^6 + 61.50 \left(\frac{b}{W}\right)^8
\]

(9)

Nayak [18] determined the stiffness changes in a structure because of presence of crack in terms of crack depth. Therefore, spring constant in terms of crack depth is given by;

\[
k_{\text{crack}} = \frac{Ewh^2}{18\pi L_0^2(1 - \vartheta^2)g(\alpha)}
\]

(10)

Where; \(L_0\) - distance of crack from the left end, \(g(\alpha)\) - function representing crack depth to plate thickness ratio \((a - \frac{h}{h})\) and is obtained from [18]:

\[
g(\alpha) = 19.6(\alpha)^8 - 40.7556(\alpha)^9 + 47.1063(\alpha)^8 - 33.0351(\alpha)^7 + 20.2918(\alpha)^6 - 9.0736(\alpha)^5 + 4.5948(\alpha)^4 - 1.0533(\alpha)^3 + 0.272(\alpha)^2
\]

(11)

In the present research, stiffness of crack, formulated by Okamura et. al. [16] is modified by introducing a term \(L_0^2\) in the denominator to represent location of crack in the laminate (12).
The complete effect of crack location, depth and length is incorporated into a single equation by adding (10) and (12)

\[ k_w = \frac{Ew}{18L^2(1-\nu^2)} \left( \frac{h}{\pi b} + \frac{w}{4F} \right) \]  

where, \( k_w \) - Equivalent crack stiffness

Length of crack is varied from 15\( mm \) to 25\( mm \) with an increment of 5\( mm \) in each step and stiffness of laminate is calculated.

### III. FREQUENCY RESPONSE OF BOLTED CRACKED LAP GEOMETRY UNDER SHOCK LOAD

Current investigation is intended to determine the stability of structure by analyzing its frequency response results. So, the shock load is \( F(t) = F_0 e^{\omega t}/(1+\sin \omega t) \) for \( t > 0 \), is applied on the surface of plate. \( F_0 \) is the peak load and \( t_0 \) is the initial time and in this case, it is 0.1sec. The displacement, \( x_1(t) \) & \( x_2(t) \), response of bolted composite lap structure to the applied shock load is assumed as;

\[ x_1(t) = X_1 e^{-\omega t}(1+\sin \omega t) \]  
\[ x_2(t) = X_2 e^{-\omega t}(1+\sin \omega t) \]  

Substituting displacement responses from (14) & (15) into (1) & (2) and the value of sine & cosine functions are maximum at resonance, we get;

\[ X_1 \left[ -m_1 \omega^2 - \frac{2m_1 \omega}{t_0} + \frac{2m_1}{t_0} + 2(k_1 + k_2) \right] - X_2(2k_2) = 2F_0 \]  
\[ X_2 \left[ -m_2 \omega^2 - \frac{2m_2 \omega}{t_0} + \frac{2m_2}{t_0^2} + 2k_2 \right] - X_1(2k_2) = 0 \]  

Equations (16) & (17) can be represented in matrix form;

\[
\begin{bmatrix}
- m_1 \omega^2 - \frac{2m_1 \omega}{t_0} + \frac{2m_1}{t_0} + 2(k_1 + k_2) \\
- m_2 \omega^2 - \frac{2m_2 \omega}{t_0} + \frac{2m_2}{t_0^2} + 2k_2
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix}
= \begin{bmatrix}
2F_0 \\
0
\end{bmatrix}
\]  

For nontrivial solution to obtain frequency response of system, the determinant of coefficient of \( X_1 \) and \( X_2 \) must be zero, we get,

\[
\omega^4(m_1m_2) + \omega^2 \left( \frac{4m_1m_2}{t_0^2} + 4m_1k_2 + 4k_1k_2 \right) - \omega^2 \left( \frac{2m_1m_2}{t_0} + 2(k_1 + k_2)m_2 \right) - \omega \left( \frac{8m_1m_2}{t_0^2} + 4m_1k_2 + 4(k_1 + k_2)m_2 \right) + \omega \left( \frac{8m_1m_2}{t_0} + 4m_1k_2 + 4(k_1 + k_2)m_2 \right) - 4k_2^2 = 0
\]  

Equation (19) is known as the frequency or characteristic equation of the composite bolted lap structure. The frequency response equations for bolt composite plate assembly with and without crack are worked out in order to acquire the fundamental frequencies.

Results with cracks located at 100\( mm \) and 150\( mm \) distance from left end and un-cracked bolted geometries are shown in Table 2. When the crack is brought closer to the region of stress concentration it causes a decrease in frequency in single, double and triple bolt geometry. The effect of increase in crack length on frequency is more pronounced when crack is located at 150\( mm \) distance from the fixed end and is closer to the bolt(s).
### Table 2. Analytical frequency response results with and without crack

| Crack Length | 1 Bolt (Hz) | 2 Bolts (Hz) | 3 Bolts (Hz) |
|--------------|-------------|--------------|--------------|
| 0 (mm)       | 158.716     | 158.55       | 158.385      |
| b. Crack located at 100mm distance from the fixed end |
| 15 (mm)      | 149.0428    | 148.8876     | 148.617      |
| 20 (mm)      | 142.7143    | 142.5654     | 142.4159     |
| 25 (mm)      | 135.7193    | 135.5775     | 135.4352     |
| c. Crack located at 150 mm distance from fixed end |
| 15 (mm)      | 139.076     | 138.9307     | 138.6623     |
| 20 (mm)      | 127.9502    | 127.6164     | 127.4156     |
| 25 (mm)      | 117.5307    | 117.1901     | 116.9941     |

### IV. FINITE ELEMENT SIMULATION USING ANSYS

In order to validate the proposed approximate analytical model, 3D finite element cracked single, double and triple bolted lap geometries are modeled in ANSYS. The carbon fiber / epoxy composite laminate plates have a quasi-isotropic lay-up with stacking sequence of $[45/0/-45/90]_s$ and material designation HTA/6376, bolted with aerospace grade titanium bolts. In order to apply the fixed boundary conditions the translational and rotational Degrees of Freedom (DoF) of all nodes at both ends of lap geometry are completely constrained. Homogenized laminate material properties (Table 1) are used for modeling of composite laminates. The forcing function i.e. the shock load is applied by keeping in account the transient loading that is applied to the top surface of whole geometry for duration of 1s. The initial time at which the shock wave strikes the target is 0.1 seconds. Four different types of element i.e. tet10, hex20, wed15 and quad4 are used for meshing of bolted plate configuration out of which quad 4 has the major contribution to keep aspect ratio almost equal to 1.

Transverse straight line cracks of length 15mm, 20mm and 25mm are introduced at a distance of 100mm and 150mm from the fixed end in composite lap configurations. The material is carbon fiber / epoxy composite with laminate plates having aerospace grade titanium bolts. Both ends of lap geometry are completely fixed. Laminates are modeled considering homogenized material properties. The shock load is applied on the top surface of whole geometry for the duration of 1s with initial target time as 0.1 seconds. Results of FE simulations for one and three bolt geometries with cracks of length 15 & 25mm located at 100 & 150mm distance from fixed left end in single and triple bolt geometries are shown in Figure 5. It is obvious that presence of crack has caused loss of stiffness of composite plate leading to decrease in frequency.

### V. RESULTS AND DISCUSSION

Figure 6 shows comparison of FE simulations results for crack of different lengths located at different locations in single, double, and triple bolt geometries. Discussed below are the outcomes drawn after observing the plots:

- The increase in number of bolt holes lead to decrease in frequency. No, doubt the bolts are inserted in holes but the regions of stress concentration generated around each hole lead to decrease in frequency. Therefore, the lowest frequency response is attained for geometry having 3 bolts i.e. 158.385Hz for analytical results.
- For single bolt geometry having either of the following crack lengths 15mm, 20mm or 25mm, the lowest frequency response is acquired for crack length of 25mm. This showed that decrease in stiffness of structure because of damages in the form of crack lead to decrease in frequency.
- Bringing the crack closer to the region of stress concentration causes a pronounced decrease in frequency. The frequency response for single bolt geometry for crack of length 25mm located at a distance of 100mm (135.7193Hz /124.8Hz) from fixed end is higher as compared to the same geometry with crack located at a distance of 150mm (117.5307Hz / 113.17Hz) from the fixed end.
Crack length 15 mm at 100mm distance from fixed end

Crack length 25 mm at 100mm distance from fixed end

Crack length 15 mm at 150mm distance from fixed end

Crack length 25 mm at 150mm distance from fixed end

Figure 5. Fundamental frequency results for different crack locations (a) 1-bolt (b) 3-bolt
• The highest frequency response is acquired for single bolt geometry without crack i.e. 158.716 Hz. The frequency is maximum for no crack condition and it continuously decreases with increase in crack length.

The error percentage between analytical and ANSYS FE results is almost 13.5%. The main factors contributing to this discrepancy are:

a. In FE results factors like meshing, numerical techniques used and element type etc. play a vital role. However, in approximate analytical modeling characteristic equation, containing stiffness of different regions of bolted geometry affects frequency of the whole system.

b. In FE analysis, an adhesive bond is between the region of overlap, lower surface of bolt head & upper surface of top plate and in between top face of lower plate & washer/nut whereas stiffness of adhesive is ignored in approximate analytical modeling.

c. In approximate analytical modeling, homogenized laminate modulus existing in the direction of load application i.e. z-direction is considered in the formulation for stiffness of laminate while in FE analysis the composite plates are completely modeled by defining orientation of fibers in each layer.

![Image](a)
![Image](b)
![Image](c)

Figure 6. Analytical & FE frequency comparisons for two crack locations

• The overall analysis and comparison of frequency responses for single, double and triple bolt geometries show that the least response is acquired for crack of length 25 mm located at 150 mm distance from the fixed end in triple bolt geometry i.e. 107.09 Hz. This frequency response is lowest among all of the configurations considered in this study.

• The main contributing factors to cause decrease in stiffness and frequency are increase in crack length, increase in the number of bolt holes and placement of crack closer to the regions of stress concentrations

VI. CONCLUSION

An approximate analytical model and 3D finite element analysis (FEA) are presented, to determine the effect of existence of crack and application of shock loading on vibration characteristics and stiffness of composite laminates. The problem geometry consists of two overlapping flat bolted composite plates. The physical change is set up in the system by varying the number of bolts and introducing cracks of different lengths at different locations from the fixed end of the plate. The frequency analysis of the test specimen is performed when it is subjected to shock.
load. An analytical model of defined geometric configuration is approximated by mass-spring system having the effects of stiffness of clamped region, bolt and composite laminates. Crack introduced in composite plate is modeled as a linear spring. A modified crack stiffness formula is developed by incorporating three important parameters i.e. crack length, crack depth and crack location in single equation. The crack stiffness formulations are introduced into analytical model to determine the effect of presence of crack on stiffness and frequency characteristics. The analytical frequency responses of cracked and un-cracked geometry are compared with ANSYS FE simulations to check the validity of the developed analytical model. It is concluded that decrease in stiffness leads to decrease in frequency of system i.e. frequency response can be considered as a means of detecting changes in structural stiffness. The main contributing factors to cause decrease in stiffness and frequency are increase in crack length, increase in the number of bolt holes and placement of crack closer to the regions of stress concentrations.

REFERENCES

[1] Tate M.B., "Preliminary investigation of the loads carried by individual bolts in bolted joints," 1946.
[2] Nelson W.D., B.L. Bunin, and L.J. Hart-Smith, "Critical Joints in Large Composite Aircraft Structure," DTIC Document, 1983.
[3] McCarthy M., C. McCarthy, and G. Padhi, "A simple method for determining the effects of bolt–hole clearance on load distribution in single-column multi-bolt composite joints," Composite Structures, 73(1): p. 78-87, 2006.
[4] McCarthy C.T. and P.J. Gray, "An analytical model for the prediction of load distribution in highly torqued multi-bolt composite joints," Composite Structures, 93(2): p. 287-298, 2011.
[5] Gray P.J. and C.T. McCarthy, "An analytical model for the prediction of through-thickness stiffness in tension-loaded composite bolted joints," Composite Structures, 94(8): p. 2450-2459, 2012.
[6] Barrois W., "Stresses and displacements due to load transfer by fasteners in structural assemblies," Engineering fracture mechanics, 10(1): p. 115-176, 1978.
[7] Gray P. and C. McCarthy, "A highly efficient user-defined finite element for load distribution analysis of large-scale bolted composite structures," Composites Science and Technology, 71(12): p. 1517-1527, 2011.
[8] Soykasap Ö., Z. Mecitoğlu, and O. Borat, "Dynamic Response of Composite Cylindrical Shells to Shock Loading. Mathematical and Computational Applications," 1: p. 85-96, 1996.
[9] Upadhyay A., R. Pandey, and K. Shukla, "Nonlinear dynamic response of laminated composite plates subjected to pulse loading," Communications in Nonlinear Science and Numerical Simulation, 16(11): p. 4530-4544, 2011.
[10] Vaziri A., Z. Xue, and J.W. Hutchinson, "Performance and failure of metal sandwich plates subjected to shock loading," Journal of Mechanics of Materials and Structures, 2(10): p. 1947-1963, 2007.
[11] Nemes J. and P. Randles, "Modelling the Response of Thick Composite Materials Due to Axisymmetric Shock Loading," DTIC Document, 1991.
[12] Ghoneam S., "Dynamic analysis of open cracked laminated composite beams," Composite Structures, 32(1): p. 3-11, 1995.
[13] K, M.S.C., "Characterization of damage progression in layered composites," Journal of Sound and Vibration, 152: p. 177-179, 1992.
[14] Abd El-Hamid Hamada A., "An investigation into the eigen-nature of cracked composite beams," Composite Structures, 38(1): p. 45-55, 1997.
[15] Zak A., M. Krawczuk, and W. Ostachowicz, "Numerical and experimental investigation of free vibration of multilayer delaminated composite beams and plates," Computational Mechanics, 26(3): p. 309-315, 2000.
[16] Okamura H., "A cracked column under compression," Engineering fracture mechanics, 1(3): p. 547-564, 1969.
[17] Krawczuk M. and W. Ostachowicz, "Modelling and vibration analysis of a cantilever composite beam with a transverse open crack," Journal of sound and vibration, 183(1): p. 69-89, 1995.
[18] Nayak S.S., "Vibrational analysis of a simply supported beam with crack," 2013.
[19] Guo Z. and Z. Sun, "Multiple cracked beam modeling and damage detection using frequency response function," Structural Longevity, 5(2): p. 97-106, 2011.
[20] McCarthy M., et al., "Three-dimensional finite element analysis of single-bolt, single-lap composite bolted joints: part I—model development and validation," Composite Structures, 71(2): p. 140-158, 2005.
[21] Nassar S.A. and A. Abboud, "An improved stiffness model for bolted joints," Journal of Mechanical Design, 131(12): p. 121001, 2009.