Hilbert schemes of lines and conics
and automorphism groups of Fano threefolds

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Abstract. We discuss various results on Hilbert schemes of lines and conics and automorphism groups of smooth Fano threefolds of Picard rank 1. Besides a general review of facts well known to experts, the paper contains some new results, for instance, we give a description of the Hilbert scheme of conics on any smooth Fano threefold of index 1 and genus 10. We also show that the action of the automorphism group of a Fano threefold $X$ of index 2 (respectively, 1) on an irreducible component of its Hilbert scheme of lines (respectively, conics) is faithful if the anticanonical class of $X$ is very ample except for some explicit cases.

We use these faithfulness results to prove finiteness of the automorphism groups of most Fano threefolds and classify explicitly all Fano threefolds with infinite automorphism group. We also discuss a derived category point of view on the Hilbert schemes of lines and conics, and use it to identify some of them.

Keywords and phrases: Fano variety, Hilbert scheme, automorphism group, line, conic, derived category

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