Towards a performance bound on MIMO DFRC systems

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Abstract—It is a fundamental problem to analyse the performance bound of multiple-input multiple-output (MIMO) dual-functional radar-communication (DFRC) systems. To this end, we derive a performance bound on the communication function under a constraint on radar performance. To facilitate the analysis, we consider a toy example, in which there is only one down-link user with a single receive antenna and one radar target. In such a simplified case, we obtain an analytical expression for the performance bound and the corresponding waveform design strategy to achieve the bound. The results reveal a tradeoff between communication and radar performance, and a condition when the transmitted energy can be shared between these two functions.

Index Terms—DFRC, MIMO, performance bound, radar signal-to-noise ratio (SNR), channel capacity

I. INTRODUCTION

There is growing demand for implementing both radar and communication in one system [1]–[4] due to their similarities in both signal processing algorithm and hardware architecture [2, 3]. To this end, dual-functional radar-communication (DFRC) technology has attracted a lot of attention, as DFRC design reduces system overhead and saves spectrum resources [1]–[3]. Among typical DFRC schemes [4]–[9], multiple-input multiple-output (MIMO) DFRC is of great significance because of the benefits introduced by transmit diversity [9]–[11]. Therefore, in this paper we analyse a fundamental performance bound of MIMO DFRC systems.

Many previous works have shown that there exists performance tradeoffs between radar and communication in a DFRC system [4, 12]. However, the results are mostly numerical and are achieved under specific signaling and encoding schemes. Just as researchers study channel capacity as a universal performance bound for pure communication systems [13]–[15], it is important to find a fundamental performance bound for a MIMO DFRC system.

While it is generally difficult to simultaneously analyse the performance bounds of radar and communication, a common approach in existing works is to calculate the theoretical performance bound of one function while constraining the other [12], [16], [17]. In [16], the radar performance limit is considered under a communication requirement: the transmit signal of MIMO DFRC system should be carefully designed such that the down-link user receives the exact desired symbols. However, such strict signalling strategy causes radar performance degradation. Therefore, the MIMO DFRC system considered in [16] does not achieve an optimal performance tradeoff between radar and communication. The authors in both [12] and [17] use channel capacity to evaluate the communication performance limit, not restricted to a certain signalling scheme, under some constraint on the radar performance. However, their performance bounds are obtained by solving complex optimization problems.

In this paper, we aim to provide an analytical expression of the fundamental performance bound of a MIMO DFRC system, such that the tradeoff between radar and communication functions is intuitively revealed. To facilitate the analysis, we consider a toy example in which there is only one down-link user with a single receive antenna and one radar target. Taking achievable communication rate and radar signal-to-noise ratio (SNR) as the communication and radar performance metrics, respectively, we formulate an optimization problem by constraining the radar SNR and optimizing communication performance. We obtain an analytical solution, which implies an optimal transmit design strategy that achieves the corresponding theoretical performance limit.

The remainder of this paper is organized as follows. Section II introduces the system model and formulates a performance bound analysis as an optimization problem. Section III gives an analytical expression of the performance bound. Simulations are performed to verify the analysis in Section IV. Section V draws conclusions.
A complex Gaussian distribution with mean \( \mu \) of the AWGN by setting additive white Gaussian noise (AWGN) of the communication vectors, boldface uppercase letters for matrices. Superscripts \( \cdot^H \) and \( \cdot^\ast \) represent Hermitian transpose and conjugate respectively, and \( \text{tr} (\cdot) \) stands for the trace of a matrix. The \( N \)-dimensional complex Euclidean space is expressed as \( \mathbb{C}^N \). A complex Gaussian distribution with mean \( \mu \) and covariance \( \Sigma \) is expressed as \( \mathcal{C}N (\mu, \Sigma) \). The statistical expectation is represented by \( E \{ \cdot \} \). \(| \cdot | \) and \( \| \cdot \|_2 \) denote absolute value and Euclidean norm, respectively.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a theoretical performance bound of a MIMO DFRC system. To this end, we first introduce the system model, the communication and radar performance metrics as well as the transmit power constraint. Then, we formulate the fundamental performance bound as an optimization problem.

A MIMO DFRC system simultaneously performs MIMO communication and MIMO radar functions, whose waveform is optimized to meet requirements of both radar and communication \( [2] \). Denote by \( M \) the number of transmit antennas, and by \( x (t) \in \mathbb{C}^M \) the transmit waveform. As illustrated in Fig. 1 to facilitate the theoretical analysis, in this paper we consider a toy example where there is only one communication user with a single receive antenna, one point-like radar target located at angle \( \theta \) and no clutter in space. In addition, we assume that the transmit signal is narrow-band and the system uses the same transmit antennas to receive radar returns.

A. Single-user MIMO communication performance

The received signal of the communication user \( y_C (t) \in \mathbb{C} \) is expressed as

\[
y_C (t) = h^H x (t) + n_C (t), \tag{1}
\]

where \( h \in \mathbb{C}^M \) is the channel vector, and \( n_C (t) \in \mathbb{C} \) is the additive white Gaussian noise (AWGN) of the communication receiver. Without loss of generality, we normalize the power of the AWGN by setting \( n_C (t) \sim \mathcal{C}N (0, 1) \).

The achievable rate is a fundamental bound of communication performance. It is determined by the covariance matrix of the transmit signal, given by

\[
R = E \{ x (t) x^H (t) \} \in \mathbb{C}^{M \times M}. \tag{2}
\]

The achievable rate from the transmitter to the communication user is calculated as \( [13] \)

\[
C = \log \left( 1 + \frac{h^H R h}{\sigma^2} \right) = \log \left( 1 + h^H R h \right), \tag{3}
\]

where \( \sigma^2 \) is the power of the AWGN, assumed to be 1.

B. MIMO radar performance

Given the target direction \( \theta \), the signal impinging on the target is \( a_t^H (\theta) x (t) \), where \( a_t (\theta) \in \mathbb{C}^M \) is the transmit steering vector. For a uniform linear array (ULA), \( a_t \) is given by

\[
a_t (\theta) = \begin{bmatrix} 1 & e^{j2\pi \frac{d}{\lambda} \sin \theta} & \cdots & e^{j2\pi (M-1) \frac{d}{\lambda} \sin \theta} \end{bmatrix}^H, \tag{4}
\]

where \( d \) is the interval between adjacent antennas and \( \lambda \) is the wavelength. Then the received signal \( y_R (t) \in \mathbb{C}^M \) is expressed as

\[
y_R (t) = a_0 a_r^H (\theta) a_t^H (\theta) x (t) + n_R (t), \tag{5}
\]

where \( a_0 \) is the target amplitude, \( a_r (\theta) \in \mathbb{C}^M \) is the receive steering vector, \( n_R (t) \in \mathbb{C}^M \) is the AWGN of the radar receiver, i.e. \( n_R (t) \sim \mathcal{C}N (0, I_M) \). We have \( a_r (\theta) = a_t (\theta) \) if we use the same antenna array to receive radar returns. Here we also note that the Doppler frequency and time-delay of the target echo are omitted for simplicity. The received radar echo is beamformed to enhance the SNR, yielding

\[
r (t) = w^H y_R (t) = a_0 w^H a_r^H (\theta) a_t^H (\theta) x (t) + w^H n_R (t), \tag{6}
\]

where \( w \in \mathbb{C}^M \) is the receive beamforming vector.

We use SNR to evaluate the radar performance, which is given by \( [18] \)

\[
\text{SNR} = E \left\{ \frac{|a_0 w^H a_r^H (\theta) a_t^H (\theta) x (t)|^2}{|w^H n_R (t)|^2} \right\}.
\]

Under the constraint \( \| w \|_2^2 = 1 \), the maximal SNR is achieved when \( w = \frac{a_r}{\| a_r \|_2} \). Plugging it into (7) we have

\[
\text{SNR} = \frac{\alpha_0^2 \| a_r \|_2^2 a_t^H R a_t}{\| a_r \|_2^2}. \tag{8}
\]

C. Transmit power constraint

We consider sum-power constraint

\[
E \left\{ \| x (t) \|_2^2 \right\} \leq P, \tag{9}
\]

where \( P \) is the maximal average transmit power. By substituting (2) into (9), we rewrite the power constraint with respect to \( R \) as

\[
\text{tr} (R) \leq P. \tag{10}
\]

Notation: We use boldface lowercase letters for column vectors, boldface uppercase letters for matrices. Superscripts \( \cdot^H \) and \( \cdot^\ast \) represent Hermitian transpose and conjugate respectively, and \( \text{tr} (\cdot) \) stands for the trace of a matrix. The \( N \)-dimensional complex Euclidean space is expressed as \( \mathbb{C}^N \).
D. Problem formulation

We formulate the fundamental performance bound of a MIMO DFRC system as an optimization problem. To this end, we calculate the theoretical performance bound of one function while constraining the other. Particularly, in this paper, we calculate the communication capacity under the SNR constraint of the radar function. This yields the optimization problem

\[
\begin{align*}
\max_{R \succeq 0} & \quad \log (1 + h^H R h), \\
\text{s.t.} & \quad \alpha^2 ||a||^2 R \alpha \geq \text{SNR}_0, \\
& \quad \text{tr}(R) \leq P,
\end{align*}
\]

where SNR\(_0\) is the allowed minimal radar SNR.

Since the objective function \((11a)\) monotonically increases with \(h^H R h\), which is the power of the signal transmitted to the communication user, maximizing \((11a)\) is equivalent to maximizing \(h^H R h\). The radar SNR is directly proportional to \(a^H R \alpha\), which is the power of the signal impinging on the radar target. Hence the constraint \((11b)\) can be rewritten as \(a^H R \alpha \geq \frac{\text{SNR}_0}{\alpha^2 ||a||^2} = \gamma\), where \(\gamma\) is the minimal power impinging on the radar target. In addition, since larger transmit power always leads to higher channel capacity, we let the sum-power constraint \((11c)\) hold with equality. Therefore, optimization problem \((11)\) is equivalently expressed as

\[
\begin{align*}
\max_{R \succeq 0} & \quad h^H R h, \\
\text{s.t.} & \quad a^H R \alpha \geq \gamma, \\
& \quad \text{tr}(R) = P.
\end{align*}
\]

III. Beamforming Design and Performance Analysis

By solving the optimization problem \((12)\), we obtain the channel capacity and the corresponding waveform to achieve it. In this section, we provide the detailed derivations, which yield analytical expressions of the solution, offering insight into the MIMO DFRC system.

A. Optimal beamforming design

Note that the optimization problem \((12)\) is a semi-definite programming (SDP) problem with two linear constraints. According to \(15\), there exists an optimal \(R\) with rank one. Thus, we rewrite \(R\) as

\[
R = cc^H,
\]

where \(c \in \mathbb{C}^M\). Substituting \((13)\) into \((12)\) yields

\[
\begin{align*}
\max_c & \quad ||c^H h||^2, \\
\text{s.t.} & \quad ||c^H a\alpha||^2 \geq \gamma, \\
& \quad c^H c = P.
\end{align*}
\]

Solving the Karush-Kuhn-Tucker (KKT) conditions of \((14)\), we find that \(c\) is expressed as

\[
c = ah + ba^t,
\]

where \(a, b \in \mathbb{C}\) are given by

i) When \(0 \leq \gamma < \frac{P||h^H a\alpha||^2}{||h||_2^2}\),

\[
\begin{align*}
|a| = \sqrt{\gamma} ||h^H a\alpha|| \rho, \\
|b| = 0,
\end{align*}
\]

where the phase of \(a\) is arbitrary.

ii) When \(\frac{P||h^H a\alpha||^2}{||h||_2^2} \leq \gamma \leq P||a^t\alpha||^2\),

\[
\begin{align*}
|a| = \eta, \\
|b| = \sqrt{\gamma} - \frac{||h^H a^t||}{||a^t\alpha||^2} \eta
\end{align*}
\]

where

\[
\eta = \sqrt{\frac{P||a^t\alpha||^2 - \gamma}{||h||_2^2||a^t\alpha||^2 - ||h^H a^t||^2}}
\]

and the phases of \(a\) and \(b\) should satisfy

\[
\arg(a) - \arg(b) = \arg(h^H a^t)
\]

if \(h^H a^t \neq 0\) or can be arbitrary if \(h^H a^t = 0\).

iii) When \(\gamma > P||a^t\alpha||^2\), there is no feasible solution.

The derivation is provided in Appendix A.

The formation of \(c\) indicates that the design of the optimal waveform is actually power allocation between the radar target and the communication user. The coefficients \(a\) and \(b\) represent the amplitudes of resources allocated for communication and radar sensing respectively. In case (i), the requirement for the radar sensing SNR is low and \(|b| = 0\). In this case, we do not need to particularly allocate power to the radar target, as the signal transmitted to the communication user already sheds enough power on the radar target to meet the SNR requirement. In case (ii), the requirement for the radar sensing SNR is higher. In this case, as the SNR requirement increases, \(\gamma\) increases, thereby \(|a|\) decreases and \(|b|\) increases, which means that we need to allocate more power to radar sensing. When \(\gamma = P||a^t\alpha||^2\), \(|a| = 0\), the multi-antenna transmitter works like a phased-array radar and the radar SNR achieves its upper bound. In case (iii), there is no feasible solution, since we can only achieve limited radar SNR with limited transmit power.

When the radar target and the communication user are in the same direction, which means that \(a^t\) is parallel to \(h\), the communication waveform also serves as the radar sensing waveform, and there is no need to allocate the transmit power. When \(a^t\) is orthogonal to \(h\), radar sensing and communication functions operate independently without any energy sharing between each other. The expressions of the power allocated for communication and radar sensing are \(|a|^2||h||^2\) and \(|b|^2||a^t\alpha||^2\) respectively, which satisfy the equation \(|a|^2||h||^2 + |b|^2||a^t\alpha||^2 = P\).
B. Channel capacity analysis

By substituting the optimal solution $R$ into (11a), we obtain the channel capacity with respect to radar SNR threshold $\gamma$ as follows:

i) When $0 \leq \gamma < \frac{P|h^H\alpha_t|^2}{||\alpha_t||^2}$,

$$C = \log \left(1 + P \parallel h \parallel^2\right).$$

The channel capacity is constant.

ii) When $\frac{P|h^H\alpha_t|^2}{||\alpha_t||^2} \leq \gamma \leq P ||\alpha_t||^2$,

$$C = \log \left(1 + \left(\frac{\sqrt{\gamma} |h^H\alpha_t + \sqrt{\beta}|}{||\alpha_t||^2}\right)^2\right),$$

where

$$\beta = \left(P ||\alpha_t||^2 - \gamma\right) \left(||h||^2 ||\alpha_t||^2 - |h^H\alpha_t|^2\right).$$

iii) When $\gamma > P ||\alpha_t||^2$, the transmit power is not enough to meet the radar SNR requirement.

In case (i), since there is no dedicated radar signal (the communication signal simultaneously serves as a probing function), the channel capacity is constant and equals the achievable capacity of the pure MIMO communication system without radar function. In case (ii), we see that $\frac{C}{R} \leq 0$, which means the channel capacity is negatively correlated with the threshold $\gamma$, which indicates the performance tradeoff between radar sensing and communication.

IV. NUMERICAL RESULTS

In this section, we demonstrate the performance of MIMO DFRC systems via numerical simulations. In the simulations, we consider a ULA transmitter with half-wavelength element spacing $d = \frac{\lambda}{2}$, and let the number of the transmit antennas $M = 10$. The radar target is located at angle $\theta = -30^\circ$. The communication signal is transmitted directly from the transmitter to the communication user. Given the communication user direction $\theta_C$, the channel vector $h$ is given by

$$h = \left[1 \ \ e^{j \pi \sin \theta_C} \ \ \cdots \ \ e^{j 9 \pi \sin \theta_C}\right]^H.$$

In the simulation, we use SNR loss to represent the SNR threshold in (23), given by

$$\text{SNR loss} = 10 \log_{10} \left(\frac{\text{SNR}_0}{\text{SNR}_{\text{MAX}}}\right),$$

where $\text{SNR}_{\text{MAX}} = 10^{\frac{20}{10}} P ||\alpha_t||^2$ is the maximal achievable SNR under the sum-power constraint.

We perform the first simulation to show the channel capacity versus radar SNR loss with $\theta_C$ set to $-30^\circ$, $0^\circ$ and $30^\circ$. The results are shown in Fig. 2. Since both (11a) and (12a) are convex and can be solved by the CVX, a package for specifying and solving convex programs [19], [20], we also compare the analytical expressions of the optimal solutions given in Section III with the CVX results, denoted by ‘analytical’ and ‘CVX’, respectively. We find that for the same $\theta_C$, the CVX and analytical expressions yield the same curve, verifying the correctness of the derivation in Section III.

From Fig. 2, the channel capacity increases with the radar SNR loss when the communication user and the radar target are in different directions, $\theta \neq \theta_C$, while the capacity is fixed when $\theta = \theta_C$. We explain the phenomenon under different locations of down-link user and target.

- When $\theta = -30^\circ$ and $\theta_C = 30^\circ$, it holds that $h^H\alpha_t = 0$. In this case, there is no energy sharing between radar and communication functions. As a result, the channel capacity reduces to 0 when the radar SNR loss is 0, because all the transmit energy is allocated to the radar. As the radar SNR loss becomes higher, the channel capacity increases and reaches its upper bound, the channel capacity of a communication-only system with all the transmit power allocated to communication.

- When $\theta = \theta_C$, which means that $\alpha_t$ is parallel to $h$, the transmit power is shared between these two functions. Consequently, by allocating all the transmit power in the desired direction, the channel capacity is fixed and achieves the upper bound, and the radar SNR is also maximized simultaneously.

- When $\theta_C = 0^\circ$, the steering vector $\alpha_t$ is neither parallel nor perpendicular to $h$. Part of the transmit power for communication is also used for probing. Therefore, even when the SNR loss is strictly 0, the capacity is still positive. The capacity curve stays between those of $\theta_C = -30^\circ$ and $30^\circ$.

In the second simulation, we show the transmit beam patterns for different radar SNR losses in Fig. 3a and 3b where $\theta_C$ are set to $-30^\circ$ and $0^\circ$, respectively. In Fig. 3a when the communication user and radar target are in the same direction, $\theta = \theta_C$, the transmit beam pattern is towards $\theta$ for different radar SNR losses. In this case, there is no power allocation between radar and communication, because the communication signal simultaneously serves for probing. In Fig. 3b when $\theta \neq \theta_C$, there are two beams towards $\theta$ and $\theta_C$, respectively, indicating that the transmit power is allocated between radar and communication functions. As the radar SNR
loss increases, less power is allocated to the radar target.

We note that our results are different from those in [16], which states that there is SNR loss for radar when the communication user and the radar target are in the same direction. The reason is that in this paper we use capacity to evaluate the communication performance, not restricting to using a certain coding strategy for communication. However, the signalling scheme in [16] constrains that the down-link user receives desired signals, which causes SNR loss for radar.

V. CONCLUSION

In this paper, we consider a toy example of a MIMO DFRC system where there is only one down-link user with a single receive antenna and one radar target. We take achievable communication rate as the communication performance metric, and radar SNR as the radar performance metric. By constraining radar performance and optimizing communication performance, we formulate an optimization problem to study the performance bound of the system. We find the analytical expressions of the optimal waveform design and corresponding channel capacity, which facilitate the discussion on the performance bound of MIMO DFRC systems. The DFRC system essentially allocates transmit power between radar target and down-link users, revealing the performance tradeoff between these two functions.

APPENDIX A

SOLVING THE OPTIMIZATION (14)

Before solving (14), we analyze the feasibility of the optimization problem first. According to (14c), we have

$$a_t^H c c_t^H a_t \leq \|c_t\|_2^2 \|a_t\|_2^2 = P \|a_t\|_2^2,$$

where the equivalence is achieved when $c$ is parallel to $a_t$. Thus the $\gamma$ in (14b) must satisfy that

$$\gamma \leq P \|a_t\|_2^2,$$

Otherwise there is no feasible solution for (14), which proves the result of case (iii) in Section III-A.

Since (14) satisfies the Slater’s condition, we solve it by considering its KKT conditions.

The Lagrange function of (14) is given by

$$L(c, c^*, \lambda, \mu) = -h^H c c^H h + \lambda \left( \gamma - a_t^H c c_t^H a_t \right) + \mu \left( \text{tr}(cc^H) - P \right),$$

where $\lambda$ and $\mu$ are dual variables. Then its KKT conditions are

$$\begin{align*}
\frac{\partial L}{\partial c} &= -h^H c h - \lambda a_t^H c a_t + \mu c = 0, \\
\text{tr}(cc^H) - P &= 0,
\end{align*}$$

$$\begin{align*}
\gamma - a_t^H c c_t^H a_t &\leq 0, \\
\lambda &\geq 0, \\
\lambda \left( \gamma - a_t^H c c_t^H a_t \right) &= 0.
\end{align*}$$

To solve (28), we first analyse the form of the optimal $c$. Rewrite (28a) as

$$c = \frac{h^H c}{\mu} h + \frac{\lambda a_t^H c}{\mu} a_t,$$

indicating that $c$ is a linear combination of $h$ and $a_t$. Assume that

$$c = ah + ba_t.$$  

We then calculate $a$ and $b$ by substituting (30) into (28).

According to (28a), at least one of $\lambda$ and $\gamma - a_t^H c c_t^H a_t$ equals 0. Consider the case that $\lambda = 0$. Then (29) becomes

$$c = \frac{h^H c}{\mu} h.$$  

Substituting (30) into (28b) and (31) yields

$$\begin{align*}
|a|^2 &h^H h + a^* h^H a_t + ab^* a_t^H h + |b|^2 a_t^H a_t = P, \\
|a|h + ba_t &= \frac{a h^H h + bh^H a_t}{\mu} h.
\end{align*}$$

According to (32b) we find that

$$b = 0.$$  

By substituting (33) into (32a) we obtain that

$$|a| = \frac{\sqrt{P}}{\|h\|_2}.$$
where the phase of $a$ is arbitrary. Substitute the solution of $c$ into (38). We then find that
\[
\gamma \leq \frac{P|H^*a_t|^2}{\|h\|^2},
\]
which proves the result of case (i) in Section III-A.

Next, consider the case that
\[
\gamma - a_t^Hcc^H a_t = 0.
\]
By substituting (30), we rewrite (36) and (29) as
\[
\begin{aligned}
|a|^2a_t^Hhh^* a_t + a^* b a_t^H a h^* a_t \\
+ ab^* a_t^H h a_t^* a_t + |b|^2 a_t^H a_t a_t^H a_t = \gamma.
\end{aligned}
\]
Combining (32a) and (37a) we obtain that
\[
|a| = \eta := \sqrt{\frac{P\|a\|^2 - \gamma}{\|h\|^2 |a_t|^2 - |H^*a_t|^2}}.
\]
From (37b) we find that
\[
\begin{aligned}
\arg(a) &= \arg(a h^H h + b h^H a_t), \\
\arg(b) &= \arg(a a_t^H h + b a_t^H a_t).
\end{aligned}
\]
This infers the the relationship between the phases of $a$ and $b$. When $h^H a_t = 0$, (39) always holds and the phases of $a$ and $b$ are arbitrary. When $h^H a_t \neq 0$, we have
\[
\arg(a) - \arg(b) = \arg(h^H a_t).
\]
This results from the facts that $h^H h$ in (39a) is a real number and $\arg(a) = \arg(a h^H h)$, which implies $\arg(a) = \arg(b h^H a_t)$. Similarly, $\arg(b) = \arg(a a_t^H h)$.

Substituting (38) and (39) into (32a) yields
\[
|b| = \frac{\gamma}{\|a_t\|^2} - \frac{|H^*a_t|^2}{\|a_t\|^2} \eta.
\]
In addition, since $|b| \geq 0$, we have $\gamma \geq \frac{P|H^*a_t|^2}{\|h\|^2}$. From (38), (40) and (41), we prove the result of case (ii) in Section III-A.

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