Density-optimized Intersection-free Mapping and Matrix Multiplication for Join-Project Operations (extended version)

Zichun Huang, Shimin Chen*
SKL of Computer Architecture, ICT, CAS
University of Chinese Academy of Sciences

ABSTRACT
A Join-Project operation is a join operation followed by a duplicate eliminating projection operation. It is used in a large variety of applications, including entity matching, set analytics, and graph analytics. Previous work proposes a hybrid design that exploits the classical solution (i.e., join and deduplication), and MM (matrix multiplication) to process the sparse and the dense portions of the input data, respectively. However, we observe three problems in the state-of-the-art solution: 1) The outputs of the sparse and dense portions overlap, requiring an extra deduplication step; 2) Its table-to-matrix transformation makes an over-simplified assumption of the attribute values; and 3) There is a mismatch between the employed MM in BLAS packages and the characteristics of the Join-Project operation.

In this paper, we propose DIM³, an optimized algorithm for the Join-Project operation. To address 1), we propose an intersection-free partition method to completely remove the final deduplication step. For 2), we develop an optimized design for mapping attribute values to natural numbers. For 3), we propose DenseEC and SparseBMM algorithms to exploit the structure of the Join-Project for better efficiency. Moreover, we extend DIM³ to consider partial result caching and support Join-aggregate queries, including Join-Aggregate and MJP (Multi-way Joins with Projection). Experimental results using both real-world and synthetic data sets show that DIM³ outperforms previous Join-Project solutions by a factor of 2.3x-18x. Compared to RDBMSs, DIM³ achieves orders of magnitude speedups.

1 INTRODUCTION
A Join-Project operation is a join operation followed by a duplicate eliminating projection operation [1]. Given two tables \( R(x, y) \) and \( S(z, y) \), the Join-Project operation can be written as follows:

\[
P_{x,z}(R(x, y) \bowtie_y S(z, y))
\]

It joins \( R \) and \( S \) with \( y \) as the join key, then projects and deduplicates \((x,z)\) tuples. \( \bowtie \) denotes the duplicate eliminating projection.

Example: In the HetRec2011 data set [7], \( U(\text{userID}, \text{bookID}, \text{tagID}) \) table contains tags given by users to books that they read, and \( B(\text{bookID}, \text{tagID}, \text{weight}) \) table records books and their possible tags with weights. The following SQL query recommends books to users based on the tags recorded in users’ reading history:

```sql
SELECT DISTINCT U.userID, B.bookID
FROM U, B
WHERE U.tagID = B.tagID
```

This Join-Project operation joins \( U \) and \( B \) on tagID, then projects and deduplicates using SELECT DISTINCT. The results can be stored by the application for quick user-specific recommendations.

The Join-Project operation is used in a large variety of applications [10], including entity matching, set analytics, and graph analytics. The above is an example of entity matching. Similar examples include finding users who have seen the same movies in the MovieLens data set [15], and discovering co-authors in the DBLP data set [43]. Moreover, if tuple \((x,y)\) represents that set \( x \) contains element \( y \), then the Join-Project operation using \( y \) as the join key obtains all the pairs of sets that intersect with each other. Furthermore, if we interpret tuple \((x,y)\) as an edge between two vertices \( x \) and \( y \) in a graph, then the Join-Project operation can be used to compute all pairs of vertices that are indirectly connected.

1.1 Previous Solutions

Classical Solution. The classical solution to compute the Join-Project in RDBMSs is to first perform the join operation [2, 23, 33], then deduplicate the projected join results using hash tables [24] or other types of indices [14, 17, 25]. The time complexity of the classical solution is \( \Theta(|R| + |S| + |OUT_j|) \), where \(|R|, |S|, \) and \(|OUT_j|\) denote the sizes of input table \( R \), table \( S \), and the join results before deduplication, respectively. This cost is reasonable when the number of duplicates is low. However, the solution is less efficient when \(|OUT_j|\) is much larger than the size \(|OUT_{TP}|\) of the final results after deduplication. For example, \(|OUT_{TP}|\) is 3.7x as large as \(|OUT_{TP}|\) in the HetRec2011 example, while \(|OUT_{TP}|\) is 24x larger than \(|OUT_{TP}|\) in the MovieLens data set. Let \(|X|, |Y| \) and \(|Z|\) denote the number of distinct values in column \( x \), \( y \), and \( z \) in Eqn 1, respectively. Consider the case where \(|X|\) \(=|Y|\) \(=|Z|\) \(\approx n\). \(|OUT_{TP}|\) can be \( O(n^3) \) in the worst case, while \(|OUT_{TP}|\) is only \( O(n^2) \). In other words, the classical solution can spend a lot of time generating the \(|OUT_{TP}|\) join results and then processing them to remove a large number of duplicates.

Matrix Multiplication. Alternatively, the Join-Project operation can be computed using MM. The basic idea is to represent tables \( R(x,y) \) and \( S(z,y) \) as two matrices \( R^{x\times y} \) and \( S^{y\times z} \). Specifically, \( R_{i,j} = 1 \) if and only if tuple \((x_i,y_j)\) \(\in R \) (similarly for \( S \)). Then the multiplication of \( R^{x\times y} \) and \( S^{y\times z} \) gives matrix \( C^{x\times z} \), where \( C_{i,j} = \sum_{k=1}^{y} R_{i,k} S_{k,j} \). A non-zero element \( C_{i,j} > 0 \) in the matrix corresponds to a tuple \((x_i,z_j)\) in the final output of the Join-Project operation. Compared to the classical solution, MM performs the join and the deduplication together. There are efficient MM implementations in BLAS (Basic Linear Algebra Subprograms) packages with advanced techniques [9, 26, 37]. Moreover, there are sub-cubic MM algorithms in theory. The best known is the Coppersmith-Winograd algorithm with \( O(n^{2.373}) \) complexity [12].

Hybrid Solution. Recent studies [1, 10] combine the classical solution and the MM solution based on the observation that the classical
solution performs better when the data is sparse, while MM performs better when the data is dense. Amossen et al. [1] propose an algorithm to partition the data into dense and sparse parts according to degrees of x, y, and z. Then dense MM is employed for the dense parts and the classical solution is used for the remaining parts.

Deep, Hu, and Koutris [10] correct errors in the cost analysis of [1] and implement the algorithm for experimental comparison. We call this state-of-the-art algorithm DHK. DHK has been shown to significantly outperform the classical solution. However, the following problems reduce the efficiency and practicality of DHK:

- **Overlapping outputs between sparse and dense parts:** The Join-Project results computed from the sparse and from the dense parts can overlap. Consequently, a final deduplication step is necessary to deduplicate the overlapping results. Figure 1 shows the breakdown of the run time of DHK for five data sets (cf. Section 5). For all these data sets, DHK partitions the input data into dense and sparse parts. It performs the final deduplication step, incurring significant overhead.

- **Over-simplified assumption for table-to-matrix transformation:** The DHK implementation assumes that the input columns x, y, and z contain consecutive natural numbers starting from 0. Thus, it directly uses their values as row or column ids in the matrices. However, in reality, database attribute values are rarely natural numbers. A step is missing: mapping values of input columns to consecutive natural numbers.

- **Caveats of MM Implementation:** DHK invokes MM in BLAS packages (e.g., Intel MKL [42]) as a black box. On the one hand, BLAS packages typically implement the $O(n^3)$ MM algorithm. It would be interesting to investigate sub-cubic MM algorithms. On the other hand, the MM invocation as a black box cannot exploit the characteristics of the Join-Project operations for performance improvement.

### 1.2 Our Solution: DIM$^3$

In this paper, we propose an efficient and practical Join-Project algorithm, DIM$^3$ (Density-optimized Intersection-free Mapping and Matrix Multiplication). We address the above problems as follows:

- **Intersection-free partitioning:** We propose a novel partitioning method that divides matrix $S^{xyz}$ into subsets of rows based on the density of $z_j$ rows and then chooses different evaluation strategies for the dense rows and the sparse rows. Since the results $(x_i, z_j)$ of the two parts have different $z_j$s, this method is guaranteed to be intersection-free. Hence, DIM$^3$ completely removes the final deduplication step required by DHK.

- **Optimized mapping design:** We investigate the design of the mapping step in DIM$^3$. First, we identify cases (e.g., auto increment, dictionary encoding) where columns contain roughly natural numbers and thus the mapping step can be skipped. Second, we exploit the mapping of the shared join key columns to perform a semi-join like optimization to discard tuples without matches. Finally, we design a cache-optimized hash-based algorithm to efficiently compute the mappings.

- **Optimized MM algorithms:** We obtain and evaluate an implementation of the sub-cubic Strassen algorithm [40] in a recent study [3]. However, our results show that it cannot beat Intel MKL (cf. Section 5.2). Therefore, we focus on $O(n^3)$ algorithms in DIM$^3$. For the dense rows, we observe that the computation $C_{x_i, z_j} = \sum_{k=1}^{[Y]} R_{x_i, y_k} S_{y_k, z_j}$ can stop early as soon as there is a non-zero $R_{x_i, y_k} S_{y_k, z_j}$. We design the DenseEC (Dense MM with Early stopping Checking) algorithm. For the sparse rows, we design the SparseBMM (Sparse Boolean MM) algorithm that leverages the CSR (Compressed Sparse Row) [22] format of matrix $S^{xyz}$ as a hash table on the join key $y$ with NO hash conflicts. We also introduce a way to reduce the cost for initializing the deduplication vector.

In addition to addressing the three problems, we extend the Join-Project solution in the following two directions:

- **Partial Result Caching:** We observe that in real-world data sets, computing results for different $z_j$ values often take widely different amounts of time. This motivates us to investigate if caching results for a subset of $z_j$ values is profitable. We consider caching either the original or the complement set of $x$’s given a $z_j$. The caching decision for $z_j$ is based on a score computed from the computation time and the required result space. Our experiments show that partial result caching can effectively reduce the Join-Project computation time with reasonable space cost for most data sets.

- **Support for Join-Op Queries:** We discuss different types of relational operations op, and investigate whether we can leverage the Join-Project algorithm to support Join-op queries. We study two interesting ops in depth: 1) For Join-Aggregate queries [18], we show that DIM$^3$ can be applied with simple modifications; and 2) For MJP (Multi-Way Joins with Projection) queries, a.k.a. conjunctive queries with projection [11], we develop a dynamic programming algorithm to find the optimal query plan that considers pushing down deduplication operations to after the join operations.

### 1.3 Contributions

The main contributions of this paper is threefold. First, we propose DIM$^3$ with intersection-free partitioning, optimized mapping, and DenseEC and SparseBMM algorithms to address the three problems in the state-of-the-art solution (cf. Section 2). Second, we propose partial result caching for the Join-Project algorithm (cf. Section 3) and generalize DIM$^3$ to support Join-op queries (cf. Section 4). Third, we perform extensive experimental evaluation using both real-world and synthetic data sets (cf. Section 5). Experimental results show that DIM$^3$ outperforms previous Join-Project solutions by a factor of $2.3\times18\times$. Compared to commercial and open-source RDBMSs, DIM$^3$ achieves orders of magnitude speedups.
2 DIM³ FOR JOIN-PROJECT

In this section, we first overview the DIM³ algorithm in Section 2.1. Then, we explain the components of the algorithm in detail. Specifically, we present the mapping phase in Section 2.2, intersection-free partitioning in Section 2.3, SparseBMM in Section 2.4, and DenseEC in Section 2.5. We analyze the algorithm and describe the strategy selection criteria in Section 2.6. Finally, we analyze the time complexities of the solutions in Section 2.7.

2.1 Overview

The DIM³ algorithm is depicted in Figure 2 and listed in Algorithm 1. We perform the Join-Project operation on two tables \( R(x, y) \) and \( S(z, y) \). Without loss of generality, suppose \( R \) is the larger table and \( S \) is the smaller table.

DIM³ begins by selecting either the classical or the hybrid solution for evaluating the Join-Project operation. This allows completely avoiding the mapping step, which can have significant cost for highly sparse data. DHK [10] uses a rule-of-thumb condition to choose the classical solution. In comparison, our strategy selection function \( f_1 \) makes better decisions based on the estimated run times of the classical and the hybrid solutions (cf. Section 2.6 and 5.2).

In the hybrid strategy, the first step is to map columns \( x, y, \) and \( z \) to consecutive natural numbers. In this way, an \((x, y)\) tuple in \( R \) (similarly \((y, z)\) tuple in \( S \)) can be converted to an element at row \( x \) and column \( y \) in matrix \( R^{x\times y} \). DHK [10] makes the over-simplified assumption that the input columns contain consecutive natural numbers. We delve into the design of the mapping step to provide general-purpose support for all attribute types.

The second step converts table \( R \) to CSR [22] format, then partitions \( S \) to \( S_{\text{sparse}} \) and \( S_{\text{dense}} \). We propose an intersection-free partition method so that the Join-Project results of \( S_{\text{sparse}} \) and \( S_{\text{dense}} \) can be simply combined without the final deduplication step required by DHK [10]. The partition method uses function \( f_2 \) to decide which \( z \) row is dense. \( f_2 \) will be described in Section 2.6.

The third step designs SparseBMM and DenseEC algorithms for processing the sparse and dense data, respectively. SparseBMM computes the Join-Project results on \( R \) and \( S \). DenseEC multiplies \( R \) and \( S_{\text{dense}} \) to obtain \( R \times S_{\text{dense}} \). In DenseEC, function \( f_3 \) is used to choose the best method for intersecting two bitmaps. The computation of \( f_3 \) will be discussed in Section 2.6.

The final step is to merge \( R \) and \( S \) to obtain the final results. Because of intersection-free partitioning, there is no need to perform an extra deduplication step.

2.2 Mapping

The mapping step maps columns \( x, y, \) and \( z \) to consecutive natural numbers. This can be achieved with a baseline hash-based algorithm. Given a column, the algorithm looks up the values of the column one by one in a hash table. If a value \( v \) does not exist in the hash table, it inserts \((v, x)\), the next consecutive number into the hash table. As a result, every distinct value in the column is assigned a natural number. We can repeat this algorithm for \( x, y, \) and \( z \).

This mapping algorithm can be costly. It creates three hash tables for \( x, y, \) and \( z \), and performs up to \( 2(|R| + |S|) \) hash table lookups and/or inserts. Compared to the hash join of \( R(x, y) \) and \( S(z, y) \), which performs \(|R| + |S|\) hash table accesses, the mapping algorithm pays twice as much cost for hash table visits. This can be a significant additional overhead for the Join-Project algorithm when the join result size is not much larger than the input sizes.

In the following, we consider three opportunities to optimize the baseline algorithm. Then, we extend the mapping to support a wider range of Join-Project operations.

Optimization 1: Skip Mapping. First, all, DIM³ chooses the classical solution for highly sparse data sets as shown in Figure 2. This completely avoids the mapping step. Second, it is possible to skip mapping for columns that already contain natural numbers. There are two common cases in database systems. (i) Columns declared with auto increment (e.g., AUTO_INCREMENT in Oracle, IDENTITY in SQLServer and DB2, SERIAL in PostgreSQL, AUTOINCREMENT in MySQL) contain consecutive natural numbers. (ii) String columns can be encoded by dictionary encoding [21] and stored as natural numbers in database systems (e.g., SAP HANA and MonetDB).

Optimization 2: Reduce Computation with Join Key Mapping. The naïve way to map \( y \) is to use \( R.y \cup S.y \) as the mapping input. We observe that only \( R.y \cap S.y \) contributes to the equality join results. Therefore, we can employ a semi-join like idea, and optimize the mapping of \( y \) as follows. Since \( S \) is the smaller table, we first compute the mapping of \( S.y \) using a hash table. Then we map tuples in \( R \) using the same hash table. If \( y_k \in R.y \) but \( y_k \notin S.y \), then the corresponding \( R \) tuple can be safely discarded because it does not have any matches in \( S \). Note that we choose not to pay the cost of re-scanning \( S \) to remove \( S \) tuples with \( y_k \in S.y \) but \( y_k \notin R.y \). As shown in Figure 2, the first and third \( R \) tuples are filtered out.
Optimization 3: Optimize Hash Table Performance. When a hash table is larger than the CPU cache, hash table accesses result in expensive random memory accesses with poor CPU cache behavior. A hash table visit may probe multiple locations, and dereference pointers (e.g., in the case of chained hash table), incurring significant overhead. Therefore, we employ the following designs to improve the hash table performance in the mapping algorithm. First, we estimate the hash table size for a column (e.g., based on statistics of the number of distinct keys). If the size exceeds the last-level CPU cache, we employ cache partitioning. We use the last k bits of the hash value to divide the data into 2^k partitions so that the hash table of each partition fits into the last-level CPU cache. Then, we compute the mapping for each partition. Second, we employ a linear probing hash table design to avoid pointer dereference. We tune the number of slots and the maximum probing distance to reduce the cost of hash table accesses. If no available slots are found for a given column value, we employ a hash table (Flat_hash_map [39] in our implementation) to store the overflow data.

Supporting Wider Range of Join-Project Operations. We can map not only single attribute but also multiple attributes to consecutive numbers. For instance,

\[ \Pi_{a,b,c,d,e}(R(a, b, c, d, e) \bowtie_{c,d} S(c, d, e, f)) \]

can be treated as

\[ \Pi_{x,z}(R(x,y) \bowtie_{y} S(z,y)) \]

where \( R_x = \{ a, b \} \), \( S_z = \{ c, d, e \} \), and \( R_y = \{ c, d \} \). In this way, we can support any combinations of join keys and projection columns, including operations that contain the join key in the output.

2.3 Intersection-Free Partitioning

Figure 3 compares the partition methods of DHK [10] and DIM\(^3\). DHK makes separate decisions on \( x, y \) and \( z \) according to their degrees (i.e., the number of tuples of the same attribute value). An \( (x, y) \) tuple is added to the dense part only if both its \( x \) and \( y \) attributes are considered as dense (shown as red color in the figure).

In comparison, we propose an intersection-free partition method\(^1\) as shown in Figure 3. It examines table \( S \) and ensures that all tuples with the same \( S_z \) value are in the same partition. Note that table \( R \) will not be partitioned. Essentially, we partition matrix \( S \) by its columns. Our partitioning method has the following benefits.

First, the Join-Project results generated by the sparse and dense parts do not intersect. Given a result \( (x,z) \), if \( z \) is judged as dense, this result must be generated by the dense part. Otherwise, it is generated by the sparse part. In contrast, this property does not hold in DHK. As shown in Figure 3, the result \( (x_2, z_1) \) is generated both in the dense part by joining \( (x_2, y_1) \) and \( (y_1, z_1) \), and in the sparse part by joining \( (x_2, y_3) \) and \( (y_3, z_2) \). Therefore, while DHK must deduplicate the results from the two parts, DIM\(^3\) can eliminate this final deduplication step.

Second, DIM\(^3\) may apply dense MM to more tuples. DHK considers a tuple as dense only if both its attributes are judged as dense. In comparison, DIM\(^3\) makes the partitioning decision based solely on \( S_z \). Since its dense criteria tend to be more flexible, DIM\(^3\) can employ dense MM under more scenarios, as illustrated in Figure 3.

Third, the selection based on \( z \) simplifies the Join-Project computation. In DHK, it is costly (spatially) to record the per-tuple density decisions. Therefore, DHK does not save them. Instead, when processing the sparse part, DHK uses the degree thresholds to re-compute whether a tuple is dense and should be skipped. In comparison, DIM\(^3\) avoids this complexity. As shown in Figure 2, table \( S \) is divided into the sparse and the dense parts based on \( S_z \). Hence, there is no need to re-evaluate the density criteria any more.

Finally, the cost for computing the partition decision in DIM\(^3\) is lower compared to DHK. DHK makes \(|R|+|S|\) decisions on all input tuples. In comparison, DIM\(^3\) makes \(|Z|\) decisions on \( S_z \). The number of decisions to make is much smaller. Consider the case where \( |X|=|Y|=|Z|=n \). In the worst case, the cost is \( O(n^2) \) in DHK, but only \( O(n) \) in DIM\(^3\). Moreover, DHK performs binary search to determine the density threshold, which incurs additional cost.

2.4 SparseBMM

The classical hash-based solution is often used to process the sparse part of the data. We propose a SparseBMM algorithm with two main optimization techniques, as shown in Algorithm 2.

First, we observe that hash table accesses are often one main cost of the classical hash-based computation. Interestingly, since the column values are mapped to natural numbers, the CSR (Compressed Sparse Row) [22] format of matrix \( S^{y\times z} \) is essentially a hash table on the join key \( y \) with NO hash conflicts. The original CSR structure consists of three arrays: \( Val[] \), \( Col[] \), and \( RowPtr[] \). \( Val[] \) and \( Col[] \) contain the value and the column index of non-zero elements in the matrix, respectively. \( RowPtr[] \) points to the row starts in \( Val[] \) and \( Col[] \). In the case of Join-Project, \( Val[] \) contains all 1’s and can be omitted. Therefore, we have two arrays \( Col[] \) and \( RowPtr[] \). We employ matrix \( S^{y\times z} \) as the hash table. Given \( y_k \), we locate all non-zero element \( S_{y_k,z_j} \) by visiting entries \( Col[RowPtr[y_k]] \ldots Col[RowPtr[y_k]+1]-1 \). In fact, \( RowPtr \) serves as the hash bucket header, and \( Col[RowPtr[y_k]] \ldots Col[RowPtr[y_k]+1]-1 \) contain the hash entries in bucket \( y_k \). In this way, we avoid the hash function computation and hash conflicts in common hash table designs.

Second, DHK performs deduplication using a z-vector. For each \( x_j \), it initializes the z-vector to all zeros. Then, it checks all \( (x_j, y_k) \) to compute the join results. For every result \( (x_j, z_j) \), DHK increments the corresponding element in the z-vector. Hence, the non-zero elements in the z-vector indicate the deduplicated Join-Project results. However, the initialization cost is \( O(|Z|) \), while the number of non-zero \( z \)’s can be small for the sparse part of the data. Consequently, the initialization of the z-vector is often a main cost of

\(^1\)Previous work proposes an intersection-free partition method in the context of query enumeration algorithms [11]. It divides the data into two sets with roughly equal number of join results. The method optimizes enumeration delays rather than end-to-end query run times. It is not directly applicable to the Join-Project operation.
the deduplication computation. We remove this per-x initialization cost with a monotonically increasing flag for different x. As shown in Algorithm 2, the SPA array is the z-vector. We initialize the SPA array only once before any computation. The cur_x and cur_z loops (Line 3–7) compute the join results for the given cur_x. For a newly computed join result (cur_x, cur_z), we set SPA[cur_z] to cur_x. If there are multiple duplicate (cur_x, cur_z), SPA[cur_z] is set only once for the first instance. In this way, for the next cur_x + 1, the previous content of SPA is automatically invalid. This saves the cost of initializing SPA in every cur_x loop iteration.

We consider the time and space complexity of SparseBMM. Line 5 of Algorithm 2 runs |OUT_j| times. Thus, the time complexity is Θ(|R| + |S| + |OUT_j|). While this is the same as the classical hash-based solution, SparseBMM reduces the constant factor, accelerating hash table visits and deduplication. Moreover, SparseBMM requires Θ(|R| + |S| + |Z|) space if the final output is consumed by upper level operators. While hash-based deduplication requires Θ(|OUT_j|) space for the hash table, SparseBMM allocates only Θ(|Z|) space for SPA, which is often much smaller than Θ(|OUT_j|).

2.5 DenseEC
To compute C_{x,y,z} = \sum_{k=1}^{Y} R_{x,y,k} S_{y,z,k}, standard dense MM enumerates all pairs of R_{x,y,k} and S_{y,z,k}. We observe that the computation can stop early as soon as there is a non-zero R_{x,y,k} S_{y,z,k}. Therefore, we propose a DenseEC algorithm to leverage this observation, as listed in Algorithm 3.

In Algorithm 3, the first two for-loops enumerate all the pairs of R and S and z. Line 4–11 use one of two methods to check if there is any common y between Bitmap_x and Bitmap_z. The first method uses SIMD to compute the bit-wise AND of Bitmap_x and Bitmap_z (e.g., using _mm256_testz_si256). The second method examines each y related to cur_x in Bitmap_x using a random memory access. If any common y is found, both methods stop early. Generally speaking, the SIMD method performs better when the cur_x row has a large number of y. We determine which method to use with function f_3, which will be discussed in Section 2.6.

Apart from the time saving due to early stopping, DenseEC saves memory space compared to dense MM. DenseEC represents every element as a single bit rather than a 4-byte integer or floating point value in BLAS packages. Moreover, a dense MM invocation would require space to save the temporary output matrix. In comparison, DenseEC never generates the output matrix.

2.6 Evaluation Path Selection Functions
We compute the three functions used in the DIM^2 algorithm to select different evaluation paths. Table 1 lists the symbols used in this subsection. All the listed parameters can be measured in advance.

### Strategy Selection (f_1). DHK chooses the classical solution using a rule-of-thumb condition: |OUT_j| ≤ 20 · N, where N = |R| = |S|. In comparison, DIM^3 determines whether to use the classical solution or the hybrid solution with function f_1. As the classical solution is beneficial only when the data is sparse, we compare mapping + SparseBMM and the classical solution to compute f_1. When f_1 > 0, the classical solution is faster and will be chosen.

\[
f_1(|R|, |S|, |OUT_j|) = (T_{mapping} + T_{SparseBMM}) - (T_{join} + T_{dedup})
\]

\[
= T_{mapping} + (T_{SparseBMM} - T_{join}) - T_{dedup}
\]

\[
= 2(|R| + |S|)T_{map} + |OUT_j|T_{randRW} - |OUT_j|T_{hash}
\]  (2)

Here, T_{mapping} = 2(|R| + |S|)T_{map} from Section 2.2. To estimate T_{SparseBMM} - T_{join}, we see that generating join results with CSR in SparseBMM has smaller or similar cost compared to hash joins. Hence, the difference is mainly the deduplication cost of SparseBMM. This is |OUT_j| * T_{randRW} because SparseBMM performs a random access to the SPA array per join result. Finally, T_{dedup} = |OUT_j|T_{hash} because the classical solution performs a hash table access for deduplicating every join output tuple.

### Bitmap Comparison in DenseEC (f_3). Line 4 of Algorithm 3 uses function f_3 to choose the SIMD or non-SIMD method for comparing Bitmap_x and Bitmap_z. The bits have |Y| hits. Suppose there are m_x and m_z 1’s in Bitmap_x and Bitmap_z, respectively.

In the non-SIMD method, the comparison stops as soon as a check hits a set bit in Bitmap_z. The probability that the check hits a set bit is p_x = m_x / |Y|. The number of checks follows a geometric distribution.

### Table 1: Symbols used in Section 2.6.

| Symbol    | Description                                   |
|-----------|-----------------------------------------------|
| T_{seqR}  | time for sequential memory read               |
| T_{randR} | time for random memory read                   |
| T_{randRW} | time for random memory read-modify-write      |
| T_{hash}  | time of a lookup or insertion to Plain hash table |
| T_{map}   | time to access the optimized hash table for mapping |
| T_{ECs}   | time of a non-SIMD comparison in DenseEC      |
| T_{ECd}   | time of a SIMD comparison in DenseEC          |
with probability $p_x$, and the method performs at most $m_x$ checks. Hence, the expected number of checks is calculated as:

$$\text{Check}_{\text{nonsimd}} = \sum_{i=1}^{m_x} i(1-p_x)^{i-1}p_x + m_x (1-p_x)^{m_x} = \frac{1-(1-p_x)^{m_x}}{p_x}$$

In the SIMD method, every SIMD comparison checks 256 bits in the two bitmaps. When the bits at the same position in the two bitmaps are both set, the comparison returns true and the process stops. The probability that an SIMD comparison returns true is $p_d = 1 - (1 - \frac{m_x m_d}{|S|^2})^{256}$. The number of SIMD checks follows a geometric distribution with probability $p_d$, and the method performs up to $\frac{|S|^2}{m_d}$ checks. Hence, the expected Check$_{\text{simd}} = \frac{1-(1-p_d)^{m_d}}{p_d}$.

Then, we can compute $f_3$ as follows. When $f_3 > 0$, the SIMD method is preferred. Otherwise, the non-SIMD method is selected.

$$f_3(m_x, m_d, |Y|) = \text{Check}_{\text{nonsimd}}_{\text{EC}} - \text{Check}_{\text{simd}}_{\text{EC}}$$

We can reduce the computation overhead of $f_3$ as follows. Note that $m_x$ and $|Y|$ are constants in the for-loop at line 3 of Algorithm 3. $f_3(m_x, m_d, |Y|)$ is monotonically increasing with $m_x$. Therefore, an optimization is to use binary search to find the threshold value of $m_x$, so that $f_3(m_x, m_d, |Y|) = 0$. Then, we can select the SIMD method if $m_x > m_d$. Moreover, we find that multiple R.x.’s can share the same $m_x$. Since $m_d$ is determined for a given $m_x$, we can cache the pairs of $m_x$ and $m_d$, then reuse the computed $m_d$ to avoid redundant binary searches. In this way, the time for computing all $f_3$ thresholds is at most 1.15ms for the real-world data sets in Section 5, which is less than 0.5% of the total run time.

**Dense vs. Sparse Partitions ($f_2$).** At line 6 in DIM$^3$ (Algorithm 1), intersection-free partitioning uses function $f_2$ to determine if $S.z$ column is dense or sparse.

For column $z$, the number of join results can be estimated as $m_z |\text{OUT}_j|$, where $m_z$ is the number of $S$ tuples in column $z$. We denote this value as $\text{OUT}_j$. We compute the cost of processing column $z$ using either SparseBMM and denseEC.

The cost of SparseBMM for $z$ is computed as follows:

$$T_{\text{SparseBMM}} = \frac{(2|X|+|R|)\text{seq} + 2|R|\text{randR}}{|Z|} + 2\text{OUT}_j \times (\text{seq} + \text{randR})$$

The first component computes the costs of the two for-loops at Line 2–3 amortized to one of $|Z|$ columns. The second component estimates the cost of Line 4–7. For DenseEC, we know that the cost for each pair of $x$ and $z$ from the above. Then we can sum this up to obtain $T_{\text{denseEC}} = \sum_{i=1}^{|X|} \min(\text{Check}_{\text{nonsimd}}_{\text{EC}}, \text{Check}_{\text{simd}}_{\text{EC}})$.

Finally, we can compute $f_2$ as follows. When $f_2 > 0$, we consider column $z$ as dense.

$$f_2 = T_{\text{SparseBMM}} - T_{\text{denseEC}}$$

**Effectiveness of the Functions.** Figure 4(a) compares the measured running time and the estimated running time of the classical solution and mapping+SparseBMM. In this experiment, we generate random data with column values in the range of 0 to $10^7$. We vary $|R|=|S|$ from 5 million to 60 million. The X-axis shows the join result size. The Y-axis reports the running time. From the figure, we see that the estimated times show consistent trends compared to the measured times. Function $f_1$ models the intersection of the curves for the two algorithms. We see that the intersection points of the measured and estimated time curves are very close, showing the effectiveness of $f_1$ for strategy selection.

Figure 4(b) compares the measured running time and the estimated running time of the SIMD and non-SIMD methods. From the figure, we see that (i) the estimated values are always smaller than the measured values because certain runtime overheads, such as the for-loop, are not taken into consideration; (ii) the estimated curves show similar trends as the measured curves; and (iii) the intersection points of the two sets of curves are very close.

Figure 4(c) compares the measured running time and the estimated running time of SparseBMM and DenseEC. From the figure, we see that (i) the estimated curves show similar trends as the measured curves; and (ii) the intersection points of the two sets of curves are close. This demonstrates the effectiveness of using function $f_2$ for intersection-free partitioning.

### 2.7 Time Complexity Analysis

Based on the above discussion in Section 2.6, we have the following:

$$\text{Complexity}_{\text{SparseBMM}} = \Theta(|R| + |S| + |\text{OUT}_j|)$$

$$\text{Complexity}_{\text{DenseEC nonsimd}} = \Theta(\sum_{i=1}^{|X|} \sum_{j=1}^{|Z|} T_{\text{Check nonsimd}}(x_i, z_j))$$

$$\text{Complexity}_{\text{DenseEC simd}} = \Theta(\sum_{i=1}^{|X|} \sum_{j=1}^{|Z|} T_{\text{Check simd}}(x_i, z_j))$$

Note that $T_{\text{Check nonsimd}}(x_i, z_j)$ and $T_{\text{Check simd}}(x_i, z_j)$ are the actual numbers of checks given $(x_i, z_j)$, which are determined by the input data distribution.

**Worst-Case Costs.** We compute the worst-case costs. Suppose that input tables $R$ and $S$ do not have duplicates. Then, for SparseBMM, it is clear that $|\text{OUT}_j| < |X||Y||Z|$. For DenseEC, early stopping never happens in the worst cases. Hence, $T_{\text{Check nonsimd}}(x_i, z_j) = \Theta(m_{x_i})$ and $T_{\text{Check simd}}(x_i, z_j) = \Theta(|Y|/\text{SIMD bit length})$.
Therefore, the worst case time complexities are as follows:

\[
\text{Complexity}_{\text{SparseBMM}} = O(|X||Y||Z|)
\]

\[
\text{Complexity}_{\text{DenseEC}_{\text{non-simd}}} = O(|R||Z|)
\]

\[
\text{Complexity}_{\text{DenseEC}_{\text{simd}}} = O\left(\frac{|X||Y||Z|}{\text{SIMD}_\text{bit}_\text{length}}\right)
\]

**Average-Case Costs.** It is difficult to compute the precise formula for average-case costs because the time complexities are affected by the path selection decisions and the density of the input tables. In the following, we analyze the costs under a simplifying assumption of uniform density distribution in the input tables.

We assume that the non-zeros in matrices \(R_{x,y}\) and \(S_{x,z}\) are uniformly randomly distributed, and the densities (probability) of non-zero values are \(\alpha\) and \(\beta\), respectively. Then, we have \(|OUT_j| = \alpha\beta|X||Y||Z|, m_x = \alpha|Y|\) and \(m_z = \beta|Y|\). We also assume that \(|OUT_j| > \max(|R|,|S|)|.

Since all the rows and columns of the input data are of the same density, \(f_j\) considers the entire \(S\) as dense or sparse. Similarly, \(f_j\) selects the same execution path for all pairs of \((x,z)\). That is, all data will be processed by the same algorithm. We can use the expected numbers of checks (\(\text{Check}_{\text{non-simd}}\) and \(\text{Check}_{\text{simd}}\)) computed in the previous subsection to estimate \(\text{Check}_{\text{non-simd}}\) and \(\text{Check}_{\text{simd}}\). The above formula can be rewritten as follows:

\[
\text{Complexity}_{\text{SparseBMM}} = \Theta(\alpha\beta|X||Y||Z|)
\]

\[
\text{Complexity}_{\text{DenseEC}_{\text{non-simd}}} = \Theta\left(\frac{1 - (1-\beta)^m_x}{\beta}|X||Z|\right)
\]

\[
\text{Complexity}_{\text{DenseEC}_{\text{simd}}} = \Theta\left(\frac{1 - (1-\alpha)\beta}{1 - (1-\alpha)\beta|\text{SIMD}_\text{bit}_\text{length}}|X||Z|\right)
\]

The classical hash solution shares the same complexity with \(\text{SparseBMM}\) (i.e., \(\Theta(\alpha\beta|X||Y||Z|)\)).

The complexity of dense MM is \(\Theta(|X||Y||Z|^{\omega}+3)\), where \(\omega = \min\{|X|,|Y|,|Z|\}\) [10]. Though the best known \(\omega\) value can be 2.373 [12], these theoretical MM algorithms are often considered impractical [29, 35]. And in the widely used packages for MM (e.g., Intel MKL), \(\omega = 3\).

**Estimating the Costs of Two Special Cases.** We consider two specific cases to better understand the average-case costs. First, \(\alpha = \beta = 1\). That is, the input matrices are full. In this very dense case, the time complexity of \(\text{SparseBMM}\) is \(\Theta(|X||Y||Z|)\), while the costs of \(\text{DenseEC}_{\text{non-simd}}\) and \(\text{DenseEC}_{\text{simd}}\) are both \(\Theta(|X||Z|)\). Thanks to the early stopping technique, \(\text{DenseEC}\) is significantly faster than the other algorithms (including Dense MM).

Second, \(\alpha = \beta = \frac{1}{|T|}\). This is a very sparse case. The complexity of \(\text{SparseBMM}\) is \(\Theta\left(\frac{|X||Z|}{|T|}\right)\). No-simd will be chosen for the very sparse case. The complexity of \(\text{DenseEC}_{\text{non-simd}}\) is \(\Theta(|X||Z|)\). Therefore, \(\text{SparseBMM}\) is the best solution.

3 PARTIAL RESULT CACHING

We observe that computing Join-Project results for different \(z\) values can take widely different amounts of time for real-world data sets. Figure 5 depicts the CDF (Cumulative Distribution Function) of computation cost for two representative real-world data sets. In HetRec, the top 1% of \(z\) values take 60% of the total time. In MV2, the top 10% of \(z\) values contribute to 23% of the total cost. This motivates us to study caching the results of a subset of \(z\) values to improve Join-Project performance. Note that our technique is different from materialized views [28] or query result cache [27], where the full query results are cached.

**DIM\(^3\) with Cached Partial Results.** We choose to cache results based on \(z\) values because the intersection-free partitioning method divides table \(S\) according to the \(S_z\) values. Hence, it is easy to integrate the cached partial results into the DIM\(^3\) algorithm.

Suppose \(Z_{\text{cached}}\) is the subset of \(S_z\), whose Join-Project results are cached. That is, the cached partial results are \(R_{\text{cached}} = \{ (x,z) \mid (x,z) \in \Pi_{x,z}(R(x,y) \lor_{x} S(z,y)) \land z \in Z_{\text{cached}} \}\). In Figure 2, we can simply omit any \(z \in Z_{\text{cached}}\) when generating \(S_{\text{sparse}}\) and \(S_{\text{dense}}\), then keep the other steps of DIM\(^3\) unchanged. Finally, we output \(R_{\text{cached}}\) in addition to the results computed by DIM\(^3\).

**Caching Score.** Given a caching space budget \(B\) for a Join-Project query, we rank \(z\) values for caching with the following score:

\[
\text{CacheScore}(z) = \frac{\cos t(z)}{\text{space}(z)}
\]

\(\text{space}(z)\) is the cache space required to store the \((x,z)\) results for the given \(z\). The cache content is \((z, \text{size}, x_1, x_2, ..., x_k)\). When \(k\) is small, \(\text{size} < B\), and we cache the original results. When \(k > 0.05|X|\), we store the complement set of \(x\)’s to save space, and set \(\text{size} = k - |X| < 0\). Hence, \(\text{space}(z) = 2 \times \text{min}(k, |X| - k)\).

\(\cos t(z)\) is the time for computing \((x,z)\) results for the given \(z\). This is the benefit from caching \(z\). Since the processing related to \(z\) is distributed into many inner loops in SparseBMM and DenseEC, we cannot directly measure \(\cos t(z)\). Instead, we collect statistics and estimate the cost as follows:

\[
\cos t(z) = n_{\text{parse}}(t_{\text{seq}} + t_{\text{rand}} + n_{\text{simd}}t_{\text{ECd}} + n_{\text{non-simd}}t_{\text{ECs}})
\]

where \(n_{\text{parse}}\) is the number of times that \(z\) is checked in the inner loop of the SparseBMM algorithm, \(n_{\text{simd}}\) and \(n_{\text{non-simd}}\) are the number of times that \(z\) is encountered in the SIMD and non-SIMD part of the inner loop of the DenseEC algorithm, respectively.

\(\text{CacheScore}(z)\) shows the benefit per unit space for caching \(z\). The higher the \(\cos t(z)\), the lower the \(\text{space}(z)\), the more beneficial to cache \(z\). Interestingly, \(z\) with high \(\cos t(z)\) may produce a large number of results. This actually leads to small \(\text{space}(z)\) when the complement result set is saved.

**Practical Considerations.** We discuss several practical issues for using the partial caching. First, database users can enable Join-Project caching dynamically. We see in experiments that the statistics collection and computation for \(\cos t(z)\) and \(\text{space}(z)\) do not
incurs significant overhead for \( \text{DIM}^3 \). Hence, after caching is enabled, the first run of \( \text{DIM}^3 \) can compute \( \text{CacheScore}(z) \) and populate the partial result cache. Then, subsequent runs of \( \text{DIM}^3 \) on the same tables can leverage the cached results to improve performance. Second, when the underlying tables are modified by insert/delete/update, we can simply invalidate the cache and let the next run of \( \text{DIM}^3 \) to re-populate the result cache. Note that updating the cached results (similar to the view maintenance problem [4]) is beyond the scope of this work. Finally, we can use the cached result to support filter predicates on \( x \) and/or \( z \). However, if there are filter predicates on \( y \), we have to run \( \text{DIM}^3 \) without the cached result.

4 SUPPORT FOR JOIN-OP QUERY TYPES

In the above, we focus on the Join-Project operation. Join-Project is a special case of Join-op queries, where \( \text{op} \) is a common SQL operation. An interesting question arises: Is it possible to apply \( \text{DIM}^3 \) to Join-op queries in general? We consider \( \text{op} \) other than projection in the following:

- **Group-by Aggregation**: The group-by operation inherently removes duplicates. A join followed by a group-by aggregation operation, which we call Join-Aggregate for simplicity, implicitly performs a Join-Project operation. We describe how to extend \( \text{DIM}^3 \) to support Join-Aggregate in Section 4.1.
- **Join**: Multiple join operations are followed by a duplicate eliminating projection. One strategy to evaluate this type of queries is to employ Join-Project to deduplicate the results of the last join operation. But can we do better? We consider how and when to push the deduplication down in the query plan of MJP in Section 4.2.
- **Selection or Sorting**: As Join-Selection or Join-Sorting do not require deduplication, there is no need to employ Join-Project for these types of queries in general. However, in special cases where there are too many duplicates, an alternative evaluation strategy can be more efficient. We compute Join-Project and keep the duplicate count for each generated join result tuple. Then, the selection or sorting operation processes the much smaller, deduplicated join result, thereby achieving better performance. Finally, we output the correct number of duplicates based on the per-tuple duplicate counts.
- **Intersection/Difference/Union**: First, the computation of intersection is similar to a join operation. Hence, for Join-Intersection, we can employ MJP. Second, set difference can be evaluated as a left-outer join followed by deduplication. We can modify \( \text{DIM}^3 \) to compute Outer-Join-Project for Join-Difference. To support outer-joins, \( \text{DIM}^3 \) can be extended with a bitmap for \( x(z) \). It sets a bit if the corresponding \( x \) row (z column) has generated join results. In this way, the modified \( \text{DIM}^3 \) can compute \( x(z) \) with no matches for outer-joins. Finally, Join-Union requires the deduplication of the outputs from two joins. We can push the deduplication operation down in the query plan, and apply similar considerations as in Section 4.2.

4.1 Join-Aggregate

Our \( \text{DIM}^3 \) algorithm can be applied to Join-Aggregate operations with slight modifications. Without loss of generality, we divide Join-Aggregate operations into two categories based on the group by attributes: i) group-by attributes are from both tables, and ii) group-by attributes come from one table.

**Group-by Attributes from Both Tables.** For instance, given tables \( R(x, y, z) \) and \( S(z, y, u) \), we want to compute

\[
x.z \text{aggregate}(f(R(x, y, z))) (R(x, y, z) \bowtie_y S(z, y, u))
\]

This task is similar to the original Join-Project operation. The main difference is that it computes \( \text{aggregate}(f(R(x, y, z))) \) on the join results with the same \((x, z)\) rather than deduplicating the results.

We modify \( \text{DIM}^3 \) to support this task. In SparseBMM, we change the SPA array to contain the aggregate for each \( cur_x \). SPA is initialized in each outer-loop iteration. Line 5–7 is modified to accumulate the aggregate for group \((cur_x, cur_z)\). In DenseEC, bitmaps cannot support the aggregates. Therefore, we use a plain dense MM, while computing aggregates for each pair of \((cur_x, cur_z)\).

**Group-by Attributes from One Table.** Given tables \( R(x, y) \) and \( S(z, y) \), we want to compute the following query. If the group-by attribute is from \( S \), we switch table \( R \) and \( S \):

\[
x \text{aggregate}_1(R(x, y) \bowtie_y S(z, y))
\]

We can rewrite the query as follows:

\[
x \text{aggregate}_2'(z') (R(x, y) \bowtie_y (y \text{aggregate}_1'(z) (S(z, y))))
\]

We first compute the group-by aggregate on \( S \) with \( y \) as the group-by key. If \( \text{aggregate} \) is \( \text{sum} \), \( \text{min} \), or \( \text{max} \), then \( \text{aggregate}' \) and \( \text{aggregate}'' \) are the same. For \( \text{count} \), \( \text{aggregate}' \) is \( \text{count} \) and \( \text{aggregate}'' \) is \( \text{sum} \). For avg, the \( \text{aggregate}' \) consists of both \( \text{sum} \) and \( \text{count} \). Then \( \text{aggregate}'' \) accumulates the two components, and finally computes a division to obtain the avg. We denote the resulting table as \( SG(y, z') \), where \( z' \) is the intermediate aggregate value(s) for \( y \). Note that \(|SG| = |Z|\) is often much smaller than \(|S|\). SG is highly sparse because there is no duplicate \( y \)'s in the table. Therefore, we employ the classical hash-based algorithm to join \( R \) and \( SG \) then compute the final group-by aggregates.

4.2 MJP (Multi-Way Joins with Projection)

So far, we have been focusing on Join-Project on two tables. In this subsection, we study deduplication on the results of joining multiple tables. This is also called conjunctive queries with projection [11]. For example, the line join projection with \( n \) tables is expressed as:

\[
\Pi_{x_1, x_{n+1}} (R_1(x_1, x_2) \bowtie_{x_2} R_2(x_2, x_3) \bowtie_{x_3} ... \bowtie_{x_n} R_n(x_n, x_{n+1}))
\]

Figure 6 illustrates the query plan tree for evaluating a MJP query. The deduplication operation (\Pi) can be pushed down to after each join operation. While it incurs extra overhead, deduplication reduces the intermediate result size, thereby reducing the cost of subsequent join operations. There are two baseline execution plans. The first plan computes all the joins and then deduplicates the final join results. The second plan deduplicates the results immediately after each join. However, it is easy to construct cases to show neither plan is optimal. Therefore, we need to judiciously place the deduplication operations into the query plan tree.

In the following, we develop a DP (Dynamic Programming) algorithm to find the optimal query plan. To limit the scope of our investigation, we make the following assumptions.

(1) The join order is given by the query optimizer. We focus on the deduplication placement problem. For ease of presentation, we number the tables in the join order as \( R_1, ..., R_k \).
The problem of optimizing both join order and deduplication placement is beyond the scope of this work.

(2) The query plan is in the form of a left-deep tree [20, 38]. Left-deep or right-deep trees are widely used in RDBMSs to process multi-way joins. If a solution produces left-deep trees, it is easy to modify it to support right-deep trees.

(3) Estimated $|OUT_{P_i}|$ and $|OUT_{P_{j+1}}|$, where $i=2, ..., k$, are available. (Please refer to recent work on cardinality estimation for details [6, 16, 30, 36].) Here, $|OUT_{P_i}| = |R_1 \times \cdots \times R_i|$ denotes the size of the intermediate results after joining the first $i$ tables without deduplication. $|OUT_{P_{j+1}}| = |\Pi(OUT_{P_j})|$ denotes the size of the intermediate results after deduplication. For simplicity, we define $|OUT_{P_1}| = |OUT_{P_{j+1}}| = |R_1|$.

(4) Adding deduplication to the $i$-th join reduces the final join result by a factor of $\frac{|OUT_{P_{j+1}}|}{|OUT_{P_j}|}$. That is, $|\Pi(R_1 \times \cdots \times R_i) \join R_{i+1}| \approx \frac{|OUT_{P_{j+1}}|}{|OUT_{P_j}|} R_{i+1}$. This is reasonable since the join input size to the $(i+1)$-th join is reduced by a factor of $\frac{|OUT_{P_{j+1}}|}{|OUT_{P_j}|}$. This assumption simplifies the estimation of the final result size after inserting a deduplication operation. More precise estimation requires collecting more statistics, and may incur much higher cost.

We use $DP_i$ to denote the optimal time for computing $\Pi(R_1 \times \cdots \times R_i) \join R_{i+1}$. We observe that if we already add deduplication to the $i$-th join, the deduplicated result $OUT_{P_{j+1}}$ does not change if more deduplication operations are added to the joins before the $i$-th join. Therefore, we have the following equations:

$$DP_i = \begin{cases} 0, & i = 1 \\ \min_{j=1}^{i-1} \left( DP_j + \sum_{h=j+1}^{i-1} JoinsCost_h + JCPost_h \right), & i > 1 \end{cases}$$

The formula inside $\min$ computes the case where a deduplication is added to the $j$-th join and there is no deduplication from the $(j+1)$-th to $(i-1)$-th join. The total cost of this case has three components: (i) $DP_j$, which is the optimal time for computing $OUT_{P_j}$; (ii) the cost of the subsequent joins, i.e., the $(j+1)$-th to $(i-1)$-th join; and (iii) the cost of the final join project operation. Note that when $j=1$, there is no deduplication before the $i$-th join.

Based on assumption (3), we estimate the input table size for component (ii) and (iii). Specifically, $JoinsCost_h$ is the cost of joining $\Pi(R_1 \times \cdots \times R_j) \times \cdots \times R_h$ and $R_h$. The size of the former is estimated as $\frac{|OUT_{P_{j+1}}|}{|OUT_{P_{j+1}}|} |OUT_{P_{j+1}}|$ and $R_h$. Moreover, $JCPost_h$ is the cost of the Join-Project operation at the $i$-th join. The sizes of the two input tables to the Join-Project operation are $|OUT_{P_{j+1}}| |OUT_{P_{j+1}}|$ and $|R_i|$. The intermediate join result in the Join-Project operation can be estimated as $\frac{|OUT_{P_{j+1}}|}{|OUT_{P_{j+1}}|} |OUT_{P_{j+1}}|$ and the final output size is $|OUT_{P_{j+1}}|$.

Given the sizes of the input tables, the intermediate join result, and the final output, we can use the formulas in Section 2.6 to estimate the cost of the Join-Project operation.

For a MJP query on $n$ tables, the path to get $DP_n$ in the DP process gives the optimal deduplication placement. The time complexity of this algorithm is $\Theta(n^3)$. As $n$ is often not large, the overhead of this DP algorithm is small.

5 PERFORMANCE EVALUATION

In this section, we evaluate the performance of our proposed solutions using both real-world and synthetic data sets.

5.1 Experimental Setup

Machine Configuration. All experiments are performed on a machine with Intel Core i7-9700 CPU (3.00GHz, Turbo Boost 4.70 GHz, 8 cores/8 threads, 12MB last level cache) and 32 GB RAM, running Ubuntu 18.04.5 LTS with Linux 5.4.0-81 kernel. All code is written in C/C++ and compiled with g++ 7.5.0 using -std=c++11, -O3, -mavx flags. MKL (Intel Math Kernel Library) is used in DHK. By default, we run single-threaded experiments. In the parallel experiments, we use OpenMP for parallelization.

Datasets. We use six real-world datasets with different input size, $|OUT_{P_i}|$, and $|OUT_{P_j}|$ in our experiments, as shown in Table 2. Amazon [31] data set records the frequently co-purchased products on Amazon website. The Join-Project queries help to find potential products that can be co-purchased. Slashdot [32] is a technology-related news website. The data set contains friend/foe links between users of Slashdot. The query computes the indirectly connected pairs of users. The data set and the query of HetRec [7] follow the motivating example in Section 1. MV1 and MV2 are two MovieLens [15] data sets. They contain user ratings for movies. Similarly, Jokes [13] contains user ratings for jokes. The Join-Project queries on MovieLens and Jokes data sets find users that have rated the same objects. Friendster [44] is a social network dataset that contains the retweet relationship between users. We use this data set to study MJP, which computes the multi-hop connections between users.

We also conduct experiments on the TPC-H data set with SF=10. A representative query joins LineItem and Orders, then projects on CUSTKEY and SUPPKEY to attain the purchase relationship between customers and suppliers. However, since this join is a primary-foreign key join, $|OUT_{P_i}|$ is similar to the size of LineItem. Both DIM3 and DHK choose the classical solution, showing the same performance. Hence, we do not study TPC-H further.

In contrast, $|OUT_{P_i}|$ and $|OUT_{P_j}|$ in the other data sets are much larger than their input sizes. The higher the $pnz$, the denser the dataset, and thus the more intermediate join results. As we do not assume any special sort order in the input tables of Join-Project, we randomly shuffle the data sets before the experiments.

Solutions to Compare. We compare two categories of solutions: stand-alone Join-Project algorithms, and full-fledged RDBMSs.

We compare the following stand-alone implementations. (1) Classical: The classical solution performs the Radix join [2] then a hash-based deduplication using a flat_hash_map [39]. (2) MKL: The state-of-the-art dense MM in Intel MKL is used to evaluate Join-Project. We implement the baseline mapping algorithm as described.
We find this feature is missing in the DHK code, and implement version 11.39.17: a representative analytical main memory version 10.5.11: two popular open-source RDBMSs. (8) warms up the RDBMSs so that the input tables are loaded into main memory. We run the same query on the target data set twice. The first run performs the deduplication once to show that our proposed algorithms are amenable to parallelization. Partial result caching is disable by default. Every experiment is run five times. We report the average of the five runs. For the MM component, we compare DenseEC, SparseBMM, MM in MKL, and a sub-cubic classical solution when

\[ |\text{OUT}_f| \leq 20 \cdot |\text{R}|, \text{ where } N = |\text{R}| = |\text{S}|. \]

We examine the query plans generated by the RDBMSs and see that they all essentially employ the Classical solution, i.e., a join followed by a deduplication of the intermediate results. Among the RDBMSs, MonetDB is the best when memory is sufficient. However, its performance drops sharply when the required space exceeds the memory size (for MV2 and Jokes). In the case of Jokes, all the RDBMSs fail to perform the deduplication operation entirely in memory. The queries take much longer time because of the disk I/Os for storing and accessing the intermediate results. We stop the experiment when the RDBMS runs out of memory. 'X' shows when MKL/Classical runs out of memory and fails. We label speedups of DIM\textsuperscript{3} over DHK in (a), and over the best RDBMS solution in (b).

5.2 Evaluation for Join-Project Operations

Performance of Stand-alone Solutions on Real-World Data Sets. Figure 7(a) compares DIM\textsuperscript{3} with stand-alone Join-Project algorithms on real-world data sets. We see that DIM\textsuperscript{3} achieves the best performance among all stand-alone algorithms with or without result materialization. Compared to DHK, the state-of-the-art algorithm, DIM\textsuperscript{3} achieves 2.3x–18x improvements. For all the six data sets, DIM\textsuperscript{3} chooses the hybrid strategy. In contrast, for Amazon, DHK chooses the classical solution. The speedup of DIM\textsuperscript{3} over DHK for Amazon shows that our strategy selection function \( f_3 \) is more accurate than the rule-of-thumb condition of DHK.

Moreover, DIM\textsuperscript{3} has smallest memory footprints among all algorithms, as shown in Table 3. We use ‘\texttt{/usr/bin/time -v}’ to measure the peak memory usage (i.e., maximum resident set size). Typically, the final Join-Project results will be consumed by upper-level operators, and therefore we do not allocate space for storing the final results. Compared to Classical, when the data sets are dense, DIM\textsuperscript{3} saves 80%–99% memory because it chooses the hybrid strategy and saves the space required by hash-based deduplication in Classical. The memory usage of Gemm(MKL) is mainly determined by the matrix sizes. MKL runs out of memory for the two sparsest data sets (i.e., Amazon and Slashdot). When the datasets are dense, DIM\textsuperscript{3} saves 68%–99% memory of Gemm(MKL) because DenseEC saves the space of the output matrix, and uses one bit per element rather than 4-byte integers in MKL. As for DHK, one main source of its memory usage is the input and output matrices for dense MM invocation. In comparison, DenseEC significantly saves this memory.

Comparison with RDBMSs on Real-World Data Sets. Figure 7(b) compares DIM\textsuperscript{3} with RDBMSs. We see that DIM\textsuperscript{3} outperforms all the RDBMSs. For data sets with \( |\text{OUT}_f| \gg |\text{R}| + |\text{S}| \), DIM\textsuperscript{3} achieves one to three orders of magnitudes of speedups.

We examine the query plans generated by the RDBMSs and see that they all essentially employ the Classical solution, i.e., a join followed by a deduplication of the intermediate results. Among the RDBMSs, MonetDB is the best when memory is sufficient. However, its performance drops sharply when the required space exceeds the memory size (for MV2 and Jokes). In the case of Jokes, all the RDBMSs fail to perform the deduplication operation entirely in memory. The queries take much longer time because of the disk I/Os for storing and accessing the intermediate results. We stop the

Table 2: Real-world data sets used in experiments.

| Data set      | | \( p_{nz}(R) \) | \( p_{nz}(S) \) | \( |\text{OUT}_f| \) | \( |\text{OUT}_p| \) |
|---------------|---|---------------|---------------|----------------|----------------|
| Amazon        | 1.2M | 1.2M | 0.0018% | 0.0018% | 14M   | 11M   |
| Slashdot      | 905K | 905K | 0.02%    | 0.02%    | 118M  | 81M   |
| HetRec        | 487K | 438K | 0.02%    | 0.03%    | 125M  | 34M   |
| MV1           | 500K | 500K | 2.2%     | 2.2%     | 204M  | 29M   |
| MV2           | 1M   | 1M   | 4.5%     | 4.5%     | 816M  | 35M   |
| Jokes         | 617K | 617K | 25%      | 25%      | 10B   | 622M  |
| Friendster     | 1.8B | 4.2 × 10\(^{-7}\) | – | – | – | – |

Note: \( p_{nz}(M) \) is the percentage of non-zero elements in matrix \( M \).

Figure 7: Join-Project on real-world datasets. (By default, the algorithms count then discard the result tuples. Subscript 'M' denotes a version of an algorithm that materializes the result tuples in memory. 'X' shows when MKL/Classical runs out of memory and fails. We label speedups of DIM\textsuperscript{3} over DHK in (a), and over the best RDBMS solution in (b).)
Table 3: Memory usage of stand-alone Join-Project algorithms.

|                | Amazon | Slashdot | HetRec | MV1   | MV2   | Jokes |
|----------------|--------|----------|--------|-------|-------|-------|
| DIM            | 86MB   | 64MB     | 32MB   | 39MB  | 72MB  | 47MB  |
| DHK            | 426MB  | 70MB     | 168MB  | 68MB  | 132MB | 2.4GB |
| Gemm(MKL)      | >32GB  | >32GB    | 7.1GB  | 211MB | 227MB | 2.4GB |
| Classical      | 426MB  | 3.0GB    | 1.5GB  | 789MB | 1.5GB | 24GB  |

Figure 8: Multi-threaded performance.

queries when they took longer than 1 hour. The figure reports the lower bound (i.e., 3600s) for RDBMSs on the Jokes data set.

As RDBMSs implement the classical solution, we focus on the comparison with stand-alone solutions in the rest of this subsection.

Scalability. We implement the multi-threaded DIM$^3$ with OpenMP by modifying around 20 lines of code. Specifically, we parallelize the outer for-loop in SparseBMM and DenseEC. Since every outer loop iteration focuses on a specific $x$, there is no dependence or contention among the execution of different outer-loop iterations. While more involved changes may better parallelize the algorithm, we find that such simple modifications can already achieve promising results.

We compare DIM$^3$ with DHK using 8 threads in Figure 8(a). We see that DIM$^3$ achieves 2.6×-20× speedups over DHK. Figure 8(b)–(c) show the scalability of the two algorithms. The Y-axis reports the speedup compared to the single-threaded execution of the same algorithm. We see that DIM$^3$ shows good scalability as the number of threads increases from 1 to 8.

Sensitivity Analysis with Synthetic Data Sets. We use synthetic data sets to investigate the Join-Project performance for a wide range of situations. The default parameters in these experiments are as follows. All input columns are generated with uniform distributions. $|R| = |S| = 10^3$, and $|X| = |Y| = |Z| = n = 10^4$. We study the performance impact varying $n$, data skews, and selectivity in Figure 9. Overall, we see that DIM$^3$ performs the best in all the cases.

(1) Varying $n$. Figure 9(a) varies $n$, the number of distinct $x/y/z$, while keeping the number of non-zero elements (i.e. $|R|$ and $|S|$) fixed. As $n$ increases, the input data sets become more and more sparse, and $|OUT_T|$ decreases from $10^3$ to $10^6$. Classical runs faster for more sparse data sets. Gemm(MKL) sees a sharp increase of run time as the matrix dimensions (i.e., $n$) increase. DIM$^3$ wisely selects evaluation strategies based on data density. It employs DenseEC for all data when $n$<10^4. SparseBMM when $n$ ∈ $[10^4, 4 \times 10^5]$, and the classical solution when $n$≥$4 \times 10^5$. In this way, DIM$^3$ achieves the best performance in all cases.

DHK has good performance in the middle range in Figure 9(a) and switches to Classical in the higher range. However, its performance is worse than Gemm(MKL) in the lower range. To understand the reason, we look into the DHK code and find that DHK restricts the degree thresholds (i.e., $\Delta_1$ and $\Delta_2$ [10]) to have $\Delta_2 = \frac{|R| |\Delta_1|}{|OUT_T|}$ in order to reduce the threshold search space. Unfortunately, this narrows the range of considered thresholds. For the lower range when the data is very dense, DHK fails to put all data into the dense part, and thus performs worse than Gemm(MKL). In comparison, DIM$^3$ does not suffer from this problems. DIM$^3$ makes the partition decision based on $z$, which is simple and less error-prone to compute.

(2) Varying data skews. In Figure 9(b), we generate $R.x$ and $S.z$ using the Zipf distribution, and vary the parameter $\alpha$ from 0 to 1. As $R.x$ and $S.z$ become more skewed, $|OUT_T|$ stays around $10^5$, while the deduplicated result size $|OUT_T|$ decreases from $6 \times 10^7$ to $1.4 \times 10^8$. DIM$^3$ partitions the more skewed $z$’s into the dense part as $\alpha$ increases, and achieves a speedup of $\sim 1.5x$ compared to DHK.

In Figure 9(c), we generate the join attribute, $R.y$ and $S.y$, using Zipf distribution, and vary $\alpha$ from 0 to 1. As $R.y$ and $S.y$ become more skewed, the number of intermediate join results $|OUT_T|$ increases sharply from $10^8$ to $4.3 \times 10^9$, and the data sets become increasingly dense. Classical runs slower as the data sets become denser. Gemm(MKL) sees a flat curve because the matrix dimensions are the same. DHK suffers from the threshold search space problem for very dense data sets. DIM$^3$ achieves the best performance in all cases. There is a dip in the DIM$^3$ curve at $\alpha=1$ because early stopping effectively reduces DenseEC computation in this case.

In Figure 9(d), we use R-mat [8] to generate skew data so that the two columns in a table are correlated. We use the R-mat parameters in Graph500 [34] (i.e., $a = 0.57$, $b = 0.19$, $c = 0.19$, and $d = 0.05$). We generate a directed graph with $n$ vertices and $|R|$ edges. The Join-Project finds 2-hop paths in the graph. As R-mat requires $n$ to be a power of $2$, we set $n = 2^{10}$, which is close to the default $10^5$. We vary $|R|$ from $10^6$ to $10^7$. From Figure 9(d), we see that DIM$^3$ performs significantly better than the other algorithms. As $|R|$ increases, the
data set becomes increasingly dense. Classical, Gemm(MKL), and DHK curves show similar trends as in Figure 9(c).

3) Varying selectivity. In Figure 9(e), we consider the cases where there are filtering predicates on both R.x and S.z. We vary their selectivity at the same time from 0 to 1. Figure 9(f) zooms in to the bottom left of the Figure 9(e). As the selectivity decreases, both the input sizes and the output size decrease. Hence, all the algorithms run faster. From the figure, we see that DIM\(^3\) performs the best in all cases. Note that when the selectivity is very small (less than 0.03), DIM\(^3\) switches to the classical solution. When the selectivity is less or equal than 0.2, DHK switches to the classical solution.

Effectiveness of DIM\(^3\) Components. In the following, we use synthetic data sets with the same default parameters as the above.

1) Mapping step. We compare our optimized mapping solution with the baseline using std::unordered_map, and an improved baseline using flat_hash_map. In Figure 10(a), we fix \(|R|=|S|=10^4\) while varying \(n\) from \(10^2\) to \(10^5\). The hash table size increases as \(n\). When \(n < 4 \times 10^4\), the hash table fits into the L3 cache, and thus we do not perform cache partitioning. Our solution has similar performance compared to flat_hash_map. When \(n \geq 4 \times 10^4\), we use cache partitioning to reduce expensive CPU cache misses, thereby significantly out-performing flat_hash_map. As for std::unordered_map, it is much slower because of chained hashing.

In Figure 10(b), we fix \(n=10^6\) while varying \(|R|=|S|\) from \(10^5\) to \(10^7\). The size of the hash table is fixed. The number of hash visits increases. Hence, we see the increasing trends for all curves. Overall, our optimized mapping solution performs the best.

Moreover, we compare DIM\(^3\) (with optimized mapping) and DIM\(^3\) with baseline std::unordered_map for the Amazon data set. The mapping step takes 54.9\% of the Join-Project run time for DIM\(^3\) with baseline mapping. Replacing the baseline mapping with optimized mapping, DIM\(^3\) achieves an improvement of 1.5x.

2) DenseEC. In Figure 10(c), we vary \(n\) from 1000 to 6000, where the data sets are relatively dense and thus dense MM makes sense. We compare DenseEC with dense MM implementations.

We investigate sub-cubic fast MM algorithms. While the best known \(O(n^{2.373})\) algorithm [12] is considered impractical because of its huge constant factors [29, 35], recent work aims to make the Strassen algorithm practical [3]. We obtain and evaluate this

![Figure 10: Effectiveness of DIM\(^3\) Components.](image)

![Figure 11: Partial result caching. (\* shows the point where cached result size = input data size.)](image)

STRASSEN implementation [3]. Our initial results show that it is slower than Intel MKL. One potential reason is that the original STRASSEN supports only floating point MM, while the MKL run uses integer MM. Since the STRASSEN code calls BLAS as the underlying MM sub-routine, it is straightforward to modify the code to call MKL’s integer MM. The resulting integer STRASSEN indeed runs slightly faster, but it is still slower than MKL. Figure 10(c) shows the performance of the improved integer STRASSEN.

Overall, DenseEC out-performs dense MM implementations because early stopping can effectively reduce computation cost.

3) SparseBMM. In Figure 10(d), we vary \(n\) from \(10^3\) to \(10^6\). The generated data sets become more and more sparse, and therefore sparse MM and Classical are more suitable. We compare SparseBMM with Classical and MKL’s sparse MM. From the figure, we see that SparseBMM performs the best.

We also plot the SparseBMM curve in Figure 10(c). The crossing point of SparseBMM and DenseEC is around 4000. This shows why DIM\(^3\) chooses between DenseEC and SparseBMM with function \(f_2\).

5.3 Evaluation for Partial Result Caching

We evaluate partial result caching for DIM\(^3\) using both real-world and synthetic data sets, as shown in Figure 11. The synthetic data sets use the same default parameters as in Section 5.2. Our solution retrieves cached partial results and performs JoinProject between \(R\) and \((z,y)(z,y)\in S \land z \notin Z_{cached}\). The X-axis varies the percentage of \(z\) values that are cached. The Y-axis reports either the run time normalized to DIM\(^3\) without caching, or the cached result size normalized to the size of the total JoinProject results.

Figure 11(a) and 11(b) show the run time and the cached result size on real-world data sets, respectively. For Slashdot and HetRec, the computation time distributions for \(z\) values are very skewed. The space cost is relatively high because most results are cached by their original values. Caching a relatively small percentage (e.g., 20\%) of \(z\) values can significantly speedup DIM\(^3\). On the other hand, the denser data sets (i.e., MV1, MV2, and Jokes) see less skewed computation time distributions for \(z\) values. As the number of results for the same \(z\) is often large, our technique to cache the complement set of result vectors can drastically reduce the cached result size. For
example, the space at 100% (i.e., for caching all results) is only 12%, 6%, and less than 1% of the total result sizes for MV1, MV2, and Jokes, respectively. Thus, we can use a small amount of cache space to achieve significant speedups. Overall, if we cache either 20% of z values or up to the input data size, partial result caching achieves 3.3x–20x improvements over DIM3 without caching.

Figure 11(c) shows the run time on synthetic data sets where R.x and S.z follow Zipf distribution. When Zipf parameter \( \alpha = 0 \), the data is uniformly distributed, and the benefits of caching are lost. As \( \alpha \) increases from 0.25 to 1, the computation time distribution for z values becomes more skewed and caching is more helpful. Caching only 10% of z values reduces the run time by 13% to 57%.

Figure 11(d) shows the run time on R-mat data varying the input table size \( |R| \) from \( 10^6 \) to \( 10^9 \). If we limit the cache space to the size of the input table, partial result caching reduces the run time of DIM3 by 25%–67%.

5.4 Evaluation for Join-OP Query Types

Join-Aggregate. Figure 12(a) compares the Join-Aggregate performance of DIM3 with RDBMSs and the stand-alone hash-based solution on MV1, MV2, and Jokes. We focus on the more complex case where the group-by attributes are from both tables. As described in Section 4.1, if the group-by attributes are from one table, we can rewrite the query and employ the classical solution for efficient evaluation. We compute aggregates on the ratings for the MovieLens and Jokes data sets. (We omit Slashdot and HetRec in this set of experiments because they do not have natural value columns to evaluate. We compute aggregates on the ratings for the MovieLens and Jokes data sets. (We omit Slashdot and HetRec in this set of experiments because they do not have natural value columns to evaluate.) We see that DIM3 performs the best among all solutions. Compared with the hash-based solution, DIM3 obtains a speedup of 18–24x. Compared with the fastest RDBMS solution, DIM3 obtains a speedup of at least 30x.

MJP. We evaluate the effectiveness of our proposed DP algorithm. We compare three DIM3 variants with RDBMSs: 1) DIM3(DP) runs DP to find the optimal query plan; 2) DIM3(EagerDedup) places a deduplication after every join; and 3) DIM3(LazyDedup) performs only one deduplication after all the joins.

In this set of experiments, we use the Friendster data set [44] and compute multi-hop connections between users. Since the original Friendster data set is large and very sparse, as shown in Table 2, we construct subsets of the data set using a parameter range. Given a range, we filter out any tuples whose attributes are both larger than range. From the remaining tuples, we randomly extracted 10 tables, each with \( 10^5 \) tuples, then perform a 10-way join with projection. We set ranges \( 5 \times 10^5, 7 \times 10^5 \) or \( 9 \times 10^5 \). Note that the larger the range, the sparser the input tables. For the three sub data sets, the final output size \( |OUT_P| \) is 46M, 18M, and 8M, and DIM3(DP) decides to place 4, 3, and 2 deduplication operations, respectively.

As shown in Figure 12(b), all DIM3 variants run faster than RDBMSs because of the better Join-Project performance of DIM3. Compared with the hash-based solution with EagerDedup strategy, DIM3(DP) obtains a speedup of 4.9–7.4x. When range \( 5 \times 10^5 \), DIM3(DP) gains a 10x speedup compared to LazyDedup. When range \( 7 \times 10^5 \), DP are 11% and 39% better than EagerDedup and LazyDedup, respectively. When range \( 9 \times 10^5 \), DP is 1.9x as fast as EagerDedup. However, DP is slightly (i.e., \( 5\% \)) slower than LazyDedup. In this case, DP places a deduplication after the first join. The hope is that the amount of computation in subsequent joins will be proportionally reduced, but the actual reduction is not as significant. Overall, we see that DIM3(DP) performs the best among the three variants. This confirms the effectiveness of the DP algorithm for finding the optimal plan for evaluating MJP.

6 CONCLUSION AND FUTURE WORK

In this paper, we propose DIM3 that combines intersection-free partitioning, optimized mapping, and DenseEC and SparseBMM algorithms to improve the state-of-the-art DHK solution. Moreover, we investigate partial result caching and extend DIM3 to efficiently compute Join-op queries. Our results show that DIM3 is a promising solution for the widely used Join-Project operation.

Several promising future directions remain to be explored. The first is out-of-core computation. When the input table \( R \) and \( S \) are too large to fit in the allocated memory, one way is to perform I/O partitioning for \( R \) and \( S \) according to \( R.x \) and \( S.z \), respectively. Then, we load each pair of \( R.x \) partition and \( S.z \) partition into memory and employ DIM3 to compute the results. An alternative way is to partition the tables according to the join key \( y \). However, a final deduplication step is necessary because the intermediate results from different partitions may contain duplicates.

The second direction is to study caching for multiple queries when partial result caching is enabled. In addition to traditional considerations, such as query access statistics, the cache space allocated to a Join-Project query becomes a tunable parameter. This adds a new dimension in the design of the caching strategy.

Last but not least, it is interesting to exploit new hardware to accelerate database operations. For example, GPUs (Graphics Processing Units) [5] and TPUs (Tensor Core Units) [19] can significantly accelerate matrix multiplication in Join-Project. NVMM (Non-Volatile Memory) provides an interesting combination of persistence, capacity, and performance, which can be used to speed up queries with huge memory consumption [41].
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