Do solar neutrinos constrain the electromagnetic properties of the neutrino?

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It is of great interest whether the recent KamLAND bound on the flux of electron antineutrinos from the Sun constrains the electromagnetic properties of the neutrino. We examine the efficiency of the electron antineutrino production in the solar magnetic fields, assuming the neutrinos are Majorana particles with a relatively large transition moment. We consider fields both in the radiative and convective zones of the Sun, with physically plausible strengths, and take into account the recently established values of the oscillation parameters. Our analysis shows that the production rate in question is presently unobservable. In the radiative zone, it is suppressed by the large measured value of the flavor mixing angle which eliminates the resonant level crossing. A corresponding general resonance condition, valid for large as well as small values of the mixing angle, is derived. Likewise, in the convective zone, the strength of the small-scale magnetic field is likely to be insufficient. Thus, no useful bound on the neutrino transition moment can be derived from the published KamLAND bound. KamLAND may be, however, on the edge of probing an "optimistic" scenario, making further improvements of its sensitivity desirable.

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I. INTRODUCTION

One of the most important current tasks in neutrino physics is to understand the full implications of the solar, atmospheric, and reactor neutrino data that became available in the last five years. While much of the recent effort has been focused on establishing the values of the neutrino masses and mixings, there has also been a growing realization that the same data may contain valuable information about other neutrino properties, such as its interactions with matter or with electromagnetic fields. Any measured deviation of these properties from their Standard Model (SM) values would have profound theoretical implications.

As part of this program, we wish to reexamine here the sensitivity of the recent solar neutrino measurements to the neutrino electromagnetic properties.

The idea that the neutrino may possess a relatively large magnetic (transition) moment has been the subject of intense theoretical speculations for many years. It was originally put forth in the early 1970’s, as a possible explanation to the solar neutrino deficit: the solar neutrinos, it was argued, were “disappearing” as a result of the neutrino spin precession in the solar magnetic field. Remarkably, this explanation – much improved with time – remained viable for the next three decades. Only in 2002 did the KamLAND measurement of the reactor antineutrino disappearance conclusively rule it out as the dominant mechanism of the solar neutrino conversion. The possibility of the spin precession happening at a subdominant level, however, remains of great interest to this day, as a probe of the neutrino electromagnetic properties and, at the same time, of the magnetic fields in the solar interior.

For definiteness, here we will focus on the case of Majorana neutrinos. The experimental sensitivity to magnetic spin precession in this scenario is especially high, as has been long recognized: upon the spin flip, a Majorana neutrino turns into an antineutrino and – thanks to flavor oscillations – can then be detected with some probability as an electron antineutrino, $\bar{\nu}_e$. The $\bar{\nu}_e$ flux from the Sun, in turn, is strongly constrained by Super-Kamiokande, SNO, and especially KamLAND which can detect $\nu_e \rightarrow \bar{\nu}_e$ conversion of the solar $^8$B neutrinos at the level of a few hundreds of a percent.

To understand if the KamLAND bound places a nontrivial constraint on the neutrino magnetic moment, we need to compare it to the maximum possible $\bar{\nu}_e$ flux from the Sun. This must be done taking into account (i) the recently measured values of the oscillation parameters, (ii) the bounds on the solar magnetic field strength and (iii) the existing direct bounds on the neutrino magnetic moment. We also need to consider that magnetic spin flip could occur either in the convective zone (CZ) of the Sun, which comprises the outer 30% by radius, or deeper in the radiative zone (RZ). In the first case, the neutrino traverses a region of turbulent magnetic field; in the second case, the field, if present, is smooth and frozen into stationary plasma. The physics of the neutrino evolution is quite different in the two cases and requires separate treatments.

Both the CZ and the RZ cases have been investigated in the literature and in both cases it has been concluded that it was a priori possible to have a signal greater than the current KamLAND sensitivity reach. In the case of the CZ, even a bound on the neutrino magnetic moment was claimed.

In our analysis, we reach different conclusions. In the

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case of the RZ, we find that the usual treatment, based on the $\nu_e \rightarrow \bar{\nu}_\mu$ resonance, works only for small flavor mixing. The large measured value of the solar neutrino mixing angle leads to a qualitatively new effect: the resonant level crossing disappears and no measurable magnetic spin flip can take place. A new analytical result, extending the classical “magnetic resonance” condition to large flavor mixing, is derived here for the first time.

In the case of the CZ, we consider two qualitatively different models of the magnetic field. One of these models, that of the uniform Kolmogorov turbulence, follows \cite{28, 29} and our basic treatment of the neutrino evolution in this case agrees with that work. We, however, take what we regard as more physically justifiable magnetic field parameters. For the second model, we take a scenario of isolated flux ropes as suggested by the recent solar physics research. This scenario, to the best of our knowledge, is considered in the neutrino literature for the first time. The net result is that in both cases we estimate the maximum $\bar{\nu}_e$ flux to be below the present KamLAND bound.

We begin by giving the necessary background information, including the form of the interaction (Sect. II A), the known bounds on the neutrino magnetic moment (Sect. II B), the general properties of the solar magnetic fields (Sect. II C), and the form of the neutrino oscillation Hamiltonian (Sect. II D). The case of the RZ magnetic field is treated in Sect. III after reviewing the conventional approach (Sect. III A), we argue that resonant antineutrino production is suppressed for large flavor mixing (Sect. III B), derive a corresponding general analytical condition (Sect. III C), and finally perform a direct numerical scan of the parameter space to confirm our analytical conclusions (Sect. III D). This is followed by the analysis of the CZ fields in Sect. IV where the two models of the magnetic field mentioned above are considered (in Sects. IV A and IV B). The results are summarized in Sect. V. Some additional technical details about the CZ neutrino evolution can be found in the Appendix.

II. BACKGROUND

A. Interaction

The electromagnetic moment interaction of the Majorana spinors is given by the following dimension five operator

$$L_{EM} = -\frac{1}{2} \mu_{ab} \bar{\Psi}_a \Sigma^{\mu\nu} \Psi^\dagger_b F_{\mu\nu},$$

(1)

where $\Psi_a$ and $\Psi_b$ are Majorana spinors of flavors $a$ and $b$ in the 4-component notation, and $\Sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, with $\mu$ and $\nu$ being the Lorentz indices.

It proves useful to rewrite this Lagrangian in the 2-component Weyl spinor notation. This notation, besides being simply more natural for ultrarelativistic fermions, brings considerable physical insight.

Writing the 4-component Majorana spinor $\Psi$ in the Weyl basis as $\Psi = (\chi_a, \bar{\chi}^a)^T$ and introducing $2 \times 2$ matrices $(\sigma^{\alpha\beta})_{ab} \equiv i(\sigma^\mu \bar{\sigma}^{\alpha\beta} \nu - \sigma^\nu \bar{\sigma}^{\alpha\beta} \mu)/2$, with $\sigma^\mu_{\alpha\beta} \equiv (1, \bar{\sigma})$, $\bar{\sigma}^{\alpha\beta} \nu \equiv (1, -\bar{\sigma})$ (here $\bar{\sigma}$ are the usual Pauli matrices), we rewrite Eq. (1) as

$$L_{EM} = -\frac{1}{2} \mu_{ab} (\chi^a)(\sigma^{\mu\nu})_{ab} \bar{\chi} (\chi^b) F_{\mu\nu} + h.c.$$ 

(2)

This form makes the following important physical properties of the interaction manifest:

- The interaction couples the neutrino and antineutrino states. Notice in Eq. (2) the spinors are either both “undotted” or both “dotted”. Each “undotted” spinor destroys a neutrino state or creates an antineutrino state.

- No flavor-diagonal terms. Since spinors anticommute, the $a = b$ terms in Eq. (2) vanish identically, and hence Majorana neutrinos cannot have flavor-diagonal magnetic moments \cite{33, 34}. This can be easily seen, for example, using the definition of the $(\sigma^{\mu\nu})_{ab}$ matrix given above and the identity

$$\chi^{\alpha\beta} \bar{\sigma}^\beta = \psi^\alpha \sigma^\mu \chi$$

(3)

The transition $(a \neq b)$ moments, however – which lead to a simultaneous spin and flavor change, i.e., $\nu_e \rightarrow \bar{\nu}_\mu$ – are perfectly allowed.

- The interaction with the $z$ (longitudinal) component of the magnetic field vanishes identically, while the interaction with the transverse components $\vec{B}_x, \vec{B}_y$ is nonzero. One of the two spinors in Eq. (2) has a raised index. Denoting it by $\chi \equiv \chi^a = e^{c_{\alpha a}} \bar{\chi}_a$, we write Eq. (2) in the three-dimensional notation as

$$L_{EM} = -\mu_{ab} (\chi^a) \bar{\sigma}(\nu)_b (\vec{B} - i\vec{E}) + h.c.$$ 

(4)

Plugging in $\chi_a \propto \chi^a \propto (0, 1)^T$ – which neglects terms of order $m/E_\nu$ – we find that the interaction picks the 12 component of the matrix $\sigma$, i.e., 1 for $\sigma_x, -i$ for $\sigma_y$, and 0 for $\sigma_z$.

The same conclusion can be reached by boosting the magnetic fields to the neutrino rest frame.

- The real part of $\mu_{ab}$ corresponds to the magnetic dipole moment and the imaginary part to the electric dipole moment. An ultrarelativistic neutrino precesses in an external magnetic field if either the magnetic or electric dipole moments are non-zero \cite{33, 34}. Following the convention, we will henceforth use the term “magnetic moment” to refer to both possibilities.

All these properties are well-known and are derived here for completeness.
B. Size of the transition moment

The laboratory bounds on the neutrino magnetic (transition) moment come from measuring the cross sections of $\nu e^-$ or $\bar{\nu} e^-$ scattering in nearly forward direction. The recent bound for the interaction involving the electrons of the transition moment come from measuring the cross section at the 90% confidence level \( \mu_{\text{neton}} \), where \( \mu_{\text{bind}} \equiv e/(2m_e) \) is the Bohr magneton \((m_e \text{ is the electron mass, } e \text{ is its charge})\). Stronger bounds, \( \mu < 3 \times 10^{-12} \mu_B \), exist from astrophysical considerations, particularly from the study of red giant populations in globular clusters \( [11] \). Larger values of the transition moment would provide an additional cooling mechanism and change the red giant core mass at helium flash beyond what is observationally allowed.

Even allowing for new inputs (such as updated distances to globular clusters) or refinements in the stellar models, it appears extremely unlikely that the red giant transition moment would provide an additional cooling mechanism and change the red giant core mass at helium flash beyond what is observationally allowed.

From the theoretical point of view, the neutrino does possess a nonzero transition moment, already in the SM. It is induced by loop effects: as the neutrino briefly dissociates into virtual charged particles (W and a charged lepton l), those constituent particles couple to the photon. Because the operator in Eq. (2) requires a helicity flip, and because in the SM the W boson couples only to left-handed fields, the mass insertion has to be put on the neutrino line. The resulting moment is proportional to the neutrino mass and hence is highly suppressed, \( \mu_{\nu} \sim eG_F m_\nu \sim 10^{-19}\mu_B (m_\nu/eV) \). In possible extensions of the SM, however, this need not be the case, and the effect can be proportional to the mass of the charged lepton running in the loop. For example, in the left-right symmetric model one obtains \( \mu_{\nu} \sim eG_F m_1 \sin 2\eta \), where \( \eta \) is the left-right mixing parameter in the model.

C. Solar magnetic fields

1. Convective Zone

The CZ of the Sun is known to host significant magnetic fields, which manifest themselves in a variety of effects on the solar surface (sunspots, flares, prominences, etc). These fields are created and destroyed during each solar cycle by a combination of convective motions and differential rotation. Detailed understanding of the solar cycle is still an active subject of research. Nevertheless, the following general features of the CZ fields are well established:

- The magnetic field observed in sunspots is several kG, exceeding the average surface field strength by three orders of magnitude. Sunspots usually come in pairs of opposite polarity and are thought to be manifestations of large-scale magnetic structures residing inside the CZ.
- The total toroidal flux in the CZ at sunspot maximum can be estimated from the total flux that emerges on the surface during the solar cycle, which is around \( 2 \times 10^{25} \text{ Mx} \). Since the same flux tube may be emerging more than once, \( 10^{24} \text{ Mx} \) has been argued to be the likely value \( [12] \). Averaged over the CZ, this gives a field of several kG.
- The turbulent equipartition value for the magnetic field at the base of the CZ can be expressed in terms of the density \( \rho \), the solar luminosity \( L_\odot \), and the distance to the center of the Sun \( r \): \( B_{\text{eq}} \sim \rho^{1/6} L_\odot^{1/3} r^{-2/3} \sim 10 \text{ kG} \) (e.g., \( [14] \)).
- Modern models of the CZ fields argue that fields on the order of 100 kG may exist in the CZ. For the present discussion, it is important to realize that such strong fields, if they exist, can occupy only a small fraction of the CZ volume, such as the thin shear layer near the base of the CZ (‘tachocline’), or be localized in isolated flux tubes \( [14] \). To have even half of the CZ filled with such strong fields would be contrary to the flux arguments mentioned above, as well as basic energetics considerations.

For an in-depth review of the CZ fields and further references, the reader is urged to consult an excellent review \( [14] \). For the status of the present-day simulation efforts see, e.g., \( [12] \).

2. Radiative Zone

Unlike the CZ, the RZ has no internal motions to provide an active mechanism to generate magnetic fields. On the other hand, the conductivity of the RZ plasma is very high, which opens up the possibility that the RZ could support large-scale magnetic fields left over from the early stages of the solar evolution. Let us consider both of these points in turn.

The evidence for the lack of any large-scale mixing motions in the RZ comes from several sources. First, helioseismological studies show that, unlike the CZ, the RZ rotates as a solid body. (This fact by itself, incidentally, provides a hint that at least a weak poloidal field is present in the RZ \( [10] \)). Second, solar model calculations show that the stratification (entropy gradient) in the RZ interior is quite strong, precluding large radial motions. Finally, a very impressive direct confirmation of the lack of radial mixing is provided by the observed Beryllium abundance at the solar surface \( [13] \). Be is destroyed at \( T = 3 \times 10^6 \text{ K} \), which is the temperature at \( r = 0.61 R_\odot \), \( R_\odot \) being the solar radius, \( 6.96 \times 10^8 \text{ km} \). Since Beryllium observed at the solar surface is not noticeably depleted relative to its primordial abundance, the material at \( r \sim 0.7 R_\odot \) (on the bottom of the CZ) and the material at \( r \sim 0.6 R_\odot \) have not been mixed during the lifetime of
the Sun (except for a brief period during the pre-main sequence convection stage).

While a large-scale magnetic field in the RZ is not being presently generated, it is also not destroyed by Ohmic decay over the solar lifetime, 4.7 billion years. The Ohmic decay time for large-scale field configurations is in the billions of years \[130\], in fact for the lowest toroidal mode it is about 24 billion years \[131\]. Over time, a primordial RZ field has been invoked by many authors to explain a variety of effects: from the solar neutrino problem \[132, 133\] to the so-called Princeton solar oblateness measurements \[52\].

What is important for the present analysis is that the field in the RZ, if it exists, must be (i) smooth and (ii) bounded in strength \[134\]. Smoothness follows from the fact that, because of Ohmic decay, any small-scale magnetic field \(l \lesssim R_\odot/10 - R_\odot/20\) features would have decayed away over the lifetime of the Sun. This ensures the adiabatic character of the neutrino evolution throughout the bulk of the CZ. The bounds on the field strength come from several independent arguments, such as the measurements of the solar oblateness and the stability analysis of field configurations. It follows that the amplitude of the large scale toroidal field cannot exceed \(5 - 7\) MG \[135, 136\]. Combined with the bounds on the size of the neutrino transition moment, this limits the value of the magnetic term (see below) in the neutrino oscillation Hamiltonian.

### D. Oscillation Hamiltonian

Since flavor oscillations change the neutrino (and antineutrino) flavor and the magnetic interactions convert neutrinos into antineutrinos, the evolution of the neutrino state is, in general, governed by a 6 Hamiltonian.\footnote{Historically, in discussions of spin-flavor precession (SFP) in the Sun, special attention has been given to the “SFP resonance” region \[37, 38\], defined as the region where two of the diagonal terms in the Hamiltonian \[42, 43\], namely those corresponding to the ee and \(\mu\mu\) entries, become equal. The corresponding resonance condition is}

\begin{equation}
\sqrt{2}G_F(n_e - n_\nu) = \frac{\Delta m^2}{2E_\nu} \cos 2\theta. \tag{9}
\end{equation}

\begin{equation}
\sqrt{2}G_F n_e = \frac{\Delta m^2}{2E_\nu} \cos 2\theta, \tag{10}
\end{equation}

The physical reason for the \(6 \rightarrow 4\) reduction is that the large atmospheric neutrino splitting effectively decouples two of the six states in the case of the smooth magnetic field. It should be noted, however, that the “noisy” field in the CZ enables transitions between states separated by the large splitting. Consequently, in the CZ one may need to consider the full six-neutrino problem \[54\].

#### III. RADIATIVE ZONE

**A. Conventional approach**

Historically, in discussions of spin-flavor precession (SFP) in the Sun, special attention has been given to the “SFP resonance” region \[37, 38\], defined as the region where two of the diagonal terms in the Hamiltonian \[42, 43\], namely those corresponding to the ee and \(\mu\mu\) entries, become equal. The corresponding resonance condition is
values of the oscillation Hamiltonian (5-8) for vanishing

FIG. 1: The diagonal elements of the Hamiltonian for \( \Delta m^2 = 8 \times 10^{-5} \text{eV}^2 \), \( E_\nu = 9 \text{MeV} \). The solid/dashed lines correspond to the neutrino/antineutrino entries.

B. Large mixing and the disappearance of the resonance

The above treatment relies on the notion of resonance, as given by Eq. (9). Fundamentally, this notion comes from the analysis of a two-level system, and even there it applies only in the limit of small mixing [54, 57]. We wish to examine whether it can be applied to the four-level system at hand.

Let us first briefly review the physics behind the standard small-angle MSW effect [58, 59]. Suppose we are given a two-level neutrino system with a Hamiltonian

\[
H_{2\nu} = \begin{pmatrix}
-D & d \\
-d & D
\end{pmatrix}.
\]

(12)

If \( D \gg d \), flavor oscillations are suppressed, as observed already in [58]. Indeed, the mixing angle is given by \( \tan 2\theta = d/D \) and the oscillation amplitude is proportional to \( \sin^2 \theta \). The physical idea behind the resonance is that this suppression is lifted if the diagonal splitting becomes small in some region along the neutrino trajectory, as a result of the crossing of the levels. Suppose \( |D(l)| \lesssim |d(l)| \) over a distance \( \delta l \). Then, inside this interval, \( d \) becomes the dominant part of the Hamiltonian and hence drives oscillations. If \( \delta l \) is sufficiently large, it may be possible to obtain large flavor conversion [72].

With this in mind, let us return to the Hamiltonian in Eqs. (5-8). Now, the electron neutrino state, in addition to being coupled to the \( \bar{\nu}_\mu \) state via the magnetic moment interaction, is also coupled to \( \nu_\mu \) state via the flavor oscillation term. A simple, but crucial observation is the following: for large values of the mixing angle, even if the diagonal \( e\bar{e} \) and \( \mu\mu \) terms are equal, the Hamiltonian is still not dominated by the off-diagonal magnetic terms \( \mu B \), but by the off-diagonal flavor oscillation terms \( \Delta \sin 2\theta \). Thus, there is no reason to expect that the condition in Eq. (9) should automatically lead to large \( \nu_e \rightarrow \bar{\nu}_\mu \) conversion.

This complication is, however, easily removed by going to a basis in which the large off-diagonal flavor oscillation terms are absent. Consequently, we diagonalize \( H_\nu \) and \( H_\bar{\nu} \) given in Eqs. (7) and (8) by performing separate flavor rotations on neutrinos and antineutrinos. We find

\[
H = \begin{pmatrix}
\Delta_{m1} & 0 & \mu_{e\mu} B \sin(\theta_m - \bar{\theta}_m) \\
0 & \Delta_{m2} & -\mu_{e\mu} B \cos(\theta_m - \bar{\theta}_m) \\
\mu_{e\mu} B \sin(\theta_m - \bar{\theta}_m) & -\mu_{e\mu} B \cos(\theta_m - \bar{\theta}_m) & \Delta_{m1} \sin^2(\theta_m - \bar{\theta}_m)
\end{pmatrix},
\]

(13)

magnetic field,

\[
\Delta_{m1} = \frac{A_e + A_\mu}{2} \pm \left[ \left( \Delta \cos 2\theta - \frac{A_e - A_\mu}{2} \right)^2 + \Delta^2 \sin^2 2\theta \right]^{\frac{1}{2}},
\]

(14)

\[
\Delta_{m2} = -\frac{A_e + A_\mu}{2} \pm \left[ \left( \Delta \cos 2\theta + \frac{A_e - A_\mu}{2} \right)^2 + \Delta^2 \sin^2 2\theta \right]^{\frac{1}{2}},
\]

(15)

while \( \theta_m \) and \( \bar{\theta}_m \) are the rotation angles,

\[
\tan 2\theta = \Delta \sin 2\theta / [\Delta \cos 2\theta - (A_e - A_\mu)/2],
\]

(16)

\[
\tan 2\bar{\theta} = \Delta \sin 2\bar{\theta} / [\Delta \cos 2\theta + (A_e - A_\mu)/2].
\]

(17)

The barred quantities refer to antineutrinos.
Let us first discuss the general properties of these equations. When the matter potentials are negligible, $\Delta m_1$ and $\Delta m_2$ both approach $-\Delta$, while $\Delta m_2$ and $\Delta m_3$ both approach $\Delta$. Likewise, in the same limit the rotation angles $\theta_m$ and $\theta_m$ both approach the vacuum value $\theta$. Notice that in vacuum the structure of the off-diagonal magnetic interactions in the mass basis is the same as in the flavor basis. This structure is that of an antisymmetric tensor, Eq. 10, which is invariant under an $SU(2)$ rotation applied to both indices. More generally, this is a manifestation of the fact that the operator in (2) can be written as a product of phase rotations applied to both indices. More generally, this is a manifestation of the fact that the operator in (2) can be written as a product of phase rotations applied to both indices.

For the energies at which KamLAND is sensitive to solar antineutrinos, $E_{\nu} \gtrsim 8.3$ MeV, the magnitude of $2\Delta$ is of order $\sim 8 \times 10^{-3} / (2 \times 10^7) \sim 4 \times 10^{-12}$ eV. By comparison, the value of the off-diagonal magnetic terms is at most of order $10^{-11} \mu_B \times 5 \text{ MG} \sim 3 \times 10^{-13}$ eV, or at least an order of magnitude smaller. This makes the analysis in the mass basis quite straightforward. Clearly, transitions between any two states with diagonal splitting of order $10^{-12} - 10^{-11}$ eV will be suppressed. Conversions between a pair of states with smaller diagonal splitting may take place, provided that $\theta_m$ and $\theta_m$ are sufficiently different, i.e., the matter potential is sufficiently large.

In the top panel of Fig. 2 we show the diagonal elements of the Hamiltonian Eq. (13) for the best-fit values of the oscillation parameters. Obviously, their behavior is quite different from the diagonal elements in the flavor basis, plotted in Fig. 1. In particular, in Fig. 2 there is no level crossing at $R \simeq 0.18 R_\odot$. The mass eigenvalues do not intersect anywhere inside the Sun.

Notice that the top two levels in Fig. 2 get close to each other for $r \gtrsim 0.3 - 0.4 R_\odot$. The difference $\theta_m - \theta_m$ there is small, but non-zero, and a priori could drive a transition between these two levels. The possibility of this transition requires a more detailed study.

The answer comes from considering the adiabaticity of the neutrino evolution. Consider an idealized problem: a neutrino mass eigenstate moving through a magnetic field which is adiabatically turned on and then turned off. The neutrino will be partially converted into the corresponding antineutrino state while inside the field region, but upon exiting the region will revert back to being fully a neutrino.

This idealized description applies in our case: (i) At the production point in the core, for the best-fit KamLAND mass splitting $\Delta m^2 \sim 8 \times 10^{-5} \text{ eV}^2$, the diagonal splitting dominates, suppressing the effects of the magnetic field. The neutrino is produced predominately in the heavy eigenstate (the top line in the upper panel of Fig. 3). (ii) In the region $r \sim 0.2 - 0.5 R_\odot$, the magnetic term drives partial $\nu \leftrightarrow \bar{\nu}$ conversion (transitions between the top two states in the top panel of Fig. 2). (iii) As the effects of the magnetic term are decreased (mainly due to the alignment of $\theta_m$ and $\theta_m$ at smaller densities), the $\nu \leftrightarrow \bar{\nu}$ conversion is largely undone. The adiabaticity through the bulk of the RZ zone is ensured by the physics of the RZ fields: as mentioned in Sec. 11, any structures smaller than $R_\odot/10 - R_\odot/20$ must have decayed away over the lifetime of the Sun.

This behavior is illustrated in Fig. 3 which shows that the electron antineutrino fraction $P$ indeed first grows, but then decreases to unobservable levels, as the effect of the magnetic term decreases in the outer part of the RZ. The residual nonzero value of $P$ is due to the violation of adiabaticity near the top of the RZ.

In summary, except in the limit of small mixing angle, Eq. 10 is not the right condition for describing resonant neutrino-antineutrino conversion. A correct physical criterion should be based on comparing the diagonal elements in the mass basis, not in the flavor basis. This “mass basis level-crossing” would be indeed realized in the Sun, had the value of the mixing angle been small. For the large measured value of the mixing angle, however, the level-crossing is simply absent and the efficiency of the neutrino-antineutrino conversion is very low.
be said that there is a “topological” difference between the level crossing as given by Eq. (21) disappears. It can always, the above condition can be satisfied only for

\[ n = \frac{\pi}{2} \]

\[ \Delta m^2 = \Delta m_2, \]  

and

\[ \Delta m_1 = \Delta m_1. \]  

where the indices “1” and “2” refer to the light and heavy states respectively (the “-” and “+” signs in Eqs. (18,19)).

Written out explicitly, the above two conditions read

\[
a \sqrt{2} G_F (n_e - n_n) = \left[ \left( \sqrt{2} G_F n_e / 2 + \Delta \cos 2\theta \right)^2 + \Delta^2 \sin^2 2\theta \right]^{\frac{1}{2}} - \left[ \left( \sqrt{2} G_F n_e / 2 - \Delta \cos 2\theta \right)^2 + \Delta^2 \sin^2 2\theta \right]^{\frac{1}{2}}, \tag{20}\]

where \( a \) is +1 for the condition in Eq. (18) and −1 for the condition in Eq. (19). Since in the Sun \( n_e - n_n > 0 \) always, the above condition can be satisfied only for \( a = -1 \) for \( \theta > \pi/4 \) and only for \( a = +1 \) for \( \theta < \pi/4 \). In other words, in the “dark side” only the two light levels can intersect, while in the “light side”, as preferred by the data, only the two heavy levels can.

With this in mind, let us solve Eq. (20). We find, in addition to the trivial vacuum solution, \( n_e = n_n = 0 \), a solution

\[
\frac{\Delta m^2}{2 E_\nu} = \sqrt{2} G_F (n_e - n_n) \sqrt{\frac{n_e^2 - (n_e - n_n)^2}{n_e^2 \cos^2 2\theta - (n_e - n_n)^2}}. \tag{21}\]

This is the desired general condition for having resonant antineutrino production in the solar magnetic field. Once again, for \( \theta < \pi/4 \), it solves Eq. (18), while in the opposite case it solves Eq. (19). (It is also worth reminding here that (21) has been derived assuming the neutrino is a Majorana particle.)

We indeed see that Eq. (21) and the conventionally used Eq. (9) agree only for \( \theta = 0 \) and are substantially different for large \( \theta \). In particular, as \( \theta \) is increased beyond

\[
| \cos 2\theta_{\text{crit}} | \simeq (1 - n_n/n_e), \tag{22}\]

the level crossing as given by Eq. (21) disappears. It can be said that there is a “topological” difference between the cases of small and large mixing and Eq. (22) gives the angle at which the topology changes.

The ratio \( n_n/n_e \) varies with the distance to the center of the Sun. Correspondingly, the numerical value of \( \theta_{\text{crit}} \) for which the disappearance takes place varies with \( \Delta \) (which controls the location in the Sun where the level crossing could have happened). In the central region, \( n_n/n_e \approx 1/2 \), while for the outer regions \( n_n/n_e \approx 1/6 \). The corresponding variation in \( \theta_{\text{crit}} \) is

\[
\tan^2 \theta_{\text{crit}} \sim (0.09 - 0.33). \tag{23}\]

Let us finally discuss when each of the level-crossings, (18) and (19), can affect the neutrino evolution. The answer depends on the probability of finding the initial electron neutrino in each of the two neutrino mass eigenstates. This probability, in turn, depends on the flavor mixing angle at the production point. If the matter potential \( \sqrt{2} G_F n_e \) dominates over the vacuum term \( \Delta m^2/2 E_\nu \), the mixing angle at production approaches \( \pi/2 \) and the neutrino is produced almost entirely in the heavy neutrino mass eigenstate. In the opposite limit, the neutrino is produced in the superposition of both mass eigenstates, with the probabilities \( \cos^2 \theta \) and \( \sin^2 \theta \).

The rough boundary between these two regimes for the \( \sim 10 \text{ MeV} \) \( ^8 \text{B} \) neutrinos lies near \( \Delta m^2 \approx 10^{-4} \text{ eV}^2 \), for smaller \( \Delta m^2 \) including the best-fit region of \( \Delta m^2 \approx 8 \times 10^{-5} \text{ eV}^2 \) only the condition in Eq. (18) matters, while in the opposite case both Eq. (18) and Eq. (19) do. For

\[
\Delta m^2 = \Delta m_2, \tag{18}\]

\[
\Delta m_1 = \Delta m_1. \tag{19}\]
$\Delta m^2 \gtrsim 10^{-4} \text{ eV}^2$ we can thus expect sizable antineutrino production both in the “light” and “dark” sides, while for $\Delta m^2 \ll 10^{-4} \text{ eV}^2$ the effect can happen only in the light side.

**D. Large mixing and $\bar{\nu}_e$ production: numerical treatment**

To confirm this analysis, we have carried out a numerical scan of the oscillation parameter space, in the range $0.5 \times 10^{-5} < \Delta m^2 < 3 \times 10^{-5} \text{ eV}^2$. The calculation was carried out for the realistic numerical solar density and chemical composition profiles [52]. For the magnetic field profile, we have taken the lowest toroidal Ohmic eigenmode, as described in [27] and in detail in [49]. While higher eigenmodes could also be present, the choice of the lowest mode is expected to capture the essential physics: the field is axisymmetric, smooth, and vanishes at the RZ/CZ boundary. The only free parameter describing the field in this case is the overall mode normalization. We have performed scans for three different (highly optimistic) normalization values of the magnetic field, $B_{\text{max}} = 0.5, 2, 5 \text{ MG}$, also taking a highly optimistic value of the transition moment, $\mu = 1 \times 10^{-11} \mu_B$.

The results of the scan are shown in Fig. 4. The three colored regions on the left side (small mixing) indicate the regions where the $\bar{\nu}_e$ flux for the chosen normalization values of the magnetic field would exceed the KamLAND bound. The corresponding regions in the “dark” side, $\theta > \pi/4$, are also shown. All the regions agree well with the analytical analysis of the previous Subsection.

The Figure also shows the allowed regions from the KamLAND analysis of their reactor antineutrino spectrum [14], as well as the region that gives a fit to the solar neutrino data (unfilled contours). The best-fit (“LMA-I”) region lies at the intersection of the solar and KamLAND regions and, as can be clearly seen, has no overlap with the regions where the $\bar{\nu}_e$ flux would be measurable.

We note that a similar conclusion was reached in [24],...
using, however, an analytical criterion that differs from Eq. (22). The relatively large \( \bar{\nu}_e \) flux cited in that work was obtained by assuming the field strength of 50 MG at the neutrino production point, which was only later shown to be not allowed \(^{49}\).

IV. CONVECTIVE ZONE

Compared to the case of the RZ, the case of the CZ requires a different treatment. The difference has to do with the turbulent nature of the CZ magnetic fields. Below, for completeness, we briefly review the relevant physics.

The main idea is that random variations \(^{53,64}\) of the magnetic field on scales equal to, or smaller than, the neutrino oscillation length lessen the suppression of the oscillations by a large diagonal mass splitting. While the neutrino follows slow variations with scales longer than the oscillation length \( \lambda_{osc} \sim 1/\Delta \) adiabatically, for shorter variations the adiabaticity condition is violated. The net result of the small-scale “noise” is that the neutrino state vector performs a “random walk” in the oscillation space, gradually receding from its original position.

To be specific, consider a simple model in which the magnetic field changes on scales of \( \lambda_{corr} \lesssim \lambda_{osc} \). The probability of spin flip upon traversing the first correlation cell, \( \lambda_{corr} \), is \( (\mu B \lambda_{corr})^2 \) (simple vacuum oscillation on scales smaller than the oscillation length). Since the fields in different \( \lambda_{corr} \) intervals are uncorrelated, as the neutrino traverses many domains the probabilities, not amplitudes, add up. For solar neutrinos, the experimental bound on \( \bar{\nu}_e \) flux implies \( P \ll 1 \). So long as this is satisfied, the probability of spin flip after traveling the distance \( L \) can be written as \( P \sim (\mu B \lambda_{corr})^2 L/\lambda_{corr} \).

For slow variations with \( \lambda_{corr} \gg \lambda_{osc} \), as already mentioned, the evolution is adiabatic and the conversion probability is exponentially suppressed. One can also imagine a case when the field domains are longer than \( \lambda_{osc} \), but have sharp boundaries. In such a case, the conversion probability is \( P \sim \sin^2 2\theta L/\lambda_{corr} \sim (\mu B/\Delta)^2 L/\lambda_{corr} \). We see that in any case the conversion is most efficient for \( \lambda_{corr} \sim \lambda_{osc} \sim 1/\Delta \), namely, \( P_{opt} \sim (\mu B)^2 \lambda_{osc} \).

Let us apply this to solar neutrinos. Once again, we are interested in the high-energy \(^{8}\)B neutrinos, as these are the neutrinos the KamLAND bound applies to. These neutrinos are produced almost exclusively in the heavy mass eigenstate \( \nu_2 \) and then propagate adiabatically through the RZ. In the CZ, these neutrinos encounter turbulent magnetic fields and, if the transition moment is large, a small fraction of them gets converted into the light antineutrino mass eigenstate \( \bar{\nu}_1 \). The net result is the flux of neutrinos in the heavy eigenstate with possibly a small admixture of the antineutrinos in the light mass eigenstate. The light mass eigenstate, in turn, is detected as an electron antineutrino with probability \( \cos^2 \theta \). The relevant field fluctuations are those on scales of \( \lambda_{osc} \sim 2 \times 10 \text{ MeV}/(8 \times 10^{-5}) \text{ eV}^2 \sim 3 \times 10^2 \text{ km} \). We will denote them by \( B_{\lambda_{osc}} \). Overall, we get

\[
P(\nu_e \rightarrow \bar{\nu}_e) \sim \cos^2 \theta (\mu B_{\lambda_{osc}})^2 \lambda_{osc}.
\]

The oscillation parameters are known, \( \cos^2 \theta \sim 0.7 \), \( \Delta \sim 10^{-11} \text{ eV}(10 \text{ eV}/E_\nu) \). The extent of the region of the strong random field can be estimated to be some fraction of the CZ, say \( L \sim 0.1 R_\odot \). Finally, the main issue is to estimate the size of the amplitude of the small-scale noise \( B_{\lambda_{osc}} \). Unfortunately, the theory and simulation efforts are not yet at the level to give a robust prediction of \( B_{\lambda_{osc}} \) (see \(^{45}\) for the state of the art on the simulation front). We will next consider two models for the magnetic field.

A. Model of “uniform” turbulence

A physically sensible estimate can be obtained by assuming that (i) the field on the large scales is of the order of its turbulent equipartition value; and (ii) that the small-scale noise follows some kind of a scaling law

\[
B_\lambda \propto \lambda^\alpha,
\]

typical of turbulent systems. Such approach was adopted in \(^{28,20}\) and, in our opinion, is a clear improvement over naive models of delta-correlated noise.

The largest scale of the turbulence, at which energy pumping takes place, is estimated to be of order the typical pressure scale height in the CZ. In the inner part of the CZ, the pressure scale height is of order \( L_{max} \sim 5 \times 10^4 \text{ km} \)^{14}. This number can be estimated directly from the solar model \(^{55}\), and is also consistent with the value of the mixing length \( 8 \times 10^4 \text{ km} \) at the base of the CZ given in \(^{65}\). Although the corresponding large-scale eddies reside in the interior of the CZ, they manifest themselves on the solar surface as supergranules \( (l \sim 15 \times 10^4 \text{ km}) \). These should not be confused with surface granules \( (l \approx 10^3 \text{ km}) \) whose size is dictated by the scale height near the surface.

The turbulent equipartition value for the large scale turbulent fields near the bottom of the CZ is estimated as \( B_{L_{max}} \sim \rho_{\odot}^{1/6} L_{\odot}^{1/3} r^{-2/3} \sim 10 \text{ kG} \)^{22}. Putting the numbers together, we find

\[
P(\nu_e \rightarrow \bar{\nu}_e) \sim \cos^2 \theta (\mu B_{L_{max}})^2 L_{L_{max}}.
\]
This means that, for this field model, even with the most optimistic values of the neutrino magnetic moment KamLAND will not have a chance to see antineutrinos from the CZ spin flip unless its sensitivity is improved by at least one order of magnitude.

B. Model of isolated flux tubes

The estimate just given uses a simple model for the turbulent magnetic field. It is possible that the field in the CZ instead has a “fibril” nature [44], i.e., is expelled by the turbulence and combines in isolated flux tubes. It has been argued that the total energy of the CZ (thermal + gravitational + magnetic) is reduced by the fibril state of the magnetic field by avoiding the magnetic inhibition of convection [60].

Let us then consider a model in which the neutrinos travel through spatially separated flux bundles. The conversion probability in this case depends on the transverse size of a bundle. Given that a sunspot typically contains $10^{20}$ Mx [43, 44] and assuming superequipartition field strength inside the bundle of $\sim 100$ kG, one finds the transverse size of the bundle that is actually close to optimal, $\sim 300$ km. The thickness of the flux rope comes out to be close to the oscillation length for the $^8\text{B}$ neutrinos, a remarkable coincidence indeed! Since the total toroidal flux in the CZ is estimated to be of the order of $10^{24}$ Mx [24], a given neutrino encounters only several of such flux bundles. The resulting conversion probability in the most optimistic case is then close to the KamLAND sensitivity bound, as can be easily checked.

V. CONCLUSIONS

We have shown that the large measured value of the flavor mixing angle qualitatively changes the treatment of the neutrino interaction with the magnetic field in the radiative zone of the Sun. The physics of the effect is most transparent in the basis of the mass eigenstates, rather than in the flavor basis. We have shown that in the mass basis, for the allowed values of the oscillation parameters, there is no neutrino-antineutrino level crossing and, as a result, no measurable antineutrino production in the RZ. A correct level crossing condition, Eq. (21), was derived, and a detailed numerical scan carried out to confirm this conclusion.

We emphasize that the recent determination of the neutrino oscillation parameters is crucial for reaching this conclusion: for small flavor mixing angle, it was possible to have large $\nu_e \rightarrow \bar{\nu}_\mu$ conversion even with magnetic fields an order of magnitude below the bound, $B \sim 0.5$ MG [24]. Moreover, as seen in Fig. 4, even a relatively minor shift of the best-fit point, from $\Delta m^2 \simeq 0.8 \times 10^{-4}$ eV$^2$ and $\theta \simeq 32^\circ$ to, e.g., $\Delta m^2 \simeq 2 \times 10^{-4}$ eV$^2$ and $\theta \simeq 24^\circ$, would have made the resonant antineutrino production possible. The field of solar neutrinos has truly entered the precision measurement stage!

One may wonder if our analysis is completely self-consistent. After all, the oscillation parameters we use are found assuming only the flavor oscillations and neglecting possible magnetic $\nu \leftrightarrow \bar{\nu}$ transitions. This, however, is entirely justified, because the KamLAND bound on the $\bar{\nu}_e$ flux ensures that the magnetic transitions can be present only at very small levels and thus cannot affect the fit to the oscillation parameters.

In the convective zone, the solar neutrino propagate through turbulent magnetic fields. Our estimate of the probability of the spin-flavor flip in this case gives a rate that, at present, is likewise not observable. This conclusion disagrees with earlier analyses. The source of the disagreement is not in the treatment of the neutrino evolution, but in the treatment of the solar magnetic fields, which we believe were significantly overestimated in the earlier studies.

Overall, our conclusion is that, despite achieving a clearly impressive level of sensitivity to $\bar{\nu}_e$ from the Sun, KamLAND has not yet reached the level where it would be placing a meaningful constraint on the neutrino transition moment and/or solar magnetic fields. On the other hand, KamLAND has reached the point where it may begin probing the “optimal” scenario, $\mu_\nu \sim 10^{-11} \mu_B$. Clearly, further sensitivity improvements through accumulation of additional statistics is very desirable.

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APPENDIX A: SOME TECHNICAL DETAILS ON THE CONVECTIVE ZONE

As described in Sect. 11, simple physical arguments give the following estimates for the probability of spin-flavor flip in random magnetic fields:

\[ P \sim \begin{cases} \frac{(\mu B_{\text{corr}})^2 L}{\lambda_{\text{corr}}}, & \lambda_{\text{corr}} \ll \lambda_{\text{osc}}, \text{ sharp edge} \text{ (A1)} \\ \frac{(\mu B_{\text{osc}})^2 L}{\lambda_{\text{corr}}}, & \lambda_{\text{corr}} \ll \lambda_{\text{osc}}, \text{ smooth edge} \end{cases} \text{ exp. suppressed,} \]

More generally, the probability of conversion is proportional to a Fourier component of the field correlation function \[ P \approx 2 \mu^2 L \int_0^\infty dl' \langle B(0)B(l') \rangle \cos(2\Delta l'). \] (A2)

If \( \langle B(0)B(l') \rangle \) is a slowly decaying function (compared to \( 1/\Delta \)) with a vanishing slope for small \( l' \), indicating no small-scale noise, for example \( \langle B(0)B(l') \rangle \propto \exp[-(l')^2/\lambda_{\text{osc}}^2] \), the resulting \( P \) is exponentially small, as expected for adiabatic evolution.

The quantity of interest in the isotropic turbulence model is \( B_{\lambda_{\text{osc}}} \sim B_{\lambda_{\text{max}}}(\lambda_{\text{osc}}/\lambda_{\text{max}})^\alpha \). Refs. \[28, 29\] performed an estimate based on a simple Kolmogorov turbulence. Within this model, the effects of the magnetic field on the character of the turbulence are neglected and the simple Kolmogorov scaling law for velocities is applied, \( \nu_s \sim (\epsilon \lambda)^{1/3} \), where \( \epsilon \) is the energy dissipation rate per unit mass. The field was assumed to scale with \( \lambda \) the same way as velocities, \( i.e., B_\lambda \sim \lambda^{1/3} \).

More generally, if the magnetic energy becomes comparable to the kinetic energy of the fluid, the character of the turbulence changes (see, \( \epsilon \), pp. 138-142). The rate of energy transfer to small scales is decided by the interaction of Alfvén waves in plasma. Recent investigations \[6\] argue that in the presence of magnetic fields the turbulence becomes anisotropic in \( k \) space (see \[6\], Appendix A, for a clear summary). While a detailed discussion of the latest models of magnetohydrodynamics turbulence is clearly beyond the scope of this work, the preceding example show that the exponent in the scaling law \[28\] is not known precisely. The correlation function can then be parameterized as \( \langle B(0)B(l') \rangle \propto (l'/L_{\text{max}})^\alpha \lambda_{\text{osc}}(l'/L_{\text{max}})^\alpha \), yielding upon integration

\[ P \approx 2 \mu^2 B_{\lambda_{\text{max}}}(\lambda_{\text{osc}}/L_{\text{max}})^{2\alpha} L/\Delta, \] (A3)

as expected on physical grounds.

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