Robust Adaptive Control for Uncertain Input Delay MIMO Nonlinear Non-Minimum Phase System: A Fuzzy Approach

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ABSTRACT

In this paper, the robust controller design problem of uncertain multi-input multi-output nonlinear non-minimum phase system is discussed. The nonlinear system is suffering from both uncertainty and input delay, so the controller design is difficult. The traditional stable inversion controller is utilized and extended to uncertain case. An integral of past control input is constructed and fuzzy logical system is utilized for approaching the unknown state matrix and input matrix. Then a robust adaptive control strategy is presented. Finally, a numerical simulation on vertical takeoff and landing aircraft is given to show the effectiveness of the proposed method.

INDEX TERMS

Nonlinear non-minimum phase, fuzzy logical system (FLS), parameter uncertainty, unmodelled dynamics, input delay.

I. INTRODUCTION

The dynamic characteristic of a practical servo system is often viewed as a linear function, while the real dynamic characteristic is really complex. From receiving the control order to providing a regular control input, the running time of the servo system, which is usually called input delay, is not considered in most situation. For an actual system, the processing speed is limited, so the input is usually time delay, besides, the nonlinear dynamics of the servo is complex, this can also cause time delay in input. Time delay of input may affect the control performance or even lead to unstability of a real system, so we must take it seriously in controller design [2], [3], [8].

Control for input delay system has been widely discussed for linear continuous system [37] and discrete-time system [29]. But unfortunately, most of the real system are nonlinear, so designing a controller for input delay nonlinear system is more significative. T-S model based fuzzy controller is an efficient way for this problem, and has been widely researched [9], [15], [33]. Obviously, the more complex the built T-S fuzzy model is, the better the approximation result is. But when the T-S model is complex, a feasible solution for the T-S fuzzy model is hard to be get [4]. To reduce the conservative of the designed controller, diffeomorphism coordinate transformation (DCT) based controller design methods are proposed for input delay nonlinear system. Through choosing appropriate DCT, the original nonlinear system is simplified, and then, lots of controller design strategies, like back-stepping method [23], [34], [35], sliding mode control [32] and adaptive neural network method [18] are proposed for the input delay nonlinear system. It is noteworthy that, the methods for input delay nonlinear system directly are all based on an assumption that, the nonlinear model can be completely linearization, or its internal dynamics are stable. When the internal dynamics is unstable, the above methods will lose effectiveness [4].

The internal dynamics of a lot of practical systems are unstable [5]–[7], so the controller design of such a kind of system is worthy to be studied. The minimum nonlinear system
can be converted as an inverted triangle form, and then, some powerful nonlinear control approach can be utilized directly [4]. But if it is non-minimum phase, the inverted triangle form cannot be got, then the traditional power nonlinear control strategy will lose efficacy. In this case, the control design becomes complex and challenging, and some novel control design strategy is needed.

The output tracking control of non-minimum phase system has been widely studied, and ideal internal dynamics (IID) based controller method is a widely used [10]. By choosing appropriate DCT, the partially linearized dynamics and the internal dynamics can be constructed. After computing IID, a state tracking control problem can be constructed, then a nonlinear stable inversion controller can be constructed [4]. The II D based controller strategy can track a given command accurately [25], [27], so it has been widely used to identify the internal dynamics model adopted here is not invariant. When the parameters are uncertainty, or the nonlinearity nonlinear system is extended to input delay nonlinear system [16], [28], output control of nonlinear non-minimum phase system. The constructed internal dynamics model adopted here is not a standard form [13], instead, is a common form. The input in a standard form is disappeared in the nonlinear expression, so the controller design is simple. While the common form is more complex, and the controller design is also difficult. Unfortunately, standard form is hard to be get, so a controller for nonlinear system with common form ID is more important.

The model of vertical takeoff and landing aircraft (VTOL) is non-minimum phase and the control design has been studied and a lot of results can be get in literature, such as the robust control of VTOL [24], [30], [31], fault tolerant control of VTOL [1] and so on. But the robust output tracking control for VTOL with both uncertainties and unmodelled disturbance has not been completely solved. In this case, an adaptive robust controller design method is considered in this paper, and a numerical simulation is listed to confirm its' effectiveness.

The main feature and contribution of this paper is:

1. A robust controller is designed for input delay MIMO nonlinear non-minimum phase system.
2. Not only parameter uncertainty and unmodelled dynamic, but also input delay are considered in this paper.
3. The stable inversion based nonlinear controller for exact nonlinear non-minimum phase system is extended to input delay and uncertain case.

A nonlinear model which is suffering from input delay is listed in section 2, together with the control objective of the paper. The controller design method is proposed in Section 3. Numerical simulation is developed in section 4, and we summarize this paper in Section 5.

II. PROBLEM AND OBJECTIVE

A. MODEL DESCRIPTION

For an input delay nonlinear system

\[\begin{align} 
\dot{x} &= f(x) + g(x)u(t - \tau) \\
y(t) &= h(x) = [y_1, y_2, \ldots, y_m]^T \\
r \in R^n, u(t) \in R^m, \tau \text{ is time delay which is already known and } u(t - \tau) = 0 \text{ if } t < \tau, y \in R^m \text{ is system output. Assuming that } \eta_0 (x = 0, u = 0) \text{ is a balance point} \\
\end{align}\]

\[(1)\]

where \(x(t) \in R^n, u(t - \tau) \in R^m, \tau \text{ is time delay which is already known and } u(t - \tau) = 0 \text{ if } t < \tau, y \in R^m \text{ is system output. Assuming that } \eta_0 (x = 0, u = 0) \text{ is a balance point} \]

\[\begin{align} 
\dot{y}_1^{(r_1)} &= F_1(x) + G_1(x)u(t - \tau) \\
\vdots \\
\dot{y}_m^{(r_m)} &= F_m(x) + G_m(x)u(t - \tau) \\
\dot{\eta} &= s(\zeta, \eta, u(t - \tau)) \\
\end{align}\]

\[(2)\]

\(r_1 \text{ is the differential order of } y_1, \text{ and } r_1 + r_2 + \cdots + r_m = r. \eta \in R^{n-r} \text{ is the internal state, } \zeta = [y_1, y_1^{(r_1-1)}, y_2, \ldots, y_m^{(r_m-1)}]^T \text{ represents the external state, } s(\zeta, \eta, u(t - \tau)) \text{ represents the nonlinear expression of internal dynamics. Then the partially linearized system (2) is} \]

\[\begin{align} 
\dot{\zeta} &= A_\zeta \zeta + B_\zeta v = A_\zeta \zeta + B_\zeta [F(x) + G(x)u(t - \tau)] \\
\dot{\eta} &= s(\zeta, \eta, v) \\
&= s(\zeta, \eta, (F(x) + G(x)u(t - \tau))), \\
\end{align}\]

\[(3)\]

where

\[F(x) = [F_1(x), F_2(x), \ldots, F_m(x)]^T, \]

\[G(x) = [G_1(x), G_2(x), \ldots, G_m(x)]^T, \]

\[A_\zeta = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\
0 & A_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & A_m \end{bmatrix}_{r \times r}, \]

\[s(\zeta, \eta, v) \text{ represents the non-
Based on IID, a state tracking control problem is built and bounded, then the IID of the original system can be let alone the unmodelled dynamics and input delay the control design method proposed in [12] is unsuitable, unknown matrices can guarantee the stability of (5) under unmodelled dynamics, part of matrix uncertain, the expression of (4) should be improved as
\[
\dot{v} = \mathcal{A} v + \mathcal{B} \begin{bmatrix} y^d - \psi^r \end{bmatrix} + d,
\]
where \( e = [e^T, e^T]^T, e^\varsigma = \varsigma - \varsigma^d, e^\eta = \eta - \eta^d, \varsigma^d = [y_1^d, y_1^{d(1)}, \ldots, y_1^{d(r-1)}, \eta_1^d, \ldots, y_m^{d(m-1)}]^T \) is the reference trajectory and its (r - 1) order differential, \( \eta^d \) represents IID of (1), and \( y^d = [\dot{y}^d, \dot{y}_1^{d(1)}, \ldots, \dot{y}_1^{d(r-1)}, \ldots, \dot{y}_m^{d(m-1)}]^T \) represents r order differential of the given command, \( d \) is the linearization error,
\[
\begin{align*}
\mathcal{A} & = \begin{bmatrix} A \varsigma & 0 \\ A_{\eta 1} & A_{\eta 2} \end{bmatrix}, \\
\mathcal{B} & = \begin{bmatrix} B_\varsigma \\ B_\eta \end{bmatrix}, \\
A_{\eta 1} & = \frac{\partial \varsigma(x, \hat{y}_i)}{\partial \varsigma} |_{x=0, \eta=0, v=0}, \\
A_{\eta 2} & = \frac{\partial \varsigma(x, \hat{y}_i)}{\partial \eta} |_{x=0, \eta=0, v=0}, \\
B_\eta & = \frac{\partial \varsigma(x, \hat{y}_i)}{\partial v} |_{x=0, \eta=0, v=0},
\end{align*}
\]
where \( v = F(x) + G(x)u(t) - \dot{\varsigma} \). Since \( F(x) \) and \( G(x) \) are uncertain, the expression of (4) should be improved as:
\[
\begin{align*}
\dot{v} & = (A + \Delta A) e + (B + \Delta B) \\
& \times \left( v - y^d + d \right),
\end{align*}
\]
where \( \Delta A \) is uncertain part of matrix \( A \), and \( \Delta B \) is uncertain part of matrix \( B \). Because of the existence of \( \Delta A \) and \( \Delta B \), the control design method proposed in [12] is unsuitable, let alone the unmodelled dynamics and input delay \( \tau \). In this case, the control goal of (5) is: Find a robust controller which can guarantee the stability of (5) under unmodelled dynamics, unknown matrices \( \Delta A \) and \( \Delta B \), and input delay \( \tau \).

### III. MAIN RESULT

Because of the existence of input delay \( \tau \), controller design cannot be carried out directly for (5), so a transformation is needed. Defining
\[
\epsilon_p = \int_{t-\tau}^{t} u(z)dz
\]
then a new error defined as
\[
\epsilon_s = \epsilon + (B + \Delta B) G(x) \epsilon_p
\]
is constructed.

**Assumption 1:** Through out the running time, \( \epsilon_p \) is always bounded, and can be described by
\[
\| \epsilon_p \| \leq \rho_p e.
\]
where \( \rho_p \) is a known positive scalar.

**Remark 1:** For the designed controller, \( u(t) \) can be viewed as the function of system states. If the controller is designed appropriately, it should be bounded. In this case, as the finite time integral of \( u(t) \), \( \epsilon_p \) is bounded.

From (4) and (7), the derivative of \( \epsilon_s \) is
\[
\dot{\epsilon}_s = \dot{\epsilon} + (B + \Delta B) \begin{bmatrix} \partial G(x) \partial x \end{bmatrix} \epsilon_p
\]
Considering (4) we can get
\[
\dot{\epsilon}_s = (A + \Delta A) e + (B + \Delta B) \begin{bmatrix} \partial G(x) \partial x \end{bmatrix} \epsilon_p
\]

\[
N(x) \partial G(x) \epsilon_p = \begin{bmatrix} \partial G_{11}(x) \partial x \hat{x} \\ \vdots \\ \partial G_{1m}(x) \partial x \hat{x} \end{bmatrix} \epsilon_p
\]

**Remark 2:** From (9) we can see that, \( N(x) \) is determined by \( G(x) \). Since \( G(x) \) is unknown, \( N(x) \) should be also viewed as an unknown function of \( x \).

Since parameter uncertainties and disturbances always exist, the controller of such a system should be, not only a robust controller for \( \Delta A \) and \( \Delta B \), but also an identified method for unmodelled dynamics in \( F(x) \), \( G(x) \) and \( N(x) \). As an universal approximation, fuzzy logic system (FLS) has good approximation effect, so it is utilized here. The FLS adopted in this paper has the same form of [11], then the output of FLS can be wrote as
\[
\hat{y}_i = \theta_i^T \xi(\hat{x}),
\]
where \( \theta_i^T = [\theta_i^1 \theta_i^2 \cdots \theta_i^M]^T \) are adaptive parameters, \( \xi(\hat{x}) \) is basis function. By training \( \theta_i^T \) and \( \xi(\hat{x}) \), \( F(x) \) \( G(x) \) and \( N(x) \) can be replaced by fuzzy sets:
\[
F(x|\theta_f) = \begin{bmatrix} F_1(x|\theta_f) \\ F_2(x|\theta_f) \\ \vdots \\ F_r(x|\theta_f) \end{bmatrix} = \xi_f(x) \theta_f.
\]
where $K$ and $u_h, u_s$ will be discussed in the following part. By substituting (10) into (8), we can get a new tracking expression:

$$
\dot{e}_s = (A + \Delta A) e + (B + \Delta B) \left[ Ke + \xi_f(x) \hat{\theta}_f \right] + \xi_g(x) \hat{\theta}_g u + \xi_n(x) \hat{\theta}_n + u_h + u_s + \Delta F + \Delta M + \Delta G u + d 
$$

(11)

where $\Delta F = F(x) - F(x|\theta^*_f)$, $\Delta G = G(x) - G(x|\theta^*_g)$, $\Delta N = N(x) - N(x|\theta^*_n)$, $\hat{\theta}_f = \theta^*_f - \theta_f$, $\hat{\theta}_g = \theta^*_g - \theta_g$, $\hat{\theta}_n = \theta^*_n - \theta_n$.

The following assumptions are listed for $\Delta A, \Delta B$ and $\Delta G G^{-1}(x|\theta^*_g)$:

Assumption 2: $G(x|\theta^*_g)$ is invertible.

Assumption 3: $\|\Delta G G^{-1}(x|\theta^*_g)\| \leq \kappa_G$, $\kappa_G$ is already known scalar and $\kappa_G < 1$.

Assumption 4: The uncertain matrices $\Delta A$ and $\Delta B$ are bounded and $\Delta B = \Delta B H$, $\|\Delta H\| \leq \kappa_B < 1$, where $\kappa_B$ is a known scalar.

Assumption 5: Define $M_B(x) = e_f^T P (B + \Delta B) \cdot (\Delta F + \Delta G + \Delta N + \Delta G G^{-1}(x|\theta^*_g)) \cdot (-F(x|\theta^*_f) + y^{d(r)}) + Ke - e_f^T P BR^{-1}B^T P (B + \Delta B) G(x|\theta^*_g)e_p - e_f^T P [A + \Delta A] (B + \Delta B) K (B + \Delta B) G(x|\theta^*_g)e_p$.

$M_B(x)$ is unknown and bounded, which means that, $\|M_B(x)\| \leq (\rho_0 + \rho_1 \|e\|) \|B^T P e\|$, where $\rho_0$ and $\rho_1$ are unknown scalars.

Remark 3: For a real system, $G(x)$ is invertible. As a fuzzy approximation of $G(x)$, $G(x|\theta^*_g)$ should also be invertible. So Assumption 2 is reasonable.

Remark 4: When the value of $G(x|\theta^*_g)$ is close to zero, the computing of $\frac{1}{G(x|\theta^*_g)}$ becomes an ill-conditioned inverse problem. This problem should be taken seriously, but a lot of results, such as Newton algorithm [38], iterative refinement method [39], can be used and referenced.

Remark 5: $\Delta G = G(x) - G(x|\theta^*_g)$, so $\Delta G$ is bounded. From Assumption 2, $G(x|\theta^*_g)$ is also bounded. Then $G^{-1}(x|\theta^*_g)$ is bounded and the product of $\Delta G$ and $G^{-1}(x|\theta^*_g)$ is also bounded. So Assumption 3 is reasonable.

Remark 6: Since the parameter uncertainty and unmodelled dynamics in practice is bounded, the derived uncertainty matrices $\Delta A$ and $\Delta B$ are bounded. The input matrix $B$ is full column rank, so $\Delta H = (B^T B)^{-1} B^T \Delta B$, and $\Delta H$ is bounded. So Assumption 4 is reasonable.

Remark 7: From the expression of $M_B(x)$ we can see that, $\Delta F$, $\Delta G$ and $\Delta M$ are approximation errors which can be viewed as a small amount, $G(x|\theta^*_g)$ is bounded, and $F(x|\theta^*_f)$ is Lipschitz, $e_p$ and $Ke$ are all functions of tracking error $e$, so Assumption 5 is reasonable.

Theorem 1: If Assumption 1-4 is hold, and there are matrices $P$, $K$, and scalars $\varepsilon_A$, $\varepsilon_B$, satisfying

$$
P > 0, \varepsilon_A > 0, \varepsilon_B > 0
$$

$$
P (A + BK) + (A + BK)^T P + Q + 
\frac{1}{\ell_A} \lambda_A \Delta A I + \lambda_B \Delta B e_B K^T K + 
(\varepsilon_A^{-1} + \varepsilon_B^{-1}) PP + PB \left( P^{-1} I + R^{-1} \right) B^T P < 0,
$$

(12)
where $Q > 0$ is a given matrix, $R$ is a given control gain and $\rho$ is prescribed attenuation index, then (11) is stable under the following robust controller

$$
u_h = \frac{1}{2(1 + \kappa_G)} R^{-1} B^T Pe$$  \quad (13)
$$\nu_s = -\rho_0 + \rho_1\|e\|sgn(B^T Pe)$$  \quad (14)

$\hat{\rho}_0$ and $\hat{\rho}_1$ are estimates of $\rho_0$ and $\rho_1$, and

$$\dot{\hat{\rho}}_0 = \gamma_T \Theta \hat{x}_T^T(x)B^T Pe$$  \quad (15)
$$\dot{\hat{\rho}}_s = \gamma_s \Theta u(t)(e^T PB)\hat{x}_T(x)$$  \quad (16)
$$\dot{\hat{\rho}}_s = \gamma_s \Theta e_T^T(x)B^T Pe$$  \quad (17)
$$\dot{\hat{\rho}}_0 = q_0 \|B^T Pe\|$$  \quad (18)
$$\dot{\hat{\rho}}_1 = q_1\|e\|\|B^T Pe\|$$  \quad (19)

where $\Theta = (1 + \lambda^2_{\text{max}}(B)(1 + \kappa_B))\rho_0 \rho_G$.

**Proof:** Choosing a Lyapunov function for system (11) as

$$V = \frac{1}{2} e^T Pe_s + \frac{1}{2\gamma_T} \dot{\hat{\rho}}_s^T \dot{\hat{\rho}}_s + \frac{1}{2\gamma_T} \dot{\hat{\rho}}_0^T \dot{\hat{\rho}}_0 + \frac{1}{2\gamma_1} \dot{\hat{\rho}}_1^T \dot{\hat{\rho}}_1$$

where $\gamma_T, \gamma_s, \gamma_n, q_0$ and $q_1$ are all scalars. Since $\dot{\hat{\rho}}_0 = -\dot{\hat{\rho}}_0$, $\hat{\rho}_0 = -\hat{\rho}_0$ and $\hat{\rho}_1 = -\hat{\rho}_1$, differentiating (11) we have

$$V \leq \frac{1}{2} e^T Pe_s + \frac{1}{2\gamma_T} \dot{\hat{\rho}}_s^T \dot{\hat{\rho}}_s - \frac{1}{2\gamma_T} \dot{\hat{\rho}}_0^T \dot{\hat{\rho}}_0 - \frac{1}{2\gamma_1} \dot{\hat{\rho}}_1^T \dot{\hat{\rho}}_1$$

$$\leq e^T P [A + \Delta A] + (B + \Delta B) K^T e_s$$
$$- e^T P [(A + \Delta A) + (B + \Delta B) K] (B + \Delta B) G(x|\theta_g) e_p$$
$$+ e^T P (B + \Delta B) (I + \Delta G^{-1}(x|\theta_g))u_n$$
$$+ e^T P (B + \Delta B) (I + \Delta G^{-1}(x|\theta_g))u_s$$

$$+ e^T P (B + \Delta B) \left( \Delta F + \Delta N + \Delta G^{-1}(x|\theta_g) \right)$$

$$\times (-F(x|\theta_g) - N(x|\theta_g) + \epsilon^{d_r}(r) + K e)$$

$$- \frac{1}{q_0} \hat{\rho}_0 \dot{\hat{\rho}}_0 - \frac{1}{q_1} \hat{\rho}_1 \dot{\hat{\rho}}_1$$

$$+ e^T P (B + \Delta B) \xi_g(x) \dot{\hat{\rho}}_0 - \frac{1}{\gamma_T} \dot{\hat{\rho}}_0^T \dot{\hat{\rho}}_0$$

$$+ e^T P (B + \Delta B) \xi_s(x) \dot{\hat{\rho}}_s - \frac{1}{\gamma_n} \dot{\hat{\rho}}_s^T \dot{\hat{\rho}}_s$$

$$+ e^T P (B + \Delta B) d$$

Since

$$\Delta A^T P + P \Delta A \leq \epsilon_A \Delta A^T \Delta A + \epsilon_A^{-1} PP$$

$$\leq \lambda^2_{\text{max}}(\Delta A) \epsilon_A I + \epsilon_A^{-1} PP$$

$$P \Delta BK + K^T \Delta B^T P \leq \epsilon_B K^T \Delta B K + \epsilon_B^{-1} PP$$

$$\leq \lambda^2_{\text{max}}(\Delta B) \epsilon_B K^T K + \epsilon_B^{-1} PP$$

then

$$P \{(A + \Delta A) + (B + \Delta B) K \}$$
$$+ [(A + \Delta A) + (B + \Delta B) K^T P$$
$$P \leq \lambda^2_{\text{max}}(\Delta A) \epsilon_A I + \lambda^2_{\text{max}}(\Delta B) \epsilon_B K^T K$$

$$+ \left( \epsilon_A^{-1} + \epsilon_B^{-1} \right) PP$$

$$\leq$$

Since $\|\Delta G^{-1}(x|\theta_g)\| \leq K_G < 1, \|\Delta H\| \leq \kappa_B < 1$, and

$$\nu_h = \frac{1}{2(1 + \kappa_G)(1 + \kappa_B)} R^{-1} B^T Pe_s$$

$$u_s^T (I + \Delta G^{-1}(x|\theta_g))^T (B + \Delta B)^T Pe_s$$

$$+ e^T P (B + \Delta B) (I + \Delta G^{-1}(x|\theta_g))u_n$$

$$\leq e^T P BBR^{-1}B^T Pe_s$$

$$- e^T P BBR^{-1}B^T Pe_s$$

$$\leq e^T P BBR^{-1}B^T Pe_s$$

$$\leq$$

$$\leq$$

From Assumption 4, $M_h(x)$ is bounded, $u_s = \varrho_0 + \varrho_1\|e\|sgn(B^T Pe)$, and

$$\|e_s\| \leq \|e + (B + \Delta B) G(x|\theta_g) e_p\|$$

$$\leq (1 + \lambda^2_{\text{max}}(B)(1 + \kappa_B)\rho_G \rho_p) \|e\|$$

$$\dot{\hat{\rho}}_0 = q_0 \|B^T Pe\|$$

$$\dot{\hat{\rho}}_1 = q_1\|e\|\|B^T Pe\|$$

so

$$u_s^T (I + \Delta G^{-1}(x|\theta_g))^T (B + \Delta B)^T Pe_s$$

$$+ M_h(x) - \frac{1}{q_0}\hat{\rho}_0 \dot{\hat{\rho}}_0 - \frac{1}{q_1}\hat{\rho}_1 \dot{\hat{\rho}}_1$$

$$\leq$$

$$\leq$$

$$\leq$$

$$\leq$$

$$\leq$$

$$\leq$$

Also form Lemma 1 of [19],

$$\left[ d^T (I + \Delta H)^T B^T Pe_s + e^T P B^T (I + \Delta H) d\right]$$

$$\leq \rho^2 d^T (I + \Delta H)^T (I + \Delta H) d + \rho^{-2} e^T P BBR^{-1}B^T Pe_s$$

$$\leq (1 + \kappa_B)^2 \rho^2 d^T d + \rho^{-2} e^T P BBR^{-1}B^T Pe_s$$

So

$$V \leq \frac{1}{2} e^T \left[ (A + BK) + (A + BK)^T P + \lambda^2_{\text{max}}(\Delta A) \epsilon_A I + \lambda^2_{\text{max}}(\Delta B) \epsilon_B K^T K + \left( \epsilon_A^{-1} + \epsilon_B^{-1} \right) PP$$

$$+ P BBR^{-1}B^T P + \rho^{-2} e^T P BBR^{-1}B^T Pe_s$$

$$+ \frac{1}{2} (1 + \kappa_B)^2 \rho^2 \omega^T \omega$$

$$+ \frac{1}{2} \left[ \tilde{\rho}_T^T \xi_T^T (x) (I + \Delta H)^T B^T Pe_s \right.$$
we have $\mathbf{V} \leq \left(1 + \lambda_{\text{max}}^2(B)(1 + \kappa_B)\rho \rho_p)(1 + \kappa_B)e^{T}P\mathbf{E}_j(x)\right)$ so
\[ e^T_s PB (I + \Delta H) \xi_j(x) - \frac{1}{\gamma_f} \hat{\theta}_f^T \hat{\theta}_f \leq 0 \]

Similarly,
\[ e^T_s P (B + \Delta B) \xi_j(x) \hat{\theta}_j u(t) - \frac{1}{\gamma_y} \hat{\theta}_j^T \hat{\theta}_j \leq 0 \]
from (12) we have
\[ \dot{V} \leq -\frac{1}{2} e^T_s Q e_s + \frac{1}{2}(1 + \kappa_B)^2 \rho^2 d^T d \]

Since $d$ is bounded, for $e_s$, if
\[ \|e_s\| \geq \sqrt{\lambda_{\text{max}}(Q)} \]
$\dot{V} \leq 0$ is always hold. Then for the $H_{\infty}$ performance, we have:
\[ \int_0^t e^T_s Q e_s dt \leq e^T_s(0)P e_s(0) + \frac{1}{\gamma_f} \hat{\theta}_f(0)\hat{\theta}_f(0) \]
\[ + \frac{1}{\gamma_y} \hat{\theta}_j^T \hat{\theta}_j(0) + \frac{1}{\gamma_y} \hat{\theta}_j^T \hat{\theta}_j(0) + \rho^2 \int_0^t d^T d dt. \]

The proof is completed. \hfill $\blacksquare$

Remark 8: Theorem 1 gives the designing method for input delay non-minimum phase system, but it seems a little complex. Actually, inequation (12) can be easily solved by LMI toolbox of MATLAB, while (15), (16) and (17) can be easily approached by FLS toolbox of MATLAB. So relying on existing MATLAB toolbox, Theorem 1 is easily to be implemented.

For facilitating the application of the proposed method, an overall block diagram is given in Figure 1.

In order to ensure the adaptive parameters $\theta_f$, $\theta_j$ and $\theta_m$ to be bounded, the projection algorithm is utilized here to amend the adaptive law (15), (16) and (17). Assume the constraint sets $\Omega_f$, $\Omega_j$ and $\Omega_m$ are specified as $\Omega_f \triangleq \left\{ \theta_f : \|\theta_f\| \leq \sigma_f \right\}$, $\Omega_j \triangleq \{ \theta_j : \|\theta_j\| \leq \sigma_j \}$ and $\Omega_m \triangleq \{ \theta_m : \|\theta_m\| \leq \sigma_m \}$, respectively, where $\sigma_f$, $\sigma_j$ and $\sigma_m$ are all positive constants and can be arbitrarily specified. Thus, the adaptive law (15), (16) and (17) can be modified as

**IV. SIMULATION RESULTS**

A numerical simulation example on VTOL is considered, and the nonlinear expression of VTOL is adopted here and a sketch of VTOL is given in Figure 2. The nonlinear expressions are listed here:
The physical means of the above nonlinear expression can be founded in [10], and \( \varepsilon(t) = 0.5 + 0.1 \sin x_5 \). \( \xi_1 \) and \( \xi_2 \) are additional unknown dynamics. Then (20) can be replaced by

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\end{bmatrix} = \begin{bmatrix}
x_2 \\
0 \\
x_4 \\
-1 \\
x_6 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
\varepsilon(t) \cos x_5 \\
0 \\
\cos x_5 \\
0 \\
0 \\
1 \\
\end{bmatrix} (u(t - \tau))
\]

where

\[
f(x) = \begin{bmatrix}
x_2 \\
x_4 \\
-1 \\
x_6 \\
0 \\
\varepsilon(t) \cos x_5 \\
0 \\
\cos x_5 \\
0 \\
1 \\
\end{bmatrix},
\]

\[
g(x) = \begin{bmatrix}
x_2 \\
0 \\
-1 + \cos x_5 \xi_1 + \varepsilon(t) \sin x_5 \xi_2 \\
0 \\
0 \\
0 \\
0 \\
1 \\
\end{bmatrix},
\]

\[
z = \begin{bmatrix}
\xi_1 \\
\xi_2 \\
\end{bmatrix}^T.
\]

The output of the plant is

\[
y_1 = x_1, \quad y_2 = x_3
\]

and through input/output linearization, (21) can be simplified as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\end{bmatrix} = \begin{bmatrix}
x_2 \\
\cos x_5 x_1 \\
0 \\
\cos x_5 x_1 \\
x_6 \\
0 \\
\end{bmatrix} (u(t - \tau)) + \begin{bmatrix}
\varepsilon(t) \cos x_5 \\
0 \\
\varepsilon(t) \\
0 \\
0 \\
0 \\
\end{bmatrix} (u(t - \tau))
\]

What is with mentioning, (20) is a common form, and the internal state is

\[
\eta = \begin{bmatrix}
x_5 \\
x_6 \\
\end{bmatrix}
\]

and

\[
F(x) = \begin{bmatrix}
\varepsilon(t) \cos x_5 \xi_2 - \sin x_5 \xi_1 \\
-1 + \cos x_5 \xi_1 + \varepsilon(t) \sin x_5 \xi_2 \\
-\sin x_5 \varepsilon(t) \cos x_5 \\
\varepsilon(t) \sin x_5 \\
\varepsilon(t) \sin x_5 \\
\end{bmatrix},
\]

\[
G(x) = \begin{bmatrix}
\varepsilon(t) \cos x_5 \\
\varepsilon(t) \sin x_5 \\
\end{bmatrix},
\]

\[
s(x, \xi, (F(x) + G(x)u)) = \begin{bmatrix}
x_6 \\
-\xi_2 + \frac{\sin x_5}{\varepsilon(t)} \\
0 \\
\cos x_5 \frac{\sin x_5}{\varepsilon(t)} \\
\end{bmatrix} (F(x) + G(x)u(t - \tau))
\]

Based on the analysis of [31], (21) is non-minimum phase. For the construction of IID, here we utilize the method presented in [21], and in calculation, \( \varepsilon \) is assumed to be constant and chosen as \( \varepsilon = 0.5 \). Then IID \( \eta^d = (\eta_{1}^{d}, \eta_{2}^{d})^T \triangleq (x_{4}^{d}, x_{6}^{d})^T \) can be constructed by solving the equation below:

\[
\dot{\eta}^d + c_1 \dot{\eta}^d + c_0 \eta^d = - (P_1 \dot{x}^d + P_0 x^d)
\]

where \( c_0 = 1, c_1 = 2, x^d = (0, 1)^T, (\eta_{1}^{d}, \eta_{2}^{d})^T \triangleq (x_{4}^{d}, x_{6}^{d})^T, P_0 \) and \( P_1 \) are gain matrices, and the solving details can be found in [31].

Remark 9: For an uncertain input delay nonlinear system, IID is also computed by the exact model (20) with a constant coupling coefficient \( \varepsilon \) and \( \tau = 0, \xi_1 = 0, \) and \( \xi_2 = 0, \) since IID is a ideal state for internal dynamic which we want.

Reference trajectories \( y_1^d = R \cos(\omega t), y_2^d = R \sin(\omega t) \) are chosen for simulation here, and \( \tau = 1, \omega = 0.1 \). The given trajectories and IID are shown in Figure (3).
Defining states error as

\[
\begin{align*}
e_1 &= y_1 - y_1^d = x_1 - x_1^d \\
e_2 &= y_1 - y_2^d = x_2 - x_2^d \\
e_3 &= y_2 - y_3^d = x_3 - x_3^d \\
e_4 &= y_2 - y_4^d = x_4 - x_4^d \\
e_5 &= \eta_1 - \eta_1^d = x_5 - x_5^d \\
e_6 &= \eta_2 - \eta_2^d = x_6 - x_6^d
\end{align*}
\]

then

\[
\begin{pmatrix}
\dot{e}_1 \\
\dot{e}_2 \\
\dot{e}_3 \\
\dot{e}_4 \\
\dot{e}_5 \\
\dot{e}_6
\end{pmatrix} = (A + \Delta A) \begin{pmatrix}
e_1 \\
e_2 \\
e_3 \\
e_4 \\
e_5 \\
e_6
\end{pmatrix} + (B + \Delta B)
\]

\begin{equation}
(23)
\end{equation}

where

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\cos x_5}{\varepsilon} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
\Delta A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\partial \xi_2}{\partial x_5} & 0 & \frac{\partial \xi_2}{\partial x_6}
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0 \\
\frac{\cos x_5}{0.5} & \sin x_5
\end{bmatrix},
\]

\[
\Delta B = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\cos x_5 & \frac{\cos x_5}{\varepsilon} & \sin x_5 \\
\frac{\cos x_5}{0.5} & \varepsilon & \sin x_5
\end{bmatrix}.
\]

The initial states \(x(0) = [1.5, 0, -0.5, 0.2, 0.28, 0]^T\), \(\varepsilon(t) = 0.5 + 0.2 \sin x_5\), the disturbance caused by wind \(\zeta_1\) and \(\zeta_2\) are chosen as \(\zeta_1 = 0.2 \cos x_5\), \(\zeta_2 = 0.2 \sin x_5\), the input delay \(\tau\) is already known and \(\tau = 0.15\). The initial value for \(\theta_f, \theta_g\) and \(\theta_m\) are all set to be “1”.

**Step 1:** For the approximating of unmodelled dynamics, membership functions in FLS are chosen as \(x_5\) and \(\varepsilon(t)\), and:

\[
\begin{align*}
\mu_{\tilde{F}_{i5}} &= \exp \left\{ -0.003 (x_5 - 0.01)^2 \right\} \\
\mu_{\tilde{F}_{i3}} &= \exp \left\{ -0.003 (x_5 - 0.1)^2 \right\} \\
\mu_{\tilde{F}_{i1}} &= \exp \left\{ -0.003 (x_5 + 0.01)^2 \right\} \\
\mu_{\tilde{F}_{i2}} &= \exp \left\{ -0.003 (x_5 + 0.1)^2 \right\} \\
\mu_{\tilde{F}_{i5}} &= \exp \left\{ -0.003 (\varepsilon(t) - 0.2)^2 \right\} \\
\mu_{\tilde{F}_{i3}} &= \exp \left\{ -0.003 (\varepsilon(t) - 0.5)^2 \right\} \\
\mu_{\tilde{F}_{i1}} &= \exp \left\{ -0.003 (\varepsilon(t) - 0.7)^2 \right\}
\end{align*}
\]

and the corresponding fuzzy rules are:

\(R_{ij}^{l}: \text{if } x_5\text{ is }\tilde{F}_{i5}\text{ and }\varepsilon(t)\text{ is }\tilde{F}_{j5}\text{, then }y\text{ is }F_{ij}\), where \(i = 1, 2, 3; j = 1, 2, 3; l = 1, 2, \ldots, 9\). Then \(F(x), G(x)\) and \(N(x)\) can be replaced by \(\xi_f(x)\theta_f, \xi_g(x)\theta_g, \xi_m(x)\theta_m\).

**Step 2:** From (23), \(\lambda_{\max}(\Delta A) = 0.036, \lambda_{\max}(\Delta B) = 0.71, \kappa_B = 0.5, \kappa_G = 0.3, \rho_G = 0.29, \rho_G = 0.79\). Choosing \(\Omega = 1 \times 10^{-2}, R = \text{diag}(20, 20), \varepsilon_A = 0.2, \varepsilon_B = 0.2, \rho = 0.1\), the control matrix \(K\) can be constructed and

\[
K = \begin{bmatrix}
2.11 & 7.42 & 0.92 & 2.75 & -12.45 & -8.96 \\
0.92 & 2.75 & -0.82 & -1.35 & -3.58 & -2.57
\end{bmatrix}.
\]

**Step 3:** The disturbance \(d(t)\) are chosen as the same as [10], and the parameters in theorem 1 are chosen as \(\gamma_f = 1000, \gamma_g = 10, \gamma_m = 10, q_0 = 100,\) and \(q_1 = 100.\)
The proposed controller is marked as $u_{IF}$, and is carried on VTOL suffering from input delay, uncertainties and unmodelled dynamics together with a classical stable inversion controller $u_d$ proposed in [4]. Simulation results are given in Figures (4–7), the given path and the controller response path are given in Figure (6). See the tracking performance of $u_d$ in Figure (4), (5) and (6), a tracking error appears, while the tracking error of $u_{IF}$ always keep remarkably small, then we can say that, the control effect of $u_{IF}$ is much better.

Figure (7) is the input of $u_{IF}$ and $u_d$. The estimation of $\rho_0$ and $\rho_1$ are also given in Figure (8).

V. CONCLUSION

The controller design of input delay uncertain MIMO nonlinear non-minimum phase system is discussed in this paper, and a FLS based controller is proposed. By input/output linearization, the expression of the nonlinear system is simplified, and then IID is constructed. Based IID, a state tracking problem is built. By defining a integral of past input, the input delay is transformed into a simple one, and then FLS is adopted in this paper to approach the uncertainty and unmodelled disturbance. Then the FLS adaptive fuzzy controller is constructed. Finally, a numerical simulation on VTOL is carried to test the good performance of the proposed strategy.

The uncertainty adopted in this paper is a norm bound uncertainty, and the priori information of them are assumed to be known. This is certainly not suitable for all real situations and will cause conservatism to the designed controller.

Further work is needed to design an online estimation method for the parameter uncertainty and unmodelled dynamics, and get a better robust controller. To testify the validity of the proposed method thoroughly, a real flight test of VTOL with the proposed control strategy is also need in the future.

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