Anomalous microwave emission (AME) is an important Galactic foreground of cosmic microwave background radiation. It is believed that AME arises from rotational emission by spinning polycyclic aromatic hydrocarbons in the interstellar medium (ISM). In this paper, we suppose that a new population of ultra-small silicate grains perhaps exists in the ISM, and quantify the rotational emissivity from these tiny particles and its polarization spectrum. We find that spinning silicate nanoparticles can produce strong rotational emission when the tiny grains follow a log-normal size distribution. The polarization fraction of spinning dust emission from tiny silicates increases with decreasing dipole moment per atom ($\beta$) and can reach $P \sim 20\%$ for $\beta \sim 0.1$ D at a grain temperature of 60 K. We identify a parameter space ($\beta$, $Y_{Si}$), with $Y_{Si}$ being the fraction of Si abundance in nanoparticles, in which its rotational emission can adequately reproduce both the observed AME and the polarization of the AME, without violating the observational constraints of ultraviolet extinction and polarization of starlight. Our results reveal that rotational emission from spinning silicate may be an important source of AME.

**Key words:** cosmic background radiation – diffuse radiation – dust, extinction – radiation mechanisms: non-thermal

1. INTRODUCTION

Anomalous microwave emission (AME) in the ~10–60 GHz frequency range is a new, important Galactic foreground component, which was discovered about 20 years ago (Kogut et al. 1996; Leitch et al. 1997). Two new emission processes have been proposed to explain AME, including spinning dust emission (Draine & Lazarian 1998, hereafter DL98) from rapidly spinning ultra-small grains, and magnetic dipole emission (MDE) from magnetic nanoparticles (Draine & Lazarian 1999).3

In the spinning dust emission paradigm, the emissivity is determined mostly by three principal parameters, including the permanent dipole moment, the rotation rate, and the abundance of spinning ultra-small grains (Erickson 1957; DL98; Ali-Haïmoud et al. 2009; Hoang et al. 2010, hereafter HDL10). In that sense, any nanoparticles having a permanent dipole moment would produce rotational emission because nanoparticles of any kind naturally rotate due to gas–grain collisions and absorption of ultraviolet (UV) photons.

Since the introduction of spinning dust emission, polycyclic aromatic hydrocarbons (hereafter PAHs) have been taken to be the unique carriers of rotational emission, because their existence in the interstellar medium (ISM) was well established (Leger & Puget 1984; see Tielens 2008 for a recent review). Theoretical models with spinning PAHs have proved to be successful in reproducing the observational data from both ground-based experiments (Finkbeiner et al. 2004) and satellites (Hoang et al. 2011; Ade et al. 2011, 2014). In particular, Tibbs et al. (2016) recently showed that the 1 cm CARMA data from dense cores do not follow the extrapolation of thermal dust radiation and can be fitted with spinning dust emission.

Aiming to test the hypothesis of rotation emission from spinning PAHs as a source of AME, Hensley et al. (2015) carried out an analysis using full-sky AME data from *Planck* and all-sky *WISE* data at 12 $\mu$m. Surprisingly, they found no correlation between PAH abundance and AME, casting doubt on PAHs as the underlying carriers. It has been suggested that the AME may be produced by spinning non-PAH nanoparticles (Hensley et al. 2015), or that the physics of interstellar PAHs is different from the one adopted in the spinning dust model (Hoang & Lazarian 2016). This encourages us to search for unexplored possibilities that may be important for AME.

Recently, Hoang & Lazarian (2016) quantified rotational emission from spinning iron nanoparticles that have intrinsic magnetic dipole moments due to spontaneous magnetization. They found that the rotational emissivity is one order of magnitude lower than that produced by spinning PAHs, even when the entire iron abundance is concentrated within free-flying nanoparticles. Interestingly, their detailed calculations show that the polarization of MDE from free-fliers is low, which suggests a more important role for MDE produced by free-flying nanoparticles in AME than previously thought.

This study is based on the assumption that there may exist a new population of nanoparticles in the ISM, namely silicate nanoparticles, in addition to PAHs and potential iron nanoparticles. Since nanosilicates can have permanent electric dipole moments (see Section 2), we expect that these spinning nanoparticles will produce considerable rotational emission if their abundance is sufficiently large.

The presence of ultra-small silicate grains in the ISM remains a hypothesis, in contrast to PAHs whose existence is demonstrated by the 2175 Å extinction bump and mid-infrared (IR) emission features (see Draine 2011). The first estimate by Désert et al. (1986) shows that the fraction of total Si abundance ($Si/H = 3.6 \times 10^{-5}$) contained in ultra-small grains, denoted by $Y_{Si}$, is less than 1%. However, a later analysis by Li & Draine (2001b) shows that $Y_{Si}$ can reach ~10% without...
violating the observational constraints of the UV starlight extinction and mid-IR emission. If silicate nanoparticles are indeed present in the ISM, their rotational emission will certainly contribute to the AME. The goal of this paper is to employ our advanced spinning model (HDL10; Hoang et al. 2011; hereafter HLD11) to quantify the rotational emissivity from silicate nanoparticles and to constrain its physical parameters as required to reproduce the observed AME in the ISM.

Our paper is structured as follows. In Section 2 we briefly describe the grain properties, including electric dipole moments, of silicate material. Section 3 presents an overview of the spinning dust emission mechanism. Section 4 is devoted to the description of rotational damping and excitation of silicate grains, and numerical methods. Results for spinning silicate emission and polarization spectra are presented in Section 5. An extended discussion of the implications of rotational emission from spinning silicate for AME, AME polarization, and UV polarization of starlight are presented in Section 6. The principal results are then summarized in Section 7.

2. GRAIN PROPERTIES OF ULTRA-SMALL SILICATES

2.1. Grain Shape and Size

We consider oblate spheroidal grains with moments of inertia \( I_1 > I_2 = I_3 \) along the grain principal axes denoted by \( \hat{a}_1 \), \( \hat{a}_2 \), and \( \hat{a}_3 \). Let \( I_1 = I \) and \( I_2 = I_3 = I_b \). They take the following forms:

\[
I_1 = \frac{8\pi}{15} \rho a_1^4 a_2^2, \quad I_2 = \frac{4\pi}{15} \rho a_1^2 a_2^2 + a_3^2,
\]

where \( a_1 \) and \( a_2 = a_3 \) are the lengths of the semiminor and semimajor axes of the oblate spheroid with axial ratio \( s = a_1/a_2 < 1 \), and \( \rho \) is the grain material density.

The effective grain size \( a \) is defined as the radius of an equivalent sphere of the same volume, which is given by

\[
a = \left( \frac{3}{4\pi} \frac{(4\pi/3)a_1^2 a_2^2}{a_3^2} \right)^{1/3} = a_2 s^{1/3}.
\]

Assuming that the entire Si, Mg, and Fe abundance is present in dust grains to form the typical structure \( \text{MgSi}_0.7\text{Fe}_{0.3}\text{SiO}_3 \), the grain material density corresponds to \( \rho \approx 4 \text{ g cm}^{-3} \). The number of Si atoms is related to the grain size \( a \) as \( N_\text{Si} = 59.67a_3^5 \), and the total number of atoms is \( N = 7N_\text{Si} = 417.74a_3^3 \), where \( a - \gamma = a/10^{-7} \text{ cm} \).

2.2. Electric Dipole Moment

The rotational emission mechanism is built upon the assumption that nanoparticles can have non-zero dipole moments. Since the specific composition of ultra-small silicate is uncertain, its electric dipole moment is rather arbitrary. For instance, a SiO molecule can have a large electric dipole moment of \( \mu = 3.098 \text{ D} \), which corresponds to a dipole moment per atom \( \beta = 1.098 \text{ D} \). SiC has a larger dipole moment of \( \mu = 5.6 \text{ D} \). Moreover, the dipole moments of pyroxene structures, enstatite (MgSiO\(_3\)) and ferrosilite (FeSiO\(_3\)), are \( \mu = 12.2 \text{ D} \) and 9.5 D, respectively (see Saunders & Plane 2011). Thus, we expect that silicate nanoparticles can have quite a large dipole moment. Table 1 lists the electric dipole moments (\( \mu \)) of selected bonds and molecules and the dipole per atom (\( \beta \)).

Let us assume that \( N \) atoms in the grain have a random distribution, then the intrinsic dipole moment of the grain can be estimated using the random walk for the orientation of individual dipoles:

\[
\mu_i^2 = N \beta^2 = 66.84(\beta/0.4 \text{ D})^2 a_3^2 D^2.
\]

Also, when ultra-small silicate grains exhibit some asymmetric charge distribution in which the grain charge centroid is displaced from its center of mass, the net dipole moment is non-zero. Thus, the total electric dipole moment of the grain can be written as (DL98):

\[
\mu^2 = \mu_i^2 + \epsilon (Z^2),
\]

where \( (Z^2)^{1/2} \) is the rms value of the grain charge, and \( \epsilon \) is a parameter taken to be 0.1 as in DL98.

3. OVERVIEW OF THE SPINNING DUST EMISSION MECHANISM FROM WOBBLING GRAINS

Consider a grain with dipole moment \( \mu \) fixed in the grain body rotating with angular momentum \( J \). If the grain only spins around its symmetry axis \( \hat{a}_1 (\hat{a}_1 \| J) \), the spinning dipole moment emits radiation at a unique frequency \( \nu \) equal to the rotational frequency, i.e., \( \nu = \omega/2\pi \) (DL98). The power emission by the spinning grain at frequency \( \nu \) is

\[
P_{\text{ed}} = \frac{2\mu_i^2 \omega^4}{3c^3},
\]

where \( \mu_i \) is the electric dipole component perpendicular to \( \hat{a}_1 \).

For more realistic rotation with \( \hat{a}_1 \) misaligned from \( J \) (HDL10; HLD11), an isolated grain of conserved \( J \) essentially emits radiation at four frequency modes, which arises from the combination of spinning around its symmetry axis \( a_i \) and the precession of \( a_i \) around \( J \). The four frequency mode are described by (HLD11):

\[
\omega_m = \Omega (1 + (h - 1) \cos \theta), \quad \omega_n = \Omega (1 - h) \cos \theta,
\]

where \( i = 0, \pm 1 \) denotes the order of the mode, \( h = I_1/I_2 \), \( \Omega = J/I_2 \) is the angular velocity if the grain is rotating along the symmetry axis \( \hat{a}_i \), and \( \theta \) is the angle between \( \hat{a}_i \) and \( J \) (see Figure 9(a)). Here, \( \omega_m \) and \( \omega_n \) are the modes induced by the oscillation of the dipole moment component perpendicular to \( J \), and \( \omega_0 \) is the mode induced by the oscillation of the dipole component parallel to \( J \).

Due to thermal fluctuations within the grain, the angle \( \theta \) changes rapidly, which results in the wobbling of the grain with respect to \( J \) (see the next section). For the case of efficient
internal relaxation (see HLD11 for details), the thermal fluctuations of \( \theta \) can be described by a distribution function (Lazarian & Roberge 1997):

\[
     f_{\text{LTE}}(\theta, J) \propto \exp \left( -\frac{J^2}{2 I_k T_d} \left[ 1 + (h - 1)\sin^2 \theta \right] \right) \sin \theta, \tag{8}
\]

where \( T_d \) is the dust grain temperature, and \( f_{\text{LTE}}(\theta, J)d\theta \) describes the probability of finding \( \theta \) between \( \theta, \theta + d\theta \) for a given \( J \).

The grain angular momentum \( J \) changes randomly due to a variety of interaction processes (see DL98). It can be described by a distribution function \( f_J(J) \), for which \( f_J(J)dJ \) indicates the probability of finding the grain angular momentum between \( J, J + dJ \).

The rotational emissivity at observation frequency \( \nu \) \((\omega/(2\pi))\) from a grain of size \( a \) is obtained by integrating the power emission over the angular momentum distribution and summing over all emission modes:

\[
     j_c^a \equiv \frac{1}{2} f_J(l_j/\hbar) \frac{2 \mu^2}{3 c^3} \omega^4 (\sin^2 \theta) + \frac{1}{2} \frac{\mu^2}{6 c^3} \omega^4 \int_{-1}^{1} (1 + \cos \theta_m J) pdf_m(\omega J)f_J(J)dJ
     + \frac{1}{2} \frac{\mu^2}{6 c^3} \omega^4 \int_{-1}^{1} (1 - \cos \theta_m J) pdf_m(\omega J)f_J(J)dJ
     + \frac{1}{2} \frac{\mu^2}{3 c^3} \omega^4 \int_{0}^{1} \sin^2 \theta pdf_m(\omega J)f_J(J)dJ, \tag{9}
\]

where \( \mu_{\parallel} = \mu \cos \theta, \mu_{\perp} = \mu \sin \theta \), the lower and upper limits of \( J \) are determined by \( \cos \theta = -1 \) and 1 from setting \( \omega_k \) equal to the observed frequency \( \omega \cos \theta_m = (-\omega/\Omega + h)/(h - 1) \), \( \cos \theta_m = (\omega/\Omega - h)/(h - 1) \) (using Equation (6)), and \( pdf_m(\omega J) \) is the probability of finding the emission at frequency \( \omega \) by mode \( \omega \), with \( k = m_{\perp} \pm 1, m_{\parallel} \).

It is straightforward to obtain \( J_i = l_j/\omega(2h - 1) \) and \( J_o = l_j/\omega \) for the \( m_{\perp} \) mode, \( J_o = l_j/\omega(2h - 1) \) and \( J_i = l_j/\omega(h - 1) \) for the \( m_{\parallel} \) mode, and \( J_i = l_j/\omega(h - 1) \) and \( J_o = \infty \) for the \( n_1 \) mode.

The probability \( pdf_\omega(\omega J) \) is given by (HLD11):

\[
     pdf_{m_{\perp}} = f_{\text{LTE}}(\theta, J) \frac{1}{\Omega(h - 1)}, \tag{10}
     pdf_{m_{\parallel}} = f_{\text{LTE}}(\theta, J) \frac{1}{\Omega(h - 1)}. \tag{11}
\]

where \( f_{\text{LTE}} \) is given by Equation (8). It is easy to see that the second and third term in Equation (9) are the same because \((1 + \cos \theta_{m_{\perp}})^2 = (1 - \cos \theta_{m_{\perp}})^2 \).

The rotational emissivity from all spinning ultra-small grains is calculated by integrating over the grain size distribution \( dn/da \):

\[
     j_c^a = \int_{a_{\min}}^{a_{\max}} j_c(a) \frac{dn}{n_H da}, \tag{12}
\]

where \( a_{\min} = 0.35 \) nm and \( a_{\max} = 10 \) nm are assumed, as in previous works (DL98; HLD10).

The calculations of rotational emissivity depend essentially on the dipole moment \( \mu \), the distribution function \( f_J \), and \( dn/da \). In the next section, we will discuss spinning silicate grains in a bit more detail.

It should be noted that, for very small grains (smaller than \( \sim 2 \) nm), the grain temperature has strong fluctuations due to absorption of UV photons (Greenberg 1968; Draine & Anderson 1985). As a result, they cannot be characterized by a single equilibrium temperature and are described by a temperature distribution function (Draine & Li 2001). The temperature fluctuations induce wobbling of the grain axes with respect to the angular momentum and modify the spinning dust spectrum. Thus, we must convolve \( j_c(\omega) \) with the temperature distribution function to obtain the average rotational emissivity (HLD11). As shown in HLD11, the average rotational emissivity can be approximated by the emissivity at a single temperature \( T_d \sim 60 \) K. Although the study in HLD11 deals with spinning PAHs, it is presumably applicable to nanosilicates. Therefore, in this paper, we adopt \( T_d = 60 \) K for nanoparticles, unless explicitly specified otherwise.

4. ROTATIONAL DYNAMICS AND NUMERICAL METHOD

The discussion of the rotational dynamics of silicate nanoparticles in this section is in analogy to that of spinning PAHs, which is presented in detail in DL98 and HLD10. Below, we provide a brief description for reference.

4.1. Rotational Damping and Excitation Coefficients

Rotational damping and excitation for dust grains, in general, arise from collisions between grains and gaseous atoms followed by the evaporation of atoms/molecules from the grain surface, absorption of starlight, and IR emission (DL98; HLD10). If the grain possesses an electric dipole moment, the distant interaction of the grain electric dipole with passing ions results in an additional effect, namely plasma drag. The rotational damping and excitation of these processes are described by the dimensionless damping coefficient \( F \) and excitation coefficient \( G \), respectively (see HLD10 for more detail).

For calculations of the \( F \) and \( G \) coefficients, one needs to know the charge distribution \( f_Z \) of nanosilicates. As in Hoang & Lazarian (2012), we find \( f_Z \) by solving the ionization equilibrium equations that take into account collisional charging (Draine & Sutin 1987) through sticking collisions with electrons and ions, and charging by photoemission (Weingartner & Draine 2001). The dielectric function of astronomical silicate is adopted for nanosilicates. It is noted that, for nanoparticles, the dielectric function of charged particles will be different from that of neutral particles (Li & Draine 2002), yet a detailed treatment of this effect is beyond the scope of this paper.

Moreover, rotational emission by an electric dipole moment results in rotational damping. The characteristic damping time due to dipole emission \( \tau_{ed} \) varies as \( \beta^2 a^7 \) (see Appendix A). We assume that the electric dipole is assumed to be in the plane perpendicular to the grain symmetry axis. It is noted that, as found in HLD10, the difference with the result from the case of isotropic orientation is small, within 10\%, because strong internal fluctuations average out the dipole orientation with \( J \).

Figure 1 shows the obtained values of \( F \) and \( G \) for various processes for silicate grains in the cold neutral medium (CNM; gas density \( n_H = 30 \) cm\(^{-3}\), ionization fraction \( x_H = 0.0012 \).
and gas temperature $T_{\text{gas}} = 100$ K computed for an oblate spheroidal grain rotating along its symmetry axis. As shown, IR emission is dominant for rotational damping of small grains ($a < 0.05 \mu m$), and neutral collisions are dominant for rotational damping/excitation for larger grains. Ion collisions dominate rotational excitation for the smallest silicate grains (i.e., $a < 10 \AA$) in the CNM because such grains tend to have negative average charge (see Weingartner & Draine 2001).

4.2. Numerical Method: Langevin Equations

Following our previous works (HDL10, HLD11), to find the accurate distribution of grain angular momentum, we numerically solve the Langevin equations for the evolution of $J$ in time in an inertial coordinate system, denoted by unit vectors $\hat{\mathbf{e}}_1$, $\hat{\mathbf{e}}_2$, and $\hat{\mathbf{e}}_3$, where $\hat{\mathbf{e}}_1$ is chosen to be parallel to the ambient magnetic field $B$ (see Figure 9(b)). The Langevin equations read

$$dI_i = A_i dt + \sqrt{B_i} dW_i \quad \text{for } i = 1, 2, 3, \quad (13)$$

where $dW_i$ are the random variables drawn from a normal distribution with zero mean and variance $\langle dW_i^2 \rangle = dt$, and $A_i = \langle \Delta I_i / \Delta t \rangle$ and $B_i = \langle (\Delta I_i)^2 / \Delta t \rangle$ are the damping and excitation coefficients defined in the $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3)$ frame.

The damping and excitation coefficients in a frame of reference fixed to the grain body, $A_i^b$ and $B_i^b$, are related to the dimensionless damping and excitation coefficients $(F$ and $G)$ as follows:

$$A_i^b = -\frac{J_i^b}{\tau_{\text{gas},i}} = -\frac{J_i^b}{\tau_{\text{H},i}} F_{\text{tot},i}, \quad (14)$$

$$B_{11}^b = B_1 = \frac{2I_i k_B T_{\text{gas}}}{\tau_{\text{H},i}} G_{\text{tot},1}, \quad (15)$$

$$B_{22}^b = B_2 = \frac{2I_i k_B T_{\text{gas}}}{\tau_{\text{H},\perp}} G_{\text{tot},2}, \quad (16)$$

where $\tau_{\text{H},i}$, $\tau_{\text{H},\perp}$ are the rotational damping due to gas collisions for rotation parallel and perpendicular to the symmetry axis (see Appendix A), $F_{\text{tot},i}$ and $G_{\text{tot},i}$ for $i = 1, 2, 3$ (or $\perp, \parallel$) are the total damping and excitation coefficients from various processes, and $\tau_{\text{gas},i} = \tau_{\text{H},i}/F_{\text{tot},i}$. Finally, $A_i$ and $B_i$ are obtained by using the transformation of coordinate systems for $A_i^b$, $B_i^b$ from the $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3)$ frame to the $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3)$ frame (see HLD10; Hoang et al. 2014).

To study the effect of magnetic relaxation on the alignment of nanoparticles, we incorporate a damping term $-J_{2,3}/\tau_m$ to $A_{2,3}$ and an excitation term $B_{m,2} = B_{m,3} = \bar{\epsilon}_m T_{\text{gas}}/\bar{\Omega}$ to $B_{2,2}$ and $B_{3,3}$, respectively (see Appendix B). In the dimensionless units of $J' = J/\bar{\Omega} \tau_m$ and $\epsilon' = \epsilon/\tau_{\text{H},||}$ with $\bar{\Omega}$ being thermal angular velocity (Equation (27)), Equation (13) becomes

$$dJ_i' = A_i' dt' + \sqrt{B_i'} dW_i' \quad \text{for } i = 1, 2, 3, \quad (17)$$

where $\langle dW_i'^2 \rangle = dt'$ and

$$A_i' = -J_i' \frac{1}{\tau_{\text{gas},i}} + \delta_i (1 - \delta_i) \frac{2 J_i^3}{3 \tau_{\text{ed},i}'}, \quad (18)$$

$$B_{ii}' = \frac{B_i}{2 \bar{\epsilon}_i \tau_{\text{H},\parallel}} - \bar{\epsilon}_i (1 - \delta_i). \quad (19)$$

Above, $\delta_i = 1$ for $i = 1$ and $\delta_i = 0$ for $i \neq 1$, and

$$\tau_{\text{gas},i}' = \tau_{\text{gas},i} / \tau_{\text{H},||}, \quad \tau_{\text{ed},i}' = \tau_{\text{ed},i} / \tau_{\text{H},||}, \quad (20)$$

where $\tau_{\text{gas},i}$ and $\tau_{\text{ed},i}$ are the effective damping times due to dust–gas interactions and electric dipole emission (see Equation (E4) in HDL10).

To numerically solve the Langevin Equation (17), we use the second-order integrator, for which the angular momentum component $J_i \equiv J_i'$ at iterative step $n + 1$ is evaluated as follows:

$$J_{i,n+1} = J_{i,n} - \gamma_{i,n} \Delta h_i + \sqrt{\Delta h_i} \sigma_{i,n}$$

$$\cdot \sqrt{h_i \sigma_{i,n}} - \gamma_i A_{i,n} - \gamma_i \mathbf{A}_{i,n} \mathbf{B}_{i,n}, \quad (21)$$

where $h$ is the timestep, $\gamma_i = 1/J_{\text{gas},i}' + \delta_i (1 - \delta_i)$, $\gamma_{i,n} = 2/(3 \tau_{\text{ed},i}' \tau_{\text{gas},i}')$, $\sigma_{i,n} = \sqrt{\tau_{i,n}'} B_{ii}'$, and

$$A_{i,n} = -\frac{\Delta h_i^2}{2} \gamma_{i,n} J_{i,n} + \sigma_i h_i^3/2 g(\xi_{i,n}, \eta_{i,n}) - \eta_{i,n}^3/2, \quad (22)$$

$$B_{i,n} = j_{i,n}^3 h - 3 \eta_{i,n}^3 h^2/2 - 3 j_{i,n}^3 \gamma_{i,n} h^2/2$$

$$+ 3 j_{i,n}^2 \gamma_{i,n} h^3/2 (\xi_{i,n}, \eta_{i,n}), \quad (23)$$

with $\eta_{i,n}$ and $\xi_{i,n}$ being independent Gaussian variables with zero mean and unit variance and $g(\xi_{i,n}, \eta_{i,n}) = \eta_{i,n}^3/2 + \eta_{i,n}^3/2 \sqrt{3}$ (see Hoang & Lazarian 2016 for details).

For silicate nanoparticles and PAHs with efficient rotational emission, the timestep $h$ is determined essentially by the two timescales, the shortest one electric dipole damping and the long one gas damping time. Thus, $h$ is chosen such that $h = 0.01 \min [1/F_{\text{tot},i}, 1/G_{\text{tot},i} \tau_{\text{ed},i} / \tau_{\text{H},i} || / \tau_{\text{gas},i}] / \bar{\epsilon}_i$. A fixed integration time $T = 10^3 \bar{\epsilon}_{i,\parallel}$ is chosen, which ensures that $T$ is much larger than the longest dynamical timescale to provide good statistical calculations of the degrees of grain alignment. The total number of timesteps $N_{\text{step}}$ for a given size is evaluated by $T/(\bar{\epsilon}_{i,\parallel} / h)$. As usual, the initial grain angular momentum is assumed to have random orientation in the space and magnitude $J = \bar{\Omega} \epsilon_{i,\parallel}$ (i.e., $j = 1$). The solutions $J_1$, $J_2$, $J_3$ at each timestep are tabulated to find the grain angular momentum.
distribution function $f_j$ and degree of alignment (see the next section).

### 4.3. Degrees of Grain Alignment

Let $Q_X = \langle G_X \rangle$ with $G_X = (3 \cos^2 \theta - 1)/2$ be the degree of internal alignment of the grain symmetry axis $\hat{a}$ with $J$, and let $Q_J = \langle G_J \rangle$ with $G_J = (3 \cos^2 \zeta - 1)/2$ be the degree of external alignment of $J$ with $B$. Here $\theta$ is the angle between $\hat{a}$ and $J$, and $\zeta$ is the angle between $J$ and $B$ (see Figure 9). The angle brackets denote the average over the ensemble of grains. The net degree of alignment of $\hat{a}$ with $B$, namely the Rayleigh reduction factor, is defined as $R = \langle G_X G_J \rangle$ (Greenberg 1968).

The angular momentum $J$ and the angle $\zeta$ obtained from the Langevin equations are employed to compute the degrees of alignment, $Q_J$ and $R$. For instance, the Rayleigh reduction factor $R$ is calculated as follows:

$$R \equiv \frac{\sum_{n=0}^{N_{\text{step}}} G_X(\cos^2 \theta) G_J(\cos^2 \zeta)}{N_{\text{step}}},$$

where $G_X$ can be replaced by $q_X(L_0) = \int_0^\pi G_X(\cos^2 \theta) f_{\text{LTE}}(\theta, L_0) d\theta$ in the case of fast internal relaxation (see e.g., Roberge & Lazarian 1999).

### 5. ROTATIONAL EMISSIVITY FROM SPINNING ULTRA-SMALL SILICATES AND POLARIZATION

#### 5.1. Grain Size Distribution

Following Li & Draine (2001b), we assume that ultra-small silicate grains follow a log-normal size distribution:

$$\frac{1}{n_{\text{HI}}} \frac{dn}{da} = \frac{B}{a^4} \exp \left(-0.5 \left( \frac{\log(a/a_0)}{\sigma} \right)^2 \right),$$

where $a_0$ and $\sigma$ are the model parameters, and $B$ is a constant determined by $Y_{\text{Si}}$.

The peak of the mass distribution $a^3dn/d\ln a$ occurs at $a_p = a_0 e^{3\sigma^2}$. For different models shown in Table 1 of Li & Draine (2001b) with $a_0 = 3\times6$ Å and $\sigma = 0.3-0.6$, the upper limits for $Y_{\text{Si}}$ for amorphous ultra-small silicate grains is $Y_{\text{Si}}^\text{max} = 20, 20, 20, 30\%$ (Li & Draine 2001b).

Figure 2 shows the size distribution for different model parameters with $Y_{\text{Si}} = 1\%$–20\%, where we have added the power law term $A_{\text{MRN}}a^{-3.5}$ (Mathis et al. 1977) with $A_{\text{MRN}} = 10^{-25.11} \text{ cm}^{-2} \text{ Hz}^{-1}$ for silicate grains larger than 20 nm, although the latter, slowly rotating grains, mostly do not contribute to rotational emission.

#### 5.2. Rotational Emissivity and Polarization

Using the angular momentum distribution obtained from the Langevin equation and $dn/da$ from Equation (25), we can compute rotational emissivity by spinning silicate using Equation (12). We consider oblate spheroidal grains with axial ratio $r = 2$ for a typical model. To explore the importance of spinning silicate, we consider a model of the ISM with typical physical parameters characterized by the CNM and adopt a typical magnetic field strength $B = 10 \mu G$.

In addition to the rotational emissivity computed by Equation (12), we are also interested in the polarized emissivity, which can be calculated as:

$$\frac{q_p}{n_{\text{HI}}} = \int_{a_{\text{min}}}^{a_{\text{max}}} Q_J(a) \cos^2 \gamma_B j_p(a) \frac{dn}{n_{\text{HI}} da},$$

where $Q_J$ is the degree of alignment of grain size $a$, and $\gamma_B$ is the angle between $B$ and the plane of the sky.

Figure 3 shows spinning dust emissivity for the different size distribution parameters with $Y_{\text{Si}} = Y_{\text{Si}}^\text{max}$. Here we take $Y_{\text{Si}} = 20/20/30\%$ for $\sigma = 0.3, 0.4$ and 0.6 estimated from Li & Draine (2001b). A range of the dipole moment $\beta$ from 0.1 to 1D is considered. As $\beta$ increases, the peak emissivity increases while the peak frequency decreases as a result of the stronger rotational damping by electric dipole emission. In addition, for a fixed $a_0 = 0.3$ nm, the peak emissivity decreases as $\sigma$ increases from 0.3 to 0.6 due to the increase in the peak size of nanoparticles $a_p$. The peak emissivity is slightly larger for the case $T_e = 60$ K due to the additional excitation arising from magnetic fluctuations (see Appendix B).

Figure 4 shows the polarization fraction of the rotational emission ($P = 100q_p/j_p$) for similar realizations as in Figure 3. The magnetic field direction is assumed to be in the plane of the sky ($\gamma_B = 0$). Colder spinning silicates tend to generate stronger polarized emission due to weaker internal thermal fluctuations. Interestingly, we can see that the larger dipole results in a smaller polarization fraction. This is because the larger dipole induces faster electric dipole damping. This substantially lowers the rms angular momentum of grains and reduces the degree of grain alignment, according to the magnetic alignment mechanism (see, e.g., Lazarian et al. 2015). For considerable values of $\beta \gtrsim 0.4$ D, the peak polarization fraction is essentially below 10\%. However, for $\beta < 0.4$ D, the peak polarization can reach 5\%–15\% for $\nu = 20-60$ GHz.

### 6. DISCUSSION

#### 6.1. Can Rotational Emission from Spinning Silicate Reproduce the AME?

Spinning dust emission is a new mechanism to produce microwave rotational emission, which essentially depends on the permanent dipole moment, rotation rate, and the abundance of ultra-small grains (DL98; HDL10). In that sense, any nanoparticles owning a permanent dipole moment will produce rotational emission, because nanoparticles of any kind always...
spin due to gas–grain collisions and UV photon absorption. Previous works have quantified rotational emission by spinning PAHs (DL98; HDL10; HLD11) and spinning iron nanoparticles (Hoang & Lazarian 2016). In this paper, we have quantified rotational emission from a potential dust population, so-called silicate nanoparticles. Assuming a log-normal size distribution for silicate nanoparticles with maximum Si abundance contained in ultra-small grains \( Y_{\text{Si}}^{\text{max}} = 20\% \), we find that the spinning dust emissivity can span \( j_r/\nu T_\text{d} \sim 10^{-18} - 10^{-16} \text{Jy cm}^2/\text{sr}/\text{Hz} \), which can even exceed the AME level of \( j_{\text{obs}}/\nu T_\text{d} \sim 10^{-17} \text{Jy cm}^2/\text{sr}/\text{Hz} \) from the ISM.

To see with what value of \( Y_{\text{Si}} \) the spinning nanosilicate can reproduce the AME, in Figure 5 we plot \( j_r \) calculated for \( Y_{\text{Si}} = 1\%, 5\%, 10\%, 20\% \), and selected \( \beta = 0.2, 0.4, 0.8, 1 \) D. Since nanoparticles are hotter than big grains, we take \( T_\text{d} = 60 \text{ K} \) as in previous works (HLD10; HLD11). The observational data from various instruments extracted from Finkbeiner et al. (2004) are shown by the symbols, and the solid lines are the total spinning dust plus thermal dust emissivity described by a modified blackbody \( j_{\text{bd}} = j_{\nu,\text{GB}} (\nu/\nu_\text{GB})^{-1.7} \), with \( j_{\nu,\text{GB}} = 0.8 \text{ MJy/sr} \). It can be seen that spinning nanosilicate with \( Y_{\text{Si}} < 10\% \) and \( \beta > 0.2 \) D (Figures 5(b)–(d)) can successfully reproduce the observed

![Figure 3](image1.png)

**Figure 3.** Spinning dust emissivity by ultra-small silicates computed for the different values of \( \beta \). Results for \( \sigma = 0.3, 0.4, 0.6 \) are shown in ((a), (d)), ((b), (e)), and ((c), (f)), respectively. Two temperatures \( T_\text{d} = 20 \text{ K} \) (upper panels) and 60 K (lower panels) are considered.

![Figure 4](image2.png)

**Figure 4.** Same as Figure 3, but for the polarization spectra of rotational emission. The polarization fraction varies significantly with the value of \( \beta \). It is lower for the higher temperature case.
AME, in terms of both peak emissivity and frequency. Nanosilicates with larger values of $\beta$ require a lower abundance of $Y_{Si}$ to reproduce the AME.

It is noted that both the Wilkinson Microwave Anisotropy Probe (WMAP) and Planck all-sky fitting to the AME requires two spinning dust components, one with a low peak frequency and the other with a higher peak frequency (Ade et al. 2015). In HLD11, it is successfully reproduced by rotational emission from spinning PAHs from the CNM and a warm ionized medium (see also Ade et al. 2015). In light of this study, two components perhaps arise from spinning dust in the same phase but with two separate dust populations, namely spinning PAHs and spinning ultra-small silicate. In a future paper, we will carry out fitting rotational emission from spinning PAH and spinning silicate to the AME obtained from Planck and QUIJOTE (Génova-Santos et al. 2015). It is worth mentioning that the dust model of Li & Draine (2001a) and Draine & Li (2007) contains two size components for PAHs with peaks at 0.6 and 2 nm. The latter is expected to contribute little to the AME in the 10–60 GHz frequency range because of its slower rotation.

6.2. Constraints from the AME Polarization

We have found that the polarization fraction of rotational emission by ultra-small silicate grains varies significantly with electric dipole moment $\beta$, and increases with decreasing $\beta$. Thus, the AME polarization can be used to constrain the value of $\beta$ because the polarization fraction is independent of $Y_{Si}$.

Figure 6 shows the polarization fraction for the different $\beta$ overplotted with the AME polarization data observed by various experiments. As shown, the predicted polarization for spinning silicate with $\beta < 0.2$ D exceeds most of the observed data, whereas spinning silicate with $\beta \geq 0.2$ D can adequately reproduce the level of the observed AME polarization.

It is also cautious that the polarization of MDE from free-flying iron nanoparticles is found to be within the AME.
polarization, therefore, the constraint by the AME polarization is not so restrictive.

6.3. Constraints from the UV Polarization of Starlight

Interstellar polarization of starlight was discovered more than 60 years ago (Hall 1949; Hiltner 1949). It is now becoming established that the interstellar polarization is produced by aspherical grains aligned with the magnetic field (see Lazarian et al. 2015 for a review). For sub-micron grains, the leading mechanism of grain alignment is radiative torque alignment, which was first introduced by Dolginov & Mitrofanov (1976), numerically demonstrated in Draine & Weingartner (1996), and analytically modeled by Lazarian & Hoang (2007). An unified model of grain alignment for interstellar grains with magnetic inclusions is presented in Hoang & Lazarian (2016), where the joint effect of magnetic relaxation and radiative torques can produce perfect grain alignment. For nanoparticles, paramagnetic relaxation (David & Greenstein 1951; Jones & Spitzer 1967; Roberge & Lazarian 1999) can induce weak alignment (Hoang et al. 2014). It is quantified in Hoang et al. (2014) that small grains dominate the UV starlight polarization. Therefore, the presence of interstellar silicate nanoparticles will obviously affect the UV polarization.

To see for which values of $Y_t$ and $\beta$ the predicted polarization by silicate nanoparticles does not violate the observed UV polarization, we compute the polarization curves of starlight for different values of $Y_t$ and $\beta$. To this end, we first adopt the best-fit model for the ISM from Hoang et al. (2014) that reproduces the “observed” polarization curve described by the Serkowski law (Serkowski et al. 1975) with $\lambda_{\text{max}} = 0.55 \mu m$ and the extinction curve. Then, we introduce a population of silicate nanoparticles with the size distribution (Equation (25)) and the alignment degree computed in Section 4. The final polarization curve is then computed using Equation (14) in Hoang et al. (2013) (see also Hoang et al. 2014).

The obtained results are shown in Figure 7, where the symbols show the Serkowski law. The UV polarization tends to increase with increasing $Y_t$, as expected. The case with $\beta < 0.2$ D easily exceeds the UV polarization for abundance $Y_t = 5\%$, which can be ruled out. Moreover, the case of larger $\beta$ ($\beta \geq 0.4$ D) and $Y_t \leq 10\%$ has only a small impact on the increase in the UV polarization.

Parameter space for spinning silicate that does not violate current observational constraints is sketched in Figure 8, where $\beta > 0.2$ D and $Y_t \leq 10\%$. It is noted that the constraint $Y_t < 10\%$ is consistent with the estimate based on the UV extinction and mid-IR emission by Li & Draine (2001b).

6.4. Comparison with Previous Spinning Dust Studies

Previous spinning dust models (DL98; Ali-Haïmoud et al. 2009; Silsbee et al. 2011; HDL10; HLD11) dealt with spinning PAHs, because they are a well-established dust population in the ISM (see Tielens 2008). The last spinning dust model (HDL10; HLD11) treats all known important physical effects, including grain wobbling due to thermal fluctuations within the grain, transient spin-up by single-ion collisions. In particular, the model can be applied to spinning dust grains of arbitrary shape and arbitrary temperature (HLD11). Since the physics of spinning silicate (iron) is essentially analogous to that of spinning PAHs, it is natural to use our advanced spinning dust model for modeling emission from spinning non-PAHs.

Finally, the spinning dust model developed in HDL10 and HLD11 is the unique model that can predict self-consistently the polarization level of spinning dust emission (see also Hoang et al. 2013, 2014); the spinning models from other groups only provide rotational emissivity. Therefore, our spinning dust model can be used to constrain the dipole moment and magnetic properties of nanoparticles using the AME polarization data.

7. SUMMARY

In this paper, we have obtained the principal results as follows:
1. We have quantified rotational emission from spinning ultra-small silicate grains for different grain size distribution parameters, various fractions of total Si abundance contained in ultra-small grains $Y_S$, and a wide range of the dipole moment per atom $\beta$.

2. The polarization fraction of spinning silicate emission is found to increase with decreasing value of $\beta$. The polarization fraction is below 5% for the dipole moments $\beta > 0.4$ D, but it can reach ~20% for $\beta \sim 0.1$ D assuming $T_d = 60$ K.

3. We have identified that the presence of silicate nanoparticles with $Y_S \leq 10\%$ and the dipole moment $\beta \geq 0.4$ D does not violate the UV polarization of starlight and the observed polarization of the AME. With this realization ($Y_S \leq 10\%$, $\beta \geq 0.4$ D), rotational emission from spinning nanosilicate can adequately reproduce the observed AME from the ISM.

4. Our quantitative results suggest that rotational emission from spinning silicate nanoparticles may be an important source of AME. Future observations searching for signatures of nanosilicates from the regions with prominent AME would be valuable in understanding the carrier of AME.

There are several outstanding questions on the problem of spinning nanosilicates, including the dipole moment and the abundance of nanoparticles $Y_S$. In particular, unlike PAHs that have been identified through mid-IR emission features, the signature of silicate nanoparticles is not yet clear. Further studies looking for signatures of silicate nanoparticles are necessary for better understanding of the AME. Lastly, since this paper is intended to explore the importance of rotational emission from spinning silicate, we have presented calculations only for a standard model of the ISM with typical physical parameters characterized by the CNM. More calculations for the different phases of the ISM and fitting to observational data will be presented in a future paper.

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**APPENDIX A**

**ROTATIONAL DYNAMIC TIMESCALES**

The thermal angular velocity of a grain around its symmetry axis (of rotational energy $k_B T_{\text{gas}}$) is

$$\omega_T = \left( \frac{2 k_B T_{\text{gas}}}{I_{\parallel}} \right)^{1/2} \simeq 3.3 \times 10^5 a^{-5/2} s^{-1/2} \rho^{-2/3} s^{-1},$$  \hfill (27)

where $\rho = \rho / 3$ g cm$^{-3}$, $a_{-5} = a / 10^{-5}$ cm, and $T_{\text{gas}} = T_{\text{gas}} / 100$ K.

The characteristic damping times of an oblate spheroidal grain for rotation along the directions parallel and perpendicular to the grain symmetry axis $\hat{a}$ are, respectively, given by (Lazarian 1997)

$$\tau_{d,\parallel} = \frac{3 I_{\parallel}}{4 \pi n_{\text{H}} m_{\text{H}} v_{\text{th}} a_{-5}^4 \Gamma_{\parallel}}, \quad \tau_{d,\perp} = \frac{3 I_{\perp}}{4 \pi n_{\text{H}} m_{\text{H}} v_{\text{th}} a_{-5}^4 \Gamma_{\perp}},$$  \hfill (28)

where $\Gamma_{\parallel} = \Gamma_{\perp}$ and $\tau_{d,\parallel} = \tau_{d,\perp}$ with $z$ the grain symmetry axis, and $x$ and $y$ the axes perpendicular to the symmetry axis. In the above equation, $n_{\text{H}}$ is the gas density, $m_{\text{H}}$ is the hydrogen mass, and $v_{\text{th}} = (2 k_B T_{\text{gas}} / m_{\text{H}})^{1/2}$ is the thermal speed of hydrogen. Above, $\Gamma_{\parallel}$ and $\Gamma_{\perp}$ are geometrical factors, which are given by (Roberge et al. 1993):

$$\Gamma_{\parallel} = \frac{3}{16} \left[ 3 + 4(1 - e^2) g(e) - e^{-2}(1 - (1 - e^2)^2) g(e) \right],$$  \hfill (29)

$$\Gamma_{\perp} = \frac{3}{32} \left[ 7 - e^2 + (1 - e^2)^2 g(e) + (1 - 2e^2)(1 + e^2[1 - (1 - e^2)^2] g(e)) \right].$$  \hfill (30)

where $e = \sqrt{1 - s^2}$ and

$$g(e) = \frac{1}{2e} \ln \left( \frac{1 + e}{1 - e} \right).$$  \hfill (31)

The limiting values are $\Gamma_{\parallel} = \Gamma_{\perp} = 1$ for $e = 0$ (i.e., spherical grains), and $\Gamma_{\parallel} = \Gamma_{\perp} = 3/8$ for $e = 1$.

For the typical parameters of the ISM, Equations (28) become

$$\tau_{d,\parallel} \simeq 6.58 \times 10^4 \rho^2 s^{2/3} a_{-5}^{-1} I_{\text{gas}}^{-1/2} \Gamma_{\parallel}^{-1} \text{year},$$

$$\tau_{d,\perp} \simeq 4.11 \times 10^4 \rho^2 s^{2/3} \left( \frac{1 + s^2}{1.25} \right) a_{-5}^{-1} I_{\text{gas}}^{-1/2} \Gamma_{\perp}^{-1} \text{year},$$

where $s = s/0.5$ and $I_{\text{gas}} = n_{\text{H}} / 30$ cm$^{-3}$.

Likewise, the characteristic damping times due to the rotational dipole emission can be rewritten as (see HDL10):

$$\tau_{d,\parallel} = \frac{3 I_{\parallel} c^3}{6 k_B T_{\text{gas}} \mu_{\parallel}^2}, \quad \tau_{d,\perp} = \frac{3 I_{\perp} c^3}{6 k_B T_{\text{gas}} \mu_{\perp}^2},$$  \hfill (33)

where $\mu_{\parallel}$ and $\mu_{\perp}$ are the components of the dipole moment $\mu$ parallel and perpendicular to the grain symmetry axis.

**APPENDIX B**

**MAGNETIC ALIGNMENT OF THERMALLY ROTATING GRAINS**

Since we are also interested in polarization of spinning dust emission, it is worth briefly summarizing the magnetic properties of silicate material and magnetic alignment.

Silicate material is an ordinary paramagnetic material. The zero-frequency susceptibility is given by

$$\chi(0) = 0.042 f_p \left( \frac{15 \text{ K}}{T_d} \right),$$  \hfill (34)

where $f_p$ is the fraction of paramagnetic atoms (i.e., atoms with partially filled shells) in the grain (see Draine & Weingartner 1996 and references therein). For the presented calculations, a typical value $f_p = 0.1$ is assumed for the ordinary paramagnetic material of silicate.
Assuming the critically damped condition (Draine & Lazarian 1999), the frequency-dependence imaginary part of susceptibility $\chi_2(\omega)$ reads

$$\chi_2(\omega) = \frac{\chi(0)\omega\tau}{1 + (\omega\tau/2)^2},$$  \hspace{2cm} (35)$$

where $\tau = \tau_2 \sim 2.9 \times 10^{-12} / f_p$ s is the spin–spin relaxation time (see Hoang et al. 2014).

Davis & Greenstein (1951) suggested that a paramagnetic grain rotating with angular velocity $\omega$ in an external magnetic field experiences paramagnetic relaxation, which dissipates the grain rotational energy into heat. This results in the gradual alignment of $\omega$ and $J$ with the magnetic field at which the rotational energy is minimum.

The characteristic timescale for the magnetic alignment of $J$ with $B$ is given by

$$\tau_m = \frac{l_I}{K(\omega)B^2} = \frac{2\rho a^2 s^{-2/3}}{5K(\omega)B^2},$$  \hspace{2cm} (36)$$

where $K(\omega) = \chi_2(\omega)/\omega$, and $l_I$ from Equation (1) has been used.

To describe the effect of grain alignment by magnetic relaxation against the randomization by gas atoms, a dimensionless parameter $\delta_m$ is usually used:

$$\delta_m = \frac{\tau_t}{\tau_m} \approx 0.28 \frac{a^{-1} v^2/3 \hat{B}^2 (K(\omega)/10^{-13} s^{-1})}{\bar{\eta}_I T_{\text{gas}} l_I},$$  \hspace{2cm} (37)$$

where $\eta_{II}$ is the gaseous damping for the rotation around the grain symmetry axis, and $\hat{B} = B/10 \mu G$.

The fluctuation terms associated with the magnetic dissipation are

$$B_{m,xx} = B_{m,yy} = \frac{T_d}{T_{\text{gas}}} \delta_{m},$$  \hspace{2cm} (38)$$

and $B_{m,zz} = 0$.

REFERENCES

Ali-Haïmoud, Y., Hirata, C. M., & Dickinson, C. 2009, MNRAS, 395, 1055

Ade, P. A. R., Aghanim, N., Arnaud, M., (Planck Collaboration), et al. 2011, A&A, 536, A20

Ade, P. A. R., Aghanim, N., Alves, M. I. R., et al. (Planck Collaboration) 2014, A&A, 566, 55

Ade, P. A. R., Alves, M. I. R., Aniano, G., (Planck Collaboration), et al. 2015, A&A, 576, A107

Battistelli, E. S., Rebolo, R., Rubiño-Martín, J. A., et al. 2006, ApJL, 645, L141

Davis, L. J., & Greenstein, J. L. 1951, ApJ, 114, 206

Désert, F.-X., Bourlanger, F., Leger, A., Puget, J.-L., & Sellgren, K. 1986, A&A, 159, 328

Dickinson, C., Peel, M., & Vidal, M. 2011, MNRAS, 418, L35

Dolginov, A. Z., & Mitrofanov, I. G. 1976, Ap&SS, 43, 291

Draine, B. T. 2011, Physics of the Interstellar and Intergalactic Medium (Princeton, NJ: Princeton Univ. Press)

Draine, B. T., & Anderson, N. 1985, ApJ, 292, 494

Draine, B. T., & Lazarian, A. 1998, ApJ, 508, 157

Draine, B. T., & Lazarian, A. 1999, ApJ, 512, 740

Draine, B. T., & Li, A. 2001, ApJ, 551, 807

Draine, B. T., & Li, A. 2007, ApJ, 657, 810

Draine, B. T., & Sutin, B. 1987, ApJ, 320, 803

Draine, B. T., & Weingartner, J. C. 1996, ApJ, 470, 551

Ericksen, W. C. 1957, ApJ, 126, 480

Finkbeiner, D. P., Langston, G. L., & Minter, A. H. 2004, ApJ, 617, 350

Génova-Santos, R., Martin, J. A. R., Rebuli, R., & et al., 2015, MNRAS, 452, 4169

Greenberg, J. M. 1968, in Nebulae and Interstellar Matter, ed. B. M. Middlehurst, & L. H. Aller (Chicago, IL: Univ. Chicago Press), 221

Hall, J. S. 1949, Sci, 109, 166

Hensley, B. S., Draine, B. T., & Meisner, A. M. 2015, arXiv:1505.02157

Hiltner, W. A. 1949, Sci, 109, 165

Hoang, T., Draine, B. T., & Lazarian, A. 2015, in Polarimetry of Stars and Planetary Systems, ed. L. Kolokolova, J. Hough, & A.-C. Lesvasseur-Regourd (New York: Cambridge Univ. Press), 81

Hoang, T., & Lazarian, A. 2011, ApJ, 741, 87

Hoang, T., Lazarian, A., & Martin, P. G. 2013, ApJ, 779, 152

Hoang, T., Lazarian, A., & Martin, P. G. 2014, ApJ, 790, 6

Jones, R. V., & Spitzer, L. 1967, ApJ, 147, 943

Kogut, A., Banday, A. J., Bennett, C. L., et al. 1996, ApJL, 464, L5

Lazarian, A. 1997, MNRAS, 288, 609

Lazarian, A., & Anderson, B. G., & Hoang, T. 2015, in Polarimetry of Stars and Planetary Systems, ed. L. Kolokolova, J. Hough, & A.-C. Levasseur-Regourd (New York: Cambridge Univ. Press), 81

Lazarian, A., & Hoang, T. 2007, MNRAS, 378, 910

Lazarian, A., & Roberge, W. G. 1997, ApJ, 484, 230

Leger, A., & Puget, J.-L. 1984, A&A, 137, L5

Leitch, E. M., Readhead, A. C. S., Pearson, T. J., & Myers, S. T. 1997, ApJL, 486, L23

Li, A., & Draine, B. T. 2001a, ApJ, 554, 778

Li, A., & Draine, B. T. 2001b, ApJL, 550, L213

Li, A., & Draine, B. T. 2002, ApJ, 564, 803

López-Caraballo, C. H., Rubiño-Martín, J. A., Rebuli, R., & Génova-Santos, R. 2011, ApJ, 729, 25

Mason, B. S., Robishaw, T., Heiles, C., Finkbeiner, D., & Dickinson, C. 2009, ApJ, 697, 1187

Mathis, J. S., Rumpl, W., & Nordsieck, K. H. 1977, ApJ, 217, 425

Roberge, W. G., Degrassi, T. A., & Flaherty, J. E. 1993, ApJ, 418, 287
Roberge, W. G., & Lazarian, A. 1999, MNRAS, 305, 615
Rubíño-Martín, J. A., López-Caraballos, C. H., Génova-Santos, R., & Rebolo, R. 2012, AdAst, 2012, 1
Saunders, R. W., & Plane, J. M. C. 2011, Icar, 212, 373
Serkowski, K., Mathewson, D. S., & Ford, V. L. 1975, ApJ, 196, 261
Silsbee, K., Ali-Haïmoud, Y., & Hirata, C. M. 2011, MNRAS, 411, 2750
Tibbs, C. T., Paladini, R., Cleary, K., et al. 2016, MNRAS, 456, 2290
Tielens, A. G. G. M. 2008, ARA&A, 46, 289
Weingartner, J. C., & Draine, B. T. 2001, ApJS, 134, 263