Measurement of Trustworthiness of the Online Reviews

Dipankar Das

Abstract

In electronic commerce (e-commerce) markets, a decision-maker faces a sequential choice problem. Third-party intervention is essential in making purchase decisions in this choice process. For instance, while purchasing products/services online, a buyer’s choice or behavior is often affected by the overall reviewers’ ratings, feedback, etc. Moreover, the reviewer is also a decision-maker. After purchase, the decision-maker would post their reviews for the product online. Such reviews would affect the purchase decision of another potential buyer, who would read the reviews before conforming to their final purchase. The question that arises is how trustworthy are these review reports and ratings? The trustworthiness of these review reports and ratings is based on whether the reviewer is rational or irrational. Indexing the reviewer’s rationality could be a way to quantify a reviewer’s rationality, but it does not communicate the history of their behavior. In this article, the researcher aims to derive a rationality pattern function formally and, thereby, the degree of rationality of the decision-maker or the reviewer in the sequential choice problem in the e-commerce markets. Applying such a rationality pattern function could make quantifying the rational behavior of an agent participating in the digital markets easier. This, in turn, is expected to minimize the information asymmetry within the decision-making process and identify the paid reviewers or manipulative reviews.

keyword: Sequential Choice Problem, Rationality, Pattern, Graph, E-Commerce, Information.

JEL Classification Code: D

1 Introduction

The present paper, for the first time, gives a measure to understand the irrational online reviews of consumers. The review comments are about voting against or in favor of the product. Hence, eliminating faulty review comments is essential in selling the product, setting the prices and advertisements, determining the market-clearing prices, and the industry competition. Choice models, graph theories, and fuzzy logic are used in the paper to measure the degree of rationality of the reviewers. Market analysts, marketing managers, and platform economy agents are the intended audiences.

Third-party intervention in the electronic-commerce(e-commerce) markets plays a vital role in purchasing decisions. By that, the researcher implies reviewer’s ratings, comments, etc., that could affect a buyer’s behavior. The decision to buy a product from an array or list of options often depends upon the number of positive
vis-à-vis negative customer comments corresponding to each option. Product ranking through online reviews based on evidential reasoning theory and stochastic dominance has been studied in [Qin and Zeng, 2022]. Reviewer’s attitude towards a specific topic, or a product, is positive, negative, or neutral has been studied in [Punetha and Jain, 2023]. The literature proposes that it is essential to quantify the trustworthiness of the reviewer’s feedback [Huang and Chen, 2006; Chen et al., 2017; Filieri, 2016; Racherla and Friske, 2012; Utz et al., 2012; Song et al., 2020]. Ratings also play an essential role, such as an average number of ratings [Khopkar and Nikolaev, 2017; Saumya et al., 2018; Xu et al., 2012]. Distribution of online ratings plays a crucial informational role in e-commerce platforms [Etumnu et al., 2020]. Many papers have doubted the reviewer’s ratings and comments and tried quantifying the impacts, including [Koh et al., 2010; Ma et al., 2019; Malik and Hussain, 2018; Pan et al., 2018]. Some studies have studied linguistics and psychology and combined them with the features of online reviews; such papers are [Hong et al., 2020; Cui et al., 2020]. Mining online reviews have become an essential tool for identifying consumer behavior and the innovation direction of products. It is difficult for producers and consumers to analyze and extract relevant opinions from many online reviews effectively. To overcome this, a product ranking method that combines feature–opinion pairs mining and interval-valued Pythagorean fuzzy sets was proposed in a study [Fu et al., 2020]. Even though a set of studies exists, these studies could not identify the route of a rational reviewer or their trustworthiness. No such standard theoretical and quantifiable methods are available. Notably, the reviewer and the decision-maker are both the same person. After a particular purchase, the decision-maker provides review comments on the digital platform. But, these review comments do not reveal the rationality of the decision-maker. The condition of rationality interprets the decision-maker’s behavior. The review reports of a rational decision-maker significantly affect the intended decision-maker, who reads the reviews before purchasing. Indexing the reviewer’s rationality could be a way to quantify it, but indexing does not convey the history of an agent’s behavior. A consumer could often build up ideas about a product from YouTube review videos (cosmetics, electronic goods, salon services, etc.). But how does a potential buyer determine the rationality of the YouTuber? One way to determine the extent of the YouTuber’s rationality would be to review their past uploads. If any of the YouTuber’s past uploads match the buyer’s purchasing experience, the buyer could also rely on the YouTuber for future purchases. This paper works with the idea that if it is found that the YouTuber is rational, then the uploaded videos would get the highest ratings and viewers. This paper gives a standard theory and quantifiable ideas by measuring the new rationality axiom and information indexing about a product and by measuring the reviewer’s/consumer’s consistency or rationality by using the pattern of past behavior. All reviewers are also consumers on any given platform, such as Amazon. Therefore, if it is possible to extract the system data of the records, it would be easier to identify the rationality of a consumer/reviewer. Rubinstein & Salant, [Rubinstein and Salant, 2006] explain a model of choice from lists where the agent does not have a comprehensive set of elements before him. Instead, the elements come to their mind sequentially. When an agent starts selecting from the list or the sequence since, each option carries some information about different objects. Consider a customer selecting an electronic gadget (say, a cell phone) from the list of cell phones on an online retail platform (say, Amazon), and the product carries an average rate from reviewers. The concerned agent reads this information and stops searching for the product for which the information is the maximum (which has
been tested in the paper). There are different information indexing on different digital platforms. Netflix uses a matching index (in percentages), where a particular movie or web series matches with the agent’s preference expressed in percentages. Other platforms use an average of the reviewers’ ratings on a five-point scale. The question lies in finding a scientific method to prepare these indices. Will the indices be adjusted according to the rational judgment of the agent or reviewers? In the e-commerce market, reviewers’ reports and ratings are common, but it does not reveal the trustworthiness of the reviewers. And the only way to measure trustworthiness is to measure the degree of rationality of the reviewers. If the pattern of rationality could be attached to the comments, then a potential buyer would take the decision-making with full information. The information index could be measured correctly. The present paper formally derives the measure of the pattern of the rationality of an agent in the e-commerce markets.

Given the existing indices (as mentioned in the above paragraph), the question arises- What are the prerequisites for an agent to believe the reviewers’ reports and their ratings? Are they rational individually and collectively so that the ratings can be taken confidently? Information about the consistency of the reviewer is missing. For example, if a reviewer gives positive (negative) feedback for a particular product, it must imply that the reviewer consistently prefers (rejecting) that object over other available options. In other words, the reviewer’s preference is transitive and acyclic at any given time. Whereas, if the reviewer gives negative feedback about a product and then purchases the product at any given time, they display inconsistent preference towards that object. Would this be an irrational behavior? Measuring rationality across time is dynamic. It is essential, yet undefined, to derive the rationality pattern dynamically. To begin with, each reviewer has two membership functions: the degree of consistency and the degree of inconsistency. If these two can be added along with the aggregate preferences dynamically, then the sequential choice problem would be free from asymmetric and incomplete information.

Moreover, a reviewer gives a review for a particular object. Hence, the rationality of that reviewer towards that object is essential. The paper derives the pattern of the rationality function on a specific thing and the overall rationality in a choice problem in different combinations of objects. Here lies the source of measuring the degree of rationality.

The design of the paper is as follows. In the first segment, the paper derives the choice problem in the e-commerce market, followed by a discussion on how to read information and index the available information for an object coming sequentially in the second segment. The third segment narrates a detailed theory of the rationality pattern function of an agent. The paper concludes by propounding a measure of the degree of rationality to a particular object and proposing an overall rationality measure.

1.1 Statement of Intended Contribution

Let’s start with a hypothetical example in Table: Review Comments and Ratings of the Given Decision Maker, to be analyzed in the recent article. The example is based on real-life practices of one agent and two periods model of choice. For reference see in Figure 1, Figure 2. The real-life review ratings with comments are given in Figure 1 & in Figure 2 as examples.

A decision-maker who has taken decisions to select one object from the set $X$ of four objects $X = \{M, N, V, Z\}$
in two periods \( t = 1, 2 \). Now, the article assumes that the two preference patterns given have been extracted from the model of the choice process, as mentioned in section 3.1. The preference pattern of the given decision maker at time \( t = 1 \) can be written as \( M \rightarrow N \rightarrow V \rightarrow Z \), and the preference pattern at time \( t = 2 \) can be written as \( Z \rightarrow V \rightarrow N \rightarrow M \). Here (\( \rightarrow \)) means preferred to. These two are known and can be extracted from history. The decision-maker also gives reviews in words with ratings on a five-star scale for M, N, V, and Z objects, respectively, in period \( t = 2 \) as below in Table: Review Comments and Ratings of the Given Decision Maker.

| Commodity/Object type | Review Comments | Review Ratings | Degree of Trustworthiness |
|-----------------------|-----------------|----------------|---------------------------|
| M                     | Bad product     | ⭐⭐⭐⭐⭐          | ?                         |
| N                     | Not so good product | ⭐⭐⭐⭐          | ?                         |
| V                     | Relatively good product | ⭐⭐⭐⭐          | ?                         |
| Z                     | Premium product  | ⭐⭐⭐⭐⭐          | ?                         |
| Or, Z                 | Not a Premium product | ⭐⭐⭐⭐⭐          | ?                         |

Hence, the question is to measure the degree of trustworthiness of this hypothetical decision maker (or reviewer). The last column needs to include the true value of the degree of trustworthiness of each review comment and the ratings. For example, for object \( N \), the reviewer’s comment is Not so good product with a two-star rating. But how far these comments and ratings can be trusted? This is not known. Hence, the last column has a question mark (?). This is why question marks are there for each review’s comments and rating. If this degree of trustworthiness is attached to it, then the decision-making of the new buyers will be error-free and rational with complete information. The article derives the method of measuring the degree of trustworthiness of the individual.
review comments and ratings.

To understand why this measure of trustworthiness is required, consider an example. A seller knows an object has demand but posts it at a price higher than the estimated private value. The potential buyer may look for a price reduction. But if there are review ratings and comments in favor of the object relative to the available substitutes by previous users, the potential buyer would be tempted to buy the thing at the offered price. This is even at a price higher than the available alternatives. This implies that review ratings and comments can change private values further [Das, 2021].

The digital markets, both formal and informal, have an inbuilt system of getting live reactions from past users of the object being posted for sale. Consumers need complete information about the prices of goods. Still, their information could be better about the quality variation of objects simply because the latter statement is more difficult to obtain. The buyer can also buy the thing at a higher price than the private value. Without any other information, the consumer would not know if he was better off experimenting with low- or high-priced brands. Consumer behavior is also relevant to determining monopoly power in consumer industries [Nelson, 1970].

The information problem is to evaluate the utility of each option. Search plays an important role here. Search to include any way of assessing these options subject to two restrictions: (1) the consumer must inspect the option, and (2) that inspection must occur before purchasing the brand. Stigler has developed a theory of search already [Stigler, 1961]. The model is appropriate for the following conditions. Suppose a consumer has to decide on the number of searches he will undertake before searching. After searching, he can choose the best from the set of alternatives he has examined. Assume further that he must search by random sampling and that he knows the form of the probability distribution of his options. After using a brand, its price and quality can be combined to give us posterior estimates of the utility of its purchase. Digital markets today have been able to eliminate these shortcomings and have been able to provide posterior estimates of the utility of its purchase. This is generated in the form of customer reviews, ratings, reactions, comments, etc... Not only that, this posterior estimate of the utility is not constant but is changing sequentially. Therefore, the expected
value is also changing. As a result, the total value is also changing. The buyer can buy the object at a higher price than the private value if the posterior estimate of the utility is in increasing order. The present article tries to provide a new measure to it. A set of paid review rations are also present. To eliminate these artificial review rations and comments, it is required to know the trustworthiness of each review. Hence, the proposed measure will generate a strong belief in the posterior estimate of the utility. The present article derives the last column “Degree of Trustworthiness” of the reviewer for each object and reviews comments and ratings given the past two periods’ preference patterns are known. In section 3.6, the complete table has been prepared with this new information of “Degree of Trustworthiness”, along with the interpretation.

1.1.1 Objective of the Study

The review style is presented here interns of review comments and star (∗) ratings on a five-point scale, etc...So, first, the paper shows the decision-making process of an agent in period $t = 1$ and after that, in period $t = 2$ justifying with the review reports given by the agent himself. A detailed analysis of the preference patterns has been discussed after that. These data in their present form need to provide more information to the new decision-maker who plans to buy an object from these four (from Table 1). The previous decision-maker or the reviewer could be wrong or could be irrational. How would the new decision-maker judge this? This is undefined here. Here, the essential facts or information about the reviewer are missing, such as the trustworthiness of the reviewer or the previous decision-maker. Whether they are rational, trustworthy, or not? And to what extent? How to get there? The following sections have given a method to identify the pattern of the rationality of the reviewer based on their past decisions, viz, in periods $t = 1$ & $t = 2$. The present paper explains how to assess trustworthiness, identify the rationality pattern, and measure the degree of rationality of the reviewer. In short, the paper describes how to determine these review ratings and comments based on the past two periods’ preferences of this hypothetical decision-maker/reviewer.

2 Related Literature

Despite the explosive growth of electronic commerce, very little is known about how consumers make purchase decisions in such settings. While making purchase decisions, consumers often cannot evaluate all available alternatives in great depth and, thus, tend to use two-stage processes to reach their decisions. At the first stage, consumers typically screen many available products and identify a subset of the most promising alternatives [Häubl and Trifts, 2000]. This means a sequential choice problem is there. Consumers have limited cognitive resources and may be unable to process the potentially vast amounts of information about these alternatives. Consumer trust plays an important role in online commerce. Then, what makes customers return to an e-vendor? This question has been answered in [Gefen et al., 2003]. It has been explained that trust-building mechanisms should be there [Awad and Ragowsky, 2008]. Research on user trustworthiness on social networks is gaining attraction, with many interesting studies conducted in recent years. A recent paper has studied a systematic review of it and its future directions in [Alkhamees et al., 2021]. Creating a trust-based connection with
customers is a primary benefit that is nearly as important as the technical attributes of the website, such as usefulness. Perfect competition is rarely seen in practice, where market imperfections mitigate such factors as perfect information, market prices, and features of commodities. Combined with the criteria for product selection, the multiple sources of information available through the e-commerce channel can reduce consumers’ search costs and support their intelligence phase, and, subsequently, they can lead to the development of a plan to evaluate the alternatives available for making the decision. Online support for gathering information leads to better development of criteria for evaluating decision alternatives. Recognizing the need for information gathering (intelligence phase), the channel owner can provide links to an independent comparison and rating website, thus supporting the consumer’s decision-making process [Kohli et al., 2004]. In this respect, review ratings and review comments play important roles. But a correlation between good reviews and high demand may be spurious, induced by an underlying correlation with unobservable quality signals [Reinstein and Snyder, 2005]. The author has explained that an early positive review increases the consumers’ attention, and the influence effect differs across categories. The results suggest that expert reviews can be an important mechanism for transmitting information about goods of uncertain quality [Reinstein and Snyder, 2005]. A theoretical analysis of the impact of such behavior on firm profits and consumer surplus has been done in [Dellarocas, 2006]. The study shows that, in most cases, all firms will manipulate their online ratings. This implies that consumers should expect a certain amount of hype to be present in most online forums and must learn to compensate for it (by properly deflating what they see and read in such forums) when making inferences from such information. The online marketplace should have a mechanism to remove the problem of the market of lemons. Because numerical ratings do not convey much information beyond text comments, feedback forum designers could attempt to either codify and summarize all of the sellers’ text comments or enable buyers to report their past experiences in terms of meaningful and quantifiable categories [Pavlou and Dimoka, 2006]. Here is the question of how to identify the past behavior of the reviewer. A study finds that dispersion of ratings is positively correlated with sales growth and that the mean of the high end of the set of ratings is positively correlated with growth [Clemons et al., 2006]. A normative model to address several important strategic issues related to consumer reviews has been studied in [Chen and Xie, 2008]. The impact of changes in ranking on product adoption and the lack of impact of product reviews on the adoption of popular products studied in [Duan et al., 2009]. An interesting study has been done, where it has been shown that online reviewers are competing to get attention as a scarce resource. This study tries to understand how online users, especially online reviewers, compete for scarce resources and attention when writing online reviews [Shen et al., 2013]. We still have a limited understanding of the individual’s decision to contribute these opinions. The selection and adjustment effects that influence the evaluation decision have been studied here in [Moe and Schweidel, 2012]. The type of product moderates the effect of review extremity and depth on the helpfulness of a review [Mudambi and Schuff, 2010]. One can learn from and be affected by other consumers’ opinions and others’ actual purchase decisions. Opinions or word of mouth have been studied in [Chen et al., 2011]. Social influence from others’ online ratings has been studied in [Sridhar and Srinivasan, 2012]. Consumers use reviewer disclosure of identity-descriptive information to supplement or replace product information when making purchase decisions and evaluating the helpfulness of online reviews. The paper studies in [Forman et al., 2008] and suggests that online retailers may be able to
increase sales on their sites by taking actions to encourage reviewers to reveal more identity descriptive content about themselves. Understanding the relationship between firms’ promotional marketing and word of mouth in the context of a third-party review platform has been studied in [Lu et al., 2013]. Online reviews should create a producer and consumer surplus by improving the ability of consumers to evaluate unobservable product quality. Fake or “promotional” online reviews are also there. Suboptimal choices and consumers’ mistrust reviews are the two important impacts. These are due to Fake or “promotional” online reviews. A methodology for empirically detecting review manipulation has been given in [Mayzlin et al., 2014]. However, no such methods are available to understand the trustworthiness of the reviewer. The present article tries to give a method of indexing and identifying the pattern of online reviewers.

3 Model

The model is based on the sequential choice problem derived here with the help of Theorem 1. First, the article set the sequential choice process in e-commerce, and then using hypothetical review data, the rationality pattern function and the degree of trustworthiness/ degree of rationality have been derived. In the end, a standard algorithm has been proposed.

Theorem 1 There exists a two-way consistency of the choice problem. The choice would be the same if the decision-maker moved from a large set to a small set and from a small one to a large one. This happens provided the decision-maker can interpret the information correctly for each set.

The proof is in the appendix.

Remarks 1 This Theorem 1 indirectly says that the decision-maker is deciding on a reference. It can be achieved in both ways, from a large or a small set, if the reference point is in both sets. A study shows that 53% of the products have a bimodal and non-normal distribution. For these products, the average score does not necessarily reveal the product’s true quality and may provide misleading recommendations. It shows that the distribution of the Ratings on Amazon.com fitted with a U-shaped curve for a Music CD. However, when they asked an unbiased population of 66 students to test the product whose review distribution is shown on the left, they obtained a distribution close to a Gaussian [Hu et al., 2006]. It means there is a problem with the rationality condition. More consumers are leaving extreme reviews than consumers leaving ordinary reviews. Therefore, the average of the ratings does not reflect the aggregated opinion of all the consumers; instead, it is a compromise of the two extreme opinions. The average score of the online reviews does not reveal the “true” product quality since the consumers’ opinions do not converge to or concentrate on the mean. A bimodal distribution is far more general. Theorem 1 takes care of this problem of rationality condition.

3.1 The Choice Process

This paper puts forth a measure to identify a decision-maker’s preferences and rationality pattern so that a new decision-maker can compare their perceived and actual notions about the former decision-maker who is now the
reviewer. The information indices should showcase buyers’ as well as reviewers’ behaviors. To start measuring rationality, the first one needs to know how the decision set has been prepared for measuring rationality. Buying an object from an online/digital retail service provider is as follows. Initially, an agent opens an account with online service providers/platforms like Flipkart, Amazon, etc. Initially, the agent must provide basic information and preferences while creating a profile. When the agent decides to purchase an object, they select the product catalog and choose a particular thing. A new agent first searches a few objects and starts filtering based on preferences and budget constraints. After filtering, the attainable set is ready before the agent and appears as a sequence. This sequence acts like a list. There could be more than one list of a given feasible/attainable set but with different orders. The agent selects from the list and creates a wish list. Pay has been made at this stage. From the wish list, the agent again shortlists and creates a new list called 'add to the cart. The final choice would be made from the cart, and payment would be made. Therefore, there are four steps in buying: creating an attainable set, making a wish list, building the cart, finalizing the object, and making payment. Therefore, there are four sets. Each choice/buying process is time-dependent. These sets are visible, and it would be easy to measure an agent’s rationality if the buying process pattern could be analyzed. In a planned system, all of these choice processes are recorded.

**Definition 1** Object: An object $x$ is a collection of finite $n$ attributes or dimensions. Hence the object is treated as a vector point in $n$-tuple in $\mathbb{Z}_n^+$, i.e. $x_i \in \mathbb{Z}_n^+$. The object could be a personal computer(PC)/laptop, etc... A PC is a collection of different attributes, say, for example, Operating systems, RAM, Screen, Hard Disk, SSD, Battery, Weight, Color, Slim, etc... A movie is an object. The attributes are Romance, Action, Drama, Thriller, Acting, Casting, Music, Political, Social, Child, Art, Outdoor Location, Director, VGA, HD, Animation, Graphics, etc... The combination of these attributes creates an object, say $x$. These attributes are in non-negative discrete/integer spaces. For example, SSD is available for either 128GB or, 256GB or 512GB or 1024GB, etc., but not between any pair. The space is $\mathbb{Z}_n^+$ for $n$ number of attributes. For any two alternatives $x_i$ & $x_j \in \mathbb{Z}_n^+$ and for any $0 < \alpha < 1$, $[\alpha x_i + (1 - \alpha)x_j] \notin \mathbb{Z}_n^+$. The space $\mathbb{Z}_n^+$ is a discrete and weak convex. For example, for $0 \leq \alpha \leq 1$, the convex combination for 128 GB and 256 GB is possible, i.e., GB for $\alpha = 0$ and 256 GB for $\alpha = 1$, but not for $0 < \alpha < 1$. The stages of the choice process are given below.

**Definition 2** Choice Problem: Let the set of alternatives are in $X = \{x_1, ..., x_n\}$. Therefore, the choice problem is

$$C : X \rightarrow X$$

Where, $X_i \subseteq 2^n$. $2^n$ is the set of all subsets. The preference relation on $X$ is reflexive, transitive, and asymmetric, and the relation is $R = "better than or equal to"$ i.e. $\succsim$. The choice problem is any subset $X_i$, and the choice function is $C_\succsim(X_i) \rightarrow X$.

Here $X_i = \{S, A, W, X\}$ is the set of subsets for $X = \{x_1, ..., x_k\}$ such that $S \subseteq A \subseteq W \subseteq X$. So, there are four steps. First, the decision maker selects $X$; in the second stage, the decision maker selects the smaller set $W \subseteq X$;
third, the decision maker decides the subset $A \subseteq W$ and at the end, the final choice set $S \subseteq A$. These steps are given below.

**STAGE 1**

Creation of an attainable set $X \subseteq \mathbb{Z}_n^+$. This is a first-level shorting. Here, the agent first filters and sets the requirements. It represents the agent’s extraneous constraint, such as budget constraint or some required attributes that make them willing to buy the agent. The attainable set derives by solving a system of constraints;

$$AY \leq B$$  \hspace{1cm} (1)

Where $A$ is a matrix of coefficients, $B$ is the column of the constraints, and $Y$ is the column vector of all $n$ attributes that are required collectively to represent an object here. The feasible region/solution set is compact and has a discrete convex polyhedron. Each vector point in this region represents an object. The possible options would be visible by matching these vector points in the next step. The solution of equation (1) is the set $X \subseteq \mathbb{Z}_n^+$.

**STAGE 2**

Making a wish-list $W$ from the attainable set $X$, such that $W \subseteq X$. This is a second-level shorting. Let there be $n$ attributes. So, the vector $x_i \in X \subseteq \mathbb{Z}_n^+$; where $x_i$ is the $i^{th}$ vector; $i = 1, 2, ..., k$. There are finite $k$ vector points in $X$. All the $k$ vectors are in the achievable set $X \subseteq \mathbb{Z}_n^+$. The consumer/agent faces a sequence of alternatives in the form of a list [Rubinstein and Salant, 2006]. The agent faces a sequence of vectors of different combinations of attributes. Each list is satisfied by the constraints set, as explained in Stage 1. So, each agent faces a budget line for the different vectors. Hence, the question is, if each vector carries the same level of income/expenditure, then where do they stop? Which one should you select and stop searching for the next sequences?

**Definition 3** Choice Function from the Lists: Let $X \subseteq \mathbb{Z}_n^+$ be a finite set of vectors where. Let $\Gamma$ be the set of all lists. A choice function from lists $D : \Gamma \rightarrow X$ is a function that assigns to every list $L = \{x_1, ..., x_k\}$ a single vector $D(L)$ from the set $X = \{x_1, ..., x_k\}$.

The service provider creates a set of lists based on the set $X$. Let $X \subseteq \mathbb{Z}_n^+$ be a finite set of vectors. Let $\Gamma$ be the set of all lists. A choice function from lists

$$f_1 : \Gamma \Rightarrow W$$  \hspace{1cm} (2)

is a function that assigns to every list $L_i \in \Gamma$ and $L_i = \{x_1, ..., x_k\}$ the agent selects a set of vectors/objects $f_1(L)$ from the set $X = \{x_1, ..., x_k\}$ and creates a wish-list $W$. Here the mapping notation $\Rightarrow$ means the set-valued function.

**STAGE 3**

Creation of an add to cart set $A$ from $W$, such that $A \subseteq W$. This is a third-level shorting. This is also a set-valued map as no payment is there at this stage, so the agent can

10
select more than one object at this stage and makes a small list \( A \) where \( A \subseteq W \subseteq X \subseteq \mathbb{Z}_+^n \). The choice function is \( f_2 \).

\[
f_2 : A \Rightarrow W \tag{3}
\]

**STAGE 4**

Final choice from the cart and making payment, \( S \subseteq A \subseteq W \subseteq X \subseteq \mathbb{Z}_+^n \).

The paper assumes that the agent buys only one object, but it can be more than one or in a bundle. The choice function is given below. If selects more than one

\[
f_3 : S \Rightarrow A \tag{4}
\]

If one selects one object, then

\[
f_4 : S \rightarrow A \tag{5}
\]

The final choice is to say \( x_i \) using equation (5) or a set \( S \) where \( x_i \in S \) using equation (4).

**Definition 4** Rational Choice: The decision maker/agent has a strict preference relation, i.e., complete, asymmetric, and transitive \( \succ \) over \( X \) and chooses the \( \succ \)-best element from every list.

The rationality conditions states that the final choice say \( x_i \) or a set \( S \) where \( x_i \in S \) fulfills the condition that \( C(S) = C(A) = C(W) = C(X) \) for \( S \subseteq A \subseteq W \subseteq X \subseteq \mathbb{Z}_+^n \). The discrete space \( \mathbb{Z}_+^n \) guarantees that the attainable set would be countable few; hence it is bounded. And this should be true in all the times i.e. \( C_t(S) = C_t(A) = C_t(W) = C_t(X) \). Collecting the data for all the transactions at different times can be measured by the degree of rationality by identifying the pattern.

### 3.2 Single Agent and Two Periods Model of the Rationality Pattern

Section 3.1 gives a theoretical idea of a sequential choice problem in the e-commerce markets. The present section gives a step-by-step approach to measuring/identifying the rationality pattern of a decision maker in the sequential choice problem in the e-commerce markets. This section is based on the hypothetical example explained in the problem statement section 1.1 and in Table 1 of a decision-maker who has made decisions to select one object from the set of four objects \( X = \{M, N, V, Z\} \) in two periods \( t = 1 \& 2 \). The preference pattern at time \( t = 1 \) and \( t = 2 \) are given as \( M \rightarrow N \rightarrow V \rightarrow Z \& Z \rightarrow V \rightarrow N \rightarrow M \) respectively. The decision-maker also gives reviews in words with ratings on a five-star scale for M, N, V, and Z objects, respectively, in period \( t = 2 \) as given in Table 1. The analysis of preference patterns in each period has been discussed first. After considering both periods together, an aggregate preference pattern has been derived.

#### 3.2.1 Preference in Period \( t = 1 \) and the Pattern

This section analyzes the preference pattern of the agent in period \( t = 1 \) on the set \( X = [M, N, V, Z] \). The choice problems for the set \( X = [M, N, V, Z] \) and the different steps of the choice process at time \( t = 1 \) are extracted,
say, for example, from the system using the choice steps have been discussed in section 3.1 below. 

\[ X^T \subseteq Z_n^d = (M \ N \ V \ Z); f_1 = W^T = (M \ N \ V); f_2 = A^T = (M \ N) \]

\[ \& f_3 = S^T = (M) \]

The sets \( X^T; W^T; A^T; S^T \) have been calculated using the four steps in section 3.1. Here, \( S^T \subseteq A^T \subseteq W^T \subseteq X^T \). The \( k \) number of objectives can be arranged in \( k \) places in \( k! \) ways. The entire preference analysis follows only one (i.e., fixed) order as given in the present case of \( k = 4; \) i.e., \( X = \{M, N, V, Z\} \). Here (\( \rightarrow \)) means "preferred to." Comparing \( S^T \) and \( A^T \), the preference derives as \( M \rightarrow N \) because \( N \in A \& N \notin S \). Comparing \( A^T \& W^T \) the preference relation derives as \( M \rightarrow V \& N \rightarrow V \). Comparing \( W^T \& X^T \) the preference relation derives as \( M \rightarrow Z; N \rightarrow Z \& V \rightarrow Z \). Therefore, the complete relation can be written as \( M \rightarrow N \rightarrow V \rightarrow Z \).

Due to transitivity and acyclic preference, the final choice is \( M \rightarrow Z \).

The matrix \( U_{t=1} \) of this preference relation is given below based on the binary relation, i.e., if preferred, then one and zero otherwise.

\[
U_{t=1} = \begin{bmatrix}
M & N & V & Z \\
M & 0 & 1 & 1 & 1 \\
N & 0 & 0 & 1 & 1 \\
V & 0 & 0 & 0 & 1 \\
Z & 0 & 0 & 0 & 0
\end{bmatrix}
\]

The outdegrees of the four vertices \( [M, N, V, Z] \) of the graph \( U_{t=1} \) are \( [3, 2, 1, 0] \). This means \( M \) carries the highest degree. This will be used as a pattern in the subsequent analysis. This is a non-negative square matrix and an upper triangular matrix as well. If considering only the elements above the diagonal and starting at the left of the first row, then obtain the vector of the preference \( P \).

\[
P = \begin{bmatrix}
1 & M \rightarrow N \\
1 & N \rightarrow V \\
1 & V \rightarrow Z \\
1 & M \rightarrow V \\
1 & N \rightarrow Z \\
1 & M \rightarrow Z
\end{bmatrix}
\]

Hence, the final choice is \( M \rightarrow Z \), i.e., \( M \). The \( U_t \) is time-dependent. The graph of this choice, \( U_t \), is shown in figure 3 in which no cycle exists. The preference graph \( U_t \) is also time-dependent. The individual graph may be acyclic, but the collective graph might not be the case of free from the cycle. The system will have a sequence of preference graphs, i.e., \( [U_t]^{m}_{t=1} \). The time has been set to a finite upper bound \( m \), i.e., \( t \in [1, m] \) to derive collective graphs. The Graph \( U_{t=1} \) can be written in vector form as

\[
P_{t=1}^T = [0111001100010000]
\]
The transpose of $P$ is $P^T$. The pattern $P^T$ has two sub-patterns $P^T_{0/t=1}$ & $P^T_{1/t=1}$ as

$$P^T_{0/t=1} = [[0], [00], [000], [0000]]$$

and

$$P^T_{1/t=1} = [[111], [11], [1]]$$

[When the choice problem is moving from the large set to the small set]

According to Theorem 1, rationality also follows the following pattern.

$$P^T_{1/t=1} = [[1], [111], [111]]$$

[When the choice problem is moving from the large set to the small set]

**Remarks 2** For a rational choice, these two patterns must select the same object at which the information index, i.e., $H$, is higher (using Lemma 2). If any choice problem is rational in terms of transitive and acyclic, then that should follow the order of the pattern $P^T$. Here 1 means preferred to, and 0 means not preferred to. Any change in this pattern signifies that the choice problem is inconsistent/ irrational. Correct measurement of the $H$ index confirms the pattern.

### 3.2.2 Preference in Period $t = 2$ and the Pattern

This section analyzes the preference pattern of the agent in period $t = 2$ on the set $X = [M, N, V, Z]$. Now consider the preference graph $U_t$ say $U_{t=2}$ for $t = 2$. Here, say the preference is just the opposite given that the $U_{t=1}$ has happened, then the graph and the $P^T$ would be the complete relation and can be written as $Z \rightarrow V \rightarrow N \rightarrow M$. Due to transitivity and acyclic preference, the final choice is $Z \rightarrow M$. The preference matrix $U_{t=2}$ is given below.

\(^1\)H index has been defined in detail in the appendix.
The outdegrees of the four vertices \([M, N, V, Z]\) graph \(U_{t=2}\) are \([0, 1, 2, 3]\). This means \(Z\) carries the highest degree.

The Graph \(U_{t=2}\) can be written in vector form as

\[
P_{t=2}^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}
\]

According to Theorem 1 of rationality, the pattern \(P_{t=2}^T\) also follows the following pattern as \(P_{t=1}^T\).

\[
P_{t=1}^T = \begin{bmatrix} 1 \\ 11 \\ 111 \end{bmatrix}
\]

Remarks 3 Comparing the two rationality patterns, it can be written that the two sets \(P_{t=1}^T = \{1, 11, 111\} = \{111, 11, 1\}\) and \(P_{t=2}^T = \{1, 1, 111\} = \{111, 11, 1\}\) are carrying the same rationality behavior in any given time \(t\). Therefore, \(P_{t=1}^T = P_{t=2}^T\) for any two different choices (i.e., objects) in two different times, i.e., in any static choice. Therefore, either of the two can be used to represent the rationality pattern for any given time. But these are not the same rationality conditions for the two different times taken together in a single graph for any given object. This has been explained below.

3.2.3 Joint Preference for the Period \(t = 1&2\) Together and the Pattern

This section analyzes the aggregate preference pattern of the agent in period \(t = 1&2\) on the set \(X = [M, N, V, Z]\).

The Graph \(U_{t=1}&U_{t=2}\) can be written jointly in vector form as

\[
P_{t=1+2}^T = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}
\]

Hence the aggregate pattern of \(U_{1}&U_{2}\) can be written as a matrix form \(U_{1+2}\) and in figure 4.

\[
U_{1+2} = \begin{bmatrix} M & N & V & Z \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}
\]

The outdegrees of the four vertices \([M, N, V, Z]\) graph \(U_{1+2}\) are \([3, 3, 3, 3]\). This means all the vertices carry the highest outdegree and indegree, i.e... In this situation, the derivation of consistency level is difficult and has been explained in a dynamic sense in this paper. The cycles are present in this joint graph. The paper gives a measure
to identify the rationality pattern and the degree of it if these two graphs, viz. \( U_{t=1} \& U_{t=2} \) are present. The Graph \( U_{1+2} \) can be written in vector form as

\[
P_{1+2}^T = [011101111011110]
\]

Or, segregating patterns can be written as below.

\[
P_{1+2}^T = [0, 111; 1, 011; 11, 011; 111, 0]
\]

**Remarks 4** The pattern for periods one and two should carry all the information, including pairwise comparisons. The pattern, \( P_{1+2}^T \) and \( P_{1+2}^T \ast \), both represent the joint preferences of periods one and two. But the pattern \( P_{1+2}^T \) doesn’t carry all the information, including the pairwise comparison, because the graph includes cycles. Hence, in the subsequent sections, the pattern \( P_{1+2}^T \ast \) has been considered to analyze the pattern of rationality. Here are two important concepts to read the pattern: “Stop” & “Run.”

**Definition 5** The “Stop” and the “Run”: The qualitative variables ‘stop’ and ‘run’ are represented here by a set of binary numbers, \( \{0, 1\} \). The pattern \( P_{1+2}^T \ast \) can be interpreted by considering 0 as a stop and ‘1’ as a run. ‘Stop’ means ‘preferred to,’ and ‘run’ means ‘not preferred to.’ The general way to write the typical pattern for \( \sum_t \) periods is \( P_{1+2}^T \).

**Numerical Calculation 1** Therefore the general joint pattern is

\[
P_{1+2}^T = [011101111010100001001101110].
\]

There is a one-to-one correspondence; from \[MNVZMNVZMNVZMNVZMNVZMNVZMNV\] to \[01110110010100100000010011001110]\.

**Remarks 5** In the pattern of two periods, i.e., \( P_{1+2}^T \), the first binary value is 0, and that is for \( M \). It means stop here and start counting the runs for \( M \). The object \( M \) is preferred to the object \( N, V, Z \), i.e., three runs. Because
the next binary numbers ’1, 1, 1’ are for N, V, Z respectively. As a result, the objects N, V, Z are not preferred to any of the objects in the set X. This means for the pattern \( P_{2}^{2} \), it starts with zero, so first is stop, then run is up to three ’111’s’ until zero occurs, then again stop and so on. For four consecutive zeros for four objects, the number of runs would be 0 because, in this example, the number of objects is 4. Now, create two sets, one with the stop and one with the run. The run says, for example, ’111’ means an object has been selected over three other objects in the set X. The same thing is true for ’11’, i.e., an object has been selected over two other objects, etc...So, the first is to stop for M, and there are three immediate three runs. It means M is preferred to the next three objects. N, V, Z. The next is stop and is for M again, but the primary point stops. It suggests we must stop here to count runs for M. The zero here is for N, and now we can start counting the runs for N if possible, and there are two runs for N. After that, no runs can be counted for M, N, but the run is one for me. This is how the frequency of runs or stops can be computed for the set of four objects for two periods. This is given below:

\[
\begin{bmatrix}
M & N & V & Z & M & N & V & Z \\
3 & 2 & 1 & 0 & 0 & 1 & 2 & 3
\end{bmatrix}
\]

This frequency is interpreted as the total number of runs for each object. For M, the total number of runs is 3. This idea of stop and run will be used in the following sections to analyze the patterns of choice and derivation of the rationality pattern function. And, indeed, complete irrationality is not possible (Lemma 1). Given these two graphs \( U_{t=1}, U_{t=2} \) and the joint graph \( U_{1+2} \), the next sections explain how to identify the rationality pattern of the decision maker and the degree of it.

**Lemma 1** The complete irrational agent is not possible

The proof is in the appendix.

**Remarks 6** Lemma 1 suggests that each agent possesses some information in hand.

### 3.3 The Rationality Pattern Function and the Measurement

This section formally derives the rationality pattern function for a particular object. When a reviewer reviews a particular object, their comments should be supported by their preference pattern for that object only and not on their entire choice problem or buying behavior. The paper uses the idea of automata to give a mathematical structure. According to the theory of automata \( \sum_{1}^{1} = \{0, 1\} \), is the set of binary alphabets. Using these alphabets, different strings or patterns can be created [Hopcroft et al., 2001]. Here 0 means not preferred, and preferred means 1. The empty string is denoted by \( \sum^{0} = \{\epsilon\} \). This means no revealed preferences have been made. The pairwise comparison is denoted by a string/pattern \( \sum^{2} = \{00, 01, 10, 11\} \), i.e. \( 2^{n} \) possibilities where \( n = 2 \). Here, the string is treated as a pattern. This means the set of all combinations of preferred and non-preferred patterns for two objects’ cases. For example, the two objects can be preferred or not in four different ways, i.e., [0, 1] means that the first object is not preferred over the two objects, and the second object is preferred over the two. When \( n = 3 \) then the pattern is \( \sum^{3} = \{000, 001, 010, 100, 101, 110, 011, 111\} \) i.e. \( 2^{3} = 8 \) possibilities. The possible compression or the preference order for \( 2^{4} \) is
The membership function or the degree of rationality is given by: 

\[ \text{Degree of rationality} = \frac{1}{2^i + 2} \]

The complete (adding) pattern is 

\[ P = \{1, 1, 1, 1\} \] for a set of four objects in \( X \). This means the agent prefers the selected object consistently in the large and smaller sets, respectively. This signifies that \( L_R \subseteq \sum^+ \) or \( L_R \subseteq \sum^* \). Here, we consider that the agent buys something through pairwise comparison, so \( \sum^+ \) has been considered throughout the paper. Using the past behavior data of any agent, the \( \sum^+ \) can be easily calculated. Then, the next task would be to identify the rationality pattern individually for any particular object. \( L_R \) is the rationality pattern for a single-time buying behavior. What is it for repeated buying behavior? i.e. What would be the rational preference pattern \( L_R \) for \( \sum^+ \) Let the repetitions be allowed in preference order in \( \sum^+ \) and continue to the present example of four objects in \( X \). Finally, an object will be selected if the pattern is \( 111' \) over three other objects in a set of four objects. So, the first task would be to identify the object for which it is true. The rationality lies in searching whether that object includes in any pairwise comparisons, i.e., when an object has been selected over three other objects, i.e., \( 11' \) then that object should be in \( 1111', 11111', 111111', \ldots \). This is based on our Theorem 1; where an agent is moving from a small set to the large set and shows consistency in the choice problem. This means the selected object has been preferred when there are two, three, four, etc… Therefore, the rationality string is to identify the pattern as 

\[ L_R/\sum^+ = \{1, 11, 111, 1111, \ldots\} \]

for a repetitive preference order pattern of a period of more than one, i.e., \( t > 1 \). Here, \( \{1\} \) means the object has been preferred over only one remaining alternative; \( \{11\} \) means the object has been preferred over only two other remaining alternatives; \( \{111\} \) means the object has been preferred over only three other remaining alternatives; and so on. If this pattern (string) is true for any object, then it is true that the agent prefers the same object consistently. This means when there are only two objects, then the object has been preferred, i.e., \( '1' \); when there are three objects, then also the object has been preferred, i.e., \( '11' \) and so on for a set of \( n \) objects. Therefore, two important rationality patterns have been derived here. They are as follows.

\[ L_R = \{111, 11, 1\} \] for \( n = 4; t = 1 \)

\[ L_R/\sum^+ = \{1, 11, 111, 1111, \ldots\} \] for \( n \geq 4; t > 1 \)

The membership function or the degree of rationality is given by: 

\[ \text{Degree of rationality} = \frac{1}{2^i + 2} \]

The numerical calculation 2 in our problem in question; where, \( X = [M, N, V, Z] \) say for example the two preference patterns for \( t = 1, 2 \) & \( n = 4 \) are, 

\[ \sum^+_{t=1} = \{0, 111, 00, 11, 000, 1, 0000\} \]

\[ \sum^+_{t=2} = \{0000, 1, 0000, 11, 000, 1010, 0111, 1110, 1111\} \]

The complete (adding) pattern is 

\[ \sum^+_{t=1+2} = \{0, 111, 00, 11, 000, 1, 0000; 0000, 1, 000, 11, 00, 111, 0\} \]

So, identify the runs for each object where the object has been revealed preference. 

\[ \omega_M = \{111\}; \omega_Z = \{111\}; \omega_N = \{11, 1\}; \omega_V = \{1, 11\} \] using matrices, \( U_{t=1} \& U_{t=2} \).
Remarks 7 Therefore, as we have seen, the two patterns are completely different in their choice. In the first one, i.e., at $t = 1$, M has been selected, but at $t = 2$, i.e., Z. So for the two objects, it shows inconsistency. But what about for the other objects viz. V and Z? Which $\omega_i$ matches with any of the sub-patterns of $L_{R/1}^1$? To answer this question, first, we have to understand the sub-patterns. For $k$ number of runs in $L_{R/1}^1$, total number of sub-patterns are $2^k$. But here, we must consider only the order subsets that consist of at least two runs. Moreover, the order and the runs would be in ascending order. This means $\{1, 11\} \neq \{11, 1\}$. $\{1, 11\}$ means the preference did not change in the presence of a new object, and $\{11, 1\}$ means the preference did not change due to the absence of any existing object (Lemma 2). If these two choices are the same, then the answer for the question “Which $\omega_i$ matches with any of the sub-patterns of $L_{R/1}^1$? is $\omega_V$ because $\omega_V \subseteq L_{R/1}^1$. $\subseteq$ is defined as the subset of the sub-pattern as discussed above. Numerically, the degrees for each object have been measured in the next sections.

Lemma 2 For a fixed number of objects across time $t$ i.e. $n$ strong consistency implies $\{1, 11\}$ and weak consistency implies $\{11, 1\}$.

The proof is in the appendix.

Remarks 8 Lemma 2 is based on Theorem 1. It is obvious from Theorem 1. let $X_1 = \{1, 11\}$ and $X_2 = \{11, 1\}$. This means the agent selects the object $x_i \in X$ when there were only two objects, i.e., $n = 2$ and appear sequentially in ascending order in pattern $X_1$ and also selects the same object $x_i \in X$ when $n = 2$ and appear sequentially in descending order in pattern $X_2$. In other words, the agent could measure the information index correctly for the object $x_i \in X$ as it would happen in pattern $X_2$.

3.3.1 Rationality Pattern Function

Definition 6 For a set of feasible objects $X$, the rationality outcome set is the total number of pattern sets available for $t'$ periods. Therefore, the rationality outcome set $\tau$ would be below.

Rationality Outcome Set, when $n$ is fixed.

$$\tau = [L_{R/1}^1 \times L_{R/1}^2 \times \ldots \times L_{R/1}^t, \{n - 1 - n + 1 + 1\}, \ldots \{n - 1 - k\}, \ldots \{n - 2\}, \{n - 1\}]$$

Where $n$ is the number of objects in $X$. Here $n$ is the number of 1s, i.e., if $n = 2$, it will be $\{11\}$. When $n$ is not fixed and is changing concerning time $t$, then $\sum^+ \tau$ will be replaced by $\sum^+ \tau$. This means some objects may not be in the choice set $X$; hence, that object’s set point is null. This adjusts the exclusion and inclusion of the old and new objects at different times, respectively.

$$\tau = [L_{R/1}^1 \times L_{R/1}^2 \times \ldots \times L_{R/1}^t] \quad (6)$$

This equation is the general condition of the rationality outcome set.

Numerical Calculation 3 When $t = 2$; then for a set $X$ of four objects space i.e. $n = 4$ for all $t$; $\tau$ would be

$$\tau = [L_{R/1}^1 \times L_{R/1}^2, \{n - 1 - n + 1 + 1\}, \ldots \{n - 1 - k\}, \ldots \{n - 2\}, \{n - 1\}]$$

$$= \{[1, 11, 111] \times \{1, 11, 111\}, \{1\}, \{11\}, \{111\}\}$$

$$\tau = [[1, 1], \{1, 11\}, \{1, 111\}, \{11, 1\}, \{11, 11\}, \{11, 111\}]$$
Definition 7 The Rationality Pattern Function is represented by

\[ \omega_i : X \equiv \tau \]  

Where, \( X = \{1, 2, ...i, ..., n\}; i = \text{object}; \omega_i \text{ is the rationality pattern for the } i^{th} \text{ object}; \tau = \text{rationality outcome set} \).

Let us take an example for \( t = 2 \) and fixed \( n = 4 \) so, \( r = s = 0 \) then calculate of \( \omega_i \). To give a numerical example, a fixed \( n \) is assumed.

Numerical Calculation 4 When \( t = 2, n = 4 \); \( X = \{M, N, V, Z\} \), then for an order \( X = \{M, N, V, Z\} \),

\[
\sum_{t=1}^{+} = \sum_{t=1}^{+} \times t
\]

\[
= [0, 1, 1, 1; 0, 0, 0, 1; 1, 0, 0, 0; 0, 0, 0; 1, 0, 0; 1, 1, 0; 1, 1, 1, 0].
\]

There is a one-to-one correspondence; from

\[ [MNVZMNVZMNVZMNVZMNVZMNVZMNVZ] \]

to \( [011100110000000001000110011001] \). This means for the first four elements \( [M = 0, N = 1, V = 1, Z = 1] \) and for the second four elements \( [M = 0, N = 0, V = 1, Z = 1] \), the subsequent sequences of four elements also follow the same order. Now for example derive one functional value \( \omega_i \) for the set \( X \) and explain why.

Remarks 9 What is functional value of \( \omega_M \)? The answer is : \( \omega_M = \{111\} \). This has been calculated as there are four objects, so we must consider four patterns at a time, i.e., \( 0, 1, 1, 1; 0, 0, 0, 1; 1, 0, 0, 0; 0, 0, 0; 1, 0, 0; 1, 1, 0; 1, 1, 1, 0 \). The first four patterns state that \( M \) is preferred over the other three objects. The second pattern states that \( N \) is preferred over \( V \& Z \), i.e. \( 0, 0, 1, 1 \). This would continue till all the objects have been considered once, i.e., for \( t = 1 \). The process will continue and again start from the first object \( M \), i.e. when \( t = 2 \). The pattern \( 0, 0, 0, 0 \) means \( M \) is not preferred over the other three objects.

The rules are given below.

(i) Count the number of runs, i.e. 1s if the object is at 0. Stop counting runs if the next binary value is 0.

(ii) If there are four objects, then for a set of four zeros, the number of runs is 0.

(iii) Counting runs are not allowed if the object is at 1.

From the pattern \( [011100110000000001000110011001] \), the first value is 0. Therefore, we can start counting several runs next to that 0. After the third 1, the next value is 0, so we have to stop here. Hence, the run is \([111] \) for that first 0, i.e., \( M \). Then start counting again from that 0 to count runs. But after that 0, the next value is 0, so we cannot count runs. After that, the following two values are 1, 1, so the run is 11 for the sixth zero, i.e. \( N \). After that, there are three zeros, so for the ninth and tenth zeros, no run is there, but for the eleventh zero, the run is 1, which is for the object \( V \). Now, there are two sets of four zeros. For the first four zeros, the run is 0; for the second four zeros, the run is 0. This will continue, and we would get \([111, 11, 1, 0, 0, 1, 11, 11]\) for \( M, N, V, Z, M, N, V, Z \) respectively. This means \( M = [111]; N = 11; V = 1, Z = 0; M = 0, N = 1, V = 11, Z = 111 \). Now we can
identify the value of $\omega_M = [111]; \omega_N = [11, 1]; \omega_V = [1, 11]; Z = [111]$. The calculated values of $\omega_i$ can be matched with the set $\tau$ to identify the rationality pattern.

### 3.3.2 Exclusion and Inclusion of Objects in the Choice Set X in Different Times

When a new object has been included in $X$, it should be added at the end of the set $X$, and exclusion would not change the order but add $\epsilon$ to that place. So that inclusion and exclusion would not change the previous order. Inclusion and exclusion of an object will alter the set $\tau$, the ranking of the rationality pattern, and the degree of rationality. For example, when $L^{1}_{R/\Sigma^*} = \{1, 11, 111\}$ and $L^{2}_{R/\Sigma^*} = \{1, 11, 111, 1111\}$. Therefore, $\tau = \{1, 11, 111, \epsilon\} \times \{1, 11, 111, 1111\}$. And, when, $L^{1}_{R/\Sigma^*} = \{1, 11, 111\} \& L^{2}_{R/\Sigma^*} = \{1, 11\}$; then $\tau = \{1, 11, 111\} \times \{1, 11, \epsilon\}$.

### 3.3.3 Degree of the Rationality Pattern

The degree of rationality is different for different objects. If the reviewer gives any review for any object, then the degree of rationality would depend on the preference pattern for that object only. If the agent gives a good review for the object $V$, then it should be supported by the preference pattern of $V$ only. Hence we see that $\omega_V \subseteq L^{1}_{R/\Sigma^*}$. This should have a membership grade $\mu_{\omega_i}$ of $\omega_V$ in $L^{1}_{R/\Sigma^*}$. This would be called the degree of rationality. On the other hand, if the agent gives an excellent review for $M$, then this does not support that $\omega_M \subseteq L^{1}_{R/\Sigma^*}$ or $\neg[\omega_M \subseteq L^{1}_{R/\Sigma^*}]$. But if it gives a negative review, that would match that $\neg[\omega_M \subseteq L^{1}_{R/\Sigma^*}]$. Then, the review information would be correct to make a decision.

**Definition 8** The degree of rationality pattern is given formally by the following function:

$$\mu_{\omega_i} : \omega_i \rightarrow (0, 1) \quad (8)$$

Where, $\mu_{\omega_i}$ is the degree to which the pattern of $\omega_i$ for each object $i$ belongs to the set $L^{1}_{R/\Sigma^*}$. The reason behind the range of the degree, i.e., $\mu_{\omega_i} \in (0, 1]$, is due to Lemma 1. This states that a completely irrational agent is not possible.

### 3.3.4 Calculation of $\mu_{\omega_i}$

The calculation of the degree to which the pattern of $\omega_i$ for each object $i$ belongs to the set $L^{1}_{R/\Sigma^*}$ or in a board sense in $\tau$ table for each pattern is to be calculated. First, the degree of membership for each pattern in $\tau$ is to be calculated. After identifying the pattern for each object degree, the actual data will be derived from the $\tau$ degree table. Say from the above numerical calculations 3, the $\tau$ can be written as

$$\tau = \{[1, 1], [1, 2], [1, 3], [2, 1], [2, 2], [2, 3], [3, 1], [3, 2], [3, 3], \{1\}, \{2\}, \{3\} \}.$$  

Or,

$$\tau = \{[1, 1], [1, 2], [1, 3], [2, 1], [2, 2], [2, 3], [3, 1], [3, 2], [3, 3], [\epsilon, 1], [\epsilon, 2], [\epsilon, 3] \{1, \epsilon\}, [2, \epsilon], [3, \epsilon]\}.$$  

That is for $\{1\} = \{1\}, \{11\} = \{2\}, \{111\} = \{3\}$ etc... These are the outdegrees for the $i^{th}$ vertex. The meaning
of the pair \{i, j\} in \(\tau\) is for two-period patterns, the agent prefers the object over \(i^\text{th}\) number of objects in period \(t=1\), and the agent prefers the object over \(j^\text{th}\) number of objects in period \(t=2\). The individual values viz. \{i\}, \{j\} represent either \{\epsilon, j\} and \{i, \epsilon\}. \{i, \epsilon\} means the object was selected over \(i^\text{th}\) number of objects in time \(t=1\) but now at time \(t=2\) it has eliminated from the consideration or it becomes the worst object. \{\epsilon, j\} means the object was not selected over some objects in time \(t=1\). Still, at time \(t=2\), it has been selected over \(j\) number of objects for consideration at time \(t=2\), or it becomes a superior element over some objects that were not at time \(t=1\).

From numerical calculation 4 the actual pattern is

\[
\sum_{i+2}^{+} = \sum_{\sum_{i}^{+} = 1}^{t} = [0, 1, 1; 0, 1, 1; 0, 0, 0; 0, 0, 0; 0, 1, 0, 0; 1, 1, 0; 1, 1, 1, 1, 0].
\]

It can be written as

\[
\sum_{i+2}^{+} = \sum_{\sum_{i}^{+} = 2}^{t} = [0, 3; 0, 0, 2; 0, 0, 0, 1; 0, 0, 0, 0; 0, 0, 0, 0; 1, 0, 0, 0; 2, 0, 0, 3; 0, 0, 0].
\]

Or, \(\sum_{i+2}^{+} = \sum_{\sum_{i}^{+} = 2}^{t} = \begin{bmatrix} M & N & V & M & N & V & Z \\ 3 & 2 & 1 & 0 & 0 & 1 & 2 & 3 \end{bmatrix}\)

Therefore, \(\omega_{M} = \{3, 0\} = \{3, \epsilon\} = \{111\}\) (M was superior at time \(t=1\) over three objects; but now at time \(t=2\), it becomes inferior) \(\omega_{N} = \{2, 1\} = \{11, 1\}\) (N was superior at time \(t=1\) over two objects; but now at time \(t=2\), it becomes superior over only one object) \(\omega_{V} = \{1, 2\} = \{1, 11\}\) (V was superior at time \(t=1\) over one object; but now at time \(t=2\), it becomes superior over two objects) \(\omega_{Z} = \{0, 3\} = \{\epsilon, 3\} = \{111\}\) (Z was superior at time \(t=1\) over zero object; but now at time \(t=2\), it becomes superior over three objects). Therefore, \(\sum_{i+2}^{+} \subseteq \tau\) for \(t = 2\) and \(n = 4\). Therefore, a table of membership grades is required for the set \(\tau\) for each member and different \(t\) and \(n\). From the membership table of \(\tau\), the degree of membership of each \(\omega_{i}\), can be easily calculated for different \(t\&n\). Interpreting the reviewer’s comments using hypothetical review comments with these patterns attached to them will be clear.

### 3.4 Calculation of the Rationality Outcomes Table for the Set \(\tau\) and the Degree

Given the general condition set \(\tau\) for \(t = m\) finite time periods and finite number of objects \(n_{t}\) in \(X\),

\[
\tau = [L_{R/\sum}^{1} \times L_{R/\sum}^{2} \times \ldots \times L_{R/\sum}^{t}]
\]

it is clear that the degree of rationality is \(\rho_{\omega_{i}} \in (0, 1]\). Complete rationality is possible and would be ranked one, but complete irrationality is impossible (from Lemma 1); therefore, the lower bound is not converging to zero.

This has been explained below, showing only the pattern’s degree or rank with an upper bound, i.e., 1.

#### 3.4.1 Rationality Ranking in \(\tau\)

To derive a table of the degree of membership for the set \(\tau\), it is required to rank them first. When \(t = 2\) then each element in \(\tau\) represents an ordered pair i.e., \{i, j\} \(\in \tau\). Where, \(i\) is for \(t = 1\) and \(j\) is for \(t = 2\). Likewise, when \(t = 3\) then \{i, j, k\} \(\in \tau\); where, \(i\) is for \(t = 1\), \(j\) is for \(t = 2\) and \(k\) is for \(t = 3\). This can be extended for \(t = m\) a finite number of times. The rationality relation in \(\tau\) is denoted by let \(T_{\omega}\). Such as;

\[
i \preceq j \text{ (read as "i precedes j")}
\]
\[ i \prec j \text{(read as "i strictly precedes j")} \]

\[ i \succeq j \text{(read as "i dominates j")} \]

\[ i \succ j \text{(read as "i strictly dominates j")} \]

The reflexive relation carries the highest rank in terms of rationality for finite \( n \) objects across time \( t \). For example, for \( t = 2 \) & \( n = 4 \), the pattern, \( T_{\prec} = \{[1, 1];[11, 11];[111, 111]\} \) carry highest rank of rationality. The degree of this reflexive relation is one.

**Definition 9** The highest rank of rationality is when the number of objects is the same across time, i.e., \( \bar{n} \).

This means \( T_{\prec} \) is said to be a reflexive relation on \( L_{R/\Sigma^*}^t \), that is from the set \( L_{R/\Sigma^*}^t \) to itself, if for every \( i, j \in L_{R/\Sigma^*}^t \); \{i, j\} \in T_{\prec} \) where \( i = j \).

**Remarks 10** This means the agent is dynamically maintaining the same pattern, where the pattern does not show increasing or decreasing patterns. Therefore, it shows the highest level of consistency over the same set of objects.

If the number of objects \( n_t \) is increasing concerning time \( t \), then the highest level of consistency would be an increasing order. Therefore, the rank definition is below.

**Definition 10** The second highest rank of rationality is when the number of objects increases across time, i.e., \( n_t \uparrow \).

**Remarks 11** This means \( T_{\prec} \) is said to be an asymmetric relation on \( L_{R/\Sigma^*}^t \), that is from the set \( L_{R/\Sigma^*}^t \) to itself then \( T_{\prec} \) is said to be the second highest ranked pattern if for every \( i, j \in L_{R/\Sigma^*}^t \); \{i, j\} \in T_{\prec} \) where \( i \prec j \) but \( \neg[i \succ j] \).

On the other hand, when \( n_t \) is decreasing concerning time \( t \), then the highest consistency over time would be below.

**Definition 11** The third highest rank of rationality is when the number of objects decreases across time, i.e., \( n_t \downarrow \).

**Remarks 12** This means \( T_{\prec} \) is an asymmetric relation on \( L_{R/\Sigma^*}^t \), that is from the set \( L_{R/\Sigma^*}^t \) to itself then \( T_{\prec} \) is said to be the third highest ranked pattern if for every \( i, j \in L_{R/\Sigma^*}^t \); \{i, j\} \in T_{\prec} \) where \( i \succ j \) but \( \neg[i \prec j] \).

**Definition 12** The fourth highest rank of rationality is when the number of objects changes (i.e., increasing and decreasing with or without any order) across time, i.e., \( n_t \uparrow \downarrow \).

**Remarks 13** This means \( T_{\prec} \) is symmetric relation on \( L_{R/\Sigma^*}^t \), that is from the set \( L_{R/\Sigma^*}^t \) to itself then \( T_{\prec} \) is said to be highest ranked pattern if for every \( i, j \in L_{R/\Sigma^*}^t \); \{i, j\} \in T_{\prec} \) where \( i \prec j \& i \succ j \) both.

Different combinations carry different rationality patterns for the combinations of \( t \& n_t \). A zero rationality pattern is not possible. A complete table is required to find the degree for the different parameters viz. \( t; n_t \). This is related to the concept of value judgment. The identification of a value judgment ranking of the different combinations would be required to derive the table. The calculation of the complete membership table will be considered in our future work.

22
3.4.2 Calculation of the Table $\tau$

The rationality ranking table has been calculated to interpret the given problem where $t = 1, 2; n = 4$, i.e., $X = \{M, N, V, Z\}$. A separate table has to be constructed for different combinations of $t, n$ & $X$. For a set of $n$ objects, and $t = 1, 2$ periods the total number of increasing mapping of $X$ onto $L_{R/P}^t$ would be $(n-1)! + \frac{[n]^t}{t!} = 16$. Where $[n]^t = n(n+1)...(n+t-1), the set L_{R/P}^t$ be the set of numbers $\{\epsilon\}, \{1\}, \{11\}, \{111\}, .., \{n\}$, ordered so that $\{\epsilon\} < \{1\} < \{11\} < \{111\} < ... < \{n\}$. A pattern of length $t$ is increasing if $\{\epsilon\} < \{1\} \leq \{11\} \leq \{111\} \leq ... \leq \{n\}$. Therefore, the increasing length pattern $t = 2$ is given in the following matrix. This idea of pattern has been taken from [Berge, 1971].

\[
T = \begin{bmatrix}
A & 32 & 21 & 1\epsilon \\
B & 31 & 2\epsilon \\
C & 3\epsilon \\
D & \epsilon\epsilon & 11 & 22 & 33 \\
E & \epsilon1 & 12 & 23 \\
F & \epsilon2 & 13 \\
G & \epsilon3 \\
\end{bmatrix}
\]

According to $(n-1)! + \frac{[n]^t}{t!} = 16; (n-1)! = 6$ are for the irrational decreasing order and $\frac{[n]^t}{t!} = 4 \times 5 = 10$ are for the rational increasing order.

**Remarks 14** The matrix has to read in order like below.

The irrationality in decreasing order.

\[
\{32\} \leq \{31\} \leq \{3\epsilon\} \leq \{21\} \leq \{2\epsilon\} \leq \{1, \epsilon\}
\]

The rationality is in increasing order.

\[
\{\epsilon\epsilon\} \leq \{\epsilon1\} \leq \{\epsilon2\} \leq \{\epsilon3\} \leq \{11\} \leq \{12\} \leq \{13\} \leq \{22\} \leq \{23\} \leq \{33\}
\]

3.4.3 Degree of Membership and Overall Rationality Measure

From the table $T$, the membership of each pattern can be categorized as follows. Each row carries the same membership grades, but each column carries different membership grades. For example, $\mu(32) = \mu(21) = \mu(1\epsilon)$ and $\mu(32) \neq \mu(31) \neq (3\epsilon)$. From the matrix $T$ and the rationality rankings, it is clear that the rationality pattern follows Binomial Distribution, as in the figure below.
Remarks 15 The above binomial tree has been calculated using the concept of continuity. For example, the difference between $3 + \epsilon$ is two therefore, $(3, \epsilon) & (\epsilon, 3)$ carry the same difference. Therefore, $C & G$ carries the same difference. The other bars are also carrying the same viz. $B & A; D & E$. The bar $D$ carries the highest degree because the difference is zero. It means the behavior change is nil. $D$ has divided the degree into two parts. The parts consisting of $C, B, A$ are called the irrational zone, and the part consisting of $E, F, G$ is called a rational zone. The probability of rationality is $p$, and irrationality is $q = (1 - p)$. The total number of objects is $n$. Then the probability that out of $n$ objects, any $r$ object/s would be in the rationality zone in any graph of $t$ periods would be,

$$f(r) = \binom{n}{r} p^r q^{n-r}$$  \hspace{1cm} (9)

From our example, in the graph of $t = 1, 2; n = 4$ the actual choice is given by,

$$\sum_{t=2}^{t} \sum_{r} = \begin{bmatrix} M & N & V & Z & M & N & V & Z \\ 3 & 2 & 1 & 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

The patterns are $M = (3, \epsilon); N = (2, 1); V = (1, 2); Z = (\epsilon, 3)$. From the pattern it is clear that $M = (3, \epsilon); N = (2, 1)$ fall in the irrational zone and $V = (1, 2); Z = (\epsilon, 3)$ fall in the rational zone. Therefore, there are two options, viz. rational and irrational zones, hence $p = q = \frac{1}{2}$. Therefore, the probability function is,

$$f(r) = \binom{4}{r} \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{4-r}$$

Remarks 16 $f(r = 1)$ is the probability that any one object falls in the rationality zone. Likewise, for any $r$, $f(r)$ can be calculated. This will serve an agent’s rationality and irrationality in any choice problem. Moreover, the degree of each pattern can be calculated using the above binomial tree using any membership function.

3.4.4 Membership Function

The binomial tree can be arranged as a frequency distribution in Table 11. The degree of membership of each $A, B, C, D, E, F, G$ has been calculated using the following membership function.

$$\mu_x = \frac{x - \min}{\max - \min}$$  \hspace{1cm} (10)
Table 1: Frequency Distribution.

| Pattern Set(x) | Frequency(f) |
|----------------|--------------|
| A              | 3            |
| B              | 2            |
| C              | 1            |
| D              | 4            |
| E              | 3            |
| F              | 2            |
| G              | 1            |

Therefore, \( \mu_C = \mu_G = \frac{1 - 1}{4 - 1} = 0; \mu_B = \mu_F = \frac{2 - 1}{4 - 1} = 0.33; \mu_A = \mu_E = \frac{3 - 1}{4 - 1} = 0.67; \) & \( \mu_D = \frac{4 - 1}{4 - 1} = 1 \)

This could have been done using any suitable membership function.

3.5 Algorithm

The formal algorithm is as follows.

(i) Derive the rationality outcome set \( \tau \) based on the set \( X_t, t \& n_t \) of highest number of observations and for each object \( i \).

(ii) Derive the actual and the complete preference set for all the transactions, i.e. \( \sum^* \).

(iii) Derive \( \omega_i \) for each object \( i \).

(iv) Match each \( \omega_i \) with the pattern in \( \tau \).

(v) Find the membership \( \mu_{\omega_i} \) of \( \omega_i \) from the table of \( \tau \).

3.6 Analysis of the Rationality Rankings

This section presents complete information about the reviewers’ review comments and ratings, including trustworthiness in Table: Review Comments, Ratings, and the degree of Trustworthiness of the Given Decision Maker.

Moreover, the actual interpretations have been explained. If the agent in the above example gives reviews for M, N, V, and Z, respectively, as in section 1.1, then the above rationality pattern will work as below:
| Commodity/Object Type | Review Comments                | Review Ratings | Degree of Trustworthiness |
|-----------------------|--------------------------------|----------------|--------------------------|
| M                     | Bad product                    | ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ | 0.33                     |
| N                     | Not so good product            | ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ | 0.67                     |
| V                     | Relatively good product        | ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ | 0.67                     |
| Z                     | Premium product                | ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ | 0.33                     |
| Or, Z                 | Not a Premium product          | ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ | 0.33                     |

Remarks 17

(i) For M: Review comments- Bad product- ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ $\omega_M = \{3, \epsilon\}/\mu_{\omega_M} = 0.33$

Interpretation of the review report and the about the reviewer: The pattern $\omega_M = \{3, \epsilon\}$ says that the reviewer completely rejects the object the second time. Though the reviewer is trustworthy, as the review report matches the pattern, the degree $\mu_{\omega_M} = 0.33$ states that his consistency is poor. Because the 3 means the object has been preferred over three other alternatives the first time but dropped suddenly without maintaining continuity. This confirms that the first choice was not a completely rational decision. The decision-maker drops the object completely once s/he realizes the same.

(ii) For N: Review comments- Not so good product- ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ $\omega_N = \{2, 1\}/\mu_{\omega_N} = 0.67$

Interpretation of the review report and the about the reviewer: The pattern $\omega_N = \{2, 1\}$ says that the reviewer did not reject the object the second time completely. The reviewer is trustworthy as the review report matches the pattern with degree $\mu_{\omega_N} = 0.67$. This time, the reviewer maintains continuity. Hence, the reviewer, though not completely rational, has better consistancy.

(iii) For V: Review comments- Relatively good product- ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ $\omega_V = \{1, 2\}/\mu_{\omega_V} = 0.67$

Interpretation of the review report and the about the reviewer: The pattern $\omega_V = \{1, 2\}$ says that the reviewer still considers the object the second time, and confidence has been increased. The reviewer is trustworthy as the review report matches the pattern with degree $\mu_{\omega_V} = 0.67$. Like case (ii), this time, the reviewer maintains continuity. Hence, the reviewer, though not completely rational, has better consistancy.

(iv) For Z: Review comments- Premium product- ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ $\omega_Z = \{\epsilon, 3\}/\mu_{\omega_Z} = 0.33$

Interpretation of the review report and the about the reviewer: The pattern $\omega_Z = \{\epsilon, 3\}$ says that the reviewer did not consider the object the first time and in the second time this was the final demand. The reviewer is trustworthy as the review report matches the pattern with degree $\mu_{\omega_Z}$. However, the reviewer did not maintain the continuity because the reviewer jumped from zero preference to a preference over three objects. Hence, the review is correct, but the degree of rationality will be lower, i.e.0.33., Or,

(v) For Z: Review comments- Not a Premium product- ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ $\omega_Z = \{\epsilon, 3\}/\mu_{\omega_Z} = 0.33$

Interpretation of the review report and the about the reviewer: The pattern $\omega_Z = \{\epsilon, 3\}$ says that the reviewer did not consider the object the first time and in the second time this was the final demand. The reviewer is trustworthy as the review report matches the pattern with degree $\mu_{\omega_Z} = 0.33$. 

26
The last two review comments are interesting to interpret. With the pattern and the degree, the trustworthiness of the reviewer would be easier to judge. In all four situations, the reviewer for the comments (iii) carries the highest rationality rankings due to Theorem 1. This will be further reflected by the respective degree of rationality. The differences between the set of comments without a pattern and with a pattern. Attaching these allows the potential buyer to become perfectly informed about the reviewers and their trustworthiness.

**Calculations of the Overall Rationality of the Agent in Question**

The probability that the agent selects one object with any of the rationality patterns, i.e., $f(r = 1) = \left(\frac{4}{2}\right)\left(\frac{1}{2}\right)^{4-1} = 0.25$ and $f(r \leq 1) = 0.31250000; f(r > 1) = 0.68750000; f(r < 1) = 0.06250000; f(r \geq 1) = 0.93750000$, etc...

**4 Conclusions and Limitations**

The paper gives an analytical tool to measure rationality using a rationality pattern function and to identify the trustworthiness/consistency level of the third party involved in the online/electronic commerce markets. The paper has a few limitations. These are- (i) The agent is allowed to select only one object from the set $A$ or the set $S$ consists of only one thing, (ii) the agent is buying only for themselves from their account, and (iii) If the set $X$ consists of more than four objects, then any two objects would not be compared because a four-step choice problem has been considered. Including more objects while creating lists by the consumer could provide a future scope of study through an extension of the model, where $n > 4&t > 2$. If there are more than four objects, then there must be an option for the decision-maker to create extra choice sets/sub-choice sets accordingly.

**Conflict of interest statement**:

On behalf of all authors, the corresponding author states that there is no conflict of interest.

**Appendices**

**Rationality and the Condition of Consistency**

Rationality is the art of making logical decisions. Identification of a decision-making process or the mapping pattern from the domain to the action space depicts rationality. In each stage, the choice of the actions or the strategy profile should follow perfect Bayesian equilibrium. Herbert Simon introduces the concept called bounded rationality[Simon, 1997]. It implies that it is difficult to have all information for a person. According to Tsang [Tsang, 2008] human beings are not fully rational. So, there is a degree of rationality to a situation that is only true if rationality is divisible. Blin et al. [Blin et al., 1973] derive a fuzzy measure of the pattern of rationality. Other related works on measurement of rationality are [Apesteguia and Ballester, 2015];[Dean and Martin, 2016];[Schick, 1963].

The preference relations, as revealed by choice, must be acyclic [Samuelson, 1938];[Houthakker, 1950];[Richter, 1966]. This acyclicity has been tested non-parametric way in [Afriat, 1967];[Afriat, 1973];[Varian, 1982]. The other
two measures are in [Houtman and Maks, 1985];[Echenique et al., 2011]. But it provides no information as to whether choices that contain revealed preference cycles are close to being rational or not. Measuring rationality in the sequential choice problem is common today in the e-commerce market. All of these measures are based on the expenditure side. These measures do not consider the pattern of choice irrespective of budget. This means rationality follows some pattern of choice. Therefore, the rationality condition has to be re-defined and re-measured. The present study is based on the condition of rationality as follows in Theorem 1.

**Theorem 1** There exists a two-way consistency of the choice problem. The choice would be the same if the decision-maker is moving from a large set to a small set and from a small set to a large set. This happens provided the decision-maker can interpret the information correctly for each set.

**Proof:** Theorem 1 indirectly says that the decision-maker is deciding on a reference. It can be achieved in both ways; from a large set or a small set if the reference point is there in both of the sets.

This condition of rationality is supported by the condition given by Arrow [Arrow, 1959]. If some elements are chosen out of the set say $X_2$ and then the range of alternatives is narrowed to $X_1$ but still contain some previously chosen elements, no previously unchosen element becomes chosen and no previously chosen element becomes unchosen. This means For any $X_1 \& X_2 \in X$, the following condition holds for a rational agent.

$$[X_1 \subseteq X_2] \implies [X_1 \cap C(X_2) = \emptyset] \lor [X_1 \cap C(X_2) = C(X_1)].$$

Let the set of alternatives are in $X = \{x_1, ..., x_n\}$. Therefore, the choice problem is

$$C : X_1 \rightarrow X$$

Where, $X_i \subseteq 2^n.2^n$ is the set of all subsets. The preference relation on $X$ is reflexive, transitive, and asymmetric and the relation is $R = \text{"better than or equal to"}. \Rightarrow$. The choice problem is any subset $X_i$ and the choice function is $C_\succ_i (X_i) \rightarrow X$. Let there are two choice problems; $X_1 \& X_2 \subseteq X$. According to the classical theory of rationality $C$ satisfies the consistency condition iff ; $\forall X_1 \subseteq X_2 \subseteq X \text{if } C(X_2) \in X_1 \text{then } C(X_1) = C(X_2)$. This definition of rationality assumes that the decision maker is moving from a large set $X_2$ to a smaller set $X_1$. But what if the decision maker is moving from a smaller set to a larger set i.e. $X_1$ to $X_2$? The conditions would be as below:

$$\forall X_1 \subseteq X_2 \text{Xif } C_\succ_i(X_2) \in X_1 \text{then } C_\succ_i(X_1) = C_\succ_i(X_2) \quad (2)$$

$$\forall X_1 \subseteq X_2 \text{Xif } C_\succ_i(X_1) \in X_2 \text{then } C_\succ_i(X_2) = C_\succ_i(X_1) \quad (3)$$

This means there must be a two-way consistency. Let an example clarify the fact. □

**Example 1**

Let $X = \{x_1, x_2, x_3\}, X_1 = \{x_1\}$ and $X_2 = \{x_1, x_3\}$. According to the first criteria i.e. (7); the agent selects first from the set $X_2$ i.e. $C_\succ(x_1, x_3) = x_1$ and thereafter from $X_1$ i.e. $C_\succ(x_1) = x_1$. Therefore, $C_\succ(X_2) = C_\succ(X_1) = x_1$. According to the second criterion i.e. (8) if the agent starts selecting from the set $X_1$ and thereafter from $X_2$ then the condition of equality may not hold and it would depend on the pairwise comparison between $x_1, x_3$, that is not known from the first choice. This means either; $C_\succ(x_1) = x_1$, and $C_\succ(x_1, x_3) = a$ or $C_\succ(x_1) = x_1$, and $C_\succ(x_1, x_3) = x_3$. Therefore, more information should be included in the above classical
condition of rationality. When the agent moves from a larger set to a smaller set then the process carries some information. But in the reverse case loss of information is there.

Example 2

An agent is taking a decision in $S_1 \subseteq S_2 \subseteq S_3 \subseteq S_4 \subseteq ... S_n \subseteq R^n$. The agent is deciding on a smaller set to a larger set. This means the choice from each list under each set should be equal i.e. $C(L_1 \in S_1) = ... = C(L_n \in S_n) = Y^n = [v_1, ..., v_n]$. The set $Y^n = [v_1, ..., v_n]$ is a linearly dependent vectors, i.e. $C(L_1 \in S_1) = v_1, C(L_2 \in S_2) = v_2 = \lambda_1 v_1, ..., C(L_n \in S_n) = v_n = \lambda_n v_1$ for $\lambda_1...\lambda_n > 0$. The set of reference points are $Y^n = [v_1, v_1, ..., v_1]$ i.e. $Y^n = [v_1, ..., v_n]$ Therefore, the choice would be such that, for a one-dimensional case

\[
\text{[for any two subsets } X_i \subseteq X_j \subseteq X]\], \[C\{N/(\delta, x^*) \cap X_i\} = C\{N/(\delta, x^*) \cap X_j\} = x^* \in N(\delta, x^*) \quad (4)
\]

and for n-tuple

\[
\text{[for any two subsets } X_i \subseteq X_j \subseteq X]\], \[C\{B_r/(x^*) \cap X_i\} = C\{B_r/(x^*) \cap X_j\} = x^* \in B_r(x^*) \quad (5)
\]

This means,

\[
\forall X_1 \subseteq X_2 \subseteq X \Rightarrow \text{if } C_{\phi}(X_2) \in X_1 \text{ then } C_{\phi}(X_1) = a \text{ (say for example)} \quad (6)
\]

And

\[
\forall X_1 \subseteq X_2 \subseteq X \Rightarrow \text{if } C_{\phi}(X_1) \in X_2 \text{ then } C_{\phi}(X_2) = \lambda a \text{ (\because } \lambda > 0) \quad (7)
\]

So that the set, $[a, \lambda a]$ are dependent to each other and $C_{\phi}(X_1) = C_{\phi}(X_2)$.

When an agent is moving from a small set to a large set i.e. $X_1 \subseteq X_2 \subseteq X$ then s/he prepares an index mentally for the individual objects. S/he selects the object that carries the highest information. If the information is correctly assessed then the index becomes correct and the loss of information can be minimized. One of the valuable imperfect and trust indicator information is the reviewer comments and the ratings. If these could be assessed correctly Theorem 1 will be fulfilled. This mental indexing process has been discussed first and thereafter measure of the reviewers’ rationality has been provided at the end.

Information Indexing

When an agent starts selecting the alternative/element/object from the list, then the agent tries to match the predetermined attributes/messages with the actual messages that each object carries. The degree of fulfillment, degree of non-fulfillment, and degree of indeterminacy of those messages play an important role in selecting the element from the given list. Say the set of messages to buy a cell phone are; (i) product information, (ii) customers’ reviews, (iii) after-sale services, etc. If the agent gets perfect knowledge/messages/information about each of the attributes, then the product with the highest information will be used as a satisfactory threshold and the searching process will stop either to it or near it. The task of selecting alternatives will be easier if the alternatives carry this information. The agent tries to minimize the degree of indeterminacy and select the alternative that carries the highest information. The search process will continue until that alternative, carries the highest information.
The idea of this type of choice function from lists is derived here with the help of Intuitionistic fuzzy sets given by Atanassov. [Atanassov, 1986]. For each element in a list, the customer calculates the degree of information. This information is attached to each of the elements and the sequence continues until the information reaches its optimum point.

Let a set \( E : x \in E \) of attributes be fixed. For example, \( E = \{ x_1, x_2, x_3 \} \) where, say \( x_1 \) is product information, \( x_2 \) is customer review, \( x_3 \) is after sale services etc.

**Definition 1** Intuitionistic fuzzy set (IFA): \( X^* \) in \( E \) is an object having the form
\[
X^* = \{ x_i, \mu_X(x_i), \nu_X(x_i) | x_i \in E \}
\]
where the functions \( \mu_A(x_i) : E \rightarrow I = [0, 1] \) and \( \nu_X(x_i) : E \rightarrow I = [0, 1] \) define the degree of membership and non-membership, respectively of the element \( x \in E \) to the set \( A \) & for every \( x \in E, 0 \leq \mu_X(x_i) + \nu_X(x_i) \leq 1 \). The rest part \( \pi_X(x_i) = 1 - \mu_X(x_i) - \nu_X(x_i) \) is called the in-deterministic part of \( x \) and \( 0 \leq \pi_X(x_i) \leq 1 \).

**Definition 2** Choice function from lists Let \( X \) be a finite set of \( N \) elements; \( X = \{ x_1, ..., x_k, ..., x_N \} \). Let \( \Gamma \) be the set of all lists. A choice function from lists \( D : \Gamma \rightarrow X \) is a function that assigns to every list \( L = \{ x_1, ..., x_k \} \) a single element \( D(L) \) from the set \( \{ x_1, ..., x_k \} \).

**Definition 3** Intuitionistic fuzzy choice function from lists: Let \( X \) be a finite set of \( N \) elements. Let \( \Gamma_{IFS} \) be the set of all lists with IFS grades of messages. Each element in \( X \) carries the information of the IFS. A choice function from lists \( D : \Gamma_{IFS} \rightarrow X \) is a function that assigns to every list \( L_{IFS} = \{(x_1, \mu_1, \nu_1), ..., (x_j, \mu_j, \nu_j), ..., (x_k, \mu_k, \nu_k)\} \) a single element \( D(L_{IFS}) \) from the set \((x_1, \mu_1, \nu_1), ..., (x_j, \mu_j, \nu_j), ..., (x_k, \mu_k, \nu_k)\). Each element \( x_i \) carries two messages viz. the degree of membership \( \mu_i \) and non-membership \( \nu_i \).

**Measurement of the Information from Messages**

If the number of messages in the set is finite then this number or any monotonic function of this number can be regarded as a measure of the information produced when one message is chosen from the set, all choices being equally likely. The measure of information has been done using the expression given by Shannon, [Shannon, 1948] in the article the mathematical theory of communication. Suppose we have a set of possible events whose probabilities of occurrence are \( p_1, ..., p_n \) then the measure of information is \( H = - \sum p_i \log p_i \). Here we are choosing only between two possible messages; viz. degree of membership \( p_1 \) and non-membership \( p_2 \). In the Intuitionistic fuzzy set, [Atanassov, 1986] the grades fulfill axioms of probability and the possibility is the upper limit of probability [Zadeh, 1978]. Hence, \( \mu_i, \nu_i \) for the \( i^{th} \) element can be replaced by \( p_1 \& p_2 \) respectively. Here agents are choosing only between two possible messages, whose possibilities are \( p_1 \& p_2 \). From the definition, we know that \( 0 \leq \mu_X(x_i) + \nu_X(x_i) \leq 1 \). Hence, when \( \pi_X(x_i) = 0 \) then \( p_1 + p_2 = 1 \). \( H \) has its largest value, namely one when \( p_1 = p_2 = \frac{1}{2} \); that is to say when one is completely free to choose between the two messages. Just as soon as one message becomes more probable that the other (say \( p_1 > p_2 \)) the value of \( H \) decreases and when one message is very probable (say \( p_1 = 1 \& p_2 = 0 \)), the value of \( H \) is very small (almost zero).

Searching will be optimum for an element where information is maximum. For a choice function \( D : \Gamma_{IFS} \rightarrow X \);
such element, then it would be called choice correspondence. The element $D: \Gamma \times X$ would be chosen with the value of the H will be highest, as the degree of indeterminacy will be zero i.e. $\pi_i = 0$. If we have more than one such element, then it would be called choice correspondence. The element $x_i$ with $(\mu_i = \nu_i)$ might not be present in $X$. Then the neighborhood of this element would be considered as the stopping point or the optimum.

Partition of Independence (PI)

$D: \Gamma_{IFS} \rightarrow X$ fulfills property PI as for every pair of disjoint lists $\Gamma_{IFA_1}, \Gamma_{IFA_2} \in \Gamma_{IFS}$, we have $D((\Gamma_{IFA_1}, \Gamma_{IFA_2})) = D(D(\Gamma_{IFA_1}), D(\Gamma_{IFA_2}))$. This means:

$D((x_1, \mu_1, \nu_1), \ldots, (x_k, \mu_k, \nu_k)) = D(D(\ldots D((x_1, \mu_1, \nu_1), (x_2, \mu_2, \nu_2)), (x_3, \mu_3, \nu_3)), \ldots (x_k, \mu_k, \nu_k))$.

The operation starts by computing $D((x_1, \mu_1, \nu_1), (x_2, \mu_2, \nu_2))$, then compare the winner with $(x_3, \mu_3, \nu_3)$ to obtain $D((x_1, \mu_1, \nu_1), (x_2, \mu_2, \nu_2), (x_3, \mu_3, \nu_3))$, and so on, until $D(\ldots, (x_k, \mu_k, \nu_k))$ is compared with $(x_k, \mu_k, \nu_k)$ to obtain the chosen element where $(\mu_i = \nu_i)$ for $x_i$.

List Independence of Irrelevant Alternatives (IIA)

$D: \Gamma_{IFS} \rightarrow X$ fulfills IIA as it fulfills PI (using Proposition 1 given by Rubinstein & Salant (2006)). This information index largely depends on the reviewer comments and the ratings. If a degree of rationality to this information provider attaches then the decision-making will be without loss of information.

Reviewers’ comments are the important determinants of index $H$. Any false or unreliable information can cause a poor calculation of the index as a result the choice process would be based on irrational information. The next section formally derives the rationality pattern of a reviewer based on his/her past behavior. Inclusion of the rationality pattern and the degree to which any review comments and ratings will improve the index $H$ and make the choice problem error-free.

Lemma 1 A completely irrational agent is not possible.

Proof: The proof is obvious because the pattern $P^T_{i+2}$ has one of the three order elements of $P^T_{i+1}$ and $P^T_{i+2}$ i.e. for example, $\{111\} \in P^T_{i+1}$ and $\{111\} \in P^T_{i+2}$. Here, $\{111\} \in P^T_{i+1}$ and $\{111\} \in P^T_{i+2}$. 

Lemma 2 For a fixed number of objects across time $t$ i.e. $n$ strong consistency implies $\{1, 11\}$ and weak consistency implies $\{11, 1\}$

Proof: It is obvious from Theorem 1. let $X_1 = \{1, 11\}$ and $X_2 = \{11, 1\}$. Then the following two equations are true.

$\forall X_1 \subseteq X_2 \subseteq X_{ifs}(X_2) \in X_1 then C_{ifs}(X_1) = C_{ifs}(X_2)$

$\forall X_1 \subseteq X_2 \subseteq X_{ifs}(X_1) \in X_2 then C_{ifs}(X_2) = C_{ifs}(X_1)$

Moreover, information in the pattern $X_1$ is equal to $X_2$. This means,

$[C_{ifs}(X_1/H)] = [C_{ifs}(X_2/H)] = [x_i/H]$
This means the agent selects the object $x_i \in X$ when there were only two objects i.e. $n = 2$ and appear sequentially in ascending order in pattern $X_1$ and also selects the same object $x_i \in X$ when $n = 2$ and appear sequentially in descending order in pattern $X_2$. In other words, the agent could measure the information index correctly for the object $x_i \in X$ as it would happen in pattern $X_2$. □

References

[Afriat, 1967] Afriat, S. N. (1967). The construction of utility functions from expenditure data. *International economic review*, 8(1):67–77.

[Afriat, 1973] Afriat, S. N. (1973). On a system of inequalities in demand analysis: an extension of the classical method. *International economic review*, pages 460–472.

[Alkhamees et al., 2021] Alkhamees, M., Alsaleem, S., Al-Qurishi, M., Al-Rubaian, M., and Hussain, A. (2021). User trustworthiness in online social networks: A systematic review. *Applied Soft Computing*, 103:107159.

[Apesteguia and Ballester, 2015] Apesteguia, J. and Ballester, M. A. (2015). A measure of rationality and welfare. *Journal of Political Economy*, 123(6):1278–1310.

[Arrow, 1959] Arrow, K. J. (1959). Rational choice functions and orderings. *Economica*, 26(102):121–127.

[Atanassov, 1986] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1):87 – 96.

[Awad and Ragowsky, 2008] Awad, N. F. and Ragowsky, A. (2008). Establishing trust in electronic commerce through online word of mouth: An examination across genders. *Journal of management information systems*, 24(4):101–121.

[Berge, 1971] Berge, C. (1971). Principles of combinatorics. *New York*, page 176.

[Blin et al., 1973] Blin, J.-M., Fu, K.-S., Whinston, A. B., and Moberg, K. B. (1973). Pattern recognition in micro-economics. *Journal of Cybernetics*, 3(4):17–27.

[Chen et al., 2017] Chen, L., Jiang, T., Li, W., Geng, S., and Hussain, S. (2017). Who should pay for online reviews? design of an online user feedback mechanism. *Electronic Commerce Research and Applications*, 23:38–44.

[Chen et al., 2011] Chen, Y., Wang, Q., and Xie, J. (2011). Online social interactions: A natural experiment on word of mouth versus observational learning. *Journal of marketing research*, 48(2):238–254.

[Chen and Xie, 2008] Chen, Y. and Xie, J. (2008). Online consumer review: Word-of-mouth as a new element of marketing communication mix. *Management science*, 54(3):477–491.

[Clemons et al., 2006] Clemons, E. K., Gao, G. G., and Hitt, L. M. (2006). When online reviews meet hyperdifferentiation: A study of the craft beer industry. *Journal of management information systems*, 23(2):149–171.
[Cui et al., 2020] Cui, Y., Mou, J., Cohen, J., Liu, Y., and Kurcz, K. (2020). Understanding consumer intentions toward cross-border m-commerce usage: A psychological distance and commitment-trust perspective. *Electronic Commerce Research and Applications*, 39:100920.

[Das, 2021] Das, D. (2021). Adaptive price mechanism and a sequential reverse negotiation model in social commerce. *Available at SSRN 3896318*.

[Dean and Martin, 2016] Dean, M. and Martin, D. (2016). Measuring rationality with the minimum cost of revealed preference violations. *Review of Economics and Statistics*, 98(3):524–534.

[Dellarocas, 2006] Dellarocas, C. (2006). Strategic manipulation of internet opinion forums: Implications for consumers and firms. *Management science*, 52(10):1577–1593.

[Duan et al., 2009] Duan, W., Gu, B., and Whinston, A. B. (2009). Informational cascades and software adoption on the internet: an empirical investigation. *MIS quarterly*, pages 23–48.

[Echenique et al., 2011] Echenique, F., Lee, S., and Shum, M. (2011). The money pump as a measure of revealed preference violations. *Journal of Political Economy*, 119(6):1201–1223.

[Etumnu et al., 2020] Etumnu, C. E., Foster, K., Widmar, N. O., Lusk, J. L., and Ortega, D. L. (2020). Does the distribution of ratings affect online grocery sales? evidence from amazon. *Agribusiness*.

[Filieri, 2016] Filieri, R. (2016). What makes an online consumer review trustworthy? *Annals of Tourism Research*, 58:46–64.

[Forman et al., 2008] Forman, C., Ghose, A., and Wiesenfeld, B. (2008). Examining the relationship between reviews and sales: The role of reviewer identity disclosure in electronic markets. *Information systems research*, 19(3):291–313.

[Fu et al., 2020] Fu, X., Ouyang, T., Yang, Z., and Liu, S. (2020). A product ranking method combining the features–opinion pairs mining and interval-valued pythagorean fuzzy sets. *Applied Soft Computing*, 97:106803.

[Gefen et al., 2003] Gefen, D., Karahanna, E., and Straub, D. W. (2003). Trust and tam in online shopping: An integrated model. *MIS quarterly*, pages 51–90.

[Häubl and Trifts, 2000] Häubl, G. and Trifts, V. (2000). Consumer decision making in online shopping environments: The effects of interactive decision aids. *Marketing science*, 19(1):4–21.

[Hong et al., 2020] Hong, W., Yu, Z., Wu, L., and Pu, X. (2020). Influencing factors of the persuasiveness of online reviews considering persuasion methods. *Electronic Commerce Research and Applications*, 39:100912.

[Hopcroft et al., 2001] Hopcroft, J. E., Motwani, R., and Ullman, J. D. (2001). Introduction to automata theory, languages, and computation. *Acm Sigact News*, 32(1):60–65.

[Houthakker, 1950] Houthakker, H. S. (1950). Revealed preference and the utility function. *Economica*, 17(66):159–174.
[Houtman and Maks, 1985] Houtman, M. and Maks, J. (1985). Determining all maximal data subsets consistent with revealed preference. *Kwantitatieve methoden*, 19(1):89–104.

[Hu et al., 2006] Hu, N., Pavlou, P. A., and Zhang, J. (2006). Can online reviews reveal a product’s true quality? empirical findings and analytical modeling of online word-of-mouth communication. In *Proceedings of the 7th ACM conference on Electronic commerce*, pages 324–330.

[Huang and Chen, 2006] Huang, J.-H. and Chen, Y.-F. (2006). Herding in online product choice. *Psychology & Marketing*, 23(5):413–428.

[Khopkar and Nikolaev, 2017] Khopkar, S. S. and Nikolaev, A. G. (2017). Predicting long-term product ratings based on few early ratings and user base analysis. *Electronic Commerce Research and Applications*, 21:38–49.

[Koh et al., 2010] Koh, N. S., Hu, N., and Clemons, E. K. (2010). Do online reviews reflect a product’s true perceived quality? an investigation of online movie reviews across cultures. *Electronic Commerce Research and Applications*, 9(5):374–385.

[Kohli et al., 2004] Kohli, R., Devaraj, S., and Mahmood, M. A. (2004). Understanding determinants of online consumer satisfaction: A decision process perspective. *Journal of Management Information Systems*, 21(1):115–136.

[Lu et al., 2013] Lu, X., Ba, S., Huang, L., and Feng, Y. (2013). Promotional marketing or word-of-mouth? evidence from online restaurant reviews. *Information Systems Research*, 24(3):596–612.

[Ma et al., 2019] Ma, H., Kim, J. M., and Lee, E. (2019). Analyzing dynamic review manipulation and its impact on movie box office revenue. *Electronic Commerce Research and Applications*, 35:100840.

[Malik and Hussain, 2018] Malik, M. and Hussain, A. (2018). An analysis of review content and reviewer variables that contribute to review helpfulness. *Information Processing & Management*, 54(1):88–104.

[Mayzlin et al., 2014] Mayzlin, D., Dover, Y., and Chevalier, J. (2014). Promotional reviews: An empirical investigation of online review manipulation. *American Economic Review*, 104(8):2421–55.

[Moe and Schweidel, 2012] Moe, W. W. and Schweidel, D. A. (2012). Online product opinions: Incidence, evaluation, and evolution. *Marketing Science*, 31(3):372–386.

[Mudambi and Schuff, 2010] Mudambi, S. M. and Schuff, D. (2010). Research note: What makes a helpful online review? a study of customer reviews on amazon. com. *MIS quarterly*, pages 185–200.

[Nelson, 1970] Nelson, P. (1970). Information and consumer behavior. *Journal of political economy*, 78(2):311–329.

[Pan et al., 2018] Pan, X., Hou, L., Liu, K., and Niu, H. (2018). Do reviews from friends and the crowd affect online consumer posting behaviour differently? *Electronic Commerce Research and Applications*, 29:102–112.
[Pavlou and Dimoka, 2006] Pavlou, P. A. and Dimoka, A. (2006). The nature and role of feedback text comments in online marketplaces: Implications for trust building, price premiums, and seller differentiation. Information Systems Research, 17(4):392–414.

[Punetha and Jain, 2023] Punetha, N. and Jain, G. (2023). Bayesian game model based unsupervised sentiment analysis of product reviews. Expert Systems with Applications, 214:119128.

[Qin and Zeng, 2022] Qin, J. and Zeng, M. (2022). An integrated method for product ranking through online reviews based on evidential reasoning theory and stochastic dominance. Information Sciences, 612:37–61.

[Racherla and Friske, 2012] Racherla, P. and Friske, W. (2012). Perceived ‘usefulness’ of online consumer reviews: An exploratory investigation across three services categories. Electronic Commerce Research and Applications, 11(6):548–559.

[Reinstein and Snyder, 2005] Reinstein, D. A. and Snyder, C. M. (2005). The influence of expert reviews on consumer demand for experience goods: A case study of movie critics. The journal of industrial economics, 53(1):27–51.

[Richter, 1966] Richter, M. K. (1966). Revealed preference theory. Econometrica: Journal of the Econometric Society, pages 635–645.

[Rubinstein and Salant, 2006] Rubinstein, A. and Salant, Y. (2006). A model of choice from lists. Theoretical Economics, 1(1):3–17.

[Samuelson, 1938] Samuelson, P. A. (1938). A note on the pure theory of consumer’s behaviour. Economica, 5(17):61–71.

[Saumya et al., 2018] Saumya, S., Singh, J. P., Baabdullah, A. M., Rana, N. P., and Dwivedi, Y. K. (2018). Ranking online consumer reviews. Electronic Commerce Research and Applications, 29:78–89.

[Schick, 1963] Schick, F. (1963). Consistency and rationality. The Journal of philosophy, 60(1):5–19.

[Shannon, 1948] Shannon, C. E. (1948). A mathematical theory of communication. Bell system technical journal, 27(3):379–423.

[Shen et al., 2013] Shen, W., Hu, Y., and Rees, J. (2013). Competing for attention: An empirical study of online reviewers’ strategic. Technical report, Working paper.

[Simon, 1997] Simon, H. A. (1997). Models of bounded rationality: Empirically grounded economic reason, volume 3. MIT press.

[Song et al., 2020] Song, W., Li, W., and Geng, S. (2020). Effect of online product reviews on third parties’ selling on retail platforms. Electronic Commerce Research and Applications, 39:100900.

[Sridhar and Srinivasan, 2012] Sridhar, S. and Srinivasan, R. (2012). Social influence effects in online product ratings. Journal of Marketing, 76(5):70–88.
[Stigler, 1961] Stigler, G. J. (1961). The economics of information. *Journal of political economy, 69*(3):213–225.

[Tsang, 2008] Tsang, E. P. (2008). Computational intelligence determines effective rationality. *International Journal of Automation and Computing, 5*(1):63–66.

[Utz et al., 2012] Utz, S., Kerkhof, P., and Van Den Bos, J. (2012). Consumers rule: How consumer reviews influence perceived trustworthiness of online stores. *Electronic Commerce Research and Applications, 11*(1):49–58.

[Varian, 1982] Varian, H. R. (1982). The nonparametric approach to demand analysis. *Econometrica: Journal of the Econometric Society, pages* 945–973.

[Xu et al., 2012] Xu, K., Guo, X., Li, J., Lau, R. Y., and Liao, S. S. (2012). Discovering target groups in social networking sites: An effective method for maximizing joint influential power. *Electronic Commerce Research and Applications, 11*(4):318–334.

[Zadeh, 1978] Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. *Fuzzy sets and systems, 1*(1):3–28.