Fat penguins and imaginary penguins in perturbative QCD

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Abstract

We evaluate $B \rightarrow K\pi$ decay amplitudes in perturbative QCD picture. It is found that penguin contributions are dynamically enhanced by nearly 50% compared to those assumed in the factorization approximation. It is also shown that annihilation diagrams are not negligible, and give large strong phases. Our results for branching ratios of $B \rightarrow K\pi$ decays for a representative parameter set are consistent with data.

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Factorization assumption (FA) for nonleptonic two-body $B$ and $D$ meson decays pioneered by Stech and his collaborators [1] has been extremely successful. It gives correct order of magnitude for branching ratios of most two-body $B$ meson decays. Why does it work so well? Recent CLEO data of $B \to \pi \pi$ and $B \to K \pi$ branching ratios [2,3] require not only order-of-magnitude predictions but quantitative predictions for these decay modes. As asymmetric $B$ factories, which are eventually capable of producing almost $10^8 B$'s per year, have started their operation, quantitative theoretical understanding will allow us to extract CP phases hidden in the above branching ratios. How can we go beyond FA?

Let us see what QCD can say about these questions. The fact that 5 GeV of energy is released and shared by two light mesons suggests that the basic interaction in two-body $B$ meson decays is mainly short-distance. In this letter we shall attempt to compute these decay amplitudes using as much information from the underlying theory, QCD, as possible. Our method is perturbative QCD (PQCD) factorization theorem, which has been worked out by Li and his collaborators [4–7] based on the formalism developed by Brodsky and Lapage [8] and by Botts and Sterman [9].

Consider the specific $\bar{B}^0 \to K^- \pi^+$ decay amplitude shown in Fig. 1, where the $b \to s\bar{u}u$ decay occurs. The pair of the $s$ and $\bar{u}$ quarks fly away and form the $K^-$ meson. The spectator $\bar{d}$ quark of the $\bar{B}^0$ meson is more or less at rest and the $u$ quark is flying away. The probability that a quark and an antiquark with large relative velocity form the $\pi^+$ meson is suppressed by the pion wave function. How big is the suppression? It depends on the functional form of the wave function. It is safe to say that this suppression from the wave function is of the form $(\Lambda_{QCD}/M_B)^n$, where $n$ is likely to be large. Therefore, we expect that dominant contributions to the $\bar{B}^0 \to K^- \pi^+$ decay come from the process, where a hard gluon is exchanged so that $\bar{d}$ quark momentum and $u$ quark momentum are aligned to form the pion. The rectangular dotted boxes in Fig. 1 enclose the part of interaction which is hard. The blobs represent wave functions giving amplitudes for a quark and an antiquark to form a meson.

For the $B$ meson mass $M_B \gg \Lambda_{QCD}$ and the kaon and pion masses $M_K \sim M_\pi \sim 0$, the $\bar{B}^0 \to K^- \pi^+$ decay amplitude is then written as a convolution of four factors,

$$M = \int d[x] d[b] \phi_B(x_1, b_1) \phi_K(x_2, b_2) \phi_\pi(x_3, b_3) H([x], [b], M_B),$$

(1)

where the wave functions $\phi_{B,K,\pi}$ for the $\bar{B}^0$, $K^-$, $\pi^+$ mesons absorb nonperturbative dynamics of the process, the hard amplitude $H([x], [b], M_B)$ can be calculated in perturbation theory, $[x]$ is a shorthand for the momentum fractions $x_1$, $x_2$, and $x_3$ associated with the $\bar{d}$ quark in the $\bar{B}^0$ meson, the $u$ quark, and the $\bar{d}$ quark in the $\pi^+$ meson, respectively, and $[b]$ is a shorthand for the two-dimensional vectors, i.e., the transverse extents, $b_1$, $b_2$, and $b_3$ of the $\bar{B}^0$, $K^-$ and $\pi^+$ mesons, respectively.

Here are some important questions:
1. Is $H([x],[b],M_B)$ really dominated by short-distance contributions?

2. Figure 1(a) is factorizable, since it can be written in terms of the $B \to \pi$ transition form factor $F_{B\pi}$ and the kaon decay constant $f_K$. This is the amplitude considered in FA. FA assumes that a nonfactorizable amplitude from Fig. 1(b), which can not be written in terms of a form factor and a decay constant, is negligible compared to Fig. 1(a).

Are nonfactorizable amplitudes negligible compared to factorizable ones? We have made a numerical study of this issue. While it is important to always check the relative magnitudes, we have found that nonfactorizable contributions are usually less than few percents of factorizable contributions. This is the reason FA has been so successful. However, there are exceptions: (1) In $B \to D\pi$ decays some nonfactorizable contributions can reach as much as 30% of factorizable ones. Actually, the experimental fact that the ratio $a_2/a_1 \sim 0.2$ in the Bauer-Stech-Wirbel model [1] requires large nonfactorizable contributions. (2) In $B \to J/\psi K^{(*)}$ decays nonfactorizable and factorizable contributions are of the same order of magnitude [7].

3. It is well known [10] that the role of penguins is essential for explaining the observed $B \to K\pi$, $\pi\pi$ branching ratios. How big are penguin amplitudes? We shall show below that penguin amplitudes can be dynamically enhanced by 50% in PQCD compared to those assumed in FA.

4. Are annihilation diagrams in Fig. 2 really negligible?

5. How big are final-state-interaction (FSI) effects? It is impossible to compute FSI phases in FA. Effects from infinite soft gluon exchanges among mesons in two-body $B$ meson decays have been analyzed quantitatively by means of renormalization-group methods and found to be small [11]. This observation implies that effects from exchange of soft objects between the two final-state mesons are also small. Where then do strong phases come from? We shall show that contrary to common belief, annihilation diagrams are important, and in fact, they contribute large strong phases.

We present the factorizable PQCD amplitudes $F_{e_1}$, $F_{e_4}^P$, and $F_{e_6}^P$ corresponding to Fig.1(a) and $F_a$, $F_{a_4}^P$, and $F_{a_6}^P$ corresponding to Fig.2(a) from the four-quark operators $O_{1,2}$, $O_{3,4,9,10}$, and $O_{5,6,7,8}$, respectively [12],

$$F_{e_1}^P = 16\pi C_F M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1)$$
\[\times \{ [(1 + x_3)\phi_\pi(x_3) + r_\pi(1 - 2x_3)\phi'_\pi(x_3)] E_{e_4}(t_e^{(1)}) h_e(x_1, x_3, b_1, b_3, M_B) \]
\[+ 2r_\pi \phi'_\pi(x_3) E_{e_4}(t_e^{(2)}) h_e(x_3, x_1, b_3, b_1, M_B) \}, \] (2)
\[ F_{6e}^P = 32\pi C_F M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \]
\[ \times r_K [\phi_\pi(x_3) + r_\pi (2 + x_3) \phi'_\pi(x_3)] E_{a6} (t_e^{(1)}) h_e(x_1, x_3, b_1, b_3, M_B) \]
\[ + [x_1 \phi_\pi(x_3) + 2r_\pi (1 - x_1) \phi'_\pi(x_3)] E_{a6} (t_e^{(2)}) h_e(x_1, x_3, b_1, b_3, M_B) \}, \]
\[ F_{a4}^P = 16\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \]
\[ \times \{ [r_\pi x_3 \phi_K(x_2) \phi'_\pi(x_3) - 2r_\pi r_K (1 + x_3) \phi'_K(x_2) \phi'_\pi(x_3)] E_{a4} (t_a^{(1)}) h_a(x_2, x_3, b_2, b_3, M_B) \]
\[ + [x_2 \phi_K(x_2) \phi_\pi(x_3) + 2r_\pi r_K (1 + x_2) \phi'_K(x_2) \phi'_\pi(x_3)] E_{a4} (t_a^{(2)}) h_a(x_2, x_3, b_2, b_3, M_B) \} \]
\[ F_{a6}^P = 32\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \]
\[ \times \{ [r_\pi x_3 \phi_K(x_2) \phi'_\pi(x_3) - 2r_\pi r_K (1 + x_2) \phi'_K(x_2) \phi'_\pi(x_3)] E_{a6} (t_a^{(1)}) h_a(x_2, x_3, b_2, b_3, M_B) \]
\[ + [2r_\pi r_K (x_2) \phi'_K(x_2) \phi_\pi(x_3) + r_K x_2 \phi'_K(x_2) \phi'_\pi(x_3)] E_{a6} (t_a^{(2)}) h_a(x_2, x_3, b_2, b_3, M_B) \} \],

\( C_F \) being a color factor. The expression of \( F_e \) (\( F_a \)) for the \( O_{1,2} \) contributions is the same as \( F_{6e}^P \) (\( F_{a4}^P \)) but with the Wilson coefficient \( a_1(t_e) \) (\( a_1(t_a) \)). The hard functions \( h \)’s in Eqs (2)-(5) are given by

\[ h_e(x_1, x_3, b_1, b_3, M_B) = K_0 (\sqrt{x_1 x_3} M_B b_1) \]
\[ \times \left[ \theta(b_1 - b_3) K_0 (\sqrt{x_3} M_B b_1) I_0 (\sqrt{x_3} M_B b_3) + (b_1 \leftrightarrow b_3) \right] \]
\[ h_a(x_2, x_3, b_2, b_3, M_B) = \left( \frac{i\pi}{2} \right)^2 H_0^{(1)} (\sqrt{x_2 x_3} M_B b_2) \]
\[ \times \left[ \theta(b_2 - b_3) H_0^{(1)} (\sqrt{x_3} M_B b_2) J_0 (\sqrt{x_3} M_B b_3) + (b_1 \leftrightarrow b_3) \right] \].

The evolution factors

\[ E_{a1}(t) = \alpha_s(t) a_1(t) \exp[-S_B(t) - S_\pi(t)], \quad E_{a4}(t) = \alpha_s(t) a_4(t) \exp[-S_K(t) - S_\pi(t)], \]

arise from the summation of infinite infrared gluon emissions that give double (Sudakov) logarithms and single logarithms connecting the hard scales \( t \) and the characteristic scales \( 1/b \) of the wave functions. For the explicit expressions of the Sudakov exponents \( S_B \), \( S_K \), and \( S_\pi \), refer to [3]. The hard scales \( t \) are chosen as the virtualities of internal particles in hard \( b \) quark decay amplitudes,

\[ t_e^{(1)} = \max(\sqrt{x_3} M_B, 1/b_1, 1/b_3), \quad t_e^{(2)} = \max(\sqrt{x_1} M_B, 1/b_1, 1/b_3), \]
\[ t_a^{(1)} = \max(\sqrt{x_3} M_B, 1/b_2, 1/b_3), \quad t_a^{(2)} = \max(\sqrt{x_2} M_B, 1/b_2, 1/b_3). \]

It has been shown that this choice minimizes higher-order corrections to exclusive QCD processes [11]. Equation (3) is consistent with the fact that the hard scales \( t \) and the evolution effects related to running of \( t \) should be process-dependent. The Wilson coefficients are

\[ a_1 = C_2 + \frac{C_1}{N_c}, \quad a_4(6) = C_{4(6)} + \frac{C_{3(5)}}{N_c} + \frac{3}{2} C_q \left( C_{10(8)} + \frac{C_{9(7)}}{N_c} \right), \]

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for the tree and the \((V - A)(V + A)\) penguins, respectively, \(N_c\) being the number of colors. \(F_{\epsilon^{(a)0}}\) has a different integrand from \(F_{\epsilon^{(a)4}}\), reflecting the different helicity structures.

The factors \(r_\pi\) and \(r_K\),

\[
r_\pi = \frac{m_{0\pi}}{M_B}, \quad m_{0\pi} = \frac{M_\pi^2}{m_u + m_d}; \quad r_K = \frac{m_{0K}}{M_B}, \quad m_{0K} = \frac{M_K^2}{m_s + m_d},
\]

are associated with the normalizations of the pseudoscalar wave functions \(\phi'\), where \(m_u\), \(m_d\), and \(m_s\) are the current quark masses of the \(u\), \(d\) and \(s\) quarks, respectively, and \(M_\pi\) and \(M_K\) the pion and kaon masses, respectively. The pseudovector and pseudoscalar pion wave functions \(\phi_\pi\) and \(\phi'_\pi\) are defined in terms of matrix elements of nonlocal operators \(\langle 0|\bar{d}\gamma_5 u|\pi\rangle\) and \(\langle 0|\bar{d}\gamma_5 u|\pi\rangle\), respectively. The kaon wave functions \(\phi_K\) and \(\phi'_K\) possess similar definitions.

We employ the following set of meson wave functions as an illustrative example:

\[
\begin{align*}
\phi_B(x,b) &= N_B x^2 (1-x)^2 \exp \left[ -\frac{1}{2} \left( \frac{x M_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b^2}{2} \right], \\
\phi_\pi(x) &= \frac{3}{\sqrt{2N_c}} f_\pi x (1-x) [1 + 0.8(5(1-2x)^2 - 1)] , \\
\phi_K(x) &= \frac{3}{\sqrt{2N_c}} f_K x (1-x) [1 + 0.51(1-2x) + 0.3(5(1-2x)^2 - 1)] , \\
\phi'_K(x) &= \frac{3}{\sqrt{2N_c}} f_\pi x (1-x), \quad \phi'_K(x) = \frac{3}{\sqrt{2N_c}} f_K x (1-x),
\end{align*}
\]

with the shape parameter \(\omega_B = 0.3\) GeV and the decay constants \(f_\pi = 130\) MeV and \(f_K = 160\) MeV. The normalization constant \(N_B\) is related to the \(B\) meson decay constant \(f_B = 190\) MeV via \(\int \phi_B(x,b=0)dx = f_B/(2\sqrt{2N_c})\). \(\phi_K\) is derived from QCD sum rules [17]. All other meson wave functions and \(f_B\) are determined from the data of the \(B \rightarrow D\pi, \pi\pi\) decays and of the pion form factor [12]. Note that we have included the intrinsic \(b\) dependence for the heavy meson wave function \(\phi_B\) but not for the light meson wave functions \(\phi_\pi\) and \(\phi_K\). It has been shown that the intrinsic \(b\) dependence of the light meson wave functions, resulting in only 5% reduction of the predictions for the form factor \(F^{B\pi}\), is not important [4]. We do not distinguish the pseudovector and pseudoscalar components of the \(B\) meson wave functions under the heavy quark approximation.

- **Is PQCD legitimate?** We show that PQCD allows us to compute two-body decay amplitudes by examining where dominant contributions to the form factor \(F^{B\pi}\) come from. Figure 3 displays the fractional contribution as a function of \(\alpha_s(t)/\pi\). It is observed that 97% of the contribution arises from the region with \(\alpha_s(t)/\pi < 0.3\). Therefore, our PQCD results are well within the perturbative region. This analysis implies that \(H([x],[b],M_B)\) is dominated by short-distance contributions, contrary to the viewpoint of Beneke, Buchalla, Neubert, and Sachrajda (BBNS) in [18], and that the physical picture we have given for what is happening in Fig. 1 is indeed valid.
• Fat penguins in PQCD: Let us have a careful look at the matrix elements of the penguin operators. It is noticed that unlike $C_2$, $C_4$ and $C_6$ have a steep $\mu$ dependence. In FA, amplitudes depend on the matching scale. Normally, it is taken to be $m_b/2$, $m_b$ being the $b$ quark mass, but there is no theoretical basis for this choice. One of the main advantages in PQCD is that it provides a prescription for choices of the hard scale $t$: $t$ should be chosen as the virtuality of internal particles in a hard amplitude in order to decrease higher-order corrections. A good fraction of contributions then come from $t < m_b/2$, and penguin contributions are enhanced. Numerically, this enhancement is given by:

$$\frac{(F_{P_6})_{\text{PQCD}}}{(F_{P_6})_{\text{FA}}} = 1.6,$$

$$\frac{(F_{P_4})_{\text{PQCD}}}{(F_{P_4})_{\text{FA}}} = 1.4,$$

$$\frac{(F_{\ell})_{\text{PQCD}}}{(F_{\ell})_{\text{FA}}} = 1.0,$$  \hspace{1cm} (16)

where $(F)_{FA}$ represent the form factors evaluated in PQCD but with the Wilson coefficients $C(t)$ set to $C(M_B/2)$. Equation (16) shows that penguin contributions are dynamically fattened by about 50%, and that the tree amplitudes from $O_{1,2}$ remains invariant. Other sources of penguin enhancement are referred to [12].

The enhancement due to the increase of $C_6(t)$ with decreasing $t$ makes us worry that the contribution from the small $t$ region may be important. This will invalidate the perturbative expansion of $H([x], [b], M_B)$. As a check, we examine the fractional contribution to $F_{P_6}$ as a function of $\alpha_s(t)/\pi$. The results, similar to Fig. 3, indicate that about 90% (80%) of the contribution comes from the region with $\alpha_s(t)/\pi < 0.3$ (0.2). Therefore, exchanged gluons are still hard enough to guarantee the applicability of PQCD.

We emphasize that the penguin enhancement is crucial for the simultaneous explanation of the $B \rightarrow K \pi$, $\pi\pi$ data using a unitarity angle $\phi_3 \sim 90^\circ$ [12,13]. It has been shown [19] that a simultaneous understanding of the data $R = Br(B^0 \rightarrow K^+\pi^-)/Br(B^+ \rightarrow K^0\pi^+) \sim 1.0$ and $Br(B^0 \rightarrow \pi^+\pi^-) \sim 4.3 \times 10^{-6}$ is difficult in FA. The former indeed leads to $\phi_3 \sim 90^\circ$. However, the latter leads to $\phi_3 \sim 130^\circ$, even if $m_0$ is stretched to $m_0 \sim 4$ GeV corresponding to $m_4 = 2m_u = 3$ MeV.

• Imaginary annihilation penguins: There has been a widely spread folklore that the annihilation diagrams give negligible contribution due to helicity suppression, just as in $\pi \rightarrow e\nu$ decay. That is, a left-handed massless electron and a right-handed antineutrino can not fly away back to back because of angular momentum conservation. However, this argument does not apply to $F_{P_6}$. A left-handed quark and a left-handed antiquark, for which helicities are dictated by the $O_6$ operator, can indeed fly away back to back because of angular momentum conservation. These behaviors have been reflected by Eqs. (4) and (5): Eq. (4) vanishes exactly, if the kaon and pion wave functions are identical, while the two terms in Eq. (5) are constructive. Numerical results in Table I show that the strong phase associated with $F_{P_6}$ is nearly 90$^\circ$. The large absorptive part arises from cuts on the intermediate state $(sd\bar{u})$ in the decay $B^0 \rightarrow s\bar{d} \rightarrow K^-\pi^+$ shown in Fig. 2. The intermediate state $(s\bar{d})$ can be regarded as being highly inelastic, if expanded in terms of hadron states.
On the issue of FSI, Suzuki has argued that the invariant mass of the $s\bar{d}$ pair in Fig. 2 is of order $(\Lambda_{QCD}M_B)^{1/2} \sim 1.2$ GeV [20]. Hence, the $B \to K\pi$ decays are located in the resonance region and their strong phases are very complicated. We have computed the average hard scale of the $B \to K\pi$ decays, which is about 1.4 GeV, in agreement with the above estimate. Since the outgoing $s\bar{d}$ pair should possess an invariant mass larger than 1.4 GeV, the processes are in fact not so close to the resonance region. We could interpret that the $B \to K\pi$ decays occur via a six-fermion operator within space smaller than $(1/1.4)$ GeV$^{-1}$. Though they are not completely short-distance, the fact that over 90% of contributions come from the $x$-$b$ phase space with $\alpha_s(t)/\pi < 0.3$ allows us to estimate the decay amplitudes reliably. We believe that the strong phases can be computed up to about 20% uncertainties, which result in 30% errors in predictions for CP asymmetries.

- **Br($B \to K\pi$):** We present PQCD results of various $B \to K\pi$ branching ratios in Table II, which are well consistent with the CLEO data [3]. These results are meant to be an example for a representative parameter set such as the wave functions in Eqs. (12)-(15), which are determined from the best fit to the data of the $B \to D\pi, \pi\pi$ and of the pion form factor, and $m_{0\pi} = 1.4$ GeV and $m_{0K} = 1.7$ GeV [12]. When all other two-body decay modes are considered, we shall present an exhaustive study of the entire parameter space allowed by data uncertainties.

- **Comparison with the BBNS approach:** Here we compare our approach to exclusive nonleptonic $B$ meson decays with the BBNS approach [18]. The differences between the two approaches are briefly summarized below. For more details, refer to [21].

As stated before, the PQCD theory for exclusive processes was first formulated by Brodsky and Lepage [8]. This formalism has been criticized by Isgur and Llewellyn-Smith [22], since involved perturbative evaluations, based on expansion in terms of a large coupling constant in the end-point region of momentum fractions, are not reliable. Li and Sterman [23] pointed out that Sudakov resummation of large logarithms associated with parton transverse momenta, which was worked out by Botts and Sterman [9], causes suppression in the end-point region. This suppression is strong enough to render PQCD analyses self-consistent at energy scales of few GeV. The above approach was then extended by Li and his collaborators to exclusive $B$ meson decays in the heavy meson limit [11][7], where Sudakov suppression, cutting off the infrared singularity in heavy-to-light transition form factors [21], is even more important. Our calculation of two-body $B$ meson decays has followed the formalism developed by the above authors. Therefore, the $B$-to-light-meson transition form factors at maximal recoil are calculable in PQCD. In the BBNS approach, because Sudakov suppression is not considered, the transition form factors are not calculable and must be treated as inputs.

Another crucial difference is that in the PQCD formalism annihilation diagrams are of the same order as factorizable diagrams in powers of $1/M_B$, which are both $O(1/(M_B\Lambda_{QCD}))$. 
The BBNS approach follows FA, in which it has been assumed that factorizable contributions, being \( O(1/\Lambda_{QCD}^2) \), are leading, and all other contributions such as annihilation and nonfactorizable diagrams, being \( O(1/(M_B\Lambda_{QCD})) \), are next-to-leading. Factorizable and nonfactorizable contributions are considered by BBNS, but annihilation are not. The difference is again traced back to the inclusion of parton transverse momenta and of the Sudakov form factor in our calculation. With Sudakov suppression, gluon exchanged in factorizable diagrams are as hard as those in annihilation diagrams. Because of parton transverse momenta, the internal particles in hard amplitudes may go onto the mass shell at nonvanishing momentum fractions \[21\]. As a consequence, annihilation diagrams lead to large imaginary contributions, whose magnitudes are comparable to factorizable ones, and to large CP asymmetries in the \( B \rightarrow K\pi \) decays.

We emphasize that annihilation diagrams are indeed subleading in the PQCD formalism as \( M_B \rightarrow \infty \). This can be easily observed from the hard functions in Eqs. (6) and (7). When \( M_B \) increases, the \( B \) meson wave function in Eq. (12) enhances contributions to \( h_e \) from smaller momentum fraction \( x_1 \) as expected. However, annihilation amplitudes proportional to \( h_a \), being independent of \( x_1 \), are relatively insensitive to the variation of \( M_B \). Hence, factorizable contributions become dominant and annihilation contributions are subleading in the \( M_B \rightarrow \infty \) limit. For \( M_B \sim 5 \text{ GeV} \), our calculation shows that these two types of contributions are comparable.

We have shown that PQCD allows us to compute matrix elements of various four-quark operators. While FA gives reliable estimates for \( O_{1,2} \), since their Wilson coefficients are nearly constant in the hard scale \( t \), matrix elements of the penguin operators are another story. We have observed that PQCD results are larger than FA results by about 50% for the penguin operators, because of the \( t \) dependence of the Wilson coefficients. With the penguin enhancement in PQCD, the CLEO data of the \( B \rightarrow K\pi, \pi\pi \) branching ratios can be understood in a more self-consistent way. We have also pointed out that penguin annihilation diagrams are not negligible as claimed in FA. In fact, they contribute large strong phases, which are essential for predictions of CP asymmetries.

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REFERENCES

[1] M. Bauer, B. Stech, M. Wirbel, Z. Phys. C 34, 103 (1987); Z. Phys. C 29, 637 (1985).
[2] CLEO Coll., Y. Kwon et al., hep-ex/9908039.
[3] CLEO Coll., D. Cronin-Hennessy et al., hep-ex/0001010.
[4] H-n. Li and H.L. Yu, Phys. Rev. Lett. 74, 4388 (1995); Phys. Lett. B 353, 301 (1995); Phys. Rev. D 53, 2480 (1996).
[5] H-n. Li, Phys. Rev. D 52, 3958 (1995).
[6] C.H. Chang and H-n. Li, Phys. Rev. D 55, 5577 (1997).
[7] T.W. Yeh and H-n. Li, Phys. Rev. D 56, 1615 (1997).
[8] G. P. Lepage and S. Brodsky, Phys. Rev. D 22, 2157 (1980).
[9] J. Botts and G. Sterman, Nucl. Phys. B225, 62 (1989).
[10] For example, see I.I. Bigi and A.I. Sanda, CP Violation (1999, Cambridge).
[11] H-n. Li and B. Tseng, Phys. Rev. D 57, 443 (1998).
[12] Y.Y. Keum, H-n. Li and A.I. Sanda, hep-ph/0004173, to appear in Phys. Rev. D.
[13] C. D. Lü, K. Ukai, and M. Z. Yang, hep-ph/0004213.
[14] We thank H.Y. Cheng for a discussion on this point.
[15] B. Melic, B. Nizic and K. Passek, Phys. Rev. D 60, 074004 (1999).
[16] M. Bauer and M. Wirbel, Z. Phys. C 42, 671 (1989).
[17] P. Ball, JHEP 9809, 005 (1998).
[18] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); hep-ph/0006124.
[19] W.S. Hou, J.G. Smith, and F. Würthwein, hep-ph/9910014.
[20] M. Suzuki, hep-ph/0001170.
[21] Y.Y. Keum and H-n. Li, hep-ph/0006001, to appear in Phys. Rev. D.
[22] N. Isgur and Llewellyn-Smith, Nucl. Phys. B317, 526 (1989).
[23] H-n. Li and G. Sterman, Nucl. Phys. B381, 129 (1992).
### TABLE I. Amplitudes for the $B^0 \to K^-\pi^+$ decay in units of $10^{-2}$ with $F^P_e = F^P_{e4} + F^P_{e6}$ and $F^P_a = F^P_{a4} + F^P_{a6}$.

| Amplitude | Real part | Imaginary part |
|-----------|-----------|----------------|
| $F^P_e$   | 71.60     | 0              |
| $F^P_e$   | -6.18     | 0              |
| $F^P_a$   | 0.30      | 2.58           |

### TABLE II. PQCD predictions of branching ratios for a representative parameters set.

| Branching ratio | PQCD prediction $(10^{-6})$ | CLEO data (average) $(10^{-6})$ |
|-----------------|-----------------------------|---------------------------------|
| Br($B^+ \to K^0\pi^+$) | 21.72                       | 18.2$^{+4.6}_{-4.0}$ $\pm$ 1.6 |
| Br($B^- \to \bar{K}^0\pi^-$) | 21.25                       | 18.2$^{+4.6}_{-4.0}$ $\pm$ 1.6 |
| Br($B^0 \to K^+\pi^-$) | 24.19                       | 17.2$^{+2.5}_{-2.4}$ $\pm$ 1.2 |
| Br($\bar{B}^0 \to K^-\pi^+$) | 16.84                       | 17.2$^{+2.5}_{-2.4}$ $\pm$ 1.2 |
| Br($B^+ \to K^+\pi^0$) | 14.44                       | 11.6$^{+3.9+1.4}_{-2.7-1.3}$ |
| Br($B^- \to K^-\pi^0$) | 10.65                       | 11.6$^{+3.9+1.4}_{-2.7-1.3}$ |
| Br($B^0 \to K^0\pi^0$) | 11.23                       | 14.6$^{+5.9+2.4}_{-5.1-3.3}$ |
| Br($\bar{B}^0 \to \bar{K}^0\pi^0$) | 11.84                       | 14.6$^{+5.9+2.4}_{-5.1-3.3}$ |
Figure Captions

Fig. 1: (a) Factorizable and (b) nonfactorizable tree or penguin contributions.

Fig. 2: (a) Factorizable and (b) nonfactorizable annihilation contributions. A cut on the $s\bar{d}$ quark lines corresponds to the absorptive part.

Fig. 3: Fractional contribution to the $B \to \pi$ transition form factor $F_{B\pi}$ as a function of $\alpha_s(t)/\pi$. 
Figure 1:

Figure 2:
Figure 3: