Dark Z Implication for Flavor Physics

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Dark $Z$/dark photon ($Z'$) is one candidate of dark force carrier, which helps to interpret the properties of dark matter (DM). Other than traditional studies of DM including direct detection, indirect detection and collider simulation, in this work we take flavor physics as a complementary approach to investigate the features of dark matter. We give an exact calculation of the new type of penguin diagram induced by $Z'$ which further modifies the well-known $X, Y, Z$ functions in penguin-box expansion. The measurement of rare decays $B \rightarrow K^{(*)}\mu^+\mu^-$ and $B_s \rightarrow \mu^+\mu^-$ at LHC, together with direct CP violation $\varepsilon'/\varepsilon$ in $K \rightarrow \pi\pi$ as well as $K_L \rightarrow \mu^+\mu^-$, are used to determine the parameter space. The size of coupling constant, however, is found to be $O(1)$ which is much weaker than the known constraints.
I. INTRODUCTION

Dark matter (DM) constitutes about 27% of the energy-matter budget of universe, significantly more than 5% of baryonic matter[1]. Nevertheless, the exact nature of DM is kept mysterious so far. Yet it is still unclear whether DM can be described by scalar, fermion, vector or even graviton.

Among various DM candidates, dark photon has been the one of particular interest. The idea was initiated in 1980s[2], and developed in recent years[3][4]. Suppose there is an extra U(1)D group besides SM gauge group, under which all the SM interactions are invariant. The gauge boson of U(1)D, named dark photon, interacts with SM U(1)Y gauge boson via a kinetic mixing term. It helps to explain the astrophysical observation of positron excesses[5], as well as other astrophysical phenomenology such as supernova bounds[4]. The direct search of dark photon, for example XENON 100, has put a very strong constraint[6]. For e⁺e⁻ collider, a recent search performed at BaBar shows a null result, neither finds nor rules out dark photon[7]. A further experiment, the Heavy Photon Search (HPS) experiment located at Jefferson Lab[8], is designed to search dark photon in the mass range 20 MeV to 1 GeV as well as the related coupling. However, it has been realized recently that a simple dark photon is not favored by 3.6σ deviation of muon anomalous magnetic moment[9]. Given the fact that dark photon model is the extreme case when the parameter to describe Z-Z’ mass mixing is closed in a more generic dark Z model, it is necessary to extend dark photon to dark Z which is the working frame of this paper.

Flavor physics is not only taken as a platform for precise physics of SM, but also plays a role in indirect search of new physics (NP) beyond Standard Model (SM). Great progresses have already been made since LHC runs. For example, it was hoped for decades that NP might exist in zero crossing point of new physics (NP) beyond Standard Model (SM). Great progresses have already been made since LHC runs. For example, it was hoped for decades that NP might exist in zero crossing point of new physics (NP) beyond Standard Model (SM). Great progresses have already been made since LHC runs. For example, it was hoped for decades that NP might exist in zero crossing point of new physics (NP) beyond Standard Model (SM). Great progresses have already been made since LHC runs. For example, it was hoped for decades that NP might exist in zero crossing point of new physics (NP) beyond Standard Model (SM). Great progresses have already been made since LHC runs. For example, it was hoped for decades that NP might exist in zero crossing point of new physics (NP) beyond Standard Model (SM). Great progresses have already been made since LHC runs. For example, it was hoped for decades that NP might exist in zero crossing point of new physics (NP) beyond Standard Model (SM). Great progresses have already been made since LHC runs. For example, it was hoped for decades that NP might exist in zero crossing point of new physics (NP) beyond Standard Model (SM). Great progresses have already been made since LHC runs.

The paper is organized as follows. In section 2, we will briefly set up the dark Z model. An exact result of Z’ penguin and further modifications to X, Y, Z functions are given in Section 3. Some typical processes which are affected by the Z’ including the recently measured B → K⁺μ⁺μ⁻ and B_s → μ⁺μ⁻, as well as ε'/ε and K_L → μ⁺μ⁻ are discussed in section 4, also the relevant formulas are given therein. In section 5 the obtained numerical results are shown, based on which we will make a discussion. The details of Z’ penguin calculation can be found in appendix.

II. THE MODEL

Suppose there exits an extra U(1) group, other than SM U(1), what it brings in phenomenology is an interesting question. It was considered how electromagnetic charge is shifted by this extra U(1) group in the initial paper[2]. Until recently it becomes popular to take this U(1) gauge boson as a DM candidate.

Under the dark group, notated as U(1)D, all the SM interactions are invariant. The connection between dark photon with SM particles is from a kinetic mixing term, leading to the effective Lagrangian[4]

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{\epsilon}{2 \cos \theta_W} B_{\mu\nu} Z^{\prime \mu\nu} - \frac{1}{4} Z^{\prime \mu\nu} Z^{\prime \mu\nu},$$  

where $\theta_W$ is Weinberg angle, $Z'$ and $B$ are dark photon and SM $B$ field with the corresponding field strength

$$\dot{B}_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}, \quad \dot{Z}'_{\mu\nu} = \partial_{\mu} Z'_{\nu} - \partial_{\nu} Z'_{\mu}. $$  

(2)

and the mixing of gauge bosons is mimicked by parameter $\epsilon$, which is supposed to be small and need to be determined. The convention above, in gauge interaction state, is not diagonalized. By redefining fields as,

$$\begin{pmatrix} Z' \\ B \end{pmatrix} = \begin{pmatrix} \sqrt{1 - \frac{\epsilon^2}{\cos^2 \theta_W}} & 0 \\ -\frac{\epsilon}{\cos \theta_W} & 1 \end{pmatrix} \begin{pmatrix} Z' \\ B \end{pmatrix}$$  

(3)

the Lagrangian is then rotated to a diagonal form

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\prime \mu\nu}. $$  

(4)
The new interaction brought in by NP will modify parts/all of these SM penguin and box contribution. Within the contributing to FCNC can be classified to three point penguin diagram and four point box diagram. If NP exists, small, (for example, see [4]). Now combine together the two rotations eq. (3) and eq. (6), the modifications to photon dark \( Z \)

changing neutral current (FCNC) processes. In this section, we will investigate these processes of meson physics in dark photon model. It is known in SM FCNC processes are induced at loop level. Conventionally the Feynman diagrams in which the SM electrical current and weak neutral current are other gauge bosons, it has both “\( Z \)-component” and “\( A \)-component” sized by \( \epsilon \) and \( \epsilon_Z \) respectively.

Formally the shift of neutral fields in dark photon model is characterised by two independent parameters \( \epsilon \) and \( \epsilon_Z \), respectively. In fact \( \epsilon_Z \) also has a lengthy analytical expression based on detailed model. The equivalent \( \epsilon_Z \) defined to replace the complicated structure brings the convenience. The dark \( Z \) field here and also hereafter is denoted as \( Z' \), with original SM field denoted with a hat. The two coupling constants together with \( Z' \) mass constitute the unique 3 model parameters of dark \( Z \), which could be measured in experiments.

Apparently the interactions between \( Z' \) and SM particles are simply induced by the shifted neutral gauge field. Explicitly, we show how \( Z' \) couples to SM fermions,

\[
\mathcal{L}_{Z'ff} = - \left( \epsilon e J_{em}^\mu + \epsilon_Z \frac{g}{2 \cos \theta_W} J_{NC}^\mu \right) Z'^\mu
\]

in which the SM electrical current and weak neutral current are

\[
J_{em}^\mu = Q_f \bar{f} \gamma^\mu f,
J_{NC}^\mu = (T_{3f} - 2Q_f \sin^2 \theta_W) \bar{f} \gamma^\mu f - T_{3f} \bar{f} \gamma^\mu \gamma_5 f,
\]

where \( f \) stands for fermions with corresponding electric charge \( Q_f \), isospin \( T_{3f} = \pm \frac{1}{2} \). With both vector coupling and axial-vector coupling, \( Z' \) behaves as a light version of \( Z \) and heavier version of photon. For the coupling of \( Z' \) and other gauge bosons, it has both “\( Z \)-component” and “\( A \)-component” sized by \( \epsilon \) and \( \epsilon_Z \) respectively.

III. THE \( Z' \) EFFECT IN FCNC PROCESSES

As current energy frontier, the LHC brings plentiful opportunities for flavour physics which dominated by flavour changing neutral current (FCNC) processes. In this section, we will investigate these processes of meson physics in dark \( Z \) model. It is known in SM FCNC processes are induced at loop level. Conventionally the Feynman diagrams contributing to FCNC can be classified to three point penguin diagram and four point box diagram. If NP exists, the new interaction brought in by NP will modify parts/all of these SM penguin and box contribution. Within the \( Z' \) model working frame, however, this modification is only applied in photon penguin and \( Z \) penguin, keeping box diagram contribution unchanged. To make the new effect more distinguishable, we extract these modifications alone and name it as \( Z' \)-penguin specifically.
A. $Z'$ penguin

We take $b \to sZ'$ as an example, noting similar result can be applied to $b \to dZ'$ and $s \to dZ'$ when necessary conditions are satisfied.

In Feynman-t’Hooft gauge there are totally 10 Feynman diagrams giving contributions to $b \to sZ'$. During our realistic calculation, we group two of the external leg corrections and replace them by an effective vertex shown in Fig. 1 with $Z'$ inserted in either two legs. The four self-energy diagrams, effectively two, are then shown as (g) in Fig. 2 together with the remaining six ordinary three point diagrams given as (a) to (f).

![Fig. 1. The effective vertex for $b \to s$ transition.](image1)

![Fig. 2. The Feynman diagrams contributing to $b \to sZ'$ in Feynman-t’Hooft gauge. Fig. (a) to (f) are three point diagrams while Fig. (g) is from the correction to external leg where the effective vertex denoted by a cross is explained in Fig. 1.](image2)

As mentioned above the full result can be decomposed into “$A$ component” (or $\epsilon$ component) and “$Z$ component” (or $\epsilon_Z$ component). The $\epsilon$ component is same as $b \to s\gamma$ while the latter one is similar to $b \to sZ$. For photon penguin, there are two types of effective vertices corresponding to real and virtual photon. In below since we focus on semileptonic processes thus only the virtual photon vertex is taken into account. In order to keep the final result the same structure as photon penguin, the $\epsilon_Z$ component differs from SM Z penguin by neglecting dipole term contribution. An exact calculation, shown in appendix A gives the $Z'$ penguin vertex as

$$\pi^{\mu}b_{Z'} = i\lambda_i \frac{G_F}{\sqrt{2}} \frac{\epsilon}{8\pi^2} H_0(x_i) \pi(q^2 m^2 - q^2 \theta)(1 - \gamma_5)b,$$

in which $\lambda_i = V_{ib}V^*_{is}$ ($i = u, c, t$), $x_i = \frac{m^2_{V_i}}{m^2_{W}}$ and $q$ is outgoing momentum carried by gauge boson. The vertex function $H_0(x_i)$ consisting of photon component function $D_0(x_i)$ and newly calculated $Z$ component function $\tilde{D}_0(x_i)$, are characterised by $\epsilon$ and $\epsilon_Z$, giving

$$H_0(x) = \epsilon D_0(x) + \epsilon_Z \tilde{D}_0(x)$$

$$D_0(x) = -\frac{4}{9} \ln x + \frac{-19x^3 - 25x^2}{36(x - 1)^3} + \frac{x^2(5x^2 - 2x - 6)}{18(x - 1)^4} \ln x$$

$$\tilde{D}_0(x) = -\frac{1}{8w^3c^4} \left[ \frac{34x^3 - 141x^2 + 147x - 58}{216(-1)^3} + \frac{(-3x^4 + 18x^3 - 27x^2 + 19x - 4)\ln x}{36(x - 1)^4} \right]$$

$$+ c^3 \left[ \frac{-47x^3 + 237x^2 - 312x + 104}{108(x - 1)^3} + \frac{(3x^4 - 30x^3 + 54x^2 - 32x + 8)\ln x}{18(x - 1)^4} \right],$$

with $s_W = \sin \theta_W$, $c_W = \cos \theta_W$. Note during evaluating, the light down type quark is supposed to be massless and the momentum transfer is small, comparable to light quark mass. We also assume the light $Z'$ mass smaller than the mass threshold for muon pair production, which guarantees no $Z'$ resonance is produced when final state of charged lepton is muon. Also in this work we will not touch electron and neutrino as the lepton final state for the production of $Z'$ when $m_{Z'}$ is above electron threshold. Nevertheless, it is safe to neglect $m_{Z'}$ in this paper.
In the state-of-the-art effective Hamiltonian approach, physical observables can be factorized into short distance (SD) and long distance (LD) contribution. The SD part is treated in perturbative theory while LD hadronic matrix resorts to various methods including lattice QCD. The SD contribution gives various combination of penguin diagrams and leads to he so-called $X, Y, Z$ functions, which also depends on both the theoretical working frame and the calculated physical observable. For example, $Y$ is to characterize the Wilson coefficient in the effective Hamiltonian of $B_q \rightarrow \ell^+\ell^-$, and for a more complicated process $B \rightarrow K^\ast \mu^+\mu^-$ more functions are involved. In the frame of $Z'$ model, we have already discussed a new $Z'$ penguin in previous sector. Before combining detailed phenomenology, to include $Z'$ contribution in standard $X, Y, Z$ functions systematically is now our target.

In SM the $X, Y, Z$ functions are obtained via comibing $bs\gamma$, $bsZ$, box diagram vertex (see [13]), and now we need to contain $bsZ'$ vertex. The amplitudes of $b \rightarrow s\ell (or \bar{s}b \rightarrow \bar{\ell}\ell)$ mediated by different gauge bosons are in the form of

$$iM_{\gamma} = i\lambda_b \frac{G_F}{\sqrt{2} \pi} D_0(x_b) (\bar{s}b) \gamma_{-A}(\bar{\ell}\ell)_V$$

$$iM_{Z} = i\lambda_b \frac{G_F}{\sqrt{2} \pi s_W} 2C_0(x_b) \left[ v_f (\bar{s}b) \gamma_{-A}(\bar{\ell}\ell)_V - a_f (\bar{s}b) \gamma_{-A}(\bar{\ell}\ell)_A \right]$$

$$iM_{Z'} = i\lambda_b \frac{G_F}{\sqrt{2} \pi s_W} 2C_0(x_b) \left[ \frac{4c_W s_W}{4c_W C_0(x_b)} v'_f (\bar{s}b) \gamma_{-A}(\bar{\ell}\ell)_V - \frac{4c_W s_W}{4c_W C_0(x_b)} a'_f (\bar{s}b) \gamma_{-A}(\bar{\ell}\ell)_A \right]$$

with $L = (\ell, \nu)$ and the coupling of $Zff$ and $Z'ff$

$$v_f = T_{3f} - 2Q_f s_W^2, \quad a_f = T_{3f},$$

$$v'_f = \epsilon \cdot 2Q_f s_W c_W + \epsilon_Z (T_{3f} - 2Q_f s_W^2), \quad a'_f = \epsilon_Z T_{3f}.$$  (13)

For convenience $Z$ and $Z'$ contribution can be put together in a compact form by showing exact final state

$$iM_{ZZ'}^\ell = i\lambda_b \frac{G_F}{\sqrt{2} \pi s_W} C_0(x_b) \left[ (-1 + 4s_W^2)(1 + \delta_1) (\bar{s}b) \gamma_{-A}(\bar{\ell}\ell)_V + (1 + \delta_2) (\bar{s}b) \gamma_{-A}(\bar{\ell}\ell)_A \right]$$

$$iM_{ZZ'}^\nu = i\lambda_b \frac{G_F}{\sqrt{2} \pi s_W} C_0(x_b) \left[ (1 + \delta_2) (\bar{s}b) \gamma_{-A}(\bar{\nu}\nu)_V - (1 + \delta_2) (\bar{s}b) \gamma_{-A}(\bar{\nu}\nu)_A \right]$$  (14)

in which we have introduced two parameters

$$\delta_1 = \frac{s_W H_0(x_b)}{4c_W C_0(x_b)} \left[ \frac{4c_W s_W}{1 - 4s_W^2} + \epsilon_Z \right], \quad \delta_2 = \epsilon_Z \frac{s_W H_0(x_b)}{4c_W C_0(x_b)}.$$  (15)

Incorporating box diagram contribution, the Wilson coefficients of operators $Q^\ell_9 = (\bar{s}b) \gamma_{-A}(\bar{\ell}\ell)_V$, $Q^{\ell}_{10} = (\bar{s}b) \gamma_{-A}(\bar{\ell}\ell)_A$ as well as quark-neutrino operator $Q^{\nu}_{10} = (\bar{s}b) \gamma_{-A}(\bar{\nu}\nu)_A$ can be explicitly extracted as

$$C^\ell_9 = \lambda_b \frac{G_F}{\sqrt{2} \pi s_W} \left[ s_W \cdot 4 (Z_0(x_b) + \Delta Z_0(x_b)) - (Y_0(x_b) + \Delta Y_0(x_b)) \right]$$

$$C^{\ell}_{10} = \lambda_b \frac{G_F}{\sqrt{2} \pi s_W} \left[ Y_0(x_b) + \Delta Y_0(x_b) \right]$$

$$C^{\nu}\nu = \lambda_b \frac{G_F}{\sqrt{2} \pi s_W} \left[ X_0(x_b) + \Delta X_0(x_b) \right]$$  (16)

in which the $X, Y, Z$ functions are given in the combination of penguin and box diagrams $X_0(x) = C_0(x) - 4B_0(x), Y_0(x) = C_0(x) - B_0(x), Z_0(x) = C_0(x) + \frac{4}{3} B_0(x)$. The corresponding corrections to $X, Y, Z$ are then given as

$$\Delta Y_0(x) = \delta_1 C_0(x), \quad \Delta Z_0(x) = \delta_1 C_0(x),$$

$$\Delta X_0(x) = \delta_2 C_0(x), \quad \Delta Y_0(x) = \delta_2 C_0(x).$$  (17a)

Especially, we find that the modification to $Y$ has two types due to different Dirac structure of lepton pair in the operators, while modification to $X$ and $Z$ function are in a fixed way. The modifications can be written in a more
explicit form as

\[ \Delta X_0(x) = \frac{1}{4} t_W \epsilon_Z H_0(x) \]  

\[ \Delta Y_A(x) = \frac{1}{4} t_W \epsilon_Z H_0(x) \]  

\[ \Delta Y_V(x) = \left[ \epsilon \frac{s_W^2}{1 - 4s_W^2} + \epsilon_Z \frac{t_W}{4} \right] H_0(x) \]  

\[ \Delta Z_0(x) = \left[ \epsilon \frac{s_W^2}{1 - 4s_W^2} + \epsilon_Z \frac{t_W}{4} \right] H_0(x) \]  

In the limit of \( \epsilon \to 0, \delta_1 = \delta_2 \), the modifications are identical. However, if \( \epsilon_Z \to 0 \) (exactly dark photon model case), leading to \( \Delta X_0 = \Delta Y_A = 0 \), then the phenomenology is much more tedious.

### IV. PHENOMENOLOGY

The physical observables can be classified into two types in dark \( Z \) model. One type relating to box diagram, like the mass difference of neutral meson, is not modified by \( Z' \). The other one involving photon and \( Z \) penguins, such as the direct CP violation in \( K \to \pi \pi \), does change. In this section, we will choose several typical processes to see a generic effect of \( Z' \) on flavour physics.

#### A. \( B_q \to \mu^+ \mu^- \)

It has been hoped for decades that NP might be unfolded in rare decay \( B_s \to \mu^+ \mu^- \). However, no hint of NP appeared in \( B_s \to \mu^+ \mu^- \) mode from LHC Run I data, given by the full combination results of CMS and LHCb \[10\].

\[
\mathcal{B}(B_s \to \mu^+ \mu^-) = (2.8^{+0.7}_{-0.6}) \times 10^{-9} \\
\mathcal{B}(B_d \to \mu^+ \mu^-) = (3.9^{+1.6}_{-1.4}) \times 10^{-10}.
\]  

(19a)  

(19b)

Though \( B_s \to \mu^+ \mu^- \) turns out to be SM-like, there remains a hope for NP in the much rarer mode \( B_d \to \mu^+ \mu^- \) (for example, see \[17\]).

Due to the precise measurement of decay \( B_s \to \ell^+ \ell^- \) is now realistic, one should consider the effect of sizeable width difference \( \Delta \Gamma_s \) in \( B_s^0-\bar{B}_s^0 \) oscillation. The theoretical formula has to be corrected to compare with measured branching ratio \[18, 19\] which is denoted with a bar,

\[
\mathcal{B}(B_s^0 \to \ell^+ \ell^-) = \left[ 1 - \frac{y_s^2}{1 + A_{\Delta \Gamma}^{\ell^+ \ell^-} y_s} \right] \overline{\mathcal{B}}(B_s^0 \to \ell^+ \ell^-) 
\]  

(20)

where \( y_s = \frac{\Delta \Gamma}{2 \Gamma_s} = \frac{\Gamma_s^{(\ell^+ \ell^-)}}{2 \Gamma_s}, \ A_{\Delta \Gamma}^{\ell^+ \ell^-} = \frac{R_s^{\ell^+ \ell^-} - R_s^{\ell^+ \ell^-}}{R_s^{\ell^+ \ell^-} + R_s^{\ell^+ \ell^-}}. \) It is known \( A_{\Delta \Gamma}^{\ell^+ \ell^-} = 1 \) in SM \[19\], thus

\[
\mathcal{B}(B_s^0 \to \ell^+ \ell^-) = (1 - y_s^2) \overline{\mathcal{B}}(B_s^0 \to \ell^+ \ell^-). 
\]  

(21)

The (uncorrected) SM branching ratio of \( B_q \to \ell^+ \ell^- \) is induced by \( Z \) penguin and hence depends on \( Q_{10} \), (see ref. \[23\]). Now incorporating \( Z' \)-penguin, which gives a similar component as \( Z \), leads to

\[
\mathcal{B}(B_q \to \ell^+ \ell^-) = \tau(B_q) \frac{G_F^2}{4 \pi} \left( \frac{\alpha}{4 \pi s_{W}^2} \right)^2 f_B^2 m_B^2 m_{\ell} \left[ 1 - \frac{4m_B^2}{m_B^2} \eta_{\text{eff}} \right] \left| \lambda_i \left( Y_0(x_i) + \Delta Y_A(x_i) \right) \right|^2
\]  

(22)

with \( \eta_{\text{eff}} = 0.9882 \pm 0.0024 \) which takes into account NNLO QCD correction and NLO electroweak correction \[22\]. Apparently the reason why only \( \Delta Y_A \) contributes, is exact with the same reason why photon penguin contribution vanishes.
The branching ratio from SD (see ref. [27]) to SD is based on QCDF. It is not necessary to repeat the whole long story in this work. Instead, we would like to simply focus on C9 and C10 to see how the data constrain the NP parameter space. In dark Z model, the modification of C9 and C10 due to the dark Z contribution is

\[ \Delta C_9(x_i) = 4\Delta Z_0(x_i) - \frac{\Delta Y_V(x_i)}{s_W^2} \]  
\[ \Delta C_{10}(x_i) = \frac{\Delta Y_A(x_i)}{s_W^2} \]  

Driven by new data many efforts have been made to extract the information hidden inside the two coefficients. In [24] a model-independent fit was taken based on \( B \to X_s \ell^+ \ell^- , B \to X_s \gamma , B \to K^{*} \gamma \) and \( B \to K^* \mu^+ \mu^- \), we will adopt their constraints on \( \Delta C_9 \) and \( \Delta C_{10} \) at 2\( \sigma \) [25]

\[ -1.5 < \text{Re}(\Delta C_9) < 1.2, \quad -2.8 < \text{Im}(\Delta C_9) < 2.8 \]  
\[ -1 < \text{Re}(\Delta C_{10}) < 1.5, \quad -3 < \text{Im}(\Delta C_{10}) < 3. \]

In our scenario, the NP only exists in the change of real part of \( C_9/10 \).

\[ B(K_L \to \mu \bar{\mu}) \]

The branching ratio of \( K_L \to \mu \bar{\mu} \) contains LD and SD contribution. The calculation of LD contribution remains a challenge in theory. Combining a latest theoretical LD estimation and experimental bound [26] [25], the constraint to SD is

\[ B(K_L \to \mu^+ \mu^-)_{SD} \leq 2.5 \times 10^{-9} \]  

The branching ratio from SD (see ref. [27])

\[ B(K_L \to \mu^+ \mu^-)_{SD} = \kappa_\mu \left[ \frac{\text{Re}(\lambda_c)}{|V_{us}|} P_c(Y_K) + \frac{\text{Re}(\lambda_t)}{|V_{us}|} \eta_Y Y_0(x_t) \right]^2 \]

with \( \lambda_q = V^*_{qs}V_{qt}(q = c, t) \) and \( \kappa_\mu = (2.009 \pm 0.017) \times 10^{-9} \left( \frac{|V_{us}|}{0.225} \right)^8, P_c(Y_K) = (0.115 \pm 0.018) \left( \frac{0.225}{|V_{us}|} \right)^8 \) [28], QCD correction factor \( \eta_Y = 1.012 \) [29], now is modified as

\[ B(K_L \to \mu^+ \mu^-)_{SD} = \kappa_\mu \left[ \frac{\text{Re}(\lambda_c)}{|V_{us}|} P_c(Y_K) + \frac{\text{Re}(\lambda_t)}{|V_{us}|} \eta_Y \left( Y_0(x_t) + \Delta Y_A(x_t) \right) \right]^2, \]  

by including the dark Z contribution.

\[ \varepsilon' / \varepsilon \]

Historically two approaches, operator production expansion (OPE) method and penguin-box expansion (PBE) method, are adopted for the study of direct CP violation in \( K \to \pi \pi \), which involves all the QCD penguin and electroweak penguin in SM. For the phenomenology study here, we make use of the simple analytical formula based on the PBE method [30]. By modifying relevant parts due to the dark Z effect, an updated formula [27] to depicted \( \varepsilon' / \varepsilon \) is

\[ \text{Re} \varepsilon' \varepsilon = a \text{Im}(\lambda_t) \cdot F(x_t) \]  

(27)
FIG. 3. The allowed $\epsilon - \epsilon_Z$ parameter space by various experiments if $m_{Z'}$ is less than $2m_\mu$ and hence ignored. The meaning of colours are given as: grey stands for $K_L \to \mu^+\mu^-$, orange represents $B_s \to \mu^+\mu^-$, green denotes $\Delta C_{10}$, blue stands for $\Delta C_9$ while the yellow is one example of $\epsilon'/\epsilon$ with $(R_6, R_8) = (0.7, 1.1)$ and the black dot remarks SM case.

where $F(x)$ is given by

$$F(x) = P_0 + P_X [X_0(x) + \Delta X(x)] + P_Y [Y_0(x) + \Delta Y(x)] + P_Z [Z_0(x) + \Delta Z] + P_E E_0(x)$$  \hspace{1cm} (28)

and the factor $a = 0.92 \pm 0.03$ \cite{31}, which takes into account the correction due to $\Delta I = 5/2$ transitions \cite{32}. Note the dark $Z$ modifies most parts of SM $F$ function but keep the gluon penguin vertex $E(x)$ unchanged. The coefficients $P_i (i = 0, X, Y, Z, E)$ are given in terms of $R_6, R_8$

$$P_i = r_0^{(i)} + r_6^{(6)} R_6 + r_8^{(8)} R_8.$$  \hspace{1cm} (29)

We adopt their numerical values for $\alpha_s(M_Z) = 0.1185$ \cite{33} given in Table 1 of Ref. \cite{31}. For the nonperturbative parameters, we adopt the value $R_8 = 0.6, R_6 = 1.1$. The former one is obtained from lattice \cite{34}, with the translation by Ref. \cite{31}. But a reliable lattice result for $R_6$ is still lack, here we choose 10% deviation from large $N$ result. The experimental value with 1σ error for $\epsilon'/\epsilon$ is

$$\frac{\epsilon'}{\epsilon} \simeq \text{Re} \left( \frac{\epsilon'}{\epsilon} \right) = (1.66 \pm 0.23) \times 10^{-3},$$  \hspace{1cm} (30)

taken form PDG \cite{32}.

V. RESULTS AND DISCUSSION

There are limited three free parameters in dark $Z$ model, $m_{Z'}, \epsilon$ and $\epsilon_Z$. In our working scenario, the light $Z'$ mass is ignored thus it might be promising to determine the remaining two by above observables. We take the global fit of Wolfenstein parametrization of CKM matrix as input since $Z'$ does not change SM flavour structure. The other related input parameters have been given herebefore. Combining $B_s \to \mu^+\mu^-$, $K_L \to \mu^+\mu^-$ and $\Delta C_9, \Delta C_{10}$ from a global fit of $B \to K^{(*)}\mu^+\mu^-$ and so on, we plot allowed parameter space in $(\epsilon, \epsilon_Z)$ plane shown in Fig.4.

The ranges of $\epsilon$ and $\epsilon_Z$ are both shown in the section $(-10, 10)$. Due to its large uncertainty SD of $K_L \to \mu^+\mu^-$, as presented in grey region, gives a pretty wide band with $\frac{3}{5}$ slope. The green band with the same slope and less width of grey one, together with the gap out of this band surrounded by a set of hyperbolic curve, is resulted from $\Delta C_{10}$. Another parameter obtained from $B \to K^{(*)}\mu^+\mu^-$, namely, $\Delta C_9$, constrains parameters in blue colour, which is restricted in two sets of hyperbolic curves and extended to slope $\frac{3}{5}$ direction as well as near vertical direction. Apparently the latter part is excluded by $K_L \to \mu^+\mu^-$ (or $C_{10}$) while the former one is embedding in green area. Remarkably in orange, the important $B_s \to \mu^+\mu^-$ also embeds in $\Delta C_{10}$ but somehow has an overlap part with $\Delta C_9$. In principle, $B_d \to \mu^+\mu^-$ can also be included. Considering its uncertainty is larger than $B_s \to \mu^+\mu^-$, the allowed region is then also wider than current orange area thus we do not show it. Here we also add in a typical constraint
from $\epsilon'/\epsilon$ by fixing its non-perturbative parameter $R_6$ and $R_8$. Showing as two narrow hyperbolic curves in yellow, it looks $\epsilon'/\epsilon$ could give a very strict constraint. However, due to the large uncertainty of the two non-perturbative parameters, especially $R_6$, one cannot take the direct CP violation of kaon too seriously. Nevertheless, once $R_6$ is fixed within certain precisement in the future, $\epsilon'/\epsilon$ can be taken as an important discrimination to further constrain the parameter.

Aiming at the determination of parameter space of dark $Z$ model, we reach the allowed $(\epsilon, \epsilon_Z)$ in the narrow linear region

$$\epsilon_Z = \frac{3}{5} \epsilon \tag{31}$$

and especially $-2 < \epsilon < 2$, $-1 < \epsilon_Z < 1$ are favored. In other words, the order of mixing parameters is constrained to be $O(1)$. Generally speaking, more FCNC processes can be considered to determine the bound, but we can believe $O(1)$ should be the typical value from flavour physics.

Many other works have already put constraints on dark photon model. Though it is not exact same model as our working scenario, the obtained mixing angle in dark photon still enlightens parameters in dark $Z$ model. For example, based on supernova 1987A the limit on mixing angle could be $O(10^{-12.5})$ for $m_{Z'} > 2m_e$ [7]. A recent work from direct detection experiments such as XENON10 and XENON100, for the absence of an ionization signal, puts a even more stringent limit on $\epsilon$ down to $O(10^{-15})$. One may expect if $\epsilon_Z$ is added, the bound of $\epsilon$ will also be changed from astrophysical observables and direct detection experiments. Meanwhile, the working scenario in flavour physics can also be modified to allow $Z'$ resonance production [4], leading to a possible different constraint on mixing parameters. In any case, given the bound from flavour physics in current scenario $O(1)$, one may not expect a dramatical change (a more than 10 orders of magnitude) happens unnaturally.

In the existence of dark $Z$, we have investigated its effect in flavour physics especially by connecting the effect to the newly measured processes at LHC. However, the obtained bound, $O(1)$, may not compete with the corresponding one in traditional dark matter study.

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Appendix A: The calculation of $Z'$ penguin

The effective $bsZ'$ vertex is in the form of

$$\Gamma_\mu(k_1, k_2) = \epsilon \lambda_i \frac{g_2^2}{2(4\pi)^2 m_W^2} \left[ F_1 (q_\mu q^2 \gamma_\mu) \mathbb{P}_L + F_2 i \sigma_{\mu\nu} q^\nu m_b \mathbb{P}_R \right], \tag{A1}$$

which satisfies Ward Identity approximately in the limit of $m_{Z'} \to 0$. In the phenomenology study of this work, we only focus on $F_1$ contribution, which has the relation with $H_0$ in section III.A

$$H_0(x) = -2F_1(x). \tag{A2}$$

For convenience, in following calculation the effective vertex $\Gamma_\mu$ can be rewritten as

$$\Gamma^\mu(k_1, k_2) = \epsilon \lambda_i \frac{g_2^2}{2(4\pi)^2 m_W^2} \left[ c_b T^\mu_L + c_s T^\mu_b + c_s T^\mu_s \right] \tag{A3}$$

by introducing $c_b = F_1 + F_2, c_s = F_2 - F_1, c_L = -F_2$ and $T^\mu_L = m_b^2 \gamma^\mu \mathbb{P}_L, T^\mu_b = \bar{k}_1 k^\mu \mathbb{P}_L, T^\mu_s = \bar{k}_1 k^\mu \mathbb{P}_L$. Particularly we have

$$F_1 = \frac{1}{2} (c_b - c_s). \tag{A4}$$

In the calculation, the $F_1$ function may be projected to

$$F_1 = \epsilon F_1|_\gamma + \epsilon_Z F_1|_Z \tag{A5}$$

Following we will evaluate $F_1$ component by component.
1. The $\epsilon$ component

The contribution of $\epsilon$ component is exactly same as $b \to s \gamma$ case. Here we recalculate the photon penguin contribution. In Feynman-t’Hooft gauge, with the approximation $m_s = 0, q^2 = 0$ and making use of quark on-shell condition as well as unitary CKM relation, we have the total contribution as

$$i\Gamma^\mu|_\gamma = \frac{eg_\mu}{2(4\pi)^2} \sum_j M_j^\mu|_\gamma$$

(A6)

with the individual amplitude from each diagram

$$M_\alpha|_\gamma = -\frac{4}{3} \left\{ \left( -2C_{00} + m_3^2 C_0 - m_3^2 (C_0 + 2C_1 + C_2 + C_{11} + C_{12}) \right) T_L + 2(C_{11} + C_1) T_b + 2(C_0 + C_1 + C_2 + C_{12}) T_s \right\}$$

(A7)

$$M_\beta|_\gamma = -\frac{2m_3^2}{3m_W^2} \left\{ \left( -2C_{00} + \frac{1}{2} m_3^2 C_0 - m_3^2 (C_{11} + C_1 + C_{12} + C_0) \right) T_L + 2C_1 T_b + 2(C_{12} - C_2) T_s \right\}$$

$$M_\alpha|_\gamma = -\left[ 12C_{00} + m_3^2 (3C_1 + 2C_2 + 2C_{11} + 2C_{12}) \right] T_L + (4C_{11} + 2C_1) T_b + (4C_{21} - 2C_1 - 2C_2) T_s$$

$$M_\alpha|_\gamma = -\frac{m_3^2 C_0 T_L}{m_W^2}$$

in which $B_i, C_j$ ($i = 0, 1, j = 0, 1, 2, 11, 12, 21, 22$) are Passarino-Veltman integrals, and the positions for variables are assigned as $B(m_3^2, m_W^2, m_2^2), C_{a,b}(m_3^2, 0, 0, m_W^2, m_2^2, m_2^2)$ for $a, b$ case, and $C_{c,d,e,f}(m_3^2, 0, 0, m_W^2, m_2^2, m_2^2)$ for $c, d, e, f$ case. Note in the calculation, the light down type quark contribution in Goldstone-quark-quark vertex cannot be neglected.

The PV functions can be reduced into basic scalar function $B_0, C_0$, and we further perform Taylor expansion up to $m_3^4$. Sum up all the contribution together, we have

$$i\Gamma^\mu|_\gamma = \frac{eg_\mu}{2(4\pi)^2} \sum_j M_j^\mu|_\gamma$$

(A8a)

$$c_L|_\gamma = \frac{x_t^2 (2 - 3x_t \ln x_t)}{2(x_t - 1)^4} \left\{ \frac{1}{3} - 2\Delta \right\} \frac{T_L}{m_b^2} + c_L T_L + c_b T_b + c_s T_s$$

(A8b)

$$c_b|_\gamma = \frac{(3x_t^4 - 3x_t^3 + 36x_t^2 - 32x_t + 8) \ln x_t}{18(x_t - 1)^4} + \frac{19x_t^3 - 222x_t^2 + 165x_t - 34}{108(x_t - 1)^3}$$

(A8c)

$$c_s|_\gamma = \frac{(-3x_t^4 + 57x_t^3 - 72x_t^2 + 32x_t - 8) \ln x_t}{18(x_t - 1)^4} + \frac{113x_t^3 - 696x_t^2 + 789x_t - 242}{108(x_t - 1)^3}$$

(A8d)

The first term in eq. (A8a) gives zero contribution after applying the unitary triangle relation $\sum \lambda_i \cdot \text{constant} = 0$. Also we have

$$c_b|_\gamma - c_s|_\gamma = \frac{(3x_t^4 - 30x_t^3 + 54x_t^2 - 32x_t + 8) \ln x_t}{9(x_t - 1)^4} + \frac{-47x_t^3 + 237x_t^2 - 312x_t + 104}{54(x_t - 1)^3}$$

$$-2 \left( D_0(x_t) + \frac{26}{27} \right)$$

(A9)

in which $D_0(x_t)$ is vertex function of virtual photon in $b \to s \gamma$. Note in my calculation, the $D_0, D'_0$ differs from the one in [13], up to a minus sign, for the different convention in QED vertex.
2. The \( \epsilon_2 \) component

The total contribution for \( b \rightarrow sZ \) is

\[
i\Gamma_{\mu}\bigg|_Z = i \frac{g_3^2}{2(4\pi)^2} \frac{g_2}{c_W} \lambda_i \sum_j \Gamma_{j\mu}\bigg|_Z
\]

(A11)

Compared with \( b \rightarrow s\gamma \) case, we only need to recalculate Fig.(a) and Fig.(b),

\[
\Gamma_{a\mu}\bigg|_Z = \frac{2}{3} s_W^2 \cdot 2 \left\{ -2C_{00} + m_t^2 C_0 - m_t^2 (C_{11} + C_{12} + 2C_1 + C_2 + C_0) \right\} T_b^\mu
\]

\[
+ 2(C_{11} + C_1) T_b^\mu + 2(C_{12} + C_2 + C_1 + C_0) T_s^\mu \right\}
\]

\[
- \frac{1}{2} \cdot 2 \left\{ -2C_{00} - m_t^2 (C_{11} + C_{12} + 2C_1 + C_2 + C_0) \right\} T_L^\mu
\]

\[
+ 2(C_{11} + C_1) T_b^\mu + 2(C_{12} + C_2 + C_1 + C_0) T_s^\mu \right\}
\]

(A12a)

\[
\Gamma_{b\mu}\bigg|_Z = \frac{2}{3} s_W^2 \cdot \frac{m_t^2}{m_W} \left\{ -2C_{00} + \frac{1}{2} + m_t^2 C_0 - m_t^2 (C_{11} + C_{12} + C_1 + C_0) \right\} T_L^\mu
\]

\[
+ 2C_{11} T_b^\mu + 2(C_{12} - C_2) T_s^\mu \right\}
\]

(A12b)

while the other contributions are obtained by the following replacement

Fig.(c) : \( e \rightarrow g_2 c_W \)

Fig.(d) : \( e \rightarrow g_2 \frac{1 - 2s_W^2}{2c_W} \)

Fig.(e), (f) : \( e \rightarrow \frac{g_2 s_W^2}{c_W} \)

Fig.(g) : \( -Q_b \rightarrow -\left( \frac{1}{2} + \frac{1}{3} s_W^2 \right) \)

(A13)

with the same PV function convention as in \( b \rightarrow s\gamma \) case.

In this work, since we only consider the \( (g_\mu \tilde{q}^2 - q^2 \gamma_\mu) \) term, thus only the coefficients of \( T_b^\mu \) and \( T_s^\mu \) are of interest. Especially they are listed individually,

\[
\left[ \frac{-5x_t^2 + 22x_t - 5}{18(x_t - 1)^3} \right] + \frac{(1 - 3x_t) \ln x_t}{3(x_t - 1)^4} + \frac{s_W^2}{s_W} \left( \frac{2(5x_t^2 - 22x_t + 5)}{27(x_t - 1)^3} + \frac{4(3x_t - 1) \ln x_t}{9(x_t - 1)^4} \right) \right] T_b^\mu
\]

\[ + \left[ \frac{20x_t^2 - 7x_t - 7}{18(x_t - 1)^3} \right] - \frac{(6x_t^2 - 6x_t + 1) \ln x_t}{3(x_t - 1)^4} \right] T_s^\mu
\]

\[ + s_W^2 \left( \frac{2(20x_t^2 - 7x_t - 7)}{27(x_t - 1)^3} \right) + \frac{(6x_t^2 - 6x_t + 1) \ln x_t}{9(x_t - 1)^4} \right] T_s^\mu
\]

\[
\left[ \frac{-17x_t^2 - 8x_t + 1}{18(x_t - 1)^3} \right] + \frac{x_t(x_t - 3)}{4(x_t - 1)^2} + \frac{x_t \ln x_t}{9(x_t - 1)^4} \right] T_b^\mu
\]

\[ + s_W^2 \left( \frac{x_t(11x_t^2 - 43x_t + 38)}{27(x_t - 1)^3} - \frac{2(3x_t - 4) \ln x_t}{9(x_t - 1)^4} \right] T_s^\mu
\]

\[
\left[ \frac{-38x_t^2 + 43x_t - 11}{9(x_t - 1)^3} \right] + \frac{2x_t^2(4x_t - 3) \ln x_t}{3(x_t - 1)^4} \right] T_b^\mu
\]

\[ + s_W^2 \left( \frac{-2x_t^2(4x_t - 3) \ln x_t}{3(x_t - 1)^4} + \frac{38x_t^3 - 43x_t + 11}{9(x_t - 1)^4} \right] T_s^\mu
\]
Sum up together, we have

\[
\begin{align*}
| c_b |_Z = & \left[ \frac{-35x_t^3 + 156x_t^2 - 213x_t + 20}{216(x_t - 1)^3} + \frac{(-3x_t^4 + 9x_t^3 - 18x_t^2 + 28x_t - 4) \ln x_t}{36(x_t - 1)^4} \right] \\
+ & \frac{x_t^3}{36(x_t - 1)^3} W^2 \left( \frac{19x_t^3 - 222x_t^2 + 165x_t - 34}{108(x_t - 1)^3} + \frac{(3x_t^4 - 3x_t^3 + 36x_t^2 - 32x_t + 8) \ln x_t}{18(x_t - 1)^4} \right)
\end{align*}
\]

(A16)

\[
\begin{align*}
| c_s |_Z = & \left[ \frac{-103x_t^3 + 438x_t^2 - 507x_t + 136}{216(x_t - 1)^3} + \frac{(3x_t^4 - 27x_t^3 + 36x_t^2 - 10x_t + 4) \ln x_t}{36(x_t - 1)^4} \right] \\
+ & \frac{x_t^3}{36(x_t - 1)^3} W^2 \left( \frac{113x_t^3 - 696x_t^2 + 780x_t - 242}{108(x_t - 1)^3} + \frac{(-3x_t^4 + 57x_t^3 - 72x_t^2 + 32x_t - 8) \ln x_t}{18(x_t - 1)^4} \right)
\end{align*}
\]

(A17)

Now we obtain the \( \hat{D}_0 \) function as

\[
\begin{align*}
| c_b |_Z - | c_s |_Z = & \left[ \frac{34x_t^4 - 141x_t^3 + 147x_t^2 - 58}{216(x_t - 1)^3} + \frac{(-3x_t^4 + 18x_t^3 - 27x_t^2 + 19x_t - 4) \ln x_t}{36(x_t - 1)^4} \right] \\
+ & \frac{x_t^3}{36(x_t - 1)^3} W^2 \left( \frac{-47x_t^4 + 237x_t^3 - 312x_t + 104}{108(x_t - 1)^3} + \frac{(3x_t^4 - 30x_t^3 + 54x_t^2 - 32x_t + 8) \ln x_t}{18(x_t - 1)^4} \right).
\end{align*}
\]

(A18)

Take into account \( D_0 \) and \( \hat{D}_0 \) together, the vertex function for \( Z' \) penguin \( H_0 \) is produced.

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