Exclusive production of vector mesons $\rho$, $\rho'$ and $\rho''$ by real and virtual photons

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We present a non-perturbative QCD calculation of high-energy diffractive photo- and leptoproduction of vector mesons $\rho$, $\rho'$ and $\rho''$ on a nucleon. The initial photon splits up in a $q\bar{q}$-dipole and transforms into a vector meson by scattering on the quark-diquark nucleon. The dipole-dipole scattering amplitude is provided by the non-perturbative model of the stochastic QCD vacuum, the wave functions result from considerations on the light-cone. We assume the physical $\rho'$- and $\rho''$-states to be mixed states of an active $2S$-excitation and a rest whose coupling to the photon is suppressed. We obtain good agreement with the experimental data and get an understanding of the markedly different spectrum in the $\pi^+\pi^-$-invariant mass for photoproduction and $e^+e^-$-annihilation.

1. Introduction

Diffractive scattering processes are characterized by small momentum transfer, $t \lesssim 1$ GeV$^2$, and thus governed by non-perturbative QCD. To get more insight in the physics at work we investigate exclusive vector meson production by real and virtual photons. In this note we summarize recent results from Ref. [1] on $\rho$-, $\rho'$- and $\rho''$-production. In Refs [1,2] we have developed a framework which we here can only flash.

We consider high-energy diffractive collision of a photon which dissociates into a $q\bar{q}$-dipole and transforms into a vector meson with a proton in the quark-diquark picture which remains intact. The scattering $T$-amplitude can be written as an integral of the dipole-dipole amplitude and the corresponding wave functions. Integrating out the proton side, we have

$$T^\lambda_V(s,t) = i s \int \frac{dzd^2r}{4\pi} \psi^\dagger_V(\lambda) \psi(\gamma(Q^2,\lambda))(z,r) \times J_p(z,r,\Delta T),$$  \hspace{1cm} (1)

where $V(\lambda)$ stands for the final vector meson and $\gamma(Q^2,\lambda)$ for the initial photon with definite helicities $\lambda$ (and virtuality $Q^2$); $z$ is the light-cone momentum fraction of the quark, $r$ the transverse extension of the $q\bar{q}$-dipole. The function $J_p(z,r,\Delta T)$ is the interaction amplitude for a dipole $\{z,r\}$ scattering on a proton with fixed momentum transfer $t = -\Delta^2_T$; for $\Delta T = 0$ due to the optical theorem it is the corresponding total cross section (see below Eq. (3)). It is calculated within non-perturbative QCD: In the high-energy limit Nachtmann [3] derived a non-perturbative formula for dipole-dipole scattering whose basic entity is the vacuum expectation value of two lightlike Wilson loops. This gets evaluated in the model of the stochastic QCD vacuum [4].

2. The model of the stochastic vacuum

Coming from the functional integral approach the model of the stochastic QCD vacuum [3] assumes that the non-perturbative part of the gauge field measure, i.e. long-range gluon fluctuations that are associated with a non-trivial vacuum structure of QCD, can be approximated by a stochastic process in the gluon field strengths with convergent cumulant expansion. Further assuming this process to be gaussian one arrives at a description through the second cumulant $\langle g^2 F_{\mu\nu}^A(x;x_0) F_{\rho\sigma}^A(x';x_0) \rangle$ which has two Lorentz
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The underlying mechanism of (interacting) gluonic strings also shows up in the scattering of two fining. Whereas the $D$-structure large contributions show up also from between the endpoints. This is to be interpreted as interaction with the gluonic string between the quark and antiquark.

3. Physical states $\rho$, $\rho'$ and $\rho''$

Analyzing the $\pi^+\pi^-$-invariant mass spectra for photoproduction and $e^+e^-$-annihilation Donnachie and Mirzaie [8] concluded evidence for two resonances in the 1.6 GeV region whose masses are compatible with the $1^{--}$ states $\rho(1450)$ and $\rho(1700)$, cf. Fig. [3]. We make as simplest ansatz

\[ |\rho(770)\rangle = |1S\rangle, \]
\[ |\rho(1450)\rangle = \cos \theta \ |2S\rangle + \sin \theta \ |rest\rangle, \]
\[ |\rho(1700)\rangle = -\sin \theta \ |2S\rangle + \cos \theta \ |rest\rangle, \]

(2)

where $|rest\rangle$ is considered to have $|2D\rangle$- and hybrid component whose couplings to the photon both are suppressed, see Ref. [6,8]. With our convention of the wave functions the relative signs $\{+, -, +\}$ of the production amplitudes of the $\rho$, $\rho'$- and $\rho''$-states in $e^+e^-$-annihilation determine the mixing angle as $\theta = 41.2^\circ$. With this value and the branching ratios in $\pi^+\pi^-$-annihilation determined by small dipole sizes. With this value and the branching ratios in $\pi^+\pi^-$-annihilation determined by small dipole sizes.

![Figure 1. Interaction amplitude (arbitrary units) of two colour dipoles as function of their impact (units of correlation lengths $\alpha$). One large $q\bar{q}$-dipole of extension $12\alpha$ is fixed, the second small one of extension $1\alpha$ is, averaged over all its orientations, shifted along on top of the first one. For the $D_1$-tensor structure of the correlator there are only contributions when the endpoints are close to each other, whereas for the $D$-structure large contributions show up also from between the endpoints. This is to be interpreted as interaction with the gluonic string between the quark and antiquark.](image)

![Figure 2. Mass spectra of $e^+e^-$-annihilation on the proton: The interference in the 1.6 GeV region is destructive and constructive, respectively, and shows specific amplitude signs patterns.](image)
4. Light-cone wave functions

In the high-energy limit the photon can be identified as its highest Fock $q\bar{q}$-component. The vector meson wave function arises by distributing this $q\bar{q}$-dipole $\{z, r\}$.

**Photon.** With mean of light-cone perturbation theory (LCPT) we get explicit expressions for both longitudinal and transverse photons. The photon transverse size which we will see to determine the $T$-amplitude is governed by the product $\varepsilon r$, $\varepsilon = \sqrt{z^2 Q^2 + m^2}$. For high $Q^2$ longitudinal photons dominate by a power of $Q^2$; their $z$-endpoints being explicitly suppressed, LCPT is thus applicable. For moderate $Q^2$ also transverse photons contribute which have large extensions because endpoints are not suppressed. For $Q^2$ smaller than 1 GeV$^2$ LCPT definitively breaks down. However, it was shown $^9$ that a quark mass phenomenologically interpolating between a zero valence and a 220 MeV constituent mass astonishing well mimics chiral symmetry breaking and confinement. Our wave function is thus given by LCPT with such a quark mass $m(Q^2)$, for details cf. Refs $^1,2$.

**Vector mesons.** The vector mesons wave functions of the 1$S$- and 2$S$-states are modelled according to the photon. We only replace the photon energy denominator $(\varepsilon^2 + k^2)^{-1}$ by a function of $z$ and $k$ for which ansätze according to Wirbel and Stech $^4$ are made; for the "radial" excitation we account by both a polynomial in $z^2$ and the 2$S$-polynomial in $k^2$ of the transverse harmonic oscillator. The parameters are fixed by the demands that the 1$S$-state reproduces $M_p$ and $f_p$ and the 2$S$-state is both normalized and orthogonal on the 1$S$-state. Details again in Ref. $^1$.

5. Results

Before presenting our results we remark that all calculated quantities are absolut predictions. The cross sections are constant with total energy $s$ due to the eikonal approximation applied and refer to $\sqrt{s} = 20$ GeV where the proton radius as the third parameter of the model of the stochastic QCD vacuum is fixed; its two other parameters, the gluon condensate $\langle g^2FF \rangle$ and the correlation length $a$, are determined by matching low-energy and lattice results, cf. Ref. $^1$.

In Fig. 3 we display both the functions

\[ J_p^{(0)}(z, r) := \int_0^{2\pi} \frac{d\varphi}{2\pi} J_p(z, r, \Delta_T = 0) \] (3)

and

\[ r\psi_{\gamma}(z, r) \] (4)

which together, according to Eq. $^1$, determine the leptoproduction amplitude. The curve for photoproduction of the transverse state strikingly
shows how the outer positive region of the wave functions effective overlap $r\psi_\lambda^\dagger V_{\psi}(Q^2,\lambda)(r)$ wins over the inner negative part due to the strong rise with $r$ of the dipole-proton interaction amplitude $J_p^{(0)}$. This rise itself is a consequence of the string interaction mechanism discussed above. Note, that to the cross section dipole sizes up to 2.5 fm contribute significantly.

In Fig. 3 we display our predictions for differential cross sections as a function of $-t$, the invariant momentum transfer squared. The curves follow roughly exponential behaviour with a slight upward curvature at larger values of $-t$. Due to the nodes in the wave functions for the $2S$-states in addition dips occur due to cancellations.

Our results for integrated elastic cross sections as functions of $Q^2$ are given in Fig. 4. For the $\rho$-meson our prediction is about 20−30% below the E665-data [11]. However, we agree with the NMC-experiment [12] which measures some definite superposition of longitudinal and transverse polarization, see Table 3 in Ref. [1]. For the $2S$-state we predict a marked structure due to the nodes of the wave function whose explicit shape, however, strongly depends on the parametrization of the wave functions.

In Fig. 5 we display the ratio $R_{LT}(Q^2)$ of longitudinal to transverse cross sections and find good agreement with experimental data for the $\rho$-state. For the $2S$-state we again predict a marked structure which is very sensitive to the node positions in the wave functions.

Finally, in Fig. 6 we confront with experimental data our calculation of the ratio $R_\pi(Q^2)$ of $2\pi^+2\pi^-$-production via $\rho'$ and $\rho''$ to $\pi^+\pi^-$-production via $\rho$ (for explicit definition see Ref. [1]).
\( \frac{d\sigma}{dt} [\mu b \times \text{GeV}^{-2}] \)

\( Q^2 = 1/4 \text{ GeV}^2 \)
\( Q^2 = 2 \text{ GeV}^2 \)
\( Q^2 = 10 \text{ GeV}^2 \)
\( Q^2 = 20 \text{ GeV}^2 \)

\( -t [\text{GeV}^2] \)

Figure 7. Differential cross sections as a function of \( -t \) for the \( \rho \)-meson and the 2S-state (upper and lower plots) for both longitudinal and transverse polarization (left and right). The curves with increasing dash sizes refer to \( Q^2 = 0, 1/4, 2, 10, 20 \text{ GeV} \). For the 2S-state the node in the wave function has a strong influence on the \( t \)-dependence.

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