MOCK OBSERVATIONS OF BLUE STRAGGLERS IN GLOBULAR CLUSTER MODELS

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ABSTRACT

We created artificial color-magnitude diagrams of Monte Carlo dynamical models of globular clusters, and then used observational methods to determine the number of blue stragglers in those clusters. We compared these blue stragglers to various cluster properties, mimicking work that has been done for blue stragglers in Milky Way globular clusters to determine the dominant formation mechanism(s) of this unusual stellar population. We find that a mass-based prescription for selecting blue stragglers will choose approximately twice as many blue stragglers than a selection criterion that was developed for observations of real clusters. However, the two numbers of blue stragglers are well-correlated, so either selection criterion can be used to characterize the blue straggler population of a cluster.

We confirm previous results that the simplified prescription for the evolution of a collision or merger product in the BSE code overestimates the lifetime of collision products. Because our observationally-motivated selection criterion does not include the brightest collision products, we show that our model blue stragglers follow the same trends with cluster properties (core mass, binary fraction, total mass, collision rate) as the true Milky Way blue stragglers. The total number of blue stragglers in globular clusters is determined mainly by the properties and numbers of binary stars, and not the collision rate, even though the blue stragglers are formed through binary-mediated collisions.

Keywords: globular clusters: general – stellar dynamics – blue stragglers – binary stars

1. INTRODUCTION

Blue stragglers are main sequence stars that are brighter and bluer than the main sequence turnoff in their environment. They are found in open and globular clusters (de Marchi et al. 2006; Moretti, de Angeli, & Piotto 2008), in dwarf galaxies (Mapelli et al. 2007), and even in the field (Preston & Sneden 2000). Since most of these environments do not contain enough gas to support a current or recent burst of star formation, the expectation is that blue stragglers are formed through some interaction which adds mass to a normal main sequence stars. The two dominant formation mechanisms are expected to be stellar collisions and binary mass transfer. In dynamically active environments such as globular clusters, we also expect that both of these mechanisms could be moderated by dynamical interactions. For example, collisions can occur during close interactions between pairs of binary stars; close encounters could also modify binary orbits so that mass transfer happens either sooner or later than would occur in an unperturbed situation. Despite many studies to try to disentangle these effects, we have not been able to convincingly determine which process(s) are dominant in globular clusters.

Based on calculated of predicted collision rates, it was expected that collisions were most important in the cores of GCs (e.g. Leonard 1989). There was even evidence from dynamical models using a static cluster background that collisions dominated in the cores while binary coalescence was more important in the outskirts (Mapelli et al. 2006). However, a survey of blue stragglers in globular clusters found no correlation between the fraction of blue stragglers and the collision rate in clusters (Davies et al. 2004), and in fact the two quantities are slightly anti-correlated. A more detailed look at the same survey (Knigge et al 2009) confirmed the lack of correlation between blue straggler number and collisional properties, and found that the best correlation was in fact with core mass. The inference is that the binary population of the cluster (which scales with core mass) is the dominant driver of blue straggler population in a cluster.

At the same time, various groups were modeling popu-
lutions of blue stragglers in clusters. The first large-scale N-body model to include stellar and binary evolution and a population of primordial binaries, and to specifically look at the resultant blue straggler population, was the work of Hurley et al. (2003) for the open cluster M67. He found that both dynamics and binary evolution were important in creating the present-day blue straggler population. Similar models were calculated for a slightly older open cluster, NGC 188 (Geller, Hurley, & Mathieu 2010). Particular attention was paid to the initial conditions for the binary population, and the authors found that the models underproduced blue stragglers formed via mass transfer. They suggest that the criteria for invoking a common envelope phase during mass transfer may be too strict. Since direct star-by-star N-body models for globular clusters are still prohibitively expensive, Leigh, Sills, & Knigge (2011) used a simple analytic prescription for binary and single star encounter rates and the expected number of binary stars. They found that in order to match the observed numbers of blue stragglers in globular clusters, binary evolution should dominate over collisional processes. Most recently, Monte Carlo dynamical models of globular cluster evolution have started to include stellar and binary evolution, and therefore can follow the creation of blue stragglers (Hypki & Giersz 2013; Chatterjee et al. 2013).

To date, however, there has been a fundamental disconnect between the theoretical and observational investigations of blue stragglers in clusters. Theoretical blue stragglers are identified by their mass compared to the turnoff mass. Observational blue stragglers are chosen simply by their position in the color-magnitude diagram of a cluster. Many of the studies quoted above chose the blue stragglers in only one color band. That band is typically V-I, which is not ideal for selecting hot stellar populations as shown by Ferraro et al. (1997). Therefore, there is always an uncertainty in the observed blue straggler populations — are they all really main sequence stars, more massive than the turnoff, or are we contaminated by chance superpositions, blends of binary stars, and photometric errors or anomalous populations? Sills et al. (2000) showed that even using three photometric bands (U, B and V) to select blue stragglers in 47 Tucanae resulted in 8% of the objects from the B-V color-magnitude diagram to be rejected as true blue stragglers, since they were not in the correct part of the color-magnitude diagram in both U-B and B-V colors.

We can now address this issue, however, by looking at the populations of model blue stragglers in the suite of cluster models from Monte Carlo modeling. In this paper, we “observe” the blue stragglers in the simulated clusters presented in Chatterjee et al. (2013) (Paper I). We determine the blue stragglers and other cluster properties from the color-magnitude diagrams, using methods that are as close as possible to the various observational groups who do this for real clusters. We wish to determine if there are any observational biases which affect the conclusions from those groups. We also wish to determine if the theoretical models have any short-comings that renders any comparison to observations invalid.

In section 2, we outline our method for “observing” the blue stragglers in the model clusters. Section 3 gives detailed comparisons between our model blue stragglers and a number of significant observational results about blue stragglers from the past decade. In section 4, we present our conclusions and discuss future directions.

2. METHODS

A large collection of models of globular clusters was constructed using the Cluster Monte Carlo (CMC) code (Joshi, Rasio, & Portegies Zwart 2000; Joshi, Nave, & Rasio 2001; Fregeau et al. 2003; Fregeau & Rasio 2007; Chatterjee et al. 2013; Umbreit et al. 2012). These models are described in detail in Chatterjee et al. (2013). They include may physical processes such as two body relaxation, physical collisions, binary-mediated scattering. Binary and single star evolution is treated using BSE (Hurley, Tout, & Pols 2002). A number of initial cluster parameters were explored, including binary fraction, initial virial radii, initial number of stars, and initial concentration. The clusters were evolved to an age of 12 Gyr. We have a total of 126 models, which span a range of final properties such as total mass, core mass, and core radius. The blue straggler populations in these models were analyzed in detail in Paper I. In that paper, the selection of blue stragglers was done based on non-observational properties; now we wish to test that method against observational selection.

In order to compare the model blue stragglers with the observed population of blue stragglers in real Milky Way clusters, we converted the stellar luminosities and effective temperatures to HST/ACS F606W and F814W magnitudes. We used the color transformation program of Dotter et al. (2008), modified to include the ACS filters (Dotter, personal communication). These are the same filters that are used in the ACS Survey of Galactic Globular Clusters (Sarajedini et al. 2007) and therefore are those used by Leigh, Sills, & Knigge (2011) to select blue stragglers and other stellar populations in a self-consistent way from color-magnitude diagrams. We applied the selection criteria of Leigh, Sills, & Knigge (2011) to our model clusters. These criteria have some free parameters, namely the location of the turnoff and of the horizontal branch, which were determined from the color-magnitude diagrams. A color-magnitude diagram of one of our models is shown in Figure 1. This is the same model cluster whose HR diagram is shown in figure 1 of Paper I. The observational blue straggler selection box is shown, and all blue stragglers selected by this technique are shown as solid squares. The circled stars are objects which meet the theoretical selection criteria of Paper I: they are main sequence (core-hydrogen burning) stars with masses greater than 1.1 times the current turnoff mass (0.835 $M_{\odot}$ in this model cluster). For binary systems, either of the components had to meet these criteria.

It is encouraging that the majority of the objects inside the selection box are chosen by both methods. In some clusters, we see one or two stars which are brighter than the majority of the blue stragglers which are not selected by the theoretical criteria. These are stars which do have masses greater than 1.1 times the turnoff mass, but are not core hydrogen burning. They are former blue stragglers who are currently traversing the Hertzsprung gap. Because this phase is short-lived, their contamination of the blue straggler population is small, and one could argue that we should include them in our count because
they do trace the blue straggler formation efficiency as much as the true, main sequence blue stragglers.

The objects outside the observational selection box are also instructive. First, there are blue stragglers fainter than our selection box. These blue stragglers have masses which are only slightly larger than the cutoff mass. These objects are also seen in real color-magnitude diagrams of clusters, but are confused with blends of binary systems (the small dots that are not circled in that region of Figure 1), and any photometric error will also broaden the main sequence turnoff in this region. Removing these objects from any standardized selection of blue stragglers is sensible as the confusion between the three contributors is large.

The model clusters show a group of blue stragglers which are even bluer than the blue straggler selection box. These objects do not appear in the color-magnitude diagrams of the real clusters in the ACS Survey. Their presence in the model color-magnitude diagrams is an artifact of the treatment of mergers within BSE. When two stars merge, BSE assigns a homogeneous composition to the remnant which is based on the amounts of hydrogen and helium present in the two parent stars. This fully mixed product has a long lifetime because hydrogen in brought to the interior, and is blue because of the enhanced surface helium abundance. At various times in the past, both collision products (Benz & Hills 1992) and binary coalescence products (Lu, Deng, & Zhang 2010) have been assumed to be fully mixed. More recent models (Lombardi, Rasio, & Shapiro 1993; Sills et al. 2001; Glebbeek & Pols 2008) have shown that in fact collision products retain a strong memory of the chemical profiles of their parents. Models of binary coalescence (Chen & Han 2008) are also inconsistent with full mixing during the merging process. The lack of real blue stragglers to the blue of the Leigh, Sills, & Knigge (2011) selection box, combined with their presence in our model clusters, suggests that the detailed models are correct and no formation mechanism produces fully mixed blue stragglers. The error is more severe for more massive blue stragglers, which come from more evolved parents and should therefore have very short main sequence lifetimes (Sills et al. 1997). Applying a correction factor as proposed by (Glebbeek & Pols 2008) reduces the lifetime of the merger products. The most massive merger products then evolve away from the main sequence and would no longer appear as blue stragglers in the simulations.

There are also objects that lie above the subgiant branch and between the blue stragglers and the giant branch. These are binary stars, which contain a main sequence star (the blue straggler) plus an RGB star, a Hertzsprung gap star, or a helium white dwarf. Their positions in the color-magnitude diagram are dominated by the light of the other object in the system rather than that blue straggler. In the simulation shown in figure 1 the blue straggler - RGB systems are the two objects near the RGB at a magnitude of ~ 1. A Hertzsprung gap system is just outside the selection box on the bright side, and the object just outside the selection box nearest the turnoff is a blue straggler - helium WD binary. By choosing this specific observational selection box, we are removing legitimate blue stragglers from our consideration because they are in a binary system with something quite bright. Any conclusions which are drawn from these sections will be biased against blue stragglers that are formed in recent mass transfer events, which means that we are underestimating the importance of binary stars in blue straggler creation.

3. RESULTS

In this section, we will compare the number of “observationally-selected” model blue stragglers (i.e. those in the selection box shown in Figure 1) to a variety of cluster properties, guided by papers that have done this for real clusters over the past decade. The cluster properties (total mass, core mass, etc.) for the model clusters are those calculated as described in Chatterjee et al. (2013). In particular, the core properties were determined by creating artificial surface brightness profiles and determining the core radius from those, rather than using the standard dynamical definition of core radius. Therefore, we have “observational” properties for the models which can be directly compared to real clusters. The observed number of blue stragglers are taken from Leigh, Sills, & Knigge (2011), and the observed cluster properties are taken from the Harris catalogue (Harris 1996, 2010 revision).

In Figure 2, we plot the number of model blue stragglers selected in two different ways: theoretically selected numbers on the x-axis vs “observationally” selected on the y-axis. The theoretical selection method chooses about twice as many blue stragglers as the method which uses the selection box in the color-magnitude diagram, as expected from Figure 1. However, the correlation is quite good. Therefore, we confirm that the observational selection procedure is robust and can be used to determine population sizes, and certainly can be used to look at correlations between blue straggler populations and cluster properties for the models which can be directly compared to real clusters. The observed number of blue stragglers are taken from Leigh, Sills, & Knigge (2011), and the observed cluster properties are taken from the Harris catalogue (Harris 1996, 2010 revision).
properties.

We looked at the formation history of the blue stragglers inside and outside the observational selection box, to see if the observational box is preferentially choosing blue stragglers made in a particular way. We found that there is no clear bias in the selection procedure. Collisional blue stragglers have, on average, the same fraction in the model clusters using either criterion, with a small scatter. Mass transfer binaries are also selected with approximately the right fraction, although in this case, there is a larger spread from model to model, with a few models having almost all their mass transfer systems inside the observational selection box and others with almost none inside the observational box. We conclude that the observational selection criterion samples the blue stragglers created from all formation channels without any clear bias, if a large enough sample of clusters is considered.

In the following sections, we restrict ourselves to blue stragglers found in the core of the clusters, so that we can make a direct comparison to the observed blue stragglers found in [Leigh, Sills, & Knigge, 2011].

3.1. The blue straggler-cluster property correlations

A leap forward in the study of blue straggler populations came from the HST/WFPC2 survey of globular clusters [Piotto et al. 2002]. This was the first detailed and self-consistent look at the cores many globular clusters at once (74 in this case), and a substantial population of blue stragglers was found in every cluster. A subsequent paper [Davies et al. 2004] tested the prediction that the number of blue stragglers should be correlated with the collision rate in the cluster. As a control, they also compared the number of blue stragglers to the total mass in the cluster. Surprisingly, the correlation with total mass was stronger than the correlation with collision rate, and if anything, there was a weak anti-correlation with collision rate. These results have subsequently been confirmed by various other authors [Leigh, Sills, & Knigge, 2007; Knigge et al. 2009]. In Figure [3], we plot the blue stragglers in the core vs total cluster mass, with the models in solid squares and the Milky Way clusters in open circles. The models have a restricted range of total mass compared to the observations, but in the regions where they overlap, we predict approximately the correct number of core blue stragglers. In agreement with all previous observations, we have more blue stragglers in more massive clusters.

Figure [4] shows the number of core blue stragglers vs the collisional parameter $\Gamma$. Following [Leigh, Sills, & Knigge, 2007], we calculate $\Gamma$ under the assumption that the cluster is well-fit by a King model, so that the collision rate is proportional to $\rho_0^2 r_c^3 / \sigma$, where $\rho_0$ is the central density, $r_c$ is the core radius, and $\sigma$ is the central velocity dispersion [Pooley & Hut, 2006]. Again, while our model clusters span a narrower range in $\Gamma$ than the observations, the model results are consistent with them. A more detailed discussion of the relationship between $\Gamma$ and the blue straggler number for the entire cluster, not just the core, can be found in Paper I.

The tightest observed correlation between blue straggler populations and cluster properties is between core blue straggler number and core mass, first identified in [Knigge et al. 2009]. They found a tight but sub-linear correlation, which is reproduced in Figure [5]. Our model clusters match this correlation very well. [Knigge et al. 2009] also performed a detailed multi-component fit to the independent cluster parameters of core radius, central density, and central velocity dispersion [Pooley & Hut, 2006]. Again, our models match the data very well. The interpretation of these results, specifically the sub-linearity of the correlation, was that the blue stragglers were produced by a process which predominantly depended on binary
stars, and did not depend on collisions. We know the
detailed formation history of each blue straggler in the
models, which are discussed in depth in Paper I. The
majority of blue stragglers are produced in binary-mediated
collisions. That is to say, a binary star has a strong in-
teraction with a single star or another binary star, and
during the course of that interaction, two of the stars
physically collide. Both binary stars and collisions are
involved, which means we cannot easily separate the two
formation mechanisms.

In Leigh et al. (2013), the number of blue stragglers
was compared to the number of binary stars in the core.
The number of binary stars was approximated by mul-
tiplying the binary fraction by the core mass, and we
have done the same thing here. The binary fractions
for the models were determined using a technique that
mimics the selection criteria of Milone et al. (2012): we
counted the number of main sequence stars between the
turnoff and 4 magnitudes fainter, and determined from
the models how many of those were binaries with a mass
ratio larger than $q = 0.5$. We then doubled that number
to get the predicted number of binaries of all mass ratios,
as done by Milone et al. (2012). The agreement between
the observations and the models is excellent. Since we
know that most of the blue stragglers are produced in
binary-mediated collisions, it appears that the driving
factor in determining whether interactions will produce
blue stragglers is dominated by the properties of the bi-
naries, not the number of collisions. Only those interac-
tions with the right combinations of binary masses and
orbital parameters will produce blue stragglers. These
results suggest that we could use blue stragglers to con-
strain the distribution of binary properties (mass ratios,
eccentricities, semi-major axes) in clusters, but that we
cannot use them to probe the collisional history of a cluster.

3.2. Other comparisons

With these model globular clusters, we can investi-
gate other predictions from past theoretical studies, or
compare our “observed” blue straggler population to real
populations in clusters.

The seminal paper on collisionally-produced blue
stragglers, Leonard (1989), made the simple assumption
that half of all blue stragglers were produced in the cores
of globular clusters, and that the other half were found
We find that the algorithm for treating stellar collisions and mergers in the stellar/binary evolution software BSE over-produces the number of bright blue stragglers compared to observations. We confirm that this is a result of the mixing prescription in BSE, and reaffirm that the correction suggesting by Glebbeek & Pols (2008) is more appropriate for correctly modeling blue stragglers. We find that the numbers of blue stragglers selected using observational techniques correlates well with the number selected using a simple mass cutoff. Therefore, we are comfortable using either selection method to study blue straggler populations.

We investigate the correlations between “observationally” selected blue straggler and various cluster properties (total mass, core mass, collision rate, binary fraction) and find that the model blue stragglers are consistent with the observed populations. Specifically, we find a correlation with cluster mass and a tighter correlation with core mass, a weak anti-correlation with collision rate, and a strong correlation with number of binary stars. The interpretation of these results has been that blue stragglers are not collision products but are formed through binary evolution. In the models, the blue stragglers are in fact created in binary-mediated collisions. We need to reconcile this contradiction.

Our understanding of the relationship between the collisional parameter $\Gamma$ and binary interactions is guided by the work of Leonard (1989). He starts with a derivation based on a single-single collisions, and calculates the gravitationally focussed cross section for single stars. He then replaces the radius of the star with the semi-major axis of the binary in this calculation to determine the likelihood of a strong interaction between a binary system and another object. These calculations do predict the collision rates of various objects in a cluster, under the assumption that there is a typical stellar mass, binary mass, stellar radius, and binary semi-major axis. However, they do not take into account the possible outcomes of such interactions. For example, binary-single encounters can produce 11 possible outcomes: preservation, ionization, exchanges, and mergers with different combinations of stars involved (McMillan & Hut 1996) and binary-binary encounters are even more complicated. Only some of those interactions will produce a blue straggler, and the probability of those interactions occurring depend on the properties of the binaries and single stars involved. Even if we have a triple merger, but with three 0.2 M$\odot$stars, it will not produce a blue straggler since the mass will still be less than the turnoff mass. Therefore, we conclude that the simple approximations of collision rates in clusters, particularly for binary stars, are not suitable for determining the number of a particular subset of the interactions that occur. To put it another way, the probability that a blue straggler will be formed during a binary-single interaction is more strongly dependent on the binary properties than on the simple collision rate. More accurate analytic calculations for these predictions should, at the very least, include the range of binary properties in the collision rate calculation, and should also include factors which take into account the likelihoods of the various appropriate formation mechanisms (mergers compared to ionizations, for example).

There are other populations in clusters which are expected to be formed through collisions. In particular,

![Figure 7](image_url)
low-mass X-ray binaries and cataclysmic binaries are thought to be created when a compact object (a neutron star or a white dwarf) acquires a new binary companion through an exchange interaction or tidal capture, or collides with a giant star. Both these populations do show a correlation with collision rate in globular clusters (Pooley et al. 2003; Pooley & Hut 2006; Bahramian et al. 2013). We predict that the difference between these populations and the blue straggler populations is that there is a smaller range of binary properties which can produce these populations, so the assumption that the production mechanism is a simple factor of the average encounter rate is more appropriate than for blue stragglers.

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