Stiffness and damping effects of the rhombic piezoelectric stack transducer with a negative capacitance shunt

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Abstract. The piezoelectric shunt damping (PSD) is widely utilized to control structural vibrations based on its damping performance. Unlike previous studies, this work considers the stiffness and damping effects of a rhombic piezoelectric stack transducer with a negative capacitance shunt. The piezoelectric shunt stiffness (PSS) and the PSD concepts are proposed and theoretically modelled to investigate the stiffness and damping performance. The piezoelectric stack transducer with a negative capacitance can achieve both negative and positive stiffnesses to tune the natural frequency of the host structure. The root locus methods are utilized to judge the stability of the system. Both numerical simulation and experimental results demonstrate that the negative stiffness and the positive stiffness of the system greatly depend on the negative capacitance, while the damping depends on the resistance.

1. Introduction
The PSD has been widely investigated since Forward [1] carried out a preliminary experimental examination of feasibility of using external electronic circuits to suppress vibrations in optical systems. Some passive resonant shunts have been investigated in depth [2-4]. However, they are sensitive to the natural frequency and external loads [5].

The active shunt needs an external electrical energy to drive, and it is able to overcome the dependency of the natural frequency. In principle, the piezoelectric transducer is electrically equivalent to an inherent capacitance and a current source. The negative capacitance can cancel the inherent capacitance of the piezoelectric transducer to realize broadband vibration control [6]. A negative impedance converter (NIC) can be used to construct a negative capacitance, a negative resistance [7] and a negative inductance [8]. Neubauer et al. [9] studied the effect of the negative capacitance shunted piezoelectric transducers in damping and absorbing systems. Manzoni et al. [10] discussed the values of the electric elements composing of the negative capacitance to improve vibration reduction efficiency and to avoid instability at low frequencies. Mokrani et al. [11] utilized a synthetic negative capacitor to reduce structural vibration. Beck et al. [12] suggested that the negative capacitance shunt can modify the effective modulus of the piezoelectric transducer, and they discussed the power output and efficiency of the negative capacitance. Han et al. [13] proposed an adaptive shunt that consists of a switched inductance resistance network connected in parallel with a negative capacitance to improve the damping performance. Gripp et al. [14] described an adaptive resonant piezoelectric vibration absorber enhanced by a synthetic negative capacitance.

Theses researches on the negative capacitance shunt mainly using piezoelectric patches to control the structural vibration [15, 16]. Preumont [17] and Marneffe [18] employed the negative capacitance shunted piezoelectric stack for the vibration control of a truss structure. Beck et al. [12] suggested that a careful choose of the negative capacitance can produce the stiffness effect. Manzoni et al. [10] found...
that a frequency shift phenomenon appeared in negative capacitance shunted piezoelectric patches. However, in the past and existing research efforts, few studies discussed the stiffness of the PSD, especially for piezoelectric stack transducers. The corresponding stiffness performance has not yet been investigated by far. Therefore, this paper proposes the controlled stiffness concept using the negative capacitance based the PSD. The theoretical model is established, and the PSS and the PSD are proposed. Experiments are carried out to verify the concept and the theoretical analysis.

2. Modelling of electromechanical system

Figure 1 shows a rhombic piezoelectric stack transducer with a negative impedance shunt circuit. The rhombic frame magnifies the output displacement of the piezoelectric transducer [19]. The piezoelectric stack transducer connects to the NIC shunt that is constructed by an operational amplifier [6, 18]. Figure 2(a) presents the electrical model of the piezoelectric stack transducer with the negative resistance shunt, and the simplified model is shown in Figure 2(b). The governing equations are shown as [20, 21]:

\[
mx' + cx + (k + K_a)x - \theta V = F, \tag{1}
\]

\[
C_p V + \theta x + I_s = 0, \tag{2}
\]

where \(\theta\) and \(C_p\) are the electromechanical coupling coefficient and the inherent capacitance of the stack, respectively. \(I_s\) is the current flowing in the circuit. \(m\), \(c\) and \(k\) are the mass, the damping and the stiffness, respectively. The structural damping coefficient of the frame is represented by \(c = 2\zeta\omega_n / m\), and the damping ratio \(\zeta\) can be selected between 0.5% and 1%. \(V\) is the voltage across the load resistance.

![Figure 1](image1.png)

**Figure 1.** Model of a rhombic piezoelectric stack transducer with the NIC. \(R_s\) is the adjustable impedance of the shunt. \(Z_1\), \(Z_2\) and \(Z_s\) are the impedances.

![Figure 2](image2.png)

**Figure 2.** Electrical equivalent model of the piezoelectric stack transducer with the hybrid shunt, (a) full model, and (b) equivalent model.
3. Numerical simulation

Table 1 shows the parameters of the piezoelectric stack and the hybrid shunts. Figure 3 represents the frequency response of the piezoelectric stack transducer with the negative capacitance shunt. A positive capacitance can increase the natural frequency of the transducer. When $R_s = 10$ kΩ, the amplitude approximates to the uncontrolled condition. With $R_s$ decreases, the amplitude decreases while the natural frequency increases. When $C_s = 2$ μF, this hybrid shunt can produce the negative stiffness effect, and the natural frequency of the system is decreased. In this case, the amplitude decreases along with the increase of $R_s$. If this transducer is employed as an isolator, the negative PSS is a better choice. If someone just wants to avoid the resonance of the system, both the positive and negative stiffnesses are acceptable.

**Table 1. Parameters of the transducer and the negative capacitance shunt.**

| Parameters (Unit)                | Value       |
|----------------------------------|-------------|
| Capacitance of the stack, $C_p$ (uF) | $1.478 \times 10^{-6}$ |
| $R$ (Ω)                          | $1 \times 10^6$ |
| $T_R$                            | $1$         |
| $L_s$ (mH)                       | $10$        |
| Mass, $m$ (kg)                   | $0.1$       |
| Test natural frequency of transducer, $f_n$ (Hz) | $154.9$ |

![Figure 3. Frequency response of the piezoelectric stack transducer with the negative capacitance.](image)

4. Experimental verification

4.1. Experiment setup

Experiments are carried out to verify the stiffness and damping effects of the piezoelectric stack transducer with the negative capacitance shunt circuit in simulations and the theoretical analysis. A rhombic frame prototype is manufactured with the Aluminium material, and the piezoelectric stack is P-885.51 from PI cooperation. The negative capacitance shunt circuit is constructed by a breadboard. A DC source is utilized to power up OPA 445. Figure 4 is the schematic of the experiment. The base disturbance is generated from an exciter which is a closed loop system.
4.2. Results and discussion

Figure 5 shows the frequency response of the piezoelectric stack transducer under different \( C_s \) and \( R_s \). When \( C_s \) is 1.32 \( \mu \)F, the natural frequency increases from 400.2 Hz to 404.5 Hz. It means that the negative capacitance shunt provides a positive dynamic stiffness to the transducer. When \( R_s \) increases, the PSD increases which reduces the response of the transducer. When \( C_s \) is 1.65 \( \mu \)F, the natural frequency of the transducer decreases to 395.7 Hz that implies a negative stiffness is produced. In this case, the amplitude is decreased dramatically. The results qualitatively prove the stiffness and damping effects of the piezoelectric stack transducer with the negative capacitance shunt in simulations. The shunt circuit is constructed by a breadboard that may have a non-negligible impedance, which will influence the accuracy of the sample experiment.

5. Conclusion

This study discusses the stiffness and damping effects of a rhombic piezoelectric stack transducer with the negative capacitance. The governing equations of the presented electromechanical system are established. In the frequency domain, the piezoelectric shunt stiffness and the piezoelectric shunt damping are proposed to analyse the stiffness and damping effects of the negative capacitance shunted transducer. The root locus method is employed to graphically judge the stability of the system. More importantly, experiments are designed to verify this stiffness phenomenon. The results demonstrate that the piezoelectric stack transducer with hybrid shunts can produce both the positive and negative stiffness performances. The adjustable resistance can be employed to change the damping of the system. The negative stiffness effect requires a careful choice of the negative capacitance to sustain the stability of the system.
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