Optimal Substation Placement in Radial Distribution Network Considering Load Fluctuation

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Abstract. It is an important aim of substation placement to reduce the energy losses in feeder lines. The energy losses mainly depend on the sizes of load, lengths of feeder lines and load fluctuation. Load fluctuation is ignored in the existed publications. In this paper, an optimization method of substation placement considering the load fluctuation is proposed to achieve the minimum total costs of energy losses in feeder lines. In the proposed method, the concept of loss equivalent duration under maximum load is introduced to quantitatively assess the impact of load fluctuation on the energy losses. Numerical results demonstrate the effectiveness of the proposed method.

Introduction

Optimizing substation placement is very important for distribution network planning, which has impact on the economy and reliability of the entire distribution network construction. A new model for large distribution systems is proposed in the [1]. The model considers the fixed and the variable costs of substations and feeders as well as the cost of the power losses. The [2] optimizes distribution substation locations by using operations research methods. An application of recent advances in optimization techniques for distribution substation siting and radial feeder planning is presented in the [3]. The maximum benefit is easily obtained based on the method. The [4] introduce an algorithm for large radial distribution systems. Other method applied to distribution system planning in the [5-12].

The substations are generally placed close to the load center. The center of gravity method, which is usually referred as the location-allocation method [13], is widely used to find the load center. Generally, load-distance products (LDS) is assumed to have an approximatively linear relationship with the power losses in feeder lines. The center of gravity method usually aims to obtain the minimum sum of LDS, and the optimal solution of this method is given in [14]. An improved method, known as L2DS, where the square of active power is used instead of active power, is given in [15]. The solution proposed in [15] is closer to the heaviest load point, and the active power losses in feeder lines are smaller. In the existed publications, maximum load is usually used to optimize the location of substations without considering load fluctuation. In this study, considering load fluctuation, the concept of loss equivalent duration under maximum load is introduced. Based on the concept, an optimization method of substation placement is proposed.

The Formulation of the Proposed Method

The energy losses in feeder lines can usually be calculated by (1).

\[ W_{\text{Loss}} = \Delta P_{\text{av}} T \]

where \( W_{\text{Loss}} \) denotes the annual energy losses in feeder lines, \( \Delta P_{\text{av}} \) denotes the average power losses in a year, and \( T \) is treated as 8760 hours.
Using the maximum load to calculate the energy losses, the concept of loss equivalent duration under maximum load $\tau$, is defined in (2). If the transmitted power in feeder lines maintains the maximum load, the electrical energy losses in a period of $\tau$ equal to the real electrical energy losses in the whole year.

$$W_{loss} = \Delta P_{\text{max}} \tau$$  \hspace{1cm} (2)

where $\Delta P_{\text{max}}$ denotes the power loss at the maximum load.

In order to get the value of $\tau$, (1) can be transformed into (3).

$$W_{loss} = \Delta P_{\text{av}} T = \Delta P_{\text{max}} \frac{\Delta P_{\text{av}}}{\Delta P_{\text{max}}} T = \Delta P_{\text{max}} \tau FT$$  \hspace{1cm} (3)

where $F (\Delta P_{\text{av}}/\Delta P_{\text{max}})$ denotes the loss factor.

According to [16], loss factor can be determined by the Buller and Woodrow formula [16]. Comparing (2) and (3), $\tau$ can be obtained by (4), (5).

$$\tau = FT = 8760 \cdot [cf + (1-c)f^2]$$  \hspace{1cm} (4)

$$f = \frac{P_{\text{av}}}{P_{\text{max}}}$$  \hspace{1cm} (5)

where $c$ denotes an empirically determined constant usually between 0.15 and 0.3, $f$ denotes the load factor, $P_{\text{av}}$ denotes the average load, and $P_{\text{max}}$ denotes the maximum load.

The introduction of $\tau$ changes the fluctuant load into a constant. Using $\tau$ to calculate energy losses, the costs of energy losses in feeder lines can be obtained by (6), (7).

$$C_{\text{cost}} = \lambda W_{loss} = \lambda \Delta P_{\text{max}} \tau = \frac{\lambda R_0}{U^2} \sum_{i=1}^{n} \frac{P_i^2 \tau_i L_i}{\cos^2 \phi_i}$$  \hspace{1cm} (6)

$$L_i = [(x_i - x)^2 + (y_i - y)^2]^{1/2}$$  \hspace{1cm} (7)

where $C_{\text{cost}}$ denotes the annual costs of energy losses in feeder lines, $\lambda$ denotes the energy cost per kWh, and $n$ denotes the number of load points; $P_i$ denotes the maximum active power at $(x_i, y_i)$, $\cos \phi_i$ denotes the power factor at $(x_i, y_i)$, $\tau_i$ denotes the loss equivalent duration under maximum load at $(x_i, y_i)$, $(x, y)$ denotes the load center, and $L_i$ denotes the length of feeder line between $(x_i, y_i)$ and $(x, y)$; $R_0$ denotes the resistance per unit feeder length, and $U$ denotes the line voltage of load.

It can be seen from (6) that $P_i^2 \tau_i L_i/\cos^2 \phi_i$ can be regarded as an index of the costs of energy losses in feeder lines. Therefore, the optimization model of gravity-center method is proposed as follows:

$$\min Z = \sum_{i=1}^{n} \left( \frac{P_i}{\cos \phi_i} \right)^2 \tau_i L_i$$  \hspace{1cm} (8)

Compared with the methods in Table 1, the proposed method retains the advantage of [15], where $L^2$DS is used instead of LDS considering the non-uniform distribution of load. In addition, load fluctuation and the difference of power factor are also considered in the proposed method.

Table 1. The gravity-center methods of substation placement in [14] and [15].

| Methods | Method in [14] | Method in [15] |
|---------|---------------|---------------|
| $\min Z = \sum_{i=1}^{n} P_i L_i$ | $\min Z = \sum_{i=1}^{n} P_i^2 L_i$ |

To get the optimal solution of the proposed method, (8) can be transformed into (9).
\[ \min Z = \sum_{i=1}^{n} w_i L_i \]  \hspace{1cm} (9)

where \( w_i \left( P_i^2 \tau_i \cos^2 \phi_i \right) \) denotes the weight coefficient.

The partial differential equations of (9) are depicted in (10), (11).

\[ \frac{\partial Z}{\partial x} = \frac{\sum_{i=1}^{n} w_i (x - x_i)}{L_i} = 0 \]  \hspace{1cm} (10)

\[ \frac{\partial Z}{\partial y} = \frac{\sum_{i=1}^{n} w_i (y - y_i)}{L_i} = 0 \]  \hspace{1cm} (11)

From (10), (11), the optimal solution of the proposed method can be obtained by (12), (13). Here, an iterative algorithm proposed in [14] is adopted to solve the equations.

\[ x = \frac{\sum_{i=1}^{n} w_i x_i / L_i}{\sum_{i=1}^{n} w_i / L_i} \]  \hspace{1cm} (12)

\[ y = \frac{\sum_{i=1}^{n} w_i y_i / L_i}{\sum_{i=1}^{n} w_i / L_i} \]  \hspace{1cm} (13)

**Case Study**

The proposed method was verified through a test system with sixteen load points in Fig. 1. To simplify the calculation, these load points are divided into four small load regions using minimum-circle-cover method [16]. The parameters of each small load region are given in Table. 2.

![Figure 1. A test system with sixteen load points for case study.](image)

| Table 2. The parameters of each small load region. |
|-------------------------------------------------|
| Region | Region A | Region B | Region C | Region D |
| Coordinates (m) | (100,500) | (300,200) | (500,700) | (600,400) |
| Maximum active power (kW) | 30 | 10 | 20 | 20 |
| Loss equivalent duration (h) | 2200 | 1800 | 2000 | 2000 |
| Power factor | 0.85 | 0.95 | 0.9 | 0.9 |

The coordinates of substation are then obtained by the methods in [14], [15] and this study, respectively. In order to get the annual total costs of the energy losses in feeder lines, the length,
cross-section (S) and resistance of each feeder line are calculated in Table. 3. The results show that the annual total costs of energy losses in feeder lines can be reduced by the proposed method compared with the methods in [14] and [15].

| Growth parameters   | Regression equation | F    | R²  |
|---------------------|---------------------|------|-----|
| Stem height (SH)    | \( y_{SH} = 136.87 + 18.41x_1 + 15.42x_2 + 10.20x_3 + 7.64x_1x_2 - 16.60x_1^2 - 11.03x_2^2 \) | 19.86 | 0.95 |
| Leaf area (LA)      | \( y_{LA} = 2805.75 + 572.48x_1 + 464.95x_2 + 339.19x_3x_2 + 221.72x_1x_3 - 335.46x_1^2 - 359.92x_2^2 - 419.54x_3^2 \) | 31.02 | 0.97 |
| Net photosynthesis rate (PN) | \( y_{PN} = 17.06 + 1.32x_1 + 0.77x_2 + 0.70x_3 - 0.72x_1^2 - 0.81x_2^2 - 0.72x_3^2 \) | 6.65  | 0.86 |
| Biomass yield (BY)  | \( y_{BY} = 55.66 + 11.17x_1 + 6.55x_2 + 4.08x_3 + 5.67x_1x_2 - 5.58x_1^2 - 7.15x_2^2 - 10.66x_3^2 \) | 13.87 | 0.93 |

**Conclusion**

In this study, to achieve the minimum energy losses in feeder lines, an optimization method of substation placement considering load fluctuation is proposed. The concept of loss equivalent duration under maximum load is introduced to assess the impact of load fluctuation on the energy losses. The proposed method has been verified in a test system.

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