Drift instability grow rates in non-ideal inhomogeneous bi-dust plasmas

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Abstract. Dust Acoustic Wave propagation in unmagnetized inhomogeneous dusty plasmas has been extensively studied, considering non-ideal Padé type state equation. In this paper, the drift instability is analyzed in a weakly magnetized dusty plasma including inhomogeneities of the type called gradient instabilities considering two kind of dust particles. A fully kinetic approach is achieved using a dispersion relation in a polynomial form through a multipole approximation. The drift instability for the dust acoustic mode is extensively analyzed including the non-ideal effect in dusty plasmas, using the state equation introduced by Ree and Hoover. These unstable modes are well discriminated and treated as a function of the density gradient and dust grain radius. Temperature gradients and charging processes will be ignored.

1. Introduction
Plasma inhomogeneities across the magnetic field in the presence of finite - size charged grains cause a wide class of instabilities of an inhomogeneous dusty plasma called gradient instabilities. Such instabilities can be studied in the approximations of magnetic fields with parallel, straight field lines in order to simplify our treatment. We look for instabilities in the very low frequency regime where a new spectrum of instabilities and waves appears, induced by the dust collective dynamics: Dust-Acoustic-Waves (DAWS), Dust-Ion-Acoustic-Waves (DIAWS), etc. The frequency of DAWS are around 10 Hz as determined in the laboratory, and even below that in astrophysical plasmas [1],[2]. In the case that grains are in the micron range we expect a non-ideal behavior due to the fact that the particulate are highly charged and intermolecular forces play certainly an important role. In order to discuss this problem we compare the ideal properties with the simple hard-core model and in a following work we will use a better model by considering a square-well model from Ree and Hoover defined by a new state equation

\[
p = n k_B T (1 + b_0 n + a_1 b_0 n + a_2 b_0^2 n^2 / (1 - b_1 b_0 n + b_2 b_0^2 n^2)).
\] (1)

Here \( b_0 = 2 \pi \sigma^3 / 3 \) with \( \sigma \) the grain radius.
2. Theoretical model

In this paper we introduce a new numerical treatment in combination with a more realistic formulation of the state equation to simulate weak non ideal effects in order to analyze inhomogeneous Vlasov-Dusty Plasma systems where a linearized dispersion relation is obtained. Due to the low frequency range enough energy can be transferred from the particle to the wave and instabilities can be generated. In order to get an adequate linear dispersion relation with a magnetic field given by $\mathbf{B} = B_0 \mathbf{k}$ for maxwellian multi-species plasmas (electron, ion and dust), we introduce the well known and very accurate multipolar approximation for the $Z$ dispersion function.

In the presence of a magnetic field we have the distribution function of the specie $\alpha$, as solution of the kinetic equation.

$$\frac{df_\alpha}{dt} = \frac{q_\alpha}{m_\alpha} \mathbf{\nabla} \phi \cdot \frac{\partial f_{0\alpha}}{\partial \mathbf{v}}$$

(2)

given by

$$f(r, v, t) = \frac{q_\alpha}{m_\alpha} \int_{-\infty}^{t} \exp[i\omega(t-t')]\mathbf{\nabla} \phi(r(t')) \cdot \frac{\partial f_{0\alpha}}{\partial \mathbf{v}} dt'$$

(3)

where $\alpha = e, i, d_1, d_2$. Now, the dispersion relation in terms of the dielectric susceptibilities in the low frequency approximation ($\omega, k_z v T \ll \omega_c$) is

$$1 + \sum \chi_\alpha = 0$$

(4)

with:

$$\chi_{0\alpha} = \frac{1}{(k\lambda D_\alpha)^2} \left[ 1 + l_\alpha \frac{\omega}{\sqrt{2k_z}} Z(\xi_\alpha) \mathbf{I}_{0\alpha} e^{-z_\alpha} \right]$$

(5)

Taking into account the low frequency range, enough energy could be transferred from the particle to the wave inducing instabilities. As an adequate linear dispersion relation with a magnetic field defined by $\mathbf{B} = B \mathbf{k}$ for Maxwellian multi-species plasmas, we introduce the well known and very accurate multipolar approximation for the $Z$ dispersion function neglecting temperature gradients (i.e. $\frac{\partial T}{\partial x} = 0$).

Further, in order to simplify our expressions, we need:

$$\frac{d}{dT}(\frac{1}{v_\alpha}) = -\frac{m_\alpha k_y T_{\alpha}^3}{2 T_{\alpha}^{3/2}}; \quad \frac{d z_\alpha}{dT} = \frac{k_y^2 T_{\alpha}}{m_\alpha \omega_{D\alpha}}; \quad \frac{d \xi_\alpha}{dT} = -\frac{\omega}{\sqrt{2k_z v T_{\alpha}}}$$

(6)

Now, using the following identity for the dispersion function $Z$

$$Z' = -2[1 + \xi_\alpha Z(\xi_\alpha)]$$

(7)

we obtain after several cumbersome algebraic manipulations the general dielectric susceptibilities in the form

$$\chi_{0\alpha} = \frac{1}{(k\lambda D_\alpha)^2} \left[ 1 + \frac{\omega Z I_{0\alpha} e^{-z_\alpha}}{\sqrt{2k_z v T_{\alpha}}} \left\{ 1 - \frac{k_y T_{\alpha}}{m_\alpha \omega_{D\alpha}} \left( \frac{n_{0\alpha}'}{n_{0\alpha}} \right) \right\} + T_{\alpha}' \left( -\sqrt{\frac{m_\alpha v T_{\alpha}}{T_{\alpha}^3}} \frac{Z' \xi'}{Z} + \frac{I_{0\alpha}'}{I_0} - z'_\alpha \right) \right]$$

(8)
with \( \alpha = e, i, d1, d2 \),

In this work we neglect temperature gradients, as was mentioned before, then we have in such cases only the following expression

\[
\chi_\alpha = \frac{1}{(\kappa \lambda_D \alpha)} \left[ 1 + \frac{\omega Z I_{\alpha} e^{-\alpha}}{\sqrt{2} k_z v_T} \left\{ 1 - \frac{k_y T_\alpha n_{\alpha}'}{m_\alpha n_{0 \alpha}} \right\} \right] \tag{9}
\]

In order to put our dispersion relation in a dimensionless form, we introduce the following suitable definitions:

\[
\begin{array}{c|c|c|c|c}
\lambda_D \alpha & K & \xi_\alpha & \mu_\alpha & \Theta_\alpha \\
\hline
\sqrt{\frac{T_\alpha}{n_{0 \alpha} Z_{\alpha} e^2}} & \kappa \lambda_D & \omega & \frac{m_\alpha}{m_0} & \frac{T_\alpha}{T_i} \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
\omega_0 \alpha & \kappa \lambda_D \alpha & \Omega & \Omega_0 \alpha & U_{\alpha} \\
\hline
\frac{z_\alpha e B}{m_\alpha} & K \sqrt{\frac{\sigma_\alpha}{\mu_0}} & \omega & \frac{w_0}{w_{\alpha j}} & \frac{v_T a}{c_s j} \\
\end{array}
\]

Now, using these results and assuming that \( \omega \ll \omega_0 i \ll \omega_0 d \) we can write down (9) as

\[
1 + \chi_0 e + \chi_0 i + \chi_0 d_1 + \chi_0 d_2 = 0 \tag{10}
\]

with

\[
\chi_0 \alpha = \frac{\mu_\alpha}{K^2 Theta_\alpha} \left[ 1 + \frac{\Omega Z I_{0 \alpha} e^{-\alpha}}{\sqrt{2} K z U_{\alpha}} \left\{ 1 - \frac{K y T_\alpha^2}{\Omega_0 \alpha \Lambda n_{\alpha}} \right\} \right] \tag{11}
\]

Now it is very convenient to define the following relations [4]:

\[
\frac{1}{L_p} = \frac{\nabla p_d}{p_d}; \quad \frac{1}{L_n} = \frac{\nabla n_d}{n_d} \tag{12}
\]

\[
L_p = \frac{p_d}{L_n} = \frac{\alpha_1 n_d + \alpha_2 n_d^2 + \alpha_3 n_d^3 + \alpha_4 n_d^4}{1 + \beta_1 n_d + \beta_2 n_d^2} \frac{1}{L_n} \tag{13}
\]

where, \( d = d1, d2 \) and the coefficients of equation (13) are given in table 1.

| Nr. | Coefficients | Value |
|-----|--------------|-------|
| 1   | \( \alpha_1 \) | 1.000000 \( k_B T_d \) |
| 2   | \( \alpha_2 \) | 0.438507 \( k_B T_d b_0^4 \) |
| 3   | \( \alpha_3 \) | 0.14482 \( k_B T_d b_0^2 \) |
| 4   | \( \alpha_4 \) | 0.017329 \( k_B T_d b_0^4 \) |
| 5   | \( \beta_1 \) | -0.561493 \( b_0 \) |
| 6   | \( \beta_2 \) | 0.081313 \( b_0^2 \) |

Now, introducing the multipolar approximation to \( Z \) we can get a rational polynomial expression in the form [5]

\[
\sum_j a_j \Omega_i / \sum_j b_j \Omega_j = 0 \tag{14}
\]
where coefficients $a_i$ and $b_i$ of the system parameters. To solve this relation is very easy to find roots of the numerator. Analysis of of the solutions region permit us to find the gain $\gamma = Im(\Omega)$ in function of $1/K_y$ or simply versus $K_y$ as shown in figures 1 and 2.

3. Results and conclusions

The quasi-neutrality equation for dusty plasmas can be simplified due to the high state of charge of the dust particulate to the more simple relation

$$n_{oi} = Z_D n_{od} + Z_D n_{od1} + n_{oe}$$

(15)
and the electron susceptibility can be neglected in the dispersion relation. The range of the main parameters in the study of the low frequency oscillation of dust particulate is established by the approximations that conduced to the simplified dispersion relation

\[ \Omega, K_z U_d \ll \Omega_{cd} \]  \hspace{1cm} (16)

Non negligible unstable dust oscillations \((\text{Im}(\Omega) > 0)\) are found for \(\Omega_{cd} \simeq 10^{-1}, K_z U_d \simeq 10^{-2}\). For slightly inhomogeneous plasmas, the shape of the dust instability \((\text{Im}(\Omega))_{\text{max}}\) curve as function of the perpendicular to magnetic field wavelength \((1/K_y)\) is similar to that for ions, previously studied [6]. The maximum value of the instability increases slightly with the particle radius \(r_d\) and decreases towards smaller radius shifting to the ideal region. A widening of the growth rate profile is to observe in the region of smaller radius. For typical laboratory weakly dusty plasma \((n_d \sim 10^4 m^{-3}, Z_D \sim 10^3)\) the instability of dust acoustic or electrostatic waves is narrower and smaller than that for ions. In figure 1 the maximum of the instability corresponds to the typical shape of instability of weakly inhomogeneous dusty plasmas and density gradients lengths of \(\Lambda_p \equiv \Lambda \leq 1.5 \times 10^1\), while for the figure 2 the region of instability appears for density gradient lengths of the order of \((\Lambda_p \equiv \Lambda \leq 2.5 \times 10^1)\). The introduction of a new dust particle reduce the the growth rate change and the achieved maxima are very similar by changing the radius.

\[ r_d \ll \lambda_{Di} \]  \hspace{1cm} (17)

necessary condition for the exhibition of dust acoustic waves.

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