The recently discovered material Co$_3$Sn$_2$S$_2$ shows an impressive behavior of the quantum anomalous Hall (QAH) conductivity driven by the interplay between ferromagnetism in the z direction and antiferromagnetism in the xy plane [1]. Motivated by these facts, we will show how we can describe qualitatively such a correlation between magnetic and topological properties. In our model [2], the magnetism of Co atoms is described through localized spin-1/2’s, reflecting the strong Hubbard interaction, and the low-energy bands are in agreement with ab initio calculations on Co$_3$Sn$_2$S$_2$ established in the ferromagnetic phase. Also, we include conduction electrons which are coupled to the localized spin-1/2’s through a strong Hund's coupling. The spin-orbit coupling results in topological low-energy bands. For 2/3 on-site occupancy, we find a topological transition from a QAH ferromagnetic insulating phase with Chern number one to a quantum spin Hall (QSH) antiferromagnetic phase. The QAH phase is metallic when slightly changing the on-site occupancy. To account for temperature effects, we include fluctuations in the direction of the Hund's coupling. We show how the Hall conductivity can now smoothly evolve when spins develop a 120° antiferromagnetism in the xy plane and can synchronize with the ferromagnetic fraction.

[1] Z. Guguchia, et al., Nat. Commun. 11, 559 (2020).
[2] Julian Legendre and Karyn Le Hur, Phys. Rev. Research 2, 022043(R) (2020).

Figure 1: Magnetic and topological transition as a function of the effective parameter $J_{xy}$ of our model. $C$ is the Chern number, the symbol $\langle\ldots\rangle$ refers to an ensemble-average value.