Thin film evaporation model for two-phase capillary heat transfer devices: examination of boundary conditions and vapour pressure gradient

S Ahmed\textsuperscript{1} and M Pandey\textsuperscript{1*}

\textsuperscript{1}Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Assam, 781039, India

Abstract. Thin film evaporation model has been used by several researchers to study the transport phenomena of two-phase capillary devices, such as heat pipes and capillary pumped loops. The present work is focused on mathematical modeling of the evaporation phenomena in such devices. Liquid-vapour interface in the evaporator is modelled using a thin film model and the lubrication approximation, along with a slip boundary condition at the wall and a shear boundary condition at the interface. Different models are used for the evaporating mass flux and the vapour pressure gradient at the liquid-vapour interface is considered. An attempt is also made to determine the non-evaporating film thickness that satisfies the underlying physics. The film thickness profile and the pressure components are obtained by numerical simulations. It is found that the choice of the evaporating mass flux model has a significant effect on the results, and is very important for heat transfer characterization.

1. Introduction

Two-phase Capillary Devices are those devices which use the capillary force to pump working fluid along their system. These devices work independently of gravity and their performance is not affected by their orientation. Examples of such devices include those which come under heat pipe family and capillary pumped loop. Their components consist of an evaporator, condenser, wick (to provide capillary support) and transport channels. Heat is applied in the evaporator where the working fluid turns into vapor and reaches the condenser via transport channels. In condenser, heat is rejected and the vapor turns into liquid which is then returned to the evaporator via the wick. The wick consists of either porous material (e.g. sintered metal powder) or microchannels. When the heated working fluid wets the wall of these channels, the extended meniscus (figure 1) can be divided into three regions:

* corresponding author: manmohan@iitg.ac.in
The capillary region where the capillary force dominates. It is also called intrinsic meniscus.

The thin film region where maximum amount of heat transfer occurs due to low thermal resistance. It is dominated by both capillary and disjoining pressure.

The adsorbed region (also known as equilibrium thin film region) where no evaporation takes place due to high disjoining pressure.

Dergajuin laid down the foundation stone for the study of ultra-thin films. He defined the concept of disjoining pressure as “The difference between the pressure in a region of a phase adjacent to a surface confining it, and the pressure in the bulk of this phase”. Since then, researchers have carried out both experimental and theoretical studies. Among them, the work of Wayner Jr. and Coccio [1] deserves special mention as it was in their experimental study, that they postulated the existence of a thin film adjacent to the intrinsic meniscus which has been later verified by several researchers. Wayner et al. [2] further simplified the numerical model by implementing the lubrication approximation and simplifying the Schrage correlation for calculating interfacial mass transfer. This later became popularly known as Wayner’s Model. Wang et al. [3] studied heat transfer characteristics of both thin film and micro region. They used both Schrage model and Wayner model in the calculation of evaporating mass flux. They concluded that thin film profile generated by Wayner model agrees with that of Schrage model upto superheat of 5 K. However, for high superheat, Schrage model has to be used. Apart from that, they also studied the effect of capillary pressure in their role in suppression of evaporation. Hanchak et al. [4] implemented a better disjoining pressure model which was based on a retarded van der Waals interaction. They also carried out experimental verification of their numerical model. They applied reflectometry to measure the thickness of thin film of n-octane on silicon wafer. Jasvanth et al. [5] carried out a comparative study of evaporating thin film for different working fluid in a loop heat pipe. They confirmed that heat transfer coefficient of a loop heat pipe directly depends on fluid figure of merit (namely interline heat flow parameter [6–8] and heat pipe figure of merit, shown by equation (1)).
Du and Zhao [9] worked on getting more accurate solution by taking improved boundary conditions. Later on, Akkus and Dursunkaya [10] extended their work by starting the calculation from intrinsic meniscus. In another work, Akkus et al. [11] developed a two dimensional solution methodology for the thin film model. Fu et al. [12] considered the bulk flow between the vapour and interface phase. They directly solved the momentum conservation equation instead of lubrication approximation in their analysis. They also studied the effect of accommodation coefficient and concluded that it significantly affects the important parameters like interfacial temperature, heat flux, mass flow rate etc. but has no effect on the thin film profile.

Although researchers came up with numerous different strategies to improve the model, they had one thing in common. They assumed uniform vapour pressure, and, as a consequence, ignored the vapour pressure gradient. However, Park et al. [13] had considered the vapour pressure gradient and proved that it affects the thickness of thin film. They also incorporated slip boundary condition in their model.

In the present study, effect of vapour pressure gradient is investigated using different evaporation flux model. Apart from that, issues associated with boundary conditions is examined and a simple yet effective solution is put forward. Thus, this work can be considered an extension of Wang et al. [3], Park et al. [13], and Kou et al. [14] and lays down foundation stone for our future work [15, 16].

2. Mathematical model

Figure 2 shows the thin film region in details. To simplify the model, following assumptions are made:

i. Two-dimensional and steady-state laminar flow
ii. Pressure is a function of x-coordinate only.
iii. Incompressible liquid and vapour state
iv. Uniform wall temperature
v. Working fluid having constant properties and surface tension
vi. Negligible convective terms in the momentum equation
vii. Evaporation is restricted to thin film region only

2.1. Governing equations

In the thin film, both capillary and disjoining force act. Thus, the pressure difference is balanced out by both capillary and disjoining forces, which is shown in equation (2).

\[ P_v - P_l = P_c + P_d \]  \hspace{1cm} (2)
Here

\[ P_c = \sigma \left( \frac{d^2 \delta}{dx^2} \right) \left\{ 1 + \left( \frac{d\delta}{dx} \right)^2 \right\}^{3/2} \] (3)

\[ P_d = \frac{\bar{A}}{\delta^3} \] (4)

Differentiating equations (2), (3) and (4) the following equation is obtained:

\[ \frac{d^3 \delta}{dx^3} = \frac{3}{1 + \left( \frac{d\delta}{dx} \right)^2} \left( \frac{d^2 \delta}{dx^2} \right)^2 \left( \frac{dP_l}{dx} - \frac{dP_v}{dx} + 3 \frac{\bar{A}}{\delta^4} \frac{d\delta}{dx} \right) \] (5)

The equation for liquid pressure gradient (6) and vapour pressure gradient (7) is obtained from the lubrication approximation and momentum equation respectively:

\[ \frac{dP_l}{dx} = \mu_l \left( \frac{d^2 u_l}{dy^2} \right) \] (6)

\[ \frac{dP_v}{dx} = -\mu_v \left( \frac{d^2 u_v}{dy^2} \right) \] (7)

Equations (6) and (7) are integrated. Boundary conditions used are:

- **For equation (6):** Here, shear and slip boundary conditions are applied.
  
i. At \( y = \delta \), \( \frac{du_l}{dy} = \frac{\tau_l}{\mu_l} \)
  
ii. At \( y = 0 \), \( u_l = \beta \frac{du_l}{dy} \)

Here \( \beta = 5 \times 10^{-9} m \) [13].

- **For equation (7):**
  
i. At \( y = H/2 \), \( \frac{du_v}{dy} = 0 \)
  
ii. At \( y = \delta \), \( u_v = u_\delta \)

This gives:

\[ u_l = \frac{1}{2\mu_l} \frac{dP_l}{dx} + \frac{\tau_l}{\mu_l} (y + \beta) \] (8)

\[ u_v = u_\delta - \frac{1}{2\mu_v} \frac{dP_v}{dx} + (y + \delta - H) \] (9)

The value of \( u_\delta \) can be found out from equation (8). Now, we know that:

\[ \dot{m}_l = \int_0^\delta \rho_l u_l dy \] (10)

\[ \dot{m}_v = \int_0^{H/2} \rho_v u_v dy \] (11)

Substituting the values of \( u_l \) and \( u_v \) in equations (10) and (11) respectively:

\[ \frac{dP_l}{dx} = -\frac{3v_l}{\delta^2 (\delta + 3\beta)} \left\{ \dot{m}_l - \frac{\tau_l \delta (\delta + \beta)}{v_l} \right\} \] (12)

\[ \frac{dP_v}{dx} = -\frac{24v_v}{H^3 + 6H \delta (\delta - H)} \left\{ \dot{m}_v - \rho_v u_\delta \frac{H}{2} \right\} \] (13)

Now at the liquid-vapour interface, the shear stress for vapour is equal to the shear stress for liquid.

\[ \therefore \tau_l = \mu_l \left( \frac{\partial u_l}{\partial y} \right)_{y=\delta} = \mu_v \left( \frac{\partial u_v}{\partial y} \right)_{y=\delta} \]
Table 1. List showing different values of first and second derivative of \( \delta \) from literature

| Paper | \( \frac{d\delta}{dx} \) | \( \frac{d^2\delta}{dx^2} \) |
|-------|----------------|----------------|
| [13]  | 0              | 0              |
| [18]  | 0              | \( 5 \times 10^6 \) |
| [4]   | 0              | Far-field boundary condition |
| [3],[14] | slightly greater than zero | Far-field boundary condition |

Substituting \( \tau_i \) at equations (12) and (13),

\[
\frac{dP_l}{dx} = -\frac{3v_l}{\delta^2(\delta + 3\beta)} \left( \dot{m}_l + \frac{\delta}{v_l} \left( \frac{\delta}{2} + \beta \right) \left( \delta - \frac{H}{2} \frac{dP_l}{dx} \right) \right) 
\]

\[
\frac{dP_v}{dx} = -\frac{24v_v}{H^3 + 6H\delta(\delta - H)} \left[ \dot{m}_v + \rho_v \frac{H}{2} \frac{1}{2\mu_i} \frac{dP_l}{dx} \right] 
\]

\[
+ \frac{1}{2\mu_i} (\delta + \beta)(2\delta - H) \frac{dP_v}{dx} \quad (14)
\]

Mass flow rates of liquid and vapour are given by:

\[
\dot{m}' = -\frac{d\dot{m}_l}{dx} = \frac{d\dot{m}_v}{dx} 
\]

(16)

Two different correlations are used to evaluate \( \dot{m}' \):

- **Linear Model:**

\[
\dot{m}'' = \frac{q''}{\lambda} 
\]

(17)

- **Wayner Model** (from Wayner et al. [2]):

\[
\dot{m}'' = \frac{2\dot{\delta}}{2 - \dot{\delta}} \left( \frac{\bar{M}}{2\pi R} \right)^{1/2} \left[ a(T_{lv} - T_v) - b(P_v + P_d) \right] 
\]

(18)

Here, the working fluid considered is a non-polar liquid. Thus, \( \sigma^v \) is taken as unity. Correlation of the other parameters are given below:

\[
T_{lv} = T_w - \frac{\delta\dot{m}''\lambda}{k_l} 
\]

(19)

\[
a = \left( \frac{P_v\bar{M}\lambda}{\bar{R}T_vT_{lv}^\frac{3}{2}} \right) 
\]

(20)

\[
a = \left( \frac{V_lP_v}{\bar{R}T_{lv}^3/2} \right) 
\]

(21)

Equation (19) is the same equation which Moosman and Homsy [17] used to modify the Wayner’s Model by eliminating \( T_v \).

2.2. **Boundary conditions**

So, the equations to be solved are: (5), (14), (15) and (16). Boundary conditions (@x = 0) consists of:

\[
\delta = \delta_0, \quad \frac{d\delta}{dx} = 0, \quad \frac{d^2\delta}{dx^2} = 0, \quad \dot{m}_l = 0, \quad \dot{m}_v = \dot{m}_{l(\sigma = L)}, \quad P_v = P_{sat},
\]
Figure 3. Comparision of: (a) thin-film thickness, (b), Liquid Pressure and (c) Disjoining Pressure, at x = 0 calculated from ‘Wayner’ and ‘Clausius-Clapeyron’ method for different working fluid at same saturation temperature and superheat
Figure 4. Effect of superheat on the mass flow rate of vapour

Figure 5. Effect of superheat on the thin-film thickness: (a) Overall variation, (b) Initial variation
Figure 6. Effect of superheat on the liquid pressure

Figure 7. Effect of superheat on the liquid pressure

Figure 8. Variation of different pressure components along the meniscus length
\[ P_l = P_v - \frac{\bar{A}}{\delta^3} - \sigma \frac{d^2 \delta}{dx^2} \left\{ 1 + \left( \frac{d\delta}{dx} \right)^2 \right\}^{3/2} \]

Table 1 shows different values of first and second derivative of film thickness available in literature. As can be seen from the table their values are taken from Akkus and Dursunkaya [10] and Park et al [13], although Du and Zhao [9] have advised not to use the same. They are still used because it is relatively easy to implement them as compared to far field boundary conditions.

The mass flow rate of vapour should always be zero at the end of the thin film region [13, 19] and the mass flow rate of liquid is found out by ignoring the vapour pressure gradient while running the calculation. Whatever the liquid flow rate that we get is substituted into the vapour flow rate at \( x = 0 \).

\( \delta_0 \) is calculated by any of the following two ways:

1. **Using Wayner’s mass flux model:** Here, \( \dot{m}'' = 0 \) and \( T_v = T_w \) are substituted in equation (18) (At \( x = 0 \), there is no heat transfer, so, the mass flux is taken as zero and since the film is very thin here, the interface temperature is almost the same as the wall temperature). Thus, the following expression is derived:

\[
\delta_0 = \left( \frac{\bar{A} T_v}{\rho_v \lambda (T_w - T_v)} \right)^{1/3} \tag{22}
\]

In this study, this method will be termed Wayner Method.

2. **Using Clausius–Clapeyron equation:** Here, the values of \( \dot{m}'' \) and \( T_v \) at \( x=0 \) are substituted in Clausius–Clapeyron equation and the following expression is derived:

\[
\delta_0 = \left( \frac{\bar{A} T_v}{\rho_v \lambda (T_w - T_v)} \right)^{1/3} \tag{23}
\]

This method will be termed Clausius–Clapeyron Method.

As can be seen from the system of boundary conditions, proper calculation of \( \delta_0 \) is crucial for determination of liquid pressure at \( x = 0 \) and thus for overall solution. Figure 3 compares the various parameters at \( x = 0 \) calculated from the two methods. Working fluid considered are some of the common fluid used in these problems i.e octane, ammonia, acetone and pentane. The saturation temperature was kept at a constant value of 30°C and superheat of 1K. From figure 3(a), it is clear that \( \delta_0 \) calculated from equation (22) is very low. This gives negative liquid pressure (figure 3(b)) which does not satisfy the underlying physics of the problem. The same problem was also reported by Wang et al. [20]. Although various researchers ([3], [4]) have advised to take a slightly higher value than calculated, the problem still persist. Equation (23) on the other hand, gives high value of \( \delta_0 \). This gives relatively thick film at \( x = 0 \) which leads to low value of disjoining pressure (figure 3(c)). This also goes against the nature of the problem. Simplest solution is to take a value in between the values calculated from equation (22) and (23). In this paper, an attempt is made to define \( (\delta_0)_{\min} \) and \( (\delta_0)_{\max} \) which would satisfy the physics of thin films. As can be seen from figure 2, \( x = 0 \) point lies in between the adsorbed region and the thin film region. As such at that point evaporation has not started yet and disjoining pressure still dominates. Thus, the governing equation here is a slightly modified form of equation (2).

\[
P_v - P_l = P_d
\]

\[
\Rightarrow P_v - P_l = \frac{\bar{A}}{\delta_0^3} \tag{24}
\]

Now, as \( \delta_0 \rightarrow (\delta_0)_{\min} \), \( P_v - P_l \rightarrow P_v \). So, equation (24) becomes:

\[
(\delta_0)_{\min} = \left( \frac{\bar{A}}{P_v} \right)^{1/3} \tag{25}
\]
(δ₀)_{max} is assigned the value calculated from equation (23). Next, average or any value between (δ₀)_{min} and (δ₀)_{max} is taken to be δ₀.

3. Results
The system of equations is solved taking octane as working fluid and saturation temperature, 70°C. To verify the results, mass flow rate of vapour and thin film profile are plotted for different superheat. As stated earlier, mass flow rate of vapour is always zero at the end of the thin film region. Figure 4 clearly exhibit such a trend. Figure 5 shows that the thin film thickness increases with increase in superheat (both overall and at the onset of thin-film region) but its length decreases which agrees with the results found in the literature. The reason is that high superheat increases the rate of evaporation and thus more liquid is pumped into the thin film. This also increases the liquid pressure as is clear from figure 6.

Figure 7 shows how the vapour pressure gradient affects the profile of thin film for different evaporating mass flux model. As can be clearly seen from the figure, vapour pressure gradient plays a negligible role in the thin film region for capillary cooling devices. This can also be confirmed from Figure 8 where effect on different components of pressure (capillary, disjoining, liquid and vapour) along the film length is shown. The figure clearly shows that the disjoining pressure decreases and the capillary pressure increases as the film progresses from the thin film region to the capillary region as expected. However, the vapour pressure remains constant throughout the extended meniscus. As such the vapour pressure gradient can be neglected. This justifies the assumption of constant vapour pressure by most of the researchers.

4. Conclusion
Following are the conclusions:
   i. δ₀ calculated from both equations (22) and (23) do not satisfy the physics of the problem and may give unrealistic solution.
   ii. The vapour pressure is uniform throughout the extended meniscus of two phase capillary devices and hence its gradient can be neglected.
   iii. A proper choice of the evaporating mass flux model is critical in the analysis of evaporating thin films of two-phase capillary devices, which is very important for predicting their heat transfer characteristics.

Nomenclature

**General Symbols**

| Symbol | Description |
|--------|-------------|
| A | Dispersion constant (J) |
| M | Molecular mass (kg/kmol) |
| R | Universal gas constant (J/kmol·K) |
| H | Channel height (m) |
| k | Thermal conductivity (W/mK) |
| L | Length (m) |
| m'''' | Interface net mass flux (kg/m²s) |
| m | Mass flow rate (kg/ms) |
| P | Pressure (Pa) |
| q'''' | Heat flux (W/m²) |
| T | Temperature (K) |

**Greek Symbols**

| Symbol | Description |
|--------|-------------|
| β | Slip coefficient |
| δ | Thin film thickness (m) |
| λ | Latent heat of vaporisation (J/kg) |
| μ | Coefficient of viscosity (Pa·s) |

**Subscripts**

| Symbol | Description |
|--------|-------------|
| c | Capillary |
| cc | Clausius-Clapeyron |
| d | Disjoining |
| l | Liquid |
| lv | Liquid-vapour interface |
| max | Maximum |
| min | Minimum |
| sat | Saturation |
| v | Vapour |
| w | Wall |
| way | Wayner |

**Abbreviations**

| Symbol | Description |
|--------|-------------|
| FoM | Figure of merit |
\( \rho \) Density (kg/m\(^3\)) \hspace{1cm} \text{HP} \hspace{1cm} \text{Heat pipe}

\( \sigma \) Surface tension (N/m) \hspace{1cm} \text{IL} \hspace{1cm} \text{Interline}

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