The Vertical Dynamics of the Car, Taking into Account the Irregularity of the Track Gauge

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Abstract—The paper deals with the vertical dynamics of the car to ensure the safety of trains, the reliability of the cars and the track. It is accepted that the dynamic model of the car has five degrees of freedom. The complex oscillatory process is divided into separate components: vertical, transverse and longitudinal horizontal. It is taken into account that the spring-suspended weight of the car can also make angular oscillations. The equations of motion of a mechanical system are composed using the second-kind Lagrange equations by first generating the expressions for potential and kinetic energy. Since one of the most important problems in the dynamics of rolling stock is the study of forced oscillations of railway vehicles caused by irregularities of rails, therefore, the formulas of Professor N.N. Kudryavtsev were used as a kinematic effect on bogies. In the model being developed, the two wheels of the car trolley will be considered as one, then the equivalent perturbation is the average value of the perturbations transmitted to each wheel, that is, the pitching of the bogies can be neglected. Isolated perturbations are determined using reasoning about the time of passage of the wheels over the track irregularity. The obtained differential equations are rather complicated for an analytical solution. Therefore, to find solutions, the MathCAD mathematical software package was used. It provides a set of built-in functions for the numerical solution of differential equations. The mass-inertial characteristics and geometrical dimensions of the gondola car of model 12-132 are taken as input data. The speed of movement of the car and the length of the irregularities varied in a fairly wide range. As a result of numerical modeling, graphs of linear and angular oscillations versus irregularity lengths were obtained. It was found that the amplitude of oscillations varies within certain limits from 0 to 0.007 (vertical oscillations) and from $-10^{-4}$ to $+10^{-4}$ rad. (for the speed of 20 m/s and the length of the irregularity of 25 m).

Keywords—track gauge; vertical dynamics; safety of trains; dynamic model; oscillatory process

I. INTRODUCTION

The processes of interaction between cars and railways are challenging to study [1, 2]. The results of the research of their interaction are necessary to create reliable and durable designs of cars and tracks, determine the regulations of their arrangement, rules of repair and technical maintenance [3].

Ensuring the safety of train traffic [4], the reliability of the car fleet and the railway track, in case of maximum performance, minimal labor and energy resources, cannot be successfully implemented without knowing the processes of interaction between rolling stock and infrastructure [5], which are ultimately reduced to interconnected random oscillations of various elements of cars and track. As a result, significant residual deformations of the track, loss of stability of cars on rails, the breakdowns of individual elements and parts due to their fatigue or fragility may occur.

Studies of flat and spatial oscillations of cars, as they move along determinate irregularities of the track, are the scientific basis for the rational designing and operation of the car fleet and railway track, as well as a means of intensifying their use,
increasing the track capacity and turnover, ensuring traffic safety from the viewpoint of steady position of wheels on the rail and the car body transverse tipping stability when driving along curved sections of the track.

Under real-life conditions, the rails and wheels have irregularities on the rolling surface, as well as other technological features, as a result, oscillations occur in the elements of the railway track and rolling stock.

One of the most important problems in the dynamics of rolling stock is the study of forced oscillations of railway vehicles [6-11], caused by uneven rails.

When studying the processes of interaction between rolling stock and railway track, oscillations of cars, track and dynamic forces, developing in a single dynamic system “car-track” are investigated. For a theoretical study of oscillations of this type, it is customary to build computational schemes and models in which this complex oscillatory process is divided into separate components: vertical, transverse, and longitudinal horizontal. With that, it should be taken into account that the spring-suspended weight of the car can also execute angular oscillations.

II. CONSTRUCTION OF A MATHEMATICAL MODEL

We consider the mechanical scheme of the car, (Figure 1 a shows the right rail in the direction of travel, and Figure 1 b shows the left rail and the left side of the suspension. Points B1-B4 are attachment points for suspensions), consisting of the body and bogies. We assume that the car body and bogies execute vertical and angular oscillations (bouncing, pitching and rolling).

When considering a dynamic model of a freight car, we take vertical irregularities in the form of determinate periodic perturbations with a period equal to twice the length of the irregularity. In particular, this law can be adopted in the following form (N.N. Kudryavtsev’s model of irregularity):

\[
f(t) = |A_1 \sin \alpha t + A_2 \sin 3\alpha t|,
\]

for the right rail;

\[
f(t) = |A_1 \sin \alpha t + A_3 \sin 3\alpha t|,
\]

for the left rail,

where \(\alpha = \frac{\pi v}{l_1}\), \(\alpha_3 = \frac{\pi v}{l_2}\) (2\(l_1\) and 2\(l_2\) is the irregularity length, \(v\) is the speed of the car), coefficients \(A_1\), \(A_2\), \(A_3\) and \(A_4\) are selected depending on the type and condition of the track.

In the model being developed, the two wheels of the car bogie will be considered as one, then the equivalent perturbation is the average value of the perturbations transmitted to each wheel, that is, the pitching of the bogies can be neglected.

Isolated perturbations, as shown in Fig. 1, can be determined using the following reasoning. The second wheel passes the same point of irregularity of the track, which at the moment of time \(t\) the first wheel passes at the moment of time \(t + \frac{L_{21} + L_{22}}{v}\). Thus:

\[
z_{e1} = f_1(t), \quad z_{e2} = f_1\left(t + \frac{L_{21} + L_{22}}{v}\right).
\]

The third wheel will pass the same point of irregularity at the moment of time,

\[
t_3 = t + \left(L_4 + L_2\right) + \left(L_{21} - L_{31}\right) \frac{v}{L_2},
\]

and the fourth at the time of

\[
t_4 = t + \left(L_4 + L_2\right) - \left(L_{21} + L_{32}\right) \frac{v}{L_2}.
\]

Similar expressions can be written from the fifth wheel to eighth one.

Finally we get:

\[
\begin{align*}
z_{e3} &= f_2\left(t + \frac{L_1 + L_2}{v}\right),
\end{align*}
\]

\[
\begin{align*}
z_{e4} &= f_2\left(t + \frac{L_1 + L_2 + L_{21} - L_{31}}{v}\right),
\end{align*}
\]

\[
\begin{align*}
z_{e5} &= f_2(t),
\end{align*}
\]

\[
\begin{align*}
z_{e6} &= f_2\left(t + \frac{L_{21} + L_{22}}{v}\right),
\end{align*}
\]

\[
\begin{align*}
z_{e7} &= f_2\left(t + \frac{L_1 + L_2}{v}\right) + \left(L_{21} - L_{31}\right),
\end{align*}
\]

\[
\begin{align*}
z_{e8} &= f_2\left(t + \frac{L_1 + L_2}{v}\right) + \left(L_{21} + L_{32}\right).
\end{align*}
\]

The equivalent perturbations for the bogies will be given in the form:

\[
\begin{align*}
\eta_1 &= \frac{z_{e1} + z_{e2}}{2}, \quad \eta_2 = \frac{z_{e3} + z_{e4}}{2},
\end{align*}
\]

\[
\begin{align*}
\eta_3 &= \frac{z_{e5} + z_{e6}}{2}, \quad \eta_4 = \frac{z_{e7} + z_{e8}}{2}.
\end{align*}
\]

In this regard, the bogies will only execute vertical (bouncing) vibrations.

In accordance with (3) we have:

\[
\begin{align*}
\eta_1 &= \frac{z_{e1} + z_{e2}}{2}, \quad \eta_2 = \frac{z_{e3} + z_{e4}}{2},
\end{align*}
\]

\[
\begin{align*}
\eta_3 &= \frac{z_{e5} + z_{e6}}{2}, \quad \eta_4 = \frac{z_{e7} + z_{e8}}{2}.
\end{align*}
\]
where

\[ \dot{z}_{41} = \{ A_1 \cos \omega t + A_2 \cos 3\omega t \} \times \]
\[ \times \text{sgn} \{ A_1 \sin \omega t + A_2 \sin 3\omega t \}, \]

\[ \dot{z}_{42} = \{ A_1 \cos \omega t + A_2 \cos 3\omega t \} \times \]
\[ \times \text{sgn} \{ A_1 \sin \omega t + A_2 \sin 3\omega t \}, \]

\[ \dot{z}_{43} = \{ A_1 \omega_1 \cos \omega t_1 + A_2 \omega_1 \cos 3\omega t_1 \} \times \]
\[ \times \text{sgn} \{ A_1 \sin \omega t_1 + A_2 \sin 3\omega t_1 \}, \]

\[ \dot{z}_{44} = \{ A_1 \cos \omega t_4 + A_2 \cos 3\omega t_4 \} \times \]
\[ \times \text{sgn} \{ A_1 \sin \omega t_4 + A_2 \sin 3\omega t_4 \}, \]

\[ \dot{z}_{45} = \{ A_1 \cos \omega t_5 + A_2 \cos 3\omega t_5 \} \times \]
\[ \times \text{sgn} \{ A_1 \sin \omega t_5 + A_2 \sin 3\omega t_5 \}, \]

\[ \dot{z}_{46} = \{ A_1 \omega_1 \cos \omega t_6 + A_2 \omega_1 \cos 3\omega t_6 \} \times \]
\[ \times \text{sgn} \{ A_1 \sin \omega t_6 + A_2 \sin 3\omega t_6 \}, \]

\[ \dot{z}_{47} = \{ A_1 \cos \omega t_7 + A_2 \cos 3\omega t_7 \} \times \]
\[ \times \text{sgn} \{ A_1 \sin \omega t_7 + A_2 \sin 3\omega t_7 \}, \]

\[ \dot{z}_{48} = \{ A_1 \omega_1 \cos \omega t_8 + A_2 \omega_1 \cos 3\omega t_8 \} \times \]
\[ \times \text{sgn} \{ A_1 \sin \omega t_8 + A_2 \sin 3\omega t_8 \}. \]

To study the vibrations of the sprung parts of the car, the following designations are used:

- \( m_k \), \( m_{11} \), \( m_{12} \) - the mass of the body and bogies, respectively;
- \( c_{11} \), \( c_{12} \), \( c_{13} \), \( c_{14} \) - the vertical stiffness of the central suspension of the bogie;
- \( c_{21} \), \( c_{22} \), \( c_{31} \), \( c_{32} \), \( c_{41} \), \( c_{42} \), \( c_{51} \), \( c_{52} \) - the vertical stiffness of the axle box suspension of the wheelset;
- \( \beta_{11} \), \( \beta_{12} \), \( \beta_{13} \), \( \beta_{14} \) - damping coefficients of the central suspension of the bogie;
\[
\begin{align*}
\beta_1, \beta_2, \beta_3, \beta_4, \beta_a, \beta_b, \beta_1, \beta_2 - & \text{ the damping coefficients of the axle box suspension of the wheelset;}
\end{align*}
\]
\[
\begin{align*}
z_K, z_{11}, z_{12} - & \text{ the current vertical movement of the center of gravity, respectively, of the body, the first and second bogies;}
\end{align*}
\]
\[
\begin{align*}
L_1 + L_2 - & \text{ the longitudinal base of the body,}
\end{align*}
\]
\[
\begin{align*}
b_1 + b_2 - & \text{ the transverse base of the body.}
\end{align*}
\]

Writing the kinetic, potential energy, the dissipation function and using the Lagrange equations of the second kind, we obtain the differential equations of forced oscillations:

\[
m_k \ddot{z}_k + \left( c_{11} + c_{12} + c_{13} + c_{14} \right) z_k +
+ (c_{11}L_2 - c_{12}L_2 + c_{13}L_4 - c_{14}L_2) \ddot{\phi}_k +
+ (-c_{11}b_1 - c_{12}b_1 + c_{13}b_2 + c_{14}b_2) \ddot{\psi}_k -
- c_{11}z_{11} - c_{12}z_{12} - c_{13}z_{13} - c_{14}z_{14} +
+ (\beta_1L_1 - \beta_2L_2 + \beta_3L_1 - \beta_4L_2) \ddot{\phi}_k +
+ (-\beta_1b_1 - \beta_2b_1 + \beta_3b_2 + \beta_4b_2) \ddot{\psi}_k -
- \beta_1z_{11} - \beta_2z_{12} - \beta_3z_{13} - \beta_4z_{14} = 0;
\]
\[
I_y \dddot{\phi}_k + \left( c_{11}L_1 - c_{12}L_2 + c_{13}L_4 - c_{14}L_2 \right) \dot{z}_k +
+ (c_{11}L_2 + c_{12}L_1 + c_{13}L_2 - c_{14}L_1) \dot{\phi}_k +
+ (c_{11}b_1 + c_{12}b_1 + c_{13}b_2 + c_{14}b_2) \dot{\psi}_k +
+ (-c_{11}z_{11} - c_{12}z_{12} - c_{13}z_{13} + c_{14}z_{14} +
+ (\beta_1L_1 - \beta_2L_2 + \beta_3L_1 - \beta_4L_2) \dot{\phi}_k +
+ (-\beta_1b_1 - \beta_2b_1 + \beta_3b_2 + \beta_4b_2) \dot{\psi}_k -
- \beta_1z_{11} - \beta_2z_{12} - \beta_3z_{13} - \beta_4z_{14} = 0;
\]
\[
I_y \dddot{\psi}_k + \left( c_{11}L_1 - c_{12}L_2 + c_{13}L_4 - c_{14}L_2 \right) \dot{z}_k +
+ (c_{11}L_2 + c_{12}L_1 + c_{13}L_2 - c_{14}L_1) \dot{\phi}_k +
+ (c_{11}b_1 + c_{12}b_1 + c_{13}b_2 + c_{14}b_2) \dot{\psi}_k +
+ (-c_{11}z_{11} - c_{12}z_{12} - c_{13}z_{13} + c_{14}z_{14} +
+ (\beta_1L_1 - \beta_2L_2 + \beta_3L_1 - \beta_4L_2) \dot{\phi}_k +
+ (-\beta_1b_1 - \beta_2b_1 + \beta_3b_2 + \beta_4b_2) \dot{\psi}_k -
- \beta_1z_{11} - \beta_2z_{12} - \beta_3z_{13} - \beta_4z_{14} = 0;
\]
\[
m_{12} \dddot{z}_{12} - c_{12}z_{12} + c_{12}L_2 \ddot{\phi}_k + c_{12}b_2 \ddot{\psi}_k +
+ (c_{11} + c_{12} + c_{13} + c_{14}) z_{12} + c_{12}L_2 \ddot{\phi}_k + c_{12}b_2 \ddot{\psi}_k +
+ (\beta_1 + \beta_2 + \beta_3 + \beta_4) z_{12} = (c_{11} + c_{12}) \eta_1 + (c_{13} + c_{14}) \eta_3;
\]

**III. NUMERICAL SOLUTION**

It is rather difficult to obtain an analytical solution of this system of equations, even if we use the principal coordinates [6]. Therefore, we use the MathCAD mathematical software package to study it. It provides a set of built-in functions for the numerical solution of differential equations.

As an example, we accept the mass-inertial characteristics and geometrical dimensions of the gondola car of model 12-132. We assume that damping is small; therefore, we will not take it into account.

The speed of the car was taken equal to 10, 20 and 25 m/s, and the length of the irregularities was equal to 12.5, 25 and 50 m.

Fig. 2 - 4 show body oscillations with a length of irregularity 2l1 = 2l2 = 50 m and speed of movement \( v = 15 \) m/s.

Figure 5 - 7 show body oscillations with a length of irregularity 2l1 = 2l2 = 25 m and speed of movement \( v = 15 \) m/s.
IV. Conclusion

1. For the considered mechanical system, the transition from seven degrees of freedom to five is justified (the equivalent perturbation is the average value of the perturbations).

2. As with a system with three degrees of freedom, the frequency of oscillations of the body increases with increasing speed.

3. The amplitude of body oscillations increases with increasing speed and length of irregularities.

4. It was found that the amplitude of oscillations varies within certain limits from 0 to 0.007 (vertical oscillations) and from $-10^{-4}$ to $+10^{-4}$ rad. for the angular oscillations (the speed is 20 m/s and the length of the irregularity is 25 m).

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