A recent interesting paper [1] revisits the problem of (in)stability of AdS under spherically symmetric massless scalar field perturbations, first studied in [2]. The authors claim that the set of initial data which do not trigger instability is larger than originally envisioned in [2], in particular it comprises small amplitude two-mode initial data with energy equally distributed among the modes. To support this claim, the long time numerical evolution of these data was shown to be stable against black hole formation. The aim of this comment is to demonstrate that this numerical result is incorrect. Hereafter, we use the notation and references to equations of [1].

Using our code (see [3] for the detailed description), we solved the system of equations (2-4) for the two-mode initial data (20) with \( \kappa = 3/5 \) and \( \varepsilon = 0.09 \) used in Fig. 3 in [1]. The comparison of our result with the one of [1] is shown in Fig. 1 which depicts the upper envelope of the quantity \( \Pi^2(t,0) \) (related linearly to the Ricci scalar at the origin).

To feel confident that our computation is correct (as opposed to the one of [1]), we have validated it by convergence tests. The evidence for the expected fourth-order convergence is shown in Fig. 2.

![Fig. 1: The upper envelope of \( \Pi^2(t,0) \) for solutions starting from the two-mode initial data (20) with \( \kappa = 3/5 \) and \( \varepsilon = 0.09 \). Superimposed (red curve) is the numerical result of [1].](image1)

![Fig. 2: The late time convergence test for the simulation shown in Fig. 1 is depicted in blue. The convergence factor for the solution \( \Phi_{N} \) computed on the grid of size \( 2^N \) is defined by \( Q_N = \log_2 \left( \frac{||\Phi_N - \Phi_{N-1}||}{||\Phi_{N-1} - \Phi_{N-2}||} \right) \), where \( || \cdot || \) is the spatial \( \ell_2 \)-norm. The convergence tests for coarser grids are added in order to illustrate the degradation of convergence caused by an insufficient spatial resolution.](image2)

We stress that in numerical simulations of turbulent phenomena the convergence test is an indispensable tool of verifying whether small spatial scales are properly resolved. We suspect that the numerical solution depicted by the red curve in Fig. 1 suffered from the gradual loss of spatial resolution (presumably due to a too coarse grid or/and ineffective adaptive mesh refinement) and, as a result of that, the simulation stepped over the collapse and went off track. Unfortunately, the ‘visual’ convergence test shown in Fig. 3 of the Supplementary Material to [1] was stopped much too early to spot the loss of resolution.

In conclusion, contrary to the claim made in [1], the question of existence of a threshold for black hole formation in the evolution of the two-mode initial data (20) remains open. The resolution of this question seems very challenging because the computational cost of simulations rapidly increases with \( 1/\varepsilon \).

**Acknowledgments:** We thank the authors of [1] for sending us the data file for the red curve in Fig. 1. This work was supported in part by the NCN grant NN202 030740.
[1] V. Balasubramanian, A. Buchel, S.R. Green, L. Lehner, S.L. Liebling, *Holographic Thermalization, stability of AdS, and the Fermi-Pasta-Ulam-Tsingou paradox*, Phys. Rev. Lett. 113, 071601 (2014)

[2] P. Bizoń, A. Rostworowski, *On weakly turbulent instability of anti-de Sitter space*, Phys. Rev. Lett. 107, 031102 (2011)

[3] M. Maliborski, A. Rostworowski, *Lecture Notes on Turbulent Instability of Anti-de Sitter Spacetime*, International Journal of Modern Physics A, Vol. 28, 1340020 (2013)