Zeros of the order parameter of \( d_{x^2-y^2} \) superconducting film in the presence of uniform current

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We show that additional \( p \)-wave component is generated in the pure \( d \)-wave superconductor in the presence of the uniform current. When the current flows in the antinodal direction the spectrum has a gap over all Fermi surface. If the current flows in the nodal direction, the gap opens only in the direction parallel to the current. The correction to the current due to the \( p \)-wave component of the order parameter is linear in vector potential. We show, that the spin-orbit coupling is responsible for the additional component of the order parameter.

Recently it was proposed that classification of the order parameter in the ferromagnetic superconductor may be performed in accordance with corepresentations of nonunitary magnetic group \([1–3]\). Similar approach was used in Ref. \([4]\) to consider the effect of broken time-reversal symmetry at \( T^* \) on the superconducting state in high-\( T_c \) superconductors. Here we would like to extend this type of analysis for the case of a \( d \)-wave superconductor with uniform current or/and the external magnetic field.

There is a common belief that the high temperature superconductors have nontrivial order parameter transforming as \( B_{1g} (x^2 - y^2) \) representation of \( D_{4h} \) point group. When external magnetic field is applied the symmetry of the order parameter is reduced \([5,6]\). As a result, for magnetic field directed along the \( c \)-axis the new order parameter has the form: \( d_{x^2-y^2} + i d_{xy} \). Furthermore, additional component of the order parameter may be induced by the surface. Since any surface breaks inversion symmetry, additional \( p \) wave component is generated near the surface due to the surface induced spin orbit coupling \([7]\). In the following we show, that a constant superconducting current reduces the symmetry of the order parameter to \( d_{x^2-y^2} + ip_x \) when the current is directed along the \( x \)-axis. Such form of the order parameter is consistent with the group theory analysis and may be generated by the spin-orbit coupling. We will construct the corresponding invariant terms in the free energy and show how \( p \)-wave is generated from the microscopic BCS theory.

Since a constant superfluid current is a real vector that changes sign under time reversal transformation, the total \( D_{4h} \) group will be reduced to nonunitary (magnetic) group \( D_{2h} (C_{2v}) \). We consider for simplicity that superfluid current is directed along the \( x \)-axis. Unitary subgroup \( C_{2v} \) has four elements \( E, C_{2x}, \sigma_x, \sigma_y \) and four representations \( A_1, A_2, B_1, B_2 \). In accordance with the general theory of corepresentations \([8]\) all 4 irreducible representations belong to the class ‘a’. This means that each irreducible representation generates one nonequivalent corepresentation of the nonunitary group. All corepresentations are one dimensional and are listed in the Table 1 together with the basis functions.

| \( E \) | \( C_{2x} \) | \( \sigma_y \) | \( \sigma_y \) | \( RI \) | \( R\sigma_x \) | \( RC_{2x} \) | \( RC_{2y} \) | basis functions |
|---|---|---|---|---|---|---|---|---|
| \( A_1 \) | 1 | 1 | 1 | 1 | \( \epsilon \) | \( \epsilon \) | \( \epsilon \) | \( \epsilon \) | \( k_x^2 - k_y^2 + i\sigma_3 k_y \) |
| \( A_2 \) | 1 | 1 | -1 | -1 | \( \epsilon \) | \( \epsilon \) | -\( \epsilon \) | -\( \epsilon \) | \( k_y k_x + i\sigma_1 k_x + i\sigma_2 k_y \) |
| \( B_1 \) | 1 | -1 | 1 | -1 | -\( \epsilon \) | \( \epsilon \) | -\( \epsilon \) | -\( \epsilon \) | \( k_y k_x + i\sigma_3 k_y \) |
| \( B_2 \) | 1 | -1 | -1 | -1 | \( \epsilon \) | \( \epsilon \) | \( \epsilon \) | \( \epsilon \) | \( k_y k_x + i\sigma_1 k_x + i\sigma_2 k_y \) |

Here \( a \) and \( b \) are real numbers and \( \epsilon = \exp(2i\phi) \) where \( \phi \) is real. Following Ref. \([2]\) we introduced factor 2 in the exponent. To clarify the derivation of basis functions in Table 1 we construct the basis function for the \( A_1 \) corepresentation. It is easy to see that \( \exp(-i\phi)(k_x^2 - k_y^2) \) transforms as the \( A_1 \) representation of the unitary subgroup as well as corresponding corepresentation of the nonunitary group. Since the group in the presence of uniform current does not have the center of inversion, a mixture of singlet and triplet order parameter is possible. The general expression for the triplet part of the order parameter has the following form:

\[
\Psi_1 = \hat{x} f_x(k_x, k_y) + \hat{y} f_y(k_x, k_y) + \hat{z} f_z(k_x, k_y)
\]

where \( \hat{x}, \hat{y}, \hat{z} \) are spin unit vectors in the \( a, b, c \) axis directions respectively. The functions \( f_{x,y,z}(k_x, k_y) \) are odd with respect to the \( k \to -k \) transformation. Applying the \( \sigma_h \) operation to \( f_{x,y,z}(k_x, k_y) \) we obtain that \( f_{x,y}(k_x, k_y) = -f_{x,y}(k_x, k_y) \) for \( A_1 \) and \( B_1 \) representations. Therefore \( f_{x,y}(k_x, k_y) = 0 \) for \( A_1 \) and \( B_1 \) cases. Applying all operations from Table 1 to the remaining function \( f_z(k_x, k_y) \) we obtain the following set of equations:

\[
f_z(k_x, -k_y) = -f_z(k_x, k_y)
\]
the gap function has the form:
\[ f_x(-k_x, -k_y)^* = \epsilon f_x(k_x, k_y) \]
\[ f_z(-k_x, k_y)^* = -\epsilon f_z(k_x, k_y) \]  
(2)

It is easy to see that the function
\[ \Psi(k_x, k_y) = \exp(-i\phi)(d_{x-2}y^2 + i\eta p_x) \]  
(3)

where \( p_x = \hat{z}k_y \) satisfies all the equations and transforms as the \( A_1 \) corepresentation of the nonunitary group \( D_{2h}(C_{2v}) \). This means that the uniform current in the \( x \) direction generates the \( p_x \) wave contribution to the gap function. The gap function in that case is defined as \( \Delta(k_x, k_y) = i(d_{x-2}y^2 + i\eta\sigma_3 k_y)\sigma_2 \). Since \( \Delta \hat{\Delta} \propto \hat{\sigma}_0 \) the corresponding phase is unitary and the gap in the excitation spectrum is determined by \( |d_{x-2}y^2|^2 + \eta^2 k_y^2 \) [9]. As a result, the gap is fully developed over all Fermi surface.

Let us show that the term in the free energy:
\[ i\eta' \left( v_x p_x - v_y p_y \right) d_{x-2}y^2 + c.c., \]  
(4)

is an invariant of the group \( D_{4h} \). Here \( \eta' \) is a real constant and \( (v_x, v_y) \) are components of the superfluid velocity. Indeed, \( (v_x, v_y) \) and \( (p_x, p_y) \) transform as the \( E_u \) representations of the group \( D_{4h} \). Since the direct product \( E_u \times E_u = A_{1g} + A_{2g} + B_{1g} + B_{2g} \) contains \( B_{1g}(d_{x-2}y^2) \) representation, Eq.(4) represents the true scalar. The presence of \( i \) in Eq.(4) is important since the superfluid current is antisymmetric with respect to the time reversal symmetry.

The existence of the \( p \) component of the order parameter in the presence of the supercurrent may be seen if we write relevant Lifshitz invariant in the free energy [10]:
\[ F = F_d + i\eta'' \left( \psi_d^*(D_x p_x - D_y p_y) - \psi_d(D_x^* p_x^* - D_y^* p_y^*) \right) + \alpha_p(|p_x|^2 + |p_y|^2) \]  
(5)

where \( D_l = -i\nabla_l - 2eA_l \) \( (l = x, y, z) \), \( A \) is a vector potential, and \( F_d \) is the free energy for the \( d \)-wave superconductor without current. Assuming that \( \psi_d = \psi_0 \exp(i m v x) \) and \( (p_x, p_y) = (p_x, 0) \exp(i m v x) \) we obtain a contribution to the free energy, which is similar to Eq.(4). By minimizing Eq.(5) with respect to \( p_x \) we obtain the amplitude of the \( p \)-component of the order parameter: \( p_x = i\eta'' m v \psi_d/\alpha_p \).

When the current is flowing in the nodal direction, \( B_{1g}(x^2 - y^2) \) representation of the \( D_{4h} \) group is no longer \( A_1 \) corepresentation of the reduced nonunitary group. Group theoretical analysis for this case is presented in Table 2. It should be pointed out that this case is similar to the case II considered in the Ref. [4].

**Table 2. Corepresentations of the nonunitary group, when current flows in the nodal direction.**

| \( E \) | \( U_{xy} \) | \( \sigma_h \) | \( \sigma_{xy} \) | \( RI \) | \( R\sigma_{xy} \) | \( RC_{2z} \) | \( RU_{xy} \) | basis functions |
|------|----------|---------|---------|------|---------|-------|---------|------------------|
| \( A_1 \) | 1 | 1 | 1 | 1 | \( \epsilon \) | \( \epsilon \) | \( \epsilon \) | \( \epsilon \) | \( k_x^2 + k_y^2 + i\alpha_3(k_x - k_y) \) |
| \( A_2 \) | 1 | 1 | -1 | -1 | \( \epsilon \) | \( \epsilon \) | \( -\epsilon \) | \( -\epsilon \) | \( (k_x - k_y)k_z + i\alpha_1(k_y + k_z) \) |
| \( B_1 \) | 1 | -1 | 1 | -1 | \( \epsilon \) | \( -\epsilon \) | \( \epsilon \) | \( -\epsilon \) | \( k_x^2 - k_y^2 + i\alpha_3(k_z + k_y) \) |
| \( B_2 \) | 1 | -1 | -1 | 1 | \( \epsilon \) | \( -\epsilon \) | \( -\epsilon \) | \( \epsilon \) | \( (k_x + k_y)k_z + i\alpha_1(k_x - k_y) \) |

Similarly to the case when the current flows in the \( x \) direction, the corresponding phase is unitary: \( \Delta \hat{\Delta} \propto \hat{\sigma}_0 \), and the gap function has the form: \( |d_{x-2}y^2|^2 + a^2(k_x + k_y)^2 \). Contrary to the previous case, however, gap opens only in the direction parallel to the current, and the spectrum remains gapless in the direction perpendicular to the current [11].

Let us now discuss the correction to the current when the additional component of the order parameter is generated. It is easy to see from Eq.(5) that the additional contribution to the current is given by:
\[ j_x = j_{dx} + 2i\eta'' (\psi_d^* p_x - \psi_d p_x^*) \]  
(6)

Substituting \( p_x = i\eta'' m v \psi_d/\alpha_p \) to the last equation and taking into account that in the quasiclassical limit \( m v = -2eA \), correction to the supercurrent has the form:
\[ j = j_d + 4e^2 \eta'' \alpha^2 A |\psi_d|^2/\alpha_p \]  
(7)

At this point we should point out that usually the nonlinear London equation is discussed in the case of the \( d \)-wave superconductors with the nodes in the spectrum [11]. Nonlinear corrections to the London equation appear due to the
Doppler shift of the spectrum in the presence of the superflow. As a result, in some regions of the Fermi surface the excitation energy becomes negative leading to the finite quasiparticle current. However, as we have shown here, the generation of the $p$-wave component of the order parameter in the presence of the superflow leads to the correction to the supercurrent which is linear in the vector potential. Opening of the $p$ wave component of the order parameter leads to the linear in current correction to the penetration depth and could be detected experimentally.

At the end we suggest one possible microscopic mechanism, which causes the appearance of the $p$-component of the order parameter in the presence of uniform current. We assume that current flows along the $x$ axis. Spin-orbit coupling in that case could be written as

$$H_{so} = i\gamma \sum_k \psi_k^\dagger [\mathbf{v} \times \partial_k]_z \sigma_3 \psi_k$$

where $v$ is the superfluid velocity and $\gamma$ is the spin-orbit coupling constant. Following Balatsky [5] we can calculate the correction to the anomalous Green function $\hat{F}(k, \omega) = \Delta_0(k)/D(k, \omega)$, where $D(k, \omega) = \omega^2 + \xi(k)^2 + |\Delta_0(k)|^2$, using perturbation theory due to the interaction $H_{so}$. The first correction to $\hat{F}^0(k, \omega)$ is:

$$\delta \hat{F}(k, \omega) = i\gamma v \hat{G}^0(k, \omega) \partial_{k_y} \sigma_3 \hat{F}^0(k, \omega)$$

$$= i\gamma v \left(\frac{i\omega - \xi(k)}{D(k, \omega)^2}\sin(k_y)\sigma_2\right)$$

$$= -i\gamma v \left(\frac{\omega + \xi(k)}{D(k, \omega)^2}\sin(k_y)\right)$$

(8)

Here we take into account that $\hat{\Delta}_0(k) = \Delta_0(\cos(k_x) - \cos(k_y))\sigma_2$, and $\hat{G}^0(k, \omega) = (i\omega - \xi(k))\sigma_0/D(k, \omega)$. To estimate the $p$-wave correction to the order parameter we assume a repulsive separable interaction in the following form: $V_g(k, k') = V_g \sin(k_y)\sin(k_y')$. As a result, the correction to $\Delta_0(k)$ is given by:

$$\Delta_1(k) = T \sum_{\omega, k} V_g \sin(k_y)\sin(k_y')\delta \hat{F}(k', \omega)$$

$$= -iC\gamma v \frac{\Delta_0}{E_F} N_0 V_g \sin(k_y)\sigma_1$$

(9)

Here $C$ is a real constant of the order of 1, and $N_0$ is the density of states at the Fermi surface.

In conclusion, we have shown that additional $p$ wave component appears in the case of $d$-wave superconductor in the presence of uniform supercurrent. This effect leads to the additional contribution to the current linear in the vector potential. It is shown that spin orbit coupling is responsible for this effect.

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