Concentrated Document Topic Model

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Abstract

We propose a Concentrated Document Topic Model (CDTM) for unsupervised text classification, which is able to produce a concentrated and sparse document topic distribution. In particular, an exponential entropy penalty is imposed on the document topic distribution. Documents that have diverse topic distributions are penalized more, while those having concentrated topics are penalized less. We apply the model to the benchmark NIPS dataset and observe more coherent topics and more concentrated and sparse document-topic distributions than Latent Dirichlet Allocation (LDA).

1 Introduction

Probabilistic topic modeling is a popular method to cluster data into different groups and reduce the dimension. It has been applied in many different areas (Blei and Lafferty, 2007; Huang et al., 2017; Maier et al., 2018; Reisenbichler and Reutterer, 2019; Lei et al., 2020; Fei-Fei and Perona, 2005; Liu et al., 2016; González-Blas et al., 2019). Much attention in previous studies are focused on improving the estimated topic quality (Griffiths et al., 2005; Wallach et al., 2009; Das et al., 2015; Shi et al., 2017; Xu et al., 2018). And little is paid on the document-topic distributions. Yet in real world applications, the document-topic distributions are no less, if not more, important than the topic contents, i.e. the topic-word distributions. For example, in an information retrieval task, the document-topic distributions decide how accurately the relevant documents can be retrieved for a given topic. Existing probabilistic topic models make little assumptions on the document-topic distribution and rely on the posterior maximization. For example, Latent Dirichlet Allocation assumes that the document-topic distribution is drawn from a Dirichlet distribution and topics are drawn from it following a multinomial distribution. As a result of the minimal number of assumptions, the estimated document-topic distribution can take any form, as long as they are proper multinomial parameters, i.e. elements are non-negative and sum up to 1. One extreme case is that a document contains significant portions of all topics. This does not inline with the reality, in which documents only contain a few topics. In the information retrieval task, this document will not be accurately mapped to the users’ query. To fill this gap, we propose a Concentrated Document Topic Model (CDTM). Our proposed CDTM is able to produce concentrated and sparse document-topic distributions. In particular, we add the entropies of the document-topic distributions to the model posterior,
which encourages low entropy values, thus encouraging concentrated and sparse document-topic distributions.

Two earliest work in the field is Probabilistic Latent Semantic Indexing (PSLI) \cite{Hofmann1999} and Latent Dirichlet Allocation (LDA) \cite{Blei2003}. One difference of the two models is that PSLI assumes that every document has only one topic, while LDA allows multiple topics in a document. LDA assumes the document-topic parameter is drawn from a Dirichlet prior. Thus the topics are almost independent of each other. \cite{Blei2007} extend LDA to a correlated topic model. Instead of the Dirichlet distribution, it assumes the document-topic parameter is from a logistic normal distribution. The covariance matrix is used to model the topic correlation inside the documents. Their empirical experiment shows that the correlated topic model fits better than the LDA and supports more topics than LDA, i.e. the log probability of LDA on test dataset peaks near 30 topics, while that of the correlated topic model is 90 topics. \cite{Wallach2009} investigate the effect of asymmetric priors on the performance of LDA. The evaluation metric is the log-probability per word in the training dataset and probability on the test dataset. Their study shows that the combination of asymmetric prior on document-topic parameters and symmetric prior on topic-words parameters produces the best results on several benchmark dataset. They also provide a possible explanation of the superior performance of the combination. Namely, the asymmetric prior on document-topic parameters serves to share the common words across different documents and the symmetric prior on topic-word parameters serves to distinguish different topics. Their study in some way supports our proposed method.

Our proposed CDTM make use of entropy \cite{Shannon1948} to measure the topic concentration. Entropy is widely used in estimating the functional form of density. \cite{Zellner1988} propose to estimate the density by maximizing the entropy while subjecting to moments constraints. They show that the estimates takes an exponential polynomial form where the coefficients are numerically computed using Taylor series expansion and Newton’s method. \cite{Ryu1993} extends the Maximum Entropy (ME) density \cite{Zellner1988} to a flexible ME density by replacing the moments constraints with constraints on known functions and the ME regression functions by replacing the density with the regression function. They show that several well-known econometric functions, e.g. exponential polynomial, Cobb-Douglas, translog, generalized Leontief, Fourier flexible form can be derived using this approach. \cite{Park2009} combines the ME density estimator with ARCH series models \cite{Engle1982}. The moment constraints are extended to constraints with additional parameters. Several moment functions are used to capture excess kurtosis, asymmetry and high peakedness in financial data. Entropy has also been used as a penalization term. \cite{Gomes2007} use the entropy penalty as a regularization when estimating the viscosity solution of the Hamilton-Jacobi equation. \cite{Koltchinskii2009} theoretically study the sparsity estimation in convex hulls using entropy penalization. They show that the ‘approximate sparsity’ of the solution to the theoretical risk minimization problem under the entropy penalization implies the ‘approximate sparsity’ of its counterparty in the empirical risk minimization problem. They also explore various bounds on the excess risk of the empirical solution. A similar idea is explored by \cite{Koltchinskii2011} when estimating low-rank matrix. Instead of Shannon entropy, they use the von Neumann entropy as the penalty term. They show that when the target matrix is nearly low-rank, the empirical estimator is well approximated by low-rank matrices and $L_2$ error can be controlled in terms of the ‘approximate rank’ of the
target matrix. Entropy penalty is also widely used in deep learning and reinforcement learning. Williams and Peng (1991) and Mnih et al. (2016) find that penalizing the low entropy of the policy improves the reinforcement learning exploration by discouraging premature convergence to suboptimal deterministic policies. The entropy penalty is also used by Luo et al. (2017) to train an online sequence-to-sequence model. The entropy of the emission policy was added in the objective function. Pereyra et al. (2017) add the entropy of the output to the objective function. They test the proposed method on six common benchmarks, e.g. image classification, machine translation, etc and find that the penalty improves the state-of-art models across benchmarks without modifying existing hyperparameters.

The rest of this paper is organized in the following way. In section 2, we provide details of the proposed method and the algorithms to estimate the parameters. In section 3, we apply the proposed method to the public NIPS dataset. The results show that the proposed method improves the topic coherence and encourages concentration and sparsity on the document-topic distributions. We conclude the paper in section 4.

2 Method

The model setup is the same as LDA (Blei et al., 2003). Namely, given a corpus \( C \), we assume it contains \( K \) topics. Every topic \( \eta_k \) is a multinomial distribution on the vocabulary. Every document \( d \) contains one or more topics. The topic proportion in each document is governed by the local latent parameter document-topic \( \theta \), which has a Dirichlet prior with hyperparameter \( \zeta \). Every word in document \( d \) is generated from the contained topics as follows:

- for every document \( d \in C \), its topic proportion parameter \( \theta \) is generated from a Dirichlet distribution, i.e. \( \theta \sim Dir(\zeta) \).
- for every word in the document \( d \),
  - a topic \( Z \) is first generated from the multinomial distribution with parameter \( \theta \), i.e. \( Z \sim Multinomial(\theta) \)
  - a word \( w \) is then generated from the multinomial distribution with parameter \( \eta_Z \), i.e. \( w \sim Multinomial(\eta_Z) \)

The graphical representation is shown in Figure 1. The outer rectangle represents the document-level and the inner rectangle represents the word-level. \( \zeta \) and \( \eta \) are global parameters, i.e. shared by all the documents. \( \theta \) and \( Z \) are local latent variables. A complete Bayesian approach further assumes that topics \( \eta_1, \ldots, \eta_K \) are generated from a Dirichlet prior with hyperparameter \( \beta \). Here we use this formulation as in Blei et al. (2003) for the ease of adding a penalty.

The latent parameters in CDTM are estimated by maximizing the following penalized posterior.

\[
\max \ p(\theta, Z, \zeta, \eta | W) \prod_{d=1}^{D} \exp(-\lambda_d H(\theta_d)) \tag{1}
\]

where \( D \) is the number of documents in the corpus, \( H(\theta_d) = -\sum_{i=1}^{K} \theta_{di} \log \theta_{di} \) is the entropy...
Figure 1: The graphical representation. The outer box represents the document level. The inner rectangle represents the word level.

\[ p(\theta, Z, \zeta, \eta|W) = \prod_{d=1}^{D} p(\theta_d|\zeta) \prod_{n=1}^{N_d} p(Z_{dn}|\theta_d)p(w_{dn}|Z_{dn}, \eta) \]  

is the posterior and \( N_d \) is the number of words for document \( d \). In the model, documents are assumed to be exchangeable. In the following discussion, we omit the subscript \( d \) and limit the discussion for a single document. When a document only contains a couple of concentrated topics, the entropy \( H(\theta) \) would be small and penalty is close to 1. On contrary, when the document topics are equally distributed across all the topics, the entropy would achieve its largest value and the penalty value is closer to 0. Therefore, this productive penalty encourages concentrated and sparsity in document-topic distributions and penalizes diverse and equally distributed document-topic parameters.

The posterior \( p(\theta, Z, \zeta, \eta|W) \) is intractable. We maximize the Evidence Lower Bound (ELBO) (Blei et al., 2003, 2017)

\[ L(\zeta, \eta, \gamma, \phi) = E_q[\ln p(\theta, Z, W|\zeta, \eta) - \lambda H(\theta)] - E_q[\ln q(\theta, Z|\gamma, \phi)] - E_q[\ln q(\theta|\gamma)] + E_q[-\lambda H(\theta)] \]

where \( p(.) \) is the single document version of equation 2 and \( q(\theta, Z|\gamma, \phi) \) is the mean-field variational distribution

\[ q(\theta, Z|\gamma, \phi) = q(\theta|\gamma) \prod_{n=1}^{N} q(Z_n|\phi_n) \]
where $N$ is the number of words in a document, $q(\theta|\gamma) \sim \text{Dirichlet}(\gamma)$, and $q(Z_n|\phi_n) \sim \text{Multinomial}(\phi_n)$. $E_q$ represents the expectation under the variational distribution. The variational distribution decouples the $\theta$ and $Z_n$, and simplifies the intractable computation.

The ELBO $L(.)$ is maximized in an 'EM'-like steps. In the E-step, we maximize the ELBO w.r.t. the local latent parameters $\gamma, \phi$ for every document, while conditioning on the latent parameters $\zeta, \eta$. In the M-step, we maximize the ELBO w.r.t. to global latent parameters $\zeta, \eta$ while conditioning on the local latent parameters $\gamma, \phi$. Since the penalty is a function of document-topic distribution $\theta$, it doesn’t affect the estimation of $\phi$, which will be the same as that in Blei et al. (2003),

$$
\phi_{ni} \propto n_{iw} \exp E_q[\log(\theta_i)|\gamma]
$$

where

$$
\exp E_q[\log(\theta_i)|\gamma] = \Psi(\gamma_i) - \Psi\left(\sum_{l=1}^{K} \gamma_i\right)
$$

where $\Psi$ is the digamma function.

Taking out all the terms containing $\gamma$ from the ELBO, we have the following function

$$
L_{\gamma} = \sum_{i=1}^{K} (\Psi(\gamma_i) - \Psi(\sum_{l=1}^{K} \gamma_l))(\zeta_i + \sum_{n=1}^{N} \phi_{ni} - \gamma_i) - \log \Gamma(\sum_{l=1}^{K} \gamma_l) + \sum_{l=1}^{K} \log \Gamma(\gamma_l) + \lambda \left( \sum_{i=1}^{K} \gamma_l \Psi(\gamma_l)/ \sum_{l=1}^{K} \gamma_l - \Psi(\sum_{l=1}^{K} \gamma_l) + (K-1)/\sum_{l=1}^{K} \gamma_l \right)
$$

where $\Gamma$, $\Psi$ are the gamma and digamma function, $K$ is the number of topics, $N$ is the number of words in the document. The last term is $E_q[-\lambda H(\theta)]$, which is the expected entropy under the variational distribution (see appendix for the derivation) and not separable. There is no closed-form solution for equation (3). We use the coordinate descent algorithm to estimate the document-topic distribution parameter $\gamma$ for every document.

$$
\gamma_{i+1}^{s+1} := \text{argmin} -L_{\gamma}(\gamma_1^s, \ldots, \gamma_{i-1}^s, x, \gamma_i^{s+1}, \ldots, \gamma_K^s), i = 1, \ldots, K
$$

(4)

The minimization can be solved using Newton’s method.

$$
\gamma_{i+1}^t := \gamma_i^t - \alpha \frac{L'_{\gamma_i}}{L''_{\gamma_i}}
$$

(5)

where

$$
L'_{\gamma_i} = \Psi'(\gamma_i)(\zeta_i + \sum_{n=1}^{N} \phi_{ni} - \gamma_i) - \Psi'(\sum_{l=1}^{K} \gamma_l) \sum_{l=1}^{K} (\zeta_i + \sum_{n=1}^{N} \phi_{nl} - \gamma_l) + \lambda \left( (\Psi(\gamma_i) + \gamma_i \Psi'(\gamma_i)) / \sum_{l=1}^{K} \gamma_l - \sum_{l=1}^{K} \gamma_l \Psi(\gamma_l)/(\sum_{l=1}^{K} \gamma_l)^2 - \Psi'(\sum_{l=1}^{K} \gamma_l) + (K-1)/(\sum_{l=1}^{K} \gamma_l)^2 \right)
$$

is the partial derivative of $L_{\gamma}$ with respect to $\gamma_i$ and

$$
L''_{\gamma_i} = \Psi''(\gamma_i)(\zeta_i + \sum_{n=1}^{N} \phi_{ni} - \gamma_i) - \Psi''(\sum_{l=1}^{K} \gamma_l) \sum_{l=1}^{K} (\zeta_i + \sum_{n=1}^{N} \phi_{nl} - \gamma_l) + \Psi'(\sum_{l=1}^{K} \gamma_l) + \lambda \left( 2 \Psi'(\gamma_i) + \gamma_i \Psi''(\gamma_i) / \sum_{l=1}^{K} \gamma_l - 2(\Psi(\gamma_i) + \gamma_i \Psi'(\gamma_i))/(\sum_{l=1}^{K} \gamma_l)^2 + 2(K-1 + \sum_{l=1}^{K} \gamma_i \Psi(\gamma_i))/(\sum_{l=1}^{K} \gamma_i)^3 - \Psi''(\sum_{l=1}^{K} \gamma_l) \right)
$$
is the second partial derivative of $L_{[\gamma]}$ with respect to $\gamma_i$. The step $\alpha$ can be found using backtracking line search. The complete algorithm for the e-step for a document is given in Algorithm [1].

In the M-step, we maximize the ELBO with respect to $\eta$ and the updating equation is

$$\eta_{ij} \propto \sum_{d=1}^{D} \sum_{n=1}^{N_d} \phi_{dni} w^j_{dn}$$

**Result:** Update the words topic assignment parameter $\phi$ and the document topic parameter $\gamma$ for a document $d$

Initial the $\gamma_i = \alpha_i + N/K, \forall i = 1, \ldots, K$ and $\phi^0_{ni} = 1/K, \forall n = 1, \ldots, N, i = 1, \ldots, K$; Choose the stopping criterion $\epsilon$ and line searching parameter $\alpha \in (0, 0.5)$, $\rho \in (0, 1)$ ;

while Not converge do
  for word $n, n = 1, \ldots, N$ do
    for document $i, i = 1, \ldots, K$ do
      $\phi^{t+1}_{ni} := \eta_{wn} \exp E_q[\log(\theta_i)|\gamma]$
      end
    Normalize $\phi^{t+1}_n$ so that its sum equals to 1.
  end
  for topic $i, i = 1, \ldots, K$ do
    while Not converge do
      Compute the Newton step $\Delta \gamma_i$: $\Delta \gamma_i := -L'_\gamma / L''_{\gamma_i}$;
      if $|\Delta \gamma_i| < \epsilon$ then
        Stop updating $\gamma_i$
      else
        Find step size $\alpha$ by backtracking line search:
        Initialize the step size $\alpha := 1$;
        while $-L(\gamma_i + \alpha \Delta \gamma_i) > -L(\gamma_i) - \delta \alpha L'_\gamma \Delta \gamma_i$ do
          $\alpha := \rho \alpha$
        end
        $\gamma_i := \gamma_i + \alpha \Delta \gamma_i$;
      end
  end
-end

Algorithm 1: Variational E-step

3 Real Data Application

To test the empirical performance of our proposed method, we apply LDA and CDTM to the NIPS dataset, which consists of 11,463 words and 7,241 NIPS conference paper from 1987 to 2017. The data is randomly split into two parts: training (80%) and testing (20%). We select the number of topics being 10 from $\{5, 10, 15, 20, 25, 30\}$ for LDA using cross-validation with perplexity as the evaluation measure on the training dataset (Blei et al., 2003). For CDTM, we use homogeneous hyperparameters in the experiment, i.e. $\lambda_d = \lambda, \forall d \in \{1, \ldots, D\}$. The
model is flexible to use any other weighting schemes, such as a series of increasing $\lambda$s for the increasing entropy values.

We use cross-validation to select the penalty weight $\lambda = 35$ from $\{25, 30, 35, 40, 45\}$. The evaluation metric is the topic coherence score $C_V$ (Röder et al., 2015), which has been shown achieving the highest correlation with all available human topic ranking data (Röder et al., 2015; Syed and Spruit, 2017). The $C_V$ coherence score is calculated as follows. The top $N$ words of each topic are selected as the representation of the topic, denoted as $W = \{w_1, \ldots, w_N\}$. Each word $w_i$ is represented by an $N$-dimensional vector $v(w_i) = \{NPMI(w_i, w_j)\}_{j=1,\ldots,N}$, where $j$-th entry is the Normalized Pointwise Mutual Information (NPMI) between $w_i$ and $w_j$. A sliding window of size 110, which is the default value in the Python package ‘gensim’ and robust for many applications, is used to create pseudo-document and estimate the probabilities. The purpose of the sliding window is to take the distance between two words into consideration. For each word $w_i$, a pair is formed $(v(w_i), v(W))$. A cosine similarity measure $\phi_i(v(w_i), v(W)) = \frac{v(w_i)^T v(W)}{\|v(w_i)\| \|v(W)\|}$ is then calculated for each pair. The final $C_V$ score for the topic is the average of all $\phi_i$s.

We then refit both LDA and CDTM to the whole training dataset and use the test dataset to calculate the coherence score $C_V$ as a final evaluation of the out-of-sample performance. The results are shown in Table 2. Overall the $C_V$ score of LDA topics is 0.50 and that of CDTM is 0.52, a 5% improvement. We then compute the entropy of the document-topic distributions and plot its histogram in figure 2. The summary statistics of the entropy values is shown in Table 1. From both the plot and summary statistics, we observe that the entropy of the document-topic distributions of CDTM is on average smaller (1.08) than that of LDA (1.24), indicating that overall the document-topic is more concentrated in CDTM than LDA.

We present a specific example: the paper titled ‘Connection Topology and Dynamics in Lateral Inhibition Networks’ (Syed and Spruit, 2017). This paper studies dynamics of modeling the lateral inhibition, a topic in neurobiology. A paragraph from the paper is quoted below.

In this paper we study the dynamics of simple neural network models of lateral inhibition in a variety of two-dimensional connection schemes. The lattice structures we studied are shown in Fig. 1. Two-dimensional lattices are of particular importance to artificial vision systems because they allow an efficient mapping of an image onto a network and because they are well-suited for implementation in VLSI circuitry. We show that the stability of these networks depends sensitively on such design considerations as local connection topology, neuron self-coupling, the steepness or gain of the neuron transfer function, and details of the network dynamics such as connection delays for continuous-time dynamics or update rule for discrete-time dynamics.

LDA assigns two significant topics to this paper: 0.39 to topic Neural Network and 0.42 to topic Neurobiology; while CDTM assigns 0.93 to topic Neurobiology. Although this paper have words neural network, it has nothing to do with the modern computer science neural network with hidden layers, training etc. In fact, the paper doesn’t even contain words like learning,
layer, training, hidden, error, weight etc. LDA wrongly assigns 0.39 to topic Neural Network. Moreover, the sum of the two main topic probabilities in LDA is 0.81, which is smaller than the 0.93 assigned by CDTM.

|       | Mean | Variance | Skewness | Kurtosis |
|-------|------|----------|----------|----------|
| LDA   | 1.24 | 0.12     | -0.48    | 0.17     |
| CDTM  | 1.08 | 0.13     | -0.43    | 0.01     |

Table 1: Summary statistics of the entropies of document-topic distributions.

4 Conclusion

We propose a Concentrated Document Topic Model (CDTM), which is able to produce concentrated and sparse document-topic distributions. In particular, we add an exponential negative entropy of the document-topic parameters as a productive penalty to the posterior. Entropy is used to measure the topic concentration. Concentrated document-topic distributions have low entropy values and thus are penalized less, while the more uniformly distributed document-topic distributions have high entropy values and thus are penalized more. We then apply our proposed model to the NIPS dataset and observe improved topic coherence scores and more concentrated document-topic distributions. Our proposed model could potentially improve the information retrieval accuracy comparing to LDA.
Table 2: Top 20 words of the topics estimated by LDA and CDTM.
Appendices

A Expectation of penalty under the variational distribution

In this section, we derive the equation of the expectation of the penalty term under the variational distribution. We first find the equation for $E_q[\theta_i \log \theta_i]$.

$$E_q[\theta_i \log \theta_i] = \int \theta_i \log \theta_i \frac{\Gamma(\sum_{l=1}^{K} \gamma_i^l)}{\prod_{l=1}^{K} \Gamma(\gamma_i^l)} \theta_i^{\gamma_i^l - 1} d\theta$$

$$= \frac{\gamma_i}{\sum_{l=1}^{K} \gamma_i^l} \int \log \theta_i \frac{\Gamma(\sum_{l=1}^{K} \gamma_i^l)}{\prod_{l=1}^{K} \Gamma(\gamma_i^l)} \theta_i^{\gamma_i^l - 1} d\theta$$

$$= \frac{\gamma_i}{\sum_{l=1}^{K} \gamma_i^l} [\Psi(\gamma_i') - \Psi(\sum_{l=1}^{K} \gamma_i^l)]$$

$$= \frac{\gamma_i}{\sum_{l=1}^{K} \gamma_i^l} [\Psi(\gamma_i) + 1/\gamma_i - \Psi(\sum_{l=1}^{K} \gamma_i) - 1/\sum_{l=1}^{K} \gamma_i]$$

where in the second equality, $\gamma_i' = \gamma_i + 1$ and $\gamma_j' = \gamma_j, \forall j \neq i$, and we make use of $\Gamma(x + 1) = x\Gamma(x)$. The third equality follows from [Blei et al.] (2003). Then the expectation of the penalty term under the varitaional distribution takes following form

$$E_q[-\lambda H(\theta)] = \lambda \sum_{i=1}^{K} E_q[\theta_i \log \theta_i]$$

$$= \lambda \left( \frac{\sum_{i=1}^{K} \gamma_i \Psi(\gamma_i)}{\sum_{i=1}^{K} \gamma_i} - \Psi(\sum_{i=1}^{K} \gamma_i) + \frac{K - 1}{\sum_{i=1}^{K} \gamma_i} \right)$$

References

D. Blei, A. Ng, and M. Jordan. Latent dirichlet allocation. *Journal of Machine Learning Research*, 3(Jan):993–1022, 2003.

D. M. Blei and J. D. Lafferty. A correlated topic model of science. *The Annals of Applied Statistics*, pages 17–35, 2007.

D. M. Blei, A. Kucukelbir, and J. D. McAuliffe. Variational inference: A review for statisticians. *Journal of the American statistical Association*, 112(518):859–877, 2017.

T. Bollerslev. Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3):307–327, 1986.

R. Das, M. Zaheer, and C. Dyer. Gaussian lda for topic models with word embeddings. In *Proceedings of the 53rd Annual Meeting of the Association for Computational Linguistics and the 7th International Joint Conference on Natural Language Processing (Volume 1: Long Papers)*, pages 795–804, 2015.
R. F. Engle. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the Econometric Society*, pages 987–1007, 1982.

L. Fei-Fei and P. Perona. A bayesian hierarchical model for learning natural scene categories. In *Computer Vision and Pattern Recognition, 2005. CVPR 2005. IEEE Computer Society Conference on*, volume 2, pages 524–531. IEEE, 2005.

D. A. Gomes and E. Valdinoci. Entropy penalization methods for hamilton–jacobi equations. *Advances in Mathematics*, 215(1):94–152, 2007.

C. B. González-Blas, L. Minnoye, D. Papasokrati, S. Aibar, G. Hulselmans, V. Christiaens, K. Davie, J. Wouters, and S. Aerts. Cistopic: cis-regulatory topic modeling on single-cell atac-seq data. *Nature methods*, 16(5):397–400, 2019.

T. L. Griffiths, M. Steyvers, D. M. Blei, and J. B. Tenenbaum. Integrating topics and syntax. In *Advances in Neural Information Processing Systems*, pages 537–544, 2005.

T. Hofmann. Probabilistic latent semantic indexing. In *Proceedings of the 22nd Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, pages 50–57. ACM, 1999.

A. H. Huang, R. Lehavy, A. Y. Zang, and R. Zheng. Analyst information discovery and interpretation roles: A topic modeling approach. *Management Science*, 64(6):2833–2855, 2017.

V. Koltchinskii et al. Sparse recovery in convex hulls via entropy penalization. *The Annals of Statistics*, 37(3):1332–1359, 2009.

V. Koltchinskii et al. Von neumann entropy penalization and low-rank matrix estimation. *The Annals of Statistics*, 39(6):2936–2973, 2011.

H. Lei, Y. Chen, and C. Y.-H. Chen. Investor attention and topic appearance probabilities: Evidence from treasury bond market. *Available at SSRN 3646257*, 2020.

L. Liu, L. Tang, W. Dong, S. Yao, and W. Zhou. An overview of topic modeling and its current applications in bioinformatics. *SpringerPlus*, 5(1):1608, 2016.

Y. Luo, C.-C. Chiu, N. Jaitly, and I. Sutskever. Learning online alignments with continuous rewards policy gradient. In *2017 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 2801–2805. IEEE, 2017.

D. Maier, A. Waldherr, P. Miltner, G. Wiedemann, A. Niekler, A. Keinert, B. Pfetsch, G. Heyer, U. Reber, T. Häussler, et al. Applying lda topic modeling in communication research: Toward a valid and reliable methodology. *Communication Methods and Measures*, 12(2-3):93–118, 2018.

V. Mnih, A. P. Badia, M. Mirza, A. Graves, T. Lillicrap, T. Harley, D. Silver, and K. Kavukcuoglu. Asynchronous methods for deep reinforcement learning. In *International conference on machine learning*, pages 1928–1937, 2016.

S. Y. Park and A. K. Bera. Maximum entropy autoregressive conditional heteroskedasticity model. *Journal of Econometrics*, 150(2):219–230, 2009.
G. Pereyra, G. Tucker, J. Chorowski, L. Kaiser, and G. Hinton. Regularizing neural networks by penalizing confident output distributions. arXiv preprint arXiv:1701.06548, 2017.

M. Reisenbichler and T. Reutterer. Topic modeling in marketing: recent advances and research opportunities. Journal of Business Economics, 89(3):327–356, 2019.

M. Röder, A. Both, and A. Hinneburg. Exploring the space of topic coherence measures. In Proceedings of the Eighth ACM International Conference on Web Search and Data Mining, pages 399–408, 2015.

H. K. Ryu. Maximum entropy estimation of density and regression functions. Journal of Econometrics, 56(3):397–440, 1993.

C. E. Shannon. A mathematical theory of communication. The Bell system technical journal, 27(3):379–423, 1948.

B. Shi, W. Lam, S. Jameel, S. Schockaert, and K. P. Lai. Jointly learning word embeddings and latent topics. In Proceedings of the 40th International ACM SIGIR Conference on Research and Development in Information Retrieval, pages 375–384, 2017.

S. Syed and M. Spruit. Full-text or abstract? examining topic coherence scores using latent dirichlet allocation. In 2017 IEEE International Conference on Data Science and Advanced Analytics (DSAA), pages 165–174. IEEE, 2017.

H. M. Wallach, D. M. Mimno, and A. McCallum. Rethinking lda: Why priors matter. In Advances in Neural Information Processing Systems, pages 1973–1981, 2009.

R. J. Williams and J. Peng. Function optimization using connectionist reinforcement learning algorithms. Connection Science, 3(3):241–268, 1991.

H. Xu, W. Wang, W. Liu, and L. Carin. Distilled wasserstein learning for word embedding and topic modeling. In Advances in Neural Information Processing Systems, pages 1716–1725, 2018.

A. Zellner and R. A. Highfield. Calculation of maximum entropy distributions and approximation of marginalposterior distributions. Journal of Econometrics, 37(2):195–209, 1988.