Optimization of the manned aircraft pitch angle control loop with actuator rate limitation and nonlinear correction

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Abstract. In the paper a flight control system for a piloted aircraft on the pitch angle is considered, taking into account the rate saturation of the aircraft controlling surface. To prevent occurrence of the non-linear pilot-involved oscillations (PIO), a nonlinear correcting device is introduced into the control loop. On the basis of the pilot’s adaptation to control tasks variations, his/her mathematical model parameters are optimized and the choice of the nonlinear correcting device parameters is proposed. The sensitivity functions of the non-linear system are calculated for harmonic and square-wave reference signals. The results of the optimized system simulations are presented demonstrating good performance of the closed-loop system with nonlinear correction.

1. Introduction
When performing flight control tasks, the pilot continuously forms and performs control actions. The pilot adjusts the characteristics of his actions purposefully, serving as a controlling element, see Refs. [1–3]. The parameters of the pilot behavior model depend on the aircraft dynamics in the given flight conditions and the frequency bounds of the external actions, such as disturbances or the reference path, and the extensive simulations for various conditions can reveal undesirable system behavior or abnormal pilot-aircraft interactions such as pilot-involved oscillations (PIOs), see Refs. [4–10]. Due to the pilot adaptation property, the characteristics of the aircraft-pilot system remain relatively unchangeable. This suggests that when tracking, the pilot seeks to optimize the properties of the closed-loop aircraft-pilot system. Even if the aircraft model for a certain flight conditions and small attack and sideslip angles may be considered as a linear one, influence of the magnitude and rate saturation of the controlling surface servos can not be ignored in the cases of aggressive maneuvers. Of greatest interest is the drive speed nonlinearity of the saturation type. When maximum angular deflection rate is reached, the speed saturation effect causes a significant phase shift in the control loop, which, in turn, can lead to nonlinear oscillations and stability loss of the attitude. For phase correction of the system under study, the nonlinear correcting unit (NCU), Refs. [10–15] is employed. Creating an optimal system pilot control is one of the ways for preventing the occurrence of non-linear hidden oscillations in
a closed-loop control system [3]. In the present work, as a criterion optimality, it is proposed to choose the system settling time and examining the optimized system behavior for various values of the drive speed saturation. the reminder of the paper is organized as follows. Aircraft and servo system dynamics models are described in Sec. 2. Section 3 gives the optimization problem statement. The objective function calculation is considered in Sec. 4, and the optimization results are presented in Sec. 5. Concluding remarks and the future work intentions are given in Sec. 7.

2. Aircraft and servo system dynamics model
In the paper the longitudinal short-period mode of the aircraft motion on pitch \(\theta\) is considered. Following Ref. [16] the aircraft transfer function from elevator deflection \(\delta_e(t)\) (with respect to the trim value) to pitch angle \(\theta(t)\) is taken as

\[
W^q_{\delta_e}(s) = \frac{29.11(s + 4.356)}{s(s^2 + 7.358s + 25.65)},
\]

where \(s \in \mathbb{C}\) denotes Laplace transform variable.

The elevator servo system is considered that has the dead time as \(\tau_p = 0.2\) s and the angular speed limitation of \(\bar{\delta}\), varying at the different examinations of the system performance. More specifically, it is described by the following equations:

\[
\begin{align*}
\hat{\delta}_e(t) &= \text{sat}_\delta \left( \frac{1}{T_a} e_\delta(t) \right), \\
e_\delta(t) &= u(t - \tau_a) - \delta_e(t),
\end{align*}
\]

where \(T_a\) denotes the actuator time constant. It is taken as \(T_a = 0.076\) s, Ref. [16].

To maintain acceptable controllability characteristics of the aircraft, the pitch damper is introduced in the form of negative feedback from the pitch velocity \(q(t)\) to the control signal of the elevator \(u(t)\) with a certain feedback gain \(k_q\), so that

\[
u(t) = \tilde{u}(t) - k_q q(t),
\]

where \(q(t)\) stands for the pitch rate in the body-axes frame, and \(\tilde{u}(t)\) denotes the pilot’s command \(u_p\), or the output of the sequential correcting device with the pilot’s command \(u_p\) as an input. In the sequel, \(k_q = 0.15\) s is taken. The pitch rate contour step response, obtained for the linear model (avoiding the actuator rate saturation) is pictured in Fig. 1.

![Figure 1. Step response of pitch rate loop (1)–(3) (linear model).](image-url)
3. Optimization problem statement

Consider the nonlinear system model of the form:

$$\dot{x}(t) = f(x, r), \quad y(t) = h(x, r), \quad (4)$$

where $x(t) \in \mathbb{R}^n$ denotes the state-space vector, $y \in \mathbb{R}^1$ stands for the scalar system output, $r(t)$ is the command (reference) signal. Define the control error $e(t)$ as $e(t) = r(t) - y(t)$. Let for $r(t) = A \sin(\omega t)$ system (4) be uniformly convergent. Then, as follows from Refs. [17–21], system (4) possesses a unique periodic solution $\bar{x}(t)$ with output $\bar{y}(t)$ and control error $\bar{e}(t)$.

Let us employ the structural approach to describing the pilot model, which reflects the processes of the information processing and developing the action strategies. This approach consists of the following steps: pilot dynamics description; choice of the optimization criterion; objective function calculation. These steps are described below in more detail.

3.1. Pilot dynamics description

The pilot dynamics are usually represented by the following transfer function Refs. [1, 2, 22–25]:

$$W_p(s) = \frac{K_p T_L s + 1}{T_p s + 1} e^{-\tau_p s}, \quad (5)$$

where $K_p$ is the “pilot’s gain”; $T_L, T_p$ are the lead and lag time constants, introduced by the pilot (respectively); $\tau_p$ denotes the dead time value, taking into account the pilot’s muscular delay, delay in perception and his/her actions formation in the central nervous system. For the aircraft with the good flight qualities, model (5) parameters should belong to the following intervals, see Refs. [1, 26]: $T_L, T_p \in (0, 1] s$, $\tau_p \in [0.1, 0.3] s$.

3.2. Choice of optimization criterion

In this study, the problem of optimizing the control of the movement of a manned aircraft is considered as minimizing the transition time $t^*$. It can be approximately taken that $t^*$ is inversely proportional to the cutoff frequency of the open-loop system $\omega_c$, therefore the objective function $J = \omega_c$ can be taken as a variable to be maximized. The following variables of the pilot model (5) are chosen as the optimization parameters: $T_L, T_p, K_p, \tau_p$.

Due to the presence of restrictions on some variables, the problem under consideration belongs to the conditional optimization one, Refs. [27, 28]. The penalty optimization is used to cope with the constrained optimization problem: the penalty functions are introduced whose values on a point increases sharply as the point goes out of the prescribed feasible region. In this study, the following inequality-type constraints are used.

- $0 < T_L \leq 1$, $0 < T_p \leq 1$, $K_p > 0$, $\tau_p$ are taken from the feasible region $T_L, T_p \in (0, 1] s$, $\tau_p \in [0.1, 0.3] s$;
- degree of stability $\eta$ should be positive. It is calculated as $\eta = - \max_{i=1,...,n} (\text{Re} s_i)$, where $s_i$ denote the poles of the closed-loop system transfer function. Due to the presence of the dead time in pilot’s model (5) and the servosystem model as well, the overall system has an infinite high order, however, employing Padé $(5, 5)$ approximation of the delay transfer function (cf. Refs. [29, 30])

$$e^{-\tau s} \approx -\frac{\lambda^5 + 30\lambda^4 - 420\lambda^3 + 3360\lambda^2 - 1.512\times 10^4 \lambda + 3.024\times 10^4}{\lambda^5 + 30\lambda^4 + 420\lambda^3 + 3360\lambda^2 + 1.512\times 10^4 \lambda + 3.024\times 10^4}, \quad \text{where} \quad \lambda = \tau s, \quad (6)$$

makes it possible to reduce the system order to the finite one (despite large). The adopted finite-order approximation of the overall system is of order as $n = 23$;
- index of oscillation $M$ (the same as $H_{\infty}$-gain) should not exceed 1.25.
in the case if current point \((T_L, T_P, K_p, \tau_p)\) is feasible, objective function \(J\) is set to \(J = -\omega_c\).

In the present work, the standard MATLAB unconstrained multivariable function minimization routine \texttt{fminsearch}, using derivative-free simplex search Nelder-Mead method, see Refs. [31, 32], is employed.

4. Objective function calculation

Objective function \(J = -\omega_c\), calculation for each optimization iteration, governed by \texttt{fminsearch} procedure consists of the following steps.

(i) checking feasibility of current values \(T_L\), \(T_P\), \(K_p\), \(\tau_p\). If any of them is not feasible, the barrier function is involved, and the calculation step is terminated;

(ii) for given \(\tau = \tau_p\) the coefficients of Padé approximation (6) for \(W_{\eta_p}(s) \approx e^{\tau_p s}\) are found;

(iii) for given \(\tau = \tau_0\) the coefficients of Padé approximation (6) for \(W_{\eta_0}(s) \approx e^{\tau_0 s}\) are found;

(iv) transfer function \(W_{\delta_0}^q(s)\) is found from (1);

(v) transfer function \(W_{\delta_0}^e(s) = \frac{1}{T_0 s + 1} e^{-\tau_0 s}\) of actuator (2) linearized model is found;

(vi) numerator \(B(s)\) and denominator \(A(s)\) coefficients of open-loop transfer function \(W(s)\) are calculated by the following relations:

\[
W_{\delta_0}^q(s) = \frac{1}{s} W_{\delta_0}^d(s), \quad \Phi_{\delta_0}^q(s) = W_{\delta_0}^q(s) \frac{1 + k_0 W_{\delta_0}^p(s)}{1}, \quad W(s) = \frac{B(s)}{A(s)} = W_p(s) \Phi_{\delta_0}^q(s); \quad (7)
\]

(vii) the characteristic polynomial of the closed-loop system model id found as \(D(s) = A(s) + B(s)\) and its roots \(r_i, i = 1, \ldots, 23\) are found by means of the standard MATLAB routine \texttt{roots};

(viii) closed-loop system transfer function from reference signal \(\theta^*\) to pitch angle \(\theta\) is calculated as \(\Phi_{\theta_p}^q(s) = \frac{B(s)}{A(s)}\) and the corresponding index of oscillation \(M\) is found by the standard MATLAB routine \texttt{norm(}., \texttt{’inf’})\). If \(M \geq 1.25\) then objective function \(J\) is set to \(J = 10^3 M\) and the current calculation step of \(J\) is terminated;

(ix) Cutoff frequency \(\omega_c\) is found as the maximal value of \(\omega \in [0.1, 100]\) for what the open-loop transfer function magnitude frequency response \(|W(i\omega)|\) is equal to or greater than 1. For computations, 5000 points of the frequency inside the interval \([0.1, 100]\) rad/s are logarithmically chosen;

(x) degree of stability \(\eta = -\max_{1 \leq i \leq 23} (\text{Re} r_i)\) is found and the condition \(\eta > 0\) is checked. If this condition fails, then \(J\) is set to \(J = -10^{15} \eta\) and the calculation step is terminated. Otherwise, \(J\) is set to \(J = -\omega_c\) and the calculation of the objective function for a given set of optimized parameters \(T_L\), \(T_P\), \(K_p\), \(\tau_p\) is successfully finished.

5. Optimization results

The following starting point was chosen for optimization: \(T_L = 0.5\) s, \(T_P = 0.5\) s, \(K_p = 1.75\), \(\tau_p = 0.2\) s. It turned out that this point corresponds to an unstable closed-loop system. The optimization procedure leads to the following values of the system parameters: \(T_L = 0.49\) s, \(T_P = 0.60\) s, \(K_p = 0.60\), \(\tau_p = 0.18\) s. For the parameters obtained cutoff frequency \(\omega_c = 1.64\) rad/s, \(M = 1.25\). Bode diagram of the optimized system is pictured in Fig. 2.

It is seen that gain stability margin \(Gm = 5.81\) dB, phase stability margin \(Pm = 49.8\) deg, which indicates a good quality of the closed-loop system.
6. Nonlinear correction in the pilot–aircraft control loop

It should be noted, however, that the results obtained refer to a linear system for which saturation of the actuator speed is not taken into account. Presence of the rate saturation can drastically change the flight control system performance, as demonstrated by the plots in Fig. 3, where the time histories of tracking the square-wave reference signal $\theta^*(t)$ for system without the actuator rate saturation in comparison with the rate saturated case in 6.0 deg/s are plotted. The second case can be treated as the PIO emergence.

![Bode Diagram](image)

**Figure 2.** Bode diagram of the optimized system (linear model).

![Time histories](image)

**Figure 3.** Time histories of tracking the square-wave reference signal $\theta^*(t)$. $\theta_1$ – unsaturated (linear) actuator, $\theta_2$ – actuator rate saturation of 6.0 deg/s.

The occurrence of a PIO is usually interpreted as a result of a negative phase shift caused by saturation of the actuator motion in magnitude and/or speed. Therefore, the introduction
of a positive phase shift could improve the system behavior, however limitations of the linear correction follow from a tight relationship between magnitude and phase frequency responses. Therefore linear compensation is highly restricted. This leads to an idea of employing the nonlinear phase shift compensation, instead of linear correction.

6.1. Nonlinear dynamical corrective devices

Limitations of the linear correction follow from a tight relationship between magnitude and phase frequency responses. Therefore linear compensation is highly restricted. This leads to an idea of employing the nonlinear phase shift compensation, instead of linear correction, see Refs. [10, 14, 33–40].

Nonlinear Corrective Devices (NCD) make it possible to change the phase-frequency and amplitude-frequency responses independently, Refs. [11–13, 41, 42]. For Pseudo-linear corrective devices (PLCD), frequency characteristics do not depend on the input signal magnitude (but on its frequency only).

The following Nonlinear Phase Predicting Filter (NPPF) is employed in the present study:

\[ y = k|u| \text{sign}(x), \]
\[ A(p)x = B(p)u, \]

where \( p = d/dt \), polynomials \( A(p) \), \( B(p) \) are chosen ensuring positive phase shift for frequency response of \( W(i\omega) = B(i\omega)/A(i\omega) \) for all \( \omega > 0, \omega \in \mathbb{R}, i^2 = -1 \).

The first-order lead-lag filter with transfer function

\[ W(s) = \frac{T_2s + 1}{T_1s + 1}, \]

is chosen, where \( 0 < T_1 < T_2 \) are design parameters. Define \( \nu = T_1/T_2 \). The describing functions (harmonic balance) method (see Refs. [14, 19, 20, 36, 42–45]) gives the following equivalent phase shift of NPPF (8)–(10):

\[ \varphi(\omega) = \arctan \omega T_2 - \arctan \omega T_1 > 0 \quad \forall \omega > 0. \]

Harmonic linearization gains for NPPF are as follows, see Refs. [14, 36, 42]:

\[ a = \frac{k}{\pi}(\pi - 2\varphi + \sin 2\varphi), \]
\[ b = \frac{k}{\pi}(1 - \cos 2\varphi). \]

The wider the bandwidth \( \nu \) is, the greater is the positive phase lag. Transmission gain \( A = \sqrt{a^2 + b^2} \) is close to 1 and its influence can be neglected.

6.2. Comparative results

The closed-loop aircraft control system with NPPF (8)–(10), which was sequentially inserted into the break between the pilot and the servo (signal \( \tilde{u}(t) \) in (3)) was examined by the simulations. The following parameters were taken for the simulations: \( T_L = 0.49 \text{ s}, T_P = 0.60 \text{ s}, K_p = 0.60, \tau_p = 0.18 \text{ s} \) (they are the same that were obtained by the optimization procedure), the actuator rate bound \( \text{sat}_\delta \) in (2) was set to \( \text{sat}_\delta = 6.0 \text{ deg/s} \). The NPPF (8)–(10) parameters are taken as follows: \( k = 1 \), time constants \( T_1, T_2 \) are chosen satisfying the inequality \( 1/T_2 < \omega_c < 1/T_1 \). Namely, \( T_1 = 0.08 \text{ s}, T_2 = 2.5 \text{ s} \) in (10) are taken (this corresponds the NPPF bandwidth \( \nu = 31 \)).
6.2.1. Tracking the square-wave reference signal. Time histories of tracking the square-wave reference signal in the case of the actuator rate saturation as 6.0 deg/s for systems with and without NPPF are plotted in Fig. 4. The plots demonstrate PIO alleviation by means of NPPF augmentation.

![Figure 4](image-url)  
*Figure 4.* Time histories of tracking the square-wave reference signal $\theta^*(t)$ for systems with $(\theta_1)$ and without $(\theta_2)$ NPPF.

6.2.2. Tracking the harmonic reference signal. Time histories of tracking the harmonic reference signal $\theta^*(t) = 50 \sin t$ deg are plotted in Fig. 5 for the cases of: the presence of NPPF correction and the actuator rate saturation 6 deg/s; absence of correction and the saturated actuator; linear system without actuator saturation. The plots show that absence of the NPPF correction for the saturated actuator case leads to unacceptable system output, whilst the NPPF correction makes system output closer to that of the ‘ideal’ linear unsaturated system.

6.2.3. Sensitivity functions. The performance analysis of linear control systems is essentially based on frequency domain characteristics such as sensitivity function $S(i\omega)$ and complementary sensitivity function $T(i\omega)$. The generalized versions of these functions are defined in Refs. [20] for nonlinear Lur’e systems, possessing the convergence property, cf. Ref. [46].

Let the nonlinear system be modeled as

$$\dot{x}(t) = f(x, r), \quad y(t) = h(x, r), \quad e(t) = r(t) - y(t),$$

(14)

where $x(t) \in \mathbb{R}^n$ is a state vector, $y(t) \in \mathbb{R}$ is the system output, $r(t)$ is the reference signal, $e(t)$ is the reference error. Assume that $r(t) = a \sin(\omega t)$, where $a$ is the amplitude and $\omega$ is the frequency of the input (reference) signal, and that the system (14) is uniformly convergent in this class of inputs. Then there exists an unique steady-state $\frac{2\pi}{\omega}$-periodic solution $\bar{x}(t)$ of (14) with the corresponding response $\bar{y}(t)$ and $\bar{e}(t) = r(t) - \bar{y}(t)$ Refs. [47, 48].

**Definition** Ref. [20]: The functions

$$S(a, \omega) = \|\bar{e}\|_2 / \|r\|_2, \quad T(a, \omega) = \|\bar{y}\|_2 / \|r\|_2$$

where $\|z\|_2 = \left(\frac{\omega}{2\pi} \int_{0}^{2\pi/\omega} z(\tau)^2 d\tau\right)^{1/2}$, are called, respectively, the generalized sensitivity and the generalized complementary sensitivity functions of the convergent system (14). For the
Figure 5. Time histories of tracking the harmonic reference signal $\theta^*(t)$. 1) NPPF correction; 2) reference signal $\theta^*$; 3) no correction; 4) linear unsaturated system.

In the linear case, the functions $S(a, \omega)$ and $T(a, \omega)$ coincide with customary amplification frequency characteristics $|S(i\omega)|$, $|T(i\omega)|$, respectively. For nonlinear systems, the functions $S(a, \omega)$ and $T(a, \omega)$ depend not only on the excitation frequency $\omega$, but also on the amplitude $a$. These generalized sensitivity functions may be evaluated numerically based on system (14) simulation with given harmonic input $r(t)$. Significant reducing the computation costs may be achieved by using the generalization of the describing functions (harmonic linearization) method on nonautonomous convergent systems given in Ref. [44].

The sensitivity functions for the reference signal magnitude 30 deg and the cases of: 1) no correction; 2) NPPF correction; 3) linear unsaturated system are depicted in Fig. 6, confirming efficiency of the NPPF correction.

Figure 6. Sensitivity functions for the cases of: 1) no correction; 2) NPPF correction; 3) linear unsaturated system.
7. Conclusions
In the paper the longitudinal short-period mode of the piloted aircraft motion on pitch $\theta$ is considered. The focus of the work attention is the possibility of the PIO emergence due to the actuator rate saturation. For alleviation of the PIO, a nonlinear correcting device is introduced into the control loop. The pilot-in-the-loop mathematical model parameters are found by means of the optimization procedure and the choice of the nonlinear correcting device parameters is proposed. The time histories of the square-wave and harmonic reference signals are found and their comparative analysis is given. The generalized results are presented in the form of the sensitivity functions. The results of the optimized system simulations are presented demonstrating good performance of the closed-loop system with nonlinear correction.

The future work is intended to examine more complex methods of the nonlinear correction, including the Implicit Reference Model adaptive control of Refs. [49–51].

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