Neutrino emissivity from Goldstone boson decay in magnetized neutron matter

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Neutron matter at densities somewhat above nuclear densities is believed to be superfluid due to the condensation of neutron pairs in the $^3P_2$ channel. This condensate breaks rotational symmetry spontaneously and leads to the existence of Goldstone bosons (angulons). We show that the coupling to magnetic fields mediated by the magnetic moment of the neutron makes angulons massive and capable of decaying into a neutrino-antineutrino pair. We compute the rate for this process and argue they become competitive with other cooling processes for temperatures around $10^7 K$ as long as the interior magnetic field of the star is in the $B \approx 10^{15} G$ range or above.

I. INTRODUCTION

The cooling of pulsars, either after their formation or after an accretion episode, is an important probe of their interior. Contrary to the star’s mass and radius relation, determined by the equation of state of dense matter, cooling process are sensitive to the effective degrees of freedom in the star core. The temperatures at which neutron stars are typically found are much smaller than the Fermi energies and are only capable of exciting the low lying modes. It is the decay and interactions of these modes that control the neutrino emission process leading to loss of energy and cooling of the star. The properties of the low lying degrees of freedom are very sensitive to the thermodynamic phase realized in the star: the low lying modes of quark matter are different from the ones in neutron matter and both are sensitive to neutron, proton and quark pairing. Therefore, the comparison of neutron star cooling curves with theoretical predictions based on different models of dense matter is a way of finding out which thermodynamic phase is actually realized at the core of the star and can even distinguish between two phases whose equations of state are very similar. The recent observations of neutron stars with mass above two solar masses [1], [2] strongly suggest that ”exotic” phases – those made up of other particles besides neutron, protons and electrons – are not present in neutron stars. It is then natural to focus the attention onto non-exotic phases. The theoretical expectation, based on vacuum phase shifts and model calculations, is that neutron matter is paired at the relevant densities, with the pairing occurring in the $^1S_0$ channel at lower densities and on the $^3P_2$ channel at higher densities. The protons, due to their lower density, are expected to be paired in the $^1S_0$ channel. The condensation of neutrons in the $^3P_2$ channel leads to the appearance of a unique class of low energy excitations. As the condensate is a spin 2 object, its orientation in space defines a special frame and breaks rotational symmetry spontaneously. As a consequence, one expects Goldstone bosons (named “angulons”), which are ungapped scalar excitations, to exist in the $^3P_2$ phase. This observation was made some time ago in [3] where one of the variants of the $^3P_2$ phase was considered. The recent observation of rapid cooling of the neutron star Cassiopeia A [4] has renewed interest in the phenomenon of neutron pairing and condensation as the phenomenon was explained [5] [6] [7] as a result of neutrino emission in the process of $^3P_2$ Cooper pair breaking and formation (PBF) [8] [9]. The PBF process is only effective at temperatures close to the critical temperature where unpaired neutrons exist in substantial numbers. At lower temperatures, processes involving angulons are likely to dominate. In [3] an estimate was made for the bremsstrahlung of neutrino pairs following a angulon-angulon collisions. The emissivity was found to be proportional to $T^3$ and small for most relevant densities and temperatures. Angulon decay into neutrino pairs is kinematically forbidden as the angulon momentum is space-like. The main point we make of the present paper is that the dispersion relation of the angulons is changed due to the presence of strong magnetic fields and one of them develops a gap of the order of $eB/M$ ($B$ is the magnetic field, $e$ the electron charge and $M$ the neutron mass, corrected by Fermi liquid effects). The gapped angulon is then kinematically allowed to decay into a neutrino pair.

In order to proceed with the calculation of the emissivity due to magnetic field catalyzed angulon decay we first review some angulon properties derived in [10] where the low-energy effective theory for the $^3P_2$ phase was developed. The core of a neutron star is expected to have densities above the nuclear matter saturation density at a temperature well below the Fermi momentum of the neutrons and the attractive force between neutrons near the Fermi surface leads to Cooper pair formation. At moderate densities, $3 \times 10^{12} \text{g/cm}^3 \leq \rho \leq 10^{14} \text{g/cm}^3$ the neutrons form s-wave Cooper pairs while further inside the core, at even higher densities ranging from $1.5 \times 10^{14} \text{g/cm}^3$ to $10^{15} \text{g/cm}^3$.
neutrons undergo triplet ($^3P_2$) pairing due to short range spin orbit interaction [11]. The order parameter for the triplet condensed phase is given by

$$\langle n^T \sigma_2 \sigma_i \bar{n} \rangle = \Delta_{ij} e^{i\alpha},$$

where $\Delta_{ij}$ is a symmetric traceless matrix [12, 13]. Here, $n$ is the neutron field, $\sigma$ are Pauli spin matrices and $\alpha$ is an arbitrary phase. Different forms of $\Delta_{ij}$ break rotational symmetry in different ways and lead to different angulon properties. Near the critical temperature, where Ginzburg-Landau arguments are valid, the condensate in the energetically favorable ground state is [13]

$$\Delta = \Delta_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & -1 - r \end{pmatrix},$$

with $r = -1/2$ (assuming certain parameters are not too different from the BCS values) and it has been argued this pattern is stable as the temperature is lowered [14]. It will be an assumption of our calculation that the condensate with the energetically favorable ground state is $\bar{n}$ is an arbitrary phase. Different forms of $\Delta_{ij}$ have the same form as the angulon coupling to the spatial part of the $Z_0$ boson worked out in [10], from which we can read off the terms proportional to $B$ in eq. 4.

The condensate in eq. 1 breaks spontaneously the rotation and phase invariance symmetries $SO(3) \times U(1)$ down to the subgroup $O(2) \times Z_2$ composed of rotations around the $z$-axis (direction of the magnetic field) and rotations of the phase of the neutron field by $\pi$ which leave the condensate in eq. 1 invariant. As a consequence we expect two Goldstone bosons corresponding to rotations of the condensate around the $x$ and $y$ axis. Since, by rotational symmetry, long wavelength oscillations of the condensate in these directions cost no energy, the quantized modes will be ungauged, in accordance with the Goldstone theorem. These oscillations can be parametrized by the fields $\alpha_1$ and $\alpha_2$ defined by

$$\Delta(x) = e^{i(J_1 \alpha_1 + J_2 \alpha_2)/f} - \Delta e^{-i(J_1 \alpha_1 + J_2 \alpha_2)/f},$$

where $J_1, J_2$ are the $3 \times 3$ matrix generating rotations around the $x$ and $y$ axis. The effective theory for the angulons was derived in [10] under mild assumptions. Due to the lack of rotational symmetry the explicit expression of even the lowest order terms (in powers of $\alpha_1$ and in the number of derivatives), is unenlightening and can be found in [10].

For our purposes only two terms are important. The first is the leading (two derivatives) term quadratic in $\alpha_1$ in the presence of a magnetic field which, in momentum space reads

$$S = \int \frac{d^4p}{(2\pi)^4} \left( \alpha_1(p) \alpha_2(p) \right) \left( \frac{a p_x^2 + v_F^2(p_x^2 + p_y^2 + b p_z^2)}{c v_F p_x p_y - i \frac{g s N B_{pa}}{2M}} + \frac{c v_F p_x p_y - i \frac{g s N B_{pa}}{2M}}{a p_0^2 + v_F^2(p_x^2 + p_y^2 + b p_z^2)} \right) \alpha_1(-p) \alpha_2(-p),$$

where $a, b, c, d, e$ are given by [10]

$$a = 3 + \frac{\pi}{\sqrt{3}} \approx 4.81, \quad b = \frac{3}{2} + \frac{\pi}{9\sqrt{3}} \approx -1.30, \quad c = -\frac{4\pi}{3\sqrt{3}} \approx -2.42,$$

$$d = \frac{3}{2} + \frac{2\pi}{9\sqrt{3}} \approx -1.10, \quad e = \frac{3}{2} + \frac{14\pi}{9\sqrt{3}} \approx -1.32,$$

$v_F$ is the Fermi velocity of the neutrons, $B$ is the magnetic field which points along the $z$ direction, $\epsilon$ is the charge of an electron and $g_N$ and $M$ are the magnetic moment and mass of the neutrons respectively, including Fermi liquid corrections. The symbol $\epsilon$ has been used to denote both a low energy constant and the electric charge. However, it will always be clear from the context which of the two quantities the symbol stands for. The coupling of the angulons to the magnetic field can be obtained from the results in [10] by noticing that magnetic fields couple to neutron through their magnetic moment:

$$\mathcal{L}_{B-a} = \frac{eg}{2M} n^\dagger \mathbf{S} \cdot \mathbf{B} n,$$

where $g = -1.913$ is the neutron anomalous magnetic moment (in unit of the nuclear magneton). This coupling has the same form as the angulon coupling to the spatial part of the $Z_0$ boson worked out in [10], from which we can read off the terms proportional to $B$ in eq. 4.
FIG. 1: Feynman diagram showing the massive angulon (dashed line) decay into a neutrino pair (solid line). The wavy line represents a $Z_0$

In order to diagonalize the action we introduce new fields $\beta_1$ and $\beta_2$

$$\sqrt{a}\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \quad (9)$$

in terms of which the quadratic part of the action reads

$$S = \int \frac{d^4p}{(2\pi)^4} \begin{pmatrix} \beta_1(p) \\ \beta_2(p) \end{pmatrix} \begin{pmatrix} (p_0 - \xi_1)(p_0 - \xi_2) & 0 \\ 0 & (p_0 + \xi_1)(p_0 + \xi_2) \end{pmatrix} \begin{pmatrix} \beta_1(-p) \\ \beta_2(-p) \end{pmatrix} \quad (10)$$

where, $\xi_1$ and $\xi_2$ determine the dispersion relation of the modes:

$$\xi_1 = A + \sqrt{A - B}, \quad (11)$$
$$\xi_2 = A - \sqrt{A - B} \quad (12)$$

where

$$A = \frac{1}{2} \left( \frac{e g N B}{2 M a} \right)^2 - \frac{c + d}{2a} v_F^2 (p_x^2 + p_y^2) - \frac{b}{a} v_F^2 p_z^2 \quad (13)$$

and

$$B = \frac{cd}{a^2} v_F^4 (p_x^4 + p_y^4) + \frac{bc + bd}{a^2} v_F^4 (p_x^2 + p_y^2) p_z^2 + \frac{b^2}{a^2} v_F^4 p_z^4 + \frac{c^2}{a^2} v_F^4 p_z^2 p_y^2 + \frac{d^2}{a^2} v_F^2 p_z^2 p_y^2 - \frac{e^2}{a^2} v_F^2 p_z^2 p_y^2. \quad (14)$$

We notice that the presence of the magnetic field turned one of the Goldstone bosons into a massive mode while the remaining massless mode has now a quadratic dispersion relation at small momenta. This is in accord to the generalized Goldstone theorem valid in the absence of Lorentz symmetry [15].

The second relevant term of the effective action is the coupling of the angulons to the electroweak $Z_0$ gauge boson [10]:

$$\mathcal{L} = C_A g f (Z_0^0 \partial_0 \alpha_2 - Z_1^0 \partial_0 \alpha_1) \quad (15)$$
FIG. 2: \( h(x) \) as a function of \( x = \frac{\cos Bg}{2MaT} \) (solid line) and its analytic approximation \( h(x) \approx 0.000042 \ x^7 e^{-x} \) (dashed line).

where \( f^2 = \frac{Mk_F}{\sqrt{2}} \), \( k_F \) is the neutron Fermi momentum, \( C_A^2 = C_A^2 \frac{G_FM_Z^2}{2\sqrt{2}} \) with \( C_A^2 \sim 1.1 \pm 0.15 \), \( G_F \) the Fermi constant and \( M_Z \) the \( Z_0 \) boson mass.

Finally, the coupling between the gauge boson and neutrinos is well known [16]:

\[
\mathcal{L}_{Z-\nu} = \frac{gZ_{\mu}}{\cos \theta_W} \left( \frac{1}{4} \bar{\nu} \Gamma^\mu (1 - \Gamma^5) \nu \right).
\] (16)

II. EMISSIVITY

The tree level contribution to the massive angulon decay is given by the diagram in fig. 1. Using appropriate normalization for the states, the amplitude for this process can be written as

\[
A = \left( \bar{u}_s(p) \frac{\gamma^1(1 - \gamma^5)}{2} v_l(p') c_{11} - \bar{u}_s(p) \frac{\gamma^2(1 - \gamma^5)}{2} v_l(p') c_{21} \right) \frac{C_A g_{fg}}{\sqrt{\alpha \cos \theta_W}} \frac{k_0}{M_Z^2} \frac{(2\pi)^4 \delta^4(p + p' - k)}{V^{3/2} \sqrt{2\xi_k^2 w_p^2 w_{p'}}} \] (17)

where \( p \) and \( p' \) are the momenta of the outgoing neutrinos and \( k \) is the momentum of the angulon \( \beta_1 \). The on-shell conditions for the external legs are

\[
k_0 = \xi_1 \\
p_0 = w_p = |p| \\
p'_0 = w'_p = |p'|
\] (18)

as the neutrinos are taken to be massless.

The decay rate is defined by

\[
\Gamma = N \sum_{\text{neutrino momenta and helicities}} \frac{|A|^2}{\tau}
\] (19)

where \( \tau \) is time over which the interaction is on (to be taken to infinity at the end of the computation) and \( N = 3 \) is the number of neutrino flavors. The square of the amplitude brings two four dimensional \( \delta \)-functions and one of them can be replaced by \( V \tau \) (\( V \) is the volume of the space). After performing the phase space integral over one of the outgoing neutrino momentum we end up with

\[
\Gamma = N \int \frac{d^3p}{(2\pi)^3} \frac{2\pi \delta(p_0 + p'_0 - k_0)}{2\xi_k^2 w_p^2 w_{p'}} 2|P_1|c_{11}|^2 + |P_2|c_{11}|^2 - |P_1c_{11} e_{21}|^2 - |P_2 c_{11} e_{21}^*|^2 \frac{C_A g_{fg}}{\sqrt{\alpha \cos \theta_W}} \frac{k_0}{M_Z^2} \] (20)

where,

\[
P_1 = p_0p_0' + p_1p_1' - p_2p_2' - p_3p_3' \\
P_2 = p_0p_0' + p_2p_2' - p_1p_1' - p_3p_3' \\
P_{12} = p_1p_2' + p_2p_1' + i(p_0p_3' - p_3p_0')
\] (21)
The factor of which is a function of the dimensionless quantity \( B_{\text{eq}} \)
and plot it as a function of \( T \) with \( -\beta \).

In order to write \( Q \) as a dimensionless integral, we normalize all our symbols
with respect to \( T^3 \) where \( \lambda \) is the mass dimension of the quantity
the symbol stands for. In terms of the new dimensionless symbols, which we distinguish from the unnormalized ones by a tilde, the emissivity can be expressed as

\[
Q = N \int T^5 \frac{d^3 k d^3 \tilde{p}}{(2\pi)^6} \delta \left( \tilde{p}_0 + \tilde{p} \right) \left( \tilde{\rho}_0 + \tilde{\rho} - \tilde{\kappa}_0 \right)
\]

\[
\frac{2(\tilde{P}_1 |c_{11}|^2 + \tilde{P}_2 |c_{11}|^2 - \tilde{P}_{12} c_{11} c_{21} - \tilde{P}_{12}^* c_{11} c_{21})}{2\tilde{w}_k 2\tilde{w}_p 2\tilde{w}'_p} \left( \frac{C_A g \tilde{g}}{2\sqrt{\alpha} \cos(\theta_W')} \right)^2 \frac{\tilde{k}_0}{\tilde{k}_0 - 1}.
\]

The factor of \( \left( \frac{C_A g \tilde{g}}{2\sqrt{\alpha} \cos(\theta_W')} \right)^2 \frac{1}{M^2} \) can be written as \( C_A g \tilde{g} \frac{G_F^2 \tilde{M}^2 \tilde{k}_F^2}{} \) and plugging this back in (23) we get,

\[
Q = NC_A^2 \frac{81}{128 \pi} \tilde{G}_F^2 \tilde{M} \tilde{k}_F \int T^5 \frac{d^3 k d^3 \tilde{p}}{(2\pi)^6} \delta \left( \tilde{p}_0 + \tilde{p} \right) \left( \tilde{\rho}_0 + \tilde{\rho} - \tilde{\kappa}_0 \right)
\]

\[
2(\tilde{P}_1 |c_{11}|^2 + \tilde{P}_2 |c_{11}|^2 - \tilde{P}_{12} c_{11} c_{21} - \tilde{P}_{12}^* c_{11} c_{21}) \frac{1}{2\tilde{w}_k 2\tilde{w}_p 2\tilde{w}'_p} \frac{\tilde{k}_0^3}{\tilde{k}_0 - 1}.
\]

\[
= G_F^2 M k_F T^7 h \left( \frac{e g N B}{2 \alpha M T} \right)
\]

where \( h(x) \) is given by the dimensionless integral

\[
h = NC_A^2 \frac{81}{128 \pi} \int d^3 k d^3 \tilde{p} \delta \left( \tilde{p}_0 + \tilde{p} \right) \left( \tilde{\rho}_0 + \tilde{\rho} - \tilde{\kappa}_0 \right)
\]

\[
2(\tilde{P}_1 |c_{11}|^2 + \tilde{P}_2 |c_{11}|^2 - \tilde{P}_{12} c_{11} c_{21} - \tilde{P}_{12}^* c_{11} c_{21}) \frac{1}{2\tilde{w}_k 2\tilde{w}_p 2\tilde{w}'_p} \frac{\tilde{k}_0^3}{\tilde{k}_0 - 1},
\]

which is a function of the dimensionless quantity \( x = \frac{B_{\text{eq}}}{2 M^2 T} \) alone. We compute the integral in equation 26 numerically
and plot it as a function of \( \frac{B_{\text{eq}}}{2 M^2 T} \) in Fig. 2. It turns out that the function \( h(x) \) is very well approximated by

\[
h(x) \approx 0.000042 x^7 e^{-x}
\]

Fig. 3 is a plot of corresponding neutrino emissivity as a function of temperature for three different magnetic fields.
III. DISCUSSION

Our main result is the neutrino emissivity due to the decay of angulons in the presence of a magnetic field as shown in eq. 29 and depicted in Fig. 3. We provided a simple, precise analytic form in eqs. 29 and 27. The calculation involved a number of approximations, all well controlled and precise enough for the application to the cooling of neutron stars. The first is in the derivation of the parameters of the effective theory whose validity is discussed at length in [10]. Higher orders in the low momentum expansion, either from terms in the effective theory with more terms or from loops are suppressed by factors of $(T/\Delta_0)^2$ and are small at temperatures well below the critical one. Effects arising from a possible finite temperature mass the angulons may acquire [17] belongs to this category. In addition, we use $r = -1/2$ for our calculations. A different value of $r$ would mean a different pattern of symmetry breaking. Generically, the rotation group would be broken down to the discrete subgroup $(Z_2)^3$ of inversion along the principal axis of $\Delta_{ij}$ and three Goldstone bosons would exist. However, only one of them would acquire a mass due to the interaction with the magnetic field and our calculation would be changed just by a small shift of the angulon mass. For magnetic fields in excess of $B \approx 10^{17} G$ the phase with $r = -1$ is expected to be favored [11]. This phase is qualitatively distinct from the other because the neutron are gapless along a certain direction in space. Those ungapped neutron may undergo beta decay and provide a source of neutrinos suppressed only by the restricted phase space of ungapped neutrons. Our calculation will still hold for the angulon part but, in this case, it has to be supplemented by the ungapped neutron part.

The emissivity rates from PBF and angulon decay processes are not to be directly compared. This is because the PBF is, at any given time, effective only on a shell of the star where the temperature is near the critical temperature (which is density dependent). On the other hand, due to the proton superconductivity, magnetic fields are believed to be confined to flux tubes and thus angulons can decay only inside the flux tubes or in their immediate vicinity. The comparison between the angulon and PBF emissivities is further complicated by the uncertainty on the value of the gap (which affects the PBF primarily) and on the value of the magnetic fields in the core (which affect the angulon rate). Still, it is instructive to look at their relative numerical values. The emissivity of neutrinos in PBF processes is given by [8][9]

$$Q_{PBF} = \frac{4G_F^2\pi kF7NF}{15\pi^5} \frac{\Delta}{T}$$

where $F$ is a function of ratio of the gap $\Delta$ to temperature peaking at $T \sim \Delta$ and decaying exponentially at smaller temperatures. On the other hand the massive angulon decay gives rise to an emissivity equal to

$$Q_{ang} = G_F^2\pi kT^7 h \left(\frac{Beg_N}{2aMT}\right),$$

where the function $h(x)$ peaks at $x \sim 7$ and decays exponentially at larger values of $x$ and as $\sim x^7$ at small $x$. For temperatures near the gap value $T \sim \Delta_0$ the PBF process is much larger than the angulon emissivity (assuming $\Delta_0 \gg eB/M$). At lower temperatures, around $T \sim eB_{15}/M \approx 3 \times 10^7 KB_{15}$, the angulon emissivity is larger than the one from PBF. For temperatures smaller than that, the angulon process still dominates but the phenomenological interest of these rates is small as it is difficult to observe stars so cold.

Since the angulon decay process can occur only in regions of high magnetic field it is important to have an estimate of the volume fraction of the star that are close enough to magnetic flux tubes. Each flux tube carries a flux quantum equal to $\Phi_0 = \pi/e$. Assuming a dipole form for the magnetic field inside of the star, the total flux crossing the star is $\Phi = \pi R_{\text{star}}^2 B_{\text{star}}$ (where $R_{\text{star}}$ and $B_{\text{star}}$ are the radius and average magnetic field in the interior of the star). The number of flux tubes then will be of the order of $N \approx \Phi/\Phi_0 \approx R_{\text{star}}^2 eB_{\text{star}}$ and they will be separated by an average distance of $L \approx \sqrt{\pi R_{\text{star}}^2/N} \approx \sqrt{\pi/(eB_{\text{star}})}$. The magnetic fields extends around a flux tube to a distance of the order of the penetration length $\lambda = \sqrt{m/(4\pi n_p e^2)}$, where $n_p$ is the proton density. Thus, the fraction of the star volume with sizable magnetic fields is of the order of $(\lambda/L)^2 \approx 0.04B_{15}(0.1n_0)/n_p$, where $B_{15} = B/10^{15} G$, $n_p$ is the proton density and $n_0 = 0.16 fm^{-3}$ the nuclear saturation density. Clearly only in stars with very large magnetic fields the angulon decay mechanism may be relevant. Magnetars form a class of neutron stars where fields of this order are known to exist but ordinary neutron stars, with much smaller long range magnetic fields may have magnetic fields of this order in their interior. It is clear, however, that a proper assessment of the angulon decay mechanism on the cooling curves can only be done with a realistic cooling code.
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