Coupled channel approach to the structure of the $X(3872)$

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Abstract

We have performed a coupled channel calculation of the $1^{++} c\bar{c}$ sector including $q\bar{q}$ and $DD^*$ molecular configurations. The calculation was done within a constituent quark model which successfully describes the meson spectrum, in particular the $c\bar{c} 1^{--}$ sector. Two and four quark configurations are coupled using the $^3P_0$ model.

The elusive $X(3872)$ meson appears as a new state with a high probability for the $DD^*$ molecular component. When the mass difference between neutral and charged states is included a large $D^0D^{*0}$ component is found which dominates for large distances and breaks isospin symmetry in the physical state. The original $c\bar{c}(2^3P_1)$ state acquires a sizable $DD^*$ component and can be identified with the $X(3940)$. We study the $B \to K\pi^+\pi^-J/\psi$ and $B \to KD^0D^{*0}$ decays finding a good agreement with Belle and BaBar experimental data.

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I. INTRODUCTION.

In the last years a number of exciting discoveries of new hadron states have challenged our description of the hadron spectroscopy. One of the most mysterious states is the well established \( X(3872) \). It was first discovered by the Belle Collaboration in the \( J/\psi\pi\pi \) invariant mass spectrum of the decay \( B^+ \rightarrow K^+\pi^+\pi^-J/\psi \). Its existence was soon confirmed by BaBar, CDF and D0 Collaborations. The world average mass is \( M_X = 3871.2 \pm 0.5 \text{MeV} \) and its width \( \Gamma_X < 2.3 \text{MeV} \). The measurements of the \( X(3872) \rightarrow \gamma J/\psi \) decay \([2, 6]\) implies an even \( C \)-parity. Moreover angular correlation between final state particles in the \( X(3872) \rightarrow \pi^+\pi^-J/\psi \) decay measured by Belle \([5]\) suggests that the \( J^{PC} = 0^{++} \) and \( J^{PC} = 0^{+-} \) may be ruled out and strongly favors the \( J^{PC} = 1^{++} \) quantum numbers although the \( 2^{++} \) combination cannot be excluded. A later analysis by CDF Collaboration \([7]\) of the same decay is compatible with the Belle results and concludes from the dipion mass spectrum that the most likely quantum numbers should be \( J^{PC} = 1^{++} \) but cannot totally exclude the \( J^{PC} = 2^{--} \) combination. These conclusions were confirmed by a new CDF analysis of the decay \( X(3872) \rightarrow \pi^+\pi^-J/\psi \) followed by \( J/\psi \rightarrow \mu^+\mu^- \) excluding all the other possible quantum numbers at 99.7% confidence level \([8]\). However the small phase space available for the decay \( X(3872) \rightarrow D^0\bar{D}^0\pi^0 \) observed by Belle \([9]\) discards the \( J = 2 \) leaving the \( 1^{++} \) assignment as the most probable option.

In the \( 1^{++} \) sector the only well established state in the PDG \([10]\) is the \( \chi_{c1}(1P) \) with a mass \( M = 3510.66 \pm 0.07 \text{MeV} \). The first excitation is expected around 3950 MeV. In this energy region Belle has reported the observation of three resonant structures denoted by \( X(3940) \), \( Y(3940) \) and \( Z(3930) \). The last one was observed by Belle in the \( \gamma\gamma \rightarrow D\bar{D} \) reaction \([11]\) and is already included in the PDG as the \( \chi_{c2}(2P) \). The \( X(3940) \) has been seen as a peak in the recoiling mass spectrum of \( J/\psi \) produced in \( e^+e^- \) collision. Its main decay channel is \( DD^* \). \( Y(3940) \) appears as a threshold enhancement in the \( J/\psi\omega \) invariant mass distribution of the \( B \rightarrow J/\psi\omega K \) decay \([13]\).

The relative decay rates outlines a puzzling structure for the \( X(3872) \). The \( \gamma J/\psi \) and \( \gamma \psi' \) decay rates \([14]\)

\[
\frac{X(3872) \rightarrow \gamma J/\psi}{X(3872) \rightarrow \pi^+\pi^-J/\psi} = 0.33 \pm 0.12
\]

\[
\frac{X(3872) \rightarrow \gamma \psi'}{X(3872) \rightarrow \pi^+\pi^-J/\psi} = 1.1 \pm 0.4
\]

(1)
suggest a $c\bar{c}$ structure whereas the $X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi$ decay mode

$$\frac{X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi}{X(3872) \rightarrow \pi^+\pi^- J/\psi} = 1.0 \pm 0.4 \pm 0.3$$

indicates a very different one \[15\]. The dipion mass spectrum in the $\pi^+\pi^- J/\psi$ channel shows that the pions come from the $\rho^0$ resonance. On the other hand the $\pi^+\pi^-\pi^0$ mass spectrum has a strong peak around 750 $MeV$ suggesting that the process is dominated by a $\omega$ meson. Thus the ratio $R \sim 1$ indicates that there should be an isospin violation incompatible with a traditional charmonium assumption.

Concerning the mass value, in 2006 Belle measured \[8\] an enhancement in the $D^0 D^0 \pi^0$ channel just above the $D^0 D^*^0$ threshold using the $B^+ \rightarrow K^+ D^0 D^0 \pi^0$ decay. The amazing aspect of this enhancement is that it appears at $M_X = 3875.2 \pm 0.7^{+0.3}_{-1.6} \pm 0.8 MeV$ just 3 $MeV$ above the $M_X$ world average mass value. This fact triggered a new discussion about the possibility of two different charmonium like states. The Belle mass value was confirmed later by the BaBar Collaboration \[16\]. Last year the Belle Collaboration announced a new measurement of the $B \rightarrow K D^0 D^0 \pi^0$ decay \[14\] with a lower position of the $X(3872)$ peak in $M_X = 3872.6^{+0.5}_{-0.4} \pm 0.4 MeV$. New data of the $\pi^+\pi^- J/\Psi$ decay has been also recently reported by the Belle \[18\], BaBar \[19\] and CDF \[20\] Collaborations, confirming a mass value in agreement with the world average.

The $X(3872)$ mass is difficult to reproduce by the standard quark models (see Ref. \[21\] for a review). The state appears to be too heavy for a $1D$ charmonium state and too light for a $2P$ charmonium one. Moreover no four-quark bound state configurations have been found in this mass region which rules out the possibility that this particle was a compact tetraquark system \[22, 23\].

An important property of the $X(3872)$ is that its mass is extremely close to the $D^0 D^{*0}$ threshold with a difference using PDG values given by $-0.6 \pm 0.6 MeV$. The proximity of the $D^0 D^{*0}$ threshold made the $X(3872)$ a natural candidate to a $C = + D^0 D^{*0}$ molecule. The hypothesis of a $DD^*$ molecule mainly bound by pion exchange has been suggested by several authors \[24\]. In particular, in Ref. \[25\] it is argued that the $X(3872)$ is a $J^{PC} = 1^{++} D^0 D^{*0}$ molecule stabilized by admixture of $\rho J/\psi$ and $\omega J/\psi$ states. The author shows that pion exchange alone can not bind the molecule being the combined effect of pion exchange and coupled channels responsible for that. The $D^0 D^{*0}$ component dominates the wave function at the experimental binding becoming all other contributions small.
The molecular interpretation runs into trouble when it tries to explain the high $\gamma\psi'$ 
decay rate. For a molecular state this can be only proceed through annihilation diagrams
and hence is very small.

This puzzling situation suggests for the $X(3872)$ state a combination of a $2P\ c\bar{c}$ state and
a weakly-bound $D^0 D^{*0}$ molecule [13, 14]. The experimental assignment $J^{PC} = 1^{++}$ favors
this conclusion because it allows the molecule to be in a relative $S$-wave state whereas the
соrresponding $c\bar{c}$ should be in a relative $P$-wave state. Then the masses of the additional
light quarks are compensated by the angular momentum excitation and both configurations
may be almost in the same mass region. Similar behavior has been already observed in
the open charmed sector [26]. Recently Zhang et al. [27] have analyzed, using the coupled
channel Flatté formula, the $B \to KD^0D^{*0}$ [17] and $B \to K\pi^+\pi^-J/\Psi$ [18] Belle data. They
found that a third sheet pole close, but below, the $D^0D^{*0}$ threshold is needed to describe
the data, which supports the idea of the $X(3872)$ as a mixed state of $\chi'_{c1}$ and $D^0D^{*0}$ components.
An updated Flatté analysis of the same data together with the new BaBar data of the same
reactions [16, 19] has been performed in Ref. 28 assuming a mechanism for the $X(3872)$
production via the charmonium components. The authors conclude that the data clearly
indicates a sizable $c\bar{c}\ 2^{3}P_1$ component in the $X(3872)$ wave function. Finally Dong et al. [29]
show in their analysis of the $J/\psi\gamma$ and $\psi(2S)\gamma$ decay modes of the $X(3872)$ that the large
value of the ratio $BR(X(3872) \to J/\psi\gamma)/BR(X(3872) \to \psi(2S)\gamma)$ measured by the BaBar
Collaboration provide a constraint on the value of the $c\bar{c}$ component in the $X(3872)$. From
the experimental values they deduce a small admixture of the $c\bar{c}$ component

Having in mind these evidences, in this paper we perform a microscopic coupled channel
calculation of the $1^{++}$ sector including both $c\bar{c}$ and $DD^*$ states. The calculation is done
in the framework of a constituent quark model widely used in hadronic spectroscopy. The
paper is organized as follows. In the next section we review the main ingredients of our
model. Section III is devoted to discuss the numerical procedures and the results . Finally
we summarize the main findings of our work in the last section.
II. THE MODEL.

A. The constituent quark model

The constituent quark model used in this work has been extensively described elsewhere \cite{30, 31} and therefore we will only summarize here its most relevant aspects. The model is based on the assumption that the light constituent quark mass appears as a consequence of the spontaneous breaking of the chiral symmetry at some momentum scale. As a consequence the quark propagator gets modified and quarks acquire a dynamical momentum dependent mass. The simplest Lagrangian must therefore contain chiral fields to compensate the mass term and can be expressed as \cite{32}

\[
\mathcal{L} = \bar{\psi} \left( i \partial \! \! \! / - M(q^2) U^{\gamma_5} \right) \psi
\]

where \( U^{\gamma_5} = \exp(i \pi^a \lambda^a \gamma_5 / f_\pi) \), \( \pi^a \) denotes nine pseudoscalar fields \((\eta_0, \pi, K, \eta_8)\) with \( i = 1, \ldots, 4 \) and \( M(q^2) \) is the constituent mass. This constituent quark mass, which vanishes at large momenta and is frozen at low momenta at a value around 300 MeV, can be explicitly obtained from the theory but its theoretical behavior can be simulated by parameterizing \( M(q^2) = m_q F(q^2) \) where \( m_q \simeq 300 \) MeV, and

\[
F(q^2) = \left[ \frac{\Lambda^2}{\Lambda^2 + q^2} \right]^{1/2}.
\]

The cut-off \( \Lambda \) fixes the chiral symmetry breaking scale.

The Goldstone boson field matrix \( U^{\gamma_5} \) can be expanded in terms of boson fields,

\[
U^{\gamma_5} = 1 + \frac{i}{f_\pi} \gamma^5 \pi^a \pi^a - \frac{1}{2 f_\pi^2} \pi^a \pi^a + \ldots
\]

The first term of the expansion generates the constituent quark mass while the second gives rise to a one-boson exchange interaction between quarks. The main contribution of the third term comes from the two-pion exchange which has been simulated by means of a scalar exchange potential.

In the heavy quark sector chiral symmetry is explicitly broken and this type of interaction does not act. However it constrains the model parameters through the light meson phenomenology and provides a natural way to incorporate the pion exchange interaction in the \( DD^\ast \) dynamics.
Beyond the chiral symmetry breaking scale one expects the dynamics to be governed by QCD perturbative effects. They are taken into account through the one gluon-exchange interaction \[\mathcal{L}_{gqq} = i\sqrt{4\pi\alpha_s} \bar{\psi}\gamma_\mu G^\mu_c \lambda_c \psi,\] derived from the lagrangian

where $\lambda_c$ are the SU(3) color generators and $G^\mu_c$ the gluon field.

The other QCD nonperturbative effect corresponds to confinement, which prevents from having colored hadrons. Such a term can be physically interpreted in a picture in which the quark and the antiquark are linked by a one-dimensional color flux-tube. The spontaneous creation of light-quark pairs may give rise at same scale to a breakup of the color flux-tube. This can be translated into a screened potential \[V_{CON}(\vec{r}_{ij}) = \{-a_c (1 - e^{-\mu_c |r_{ij}|}) + \Delta\} \langle \vec{\lambda}_i \cdot \vec{\lambda}_j \rangle\] in such a way that the potential saturates at the same interquark distance.

where $\Delta$ is a global constant to fit the origin of energies. At short distances this potential presents a linear behavior with an effective confinement strength $a = -a_c \mu_c (\vec{\lambda}_i \cdot \vec{\lambda}_j)$ while it becomes constant at large distances. It has been shown that this form of the potential is important to explain the huge degeneracy observed in the high excited light meson spectrum and turns out to be very important for the correct assignment of $J^{PC} = 1^{--}$ charmonium states. Explicit expressions for all these interactions are given in [37].

All these ingredients are needed to explain the hadronic phenomenology. Apart from the obvious confinement potential, gluon exchange is demanded from the hyperfine splitting in charmonium. Moreover, pion exchange is one of the best established interaction in nature being its parameters constraint by a huge amount of experiments. When Goldstone boson exchanges are considered at the quark level together with the OGE the possibility of double counting emerges. This problem has been studied in the literature concluding that the pion can be safely exchanged together with the gluon [38].

Constituent quark models are also criticized because they only incorporate a limited sector of the Fock space. In particular its applicability to high excited states may be questionable as more thresholds open up. In our case the parameters of the model has been fixed in the low lying part of the spectrum where these effects are more easily incorporated into
them. Furthermore the main contribution of the open channels are taken into account by the screened confinement potential.

B. The coupled channel approach

To model the $1^{++}$ $c\bar{c}$ system we assume that the hadronic state is

$$|\Psi\rangle = \sum_{\alpha} c_{\alpha}|\psi_{\alpha}\rangle + \sum_{\beta} \chi_{\beta}(P)|\phi_{M_1}\phi_{M_2}\beta\rangle$$

(8)

where $|\psi_{\alpha}\rangle$ are $c\bar{c}$ eigenstates of the two body Hamiltonian, $\phi_{M_i}$ are $c\bar{n}$ ($\bar{c}n$) eigenstates describing the $D$ ($\bar{D}$) mesons, $|\phi_{M_1}\phi_{M_2}\beta\rangle$ is the two meson state with $\beta$ quantum numbers coupled to total $J^{PC}$ quantum numbers and $\chi_{\beta}(P)$ is the relative wave function between the two mesons in the molecule. As we always work with eigenstates of the $C$-parity operator we use the usual notation in which $DD^*$ is the right combination of $D\bar{D}^*$ and $D^*\bar{D}$.

The coupling between the two sectors requires the creation of a light quark pair $n\bar{n}$. Similar to the strong decay process this coupling should be in principle driven by the same interquark hamiltonian which determines the spectrum. However Ackleh et al. [39] have shown that the quark pair creation $^3P_0$ model [40], gives similar results to the microscopic calculation. The model assumes that the pair creation Hamiltonian is

$$\mathcal{H} = g \int d^3x \bar{\psi}(x)\psi(x)$$

(9)

which in the non-relativistic reduction is equivalent to the transition operator [41]

$$T = -3\sqrt{2}\gamma' \sum_{\mu} \int d^3p d^3p' \delta^{(3)}(p + p') \left[ \mathcal{Y}_1 \left( \frac{p - p'}{2} \right) b^\dagger_\mu(p)d^\dagger_\mu(p') \right]$$

(10)

where $\mu$ ($\nu = \bar{\mu}$) are the quark (antiquark) quantum numbers and $\gamma' = 2^{5/2}\pi^{1/2}\gamma$ with $\gamma = \frac{g}{2m}$ is a dimensionless constant that gives the strength of the $q\bar{q}$ pair creation from the vacuum. From this operator we define the transition potential $V_{\beta\alpha}(P)$ within the $^3P_0$ model as [42]

$$\langle \phi_{M_1}\phi_{M_2}\beta|T|\psi_{\alpha}\rangle = PV_{\beta\alpha}(P) \delta^{(3)}(\vec{P}_{\text{cm}})$$

(11)

where $P$ is the relative momentum of the two meson state.
Using the wave-function from Eq. (8) and the coupling Eq. (11) we arrive to the coupled equations

\[ M_\alpha c_\alpha + \sum_\beta \int V_{\alpha\beta}(P)\chi_\beta(P) P^2 dP = E c_\alpha \]

\[ \sum_\beta \int H_{\beta'\beta}^{M_1 M_2}(P', P)\chi_\beta(P) P^2 dP + \sum_\alpha V_{\beta'\alpha}'(P')c_\alpha = E\chi_{\beta'}(P') \]

(12)

where \( M_\alpha \) are the masses of the bare \( c\bar{c} \) mesons and \( H_{\beta'\beta}^{M_1 M_2} \) is the RGM Hamiltonian for the two meson states obtained from the \( q\bar{q} \) interaction.

Solving the coupling with \( c\bar{c} \) states we finally end up with a Schrödinger type equation for the relative wave function of the two meson state

\[ \sum_\beta \int \left( H_{\beta'\beta}^{M_1 M_2}(P', P) + V_{\beta'\beta}'(P', P) \right)\chi_\beta(P) P^2 dP = E\chi_{\beta'}(P') \]

(13)

where

\[ V_{\beta'\beta}'(P', P) = \sum_\alpha \frac{V_{\beta'\alpha}'(P')V_{\alpha\beta}(P)}{E - M_\alpha} \]

(14)

is an effective interaction between the two mesons due to the coupling with intermediate \( c\bar{c} \) states.

In this way we study the influence of the \( c\bar{c} \) states on the dynamics of the two meson states. This is a different point of view from the usually found in the literature where the influence of two meson states (in general without meson-meson interaction) in the mass and width of \( c\bar{c} \) states is studied [42]. Our approach allows to generate new states through the meson-meson interaction due to the coupling with \( c\bar{c} \) states and to the underlying \( q\bar{q} \) interaction. As we will see the renormalization effects of the \( c\bar{c} \) mass due to this channel is small.

The \( c\bar{c} \) probabilities are given by

\[ c_\alpha = \frac{1}{E - M_\alpha} \sum_\beta \int V_{\alpha\beta}(P)\chi_\beta(P) P^2 dP \]

(15)

with the normalization condition \( 1 = \sum_\alpha |c_\alpha|^2 + \sum_\beta \langle \chi_\beta | \chi_\beta \rangle \).

C. Flatté parametrization

In order to compare the predictions of our model with the recent Belle and BaBar experimental data we obtain from Eq. (12) a Flatté-like parametrization of the \( DD^* \) near threshold amplitude following Ref. [43]. We remind here the main ideas.
From Eq. (12), and neglecting the $DD^*$ interaction, one can easily derive the $DD^*$ scattering amplitude

$$F_{DD^*}^\beta(P, P; E) = -\pi \mu \sum_\alpha \frac{V_{\beta\alpha}^2(P)}{E - M_\alpha + g_{DD^*}^\alpha(E)}$$

where the function $g_{DD^*}^\alpha(E)$ is given by

$$g_{DD^*}^\alpha(E) = \sum_\beta \int \frac{V_{\beta\alpha}^2(P)}{\frac{P^2}{2\mu} + M_D + M_{D^*} - E - i0^+} P^2 dP.$$ 

For small binding energies $\epsilon = M_D + M_{D^*} - E$ it can be expanded as

$$g_{DD^*}^\alpha(E) = \bar{E}_{DD^*}^\alpha + \frac{i}{2} \Gamma_{DD^*}^\alpha + \mathcal{O}(4\mu^2\epsilon/\Lambda^2)$$

where

$$E_{DD^*}^\alpha = 2\mu \sum_\beta \int_0^\infty V_{\beta\alpha}^2(P) dP$$

$$\Gamma_{DD^*}^\alpha = 2\pi \mu \sum_\beta V_{\beta\alpha}^2(0) P$$

and $\Lambda \gg \epsilon$ is the characteristic scale of the $V_{\alpha\beta}$ production amplitude which may correspond to the scale of the quark wave function and it’s assume to be much bigger than the binding energy of the physical state.

A straightforward generalization to include the $DD^*$ charged states and other channels gives the expression for the near threshold $DD^*$ scattering amplitude

$$F_{DD^*} = -\frac{1}{2P} \frac{\Gamma_{DD^*}}{E - E_f + \frac{i}{2}(\Gamma_{DD^*}^0 + \Gamma_{DD^*}^{+0} + \Gamma_{DD^*}^{-0} + \Gamma(E)) + \mathcal{O}(4\mu^2\epsilon/\Lambda^2)}$$

where $\Gamma(E)$ accounts for the width due to other processes different from the opening of the near $DD^*$ threshold. Eq. (21) corresponds to a Flatté parametrization with

$$D(E) = E - E_f + \frac{i}{2}(\Gamma_{DD^*}^0 + \Gamma_{DD^*}^{+0} + \Gamma_{DD^*}^{-0} + \Gamma(E)) + \mathcal{O}(4\mu^2\epsilon/\Lambda^2).$$

Now assuming, as in Ref. [28], that the short range dynamics of the weak $B \to KX(3872)$ transition can be absorbed into a coefficient $B$ we are able to write the differential rates in the Flatté approximation as

$$\frac{dBr(B \to KD^0D^*)}{dE} = B \frac{1}{2\pi} \frac{\Gamma_{DD^*}^0(E)}{|D(E)|^2}.$$
The analysis of the $B \to KX(3872) \to K\pi^+\pi^- J/\psi$ data is more involved because we have to calculate the $DD^* \to \pi^+\pi^- J/\psi$ transition amplitude.

This can consistently be done in our formalism assuming that the process takes place through the $DD^*$ components of the $X(3872)$ which decays in $\rho J/\psi$ and then into the final $\pi^+\pi^- J/\psi$ states. The decay width of the process is given by

$$\Gamma_{\pi^+\pi^- J/\psi} = \sum_{JL} \int_0^{k_{max}} dk \frac{\Gamma_\rho}{(M_X - E_\rho - E_{J/\psi})^2 + \frac{\Gamma_\rho^2}{4}} \left| \mathcal{M}_{X\to \rho J/\psi}^{LL}(k) \right|^2. \quad (24)$$

The amplitude $\mathcal{M}_{X\to \rho J/\psi}^{LL}$ is calculated in our model by the rearrangement diagrams of Fig. 1, averaged with the $DD^*$ component of the $X(3872)$ wave function. The rearrangement diagrams are calculated following Ref. [44]. The amplitude is given by

$$\mathcal{M}_{fi} = \sum_{i=a,a; j=b,b} \mathcal{M}_{ij} \quad (25)$$

where

$$\mathcal{M}_{ij}(\vec{P}', \vec{P}) = \langle \phi_{M'_1} \phi_{M'_2} | H_{ij}^O | \phi_{M_1} \phi_{M_2} \rangle \langle \xi_{M'_1 M'_2}^{SFC} | O_{ij}^{SFC} | \xi_{M_1 M_2}^{SFC} \rangle \quad (26)$$

and the orbital part can be written as (e.g. for the case $(ij) = (ab)$)

$$\langle \phi_{M'_1} \phi_{M'_2} | H_{ij}^O | \phi_{M_1} \phi_{M_2} \rangle = \int d^3P_{M'_1}d^3P_{M'_2}d^3P_{M_1}d^3P_{M_2} \phi_{M'_1}^{*}(P_{M'_1})\phi_{M'_2}^{*}(P_{M'_2})\delta(\vec{P}_{M'_2} - \vec{P}_{M_1})$$

$$\delta(\vec{P}_{M'_2} - \vec{P}_{M_2} - (\vec{P}' - \vec{P}))H(-\frac{1}{2}(\vec{P}_{M_1} + \vec{P}_{M_2}) + \vec{P}_{M'_1} + \frac{1}{2}(\vec{P}' - \vec{P}))$$

$$\phi_{M_1}(P_{M_1})\phi_{M_2}(P_{M_2}). \quad (27)$$

FIG. 1: Diagrams included in the quark rearrangement process $DD^* \to \rho J/\psi$. 
The spin-flavor-color matrix elements are taken from Ref. [44].

Once the decay width $\Gamma_{\pi^+\pi^-J/\psi}$ is calculated, the differential rate is given by

$$
\frac{dBr(B \to K\pi^+\pi^-J/\psi)}{dE} = B \frac{1}{2\pi} \left| \frac{\Gamma_{\pi^+\pi^-J/\psi}(E)}{|D(E)|^2} \right|^2.
$$

In order to compare with the experimental data we determine the number of events distributions from the differential cross section

$$
N_{\pi\pi J/\psi}^{Belle}(E) = 2.5[\text{MeV}] \left( \frac{131}{8.3 \times 10^{-6}} \right) \frac{dBr(B \to K\pi^+\pi^-J/\psi)}{dE} \tag{29}
$$

$$
N_{D^0\bar{D}^0\pi^0}^{Belle}(E) = 2.0[\text{MeV}] \left( \frac{48.3}{0.73 \times 10^{-4}} \right) \frac{dBr(B \to KD^0\bar{D}^0\pi^0)}{dE} \tag{30}
$$

$$
N_{\pi\pi J/\psi}^{BaBar}(E) = 5[\text{MeV}] \left( \frac{93.4}{8.4 \times 10^{-6}} \right) \frac{dBr(B \to K\pi^+\pi^-J/\psi)}{dE} \tag{31}
$$

$$
N_{D^0\bar{D}^*0\pi^0}^{BaBar}(E) = 2.0[\text{MeV}] \left( \frac{33.1}{1.67 \times 10^{-6}} \right) \frac{dBr(B \to KD^0\bar{D}^*0\pi^0)}{dE}. \tag{32}
$$

In all reactions a background is taken into account modelled as in Ref. [28]. For the $B \to KD^0\bar{D}^0\pi^0$ the $D^0D^{*0}$ signal interferes with the background and so a phase $\phi_{Belle} = 0^0$ and $\phi_{BaBar} = 324^0$ have been introduced. Also the experimental branching ratio $B(D^{*0} \to D^0\pi^0) = 0.62$ is introduced. We use a value for $B = 3.5 \times 10^{-4}$ which is in the order of the one used in Ref. [28].

III. RESULTS

A. Numerical methods

To found the quark-antiquark bound states we solve the Schrödinger equation using the Gaussian Expansion Method [45]. In this method the radial wave functions solution of the Schrödinger equation are expanded in terms of basis functions

$$
R_\alpha(r) = \sum_{n=1}^{n_{\text{max}}} b_n^\alpha \phi_{nl}^G(r) \tag{33}
$$

where $\alpha$ refers to the channel quantum numbers. The coefficients $b_n^\alpha$ and the eigenenergy $E$ are determined from the Rayleigh-Ritz variational principle

$$
\sum_{n=1}^{n_{\text{max}}} \left[ (T_{n'n}^{\alpha} - EN_{n'n}^{\alpha}) b_n^{\alpha} + \sum_{\alpha'} V_{n'n}^{\alpha\alpha'} b_n^{\alpha'} \right] = 0 \tag{34}
$$
where the operators $T_{\alpha n' n}^\alpha$ and $N_{\alpha n' n}^\alpha$ are diagonal and the only operator which mix the different channels is the potential $V_{\alpha n' n}^{\alpha \alpha'}$.

To solve the four body problem we also use the gaussian expansion of the two body wave functions obtained from the solution of the Schrödinger equation. This procedure allows us to introduce in variational way possible distortions of the two body wave function within the molecule. Using these wave functions Eq. (13) reduces to a matrix equation by Gauss integration.

A crucial problem of the variational methods is how to choose the radial functions $\phi_{nl}^G(r)$ in order to have a minimal, but enough, number of basis functions. Following [45] we employ gaussians trial functions whose ranges are in geometric progression. The geometric progression is useful in optimizing the ranges with a small number of free parameters. Moreover the distribution of the gaussian ranges in geometric progression is dense at small ranges, which is well suited for making the wave function correlate with short range potentials. The fast damping of the gaussian tail is not a real problem since we can choose the maximal range much longer than the hadronic size.

B. Results

The calculation is parameter free since all the parameters are taken from the previous calculation [31, 37] including the $\gamma = 0.26$ parameter in Eq. (10). This value was fitted to the reaction $\psi(3770) \rightarrow DD$ which is the only well established charmonium strong decay. This way to determine the value of $\gamma$ might overestimate it since the $\psi(3770)$ is very close to the $DD$ threshold and FSI effects, which were not included, might be relevant [46].

We first perform an isospin symmetric calculation including $^3S_1$ and $^3D_1$ $DD^*$ partial waves and taking the $D$ and $D^*$ masses as average of the experimental values between charged states. If we neglect the coupling to $c\bar{c}$ states we don’t get a bound state for the $DD^*$ molecule in the $1^{++}$ channel, neither in the $I = 0$ nor in the $I = 1$ channels. The interaction coming from OPE is attractive in the $I = 0$ channel but not enough to bind the system, even allowing for distortion in the meson states.

Now we include in the $I = 0$ channel the coupling to $c\bar{c}$ states. The most relevant are the
TABLE I: Masses and channel probabilities for the three states in three different calculations. The first three states are found when we perform and isospin symmetric calculation with a value of $\gamma$ fit to the decay $\psi(3770) \rightarrow DD$. The second three states shows the effect of isospin breaking in the $DD^*$ masses. The last three states correspond to a value of $\gamma = 0.19$ that fits the experimental mass of the $X(3872)$. The probability is shown as zero when it is less than 0.5%.

$$
\begin{array}{cccccc}
M (MeV) & c\bar{c}(1^3P_1) & c\bar{c}(2^3P_1) & D^0D^{*0} & D^\pm D^{*\mp} \\
3936 & 0\% & 79\% & 10.5\% & 10.5\% \\
A 3865 & 1\% & 32\% & 33.5\% & 33.5\% \\
3467 & 95\% & 0\% & 2.5\% & 2.5\% \\
3937 & 0\% & 79\% & 7\% & 14\% \\
B 3863 & 1\% & 30\% & 46\% & 23\% \\
3467 & 95\% & 0\% & 2.5\% & 2.5\% \\
3942 & 0\% & 88\% & 4\% & 8\% \\
C 3871 & 0\% & 7\% & 83\% & 10\% \\
3484 & 97\% & 0\% & 1.5\% & 1.5\% \\
\end{array}
$$

The results of this calculation are shown in part A of Table II. We find an almost pure $c\bar{c}(1^3P_1)$ state with mass $3467\, MeV$ which we identify with the $\chi_{c1}(1P)$ and two states with significant molecular admixture. One of them with mass $3865\, MeV$ is almost a $DD^*$ molecule bound by the coupling to the $c\bar{c}$ states. The second one, with mass $3936\, MeV$, is a $c\bar{c}(2^3P_1)$ with sizable $DD^*$ component. We assign the first state to the $X(3872)$, being the second one a candidate to the $X(3940)$. We have also analyzed the effect of higher bare $c\bar{c}$ states finding a negligible effect on the mass and probabilities that will not change the above numbers.

Coexistence of the $\omega J/\psi(I = 0)$ and $\rho J/\psi(I = 1)$ decay modes strongly suggest a large isospin mixing. However the relative branching fraction of both modes can be misleading with respect to the absolute magnitude of the isospin mixing in $X(3872)$ due to the phase
space suppression of the $\omega J/\psi$ channel against the $\rho J/\psi$ one. In fact if we assume that $X(3872)$ is a $D^0D^{*0}$ molecule, the ratio $\frac{B(X(3872)\rightarrow\pi^+\pi^-\pi^0 J/\psi)}{B(X(3872)\rightarrow\pi^+\pi^- J/\psi)}$ would be a factor 20 smaller than the experiment due to the different phase space.

It is clear that we need charged components in the wave function but with a different weight with respect to the neutral component. This rules out the intuitive idea of the dominance of the loosely bound neutral component. The clarification of this puzzle has been nicely done in Ref [47].

To introduce the isospin breaking in our calculation we turn to the charge basis instead of the isospin symmetric basis with the transformation

$$|D^\pm D^{*\mp}\rangle = \frac{1}{\sqrt{2}}(|DD^{*}I=0\rangle - |DD^{*}I=1\rangle)$$  \hspace{1cm} (36)

$$|D^0 D^{*0}\rangle = \frac{1}{\sqrt{2}}(|DD^{*}I=0\rangle + |DD^{*}I=1\rangle)$$  \hspace{1cm} (37)

writing our isospin symmetric interaction on the charged basis. We now explicitly break isospin symmetry taking the experimental threshold difference into account in our equations and solving for the charged and neutral components. Of course, if we don’t break it explicitly we recover our previous result as a bound state in the $I=0$ sector. Now we get again three states being the main difference in the $DD^*$ molecular component. The masses and channel probabilities are shown in part B of Table I. We now get a higher probability for the $D^0D^{*0}$ component although the isospin 0 component still dominates with a 66% probability and a 3% for isospin 1.

Having in mind that the $^3P_0$ model is probably too naive and we might be overestimating the value of $\gamma$, we show in Fig. 2 the variation of the $X(3872)$ mass with it. We can see that it is possible to get the experimental binding energy with a fine tune of this parameter. Using 0.6 $MeV$ as the binding energy we get a value of $\gamma = 0.19$, 25% smaller than the original. The results are shown in part C of Table I. Now the $D^0D^{*0}$ clearly dominates with a 83% probability giving a 70% for the isospin 0 component and 23% for isospin 1. Of course, as the isospin breaking is a threshold effect [25], it grows as we get closer to it as can be seen in Fig. 3 where we show the probabilities of the different components for the state $X(3872)$.

In Fig. 4 we compare our results with the $B \rightarrow KD^0\bar{D}^{0}\pi^0$ data from Belle (a) and $B \rightarrow KD^0\bar{D}^{*0}$ data from BaBar (b). The same comparison is done in Fig. 5 for the $B \rightarrow K\pi^+\pi^- J/\Psi$ data from Belle (a) and BaBar (b). In all figures the dashed lines shows the
FIG. 2: Mass of the $X(3872)$ as a function of the strength $\gamma$ of the $^3P_0$ model. The isospin symmetric calculation is shown in figure (a) and the isospin breaking in figure (b). Dotted lines show the threshold positions for the $DD^*$ average in figure (a) and $D^0D^{*0}$ and $D^\pm D^{*\mp}$ in (b). The solid lines shows the full result and the dashed lines turning off the $DD^*$ interaction.

results without resolution functions. The solid line gives the result using the resolution functions as in Ref. [28]. All the resolution functions are those given by Belle [17] and BaBar [19] collaboration with the exception of the BaBar $DD^*$ resolution where we use the prescription from Ref. [28].

We find a good description of the Belle $B \to K D^0 D^{*0}$ data whereas the agreement is poor in the case of the BaBar data. It is important to notice that in the Belle analysis the mass of the $X$ appears as 3872 $MeV$ while in the BaBar data the resonance is located 3 $MeV$ above. The BaBar mass value does not coincides with the mass of the $X$ obtained in our calculation which may be the reason for the disagreement.

The $B \to K \pi^+ \pi^- J/\Psi$ data are equally well described for the Belle and BaBar experiments. In this case both Collaborations give similar values for the mass of the resonance, namely 3871.4 $MeV$, which are in much better agreement with our result.
FIG. 3: Probability (in %) of different components as a function of the binding energy when we vary the $\gamma$ parameter of the $^3P_0$ model. The solid line gives the $D^0D^{*0}$ probability, the dashed-dotted the $D^\pm D^{*\mp}$, the dashed the $c\bar{c}(2^3P_1)$ and the dotted the $c\bar{c}(1^3P_1)$.

IV. SUMMARY.

As a summary, we have shown that the $X(3872)$ emerges in a constituent quark model calculation as a dynamically generated mixed state of a $DD^*$ molecule and $\chi_{c1}(2P)$. Although the $c\bar{c}$ mixture is less than the 10% it is important to bind the molecular state. This result is in agreement with the analysis of Ref [29]. The proposed structure allows to understand simultaneously the isospin violation showed by the experimental data and the radiative decay rates. Furthermore, we have demonstrated that this solution explain the new Belle data in the $D^0D^0\pi^0$ and $\pi^+\pi^-J/\Psi$ decay modes and the $\pi^+\pi^-J/\Psi$ BaBar data. The original $\chi_{c1}(2P)$ state acquires a significant $DD^*$ component and can be identified with the $X(3940)$.

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FIG. 4: Number of events for the decay $B \rightarrow KD^0\bar{D}^0\pi^0$ measured by Belle (a) and for the decay $B \rightarrow KD^0D^*0$ measured by BaBar (b). The solid and dashed lines shows the results from our model with and without the resolution functions as explained in the text.

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FIG. 5: Number of events for the decay $B \to K\pi^+\pi^- J/\Psi$ measured by Belle (a) and by BaBar (b). The solid and dashed lines shows the results from our model with and without the resolution functions as explained in the text.

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