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Optimal parameter estimation method for different types of resonant liquid sensors

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Abstract

The performance of a recently introduced method to estimate the resonance parameters from complex spectral data and the error propagation for viscosity and density parameters of liquids are examined. The method is known to produce excellent results when used for piezoelectric and Lorentz force actuated sensors for single port or two-port devices. The method is also suited for very low quality factors (< 10) which extends the usable measurement range of many sensor concepts, especially for fluid sensing applications. Generally valid expressions for the measurement accuracy are stated and compared to measurement results obtained with a piezoelectric tuning fork and a Lorentz force actuated and inductively read out platelet sensor.

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1. Introduction

To determine viscosity and density of liquids, resonant sensing principles are widely used [1]. In our approach, the complex frequency spectra of steady-state oscillations are analyzed. For a given sensor principle, the relation between the resonance parameters (quality factor $Q$ and resonance frequency $f_r$) and the fluid parameters (viscosity $\eta$ and density $\rho$) can be determined mathematically or experimentally. With an estimate of the measurement noise on the acquired spectra, the lowest error bound can be determined for the resonance parameters and subsequently for the physical parameters of interest. It can thus be verified at an early design stage whether or not the specific sensor system allows to measure $\rho$ and $\eta$ with the required accuracy, when all other adverse effects like thermal and long-term drifts are rendered negligible. In this paper, a piezoelectric tuning fork (PTF) with resonance frequency of 32.768 kHz, as used in electronic

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watches, and the double platelet (DP) sensor described in Abdallah et al. [2] are considered. The resonance parameter estimation algorithm from Niedermayer et al. [3] processes the complex recorded frequency spectrum efficiently. The flexible sensor model from Heinisch et al. [4] relating \( f_r \) and \( Q \) to \( \rho \) and \( \eta \) is used to determine the error propagation of the measurement noise.

2. Sensor Model

The sensors mentioned above (and many others, see [4]) can either be represented by the left or right equivalent circuit in Fig. 1. The resonance estimator [3] processes the admittance \( Y(f) \) or the mechanical resonator \( V(f) \). The effects of the shunt capacitance \( C_0 \), the electrical fluid loading and calibration errors, summarized in \( \tilde{Y}_0 \) and \( \phi \), are considered spurious. The Lorentz force actuated and inductively read out sensors can be represented by the circuit on the right. Spurious inductive crosstalk and wire resistance are modeled by \( L_0 \) and \( R_0 \). The coupling between electrical and mechanical part is realized by an ideal transformer with turns ratio \( n \), which depends on the external magnetic field. Due to the similarity of the frequency response functions, the same resonance estimation algorithm can be used when the admittance \( Y(f) \) is processed for piezoelectric sensors and the impedance \( Z(f) \) for the inductive sensors.

\[
Y(f) = \frac{i(f)}{Y(f)} = \left( \frac{\tilde{Y}}{1 + iQ (\frac{f}{f_r} - \frac{4}{f_f})} + Y_B + 2\pi f C_0 \right) e^{i\phi}
\]

\[
f_r = 2\pi \sqrt{\frac{C_m}{L_m}} , \quad Q = \sqrt{\frac{C_m}{L_m} / R_m}
\]

\[
Z(f) = \frac{V(f)}{i(f)} = \left( \frac{\tilde{Z}_0^{-2}}{1 + iQ (\frac{f}{f_r} - \frac{4}{f_f})} + R_0 + 2\pi f L_0 \right) e^{i\phi}
\]

\[
f_r = 2\pi \sqrt{\frac{L_m C_m}{R_m}} , \quad Q = R_m \sqrt{\frac{C_m}{L_m}}
\]

Figure 1: The left figure shows an equivalent circuit valid for PTF and micro balances (QCM). The effects of the shunt capacitance \( C_0 \), the electrical fluid loading and calibration errors, summarized in \( \tilde{Y}_0 \) and \( \phi \), are considered spurious. The Lorentz force actuated and inductively read out sensors can be represented by the circuit on the right. Spurious inductive crosstalk and wire resistance are modeled by \( L_0 \) and \( R_0 \). The coupling between electrical and mechanical part is realized by an ideal transformer with turns ratio \( n \), which depends on the external magnetic field. Due to the similarity of the frequency response functions, the same resonance estimation algorithm can be used when the admittance \( Y(f) \) is processed for piezoelectric sensors and the impedance \( Z(f) \) for the inductive sensors.

(trans)admittance spectrum \( Z(f) \) whichever is adequate. Therefore, the following considerations are restricted to piezoelectric sensors, for convenience. The effect of the spurious elements \( (\tilde{Y}_B, C_0, L_0, R_0, e^{i\phi}) \) is estimated and compensated, such that \( f_r \) and \( Q \) of the motional part \( (C_m, L_m, R_m) \) can be determined. For given complex noise \( \xi \) on the recorded spectra (mean free, with variance \( \text{var} \{ \xi \} = \sigma^2 \) and \( \text{var} \{ \text{Im} \{ \xi \} \} = \text{var} \{ \text{Re} \{ \xi \} \} = \sigma^2/2 \)), the parameter noise on the estimates of \( f_r \) and \( Q \) is governed by

\[
\text{var} \{ \hat{f}_r \} \approx q \frac{2 f_r^2}{M Q^2} \text{SNR}^{-2} , \quad \text{var} \{ \hat{Q} \} = q \frac{8 Q^2}{M} \text{SNR}^{-2} , \quad \text{SNR} = \tilde{Y} / \sigma ,
\]

where \( M \) and \( q \) denote the number of sampling points and a factor depending on the distribution of sample points around the locus plot (see Fig. 2 left), respectively. A signal-to-noise ratio (SNR) can be defined as ratio between locus diameter \( \tilde{Y} \) and the standard deviation of the noise \( \sigma \). The lowest limit\(^2 2 \) \( q = 1 \) is only obtained for a uniform distribution of sampling points on the locus circle. For the more practical

\(^1\text{Estimated values are marked with a hat.}\)

\(^2\text{It can be shown that this limit is equal to the Cramér-Rao lower bound for } Q \text{ and very close to it for } f_r.\)
distribution with constant frequency increment, \( q \) can be obtained from Fig. 2 (right). According to [4], the relation between resonance parameters and fluid parameters is given by

\[
f_r = \left( c_1 + c_2 \rho + c_3 \sqrt{\eta} \right)^{-1/2} \quad \text{and} \quad Q = \left( d_1 + d_2 \eta + d_3 \sqrt{\rho} \right)^{-1},
\]

where \( c_1 \ldots c_3, d_1 \ldots d_3 \) denote sensor calibration factors. In [4] is shown, that this model is accurate for in-plane oscillators, vibrating cylinders, spheres and tuning forks over a wide range of fluid parameters. It is assumed that the exact values for \( c_1 \ldots c_3, d_1 \ldots d_3 \) were determined by calibration measurements. The fluid parameters are obtained by inverting Eq. 1 using a non-linear least squares estimator\(^3\). Based on the variances in Eq. 1, the covariance matrix of the estimated density and viscosity can be stated by

\[
\operatorname{cov} \{ \rho, \eta \} \approx 32q^{\frac{\text{SNR}}{M Q^2}} K P K^T \quad \text{with} \quad P = \begin{bmatrix} f_r^{-4} & 0 \\ 0 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 2 c_2 + c_3 \sqrt{\eta/\rho} & c_3 \sqrt{\rho/\eta} \\ d_3 \sqrt{\eta/\rho} & 2 d_2 + d_3 \sqrt{\rho/\eta} \end{bmatrix}^{-1}.
\]

With Eq. 3, the noise transfer to \( \eta \) and \( \rho \) can be determined for a given sensor and known SNR.

### 3. Results and Conclusions

In Fig. 3, the estimated resonance parameters for a PTF (see Fig. 4 upper left) immersed in 1-pentanol (\( \rho = 810.9 \, \text{kg/m}^3, \eta = 3.44 \, \text{mPas}, \vartheta = 25 \pm 0.02 \, \text{°C} \)) is shown. The measurement was repeated for 2594 times with \( M = 101 \) points for each spectrum. The relative measurement span was \( b = 1.77 \) and therefore \( q \approx 1.4 \) (see Fig. 2 right). With the estimated SNR the \( \pm 2\sigma \) bounds where determined using Eq. 3. Good agreement is observed and therefore, the randomness in the parameters can be fully attributed to the noise on the spectra recorded by the impedance analyzer.

In Fig. 4, the errors on density and viscosity due to the noisy estimates of the resonance parameters are shown for the PTF and a DP sensor (see Fig. 4 upper right). Both sensors where calibrated with 3 fluids

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\(^3\)Note that for good numerical stability prescaling should be considered, e.g., measuring \( f \) in kHz, \( \eta \) in mPas and \( \rho \) in g/cm³.
each and $M = 101$ frequency points per spectrum. The error propagation is determined and given as relative standard deviations. Due to the different flow profiles, the PTF is more sensitive to density than the DP sensor. For the viscosities the situation is reversed. The relatively low SNR of the PTF measurements is due to the high impedance of the PTF and can be increased, e.g., by using a matching transformer. Since the error propagation is a physical fact, the noise performance can only be improved by reducing the SNR (e.g., by band limiting, longer averaging times for the acquisition of the spectra), sensor redesign or by employing it in a more sensitive parameter range.

Figure 3: The black curves show the resonance parameters of 2594 repeated measurements (impedance analyzer Agilent 4294A) of a quartz crystal tuning fork (32.768 kHz) over 16 hours. The green line is the moving average over 100 measurements. The red curves show the interval ±2σ (95.45% confidence) determined from the measurement noise around the moving average.

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