Calculating the renormalisation group equations of a SUSY model with Susyno

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Abstract

Susyno is a Mathematica package dedicated to the computation of the 2-loop renormalisation group equations of a supersymmetric model based on any gauge group (the only exception being multiple U(1) groups) and for any field content.

1 Introduction

Supersymmetry (SUSY) is an elegant and well known extension of the Standard Model (SM) which aims at solving or softening some of the SM theoretical and experimental shortcomings. Supersymmetric models based on Grand Unified Theories (GUTs) offer the appealing feature of being described by a unique gauge group and coupling constant; furthermore, the potential new superfields, as well as their dynamics, can provide answers to fundamental problems. For example SO(10) based models offer a natural explanation for neutrino masses and mixings via a type-I seesaw. This occurs since the spinor representation of the group contains a SM singlet superfield (with the appropriate quantum numbers). Even larger and/or more complex groups are often considered, for example $E_6$ in string inspired models.

The analysis of the theoretical and phenomenological implications of SUSY GUT models requires a careful study of the evolution of the fundamental parameters from the high-energy scale down to the electroweak scale, at which observables are computed and constraints applied. As such, knowledge of the renormalisation group equations (RGEs) is necessary. Although the RGEs of several models (e.g., MSSM, NMSSM) are already known [1, 3], for other SUSY extensions of the SM complicated general equations must be used [1, 2].

Here we describe Susyno, a Mathematica-based package that addresses this issue. The program takes as input the gauge group, the representations (i.e., the chiral superfield content), the number of flavours/copies\(^1\) of each representation, and any abelian discrete symmetries (e.g., R-parity). Susyno then constructs the most general superpotential and soft SUSY breaking Lagrangian consistent with the field content and symmetries imposed. Once these elements have been derived, Susyno then computes the 2-loop $\beta$-functions of all the parameters of the model, which is its main output.

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\(^1\)Throughout this document, the word *flavours* is meant to convey the sense of *copies* or *repetitions* of a given representation/field.
There is another Mathematica package, *SARAH*, which works in a similar way for models based on SU($n$) gauge factor groups. However, *Susyno* is prepared to accept as working input any gauge group that does not contain more that one U(1) factor.

This document is organised as follows. Section 2 explains how to install the program and run a first simple example (MSSM based). Sections 3 and 4 explain how to prepare the input and how to read and interpret the output, also using as examples the MSSM case. Finally, Section 5 summarises the tests conducted to validate the code.

# 2 Installation and quick start

*Susyno* works on Windows, Linux and Mac OS provided that Mathematica 7 (or a latter version) is installed. The program is obtainable from

http://web.ist.utl.pt/renato.fonseca/susyno.html

The files *LieGroups.m*, *SusyRGEs.m* and *Susyno.m* are the core of the program. These and other auxiliary files can be found inside the folder *Susyno*, which must be extracted from the downloaded *Susyno-1.0.tar* file to a location that is visible to Mathematica. Typing `$Path` in Mathematica will show a complete list of acceptable locations. A good choice is to place the whole folder (not just its contents) in

*(Mathematica base directory)/AddOns/Applications*

(note that in a Windows system the slashes “/” must be replaced by backslashes “\”). The package can be loaded by typing

```mathematica
<< Susyno`
```

in Mathematica’s front end. A text message is returned, informing that a built-in help system provides a detailed description of the program and its functions (see Appendix A). A tutorial is also included.

The *Susyno* lines below allow a simple and easy first run: the example consists in a possible way of writing the MSSM input.

```mathematica
groups = {SU2, SU3};

normalization = Sqrt[3/5];
Q = {1/6 normalization, {1}, {1, 0}};
u = {-2/3 normalization, {0}, {0, 1}};
d = {1/3 normalization, {0}, {0, 1}};
L = {-1/2 normalization, {1}, {0, 0}};
e = {normalization, {0}, {0, 0}};
Hu = {1/2 normalization, {1}, {0, 0}};
Hd = {-1/2 normalization, {1}, {0, 0}};
```

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2 Multiple U(1) groups would lead to “U(1) mixing” and at present there are still no complete published results for this case [5].
content = {Q, u, d, L, e, Hu, Hd};

nFlavours = {3, 3, 3, 3, 3, 1, 1};
discreteSym = {-1, -1, -1, -1, -1, 1, 1};

BetaFunctions2L[groups, content, nFlavours, discreteSym]

Evaluation of this simple code generates the 2-loop $\beta$-functions of the model (MSSM in this case). Notice that no external input or output files are used - everything happens on Mathematica’s front end.

3 Defining a model

A SUSY model contains two building blocks: a superpotential and a soft SUSY breaking Lagrangian. For a simple gauge group, a general superpotential can be written as

$$ W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j + L^i \Phi_i , $$ (1)

with $\Phi$ denoting a chiral superfield, and where $Y$, $\mu$ and $L$ are dimensionless, mass, and mass$^2$ parameters, respectively. A generic soft SUSY breaking Lagrangian reads

$$ -\mathcal{L}_{soft} = \left( \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + s^i \phi_i + \text{h.c.} \right) + \left( m^2 \right)^i_j \phi_i \phi^*_j + \left( \frac{1}{2} M \lambda \lambda + \text{h.c.} \right) $$ (2)

where $\lambda$ is a gaugino field and $\phi$ denotes scalar fields. $M$ ($m^2$) correspond to gaugino (scalar) soft breaking masses, while $h$, $b$ and $s$ denote trilinear, bilinear and linear soft breaking parameters.

**Susyno** works as follows: it requires as input the gauge group, the representations/fields, the number of flavours of each representation/field and the discrete abelian symmetries (if any is present) of the model. With this information the program then builds internally the superpotential and the soft SUSY breaking Lagrangian using an algorithm to automatically name the parameters of the model (see Section 4).

Assuming that this information is summarised in `groups`, `content`, `nFlavours` and `discreteSym`, the user must then call the function `BetaFunctions2L` (or `BetaFunctions1L` for 1-loop results) as follows:

BetaFunctions2L[groups, content, nFlavours, discreteSym]

We shall focus first on each of these arguments. The syntax of the functions will be discussed at the end of this section.

3.1 Gauge group

The program needs a complete list of all the simple Lie groups of the model$^3$. For the MSSM (for completeness, the superpotential and SUSY soft breaking Lagrangian of the MSSM are presented in Appendix B) this would correspond to

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$^3$We emphasise here that we are actually dealing with algebras, not groups. Nevertheless, we will adopt the common practice in high-energy physics of using *group* for both these concepts.
groups = \{\text{SU2, SU3}\};

Any simple group can be given as a factor: the simple gauge factor groups, as well as their corresponding \textit{Susyno} input are collected in Table 1.

| Simple gauge factor group | Susyno input          |
|---------------------------|-----------------------|
| SU\((n)\)                | SU2, SU3, SU4, SU5, ...|
| SO\((n)\)                | SO3, SO5, SO6, SO7, ...|
| Sp\((2n)\)               | SP2, SP4, SP6, ...    |
| G\(_2\)                  | G2                    |
| F\(_4\)                  | F4                    |
| E\(_6\), E\(_7\), E\(_8\) | E6, E7, E8           |

Table 1: Simple gauge factor groups

Notice that despite its generality, \textit{Susyno} only assigns to variables a finite number of groups (for instance variable \textit{SO100} is not set). Should this ever become a problem, there is an easy method of circumventing it, discussed in \textbf{Appendix C}.

Since it is not a simple group, U(1) is a special case and should not be included in the list of simple factor groups. The information regarding any U(1) factors present in the gauge group of the model is given through the number of hypercharges of the fields (see below).

\subsection*{3.2 Representations/fields: the content of the model}

As mentioned before, \textit{Susyno} is designed to accept an arbitrary field content. An input must be provided in the form of a list

\texttt{content=\{field1,field2,...\};}

containing all the gauge group irreducible representations present in the model. It is important to stress that the ordering of the fields in \texttt{content} is arbitrary, and decided by the user. However, once \texttt{content} has been read by \textit{Susyno}, the user must consistently adhere to the chosen ordering when inputting lists composed of field attributes (for example the number of flavours, as will be discussed in the following subsection). Each \texttt{field} should be specified as

\texttt{field=\{hChrg1,hChrg2,...,hChrgM,rep_simplegroup1,rep_simplegroup2,...\};}

The first entries correspond to the hypercharges of \texttt{field} (if any)\footnote{Notice that, while it is possible to have more than one hypercharge (i.e. more than one U(1) factor group), the formalism of \textit{Susyno} is based on \cite{11,12}, which do not include the effects of a possible U(1) mixing.}, which are just real numbers. After the hypercharges one must declare the representations of \texttt{field} under each of the simple gauge factor groups mentioned above. These representations must be specified by their Dynkin coefficients (see \textbf{Appendix C} for details). In Table 2 we list some of the representations of SU(2), SU(3), SU(5) and SO(10) (Dynkin coefficients and corresponding dimensions).

To understand how the MSSM was specified in the example of Section 2 we just need the following information from Table 2:

- the Dynkin coefficients of the trivial and fundamental representations of SU(2): \{0\} and \{1\};
| Group | Representation | Dynkin coefficients | Dimension (Name) |
|-------|----------------|---------------------|------------------|
| SU(2) | SU(2) | {0} | 1 (Trivial/Singlet) |
|       |       | {1} | 2 (Fundamental/Doublet) |
|       |       | {2} | 3 (Adjoint/Triplet) |
| SU(3) | SU(3) | {0,0} | 1 (Trivial/Singlet) |
|       |       | {1,0} | 3 (Fundamental) |
|       |       | {0,1} | 3 (Anti-fundamental) |
|       |       | {1,1} | 8 (Adjoint) |
| SU(5) | SU(5) | {0,0,0} | 1 (Trivial/Singlet) |
|       |       | {1,0,0} | 5 (Fundamental) |
|       |       | {0,0,1} | 5 (Anti-fundamental) |
|       |       | {0,1,0} | 10 |
|       |       | {2,0,0} | 15 |
|       |       | {0,0,2} | 15 (Adjoint) |
|       |       | {1,0,1} | 24 (Adjoint) |
| SO(10) | SO(10) | {0,0,0,0} | 1 (Trivial/Singlet) |
|       |       | {1,0,0,0} | 10 (Fundamental) |
|       |       | {0,0,0,1} | 16 (Spinor) |
|       |       | {0,0,1,0} | 16 (Spinor’s conjugate) |
|       |       | {0,1,0,0} | 45 (Adjoint) |
|       |       | {2,0,0,0} | 54 |
|       |       | {0,0,1,0} | 120 |
|       |       | {0,0,0,2} | 126 |
|       |       | {0,0,2,0} | 126 |

Table 2: List of some of the representations of SU(2), SU(3), SU(5) and SO(10)

- the Dynkin coefficients of the trivial, fundamental (3) and anti-fundamental (3) representations of SU(3): {0,0}, {1,0}, {0,1}.

For the MSSM each field must then be cast in the format

field={U1_charge,SU(2)_rep,SU(3)_rep};

Further normalising the hypercharges with the usual $\sqrt{\frac{3}{5}}$ factor (from an embedding of the MSSM in an SU(5) based model), we can then write the following:

```plaintext
normalization= Sqrt[3/5];
Q = {1/6 normalization, {1}, {1, 0}};
u = {-2/3 normalization, {0}, {0, 1}};
d = {1/3 normalization, {0}, {0, 1}};
L = {-1/2 normalization, {1}, {0, 0}};
e = {normalization, {0}, {0, 0}};
Hu = {1/2 normalization, {1}, {0, 0}};
Hd = {-1/2 normalization, {1}, {0, 0}};
content = {Q, u, d, L, e, Hu, Hd};
```
We must emphasise here that although the user is free to choose the ordering of the simple factor groups, SU(2) and SU(3), please bear in mind that once this is set (e.g., \( \text{groups} = \{\text{SU2}, \text{SU3}\} \)) one must adhere to the (user-established) convention, and define the representations of the fields accordingly:

\[
\text{field} = \{\text{U1\_charge}, \text{SU(2\_rep)}, \text{SU(3\_rep)}\};
\]

Also notice that \texttt{Susyno} accepts any choice for the normalisation of the hypercharges.

3.3 Number of flavours and abelian discrete symmetries

\texttt{Susyno} needs two more input lists: one containing the number of flavours of each field and another defining its abelian discrete symmetries. As previously mentioned, the attributes of the fields listed in both \texttt{nFlavours} and \texttt{discreteSym} must follow the field ordering established by \texttt{content}. In our MSSM example we used the ordering \{Q, u, d, L, e, Hu, Hd\}, and therefore

\[
\text{nFlavours} = \{3, 3, 3, 3, 1, 1\};
\]

\[
\text{discreteSym} = \{-1, -1, -1, -1, -1, 1, 1\};
\]

In this particular case, it is clear that the discrete symmetry imposed corresponds to R-parity. For the most general (R-parity violating) MSSM we have

\[
\text{discreteSym} = \{1, 1, 1, 1, 1, 1, 1\};
\]

Let us consider another example: for instance, if we were to modify the MSSM to include \( m \) copies of \( H_u \) and \( H_d \), we would write

\[
\text{nFlavours} = \{3, 3, 3, 3, m, m\};
\]

Both these lists can be empty: if the flavour list is \{\}, the program will assign by default the symbolic values \{nf[1], nf[2], \ldots\}; if the discrete symmetry list is \{\}, \texttt{Susyno} will consider it to be \{1, 1, \ldots\}, meaning it will assume that all fields transform trivially.

3.4 Calling the functions \texttt{BetaFunctions1L}, \texttt{BetaFunctions2L}

The four lists discussed so far are the arguments of \texttt{BetaFunctions1L} or \texttt{BetaFunctions2L},

\[
\text{BetaFunctions1L[groups, content, nFlavours, discreteSym, <print>]}
\]

\[
\text{BetaFunctions2L[groups, content, nFlavours, discreteSym, <print>]}
\]

These functions compute the one- and two-loop RGEs of the model (see Appendix D). The optional argument \(<\text{print}>\) can be set to \texttt{True} (default value) or \texttt{False}. If \(<\text{print}>\)=\texttt{True} the results are printed on the screen but the functions themselves do not return a value. On the contrary, if \(<\text{print}>\)=\texttt{False} there is no printing and the functions now return the results as a list. Assuming that the model contains parameters named \(X_1, X_2, \ldots\), the format of this output list is

\[
\{\{X_1, \beta_X^{(1)}\}, \{X_2, \beta_X^{(1)}\}, \ldots\}
\]

(BetaFunctions1L)
\[
\{X_1, \beta_{X_1}^{(1)}, \beta_{X_1}^{(2)}, \ldots \}, \{X_2, \beta_{X_2}^{(1)}, \beta_{X_2}^{(2)}\}, \ldots \}
\]

that is, a set of sublists each containing a parameter \(X_i\), and the one-loop (one- and two-loop) beta functions for its RG evolution: \(\beta_{X_i}^{(1)}(\beta_{X_i}^{(1)}, \beta_{X_i}^{(2)})\).

If for some reason the results are to be further processed with Mathematica, then the best choice is clearly to set \(<\text{print}>\) = \text{False}.

4 The output of Susyno

Once all the input is provided, Susyno automatically builds the Lagrangian of a model, neither asking for "names" of fields nor "names" of parameters. On the one hand, inputting a model becomes very easy, since it is not even necessary to know the exact number of its parameters. On the other hand, this renders the output hard to read (and hence not particularly user-friendly), since the names of the parameters are chosen by the program.

Still, provided we know the conventions used by the program, the notation used by Susyno can be manually changed by the user (after the code is executed). We notice that in the present version of Susyno there are no custom built-in functions to export the results. Users who wish to do so must do it manually or with the help of Mathematica’s built-in functions CForm and FortranForm.

4.1 Naming of parameters

Susyno assigns names to the parameters of a model in such a way that the user can identify which representations/fields they are multiplying:

- \(y[field1, field2, field3, InvIndex, <flav1>, <flav2>, <flav3>]\)
- \(\mu[field1, field2, InvIndex, <flav1>, <flav2>]\)
- \(l[field1, InvIndex, <flav1>]\)
- \(h[field1, field2, field3, InvIndex, <flav1>, <flav2>, <flav3>]\)
- \(b[field1, field2, InvIndex, <flav1>, <flav2>]\)
- \(s[field1, InvIndex, <flav1>]\)
- \(m2[field1, field2, InvIndex, <flav1>, <flav2>]\)

A few comments concerning the above (output) tensors are in order:

- \(y, \mu, l, h, b, s\) and \(m2\) can be easily identified with the different types of couplings and dimensionful parameters of the superpotential and the soft SUSY breaking Lagrangian;

- \(field1, field2, field3\) are the indices of the fields entering a given coupling, or soft breaking term. For example, with content = \{Q, u, d, L, e, Hu, Hd\} we have \(Q=1, u=2, d=3, L=4, e=5, Hu=6, Hd=7\). The up-quark Yukawa couplings would then be \(y[1, 2, 6, \ldots]\);

- There is the possibility that the product of 3 representations, \(R_1 \otimes R_2 \otimes R_3\), contains more than one invariant. Therefore we need \(\text{InvIndex}=1, 2, \ldots\) to distinguish them. This is rare though, so in most cases (e.g., the MSSM) \(\text{InvIndex}=1\) for all parameters. Notice that in linear and bilinear terms, \(R_1\) and \(R_1 \otimes R_2\), this problem does not arise since there is at most one invariant. However, for consistency Susyno still uses \(\text{InvIndex}\) in this case, although setting its value to 1;
• `<flav1>, <flav2>, <flav3>` are the flavour indices of `field1, field2, field3`. If any of these fields has only one flavour, the corresponding index is omitted. Consider again the example of the up-quark Yukawa couplings: we would have `y[1,2,6,1,i,j,k]` where `i = flavour of Q, j = flavour of u, k = flavour of Hu`. Yet Hu only has one flavour so the correct parameter name is `y[1,2,6,1,i,j]`.

Additionally, there are also the coupling constants and the gaugino masses:

```
g[1], g[2], ...
M[1], M[2], ...
```

A list containing the matching of MSSM’s standard notation parameters and Susyno assignments can be found in Appendix B.

### 4.2 The full Lagrangian as used by the program

Knowing the full Lagrangian can be important. As an example, consider a possible normalisation issue: how can we be sure that Susyno is correctly taking `\( \mu H_u \cdot H_d \)` as `\( \mu [6,7,1] H_u \cdot H_d \)`, and not `-\( \mu [6,7,1] H_u \cdot H_d \)`, `\( 2\mu [6,7,1] H_u \cdot H_d \)` or any other multiple? The only way to solve such problems related to the normalisation of the parameters is to check the Lagrangian. This can be done using the `ShowLagrangian` function:

```
ShowLagrangian[groups, content, nFlavours, discreteSym, <print>]
```

The first four arguments are the same as for the `BetaFunctions1L` and `BetaFunctions2L` functions. As for the last one, setting `<print>=True` (default value) will print the different parts of `W` and `L_soft`:

- `\( \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k )` (Y part), `\( \frac{1}{2} \mu^{ij} \Phi_i \Phi_j \)` (\( \mu \) part), `\( L^i \Phi_i \)` (L part);
- `\( \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k \)` (H part), `\( \frac{1}{2} b^{ij} \phi_i \phi_j \)` (B part), `\( s^i \phi_i \)` (S part) and `\( (m_2^2)^i_j \phi_i \phi_j^* \)` (M2 part).

If `<print>=False` no printing is done; instead the function returns the list of tensors `{ `Y, \( \mu, L, h, b, s, m_2^2 `)`. Note that Susyno never expands the flavour indices of these tensors so the first, second and third indices are assumed to carry symbolic flavours `i` (or none), `j` (or none), `k` (or none)\(^5\).

`ShowLagrangian` is completely independent of `BetaFunctions1L` and/or `BetaFunctions2L`, and so it can be called at any time to generate the superpotential and soft SUSY breaking Lagrangian of a given model.

Finally, notice that in order to build \( \mathcal{L} \) it is necessary to assume a particular basis for the representations/fields of the model. Quantities such as the \( \beta \)-functions clearly do not depend on this choice. However, it is important to know that Susyno can use unconventional basis for the representations. The following is a good example. According to the program, two fields, `A=\{a[1], a[2], a[3]\}` and `B=\{b[1], b[2], b[3]\}`, transforming as `3` and `\( \bar{3} \)` of SU(3) will form the invariant `a[1]b[3] - a[2]b[2] + a[3]b[1]`, not `a[1]b[1] + a[2]b[2] + a[3]b[3]`. Both are valid expressions as

\(^5\)These flavour indices do not show up if the particular entry of the tensors we are considering is unflavoured. For example the entry of the `Y` tensor related to the fields `Q, d, Hd` will depend on `i` (flavour of the first field, `Q`) and `j` (flavour of the second field, `d`) but not on `k` since the third field (`Hd`) is unflavoured. Notice also that exchanging for example `Hd` with `Q` corresponds to having a different entry of the `Y` tensor: this is because the flavour of `Q` (now the third field) is assumed to be symbolically `k` instead of `i`. 
long as they are used in a consistent way - it is just a matter of the choice of basis. Thus, when analysing the output of ShowLagrangian we must distinguish between irrelevant basis choices and relevant normalisation conventions for the parameters. The $\beta$-functions are only sensitive to the latter ones.

5 Tests/validation of Susyno

The output of Susyno was confronted with the analysis of some models available in the literature. In particular, the RGEs generated by Susyno were compared with the results of [1] (MSSM), [6] (R-parity violating MSSM), [3] (general NMSSM) as well as [7] (SU(5)-based models).

In general the program’s RGEs are consistent with the results collected in the above publications, but in some cases differences were found. These have been collected in the program’s webpage.

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Appendix A  List of available functions

As in most programs, the code Susyno is spread over many internal functions. Due to their nature, some of these functions may be useful on their own, and they were thus built in a user-friendly way, and are documented.

Below is the list of functions that can be called directly by the user in Mathematica’s front-end, followed by a brief description. The built-in help system describes in detail how to use them.

- **Adjoint** - Computes the Dynkin coefficients of the adjoint representation of a group;
- **BetaFunctions1L** - Computes the 1-loop $\beta$-functions of a SUSY model;
- **BetaFunctions2L** - Computes the 2-loop $\beta$-functions of a SUSY model;
- **CanonicalForm** - Simplifies an expression written in Einstein’s notation;
- **Casimir** - Computes the quadratic Casimir of a representation;
- **CM** - Computes the Cartan matrix of any simple group;
- **DimR** - Computes the dimension of a representation;
- **Invariants** - Computes (in a given basis) the invariant combination(s) of a product of one, two or three representations;
- **ListContract** - Efficiently calculates traces of multi-index sparse tensors;
- **PositiveRoots** - Computes the positive roots of a group;
- **ReduceRepProduct** - Decomposes a direct product representation in its irreducible parts [8];
- **RepMatrices** - Computes (in a given basis) the explicit representation matrices;
- **ShowLagrangian** - Generates the Lagrangian ($W$ and $L_{soft}$) of a SUSY model;
- **Weights** - Computes the weights of a representation, including degeneracy.

Appendix B  Parameters of the MSSM

For completeness, we include here the superpotential and soft SUSY breaking Lagrangian of the MSSM, following the conventions of [1].

\[
W = \hat{u}Y_u\hat{Q}\cdot\hat{H}_u + \hat{d}Y_d\hat{Q}\cdot\hat{H}_d + \hat{e}Y_e\hat{L}\cdot\hat{H}_d + \mu\hat{H}_u\cdot\hat{H}_d
\]

\[
-L_{soft} = \left(\hat{u}h_u\hat{Q}\cdot\hat{H}_u + \hat{d}h_d\hat{Q}\cdot\hat{H}_d + \hat{e}h_e\hat{L}\cdot\hat{H}_d + BH_u\cdot H_d + \text{h.c.}\right)
\]

\[
+\hat{Q}\hat{L}^\dagger m^2_{\hat{Q}} + \hat{L}\hat{L}^\dagger m^2_{\hat{L}} + \hat{u}\hat{u}^\dagger \hat{u}^\dagger + \hat{d}\hat{d}^\dagger \hat{d}^\dagger + \hat{e}\hat{e}^\dagger \hat{e}^\dagger + m^2_{\hat{H}_u}H^1_uH_u + m^2_{\hat{H}_d}H^1_dH_d
\]

\[
+ \left(\frac{1}{2}M_a\lambda_a\lambda_a + \text{h.c.}\right)
\]

In the above, and as usual, hats denote superfields and tildes the scalar component of each chiral superfield. The contraction of two SU(2) doublets (denoted by $\cdot$) follows the convention $H_u\cdot H_d = H^+_uH^-_d - H^0_uH^0_d$. In addition to the parameters entering $W$ and $L_{soft}$, one still has the gauge couplings $g_a$. Table 1 summarises the matching of the physical notation of Eqs. (3,4) with the one used by Susyno.
| Parameter | Susyno’s notation |
|-----------|-------------------|
| $g_1$, $g_2$, $g_3$ | $g[1]$, $g[2]$, $g[3]$ |
| $M_1$, $M_2$, $M_3$ | $M[1]$, $M[2]$, $M[3]$ |
| $(Y_u)_{ij}$ | $y[1,2,6,1,j,i]$ |
| $(Y_d)_{ij}$ | $y[1,3,7,1,j,i]$ |
| $(Y_e)_{ij}$ | $y[4,5,7,1,j,i]$ |
| $\mu$ | $\mu[6,7,1]$ |
| $(h_u)_{ij}$ | $h[1,2,6,1,j,i]$ |
| $(h_d)_{ij}$ | $h[1,3,7,1,j,i]$ |
| $(h_e)_{ij}$ | $h[4,5,7,1,j,i]$ |
| $B$ | $b[6,7,1]$ |
| $(m_{Q}^2)_{ij}$ | $m2[1,1,1,j,i]$ |
| $(m_{u}^2)_{ij}$ | $m2[2,2,1,i,j]$ |
| $(m_{d}^2)_{ij}$ | $m2[3,3,1,i,j]$ |
| $(m_{L}^2)_{ij}$ | $m2[4,4,1,j,i]$ |
| $(m_{E}^2)_{ij}$ | $m2[5,5,1,i,j]$ |
| $m_{H_u}^2$ | $m2[6,6,1]$ |
| $m_{H_d}^2$ | $m2[7,7,1]$ |

Table 1: Parameters of the MSSM assuming the field ordering \{Q,u,d,L,e,Hu,Hd\} and the simple factor group ordering \{SU2,SU3\}.

Appendix C  More on groups and representations

A simple complex Lie algebra is specified by a Cartan matrix \cite{9} which is a square matrix with diagonal entries all equal to 2 and off-diagonal entries equal to 0, -1, -2 or -3. For example, the Cartan matrix of SO(10) is

$$
\begin{pmatrix}
2 & -1 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 \\
0 & -1 & 2 & -1 & -1 \\
0 & 0 & -1 & 2 & 0 \\
0 & 0 & -1 & 0 & 2 \\
\end{pmatrix}
$$

(5)

The Cartan matrix is the ingredient required by Susyno whenever a group is being asked as input. Although the user can manually provide the Cartan matrices, the program already has a built-in function, \texttt{CM}, that can compute them for any simple group. The syntax is

\texttt{CM["group name",familyIndex]}

where “\texttt{group name}” can be “SO”, “SU”, “SP”, “G”, “F” or “E” and \texttt{familyIndex} should be an integer. For example

\texttt{CM["SO",10]}

will return the above 5 × 5 Cartan matrix. This function will work for virtually any simple group. However, this is still a heavy syntax, so the program assigns to some variables (SU2, SU3, SU5, SO10, etc.) the corresponding Cartan matrices (e.g., SO10 is the same as \texttt{CM["SO",10]}). For
the case of SO\((n)\), SU\((n)\) with \(n > 32\) and Sp\((2m)\) with \(m > 16\) no variables were set, and the CM function should be used.

As mentioned in the text, Susyno requires that each field be specified as a list of hypercharges and representations under the simple gauge factor groups. We will focus now on this last element: representations of simple groups.

A representation of a (simple) group is often labeled by its dimension (e.g., 3 of SU\((3)\), 54 of SO\((10)\), ...) but this can be ambiguous. To avoid the problem, we should use Dynkin coefficients which are a list of \(n\) non-negative integers. This number \(n\) is known as the group rank and is equal to the size of the (square) Cartan matrix of the group. For instance, since the Cartan matrix of SO\((10)\) is \(5 \times 5\), all representations of the group are of the form

\[\{i_1, i_2, i_3, i_4, i_5\}\]

\((i_1, \ldots, i_5\) are non-negative integers).

There are functions in Susyno that compute properties of the representations (e.g., DimR calculates the dimension of a representation, ReduceRepProduct reduces products of representations) and they are documented in the built-in help files (see also Appendix A). These should be enough to allow recognizing a representation by its Dynkin coefficients; however, should the user wish to consult lists of representations, these are available in the literature [10]. Some of the most frequently used representations were shown in table 2.

### Appendix D  Renormalisation group equations of a general model

Susyno uses the generic two loop RGEs of [1] [2]. For completeness these results are reproduced here (for a model based on a simple gauge group \(G\)).

In general a parameter \(X\) evolves according to the equation

\[
\frac{d}{dt}X = \frac{1}{16\pi^2}\beta_X^{(1)} + \frac{1}{(16\pi^2)^2}\beta_X^{(2)}
\]

where \(t = \log Q\) (\(Q\) being the renormalisation scale parameter). The \(\beta\)-functions for the several parameters (gauge coupling constants, superpotential parameters and SUSY soft breaking parameters) are as follows.

**Coupling constants**

\[
\beta_g^{(1)} = g^3 \left[ S(R) - 3C(G) \right]
\]

\[
\beta_g^{(2)} = g^3 \left\{ -6 \left[ C(G) \right]^2 + 2C(G)S(R) + 4S(R)C(R) \right\} - g^3 \frac{C(k)}{d(G)} \gamma_{ijk} Y_{ijk}
\]
Superpotential parameters

\[
\begin{align*}
\left[ \beta^{(1)}_Y \right]_{ijk}^{ij} &= Y_{ijp} \gamma_{(1)k} + (k \leftrightarrow i) + (k \leftrightarrow j) \\
\left[ \beta^{(2)}_Y \right]_{ijk}^{ij} &= Y_{ijp} \gamma_{(2)k} + (k \leftrightarrow i) + (k \leftrightarrow j)
\end{align*}
\]

\[
\begin{align*}
\left[ \beta^{(1)}_\mu \right]_{ij}^{ij} &= \mu_{ip} \gamma_{(1)j} + (j \leftrightarrow i) \\
\left[ \beta^{(2)}_\mu \right]_{ij}^{ij} &= \mu_{ip} \gamma_{(2)j} + (j \leftrightarrow i)
\end{align*}
\]

\[
\begin{align*}
\left[ \beta^{(1)}_L \right]_{ij}^{ij} &= L_P \gamma_{(1)ij} \\
\left[ \beta^{(2)}_L \right]_{ij}^{ij} &= L_P \gamma_{(2)ij}
\end{align*}
\]

where

\[
\begin{align*}
\gamma^{(1)i}_j &= \frac{1}{2} Y_{ipq} Y^{jpq} - 2 g^2 \delta^i_j C (i) \\
\gamma^{(2)i}_j &= -\frac{1}{2} Y_{imn} Y^{npq} Y^{mrj} + g^2 Y_{ipq} Y^{jipq} \left[ 2 C (p) - C (i) \right] \\
&\quad + 2 \delta^i_j g^4 \left[ C (i) S (R) + 2 C (i)^2 - 3 C (G) C (i) \right]
\end{align*}
\]

Soft SUSY breaking Lagrangian parameters

\[
\begin{align*}
\beta^{(1)}_M &= g^2 \left[ 2 S (R) - 6 C (G) \right] M \\
\beta^{(2)}_M &= g^4 \left[ -24 C (G)^2 + 8 C (G) S (R) + 16 S (R) C (R) \right] M + 2 g^2 \frac{C (k)}{d (G)} \left( h_{ijk} - MY_{ijk} \right) Y_{ijk}
\end{align*}
\]

\[
\begin{align*}
\left[ \beta^{(1)}_h \right]_{ijk} &= \frac{1}{2} h_{ijl} Y_{lmn} Y^{mnl} + Y_{ijl} Y_{lmn} h^{mnl} - 2 \left( h_{ijk} - 2 M Y_{ijk} \right) g^2 C (k) + (k \leftrightarrow i) + (k \leftrightarrow j)
\end{align*}
\]

\[
\begin{align*}
\left[ \beta^{(2)}_h \right]_{ijk} &= -\frac{1}{2} h_{ijl} Y_{lmn} Y^{mnpq} Y^{mrp} - Y_{ijl} Y_{lmn} Y^{mnpq} h^{mpq} + Y_{ijl} Y_{lmn} h^{npq} Y_{ppq} Y^{mrk} \\
&\quad + \left( h_{ijl} Y_{pqk} + 2 Y_{ijl} Y_{lpq} h^{lkq} - 2 M Y_{ijl} Y_{lpq} \right) g^2 \left[ 2 C (p) - C (k) \right] \\
&\quad + \left( 2 h_{ijk} - 8 M Y_{ijk} \right) g^4 \left[ C (k) S (R) + 2 C (k)^2 - 3 C (G) C (k) \right] + (k \leftrightarrow i) + (k \leftrightarrow j)
\end{align*}
\]
\[
\left[\beta_b^{(1)}\right]_{ij} = \frac{1}{2} b^{ij} Y_{mn} Y^{mn} + \frac{1}{2} Y^{ij} Y_{mn} b^{mn} + \mu^{ij} Y_{mn} h^{mn} - 2 b^{ij} - 2 M \mu^{ij} \right) g^2 C(i) + (i \leftrightarrow j)
\]

\[
\left[\beta_b^{(2)}\right]_{ij} = -\frac{1}{2} b^{ij} Y_{mn} Y^{pq} Y_{pq} Y^{mn} - \frac{1}{2} Y^{ij} Y_{mn} b^{nr} Y_{pq} Y^{pq} - \frac{1}{2} Y^{ij} Y_{mn} \mu^{nr} Y_{pq} h^{pq} - \mu^{ij} Y_{mn} h^{pq} Y_{pq} Y^{mn} - \frac{1}{2} Y^{ij} Y_{mn} Y^{npq} Y_{npq} h^{mn} + 2 Y^{ij} Y_{lpq} (b^{pq} - \mu^{pq}) g^2 C(p)
\]

\[
+ \left( b^{ij} Y_{lpq} Y^{pq} + 2 \mu^{ij} Y_{lpq} h^{pq} - \mu^{ij} Y_{lpq} Y^{pq} M \right) g^2 [2C(p) - C(i)]
\]

\[
+ (2\delta^{ij} - 8 \mu^{ij}) M \right) g^4 [C(i)S(R) + 2C(i)^2 - 3C(G)C(i)] + (i \leftrightarrow j)
\]

\[
\left[\beta_s^{(1)}\right]_{i} = \frac{1}{2} Y^{ipq} Y_{pqrs} + \mu^{ipq} b^{rs} + 2 Y^{imm} (m^2)_m h_{ml} + h^{ipq} b_{pq}
\]

\[
\left[\beta_s^{(2)}\right]_{i} = 2 g^2 C(q) Y^{ipq} Y_{pqrs} - \frac{1}{2} Y^{imn} Y_{npq} Y^{pqm} Y_{mrl} s^l - 4 g^2 C(q) (Y^{ipq} M - h^{ipq}) Y_{pqL}
\]

\[
- (Y^{imn} Y_{npq} Y^{pqm} Y_{mrl}) L^l - 4 g^2 C(q) \mu^{ipq} (\mu^{ipq} M - \mu^{pq})
\]

\[
- \mu^{ij} (Y_{mn} h^{pq} Y_{pq} \mu^{mn} + Y_{imn} Y_{npq} Y_{pq} \mu^{ln}) + 4 g^2 C(q) [2Y^{ipq} \mu^{pq} |M|^2 - Y^{ipq} b_{pq} M
\]

\[
- 2Y^{ipq} |Y_{pq}| + \mu^{ipq} b_{pq} + 2 Y^{prq} (m^2)_r \mu^{rq} - Y^{ipq} (m^2)_{pq} \mu^{pq} - Y^{imm} Y_{npq} h^{pq} \mu_{lm}
\]

\[
- Y^{imm} h_{npq} h^{pq} \mu_{lm} - Y^{imm} h_{npq} Y^{pq} \mu_{lm} - Y^{imm} (m^2)_m \mu_{tr} Y^{trpq} Y_{pq}
\]

\[
- Y^{imm} Y_{npq} (m^2)_r \mu_{rm} - Y^{imm} (m^2)_n Y_{lpq} Y^{pq} \mu_{lm} - 2 Y^{imm} Y_{npq} (m^2)_r Y^{trpq} \mu_{lm}
\]

\[
\left[\beta^{(1)}_{m^2}\right]_{i} = \frac{1}{2} Y_{lpq} Y^{pqn} (m^2)_n + \frac{1}{2} Y_{lpq} Y^{pqn} (m^2)_r + 2 Y_{lpq} Y^{prq} (m^2)_{pq} + h^{ipq} h^{jq}
\]

\[
- 8 \delta^{ij} |M|^2 g^2 C(i) + 2 g^2 t^A_j Tr (t^A m^2)
\]

\[
\left[\beta^{(2)}_{m^2}\right]_{i} = -\frac{1}{2} (m^2)_i Y_{lpq} Y^{pqn} - \frac{1}{2} (m^2)_i Y_{lpq} Y^{pqn} - Y_{lm} Y^{jmn} (m^2)_i Y_{npq} Y^{rpmq}
\]

\[
- Y_{lm} Y^{jmn} (m^2)_n Y_{npq} Y^{pqm} - 2 Y_{lm} Y^{jmn} Y_{npq} Y^{rpmq} (m^2)_r
\]

\[
- Y_{lm} Y^{jmn} h_{npq} h^{pq} - h_{lm} h^{ipq} Y_{npq} Y^{pqm} - h_{lm} h^{ipq} Y_{npq} Y^{pqm} - Y_{lm} h_{npq} h^{pq}
\]

\[
+ \left[ (m^2)_i Y_{lpq} Y^{jq} + Y_{ipq} Y^{jq} (m^2)_q + 4 Y_{lpq} Y^{jq} (m^2)_q + 2 h^{ipq} h^{jq} - 2 h^{ipq} Y^{jq} M
\]

\[
- 2 Y_{lpq} h^{jq} M^* + 4 Y_{lpq} Y^{jq} |M|^2 \right] g^2 (p) + C(q) - C(i) - 2 g^2 t^A_j (t^A m^2)_i Y_{lpq} Y^{jq} + 8 g^2 t^A_j Tr (t^A m^2)_i Y_{lpq} Y^{jq} + 8 \delta^{ij} g^4 C(i) \right\} Tr (S(i) m^2) - C(G) |M|^2
\]

A few comments and clarifications are still in order concerning the variables appearing in the different \(\beta\)-functions:

- \(Y_{ijk} = (Y^{ijk})^*\), \(h_{ijk} = (h^{ijk})^*\), \(\mu_{ij} = (\mu^{ij})^*\) and \(b_{ij} = (b^{ij})^*\);

- \(d(G)\) = Dimension of the adjoint representation of group \(G\);

- \(C(i)\) = Quadratic Casimir invariant of the representation of the chiral superfield with index \(i\);
• \( C(G) \) = Quadratic Casimir invariant of the adjoint representation of group \( G \);

• \( S(R) \) = Dynkin index summed over all chiral multiplets. However \( S(R)C(R) \) should be interpreted as the sum of Dynkin indices weighted by the quadratic Casimir invariant;

• \( t^A \) = Representation matrices under the gauge group \( G \). Terms with \( t^A \) are only relevant for \( U(1) \) groups;

• In \( \beta^{(1)}_{m^2} \) and \( \beta^{(2)}_{m^2} \), the traces should be understood as traces over all chiral superfields.

If the gauge group is not simple but rather a direct product of simple groups and \( U(1) \) factors, the equations shown here must be adapted. The modifications are fairly straightforward, and can be found in [1].

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