Central soft production of hadrons in $pp$ collisions

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Abstract

The high-energy behaviour of soft scattering observables such as total cross sections, elastic scattering at small momentum transfer, diffractive dissociation and central production have been described successfully in the context of Regge theory, with the same basic structure holding as energies have increased. For elastic scattering and diffractive dissociation the defining energies were those of the ISR, the $SpS$ collider and the Tevatron. The elastic scattering data from the LHC demonstrate the continuing applicability of Regge theory. Preliminary data on diffraction dissociation promise to add to our understanding and now, for the first time, we can expect to test fully these concepts in central production. Although the latter is the principal objective of this discussion, understanding the first two is an essential prerequisite as they define the formalism and establish parameters.

1 Elastic Scattering

Regge theory \[1, 2\] gives a unified description of soft hadronic processes at high energy, in particular elastic scattering at small momentum transfer and total cross sections \[3\]. It provides a simple quantitative description of the combined effect of multiple particle exchanges and has two basic components: the exchange of families of mesons, i.e. $q\bar{q}$ states, and the pomeron that may perhaps be \[4\] the exchange of a family of gluonic $gg$ states. The meson families $\mathcal{R}$ lie on trajectories that are to a very good approximation linear,

$$\alpha_{\mathcal{R}}(t) = \alpha_{\mathcal{R}}(0) + \alpha'_{\mathcal{R}}t$$

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with $\alpha_{I R}(0) \approx 0.5$. They have either $C = -1$ ($\rho, \omega, \cdots$) or $C = +1$ ($f_2, a_2, \cdots$). As they contribute terms to the total cross section that behave as $s^{\alpha_{I R}(0)-1}$, which is approximately $1/\sqrt{s}$, by themselves they would be at variance with observation as all hadronic total cross sections increase with $s$ at high energy. The pomeron $IP$ was introduced initially to resolve this problem, with a trajectory $\alpha_{IP}$ such that $\alpha_{IP}(0) = 1 + \epsilon_{IP}$ with $\epsilon_{IP} > 0$. As $pp$ and $\bar{p}p$ total cross sections seem to have the same rise with energy at high energy, it has $C = +1$. If its trajectory too is approximately linear, then

$$\alpha_{IP}(t) = 1 + \epsilon_{IP} + \alpha'_{IP}t.$$  

(2)

The contribution to the elastic amplitude $AB \rightarrow AB$ from the exchange of a reggeon with trajectory $\alpha(t)$ is

$$\beta_A(t)\beta_B(t) \xi^\pm(t) \left(\frac{s}{s_0}\right)^{\alpha(t)} s_0 = 1/\alpha'.$$

(3)

Here the signature factor $\xi^\pm(t)$ is

$$\xi^\pm(t) = 1 \pm e^{-i\pi\alpha(t)}$$

(4)

according to whether the exchange is of $C$-parity $\pm 1$. The theory does not tell us the coupling $\beta(t)$ of a reggeon to a hadron, but data suggest [2] that in each case a good approximation is to take it proportional to the hadron’s electromagnetic form factor, the Dirac form factor in the case of a proton or antiproton. The theory also does not identify the fixed scale $s_0$ but the choice $s_0 = 1/\alpha'$, inspired by a clever model introduced long ago by Veneziano [5] is successful for fitting data [3].

In our first fit [6] to total-cross-section data we concluded that $\alpha'_{IP} = 0.25$ GeV$^{-2}$, and that $\epsilon_{IP}$ should be close to 0.08, though a value closer to 0.096 has been claimed more recently [7]. These are effective values, representing not just the exchange of a single pomeron, but also double or more exchanges. This approach works well for describing total cross sections and elastic scattering at not-too-large $t$, but to extend the analysis to larger $t$ it is necessary to include explicitly at least the double exchange $IPIP$. We have concluded [3] that this yields

$$\alpha_{IP} = 1.110 + 0.165t$$

(5)

for the contribution from single-$IP$ exchange.

The term $IPIP$ is of particular significance as it helps explain the remarkable dip structure seen in the $pp$ elastic scattering differential cross section [5]. The contribution to the amplitude from $IPIP$ exchange has energy dependence $s^{\alpha_{IP}(t)}$ divided by some function of log $s$ where, for a linear pomeron trajectory [2],

$$\alpha_{IP}(t) = 1 + 2\epsilon_{IP} + \frac{1}{2}\alpha'_{IP}t.$$  

(6)

The double-pomeron-exchange contribution is thus flatter in $t$ than single exchange, and so its relative importance becomes greater as one goes away from $t = 0$. It increases more rapidly with energy at $t = 0$ than the single exchange, and it becomes steeper more slowly, so that the $t$-value beyond which it becomes important decreases as the energy increases. Interference between the single and double pomeron exchanges is destructive so the effect is to slow down the increase of the total cross section with increasing energy.
It also helps to generate the dip and move it to smaller \( t \) as the energy increases, in agreement with experiment. A fundamental property of elastic-scattering amplitudes is that, if they vary with energy at any value of \( t \), they must have complex phase at that \( t \). This is the reason for the signature factor \( \xi^\pm(t) \) in (3). Therefore modelling the dip structure in detail is complicated as it requires the simultaneous near-vanishing of both the real and imaginary parts of the amplitude. The \( IP \) and \( IP' \) terms combine to cancel the imaginary part but, except perhaps at low energies where the meson exchanges can contribute significantly, an additional term is required to reduce the real part. This can be achieved by introducing triple-gluon exchange, since this appears [9] to dominate the elastic amplitude at large values of \( t \). In this way, one can obtain an excellent description [3] of elastic \( pp \) and \( \bar{p}p \) scattering from \( \sqrt{s} = 20 \) GeV to 8 TeV.

2 Diffractive Dissociation

We now consider the single-particle inclusive cross section

\[
A(p_1) + B(p_2) \rightarrow A(p'_1) + X
\]

at high energy and small momentum transfer \( t = (p'_1 - p_1)^2 \) with the mass \( M_X \) of the state \( X \) large compared with that of the initial hadrons. Then the four-momentum \( p'_1 \) is almost in the same direction as \( p_1 \) and we may write \( p'_1 \sim (1 - \xi)p_1 \) with \( \xi \) small. \( \xi \) is the fractional longitudinal momentum loss of the initial hadron – not to be confused with the signature factor [4]. When \( \xi \) is sufficiently small the process is dominated by pomeron exchange, illustrated in figure 1(a). This may be thought of as a pomeron being “radiated” by particle \( A \) which then interacts with particle \( B \) to produce the system \( X \).

An inclusive diffractive experiment sums over all possible systems \( X \) and so measures the total cross section \( \sigma_{PB}(M_X^2, t) \) for a pomeron of squared mass \( t \) scattering on particle \( B \). By a generalisation of the optical theorem, this is related to the imaginary part of the forward amplitude for \( PB \) elastic scattering.
The squared centre-of-mass energy corresponding to this amplitude is

\[ M_X^2 = (\xi p_1 + p_2)^2 \sim \xi s \]  

and the differential cross section is given by

\[ \frac{d^2\sigma}{dtd\xi} = D^{P/A}(t, \xi) \sigma^{PB}(M_X^2, t) \]  

where \( D^{P/A}(t, \xi) \) is the pomeron flux and \( \sigma^{PB}(M_X^2, t) \) is the total \( \Pi^B \) cross section. In the notation of (3) the pomeron flux factor when particle A is a proton or antiproton is

\[ D^{P/A}(t, \xi) = \left[ \frac{\beta_{P}(t)}{4\pi^2} \xi^{1-2\alpha_P(t)} \right]^2 \]  

(10)

The pomeron-exchange term (9) has the structure shown in figure 1(b), in which three pomerons are coupled together. The lower one carries zero momentum and the upper two pomerons each carry momentum transfer \( t \). In practice, unless \( M_X \) is extremely large, it is necessary to include meson exchanges \( \Pi \) in \( \sigma^{PB}(M_X^2, t) \), so it has the form

\[ \sigma^{PB}(M_X^2, t) = X^{PB}(t) (\alpha'_P M_X^2)^{\alpha_P(0)-1} + Y^{PB}(t) (\alpha'_P M_X^2)^{\alpha_P(0)-1} \]  

(11)

where the second term represents \( f_2, a_2 \) exchanges. Further, unless \( \xi \) is very small, we must include terms in which either or both of the upper pomerons are replaced with meson exchanges. Thus we need a whole series of terms

\[ \Pi \Pi f_2 \Pi f_2 f_2 \omega \]  

(12)

with additional contributions also from \( \rho \) and \( a_2 \) exchange, and when \( |t| \) is of the order of \( m_\pi^2 \) one must also take account of pion exchange.

A term \( (a_c^b) \) contributes to \( d^2\sigma/dt d\xi \)

\[ f_a^A(t) f_b^A(t) f_c^B(0) G_{c/b}(t) \xi^{i(\phi(\alpha_c(t)) - \phi(\alpha_b(t))}(\alpha'_c M_X^2)^{\alpha_c(0)-1} \]  

(13)

Here, \( f_a^A(t) \) and \( f_b^A(t) \) are the couplings of the reggeons \( a \) and \( b \) to the hadron \( A \), while \( f_c^B(t) \) is that of the reggeon \( c \) to the hadron \( B \). \( G_{c/b}(t) \) is the triple-reggeon vertex and \( \phi(\alpha(t)) \) is the phase arising from the signature factor (4) associated with the trajectory \( \alpha(t) \). The complex exponential is replaced with

\[ 2\cos(\phi(\alpha_1(t)) - \phi(\alpha_2(t))) \]  

when we add the term \( (a_c^b) \). Usually, for simplicity, only terms of the form \( (a_c^b) \) are considered, in which case each term contributes

\[ f_a^A(t) f_b^A(t) f_c^B(0) G_{c/b}(t) \xi^{i(\alpha_c(t)) - 2\alpha_a(t)}(\alpha'_c M_X^2)^{\alpha_c(0)-1} \]  

(14)

to \( d^2\sigma/dt d\xi \).

Figure 1(d) needs adding to it terms in which the single exchanges are replaced by double and higher exchanges. As we have explained above, it is assumed that this can be modelled well by using an effective value for \( \alpha_P(0) \) less than
Figure 2: Integrated diffractive dissociation cross section for $\xi \leq 0.05$ in $pp$ and $\bar{p}p$ interactions. The data are from fixed-target experiments [10, 11, 12], the ISR [13, 14], the $Sp\bar{p}S$ [15], the Tevatron [16, 17] and the LHC [18]. The latter have been extrapolated to $\xi = 0.05$ to match the lower-energy data.

1.1 rather than that given in [15]. However, this does not take account of additional exchanges in figure 1b, directly between the initial or final hadrons. Their importance is not known.

The “total diffractive cross section”

$$\sigma^{\text{Diff}}(s) = \int_{t_{\text{min}}}^{t_{\text{max}}} dt \int_{\xi_{\text{min}}}^{\xi_{\text{max}}} d\xi \frac{d^2\sigma}{dt d\xi}$$

is frequently quoted, although it is not a total cross section in the usual sense as it is integrated over finite ranges of $t$ and $\xi$. Usually the single-arm cross section is multiplied by a factor of 2 to include the contribution

$$A(p_1) + B(p_2) \rightarrow X + B(p_2')$$

with the same $t$ and $\xi$.

According to [15], for a fixed lower-limit of $\xi$ the cross section rises as $(\alpha'_{P} s)^{\alpha'_{P}(0)-1}$ at sufficiently large $M_X$, while for smaller values of $M_X$ there will also be a contribution $(\alpha'_{P} s)^{\alpha'_{P}(0)-1}$ which decreases with increasing $s$. But if the integration is down to fixed $M_X^2 = \xi_{\text{min}}s$, the cross section rises even more rapidly, because as $s$ increases the integration extends down to increasingly smaller values of $\xi$. Note, however, that for [15] to be applicable $M_X$ must be larger than a few GeV. Below that we have no theory, though at low energies we do know that the low-mass region is dominated by baryon resonances.

Data for the $pp \rightarrow pX$ and $\bar{p}p \rightarrow \bar{p}X$ cross sections are shown in figure 2 with the factor of 2 included. For want of better information, the curve shown behaves as $(\alpha'_{P} s)^{\alpha'_{P}(0)-1}$.

The different experiments correspond to varying lower limits on $M_X$ and, as we have said, we do not know how to correct for that. At $\sqrt{s} = 546$ GeV the
Table 1: Data [18, 20, 21] for integrated cross sections at 7 TeV over the mass ranges shown. The error given on the TOTEM data [21] is a notional 20%.

| Source | Cross Section (mb) | Mass Range (GeV) |
|--------|--------------------|------------------|
| ALICE  | $14.9^{+3.4}_{-5.3}$ | $M_X < 200$      |
| CMS    | $4.27 \pm 0.04^{+0.65}_{-0.58}$ | $12 < M_X < 394$ |
| TOTEM  | $3.3 \pm 0.7$       | $8 < M_X < 350$  |
| TOTEM  | $6.5 \pm 1.3$       | $6.5 < M_X < 1100$ |

integrated UA4 [15] cross section for $\xi < 0.05$ is $9.4 \pm 0.7$ mb and for $M_X > 4$ GeV and $\xi < 0.05$ it is $6.4 \pm 0.5$ mb. So the cross section for $M_X < 4$ GeV is $3.0 \pm 0.8$ mb, that is about one third of the fully integrated cross section at that energy. For $M_X < 3.4$ GeV TOTEM [19] give a cross section of $(2.62 \pm 2.17)$ mb and an upper limit of 6.31 mb at 95% confidence level. So the energy dependence of the baryon-resonance contribution to single diffraction remains an open question.

Data from the LHC at 7 TeV are shown in table 1. The plot of figure 2 includes ALICE data [18] at $\sqrt{s} = 0.9, 2.7$ and 7 TeV, extrapolated by the experimentalists to $\xi_{\text{max}} = 0.05$. The CMS data [20] and the preliminary TOTEM data [21] in the table cannot be compared with those from ALICE because, as we have explained, the correction from adding in the contributions from smaller values of $M_X$ cannot be calculated. There seems no reason yet to agree with claims [22, 23, 24] that the CMS and TOTEM data call for significant modification to pomeron exchange. Although, as we have explained above, these are partly taken into account by using an effective trajectory $\alpha_P(t)$, the low-mass region needs to be understood properly before definite conclusions can be drawn.

3 Central production

There are various kinds of central production events:

- strictly exclusive production of a hadron or of a resonance
- completely inclusive central production of a hadron or of a resonance
- semi-exclusive production of a system of hadrons or resonances
- inclusive central production with rapidity gaps

We describe the theory for each of these types of event. They have different energy dependences. Our discussion is restricted to soft collisions, where the centrally-produced hadron is light and has small $p_T$. For a recent discussion of hard collisons, see [25].

3.1 Strictly exclusive production

For this kind of event, $AB \rightarrow A'B'H'B'$, one calculates an amplitude, and squares it to obtain a differential cross section. The amplitude depends on five independent
variables, for example
\[ s = (p_1 + p_2)^2 \quad s_1 = (p'_1 + p_H)^2 \quad s_2 = (p_H + p'_2)^2 \quad t_1 = (p'_1 - p_1)^2 \quad t_2 = (p'_2 - p_2)^2 \] (17)

In the case of central production at high energy, both \( s_1 \) and \( s_2 \) will be large and the amplitude is a sum of terms of the type shown in figure 3a, where \( H \) is the hadron or resonance being produced. Each of the zigzag lines \( a, b \) represents any of the exchanges \( P, \rho, \omega, f_2, a_2 \) etc and the contribution to the amplitude from the exchanges of reggeons \( a, b \) is

\[
\beta_{AA}(t_1) \xi_a(t_1) \beta_{BB}(t_2) \xi_b(t_2) f_{abH}(\eta, t_1, t_2) \alpha_a(t_1) \alpha_b(t_2) \alpha(t_2) \] (18)

Here \( \eta = s_1 s_2/(s M_H^2) \). The functions \( \beta(t) \) are the same as occur in the contribution \( \alpha \) to the elastic scattering amplitude, but this information is not very useful because not a great deal is known about the (complex) function \( f_{abH}(\eta, t_1, t_2) \) that couples the reggeons to the hadron \( H \) – its dependence on \( t_1 \) and \( t_2 \) is completely unknown and its \( \eta \) dependence is known \( \alpha \) only for small \( \eta \).

In most high-energy events \( t_1 \) and \( t_2 \) will be small. Then, with \( \xi_1, \xi_2 \) the fractional longitudinal losses of the initial hadrons,

\[ s_1 \sim s \xi_2 \quad s_2 \sim s \xi_1 \quad \xi_1 \xi_2 s \sim M_H^2 \quad \eta \sim 1 \] (19)

Hence dependence on one of the variables, say the angle between the final-state particles \( A' \) and \( B' \) has disappeared, and the energy dependence of the amplitude \( \alpha \) is given by the factor

\[
(\alpha'_a \xi_2 s)^{\alpha_a(t_1)} (\alpha'_b \xi_1 s)^{\alpha_b(t_2)} = (\alpha'_a \xi_2 s)^{\alpha_a(t_1)} (\alpha'_b M_H^2 / \xi_2)^{\alpha_b(t_2)} \] (20)

To obtain the energy dependence of the cross section, one has to square this and apply

\[
\int_{M_H^2/(s \xi_{max})}^{\xi_{max}} d\xi_2/\xi_2. \] (21)

The result is

\[
\frac{1}{2(\alpha_a(t_1) - \alpha_b(t_2))} \left\{ (\alpha'_a s)^{2\alpha_a(t_1)} (\alpha'_b M_H^2)^{\alpha_b(t_2)} (\xi_{max})^{2(\alpha_a(t_1) - \alpha_b(t_2))} - (\alpha'_b s)^{2\alpha_b(t_2)} (\alpha'_a M_H^2)^{2\alpha_a(t_1)} (\xi_{max})^{2(\alpha_a(t_2) - \alpha_b(t_1))} \right\} \] (22)
So, up to logarithmic factors, the $PP$, $PR$ and $RRP$ term give a contribution increasing as $s^{2\epsilon_P}$, with $\epsilon_P$ given in [2], while for the $RRR$ term the power is close to -1. However, when we square the amplitude there are also interference terms [27], including one behaving approximately as $1/\sqrt{s}$.

The principal reason for studying central production of mesons in high energy $pp$ collisions is to search for glueballs, mesons consisting of two or three “constituent” gluons. Reactions of the type $pp \rightarrow pX^n p$, in which the exchanges $a$ and $b$ of figure 3 are both pomerons, are thought most likely to produce glueballs, as the pomeron is believed to be primarily a gluonic system [4].

States with two constituent gluons necessarily have $C = +1$. The lowest-mass states are expected to be in a relative $S$-wave and have the quantum numbers $J^{PC} = 0^{++}, 2^{++}$ and $0^{-+}$. Three-gluon systems can have both $C = +1$ and $C = -1$ and for the ground states with a relative $S$-wave the quantum numbers are $J^{PC} = 1^{++}, 1^{+-}, 1^{-+}$ and $3^{--}$. Conventional $q\bar{q}$ states exist in all of these $J^{PC}$ states which presents a problem as there is naturally mixing between the two systems. The presence of glueballs has to be inferred by an excess of states with specific glueball quantum numbers.

Masses of glueballs have been estimated in quenched lattice gauge theory [28, 29, 30] and the results are given in table 2.

The $2^{++}$ and $0^{-+}$ are at the limit of current meson spectroscopy, so experimental and theoretical emphasis has been on the scalar mesons $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. In terms of the standard $q\bar{q}$ model assignments for the light mesons there is one too many isoscalar scalars in the 1300 to 1700 MeV mass region and this has been attributed to mixing with a scalar glueball. However it has been argued that the $f_0(1370)$ may not exist, although this has been strongly contested and the situation remains unclear [31]. Removing this ambiguity and extending meson spectroscopy to higher mass are key to unravelling the glueball question.

Preliminary data from ALICE [32] on $\pi^+\pi^-$ production in $pp$ collisions at $\sqrt{s} = 7$ TeV with a large double gap in pseudorapidity show dominant peaks in the $\pi^+\pi^-$ mass distribution associated with the $f_0(980)$ and $f_2(1270)$. While it is tempting to conclude that this result illustrates double-pomeron exchange preferentially selecting isoscalar states, without knowing the detailed kinematics this is somewhat premature. The $f_2(1270)$ is a well-established $q\bar{q}$ state and the $f_0(980)$ is not generally considered as a gluonic system. Further there is no evidence for the $f_0(1500)$ which is the scalar meson thought most likely to have a large gluonic component. Given the discussion after [27] it may be that the $RRP + PRR$ terms are responsible.

A detailed review of the present status of glueballs can be found in [33].
A second reason for studying exclusive meson production is to continue the search for evidence of the elusive odderon, the possible C = P = −1 partner of the C = P = +1 pomeron. The odderon has a long history \[34\], but unambiguous odderon effects have never been observed. It has been suggested \[35\] that the odderon could be observed in high-energy pp interactions through exclusive central production of C = −1 mesons, for example the J/ψ. The proposed mechanism is figure 3 with reggeon a the pomeron and reggeon b the odderon. Note, however, that if we take b to be the ω trajectory, or even the J/ψ, the energy dependence would be the same. Which of the choices dominates is determined by the relative magnitudes of the couplings to the J/ψ and the lower proton.

As an alternative to an exclusive process, the difference in inclusive production of particles and antiparticles in the central region has been proposed \[36\] and present data analysed, without success. Reasons for the missing odderon are given in \[37\], including its weak coupling to the nucleon at small t. A further complication arises in lattice gauge theory \[4\] as it predicts that the odderon trajectory, although it has a slope similar to that of the pomeron, \(\epsilon_{\text{Odd}}\) is negative so the odderon has an intercept that is less than one.

### 3.2 Completely inclusive production

In this case, \(AB \rightarrow HX\), the diagram is figure 3b. Each reggeon carries zero 4-momentum. The vertical line indicates that, according to the generalised optical theorem \[38\], to calculate the inclusive differential cross section one has to take the discontinuity in the variable \((p_1 + p_2 - p_H)^2\).

Define the momentum transfers
\[
\tau_1 = (p_1 - p_H)^2 \quad \tau_2 = (p_2 - p_H)^2
\]
and let the fraction of the initial centre-of-mass-frame momentum of hadron \(p_1\) carried by \(H\) be \(x\). Then figure 3b contributes
\[
\frac{d^2\sigma}{d\log x \, dP_H^2} = \frac{\beta_{Aa}(0)\beta_{Bb}(0)V_{abH}}{4\pi^3 s} \left( -\alpha'_a \tau_1 \right)^{\alpha(0)}(0) \left( -\alpha'_b \tau_2 \right)^{\alpha(0)} \]

where we have used \(d^3P_H/E_H \sim \pi dx dP_H^2 / x\). The coupling \(V_{abH}\) of the two reggeons to the hadron \(H\) is a constant since, as the reggeons carry zero 4-momentum, the only Lorentz invariant at the central vertex is \(P_H^2\).

If the exchanges a and b are the same, and also the initial hadrons have the same mass \(m\),
\[
\left( -\alpha' \tau_1 \right)^{\alpha(0)}(0) \left( -\alpha' \tau_2 \right)^{\alpha(0)} \sim \left( \alpha'(m_H^2 + p_T^2 + x^2 m^2) \right)^{\alpha(0)} \]

### 3.3 Semi-exclusive production

Figure 4a shows the particle \(H\) in figure 3(a) replaced with a cluster of particles to give the process
\[
A(p_1) + B(p_2) \rightarrow A(p'_1) + B(p'_2) + X
\]
Figure 4: (a) The inclusive process (26) for the case where both initial particles lose very little momentum. (b) The squared amplitude of (a) summed over $X$ when the invariant mass of the system $X$ is large – the vertical line indicates that the discontinuity must be taken in the variable $M_X^2$.

For most events both $t_1$ and $t_2$ will be small and if both initial particles lose a small amount of energy $p_1' \sim (1 - \xi_1)p_1$ and $p_2' \sim (1 - \xi_2)p_2$ with $\xi_1, \xi_2 \ll 1$. If $\xi_1$ and $\xi_2$ are sufficiently small then both reggeons will be pomerons. If we square the amplitude and sum over all possible systems $X$ then

$$
\frac{d^4\sigma}{dt_1dt_2d\xi_1d\xi_2} = D^{P/A}(t_1, \xi_1)D^{P/B}(t_2, \xi_2)\sigma^{PP}(M_X^2, t_1, t_2)
$$

(27)

where on the right-hand side the first two terms are the pomeron-flux factors of (8), and $\sigma^{PP}(M_X^2)$ is the total cross section for pomeron-pomeron scattering with $M_X^2 = \xi_1\xi_2s$. If $M_X^2$ is sufficiently large then only pomeron exchange between the pomerons need be included so that the right-hand side of (27) corresponds to figure 4(b). This contains two triple-pomeron vertices and the diagram factorises, allowing the cross section (27) to be written as

$$
\frac{d^2\sigma(s)}{dt_1d\xi_1} = \frac{d^2\sigma(s)}{dt_1d\xi_1} \frac{d^2\sigma}{d\xi_2} \sigma^{PP}(s)
$$

(28)

Here $\sigma^{PP}(s)$ is the pomeron-exchange contribution to the $pp$ total cross section.

This formula still applies when either or both of the upper reggeons in figure 4(b) for the first two factors on the right-hand side represent non-pomeron exchanges. Indeed, unless $\xi$ is extremely small both terms are required at high energy to describe the double distribution $d^2\sigma/dtd\xi$. These are $BPBP$ and $BRBP$. (As we have said before, the interference terms $BRBP$ and $FRBP$ are usually omitted for simplicity.) At fixed $\xi$, both have the same $s$ dependence as can be seen from (13); however they have very different $\xi$ dependence. At small $t$, $BPBP$ behaves approximately as $1/\xi$ and $BRBP$ behaves approximately as a constant. As $t$ increases the $\xi$ dependence of the $BPBP$ term changes slowly because of the small slope of the pomeron trajectory. However the much larger slope of the nonleading trajectory $BR$ causes a rapid change in the $\xi$ dependence of the $BRBP$ term so that it behaves approximately as $\xi^{1.3}$ at $t = -1 \text{ GeV}^2$. 

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Figure 5: The double differential cross section \(d^2\sigma/dtd\xi\) in \(\bar{p}p\) interactions at (a) \(\sqrt{s} = 1800\) GeV, \(t = -0.05\) GeV\(^2\) and (b) \(\sqrt{s} = 540\) GeV (circles) and 630 GeV (squares), \(t = -0.95\) GeV\(^2\). The data are from \([39, 40, 41]\), the dashed line is the \(P^2P\bar{P}\) contribution, the dotted line the \(R^2R\bar{P}\) contribution and the solid line their sum. The units are mb GeV\(^{-2}\).

Figure 6: (a) Data \([20]\) for \(\xi d\sigma/d\xi\) at \(\sqrt{s} = 7\) TeV; the units are mb. (b) \(\xi_1 \xi_2 d^2\sigma/d\xi_1 d\xi_2\) at \(\sqrt{s} = 7\) TeV, for \(\xi_2 = 10^{-5}\) (top curve), \(10^{-4}\) (middle curve) and \(10^{-3}\) (bottom curve); the units are mb.

We illustrate this explicitly with the CDF data \([39]\) for \(d^2\sigma/dtd\xi\) at \(\sqrt{s} = 1800\) GeV, \(t = -0.05\) GeV\(^2\) and the UA4 and UA8 data at \(t = -0.95\) GeV\(^2\) and \(\sqrt{s} = 546\) and 630 GeV respectively \([40, 41]\). For the \(\xi\) dependence we use the pomeron and \(C = +\) reggeon parameters of \([6]\), though different choices give qualitatively the same outputs. The result is shown in figure 5, the normalisation of the two terms in each case being adjusted to give a reasonable description of the data. At \(t = -0.05\) GeV\(^2\) pomeron dominance of \(d^2\sigma/dtd\xi\) only occurs for \(\xi \lesssim 0.004\). However at \(t = -0.95\) GeV\(^2\) it occurs for \(\xi \lesssim 0.03\).

An estimate of \(d^3\sigma/dt_1 dt_2 d\xi_1 d\xi_2\) at LHC energies can be made by combining the single-diffraction TOTEM data \([21]\) for \(d^2\sigma/dt\) for \(8 < M_X < 350\) with the CMS data \([20]\) for \(d^2\sigma/dtd\xi\) over essentially the same mass range: see table 1. The corresponding values of \(\xi\) are sufficiently small, \(\xi \lesssim 0.003\), to ensure triple-
pomeron dominance of the single-diffractive cross section. The exponential slope of the TOTEM data over this mass range is $8.5 \text{ GeV}^2$, giving a mean value of $t = -0.082 \text{ GeV}^2$ which can be used to calculate the shape of $d^2\sigma/dtd\xi$. The result is shown in figure 6a, with the normalisation adjusted to fit the data. The result of applying (28) is given in figure 6b.

3.4 Inclusive central production with rapidity gaps

If the initial hadron $p_1$ loses only a small fraction $\xi_1$ of its initial momentum the mechanism is that of figure 7a. If $\xi_1$ is extremely small, energy conservation will demand that there be a rapidity gap between $p'_1$ and the rest of the final-state particles. Whether or not that is the case, if $\xi_1$ is small enough and the total energy is high enough, the dominant contribution will come from all the reggeons being pomerons and factorisation will apply:

$$\frac{d^4\sigma(s)}{dt_1 d\xi_1 d\log x dP_T^2} = \frac{d^2\sigma(s)}{dt_1 d\xi_1} \frac{d^2\sigma(s)}{d\log x dP_T^2} \frac{1}{\sigma_{\text{Tot}}(s)}$$

(29)

where the first factor on the right-hand side is that of (9) and the second that of (24).

Similarly, if both initial hadrons lose only a very small fraction of their initial momenta,

$$\frac{d^6\sigma(s)}{dt_1 d\xi_1 dt_2 d\xi_2 d\log x dP_T^2} = \frac{d^4\sigma(s)}{dt_1 d\xi_1 dt_2 d\xi_2} \frac{d^2\sigma(s)}{d\log x dP_T^2} \frac{1}{\sigma_{\text{Tot}}(s)}$$

(30)

where the first factor on the right-hand side is that of (28).
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