Imbedding nonlocality in a relativistic chronology

Antoine Suarez∗
Center for Quantum Philosophy, P.O. Box 304, CH-8044 Zurich, Switzerland

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Abstract: An alternative description imbedding nonlocality in a relativistic chronology is proposed. It is argued that vindication of Quantum Mechanics in experiments with moving beam-splitters would mean that there is no real time ordering behind the nonlocal correlations

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The relationship between Quantum Mechanics and Relativity has been object of vast analysis since John Bell showed that: a) if one only admits relativistic local causality (causal links with \( v \leq c \)), the correlations occurring in two-particle experiments should fulfill clear locality conditions (“Bell’s inequalities”), and b) for these experiments Quantum Mechanics bears predictions violating such locality criteria (“Bell’s theorem” \[1\]). Bell-type experiments conducted in the past two decades, in spite of their loopholes, suggest a violation of local causality: statistical correlations are found in space-like separated detections; violation of Bell’s inequalities ensure that these correlations are not pre-determined by local events. Nature seems to behave nonlocally, and Quantum Mechanics predicts well the observed distributions. Nevertheless, nonlocality (“Bell influences”) cannot be used for faster-than-light communication.

Nonlocal correlations cannot just appear by chance: they require an ordering of the events, causality in some sense. But we use to think about causality as related to some temporal sequence. Therefore, also taking nonlocality for granted, the important question remains: is there a time ordering behind the nonlocal correlations?

Bohm’s theory proposes to imbed Quantum Mechanics in a preferred frame or absolute time, in which one event is caused by some earlier event \[2\]. The theory does not make predictions conflicting with Quantum Mechanics but is rather a particular interpretation of it. However, it does not tell us how to trace this frame \[1\], so that one sees no mean to decide whether the bohmian “quantum ether” has any physical reality at all.

Recent work shows that it is possible to imbed nonlocality in a real relativistic time ordering, providing one gives up Quantum Mechanics in a new type of experiments involving moving devices. This happens within Multisimultaneity, a nonlocal description using many frames to establish the cause-effect links \[1, 2\]. More specifically these frames are supposed to be those of the beam-splitters (“choice-devices”) \[6\]. Within each frame the links always correspond to a well defined chronology, one event never depending on some future event. Multisimultaneity has already been developed in the context of 2-particle experiments with moving beam-splitters. In this article we implement it in 3-particle ones, and discuss the meaning of a possible vindication of Quantum Mechanics in tests using moving beam-splitters.

In Fig. 1 (a) is sketched the schema of a source \( S \) capable of producing maximally energy-time entangled photon triplets \[6\]. Photons coming from the pulsed pump laser \( S_1 \) reach the nonlinear crystal \( C_1 \) either by path \( l_1 \) or path \( s \), and those from the pulsed pump \( S_2 \) reach the nonlinear crystal \( C_2 \) either by \( l_2 \) or \( s \). At \( C_1 \) and \( C_2 \) twin-photons are created by parametric down-conversion. Two output beams, one of \( C_1 \) and one of \( C_2 \), are directly guided to the beam-splitters \( CD_{10} \), respectively \( CD_{20} \), and as represented in Fig. 1 (b) illuminate two interferometers which use moving choice-devices \( CD_{11} \) and \( CD_{21} \). The other two beams are led to interfere into beam-splitter \( BS \). One of the output ports of \( BS \) is monitored by detector \( D \), and the photons leaving by the other are guided to beam-splitter \( CD_{30} \) and, as represented in Fig. 1 (b), illuminate a third interferometer using a resting beam-splitter \( CD_{31} \). The location of this device can be adjusted by means of delay line DL.

All \( CD_i \) \((i \in \{1, 2, 3\}, l \in \{0, 1\})\) are assumed to be 50-50 beam-splitters. The two output ports of each \( CD_{11} \) are monitored by detectors \( D_i(\sigma) \) \((\sigma \in \{+, -\})\). The short arms of the two interferometers within the source

∗suarez@leman.ch
S, and the short arms of the interferometers 1 and 2, are all of them supposed to be equal in length. The arms \( l_1 \) and \( l_2 \) of interferometer 3 are equal to the arms \( l_1 \) respectively \( l_2 \) within the source \( S \). The phase parameters are labeled \( \varphi_1, \varphi_2, \alpha, \beta, \) and \( \gamma \).

We consider only the cases in which detector \( D \) registers a photon traveling by one of the short arms, and there is one photon in each of the three output ports leading to \( CD_{10}, CD_{20}, CD_{30} \). We assume the pumps \( S_1 \) and \( S_2 \) to work well synchronized so that the detected photon triplets signals in \( D_1(\sigma) \) will exhibit the same time difference for the paths \((S_1s_1, S_2s_2, S_2l_2l_1)\) and \((S_1l_1s, S_2sl_2, S_1l_1l_2)\), where the first path expression denotes that: photon 1 comes from pump \( S_1 \), travels path \( s \) within the source, and then \( l_1 \) in interferometer 1; photon 2 comes from pump \( S_2 \), travels first path \( s \) within the source, and then \( l_2 \) in interferometer 2; photon 3 comes from pump \( S_2 \), travels path \( l_2 \) within the source, and then \( l_1 \) of interferometer 3; and the second path expression has a similar meaning [3]. Because of the movement of \( CD_{11} \) and \( CD_{21} \) the frequency of the reflected photons is Doppler-shifted by an amount \( \omega_d \), but the setup in Fig. 1 (b) is arranged so that the total frequency shift for each of the two paths is the same. Therefore detection of the triplets traveling the paths \((S_1s_1, S_2s_2, S_2l_2l_1)\) and \((S_1l_1s, S_2sl_2, S_1l_1l_2)\) will exhibit interferences.

According to Quantum Mechanics, beyond the devices \( CD_{10}, CD_{20} \) and \( CD_{30} \) the three particles are in a GHZ-state of the form:

\[
|\psi\rangle = \frac{1}{\sqrt{2}}(|S_1s_1l_1S_2s_2l_2l_1\rangle + e^{i\phi}|S_1l_1s_1S_2s_2l_2l_1\rangle) \quad (1)
\]

where \( \phi \) is a phase factor.

Independently of any timing it holds that:

\[
Pr^{QM}(\rho, \sigma, \omega) = \frac{1}{K} |A(S_1s_1\rho, S_2s_2\sigma, S_2l_2l_1\omega) + A(S_1l_1\rho, S_2s_2\sigma, S_1l_1l_2\omega)|^2 \quad (2)
\]

where \( P^{QM}(\rho, \sigma, \omega) \) denotes the joint probability of getting the outcome \( D_1(\rho), D_2(\sigma), D_2(\omega) \); \( A(path\ \rho, path\ \sigma, path\ \omega) \) the corresponding probability amplitudes for the paths and outcome triplets specified within the parentheses; and \( K \) is a normalization factor.

Substituting the amplitudes into Eq. (2) yields the following values for the conventional joint probabilities:

\[
Pr^{QM}(\rho, \sigma, \omega) = \frac{1}{8}[1 + \rho \sigma \omega \sin(\alpha - \beta - \gamma + \varphi_2 - \varphi_1)] \quad (3)
\]

Eq. (3) yields the following correlation coefficient:

\[
E^{QM} = \sum_{\rho, \sigma, \omega} \rho \sigma \omega P^{QM}(\sigma, \omega) / \sum_{\rho, \sigma, \omega} P^{QM}(\rho, \sigma, \omega) = \sin(\alpha - \beta - \gamma + \varphi_2 - \varphi_1) \quad (4)
\]

We implement now the principles and rules of Multisimultaneity in the context of 3-particle experiments.

We denote \( T_{ij} \) the time at which the choice between reflection and transmission occurs at device \( CD_{11} \). In expressions like \( (T_{11} < T_{12}) \), the subscript \( i \) after the parenthesis denotes that all times within the parentheses are measured in the inertial frame defined through the velocity of choice-device \( CD_{11} \) at the instant of the choice in this device. If it holds that \( (T_{11} < T_{21}) \) and \( (T_{11} < T_{31}) \), for \((i, j, k \in \{1, 2, 3\}, i \neq j \neq k)\), the choice at \( CD_{11} \) is said to occur with “before” timing, and labeled \( b_{ij} \). If it holds that \( (T_{11} > T_{21} > T_{31}) \), the choice at \( CD_{11} \) is said to occur with “after” timing with relation to the choice at \( CD_{k1} \), and labeled \( a_{ij[i|k]} \). If it holds that \( (T_{11} > T_{21} > T_{31}) \), the choice at \( CD_{11} \) is said to occur with “after” timing with relation to \( CD_{11} \) and \( CD_{k1} \), or simply with after timing, and labeled \( a_{ij} \). A before-choice at \( CD_{11} \) carrying out the value \( \rho \) is denoted \( b_{\rho} \), and an after-choice at \( CD_{11} \) carrying out the value \( \rho \) is denoted \( a_{ij} \).

The main Principles of Multisimultaneity are the following two:

**Principle 1**: The values \( b_{11\rho} \) of particle \( i \) do not depend on the values the other particles may produce.

**Principle 2**: The values \( a_{11\rho} \) involve nonlocal causal links, and depend on the values the other particles may produce.

Regarding **Principle 2**, in after-after timings it would obviously be absurd to assume together that \( a_{11\rho} \) depends on \( a_{11\sigma} \), and \( a_{11\sigma} \) on \( a_{11\rho} \). Therefore we assume that the outcomes particle \( i \) produces in after choices at \( CD_{11} \) do not depend on the outcomes the other particles \( j \) and \( k \) may actually produce in after choices but on the outcomes they would have produced if the choices at \( CD_{11} \) and \( CD_{k1} \) would have been before ones.

We denote \( Pr(C) \) the probability that a photon triplet belongs to the class traveling path \( C \), \( C \in \{S_1s_1, S_2s_2, S_2l_2l_1, S_1l_1s, S_2sl_2, S_1l_1l_2\} \); Expressions like \( Pr(b_{11\rho}, a_{12[i|1][1]} \sigma, a_{11\omega}) \) denote the probabilities of getting the indicated values. \( P(b_{11\rho})(C) \) the conditional probability that photon \( i \) leaves \( CD_{11} \) by output port \( \rho \) after a before-choice, providing the pair travels path \( C \); expressions as \( P(a_{12[i|1][1]}(\rho, a_{11\rho}, a_{12[i|1]}(\rho, \sigma, \omega)) \) mean the conditional probability that photon \( i \) leaves \( CD_{11} \) by output port \( \rho \) after an after-choice providing photon \( j \) would have left \( CD_{11} \) by output port \( \sigma \) in a before-choice, and photon \( k \) \( CD_{k1} \) by output port \( \omega \) in an after-choice with relation to \( CD_{11} \).

The rule to calculate the joint probabilities for the outcomes at \( CD_{11}, CD_{21} \) and \( CD_{31} \), with all choices occurring under before timing, follows straightforwardly from **Principle 1** above, and is given by the expression:

\[
Pr(b_{11\rho}, b_{21\sigma}, b_{31\omega}) = \sum_{C} Pr(C) Pr(b_{11\rho}(C)) Pr(b_{21\omega}(C)) Pr(b_{31\omega}(C)) \quad (5)
\]
For the different paths it holds that:

\[ Pr(C) = \frac{1}{|K|} |A(S_1s_1, S_2s_2, S_3s_3)|^2 \]

\[ = \frac{1}{|K|} |A(S_1s_1, S_2s_2, S_3s_3)|^2 = \frac{1}{2} \quad (6) \]

And for the \( b_{1\rho} \) values, \( i \in \{1, 2, 3\}, \rho \in \{+, -\} \), one is led to the following relations:

\[ Pr(b_{1\rho}|C) = |A(s\rho)|^2 = |A(l\rho)|^2 = |A(l\rho')|^2 = \frac{1}{2} \quad (7) \]

Substituting (6) and (7) into (5) yields:

\[ Pr(b_{1\rho}, b_{2\sigma}, b_{3\omega}) = \frac{1}{8} \quad (8) \]

For each choice-device with input ports \( (p, q) \in \{(l_1, s), (l_2, s), (l_1, l_2)\} \) and output ports \( (+, -) \), the path amplitudes fulfill the relation:

\[ A(p +)A^*(q +) + A(p -)A^*(q -) = 0 \quad (9) \]

Relation (3) implies that:

\[ \sum_\omega Pr_{QM}(\rho, \sigma, \omega) \]

\[ = \sum_\omega |A(S_1s_1\rho, S_2s_2\sigma, S_3s_3\omega)|^2 + |A(S_1s_1\rho, S_2s_2\sigma, S_1l_1\omega)|^2 \]

\[ = |C|^2|A(l_1\rho)|^2|A(s\sigma)|^2 + |C|^2|A(s\rho)|^2|A(l_2\sigma)|^2 \]

\[ = \sum_C Pr(C) Pr(b_{1\rho}|C) Pr(b_{1\omega}|C) = Pr(b_{1\rho}, b_{1\omega}) \quad (10) \]

and similar equalities for summations over \( \rho \) and \( \sigma \).

Relation (10) means that for the experiment we are considering the quantum mechanical marginals can be described as though the involved choices would be before ones, i.e. the values \( a_{1[k\rho]} \) behave as \( b_{1\rho} \) ones.

Consider first the type of timing that practically result when all choice-devices are at rest, i.e. \( (b_{1\rho}, a_{j1[1]}\sigma, a_{k1}\omega) \). For these timings we assume that Multisimultaneity reproduces the predictions of Quantum Mechanics, so that:

\[ Pr(b_{1\rho}, a_{j1[1]}\sigma, a_{k1}\omega) = Pr_{QM}(\rho, \sigma, \omega) \quad (11) \]

From Principle 2 above it follows that:

\[ Pr(b_{1\rho}, a_{j1[1]}\sigma, a_{k1}\omega) \]

\[ = \sum_C Pr(C) Pr(b_{1\rho}|C) Pr(a_{j1}\sigma|b_{1\rho}) Pr(a_{k1}\omega|b_{1\rho}, a_{j1[1]}\sigma) \quad (12) \]

Summing over \( \omega \) in (12), and taking account of (11) and (6) one is led to:

\[ \sum_\omega Pr(b_{1\rho}, a_{j1[1]}\sigma, a_{k1}\omega) = \sum_\omega Pr_{QM}(\rho, \sigma, \omega) \]

\[ = \sum_C Pr(C) Pr(b_{1\rho}|C) Pr(a_{j1}\sigma|b_{1\rho}) Pr(a_{k1}\omega|b_{1\rho}, a_{j1[1]}\sigma) \quad (13) \]

From (12) and (13) it follows that:

\[ Pr(a_{1[1]}\rho, a_{2[1]}\sigma, b_{3\omega}) = Pr(b_{1\rho}, b_{1\sigma}) \]

\[ = \sum_\omega Pr_{QM}(\rho, \sigma, \omega) \]

\[ = 0 \quad (14) \]

Since by setting \( CD_{j1} \) in movement one could change instantaneously the timing \( (b_{1\rho}, a_{j1[1]}\sigma, a_{k1}\omega) \), property (3) prevents that this action can be used to produce superluminal signaling. For experiments that don’t fulfill (3) Multisimultaneity can be conveniently completed so that superluminal signaling remains forbidden.

Consider secondly the experiment of Fig. 1 (b) conducted with timing \( (a_{1[1]}[3], a_{2[1]}[3], b_{31}) \). Since as stated above the values \( a_{1[1]}[3] \) behave as \( b_{1\rho} \) ones, taking account of (3) one is led to:

\[ Pr(a_{1[1]}[1]\rho, a_{2[1]}[1]\sigma, b_{31}\omega) = Pr(b_{1\rho}, b_{21}\omega, b_{31}\omega) = \frac{1}{8} \quad (15) \]

And (13) yields the following correlation coefficient:

\[ E^{abcd} = \frac{\sum_{\rho, \sigma, \omega} Pr(b_{1\rho}, b_{21\omega}, b_{31\omega})}{\sum_{\rho, \sigma, \omega} Pr(b_{1\rho}, b_{21\omega}, b_{31\omega})} = 0 \quad (16) \]

Consider thirdly the timing \( (a_{1\rho}, a_{21\sigma}, b_{31\omega}) \). Applying Principle 2 one gets:

\[ Pr(a_{1\rho}, a_{21\sigma}, b_{31\omega}) \]

\[ = \sum_{C, \rho'} Pr(C) Pr(b_{1\rho'}|C) Pr(b_{21\omega'}|C) Pr(b_{31\omega}) \]

\[ \times Pr(a_{1\rho}|b_{21\omega'}, b_{31\omega}) Pr(a_{21\sigma}|b_{1\rho}, b_{31\omega}) \quad (17) \]

Substituting (3), (6) and (7) into (17) gives:

\[ Pr(a_{1\rho}, a_{21\sigma}, b_{31\omega}) = \frac{1}{8} \quad (18) \]

Eq. (18) yields the following correlation coefficient:

\[ E^{aba} = \frac{\sum_{\rho, \sigma, \omega} Pr(a_{1\rho}, a_{21\sigma}, b_{31\omega})}{\sum_{\rho, \sigma, \omega} Pr(a_{1\rho}, a_{21\sigma}, b_{31\omega})} = 0 \quad (19) \]

Consider finally the timing \( (a_{1\rho}, a_{21\sigma}, a_{31\omega}) \). Principle 2 leads to the following rule:
To have the timing \((b_{11}, b_{21}, b_{31})\) the direction of movement is that indicated in Fig. 1 (b), and for timing \((a_{11}, a_{21}, b_{31})\) the reverse, and moreover, in both cases, \(CD_{31}\) has to be set so that the arrival of particle 3 at \(CD_{31}\) in the laboratory frame occurs after the arrivals of particle 1 and 2 at \(CD_{11}\), respectively \(CD_{21}\): to reverse the direction of movement indicated in Fig. 1; and to orient \(CD_{31}\), \(i \in \{1, 2\}\) till to get the sound wave propagating towards \(CD_{31}\). Assumed a value of 90° for the angle \(CD_{11}-CD_{31}-CD_{21}\), and distances \(CD_{11}-CD_{31}\), \(CD_{21}-CD_{31}\) of about 72 m, then one has a distance \(CD_{11}-CD_{21}\) of 102 m, and velocity components of 1.77 km/s in the direction \(CD_{11}-CD_{21}\), i.e. values which fulfill the condition (23).

In summary, we have shown that a description imbedding nonlocal causality in a relativistic time ordering is possible for experiments involving more than 2 particles. As far as one aims nonlocal causal descriptions based on relativistic (real) timings, it is natural to assume that each choice involves all (local and nonlocal) information that is available within the inertial frame of the choice-device at the instant the particle arrives. Then the basic principle of any relativistic nonlocal description is the following one: in experiments in which all choices take place under before timing the nonlocal correlations should disappear. By contrast Quantum Mechanics predicts such correlations independently of any timing. Therefore, experiments using only before timings can be considered a criterion allowing us to decide whether or not the nonlocal physical reality can be imbedded into a relativistic chronology.

Prevaling of Quantum Mechanics in forthcoming Bell experiments with moving choice-devices would support the view that the nonlocal correlations are caused independently of any real chronology \(\Box\). In this sense Quantum Mechanics can be considered a causal description which is both nonlocal and nontemporal.

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