Anderson cross-localization

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We report Anderson localization in two-dimensional optical waveguide arrays with disorder in waveguide separation introduced along one axis of the array, in an uncorrelated fashion for each waveguide row. We show that the anisotropic nature of such disorder induces a strong localization along both array axes. The degree of localization in the cross-axis remains weaker than that in the direction in which disorder is introduced. This effect is illustrated both theoretically and experimentally.

Anderson localization was predicted in solid state physics upon consideration of evolution of particles in a disordered infinite medium [1]. Anderson localization appears due to transformation of infinitely extended eigenmodes of the system (Bloch waves) into exponentially localized modes in the presence of disorder [2], and it is a universal concept applicable to a variety of physical systems [3], including optics [4-7]. A breakthrough was the optical analogy of Anderson localization was predicted in solid state physics [4-7]. A breakthrough was the optical analogy of Anderson localization [4-7].

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level \( S_\eta \), but the averaging of output intensity distributions was performed using an excitation of different waveguides. Averaging over different waveguides (that is used in experiment) provides absolutely the same result as averaging over the ensemble of array realizations (Fig. 1), as it was recently confirmed in 1D arrays [13].

The output intensity distributions obtained by averaging over the ensemble of \( n = 10^3 \) arrays are shown in Fig. 2 (right column) for linear case, when \( \Delta \rightarrow 0 \). At \( S_\eta = 0.0 \) (first row) one observes regular discrete diffraction. For small disorder levels \( S_\eta \leq 0.2 \) (second row) the output pattern shows a tendency for contraction: it is more confined in the horizontal direction, in which disorder is acting, and bright spots appear in the center; i.e., in this regime \( I_{\eta\eta} \) represents a superposition of a localized central part and several side lobes stemming from discrete diffraction. The transition to localization is apparent already at moderate disorder levels \( S_\eta = 0.4 \) (third row), when the diffraction side lobes almost vanish and only the localized central part featuring exponential tails remains. The localization is further enhanced with an increase of the disorder level up to \( S_\eta = 0.6 \) (fourth row).

The central result of our simulations is that increasing the disorder level causes simultaneous and almost equally strong Anderson localization along both \( \eta \) and \( \zeta \) axes (Fig. 2), despite the fact that disorder is effectively 1D (only \( \eta \) coordinates of waveguides fluctuate). The localization along the \( \zeta \) axis builds up with \( S_\eta \) much faster than expected considering that only small disorder generated by the disorder anisotropy acts in this direction. Such a "cross-localization" effect arises because introducing a 1D disorder in the separation between sites in any multidimensional system modifies the site-to-site separation also in all orthogonal directions, affecting the coupling in these directions. The almost equal degree of localization in \( \eta \) and \( \zeta \) directions is particularly surprising taking into account that for \( S_\eta \ll 1 \) the variation of the horizontal waveguide separation \( -S_\eta \) causes much smaller variation of the distance between waveguides in the neighboring rows \( -S_\zeta^2 / 2d_\zeta \), which intuitively should result in much weaker localization along the \( \zeta \) axis. Nevertheless, a comparable localization in \( \eta \) and \( \zeta \) directions was obtained already at \( S_\eta \approx 0.3 \). The same effect is observed in arrays with circular waveguides, i.e. the enhanced localization in \( \zeta \) direction is not due to elliptical waveguides.

This behavior was proven in our experiments. We fabricated arrays with \( 21 \times 21 \) single-mode waveguides for several disorder levels \( S_\eta \) using the laser direct-writing technology (for the fabrication parameters see, e.g., [12]). A central section from the microscope images of waveguide arrays are shown in Fig. 2 (left column) for different disorder levels. The fluctuations in waveguide positions along the \( \eta \) axis increase with growth of \( S_\eta \), whereas spacing along the \( \zeta \) axis remains constant. Due to the fabrication procedure the waveguides are elliptical with \( w_\eta < w_\zeta \), but the distances between their centers \( d_\eta < d_\zeta \) were also adjusted in such a way that in regular arrays the coupling constants in \( \eta \) and \( \zeta \) directions are approximately equal.

To measure the output intensity distributions the linearly polarized beam at the wavelength \( \lambda = 633 \text{ nm} \) was launched into the selected waveguide using a fiber laser and projected on a camera with 4× objective. The averaged output intensity distributions are obtained by exciting well-separated waveguides in the disordered arrays with different disorder levels \( S_\eta \). To collect a reliable statistics we excite 25 different waveguides, and upon averaging the centers of corresponding output distributions are adjusted in accordance with the position of excited waveguides. In all cases the excited waveguides were selected sufficiently far from the boundaries of arrays to
avoid surface effects [12]. The experimental patterns shown in Fig. 2 (middle column) demonstrate a transition from discrete diffraction to strongly localized distributions with increase of disorder level. Notice a comparable degree of localization in the vertical and horizontal directions. Whereas averaged intensity distributions are localized and feature exponential tails, the individual output distributions can be weakly or strongly localized (Fig. 3) depending on which eigenmodes are excited in each realization. Subsequent beam evolution is dictated by beatings between excited eigenmodes.

![Fig. 3](color online) Experimental output intensity distributions for excitation of different waveguides of disordered array with $S_\eta = 0.4$ resulting in the formation of weakly (left and center) and strongly localized (right) patterns.

In order to compare the degrees of localization of the averaged output intensity distributions along $\eta$ and $\zeta$ axes we calculated the slopes $\alpha_\eta, \alpha_\zeta$ (or exponential decay rates) of the inner linear parts of the $\ln[I_n(\eta, \zeta)]$ and $\ln[I_n(\eta = 0, \zeta)]$ dependencies. The theoretical and experimental exponential decay rates are shown as functions of $S_\eta$ in Figs. 4(a) and 4(b), respectively. In both experiment and theory we calculated $\alpha_\eta, \alpha_\zeta$ values starting from $S_\eta = 0.2$ (this is the minimum disorder level for which the appearance of an exponentially decaying inner part is ensured). The decay rates $\alpha_\eta, \alpha_\zeta$ are comparable for any disorder level, but the localization along $\eta$-direction - in which the disorder is acting - is always slightly better. Both $\alpha_\eta$ and $\alpha_\zeta$ increase monotonically with $S_\eta$.

Finally, we study the impact of nonlinearity on ACL. To this end, we calculate the inverse value of the averaged output form-factor $\chi^{-1} = \frac{1}{S_\eta} \sum_{A} \int |\eta|^2 d\eta d\zeta$, where $U = |\eta|^2 d\eta d\zeta$ is the total power at $S_\eta = 0.4$ for $n = 10^3$ array realizations as a function of input beam amplitude $A$. The quantity $\chi^{-1}$ provides the information about the width of the central “localized” portion of the output wavepacket and disregards small-amplitude diffracting waves. First, $\chi^{-1}$ slightly increases with increasing peak amplitude (i.e., initially focusing nonlinearity can even provide a small delocalization), but when $A$ reaches a certain threshold level the width of localized region rapidly decreases indicating that on the length of our sample the focusing nonlinearity stimulates localization [Fig. 4(c)]. For high peak amplitudes $\chi^{-1}$ slowly decreases, but in this case almost all light is localized in the excited guide and disorder is too weak to compete with nonlinearity.

Summarizing, we demonstrated that uncorrelated off-diagonal 1D disorder in 2D waveguide array results in Anderson cross-localization in both transverse directions. Such anisotropic off-diagonal disorder couples both transverse dimensions so that the localization along the direction in which the effective disorder is only very weak builds up remarkably fast.

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