Love type waves in a dry sandy layer lying over an isotropic elastic half-space with imperfect interface

Shikha Deep

1Department of Mathematics, School of Chemical Engineering and Physical Sciences, Lovely Professional University, Phagwara, Punjab, 144411, India

Email: shikhathakur493@gmail.com

*Corresponding author: vikassharma10a@yahoo.co.in, vikas.sharma@lpu.co.in

Abstract. The present problem is a precise approach for the investigation of Love type waves propagation in a new geometric scheme, which consist a dry sandy layer resting over an isotropic elastic half-space. An analytical approach has been used to derive secular equation for the propagation of Love type waves. Dispersion equation for Love type waves is derived in the given geometry by considering that upper dry sandy layer is imperfectly bonded to lower isotropic half-space. The effects of various parameters such as sandiness parameter, imperfect interface and thickness of the layer are presented on the propagation of Love type waves through graphical representations.

Keywords. Love wave, Dry Sandy medium, Isotopic medium, Imperfect interface.

1. Introduction

Earth is considered as one of the most complex structured bodies, as it contains various rocks and many materials in different strata of earth. The theoretical study of seismic waves is crucial, as it reveals interesting facts about the inner structure of earth and about the presence of various valuable materials, rocks deposits etc. Love type waves are surface waves which are horizontally polarised involving only one displacement component. These waves travel in layered structure involving finite thicker layer resting over a half-space. These waves are detected during earthquakes and underground explosions. Love waves are also used in long-range non-destructive testing techniques (Qingzeng et al. [1]).

Various layers of different materials inside earth are not perfectly bonded to each other and interfacial conditions prevailing between layers can affect the propagation of seismic waves. Many researchers have studied problems of wave propagation by using different interfacial conditions among layers such as Wolf [2] discussed about propagation of Love waves in irregular layered bodies. Hua et al. [3] investigated the propagation of Love waves in layered graded composite structures with imperfect interface and studied effects of imperfect interfacial conditions on the phase velocity of Love waves. Singh [4] studied dispersion of Love waves in corrugated layered media with irregular boundaries. They depicted the effects of irregular boundary surfaces on group and phase velocities of Love waves. Wang and Zhao [5] researched about the influence of interfacial defect and thickness ratio on propagation of Love waves in double layered piezoelectric/elastic composite plates. Sharma and Kumar [6] studied about propagation of SH waves in a viscoelastic layer, which is imperfectly attached to a couple stress substrate. They presented the effects of various parameters of the problem.
on phase and damping velocities of SH waves. Kumar et al. [7] investigated the propagation of shear waves in initially stressed piezoelectric layer which is imperfectly bonded with a micropolar half space. Aki and Larner [8] investigated elastic wave field in layered medium having an irregular interface due to incident plane SH waves.

Researchers have investigated about wave propagation in a layered media involving isotropic elastic medium such as Kumari et al. [9] investigated about impact of heterogeneity on the propagation of Love wave in layered isotropic structure. They presented the effects of width ratio on the phase velocity of Love waves. Kumar and Mandal [10] discussed about the generation of SH waves due to stress discontinuity in an isotropic half-space lying below an isotropic layer. Kundu et al. [11] studied about SH waves in an isotropic layer bounded between an initially stressed orthotropic layer and a heterogeneous elastic substrate. They showed the effects of inhomogeneity and initial stresses on propagation behaviour of SH waves. Dutta [12] discussed about propagation of Love type waves in non-homogeneous stratum which is bounded between two semi-infinite isotropic media by taking the different cases of rigidity and density. Singh et al. [13] studied the propagation behaviour of shear surface waves in a heterogeneous layer which cladded with piezoelectric stratum and an isotropic substrate. They presented the effects of width ratio of layers, piezoelectric constants, dielectric constants and heterogeneity parameter on the propagation of shear waves. Ejaz and Shams [14] investigated about the propagation of Love waves incompressible elastic materials with homogeneous initial stress and they considered that layer and compressible half-space are both initially stressed.

Earth cannot be treated as a homogeneous and isotropic medium. Soil can be considered as sandy in nature. Weiskopf [15] defined the mechanics of dry sandy layer and presented an appropriate relationship for dry sandy soil as \( \frac{E}{\mu D} = 2\eta(1 + \sigma) \), where \( \eta \) is termed as sandiness parameter and \( \eta \) is taken greater than one for dry sandy medium and \( \eta = 1 \) corresponds to the case of elastic medium, where \( \mu D \) is modulus of rigidity, \( E \) is Young’s modulus, and \( \sigma \) is Poisson’s ratio. Researchers have used dry sandy medium to solve problems of wave propagation such as Gupta and Ahmed [16] researched about propagation of Love waves in dry sandy layer bounded between fiber-reinforced layer and a pre-stressed porous half-space. They showed that sandiness parameter, thickness of layer, porosity, pre-stresses and reinforcement parameter affect the propagation of Love waves. Pal and Ghorai [17] presented the propagation behaviour of Love waves in a prestressed sandy layer resting over an anisotropic porous substrate under gravity and studied the effects of porosity and gravity on the behaviour of Love waves. Pal et al. [18] studied the propagation of surface waves in a sandy layer sandwiched between liquid saturated porous half-space and a liquid layer. They revealed the effects of thickness of layer and sandiness parameters on the propagation of surface waves. The propagation of Love waves in a viscoelastic sandy layer lying over an initially stressed orthotropic half-space was discussed by Pandit and Kundu [19] and they examined the effects of initial stress, viscoelastic parameter and sandiness parameter on the behaviour of Love waves. Tomar and Kaur [20] studied about reflection and transmission of SH-waves at a corrugated interface between dry sandy layer and an anisotropic half-space. Bayones [21] discussed the effects of gravity field and initial stresses on the propagation of Shear waves in anisotropic sandy medium.

This paper investigates about the behaviour of Love type waves propagating through a finite thicker dry sandy layer lying over an isotropic half-space. Both media are imperfectly bonded to each other at the interfacial surface. The geometrical model of the problem is taken to make it consistent with the actual reality of earth. Influence of various parameters involved in the problem is studied on the behaviour of Love type waves.

2. Formulation and solution of the problem
Consider Love waves propagation in a dry sandy layer lying over an isotropic half-space. It is assumed that two media are imperfectly bonded to each other. The origin of coordinate system \((x, y, z)\) is
assumed at the interface, $z$ -axis is pointing vertically downwards into the half-space and $x$ - axis is pointing along the propagation of wave as shown in Figure 1. For propagation of Love waves, it is assumed that displacement components for dry sandy layer and for isotropic half-space are independent of $y$-coordinate that is $\frac{\partial}{\partial y} \equiv 0$ and

$$u^D = w^D = 0, \quad u^I_1 = w^I_1 = 0, \quad v^D = v^D(x, z, t) \quad \text{and} \quad v^I_1 = v^I_1(x, z, t)$$

(1)

where $(u^D, v^D, w^D)$ are displacement components for dry sandy layer and $(u^I_1, v^I_1, w^I_1)$ are the displacement components for isotropic half-space.

![Figure 1. Schematic configuration of the problem.](image)

2.1. Dry sandy layer

The equation of motion for dry sandy layer is given as (Kar et al. [22])

$$\frac{\mu^D}{\eta} \left( \frac{\partial^2 u^D}{\partial x^2} + \frac{\partial^2 v^D}{\partial y^2} \right) = \rho^D \frac{\partial^2 v^D}{\partial t^2}$$

(2)

The stress - strain relationship for dry sandy medium is

$$\tau^D_{ij} = \lambda^D \Delta \delta_{ij} + 2 \frac{\mu^D}{\eta} e_{ij}$$

(3)

where $\lambda^D$ and $\mu^D$ are Lame’s constants, $\rho^D$ is density of material, $\eta$ is sandiness parameter, $\delta_{ij}$ is Kronecker’s delta, $\tau^D_{ij}$ are stress components and $e_{ij} = \frac{1}{2} \left( \frac{\partial u^D}{\partial x_j} + \frac{\partial u^D}{\partial x_i} \right)$ are components of strain tensor $\Delta = e_{11} + e_{22} + e_{33}$ is volumetric strain, where $i, j = 1, 2, 3$.

Using above mentioned conditions for the propagation of Love waves, the non-vanishing stress components are

$$\tau^D_{12} = \frac{\mu^D}{\eta} \frac{\partial u^D}{\partial x} = \tau^D_{21} \quad \text{and} \quad \tau^D_{23} = \frac{\mu^D}{\eta} \frac{\partial v^D}{\partial z} = \tau^D_{32}$$

(4)
Assume the solution of equation (2) as
\[ v^D(x, z, t) = g(z)e^{ik(x-ct)} \]  
(5)
where \( k \) is wave number and \( c \) is phase velocity.

Using equation (5) in equation (2), we obtain
\[ \frac{d^2g}{dx^2} + S^2k^2g(z) = 0 \]  
(6)
Solution of above differential equation becomes
\[ g(z) = A_1e^{ikSz} + A_2e^{-ikSz} \]  
(7)
Therefore, displacement component for dry sandy layer becomes
\[ v^D(x, z, t) = (A_1e^{ikSz} + A_2e^{-ikSz})e^{ik(x-ct)} \]  
(8)
where \( S = \sqrt{\frac{c^2\eta}{\beta^2} - 1} \), \( \beta^2 = \frac{\mu}{\rho\mu} \).

2.2. Isotropic elastic half-space
The governing equations of motion for homogeneous isotropic elastic medium are given as
\[ \frac{\partial \sigma_{11}^l}{\partial x} + \frac{\partial \sigma_{12}^l}{\partial y} + \frac{\partial \sigma_{13}^l}{\partial z} = \rho_1^l \frac{\partial^2 u_1^l}{\partial t^2} \]
\[ \frac{\partial \sigma_{21}^l}{\partial x} + \frac{\partial \sigma_{22}^l}{\partial y} + \frac{\partial \sigma_{23}^l}{\partial z} = \rho_1^l \frac{\partial^2 v_1^l}{\partial t^2} \]
\[ \frac{\partial \sigma_{31}^l}{\partial x} + \frac{\partial \sigma_{32}^l}{\partial y} + \frac{\partial \sigma_{33}^l}{\partial z} = \rho_1^l \frac{\partial^2 w_1^l}{\partial t^2} \]  
(9)
The stress - strain relationship for an isotropic medium is
\[ \sigma_{ij}^l = \lambda_1^l \Delta \delta_{ij} + 2\mu_1^l \epsilon_{ij} \]  
(10)
where \( \sigma_{ij}^l \) are stress components, \( \lambda_1^l \) and \( \mu_1^l \) are Lame’s constants, \( \rho_1^l \) is density of material, \( \epsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i^l}{\partial x_j} + \frac{\partial u_j^l}{\partial x_i} \right) \) are strain components, \( \Delta = \epsilon_{11} + \epsilon_{22} + \epsilon_{33} \) is volumetric strain and \( \delta_{ij} \) is Kronecker’s delta, where \( i, j = 1, 2, 3 \).

Using above mentioned conditions for the propagation of Love waves, the non-vanishing stress components are
\[ \sigma_{12}^l = \mu_1^l \frac{\partial v_1^l}{\partial x} = \sigma_{21}^l \text{ and } \sigma_{23}^l = \mu_1^l \frac{\partial v_1^l}{\partial z} = \sigma_{32}^l \]  
(11)
The only non-vanishing equation of motion from equation (9) is
\[ \frac{\partial^2 v_1^l}{\partial x^2} + \frac{\partial^2 v_1^l}{\partial z^2} = \frac{1}{\beta_1^l} \frac{\partial^2 v_1^l}{\partial t^2} \]  
(12)
Assume the solution of equation (12) as
\[ v_1^l(x, z, t) = h(z)e^{ik(x-ct)} \]  
(13)
where \( k \) is wave number and \( c \) is phase velocity.

Equation (12) becomes
\[ \frac{d^2 h}{dz^2} - P^2 k^2 h(z) = 0 \]  
(14)
Solution of above differential equation becomes

\[ h(z) = B_1 e^{kPz} + B_2 e^{-kPz} \]  \tag{15}

As \( z \to \infty \), \( h(z) \) must vanish, so to make this happen, we take \( B_1 = 0 \), hence solution given in equation (15) becomes

\[ h(z) = B_2 e^{-kPz} \]

Hence, the displacement component for isotropic elastic half-space is

\[ v_1(x, z, t) = B_2 e^{-kPz} e^{ik(x-ct)} \]  \tag{16}

where \( P = \sqrt{1 - c^2/\rho^2_1} \), \( \rho^2_1 = \mu_1/\rho_1 \).

3. Boundary conditions

(i) Top surface of the dry sandy layer must be free from stresses that is \( \tau_{23}^D = 0 \), at \( z = -H \).

(ii) Since interface between two media is imperfect in nature, so difference in displacement fields must be proportional to stress vector that is \( \tau_{23}^D = G(v_1^I - v^D) \), at \( z = 0 \), where \( G \) measures degree of imperfection at the interface.

(iii) The stresses must be continuous at the interface between dry sandy layer and an isotropic half-space that is \( \tau_{23}^D = \sigma_{23}^I \) at \( z = 0 \).

Using above stated boundary conditions, we get following equations

\[ (e^{-ikSH})A_1 - (e^{ikSH})A_2 = 0 \]  \tag{17}

\[ \left( \frac{i k \mu DS}{\eta} + G \right) A_1 + \left( -\frac{i k \mu DS}{\eta} + G \right) A_2 - (G)B_2 = 0 \]  \tag{18}

\[ \left( \frac{i \mu DS}{\eta} \right) A_1 + \left( -\frac{i \mu DS}{\eta} \right) A_2 + (\mu_1^I P)B_2 = 0 \]  \tag{19}

For non-trivial solution of equations (17), (18) and (19), determinant of coefficients of \( A_1 \), \( A_2 \) and \( B_2 \) must vanish,

\[ \begin{vmatrix} e^{-ikSH} & -e^{-ikSH} & 0 \\ \frac{i k \mu DS}{\eta} + G & -\frac{i k \mu DS}{\eta} + G & -G \\ \frac{i \mu DS}{\eta} & -\frac{i \mu DS}{\eta} & \mu_1^I P \end{vmatrix} = 0 \]  \tag{20}

By solving determinant, we get following dispersion equation for Love waves propagation.

\[ \tan \left( \frac{\omega}{c} \sqrt{\frac{\rho_1}{\beta_1} - 1} \right) = \frac{\mu_1^I}{\sqrt{1 - \frac{c^2}{\beta_1^2}}} \]  \tag{21}

where \( \omega = kc \), \( \omega \) is the angular frequency.

Subcase

If \( G \to \infty \) then the interfacial surface between dry sandy layer and isotropic half-space will be perfectly bonded and, in that case, dispersion equation (21) becomes
\[
\tan \left( \frac{\omega}{c} H \sqrt{\frac{\epsilon^2 \eta}{\beta^2} - 1} \right) = \frac{\mu_1^I \sqrt{1 - \frac{c^2}{\beta^2} \eta^2}}{\mu_1^D \sqrt{\frac{c^2}{\beta^2} \eta^2 - 1}}
\]

4. Numerical results and discussion

We are considering the following data to depict the results in the form of graphical illustrations

(i) Dry sandy layer (Gubbins [23])
\[\mu^D = 6.54 \times 10^{10} N/m^2, \rho^D = 3409 kg/m^3.\]

(ii) Isotropic half-space (Gubbins [23])
\[\mu_1^I = 7.10 \times 10^{10} N/m^2, \rho_1^I = 3321 kg/m^3\]

4.1. Effects of sandiness parameter

To demonstrate the effects of sandiness parameter on the phase velocity of Love wave, graphs are provided by taking five different values of sandiness parameter as \(\eta = 1\) (isotropic elastic material), 1.6, 2.1, 2.6, 3.1. Figure 2 shows the impact of sandiness parameter, when the interface between dry sandy layer and isotropic half-space is imperfect \((G = 30.5 \times 10^9)\) and the value of thickness of layer is \(H = 0.04m\). From the graph it is found that phase velocity is decreasing with the increase in the value of sandiness parameter. Figure 3 is again plotted for five different values of sandiness parameter as mentioned above, when both media are perfectly attached to each other \((G \rightarrow \infty)\) and thickness of layer is taken as \(H = 0.04m\). Trends found in figure 3 are similar to the trends seen in figure 2. From both the figures it can be concluded that sandiness parameter works against the phase velocity of Love waves.

![Figure 2](image-url)

Figure 2. Normalized phase velocity \((c/\beta)\) versus normalized wave number \((kH)\) for different values of sandiness parameter \((\eta)\) when two media are imperfectly attached.
4.2. Effects of imperfectness parameter

The role of degree of imperfectness parameter ($G$) on the propagation of Love waves is depicted by taking five different values of imperfectness parameter as $G = 30.5 \times 10^9, 10 \times 30.5 \times 10^9, 40 \times 30.5 \times 10^9, 100 \times 30.5 \times 10^9$ and $G \to \infty$. The value of thickness of layer is kept as $H = 0.04m$ and sandiness parameter is $\eta = 1.2$. From figure 4 it can be seen that phase velocity is increasing as the value of $G$ is increasing. As $G$ is increasing the bonding among layer and half space is getting better and as $G \to \infty$, dry sandy layer and isotropic half-space are perfectly bonded to each other. The phase velocity is highest when both media are perfectly bonded to each other. Figure 5 is again plotted for showing the results of imperfectness parameter on phase velocity by taking five different values of degree of imperfectness parameter as mentioned above, but herein the value of sandiness parameter is kept as $\eta = 1$, which corresponds that is layer is also isotropic in nature. Results found in figure 5 are similar to results found in figure 4.

4.3. Effects of thickness of layer

For observing the effects of thickness of layer on the propagation of Love waves, graphs are provided by taking five different values of thickness parameter as $H = 0.04m, 0.08m, 0.15m, 0.21m, 0.26m$. Figure 6 shows the impacts of thickness of layer when the interface between the dry sandy layer and isotropic half-space is imperfect ($G = 30.5 \times 10^9$). Here, the value of the sandiness parameter is taken as $\eta = 1.2$. It can be observed that phase velocity of Love waves is increasing as thickness of the layer is increasing.
Figure 4. Normalized phase velocity \((c/\beta)\) versus normalized wave number \((kh)\) for different values of \(G\), when sandiness parameter \(\eta = 1.2\).

Figure 5. Normalized phase velocity \((c/\beta)\) versus normalized wave number \((kh)\) for different values of \(G\), when sandiness parameter \(\eta = 1\) (dry sandy layer is isotropic).
Figure 6. Normalized phase velocity \((c/\beta)\) versus normalized wave number \((kH)\) for different values of thickness of layer \((H)\), when two media are perfectly attached

5. Conclusion

We have investigated the propagation of Love waves in dry sandy layer lying over an isotropic half-space. The layer is assumed to be imperfectly bonded with the half-space. Following conclusions are drawn from the present analysis.

1. The impacts of sandiness parameter \((\eta)\) of dry sandy layer are quite interesting and it is observed that the phase velocity is decreasing with the increase in the value of sandiness parameter. Similar results are found in both the cases, when two media are perfectly bonded or imperfectly bonded to each other.

2. The imperfectness parameter \((G)\) which measures the degree of imperfectness at the interface between two media affects the phase velocity of Love waves. It is seen that phase velocity is increasing as the value of \(G\) is increasing. Phase velocity is highest when two media are perfectly bonded to each other.

3. The impacts of thickness of layer are depicted on phase velocity and it is found that phase velocity increases as the value of the thickness parameter is increasing.

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