Quasiclassical approach to impurity effect on magnetooscillations in 2D metals.

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(January 2, 2022)

Abstract

We develop a quasiclassical method based on the path integral formalism, to study the influence of disorder on magnetooscillations of the density of states and conductivity. The treatment is appropriate for electron systems in the presence of a random potential with large correlation length or a random magnetic field, which are characteristic features of various 2D electronic systems presently studied in experiment. In particular, we study the system of composite fermions in the fractional quantum Hall effect device, which are coupled to the Chern–Simons field and subject to a long–range random potential.

PACS numbers: 72.15.Lh, 71.35.Hc, 03.65.Sq
I. INTRODUCTION.

In this article we develop a quasiclassical method for studying the influence of disorder on magnetooscillations in 2D electronic systems. Our original motivation was the problem of quantum particles in random magnetic fields which has attracted considerable interest during the last few years. Only quite recently realizations of a random magnetic field acting on a 2D electron gas have been prepared and investigated. One possible way of generating a random magnetic field is to use a type II disordered superconductor with randomly pinned flux lines in an external magnetic field as the substrate for the 2D electron gas [1]. Alternatively, one may use a magnetically active substrate such as a demagnetized ferromagnet with randomly oriented magnetic domains [2].

The main interest in the random magnetic field problem derives, however, from effective field theories of interacting electron systems, for which the interaction may be shown to be mediated by fictitious gauge fields. The first example of this class is the gauge field theory of high–$T_c$ superconductivity compounds [3–7]. There the gauge fields arise as a tool to implement the projection of the Hilbert space onto a subspace of states without double occupancy of lattice sites. These gauge fields are long–range correlated, with correlation function diverging as $1/q^2$ for wavevector $q \to 0$. This corresponds to a short–range correlated random magnetic field. The second example is provided by the composite fermion picture of the fractional quantum Hall effect [8–11]. There the electrons are represented by fermions with an even number of magnetic flux tubes attached. At half-filling of the lowest Landau band, the average field due to the flux tubes cancels the external magnetic field, leaving the problem of fermions moving in a random magnetic field generated by the flux tubes. The correlations of the corresponding vector potential are also long-ranged.

For a short-range correlated magnetic field, implying a long-range correlated vector potential, the total scattering rate of a charged quantum particle diverges in Born approximation [5,12]. This is due to the divergence of the differential scattering cross-section for forward scattering. The usual self-consistent treatment of strong scattering has been shown to be insufficient [13], as it violates gauge invariance. The dominance of small angle scattering calls for a quasiclassical description. We therefore employ the quasiclassical approximation method for the path integral representation of this problem. An additional advantage of the path integral formalism is the explicit gauge invariance, in contrast to the usual perturbation theory methods.

For the 2D GaAs – AlGaAs heterostructures on which the fractional quantum Hall effect (FQHE) [14] is observed, the quasiclassical treatment is appropriate for the following reasons. In these systems, the donors are located in a remote layer separated by a large distance $d_s \sim 50 \div 80\text{nm}$ from the electron gas plane. Thus, the random potential created by these impurities has a large correlation length $\sim d_s$, and the small–angle scattering dominates, which can be properly described within the quasiclassical approximation. In a strong magnetic field, such that the Landau level filling factor $\nu$ is close to $1/2$, a statistical transformation can be applied as mentioned above, converting the electrons into so-called composite fermions [8–10]. Then a random magnetic field appears, on top of the smooth random potential. The quasiclassical path integral approach allows to study the effect of both types of random field on equal footing.

The outline of the paper is as follows. In section 2 we use the path integral formalism
to calculate the total and the transport scattering rates in long–range correlated random potentials. We check that the obtained results are in agreement with the conventional perturbation theory. In section 3 we study the influence of a long–range random potential on the oscillations of the density of states and the conductivity in a magnetic field. In section 4 we generalize the results of the two preceding sections on the case of a random magnetic field (rather than random potential). Finally, in section 5 we apply the above formalism to the system of composite fermions in the FQHE device near \( \nu = 1/2 \), where both long–range random potential and random magnetic field are important. Section 6 contains a discussion of the results and conclusions.

Some of the results of this article have been published in the form of short communications \([13,15]\).

II. SCATTERING RATES IN LONG–RANGE RANDOM POTENTIAL FROM THE PATH INTEGRAL FORMALISM.

In this section we develop the quasiclassical path integral formalism and apply it to the calculation of total and transport scattering rates in a random potential with large correlation length. These results can also be obtained within conventional perturbation theory, so that this section has mainly methodological character. It is instructive, however, to see how the quasiclassical treatment reproduces results of the perturbation theory, in the case when both are applicable.

We consider a quantum particle of mass \( m \) and energy \( E \) moving in 2D in a static random potential \( U(r) \) with Gaussian distribution characterized by the correlation function

\[
\langle U(r)U(r') \rangle = W(|r - r'|) \tag{1}
\]

We assume \( W(|r|) \) to be a smooth function of \( r \) depending on the absolute value \( r = |r| \) only, with a characteristic length scale \( \xi \gg \lambda \), where \( \lambda = (2mE)^{-1/2} \) is the wavelength (we set \( \hbar = 1 \) throughout the paper). We review first results of the standard perturbation theory. The total scattering rate can be found in the Born approximation as

\[
1/\tau_s = 2\pi \int (dp_1) \tilde{W}(|p - p_1|) \delta(\epsilon_p - \epsilon_{p_1}) , \tag{2}
\]

where \((dp) = d^2p/(2\pi)^2; \ \epsilon_p = p^2/2m\) and

\[
\tilde{W}(p) = \int d^2r \ W(r) \exp(-ipr) \tag{3}
\]

Eq.(2) can be transformed as follows

\[
1/\tau_s = 2\pi N(E) \int_0^{2\pi} d\phi \frac{d\phi}{2\pi} \tilde{W}(2p_F \sin \frac{\phi}{2})
\]

\[
= 2\pi N(E) \int_0^{2\pi} d\phi \int_0^\infty dr J_0 \left(2p_F r \sin \frac{\phi}{2}\right) W(r)
\]

\[
= (2\pi)^2 N(E) \int_0^\infty dr J_0^2(p_F r) W(r) , \tag{4}
\]
where \( N(E) = m/2\pi \) is the density of states (DOS) on the Fermi surface and \( p_F = (2mE)^{1/2} \) is the Fermi momentum. The above assumption of the long–range character of \( W(r) \), \( \xi \gg \lambda \), allows to approximate the Bessel function in (3) by its asymptotic expression, yielding

\[
1/\tau_s = \frac{2}{v_F} \int_0^\infty dr W(r),
\]

with \( v_F = p_F/m \) being the Fermi velocity. This result holds under the condition of applicability of the Born approximation, which is

\[
v_F \tau_s \gg \xi
\]

Expression for the transport scattering rate \( 1/\tau_{tr} \) differs from that for \( 1/\tau_s \) by an additional factor \((1 - \cos \phi)\). We have thus, in full analogy with eq.(4),

\[
1/\tau_{tr} = \frac{2\pi^2}{p_F^2} N(E) \int dr J_0^2(p_F r) \left( \frac{d}{dr} \frac{d}{dr} W(r) \right).
\]

For a long range potential, \( \xi \gg \lambda \), we find thus

\[
1/\tau_{tr} = \frac{m}{p_F^3} \int_0^\infty dr \left[ W''(r) + \frac{W'(r)}{r} \right]
\]

In the last line we used the assumption that \( W(r) \) is an analytic function of \( r \) at \( r = 0 \), so that \( W'(0) = 0 \).

In the remaining part of this section we rederive eqs. (5), (9) in the path integral approach [16]. To calculate \( \tau_s \), we consider the (retarded) single–particle Green function

\[
G_R(0, R; T) = \int_{r(0)=0}^{r(T)=R} D\mathbf{r}(t) \exp \left\{ i \int_0^T dt \frac{m\dot{\mathbf{r}}^2}{2} - U(\mathbf{r}) \right\}
\]

After averaging over the disorder (which we will denote by angular brackets), we get

\[
\langle G_R(0, R; T) \rangle = \int_{r(0)=0}^{r(T)=R} D\mathbf{r}(t) \exp \left\{ i \int_0^T dt \frac{m\dot{\mathbf{r}}^2}{2} - \frac{1}{2} \int_0^T \int_0^T dt dt' W[\mathbf{r}(t) - \mathbf{r}(t')] \right\}
\]
In the absence of the second term in the exponent, eq.(11) describes the free particle Green function \( \langle G_R^{(0)}(0, R; T) \rangle \), which can be found exactly by the saddle-point method. The saddle point is given by the classical trajectory \( \mathbf{r}(t) = \mathbf{v} t; \mathbf{v} = \mathbf{R}/T \), which yields in 2D

\[
\langle G_R^{(0)}(0, R; T) \rangle = \frac{m}{2\pi i T} e^{imR^2/2T} \tag{12}
\]

We use now the fact that the correlator \( W(\mathbf{r}) \) is smooth, and thus the full expression (11) can still be evaluated quasiclassically. Furthermore, we assume the random potential to be relatively weak, so that the second term in the action in (11) can be considered as a perturbation. Formally, this means \( E\tau_s \gg 1 \). Then the effect of the random potential term is given by its value on the saddle-point trajectory. Assuming now \( v_F \tau_s \gg \xi \), as in eq.(7), we get

\[
\langle G_R(R; T) \rangle = \langle G_R^{(0)}(0, R; T) \rangle \exp \left( -T \int_0^\infty \frac{dr}{v_F} W(r) \right)
\]

\[
= \langle G_R^{(0)}(0, R; T) \rangle \exp \left( -T/2\tau_s \right), \tag{13}
\]

with \( \tau_s \) as in eq.(3). We find therefore that the single particle relaxation rate can be simply found by evaluating the random potential–induced term in the action on a classical trajectory.

As we will see now, calculation of the transport time in this formalism is much more elaborate. We start from the Kubo formula for the conductivity

\[
\sigma = -\frac{e^2}{4\pi} V \langle [\mathbf{v}_x G_R(E) - G_A(E)] \mathbf{v}_x [G_R(E) - G_A(E)] \rangle \tag{14}
\]

where \( V \) is the system volume, \( \mathbf{v}_x \) is velocity operator and \( G_R, G_A \) denote retarded and advanced Green functions respectively. As usual, the leading contribution is given by the terms \( \sim G_R G_A \), which can be rewritten in time representation as

\[
\sigma = \frac{e^2}{2\pi} \int_0^\infty dt_1 \int_0^\infty dt_2 \int d^2R \langle \mathbf{v}_x G_R(0, R; T_1) \mathbf{v}_x G_A(R, 0; -T_2) \rangle e^{iE(T_1 - T_2)} \tag{15}
\]

The product of the Green function can be expressed in terms of the path integral, in analogy with eq.(11) (we omit here the vertex velocity operators for simplicity and restore them in the end of calculation):

\[
\langle G_R(0, R; T_1)G_A(R, 0; -T_2) \rangle = \int_{\mathbf{r}_1(0) = 0}^{\mathbf{r}_1(T_1) = R} \exp \left\{ i \int_0^{T_1} dt \frac{m\mathbf{r}_1^2}{2} - i \int_0^{T_2} dt \frac{m\mathbf{r}_2^2}{2} \right\} 
\]

\[
-\frac{1}{2} \int_0^{T_1} \int_0^{T_1} dt \, dt' \, W(\mathbf{r}_1(t) - \mathbf{r}_1(t')) - \frac{1}{2} \int_0^{T_2} \int_0^{T_2} dt \, dt' \, W(\mathbf{r}_2(t) - \mathbf{r}_2(t')) + \int_0^{T_1} \int_0^{T_2} dt \, dt' \, W(\mathbf{r}_1(t) - \mathbf{r}_2(t')) \tag{16}
\]

One may attempt to evaluate eq.(16) by the same simple saddle-point method, which led us from eq.(11) to eq.(12). The saddle point trajectory for the action in (16) is
\[ r_1^c(t) = \frac{t}{T_1} R ; \quad r_2^c(t) = \frac{t}{T_2} R \]

However, this does not give the leading contribution to \( S_{\text{kin}} \). In mathematical terms, the failure of the usual saddle point approximation is related to the fact that for \( T_1 \) and \( T_2 \) close to each other (this condition is implied by the factor \( \exp\{iE(T_1 - T_2)\} \) in eq. \((13)\)) the impurity-induced terms in the action in \((13)\) nearly cancel each other on this trajectory. Moreover, this happens always, when \( r_1 \) and \( r_2 \) traverse the same trajectory in the same time, so that the vicinities of all such trajectories may be expected to be relevant. Physically speaking, this is because the conductivity is determined by the time scale in which the direction of velocity of the particle changes by an angle of order \( \pi \). Since in a long-range potential small–angle scattering is typical, this happens after many scattering events. The corresponding classical trajectory is smooth, but globally is typically very different from a straight line. These considerations suggest to study contributions from a class of paths \( r_1(t) \) and \( r_2(t) \), which fluctuate by a small amount about a common smooth, but otherwise arbitrary, trajectory \( \rho(t) \), traversed with a constant (by absolute value) velocity \( v_T \). Though these trajectories are not really saddle points of the action, they are “nearly saddle points” and can be expected to dominate the path integral. Thus, we represent the two paths \( r_1(t) \) and \( r_2(t) \) in \((16)\) in the form

\[
\begin{align*}
  r_1(t_1) &= \rho(t) + \frac{1}{2} r_-(t) ; \\
  r_2(t_2) &= \rho(t) - \frac{1}{2} r_-(t) ; \\
  t_1 &= t \frac{T_1}{t_+} = t + t_-/2 t_+ ; \\
  t_2 &= t \frac{T_2}{t_+} = t - t_-/2 t_+ ,
\end{align*}
\]

where we define \( t_+ = (T_1 + T_2)/2 \) and \( t_- = T_1 - T_2 \). The variable \( t \) in eq.\((18)\) is confined to the interval \([0, t_+]\). Note that the typical values of \( t_- \) and \( t_+ \) are expected to be \( t_- \sim 1/E \) and \( t_+ \sim \tau_r \), so that \( t_- \ll t_+ \). Now we expand the action \((16)\) in \( r_-(t) \). The leading contribution from the kinetic energy terms reads:

\[
S_{\text{kin}} = -i \int_0^{t_+ + t_-/2} dt_1 \frac{m}{2} \dot{r}_1^2 + i \int_0^{t_+ - t_-/2} dt_2 \frac{m}{2} \dot{r}_2^2 \\
= -i \frac{m t_+}{2} \int_0^{t_+} dt \dot{\rho}^2 + \delta S_{\text{kin}} ; \\
\delta S_{\text{kin}} \simeq -im \int_0^{t_+} dt \dot{\rho}(t) \dot{r}_-(t)
\]

For the disorder–induced terms in the action \((16)\) we use the Taylor expansion

\[
W(|\rho + \delta \bar{r}|) \simeq W(\rho) + \partial_i W(\rho) \delta r_i + \frac{1}{2} \partial_i \partial_j W(\rho) \delta r_i \delta r_j ; \\
\partial_i W(\rho) = W'(r) \frac{r_i}{r} ; \\
\partial_i \partial_j W(\rho) = W''(r) \frac{r_i}{r} \left( \delta_{ij} - \frac{r_i r_j}{r^2} \right) + W'''(r) \frac{r_i r_j}{r^2} ,
\]

with \( r = \rho(t) - \rho(t') \) and \( \delta r = \frac{1}{2}[\pm \bar{r}_-(t) \pm r_-(t')] \). Then the last three terms in \((16)\) combine to
\[ \delta S_W \simeq -\frac{1}{2} \int_0^{t+} \int_0^{t-} dt \, dt' \frac{W''[\rho(t) - \rho(t')]}{[\rho(t) - \rho(t')]} r_+^+(t) r_-^-(t') \]
\[ - \frac{1}{2} \int_0^{t+} \int_0^{t-} dt \, dt' W''[\rho(t) - \rho(t')] [r_+^+(t) r_-^-(t') \quad (21) \]

where we separated the fluctuations \( r_-(t) \) into the longitudinal \( r_-^L(t) \) and transverse \( r_-^+(t) \) parts with respect to the direction of velocity \( \dot{\rho}(t) \). As is expected on physical grounds and will be justified below, the relevant trajectories have nearly constant absolute value of velocity \( |\dot{\rho}(t)| = v \). We also expect, in view of eq. (19), the fluctuations \( r_-(t) \) to be varying slowly on the time scale \( \xi/v \). Then eq. (21) reduces to

\[ \delta S_w \simeq \frac{1}{v} w_0^+ \int_0^{t+} dt r_+^+(t) r_-^-(t) , \quad (22) \]

with \( w_0^+ = -\int_0^\infty \frac{W''(r)}{r} \, dr \) and assuming again \( \int W''(r) \, dr = 0 \). Finally, substituting (19), (22) into eq. (15), we get

\[ \sigma = \frac{e^2}{4\pi} \int_0^{t+} dt_+ \int_-^{t-} dt_- \int d^2 \rho \left[ \int D\rho(t) \int Dr_-^+(t) \int Dr_-^+(t) \dot{\rho}(0) \dot{\rho}(t_-) \right] \]
\[ \times \exp \left\{ -\frac{i m t_-}{2 \, t_+} \int_0^{t+} dt \dot{\rho}^2 - im \int_0^{t+} dt \dot{\rho}(t) \dot{\rho}_-(t) - \frac{1}{v} w_0^+ \int_0^{t+} dt r_-^+(t) r_-^-(t) \right\} \]
\[ \quad (23) \]

We integrate first over the longitudinal fluctuations \( r_-^L(t) \). This produces the \( \delta \)-function \( \Pi \delta(\dot{\rho}(t)) \), thus restricting the integral to the trajectories \( \rho(t) \) with constant velocity \( v = |\dot{\rho}(t)| \), as was expected. Further, the integral over \( t_- \) gives then \( \delta(E - mv^2/2) \), fixing the value of \( v \). Now we take the integral over \( r_-^L(t) \):

\[ \int Dr_-^L(t) \exp \left\{ \int_0^{t+} dt \dot{\rho}(t) \dot{\rho}_-(t) - \frac{1}{v} w_0^+ \int_0^{t+} dt r_-^+(t) r_-^-(t) \right\} \]
\[ \sim \exp \left\{ -\frac{m^2 v_F^2}{4 w_0^+} \int dt \dot{\rho}^2(t) \right\} \]
\[ = \exp \left\{ -\frac{m^2 v_F^2}{4 w_0^+} \int dt \dot{\phi}^2(t) \right\} , \quad (24) \]

where \( \phi \) is the polar angle of the velocity vector \( \mathbf{v} \). The action (24) corresponds to the Fokker–Planck (diffusion) process for the angle \( \phi \):

\[ \langle \dot{\phi}(t) \dot{\phi}(t') \rangle = \frac{2 w_0^+}{m^2 v_F^2} \delta(t - t') , \quad (25) \]

or, in discrete version,

\[ \langle \delta \phi^2 \rangle = \frac{2 w_0^+}{m^2 v_F^2} \delta t \]
\[ \quad (26) \]

The corresponding Fokker–Planck equation for the distribution function \( P(\phi, t) \) reads
\[
\frac{w_0^1}{m^2 v_F^3} \frac{\partial^2}{\partial \phi^2} P(\phi, t) = \frac{\partial}{\partial t} P(\phi, t)
\]

Thus,

\[
\sigma = e^2 N(E) \frac{v_F^2}{2} \int dt_+ \int d\phi \cos \phi P(\phi, t_+)
\]

where \(P(\phi, t)\) satisfies eq.(27) and the boundary condition \(P(\phi, 0) = \delta(\phi)\). To fix the normalization in eq.(28), we have exploited the condition of the particle number conservation:

\[
\int d^2 R \int_{-\infty}^{\infty} dt - e^{iEt} - \langle G_R(0, R; T_1)G_A(R, 0; -T_2) \rangle = 2\pi N(E).
\]

The solution of eq.(27) has the form

\[
P(\phi, t) = \sum_{m=-\infty}^{\infty} \exp \left\{ i m \phi - \frac{m^2 v_F^3}{t_0^1} t \right\}.
\]

We get therefore \(\sigma = e^2 N(E) v_F^2 \tau_{tr}/2\), with

\[
\frac{1}{\tau_{tr}} = \frac{w_0^1}{m^2 v_F^3},
\]

in precise agreement with eq.(31).

We have seen therefore how the quasiclassical treatment of impurity potential reproduces the results of perturbation theory. In the next section we apply the method to the situation when the perturbation theory breaks down.

**III. MAGNETOOSCILLATIONS IN THE PRESENCE OF LONG–RANGE RANDOM POTENTIAL.**

We consider a 2D gas of charged particles subject to a uniform magnetic field \(B\) and a smooth random potential \(U(\mathbf{r})\) defined by eq.(1). We will assume the impurity scattering to be relatively weak, so that \(\omega_c \tau_{tr} \gg 1\), where \(\omega_c = eB/mc\) is the cyclotron frequency. Our quasiclassical treatment will be valid for a random potential with correlation length

\[
\xi \gg l_B,
\]

where \(l_B = (c/eB)^{1/2}\) is the magnetic length. Eq.(32) is opposite to the condition of applicability of the self-consistent Born approximation \[17\] \[19\]. The de Haas–van Alphen oscillations (dHvAO) of the density of states (DOS) were studied in this regime in \[18\] by approximate summation of all orders of perturbation theory. We will demonstrate that this can be achieved in a more elegant way from the path integral formalism. We will also show that the Shubnikov–de Haas oscillations (SdHO) of conductivity can be described in this way, as well.

We consider first the DOS, which can be found from the single–particle Green function \[11\] as
\[ \rho(E) = -\frac{1}{\pi} \text{Im} G_R(E) ; \]

\[ G_R(E) = \int_0^\infty \langle G_R(0; T) \rangle e^{iET} dT \]  

(33)

In the quasiclassical approximation, the Green function \( G_R(E) \) can be represented as a sum over closed classical orbits [20]

\[ G_R(E) = \frac{m}{2} \left\{ \frac{1}{\pi} \ln(-E) - i \sum_\beta D_\beta \exp iS_\beta(E) \right\} , \]

(34)

where \( \beta \) labels the orbits, \( D_\beta \) is a factor originating from the path integration over the vicinity of the classical orbit \( \beta \), and \( S_\beta(E) \) is an action in the energy representation. In the absence of the random potential, we would have just a free particle in uniform magnetic field.

The classical trajectories are then the cyclotron orbits with radius \( R_c = v_F/\omega_c \). They can be classified by the winding number \( k \), specifying the number of times the orbit is traversed. Eq. (34) takes then the form

\[ G_R^{(0)}(E) = \frac{m}{2} \left[ \frac{1}{\pi} \ln(-E) - 2i \sum_{k=1}^\infty \exp \left\{ 2\pi k i \left[ \frac{E}{\omega_c} + \frac{1}{2} \right] \right\} \theta(E) \right] , \]

(35)

where \( \theta(E) \) is the step function. In particular, it is easy to check by using the Poisson resummation formula that (35) gives the correct expression for the DOS in terms of the sum over Landau levels:

\[ \rho(E) = \frac{1}{2\pi l_B^2} \sum_{N=0}^\infty \delta[E - \omega_c(N + 1/2)] \]

(36)

Since the impurity scattering is assumed to be weak, we neglect its influence on the classical trajectories and on the prefactor \( D_\beta \), in full analogy with calculation of \( \tau_s \) in Section 2. This gives, instead of (35),

\[ G_R(E) = \frac{m}{2} \left[ \frac{1}{\pi} \ln(-E) - 2i \sum_{k=1}^\infty \exp \left\{ 2\pi k i \left[ \frac{E}{\omega_c} + \frac{1}{2} \right] - S_W k^2 \right\} \theta(E) \right] , \]

(37)

where \( S_W \) is given by the second term in the action in (11) evaluated on a cyclotron orbit of winding number \( k = 1 \). It is easily found to be equal to

\[ S_W = \frac{1}{2v_F^2} \int dr \int dr' W(|r - r'|) \]

\[ = \frac{1}{2v_F^2} \int \langle dq \rangle \tilde{W}(q)[2\pi R_c J_0(qR_c)]^2 \]

\[ = \frac{\pi}{\omega_c^2} \int dq \langle \tilde{W}(q) J_0^2(qR_c) \rangle \]  

(38)

If the correlation length \( \xi \) of the random potential satisfies the condition \( \xi \ll R_c \), \( S_W \) takes the form
\[ S_W = \frac{1}{\omega_c v_F} \int_0^\infty dq \tilde{W}(q) = \frac{2\pi}{v_F \omega_c} \int_0^\infty dr W(r) = \frac{\pi}{\omega_c \tau_s}, \] 

(39)

with \( \tau_s \) as found in section 1, see eq.(5). We get then for the DOS at \( E > 0 \)

\[ \rho(E) = \frac{m}{2\pi} \left[ 1 + 2 \sum_{k=1}^\infty (-1)^k \cos \left( 2\pi k \frac{E}{\omega_c} \right) \exp \left( -k^2 \frac{\pi}{\omega_c \tau_s} \right) \right], \] 

(40)

or after resumming by the Poisson formula,

\[ \rho(E) = \frac{1}{2\pi l_B^2} \sum_{N=0}^\infty \sqrt{\frac{\tau_s}{\omega_c}} \exp \left\{ -\frac{\pi \tau_s}{\omega_c} (E - \omega_c (N + 1/2))^2 \right\} \] 

(41)

This formula shows that the Landau levels acquire a Gaussian form. They are well resolved if \( \omega_c \tau_s \gg 1 \). In the opposite case, \( \omega_c \tau_s \ll 1 \), the representation (40) is appropriate, where all harmonics except the first one can be omitted:

\[ \rho(E) = \frac{m}{2\pi} + \rho_{osc}(E); \]

\[ \rho_{osc} \approx -\frac{m}{\pi} \cos(2\pi E/\omega_c) e^{-\pi/\omega_c \tau_s} \] 

(42)

The dependence of the amplitude of oscillations on the magnetic field has the same form \( \sim \exp(-\pi/\omega_c \tau_s) \) as for the short range potential, so that if one plots \( \log \rho_{osc} \) versus \( 1/B \) (so-called Dingle plot [21]), one expects to get a linear behavior.

In the case of ultra-long-range potential with \( \xi \gg R_c \), we find \( S_W = 2\pi^2 W(0)/\omega_c^2 \). The Landau levels have again a Gaussian shape:

\[ \rho(E) = \frac{1}{2\pi l_B^2} \sum_{N=0}^\infty \sqrt{2\pi W(0)} \exp \left\{ -\frac{1}{2W(0)} [E - \omega_c (N + 1/2)]^2 \right\}, \] 

(43)

which in this case reflects their inhomogeneous broadening. The oscillating part of the DOS in eq.(12), \( \rho_{osc} \), is now equal to

\[ \rho_{osc} \approx -\frac{m}{\pi} \cos(2\pi E/\omega_c) e^{-2\pi^2 W(0)/\omega_c^2}, \] 

(44)

so that the Dingle plot is expected to be quadratic.

As we have already mentioned, the above results for the DOS oscillations in a long range potential were obtained in [13] by resummation of the perturbation theory expansion [22]. Besides being simpler and physically more transparent, the present derivation has the advantage that it can be straightforwardly generalized to SdHO of conductivity or to the case of random magnetic field.

The conductivity oscillations were recently considered in the framework of the path integral approach for the case of short–range random potential (\( \tau_{tr} = \tau_s \)) in [23,24]. The authors of these papers started from the Kubo formula (15), representing each Green function as a sum over classical trajectories in uniform magnetic field and taking scattering into account by including the factor \( \exp(-t/\tau_s) \). A trajectory is characterized by a number of cyclotron revolutions \( k \). Therefore, eq.(15) is reduced to a double sum over the winding numbers \( k_R, k_A \). The non-oscillating contribution to the conductivity corresponds then to
the terms with $k_R = k_A$, whereas the $n$-th harmonic of the oscillations is described by the terms with $|k_R - k_A| = n$. This holds for our problem of smooth random potential as well. The only difference is that the relevant trajectories are now not exactly cyclotron circles, but rather are very close to them, however drifting a little from one revolution to another. The typical total number of revolutions of each trajectory is of order of $\omega_c \tau_{tr} \gg 1$; in this time the shift of the center of cyclotron movement for a typical trajectory is of order $R_c$.

This is completely analogous to the consideration of conductivity in zero magnetic field in section 2, where the characteristic trajectories dominating the path integral were smooth (i.e. locally close to straight lines), but not really straight lines.

The non-oscillating contribution $\sigma_0$ to the conductivity is given by the trajectories for $G_R$ and $G_A$ having equal number of cyclotron revolutions and following closely each other within a distance $\ll \xi$, as in section 2. Then the impurity–induced terms in (16) cancel each other again to a great extent, leading to $\tau_{tr} \gg \tau_s$. The exact evaluation of the conductivity $\sigma_0$ is however much more complicated in the present situation, since the trajectories return to positions close to the preceding ones after 1, 2, … revolutions and may interact through the impurity correlator. This may lead to a deviation of $\tau_{tr}$ from its value calculated in zero magnetic field in section 2. We leave this problem aside in the present article and concentrate on the exponential damping of the oscillations by disorder.

The leading contribution to oscillations (the first harmonic) corresponds to the case when one of the trajectories has one extra cyclotron revolution as compared to another one. Then the contribution to the effective action in (16) from this part of the trajectory is not cancelled and leads to a suppression of the amplitude of oscillations. We get

$$\sigma = \sigma_0 + \sigma_1 \cos(2\pi E/\omega_c)e^{-S_W} + \ldots,$$

(45)

with $S_W$ given by eq.(38). Thus, impurity scattering leads to the same $e^{-S_W}$ exponential damping of the SdHO, as for the DOS oscillations. Depending on the relation between $\xi$ and $R_c$, this factor takes the form

$$e^{-S_W} = \begin{cases} 
    e^{-\pi/\omega_c \tau_s}, & \xi \ll R_c \\
    e^{-2\pi^2 W(0)/\omega_c^2}, & \xi \gg R_c
\end{cases}$$

(46)

as in eqs. (12), (14).

IV. CONDUCTIVITY AND MAGNETOOCSILLATIONS IN RANDOM MAGNETIC FIELD.

In this section we consider the oscillations of DOS and conductivity in the case when the carriers are subject to a random magnetic field rather than to a random potential. The properties of a quantum particle moving in a random magnetic field have been intensively discussed in the recent literature [12,25–35]. We are aware by now of three completely different physical systems, for which the random magnetic field problem is relevant. In the first class of system the random magnetic field is generated by a substrate on top of which the 2D electron gas is placed. Realizations of the substrate are a type II superconductor with randomly pinned flux lines [1], or a demagnetized ferromagnet with randomly oriented domains [2]. The second is the state with spin–charge separation of high–$T_c$ superconducting
materials \([3, 4]\). To describe it, one introduces a fictitious \(U(1)\) gauge field interacting with charge carriers. Here, the transverse (magnetic) component of the gauge field is the most important. The existence of gauge field fluctuations in high-\(T_c\) compounds can be inferred \([7]\) from experimental observations of unusual weak localization corrections to the magnetoconductance \([3]\). Finally, the third application concerns the FQHE system in the vicinity of \(\nu = 1/2\) filling factor of the Landau level. This will be studied in more detail in the next section.

We consider a Gaussian distributed random magnetic field \(h(\mathbf{r})\) (perpendicular to the 2D plane) with the correlator

\[
\left( e c \right)^2 \langle h(\mathbf{r}) h(\mathbf{r}') \rangle = \Gamma \delta^{(2)}(\mathbf{r} - \mathbf{r}')
\]

(47)

We will assume the weak disorder case, which means \(\Gamma \ll \hbar E_F\). Let us first reconsider the evaluation of scattering rates in section 2 for this type of disorder. For the one-particle Green function we have, instead of (11):

\[
\langle G_R(0, R; T) \rangle = \int_{r(0)=0}^{r(T)=R} \exp \left\{ i \int_0^T dt \frac{m \dot{\mathbf{r}}_1^2}{2} - i \int_0^T dt \frac{m \dot{\mathbf{r}}_2^2}{2} \right\}
\]

(48)

where \(a(\mathbf{r})\) is a vector potential corresponding to the magnetic field \(h(\mathbf{r})\). However, the correlator \(\langle a_i a_j \rangle\) is gauge-dependent, so that eq. (48) does not allow to define meaningfully the single particle relaxation time \(\tau_s\), as it was possible for the random potential case. We will discuss the problem of definiton of \(\tau_s\) below.

For the transport time, the problem of gauge invariance does not apply. The analogue of eq. (10) reads

\[
\langle G_R(0, R; T_1) G_A(0, 0; -T_2) \rangle
\]

\[
= \int_{r_1(0)=0}^{r_1(T_1)=R} DR_1(t) \int_{r_2(0)=0}^{r_2(T_2)=R} DR_2(t) \exp \left\{ i \int_0^{T_1} dt \frac{m \dot{\mathbf{r}}_1^2}{2} - i \int_0^{T_2} dt \frac{m \dot{\mathbf{r}}_2^2}{2} + \frac{1}{2} \left( e c \right)^2 \int_0^{T_1} dt \int_0^{T_1} dt' \dot{\mathbf{a}}_1(t) \dot{\mathbf{a}}_1(t') \langle a_i(\mathbf{r}_1(t)) a_j(\mathbf{r}_1(t')) \rangle \right. \\
- \frac{1}{2} \left( e c \right)^2 \int_0^{T_2} dt \int_0^{T_2} dt' \dot{\mathbf{a}}_2(t) \dot{\mathbf{a}}_2(t') \langle a_i(\mathbf{r}_2(t)) a_j(\mathbf{r}_2(t')) \rangle \right. \\
+ \left. \left( e c \right)^2 \int_0^{T_1} dt \int_0^{T_2} dt' \dot{\mathbf{a}}_1(t) \dot{\mathbf{a}}_2(t') \langle a_i(\mathbf{r}_1(t)) a_j(\mathbf{r}_2(t')) \rangle \right\}
\]

(49)

where we used the Stokes theorem to rewrite the effective action in the explicitly gauge-invariant form. Here \(s_{no}\) is so-called non-oriented, or Ampèrean, area inside the closed curve formed by the two trajectories \(\mathbf{r}_1(t)\) and \(\mathbf{r}_2(t)\) \([8]\). We recall, in particular, that the non-oriented area enclosed by \(k\) windings of a trajectory is the geometrical area multiplied by
We repeat now the derivation of $\tau_{tr}$ performed in section 2. For the disorder-induced part of the action we find, instead of (22),

$$\delta S_W = \frac{v \Gamma}{2} \int_0^{t_r} dt |r^{-}_-(t)|$$

Integral over the fluctuations $r_-(t)$, eq.(24), now has the following form:

$$\int Dr^\perp_-(t) \exp\{-im \int dt [\dot{v}]^\perp_+ r^\perp_- - \frac{\Gamma v}{2} \int dt |r_-|\}
= \prod_{\delta t_i} \int dr^\perp_-(i) \exp \left\{ \delta t_i \left( -im [v(t_i)]^\perp_+ r^\perp_-(i) - \frac{\Gamma v}{2} |r^-_-(i)| \right) \right\}
\prod_{\delta t_i} \frac{\Gamma v \delta t_i}{[(\Gamma v/2)\delta t_i]^2 + m^2[\delta v(t_i)]^2}$$

This implies that the scattering angle $\delta \phi \simeq |\delta v|^\perp / v$ obeys the Cauchy distribution

$$P(\delta \phi) = \frac{1}{\pi} \frac{(\Gamma/2m)\delta t}{(\Gamma/2m)^2(\delta t)^2 + (\delta \phi)^2}$$

The Boltzmann equation corresponding to eq.(52) is found to be

$$\frac{\partial P(\phi, t)}{\partial t} = \int d\phi' w(\phi - \phi') [P(\phi', t) - P(\phi, t)] ,$$

with

$$w(\phi) = \frac{1}{\pi} \frac{\Gamma}{2m} \frac{1}{\phi^2} , \quad \phi \ll \pi$$

This can be easily checked by solving eq.(53) by means of the Fourier transform in the $\phi$–space. The quasiclassical method describes correctly only the small angle scattering, so it is able to give an expression for the differential scattering rate $w(\phi)$ for $\phi \ll \pi$ only. The transport scattering rate $1/\tau_{tr} = \int d\phi w(\phi)(1 - \cos \phi)d\phi$ can be found in this way up to a numerical coefficient only: $1/\tau_{tr} \sim \Gamma/m$. The exact value can be found from the perturbation theory calculation, which gives [12]

$$w(\phi) = \frac{\Gamma}{8\pi m} \cot^2 \phi/2 ,$$

in full agreement with eq.(54). Consequently,

$$1/\tau_{tr} = \Gamma/4m$$

As we can see from eqs.(54), (55), the total scattering rate $1/\tau_s = \int d\phi w(\phi)$ diverges at $\phi \rightarrow 0$. This is related to the fact that the contribution $S_h$ to the action from the random magnetic field is proportional to the area, rather than to the length, of the trajectory. We will see below, when studying dHvAO and SdHO, that the cyclotron motion of the particle provides a natural regularization of this divergency. However, this regularization
is determined by the geometry of the experiment considered. Thus, the single particle relaxation time in the random magnetic field is dependent on the geometry of the problem.

We turn now to the consideration of magnetooscillations in the case when a uniform magnetic field $B$ is applied in addition to the random one. As in section 3, we will assume a relatively strong field, meaning $\omega_c \tau_{tr} \gg 1$, or according to eq.(56), $m \omega_c \gg \Gamma$. In full analogy with eq.(40), the DOS can then be written as

$$ρ(E) = \frac{m}{2\pi} \left[ 1 + 2\sum_{k=1}^{\infty} (-1)^k \cos \left( 2\pi k \frac{E}{\omega_c} \right) \exp \left( -k^2 S_h \right) \right],$$

(57)

with $S_h$ being the random magnetic field action on a simple (winding number 1) cyclotron orbit. For a $\delta$–like correlated magnetic field, eq.(47), it is equal to

$$S_h = \frac{1}{2} \Gamma s_{no} = \frac{1}{2} \Gamma \pi R_c^2 = \pi \Gamma / m \omega_c^2$$

(58)

Resumming eq.(57) with the help of the Poisson formula, we find

$$ρ(E) = \frac{1}{2\pi B} \sum_{N=0}^{\infty} \sqrt{\frac{m}{\Gamma E}} \exp \left\{ -\frac{\pi m}{\Gamma E} \left[ E - \omega_c (N + 1/2) \right]^2 \right\}$$

(59)

As in eq.(41), the Landau levels have a Gaussian shape. However, in contrast to the case of random potential, their width does not increase with the magnetic field, but is instead proportional to $E^{1/2}$. When the oscillations are relatively weak, they can be characterized by the first harmonic with an amplitude

$$ρ_{osc} \simeq -\frac{m}{\pi} \cos \left( \frac{2\pi E}{\omega_c} \right) e^{-\pi E \Gamma / m \omega_c^2}$$

(60)

These results for the amplitude of oscillations can be generalized to the SdHO of conductivity, as in the preceding section. We get again eq.(45), with $S_W$ replaced by $S_h$, eq.(58).

We can also consider a random magnetic field with finite correlation length $\xi$ and correlator

$$\langle h(r) h(r') \rangle = U(|r - r'|)$$

(61)

We find then

$$ρ_{osc}, \sigma_{osc} \propto e^{-S_h};$$

$$S_h = \frac{1}{2} \int_{|r| \leq R_c} d^2 r \int_{|r'| \leq R_c} d^2 r' U(|r - r'|) = \pi R_c^2 \int_0^\infty dq \tilde{U}(q) J_1(q R_c)$$

$$\simeq \begin{cases} \frac{\pi E}{m \omega_c^2}, & R_c \gg \xi \\ 2\pi^2 E^2 U(0), & R_c \ll \xi \end{cases}$$

(62)

where $\tilde{U}(q)$ is the Fourier transform of $U(r)$. Therefore, the Dingle plot will be quadratic for $R_c \gg \xi$ and quartic in the opposite case.
V. MAGNETOOSCILLATIONS NEAR THE $\nu = 1/2$ FILLING FACTOR OF THE LANDAU LEVEL.

In this section, we consider a realistic model describing the electron gas in GaAs – AlGaAs heterostructures where the FQHE is observed. The system is formed by the 2D electron gas of density $n_e$ and by the positively charged impurities located in a layer separated by a large distance $d_s$ from the electron plane. Each impurity creates a potential of the form

$$\int (dq) v_0(q) e^{i\mathbf{q}(\mathbf{r}_i - \mathbf{r})}; \quad v_0(q) = \frac{2\pi e^2}{\epsilon q} e^{-qd_s}, \quad (63)$$

where $\mathbf{r}_i$ is the projection of the impurity position to the 2D plane and $\epsilon$ is the dielectric constant. We briefly consider first the magnetooscillations in this system in low magnetic fields. The charge carriers are then characterized by their lattice mass $m_b$, which for GaAs is equal to $m_b \simeq 0.07m_e$, $m_e$ being the free electron mass. The potential $v_0(q)$, eq.(63), gets screened by the 2D electron gas into

$$v(q) = \frac{2\pi}{m_b} e^{-qd_s}. \quad (64)$$

When writing eq.(64), we assumed that the DOS determining the screening is practically constant: $\rho \approx m_b/2\pi$. Taking into account the magnetooscillations of the DOS here would lead to a non-linear screening, see [37]. However, since our primary interest is in the region of large $p$ where the amplitude of oscillations is small, we can neglect this non-linear effect. We also assume the impurity positions to be uncorrelated (see, however, the discussion in the next section). Then we find that the total random potential of the impurities is described by the correlator $W(r)$, see eq.(1), with the Fourier transform

$$\tilde{W}(q) = n_i \left(\frac{2\pi}{m_b}\right)^2 e^{-2qd_s}. \quad (65)$$

According to section 3, we find then for the amplitude of oscillations

$$\rho_{osc}, \sigma_{xx}^{osc} \propto -\cos \left(\frac{2\pi^2 n_e \epsilon}{eB} \right) e^{-S_W}, \quad (66)$$

with $S_W$ given by eq.(38). In particular, for $R_c \gg d_s$ we find

$$\rho_{osc}, \sigma_{xx}^{osc} \propto -\cos \left(\frac{2\pi^2 n_e \epsilon}{eB} \right) \exp(-\pi/\omega_c \tau_0);$$

$$\frac{1}{\tau_0} = \frac{1}{\pi v_F} \int dq \tilde{W}(q) = \frac{n_i}{m_b d_s} \left(\frac{2\pi}{n_e}\right)^{1/2}, \quad (67)$$

whereas for $R_c \ll d_s$

$$\rho_{osc}, \sigma_{xx}^{osc} \propto -\cos \left(\frac{2\pi^2 n_e \epsilon}{eB} \right) \exp \left[-\left(\frac{\pi}{\omega_c \tau_0^*}\right)^2\right];$$

$$\frac{1}{\tau_0^*} = \left[2W(0)\right]^{1/2} = \frac{1}{m_b d_s} (\pi n_i)^{1/2}. \quad (68)$$
For the systems under consideration the usual assumption is that concentrations of donors and charge carriers coincide: \( n_e = n_i \). Then the condition of weak oscillations \( \omega_c \tau_0 \ll 1 \) reduces to \( R_c \gg d_s \). Therefore, in the region of small oscillations (where our derivation is justified), eq.(67) has to be applied.

Now we consider the oscillations in strong magnetic field, near half filling of the Landau level: \( \nu = 2\pi cn_e/eb \simeq 1/2 \). It was observed experimentally that in this region the longitudinal resistivity shows oscillations with minima at \( \nu = p/(2p \pm 1) \), very much reminiscent to its behavior in low magnetic fields where conventional SdHO take place. This feature did not find an explanation within the original hierarchy theory of the FQHE \([38,39]\). To explain it, Jain \([8]\) proposed a concept based on converting the electrons into composite fermions by attaching to them an even number of flux quanta. A field-theoretical formalism based on the Jain’s idea was developed by Lopez and Fradkin \([9]\). In this approach, the statistical transformation of electrons into composite fermions is implemented by introducing a Chern–Simons (CS) gauge field interacting with electrons.

Following a similar approach Halperin, Lee and Read \([10]\) developed a theory for the half filled Landau level (see also \([11]\)). This theory gives an explanation for many experimentally observed properties of the \( \nu = 1/2 \) state, such as a non-zero value of the longitudinal resistivity, an anomaly in the surface acoustic wave propagation \([10]\), and a dimensional resonance of the composite fermions \([11]\). It predicts the formation, at half filling, of a metallic state with well defined Fermi surface. From this point of view, the \( \nu = p/(2p \pm 1) \) series can be considered as the usual \( \nu = p \) Shubnikov–de Haas oscillations (SDHO) for the composite fermions, providing an explanation for the prominence of the above FQHE states.

Details of the CS gauge field formalism can be found e.g. in \([9,10]\) and are not presented here. The statistical transformation attaches to each electron an even number \( \tilde{\phi} \) of flux quanta of the CS gauge field. To describe the vicinity of the \( \nu = 1/2 \) state, we take \( \tilde{\phi} = 2 \); the same formalism with \( \tilde{\phi} = 4 \) can be applied to the \( \nu = 1/4 \) state. In the mean field approximation, the statistical magnetic field \( B_{1/2} = 4\pi cn_e/e \) cancels exactly the externally applied field \( B \) at \( \nu = 1/2 \). When the filling factor \( \nu \) is tuned away from \( \nu = 1/2 \), the effective uniform magnetic field is equal to \( B_{\text{eff}} = B - B_{1/2} \). For \( \nu \) close to \( 1/2 \), the number of filled Landau levels of composite fermions \( p \gg 1 \), so that the problem can be considered quasiclassically.

Although a static impurity creates a scalar potential \((63)\) only, it acquires also a vector component due to screening by fermions and mixing with the CS field. In the random phase approximation one gets

\[
a_\mu = \left( \delta_\mu^\rho - U_{\mu\nu}K^{\nu\rho} \right)^{-1} a^{(0)}_\rho, \tag{69}
\]

where we united scalar \( a_0 \) and vector \( a \) potentials in a covariant vector \( a_\mu \); the vector \( a^{(0)}_\rho \) represents the bare impurity potential, eq.(63) and therefore has only \( \rho = 0 \) non-zero component. The tensors \( U_{\mu\nu} \) and \( K^{\nu\rho} \) represent the bare gauge field propagator and the current-density response tensor of the composite fermions, respectively.

To evaluate eq.(69) we use the Coulomb gauge \( \text{div}a = 0 \), go to the momentum space and choose the momentum \( q \) to be directed along the \( x \)-axis: \( q_x = q, q_y = 0 \). Then \( a_\mu \) has only 2 non-zero components corresponding to \( \mu = 0, y \), and both \( K \) and \( U \) become \( 2 \times 2 \) matrices \([10]\):
\[ K^{\mu\nu}(q) = \begin{pmatrix} -m^*/2\pi & -iq\sigma_{xy} \\ iq\sigma_{xy} & \chi q^2 - 2i\omega n_e/qk_F \end{pmatrix} \]
\[ U_{\mu\nu}(q) = \begin{pmatrix} v(q) & 2\pi i\tilde{\phi}/q \\ -2\pi i\tilde{\phi}/q & 0 \end{pmatrix} \]
\[ a^{(0)}_\mu(q) = \begin{pmatrix} v_0(q)e^{-iqr} \\ 0 \end{pmatrix} \]

(70)

where \( m^* \) is the effective mass of fermions, \( \chi = 1/24\pi m^* \) is the magnetic susceptibility, \( v(q) = 2\pi e^2/(\epsilon q) \) is the Coulomb propagator, and \( \sigma_{xy} \) is the Hall conductivity of composite fermions.

Substituting (70) in (69), we find
\[ a_\mu(q) = v_0(q)e^{-iqr} \left( \frac{v^*(q)}{2\pi m^*} + \left( \frac{\tilde{\phi} + 1}{i\tilde{\phi}m^*} \right) \right), \]

(71)

where \( s = 2\pi\sigma_{xy} \approx p \) in the limit \( \omega_c\tau \gg 1 \).

Depending on relations between parameters of the problem, one can find various regimes of behavior of the oscillations amplitude. We will concentrate on a regime which is the most relevant to the experiment. Let us compare the first and the second term in denominator of (71). As we will see below, the typical momenta are \( q \sim (2d_s)^{-1} \), and we get for \( \tilde{\phi} = 2 \)
\[ \frac{m^*v(q)/2\pi}{(2s)^2} = \frac{m^*e^2}{4eqs^2} \sim \frac{m^*e^2 k_F d_s}{ek_F 2p^2} \sim \frac{50}{p^2}, \]

(72)

where \( k_F = \sqrt{4\pi n_e} \), and we used typical experimental parameters \( n_e = 1.1 \cdot 10^{11}\text{cm}^{-2}, \)
\( d_s = 80\text{nm}, \) and the experimentally estimated value for the ratio \( m^*e^2/(ek_F) \sim 10 \). For the not too large \( p \) we are interested in, it is thus a reasonable approximation to neglect all but the first term in the denominator of (71). This gives
\[ a_\mu(q) = \frac{2\pi}{m^*} \tilde{\phi} e^{-iqr} e^{-q d_s} \left( \frac{p}{im^*} \right), \]

(73)

The random field action \( S_r \) (analogous to \( S_W \) in section 3 or \( S_h \) in section 4) is given by
\[ S_r = \frac{1}{2} \left\langle \left( \int a_\mu dr^\mu \right)^2 \right\rangle = \frac{1}{2} \left\langle \left( \int a_0 dt - \int a dr \right)^2 \right\rangle, \]

(74)

where the integration goes around a cyclotron orbit. Averaging over the impurity configurations, we find
\[ S_r = (2\pi\tilde{\phi})^2 n_i \]
\[ \times \int (dq)e^{-2qd_s} \left| \frac{p}{k_F} \int dl e^{-iqr} + \int d^2r e^{-iqr} \right|^2 \]
\[ = n_i (4\pi^2 \tilde{\phi} R_c)^2 \int (dq)e^{-2qd_s} \left| \frac{p}{k_F} J_0(qR_c) + \frac{1}{q} J_1(qR_c) \right|^2 \]

(75)

17
Here $\oint dl$ means integration along the cyclotron orbit and corresponds to the electric field contribution, whereas $\int d^2r$ goes over the area surrounded by the orbit and describes the magnetic field contribution. Taking into account that $R_c^2 = p^2/(\pi n_e)$, we have $R_c/2d_s = p/\sqrt{4\pi n_e d_s^2} \sim p/10 < 1$. Thus for relevant momenta $q \sim 1/(2d_s)$ and level numbers $p$, $qR_c \ll 1$ is a reasonable approximation. In this case eq.(75) reduces to

$$S_r = \pi^3 \phi^2 n_i \frac{R_c^4}{d_s^2} = \frac{n_e \pi \phi^2}{n_e n_i d_s^2} \left( \frac{2\pi n_e}{m^* \omega_c} \right)^4. \tag{76}$$

Note that electric and magnetic field fluctuations give equal contributions in this limit. According to sections 3,4, this gives for the oscillating part of the conductivity:

$$\sigma_{xx}^{osc} \propto -\cos \left( \frac{4\pi^2 n_e c}{eB_{eff}} \right) \exp \left[ -\left( \frac{\pi}{\omega_c \tau_1^*/2} \right)^4 \right] \tag{77}$$

where we introduced a parameter $\tau_1^*$ which is given according to eq.(76) by

$$\tau_1^* \simeq \frac{m^*}{2} \left( \frac{d_s^2}{4\pi n_i n_e^2} \right)^{1/4}. \tag{78}$$

Let us briefly consider now effect of finite temperature $T$. First of all, the SdHO are then suppressed by the usual factor $D_T = (2\pi^2 T/\omega_c)/\sinh(2\pi^2 T/\omega_c)$ originating from the Fermi distribution [43]. In addition, the fermions are scattered by the thermal fluctuations of the gauge field. The propagator of gauge field fluctuations is given by

$$D_{\mu\nu}(q, \omega) = U_{\mu\rho}(q) \left( \delta_{\rho\nu} - K^o\chi(q, \omega)U_{\chi\nu}(q) \right)^{-1}. \tag{79}$$

In particular for the $D_{11}$ component determining the magnetic field fluctuations, we get

$$D_{11}(q, \omega) = \frac{1}{2\pi m^*} \left[ \frac{1}{12} + \left( s + \frac{1}{\phi} \right)^2 \right] + \frac{\nu(q)}{(2\pi \phi)^2}. \tag{80}$$

In the quasistatic approximation we find

$$\langle A_1 A_1 \rangle_q = \int \frac{d\omega}{2\pi} \frac{2T}{\omega} \text{Im} D_{11} = \frac{T}{\chi q^2} \tag{81}$$

and consequently for the amplitude of magnetic field fluctuations

$$\langle hh \rangle_q = T/\tilde{\chi}. \tag{82}$$

If $\omega_c \tau_t \gg 1$, we have $s \simeq p$ and $\tilde{\chi} = 12p^2 \chi$. Therefore, the effective magnetic susceptibility $\tilde{\chi}$ much exceeds its bare value $\chi$, leading to a strong suppression of the fluctuations (81), (82). A similar suppression of the gauge field fluctuations in external magnetic field was found in [14] where the magnetoconductivity of the doped Mott insulators was studied. The contribution of the fluctuations (82) to the random field action $S_r$, eq.(74), is
\[ S_r^{(T)}(T) = \frac{T}{\chi} \pi R_c^2 \approx \frac{2\pi^2 T}{p \omega_c}, \]  

(83)
i.e. is small at \( p \gg 1 \) compared to the standard term \( -\ln D_T \) and decreases with \( p \).

The slope of the dependence of \( \ln \rho_{osc} \) on \( 1/B \) at \( T \approx \omega_c \) is conventionally used to extract a value of the effective mass \( m^* \) \([21]\). This procedure is based on the assumption that \( \rho_{osc} \propto D_T \), so that \( \ln \rho_{osc} = 2\pi^2 T m^*/eB + const. \) The additional attenuation factor \( \exp(-S_r^{(T)}) \), eq. (83), would then lead to a fictitious \( 1/p \) correction to the effective mass resulting in its apparent decrease with \( p \) at moderately large \( p \).

Let us compare our findings with available experimental results. In Fig.1 we present low-temperature experimental data for the amplitude of \( \rho_{osc} \) from \([12]\) \( (T = 0.19K, B_{\text{eff}} > 0) \). It is seen that they can be fitted well by \( \exp[-(\pi/\omega_c m^*)^4] \), whereas a conventional \( \exp(-\pi/\omega_c \tau) \) fit is much worse. Therefore the data apparently show the behavior \( \ln \rho_{osc} \propto 1/\omega_c^4 \) predicted by eq. (77). The value of the parameter \( \tau_{1/2}^* \) which is found from such a fit is \( \tau_{1/2}^* = 16 \cdot 10^{-12}s \) At the same time the theoretical estimate according to eq. (78) (with use of the parameters of \([12]\)) gives \( \tau_{1/2}^* \approx 2.4 \cdot 10^{-12}s \) if one uses the experimental value of the effective mass \( m^* = 0.7m_c \). A similar discrepancy is found for the low-field relaxation time: eq. (67) for \( m_b = 0.07m_e \) gives \( \tau_0^* \approx 0.6 \cdot 10^{-12}s \), whereas the value quoted in \([12]\) is \( \tau_0 = 9 \cdot 10^{-12}s \). We note also that the theoretically estimated values for the transport relaxation rate at \( \nu = 1/2 \) are typically 4 times greater than extracted from experimental mobilities \([10,42]\). Therefore the theory seems to overestimate relaxation rates systematically. This situation has been discussed previously \([13,40]\). The considerable increase of relaxation times was attributed to the correlations in positions of charged impurities due to their mutual Coulomb interaction \([40]\). We still encounter a problem, however, when trying to explain the above discrepancy in the value of \( \tau_{1/2}^* \) in this way. The correlations between impurities lead to a damping of the correlator \( \bar{W}(q) \), eq. (65) by a certain \( q \)-dependent factor. Since the mechanism of suppression of SdHO and the characteristic momenta are the same near \( B = 0 \) and \( \nu = 1/2 \), we expect a suppression of the action \( S_W \) in (66) and \( S_r \) in (73) by roughly the same factor. For the case of low fields this factor is \( \tau_0(\text{exp.})/\tau_0(\text{theor.}) \approx 15 \). At the same time, to reconcile the experimental data in the vicinity of \( \nu = 1/2 \) with eq. (78), we need this factor to be \( [\tau_{1/2}^*(\text{exp.})/\tau_{1/2}^*(\text{theor.})]^4 \approx 2300 \), i.e 150 times larger! It is not clear to us, what could be the source of such a drastic weakening of the random fields.

More recently, experimental data \([45]\) on SdHO near \( \nu = 1/2 \) on a better quality sample have been published. The obtained Dingle plot \([\text{Fig.3(a) of Ref. } [45]\]) is again highly nonlinear and can be well fitted by our formula (77). The fit yields in this case the value \( \tau_{1/2}^* = 9.5 \cdot 10^{-12}s \), whereas the theoretical estimate according to eq. (78) using the appropriate values of \( n_e, d_s \) and \( m^* \) gives \( \tau_{1/2}^* = 2.0 \cdot 10^{-12}s \). The discrepancy is somewhat smaller than for the sample from \([12]\) but still very large.

Now we discuss experimental data at higher temperatures. As a result of their analysis, an experimental value of the effective mass as a function of \( p \) was obtained in \([12,47,48]\). At moderately large \( p \) \( m^* \) was found to be slowly decreasing with \( p \) in agreement with our results. For larger \( p \), a sharp increase of \( m^* \) was observed in \([48]\), the origin of which is not clear to us. Let us note that the suggestion made in \([48]\) to explain the non-linearity of the Dingle plot by this variation of the effective mass is not supported by our results, since \( m^* \) drops out from eq. (77).
VI. CONCLUSIONS.

We have shown in this article that the path integral formalism in the quasiclassical approximation allows one to study the density of states and the conductivity of a system of charge carriers scattered by a long-range correlated random potential or a random magnetic field. We focussed attention on the oscillatory behavior of these quantities as a function of an applied magnetic field. For a random potential of correlation length $\xi$, we found the magnetooscillations in both quantities to be attenuated exponentially, the exponent being $\propto \omega_c^{-1}$ for $R_c \gg \xi$ and $\propto \omega_c^{-2}$ for $R_c \ll \xi$, where $\omega_c$ is the cyclotron frequency and $R_c$ the cyclotron radius in the external magnetic field. In the case of a random magnetic field of correlation length $\xi_B$, the magnetooscillations are again exponentially damped, this time with exponent $\propto \omega_c^{-2}$ for $R_c \gg \xi_B$ and $\propto \omega_c^{-4}$ for $R_c \ll \xi_B$. Our results show that the amplitude of magnetooscillations as a function of the magnetic field can be used to identify the scattering potential or random magnetic field.

Finally, we considered the FQHE system near $\nu = 1/2$ in the composite fermion picture. Here the Chern–Simons field of the flux tubes attached to every fermion gives rise to static random field fluctuations at impurity sites, where fermions may be trapped. For experimentally relevant values of input parameters, the damping of magnetooscillations is found to be proportional to $\exp\left[-(\pi/\omega_c \tau_\nu^{1/2})^4\right]$. This is in good agreement with experimentally obtained highly non-linear Dingle plots which can be very well fitted by a quartic dependence. However, the experimental value of the parameter $\tau_\nu^{1/2}$ is about 8 times greater than our theoretical estimate. This means that the random fields are much weaker than we expect them to be. Taking into account correlations between impurities seems not to resolve this discrepancy, as we discussed in the end of the preceding section. It is not clear to us at present whether this additional weakening of random fields can be explained within the composite fermions theory or else implies a certain inconsistency of this theory.

This work was initiated by the late Arkady Aronov, and was partially done in collaboration with him. We very much regret that he did not live to see the completion of the work. We are grateful to Yehoshua Levinson for useful discussions. This work was supported by the Alexander von Humboldt Stiftung (A.D.M.), SFB 195 der Deutschen Forschungsgemeinschaft (A.D.M. and P.W.) and by the German-Israel Foundation for Research (E.A.)
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FIGURES

FIG. 1. Dingle plot. Logarithm of normalized amplitude of resistivity oscillations \( \ln(D_T \rho^{osc}/4\rho) \), with \( D_T = \sinh(2\pi^2 T/\omega_c)/(2\pi^2 T/\omega_c) \), as a function of inverse effective magnetic field \( B_{eff}^{-1} \). Experimental data from [42] (squares) and [48] (circles) are presented, as well as their fits with eq.(77).
\[ \ln \left( \frac{D_T \rho_{\text{osc}}}{4\rho} \right) \]

fit \(-0.6 - 12.2 B^{-4}\)

\(-0.1 - 0.033 B^{-4}\)

- experimental data [48]
- experimental data [42]