Abductive and Consistency-Based Diagnosis Revisited: a Modeling Perspective

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Abstract
Diagnostic reasoning has been characterized logically as consistency-based reasoning or abductive reasoning. Previous analyses in the literature have shown, on the one hand, that choosing the (in general more restrictive) abductive definition may be appropriate or not depending on the content of the knowledge base (Console & Torasso 1991), and, on the other hand, that, depending on the choice of the definition the same knowledge should be expressed in different form (Poole 1994).

Since in Model-Based Diagnosis a major problem is finding the right way of abstracting the behavior of the system to be modeled, this paper discusses the relation between modeling, and in particular abstraction in the model, and the notion of diagnosis.

Introduction
Several characterizations have been given for Model-Based Diagnosis (Hamscher, Console, & J. de Kleer 1992). All approaches assume that a model of the system to be diagnosed is available: either a model of the correct behavior of the system, or a model of its abnormal behavior, or both.

Diagnostic reasoning has been characterized as a form of nonmonotonic reasoning: either as consistency-based reasoning, or abductive reasoning. In the first case a set of assumptions of correct behavior must be rejected in order to restore consistency with (abnormal) observations; in the second case, a set of assumptions of abnormal behavior must be introduced to entail the abnormal observations.

In (Console & Torasso 1991) the two definitions are shown to be two extremes of a spectrum whose intermediate points may also be relevant, depending on the assumptions about the completeness of the model. Poole also pointed out (Poole 1994) the importance of the representation problem for logic-based diagnosis, i.e. what has to be represented as the modeled system in order to use the different conceptualizations.

In spite of such previous work, several confusions remain in the field, for example, the confusion of declarative issues with computational issues, such as backward vs forward chaining along the model, and the lack of acknowledgement that in some significant cases the approaches are equivalent.

Moreover, in Model-Based Diagnosis a common view is that modeling is the problem; in particular, any model is an abstraction and the problem is in finding the right way of abstracting the behavior of the system to be modeled. This is a particularly significant issue since one of the claimed advantages of model-based systems is that they can rely on the same model of the system for different reasoning tasks, e.g. planning, diagnosis, configuration, reconfiguration after failure; but, unfortunately, which is the right abstraction, and then the right model, may depend on the task.

This paper, based on a general notion of prediction, illustrates, summarizing and complementing several views in the literature, how the appropriate notion of diagnosis and explanation depends on the predictiveness of the model. In particular, for deterministic models abduction and consistency-based explanation are equivalent, while for nondeterministic models, even if at first sight consistency seems to be the one providing the correct diagnoses, abduction can usually be adapted to provide the same correct diagnoses with a better (at least for someone) notion of explanation.

Basic definitions
In this paper we assume to rely on a component-based model of the system to be diagnosed; such a model describes the normal and/or abnormal behavior of the system in terms of the normal and/or abnormal behavior of its components. In the classical Reiter’s approach (Reiter 1987) there was no distinction between different abnormal behaviors, and no model or constraints for the abnormal behavior of a component; in later papers (de Kleer & Williams 1989, Struss & Dressler 1989) the concept of behavioral mode was introduced, and we similarly assume that:

- the system is composed of a set COMPS of components;
- each component has different, mutually exclusive, behavioral modes, typically one normal mode and several abnormal modes.
In the classical example of combinatorial circuits, a component could be an AND gate and one of its (abnormal) behavioral modes could be “stuck-at-0”.

In a logical representation of the model, the fact that AND gate \( a \) is in mode \( \text{stuck} - at - 0 \) would be represented with the atomic formula \( \text{stuck} - at - 0(a) \), which would occur in the formulae defining or constraining such behavioral mode, in this case \( \text{andgate}(a) \wedge \text{stuck} - at - 0(a) \rightarrow \text{output}(a, 0) \).

Here we do not assume that the model is represented in logic, it could also be, e.g., a set of qualitative equations. In this case the equation for the ok mode of the AND gate would be \( \text{out}(a) = \text{and}(\text{inp}(1)(a), \text{inp}(2)(a)) \), with an appropriate definition of the function \( \text{and} \), while the equation corresponding to the \( \text{stuck} - at - 0 \) mode would be \( \text{out}(a) = 0 \).

In any case we are interested in the notion of parameter in the model. In a model including AND gates, a parameter could be, e.g., the output of AND gate \( a \), which in logic would be the lambda expression \( \lambda x.\text{output}(a, x) \) while in an equational model would be the variable \( \text{out}(a) \). A subset of the parameters is the set of observable parameters. Each parameter has a domain of possible values; the parameter in the gate example has domain \{0, 1\} and in general, in the qualitative models commonly used in Model-Based Diagnosis, the domain would be finite. The granularity of such a domain is a major modeling choice, as, in general, is the choice of the appropriate qualitative abstraction; a first step in providing support for this is given in (Struss & Saftlenbacher 1993).

As mentioned above, we do not make unnecessary restrictions on the way the model is described and, therefore, on which is the basic inference mechanism. E.g. the model could be a set of logical formulae, with entailment as the inference mechanism, or a set of equations or (more generally) constraints on finite domains, in which case constraint propagation would be the inference mechanism. What the model is required to provide is a notion of prediction relating component behavior to observations as described in the following.

A diagnostic problem is characterized by a set of observations, i.e. an assignment of values to some or all the observable parameters. As in (Console & Torasso 1991), we distinguish between a set \( \text{CXT} \) of “contextual” observations (i.e. “inputs” to the system, e.g. to a circuit) and a set \( \text{OBS} \) of observations to be explained by a diagnosis.

A mode assignment is an assignment of one behavior mode to each component in \( \text{COMP} \).

We assume that the way the system is modeled provides a notion of prediction, i.e. states whether a mode assignment \( F \) predicts the set \( S \) of values for parameter \( p \) in context \( \text{CXT} \). This means that, in context \( \text{CXT} \) and given the assumptions \( F \) on the behavior of components, the model of the system implies that \( p \) takes one of the values in \( S \).

For example, in a logical framework, where the system is modeled in a set \( \text{MODEL} \) of logical formulae, for a finite set \( S = \{ v_1, \ldots, v_n \} \) of values, \( F \) predicts values \( S \) for \( p \) in context \( \text{CXT} \), if \( \text{MODEL} \cup F \cup \text{CXT} \models p(v_1) \lor \ldots \lor p(v_n) \) and the same condition does not hold for any \( S' \subset S \), since we are interested in the most specific prediction.

In an equational model, given the equations \( \text{MODEL} \) representing the system, and the equations \( F \) and \( \text{CXT} \) corresponding to the mode assignment and the context, the prediction for \( p \) will be the set \( S \) of values parameter \( p \) takes in the solutions of the system of equations \( \text{MODEL} \cup F \cup \text{CXT} \).

A particularly significant case is of course the one where \( S \) is a singleton, i.e. the model is able to predict an exact value for the parameter. We will refer to this case as a deterministic prediction. For example, any mode assignment that gives the “stuck-at-0” mode to the andgate \( A_1 \) predicts the value 0 for the output of \( A_1 \), while of course for the “ok” mode the prediction will depend on the context and on the mode of other components.

At the other extreme is the case of a fault making no prediction on a parameter \( p \), which we intend to coincide with the case where the prediction is the whole domain of the parameter.

An observation on a parameter \( p \) will, in general, be a set of values \( O \); in a precise observation (at least as precise as the domain granularity of \( p \)) such a set will be a singleton.

**Definition 1** Given the prediction \( S \) on parameter \( p \) of a mode assignment \( F \) in context \( \text{CXT} \) and the observation \( O \) for \( p \), we say that:

- the prediction is consistent with the observation if \( S \cap O \neq \emptyset \)
- the prediction implies the observation if \( S \subseteq O \)

Obviously, if prediction \( S \) implies \( O \) then it is consistent with it.

Moreover, in the particular case of a precise observation \( O = \{ v \} \), a consistent prediction must include \( O \), while to imply \( O \) a prediction must coincide with \( O \), i.e. predicting value \( v \) for \( p \).

In (Console & Torasso 1991), a spectrum of definitions of diagnoses is introduced, i.e. a definition with a parameter \( \text{OBS}^+ \subseteq \text{OBS} \) representing the subset of the observations to be explained abductively, while for all (other) observation consistency is required. In the terminology introduced above, such a definition can be reformulated as follows.

**Definition 2** Given diagnostic problem characterized by observations \( \text{CXT} \) and \( \text{OBS} \), and a subset \( \text{OBS}^+ \) of \( \text{OBS} \), a diagnosis is a mode assignment \( F \) such that

1. the predictions of \( F \) are consistent with \( \text{OBS} \)

2. Since, as noticed above, we consider no prediction as predicting the whole domain of \( p \), \( S \) cannot be empty.
2. the predictions of F imply observations OBS

If OBS
+ = ∅ this definition is consistency-based diagnosis, if OBS
+ = OBS it is abductive diagnosis (thus imposing, in general, a stronger requirement than consistency-based diagnosis, reducing therefore the set of diagnoses), and all intermediate choices are possible; in (Console & Torasso 1991) some guidelines are given for this choice, based on the completeness of the model, i.e. on the fact that all the possible explanations for the observations have been provided on the model; this corresponds to the idea of “anticipating explanations” in (Poole 1990), and will be referred here as backward completeness for the reasons that will be clear in the following.

Fully predictive models

In the literature there are results on conditions that make abductive and consistency based diagnoses coincide (Konolige 1992; Poole 1994), but such results are only formulated for logical representations where predictions are truth values (i.e. a prediction on p is either entailing p or entailing ¬p).

In the context of the previous section, we introduce the notion of a model whose prediction is deterministic on all observable parameters.

Definition 3 A model is fully predictive if for any context CXT, for any mode assignment F, for any observable parameter, F makes a deterministic prediction on p in context CXT.

Of course this condition can hardly be met if the domain of a parameter is the set of real numbers, but it is more sensible for e.g. the binary domain of combinatorial circuits and for qualitative abstractions of domains (moreover, in some cases a model can be sensibly expressed in a form that makes it fully predictive, as we will discuss in the next sections).

This condition corresponds to another notion of completeness of the model: a model where the consequences of assumptions can be given non-ambiguously, and, moreover, all the consequences of assumptions have been written down: therefore we can refer to it as forward completeness, as opposed to backward completeness mentioned above.

A trivial result is the following.

Property 1 For a fully predictive model, the choice of OBS
+ in definition 3 is irrelevant, i.e. abductive and consistency-based diagnosis coincide.

Proof. For a fully predictive model, a prediction on p is a singleton {v}. If it is consistent with the observation, such observation on p must be a set S including v (or the set {v} itself). Therefore the prediction implies the observation in p.

Nondeterministic models

Even with the underlying assumption that the system to be diagnosed behaves, at a macroscopic level, deterministically, a model cannot be assumed to be able to predict observations with infinite precision. This is the reason why we mentioned above that fully predictiveness can be too restrictive if the domain of observations are the real numbers.

Models are, usually, a convenient abstraction of a system, and qualitative abstractions of real values have emerged in AI as a meaningful and (sometimes) useful abstraction. Such an abstraction may have pragmatic value, allowing to represent a whole class of possible worlds at a time, and also some cognitive value, since humans (including, sometimes, scientists and engineers) are able to perform qualitative inferences or even tend to reason qualitatively, at least in some stage in the analysis of a system.

Our view is therefore that the system behaves deterministically; it is abstraction which makes the model nondeterministic. Sometimes this abstraction is due to lack of knowledge, sometimes it is just a modeling convenience. For example, we introduce the “flat” fault mode for a battery to include a whole range of values for its voltage, which would avoid predicting exactly the voltage of a flat battery (we will return to this example below).

The problem of abductive hypotheses being insufficiently predictive to entail observations has been discussed early in (Kautz 1991) in the framework of plan recognition, proposing as a simple example the fact that the plan of getting food does not explain why the agent, whose plan we are trying to recognize, is going to a specific supermarket: the plan will presumably only entail going to some supermarket. For this reason Kautz rejects the explicit use of abduction and relies on a form of closure to achieve the intended explanation. Such a closure is similar to the ones in (Console, Theseider Dupré, & Torasso 1991; Konolige 1992; Poole 1994) (and also explanatory closure in (Reiter 1991) which in fact provide results showing the equivalence of abdution to deduction (and consistency-based reasoning is a form of deduction) under an appropriate closure.

A different solution can be given based on the following idea in (Hobbs et al. 1993). Given an entity, e.g. lube oil, more specific than another, e.g. fluid, (so that lube_oil(x) → fluid(x)), if we want to explain fluid(a), but our assumption only entails fluid(a), we rely on transforming the implication

3I.e. the one of classical physics, rather than quantum physics.
4A faulty system can be intermittently faulty, or, more generally, its behavior could be time-varying. Here we do not consider the temporal dimension — see (Brusoni et al. 1998) for the different ways of taking it into account — and we mean that the system behaves deterministically in a time interval during which it is in the same behavioral mode.
5Dating back to 1987 as a Technical Report.
lube_oil(x) → fluid(x) into the equivalence fluid(x) ∨ etc(x) → lube_oil(x), where etc is assumable — corresponding to the assumption that the fluid happens to be lube oil. Assuming etc(a) allows entailing fluid(a).

Similar ideas are part of the representation methodology in (Poole 1990), in particular the idea of “anticipating explanations” which is also combined with the idea of “parametrizing assumptions”. The latter is used for example to represent the model of a flat battery with

\[
\text{battery}(B) \land \text{flat}(B,V) \rightarrow \text{voltage}(B,T)
\]

where flat is assumable, so that e.g. voltage(b1, 0.8), given battery(b1), is explained by flat(b1, 0.8) which is also consistent with the constraint. This achieves the result of being able to describe concisely a class of faults as well as reasoning, when necessary, on a specific instance of the class.

The parametrized assumption methodology has been adopted e.g. in (Ng & Mooney 1992) for the same reason, and in (Brusoni et al. 1998) for temporal constraints between the temporal extent of “causes” and “effects”, or, in general, explanans and explanandum. In that case, in fact, the same problem arises unless the temporal extent of the cause uniquely determines the one of the effect.

In general, this problem arises with any case where there are non-functional constraints between parameters of the explanans and those of the explanandum, i.e. the constraints do not allow to entail exact values for the latter even given exact values for the former.

In (Cordier 1998), however, the parametrizing assumptions methodology is rejected due to the potential proliferation of hypotheses, and a notion of explanation is introduced where “explaining” is not entailing, in particular, having a prediction which is an abstraction of the observation is sufficient. Further, a notion of conditional explanation is introduced where prediction and observation must have something that is more specific than both (e.g. A for A ∨ B and A ∨ C). In terms of predicting and observing values for parameters (as in the definitions above) this coincides with prediction and observation having an intersection, i.e. with consistency-based explanation.

### Qualitative explanation

In this section we discuss abductive and consistency-based explanations in the context of a representational abstraction we have used in recent years for modeling systems in the Vehicle Model-Based Diagnosis (VMBD) project: qualitative deviations (Malik & Struss 1996; Cascio et al. 1999; Theseider Dupré & Panati 1998).

The system is modeled in terms of differential equations that include appropriate parameters for components, whose values correspond to different (correct or faulty) behavior modes of the component. From these equations, corresponding equations for qualitative deviations are derived:

- for each variable x, its deviation \( \Delta x(t) \) is defined as \( \Delta x(t) = x(t) - x_{\text{ref}}(t) \), where \( x_{\text{ref}}(t) \) is a reference behavior (one choice is to consider the correct behavior of all the system as the reference behavior);
- from any equation \( A = B \), the corresponding equation \( \Delta A = \Delta B \) is derived;
- finally, the corresponding sign equation \( [\Delta A] = [\Delta B] \) is derived; it equates the signs of the two deviations. There are rules for expressing this equation in terms of signs of deviations of individual variables rather than expressions.

These models are useful to express concisely a number of dependencies. For example, for a tank containing a liquid, with an input flow \( in \) and an output flow \( out \), the equation

\[
\partial \Delta \text{level} = [\Delta \text{in}] \circ [\Delta \text{out}]
\]

where \( \partial \Delta \text{level} \) is the sign of the derivative of the deviation of level and \( \circ \) is subtraction in the sign algebra, expresses how the level of liquid deviates from the expected value in a wide (and exhaustive) range of cases. In particular, the reference behavior of the system need not be a stable state: even in such a behavior, flows and level may be continuously varying. What the model states is, for example, that:

- starting from the reference behavior, the level does not deviate from it, if the flows do not;
- if the outflow has no deviation and the inflow has a negative deviation, the level will start deviating negatively — this includes the cases where it decreases instead of being constant, decreases more than expected, increases less than expected, becomes constant instead of increasing, and decreases instead of increasing.

But what if the outflow also has a negative deviation? Subtracting a negative number from another can give a positive, negative or zero result. Again, the qualitative abstraction makes the model nondeterministic, at least in some case.

We do not discuss here an appropriate notion of diagnosis for dynamic systems (see (Theseider Dupré & Panati 1998)), nor a complete model for a system including the tank, but suppose that the inflow is due to a pump (so that a pump fault makes the inflow negatively deviated) and the outflow is regulated by a control system.

If the pump fault occurs, there is a negative deviation of the inflow, so a negative deviation of the level’s derivative, and then of the level; then (we omit the model) the control system reacts with a negative deviation of the outflow, and, given the qualitative model,\(^6\) due to the qualitative abstraction, the exact form of the quantitative equation is not necessary to build the qualitative model; it could however be necessary for fault detection.
any result is possible for the level deviation’s derivative. Therefore the fault will not entail any observation on the further trend of the level; this means that it would not be an abductive diagnosis for any observation, while it would be a consistency-based diagnosis for any observation. But would it be considered a good explanation?

We can do something better without abandoning the convenience of qualitative abstractions and using abduction. Suppose e.g. we observe that the level deviation’s derivative changes sign: the pump fault is consistent, but does not predict this; what is the least prescriptive assumption that predicts this? The assumption that $\Delta \text{out} < \Delta \text{in}$ (which for absolute values means $|\Delta \text{out}| > |\Delta \text{in}|$), so that $[\Delta \text{in}] \oplus [\Delta \text{out}]$ gives ‘+’. A natural language expression of this explanation is that the control system is compensating for the fault. Notice that the assumption $\Delta \text{out} < \Delta \text{in}$ is still a qualitative assumption and could correspond to a natural way of interpreting what is going on in the system.

Thus, in general, for all cases where the result of sign operators is ambiguous, we can introduce assumptions that make the result unambiguous, e.g. for the sum $[a] \oplus [b]$ where $a$ and $b$ have opposite signs, we can use the assumptions $[a] < [b]$, $[a] = [b]$, $[a] > [b]$ to predict values ‘-’, ‘0’, ‘+’ respectively.

Conclusions

In this paper we have reviewed several points of view in the literature on the relation of the notion of diagnosis with properties of the model of the system to be diagnosed, in particular, properties related to the abstractions in the model with respect to the real system.

We have pointed out that ideas that have appeared in the literature on this subject can be successfully integrated with modeling abstractions that have become widely used more recently.

The problem of identifying the right modeling abstraction for a task is far from being solved, but some work on this is being done in the model-based reasoning community (see e.g. [Brusoni et al. 1998] for the temporal dimension in diagnosis) and we expect in the near future more work in this direction, starting e.g. from the results in [Struss & Sachenbacher 1993].

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