Numerical Study for Vibration Analysis of Hole Flanged Rectangular Plate

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Abstract. Plates with flange are used in many engineering structures such as aircrafts, ships, and mechanical devices. In this paper, a numerical study for the vibration analysis of fully clamped rectangular Al-1100 alloy plates with different flange angles and different flange inner diameters was worked out. SolidWorks 2014 Simulation add-in was used to perform the analysis. It was found that increasing the flange angle and flange inner diameter increases the fundamental frequency and decreases the flange thickness. Adding the flange can increase the fundamental frequency up to (27%).

Keywords: Hole flanging, rectangular plate, fully clamped, natural frequency, numerical modeling

1. Introduction
Hole flanging is a widely used process in automotive industry. It is used to produce a rounded-lip in the sheet metal part that has a preformed hole as shown in Figure 1. In this process, the punch pushes the blank into the die causing a flange (lip) to be formed. These flanges are often formed to increase the bearing surface and strengthen the plate. Examples of practical applications of this process include locating bosses, providing additional length for tapped threads, assembling mating parts, provide openings in tubes for heat transfer, strengthen the airplane wings.

Figure 1. Conventional hole flanging process [1] (a) Before flange forming, (b) after flange forming

Researchers studied most of the hole flanging process parameters; Y. Huang & K. Chien [2] found that the punch profile affects the finished shape of the flange and the punch load. D. I. Hyun et al [3]
analyzed the fracture and lip shape to estimate the hole flangeability of high strength steel plates. A. Krichen et al [4] used the FE simulation to investigate the effect of the blank holding; they concluded that maximum punch load is not affected by the blank holding conditions, while the shape of the flange is affected by it. G. Centeno et al [5] found that the pre-cut hole has no influence on conical hole flanging produced by Single Point Incremental Forming. Y. Dewang et al [6] found in their brief review that most of the researchers used steel as the workpiece material, and for FEM modelling of the process, they found that the common conditions were assuming axisymmetric half and quarter geometry with quadrilateral meshing elements.

Free vibration of plates with opening had also been studied. M. K. Kwak and S. Han [7] proposed the Independent Coordinate Coupling Method to perform vibration analysis for rectangular plates with hole. A. Merneedi et al [8] found in their numerical study that the fundamental frequency is affected by number of holes and their position. G. Qing et al [9] developed a mathematical model for the free vibration of stiffened plates. D. Ou & C. Mak [10] introduced a finite element model to analyze the free vibration of stiffened rectangular plates; they found that the natural frequency is influenced by the stiffener location.

Most of the previous work regarding stiffened plates was limited to study plates with stiffeners that are arranged in longitudinal or orthogonal manner. In the present work, the flanged plate was considered to be used inside the plane wing structure. The initial hole in the plate helps in reducing the structure weight, while the flange was considered as a stiffener located along the circumference of the hole. The flange acts as a circular stiffener located along the circumference of the circular hole. For this purpose, we introduced a numerical study which uses SolidWorks 2014 Simulation add-in to perform Vibration analysis to study the frequency response of hole flanged plates. The effect of flanged angle and flanged inner diameter on the frequency response, which has not been investigated by other researchers, was considered here.

2. Theory
For a thin plate having constant thickness $h$, free harmonic vibration is defined by [11]:

$$\nabla^4 w + \frac{\rho h}{D} \frac{\partial^2 w}{\partial t^2} = 0$$

(1)

$$D = \frac{Eh^3}{12 (1 - \nu^2)}$$

For a fully clamped rectangular thin plate of uniform thickness, solving equation (1) will give the natural frequency formula as [12]:

$$f_{ij} = \frac{\lambda_{ij}^2 h}{2 \pi a^2} \sqrt{\frac{E}{12 \rho (1 - \nu^2)}} ; \ i = 1,2,3 \ldots ; j = 1,2,3 \ldots$$

(2)

$f$ = Natural frequency (Hertz)

$\lambda_{ij}^2$ = dimensionless frequency parameter

$I_{ij}$ = number of half-waves in mode shape along horizontal and vertical axes respectively

$h$ = Plate Thickness (mm)

$a$ = Plate side (mm)

$E$ = Young’s Modulus (N/mm²)

$\rho$ = Mass Density (Kg/m³)

$\nu$ = Poisson’s Ratio

$D$ = flexural rigidity (N.m²)

A, B = Constants

$\omega$ = natural frequency (rad/sec)

For a rectangular plate containing a hole located at its center, the non-dimensional eigenvalue problem expressed as [7]
\[ [\bar{K}_{cr} - \bar{\omega}^2 \bar{M}_{cr}] A = 0 \]  

(3)

\( \bar{K}, \bar{M} \) represent the non-dimensional stiffness and mass matrices. Hegarty and Armin [13] showed that the plate frequency depends on hole size and Poisson’s ratio. Increasing Poisson’s ratio increases the plate lateral strain energy which in turn increases the frequency due to increasing the total potential energy. Therefore, we interpolated their results for Poisson’s ratio equal to 0.33 and obtained the curve shown in figure 2 which can be fitted to equation (4) that represents the frequency variation with hole ratio for the clamped rectangular plate.

\[ \lambda^2 = 1686.3 X^3 - 262.26 X^2 - 6.6752 X + 221.46 \]  

(4)

Where X represents the hole size ratio (hole diameter/plate length).

For a rectangular plate with rectangular stiffener, the natural frequency formula is [14],

\[ \Omega = \sqrt{\frac{\rho h \omega^2 a^4}{D}} \]  

(5)

3. Numerical Modeling

The rectangular plate was modeled as a planar surface with side length \( a = b = 200 \text{mm} \) and thickness of 1mm. Plate material was AL1100 (defined as custom material with Young’s Modulus=69 GPa, Poisson’s Ratio=0.33, Density=2700 Kg/m\(^3\)). All four sides of the plate were fixed as shown in Figure 3a. A frequency convergence study for the first mode of vibration was conducted and it was decided to mesh the plate with solid mesh type, element size = 1.3mm which results in No. of nodes = 95481 and No. of elements = 47432. To study the effect of hole diameter size, five plates were modeled with (10, 20, 30, 40, and 50mm) hole located at the center of the plate (Figure 3b). To analyze the effect of the flange angle on the frequency response, plates with 40mm initial hole were used to model plate with 50mm flange diameter at different flange angles (10 to 90 degree) as shown in Figure 3b. To analyze the effect of the flange inner diameter, plates with 40mm initial hole were used to model plates with different flange diameters (54, 58, 60, 62, 66, and 70mm).
4. Results and discussion

4.1. Plate without hole

The first six mode shapes resulting from the simulation are shown in figure 4. To validate SolidWorks Simulation code, the first six frequencies were calculated using equation (2) and are listed in Table 1.

| Mode No. | $\lambda^2 [12]$ | Theoretical Frequency (Hz) | Simulated Frequency (Hz) | % Error |
|----------|------------------|----------------------------|--------------------------|---------|
| $f_{11}$ | 35.99            | 221.376                    | 221.339                  | 0.017   |
| $f_{21}$ | 73.41            | 451.733                    | 451.414                  | 0.071   |
| $f_{12}$ | 73.41            | 451.733                    | 451.416                  | 0.070   |
| $f_{22}$ | 108.3            | 666.158                    | 665.561                  | 0.09    |
| $f_{31}$ | 131.6            | 809.478                    | 809.250                  | 0.028   |
| $f_{13}$ | 132.2            | 813.168                    | 813.090                  | 0.01    |
It can be noted from Table 1 that the error of simulation is less than 1% which is within the acceptable engineering range. These results confirm and validate the accuracy of the computational code used by SolidWorks Simulation add-in and provide the basis to use this code for further simulation on complicated structures such as plate with stiffened flange.

4.2. Rectangular plate with hole

For different hole diameter to plate side length ratio, the theoretical frequency results when using the interpolated equation (4) and the simulated frequency are listed in table 2.

| Diameter (mm) | 2R/a | $\lambda^2$ | Theoretical Frequency (Hz) | Simulated Frequency (Hz) | % Error |
|---------------|------|-------------|----------------------------|--------------------------|---------|
| 10            | 0.05 | 35.88       | 220.699                    | 220.727                  | 0.013   |
| 20            | 0.10 | 35.74       | 219.838                    | 219.804                  | 0.016   |
| 30            | 0.15 | 35.81       | 220.269                    | 220.200                  | 0.031   |
| 40            | 0.20 | 36.27       | 223.098                    | 223.210                  | 0.050   |
| 50            | 0.25 | 37.35       | 229.742                    | 229.724                  | 0.008   |

$R =$ hole radius (mm)

When the hole diameter is small (10mm & 20mm) compared with plate length, it will act as a crack. The surface area of the crack relieves the lateral strain energy and therefore the fundamental frequency will decrease. When the hole diameter increases more, the effect of plate mass become more dominant than the strain relief. Therefor increasing the hole diameter will decreases the mass of the plate and this causes the frequency to increase.

4.3. Plate with variable flange angle

A plate with initial hole diameter of 40mm was used to obtain a flange with 50mm inner diameter. The results of the first six natural frequencies when changing the flange angle are listed in table 3. It can be noted that increasing the flange angle increases the fundamental frequency. Increasing the flange angle increases the damage in the flange bent and this results in increasing the lateral strain energy which in turn increases the fundamental frequency. The first six mode shapes of flange angle equal to 60° are shown in figure 5.

| Diameter (mm) | 2R/a | $\lambda^2$ | Theoretical Frequency (Hz) | Simulated Frequency (Hz) | % Error |
|---------------|------|-------------|----------------------------|--------------------------|---------|
| 10            | 0.05 | 35.88       | 220.699                    | 220.727                  | 0.013   |
| 20            | 0.10 | 35.74       | 219.838                    | 219.804                  | 0.016   |
| 30            | 0.15 | 35.81       | 220.269                    | 220.200                  | 0.031   |
| 40            | 0.20 | 36.27       | 223.098                    | 223.210                  | 0.050   |
| 50            | 0.25 | 37.35       | 229.742                    | 229.724                  | 0.008   |

4.4. Plate with variable flange diameter

A plate with initial hole diameter of 40mm was used to obtain flanges with different inner diameter at different flange angles. It was noticed that increasing flange diameter decreased the flange thickness as shown in figure 6. The flange acts as a stiffener; therefore, increasing the flange inner diameter increases its height and this increases the flexural rigidity of the plate which in turn increases the fundamental frequency of the plate as shown in figure 7.

| Diameter (mm) | 2R/a | $\lambda^2$ | Theoretical Frequency (Hz) | Simulated Frequency (Hz) | % Error |
|---------------|------|-------------|----------------------------|--------------------------|---------|
| 10            | 0.05 | 35.88       | 220.699                    | 220.727                  | 0.013   |
| 20            | 0.10 | 35.74       | 219.838                    | 219.804                  | 0.016   |
| 30            | 0.15 | 35.81       | 220.269                    | 220.200                  | 0.031   |
| 40            | 0.20 | 36.27       | 223.098                    | 223.210                  | 0.050   |
| 50            | 0.25 | 37.35       | 229.742                    | 229.724                  | 0.008   |

$R =$ hole radius (mm)

Table 2. Theoretical and simulated Frequency results for fully clamped plate (with hole)

Table 3. Frequency results for fully clamped rectangular plate at different flange angle (Hole diameter = 40mm, flange inner diameter 50mm)
Figure 5. Natural frequencies of the fully clamped plate with flange (hole diameter = 40mm, flange diameter = 50mm, flange angle = 60°)

Figure 6. Variation of flange thickness with flange angle at different flange inner diameter

Figure 7. Variation of the first natural frequency with flange angle at different flange inner diameter
It was observed that the variation in frequency is quite small in the range (50 to 80 degree) for different flange inner diameter. The fundamental frequency was increased by (17% to 27%) for flange inner diameter (50 to 70mm) respectively. So it can be said that when strengthening the airplane wings by adding a rib that contains a flanged hole, this flange can be produced within the range (50 to 80 degree) to maintain the fundamental frequency at its upper limits.

5. Conclusions

- Increasing the flange angle and flange inner diameter increases the fundamental frequency.
- Increasing the flange angle and flange inner diameter decreases the flange thickness.
- Frequency for flange angle within the range (50 to 80 degree) can be considered as constant.
- SolidWorks Simulation add-in can be used effectively to simulate complicated structures such as plate with stiffened flange.

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