Distributed call-by-value machines

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Abstract

We present a new abstract machine, called DCESH, which describes the execution of higher-order programs running in distributed architectures. DCESH implements a generalised form of Remote Procedure Call that supports calling higher-order functions across node boundaries, without sending actual code. Our starting point is a variant of the SECD machine that we call the CES machine, which implements reduction for untyped call-by-value PCF. We successively add the features that we need for distributed execution and show the correctness of each addition. First we add heaps, forming the CESH machine, which provides features necessary for more efficient execution, and show that there is a bisimulation between the CES and the CESH machine. Then we construct a two-level operational semantics, where the high level is a network of communicating machines, and the low level is given by local machine transitions. Using these networks, we arrive at our final system, the distributed CESH machine (DCESH). We show that there is a bisimulation relation also between the CESH machine and the DCESH machine. All the technical results have been formalised and proved correct in Agda, and a prototype compiler has been developed.
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1 Seamless computing

Suppose we need to program a system in which the function \( F \) runs on node \( A \) in a distributed system, for instance because \( F \) depends on a local resource residing on node \( A \). Suppose further that we need to write a program \( G \), running on node \( B \), that uses \( F \). How to achieve this depends on what programming language or library for distributed computing we choose. One of the most prominent ways to do it is using message passing, for instance with the Message-Passing Interface [1]. This involves writing \( F \) and \( G \) as separate processes, and explicitly constructing messages that are sent between them.

Suppose now that our specification changes: A part \( F' \) of \( F \) actually needs to run on a third node \( C \). Using conventional languages or libraries, this means that we have to rewrite big parts of the program since a substantial part of it deals with the architecture-specific details of the problem. Languages with support for Remote Procedure Calls [2] can help mitigate this, since such a call has the same syntax as a local procedure call, but will not work if \( F' \) is a higher-order function that is invoked with a function as its argument. In previous papers [3, 4] we suggest the following alternative way to express the two programs above:

\[
\begin{align*}
\text{let } F &= \{ \ldots \ F' \ldots \} @ A \text{ in } \{ G \} @ B \\
\text{let } F &= \{ \ldots \ (F') @ C \ldots \} @ A \text{ in } \{ G \} @ B
\end{align*}
\]

Here we write the whole program as if it was running on a single computer, and use pragma-like annotations, written \( \{ x \} @ A \), to indicate the node of execution. We call such annotations locus specifiers. The compiler uses the annotations to automatically handle architecture-specific details like communication. We call this seamless computing. A key feature is full support for higher-order functions, even across node boundaries, without sending actual code (in contrast to e.g. Remote Evaluation [5]). This is important for full generality, since it is not always the case that all code is meaningful on all nodes (for example because of resource locality or platform differences).

Our previous work enables writing these programs but uses an execution model based on game semantics that is vastly different from conventional compilation techniques. In this paper we instead develop an approach which is a conservative extension of existing abstract machines. This means that the vast literature on compiler optimisation more readily applies, and makes it possible to interface with legacy code. The key idea in this work, like in our previous work, is that computational phenomena like function calls can be subsumed by simple communication protocols. We assume that a run-time infrastructure can handle system-level aspects associated with distribution such as failure, load balancing, global reset, and so on.

Technical outline To achieve the goal of an abstract machine for seamless computing, we make gradual refinements to a machine, based on Landin’s SECD machine [6], that we call the CES machine (Sec. 2). The first change is to add heaps (Sec. 3.1) for dynamically allocating closures, forming the CESH machine (Sec. 3).
which provides features necessary for more efficient execution, and we show the CES and CESH machines to be bisimilar (Sec. 3.2). We then add communication primitives (synchronous and asynchronous) by defining a general form of networks of nodes that run an instance of an underlying abstract machine (Sec. 4). Using these networks, we illustrate the idea of subsuming function calls by communication protocols by constructing a degenerate distributed machine, DCESH₁ (Sec. 5), that decomposes some machine instructions into message passing, but only runs on one node. Finally, the main contribution is the fully distributed CESH machine (DCESH, Sec. 6), which is shown to be bisimilar to the CESH machine (Sec. 6.1).

**Formalisation in Agda** The theorems that we present in this paper have been proved correct in Agda [7], an interactive proof assistant and programming language based on intuitionistic type theory. The definitions and proofs in this paper are intricate and often consist of many cases, so carrying them out manually would be error-prone and arduous. Agda has been a helpful tool in producing these proofs, and also allows us to easily play with alternative definitions (even wrong ones). To eliminate another source of error, we do not adopt the usual practice of writing up the results in informal mathematics; in fact, the paper is built from a collection of literate Agda source files and the code blocks come directly from the formalisation. Although our work is not about Agda per se, we believe that this presentation is beneficial also to you, the reader, since you can trust that the propositions do not contain mistakes. Since Agda builds on a constructive foundation, it also means that the formalisation of an abstract machine in Agda can act as a verified prototype implementation.

**Syntax and notation for code** We assume a certain familiarity with the syntax of Agda, but since it is close to that of several popular functional programming languages we believe that this will not cause much difficulty for the audience. We will use ⋆ for the type of types. We will use implicit parameters, written e.g. \( f : \{ A : \star \} \to ... \) which means that \( f \) takes, as its first argument, a type \( A \) that does not need to be explicitly spelled out when it can be inferred from other arguments. We will sometimes use the same name for constructors of different types, and rely on context for disambiguation. Constructors will be written in bold face and keywords underlined. We make liberal use of Agda’s ability to define mixfix operators like if0_\text{then}_\text{else}_, which is a constructor that accepts arguments in the positions of the underscores, as in if0 \( b \) then t else \( f \).

This paper is organised as follows, where the arrows denote dependence, the lines with ~ symbols bisimulations, and the parenthesised numerals section numbers:
2 The CES machine

Our goal is to make a compiler for a programming language with locus specifiers that is based on conventional compilation techniques. A very common technique is the usage of abstract machines to describe the evaluation at a level low enough to be used as a basis for compilation. The starting point for our work is based on a variation of Landin’s well-studied SECD machine [6] called Modern SECD [8]. Modern SECD itself can be traced back to the SECD machine of Henderson [9], in that both use bytecode for the control component of the machine (and so use explicit return instructions); and to the CEK machine of Felleisen [10], in that they both place the continuations that originally resided in the dump (the D component) directly on the stack (the S component), simplifying the machine configurations.

We choose to call this variation the CES machine because of its three configuration constituents. This machine is important for us since it will be used as the specification for the elaborated machines that we later construct. We will show that their termination and divergence behaviour is the same as that of CES by constructing bisimulation relations.

A CES configuration (Config) is a tuple consisting of a fragment of code (Code), an environment (Env), and a stack (Stack). Evaluation begins with an empty stack and environment, and then follows a stack discipline. Sub-terms push their result on the stack so that their super-terms can consume them. When (and if) the evaluation terminates, the program’s result is the sole stack element.

Source language We show how to compile untyped call-by-value PCF [11]. The source language has constructors for lambda abstractions (λ t), applications (t $ t’), and variables (var n). Our representation uses De Bruijn indices [12], so a variable is simply a natural number.

```
data Term : * where
  λ_ : Term → Term
  _$_ : (t’ : Term) → Term
  var : N → Term
```

We also have natural number literals, binary operations on them, and conditionals:

```
tit : N → Term
op : (f : N → N → N) (t t’ : Term) → Term
```
if0_then_else_ : (b t f : Term) → Term

The language can be thought of as an intermediate representation for a compiler which may expose a more sugary front-end language. Because it is untyped, we can express fixed-point combinators without adding additional constructors.

We define the bytecode, Code, that the machine will operate on. A fragment of Code is a list of instructions, Instr, terminated by END, RET, or a conditional COND which has code fragments for its two branches:

```
mutual
data Instr : ★ where
  VAR  : N    → Instr
  CLOS : Code → Instr
  APPL : Instr
  LIT  : N    → Instr
  OP   : (N → N → N) → Instr

data Code : ★ where
  _;_  : Instr → Code → Code
  COND : Code → Code → Code
  END  : Code
  RET  : Code
```

The main work of compilation is done by the function compile', which takes a term t to be compiled and a fragment of code c that is placed after the instructions that the compilation emits.

```
compile' : Term → Code → Code
compile' (λ t) c = CLOS (compile' t RET); c
compile' (t $ t') c = compile' t (compile' t' (APPL ; c))
compile' (var x) c = VAR x; c
compile' (lit n) c = LIT n; c
compile' (op f t t') c = compile' t' (compile' t (OP f ; c))
compile' (if0 b then t else f) c =
  compile' b (COND (compile' t c) (compile' f c))
```

It should be apparent that the instructions correspond closely to the constructs of the source language but are sequentialised. Compilation of a term is simply a call to compile', terminated by END:

```
compile : Term → Code
compile t = compile' t END
```

**Example 2.1 (codeExample).** The term $(\lambda x. x) (\lambda x y. x)$ is compiled as follows:

```
compile ((\lambda var 0) $ (\lambda (\lambda var 1))) =
  CLOS (VAR 0 ; RET);
  CLOS (CLOS (VAR 1 ; RET) ; RET) ; APPL ; END
```

Compilation first emits two CLOS instructions containing the code of the function and its argument. The APPL instruction is then used to perform the actual application.
We mutually define values, closures and environments. A closure is a code fragment paired with an environment. A value is either a natural number literal or a closure. Since we are working in a call-by-value setting an environment is a list of values.

```
mutual
  Closure = Code × Env
  data Value : * where
    nat : N → Value
    clos : Closure → Value
  Env = List Value
```

A stack is a list of stack elements, defined to be either values or continuations (represented by closures):

```
data StackElem : * where
  val : Value → StackElem
  cont : Closure → StackElem
  Stack = List StackElem
```

A configuration is, as stated, a tuple consisting of a code fragment, an environment and a stack:

```
Config = Code × Env × Stack
```

Fig. 1 shows the definition of the transition relation for configurations of the CES machine. The Agda syntax may require some further explanation: The instructions’ constructor names are overloaded to also act as constructors for the relation; their usage will be disambiguated by context. We use implicit arguments, written in curly braces, for arguments that can automatically be inferred and do not need to be spelled out explicitly. The type of propositional equality is written \_≡\_.

The stack discipline becomes apparent in the definition of the transition relation. When e.g. VAR is executed, the CES machine looks up the value of the variable in the environment and pushes it on the stack. A somewhat subtle part of the relation is the interplay between the APPL instruction and the RET instruction. When performing an application, two values are required on the stack, one of which has to be a closure. The machine enters the closure, adding the value to the environment, and pushes a return continuation on the stack. Looking at the compile function, we see that the code inside a closure will be terminated by a RET instruction, so once the machine has finished executing the closure (and thus produced a value on the stack), that value is returned to the continuation.

**Example 2.2.** We trace the execution of codeExample defined above, which exemplifies how returning from an application works. Here we write a \(_\xrightarrow{\text{CES}}_{\text{x}} b\) meaning that the machine uses rule \(x\) to transition from \(a\) to \(b\).

```
let \(c_1\) = VAR 0 ; RET
\(c_2\) = CLOS (VAR 1 ; RET) ; RET
\(c_{11}\) = val (clos \((c_1,[]))\); \(c_{12}\) = val (clos \((c_2,[]))\)
```
data _ → _ : Rel Config Config where

VAR : ∀{n c e s v} → lookup n e ≡ just v → (VAR n; c, e, s) → (c, e, val v :: s)
CLOS : ∀{c' c e s} → (CLOS c'; c, e, s) → (c, e, val (clos c', e) :: s)
APPL : ∀{c e v c' e'} → (APPL ; c, e, val v :: val (clos c', e') :: s) → (c' :: e' :: cont (c, e) :: s)
RET : ∀{e v c e'} → (RET, e, val v :: cont (c, e') :: s) → (c, e', val v :: s)
LIT : ∀{n c e s} → (LIT n; c, e, s) → (c, e, val (nat n) :: s)
OP : ∀{f c e n1 n2 s} → (OP f ; c, e, val (nat n1) :: val (nat n2) :: s) → (c, e, val (nat (f n1 n2)) :: s)
COND-0 : ∀{c c' e s} → (COND c c', e, val (nat 0) :: s) → (c, e, s)
COND-1+n : ∀{c c' e n s} → (COND c c', e, val (nat (1 + n)) :: s) → (c', e, s)

Figure 1: The definition of the transition relation of the CES machine.
The final result is therefore the second closure, cl₂.

Lemma 2.3 (determinismCES). \( \Rightarrow \) is deterministic. In the formalisation this means that we can construct the following term:

\[
\text{determinism}_{\text{CES}} : \quad \Rightarrow \quad \text{is-deterministic}
\]

where the type _is-deterministic_ is defined as follows:

\[
\text{is-deterministic} : \quad \{ \text{A B} : \star \} \rightarrow \text{Rel} \; \text{A B} \rightarrow \star
\]

\[
\text{R is-deterministic} = \quad \forall \{ a b b' \} \rightarrow \text{R} \; a \; b \rightarrow \text{R} \; a \; b' \rightarrow b \equiv b'
\]

This is a key property that is useful when constructing a compiler implementation of the CES machine. Note that we write the name of the definition containing this proof in parentheses.

The CES machine terminates with a value \( v \), written \( \text{cfg} \downarrow_{\text{CES}} \) \( v \) if it, through the reflexive transitive closure of \( \Rightarrow \), reaches the end of its code fragment with an empty environment, and \( v \) as its sole stack element.

\[
\text{cfg} \downarrow_{\text{CES}} \quad \Rightarrow \quad \text{cfg} \downarrow_{\text{CES}} \; v = \text{cfg} \; \Rightarrow^* \; (\text{END},[],\text{val} \; v :: [])
\]

where the reflexive transitive closure of a relation is defined as follows:

\[
\text{data} \quad _* : \quad \{ \text{A} : \star \} \rightarrow \text{Rel} \; \text{A A} \rightarrow \star
\]

\[
\text{where}\quad \text{[ ]} : \quad (R^*) \; a \; a
\]

\[
\text{_::_} : \quad \{ \text{b c} : \text{A} \} \rightarrow \text{R} \; a \; b \rightarrow (R^*) \; b \; c \rightarrow (R^*) \; a \; c
\]

It terminates, written \( \text{cfg} \downarrow_{\text{CES}} \) if there exists a value \( v \) such that it terminates with the value \( v \). The Agda syntax for the existential quantifier normally written as \( \exists x. P(x) \) is \( \exists \lambda x \rightarrow P x \). Using this syntax, the definition of termination with value \( v \) is:

\[
\text{cfg} \downarrow_{\text{CES}} \; = \exists \lambda v \rightarrow \text{cfg} \; \downarrow_{\text{CES}} \; v
\]

It diverges, written \( \text{cfg} \uparrow_{\text{CES}} \) if it is possible to take another step from any configuration reachable from the reflexive transitive closure of \( \Rightarrow \).
where
\[
\uparrow : \{ A : \star \} (R : \text{Rel \, A \, A}) \rightarrow A \rightarrow \star \\
\uparrow R a = \forall b \rightarrow (R^*) a b \rightarrow \exists \lambda c \rightarrow R b c
\]

3 CESH: A heap machine

In a compiler implementation of the CES machine targeting a low-level language, closures have to be dynamically allocated in a heap. However, the CES machine does not make this dynamic allocation explicit. In this section, we try to make it explicit by constructing the CESH machine, which is a CES machine with an extra heap component in its configuration.

While heaps are not strictly necessary for a presentation of the CES machine, they are of great importance to us. The distributed machine that we will later define needs heaps for persistent storage of data, and the CESH machine forms an intermediate step between that and the CES machine. Another thing that can be done with heaps is to implement general recursion, without using fix-point combinators, by constructing circular closures.

A CESH configuration is defined as \( \text{Config} = \text{Code} \times \text{Env} \times \text{Stack} \times \text{Heap} \times \text{Closure} \), where Heap is a type constructor for heaps parameterised by the type of its content. Closures, values and environments are again mutually defined. Now a closure value is simply represented by a pointer:

\[
\text{ClosPtr} = \text{Ptr} \\
\text{mutual} \\
\text{Closure} = \text{Code} \times \text{Env} \\
\text{data} \text{Value} : \star \text{ where} \\
\text{nat} : \mathbb{N} \rightarrow \text{Value} \\
\text{clos} : \text{ClosPtr} \rightarrow \text{Value} \\
\text{Env} = \text{List} \text{Value}
\]

We leave the stack as in the CES machine (even though we could, in principle, change the continuations, currently represented by closures, to pointers as well – we do not do this since it is not necessary for our purposes).

\[
\text{data} \text{StackElem} : \star \text{ where} \\
\text{val} : \text{Value} \rightarrow \text{StackElem} \\
\text{cont} : \text{Closure} \rightarrow \text{StackElem} \\
\text{Stack} = \text{List} \text{StackElem}
\]

Fig. 2 shows the definition of the transition relation of the CESH machine. It is largely the same as that of the CES machine, but with the added heap component.
data \(-\rightarrow\) CESH : Rel Config Config where

CLOS : \(\forall \{c' c e s h\} \rightarrow \textbf{let} \ (h', \text{ptr}_{\text{cl}}) = h \triangleright (c', e) \textbf{in} (\text{CLOS} \ c'; c, e, s, h) \rightarrow (c, e, \text{val} (\text{clos} \text{ptr}_{\text{cl}}) :: s, h')\)

APPL : \(\forall \{c e v \text{ptr}_{\text{cl}} c' e' s h\} \rightarrow (\text{APPL} ; c, e, \text{val} v :: (\text{clos} \text{ptr}_{\text{cl}}) :: s, h) \rightarrow (c', v :: e', \text{cont} (c, e) :: s, h)\)

VAR : \(\forall \{n c e s h v\} \rightarrow (\text{VAR} n ; c, e, s, h) \rightarrow (\text{var} n :: v :: s, h)\)

RET : \(\forall \{e v c e' s h\} \rightarrow (\text{RET} ; e, \text{val} v :: (\text{cont} (c, e') :: s, h) \rightarrow (c', e', \text{var} v :: s, h)\)

LIT : \(\forall \{l c e s h\} \rightarrow \text{LIT} l ; c, e, s, h \rightarrow \text{LIT} l :: \text{val} l :: s, h\)

OP : \(\forall \{f c e l_1 l_2 s h\} \rightarrow \text{OP} f ; c, e, \text{val} (\text{natt} l_1) :: \text{val} (\text{natt} l_2) :: s, h \rightarrow (c, e, \text{val} (\text{natt} l_1) :: s, h)\)

CON-0 : \(\forall \{c c' e s h\} \rightarrow (\text{COND} c c', e, \text{val} (\text{natt} 0) :: s, h) \rightarrow (c, e, s, h)\)

CON-1+n : \(\forall \{c c' e s h\} \rightarrow (\text{COND} c c', e, \text{val} (\text{natt} (1 + n)) :: s, h) \rightarrow (c', e, s, h)\)

Figure 2: The definition of the transition relation of the CESH machine.
The difference appears in the CLOS and APPL instructions. To build a closure, the machine allocates it in the heap using the \_◮_ function, which, given a heap and an element, gives back an updated heap and a pointer to the element. When performing an application, the machine has a pointer to a closure, so it looks it up in the heap using the \_!_ function, which, given a heap and a pointer, gives back the element that the pointer points to (if it exists).

**Lemma 3.1** (determinism\_CESH\_). \_----- \_CESH \_ is deterministic, i.e. we can define the following term:

\[
\text{determinism}_{\text{CESH}} : \_----- \_\text{is-deterministic} \]

We define what it means for a CESH configuration \( \text{cfg} \) to terminate with a value \( v \) (\( \text{cfg} \_\text{CESH} v \)), terminate (\( \text{cfg} \_\text{CESH} \)), and diverge (\( \text{cfg} \_\text{CESH} \)). These are analogous to the definitions for the CES machine, with the difference that the CESH machine is allowed to terminate with any heap since it never deallocates anything:

\[
\begin{align*}
\_\_\text{CESH} \_ & : \text{Config} \to \text{Value} \to \star \\
\text{cfg} \_\text{CESH} v & = \exists \lambda h \to \text{cfg} \_\text{CESH} v \_\text{CESH} (\text{END}, [], [\text{val} v], h) \\
\_\text{CESH} & : \text{Config} \to \star \\
\text{cfg} \_\text{CESH} = \exists \lambda v \to \text{cfg} \_\text{CESH} v \\
\_\text{CESH} & : \text{Config} \to \star \\
\_\text{CESH} = \_\text{CESH} \\
\_\text{CESH} & = \_\text{CESH} \\
\_\text{CESH} & = \_\text{CESH} \\
\end{align*}
\]

### 3.1 An interface for heaps

In this section we formally define an abstract interface for the type constructor \( \text{Heap} \) and its associated functions that we use to model the dynamic memory allocation that we will need in our system. We will leave the details unspecified, requiring instead that it captures certain algebraic properties that should be fulfilled by any reasonable implementation.

The type \( \text{Heap} A \) models heaps with memory cells of type \( A \), and \( \text{Ptr} \) pointers into some heap. We require the existence of a heap \( \emptyset \) without any other requirements.

\[
\begin{align*}
\text{abstract} & \quad \text{Heap} : \star \to \star \\
& \quad \text{Ptr} : \star \\
& \quad \emptyset : \{ A : \star \} \to \text{Heap} A \\
\end{align*}
\]

We need to be able to attempt to lookup (or dereference) pointers, and allocate new items. Allocating gives back a new heap and a pointer.

\[
\begin{align*}
\_ & : \{ A : \star \} \to \text{Heap} A \to \text{Ptr} \to \text{Maybe} A \\
\_◮_ & : \{ A : \star \} \to \text{Heap} A \to A \to \text{Heap} A \times \text{Ptr} \\
\end{align*}
\]
We require the following relationship between dereferencing and allocation: if we dereference a pointer that was obtained from allocating a memory cell with value \( x \), we get \( x \) back:

\[
\text{let} \ (h', \text{ptr}) = h \triangleright x \text{ in } h'!\text{ptr} \equiv \text{just } x
\]

We define a preorder \( \sqsubseteq \) for sub-heaps. The intuitive reading for \( h \sqsubseteq h' \) is that \( h' \) can be used where \( h \) can, i.e. that \( h' \) contains at least the allocations of \( h \).

\[
\begin{align*}
\sqsubseteq & \quad : \{ A : \star \} \to \text{Heap} A \to \text{Heap} A \to \star \\
h_1 \sqsubseteq h_2 & \quad = \forall \text{ptr } \{ x \} \to h_1!\text{ptr} \equiv \text{just } x \to h_2!\text{ptr} \equiv \text{just } x \\
\sqsubseteq\text{-refl} & \quad : \{ A : \star \} \{ h : \text{Heap} A \} \to h \sqsubseteq h \\
\sqsubseteq\text{-refl} \ h \ \text{ptr eq} & \quad = \text{eq} \\
\sqsubseteq\text{-trans} & \quad : \{ A : \star \} \{ h_1, h_2, h_3 : \text{Heap} A \} \\
& \quad \to h_1 \sqsubseteq h_2 \to h_2 \sqsubseteq h_3 \to h_1 \sqsubseteq h_3 \\
\sqsubseteq\text{-trans} \ h_1 \sqsubseteq h_2, h_2 \sqsubseteq h_3 \ \text{ptr eq} & \quad = h_2 \sqsubseteq h_3 \ \text{ptr eq} (h_1 \sqsubseteq h_2 \ \text{ptr eq})
\end{align*}
\]

Our last requirement is that allocation does not overwrite any memory cells that were previously allocated (\( \text{proj}_1 \) means first projection):

\[
\text{abstract} \ h \triangleright x : \{ A : \star \} \{ h : \text{Heap} A \} \{ x : A \} \to h \sqsubseteq \text{proj}_1 (h \triangleright x)
\]

### 3.2 Correctness

To show that our definition of the machine is correct, we construct a bisimulation between the CES and CESH machines. Since the configurations of the machines are very similar, the intuition for the relation that we construct is simply that it is almost equality. The only place where it is not equality is for closure values, where the CESH machine stores pointers instead of closures directly. To construct a relation for closure values we need to parameterise it by the heap of the CESH configuration, and then make sure that the closure pointer points to a closure related to the CES closure.

Formally, the relation is constructed separately for the different components of the machine configurations. Since they run the same bytecode, we let the relation for code be equality:

\[
\begin{align*}
\mathcal{R}_{\text{Code}} & \quad : \text{Rel} \ \text{Code} \ \text{Code} \\
\mathcal{R}_{\text{Code}} \ c_1 c_2 & \quad = c_1 \equiv c_2
\end{align*}
\]

We forward declare the relation for environments and define the relation for closures, which is simply that the components of the closures are related. Since we have used the same names for some of the components of the CES and CESH machines, we qualify them, using Agda’s qualified imports, by prepending CES. and CESH. to their names. These components may contain values, so we have to parameterise the relations by a closure heap (here \( \text{ClosHeap} = \text{Heap} \text{CESH.Closure} \)).
Two values are unrelated if they do not start with the same constructor. When they do start with the same constructor, there are two cases: If the two values are number literals, they are related if they are equal. If the two values are a CES closure and a pointer, we require that

\[ \text{the pointer points to a CESH closure that is related to the}
\]

CES closure.

Two environments are related if they have the same list spine and their values are pointwise related.

Two configurations are related if their components are related. Here we pass the heap of the CESH configuration as an argument to the environment and stack relations.
Lemma 3.2 (HeapUpdate.config). Given two heaps $h$ and $h'$ such that $h \subseteq h'$, if $R_{cfg} (c, e, s, h)$, then $R_{cfg} (c, e, s, h')$.

$$
\text{config} : \forall \text{cfg } c e s \rightarrow R_{cfg} (c, e, s, h) \rightarrow R_{cfg} (c, e, s, h')
$$

Theorem 3.3 (simulation). $R_{cfg}$ is a simulation relation.

$$
\text{simulation} : \text{Simulation} \; \text{CES} \; \text{CESH} \; R_{cfg}
$$

where

$$
\text{Simulation} \; \text{CED} \; \text{CED} \; _R = \forall \; a \; a' \; b \rightarrow \\
\quad a \rightarrow a' \rightarrow a \; R \; b \rightarrow \exists \; \lambda \; b' \rightarrow b \rightarrow b' \times a' \; R \; b'
$$

Proof. By cases on the CES transition. In each case, the CESH machine can make analogous transitions. Use HeapUpdate.config to show that $R_{cfg}$ is preserved. \qed

We call a relation a presimulation if it is almost, but not quite, a simulation:

$$
\text{Presimulation} \; \text{CED} \; \text{CED} \; _R = \forall \; a \; a' \; b \rightarrow \\
\quad a \rightarrow a' \rightarrow a \; R \; b \rightarrow \exists \; \lambda \; b' \rightarrow b \rightarrow b' \times a' \; R \; b'
$$

Theorem 3.4 (presimulation). The inverse of $R_{cfg}$ is a presimulation.

$$
\text{presimulation} : \text{Presimulation} \; \text{CED} \; \text{CED} \; (R_{cfg}^{-1})
$$

Lemma 3.5 (presimulation-to-simulation). If $R$ is a simulation between relations $\rightarrow$ and $\rightarrow'$, $R^{-1}$ is a presimulation, and $\rightarrow'$ is deterministic at states $b$ related to some $a$, then $R^{-1}$ is a simulation.

$$
\text{presimulation-to-simulation} : (\_R \_ : \text{Rel } A B) \rightarrow \text{Simulation} \rightarrow \rightarrow' \_R \_ \rightarrow \text{Presimulation} \rightarrow' \rightarrow (\_R \_^{-1}) \rightarrow (\forall \; a \; b \rightarrow a \; R \; b \rightarrow \rightarrow' \; \text{is-deterministic-at } b) \rightarrow \text{Simulation} \rightarrow \rightarrow' \rightarrow (\_R \_^{-1})
$$

where \_is-deterministic-at\_ is a weaker form of determinism:

$$
\text{is-deterministic-at} \_ : (R : \text{Rel } A B) \rightarrow A \rightarrow \star \\
\_R \_ \; \text{is-deterministic-at } a = \forall \{ b, b' \} \rightarrow a \; R \; b \rightarrow a \; R \; b' \rightarrow b \equiv b'
$$

\footnote{In the actual implementation, this is inside a local module HeapUpdate, parameterised by $h$ and $h'$ and their relation, together with similar lemmas for the constituents of the machine configurations.}
Theorem 3.6 (bisimulation). $R_{Cfg}$ is a bisimulation.

\[
\begin{align*}
\text{bisimulation} : \text{Bisimulation} &\xrightarrow{CES} \text{CESH} &\xrightarrow{R_{Cfg}} \\
\end{align*}
\]

where

Bisimulation $\xrightarrow{\cdot} R = \text{Simulation} \xrightarrow{\cdot} \text{Simulation} \xrightarrow{\cdot} R^{-1}$

Proof. Theorem presimulation-to-simulation applied to determinism$_{CESH}$ and simulation implies that $R_{Cfg}^{-1}$ is a simulation, which together with simulation shows that $R_{Cfg}$ is a bisimulation.

Corollary 3.7 (termination-agrees, divergence-agrees). In particular, a CES configuration terminates with a natural number $n$ (diverges) if and only if a related CESH configuration terminates with the same number (diverges):

\[
\begin{align*}
\text{termination-agrees} : \forall \text{cfg}_1 \text{cfg}_2 \rightarrow \\
R_{Cfg} \text{cfg}_1 \text{cfg}_2 \rightarrow \text{cfg}_1 \xrightarrow{CES} \text{nat } n \leftrightarrow \text{cfg}_2 \xrightarrow{CESH} \text{nat } n \\
\text{divergence-agrees} : \forall \text{cfg}_1 \text{cfg}_2 \rightarrow \\
R_{Cfg} \text{cfg}_1 \text{cfg}_2 \rightarrow \text{cfg}_1 \xrightarrow{CES} \leftrightarrow \text{cfg}_2 \xrightarrow{CESH}
\end{align*}
\]

These results are of course not useful until we can show that there are configurations in $R_{Cfg}$. One such example is the “initial” (mostly empty) configuration for a fragment of code:

\[
\begin{align*}
\text{initial-related} : \forall \text{c} \rightarrow R_{Cfg} (\text{c},[\ ],[\ ]) &\xrightarrow{R_{Cfg}} (\text{c},[\ ],[\ ],\emptyset) \\
in\text{itial-related} \text{c} &\xrightarrow{\cdot} \text{refl}, \text{tt}, \text{tt}
\end{align*}
\]

4 Synchronous and asynchronous networks

Since we are later going to define two distributed abstract machines, it would save us some work if we could make a network model that is general enough to be used for both. In this section we will define models for synchronous and asynchronous networks, that are parameterised by an underlying labelled transition system. Both kinds of networks are modelled by two-level transition systems, which is common in operational semantics for concurrent and parallel languages. The idea is that the global level describes the transitions of the system as a whole, and the low level the local transitions of the nodes in the system. Synchronous communication is modelled by rendezvous, i.e. that two nodes have to be ready to send and receive a message at a single point in time. Asynchronous communication is modelled using a “message soup”, representing messages currently in transit, that nodes can add and remove messages from, reminiscent of the Chemical Abstract Machine [13].
We construct an Agda module Network, parameterised by the underlying transition relation, \( \_ \vdash \_ \to \_ \to \_ \to \_ \): Node \to Machine \to Tagged Msg \to Machine \to \dagger \). The sets Node, Machine, and Msg are additional parameters. Elements of Node will act as node identifiers, and we assume that these enjoy decidable equality. If we were using MPI, they would correspond to the so called node ranks, which are just machine integers. The type Machine is the type of the nodes’ configurations, and Msg the type of messages that the machines can send. The Node in the type of \( \_ \vdash \_ \to \_ \to \_ \to \_ \to \_ \) means, intuitively, that the configuration of a node knows about and can depend on its own identifier. The type constructor Tagged is used to separate different kinds of local transitions: A Tagged Msg can be silent (i.e. a \( \tau \) transition), send msg, or receive msg (for msg : Msg).

```agda
definition update (n n’ : Node) (r : Dec (n ≡ n’))
  where
  update nodes n m n’ | yes p = m
  update nodes n m n’ | no ¬p = nodes n’
```

Fig. 3 shows the definition of the transition relation for synchronous and asynchronous networks.

There are two ways for a synchronous network to make a transition. The first, silent-step, occurs when a machine in the network makes a transition tagged with silent, and is allowed at any time. The second, comm-step, is the aforementioned rendezvous. A node \( s \) first takes a step sending a message, and afterwards a node \( r \) takes a step receiving the same message. Note that \( s \) and \( r \) are not necessarily different, i.e. nodes can send messages to themselves. Asynchronous networks only have one rule, step, which can be used if a node steps with a tagged message that “agrees” with the list of messages in transit. The definition is fairly involved, but the intuition is that if the node receives a message, the message has to be in the list beforehand the transition. If the node sends a message, it has to be there after.
Figure 3: The definition of the transition relations for synchronous and asynchronous networks.
takes a *silent* step, the list stays the same before and after. This is what the usage of the detag function, which creates lists of input and output messages from a tagged message, achieves:

\[
\text{detag} : \{A : \star\} \rightarrow \text{Tagged } A \rightarrow \text{List } A \times \text{List } A
\]

\[
\text{detag silent} = \;[\;]\;[\;]
\]

\[
\text{detag (send } x) = \;[\;]\;[\;]
\]

\[
\text{detag (receive } x) = \;[\;]\;[\;]
\]

**Lemma 4.1.** If we have a synchronous transition from \( a \rightarrow b \), then we have one or more asynchronous transitions from \( (a,[]) \rightarrow (b,[]) \), as follows:

\[
\begin{align*}
\text{Sync} & \rightarrow \text{Async}^+ : \forall \{a, b\} \rightarrow a \quad \text{Async}^+ b \rightarrow (a,[]) \rightarrow^+ (b,[]) \\
\text{Sync} & \rightarrow \text{Async}^+ (\text{silent-step } s) = \text{step } [\;][][\;] s \\
\text{Sync} & \rightarrow \text{Async}^+ (\text{comm-step } s_1 s_2) = \text{step } [\;][\;][\;] s_1 : [\;][\;][\;] s_2
\end{align*}
\]

where \( ^+ \) is defined as follows:

\[
\begin{align*}
\text{data}_{-^+} [A : \star] (R : \text{Rel } A A) (a : A) : A \rightarrow \star \text{ where} \\
[\;] & : \{b : A\} \rightarrow R a b \rightarrow (R^+) a b \\
\text{data}_{-^+} & : \{b c : A\} \rightarrow R a b \rightarrow (R^+) b c \rightarrow (R^+) a c
\end{align*}
\]

We can thus say that asynchronous networks subsume synchronous networks. Going in the other direction is not possible in general, but for some specific instances of the underlying transition relation it is, as we will see later.

## 5 DCESH\(_1\): A degenerate distributed machine

In higher-order distributed programs containing locus specifiers, we will sometimes encounter situations where a function is not available locally. For example, when evaluating the function \( f \) in the term \( (f@A) (g@B) \), we may need to apply the remotely available function \( g \). As stated in the introduction, our general idea is to do this by decomposing some instructions into communication. In the example, the function \( f \) may send a message requesting the evaluation of \( g \), meaning that the APPL instruction is split into a pair of instructions: APPL-send and APPL-receive.

This section outlines an abstract machine, called DCESH\(_1\), which decomposes all application and return instructions into communication. The machine is degenerate, because it runs as the sole node in a network and sends messages to itself, but illustrates this decomposition, which will be used in the fully distributed system.

A configuration of the DCESH\(_1\) machine (Machine) is a tuple consisting of a possibly running thread (Maybe Thread), a closure heap (Heap Closure), and a “continuation heap” (Heap (Closure \( \times \) Stack)). Since the current work does not support parallelism,
we have at most one thread running at once. The thread resembles a CES configuration,

\[ \text{Thread} = \text{Code} \times \text{Env} \times \text{Stack} \]

but stacks are defined differently. A stack is now a list of values paired with an optional pointer (pointing into the continuation heap),

\[ \text{Stack} = \text{List Val} \times \text{Maybe ContPtr} \]

(\text{where ContPtr is a more descriptive synonym for Ptr}). The intuition here is that when performing an application, when CES would push a continuation on the stack, the DCESH\textsubscript{1} machine is going to stop the current thread and send a message, which means that it has to save the continuation and the remainder of the stack in the heap for them to persist the thread’s lifetime.

The optional pointer in Stack is to be thought of as being an element at the \textit{bottom} of the list of values. Comparing it to the definition of the CES machine, where stacks are lists of either values or continuations (which were just closures), we can picture their relation: Whereas the CES machine stores the values and continuations in a single, contiguous stack, the DCESH\textsubscript{1} machine stores first a contiguous block of values until reaching a continuation, at which point it stores (just) a pointer to the continuation closure and the rest of the stack.

The definition of closures, values, and environments are otherwise just like in the CESH machine.

\[
\begin{align*}
\text{ClosPtr} &= \text{Ptr} \\
\text{mutual} & \\
\text{Closure} &= \text{Code} \times \text{Env} \\
\text{data Val} : \ast & \text{ where} \\
\text{nat} : \mathbb{N} & \rightarrow \text{Val} \\
\text{clos} : \text{ClosPtr} & \rightarrow \text{Val} \\
\text{Env} &= \text{List Val} \\
\text{ClosHeap} &= \text{Heap Closure} \\
\text{ContPtr} &= \text{Ptr} \\
\text{Stack} &= \text{List Val} \times \text{Maybe ContPtr} \\
\text{ContHeap} &= \text{Heap} (\text{Closure} \times \text{Stack}) \\
\text{Thread} &= \text{Code} \times \text{Env} \times \text{Stack} \\
\text{Machine} &= \text{Maybe Thread} \times \text{ClosHeap} \times \text{ContHeap}
\end{align*}
\]

The machine communicates with itself using two kinds of messages, \texttt{APPL} and \texttt{RET}, corresponding to the instructions that we are replacing with communication.

\[
\begin{align*}
\text{data Msg} : \ast & \text{ where} \\
\text{APPL} : \text{ClosPtr} \rightarrow \text{Val} \rightarrow \text{ContPtr} \rightarrow \text{Msg} \\
\text{RET} : \text{ContPtr} \rightarrow \text{Val} \rightarrow \text{Msg}
\end{align*}
\]

Fig. 4 defines the transition relation for the DCESH\textsubscript{1} machine, written \( m \xrightarrow{t \text{msg}} m' \) for a tagged message \( t \text{msg} \) and machine configurations \( m \) and \( m' \). Most transitions are the same as in the CESH machine, just framed with the additional heaps and the just meaning that the thread is running. The interesting rules are the decomposed application and return rules. When an application is performed, an \texttt{APPL} message containing a pointer to the closure to apply, the argument value and a pointer to a return continuation (which is first allocated) is sent, and the thread is stopped (represented by the \texttt{nothing}). The machine can receive an application message if the
data _ → _ : Machine → Tagged Msg → Machine → ⋆ where

VAR
∀{n c e s r h cl v} → just v →
just (VAR n; c, e, s, r, h cl h cl h cnt) → silent

CLOS
∀{c' c e s r h cl v} → just (h cl ptr cl) = h cl in (c', e)

just (CLOS c'; c, e, s, r, h cl h cnt) → silent

APPL-send
∀{c e v ptr cl s r h cl h cnt} → let (h' cl ptr cl) = h cl in (c, e, s, r) in

just (APPL ; c, e, v :: clos ptr cl :: s, r, h cl h cnt)

APPL-receive
∀{h cl h cnt ptr cl v ptr cnt c e} → h cl ! ptr cl ≡ just (c, e)

nothing, h cl h cnt

just (c, e, clos ptr cl :: s, r, h cl h cnt)

RET-send
∀{e v ptr cl h cl h cnt} →

nothing, h cl h cnt

just (RET, e, v :: [] : just ptr cnt, h cl h cnt)

RET-receive
∀{h cl h cnt ptr cl v e s r} → h cl ! ptr cnt ≡ just ((c, e), s, r)

nothing, h cl h cnt

just (c, e, v :: s, r, h cl h cnt)

COND-0
∀{c c' e s r h cl h cnt}

just (COND c c', e, nat 0 :: s, r, h cl h cnt)

nothing, h cl h cnt

COND-1+n
∀{c c' e s r h cl h cnt}

just (COND c c', e, nat (1 + n) :: s, r, h cl h cnt)

nothing, h cl h cnt

LIT
∀{l c e s r h cl h cnt}

just (LIT l; c, e, s, r, h cl h cnt)

nothing, h cl h cnt

OP
∀{f c e l1 l2 r h cl h cnt}

just (OP f; c, e, nat l1 :: nat l2 :: s, r, h cl h cnt)

nothing, h cl h cnt

Figure 4: The definition of the transition relation of the DCESH₁ machine.
thread is not running. When that happens, the closure pointer is dereferenced and entered, adding the received argument to the environment. The stack is left empty apart from the continuation pointer of the received message. When returning from a function application, the machine sends a return message containing the continuation pointer and the value to return. On the receiving end of that communication, it dereferences the continuation pointer and enters it, putting the result value on top of the stack.

**Example 5.1.** We show what happens when we have instantiated the asynchronous networks of the Network module with this transition relation, using the unit (one-element) type for the Node set. Once again we trace the execution of our running example, codeExample. For readability, we write heaps with pointer mappings like \{ptr -> element\}. The last list shown in each step is the message list of the asynchronous network.

\[
\begin{align*}
\text{let } & h_{\text{cl}} = \{\text{ptr}_1 \mapsto (c_1, []), \text{ptr}_2 \mapsto (c_2, [])\} \\
& h'_{\text{cl}} = \{\text{ptr}_1 \mapsto (c_1, []), \text{ptr}_2 \mapsto (c_2, []), \text{ptr}_2 \mapsto (c_2, [])\} \\
& h_{\text{cnt}} = \{\text{ptr}_\text{cnt} \mapsto (\text{END}, [\text{end}], [\text{nothing}], \emptyset, \emptyset, [])\} \\
\in & (\text{just } (\text{CLOS } c_1 ; \text{CLOS } c_2 ; \text{APPL } ; \text{END } [\text{end}], [\text{nothing}], h_{\text{cl}}, \emptyset, [])) \\
\rightarrow & (\text{step } \text{CLOS}) \\
& (\text{just } (\text{CLOS } c_2 ; \text{APPL } ; \text{END } [\text{end}], [\text{clos } \text{ptr}_1], [\text{nothing}], h'_{\text{cl}}, \emptyset, [])) \\
\rightarrow & (\text{step } \text{CLOS}) \\
& (\text{just } (\text{APPL } ; \text{END } [\text{end}], [\text{clos } \text{ptr}_2, \text{clos } \text{ptr}_1], [\text{nothing}], h'_{\text{cl}}, \emptyset, [])) \\
\rightarrow & (\text{step } \text{APPL}-\text{send}) \\
& (\text{nothing}, h'_{\text{cl}}, h_{\text{cnt}}), [\text{APPL } \text{ptr}_1 (\text{clos } \text{ptr}_2) \text{ptr}_\text{cnt}] \\
\rightarrow & (\text{step } \text{APPL}-\text{receive}) \\
& (\text{just } (\text{VAR } 0 ; \text{RET } [\text{clos } \text{ptr}_2], [\text{nothing}], h'_{\text{cl}}, h_{\text{cnt}})) \\
\rightarrow & (\text{step } \text{VAR-refl}) \\
& (\text{just } (\text{RET } [\text{clos } \text{ptr}_2], [\text{nothing}], h'_{\text{cl}}, h_{\text{cnt}})) \\
\rightarrow & (\text{step } \text{RET}-\text{send}) \\
& (\text{nothing}, h'_{\text{cl}}, h_{\text{cnt}}), [\text{RET } \text{ptr}_\text{cnt} (\text{clos } \text{ptr}_2)] \\
\rightarrow & (\text{step } \text{RET}-\text{receive}) \\
& (\text{just } (\text{END } [\text{end}], [\text{clos } \text{ptr}_2], [\text{nothing}], h'_{\text{cl}}, h_{\text{cnt}})) \\
\end{align*}
\]

Comparing this to Example 2.2 we can see that an APPL-send followed by an APPL-receive amounts to the same thing as the APPL rule in the CES machine, and similarly for the RET instruction.

## 6 DCESH: The distributed CESH machine

We have so far seen two extensions of the CES machine. We have seen CESH, that adds heaps, and DCESH₁, that decomposes instructions into communication in a
degenerate network of only one node. Our final extension is a machine, DCESH, that supports multiple nodes. The main problem that we now face is that there is no centralised heap, but each node has its own local heap. This means that, for supporting higher-order functions across node boundaries, we have to somehow keep references to closures in the heaps of other nodes. Another problem is efficiency; we would like a system where we do not pay the higher price of communication for locally running code. The main idea for solving these two problems is to use remote pointers, $\text{RPtr} = \text{Ptr} \times \text{Node}$, pointers paired with node identifiers signifying on what node's heap the pointer is located. This solves the heap problem because we always know where a pointer comes from. It can also be used to solve the efficiency problem since we can choose what instructions to run based on whether a pointer is local or remote. If it is local, we run the rules of the CESH machine. If it is remote, we run the decomposed rules of the DCESH$_1$ machine.

The final extension to the term language and bytecode will add support for locus specifiers.

```haskell
data Term : * where ...
  _@_ : Term → Node → Term

data Instr : * where ...
  REMOTE : Code → Node → Instr
```

The locus specifiers, $t \ @ i$, are taken to mean that the term $t$ should be evaluated on node $i$. For simplicity, we assume that the terms $t$ in all locus specification sub-terms $t \ @ i$ are closed. This is a reasonable assumption, since a term where this does not hold can be transformed into one where it does with roughly similar behaviour, using e.g. lambda lifting [14][2]. The $\text{REMOTE} \ c \ i$ instruction will be used to start running a code fragment $c$ on node $i$ in the network. We also extend the compile’ function to handle the new term construct:

```haskell
compile’ : Term → Code → Code ...

compile’ (t \ @ i) c = REMOTE (compile’ t RET) i ; c
```

Note that we reuse the RET instruction to return from a remote computation.

Once again we assume that we are given a set Node with decidable equality:

```haskell
module DCESH (Node : *)

  (_≡_ : (n n’ : Node) → Dec (n ≡ n’))

where
```

The intended meaning of a remote pointer RPtr is that it is a pointer located in the heap of the given node. We assume once again that the set Node has decidable equality.

---

$^2$Transform every sub-term $t \ @ i$ to $t’ = (\lambda f v t. t) \ @ i (fv t)$. These have "roughly similar behaviour" in that the semantics of $t$ and $t’$ are identical under the assumption that locus specifiers do not change the meaning of a program.
equality meaning that we can, for instance, determine if an RPtr is remote or local. This generalises the DCESH$_1$ machine, since we can now hold pointers pointing to something in the heap of another node’s machine.

\[ \text{RPtr} = \text{Ptr} \times \text{Node} \]
\[ \text{ClosPtr} = \text{RPtr} \]

The definition of closures, values, environments and closure heaps are the same as in the CESH machine, but using RPtr instead of Ptr for closure pointers.

\[
\text{mutual} \\
\quad \text{Closure} = \text{Code} \times \text{Env} \\
\quad \text{data Value : \* where} \\
\quad \quad \text{nat} : \mathbb{N} \rightarrow \text{Value} \\
\quad \quad \text{clos} : \text{ClosPtr} \rightarrow \text{Value} \\
\quad \text{Env} = \text{List Value} \\
\quad \text{ClosHeap} = \text{Heap Closure}
\]

The stack combines the functionality of the CES(H) machine, permitting local continuations, with that of the DCESH$_1$ machine, making it possible for a stack to end with a continuation on another node. A stack element is a value or a (local) continuation signified by the \text{val} and \text{cont} constructors. A stack (Stack) is a list of stack elements, possibly ending with a (remote) pointer to a continuation.

\[
\text{data StackElem : \* where} \\
\quad \text{val} : \text{Value} \rightarrow \text{StackElem} \\
\quad \text{cont} : \text{Closure} \rightarrow \text{StackElem} \\
\quad \text{ContPtr} = \text{RPtr} \\
\quad \text{Stack} = \text{List StackElem} \times \text{Maybe ContPtr} \\
\quad \text{ContHeap} = \text{Heap (Closure} \times \text{Stack)}
\]

Threads and machines are defined like in the DCESH$_1$ machine.

\[
\text{Thread} = \text{Code} \times \text{Env} \times \text{Stack} \\
\text{Machine} = \text{Maybe Thread} \times \text{ClosHeap} \times \text{ContHeap}
\]

The messages that DCESH can send are those of the DCESH$_1$ machine but using remote pointers instead of plain pointers, plus a message for starting a remote computation, REMOTE c i rptr$_{cnt}$.

\[
\text{data Msg : \* where} \\
\quad \text{REMOTE} : \text{Code} \rightarrow \text{Node} \rightarrow \text{ContPtr} \rightarrow \text{Msg} \\
\quad \text{RET} : \text{ContPtr} \rightarrow \text{Value} \rightarrow \text{Msg} \\
\quad \text{APPL} : \text{ClosPtr} \rightarrow \text{Value} \rightarrow \text{ContPtr} \rightarrow \text{Msg}
\]

Note that sending a REMOTE message amounts to sending code in our formalisation, which is something that we said that it would not do. However, because no code is generated at run-time, every machine can be “pre-loaded” with all the bytecode it needs, and the message only needs to contain a reference to a fragment of code.
Figure 5: The definition of the transition relation of the DCESH machine.
Fig. 5 defines the transition relation of the DCESH machine, written \( i \vdash m \xrightarrow{\text{tmsg}} m' \) for a node identifier \( i \), a tagged message \( \text{tmsg} \) and machine configurations \( m \) and \( m' \). The parameter \( i \) is taken to be the identifier of the node on which the transition is taking place. Most instructions are similar to those of the CESH machine but adapted to this new setting, using remote pointers. The following function is used to allocate a pointer in a heap on a node \( i \), yielding a new heap and a remote pointer (pointing to the node \( i \)):

\[
\begin{align*}
&\vdash \_\Rightarrow (\_ : \{ \text{A} : \ast \}) \rightarrow \text{Node} \rightarrow \text{Heap A} \rightarrow \text{A} \rightarrow \text{Heap A} \times \text{RPtr} \\
&i \vdash h \Rightarrow x = \text{let} (h', \text{ptr}) = h \Rightarrow x \text{ in } h', (\text{ptr}, i)
\end{align*}
\]

When an application occurs and the closure pointer is on the current node, \( i \), the machine dereferences the pointer and enters it locally. If there is a local continuation on the stack and the machine is to run the return instruction, it also works just like the original CES machine. When starting a remote computation, the machine allocates a continuation in the heap and sends a message containing the code and continuation pointer to the remote node in question. Afterwards the current thread is stopped. On the receiving end of such a communication, a new thread is started, placing the continuation pointer at the bottom of the stack for the later return to the caller node. To run the apply instruction when the function closure is remote, i.e. its location is not equal to the current node, the machine sends a message containing the closure pointer, argument value, and continuation, like in the DCESH machine. On the other end of such a communication, the machine dereferences the pointer and enters the closure with the received value. The bottom remote continuation pointer is set to the received continuation pointer. After either a remote invocation or a remote application, the machine can return if it has produced a value on the stack and has a remote continuation at the bottom of the stack. To do this, a message containing the continuation pointer and the return value is sent to the location of the continuation pointer. When receiving a return message, the continuation pointer is dereferenced and entered with the received value.

Now that we have defined the transition relation for machines we instantiate the Network module with the \( \rightarrow \text{Machine} \) relation.

\[
\begin{align*}
\text{open import Network Node } _\Rightarrow _\text{public} \rightarrow \text{Machine}
\end{align*}
\]

From here on SyncNetwork and AsyncNetwork and their transition relations will thus refer to the instantiated versions.

An initial network configuration, given a code fragment \( c \) and a node identifier \( i \), is a network where only node \( i \) is active, ready to run the code fragment:

\[
\begin{align*}
\text{initial-network}_{\text{Sync}} : \text{Code} \rightarrow \text{Node} \rightarrow \text{SyncNetwork} \\
\text{initial-network}_{\text{Sync}} \ c \ i = \text{update} (\lambda i' \rightarrow (\text{nothing}, \emptyset, \emptyset)) \\
i (\text{just} (c, [\], [\], \text{nothing}, \emptyset, \emptyset))
\end{align*}
\]

An initial asynchronous network configuration is one where there are no messages in message list.
Lemma 6.1. The communication that the local step relation enables is **point-to-point**: if two nodes can receive the same message, then they are the same:

\[
\begin{align*}
\forall i_1 i_2 (ms : SyncNetwork) & \rightarrow (m_1 \rightarrow m_2) \\
i_1 \vdash ms i_1 & \rightarrow m_1 \\
i_2 \vdash ms i_2 & \rightarrow m_2 \\
i_1 \equiv i_2
\end{align*}
\]

We call a machine **inactive** if its thread is not running, i.e. it is equal to **nothing**.

\[
\text{inactive} : \text{Machine} \rightarrow * \\
\text{inactive}(t, \_ ) = t \equiv \text{nothing}
\]

Lemma 6.2 (**determinism**). If all nodes in a synchronous network except one are inactive, then the next step is deterministic.

\[
\forall \text{nodes} i \rightarrow \text{all nodes except } i \text{ are inactive} \rightarrow \text{is-deterministic-at nodes}
\]

where

\[
\begin{align*}
\text{all_except_are_} & : [A \land B : * ] \rightarrow (A \rightarrow B) \\
\text{all f except i are P} & = \forall i' \rightarrow i' \neq i \rightarrow P (f i')
\end{align*}
\]

Lemma 6.3 (**→∗-to- →∗**). If all nodes in a synchronous network except one are inactive and the network takes one or more steps asynchronously from and to configurations without any messages in the air, then that transition can also be done synchronously.

\[
\begin{align*}
\text{→∗-to-} & : \forall \{\text{nodes nodes'}\} i \rightarrow \text{all nodes except } i \text{ are inactive} \rightarrow \\
\text{→∗} & : \text{nodes, [ ]} \rightarrow \text{nodes} \rightarrow \text{nodes'}
\end{align*}
\]

This is a key result because it means that it does not matter whether we choose to look at synchronous or asynchronous networks for single threaded computations. With this result in place, we will from now on focus on the simpler synchronous networks.

We define what it means for a synchronous DCESH network nodes to **terminate with a value** \(v\) \(\downarrow_{\text{Sync}} v\), **terminate** \(\downarrow_{\text{Sync}}\), and **diverge** \(\uparrow_{\text{Sync}}\). A
network terminates with a value \( v \) if it can step to a network where only one node is active, and that node has reached the END instruction with the value \( v \) on top of its stack. The other definitions are analogous to those of the CES(H) machine.

\[
\text{Sync} : \text{Value} \rightarrow \star
\]

\[
\exists \lambda \text{i} \rightarrow \text{all nodes' except i are inactive} \times \exists \lambda \text{heaps} \rightarrow \text{nodes' i} \equiv (\text{just (END,}[,\text{],val v :: [,}\text{],nothing)},\text{heaps})
\]

\[
\text{Sync} : \forall \text{nodes} \rightarrow \star
\]

\[
\text{nodes} \downarrow \text{Sync v} = \exists \lambda \text{nodes'} \rightarrow \text{nodes} \downarrow \text{Sync v}
\]

\[
\text{Sync} : \forall \text{nodes} \rightarrow \star
\]

\*

6.1 Correctness

To prove the correctness of the machine, we will now establish a bisimulation between the CESH and the DCESH machines.

To simplify this development, we extend the CESH machine with a rule for the REMOTE \( c \ i \) instruction so that both machines run the same bytecode. This rule is almost a no-op, but since we are assuming that the code we run remotely is closed, the environment is emptied, and since the compiled code \( c \) will end in a RET instruction a return continuation is pushed on the stack.

\[
\text{data} \rightarrow \text{Config Config where}
\]

\[
\text{REMOTE} : \forall \{c' i c e s h\} \rightarrow
\]

\[
(\text{REMOTE c' i ; c,e,s,h}) \rightarrow \text{CESH (c',}[\text{,cont (c,e :: s,h})}
\]

The intuition behind the relation that we are to construct should be similar to the intuition for the relation between CES and CESH configurations, i.e. that it is almost equality, but since values may be pointers to closures, we need to parameterise it by heaps. The problem now is that both machines use pointers, and the DCESH machine even uses remote pointers and has two heaps for each node. This means that we have to parameterise the relations by all the heaps in the system.

As before, two fragments of code are related if they are equal.

\[
R_{\text{Code}} : \text{Rel Code Code}
\]

\[
R_{\text{Code}} c_1 c_2 = c_1 \equiv c_2
\]

We define the type of the extra parameter that we need as a synonym for an indexed family of the closure and continuation heaps (here \( \text{DCESH.ContHeap} = \text{Heap (DCESH.Closure} \times \text{DCESH.Stack)} \)): 28
Heaps = Node → DCESH.ClosHeap × DCESH.ContHeap

Simply following the recipe that we used for the relation between the CES and the CESH machines would not prove effective this time around. When we constructed that, we could be sure that there would be no circularity, since it was constructed inductively on the structure of the CES configuration. But now both systems, CESH and DCESH, have heaps where there is a potential for circular references (e.g. a closure, residing in a heap, whose environment contains a pointer to itself), so a direct structural induction cannot work. This is perhaps the most mathematically (and formally) challenging point of the paper. To fix this we parameterize the affected relation definitions by a natural number rank, which records how many times pointers are allowed to be dereferenced, in addition to the heap parameters.

The relation for environments and closures is as before, but with the additional parameters.

\[ R_{\text{Env}} : \mathbb{N} \to \text{CESH.ClosHeap} \to \text{Heaps} \to \text{Rel CESH.Env} \to \text{DCESH.Env} \]

\[ R_{\text{Clos}} : \mathbb{N} \to \text{CESH.ClosHeap} \to \text{Heaps} \to \text{Rel CESH.Closure} \to \text{DCESH.Closure} \]

\[ R_{\text{rptr}}_{\text{cl}} \text{ rank h hs } (c_1, e_1) (c_2, e_2) = R_{\text{Code}} c_1 c_2 \times R_{\text{Env}} \text{ rank h hs } e_1 e_2 \]

The relation for closure pointers is where the rank is used. If the rank is zero, the relation is trivially fulfilled. If the rank is non-zero, it makes sure that the CESH pointer points to a closure in the CESH heap, that the remote pointer of the DCESH network points to a closure in the heap of the location that the pointer refers to, and that the two closures are related:

\[ R_{\text{rptr}}_{\text{cl}} : \mathbb{N} \to \text{CESH.ClosHeap} \to \text{Heaps} \to \text{Rel CESH.ClosPtr} \to \text{DCESH.ClosPtr} \]

\[ R_{\text{rptr}}_{\text{cl}} (1 + \text{rank}) \text{ h hs } \text{ptr}_1 (\text{ptr}_2, \text{loc}) = \]

\[ \exists \lambda_2 \lambda c_1 c_2 \to \text{h} ! \text{ptr}_1 \equiv \text{just } c_1 \times \]

\[ \lambda \text{proj}_1 (\text{hs loc} ! \text{ptr}_2) \equiv \text{just } c_2 \times \]

\[ R_{\text{Clos}} \text{ rank h hs } c_1 c_2 \]

The relation for values is also as before, but with the extra parameters.

\[ R_{\text{Val}} : \mathbb{N} \to \text{CESH.ClosHeap} \to \text{Heaps} \to \text{Rel CESH.Value} \to \text{DCESH.Value} \]

\[ R_{\text{Val}} \text{ rank h hs } (\text{nat } n_1) (\text{nat } n_2) = n_1 \equiv n_2 \]

\[ R_{\text{Val}} \text{ rank h hs } (\text{nat } _) (\text{clos } _) = \bot \]

\[ R_{\text{Val}} \text{ rank h hs } (\text{clos } _) (\text{nat } _) = \bot \]

\[ R_{\text{Val}} \text{ rank h hs } (\text{clos } \text{ptr}) (\text{clos } \text{rptr}) = R_{\text{rptr}_{\text{cl}}} \text{ rank h hs } \text{ptr} \text{ rptr} \]

The relation for environments is also as before, but included for completeness:

\[ R_{\text{Env}} \text{ rank h hs } [ ] [ ] = \top \]

\[ R_{\text{Env}} \text{ rank h hs } [x :: e_1] [ ] = \bot \]

\[ R_{\text{Env}} \text{ rank h hs } [x :: e_1] [x :: e_2] = R_{\text{Val}} \text{ rank h hs } x_1 x_2 \times R_{\text{Env}} \text{ rank h hs } e_1 e_2 \]

The relation for stack elements is almost as before, but now requires that for any natural number rank, i.e. for any finite number of pointer dereferencings, the relations hold:
The relation for stacks now takes into account that the DCESH stacks may end in a pointer representing a remote continuation. It makes sure that the pointer points to something in the continuation heap of the location of the pointer, related to the CESH stack element.

Finally, a CESH configuration and a DCESH thread are related if the thread is running and the constituents are pointwise related:

A configuration is related to an asynchronous DCESH network if the network has exactly one running node, i, that is related to the configuration, and there are no messages in the message soup:

A configuration is related to a synchronous DCESH network if it is related to the asynchronous network gotten by pairing the synchronous network with an empty list of messages:
\( \text{RSync} : \text{Rel Config SyncNetwork} \)
\( \text{RSync \ cfg nodes} = \text{RAsync \ cfg \ (nodes[,])} \)

We order heaps of a DCESH network pointwise as follows (called \( \subseteq s \) since it is the “plural” of \( \subseteq \)):

\[
\subseteq s : (hs \hs' : \text{Heaps}) \rightarrow \star
\]

\[
hs \subseteq hs' = \forall i \rightarrow \text{let } (h_{cl}, h_{cnt}) = hs i \\
(h_{cl}', h_{cnt}') = hs' i \\
in h_{cl} \subseteq h_{cl}' \times h_{cnt} \subseteq h_{cnt}'
\]

\( \subseteq s - \text{refl} : (hs : \text{Heaps}) \rightarrow hs \subseteq hs \)
\( \subseteq s - \text{refl} \hs \hs' = \forall i \rightarrow \text{let } (h_{cl}, h_{cnt}) = hs i \\
in h_{cl} \subseteq h_{cl}' \times h_{cnt} \subseteq h_{cnt}'
\]

\( \subseteq s - \text{trans} : (hs_1, hs_2, hs_3 : \text{Heaps}) \rightarrow \\
hs_1 \subseteq hs_2 \rightarrow hs_2 \subseteq hs_3 \rightarrow hs_1 \subseteq hs_3
\]

\[
\text{Lemma 6.4} \text{ (HeapUpdate.env, HeapUpdate.stack). Given CESH closure heaps } h \text{ and } h' \text{ such that } h \subseteq h' \text{ and families of DCESH heaps } hs \text{ and } hs' \text{ such that } hs \subseteq hs', \text{ then we can prove the following:}
\]

\[
\text{env} : \forall \{ n \} e_1 e_2 \rightarrow R_{\text{Env}} n h hs e_1 e_2 \rightarrow R_{\text{Env}} n h' hs' e_1 e_2
\]

\[
\text{stack} : \forall s_1 s_2 \rightarrow R_{\text{Stack}} h hs s_1 s_2 \rightarrow R_{\text{Stack}} h' hs' s_1 s_2
\]

\[\text{Lemma 6.5 (simulationSync). } R_{\text{Sync}} \text{ is a simulation relation.} \]

\[
\text{simulationSync} : \text{Simulation} \Rightarrow_{\text{CESH}} \Rightarrow_{\text{Sync}} \Rightarrow_{R_{\text{Sync}}}
\]

\[\text{Proof. By cases on the CESH transition. In each case, the DCESH network can make analogous transitions. Use the HeapUpdate lemmas to show that } R_{\text{Sync}} \text{ is preserved.} \]

\[\text{Theorem 6.6 (presimulationSync). The inverse of } R_{\text{Sync}} \text{ is a presimulation.} \]

\[
\text{presimulationSync} : \text{Presimulation} \Rightarrow_{\text{Sync}} \Rightarrow_{\text{CESH}} \Rightarrow_{(R_{\text{Sync}})^{-1}}
\]

\[\text{Theorem 6.7 (bisimulationSync). } R_{\text{Sync}} \text{ is a bisimulation.} \]

\[
\text{bisimulationSync} : \text{Bisimulation} \Rightarrow_{\text{CESH}} \Rightarrow_{\text{Sync}} \Rightarrow_{R_{\text{Sync}}}
\]

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Proof. Theorem presimulation-to-simulation applied to determinism\textsubscript{Sync} and simulation\textsubscript{Sync} implies that \( R_{\text{Sync}}^{-1} \) is a simulation, which together with simulation\textsubscript{Sync} shows that \( R_{\text{Sync}} \) is a bisimulation. \( \square \)

**Corollary 6.8** (termination-agrees\textsubscript{Sync}, divergence-agrees\textsubscript{Sync}). In particular, a CESH configuration terminates with a natural number \( n \) (diverges) if and only if a related synchronous DCESH network terminates with a natural number \( n \) (diverges).

\[
\begin{align*}
\text{termination-agrees}_\text{Sync} : & \forall \text{cfg nodes } n \rightarrow R_{\text{Sync}} \text{cfg nodes } \rightarrow \\
& \text{cfg } \downarrow_{\text{CESH nat n}} \leftrightarrow \text{nodes } \downarrow_{\text{Sync nat n}} \\
\text{divergence-agrees}_\text{Sync} : & \forall \text{cfg}_1 \text{cfg}_2 \rightarrow R_{\text{Sync}} \text{cfg}_1 \text{cfg}_2 \rightarrow \\
& \text{cfg}_1 \uparrow_{\text{CESH}} \leftrightarrow \text{cfg}_2 \uparrow_{\text{Sync}}
\end{align*}
\]

We also have that initial configurations are in \( R_{\text{Sync}} \):

\[
\begin{align*}
\text{initial-related}_\text{Sync} : & \forall \text{c i } \rightarrow R_{\text{Sync}} (c,[\ ],[\ ],\emptyset) \\
& \text{(initial-network}_{\text{Sync c i})}
\end{align*}
\]

These final results complete the picture for the DCESH machine. We have established that we get the same final result regardless of whether we choose to run a fragment of code using the CES, the CESH, or the DCESH machine.

## 7 Related work

There is a multitude of programming languages and libraries for distributed computing. We focus mostly on those with a functional flavour. For surveys, see \cite{15,16}. Broadly speaking, we can divide them into those that use some form of explicit message passing, and those that have more implicit mechanisms for distribution and communication.

**Explicit** A prime example of a language for distributed computing that uses explicit message passing is Erlang \cite{17}. Erlang is a very successful language used prominently in the telecommunication industry. Conceptually similar solutions include MPI \cite{1} and Cloud Haskell \cite{18}. The theoretically advanced projects Nomadic Pict \cite{19} and the distributed join calculus \cite{20} both support a notion of mobility for distributed agents, which enables more expressivity for the distribution of a program than the fairly static networks that our work uses. In general, explicit languages are well-proven, but far away in the language design-space from the seamless distributed computing that we envision because they place the burden of explicit communication on the programmer.
Implicit. Our work can be seen as a generalised Remote Procedure Call (RPC) [2]. In loc. cit. it is argued that emulating a shared address space is infeasible since it requires each pointer to also contain location information, and that it is questionable whether acceptable efficiency can be achieved. These arguments certainly apply to our work, where we do just this. With the goal of expressivity in mind, however, we believe that we should enable the programmer to write the potentially inefficient programs that (internally) use remote pointers, because not all programs are performance critical. Furthermore, using a tagged pointer representation [21] for closure pointers means that we can tag pointers that are remote, and pay a very low, if any, performance penalty for local pointers.

Remote Evaluation (REV) [5] is another generalisation of RPC, siding with us on enabling the use of higher-order functions across node boundaries. The main differences between REV and our work is that REV relies on sending code and that it has a more general distribution mechanism.

The well-researched project Eden [22], which builds on Haskell, is a semi-implicit language. Eden allows expressing distributed algorithms at a high level of abstraction, and is mostly implicit about communication, but explicit about process creation. Eden is specified operationally using a two-level semantics similar to ours.

Hop [23], Links [24], and ML5 [25] are examples of so called tierless languages that allow writing (for instance) the client and server code of web applications in unified languages with more or less seamless interoperability between them. We believe that our work shows how a principled back-end and semantics can work for such languages.

8 Conclusion and further work

We have seen the definition and correctness proofs of DCESH, a distributed abstract machine. Previously we have argued that distributed and heterogeneous programming would benefit from languages that are architecture-independent, using compilation based on the idea of seamless computing [3]. This would allow the programmer to focus on solving algorithmic problems without having to worry about the low-level details of the underlying computational system. Our previous work shows how to achieve this, but is very different from conventional compilation techniques, relying on game semantics. This means that the vast literature on compiler optimisation does not generally apply to it, and that it is difficult to interface with legacy code. We believe that the current work alleviates these issues, since it shows a way to do distributed execution as a conservative extension of existing abstract machines. Additionally, DCESH adds very little overhead, if any, for local execution, while permitting any sub-terms to be seamlessly distributed.

Implementation. An implementation of the DCESH machine can be constructed by either a bytecode interpreter or compiling the bytecode into a low-level language.
by macro expansion. We have a prototype implementation that does the latter, illustrating the potential for using DCESH as a basis for a usable compiler.

**Outstanding questions**

- Do the proofs generalise to a language with parallelism?

- Can we efficiently do distributed garbage collection \[^20]\? This is necessary, since DCESH, in contrast to our previous work, never reclaims heap garbage. It would also be interesting to find out if parts of programs can use local garbage collection for better performance.

- Can we find a way to express more complicated distribution patterns than those made possible by locus specifiers? From our experience, locus specifiers are excellent for simple programs (especially those with client-server disciplines), but due to the static nature of the specifiers, it is hard to express dynamic distributed algorithms. We believe that our work can be extended with dynamic locus specifiers to handle this. A simple first step would be to add support for compiling parts of a program for more than one node at a time, making it possible to pass (references to) functions already existing on some remote node to it.

- Can we add support for sending code code (like REV \[^5]\) when the code is location-independent?

Two other language features that our abstract machines currently do not handle, but that we would like to implement are abstract data types and mutable references.

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**References**

[1] W. D. Gropp, E. L. Lusk, and A. Skjellum, *Using MPI: portable parallel programming with the message-passing interface*. MIT Press, 1999, vol. 1.

[2] A. Birrell and B. J. Nelson, “Implementing Remote Procedure Calls,” *ACM Trans. Comput. Syst.*, vol. 2, no. 1, pp. 39–59, 1984.
[3] O. Fredriksson and D. R. Ghica, “Abstract Machines for Game Semantics, Revisited,” in 28th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2013, New Orleans, LA, USA, June 25-28, 2013. IEEE Computer Society, 2013, pp. 560–569.

[4] ———, “Seamless Distributed Computing from the Geometry of Interaction,” in Trustworthy Global Computing - 7th International Symposium, TGC 2012, Newcastle upon Tyne, UK, September 7-8, 2012, Revised Selected Papers. Springer, 2012, pp. 34–48.

[5] J. W. Stamos and D. K. Gifford, “Remote evaluation,” ACM Trans. Program. Lang. Syst., vol. 12, no. 4, pp. 537–565, 1990.

[6] P. J. Landin, “The mechanical evaluation of expressions,” Computer Journal, vol. 6, no. 4, pp. 308–320, Jan. 1964.

[7] U. Norell, “Towards a practical programming language based on dependent type theory,” Ph.D. dissertation, Chalmers Uni. of Tech., 2007.

[8] X. Leroy, “MPRI course 2-4-2, part II: abstract machines,” 2013-2014. [Online]. Available: http://gallium.inria.fr/~xleroy/mpri/progfunc/

[9] P. Henderson, Functional programming - application and implementation, ser. Prentice Hall International Series in Computer Science. Prentice Hall, 1980.

[10] M. Felleisen and D. P. Friedman, “Control operators, the SECD-machine, and the lambda-calculus,” in IFIP TC 2/WG 2.2, Aug. 1986.

[11] G. D. Plotkin, “LCF Considered as a Programming Language,” Theor. Comput. Sci., vol. 5, no. 3, pp. 223–255, 1977.

[12] N. G. de Bruijn, “Lambda calculus notation with nameless dummies, a tool for automatic formula manipulation, with application to the church-rosser theorem,” Indagationes Mathematicae, pp. 381–392, 1972.

[13] G. Berry and G. Boudol, “The Chemical Abstract Machine,” in Conference Record of the Seventeenth Annual ACM Symposium on Principles of Programming Languages, San Francisco, California, USA, January 1990. ACM Press, 1990, pp. 81–94.

[14] T. Johnson, “Lambda Lifting: Transforming Programs to Recursive Equations,” in FPCA, 1985, pp. 190–203.

[15] P. W. Trinder, H.-W. Loidl, and R. F. Pointon, “Parallel and Distributed Haskells,” J. Funct. Program., vol. 12, no. 4&5, pp. 469–510, 2002.

[16] H.-W. Loidl, F. Rubio, N. Scaife, K. Hammond, S. Horiguchi, U. Klusik, R. Loopen, G. Michaelson, R. Pena, S. Priebe, Á. J. R. Portillo, and P. W. Trinder, “Comparing Parallel Functional Languages: Programming and Performance,” Higher-Order and Symbolic Computation, vol. 16, no. 3, pp. 203–251, 2003.
[17] J. Armstrong, R. Virding, and M. Williams, *Concurrent programming in ERLANG*. Prentice Hall, 1993.

[18] J. Epstein, A. P. Black, and S. L. P. Jones, “Towards Haskell in the cloud,” in *Proceedings of the 4th ACM SIGPLAN Symposium on Haskell, Haskell 2011, Tokyo, Japan, 22 September 2011*. ACM, 2011, pp. 118–129.

[19] P. T. Wojciechowski and P. Sewell, “Nomadic pict: language and infrastructure design for mobile agents,” *IEEE Concurrency*, vol. 8, no. 2, pp. 42–52, 2000.

[20] C. Fournet, G. Gonthier, J.-J. Lévy, L. Maranget, and D. Rémy, “A calculus of mobile agents,” in *CONCUR*, ser. Lecture Notes in Computer Science, U. Montanari and V. Sassone, Eds., vol. 1119. Springer, 1996, pp. 406–421.

[21] S. Marlow, A. R. Yakushev, and S. L. P. Jones, “Faster laziness using dynamic pointer tagging,” in *Proceedings of the 12th ACM SIGPLAN International Conference on Functional Programming, ICFP 2007, Freiburg, Germany, October 1-3, 2007*. ACM, 2007, pp. 277–288.

[22] R. Loogen, Y. Ortega-Mallén, and R. Peña-Mari, “Parallel functional programming in Eden,” *J. Funct. Program.*, vol. 15, no. 3, pp. 431–475, 2005.

[23] M. Serrano, E. Gallesio, and F. Loitsch, “Hop: a language for programming the web 2.0,” in *Companion to the 21th Annual ACM SIGPLAN Conference on Object-Oriented Programming, Systems, Languages, and Applications, OOPSLA 2006, October 22-26, 2006, Portland, Oregon, USA*. ACM, 2006, pp. 975–985.

[24] E. Cooper, S. Lindley, P. Wadler, and J. Yallop, “Links: Web Programming Without Tiers,” in *Formal Methods for Components and Objects, 5th International Symposium, FMCO 2006, Amsterdam, The Netherlands, November 7-10, 2006, Revised Lectures*. Springer, 2006, pp. 266–296.

[25] T. M. VII, K. Crary, and R. Harper, “Type-Safe Distributed Programming with ML5,” in *Trustworthy Global Computing, Third Symposium, TGC 2007, Sophia-Antipolis, France, November 5-6, 2007, Revised Selected Papers*. Springer, 2007, pp. 108–123.

[26] D. Plainfossé and M. Shapiro, “A Survey of Distributed Garbage Collection Techniques,” in *Memory Management, International Workshop IWMM 95, Kinross, UK, September 27-29, 1995, Proceedings*. Springer, 1995, pp. 211–249.