Lorentz Invariance of the Pure Spinor BRST Cohomology for the Superstring

Nathan Berkovits
Instituto de Física Teórica, Universidade Estadual Paulista
Rua Pamplona 145, 01405-900, São Paulo, SP, Brasil

Osvaldo Chandía
Facultad de Física, Pontificia Universidad Católica de Chile
Casilla 306, Santiago 22, Chile

In a previous paper, the BRST cohomology in the pure spinor formalism of the superstring was shown to coincide with the light-cone Green-Schwarz spectrum by using an $SO(8)$ parameterization of the pure spinor. In this paper, the $SO(9,1)$ Lorentz generators are explicitly constructed using this $SO(8)$ parameterization, proving the Lorentz invariance of the pure spinor BRST cohomology.
1. Introduction

Recently, the superstring was covariantly quantized using the BRST operator $Q = \int \lambda^\alpha d_\alpha$ where $d_\alpha$ is the fermionic Green-Schwarz constraint and $\lambda^\alpha$ is a pure spinor satisfying

$$\lambda^\alpha \gamma^\mu_{\alpha\beta} \lambda^\beta = 0$$

for $\mu = 0$ to 9 \[1\]. In order to prove equivalence of the cohomology of $Q$ with the light-cone Green-Schwarz spectrum, it was useful to solve the pure spinor constraint of \[1.1\] using an $SO(8)$ parameterization of $\lambda^\alpha$ and rewrite $Q$ in terms of unconstrained variables \[2\]. This $SO(8)$ parameterization of $\lambda^\alpha$ is more complicated than the $U(5)$ parameterization of \[1\] since it involves an infinite number of gauge degrees of freedom. However, it was necessary for the cohomology computation since the $U(5)$ parameterization becomes singular at certain values of $\lambda^\alpha$.

In this paper, $SO(9,1)$ Lorentz generators will be explicitly constructed out of these unconstrained $SO(8)$ variables, thereby proving Lorentz invariance of the cohomology computation. Although part of this construction already appeared in \[2\], the most complicated Lorentz generator, $M^j^-$, was left incomplete. As will be shown here, verifying that $M^j^-$ satisfies $[M^j^-, M^k^-] = [M^j^-, Q] = 0$ involves several rather impressive cancellations.

2. $SO(8)$ Variables

As discussed in \[2\], the pure spinor constraint of \[1.1\] for $\lambda^\alpha$ can be solved in terms of $SO(8)$ variables $s^a$ and $v^j$ satisfying $s^a s^a = 0$ as

$$(\gamma^+ \lambda)^a = s^a, \quad (\gamma^- \lambda)^\dot{a} = \sigma_{j\dot{a}a} v^j s^a,$$

where $\gamma^\pm = \frac{1}{2}(\gamma^0 \pm \gamma^9)$, $\sigma_{j\dot{a}a}$ are $SO(8)$ Pauli matrices satisfying $\sigma_{(j\dot{a})} \sigma^{(k\dot{b})} = 2\delta_{jk}\delta^{\dot{a}\dot{b}}$, and $(j, a, \dot{a}) = 1$ to 8 are $SO(8)$ vector, chiral and anti-chiral indices. The gauge invariance $\delta v^j = \sigma_{j\dot{a}a} s^a \epsilon^\dot{a}$ of the parameterization of \[2.1\] leads to an infinite chain of ghosts-for-ghosts $(s^a, v^j_M, t^\dot{a}_M)$ for $M = 0$ to $\infty$, and their conjugate momenta $(v^a, w^j_M, u^\dot{a}_M)$, where $v^0 = v^j_M$ of \[2.1\], $(s^a, v^a, v^j_M, w^\dot{a}_M)$ are bosons and $(t^\dot{a}_M, u^\dot{a}_M)$ are fermions. Also, the condition $s^a s^a = 0$ can be treated as a BRST constraint by introducing the fermionic ghost and anti-ghost $(b, c)$.
In terms of these unconstrained $SO(8)$ variables, it was shown in [2] that the BRST operator $Q = \int \lambda^\alpha d_\alpha$ can be rewritten as

$$Q' = \int (s^a G^a - b s^a s^a + cT)$$

(2.2)

where

$$G^a = (\gamma^- d)^a + \sigma^a_j v^j_0 (\gamma^+ d)^a + \hat{G}^a,$$

(2.3)

$$T = \frac{1}{2} \Pi^\mu + v^j_0 \Pi^j + \frac{1}{2} v^j_0 v^j_0 \Pi^+ + t^\alpha_0 (\gamma^+ d)^\alpha + \hat{T},$$

$$\hat{G}^a = \sigma^a_j \sum_{M=0}^\infty (u^j_M t^\alpha_M + v^j_M v^j_{M+1} u^\alpha_M),$$

$$\hat{T} = \sum_{M=0}^\infty (v^j_M v^j_{M+1} + t^\alpha_{M+1} u^\alpha_M),$$

(2.4)

and $d_\alpha$ and $\Pi^\mu$ are the fermionic and bosonic super-Poincaré covariant momenta. $SO(9,1)$ Lorentz generators will now be defined which commute with $Q'$, proving the Lorentz invariance of the BRST cohomology.

3. $SO(9,1)$ Lorentz Generators

The $SO(9,1)$ Lorentz generators will be defined as

$$M^{\mu\nu} = \int (L^{\mu\nu} + N^{\mu\nu})$$

(3.1)

where $L^{\mu\nu} = x^\mu \partial x^\nu + \frac{1}{2} \theta \gamma^{\mu\nu} p$ is constructed in the usual manner from the $(x^\mu, \theta^\alpha, p_\alpha)$ superspace variables and $N^{\mu\nu}$ is constructed from the unconstrained $SO(8)$ variables of section 2. It will now be shown that

$$N^{jk} = \frac{1}{2} s^a (\sigma^{jk})_{ab} r^b + \sum_{M=0}^\infty [(v^j_M w^k_M) + \frac{1}{2} t^\alpha_M (\sigma^{jk})_{ab} u^\alpha_M],$$

(3.2)

$$N^{j+} = w^j_0,$$

$$N^{+-} = bc - \frac{1}{2} s^a r^a + \sum_{M=0}^\infty [(M+1) v^j_M w^k_M + (M + \frac{3}{2}) t^\alpha_M w^\alpha_M],$$

$$N^{j-} = -3 \partial v^j_0 - v^k_0 N^{jk} - v^j_0 N^{+-} - \frac{1}{2} v^k_0 v^k_0 w^j_0 + v^j_0 v^k_0 w^k_0 + \frac{1}{2} c \sigma^{jk} t^\alpha_0 r^\alpha$$

(3.3)

$$+ \sum_{M,N=1} A^{MNj} v^j_M v^l_N u^m_{M+N} + \sum_{M=1} \sum_{N=0} B^{MNj} v^j_M t^\alpha_N u^\alpha_{M+N} + \sum_{M,N=0} C^{MNj} t^\alpha_M t^\beta_N w^k_{M+N+1},$$

2
satisfy the \(\text{SO}(9,1)\) current algebra

\[
N^{\mu\nu}(y)N^{\rho\sigma}(z) \rightarrow \frac{\eta^{\rho\nu}N^{\mu\sigma}(z) - \eta^{\sigma\nu}N^{\mu\rho}(z)}{y - z} - 3\frac{\eta^{\rho\nu}(y)}{(y - z)^2}
\]

where the constant \(\text{SO}(8)\)-covariant coefficients \((A_{k\ell m}^{MNj}, B_{k\ell ab}^{MNj}, C_{\ell\ell abk}^{MNj})\) will be determined in section 4 by requiring that \([\int M^j, Q'] = 0\).

To show that \(N^{\mu\nu}\) satisfies (3.4), the free-field OPE’s

\[
r^a(y)s^b(z) \rightarrow \frac{\delta^{ab}}{y - z}, \quad w_N^j(y)v_N^k(z) \rightarrow \frac{\delta^{jk}\delta_{MN}}{y - z}, \quad u_M^a(y)t_N^b(z) \rightarrow \frac{\delta^{ab}\delta_{MN}}{y - z},
\]

will be used. The only non-trivial part of checking the current algebra involving \((N^{jk}, N^{j+}, N^{+-})\) are the double poles of \(N^{jk}\) with \(N^{jk}\) and \(N^{+-}\) with \(N^{+-}\). The double pole of \(N^{jk}\) with \(N^{jk}\) gets a contribution of +2 from the first term and

\[
+2 - 2 + 2 - 2 + ... = 2 \sum_{M=0}^{\infty} (-1)^M = 2 \lim_{x \to 1} \sum_{M=0}^{\infty} (-x)^M = 2 \lim_{x \to 1} (1 + x)^{-1} = 1
\]

from the other terms, which sums to +3 as desired. The double pole of \(N^{+-}\) with \(N^{+-}\) gets a contribution of +1 from the first term, –2 from the second term, and

\[
-2(2^2 - 3^2 + 4^2 - 5^2 + ...) = -2 - 2 \sum_{M=0}^{\infty} M^2(-1)^M = -2 - 2 \lim_{x \to 1} \sum_{M=0}^{\infty} M^2(-x)^M = 1
\]

from the remaining terms. But by taking derivatives of \(\sum_{M=0}^{\infty} (-x)^M = (1 + x)^{-1}\), one finds

\[
\lim_{x \to 1} \sum_{M=0}^{\infty} M^2(-x)^M = \lim_{x \to 1} (2(1 + x)^{-3} - 3(1 + x)^{-2} + (1 + x)^{-1}) = 0,
\]

so the \(N^{+-}\) double poles sum to –3 as desired.

To check the current algebra involving \(N^{j-}\), it is convenient to define

\[
N^{j-} - \Lambda^{j-} = -3\partial v_0^j - v_0^k \Lambda^{jk} - v_0^j \Lambda^{+-} + \frac{1}{2} v_0^k v_0^j w_0^j - v_0^j v_0^k w_0^k + \frac{1}{2} c_{\sigma\rho} a_\sigma^j t_0^\rho r^a
\]

\[
\equiv a_1^j + a_2^j + a_3^j + a_4^j + a_5^j + a_6^j,
\]

where \(\Lambda^{j-}\) is the second line of \(N^{j-}\) in (3.3) and where

\[
\Lambda^{jk} = N^{jk} - v_0^{[j} w_0^{k]}, \quad \Lambda^{+-} = N^{+-} - v_0^k w_0^k
\]
are the terms in $N^{jk}$ and $N^{+-}$ which do not involve $v_0^j$. Since $\Lambda^{j-}$ does not involve $v_0^j$, one can easily verify that $N^{j-}$ with $(N^{kl}, N^{k+}, N^{+-})$ satisfies the current algebra of (3.11). As usual when constructing Lorentz generators out of light-cone variables, the most difficult part of the current algebra to check is that $N^{j-}(y)N^{k-}(z)$ has no singularity. This will be done by first showing no singularity in $(N^{j-} - \Lambda^{j-})(y)(N^{k-} - \Lambda^{k-})(z)$, then by showing no singularity in $(N^{j-} - \Lambda^{j-})(y)\Lambda^{k-}(z) + \Lambda^{j-}(y)(N^{k-} - \Lambda^{k-})(z)$, and finally by showing no singularity in $\Lambda^{j-}(y)\Lambda^{k-}(z)$.

To show that $(N^{j-} - \Lambda^{j-})(y)(N^{k-} - \Lambda^{k-})(z)$ has no singularity, one can use

$$\Lambda^{jk}(y)\Lambda^{lm}(z) \rightarrow \frac{\delta^{[jk} \Lambda^{m]}(z) - \delta^{m[k} \Lambda^{j]}l(z)}{y - z} - \frac{\delta^{m[j} \delta^{k]l}}{(y - z)^2}, \quad \Lambda^{+-}(y)\Lambda^{+-}(z) \rightarrow \frac{5}{(y - z)^2},$$

to compute that

$$a_2^j(y)a_5^k(z) \rightarrow \frac{1}{(y - z)^2} [\delta^{jk} v_0^j v_0^k - v_0^j v_0^k],$$

$$a_3^j(y)a_3^k(z) \rightarrow \frac{5}{(y - z)^2} v_0^j v_0^k + \frac{5}{(y - z)} \partial v_0^j v_0^k,$$

$$a_4^j(y)a_4^k(z) \rightarrow -\frac{1}{(y - z)^2} v_0^j v_0^k + \frac{1}{(y - z)} [-v_0^j \partial v_0^k + \frac{1}{2} v_0^l v_0^k v_0^j w_0^l],$$

$$a_5^j(y)a_5^k(z) \rightarrow -\frac{11}{(y - z)^2} v_0^j v_0^k - \frac{1}{(y - z)} [v_0^j \partial v_0^k + 10 v_0^k \partial v_0^j],$$

$$a_1^j(y)a_4^k(z) \rightarrow -\frac{3}{(y - z)^2} \delta^{jk} v_0^l v_0^l - \frac{3}{(y - z)} \delta^{jk} v_0^l \partial v_0^l,$$

$$a_1^j(y)a_5^k(z) \rightarrow \frac{6}{(y - z)^2} v_0^j v_0^k + \frac{3}{(y - z)} [v_0^j \partial v_0^k + v_0^k \partial v_0^j],$$

$$a_2^j(y)a_4^k(z) \rightarrow \frac{1}{(y - z)} v_0^j v_0^l \Lambda^{jk},$$

$$a_2^j(y)a_5^k(z) \rightarrow \frac{1}{(y - z)} v_0^j \Lambda^{k[l} v_0^l,$$

$$a_4^j(y)a_5^k(z) \rightarrow \frac{2}{(y - z)^2} [\delta^{jk} v_0^l v_0^l + v_0^j v_0^k]$$

$$+ \frac{1}{(y - z)} [2 \delta^{jk} v_0^l \partial v_0^l + 2 v_0^k \partial v_0^j - \frac{1}{2} v_0^l v_0^k v_0^j w_0^l],$$

$$a_2^j(y)a_6^k(z) \rightarrow \frac{1}{2} \frac{1}{(y - z)} v_0^j \sigma^{k]}_{\alpha \alpha'} c t^{\alpha}_0 r^a,$$

$$\text{(3.11)}$$
$a^j_{(3)}(y)a^k_{(6)}(z) \rightarrow -\frac{1}{2} \frac{1}{(y-z)} v^j_0 \sigma^k_{a\dot{a}} c t^{\dot{a}}_0 r^a$

where all functions on the right-hand side of (3.11) are evaluated at $z$ and $\Lambda^{\mu\nu}$ and $a^j_1$ are defined in (3.10) and (3.9). Furthermore, one can check that

$$a^j_1(y)a^k_1(z), \quad a^j_6(y)a^k_6(z), \quad a^j_{(1)}(y)a^k_{(2)}(z), \quad a^j_{(1)}(y)a^k_{(3)}(z), \quad a^j_{(2)}(y)a^k_{(3)}(z),$$

$$a^j_{(3)}(y)a^k_{(4)}(z), \quad a^j_{(3)}(y)a^k_{(5)}(z), \quad a^j_{(4)}(y)a^k_{(6)}(z), \quad a^j_{(5)}(y)a^k_{(6)}(z)$$

have no singularities. One can now easily sum the OPE’s of (3.11) to show that $\Lambda^j_0$ defined in (2.4). So $\Lambda^j_0 = \Lambda^{j-}(y)\Lambda^{k-}(z)$ has no singularity. Furthermore, one can check that $\Lambda^j_0 \rightarrow 0$ implies that $\Lambda^j_0 \rightarrow 0$. Finally, it will be shown that $\Lambda^j_0 \rightarrow 0$. The only contribution comes from

$$(a^j_2 + a^j_3)(y)\Lambda^{k-}(z) + \Lambda^j_0(y)(a^k_2 + a^k_3)(z) \rightarrow (\frac{1}{y-z} + \frac{1}{z-y})(\delta^{jk} v^l_0 \Lambda^{l-} - \nu^j_0 \Lambda^{k-}), \quad (3.12)$$

which has no singularity. Finally, it will be shown that $\Lambda^j_0 \rightarrow 0$. The only contribution comes from

From the explicit form of $\Lambda^j_0$ in the second line of (3.3), one can check that $\Lambda^j_0 \rightarrow 0$ where $R^{jk}$ is cubic in the $(v^j_M, t^\dot{a}_M)$ variables, linear in the $(w^j_M, u^\dot{a}_M)$ variables, and does not involve $w^j_0$ or $u^\dot{a}_0$. As will be shown in section 4, $\tilde{Q}(\int N^j) = 0$ where $\tilde{Q} = \int (c T + s^a \tilde{G}^a - s^a s^a b)$ and $T$ and $\tilde{G}^a$ are defined in (2.4). So $[\int N^j, \int N^{k-}] = \int R^{jk}$ implies that $\tilde{Q}(\int R^{jk}) = 0$. But since $R^{jk}$ does not involve $w^j_0$ or $u^\dot{a}_0$, $\tilde{Q}(\int R^{jk}) = 0$ implies that $R^{jk} = 0$. To prove this, note that

$$0 = \tilde{Q}([\int v^j_0, \int R^{kl}]) = [\int (c v^j_1 + s^a \sigma^j_{a\dot{a}} t^{\dot{a}}_0), \int R^{kl}] = [\int c v^j_1, \int R^{kl}], \quad (3.13)$$

$$0 = \tilde{Q}([\int t^\dot{a}_0, \int R^{jk}]) = [\int (c t^\dot{a}_1 + s^a \sigma^j_{a\dot{a}} v^j_1), \int R^{kl}] = [\int c t^\dot{a}_1, \int R^{kl}],$$

which implies that $R^{jk}$ does not involve $w^j_0$ or $u^\dot{a}_0$. Similarly, one can argue that if $R^{jk}$ is independent of $w^j_N$ and $u^\dot{a}_N$, then it is independent of $w^j_{N+1}$ and $u^\dot{a}_{N+1}$. So $R^{jk} = 0$, which completes the proof that $N^j(y)N^{k-}(z)$ has no singularity.
4. Lorentz Invariance of BRST Operator

In this section, the BRST operator $Q'$ of (2.2) will be shown to be Lorentz invariant for a certain choice of the coefficients $A_{kln}^{MNj}$, $B_{k\bar{a}b}^{MNj}$ and $C_{\bar{a}\bar{b}k}^{MNj}$ of (3.3). Under commutation with $M^{\mu\nu}$ of (3.1), $[s^a, \sigma^a_j v_0^j s^a + ct^a_0]$ transform as the sixteen components of an SO(9,1) spinor and $[-\frac{1}{2}(c + cv^k_0v^k_0), cv^j_0, -\frac{1}{2}(c - cv^j_0v^k_0)]$ transform as the ten components of an SO(9,1) vector, so the terms $[s^a(\gamma^a d^a + (\sigma^a_j s^a v^j_0 + ct^a_0)(\gamma^+ d^a)]$ and $[\frac{1}{2}c\Pi^+ + cv^j_0\Pi^j + \frac{1}{2}cv^k_0v^k_0\Pi^+]$ in $Q'$ are easily seen to be Lorentz invariant.

Therefore, $Q'$ is Lorentz invariant if $[\int N^{\mu\nu}, \hat{Q}] = 0$ where $\hat{Q} = \int (c\hat{T} + s^a\hat{G}^a - s^a s^a b)$. One can easily check that $[\int N^{j+}, \hat{Q}] = 0$ and $[\int N^{jk}, \hat{Q}] = 0$, so the only remaining question is if one can define the coefficients in $\Lambda^{j-}$ such that $[\int N^{j-}, \hat{Q}] = 0$. Using the OPE’s of (3.3), it is straightforward to compute that

$$[\int (N^{j-} - \Lambda^{j-}), \hat{Q}] = \int (cE^j + s^a F^{aj}) \quad \text{where} \quad (4.1)$$

$$E^j = 6\partial v^1_j + v^k_1 \sum_{M=1}^{\infty} [(M + 1)\delta^{jk}v^l_M w^l_M + v^{[j}_M w^{k]}_M] \quad (4.2)$$

$$+ v^k_1 \sum_{M=0}^{\infty} [(M + \frac{3}{2})\delta^{jk}t^a_M u^a_M + \frac{1}{2}\sigma^{jk}_{\bar{a}b} t^a_M u^b_M] + \frac{1}{2}(\sigma^{jk}_{a\bar{b}} t^a_M u^b_M \sum_{M=0}^{\infty} [w^{k}_M t^{b}_M + v^{k}_M + v^{k+1}_M u^{b}_M],$$

$$F^{aj} = 3\sigma^{jk}_{aa} \partial t^a_M + \sigma^{jk}_{a\bar{b}} t^a_M \sum_{M=1}^{\infty} [(M + 1)\delta^{jk}v^l_M w^l_M + v^{[j}_M w^{k]}_M]$$

$$+ \sigma^{jk}_{b\bar{c}} t^a_M \sum_{M=0}^{\infty} [(M + \frac{3}{2})\delta^{jk}t^b_M u^b_M + \frac{1}{2}\sigma^{jk}_{\bar{a}b} t^b_M u^c_M].$$

So one needs to define the coefficients $(A^{MNj}_{k\bar{a}b}, B^{MNj}_{k\bar{a}b}, C^{MNj}_{\bar{a}b})$ such that

$$[\hat{Q}, \int \Lambda^{j-}] = \int (cE^j + s^a F^{aj}). \quad (4.3)$$

By requiring that both sides of (4.3) coincide for all terms involving either $v^j_1$ or $t^a_M$, one learns that

$$A^{1j}_{k\bar{a}b} = A^{1j}_{k\bar{a}b} = -\frac{1}{4}(M + 1)(M + 2)\delta^{jk}\delta^{lm} - \frac{1}{2}(M + 2)\delta^{jl}\delta^{km} + \frac{1}{2}(M + 1)\delta^{jm}\delta^{kl}, \quad (4.4)$$

$$B^{1j}_{k\bar{a}b} = -\frac{1}{2}(M + 2)^2\delta^{jk}\delta_{ab} - \frac{1}{2}(M + 2)\sigma^{jk}_{ab},$$
\[ B_{k\bar{a}b}^{M_{0j}} = -\frac{1}{2}(M + 3)\delta^j_k \delta_{\bar{a}b} - \frac{1}{2}(M + 1)\sigma^j_k, \]
\[ C_{\bar{a}bk}^{0Mj} = -C_{\bar{b}ak}^{M0j} = -\frac{1}{4}M\delta^j_k \delta_{\bar{a}b} - \frac{1}{4}(M + 2)\sigma^j_k. \]

The only non-trivial check is that the terms \( \int c\partial v^j \) and \( 3\int s^a\sigma_{\bar{a}a}^j \partial t^\bar{a}_0 \) on the right-hand side of (4.1) are correctly produced by \( [\widehat{Q}, \Lambda^j^-] \). The first term is obtained from

\[ \int c\partial v^j(2\sum_{M=1}^\infty A_{k\bar{l}l}^{Mj} - \sum_{M=0}^\infty B_{k\bar{a}a}^{Mj}) \]

\[ = (\int c\partial v^j)(2\sum_{M=1}^\infty (-2(M + 1)(M + 2) - \frac{1}{2}(M + 2) + \frac{1}{2}(M + 1)) - \sum_{M=0}^\infty (-4(M + 2)^2)) \]

\[ = (\int c\partial v^j)(-\sum_{M=1}^\infty (2M + 3)^2 + \sum_{M=0}^\infty (2M + 4)^2) \]

\[ = (\int c\partial v^j)(\sum_{M=4}^\infty M^2(-1)^M = (\int c\partial v^j)(6 + \sum_{M=0}^\infty M^2(-1)^M) = 6\int c\partial v^j \]

using the result of (3.8). The second term is obtained from

\[ \int (s^a\partial t^\bar{a}_0)\sigma_{\bar{a}a}^j (\sum_{M=1}^\infty B_{k\bar{a}a}^{M0j} - 2\sum_{M=0}^\infty C_{\bar{a}bk}^{0Mj}) \]

\[ = \int (s^a\sigma_{\bar{a}a}^j \partial t^\bar{a}_0)(\sum_{M=1}^\infty (-\frac{1}{2}(M + 3) - \frac{7}{2}(M + 1)) - 2\sum_{M=0}^\infty (-\frac{1}{4}M - \frac{7}{4}(M + 2)) \]

\[ = \int (s^a\sigma_{\bar{a}a}^j \partial t^\bar{a}_0)(3 + \lim_{x\to 1} \sum_{M=0}^\infty (-2M - 1)(-x)^M) \]

\[ = \int (s^a\sigma_{\bar{a}a}^j \partial t^\bar{a}_0)(3 + \lim_{x\to 1} (-\frac{2}{(1 + x)^2} + \frac{1}{1 + x})) = 3\int s^a\sigma_{\bar{a}a}^j \partial t^\bar{a}_0 \]

where we used that \((1 + x)^{-2} = -\partial x(1 + x)^{-1} = \sum_{M=0}^\infty M(-x)^{M-1} \).

Finally, the remaining coefficients in \( \Lambda^j^- \) can be determined inductively by requiring that all terms in \( [\widehat{Q}, \Lambda^j^-] \) either involve \( v^j \) or \( t^\bar{a}_0 \). This implies that

\[ A_{k\bar{l}m}^{MNj} = A_{k\bar{l}m}^{M(N-1)j} + A_{k\bar{l}m}^{(M-1)Nj} \quad \text{for } N, M > 1, \]

\[ B_{k\bar{a}b}^{MNj} = B_{k\bar{a}b}^{M(N-1)j} + B_{k\bar{a}b}^{(M-1)Nj} \quad \text{for } M > 1 \text{ and } N > 0, \]

\[ C_{\bar{a}bk}^{MNj} = C_{\bar{a}bk}^{M(N-1)j} + C_{\bar{a}bk}^{(M-1)Nj} \quad \text{for } M, N > 0. \]
Acknowledgements: NB would like to thank CNPq grant 300256/94-9, Pronex 66.2002/1998-9 and FAPESP grant 99/12763-0 for partial financial support. OC would like to thank FONDECYT grant 3000026 for financial support. This research was partially conducted during the period that NB was employed by the Clay Mathematics Institute as a CMI Prize Fellow.
References

[1] N. Berkovits, “Super-Poincaré Covariant Quantization of the Superstring,” JHEP 04 (2000) 018, [hep-th/0001035]

[2] N. Berkovits, “Cohomology in the Pure Spinor Formalism for the Superstring,” JHEP 09 (2000) 046, [hep-th/0006003]