We show that a measurement of the reaction energy ($\sqrt{s}$) dependence of relative hadron resonance yields in heavy ion collisions can be used to study the phase structure of the dense strongly interacting matter created in these collisions, and investigate the origin of the trends observed in the excitation functions of certain soft hadronic observables. We show that presence of chemical nonequilibrium in light quark abundance imparts a characteristic signature on the energy dependence of resonance yields, that differs considerably from what is expected in the equilibrium picture.

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I. INTRODUCTION

The exploration of the properties of strongly interacting, dense quark-gluon matter, and specifically, of the equation of state, transport coefficients, degree of equilibration, and phase structure, and the dependence of these on the energy and system size is one of the main objectives of heavy ion research program. A natural approach to these challenges is the study of soft particle multiplicities produced in these reactions. This provides information about the system properties when these particles are created (chemical freeze-out conditions), as well as about bulk matter properties (e.g. entropy) which can probe deep into the birth history of the fireball.

Statistical mechanics techniques have in this context a long and illustrious history [1, 2, 3, 4]. The systematic and quantitative comparison of data to the statistical hadronization (SH) model is, however, a comparatively recent field [2, 3, 4, 7, 8, 9, 10, 11]. A consensus has developed that the SH model can indeed fit most, if not all particle yields measured at experiments conducted at a wide range of energies. Measurements conducted at the GSI Schwerionen Synchrotron (SIS), BNLS’s Alternating Gradient Synchrotron (AGS), CERN’s Super Proton Synchrotron (SPS), and BNL’s Relativistic Heavy Ion Collider (RHIC) have successfully been analyzed using SH ansatze.

When this consensus is considered more carefully, we see that in technical detail the applied SH models differ regarding the chemical equilibration condition that is presumed. As a result, it has not as yet been possible to agree statistical physics, if any, is responsible for the striking trends observed in the energy dependence of some observed hadronic yields.

In this paper we will indicate that further progress can be made with help of hadron resonances. Hadron resonances, such as e.g. $\Delta^{++}$ excitation of the proton $p$ differ typically from the “stable” particle by internal structure, rather than chemical quark content. Hence within the SH approach their production is mostly controlled by the “temperature” $T$ at which they are created.

II. EQUILIBRIUM AND NON-EQUILIBRIUM FREEZE-OUT CONDITION

In the SH model there are two types of chemical equilibrium [2]: all models assume relative chemical equilibrium, but some also assume absolute chemical equilibrium which implies the presence of just the right abundances of valance up, down, and even strange quark pairs. There are qualitative differences in the results obtained in the description of hadron production with or without using the hypothesis of absolute chemical equilibrium: if the system of produced hadrons is considered to be in absolute chemical equilibrium, then at highest heavy ion reaction energy one obtains chemical freeze-out temperature $T \sim 160 – 170$ MeV. Values as low as $T \sim 50$ MeV are reported at lowest reaction energies available.

The energy dependence of the freeze-out temperature than follows the trend indicated in panel (a) of figure [III] as the collision energy increases, the freeze-out temperature increases and the baryonic density (here baryonic chemical potential $\mu_B$) decreases [8]. An increase of freeze-out temperature with $\sqrt{s}$ is expected on general grounds, since with increasing reaction energy a greater fraction of the energy is carried by mesons created in the collision, rather than pre-existing baryons [19].

Further refinements in the approach described above are often implemented and could be of relevance:

- Allowance for strangeness chemical nonequilibrium is necessary to obtain a good description of strange particle yields [13, 14, 15] at low $\sqrt{s}$. This is accomplished by introducing strangeness phase space occupancy $\gamma_s$.
FIG. 1: (Color online) Dependence of freeze-out temperature $T$ and baryo-chemical potential $\mu_B$ on reaction energy in the Equilibrium (panel (a), [8]) and non-equilibrium (panel (b), [9]) freeze-out models. The direction of the arrow corresponds to increasing $\sqrt{s}$. The equilibrium dependence of $T$ and $\mu_B$ in the panel (a) is not significantly altered by the introduction of the fitted phase space occupancy $\gamma_s$ and/or the implementation of the Canonical ensemble for strangeness. The “star” in panel (b) corresponds to the point where the transition to the supercooled regime occurs and the phase space changes from chemically under-saturated ($\gamma_q < 1$) to chemically over-saturated ($\gamma_q > 1$). This point also corresponds to the energy of the “kink” and the tip of the “horn”.

- At smaller reaction energies and for smaller reaction system sizes, it is likely that the fireball is well away from the thermodynamic limit. In this case canonical treatment of strangeness is applied [16, 17]. These effects do not materially alter the behaviour of temperature and chemical potential shown in the panel (a) of Fig. 1.

What is most striking in these results is that there is no sign of any structure when the reaction energy varies. However, there are non-continuous features in the energy dependence of hadronic observables, such as the “kink” in the multiplicity per number of participants and the “horn” [20, 21, 22] in certain particle yield ratios. An effort was made to interpret this in terms of a shift from baryon to meson dominance [22] of the hadron yields. However no matter how hard one tries, in the chemical equilibrium model even the simple observable like $K^+/\pi^+$ remains a smooth function of reaction energy, in contrast to the experimental results. Introduction of $\gamma_s$ and deviations from the thermodynamic limit, while they help in bringing some of the model predictions closer to the data, has so far not managed to reproduce the sharpness of features such as the kink and the horn.

Non-monotonic behaviour of particle yield ratios could indicate a novel reaction mechanism, e.g. onset of the deconfinement phase [21]. In such a situation, the smoothness of the chemical freeze-out temperature dependence on energy would be surprising, since it would imply that at all energies, from about 1 $A$ GeV at SIS, to the highest RHIC values, there is no change in either the fireball evolution dynamics, nor any other imprint from the deconfined phase on the freeze-out condition, which, however is visible in the strangeness and entropy yield that $K^+$ and, respectively, $\pi^+$ represent.

Furthermore, we note that the fireball of hadronic matter formed is a relatively small system, expanding rapidly, with its content undergoing a phase transformation, or even phase transition. In this complex and rapidly evolving circumstance, one could imagine that the absolute chemical equilibrium, not always, or even ever, holds. In particular, if the expanding system undergoes a fast conversion from a Quark Gluon Plasma (QGP) to hadrons, chemical non-equilibrium [9], and super-cooling [23, 24] go hand in hand, due to entropy and flavor conservation requirements.

One can look at this situation again removing the hypothesis of absolute chemical equilibrium among hadrons produced. The systematic behaviour of $T$ with energy in this case is quite different [9], as is shown in panel (b) of figure 1. The two higher $T$ values at right are for 20 (lowest SPS) and (most to right) 11.6 $A$ GeV (highest AGS) reactions. In these two cases the source of particles is a hot chemically under-saturated ($T \sim 170$ MeV) fireball. Such a system could be a conventional hadron gas fireball that had not the time to chemically equilibrate. Other options were considered in Refs. [8, 25], such as a phase of constituent massive quarks.
Following the thick arrow in panel (b) of figure 1, we note that somewhat smaller temperatures are found with further increasing heavy ion reaction energies. Here it is possible \( \gamma_1 \geq 1 \) to match the entropy of the emerging hadrons with that of a system of nearly massless partons when one considers supercooling to \( T \sim 140 \text{ MeV} \), while both light and strange quark phase space in the hadron stage acquire significant over-saturation with the phase space occupancy \( \gamma_{u,d} > 1 \) and at higher energy also \( \gamma_s > 1 \). A drastic change in the non-equilibrium condition occurs near 30 A GeV, corresponding to the dip point on right in panel (b) of the figure 1 (marked by an asterisk). At heavy ion reaction energy below (i.e. to right in panel (b) of figure 1), hadrons have not reached chemical equilibrium, while at this point, as well as, at heavy ion reaction energy above (i.e. at and to left in panel (b) of figure 1), hadrons emerge from a much denser and chemically more saturated system, as would be expected were QGP formed at and above 30 A GeV. This is also the heavy ion reaction energy corresponding to the “kink”, which tracks the QGP’s entropy density (higher w.r.t. a hadron gas), and the peak of the “horn” \( \gamma_2 \), which tracks the strangeness over entropy ratio (also higher w.r.t. a hadron gas).

Concluding this discussion, comparing panel (a) with (b) we see in figure 1 a quite different behaviour. On panel (a), for the chemical equilibrium model, with increase of the collision energy following the black arrow, we see monotonic increase of the chemical freeze-out temperature, with no hint of new physics in a wide range of heavy ion collision energies spanning the range of SIS, AGS, SPS and RHIC. On panel (b), we see that when relative and absolute chemical equilibrium is considered \( \gamma_1 \geq 1 \), with yields of individual hadrons satisfying the relative, but not the absolute chemical equilibrium, the experimental particle yield data is best described with a temperature profile as function of reaction energy which is not monotonic. There is a minimum value of \( T \), at the point when the rapid change of the chemical composition of produced hadrons is occurring. This is clearly suggestive of a change in the reaction mechanism.

The main reason for the wider acceptance of the equilibrium approach \( \gamma_1 = 1 \) is its greater simplicity, there are fewer parameters. Moreover, considering the quality of the data the non-equilibrium parameter \( \gamma_q \) is not necessary to pull the statistical significance above it’s generally accepted minimal value of 5 %. On the other hand, the parameters \( \gamma_{u,d} \) and \( \gamma_s \) were introduced on physical grounds \( [12, 18, 22] \), thus these are not arbitrary fit parameters. Moreover, these parameters, when used in a statistical hadronization fit, converge to theoretically motivated values. They also help to explain the trends observed in the energy dependence of hadronic observables. Finally, \( \gamma_q > 1 \) in AA reactions describes the enhancement of the baryon to meson ratio yield at RHIC, as compared to elementary interactions, which dynamically arises in the recombination hadronization at fixed hadronization temperature.

III. RESONANCE RATIOS AS CHEMICAL FREEZE-OUT TEMPERATURE PROBES

In this paper, we propose the energy dependence of the resonance yields as a possible experimental observable, capable to discriminate the two scenarios, chemical equilibrium and non-equilibrium, and thus to establish the need to use \( \gamma_q \) in statistical hadronization analysis of experimental data.

Many strong interaction resonances, a set we denote by the collective symbol \( R^* \) (such as \( K^{*0}(892), \Delta(1232), \Sigma^*(1385), \Lambda^*(1520), \Xi^*(1530) \) \( [20] \) carry the same valance quark content as their ground-state counter-parts \( R \) (corresponding: \( K, N, \Sigma, \Lambda, \Xi \)), \( R^* \) typically decay by emission of a pion, \( R^* \rightarrow R + \pi \). Considering the particle yield ratio \( R^*/R \) in the Boltzmann approximation (appropriate for the particles considered), we see that all chemical conditions and parameters (equilibrium and non-equilibrium) cancel out, and the ratio of yields between the resonance and it’s ground state is a function of the masses, and the freeze-out temperature, with second order effects coming from the cascading decays of other, more massive resonances \( \Xi^* \)\( [33, 34] \):

\[
\frac{N_{R^*}}{N_R} \approx \frac{g_R W(m_R)}{g_R W(m_{R^*})} + \sum_{j \rightarrow R} b_{jR} g_j W(m_j) \quad (1)
\]

where \( W(x) = x^2 K_2(x) \) is the (relativistic) reduced one particle phase space, \( K_2(x) \) being a Bessel function, \( g_j \) is the quantum degeneracy, and \( b_{ji} \) is the branching ratio of resonance \( j \) decaying into \( R \).

When we study the results arising from Eq. (1), we consider only strong decay contributions, weak decay feed-down, such as \( \Lambda \rightarrow p, \Sigma \rightarrow p, \Xi \rightarrow \Lambda, \) and \( \Omega \rightarrow \Xi \) has to be eliminated from the data sample. Given that existing SPS \( [27] \) and RHIC \( [28] \) experiments, as well as the planned Compressed Baryonic Matter (CBM) experiment \( [29] \) have both a tracking resolution permitting precise primary vertex cuts (weak decay tracks originate from points well away from the primary vertex), as well as a momentum resolution capable of identifying resonances \( [30, 31, 32] \), this requirement is realistic.

Because of the radically different energy dependence of freeze-out temperature in the scenarios of \( \delta \) and \( \gamma \), seen in figure 1, the prediction for the resonance ratios Eq. (1) vary greatly between these two scenarios. In the equilibrium scenario the temperature goes up with heavy ion reaction energy, and thus the resonance abundance should go smoothly up for all resonances. On the other hand, the nonequilibrium scenario, with a low temperature arising only in some limited reaction energy domain, will lead to resonance abundance which should have a clear dip at that point, but otherwise remain relatively large.

We have evaluated several resonance relative ratios shown in figure 2 within the two scenarios, using the statistical hadronization code SHARE \( [33, 34] \). For the non-equilibrium scenario, we have used the parameters given...
in [8], table I. For the equilibrium scenario, we used the parametrization given in [8] figures 3 and 4. In the latter case, the strangeness and isospin chemical potentials $\mu_s$ and $\mu_{I3}$ were obtained by requiring that net strangeness be zero, and net charge per baryon in all particles produced be the same as in the colliding system. We have performed spot checks of the validity of the statistical parameters used and found that under the assumptions made these are the best parameter sets.

As seen in figure 2, the expected trend with $T$ is apparent in all considered resonance ratios, though in cases where the mass difference is large, the effect is much more pronounced than in some others. Indeed, for many of the ratios we present the experimental error may limit the usefulness of our results, however, the opposite energy dependence may prove to be helpful in discriminating the behaviour. In principle we have presented 12 different ratios, though in some cases the difference between particle and antiparticle ratio is small. It is smallest in cases when the baryochemical potential has neither a direct, nor a large indirect influence, and the difference in light quark flavor content is the smallest.

It is interesting to note here that the often ignored quark flavor effect (isospin effect) is responsible for much of the difference between particle and antiparticle ratios. This is at first a counter intuitive result, but it can be understood in quantitative simple manner. We recall the relation $u/d \propto \lambda_{I3} \propto d/u$ where $u, d$ refer to the yield of up, and, respectively, down valence quarks. In cases, such as e.g. $\Sigma^* \rightarrow \Lambda$ the $u, d$ valance quark content is different for the $R^*$ and $R$ particles, leading to $\lambda_{I3}$ dependence of the $R^*/R$ ratio. High mass cascading resonances, where the strangeness content can be different (e.g. $\Xi(1690) \rightarrow K\Sigma$), are a further source of difference between particle and antiparticle resonance ratios, especially so in the high baryo-chemical potential regime.

Many of the experimental data points needed in the analysis presented in [8] and [9] are to this day still preliminary. This means that some of the results we rely on could in the end be somewhat different. However, because of the cancellation (to a good approximation) of the baryo- and strangeness chemical potentials, the qualitative prediction for the energy dependence of the resonance yields within the two models is robust. Namely, within the chemical equilibrium model the temperature of chemical freeze-out must steadily increase and so does the $R^*/R$ ratio. For the chemical non-equilibrium model the $R^*/R$ dip primarily relies on the response of $T$ to the degree of chemical equilibration: prior to chemical equilibrium for the valance quark abundance, at a relatively low reaction energy, the freeze-out temperature $T$ is relatively high. At a critical energy, $T$ drops as the hadron yields move to or even exceed light quark chemical equilibrium, yet reaction energy is still not too large, and thus the baryon density is high and meson yield low. As reaction energy increases further, $T$ increases and the $R^*/R$ yield from that point on increases. The drop in $R^*/R$ at critical $T$, would be completely counter-intuitive in an equilibrium picture. It would hence provide overwhelming evidence that non-equilibrium effects such as supercooling, where such a drop would be expected, are at play. We further argue that such a drop can not be reproduced by resonance rescattering/regeneration.

Pseudo-elastic processes such as $R^n \rightarrow R^* \rightarrow R\pi$ and post-decay $R^* \rightarrow R\pi$ scattering of decay products in matter could potentially considerably alter the observable final ratio of detectable $R^*$ to $R$. The combined effect of rescattering and regeneration has not been well understood. We have argued that the formation of additional detectable resonances is negligible [35], while scattering of decay products can decrease the visible resonance yields except for sudden hadronization case. Other groups have studied this in quantitative manner.

Assuming a long lived hadron phase, the energy dependence of most of the resonance ratios considered here has been calculated in a hadronic quantum molecular dynamics model. The result (figure 7 and 8 in [36]) is qualitatively similar to the chemical equilibrium results for resonance ratios, we see a smooth rise with energy. Thus, in the case of chemical equilibrium, with a considerable separation between chemical and thermal freeze-out inherent in Ref. [35], rescattering and regeneration will affect the quantitative $R^*/R$ ratio, but will not alter the dependence on heavy ion reaction energy shown in figure 2. On the other hand, chemical nonequilibrium implies absence of a long lived hadron phase. Because of this, the calculation [36] would not be applicable and the resonance abundance should be in closer quantitative, as well as qualitative agreement, with the predictions of figure 2. Rescattering and regeneration, therefore, should not alter the predicted pattern of either the equilibrium model, where all $R^*/R$ should rise with energy, or the non-equilibrium model, where at the critical energy all $R^*/R$ should experience a dip.

If both of these predictions prove inaccurate, and the $R^*/R$ abundance turns out to be resonance specific with no uniform rises and dips as function of energy, this would signify that freeze-out is determined by reaction kinetics rather than thermodynamic conditions... In this case, $R^*$ abundance is determined more by $\Gamma_{R^*}\tau$, where $\tau$ is the lifetime of the fireball, than by $m_{R^*}/T$.

IV. CONCLUSIONS

We have argued that a measurement of the energy dependence of ratios such as $K^*/K$, $\Delta/\rho$, $\Sigma^*/\Lambda$, $\Lambda(1520)/\Lambda$, $\Xi^*/\Xi$ and other such ratios might be instrumental in clarifying the freeze-out conditions in heavy ion collisions, especially at low reaction energy. A resonance abundance monotonically rising with energy from the AGS energy range would suggest that the best statistical description of heavy ion data is based on chemical equilibrium, and that as collision energy increases, freeze-out temperature rises monotonically. If, on the other hand, resonance abundance shows a consistent dip, possibly at
FIG. 2: (Color online) Ratio of resonance to the stable particle. Thick lines for particles with strange quark content, thin lines for particles with anti-strange quark content, as a function of energy. Solid black lines refer to the equilibrium fits ($\gamma_{q,s} = 1$), with the parameters for AGS and SPS energies taken from [8]. Dashed red lines refer to non-equilibrium fits ($\gamma_{q,s}$ fitted), with the best fit parameters for AGS and SPS energies taken from [9].
the energy coinciding with the other non-monotonic features recently observed in particle yields and ratios \cite{20}, it would be a strong evidence that what we are seeing is, at and above this dip, a freeze-out from a supercooled high entropy density phase. GT would like to thank C. Gale and S. Jeon for helpful discussions. Work supported in part by grants: from the Natural Sciences and Engineering research council of Canada, the Fonds Nature et Technologies of Quebec, the U.S. Department of Energy DE-FG02-04ER413, and the Tomlinson foundation.

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