Medium Baseline Reactor Neutrino Experiments
with 2 Identical Detectors

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Abstract

In the next 10 years medium baseline reactor neutrino experiments will attempt to determine the neutrino mass hierarchy and to precisely measure $\theta_{12}$. Both of these determinations will be more reliable if data from identical detectors at distinct baselines are combined. While interference effects can be eliminated by choosing detector sites orthogonal to the reactor arrays, one of the greatest challenges facing a determination of the mass hierarchy is the detector’s unknown energy response. By comparing peaks at similar energies at two identical detectors at distinct baselines, one eliminates any correlated dependence upon a monotonic energy response. In addition, a second detector leads to new hierarchy-dependent observables, such as the ratio of the locations of the maxima of the Fourier cosine transforms. Simultaneously, one may determine the hierarchy by comparing the $\chi^2$ best fits of $\Delta M^2_{32}$ at the two detectors using the spectra associated to both hierarchies. A second detector at a distinct baseline also breaks the degeneracy between $\theta_{12}$ and the background neutrino flux from, for example, distant reactors and increases the effective target mass, which is limited by current designs to about 20 kton/detector.

May 5, 2014

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1 Motivation

10 years ago Petcov and Piai suggested that a medium baseline reactor neutrino experiment can determine the neutrino mass hierarchy [1] which is manifested as a subtle shift in the locations of peaks in the neutrino spectrum resulting from 1-3 oscillations. Early studies of this proposal found that it requires an unparalleled precision and an enormous detector [2, 3]. The situation changed with the recent demonstrations [4, 5, 6] that 1-3 oscillations are as much as an order of magnitude larger than had been believed just a year earlier. Using the new value of $\theta_{13}$ the analysis of Ref. [7] found that the hierarchy could now be determined with a 20 kton detector, which is about the largest that a sufficiently precise detector consisting of two concentric spheres can be. However it found that, with detectors at the locations proposed in Refs. [8, 9] neutrinos arriving from reactors at multiple baselines would erase the low energy 1-3 oscillations, diminishing the significance of a determination of the hierarchy. This problem could be resolved if the detector is placed perpendicular to a reactor array, but at the cost of using the flux from one reactor array instead of two and so increasing the statistical errors. Another study [10] found that a determination of the hierarchy requires a determination of the nonlinear energy response of the detectors to a better precision than has ever been achieved.

Just as Daya Bay and RENO were able to significantly reduce their systematic errors by relying only upon the relative flux observed at distinct baselines, in the present note we will discuss how certain relative measurements with two identical detectors at distinct baselines are insensitive to the detector’s correlated energy response, as has been suggested in Refs. [7, 11, 12, 13]. For example we will show that the sign of the energy difference between two peaks in the spectra observed at the two detectors can provide a determination of the neutrino mass hierarchy which is independent of the correlated energy responses of the detectors. We also will introduce other 2 additional two-detector observables which are sensitive to the hierarchy and reasonably insensitive to the detector’s correlated energy response.

While it may be difficult to build a sufficiently precise single detector with target mass larger than 20 kton, a two detector design can provide 40 kton of target mass. A second detector also yields new observables which can be used to determine the hierarchy. For example, the ratio of the locations of the global maxima of the Fourier cosine transforms is not only sensitive to the hierarchy, but also reasonably nondegenerate with the observables defined in Refs. [15, 12]. In addition one may use a $\chi^2$ fit to the spectra with both hierarchies
to determine $\Delta M_{32}^2$ at both detectors, the correct hierarchy yields the most compatible values for $\Delta M_{32}^2$. Both of these observables are robust in that they are independent of the overall normalization of the detector’s energy response.

We will begin in Sec. 2 with a review of reactor neutrino oscillations and how they are affected by the hierarchy. In Sec. 3 we will explain how a comparison of peak locations at similar energies but distinct baselines may be used to determine the neutrino mass hierarchy with two detectors that have a potentially large unknown correlated nonlinear energy response. In Sec. 4, we show that the neutrino mass hierarchy can be determined from the ratio of the oscillation frequencies at the two detectors. In Sec. 5 we describe how a $\chi^2$ fit of $\Delta M_{32}^2$ at both detectors provides yet another determination of the hierarchy. In Sec. 6 we explain that the presence of a second detectors can break the degeneracy between the background electron antineutrino flux and $\theta_{12}$. Finally in Sec. 7 we will discuss future directions, including the incorporation of these ideas in a more complete simulation of a reactor experiment.

2 The electron survival probability

Consider for simplicity a single reactor and two detectors at baselines $L$ and $3L/2$. Optimal values of $L$ will be between 30 and 40 km. The electron neutrino weak interaction eigenstate $|\nu_e\rangle$ is not an energy eigenstate $|k\rangle$, but it can be decomposed into a real sum of energy eigenstates

$$|\nu_e\rangle = \cos(\theta_{12})\cos(\theta_{13})|1\rangle + \sin(\theta_{12})\cos(\theta_{13})|2\rangle + \sin(\theta_{13})|3\rangle.$$  \hspace{1cm} (2.1)

In the relativistic limit, after traveling a distance $L$, the survival probability of a coherent electron (anti)neutrino wavepacket with energy $E$ can be expressed in terms of the mass matrix $M$

$$P_{ee} = |\langle \nu_e | \exp \left( i \frac{M^2 L}{2 E} \right) | \nu_e \rangle |^2 \hspace{1cm} (2.2)$$

$$P_{12} = \sin^2(2\theta_{12})\cos^4(\theta_{13})\cos \left( \frac{\Delta M_{21}^2 L}{2E} \right), \quad P_{13} = \cos^2(\theta_{12})\sin^2(2\theta_{13})\cos \left( \frac{\Delta M_{31}^2 L}{2E} \right)$$

$$P_{23} = \sin^2(\theta_{12})\sin^2(2\theta_{13})\cos \left( \frac{\Delta M_{32}^2 L}{2E} \right)$$

where $\Delta M_{ij}^2$ is the mass squared difference of mass eigenstates $i$ and $j$. 

2
3 Comparing peak locations

3.1 The peak location

The survival probability at large distances is dominated by 1-2 oscillations, described by $P_{12}$ with a fine structure of smaller $P_{13}$ oscillations which are slightly perturbed by the yet smaller $P_{23}$ oscillations. This fine structure is used to determine the hierarchy as can be seen in Fig. 1. If $P_{23}$ were constant, then the fine structure peaks would be determined entirely by $P_{13}$ and so would be periodic in $L/E$-space. The $n$th peak in the neutrino spectrum\(^1\), corresponding to neutrinos that have oscillated $n$ times, would be located at the energy

$$E^{(0)}_n(L) = \frac{|\Delta M^2_{31}|L}{4\pi n}. \quad (3.1)$$

However the $P_{23}$ oscillations deform this periodicity, shifting the $n$th peak to the energy

$$E_n(L) = \frac{|\Delta M^2_{31}|L}{4\pi (n \pm \alpha_n)}. \quad (3.2)$$

\(^1\)The zeroth peak corresponds to infinite energy, at which neutrinos have not oscillated. Higher $n$ corresponds to lower neutrino energy. The first few peaks are invisible at medium baselines due to the low reactor flux at the corresponding high energies.
where the plus (minus) sign corresponds to the normal (inverted) hierarchy and the perturbations $\alpha_n$ can be determined using Eq. (2.2). The $\alpha_n$ depend weakly upon unknown neutrino mixing parameters, but as a determination of the hierarchy is equivalent to a determination of the sign with which $\alpha_n$ enters (3.2), this small uncertainty in its value will be irrelevant.

3.2 Comparing the 10th and 15th peaks

It follows from Eq. (3.2) that the ratio of the energy of the 15th peak at $3L/2$ to that of the 10th peak at $L$ is

$$\frac{E_{15}(3L/2)}{E_{10}(L)} = \frac{3(10 \pm \alpha_{10})}{2(15 \pm \alpha_{15})} \sim 1 \pm \frac{3\alpha_{10} - 2\alpha_{15}}{30}. \tag{3.3}$$

For ease of comparison with previous studies we will use old values of the neutrino mass matrix parameters

$$\Delta M_{21}^2 = 7.59 \times 10^{-5} \text{ eV}^2, \quad |\Delta M_{32}^2| = 2.4 \times 10^{-3} \text{ eV}^2, \quad \sin^2(2\theta_{12}) = 0.8675, \quad \sin^2(2\theta_{13}) = 0.092. \tag{3.4}$$

Then the relevant $\alpha_n$ can be read from Fig. 1 of Ref. [12]

$$\alpha_{10} = 0.072, \quad \alpha_{15} = 0.035 \tag{3.5}$$

which can be substituted into Eq. (3.3) to yield the energy ratio

$$\frac{E_{15}(3L/2)}{E_{10}(L)} = 1.000 \pm 0.005 \tag{3.6}$$

where again the positive (negative) sign corresponds to the normal (inverted) hierarchy.

Therefore we have learned that in the case of the normal (inverted) hierarchy the energy of the 15th peak at the far detector will be 0.5% higher (lower) than that of the 10th peak at the near detector. A 0.5% difference is small and difficult to measure with limited statistics and detector response. By the 17th peak this difference increases to 1%, but the detector resolution is reduced at these energies. However the important point is that since the energy response of the detector is monotonic with respect to the true energy, the observed energy of the far detector peak will be greater (less) than the observed energy at the near detector if and only if the true energy is greater (less) which indicates a normal (inverted) hierarchy. Thus this determination of the hierarchy is independent of the unknown correlated nonlinear energy response of the detector.

3.3 How to find the 10th and 15th peaks

There is however one complication. In order to use this technique, one needs to be able to identify the 15th peak at the far detector and the 10th peak at the near detector. How
can this be done? The positions of these peaks depend on $|\Delta M_{32}^2|$ whose measurement at MINOS has error bars only half as large as the distance between the 15th and 16th peaks.

To find these peaks, we suggest that one begin with the 6th peak at the near detector. This is easy to find, using Eq. (3.2) it lies at an energy of

$$E_6(L) = \frac{|\Delta M_{31}^2|L}{4\pi(6 \pm \alpha_6)} \sim \frac{|\Delta M_{31}^2|L}{4\pi(6 \pm 0.052)}.$$ (3.7)

The relative energy difference between the 6th and 7th peak is 17%, about four times larger than the error with which $|\Delta M_{32}^2|$ is known from MINOS. To better estimate the errors, one can use the asymptotic form of $\alpha_n$ at low values of $n$ [18]

$$E_n(L) = \frac{\Delta M_{\text{eff}}^2 L}{24\pi}.$$ (3.8)

where

$$\Delta M_{\text{eff}}^2 = \cos^2(\theta_{12})|\Delta M_{31}^2| + \sin^2(\theta_{12})|\Delta M_{32}^2|.$$ (3.9)

The combination $\Delta M_{\text{eff}}^2$ is determined by MINOS with an error of about 5%. Thus the 6th peak at the near detector can be distinguished from the 7th peak at the 3$\sigma$ level. Including a nonlinear response of 2 to 3 percent the reliability of this determination is reduced to the 2$\sigma$ level.

Fortunately, the determination can be made more precise. For one, NO$\nu$A is likely to measure $|\Delta M_{32}^2|$ more precisely before Daya Bay II produces results. More importantly, while the 6th peak at the near detector corresponds to the 9th peak at the far detector, the 5th and 7th peaks both correspond to minima. Thus a misidentification of the 6th peak at the near detector can be discovered if, at the same energy, there is a minimum in the far detector flux. Again these relative comparisons between the near and far detector flux are independent of the unknown nonlinear response to the extent that the near and far detectors are identical.

Thus it seems possible to identify the sixth peak of the near detector with about 4$\sigma$ of certainty. To find the tenth peak, one then needs to count peaks. One knows the true energy difference between peaks from Eq. (3.2), although this can be somewhat distorted by nonlinearities. While counting peaks at the far detector may be difficult in practice due to the low flux, counting peaks at the near detector at baselines of less than 40 km is feasible, although perhaps less reliable than a method which uses all of the intermediate information like a $\chi^2$ fit.

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2The hierarchy makes little difference at such low values of $n$ because $\alpha_n$ is very close to linear, indeed $\alpha_9/\alpha_6$ is 1.42.
Figure 2: Theoretical spectra of 6 years of neutrinos at 20 kton detectors at baselines 36 km (black) and 54 km (blue) from a 23.2 GW reactor. The vertical axis is the number of neutrinos expected in each 30 keV bin, although the far detector flux has been tripled to render its features visible in this figure. A detector resolution of $2.5\%/\sqrt{E/\text{MeV}}$ is assumed. The 6th peak at the near detector lies at virtually the same energy as the 9th peak at the far detector. However the 15th far peak is at a slightly lower energy than the 10th near peak, indicating an inverted hierarchy.

3.4 The Procedure

Summarizing, the determination of the hierarchy is a four step process:

**Step 1:** Use Eq. (3.2) to identify the 6th peak at the near detector.

**Step 2:** Check that at the same energy one finds a peak at the far detector, if not, the sixth peak at the near detector is the second nearest peak to the result of Eq. (3.2).

**Step 3:** Using the expected distance between peaks from Eq. (3.2) and the location of the sixth peak found above, count peaks to find the tenth peak at the near detector.

**Step 4:** The measured energy of the 15th peak at the far detector will either be 0.5% higher or lower than the measured energy of the 10th peak at the near detector. If it is higher (lower) than the neutrino mass hierarchy is normal (inverted).

These steps are illustrated on the theoretical spectra, which exclude statistical errors, in Fig. 2. This figure contains the expected spectra at 36 and 54 km in the case of the inverted hierarchy assuming an optimistic energy resolution. The nonlinear detector response is not included in this figure. To see the effects of statistical errors, we have included the results of a simulation with the same parameters in Fig. 3. Here we have considered 6 years live time of neutrinos arising from a reactor complex with a thermal capacity of 23.2 GW, corresponding
Figure 3: Typical simulated spectra of 6 years of neutrinos at 20 kton detectors at baselines 36 km (black) and 54 km (blue) from a 23.2 GW reactor. The red curves are the theoretical spectra of Fig. 2. The vertical axis is the number of neutrinos expected in each 30 keV bin, although the far detector flux normalization has been rescaled by a factor of 3 in the figure to render its features visible. A detector resolution of $2.5\% / \sqrt{E/\text{MeV}}$ is assumed. While the peaks discussed in this paper are visible, their positions are difficult to determine by eye to within the required 0.5%.

roughly to the pairs of reactors at Daya Bay, Ling Ao I and Ling Ao II plus the planned pair Ling Ao III. Each detector is assumed to be 20 ktons. While the construction of a Daya Bay II like detector, made from concentric spherical shells, larger than 20 ktons seems difficult if not impossible, the construction of 2 detectors allows an effective target mass of 40 ktons and so, in addition to helping with the nonlinearity problem, also decreases the fractional statistical errors in the neutrino flux. We have simulated detector locations perpendicular to the reactor array\textsuperscript{3}, so that the baselines to various reactors are essentially identical, thus avoiding the interference effects of Ref. [12].

4 Comparing oscillation frequencies

We have seen that a comparison of individual peak energies can be independent of the correlated detector nonlinear energy response. In practice, given the long baselines which are necessary, a determination of the hierarchy will be strongly limited by statistics. Therefore the best determination of the hierarchy uses information from the entire spectrum, not just from a few peaks. While the determination described above can and should be applied to

\textsuperscript{3}In the case of Daya Bay this would correspond roughly to HuaShan and GuanYin mountain park respectively.
every visible peak, the highest sensitivity to the hierarchy arises at low energies where the individual peaks are difficult to identify.

There are two known ways to take advantage of the information stored in the entire spectrum, including the invisible peaks. One can use a Fourier transform to sum them together but a different direction to Ref. [17] or one may perform a $\chi^2$ analysis. In the next two sections we will describe new, hierarchy-dependent observables which are provided by a second detector using these two methods respectively.

### 4.1 The oscillation frequencies

The Fourier cosine transform of the observed neutrino spectrum $\Phi(L/E)$ at energy $E$ at the $i^{th}$ detector, depicted in Fig. 4 in the case of a $L = 60$ km baseline, is

$$F(k) = \int d\left( \frac{L}{E} \right) \Phi(L/E) \cos \left( \frac{kL}{2E} \right). \quad (4.1)$$

This transform exhibits a global maximum at a frequency $\Delta M^2_i$ near $|\Delta M^2_{31}|$ [15]. The location of this maximum can be determined using the results of Ref. [12].

At baselines of 30 km or less, neutrinos of high enough energy to be observed using inverse $\beta$ decay have oscillated less than 10 times. These first oscillations occur with a frequency determined entirely by $\Delta M^2_{\text{eff}}$ [18] and not by the hierarchy. The maximum of the Fourier cosine transform of the near detector spectrum lies at just this frequency

$$\Delta M^2_{\text{near}} = \Delta M^2_{\text{eff}} = \cos^2(\theta_{12}) |\Delta M^2_{31}| + \sin^2(\theta_{12}) |\Delta M^2_{32}|. \quad (4.2)$$

On the other hand, at a baseline of 60 km one can in principle detect at least the first 16 oscillations. The average $L/E$ frequency of the first 16 oscillations is $|\Delta M^2_{31}|$ therefore at a far detector the maximum of the Fourier cosine transform lies at approximately

$$\Delta M^2_{\text{far}} \sim |\Delta M^2_{31}|. \quad (4.3)$$

The precise location of the maximum depends on just how many peaks can be discerned at the detector. At low energies the detector’s energy resolution leads to a decrease in the peak amplitudes, and the result is that the low energy peaks contribute less to the nonzero frequency part of the Fourier transform. The low energy peaks of the spectrum drive the maximum of the Fourier transformed spectrum from $\Delta M^2_{\text{eff}}$ towards and possibly beyond $|\Delta M^2_{31}|$. Thus the better the energy resolution, the further $\Delta M^2_{\text{far}}$ will be from $\Delta M^2_{\text{eff}}$.

The hierarchy is determined by the ratio

$$f_{\text{dis}} = \frac{\Delta M^2_{\text{near}}}{\Delta M^2_{\text{far}}} \sim \frac{\Delta M^2_{\text{eff}}}{|\Delta M^2_{31}|} = 1 + \sin^2(\theta_{12}) \left( \frac{|\Delta M^2_{31}| - |\Delta M^2_{32}|}{|\Delta M^2_{31}|} \right). \quad (4.4)$$
Roughly speaking, a value greater than 1 indicates the normal hierarchy and less than one indicates the inverted hierarchy. In practice the resolution determines an overall shift in $\Delta M^2_{\text{far}}$ and so the threshold value will not be precisely equal to 1, but can be determined for example via simulations.

4.2 Results

As an illustration of this method, consider the neutrinos from a set of reactors with a total thermal capacity of $\sim$32 GW all of which are 30 km from a 20 kton near detector and 60 km from an identical far detector. We have simulated 500 experiments assuming each hierarchy, with a detector visible energy resolution of $3\%/\sqrt{E/\text{MeV}}$. We have assumed 50k events at the far detector and 200k at the near site. This corresponds to roughly $\sim$3 years live time$^4$, neglecting the decrease in total flux due to 1-2 oscillations. We have not considered the unknown nonlinear detector response and errors here.

The distribution of $f_{\text{dis}}$ is shown in Fig. 5. The dashed line is the identification cut, which appears to be very close to 1. The resulting probabilities of success are summarized in Table. 1, where they are compared with the peak-valley (RL+PV) analysis of Ref. [3] using one or two 20 kton detectors at 60 km. While there is no reason to expect a perfect symmetry, the large asymmetry between the chance of success in the case of the two hierarchies may

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$^4$The large backgrounds expected at such an experiment [19] may require severe cuts and thus an imperfect detector efficiency, as a result 3 years of live time may require appreciably more than 3 years of running at sites with only 500 meters of rock overburden.
Figure 5: Histogram of the number of simulated experiments yielding each value of $f_{\text{dis}}$ for a near baseline of $L_{\text{near}} = 30$ km and a far baseline of $L_{\text{far}} = 60$ km, Blue curve: normal hierarchy; Red curve: inverted hierarchy. The dashed line is the threshold which yields the hierarchy determination.

be an artifact of the samples of experiments considered\(^5\).

In the third row of Table. 1 we consider a combination of the 20 kton 60 km RL+PV analysis and the oscillation frequency analysis presented here. This combination, which uses a 20 kton detector at 30 km and at 60 km, significantly outperforms the two 20 kton detectors at 60 km analyzed in the fourth row. The strong improvement attained by combining the RL+PV and oscillation frequency methods is a result of the fact that they are only weakly degenerate, as can be seen in Fig. 6.

As our method relies only upon the relative positions of the peaks in the Fourier transformed spectra, it is not affected by the uncertain value of $|\Delta M^2_{32}|$ [17]. Furthermore, it was shown in Ref. [20] that the Fourier analysis is not affected by a rescaling of the energy and hardly affected by a constant energy shift, and therefore is robust with respect to a non-linear energy response of the form $E_{\text{rec}} = a \times E + b$ and even reasonably independent of the reactor flux model. We also have confirmed that the oscillation frequency method is more sensitive to statistics especially at the far site, for example if we have more than 100k events at far site then the oscillation frequency method outperforms the Fourier peak-valley analysis.

\(^5\)In Table. 1, we used the RL+PV method with a threshold values of $-0.014$ and $-0.04$ respectively for 1 or 2 detectors at 60km (the first and fourth rows). These thresholds have been chosen to yield the maximum average probabilities of successfully determining the hierarchy in a sample of experiments with an equal number of NH and IH cases. In the second row of the table, the threshold used for $f_{\text{dis}}$ is 1, and in the third row the threshold for $RL + PV$ is $37.5 \times f_{\text{dis}} - 37.55$. 

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| Method                  | NH(%) | IH(%) |
|------------------------|-------|-------|
| RL+PV                  |       |       |
| 20kton×3years          | 85.2  | 89.7  |
|                        |       |       |
| \( f_{\text{dis}} \)  |       |       |
| Oscillation frequency  | 89.2  | 90.0  |
|                        |       |       |
| RL+PV\&\( f_{\text{dis}} \) |       |       |
| Combination            | 96.9  | 95.3  |
|                        |       |       |
| RL+PV                  |       |       |
| 20kton×2×3years        | 93.2  | 93.2  |

Table 1: The probability of successfully determining hierarchy with 20 kton/detector and 3 years live time using the Fourier peak-valley analysis (RL+PV) at a 60 km baseline detector, the oscillation frequency method (\( f_{\text{dis}} \)) using a 30 km and a 60 km detector, a combination of both (RL+PV\&\( f_{\text{dis}} \)) and finally only RL+PV but with 2 detectors at 60 km.

Figure 6: The correlation between the Fourier Transform peak-valley analysis (RL+PV method) and the oscillation frequency analysis (\( f_{\text{dis}} \)).
Figure 7: The bottom panels are the fit values of $\Delta M^2_{32}$ at various baselines assuming one million events. The top panels are the corresponding values of $\chi^2_{\text{min}}$. left: simulated data using the normal hierarchy, blue: fit with NH; Red: fit with IH; right: simulated data using the inverted hierarchy, blue: fit with NH; Red: fit with IH.

5 Comparing fitted $\Delta M^2_{32}$

As has been shown in Ref. [21], if interpreted correctly, a $\chi^2$ analysis can be used to determine the neutrino mass hierarchy at a reactor experiment. As it uses all of the information available in the spectrum, it can potentially provide a more robust determination of the hierarchy than a Fourier analysis, as was seen for example in the simulations of Ref. [10].

The $\chi^2$ statistic corresponding to the simulated spectrum $N_{\text{spectrum},i}$ is defined to be

$$\chi^2 = \sum_{i}^{n} \frac{(N_{\text{spectrum},i} - N_{\text{fitted},i})^2}{N_{\text{spectrum},i}}$$

(5.1)

where $N_{\text{fitted},i}$ is the best fit spectrum with a given hierarchy. For simplicity we will only fit the parameter $|\Delta M^2_{32}|$. Here the index $i$ labels the 50 keV bins. The visible energy resolution is taken to be $3%/\sqrt{E/\text{MeV}}$.

We have found the value of $|\Delta M^2_{32}|$ which minimizes the $\chi^2$ value of fits to both hierarchies at various baselines. As can be seen in Fig. 7, while a fit to the correct hierarchy, by construction, correctly yields the simulated value $|\Delta M^2_{32}| = 2.4 \times 10^{-3} \text{ eV}^2$, a fit of the NH (IH) simulated data to the theoretical IH (NH) spectrum yields a best fit for $|\Delta M^2_{32}|$ which depends upon the baseline. This strong baseline dependence of the best fit persists even at short baselines, where fits to both hierarchies yield comparable values of $\chi^2$.

This observation may be used to determine the hierarchy as follows. One may use a $\chi^2$ fit of the spectra observed at two different baselines, assuming both hierarchies, to generate 4
values of $|\Delta M^2_{32}|$. The values generated by the correct hierarchy at the various detectors will all agree, whereas those generated by the wrong hierarchy will depend upon the baseline. For example, in the case simulated here the values of $|\Delta M^2_{32}|$ resulting from these fits are summarized in Table 2. Here we have also included the corresponding projected fits from the Daya Bay experiment’s far detectors with simulated data, which similarly yield a hierarchy dependent fit at an effective baseline. The hierarchy is determined from the relative values of $|\Delta M^2_{32}|$, which in the case of identical detectors is less sensitive to the correlated nonlinear energy response.

6 Background Flux and $\theta_{12}$

The existence of a second detector at a distinct baseline can also help with another of Daya Bay II’s goals [8], the determination of the mixing angle $\theta_{12}$. This is determined by ignoring the small 1-3 oscillations in the observed oscillated reactor neutrino spectrum, and measuring the depth of the flux minimum at the energy corresponding to the 1-2 oscillation maximum. A precise determination of $\theta_{12}$ can provide a powerful test of the unitarity of the neutrino mass matrix, providing an indirect probe of various sterile neutrino models [22].

Proposed sites for the construction of medium baseline reactor experiments are in China’s Guangdong province and in South Korea, both of which are experiencing the rapid construction of nuclear reactors. Only the nearest reactors can effectively be used to determine the hierarchy, more distant reactors provide backgrounds. The Daya Bay II location of Ref. [8] may have a large background from the proposed reactor at Lufeng, and the perpendicular locations proposed in Ref. [7] may have backgrounds from the proposed reactor at Huidong.
As described in Ref. [12], while the background reactors are twice as far as the foreground reactors, they may actually provide more flux at the 1-2 minimum than the foreground reactors because at that energy the background reactors are at their 1-2 minimum. In addition, the other sites proposed for Daya Bay II, in western Guangdong, along with all possible sites for Reno 50 [23] will have to contend with background reactors 200 km away, whose contribution to the flux at the 1-2 maximum is nonetheless appreciable [13].

Can this background flux be simply subtracted away? Of course its contribution to the statistical error cannot be removed [24]. But also it contributes a systematic error, as the overall reactor flux normalization is poorly understood [25] and so the expected background flux cannot be reliably determined. With the vast array of experiments underway it will no doubt be measured more precisely before a medium baseline experiment is built, but nonetheless the overall reactor flux normalization is unlikely to be measured as precisely as the 0.63% desired precision for the measurement of $\sin^2(2\theta_{12})$ at Daya Bay II [19].

If the reactor flux normalization is increased, then the background flux at the 1-2 maximum will be increased and so the measured value of $\theta_{12}$ will decrease. This leads to a degeneracy between $\theta_{12}$ and the overall reactor flux normalization. This degeneracy will be broken if there are multiple detectors at distinct baselines, because the relative flux deficit at the 1-2 maximum measured by each detector will be a distinct combination of the reactor flux normalization and the disappearance due to 1-2 oscillations. Thus systematic errors due to the unknown reactor flux normalization will be decreased. For example, the detectors described above at 36 km and 54 km perpendicular to the Daya Bay complex would satisfy both of these criteria.

7 Discussion

In this note we have introduced several multidetector observables which are sensitive to the hierarchy. The definitions chosen have not yet been optimized. Thus while the chance of success based on these techniques is overestimated by the fact that various backgrounds and systematics have not been included in our analysis, it is also underestimated as the analyses considered can easily be improved.

For example, consider the oscillation frequency method introduced in Sec. 4. While the fine structure oscillation frequency observed at the near detector is everywhere $\Delta M^2_{\text{eff}}$, that observed at the far detector is energy dependent. It will be $\Delta M^2_{\text{eff}}$ at high energies, and it will deviate from $\Delta M^2_{\text{eff}}$ by as much as 3% near the 1-2 oscillation maximum. The Fourier transform maximum that we have defined in Eq. (4.4) is only sensitive to the average
frequency, and so is only sensitive to the hierarchy at the 1% level, as can be seen in Fig. 5. Therefore a higher sensitivity may be obtained by Fourier transforming only the part of the spectrum which is the most sensitive to the hierarchy, the low-mid energies corresponding to the 1-2 oscillation maximum. Such a restricted Fourier transform may be obtained by using a weighted Fourier transform of the kind introduced in Ref. [26] and used in Ref. [7]. In Ref. [26] it was shown that such weights are anyway necessary to remove a spurious dependence upon the high energy tail of the neutrino spectrum, which is poorly understood, irrelevant to the hierarchy and a factor of 2-3 times lower than that given by the quadratic fit spectrum [27] used in studies such as this one and Refs. [15, 3].

Future directions

The simulations of Ref. [7] show that, even if the detector response is perfectly understood, the chance of determining the hierarchy in 6 years with a single 20 kton detector in Guangdong province appears to be limited to about 96% using a Fourier analysis, although this can be somewhat improved using a $\chi^2$ analysis [10]. However in this note we have restricted our attention to possible solutions to the nonlinearity problem, considering an idealized situation in which the baselines can be chosen at will, there are no backgrounds and the neutrino flux arises from a single reactor, thus avoiding multiple baseline interference effects. In this context we can in principle determine whether the seemingly unachievable experimental requirements that Ref. [10] found for the understanding of the nonlinear response at a single detector also apply to multiple detectors, or if instead a multidetector setup can determine the hierarchy in the presence of an unknown nonlinear response larger than that allowed in a single detector setup.

In practice statistical errors are a serious problem, and so an optimal $\chi^2$ fit should introduce pull parameters for the various sources of nonlinearity together with different penalty terms for correlated and uncorrelated errors. The uncorrelated errors are expected to be subdominant. The correlated errors on the other hand, as we have argued in this note, are not necessarily a serious obstruction to the determination of the hierarchy at a multidetector experiment.

Acknowledgement

The authors are honored to express their gratitude to Sören Jetter, Liang Zhan and especially Xin Qian for useful discussions and explanations. JE is supported by the Chinese Academy of Sciences Fellowship for Young International Scientists grant number 2010Y2JA01. EC and XZ are supported in part by the NSF of China.
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