Strangeness Contribution to the
Polarized Nucleon Structure Function $g_1(x)$

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ABSTRACT

The three flavor version of the Nambu–Jona–Lasinio chiral soliton model for baryons is employed to calculate the twist–2 contribution to the polarized nucleon structure function $g_1(x)$. In particular the role of the strange quark degree of freedom as a collective excitation of the chiral soliton is investigated in the context of flavor symmetry breaking. The model prediction for $g_1(x)$ refers to a low momentum scale $Q_0^2$. The leading order corrections to the scale dependence is computed along the QCD evolution program allowing to compare with data from SLAC.

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1. Introduction

The investigation of the strange quark contribution to the polarized structure function is mainly motivated by the empirical results obtained in the context with the proton spin puzzle, cf. ref [1] for a review. In this context one considers nucleon matrix elements of axial currents, which are obtained as the zeroth moment of polarized structure functions. The puzzle firstly refers to the smallness of the observed nucleon matrix element of the axial singlet current. Already early studies [2] in the Skyrme model (the simplest version of a chiral soliton model) indicated that chiral soliton models are capable of reproducing that result. The data analysis secondly revealed that the strange quark might make a sizable contribution to the axial singlet charge of the nucleon; up to a third of that of the down quark. This surprising result was obtained using SU(3) symmetric baryon wave–functions; the inclusion of symmetry breaking effects reduces this ratio. Certainly it is interesting to compute the full dependence of the corresponding structure functions on the Bjorken variable. At this point the collective approach to incorporate strange degrees of freedom in chiral soliton models [3] becomes very attractive. It not only allows one to account for such symmetry breaking effects in the nucleon wave–function [5] but in particular to make explicit the strange quark contribution to nucleon structure functions. This is a major advantage over other low energy models for the nucleon like e.g. the MIT bag model [6, 7].

The polarized structure function $g_1$ is extracted from the hadronic tensor for electron–nucleon scattering,

$$W_{\mu\nu}(q) = \frac{1}{4\pi} \int d^4\xi \ e^{iq\cdot\xi} \langle N | [J_\mu(\xi), J'_\nu(0)] | N \rangle , \quad (1)$$

where $J_\mu = \bar{q}(\xi) \gamma_\mu Q q(\xi)$ is the electromagnetic current with $Q = \text{diag} \left( \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3} \right)$ being the quark charge matrix. The nucleon state, denoted by $|N\rangle$ in eq (1), is characterized by its momentum ($P_\mu$) and spin ($S_\mu$). The polarized structure function $g_1$ parameterizes the longitudinal part of the antisymmetric piece,

$$W^{(A)}_{\mu\nu}(q) = (W_{\mu\nu} - W_{\nu\mu})/2i = i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda M_N}{P \cdot q} \left\{ g_1(x, Q^2) S^\sigma + \text{transverse part} \right\} . \quad (2)$$

Here $q_\mu$ refers to the momentum transferred to the nucleon by the virtual photon with $Q^2 = -q^2$. The Bjorken variable is defined as $x = Q^2/P \cdot q$. We are interested in the leading twist contribution to $g_1$. It is extracted from the hadronic tensor by assuming the Bjorken limit which corresponds to the kinematical regime

$$q_0 = |q| - M_N x \quad \text{with} \quad |q| \to \infty \quad \text{and} \quad x \ \text{fixed} . \quad (3)$$

In this limit eq (2) is straightforwardly inverted yielding

$$g_1(x) = \frac{M_N}{2} \ e^{i\omega q_0} \ W^{(A)}_{\mu\nu}(q) \quad (4)$$

for the kinematical conditions $q = (q^0, 0, 0, |q|)$, $P = (M_N, 0, 0, 0)$ and $S = (0, 0, 0, 1)$, i.e. the nucleon rest frame with the nucleon spin and the transferred momentum being aligned, hence the notion longitudinal.

\[1\] Cf. ref [4] for a recent review on soliton models in flavor SU(3).
In chiral soliton models strange quark effects enter the structure functions in two ways, firstly there are direct contributions to the currents \( J_\mu \) which do not exist in the two flavor model and secondly the structure of the nucleon state \( |N> \) changes when generalizing from flavor SU(2) to SU(3). For the present study we will consider the Nambu–Jona–Lasinio (NJL) model \[8\] in its bosonized form \[9\] which is well known to possess a chiral soliton solution \[10\]. For comprehensive lists of references we refer to recent reviews \[11, 12\]. In this model the defining Lagrangian contains only quark fields hence all quantities can formally be expressed in terms of these fields. In particular the formal expression for the current is that of a free Dirac theory which makes the commutator in eq (1) feasible. It is this feature which actually allows us to compute structure functions from a chiral soliton. In other soliton models, which do not possess such a clear connection to the quark flavor dynamics, the computation of structure functions seems infeasible due to the complicated structure of the current operator. In the two flavor version of the model various nucleon structure functions have been discussed \[13, 14, 15, 16, 17, 18\] while the present study represents the first attempt to consider the effects of strange quarks.

The extension to flavor SU(3) is particularly interesting because the calculation \[13\] of \( g_1 \) in the two flavor model already yielded reasonable agreement with experiment. On the other hand one expects strange degrees of freedom to have non–negligible impact on nucleon properties.

2. The NJL Chiral Soliton

In the three dimensional flavor space with up, down and strange quarks the NJL model Lagrangian reads

\[
\mathcal{L} = \bar{q} (i\hat{\partial} - m^0) q + 2 G_{NJL} \sum_{i=0}^{8} \left( \frac{\lambda^i}{2} q^2 + \frac{i\gamma_5}{2} \lambda^i q \right) .
\]

Here \( q, \hat{m}^0 = \text{diag}(m^0_u, m^0_d, m^0_s) \) and \( G_{NJL} \) denote the quark field, the current quark mass matrix and a dimensionful coupling constant, respectively. In what follows we will assume the isospin limit \( m^0_u = m^0_d \). Of course, we will consider flavor symmetry breaking by allowing the strange current quark mass to be different, i.e. \( m^0_s \neq m^0_u \).

Using functional integration techniques we obtain the bosonized version of the NJL model action \[3\]

\[
\mathcal{A}[\mathcal{M}] = \text{Tr} \log(i\hat{D}) + \frac{1}{4G_{NJL}} \int d^4 x \text{ tr} \left\{ m^0 (\mathcal{M} + \mathcal{M}^\dagger) - \mathcal{M} \mathcal{M}^\dagger - (m^0)^2 \right\} ,
\]

\[
D = i\hat{\partial} - (\mathcal{M} + \mathcal{M}^\dagger) - \gamma_5 (\mathcal{M} - \mathcal{M}^\dagger) .
\]

The composite scalar (\( S \)) and pseudoscalar (\( P \)) meson fields are contained in \( \mathcal{M} = S + iP \), and appear as quark–antiquark bound states. Apparently \( \mathcal{M} \) represents a 3 \( \times \) 3 matrix field in flavor space which behaves as the sum of scalar (\( S \)) and pseudoscalar (\( P \)) quark bilinears under chiral transformations. For regularization, which is indicated by the cut–off \( \Lambda \), we will adopt the proper–time scheme \[19\]. The free parameters of the model are the current quark mass matrix \( m^0 \), the coupling constant \( G_{NJL} \) and the cut–off \( \Lambda \). The equation of
motion for the scalar field $S$ may be considered as the gap–equation for the order parameter $\langle \bar{q} q \rangle$ of chiral symmetry breaking. This equation relates the vacuum expectation value, $\langle \mathcal{M} \rangle = M = \text{diag}(M_u, M_d = M_u, M_s)$ to the model parameters $\hat{m}^0$, $G_{NJL}$ and $\Lambda$. For apparent reasons $M$ is called the *constituent* quark mass matrix. The occurrence of this vacuum expectation value reflects the spontaneous breaking of chiral symmetry and causes the pseudoscalar fields to emerge as (would–be) Goldstone bosons. At this stage we expand $\mathcal{A}$ to quadratic order in $\mathcal{P}$ around $M$. Then the model parameters are related to physical quantities like the pion mass, $m_\pi = 135\text{MeV}$ and the pion decay constant, $f_\pi = 93\text{MeV}$. In account of the gap–equation this leaves one undetermined parameter which we choose to be the up constituent quark mass $M_u$ [9]. Typical values are $M_u = 350 \sim 450\text{MeV}$. The kaon mass $m_K = 495\text{MeV}$ allows us to fix the strange current quark mass subject to the gap–equation in the strange sector, also determines the strange constituent quark mass $M_s$. Using the typical values for $M_u$ the model underestimates the kaon decay constant $f_K = 114\text{MeV}$ by about 10–15% [20].

Turning to the baryon sector of the model we adopt the hedgehog *ansatz* for the meson fields $\mathcal{M} = \xi M \xi$

\[
\xi = \xi_H(r) = \exp \left( \frac{i}{2} \mathbf{\tau} \cdot \mathbf{\hat{r}} \Theta(r) \right)
\]

in order to determine the chiral soliton. For static meson configurations as (8) it is straightforward to deduce the classical energy $E[\Theta]$ functional associated with the action (8) [21]

\[
E[\Theta] = \frac{N_C}{2} \epsilon_v \left( 1 + \text{sgn}(\epsilon_v) \right) + \frac{N_C}{2} \int_{1/\Lambda^2}^{\infty} ds \frac{1}{\sqrt{4\pi s^3}} \sum_\nu \exp \left( -s \epsilon_\nu^2 \right) + \frac{m_\pi^2 f_\pi^2}{4} \int d^3 r \left( 1 - \cos \Theta(r) \right).
\]

Here $\epsilon_\mu$ refer to the energy eigenvalues of the Dirac Hamiltonian

\[
h_0 = \alpha \cdot p + M_u \beta \exp \left( i \gamma_5 \mathbf{\tau} \cdot \mathbf{\hat{r}} \Theta(r) \right) \hat{T} + M_s \beta \hat{S}
\]

which is derived from the operator (7) $D = \beta (i \partial_t - h_0)$ upon substituting the hedgehog *ansatz* (8). From the appearance of the strange and non–strange projectors $\hat{S} = \text{diag}(0,0,1)$ and $\hat{T} = \text{diag}(1,1,0)$ we observe that the strange quarks are not effected by the hedgehog field. In eq (4) the subscript “$v$” denotes the valence quark level. This state is the distinct level bound in the soliton background, *i.e.* $-m < \epsilon_v < m$. Similar to the energy functional (8) other quantities also separate into contributions associated with the explicit occupation of the valence level and a (regularized) piece due to the vacuum being polarized by the meson fields. The chiral soliton, $\Theta(r)$, is finally obtained by self–consistently extremizing $E[\Theta]$ [10].

**3. The Nucleon State in Flavor SU(3)**

States with nucleon quantum numbers are generated from the static configuration (8) by introducing collective coordinates for the large amplitude fluctuations of the soliton [22, 3]. This approach may be viewed as an approximation to the unknown time dependent solution to the equations of motion for the meson fields. Subsequently these coordinates are treated
quantum–mechanically. The large amplitude fluctuations are associated with the rotations in flavor space because the spatial rotations can be absorbed into the former as a consequence of the hedgehog ansatz \(8\). For the two–flavor NJL chiral soliton model this quantization approach has been performed in ref \(21\). In the more involved case of three flavors we have \(20\)

\[
\xi(r, t) = A(t) \xi_H(r) A^\dagger(t) ,
\]

where \(A(t)\) is a \(3 \times 3\) matrix in flavor space. Substituting this configuration and transforming to the flavor rotating frame \(q' = A(t)q\) reveals that the eigenvalues of the modified Dirac Hamiltonian

\[
h' = h_0 + h_{\text{rot}} + h_{\text{SB}}
\]

with

\[
\begin{align*}
    h_{\text{rot}} &= -iA(t) \frac{d}{dt} A^\dagger(t) = \frac{1}{2} \sum_{a=1}^{8} \lambda_a \Omega^a \\
    h_{\text{SB}} &= \frac{1}{\sqrt{3}} (M_u - M_s) T \beta \sum_{i=1}^{3} D_{8i} \lambda_i + \sum_{\alpha=4}^{7} D_{8\alpha} \lambda_\alpha + \left( D_{88} - 1 \right) \lambda_8 \right) T^\dagger
\end{align*}
\]

enter the functional trace \(6\) \(20\). Besides the angular velocities \(\Omega^a\) also the adjoint representation of the collective coordinates \(D_{ab} = (1/2)\text{tr}(\lambda_a A \lambda_b A^\dagger)\) have been used to simplify the additional parts of the Dirac Hamiltonian. For convenience the chiral rotation \(T = \frac{1}{2} (\xi_H^\dagger + \xi_H)/2 + \gamma_5 (\xi_H^\dagger - \xi_H)/2\) has been introduced in eq \(14\). Using these definitions it is straightforward to extract the Lagrangian \(L(A, \Omega^a)\) for the collective coordinates from the action functional \(6\). As the rotations \(11\) are assumed to proceed adiabatically the expansion of the action functional is terminated at quadratic order in \(\Omega^a\). In addition the functional trace in \(6\) is expanded in the difference \(M_s - M_u\) which measures the flavor symmetry breaking originating from different constituent quark masses\(2\). Essentially this represents an expansion in \(h_{\text{rot}} + h_{\text{SB}}\).

By Legendre transformation to the SU(3) right generators, \(R_b = -\partial L(A, \Omega^a)/\partial \Omega^b\) the Hamilton operator in the space of the collective coordinates is deduced. It has the form \(4\)

\[
H(A, R_a) = E + \frac{1}{2} \left[ \frac{1}{\alpha^2} - \frac{1}{\beta^2} \right] \sum_{i=1}^{3} R_i^2 + \frac{1}{2} \beta^2 \sum_{a=1}^{8} R_a^2 - \frac{3}{8} \beta^2 \\
+ \frac{\alpha}{2} \sum_{i=1}^{3} D_{8i} (2R_i + \alpha_1 D_{8i}) + \sum_{\alpha=4}^{7} D_{8\alpha} (2R_\alpha + \beta_1 D_{8\alpha}) + \frac{1}{2} \gamma (1 - D_{88}) \\
+ \frac{1}{2} \gamma_s (1 - D_{88}^2) + \frac{1}{2} \gamma_T \sum_{i=1}^{3} D_{8i} D_{8i} + \frac{1}{2} \gamma_{TS} \sum_{\alpha=4}^{7} D_{8\alpha} D_{8\alpha}
\]

(15)

together with the constraint \(R_8 = \sqrt{3}/2 \) for \(B = 1\) and \(N_C = 3\). This constraint restricts the allowed states to those with half–integer spin. The quantities \(\alpha^2, \ldots, \gamma_{TS}\) are functionals\(2\).
of the self–consistent chiral angle. For details of their evaluation and numerical results we refer to ref [20]. Here it is only important to note that for the self–consistent soliton these constants of proportionality are dominated by their valence quark contributions.

The most important feature of the collective Hamiltonian is that it can be diagonalized exactly yielding as eigenstates the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. Due to the presence of flavor symmetry breaking these are no longer pure octet (decouplet) states but acquire sizable admixture of states with identical quantum numbers in the higher dimensional SU(3) representations like $\mathbf{10}$ and $\mathbf{27}$. This diagonalization is a generalization of the Yabu–Ando approach [5] and is comprehensively described in refs [20, 4]. All nucleon matrix elements to be computed henceforth will employ these exact eigenstates.

4. Valence Quark Approximation to $g_1(x)$

In order to compute $g_1$ in the present model we have to bear in mind that we are dealing with localized field configurations. The resulting expression for the hadronic tensor in the Bjorken limit has been obtained in [7]. Its antisymmetric component becomes [7]

$$W_{\mu\nu}^{(A)}(q) = \int \frac{d^4k}{(2\pi)^4} \epsilon_{\mu\rho\nu\sigma} k^\rho \text{sgn}(k_0) \delta(k^2) \int_{-\infty}^{+\infty} dt \ e^{i(k_0+q_0)t} \times \int d^3x_1 \int d^3x_2 \exp[-i(k+q) \cdot (x_1-x_2)] \times \langle N | \left\{ \bar{\Psi}(x_1,t)Q^2\gamma^\sigma\gamma^5\Psi(x_2,0) + \bar{\Psi}(x_2,0)Q^2\gamma^\sigma\gamma^5\Psi(x_1,t) \right\} | N \rangle ,$$

(16)

where $\epsilon_{\mu\rho\nu\sigma}\gamma^\sigma\gamma^5$ is the antisymmetric combination of $\gamma_\mu\gamma_\rho\gamma_\nu$. The expression (16) is essentially obtained by applying Wick’s theorem to the current commutator in eq (1). For the intermediate quark the free correlator is substituted because in the Bjorken limit this quark is far off–shell and hence not sensitive to momenta which are of the scale as those attributed to the soliton. Finally a collective coordinate describing the position of the soliton in coordinate space is introduced and integrated over.

We have already noted that the constants of proportionality entering the collective Hamiltonian (15) are dominated by their valence quark contribution. This dominance is even more pronounced for axial matrix elements like $\langle N | \bar{q}\gamma_5\gamma_3\lambda_u q | N \rangle$ [11, 12, 23]. For these axial current matrix elements the vacuum contribution is commonly found to be 10% of the total; for some flavor combinations even less. This establishes the assumption that the vacuum contribution to the polarized structure functions is negligible and manifests itself in the valence quark approximation for the structure function $g_1(x)$. This approximation is defined by substituting the valence quark wave–function

$$\Psi_v(r,t) = e^{-i\epsilon_v t} A(t) \Phi_v(r) \quad \text{with} \quad \Phi_v(r) = \psi_v(r) + \sum_{\mu \neq v} \psi_\mu(r) \frac{\langle \mu | h_{\text{tot}} + h_{\text{SB}} | v \rangle}{\epsilon_v - \epsilon_\mu}$$

(17)

in the hadron tensor (16). The spinors $\psi_\mu(r) = \langle r | \mu \rangle$ diagonalize the classical Dirac Hamiltonian (10) yielding the eigenvalues $\epsilon_\mu$. It is important to stress that the dependence of the valence quark wave–function $\Psi_v(r,t)$ on the collective coordinates is included in the calculation of $g_1$. Only with this input it will be possible to disentangle the strange quark...
contribution. The calculation is performed by taking the Fourier transform of $\Psi_v(\mathbf{r}, t)$ which allows us to carry out the momentum integrals in eq (16). The technical details of this calculation will be presented elsewhere together with results for the unpolarized structure functions [24].

The matrix element in eq (16) between nucleon states is to be taken in the space of the collective coordinates, $A(t)$ (see eq. (11)) as the object in curly brackets is an operator in this space, which can be deduced from eq (17). Here it is important to repeat that $A(t)$ spans the three dimensional flavor space which brings the strange degrees of freedom into the picture. This is the main difference to e.g. bag model calculations where the nucleon state is considered to be a product state of three specified quarks. In the present model it is rather a collective excitation of quark fields in the background of the self–consistent soliton.

5. Projection and Evolution

In the chiral soliton model the baryons states are not momentum eigenstates causing the structure functions not to vanish exactly for $x > 1$. This short–coming is due to the localized field configuration and thus the nucleon not being a representation of the Poincaré group. As a consequence the computed structure functions are not frame independent. It turns out that in the infinite momentum frame (IMF) the structure functions indeed vanish for $x > 1$ [25, 24]. This is a consequence of the Lorentz contraction associated with the boost from the rest frame (RF) to the IMF which exactly projects onto $0 \leq x \leq 1$. For the polarized twist–2 structure function this implies

$$g_1^{(IMF)}(x) = \frac{1}{1 - x} g_1^{(RF)} \left( - \ln(1 - x) \right),$$

where $g_1^{(RF)}(x)$ denotes the structure function computed in the nucleon rest frame as discussed in the preceding sections. The transformation (18) leaves the integral over $g_1$ invariant provided that in the rest frame the integration range has been extended to infinity.

The chiral soliton model is considered to approximate QCD at a low momentum scale $Q_0^2$ whence the result $g_1^{(IMF)}(x)$ should be interpreted as $g_1(x, Q_0^2)$. It should be noted that $Q_0^2$ is a new parameter to the model. In order to compare with experimental data it is mandatory to evolve this structure function to $g_1(x, Q^2)$ according to the DGLAP procedure [27]. Here $Q^2$ is the momentum scale set by the experiment. In order to apply this procedure we first have to separate the flavor singlet (0) and non–singlet ($ns$) contributions to $g_1(x, Q_0^2)$ as the former mixes with the gluon contribution $g(x, Q^2)$ upon evolution. This separation is straightforwardly accomplished by substituting the appropriate flavor matrix for $Q_2$ in eq (16). The leading order evolution equations read in the case of three flavors and with $t = \ln(Q^2/\Lambda_{QCD}^2)$ as well as $\alpha_{QCD} = 4\pi/9t$,

$$\frac{dg_1^{(ns)}(x, t)}{dt} = \frac{\alpha_{QCD}}{2\pi} \int_x^1 \frac{dy}{y} P_{qq} \left( \frac{x}{y} \right) g_1^{(ns)}(y, t),$$

(19)

$$\frac{dg_1^{(0)}(x, t)}{dt} = \frac{\alpha_{QCD}}{2\pi} \int_x^1 \frac{dy}{y} \left\{ P_{qq} \left( \frac{x}{y} \right) g_1^{(0)}(y, t) + 6P_{qg} \left( \frac{x}{y} \right) g(y, t) \right\},$$

(20)

$$\frac{dg(x, t)}{dt} = \frac{\alpha_{QCD}}{2\pi} \int_x^1 \frac{dy}{y} \left\{ P_{gg} \left( \frac{x}{y} \right) g(y, t) + P_{qg} \left( \frac{x}{y} \right) g_1^{(0)}(y, t) \right\}.$$
Table 1: Zeroth moments of the polarized structure function $g_1$. Also given is the gluon component at $Q^2 = 3.0\text{GeV}^2$, $\Delta G = 2 \int_0^1 dx g(x)$.

| $M_a (\text{MeV})$ | $\Delta u$ | $\Delta d$ | $\Delta s$ | $\Delta G$ |
|-------------------|------------|------------|------------|------------|
| 400               | 0.64       | −0.14      | −0.01      | 0.23       |
| 450               | 0.60       | −0.16      | −0.02      | 0.21       |

The splitting functions $P_{ij}$ are listed in ref \[28\]. Here it suffices to note that the integrals $\int_0^1 dz P_{qq}(z)$ and $\int_0^1 dz P_{qg}(z)$ vanish at leading order. As a result the zeroth moments

$$\Delta q = 2 \int_0^1 dx g_1^{(q)}(x,t) \quad \text{for} \quad q = u, d, s$$  \hspace{1cm} (22)

do not depend on the momentum scale. For the evolution program being applicable we have to assume that $g(x,Q^2_0) = 0$ which implies that there are no soft gluons at the model scale while the effects attributed to hard gluons are approximated by the contact interaction in \[\text{(3)}\]. After integrating the differential equations \[\text{(19)}\)–\[\text{(21)}\) from $Q^2_0$ to $Q^2$ the singlet and non–singlet flavor components are superposed to yield the desired flavor combination.

6. Numerical Results

In a first step we determine the model scale $Q^2_0$. For this purpose we perform the evolution for the unpolarized structure functions which enter the Gottfried sum rule. As this is a non–singlet combination a simplified evolution equation like \[\text{(19)}\) applies because gluon degrees of freedom do not contribute. We vary the lower boundary $Q^2_0$ when integrating the evolution equations until maximal agreement with the experimental data available at the upper boundary $Q^2 = 5\text{GeV}^2$ is obtained. Since details of that calculation will be presented elsewhere \[24\) we content on quoting the result $Q^2_0 = 0.4\text{GeV}^2$. This is identical to the value found for the two flavor model \[13\). Naïvely one would expect that the two and three flavor models would give the same result because the strange degree of freedom cancels in this particular combination of structure functions. However, in the two and three flavor models the quark wave–functions are different as can easily be seen from eq \[\text{(17)}\); in the considerably simpler two flavor version the only contributing perturbation is $\sum_{a=1}^3 (\lambda_a/2)\Omega^a$, to be contrasted with eqs \[\text{(12)}\)–\[\text{(14)}\).

All presented results for the polarized structure function at the low momentum scale will be taken in the infinite momentum frame \[\text{(18)}\). In table \[\text{I}\) we show the zeroth moments \[\text{(22)}\) of the polarized structure functions. In ref \[23\) the vacuum contributions to these moments were computed. For the sum of valence and vacuum parts those authors give\[1\)

$\Delta u = 0.64$, $\Delta d = −0.24$ and $\Delta s = −0.02$ using $M_u = 423\text{MeV}$. Apparently the vacuum contribution to the zeroth moment is small in accord with our valence quark approximation to the polarized structure functions. We also note that due to the deviation from SU(3)

\[3\)In the present calculation PCAC violating $1/N_C$ contributions to the non–singlet combinations have been ignored. Hence we have to compare our results with the entry “NJL(scalar)” of table X in ref \[23\). In ref \[23\) a different regularization scheme to determine the chiral angle and a different expansion scheme of the fermion determinant were used. This might as well make the (small) differences in the zeroth moments.
symmetric baryon wave–functions the quantities shown in table I should not be related to the $F$ and $D$ parameters determined from semi–leptonic hyperon decays.

In figure I the main result of our calculation is displayed: the strangeness contribution $g_1^{(s)}$ to the polarized nucleon structure function $g_1$. Apparently the smallness of $\Delta s$ does not necessarily transfer to an overall negligible $g_1^{(s)}$. A cancellation between positive and negative parts is not excluded. This effect is not altered by the DGLAP evolution. We furthermore compare to the strange quark contribution obtained with a pure octet nucleon wave–function, i.e. with the symmetry breaking effects omitted when diagonalizing (15). Apparently the incorporation of symmetry breaking effects in the nucleon wave–function via exact diagonalization of the collective Hamiltonian yields significantly less pronounced strange quark contributions. This is not unexpected in the collective approach [4].

In figure 2 we compare the two and three flavor model predictions for the electromagnetic flavor combination $Q^2$ for which data from SLAC are available at $Q^2 = 3 GeV^2$ [29]. We recognize that the inclusion of the strangeness degree of freedom yields only minor changes. For $M_u = 400 MeV$ the results obtained in the two and three flavor models are almost indistinguishable. In particular both versions reasonably reproduce the SLAC data [29]. The small change of $g_1(x)$ when generalizing to flavor SU(3) is unexpected, after all there are at least two significant effects associated with this generalization. First, strange quarks appear explicitly, second in SU(3) the Clebsch–Gordan coefficients are different and so are the

\[ ^4 \text{Strictly speaking } F \text{ and } D \text{ parameters are only well–defined in a flavor symmetric formulation.} \]
nucleon matrix elements of the collective operators\footnote{The inclusion of symmetry breaking effects within the Yabu–Ando approach drives these matrix elements towards their SU(2) values.}. Apparently these two effects partially cancel each other.

In figure 3 we show our prediction for the gluon component of the polarized structure function at the experimental scale $Q^2 = 3\text{GeV}^2$. We remind the reader of the assumption that at the model scale ($Q_0^2 = 0.4\text{GeV}^2$) this component is taken to be zero, \textit{i.e.} this component is solely due to radiation and absorption of soft gluons. We remark that the singularity at $x \to 0$ is weaker than $1/x$ since $xg(x) \to 0$ in this limit.

7. Conclusions

In this letter we have presented some first results for strange quark contributions to nucleon structure functions. In particular we have concentrated on the twist–2 piece of the polarized structure function $g_1$. On the whole we find the surprising result that the contribution of the strange degrees of freedom do not significantly alter the results of the pure two flavor model. In both the two and three flavor versions of the NJL chiral soliton model we obtain reasonable agreement with experimental data for the polarized nucleon structure function $g_1$ when the scale dependence is accounted for according to the DGLAP scheme.

To calculate various flavor components of $g_1$ at the low momentum scale we have employed the valence quark approximation to the NJL chiral soliton model in flavor SU(3). Although the valence quark contribution is known to strongly dominate the static axial properties for the self–consistent chiral soliton, in the unpolarized case some conceptual requirements like positivity are slightly violated by this approximation \footnote{The inclusion of symmetry breaking effects within the Yabu–Ando approach drives these matrix elements towards their SU(2) values.}. Hence the full calculation would

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Figure2a}
\includegraphics[width=0.4\textwidth]{Figure2b}
\caption{Comparison of the two and three flavor calculation of the polarized structure function $g_1$ for the proton in the infinite momentum frame. Also shown is the leading order QCD evolution to $Q^2 = 3\text{GeV}^2$. Left panel: $M_u = 400\text{GeV}$, right panel: $M_u = 450\text{GeV}$. Data are taken from ref \cite{29}.}
\end{figure}
require to include the effects attributed to the polarized vacuum as well. As discussed, we do not expect them to have significant impact on the result. In addition an extensive next–to–leading order DGLAP evolution could be performed for the twist–2 structure functions under consideration. As the main effect we would expect a variation of the model scale $Q_0^2$, which anyhow is a free parameter.

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