Classical radiation effects on relativistic electrons in ultraintense laser fields with circular polarization

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Abstract. The propagation of a relativistic electron with initial energy \(\gtrsim 100\) MeV in a number of simple one-dimensional laser field configurations with circular polarization is studied by solving the relativistic equation of motion in the Landau–Lifschitz approach to account for the radiation friction force. The radiation back-reaction on the electron dynamics becomes visible at dimensionless field amplitudes \(a \gtrsim 10\) at these high particle energies. Analytical expressions are derived for the energy and the longitudinal momentum of the electron, the frequency shift of the light scattered by the electron and the particle trajectories. These findings are compared with the numerical solutions of the basic equations. A strong radiation damping effect results in reduced light scattering, forming at the same time a broad quasi-continuous spectrum. In addition, the electron dynamics in the strong field of a quasistationary laser piston is investigated. Analytical solutions for the electron trajectories in this complex field pattern are obtained and compared with the numerical solutions. The radiation friction force may stop a relativistic electron after propagation over several laser wavelengths at high laser field strengths, which supports the formation of a stable piston.

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1. Introduction

The interaction of relativistic electron beams with intense laser pulses offers promising sources of electromagnetic radiation in the x-ray and γ-ray domains. The elementary process is the nonlinear Thomson scattering of an intense periodic electromagnetic field by an electron [1–4]. It has been comprehensively studied for several simple interaction geometries including propagating and standing waves of linear and circular polarization. However, these earlier studies did not account for the deceleration in electron motion in high-intensity fields. At laser intensities exceeding $10^{20}$ W cm$^{-2}$, radiation effects are starting to play an important role in the interaction of relativistic electrons with intense electromagnetic waves. These phenomena were discussed in numerous theoretical papers, see for example [5–9, 11–14, 26], by including the radiation friction force in the equation of electron motion. As the classical Lorentz–Abraham–Dirac (LAD) equation [15, 16] suffers from spurious divergent solutions, an iterative approach by Landau and Lifshitz (LL) [17–19] has been used in these studies. Recently, a new renormalized form of the LAD equation was proposed in [20, 21], which conserves the generalized momentum of the electron and avoids runaway solutions also in very strong fields.

The classical approach to the radiation reaction force is limited by the condition that the electric field $E_e$, ‘seen’ by an electron in its instantaneous reference frame, is smaller than the Schwinger field, $E_S = m^2 c^3 / e \hbar$. The radiation reaction effect in stronger fields, where $\gamma \gtrsim 1$, is described by the emission probability in the framework of quantum electrodynamics (QED) [22]. It allows analysis of the electron motion and photon emission in fields with intensities $\gtrsim 10^{23}$ W cm$^{-2}$. An extended procedure, which applies the probability rates for photon emission by ultrarelativistic electrons in intense electromagnetic fields and of direct pair creation by hard photons in a Monte-Carlo approach, was developed in [23, 24]. The absorption of an

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ultraintense laser field when driving an electron–positron cascade was self-consistently treated numerically in [25]. The influence of the radiation friction effect on the spectral properties of Compton scattered laser light is analyzed in [26, 27].

In this paper, we investigate the dynamics of electrons with modest energies of several hundred MeV or less in weaker fields, with the QED-strength parameter $\chi \ll 1$. Therefore, the classical description of the electron motion in the LL approximation and of the related radiation emission is used. We restrict our study to the interaction of a single electron with given electromagnetic fields, which is an indispensable step toward the understanding of the collective electron interaction with intense laser pulses.

The propagation of a relativistic electron in several simple field configurations such as a counterpropagating or a comoving electromagnetic wave or two superimposed waves moving in opposite directions at laser intensities below $10^{23}$ W cm$^{-2}$ is studied. The latter configuration includes the special case of a standing wave, as well as the scheme of radiation pressure ion acceleration in a quasistationary laser piston driven by an ultraintense circularly polarized laser pulse [28–32]. Particle-in-cell (PIC) simulations of the piston model including the radiation damping effect show a remarkable reduction of the number of hot electrons in the laser field behind the piston. This observation motivates the necessity to understand the detailed dynamics of these electrons, which leave the piston region at high energies in the direction toward the incident laser wave. Entering the intense field of two counterpropagating waves, they experience radiation friction and may be stopped finally, if the electromagnetic field is sufficiently strong.

Besides the analysis in [3], the electron motion in two counterpropagating strong waves was investigated in [33–35]. While the first work predicted the effect of stochastic electron heating in such a field configuration, the latter two demonstrate efficient pair creation and $\gamma$-ray emission in the wave nodes at laser intensities well below the Schwinger limit. An enhancement of pair production in counterpropagating laser beams due to straggling effects in the electron motion caused by the recoil in photon emission was observed in PIC and Monte-Carlo simulations [36, 37].

Whereas the laser piston model [28, 29] was strongly motivated by the search for an efficient scheme for ponderomotive ion acceleration to enable Ion Fast Ignition in Inertial Fusion Energy production [38, 39], and therefore it was limited to laser intensities of several $10^{22}$ W cm$^{-2}$, latest publications [35–37] extended the range of field strengths to much higher values, where dense electron–positron plasmas and strong bursts of $\gamma$-rays may be produced. These new developments provide an insight into high-energy astrophysical phenomena.

The electron–positron pair creation may be already observed at weaker laser fields in the interaction with extremely high-energy electrons. In the experiment [40], a terawatt laser pulse was sufficient to provide the parameter $\chi \approx 0.4$ in the interaction with a counterpropagating electron beam of the energy $\approx 47$ GeV. An experiment with the same electron beam but a laser intensity of $5 \times 10^{22}$ W cm$^{-2}$ would make it possible to reach $\chi \sim 90$ and to obtain multiple pair production [41].

Our paper is organized as follows. Section 2 presents the basic equations describing the propagation of a relativistic electron in a number of given field configurations. The corresponding results of the numerical integration of these equations are discussed in section 3 together with analytical solutions of the set of shortened basic equations, in which the radiation friction force is reduced to its strongest term ($\sim \gamma^2$). This approximation is justified in the case of high-energy electrons. The observed estimates for electron parameters, like its longitudinal momentum and its energy as functions of the propagation distance, characteristics of the
electron orbits and of the emitted radiation spectra, are compared with the numerical results. The case of a quasistationary piston is treated in section 4. This analysis confirms our previous estimate [29] for the stopping range of the high-energy electrons propagating against the incident laser wave. Section 5 contains the concluding remarks.

2. Interaction scheme and basic equations

The choice of basic interaction geometry is motivated by ponderomotive plasma acceleration. Here, the ultraintense laser pulse is reflected from the overdense plasma layer, and the reflecting boundary surface is moving inside under the action of the radiation pressure gradient. The structure of the reflecting layer—the laser piston—was described in [29]. A relativistic electron, which escapes the electron sheath of the laser piston, starts to move in the electromagnetic fields of the counterpropagating incident and copropagating reflected planar laser waves as schematically shown in figure 1. As already mentioned, we restrict the analysis to electron and laser parameters, which ensures a classical description of the laser–electron interaction. We also neglect the existence of a residual plasma cloud behind the piston, as it was observed in PIC simulations, as well as multidimensional effects. Because of the piston motion with velocity $\beta_f = \tilde{v}_f/c$, where $c$ is the speed of light in vacuum, the frequency of the reflected laser light is Doppler downshifted by the factor $M_f = (1 - \beta_f)/(1 + \beta_f)$. In addition to the ponderomotive potential of the superimposed laser fields, the electron trajectories will also be affected by the accelerating charge separation field in the ion layer of the piston.

2.1. Equation of motion with a reactive force

At sufficiently high laser wave amplitudes, the relativistic electron will not only scatter the laser light but also emit high-energy photons. To account for this radiation friction of an intense electromagnetic field, a four-vector force $g^I$ is added to the right side of the relativistic equation

$\mathbf{F} = m \ddot{\mathbf{v}} + q \mathbf{E} + q \mathbf{v} \times \mathbf{B} + g^I$
of particle motion written in covariant form in cgs units [17–19]:

\[
mc \frac{du^i}{dt'} = -e \frac{c}{\gamma} F^{ik} u_k + g^i, 
\]

where \( t' \) is the electron proper time, \( e \) is the elementary charge and \( m \) denotes the rest mass of the electron.

The Lorentz force is determined by the product of the electromagnetic field tensor \( F^{ik} = \frac{\partial A^k}{\partial x^i} - \frac{\partial A^i}{\partial x^k} \) and the four-velocity in its covariant form, \( u_i = (\gamma, -\gamma \vec{\beta}) \). Here, \( A^i \) is the four-potential \((\phi, \vec{A})\) of the electromagnetic field, \( x^i = (ct, \vec{x}) \) is the four-radius, and the normalized energy of the particle is expressed by \( \gamma = 1/\sqrt{1 - \beta^2} \), where \( \beta = \vec{v}/c \) denotes the normalized electron velocity. The radiation reaction in the generalized notation of Abraham reads [15] (see also [19])

\[
g^i = \frac{2}{3} \frac{e^2}{c} \left( \frac{d^2 u^i}{dt^2} - u^l u^k \frac{d^2 u_k}{dt^2} \right),
\]

and satisfies the relation \( g^i u_i = 0 \), which must be fulfilled for any force four-vector. Using the classical treatment of Landau and Lifshitz [17], who suggested an interaction procedure for the replacement of the higher-velocity derivatives in (2.2) by the corresponding expressions containing the electromagnetic field strengths, we obtain

\[
g^i = \frac{2}{3m^3 c^5} \frac{\partial A^k}{\partial x^i} u_k u^i - \frac{2}{3m^2 c^3} F^{il} F_{kl} u^k + \frac{2}{3m^2 c^3} (F_{kl} u^l) (F^{km} u_m) u^i. \tag{2.3}
\]

This presentation allows us to make an estimate for electrons with velocities close to \( c \). In that ultrarelativistic case, the third term in (2.3) becomes the dominant contribution. Then for a particle velocity directed along the \( x \)-axis, the relevant component of the self-force is approximately

\[
f_x \simeq -\frac{2}{3m^2 c^3} \frac{e^4}{c^2} \gamma^2 \left[ (E_y - H_z)^2 + (E_z + H_y)^2 \right]. \tag{2.4}
\]

Obviously, it is proportional to the energy square of the ultrarelativistic particle.

### 2.2. Validity criteria

Discussing the constraints for the application of (2.1) in classical electrodynamics [17–19], we recall that the expression for the radiation damping of the electron motion in an external field, \( \vec{f} = 2e \vec{v} / 3c^3 \), is correct in the proper frame of the particle.

In a reference frame, where the velocity of the electron is small, its equation of motion including the radiation friction force can be expressed in the form \( m \vec{\dot{v}} = -e \vec{E} - \frac{e}{c} (\vec{v} \times \vec{H}) + \frac{2e^2 \vec{v}}{mc^2} \). The radiation damping term must be small in comparison with the external field. The second time derivative of the electron velocity in its proper frame reads \( \vec{\ddot{v}} \approx -\frac{e}{m} \vec{E} + \frac{e^2 \vec{v}}{mc^2} (\vec{E} \times \vec{H}) \), after applying \( \vec{\dot{v}} = -e \vec{E} / m \) and neglecting here the friction force. To ensure the smallness of the self-force as mentioned above, two conditions must be fulfilled. They follow from the expression for \( \vec{\ddot{v}} \) inserted in the relation for the friction force. The comparison of both terms with the external force, \( -e \vec{E} \), gives the requirements

\[
\lambda \gg \frac{e^2}{mc^2} = r_e, \quad H \ll \frac{m^2 c^4}{e^3} = \frac{e}{r_e^2}, \tag{2.5}
\]
where the wavelength $\lambda$ of the electromagnetic field and the field strength $H$ are related to the proper frame of the particle. These conditions mean that $\lambda$ must be much larger than the classical electron radius $r_e$ and the field must be small in comparison with the field strength $e/r_e^2$.

However, the classical electrodynamic approach fails much earlier due to quantum effects—at dimensions of the order of the Compton length $\lambda_C = \hbar/mc$ and at field strengths of the Schwinger limit, $E_S = m^2 c^3/e \hbar$. Both limits are stronger by about two orders of magnitude in comparison with the values given in (2.5). As qualitative estimates [23] and Monte-Carlo simulations of electron–positron–photon cascades in circularly polarized ultrahigh fields [24] have shown, such QED effects may already appear at laser field strengths exceeding the threshold value $E_{cr} = E_S e^2/\hbar c \approx 10^{14} \text{ Vcm}^{-1}$, which is by two orders of magnitude smaller than the Schwinger field $E_S$. A similar estimate was reported in [7], discussing the effect of radiation back-reaction on the relativistic electron motion in ultrahigh laser fields.

For the field amplitude $F$ in the laboratory frame, wherein the particle moves toward the wave with the relativistic factor $\gamma$, the condition $\chi < 1$ transforms to $\gamma F \ll E_S$. Although $F$ will comply with this relation, the friction force in the ultrarelativistic case, which is of the order of $e^2 F^2 \gamma^2$ (see (2.4)), may be very large in comparison with the common Lorentz force, if $\gamma \gg \hbar / e^2$.

The limits of the LL approximation in strong electromagnetic fields were also discussed recently in [42].

2.3. Equation of motion and external forces in three-dimensional form

To solve the four-momentum equation (2.1) for a certain field configuration, we split it into separate equations for the three-vector momentum and for the energy balance and introduce the common normalizations:

$$\hat{x}_a = k_1 \bar{x}_a \text{ with } k_1 = \frac{\omega_1}{c}, \quad \hat{t} = \omega_1 t, \quad \hat{E} = \frac{e \bar{E}}{m \omega_1 c}, \quad \hat{H} = \frac{e \bar{H}}{m \omega_1 c},$$

with the carrier frequency of the incident laser wave $\omega_1$. Hereafter, we skip the hats and use the abbreviations $\vec{F}_L = -\bar{E} - \vec{p} \times \bar{H}, \ d/dt = \partial / \partial t + \beta_a \partial / \partial x_a$ and $\tau_R = 2k_1 r_e / 3$. For a laser wavelength $\lambda_1 = 0.8 \mu\text{m}$, we estimate the dimensionless damping constant $\tau_R \approx 1.5 \times 10^{-8}$.

Inserting the tensor components $F_{ik}$ in (2.1), we obtain

$$\frac{d(\gamma \vec{p})}{dt} = \vec{F}_L + \tau_R \left\{ \gamma \frac{d\vec{E}}{dt} + \gamma \vec{p} \times \frac{d\vec{H}}{dt} + (\vec{p} \cdot \bar{E}) \bar{E} + \vec{F}_L \times \vec{H} + \gamma^2 \vec{p} \left[ (\vec{p} \cdot \bar{E})^2 - \vec{F}_L^2 \right] \right\}, \quad (2.7)$$

$$\frac{d\gamma}{dt} = \vec{p} \cdot \bar{E} + \tau_R \left\{ \gamma \vec{p} \cdot \frac{d\vec{E}}{dt} + \vec{F}_L \cdot \bar{E} + \gamma^2 \left[ (\vec{p} \cdot \bar{E})^2 - \vec{F}_L^2 \right] \right\}. \quad (2.8)$$

In planar geometry, the laser field may be described by the vector potential only. To simplify the further discussion, the field amplitude is assumed to be constant in time. In addition, we suppose a circular polarization throughout the paper. The electron motion in the laser field with linear polarization was considered in [8, 14], for example.

For the incident wave we write in normalized units

$$\vec{a}_1 = a_1 (\hat{e}_y \sin \varphi_1 + \hat{e}_z \cos \varphi_1), \quad (2.9)$$
with the phase $\varphi_1 = t + x$. Here and below the electron is injected in the direction of the $x$-axis as illustrated in figure 1. The normalized vector potential $a_1$ is related to the intensity as $a_1 = \sqrt{I_1/(mn_e c^3)}$, where $n_e$ names the electron critical density.

The energy radiated per frequency $\omega$ into the solid angle $d\Omega = \sin \theta d\theta d\phi$ (with the azimuthal angle $\theta$) is determined as

$$\frac{d^2I}{d\omega d\Omega} = \frac{c}{4\pi^2} |\tilde{E}_\omega|^2 R^2,$$

where $\tilde{E}_\omega = \int_{-\infty}^{\infty} \tilde{E} \exp(i\omega t) dt$ is calculated for the strength of the radiation field $\tilde{E}$ obtained from Lienhard–Wiechert potentials, and $R$ is the distance from the observation point to the particle. If we describe the emitted radiation in the

2.4. Classical radiation emission

The basic radiation characteristics for an electron moving in an external electromagnetic wave follow from the expression for the four-momentum $\Delta P^i$ of this radiation that is defined as

$$\Delta P^i = -\frac{2e^2}{3c} \int \frac{du^k}{dt} \frac{du_k}{dt} dx^i,$$

where cgs units are supposed. Its time component gives the total emitted energy $\Delta \epsilon_{rad} = c \Delta P^0$. The power radiated by a single electron is determined as $P_{rad} = d(\Delta \epsilon_{rad})/dt$.

For the relativistic electron motion in a circularly polarized plane laser wave with constant field amplitude, this power also remains constant, as long as the electron energy does not decay due to radiation damping. Thereby, it depends quadratically on the field strength as well as on the electron energy, $P_{rad} = \tau_R \hbar_0 a_1^2 [2]$, where $P_{rad}$ is normalized to $mc^2\omega_1$. In the case of a counterpropagating wave, the parameter $h_0$ denotes the corresponding integral of motion, $h_0 \equiv \gamma + u_x = \gamma(0) + u_x(0) = \sqrt{\gamma_0^2 + a_1^2 + \gamma_0 \beta_0}$, which accounts for the transverse momentum $a_1$ that the electron gets in addition to the initial longitudinal momentum $\gamma_0 \beta_0$, as soon as it experiences the intense field.

A detailed discussion of the radiation characteristics for the different interaction schemes, which we are going to analyze in this paper, will be given in a separate publication. In this paper, we demonstrate only a few numbers of spectral data in relation to the discussed electron dynamics, starting with a brief recall of several basic statements.

The energy radiated per frequency $\omega$ is determined as

$$\frac{d^2I}{d\omega d\Omega} \propto \left|\tilde{E}_\omega\right|^2 R^2,$$

where $\tilde{E}_\omega$ is the Fourier component of the emitted radiation field obtained from Lienhard–Wiechert potentials, and $R$ is the distance from the observation point to the particle. If we describe the emitted radiation in the
form of plane waves (far field approximation), we find the relation written in dimensionless form
\[
\frac{d^2 I}{d\omega \, d\Omega} = \frac{3 \tau_R}{2\pi} \omega^2 \left| \int_{-T/2}^{T/2} \left[ \vec{n} \times (\vec{n} \times \vec{\beta}) \right] e^{i\omega (t-r/c)} \, dt \right|^2,
\]
(2.14)
where the energy, frequency, solid angle, time and radius are normalized to \(mc^2, \omega_1, 4\pi, \omega_1^{-1}\) and \(k_1^{-1}\), respectively, \(T\) is the normalized interaction time, and \(\vec{n}\) represents the unit vector directed from the electron toward the observer [17, 18].

In contrast to the direct integration in (2.14), which requires a high temporal resolution in the calculation of the particle trajectory for the definition of the high-frequency part of the radiation spectrum, alternative algorithms were developed in [44] and [45]. The first of these assumes a quadratic interpolation of the numerically calculated particle trajectory, which reduces the integration problem to the solution of Fresnel integrals at each time step. The second approach is based on the concept of photon formation length and allows us to calculate spectra from highly relativistic particles, avoiding artifacts from the limited time resolution of the particle trajectory. Since we are restricting the discussion of spectral features in this paper to lower-order harmonics, the direct integration of the fields from the Lienard–Wiechert potentials (2.14) provides a sufficient precision at relatively low computational expenses.

The asymptotic radiation spectrum (2.14) for large harmonic numbers \(n \gg 1\) has the same form as the synchrotron spectrum emitted by an electron, which moves in an instantaneously circular orbit [17, 18]. It reads
\[
\frac{d^2 I}{d\omega \, d\Omega} \simeq \frac{18 \tau_R}{\pi} N_0 \gamma^2 \xi^2 K_{2/3}(\xi).
\]
(2.15)
This relation describes the spectral intensity emitted under the optimum angle \(\theta_0 \simeq 2/\sqrt{M_0}\) to the \(x\)-axis, while the electron traverses \(N_0\) laser wavelengths [2]. The relativistic Doppler upshift factor for the radiation scattered at this angle is \((M_0 + 1)/2\). The shift parameter \(M_0 = (1 + \beta_s)/(1 - \beta_s) \equiv h_0/(1 + a_1^2)\), with \(h_0 = \gamma_0(\sqrt{1 + a_1^2/\gamma_0^2} + \beta_0)\), characterizes the frequency of the radiation scattered under the angle \(\theta = 0\). The \(\gamma\)-factor in (2.15) is determined as \(\gamma = a_1 (M_0 + 1)/(2 \sqrt{M_0})\). Equation (2.15) contains the modified Bessel function \(K_{2/3}(\xi)\), which depends on the frequency \(\omega\) normalized to a characteristic roll-off frequency \(\omega_m = n_m (M_0 + 1) \omega_1/2\), beyond which the spectral distribution exponentially decays.

With the corresponding cut-off harmonic order, \(n_m \simeq 3a_1^3\), we obtain the expression
\[
\omega_m \simeq \frac{3}{2} a_1^3 M_0 \omega_1 \simeq \frac{3}{2} a_1 h_0^3 \omega_1
\]
(2.16)
for large numbers \(M_0\), which equals the result from [17] for the radiation emission of an ultrarelativistic electron moving along a circular trajectory in an intense electromagnetic field.

After integration of (2.15) over the solid angle we get an expression for the radiation energy emitted per frequency
\[
\frac{dI}{d\omega} \simeq 3\sqrt{3} \tau R N_0 \gamma \frac{\omega}{\omega_m} \int_{2\omega_0/\omega_m}^{\infty} K_{5/3}(\xi) \, d\xi
\]
(2.17)
in units \(mc^2/\omega_1\). The integration of this function over frequency gives the dependence of radiation energy on \(\omega/\omega_m\). Using the final value of this function at large arguments \(\omega/\omega_m \gtrsim 1\), we estimate the total amount of the emitted energy.

For the parameters \(a_1 = 50\) \((I_1 \simeq 10^{22} \text{Wcm}^{-2})\), \(\gamma_0 = 300\), we find \(n_m \simeq 3.75 \times 10^5\), \(M_0 \simeq 146\), \(\gamma \simeq 304\). The cut-off frequency becomes \(\omega_m \simeq 2.75 \times 10^7 \omega_1\), which corresponds to a
photon energy of \(\approx 43\ \text{MeV}\) in the case of a laser wavelength \(\lambda_1 = 0.8\ \mu\text{m}\). Traversing the distance of \(22\ \lambda_1\) in a counterpropagating laser wave, the electron scatters \(\approx 2.8\ \text{GeV}\) of radiation energy, as far as the radiation damping effect is neglected. The estimated scattering radiation power is approximately \(2.5\ \text{kW}\). With the account for radiation friction, the final number for radiated power becomes significantly smaller, and the accumulated radiation energy will be reduced by one order of magnitude, as the corresponding numerical simulation has shown.

3. Numerical integration of basic equations and analytical estimates for the electron motion in simple field configurations

The numerical integration of the system (2.7) and (2.8) for an electron with the initial energy \(\gamma_0\) and the longitudinal momentum \(u_x(0) = \beta_0 \gamma_0 = \sqrt{\gamma_0^2 - 1}\) (prior to the propagation in the electromagnetic field) is conducted for the following interaction schemes: (i) the electron scatters a counterpropagating wave, \(a_1 (\vec{e}_y \sin \varphi + \vec{e}_z \cos \varphi)\), with \(\varphi = t + x\); (ii) the electron copropagates with the wave \(a_1 (\vec{e}_y \sin \varphi - \vec{e}_z \cos \varphi)\), where \(\varphi = t - x\); (iii) the electromagnetic field consists of a superposition of two counterpropagating laser waves with the same constant amplitude \(a_1\) and carrier frequency \(\omega_1\); (iv) both counterpropagating waves with identical amplitudes and frequencies have a relative phase shift of \(\pi\) and show the standing wave pattern.

The set of equations (2.7) and (2.8) together with the laser waves defined in (2.9) and (2.11) were integrated for a single electron with the given initial energy \(\gamma_0\) and the longitudinal momentum \(u_0 = \beta_0 \gamma_0\) using an adaptive fourth-order Runge–Kutta scheme [43].

In parallel, we analyze the following system of reduced equations—with the leading terms on the right-hand side proportional to the factor \(\gamma^2\):

\[
\frac{d(\gamma \tilde{\beta}_\perp - \tilde{a})}{dt} = \tau_\mathcal{R} \tilde{\beta}_\perp \gamma^2 \mathcal{R},
\]

\[
\frac{du_x}{dt} = -\tilde{\beta}_\perp \frac{\partial \tilde{a}}{\partial x} + \tau_\mathcal{R} \beta_x \gamma^2 \mathcal{R},
\]

\[
\frac{d\gamma}{dt} = \tilde{\beta}_\perp \frac{\partial \tilde{a}}{\partial t} + \tau_\mathcal{R} \gamma^2 \mathcal{R},
\]

where

\[
\mathcal{R} = -\left[\left(\frac{d\tilde{a}}{dt}\right)^2 + \left(\tilde{\beta}_\perp \frac{\partial \tilde{a}}{\partial x}\right)^2 - \left(\tilde{\beta}_\perp \frac{\partial \tilde{a}}{\partial t}\right)^2\right].
\]

Equation (3.1) describes an additional transverse momentum, \(\Delta \tilde{u}_\perp = \gamma \tilde{\beta}_\perp - \tilde{a}\), which appears due to the action of the radiation friction force. As we will show below, the excitation of this momentum will reduce the electron energy loss due to radiation damping in a counterpropagating laser wave. If the electron moves in a configuration of two waves propagating in opposite directions, the induced momentum \(\Delta \tilde{u}_\perp\) may cause a re-acceleration of the electron until a stationary propagation regime has been established. Moreover, in the interaction with a single copropagating wave at certain initial parameters the electron can continuously gain energy together with an increase of its longitudinal momentum.

Equations (3.1)–(3.3) may be integrated analytically for simple electromagnetic field configurations as far as \(|\Delta \tilde{u}_\perp/\tilde{a}| \ll 1\) and \(\gamma\) and \(\beta_x\) are slowly changing functions on the time scale of a laser period.
3.1. Interaction with a counterpropagating laser wave

Firstly, we revisit the well-known case of electron propagation in a counterstreaming laser wave. However, in contrast to the process of nonlinear Thomson scattering at relatively low laser field amplitudes, \( a_1 \gtrsim 1 \), where the scattered photons get maximum frequency upshift with the factor \( M_0 \simeq (1 + \beta_0)^2/\gamma_0^2 \), we analyze the interaction in high electromagnetic fields, where the influence of radiation friction force on classical electron motion becomes significant.

When we solve the equation of motion of the electron in the wave frame, two integrals of motion can be determined at negligible radiation friction: (i) the conservation of the canonical transverse momentum, \( \vec{u}_\perp = \vec{a} \), and (ii) the conservation of energy, \( \gamma + u_x = h_0 \). The latter constant may be expressed as \( h_0 = \gamma(0) + u_x(0) = \gamma_0(\sqrt{1 + a_1^2/\gamma_0^2} + \beta_0) \), as already used in section 2.4. For laser intensities \( a_1^2 \ll \gamma_0^2 \), the parameter \( h_0 \) can be written in the reduced form, \( h_0 \simeq \gamma_0 (1 + \beta_0) \). The drift velocity behaves as \( \beta_x = u_x/\gamma = \beta_0/\sqrt{1 + a_1^2/\gamma_0^2} \). In terms of the Doppler upshift factor \( M_0 \) for the backscattered laser light it can also be expressed in the form \( \beta_x = (M_0 - 1)/(M_0 + 1) \).

At sufficiently high laser amplitudes and initial electron energies \( a_1 \sim \gamma_0 \sim 100 \), radiation friction becomes important, and these integrals of motion do not exist anymore. Since the field intensity \( a^2 \) in case of the vector potential \( \vec{a} = a_1 (\vec{e}_x, \sin \varphi + \vec{e}_z \cos \varphi) \) with the phase \( \varphi = t + x \) is constant, \( a^2 = a_1^2 \), we get the expression \( \mathcal{R} = - \left( \frac{\partial \vec{a}}{\partial \vec{x}} \right)^2 = - a_1^2 (1 + \beta_x)^2 \), which has to be inserted into (3.1)–(3.3).

The approximate solution of (3.1), supposing small changes in \( \gamma \) and \( \beta_x \) over a laser period, gives the induced transverse momentum \( \Delta \vec{u}_\perp = - \tau_R a_1^4 \gamma (1 + \beta_x) [\vec{e}_x (1 - \cos \varphi) + \vec{e}_z \sin \varphi] \). For parameters \( a_1 \sim \gamma \lesssim 100 \), it has a small magnitude, \( |\Delta \vec{u}_\perp|/a_1 \sim \tau_R a_1^2 \gamma \lesssim 10^{-2} \).

As long as the longitudinal velocity \( \beta_x \) does not decrease significantly, (3.2) can be rewritten as

\[
\frac{du_x}{dt} = - 2 \tau_R a_1^4 \left( \frac{2u_x^2}{a_1^2} + 1 \right),
\]

supposing \( \beta_x \sim 1 \) and applying the term \( \Delta \vec{p}_\perp \partial \vec{a}/\partial \vec{x} \simeq 2 \tau_R a_1^4 \) averaged over the laser field oscillation. The right-hand side of (3.5) is always negative, and we find a solution, where the particle longitudinal momentum decreases with time as

\[
u_x = \frac{u_x(0) - \frac{a_1^2}{\sqrt{2}} \tan \kappa t}{1 + \frac{\sqrt{2}}{a_1^2} u_x(0) \tan \kappa t},
\]

with the initial momentum \( u_x(0) = \beta_0 \gamma_0 \) and the coefficient \( \kappa = 2 \sqrt{2} \tau_R a_1^3 \).

Equation (3.3) in the same approximation reads

\[
\frac{d\gamma}{dt} = - 2 \tau_R a_1^4 \left( \frac{2\gamma^2}{a_1^2} - 1 \right).
\]

Since \( \gamma^2 = (1 + a_1^2)/(1 - \beta_x^2) > a_1^2 \), the right-hand side of this equation is invariably negative. Integrating it, we obtain the energy dependence on time,

\[
\gamma = \frac{\gamma(0) + \frac{a_1^2}{\sqrt{2}} \tan \kappa t}{1 + \frac{\sqrt{2}}{a_1^2} \gamma(0) \tan \kappa t},
\]
Figure 2. The laser field intensity $a^2$ and the ponderomotive force $d\gamma/dx$, ‘seen’ by the electron, its $\gamma$-factor and the longitudinal momentum $u_x$ as functions of the normalized propagation distance are plotted in the upper part of the panels. The corresponding electron velocity components are given in the lower parts. Panel (a) contains simulation results without friction force, which is taken into account in (b). Laser amplitude: $a_1 = 50$, initial electron energy: $\gamma_0 = 300$. The dashed green curves are determined by (3.6), (3.8) and by the ratio of both functions ($\beta_x = u_x/\gamma$), respectively.

with the initial energy value $\gamma(0) = \sqrt{\gamma_0^2 + a_1^2}$ and the same coefficient $\kappa$ as in (3.6). For ultrarelativistic electron energies $\gamma_0 \gg a_1$, the simpler relation $\gamma \approx \gamma(0)/[1 + 4 \tau_R a_1^2 \gamma(0) t]$ will be found [8]. In this case, the additional transverse momentum $\Delta \vec{u}_\perp$ becomes insignificant.

The analytical estimates in comparison with results of the numerical integration of (2.7) and (2.8) are presented in figure 2 (depicted by green dashed curves) for the propagation of an electron with initial energy of 150 MeV ($\gamma_0 = 300$) in a wave with constant field amplitude $a_1 = 50$ ($I_L \approx 10^{22}$ W cm$^{-2}$). Panel (a) shows the situation without radiation damping, panel (b) demonstrates the influence of radiation friction force on electron motion. The corresponding analytical solutions reproduce the decay of longitudinal electron momentum and of its $\gamma$-factor as well as the reduction of electron longitudinal velocity. The appearing gradient force $d\gamma/dx$ always has a negative sign and causes continuous retardation of the electron, which is attended by a decrease of the scattered radiation energy.

This change in radiation power $P_{rad}$ and its time integral $\Delta \varepsilon_{rad}$ are shown in figure 3 together with the dynamical characteristics of the electron, $u_x$ and $\gamma$, for several field amplitudes and initial electron energies $\gamma_0$. The influence of radiation friction proves to be marginal at laser amplitudes $a_1 \sim 10$ and initial electron energies of the order of tens of MeV (see panel(a)), whereas the radiation power decreases remarkably for electron energies $\gamma_0 \gtrsim 100$. Finally, in high-intensity fields, $I_L \gtrsim 10^{22}$ W cm$^{-2}$, it starts to drop dramatically, and the photon energy
Figure 3. The power and energy of scattered radiation, the longitudinal momentum \( u_x \) and the \( \gamma \)-factor of the electron versus the normalized distance of propagation in an intense counterpropagating plane laser wave. The relevant input quantities are given in the titles of the panels. The numerical data without the account for radiation damping are depicted by dotted lines, the dashed green curves in panels (b) and (c) display the analytical estimates described in the text.

emitted during 20 laser periods becomes ten times less than without the account for radiation damping.

Applying the time derivatives of the electron momentum components and of its \( \gamma \)-factor, taken from (3.1)–(3.3) for the given case, we derive an analytical expression for the radiation power (in units \( mc^2 \omega_1 \)),

\[
P_{rad} = \tau_R \gamma^2 \left[ \left( \frac{du}{dt} \right)^2 - \left( \frac{d\gamma}{dt} \right)^2 \right] \approx \tau_R a_1^2 (\gamma + u_x)^2.
\]

The right term is identical with the formula \( P_{rad} = \tau_R a_1^2 h_0^2 \) at negligible radiation friction [2], where \( \gamma + u_x \) is an integral of motion. For sufficiently large \( \gamma_0 \) and comparatively small laser fields \( a_1^2/\gamma_0^2 \ll 1 \), we have \( h_0 \simeq 2\gamma_0 \). With help from the analytical estimates (3.6), (3.8) for \( u_x(t) \) and \( \gamma(t) \), respectively, the approximate result

\[
P_{rad}(t) \simeq \frac{4\tau_R a_1^2 \gamma_0^2}{(1 + 4 \tau_R a_1^2 \gamma_0 t)^2}
\]

may be deduced. Its time integral gives the emitted radiation energy \( \Delta E_{rad} \). Panels (b) and (c) of figure 3 show good agreement of these analytical expressions with numerical results.

Next, we study the electron trajectory in the circularly polarized electromagnetic wave—first without the account for radiation friction. In contrast to [2, 14], we do not solve the equation for the electron trajectory in the frame of the running wave, but transform it to the proper frame of the relativistic electron, where it reads

\[
\gamma' \frac{d\vec{r}'}{dt'} = \vec{a} = a_1 (\vec{e}_y \sin \phi' + \vec{e}_z \cos \phi'),
\]
with \( \psi' = \sqrt{M_0} \, t' \) in the plane \( x' = 0 \), expressed through the frequency upshift factor \( M_0 = (1 + \beta_x)/(1 - \beta_x) \). Because of the integral of motion, \( \gamma' + u'_x = \gamma' = \sqrt{1 + a_1^2} \), equation (3.11) can be integrated immediately giving the relations

\[
y(t') = \frac{a_1}{h_0}(1 - \cos \psi'), \quad z(t') = \frac{a_1}{h_0} \sin \psi',
\]

where the initial conditions \( y(0) = z(0) = 0 \) were assumed. The frequency factor in the oscillation amplitudes, \( \sqrt{M_0} \), was replaced by the equivalent term \( h_0/\sqrt{1 + a_1^2} \), with \( h_0 = \gamma_0(\sqrt{1 + a_1^2}/\gamma_0 + \beta_0) \). The expression for the radius of the circular orbit (3.12), \( \rho = \frac{a_1}{h_0} \), is identical with [2]. However, its value is strongly reduced at high field strengths due to the large transverse momentum of the electron.

For strong radiation damping, simple integration of the trajectory equation becomes impossible, since the \( \gamma \)-factor now changes. The circular trajectory transforms into a helix with increasing radii of consecutive loops due to decreasing electron energy.

Figure 4. Electron trajectories with radiation damping (blue helix) and without it (black circles) for the parameter sets: (a, d) \( a_1 = 10, \, \gamma_0 = 30 \), (b, e) \( a_1 = 10, \, \gamma_0 = 300 \) and (c, f) \( a_1 = 50, \, \gamma_0 = 300 \). The upper panels contain projections of the trajectories into a plane normal to the propagation direction \( x \), the lower ones demonstrate electron motion in plane \( xy \).

The influence of radiation friction on electron trajectory is illustrated in figure 4. The projection of the particle orbit in the \( yz \)-plane is plotted as a function of time in figures 4(a)–(c). The corresponding field amplitudes are given in the titles. The radii of the circular orbits \( \rho = 0.16, 0.017, 0.08 \) obtained in simulations without radiation damping for different sets of parameters \( (a_1, \gamma_0) \) are identical with the estimated numbers \( \rho = \frac{a_1}{h_0} \). Figures 4(d)–(f) show the drift motion of the electron for the same sets of parameters. The normalized drift velocity

\( a_0 = 10, \, \gamma_0 = 30 \)
\( a_0 = 50, \, \gamma_0 = 300 \)

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Figure 5. Spectral intensity distributions at two observation angles. Red and orange lines display simulation results without radiation friction, blue and cyan curves show the radiation damping effect. The backward scattered radiation at θ = 0 (along the x-axis) contains only the fundamental (red and blue lines). Multiple harmonics are plotted for the optimum observation angle θ = θ₀ (orange and cyan lines). Initial parameters a₁ and γ₀ are given in the subtitles of the panels.

βₓ = (M₀ − 1)/(M₀ + 1) is in all cases with or without weak radiation friction close to unity, since the corresponding frequency upshifts are large. At significant radiation damping, the electron drift starts to decrease (see figure 4(f)). In comparison with the higher-intensity case (c), the effect of this force is strongly reduced at lower values of field strength, e.g. a₁ = 10. For a smaller initial electron energy, γ₀ = 30, it becomes almost negligible.

Finally, we briefly comment on the lower-frequency part of the corresponding radiation spectra, which was calculated with the help of (2.14) and is displayed in figure 5. The initial electron energy was taken γ₀ = 30 in figure 5(a) or 300 in figures 5(b) and (c). First, we will discuss the spectral intensity distributions evaluated without the account for radiation damping. As stated in earlier publications and recalled above, the corresponding frequency upshift factor M₀ amounts to h₂₀/(1 + a₁²). The frequencies in all demonstrated spectra are normalized to the corresponding upshift factors, M₀ = 37.6, 3566.3, 146, respectively for parameter sets (a)–(c), and the spectral intensities have the same scale in all panels. At the azimuthal observation angle θ = 0 (backscattering along the x-axis), the spectra contain independently on the values of the laser field amplitude or the initial electron energy only the first harmonic. The relative linewidth δω/ω₁ = 1/N₀ is determined by the duration of the interaction process expressed in laser periods. In our case, it should be slightly smaller than 0.05, what follows also from figure 4. The comparison of the intensity ranges in cases with equal field amplitudes reminds one about the quadratic dependence on electron energy mentioned in section 2.4.

If radiation damping becomes significant, the intensity of short-wavelength radiation remarkably decreases at high γ₀ (in panel (b) by approximately one order of magnitude), and the spectral lines are downshifted and broadened—in agreement with earlier results [8, 9]. At lower electron energy, γ₀ = 30, we observe only a weak influence of the emission by the electron along its propagation direction. In comparison, for the ten times larger electron energy, the line
broadening increases by a factor of 10. For a higher laser field, e.g. in panel (c), this effect is further reinforced.

As soon as we detect the radiation field at an azimuthal angle $\theta > 0$, harmonics with frequencies $\omega_n = n M_0 \omega_1 / (1 - \beta_1 M_0 (1 - \cos \theta))$, where $\beta_1 = (M_0 - 1) / 2 M_0$, will appear in the spectrum, as predicted by analytical theory for the electron motion without radiation damping [2]. Several lower-order harmonics, which were observed at the scattering angle, $\theta_0 \approx 2 / \sqrt{M_0}$, are depicted in figure 4 by orange lines (without radiation friction) or by cyan curves with the account for radiation damping. The comparison of the spectral intensities for cases with ten times different electron energy and equal laser intensity again shows the scaling with $\gamma_0^2$, whereas their dependence on the field amplitude scales approximately as $a_1^{-2}$, as we may conclude from panels (b) and (c). Further, in agreement with analytical theory, the linewidth of higher harmonics $\omega_n$ will be reduced by the factor $1/n$. The expression $\theta_0 = 2 / \sqrt{M_0}$ marks the angle of optimum scattering at high field strengths $a_1$, where the corresponding harmonic frequencies $\omega_n \approx n M_0 / 2$, as was already recalled in section 2.4. This behavior is nicely reproduced in panel (c) (see orange lines). For the spectral cutoff in case (c) we remind the reader about the relation (2.16) applied in section 2.4, where we obtained the value $\omega_m \approx 2.7 \times 10^7 \omega_1$, which is equivalent to a photon energy of about 40 MeV. At five times lower field amplitude the maximum photon energy amounts to 8 MeV only.

Concerning the influence of friction force on the spectrum at $\theta > 0$, it shows up in the form of a slightly increased line downshift and broadening in case (a). This behavior becomes more pronounced with higher harmonic order. At the larger electron energy $\gamma_0 = 300$, the spectrum at angles $\theta > 0$ dramatically changes and takes the form of a quasi-continuum as observed before in [8]. This is a consequence of continuous decrease of frequency upshift with drop of electron energy. Moreover, since the effect of the radiation friction increases in higher laser fields, the electron loses its energy faster and the scattering efficiency falls off.

### 3.2. Electron motion in a comoving wave

When a relativistic electron with initial energy $\gamma_0$ and momentum $u_x(0) = \beta_0 \gamma_0$ enters the circularly polarized laser field in the same direction as the wave propagates, the equation for longitudinal momentum reveals the constant of motion $\gamma - u_x = h_0$ in the wave frame as long as radiation friction is insignificant. Similar to the case of a counterpropagating wave, the electron gains a large transverse momentum from the laser field, which equals the field strength, so that its $\gamma$-factor becomes $\gamma(0) = \sqrt{\gamma_0^2 + a_1^2}$. The constant $h_0$ takes the form $h_0 = \gamma_0 \sqrt{1 + a_1^2 / \gamma_0^2 - \beta_0}$, and the electron drift velocity becomes the same as in the case with the counterpropagating wave, $\beta_x = \beta_0 / \sqrt{1 + a_1^2 / \gamma_0^2}$.

The radiation damping in the electromagnetic wave $\vec{a} = a_1 (\vec{e}_x \sin \phi - \vec{e}_z \cos \phi)$, with phase $\varphi = t - x$, is defined by the relation $R = -a_1^2 (1 - \beta_x)^2$, since field intensity again equals $a_1^2$. The integration of (3.1) in an approximation similar to that of the previous subsection gives additional transverse momentum component $\Delta \vec{u}_x = -\tau_R a_1^2 \gamma (1 - \beta_x) [\vec{e}_z (1 - \cos \phi) - \vec{e}_x \sin \phi]$, which is relatively small due to the factor $(1 - \beta_x)$.

Equation (3.2) takes the approximate form

$$\frac{du_x}{dt} = \tau_R a_1^2 (1 - \beta_x)^2 \gamma^2,$$

(3.13)
Figure 6. The laser field component $a_y$ and the ponderomotive force $d\gamma/dx$, its $\gamma$-factor and the longitudinal momentum $u_x$ as functions of the normalized propagation distance are plotted in the upper part of panels (a–c). The corresponding electron velocity components are given in the lower part. Panel (a) contains simulation results without friction force, which is taken into account in (b, c). Initial parameters are in (a, b): $a_1 = 50$, $\gamma_0 = 10$, in (c): $a_1 = 100$, $\gamma_0 = 30$. The half-cycles of the gradient force $d\gamma/dx$ are filled to the baseline as a guide for the eye. The green dashed curves in panels (b) and (c) show the analytical estimates (3.14), (3.16) and their ratio $\beta_x = u_x/\gamma$.

supposing $1 - \beta_x \ll 1$. The derivative is now positive, and the integration of this equation gives the expression

$$u_x = u_x(0) \left[ 1 + 3\tau_R \left( \frac{a_1^2}{u_x(0)} \right) \right]^{1/3}. \tag{3.14}$$

From the approximate energy balance equation

$$\frac{d\gamma}{dt} = \tau_R a_1^2 (1 - \beta_x)^2 \gamma^2, \tag{3.15}$$

which indicates the same acceleration process, the dependence

$$\gamma = \gamma(0) \left[ 1 + 3\tau_R \left( \frac{a_1^2}{\gamma(0)} \right) \right]^{1/3} \tag{3.16}$$

may be obtained. Knowing the result for $\gamma$, one could also determine the longitudinal momentum as $u_x(t) = \sqrt{u_x^2(0) + \gamma^2(t) - \gamma^2(0)}$, which follows immediately from the combination of (3.13) and (3.15) in the approximation $\beta_x \sim 1$.

These analytical dependencies (green dashed lines) together with the numerical solutions of (2.7) and (2.8) are depicted in figure 6 for two different initial electron energies (a) $\gamma_0 = 10$, (b) 30 and field amplitudes $a_1 = 50$ and 100, respectively. The analytical estimates are close to
Figure 7. The longitudinal momentum $u_x$ and the $\gamma$-factor of the electron as well as the power and energy of the radiation scattered during its drift in a copropagating intense plane laser wave. The relevant input quantities are given in the panel titles. The numerical data without the account for radiation damping are depicted by dotted lines, and the dashed green curves in panels (b) and (c) display the analytical estimates derived in the text.

the numerical results. Although the gradient force caused by the radiation friction oscillates, the positive half-periods clearly dominate. This is especially obvious in the case of the larger vector potential $a_1 = 100$ ($I_L \simeq 4 \times 10^{22}$ Wcm$^{-2}$). The acceleration effect becomes relatively small at higher values $\gamma_0$, as is shown in figure 7(c).

Figure 7 demonstrates the influence of friction effect on radiation power and energy. Generally, these values are clearly smaller than in the interaction with a counterpropagating wave because of the reduced Doppler frequency shift. In analogy with (3.9), the expression for radiation power reads

$$P_{rad} = \tau_R \gamma^2 \left[ \left( \frac{du}{dt} \right)^2 - \left( \frac{d\gamma}{dt} \right)^2 \right] \approx \tau_R a_1^2 (\gamma - u_x)^2.$$  (3.17)

For negligible radiation damping, we obtain the expression $P_{rad} = \tau_R a_1^2 h_0^2$ [2], with the corresponding integral of motion $h_0 = \gamma_0 \sqrt{1 + a_1^2/\gamma_0^2 - \beta_0}$.

In the case of $a_1 = \gamma_0 = 30$, the laser field is not strong enough to induce a sufficiently large friction force. The acceleration effect due to radiation friction at such laser amplitudes and initial electron energies in the range of several tens of MeV is weak. It is even smaller for larger drift velocities. Therefore, the electron dynamics in schemes of direct laser acceleration by a wave, which is channeling in a near-critical plasma [46] or of wakefield acceleration in the bubble regime [47, 48], which are currently studied in this parameter range, will not be distinctly altered by radiation friction in the driving laser wave.

The example in figure 7(c) also exhibits a marginal acceleration effect in spite of the high field amplitude. The reason for this behavior is the large longitudinal momentum of the electron, which provides a motion almost in phase with the laser field. Setting the coefficient $0.75 \tau_R (a_1^2/\gamma(0))^2 = 10^{-3}$ in the expression for the electron $\gamma$-factor (3.16) as the ‘threshold’ for
a distinct acceleration effect in the comoving laser wave, we derive the approximate condition 
\( (a_1)^2 \min \gtrsim 45 \gamma(0) \). It offers minimum field amplitudes for the examples demonstrated in panels (b) and (c) of figure 6 and in figure 7(b), which are below the chosen input values. In contrast, the initial \( \gamma \)-factors in the cases (a) and (c) of figure 7 result in numbers \( (a_1)_{\min} \), which exceed the input vector potentials.

Concerning the electron trajectory, we proceed in a similar way as in the previous subsection, whereby the laser light is now Doppler downshifted, \( \omega' = \sqrt{M_0} \omega_1 = \sqrt{(1 - \beta_x)/(1 + \beta_x)} \omega_1 \) in the proper frame of the electron. At negligible radiation damping, we gain the relations

\[
\begin{align*}
y(t') &= a_1 h_0 (1 - \cos \varphi'), \quad z(t') = -\frac{a_1}{h_0} \sin \varphi', \\
y(0) &= z(0) = 0.
\end{align*}
\]

(3.18)

Since the parameter \( h_0 \) is defined as \( h_0 = \gamma_0(1 + a_1^2/\gamma_0^2 - \beta_0) \), the circular orbit becomes much larger than in the interaction with a counterpropagating wave.

Figure 8 illustrates the projections of the electron motion on a plane normal to its longitudinal velocity component \( \beta_x \) (upper panels) and in the plane \( xy \) (lower panels). Applying relation (3.18), we estimate the radii of the circular orbits, \( \rho = 1.22, 1.34, 6.16 \) for the initial parameters in (a)–(c), respectively, in perfect agreement with the numerical data. At high electron energies (see figures 8(c) and (f)), the influence of radiation friction is very small,
although the laser wave has a large intensity. The increase of drift velocity is only marginal in this case, whereas the panels (d) and (e) show a remarkable acceleration. A similar effect of electron acceleration in an intense laser wave was reported in [14].

The analysis of radiation spectra shows similar features to those in the previous subsection, however, with a significant difference in the frequency shift. In a propagation direction of the electron (and of the comoving wave) we see only the laser frequency itself. The same situation was observed in the case of a counterpropagating laser wave under the angle \( \theta = \pi \), as expected. Looking in the direction of the laser light propagation, the fundamental is downshifted by the factor \( M_0 = (1 - \beta_\gamma)/(1 + \beta_\gamma) = h_0^2/(1 + a_l^2) \). At observation angles different from \( \theta = 0, \pi \), higher harmonics appear. With significant radiation friction, the line spectrum will be transformed into a quasi-continuous spectrum again.

3.3. Two identical waves with opposite propagation directions

The next case we are going to analyze is the electron motion in the superposition of two laser waves with identical frequencies \( \omega_1 \) and field amplitudes \( a_1 \) but opposite propagation directions. The sum vector potential is defined as

\[
\vec{a} = a_1 \left[ \vec{e}_y (\sin \varphi_1 + \sin \varphi_2) + \vec{e}_z (\cos \varphi_1 - \cos \varphi_2) \right] = 2a_1 \sin t (\vec{e}_y \cos x - \vec{e}_z \sin x),
\]

with the phases \( \varphi_1 = t + x \) and \( \varphi_2 = t - x \). The intensity \( a^2 = 4a_1^2 \sin^2 t \) as well as the square of the time derivative \( (\frac{d\vec{a}}{dt})^2 = 4a_1^2 (\cos^2 t + \beta_\gamma^2 \sin^2 t) \) are now oscillating functions of time.

If the radiation damping is negligible, the transverse momentum equals the field vector \( \vec{u}_\perp = \vec{a} \). The longitudinal momentum will be constant, since \( \frac{d(a^2)}{dx} = 0 \). So, we have \( u_x(t) = u_x(0) = \beta_0 \gamma_0 \). For the energy balance we find the equation

\[
\frac{dy}{dt} = \frac{2a_1^2 \sin 2t}{\gamma},
\]

which can be easily integrated giving the expression

\[
y(t) = \gamma_0 \sqrt{1 + \frac{4a_1^2}{\gamma_0^2} \sin^2 t}.
\]

The longitudinal electron velocity follows then as \( \beta_x(t) = u_x(0)/\gamma(t) \).

Since the \( \gamma \)-factor oscillates in time, a second integral of motion, \( \gamma \pm u_x = \text{const} \) does not exist. For the time-averaged functions under the assumption \( a_1^2 \ll \gamma_0^2 \), we get the expressions

\[
\overline{\gamma} \approx \gamma_0 (1 + a_1^2/\gamma_0^2), \quad \overline{\beta_x} \approx \beta_0 (1 - a_1^2/\gamma_0^2).
\]

Figure 7(a) shows the numerically determined momentum component \( u_x \) of the electron, its energy, the field intensity and the ponderomotive force without the account for radiation damping as functions of the propagation distance \( x \) of the particle. The field amplitudes were chosen \( a_1 = a_2 = 50 \), the initial electron energy is \( \gamma_0 = 300 \). The characteristics of the interaction reproduce the constant momentum \( u_x \) and the oscillating electron energy (3.21), which is caused by the oscillating ponderomotive force, \( \frac{dy}{dx} = 2a_1^2 \sin(2t)/\beta_0 \gamma_0 \).
The radiation friction leads to an additional transverse momentum component \( \Delta \vec{u}_\perp \), similar to the previously studied one-wave cases. With the functional coefficient \( R = -4a_1^2\beta_s^2 \), supposing \( a_1 \ll \gamma_0 \), this momentum obeys the equation
\[
\frac{d\Delta \vec{u}_\perp}{dt} = -4 \tau_R a_1^2 \beta_s^2 \gamma (\vec{a} + \Delta \vec{u}_\perp).
\]
(3.23)

The numerical integration of (2.7) and (2.8) has shown that the momentum component \( \Delta \vec{u}_\perp \) may not be neglected in (3.23) compared to the field vector \( \vec{a} \) in the present interaction—in contrast to the one-wave cases. This circumstance hampers the derivation of simple estimates like in the previous subsections.

In the following analysis, we demonstrate the different action of two electromagnetic waves, which move in opposite directions, on the relativistic electron motion. Thereby, we restrict the discussion to cases with weak deviations of \( \beta_s \) from unity, \( \Delta \beta_s = 1 - \beta_s \ll 1 \), and take advantage of the fact that there is a big difference in the amplitudes of low- and high-frequency contributions to the momentum \( u_\perp \) and the electron energy, as it also follows from the numerical solution (see panel (b) of figure 9). The essential parameter, which characterizes the influence of radiation friction on the electron dynamics, will be determined below as \( \tau = 4\tau_R \gamma^3 \).

Coupling equations (3.2) and (3.3), we find the relation
\[
\frac{d\Delta \beta_z}{dt} = 2a_1^2 \gamma (1 - \Delta \beta_s) \sin 2t + \frac{\Delta \vec{u}_\perp}{\gamma^2} \frac{d\vec{a}}{dt}.
\]
(3.24)

At large \( \gamma \)-factors, \( \gamma^2 \gg a_1^2 \), we get the approximate solution \( \Delta \beta_s \equiv d\varphi_2/dt \simeq (a_1^2/\gamma^2)[1 - \cos 2t] \), where we have retained the slowly changing part of the electron energy. Consequently, the phase of the comoving wave may be interpreted as the sum of a low-frequency term \( \varphi_2 \) and a rapidly oscillating function, \( \tilde{\varphi}_2 = -a_1^2/(2\gamma^2) \sin 2t \).

Next, we average (3.23) as well as the corresponding equations for \( u_\pm \), and \( \gamma \) over the period of the high-frequency oscillations, which are intrinsically driven by the counterpropagating wave. The appearing functions \( \sin \varphi_2 \) and \( \cos \varphi_2 \) may be rewritten as
\[
\begin{align*}
\left( \frac{\sin \varphi_2}{\cos \varphi_2} \right) &= J_0 \left( \frac{a_1^2}{2\gamma^2} \right) \left( \frac{\sin \varphi_2}{\cos \varphi_2} \right),
\end{align*}
\]
(3.25)

where the well-known Bessel identity \( e^{in\sin \nu} = \sum_{n=-\infty}^{\infty} J_n(\mu)e^{in\nu} \) was applied and all oscillating terms of the sum disappeared after the averaging procedure. For small arguments \( a_1^2/\gamma^2 \ll 1 \), the remaining lowest-order Bessel function is close to unity and will be skipped afterwards.

Renormalizing the slowly varying functions, \( (\Delta u_\pm) = \Delta u_\pm /a_1 \), \( \gamma_s = \sqrt{\gamma}/\gamma_0 \), as well as the time variable, \( \sigma = a_1^2/\gamma_0^2 t \), we obtain the equations
\[
\begin{align*}
\frac{d(\Delta u_\pm)}{d\sigma} &= -\tau \gamma_s (\sin \varphi_2 + (\Delta u_\pm)), \\
\frac{d(\Delta u_z)}{d\sigma} &= \tau \gamma_s (\cos \varphi_2 - (\Delta u_z)), \\
\frac{dy_s}{d\sigma} &= \frac{1}{\gamma_s} [(\Delta u_\pm) \cos \varphi_2 + (\Delta u_z) \sin \varphi_2 - \tau \gamma_s^2], \\
\frac{d\varphi_2}{d\sigma} &= \frac{1}{\gamma_s^2},
\end{align*}
\]
(3.26)
Figure 9. The field intensity $a^2$ of the superposition of two identical but counterpropagating laser waves, the ponderomotive force $d\gamma / dx$, the $\gamma$-factor and the longitudinal momentum $u_x$ of the electron as functions of the normalized propagation distance are plotted in the upper part of panels (a, b). The corresponding electron velocity components are given in the lower part. Panel (a) contains simulation results without the friction force, which is taken into account in (b). Initial parameters are: $a_1 = a_2 = 50, \gamma_0 = 300$. The halfcycles of the gradient force $d\gamma / dx$ are filled to the baseline to guide the eye. The analytical solutions of the equation of motion including the radiation friction force derived for the early interaction times are shown by green dashed curves in (b). The dashed dark-green curve displays the numerical result for the longitudinal momentum in the case $a_1 = 50, \ a_2 = 10, \ \gamma_0 = 300$, whereas the magenta line in the same panel is the integration result of equations (3.1)–(3.3) averaged over the laser period (system (3.26)). Further details are given in the text.

with the friction coefficient $\tau = 4\tau_R \gamma_0^3$. The longitudinal momentum $(u_x)_s = \gamma_s \beta_x$ obeys the same equation as the slowly varying part of the $\gamma$-factor, since we assumed $\Delta \beta_x \ll 1$. The numerical solution of the system (3.26) is shown by the magenta curve in figure 9(b). It describes the re-acceleration of the electron in the field of the comoving wave sufficiently well. At later instances the $\gamma$-factor is again decreasing with a rather small derivative.

In (3.26), we recognize the straight effect of two important parameters for the given approximation. The characteristic time scale of essential changes in the momenta and the electron energy is given by the ratio $(\gamma_0/a_1)^2$. Since we suppose small numbers $a_1/\gamma_0$, these changes evolve over tens of laser periods in the case shown in figure 9. At higher laser field amplitudes they will proceed faster. The second coefficient, $\tau = 4\tau_R \gamma_0^3$, controls the contribution of the friction force as already mentioned above. It depends only on the initial electron energy.

The early phase of electron motion may be estimated analytically, as long as we neglect the transverse momentum $\Delta \vec{u}_\perp$ in (3.2) and (3.3). The corresponding solution for the longitudinal
momentum is
\[ u_x = \frac{\beta_x \gamma_0}{1 + \kappa x}, \quad \kappa = 4\tau_0^2 a_1^2 \beta_0 \gamma_0. \] (3.27)

Next, we substitute this relation into (3.3) and evaluate the monotonic energy decrease. We get the expression
\[ \gamma^{(1)} = -\frac{\beta_x \gamma_0}{\beta_x} \frac{\kappa x}{1 + \kappa x}. \] (3.28)

with the same coefficient \( \kappa \) as in (3.27). The full solution for electron energy under the assumption of a negligible transverse momentum \( \Delta u_\perp \) can be approximated as \( \gamma = \gamma^{(0)} + \gamma^{(1)} \), where the first contribution equals the oscillating energy of the electron motion without radiation friction (3.21). The estimates (3.27) and (3.28) are displayed by green dashed curves in figure 9(b). We may conclude that the early interaction phase is dominated by the counterpropagating wave, which causes a strong retardation of the electron. With ongoing time, the slowly evolving re-acceleration in the comoving wave may balance the energy loss (at about four laser periods for the chosen initial parameters in figure 9) and even overcome it later on.

If the comoving wave is remarkably weaker than its counterpart, the electron may not be re-accelerated. This situation is also shown in figure 9(b) by the dashed dark-green curve, where the spatial evolution of the longitudinal momentum, \( u_x(x) \), is plotted for the parameters \( a_1 = 50, a_2 = 10 \). This dependence can be described analytically by (3.27). The particle stops its positive drift motion after propagating about 30 laser wavelengths. In the opposite case, \( a_1 = 10, a_2 = 50 \), the electron drift velocity slowly decreases first by only a small amount. Afterwards, the electron is strongly accelerated reaching values \( u_x \simeq \gamma \simeq 500 \), when crossing the distance of 100 \( \lambda_1 \).

A special regime is obtained in general at equal field amplitudes \( a_1 = a_2 \), where the momentum \( u_x \) and the energy \( \gamma \) decrease very slowly over hundreds of laser periods. The corresponding decline becomes more significant for larger values \( a_1/\gamma_0 \).

Finally, we infer analytical expressions describing the electron trajectory at negligible radiation damping. Again, we analyze the particle motion in its proper frame. The field components at \( a_1 = a_2 \) take the form
\[ a_y = a_1 \left( \sin \varphi'_1 + \sin \varphi'_2 \right), \quad a_z = a_1 \left( \cos \varphi'_1 - \cos \varphi'_2 \right), \] (3.29)

with the phases \( \varphi'_1 = \sqrt{M_i} t', \varphi'_2 = t'/\sqrt{M_i} \) in the plane \( x' = 0 \), where we introduced the approximate frequency upshift factor \( M_i = (1 + \beta_2)/(1 - \beta_1) \). The mean drift velocity \( \bar{\beta}_x \) is given by (3.22). The equation for the transverse part of the electron orbit obeys the simple equation
\[ \frac{d\vec{r}_\perp}{dt} = \bar{\beta}_\perp = \frac{\vec{a}}{\gamma'}, \] (3.30)

considering the integral of motion \( \vec{u}_\perp = \vec{a} \).

If we integrate this equation keeping only the rapidly oscillating terms with the phase \( \varphi'_1 \), which describe the interaction with the counterpropagating wave, we obtain the relations
\[ y = \frac{a_1}{\sqrt{M_i}} (1 - \cos \varphi'_1), \quad z = \frac{a_1}{\sqrt{M_i}} \sin \varphi'_1. \] (3.31)
Figure 10. Electron motion in the field of two identical counterpropagating laser waves: (a) spectral intensities of the radiation emitted in the direction of the electron drift motion \( (\theta = 0) \) without the radiation damping effect (red lines) and with this effect taken into account (blue dotted curves). (c) Transverse projection of the electron trajectories with radiation damping (blue helix) and without it (black curve). In addition, analytical solutions (3.31) and (3.32) are plotted by a green curve. The corresponding dependencies for the electron interaction with a standing wave are shown in (b) and (d), respectively (see next subsection). Initial parameters: \( a_1 = a_2 = 50 \), \( \omega_1 = \omega_2 \), \( \gamma_0 = 300 \).

They contain the mean \( \gamma \)-factor, \( \gamma' \simeq \sqrt{1 + 2a_1^2} \) and fulfill the initial conditions \( y_r(0) = z_r(0) = 0 \). Averaging (3.30) over the period of fast motion and integrating, we derive subsequent expressions for low-frequency oscillations in the electron trajectory

\[
y_s = \frac{a_1}{\gamma'} \sqrt{M_i} \left( 1 - \cos \varphi'_2 \right), \quad z_s = - \frac{a_1}{\gamma'} \sqrt{M_i} \sin \varphi'_2,
\]

where the initial conditions \( y_s(0) = z_s(0) = 0 \) were supposed. These oscillations are driven by the comoving wave. Because of the factor \( \sqrt{M_i} \) in the denominator of the amplitude of these oscillations, the corresponding excursions will be by \( M_i \) times larger than those given in (3.31).

Figure 10, panel (c), shows the projection \( y(z) \) of the trajectory of an electron with initial energy \( \gamma_0 = 300 \) in the field of the two waves, where each laser has the constant vector potential \( a_1 = 50 \). In addition to the numerical result for the case without radiation damping (black curve), the corresponding analytical estimate is depicted (green curve), which contains both components—the slow circular motion (3.32) plus the much faster oscillations (3.31) with a smaller radius. The estimated radii, \( \rho_s = 6, \rho_r = 0.08 \), fit well to the numerical result. The blue curve exhibits the orbit calculated numerically with the account for radiation friction.

In figure 10(a), we display the spectral intensity of the radiation emitted in the drift direction of the electron \( (\theta = 0) \) with and without radiation damping for the same initial
parameters as in panel (c). In contrast to the simulations for the one-wave schemes at neglected radiation damping, the backscattered spectrum contains numerous narrow harmonics with the Doppler upshifted frequencies, \( \omega_n = nM_i\omega_1 \). The upshift estimate \( M_i \approx 74 \), which was used to normalize the calculated spectra in panel (a), provides right positions of the lowest harmonics in the spectrum. The increasing discrepancy at higher-order harmonics, however, indicates a small underestimation of the factor \( M_i \) given above.

If radiation damping is included in the numerical integration of the equation of electron motion, the harmonics are remarkably downshifted, broadened and overlap at higher orders, as was already observed in one-wave interactions. In the range of the lowest harmonics, we obtain an approximately two times smaller factor \( M_i \) due to the influence of radiation friction. For the given initial data, the corresponding spectral intensity in the region of lower harmonics is about one order of magnitude smaller than in the case of negligible radiation friction. This is mainly the result of the decreasing scattering cross-section due to electron energy decay.

3.4. Standing wave case

As a final example for the interaction of a relativistic electron with a simple field configuration we study its motion in the superposition of an incident counterpropagating laser wave, which is reflected by an ideal static mirror. The superposition of both waves with identical frequencies \( \omega_1 \) and field amplitudes \( a_1 \) forms a standing wave. The sum vector potential is defined as

\[
\vec{a} = a_1 \left[ \vec{e}_y (\sin \varphi_1 - \sin \varphi_2) + \vec{e}_z (\cos \varphi_1 - \cos \varphi_2) \right] = 2a_1 \sin x (\vec{e}_y \cos t - \vec{e}_z \sin t),
\] (3.33)

with the phases \( \varphi_1 = t + x \) and \( \varphi_2 = t - x \).

The intensity \( a^2 = 4a_1^2 \sin^2 x \) as well as the square of the time derivative \( \left( \frac{d\vec{a}}{dt} \right)^2 = 4a_1^2 (\sin^2 x + \beta_x^2 \cos^2 x) \) oscillate in \( x \).

In the case of negligible radiation damping, the transverse momentum of the electron is determined by the field amplitude, \( \vec{u}_\perp = \vec{a} \). However, the longitudinal momentum component is not constant as in the previous example, but it equals

\[
u_x = \nu_x(0) \sqrt{1 - \frac{4a_1^2}{\nu_x^2(0)} \sin^2 x}, \quad \nu_x(0) = \beta_0 \gamma_0.
\] (3.34)

Since the vector \( \partial \vec{a} / \partial t \) is oriented perpendicularly to the field vector \( \vec{a} \), the electron energy is conserved, \( \gamma(x) = \gamma(0) = \gamma_0 \), and the longitudinal velocity obeys the expression

\[
\beta_x = \beta_0 \sqrt{1 - \frac{4a_1^2}{\nu_x^2(0)} \sin^2 x}.
\] (3.35)

The analytical solutions reproduce numerical data, which are plotted in figure 9, panel (a). If we average functions (3.34), (3.35) over the laser period, the following expressions will be obtained:

\[
\vec{\nu}_x \approx \nu_x(0) \left( 1 - a_1^2 / \gamma_0^2 \right), \quad \vec{\beta}_x \approx \beta_0 \left( 1 - a_1^2 / \gamma_0^2 \right),
\] (3.36)

whereby the assumption \( a_1^2 \ll \gamma_0^2 \) was used.

The radiation friction gives rise to an additional transverse momentum \( \Delta \vec{u}_\perp \). It obeys the same approximate equation (3.23) as in the case of two counterpropagating identical waves.
because of a similar functional coefficient \( \mathcal{R} = -4a_1^2 \). In analogy with section 3.3, we derive a reduced system of equations

\[
\begin{align*}
\frac{d(\Delta u_x)}{d\sigma} &= \tau \gamma_s \{ \sin \varphi_2 - (\Delta u_x) \} , \\
\frac{d(\Delta u_z)}{d\sigma} &= \tau \gamma_s \{ \cos \varphi_2 - (\Delta u_z) \} , \\
\frac{d\gamma_s}{d\sigma} &= -\frac{1}{\gamma_s} (\Delta u_x) \cos \varphi_2 - (\Delta u_z) \sin \varphi_2 - \tau \gamma_s^2 , \\
\frac{d\varphi_2}{d\sigma} &= \frac{1}{\gamma_s^2} ,
\end{align*}
\tag{3.37}
\]

for the slowly varying components of the excited transverse momentum, \((\Delta u_{x,z}) = \Delta u_{x,z}/a_1\), and the electron energy, \(\gamma = \tilde{\gamma}/\gamma_0\), as functions of the renormalized time \(\sigma = a_1^2/\gamma_0^2 t\). Solving this simplified system numerically, e.g. for the initial parameters \(a_1 = a_2 = 50\) and \(\gamma_0 = 300\), the result is sufficiently close to the exact solution of the basic equations (2.7) and (2.8), as shown in figure 11(b). The approximate numerical result is displayed by the magenta curve and reproduces the delayed re-acceleration of the electron in the field of the comoving wave well.

Let us remember that (3.37) was derived for small values \(a_1/\gamma\), and consequently \(\beta_s \approx 1\). In this approximation, the spatial coordinate of electron motion depends almost linearly on time, and the cases analyzed in sections 3.3 and 3.4 should behave in a similar way. In general, however, the drift velocity of the electron in a standing wave depends directly on the spatial coordinate. In contrast to (3.24), it obeys the equation

\[
\frac{d\Delta \beta_x}{dt} = \frac{2a_1^2}{\gamma^2} \sin 2x + \frac{\Delta \bar{u}_\perp}{\gamma^2} \frac{d\bar{a}}{dt} ,
\tag{3.38}
\]

with \(x = \int \beta_s t\), causing at the same time a change in the phase of the comoving wave ‘seen’ by the particle. This dephasing mechanism is responsible for observed stopping in the drift motion, which appears quickly at larger values \(a_1/\gamma_0\).

The early interaction phase can be described as analytically similar to the interaction case with two identical counterpropagating waves assuming a negligible additional transverse momentum \(\Delta \bar{u}_\perp\). From the corresponding equation for the \(\gamma\)-factor we gain the decaying solution

\[
\gamma^{(1)} = \frac{\gamma_0}{1 + \kappa x} ,
\tag{3.39}
\]

with the coefficient \(\kappa = 4\tau_\mathcal{R} a_1^2\gamma_0/\beta_x\). Using this result, we integrate the relevant equation for the longitudinal momentum and find the solution

\[
u_x = \beta_0\gamma_0 \sqrt{1 - \frac{4a_1^2}{(\beta_0\gamma_0)^2} \sin^2 x - \frac{4\kappa x \gamma_0}{1 + \kappa x} ,
\tag{3.40}
\]

with the same coefficient \(\kappa\) as in (3.39) and the mean drift velocity (3.36). The relations (3.39) and (3.40) are depicted by dashed green curves in figure 11(b).

In an analytical description of the electron trajectories, supposing negligible radiation damping, we can proceed similarly to section 3.3. Solving the differential equation for the transverse radius vector of the electron orbit (3.30) when keeping the fast oscillation terms in the field components,

\[
a_y = a_1 (\sin \varphi'_1 - \sin \varphi'_2) , \quad a_z = a_1 (\cos \varphi'_1 - \cos \varphi'_2) ,
\tag{3.41}
\]

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Figure 11. The field intensity $a^2$ in a standing wave, the ponderomotive force $d\gamma/\text{dx}$, its $\gamma$-factor and the longitudinal momentum $u_x$ as functions of the normalized propagation distance are plotted in the upper part of panels (a, b). The corresponding electron velocity components are given in the lower part. Panel (a) contains simulation results without friction force, which is taken into account in (b). The initial parameters are: $a_1 = a_2 = 50$, $\gamma_0 = 300$. The halfcycles of the gradient force $d\gamma/\text{dx}$ are filled to the baseline to guide the eye. The analytical solutions of the equation of motion including the radiation friction force derived for the early interaction times are shown by green dashed curves in (b). The magenta line in the same panel results from the numerical integration of equations (3.1)–(3.3) averaged over the laser period (system (3.37)). Further details are given in the text.

the corresponding expressions for the rapidly changing transverse coordinates read

\[ y_t = \frac{a_1}{\gamma'} \sqrt{M_i} (1 - \cos \varphi'_1), \quad z_t = \frac{a_1}{\gamma'} \sqrt{M_i} \sin \varphi'_1, \]  

with the approximate phase $\varphi'_1 = \sqrt{M_i} t'$, the averaged $\gamma$-factor $\gamma' = \sqrt{1 + 2a_1^2}$ and the initial conditions $y_t(0) = z_t(0) = 0$. The introduced frequency upshift factor $M_i = (1 + \beta_x)(1 - \beta_x)$ is determined by the mean electron drift velocity given in (3.36) and will deviate from $M_i$ because of different longitudinal velocities. However, this discrepancy should be small in the cases analyzed here, since we always assume that $a_1^2 \ll \gamma_0^2$. The expressions for slow contributions read

\[ y_s = -\frac{a_1}{\gamma'} \sqrt{M_i} (1 - \cos \varphi'_2), \quad z_s = -\frac{a_1}{\gamma'} \sqrt{M_i} \sin \varphi'_2, \]  

where $\varphi'_2 = t'/\sqrt{M_i}$ and $y_s(0) = z_s(0) = 0$ are assumed. This result differs from (3.32) in the sign of the $y$-component due to the different helicity of the comoving (in respect to the electron) waves in sections 3.3 and 3.4.
Figure 8, panels (b, d) demonstrates the numerical and analytical results for the spectral intensities and the electron trajectories in the case of a standing wave with the common amplitude of both fields, \( a_1 = 50 \), and an initial electron \( \gamma \)-factor of 300. The estimated radii of the slow and fast motions in the plane \( yz \), \( \rho_s = 5.9, \rho_f = 0.08 \), are close to the numerically obtained values. We also see opposite helicity in the slow component of the electron motion, resulting in the opposite sign of the \( y \)-component of the trajectory in contrast to the situation shown in panel (c). The harmonic spectrum, which is now normalized by the upshift factor \( M_s \simeq 70 \), has approximately the same magnitude as in the case of the counterpropagating identical waves. The factor \( M_s \) will be reduced by approximately two times when radiation damping is taken into account, and the spectrum becomes a quasi-continuum in the range of higher-order harmonics.

Comparing the results of sections 3.3 and 3.4 (see figures 9 and 11), we note that the interaction at the field amplitudes \( a_1 = a_2 = 50 \) and the initial electron \( \gamma \)-factor \( \gamma_0 = 300 \) looks quite similar in both field configurations. However, if we analyze the solution of the system (2.7) and (2.8) in the standing wave case over the long propagation distance \( x = 100 \lambda_1 \), we observe the stopping of the electron drift motion at about 90 laser wavelengths—in contrast to the much slower decay of the longitudinal momentum \( u_x \) of the electron in its quasistationary interaction with the two identical waves. This difference becomes more and more pronounced at higher ratios \( a_1/\gamma_0 \), where the drift motion of the electron will be stopped at much smaller propagation distances. In contrast, if we follow the motion of an electron with \( \gamma_0 = 300 \) in the superposition of two waves \( a_1 = 60, a_2 = 90; \omega_1 = \omega_2 \) with the phase shift \( \pi \) like in a standing wave, we observe during the propagation over the first 100 laser wavelengths a rather stable quasistationary regime with electron energy \( \gamma \approx 270 \). In this case, the strong re-accelerating ponderomotive force of the comoving wave will balance the action of the friction force in the counterpropagating wave.

Supposing a standing wave field configuration with \( a_1 = a_2 = 100 \), the corresponding lower-order harmonic spectra and trajectories are depicted in figure 12. Since the Doppler upshift factors are much smaller now, \( M_i = 20, M_s = 16 \), the radii of the slow transverse excursions in the solution without radiation damping are approximately two times smaller than in the cases with \( a_1 = 50 \), but the rapid oscillations have a two times larger amplitude. Concerning the changes in the spectral intensities with increasing field strengths at neglected radiation friction, we obtain a remarkable reduction, which is largest for the standing wave interaction.

The most obvious difference, however, appears with the account for radiation damping. In the field of the two counterpropagating identical laser waves, the electron motion runs into the quasistationary regime already mentioned in section 3.3. We see this in the transverse projection of the trajectory shown in figure 12, panel (c). It demonstrates the approximate balance between the decay of electron energy in the interaction with the counterpropagating wave and re-acceleration due to the comoving wave. The corresponding spectrum in the demonstrated lower-order range (blue dotted lines) is about ten times weaker than in the case without friction (see panel (a)). For the electron motion in the standing wave with the same field amplitudes, this balance of the counteracting forces does not form up. The longitudinal momentum of the electron approaches zero after several wavelengths, and the respective spectral intensities drop by another factor of ten (see figure 12(b)), because of the smaller interaction time we have taken into account.
4. Electron motion in the vacuum field of a quasistationary laser piston

Now we return to the interaction scheme shown in figure 1, where the electron moves in the field of two ultraintense counterpropagating plane waves, which are again circularly polarized but have different amplitudes and frequencies because of the motion of the reflecting overcritical plasma.

4.1. Stationary approximation

In case of a constant incident laser intensity and constant plasma density, this motion will be stationary in a first approximation, with the constant velocity $\beta_l = v_l/c$ in the propagation direction of the incident wave $\vec{a}_1$. Nonstationary effects discussed in [29, 30] will be neglected in this subsection. The piston velocity depends only on the laser intensity and the plasma ion density: $\beta_l = B/(1 + B)$, with the piston parameter $B = a_1 \sqrt{\frac{m_i n_c}{m_i n_{0i}}}$, where $m_i$ and $n_{0i}$ are the ion mass and their initial number density, respectively. For the following discussion we suppose a laser wave with $a_1 = 100$, illuminating an overcritical deuterium plasma with $m_i = 2m_p$ and $n_{0i} = 20n_c$. The resulting piston velocity becomes $\beta_l = 0.27$ and the reflection coefficient...
without | with radiation damping

\[
\begin{align*}
\text{Figure 13.} & \quad \text{Electron motion in the field behind a stationary laser piston. The field intensity } a^2, \text{ the ponderomotive force } d\gamma/dx, \text{ its } \gamma\text{-factor and the longitudinal momentum } u_x \text{ as functions of the normalized propagation distance are plotted in the upper part of panels (a, b). The corresponding electron velocity components are given in the lower part. Panel (a) contains simulation results without friction force, which is taken into account in (b). Initial parameters are: } a_1 = 100, \gamma_0 = 300. \text{ The amplitude of the reflected wave is } a_2 = \sqrt{R} a_1, \text{ with the reflection coefficient } R = 0.58 \text{ found in PIC simulations [29]. The halfcycles of the gradient force } d\gamma/dx \text{ are filled to the baseline to guide the eye. For comparison, the same functions, which were obtained in the case of a standing wave with the amplitudes } a_1 = a_2 = 100, \text{ are depicted in panel (c).}
\end{align*}
\]

amounts to } R = 0.58. \text{ The frequency of the reflected laser wave will be Doppler downshifted with the factor } M_f = R = (1 - \beta_f)/(1 + \beta_f). \text{ Its amplitude } a_2 \text{ is reduced by the coefficient } \sqrt{R} \text{ in comparison with } a_1.

In the following, we will analyze the dynamics of a relativistic electron, which enters the vacuum field region with a high energy } \gamma_0 \gg 1 \text{ and moves toward the incident wave } \vec{a}_1. \text{ We are going to present results from the numerical integration of equations (2.7)–(2.8) supposing the electromagnetic fields (2.9) and (2.10).}

Figure 13, panels (a) and (b) show the field intensity } a^2, \text{ the ponderomotive force } d\gamma/dx, \text{ the longitudinal momentum and the energy of the electron versus the propagation distance with and without the friction force included, respectively. In addition, the case of the electron motion in a standing wave with the same incident field amplitude is shown in panel (c). The initial electron energy (before escaping the plasma) was chosen as } \gamma_0 = 300. \text{ As we can see in panels (b) and (c), the electron will be stopped at a propagation distance of approximately three laser wavelengths. We recognize the strong ponderomotive force, which becomes more and more nonlinear. The simulation was finished as soon as the longitudinal velocity component } \beta_x \text{ approached zero.}
Figure 14. Electron motion in the ultrastrong field of a standing wave (a, c) or behind a stationary laser piston (b, d). Spectral intensities of the radiation emitted in the direction of the electron drift motion ($\theta = 0$) without the radiation damping effect (red lines) and accounting for this effect (blue dotted curves) are plotted in the upper panels (a) and (b), the transverse projections of the electron trajectories with radiation damping (blue curves) and without it (black curves) are depicted in the lower panels (c, d). The initial electron energy is $\gamma_0 = 300$, and the laser field strengths are given in the panel titles. In comparison with numerical data, the dashed green curves display the analytical relations (3.42), (3.43) in (c) and (4.2), (4.3) in panel (d).

It is interesting to study the electron trajectories and the related radiation spectra. The spectral intensities of the first ten harmonics emitted in the drift direction of the electron along the $x$-axis (toward the incident wave) at disregarded radiation damping are displayed in figure 14(a) for the standing wave case, and in panel (b) for the laser piston. The spectral intensity scale is the same as in the previously shown spectra in figures 5, 10 and 12. We identify that the radiation intensity in the shown frequency range scattered by an electron, which escaped the piston, is approximately five times higher than that from an electron moving in a comparable standing wave. The spectra of both demonstrated cases are normalized with help from the corresponding upshift factors. We recall that the factor $M_s$ was analytically estimated as $M_s = (1 + \overline{\beta}_x)/(1 - \overline{\beta}_x)$, with $\overline{\beta}_x = \beta_0 \sqrt{1 - 2\alpha_1^2/\gamma_0^2}$ and amounts to 16 for the initial parameters used in figure 14.
With a similar argumentation as in section 3.4, one derives an approximate expression for the upshift factor $M_p$, which characterizes the spectra in the piston scheme as far as we do not account for radiation friction. At the same time we also obtain an estimate for the electron trajectory in the absence of radiation friction. We analyze the electron motion in its proper frame. The field components are

$$a_y = a_1 (\sin \varphi'_1 - \sqrt{M_f} \sin \varphi'_2), \quad a_z = a_1 (\cos \varphi'_1 - \sqrt{M_f} \cos \varphi'_2),$$

(4.1)

with the approximate phases $\varphi'_1 = \sqrt{M_p} t', \varphi'_2 = M_f / \sqrt{M_p} t'$ in the plane $x' = 0$. Since the mean intensity in the electromagnetic field behind the piston can be estimated as $\bar{a}^2 = a_1^2 (1 + M_f) = 2a_1^2/(1 + \beta_f)$, we find the corresponding value for the average drift velocity $\bar{\beta}_{x,p} = \beta_0 \sqrt{1 - 2a_1^2/(1 + \beta_f)}$, which determines the Doppler upshift factor $M_p = (1 + \bar{\beta}_{x,p})/(1 - \bar{\beta}_{x,p})$. For the chosen piston parameters in figure 14 it becomes equal to 20.8. This value was used to normalize the scattering spectra in panel (b).

In analogy with (3.42), the rapidly oscillating components in the transverse projection of the electron trajectory read

$$y_t = a_1 \frac{1}{\gamma'} \sqrt{M_p} (1 - \cos \varphi'_1), \quad z_t = a_1 \frac{1}{\gamma'} \sqrt{M_p} \sin \varphi'_1,$$

(4.2)

where the mean $\gamma$-factor is approximated by $\gamma' = \sqrt{1 + 2a_1^2/(1 + \beta_f)}$ and the initial conditions $y_t(0) = z_t(0) = 0$ were assumed.

If we perform the same averaging procedure as in the previous section and integrate the differential equations for the transverse orbital coordinates $y$ and $z$, the following solutions for the slow oscillations,

$$y_s = a_1 \frac{1}{\gamma'} \sqrt{M_p} (\cos \varphi'_2 - 1), \quad z_s = -a_1 \frac{1}{\gamma'} \sqrt{M_p} \sin \varphi'_2,$$

(4.3)

can be obtained at the initial condition $y_s(0) = z_s(0) = 0$.

The analytical estimate for the slow orbital oscillations together with fast components (4.2) are plotted in figure 14(d) by a dashed green curve. The agreement with the numerically evaluated transverse orbit is excellent especially in the early phase of the interaction. Finally, we note the common feature of quasi-continuum and strongly reduced spectral intensities in the case of significant radiation damping during electron motion.

4.2. Account for the charge separation field in the ion layer and piston nonstationarity

PIC simulations, described, e.g. in [29, 30], have shown that the propagation of an ultraintense laser wave with a constant amplitude, which becomes reflected by an overcritical plasma slab with initially constant ion density, is indeed not a completely stationary process. At the chosen amplitude $a_1 = 100$ of the incident laser pulse, the reflection coefficient of the plasma target with initial density $n_{0i} = 20n_e$ oscillates with a period of approximately four times the laser period in the interval $[R - \Delta R, R]$, where $\Delta R \simeq 0.3$. It is obvious that this modulation will induce a periodic change of the Doppler shift in the reflected wave and therefore effect the motion of the electron. In the numerical study of this special case, we used the mentioned PIC result from [29] to modify the stationary piston model analyzed in section 4.1.
Figure 15. Electron motion in the ultrastrong field behind a quasistationary laser piston with an oscillating reflection coefficient. Spectral intensities of the radiation emitted in the direction of the longitudinal electron drift motion ($\theta = 0$) without the radiation damping effect (red lines) and accounting for this effect (blue curves) are plotted in the upper panels (a) and (b), while the transverse projections of the electron trajectories with radiation damping (blue curves) and without it (black curves) are depicted in the lower panels (c, d). Data from simulations accounting for the electrostatic charge separation field in the ion layer are demonstrated in panels (b, d). The initial electron energy (prior to the interaction with the laser waves) amounts to $\gamma_0 = 300$, and the incident laser field strength was chosen as $a_1 = 100$. The reflection coefficient oscillates with the amplitude $\Delta R = 0.3$ below the value for the ideal mirror $R = 0.58$ at the given laser intensity and the ion density $n_0 = 20n_c$. The electrostatic field data were used as an additional input calculated separately in [29]. The frequency shift factors used for normalization are explained in the text.

Figure 15(a) presents the simulation results for the spectral intensities in the lowest ten harmonics accounting for radiation friction (blue curves) and without it (red lines). The spectra are normalized with the frequency shift factor $M_p$ of the corresponding stationary piston and the intensity scale is identical with that used in figure 14. Comparing the spectrum at neglected radiation damping in figure 15(a) with that in figure 14(b), we note the broadening of the lines preferably to the lower-energy side. The spectral intensities become smaller by a factor of five due to the frequency modulation in the comoving wave. The projection of the trajectory in the $yz$-plane shows a transverse drift of the electron motion.
Figure 16. The Doppler shift factors $M_f$ of the laser light reflected from the piston (green curve) and $M_p$ for the radiation scattered by the electron (orange line) are plotted depending on the piston velocity. In addition, the corresponding dependences are shown for the mean longitudinal momentum component of the electron, for its energy in the proper frame and for the amplitudes of the slow and rapid oscillations in the electron trajectory. The initial laser and piston parameters are: $a_1 = 100$, $n_{0i} = 20n_c$, $\gamma_0 = 300$. The maximum reflectivity case (stationary piston) at $\beta_f \simeq 0.27$ as well as the position corresponding to the minimum of the oscillating reflectivity are marked by vertical dotted lines, and the box indicates the range of the reflection coefficient variation. The dot-dashed curves demonstrate the dependencies $M_p(\beta_f)$ (orange) and $\rho_s(\beta_f)$ (blue) for the initial electron energy $\gamma_0 = 430$.

It is useful to analyze the dependence of various factors in (4.2) and (4.3), which determine the frequency shift and consequently maximum excursion of the electron in the $yz$-plane. For this approximate analysis we use the mean values of laser intensity. As it follows from figure 16, a decrease of the piston reflection coefficient $R$, which is identical with the Doppler shift of the reflected laser photons in the laboratory frame, means an ascending piston velocity $\beta_f$. This velocity increase causes an almost linear growth of the Doppler shift factor $M_p$ in the scattered light and a more gentle decrease of factor $\gamma'$. The amplitudes of the orbital coordinates, $\rho_s$ and $\rho_r$ behave quite differently, and whereas the slow oscillation amplitude may change very strongly at large piston velocities, the excursion of fast motion stays almost independent as becomes apparent from figure 16. This behavior is coherent, since slow oscillations are determined by the reflected wave with its varying amplitude and frequency. The rapid oscillations, however, are driven by the incident counterpropagating wave with constant parameters. In the interval
of the varying reflection coefficient, \( \Delta R = \Delta M_f = [R, R - \Delta R] \) (green box), which we have applied in the numerical analysis, the changes of \( \beta_f \) and \( \rho_s \) are moderate.

With the account for radiation damping, when the particle is stopped very quickly, the oscillations in piston reflection will not influence the electron motion significantly because of the relatively large period of these oscillations (compare the blue curves in figure 14(d) and in figure 15(c)).

Due to the fact that the laser piston contains an ion layer with a large electrostatic field, which will be permanently excited by the ponderomotively driven charge separation, the electrons escaping from the electron layer will gain additional energy and longitudinal momentum when they cross this field. Consequently, one should observe an increase of the Doppler shift in the frequency of the emitted radiation. Panel (b) of figure 15 confirms this suggestion, since both spectra shown in this panel are normalized with the frequency shift factor \( M_{pe} \simeq 45 \). The energy of the electron at the beginning of its excursion into the vacuum field with the constant mean intensity \( \sigma^2 = 2a_1^2/(1 + \beta_f) \) amounts to \( \approx 376 \), when we assume the minimum piston velocity to be \( \beta_f \simeq 0.27 \). After crossing the charge-separation field, the energy finally becomes \( \gamma \simeq 450 \), as we observed in the numerical solution. From that number we deduce \( \gamma_0 = \sqrt{\gamma^2 - \sigma^2} \simeq 430 \), which results in the factor \( M_{pe} \) mentioned above.

In general, the piston oscillations make the spectra more complex like the radiation friction force does. The latter effect normally produces a quasi-continuum with strongly reduced intensity at sufficiently high field amplitudes, as we can conclude from all of the corresponding spectral distributions, which are presented in figures 14 and 15.

5. Conclusion

We recalled details of the electron motion in a counterpropagating electromagnetic wave extending our study to high field amplitudes, where radiation friction force alters electron dynamics significantly. The entailed decrease of electron energy shows up in helical trajectories with increasing radius and in line broadening and frequency downshifts of lines up to an overlap of the neighboring harmonics. Because of this energy loss, the cut-off frequencies of the spectra are also being reduced.

When a relativistic electron copropagates with an intense electromagnetic wave, it can be accelerated in case of a sufficiently large friction force. The corresponding condition for the minimum laser field amplitude at a given initial electron energy was found. For both interaction schemes, analytical estimates describing the time dependence of the electron energy, its longitudinal momentum, the radiation power and energy during propagation in the ultraintense waves were derived, which agree with the results of the numerical integration of the relativistic equation of motion including radiation friction.

The possible energy gain from a comoving wave at significant radiation friction was also obtained in the electron dynamics of the interaction with two laser waves running in opposite directions. For these more complicated interaction schemes, analytical relations were found for the early phase of the electron propagation up to the point, when the slowly evolving re-acceleration effect in the comoving wave balances the retardation of the electron in the counterpropagating wave. If the radiation friction is negligible, the electron trajectories can be described analytically in good agreement with the numerical findings. Under the same
assumption, the correct frequency shift factors could be deduced reproducing the numerically determined harmonic spectra of scattered laser light.

In the special cases of two counterpropagating waves with equal amplitudes and frequencies—either two identical waves with equal helicity or a standing wave—the numerical results reveal very similar spectra of the scattered light at initial parameters \(a_1 = 50\), \(\gamma_0 = 300\), for example, where \(a_1/\gamma_0 < 1\). Also, the transverse excursion widths of the electron are close to each other except the rotational direction due to the reverse helicities of the comoving waves in both cases. At higher values \(a_1/\gamma_0\), however, the trajectories as well as the spectral properties may change remarkably over the investigated propagation distances \(x \simeq 100 \lambda_1\). In contrast to the motion in two superimposed identical waves, where a quasistationary regime of electron propagation with a very slow temporal decrease of longitudinal momentum and energy was observed at ratios \(a_1/\gamma_0 < 1\) as well as for \(a_1/\gamma_0 \geq 1\), the particle will be stopped in a standing wave with an amplitude \(a_1 \gtrsim \gamma_0\) after traversing several laser wavelengths. Consequently, the spectral intensities are stronger by an order of magnitude in the case of the identical waves, where the transverse projection of the electron trajectory consists of a circular slow motion overlaid by rapid oscillations with much smaller radius.

The complex electron motion in the electromagnetic field of a quasistationary laser piston is similar to the dynamics in an adequate standing wave, where the reflecting surface remains at rest. Without the account for radiation damping, the electron trajectory as well as the Doppler frequency shift of the emitted harmonics can be described analytically in sufficient agreement with the numerical results. Since the piston model serves as a promising scheme to accelerate ponderomotively a large number of ions to high energies, a maximum charge-separation field is desirable, which can be depleted by multiple hot electrons escaping the piston region. Our numerical studies confirmed that those electrons can be efficiently stopped in high laser fields due to strong radiation damping.

The analytical expressions for electron trajectory and the frequency shift of scattered radiation have been discussed in dependence on the piston velocity. This simple theory helps to understand the influence of numerically observed oscillations in the reflection coefficient of the quasistationary piston on electron dynamics. The periodic reduction of the amplitude of the wave comoving with the escaped electron causes line broadening of the emitted harmonics and alters the electron trajectory introducing a transverse shift, if the radiation damping effect is not taken into account. At a significant friction force, however, the influence of piston oscillations is only marginal.

With the account for the electrostatic field in the ion layer of the piston, which is induced by charge separation due to the strong ponderomotive pressure of the laser field, an escaping electron gains additional energy crossing the ion layer. This post-acceleration process causes a slightly larger stopping range as well as a larger frequency shift and higher intensities of the laser harmonics in the spectrum of scattered laser light.

As already stated in the Introduction section, a classical description of radiation emission was used throughout the paper. Comparative simulations with the account for quantum processes have shown that this is indeed justified in the range of applied initial parameters, where the \(\chi\)-parameter stays well below unity. Only at the highest vector potentials \(a_1 \sim 100\) and at electron energies of several hundreds of MeV, which we have assumed initially, could the quantum parameter reach numbers \(\chi \sim 0.1–0.2\), e.g. the demonstrated case of the laser piston. At this limit, the quantum corrections could already be significant [35].

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