A simple model of magnetically induced warps

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Abstract. Assuming the magnetic hypothesis for the formation of warps, we deduce a very simple formula for the warp curve in an idealized scenario. According to this formula the warp rises as the third power of radius in the innermost warped region, reaches the maximum slope at intermediate radii and has an asymptotic slope coincident with the direction of the extragalactic magnetic field. In most cases, however, the galaxy’s limited size prevents the observation of the full curve. Even though the model is highly simplified, it basically reproduces real warp curves, in particular the 21 cm warp curve of NGC 5907. If the magnetic model were considered to be correct, the fitting of warp curves could allow rough estimations of the strength and direction of the magnetic field. We also propose a magnetic field distribution for the outermost part of the galaxy, matching the boundary conditions of being azimuthal inside, and constant at infinity. We use this magnetic field distribution to show that the assumptions made to obtain the warp curve with our simple model cannot introduce important errors.

Key words: Galaxies: magnetic fields – Galaxies: structure

1. Introduction

The possibility of warps being induced by extragalactic magnetic fields (Battaner et al., 1990) remains a tempting explanation of this common feature of most spiral galaxies. Without intending to enter a discussion about its validity against other current models, some observational facts may produce a renewed interest in the magnetic model: a) Binney (1991) and Combes (1994) argued that the required magnetic fields were too high for the intergalactic medium. However, Kronberg (1994) reviewed evidence for magnetic fields of about 1-3 \( \mu \)G to be common in intra-cluster media. Later, Feretti et al. (1995) have found \( B < 8.3 \mu \)G in the Coma cluster, about one order of magnitude higher than the equipartition value. This is a critical point, as magnetically driven warps require magnetic strengths of this order of magnitude. Further confirmation of high intergalactic fields would support our hypothesis. We are not aware of any published alternative interpretation of the available observations of intergalactic magnetic fields. b) One of the most interesting possibilities is that of warps being normal modes of oscillation of the galactic disk (Sparke and Cassertano, 1988). But when the back reaction of the halo is taken into account this model seems to fail (Dubinski and Kuijken, 1995; Kuijken, 1997). c) Zurita and Battaner (1997) have shown the tight alignment of the warps of the three largest spirals in the Local Group. Such an alignment is easily understood in the magnetic model, and probably also in the model by Kahn and Woltjer (1959).

In this short paper we obtain a simple formula describing the warp curve [\( \text{mean } z \text{ versus } x \)] when the magnetic hypothesis is adopted. The coordinate \( x \) is contained in the inner unwarped disc plane and is perpendicular to the line of nodes, assumed to be untwisted for simplicity. Very simple models for idealized scenarios are specially appropriate for exploratory hypotheses and remain useful even when more sophisticated computations are available.

2. The shape of magnetically induced warps

According to Battaner et al. (1990), the vertical magnetic force which induces the warp is of the order \( B_y B_z / 8\pi L \), where \( B_y \) is the component of the extragalactic field in the equatorial plane of the galaxy perpendicular to the line of nodes, \( B_z \) is the component in the direction of the rotation axis of the galaxy and \( L \) is an undetermined characteristic length. To deduce this expression, we assume a constant mean value of the magnetic field strength in a large characteristic length connecting the inner disc and the intergalactic medium. This seems to be an extremely simple assumption, but it will be shown later that it does not introduce a quantitatively important error. Let us assume in this simplified model that these quantities \( B_y, B_z, \) and \( L \) are constants in the region of interest.

The characteristic length \( L \) would depend on the degree of ionization which in turn depends on the galactocentric radius. Let us however assume that at any radius the degree of ionization is large enough as to assure in-
finite conductivity and frozen-in magnetic field lines. In the inner part of the disc this force is negligible compared with the gravitational force, but it becomes increasingly important towards the outer parts of the disc.

The expression is valid for a point in the equatorial plane of the galaxy. For a point above this plane the vertical force becomes

\[ F_z = \frac{B^2}{16\pi L} \frac{\sin(2(\alpha - \beta))}{r^3} \cos \beta \] (1)

where \( \tan \beta = z/x \) and \( \tan \alpha \) is the slope of the direction of the magnetic field in a \([x, z]\) diagram. It is easily derived that this expression is equivalent to

\[ F_z = \frac{B^2}{16\pi L} \frac{x(x^2 - z^2)}{r^3} \sin(2\alpha) - \frac{2zx^2}{r^3} \cos(2\alpha) \] (2)

where \( r = (x^2 + z^2)^{1/2} \) is the radial coordinate in the \([x, z]\) plane.

In this simple model we assume that the gravitational potential is that of a point mass in the centre of the galaxy, as a simplifying assumption for the outer part of a disk not embedded in a massive halo. This potential has already been used before, for instance by Cuddeford and Binney (1993). Equilibrium in the vertical direction gives

\[ \frac{\partial p}{\partial z} + \rho GMr^{-3}z = \frac{B^2}{16\pi L} \left( \frac{x(x^2 - z^2)}{r^3} \sin(2\alpha) - \frac{2zx^2}{r^3} \cos(2\alpha) \right) \] (3)

A full solution for the distribution of the gas in the combined magnetic-gravity force field would reproduce the whole geometry of the warp. This full solution is beyond the scope of this paper. In any case, it would be interesting to, simply by precisely define the warp curve. In the inner unwarped region, \( \frac{\partial p}{\partial z} = 0 \) at \( z = 0 \), in the galactic plane. Let us therefore define the warp curve as the locus of points where \( \frac{\partial p}{\partial z} = 0 \). We adopt an exponential law for the disc density with length scale \( R \), i.e. \( \rho = \rho_0 e^{-z/R} \).

Using \( R \) as length unit for \( x \) and \( z \), we obtain for the warp curve

\[ z = -e^{-x} - 2kx^2 \cos(2\alpha) + \left( e^{-x} + 2kx^2 \cos(2\alpha) \right)^2 + 4k^2x^4 \sin(2\alpha) \right)^{1/2} \left( 2kx \sin(2\alpha) \right)^{-1} \] (4)

where

\[ k = \frac{R^2 B^2}{16\pi L \rho_0 GM} \] (5)

is one of the adjustable parameters which compares the extragalactic magnetic energy density with the gravitational energy density. The other adjustable parameter is \( \alpha \) which specifies the direction of the extragalactic magnetic field. The warp curve is therefore defined with just two free parameters: \( k \) and \( \alpha \).

For small values of \( x \) a series expansion gives

\[ z = k \sin(2\alpha)x^3 \] (6)

which is a very simple expression for small warps. This simple formula illustrates the fact that the maximum efficiency in producing warps is obtained for \( \alpha = 45^\circ \), as in Battaner et al. (1990).

For very large values of \( x \), we obtain

\[ z = \tan(\alpha)x \] (7)

i.e. the slope of the warp curve matches the direction of the magnetic field at large radii.

In Fig. 2 we plot the obtained curves for \( k = 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7} \) for \( \alpha = \pi/4 \) and \( \alpha = \pi/6 \). The curves for \( \alpha = \pi/4 \) seem to be unrealistic at first glance. Note, however, that in practice values at \( x >> 6 \) are unobservable (or the galaxy simply does not exist at these radii). For instance if the \( [\alpha = \pi/4, k = 10^{-5}] \) curve is truncated at \( x \approx 6 \), the obtained warp curve becomes quite familiar. We reproduce the curve for larger values of \( x \) and \( z \) in order to see the region where the slope becomes equal to the direction of the extragalactic magnetic field, which probably takes place for radii far from observational capacities or galaxy limits.

It is worth noting that, for \( \alpha \leq \pi/4 \), there is a change in slope. It is higher at intermediate regions before reaching its asymptotic value \( \tan \alpha \). This is a common feature of real warps, and can be even be directly observed in the early contour maps of NGC 5907 and NGC 4565 by Sancisi (1976).

Figure 2 reproduces the observational curve for one of the best known prominent warps, in NGC 5907, adopted from Sancisi (1976). We cannot exclude that the warp of this galaxy is due to other mechanisms, but we choose this warp because it is one of the most representative and is studied very often. This figure also reproduces a fitting to the model, with parameters \( k = 8 \times 10^{-5} \) and \( \alpha = 20^\circ \). It is not straightforward to deduce the magnetic field strength from the value of \( k \), mainly because \( L \) is an equivalent undetermined quantity. For \( L = 1 \) kpc, \( \rho_0 = 1.7 \times 10^{-24} \) g cm\(^{-3} \), \( M = 2 \times 10^{11} M_\odot \) and \( R = 4 \) kpc, it is obtained that values of \( k \) in the range \( 10^{-4} \) to \( 10^{-6} \) correspond to field strengths between 3 \( \mu G \) and 0.3 \( \mu G \), in agreement with reported measurements by Kronberg (1994). Probably better results would probably be obtained with more realistic calculations, but given the present, still exploratory, character of the magnetic model it is preferable to deal with idealized systems. The noticeable fitting of the NGC 5907 warp curve suggest that the magnetic model and the approximations considered here are not unreasonable.

### 3. Magnetic field distribution in the outer disc

We have studied some magnetic effects in the density distribution of the outermost disc without considering in detail the magnetic field distribution in this region. This was made on one hand because this distribution is unknown,
Fig. 1. Warp curves for magnetically induced warps. Values of $k$ are indicated beside each curve and the value of $\alpha$ is indicated at the top of the plot.

Fig. 2. Warp curve for NGC 5907. Open symbols are experimental data adopted from Sancisi (1976) and the solid line corresponds to the model for $k = 8 \times 10^{-5}$ and $\alpha = 20^\circ$.

and on the other hand to produce an exploratory simplified model. Now, we propose a magnetic field distribution and use it to estimate the error of our assumption for the specific case of $\alpha = 45^\circ$, small warps and large galactocentric radii.

As no radio continuum measurements are available to deduce the magnetic field in this region, we must just assume it. For this task we have not so many degrees of freedom, because our field must fulfill two restrictions: a) Boundary conditions: it should be azimuthal ($B_\phi$) at the inner radius $L_1$, and should become constant at infinity. b) The field must also have vanishing divergence. We thus solve the general problem of connecting both, the disc magnetic field and the extragalactic magnetic field in a way that could be adopted in other problems not associated with the particular case of warps. Outside the galaxy the field could have any direction. To consider a general case we take $B_x = B_y = B_\infty/\sqrt{2} = A$ outside, but $B_y = 0$, due to a proper choice of the $y$ axis. Let us define:

$$B_{1y} = -A \frac{x y}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$B_{1x} = \frac{A}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \left( y^2 - \sigma^2 \right) + A$$

$$B_{2x} = B_\phi \frac{y}{|x|} \left[ 1 - \left( \frac{y}{x} \right)^2 \right]^{-1/2} \frac{L_2 - (x^2 + y^2)^{1/2}}{L_2 - L_1}$$

$$B_{2y} = -B_\phi \frac{x}{|x|} \left[ 1 - \left( \frac{x}{y} \right)^2 \right]^{-1/2} \frac{L_2 - (x^2 + y^2)^{1/2}}{L_2 - L_1}$$

We then consider three regions:

1. Inner disc, for $0 < r < L_1$. We do not propose any field distribution for this region, as our model only considers the periphery of the galaxy.

2. Outer disc, for $L_1 < r < L_2$, taking $L_2$ as the radius where the galaxy’s edge is assumed to be. In this region,

$$B_x = B_{1x} + B_{2x}$$

$$B_y = B_{1y} + B_{2y}$$

3. Nearby intergalactic space, for $r > L_2$. We propose in this region,

$$B_x = B_{1x}$$

$$B_y = B_{1y}$$
It can be easily checked that the divergence of this two-
dimensional field is zero. In Fig. 3 we plot this magnetic
field distribution. The magnetic field strength along $x$
is plotted in Fig. 4. This distribution has no jump (only the
first derivative is not a continuous function). In Fig. 4
there is an apparent jump at $L_1$, but it only means that
we are not interested in the inner disc.

Some of the formulae above would become clearer in
polar coordinates, but cartesian coordinates are the nat-
ural ones outside the galaxy, and a choice covering the
whole region is preferable.

For $r > L_2$ we adopt $B_z = A$.

In this case, in the direction of the warp plane (per-
pendicular to the line of nodes and to the galactic plane)
\[
\frac{\partial B_z}{\partial x}(y = 0) = \frac{A}{L_2} 
\] (17)
as in Battaner, Florido and Sánchez-Saavedra (1990). The
three-dimensional picture of the magnetic field distribu-
tion can be appreciated by combining figure 3 and eq.
(16). The result is depicted in figure 5, where the inner
azimuthal field lines have not been plotted.

Let us now consider a second model, with the simplify-
ing assumption of $\alpha = 45^\circ$ and $z_{\text{warp}} \ll x_{\text{warp}}$, but taking
into account the above proposed magnetic field configura-
tion.

The vertical force will be:
\[
F_z = \frac{1}{4\pi} B_z \frac{\partial B_z}{\partial x} 
= \frac{1}{4\pi} \left( \frac{A}{\sigma^2} e^{-\frac{x^2+y^2}{\sigma^2}} (y^2 - \sigma^2) + A \right. \\
+ B_x \frac{y}{|x|} \left. \left[ 1 - \left( \frac{y}{x} \right)^2 \right]^{-1/2} \right) \\
\frac{L_2 - (x^2 + y^2)^{1/2}}{L_2 - L_1} \frac{A}{L_2^2} (x^2 + y^2)^{-1/2} 2x 
\] (18)
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And for \( y = 0 \) (warp plane)

\[
F_z = \frac{A^2}{4\pi L_2} \left( 1 - e^{-\frac{x^2}{2\sigma^2}} \right)
\]  

(19)

The equation of vertical equilibrium becomes

\[
\frac{\partial p}{\partial z} + \rho G M r^{-3} z = \frac{B_\infty}{8\pi L_2} \left( 1 - e^{-\frac{x^2}{2\sigma^2}} \right)
\]  

(20)

To calculate the warp curve, we use \( \frac{\partial p}{\partial z} = 0 \) as before. We now find for \( z \) small (so that \( r \approx x \))

\[
z = \frac{B_\infty}{8\pi L_2 G M \rho_0} e^\frac{x}{r} \left( 1 - e^{-\frac{x^2}{2\sigma^2}} \right)
\]  

(21)

With the same assumptions the first model would give

\[
z = \frac{B_\infty}{8\pi L_2 G M \rho_0} e^\frac{x}{r}
\]  

(22)

We note, however, that the last exponential function in eq. (21) is not very important. For \( x = \sigma \), we have \( e^{-\frac{x^2}{2\sigma^2}} = 0.61 \); for \( x = 2\sigma \), we have 0.14; for \( x = 3\sigma \), we have only 0.01. Therefore, we conclude that the use of the first model is fully justified.

4. Conclusions

In order to study the influence of intergalactic magnetic fields on the warping of galactic discs, it is necessary to know their distribution in the outermost disc, \( B(x, y, z) \). At present, no observations are available to determine the field at very large galactocentric radii. However, we have proposed a field distribution which should not differ very much from the real one, as the boundary conditions and the condition \( \nabla \cdot B \) are in practice very restrictive. Magnetic field lines must be very similar to those presented in our figures 3, 4 and 5.

We have shown that the detailed knowledge of the magnetic field lines in the periphery of the galaxy is relatively unnecessary to predict reasonable warp curves. More sophisticated calculations should be required in the future, but considering the present state of our understanding of warps, a simplified model has more advantages. The magnetic model of warps provides curves in good agreement with real warps. Under the magnetic hypothesis, the fitting of the warp geometry could provide information about two important cosmological parameters: the direction and the strength of the large scale extragalactic magnetic field.

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