EXACT STRING-THEORY INSTANTONS BY
DIMENSIONAL REDUCTION

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ABSTRACT

We identify exact gauge-instanton-like solutions to (super)-string theory using
the method of dimensional reduction. We find in particular the Polyakov
instanton of 3d QED, and a class of generalized Yang-Mills merons. We
discuss their marginal deformations, and show that for the 3d instanton they
 correspond to a dissociation of vector- and axial- magnetic charges.
1 Introduction

A key tool for understanding non-perturbative effects in field theory has been the study of solitons and instantons. For a given Lagrangian, it is usually straightforward to find the classical solutions, calculate their mass or action, zero modes etc. It is of course a harder task to derive physical consequences, such as the formation of fermionic condensates. In string theory, on the other hand, just finding the relevant classical solutions has proved to be a non-trivial exercise [1]. Indeed, most of the solutions of the effective low-energy Lagrangian are modified by higher-order in $\alpha'$ corrections, while the known exact conformal models have often an obscure space-time interpretation. This letter is meant as an addition to the world-sheet versus space-time dictionary. Our main observation is that by means of a dimensional reduction one can identify certain instantons, merons, monopoles and other gauge- (pseudo)particles, with combinations of WZW and Feigin-Fuchs models, and hence with exact solutions to the $\beta$-function equations, at least to all orders in the $\alpha'$ expansion. We also study the exact marginal deformations of these solutions. These solutions could be important for studying gaugino condensation and the ensuing possible breaking of space-time supersymmetry, but we will not address this issue here.

2 The Instanton of 3d QED

Dimensional reduction was used in the past to reinterpret gravitational instantons as Kaluza-Klein monopoles [2]. In string-theory it has been used to identify the $SU(1,1)$ WZW model with electrovac solutions of gauged supergravity [3], and more recently the $SU(2)$ WZW plus either the $SU(1,1)/U(1)$ GKO coset or Feigin-Fuchs models, with axionic instantons and their related monopoles [4] as well as with magnetically charged 4d black holes [5]. Let us illustrate the argument with the closely-related example of the instanton in 3d compact electrodynamics,

$$F_{ij} = q\epsilon_{ijk} x^k |x|^3.$$ (1)

This solves the flat-space (Euclidean) Maxwell equations, and satisfies the Bianchi identity everywhere except at the origin of coordinates. Appended with an appropriate dilaton background,

$$\Phi = -\frac{1}{2} log|\mathbf{x}|^2 + log\left(\frac{q}{\sqrt{2}}\right),$$ (2)

it also solves the leading-order $\beta$-function equations, derived from the effective 3d string Lagrangian

$$L_{eff}^{(3)} \propto \int d^3 \mathbf{x} \sqrt{g} \left[-R + (\partial \Phi)^2 + \frac{1}{4} e^{-2\Phi} F_{ij} F^{ij} - \frac{2}{3\alpha'} \delta c e^{2\Phi}\right],$$ (3)

\footnote{A factor-of-two mistake in the $\delta c$ term has propagated in much of the literature.}
with $\delta c = 0$. Note that the “central-charge deficit” $\delta c$, depends in general on the details of the compactification from the critical down to three dimensions. It is straightforward to check that the total energy-momentum tensor of the above monopole- and dilaton-backgrounds vanishes, so that the 3d metric stays flat.

We will now show that the above backgrounds correspond precisely to an $SU(2)$ WZW plus a Feigin-Fuchs model on the world-sheet. This 3d instanton is therefore an exact solution of bosonic-string theory, as well as a reinterpretation of the much-discussed (singular) semi-wormhole \[^{\text{[6][7][1]}}\]. Indeed, the corresponding $\sigma$-model Lagrangian in conformally flat coordinates reads

$$\mathcal{L}_\sigma = \frac{k}{4} \int \frac{d^2 \xi}{4 \pi} \left[ \partial \alpha \bar{\partial} \alpha + \partial \beta \bar{\partial} \beta + \partial \gamma \bar{\partial} \gamma + 2 \cos \beta \, \partial \alpha \bar{\partial} \gamma \right] + \int \frac{d^2 \xi}{4 \pi} \left[ \partial \bar{\gamma} \bar{\partial} \bar{\gamma} + \sqrt{g} R^{(2)} Q \right]. \tag{4}$$

Here $\tilde{r}$ is the Feigin-Fuchs field, $\alpha \in [0, 4\pi]$, $\beta \in [0, \pi]$ and $\gamma \in [0, 2\pi]$ are Euler angles parametrizing the $SU(2)$ group manifold with a non-standard choice of ranges, and unitarity forces $k$ to be a positive integer. Our notation is as follows: $\xi^a$ are the conformally-flat coordinates, $z = (\xi^1 + i \xi^2)/2$ and we have set the Regge slope $\alpha' = 1$. The central charge of the above conformal model is $c = 1 + \frac{3}{2} Q^2 + 3k/(k + 2)$. We can read off the $\sigma$-model backgrounds by comparing eq. (4) to the generic form

$$\mathcal{L}_\sigma = \int \frac{d^2 \xi}{4 \pi} \left[ (G_{IJ} + B_{IJ}) \partial X^I \partial X^J + \sqrt{g} R^{(2)} \Phi(X) \right], \tag{5}$$

where $X^I$ ($I = 1, ..., 4$) denote collectively the coordinate fields. To make contact with the 3d instanton we will now view the angle $\alpha$ as a compact internal coordinate, and make a Kaluza-Klein decomposition of the metric and antisymmetric tensors \[^{3}\]:

$$G_{IJ} = \begin{pmatrix} \tilde{g}_{ij} + v a_i a_j & v a_i \\ v a_j & v \end{pmatrix}, \quad B_{IJ} = \begin{pmatrix} b_{ij} + b_i a_j - a_i b_j & b_i \\ -b_j & 0 \end{pmatrix}. \tag{6}$$

Under this reduction, the reparametrization- and antisymmetric-tensor invariances in the compact dimension, descend to a vector- and an axial-$U(1)$ gauge symmetry. The merit of the decomposition above is to simplify the corresponding transformation laws. It is indeed straightforward to check that the $\sigma$-model action is invariant (up to boundary terms) under the following transformations: $(\delta a_i = \partial \Lambda_{vec})$ and $(\delta b_i = \partial \Lambda_{ax} ; \delta b_{ij} = \Lambda_{ax} F_{ij}(a))$. Note in passing that the gauge-invariant antisymmetric-tensor field strength in the reduced dimensions reads $H_{ijk} = \partial_i b_{jk} - b_i F_{jk}(a) +$ cyclic perms.

In the case that interests us we can set $v = 1$ by rescaling the internal coordinate $\alpha \rightarrow 2\alpha/\sqrt{k}$, and work with the chiral gauge fields $A_i = (a_i - b_i)/\sqrt{2}$ and $\tilde{A}_i = (a_i + b_i)/\sqrt{2}$. We will furthermore go to the Einstein-frame metric $g_{ij} = e^{-2\Phi/(d-2)} \tilde{g}_{ij}$, in terms of which the effective low-energy Lagrangian is given by eq. (3). Defining flat polar coordinates: $r \equiv e^{-\Phi(\vec{r})}/Q$, $\theta \equiv \beta$, $\phi \equiv \gamma$ we can finally identify the backgrounds of

\[^{3}\]We use lower-case indices to label the space after dimensional reduction.
the $\sigma$-model (4) with the low-energy solution, eqs. (1,2). Indeed, the background of the $A_i$ gauge field is precisely the instanton with charge
\[ q = \sqrt{k/2}, \tag{7} \]
while $\tilde{A}_i = 0$ and the Einstein-frame metric is flat provided we choose $Q^2 = \frac{4}{k} + O(\frac{1}{k^2})$. This choice implies that $\delta c \equiv c - 4$ vanishes to one-loop order, in accordance with the effective field-theory argument. Of course, the $\sigma$ model, eq. (4), stays conformally invariant for arbitrary values of $Q$. The corresponding solution is an instanton in curved 3$d$ space:
\[ ds^2 = dr^2 + \frac{Q^2 k}{4} r^2 d^2 \Omega, \tag{8} \]
where $d^2 \Omega$ is here the distance on $S^2$.

Note that the radius of the compact internal dimension is $R = \sqrt{k}$ \[\text{¶}\], so that the spectrum of electric charges is $e_{nm} = (n/(\sqrt{2k}) + m\sqrt{\frac{k}{2}})$ with $n, m$ integers. The ”magnetic” charge, eq. (7), is thus the minimum one allowed by the Dirac quantization condition: $2qe \in \mathbb{Z}$ for all electric charges $e$ of the theory. The Dirac quantization condition is furthermore equivalent to the unitarity constraint, $k = R^2 \in \mathbb{Z}$, of the $WZW$ model \[\parallel\]. We could interpret this constraint as saying that instantons of a $U(1)$, coming from a purely holomorphic sector of the string, do not exist in general due to the presence of irrationnally-related charges.

The analysis above is a small variation of the one used in refs. \[\text{3} \] \[\text{4}\] to identify two other classes of string-theory solutions. The first are the electrovac solutions to gauged supergravities, discovered by Freedman and Gibbons \[\text{9}\], and characterized a priori by both magnetic and electric graviphoton backgrounds. The solution with vanishing magnetic field, which is distinguished by the existence of $N = 2$ unbroken space-time supersymmetries, corresponds precisely to a $SU(1, 1)$ $WZW$ model plus extra free coordinate fields \[\text{3}\]. Replacing the latter by a $SU(2)$ $WZW$ model, one can in fact reproduce the entire set of the Freedman-Gibbons solutions, including those that break space-time supersymmetry with a non-vanishing magnetic field. The second class of interesting solutions are the magnetically-charged $4d$ dilatonic black holes \[\text{10}\]. Limiting cases of these solutions are obtained \[\text{3}\] if one adds to the $\sigma$ model, eq. (4), a free time-like coordinate $t$, or if one replaces the $(\tilde{r}, t)$ system by the exact 2$d$ black hole \[\text{11}\]. In \[\text{3}\] a left orbifold of the $SU(2)$ WZW model was also considered. Its effect is to make the compactification radius of the angle $\alpha$ in (4) equal to $4\pi/N$, where $N$ is a divisor of $k$.\[\text{¶}\]

\[\text{¶}\] Had we not identified the corresponding CFT, we could thus not ascertain its existence from the effective 3$d$ action since a fourth dimension decompactifies in the weak-field limit.\[\text{∥}\] This is another facet of an argument originally due to Rohm and Witten \[\text{8}\].
3 Reduction of general WZW models

Generalizing the above procedure, one may decompose a WZW model on an arbitrary group manifold $G$, into a $H_L \times H_R$ current algebra, a $G/H$ coset manifold treated as part of non-compact space, and background gauge and antisymmetric tensor fields [3]. We will write the group elements as $\hat{g} = h(y)g(x)$ where the $y$ coordinates parametrize the $H$-subgroup manifold, while the $x$ coordinates parametrize the right coset $G/H$. An essential ingredient of the reduction is the Polyakov-Wiegmann formula

$$I(hg) = I(h) + I(g) + \int \frac{d^2\xi}{2\pi} \text{tr}[h^{-1}\partial h \tilde{g} gg^{-1}] ,$$  \hspace{1cm} (9)

where $I(g)$ is proportional to the WZW action of a simple group $G$,

$$I(g) = \int \frac{d^2\xi}{4\pi} \text{tr}[g^{-1}\partial gg^{-1}\tilde{g}] - i \int B \frac{d^3\xi}{6\pi} \epsilon^{abc} \text{tr}[g^{-1}\partial_agg^{-1}\partial_bgg^{-1}\partial_cg] ,$$  \hspace{1cm} (10)

with $B$ being as usual a solid ball whose boundary is the Euclidean 2$d$ world sheet. If the traces are taken in some $R$ representation of the group, the correctly normalized WZW action is

$$S_k^{WZW} = -\kappa I(g) , \hspace{1cm} \kappa \equiv \frac{k}{4} \frac{c_G d_G}{h_{GGR} d_R}$$  \hspace{1cm} (11)

where $k \in Z$ is an integer, $d_R (d_G)$ and $c_R (c_G)$ are the dimension and quadratic Casimir of the $R$ (adjoint) representations respectively, and $\tilde{h}_G$ is the dual Coxeter number: $\tilde{h}_G = n$ for $SU(n)$ and $\tilde{h}_G = n - 2$ for $SO(n)$.

The three terms in eq. (9) above correspond to a metric and antisymmetric tensor background on the coset space, a $H_L \times H_R$ current algebra, and a background $H_L$ gauge field. In order to read off these backgrounds we will use again gauge invariance as a guide. Our starting point is the vacuum consisting of flat space-time $(x^\mu)$ plus the $H_L \times H_R$ current algebra. Under a gauge transformation $(\upsilon(x), \bar{\upsilon}(x))$, the corresponding left- and right-gauge fields transform as follows

$$A_\mu \rightarrow \upsilon (A_\mu - \upsilon^{-1} \partial_\mu \upsilon)^{-1}$$

$$\bar{A}_\mu \rightarrow \bar{\upsilon} (\bar{A}_\mu - \bar{\upsilon}^{-1} \partial_\mu \bar{\upsilon})^{-1} ,$$  \hspace{1cm} (12)

where with our conventions the gauge-field strengths read: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$, with a similar expression for $\bar{F}_{\mu\nu}$. Next let us define the Lie-algebra valued vector field on the world sheet [2]

$$J_a \equiv h^{-1} \partial_a h + (h^{-1} \bar{A}_a h - A_a) \partial_\mu x^\mu ,$$  \hspace{1cm} (13a)

and its conjugate

$$\bar{J}_a \equiv h J_a h^{-1} .$$  \hspace{1cm} (13b)

**To avoid confusion we stress that $h(\xi)$ and $x^\mu(\xi)$ are string coordinates, while $\upsilon(x)$ and $\bar{\upsilon}(x)$ stand for target-space gauge transformations.**
Under a target-space gauge transformation and a simultaneous change of string coordinates,
\[ h \rightarrow \bar{v}(x) \ h \ u^{-1}(x) \],
these vector fields transform homogeneously:
\[ J_a \rightarrow \bar{v}J_a \bar{v}^{-1}; \ \bar{J}_a \rightarrow \bar{v}\bar{J}_a \bar{v}^{-1}. \]  

We may then write the following \(\sigma\)-model action to describe the interaction of a string with arbitrary metric, antisymmetric-tensor and gauge-field backgrounds:
\[ S = \int \frac{d^2 \xi}{4\pi} \tilde{g}_{\mu\nu} \partial x^\mu \partial x^\nu + i \int \frac{d^3 \xi}{12\pi} \epsilon^{abc} \hat{H}_{\mu\nu\rho} \partial_a x^\mu \partial_b x^\nu \partial_c x^\rho - \kappa \int \frac{d^2 \xi}{4\pi} tr(J_z \bar{J}_z) \]
\[ + i\kappa \int \frac{d^3 \xi}{6\pi} \epsilon^{abc} \left[ tr(J_a \bar{J}_b J_c) - \frac{3}{2} tr\left( F_{\mu\nu} J_c + \bar{F}_{\mu\nu} \bar{J}_c \right) \right] \partial_a x^\mu \partial_b x^\nu \]
\[ + \kappa \int \frac{d^3 \xi}{12\pi} \epsilon^{abc} \left[ \hat{H}_{\mu\nu\rho} + \kappa CS_{\mu\nu\rho}(A) - \kappa CS_{\mu\nu\rho}(\bar{A}) \right] \partial_a x^\mu \partial_b x^\nu \partial_c x^\rho + \kappa \int \frac{d^3 \xi}{12\pi} \epsilon^{abc} \left[ tr(J_a \bar{J}_b J_c) - \frac{3}{2} tr\left( F_{\mu\nu} J_c + \bar{F}_{\mu\nu} \bar{J}_c \right) \right] \partial_a x^\mu \partial_b x^\nu \partial_c x^\rho, \]
\[(16)\]

Assuming \(\tilde{g}_{\mu\nu}\) and \(\hat{H}_{\mu\nu\rho}\) do not transform, this action is manifestly gauge-invariant. Furthermore, it reduces to \(-\kappa I(h) + \int (\partial x)^2\) for vanishing background deformations. To prove however that it is well-defined, we must still show that the 3d integrand is a total divergence. This requirement is what fixed in particular the relative coefficient of the last term of the action. After a tedious but straightforward calculation, one may put eq. (16) in the form:
\[ S = -\kappa I(h) + \int \frac{d^2 \xi}{4\pi} \left[ g_{\mu\nu} - \kappa tr(A_\mu A_\nu + \bar{A}_\mu \bar{A}_\nu - 2 \bar{A}_\mu A_\nu h h^{-1}) \right] \partial x^\mu \partial x^\nu \]
\[ + \kappa \int \frac{d^3 \xi}{2\pi} \left[ tr(A_\mu h^{-1} \partial h) \partial x^\mu - tr(\bar{A}_\mu \partial h h^{-1}) \partial x^\mu \right] \]
\[ + i \int \frac{d^3 \xi}{12\pi} \epsilon^{abc} \left[ \hat{H}_{\mu\nu\rho} + \kappa CS_{\mu\nu\rho}(A) - \kappa CS_{\mu\nu\rho}(\bar{A}) \right] \partial_a x^\mu \partial_b x^\nu \partial_c x^\rho , \]
\[(17)\]

where the Chern-Simmons three-form is
\[ CS_{\mu\nu\rho}(A) \equiv tr(A_\mu F_{\nu\rho} - \frac{1}{3} A_\mu [A_\nu, A_\rho] + cyclic \ perms) . \]
\[(18)\]

It is now clear that for the action to be well-defined, we must demand that the three-form in square brackets be exact:
\[ \hat{H}_{\mu\nu\rho} + \kappa CS_{\mu\nu\rho}(A) - \kappa CS_{\mu\nu\rho}(\bar{A}) \equiv H_{\mu\nu\rho} \equiv \partial_\mu b_{\nu\rho} + cyclic \ perms . \]
\[(19)\]

We have thus rederived from the \(\sigma\)-model the Green-Schwarz mechanism of anomaly cancellation: the antisymmetric-tensor field \(b_{\mu\nu}\) must transform so as to make the generalized field strength \(\hat{H}_{\mu\nu\rho}\), through which it enters in the effective low-energy Lagrangian, gauge-invariant. A similar derivation was given before for the heterotic string by Hull and Witten [12]. In their case the Chern-Simmons transformation law came from the gauge anomaly of the fermions, while in our case it comes from the WZW term of the action.
Eq. (17) is our basic formula, which allows the identification of backgrounds for any
group-manifold compactification††. There is in fact an extra gauge-invariant term that
we could have added to the \( \sigma \)-model action. It is

\[
S' = \int \frac{d^2 \xi}{4\pi} \Phi_{\alpha \bar{\beta}}(x) \text{tr}(J_z T^\alpha) \text{tr}(\bar{J}_\bar{z} T^\bar{\beta})
\]

(20)

where \( T^\alpha \) are the group generators, and the scalar \( \Phi_{\alpha \bar{\beta}} \) transforms in the \((\text{adj, adj})\) representation of the gauge group. This term corresponds to space-dependent deformations of
the metric and antisymmetric tensor in the compact directions. Comparing the Polyakov-
Wiegmann formula with the generic form (17) and (20), one sees immediately that for
the right-coset reductions of \( WZW \) models \( \Phi_{\alpha \bar{\beta}} = \bar{A}_\mu = 0 \). The only non-trivial back-
grounds are therefore \( A_\mu, b_{\mu \nu} \) and the space-time metric which, as can be verified easily,
is always the metric of the symmetric space \( G/H \). Let us however point out that other
coset reductions are sometimes possible. Thus, if \( H \simeq H_1 \times H_2 \) is not semisimple, the
decomposition \( \hat{g} = h_1 g h_2 \) will lead to both \( \Phi_{\alpha \bar{\beta}} \) and \( \bar{A}_\mu \) backgrounds.

4 Yang-Mills merons

We will now apply this procedure to the simplest example with non-abelian gauge
fields, namely the \( SU(2)^{k_1} \times SU(2)^{k_2} \) \( WZW \) model reduced so that a diagonal subgroup
\( H \) is internal space ‡‡. Let \((g_1, g_2) \equiv (h, gh)\) be the corresponding decomposition of the
\( SU(2) \times SU(2) \) group manifold, where \( h \) and \( g \) are independent \( 2 \times 2 \) unitary matrices.
We will parametrize these latter as follows: \( h = y^0 1 + i \bar{\sigma} \cdot \vec{y} \) and \( g = w^0 1 + i \bar{\sigma} \cdot \vec{w} \),
where \( (y^0)^2 + \vec{y} \cdot \vec{y} = (w^0)^2 + \vec{w} \cdot \vec{w} = 1 \), and \( \sigma^i \) are the Pauli matrices normalized so
that \( \sigma^i \sigma^j = \delta^i_j + i \epsilon^{ijk} \sigma^k \). Using the Polyakov-Wiegmann formula, we can write the WZW
action of the model:

\[
-S_{WZW} = \frac{k_1 + k_2}{2} I(h) + \frac{k_2}{2} I(g) + k_2 \int \frac{d^2 \xi}{4\pi} \text{tr}[h^{-1} \partial h \bar{\partial} \bar{g} g^{-1}] .
\]

(21)

Comparing with eq. (17), we can read off the following non-vanishing backgrounds:

\[
\bar{g}_{ij} = \frac{k_1 k_2}{k_1 + k_2} \left( \delta_{ij} + \frac{w^i w^j}{(w^0)^2} \right) ,
\]

\[
H_{ijl} = 2k_2 \epsilon_{ijl} / w^0 ,
\]

\[
A_i = -\frac{k_2}{k_1 + k_2} \partial_i g(w) g(w)^{-1} .
\]

(22)

Note that the metric and antisymmetric-tensor field strength are proportional, respec-
tively, to the metric and volume form on \( S^3 \), as for a simple \( SU(2) \) \( WZW \) model. The
radius square of the sphere need not, however, here be an integer.

†† Explicit formulae for generic dimensional reductions can be found in refs. [13].
‡‡ Had we chosen \( H \) as one of the two \( SU(2) \) groups, the resulting gauge field background would be a
pure gauge.
As in the case of the 3d instanton, we can again add an extra Feigin-Fuchs coordinate $r$, with background charge
\[ Q^2 = \frac{4(k_1 + k_2)}{k_1 k_2}, \]
so as to render the 4d Einstein-frame metric flat. Going to the flat Cartesian coordinates
\[ x^\mu = \frac{2}{Q} e^{-\Phi/2} u^\mu, \]
and trading the antisymmetric tensor for a pseudo-scalar axion field through the duality transformation
\[ \hat{H}_{\mu\nu\rho} \equiv H_{\mu\nu\rho} + \frac{k_1 + k_2}{2} CS_{\mu\nu\rho}(A) = \epsilon_{\mu\nu\rho\sigma} e^{2\Phi} \partial^\sigma b, \]
we obtain:
\[ g_{\mu\nu} = \delta_{\mu\nu}, \Phi = -\log|x|^2 - \log \frac{Q^2}{4}, \]
\[ b = \frac{k_1 - k_2}{k_1 k_2} |x|^2, \]
while the gauge-field in these coordinates becomes
\[ A_\mu = -i \frac{e^a}{2} A^a_\mu, \quad A^a_\mu = \frac{2k_2}{k_1 + k_2} \frac{\epsilon_{\mu\nu}}{|x|^2}. \]
where $\epsilon_{\mu\nu}$ are the well-known $\eta$-symbols describing the projection of SO(4) into a left SU(2) subgroup [14].

It is straightforward to check that the above backgrounds solve the field equations derived from the effective 4d Euclidean Lagrangian
\[ \mathcal{L} = \int d^4x \sqrt{g} \left[ -R + \frac{1}{2} (\partial\Phi)^2 - \frac{1}{2} e^{2\Phi} (\partial b)^2 + \frac{1}{4g^2} e^{-\Phi} F^a_{\mu\nu} F^a_{\mu\nu} \right. \]
\[ \left. + \frac{1}{4g^2} b F^a_{\mu\nu} \ast F^a_{\mu\nu} - \frac{2}{3} \delta c e^\Phi \right]. \]
Here $\ast F^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^a_{\rho\sigma}$, $g^2 = 2/(k_1 + k_2)$ is the SU(2) coupling constant for zero value of the dilaton, and the central-charge deficit is fixed from the dilaton equation to be:
\[ \delta c = 3g^2 = 6/(k_1 + k_2). \]
This is of course consistent with the conformal-field-theory result in the $k_1, k_2 \to \infty$ limit. Note that a rotation of the axion, $b \to ib$, makes its kinetic term positive definite and the $F \ast F$ term purely imaginary. One should however keep in mind that the Euclidean functional integral must be defined in terms of the fundamental field $b_{\mu\nu}$, so that a priori the relevant saddle points of (28) are real. Note also that the constant part of the dilaton, which was a free parameter of the conformal model, has been here fixed by requiring that the space metric be $\delta_{\mu\nu}$. 

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We may choose the remaining two free parameters to be the gauge coupling $g$, and 
\[ \lambda = \frac{k_2}{k_1 + k_2}, \]
so that the gauge-field background reads
\[
A^a_\mu = 2\lambda \frac{\eta^{a}_{\mu\nu} x^\nu}{|x|^2}.
\]
(30)

This is a generalization of a well-known solution of pure Yang Mills theory, known as the meron \[13\]. The pure Yang-Mills meron has $\lambda = \frac{1}{2}$, and is characterized by a topological charge $q = \frac{1}{2}$ residing at the origin of coordinate space. We can obtain the charge of our generalized merons by integrating the Chern-Simmons form on a three-sphere around the origin, with the result
\[
q(\lambda) = -\frac{1}{48\pi^2} \int_{S^3} \epsilon_{\mu\nu\rho\sigma} CS^{\mu\nu}\hat{x}^\rho d^3x = \lambda^2(3 - 2\lambda).
\]
(31)

This calculation appears to be incompatible with the axion equation
\[ \partial^\mu (e^{2\Phi} \partial_\mu b) + \frac{1}{4g^2} F^* F = 0, \]
but the equation yields $\lambda(\lambda - 1)(2\lambda - 1) + \lambda^2(3 - 2\lambda) \equiv \lambda = 0$, which is not satisfied in general. This is however, not surprising: although our solution is exact, the validity of the low-energy equations of motion clearly breaks down at $x = 0$.

Demanding unitarity of the WZW model, imposes a quantization condition on $\lambda$ and on the corresponding topological charge. If we let $k \equiv k_1 + k_2$ be the level of the (internal) current algebra, and $n \equiv k_1 - k_2$, then the spectrum of allowed charges is
\[
q_n = \frac{1}{4}(1 - \frac{n}{k})^2(2 + \frac{n}{k}) \quad (n = -k, \ldots, k).
\]
(32)

Note that the higher the level $k$ of the current algebra, the richer the spectrum of merons. Note also that the conjugation $g \to g^\dagger$ transforms the meron to an anti-meron, and that a meron with parameter $\lambda$ transforms under the (singular) gauge transformation $h \to hg^\dagger$ to an antimeron with parameter $1 - \lambda$. A simple consequence of this fact is that $q(\lambda) + q(1 - \lambda) = 1$. Note finally that for $\lambda = 1$, $A_\mu$ is the field of an instanton \[16\], but in the zero-size limit in which it is a pure (singular) gauge. The string solution has, however, non-trivial dilaton and axion backgrounds, and is in fact the well-known semi-wormhole \[3\] [4] [1].

Despite much discussion in the literature, the physical interpretation of merons remains still unclear. They suffer from a singularity at $x = 0$, and an infrared-divergent action, but are in this respect similar to vortices in the 2d XY model. This analogy suggests that they could be instrumental for confinement in four dimensions \[17\]. They have been also interpreted as half instantons and as short-lived monopoles. In pure Yang-Mills theory they survive unchanged in any conformally flat isotropic space \[4\]. By relaxing the condition (23) on the background charge of the dilaton, we can also extend the string-theory merons to any conformally-flat space of the form $g_{\mu\nu} \propto |x|^{2\alpha - 2}\delta_{\mu\nu}$. The axion

\footnote{Callan and Wilczek \[18\] have made the interesting suggestion of using such a negative-curvature space as a gauge-invariant infrared regulator. Let us note that this role could be also played by a dilaton background, that could suppress strong fluctuations outside some finite region.}
and dilaton backgrounds in this case read $b \propto e^{-\phi} \propto |x|^{2\alpha}$, where at the level of the \(\beta\)-function equations, \(\alpha\) is an arbitrary continuous parameter of the solution.

Before closing this section let us point out that higher-dimensional generalizations of the 3\(d\) QED instanton, and the 4\(d\) Yang-Mills meron can be obtained easily through the dimensional reduction \(SO(N+1) \to SO(N)\). And that another class of CFTs, which contains current algebra and can be thus used for non-abelian compactifications, are gauged WZW models \(G/H\) where \(H\) is not maximal. It can be shown [19] that these models have chiral \(H'\) currents, for any subgroup \(H'\) commuting with \(H\). Dimensional reduction can be thus performed also here but we will not elaborate this case further.

5 Deformations

The conformal models of the previous sections have continuous deformations or moduli. From the lower-dimensional perspective, these deformations will, in general, involve massive Kaluza-Klein excitations. One way to ensure that only massless lower-dimensional backgrounds are present, is to consider perturbations that respect the \(H_R\) isometry of the WZW model. Changing the background charge of the Feigin-Fuchs field is one trivial example of this type. Here we want to discuss deformations of the WZW model itself. Deformations which mix the Feigin-Fuchs and WZW coordinates would be very interesting, since they could modify non-trivially the radial dependence of the backgrounds, but we do not know how to handle them analytically at present.

The generic exactly marginal perturbation of the WZW model is generated by the Cartan currents, \(S_{\text{pert}} \sim C_{\alpha \beta} \int J^\alpha \bar{J}^\beta\). Such deformations a priori break the local \(G_L \times G_R\) symmetry down to \(U(1)^{\text{rank}_C}_L \times U(1)^{\text{rank}_C}_R\), but we can chose to leave a larger subgroup of the current algebra unbroken. The corresponding deformed \(\sigma\)-model action is known explicitly [20, 21] only for perturbations \(S_{\text{pert}} \sim \int J \bar{J}\), where \(J\) corresponds to a single Cartan generator \(T\) of the group. Let us normalize this (antihermitean) generator so that \(\text{tr}(TT) = -\frac{1}{2}\), and parametrize the group manifold as follows: \(g = e^{T \theta} \tilde{g}(w) e^{T \bar{\theta}},\) with \(w^I\) the (\(\text{dim}G - 2\)) remaining coordinates. Using the Polyakov-Wiegmann formula we can write the WZW action in the form:

\[-\kappa I(g) = -\kappa I(\tilde{g}) + \frac{\kappa}{2} \int \frac{d^2 \xi}{4\pi} \left( \partial \theta_1 \bar{\partial} \theta_1 + \partial \theta_2 \bar{\partial} \theta_2 \right) +
\]

\[+ \frac{\kappa}{2} \int \frac{d^2 \xi}{2\pi} \left( \Sigma(w) \partial \theta_2 \bar{\partial} \theta_1 + \Gamma_1^1(w) \partial w^I \bar{\partial} \theta_1 + \Gamma_2^2(w) \partial \theta_2 \bar{\partial} w^I \right)\]

\[(33)\]

where here

\[\Gamma_1^1(w) \partial w^I = -2\text{tr}(T \tilde{g} \tilde{g}^{-1}),\]

\[\Gamma_2^2(w) \partial w^I = -2\text{tr}(T \tilde{g}^{-1} \bar{\partial} \tilde{g}),\]

\[\Sigma(w) = -2\text{tr}(\tilde{g}^{-1} T \bar{\partial} \tilde{g} T).\]

\[(34)\]

\(\dagger\)For special values of the level there may be more marginal directions.
This action is manifestly invariant under $\theta_1 \to \theta_1 + \epsilon(z)$ and $\theta_2 \to \theta_2 + \epsilon(\bar{z})$. The corresponding left and right chiral currents are

$$J = \frac{\kappa}{2} \left( \partial \theta_1 + \Sigma(w) \partial \theta_2 + \Gamma^1_i \partial w^i \right)$$  \hspace{1cm} (35a)

and

$$J = \frac{\kappa}{2} \left( \bar{\partial} \theta_2 + \Sigma(w) \bar{\partial} \theta_1 + \Gamma^2 \bar{\partial} w^I \right).$$  \hspace{1cm} (35b)

The marginal perturbation $S_{pert} \sim \int J \tilde{J}$ then leads to the following continuous family of conformally-invariant $\sigma$-models [21]:

$$S(\kappa, \zeta) = -\kappa I(\tilde{g}) + \frac{\kappa}{2} \int \frac{d^2 \xi}{4\pi} \left[ \partial \phi_1 \bar{\partial} \phi_1 + \partial \phi_2 \bar{\partial} \phi_2 + 2 \frac{1 + \Sigma - \zeta^2 (1 - \Sigma)}{1 + \Sigma + \zeta^2 (1 - \Sigma)} \partial \phi_2 \bar{\partial} \phi_1 + \ight.$$  

$$+ \frac{2\zeta}{1 + \Sigma + \zeta^2 (1 - \Sigma)} \left( \Gamma^1_i \partial w^I \bar{\partial} \phi_1 + \Gamma^2_i \partial \phi_2 \bar{\partial} w^I \right) + \frac{\zeta}{1 + \Sigma + \zeta^2 (1 - \Sigma)} \partial w^I \bar{\partial} w^I \right]$$  \hspace{1cm} (36)

where

$$\phi_1 = \frac{1}{2} \left( \zeta + \frac{1}{\zeta} \right) \theta_1 + \frac{1}{2} \left( \zeta - \frac{1}{\zeta} \right) \theta_2 , \hspace{0.5cm} \phi_2 = \frac{1}{2} \left( \zeta - \frac{1}{\zeta} \right) \theta_1 + \frac{1}{2} \left( \zeta + \frac{1}{\zeta} \right) \theta_2 ,$$  \hspace{1cm} (37)

and there is also a non-trivial dilaton background

$$\Phi(w) = - \log \left[ \zeta (1 - \Sigma) + (1 + \Sigma) / \zeta \right].$$  \hspace{1cm} (38)

Clearly, the original $U(1)_L \times U(1)_R$ symmetry of the unperturbed model survives for arbitrary $\zeta$, even though the currents themselves get modified:

$$J(\zeta) = \frac{\kappa}{2} \left( \partial \phi_1 + \frac{1 + \Sigma - \zeta^2 (1 - \Sigma)}{1 + \Sigma + \zeta^2 (1 - \Sigma)} \partial \phi_2 + \frac{\zeta}{1 + \Sigma + \zeta^2 (1 - \Sigma)} \Gamma^1_i \partial w^I \right),$$  \hspace{1cm} (39a)

$$\tilde{J}(\zeta) = \frac{\kappa}{2} \left( \bar{\partial} \phi_2 + \frac{1 + \Sigma - \zeta^2 (1 - \Sigma)}{1 + \Sigma + \zeta^2 (1 - \Sigma)} \bar{\partial} \phi_1 + \frac{\zeta}{1 + \Sigma + \zeta^2 (1 - \Sigma)} \Gamma^2 \bar{\partial} w^I \right).$$  \hspace{1cm} (39b)

It can be verified easily that $S(\kappa, \zeta)$ reduces to the WZW action at $\zeta = 1$, and that furthermore

$$S(\kappa, \zeta + \delta \zeta) = S(\kappa, \zeta) + \frac{2\delta \zeta}{\pi \kappa \zeta} \int J(\zeta) \tilde{J}(\zeta) + O(\delta \zeta^2),$$  \hspace{1cm} (40)

as claimed.

Let us restrict now our attention to the simplest case of $SU(2)$. Comparing eqs. (4) and (33) we can identify $\theta_1$, $\theta_2$ and $w$ with the Euler angles $\alpha$, $\gamma$ and $\beta$, so that $\Gamma^1 = \Gamma^2 = 0$, and $\Sigma = \cos \beta$. The action of the deformed model reads

$$S(k, \zeta) = \frac{k}{4} \int \frac{d^2 \xi}{4\pi} \left[ \partial \phi_1 \bar{\partial} \phi_1 + \partial \phi_2 \bar{\partial} \phi_2 + \partial \beta \bar{\partial} \beta + 2F(\beta) \partial \phi_2 \bar{\partial} \phi_1 \right],$$  \hspace{1cm} (41)

with

$$F \equiv \frac{1 + \cos \beta - \zeta^2 (1 - \cos \beta)}{1 + \cos \beta + \zeta^2 (1 - \cos \beta)}. $$  \hspace{1cm} (42)
One can think of the $SU(2)$ WZW model as constructed from a free boson and order-$k$ parafermions, with the actions of a discrete $Z_k$ symmetry identified. The above family of deformed models corresponds in this language to the continuous variation of the radius of the free boson \[21\].

In order to read off from the action (41) the deformation of the 3d instanton background of section 2, we must still decide what combinations of $\phi_1$ and $\phi_2$ will stand for the internal compact dimension, and for the polar angle of ”real space”. Since, up to a gauge transformation, these must be independently periodic, we should a priori identify them with the Euler angles $\alpha$ and $\gamma$, related to $\phi_1$ and $\phi_2$ through eq. (37). We will work for convenience with the rescaled angle $\sqrt{k}\alpha/2 \in [0, 2\pi \sqrt{k}]$. We will furthermore drop the radial Feigin-Fuchs coordinate and the rescaling to the Einstein-frame metric, since they will play no role in the discussion that follows. Comparing (41) with eqs. (5) and (6) we then find the following backgrounds for the scalar, the vector and axial gauge fields, and the 2d metric:

\[ v = \frac{\zeta^2 (1 + \cos \beta) + (1 - \cos \beta)}{(1 + \cos \beta) + \zeta^2 (1 - \cos \beta)}, \quad (43) \]

\[ a_\gamma = \sqrt{k} \frac{1 + \cos \beta}{(1 + \cos \beta) + (1 - \cos \beta)/\zeta^2}, \quad (44) \]

\[ b_\gamma = \sqrt{k} \frac{1 + \cos \beta}{(1 + \cos \beta) + \zeta^2 (1 - \cos \beta)}, \quad (45) \]

and

\[ ds^2 = \frac{k}{4}\left[d\beta^2 + \frac{1 - F^2}{v} d\gamma^2\right]. \quad (46) \]

We have here used gauge transformations to make $a_\gamma$ and $b_\gamma$ vanish at the south pole. Using the expansions $F \simeq 1 - \zeta^2 \beta^2/2$ and $F \simeq -1 + \zeta^2 (\pi - \beta)^2/2$ near the north and south poles of the deformed sphere, one can verify easily that the metric (46) has no conic singularities at these points. Furthermore $a_\gamma \simeq b_\gamma \simeq \sqrt{k}$ near the north pole, so that their Dirac string singularities are indeed unobservable \[11\].

It is in fact possible to demonstrate that the choice of periodicities for the angles $\phi_1$ and $\phi_2$, can be uniquely fixed by requiring the absence of conic and observable Dirac-string singularities for the $\sigma$-model backgrounds. Indeed, suppose

\[ \phi_1 = a_1 \tilde{\alpha} + c_1 \tilde{\gamma} \quad ; \quad \phi_2 = a_2 \tilde{\alpha} + c_2 \tilde{\gamma} \quad (47) \]

were some arbitrary linear combinations of the compact coordinate $\tilde{\alpha} \equiv \tilde{\alpha} + 4\pi$ and the polar angle $\tilde{\gamma} \equiv \tilde{\gamma} + 2\pi$. After some straightforward but lengthy algebra, one finds that the 2d metric is free of conic singularities provided

\[ \frac{c_1 + c_2}{(a_1 + a_2)^2}(a_1^2 c_2 + a_2^2 c_1) - c_1 c_2 = \frac{1}{\zeta^2}, \quad (48) \]

\[ \text{Another way of saying this for the axial field, is that the deformed volume form } F_{\beta\gamma}(b)d\alpha d\beta d\gamma \text{ has a } \zeta \text{-independent normalization.} \]
and
\[ \frac{c_1 - c_2}{(a_1 - a_2)^2} (a_2^2 c_1 - a_1^2 c_2) + c_1 c_2 = \zeta^2, \] (49)
and that, assuming continuity in \( \zeta \), the Dirac quantization conditions for the gauge fields read
\[ \frac{c_1 + c_2}{a_1 + a_2} - \frac{c_1 - c_2}{a_1 - a_2} = 2, \] (50)
and
\[ a_2 c_1 - a_1 c_2 = -1. \] (51)
Modulo a gauge transformation \( \tilde{\alpha} \to \tilde{\alpha} + e\tilde{\gamma} \), the unique solution to the above constraints is given by the linear combinations (37), where \( \tilde{\alpha} \) and \( \tilde{\gamma} \) are identified with \( \theta_1 \) and \( \theta_2 \), i.e. precisely with the Euler angles.

The meaning of this result is the following: the perturbative \( \beta \)-function equations are satisfied for any choice of the periodicity-lattice in the \( (\phi_1, \phi_2) \) plane. Non-perturbative effects on the world-sheet will however break this continuous degeneracy, and it is reasonable to assume that the allowed periodicities should be the ones corresponding to smooth backgrounds. From the string-field-theory point of view, the degeneracy above corresponds to continuous global symmetries of the effective low-energy action, which are broken by the massive Kaluza-Klein excitations. To be more precise, consider the effective action of a string compactified on a circle from three down to two dimensions:
\[ S_{\text{eff}} \propto \int d^2 x \ e^{-\Phi} \sqrt{g \bar{v}} \left[ -R - (\partial \Phi)^2 + \frac{2}{\sqrt{v}} \Delta \sqrt{v} + \frac{v}{4} F_{\mu \nu}(a)^2 + \frac{1}{4v} F_{\mu \nu}(b)^2 \right]. \] (52)
The field equations derived from this action have two manifest scaling symmetries: (i) \( v \to \lambda^2 v, a_\mu \to \frac{1}{\lambda} a_\mu \) and \( b_\mu \to \lambda b_\mu \), and (ii) \( g_{\mu \nu} \to \lambda^2 g_{\mu \nu}, a_\mu \to \lambda a_\mu \) and \( b_\mu \to \lambda b_\mu \), which can be used to transform one solution to another. The first transformation changes the size of the compact dimension, and boosts the vector and axial magnetic charges of the soliton. If we were dealing with the "vacuum", this would of course be an exact marginal deformation. In our case, on the other hand, only combinations of (i) and (ii) which amount to discrete rescalings of \( k \) (so that \( k \in \mathbb{Z} \) always) take us from one acceptable solution to another. We can say that the presence of "matter" has removed the continuous degeneracy of the vacuum! Strictly speaking we have, however, traded this degeneracy for a new continuous parameter, \( \zeta \). The nature of the \( \zeta \) deformation is however different. At \( \zeta = 1 \) the vector and axial magnetic fluxes are distributed uniformly over the sphere. As \( \zeta \) is being increased, the total magnetic charges do not change, but their fluxes start concentrating near the south and north poles, respectively. At the same time they deform in their vicinity the radius field so as to minimize their energy. We can thus describe this deformation as the dissociation, rather than boosting of magnetic charges. These

\^Orbifold singularities teach us however that there could be exceptions to this rule. The reader may also object that the solutions of the previous sections have singularities at the origin. These can be attributed to the Feigin-Fuchs model whose consistency as a full-fledged CFT remains indeed to demonstrate. There could in particular exist quantization conditions for the background charge \( Q \). Thus the 3d instanton and 4d merons are strictly speaking solutions to all orders in the \( \alpha' \) expansion.
effects are very much reminiscent of stringy bags [22], and it is intriguing to explore if the analogy can be pushed further.

6 Concluding Remarks

One of the motivations for this work is the hope of getting some handle on the most popular scenario for supersymmetry breaking in superstrings, which relies on the condensation of gauginos [23]. In field theory one can compute condensates in terms of fermionic zero modes in instanton backgrounds. Identifying exact gauge-instanton-like solutions of the $\beta$-function equations, is thus a natural first step before attacking the problem at the string-theory level. The solutions presented in this paper can be extended easily to the type-II and heterotic-string case. The basic ingredient is the N=1 supersymmetric version of the WZW model [24], which contains $\text{dim}G$ free fermions in addition to the bosonic group coordinates. In the case of the heterotic string, the left-moving fermions of the super-WZW model can be interpreted as part of the 32 fermions that generate the gauge group at the critical dimension, or more generally as part of the $(c,\bar{c}) = (26 - d, 15 - \frac{3}{2}d)$ conformal field theory, after compactification to $d$ dimensions. We should also point out that some of these solutions admit extended world-sheet supersymmetries. This is for instance the case for the semi-wormhole or $U(1)$ instanton of section 2, if one chooses the Feigin-Fuchs charge $Q$ so as to make the central charge $c = 4$ exactly [1][25].

In conclusion let us, however, note that the hard questions are still ahead of us. First, the exact solutions found here have no scale, and are neither asymptotically flat nor smooth at the origin. It would be particularly intriguing to have exact deformations that mix the Feigin-Fuchs and WZW models and introduce a scale. Second, we do not know the (analog of) the action, collective coordinates and normalizable fermionic zero modes in these backgrounds. And more generally how to use such solutions in a ”string-field” functional integral. We hope some progress will be made on these issues in the near future.

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