 Decomposing Impact on Longitudinal Outcome of Time-varying Covariate into Trait Effect and State Effect

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ABSTRACT

Developmental processes are often associated with each other over time; therefore, examining such associations and understanding the joint development of multiple processes is of interest. One statistical method is the latent growth curve model (LGCM) with a time-varying covariate (TVC), which estimates the effect on a longitudinal outcome of a TVC while simultaneously modeling change in the longitudinal outcome. However, this existing model does not allow the TVC to predict variation in the random growth coefficients. Our study proposes decomposing the effect of a TVC into trait and state effects to address this limitation. Specifically, we proposed three methods to decompose the impact of a TVC. In all three methods, we consider the baseline value of a TVC as the trait feature, and by regressing random intercepts and slopes on the baseline value, we obtain trait effects. Meanwhile, we characterize (1) the interval-specific slopes, (2) the interval-specific changes, or (3) the change from baseline at each measurement occasion of the TVC as the state feature in three methods, respectively. We obtain state effects by regressing the longitudinal outcome on such state features. We demonstrate the proposed methods using simulation studies and real-world data analyses, assuming the longitudinal outcome takes a linear-linear functional form. Based on the simulation results, the LGCM with a TVC breaking into the baseline value and interval-specific slopes or changes can produce unbiased and precise estimates with target confidence intervals. We provide OpenMx and Mplus 8 code for three methods with commonly used linear and nonlinear functions.

Keywords Longitudinal Process with Time-varying Covariates · Trait Effect · State Effect · Simulation Study · Individual Measurement Occasions

Longitudinal data analysis is a valuable tool for examining between-individual differences in within-individual changes in multiple disciplines, such as psychology, education, behavioral sciences, and biomedical sciences. In studies with a longitudinal design, two or more sets of repeated measurements of interest are collected together. For example, when assessing intelligence development, records of the academic performance of multiple disciplinary subjects are often collected over time for a holistic evaluation. Similarly, a clinical trial often measures biomarkers and patient-reported outcomes (PROs) repeatedly to assess treatment effects in the biomedical field. One exciting research topic is to study two or more longitudinal variables simultaneously, aiming to understand the development of each process and the joint development of these processes.

Earlier studies have demonstrated multiple statistical models in the latent growth curve modeling (LGCM) framework to analyze joint developments. For example, McArdle (1988) proposed a parallel process and correlated growth model, also referred to as a multivariate growth model (MGM) in Grimm (2007), to explore two or more longitudinal variables simultaneously by estimating intercept-intercept and slope-slope covariances of two linear longitudinal processes. More recent studies then extended such MGMs with the linear trajectory to investigate the association between multiple nonlinear longitudinal processes (Blozis, 2004; Blozis et al., 2008; Liu and Perera, 2021) and the heterogeneity of

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such associations (Liu and Perera, 2022c). Assuming that the relationship between-construct growth factors may be uni-directional rather than bi-directional, researchers have also proposed analyzing joint development by constructing longitudinal mediation models in the LGCM framework for linear (Cheong et al., 2003; Soest and Hagtvet, 2011; MacKinnon, 2008) and nonlinear longitudinal processes (Liu and Perera, 2022a). Technical details and applications of these two types of statistical models for multiple longitudinal processes can be found in the above publications.

An additional model to examine joint development is the LGCM with a time-varying covariate (TVC), which estimates the effect on a longitudinal outcome of the TVC while simultaneously modeling change patterns in the longitudinal outcome. Compared to the LGCM with a time-invariant covariate (TIC), in which the TIC is able to explain the variance of growth factors such as intercepts and slopes, the existing LGCM with a TVC allows for more efficient use of data yet does not allow the covariates to predict the variation in the random intercepts and slopes. In the presented study, we propose decomposing the effect of a TVC into trait and state effects to address this limitation. Specifically, we decompose a TVC to its baseline value that describes its trait and characteristics that help capture its state over time. Such characteristics could be the slope or amount of change in each interval or the value of change-from-baseline at each measurement occasion. Each set of the three characteristics and the baseline value can capture the change patterns of growth curves. Therefore, we propose regressing the growth factors of a longitudinal outcome on the baseline value of a TVC to examine the trait effect while regressing the measurements of the longitudinal outcome on each set of the three characteristics to investigate the state effect.

The rest of this article is organized as follows. First, we introduce existing LGCMs with covariates that could be TICs or TVCs, and a modeling framework that helps decompose a TVC into a trait feature and a state feature. We then describe the model specification and estimation of each of the three proposed methods. In the following section, we show the design of a Monte Carlo simulation to evaluate the proposed methods. Specifically, we present performance measures, including the relative bias, the empirical standard error (SE), the relative root-mean-squared-error (RMSE), and the empirical coverage probability (CP) for a nominal 95% confidence interval of parameters of interest. We analyze a real-world dataset in the Application section to demonstrate the existing and proposed methods. Finally, we frame the Discussion section with practical considerations, methodological considerations, and future directions.

Introduction

In this section, we first introduce how to add a covariate, which could be either a TIC or a TVC, when analyzing longitudinal records. We then describe the latent change score modeling (LCSM) framework (Zhang et al., 2012; Grimm et al., 2013b,a) with the novel specification proposed by Liu and Perera (2022b), which captures the change patterns of a longitudinal variable by estimating the baseline value (a trait feature) and the interval-specific slopes (a set of state features). We also depict how to obtain the other two possible types of state features, the amount of change in each time interval and the change-from-baseline value at each measurement occasion, by modifying the specification in Liu and Perera (2022b) to include additional latent variables for the interval-specific changes or the change-from-baseline values.

Existing Latent Growth Curve Model with Covariates

The latent growth curve modeling (LGCM) framework has been widely used to examine individual changes over time and potential differences across such individual changes. In the LGCM framework, within-individual changes are analyzed through a set of growth factors that together define the growth curve. For example, one can use an intercept and a slope to define a linear change pattern. Between-individual differences are captured by the variances of the growth factors. LGCMs allow for adding TICs (Jöreskog and Goldberger, 1975; McArdle and Epstein, 1987) to explain the variability of growth factors. Like independent variables in a regression model, TICs can either be continuous or categorical so that one may assess the conditional means and variance-covariance matrix of growth factors of the longitudinal outcome. As its name suggests, TIC is an individual-level variable whose value does not change over time, such as biological sex, experimental condition, or individual-level assessments only recorded at baseline. We obtain insights on possible reasons that account for between-individual differences in trajectories by examining how the between-individual differences in the growth factors are related to these TICs.

Covariates in a longitudinal study are not necessary TICs. Researchers can also collect covariates of interest repeatedly over the entire duration; that is, covariates can also be TVCs. For example, researchers may also want to collect a TVC such as attentional focus or learning approach that is assumed to affect academic performance over time in a study to evaluate intelligence development. Grimm (2007) proposed using LGCMs with a TVC to analyze two longitudinal variables simultaneously where the primary process is viewed as the longitudinal outcome while the other one is considered as the TVC (also referred to as the secondary process in the original publication) and demonstrated the proposed model by analyzing how the depression process affects the achievement process. In this model, each
measurement of the primary process is regressed on the corresponding measure of the secondary process. Therefore, this model allows for examining two longitudinal processes from at least two perspectives. On the one hand, it enables one to investigate how the secondary process relates to the primary process while controlling for between-individual differences in within-individual changes of the primary process. On the other hand, it allows one to examine how the longitudinal outcome changes over time when controlling for TVC.

The LGCM with a TVC proposed by Grimm (2007) can fit in the mixed-effects modeling framework and the structural equation modeling (SEM) framework, with the models constructed in the SEM framework being able to provide more insights due to the flexibility of the SEM framework. For example, the model in the SEM framework allows for variant effects of a TVC over time. It also enables one to estimate the covariances between the TVC and the growth factors of the longitudinal outcome (Grimm, 2007). However, the model also has limitations. First, the full model carries many parameters since there is no restricted structure on the TVC. Therefore, the mean vector, variance-covariance matrix, and TVC residuals must be estimated. The covariances among the TVC are impossible to estimate under some challenging conditions. One remedy for this is to assume the covariances to be zero, yet this assumption may not be valid if the TVC is expected to be stable to some extent (Grimm et al., 2016, Chapter 8). Second, the model does not allow the TVC to predict variation in the growth factors of the longitudinal outcome. Third, the insight regarding the TVC effects from this existing model is limited to how the absolute values of the TVC affect the absolute values of the longitudinal outcome. However, in practice, researchers may also want to investigate how the change in a TVC affects the absolute value of the longitudinal outcome over time. In the present study, we propose three possible methods to decompose a TVC into a trait feature and a set of state features. Such TVC decompositions allow for addressing the above three challenges.

**Modeling Framework Used for Decomposition of Time-Varying Covariate**

Liu and Perera (2022b) proposed a new specification for the LGSM framework. In addition to allowing researchers to build up a LGSM in the framework of individual measurement occasions, a feature of this new specification is that it allows for the estimation of the baseline value and individual interval-specific slopes. This characteristic provides a natural way to decompose a longitudinal variable into a trait and a collection of state features. In addition, this novel specification can be modified to allow for the derivation of other types of state features, such as individual interval-specific changes and individual change-from-baseline values, based on the estimated individual interval-specific slopes. Employing the LCSM with this novel specification and these possible modifications to describe a TVC decreases the number of parameters by fixing its structure. More importantly, the trait and state features of the TVC are decomposed. The estimated individual baseline value is then viewed as a predictor of growth factors of the longitudinal outcome similar to a TIC. Meanwhile, the estimated individual interval-specific slopes, individual interval-specific changes, or individual change-from-baseline values are regarded as possible predictors of the longitudinal outcome. Although LCSMs with the novel specification enable one to estimate change patterns for nonparametric and parametric nonlinear trajectories, we only introduce the novel specification for nonparametric curves in this section, as the growth coefficients obtained from nonlinear parametric curves are out of interest in this study. The novel specification for nonlinear parametric curves can also be employed to examine the change patterns of a TVC, which will be described in more detail in the Discussion section.

The nonparametric LCSM can also be viewed as the latent basis growth model (LBGM) fit in the LCSM framework (Liu and Perera, 2022b). A LGCM contains two growth factors: an intercept and a shape factor. There are multiple ways to scale the shape factor. Following Liu and Perera (2022b), we view the slope in the first time interval as the shape factor in this study; therefore, the slope in each of the other time intervals can be viewed as the product of the shape factor and the corresponding relative rate. A path diagram of the model with six repeated measures is provided in Figure 1, where we use $x_{j}^{*}$ to represent repeated measures of a longitudinal variable since they are observed measurements of a TVC in this project. As shown in Figure 1, the observed value is the sum of the corresponding latent true score (i.e., $x_{j}^{*}$) and a residual (i.e., $\epsilon_{x_{j}}$) at each measurement time $t_{j}$. At baseline, the true score is $x_{j}^{*}$. At each post-baseline, the true score at time $t_{j}$ is a linear combination of the score at the previous time point $t_{j-1}$ and the amount of true change from time $t_{j-1}$ to $t_{j}$, which is represented by the product of the time interval from $t_{j-1}$ to $t_{j}$ (i.e., $t_{j} - t_{j-1}$) and the slope (i.e., $dx_{j}$) in the interval. In addition, the slope in each interval is the product of the shape factor (i.e., $\eta_{j}^{*}$) and the corresponding relative rate $\gamma_{j-1}$ as demonstrated in the figure. The parameters needed to be estimated in the model specified in Figure 1 include the mean vector and variance-covariance matrix of the two growth factors, relative rates, and residuals over time. Another characteristic of the novel specification is that it allows one to estimate those parameters in the framework of individual measurement occasions by the ‘definition variables’ approach (Mehta and West, 2000; Mehta and Neale, 2005; Sterba, 2014). The ‘definition variables’ are observed variables that adjust model coefficients to individual-specific values. As shown in Figure 1, the ‘definition variables’ in the novel specification are individual time intervals.
A LBGM with the novel specification contains additional latent variables, such as true scores (i.e., $x^*_j$) at each measurement occasion and interval-specific slopes (i.e., $dx_j$), other than the two growth factors. However, these additional latent variables are not freely estimable; instead, they are derived from other parameters. Such non-estimable latent variables are allowed to be added into paths and then serve as predictors in a model of SEM framework. Therefore, the model specified in Figure 1 then allows for separately estimating the initial status and the interval-specific slopes. We then view the initial status as the trait feature while the slopes as the set of state features. We then examine the trait effect by regressing the growth factors of the longitudinal outcome on the TVC estimated baseline value and assess the state effect by regressing each measurement of a longitudinal outcome on the slope of the previous time interval of the TVC.

The model in Figure 1 can be modified by adding additional latent variables to allow for different types of state features. In Figure 2a, we add $\delta x_j$ to represent the amount of change in the time interval from $t_{j-1}$ to $t_j$. Similarly, in Figure 2b, we add $\Delta x_j$ to represent the change-from-baseline at time $t_j$. Similar to the true scores $x^*_j$ and slopes $dx_j$, $\delta x_j$ and $\Delta x_j$ are derived from other parameters instead of being freely estimated. They can also be included in paths and viewed as predictors in a model. Therefore, we have three possible methods to decompose a TVC. In all three methods, the estimated TVC baseline value is the trait feature, while the interval-specific slopes or changes or the change-from-baseline values are three possible types of state features. In this project, we demonstrate the proposed methods in the framework of individual measurement occasions by following multiple existing studies that illustrate this framework for LGCMs (Sterba, 2014; Liu et al., 2021) and LCSMs (Grimm and Jacobucci, 2018; Liu and Perera, 2022b) to avoid potential inadmissible solutions (Blozis and Cho, 2008).

### Method

#### Decomposition of Time-varying Covariate

This section introduces the three possible ways to decompose a TVC into a trait and a set of state features. We start with the novel specification for the latent basis growth model (LBGM) introduced in Liu and Perera (2022b), which allows for a decomposition of a TVC into an initial status (i.e., a trait feature) and interval-specific slopes (i.e., a collection of state features). Similar to multiple earlier studies, such as McArdle (2001) and Grimm et al. (2016, Chapter 11), the LBGM with $J$ measurements is viewed as a linear piecewise function with $J - 1$ segments in Liu and Perera (2022b). For the $i^{th}$ individual, the model can be specified as

$$
\begin{align*}
  x_{ij} &= x^*_{ij} + \epsilon_{ij}^{[x]}, \\
  x^*_{ij} &= \begin{cases} 
  \eta_{0i}^{[x]}, & \text{if } j = 1 \\
  x^*_{i(j-1)} + dx_{ij} \times (t_{ij} - t_{i(j-1)}), & \text{if } j = 2, \ldots, J 
  \end{cases} \\
  dx_{ij} &= \eta_{1i}^{[x]} \times \gamma_{j-1} 
\end{align*}
$$

Equations 1, 2 and 3 together define a LBGM. In Equation 1, $x_{ij}$, $x^*_{ij}$, and $\epsilon_{ij}^{[x]}$ are the observed measurement, latent true score, and residual of individual $i$ at time $t_j$, respectively. Equation 2 demonstrates that the latent true scores have different expressions at baseline (i.e., $j = 1$) and post-baseline (i.e., $j \geq 2$). At baseline, the true score is the growth factor indicating the intercept (i.e., $\eta_{0i}^{[x]}$). At each post-baseline, the true score at time $t_j$ is a linear combination of the score at the prior time point $t_{j-1}$ and the amount of true change from time $t_{j-1}$ to $t_j$ that is the product of the time interval (i.e., $t_j - t_{j-1}$) and the slope (i.e., $dx_{ij}$) in that interval. In Equation 3, the slope in each interval is further expressed by the product of the slope of the first interval (i.e., the shape factor $\eta_{1i}^{[x]}$) and the corresponding relative rate (i.e., $\gamma_{j-1}$).
The model specified in Equations 1-3 can be modified to allow for adding additional latent variables to indicate the interval-specific changes, as shown in Equations 4 and 5

\[
x_{ij}^{*} = \begin{cases} 
\eta_{0i}, & \text{if } j = 1 \\
\eta_{0i} + \delta x_{ij}, & \text{if } j = 2, \ldots, J,
\end{cases}
\]

(4)

\[
\delta x_{ij} = dx_{ij} \times (t_{ij} - t_{i(j-1)}) \quad (j = 2, \ldots, J),
\]

(5)

where \(\delta x_{ij}\) indicates the amount of change from \(t_{j-1}\) to \(t_{j}\) of the \(i^{th}\) individual. Therefore, Equations 1, 4, 5 and 3 together define a LBGM specified in Figure 2a. It is clear that the only difference between this modified specification and the original specification proposed in Liu and Perera (2022b) is that interval-specific changes are explicitly added into the model specified by Equations 1, 4, 5 and 3 so that these interval-specific changes are allowed to be predictors in the SEM framework.

Similarly, we modify the model specified in Equations 1-3 to include additional latent variables to indicate the change-from-baseline values shown in Equations 6 and 7

\[
x_{ij}^{*} = \begin{cases} 
\eta_{0i}, & \text{if } j = 1 \\
\eta_{0i} + \Delta x_{ij}, & \text{if } j = 2, \ldots, J.
\end{cases}
\]

(6)

\[
\Delta x_{ij} = \Delta x_{i(j-1)} + dx_{ij} \times (t_{ij} - t_{i(j-1)}) \quad (j = 2, \ldots, J),
\]

(7)

in which \(\Delta x_{ij}\) indicates the change-from-baseline at time \(t_{j}\) for individual \(i\), which is a linear combination of the change-from-baseline at the earlier time point \(t_{j-1}\) and the amount of change from \(t_{j-1}\) to \(t_{j}\). Equations 1, 6, 7 and 3 together define a LBGM specified in Figure 2b. The true score at each time point \(t_{j}\) can be expressed as the sum of the baseline value and the corresponding change-from-baseline. With explicit inclusion in the model specification, these change-from-baseline values are allowed to be predictors.

The novel specification proposed in Liu and Perera (2022b) and the two possible modifications can be expressed as the same matrix form, which is only related to the freely estimable parameters in the model,

\[
x_i = \Lambda_i^{[x]} \times \eta_i^{[x]} + \epsilon_i^{[x]}
\]

in which \(x_i\) is a \(J \times 1\) vector of the repeated measurements of the TVC of individual \(i\) (where \(J\) is the number of measurement occasions), \(\eta_i^{[x]}\) is a \(2 \times 1\) vector of the growth factors of the TVC, representing the initial status and the slope of the first time interval of individual \(i\), respectively. Moreover, \(\Lambda_i^{[x]}\) is a \(J \times 2\) matrix of the corresponding factor loadings,

\[
\Lambda_i^{[x]} = \begin{pmatrix}
1 & 0 \\
1 & \sum_{j=2}^{J} \gamma_{j-1} \times (t_{ij} - t_{i(j-1)})
\end{pmatrix},
\]

of which the elements of the first column are 1 since they are the factor loadings of the TVC intercept. The \(j^{th}\) element of the second column is the cumulative value of the relative rate over time to \(t_j\), so its product with \(\eta_{i{[x]}}\) represents the value of change-from-baseline at time \(t_j\). In addition, \(\epsilon_i^{[x]}\) is a \(J \times 1\) vector of residuals of individual \(i\). The growth factors \(\eta_i^{[x]}\) can be further written as

\[
\eta_i^{[x]} = \mu_i^{[x]} + \zeta_i^{[x]},
\]

where \(\mu^{[x]}\) is the mean vector of the TVC growth factors, and \(\zeta_i^{[x]}\) is the vector of deviations of the \(i^{th}\) individual from the corresponding growth factor means. More technical details can be found in Liu and Perera (2022a).

Model Specification of Latent Growth Curve Model with Decomposed Time-varying Covariate

This section presents the model specification of the proposed LGCMs with a decomposed TVC, in which the growth factors of the longitudinal outcome are regressed on the trait feature, while each post-baseline outcome measure is regressed on the corresponding value of each of the three types of state features. In this section, we do not pre-specify any functional form for the longitudinal outcome; instead, we provide a general model specification that can be adapted to any growth curve function.
For the \(i^{th}\) individual, the proposed LGCMs with a TIC and a TVC that is decomposed into its baseline value and interval-specific slopes can be expressed as

\[
\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{pmatrix} \Lambda_{i}^{[x]} & 0 \\ 0 & \Lambda_{i}^{[y]} \end{pmatrix} \times \begin{pmatrix} \eta_{i}^{[x]} \\ \eta_{i}^{[y]} \end{pmatrix} + \kappa_1 \times \begin{pmatrix} 0 \\ dx_i \end{pmatrix} + \begin{pmatrix} e_{i}^{[x]} \\ e_{i}^{[y]} \end{pmatrix},
\]

(8)

where \(y_i\) is a \(J \times 1\) vector of the repeated measures of the \(i^{th}\) individual (in which \(J\) is the number of measurement occasions), \(\eta_{i}^{[y]}\) is a \(K \times 1\) vector of growth factors (where \(K\) is the number of growth factors of the longitudinal outcome), \(\Lambda_{i}^{[y]}\) is a \(J \times K\) matrix of the corresponding factor loadings (in which the subscript \(i\) indicates that the model is constructed with individual measurement occasions), and \(e_{i}^{[y]}\) is a \(J \times 1\) vector of residuals of individual \(i\). In addition, \(dx_i\) is a \(J \times 1\) vector of interval-specific slopes of the TVC, which can be further expressed as \(dx_i = (0 \ \delta x_{i2} \ \delta x_{i3} \ldots \ \delta x_{ij})\), of which the first element is \(0\) and \(\delta x_{ij}\) is the slope in the \((j-1)^{th}\) time interval of the \(i^{th}\) individual. Therefore, in Equation 8, \(\kappa_1\) is the state effect of the TVC, which indicates how the value of \(y_i\) at \(t_j\) is affected by the slope in the previous time interval (i.e., from \(t_{j-1}\) to \(t_j\)). In the above equation, \(0\) is a \(J \times 1\) vector, and other notations have the same definition as such in previous equations.

We then further regress the growth factors on the TIC and the latent true score of the initial status (i.e., the growth factor that indicates the intercept or the true baseline value) of the TVC

\[
\eta_{i}^{[y]} = \alpha_{[y]} + (\beta_{TIC} \ \beta_{TVC}) \times \begin{pmatrix} X_i \\ \eta_{0i} \end{pmatrix} + \xi_{i}^{[y]},
\]

(9)

in which \(\alpha_{[y]}\) is a \(K \times 1\) vector of growth factor intercepts, \(\beta_{TIC}\) is a \(K \times 1\) vector of regression coefficients from the TIC to the growth factors, and \(\beta_{TVC}\) is a \(K \times 1\) vector of regression coefficients from the latent true score of the initial status of the TVC to the growth factors. In addition, \(X_i\) is the TIC value and \(\eta_{0i}\) is the TVC initial status of the \(i^{th}\) individual, and \(\xi_{i}^{[y]}\) is a \(K \times 1\) vector of deviations of the individual \(i\) from the conditional means of growth factors.

Similarly, the proposed LGCMs with a TIC and a TVC with the other two types of decomposition for individual \(i\) can be expressed in Equations

\[
\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{pmatrix} \Lambda_{i}^{[x]} & 0 \\ 0 & \Lambda_{i}^{[y]} \end{pmatrix} \times \begin{pmatrix} \eta_{i}^{[x]} \\ \eta_{i}^{[y]} \end{pmatrix} + \kappa_2 \times \begin{pmatrix} 0 \\ \delta x_i \end{pmatrix} + \begin{pmatrix} e_{i}^{[x]} \\ e_{i}^{[y]} \end{pmatrix},
\]

(10)

and

\[
\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{pmatrix} \Lambda_{i}^{[x]} & 0 \\ 0 & \Lambda_{i}^{[y]} \end{pmatrix} \times \begin{pmatrix} \eta_{i}^{[x]} \\ \eta_{i}^{[y]} \end{pmatrix} + \kappa_3 \times \begin{pmatrix} 0 \\ \Delta x_i \end{pmatrix} + \begin{pmatrix} e_{i}^{[x]} \\ e_{i}^{[y]} \end{pmatrix},
\]

(11)

respectively, where \(\delta x_i\) is a \(J \times 1\) vector of interval-specific changes of TVC, which can be further expressed as \(\delta x_i = (0 \ \delta x_{i2} \ \delta x_{i3} \ldots \ \delta x_{ij})\), while \(\Delta x_i\) is a \(J \times 1\) vector of change-from-baseline values of TVC, which can be further expressed as \(\Delta x_i = (0 \ \Delta x_{i2} \ \Delta x_{i3} \ldots \ \Delta x_{ij})\). Similar to \(dx_i\), the first element of the two vectors is \(0\), and \(\delta x_{ij}\) and \(\Delta x_{ij}\) are the amount of change in the \((j-1)^{th}\) time interval and the change-from-baseline at \(j^{th}\) time point of individual \(i\), respectively. Therefore, both \(\kappa_2\) and \(\kappa_3\) can be interpreted as a state effect of the TVC, which demonstrates how the value of \(y_i\) at \(t_j\) is affected by the amount of change in the previous time interval and change-from-baseline at \(t_j\), respectively. We can further regress \(\eta_{i}^{[y]}\) in Equations 10 and 11 on the TIC and the latent true score of the TVC baseline value as we did for \(\eta_{i}^{[y]}\) in Equation 8. The regressions of \(\eta_{i}^{[y]}\) have the same expression as Equation 9.

In practice, the longitudinal outcome \(y_i\) could take either a linear or nonlinear function, depending on the trajectory shape demonstrated by raw data and the research questions of interest. In Table 1, we list functional forms, growth factors and corresponding interpretations, and factor loadings for the linear growth curve and four commonly used nonlinear trajectories.

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Insert Table 1 about here
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**Model Estimation**

This section describes how to estimate LGCMs with a decomposed TVC. In order to simplify the estimation, we make the following assumptions. First, the TIC and the growth factors of TVC are normally distributed; that is,
When designing the simulation, we decide the number of repetitions $S = 1,000$ with an empirical method suggested by Morris et al. (2019). Specifically, we performed a pilot study and noticed that the standard errors of all coefficients were less than 0.15 except for the initial status of the TVC and the longitudinal outcome, which requires at least 900
repetitions to keep the Monte Carlo standard error of the bias less than 0.15². We then proceeded with the simulation with 1,000 replications for more conservative consideration.

**Design of Simulation Study**

All the conditions we considered for all three decomposition methods in the simulation design are provided in Table 3. In general, we manipulated the conditions supposed to affect TVC effects but fixed the rest to limit the size of the simulation conditions in this project. The number of measurement occasions is important when examining longitudinal processes since a longitudinal model presumably performs better under conditions with more repeated records. We are interested in investigating whether the number of repeated measures affects the TVC effects via this simulation study. To this end, two levels of repeated measurements, six and ten waves, were selected in the design. The condition with six measures was selected for model identification purposes (Bollen and Curran, 2005; Liu et al., 2021), while the other condition with ten measurements was picked as we wanted to see whether a longer study duration would improve model performance. In addition, we set individual measurement occasions by allowing a ‘medium’ time window (−0.25, +0.25) around each wave following Coulombe et al. (2015).

In the simulation design, we considered standardized TIC and fixed the distribution of the growth factors of the longitudinal outcome and the TVC since the examination of how the trajectory shapes affect model performance is out of the interest of this project. We considered a midway knot for the trajectories of the longitudinal outcome. The correlation between the TIC and trait feature of the TVC was set as 0.3. Three levels of trait effects were included in the simulation design to allow the TIC and the trait feature of the TVC to account for 13% or 26% variability of the growth factors of the longitudinal outcome. Specifically, all coefficients from the TVC trait feature were set as 0, and the TIC was set to explain 13% variability of the growth factors of the longitudinal outcome in the first scenario. In the second and third scenarios, they together accounted for 13% or 26% variability, respectively. Moreover, four levels of state effects were considered for each of the three methods. With these conditions of the regression coefficients, we aimed to check if the proposed methods could detect trait and state effects. In addition, we set the residual covariance between the longitudinal outcome and the TVC to a moderate level (i.e., the correlation was set as 0.3) and two levels of sample size (n = 200 or 500).

**Data Generation and Simulation Step**

The simulation study for each condition of each method listed in Table 3 was carried out in the following steps:

1. Generate the TIC, the growth factors of the TVC, and the growth factors of the longitudinal outcome for a LGCM with a decomposed TVC using the R package MASS (Venables and Ripley, 2002),
2. Generate a time structure with J waves \( t_j \) (\( J = 6 \) or 10) and allow for disturbances around each wave \( t_{ij} \sim U(t_j - \Delta, t_j + \Delta) \) (\( \Delta = 0.25 \)) to have individual measurement occasions,
3. Calculate factor loadings for the longitudinal outcome and the TVC, which are functions of the individual measurement occasions and additional growth coefficient(s) (i.e., the knot for the longitudinal outcome and the relative rates of the TVC),
4. Calculate a set of state features of the TVC, which is the interval-specific slopes (from the slope in the first time interval and relative rates), the interval-specific changes (from the interval-specific slopes and individual time intervals), or the change-from-baseline (from the accumulative interval-specific changes) for the three methods, respectively,
5. Calculate the true values of the repeated measures for the TVC and the longitudinal outcome: the former is based on its growth factors and factor loadings, while the latter is based on its growth factors, factor loadings, and a set of state features of the TVC, then add the residual matrix on the longitudinal outcome and the TVC,
6. Implement the LGCM with each decomposition method, estimate the parameters, and construct the corresponding 95% Wald confidence intervals,
7. Repeat the above steps until achieving 1,000 convergent solutions.

²Bias is the most important performance metric in a simulation study, and the equation of its Monte Carlo standard error is 

\[ \text{Monte Carlo SE(Bias)} = \sqrt{\text{Var}(\hat{\theta})} / S \] (Morris et al., 2019).
Results

We summarize simulation results in this section. We first examined the convergence\(^3\) rate of each LGCM with a decomposed TVC. All three proposed methods with a linear-linear piecewise longitudinal outcome converged well since they reported a 100% convergence rate for all conditions listed in Table 3.

We then examined the performance metrics of each parameter for three decomposition methods under all conditions, including relative bias, empirical SE, relative RMSE, and empirical coverage of the nominal 95% confidence interval. Given the size of the simulation conditions and the number of parameters, we first examine the summary statistics of each of the four performance metrics for each parameter of each decomposition method. Specifically, we calculated each performance metric across 1000 replications under each condition and summarized the results across all conditions into the corresponding median and range. Based on our simulation results, the first two decomposition methods performed well and could generate unbiased and accurate point estimates with target probabilities. The summary statistics of the four performance measures of these two methods are provided in Tables S1 and S2 in the online supplementary document, respectively.

However, the estimates of mean values of the slopes of the longitudinal outcome, the trait effects on these two slopes, and the state effect of the third decomposition method demonstrated some bias (i.e., the relative bias could achieve 10%, see Table S3), leading to inferior performance in relative RMSE and coverage probability. We then plotted the relative biases of these parameters stratified by the number of repeated measures, the sample size, the size of trait and state effects, and the residual variance of the longitudinal outcome in Figures S1-S5. From those figures, we noticed that such biased estimates are produced under conditions with the shorter study duration (i.e., \(J = 6\)). In addition, the estimate of the state effect was more biased under the conditions with the larger trait effect or smaller state effect; the latter might be attributed to the small population value (i.e., \(\kappa_3 = 0.2\)).

One possible explanation for the poor performance of the third decomposition method is its large size state feature (i.e., the change-from-baseline) compared to the state features in the other two methods (i.e., the interval-specific slopes or changes). As stated earlier, we regress the longitudinal outcome on such state characteristics; therefore, this large state feature could squeeze the size of the state effect while inflating the estimates of coefficients related to growth factors of the longitudinal outcome, especially under conditions with a shorter study duration. This may also explain the more biased estimate of the mean value of the second slope of the longitudinal outcome since the state feature is even larger during the later stage.

Therefore, based on our simulation study, the first two decomposition methods performed satisfactorily under all examined conditions, while the third model only worked well under some mild conditions, suggesting that the third method requires careful data preprocessing and model interpretation in practice, which we will demonstrate in the Application section.

Application

This section demonstrates how to employ the proposed TVC decomposition methods to analyze bivariate longitudinal variables, which are viewed as a TVC and a longitudinal outcome, respectively, with baseline characteristics (i.e., TICs). The application section has two goals. The first goal is to provide feasible recommendations for employing the proposed TVC decomposition methods to answer specific research questions. The second goal is to show how the inclusion of a TVC when modeling a LGCM affects the estimation of growth factors in real-world practice. To realize this aim, we built up LGCMs with the three proposed methods and the existing LGCM with a TVC. We randomly selected 400 students from the Early Childhood Longitudinal Study, Kindergarten Cohort: 2010-2011 (ECLS-K: 2011) with non-missing records of repeated reading and mathematics assessment with baseline teacher-reported inhibitory control for this application\(^4\).

ECLS-K: 2011 is a longitudinal study that starts from the 2010-2011 school year and collects records from children enrolled in approximately 900 kindergarten programs across the United States. In the survey, students’ reading and mathematics abilities were assessed in nine waves: each semester in kindergarten, first and second grade, followed by only the spring semester in third, fourth, and fifth grade. As pointed out by Lê et al. (2011), only about one-third of students were evaluated in the 2011 and 2012 fall semesters. In this analysis, we used the age-in-month for each wave so that each student had individual measurement occasions.

---

\(^3\)Convergence of this presented project is defined as arriving at the OpenMx status code 0 that suggests a successful optimization.

\(^4\)ECLS-K: 2011 contains \(n = 18174\) participants. There are \(n = 3144\) students after removing rows with missing values (i.e., records with any of \(\text{Na}_N/ - 9/ - 8/ - 7/ - 1\)).
We fit the LGCMs with a decomposed TVC and the existing LGCM with a TVC to analyze how students’ baseline teacher-reported inhibitory control and reading ability affect their development of mathematics ability with the assumption that the mathematics trajectories take the bilinear spline function with an unknown fixed knot. We standardized the TIC, baseline teacher-reported inhibitory control. For the TVC, reading achievement scores over time, we first calculated the mean and variance of the baseline reading ability and then standardized the ability at each wave using the baseline mean and variance. All four models converged, and the estimated likelihood, Akaike information criterion (AIC), Bayesian information criterion (BIC), residual variance, and the number of parameters of each model are provided in Table 4. In addition to these four models with a TVC, we constructed two reference models, the LGCM and LGCM with a TIC only, and the corresponding model summary is also included in Table 4. From the table, we noticed that adding a TVC into a LGCM increased the estimated likelihood, and then AIC and BIC, but did not affect the residual variance of mathematics growth curves meaningfully. Among four models with a TVC, the LGCM with decomposed TVC into a trait feature and change-from-baseline values had the smallest estimated likelihood, AIC, and BIC, suggesting that this model fit the raw data best from the statistical perspective.

To examine how the inclusion of a TVC affects the estimation of the growth factors of mathematics achievement, we plotted a model-implied curve on the smooth line of the development of mathematics ability for each of the six models in Figure 3. The figure shows that the estimated trajectory from the models without a TVC fits the smooth line. The development of mathematics ability can be viewed in two stages: the growth in the early stage is relatively rapid and then slowed down substantially since around Month 100. The growth factors of mathematics development from the models with a TVC were somewhat underestimated. It is within our expectations to see since allowing a TVC to account for the variability of the longitudinal outcome would squeeze the estimation of the growth factors. We also notice that such effects on the growth factors were more evident in the later stage. It is not surprising. The increased reading ability over time led to an increasing influence on the estimation. However, such squeezing effects were not meaningful in the LGCM with a TVC that is decomposed into the baseline value and interval-specific slopes or changes. We then list the estimates of these two models in Tables 5 and 6, respectively.

Table 5 shows that, for the development of mathematics ability, the estimated knot was around 9-year old and that the pre-and post-knot growth rates were 1.62 and 0.67, respectively. As stated earlier, we standardized reading scores at each wave with the baseline mean and variance. The estimated mean of the baseline score was 0.06, which is within our expectation since the score at baseline has been centered to 0 when performing the standardization. The estimated mean of the slope during the first interval was 0.17, indicating that the standardized reading ability increased 0.17 per month during the first time interval, based on which we can also calculate the unstandardized growth rate if it is of research interest. The other interval-specific slopes can be calculated from the slope in the first interval and the corresponding relative rates. For example, the slope during the second time interval was 0.14 (i.e., 0.17 × 0.81). More detailed interpretations of the growth coefficients for the TVC and the longitudinal outcome can be found in earlier research works, such as Liu et al. (2021) and Liu and Perera (2022b).

After decomposing the reading ability development, we can also estimate the covariance between the TIC and the TVC trait feature, which was 0.25 in this application. This suggests that teacher-reported inhibitory control and baseline reading ability were positively associated. In addition, one standardized unit increase in the baseline reading score resulted in 0.07 and 0.09 increase in the initial status and the first slope of the mathematics ability development. The estimated state effect was 27.37, indicating that, for example, one unit increase in the slope of standardized reading ability development in the first grade spring semester led to 27.37 increase in mathematics final scores of that semester.

The estimates from the model with the decomposed TVC into the trait feature and interval-specific changes are very similar to those shown in Table 6, except for the state effect. The state effect of this model was 4.37, suggesting that, for
example, one unit increase in the change of standardized reading ability development in the first grade spring semester resulted in 4.37 increase in mathematics final scores of that semester.

**Discussion**

This article proposes three methods to decompose a TVC to evaluate the trait and state effects separately. Specifically, we view the baseline value as the trait feature and either the interval-specific slopes or changes or change-from-baseline values as a set of the state features. We evaluated the three decomposition methods through extensive simulation studies by fitting LGCMs with a decomposed TVC, assuming that the longitudinal outcome takes the bilinear spline function with an unknown knot. Based on our simulation studies, the LGCMs with the first two decomposition methods are capable of estimating the parameters of interest unbiasedly and accurately with generating target coverage probabilities, though the estimates from the model with the third method might be biased under some challenging conditions, such as the conditions with a shorter duration. The patterns we observed in the simulation study were also supported by real-world data analysis: the underestimation of the growth factors of the longitudinal outcome with the first two decomposition methods was negligible, while the discrepancy of the third method is relatively meaningful.

**Practical Considerations**

We provide a set of recommendations for empirical researchers based on the simulation study and real-world data analysis in this section. First, we proposed three TVC decomposition methods in this article; therefore, along with the existing method, four approaches enable one to add a TVC when fitting a LGCM. It is not our aim to demonstrate that a decomposition method is better than the existing method or that one decomposition method is universally preferred. The selection of methods should be based on research objectives and specific research questions. For example, if the research interest is to estimate the trait and state effects separately, the proposed decomposed methods are great candidate models. The decomposition methods that produce the trait effect and interval-specific intervals or changes as the state effect are good options if one aims to obtain unbiased estimates and growth factors that reflect developmental processes. Additionally, the estimated effect size of the state effect and the corresponding interpretation are different across methods, as demonstrated in the Application section. This also needs to be considered when selecting a method. For example, if the research question is to understand how the growth rate of reading ability affects mathematics achievement, the decomposition method with the interval-specific slopes is a great candidate model. Similarly, the method with the interval-specific changes helps examine how the amount of change in reading ability influences mathematics ability.

Second, based on the results of real-world data analysis, a good index value (e.g., AIC or BIC) of a model does not guarantee that the estimates are unbiased or that the estimated parameters from the model have meaningful interpretations to reflect true developmental processes. For example, the LGCM with the third decomposition method has the smallest AIC and BIC yet underestimates slopes, especially the post-knot slope of the mathematics development. Therefore, we recommend fitting LGCMs without a TVC as reference models, as we did in the Application section. As demonstrated by the simulation study in earlier studies such as Liu et al. (2021) and the example in the Application section, the LGCMs with no TVC are capable of capturing the underlying change patterns of the longitudinal outcome. Such models help us understand the discrepancy, if any, between the LGCM with a TVC and the change patterns demonstrated by the raw data.

Third, we often standardize a covariate when including it in a model so that the estimated effects are comparable across covariates. However, the standardization of a TVC should be careful since it is important to keep the change patterns and the variance-covariance structure of the TVC. One way is to standardize the TVC at each wave using the same mean and variance. For example, we scale the TVC across time with the mean and variance of its baseline values. After fitting the model, this standardization strategy also enables one to transform the estimated results into the effects in the original scale for interpretation purposes.

Last, as shown in the simulation study, the estimates of coefficients related to the slopes and the state effect from the LGCM with a TVC decomposing into a trait feature and change-from-baseline values demonstrated some bias, especially under challenging conditions. The interpretation of the mean values of slopes, trait effects on the slopes, and the state effect need to be cautious as the coefficients related to the slopes could be overestimated while the state effect could be underestimated based on the simulation study.

**Methodological Considerations and Future Directions**

There are several directions to consider for future studies. First, the inclusion of a TVC when fitting a LGCM divides the variability of the longitudinal outcome into three parts, the variability explained by its growth factors and by the
TVC, as well as the unexplained part. Therefore, it is not unexpected that including a TVC in the model could squeeze the estimates of the growth factors and that such influences could change with the size of state features. This helps explain the inferior performance of the third decomposition method where change-from-baseline values, which are larger than the interval-specific slopes or changes and accumulate over time, serve as the state features. In the proposed methods, we assume that the state effect is homogeneous over time to simplify model specification and allow for using these methods in the mixed-effects modeling framework. Relaxing this assumption may improve the performance of the third method since, intuitively, the squeezing impacts on the growth factors of the state features may be alleviated if the state effect is allowed to be adjusted with the magnitude of the corresponding state feature. The examination of the methods with a relaxed assumption is out of the scope of the present project, but it could be a future direction.

Second, the present study assumes the TVC takes a nonparametric functional form, of which only the interval-specific slopes are of interest. In addition to the new specification for the nonparametric function, Liu and Perera (2022b) included this novel specification for multiple parametric nonlinear trajectories such as quadratic and negative exponential functions. Other than the interval-specific slopes, these parametric functions also allow for the examination of growth coefficients, such as growth acceleration or growth capacity. Therefore, these nonlinear parametric functional forms can also be used to describe the change patterns of a TVC if the evaluation of these coefficients of the TVC is also of research interest. The proposed methods can also be extended accordingly.

Third, we performed the simulation study for the proposed methods with the assumption that the longitudinal outcome takes a linear-linear functional form since we aim to demonstrate how the state features affect the estimation of the growth factors of the longitudinal outcome in the earlier and later stage separately. Though a LGCM with a decomposed TVC where the longitudinal outcome takes other functions is likely to demonstrate very similar patterns that we observed from the project, one can also perform additional simulations to answer specific research questions, such as how the state features affect the estimation of growth acceleration.

Fourth, we illustrated the proposed methods with the same time structure for the TVC and the longitudinal outcome. However, it is possible to extend the methods for a bivariate longitudinal process in which one variable has fewer measurements. Last, it is also possible to extend the proposed methods to analyze a bivariate longitudinal process with dropout under the assumption of missing at random thanks to the FIML technique used in model estimation.

Concluding Remarks

To summarize, this article proposes three methods to decompose a TVC into a trait feature and a set of state features. Specifically, we view the baseline value as the trait feature and the interval-specific slopes or changes, or change-from-baseline values as a possible set of the state features. With the proposed methods, we are able to evaluate the trait effect and state effect of a TVC separately. As discussed above, the proposed methods can be further extended in practice and further examined in methodology.

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Appendix A  Mean vector and Variance-covariance Matrix of the Longitudinal Variables

For the LGCM with a TVC that is decomposed to the baseline value and interval-specific slopes, the expected mean vector and variance-covariance structure of the TVC (\(x_i\)) and the longitudinal outcome (\(y_i\)) for the \(i^{th}\) individual can be expressed as

\[
\mu_i = \begin{pmatrix} \mu_i^{[x]} \\ \mu_i^{[y]} \end{pmatrix} = \begin{pmatrix} \Lambda_i^{[x]} \\ 0 \end{pmatrix} \times \begin{pmatrix} \mu_{\eta_i}^{[x]} \\ \mu_{\eta_i}^{[y]} \end{pmatrix} + \kappa_1 \times \begin{pmatrix} 0 \\ \mu_{dx} \end{pmatrix},
\]

and

\[
\Sigma_i = \begin{pmatrix} \Sigma_i^{[x]} & \Sigma_i^{[xy]} \\ \Sigma_i^{[xy]} & \Sigma_i^{[y]} \end{pmatrix} = \begin{pmatrix} \Lambda_i^{[x]} \\ 0 \end{pmatrix} \times \begin{pmatrix} \Phi_i^{[x]} \\ 0 \end{pmatrix} \times \begin{pmatrix} \Lambda_i^{[x]} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \kappa_1 \phi_{dx} \end{pmatrix} + \begin{pmatrix} \theta_i^{[x]} I \\ \theta_i^{[xy]} I \\ \theta_i^{[y]} I \end{pmatrix},
\]

respectively, where \(\mu_{dx}\) and \(\phi_{dx}\) are the means and variances of the interval-specific slopes, which are not freely estimable parameters in the proposed model. In practice, such parameters can be created by the function \(mxAlgebra()\) in \(OpenMx\) and the estimates are stored in the corresponding objective while the standard errors are usually evaluated by the function \(mxSE()\). In addition, \(\mu_{\eta_i}^{[y]}\) and \(\text{Var}(y)\) are the conditional mean vector and variance-covariance matrix of the growth factors of the longitudinal outcome on the TIC and the true score of the TVC initial status, which can be further expressed as

\[
\mu_{\eta_i}^{[y]} = \alpha^{[y]} + (\beta_{\text{TIC}} \quad \beta_{\text{TVC}}) \times \left( \begin{pmatrix} \mu_x \\ \mu_{\eta_i}^{[x]} \end{pmatrix} \right),
\]

and

\[
\text{Var}(y) = \Psi^{[y]} + (\beta_{\text{TIC}} \quad \beta_{\text{TVC}}) \times \left( \begin{pmatrix} \phi_x \\ \phi_{\eta_i}^{[x]} \end{pmatrix} \sqrt{\rho_{\text{BL}}} \begin{pmatrix} \phi_x \\ \phi_{\eta_i}^{[x]} \end{pmatrix} \right) \times (\beta_{\text{TIC}} \quad \beta_{\text{TVC}})^T,
\]

respectively, where \(\mu_{\eta_i}^{[x]}\) and \(\phi_{\eta_i}^{[x]}\) are the mean and variance of the TVC initial status, and \(\rho_{\text{BL}}\) is the correlation between the TIC and the true value of the TVC initial status.
For the LGCM with a TVC that is decomposed into the baseline value and interval-specific changes, the expected mean vector and variance-covariance structure of the TVC ($x_i$) and the longitudinal outcome ($y_i$) for the $i^{th}$ individual can be expressed as

$$
\mu_i = \left( \begin{array}{c} \mu_i^{[x]} \\ \mu_i^{[y]} \end{array} \right) = \left( \begin{array}{cc} \Lambda_i^{[x]} & 0 \\ 0 & \Lambda_i^{[y]} \end{array} \right) \times \left( \begin{array}{c} \mu_i^{[x]} \\ \mu_i^{[y]} \end{array} \right) + \kappa_2 \times \left( \begin{array}{c} 0 \\ \mu_i^{\delta x} \end{array} \right),
$$

and

$$
\Sigma_i = \left( \begin{array}{cc} \Sigma_i^{[x]} & \Sigma_i^{[xy]} \\ \Sigma_i^{[xy]} & \Sigma_i^{[y]} \end{array} \right) = \left( \begin{array}{cc} \Lambda_i^{[x]} & 0 \\ 0 & \Lambda_i^{[y]} \end{array} \right) \times \left( \begin{array}{cc} \Phi_i^{[x]} & 0 \\ 0 & \text{Var}(y) \end{array} \right) \times \left( \begin{array}{cc} \Lambda_i^{[x]} & 0 \\ 0 & \Lambda_i^{[y]} \end{array} \right)^{T} + \left( \begin{array}{cc} 0 & 0 \\ 0 & \kappa_2^2 \phi_i^{\delta x} \end{array} \right) + \left( \begin{array}{cc} \theta_i^{[x]} & \theta_i^{[xy]} \\ \theta_i^{[xy]} & \theta_i^{[y]} \end{array} \right),
$$

respectively, where $\mu_i^{\delta x}$ and $\phi_i^{\delta x}$ are the means and variances of the interval-specific changes, which are not freely estimable parameters and can be derived using the function $mxAlgebra()$. $\mu_i^{[y]}$ and Var($y$) have the same definitions as the previous equations.

For the LGCM with a TVC that is decomposed into the baseline value and change-from-baseline values, the expected mean vector and variance-covariance structure of the TVC ($x_i$) and the longitudinal outcome ($y_i$) for the $i^{th}$ individual can be expressed as

$$
\mu_i = \left( \begin{array}{c} \mu_i^{[x]} \\ \mu_i^{[y]} \end{array} \right) = \left( \begin{array}{cc} \Lambda_i^{[x]} & 0 \\ 0 & \Lambda_i^{[y]} \end{array} \right) \times \left( \begin{array}{c} \mu_i^{[x]} \\ \mu_i^{[y]} \end{array} \right) + \kappa_3 \times \left( \begin{array}{c} 0 \\ \mu_i^{\Delta x} \end{array} \right),
$$

and

$$
\Sigma_i = \left( \begin{array}{cc} \Sigma_i^{[x]} & \Sigma_i^{[xy]} \\ \Sigma_i^{[xy]} & \Sigma_i^{[y]} \end{array} \right) = \left( \begin{array}{cc} \Lambda_i^{[x]} & 0 \\ 0 & \Lambda_i^{[y]} \end{array} \right) \times \left( \begin{array}{cc} \Phi_i^{[x]} & 0 \\ 0 & \text{Var}(y) \end{array} \right) \times \left( \begin{array}{cc} \Lambda_i^{[x]} & 0 \\ 0 & \Lambda_i^{[y]} \end{array} \right)^{T} + \left( \begin{array}{cc} 0 & 0 \\ 0 & \kappa_2^2 \phi_i^{\Delta x} \end{array} \right) + \left( \begin{array}{cc} \theta_i^{[x]} & \theta_i^{[xy]} \\ \theta_i^{[xy]} & \theta_i^{[y]} \end{array} \right),
$$

respectively, where $\mu_i^{\Delta x}$ and $\phi_i^{\Delta x}$ are the means and variances of the interval-specific changes, which are not freely estimable parameters and can be derived using the function $mxAlgebra()$. $\mu_i^{[y]}$ and Var($y$) have the same definitions as the previous equations.
Figure 1: Path Diagram of Latent Basis Growth Model with Novel Specification
Note: boxes=manifested variables, circles=latent variables, single arrow=regression paths; doubled arrow=(co)variances; triangle=constant; diamonds=definition variables.
In the model, $\gamma_1$ is set as 1 for model identification considerations.
Figure 2: Modified Path Diagram of Latent Basis Growth Model with Novel Specification
Note: boxes=manifested variables, circles=latent variables, single arrow=regression paths; doubled arrow=(co)variances; triangle=constant; diamonds=definition variables.
In the model, $\gamma_1$ is set as 1 for model identification considerations.

Figure 3: Model Implied Trajectory and Smooth Line of Mathematics Performance
Note: BLSGM stands for bilinear spline growth model with an unknown fixed knot.
Table 1: Summary of Linear and Commonly Used Nonlinear Functional Forms

| Growth Factors | Factor Loadings | Interpretation of Growth Factors and Additional Coef. |
|----------------|-----------------|-------------------------------------------------------|
| Linear Function: \( y_{ij} = \eta_{0i}^{[y]} + \eta_{1i}^{[y]} \times t_{ij} + \epsilon_{ij}^{[y]} \) | \( A_i^{[y]} = \begin{pmatrix} 1 & t_{ij} \end{pmatrix} \) | \( \eta_0^{[y]} \): the initial status \( \eta_1^{[y]} \): the linear component of change |
| Quadratic Function: \( y_{ij} = \eta_{0i}^{[y]} + \eta_{1i}^{[y]} \times t_{ij} + \eta_{2i}^{[y]} \times t_{ij}^2 + \epsilon_{ij}^{[y]} \) | \( A_i^{[y]} = \begin{pmatrix} 1 & t_{ij} & t_{ij}^2 \end{pmatrix} \) | \( \eta_0^{[y]} \): the initial status \( \eta_1^{[y]} \): the linear component of change \( \eta_2^{[y]} \): the quadratic component of change |
| Negative Exponential Function: \( y_{ij} = \eta_{0i}^{[y]} + \eta_{1i}^{[y]} \times (1 - \exp(-b \times t_{ij})) + \epsilon_{ij}^{[y]} \) | \( A_i^{[y]} = \begin{pmatrix} 1 & \exp(-b \times t_{ij}) \end{pmatrix} \) | \( \eta_0^{[y]} \): the initial status \( \eta_1^{[y]} \): the change from initial status to asymptotic level \( b \): the log-ratio of rate-of-change at \( t_{ij} \) to that at \( t_{ij(j-1)} \) |
| Jenss-Bayley Function: \( y_{ij} = \eta_{0i}^{[y]} + \eta_{1i}^{[y]} \times (\exp(c \times t_{ij}) - 1) + \epsilon_{ij}^{[y]} \) | \( A_i^{[y]} = \begin{pmatrix} 1 & \exp(c \times t_{ij}) \end{pmatrix} \) | \( \eta_0^{[y]} \): the initial status \( \eta_1^{[y]} \): the slope of the linear asymptote \( c \): the log-ratio of acceleration at \( t_{ij} \) to that at \( t_{ij(j-1)} \) |
| Bilinear Spline Function with a Fixed Knot \( c \) : \( y_{ij} = \begin{cases} \eta_{0i}^{[y]} + \eta_{1i}^{[y]} \times t_{ij} + \epsilon_{ij}^{[y]}, & t_{ij} < \gamma \\ \eta_{0i}^{[y]} + \eta_{1i}^{[y]} \times \gamma + \eta_{2i}^{[y]} \times (t_{ij} - \gamma) + \epsilon_{ij}^{[y]}, & t_{ij} \geq \gamma \end{cases} \) | \( A_i^{[y]}' = \begin{pmatrix} 1 & t_{ij} - \gamma & |t_{ij} - \gamma| \end{pmatrix} \) | \( \eta_0^{[y]} \): the initial status \( \eta_1^{[y]} \): the slope of the first linear piece \( \eta_2^{[y]} \): the slope of the second linear piece \( \gamma \): the transition time from 1\textsuperscript{st} linear piece to 2\textsuperscript{nd} linear piece (i.e., knot) |

\( a \) Growth factors \( \eta_i^{[y]} \) is a \( K \times 1 \) matrix of growth factors, where \( K \) is the number of growth factors.

\( b \) Factor loadings \( A_i^{[y]} \) is a \( J \times K \) matrix of factor loadings, where \( K \) is the number of growth factors and \( J \) is the number of repeated measures.

\( c \) The coefficient \( b, c, \) and \( \gamma \) can also be at the individual level and be viewed as an additional growth factor in the negative exponential function, Jenss-Bayley function, and bilinear spline function, respectively. Growth curves with such additional growth factors can be modeled through the Taylor series expansion in the structural equation modeling framework. Technical details can be found in earlier studies such as Preacher and Hancock (2015), Liu et al. (2021) and Grimm et al. (2016, Chapter 11).

\( d \) The bilinear spline function, also referred to as the linear-linear functional form, is widely employed to describe a longitudinal process in multiple domains, such as earlier-stage and later-stage growth in intellectual development or short-term recovery period and long-term recovery period in psychotherapy. The transition time from the first to the second stage can be viewed as a free parameter. With the reparameterization shown in the table \( (\eta_{i}^{[y]'}) \) and \( A_i^{[y]}' \) are the reparameterized growth factors and corresponding factor loadings), we can unify the expression pre- and post-knot and fit the model in the structural equation modeling framework. More technical details can be found in Liu et al. (2021).
Table 2: Performance Measures for Evaluating an Estimate (\(\hat{\theta}\)) of Parameter (\(\theta\))

| Criteria          | Definition                                      | Estimate                                                                 |
|-------------------|-------------------------------------------------|--------------------------------------------------------------------------|
| Relative Bias     | \(E_{\theta}(\hat{\theta} - \theta)/\theta\) | \(\sum_{s=1}^{S}(\hat{\theta}_s - \theta)/\theta S^b\)                  |
| Empirical SE      | \(\sqrt{Var(\hat{\theta})}\)                   | \(\sqrt{\sum_{s=1}^{S}(\hat{\theta}_s - \hat{\theta})^2/(S - 1)}\)    |
| Relative RMSE     | \(\sqrt{E_{\theta}(\hat{\theta} - \theta)^2/\theta}\) | \(\sqrt{\sum_{s=1}^{S}(\hat{\theta}_s - \theta)^2/S/\theta}\)          |
| Coverage Probability | \(Pr(\hat{\theta}_{lower} \leq \theta \leq \hat{\theta}_{upper})\) | \(\sum_{s=1}^{S}I(\hat{\theta}_{lower,s} \leq \theta \leq \hat{\theta}_{upper,s})/S\) |

\(\hat{\theta}_s\): the estimate of \(\theta\) from the \(s^{th}\) replication

\(S\): the number of replications and set as 1,000 in our simulation study

\(\hat{\theta}\): the mean of \(\hat{\theta}_s\)’s across replications

\(I()\): an indicator function
Table 3: Simulation Design for TVC Decomposition Methods with Individual Measurement Occasions

| Sample size | \( n = 200,000 \) |
|-------------|-------------------|
| Study wave \( (t_j) \) | 6 equally-spaced: \( t_j = 0, 1, \ldots, J - 1 \) \((J = 6)\) 10 equally-spaced: \( t_j = 0, 1.00, \ldots, J - 1 \) \((J = 10)\) |
| Individual time \( (t_{ij}) \) | \( t_{ij} \sim U(t_j - \Delta, t_j + \Delta) \) \((\Delta = 0.25)\) |

**Parameters of the growth curves of the longitudinal outcome**

Mean vector

\[
\begin{pmatrix}
\mu_{t_0}^{(y)} \\
\mu_{t_1}^{(y)} \\
\mu_{t_2}^{(y)} \\
\mu_{t_3}^{(y)} \\
\mu_{t_4}^{(y)} \\
\mu_{t_5}^{(y)} \\
\mu_{t_6}^{(y)} \\
\mu_{t_7}^{(y)} \\
\mu_{t_8}^{(y)} \\
\mu_{t_9}^{(y)} \\
\mu_{t_{10}}^{(y)} \\
\mu_{t_{11}}^{(y)} \\
\mu_{t_{12}}^{(y)} \\
\mu_{t_{13}}^{(y)} \\
\mu_{t_{14}}^{(y)} \\
\mu_{t_{15}}^{(y)} \\
\mu_{t_{16}}^{(y)}
\end{pmatrix} = (100, 5, 1.8)
\]

Variance-covariance matrix

\[
\begin{pmatrix}
\psi_{00}^{(y)} & \psi_{01}^{(y)} & \psi_{02}^{(y)} \\
\psi_{10}^{(y)} & \psi_{11}^{(y)} & \psi_{12}^{(y)} \\
\psi_{20}^{(y)} & \psi_{21}^{(y)} & \psi_{22}^{(y)}
\end{pmatrix} = 
\begin{pmatrix}
25 & 1.5 & 1.0 \\
1.5 & 1.0 & 0.3 \\
1.0 & 0.3 & 1.0
\end{pmatrix}
\]

Knot location

\( \gamma = 2.5 \) when \( J = 6 \)

\( \gamma = 4.5 \) when \( J = 10 \)

Residual variance

\( \theta_{\tau}^{(y)} = 1 \) or \( 2 \)

**Parameters of the growth curves of the time-varying covariate**

Mean vector

\[
\begin{pmatrix}
\mu_{t_0}^{(\tau)} \\
\mu_{t_1}^{(\tau)} \\
\mu_{t_2}^{(\tau)} \\
\mu_{t_3}^{(\tau)} \\
\mu_{t_4}^{(\tau)} \\
\mu_{t_5}^{(\tau)} \\
\mu_{t_6}^{(\tau)} \\
\mu_{t_7}^{(\tau)} \\
\mu_{t_8}^{(\tau)} \\
\mu_{t_9}^{(\tau)} \\
\mu_{t_{10}}^{(\tau)} \\
\mu_{t_{11}}^{(\tau)} \\
\mu_{t_{12}}^{(\tau)} \\
\mu_{t_{13}}^{(\tau)} \\
\mu_{t_{14}}^{(\tau)} \\
\mu_{t_{15}}^{(\tau)} \\
\mu_{t_{16}}^{(\tau)}
\end{pmatrix} = (10, 5)
\]

Variance-covariance matrix

\[
\begin{pmatrix}
\psi_{00}^{(\tau)} & \psi_{01}^{(\tau)} \\
\psi_{10}^{(\tau)} & \psi_{11}^{(\tau)}
\end{pmatrix} = 
\begin{pmatrix}
16 & 1.2 \\
1.2 & 1.0
\end{pmatrix}
\]

Relative Rate-of-Change\(^a\)

6 waves: \( \gamma_1 = 1.0 \) (fixed), \( \gamma_2/3/4/5 = 0.9/0.8/0.7/0.6 \)

10 waves: \( \gamma_1 = 1.0 \) (fixed), \( \gamma_2/3/4/5/6/7/8/9 = 0.9/0.8/0.7/0.6/0.5/0.4/0.3/0.2 \)

Residual variance

\( \theta_{\tau}^{(\tau)} = 1 \)

**Parameters related to time-invariant covariate and trait feature**

Correlation\(^b\)

\( \rho_{BL} = 0.3 \)

Coeff. to growth factors

Time-invariant covariate explains 13% variability of growth factors

Trait feature explains 0% variability of growth factors

Time-invariant covariate explains 3% variability of growth factors

Trait feature explains 7% variability of growth factors\(^b\)

Time-invariant covariate explains 6% variability of growth factors

Trait feature explains 14% variability of growth factors\(^b\)

Variables

\( X \sim N(0, 1^2) \)

**Parameters related state feature**

Decomposition method 1

\( \kappa_1 = 0 \) or 0.2 or 0.4 or 0.6

Decomposition method 2

\( \kappa_2 = 0 \) or 0.2 or 0.4 or 0.6

Decomposition method 3

\( \kappa_3 = 0 \) or 0.2 or 0.4 or 0.6

**Other parameters**

Residual covariance

\( \theta_{\tau}^{(y)} = 0.3 \times \sqrt{\theta_{\tau}^{(\tau)} \times \theta_{\tau}^{(y)}} \)

\(^a\) Relative rate-of-change is defined as the absolute rate-of-change over the shape factor.

\(^b\) The correlation between the time-invariant covariate and the trait feature is set as 0.3 so that they together explain 13% and 26% in the second and third conditions of the coefficients to growth factors of the longitudinal outcome, respectively.

Table 4: Summary of Model Fit Information For the Models

| Model | -2ll | AIC | BIC | # of Para. | Math Res. |
|-------|------|-----|-----|-----------|-----------|
| BLSGM | 25332.96 | 25335 | 25399 | 11 | 35.88 |
| BLSGM w/ a TIC | 26451.70 | 26484 | 26548 | 16 | 35.88 |
| BLSGM w/ a TIC and a TVC | 37039.45 | 38009 | 38149 | 35 | 34.36 |
| BLSGM w/ a TIC and a Decomposed TVC (w/ Interval-specific Slopes) | 34097.35 | 34167 | 34307 | 35 | 33.56 |
| BLSGM w/ a TIC and a Decomposed TVC (w/ Interval-specific Changes) | 34096.30 | 34166 | 34306 | 35 | 33.52 |
| BLSGM w/ a TIC and a Decomposed TVC (w/ Change-from-baseline Values) | 33771.76 | 33842 | 33981 | 35 | 33.54 |

\(^a\) BLSGM stands for bilinear spline growth model with an unknown fixed knot.
Table 5: Estimates of BLSGM\(^a\) with a TIC and a Decomposed TVC with Interval-specific Slopes

| Para. | Estimate (SE) | P value | Para. | Estimate (SE) | P value |
|-------|---------------|---------|-------|---------------|---------|
| Parameters of TVC | Parameters of Outcome |
| \(\mu_0\) | 0.0615 (0.0607) | 0.3392 | \(\mu_0\) | 23.3945 (0.6212) | < 0.0001* |
| \(\mu_{10}\) | 1.677 (0.0661) | < 0.0001* | \(\mu_{10}\) | 1.6274 (0.0153) | < 0.0001* |
| \(\phi_{10}\) | 1.1055 (0.0031) | 0.3392 | \(\phi_{10}\) | 0.6389 (0.0201) | < 0.0001* |
| \(\phi_{01}\) | -0.0023 (0.0115) | 0.1236 | \(\psi_{01}\) | 71.3255 (7.2811) | < 0.0001* |
| \(\phi_{11}\) | 0.0004 (0.0001) | < 0.0001* | \(\psi_{11}\) | 0.0647 (0.1493) | 0.7722 |
| \(\gamma_3\) | 0.8135 (0.0535) | < 0.0001* | \(\psi_{02}\) | -1.2127 (0.1828) | < 0.0001* |
| \(\gamma_4\) | 1.4573 (0.0610) | < 0.0001* | \(\psi_{11}\) | 0.0501 (0.0058) | < 0.0001* |
| \(\gamma_5\) | 0.5995 (0.0421) | < 0.0001* | \(\psi_{12}\) | -0.0863 (0.0056) | 0.2928 |
| \(\gamma_6\) | 0.8381 (0.0446) | < 0.0001* | \(\psi_{21}\) | 0.0660 (0.0100) | < 0.0001* |
| \(\gamma_7\) | 0.3531 (0.0240) | < 0.0001* | \(\gamma_{22}\) | 107.1501 (0.0541) | < 0.0001* |
| \(\gamma_8\) | 0.3393 (0.0233) | < 0.0001* | \(\beta_{[t]}\) | 33.5570 (0.9832) | < 0.0001* |
| \(\gamma_9\) | 0.3035 (0.0230) | < 0.0001* |

\([\beta]\) indicates statistical significance at 0.05 level.

Parameters of TIC

| \(\mu_x\) | 0.0000 (0.0000) | > 0.9999 |
| \(\phi_x\) | 0.9975 (0.0705) | < 0.0001* |

Cov between TIC and TVC Trait

| \(\text{cov}_{\text{BL}}\) | 0.2506 (0.0608) | < 0.0001* |

Residual Covariance

| \(\beta_{[t]}^{[x]}\) | 0.6804 (0.0700) | < 0.0001* |

\(\beta_{[t]}^{[x]}\) is the coefficient of TIC with a TIC and a Decomposed TVC with Interval-specific Slopes.

Table 6: Estimates of BLSGM\(^a\) with a TIC and a Decomposed TVC with Interval-specific Changes

| Para. | Estimate (SE) | P value | Para. | Estimate (SE) | P value |
|-------|---------------|---------|-------|---------------|---------|
| Parameters of TVC | Parameters of Outcome |
| \(\mu_0\) | 0.0615 (0.0607) | 0.3392 | \(\mu_0\) | 23.3945 (0.6212) | < 0.0001* |
| \(\mu_{10}\) | 1.677 (0.0661) | < 0.0001* | \(\mu_{10}\) | 1.6274 (0.0153) | < 0.0001* |
| \(\phi_{10}\) | 1.1055 (0.0031) | 0.3392 | \(\phi_{10}\) | 0.6389 (0.0201) | < 0.0001* |
| \(\phi_{01}\) | -0.0023 (0.0115) | 0.1236 | \(\psi_{01}\) | 71.3255 (7.2811) | < 0.0001* |
| \(\phi_{11}\) | 0.0004 (0.0001) | < 0.0001* | \(\psi_{11}\) | 0.0647 (0.1493) | 0.7722 |
| \(\gamma_3\) | 0.8135 (0.0535) | < 0.0001* | \(\psi_{02}\) | -1.2127 (0.1828) | < 0.0001* |
| \(\gamma_4\) | 1.4573 (0.0610) | < 0.0001* | \(\psi_{11}\) | 0.0501 (0.0058) | < 0.0001* |
| \(\gamma_5\) | 0.5995 (0.0421) | < 0.0001* | \(\psi_{12}\) | -0.0863 (0.0056) | 0.2928 |
| \(\gamma_6\) | 0.8381 (0.0446) | < 0.0001* | \(\psi_{21}\) | 0.0660 (0.0100) | < 0.0001* |
| \(\gamma_7\) | 0.3531 (0.0240) | < 0.0001* | \(\gamma_{22}\) | 107.1501 (0.0541) | < 0.0001* |
| \(\gamma_8\) | 0.3393 (0.0233) | < 0.0001* | \(\beta_{[t]}\) | 33.5570 (0.9832) | < 0.0001* |
| \(\gamma_9\) | 0.3035 (0.0230) | < 0.0001* |

Parameters of TIC

| \(\mu_x\) | 0.0000 (0.0000) | > 0.9999 |
| \(\phi_x\) | 0.9975 (0.0705) | < 0.0001* |

Cov between TIC and TVC Trait

| \(\text{cov}_{\text{BL}}\) | 0.2506 (0.0608) | < 0.0001* |

Residual Covariance

| \(\beta_{[t]}^{[x]}\) | 0.6804 (0.0700) | < 0.0001* |

\(\beta_{[t]}^{[x]}\) is the coefficient of TIC with a TIC and a Decomposed TVC with Interval-specific Changes.

\(\beta_{[t]}^{[x]}\) indicates statistical significance at 0.05 level.

\(\beta_{[t]}^{[x]}\) is the coefficient of TVC with a TIC and a Decomposed TVC with Interval-specific Changes.

\(\beta_{[t]}^{[x]}\) indicates statistical significance at 0.05 level.