Determination parameter of exponential function based positive number

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Abstract. The relationship between variables can be made a model or function. The use of the function is as a tool for making a prediction of the response, provided the value of the predictor is known. A function has independent variable, dependent variable, and parameter. The function discussed is an exponential function with base number in positive number. Furthermore, it is developed in the form of the exponential function with one parameter, the exponential function with two parameters and the exponential regression model. To create a function required some parameters. We need a certain method to find parameter. The accuracy of obtained function model shows the goodness of the function in approaching the distribution of the data. The method used to find parameter using the sum square error and logarithm function. Using logarithm function produces a linear function. Computation of parameters using R and Matlab programs. To measure the value of model accuracy using the mean square error. The purpose of this study is to find parameter of the exponential function with one parameter, the exponential function with two parameters and the exponential regression model. The presentation of the result of this discussion is in form of table and picture in the hope that it is easy to understand and interesting. The output of the exponential function with one parameter shows that the base value is 1.050021, the parameter value α is 7.969737 and the prediction accuracy value is 99.996%. The exponential function with two parameters shows that the base value is 0.5, the parameter value α is 7.969737, and the parameter value β is -0.071, while for the base value is 10, the parameter value α is 7.969737, and the parameter value β is 0.02. The prediction accuracy of the exponential function with two parameters is 99.996%. While the exponential regression model shows that the base value is 0.5, the parameter value α is 128.336, and the parameter value β is -55.828. The prediction accuracy of the exponential regression model is 84.8%.

1. Introduction
The relationship between variables can be made a model in the form of model or function. A function is a rule that pairs each member of the domain to one and only one member of the codomain. The use of the function is as a tool for making predictions of value of response variable, provided the value of the predictor variable is known. Generally, a function consist of independent variable (predictor), dependent variable (response) and parameters [14,16]. The primitive function discussed in this paper is exponential function. Furthermore, the exponential function will develop into the exponential function with one parameter, the exponential function with two parameters, and the exponential regression function. These functions have their own properties and characteristics. To create a function, the parameters must be determined. We need a certain suitable method to find parameters.
The method was tried out on the data under investigation. The accuracy of the obtained function model shows the goodness of function in approaching the data distribution being tested. The higher of prediction accuracy, the more valid the function is to be used as a model of data distribution [4,13].

The exponential function is a function that has a predictor variable as a power or exponential of a base number. The base number under investigation is a positive number. The exponential function with one parameter is an exponential function multiplied by a parameter. The property of the multiplier is that if the multiplier is less than one, then the function is shrinking. If the multiplier is equal to one, then the function is equal, and if the multiplier is greater than one, the function is enlarged. The exponential function with two parameters is an exponential function with one parameter, but is extended by a multiplier on its independent variable. The exponential regression function is an exponential function with one parameter plus a parameter. So the exponential regression function has two parameters.

Several papers that have discussed exponential function are [15] wrote the construction of the exponential function of a real exponent from the definition of power of a real number, its properties, convergence, and limit of a function. In [8] wrote about the benefits of using a transformation approach in the teaching and learning of exponential and logarithm function. The finding indicated that the interventional strategy had an impact on the academic performance of the learner. In [1] apply exponential growth regression modeling using SAS on the Biostatistics field. This method is a combination of two major techniques, which include bootstrapping and fuzzy regression for the exponential growth model. On [5] use an exponential model for survival analysis using R software that the survival function will be estimated using a parametric model based on imputation techniques in the present of partly interval censored and simulation data. The exponential distribution is used extensively in the field of life-testing. Estimation of parameters is revisited in two-parameter exponential distribution [12]. By [6] discusses the generalized exponential distribution with three parameters. These parameters be estimated by the maximum likelihood method. The sufficient condition for the existence of a unique solution of parameters obtained by the moment method. The exponentiated power exponential regression model be applied in nursing data. It shows that the new regression model can be applied to dispersion data since it represents a parametric family of models that includes as sub-models some widely-known regression models [11].

The method used to find parameters is using the sum square error. But not every function can be completed directly from this method. Because there are some equations that be produced, they are open form equations. Therefore, additional methods are needed so that the parameters of the function can be found. Another method used is the logarithm function. Because the logarithm function is the inverse function of the exponential function [14,16]. The resulting function is a linear function. We use software R and Matlab. The purposes of the research are to find the parameters of the exponential function with one parameter, the exponential function with two parameters, the exponential regression model, and measure the value of the accuracy model using the mean square error (MSE). The presentation of the results of the discussion is in the form by table and picture in hope that it is easy to understand and interesting.

2. Material and methods

2.1. Exponential function with one parameter

Before discussing further, we will explain the basic of the exponential function. The exponential function with a basis for a positive constant and any real number \( x \), written as

\[
y = a^x
\]  

(1)

The function is said to be an exponential function because the variable \( x \) is an exponential [14, 16]. There are three kinds of possibilities in the exponential function, namely

a. For \( 0 < a < 1 \), the exponential function is a descending function.
b. For \( a = 1 \), the exponential function is a constant function.

c. For \( a > 1 \), the exponential function is an ascending function.

Common bases are 10 and \( e \). The decreasing and ascending exponential function is a bijection function, so it has an inverse function called a logarithm function with a base number \( a \) [14]. The first to discuss is the exponential function with one parameter and base \( a \) positive number. The model is as follows [4,13]  
\[
y = a^x + e
\]

where \( a \) : parameter of function, \( a \) : base number, \( x \) : independent variable, \( y \) : dependent variable, and \( e \) : error. To get this model, it is necessary to estimate the value of parameter \( a \). The parameter \( a \) can be estimated using the sum squares error. The steps are as follows. First, the error is determined, that is \( e = y - a^x \). Then the error square becomes \( e^2 = (y - a^x)^2 \). If we use exponential model to a set of paired observations, that is \( \{x_1, y_1\}, \{x_2, y_2\}, \ldots, \{x_n, y_n\} \). The mathematical expression for the sum square error is  
\[
Sr = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - a^{x_i})^2
\]

To get an estimate of \( a \) value, then \( Sr \) is necessary differentiated with respect to that parameter \( a \).  
\[
\frac{\partial Sr}{\partial a} = -2\sum_{i=1}^{n} (y_i - a^{x_i})(a^{x_i}).
\]

This result in order to \( Sr \) is minimum, the set of this derivatives must equal to zero. If this is done, the equation can be expressed as  
\[
\sum_{i=1}^{n} (y_i - a^{x_i}) = 0.
\]

This equation could be simplest and then it finds parameter that is  
\[
\alpha = \left( \sum_{i=1}^{n} y_i a^{x_i} \right) \left( \sum_{i=1}^{n} a^{2x_i} \right)^{-1}
\]

The difficulty with this method is that the base value \( a \) must be known. So it uses the trial and error method to get parameter with the smallest error. An alternative to this method that it can produce exact parameter values by a linear transformation.

The method for the concept of linear transformation is described below. If the estimate of equation (2) is known, then the equation is described using the logarithm function. The result is \( \log y = x \log a + \log \alpha \). This equation is constructed into a linear equation  
\[
Y = Ax + B
\]

where \( A = \log a, B = \log \alpha \) and \( Y = \log y \). Based on equation (5), we can make an error \( e = Y - Ax - B \). Next, the error square is determined \( e^2 = (Y - Ax - B)^2 \). If we use the exponential model to a set of paired observations, that is \( \{x_1, y_1\}, \{x_2, y_2\}, \ldots, \{x_n, y_n\} \). The sum square error of linear transformation is  
\[
Srt = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - Ax_i - B)^2
\]

To get an estimate of the values \( A \) and \( B \), so \( Srt \) is necessary to differentiate with respect to parameters \( A \) and \( B \), respectively  
\[
\frac{\partial Srt}{\partial A} = -2\sum_{i=1}^{n} (Y_i - Ax_i - B)(x_i) \quad \text{and} \quad \frac{\partial Srt}{\partial B} = -2\sum_{i=1}^{n} (Y_i - Ax_i - B).
\]
These derivatives must equal to zero in order $Sr_t$ in a minimum value. If this is done, the equations can be expressed as $\sum_{i=1}^{n}(x_iY_i - Ax_i^2 - Bx_i) = 0$ and $\sum_{i=1}^{n}(Y_i - Ax_i - B) = 0$. We can express the equations as a set of two simultaneous linear equation with two parameters A and B. The equations are

$$A \sum_{i=1}^{n} x_i^2 + B \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i Y_i \quad \text{and} \quad A \sum_{i=1}^{n} x_i + Bn = \sum_{i=1}^{n} Y_i.$$  

These equations are called the normal equations. They can be solved simultaneously for $A$,

$$A = \left( n \sum_{i=1}^{n} x_i^2 - \left( \sum_{i=1}^{n} x_i \right)^2 \right) \left( \sum_{i=1}^{n} x_i Y_i - n \sum_{i=1}^{n} x_i \sum_{i=1}^{n} Y_i \right)^{-1}$$  

This result can be used in conjunction with equation (7) to solve for $B$,

$$B = \bar{Y} - A \bar{x}$$  

where $\bar{Y}$ is the means of $Y$, and $\bar{x}$ is the means of $x$.

If the values of $A$ and $B$ are known, the base number $a$ and the parameter value $\alpha$ can be determined. To determine the accuracy of predictions using the mean square error (MSE). The formula for MSE is

$$MSE = \overline{e^2}, \quad \text{where} \quad \overline{e^2} : \text{the mean of square error.}$$  

2.2. Exponential function with two parameters

The next discussion is the exponential function with two parameters on basis $a$. This model could be constructed [4, 13] $y = \alpha a^{\beta x} + e$ (11)

where $\alpha, \beta$: parameters of function, $a$: base number, $x$: independent variable, $y$: dependent variable, and $e$: error. To solve this model, we will try to use the concept of the sum square error to estimate the value of the parameters $\alpha$ and $\beta$. The steps are as follows. First, the error is determined, that is $e = y - \alpha a^{\beta x}$. Next, the error square is determined by $e^2 = (y - \alpha a^{\beta x})^2$. If we use the exponential model to a set of paired observations $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$. The equation for the sum square error is

$$Sr = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left( y_i - \alpha a^{\beta x} \right)^2$$  

To get an estimate of the $\alpha$ and $\beta$ values, then $Sr$ is necessary to differentiate with respect to these parameters $\alpha$ and $\beta$, respectively. The result equations are

$$\frac{\partial Sr}{\partial \alpha} = -2 \sum_{i=1}^{n} \left( y_i - \alpha a^{\beta x} \right) (a^{\beta x}) \quad \text{and} \quad \frac{\partial Sr}{\partial \beta} = -2\alpha \sum_{i=1}^{n} \left( y_i - \alpha a^{\beta x} \right) (x, a^{\beta x} \ln a).$$  

This result in order $Sr$ is minimum, set this derivatives must equal to zero. If this is done, the equations can be expressed as $\sum_{i=1}^{n} \left( y_i a^{\beta x} - \alpha a^{2\beta x} \right) = 0$ and $\alpha \ln a \sum_{i=1}^{n} (x_i y_i a^{\beta x} - \alpha x_i a^{2\beta x}) = 0$.

Because the above equations have parameters in joining each other. They are difficult to be solved. Alternative method by linear transformation method with the logarithm function. The trick is as follows. We find the estimate of equation (11). The model of the exponential function with two
parameters is converted to \( \log y = x\beta \log a + \log \alpha \). This equation is formed into a linear equation, i.e. \( Y = Cx + B \), where \( C = \beta \log a \), \( B = \log \alpha \) and \( Y = \log y \)  
Equation (13) is used to find the parameters \( \alpha \) and \( \beta \). If this is done, the equations can be expressed as follow 
\[
\sum_{i=1}^{n} \left( x_i \log \alpha - C x_i - B \right) = 0 \quad \text{and} \quad \sum_{i=1}^{n} \left( Y_i - C x_i - B \right)^2 = 0.
\]

To get an estimate of the values of \( C \) and \( B \), then the normal equations can be expressed as follow 
\[
C \sum_{i=1}^{n} x_i^2 + B \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i Y_i \quad \text{and} \quad C \sum_{i=1}^{n} x_i + B n = \sum_{i=1}^{n} Y_i
\]

The following is a discussion of the exponential regression model with two parameters on basis \( a \). The regression exponential model is presented by [4, 13]: 
\[
y = \alpha + \beta a^x + e
\]

\( \alpha, \beta \): parameters of the model, \( a \): base number, \( x \): predictor variable, \( y \): response variable, and \( e \): error. We want to determine these parameters. The steps are as follows. First, the error is determined by 
\[
e = y - \alpha - \beta a^x.
\]

If we use exponential model to a set of paired data \( (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n) \). Formula of the sum square error that is

\[
Sr = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left( y_i - \alpha - \beta a^x \right)^2
\]

To get an estimate of the \( \alpha \) and \( \beta \) values, equation (19) is necessary to differentiate with respect to these parameters. The results are
\[ \frac{\partial Sr}{\partial \alpha} = -2\sum_{i=1}^{n} (y_i - \alpha - \beta a_i^{x_i}) \text{ and } \frac{\partial Sr}{\partial \beta} = -2\sum_{i=1}^{n} (y_i - \alpha - \beta a_i^{x_i})(a_i^{x_i}). \]

This result in order above equations are minimum, then set this derivatives must be equal to zero. If this is done, the equations can be expressed become

\[ \sum_{i=1}^{n} (y_i - \alpha - \beta a_i^{x_i}) = 0 \text{ and } \sum_{i=1}^{n} (y_i a_i^{x_i} - \alpha a_i^{x_i} - \beta a_i^{2x_i}) = 0. \]

We can express the equations as a set of two simultaneous nonlinear equation with two parameters \( \alpha \) and \( \beta \), that are

\[ \alpha n + \beta \sum_{i=1}^{n} a_i^{x_i} = \sum_{i=1}^{n} y_i \text{ and } \alpha \sum_{i=1}^{n} a_i^{x_i} + \beta \sum_{i=1}^{n} a_i^{2x_i} = \sum_{i=1}^{n} Y_i a_i^{x_i}. \]

These normal equations could be solved simultaneously for \( \alpha \) and \( \beta \). The parameter formulas are

\[ \alpha = \left( \sum_{i=1}^{n} a_i^{2x_i} \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} a_i^{x_i} \sum_{i=1}^{n} Y_i a_i^{x_i} \right) \left( \sum_{i=1}^{n} a_i^{2x_i} - \left( \sum_{i=1}^{n} a_i^{x_i} \right)^2 \right)^{-1} \] (20)

\[ \beta = \left( n \sum_{i=1}^{n} y_i a_i^{x_i} - \sum_{i=1}^{n} a_i^{x_i} \sum_{i=1}^{n} y_i \right) \left( n \sum_{i=1}^{n} a_i^{2x_i} - \left( \sum_{i=1}^{n} a_i^{x_i} \right)^2 \right)^{-1} \] (21)

Based on equations (20) and (21), respectively the parameters of the exponential regression model can be immediately determined, provided that the base number \( a \) is known. The accuracy of prediction using the mean square error.

3. Result and discussion

The discussion of this paper is to apply the three kinds of the exponential functions to the distribution of data. The exponential functions with one and two parameters are applied to the distribution of time data and the concentration of carbon monoxide (CO) gas at a given location. While the exponential regression model is applied to the distribution of time data and gas concentration in an experiment. The presentation is done computationally using the R and Matlab programs. Analysis of processed results using statistical concepts as discussed in material and method. The parameter results are visualized in the form of a scatter diagram.

3.1. Application of exponential function with one parameter

We will break down the exponential function with one parameter. It will be found by the logarithm function based on number 10. This function is tried to the following data. Let there are data about time \( X \) (hour) and concentration of carbon monoxide (CO) gas \( Y \) in parts per million (ppm) at the place as in table 1 [13].

| Table 1. Data of time and concentration CO gas |
|-----------------------------------------------|
| X   | 0   | 1   | 2   | 3   | 4   | 5   | 6   |
| Y   | 8.0 | 8.2 | 8.8 | 9.5 | 9.7 | 10  | 10.7|

We want to find a relation model between \( Y \) with respect to \( X \). First, the exponential function be changed by logarithm function. The result is the linear function. The linear function as equation (4). The computation result is tabulated in table below [3,7].

| Table 2. Result of the parameter of linearization function |
|------------------------------------------------------------|
| Parameter | Estimate     | t-value | \( \text{Pr(>|t|)} \) | Sign |
|-----------|--------------|---------|------------------------|------|
| B         | 0.901444     | 171.34  | 1.28e-10               | ***  |
| A         | 0.021198     | 14.53   | 2.79e-05               | ***  |
According to table 2, it means that the parameters $A = 0.021198$ and $B = 0.901444$. The sign is three asteric. They have meaning that the significant of parameters are 0.001 (excellent). The estimated value of two parameters is very appropriate in building the model. To find out the accuracy of the estimated parameter values and the resulting data distribution is shown in the figure below [3,7].

![Figure 1](image1.png)

**Figure 1.** Linear relation about time and logarithm of gas concentration.

In figure 1, there are points that describe the successive pairing between time and gas concentration (Data 1). The linear line describes the linear relationship between gas concentration and time (Data 2). It can be seen that the distribution of all the points with the prediction line is very close. This shows that the significance of the parameter is very high. In addition, it is also proven that the prediction error of parameter values is below 0.1%. So the level of predictability is very high. Next, determination of the base number, namely $a = 1.050021$, and the function parameter, which is $\alpha = 7.969737$. Therefore we can construct the exponential function with one parameter, namely

$$\hat{y} = 7.969737 (1.050021)^x$$

(22)

This function is useful for predicting the distribution of CO gas concentrations at that place for another time. To measure the accuracy of prediction of the model on the data distribution using the MSE. The MSE of this process is 0.000042586, so it can be said that the accuracy of the model prediction is 99.996%. The image for the original model is visualized in figure 2 as follows [3,7].

![Figure 2](image2.png)

**Figure 2.** The exponential function between time and concentration gas.
Figure 2 shows the points depicting the successive pairs between time and gas concentration (Data 1). The nonlinear line represents the exponential relationship between time and gas concentration (Data 2). It can be seen that the distribution of all that points with the prediction line is very close, too. So the level of prediction correctness is very high, which implies that estimation of this model is suitable to be used for predicting problems.

3.2. Application of Exponential Function with Two Parameters

The application of the exponential function with two parameters is applied to data as written in table 1. We want to find the relationship between time and concentration of CO gas in the place. Based on the theoretical description that the sum square error cannot be used directly to the model, because the resulting normal equation is an equation in an open form. Therefore, this exponential function is solved by a linear function using a logarithm function. Function parameters are searched using the help of linear model function and R programming, the result is shown in table 3 [3, 7].

| Parameter | Estimate | $t$-value | Pr(>|$t$|) | Sign |
|-----------|----------|-----------|-----------|------|
| B         | 0.901444 | 171.34    | 1.28e-10  | ***  |
| C         | 0.021198 | 14.53     | 2.79e-05  | ***  |

According to table 3, it means that parameters C = 0.021198 and B = 0.901444. The sign is three asteric. They have meaning that significant of parameters are 0.001 (excellent). Conclusion is that estimated value of two parameters is very good in making model. To find out the accuracy of estimated parameter value and the resulting data distribution is shown in the figure below [3, 7].

In figure 3, there are points that describe the successive pairs between time and gas concentration (Data 1). The linear line describes the linear relationship between gas concentration and time (Data 2). The distribution of all the points with a prediction line is near. This indicates that the significance of the parameter is excellent. Besides that, the prediction error of parameter value is below 0.1%. So the level of predictability is very high. To measure the accuracy of prediction of the model on data distribution using the MSE. The MSE of this process is 0.000042586, so it can be said that the accuracy of model prediction is 99.996%.

Based on the computation result that $B = 0.901444$, then the value of $\alpha = 7.969737$. Meanwhile, to get parameter value $\beta$, it is necessary to determine the base value $a$. For the sample, the base value be taken at $a = 0.5$ and $a = 10$. If the value $a = 0.5$, the value $\beta = -0.071$, while if the value $a = 10$, the value $\beta = 0.02$. Therefore the exponential function with two parameters can be made, namely
\[ \hat{y} = 7.969737 \times (0.5)^{-0.071x} \]  

(23)

and

\[ \hat{y} = 7.969737 \times (10)^{0.02x} \]  

(24)

This function is useful for predicting the distribution of CO gas concentration at that place for the future. Complete depiction of two processed functions is shown in the following figure [2,9,10].

**Figure 4.** Exponential functions have base numbers both 0.5 and 10.

In figure 4, there are points that describe the successive pairing between time and gas concentration (data 1). There are two nonlinear lines (data 1 and data 2), respectively, namely nonlinear line of the exponential function with two parameters for \( a = 0.5 \) and \( a = 10 \). It can be seen that the distribution of all the points with a predicted line is rather close. Therefore, the level of predictability is very high, which implies that the estimation of this model is suitable for prediction at other chance.

### 3.3. Application of Exponential Regression Model

Application of the exponential regression model to data about time X (hours) and gas concentration Y (ppm) in a research [13]. Variable predictor X need not random variable, but variable response Y must be a random variable. The response variable Y is a random variable because it is a function, has the same probability in chance, its value can not be predicted as a result from the experiment. The data is summarized in table 4.

**Table 4.** Data of time and gas concentration

| X  | 0  | 1  | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Y  | 77.6 | 94.9 | 107.6 | 116.4 | 122.8 | 127.1 | 129.6 | 130.3 | 130.8 | 131.2 | 131.6 |

To find the relationship between time and gas concentration, the model is approached by equation (18). Based on equations (20) and (21) with the base value is taken on 0.5, the results are obtained as written in table 5, namely [3,7].

**Table 5.** Result of parameters of exponential regression function

| Parameter | Estimate | \( t \)-value | \( \text{Pr}(>|t|) \) | Sign |
|-----------|----------|---------------|----------------|------|
| \( \alpha \) | 128.336 | 84.33         | 2.35e-14        | ***  |
| B         | -55.828  | -12.77        | 4.52e-07        | ***  |
According to table 5, it means that parameters $\alpha = 128.336$ and $\beta = -55.828$. The sign is three asteric. They have meaning that significant of parameters are 0.001 (excellent). The estimated values of both parameters are very appropriate for modeling. The exponential regression function is

$$\hat{y} = 128.336 - 55.828 (0.5)^t$$

To find out the accuracy of the estimated parameter values and resulting data distribution is shown in the figure below [3, 7].

![Figure 5. Exponential regression model distribution.](image)

Figure 5 shows that there are some points that represent a sequence pair between time and gas concentration (Data 1). The nonlinear line represents nonlinear relationship between gas concentration and time (Data 2). It can be seen that distribution of all the points with the prediction line is near. This indicates that significance of that parameter is good. In addition, it is also proven that the prediction error of parameter values is below 0.1%, then level of predictability is high. To measure accuracy of model prediction on the data distribution using MSE. The MSE of computational data processing is 0.152, so it can be informed that accuracy of model prediction is 84.8%.

4. Conclusion
The exponential function based on positive number can be decreasing function, constant function, and increasing function. For exponential function with one parameter when solved directly using the least square method, the base number needs to be determined first. Because the normal equations are opened form. The solution is produce an exact base value, it is necessary to use the transformation of the logarithm, then only use the least square method. The exponential function with two parameters in order for the parameters to be solved first needs to know its basis. Determination of parameters for the exponential regression model can be directly used in the least square method as long as the base number is determined first. Selection of base number using the trial and error method. The computation of the exponential regression model uses the nonlinear square function. The prediction accuracy of the parameters can use the mean square error. The output for the exponential function with one parameter shows that the base value is 1.050021, the parameter value $\alpha$ is 7.969737, and the prediction accuracy value is 99.996%. The exponential function with two parameters shows that the base value is 0.5, the parameter value $\alpha$ is 7.969737, and the parameter value $\beta$ is -0.071, while for the base value is 10, the parameter value $\alpha$ is 7.969737, and the parameter value $\beta$ is 0.02. The prediction accuracy of the exponential function with two parameters is 99.996%. While the exponential
regression model shows that the base value is 0.5, the parameter value $\alpha$ is 128.336, and the parameter value $\beta$ is -55.828. The prediction accuracy of the exponential regression model is 84.8%.

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