Model of fuzzy ultra-acoustic diagnostics of nanocomposite functionally graded plate constructions in mechanical engineering

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Abstract. A description of a numerical-analytical technique for obtaining fuzzy estimates of the physical-mechanical parameters of plates that are exponentially inhomogeneous in thickness from functionally gradient transversely isotropic nanocomposite materials intended for use in mechanical engineering and aerospace technologies is given. The technique is based on the use of uncertain experimental data of measuring the lengths of normal shear elastic waves with different frequencies for different modes of the dispersion spectrum of the plate under consideration having interpretation in form of fuzzy sets. For obtaining fuzzy estimates of physical-mechanical parameters the transition to fuzzy-multiple arguments in the analytical relationships between the physical-mechanical parameters of functionally gradient transversely isotropic nanocomposite materials and the characteristics of ultrasonic measurements are used.

1. Introduction

The use of modern functional-gradient anisotropic nanocomposite materials [1-3] in plate structures is one of the new important areas in mechanical engineering and aerospace technologies. The efficiency of using plate constructions made of such materials, including materials with an exponential type of heterogeneity of deformation properties [4-6], largely depends on the accuracy of determining their real physical and mechanical parameters. One of the possible promising methods for identifying these characteristics is the use of various types of ultrasound diagnostics [7] data of transversely isotropic nanocomposite plates with inhomogeneity along thickness. In particular, it is possible to use the experimental data of fuzzy estimation of the length of normal elastic shear waves and the experimental values of the critical frequencies for various guided wave modes in the plates under consideration [16].

Creating a theoretical-experimental model of fuzzy estimation of the physical-mechanical parameters of transversely isotropic functionally-gradient nanocomposite plate structures with exponential heterogeneity along thickness is an urgent problem in improving methods for calculating the strength and reliability of engineering and aerospace structures. To solve it, this paper proposes a numerical-analytical method for obtaining fuzzy estimates for uncertain physical and mechanical...
parameters. This method is based on the transition to fuzzy-sets arguments in the obtained analytical relations between the physical and mechanical parameters of functionally-gradient transversely isotropic nanocomposite materials and the characteristics of ultrasonic measurements [7]. It is also applied a modified form of the heuristic principle of generalization in the theory of fuzzy computing [8-12, 17]. An example of the implementation of the developed methodology is given. For this, in paper an analytical solution to the problem of the propagation of guided elastic shear waves in a functionally gradient transversely isotropic deformable layer with elastic and density modules that vary exponentially along the transverse coordinate is constructed.

2. Obtaining of calculations relations for the length and critical frequencies of guided shear waves in a transversely isotropic functionally-gradient layer

The problem of the propagation of a plane stationary guided SH wave with a cyclic frequency \( \omega \) and wave number \( k \) along the coordinate direction in a transversally isotropic functionally-gradient elastic layer occupying the region \( V = \{ x_1, x_2 \} \in \mathbb{R}^2, x_3 \in [-h, h] \) is considered. The anisotropy axis of the layer material is oriented along the coordinate \( x_3 \). The boundary surfaces of the layer \( x_3 = \pm h \) are free. In the case under consideration, the field of wave elastic displacements is characterized by a single component \( u_2(x_1, x_2, t) = \phi(x_3) \exp(-i(\omega t - k x_1)) \). The equation of wave motions and relations between the nonzero components of the stress tensor and tensor of deformations under formulated assumptions respectively have the form

\[
\sigma_{21} = c_{66} \exp(\lambda x_3) \cdot \partial_t u_2, \quad \sigma_{23} = c_{44} \exp(\lambda x_3) \cdot \partial_3 u_2, \quad \partial_t \sigma_{21} + \partial_3 \sigma_{23} - \rho \exp(\lambda x_3) \cdot \partial^3_t u_2 = 0
\]  

where \( c_{44}, c_{66} \) and \( \rho \) are, respectively, the parameters of elasticity and density for a transversely isotropic material of the layer; \( \lambda \) - is parameter of transverse exponential heterogeneity of the material of the layer; \( \partial_j = \partial / \partial x_j, \quad \partial_t = \partial / \partial t \) are the operators of partial differentiation.

The problem of determination \( u_2(x_1, x_2, t) \) reduces to integrating an ordinary differential equation for a function \( \phi(x_3) \) whose solution has the form

\[
\phi(x_3) = d_+ \exp(\gamma_+ x_3) + d_- \exp(\gamma_- x_3), \quad \gamma_\pm = - (\lambda / 2) \pm (\lambda / 2 - \beta^2)^{1/2},
\]

where \( d_+, d_- \) - are arbitrary constant coefficients. Further, from the boundary conditions on the free flat faces of the layer \( (\phi(x_3))_{x_3=\pm h} = 0 \), a system of homogeneous algebraic equations for constants \( d_\pm \) follows.

The general dispersion relation for the investigated guided waves is the equality to zero of the determinant of the resulting algebraic system. It may initially be written as

\[
\omega = [\zeta_{66} k_p^2 + \zeta_{44} (\tau + \Delta_p)]^{1/2}, \quad \zeta_{ij} = \rho c_{ij}, \quad \tau = \lambda^2 / 4, \quad \Delta_p = (p \tau / (2h))^2
\]

and converted into an analytic relation for the wavelength parameter \( \delta_p \) for mode of guide wave with the number \( p \) (\( p = 0, \infty \)) from the investigated dispersion spectrum

\[
\delta_p = (4\pi^2 \zeta_{66} / (\omega^2 - \zeta_{44} (\tau + \Delta_p)))^{1/2}
\]

It is assumed that for guided normal waves from the lowest mode of the spectrum \( p = 0 \) with two frequencies \( \omega_0 \) and \( \omega_2 \) \((\omega_0 < \omega_2)\), their length \( \delta_0 \) and \( \delta_0 \) \((\delta_0 \geq \delta_0)\) parameters can be determined experimentally. For a wave with a frequency \( \omega_3 \) from the lowest mode of the spectrum \( p = 0 \) and a
wave with a frequency $\omega_4$ from the mode of the spectrum $p = 1$ ($\omega_4 > \omega_0$), the parameters of their length $\delta_{03}$ and $\delta_{14}$ ($\delta_{14} > \delta_{03}$) can also be determined accordingly by experiment. In addition, the plate mass density parameter $m$, which is related to the parameter $\rho$ in the case under consideration by the ratio $\rho = m \lambda / (2sh(\lambda h))$, can be experimentally measured.

With these hypotheses for parameters $\zeta_{66}$, $\zeta_{44}$ and $\tau$ its representations can be written

$$
\zeta_{66} = (\omega_2^2 - \omega_0^2)/(4\pi^2(\delta_{02}^2 - \delta_{01}^2)),
\zeta_{44} = (\gamma_4 - \gamma_3)/(\Delta_1 - \Delta_0) = (\omega_2^2 - \omega_0^2 + 4\pi^2(\delta_{03}^2 - \delta_{14}^2)/(4\pi^2(\delta_{02}^2 - \delta_{01}^2)))/(\Delta_1 - \Delta_0), \quad \gamma_4 = \omega_2^2 - 4\pi^2\zeta_{66}\delta_{01}^2.
$$

$$
\tau = \gamma_{44}^{-1} - \Delta_0 = (\omega_2^2 - 4\pi^2(\omega_2^2 - \omega_1^2)/(4\pi^2(\delta_{02}^2 - \delta_{01}^2)))\delta_{01}^2/(\omega_2^2 - \omega_0^2 + 4\pi^2(\delta_{02}^2 - \delta_{14}^2)))/(\Delta_1 - \Delta_0) - \Delta_0.
$$

Thus, for the parameters $\lambda$, $\rho$, $\zeta_{66}$, $\zeta_{44}$ to be identified explicit analytical representations with exogenous parameters $\omega_0, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m$ are obtained:

$$
\lambda = \Phi_1(\omega_0, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) = 2(\omega_2^2 - 4\pi^2
$$

$$
/((\omega_2^2 - \omega_0^2)/(4\pi^2(\omega_2^2 - \omega_1^2))\delta_{01}^2)))/((\omega_2^2 - \omega_0^2 + 4\pi^2(\delta_{02}^2 - \delta_{14}^2)))/(\Delta_1 - \Delta_0) - \Delta_0)^{1/2} /\]

$$
\rho = \Phi_2(\omega_0, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) = 2m(\omega_2^2 - 4\pi^2
$$

$$
/((\omega_2^2 - \omega_0^2)/(4\pi^2(\omega_2^2 - \omega_1^2))\delta_{01}^2)))/((\omega_2^2 - \omega_0^2 + 4\pi^2(\delta_{02}^2 - \delta_{14}^2)))/(\Delta_1 - \Delta_0) - \Delta_0)^{1/2} /\]

$$
\zeta_{66} = \Phi_3(\omega_0, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) = (\omega_2^2 - \omega_0^2)/(4\pi^2(\delta_{02}^2 - \delta_{01}^2))
$$

$$
2m(\omega_2^2 - 4\pi^2 (\omega_2^2 - \omega_0^2)/(4\pi^2(\omega_2^2 - \omega_1^2))\delta_{01}^2)))/((\omega_2^2 - \omega_0^2 + 4\pi^2(\delta_{02}^2 - \delta_{14}^2)))/(\Delta_1 - \Delta_0) - \Delta_0)^{1/2} /\]

$$
\zeta_{44} = \Phi_4(\omega_0, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) = (\omega_2^2 - \omega_0^2 + 4\pi^2(\delta_{02}^2 - \delta_{14}^2))
$$

$$
/((\omega_2^2 - \omega_0^2)/(4\pi^2(\omega_2^2 - \omega_1^2))\delta_{01}^2)))/(\Delta_1 - \Delta_0) - \Delta_0)^{1/2} /\]

3. Algorithm for obtaining of fuzzy estimates for identifiable physical and mechanical parameters of the plate.
In obtaining estimates for the identifiable physical and mechanical parameters of the plate, the hypothesis of the existence of scatter errors of the experimental parameter values $\omega_0, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, \lambda, m$ in the described model is used. It is assumed that these indefinite exogenous parameters are described by convex normal fuzzy sets $\bar{\omega}_0, \bar{\omega}_2, \bar{\omega}_3, \bar{\omega}_4, \bar{\delta}_{01}, \bar{\delta}_{02}, \bar{\delta}_{03}, \bar{\delta}_{14}, \bar{m}$
with corresponding membership functions \( \mu_{\tilde{o}_j} (j = \overline{1,4}) \), \( \mu_{\tilde{\delta}_j} (j = \overline{1,3}) \), \( \mu_{\tilde{\delta}_{14}} \), \( \mu_{\tilde{m}} \). The parameter \( h \) is considered as a clearly defined value. The introduced fuzzy sets can be represented in the form of superposition of a subset of \( \alpha \)-level:

\[
\tilde{o}_j = \bigcup_{a \in [0,1]} [\tilde{o}_{ja}] \quad (j = \overline{1,4}); \quad \tilde{\delta}_j = \bigcup_{a \in [0,1]} [\tilde{\delta}_{ja}] \quad (j = \overline{1,3}); \quad \tilde{m} = \bigcup_{a \in [0,1]} [\tilde{m}_a].
\]

(7)

Application of the modified \( \alpha \)-level form of the heuristic principle of generalization [8-14] to clear analytical representations for identifiable endogenous parameters \( \lambda \), \( \rho \), \( \xi_{66} \), \( \xi_{44} \) of the model under consideration allows us to obtain their descriptions in the form of fuzzy sets \( \tilde{\lambda} \), \( \tilde{\rho} \), \( \tilde{\xi}_{66} \), \( \tilde{\xi}_{44} \) decompositions into subsets of the \( \alpha \)-level:

\[
\begin{align*}
\tilde{\lambda} & = \bigcup_{a \in [0,1]} [\tilde{\lambda}_a] \quad \tilde{\rho} = \bigcup_{a \in [0,1]} [\tilde{\rho}_a] \quad \tilde{\xi}_{66} = \bigcup_{a \in [0,1]} [\tilde{\xi}_{66,a}] \quad \tilde{\xi}_{44} = \bigcup_{a \in [0,1]} [\tilde{\xi}_{44,a}]; \\
\tilde{\lambda}_a & = \inf \{ \Phi_{\lambda} (\omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) \} ; \\
\tilde{\rho}_a & = \inf \{ \Phi_{\rho} (\omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) \} ; \\
\tilde{\xi}_{66,a} & = \inf \{ \Phi_{\xi_{66}} (\omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) \} ; \\
\tilde{\xi}_{44,a} & = \inf \{ \Phi_{\xi_{44}} (\omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) \} .
\end{align*}
\]

(8)
\[ \tilde{c}_{44} = \sup_{\omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m} \{ \Phi_{44}(\omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) \} \]

The implementation scheme of the developed methodology is presented for the general case of a description of the scatter of parameter \( \omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, m \) values by normal fuzzy trapezoidal intervals \( \tilde{\omega}_1, \tilde{\omega}_2, \tilde{\omega}_3, \tilde{\omega}_4, \tilde{\delta}_{01}, \tilde{\delta}_{02}, \tilde{\delta}_{03}, \tilde{\delta}_{14}, \tilde{m} \) [9-14]. In special cases, a certain subset of parameters can be considered as clearly defined values without scatter. It is taken into account that for an arbitrary normal fuzzy trapezoidal interval \( \tilde{\gamma} = (\gamma_1, \gamma_2, \gamma_3, \gamma_4) \) a representation \( \tilde{\gamma} = \bigcup_{\alpha \in [0,1]} [\gamma_2, \gamma_3] \) can be written, where \( \gamma_2 = (1-\alpha)\gamma_1 + \alpha \gamma_2, \gamma_3 = \alpha \gamma_3 + (1-\alpha)\gamma_4 \).

In expressions (9), a certain number of variable parameters can be reduced, since analytical representations of partial derivatives with respect to some subsets of arguments for functions \( \Phi_{44}(\omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) \), \( \Phi_{44}(\omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, h, m) \), \( \Phi_{44}(\omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, \omega_1) \), \( \Phi_{44}(\omega_1, \omega_2, \omega_3, \omega_4, \delta_{01}, \delta_{02}, \delta_{03}, \delta_{14}, \omega_4) \) can be obtained. The properties of sign-definiteness for this partial derivatives can be determined in the entire range of admissible values of the arguments and the modification of the relations (9) based on the corresponding variant of the heuristic principle of generalization [12] can be written.

In particular, from the calculated ratios of the model for a plate with a clear value of the thickness parameter \( h = 1.0 L \), with clear frequency parameters \( \omega_1 = 0.5 \omega_c \), \( \omega_2 = 1.0 \omega_c \), \( \omega_3 = 1.5 \omega_c \), \( \omega_4 = 2.0 \omega_c \) and values of fuzzy-interval experimental data

\[
\begin{align*}
\tilde{\omega}_{01} &= (20.51L, 20.52L, 20.53L, 20.54L); \\
\tilde{\omega}_{02} &= (9.14L, 9.17L, 9.18L, 9.19L); \\
\tilde{\omega}_{03} &= (5.97L, 6.01L, 6.04L, 6.05L); \\
\tilde{\omega}_{04} &= (7.31L, 7.32L, 7.33L, 7.34L); \\
\tilde{\omega}_{14} &= (1.96m, 1.99m, 2.01m, 2.03m); \\
\omega_c &= 10^3 \text{ [rad/m]; } l_c = 1 \text{ [m]; } m_c = 10^3 \text{ [kg]; } \omega_c \text{ is the frequency of pure oscillations, } l_c \text{ and } m_c \text{ are the characteristics of mass and area of the plate.}
\end{align*}
\]

Membership functions calculated for the described methodology of obtaining of endogenous fuzzy estimates \( \tilde{\omega}, \tilde{\rho}, \tilde{\omega}_{66}, \tilde{\omega}_{44} \) are given in the corresponding dimensions in figures 1 - 4.

**Figure 1.** Membership function for \( \tilde{\omega} \).

**Figure 2.** Membership function for \( \tilde{\rho} \).
Figure 3. Membership function for $\tilde{c}_{66}$.

Figure 4. Membership function for $\tilde{\rho}$.

The presented estimates describe normalized relative indicators of the degree of confidence that the physical-mechanical parameters of functionally-gradient transversely isotropic nanocomposite materials with exponentially inhomogeneity along thickness of plate will take appropriate values when taking into account the given scatter errors in the values of the exogenous parameters of the model under consideration.

4. Conclusion
The result of the research presented is the development of a numerical-analytical method for obtaining estimates for the values of the physical-mechanical parameters of exponentially-inhomogeneous along thickness plates from functionally gradient transversely isotropic nanocomposite materials intended for use in mechanical engineering and aerospace technologies. The technique is based on the use of uncertain experimental data of measuring the lengths of guided shear elastic waves with different frequencies from the lowest modes of the dispersion spectrum for the plate under consideration, having a fuzzy-sets interpretation. To obtain fuzzy estimates of the physical-mechanical parameters be used the transition to fuzzy-multiple arguments in the analytical relationships between the physical-mechanical parameters of the functionally gradient transversely isotropic nanocomposite materials and the characteristics of ultrasonic measurements. A modified form of the heuristic principle of generalization in the theory of fuzzy computing is also used. An example of the implementation of the developed technique is presented.

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