Status of Microscopic Modeling of Black Holes by $D1 - D5$ System *

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ABSTRACT

We briefly review the microscopic modeling of black holes as bound states of branes in the context of the soluble $D1 - D5$ system. We present a discussion of the low energy brane dynamics and account for black hole thermodynamics and Hawking radiation rates. These considerations are valid in the regime of supergravity due to the non-renormalization of the low energy dynamics in this model. Using Maldacena duality and standard statistical mechanics methods one can account for black hole thermodynamics and calculate the absorption cross section and the Hawking radiation rates. Hence, at least in the case of this model black hole, since we can account for black hole properties within a unitary theory, there is no information paradox.

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1 Black Holes, QFT and Information Puzzle

One of the most important aspects of string theory is that gravity is a prediction of string theory [1], [2], [3]. Since string theory is consistent with quantum mechanics (in particular it is unitary and finite) it is widely believed that it is also a consistent theory of quantum gravity and hence in principle should be able to resolve the conundrums of general relativity.

One of these conundrums goes by the name of the “Information Puzzle”, which is intimately tied to the fact that black holes have an event horizon. In the classical theory the horizon is a one way gate, in the sense that once a particle is inside it cannot get out because of the causal structure of the black hole space time. In the quantum theory [4] black holes radiate and the radiation (at least in the semi-classical calculation valid for large mass black holes) is supposed to be exactly thermal\(^2\). The radiation is characterized by the Hawking temperature

\[
T_H = \frac{\hbar \kappa}{2\pi}
\]  

(1)

\(\kappa\) is surface gravity (acceleration due to gravity felt by a static observer) at the horizon of the black hole. For a Schwarzschild black hole:

\[
\kappa = \frac{1}{4G_N M}
\]  

(2)

Black holes are also characterized by an entropy proportional to the area of the even horizon

\[
S_{bh} = a A_h, \quad a = \frac{c^3}{4G_N \hbar}
\]  

(3)

The constant of proportionality \(a\) is determined using the result from thermodynamics
\(TdS = dM\). Eqn. (3) is the celebrated formula of Bekenstein and Hawking [5]. Hawking also gave a formula for the decay rate of a black hole in terms of the absorption cross section \(\sigma_{abs}(\omega)\):

\[
\Gamma_H = \sigma_{abs}(\omega)(e^{\omega/T_H} - 1)^{-1} \frac{d^3k}{(2\pi)^4}
\]  

(4)

Hawking radiation as calculated in semi-classical general relativity is described by a mixed state. If this were exactly true it would be in conflict with the known principles of quantum mechanics. This conundrum is called the information puzzle.

\(^2\)The causal structure of the horizon plays an important role in this derivation
The information puzzle would cease to exist if one could show that Hawking radiation is similar to radiation from a standard black body which is describable by unitary quantum mechanical evolution and the actual wave function of the black body, although hugely complicated, could be discerned from subtle correlations existing among the radiated particles that come out. It turns out to be difficult to calculate such correlations in the case of Hawking radiation in the standard framework of general relativity. Such a calculation would require a good quantum theory of gravity where controlled approximations are possible.

2 String Theory Framework for Black Holes

String theory provides a calculable quantum theory of gravity. The theory, as we know it, is unitary and hence it can attempt to explain black hole thermodynamics in terms of the known principles of statistical mechanics. This means that in string theory a black hole should be described by a density matrix:

\[
\rho = \frac{1}{\Omega} \sum_i |i\rangle \langle i| \\
S = \ln \Omega
\]  

(5)

where \(|i\rangle\) is a micro-state.

In such a framework Hawking radiation is no different from the radiation emitted by burning a piece of wood. The thermal description is a useful gross description in which one averages over a large ensemble of states. In such a framework the Bekenstein-Hawking formula is the same as Boltzmann’s formula and the Bose factor in the formula for the decay rate corresponds to the statistical average of the occupation number at the frequency \(\omega\).

Ingredients of String Calculation

We enumerate the basic ingredients we need to do the string theory calculation:

1. We need a microscopic model of the black hole and the effective Lagrangian of the low energy excitations of this model at strong coupling.
2. In order to calculate the Hawking process we need the interaction of the effective
degrees of freedom with the supergravity modes which are radiated by the black
hole.

3. Once we can calculate transition amplitudes in a unitary theory between black
hole states we can derive black hole thermodynamics using the micro-canonical or
canonical ensemble.

The model which allows string theory calculation under controlled approximations, is
the near extremal 5-dim. black hole of type II-B string theory compactified on a 4-torus
(or $K_3$) [7]. This black hole has a very small temperature, so that the thermal wavelength
is much larger than the typical gravitational radius of the black hole.

3 The Near Extremal Black Hole

We now elaborate a bit more about the black hole we are modeling. The near extremal
black hole solution of II-B string theory compactified on $T^4 \times S^1$ preserves none of the
original 32 supersymmetries of the type IIB theory. It has a non-zero Hawking tempera-
ture and a positive specific heat (unlike the Schwarzschild black hole). We want to model
the thermodynamics of this solution in string theory.

The solution involves the metric, the dilaton and the Ramond-Ramond 2-form $C^{(2)}$.
The coordinates $x^i, i = 1, 2, 3, 4$ are non-compact. $x^5$ is periodically identified with period
$2\pi R_5$ and directions $x^6, \ldots, x^9$ are compactified on a torus $T^4$ of volume $V_4$, $R_5 \gg (V_4)^{1/4}$.

This solution is parameterized by six independent quantities: $r_1, r_5, r_0, \sigma, R_5$ and $V_4$
(notations defined in [11]). These are related to the number $Q_1$ of D1-branes, $Q_5$ of D5-
branes and Kaluza-Klein momentum N to be distributed in both directions around the
$x_5$ direction. $r_0$ is the non-extremality parameter.

The entropy and mass of the black hole are well known. The near extremal black
hole has a small Hawking temperature and unlike the Schwarzschild black hole a positive
specific heat.

The classical solution is relevant in the quantum theory only if quantum loops are
suppressed: $g_s \to 0$. Hence we require $g_s Q_1, g_s Q_5$ to be held fixed and we are dealing
with the large $Q_1, Q_5$ limit. For a macroscopic black hole the horizon area is much larger
than the string length \( l_s = \sqrt{\alpha'} \). This implies \( g_s Q_1 \gg 1, \ g_s Q_5 \gg 1, \ g_s^2 N \gg 1 \). Since, \( g_s Q_1, g_s Q_5 \) correspond to the effective open string coupling constants, a macroscopic black hole exists at strong coupling!

4 Absorption Cross Section, Decay Rate

Using the black hole solution one can calculate by solving the wave equation the absorption cross section for various particles. The simplest one to do are the so called minimal scalars because they only couple to the background Einstein metric and their wave equation is

\[
D_\mu \partial^\mu \varphi = 0
\]  

(6)

The s-wave absorption cross section is given by

\[
\sigma_{abs} = 2\pi^2 r_1^2 r_5^2 \frac{\pi \omega}{2} \frac{\exp(\omega/T_H) - 1}{(\exp(\omega/2T_R) - 1)(\exp(\omega/2T_L) - 1)}
\]  

(7)

In the \( \omega \to 0 \) limit, one gets

\[
\sigma_{abs} = A_h
\]  

(8)

where \( A_h \) denotes the area of the event horizon. The decay rate is

\[
\Gamma_H = \sigma_{abs} (e^{\omega/T_H} - 1)^{-1} \frac{d^4 k}{(2\pi)^4}
\]  

(9)

The absorption cross section of higher partial waves vanishes in the \( \omega \to 0 \) limit.

5 Microscopic Model: D1-D5 System

We begin with the theory of the \( Q_5 \) D5 branes along the compact coordinates \( x_i, i = 5, 6, 7, 8, 9 \). The low energy degrees of freedom of this system are described by an \( N = 2 \) \( U(Q_5) \) gauge theory in 6 dimensions.

In this gauge theory the configurations which break the 16 supersymmetries to 8 are instantons. Along the \( x_5 \) direction this is a string like configuration and in fact it is to be identified with \( Q_1 \) D1 branes, if the instanton charge is \( Q_1 \). The instantons are characterized by moduli whose variation does not change the action of SYM\(_6\). Promoting these moduli to slowly varying functions of \( x_5, t \) we obtain the motions of the D1 branes inside the 5-branes. These represents the low lying collective modes of the D1, D5 system.
It turns out that the moduli space $\mathcal{M}$, of instantons on $T^4$, is the Hilbert Scheme of the symmetric product $(\tilde{T}^4)^{Q_1Q_5}/S(Q_1Q_5)$. ($\tilde{T}^4$ can be different from the compactification torus $T^4$.)

Our attitude will be to consider the sigma model on $\mathcal{M}$, as a resolution of the sigma model on the orbifold $(\tilde{T}^4)^{Q_1Q_5}/S(Q_1Q_5)$. $\mathcal{M}$ is a hyper-kahler manifold and hence one can define a $N = (4, 4)$ SCFT. We can explicitly construct the $N = (4, 4)$ orbifold SCFT. The 4 marginal operators of the SCFT are the blowing up modes of the orbifold. Before we summarize the SCFT we list a few points regarding the validity of our considerations in the strong coupling region where $g_s Q_1 Q_5 \gg 1$.

- The instanton equation is derived as a condition from supersymmetry and it is independent of the coupling constant and $\alpha'$.

- The moduli space and the corresponding sigma model does not receive any corrections in the string coupling, because the hypermultiplet moduli space does not get renormalized by the interactions. This fact is crucial because it says that the SCFT that we found at weak coupling is valid at strong coupling. Hence we can use it to make comparisons with supergravity calculations of the entropy and temperature of the black hole and the Hawking rates corresponding to particles which are in the short multiplets of the $N = 4$ superconformal algebra.

### 6 N=4 SCFT on Sym($\tilde{T}^4$)

The $N = (4, 4)$ SCFT on Sym($\tilde{T}^4$) is described by the free Lagrangian

$$S = \frac{1}{2} \int d^2z \left[ \partial x^i_A \partial x_{i,A} + \psi^i_A(z) \bar{\partial} \psi^i_A(z) + \bar{\psi}^i_A(\bar{z}) \partial \bar{\psi}^i_A(\bar{z}) \right]$$  \hspace{1cm} (10)

Here $i$ runs over the $\tilde{T}^4$ coordinates 1,2,3,4 and $A = 1, 2, \ldots, Q_1 Q_5$ labels various copies of the four-torus. The symmetric group $S(Q_1Q_5)$ acts by permuting the copy indices.

The central charge of the SCFT is $c = 6Q_1 Q_5$. The N=4 algebra contains the global supergroup $SU(1, 1|2) \times SU(1, 1|2)$ which contains the bosonic subgroup $SL(2, R) \times SU(2)_R$. The subscript R indicates the R-symmetry. The theory also has an additional global symmetry group $SO(4)_I$. 


SU(1,1|2) has 8 real supercharges and hence the total number of supercharges is 16. This is in contrast to the fact that the blackhole solution even with KK charge \( N = 0 \) had only 8 SUSYS! This puzzle was resolved by Maldacena [15] who showed that the relevant supergravity solution is the so called near horizon geometry viz. \( AdS_3 \times S^3 \times \tilde{T}^4 \).

The conformal and R-symmetries of the SCFT become isometries of the near horizon geometry.

### 7 Matching SCFT Operators and SUGRA Moduli using Maldacena duality

The duality conjecture of Maldacena [15] as far as the symmetries are concerned states that the \( SU(1,1|2) \times SU(1,1|2) \) isometries of the near horizon geometry are matched with the global symmetries of the \( \mathcal{N} = (4,4) \) SCFT on \( \mathcal{M} \). Further the \( SO(4)_I \) algebra of \( T^4 \) is identified with \( SO(4)_I \) algebra of \( \tilde{T}^4 \). It is important to note that even though \( SO(4)_I \) is not a symmetry of \( T^4 \) it is useful to classify the low energy states.

The representations of \( \mathcal{N} = (4,4) \) can be classified in terms of the chiral primary operators which are specified by \( h = j \) (i.e, conformal dimension = spin). Similarly \( \bar{h} = \bar{j} \). The chiral primary is denoted by \( (2h + 1, 2h' + 1)_S \). For each chiral primary there corresponds a short multiplet obtained by the action of the global supercharges \( G_{1/2}^{11}, G_{-1/2}^{22} \).

All the chiral primaries for the orbifold conformal field theory (OCFT) on \( \mathcal{M} \) can be explicitly constructed by the product of the chiral primaries corresponding to the cohomology of the diagonal \( \tilde{T}^4 \) (the sum of all copies of \( \tilde{T}^4 \)) and the various k-cycle chiral primaries. The latter are in one-to-one correspondence with the cyclic subgroups of \( S(Q_1Q_5) \) which are characterized by the length \( k \) of the cycle. \( k = 1, 2, \ldots Q_1Q_5 \). The conformal dimension and spin of the chiral primary is \( (k - 1)/2, (k - 1)/2 \). Note that \( k_{max} = Q_1Q_5 \). This upper bound is called the stringy exclusion principle.

The SCFT under consideration has 5 \((2,2)\) short multiplets. These chiral primaries constitute 20 relevant operators of the SCFT. In the OCFT these operators belong to the the cyclic subgroup of length 2. 4\((2,2)\) short multiplets come from the untwisted sector and one from the \( Z_2 \) twisted sector. The corresponding top components of the \((2,2)\) short
multiplets are the 20 marginal operators. The 4 marginal operators that come from the $Z_2$ twisted sector are the 4 blowing up modes of the OCFT. One can show that the 20 marginal operators of the SCFT define the Zamolodchikov metric of the coset $\frac{SO(4,5)}{SO(4) \times SO(5)}$. The number of marginal operators is a topological invariant. This is because the number of chiral primaries with $(j_R, \tilde{j}_R) = (m,n)$ is the Hodge number $h_{2m,2n}$ of the target space $M$ of the SCFT. As far as the quantum numbers of the 20 marginal operators are concerned a distinction can be made only on the basis of their $SO(4)$ quantum numbers (See table below).

The SUGRA moduli in the near horizon geometry are classified using the formula:

$$h + \bar{h} = 1 + \sqrt{1 + m^2}$$  \hspace{1cm} (11)

and the global symmetry $SO(4) = SU(2)_I \times SU(2)_{\bar{I}}$. We see that marginal operators correspond to massless excitations in SUGRA. The 20 massless scalars (see table below) form the coset $\frac{SO(4,5)}{SO(4) \times SO(5)}$. Once again the distinct quantum numbers come from the global $SO(4)$ symmetry.

The matching of the marginal operators of the SCFT and the SUGRA moduli is summarized in the table below:

| Operator | Field | $SU(2)_I \times SU(2)_{\bar{I}}$ |
|----------|-------|----------------------------------|
| $\partial x^i_A(z) \bar{\partial} x^j_A(z) - 1/4 \delta^{ij} \partial x^k_A \bar{\partial} x^k_A$ | $h_{ij} - 1/4 \delta_{ij} h_{kk}$ | $(3,3)$ |
| $\partial x^i_A(z) \bar{\partial} x^j_A(z)$ | $b_{ij}'$ | $(3,1) + (1,3)$ |
| $\partial x^i_A(z) \bar{\partial} x^i_A(z)$ | $\phi$ | $(1,1)$ |
| $\mathcal{T}^1$ | $b_{ij}^+$ | $(1,3)$ |
| $\mathcal{T}^0$ | $a_1 C_0 + a_2 C_{6789}$ | $(1,1)$ |

$\mathcal{T}^1$ and $\mathcal{T}^0$ are twisted sector operators. $b_{ij}^+$ is the self dual part of $B_{NS}$. It is the modulus that leads to stable (non-marginal) bound states. $C_0$ and $C_{6789}$ are Ramond fields. In order to make a precise matching of the above we have related the blowing up modes of the SCFT coming from the twisted sector with the stabilizing moduli in supergravity.

8 Maximally Twisted Sector and Black Hole Hilbert Space

We now investigate the Hilbert space of the black hole states. The longest cyclic subgroup of $S(Q_1 Q_5)$ has length $Q_1 Q_5$ and leads to the the maximally twisted sector of the orbifold
SCFT, which is characterized by a chiral primary \( ((Q_1Q_5-1)/2, (Q_1Q_5-1)/2) \). Since this belongs to the longest cyclic subgroup there are no chiral primaries of higher spin. The presence of this twist field leads to twisted boundary conditions for the basic coordinates of the orbifold.

\[
X_A(e^{2\pi i z}, e^{-2\pi i \bar{z}}) = X_{A+1}(z, \bar{z})
\]

(13)

This implies that the momentum \( n_L, n_R \) in the twisted sector is quantized in units of \( 1/(Q_1Q_5) \), and hence the momentum quantum number can go up to an integer multiple of \( (Q_1Q_5) \).

The bh micro-states are defined by the level conditions

\[
L_0 = N_L \quad \bar{L}_0 = N_R
\]

(14)

It reflects the fact that the general non-extremal black hole will have Kaluza-Klein excitations along both the directions on the \( S^1 \).

The leading order (in large \( Q_1Q_5 \)) entropy formula corresponding to these level conditions is

\[
S = 2\pi \sqrt{n_L} + 2\pi \sqrt{n_R}
\]

(15)

\( n_L = Q_1Q_5N_L \) and \( n_R = Q_1Q_5N_R \). This also turns out to be the contribution to the entropy from the maximally twisted sector. Hence the leading contribution to the black hole entropy comes from the twisted sector. Hence we can assert that to leading order in \( Q_1Q_5 \), the black hole micro-states reside in the maximally twisted sector. The entropy formula readily enables a calculation of the Hawking temperature which agrees with SUGRA. The Hawking temperature is independent of the string coupling!

9 Hawking Radiation

Absorption cross-section of a supergravity fluctuation \( \delta \phi \) is related to the thermal Green’s function of the corresponding operator \( \mathcal{O} \) of the \( \mathcal{N} = (4, 4) \) SCFT on the orbifold \( \mathcal{M} \).

The absorption of a quantum \( \delta \tilde{\phi} = \kappa_5 e^{-ipx} \) corresponding to the operator \( \mathcal{O} \) is calculated using the Fermi’s Golden Rule and is related to the discontinuity of the thermal Green’s function \( \mathcal{G}(-i\tau, x) \) of the operator \( \mathcal{O} \). The Green’s function \( \mathcal{G} \) is determined by the two-point function of the operator \( \mathcal{O} \). This is in turn determined by conformal
dimension \((h, \bar{h})\) of the operator \(O\) and the normalization \(C_O\) of the two-point function.

\[
\sigma_{abs} = \frac{\mu^2 \kappa_2^2 L}{F} \int dt \, dx (G(t - i\epsilon, x) - G(t + i\epsilon, x)) \tag{16}
\]

Hence

\[
\sigma_{abs} = \frac{\mu^2 \kappa_2^2 L C_O (2\pi T_L)^{2h-1}(2\pi T_R)^{2\bar{h}-1} e^{\beta \cdot p/2} - (-1)^{2h+2\bar{h}} e^{-\beta \cdot p/2}}{2 \left| \Gamma(h + i \frac{p_+}{2\pi T_L}) \Gamma(h + i \frac{p_-}{2\pi T_R}) \right|^2} \tag{17}
\]

How does one fix the coefficient \(C_O\) within the microscopic theory? Presently we do not know how to do this. All that one can say is that this coefficient which is the normalization of the SCFT operators undergoes no renormalization and it is independent of the string coupling. We can determine this number by using the AdS/CFT correspondence.

The formula (18) can be applied to the minimal scalars and the coefficient \(C_O\) can be fixed by matching the zero temperature 2-point function of the minimal scalar \(h_{67}\) corresponding to the fluctuation of the metric of \(T^4\). \(C_O\) is the same for all the minimal scalars because the corresponding operators of the SCFT define the Zamolodchikov metric of the coset \(\frac{SO(4, 5)}{SO(4) \times SO(5)}\). \(C_O\) is then the relative normalization between the Zamolodchikov metric and the metric in supergravity of the same coset.

In this way we find the minimal scalar absorption cross section.

\[
\sigma_{abs} = 2\pi^2 r^4 \gamma \frac{\pi \omega}{2} \frac{\exp(\omega/T_H) - 1}{(\exp(\omega/2T_R) - 1)(\exp(\omega/2T_L) - 1)} \tag{18}
\]

Thus the SCFT calculation and the supergravity calculation of the absorption cross-section agrees exactly with the semiclassical result.

**Moduli and Hawking Radiation**

In the semiclassical calculation it is clear that absorption cross section is independent of the presence of vevs of the massless fields. It is possible to show that \(\square\) the same is true in the SCFT.

10 What’s new

It needs a lot of work to be able to correctly calculate the absorption crosssection and the Hawking rates which agree with the semi-classical supergravity calculations. The string
theory calculations were originally done in \cite{9,10} and were based on a model that was physically motivated by string dualities. In particular the calculation in \cite{10} based on the DBI action reproduced even the exact coefficient that matched with the semi-classical answer for the absorption cross section of the minimal scalars. However this method did not work when applied to the fixed scalars \cite{12}. This fact was very discouraging because it meant the absence of a consistent starting point for string theory calculations. The discovery of Maldacena \cite{15} finally enabled the string theory calculations \cite{13} because it was able to make a precise connection of the near horizon geometry with the infra-red fixed point theory of brane dynamics.

11 Open problems

Let us conclude by stating some interesting problems:

- How does one formulate the effective long wave length theory of the non-supersymmetric black holes?

- How does one derive space-time from brane theory? In particular is there a way of deducing $AdS_3 \times S^3$ (the infinitely stretched horizon) as a consequence of brane dynamics? The method of co-adjoint orbits applied to the SCFT is a promising approach. And what about the black hole horizon itself? These questions are intimately tied to explaining the geometric Bekenstein-Hawking formula or in other words understand the holographic principle \cite{16}.

- The D1/D5 system has relevant perturbations. It would be interesting to understand to study the holographic renormalization group in this situation. What is the end point of the RG flow?

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