An integrated energy conservation model in subway systems

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Abstract. Energy conservation is a hot topic in subway systems in recent years. Trains' traction energy consumption takes a great proportion of total energy consumption. To reduce it, this paper proposes an integrated optimization problem, which simultaneously optimizes the speed profile and running time at each subway inter-station. A mathematical model is formulated, and to solve it, an Artificial Bee Colony (ABC) algorithm is designed. Numerical examples based on the actual data from a subway system are simulated to prove the effectiveness of the proposed method.

1. Introduction

Energy conservation in subway systems is a hot research topic nowadays. As traction energy takes a great proportion, its reduction has attracted much attention [1–3]. The research on traction energy reduction is divided into two levels in the research history. The first one focuses on optimizing the speed profile of a train at a given inter-station with given running time. Based on this, the second one aims to optimize the allocation of running time at all inter-stations with the constraint of given travel time.

Ishikawa [4] first studied an optimal control problem in a straight level track. Howlett et al. [5] studied an optimization problem in a track with steep grades. Khmelnitsky [6] proved that the most energy-efficient driving strategy is consisted of 4 phases, i.e., maximum acceleration, cruising, coasting and maximum braking ones. Chuang et al. [7] optimized coasting speeds of a train, with the aim to reduce its energy consumption and passengers’ travel time.

All the aforementioned studies aim to obtain the optimal speed profiles with the constraint of given running time. But for a subway system, which is comprised of several inter-stations and stations, if the distribution of running time at the inter-stations is slightly adjusted, the total traction energy consumption may be reduced correspondingly. As the energy-efficient speed profile is closely related with the running time at an inter-station, they should be optimized at the same time to further reduce traction energy demand. But the research on their integrated optimization is relatively rare. Huang et al. [8] propose a two-level programming model to optimize traction energy consumption at multiple inter-stations. Albrecht et al. [9] review the problem of searching for the energy-efficient speed profiles for a train travelling on a track with steep grades considering a given travel time.

Integrated optimization of speed profiles and timetable is adopt in this work to reduce the traction energy demand. Then, to solve the non-linear optimization problem, an Artificial Bee Colony (ABC) algorithm is specially designed.
2. Mathematical Model

As shown in Fig. 1, suppose there are totally Y, N and L stations, platforms and inter-stations in a subway system, respectively. We label each station, platform and inter-station with \( y, n \in \{1, 2, \ldots, Y\} \), \( n \in \{1, 2, \ldots, N\} \), and \( l \in \{1, 2, \ldots, L\} \), respectively. Note that there are two platforms at each station, except the terminal station \( Y \) [10–12]. Thus, \( N = 2Y - 1 \). There is only one platform in terminal station \( Y \), labeled as platform \((N + 1)/2\). The platforms in station \( y \in \{1, 2, \ldots, Y - 1\} \) are labeled as platforms \( n \) and \( N - n + 1 \), respectively, by noting \( y = n \). The inter-station connecting platforms \( n \) and \( n + 1 \) is labeled as inter-station \( l \), by noting \( l = n \). Thus, we have \( L = N - 1 = 2Y - 2 \).

![Figure 1. Train travel process](image)

![Figure 2. Train running process at an inter-station](image)

Every train runs from platform 1, through each inter-station, and it arrives station \( Y \), where it turns around to the other direction. Then it runs until it arrives at platform N. According to [6], an energy-efficient speed profile is comprised by maximum traction, coasting and braking phases in a subway system. Thus, a coasting-control method is adopted in this work. A train departs from a platform with a traction phase, then it turns into a coasting phase when its speed reaches a predefined speed threshold (retraction speed threshold \( v' \)). Thereafter, when its speed reaches another predefined speed threshold (coasting speed threshold \( v'' \)), it turns into a traction phase again. Thus, the traction phase and coasting phase appears in turn, until the train speed reaches a speed limit to brake. Then, it turns into a braking phase and finally stops at the next platform. The detailed processes are illustrated in Fig. 2. A red dotted line represents a traction phase, a green solid one represents a braking phase and a purple dashed one represents a coasting phase.

From the above, we can obtain all the key time points for each train via the following procedure:

1) determine the time of the other key points for train \( i \) at inter-station \( l \) by that train’s start time at that inter-station:

\[
t^0_{il} = t^{u-1}_{il} + r^l_1, \quad u \in \{2, \ldots, U\}, \gamma = u - 1
\]

where \( \gamma \) is the running phase index, \( r^l_1 \) is the running time of phase \( \gamma \), \( t^0_{il} \) is the time of the \( u \)th turning point for train \( i \) at inter-station \( l \).

2) determine \( t^l_{il} \) by \( t^0_{il} \), i.e.,

\[
t^l_{il} = \begin{cases} 
\hat{t}^l_{ij} + x_{in} & n \in \{1, 2, \ldots, N-1\} \setminus \{\frac{N + 1}{2}\} \\
\hat{t}^l_{ij} + x_{in} + \Phi & n = \frac{N + 1}{2}
\end{cases}
\]

where \( \gamma \) is the running phase index, \( r^l_1 \) is the running time of phase \( \gamma \).

3) determine \( t^l_{i,0} \) by \( t^0_{i,0} \), i.e.,

\[
t^l_{i,0} = t^l_{i,1} + h_{i-1} = t^l_{i,0} + \sum_{j=1}^{i-1} h_j, \quad i \in \{2, \ldots, I\}
\]

where \( h_j \) is the headway time between train \( i \) and its following one \( i + 1 \).

From Fig. 2, running time \( r_i \) at inter-station \( l \) is obtained as \( r_i = \sum_{j=1}^{l-1} r^l_j \).
Travel time $\tau_j$ is obtained as:

$$ t_j = \sum_{a=1}^{N-1} x_{j,a} + \sum_{i=1}^{L} r_i^j + \sum_{l=1}^{J} F_l. $$

For convenience, we labeled all the traction phases and braking phases respectively, as follows:

$$ \Gamma_{\gamma,i}(t) = \begin{cases} 1 & t \in [t_{\gamma,i}^{U}, t_{\gamma,i}^{U+1})u \mod 2 = 1, u \in \{1, 2, \cdots, U - 2\} \\ 0 & \text{otherwise} \end{cases} \quad (4) $$

$$ \Gamma_{\gamma,c}(t) = \begin{cases} 1 & t \in [t_{\gamma,c}^{U}, t_{\gamma,c}^{U+1})u \mod 2 = 0, u \in \{2, 3, \cdots, U - 2\} \\ 0 & \text{otherwise} \end{cases} \quad (5) $$

The acceleration $a_i^\gamma(t)$ of a train at inter-station $l$ is determined by its running phase $\gamma$, i.e., when it is in a traction phase, its acceleration is determined by traction force $M$ and running resistance $R$; when it is in a braking phase, its acceleration is determined by braking force $B$ and running resistance; when it is in a coasting phase, its acceleration is determined by running resistance. It is shown as follows:

$$ a_i^\gamma(t) = \begin{cases} \frac{M - R_{\gamma,i}}{m} & \Gamma_{\gamma,i}(t) = 1 \\ \frac{-B - R_{\gamma,i}}{m} & t \in [t_{\gamma,i}^{U}, t_{\gamma,i}^{U+1}) \mod 2 = 1, u \in \{1, 2, \cdots, U - 2\} \\ R_{\gamma,i} & \Gamma_{\gamma,i}(t) = 1 \\ 0 & \text{otherwise} \end{cases} \quad (6) $$

Note that phase $\gamma$ of train $i$ at inter-station $l$ starts from time $t_{\gamma,i}^{U}$ to $t_{\gamma,i}^{U+1}$, where $\gamma = u$. When $t \in [t_{\gamma,i}^{U}, t_{\gamma,i}^{U+1}]$ train $i$ is in a braking phase at inter-station $l$. When it dwells at a platform, its acceleration is 0. Train running resistance $R$ is determined by basic running resistance and gradient.

Train speed $v_i(t)$ is determined as follows:

$$ v_i(t) = \begin{cases} 0 & t \in [t_{\gamma,i}^{U}, t_{\gamma,i}^{U+1}] \\ v_i^U + a_i^\gamma(t - t_{\gamma,i}^{U}) & t \in [t_{\gamma,i}^{U+1}, t_{\gamma,i}^{U+1}] \end{cases} \quad (7) $$

where $\gamma = u \in \{1, 2, \cdots, U - 1\}$. Thus, the kinetic energy of each train at any time is determined. Note that train speed at the time point when it turns into a coasting phase and re-enters a traction phase are all predetermined, i.e.,

$$ v_i(t = t_{\gamma,i}^{U+1}) = \begin{cases} v_i^U + a_i^\gamma(t - t_{\gamma,i}^{U}) & t \in [t_{\gamma,i}^{U+1}, t_{\gamma,i}^{U+1}] \mod 2 = 0, u \in \{2, 3, \cdots, U - 2\} \\ v_i^U & t \in [t_{\gamma,i}^{U+1}, t_{\gamma,i}^{U+1}] \mod 2 = 1, u \in \{2, 3, \cdots, U - 2\} \end{cases} \quad (8) $$

In addition, according to Newton's law of motion, a train's position change is determined by its speed, i.e.,

$$ s_i(t) = \int_{t^{\gamma,i}}^{t} v_i(t)dt, \quad (9) $$

where $s_i(t)$ is the position of train $i$ at time $t$. Thus, the running distance at inter-station $l$ can be obtained as:

$$ S_l = s_l(t_{\gamma,i}^{U}) - s_l(t_{\gamma,i}^{U+1}) \quad (9) $$

The electrical energy demand for accelerating a train is determined as follows:

$$ E_{\gamma,i}(t) = \begin{cases} \frac{1}{2} \left( (v_i^U(t))^2 - (v_i^U)^2 \right) & \Gamma_{\gamma,i}(t) = 1 \\ 0 & \text{otherwise} \end{cases} \quad (10) $$

where $\psi_i = m/2\eta$ is a constant, $m$ represents a train's mass, $\eta$ is the conversion efficiency from electrical energy to a train's kinetic energy.

Thus, the total traction energy demand $E^a$ is:
Based on the above analysis, we formulate an integrated optimization model to reduce trains’ traction energy consumption, as follows:

\[
E^a = \sum_{i=1}^{L} \sum_{l=1}^{L} E^a_{ij}(t)
\]  

(11)

where

\[
E^a = g(v^a, r)
\]

(12)

\[
V^c < v^a < V^r, \forall l \in \{1, 2, \cdots, L\} \tag{13}
\]

\[
V^c < v^a < V^r, \forall l \in \{1, 2, \cdots, L\} \tag{14}
\]

\[
v^a < v^a, \forall l \in \{1, 2, \cdots, L\} \tag{15}
\]

\[
V_{(x_o)} < V_{(x_o)}, t \in [\mu_{(x_o)}, U_{(x_o)}] \tag{16}
\]

\[
\sum_{l=1}^{L} x_{il} + \sum_{m=1}^{N} x_{i,m} + F \text{ N} l, i, j \in \{1, 2, \cdots, L\} \tag{17}
\]

\[
v^a, v^a, r, Z^a, l \in \{1, 2, \cdots, L\} \tag{18}
\]

where \(v^a = (v^a_1, v^a_2, \cdots, v^a_i, \cdots, v^a_L)\) and \(r = (r_1, r_2, \cdots, r_L)\) in (12) are the decision vectors, where \(v^a_i = (v^a_i, v^a_i)\). Objective (12) aims to minimize trains’ traction energy demand. Constraints (13) and (14) ensure that the coasting and re-traction speed thresholds are within a predetermined bound, respectively. (15) ensures the re-traction speed threshold is less than the coasting speed threshold. (16) ensure the safety of each train, i.e., its speed is less than the speed limit at any time or position. (17) ensures that the accuracy of a train, i.e., its travel time is not greater than the current timetable. (18) ensures the decision variables are positive integers.

3. Resolution method

A train’s traction energy and running time at an inter-station are both determined by a speed profile, and different speed profiles may have identical running time but different traction energy consumption. Furthermore, the coasting and re-traction speed thresholds are both required to be integers. Thus, the problem is solved via the following procedures:

**Algorithm 1 Obtain the Pareto optimal solutions at a section**

**Input:** \(V^c, V^r, V^a, V^a\)

**Output:** \(\{r_1, E_1, V^c_1, V^r_1\}\)

1: \(i = 0; \) %used to record the number of solutions
2: \(V^c_1 = V^c \) to \(V^r \) do
3: \(V^r_1 = V^c \) to \(V^a \) do
4: \(i = i+1; \)
5: \(r_1 = r_1; \)
6: \(V^c_1 = V^c_1; \)
7: \(V^r_1 = V^r_1; \)
8: \(E_1 = E_1; \)
9: \(r_1 = r_1; \)
10: end for
11: \%pick the Pareto optimal solutions for a section
12: \(i = 1 \) to \(i \) do
13: \(a = \min\{r_1\}; \) %Find the minimum \(r_1\) in \(\{r_1\}\)
14: \(\{(V^c_1, V^r_1, b)\} = \) all solutions and their corresponding energy whose running time equals \(a; \)
15: Choose the solution whose energy is the minimum in \(\{(V^c_1, V^r_1, b)\};
16: Record the chosen solution, its running and energy as a Pareto best one for Section \(i; \)
17: Output all the recorded running time, energy and their corresponding speed profiles as the Pareto front and Pareto optimal solutions.
1) Use an exhaustive search method to find the possible combinations of $v_i$ and $v_o$ for inter-station $l$.
2) Pick out the Pareto optimal solutions that have a minimum running time and traction energy demand for inter-station $l$. Its detailed procedure is shown in Algorithm 1.
3) Apply ABC to reallocate the running time. The detailed procedure can be referred to [10, 11].

4. Experimental results and analysis

Numerical examples are simulated with the real data obtained from a subway system in China. Table 1 shows its actual running time at an inter-station and dwell time at a platform, and Table 2 shows the other parameters of this line.

Table 1. Actual running time and dwell time of the experimental subway line

| DP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| $x_i$ (s) | 30 | 30 | 30 | 30 | 25 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 |
| AP | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| $r_i$ (s) | 129 | 98 | 117 | 135 | 139 | 84 | 128 | 141 | 136 | 124 | 83 | 140 | 132 | 117 | 96 | 119 |
| $r_o$ (s) | 119 | 88 | 107 | 125 | 129 | 74 | 118 | 131 | 126 | 114 | 73 | 130 | 122 | 107 | 86 | 109 |
| $r_i$ (s) | 139 | 108 | 127 | 145 | 149 | 94 | 138 | 151 | 146 | 134 | 93 | 150 | 142 | 127 | 106 | 129 |

Table 2. Other Parameters of the experimental subway line

| Parameter | Value | Parameter | Value | Parameter | Value | Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|-----------|-------|-----------|-------|-----------|-------|
| $I$       | 131   | $L$       | 9     | $Y$       | 9     | $N$       | 17    | $m$       | 287080 |
| $h$ (s)   | 482   | $\Phi$ (s) | 188   | $\tau$ (s) | 2576  | $M$ (N)   | 0.8   | $B$ (N)   | 0.6   |
| $V_i$ (km/h) | 79    | $V_i^c$ (km/h) | 50    | $V_i^a$ (km/h) | 78    | $V_i^u$ (km/h) | 25    | $\eta$ | 0.95 |

By applying the proposed solution method, the Pareto front of each inter-station is obtained first. To save pages, Inter-station 8 is chosen as examples to illustrate the results, as shown in Fig 3. It is easy to see that several potential solutions are available for an inter-station. There are also several possible solutions with the same running time as the current timetable, and the solution in the current timetable is not the optimal one on traction energy consumption. Points on the Pareto front are recorded as the basis for the reallocation of running time, as shown by the blue stars in Fig. 3.

Figure 3. Traction energy vs. running time at an inter-station

By using a Pareto optimal solution with the same running time as the current timetable at each inter-station, we are able to reduce traction energy consumption to 801.81 (kWh), which means 6.94% energy is saved compared to the current timetable.

Figure 4. Running time optimization process
Then, ABC is used to optimize the running time. The optimized running time and its corresponding traction energy consumption are shown in a red diamond in Fig. 3, with inter-stations 8 as an example. The optimization process is shown in Fig. 4, traction energy is reduced to 790.97 (kWh) finally. It is clear that ABC can obtain a good result and it converges quickly.

To sum up, traction energy is reduced dramatically after the first step of the proposed method, then it is reduced to the greatest extent after the second step, which proves its effectiveness.

5. Conclusion

An integrated optimization problem is proposed to reduce traction energy consumption in subway systems, where speed profile and running time are simultaneously optimized. An exhaustive search method and ABC are combined to solve the problem. Numerical examples based on the actual data obtained from a subway system are executed to show the effectiveness of the method. In the future, we plan to extend the problem by considering regenerative energy utilization to further saving energy.

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