We study a model, in which lepton number violation is solely triggered by a dimension 4 hard breaking term in the scalar potential. A minimal model which contains a SU(2) triplet with hypercharge $Y = 2$, and a pair of singlet doubly charged scalar fields in addition to the Standard Model (SM) Higgs doublet is constructed. The model is technically natural in the sense that lepton number violation is preserved in the limit that the hard term vanishes. SM phenomenology restricts the vacuum expectation value of the triplet scalar field $v_T < 5.78$ GeV. Neutrino masses controlled by $v_T$ are generated at the two loop level and are naturally to the sub-eV range. In general they exhibit normal hierarchy structure. Here the neutrino mass term does not dominate neutrinoless double beta decays of nuclei. Instead the short distance physics with doubly charged Higgs exchange gives the leading contribution. We expect weak scale singly and doubly charged Higgs bosons to make their appearances at the LHC and the ILC.

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It is now generally accepted that the three active left-handed neutrinos of the Standard Model have very small masses. However, the origin of this is still unknown. The orthodox view is they are arise from the seesaw mechanism employing one or more very massive right-handed neutrinos (> $10^{12}$ GeV). While this can be elegantly incorporated in some Grand Unified Theories such as SO(10) direct experimental tests of the mechanism is difficult if not impossible. There are attempts to lower the seesaw mass scale to the TeV range, but this usually entails complicated structures in the right-handed neutrinos with fine tuning embedded.

In this paper we investigate an alternative approach in which no right-handed neutrinos are involved. We follow the proposal first investigated in [1] that active neutrino masses arise from an extended Higgs sector of the SM without extending the gauge group. In this prototype construction a SU(2) singlet Higgs field with non-trivial hypercharge was used. Then doubly charged scalars were added later [2] to give more realistic neutrino masses. Similarly, we extend the Higgs sector by adding one SU(2) singlet and one triplet fields. We also postulate that the only lepton number violating interaction are of dimension four and resides in the extended Higgs potential. Together with additional Yukawa terms that are allowed by the symmetry of the model we can generate light neutrino masses without adding singlet fermions. Since there are no right-handed neutrinos in our construction, active neutrino masses must necessarily be of Majorana type.

Next we discuss the construction of our minimal model. Group theory dictates that SU(2) singlets and/or triplets be involved. Indeed we introduce a triplet $T(1,2)$ and a singlet $\Psi(0,4)$, where the bracket denotes SU(2) x U(1) quantum numbers. Then $T$ consists the fields $T^0, T^\pm, T^{\pm\pm}$ and $\Psi$ are doubly charged scalar fields $\Psi^{\pm\pm}$. We also assign lepton number $L = 2$ to $\Psi$ whereas $T$ fields carry no lepton number. With these ingredients the most general renormalizable potential is given by

$$V(\phi, T, \psi) = -\mu^2\phi^\dagger\phi + \lambda_\phi(\phi^\dagger\phi)^2 - \mu_T^2 Tr(T^\dagger T) + \lambda_T Tr(T^\dagger T) + m^2\Psi^\dagger T^\dagger T \Psi^\dagger + \kappa_1 Tr(\phi^\dagger T^\dagger T) + \kappa_2 \phi^\dagger T^\dagger T \phi + \kappa_3 \phi^\dagger T^\dagger T \phi + \rho Tr(T^\dagger T \Psi^\dagger \Psi) + \left[ \lambda(\phi^\dagger T^\dagger T \phi) - M(\phi^\dagger T^\dagger T \phi) + h.c. \right], \tag{1}$$

where $\phi$ denotes the SM Higgs doublet field. The lepton number violation is given by the last term in Eq. (1). In order to induce spontaneous symmetry breaking we take $\mu^2, \mu_T^2$ both positive. Minimizing the potential gives us the vacuum expectation values: $\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$ and $\langle T^0 \rangle \equiv \frac{v_T}{\sqrt{2}}$.

After SSB the W and Z bosons pick up masses at the tree level given by $M_W^2 = \frac{g^2}{4}(v^2 + 2v_T^2)$ and $M_Z^2 = \frac{g^2}{4 \cos^2 \theta_W} (v^2 + 4v_T^2)$ where we have used standard notations. The tree level relation $\cos \theta_W = g \sin \theta_W$ also holds. To get an estimate on $v_T/v$ we use the limit from the Particle Data Group $\rho = 1.002^{+0.007}_{-0.009}$ [3] and the W mass from the Fermi coupling. This gives approximately $v_T < 5.78$ GeV.

As we shall see next this is a controlling scale for neutrino masses. As can be seen later, the dimension three soft term also does not contribute to neutrino masses and by assuming no fortuitous cancellations in the minimal conditions we obtain $M \gtrsim 7.5$ TeV.

It is also clear from Eq. (1) that there are mixing among various Higgs fields. In particular $\Re \phi^0, \Re T^0$ pair will mix to give two physical neutral scalars $h^0, P^0$. The pair $3\Re \phi^0, 3\Re T^0$ will mix with one combination eaten by the Z boson leaving a physical psuedoscalar...
\[ T^a_\alpha. \] Similarly, for the charged states \( \phi^\pm, T^\pm \) one combination will be eaten by the \( W \) bosons leaving only a pair of singly charged \( P^\pm \) scalars. Finally the weak eigenstates \( T^{\pm\pm}, \Psi^{\pm\pm} \) will also mix to form physical states \( P_1^{\pm\pm}, P_2^{\pm\pm} \) of masses denoted by \( M_1, M_2 \) respectively and mixing angle \( \omega \). All the masses and mixing angles are free parameters and we can use them to replace the parameters in \( V(\phi, T, \Psi) \). They are to be determined experimentally.

Besides the usual Yukawa interactions of \( \phi \) with the fermions we also add the term \( Y_{ab} \sum R^b R^a \Psi \), which is lepton number conserving in our scheme and is allowed. Here \( a, b \) are family indices. On the other hand, the term \( L \) \( LT \) where \( L \) denotes a SM lepton doublet violates lepton number and is disallowed [6]. The absence of this variant derivatives of \( \phi, T, \Psi \) where the physical states. Clearly, applying these rotations in general does not diagonalize \( Y_{ab} \). Hence, we expect flavor violating couplings \( Y'_{ab} \) between families of right-handed leptons and the physical \( P^{++} \) states. Thus, the decay modes such as \( P^{++} \rightarrow \mu^+ e^+ \) must occur in general. On the other hand, \( P^\pm \) coupling to fermions is similar to SM but is scaled by \( (v_T/v) \).

Now we can calculate the active neutrino mass matrix. The leading contribution is given by the 2-loop Feynman diagram depicted in Fig. 1. We find

\[
\begin{align*}
\langle m_{\nu} \rangle_{ab} & \simeq \sqrt{2} g^4 m_a m_b v_T Y_{ab} \cos \omega \sin \omega \\
& \quad \times \left[ I(M^2_W, M^2_1, m_a, m_b) - I(M^2_W, M^2_2, m_a, m_b) \right] \\
& \equiv I(M^2_W, M^2_i, m_a^2, m_b^2) = \\
& \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m_b^2 - M^2_W} \frac{1}{q^2 - M^2_i} \\
& \quad \times \frac{1}{1} \frac{1}{q^2 - m_b^2(k - q)^2 - M^2_i}.
\end{align*}
\]

There is a generalized G.I.M. mechanism at work here. This can clearly be seen in the limit \( M_{1,2} > M_W \).

\[
I(M^2_W, M^2_i, 0, 0) \sim \frac{1}{(4\pi)^4} \frac{1}{M^2_W} \ln^2 \left( \frac{M^2_W}{M^2_i} \right). \tag{4}
\]

It is interesting to note that the \( m_{\nu} \) mass matrix is independent of the singly charged Higgs at the leading order. The lepton violating parameter is now hidden in the mixing angle \( \omega \). In the limit \( \lambda \to 0 \) then \( \omega \to 0 \) and lepton number is restored and neutrinos remain massless. This can also be seen by expressing Fig. 1 in weak eigenstates. The neutrino mass matrix is now proportional to the form

\[
\begin{pmatrix}
\chi^2 e_{ee} & \chi e_{e\tau} Y_{e\tau} & \chi e_{\tau\tau} Y_{\tau\tau} \\
\chi e_{e\tau} Y_{e\tau} & \chi^2 e_{\mu\mu} & \chi e_{\mu\tau} Y_{\mu\tau} \\
\chi e_{\tau\tau} Y_{\tau\tau} & \chi e_{\mu\tau} Y_{\mu\tau} & \chi^2 e_{\tau\tau}
\end{pmatrix}, \tag{5}
\]

where \( \chi_0 = m_2^2/M^2_W \) for \( a = e, \mu, \tau \). Barring fine tuning of \( Y_{ab} \) this is of the normal hierarchy type since the first row and column is much smaller than the rest due to the electron mass. Before we embark on an estimate of the size of the neutrino mass we list the experimental limits on the couplings \( Y_{ab} \). For flavor diagonal terms the most stringent ones are for the muon and electron sectors

\[
\begin{align*}
Y^2_{ee} & \lesssim 9.7 \times 10^{-6} \text{GeV}^{-2} M^2_{P^{--}}, \\
Y_{ee} Y_{\mu\mu} & \lesssim 2.0 \times 10^{-7} \text{GeV}^{-2} M^2_{P^{--}}, \\
Y^2_{\mu\mu} & \lesssim 2.5 \times 10^{-5} \text{GeV}^{-2} M^2_{P^{--}},
\end{align*}
\]

where \( M^2_{P^{--}} = \sin^2 \omega M^2_{1^{--}} + \cos^2 \omega M^2_{2^{--}} \) and we have updated the limits as given in [8]. The corresponding limits for \( \tau \) couplings are much less stringent. These limits are to the masses of the doubly charged scalars and their mixings. If we assume that the lighter one is a 500 GeV particle and the mixing is small, then we get \( Y_{ee} \lesssim 1.6 \).

For non-diagonal terms the limits are more stringent from \( \mu \rightarrow 3e \) and \( \mu \rightarrow e\gamma \) decays [8]. Explicitly

\[
\begin{align*}
Y_{e\mu} Y_{ee} & \lesssim 6.6 \times 10^{-11} \text{GeV}^{-2} M_{P^{--}}^2, \\
Y_{\mu\mu} Y_{ee} & \lesssim 1.5 \times 10^{-9} \text{GeV}^{-2} M_{P^{--}}^2,
\end{align*}
\]

where the second one is obtained from \( \mu \rightarrow e\gamma \), which is not as tight since it is a 1-loop process [8]. Similar constraints on the \( \tau \) are much weaker and are given below:

\[
\begin{align*}
Y_{e\tau} Y_{ee} & < 3.0 \times 10^{-8} \text{GeV}^{-2} M_{P^{--}}^2, \\
Y_{\tau\mu} Y_{e\tau} & < 3.0 \times 10^{-8} \text{GeV}^{-2} M_{P^{--}}^2, \\
Y_{\tau\mu} Y_{\mu\mu} & < 2.9 \times 10^{-8} \text{GeV}^{-2} M_{P^{--}}^2, \\
Y_{\tau\mu} Y_{ee} & < 2.9 \times 10^{-8} \text{GeV}^{-2} M_{P^{--}}^2. \tag{8}
\end{align*}
\]

We can now estimate the size of the \( \tau\tau \) element of \( m_{\nu} \). Explicitly,

\[
\langle m_{\nu} \rangle_{\tau\tau} \sim 0.55 \left( \frac{Y_{\tau\tau}}{T} \right) \left( \frac{\sin \omega}{0.1} \right) \left( \frac{v_T}{6 \text{GeV}} \right) \text{eV} \tag{9}
\]
where we used $M_1 = 0.5, M_2 = 1\text{TeV}$. This is well within the experimental bounds from tritium beta decay \cite{10}. We recapitulate the physics for the smallness of $m_\nu$. It arises firstly from a low triplet scale as required by phenomenology, secondly from the 2-loop factor and thirdly from the helicity flips in the internal charged lepton, which introduce two lepton masses, both of which are small. Clearly, the $ee$ element will be much smaller due to the electron mass suppression. Thus, the effect from $(m_\nu)_{ee}$ will not be detectable in the next round of neutrinoless double beta decay, $0\nu\beta\beta$, of nuclei experiments.

Next we examine the short distance physics itself in $0\nu\beta\beta$. The tree level Feynman diagram is depicted in Fig. (2). The amplitude is given by

\[
A_{tree} \sim \frac{g^4 Y_{ee}}{8\sqrt{2}} v_T \sin \omega \cos \omega \frac{1}{M_W^2} \left( \frac{1}{M_1^2} - \frac{1}{M_2^2} \right) \tag{10}
\]

On the other hand, the amplitude from neutrino mass is given by \cite{11}

\[
A_\nu \sim \frac{g^4}{16 M_W^2} \frac{m_{\nu ee}}{\langle p^2 \rangle} \tag{11}
\]

where $\langle p^2 \rangle \sim 0.01\text{GeV}^2$. From Eqs. (24) we can see clearly that $A_{tree} >> A_\nu$. This can also be seen diagrammatically. Fig. (2) can be obtained from Fig. (1) by cutting the internal lepton line and attaching the $W$ propagators to quarks. In doing so, the loop factors and the helicity flips, which are essential for $m_\nu$, no longer apply. Thus, the direct short distance physics is more important here, although the origins for $0\nu\beta\beta$ and $m_\nu$ come from the same source in the Higgs potential.

To understand the connection between neutrino mass and $0\nu\beta\beta$ decays better we examine a representative model in which TeV scale right-handed Majorana neutrinos are invoked to generated small masses to the active neutrinos via the seesaw mechanism. In this case it is found that the exchange of active neutrino contribution dominates the $0\nu\beta\beta$ amplitudes \cite{12}. Notice now $(m_\nu)_{ab}$ is generated at the tree level. A second example is given in \cite{13, 14} where a triple Higgs similar to our $T$ is introduced together with lepton violation in $\text{LLT}$ term. In this model $m_\nu$ is generated at tree level and the effective $W - W$-doubly charged Higgs coupling is proportional to $m_\nu$, allowing the neutrino mass exchange to dominate in $\nu\nu\beta\beta$ process. Our model gives an example, in which the short distance physics is now the leading term for $0\nu\beta\beta$ decays, since it is a tree level process. On the other hand, neutrino masses are dynamically generated in higher loops as well as Yukawa suppressed. Thus, we conclude that in general one cannot extract the value of the neutrino mass from the observation of $0\nu\beta\beta$ decays aside from nuclear physics uncertainties. However, a positive signal from these reactions clearly indicates that new physics of total lepton number violation is at work. To determine the source, be it heavy Majorana neutrinos or lepton violation in the scalar sector as in our model or other models, will require further experimentation. Indeed, current limit on this decay will set $Y_{ee} < 0.25$ for $M_1 = 500\text{GeV}$ and the parameters we used above.

One of the most spectacular signal of the model we presented is the production of the doubly charged scalars $P_{1,2}^{\pm\pm}$. At the LHC they can be produced via the $2W$ fusion process:

\[
u + u \rightarrow d + d + P^{++} \rightarrow d + d + \tau^+ \mu^+ \tag{12}
\]

and

\[
d + d \rightarrow u + u + P^{--} \rightarrow u + u + \tau^- \mu^- . \tag{13}
\]

The mechanism is the same as that depicted in Fig. (2) with obvious changes. The signature for both is a pair of jets plus two resonating leptons. All six combinations of lepton pair flavors are possible if no $Y_{ab}$ vanishes. The Drell-Yan production mechanism is much smaller here, since $P^{\pm\pm}$ do not directly couple to quarks. We expect the rate for Eq. (12) to twice that of Eq. (13) due to the larger u-quark content in proton. The lepton number violation in the final state will be unmistakable. If the polarizations of the leptons can be measured, they can be used to distinguish the $P$ from other models with doubly charged Higgs bosons. Another characteristics of the model is that $P_{1,2}$ will have small branching ratios into quark pairs. The expected production cross section is $\sim 1.5\text{fb}$ for $M_1 = 500\text{GeV}$ and the discovery limit at the LHC is $\sim 1.75\text{TeV}$ \cite{15}.

At the ILC it will be best to employ the $e^-e^-$ option. The $P_{\pm\pm}$'s can be produced at rest and decay into same sign lepton pairs with various flavor combinations. The signatures are clean and will give a direct measurement of the couplings $Y_{ab}$. Whether they are observable depends crucially on $Y_{ee}$ as well as energetics. If the only option at the ILC is $e^+e^-$ then the production of $P^{--}$ can proceed via

\[e^+ e^- \rightarrow e^+ l^+ + P^{--} \tag{14}\]

with the subsequent decay of $P^{--}$ into lepton number violating channels. The signature is equally spectacular.
We emphasize that the productions of $P^{\pm\pm}$ at the LHC and ILC will be direct checks on the mechanism for $0\nu\beta\beta$ decays of nuclei, if they are observed first. They are crucial for understanding the origin of neutrino masses.

Although the singly charged scalars $P^{\pm}$ play only a sub-dominant role in neutrino masses, they are an essential part of the model. The important feature that distinguishes $P^+$ from other charged Higgs model is the universal reduction of its Yukawa coupling to fermions. In particular the $t-b-P$ coupling is given by

$$\frac{\sqrt{2}G_F}{v^2}m_b \bar{b}_L t_R P^- + h.c. \quad (15)$$

This leads to its production at the LHC being dominated by the gluon b-quark fusion process:

$$b + g \rightarrow t + P^- . \quad (16)$$

The production rate will be analogous to that of two Higgs doublet models (2HDM) with large tan $\beta \sim 40$ with appropriate modifications to couplings. For a 400 GeV $P^+$ we expect the production cross section at the LHC to be $\sim 0.1$ pb. The decay signature will depend on the mass. If it is heavy enough to decay into a t-b pair then this will be the dominant mode. Otherwise; the main fermion decay mode will be into $\tau \nu$. Interestingly the helicity of the final state $\tau$ will be exclusively right-handed. This can be used as a diagnostic tool for distinguishing $P^{\pm\pm}$ from charged Higgs in the 2HDM.

We have constructed a model of hard lepton number violation in the scalar sector by extending the SM Higgs sector with a Higgs triplet and a singlet with two units of charged. We make essential use of the coupling term $\Psi_{E_RCR}$ to generate active neutrino masses at the 2-loop level, and at the same time $0\nu\beta\beta$ decays of nuclei are induced at tree level. This construction gives an example that these latter decays probe the short-distance physics of doubly charged Higgs exchange and not the exchange of light active neutrinos. We also sketched the phenomenology of this extended Higgs model at the colliders and found it to be different from that of the 2HDM.

LHC can play a crucial role in understanding the origin of neutrino masses if their governing scale is not the GUT scale. It is also clear that flavor violating decays of charged leptons should be pushed further.

In conclusion, we note that there is another model of hard lepton number violation that does not use doubly charged Higgs bosons. This model consists of a triplet $t^+ , l^0 , t^-$ with $Y = 0$ and a singlet $S^{\pm\pm}$ with $Y = 2$. Assigning $L = 0$ to the triplet and $L = 2$ to the singlet allows the term $LLS$ and the lepton number violation being triggered by $\phi^2 t_0 S$. This resembles the original Zee model for neutrino masses [1]. Detail study of this second model will be left for a future study.

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