Tracking of an underwater source using sparse method

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Abstract. The sparse method or better known as compressed sensing (CS), is a method often used for the signal reconstruction process. This method had considered better than conventional methods because it can reconstruct a signal with a smaller amount of data. Many algorithms had used for signal reconstruction using the CS method, including l1-minimization and orthogonal matching pursuit (OMP). In this study, the two algorithms were used for signal reconstruction of underwater objects and then compared to find out which algorithm is better for the signal reconstruction of underwater objects. Comparing the two algorithms had based on parameters in the form of PSNR and RMSE against sparsity. Based on the simulations that had being done, known that the l1-minimization algorithm can reconstruct signal up to 40% sparsity. Whereas the OMP algorithm can only reconstruct signals up to 30% sparsity. PSNR and RMSE generated from the l1-minimization algorithm show that this algorithm provides better reconstruction results than OMP for underwater object signals. The results obtained show that the best tracking process is at an angle of incidence of 90°.

1. Introduction

Currently, underwater acoustics is one of the fields of science that still requires much development. Underwater acoustics is a scientific discipline that studies sound in the water, generally in the ocean. Underwater acoustics not only studies the propagation of sound in the water but also learns about how sound signals not be detected due to interference and how signal processing had carried out to separate it from interference [1]. By studying this field of science, there is much research that can be done to develop underwater acoustic technology. One of these technological developments can be utilized in defense and security, such as acoustic instrumentation as an active and passive sonar system for surveillance and tracking applications [2].

Several studies on underwater tracking had been doing before. From these studies, it has been seen that data acquisition for underwater research was made in two ways, namely direct measurements at sea and indirect measurements in the laboratory. In practice, measuring directly at sea is not an easy thing to do. Apart from requiring a lot of equipment and costs, sea measurements usually give less accurate results due to loss during data transmission from the transducer source. It occurs because the propagation of sound in the sea depends on depth, temperature, and salinity [3]. With this loss, the transducer's signal is not the
same as the signal from the source. Thus, research on how to make the received signal the same as the source signal continues. One way is to use the sparse method, also known as compressed sensing.

Compressed sensing (CS) is a new data acquisition method superior to conventional methods [4]. In the conventional method, according to [5] and [6], a minimum amount of data or samples is needed to obtain high accuracy for reconstruction, known as the Nyquist-Shannon theorem. Based on the theorem, the sampling frequency taken is twice the desired signal frequency. Whereas in CS, the sample size can be smaller if the signal is incoherent and sparse [7]. Research on the use of CS in the field of underwater acoustics has been done previously by [8]. In that study, the algorithm used for signal reconstruction using the CS method is $l_1$-minimization. While in this study, signal reconstruction was done using the CS or sparse method using the $l_1$-minimization algorithm and the Orthogonal Matching Pursuit (OMP). The results were then compared with Peak Signal to Noise Ratio (PSNR) and Root Mean Square Error (RMSE). Therefore, it has seen which algorithm has better performance for underwater signal processing and can determine the direction of the tracking process's best angle.

2. Compressed sensing

Based on the conventional method, there are two stages in processing a signal: acquisition and compression stages [9]. Data acquisition is taking data and converting obtained data from the sensor into electrical signals that are then converted into digital form to be processed by a computer. Data acquisition is carried out based on the Nyquist-Shannon theorem, where the sampling frequency is twice the highest frequency of the desired signal or to be reconstructed. After obtaining sufficient and adequate data from the data acquisition stage, the following process is compression, which removes things that are not needed or not used. So that in signal processing with conventional methods usually takes a long time for signal reconstruction because it needs to go through the acquisition and compression stages. Besides, the two stages of data processing have opposite concepts. So, we need a method to obtain fewer data used to get a compressed representation of the sought signal. This method is then known as Compressed Sensing (CS).

Compressed Sensing (CS) is a method used to reconstruct a pressed representation of a signal using a limited amount of data. Mathematically, the CS method can be said to be a method for forming or obtaining a CS matrix from a system of linear equations using a random measurement matrix. The goal is to solve the system of linear equations obtained using optimization algorithms such as the $l_1$-minimization and greedy algorithms, where the system of linear equations written as follows [10],

$$y = Ax$$  \( (1) \)

where $y$ is the measurement vector with $A$ is the CS matrix. The linear equations system in equation (1) is obtained if the random measurement matrix $\Phi$ ($K, N$) is known. Matrix $\Phi$ ($K, N$) is a set or collection of random measurements selected from signal $f$ that has length $N$, where $K$ is the number of measurements taken randomly. Then, a measurement vector $y$ formed by defining it as follows,

$$y = \Phi f$$  \( (2) \)

where $f$ in the transform domain is equivalent to

$$f = \Psi x$$  \( (3) \)

with $\Psi = \Psi(N, N)$ is an orthogonal base matrix. Thus, equation (2) then written as

$$y = \Phi \Psi x$$  \( (4) \)

$\Phi \Psi$ can be denoted by $A$. Thus, the final system of linear equations obtained as in equation (1).

For CS to reconstruct a signal, two conditions that must be qualified are the signal is incoherent and sparse. If the incoherent conditions are qualified, it is assumed that the signal has a very large number of samples in the original domain [10]. Thus, sufficient information about the signal was obtained. Sparse or sparsity is a condition in which a signal has a small number of non-zero samples in a particular domain [10]. In other words, the signal has a predominantly zero sample. If it meets this sparse condition, then a
signal can be said to have a non-zero coefficient $K \ll N$ where $K$ is the number of components or signal coefficient, and $N$ is the signal length.

Equation (1) is a system of linear equations with the solution obtained from this equation is the sparse solution. There are three possible solutions: $k > n$, $k = n$, and $k < n$. A solution for the sparse vector $x$ of size $n \times 1$ is determined that the vector $y$ is of size $k \times 1$ with the matrix $A$ is the $k \times n$ matrix.

The most common problem with the CS method is when $k < n$, an underdetermined system. So that, in principle, there will be infinite solutions. For such cases, the most commonly proposed solution is to use the $l_2$-norm minimum. So, the problem will be as follows,

$$\text{minimize} \|x\|_2^2, \text{ subject to } A x = y$$

By using Lagrange multipliers and derivatives in solving these problems, the obtained analytical solutions are as follows,

$$x = (A^T A)^{-1} A^T y$$

However, in general, the solution obtained is not sparse.

In signals, it is usually necessary to transform a set of $y$ measurements into a sparse vector $x$. Then, the vector $x$ had used to reproduce $y$ precisely, or in specific applications, it is sufficient as an approximation. As previously discussed in [11], this often occurs in the case with wavelets, where most of the information contained in the $x$ coefficient, while other coefficients ignored. Thus, it is possible to consider an approach as follows,

$$\|Ax - y\| < \varepsilon$$

where some of the coefficients of $x$ are set to zero. It is a form of the sparse approach [11].

3. Methodology

3.1. Data acquisition of source position tracking

This research conducted using secondary data. The used secondary data are data obtained from the results of studies conducted by [8]. Data acquisition for tracking the sound source's position had carried out in the research pond of the Vibration and Acoustics Laboratory. The experiment was conducted by repeated the measurement setup by Dhany et al. [12]. An illustration of the pond had shown in Figure 1(a), and the data acquisition scenario is seen in Figure 1(b).

Figure 1. (a) 3D illustration of a research pond; (b) The scenario of tracking the position of a discrete sound source with a semi-circular trajectory [12].

The hydrophones were placed at coordinates (450, 250) and were in line with the underwater speaker. The sound source in the form of a submarine had turned on in a semicircular trajectory. The generation of the sound source done discretely and with variations in the angle of incidence of $0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, $150^\circ$, $180^\circ$, $210^\circ$, $240^\circ$, $270^\circ$, $300^\circ$, and $330^\circ$. The direction of the sound source had turned on one and two are moving in a semicircular path.
The number of hydrophones used was four, and the separation distance between each hydrophone was 30 cm.

The submarine had placed hovering in the pond according to its trajectory at an angle of 0°. Then with the submarine still in place, the propeller of the submarine turned on. When the propeller had turned on, a sound was produced. Then, the sound was recorded by the hydrophone for two seconds. The process was repeated for each predetermined variation of the angle and repeated twice.

3.2. Data processing using CS
After the audio data obtained, the next step was to process the data using the CS method using Matlab software. Based on [13], there are three stages in the CS method. The first stage is the sparsity transformation stage. In general, a signal is usually not naturally sparse. Therefore, it is necessary to carry out a sparsity transformation to obtain a sparse signal. The second stage is a sampling. The third stage is a signal reconstruction using the l1-minimization algorithm and Orthogonal Matching Pursuit (OMP).

3.3. Identification of the signal quality of the reconstruction results
In identifying the quality or similarity of a reconstructed signal to the original signal, several parameters are commonly used, including Root Mean Square Error (RMSE) and Peak Signal to Noise Ratio (PSNR).

3.3.1. Root mean square error (RMSE)
Root mean square error or better known as RMSE, is a calculation used to determine the reconstructed signal's quality. RMSE calculation had done by calculating the square root value of the mean for the difference in squares between the original measured signal and the reconstructed signal on the signal of a specific size. Mathematically, RMSE had written as follows [14],

\[
RMSE = \sqrt{\frac{1}{m \times n} \sum_{a=0}^{m-1} \sum_{b=0}^{n-1} [x_{\text{org}}(a, b) - x_{\text{rec}}(a, b)]^2}
\]  

(8)

where \(x_{\text{org}}\) is the original signal, \(x_{\text{rec}}\) is the reconstructed signal, and \(m \times n\) is the size of the signal \(x\). In this study, the signal size \(x\) is the signal length (N).

To determine the reconstructed signal's quality, the smaller the RMSE value obtained, the better the signal quality. In other words, the reconstructed signal will get closer to the original signal. Conversely, if the RMSE value obtained is higher, then the quality of the reconstruction signal will get worse. It means that the reconstructed signal is not similar to the original signal.

3.3.2. Peak signal to noise ratio (PSNR)
Peak signal to noise ratio or commonly known as PSNR is a calculation that is often used in signal processing to determine the quality of a reconstructed signal. PSNR is usually in decibels (dB). The PSNR calculation had done by calculating the ratio between the maximum value of the measured signal or the original signal and the root mean square error (RMSE) value. In this case, the RMSE value had assumed to be noise. Mathematically, PSNR formulated as follows [10],

\[
PSNR = 10 \log_{10} \frac{c_{\text{max}}}{\sqrt{\frac{1}{m \times n} \sum_{a=0}^{m-1} \sum_{b=0}^{n-1} [x_{\text{org}}(a, b) - x_{\text{rec}}(a, b)]^2}}
\]  

(9)

where \(c_{\text{max}}\) is the maximum value of the original signal, \(x_{\text{org}}\) is the original signal, \(x_{\text{rec}}\) is the reconstructed signal, and \(m \times n\) is the size of the signal \(x\). In this study, the signal size \(x\) is the signal length (N).

In determining the signal quality, the higher the PSNR value, the better the reconstructed signal quality and vice versa. Based on equation (2), the PSNR value will be higher if the error is small. It means that the reconstructed signal is close to the original signal.

4. Results and discussion
For the sparse method or CS to be used for signal reconstruction, the signal must be sparse in a particular domain. In this study, a time-domain signal was used. This signal has a significant coefficient value that is
more dominant. The signal is then transformed into the frequency domain using DCT to determine the sparse of the signal. In the frequency domain, the signal coefficient becomes zero dominant. The signal can be converted into a sparse matrix to prove or clarify the sparsity of the signal. The signal needs to be multiplied by a random measurement matrix to convert it to a sparse matrix. This matrix had obtained using the randn function \((K, N)\) in Matlab, where \(K\) is the number of nonzero coefficients and \(N\) is the signal length. The randn function in Matlab itself is a function that aims to return the \(K\)-by-\(N\) matrix from normally distributed random numbers [15]. This matrix is then processed using the sparse matrix operations on the Matlab.

4.1. Validation of the reconstruction algorithm with sine signals

In this study, the two algorithms used for reconstruction need to be validated to determine whether the algorithm qualifies for reconstruction. Before using the \(l_1\)-minimization and OMP algorithms to reconstruct underwater object signals, it is necessary to carry out a simple simulation using a sine signal as proof that both algorithms qualified for the signal reconstruction process. In this simple simulation, a sine signal without noise and with a noise 10 dB generated from the Matlab had used, namely \(x = \sin (2\pi f)\) with a frequency of 10 Hz to provides preliminary information about the sine signal. Simulations performed using Fast Fourier Transform (FFT) determine whether the two signals have a frequency of 10 Hz as defined. Then, reconstruction was performed for the sine signal without noise and sine signal with noise 10 dB using the \(l_1\)-minimization algorithm and OMP.

A simulation with FFT was carried out to determine whether the two algorithms can indeed be used for reconstruction. The results show that the necessary information from the signal in the form of frequency is not lost and can be reconstructed correctly. Thus, both algorithms are proven can be used for signal reconstruction. Furthermore, both algorithms are going to be used to reconstruct a more complex signal, namely the underwater object signals.

4.2. Reconstruction results of underwater object signals

Before reconstructing the underwater object signal with the \(l_1\)-minimization and OMP algorithms, it is necessary to validate the characteristics of the signal generated by the used source. In this study, the used source has a frequency of 1000 Hz. Therefore, simulations were carried out using FFT to determine whether the signal frequency received by the hydrophone is at 1000 Hz. If the signal had been reconstructed, it still has a frequency of 1000 Hz. The result shows that when reconstructed using either \(l_1\)-minimization or OMP, necessary information from the signal is not lost or exactly reconstructed because it shows 1000 Hz, just like the original signal. Thus, both of these algorithms can be used for signal reconstruction of underwater objects.

In this study, the original signal's length from an underwater object recorded for two seconds was 44100. However, the signal length cannot be processed using Matlab because it is too large and lacks memory to process data. So, the processed signal is a recorded signal which has been cut into 0.2 seconds with a signal length of 8800. In reconstructing an underwater object signal, the used sparsity is from 10% to 90% of the signal length. Based on the simulation results, it was found that signal reconstruction can only be done up to 40% sparsity for the \(l_1\)-minimization algorithm and up to 30% sparsity for the OMP algorithm. The simulation results for the angle of incidence of 90° and each algorithm can be seen in Figure 2 and 3.

Figure 2 shows the results of signal reconstruction obtained using the \(l_1\)-minimization algorithm. Part (a) shows the reconstructed sparse signal compared to the original sparse signal at 10% sparsity. It can be seen that the reconstructed signal is not the same as the original signal. While the original signal does not have a significant coefficient value or the coefficient is zero, the reconstructed signal has a significant coefficient value for the part (c), which shows the reconstructed original signal compared to the original signal at 10% sparsity. It appears that the reconstruction can be done, and the results are not good because the reconstruction signal indicated by the orange line does not follow the original blue signal pattern properly. Part (b) shows the reconstructed sparse signal compared to the original sparse signal at 40% sparsity. It can be seen that in this sparsity, the reconstruction of the signal gave good results because it has a pattern that is almost the same as the original signal. Part (d) shows the reconstructed original signal...
compared to the original signal at 40% sparsity. It can be seen that the reconstructed signal, indicated in the orange line, produces a signal pattern that follows or is almost identical to the original signal in the blue line. So, it can be seen that in this algorithm, the best reconstruction result is at 40% sparsity.

Figure 2. Simulation results of signal reconstruction at 90°: (a) Sparse signal at 10%, (b) Sparse signal at 40%, (c) Original signal at 10%, and (d) Original signal at 40%.

Figure 3. Simulation results of signal reconstruction at 90°: (a) Sparse signal at 10%, (b) Sparse signal at 30%, (c) Original signal at 10% and (d) Original signal at 30%.
Figure 3 shows the results of signal reconstruction using the OMP algorithm. Part (a) shows the reconstructed sparse signal compared to the original sparse signal at 10% sparsity. It can be seen that the reconstruction signal is not the same as the original signal. While the original signal does not have a significant coefficient value or zero, the reconstructed signal has a significant coefficient value. Compared to Figure 2(a), the reconstruction using the $l_1$-minimization algorithm gave better results than the OMP algorithm because of the smaller signal coefficient value.

Figure 3(c) shows the reconstructed original signal compared to the original signal at 10% sparsity. It appears that the reconstruction can be done. However, the results are not good because the orange line's reconstruction signal does not follow the original blue signal pattern completely. Compared to Figure 2(c), the reconstruction using the $l_1$-minimization algorithm gave better results than the OMP algorithm because the original signal has a range of coefficient values between 0 and 1. In contrast, the OMP algorithm has coefficient values outside the range of 0 to 1.

Figure 3(b) shows the reconstructed sparse signal compared to the original sparse signal at 30% sparsity. In this sparsity, signal reconstruction gave good results because the reconstruction results have a pattern that is almost the same as the original signal than at 10% sparsity. However, compared to Figure 2(b) with the minimum sparsity used, the reconstruction using the $l_1$-minimization algorithm gave better results than the OMP algorithm because of the smaller signal coefficient value.

Figure 3(d) shows the reconstructed original signal compared to the original signal at 30% sparsity. The reconstructed signal indicated by the orange line produces a signal pattern that follows or is almost identical to the original signal in the blue line. Therefore, in this algorithm, the best reconstruction result is at 30% sparsity. However, just like at 10% sparsity, compared to the $l_1$-minimization algorithm, the average reconstructed signal with OMP gave results with coefficient values outside the range 0 to 1. While the original signal is in the range 0 to 1 and the averaged signal reconstruction using $l_1$-minimization gave the coefficient values that match the original signal.

Following the results obtained in Figure 2, for the $l_1$-minimization algorithm, the signal pattern at 40% sparsity is closer to the original signal when compared to the 10% sparsity. It proves that the reconstruction signal's quality at 40% sparsity is better than 10% sparsity. So as for the OMP algorithm, in Figures 3, it can be seen that the quality of the reconstruction signal generated at 30% sparsity is better than 10% sparsity. Parameters such as PSNR and RMSE were used to strengthen those statements.

Sparsity is the ratio between the number of non-zero coefficients (K) to the used signal length (N) [16]. Therefore, when a K-sparse of 880 was used, the reconstruction process's sample is only 10% of the original signal length. PSNR value is proportional to the sparsity value, whereas the RMSE value is inversely proportional to the sparsity value. Graphs (a) and (b) in Figure 4 shows that the highest PSNR value was obtained at the angle of incidence of 90°. In comparison, graphs (c) and (d) shows that the smallest RMSE value was obtained at an angle of incidence of 90°.

PSNR value generated by reconstruction using $l_1$-minimization has a higher value than reconstruction using OMP. RMSE value generated by the $l_1$-minimization algorithm is smaller than that of the OMP. So from these two parameters, it is known that for the sparse method or CS, the $l_1$-minimization algorithm gave better results than OMP for the signal reconstruction of underwater objects. The highest PSNR and lowest RMSE values obtained for the $l_1$-minimization and OMP algorithms are at an angle of 90°. Therefore, the angle of 90° from the direction of arrival of the source is the best tracking process for underwater objects. Also, from the results that have been obtained, OMP has the advantage of being faster than $l_1$-minimization. However, the reconstructed signal is no more accurate than $l_1$-minimization.
Figure 4. PSNR graph on an angle of incidence for (a) $l_1$-minimization and (b) OMP. RMSE graph on an angle of incidence for (c) $l_1$-minimization and (d) OMP.

5. Conclusion
Signal reconstruction using the $l_1$-minimization algorithm produced the best signal quality when the sparsity is 40% in the angle of incidence of 90° with an error of 0.009. Signal reconstruction using the OMP algorithm produced the best signal quality when the sparsity is 30% in the angle of incidence of 90° with an error of 0.019. Should be compared as a whole based on PSNR and RMSE parameters, the $l_1$-minimization algorithm gave better results than the OMP algorithm for signal reconstruction of underwater objects.

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