Design and modelling of self-excited SRG and FM-SRG for wind energy generation

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Abstract
The paper discusses the design and analysis of synchronous reluctance generator with a rating of 2.1 kW. These generators with low power ratings may be a suitable candidate for the rural electrification. In synchronous reluctance generator, the design of the rotor is the most challenging part of the design. Here, the thickness of the axial and tangential ribs plays a significant role in its performance. The design procedure presented in the paper includes the effect of stator resistance, as it cannot be neglected in the design of synchronous reluctance generator with low power ratings. The paper discusses the design procedure of synchronous reluctance generator and analyses the self-excited ferrite-magnet synchronous reluctance generator. Various parameters of the designed machine are analysed through an analytical model and Ansys Electronics Desktop software. Further, the paper also includes the experimental validation of the synchronous reluctance generator results obtained through finite element analysis. The paper also proposes formulae to approximate the minimum values of the excitation capacitor requirement, for a self-excitation of synchronous reluctance generator with inductive load.

1 | INTRODUCTION

In recent years, the evolution of renewable energy sources such as solar, hydroelectric, wind energy, biogas, and geothermal energy has gained global attention. Wind energy has a very high potential to meet global energy demand. Many rural areas located in remote places in the world have good potential for wind energy. Moreover, it is costly to provide electricity to these areas through the power grid. Such a location can use the isolated self-stand generator to produce electricity [1]. The conventional stand-alone energy generation is a synchronous generator that requires a direct current field excitation. These types of energy-generating methods are not suitable for rural electrification, as they require high maintenance. Instead, a self-stand and self-excited generating system will be more convenient for such applications [2], as depicted in Figure 1(b).

For machines like synchronous reluctance machines and induction machines, self-excitation can be realized by the interconnection of excitation capacitors across the stator terminals, allowing them to be used as self-stand generators [3–6]. The self-excited induction generator (SEIG) offers certain advantages over a conventional synchronous generator as a source of isolated power supply, such as low cost, rugged structure, reduced size, the absence of a direct current source for field excitation, and low maintenance requirements [3,6,7]. However, the generated voltage frequency is significantly influenced by the connected loads and excitation capacitor banks. A self-excited synchronous reluctance generator (SRG) includes the advantages of SEIG. Moreover, its rotor copper losses and the output frequency is marginally influenced by the variation of the load [8,11].

Only a few mathematical models have been reported to evaluate the performance of a self-excited SRG in literature. Abdel-Kader et al. attempted to develop an equivalent circuit for the SRG by extending the idea of modelling of SEIG. Yawei Wang et al. [6] also attempted the analysis and modelling of self-excited SRGs. But, they have considered the resistive model of load in their study. Moreover, they have also neglected the effects of core losses and saturation effects, which has significantly reduced the developed model’s accuracy. Rahim et al.
In Chan et al. [11,26,27], a two-axis theory was used to model and analyse a three-phase self-excited reluctance generator which supplies to an isolated inductive load. However, they did not include a simple method to estimate the excitation capacitor. Nevertheless, these research papers are mostly based on conventional salient rotor SRGs, that is, no magnetic material or bridges in the rotor. Also, very few studies have been carried out on the analysis and design of FM-SRG. Hence, there is a need to develop a better model and analysis of FM-SRG. This paper tries to address these issues.

Wind turbine modelling, design, and an analytical model in the dq rotor reference frame are developed, as shown in Figure 1(a). It also includes the inductive load to evaluate the performance of the SRG and FM-SRG. The summary of the main contributions of this paper are:

- Algorithms are developed to design, model, and also include a detailed comparison of the FM-SRG and references SRG.
- Stator winding resistance \((R_s \neq 0)\) along with ideal case \((R_s = 0)\) are considered during electromagnetic torque analysis, while the existing literature on the generator or the motor design is based only on the ideal case \((R_s = 0)\).
- Further, a new and simple method to determine the minimum capacitance required by the generator to generate electricity is developed in the present work.

This paper is organized as follows. The second section presents the mathematical modelling of the wind turbine. The third section presents the design steps and analytical modelling. Section 4 presents the experimental implementation and FEA. The technical comparison of the machines is discussed in Section 5. The steady-state performance analysis of the generator is given in Section 6. The effects of wind speed and loads on the system performance are presented in Section 7 and conclusions are provided in Section 8.

## 2  WIND TURBINE MODEL

Among renewable energy sources on the earth, the generation of electricity from wind energy is more economical and environmentally friendly. Nowadays, there is a significant expansion in the use of wind for power generation. It results in the need for developing turbines and a generator of higher power ratings. Since the present work is more focussed on the development of low-cost, high-efficiency generators for rural applications, the rating of the turbine is in the range of kilo watts. The performance coefficient of the turbine in terms of the tip speed ratio (TSR) and blade pitch angle is given as [20,25]

\[
C_p(\lambda, \alpha) = C_1 \left( \frac{C_2}{\lambda_1} - C_3 \alpha - C_4 \right) \exp \left( -\frac{C_5}{\lambda_1} \right) + C_6 \lambda, \quad (1)
\]

where

\[
\lambda_1 = \frac{(\alpha^3 + 1)(\alpha + 0.08\alpha)}{\alpha^3 - 0.0028\alpha - 0.035\lambda + 1}. \quad (2)
\]

The turbine output power is given as (watts)

\[
P_m = \frac{R^5 \pi \rho_c C_p(\lambda, \alpha) \Omega^3}{2 \lambda^3}. \quad (3)
\]

Equations (1) and (2) reflect the relationship of the TSR, power coefficient, and pitch angle. Figure 2(a) provides the curves between the TSR and the power coefficient, where \(C_1, C_2, C_3, C_4, C_5, \) and \(C_6\) are 0.219, 119, 0.4, 6, 12.5, and 0, respectively. The value of \(\lambda\) has been selected for optimal point of \(C_p\) at \(\alpha = 0\) as shown in Figure 2(a). Once the values of \(\lambda\) and \(C_p\) are found, the turbine speed and the radius have been calculated from (3) and (4). The TSR can be expressed as:

\[
\lambda = R \Omega / v_{in}. \quad (4)
\]

The torque generated by the wind turbine shafts depends on the turbine rated power output and angular velocity. It can be expressed as

\[
T_m = P_m / \Omega = \frac{R^5 \pi \rho_c C_p(\lambda, \alpha) \Omega^2}{2 \lambda^3}. \quad (5)
\]
The gearbox used in wind turbine is utilized to transfer torque to the generator shaft rotating at a higher speed. The gear ratio is expressed as:

$$G_r = \frac{\omega}{\Omega},$$  \hspace{1cm} (6)

where, \(\omega\) and \(\Omega\) are the rotational speed of the generator and turbine, respectively. Equations (1) to (6) are used to design a wind turbine that can produce an output shaft power of 2.1 kW at a rated wind speed of 10 m/s. Table 1 summarizes the designed parameters of the wind turbine. The variation of power coefficient of the wind turbine for different blade pitch angle \(\alpha\) is shown in Figure 2(a). Figure 2(b), shows the generated mechanical power at different wind velocity. From Figure 2(b), it is observed that as the wind speed increases, the rotational speed of the turbine also needs to be increased to extract maximum power out of the turbine. It is also observed that for the designed wind turbine, the maximum power attains near 35 rad/s for the mean wind speed of 10 m/s.

**3 | DESIGN AND ANALYTICAL MODELLING OF SRG**

The subsection below presents the algorithms used to design the SRG.

### 3.1 | Design algorithm

The sizing procedure of the generators starts with assigning the initial critical parameters of a wind turbine, such as speed, power, and maximum torque. These assigned parameters are used in the calculations of magnetic, geometric, and electric parameters together with the analytical model of the generators. The design of the generator starts with the desired range of the output parameters, such as stator geometry and outer rotor diameter. If any of the output parameters after the initial design is not in the desired range, the designed procedure is repeated with some modification, and this process is continued until the objectives of the design are satisfied [12–14]. The analysis of the designed SRG is done using finite element (Ansys Electronics Desktop) software. It is used to analyse the generator’s performance related to the output functions such as torque ripple, electromagnetic torque, and the magnetic properties such as magnetic field \(H\) and magnetic field density \(B\). The process ends if results obtained from the Ansys Electronic Desktop software satisfy the design requirements. Otherwise, the process is repeated by updating the assigned parameters such as current and magnetic loading, pole pitch to air gap ratio, and stack aspect ratio to obtain the proper size (see Table 2). The procedure mentioned above is done for both machines SRG and FM-SRG.

The sequence of how the algorithm is executed for the design of the SRG is summarized as follows:
TABLE 2  Assigned parameters

| Parameters | Values   | Units |
|------------|----------|-------|
| $B_m$      | [0.5, 1] | T     |
| $k_d$      | [0.6, 1] | -     |
| $k_q$      | (0, 0.4] | -     |
| $L/	au$   | [0.6, 3] | -     |
| $V_w$      | [5, 25]  | m/s   |
| $J$        | [4.5, 9] | A/mm² |
| $k_w$      | [0.4, 0.7] | - |
| $\rho_a$  | 1.2      | kg/m³ |
| $Q$        | 36       | -     |
| $P$        | 2        | -     |

Step 1. The initial data (i.e. power, torque, and speed etc.) required for the design is obtained from the assigned and the calculated value of wind turbine parameters, as given in Table 1. Initial data is fed to an analytical model to get an estimate of the initial design parameters of the machine.

Step 2. Fix/assign parameters: some parameters can be fixed during calculations, and some of them should be revised during the design process. The summary of the assigned data parameter is shown in Table 2.

Step 3. Choose the rotor’s design parameter based on power, torque, size of the generator, and torque ripple. Equations (7) and (8) are used to get the rotor design parameters such as pole pitch, the outer diameter of the rotor, and stack length.

Step 4. Approximately assign the number of poles. Then search appropriate global and local points to design the rotor flux barrier, and interpolate the points to get a smooth ribs curve. Figure 3(a) shows the step involved in the design process.

Step 5. Evaluate the performance of the generator and compare the results with the desired values.

Step 6. If the performance of the generator is close to the desired values, go to step 9, else go to step 7.

Step 7. Optimizing the performance of the generator (i.e. flux noise, smoothen the EMF, and reduction of ripple) by updating the assigned design parameters.

Step 8. Go to step 6

Step 9. The design process is completed.

The design objectives for SRG and FM-SRG which need to be satisfied are given as:

- The rating of the machines should be 2.1 kW, with electromagnetic torque $\geq 12.75$ Nm, maximum torque ripple $\leq 12\%$.
- Ferrite magnet (FM) volume is $\leq 30\%$ of total of air barriers, maximum back-emf $\geq 145$ V for SRG, and $\geq 120$ V for FM-SRG.

3.2 | Analytical model of SRG

The analytical model uses the following simplifying assumptions:

- The time harmonics in current and space harmonics in air gap flux are neglected.
- Stator core losses are considered and rotor core losses are neglected.
- The core loss resistance is assumed to be constant and has no effect on excitation.

Pole pitch $\tau$, as given in the equation below, is the main parameter to obtain the outer rotor diameter of the generator.

$$\tau = \sqrt[3]{\frac{T_m k^2 \mu_a}{B_m^2 p^2 (k_d - k_q) \sqrt{L_d / L_q} k_s (1 + k_s) \tau (I / \tau)}},$$

where $T_m$, $k_s = 1.03$, and $k_q = 1.2$ are generator mechanical torque obtained from wind turbine modelling, Carter factor, and saturation factor, respectively. The outer rotor diameter, $D_r$, and stack length, $L_s$, are given as:

$$\begin{cases}
D_r = 2Pr / \pi \\
L_s = \tau \left( L / \tau \right)
\end{cases}$$
The parameters are defined in Table 2, while the saliency ratio is defined as \( L_{d}/L_{q} = (k_{d} - k_{q})/2k_{q} \).

The design of the stator core geometry, that is, the stator slot dimensions are shown in Figure 3(b), whereas, Figure 3(a) and Figure 3(b) show in detail the structure of rotor ribs. The distance of the rotor air gap ribs from the shaft radius is shown in the Figure 3(a). It also shows the separation of the edge of the ribs along the rotor’s inner core radius with an angle of \( \alpha_{w} \). The segments/points are selected and are interpolated to get the structure of the four-pole rotor (Figure 3(b)). Every air flux barrier consists of trapezoid-shaped segments with a tangential thickness, radial thickness, and the endpoint angle at the air gap (\( \alpha_{w} \)). These parameters are designed in such a way as to ensure the structural stability at the high speed with the required electromagnetic performance. The expression for maximum rotor tips mechanical end point angle \( \alpha_{w} \), in terms of number of barriers \( k_{w} \), poles pair \( P \), and floating angle \( \beta \) is given as [16]:

\[
\alpha_{w} = (\frac{\pi}{2P} - \beta) \left( k_{w} + 1/2 \right).
\]                      (9)

Here, the floating angle \( \beta \) lies between 0° and 10° (0 to \( \pi/18 \) rad). The \( d \)-, and \( q \)-axes components of ampere-turns are related to the total slot ampere-turns through the equation, as given below:

\[
mI_{w} = \sqrt{(mI_{d})^2 + (mI_{q})^2}.
\]                      (10)

Here,

\[
\begin{cases}
mI_{d} = B_{w} \pi (1 + k_{w})_{w}L_{Space}/3 \sqrt{2(qk_{w}k_{w}\mu_{0})} \\
mI_{q} = mI_{d}\sqrt{(L_{d}/L_{q})}
\end{cases}
\]                      (11)

where, \( k_{w} \) is stator slot winding factor, the present design uses \( k_{w} = 0.9598 \). The conductor per slot turns \( m \) is one of the critical parameters in the design, and it affects the parameters such as stator resistance, leakage inductance, and machine inductances. The resistance per phase \( R_{s} \), is:

\[
R_{s} = 2(L_{w} + L_{s})Pd/m\rho_{c}/L_{m},
\]                      (12)

where, \( L_{s} \) is end winding length, \( L_{w} = \pi \tau /2 \), and \( \rho_{c} \) is copper resistivity at 120°C. The leakage inductance \( L_{jk} \) is expressed as:

\[
L_{jk} = (\epsilon_{s} + \epsilon_{r} + \epsilon_{c})2L_{phm}^{2}\mu_{r}
\]                      (13)

Here, the calculated slot permeance \( \epsilon_{s} = 1.7564 \), air gap coefficient \( \epsilon_{r} = 0.25 \), and the end winding length coefficient \( \epsilon_{c} = 0.0011685 \). The magnetizing inductance \( L_{m} \), for uniform air gap, is given as:

\[
L_{m} = 6\mu_{0}\pi LP(qk_{w}m)^{2}/\pi^{2}L_{c}(1 + k_{c}).
\]                      (14)

| Table 3 Parameters expression |
|--------------------------------|
| Parameters | SRG | FM-SRG | Units |
| \( R_{s} \) | 1.597 \times 10^{-3}(m)^{2} | 1.362 \times 10^{-3}(m)^{2} | \Omega |
| \( L_{jk} \) | 2.001 \times 10^{-6}(m)^{2} | 1.514 \times 10^{-6}(m)^{2} | H |
| \( L_{m} \) | 1.308 \times 10^{-4}(m)^{2} | 1.006 \times 10^{-4}(m)^{2} | H |
| \( L_{d} \) | 1.39 \times 10^{-4}(m)^{2} | 1.205 \times 10^{-4}(m)^{2} | H |
| \( \phi_{q} \) | 0.2616 \times 10^{-4}(m)^{2} | 0.2006 \times 10^{-4}(m)^{2} | H |

Therefore, for double layer winding, the stator resistance, leakage inductance, magnetizing inductance, \( d \)-, and \( q \)-axes inductances are the function of \( m \). By simplifying the calculation, they are expressed as in Table 3:

Using the \( d-q \)-axes rotor reference frame, as shown in Figure 1(a), the equation of SRG in the transient state is given as below [24].

\[
\begin{cases}
V_{d} = R_{s}I_{d} + \rho \lambda_{d} - \omega P \lambda_{q} \\
V_{q} = R_{s}I_{q} + \rho \lambda_{q} + \omega P \lambda_{d} \\
V_{ph} = \sqrt{V_{d}^{2} + V_{q}^{2}}
\end{cases}
\]                      (15)

at steady state,

\[
V_{ph} = \sqrt{40 \times m},
\]                      (16)

\[
\begin{cases}
\lambda_{d} = I_{d}L_{d} + \Lambda_{resi} \\
\lambda_{q} = I_{q}L_{q}
\end{cases}
\]                      (17)

The above Equations (15) are written using the motor convention. When the machine operates in generator mode, they convert mechanical power into active power but require reactive power to magnetize their magnetic field paths. While operating in motoring mode, the \( I_{d} \) and \( I_{q} \) are of the same sign, while in generating mode, they have opposite signs. Referring to the sign of \( q \)- and \( d \)-axes currents, the motoring operations are in the first and third quadrants, that is, the motoring mode is when \( I_{d} > 0 \) and \( I_{q} < 0 \) or \( I_{d} < 0 \) and \( I_{q} > 0 \); \( T_{r} > 0 \). If \( I_{d} < 0 \) and \( I_{q} < 0 \) or \( I_{d} > 0 \) and \( I_{q} > 0 \), \( T_{r} < 0 \) implies generating mode, that is, the mode of operations are in the second and fourth quadrants. The reluctance torque developed by the SRG and FM-SRG which show the above mode of operations is given in Figures 5(c), 6(a), 6(b), and 6(c).

Neglecting the effect of stator resistance, the torque equation is given as:

\[
T_{r} = \frac{3P}{2}(\lambda_{d}I_{q} - \lambda_{q}I_{d}) = \frac{3P}{2}(L_{d} - L_{q})I_{d}I_{q}.
\]                      (18)

The equation for FM-SRG is similar to that of the synchronous reluctance machine with the magnetic flux terms included in the equation based on the orientation of ferrite material along the axes. The voltage equation for FM-SRG in the rotor reference
Excitation capacitance and load modelling

Thus, the total stator iron core mass is the sum of stator yoke \((w_y)\) and teeth \((w_t)\) masses, that is, \(w_{iron} = w_y + w_t\). For this machine, the values taken by the machine parameters are \(b_1 = 1.2\) mm, \(b_2 = 1.3\) mm, \(b = 1.6\) mm, \(b = 13\) mm, \(b' = 11.81\), and \(\rho_{\mu} = 7600\) kg/m³, which represent slot opening height, wedge height, width of slot, and iron density, respectively. The same equations and parameters are applied for FM-SRG and SRG, except for the stack height \((L_s)\), that is, 65 mm and 50 mm for SRG and FM-SRG, respectively. Thus, the core losses can be estimated as follows:

\[
P_{cop} = 3R_I L_{ph}^2.
\]

For machine with low power ratings, the core losses can be determined based on machine parameters. Therefore, for the calculation of iron loss, it is important to know the mass of stator yoke \((w_y)\) and teeth \((w_t)\), which are given by the equations as given below [5].

\[
\begin{align*}
w_y &= Q \left[ b_2 b' + b_2 (r_s - b_1 + b_2) \right] \left[ 2 + b_1 (r_s - b) \right] L_{\mu} \\
w_t &= \pi \left[ \left( \frac{D_t}{2} + b_1 + b_2 + b' \right)^2 \right] L_{\mu}.
\end{align*}
\]

3.2.2 Excitation capacitance and load modelling

The inductive load \(R_L, X_L\) is connected to a capacitor bank in shunt at the stator terminals. The equation which relates the
TABLE 5 Values of polynomial coefficients evaluated from curve fit

| Polynomial coefficient ($c, d$) for SRG | Polynomial coefficient ($a, b$) for FM-SRG | Values     | Values     |
|----------------------------------------|------------------------------------------|-----------|-----------|
| $c_0$                                  | $a_0$                                    | 0.5       | 0.005     |
| $c_1$                                  | $a_1$                                    | -96.7     | 29.3      |
| $c_2$                                  | $a_2$                                    | 104.1     | 42.8      |
| $c_3$                                  | $a_3$                                    | -22.1     | -11.7     |
| $c_4$                                  | $a_4$                                    | 2.1       | 7.2       |
| $c_5$                                  | $a_5$                                    | -0.1      | -0.06     |
| $c_6$                                  | $a_6$                                    | 0.002     | 0.001     |
| $d_0$                                  | $b_0$                                    | 2.2       | -0.4      |
| $d_1$                                  | $b_1$                                    | 57        | 1.5       |
| $d_2$                                  | $b_2$                                    | -4.3      | 6.4       |
| $d_3$                                  | $b_3$                                    | 0.1       | -1.5      |
| $d_4$                                  | $b_4$                                    | -        | -7.2      |
| $d_5$                                  | $b_5$                                    | -        | 0.1       |
| $d_6$                                  | $b_6$                                    | -        | -0.004    |

Note: The unit of Equations (35) and (36) is in mWb.

stator current, load current, and terminal voltages are presented as follows:

\[
\begin{align*}
I_{dc} &= -I_d - I_{dl} \\
I_{qc} &= -I_q - I_{ql}
\end{align*}
\]  
(26)

The excitation capacitance in rotor reference frame is as follows Equation (27):

\[
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix} = \begin{bmatrix}
\rho C & \omega C \\
-\omega C & \rho C
\end{bmatrix} \begin{bmatrix}
V_{dl} \\
V_{ql}
\end{bmatrix}.
\]  
(27)

By rewriting the above equations in terms of rate of change of $d$- and $q$-axis load voltages, the equation obtained is given, as shown below:

\[
\begin{align*}
\rho V_{ql, t} &= \omega PV_{dl, t} + (-I_q - I_{ql, t})/C \\
\rho V_{dl, t} &= -\omega PV_{ql, t} + (-I_d - I_{dl, t})/C
\end{align*}
\]  
(28)

Below are the equations governing the $RL$ load model (29):

\[
\begin{align*}
I_{ql, t} &= \int \frac{1}{L} (V_{ql, t} - I_{ql, t} R_{l, t} + \omega P L I_{dl, t}) dt \\
I_{dl, t} &= \int \frac{1}{L} (V_{dl, t} - I_{dl, t} R_{l, t} + \omega P L I_{ql, t}) dt
\end{align*}
\]  
(29)

Equation (29) is obtained using a balanced $RL$ load model (i.e. $V = R_{l} I + L_{d} \frac{dI}{dt}$). Table 4 summarizes the evaluated performance parameters using the analytical model of the designed SRG and FM-SRG.

FIGURE 4 (a) The drive motor and SRG. (b) Rotor core lamination and assemble. (c) Schematic diagram of the test bench

4 RESULTS OF FEA AND ITS EXPERIMENTAL VALIDATION

The present section includes the experimental validation of the proposed design of the SRG on a prototype of the SRG, as shown in Figure 4(a). The prototype is developed as per the proposed design specifications. Figure 4(b) shows the core lamination and Figure 4(c) shows the schematic diagram of the experimental setup. The performance of the designed SRG and FM-SRG is compared using FEA.

The maximum induced electromotive forces (EMFs) in the stator winding of the 2.1 kW SRG and FM-SRG with the symmetric design are shown in Figure 5(a). The induced voltage in the machine is given by Equation (19), which clearly shows that if the effective $q$-axis flux is reduced, it leads to a decrease in induced voltage. The results obtained through FEA simulation with the same excitation current of the machines is shown in Figure 5(a). It is observed that SRG has more induced EMF as compared with FM-SRG as predicted by Equation (19).

The peak values of back EMF produced at 1500 rpm versus the magnetizing currents of SRG and FM-SRG are shown in Figure 5(b). It is observed that, for SRG, EMF starts with
FIGURE 5 (a) Machines EMF at given frequency using FEA. (b) Machines EMF curves as a function of current using FEA and experimental test. (c) Torque verses power angle curves using FEA. (d) Cogging torque of FM-SRG (without excitation) using FEA.

zero voltage, whereas for FM-SRG, with zero magnetizing current, the induced voltage is approximately 42 V (peak) due to the presence of a magnet. But the voltage at saturation for FM-SRG is approximately 20 V less than the voltage of SRG. Figure 5(b) also includes the experimentally obtained EMF by the prototype of SRG developed in the laboratory, which is observed to be very close to that of the simulated (FEA) results.

The analytical equation of electromagnetic torque of SRG is given as:

$$T_e = \frac{3P}{2(P\omega)^2 L_d} E_{ph}^2 (1 - \varphi) \left[ (\varphi - \beta^2) \sin(2\gamma) - 2\beta (1 + \varphi) \sin^2(\gamma) + 2\varphi \beta \right], \quad (30)$$

where, $\beta = R_s / (P\omega L_d)$, $\varphi = L_q / L_d$. The torque versus power angle curves, as shown in Figure 5(c), are obtained using FEA. It is observed that the torque at zero power angle (i.e. $\gamma = 0^\circ$) is positive. This can also be observed using the analytical equation of torque as given by (30), that is, $T_e(\gamma = 0) = \frac{3P E_{ph}^2}{2(P\omega)^2 L_d} (1 - \varphi) \beta [((P\omega)^2 L_d (\varphi + \beta^2)^2]$. It is also observed that the maximum torque angle decreases due to the presence of stator resistance, which cannot be neglected for machines with small ratings. However, it is observed that the torque produced by the SRG and FM-SRG for $R_s \neq 0$ are comparable with little variations. The peak torque in generating mode is achieved at an angle close to $100^\circ$ of power angle for FM-SRG, and near to $94^\circ$ for SRG as shown in Figure 5(c). For both the machines, it is observed that for $R_s \neq 0$, the torques do not attain their maximum values at power angle of $45^\circ$ (i.e. $\gamma = 45^\circ$) [5].

If the effect of stator resistance is neglected, that is, $R_s = 0$, Equation (30) is given as

$$T_e = \frac{3P}{2(P\omega)^2 L_d} E_{ph}^2 (1 - \varphi) \varphi \beta \sin(2\gamma). \quad (31)$$

The above equation, that is, (31) can be rewritten as:

$$T_e = \frac{3P}{2(P\omega)^2 (L_d - L_q)} E_{ph}^2 (I_d - I_q) \sin(2\gamma). \quad (32)$$

Since, $E_{ph} / (P\omega L_d) = I_d$ and $E_{ph} / (P\omega L_q) = I_q$, the torque equation is rewritten as

$$T_e = \frac{3P}{2} (I_d - I_q) I_d I_q \sin(2\gamma). \quad (33)$$

The maximum torque power angle, $\gamma_m$ is obtained from equation given below

$$\begin{align*}
\frac{dT_e}{d\gamma} &= 0 \\
2\alpha \omega \gamma(2\gamma) &= 0.
\end{align*} \quad (34)$$

This implies that the maximum torque is obtained at $\gamma_m = 45^\circ$ or $135^\circ$. From Equation (31) it can be seen that the torque tends to zero at $\gamma = 0^\circ$ and $\gamma = 90^\circ$. 
Further, the cogging torque for the FM-SRG is shown in Figure 5(d). While for SRG, this phenomenon is not observed. The presence of cogging torque has a negative role in power generation, and hence, the machine with less cogging torque is more suitable for wind power generation.

Figure 6(a) represents the performance of the machines in motoring and generating mode. From Figure 6(a), it is observed that the average torque is as a function of the square of stator current till the current of the machine is 5 A. However, after 5 A, the difference between \(L_d - L_q\) is approximately constant. Hence, the variation of average torque is observed to be linear. It is required to have an FM-SRG to increase maximum average torque per ampere. However, the strength of the ferrite magnetic is limited by the permissible ripple torque in its design (i.e. nearly \(\leq 12\%\) of rated torque ripple considered in the design).

Figures 6(b) and 6(c) provide the comparison of electromagnetic torque of both machines with variations of \(\gamma\). From Figures 6(b) and 6(c), comparing the torque magnitudes, it can be seen that the torque of machine with FM is about 1% higher than the machine without FM. As the magnitude of average torque increases, there is an increase in the ripple torque for both machines but it is slightly higher for the FM-SRG (see Figures 6(b) and 6(c)).

Table 6, provides a comparative analysis of the presence of ripple torque in SRG and FM-SRG with an increase of stator current. It can be seen that the ripple torque is less in SRG as compared to FM-SRG.
Figure 7(b) shows the $q$- and $d$-axes flux linkages versus the $q$- and $d$-axes current, respectively. The curves are obtained from finite element analysis (FEA), and are verified through experimental tests on the prototype SRG machine. It is observed that the estimated FEA results for the $q$- and $d$-axes flux linkages are very close to experimental results. From Figure 7(b), it is observed that $\lambda_q$ and $\lambda_d$ increase linearly when $I_d$ and $I_q$ are less than 6 A. After that, $\lambda_d$ and $\lambda_q$ become almost constant due to saturation. It is also observed that for FM-SRG, due to the presence of magnet along the $q$-axis (i.e. the magnet produces a flux of 0.1 wb, see Figure 7(a)) get saturated at a low value of flux as compared with SRG.

It is observed that the $q$- and $d$-axes flux linkages are the function of armature currents. Hence, a polynomial model is used to represent the $q$- and $d$-axes flux for SRG, as shown below:

$$\begin{align*}
\lambda_d &= c_0 T_d^0 + c_1 T_d^1 + c_2 T_d^2 + c_3 T_d^3 + c_4 T_d^4 + c_5 T_d^5 + c_6 T_d^6 + c_7 T_d^7 + \cdots + c_n T_d^n + c_0 \\
\lambda_q &= d_0 T_q^0 + d_1 T_q^1 + d_2 T_q^2 + d_3 T_q^3 + d_4 T_q^4 + d_5 T_q^5 + d_6 T_q^6 + d_7 T_q^7 + \cdots + d_n T_q^n + d_0
\end{align*}$$

(35)

Similarly, the polynomial model for FM-SRG is given by the equation below:

$$\begin{align*}
\lambda_d &= a_0 T_d^0 + a_1 T_d^1 + a_2 T_d^2 + a_3 T_d^3 + a_4 T_d^4 + a_5 T_d^5 + a_6 T_d^6 + a_7 T_d^7 + \cdots + a_n T_d^n + a_0 \\
\lambda_q &= b_0 T_q^0 + b_1 T_q^1 + b_2 T_q^2 + b_3 T_q^3 + b_4 T_q^4 + b_5 T_q^5 + b_6 T_q^6 + b_7 T_q^7 + \cdots + b_n T_q^n + b_0
\end{align*}$$

(36)

For low or near-zero value of currents ($I_d$, $I_q$), the flux linkage $\lambda_d$ is near zero, whereas $\lambda_q$ is slightly negative due to the presence of $q$-axis magnets in FM-SRG.

The polynomial coefficients $c_0 - c_n$, $d_0 - d_n$, $a_0 - a_n$, and $b_0 - b_n$ are obtained from a curve fit of the actual $q$- and $d$-axes flux linkages as obtained from Ansys Electronic Desktop software FEA. The estimated coefficients are shown in Table 5. These equations are used to estimate the minimum residual flux required for the generator to produce terminal voltage. Further analysis of the self-excited SRG is given in Section 6, for an inductive load.

The expression of the inductance along the $d$- and $q$-axes for SRG, obtained through curve fitting of the polynomial is given as:

$$\begin{align*}
L_d &= g_0 I_d^0 + g_1 I_d^1 + g_2 I_d^2 + g_3 I_d^3 + g_4 I_d^4 + g_5 I_d^5 + g_6 I_d^6 + g_7 I_d^7 + g_8 I_d^8 + g_9 I_d^9 + g_{10} \\
L_q &= b_1 I_q^1 + b_2 I_q^2 + b_3 I_q^3 + b_4 I_q^4 + b_5 I_q^5 + b_6 I_q^6 + b_7 I_q^7 + b_8 I_q^8 + b_9 I_q^9 + b_{10}
\end{align*}$$

(37)

Similarly, the inductance’s expression along the $d$- and $q$-axes for FM-SRG is given as:

$$\begin{align*}
L_d &= e_0 I_d^0 + e_1 I_d^1 + e_2 I_d^2 + e_3 I_d^3 + e_4 I_d^4 + e_5 I_d^5 + e_6 I_d^6 + e_7 I_d^7 + e_8 I_d^8 + e_9 I_d^9 + e_{10} \\
L_q &= f_0 I_q^0 + f_1 I_q^1 + f_2 I_q^2 + f_3 I_q^3 + f_4 I_q^4 + f_5 I_q^5 + f_6 I_q^6 + f_7 I_q^7 + f_8 I_q^8 + f_9 I_q^9 + f_{10}
\end{align*}$$

(38)

where Equations (37) and (38) are referred to as SRG and FM-SRG, respectively. The polynomial coefficients $g_0 - g_9$, $b_0 - b_9$, $e_0 - e_9$, and $f_0 - f_9$.
$e_0 - e_i$, and $f_0 - f_i$ are obtained from a curve fit of the actual $d$- and $q$-axes inductances as shown in Table 7.

Figures 7(c) and 7(d) show the variation of the $d$- and $q$-axes inductances versus the $q$- and $d$-axes currents, for SRG and FM-SRG, respectively. The curves are obtained from FEA, and are verified through experimental tests on the prototype SRG machine. It is observed that the estimated FEA results for the $q$- and $d$-axes inductance are very close to experimental results.

From Figure 7(c), it is observed that the $L_d$ of the SRG keeps increasing for low values of current. However, as the current becomes approximately more than 5 A, the value of $L_d$ starts decreasing and attains a constant value due to saturation. While the $L_q$ of SRG is high for the low value of current, but due to the presence of a bridge in $q$-axis, it decreases gradually with the increase of current and subsequently reaches to a constant value.

From Figure 7(d), it is observed that the $L_q$ of the FM-SRG keeps increasing for low values of current. However, as the current increases to a certain value, it starts decreasing and subsequently reaches a constant value because of the iron saturation. The $L_d$ of FM-SRG remains constant at a low value of current, whereas it starts decreasing gradually when current is more than 4 A. Because of the presence of ferrite material in the $q$-axis, there is no flux linkage in $d$-axis without excitation current. But, the $d$-axis inductance is of high value at low current and starts decreasing as current increases. It remains almost constant when the current is more than 13.5 A.

5 TECHNICAL COMPARISONS OF SRG AND FM-SRG

The compactness, reliability, and efficiency are important issues related to power generation at remote locations [17]. The compactness of the machine helps in many ways, such as easy to transport, reduces installation cost, reduces weight, and reduces the cost of other parts of the structure. To develop a low power rating machine with the high power density and high efficiency is very challenging work. The authors design a low power rating machine with high power density using FEA. The key parameters, such as volume, mass, rated power, and torque of the machines, are utilized to design the machine, where the volume of the machine $V_{ml}$, can be expressed as:

$$V_{ml} = D_o^2 L \pi / 4.$$  \hspace{1cm} (39)

There are inherent advantages in utilizing SRGs in terms of machine performance and manufacturing. It also has high efficiency and high power density. SRG and FM-SRG are designed for a power rating of 2.1 kW. By optimizing the machines’ design parameters, the final design has the stack length of 65 mm for SRG and 50 mm for FM-SRG. The other initial and final (i.e. the optimal one), such as power density ($P_d$), specific power density ($P_{sp}$), torque density ($T_d$), and specific torque ($T_{sp}$) are provided in Table 8. It can be noted that the performance in terms of torque density ($T_d$) and specific torque ($T_{sp}$) is comparable for the two machines. Moreover, SRG is more robust, and its manufacturing cost is less as compared with FM-SRG while FM-SRG is more compact for the same rating.

Table 9 provides performance comparisons between self-excited SRG and FM-SRG generators. From the comparison, as shown in Table 9, it is observed that FM-SRG is more compact as compared with SRG, but their efficiency and torque/power densities are comparable. In terms of torque ripple, SRG is better than FM-SRG. Overall, it can be said that the design of SRG is more robust and less costly, but in terms of performance, it is slightly behind FM-SRG.

The cost and robustness are the primary criteria for the suitability of the generator for rural electrification applications. Hence, the SRG may be a better choice for rural electrification.

Figure 8(a) provides magnetic field density distributions and flux lines of SRG and FM-SRG generators. While Figure 8(b) shows the compactness/volume comparison between the two machines, from Figure 8(b), it is observed that FM-SRG is more compact as compared with SRG.
6 | STEADY-STATE PERFORMANCE ANALYSIS

The self-excitation of the SRG and FM-SRG requires the presence of residual flux in the field poles. A suitable rating of excitation capacitance is required to provide the reactive power support to the machine. It is observed that the FM-SRG requires slightly small excitation voltage due to the presence of the flux of magnet in the rotor. The critical value of the excitation capacitance can be obtained through the impedance load analysis. As the impedance load analysis is similar for both machines, the present work includes the detailed performance analysis only for the SRG [18,19,23].

The inductive load \((R_L, L)\), is connected in parallel to a bank of capacitors \(C\) as shown in Figure 9. Consequently, the equivalent impedance can be expressed as:

\[
Z_{eq} = -jX_c(R_L + jX_L) / (R_L + j(X_L - X_c)).
\] (40)

Taking into account the rationalization it yields,

\[
Z_{eq} = \frac{X_c R_L X_L}{(R_L)^2 + (X_L - X_c)^2} - \frac{jX_c X_L (X_L - X_c)}{(R_L)^2 + (X_L - X_c)^2}.
\]

Defining

\[
Y = X_c / (R_L^2 + (X_L - X_c)^2),
\] (42)

then \(Z_{eq} = Y R_L X_c + j Y X_L X_c - j Y R_L^2 - j Y X_L^2\). The voltage of impedance (loads) is

\[
V = -Z_{eq} I = (-Y R_L X_c + jY X_L (X_L + X_c) + jY R_L^2 + Y R_L (X_L - X_c))(I_d + jI_q)
\]

\[
= -Y (R_L I_d X_q + I_q R_L^2 + I_q X_L^2 - I_q X_L X_c))
\]

\[
- jY (R_L I_d X_q - I_d R_L^2 - I_d X_L^2 + I_d X_L X_c).
\] (43)

Therefore, the voltage equations are given as:

\[
\begin{cases}
V_d = R_L I_d - X_L Q_d = -Y (R_L I_d X_q + R_L^2 I_q + X_L^2 I_q - X_L X_q I_c)
\end{cases}
\]

\[
V_q = R_L I_d + X_L Q_d = -Y (-R_L^2 I_d - X_L I_d)
\]

\[
+ X_L I_d + R_L I_c.
\] (44)

By elimination of \(I_d\) and \(I_q\) from Equations (44) and rearrangement of terms yield the following quadratic equation.

\[
(Y R_L X_c + R_L)^2 = (-Y(R_L^2 + X_L^2 - X_L X_c) + X_q)(Y(-R_L^2 - X_L^2 + X_L X_c) + X_q).
\] (45)

The condition required for the self-excitation of SRG is obtained by maintaining the active power balanced and equating the reactive power to zero, that is, \(P_2 = P_{cop} + P_{qext} + P_{load}\) and \(Q_c - Q_d - Q_{ext} = 0\). The equation below is obtained by substituting Equation (42) in Equation (45):

\[
\frac{X_c^2 (X_c - X_d) - X_c X_d (X_c - 2 X_d) + R_L^2 (X_c - X_d) - X_c^2 X_d}{R_L (R_L^2 + X_c^2 - 2 X_c X_d + X_c^2) + R_L X_c^2} = 0.
\] (46)
Solving (46), for $X_1$, the minimum value of the capacitor for self-excitation can be achieved at different speeds, which gives four resonant points in the self-excitation of SRG (i.e., with $X_j = \omega L_d$, $X_q = \omega L_s$ and $X_1 = 1/\omega C$). The solutions obtained are given below:

$$
\begin{align*}
X_1 &= 1/\omega C = X_L + jR_L, \\
X_1 &= 1/\omega C = X_L - jR_L,
\end{align*}
$$

(47)

$$
\begin{align*}
X_1 &= 1/\omega C = 1/ \left[ 2 \left( R_2^2 - 2R_4R_2 + R_3^2 + X_d^2 + X_LX_q + X_1X_q + X_1X_d \right) \\
&+ \left( R_4^2 \left( X_d^2 - 4X_d^2X_q - 4R_2X_dX_q + X_1X_q^2 - 2X_1X_dX_q + X_1^2X_q^2 - 2X_1^2X_dX_q - 2X_1^2X_d^2 - X_1^2X_d^2 \right) + 8R_2X_1X_d \right) \right]^{1/2} \\
X_1 &= 1/\omega C = 1/ \left[ 2 \left( R_2^2 - 2R_4R_2 + R_3^2 + X_d^2 + X_LX_q + X_1X_q + X_1X_d \right) \\
&+ \left( R_4^2 \left( X_d^2 - 4X_d^2X_q - 4R_2X_dX_q + X_1X_q^2 - 2X_1X_dX_q + X_1^2X_q^2 - 2X_1^2X_dX_q + X_1^2X_d^2 - X_1^2X_d^2 \right) + 8R_2X_1X_d \right) \right]^{1/2}
\end{align*}
$$

(48)

Since $L_d$, $L_s$, and $R_s$ are parameters of the generator that have been obtained from the designed machine as shown in Table 4 and Equation (37), therefore solving (46) gives the minimum value of the capacitive reactance required for self-excitation of the SRG generator. It is observed that while solving Equation (46), four solutions with two real roots and two imaginary roots (see Equations (47), (48), and (49)) are found. In the case of imaginary roots, no excitation is possible, and the generator cannot achieve the rated voltages (Equations (47)), while in the case of real roots, we calculate the capacitive reactance given by the roots of Equations (48) and (49). Since we are interested in the minimum value of the capacitor, the root that provides maximum reactance is considered. The plot of the estimated capacitor as a function of speed corresponding to the minimum real roots for a load of $R_L = 18$ $\Omega$, $L = 80$ mH, and $R_L = 250$ $\Omega$, $L = 30$ mH is provided in Figure 10(a).

From Figure 10(a), it is also observed that the curves are more focussed on the solution which give minimum capacitance value with the variation of the speed. Since we are interested in finding the minimum requirement of the capacitor, hence, we have discarded the other solution and use a solution with a minimum capacitor. At approximately 62.5 rad/s, the value of the capacitor used is 0.00425 F, and for 500 rad/s, it is about to 0.00018 F.

Figure 10(a) also provides a comparison of the terminal voltage versus speed with the value of minimum capacitance. It is also observed that the minimum capacitance required at a particular speed is also a function of the inductive load. This is shown by two different curves of minimum capacitance corresponding to the loads $R_L = 18$ $\Omega$, $L = 80$ mH, and $R_L = 250$ $\Omega$, $L = 30$ mH.

It can also be observed that the terminal voltage depends on the value of capacitor and speed, that is, by increasing the capacitor or the speed, the terminal voltage increases or vice versa.

For the effective use of SRG to generate power, it is important to have some ways of varying the capacitor to maintain the magnitude of the terminal voltage. An economical way is to design an algorithm that picks the turbine speed and the value of the capacitor to improve the overall efficiency of the system.

In a self-excited system, the reactive power is provided by the capacitor's bank. For inductive load, a reactive power balance is achieved between the generators, capacitors, and inductive load, as shown in the following equations.

$$
\begin{align*}
P_g &= P_{cop} + P_{iron} + P_{load},
\end{align*}
$$

(50)

where $P_g$, $P_{cop}$, $P_{iron}$, and $P_{load}$ are obtained from the FEA simulation. $P_{load}$ and $Q_j$ are calculated from the generated voltage, which is also derived from the analytical model and FEA simulation. Figure 10(b) provides the net reactive power supplied to the generator (i.e., $Q_j = Q_j - Q_0$), and the reactive power drawn by the windings of the generator (i.e., $Q_0$) with increasing load current $I_L$. For obtaining the plots, it is considered that the generator is rotating at 1500 rpm, connected with an inductive load of $R_L = 250$ $\Omega$, $L = 3$ mH. From Figure 10(b), the SRG is able to
generate power, provided \((Q_i - Q_l) \geq Q_{0i}\), thus from the plot it can be observed that for the capacitor with a capacitance of 64 \(\mu F\), the SRG will not be able to produce active power, whereas, with capacitor with capacitance 335 \(\mu F\), it will generate active power till the load current is less than approximately 13.5 A. Similarly, for \(C = 189 \mu F\), it will generate power until the load current is less than 6 A.

The similar analysis can be done for FM-SRG, only the voltage equation (44) gets slightly modified as shown below:

\[
\begin{align*}
V_d &= R_l I_d - X_{lq} I_q + \Lambda_{res} \omega = -Y R_l X_{li} I_d + X_{lq}^2 I_q - X_{li} X_{li} I_d \\
V_q &= R_l I_q + X_{lq} I_d = -Y (-R_{li}^2 I_d - X_{lq}^2 I_q + X_{li} X_{li} I_d + R_{li} X_{li})
\end{align*}
\]

(51)

### 6.1 Residual flux linkage for self-excitation

The rating of the excitation capacitor is a function of the operating speed range. It is important to have some residual voltage along with the capacitor to generate voltage to its required value. The developed prototype of SRG fails to generate a voltage in its first run, due to absence of the residual flux in the rotor core, hence, in case if there is no residual flux, it is important to magnetize the core at electric speed. In the generator, the value of air gap flux due to FM is 0.01 Wb. The residual flux linkage \(\Lambda_{res}\) and \(\Lambda_{resm}\) in the rotor core are one of the crucial factors to determine self-excitation in the generators. The residual flux linkage in air gap can be expressed as:

\[
\Lambda_{res} = E_f / \omega P,
\]

(52)

where \(\omega P\) is electrical speed, and \(E_f\) is an EMF required to magnetize the SRG core at electric speed. Meanwhile, the residual flux of FM-SRG can be calculated as:

\[
\Lambda_{resm} = E_m / \omega P,
\]

(53)

where \(E_m\) is an EMF required to magnetize the FM-SRG core at electrical speed. In the generator, the value of air gap flux due to FM is 0.1 Wb. The residual flux linkage \(\Lambda_{res}\) and \(\Lambda_{resm}\) can be determined from the EMFs at a given speed.

The capacity of the SRG with residual rotor magnetism can be examined. In the analysis, a 190 \(\mu F\) capacitors bank, without load, was used at the terminals of the SRG. The voltage/current was used to produce residual magnetism. The rotor speed was slowly changed during the analysis. It is observed that the current has the maximum value at the resonance point (i.e. \(X_r = X_l\)), which gives maximum residual flux. The current is increased until the process of self-excitation/resonance point then it starts decreasing. The equation of the excitation current, \(I_{ex}\), can be expressed as:

\[
I_{ex} = \omega P \Lambda_{res} / \left( R_i + j(X_r - X_l) \right).
\]

(54)

From Equation (54) the current at resonance \((X_r = X_l)\), \(I_r\), is defined as:

\[
I_r = \Lambda_{res} / R_i \sqrt{L_s C},
\]

(55)

where, \(X_r\) is generator reactance, \(\omega P = 2\pi f\), and at resonance point \(f = 1/2\pi \sqrt{L_s C}\), that gives \(\omega P = 1/\sqrt{L_s C}\).

The current at resonance value can be used to decide whether the SRG with certain residual flux can be self-excited or not. In fact, the current \(I_r\) produces the minimum air gap flux linkage required for self-excitation.

### 7 EFFECTS OF WIND SPEED AND LOAD ON THE SYSTEM PERFORMANCE

In this section, the simulation has been carried out in MATLAB/Simulink using the design parameters to observe the dynamic behaviour of the system under various conditions such as variation of wind speed, constant wind speed, and variation of the load for SRG and FM-SRG.

The performance of the SRG and FMSRG under fixed wind speed of 10 m/s and an inductive load \((R_L = 250 \Omega, L = 30 \text{ mH})\) with an excitation capacitor of 190 \(\mu F\) are shown in Figure 11(a). As the mechanical torque from the wind turbine is applied, the rotor speed of the generator increases to the rated speed of 1500 rpm. For constant wind speed, the voltage and current plot of SRG and FM-SRG are shown in Figure 11(a). It is observed that the steady state is reached in about 1 s for both machines.

The effect of a sudden change in wind speed is simulated to get the dynamic response of the SRG and FM-SRG. Here, the wind speed suddenly drops to 4.98 m/s. This results in the falls of the generators rotor speed from the rated values of 157.08 rad/s (1500 rpm) to 67.54 rad/s (645 rpm), as shown in Figure 11(b) for SRG and FM-SRG. Under this condition, the generated voltage and current decrease from an initial value of 149.1 V to a new value of 92.8 V for SRG. Similarly, for FM-SRG the voltage drops from 121 V to a new value of 65.5 V. The current supplied by both machines also decreases. At \(t = 6.5\) s, again, the wind speed is increased to its previous value.

The performance of the SRG and FM-SRG under the change of load are presented in Figure 11(c). The inductive loads instantly decrease from 258 \(\Omega\), 30 mH to 150 \(\Omega\), 10 mH at 5 s, and then increases back to the initial impedance load values at \(t = 7\) s. At 5 s, the reduction in both the stator currents and voltages is observed, that is, the voltage slightly changes from the steady-state value of 149.1 V for SRG, and it drops from 121 V to 118 V for FM-SRG. Similarly, the stator current decreases
FIGURE 11  (a) Wind-driven voltage, and current waves for FM-SRG and SRG at constant of $v_\omega = 10 \text{ m/s}$ with $R_L = 250 \Omega$, $L = 30 \text{ mH}$ for $C = 190 \mu \text{F}$. (b) Effects of falls in wind speed of SRG and FM-SRG from $v_\omega = 10 \text{ m/s}$ to $v_\omega = 5.4 \text{ m/s}$ with $R_L = 250 \Omega$, $L = 30 \text{ mH}$ for $C = 190 \mu \text{F}$. (c) Effects of sudden change in load of SRG and FM-SRG at constant of $v_\omega = 10 \text{ m/s}$ with $Z_L = 258 \Omega$ to 150 $\Omega$ for $C = 190 \mu \text{F}$.

FIGURE 12  Performance of voltage and current waves for FM-SRG and SRG at variable wind speed with $R_L = 250 \Omega$, $L = 30 \text{ mH}$, and $C = 190 \mu \text{F}$ whose frequency and voltage are highly influenced by changing load conditions. From Figures 11(c), it is observed that the change in load for SRG and FM-SRG slightly changes the current and voltage as compared to the SEIG.

The performance of the SRG and FM-SRG under the variable wind speed is shown in Figure 12. The inductive load is kept constant at 250 $\Omega$, 30 mH, and the wind speed is varied, as shown in Figure 12. This variation of wind speed was observed from a location in India over a period of 1 h, 40 min. But to show the effect of wind variation on the machine’s performance, the duration of variation was mapped to a time of 10 s purely for simulation. Figure 12 shows the variation in the stator current, voltage, and rotor speed. It is observed that the variation in current FM-SRG is slightly more as compared with SRG. But, as the practical generator will have a speed controller, which will regulate the rotor speed, this kind of voltage and current fluctuations will not be observed for a practical SRG or FM-SRG.

8 | CONCLUSION

The paper presents in detail the design and analytical modelling of reluctance and FM reluctance generators. The performance, such as compactness, reliability, and efficiency of these generators, is compared in the paper to determine the merits and demerits of its use in rural area electrification. The paper also proposes a simple algorithm to find the required minimum capacitance for a self-excited SRG with an inductive load connected to the machines. The simulation has been carried out in MATLAB/Simulink using the design parameters to observe the dynamic behaviour of the system under various conditions such as variation of wind speed, constant wind speed, and variation of the load for SRG and FM-SRG. It is observed that the generated

from 16.5 A to 15.2 A for SRG and from 13.5 A to 12.9 A for FM-SRG. At $t = 7$ s impedance load value is changed to its previous value. It is observed that the current and voltage increase to the initial values.

As the speed of the SRG and FM-SRG kept fixed at 1500 rpm, and thus the frequency remains the same despite the change in load. This is one of the advantages over the SEIG,
voltage increases significantly with the increase in the size of the capacitor or speed while it drops with the increase in inductive loads. To keep the voltage constant, with the change of load, it is important to continuously vary reactive power support, that is, to dynamically vary the capacitance of the capacitor.

The analysis presented in the paper also includes the effects of residual flux linkage on the excitation of reluctance generator. Further, the paper also includes the experimental validation of a few simulated results with the developed prototype of SRG.

NOMENCLATURE

- \( T_\rho \): torque density SRG
- \( T_{\rho,m} \): torque density FM-SRG
- \( \omega \): generator rated speed (mechanical)
- \( T_p \): specific torque SRG
- \( T_{\rho,m} \): specific torque FM-SRG
- \( R \): turbine blade radius
- \( P_{\rho,m} \): specific power FM-SRG
- \( P_p \): specific power SRG
- \( k_s \): stator fill factor
- \( B_{m0} \): maximum air gap flux density
- \( k_{\phi} \): ratio of \( d \)-axis inductance to magnetizing inductance
- \( k_{\lambda} \): ratio of \( q \)-axis inductance to magnetizing inductance
- \( L/R \): stack aspect ratio
- \( V_w \): mean wind speed
- \( J \): current density
- \( \rho_c \): air density
- \( P \): number of pole pairs
- \( \Omega \): turbine tip speed
- \( P_p \): power density of SRG
- \( P_{\rho,m} \): power density of FM-SRG
- \( \rho \): iron mass density
- \( \Lambda_{m} \): maximum flux linkage of ferrite magnet
- \( Q \): differential operator
- \( \eta \): number of stator slot
- \( \mu_s \): air permeability (\( \mu_s = 4\pi \times 10^{-7} \))
- \( b_{st} \): stator tooth width
- \( D_{st} \): stator air gap diameter
- \( D_{st} \): stator yoke diameter
- \( E_{pm} \): EMF induced per phase
- \( A_{m0} \): residual flux linkage require for SRG
- \( A_{pm0} \): residual flux linkage require for FM-SRG
- \( Q \): generator net reactive power
- \( Q_r \): capacitor reactive power
- \( Q_m \): winding reactive power
- \( P_{cap} \): stator copper losses
- \( P_\ell \): electromagnetic active power of the generator
- \( P_{dual} \): output active power of the generator
- \( P_{core} \): stator core losses
- \( R_s \): stator core resistance

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APPENDIX

Some important equations are placed below as:

\[ L_{md} = k_d L_m, \quad L_{mq} = k_q L_m, \]

\[ \xi = \frac{L_d}{L_q} = \frac{(k_d + k_q)}{2k_q}, \]

where, \( \xi \) is power factor.

\[ \lambda = \frac{I_d}{\tau}, \quad \tau = \tau \lambda, \]

\[ \tau = \frac{3q}{3q}, \quad q = \frac{Q}{6P}, \]

\[ A_s = \frac{mL_m}{k_u}, \]

where \( A_s \) and \( k_u \) are slot area and slot filling factor, respectively.

\[ L_d = L_{md} + L_{dkt}, \quad L_q \approx 2L_{sq}, \]

\[ e_s = 2 \frac{b'}{3(b_1 + b_2) + 2b_2} \frac{(b_1 + b_2 + b_1)}{b}, \]

\[ e_s = \frac{5b}{5b + 4b'}, \quad e_s = 0.34q \frac{L(L_m - 0.64 \tau)}, \]

\[ k_c = \frac{\tau(5b + b)}{\tau(5b + b) - b^2}, \quad k_r = \frac{B_w}{b}, \]

where \( B_w \) is average air gap flux density.