Solar Dynamo Models

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Abstract. Mean-field dynamo models and 3D magnetohydrodynamic (MHD) simulations of global-scale solar convection provide complementary insights into the origins of cyclic magnetic activity in the Sun. One particular class of mean-field dynamo models, known as the Babcock-Leighton Flux-Transport (BL-FT) modeling approach, has enjoyed much success recently in reproducing many aspects of the solar activity cycle. We review the essential ingredients of BL-FT models and we address whether these ingredients are consistent with convection simulations. Although BL-FT models appear to be robust enough to tolerate the variability of the meridional circulation found in convection simulations, they often neglect turbulent transport and dynamo processes within the bulk of the convective envelope. Current estimates of the efficiency of such processes from convection simulations suggest that this neglect is not justified.

1. Introduction

Ever since George Ellery Hale’s first measurements of magnetic fields in the solar photosphere a century ago, the question of how those fields are generated has remained at the forefront of solar physics. Solar dynamo theory remains a vibrant field, fueled by fresh insights from helioseismology and increasingly sophisticated numerical simulations. This paper is not intended as a comprehensive review. There have been many such reviews in recent years which provide an excellent overview of the key triumphs and challenges [1, 2, 3]. Here we will focus on one class of dynamo models which have received much attention recently and which show much promise. These are the so-called Babcock-Leighton Flux-Transport (BL-FT) models which will be described in §2.

BL-FT dynamo models may be regarded as mean-field models because they only solve for the axisymmetric field components while parameterizing the influence of non-axisymmetric flows and fields. Alternatively, three-dimensional (3D) magnetohydrodynamic (MHD) simulations have the advantage that turbulent flow and field components can be explicitly resolved and their role in maintaining mean fields can be assessed and quantified. However, 3D MHD simulations are more computationally expensive and do not yet have sufficient spatial resolution to capture many of the processes thought to be important in the solar dynamo such as the destabilization, rise, and emergence of concentrated toroidal flux tubes (§2).

Although they cannot achieve solar parameter regimes, current 3D MHD simulations do exhibit turbulent dynamics which can be used to validate and further develop solar dynamo theory in general and mean-field dynamo theory in particular. In this paper we will highlight several results from global 3D MHD simulations of solar convection and we will discuss what implications these have for BL-FT dynamo models. After reviewing the BL-FT paradigm in §2, we will then address the temporal variability of the meridional circulation in §3. This is
particularly relevant for BL-FT models because of the essential role played by the meridional circulation in establishing cyclic activity. In §4 and §5 we discuss the transport and generation of magnetic fields by turbulent convection and how this may not be properly accounted for in BL-FT models. We summarize our primary conclusions in §6.

2. Babcock-Leighton Flux-Transport Dynamo Models

All BL-FT dynamo models explicitly or implicitly divide the solar envelope into three distinct regions as shown in Figure 1. The precise delineation between the regions may vary among models but for illustration, we will say that the bulk of the convection zone, region II, lies between 0.72\(R\) and 0.98\(R\) where \(R\) is the solar radius. Region I then stretches from 0.98\(R\) to the photosphere and region III includes the tachocline, the overshoot region, and a small portion of the convective envelope \(r \sim 0.68R-0.72R\) (the base of the convection zone lies at \(r = 0.713R\)).

![Figure 1. Schematic diagram of the solar envelope, highlighting (I) the near-surface layers, (II) the bulk of the convection zone, and (III) the overshoot region and tachocline. In BL-FT models, poloidal field is generated in region I while toroidal field is generated primarily in region III. Coupling between regions I and III occurs through the meridional circulation (lines), turbulent diffusion, and through the non-local nature of the poloidal source term. Some toroidal field generation may also occur in region II where there is an \(\Omega\)-effect driven mainly by latitudinal shear.](image)

According to most recent dynamo models (not only BL-FT models), the generation of mean (axisymmetric) toroidal fields occurs in region III as a consequence of rotational shear (the \(\Omega\)-effect). Most models attribute toroidal field generation to vertical shear in the tachocline but in some cases, notably the Dikpati & Gilman model [6], the \(\Omega\)-effect occurs above the tachocline, near the base of the convection zone, and is attributed to latitudinal shear. As they gain in strength, toroidal fields become buoyantly unstable and rise to the surface, emerging as bipolar active regions. Although it is unclear precisely how such structures form [7], the morphology of active regions suggests that it is meaningful to consider the emergence processes in terms of isolated flux tubes which twist as they rise under the influence of the Coriolis force [8].

Photospheric observations and models of rising flux tubes further suggest that the toroidal fields responsible for bipolar active regions must be strong, \(\sim 10^4-10^5\)G in the tachocline [8]. Whether this threshold arises from the instability criterion or from a convection-induced selection

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1 The solar tachocline is a transition layer near the base of the convection zone characterized by strong radial shear in the angular velocity [4].
effect (weaker fields are shredded), it takes time for such strong fields to build up. According to BL-FT dynamo models, toroidal fields building up near the base of the convection zone are advected by a meridional circulation which is equatorward throughout region III as illustrated in Figure 1. Although its kinetic energy density is orders of magnitude smaller than the magnetic energy density, recent modeling [9] suggests that a continually maintained equatorward flow of 2-3 m s$^{-1}$ is indeed capable of transporting toroidal fields as strong as 20-30 kG through the tachocline (this is ultimately accomplished by tapping the nuclear and rotational energy of the star). This advection accounts for the equatorward migration of activity belts over the course of the solar cycle as manifested in the solar butterfly diagram.

The sense of the Coriolis-induced tilt of emerging flux tubes is such that their trailing (western) edge lies at higher latitudes relative to their leading (eastern) edge and possesses a radial field which is opposite in sign to the current polar field. According to BL-FT models, diffusion of this emerging flux by supergranulation and advection by a poleward circulation leads to reconnection across the equator and a corresponding erosion of the current polar field which ultimately leads to a global polarity reversal. This process, known as the Babcock-Leighton mechanism, has been modeled by 2D (latitude-longitude) simulations driven by photospheric magnetograms [10, 11, 12]. Although these may also be regarded as flux-transport models, the BL-FT dynamo models we focus on here are 2D in an axisymmetric sense (radius-latitude) and incorporate the Babcock-Leighton mechanism as a parameterized source term for the mean poloidal field which operates near the surface (region I).

The poloidal source term is nonlocal in the sense that its amplitude depends on the strength of the toroidal field at the base of the convection zone, implicitly representing the rise, twist, and emergence of toroidal flux tubes as discussed above. This together with the meridional circulation couples regions I and III as indicated in Figure 1. In principle, turbulent magnetic diffusion also couples regions I and III but in practice, BL-FT dynamos operate in a parameter regime where the diffusive coupling is small [14]. Thus, BL-FT dynamos operate by generating a toroidal field in region III by the Ω-effect which in turn generates poloidal field in region I via the Babcock-Leighton mechanism. Accumulation of oppositely-signed radial field in each hemisphere due to turbulent diffusion and poleward advection eventually leads to a polarity reversal. Poloidal flux near the poles is then transported from region I to region III by the meridional circulation and the process repeats. The cycle period is largely determined by the meridional flow speed [13].

BL-FT models have been intensely studied in recent years and have enjoyed remarkable success at reproducing the evolution of surface flux observed in photospheric magnetograms including the 22-year periodicity, the steady migration of active bands toward the equator, the propagation of weaker radial flux toward the poles, and the occurrence of the poloidal field reversal just as the activity in the low-latitude bands nears its peak [14, 13, 2, 6]. With additional elements such as nonlinear feedbacks, enhanced cooling in active regions, and stochastic fluctuations, BL-FT models have also reproduced more subtle features of the solar activity cycle such as torsional oscillations, chaotic modulation (e.g. grand minima), and phase locking [15, 2, 16, 17]. Although other types of dynamo models have exhibited similar features, none have yet demonstrated the robustness and flexibility of BL-FT models.

3. MC Variability

Figure 2 illustrates convective and mean flow patterns in a recent global-scale simulation of solar convection [18], obtained using the Anelastic Spherical Harmonic (ASH) code [19, 20]. The convection patterns are intricate and continually evolving but amid the turbulent fluctuations are coherent structures which establish differential rotation and meridional circulation by means of convective momentum and heat transport [21].

The angular velocity profile is roughly solar-like, decreasing monotonically toward the poles
Figure 2. (a) Convective patterns, (b) differential rotation, and (c) meridional circulation in a simulation of solar convection [18]. (a) The radial velocity near the outer boundary of the computational domain ($r = 0.98$) is shown in a Molleweide projection, with bright and dark tones denoting upflow and downflow respectively. The mean angular velocity and streamlines of the mass flux shown in (b) and (c) are averaged over longitude and time (130 days).

with nearly radial contours at mid-latitudes. The meridional circulation in this, the highest-resolution simulation of global solar convection published to date, is dominated by a single cell in each hemisphere with poleward flow in the upper convection zone and equatorward flow in the lower convection zone as in BL-FT models (Fig. 1). This is in contrast to previous simulations where artificial diffusion disrupted the delicate balance between Reynolds stresses and advection of angular momentum by the meridional circulation [21]. Smaller counter-cells near the boundaries are also evident in Figure 2 but these may be sensitive to the boundary conditions and should be interpreted with care. For example, convection simulations which include penetration into an underlying stable zone generally exhibit equatorward meridional flow throughout the overshoot region [22].

Figure 3. Still from a movie showing the time variation of the meridional circulation in a numerical simulation of solar convection (movie at [http://www.iop.org/EJ/mmedia/1742-6596/118/1/012031/]). Shown are streamlines of the mass flux, averaged over longitude, with red and blue denoting clockwise and counter-clockwise flow respectively. The inset plot shows the colatitudinal velocity (positive southward) in meters per second (horizontal axis) as a function of latitude (vertical axis) at the outer boundary. The time interval covered is about two months.

One of the robust features of such convection simulations is that the meridional circulation undergoes large fluctuations about its mean. Instantaneous fluctuations in the flow velocities are about a factor of three larger than the mean and exhibit multiple cells in latitude and radius as demonstrated in Figure 3. However, the correlation time for these fluctuations is relatively short so they largely average out over timescales of a month or more. Month-to-month variations are roughly 50% of the mean (100% peak-to-peak) as shown in Figure 4.

Given the essential role that the meridional circulation plays in BL-FT dynamo models, such temporal fluctuations are of great interest. Large fluctuations could in principle disrupt the dynamo. However, Charbonneau & Dikpati have shown that BL-FT dynamos are robust to
fluctuations in the meridional flow velocity if the correlation time for such fluctuations is less than about six months [15]. In particular, fluctuations of amplitude 300% with a correlation time of roughly one month as found in convection simulations are still consistent with the BL-FT paradigm as long as the longer-term mean circulation has a single-celled flow structure as in Figure 1. In fact, fluctuations of this order may help improve comparisons with observations; by stochastically varying the poloidal source term as well as the circulation amplitude Charbonneau & Dikpati find parameter regimes in which the cycle amplitude and duration vary in an anti-correlated manner as observed on the Sun [15].

As active latitude belts migrate toward the equator during the course the solar activity cycle, they are accompanied by alternating bands of faster and slower rotation known as torsional oscillations [4]. Rempel has reproduced such torsional oscillation patterns in a non-kinematic BL-FT dynamo model through the combined action of Lorentz forces and enhanced cooling in active regions [16]. However, he finds that torsional oscillations may be disrupted by fluctuations in the meridional flow. Furthermore, limits on the amplitude of the fluctuations based on the persistence of the torsional oscillation pattern are more stringent than those based on the persistence of cyclic activity. His model suggests that month-to-month variations of the meridional flow speed must be less than about 10 m s$^{-1}$ (20 m s$^{-1}$ peak-to-peak) in order to exhibit pronounced torsional oscillations. This appears to be marginally consistent with the variability typically found in convection simulations (cf. Fig. 4).

4. Turbulent Transport

With a few notable exceptions [23], most BL-FT dynamo models are kinematic in the sense that the differential rotation and meridional circulation are specified while the mean induction equation is solved for the axisymmetric magnetic field component. The influence of non-axisymmetric flows and fields, including but not necessarily limited to convection, is contained in the turbulent emf $\langle \mathbf{v}' \times \mathbf{B}' \rangle$ which is generally represented by means of a turbulent magnetic diffusion and a source term for the poloidal field (primes indicate fluctuations about the mean and brackets indicate averages over longitude and possibly time).

This representation can be justified if two assumptions are made, namely that the flow is kinematic such that the velocity field $\mathbf{v}$ is independent of the magnetic field $\mathbf{B}$, and that there is a separation of spatial and temporal scales between the mean and fluctuating components of $\mathbf{v}$ and $\mathbf{B}$. Under these assumptions, the turbulent emf may be expressed in terms of an $\alpha$-effect (field generation), a $\gamma$-effect (turbulent pumping), and a turbulent diffusion $\eta$ [1]. Although each

![Figure 4. Month-to-month variations in the meridional circulation for the same simulation as shown in Figure 2. The black line represents the colatitudinal velocity, $v_\theta$ versus radius at a latitude of $30^\circ$ averaged over longitude and over a time interval of four months. Red lines indicate four successive one-month averages covering the same time interval.](image-url)
Figure 5. Pumping, organization, and amplification of magnetic fields in a convective dynamo simulation which includes a tachocline of rotational shear [24]. An orthographic projection of the longitudinal field component $B_\phi$ is shown for (a) the mid convection zone and (b) the tachocline with colors indicating field polarity. In both images the color table saturates at $\pm 5500$G. (c) The angular velocity as a function of radius for selected latitudes, averaged over longitude and time. The dashed line indicates the base of the convection zone and the dotted lines indicate the horizontal surfaces shown in (a) and (b).

of these quantities is in principle a tensor, isotropy is often assumed so $\alpha$ and $\eta$ become scalar quantities.

In most BL-FT dynamo models, the $\alpha$-effect and the $\gamma$-effect due to turbulent convection are neglected. The possible role of the turbulent $\alpha$-effect is addressed in §4. Poloidal field is instead generated near the surface (region I) by the Babcock-Leighton mechanism which is often parameterized as a non-local $\alpha$-effect (2). The Babcock-Leighton source term does not necessarily imply kinematic evolution and scale separation but a local turbulent diffusion does.

Scale separation is marginally justified in convection simulations and in stellar interiors but the kinematic assumption is more difficult to justify. Recent MHD simulations have demonstrated that Lorentz forces can have a profound influence on turbulent field generation and transport even if the large-scale field is weak, with a magnetic energy far below equipartition with the kinetic energy [25, 26, 27]. Thus, although the $\alpha$, $\gamma$, and $\eta$ parameterizations provide a useful context for understanding and modeling some aspects of turbulent field generation and transport, it must be remembered that their theoretical justification is questionable. Toward the end of his illustrious career more than two decades ago, Thomas Cowling reflected that “the most serious difficulty of dynamo theory is in its appeal to turbulent diffusion” [28].

Still, some approximations are necessary in order to make progress. Thus, current BL-FT dynamo models parameterize turbulent transport as an isotropic turbulent diffusion and furthermore they assume that this diffusive transport is less efficient than the advection of flux by the meridional circulation. This is referred to as the advection-dominated regime. Choosing the circulation amplitude based on solar photospheric measurements and local helioseismic inversions then implies that the turbulent diffusivity $\eta$ must be less than about $5 \times 10^{11}$ cm s$^{-2}$ [10, 29, 30, 6, 23]. For larger values of $\eta$, diffusive coupling between regions I and III in Figure 1 disrupts the cyclic variability, particularly the equatorward migration of activity belts (butterfly diagram).

We can estimate the value of the turbulent diffusivity $\eta$ from convection simulations. For the simulation shown in Figure 2, the rms value of the fluctuating velocity components $v$ is about 100 m s$^{-1}$ in the mid convection zone whereas a characteristic length scale $L$ is of order 100 Mm
This implies a turbulent diffusion of $\eta \sim vL \sim 10^{14}$ cm s$^{-2}$, more than two orders of magnitude larger than that required by current BL-FT dynamo models.

The interpretation of this result depends somewhat on whether turbulent transport may adequately be described in terms of a turbulent diffusion. However, the same conclusion is reached by considering the energetics of the various flow components. The volume-integrated kinetic energy contained in convective motions is about 150 times that contained in the meridional circulation. This again calls into question the advection-dominated regime required by BL-FT dynamo models. Poloidal flux generated in the surface layers (region I) will likely be shredded and reprocessed by the convective motions as it is advected through the convection zone (region II) by the relatively slow meridional circulation. The question then arises: how is magnetic flux transported to region III where toroidal field generation is thought to occur? According to the BL-FT paradigm, the meridional circulation advects poloidal flux into region III at high latitudes (Fig. 1). However, turbulent compressible convection can efficiently and systematically pump magnetic flux downward where it can accumulate in an underlying stable region [31, 32, 33, 24]. Given the relatively large kinetic energy content of the convection and the unlimited range in latitude where it may operate, such turbulent pumping is likely to dominate over advective transport by the meridional circulation.

An example of turbulent pumping into a stably-stratified tachocline is shown in Figure 5. The tachocline is maintained by external forcing which suppresses rotational shear in the radiative interior and maintains a specified latitudinal entropy gradient near the base of the convection zone [24]. Within the convection zone, more than 95% of the magnetic energy is contained in the fluctuating field components. Convection pumps this field downward into the tachocline where it is organized into toroidal bands and amplified by rotational shear. This can loosely be interpreted in terms of the $\gamma$-effect but again, the flow is not kinematic so the turbulent pumping process does not necessarily depend linearly on the mean field components. Although the $\gamma$-effect is specifically concerned with mean fields, much of the flux that is transported to the tachocline is in fluctuating fields which may be locally converted to a mean toroidal flux through rotational smoothing [34]. However, this process is likely overestimated in the simulations as a result of subgrid-scale diffusion [24].

5. Turbulent Field Generation

Convection not only transports magnetic fields, but it can also amplify magnetic fields through what is known as the turbulent $\alpha$-effect [1]. In its strictest sense, this term implies kinematic flows and scale separation whereby the component of the turbulent emf which accounts for field amplification becomes linearly proportional to the mean field [1]. Whether or not this linearity is justified, we will nevertheless refer to the turbulent $\alpha$-effect in a loose sense as the generation of mean poloidal field (and possibly mean toroidal field) by convection in region II of Fig. 1. In this section we address the role of this turbulent $\alpha$-effect relative to the Babcock-Leighton (BL) source term discussed in [2].

The first issue to note in this context is that the volume and mass occupied by region I is very small. If the magnetic field were purely radial such that the field strength $B \propto r^{-2}$, then the ratio of the magnetic energy in regions II and III to that in region I would be about 20. Alternatively for a uniformly distributed horizontal field, $B \propto \rho$, this ratio becomes more than $10^5$. These estimates are crude but nevertheless they raise the question of how the Babcock-Leighton mechanism alone can determine the poloidal field polarity throughout the convection zone when the relative magnetic energy in region I is likely to be very small [28]. Put another way, can the tail wag the dog?

Clearly, this problem can be mitigated if the generation region for the poloidal field (region I) is made wider. This is in fact what is typically done in practice. Although Leighton’s original formulation [35] attributed the horizontal diffusion of magnetic flux specifically to
supergranulation which would limit region I to \( r > 0.97\text{--}0.98R \) as depicted in Figure 1, more recent BL-FT models have the poloidal source term reaching as deep as \( 0.92R \) \([13, 36, 23, 17]\). However, this is still only a fraction of the convection zone.

Due to its proximity to region III where toroidal field is generated, even a relatively weak \( \alpha \)-effect in region II may easily overwhelm a stronger, more distant poloidal source in region I \([37]\). Thus, in order for BL-FT dynamo models to operate, the generation of mean fields by convection in region II must either be extremely inefficient or it must be chaotic such that a weak but systematic BL source term might induce a periodicity, perhaps by exciting a natural resonance \([38]\).

Simulations of turbulent convection in rotating spherical shells clearly exhibit dynamo action \([39, 40, 20, 24]\). Although more than 95% of the magnetic energy is in the non-axisymmetric field components, mean fields are indeed generated. Typical amplitudes in the more recent simulations range from about 10G for the mean poloidal field to over 5 kG for the mean toroidal field (Fig. 5), which is comparable to the field strengths achieved in BL-FT dynamo models. Although peak field strengths are determined largely by saturation mechanisms and subgrid-scale diffusion, this nevertheless suggests that the neglect of the turbulent \( \alpha \)-effect is not justified by current MHD convection simulations.

Might MHD convection simulations be overestimating the efficiency of the turbulent \( \alpha \)-effect? Recent work has indeed indicated that the turbulent \( \alpha \)-effect may be readily suppressed by small-scale Lorentz forces, known as catastrophic \( \alpha \)-quenching \([26]\), or alternatively it may depend critically on the molecular diffusivity \([27]\). Either case would imply that the generation of mean fields by convective turbulence would be negligible for the parameter regimes which prevail in stellar interiors (large magnetic Reynolds numbers).

However, this work is based on idealized numerical experiments designed to address specific aspects of mean-field dynamo theory. They do not apply to the solar convection zone where kinematic parameterizations break down, where mean flows play an important role in field generation, where open boundaries allow for a continual flux of magnetic helicity and magnetic energy into the underlying tachocline and the overlying atmosphere, and where the rotating spherical-shell geometry promotes global-scale field connectivity. It is likely that such a system would indeed be capable of sustaining substantial mean fields through self-organization processes associated with the upscale transfer of magnetic helicity \([43, 41, 42]\).

It is quite possible that the turbulent \( \alpha \)-effect may peak in the upper convection zone, where the velocity amplitudes and the helicity peak in convection simulations \([22, 18]\). In this case, the distinction between the turbulent \( \alpha \)-effect and the BL source term blurs, although the latter usually implies non-locality and a narrower radial extent for region I. Flux-transport models using a local turbulent \( \alpha \)-effect confined to the upper convection zone do indeed exhibit similar behavior to BL-FT models in that the butterfly diagrams and cycle periods are sensitive to the form and amplitude of the meridional flow \([29, 30]\). However, they appear to be somewhat less robust, exhibiting solar-like behavior for only a narrow range of meridional flow speeds \([30]\).

6. Summary and Conclusion
Babcock-Leighton Flux-Transport models have been extremely useful tools to build our intuition on how the solar dynamo may operate and they are well supported by observational data. However, the observational data is not yet sufficient to rule out alternative possibilities such as

\[\text{If the upscale transfer is local in spectral space, this is referred to as an inverse cascade of magnetic helicity. However, MHD simulations with helical forcing indicate that after the initial growth phase, the transfer of magnetic helicity is nonlocal, from the forcing scale directly to the global scales. This is advantageous from the point of view of mean-field dynamo theory because it implies a scale separation which helps to validate the concept of an } \alpha \text{-effect.}\]
interface dynamo models [1, 2, 3]. Furthermore, several serious questions are still open regarding the theoretical justification of the BL-FT paradigm.

The most serious of these questions is whether the convection zone is indeed in the advection-dominated regime (§4). Convection simulations suggest that this assumption is not justified. Furthermore, the representation of turbulent transport as an isotropic turbulent diffusion is questionable because the evolution is not likely to be kinematic (§4). Even if the kinematic assumption were justified, other terms in the expansion of the turbulent emf such as the $\gamma$-effect would likely be important. However, if the convection zone were indeed in the advection-dominated regime, more sophisticated models for turbulent transport may have little effect on the essential operation of the dynamo.

Another open question is whether the Babcock-Leighton mechanism can dominate over turbulent field generation in the convection zone (§5). Some support for this comes from the observed evolution of fields from photospheric magnetograms. Although surface flux transport models do tend to accumulate a dipole moment unless steps are taken to avoid it, it is remarkable that the flux contained in emerging active regions and its subsequent dispersal is largely adequate to account for polar field reversals [11, 12]. However, this may be a surface manifestation of dynamo processes occurring deeper within the convection zone.

Despite these open issues, many aspects of global-scale convection simulations support or are at least consistent with the BL-FT paradigm. Notable among these is the form, amplitude, and variability of the meridional circulation (§3). Further comparisons between mean-field models and 3D MHD simulations are needed for the mutual benefit of both approaches.

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