A Comparative Study on Vibration-Based Damage Identification and Localization Methods

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Abstract. This paper outlines various vibration-based methods for detecting and locating damage in mechanical and civil structural systems. The basic premises of these methods are that any changes in physical characteristics (mass and stiffness) of the structure will induce detectable changes in the modal properties of the structure (natural frequency, mode shape and damping). A comparative study of eight extensively used damage identification methods is done on a simulated simply supported beam to demonstrate the validity and effectiveness. The outcomes of various methods are compared in terms of damage detection and localization under single and multiple damage scenarios.

1. Introduction

The traditional damage identification methods like visual inspection and non-destructive testing are less efficient since they require the location of damage to be easily known and readily accessible for testing purposes. Vibration-based techniques for damage identification were developed due to the need for identification of damage globally. The fundamental premise for such global identification method is that vibration characteristics such as natural frequency, mode shape and damping of the structure will be affected by the change in physical properties of the structure such as mass and stiffness. Hence, it is intuitive to examine the changes in structure’s modal properties to detect the damage [1].

The method for identifying damage can be categorized in to following four levels [2] based on the damage information provided by the methods

1) Level 1: Damage Existence Identification
2) Level 2: Damage Location Identification
3) Level 3: Damage Severity Identification
4) Level 4: Remaining Life Prediction

There is no unique vibration-based damage identification method which gives accurate results for all structural systems and levels of damage. Even though different methods yield varying results, it is expected that these results converge thereby increasing the confidence in output. In this paper, the aspects under consideration for various damage identification methods are damage existence (Level 1) and localization (Level 2). The selected methods for comparison are: MAC & COMAC methods [3];
Mode Shape Difference method [4]; Mode Shape Curvature method [5]; Cumulative Damage Factor method [6]; Damage Index method [7]; Modal Flexibility Change method [8]; and Uniform Flexibility Shape Curvature Change method [9].

2. Vibration-Based Damage Identification Methods

2.1 Modal Assurance Criterion (MAC) and Coordinate MAC (COMAC) Method

MAC is a scalar quantity having values ranging 0 to 1 which indicates the resemblance between mode shapes while COMAC values indicates a point wise change between the mode shapes and it ranges between 1 and 0.

\[
MAC = \frac{\sum_{i=1}^{m} |\phi_i^* \phi_i^T|}{\sum_{i=1}^{m} |\phi_i^T| \sum_{i=1}^{m} |\phi_i^T|}
\]

COMAC \(j\) = \[ \frac{|\sum_{i=1}^{m} (\phi_i^*)^2 - \sum_{i=1}^{m} \phi_i^2|}{\sum_{i=1}^{m} (\phi_i^*)^2} \] (2)

where \(\phi\) = mode vectors of intact beam; \(\phi^*\) = mode vectors of damaged beam; \(m\) = number of modes considered; \(i\) = mode number; and \(j\) = node number.

2.2 Mode Shape Difference Method

This traditional method makes use of the normalized absolute differences of the modal vectors of intact and damaged structures

\[
\Delta\phi_{i,j} = |\phi_i^* - \phi_j^*|
\]

\[
MSD_j = \sum_{i=1}^{m} \Delta\phi_{i,j}
\]

2.3 Mode Shape Curvature (MSC) Method

MSC is the second derivative of mode shape and it is highly sensitive to the damage in the structure. Applying the Central Difference approximation over the displacement mode shapes, Curvature at node \(j\) along the beam is given by

\[
\dot{\phi}_j = \frac{\phi_{j+1} - 2\phi_j + \phi_{j-1}}{h^2}
\]

Where \(\dot{\phi}_j\) = Curvature at node \(j\); and \(h\) = length of each element.

The absolute curvature difference between the intact and damage structure is given as

\[
\Delta MSC_j = |\phi_j^" - \phi_j^"|
\]

2.4 Cumulative Damage Factor (CDF) Method

CDF is the average of absolute difference between the intact and damaged curvature mode shapes. CDF of \(j^{th}\) node, considering ‘m’ number of modes is given by:

\[
CDF_j = \frac{1}{m} \sum_{i=1}^{m} \Delta MSC_j
\]
2.5 Damage Index Method (DI) Method

The damage index method depends on the change of strain Energy which is stored in the structure when it deforms under a particular mode shape. For a location j on the beam the change in strain energy of the ith mode, \( \phi_i(x) \), is related to changes in curvature of the mode at location j. The damage index for this location and this mode, \( \beta_{ij} \), is defined as

\[
\beta_{ij} = \frac{\int_0^L [\phi_i''(x)]^2 \, dx + \int_0^L [\phi_i''(x)]^2 \, dx}{\int_0^L [\phi_i'(x)]^2 \, dx + \int_0^L [\phi_i'(x)]^2 \, dx}
\]  

(8)

The normalized damage index \( Z_i \) is given by

\[
Z_i = \frac{\beta_i - \mu}{\sigma}
\]  

(9)

where \( \mu \) = mean of \( \beta_i \); and \( \sigma \) = standard deviation of \( \beta_i \). To reduce the problem of false alarms due to influence of higher modes, normalized mode shape curvatures are used [10].

2.6 Modal Flexibility Change Method

The flexibility matrix \( [F] \) is given by

\[
[F] = [\phi]^\top \left[ \frac{1}{\omega^2} \right] [\phi]^\top
\]  

(10)

Where \( [\phi] \) = mass orthonormalized mode shape matrix; and \( \omega \) = natural frequency.

The change in flexibility matrix for a structure can be given by

\[
[\Delta F] = [F] - [F^*]
\]  

(11)

Where \([F]\) = flexibility matrix of intact beam; and \([F^*]\) = flexibility matrix of damaged beam.

The Element with the largest absolute value in the k\textsuperscript{th} column of \( \Delta F \) is considered as the k\textsuperscript{th} element of the row vector \( [\delta_{max}] \).

\[
[\delta_{max}]_k = \max \left| [\Delta F]_k \right|
\]  

(12)

2.7 Uniform Flexibility Shape Curvature Change Method

The change in flexibility curvature is given by the equation

\[
[\Delta F''] = [F''] - [F'^*]
\]  

(13)

Where \([F']\) = Curvature of flexibility matrix of intact beam; and \([F'^*]\) = Curvature of flexibility matrix of damaged beam.

The Element with the largest absolute value in the k\textsuperscript{th} column of \( \Delta F'' \) is considered as the k\textsuperscript{th} element of the row vector \( [\delta^*_max] \).

\[
[\delta^*_max]_k = \max \left| [\Delta F'']_k \right|
\]  

(14)

3. Methodology

A simply supported beam of rectangular cross-section (0.25m (width) x 0.2m(depth)) having 4m length was selected for this study. The modulus of elasticity and density of the beam were presumed to be 32 GPa and 2500 kg/m\(^3\) respectively. MATLAB programming was used to obtain the modal properties by considering beam as 2-D Euler-Bernoulli beam. The beam was divided equally into 20 2-D beam elements (Figure 1). All the mode shapes obtained were normalized to the mass matrix. Different damage scenarios were added by reducing the stiffness i.e. modulus of elasticity of the beam elements as shown in Table 1. To obtain the assumed experimental modal parameters, modal analysis was again
done in MATLAB using the properties of damaged beam. The data obtained was further used in various damage detection algorithms to detect and locate the damage.

Figure 1: Simulated Simply Supported Beam

Table 1: Damage Scenarios

| Damage Cases | D0 | D1 | D2 | D3 | D4 | D5 | D6 |
|--------------|----|----|----|----|----|----|----|
| Damaged element | -  | 3  | 10 | 15 | 10,11 | 3,18 | 3,10,11,18 |
| Severity | -  | 10 | 50 | 90 | 20%, 30% | 40%, 30% | 20%, 40%, 30%, 30% |

4. Results and Discussions

The natural frequencies of intact and various damaged beams are given in Table 2. The reduction in natural frequencies indicates the existence of the damage but the exact location of damage is not known. COMAC values in each mode (Table 3) and MAC values of each nodes (Table 4) also serves the purpose of damage detection but gives no indication of location of damage.

Table 2: Natural Frequencies of beam under different damage scenarios

| Modes | D0  | D1  | D2  | D3  | D4  | D5  | D6  |
|-------|-----|-----|-----|-----|-----|-----|-----|
| 1     | 20.28 | 20.23 | 20.06 | 20.17 | 19.86 | 18.56 | 19.32 |
| 2     | 81.13 | 80.55 | 81.02 | 80.27 | 76.34 | 80.26 | 74.73 |
| 3     | 182.61 | 180.82 | 181.13 | 181.63 | 169.55 | 171.91 | 175.91 |
| 4     | 325.00 | 322.24 | 323.67 | 324.56 | 307.38 | 315.46 | 321.04 |
| 5     | 508.98 | 506.31 | 506.26 | 506.23 | 492.01 | 490.10 | 488.93 |

Table 3: COMAC values in different damage scenarios

| Nodes | D1  | D2  | D3  | D4  | D5  | D6  |
|-------|-----|-----|-----|-----|-----|-----|
| 1     | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2     | 1.000 | 0.997 | 0.995 | 1.000 | 0.999 | 1.000 |
| 3     | 1.000 | 0.997 | 0.993 | 1.000 | 1.000 | 1.000 |
| 4     | 1.000 | 0.998 | 0.986 | 0.999 | 1.000 | 0.999 |
| 5     | 1.000 | 0.998 | 0.982 | 0.999 | 0.999 | 1.000 |
| 6     | 1.000 | 0.996 | 0.978 | 0.999 | 0.999 | 0.999 |
| 7     | 1.000 | 0.997 | 0.966 | 0.998 | 1.000 | 0.998 |
| 8     | 1.000 | 0.997 | 0.966 | 0.999 | 1.000 | 0.998 |
| 9     | 1.000 | 0.995 | 0.943 | 0.999 | 1.000 | 0.996 |
| 10    | 1.000 | 1.000 | 0.911 | 0.999 | 1.000 | 0.998 |
| 11    | 1.000 | 1.000 | 0.927 | 1.000 | 0.999 | 1.000 |
| 12    | 1.000 | 0.994 | 0.930 | 0.999 | 0.999 | 0.998 |
| 13    | 1.000 | 0.997 | 0.912 | 0.998 | 0.998 | 0.999 |
| 14    | 1.000 | 0.997 | 0.913 | 0.999 | 0.998 | 1.000 |
| 15    | 1.000 | 0.994 | 0.928 | 0.999 | 0.997 | 0.998 |
| 16    | 1.000 | 0.997 | 0.989 | 0.999 | 0.997 | 0.999 |
| 17    | 1.000 | 0.997 | 0.980 | 0.999 | 0.996 | 0.999 |
| 18    | 1.000 | 0.998 | 0.970 | 1.000 | 0.998 | 1.000 |
| 19    | 1.000 | 0.998 | 0.964 | 1.000 | 0.999 | 1.000 |
| 20    | 1.000 | 0.996 | 0.965 | 1.000 | 0.998 | 0.999 |
Table 4: MAC values of different damage scenarios in first five modes

| Modes | D1   | D2   | D3   | D4   | D5   | D6   |
|-------|------|------|------|------|------|------|
| 1     | 1    | 1    | 0.999| 1    | 1    | 1    |
| 2     | 1    | 1    | 0.999| 1    | 0.999| 1    |
| 3     | 0.999| 0.999| 0.998| 0.999| 0.999| 0.999|
| 4     | 0.999| 0.999| 0.996| 0.999| 0.999| 0.999|
| 5     | 0.999| 0.999| 0.988| 0.999| 0.999| 0.999|

Figure 2: (a)-(f) MSD in damage scenarios D1-D6 considering first three modes

Figure 3: (a)-(f) MSC difference in first three modes for D1-D6 damage scenarios.
The absolute differences in Mode shapes (MSD) for various damage scenarios are plotted in Figure 2. It is indicated that absolute difference of mode shape is more in damaged region. This method gives many false indications in multiple damage scenarios.

The absolute differences in Mode Shape Curvatures (MSC), between intact and various damaged beams for Mode-1, Mode-2 and Mode-3 are plotted in Figures 3. For damage cases D1-D5, it was observed that at lower modes (mode 1 and mode 2) damage locations are indicated by definite peaks in graphs, but at higher modes, numerous peaks are observed leading to false positive alarms. Cumulative Damage Factors (CDF) of damaged and intact beams considering Mode-1, Mode-2 and Mode-3 are plotted in Figure 4. Both MSC and CDF method successfully predict single damage location. For Multiple damage case D6, both MSC and CDF methods do not exhibit good performance.

Figure 4: (a)-(f) CDF D1-D6 damage scenarios considering first three modes

Figure 5: (a)-(f) Damage Index $\beta$ for D1- D6 damage scenarios
The normalized damage index is shown in Figure 5. The positive values of index $\beta$ indicate the presence of damage. The results indicate that the damage index method successfully predicts damage location for single and multiple damage scenarios.

Figure 6: (a)-(f) Change in Modal Flexibility Matrix for D1- D6 damage scenarios

Figure 7: (a)-(f) Changes in Uniform Flexibility Shape Curvatures for D1- D6

The changes in flexibility between intact and different damaged beams are plotted in Figure 6. Damage locations are indicated accurately by peaks. It gave good results in double damage cases indicating damages by change in slopes. In case of multiple damages, changes in slope are not noticeable. The changes in uniform flexibility shape curvature are plotted in Figure 7. The peaks accurately indicate the location of damage for single and multiple damage scenarios.

5. Conclusions
A comparative study on simulated simply supported beam was carried out to evaluate the effectiveness of different algorithms for damage detection. The following conclusion are drawn from comparison among different damage identification methods.

1. The change in natural frequencies, MAC, COMAC and Mode shape difference methods detected the presence of damage but were unsuccessful in identifying the location of damage and hence can be classified as Level-1 methods.
2. Mode shape curvature and Curvature damage factor method could locate damages at lower modes but the results were ambiguous for higher modes. Also, both methods proved effectively only for single damage detection. For double and multiple damage scenarios both the methods do not exhibit good performance.
3. The change in Flexibility matrix method detected the location of damage for single and double damage scenarios. However, the results for multiple damage scenarios were not easily discernible.
4. Both Damage Index and uniform flexibility shape curvature change method proved to be effective in identification of damage in single and multiple damage scenarios.

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