The Numerical Simulation of a High Power Hall Effect Thruster

Lai Li\(^1\), Xi Lu\(^2\), Hulin Huang\(^1\), Xidong Zhang\(^3\), Guiping Zhu\(^1\)\(^*\)

\(^1\) College of Astronautics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
\(^2\) Shanghai Key Laboratory of Deep Space Exploration Technology, Shanghai 201109, China
\(^3\) College of Energy and Power Engineering, Nanjing Institute of Technology, Nanjing 211167, China

Corresponding author’s E-mail: zhuguiping@nuaa.edu.cn

**Abstract:** This work studied and presented a high-power Hall-effect electric propulsion thruster. The working Xenon gas is heated and ionized by nuclear energy to generate plasma which is injected into the channel. The plasma in the channel is described by two-temperature model consists of ions, electrons and atoms with the electron density of plasma above \(10^{20}\) m\(^{-3}\). The Lorentz force acting on the plasma makes the flow velocity increase obviously. The operating power of the thruster is about 20 kW. The mass flow rate of xenon is set at 1.56 g/s.

1. Introduction

The concept of electric propulsion was brought up by Tsiolkovsky since 1903 and Goddard since 1906 [1], respectively. So far the electric propulsion technology has been developed for more than a century. There are four main types of electric propulsion technology: electrothermal, electromagnetic, electrostatic and the mixed type. Electrothermal propulsion, which was not strictly an electric propulsion technology because there was no obvious plasma ionization and electromagnetic field to accelerate plasma, is not the mainstream of future development. The study of electromagnetic, electrostatic, mixed type propulsion technology [2]-[12] mainly includes two points. The main concern is the way to realize the ionization of the working fluid. The other critical issue is the approach to make the ionized plasma to be ejected from the electrical power. The current thrust of electric propulsion technology is very small, only dozens to hundreds of mN orders. The chemical propulsion can dive the satellite transfer to the requirements orbit with only short-term thrust because of its large magnitude of the thrust. However, the same orbit transferring may take a number of circle orbits by the electric propulsion, which means it needs more time to complete the task.

With the continuous developing of manned deep space exploration technology, propulsion technology is one of the most critical technologies. A manned deep space exploration task with large load quality requires large actual load and stability of spacecraft operation, which causes the complex and cumbersome of spacecraft. Therefore, it is necessary to save the propellant for the thruster and ensure the smooth work. The time of voyage is required to be as short as possible for the life needs, the safety and endurance of the astronauts. Hence, larger thrust propulsion technology is needed to shorten the transit time than a small thrust. Larger thrust can be achieved by chemical propulsion, but the specific impulse of chemical propulsion is lower and the improving space of the chemical propulsion is also small. The specific impulse of the nuclear thermal propulsion is about 2-3 times that of the chemical
propulsion [13], [14]. However, the demand for propellant is too much during the deep space exploration tasks for the high speed increment. Electric propulsion has a remarkable characteristic of high specific impulse whose thrust is proportional to the power×efficiency/specific impulse. Large power electric propulsion can achieve the needed thrust to shorten the time of orbit transfer or the space traveling.

For further optimize the configuration and performance, the Hall electric propulsion is studied in this paper. Different from the general Hall effect thruster, the magnetic flux in this study is uniform and only applied on the z-direction. Xenon is used as the working gas and assumed to be heated and pre-ionized by the nuclear energy before inflowing into the channel. The conducting flow is accelerated by the Lorenz force through the applied electric field and a magnetic field. The working flow is also heated by the Joule heating when flowing through the channel.

2. Numerical model

2.1 Numerical simulation domain

Figure 1. The model and computational domain of the calculation: (a) The structure of the simulation domain; (b) Channel cross-section.

Figure 1 illustrates the structure of the simulation domain. The thruster is an broad-leaved structure. The electrodes are placed in the front and rear ends, respectively. The throat height and the cathode inlet height are 3.55 mm (x=54.5 mm) and 9.87 mm (x=115 mm), respectively.

2.2. Mathematical model and working conditions

In our work, the working gas is assumed to be pure noble gas. The governing equations of state equations and Maxwell equations are in combination to describe the non-equilibrium plasma. The non-equilibrium plasma in the channel consists of ions, electrons and atoms under two-temperature model [15], [16]. The Navier-Stokes equations and the energy equation include Lorentz force and Joule heating terms, respectively. Large Eddy simulation (LES) is used to describe the flow. This work adopted high-temperature seed-free xenon plasma. Table 1 shows the working conditions in the simulation processes. A non-slip wall condition is applied as the boundary condition for the flow. The wall of the partial channel is electrically insulated and its temperature is set to be 300K. The boundary condition for the electrical equation is set to be on the anode and on the cathode.

| Working gas       | Xenon    |
|-------------------|----------|
| Total inflow pressure | 0.16 MPa |
| Total inflow temperature | 9000 K   |
| Inlet electron temperature | 11603K    |
| Magnetic flux density | 4T       |

2.2.1 Governing equations for MHD plasma flows.

The time dependent compressible Navier-Stokes equations coupled with Lorentz force and Joule heating terms are shown following:
Continuity equation:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \]  \hspace{1cm} (1)

Momentum equation:

\[ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = \mathbf{j} \times \mathbf{B} - \nabla p + \nabla \cdot \mathbf{\tau} \]  \hspace{1cm} (2)

Energy equation:

\[ \frac{\partial E}{\partial t} + \nabla \cdot (\mathbf{E} + p) \mathbf{u} = \mathbf{j} \cdot \mathbf{E} - \nabla \cdot \mathbf{q} + \nabla \cdot (\mathbf{r} \mathbf{u}) \]  \hspace{1cm} (3)

\[ E_s = \rho \left( e_i T + \frac{1}{2} \mathbf{u}^2 \right) \]  \hspace{1cm} (4)

where \( \rho \) is mass density, \( \mathbf{B} \) is the magnetic flux density vector, \( \mathbf{u} \) is the velocity vector of flow, \( e_i \) is the constant volume specific heat, \( T \) is the static temperature, \( \mathbf{j} \) is the current density vector, \( E_s \) is the total energy, \( p \) is the static pressure, \( \sigma \) is the electrical conductivity, \( \mathbf{q} \) is the conductive heat flux vector, \( \mathbf{\tau} \) is the viscous stress.

2.2.2. Governing equations for ionization. Conservation of ion number density:

\[ \frac{\partial n_i^+}{\partial t} + \nabla \cdot n_i^+ \mathbf{u} = n_i = k_{f_i} n_i n_e - k_{r_i} n_i^2 n_e \]  \hspace{1cm} (5)

\[ n_e = \sum_{i} n_i^+ \]  \hspace{1cm} (6)

Energy equation for electron:

\[ \frac{\partial U_e}{\partial t} + \nabla \cdot (U_e \mathbf{u}) = \frac{|j|^2}{\sigma} - p_e \nabla \cdot \mathbf{u} - 3n_e k (T_e - T) \sum_{k} \frac{m_k}{m_e} \nu_{e_k} \]  \hspace{1cm} (7)

\[ p_e = n_e k T_e, \quad U_e = \frac{3}{2} n_e k T_e + n_e^i e_i \]  \hspace{1cm} (8)

In equation (5), \( n_i^+ \) is the number density of xenon ions, \( n_i \) is the number density of electron, \( n_i \) is the number density of xenon atom, the three-body recombination rate coefficient \( k_r \) is determined by the following relations\[13\]:

\[ k_r = k_{r_{ih}} k_{r_{io}} / (k_{rh} + k_{ro}) \]  \hspace{1cm} ,

\[ k_{rh} = 1.09 \times 10^{-20} T_e^{-9/2} \]  \hspace{1cm} ,

\[ k_{ro} = \frac{1.21 \times 10^{-35}}{T_e^2} \exp \left( \frac{55300}{T_e} \right) \]  \hspace{1cm} ,

the ionization rate coefficient \( k_i \) is derived on the basis of the principle of detailed balance with the Saha equilibrium:

\[ k_i n_i n_e = k_f n_i^2 n_e, \quad k_f = k_f \frac{g_i}{g_0} \left( \frac{2 m_e k T_e}{\hbar^2} \right)^{3/2} \exp \left( - \frac{e_i}{k T_e} \right) \]  \hspace{1cm} ,

where \( g_i \) is the statistical weight of the ground state of the ion, \( g_0 \) is the statistical weight of the ground state of the neutral xenon atom, \( e_i \) is ionization potential of xenon atom\[14\], the subscripts \( e, i \) and the
superscript $^+$ denote the electrons, neutral atoms and ionized particles, respectively. In equation (7), $U_e$ is the electron energy, $\mathbf{u}_e$ is the velocity vector of electrons, $m_h$ is the mass of heavy particle, $m_e$ is the mass of electron, $p_e$ is the electron pressure. In equation (8), $k$ is Boltzmann constant, $T_e$ is electron temperature, $\epsilon_i$ is the ionization energy.

2.2.3. Governing equations for electrical-magnetic. generalized ohm’s law:

$$\mathbf{j} + \frac{\beta}{|\mathbf{B}|} \mathbf{j} \times \mathbf{B} = \sigma (\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

(9)

$$\sigma = \frac{e^2 n_e}{m_e v_{eh}}, \quad \beta = \frac{e |\mathbf{B}|}{m_e v_{eh}}$$

(10)

Maxwell equations:

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0$$

(11)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{D} = \rho_e$$

(12)

Since the charge neutrality is assumed and magnetic Reynolds number is quite small, the Maxwell equations are simplified as follows:

$$\nabla \times \mathbf{E} = 0, \quad \nabla \cdot \mathbf{j} = 0$$

(13)

Equation (9) is the general Ohm’s law, In equation (10), $\bar{v}_{eh}=\sum \frac{n_h Q_{eh} c_e}{n}$ is the average momentum transfer collision frequency for an electron $e$ with a heavy particle h, where $Q_{eh}$ is the energy-averaged momentum transfer cross section, $C_e=\sqrt{8kT_e/\pi m_e}$ is the mean electron velocity for a Maxwellian distribution, $\beta$ is Hall parameter. In equation (12), $\mathbf{E} = -\nabla \Phi$ is the electric field vector, where $\Phi$ is the electrical potential.

3. Results and Discussions

3.1 The characteristics of the plasma

**Figure 2.** The distribution of the electric power

**Figure 3.** The distribution of the electric conductivity

The energy exchange between the external system and the fluid $\mathbf{j} \cdot \mathbf{E}$ is denoted by $q_e$. Fig 2 shows the distribution of $q_e$ indicating the electric power injected into the fluid, which is about 20 kW.
The electron number density under pre-ionization of inlet is about $2.17 \times 10^{21}$ m$^{-3}$, as the velocity increases, the electron number density decreases rapidly as shown in Fig 3. The channel’s broad-leaved structure also has an influence on the distribution of electron number density. Fig 4 shows the distribution of electrical conductivity, which changes within the 6-15 range, because of inhomogeneous distribution of electrons. The Hall parameter changes at the range of 0.2-2.5, which can be seen from Figure 5.

3.2 The characteristics of the flow

There are three kinds of current in the channel, one is in the radial direction:

$$j_r = \frac{\sigma}{1 + \beta^2} \left( E_r - \beta E_\theta + \beta u_r B + u_\theta B \right)$$  \hspace{1cm} (14)

The second is in the tangential direction:

$$j_\theta = \frac{\sigma}{1 + \beta^2} \left( \beta E_r + E_\theta - u_r B + \beta u_\theta B \right)$$  \hspace{1cm} (15)

The third is in the z direction:

$$j_z = \sigma E_z$$  \hspace{1cm} (16)

Where $E_r, E_\theta$ are the component of electric field in $r, \theta$ directions, respectively; $j_r, j_\theta, j_z$ are the component of current density in radial, tangential, z directions, respectively; $u_r, u_\theta$ are the component of flow velocity in the radial, tangential directions, respectively.

The Lorentz force $F_x = j_\theta B$ is with the same direction of the radial velocity. From Figure 6, we can find the distribution and direction of Lorentz force. The Lorentz force near the anode is the strongest.
because of the relative low flow velocity and strong electric field strength. From the equation (15), the radial velocity increases because of the Lorenz force and meanwhile the electric field $E_r$ decreases, therefore, $j_o$ decreases along the flow direction downstream (Figure 7). As a result, based on the constant assumption of the magnetic flux density, the Lorenz force is also reduced even in the reverse direction (Figure 8) at the end of the channel.

![Figure 7. The distribution of the tangential current](image1)

![Figure 8. The distribution of the Lorentz force](image2)

For contrast, the radial velocity of no magnetic flow is shown in Figure 9(a). The radial velocity is accelerated by the Lorentz force obviously when the 4T external magnetic is applied from Figure 9(b). From the comparison, the radial velocity from the outlet is about 1800 m/s, which is almost two times that of no magnetic flow. The mass flow rate of the outlet in the acceleration channel is 1.56 g/s at the moment.

![Figure 9. The contours of radial velocity](image3)

4. Summary
The numerical simulation of the Hall effect electric propulsion thruster was carried out by using the high temperature xenon gas. The high density plasma from the inlet is produced by the nuclear power and maintained at an appropriate level by the input electric power, the number density of electrons is kept to the level of 1021/m³, which help to keep an adequate electrical conductivity about 5-15 S/m. In this model, the Hall parameter was about 0.2-2.5, which is high enough to maintain the Hall effect. The Lorentz force is the strongest because of the lower flow velocity and stronger electric field strength near the anode. For the xenon gas is too heavy, the acceleration is not as well as the light gas, such as krypton and argon gas. Thus, the different gas, the combination of the electric power input and magnetic flux density, thrust assessment, the internal ionization structure of the plasma still requires further research.

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