Consistent Dirac Quantization of SU(2) Skyrmion equivalent to the BFT Scheme

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ABSTRACT

In the framework of Dirac quantization, the SU(2) Skyrmion is canonically quantized to yield the modified predictions of the static properties of baryons. We show that the energy spectrum of this Skyrmion obtained by the Dirac method with the suggestion of generalized momenta is consistent with the result of the Batalin-Fradkin-Tyutin formalism.

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It is well known that baryons can be obtained from topological solutions, known as SU(2) Skyrmions, since the homotopy group \( \Pi_3(SU(2)) = \mathbb{Z} \) admits fermions [1, 2, 3]. Using the collective coordinates of the isospin rotation of the Skyrmion, Adkins et al. [4] have performed semiclassical quantization having the static properties of baryons within 30% of the corresponding experimental data. Also the chiral bag model, which is a hybrid of two different models, the MIT bag model at infinite bag radius on one hand, and the SU(3) Skyrmion model at a vanishing radius on the other hand, has enjoyed considerable success in predicting the strange form factors of baryons [4] to confirm the recent experimental result of the SAMPLE Collaboration [5].

On the other hand, in order to quantize the physical systems subjective to the constraints, the Dirac quantization scheme [6] has been used widely. First of all, string theory is known to be restricted to obey the Virasoro conditions, and thus it is quantized by the Dirac method [7]. Also, in the 2+1 dimensional O(3) \( \sigma \) model, Bowick et al. [8] have used the Dirac scheme to obtain the fractional spin.

However, whenever we adopt the Dirac method, we frequently meet the problem of the operator ordering ambiguity. In order to avoid this problem, Batalin, Fradkin, and Tyutin (BFT) developed a method [9] which converts the second-class constraints into first-class ones by introducing auxiliary fields. Recently, this BFT formalism has been applied to several interesting models [10]. Very recently, the SU(2) Skyrme model has been studied in the context of the Abelian and non-Abelian BFT formalism [11, 12]. But, there exists some inconsistency on the constraint structure.

In this paper, we will canonically quantize the SU(2) Skyrme model by using the desired Dirac quantization method, which is consistent with the BFT one. Firstly, the Dirac bracket scheme will be discussed in the framework of the SU(2) Skyrmions to quantize the baryons. The adjustable parameter will be introduced to define the generalized momenta without any loss of generality. Next, we will apply the proper BFT method to the Skyrmion to obtain the energy spectrum of the baryons by including the Weyl ordering correction. Finally, we will show that by fixing this free parameter the baryon energy eigenvalues obtained by the Dirac method are consistent with the result of the BFT formalism and modify the predictions of the baryon static properties.
Now we start with the Skyrmion Lagrangian of the form

\[
L = \int d^3r \left[ \frac{f_\pi^2}{4} \text{tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 \right],
\]

where \( f_\pi \) is the pion decay constant, \( e \) is a dimensionless parameter, and \( U \) is an SU(2) matrix satisfying the boundary condition \( \lim_{r \to \infty} U = I \), so that the pion field vanishes as \( r \) goes to infinity. For the minimum energy of the Skyrmion, one can take the hedgehog ansatz \( U_0(\vec{x}) = e^{i\tau_a \hat{x} a f(r)} \), where the \( \tau_a \) are Pauli matrices, \( \hat{x} = \vec{x}/r \) and for the unit winding number \( \lim_{r \to \infty} f(r) = 0 \) and \( f(0) = \pi \). On the other hand, since the hedgehog ansatz has maximal or spherical symmetry, it is easily seen that spin plus isospin equals zero, so that isospin transformations and spatial rotations are related to each other.

Furthermore, in the Skyrmion model, spin and isospin states can be treated by collective coordinates \( a^\mu = (a^0, \vec{a}) \) \( (\mu = 0, 1, 2, 3) \) corresponding to the spin and isospin rotations \( A(t) = a^0 + i\vec{a} \cdot \vec{\tau} \). With the hedgehog ansatz and the collective rotation \( A(t) \in \text{SU}(2) \), the chiral field can be given by \( U(\vec{x}, t) = A(t) U_0(\vec{x}) A^\dagger(t) = e^{i\tau_a R_{ab} \hat{x} b f(r)} \) where \( R_{ab} = \frac{1}{2} \text{tr}(\tau_a A \tau_b A^\dagger) \).

The Skyrmion Lagrangian is then given by \( \[1\] 

\[
L = -E + 2I \dot{\vec{a}} \cdot \dot{\vec{a}}^\mu, \tag{2}
\]

where the soliton energy and the moment of inertia are given by

\[
E = 4\pi \int_0^\infty dr r^2 \left[ \frac{f_\pi^2}{2} \left( \frac{df}{dr} \right)^2 + 2 \frac{\sin^2 f}{r^2} \right] + \frac{1}{2e^2} \frac{\sin^2 f}{r^2} \left[ 2 \left( \frac{df}{dr} \right)^2 + \frac{\sin^2 f}{r^2} \right],
\]

\[
I = \frac{8\pi}{3} \int_0^\pi dr r^2 \sin^2 f \left[ f_\pi^2 + \frac{1}{e^2} \left( \frac{df}{dr} \right)^2 + \frac{\sin^2 f}{r^2} \right]. \tag{3}
\]

Introducing the canonical momenta \( \pi^\mu = 4I \dot{a}^\mu \) conjugate to the collective coordinates \( a^\mu \) one can then obtain the canonical Hamiltonian

\[
H = E + \frac{1}{8I} \pi^\mu \pi^\mu \tag{4}
\]

and the spin and isospin operators

\[
J^i = \frac{1}{2} (a^0 \pi^i - a^i \pi^0 - \epsilon_{ijk} a^j \pi^k),
\]

\[
I^i = \frac{1}{2} (a^i \pi^0 - a^0 \pi^i - \epsilon_{ijk} a^j \pi^k). \tag{5}
\]

\(^1\)Here one can easily check that the Skyrmion Lagrangian can be rewritten as \( L = -E + 2I \dot{\vec{a}} \cdot \dot{\vec{a}}^\mu \) by defining the new variables \( \alpha^k = a^k \dot{a}^k - \dot{a}^k a^k + e^{i\tau_a R_{ab} \hat{x} b} \).
On the other hand, we have the following second-class constraints:

\[ \begin{align*}
\Omega_1 &= a^\mu a^\mu - 1 \approx 0, \\
\Omega_2 &= a^\mu \pi^\mu \approx 0,
\end{align*} \]  

(6)

to yield the Poisson algebra

\[ \Delta_{kk'} = \{\Omega_k, \Omega_{k'}\} = 2\epsilon^{kk'} a^\mu a^\mu \]  

(7)

with \(\epsilon^{12} = -\epsilon^{21} = 1\). Using the Dirac brackets defined by

\[ \{A, B\}_D = \{A, B\} - \{A, \Omega_k\} \Delta_{kk'} \{\Omega_{k'}, B\} \]

with \(\Delta_{kk'}\) being the inverse of \(\Delta_{kk'}\) and performing the canonical quantization scheme \(\{A, B\}_D \rightarrow \frac{1}{i} [A_{\text{op}}, B_{\text{op}}]\) one can obtain the operator commutators

\[ \begin{align*}
[a^\mu, a^\nu] &= 0, \\
[a^\mu, \pi^\nu] &= i(\delta^{\mu\nu} - \frac{a^\mu a^\nu}{a^\sigma a^\sigma}), \\
[\pi^\mu, \pi^\nu] &= \frac{i}{a^\sigma a^\sigma}(a^\nu \pi^\mu - a^\mu \pi^\nu)
\end{align*} \]  

(8)

with \(\pi^\mu = -i(\delta^{\mu\nu} - \frac{a^\mu a^\nu}{a^\sigma a^\sigma})\partial^\nu\), and the closed current algebra

\[ \begin{align*}
[M^\mu, M^\nu] &= \epsilon^{\mu\nu\sigma} M^\sigma, \\
[M^\mu, N^\nu] &= \epsilon^{\mu\nu\sigma} N^\sigma, \\
[N^\mu, N^\nu] &= 0
\end{align*} \]

with \(M^\mu = i\epsilon^{\mu\nu\sigma} \pi^\nu a^\sigma, N^\mu = ia^\mu\).

Now we observe that without any loss of generality the generalized momenta \(\Pi^\mu\) fulfilling the structure of the commutators are of the form

\[ \Pi^\mu = -i(\delta^{\mu\nu} - \frac{a^\mu a^\nu}{a^\sigma a^\sigma})\partial^\nu - \frac{i\epsilon a^\mu}{a^\sigma a^\sigma} \]  

(9)

\(^2\)Here one notes that, due to the commutator \(\{\pi^\mu, \Omega_1\} = -2a^\mu\), one can obtain the algebraic relation \(\{\Omega_1, H\} = \frac{1}{2\epsilon} \Omega_2\).

\(^3\)In Ref. the authors did not include the last term so that one cannot clarify the relations between the BFT scheme and the Dirac bracket one. Also one can easily see that \(\Pi^\mu\) are not the canonical momenta conjugate to the collective coordinates \(a^\mu\) any more since \(\Pi^\mu\) depend on \(a^\mu\), as expected.
with an arbitrary parameter $c$ to be fixed later. It does not also change the
spin and isospin operators (5).

On the other hand, the energy spectrum of the baryons in the SU(2)
Skyrmion can be obtained in the Weyl ordering scheme [14] where the Hamiltonian (4) is modified into the symmetric form

$$ H_N = E + \frac{1}{8L} \Pi^\mu_N \Pi^\mu_N, $$

where

$$ \Pi^\mu_N = -\frac{i}{2} \left[ (\delta^{\mu\nu} - \frac{a^\mu a^\nu}{a^\sigma a^\sigma}) \partial^\nu + \partial^\nu (\delta^{\mu\nu} - \frac{a^\mu a^\nu}{a^\sigma a^\sigma}) + \frac{2ca^\mu}{a^\sigma a^\sigma} \right]. $$

After some algebra, one can obtain the Weyl ordered $\Pi^\mu_N \Pi^\mu_N$ as follows:

$$ \Pi^\mu_N \Pi^\mu_N = -\partial^\mu \partial^\mu + \frac{3a^\mu}{a^\sigma a^\sigma} \partial^\mu + \frac{a^\mu a^\nu}{a^\sigma a^\sigma} \partial^\mu \partial^\nu + \frac{1}{a^\sigma a^\sigma} (\frac{9}{4} - c^2) $$

to yield the modified quantum energy spectrum of the baryons

$$ \langle H_N \rangle = E + \frac{1}{8L} \left[ l(l+2) + \frac{9}{4} - c^2 \right]. $$

Next, following the Abelian BFT formalism [9, 10, 11] which systematically converts the second-class constraints into first-class ones, we introduce two auxiliary fields $\Phi^i$ corresponding to $\Omega^i$ with the Poisson brackets

$$ \{ \Phi^i, \Phi^j \} = \omega^{ij}. $$

The first-class constraints $\tilde{\Omega}_i$ are then constructed as a power series of the auxiliary fields:

$$ \tilde{\Omega}_i = \sum_{n=0}^\infty \Omega^{(n)}_i, \quad \Omega^{(0)}_i = \Omega_i $$

where $\Omega^{(n)}_i$ are polynomials in the auxiliary fields $\Phi^j$ of degree $n$, to be determined by the requirement that the first-class constraints $\tilde{\Omega}_i$ satisfy an Abelian algebra as follows:

$$ \{ \tilde{\Omega}_i, \tilde{\Omega}_j \} = 0. $$

---

4 Here the first three terms are nothing but the three-sphere Laplacian [15] given in terms of the collective coordinates and their derivatives to yield the eigenvalues $l(l+2)$.

5 Due to the missing factor $a^\sigma a^\sigma$ in the denominators in Eq. (8) which is ignored in Refs. [13, 15], apart from $-c^2$ originated from the additional $c$-term in Eq. (3) we obtain the Weyl ordering correction $\frac{9}{4}$, different from the value $\frac{5}{4}$ given in Ref. [14].
Since $\Omega_i^{(1)}$ are linear in the auxiliary fields, one can make the ansatz
$$\Omega_i^{(1)} = X_{ij} \Phi^j. \quad (16)$$
Substituting Eq. (16) into Eq. (15) leads to the following relation:
$$\Delta_{ij} + X_{ik} \omega^{kl} X_{jl} = 0, \quad (17)$$
which, for the standard choice $\omega_{ij} = \epsilon_{ij}$, has a solution
$$X_{ij} = \begin{pmatrix} 2 & 0 \\ 0 & -a^\mu a^\mu \end{pmatrix}. \quad (18)$$
Substituting Eq. (18) into Eqs. (14) and (16) and iterating this procedure, one can obtain the first-class constraints
$$\tilde{\Omega}_1 = \Omega_1 + 2\Phi_1,$$
$$\tilde{\Omega}_2 = \Omega_2 - a^\mu a^\mu \Phi_2, \quad (19)$$
which yield the strongly involutive first-class constraint algebra (15). On the other hand, the corresponding involutive first-class Hamiltonian is given by
$$\tilde{H} = E + \frac{1}{8\mathcal{I}}(\pi^\mu - a^\mu \Phi^2)(\pi^\nu - a^\nu \Phi^2) \frac{a^\mu a^\nu}{a^\nu a^\nu + 2\Phi_1}, \quad (20)$$
which is also strongly involutive with the first-class constraints
$$\{\tilde{\Omega}_i, \tilde{H}\} = 0. \quad (21)$$
Here one notes that, with the Hamiltonian (20), one cannot naturally generate the first-class Gauss’ law constraint from the time evolution of the primary constraint $\Omega_1$. Now, by introducing an additional term proportional to the first-class constraints $\tilde{\Omega}_2$ into $\tilde{H}$, we obtain an equivalent first-class Hamiltonian
$$\tilde{H}' = \tilde{H} + \frac{1}{4\mathcal{I}} \Phi^2 \tilde{\Omega}_2, \quad (22)$$
which naturally generates the Gauss’ law constraint
$$\{\tilde{\Omega}_1, \tilde{H}'\} = \frac{1}{2\mathcal{I}} \tilde{\Omega}_2,$$
$$\{\tilde{\Omega}_2, \tilde{H}'\} = 0. \quad (22)$$
Here one notes that $\tilde{H}$ and $\tilde{H}'$ act on physical states in the same way since such states are annihilated by the first-class constraints. Similarly, the equations of motion for observables are also unaffected by this difference. Furthermore, if we take the limit $\Phi^i \to 0$, then our first-class system exactly returns to the original second-class one.

Now, using the first-class constraints in the Hamiltonian (21), one can obtain a Hamiltonian of the form

$$\tilde{H}' = E + \frac{1}{8L} (a^\mu a^\nu \pi^\nu \pi^\nu - a^\mu \pi^\mu a^\nu \pi^\nu).$$

(23)

Following the symmetrization procedure, the first-class Hamiltonian yields the energy spectrum with the Weyl ordering correction

$$\langle \tilde{H}'_N \rangle = E + \frac{1}{8L} [l(l+2) + 1].$$

(24)

Then, in order for the Dirac bracket scheme to be consistent with the BFT one, the adjustable parameter $c$ in Eq. (12) should be fixed with the values

$$c = \pm \frac{\sqrt{5}}{2}.$$  

(25)

Here one notes that these values for the parameter $c$ relate the Dirac bracket scheme with the BFT one to yield the desired quantization in the SU(2) Skyrmion model so that one can achieve the unification of these two formalisms.

Next, using the Weyl ordering corrected energy spectrum (24), we easily obtain the hyperfine structure of the nucleon and $\Delta$ hyperon masses to yield the soliton energy and the moment of inertia

$$E = \frac{1}{3} (4M_N - M_\Delta)$$

$$I = \frac{3}{2} (M_\Delta - M_N)^{-1}.$$  

(26)

Substituting the experimental values $M_N = 939$ MeV and $M_\Delta = 1232$ MeV into Eq. (26) and using expressions (3), one can predict the pion decay constant $f_\pi$ and the Skyrmion parameter $e$ as follows:

$$f_\pi = 63.2 \text{ MeV}, \quad e = 5.48.$$
With these fixed values of $f_\pi$ and $e$, one can then proceed to yield the predictions for the other static properties of the baryons. The isoscalar and isovector mean-square (magnetic) charge radii and the baryon and transition magnetic moments are contained in Table 1, together with the experimental data and the standard Skyrmion predictions \[1,3,16\]. It is remarkable that the effects of Weyl ordering correction in the baryon energy spectrum are propagated through the model parameters $f_\pi$ and $e$ to modify the predictions of the baryon static properties.

It seems appropriate to comment on the “non-Abelian” BFT scheme of this Skyrme model, although this scheme gives the same baryon energy eigenvalues \[12\]. This non-Abelian scheme is mainly based on the introduction of auxiliary fields satisfying

\[
\{\tilde{\Omega}_i, \tilde{\Omega}_j\} = C_{ij}^{k}\tilde{\Omega}_k, \\
\{\tilde{\Omega}_i, \tilde{H}\} = B_{ij}^{k}\tilde{\Omega}_j,
\]

(27)

where $\tilde{\Omega}_i$ and $\tilde{H}$ can be constructed as a power series of auxiliary fields as before. Then, besides $\omega^{ij}$ and $X_{ij}$ to be chosen, one should find the coefficients $C_{ij}^{k}$ further, which solve $C_{ij}^{k}\tilde{\Omega}_k = \Delta_{ij} + X_{ik}\omega^{kl}X_{jl}$ at the zeroth order of Eq. (27). Among many possible values, if one chooses $C_{12}^{1} = 2$, $\omega^{12} = -\omega^{21} = 1$, $X_{11} = -X_{22} = 1$ with the other vanishing components as in Ref. \[12\], one would have the first-class constraints having a nonlinear term of auxiliary fields as

\[
\tilde{\Omega}_1 = \Omega_1 + \Phi^1 \\
\tilde{\Omega}_2 = \Omega_2 - \Phi^2 + \Phi^1\Phi^2
\]

(28)

satisfying the constraint algebra

\[
\{\tilde{\Omega}_1, \tilde{\Omega}_1\} = \{\tilde{\Omega}_2, \tilde{\Omega}_2\} = 0, \\
\{\tilde{\Omega}_1, \tilde{\Omega}_2\} = 2\tilde{\Omega}_1.
\]

(29) (30)

Moreover, using the corresponding first-class Hamiltonian such as

\[
\tilde{H} = H - \frac{1}{8L}\pi^\mu\pi^\mu\Phi^1 + \frac{1}{2}(B_1^1\Omega_1 - \frac{1}{2L}\Omega_2)\Phi^2 \\
+ \frac{1}{2}(B_1^1 + \frac{1}{2L}\Omega_2)\Phi^1\Phi^2 + \frac{1}{8L}\varrho^\mu\varrho^\mu(1 - \Phi^1)\Phi^2\Phi^2,
\]

(31)

\footnote{For the $\Delta$ magnetic moments, we use the experimental data of Nefkens \textit{et al.} \[17\].}
we obtain

\[
\begin{align*}
\{\tilde{\Omega}_1, \tilde{H}\} &= B_1^1 \tilde{\Omega}_1, \\
\{\tilde{\Omega}_2, \tilde{H}\} &= 0,
\end{align*}
\]  

(32)

where \(B_1^1\) remains undetermined in general. This non-Abelian scheme seems to work, \(i.e.,\) the first-class Hamiltonian (31) has simple finite sums for this nonlinear theory, compared with the previous one Eq. (20), and thus it would be an adequate approach to studying such a nonlinear theory rather than the Abelian version of BFT.

However, there still exists some inconsistency in the algebraic relations, which should be resolved, even though the Hamiltonian (31) yields the same energy eigenvalues (24) as in the Abelian case. In particular, Eq. (30) in the first-class constraint algebra is not consistent in the limit of the auxiliary fields \(\Phi^i \to 0, \ i.e.,\) it does not recover the original second-class structure such as the Poisson algebra (7). This kind of situation happens again when one considers Eq. (32) obtained from the non-Abelian BFT scheme. Moreover, it does not generate the Gauss’ law constraint naturally.

In summary, we have clarified the relation between the Dirac bracket scheme and the BFT one, which has been obscure and unsettled, in the framework of the SU(2) Skyrmion model. In this approach we have introduced the generalized momentum operators with the free parameter, which is fixed to yield the consistency between these two formalisms. We have shown that one could see the effects of the Weyl ordering correction in the baryon energy spectrum propagated through the model parameters \(f_\pi\) and \(e\) in the predictions of the baryon static properties. Also, in the Abelian BFT scheme, we have obtained the Gauss’s law constraint which was not attainable in the non-Abelian BFT one. Finally, through further investigation, the SU(3) extension [18] of this analysis will be studied.

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Table 1: The static properties of baryons in the standard and Weyl ordering corrected (WOC) Skyrmions compared with experimental data. The quantities used as input parameters are indicated by *.

| Quantity | Standard | WOC | Experiment |
|----------|----------|-----|------------|
| $M_N$    | 939 MeV* | 939 MeV* | 939 MeV  |
| $M_\Delta$ | 1232 MeV* | 1232 MeV* | 1232 MeV |
| $f_\pi$  | 64.5 MeV | 63.2 MeV | 93.0 MeV  |
| $e$      | 5.44     | 5.48 |           |
| $\langle r^2 \rangle_{M,I=0}^{1/2}$ | 0.92 fm | 0.94 fm | 0.81 fm |
| $\langle r^2 \rangle_{M,I=1}^{1/2}$ | $\infty$ | $\infty$ | 0.80 fm |
| $\langle r^2 \rangle_{I=0}^{1/2}$ | 0.59 fm | 0.60 fm | 0.72 fm |
| $\langle r^2 \rangle_{I=1}^{1/2}$ | $\infty$ | $\infty$ | 0.88 fm |
| $\mu_p$  | 1.87     | 1.89 | 2.79       |
| $\mu_n$  | −1.31    | −1.32 | −1.91     |
| $\mu_{\Delta^+}$ | 3.72 | 3.75 | 4.7−6.7  |
| $\mu_{N\Delta}$ | 2.27 | 2.27 | 3.29     |
| $\mu_p - \mu_n$ | 3.18 | 3.21 | 4.70     |