On Alfvén’s hypothesis about nuclear hydromagnetic resonances

S I Bastrukov1,2, I V Molodtsova1, J W Yu1 and R X Xu1

1 State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, 100871 Beijing, People’s Republic of China
2 Data Storage Institute, A *Star, Singapore 117608, Singapore
3 Joint Institute for Nuclear Research, Dubna 141980, Russia

E-mail: bast.sergey@gmail.com

Received 9 July 2011
Accepted for publication 1 May 2012
Published 24 May 2012
Online at stacks.iop.org/PhysScr/85/065204

Abstract
The atomic nucleus capability of responding by hydromagnetic vibrations, which was considered long ago by Hannes Alfvén, is re-examined in the context of current developments in nuclear physics and pulsar astrophysics.

PACS number: 26.50+x.

1. Introduction

In the paper [1], Hannes Alfvén long ago pointed out that the electromagnetic response of an atomic nucleus should manifest features generic in hydromagnetic vibrations of an ultra-fine piece of a perfectly conducting continuous medium with frozen-in magnetic field, and made an attempt to evaluate ‘the order of magnitude of possible magnetohydrodynamic (MHD) resonance frequencies’[2]. In this paper we revisit this Alfvén proposition with a focus on the magnitude of intranuclear magnetic field, whose presence in the nucleus volume is the chief prerequisite for sustaining MHD oscillations. In approaching this issue it seems best to start with the current understanding of ‘the history of matter from Big Bang to the present’ [2], which teaches us that the nuclear material objects, both neutron stars and atomic nuclei heavier than 56Fe, are produced in magnetic-flux-conserving core-collapse supernovae. The implosive contraction of a massive main-sequence star gives birth to a neutron star—neutron-dominated mass of self-gravitating nuclear matter with a frozen-in magnetic field of an extremely large intensity. Since the r-process of explosive nucleosynthesis proceeds too in the presence of a superstrong magnetic field, it seems not inconsistent to expect that neutron-dominated mid-weight and heavy nuclei come, like neutron stars, into existence with entrapped magnetic field. In other words, the frozen-in magnetic field is the fundamental property of both neutron stars and heavy atomic nuclei.

Before embarking on a theoretical underpinning of the collective model of nuclear hydromagnetic vibrational response, we note that the basic purpose of the continuum-mechanical description of nuclear giant resonances in terms of vibrational eigenstates of an ultra-fine piece of continuous nuclear matter is to gain insight into the macroscopic properties of nuclear material. The macroscopic nature of giant resonance is determined by restoring force. The position of the energy centroid of a resonance in the nuclear spectrum is defined by the standard quantum-mechanical equation

\[ E_{GR} = \hbar \omega, \]

where \( \omega \) is the frequency of nuclear vibrations carrying information about electromagnetic and mechanical parameters of nuclear material and on the nucleus radius \( R = r_0 A^{1/3} \). The key idea is to extract the magnitude of these parameters by identifying theoretical and empirical energies of resonance under consideration. A representative example...
of such an approach is the macroscopic treatment of isoscalar giant resonances in terms of spheroidal and torsional modes of shear elastic vibrations of a solid sphere whose fundamental frequency reads \( \omega_E = \sqrt{\mu / (\rho R^2)} \), where \( \mu \) is the shear modulus of nuclear matter. This interpretation rests on the observation [20] that the energy of vibrational eigenstates, \( E_c \sim h \omega_c \sim A^{-1/3} \), has one and the same mass-number dependence as the empirical energy of giant isoscalar resonances \( E_{GR} \sim A^{-1/3} \). The main outcome of this line of argument consists in an assessment of the shear modulus of nuclear matter, \( 10^{33} < \mu < 10^{34} \text{dyn cm}^{-2} \) [21, 22], which is of particular interest in the astroseismology of neutron stars (see e.g. [22–24] and references therein). In this context, it is worth mentioning that MHD theory rests on the statement that magnetic field pervading a perfectly conducting medium imparts to it a supplementary portion of solid-mechanical elasticity [5, 6]. This suggests that the hydromagnetic vibrations in question should have some features in common with elastic vibrations of a solid sphere and, hence, manifest themselves as giant resonances of the isoscalar type. Adhering to the idea that mid-weight and heavy nuclei are produced (in the \( r \)-process of explosive nucleosynthesis) with a frozen-in magnetic field, we consider the nuclear MHD vibrations from a different angle than the work [1], with a focus not on the energy of hydromagnetic resonances but on the intensity of intranuclear magnetic field. Namely, having observed that the mass-number dependence of energy of hydromagnetic resonant excitations is similar to that for empirically established giant resonances, we show that the above line of reasoning allows one to evaluate the magnetic field magnitude.

2. Governing equations

Following the line of argument in [1], we assume that the strongly collective response of the atomic nucleus (to perturbation induced by inelastically scattered electrons or elastically scattered photons) is dominated by hydromagnetic vibrations. The MHD equations relevant to this case can be conveniently written in the form [5]

\[ \rho \delta \dot{v} = \frac{1}{c} [\delta \mathbf{j} \times \mathbf{B}], \quad \delta \mathbf{j} = \frac{c}{4\pi} [\nabla \times \delta \mathbf{B}], \quad \delta \mathbf{B} = \nabla \times [\delta \mathbf{v} \times \mathbf{B}], \quad \nabla \cdot \delta \mathbf{v} = 0. \tag{1} \]

These equations describe Lorentz-force-driven oscillations of velocity \( \delta \mathbf{v} \) of material flow coupled with fluctuations of magnetic field \( \delta \mathbf{B} \) about the immobile equilibrium state of an incompressible and perfectly conducting continuous medium of density \( \rho \) pervaded by magnetic field \( \mathbf{B} \). Taking into account that \( \delta \mathbf{v} = \mathbf{u} \) where \( \mathbf{u} \) is the field of material displacement (which is the basic variable of solid-mechanical theory of elasticity), the coupled equations (1) and (2) can be reduced to only one equation

\[ \rho \dot{\mathbf{u}} = \frac{1}{4\pi} [\nabla \times [\nabla \times (\mathbf{u} \times \mathbf{B})] \times \mathbf{B}]. \tag{3} \]

In the approximation of node-free vibrations, widely used in macroscopic models of collective nuclear dynamics, the frequency of the Alfvén hydromagnetic modes can be computed by the energy method, which rests on the integral equation of energy balance

\[ \frac{\partial}{\partial t} \int \frac{\rho \mathbf{u}^2}{2} \, dV = \frac{-1}{4\pi} \int \left[ \mathbf{B} \times [\nabla \times [\nabla \times (\mathbf{u} \times \mathbf{B})]] \right] \cdot \dot{\mathbf{u}} \, dV. \tag{4} \]

In this method, the bulk density \( \rho \) and the shape of the frozen-in magnetic field \( \mathbf{B} \) are regarded as known functions of position, \( \rho(r) = \rho f(r) \) and \( \mathbf{B}(r) = B(b(r)) \), where \( \rho = \text{constant} \) is the density in the nucleus center and the dimensionless scalar function \( f(r) \) describes the density profile, \( B = \text{constant} \) denotes the magnetic field intensity and \( b(r) \) stands for the dimensionless vector function of the spatial distribution of the field over the nucleus volume. Substitution, into the latter equation, of the following separable representation of fluctuating material displacements,

\[ \mathbf{u}(r, t) = \mathbf{a}(r) \alpha(t), \tag{5} \]

where \( \mathbf{a}(r) \) is the time-independent field of instantaneous displacements, leads to the equation for amplitude \( \alpha(t) \) describing harmonic vibrations

\[ \frac{d\mathbf{H}_A}{dt} = 0, \quad \mathbf{H}_A = \frac{M\dot{\alpha}(t)}{2} + \frac{K\alpha^2(t)}{2}, \tag{6} \]

\[ M\dot{\alpha}(t) + K\alpha(t) = 0, \quad M = \rho m, \quad K = \frac{B^2}{4\pi} k, \tag{7} \]

\[ m = \int f(r) \mathbf{a}(r) \cdot \dot{\mathbf{a}}(r) \, dV, \tag{8} \]

\[ k = \int \mathbf{a}(r) \cdot [\mathbf{b}(r) \times [\nabla \times [\nabla \times (\mathbf{a} \times \mathbf{b}(r))]]] \, dV. \tag{9} \]

The general analytic expression for the spectrum of discrete frequencies of MHD oscillations, \( \omega_k(\text{MHD}) \), can be represented as follows:

\[ \omega_k(\text{MHD}) = \sqrt{\frac{K}{M}} = \omega_k(\text{B}) \, s_k, \quad \omega_k(\text{B}) = \frac{v_A}{R}, \tag{10} \]

where \( \omega_k(\text{B}) \) stands for the Alfvén frequency which is the natural unit of frequency of MHD oscillations depending only on the field strength \( B \) and \( s_k \) is numerical spectral factor depending of multipole degree \( \ell \) of hydromagnetic oscillations. As an illustrative example relevant to the subject of this work, we present the result of calculations with frozen-in magnetic field of an axisymmetric configuration, pictured in figure 1, whose spherical components are

\[ \mathbf{b} = \left[ b_r = 0, b_\theta = 0, b_\phi(r, \theta) = \left( R^2 - r^2 \sin^2 \theta / R \right)^{1/2} \right]. \tag{11} \]

The frequency spectrum of MHD vibrations with the node-free irrotational field of instantaneous displacements

\[ \mathbf{a} = A(s) \nabla [s^p P(r)], \tag{12} \]
(with \(P_\ell(\theta)\) being a Legendre polynomial of multipole degree \(\ell\)) is given by

\[
\omega_\ell(B) = \omega_A(B) s_\ell, \quad s_\ell = \left[ \frac{(2\ell + 1)(\ell - 1)}{(2\ell - 1)} \right]^{1/2} \tag{13}
\]

The basic frequency of Alfvén oscillations \(\omega_A\) of a spherical mass \(M = (4\pi/3)\rho R^3\) can be represented in the following equivalent form:

\[
\omega_A(B) = B \sqrt{\frac{R}{3M}}. \tag{14}
\]

Making use of the standard equations for the nucleus mass \(M = mA\) (\(m\) is the nucleon mass) and radius \(R = r_0A^{1/3}\), one finds that mass-number dependence of energies (in MeV) of expected hydromagnetic resonant modes

\[
E_\ell(B) = h\omega_\ell(B) = \kappa_\ell B A^{-1/3},
\]

\[
\kappa_\ell = \hbar \left( \frac{r_0}{3m} \right)^{1/2} s_\ell. \tag{15}
\]

Equation (15) shows the fact that the nuclear hydromagnetic resonances, if they exist, are characterized by one and the same dependence of energy centroids on mass number \(A\) as empirically established giant resonances:

\[
E_{GR}(\ell) = C_\ell A^{-1/3} = \text{constant.} \tag{16}
\]

This suggests that, if some of the detected giant resonances are of a predominantly hydromagnetic nature, then from the identification of the above theoretical and experimental estimates, \(E_\ell(B)A^{1/3} = E_{GR}(\ell)A^{1/3}\), one can evaluate the magnitude of the nuclear internal magnetic field \(B\). For the giant resonant modes lying in the energy interval

\[
30 < E_{GR} A^{1/3} < 90, \quad (5 \leq E_{GR} \leq 15) \text{ (MeV)}, \tag{17}
\]

\[
4.8 \times 10^{-5} < E_{GR} A^{1/3} < 1.4 \times 10^{-4} \text{ (erg)}, \tag{18}
\]

we obtain

\[
3.0 \times 10^{17} \leq B \leq 9.0 \times 10^{17} \text{ G.} \tag{19}
\]

Remarkably, then, according to quantum chromodynamics (QCD) estimates, radius distribution of magnetic moment in a nucleon (whose origin is attributed to persistent Fermi motion of quarks) falls in the range \(0.3 < R_N < 0.6 \text{ fm}\). Taking into account that the nuclear magneton \(\mu_N = 5.05 \times 10^{-26} \text{ erg G}^{-1}\), it easy to see that the intensity of the dipole magnetic field \(B = 2\mu_N/R_N^2\) on the magnetic poles of a sphere of the above radius (spatial region occupied by quark matter) ranges in the interval \(5.0 \times 10^{16} \sim 5.0 \times 10^{17} \text{ G}\). A similar argument has been used in the paper [24] devoted to the possibility of a ferromagnetic state of superdense matter. The above estimates give an idea of the magnetic field intensity in the quark matter, which is expected to exist in the deep cores of neutron stars [25]. In the latter context, it is also noteworthy that current investigations on the search for the chiral magnetic effects lead us to the conclusion that magnetic fields of the above strength should be generated in heavy-ion collisions at intermediate energies (see, e.g., [26, 27] and references therein). It seems interesting to note that the magnetic field energy stored in a spherical volume of nuclear radius, \(R = r_0A^{1/3}\), is proportional to the mass number \(A\); namely, \(W_B \sim B^2 R^3 \sim A\), as is the case for the volume–energy term in the semi-empirical formula for nuclear binding energy. This suggests that the volume term of nuclear binding energy may be of magnetic origin, that is, due to the energy of a huge magnetic field stored in the nucleus at the stage of explosive nucleosynthesis. In this connection, it is worth noting that the synthesis of chemical elements in the presence of superstrong magnetic fields of magnetars has recently been studied in [28–31] with a remarkable conclusion that fields of the order of \(B \sim 10^{17} \text{ G}\) can substantially affect both the \(r\)-process of neutron capture and the formation of shell nuclear structure, that is, magic nuclei with enhanced stability. Finally, it may be worth mentioning the well known astrophysical argument (see, e.g., [32]) regarding the effect of a strong internal magnetic field on the star shape: the prevailing poloidal magnetic field leads to oblate deformation of the star shape, whereas the toroidal field leads to prolate deformation. From this perspective, it is not impossible to expect, therefore, that it is the superstrong internal magnetic field that plays a decisive role in the formation of equilibrium shapes of nuclei heavier than \(^{56}\text{Fe}\).

3. Summary

As a further development of the Alfvén hypothesis about the atomic nucleus capability of responding by MHD vibrations [1], we have set up a collective model providing a theoretical basis for computing the energies of nuclear hydromagnetic resonances. Central to this model, which is appropriate for nuclei heavier than \(^{30}\text{Fe}\), is the intranuclear magnetic field. This field is considered as being frozen-in the mid-weight and heavy nuclei at the stage of their formation in the \(r\)-process of explosive nucleosynthesis. The model predicts that the energy of nuclear hydromagnetic resonances is a linear function of the internal magnetic field. The mass-number dependence of energy has exactly the same shape as that for typical giant resonances. Based on this and assuming that some

---

**Figure 1.** An illustrative example of lines of frozen-in magnetic field having axisymmetric pure toroidal configuration: \(B_r = B_\theta = 0, B_r = 0, b_\ell(r, \theta) = (B/r)(R^2 - r^2 \sin^2 \theta)^{1/2}\) with \(B = \text{constant.}\)
of the observed giant resonances are predominantly of a hydromagnetic nature, we found that the intensity of the intranuclear magnetic field falls in the realm of magnetic fields of magnetars.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (grant numbers 10935001 and 10973002), the National Basic Research Program of China (grant number 2009CB824800) and the John Templeton Foundation.

References

[1] Alfvén H 1957 Phys. Rev. 107 632
[2] Arnett D 1996 Supernovae and Nucleosynthesis: An Investigation of the History of Matter from Big Bang to the Present (Princeton, NJ: Princeton University Press)
[3] Stuewer R 1994 Perspect. Sci. 2 76
[4] Frenkel V J 1974 Arch. Hist. Exact Sci. 13 1
[5] Chandrasekhar S 1961 Hydromagnetic and Hydrodynamic Stability (Oxford: Clarendon)
[6] Alfvén H and Fälthammar C-G 1963 Cosmical Electrodynamics. Fundamental Principles (Oxford: Clarendon)
[7] Fälthammar C 2007 J. Atmos. Sol.-Terr. Phys. 69 1604
[8] Bastrukov S I and Podgaign D V 1996 Phys. Rev. E 54 4465
[9] Bastrukov S, Papoyan V and Podgainy D 1996 JETP Lett. 64 637
[10] Bastrukov S, Molodtsova I, Papoyan V and Podgainy D 1997 J. Atmos. Sol.-Terr. Phys. 69 1604
[11] Bastrukov S I, Chen K-T, Chang H-K, Molodtsova I V and Podgainy D V 2009 Astrophys. J. 690 998
[12] Bastrukov S I, Chang H-K, Molodtsova I V, Wu E-H, Chen K-T and Lan S-H 2009 Astrophys. Space Sci. 323 235
[13] Bastrukov S, Molodtsova I, Takata J, Chang H-K and Xu R X 2010 Phys. Plasmas 17 112114
[14] Bastrukov S I, Yu J-W, Molodtsova I V and Xu R-X 2011 Astrophys. Space Sci. 334 150
[15] Bastrukov S I, Yu J W, Xu R X and Molodtsova I V 2011 Mod. Phys. Lett. A 26 359
[16] Bastrukov S I and Podgaign D V 1997 Phys. Lett. A 226 93
[17] Bastrukov S, Podgainen D and Molodtsa I V 1997 Similarities and Differences Between Atomic Nuclei and Clusters vol 416 ed Y Abe et al (New York: AIP) p 467
[18] Bastrukov S and Yang J 2002 Phys. Scr. 65 340
[19] Bastrukov S I 1994 Phys. Rev. E 49 3166
[20] Bertsch G F 1974 Ann. Phys. 86 138
[21] Bastrukov S, Chang H-K, Miˇ sicu Š, Molodtsova I and Podgainy D 2007 Int. J. Mod. Phys. A 22 3261
[22] Bastrukov S, Molodtsova I, Podgainy D, Miˇ sicu Š and Chang H-K 2008 Phys. Lett. B 664 258
[23] Miˇ sicu Š and Bastrukov S 2008 Fission and Properties of Neutron-Rich Nuclei ed J H Hamilton et al (Singapore: World Scientific) p 641
[24] Akhiezer A I, Laskin N V and Peletminskii S V 1996 J. Exp. Theor. Phys. 82 1066
[25] Xu R X 2009 J. Phys. G: Nucl. Part. Phys. 36 064010
[26] Skokov V, Illarionov Yu A and Toneev V D 2009 Int. J. Mod. Phys. A 24 5923
[27] Ou L and Li B-A 2011 Phys. Rev. C 84 064605
[28] Thompson T A 2003 Astrophys. J. 585 L33 998
[29] Kondratyev V N 2004 Phys. Rev. C 69 038801
[30] Kondratyev V N and Kadenko I M 2005 Mon. Not. R. Astron. Soc. 359 927
[31] Kondratyev V N, Zyzak M V and Kadenko I M 2009 Phys. Atom. Nucl. 72 1781
[32] Shulman A G 2011 Astron. Rep. 55 267
[33] Sweet P A 1961 Proc. R. Soc. Lond. A 260 160