Tunable liquid crystal astigmatic plate

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Abstract. We proposed two schemes of compact tunable liquid crystal astigmatic plate. The device is a modal spatial light modulator with a specific electrodes configuration. Numerical simulations of operation of the device showed its capability to form astigmatic wave front describing by Z_22 and Z_{2-2} Zernike polynomials and also by their superposition. Various modes of operation of LC device are considered. The capability of operation as a truncated axicon is shown.

1. Introduction

The problems of control of a wavefront shape and compensating aberrations of various types have not lost their relevance for many decades. Among the popular applications are astronomy, biomedicine, military industry, microscopy, spectroscopy, and others. For example, the authors [1, 2] solved the problem of compensating phase aberrations in digital holographic microscopy, where astigmatism occurs either due to the presence of a micro-lens used to increase the transverse resolution, or the beam-splitting cube. The astigmatism compensation is important also in light-sheet microscopy [3]. In [4], the technique to compensate wavefront aberrations in a spacecraft telescope is proposed. The task of aberrations compensation occurs when using echelle spectrometers [5] and in endoscopic applications [6]. It is clear that ordinary cylindrical lenses, mirrors and their combinations can be used to compensate aberrations, as was demonstrated by the authors [1-5], who combined the use of those optical elements with new methods of mathematical processing. But adaptive liquid crystals (LC) lenses that have been actively studied for last decades (see, for example, Rev. [7] and references in it) have some advances for dynamical wavefront control. For example, they are interesting and promising in systems where mechanical movements are undesirable. And today LC lens are actively used for aberration compensations and the capabilities of other LC devices for that purposes are studied [6, 7, 8].

LC adaptive lenses can be classified as modulators of modal type. A feature of these devices is the presence in their design of a uniform high-resistance transparent layer - a control electrode, which allows the use of a small number of electrodes and low control voltages. Along with adaptive lenses of various configurations [6, 7, 9-13] devices of this type include wavefront correctors with a tunable response function [14,15], adaptive lenses with optically controlled focal length, two-dimensional matrices of controlled microaxicons [8, 16,17], LC spiral plates [18-21], LC focusing devices [22-26]. Focusing properties of the lenses and parameters of generated optical fields can be controlled by means of variation of voltage amplitude and frequency. The change of control regimes allows forming...
tilted and astigmatic wavefronts and implementing microaxicons. Although from the point of view of functional capabilities the LC devices of modal type cannot be compared with multi-pixel LC modulators, they are of interest as relatively cheap and technologically simple adaptive devices for various practical applications in solving problems in biomedicine, astronomy, and industry.

We propose a new simple configuration of the compact tunable modal LC astigmatic plate in the report. The device allows forming and compensating an astigmatic wavefront as well as to control value and angle orientation of astigmatism by means of change of amplitudes of potentials applied to the device contacts. In contrast, for example, to wavefront correctors or a microaxial arrays, which, can realize not only astigmatism but other types of phase aberrations [8, 14, 15], the proposed phase plate has fewer contact electrodes (only 4 or 8, depending on the configuration).

2. Design of the device, principle of operation and mathematical model of LC astigmatic plate

LC astigmatic plate consists of two glass substrates with applied orienting coatings and the layer of nematic LC between them. The LC-layer thickness is maintained by spacers. Two different configurations of the device are possible – with one and two control electrodes. In the first case, only one of substrates (for example upper) is covered with transparent high-resistance coating (control electrode). Transparent low-resistance homogeneous conductive coating (ITO, for example) is applied on the opposite substrate. The scheme of the device is in figure 1a. The geometry of contact electrodes is in figure 1b. Contact electrodes (black colour) are applied to the line parts of boundaries of control electrode (grey colour).

![Figure 1. Scheme of LC astigmatic plate with one control electrode (a). Geometry of contact electrodes (b).](image1)

The scheme of the device with two control electrodes is in figure 2a. In that case, high-resistance coating is applied to both substrates. Second substrate is similar to the first one but its electrodes system is rotated 45° to the other. The positions of contact electrodes are in figure 2b.

![Figure 2. Scheme of LC astigmatic plate with two control electrodes (a).](image2)

The operational principle of the device is based on the formation of special distribution of voltage over the aperture due to the form of conductive layer and special arrangement of electrodes. The
reorientation of molecules of LC layer occurs under effect of voltage (S-effect). That results in the change of refractive index of the medium and correspondingly of distribution of phase delay introduced by LC layer into passing light wave.

The distribution of potentials on the substrates with applied high-resistance coatings is determined by Laplace equations system:

\[
\begin{align*}
\Delta_x \phi_1 &= 0; \\
\Delta_y \phi_2 &= 0.
\end{align*}
\]  

Voltage \(U\), applied to LC layer has the form:

\[
U = \phi_1 - \phi_2.
\]

Boundary conditions for contact electrodes are as follows. The potentials \(\phi_{11}\) (right), \(\phi_{12}\) (top), \(\phi_{13}\) (left), \(\phi_{14}\) (bottom) are set for the contacts of the upper substrate. And similarly for the contacts of the lower substrate: \(\phi_{21}\) (right), \(\phi_{22}\) (top), \(\phi_{23}\) (left), \(\phi_{24}\) (bottom). The normal derivative of potentials on the other parts of boundary is zero. At that

\[
\begin{align*}
\phi_{ij} &= A_{ij} \exp(i \alpha_{ij}) \\
\phi_{2j} &= A_{2j} \exp(i \alpha_{2j}),
\end{align*}
\]

where \(A_{ij}, A_{2j}\) и \(\alpha_{ij}, \alpha_{2j}\) — amplitudes and phases of corresponding potentials, \(i\) — imaginary unit, \(j = 1, 2, 3, 4\).

Formulas (1) – (3) describes LC astigmatic plate with two control electrodes (the scheme of that plate is presented in figure 2).

Distribution of the potential on the lower substrate is constant and it can be set equal to zero: \(\phi_2 = 0\), in the case of LC plate with one control electrode (the scheme is presented in figure 1). Correspondingly potentials \(\phi_{2j}\) are also equal to zero in that case.

Solving the considered boundary problem, we obtain the corresponding voltage distributions over the device aperture and phase delay introduced by the LC layer into light wave.

3. Numerical simulation results and discussion

We used the following parameters of LC device during simulations: square aperture with 1 mm size and the 20 µm- thickness of the LC layer.

At the first stage we simulated the influence of the contact electrodes size on voltage distribution. The simulation was carried out for LC astigmatic plate with one control electrode. The potentials amplitudes on the contacts were equal to \(A_{11}=A_{13}=1, A_{12}=A_{14}=0\). The form of voltage distributions over the aperture of the astigmatic plate changed depending on relationship between contact’s width and the size of the device aperture. In figure 3 one can see corresponding voltage distributions for different sizes of contact electrodes at that the central part of distributions was limited by means of circular aperture with 1 mm diameter. The obtained distributions were compared with “ideal” voltage distribution specific for astigmatic wave front described by Zernike polynomials \(Z_{22}\) with corresponding coefficient \(C_{22} = 0.5\) [27].

**Figure 3.** Voltage distribution over aperture for different width of contact electrodes: 0.1 mm (a), 0.5 mm (b), 0.9 mm (c). Voltage distribution, described by Zernike polynomials \(Z_{22}\) (d).

RMS deviation of voltage distributions of astigmatic plates from described by Zernike polynomial \(Z_{22}\) voltage distribution depending on the width of contact electrode \(l\) is in figure 4. As you can see from the figure there is an optimal width for the contact electrodes when the voltage distribution
formed over aperture is most similar to “ideal” distribution. The value of that width is 0.45 mm that means the contact sizes should be somewhat less than the half aperture size. We used the value 0.45 mm in our further simulations. At that, there is a range of acceptable values of contacts sizes for which the RMS deviation is not large (see figure 4).

Figure 4. RMS deviation of voltage distributions of astigmatic plates from “ideal” voltage distribution presented in figure 3(d) depending on the width of contact electrodes.

The profile of phase delay is determined by the value and shape of voltage that depends on potentials applied to contacts. To calculate the phase delay profile, we used experimental voltage-phase dependence normalized to the thickness of the layer. That dependence was obtained for the layer of nematic LC (BL037) of 10 µm thickness on 633 nm wavelength [21]. While modelling the device as astigmatic plate, one should choose the part of the volt-phase dependence most similar to a straight line. We used line approximation of that range in our simulations.

3.1. LC astigmatic plate with one control electrode

3.1.1. Formation of astigmatic wavefront

Distributions of voltages over the aperture (a), profile of phase delay (b), polarization interferogram (c) and intensity distribution generated in observation plane at passing of plane homogenous wave through the LC astigmatic plane (d) for values of potentials specified in table 1 are in figure 5.

Table 1. Examples of amplitudes and phases of potentials for astigmatic wave front formation.

| A_{11} (V) | \alpha_{11} | A_{12} (V) | \alpha_{12} | A_{13} (V) | \alpha_{13} | A_{14} (V) | \alpha_{14} |
|------------|-------------|------------|-------------|------------|-------------|------------|-------------|
| 2.45       | 0           | 1.90       | 0           | 2.45       | 0           | 1.90       | 0           |

Figure 5. Voltage distribution over aperture (a), profile of phase delay (b), polarization interferogram (c) and intensity distribution generated in observation plane at passing of plane homogenous wave through the LC astigmatic plane (d) for values of potentials specified in table 1. The observation plane was 20 cm from the LC device.

As shown above in the case of equality of amplitudes on opposite contacts (at that the amplitudes of neighboring contacts are not equal between themselves) the voltage distributions and accordingly distributions of phase delay have the shape of the surfaces with astigmatic profile. It is obvious that one can observe the focusing of light wave passing through that device in one direction and its defocusing in orthogonal direction. Thus, at some distance from the astigmatic plane you can observe the intensity distribution in the shape of narrow line segment, for example vertical (figure 6 d).
We can easily rotate to $90^0$ the formed distributions of the voltage and phase delay by swapping places of potentials pairs on contacts. For the considered in figure 6 case one should set the following potentials amplitudes: $A_{11} = A_{13} = 1.90$ V, $A_{12} = A_{14} = 2.45$ V. B. That leads to the formation of orthogonal distribution of intensity (horizontal line segment).

If this device is used in conjunction with a spherical lens, then at some distance $z$ from the plane of the formation of a vertical segment, the intensity distribution in the shape of a horizontal segment will be formed. This is illustrated by Figure 7, which shows the simulation results when using a lens with a focal length of 10 cm.

![Figure 6. Intensity distributions in observation planes located at different distances from lens: 6.7 cm (a), 10 cm (b), 20 cm (c).](image)

3.1.2. Astigmatic control

Changing the amplitude of the voltage on the electrodes, it is possible to control the magnitude of the phase deflection, and, accordingly, the astigmatism of the wave front (the value of the coefficient $C_2$). To increase the phase deflection, it is necessary to increase the difference of the amplitudes supplied to the contacts of mutually perpendicular directions, without breaking the ratios:

$$
\begin{align*}
A_{11} = A_{13} & \neq A_{12} = A_{14} \\
\alpha_{11} = \alpha_{12} & = \alpha_{13} = \alpha_{14}
\end{align*}
$$

At that it should be remembered that the operating voltage range is determined by the linear part of the volt-phase dependence of the crystal used. Thus, the maximal attainable value of astigmatism is determined by the type of liquid crystal used and the thickness of its layer.

Table 2 presents the values of potentials at which the value of astigmatism is maximum for this type of LC with a thickness of 20 $\mu$m, and amounted to $2\lambda$. The corresponding voltage distributions (a), phase delay (b), polarization interferogram (c) and intensity distribution in the observation plane for the case of plane homogenous wave passing through the LC astigmatism plate (d) are in Figure 7. The observation plane is 10 cm from the LC device.

![Figure 7. Voltage distribution over aperture (a), profile of phase delay (b), polarization interferogram (c) and intensity distribution generated in observation plane at passing of plane homogenous wave through the LC astigmatic plane (d) for values of potentials specified in table 2. The observation plane was at distance 10 cm from the LC device.](image)

| $A_{11}$, V | $\alpha_{11}$, $^0$ | $A_{12}$, V | $\alpha_{12}$, $^0$ | $A_{13}$, V | $\alpha_{13}$, $^0$ | $A_{14}$, V | $\alpha_{14}$, $^0$ |
|-------------|------------------|-------------|------------------|-------------|------------------|-------------|------------------|
| 2.73        | 0                | 2.73        | 0                | 2.73        | 0                | 2.73        | 0                |

![Table 2.](image)
One can see from figure 7 that the change of potentials amplitudes allowed a twofold increase in the deflection of the phase delay and, accordingly, the value of the astigmatism of the formed wavefront.

### 3.1.3. Operation regime of truncated axicon

The aim of this study is to propose maximal simple device allows to form astigmatic wave front and control it. As it was shown in previous paragraphs, to that in the proposed scheme of the device one should set potentials in accordance with formula (4). The question arises, are there any other ratios for the amplitudes and phases of the potentials that allow the formation of other types of wavefront?

It was shown by numerical simulation methods that if we set the amplitudes and phases of the potentials in accordance with formula (5), a truncated axicon can be formed.

\[
\begin{align*}
A_{11} &= A_{13} = A_{12} = A_{14}; \\
\alpha_{11} &= 0; \quad \alpha_{12} = \frac{\pi}{2}; \quad \alpha_{13} = \pi; \quad \alpha_{14} = \frac{3\pi}{2}.
\end{align*}
\]

Setting the potentials in a similar way, we used a certain analogy of the LC device considered here with the LC focusator proposed by us earlier [22, 23, 25]. You may notice that the contact electrodes of both devices are arranged in such a way that they form a square aperture. In [22, 23, 25] we have demonstrated numerically and experimentally the ability of the LC focusing device to form light rings in near diffraction zone and in some plane to focus radiation passing through it into a point spot.

The examples of that potentials are presented in Table 3. In figure 8 you can see the corresponding voltage distributions, phase delay, polarization interferogram and intensity distribution at the distance 5 cm from the LC device for different diameters of circular aperture 1 mm and 0.5 mm. The voltage used in this case varies from the value applied to the contacts to zero in the center of the aperture and so at simulation we used a piecewise continuous approximation of total voltage-phase dependence (not only its line part). Due to a threshold nature of an S-effect, a section with a constant phase delay value is observed in the center of the aperture, and as a result, a truncated quasi-axicon is obtained.

| Table 3. The examples of amplitudes and phases of potentials for implementing axicon. |
|---------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $A_{11}$, V                     | $\alpha_{11}$, $^\circ$ | $A_{12}$, V | $\alpha_{12}$, $^\circ$ | $A_{13}$, V | $\alpha_{13}$, $^\circ$ | $A_{14}$, V | $\alpha_{14}$, $^\circ$ |
| 7.07                           | 0           | 7.07        | 90          | 7.07        | 180         | 7.07        | 270         |

**Figure 8.** Voltage distribution over aperture (a, c), profile of phase delay (b, f), polarization interferogram (c, g) intensity distribution generated in observation plane at passing of plane homogenous wave through the LC device (d, h) for values of potentials specified in table 3. Circular aperture of 1 mm- diameter was used (a-d). Circular aperture of 0.5 mm- diameter limited the working area of the LC device (e-h).
As one can see from the figure, the distribution of the phase delay is obtained in the form of a truncated quadrangular pyramid (see fig. 8b, c), which top is close to a circular cone, but it is not possible to obtain a quality spot in the focusing area (see fig. 8d). To ensure the operation of the device in the mode of truncated axicon one should limited the device aperture with its central area. In this case a plane homogeneous light wave passing through the device focuses in point light spot in some plane (see fig. 8h).

Thus, the proposed LC device depending on applied voltages can implement either tunable LC astigmatic plate to form wavefront described with Zernike polynomial $Z_{22}$ (astigmatism $90^\circ$) or truncated axicon. The device allows you to control astigmatism (value of coefficient $C_{22}$) by smoothly changing voltages applied to contacts. As was mentioned above astigmatic wave front have been implemented in the study [12]. The main differs of the astigmatic plate offered by us is that the high-resistance coating and contact electrodes are applied only on one of the substrates. In addition, in [12] dual-frequency control mode is used. Our device allows working on one frequency.

### 3.2. Astigmatic plate with two control electrodes

Also we proposed the scheme with two control electrodes rotated $45^\circ$ relative to each other (see fig.2). That scheme will operates as the astigmatic plate with one control electrode and form astigmatic wave front described by Zernike polynomial $Z_{22}$ if set nonzero potentials $\phi_j$ similar the case described in 3.1 at upper substrate and zero potentials $\phi_2j$ at lower substrate.

Let us set zero potentials to the contacts of the first substrate, and nonzero potentials to the contacts of the second substrate, similar to the case of a plate with one control electrode (see p. 3.1.). In particular, the amplitudes of opposite contacts should be equal differing from the amplitudes at the neighboring contacts. In that case astigmatic wave front describes by Zernike polynomials $Z_{22}$ will be formed. The examples of possible values of amplitudes and phase of potentials for that case are in row 1 of Table 4 and corresponding distributions of voltage and phase delay, interferogram and intensity distribution in observation plane are in line 1 of figure 9.

The most interesting is the case of nonzero potentials both the first and the second substrates. Change of amplitude and phase of potentials applied to contacts allow to control astigmatism magnitude of each substrates i.e to control coefficients $C_{22}$ and $C_{22}$. And thus one could smoothly rotate the distributions to arbitrary angle and form and correct arbitrary wavefront that could be mathematically described as superposition of wavefronts described by Zernike polynomials $Z_{22}$ and $Z_{22}$ with different values of $C_{22}$ and $C_{22}$ coefficients. The examples of possible values of amplitudes and phase of potentials are in Table 4 and corresponding distributions of voltage and phase delay, interferogram and intensity distribution in observation plane in figure 9.

| Table 4. Examples of potentials amplitude and phase at upper and lower substrates for formation of astigmatic wave front. |
| A_{11}, V | A_{12}, V | A_{13}, V | A_{14}, V | A_{21}, V | A_{22}, V | A_{23}, V | A_{24}, V | \alpha_{1p},^0 | \alpha_{3p},^0 | Row number |
|----------|----------|----------|----------|----------|----------|----------|----------|-----------|-----------|-----------|
| 0.0      | 0.0      | 0.0      | 0.0      | 2.45     | 1.90     | 2.45     | 1.90     | 0         | 180       | 1         |
| 0.99     | 1.19     | 0.99     | 1.19     | 0.83     | 1.34     | 0.83     | 1.34     | 0         | 180       | 2         |
| 0.89     | 0.89     | 0.89     | 0.89     | 1.29     | 1.29     | 1.29     | 1.29     | 0         | 180       | 3         |
| 0.83     | 1.34     | 0.83     | 1.34     | 0.99     | 1.19     | 0.99     | 1.19     | 0         | 180       | 4         |

Thus, the scheme of tunable LC astigmatic plate with two control electrodes (both substrates are covered by high-resistance layers) was proposed. The advantages of that plate compared with the plate considered in paragraph 3.1 (plate with one control electrode) as well as plate described in paper [12] is the capability to form and correct wave fronts with astigmatism of arbitrary angle orientation.

Since astigmatic converters of light beams are of great interest [28–31], in the further we plan to study the capability of using proposed LC astigmatic plate for these purposes.
Figure 9. Voltage distribution over aperture (first column), profile of phase delay (second column), polarization interferogram (third column) and intensity distribution generated in observation plane at passing of plane homogenous wave through the LC astigmatic plane (fourth column) for values of potentials specified in table 4. Row number in table 4 corresponds to number of line in the figure (top down).

4. Conclusion

The results of numerical modeling indicated the ability of implementing compact tunable astigmatic plate of simple design based on nematic liquid crystals. We proposed two schemes for the device. We can realize and control astigmatic wave front which is described by Zernike polynomials $Z_{22}$ for the device with one control electrode. At certain applied voltages that device could operate as a truncated axicon.

The scheme of the LC astigmatic plate with two control electrodes (with eight contacts electrodes) allows you to control and correct any astigmatic wave front by means of amplitude and phase of potentials applied to contacts electrodes of upper and lower substrates. You can realize astigmatic wave front describing by $Z_{22}$ and $Z_{2-2}$ Zernike polynomials and also by their superposition.

The proposed LC astigmatic plate is technologically simple and relatively low cost device. It provides tuning capability without mechanical moving; the capability of the operation in a transmittance regime and as a result the compactness of optical schemes based of that modulators. Besides, the use of solid electrodes allows to form a smooth continuous profile of the phase delay and to control it smoothly. Thus, the diffraction losses can be reduced and the capability of very smoothly (theoretically continuously) control of wave front can be realized.
5. References

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