A lattice QCD perspective on weak decays of $b$ and $c$ quarks

Snowmass 2022 White Paper

Peter A. Boyle,1,2 Bipasha Chakraborty,3 Christine T. H. Davies,4 Thomas DeGrand,5 Carleton DeTar,6 Luigi Del Debbio,2 Aida X. El-Khadra,7 Felix Erben,2 Jonathan M. Flynn,8 Elvira Gámiz,9 Davide Giusti,10 Steven Gottlieb,11 Maxwell T. Hansen,2 Jochen Heitger,12 Ryan Hill,2 William I. Jay,13 Andreas Jüttner,8,14,15 Jonna Koponen,16 Andreas Kronfeld,17 Christoph Lehner,10 Andrew T. Lytle,7,* Guido Martinelli,18 Stefan Meinel,19 Christopher J. Monahan,20,21 Ethan T. Neil,5 Antonin Portelli,2 James N. Simone,17 Silvano Simula,22 Rainer Sommer,23,24 Amarjit Soni,1 J. Tobias Tsang,25 Ruth S. Van de Water,17 Alejandro Vaquero,6 Ludovico Vittorio,26 and Oliver Witzel27,†

1Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA
2Higgs Centre for Theoretical Physics, The University of Edinburgh, EH9 3FD, UK
3DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, UK
4SUPA, School of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, UK
5Department of Physics, University of Colorado, Boulder, CO 80309 USA
6Department of Physics and Astronomy, University of Utah, Salt Lake City, UT 84112, USA
7Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA
8Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK
9Theoretical Physics Department, University of Granada, E-18071 Granada, Spain
10Universität Regensburg, Fakultät für Physik, 93040 Regensburg, Germany
11Department of Physics, Indiana University, Bloomington, IN 47405, USA
12Westfälische Wilhelms-Universität Münster, Institut für Theoretische Physik, 48149 Münster, Germany
13Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
14STAG Research Centre, University of Southampton, Southampton SO17 1BJ, UK
15CERN, Theoretical Physics Department, Geneva, Switzerland
16PRISMA+ Cluster of Excellence & Institute for Nuclear Physics, Johannes Gutenberg University of Mainz, 55128 Mainz, Germany
17Fermi National Accelerator Laboratory, Batavia, IL 60510, USA
18University of Roma “La Sapienza” and INFN, Sezione di Roma, Piazzale Aldo Moro 5, 00185 Roma, Italy
19Department of Physics, University of Arizona, Tucson, AZ 85721, USA
20Department of Physics, William & Mary, Williamsburg, VA 23187, USA
21Theory Center, Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA
22Istituto Nazionale di Fisica Nucleare, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy
23John von Neumann Institute for Computing (NIC), DESY, Platanenallee 6, 15738 Zeuthen, Germany
24Institut für Physik, Humboldt-Universität zu Berlin, Newtonstr. 15, 12489 Berlin, Germany
25CP3-Origins and IMADA, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark
26Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126, Pisa, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, Largo Bruno Pontecorvo 3, I-56127 Pisa, Italy
27Center for Particle Physics Siegen, Theoretische Physik 1, Universität Siegen, 57068 Siegen, Germany (Dated: August 15, 2022)

Lattice quantum chromodynamics has proven to be an indispensable method to determine non-perturbative strong contributions to weak decay processes. In this white paper for the Snowmass community planning process we highlight achievements and future avenues of research for lattice calculations of weak $b$ and $c$ quark decays, and point out how these calculations will help to address the anomalies currently in the spotlight of the particle physics community. With future increases in computational resources and algorithmic improvements, percent level (and below) lattice determinations will play a central role in constraining the standard model or identifying new physics.

*atlytle@illinois.edu
†oliver.witzel@uni-siegen.de

arXiv:2205.15373v2 [hep-lat] 12 Aug 2022
I. INTRODUCTION

Processes involving weak decays of \( b \) or \( c \) quarks may provide a window on new physics not described by the standard model (SM) of elementary particle physics. For several years such weak decay processes have shown persistent differences of a few standard deviations between theoretical predictions of the SM and experimental measurements. The most prominent deviations, commonly referred to as \( B \) anomalies, include

- Ratios testing lepton flavor universality for tree-level decays such as \( B \to D^{(*)}\ell\nu \)
- Tests of lepton flavor universality for rare, loop-level decays such as \( B \to K^{(*)}\ell^+\ell^- \)
- Differences in certain \( q^2 \) bins/ranges for rare decay differential branching fractions e.g. \( B \to K^*\ell^+\ell^- \), \( B_s \to \phi\ell^+\ell^- \) and corresponding derived angular observables like \( P_5 \)
- Some tension in the branching fraction for the rare leptonic decay \( B_s \to \mu^+\mu^- \)
- Tension between exclusive and inclusive determinations of CKM matrix elements \(|V_{ub}|\) and \(|V_{cb}|\)

In addition, the use of QCD factorization to describe nonleptonic decays is under scrutiny due to observed large discrepancies with experimental results. Summaries and further details can be found, for example, in [1–3] and in recent reviews of lattice calculations [4–7]. Currently no single quantity is considered significant and trustworthy enough to claim a smoking gun signal for new physics. While deviations in ratios testing lepton flavor universality mostly point in the same direction and collectively favor some SM extensions over others, the tension between exclusive and inclusive determinations of Cabbibo-Kobayashi-Maskawa (CKM) matrix elements lacks a good phenomenological explanation and may hint at underestimated uncertainties. Understanding and resolving the nature of these \( B \) anomalies is the challenge for the coming years.

With ongoing and future experimental measurements from Belle II, LHCb, ATLAS, CMS, and BES III, it is critical for theoretical predictions to improve to fully leverage increased experimental precision. A key ingredient here are SM predictions for contributions due to quantum chromodynamics (QCD), which describes the strong interactions of quarks and gluons. Standard perturbative methods work reliably only at (very) high energies and truly nonperturbative concepts are required to study the low energy range. Lattice field theory (LFT) is a nonperturbative framework to study QCD processes at low as well as at high energies. Based on first principles, LFT uses the QCD Lagrangian to simulate the strong interaction using Markov chain Monte Carlo methods. After using a few experimental quantities to fix input values like bare quark masses, many predictions for QCD processes can be calculated and the accuracy of the results can be systematically improved.

Specifically, lattice QCD provides theoretical input that enables us to determine parameters of the SM such as the renormalized quark masses, as well as quantities parametrizing nonperturbative hadronic properties like decay constants, form factors, bag parameters, or the QCD contribution to lifetimes. Precise knowledge of such quantities is essential to enhance our understanding of the SM and distinguish, for example, QCD effects from new physics. In the remainder of this section, we summarize some of the major achievements of and opportunities for lattice QCD for weak \( b \) and \( c \) decays, and refer the reader to the relevant part of Section II for more details.

In the heavy quark sector, lattice determinations of the leptonic decay constants \( f_B \) and \( f_{B_s} \) are needed for SM predictions of the rare processes \( B \to \mu^+\mu^- \) and \( B_s \to \mu^+\mu^- \). Here the lattice community has managed to determine both decay constants to the sub-percent level (\( \sim 0.6\% \) for \( f_B(f_{B_s}) \) [4]), so that hadronic uncertainties are now sub-dominant to other sources of error. For \( D_{s(1)} \) mesons, \( f_D \) and \( f_{D_s} \) are used to extract \(|V_{td}|\) and \(|V_{ts}|\) from leptonic decay measurements. Here also the uncertainties in the decay constants are well below those from experiment. Further progress can be achieved by including quantum electrodynamics (QED) and strong-isospin breaking effects into the lattice calculations, and significant advances have been made in this direction [8–14]. The status and physics impact of heavy meson leptonic decays are expanded on in Section II A.

Semileptonic decay processes are critical inputs for heavy flavor studies, where lattice predictions allow for extraction of CKM matrix elements and give pure SM predictions of \( R \)-ratios and other quantities under study. The most precise exclusive determinations of \(|V_{ub}|\) and \(|V_{cb}|\) come from combining experimental and lattice results for \( B \to \pi\ell\nu \) and \( B \to D^{(*)}\ell\nu \) respectively. In recent years, LHCb has given first measurements of processes such as \( B_c \to J/\psi\ell\nu \), \( B_s \to D_s^{(*)}\ell\nu \), \( B_s \to K\ell\nu \), and heavy baryon decays. The lattice community has kept pace with theoretical calculations of these same processes. Progress and outlook for this important class of decays is explored in more detail in Section II B.

Meson mixing and lifetimes are discussed in II C. For neutral \( B \)-mixing, which is dominated by short-distance operators, lattice QCD has already delivered ratios of mass differences with precision around 1.5\%, compared to experimental uncertainties of around 0.4\%. The dominant sources of systematic error in the lattice QCD calculations
can be reduced or eliminated with modern techniques. The next five years are likely to see high-precision calculations of both bag parameters (< 1%) and for ratios (< 0.5%), bringing results to point where QED effects become important. Neutral $D$-meson mixing presents a greater challenge both experimentally and theoretically. The short-distance, CP-violating $(\Delta C = 2)$ matrix elements have already been determined via lattice QCD with roughly 5% statistical precision, comparable to experimental measurements.

Quark masses – fundamental parameters of the SM important for high precision tests of the Higgs sector – have achieved an impressive level of precision ($\lesssim 1.0(0.5\%)$ for $m_c$ ($m_b$) [4]), thanks to long-term efforts from the community. Here also calculations have reached near to the “QED wall” where electromagnetic effects must be accounted for.

Details of progress in this area are given in Section II D.

Moving beyond these “traditional” areas, members of the community have continued to innovate and expand the scope of physics accessible to lattice computation. Important examples of this relevant to studies of heavy flavor include first-principles computation of radiative decay processes (Section II E), development and implementation of theoretical machinery to handle multi-hadron states (discussed in Sections II B and II C), and exploration of methods to determine inclusive decay rates, which would be invaluable for resolving inclusive/exclusive discrepancies in determinations of CKM matrix elements (Section II F). Each of these areas herald a significant advance in our ability to calculate strong processes from first principles, and in the relevant subsections we have attempted to provide context on the remaining challenges and potential timeline to make an impact on phenomenology.

We close by briefly touching on the computational aspects needed to pursue the outlined calculations in Sec. III.

II. PROSPECTS AND CHALLENGES

Lattice calculations with charm and bottom quarks face the challenge that in order to keep discretization effects in simulations with fully relativistic actions under control, the quark mass $m_q$ must obey $m_q < a^{-1}$. Here $a^{-1}$ is the inverse lattice spacing or cutoff typically given in $[\text{GeV}]$, whereas the lattice spacing $a$ is quoted in $[\text{fm}]$. In the past, but also in certain calculations today, the large mass of charm and especially bottom quarks make it impossible to meet this requirement, forcing the use of effective actions. By now algorithmic improvements and increased computational power enable the use of a fully relativistic setup for all quarks and more fully relativistic calculations will be published in the near future. A fully relativistic setup features a simpler and more accurate handling of the renormalization, which for most calculations will be performed nonperturbatively. By combining simulations either featuring up/down quarks at their physical mass or close-to-physical mass bottom quarks, we can already today largely eliminate two major sources of uncertainty: chiral extrapolation and the need for (partly) perturbative renormalization schemes at low energies. By further decreasing the lattice spacing to $a \leq 0.044 \text{fm}$ ($a^{-1} \gtrsim 4.5 \text{GeV}$), even bottom quarks can be simulated with the same action as up/down quarks. With further improved numerical performance, fully dynamical simulations with up/down, strange, charm, and bottom quarks become possible [15], although a practical improvement due to simulating dynamical bottom quarks is most likely marginal. In addition machine learning techniques may offer new possibilities for LFT [16]. Complementary to LFT calculations would be to directly perform quantum simulations [17]. That however requires to have quantum computers with (very) many qubits and long enough coherence time.

A. Leptonic decays

Determinations of leptonic decay constants for $D_{(s)}$ [18–36], $B_{(s)}$ [22, 23, 26, 32, 35, 37–46], and $B_c$ [47, 48] mesons, obtained from 2-point lattice correlation functions at zero momentum, showcase the potential of lattice QCD calculations. Several groups have determined decay constants with high precision and a complete error budget. The agreement between the different results strengthens the credibility of lattice results overall and leads to even more precise average values presented by the Flavor Lattice Averaging Group (FLAG) [4]. Using the lattice averages for $f_D$, $f_{D_s}$, and $f_{D_0}$, together with available experimental data for the corresponding leptonic decays, provides a way of extracting the CKM matrix elements $|V_{cd}|$, $|V_{cs}|$, and $|V_{ub}|$, respectively. For all three cases lattice QCD uncertainties are well below those of experiment.

The most precise determinations of $|V_{cd}|$ at present come from combining experimental measurements of $D \rightarrow \ell \nu \ell$ with the lattice determinations of $f_D$. Until last year, the most precise values of $|V_{cs}|$ similarly came from $D_s \rightarrow \ell \nu \ell$ and $f_{D_s}$, but new lattice results for the semileptonic decay $D \rightarrow K \ell \nu$ [49] are improving on this (see Section II B). The leptonic determination of $|V_{ub}|$ is not competitive with that from semileptonic decays, but with improvements in the experimental precision expected from Belle II, it could help to shed light over the inclusive-exclusive tension in the determination of that parameter.
With precision around or below the percent level, future progress to reduce uncertainties will require electromagnetic and strong isospin breaking effects be accounted for. Lattice calculations combining QED and QCD in the heavy quark sector have already been demonstrated e.g. in the case of charmonium [50] and bottomonium [51]. Further details on radiative decays, which lift helicity suppression, and radiative corrections are presented in Sec. IIE.

Lattice determinations of neutral $B$ meson decay constants are also crucial inputs for the study of rare leptonic decays. These flavor-changing neutral current (FCNC) processes are highly suppressed in the SM, and provide important constraints on new physics. They are largely determined by the same QCD matrix elements as the decay constants, with corrections from subleading operators. The branching ratio for $B_s \rightarrow \mu^+ \mu^-$ is rather precisely determined [52–54] using the lattice input for $f_{B_s}$, and shows some tension with the current experimental result [55–57]. For $B_d \rightarrow \mu^+ \mu^-$, the theory error is larger [52, 53], but the result is consistent with the less well-determined experimental value [56, 57]. Similarly to the extraction of CKM matrix elements, in these comparisons lattice QCD inputs have now exceeded the precision of corresponding experimental measurements. Further insight on the above theory-experiment tension could be extracted from a correlated analysis with the parameters that describe $B_{(s)}$ meson mixing [58].

### B. Exclusive semileptonic decays at tree- and loop-level

Semileptonic decays provide a rich variety of hadronic systems to study many different decay processes, extract CKM matrix elements, and perform stringent tests on the SM. To extract CKM matrix elements, experimental results for tree-level branching fractions are combined with form factors calculated using lattice QCD. These combinations often provide the most precise determinations of the relevant CKM matrix elements, as for $|V_{cs,cd}^{\text{excl.}}|$, $|V_{ub,cd}^{\text{excl.}}|$, or $|V_{ub}^{\text{excl.}}|$ [4]. Both tree-level weak charged current and loop-suppressed flavor-changing neutral current (FCNC) semileptonic decays provide tests of the SM via comparison of experimental measurements and SM predictions for differential rates, angular distributions, or ratios of decay modes with the same hadronic final state but different generations of final-state leptons. These ratios test lepton flavor universality (LFU) and have received substantial attention due to few-$\sigma$ tensions between experimental and theoretical predictions for several decay channels. Several experiments have reported such ratios for tree-level decays (e.g. $B \rightarrow D^{(*)} \ell \nu$ [59–62] or $B_s \rightarrow J/\psi \ell \nu$ [63]) as well as rare loop-level $b \rightarrow s,d, \tau \ell \nu$ decays (e.g. $B \rightarrow K^{(*)} \ell^+ \ell^-$ [64–69] or $B_s \rightarrow \phi \ell^+ \ell^-$ [70, 71]) including also baryonic initial and final states ($\Lambda_b^0 \rightarrow pK^- \ell^+ \ell^-$ [72] or $\Lambda_b^0 \rightarrow \Lambda^+_c \pi^+ \nu$ [73]). On the theory side, these ratios are exceptionally clean, and reported tensions with experimental observations have increased interest in those quantities. While tensions vary for different processes, it is intriguing that these can be accounted for in a model-independent way by assuming new-physics contributions to certain Wilson coefficients of the effective weak Hamiltonian. For details see, e.g., Refs. [74–79] as well as references within. Global fits to $b \rightarrow s(ll)$ and $b \rightarrow c\tau \nu$ anomalies provide a basis to build new physics models. Candidates include, for instance, scenarios with a $Z'$ boson [80–83], leptoquarks [84–93], or scenarios related to supersymmetry (SUSY) [94–96]. For an overview and further details see Ref. [97].

To help confirm or refute the observed deviations, higher-precision calculations of semileptonic form factors are needed, with systematic and statistical uncertainties commensurate with current and upcoming experiments. From the perspective of lattice QCD, the simplest processes to compute are semileptonic decays with a pseudoscalar final state. These calculations involve two-point and three-point correlation functions at zero and non-zero momenta, which furnish the two form factors $f_1$ and $f_2$ entering at tree-level or also $f_T$ for rare loop-level decays. Calculations exist in the literature for a variety of semileptonic $B$ decays: $B \rightarrow \pi \ell \nu$ [98–102], $B \rightarrow \pi^{\pm} \ell^\mp$ [103], $B \rightarrow K^{\pm} \ell^{-}$ [104, 105], $B_s \rightarrow K \ell \nu$ [100, 106–110], $B \rightarrow D \ell \nu$ [111–113], $B_s \rightarrow D_s \ell \nu$ [108, 110, 113–116] and also for semileptonic $D$ decays: $D \rightarrow \pi \ell \nu$ [117–120], $D \rightarrow K \ell \nu$ [49, 117, 119–121]. Once the lattice form factors over the full $q^2$ range have been obtained, it is a simple post-processing task to integrate these form factors over the full $q^2$ range to obtain $R$-ratios testing LFU. Hence $R$-ratios have also been determined for processes like $B \rightarrow \pi \ell \nu$ which so far have not been reported by experiments. The extraction of $|V_{ub}^{\text{excl.}}|$ from $B \rightarrow \pi \ell \nu$, the most precise channel for that CKM parameter, has commensurate errors coming from experiment and lattice QCD form factors [4]. For $B \rightarrow D \ell \nu$ and the extraction of $|V_{cb}^{\text{excl.}}|$, experimental uncertainty presently exceeds the theoretical error from lattice QCD [4]. However, improved theoretical precision will be crucial in both modes in order to make full use of expected improvements in experimental data from Belle II. Improved precision will also be valuable for understanding the inclusive-exclusive tensions for $|V_{cb}|$ and $|V_{us}|$. Furthermore, LHCb demonstrated its capabilities to determine the ratio $|V_{ub}^{\text{excl.}}/V_{cb}^{\text{excl.}}|$ by performing a combined analysis of $B_s \rightarrow K \mu \nu$ and $B_s \rightarrow D_s \mu \nu$ [122]. With more statistics and a finer resolution of the $q^2$ bins this approach can be an interesting alternative to determine the ratio of CKM matrix elements.

In particular, the large mass of the $B_{(s)}$ meson in the initial state leads to a large allowed range of momentum transfer $q^2$ to the outgoing leptons. Maintaining statistical control, especially at low $q^2$, presents a challenge for these calculations. A common approach in the literature has been to focus on the high-$q^2$ behavior and then extend the calculation to full kinematic range using the $z$-expansion [123–126]. Recent work has revived old ideas about
using dispersive bounds [127–131] to constrain the low-$q^2$ behavior of the form factors given results at high $q^2$. Even though covering the full $q^2$ range is computationally challenging, comparing the shape of the form factors to the experimental data across $q^2$ provides further insight on the quality of our theoretical description of the experimental process. Thanks to the advances in simulating heavy flavors and due to new ensembles with finer lattice spacings, the range of directly accessible $q^2$ values is increasing. For more than a decade, the full kinematic range has been accessible to lattice QCD calculations of $D$ semileptonic decays. For heavy-to-heavy decays there has been recent progress towards the full $q^2$ range: $B_s \to D^{\ast} \ell \nu$ [115] as well as $B_c \to B_{s,d}$ [132], $B_c \to D^{\ast} \ell \nu$ and $B_c \to D_s \ell \nu$ [133]. Near-term progress on extending the $q^2$ range in lattice QCD calculations of $B$ semileptonic decays (especially $B$-to-light decays) will be key to improved determinations of CKM matrix elements and more stringent tests of the SM.

Exclusive semileptonic decays with vector final states are more challenging and for many years lattice results for heavy-to-heavy transitions were available only at zero recoil ($B \to D^* \ell \nu$ [134, 135] and $B_s \to D^*_s \ell \nu$ [116, 135]). Recently, the first lattice calculation of the form factors for $B \to D^* \ell \nu$ going beyond zero recoil was performed in Ref. [136]. These results gave the first pure-lattice calculation of the LFU ratio $R(D^*)$. Two additional and entirely independent determinations of $B \to D^* \ell \nu$ form factors at non-zero recoil are expected soon [137, 138]. Experimentally $B \to D^* \ell \nu$ is the preferred channel to extract $|V_{cb}^{\text{exp}}|$. Hence the lattice form factor data for $B \to D^* \ell \nu$ beyond zero recoil are critical to shed light on the tension between exclusive and inclusive determinations of $|V_{cb}|$, compare shapes of the form factors, and test different methods to constrain the low-$q^2$ range using more precise data at high $q^2$. Improved knowledge of the $B \to D^* \ell \nu$ form factors will also benefit the theory prediction of $R(D^*)$, which presently is in tension with the experimental value [139]. Recent results for tree-level decays with vector final states, $B_s \to D^*_s \ell \nu$ [140] and $B \to J/\psi \ell \nu$ [141], include all four form factors and directly cover most of the physically allowed $q^2$ range. Both modes provide alternative ways to extract $|V_{cb}^{\text{exp}}|$ and may provide useful insight into the theory-experiment tensions for $R(D)$ and $R(D^*)$, especially given expected experimental results from Belle II and LHCb.

One outstanding challenge for the future is including the final state’s decay width as part of the nonperturbative process in the light sector is more favorable. Once again, the kinematics in the light sector is more favorable. While such calculations have already been performed in the light sector, e.g., for $B \to K^{(*)} \ell \nu$ [157, 158], these results gave the first pure-lattice calculation of the LFU ratio. Near-term progress on extending the $q^2$ range in lattice QCD calculations of $B$ semileptonic decays (especially $B$-to-light decays) will be key to improved determinations of CKM matrix elements and more stringent tests of the SM. Progress in calculating the $K \pi$ scattering amplitude, a required input for $K \to \pi \pi$ decays, was recently developed [159, 160].

The general formalism enabling lattice studies of $1 \to 2$ hadronic processes, like $B \to K^{(*)}(\to K \pi) \ell^+ \ell^-$, has been developed in Refs. [142–151]. The formalism provides a rigorous non-perturbative relation between finite-volume Euclidean quantities calculable in lattice QCD and the physical, infinite-volume $1 \to 2$ decay amplitude. Compared to form factor calculations with single-hadron final states, $1 \to 2$ hadronic processes require conceptually different calculations and substantially larger computational effort. For a detailed discussion see Refs. [1, 2] and references therein. While such calculations have already been performed in the light sector, e.g., for $K \to \pi \pi$ [152–156], decays of $B_{s,(s)}$ and $D_{s,(s)}$ mesons are typically more challenging because the large decaying meson mass makes additional final states kinematically allowed. The level of difficulty is mainly determined by the energy of the two-hadron final state, so semi-leptonic calculations in which the leptons carry away much of the initial energy are more accessible.

In particular, processes such as $B \to K^{(*)}(\to K \pi) \ell^+ \ell^-$ [156] and $B \to \rho(\to \pi \pi) \ell \nu$ are natural starting points for multi-hadron heavy-flavor decays. Progress in calculating the $K \pi \to K \pi$ scattering amplitude, a required input for the weak decay into this final state, is reported e.g. in Refs. [157, 158].

The kinematics of purely hadronic heavy-flavor decays presents additional challenges. However, by working with an unphysical setup (e.g. heavier-than-physical $u/d$ quarks or lighter-than-physical $b/c$ quarks) the number of kinematically allowed final states can also be controlled in other channels. In this way, the methodology used for calculating $K \to \pi \pi$ can be extended in steps towards $D \to \pi \pi$, for instance. However, honest calculation of the physical process eventually requires a formalism that rigorously treats all important open channels in the decay, including four-particle states. In this vein, work is ongoing to extend the general $1 \to 2$ formalism to more particles. The approach to study weak three-hadron decays, including $K \to \pi \pi \pi$, was recently developed [159, 160].

In the future, this work may open the path to lattice calculations of more advanced phenomenologically interesting processes such as $B^0 \to D^{\ast} \{K^-, \pi^-\}$ [161, 162], or the long distance contribution to neutral $D$-meson mixing. Long-distance contributions (in the form of charm resonances) also occur in rare loop-level decays such as $B \to K^{(*)} \ell^+ \ell^-$ and $B_s \to \phi \ell^+ \ell^-$, where typically an operator product expansion (OPE) is used to express matrix elements of nonlocal operators in terms of local-operator matrix elements. In Refs. [163–165] the local matrix elements have been determined on the lattice to extract the seven form factors for $B \to K^{(*)} \ell^+ \ell^-$ and $B_s \to \phi \ell^+ \ell^-$. In this calculation the vector final state is treated as a stable particle, not accounting for the associated systematic uncertainties. Since the observed deviations between theory and experiment for certain $q^2$ bins have persisted for several years, it is of utmost importance to have well-founded theory predictions. Once again, the kinematics in the light sector is more favorable for lattice calculations, and a proper treatment of long-distance effects in rare kaon decays has been demonstrated [166–172]. Very first steps towards the direct computation of nonlocal matrix elements for $B \to K \ell^+ \ell^-$ have been
C. Meson mixing and lifetimes

Although a loop-level process, neutral $B_s$- and $B$-meson mixing is the preferred experimental channel for extracting the CKM matrix elements $|V_{us}|$ and $|V_{cd}|$. Experiments measure oscillation frequencies with high precision, and global averages [202], dominated by the latest LHCb results [203, 204], show sub-percent level uncertainties. In the SM and beyond, the hadronic contribution to these processes is governed by five local, four-fermion ($\Delta B = 2$) operators. The relevant matrix elements are calculable in lattice QCD via two-point and three-point correlation functions at zero momentum. The SU(3)-breaking ratio $\xi$ [205], formed using the ratio of $B_s$- and $B$-meson mixing parameters, is an important input for global CKM unitarity triangle fits [200, 201]. Lattice calculations of $\xi$ have reached percent-level precision [26, 37, 42, 54, 208, 210], but further progress is needed to achieve the same level of precision for the matrix elements (expressed, e.g., as “bag parameters”) of the individual mixing processes, presently determined at the few percent level. The next five years are likely to see high-precision calculations of both bag parameters ($< 1\%$) and for ratios ($< 0.5\%$), bringing results to points where QED effects become important.

At present, tensions exist among the lattice calculations for some $\Delta B = 2$ operators. Calculations by different groups employ different renormalization schemes, lattice discretizations, and numbers of dynamical quark flavors [26, 37, 42, 54, 208, 210]. Understanding and resolving these tensions is essential for answering the experimental question of whether or not new physics is present in natural $B$-meson mixing [211–213]. As precision improves, higher dimensional operators of the effective weak Hamiltonian become important, particularly for determination of the lifetime difference $\Delta \tau$, which can provide a complementary test for the SM. A pioneering study calculated the dimension-7 operators for neutral meson mixing [214], and confirmation by an independent calculation is desirable.

Neutral $D$-meson mixing offers complementary constraints on the CKM matrix. Hadronic contributions to this process enter in two classes: short-distance, CP-violating ($\Delta C = 2$) matrix elements and long-distance, CP-preserving ($\Delta C = 1$) matrix elements. The $\Delta C = 2$ matrix elements have already been determined via lattice QCD with roughly $5 – 10\%$ statistical precision [215–217], comparable to experimental measurements. Over the next five years, experimental precision is expected to improve by an order of magnitude [218]. For continued impact, improved lattice calculations are needed on the same timescale. The long-distance $\Delta C = 1$ contributions present a much harder theoretical problem, but the kinematically simpler case of kaon-mixing has been investigated [219–221]. Further development of lattice methods for multi-hadron states will be necessary for direct calculations (see remarks in the previous subsection). Support for ongoing theoretical and algorithmic work is needed to enable controlled lattice QCD calculations of the long-distance $\Delta C = 1$ matrix elements on the ten-year timescale.

B-meson lifetimes are important targets for lattice QCD. Besides the $\Delta B = 2$ operators appearing in mixing calculations of hadron lifetimes also require $\Delta B = 0$ operators. In particular, lifetime ratios provide valuable tests of expectations from heavy quark effective theory (HQET) (see [222] for a review). While the ratios $\tau(B^+) / \tau(B_s)$ and $\tau(B^0) / \tau(B_d)$ are in good agreement with the HFLAV average [202], the recent ATLAS measurement [223] deviates substantially from recent measurements by LHCb [224, 225] and CMS [220]. To bolster confidence in the theory predictions, currently dominated by QCD sum-rule calculations [211, 227], a state-of-the-art lattice calculation is desirable. Despite early attempts [228–232], no lattice calculation with a complete systematic error budget exists to date. A lattice calculation of lifetimes faces the challenge that operators of different mass dimension mix under renormalization. A breakthrough on that issue could be made by taking advantage of the gradient flow [233–236] and the concept of the short-flow-time expansion [237–240] to define a new, nonperturbative renormalization scheme [241–244]. A further challenge arises from quark-line disconnected contributions, which are notoriously hard to compute with sufficient statistical precision.
D. \( b \) and \( c \) quark masses

In addition to providing SM predictions of heavy meson and baryon properties, lattice QCD simulations are well-suited to the precision determination of charm and bottom quark masses. These fundamental parameters are needed to stringently test the Higgs sector of the SM \([245]\), by comparing Higgs couplings to \( b \) and \( c \) quarks measured in experiment with the determinations of quark masses computed via lattice QCD. The precision computation of quark masses has made good progress in recent years \([246]\), with lattice now delivering charm and bottom mass to a (sub-)percent level of precision, laying the groundwork for future experimental tests. Measurements from HL-LHC will be able to pin down coupling to bottom at the few-percent level, and first evidence of coupling to charm may also be achievable \([247, 248]\). Next generation accelerators could improve these coupling measurements to a level roughly commensurate with present lattice determinations \([245, 249]\).

There are now several different strategies for determining quark mass — among these are approaches based on moments of current-current correlators \([250, 251]\), the implementation of momentum subtraction schemes on the lattice \([252, 253]\), spectroscopy of heavy meson masses and HQET \([254, 255]\), nonperturbative HQET determinations \([256]\) and computations involving step-scaling in small volume \([257, 258]\). These methods, though all relying crucially on lattice simulation, differ substantially in approach and are hence subject to differing sources of systematic uncertainty. The good agreement amongst results from these approaches \([4]\) gives confidence in the robustness of the determinations at this level of precision. Recently, the effects of adding QED have been quantified, introducing small uncertainty. The good agreement from these approaches \([4]\) gives confidence in the robustness of the determinations at this level of precision. Moving forward, it will be important to hone the efficacy of existing strategies and also develop new ideas, while the widespread use of multiple techniques will help ensure robust error estimates as values continue to improve.

E. Radiative decays and corrections

The ability to calculate radiative decay processes from first principles is an exciting advance that offers opportunities for precision flavor physics, BSM physics, and hadronic structure. The development of lattice QCD methods to calculate radiative decay processes is relatively recent \([8]\). The general procedure for the lattice calculations has been for precision flavor physics, BSM physics, and hadronic structure. The development of lattice QCD methods to

The determination of CKM matrix elements from leptonic decays (cf. Sec. II A) at \( O(\alpha_{em}) \) requires the evaluation of amplitudes with a real photon \([8]\). Thus, this technology can directly address radiative corrections in leptonic decays and advance lattice calculations beyond the “QED wall” for these important processes. First-principles computations are in progress or planned for the structure-dependent form factors for \( B \to \ell \nu \gamma \), \( D(s) \to \ell \nu \gamma \) and \( B(s) \to \ell^+ \ell^- \gamma \), with a broad photon energy spectrum \([261]\). For \( B \) decay, an enhancement of the radiative corrections may be expected due to the nearby \( B^* \) resonance \([262]\). Currently only model-dependent predictions of the decay rates are available in the literature based on QCD factorization and sum rules \([263–274]\). A fully non-factorized, nonperturbative calculation could lead to improved precision in the determination of the corresponding CKM matrix elements.

Adding a hard photon in the final state for leptonic decay of a pseudoscalar meson lifts helicity suppression \([275]\), providing sensitivity to a larger set of operators in the weak effective Hamiltonian. For example the processes \( B^0 \to \ell^+ \ell^- \gamma \) and \( B_s \to \ell^+ \ell^- \gamma \) probe additional operators beyond those of the corresponding purely leptonic decays, which can bear on global fits for \( b \to s \ell^+ \ell^- \), and are well-suited for testing LFU with light leptons \([260]\).

Radiative processes also give important information on hadron structure. For large photon energy the process \( B \to \ell \nu \gamma \) is the cleanest probe of the first inverse moment, \( 1/\lambda_B \), of the \( B \) meson lightcone distribution amplitude, an important input for QCD factorization predictions for nonleptonic B decays \([265, 267, 277]\). Using the upper limit for \( B(B^- \to \ell^- \nu \ell \gamma, E_\gamma > 1 \text{ GeV}) \) from Belle \([278]\) or a lattice form factor calculation can constrain \( \lambda_B \) \([279]\). A similar calculation in the charm sector would allow to make comparisons with BES III results for \( D_s^+ \to e^+ \nu_e \gamma \) \([280, 281]\). An alternate approach, based on recent developments in computing \( x \)-dependent hadron structure, may also provide information on the \( B \) and \( D \) meson distribution amplitudes \([282]\).

F. Inclusive decays

SM predictions for the CKM matrix elements |\( V_{ub} \)| and |\( V_{cb} \)| have been computed based on both inclusive \([202, 283–285]\) and exclusive \([4, 98–100, 134, 136]\) decay channels. The results have exhibited a long-standing tension, with the size of the tension varying between computations. Compared to their exclusive counterparts, inclusive semileptonic decays present an additional theoretical challenge for lattice QCD. The essential difficulty is extracting Minkowski-
space spectral densities from finite-volume Euclidean correlation functions. However, novel and promising ideas [286–290] may have overcome this theoretical hurdle, paving the way for calculations of fully inclusive decay rates from lattice QCD simulations. The new method also opens the door for further applications, such as moments of the lepton energy and the hadronic invariant mass. Exploratory numerical studies now exist [286, 291, 292], raising hopes for future work with physical parameters and controlled systematic uncertainties. These calculations may play a significant role in resolving the tension between inclusive and exclusive determinations of CKM matrix elements. Methods in this entirely new direction in lattice QCD are still in the early stages of development. It is conceivable, however, that results with controlled systematics that are sufficiently precise to allow for meaningful SM tests could become available in the next decade.

III. COMPUTATIONAL RESOURCES

The calculations outlined in this white paper require post-exascale computational resources [293]. A comprehensive research program on weak $b$ and $c$ decays aimed at percent (or subpercent) precision requires gauge-field ensembles where the wavefunctions of both heavy and light degrees of freedom are well resolved and can be studied without distortion. This translates into gauge fields with both small lattice spacings $a \leq 0.044$ fm ($a^{-1} \gtrsim 4.5$ GeV) and with large physical volumes $M_f \cdot L > 4$. Such simulations require increased lattice sizes. For example, a $256^3 \times 512$ lattice at a lattice spacing $a = 0.040$ fm ($a^{-1} \approx 5$ GeV) would allow for simulation of up/down, strange, charm, and bottom quarks at their physical mass in a 10 fm box with $M_f \cdot L \approx 7$. Such an ensemble would provide sufficient physical distance between hadronic initial and final states to isolate the required matrix elements in calculations of form factors or meson mixing. Moreover, such large lattices will allow for new analysis concepts based e.g. on masterfields [294]. Simulating all quarks at their physical mass is particularly beneficial for systematic control of calculations like $B \to p\ell
\nu$, where no effective field theory is available to guide extrapolations to physical masses. The large 10 fm box suppresses finite volume effects, which is especially critical for studying processes with multiple hadrons.

Today, lattice simulations already tackle lattice sizes of $96^3 \times 192$ or $144^3 \times 288$, and research is ongoing to address algorithmic and computational challenges when simulating finer and larger lattices. Due to the algorithmic phenomenon of critical slowing down, development of new algorithms is likely needed to accelerate sampling the QCD path integral in hadronic systems with multiple length scales. On the computational side, harnessing the rapid increase of computational (GPU) performance is constrained by the stagnating network performance. Communication-suppressing algorithms [295, 296] are promising candidates to meet that challenge and with an anticipated tenfold performance increase with the next generation of machines, a $256^3 \times 512$ lattice could already become viable. Professional software support is essential to ensure that algorithmic advances are leveraged to their full potential (e.g., by using advances in generating gauge field configurations in programs that perform measurement tasks). Post-exascale computing resources, when combined with new and more precise experimental data, will enable tests of the SM in the heavy quark sector with unprecedented precision.

ACKNOWLEDGMENTS

P.A.B. and A.S. have been supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under the Contract No. DE-SC-0012704 (BNL). B.C. acknowledges support by STFC consolidated grant ST/P000681/1, the Isaac Newton Trust, and the Leverhulme Trust ECF scheme. C.T.H.D. acknowledges support from the UK Science and Technology Facilities Council grant ST/T000945/1. L.D.D., R.H. and M.T.H. are supported by the U.K. Science and Technology Facility Council (STFC) grant ST/P000630/1. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of High Energy Physics under Award Number DE-SC-0010005 (T.D. and E.T.N.). C.D. and A.V. acknowledge support by the National Science Foundation (NSF) under grant number PHY-20-13064. A.K.K. and A.T.L. acknowledge support by the U.S. Department of Energy under grant number DE-SC0015655. F.E. received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme under grant agreement No 757646. Work supported in part by SRA under Grant No. PID2019-106087GB-C21 and by Junta de Andalucía under Grants No. P18-FR-4314 (FEDER), FQM-101, and A-FQM-467-UGR18 (E.G.). S.G. acknowledges support by the U.S. Department of Energy under grant number DE-SC0010120. M.T.H. is supported by UK Research and Innovation Future Leader Fellowship MR/T019956/1. J.H. acknowledges support by the Deutsche Forschungsgemeinschaft (DFG) through the Research Training Group “GRK 2149: Strong and Weak Interactions – from Hadrons to Dark Matter”. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under grant Contract Numbers DE-SC0011090 and DE-SC0021006 (W.J.). J.K. acknowledges support by the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation program through Grant Agreement.
No. 771971-SIMDAMA. This work was supported in part by the U.S. Department of Energy, Office of Science, under Contract No. DE-AC02-07CH11359 (A.S.K., J.N.S. and R.S.V.). S.M. is supported by the U.S. Department of Energy, Office of High Energy Physics under Award Number DE-SC0009913. C.J.M. is supported in part by USDOE grant No. DE-AC05-06OR23177, under which Jefferson Science Associates, LLC, manages and operates Jefferson Lab. The project leading to this application has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 894103 (J.T.T.).

[1] Christoph Lehner et al. (USQCD), “Opportunities for Lattice QCD in Quark and Lepton Flavor Physics,” Eur. Phys. J. A 55, 195 (2019), arXiv:1904.09479 [hep-lat].

[2] P. Gambino et al., “Challenges in semileptonic B decays,” Eur. Phys. J. C 80, 966 (2020), arXiv:2006.07287 [hep-ph].

[3] Alexander Lenz, “Theory Motivation: What measurements are needed?” in 15th International Conference on Heavy Quarks and Leptons (2021) arXiv:2110.01662 [hep-ph].

[4] Y. Aoki et al., “FLAG Review 2021.,” (2021), arXiv:2111.09849 [hep-lat].

[5] Matthew Wingate, “Quark flavor physics and lattice QCD,” Eur. Phys. J. A 57, 239 (2021), arXiv:2103.17224 [hep-lat].

[6] Andrew Lytle, “Lattice $B \to D^{(*)}$ form factors, $R(D^{(*)})$, and $|V_{cb}|$,” PoS LATTICE2019, 228 (2020), arXiv:2004.01132 [hep-lat].

[7] Oliver Witzel, “Lattice QCD (focus on Charm and Beauty form factors, $R(D^*)$, $b$- & $c$-quark masses),” PoS Beauty2019, 037 (2020), arXiv:2002.01056 [hep-ph].

[8] N. Carrasco, V. Lubich, G. Martinelli, C. T. Sachrajda, N. Tantalo, C. Tarantino, and M. Testa, “QED Corrections to Hadronic Processes in Lattice QCD,” Phys. Rev. D 91, 074506 (2015), arXiv:1502.00257 [hep-lat].

[9] D. Giusti, V. Lubich, G. Martinelli, C. T. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo, and C. Tarantino, “First lattice calculation of the QED corrections to leptonic decay rates,” Phys. Rev. Lett. 120, 072001 (2018), arXiv:1711.06537 [hep-lat].

[10] M. Di Carlo, D. Giusti, V. Lubich, G. Martinelli, C. T. Sachrajda, F. Sanfilippo, S. Simula, and N. Tantalo, “Light-meson leptonic decay rates in lattice QCD+QED,” Phys. Rev. D 100, 034514 (2019), arXiv:1904.08731 [hep-lat].

[11] A. Desiderio et al., “First lattice calculation of radiative leptonic decay rates of pseudoscalar mesons,” Phys. Rev. D 103, 014502 (2021), arXiv:2006.03538 [hep-lat].

[12] R. Frezzotti, M. Garofalo, V. Lubich, G. Martinelli, C. T. Sachrajda, F. Sanfilippo, S. Simula, and N. Tantalo, “Comparison of lattice QCD+QED predictions for radiative leptonic decays of light mesons with experimental data,” Phys. Rev. D 103, 053005 (2021), arXiv:2012.02120 [hep-ph].

[13] R. Frezzotti, G. Gagliardi, V. Lubich, F. Sanfilippo, and S. Simula, “Rotated twisted-mass: a convenient regularization scheme for isospin breaking QCD and QED lattice calculations,” Eur. Phys. J. A 57, 282 (2021), arXiv:2106.07107 [hep-lat].

[14] G. Gagliardi, V. Lubich, G. Martinelli, F. Mazzetti, C. T. Sachrajda, F. Sanfilippo, S. Simula, and N. Tantalo, “Virtual photon emission in leptonic decays of charged pseudoscalar mesons,” Phys. Rev. D 105, 114507 (2022), arXiv:2202.03833 [hep-lat].

[15] Ting-Wai Chiu, “Beauty mesons in $N_f=2+1+1+1$ lattice QCD with exact chiral symmetry,” Phys. Rev. D 102, 034510 (2020), arXiv:2004.02142 [hep-lat].

[16] Denis Boyda et al., “Applications of Machine Learning to Lattice Quantum Field Theory,” in 2022 Snowmass Summer Study (2022) arXiv:2202.05838 [hep-lat].

[17] Christian W. Bauer et al., “Quantum Simulation for High Energy Physics,” (2022), arXiv:2204.03381 [quant-ph].

[18] C. Aubin et al., “Charmed meson decay constants in three-flavor lattice QCD,” Phys. Rev. Lett. 95, 122002 (2005), arXiv:hep-lat/0506030.

[19] E. Follana, C. T. H. Davies, G. P. Lepage, and J. Shigemitsu (HPQCD, UKQCD), “High Precision determination of the pi, K, D and D(s) decay constants in lattice QCD,” Phys. Rev. Lett. 100, 062002 (2008), arXiv:0706.1726 [hep-lat].

[20] B. Blossier et al. (ETM), “Pseudoscalar decay constants of kaon and D-mesons from $N_f = 2$ twisted mass Lattice QCD,” JHEP 07, 043 (2009), arXiv:0904.0954 [hep-lat].

[21] C. T. H. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na, and J. Shigemitsu, “Update: Precision $D_s$ decay constant from full lattice QCD using very fine lattices,” Phys. Rev. D 82, 114504 (2010), arXiv:1008.4018 [hep-lat].

[22] P. Dimopoulos et al. (ETM), “Lattice QCD determination of $m_s$, $f_B$ and $f_N$, with twisted mass Wilson fermions,” JHEP 01, 046 (2012), arXiv:1107.1441 [hep-lat].

[23] A. Bazavov et al. (Fermilab Lattice, MILC), “B- and D-meson decay constant predictions from three-flavor lattice QCD,” Phys. Rev. D 85, 114506 (2012), arXiv:1112.3051 [hep-lat].

[24] Y. Namekawa et al. (PACS-CS), “Charm quark system at the physical point of 2+1 flavor lattice QCD,” Phys. Rev. D 84, 074505 (2011), arXiv:1104.4600 [hep-lat].

[25] Heechang Na, Christine T. H. Davies, Eduardo Follana, G. Peter Lepage, and Junko Shigemitsu, “$|V_{cb}|$ from D Meson Leptonic Decays,” Phys. Rev. D 86, 054510 (2012), arXiv:1206.4936 [hep-lat].

[26] N. Carrasco et al. (ETM), “B-physics from $N_f = 2$ twinQCD: the Standard Model and beyond,” JHEP 03, 016 (2014), arXiv:1308.1851 [hep-lat].
Andreas Crivellin, Giancarlo D’Ambrosio, and Julian Heeck, “Explaining L
and H. Lew, and R. R. Volkas, “Simplest Z-prime model,” Phys. Rev. D 44
X. G. He, Girish C. Joshi, H. Lew, and R. R. Volkas, “NEW Z-prime PHENOMENOLOGY,” Phys. Rev. D 43, 22–24 (1991).
Xiao-Gang He, Girish C. Joshi, H. Lew, and R. R. Volkas, “Simplest Z-prime model,” Phys. Rev. D 44, 2118–2132 (1991).
Wolfgang Altmannshofer and Peter Stangl, “New physics in rare B decays after Moriond 2019,” Eur. Phys. J. C 80, 252 (2020), arXiv:1903.10434 [hep-ph].
138 (2014), arXiv:1403.1269 [hep-ph].
S. Wehle (Belle), “Lepton-Flavor-Dependent Angular Analysis of B → K^+\ell^+\ell^- decays,” JHEP 118, 111801 (2017), arXiv:1612.05014 [hep-ex].
89, 047 (2016), [Erratum: Phys.Rev.Lett. 115, 159901 (2015)], arXiv:1506.04731 [hep-ex].
Roel Aaij et al. (LHCb), “Angular analysis of the B^0 \to K^+\mu^+\mu^- decay using 3 fb^{-1} of integrated luminosity,” JHEP 02, 104 (2016), arXiv:1512.04442 [hep-ex].
J. P. Lees et al. (BaBar), “Measurement of an Excess of \bar{B} \to K^+\mu^+\mu^- decays and the \tau^b decay using three-prong \tau decays,” Phys. Rev. D 97, 072013 (2018), arXiv:1711.02505 [hep-ex].
B, “Angular analysis of the B^0 \to D^{\ast}\tau^0\bar{\nu}_\tau decays, Decays and Implications for Charged Higgs Bosons,” Phys. Rev. D 88, 072012 (2013), arXiv:1303.0571 [hep-ex].
B^0 \to \phi\mu^+\mu^- decays,” JHEP 09, 179 (2015), arXiv:1506.08777 [hep-ex].
Roel Aaij et al. (LHCb), “Branching fraction measurements of the rare B^{\pm}_s \to \phi\mu^+\mu^- and B^{0}_s \to f_2(1525)\mu^+\mu^- decays,” Phys. Rev. Lett. 127, 151801 (2021), arXiv:2105.14007 [hep-ex].
Roel Aaij et al. (LHCb), “Test of lepton universality with \Lambda^0 \to pK^-\ell^+\ell^- decays,” JHEP 05, 040 (2020), arXiv:1912.08139 [hep-ex].
Roel Aaij et al. (LHCb), “Observation of the decay \Lambda^0 \to \Lambda^+_\pi^-\pi^+,” Phys. Rev. Lett. 128, 191803 (2022), arXiv:2201.03497 [hep-ex].
B^0 \to s\ell^+\ell^- transitions in the light of recent data,” JHEP 01, 093 (2018), arXiv:1704.05340 [hep-ph].
Jason Aeberisher, Wolfgang Altmannshofer, Diego Guadagnoli, Méril Reboud, Peter Stangl, and David M. Straub, ”B-decay discrepancies after Moriond 2019,” Eur. Phys. J. C 80, 252 (2020), arXiv:1903.10434 [hep-ph].
Marco Ciuchini, António M. Coutinho, Marco Fedele, Enrico Franco, Ayan Paul, Luca Silvestrini, and Mauro Valli, “New physics in B → s\ell^+\ell^- confronts new data on Lepton Universality,” Eur. Phys. J. C 79, 719 (2019), arXiv:1903.09632 [hep-ph].
Clara Murgui, Ana Peñuelas, Martin Jung, and Antonio Pich, “Global fit to b \to c\tau\nu transitions,” JHEP 09, 103 (2019), arXiv:1904.09311 [hep-ph].
Gino Isidori, Davide Lancieri, Patrick Owen, and Nicola Serra, “On the significance of new physics in b \to s\ell^+\ell^- decays,” Phys. Lett. B 822, 136644 (2021), arXiv:2104.05631 [hep-ph].
Wolfgang Altmannshofer and Peter Stangl, “New physics in rare B decays after Moriond 2021,” Eur. Phys. J. C 81, 952 (2021), arXiv:2103.13370 [hep-ph].
X. G. He, Girish C. Joshi, H. Lew, and R. R. Volkas, “NEW Z-prime PHENOMENOLOGY,” Phys. Rev. D 43, 22–24 (1991).
Jason Aeberisher, Wolfgang Altmannshofer, Diego Guadagnoli, Méril Reboud, Peter Stangl, and David M. Straub, ”B-decay discrepancies after Moriond 2019,” Eur. Phys. J. C 80, 252 (2020), arXiv:1903.10434 [hep-ph].
Marco Ciuchini, António M. Coutinho, Marco Fedele, Enrico Franco, Ayan Paul, Luca Silvestrini, and Mauro Valli, “New physics in B → s\ell^+\ell^- confronts new data on Lepton Universality,” Eur. Phys. J. C 79, 719 (2019), arXiv:1903.09632 [hep-ph].
Clara Murgui, Ana Peñuelas, Martin Jung, and Antonio Pich, “Global fit to b \to c\tau\nu transitions,” JHEP 09, 103 (2019), arXiv:1904.09311 [hep-ph].
Gino Isidori, Davide Lancieri, Patrick Owen, and Nicola Serra, “On the significance of new physics in b \to s\ell^+\ell^- decays,” Phys. Lett. B 822, 136644 (2021), arXiv:2104.05631 [hep-ph].
Wolfgang Altmannshofer and Peter Stangl, “New physics in rare B decays after Moriond 2021,” Eur. Phys. J. C 81, 952 (2021), arXiv:2103.13370 [hep-ph].
X. G. He, Girish C. Joshi, H. Lew, and R. R. Volkas, “NEW Z-prime PHENOMENOLOGY,” Phys. Rev. D 43, 22–24 (1991).
Xiao-Gang He, Girish C. Joshi, H. Lew, and R. R. Volkas, “Simplest Z-prime model,” Phys. Rev. D 44, 2118–2132 (1991).
[84] Gudrun Hiller and Martin Schmaltz, “$R_K$ and future $b \to s \ell \ell$ physics beyond the standard model opportunities,” Phys. Rev. D **90**, 054014 (2014), arXiv:1408.1627 [hep-ph].

[85] Rodrigo Alonso, Benjamin Grinstein, and Jorge Martin Camalich, “Lepton universality violation and lepton flavor $B$-meson decays,” JHEP **10**, 184 (2015), arXiv:1505.05164 [hep-ph].

[86] Martin Bauer and Matthias Neubert, “Minimal Leptoquark Explanation for the $R_{D^{(*)}}$, $R_K$, and $(g - 2)_\mu$ Anomalies,” Phys. Rev. Lett. **116**, 141802 (2016), arXiv:1511.01900 [hep-ph].

[87] Svjetlana Fajfer and Nejc Košnik, “Vector leptoquark resolution of $R_K$ and $R_{D^{(*)}}$ puzzles,” Phys. Lett. B **755**, 270–274 (2016), arXiv:1511.06024 [hep-ph].

[88] Riccardo Barbieri, Gino Isidori, Andrea Pattori, and Fabrizio Senia, “Anomalies in $B$-decays and $U(2)$ flavour symmetry,” Eur. Phys. J. C **76**, 67 (2016), arXiv:1512.01560 [hep-ph].

[89] Bhubanjyoti Bhattacharya, Alakabha Datta, Jean-Pascal Guévin, David Londou, and Ryoutaro Watanabe, “Simultaneous Explanation of the $R_K$ and $R_{D^{(*)}}$ Puzzles: a Model Analysis,” JHEP **01**, 015 (2017), arXiv:1609.09078 [hep-ph].

[90] Dario Buttazzo, Admir Greljo, Gino Isidori, and David Marzocca, “B-physics anomalies: a guide to combined explanations,” JHEP **11**, 044 (2017), arXiv:1706.07808 [hep-ph].

[91] Andreas Crivellin, Dario Müller, and Toshihiko Ota, “Simultaneous explanation of $R(D^{(*)})$ and $b \to s \gamma^\pm \gamma^-$: the last scalar leptoquarks standing,” JHEP **09**, 040 (2017), arXiv:1703.09220 [hep-ph].

[92] Alakabha Datta, Brian Colquhoun, Shoji Hashimoto, Takashi Kaneko, and Jonna Koponen (JLQCD), “Form factors of $B \to K\ell\nu$ decays in nonperturbative HQET,” Int. J. Mod. Phys. A **32**, 183 (2015), arXiv:1507.01618 [hep-ph].

[93] Claudia Cornellia, Javier Fuentes-Martin, and Gino Isidori, “Revisiting the vector leptoquark explanation of the $B$-physics anomalies,” JHEP **07**, 168 (2019), arXiv:1903.11517 [hep-ph].

[94] Wolfgang Altmannshofer, P. S. Bhupal Dev, and Amarjit Soni, “$R_{D^{(*)}}$ anomaly: A possible hint for natural supersymmetry with $R$-parity violation,” Phys. Rev. D **96**, 095010 (2017), arXiv:1704.06659 [hep-ph].

[95] Diganta Das, Chandan Hati, Girish Kumar, and Namit Mahajan, “Scrutinizing $R$-parity violating interactions in light of $R_{K^{(*)}}$ data,” Phys. Rev. D **96**, 095033 (2017), arXiv:1705.09188 [hep-ph].

[96] Wolfgang Altmannshofer, P. S. Bhupal Dev, Amarjit Soni, and Yicong Sui, “Addressing $R_{D^{(*)}}$, $R_{K^{(*)}}$, muon $g - 2$ and ANITA anomalies in a minimal $R$-parity violating supersymmetric framework,” Phys. Rev. D **102**, 015031 (2020), arXiv:2002.12910 [hep-ph].

[97] Wolfgang Altmannshofer and Jure Zupan, “Snowmass White Paper: Flavor Model Building,” in 2022 Snowmass Summer Study (2022), arXiv:2203.07726 [hep-ph].

[98] Emel Dalig, Alan Gray, Matthew Vingate, Christine T. H. Davies, G. Peter Lepage, and Junko Shigemitsu (HPQCD), “B meson semileptonic form-factors from unquenched lattice QCD,” Phys. Rev. D **73**, 074502 (2006), [Erratum: Phys.Rev.D 75, 119906 (2007)], arXiv:hep-lat/0601021.

[99] Jon A. Bailey et al. (Fermilab Lattice, MILC), “[|$V_{ub}$| from $B \to s \ell\ell$ decays and (2+1)-flavor lattice QCD],” Phys. Rev. D **92**, 014024 (2015), arXiv:1503.07839 [hep-lat].

[100] J. M. Flynn, T. Izbuchi, T. Kawanai, C. Lehner, A. Soni, R. S. Van de Water, and O. Witzel (RBC, UKQCD), “$B \to \pi\ell\nu$ and $B_s \to K\ell\nu$ form factors and $|V_{ub}|$ from 2+1-flavor lattice QCD with domain-wall light quarks and relativistic heavy quarks,” Phys. Rev. D **91**, 074510 (2015), arXiv:1501.05373 [hep-lat].

[101] B. Colquhoun, R. J. Dowdall, J. Koponen, C. T. H. Davies, and G. P. Lepage, “$B \to \pi\ell\nu$ at zero recoil from lattice QCD with physical $u$/$d$ quarks,” Phys. Rev. D **93**, 034502 (2016), arXiv:1510.07446 [hep-ph].

[102] Brian Colquhoun, Shoji Hashimoto, Takashi Kaneko, and Jonna Koponen (JLQCD), “Form factors of $B \to \pi\ell\nu$ and a determination of $|V_{ub}|$ with Möbius domain-wall-fermions,” (2022), arXiv:2203.04938 [hep-lat].

[103] Jon A. Bailey et al. (Fermilab Lattice, MILC), “$B \to \ell\ell$ form factors for new-physics searches from lattice QCD,” Phys. Lett. B **764**, 473–479 (2017), [Erratum: Phys.Lett.B 766, 184 (2017)], arXiv:1601.04277 [hep-lat].

[104] Chris Bouchard, G. Peter Lepage, Christopher Monahan, Heechang Na, and Junko Shigemitsu (HPQCD), “Rare decay $B \to K^{*}\ell\nu$ form factors from lattice QCD,” Phys. Rev. D **88**, 054509 (2013), [Erratum: Phys.Rev.D 88, 079901 (2013)], arXiv:1306.2384 [hep-lat].

[105] Jon A. Bailey et al. (Fermilab Lattice, MILC), “$B \to K^{*}\ell\nu$ Decay Form Factors From Three-Flavor Lattice QCD,” Phys. Rev. D **93**, 025026 (2016), arXiv:1509.06235 [hep-lat].

[106] C. M. Bouchard, G. Peter Lepage, Christopher Monahan, Heechang Na, and Junko Shigemitsu (HPQCD), “$B_s \to K\ell\nu$ form factors from lattice QCD,” Phys. Rev. D **93**, 054506 (2016), arXiv:1406.2279 [hep-lat].

[107] Felix Bahr, Debashis Banerjee, Fabio Bernardoni, Anosh Joseph, Mateusz Koren, Hubert Simma, and Rainer Sommer (ALPHA), “Continuum limit of the leading-order HQET form factor in $B_s \to K\ell\nu$ decays,” Phys. Lett. B **757**, 473–479 (2016), arXiv:1601.04277 [hep-lat].

[108] Christopher J. Monahan, Chris M. Bouchard, G. Peter Lepage, Heechang Na, and Junko Shigemitsu, “Form factor ratios for $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ semileptonic decays and $|V_{ub}/V_{cb}|$,” Phys. Rev. D **98**, 114509 (2018), arXiv:1808.09285 [hep-lat].

[109] Felix Bahr, Debashis Banerjee, Fabio Bernardoni, Mateusz Koren, Hubert Simma, and Rainer Sommer, “Extraction of bare form factors for $B_s \to K\ell\nu$ decays in nonperturbative HQET,” Int. J. Mod. Phys. A **34**, 1950166 (2019), arXiv:1903.05870 [hep-lat].

[110] Alexei Bazavov et al. (Fermilab Lattice, MILC), “[|$V_{ub}$| from $B \to K\ell\nu$ decay from lattice QCD],” Phys. Rev. D **100**, 034501 (2019), arXiv:1901.02561 [hep-lat].

[111] Heechang Na, Chris M. Bouchard, G. Peter Lepage, Chris Monahan, and Junko Shigemitsu (HPQCD), “$B \to D_{s\ell}f$ form factors at nonzero recoil and extraction of $|V_{ub}|$,” Phys. Rev. D **92**, 054510 (2015), [Erratum: Phys.Rev.D 93, 119906 [hep-lat].
[112] Jon A. Bailey et al. (Fermilab Lattice, MILC), “$B \rightarrow D \ell \nu$ form factors at nonzero recoil and $|V_{cb}|$ from 2+1-flavor lattice QCD,” Phys. Rev. D 92, 034506 (2015), arXiv:1503.07237 [hep-lat].

[113] Christopher J Monahan, Heechang Na, Chr. M Bouchard, G Peter Lepage, and Junko Shigemitsu, “$B_s \rightarrow D_s \ell \nu$ Form Factors and the Fragmentation Fraction Ratio $f_\ell/f_s$,” Phys. Rev. D 95, 114506 (2017), arXiv:1703.09728 [hep-lat].

[114] Jon A. Bailey et al., “$B_s \rightarrow D_s / B \rightarrow D$ Semileptonic Form-Factor Ratios and Their Application to BR($B_s^0 \rightarrow \mu^+ \mu^-$),” Phys. Rev. D 85, 114502 (2012), Erratum: PhysRevD.86.039904 (2012), arXiv:1202.6346 [hep-lat].

[115] E. McLean, C. T. H. Davies, J. Koponen, and A. T. Lytle (HPQCD), “$B_s \rightarrow D_s \ell \nu$ Form Factors for the full $q^2$ range from Lattice QCD with non-perturbatively normalized currents,” Phys. Rev. D 101, 074513 (2020), arXiv:1906.07011 [hep-lat].

[116] Benoît Blossier, Pierre-Henri Calue, Jochen Heitger, Simone La Cesa, Jan Neuendorf, and Savvas Zafeiropoulos, “Extraction of $B_s \rightarrow D_s^{(*)}$ form factors from $N_f=2$ lattice QCD,” Phys. Rev. D 105, 054515 (2022), arXiv:2110.10061 [hep-lat].

[117] C. Aubin et al. (Fermilab Lattice, MILC, HPQCD), “Semileptonic decays of $D$ mesons in three-flavor lattice QCD,” Phys. Rev. Lett. 94, 011601 (2005), arXiv:hep-ph/0408306.

[118] Heechang Na, Christine T. H. Davies, Eduardo Follana, Jonna Koponen, G. Peter Lepage, and Junko Shigemitsu (HPQCD), “$D \rightarrow \pi \ell \nu$ Semileptonic Decays, $|V_{cd}|$ and 2$^\text{nd}$ Row Unitarity from Lattice QCD,” Phys. Rev. D 84, 114505 (2011), arXiv:1109.1501 [hep-lat].

[119] V. Lublitz, L. Riggio, G. Salerno, S. Simula, and C. Tarantino (ETM), “Scalar and vector form factors of $D \rightarrow \pi(K)|\ell\nu$ decays with $N_f=2+1+1$ twisted fermions,” Phys. Rev. D 96, 054514 (2017), Erratum: Phys.Rev.D 99, 099909 (2019), Erratum: Phys.Rev.D 100, 079901 (2019), arXiv:1706.03017 [hep-lat].

[120] V. Lublitz, L. Riggio, G. Salerno, S. Simula, and C. Tarantino (ETM), “Tensor form factor of $D \rightarrow \pi(K)|\ell\nu$ and $D \rightarrow \pi(K)|\ell\nu$ decays with $N_f=2+1+1$ twisted-mass fermions,” Phys. Rev. D 98, 014516 (2018), arXiv:1803.04807 [hep-lat].

[121] Heechang Na, Christine T. H. Davies, Eduardo Follana, G. Peter Lepage, and Junko Shigemitsu (HPQCD), “The $D \rightarrow K,\ell \nu$ Semileptonic Decay Scalar Form Factor and $|V_{cb}|$ from Lattice QCD,” Phys. Rev. D 82, 114510 (2010), arXiv:1008.4562 [hep-lat].

[122] Roel Aaj et al. (LHCb), “First observation of the decay $B_s^0 \rightarrow K^- \mu^+ \nu_\mu$ and Measurement of $|V_{cb}|/|V_{ub}|$,” Phys. Rev. Lett. 126, 081804 (2021), arXiv:2012.05143 [hep-ex].

[123] C. Glenn Boyd, Benjamin Grinstein, and Richard F. Lebed, “Constraints on form-factors for exclusive semileptonic heavy to light meson decays,” Phys. Rev. Lett. 74, 4603–4606 (1995), arXiv:hep-ph/9412324.

[124] C. Glenn Boyd, Benjamin Grinstein, and Richard F. Lebed, “Model independent determinations of $\tilde{B} \rightarrow D\bar{\nu}, D^*\bar{\nu}$ form-factors,” Nucl. Phys. B 461, 493–511 (1996), arXiv:hep-ph/9508211.

[125] Irinel Caprini, Laurent Lellouch, and Matthias Neubert, “Dispersive bounds on the shape of $\tilde{B} \rightarrow D^* \ell \nu$ form-factors,” Nucl. Phys. B 530, 153–181 (1998), arXiv:hep-ph/9712417.

[126] Claude Bourrely, Irinel Caprini, and Laurent Lellouch, “Model-independent description of $B \rightarrow \pi \ell \nu$ decays and a determination of $|V_{cb}|$, Phys. Rev. D 79, 013008 (2009), [Erratum: Phys.Rev.D 82, 099902 (2010)], arXiv:0807.2722 [hep-ph].

[127] M. Di Carlo, G. Martinelli, M. Naviglio, F. Sanfilippo, S. Simula, and L. Vittorio, “Unitarity bounds for semileptonic decays in lattice QCD,” Phys. Rev. D 104, 054502 (2021), arXiv:2105.02497 [hep-lat].

[128] G. Martinelli, S. Simula, and L. Vittorio, “Constraints for the semileptonic $B \rightarrow D(*)$ form factors from lattice QCD simulations of two-point correlation functions,” Phys. Rev. D 104, 094512 (2021), arXiv:2105.07851 [hep-lat].

[129] G. Martinelli, S. Simula, and L. Vittorio, “Exclusive semileptonic $B \rightarrow \pi \ell \nu$ and $B_s \rightarrow K \ell \nu$ decays through unitarity and lattice QCD,” (2022), arXiv:2202.10285 [hep-ph].

[130] G. Martinelli, S. Simula, and L. Vittorio, “$|V_{cb}|$ and $R(D^{(*)})$ using lattice QCD and unitarity,” Phys. Rev. D 105, 034503 (2022), arXiv:2105.08674 [hep-ph].

[131] G. Martinelli, S. Simula, and L. Vittorio, “Exclusive determinations of $|V_{cb}|$ and $R(D^*)$ through unitarity,” (2021), arXiv:2109.15248 [hep-ph].

[132] Laurence J. Cooper, Christine T. H. Davies, Judd Harrison, Javad Komijani, and Matthew Wingate (HPQCD), “$B_s \rightarrow B_{sd(\ell)}$ form factors from lattice QCD,” Phys. Rev. D 102, 014513 (2020), [Erratum: Phys.Rev.D 103, 099901 (2021)], arXiv:2003.09914 [hep-lat].

[133] Laurence J. Cooper, Christine T. H. Davies, and Matthew Wingate (HPQCD), “Form factors for the processes $B_s^+ \rightarrow D_s^+ \ell^+ \nu_\ell$ and $B_s^0 \rightarrow D_s^0 \ell^0 (\ell^\pm \nu_\ell)$ from lattice QCD,” Phys. Rev. D 105, 014503 (2022), arXiv:2108.11242 [hep-lat].

[134] Jon A. Bailey et al. (Fermilab Lattice, MILC), “Update of $|V_{cb}|$ from the $B \rightarrow D^* \ell \nu$ form factor at zero recoil with three-flavor lattice QCD,” Phys. Rev. D 89, 114504 (2014), arXiv:1403.0635 [hep-lat].

[135] Judd Harrison, Christine Davies, and Matthew Wingate (HPQCD), “Lattice QCD calculation of the $B_{sd(\ell)} \rightarrow D_{sd(\ell)}^{(*)} \ell \nu$ form factors at zero recoil and implications for $|V_{cb}|$,” Phys. Rev. D 97, 054502 (2018), arXiv:1711.11013 [hep-lat].

[136] A. Bazavov et al. (Fermilab Lattice, MILC), “Semileptonic form factors for $B \rightarrow D^* \ell \nu$ at nonzero recoil from 2 + 1-flavor lattice QCD,” (2021), arXiv:2105.14019 [hep-lat].

[137] Takashi Kanelo, “ILQCD form factors for $B \rightarrow D^{(*)}\ell\nu$,” in Challenges in Semileptonic $B$ decays, Barolo, Italy (2022).

[138] Judd Harrison, “HPQCD calculations for $H \rightarrow H$,” in Challenges in Semileptonic $B$ decays, Barolo, Italy (2022).

[139] Simone Bifani, Sébastien Descotes-Genon, Antonio Romero Vidal, and Marie-Hélène Schune, “Review of Lepton Universality tests in $B$ decays,” J. Phys. G 46, 023001 (2019), arXiv:1809.06229 [hep-ex].
[140] Judd Harrison and Christine T. H. Davies (HPQCD), “$B_s \rightarrow D^*_s\pi$ form factors for the full $q^2$ range from lattice QCD,” Phys. Rev. D 105, 094506 (2022), arXiv:2105.11433 [hep-lat].

[141] Judd Harrison, Christine T. H. Davies, and Andrew Lytle (HPQCD), “$B_s \rightarrow J/\psi$ form factors for the full $q^2$ range from lattice QCD,” Phys. Rev. D 102, 094518 (2020), arXiv:2007.06957 [hep-lat].

[142] M. Lüscher, “Volume Dependence of the Energy Spectrum in Massive Quantum Field Theories. 2. Scattering States,” Commun. Math. Phys. 155, 153–188 (1986).

[143] Martin Lüscher, “Signatures of unstable particles in finite volume.” Nucl. Phys. B 364, 237–251 (1991).

[144] Laurent Lellouch and Martin Lüscher, “Weak transition matrix elements from finite volume correlation functions,” Commun. Math. Phys. 219, 31–44 (2001), arXiv:hep-lat/0003023.

[145] C. J. David Lin, G. Martinelli, Christopher T. Sachrajda, and M. Testa, “$K \rightarrow p\pi$ decays in a finite volume,” Nucl. Phys. B 619, 467–498 (2001), arXiv:hep-lat/0104006.

[146] Norman H. Christ, Changhoan Kim, and Takeshi Yamazaki, “Finite volume corrections to the two-particle decay of states with non-zero momentum,” Phys. Rev. D 72, 114506 (2005), arXiv:hep-lat/0507009.

[147] Maxwell T. Hansen and Stephen R. Sharpe, “Multiple-channel generalization of Lellouch-Luscher formula,” Phys. Rev. D 86, 016007 (2012), arXiv:1204.0826 [hep-lat].

[148] Raúl A. Briceño, Maxwell T. Hansen, and André Walker-Loud, “Multichannel 1 → 2 transition amplitudes in a finite volume,” Phys. Rev. D 91, 034501 (2015), arXiv:1506.05695 [hep-lat].

[149] Raúl A. Briceño and Maxwell T. Hansen, “Multichannel 0 → 2 and 1 → 2 transition amplitudes for arbitrary spin particles in a finite volume,” Phys. Rev. D 92, 074509 (2015), arXiv:1502.04314 [hep-lat].

[150] Andria Agadjanov, Véronique Bernard, Ulf-G. Meißner, and Akaki Rusetsky, “The $B \rightarrow K^*\pi\mu\nu$ transitions on the lattice using form factors from lattice QCD,” Phys. Rev. D 100, 014501 (2019), arXiv:1809.03893 [hep-lat].

[151] R. Abbott et al. (RBC, UKQCD), “Direct CP violation and the $\Delta I = 1/2$ rule in $K \rightarrow \pi$ decay from the standard model,” Phys. Rev. D 102, 054509 (2020), arXiv:2004.09440 [hep-lat].

[152] T. Blum et al. (RBC, UKQCD), “Lattice determination of $I=0$ and 2 $\pi$ scattering phase shifts with a physical pion mass,” Phys. Rev. D 104, 054509 (2021), arXiv:2105.02017 [hep-lat].

[153] Judd Harrison and Christine T. Sachrajda, “Prospects for a lattice computation of rare $K\pi$ decays and extraction of the $\pi\pi$ scattering phase shifts with a physical pion mass,” Phys. Rev. D 105, 094501 (2022), arXiv:1911.04036 [hep-lat].

[154] Martin Lüscher, “Signatures of unstable particles in finite volume.” Nucl. Phys. B 364, 237–251 (1991).

[155] Laurent Lellouch and Martin Lüscher, “Weak transition matrix elements from finite volume correlation functions,” Commun. Math. Phys. 219, 31–44 (2001), arXiv:hep-lat/0003023.
Thomas Blake, Stefan Meinel, and Danny van Dyk, “Bayesian Analysis of b Quark Decays K \to \pi^+\ell^-\nu_\ell using domain wall lattice QCD with physical light quark masses,” (2022), arXiv:2202.08795 [hep-lat].

Katsumasa Nakayama, Tutomu Ishikawa, and Shojo Hashimoto (JLQCD), “Charmonium contribution to B \to K^{\ast\ast}\ell\nu_\ell: testing the factorization approximation on the lattice,” PoS LATTICE2019, 062 (2020), arXiv:2001.10911 [hep-lat].

Roel Aaij et al. (LHCb), “Determination of the quark coupling strength |V_{ub}| using baryonic decays,” Nature Phys. 11, 743–747 (2015), arXiv:1504.01568 [hep-ex].

Roel Aaij et al. (LHCb), “Differential branching fraction and angular analysis of \Lambda^0_b \to \Lambda\mu^+\mu^- decays,” JHEP 06, 115 (2015), [Erratum: JHEP 09, 145 (2018)], arXiv:1503.07138 [hep-ex].

Roel Aaij et al. (LHCb), “Measurement of the shape of the \Lambda^0_b \to \Lambda\mu^+\tau^-\nu_\tau differential decay rate,” Phys. Rev. D 96, 112005 (2017), arXiv:1709.01920 [hep-ex].

R. Aaij et al. (LHCb), “Search for the rare decay \Lambda^+_c \to p\mu^+\nu_\mu,” Phys. Rev. D 97, 091101 (2018), arXiv:1712.07938 [hep-ex].

Roel Aaij et al. (LHCb), “Angular moments of the decay \Lambda^0_b \to \Lambda\mu^+\mu^- at low hadronic recoil,” JHEP 09, 146 (2018), arXiv:1808.00264 [hep-ex].

M. Ablikim et al. (BESIII), “Measurement of the absolute branching fraction for \Lambda^+_c \to \Lambda e^+\nu_e,” Phys. Rev. Lett. 115, 221805 (2015), arXiv:1510.02610 [hep-ex].

Medina Ablikim et al. (BESIII), “Measurement of the absolute branching fraction for \Lambda^+_c \to \Lambda\mu^+\nu_\mu,” Phys. Lett. B 767, 42–47 (2017), arXiv:1611.04382 [hep-ex].

Medina Ablikim et al. (BESIII), “Measurement of the absolute branching fraction of the inclusive semileptonic \Lambda^+_c decay,” Phys. Rev. Lett. 121, 251801 (2018), arXiv:1805.09960 [hep-ex].

Y. B. Li et al. (Belle), “Measurements of the branching fractions of the semileptonic decays \Xi^{0}_{b} \to \Xi^{-}\ell^{+}\nu_\ell and the asymmetry parameter of \Xi^{0}_{b} \to \Xi^{-}\pi^{+}\pi^{-},” Phys. Rev. Lett. 127, 121803 (2021), arXiv:2103.06496 [hep-ex].

Shreyasi Acharya et al. (ALICE), “Measurement of the Cross Sections of \Xi^0_b and \Xi^+_b Baryons and of the Branching-Fraction Ratio \text{BR}(\Xi^0_b \to \Xi^-\ell^+\nu_\ell)/\text{BR}(\Xi^0_b \to \Xi^-\pi^+) \text{ in pp collisions at 13 TeV},” Phys. Rev. Lett. 127, 272001 (2021), arXiv:2105.05187 [nucl-ex].

G. Parisi, “The Strategy for Computing the Hadron Mass Spectrum,” Phys. Rept. 103, 203–211 (1984).

G. Peter Lepage, “The Analysis of Algorithms for Lattice Field Theory,” in Theoretical Advanced Study Institute in Elementary Particle Physics (1989).

Stefan Meinel, “\Lambda_c \to N form factors from lattice QCD and phenomenology of \Lambda_c \to n\ell^+\nu_\ell and \Lambda_c \to p\mu^+\nu_\mu decays,” Phys. Rev. D 97, 034511 (2018), arXiv:1712.05783 [hep-lat].

Stefan Meinel, “\Lambda_c \to \Lambda^\ast e^+\nu_\ell form factors and decay rates from lattice QCD with physical quark masses,” Phys. Rev. Lett. 118, 082001 (2017), arXiv:1611.09969 [hep-lat].

Stefan Meinel and Gumaro Rendon, “\Lambda_c \to \Lambda^{*}(1520) form factors from lattice QCD and improved analysis of the \Lambda_b \to \Lambda^{*}(1520) and \Lambda_b \to \Lambda^{*}_c (2595, 2625) form factors,” Phys. Rev. D 105, 054511 (2022), arXiv:2107.13140 [hep-lat].

Stefan Meinel and Gumaro Rendon, “Charm-baryon semileptonic decays and the strange \Lambda^* resonances: New insights from lattice QCD,” Phys. Rev. D 105, 051505 (2022), arXiv:2107.13084 [hep-ph].

Alakabha Datta, Saeed Kamali, Stefan Meinel, and Ahmed Rashed, “Phenomenology of \Lambda_b \to \Lambda_c\tau^+\nu_\tau using lattice QCD calculations,” JHEP 08, 131 (2017), arXiv:1702.02243 [hep-ph].

William Detmold, Christoph Lehner, and Stefan Meinel, “\Lambda_b \to p\ell^-\nu_\ell and \Lambda_b \to \Lambda_c\ell^-\nu_\ell form factors from lattice QCD with relativistic heavy quarks,” Phys. Rev. D 92, 034503 (2015), arXiv:1503.01421 [hep-lat].

William Detmold, C. J. David Lin, Stefan Meinel, and Matthew Wingate, “\Lambda_b \to p\ell^-\nu_\ell form factors from lattice QCD with static b quarks,” Phys. Rev. D 88, 014512 (2013), arXiv:1306.0446 [hep-lat].

Thomas Blake, Stefan Meinel, Musleem Rahimi, and Danny van Dyk, “Dispersive bounds for local form factors in \Lambda_b \to \Lambda transitions,” (2022), arXiv:2205.06041 [hep-ph].

Thomas Blake, Stefan Meinel, and Danny van Dyk, “Bayesian analysis of b \to s\mu^+\mu^- Wilson Coefficients using the Full Angular Distribution of \Lambda_b \to \Lambda(\to p\pi^-)\mu^+\mu^- Decays,” Phys. Rev. D 101, 035023 (2020), arXiv:1912.05811 [hep-ph].

William Detmold and Stefan Meinel, “\Lambda_b \to \Lambda^\ast e^+\ell^- form factors, differential branching fraction, and angular observables from lattice QCD with relativistic b quarks,” Phys. Rev. D 93, 074501 (2016), arXiv:1602.01399 [hep-lat].

William Detmold, C. J. David Lin, Stefan Meinel, and Matthew Wingate, “\Lambda_b \to \Lambda^\ast e^+\ell^- form factors and differential branching fraction from lattice QCD,” Phys. Rev. D 87, 074502 (2013), arXiv:1212.4827 [hep-lat].

Stefan Meinel and Gumaro Rendon, “\Lambda_b \to \Lambda^* (2595, 2625)\ell^-\nu_\ell form factors from lattice QCD,” Phys. Rev. D 103, 094516 (2021), arXiv:2103.08775 [hep-lat].

Stefan Meinel and Gumaro Rendon, “\Lambda_b \to \Lambda^{*}(1520)\ell^-\nu_\ell form factors from lattice QCD,” Phys. Rev. D 103, 074505 (2021), arXiv:2009.09313 [hep-lat].

Qi-An Zhang et al., “First lattice QCD calculation of semileptonic decays of charmed-strange baryons \Xi_c^\ast ,” Chin. Phys. C 46, 011002 (2022), arXiv:2103.07064 [hep-lat].

J. Charles, Andreas Hocker, H. Lacker, S. Laplace, F. R. Le Diberder, J. Malcles, J. Ocariz, M. Pivk, and L. Roos (CKMfitter Group), “CP violation and the CKM matrix: Assessing the impact of the asymmetric B factories,” Eur.
[231] Massimo Di Pierro, Christopher T Sachrajda, and Christopher Michael (UKQCD), “An Exploratory lattice study of spectator effects in inclusive decays of the $\Lambda_b$ baryon,” Phys. Lett. B 468, 143 (1999), [Erratum: Phys.Lett.B 525, 360–360 (2002)], arXiv:hep-lat/9906031.

[232] Massimo Di Pierro and Christopher T. Sachrajda (UKQCD), “A Lattice study of spectator effects in inclusive decays of B mesons,” Nucl. Phys. B 534, 373–391 (1998), arXiv:hep-lat/9805028.

[233] R. Narayanan and H. Neuberger, “Infinite N phase transitions in continuum Wilson loop operators,” JHEP 03, 064 (2006), arXiv:hep-th/0601210.

[234] Martin Lüscher, “Trivializing maps, the Wilson flow and the HMC algorithm,” Commun. Math. Phys. 293, 899–919 (2010), arXiv:0907.5491 [hep-lat].

[235] Martin Lüscher, “Properties and uses of the Wilson flow in lattice QCD,” JHEP 08, 071 (2010), [Erratum: JHEP 03, 092 (2014)], arXiv:1006.4518 [hep-lat].

[236] Martin Lüscher, “Chiral symmetry and the Yang–Mills gradient flow,” JHEP 04, 123 (2013), arXiv:1302.5246 [hep-lat].

[237] Robert V. Harlander, Yannick Kluth, and Fabian Lange, “The two-loop energy–momentum tensor within the gradient-flow formalism,” Eur. Phys. J. C 78, 944 (2018), [Erratum: Eur.Phys.J.C 79, 858 (2019)], arXiv:1808.09837 [hep-lat].

[238] Johannes Artz, Robert V Harlander, Fabian Lange, Tobias Neumann, and Mario Prausa, “Results and techniques for higher order calculations within the gradient-flow formalism,” JHEP 06, 121 (2019), [Erratum: JHEP 10, 032 (2019)], arXiv:1905.00882 [hep-lat].

[239] Jangho Kim, Thomas Luu, Matthew D. Rizik, and Andrea Shindler (SymLat), “Nonperturbative renormalization of the quark chromoelectric dipole moment with the gradient flow: Power divergences,” Phys. Rev. D 104, 074516 (2021), arXiv:2106.07633 [hep-lat].

[240] Robert V. Harlander and Fabian Lange, “Effective electroweak Hamiltonian in the gradient-flow formalism,” Phys. Rev. D 105, L071504 (2022), arXiv:2201.08618 [hep-lat].

[241] Hiroki Makino and Hiroshi Suzuki, “Lattice energy–momentum tensor from the Yang–Mills gradient flow—incursion of fermion fields,” PTEP 2014, 063B02 (2014), [Erratum: PTEP 2015, 079202 (2015)], arXiv:1403.4772 [hep-lat].

[242] Andrea Carosso, Anna Hasenfratz, and Ethan T. Neil, “Nonperturbative Renormalization of Operators in Near-Conformal Systems Using Gradient Flows,” Phys. Rev. Lett. 121, 201601 (2018), arXiv:1806.01385 [hep-lat].

[243] Asobu Suzuki, Yusuke Taniguchi, Hiroshi Suzuki, and Kazuyuki Kanaya, “Four quark operators for kaon bag parameter with gradient flow,” Phys. Rev. D 103, 034508 (2020), arXiv:2006.00999 [hep-lat].

[244] Anna Hasenfratz, Christopher J. Monahan, Matthew David Rizik, Andrea Shindler, and Oliver Witzel, “A novel non-perturbative renormalization scheme for local operators,” PoS LATTICE2021, 155 (2022), arXiv:2201.09740 [hep-lat].

[245] G. Peter Lepage, Paul B. Mackenzie, and Michael E. Peskin, “Expected Precision of Higgs Boson Partial Widths within the Standard Model,” (2014), arXiv:1404.0319 [hep-ph].

[246] Andrew T. Lytle, “$m_c$ (and $m_b$) from lattice QCD,” PoS CHARM2020, 013 (2021), arXiv:2110.15090 [hep-lat].

[247] “Snowmass White Paper Contribution: Physics with the Phase-2 ATLAS and CMS Detectors,” (2022).

[248] Andrea Dainese, Michelangelo Mangano, Andreas B. Meyer, Aleandro Nisati, Gavin Salam, and Mika Anton Vesterinen, eds., Report on the Physics at the HL-LHC, and Perspectives for the HE-LHC, CERN Yellow Reports: Monographs, Vol. 7/2019 (CERN, Geneva, Switzerland, 2019).

[249] Michael E. Peskin, “Estimation of LHC and ILC Capabilities for Precision Higgs Boson Coupling Measurements,” in Andrea Dainese, Michelangelo Mangano, Andreas B. Meyer, Aleandro Nisati, Gavin Salam, and Mika Anton Vesterinen, eds., Report on the Physics at the HL-LHC, and Perspectives for the HE-LHC, CERN Yellow Reports: Monographs, Vol. 7/2019 (CERN, Geneva, Switzerland, 2019).

[250] Andrew T. Lytle, C. T. H. Davies, D. Hatton, G. P. Lepage, and C. Sturm (HPQCD), “Determination of quark masses from $N_f = 4$ lattice QCD and the RI-SMOM intermediate scheme,” Phys. Rev. D 98, 054507 (2018), arXiv:1805.06225 [hep-lat].

[251] Katsumasa Nakayama, Brendan Fahy, and Simon Kubes (ALPHA), “Non-perturbative quark mass renormalisation and running in $N_f = 3$ QCD,” Eur. Phys. J. C 78, 387 (2018), arXiv:1802.05243 [hep-lat].

[252] Jochen Heitger, Fabian Joswig, and Simon Kubes (ALPHA), “Determination of the charm quark mass in lattice QCD with $2 + 1$ flavours on fine lattices,” JHEP 05, 288 (2021), arXiv:2101.02694 [hep-lat].

[253] D. Hatton, C. T. H. Davies, J. Koponen, G. P. Lepage, and A. T. Lytle, “Determination of $m_c/m_b$ and $m_b$ from $N_f = 4$ lattice QCD+QED,” Phys. Rev. D 103, 114508 (2021), arXiv:2102.09609 [hep-lat].
Alexander Khodjamirian, Rusa Mandal, and Thomas Mannel, “Inverse moment of the B meson distribution amplitude,” PoS LATTICE2019, 134 (2019), arXiv:1907.00279 [hep-lat].

Christopher Kane, Davide Giusti, Christoph Lehner, Stefan Meinel, and Amarjit Soni, “Controlling unwanted exponentials in lattice calculations of radiative leptonic decays,” PoS LATTICE2021, 162 (2022), arXiv:2110.13196 [hep-lat].

Danil Becirevic, Benjamin Haas, and Enni Kou, “Soft Photon Problem in Leptonic B-decays,” Phys. Lett. B 681, 257–263 (2009), arXiv:0907.1845 [hep-ph].

Enrico Lunghi, Dan Pirjol, and Daniel Wyler, “Factorization in leptonic radiative B → γℓν decays,” Nucl. Phys. B 649, 349–364 (2003), arXiv:hep-ph/0210091.

V. M. Braun and A. Khojdjamirian, “Soft contribution to B → γℓν and the B-meson distribution amplitude,” Phys. Lett. B 718, 1014–1019 (2013), arXiv:1210.4453 [hep-ph].

Yu-Ming Wang, “Factorization and dispersion relations for radiative leptonic B decay,” JHEP 09, 159 (2016), arXiv:1606.03080 [hep-ph].

Yu-Ming Wang and Yue-Long Shen, “Subleading-power corrections to the radiative leptonic B → γℓν decay in QCD,” JHEP 05, 184 (2018), arXiv:2102.13390 [hep-ph].

Yue-Long Shen, Zhi-Tian Zou, and Yan-Bing Wei, “Subleading power corrections to B → γℓν decay in PQCD approach,” Phys. Rev. D 99, 016004 (2019), arXiv:1811.09250 [hep-ph].

Christopher Kane, Christoph Lehner, Stefan Meinel, and Amarjit Soni, “Radiative leptonic decays on the lattice,” PoS LATTICE2018, 073 (2018), arXiv:1811.09250 [hep-ph].

Marc Beneke, Christoph Bobeth, and Yu-Ming Wang, “Radiative decays of heavy-light mesons and the Bγℓν decay in PQCD,” Nucl. Phys. B 650, 356–390 (2003), arXiv:hep-ph/0209216.

Marc Beneke, Christoph Bobeth, and Yu-Ming Wang, “Bd,s → γℓℓ decay with an energetic photon,” JHEP 12, 148 (2020), arXiv:2008.12494 [hep-ph].

Yue-Long Shen, Yan-Bing Wei, Xue-Chen Zhao, and Si-Hong Zhou, “Revisiting radiative leptonic B → γℓν decays,” JHEP 09, 005 (2021), arXiv:2106.13616 [hep-ph].

Marc Beneke, Christoph Bobeth, and Yu-Ming Wang, “Bd,s → γℓℓ decay with an energetic photon,” JHEP 12, 148 (2020), arXiv:2008.12494 [hep-ph].

Alexandre Carvunis, Francesco Dettori, Shireen Gangal, Diego Guadagnoli, and Camille Normand, “On the effective B meson distribution amplitude,” Phys. Lett. A 444, 2021016 (2021), arXiv:2105.13730 [hep-lat].

D. Atwood, G. Eilam, and A. Soni, “Pure leptonic radiative decays B±, D(s) → ℓγγ and the annihilation graph,” Mod. Phys. Lett. A 11, 1061–1068 (1996), arXiv:hep-ph/9411367.

S. Descotes-Genon and C. T. Sachrajda, “Factorization, the light cone distribution amplitude of the B meson and the annihilation graph,” Mod. Phys. Lett. A 11, 1061–1068 (1996), arXiv:hep-ph/9411367.

M. Beneke, V. M. Braun, Yao Ji, and Yan-Bing Wei, “Radiative leptonic decay B → γℓν with subleading power corrections,” JHEP 07, 154 (2018), arXiv:1804.04962 [hep-ph].

M. Gelb et al. (Belle), “Search for the rare decay of B → ℓ±νℓγγ with improved hadronic tagging,” Phys. Rev. D 98, 112016 (2018), arXiv:1810.12976 [hep-ex].

M. Beneke and J. Rohrwild, “B meson distribution amplitude from B → γℓν,” Phys. Rev. C 71, 1818 (2005), arXiv:1110.3228 [hep-ph].

Medina Ablikim et al. (BESIII), “Search for the radiative leptonic decay D → γℓν,” Phys. Rev. D 95, 071102 (2017), arXiv:1702.05837 [hep-ex].

Medina Ablikim et al. (BESIII), “Search for the decay D∗± → γℓ±νℓ,” Phys. Rev. D 99, 072002 (2019), arXiv:1902.03351 [hep-ex].

Martha Constantinou et al., “Lattice QCD Calculations of Parton Physics,” (2022), arXiv:2202.07193 [hep-lat].

Paolo Gambino and Christoph Schwanda, “Inclusive semileptonic fits, heavy quark masses, and Vcb,” Phys. Rev. D 89, 014022 (2014), arXiv:1307.4551 [hep-ph].

Andrea Alberti, Paolo Gambino, Kristopher J. Healey, and Soumitra Nandi, “Precision Determination of the Cabibbo-Kobayashi-MaskawaElement Vcb,” Phys. Rev. Lett. 114, 061802 (2015), arXiv:1411.6560 [hep-ph].

L. Cao et al. (Belle), “Measurements of partial branching fractions of inclusive B → Xuℓ±νℓ decays with hadronic tagging,” Phys. Rev. D 104, 012008 (2021), arXiv:2102.00020 [hep-ex].

Paolo Gambino and Shoji Hashimoto, “Inclusive Semileptonic Decays from Lattice QCD,” Phys. Rev. Lett. 125, 032001 (2020), arXiv:2005.13730 [hep-lat].

Shoji Hashimoto, “Inclusive semi-leptonic B meson decay structure functions from lattice QCD,” JHEP 2017, 053 (2017), arXiv:1703.01881 [hep-lat].

Maxwell T. Hansen, Harvey B. Meyer, and Daniel Rosbain, “From deep inelastic scattering to heavy-flavor semileptonic decays: Total rates into multihadron final states from lattice QCD,” Phys. Rev. D 96, 094513 (2017), arXiv:1704.08993 [hep-lat].

Thomas DeGrand, “Remarks about weighted energy integrals over Minkowski spectral functions from Euclidean lattice data,” Phys. Rev. D 106, 014504 (2022), arXiv:2203.04393 [hep-lat].
[291] Sandro Maechler, Paolo Gambino, and Shoji Hashimoto, “Comparison of lattice QCD results for inclusive semi-leptonic decays B mesons with the OPE,” PoS LATTICE2021, 512 (2022), arXiv:2111.02833 [hep-ph].

[292] Paolo Gambino, Shoji Hashimoto, Sandro Mächler, Marco Panero, Francesco Sanfilippo, Silvano Simula, Antonio Smecca, and Nazario Tantalo, “Lattice QCD study of inclusive semileptonic decays of heavy mesons,” JHEP 07, 083 (2022), arXiv:2203.11762 [hep-lat].

[293] Peter Boyle et al., “Lattice QCD and the Computational Frontier,” in 2022 Snowmass Summer Study (2022) arXiv:2204.00039 [hep-lat].

[294] Anthony Francis, Patrick Fritzsch, Martin Lüscher, and Antonio Rago, “Master-field simulations of O(α)-improved lattice QCD: Algorithms, stability and exactness,” Comput. Phys. Commun. 255, 107355 (2020), arXiv:1911.04533 [hep-lat].

[295] Martin Luscher, “Solution of the Dirac equation in lattice QCD using a domain decomposition method,” Comput. Phys. Commun. 156, 209–220 (2004), arXiv:hep-lat/0310048.

[296] Peter A. Boyle, Dennis Bollweg, Christopher Kelly, and Azusa Yamaguchi, “Algorithms for domain wall Fermions,” PoS LATTICE2021, 470 (2022), arXiv:2203.17119 [hep-lat].