THE PARKER INSTABILITY IN A THICK GASEOUS DISK. II. NUMERICAL SIMULATIONS IN TWO DIMENSIONS

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ABSTRACT

We present two-dimensional, ideal-MHD numerical simulations of the Parker instability in a multicomponent warm disk model. The calculations were done using two numerical codes with different algorithms, TVD and ZEUS-3D. The outcomes of the numerical experiments performed with both codes are very similar and confirm the results of the linear analysis for the undular mode derived in a previous work: the most unstable wavelength is about 3 kpc and its growth timescale is between 30 and 50 Myr (the growth rate is sensitive to the position of the upper boundary of the numerical grid). Thus, the time and length scales of this multicomponent disk model are substantially larger than those derived for thin-disk models. We use three different types of perturbations, random, symmetric, and antisymmetric, to trigger the instability. The antisymmetric mode is dominant and determines the minimum time for the onset of the nonlinear regime. The instability generates dense condensations, and the final peak column density value in the antisymmetric case, as also derived by Kim and coworkers, is about a factor of 3 larger than its initial value. These wavelengths and density enhancement factors indicate that the instability alone cannot be the main formation mechanism of giant molecular clouds in the general interstellar medium. The role of the instability in the formation of large-scale corrugations along spiral arms is briefly discussed.

Subject headings: instabilities — ISM: clouds — ISM: magnetic fields — ISM: structure — MHD

1. INTRODUCTION

The interstellar disk in our Galaxy is thicker and more magnetized than previously assumed, and the transition between the gaseous disk and the halo has a complex magnetic field structure and extended gas components, including the ionized Reynolds layer (Hoyle & Ellis 1963; Reynolds 1989). This type of extended disk also seems to be common in other spiral galaxies, and thick diffuse gas layers, with worms and dusty filaments, have been observed, for instance, in NGC 891, NGC 5775, and NGC 4302 (Dettmar 1990; Rand, Kulkarni, & Hester 1990; Rand 1995; Howk & Savage 1997). Similarly, the observed magnetic fields in edge-on galaxies can be traced well above the main gaseous disk (Hummel & Beck 1995).

Boulares & Cox (1990; hereafter BC) have incorporated the observed extended gas layers, the gradient of the magnetic field, and the cosmic-ray pressure in a model for the thick disk of the Galaxy (see also Kalberla & Kerp 1998). The existence of magnetized gas layers with large scale heights has important consequences for the structure of the gaseous disk and its response to large-scale perturbations (see Cox 1990; Martos 1993; Franco, Santillán, & Martos 1995). For instance, the total interstellar pressure in a disk with extended layers is substantially larger than that for thin-disk models. Also, the interaction between the disk and the halo is more complex: spiral density waves can produce flows, shocks, and even star formation at high Galactic latitudes (Martos & Cox 1998; Martos et al. 1999), whereas the penetration of high-velocity clouds into the disk is severely reduced by magnetic pressure and tension (e.g., Santillán et al. 1999). The stability of the system is also modified, and, depending on the temperature and B-field distributions, the equilibrium configuration of a thick disk can be Parker unstable. For the observed conditions at the solar neighborhood (BC), the linear stability analysis of a multicomponent isothermal disk indicates that the fastest growing undular mode has a wavelength of the order of 3 kpc (Kim et al. 2000a, hereafter Paper I). This is a factor of 8 larger than the one derived for the thin gaseous disk and illustrates the variations that can be expected when considering the extended gas layers.

In Paper I we derived both the dispersion relations and the final equilibrium configuration in two dimensions (2D) of the undular mode of the Parker instability. This mode generates high-density condensations at the magnetic valleys, which are eventually stabilized by magnetic tension, and the system settles down into a new final equilibrium. The linear analysis provides the initial growth rates and interdistance scales of the condensations, and the final equilibrium configuration (obtained by solving the magnetohydrostatic equations with the flux freezing condition) provides their final steady state structure. To complete the study, here we present 2D numerical simulations for the evolution of the instability. This enables us to verify the results of the linear analysis and to follow the details of the nonlinear regime until the final equilibrium is reached. As a check on the reliability of the numerical results, the numerical experiments have been performed with two different MHD numerical codes, ZEUS-3D and TVD. The results obtained with both codes are very similar and confirm the
growth rates and wavelengths of the fastest growing modes derived in Paper I. The plan of the present paper is as follows. The details of the numerical work are described in the next section. The results are described in § 3, and a brief summary and discussion are given in § 4.

2. NUMERICAL SETUP AND CODES

Since the first one-dimensional simulation of the problem (Baierlein 1983), there have been a number of numerical studies of the evolution of the Parker instability under different conditions. For instance, Matsumoto et al. (1988, 1990) and Matsumoto & Shibata (1992) have explored the nature of the undular and mixed modes for accretion disk environments with 2D and three-dimensional (3D) simulations. Also, using the same numerical code, Shibata et al. (1989a, 1989b), Kaisig et al. (1990), and Nozawa et al. (1992) have used the Parker instability as the driving mechanism for emergent flux tubes in the Sun and explained several features of the solar activity. In a different type of study, addressing the question of giant molecular cloud formation, Basu, Mouschovias, & Paleologou (1997) and Kim et al. (1998) have performed 2D and 3D simulations of the instability in a thin gaseous disk under the influence of a simplified uniform gravity. Here we follow the evolution of the undular instability in a multicomponent, isothermal and magnetized gaseous disk, with the properties of the solar neighborhood.

2.1. A Conservative Form of the Isothermal MHD Equations

We use two-dimensional Cartesian coordinates \((y, z)\), whose directions are azimuthal and perpendicular to the Galactic disk, respectively. When the vertical gravitational acceleration has only a \(z\)-component, \(g = -g(z)\hat{z}\), the conservative form of the isothermal MHD equations is (see Paper I)

\[
\frac{\partial q}{\partial t} + \frac{\partial F_x}{\partial y} + \frac{\partial F_z}{\partial z} = S, \tag{1}
\]

where

\[
S = \begin{pmatrix} 0 \\ 0 \\ -g v_z \end{pmatrix}, \tag{4}
\]

and \(a\) is the isothermal speed for all interstellar components (i.e., the velocity dispersion of all gas components is set to the same value) and the rest of the symbols have their usual meaning.

2.2. Initial Condition: The Thick Galactic Disk

As the initial condition for our numerical experiments, we introduce the Galactic disk model that was originally proposed by Martos (1993). The model is the magnetohydrostatic equilibrium of a system composed of five interstellar gas layers, with a horizontal magnetic field and a realistic gravitational acceleration. Since the detailed building procedure of the model is given in Santillán et al. (1999) and Paper I, here we give only a brief description of the model.

The two building blocks of the model are the density distribution of the multicomponent gaseous disk (BC),

\[
n(z) = 0.6 \exp \left[-\frac{z^2}{2(70 \text{ pc})^2}\right] + 0.3 \exp \left[-\frac{z^2}{2(135 \text{ pc})^2}\right] + 0.07 \exp \left[-\frac{z^2}{2(135 \text{ pc})^2}\right] + 0.1 \exp \left[-\frac{|z|}{400 \text{ pc}}\right] + 0.03 \exp \left[-\frac{|z|}{900 \text{ pc}}\right], \tag{5}
\]

and a simple fitting formula (Martos 1993) to the gravitational acceleration at the solar neighborhood (Bienaymé, Robin, & Crézé 1987),

\[
g(z) = 8 \times 10^{-6} \left[1 - 0.52 \exp \left(-\frac{|z|}{325 \text{ pc}}\right) - 0.48 \exp \left(-\frac{|z|}{900 \text{ pc}}\right)\right] \text{ cm s}^{-2}. \tag{6}
\]

The mass density stratification, including 10% He by number, is \(\rho(z) = 1.27 m_H n(z)\), where \(m_H\) is the mass of a hydrogen atom. The total (gas plus magnetic) pressure, \(P_0(z)\), is obtained from the integral of \(\rho(z)\) and \(g(z)\) with the boundary condition, \(P_0(z = 10 \text{ kpc}) = 0\). Assuming that the gas is isothermal and setting the midplane value of the magnetic field, \(B_\parallel(0) \approx 5 \mu\text{G (BC; Heiles 1996)}\), we deduce the initial field stratification, \(B(z)\), from the distribution of the total pressure \(P_0(z)\). In contrast to thin-disk models, where the magnetic-to-thermal pressure ratio \(\alpha\) is usually held constant, our isothermal thick disk has a \(z\)-dependent \(\alpha\) that increases with height (see Paper I). Thus, the high-latitude gas is largely supported by the magnetic field and is unstable to the undular mode.

The isothermal thick disk model is specified by two parameters: the effective scale height, \(H_{\text{eff}} = 166 \text{ pc}\), and the isothermal sound speed \(a = 8.4 \text{ km s}^{-1}\) (see Paper I). This
scale height is simply defined as one-half of the total disk column density divided by the density at the midplane, and the sound speed is determined by the midplane values $P_0(0)$ and $B_0(0)$. These two parameters, $H_{\text{eff}}$ and $a$, define our units of length and velocity. The time unit is defined as $H_{\text{eff}}/a = 1.9 \times 10^7$ yr, and, unless otherwise stated, all physical quantities are normalized to these three units.

2.3. Perturbations

The linear stability analysis in Paper I showed that the above equilibrium state is unstable against the undular instability when the perturbation wavelength is longer than about 1.5–1.8 kpc, depending on the location of the nodal point (i.e., the location where $v_z = 0$; see below). These critical wavelength values correspond to nodal points located between 5 and 1.5 kpc above midplane, respectively. Thus, in order to initiate the instability in the numerical experiments, the equilibrium state is perturbed with appropriate wavelengths.

We have used three kinds of perturbations. The first type is the even-parity midplane symmetric (MS) perturbation, which forms structures distributed symmetrically with respect to the midplane,

$$v_z = C \sin \left( \frac{2 \pi y}{\lambda_y} \right) \sin \left( \frac{2 \pi z}{\lambda_z} \right),$$

where $C$ is the amplitude of the vertical velocity perturbation and $\lambda_y$ and $\lambda_z$ are the horizontal and vertical wavelengths of the perturbations. The second type is the odd-parity midplane antisymmetric (MA) perturbation,

$$v_z = C \sin \left( \frac{2 \pi y}{\lambda_y} \right) \cos \left( \frac{2 \pi z}{\lambda_z} \right).$$

The initial amplitudes for both MS and MA perturbations are usually set to $C = 10^{-3} a$.

The last type is a random velocity perturbation, without any preferred wavelength or parity. The standard deviation of each velocity component is here set to $10^{-4} a$.

2.4. Boundary Conditions

In deriving the dispersion relations in Paper I, perturbations were allowed to vary periodically along the horizontal axis. The boundary conditions at midplane determined the parity of the perturbation (i.e., $v_z = 0$ at $z = 0$ for the MS perturbation and $d v_z / dz = 0$ at $z = 0$ for the MA case). However, regardless of parity, the vertical velocity perturbation at the upper (or lower) boundary was always set equal to zero. Therefore, the upper boundary becomes a nodal point.

In the numerical experiments, for consistency with the boundary conditions used in the linear analysis, we use a periodic condition at the $y$-boundaries of the computational domain and a reflecting condition at the $z$-boundaries. The reflecting condition imposes $v_z = 0$ on the vertical boundaries, which means that the boundaries mimic the nodal points of the linear analysis. Given that no flow is allowed through the $z$-boundaries and any outflow at one horizontal boundary is always compensated for by an identical inflow at the other boundary, the mass in the computational domain is conserved at all times. This allows us to easily track the evolution of the energy content of the system.

The even-parity perturbation is symmetric with respect to the midplane and has a nodal point at $z = 0$. Thus, $\lambda_z$ in equation (7) should be set equal to $2 \lambda_{\text{node}}$, and it is sufficient to use only one (the upper) hemisphere as the computational domain. For the case of an odd-parity perturbation, $\lambda_z$ in equation (8) should be set equal to $4 \lambda_{\text{node}}$, and the simulations have to be performed with both hemispheres in the computational domain (see Table 1).

For completeness, we have also performed simulations with either outflow or inflow conditions at the $z$-boundaries. The results for the fastest growing modes are similar to ones presented here, but, given that the magnetic and velocity fields change when the gas is allowed through the $z$-boundaries, some differences appear for perturbations with longer wavelengths. These results will be presented in a future communication.

2.5. Numerical Codes

The evolution of the undular instability is followed with both the isothermal MHD-TVD code described by Kim et al. (1999) and ZEUS-3D, which has been described by Stone & Norman (1992a, 1992b). The first code is based on a second-order accurate, conservative, explicit, total variation diminishing (TVD) method. The $\mathbf{V} \cdot \mathbf{B} = 0$ condition is maintained during the simulations by a scheme similar to a constrained transport (CT) scheme (Evans & Hawley 1988), which is reported in Ryu et al. (1998). The ZEUS-3D code, on the other hand, solves the ideal MHD equations on an Eulerian staggered mesh using an operator-split formalism wherein accelerations and heating are computed using centered differences followed by a conservative step. Monotonic upwind interpolation of either second or third order (van Leer or PPM advection) is used. Shocks are smeared using artificial viscosity. The CT algorithm is also adopted for the evolution of the magnetic field.

The main differences between the two codes are (1) the TVD code uses an approximate Riemann solver whereas ZEUS-3D uses a finite difference algorithm, (2) the present

| Case | Computational Domain | Resolution (cells) | Perturbation | Parity | $\lambda_y$ (H) | $\lambda_z$ (H) |
|------|----------------------|--------------------|--------------|--------|----------------|----------------|
| 1..... | $-9 \leq z \leq 9$ | 128 | No | ... | ... | ... |
| 2..... | $-9 \leq z \leq 9$ | 256 | No | ... | ... | ... |
| 3..... | $-9 \leq z \leq 9$ | 512 | No | ... | ... | ... |
| 4..... | $-9 \leq y \leq 9$, $0 \leq z \leq 9$ | $256 \times 128$ | Sinusoidal | MS | 18 | 18 |
| 5..... | $-9 \leq y \leq 9$, $-9 \leq z \leq 9$ | $256 \times 256$ | Sinusoidal | MA | 18 | 36 |
| 6..... | $-9 \leq y \leq 9$, $-12 \leq z \leq 12$ | $192 \times 256$ | Sinusoidal | MA | 18 | 36 |
| 7..... | $-27 \leq y \leq 27$, $-9 \leq z \leq 9$ | $768 \times 256$ | Random | ... | ... | ... |
version of the TVD code solves the isothermal MHD equations (without the energy term) whereas ZEUS-3D has the adiabatic MHD with $\gamma = 1.01$, and (3) ZEUS-3D allows less restrictive boundary conditions. The use of two different numerical algorithms, together with the linear analysis in Paper I, allows for a verification of the details of the numerical results. As described in the next section, and except for secondary details, the results from both codes are nearly identical. The advantage we have found in the isothermal TVD code is that it uses less CPU time per run. This is an expected advantage, however, because the code was optimized for isothermal problems and does not solve the energy equation. The important result of this exercise, then, is that both codes handle the problem in an adequate manner and with similar accuracies.

2.6. The Set of Numerical Experiments

In Table 1 we summarize the parameters of the numerical experiments presented in this paper. The simulations can be grouped into three categories. The first group is a set of $(1 + \frac{1}{2})$-dimensional "equilibrium tests" (see below), performed with different resolutions and labeled as cases 1, 2, and 3. They were designed to check whether the codes were able to maintain the initial magnetohydrostatic equilibrium over long times. Within the accuracy of the algorithm for solving the equality in equation (1), a small force unbalance is allowed at every time step, and its accumulative effect may affect the long-term evolution of the instability.

The second group of experiments is designed to compare the evolution, growth rates, and final equilibrium states for perturbations with different parities. This group is labeled as cases 4, 5, and 6. For this purpose, we have perturbed the initial state with MS and MA perturbations that have selected values of $\lambda_y$ and $\lambda_z$. The cases shown here have $\lambda_y$ equal to the most unstable wavelength, and, as stated above, $\lambda_z$ is equal to an integer number of times $z_{node}$.

The third group is illustrated with only one simulation, case 7, and is devised to pick up the value of the most unstable wavelength and the preferred parity (MS or MA) dominating the system. For this purpose, we generate random velocity perturbations in an extended computational domain.

3. RESULTS

As stated before, the coordinates $(y, z)$ represent distances along and perpendicular to the midplane, respectively. The $y$-axis is anchored at the solar Galactocentric radius and is quasi-azimuthal, defined by the tangent of the local field orientation in our Galaxy (see Heiles 1996; Vallee 1997).

3.1. Equilibrium Tests: Thick-Disk Oscillations

By $(1 + \frac{1}{2})D$ we mean that (1) the state vector $q$ in equation (1) is a function of only $z$ and $t$, so $\partial F_q/\partial y = 0$, but (2) the $y$-components of $\vec{v}$ and $\vec{B}$ survive and vary with $z$. In the isothermal TVD code, the source term of the gravitational acceleration is separated from the rest of the MHD equations by use of a splitting procedure (a detailed description of this procedure is given in Kim et al. 1999), but in ZEUS-3D the term is incorporated into the MHD equations (Stone & Norman 1992a, 1992b). Here we verify whether these different methods are able to maintain the magnetohydrostatic equilibrium over long periods of time.

After setting the initial condition described in § 2.2, we trace the maximum vertical velocity as a function of time (Fig. 1). Numerical noise is produced by slight force imbalances at every cell, with wavelengths comparable to the cell size, and propagates along the computational domain as compressive magnetosonic waves. The waves propagate through the stratified medium and reach their peak velocities near the $z$-boundaries. The response of the gaseous disk to the continuous perturbation generated by this numerical noise is to oscillate with the natural frequency of the system. Obviously, the velocities involved in the normal mode of oscillation vary with height, from the "shallow water wave" behavior near the plane to the more complex response at higher values of $z$ (see Martos 1993; Martos & Cox 1998). The harmonic motion period in the linear gravity regime (for $z$ within 170 pc from midplane) is proportional to

$$\left[\frac{\partial g(z)}{\partial z}\right]^{-1/2} \approx 1.4 \times 10^{7} \text{ yr}, \quad (9)$$

and such approximation breaks down at high latitudes. Thus, the system acts as a complex set of coupled oscillators, and a modulated pattern should appear in our thick disk. This is indeed seen in the equilibrium tests performed with both codes.

In case 1, the simulation with the coarsest resolution, the peak velocity value shows modulated oscillations with a very small amplitude during the whole evolution. The resulting amplitude decreases to even lower values as the resolution is increased in cases 2 and 3. The top panel of Figure 1 shows the results for case 1 performed with ZEUS-3D. The oscillation amplitude is always less than 1% of the isothermal sound speed, and the plot shows the modulated oscillatory pattern. The oscillation is complex, but three components with periods of about $2.8 \times 10^{7}$, $6 \times 10^{7}$, and $11 \times 10^{7}$ yr are revealed.

The lower panel of Figure 1 shows the same case performed with the isothermal TVD code. Since the TVD code is based on the Riemann solution, the gravity term should be treated separately. The code first updates the fluid variables (density, magnetic field, and velocity) without taking into account the gravitational acceleration. The force unbalance generated by this first step is then compensated for by adding the vertical momentum due to gravity. There are residuals after this second step because the code calcu-
lates the difference between two large numbers. Thus, because of a transient generated by the initial residuals, the first few velocity peaks are slightly larger than those obtained with ZEUS-3D, but the oscillation amplitude decreases below 1% of the sound speed for the rest of the evolution. Also, the plot shows a cleaner oscillation mode, with a period of about $6 \times 10^7$ yr, because a fixed velocity pattern was enforced at the inner parts of the disk (i.e., this acts like a filter; the central parts of the disk are not allowed to oscillate at their natural frequencies, and one can isolate the oscillations of the extended layers).

The main conclusion of these tests, then, is that both ZEUS and TVD preserve the equilibrium state within an accuracy of a maximum spurious velocity of less than 1% after an elapsed time of about $10^9$ yr.

3.2. Sinusoidal MS and MA Perturbations

As stated above, cases 4, 5, and 6 have been constructed to follow the evolution of the undular instability triggered by MS and MA perturbations. From Paper I, the wavelength with the maximum growth rate in our thick-disk model is about $18H_{\text{eff}}$, and this value is not sensitive to the parity of the perturbation. The three cases shown here have been done with this wavelength value but varying, aside from the perturbation parity, the domain size and grid resolution (see Table 1).

The four panels of Figure 2 show the rms value of the horizontal (dotted lines) and vertical (dashed lines) velocities as a function of time for both MS (left panels) and MA (right panels) perturbations. The solid line has a slope equal to the growth rate for the fastest growing mode with $z_{\text{node}} = 9$ derived in Paper I. The results obtained with the TVD code are displayed in the two upper panels, and the ones from ZEUS-3D are given in the lower panels. As mentioned before, the rms velocities are given in units of the isothermal sound speed (the panels have a natural logarithmic scale) and the units of time are $H_{\text{eff}}/a$. The plots show a good consistency in the results attained with both codes.

The shapes of the velocity curves suggest that the evolution of the undular instability can be divided into three well-defined stages: a linear growth, a nonlinear saturated regime, and a damping oscillatory stage. During the linear part (up to $t = 35$ for the MS mode and up to $t = 27$ for the MA mode), the rms velocities increase at a constant growth.
rate. At early times, before \( t = 20 \) for the MS mode and \( t = 15 \) for the MA mode, the vertical rms velocity simply oscillates. The horizontal rms velocity, however, grows nearly steadily from zero (after a few oscillations) to its maximum value. A close look at the slopes for the horizontal rms velocities indicates that the slope of the MA case is slightly steeper than that of the MS case. This is a manifestation that the MA mode is slightly more unstable than the MS mode (Horiuchi et al. 1988). We did not derive this in the linear analysis of Paper I but here show numerically that it is indeed the case in the next subsection. The end time for this phase can be defined as the moment when, as a result of the action of magnetic tension, the growth rate decreases abruptly to zero \((t = 35 \) for MS, and \( t = 27 \) for MA). In a more strict sense, the term "linear" should be applied to times when the velocity has only a modest increase, smaller than the sound speed. Nonetheless, the rms values for the horizontal velocity at the end of the linear stage are nearly equal to the sound speed. This implies that the "linear approximation" in this case goes well beyond its conventional limits.

The end of the linear phase marks the beginning of the nonlinear saturated stage, in which the rms velocities reach their peak values. These peak values are maintained for a brief period of time, and then the velocities drop down and begin to oscillate about a nearly constant final value. It is a matter of taste to define the end epoch of this nonlinear stage. For simplicity, we take this moment as the time when the first oscillation begins, \( t = 55 \) for the MS case and \( t = 45 \) for the MA case. This corresponds to, approximately, the moment when the gas accumulated in the "magnetic valleys," which is compressed by the ram pressure of the gas falling along the distorted field lines, begins to reexpand. After a large fraction of the halo gas has fallen into the valleys, the ram pressure decreases rapidly and the condensations readjust. The system also enters into its final equilibrium stage: as the rate of mass falling into the valley stalls, the weight of the condensations is stabilized and the tension of the distorted field lines is able to support them. Thus, the gas structures begin to stabilize in the damping oscillatory phase. As time goes by, the rms velocity gradually decreases because of numerical dissipation, and this is the reason why we use the term "damping" (see Kim et al. 1999).

Figure 3 shows snapshots of the density structures, velocity fields, and magnetic field topologies at three selected times. The snapshots obtained with both codes are nearly identical, and, for simplicity, we show only the ones obtained with the TVD simulations. The left panels show case 4 with the MS perturbation, and the right panels show case 5 with the MA mode. The velocity fields are represented with arrows, and the standard velocity vector (with size \( 2a \)) is shown at the top of the left panels. The \( B \)-field lines are chosen in such a way that the flux between consecutive lines is constant. The densities are gray scale from white to black, as the value decreases. The initial sinusoidal perturbations propagate vertically in the form of magnetosonic waves and, as stated above, reach their peak values near the \( z \)-boundaries of the computational domain (see also Fig. 1).

The two upper panels, corresponding to the end time of the linear stage \( (t = 35 \) for the MS case and \( t = 27 \) for the MA case), show that the instability is already fully developed. The midplane is slightly bent in the MA case. Later on, during the nonlinear stage, gas with supersonic velocities is continuously accumulated into the magnetic valleys (some weak shocks are generated during this stage), forming spur-like structures (see middle panels). The ram pressure of the falling gas keeps the layer thin and compressed, creating relatively large central densities. At the end of the nonlinear stage, the ram pressure drops and the overcompressed structures become thicker. The gas velocity near the edge of the spurs reverses its direction, and the system enters into the damping oscillatory phase. The structures eventually move into a quasi-static state (lower panels), which is similar to the final equilibrium state obtained in Paper I.

The runs of the different energies in the system are plotted in Figure 4. The left side corresponds to the MS case, and the right side to the MA case. The internal, kinetic, magnetic, gravitational, and heat-exchange energies are defined by

\[
E_i = \int \int \frac{3}{2} p dy dz, \quad E_k = \int \int \frac{1}{2} \rho v_x^2 + v_y^2 dy dz, \quad (10)
\]

\[
E_m = \int \int \frac{1}{8\pi} (B_x^2 + B_y^2) dy dz, \quad E_g = \int \int \rho g dy dz,
\]

\[
E_h = \int \int \rho l p dy dz, \quad (11)
\]

where \( \phi = \int_{-\infty}^{z} g(z) \) is the gravitational potential and the double integrals are performed over the whole computational domain. Because of the isothermal condition, the internal energy is just proportional to the total system mass. Since there is no mass exchange between our system and the outside, the total mass is conserved in our simulations (see solid lines, labeled \( E_i \)). We use the internal energy as the normalization unit for all energies.

Following Mouschovias, Shu, & Woodward (1974), we define the energy integral (or total energy) as

\[
E = E_i + E_m + E_g + E_h. \quad (12)
\]

When shocks appear, as in our simulations, this energy is not conserved during the evolution (see also the simulations of Matsumoto et al. 1990). The curve labeled \( E_i \) in Figure 4 is constant up to nearly the end of the linear stage because the perturbation speeds are below \( a \) at those stages. When shocks begin to form during the nonlinear regime, the total energy begins to decrease, but it levels up later, when the gas speeds fall below the sound speed and the system enters into the damping oscillatory state. This is particularly clear in the MS case. For the MA case, there are very weak shocks even at the damping oscillatory stage and the total energy curve is still decreasing at \( t = 80 \) (see lower right panel in Fig. 3).

The kinetic energy, \( E_k \), in both MS and MA cases shows small variations, even though shocks are generated during the nonlinear stage. This is due to the fact that these shocks occur in low-density regions. The heat-exchange term, \( E_h \), corresponds to the amount of work done by or on the gas (when \( \Delta E_h \) is positive, work is done by the gas; see Mouschovias 1974). The gravitational energy, \( E_g \), decreases as gas from high latitudes falls down to the magnetic valleys. Comparing the runs for \( E_g \) in both panels, one finds that a larger amount of gravitational energy is released in the MA case. This is because the MA oscillations are able to bend the midplane, and a larger amount of gas lowers its position to be collected in the high-density condensations (see Paper I).

The gravitational energy is first transformed into kinetic
energy, and then it is either dissipated in shocks or goes into magnetic field compression and tension. In general, the instability proceeds at the expense of lowering the gravitational energy (Mouschovias 1974).

To compare the final equilibrium state derived in Paper I with the late-times results of the numerical simulations, we obtain the column densities as a function of $y$. For a given horizontal location, the column density at any time $t$ is

$$N(y, t) = \int_{z(y, A[z=0, t=0])}^{z_{\text{max}}} \rho(y, z, t) dz,$$  

where the upper limit of the integral corresponds to the upper boundary of the computational domain and the lower one is the $z$-coordinate of the magnetic field line that was initially located at midplane (labeled $A[z=0, t=0]$; see Paper I). During the linear stage, the run of the column densities has only small variations. The important changes occur during the nonlinear stage, when the contrast between condensations and voids becomes maximum. Later on, during the oscillatory stage, the column densities maintain the values achieved during the end of the nonlinear regime and the values at $t = 80$, normalized to the initial
column density value \( N(t = 0) \), are plotted in Figure 5 for both MS (dotted line) and MA (dashed line) perturbations. The MA case has the larger density contrast between condensations and voids. As stated before, this is because the MA perturbations provide more room for the high-density gas by bending the midplane. At this last time, the undular instability has almost reached the new final equilibrium in both MS and MA cases, and the final column density configurations are very similar to those shown in Figure 5 of Paper I. The differences in the numerical and analytical results for the MS case are of only about 10%. The differences in the MA case are in general larger, particularly in the voids, but they are also of the order of 10% at the center of the magnetic valleys.

In Paper I, we found that the growth rate increases, up to a certain maximum value, with the location of the nodal point. This is because gravity, the driving force of the instability, increases up to a certain maximum value. To verify this dependence in the numerical experiments, we made a series of cases with different values for the \( z \)-boundaries. Here we show one simulation, case 6, with an extended vertical domain, \(-12 \leq z \leq 12\), and a sinusoidal MA perturbation. Figure 6 shows the resulting rms velocities (dotted and dashed lines as in Fig. 2) as a function of time. The solid line now has a slope equal to the growth rate corresponding to the nodal points for this case. As is clear from the figure, the growth of the linear part follows the rate predicted by the linear analysis for this case.

3.3. Random Perturbations

As a final issue of this part, we now verify the wavelength value and parity of the fastest growing mode in the thick-disk system. For this purpose we have performed simulations with random velocity perturbations, without any preferred parity or wavelength. The horizontal size of the computational domain has been extended, and the size of the simulation in case 7 is 3 times longer than in the pre-

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**Fig. 4.**—Time evolution of different energies (internal \( E_i \), kinetic \( E_k \), magnetic \( E_m \), gravitational \( E_g \), heat-exchange \( E_h \), and total \( E_t \)) for the simulations presented in Fig. 3. All energies are normalized to the internal energy. Left: MS case. Right: MA case.

**Fig. 5.**—Final column density values as a function of horizontal location, for the MS (dotted line) and MA (dashed line) cases of Fig. 3. The column densities are normalized to the initial value, \( N(t = 0) \).
previous models. We also have increased accordingly (by a factor of 3) the number of cells in the y-axis.

Figure 7 shows the run of the rms velocities for this case, and, as before, the slope of the solid line corresponds to the growth rate. The horizontal component (dotted line) has an initial adjustment and enters into the linear growth at \( t \approx 8 \). The velocity grows linearly, up to \( t \approx 45 \), with the predicted maximum growth rate. As in the previous models, the rms of the vertical component has some initial fluctuations, and then, after \( t \approx 32 \), it also grows with the predicted rate. The rest of the evolution is similar to the one already described in case 5. The only difference is that the evolutionary stages are now significantly delayed. The resulting structures are displayed in two snapshots, at \( t = 45 \) and \( t = 63 \) (the end of the linear and nonlinear stages, respectively), in Figure 8.

The series of condensations are formed at slightly different times, but they have the interdistance scale of the most unstable wavelength and the MA symmetry. This shows that, even when there was a parity degeneracy in the linear analysis, the odd-parity mode is dominant.

4. DISCUSSION

In this paper, we have confirmed numerically the results of the linear analysis obtained in Paper I for the undular
mode of the Parker instability in a thick, warm disk model of the Galaxy. The dependence of the growth rate on the position of the upper boundary of the numerical grid has been confirmed, and an explanation in terms of the action of the gravitational field is provided. The main properties of the final equilibrium configuration were also confirmed. The differences between the final states for the symmetric and antisymmetric modes were corroborated by the numerical study. The actual values of the column density distributions are slightly different, but the shapes and the final-to-initial column density ratios were confirmed. In addition, the present study also provides a demonstration that the antisymmetric mode is dominant and its signature, that of an odd parity with respect to the central Galactic plane, should be present in structures formed by the instability. In addition to the above, we are confident of the new evolutionary details of the instability in a multicomponent plane, should be present in structures formed by the instability. In addition to the above, we are confident of the new evolutionary details of the instability in a multicomponent thick disk, since we have obtained the same results with two different numerical codes (TVD and ZEUS-3D).

These new details may have important implications in our understanding of the Galactic disk. The extended vertical structure observed in our Galaxy and others, as the diffuse ionized warm layer, forces higher scale heights (for mass, pressure, and B-fields) than assumed in previous calculations of the Parker instability. The resulting thick disk alters considerably the time and length scales involved in the process, and, as long as the upper layers are not dominated by a hot coronal plasma phase, the disk is unstable to undular modes. Evidence for the pervasiveness of this hot phase is still under debate (see Cox 1995, and the conference proceedings edited by Arthur, Brickhouse, & Franco 2000), but, regardless of the actual details of the gas temperature distribution, one can explore the consequences of the instability with the present isothermal thick disk model.

In considering some of the possible applications of this work, one has to keep in mind the fact that our results refer only to a 2D analysis. The final equilibrium states, with a well-defined periodicity of troughs and crests, are unstable to 3D perturbations (Asséao et al. 1978). Nonetheless, tridimensional simulations in a thin disk (Kim et al. 1998) show that some of the 2D structuring is still present in this case. The instability in this case is a combination of two modes, interchange and undular. The interchange mode corresponds to the case in which the field lines maintain their initial direction but some flux tubes are displaced upward when they find themselves less dense than the surroundings. The undular mode (which is the one addressed in this study and is simply referred to as the Parker instability) corresponds to the case in which the field lines are wavy above the midplane and gas aggregation occurs at the magnetic valleys. Three-dimensional motions are more easily destabilized than those in two dimensions, for density fluctuations (which instigate the interchange mode) are related to the total (gas plus magnetic) pressure whereas the undular mode depends on the gas pressure alone (Hughes & Cattaneo 1987).

The most unstable growth rate of the interchange mode has a vanishing wavelength along the third dimension, which is perpendicular to both gravity and initial field directions. This fact has been known from one of the pioneering works of Parker (1967) and can limit the size and mass of the resulting structures. Our isothermal disk is unstable to the 3D interchange mode (Martos 1993), and its fastest growth rate is that of an infinitesimally small wave-length perpendicular to the 2D plane of motion considered in our calculations. As a result, and depending on the coupling of the field lines, small-scale structuring or turbulence will be also made. Additional physical effects, such as rotation, self-gravity, or variable gravity in the z direction, cannot stabilize the buckling of the field lines (Asséao et al. 1978; Elmegreen 1991). Thus, the instability will tend to proceed faster in 3D than in 2D, creating complex small-scale structuring, but some of the 2D signatures will be also present at the large, kiloparsec size scales (i.e., the large-scale undular mode, with its odd parity with respect to the central Galactic plane, will persist). The relative importance of the undular large-scale structures depends on whether one is considering arm or interarm regions. The interarm regions are sheared, and the effects of rotation in the interchange mode lead to a preferential small-scale structuring (see Kim, Ryu, & Jones 2000b). Thus, little or no undular signatures are found, and alternating dense and rarefied sheets are formed by the three-dimensional perturbations (Kim et al. 1998, 2000b). The density structures of these sheets are far from those expected for giant molecular clouds, and the enhancement factors for the vertical column density are small. Thus, it is difficult to consider the Parker instability alone as the formation mechanism of giant clouds in the general interstellar medium (the resulting 3D structures may be more reminiscent of the worms observed in atomic hydrogen; see Koo, Heiles, & Reach 1992).

In contrast, inside the arms, with a stronger gravitational field and no shear, the large-scale undular structures are clearly present along the arms (Franco et al. 2000). The spacing between the resulting condensations is similar to the most unstable mode derived here, and the antisymmetric mode creates a corrugation pattern along the spiral arm. An interesting test of this prediction is suggested by the structuring detected in the Carina arm, with a corrugation scale of 2.4 kpc (Alfaro, Cabrera-Caño, & Delgado 1992). Early 3D studies already showed that the spiral density wave triggers the instability in the thick gaseous disk (Martos & Cox 1994), and current work in 3D (Franco et al. 2000) indicates that the undular mode indeed gives rise to a corrugation, or bending, along the arm. There is good agreement with the observed corrugation length when appropriate scaling of the gravitational field toward the inner Galaxy is accounted for. This feature, along with the hydraulic jumps generated by the action of spiral density waves (Martos & Cox 1998), can create strong spatial and velocity distortions in the vicinity of spiral arms. These issues certainly require further observational studies of corrugations and the velocity fields across spiral arms (Alfaro et al. 2000).

Physics not included in this study will alter some details of the picture discussed above. For instance, the Galactic field is mostly tangled, and the random component must have a stabilizing influence (Zweibel & Kulsrud 1975). Also, self-gravity (Elmegreen 1982) and twisting of the field lines (Shibata & Matsumoto 1991; Kim et al. 2000b) will contribute to the formation of coherent, finite- (but small-) size structures, setting more realistic lower limits to the theoretical zero wavelength predicted by theory for the fastest growing interchange mode. In addition, magnetic reconnection should have played an important role at the late evolutionary stages. The twisted random field components may lead to a rapid energy release (e.g., Lazarian & Vishniac 1999) and can create additional flows and modify the final
energy budget of the process. We notice, however, that the scales of the problem addressed here are much larger than the cell size of the random component, 50–100 pc. The effect of Galactic differential rotation, as stated above, will modify the outcome of the instability (e.g., Foglizzo & Tagger 1994, 1995). Finally, the timescales for our 2D simulations are much larger than the cosmic-ray residence time of $2 \times 10^7$ yr, making a causal association between cosmic-ray escape and the Parker instability unlikely. However, again, this, as most previous issues, demands precise three-dimensional studies, such as those presented by Kim et al. (1998, 2000b), and should be addressed in the near future.

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