Features of Dempster–Shafer Theory Application in Planning of Construction Production

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Abstract. The article deals with the features of Dempster-Shafer theory application in the planning of the construction production. The numerical example of the determining of the most probable delay causes in the work performance of the project network (calendar plan) is given for the purpose of the taking them into account in the planning.

1. Introduction

One of main problems at building network model has been, and remains a quantification of resources, including temporary ones. Choice of methods and tools for evaluating resources depends on availability of information, completeness and accuracy of the original data received from contractors, decision makers and managers.

2. Purpose and problem statement

For a long time, the main way to overcome inaccuracies in the process of the network model creation was to apply a probabilistic method.

The duration of the project operations was treated as a random variable. A significant drawback of this approach was the impossibility of the analytical expression obtaining for the probability distributions and other characteristics of the project events (excluding the operations that are performed sequentially or in parallel). In addition, the statistical data for the assessing the probabilistic characteristics of project events are often not enough in the practice.

Another well-known method of the uncertainty accounting is the fuzzy set theory. The fuzzy information about the operation duration can be obtained from the expert in the case when there are no normative and statistical data. The expert assessment involves the specifying parameters in the form of the fuzzy numbers. The main difference between this approach and the probabilistic one is the creation possibility of the analytical dependencies of the project characteristics. This is extremely important for the making managerial decisions.

When using the fuzzy set theory, all the information about the different nature resources, production costs, labor intensity of the individual works should be transformed to the unified form and represented as the membership functions. The main types of the membership functions are the triangular, trapezoidal, piecewise linear, Gaussian, sigmoid and other functions. But the membership function choosing of the particular variable is a poorly formalized problem, the solution of which is based, as a rule, on the intuition and experience.

When working with the uncertainty, the alternative approach is Dempster-Shafer evidence theory, which considers a set of propositions (hypotheses) and assigns each of them a probability interval of the uncertainty (plausibility), to whom the belief degree in each proposition belongs. In addition, this approach solves the problem of the evidence measuring [2-5].
3. Fundamentals of Dempster-Shafer theory

Let the set $\Theta = \{A_1; A_2; \ldots; A_N\}$ of the mutually exclusive hypotheses (statements), which is called the area of analysis (the frame of the hypotheses), be considered. Designate the power set as $2^\Theta = \{A|A \subseteq \Theta\}$. The basic assignment of the probabilities or mass function (evidence measure) is a function that is represented as $2^\Theta$ in the $[0; 1]$ interval, so that $m(\emptyset) = 0$, where $\emptyset$ is an empty set and $\Sigma_{A \in \Theta} m(A) = 1$.

The function of the belief and interest is defined as follows:

- $Bel(A) = \sum_{B \subseteq A} m(B) \forall A \subseteq \Theta$;
- $Pl(A) = 1 - Bel(\bar{A}) = 1 - \sum_{B \subseteq \bar{A}} m(\bar{A}) = \sum_{B \cap A = \emptyset} m(B) \forall A \subseteq \Theta$.

Where: $Bel(A)$ - the mass amount of the belief in the subsets of $A$ or the belief assessment of $A$, i.e. the measure of the total belief amount in $A$ and in its subsets;

$Pl(A)$ - the mass amount of the mistrust of the subset A or interest assessment, i.e. the measure of the plausibility.

In this case, the belief interval is defined as:

- $EI(A) = [Bel(A); 1 - Bel(\bar{A})]$, i.e. $Bel(A) \leq P(A) \leq Pl(A)$;

The assessment of the doubt and ignorance is calculated by the formulas:

- $Dbt(A) = Bel(\bar{A}) = 1 - Pl(A)$;
- $Ign(A) = Pl(A) - Bel(A)$.

In this way, the beliefs in the form of X and Y subsets are combined according to Dempster’s rule both the orthogonal sum of two measures. This quantity is called the joint mass and is defined as:

- $m_1 \otimes m_2(A) = k$.
- $k = \frac{1}{1 - \Sigma_{X \cap Y = \emptyset} m_1(X)m_2(Y)}$.

Where: $k$ – the normalization constant (the measure of the conflict between two sets of the masses);

- $\Sigma_{X \cap Y = \emptyset} m_1(X)m_2(Y)$ - a conflict between two beliefs.

For an empty set $m_1 \otimes m_2(\emptyset) = 0$, $A = \emptyset$.

If $k^{-1} = 0$, then the orthogonal sum does not exist, and $m_1$ and $m_2$ measures are called completely mutually exclusive. In general, for $n$ number of $m$ mass functions in the $\Theta$ set, the conflict will be like

- $K = \Sigma_{i=1}^{n} \Sigma_{E_i = \emptyset} m_1(E_1)m_2(E_2) \ldots m_n(E_n) > 0$

and after the combination, the mass function will be

- $m(A) = (m_1 \otimes m_2 \otimes \ldots \otimes m_n)(A) = \frac{1}{1-K}$.
4. Numerical example of Dempster-Schaefer theory application in construction production

It is required to predict (identify) the most likely reasons for the delay in the work performance of the project network (calendar plan) in order to take them into account in the planning.

The initial data for the task are the views of two expert groups (to the expert opinion) about the possible reasons (hypotheses) for delaying the work execution, such as:

- $a$ – due to the dramatic changes in the weather conditions, the work duration increases by 20%;
- $b$ – due to the constrained conditions, the work duration increases by 10%;
- $c$ – due to the malfunctions in the material supply, the duration increases by 15%;
- $d$ – the human factor causes a 10% delay in the work performance.

The work takes into account the opinion (evidence) of two expert groups.

**Table 1. Initial data of mass functions.**

| Belief (Model) No. 1 | Belief (Model) No. 2 |
|----------------------|----------------------|
| 1st expert group     | 2nd expert group      |
| $m_1$ (d) = 0.3      | $m_2$ (c) = 0.2       |
| $m_1$ (c) = 0.5      | $m_2$ (b) = 0.6       |
| $m_1$ (b) = 0.2      | $m_2$ (a) = 0.2       |

Thus, at the initial stage two beliefs are combined in the form of the subsets of the hypotheses $X = \{b, c, d\}$ and $Y = \{a, b, c\}$ by Dempster’s rule:

$$m_3 \otimes m_2(Z) = \frac{\sum_{X \cap Y = Z} m_1(X)m_2(Y)}{1 - \sum_{X \cap Y = \emptyset} m_1(X)m_2(Y)}.$$

As can be seen in the tables No. 2 and No.3 (gray cells), the mass functions for which the masses are zero are not given, since they do not allow the combination to form new mass values $m_3 = m_1 \otimes m_2$ (because 0 multiplied by any number gives 0).

In the table No.4 the combining $m_1$ and $m_2$, as well as the intermediate data: $(X \cap Y)$ general subsets are defined and the product of their weight values.

**Table 2. Results of calculations to determine plausibility and interest of X.**

| $X$     | $m_1$ | Bel(X) | Dbt(Bel(X)) | Pl(X) | EI[Bel(X), Pl(X)] |
|---------|-------|--------|-------------|-------|------------------|
| $\emptyset$ | 0.2   | 0.8    | 0.8         | 0.2   | [0.2;0.2]       |
| $\emptyset$ | 0.5   | 0.5    | 0.5         | 0.5   | [0.5;0.5]       |
| $\emptyset$ | 0.3   | 0.7    | 0.7         | 0.3   | [0.3;0.3]       |
| $\emptyset$ | 0     | 0.8    | 0.8         | 0.8   | [0.8;0.8]       |
| $\emptyset$ | 0.1   | 1.0    | 1.0         | 1.0   | [1.0;1.0]       |
| $\emptyset$ | 0     | 0      | 0           | 0     | [0;0]           |
Table 3. Results of calculations to determine plausibility and interest of $Y$.

| Model No. 2 | Y          | $m_2$ | Bel(Y) | Dbt(Bel(Y)) | Pl(Y) | $EI[Bel(Y); Pl(Y)]$ |
|-------------|------------|-------|--------|-------------|-------|---------------------|
|             | $\{a\}$   | 0.2   | 0.2    | 0.8         | 0.2   | [0.2;0.2]           |
|             | $\{b\}$   | 0.6   | 0.6    | 0.4         | 0.6   | [0.6;0.6]           |
|             | $\{c\}$   | 0.2   | 0.8    | 0.2         | 0.8   | [0.2;0.2]           |
|             | $\{a;b\}$ | 0     | 0.8    | 0.2         | 0.8   | [0.8;0.8]           |
|             | $\{a;c\}$ | 0     | 0.4    | 0.6         | 0.6   | [0.4;0.4]           |
|             | $\{b;c\}$ | 0     | 0.8    | 0.2         | 0.8   | [0.8;0.8]           |
|             | $\emptyset$ | 0    | 1.0    | 0           | 1.0   | [1.0;1.0]           |

Table 4. Results of class combinations.

|          | $m_2$ (a) = 0.2 | $m_2$ (b) = 0.6 | $m_2$ (c) = 0.2 | $m_2$ (d) = 0 |
|----------|----------------|----------------|----------------|---------------|
| $m_1$(b) | 0.2            | 0.12           | 0.04           | 0             |
| $m_1$(c) | 0.5            | 0.30           | 0.10           | 0             |
| $m_1$(d) | 0.3            | 0.18           | 0.06           | 0             |
| $m_1$(a) | 0.0            | 0.12           | 0.04           | 0             |

Further we find the values:

- $\Sigma_{X|Y=\emptyset} m_1(X)m_2(Y) = 0.04 + 0.04 + 0.1 + 0.30 + 0.06 + 0.18 + 0.06 = 0.78$.

and the value of the normalization factor:

- $1 - \Sigma_{X|Y=\emptyset} m_1(X)m_2(Y) = 0.22$.

Define:

- $m_1 \otimes m_2(\{a\}) = 0$; $m_1 \otimes m_2(\{b\}) = 0.12/0.22$;
- $m_1 \otimes m_2(\{c\}) = 0.10/0.22$; $m_1 \otimes m_2(\{d\}) = 0$;
- $m_1 \otimes m_2(\emptyset) = 0$.

The above values allow you to calculate new belief values $m_3$ based on the $m_1$ and $m_2$ combinations, which are shown below (Table 5).

As a result, the intervals obtained after the combinations are formed:

- $EI(\{c; d\}) = [0.45; 0.45]$;
- $EI(\{a; b\}) = [0.54; 0.54]$.

which show that the possible causes of «a» and «b» are most likely.
| Z               | m3 | Dbt(Bel(Z)) | EI[Bel(Z), Pl(Z)] |
|-----------------|----|-------------|-------------------|
| {a}             | 0  | Z={b;c;d}   | 1.0 [0.0]         |
| {b}             | 0.54 | Z={a;c;d} | 0.45 [0.54; 0.54] |
| {c}             | 0.45 | Z={a;b;d} | 0.54 [0.45; 0.45] |
| {d}             | 0  | Z={a;b;c} | 1.0 [0.0]         |
| {a,b}           | 0  | Z={c;d}    | 0.45 [0.54; 0.54] |
| {a,c}           | 0  | Z={b;d}    | 0.54 [0.45; 0.45] |
| {a,d}           | 0  | Z={b;c}    | 1.0 [0.0]         |
| {b,c}           | 0  | Z={a;d}   | 0.0 [1.0;1.0]     |
| {b,d}           | 0  | Z={a;c}   | 0.45 [0.54; 0.54] |
| {c,d}           | 0  | Z={a;b}  | 0.54 [0.45; 0.45] |
| {a,b,c}         | 0  | Z={d}    | 0.0 [1.0;1.0]     |
| {a,b,d}         | 0  | Z={c}    | 0.45 [0.54; 0.54] |
| {a,c,d}         | 0  | Z={b}    | 0.54 [0.45; 0.45] |
| {b,c,d}         | 0  | Z={a}    | 0.0 [1.0;1.0]     |
| Ø={a,b,c,d}     | 0  | Z={ Ø } | 0.0 [1.0;1.0]     |
| Ø               | 0  | Z={ Ø } | 1.0 [0.0]         |

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