Testing the Standard Model of particle physics using lattice QCD

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Abstract. Recent advances in both computers and algorithms now allow realistic calculations of Quantum Chromodynamics (QCD) interactions using the numerical technique of lattice QCD. The methods used in so-called “2+1 flavor” lattice calculations have been verified both by post-dictions of quantities that were already experimentally well-known and by predictions that occurred before the relevant experimental determinations were sufficiently precise. This suggests that the sources of systematic error in lattice calculations are under control, and that lattice QCD can now be reliably used to calculate those weak matrix elements that cannot be measured experimentally but are necessary to interpret the results of many high-energy physics experiments. These same calculations also allow stringent tests of the Standard Model of particle physics, and may therefore lead to the discovery of new physics in the future.

1. Lattice Quantum Chromodynamics and the Standard Model
Quantum Chromodynamics (QCD), part of the Standard Model of particle physics, governs the strong nuclear interactions. It describes how quarks and gluons, the fundamental constituents of ordinary matter, bind together to form composite particles (hadrons) such as protons and neutrons, and it determines how these particles in turn interact to form atomic nuclei. An important property of QCD is that of quark confinement. One cannot detect free quarks in nature; quarks only occur in bound states as pairs (forming mesons such as pions and kaons) or triplets (forming baryons such as protons and neutrons). Thus any calculation of quark interactions must include the effects of confining quarks into hadrons. Currently the only available technique to fully include such effects is lattice QCD. Lattice calculations formulate QCD on a discrete spacetime lattice, thereby transforming the infinite-dimensional Quantum Field Theory path integral into a finite-dimensional integral that can be solved numerically with Monte Carlo methods and importance sampling. Using lattice QCD, particle physicists can calculate both fundamental parameters of the Standard Model, such as hadron masses and lifetimes, and weak interaction matrix elements needed to interpret the results of experiments at SLAC, Fermilab, and elsewhere. One of the primary goals of the US Lattice QCD (USQCD) Collaboration is therefore to “calculate these matrix elements to the accuracy needed to make precise tests of the Standard Model” [1].

2. 2004: Lattice QCD agrees with experiment
Every hadron is filled with a sea of quarks and gluons (that are constantly emitted and reabsorbed) which bind the valence quarks together. Until recently, including the full particle sea
in lattice QCD calculations was too computationally expensive, and most simply omitted it. This
procedure, called quenching, is not correct, but often produces results that are within 10-15%
of experimental values. Advances in computing power, however, now allow realistic simulations
including three types (flavors) of dynamical sea quarks. Results from these simulations agree
with experimental measurements for a wide range of meson masses, mass-splittings, and weak
matrix elements. Figure 1 shows the ratio of lattice QCD determinations of several quantities
over the experimental measurements. Agreement is therefore signaled by a ratio of one. The
left-hand side shows the ratio values for quenched lattice QCD calculations; the lattice and
experimental results disagree by as much as 15% in both the positive and negative direction. The
right-hand side shows the same ratios for “2+1 flavor” lattice QCD calculations, and everything
agrees within 1.5 \( \sigma \). This dramatic success suggests that 2+1 flavor lattice QCD calculations
now have the sources of systematic error under control.

3. 2004–2005: Lattice QCD predicts experimental results
Using lattice QCD to make predictions (\textit{i.e.} calculations that are completed before suitably
precise experimental measurements are made) allows an unbiased test of the methodology. After
the postdictions in early 2004, members of USQCD successfully predicted the mass of the \( B_c \)
meson before it was measured at Fermilab. The result is shown in Fig. 2(a) along with the
experimental measurement. USQCD members also predicted the shape and normalization
of the \( D \to K \ell \nu \) form factor before it was measured at Fermilab and KEK. Figure 2(b)
shows the lattice QCD determination of the form factor with experimental data from the Belle
Collaboration over top. These and other predictions (such as for the \( D \)-meson decay constant [8])
give confidence that 2+1 flavor lattice QCD calculations are reliable, and can be used to calculate
the weak matrix elements needed to interpret many high-energy experiments.

4. 2006 and beyond: Lattice QCD tests the Standard Model
Although the Standard Model of particle physics has successfully described all high-energy
experiments to-date, we know that it is not a complete theory-of-everything. For example, the
Standard Model cannot account for the amount of dark matter and dark energy in the universe,
Figure 2. (a) Comparison of the quenched \([3]\), \(n_f = 2 + 1 \) \([4]\) and experimental \([5]\) values of \(m_{B_c}\); the dashed line denotes the baseline \((\bar{m}_\psi + m_\Upsilon)/2\). (b) Comparison of the lattice QCD determination \([6]\) and experimental measurement \([7]\) of the \(D \rightarrow Kn\nu\) form factor shape. The orange and yellow bands show the 1- and 2-\(\sigma\) statistical errors in the lattice calculation, respectively.

nor the abundance of matter over antimatter. Thus one of the foremost goals of the elementary particle physics community is to test the Standard Model in order to determine its range of validity and to search for physics beyond the Standard Model. One way to do this is to try to create new particles directly at high-energy colliders such as the Tevatron and LHC. Another is to look for experimental results that cannot be explained by the Standard Model. This requires an extremely precise knowledge of the Standard Model predictions, many of which must be provided by lattice QCD calculations. A particularly powerful test of the Standard Model is the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) Matrix. The elements of the CKM matrix impact flavor-changing processes in which one type of quark turns into another type of quark; unitarity-violation could signal the presence of new quark flavors and/or new interactions between quarks. Because (within the Standard Model) it is a 3 \( \times \) 3 unitary matrix, products of the CKM elements form the sides of a triangle in the complex plane. The area inside the unitarity triangle (UT) is related to the amount of CP-violation in the Standard Model, and CP-violation likely played an important role in producing the universes matter-antimatter imbalance. Some constraints on the apex of the unitarity triangle come from experimental measurements alone; many, however, require combining experiment and lattice QCD. The hope is that, once experimental measurements and lattice QCD calculations are sufficiently precise, inconsistencies among the various constraints will appear and lead to the discovery of new physics. Those constraints which need lattice QCD inputs are listed in Table 1.\(^1\) Figures 3–5 show the allowed location of the unitarity triangle’s apex (circled in red) resulting from: 1) experiment alone, 2) experiment/quenched lattice QCD, and 3) experiment/2+1 flavor lattice QCD.\(^2\) The lattice constraints are complimentary to the purely experimental ones. The constraints from 2+1 flavor lattice calculations are significantly tighter than before, and work by members of USQCD will further reduce their errors in the near future.

\(^1\) The unquenched determination of \(\xi\) combines a 2+1 flavor calculation of the decay constants \([17]\) with a 2 flavor calculation of the bag parameter \([18]\).

\(^2\) These plots were generated using the publicly available CKM Fitter code \([19]\).
Table 1. Lattice QCD inputs to the CKM unitarity triangle analysis.

| UT Constraint | Lattice Quantity | Quenched | 2+1 flavor |
|---------------|-----------------|----------|------------|
| $\epsilon_K$  | $\hat{B}_K$      | 0.86 ± 0.06 ± 0.14 [9] | 0.765 ± 0.017 ± 0.040 [10] |
| $\Delta m_s$  | $f_{B_s} \sqrt{B_{B_s}}$ | 276 ± 38 MeV [11] | 281 ± 21 MeV [12] |
| $\Delta m_s$ & $\Delta m_d$ | $\xi$      | 1.24 ± 0.04 ± 0.06 [11] | 1.216 ± 0.04 [13] |
| $|V_{ub}/V_{cb}|$ | $|V_{ub}|_{excl.} \times 10^3$ | 4.1 ± 1.1 [14] | 3.65 ± 0.22 ± 0.51 [15, 16] |

Figure 3. Constraints on the apex of the CKM unitarity triangle from experimental measurements of the angles $\alpha$, $\sin(2\beta)$, and $\gamma$.

Figure 4. Constraints on the apex of the CKM unitarity triangle from both experimental measurements and quenched lattice QCD calculations of weak matrix elements.
Figure 5. Constraints on the apex of the CKM unitarity triangle from both experimental measurements and 2+1 flavor lattice QCD calculations of weak matrix elements.

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References
[1] http://www.usqcd.org.
[2] Davies C T H et al. [HPQCD, MILC and Fermilab Lattice Collaborations], Phys. Rev. Lett. 92, 022001 (2004) [hep-lat/0304004].
[3] Shanahan H P, Boyle P, Davies C T H and Newton H [UKQCD Collaboration], Phys. Lett. B 453, 289 (1999) [hep-lat/9902025].
[4] Allison I F, Davies C T H, Gray A, Kronfeld A S, Mackenzie P B and Simone J N [HPQCD and Fermilab Lattice Collaborations], Phys. Rev. Lett. 94, 172001 (2005) [hep-lat/0411027].
[5] Acosta D et al. [CDF Collaboration], Phys. Rev. Lett. 96, 082002 (2006) [hep-ex/0505076].
[6] Aubin C et al. [Fermilab Lattice, MILC and HPQCD Collaborations], Phys. Rev. Lett. 94, 011601 (2005) [hep-ph/0408306].
[7] Widhalm L et al. [Belle Collaboration], arXiv:hep-ex/0604049, arXiv:hep-ex/0510003.
[8] Aubin C et al., [Fermilab Lattice, MILC and HPQCD Collaborations], Phys. Rev. Lett. 95, 122002 (2005) [hep-lat/0506030].
[9] S. Aoki et al. [JLQCD Collaboration], Phys. Rev. Lett. 80, 5271 (1998) [arXiv:hep-lat/9710073].
[10] D. J. Antonio et al. [RBC Collaboration], arXiv:hep-ph/0702042.
[11] L. Lellouch, Nucl. Phys. Proc. Suppl. 117, 127 (2003) [arXiv:hep-ph/0211359].
[12] E. Dalgic et al., arXiv:hep-lat/061004.
[13] M. Okamoto, PoS LAT2005, 013 (2006) [arXiv:hep-lat/0510113].
[14] A. Abada, D. Becirevic, P. Boucaud, J. P. Leroy, V. Lubicz and F. Mescia, Nucl. Phys. B 619, 565 (2001) [arXiv:hep-lat/0011065].
[15] M. Okamoto et al., Nucl. Phys. Proc. Suppl. 140, 461 (2005) [arXiv:hep-lat/0409116].
[16] E. Barberio et al. [Heavy Flavor Averaging Group (HFAG) Collaboration], arXiv:0704.3575 [hep-ex].
[17] A. Gray et al. [HPQCD Collaboration], Phys. Rev. Lett. 95, 212001 (2005) [arXiv:hep-lat/0507015].
[18] S. Aoki et al. [JLQCD Collaboration], Phys. Rev. Lett. 91, 212001 (2003) [arXiv:hep-ph/0307039].
[19] Charles J et al. [CKMfitter Collaboration] Eur. Phys. J. C41, 1-131 (2005), [hep-ph/0406184], code available at: http://ckmfitter.in2p3.fr