MULTIPLICITY AND TRANSVERSE ENERGY DISTRIBUTIONS ASSOCIATED TO RARE EVENTS IN NUCLEUS-NUCLEUS COLLISIONS.

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Abstract

We show that in high energy nucleus-nucleus collisions the transverse energy or multiplicity distribution $P_C$, associated to the production of a rare, unabsorbed event C, is universally related to the standard or minimum bias distribution $P$ by the equation

$$P_C(\nu) = \frac{\nu}{<\nu>} P(\nu),$$

with $\sum P(\nu) = 1$ and $\nu \equiv E_T$ or $n$. Deviations from this formula are discussed, in particular having in view the formation of the plasma of quarks and gluons. This possibility can be distinguished from absorption or interaction of comovers, looking at the curvature of the $J/\Psi$ over Drell-Yan pairs as a function of $E_T$.

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It is clear, from theoretical models and experiment [1, 2], that particle production in hadron-hadron, hadron-nucleus and nucleus-nucleus collision is generated by superposition of elementary, particle emitting, collisions. In the Dual Parton Model (DPM) [3], for instance, the elementary collisions involve the partonic constituents of hadrons and the production of particle emitting strings.

Assuming that all the elementary collisions may be treated as equivalent, particle production fluctuations will contain a contribution from the fluctuation in the number of elementary collisions (or the number of strings) and from the fluctuation in particle production resulting from the elementary collision (string particle distribution) [4]. The fact that the width of the nucleus-nucleus particle distribution is very large and very different from the width of the $e^+e^-$ distribution or even hadron-hadron distribution, is an indication that multiparticle fluctuations in nucleus-nucleus collisions are dominated by fluctuations in the number $\nu$ of elementary collisions.

In general, we then have for the average multiplicity $< n >$ and for the square of the dispersion $D^2 \equiv < n^2 > - < n >^2$:

\[
< n > = < \nu > \bar{n}, \tag{1}
\]

and

\[
\frac{D^2}{< n >^2} = \frac{< \nu^2 > - < \nu >^2}{< \nu >^2} + \frac{1}{< \nu >} \frac{d^2}{\bar{n}^2}, \tag{2}
\]

where $\nu$ and $d$ are the average multiplicity and the dispersion of the elementary collision particle distribution, respectively.

In nucleus-nucleus high energy collisions, as the left hand side of (2) is of the order of 1 and the second term in the right hand side is of the order of $10^{-2} - 10^{-3}$, naturally a good approximation is obtained by writing

\[
\frac{D^2}{< n >^2} = \frac{< \nu^2 > - < \nu >^2}{< \nu >^2}, \tag{3}
\]

The same kind of approximation is valid for higher moments [4].

In order to obtain (1) and (3) we make the simplifying assumption

\[
P(n) = P(\nu), \quad n = \nu \bar{n}. \tag{4}
\]

Equation (4) essentially tells us that, in nucleus-nucleus interactions, the KNO distribution [5] for particles is well approximated by the KNO distribution for elementary collisions.

Note that, as the transverse energy $E_T$ is a good measure of $n$, Eq.(4), and what follows, can as well be written for $E_T$ distributions.
Let us next consider the particle distribution associated to a rare event C (Drell-Yan pairs in some mass region, \(J/\Psi\) and \(\Psi'\) production in rough approximation, \(\Upsilon\) production, some weak process, etc.) which does not suffer strong absorption. These are events of type C in the classification of [6].

If \(\alpha_C\), \(0 > \alpha_C > 1\) is the probability of event C to occur in an elementary collision and if in a nucleus-nucleus experiment \(N(\nu)\) is the number of events with \(\nu\) elementary collisions, we have

\[
N(\nu) = \sum_{i=0}^{\nu} \binom{\nu}{i} (1 - \alpha_C)^{\nu-i} \alpha_C^i N(\nu)
\]

where \((1 - \alpha_C)^{\nu}\) is the probability of C not occurring, \(\nu(1 - \alpha_C)^{\nu-1}\alpha_C\) the probability of C occurring once, etc. If event C is rare we can approximate (5) by the leading terms in \(\alpha_C\) (this is our definition of rare event):

\[
N(\nu) = (1 - \alpha_C \nu)N(\nu) + \alpha_C \nu N(\nu)
\]

where

\[
N_C(\nu) = \alpha_C \nu N(\nu)
\]

is the number of events where event C occurs. If \(N\) is the total number of events, we have

\[
\sum_{\nu} N(\nu) = N
\]

\[
\sum_{\nu} \nu N(\nu) = <\nu> N
\]

and, for the total number of events with C occurring

\[
\sum_{\nu} (\alpha_C \nu) N(\nu) = \alpha_C <\nu> N
\]

This implies, for the probability distribution

\[
P_C(\nu) = \frac{\alpha_C \nu N(\nu)}{\alpha_C <\nu> N} = \frac{\nu}{<\nu>} P(\nu)
\]

Within approximation [3] we finally obtain,
\[ P_C(n) = \frac{n}{< n >} P(n) \]  
(11)

and, equivalently,

\[ P_C(E_T) = \frac{E_T}{< E_T >} P(E_T) . \]  
(12)

Relations (11) and (12) are universal, independent of \( \alpha_C \).

Our main result, (11) and (12), depends on assumptions to be discussed now.

i) The first assumption is the dominance of fluctuations in the number of elementary collisions, as seen in (4). This dominance increases with the increase of \(< \nu >\). This means going to heavier nuclei and higher energies. Specific DPM calculations suggest that this approximation is quite reasonable \([7]\).

ii) The second assumption is the smallness of the probability of the events C to occur. One can take (3) to higher orders in \( \alpha_C \), if \( \alpha_C \) is not small enough. In that case one may still obtain a relation between \( P_C(n) \) and \( P(n) \), but it will depend on \( \alpha_C \): universality is lost.

iii) The third assumption is the assumption of linearity in (6) of the dependence of the probability of events C on \( \nu \), for previous discussion see \([8]\). Final state destructive absorption, as in \( J/\Psi \) production, for instance, will eliminate this linearity by making the effective number of collisions where event C appears smaller. This can be visualized by making in (6) the change

\[ \alpha_C \nu N(\nu) \rightarrow \alpha_C \nu^\varepsilon N(\nu), \quad \varepsilon < 1 \]  
(13)

and

\[ P_C(n) = \frac{n^\varepsilon}{< n^\varepsilon >} P(n) , \]  
(14)

or

\[ P_C(E_T) = \frac{E_T^\varepsilon}{< E_T^\varepsilon >} P(E_T) , \]  
(15)

It is clear that, as \( 0 < \varepsilon < 1 \), absorption makes the associated distribution closer to the standard distribution.

We would like, at this stage, to check the validity of (12) and (15), by comparing Drell-Yan, \( J/\Psi \) and minimum bias \( E_T \) distributions of the S-U experiment of NA38 Collaboration \([9]\).
Relation (12), for Drell-Yan production in comparison with minimum bias data is tested in Fig.1. The agreement is quite good. Best agreement, having in mind absorption, (13), is obtained for $\varepsilon_{DY} \simeq 0.95$ (not shown in the Figure). In Drell-Yan events absorption is not important.

In the case of $J/\Psi$ production, see Fig.2, the best agreement, from (15), is obtained for $\varepsilon_{J/\Psi} \simeq 0.7$. In this case absorption cannot be neglected.

One can also directly compare the $J/\Psi$ production to DY production. From (13), with $\nu \equiv E_T$,

$$N_{J/\Psi}(E_T)/N_{DY}(E_T) \sim 1/E_T^\gamma,$$

(16)

and $\gamma \equiv \varepsilon_{DY} - \varepsilon_{J/\Psi}$. As absorption in the $J/\Psi$ case is more important than in Drell-Yan case, $\gamma > 0$. This means that the ratio (16) decreases with $E_T$ (the first derivative is negative) and the curvature (the second derivative) is positive. In all calculations of $J/\Psi$ absorption, including destruction by comovers [10, 11], the tendency for a large $E_T$ saturation occurs, which implies positive curvature.

There is, however, another possibility for changing the $\nu$ linearity of events C in (5): if a transition to a quark-gluon plasma occurs. In this case the $J/\Psi$ and $\Psi'$ formation will be prevented [12]. Now, besides the change in the effective number $\nu$, it is $\alpha_C$ itself that changes, becoming a function of $\nu$ and vanishing for large values of $\nu$.

If the transition is an abrupt transition at $\nu = \nu^*(\nu^*$ being energy, nuclei and acceptance dependent) we have,

$$\alpha_{J/\Psi}(\nu) = \alpha_{J/\Psi} \quad \nu \leq \nu^*$$

$$\alpha_{J/\Psi}(\nu) = 0 \quad \nu > \nu^*$$

(17)

or, making a more reasonable gaussian approximation to (17),

$$\alpha_{J/\Psi}(\nu) = \alpha_{J/\Psi} \exp\left(-\frac{\nu^2}{\nu^*^2}\right).$$

(18)

The $J/\Psi$ to DY ratio becomes now:

$$N_{J/\Psi}(E_T)/N_{DY}(E_T) \sim \exp\left(-\frac{E_T^2}{E_T^*^2}\right),$$

(19)

to be compared to (16). There is an essential difference: if the plasma is produced the curvature in the $E_T$ dependence of the ratio $N_{J/\Psi}/N_{DY}$ is negative (for $E_T < E_T^*$).

In Fig.3 we show the NA38/50 data [13, 14] on S-U and Pb-Pb collisions without any curve, for not guiding the eye. It is clear that the S-U data can be fitted using (16), with $\gamma \equiv \varepsilon_{DY} - \varepsilon_{J/\Psi} \simeq 0.25$, and that the last four points of the Pb-Pb 158 GeV/c data can
be fitted using (19), with \( E_T \simeq 250 \text{ GeV} \). An interpretation of the Pb-Pb NA50 results, including the \( \Psi'/\Psi \) ratio, in terms of quark-gluon plasma formations \([15, 16]\) is indeed allowed by data.

Coming back to our basic relation (12) we can mention another possibility for distinguishing absorption effects from quark-gluon plasma formation.

If absorption in the form (15) dominates (or if it is absent, (12)), we have

\[
P_{J/\Psi}^{Abs}(E_T) \underset{E_T \to \infty}{\gtrsim} P(E_T), \tag{20}
\]

while if plasma is formed, (17),

\[
P_{J/\Psi}^{Plasma}(E_T) \underset{E_T \to \infty}{\ll} P(E_T), \tag{21}
\]

where \( P(E_T) \) is the normalized minimum bias distribution. The S-U data are consistent with (20), see \([4]\).

In conclusion, our main relations (11) and (12) seem to have reasonable theoretical justification and to work fairly well. They can be used to test the formation of the plasma of quarks and gluons, inequalities (20) and (21). On the other hand, in the direct comparison of \( J/\Psi \) to DY production the sign of the curvature of the \( E_T \) dependence of the ratio \( J/\Psi/DY \) may be critical, Eqs. (16) and (19).

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Figure captions

**Fig.1.** NA38 experimental associated multiplicity to Drell-Yan pairs in S-U collisions (cross points) compared to $\frac{n}{<n>} P(n)$ where $P(n)$ is the experimental multiplicity distribution of S-U collisions (squared points).

**Fig.2.** Same as fig 1 but now the cross points are the experimental associate multiplicity to $J/\Psi$ production and comparison is made with $\frac{n^ε}{<n^ε>} P(n)$, and $ε = 0.7$.

**Fig.3.** Experimental $J/\Psi$ over DY pairs as a function of $E_T$ in S-U collisions.

**Fig.4.** Experimental $J/\Psi$ over DY pairs as a function of $E_T$ in Pb-Pb collisions.
