Model-independent searches for CP violation in multi-body charm decays

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Model-independent techniques for CP violation searches in multi-body charm decays are discussed. Examples of recent analyses from BaBar and LHCb are used to illustrate the experimental challenges involved.

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1 Introduction

The recent evidence for CP violation (CPV) in singly Cabibbo suppressed $D^0$ decays to two-body final states from LHCb [1] and CDF [2] has heightened interest from theoreticians in charm physics. In order to investigate these promising hints further, multi-body decays with three or four particles in the final state will be needed. Direct CPV arises when decays with two possible routes to the same final state with different relative weak and strong phases interfere. The intermediate resonances in multi-body final states have different strong phases, and thus if there are also different weak phases, CPV is guaranteed. Moreover, in multi-body $D$ decays it is possible in principle to extract information about these strong and weak phases which remains hidden in two-body systems.

Furthermore, some of the easiest charm decays to reconstruct experimentally have three or four charged particles in the final state. For example, the most abundant flavour-tagged singly Cabibbo suppressed final state available to most experiments is the three-body $D^+ \rightarrow K^- K^+ \pi^+$ decay [3], and so, naively, this should be the most promising charm decay mode for CPV searches.

However, multi-body charm decays present several key challenges. To exploit the phase information to the full requires an amplitude model. Such models carry with them systematic uncertainties that are difficult to quantify and depend on choices made by the analysts. The properties of the many possible scalar resonances required by the amplitude models are often poorly known, and model uncertainties now approach or even exceed statistical uncertainties in many amplitude analyses (e.g. [4]). Therefore it is worth investigating model-independent techniques which are not subject to these uncertainties. With the large data samples now under study at the B-factories and LHCb, such techniques should also be faster and simpler to implement.

There are now several model-independent techniques in the experimentalists’ armoury. I will discuss recent analyses which illustrate some of these well.

2 $D^0 \rightarrow hh\pi^0$ at BaBar

One of the first model-independent searches for CPV was performed at BaBar on the $D^{*-}$-tagged decays $D^0 \rightarrow \pi^+\pi^-\pi^0$ and $D^0 \rightarrow K^+K^-\pi^0$ [5]. An amplitude analysis, a phase-space-integrated search for CPV, and two model-independent techniques were employed. In 82,468 ± 321 $\pi^+\pi^-\pi^0$ and 11,278 ± 110 $K^+K^-\pi^0$ decays, no evidence of CPV is found.

In the first search, the Dalitz plots of these decays are divided into simple grids
of bins. In each bin, the variable $S_{CP}^i$, defined as

$$S_{CP}^i = \frac{N_{D^+}^i - N_{D^-}^i}{\sqrt{(\Delta N_{D^+}^i)^2 + (\Delta N_{D^-}^i)^2}} \alpha,$$

$$\alpha = \frac{\sum_i N_{D^+}^i}{\sum_i N_{D^-}^i}$$

(1)

is calculated. Here, $N_{D^+}^i$ represents the number of $D^+$ decays observed in a given bin $i$, $\Delta N_{D^+}^i$ its uncertainty, and $\alpha$ is an overall normalisation used to cancel any production asymmetry effect. This is similar to the “Miranda” method [6].

The sum of the squares of the $S_{CP}^i$ over all bins is a $\chi^2$ for consistency with no CPV, with a number of degrees of freedom equal to the number of bins minus one (due to the overall normalisation). BaBar obtain a one-sided Gaussian confidence level for consistency with no CPV of 32.8% for $\pi^+\pi^-\pi^0$ and 16.6% for $K^+K^-\pi^0$. The distribution of $S_{CP}^i$ values over the Dalitz plots is shown in Figure 1.

Figure 1: Distribution of $S_{CP}^i$ values over the Dalitz plots: $D^0 \rightarrow \pi^+\pi^-\pi^0$ on the left, $D^0 \rightarrow K^+K^-\pi^0$ on the right.

BaBar also perform a complementary search for CPV which makes use of the angular moments of the helicity angle $\theta_H$. This is defined for decays of the type $D \rightarrow r(AB)C$ as the angle between the momentum of $A$ in the $AB$ rest frame and the direction opposite to the $D$ momentum in that same frame. The procedure is to bin the Dalitz plot projections in two-body invariant mass, then weight the efficiency-corrected number of events in a bin by the $l^{th}$ Legendre polynomial function of $\cos \theta_H$. This yields moments

$$P_0 = \frac{|S|^2 + |P|^2}{\sqrt{2}}, P_1 = \sqrt{2}|S||P|\cos \theta_{SP}, P_2 = \sqrt{\frac{2}{3}}|P|^2, \ldots$$

(2)
which allow one to separate the $S$ and $P$-wave components of the Dalitz plot. An asymmetry variable $X_{l,CP}$ is then defined as

$$X_{l,CP} = \frac{P_l - \alpha P_l}{\sqrt{\sigma^2_{P_l} + \alpha^2 \sigma^2_{S_l}}}$$

(3)

with $\alpha$ defined as before. Legendre polynomial moments of orders greater than eight are neglected. A $\chi^2$ for consistency with no CPV is calculated in the same way, this time accounting for correlations between the eight different angular moments $l$ with a correlation coefficient $\rho_{ij}$ obtained from simulation:

$$\chi^2 = \sum_{i=0}^{k} \sum_{j=0}^{7} X_i \rho_{ij} X_j$$

(4)

Here $k$ is the number of intervals of invariant mass and the number of degrees of freedom is $8k$. BaBar find the one-sided confidence level for no CPV to be 28.2% for the $\pi^+\pi^-$, 28.4% for the $\pi^+\pi^0$, 63.1% for the $K^+K^-$, and 23.8% for the $K^+\pi^0$ sub-systems. It should be noted that this procedure assumes there is no interference from crossing channels and that only $S$ and $P$ waves are present.

3 $D^+ \rightarrow K^-K^+\pi^+$ at LHCb

A similar analysis strategy was pursued with the first 35 pb$^{-1}$ of data taken at the LHCb experiment in 2010 [7]. The hadronic environment and the higher statistics present some additional challenges. For example, all multi-body analyses are subject to some extent to the variation of interaction asymmetries of particles with detector material across the Dalitz plot, because these asymmetries can depend on momentum. At LHCb the high boost tends to reduce these problems. This was demonstrated in the convenient Cabibbo-favoured control channel, $D_s^+ \rightarrow K^-K^+\pi^+$, which has similar kinematics and signal yield to the $D^+$ decay. The reconstructed $K^-K^+\pi^+$ mass distribution and $D^+ \rightarrow K^-K^+\pi^+$ Dalitz plot is shown in Figure 2. There are around 370,000 $D^+ \rightarrow K^-K^+\pi^+$ signal events with 90% purity in the 2010 dataset.

Monte Carlo simulations based on an amplitude model from CLEO-c [8] are used to develop binning schemes for the Dalitz plot with greater sensitivity to plausible types of CP violation than a simple regular grid. For example, it is found that the strong phase can change across resonances. If one bin is used for the whole resonance, the CPV would change sign within it and the $S_{CP}$ value for that bin would probably be consistent with 0. Therefore, without sacrificing model independence, the resonances are divided up into bins an appropriate way to account for the changing strong phase. With 25 bins in the Dalitz plot, the Monte Carlo studies suggest, for example, that a CP-violating phase in the $\phi(1020)$ of $5^\circ$ would be observed at the $3\sigma$ C.L. with $\sim 90\%$
probability. Other possible CPV signals are explored and, for example, the method is similarly sensitive to an $\sim 11\%$ CPV in the magnitude of the $\kappa(800)$.

The charge asymmetries in the $D_s^+ \rightarrow K^- K^+ \pi^+$ control channel, and also in the Cabibbo-favoured $D^+ \rightarrow K^- \pi^+ \pi^+$ decay, are investigated to eliminate the possibility of observing detector asymmetries as signals of CPV. No significant charge asymmetries are observed, indicating that no Dalitz-plot dependent fake asymmetries are yet detectable.

The signal mode is then binned according to four binning schemes, two of which account for the resonant structure of the decay and two of which, following the analysis at BaBar\cite{5}, do not. The $S_{CP}$ values in each bin are calculated and the resulting $\chi^2$ for consistency with no CPV is converted into a $p$-value. The $S_{CP}$ values are shown in Figure 3 and the $p$-values are tabulated in Table 1. No evidence of CPV is seen.

Figure 3: Distributions of $S_{CP}$ variable for the LHCb $D^+ \rightarrow K^- K^+ \pi^+$ Dalitz plot, in phase space (top) and as pulls (bottom) for four choices of Dalitz plot binning.
Binning | $\chi^2$/ndf | $p$-value (%) \\
---|---|---
(a) Adaptive I | 32.0/24 | 12.7 \\
(b) Adaptive II | 123.4/105 | 10.6 \\
(c) Uniform I | 191.3/198 | 82.1 \\
(d) Uniform II | 519.5/529 | 60.5 \\

Table 1: Table showing $\chi^2$/ndf and $p$-values for consistency with no CPV for the binnings shown in Figure 3

4 Four body decays

The five-dimensional phase space of four body decays presents new challenges and new opportunities. The binned search for CP violation can be adapted to four-body decays, as done recently at LHCb in around 180,000 $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ decays [9]. In four-body decays, the method of $T$-odd moments also becomes available. This has been applied in several analyses, for example by the BaBar Collaboration in their search for CPV in $D^+ \rightarrow K_S^0 K^+\pi^+\pi^-$ [10].

The procedure is to define a triple product of the momenta of three of the particles.

$$C_T = \vec{p}_{K^+}.(\vec{p}_{\pi^+} \times \vec{p}_{\pi^-})$$

The fourth momentum is completely constrained. Then one defines an asymmetry

$$A_T = \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)}$$

for $D^+$ decays and the analogous $A_T$ for $D^-$ decays. The quantity $A = \frac{1}{2}(A_T - A_T)$ is then a measure of $T$ violation. $A_T$ and $A_T$ can be individually nonzero if there are final state interactions [11], but their difference cannot, unless CP is violated. In a sample of 20,000 decays, BaBar obtain $A = (-1.2 \pm 1.0 \pm 0.46)\%$.

5 Unbinned techniques

There is a promising new technique available for unbinned model-independent searches which has not yet been applied to experimental data [12]. It is possible to compare the densities of oppositely charged points in phase space using a two-sample test. A test statistic $T$ is used to correlate the difference between $D^+$ and $D^-$ p.d.f.s, $f(\vec{x})$ and $f(\vec{x'})$,

$$T = \frac{1}{2} \int \int \left( f(\vec{x}) - \bar{f}(\vec{x}) \right) \left( f(\vec{x'}) - \bar{f}(\vec{x'}) \right) \psi(|\vec{x} - \vec{x'}|) d\vec{x} d\vec{x'}$$

(7)
To calculate $T$ from data, one then translates this to:

$$T = \frac{1}{n(n-1)} \sum_{i,j>i}^n \psi(|\vec{x}_i - \vec{x}_j|) + \frac{1}{n(n-1)} \sum_{i,j>i}^n \psi(|\vec{x}_i - \vec{x}_j|) - \frac{1}{n(n-1)} \sum_{i,j>i}^n \psi(|\vec{x}_i - \vec{x}_j|)$$

where $n$ is the number of events (c.c. events). A sensible choice for weighting function $\psi(|\vec{x}_i - \vec{x}_j|)$ is $e^{-|\vec{x}_i - \vec{x}_j|^2/2\sigma^2}$ where the choice of $\sigma$ depends on the scale of the asymmetries expected in the Dalitz plot.

The permutation test is then used to determine a $p$-value for consistency with no CPV. Many datasets where the “+” and “−” labels are assigned randomly to $D$ decays are produced, such that there are $n$ “+” decays and $n$ “−” decays in each set. Then the $p$-value is the fraction of elements in the set for which $T > T_{\text{obs}}$.

Preliminary Monte Carlo studies indicate that this method should be more sensitive than binned searches, although it could also be very computationally expensive if applied to large datasets in its most basic form.

6 Outlook

Model-independent techniques for CPV searches are likely to become more important for the very large Belle II and LHCb data samples. Whilst they are not substitutes for full amplitude analyses, they provide the most straightforward means to establish CPV in a multi-body decay. It is hoped that model-independent techniques can shed light on some of the most pressing issues in charm physics in the near future.

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