Spin-selective equilibration among integer quantum Hall edge channels

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The equilibration between quantum Hall edge modes is known to depend on the disorder potential and the steepness of the edge. Modern samples with higher mobilities and setups with lower electron temperatures call for a further exploration of the topic. We develop a framework to systematically measure and analyze the equilibration of many (up to 8) integer edge modes. Our results show that spin-selective coupling dominates even for non-neighboring channels with parallel spin. Changes in magnetic field and bulk density let us control the equilibration until it is almost completely suppressed and dominated only by individual microscopic scatterers. This method could serve as a guideline to investigate and design improved devices, and to study fractional and other exotic states.

Quantum Hall devices remain paradigmatic for research on topological systems [1]. The Hall regime is accessed with a quantizing magnetic field perpendicular to a two-dimensional electron gas (2DEG) [2,3]. Dissipationless non-equilibrium currents flow in one-dimensional chiral channels along the edge of the system in response to an external voltage [4,7], experiencing inter-edge-state scattering in the presence of a background disorder potential [8-14]. Equilibration phenomena among non-equilibrium edge currents are not yet fully understood despite the rich history of past experiments on semiconducting devices.

Haug and coworkers found length-dependent equilibration in spin-degenerate quantum Hall systems with top gates acting as partially transmitting barriers [6,8,11,12], but did not report about spin-related effects. Later, Müller found that in the presence of a background disorder potential, spin-orbit interactions mediate the equilibration between spin-polarized edge modes by allowing charge carriers to flip their spin [11,16]. The continuous advancements in material technologies thus motivated a revival of equilibration experiments [17-20].

Local probe experiments by Weis et al. already showed the complexity of the microscopic reconstruction of the edge potential [21-24]. Further details on the edge could be revealed assuming that the presence of an incompressible region of a specific filling factor between two channels implies weak equilibration.

Quantum Hall edge state equilibration experiments gradually expanded to the fractional regime too, often finding non-trivial edge reconstructions and current distributions [25-27]. Graphene is another mature platform for quantum Hall experiments unraveling the role of valley and spin degrees of freedom in equilibration phenomena [28-31].

In this manuscript, we address the question of inter-edge-mode scattering in state-of-the-art devices using electronic transport experiments. The design that we use is inspired by historically well-known experiments, where edge channels can be reflected and transmitted with barrier gates to obtain well-controlled out-of-equilibrium population of edge modes [15,16,32,33]. We study how the excitation of selected integer edge modes is redistributed as they co-propagate and extract the strength of pair-wise coupling among many channels (up to 8). We find that the spin of the modes determines the equilibration at low enough fields, spin-selectively coupling even distant channels in contrast with many findings from the past [6,8,11,12]. For larger fields the equilibration is almost completely suppressed and mesoscopic impurities dominate the weak equilibration between spin-split channels.

Our device is an MBE-grown AlGaAs heterostructure equipped with a patterned back gate located roughly 1 μm below the plane of the 2DEG [37,38]. We lithographically defined top gates as indicated in Fig. 1(a). Following the Landauer-Büttiker formalism [31,32], the number of channels transmitted by each barrier depends on the local filling factors

\[ n_{\text{I}}, n_{\text{D}} = \frac{\hbar}{eB} n_{\text{I}} \leq n_{\text{D}}, \]

where \( n_{\text{I}} \) and \( n_{\text{D}} \) are the filling factors of the 2DEG under the injector and detector gates respectively, when we fix the local densities to \( n_{\text{I}} \) and \( n_{\text{D}} \). We tune the system such that \( n_{\text{I}}, n_{\text{D}} \) and the bulk filling factor \( n_{\text{B}} \) have integer values to perform the experiments in a controlled way and suppress bulk equilibration [19]. We can route channels carrying different electrochemical potentials to flow...
after flowing through the injector barrier, the trans-mitted channels will have a different electrochemical po-tential $\mu_1$ compared to the reflected modes on the other side of the barrier, coming from the grounded contact [cf. red versus yellow and white lines in Fig. 1(a)]. However, measuring the transverse voltage $V_{xy}$ between the detector and ground will reveal no details about inter-mode coupling along the path if all channels equilibrate in the detector ($\nu_D = \nu_B$) [13]. The contact settles at the electrochemical potential

$$\mu_D = \frac{1}{\nu_D} \sum_{i=1}^{\nu_D} \mu_i,$$  \hspace{1cm} (2)$$

where $\mu_i$ is the potential of an individual channel $i$ when entering the detector. In the integer regime, all channels have transmission of one and contribute equally to the potential of the contact. When the detector barrier allows only selected modes to be transmitted ($\nu_D < \nu_B$), measurements of the transverse resistance will yield

$$R_{xy}^{\nu_D} = \frac{V_{xy}}{I_{ac}} = \frac{\hbar}{e^2} \frac{\nu_D}{\nu_B} = \frac{\hbar}{e^2} \nu_B \nu_D \sum_{i=1}^{\nu_D} \mu_i.$$  \hspace{1cm} (3)$$

The total equilibration between the channels does not depend on the specific tuning of the barriers, but rather on the edge potential along the propagation path, on the mesoscopic disorder background and on the length of co-propagation. Equilibration among channels under the detector gate does not affect the measurements [13]. If the external current is completely injected in the outermost channel ($\nu_I = 1$), an out-of-equilibrium population of spin-polarized electrons is built up. This channel can equilibrate either with other channels of the same spin polarization, or with channels of the opposite spin, if spin-flips are involved.

We devised a measurement protocol to extract the electrochemical potential of the channels at the detector. We measure $R_{xy}^{\nu_D}$ for different values of $\nu_D$ while the injector barrier is tuned to $\nu_I = 1$. A system of equations of the form of Eq. (3) with values $1 \leq \nu_B \leq \nu_D$ describes the measurements [see Fig. 2(a)]. We can solve the system to find the normalized electrochemical potentials of the channels $\tilde{\mu}_i = \mu_i / \mu_1$, with the initial conditions $\tilde{\mu}_1^I = 1$, and $\tilde{\mu}_j^D = 0$ for $j \neq 1$.

Figures 2(b)–(f) show the results of the analysis for different $\nu_I$ and in a range of magnetic fields and bulk densities. In Fig. 2(b)–(d), we observe that electrons preferentially equilibrate with states of the same spin, leaving channel 1 to occupy states in modes 3 and 5. Channels labeled with even numbers were mostly decoupled from the only initially excited channel and their potential is closer to the bottom of our energy scale.

In particular, when $\nu_B = 8$ or 6, two bundles of channels with opposite spin are resolved and well separated in energy. Even though the clear separation between the two bundles is not visible for the case of $\nu_B = 7$, also here the system favors spin-selective equilibration. The presence of reproducible fluctuations is likely due to impu-

![Figure 1](https://via.placeholder.com/150)

FIG. 1. (a) Schematic device structure and measurement setup, not to scale. A current $I_{ac} = 500$ pA flows through the device between injector and ground contacts, at potentials $\mu_I$ and $\mu = 0$ respectively. Two top gates act as barriers downstream of the injector (violet gate) and upstream of the detector (yellow gate) contacts. They are controlled via two dc voltage sources $V_1$ and $V_2$ (not shown). A side gate (green) laterally depletes the 2DEG along a length $L_{eq} = 35 \mu$m. (b) 2-terminal conductance measured through the device as a function of the left barrier gate voltage $V_1$. The right barrier gate is grounded [35]. The magnetic field $B$ and bulk density $n_B$, controlled with the back gate, are stepped together to ensure constant bulk filling factor $\nu_B = 5$. Diagonal dashed lines indicate regions of constant conductance, corresponding to a quantized local filling factor $\nu_1$ below the injector gate.

along the co-propagation path [$L_{eq}$ in Fig. 1(a)]. Measuring the longitudinal resistance across this path or the potential of the detector with respect to ground (our case), will yield information about the strength of the equilibration processes among the channels.

We measure the two-terminal conductance $G_{2T}$ as a function of the barrier voltage $V_1$ while an ac current $I_{ac}$ flows from the injector to ground and the other barrier is fully transparent. Figure 1(b) shows the result measured at $\nu_B = 5$. Plateaus of constant conductance matching integer multiples of $e^2/h$ are found as the barrier gate voltage decreases, (white dashed lines). Each diagonal feature corresponds to a fixed number of channels transmitted through the barrier region. We repeat the same experiment with the detector gate and for different bulk filling factors to observe the transmission characteristics of both barriers (not shown).

After flowing through the injector barrier, the transmitted channels will have a different electrochemical potential $\mu_1$ compared to the reflected modes on the other
FIG. 2. (a) Equilibration measurement performed at the star-shaped symbol in Fig 1(b). The red circles indicate the data points required to calculate a set of $\mu_i$ following Eq. (3). (b)–(f) Electrochemical potential $\tilde{\mu}_i$ of the modes for integer bulk filling factors $\nu_B = 8–4$ after the equilibration path. Matching labels indicate the spin and index of the channels. The back gate voltage $V_{BG}$ (controlling $n_B$) and the magnetic field $B$ are simultaneously stepped to fix $\nu_B$ during each experiment, similar to Fig 1(b). The shaded regions in figures (a)–(d) indicate the range where coupling parameters have been extracted [see text and Fig. 3(c)–(f)].

rities occurring on mesoscopic length scales, modulating the coupling between the modes [44]. If the three spin-up channels in Fig. 2(b) completely equilibrate while the others do not participate, we expect to find $\tilde{\mu}_{1,3,5} = 1/3 \approx 0.33$, a case nearly reached at the highest densities. Conversely, if all channels equilibrate, then $\tilde{\mu}_i = 1/8 \approx 0.125$ for all of them, which is nearly the case at the lowest densities.

As the number of channels in the bulk decreases with increasing external magnetic field, so does the coupling between them. Figure 2(e) ($\nu_B = 5$) shows that electrons in channels 1 and 3 are not fully equilibrating along $L_{eq}$, contrary to the cases with $\nu_B > 5$. We observe that the coupling becomes weaker for larger $B$, but spin-selective equilibration still remains the favored process. In Fig. 2(f) ($\nu_B = 4$) the coupling weakens to the point where $\tilde{\mu}_1 \approx \tilde{\mu}_0 = 1$ for the whole range. Few mesoscopic features increase the coupling between the two modes in the lowest Landau level, which requires some spin-flip mechanism. Spin-selective equilibration is not observed in this case. Performing the same experiments over a distance $L_{eq} \approx 535 \mu m$ reveals full equilibration irrespective of the spin alignment, although the innermost channel remains decoupled [43].

The density profile at the edge, sketched in the spirit of Ref. [45] in Fig. 3(a), guides us in understanding the results of Fig. 2. The edge channels represent discrete conducting regions located where the density has a non-zero gradient. Incompressible stripes with a fixed filling factor separate compressible regions that form at the edge as a result of screening and interactions in the presence of an external $B$ field [30] [45] [47]. Decreasing $\nu_B$ at
constant density, by increasing $B$, means that a smaller number of channels spans the density profile, pushing the innermost channels further into the bulk \cite{21, 26, 48}. Increasing $B$ and $n_B$ while keeping $\nu_B$ constant instead results in wider incompressible stripes and a larger separation between the channels. The magnetic length $\ell_B$, and consequently the spatial extent of the wavefunction of the edge modes, decreases for stronger fields. Since charge transfer between channels requires wavefunction overlap, a larger distance and stronger confinement can quickly suppress the tunneling probability amplitude.

Electrons can in principle tunnel from one channel to any other, conserving or flipping their spin. The energy transfer between modes can be approximated with a system of rate equations of the form \cite{43}: \[ \frac{d\mu_i}{dx} = -\frac{1}{2} \sum_{j \neq i} \gamma_{ij} (\mu_i - \mu_j). \] (4)

Here the potentials $\mu_i$ are intended to be position dependent along the equilibration path between injector and detector. The terms $\gamma_{ij} = \gamma_{ji}$ model a uniform coupling between channels $i$ and $j$ \cite{36}. These parameters encapsulate any equilibration process in our model, giving us a quantity related to the average equilibration lengths $\ell^0_{ij} = \gamma_{ij}^{-1}$ between channels $i$ and $j$. We can numerically calculate the whole set of $\gamma_{ij}$ by performing an equal amount of independent measurements at the detector, each time setting the barrier filling factors to integer values such that $\nu_I \leq \nu_D$ and $\nu_I, \nu_D < \nu_B$ \cite{43}.

Starting with $\nu_B = 8$ in Fig. 3(c), we observe that spin-conserving coupling terms dominate, while spin-flip terms can be more than one order of magnitude smaller. Spin-selective tunneling couples not only the spatially closest channels with parallel spin (channels 1 and 3), but also terms like $\gamma_{15}$ and $\gamma_{26}$ are much larger than spin-flip terms coupling nearest-neighbors. This shows that it is more likely for electrons to tunnel a larger distance without flipping their spin rather than tunneling through a thinner barrier undergoing a spin-flip event.

Increasing the field and decreasing $n_B$ at constant density progressively decouples the channels. In Fig. 3(d) and (e) we observe a reduction of the coupling, in particular of the long-distance terms $\gamma_{15}$ and $\gamma_{26}$. In Fig. 3(f) the trend continues and also short-range spin-selective coefficients decrease. Finally, for $\nu_B \leq 4$, all the integer channels are mostly decoupled, either too far removed towards the bulk or limited by the frequency of spin-flip events.

In this manuscript, we analyzed our data based on the
well-established edge channel picture of the integer quantum Hall effect, finding that channels with parallel spin selectively couple with each other, while flipping the spin of electrons is much less likely. At low enough fields, spin-conserving tunneling even couples modes separated by several compressible and incompressible stripes instead of only neighboring channels with parallel spin. In general, the equilibration process is influenced by experimental parameters, like magnetic field and temperature, and by sample properties, such as material quality and heterostructure design. Controlling the transfer of particles between channels could lead to the use of edge modes as spin rails to transport well-defined magnetic moments in quantum computation experiments. The presence of spin-selective signatures at low field would help to integrate such a technology with others that do not tolerate or require high fields.

In the fractional regime, a precise knowledge of the equilibration length is sought after to improve experiments involving interferometers and other confined systems. Anyonic statistics, the thermal conductance of exotic states and the complex edge reconstruction associated to fractional states like the fractional quantum Hall effect, finding that channels with parallel spin selectively couple with each other, while flipping the spin of electrons is much less likely. At low enough fields, spin-conserving tunneling even couples modes separated by several compressible and incompressible stripes instead of only neighboring channels with parallel spin. In general, the equilibration process is influenced by experimental parameters, like magnetic field and temperature, and by sample properties, such as material quality and heterostructure design. Controlling the transfer of particles between channels could lead to the use of edge modes as spin rails to transport well-defined magnetic moments in quantum computation experiments. The presence of spin-selective signatures at low field would help to integrate such a technology with others that do not tolerate or require high fields.

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