Hard exclusive processes involving kaons

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Abstract. Hard exclusive electroproduction of kaons as well as the kaon-induced exclusive Drell-Yan process are investigated within the handbag approach which is based on factorization in hard subprocesses and soft generalized parton distributions (GPDs). The kaon-hyperon transition GPDs occurring here are related to the proton GPDs by flavor symmetry. Like in hard processes involving pions the transversity GPDs play an important role — the transverse cross sections are larger than (or, for the Drell-Yan process, about equal to) the longitudinal ones. The evolution of the transversity GPDs is taken into account for the first time but, as the analysis reveals, it is a minor effect in the range of photon virtualities of interest. The predictions for the cross sections agree fairly well with the sparse available electroproduction data.

1 Introduction

The handbag approach to hard exclusive meson electroproduction is based on factorization of the process amplitudes in hard subprocesses and soft hadronic matrix elements, parametrized as GPDs. This factorization property has been shown to hold rigorously to leading-twist accuracy in the generalized Bjorken regime of large photon virtuality, \( Q^2 \), and large invariant mass of the hadrons in the final state, \( W \), but fixed Bjorken-\( x \), \( x_B \), and small Mandelstam-\( t \) \([1]\). From extensive experimental and theoretical investigations of hard exclusive meson electroproduction carried through over the last two decades it however turned out that the naive asymptotic result is not readily applicable in the range of kinematics accessible to current experiments. In fact, large power corrections are required to the asymptotically dominant amplitudes for longitudinally polarized photons. Moreover there are strong contributions from transversal photons which are asymptotically suppressed by \( 1/Q^2 \) in the cross sections. In some cases, as for instance for \( \pi^0 \) electroproduction \([2]\), the contributions from transversely polarized photons are even dominant.

In a series of articles we have developed a generalization of the handbag approach which allows to model these power corrections, see for instance the detailed report \([3]\). The decisive point is to retain the quark transverse momenta in the subprocess. Implicitly, this way the transverse size of the meson is taken into account. This generalized handbag approach has been applied to electroproduction of pions \([4, 5]\) as well as to \( \rho^0 \) and \( \phi \) mesons \([6]\). It turned out that the data on these processes are well fitted within this approach in a large range of kinematics. An outcome of these investigations is the extraction of a set of GPDs which subsequently allow to study the parton localization in the transverse position plane, to evaluate the parton angular momentum and, exploiting the universality properties of the GPDs, to calculate other hard exclusive processes as for instance deeply virtual Compton scattering \([7]\) or \( \omega \) production \([8]\).

In this article I am going to apply the generalized handbag approach to hard exclusive processes involving kaons. Not much has been done as yet for these processes neither theoretically nor experimentally. Only a few data on the separated electroproduction cross sections for forward emitted kaons have been measured at the Jefferson lab \([9–11]\). More data will come from the JLab experiment E12-09-011 in the near future. The kaon-induced exclusive Drell-Yan process is planned to measure at J-Parc \([12]\). Thus, it seems to be of interest and timely to study these kaon reactions in order to probe the set of extracted GPDs against kaon data and to make predictions for future experiments.

The plan of the paper is the following: In the next section the generalized handbag approach is briefly sketched and the soft input (GPDs, kaon wave functions, kaon-pole term) is represented. Results for kaon electroproduction are given in sect. 3 and compared to the data. In sect. 4 predictions for the Drell-Yan process are presented and the implications of the excitation of charmonia are examined. A summary is given in sect. 5 and in the appendix...
a method for the numerical solution of the evolution equation for the transversity GPDs is discussed.

## 2 The handbag approach

The generalized handbag approach has been described in great detail in previous work [4–6]. Therefore, only the basics facts will be sketched here. Consider the proton and the meson masses are denoted by parity conservation. The positron charge is denoted by $e_0$ and the skewness, $\xi$, is related to Bjorken-$x$, $x_B$, by

$$\xi = \frac{x_B}{2 - x_B},$$

where possible corrections of order $1/Q^2$ are ignored. The minimal value of $-t$ corresponding to forward scattering, is given by

$$t_0 = -\frac{2\xi}{1 - \xi^2} \left[ \Lambda^2 (1 + \xi) - m^2 (1 - \xi) \right].$$

The characteristic $Q$-dependencies of the amplitudes have been made explicitly in eq. (1). For dimensional reasons the mass parameter, $\mu_K$, is additionally pulled out from the convolutions, $\langle K_K \rangle$, for transversely polarized photons. This parameter is the meson mass enhanced by the chiral condensate

$$\mu_K = \frac{m_K^2}{m_u + m_s}$$

by means of the divergence of the axial vector current. The masses $m_u$ and $m_s$ are the current-quark masses of the kaon’s valence quarks. For the numerical studies to be presented below, a value of 2 GeV is used for $\mu_K$ at the initial scale $\mu_0 = 2$ GeV. Since the current-quark masses decrease with increasing scale $\mu_K$ is scale dependent. The respective anomalous dimension is $4/\beta_0 = 12/25$ for four flavors. The mass parameter, $\mu_K$, occurs since the use of the transversity or helicity-flip GPDs, $H_T$ and $E_T$, goes along with the twist-3 kaon wave function which is applied in Wandzura-Wilczek approximation. As one sees from eq. (1) the transverse amplitudes are parametrically suppressed by $\mu_K/Q$ as compared to the longitudinal amplitudes which are of twist-2 nature.

The item $\langle K_K \rangle$ in (1) denotes the convolution of a proton-hyperon transition GPD, $K$, with a subprocess amplitude, $\mathcal{H}$:

$$\langle K_K \rangle_\mu = \sum_{\lambda, \lambda'} \int_{-1}^1 dx \mathcal{H}_{0,\lambda' : \lambda}(x, \xi, Q, t = 0) K_K(x, \xi, t).$$

The labels $\lambda$ and $\lambda'$ refer to the helicities of the partons participating in the subprocess. The subprocess amplitudes are calculated with the quark transverse momenta retained in the subprocess while the emission and reabsorption of the partons from the baryons are still treated collinear to the baryon momenta. The subprocess amplitudes read

$$\mathcal{H}_{0,\lambda' : \lambda} = \int d\tau d^2 b \psi_{K, -\lambda' : \lambda}(\tau, -b, \mu_F) \times \bar{\psi}_{K, \lambda} \exp[-S(\tau, b, Q, \mu_R)]\] (6)$$

in the impact parameter space; $b$ is canonically conjugated to the quark transverse momenta. The Sudakov factor, $S$, has been calculated by Botts and Sterman [13] in next-to-leading-log approximation using resummation techniques and having recourse to the renormalization group. It takes into account the gluon radiation resulting from the separation of color charges which is a consequence of the quark transverse momenta. The Sudakov factor can be found in [14]. Its properties force the following choice of the factorization scale: $\mu_F = 1/b$. The renormalization scale is taken to be the largest mass scale appearing in the subprocess, i.e. $\mu_R = \max(\tau Q, (1 - \tau) Q, 1/b)$ ($\tau$ is the momentum fraction of the quark entering the meson). The renormalization scale also applies to the mass parameter defined in eq. (4). For the hard scattering kernel, $F$, evaluated to lowest order of perturbative QCD, it is referred to refs. [4, 6].

The last item to be explained is $\hat{\psi}_{K, \lambda' : \lambda}$. It represents the Fourier transform of a meson light-cone wave function. For longitudinally polarized photons, one has $\lambda' = \lambda$ and the distribution amplitude associated to $\hat{\psi}$, is the familiar twist-2 one. For transversal photons one has $\lambda' = -\lambda$ and a twist-3 wave function is required. The wave functions are specified in sect. 2.2. The inclusion of the quark transverse momenta and the Sudakov factor has two advantages —firstly the magnitude of the subprocess amplitudes are somewhat reduced as compared to a collinear calculation which leads to a better agreement with experiment (see
e.g. [5]) and, secondly, the infrared singularities occurring in a collinear calculation of the twist-3 subprocess amplitudes are regularized. In passing it should be noted that there are other $1/Q$ suppressed contributions to the transverse amplitudes, as for instance twist-3 GPDs in combination with leading-twist meson wave functions. Since for such contributions there is no enhancement known it seems reasonable to neglect them and to include just the combined effect of the transversity GPDs and the twist-3 meson wave functions.

2.1 The GPDs

For the process of interest in this work the proton-$\Lambda$ transition GPDs, $K_K$, occur. Flavor symmetry however relates these GPDs to the diagonal proton ones [15]

$$K_K \simeq -\frac{1}{\sqrt{6}} [2K^u_i - K^d_i - K^s_i].$$

(7)

At zero skewness, from where the construction of the GPD starts, the GPDs are splitted into valence, $\alpha_v$, and sea, $\alpha_s$, contributions. For instance, for $x > 0$,

$$\tilde{H}^a(x,0,t) = \tilde{H}^a(x,0,0,t) - \tilde{H}^a(-x,0,t) + \tilde{H}^a(-x,0,0,t)$$

(8)

and analogously for the other GPDs. On the premise of a flavor symmetric sea only valence quarks contribute to kaon electroproduction and the proton-$\Lambda$ GPDs are given by

$$K_K \simeq -\frac{1}{\sqrt{6}} [2K^u_i - K^d_i],$$

(9)

where, from now on, any flavor label stands for valence quarks.

In [5] the zero-skewness GPDs for flavor $a$ are parameterized as

$$K^a_i(z, \xi = 0, t) = K^a_i(x, \xi = t = 0) \exp[(b^a_i - \alpha^a_i \ln(x))]t.$$  

(10)

The forward limits, $\xi, t \to 0$, of the GPDs $\tilde{H}$ and $H_T$ are given by the polarized and transversity parton densities, respectively. In order to respect the Soffer bound the transversity density is parametrized as [16]

$$\delta^v = N^v_{HT} \sqrt{T} (1 - x) [q^v(x) + \Delta q^v(x)].$$

(11)

The unpolarized and polarized parton densities are taken from [17] and [18], respectively. For the $E$-type GPDs the forward limits are not accessible in deep inelastic lepton-nucleon scattering and, hence, unknown. Therefore, they are parameterized like the PDFs

$$K^a_i(x, \xi = t = 0) = N^a x^{-\alpha_i^a(0)}(1 - x)^{\beta^a_i}$$

(12)

with the additional parameters to be adjusted to the electroproduction data. The products of the zero-skewness GPDs with suitable weight functions are considered as double distributions from which the full GPDs can be calculated [19]. The parameters of the GPDs, compiled in table 1, are taken from [5]. The powers $\beta^a_i$ are set to the following values:

$$E^{n.p.}: \beta^u = 5, \quad \beta^d = 5,$$

$$E_T: \beta^u = 4, \quad \beta^d = 5.$$  

(13)

It should be noticed that two variants of the transversity GPDs are discussed in [5]. One which leads to a rather deep dip in the $n^0$ cross section for forward scattering while for the second one the dip is less deep. Since the second one is in better agreement with the $n^0$ electroproduction data [2, 20] it is used here.

As is well-known the GPDs evolve with the scale. The evolution of the helicity non-flip GPDs, $\tilde{H}$, $E^{n.p.}$, is evaluated with the help of Vinnikov’s code [21]. In contrast to previous applications of the transversity GPDs [4, 5, 22] their scale dependence is also taken into account in this work. The numerical method used to compute the evolution is described in the appendix. It turns out that the evolution of the transversity GPDs is a minor effect within the range of scales accessible to current electroproduction experiments as can be seen from fig. 1 where $E_T$ and $H_T$ are shown at the scales $\mu^2 = 2$ and 20 GeV$^2$. That the evolution is a small effect is already signaled by the anomalous dimension of the lowest moment of $E_T$ at $\xi = t = 0$, the tensor anomalous magnetic moment of the nucleon,

$$\kappa_T^\mu(\mu^2) = \int_{-1}^{1} dx E^n_T(x, \xi = t = 0, \mu^2)$$

(14)

which evolves as [23, 24]

$$\kappa_T^\mu(\mu^2) = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_0^T/\beta_0} \kappa_T^\mu(\mu_0^2)$$

(15)

with the anomalous dimension$^1$

$$\gamma^T_0/\beta_0 = C_T/\beta_0 = 4/25.$$  

(16)

The same scale dependence exhibits the tensor charge, the lowest moment of $H_T$ at $\xi = t = 0$. As an example for the

$^1$ Anomalous dimensions are quoted for four flavors, $n_f = 4$, throughout the paper.
significance of the evolution of the transversity GPDs it is noticed that the transverse cross section for $K^- p \rightarrow \gamma^* A$ which will be discussed in sect. 4, is reduced by mere 8% at a photon virtuality of 14 GeV$^2$ if the evolution is taken into account. Hence, the neglect of the evolution of the transversity GPDs in previous work [4, 5] is justified.

Since the factorization of the GPDs and the subprocess is treated collinearly, the GPDs do not know of the impact-parameter dependence in the subprocess since $b$ is integrated over. Hence, the factorization scale $\mu_F$ does not apply to the GPDs, it refers to the factorization of the soft meson wave function and the remaining hard part of the subprocess. The scale of the GPDs is therefore taken as the photon virtuality.

2.2 The meson wave functions

In contrast to the GPDs which are universal, i.e. process independent, the subprocess amplitudes depend on the meson by means of the meson wave function. For the soft twist-2 kaon light-cone wave function the following form is used:

$$\Psi_{K, -+} = 8\pi^2 \frac{f_K}{\sqrt{2N_c}} \frac{\zeta_K^2}{\tau^2} \Phi_K(\tau) \exp[-\zeta_K^2 k_\perp^2/(\tau\tau)],$$

(17)

with the distribution amplitude ($\bar{\tau} = 1 - \tau$)

$$\Phi_K(\tau) = 6\tau [1 + a_{K1} C_1^{3/2}(2\tau - 1)$$
$$+ a_{K2} C_2^{3/2}(2\tau - 1) + \ldots].$$

(18)

For the transverse size parameter of the kaon the same value as for the pion is taken

$$\zeta_K = \zeta_\pi = 0.853 \text{ GeV}^{-1}. $$

(19)

The kaon decay constant is 159 MeV [25]. The Gegenbauer coefficients, $a_{K_n}$ evolve with the scale

$$a_{K_n}(\mu_F^2) = \left( \frac{a_{K}(\mu_F^2)}{a_{K}(\mu_0^2)} \right)^{\gamma_{1}/\beta_0} a_{K_n}(\mu_0^2) \quad (20)$$

with the anomalous dimensions $\gamma_1/\beta_0 = 32/75$ and $\gamma_2/\beta_0 = 2/3$.

The kaon distribution amplitude is not well known. Fortunately, the Sudakov factor in conjunction with the hard scattering kernel suppresses the contributions from the higher Gegenbauer terms as compared to the lowest term at not too large values of the factorization scale [14].

The strength of the suppression grows with the Gegenbauer index. On the strength of this property the values of the Gegenbauer coefficients do not matter much in the calculation. Therefore, I assume that the two lowest coefficients in (18) are $a_{K1} = 0.1$ and $a_{K2} = -0.2$ at the scale 1 GeV and $a_{K_n} = 0$ for $n \geq 0$. Evolving these value to the scale $\mu_0$ used in this work, one arrives at

$$a_{K1}(\mu_0) = 0.09 \pm 0.04, \quad a_{K2}(\mu_0) = -0.16 \pm 0.07. \quad (21)$$

With these values the antistrange quark in the $K^+$ carries a smaller momentum fraction on the average than the u-quark. The value of $a_{K1}$ is compatible with results from QCD sum rules, see [26] and [27] and references therein. A value of $a_{K2}$ of about 0.2 at the scale 1 GeV is in accordance with QCD sum rules results [27] as well with the Dyson-Schwinger approach [28]. However, it is expected that, a positive $a_{K2}$ leads to an overestimate of kaon electroproduction. Kaon channels are typically suppressed by about 10% as compared to pion channels. This, for instance, can be seen in the time-like electromagnetic form factors [29] or in two-photon annihilation into pairs of mesons [30]. The value of $a_{K2}$ quoted in (21) is in agreement with the one chosen in a study of $\chi_cJ$ decays into pairs of kaons [31].

The twist-3 light-cone wave function is assumed to be

$$\Psi_{K, ++} = \frac{16\pi^{1/2}}{\sqrt{2N_c}} f_K\zeta_K^3 k_\perp \exp[-\zeta_K^2 k_\perp^2]$$

(22)

with the associated pseudoscalar twist-3 distribution amplitude $\Phi_{KP} \equiv 1$ [32]. The transverse size parameter $a_{KP}$ is set to the same value as in the pion case [33], namely 1.8 GeV$^{-1}$. It should be mentioned that there is a second two-body twist-3 wave function, the tensor one. It
has been shown \[4\], however, that its contribution to the subprocess amplitude is proportional to \(t/Q^2\) and, hence, neglected. Also neglected are possible contributions from the three-body twist-3 wave function.

### 2.3 The kaon-pole term

The kaon-pole contribution is treated as a one-boson-exchange contribution \[4\] which leads to the amplitudes

\[
M_{\text{pole}}^{\pm} = \frac{e_0 (m + m_A) \xi}{Q \sqrt{1 - \xi^2}} \frac{\rho_K}{t - m^2_K},
\]

where

\[
M_{\text{pole}}^{0+} = \frac{e_0}{Q} \sqrt{1 - t + t_0} \frac{\rho_K}{t - m^2_K},
\]

These amplitudes have to be added to those in eq. (1). The residue of the kaon pole is given by

\[
F_{\text{pole}} = \sqrt{2} g_{KN}\Lambda F_{K\Lambda}(t) Q^2 F_{K\Lambda}^2(Q^2).
\]

The kaon-baryon coupling constant is taken as

\[
g_{KN} = -14.5 \pm 1.3.
\]

This value is a combination of results quoted in \[34\] with a more recent value extracted from data on \(pp \rightarrow \Lambda\Lambda\) \[35\]. There is also a form factor for the coupling of the kaon to the baryons which is parametrized as

\[
F_{K\Lambda} = \frac{A_{\rho\Lambda} - m^2_K}{A_{\rho\Lambda} - t},
\]

where \(A_{\rho\Lambda} = 1.1 \pm 0.08\) GeV. This parameter is adjusted to the data on the longitudinal kaon electroproduction cross section (see sect. 3). The last item in (24) to be specified is the electromagnetic form factor of the kaon in the space-like region. It is parametrized as

\[
Q^2 F_{K\Lambda}^2(Q^2) = [1 + Q^2/c_{K\Lambda}^{\perp}] .
\]

The parameter \(c_{K\Lambda}^{\perp}\) is taken as \(0.5 \pm 0.04\) GeV\(^2\) in agreement with JLab data \[11\].

To leading-twist accuracy the kaon pole can be viewed as part of the GPD \(E\) \[36\]. The convolution of this GPD with a hard subprocess amplitude lead to the same longitudinal amplitudes as in (23) except that the electromagnetic form factor of the kaon is the leading-order perturbative result. This leads to an underestimate of the pole contribution to the kaon electroproduction cross section. The kaon-pole contribution (23) with the electromagnetic form factor of the kaon taken from experiment, does not follow from perturbative QCD. Hence, it is not subject to evolution and higher-order perturbative QCD.

The time-like kaon electromagnetic form factor which will be needed for the evaluation of the kaon-induced exclusive Drell-Yan process, is taken from the CLEO data \[29\]. It is parametrized as

\[
Q^2 F_{K\Lambda}^2(Q^2) = [1 + Q^2/c_{K\Lambda}^{\perp}] .
\]

With the universal GPDs at disposal and the information about the kaon, specified in sects. 2.2 and 2.3, the partial cross sections for electroproduction of kaons can be computed. The results on the longitudinal and transverse cross sections for forward going kaons are shown in fig. 2 and compared to the available data \[9–11\]. Predictions for the kinematics chosen for the Jlab E12-09-011 experiment are also displayed in this figure. Fair agreement with experiment is to be seen. The parametric uncertainties of the predicted cross sections have been estimated from errors of the various parameters discussed in sects. 2.2 and 2.3 as well as from an estimate of the uncertainties of the GPDs \[5,6,33\]. The evolution of the GPDs, the kaon distribution amplitude as well as that of the mass parameter \(\mu_K\) are taken into account. A remarkable fact is that even for forward scattering the transverse cross section is dominant in contrast to \(\pi^+\) production where, for small

![Fig. 2. The longitudinal and transversal cross sections for \(\gamma^*p \rightarrow K^+\Lambda\). The experimental data \[9–11\] are displayed by filled symbols, the theoretical results by open ones. Triangles (circles) represent the transverse (longitudinal) cross sections. Data and predictions are at the respective \(t_0\) except for \(W = 2.39\) GeV, \(Q^2 = 2.07\) GeV\(^2\) where \(t = -0.4\) GeV\(^2\).](Image 434x692 to 449x693)
−t, the longitudinal cross section is larger than the transverse one [4,38,39]. This is a consequence of the fact that the kaon pole is much further away from the physical region than the pion one. Therefore, its contribution to the cross section is suppressed although its coupling constant and form factors are very similar to those of the pion. The relative suppression of the kaon cross sections is given by

\[ \frac{d\sigma_L(K^+)}{d\sigma_L(\pi^+)} \sim \frac{(t - m_{\pi}^2)^2}{(t - m_K^2)^2} \]  

(30)

at small −t. At small skewness and t ≈ 0 the suppression factor is approximately

\[ \simeq (m_\pi/m_K)^4 = 0.63 \cdot 10^{-2}. \]  

(31)

The t-dependence of the longitudinal and transversal cross sections for the K+Λ channel are shown in fig. 3 at kinematics typical for the E12-09-11 experiment: \( W = 3.14 \text{ GeV} \) and \( Q^2 = 3.0 \text{ GeV}^2 \). The smaller contribution from the kaon pole also affects the shape of the longitudinal cross section. It starts with a dip for forward scattering. Like for \( \pi^+ \) production the longitudinal cross section is dominated by the pole contribution and the interference between the pole and \( \tilde{H} \) contributions is negative. The contribution from \( E_{\pi^-}p \) is almost negligible. The transverse cross section is dominated by the contribution from \( H_T \), the one from \( E_{\gamma}T \) only amounts to about 10%. The longitudinal-transverse and the transverse-transverse interference cross sections are also shown in fig. 3. They are markedly smaller in absolute value than the transverse cross section.

There are also interesting polarization phenomena as for instance the correlation between the helicities of the virtual photon and that of the target proton or the asymmetries measured with a transversely polarized target. In general these polarizations are very similar in size to the case of \( \pi^+ \) productions [4] but have occasionally the opposite sign.

One may also calculate the \( K^+\Sigma^0 \) channel. For a flavor symmetric sea only the d-valence quark GPDs contribute [15],

\[ K_{p\rightarrow \Sigma^0,i} \simeq \frac{1}{\sqrt{2}} K_{i}^{d}, \]  

(32)

and the coupling constant \( g_{KN\Sigma^0} \) is 3.5 [34]. All other input parameters are the same as for the \( K\Lambda \) channel. Since both the GPD as well as the coupling constant are much smaller than for the case of the \( \Lambda \), the \( K^+\Sigma^0 \) cross sections are more than an order of magnitude smaller than the ones for the kaon-\( \Lambda \) channel while the shapes of the cross sections are similar in both cases.

### 4 The kaon-induced exclusive Drell-Yan process

Next the process \( K^-(q,0)p(p,\nu') \rightarrow \gamma^*(q',\mu')\Lambda(p',\nu) \) (with \( \gamma^*(q') \rightarrow l^-(k)l^+(k') \)) will be investigated. It is treated in full analogy to the case of a pion beam [33]. Mandelstam \( s = (p + q)^2 \) as well as the photon virtuality \( Q^2 = (k + k')^2 \) are considered to be large but the time-like analogue of Bjorken-x:

\[ \tau = \frac{Q^2}{s - m^2} \]  

(33)

is assumed to be small. The skewness is related to \( \tau \) analogously to eq. (2) by

\[ \xi = \frac{\tau - \tau}{2 - \tau}. \]  

(34)
At large values of $\tau$ respective large values of $Q^2$ and fixed $s$ only large $-t$ contribute since $-t_0$ (see (3) which also holds for the Drell-Yan process) becomes large. For $-t$ larger than about 1 GeV$^2$ the GPDs are not well known. They are merely extrapolations from a region of smaller $-t$ [4,5]. Assuming factorization the Drell-Yan amplitudes $M_{\mu'\nu',0\nu}$ are expressed as convolutions of hard subprocess amplitudes and the same proton-$\Lambda$ transition GPDs as for electroproduction of kaons. The subprocess amplitudes are the $\hat{s}-\hat{u}$ crossed electroproduction subprocess amplitudes (6)

$$H^{K^-\to\gamma^*}(\hat{s},\hat{u}) = -H^{K^+\to\gamma^*}(\hat{u},\hat{s})$$

(35)

where $\hat{s}$ and $\hat{u}$ denote the subprocess Mandelstam variables. The replacement $\hat{s} \to \hat{u}$ is equivalent to replacing $\xi$ by $-\xi$ and taking the complex conjugated amplitude. Thus, the calculation of the Drell-Yan process is analogous to that of electroproduction. The only difference is that in the pole residue (24) the time-like form factor of the kaon (28) instead of the space-like one (27) is to be used.

As for electroproduction there are four partial cross sections which are defined analogously to electroproduction. The four-fold differential cross section for $K^-p \to l^-l^+\Lambda$ reads [33]

$$\frac{d\sigma}{dt\ dQ^2 d\cos \theta \ d\phi} =$$

$$\frac{3}{8\pi} \left\{ \sin^2 \theta \ \frac{d\sigma_L}{dt\ dQ^2} + \frac{1}{2} (1 + \cos^2 \theta) \ \frac{d\sigma_T}{dt\ dQ^2} + \frac{1}{\sqrt{2}} \sin (2\theta) \ \frac{d\sigma_{LT}}{dt\ dQ^2} + \sin^2 \theta \cos (2\phi) \ \frac{d\sigma_{TT}}{dt\ dQ^2} \right\}. $$

(36)

The azimuthal angle between the lepton and the hadron plane is denoted by $\phi$ while $\theta$ is the decay angle in the center of mass frame.

The longitudinal cross section which is defined by (see also [40])

$$\frac{d\sigma_L}{dt\ dQ^2} = \kappa \sum_{\nu'} |M_{\nu'\nu,0\nu}|^2$$

(37)

where the normalization factor reads

$$\kappa = \frac{\alpha_{em}}{48\pi^2} \frac{\tau^2}{Q^6},$$

(38)

are shown in fig. 4 at $s = 30$ GeV$^2$ and at $Q^2 = 4$ GeV$^2$ and integrated upon $t$. The longitudinal cross section reveals a deeper forward dip than in the space-like region. Such a dip is not seen in the case of the pion. The reason for this distinction is that the meson-pole term dominantly feeds the helicity-flip amplitude which is of course suppressed for $t \to t_0$. However, in the case of the pion and for $Q^2/s \ll 1$, this effect is hidden by a strong non-flip amplitude. For $Q^2/s \ll 1$ $t_0$ is small and, hence, $1/(t - m_\pi^2)$ becomes very large for $t \to t_0$. This can be seen from fig. 5 where the absolute values of the pole contributions to the longitudinal helicity flip and non-flip amplitudes are displayed at $s = 30$ GeV$^2$ and $Q^2 = 4$ GeV$^2$. For larger values of $Q^2/s$ $-t_0$ becomes large and one is far away from the poles in both cases. In contrast to the case of the pion [12,33,40] the results for the longitudinal cross section of the kaon-induced Drell-Yan process is not very different from a leading-twist calculation.

The transverse cross section, defined by [33]

$$\frac{d\sigma_T}{dt\ dQ^2} = \kappa \sum_{\mu = \pm 1, \nu'} |M_{\mu'\nu,0\nu}|^2$$

(39)
is extremely small, about $0.1 \text{ pb/GeV}$.

The transverse-transverse interference term is the longitudinal-transverse interference cross section. It comes from the GPD $H$ is responsible for that. The by far dominant contribution to the transverse cross section comes from the GPD $\tilde{H}$ evaluated as a one-bose exchange, is not negligible. Also shown in fig. 6 is the longitudinal-transverse interference cross section. It is very small. The transverse-interference term is extremely small, about $0.1 \text{ pb/GeV}^2$, and therefore not displayed in the figure. The definitions of the interference cross sections can be found in [33].

Despite the fairly large range of the photon virtualities considered here evolution is still a minor effect. For the longitudinal cross section this is so because the strong kaon pole term evaluated as a one-bose exchange is still not subject to evolution. The evolution of the transversity GPDs is a small effect as is discussed in sect. 2.1. Therefore, the longitudinal cross section does not change much for $Q^2$ in the range of interest. For instance, it decreases by about 8% at $Q^2 = 14 \text{ GeV}^2$ if the evolution of the transversity GPDs is taken into account as compared to the cross section obtained without evolution. Of course, for very large $Q^2$ evolution matters — the transversity GPDs evolve to zero.

One has to be aware of the excitation of the charmonium states which appear as sharp, narrow spikes in the Drell-Yan cross sections for photon virtualities near the masses of the $J/\psi$ or the $\Psi(2S)$ [41]. The spread of the beam momentum will however widen the peaks considerably. One may also detect the $J/\psi$ directly, i.e. measure the cross section of the process $K^- p \to J/\psi n$. The above calculation of the exclusive Drell-Yan process allows for an estimate of the electromagnetic contribution to this cross section. Such an estimate is beyond the scope of the present work.

5 Summary

In this article the hard exclusive electroproduction of kaons as well as the crossed process, the kaon induced exclusive Drell-Yan process, have been investigated within the handbag approach. As for the analogous processes involving pions the transversity GPDs play an important role. Their use goes along with a twist-3 meson wave function which is applied in Wandzura-Wilczek approximation. It would be interesting to go beyond this approximation and to include both the two- and three-body twist-3 contributions. In wide-angle photoproduction of pions at least these contributions play an important role [42]. In contrast to previous studies of hard processes involving pseudoscalar mesons the evolution of the transversity GPDs is taken into account. It turns out, however, that this evolution effect is small in the range of photon virtualities of interest currently.

Predictions for the various cross sections are given and compared to the available data. Fair agreement with experiment [9–11] is observed. It is to be stressed that the phenomenological input as the GPDs as well as the parameters determining the kaon wave function and the kaon-pole contribution, is taken from other sources. The only exception is the parameter $A_{pA}$ in eq. (26) which is adjusted to the $kA$ electroproduction data. At present there is no significant signal of contributions from sea quarks neither to $\pi^+$ nor to $K^+$ production. This situation may change with the advent of more and better data on these processes as, for instance, can be expected from experiments performed at the upgraded JLab. Nothing is known yet on the GPDs $\tilde{E}, \tilde{H}T$ and $\tilde{E}T$ for sea quarks. Only the forward limit of $\tilde{H}$ for sea quarks, i.e. the combination $-2(2\tilde{H}u - \tilde{H}d - \tilde{H}s)/\sqrt{6}$ can be estimated. According to the DSSV polarized parton densities [18], it is small and has a zero at $x \simeq 0.18$. Combined with a Regge-like profile function as in (10) one finds that the contribution from this GPD to the longitudinal electroproduction cross section is small well within the uncertainties of the theoretical predictions.

Appendix A. Solving the evolution equation of the transversity GPDs

The evolution of the quark transversity GPDs with the scale, $\mu^2$, is controled by the integro-differential equation

$$
\frac{d}{d \ln \mu^2} K_T(x, \xi, t, \mu^2) = -\frac{1}{2} \int_{-1}^{1} dy P(x, y, \xi) K_T(y, \xi, t, \mu^2)
$$

(A.1)
The variables $x$ following Vinnikov [21] logarithmic grids for transversity GPDs evolve independently from each other. In agreement with the corresponding gluon GPDs, quark and gluon GPDs evolve independently from each other. For quarks, the evolution kernel reads [24,43]

$$P(x, y, \xi) = \frac{\alpha_s C_F}{2\pi} \left\{ \left[ \frac{\xi + \theta(x, \xi)}{\xi} \right] \frac{\theta(x, y) - \theta(\xi, x)\theta(y, -x)}{-1} \right\} +$$

$$+ \left[ \frac{\xi - x}{\xi - y} \right] \frac{\theta(x, \xi)\theta(y, -x) - \theta(x, \xi)\theta(y, -x)}{-1} \right\} +$$

$$+ \frac{1}{2} \delta(x - y) \right\} . \tag{A.2}$$

The symbol $\left[ \ldots \right]_+$ denotes the usual plus prescription and $C_F = 4/3$. It is important to note that there is no mixing between the gluon and quark transversity GPDs evolve independently from each other. Following Vinnikov [21] logarithmic grids for $x$ and the scale $\mu^2$ are introduced

$$x_i = \delta \left( 1 - e^{-\gamma(i-2n)} \right) \text{ for } -1 \leq x \leq 0 \quad 0 \leq i \leq 2n$$

$$x_i = -\delta \left( 1 - e^{\gamma(i-2n)} \right) \text{ for } 0 < x \leq 1 \quad 2n < i \leq 4n. \tag{A.3}$$

The variables $\delta$ and $\gamma$ are defined by

$$\gamma = \frac{1}{n} \ln \frac{1 - \xi}{\xi}, \quad \delta = \frac{\xi^2}{1 - 2\xi} \tag{A.4}$$

and, for the evolution from $\mu_0$ to $\mu_1$,

$$u_j = u_0 + j\delta \quad d = \frac{u_1 - u_0}{m} \quad 0 \leq j \leq m - 1 \tag{A.5}$$

where $u = \ln \mu^2$. With the help of the $x$-grid the integral in (A.1) is replaced by a sum using Simpson’s rule.

This transforms the integro-differential equation into a system of coupled differential equations. This system is subsequently solved with the help of the 4th-order Runge-Kutta procedure. The numerical method is quite stable and for $n = 20$ and $m = 1(2)$ (for $\mu^2 = 10(20) \text{ GeV}^2$) good results have been obtained. A numerical code for calculating the evolution of the quark transversity GPDs can be obtained from the author on request.

**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Author’s comment: All data generated during this study are contained in this published article.]

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