Searching for CP violation through two-dimensional angular distributions in four-body decays of bottom and charmed baryons

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It is proposed to search for CP violation through the two-fold angular distributions in the four-body decays of bottom and charmed hadrons. The two polar angles in the two-fold angular distributions are correlated, to which the interferences of intermediate resonances are one important origin. These interferences will leave tracks in the two-fold angular distributions, with which the CP violation can be studied. Special attention is paid to the case when all the intermediate resonances are different, which is unique to four-body decays. It is suggested to look for CP violation in four-body decays such as \( \Lambda_b^0 \to p\pi^-\pi^+\pi^- \) through the analysis of the two-fold angular distributions. The method proposed in this paper is also widely applicable to other four-body decays processes.

I. INTRODUCTION

CP violation (CPV), as one of the cornerstones for the explanation of the matter-antimatter asymmetry of the Universe through baryogenesis [1], is of great importance in the baryon sector both theoretically and experimentally. Searches for CPV have been carried out in the baryonic decay channels of CP-conjugate pairs, through direct measurements of the differences between the full partial decay widths [2–5], the decay widths corresponding to part of the phase space [6, 7], the Triple-Product Asymmetries (TPAs) [8–10], and the decay asymmetry parameters [11–13], or through other indirect techniques such as the amplitude analysis [14] and the energy test method [10], etc.. Puzzling enough, although it was first discovered almost sixty years ago in the neutral-kaon decays [15], CPV has never been observed in the baryon sector, despite that so many efforts have been paid.

Contrary to the baryonic cases, CPV has been observed in the decays of \( K \) and \( D \) mesons [15, 16], and in plenty of decay channels of \( B \) mesons [17–20], some of which have quite large CP asymmetries (CPAs). One such kind of examples are the large regional CPAs observed in parts of the phase space for some three-body decays of \( B \) meson [21–24], in which the interfering effect between the nearby resonances (or the interference of resonance with the non-resonant part) is one important mechanism for the generating of large regional CPAs [25–29]. Similar to the \( B \) meson cases, multi-body decays of bottom and charmed baryons are usually dominated by various intermediate resonances. For example, \( \Lambda_b^0 \to p\pi^-\pi^+\pi^- \) is dominated by \( N(1520)^0 \) and \( \rho(770)^0 \) via “branching” decays \( \Lambda_b^0 \to N(1520)^0(\to p\pi^-)\rho(770)^0(\to \pi^+\pi^-) \) [6]. The prevalent interferences between the amplitudes corresponding to various intermediate resonances are expected to leave tracks in the angular distributions of the final particles, in which the CPV is potentially hidden.

Based on the Cabibbo-Kobayashi-Maskawa mechanism of the Standard Model [30, 31], the CPAs in some of the bottom and charmed baryon decays are expected to have similar magnitudes with those in the bottom and charmed meson decays, respectively [32–34]. However, the baryon decays usually suffer from substantial lower statistics comparing with the meson cases. Hence one of the best strategies for the CPV searching in the baryon sector is to start with the decay channels with the largest statistics. The four-body decays such as \( \Lambda_b^0 \to pK^-\pi^+\pi^- \) and \( \Lambda_b^0 \to p\pi^-\pi^+\pi^- \) are one important type that fulfill this criteria.

Although CPAs corresponding to angular distributions of final particles have been extensively investigated for the heavy baryon decays in the literature [34–47], for the four-body decay cases, however, this area is largely unexplored. As one kind of angular distribution correlated CPAs, the TPA induced CPAs have played an unique role in heavy meson and baryon decays [36, 37, 48–53]. In fact, the TPA induced CPAs are especially suitable to be studied in four-body decays of heavy baryons [34, 41]. Although it is in principle possible to study TPA and corresponding CPAs in two- or three-body decays of heavy baryons [36, 37, 51, 52], this requires, however, that the mother baryons to be polarized, which is not possible in the current stage [54–56]. In this paper, CPV corresponding to angular distributions, which complements to that corresponding to TPA, is investigated for the aforementioned four-body decays with the largest statistics.

II. THE TWO-FOLD ANGULAR DISTRIBUTIONS FOR FOUR-BODY DECAYS

To put on a more general ground, let us first consider a four-body decay process of the branching form \( H \to a(\to 12)b(\to 34) \), where \( H \) is the mother hadron, either baryon or meson, bottom or charmed, \( a \) and \( b \) are the intermediate resonances, and 1234 label the four final particles. The kinematical variables are illustrated in FIG. 1, where the Jackson convention is adopted for the reference frames [57]. In the center-of mass (c. m.) frame of \( H \), the \( z \) axis is conveniently chosen as the normal to the production plane of \( H \). The polar and azimuthal angles of the momentum of \( a \) in this frame is denoted as \( \theta \) and \( \phi \). In the c. m. frame of \( a(b) \), the \( z(a(b)) \) axis is chosen along the direction of the momentum of \( H \). The
FIG. 1. Illustration of the kinematic variables for the four-body decay $H \to a(\to 12)b(\to 34)$. The reference frames are defined according to the Jackson convention. Note that $\theta$ and $\phi$ are defined in the c. m. frame of $H$, while $\theta_{a(b)}$ and $\phi_{a(b)}$ are defined in the c. m. frame of $a(b)$. The angle $\varphi$ can be defined in any of the three frames. The invariant mass squared of 12 and 34, $s_{12}$ and $s_{34}$, which are Lorentz-invariant, are not shown in the figure.

$x_a$ and $x_b$ ($y_a$ and $y_b$) axes will be chosen (anti-)aligned with each other. The polar and the azimuthal angles of the particle 1(3) in the c. m. frame of $a(b)$ will be denoted as $\theta_{a(b)}$ and $\phi_{a(b)}$, respectively. The angle from the decay plane of the $a \to 12$ to that of the $b \to 34$, which is related to TPA, will be denoted as $\varphi^1$. One can find from FIG. 1 the relation $\varphi = 2\pi - \phi_a - \phi_b$.

The spin-averaged decay amplitude squared for unpolarized $H$, after integrating out the azimuthal angles $\phi_a$ and $\phi_b$, will take the form $^2$,

$$
\int |A|^2 d\varphi \propto \sum_{jl} \Gamma_{jl}(s_{12}, s_{34}) P_j(c_{\theta_a}) P_l(c_{\theta_b})
$$

(1)

where $P_j$ are the Legendre polynomials with $c_{\theta_a} \equiv \cos \theta_{a(b)}$, and

$$
\Gamma_{jl} = \sum_{a,a',b,b'} \langle \lambda_a | \lambda_{a'} \rangle \langle \lambda_b | \lambda_{b'} \rangle \langle \lambda_{a'} | \lambda_{b'} \rangle \langle \lambda_a | \lambda_b \rangle 
$$

(2)

$\mathcal{I}$’s are the reciprocals of the Breit-Wigner propagators, taking the form $\mathcal{I}_{a} = s_{a} - m_{a}^2 + i m_{a} \Gamma_{a}$ and $\mathcal{I}_{b} = s_{b} - m_{b}^2 + i m_{b} \Gamma_{b}$ with $s_{12}$ and $s_{34}$ the invariant mass squared of the 12 and the 34 systems, and

$$
\mathcal{W}_{j}(a.a') = \sum_{\sigma_p} (\sigma s_a + \rho s_b) \langle \sigma \sigma_a | j0 \rangle 
$$

(3)

$$
\mathcal{G}_j^{(aa')} = \sum_{\lambda_1 \lambda_2} (-)^{s_a - \lambda_{12}} \langle \sigma \lambda_{12} | j0 \rangle \mathcal{F}_{\lambda_1 \lambda_2}^{a \rightarrow 12} \mathcal{F}_{\lambda_1 \lambda_2}^{b \rightarrow 34}
$$

(4)

$$
\mathcal{G}_j^{(bb')} = \sum_{\lambda_3 \lambda_4} (-)^{s_b - \lambda_{34}} \langle \sigma \lambda_{34} | j0 \rangle \mathcal{F}_{\lambda_3 \lambda_4}^{b \rightarrow 34} \mathcal{F}_{\lambda_3 \lambda_4}^{a \rightarrow 12}
$$

(5)

with all the $\mathcal{F}$’s being the decay amplitudes in the helicity form, “$\cdots | \cdots \rangle$” being the Clebsch-Gordan coefficients, $s_{a(b)}$ and $s_{b(b)}$ being the spins of $a(b)$ and $b(b)$, $\sigma$ and $\rho$ being the helicities of $a(a)$ and $b(b)$ in the c. m. frame of $H$, $\lambda_i$ ($i = 1, \cdots, 4$) being the helicity index of particle $i$ (defined in the c. m. frame of $a$ or $b$), $\lambda_i' = \lambda_i - \lambda_i$. The summations over $a, a', b,$ and $b'$ indicate that there may be more than one resonances either for $a$ or $b$. Note that there are four kinematic variables left, which can be chosen as the invariant mass squared of 12 and 34, $s_{12}$ and $s_{34}$, the polar angles $\theta_a$ and $\theta_b$. When $s_{12}$ and $s_{34}$ are set fixed in certain ranges, we get a two-dimensional phase space (2DPS) expanded by $c_{\theta_a}$ and $c_{\theta_b}$, which is illustrated in FIG. 2.

The following remarks to the two-fold angular distributions in Eq. (1) are in order:

i) The two angles $\theta_a$ and $\theta_b$ are correlated, in the sense that $\Gamma_{jl}$ cannot be factorized into the form $\Gamma_{jl} = \xi_j \eta_l$. If $\Gamma_{jl}$ were factorized, that would mean that one can simply integrate out either $\theta_a$ or $\theta_b$ without lost of any information, so that the four-body decay will reduced effectively to three-body decays. However, this is not true, because that the amplitudes are entangled in Eq. (2). Among the origins for the entanglement of $j$ and $l$ in Eq. (2), the genuine interferences (GIs) between the amplitudes of

FIG. 2. Illustration of the 2DPS expanded by $c_{\theta_a}$ and $c_{\theta_b}$.

\footnotesize

1 For the current situation, the TPA constructed from the momenta of the final particles can be defined as $A_T \equiv N(C_T > 0) - N(C_T < 0)$, with $C_T \equiv (p_1 \times p_2) \cdot p_3$, where $p_1$ represent the momentum of particle $j$ in the c. m. of $H$. Form FIG. 1 one can see that the above definition is equivalent to $A_T = N(\sin \varphi > 0) - N(\sin \varphi < 0)$. Consequently, the aforementioned TPA $A_T$ is sometimes also called as the up-down asymmetry is some references as the decay plane of the 12 system divide the space into the “up” part ($\sin \varphi > 0$) and the “down” part ($\sin \varphi < 0$) [53, 58].

2 The expression of the spin-averaged decay amplitude squared for polarized $H$ without integrating out the azimuthal angles are presented in Appendix A.
$H \to a(\to 12)b(\to 34)$ and $H \to a'(\to 12)b'(\to 34)$, where $a \neq a'$ and $b \neq b'$, are the most special ones. The intermediate resonances of the two amplitudes are all different, hence they are unique for the four-body decays and can not be accounted for by any two- or three-body decays. In other word, GI is a two-fold interference, because it contains both the $a \leftrightarrow a'$ and $b \leftrightarrow b'$ interferences. Anyway, it would be interesting to study the correlation between $\theta_a$ and $\theta_b$ both theoretically and experimentally.

ii) From the Clebsh-Gordan coefficients in Eq. (4) one can see that $j$ takes integer values from 0 to $2 \max(a, s_a)$, where $a$ runs over all the possible resonances. Similarly, $l$ takes integer values from 0 to $2 \max(b, s_b)$. Note that the allowed values of $j$ and $l$ are determined by the spin of the resonances through the Clebsh-Gordan coefficients, not directly by the angular momentum between the final particles 1 and 2 (or 3 and 4). Though they equal with each other (the spin of the resonances and the angular momentum of 12 or 34) only when all the final particles are spin-0 ones.

iii) In principle, $CP$ can show up in any of the $\Gamma_{jl}$'s through the difference between $\Gamma_{jl}$ and its $CP$ conjugate $\Gamma_{jl}^\ast$. However, it is more likely that $CP$ shows in those $\Gamma_{jl}$'s which contain the resonant-interfering terms. This is because that there may be, between the amplitudes corresponding to different resonances, non-perturbative strong phase differences, which are crucial for the generating of large $CP$As. Consequently, it is important to find the rules whether there are resonant-interfering terms in each $\Gamma_{jl}$ or not. To this end, notice that the parity conservation, as well as the properties of the Clebsh-Gordan coefficients, imply from Eqs. (4) and (5) that $\Pi_j \Pi_{a'}(-)^j = 1$ and $\Pi_j \Pi_{b'}(-)^j = 1$, where $\Pi_j$'s represent the parities of the corresponding particles. It follows that there are selection rules (SRs) for the presence of (non-)interference terms in $\Gamma_{jl}$, which fall into four situations: (1) The non-interfering terms $(a = a', b = b')$ for $H \to ab$ will present in $\Gamma_{jl}$ if $0 \leq j \leq 2s_a$ and $0 \leq l \leq 2s_b$, and $j$ and $l$ are even. (2) The singly interfering terms between $H \to ab$ and $H \to a'b'$ will present in $\Gamma_{jl}$ if $|s_a - s_{a'}| \leq j \leq s_a + s_{a'}$, $\Pi_a \Pi_{a'}(-)^j$ is positive, $0 \leq l \leq 2s_b$, and $l$ is even. (3) The singly interfering terms between $H \to ab$ and $H \to ab'$ will present in $\Gamma_{jl}$ if $|s_b - s_{b'}| \leq l \leq s_b + s_{b'}$, $\Pi_b \Pi_{b'}(-)^l$ is positive, $0 \leq j \leq s_a$, and $j$ is even. (4) The GI terms will present in $\Gamma_{jl}$ if $|s_a - s_{a'}| \leq j \leq s_a + s_{a'}$, and $|s_b - s_{b'}| \leq l \leq s_b + s_{b'}$, and $\Pi_a \Pi_{a'}(-)^j$ and $\Pi_b \Pi_{b'}(-)^l$ are positive.

iv) One special case is when the intermediate resonances have opposite parities in each of the two decay branches. For example, suppose that there are two resonances at each decay branch of $a$ and $b$, which are $R_{a}^{(\pm)}, R_{b}^{(-)}$ and $R_{a}^{(-)}, R_{b}^{(\pm)}$, respectively, with opposite parities indicated in the superscripts. According to the SRs, the non-interfering terms, the singly interfering terms, and the GI terms will show up in $\Gamma_{jl}$ when both $j$ and $l$ even, $j$ even $l$ odd or $j$ odd $l$ even, and both $j$ and $l$ odd, respectively. Hence they are all well separated in $\Gamma_{jl}$'s.

v) Each of the dynamical parameters $\Gamma_{jl}$'s in Eq. (1) represents a degree of freedom for angular distributions, which can be expressed as

$$\Gamma_{jl} \propto \int \left| A_{jl} \right|^2 P_j(c_{\theta_a})P_l(c_{\theta_b})dc_{\theta_a}dc_{\theta_b}. \quad (6)$$

Consequently, $\Gamma_{jl}$ can be determined experimentally according to

$$\Gamma_{jl} \propto \sum_{i=1}^{N} P_j(c_{\theta_{ai}})P_l(c_{\theta_{bi}}), \quad (7)$$

where $N$ is the total event yields, and $i$ labels each event. The relative $jl$-th moment for the expansion of Eq. (1) is then defined as

$$A_{jl} \equiv \frac{\Gamma_{jl}}{\Gamma_{00}}. \quad (8)$$

The $CP$As corresponding to $A_{jl}$ can then be obtained according to

$$A_{CP}^{jl} \equiv \frac{1}{2}(A_{jl} - \overline{A_{jl}}), \quad (9)$$

for $j \neq 0$ and/or $l \neq 0$, where $\overline{A_{jl}}$ is the same with $A_{CP}^{jl}$ but for the $CP$ conjugate process $3$. vi) Although very clean from the theoretical side, the disadvantage of the $CPA$ defined in Eq. (9) is also obvious: the events located in different positions of the 2DPS are weighted differently, which complexes the experimental analysis of uncertainties. To avoid this, we use an alternative definition for $CPA$, which weights all the events equally throughout the whole 2DPS (up to a signature). To achieve this, one first notice that the contributions of each event to $\Gamma_{jl}$ are different in signature because of the factor $P_jP_l$ in Eq. (7). Hence one can introduce a twisted $\Gamma_{jl}$ according to

$$\tilde{\Gamma}_{jl} \propto \sum_{i=1}^{N} \text{sgn}(P_j(c_{\theta_{ai}})P_l(c_{\theta_{bi}})). \quad (10)$$

3 $CPAs$ are alternatively defined as $A_{CP}^{(jl)} \equiv \frac{\Gamma_{jl} - \overline{\Gamma_{jl}}}{\Gamma_{00} + \overline{\Gamma_{00}}}$. For $j = 0 = l$, it reduces to the direct $CPA$ defined by the decay width: $A_{CP}^{(00)} = A_{CP}^{\text{dir}}$, as one can see that $\Gamma_{00} = N$, and $\overline{\Gamma_{00}} = \overline{N}$. 


This is equivalent to dividing the 2DPS into \((j+1) \times (l+1)\) bins which are boarded by the zero lines of the Legendre polynomials \(P_j\) and \(P_l\), and assign the event yields of each bin by a proper signature. Eq. (10) is then equivalent to

\[
\tilde{\Gamma}_{jl} \propto \sum_j \sum_{i_1=0}^j \sum_{i_1=0}^l (-1)^{i_1+i_l} N_{i_1 i_l}, \tag{11}
\]

where \(N_{i_1 i_l}\) is the event yields of the bin \(i_1 i_l\). The twisted \(jl\)-th relative moment takes the form

\[
\tilde{A}^{jl} = \frac{\tilde{\Gamma}_{jl}}{\tilde{\Gamma}_{00}}. \tag{12}
\]

The corresponding CPA is then defined as

\[
\tilde{A}^{jl}_{\text{CP}} = \frac{1}{2} (\tilde{A}^{jl} - \overline{\tilde{A}^{jl}}). \tag{13}
\]

vii) Similar analysis is also applicable to the four-body sequential decays \(H \to a_1a, a \to 2b, b \to 34\). It turns out that the SRFs for the presence of different type of terms for the sequential decays are exactly the same with those for the branching ones. A detailed analysis of this type of decays is presented in the appendix.

III. APPLICATIONS

As an application, we propose to look for CPV in \(\Lambda^0_b \to p\pi^+\pi^-\pi^-\) through the analysis of the two-fold angular distributions in the 2DPS. Previous experiments indicate that this decay channel is dominated by quasi-two-body decay \(\Lambda^0_b \to N(1520)^0\rho(770)^0\). Hence there potentially are the \(f_0(500) - \rho(770)^0\) and \(N(1440)^0 - N(1520)^0\) interferences. Moreover, there can also be the GI terms such as the interference between \(\Lambda^0_b \to N(1520)^0 f_0(500)\) and \(\Lambda^0_b \to N(1440)^0 \rho(770)^0\). The spin-parity of these two pairs of resonances are \(0^+ - 1^-\) and \((\frac{1}{2})^+ - (\frac{1}{2})^-\), respectively. According to the SRFs, both \(j\) and \(l\) can take values 0, 1, 2. The non-interfering terms, the singly interfering terms, and the GI terms are all perfectly separated in \(\Gamma_{jl}\) because of the opposite parities of the two pairs of resonances. The non-interfering terms will show up in \(\Gamma_{jl}\) for \(j = 00, 02, 20, 22\); the singly interfering terms will show up in \(\Gamma_{jl}\) for \(j = 01, 10, 12, 21\); while the GI terms will show up in and only in \(\Gamma_{11}\). This can be represented in a matrix form as

\[
(\Gamma_{jl}) \sim \begin{pmatrix}
(N_{1440} N_{1520}) |f|^2 & (N_{1440} N_{1520}) |f|^2 \\
(f_0 |N_{1440}|^2)^2 & (f_0 |N_{1520}|^2)^2 \\
(N_{1440} N_{1520}) |\rho|^2 & (N_{1440} N_{1520}) |\rho|^2
\end{pmatrix}.
\tag{14}
\]

CPV induced by the interference of the intermediate resonances can be embedded in any of the aforementioned five \(\Gamma_{jl}\)’s for \(j = 01, 10, 12, 21\), and 11, which can be measured according to Eqs. (8) and (9), or alternatively, according to Eqs. (12) and (13).

The bin divisions for the measurements of all the nine \(\tilde{A}^{jl}\)’s are illustrated in FIG. 3. Since only \(\Gamma_{11}\) contains the GI terms for the four-body decays, it deserves special attentions. To measure it, one first needs to divide the 2DPS into four bins, which is illustrated as the one in the center of FIG. 3. The 11-th relative moment \(\tilde{A}^{11}\), which can also be called as the two-fold forward-backward asymmetry (TFFBA), is then measured according to

\[
\tilde{A}^{11} = \frac{(N_{11} - N_{21} + N_{22} - N_{12})}{N}, \tag{15}
\]

where the subscripts in the event yields denote the four quadrants according to the bin division shown in the center of FIG. 3. One immediately obtains the CPA \(\tilde{A}^{11}_{\text{CP}}\) according to Eq. (13) with \(\tilde{A}^{11}\) and \(\overline{\tilde{A}^{11}}\) at hand.

Similar 2DPS analysis can be performed in sequential decays. For example, the four-body decay \(\Lambda^0_b \to p\pi^+\pi^-\pi^-\) through the decay chains \(\Lambda^0_b \to p a_1(1260)^0\), \(a_1(1260)^- \to \rho(770)^0\pi^-, \rho(770)^0 \to \pi^+\pi^-\), and \(\Lambda^0_b \to \rho(770)^0\pi^+\pi^-\pi^-\)
CPV corresponding to the two-fold angular distributions in four-body decays is analyzed. The interferences of intermediate resonances may generate large CPAs corresponding to the two-fold angular distributions, which have never been studied experimentally. We propose to search for CPV through the analysis of the two-fold angular distributions in the decay channels such as $A^0 \rightarrow p\pi^+\pi^-\pi^+$ and $A^+ \rightarrow A^0 K^0\pi^+$, this method is quite general, which can be used in a wide class of four-body decays of bottom and charmed hadrons, and are not limited to CPV studies.

IV. SUMMARY

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Appendix A: decay amplitude squared without integrating out $\varphi$

We present here the decay amplitude squared without integrating out $\varphi$, and without the assumption of unpolarized $H$, which reads

$$|\mathcal{A}|^2 \propto \sum \gamma^{jl}_{a^i_a^j} \sigma_{a^i_a^j} \sigma_{a^i_a^j} d^j_{a^i_a^j} d^j_{a^i_a^j} \langle \theta_a \rangle \langle \theta_b \rangle e^{i\sigma_{a^i_a^j} \phi_a} e^{i\sigma_{a^i_a^j} \phi_b},$$

where

$$\sigma_j = \sum_{a^i_a^j} \sigma_{a^i_a^j} \sigma_{a^i_a^j}, \quad j \not= 1, \quad \sigma_j = \sigma_j^l = \sigma_j^r,$$

and

$$\gamma^{jl}_{a^i_a^j} = \frac{\langle \alpha \rangle \langle \beta \rangle I_{a^i_a^j} \phi^*_{b} \phi^*_{b}}{I_{a^i_a^j}},$$

with

$$\sigma_{a^i_a^j} = \langle s_a - \sigma_n \sigma_n \rangle \langle s_{a^i_a^j} \rangle \langle s_n \rangle \langle \sigma_{n} \rangle \langle \sigma_{n} \rangle,$$

where $\sigma_{a^i_a^j}$ describes the polarization of $H$ with $\sigma_{a^i_a^j} = \sigma_{a^i_a^j} - \sigma_{a^i_a^j}$. Note that we have set $\phi = 0$ in Eq. (A1). One can also set either $\varphi_a = 0$ or $\varphi_b = 0$, which we did not do here.

Concerning the polarization of $H$, the only relevant $H$ in practice are the spin-half baryons such as $\Lambda_b$. It is well known that $H$ can only polarize along the normal to the production plane due to the constraint of the parity symmetry in the producing process. The factor $P_{\sigma_{a^i_a^j}^l, \sigma_{a^i_a^j}^r}^{l_0}(\theta)$ then takes the form

$$P(\theta) = \frac{1}{2} \left( 1 + \cos \theta - \sin \theta \cos \theta \right) \frac{1}{P_z},$$

with $P_z$ the polarization of $H$ along the $z$ axis. For unpolarized $H$, $P_z = 0$, as observed for the $\Lambda_b$ case in $pp$ collision by LHCb and CMS at a few percent level [54–56], $P_{\sigma_{a^i_a^j}^l, \sigma_{a^i_a^j}^r}(\theta)$ reduce to $\delta_{a^i_a^j, \sigma_{a^i_a^j}^l} / 2$, so that $\sigma_{a^i_a^j} = \sigma_{a^i_a^j}$. The last two factors in Eq. (A1) then reduce to $e^{-i\sigma_{a^i_a^j} \varphi}$. A simultaneous analysis of the correlation of $\theta_a$, $\theta_b$, and $\varphi$ may give us deeper insights in CPV. Take again the decay $A^0 \rightarrow p\pi^+\pi^-\pi^-$ as an example, according to the SRs, the presence of the GI will also generate terms (with $j = 1$ = $l$) which are proportional to $d^j_{a^i_a^j}(\theta_a) d^j_{a^i_a^j}(\theta_b) \cos \varphi \sim \sin \theta_a \sin \theta_b \sin \varphi$ and $d^j_{a^i_a^j}(\theta_a) d^j_{a^i_a^j}(\theta_b) \sin \varphi \sim \sin \theta_a \sin \theta_b \cos \varphi$, respectively. The simultaneous analysis can be performed by fitting the aforementioned two factors.

Besides a simultaneous analysis, an analysis of solely the $\varphi$ dependence is simpler. For the two aforementioned
GI terms, the former has to do with TPA, and the related CPV can be studied accordingly. While the latter can be described by the Left-Right Asymmetry (LRA):

$$A^{LR} = \frac{N_L - N_R}{N_L + N_R},$$

(A5)

where $N_{R/L}$ are the event yields defined by $N_{L/R} \equiv N(\cos \phi \gtrless 0)$. The corresponding CPV can be studied through the LRA induced CP asymmetry

$$A^{LR}_{CP} = \frac{1}{2}(A^{LR} - \overline{A}^{LR}),$$

(A6)

where $\overline{A}^{LR}$ is the LRA of the CP conjugate process.

Appendix B: Two-dimensional angular distributions for $H \rightarrow 1\alpha(\rightarrow 2b(\rightarrow 34))$

For completeness, we present here a brief discussion on four-body sequential decays of the form $H \rightarrow 1\alpha, a \rightarrow 2b, b \rightarrow 34$. The spin-averaged decay amplitude squared can be expressed as

$$\int |\hat{A}|^2 d\phi \propto \sum_{jl} \hat{\Gamma}_{jl}(s_{234}, s_{34}) P_j(c_{\theta_a}) P_l(c_{\theta_b}),$$

(B1)

where $s_{234}$ are the invariant mass squared of the $234$ system, the angle $\theta_{a(b)}$ is the polar angle of the momentum of 2(3) in the c. m. frame of $a(b)$, where $z_a$ and $z_b$ are defined respectively in the c. m. frame of $a$ and $b$ in a similar manner with those in the main text, and

$$\hat{\Gamma}_{jl} = \sum_{a,a', b,b'} \frac{\hat{W}_j(a,a') \hat{G}^{(ab,a'b')}(l) \hat{G}^{(b'b)}(j)}{I_{a,b}^{*}I_{a',b'}^{*}},$$

(B2)

where $I_{a}^{(*)} = s_{234} - m_a^2 + im_a \Gamma_a(t)$ for now, and

$$\hat{W}_j(a,a') = \sum_{\lambda_{i,j}} (-)^{s_{a} - s_{a'}} (-)^{s_{a} - s_{a'}} \frac{i}{\lambda_{i,j}} f_{\lambda_{i,j}}^{H \rightarrow 1\alpha} f_{\lambda_{i,j}}^{H \rightarrow 1\alpha'},$$

(B3)

$$\hat{G}^{(ab,a'b')}_{jl} = \sum_{\lambda_{i,j}} (-)^{s_{a} - s_{a'} - s_{b} - s_{b'}} (-)^{s_{a} - s_{a'}} \frac{i}{\lambda_{i,j}} (s_{a} - \lambda_{j}) s_{a'} (\lambda_{j} - \lambda_{i}) |j0\rangle f_{\lambda_{i,j}}^{a \rightarrow 2b} f_{\lambda_{i,j}}^{a' \rightarrow 2b'}. $$

(B4)

Note that the azimuthal angles have also been integrated out.

Suppose that $a \rightarrow 2b$ and $b \rightarrow 34$ are strong processes, there comes the non-zeroness conditions for $\hat{G}^{(ab,a'b')}_{jl}$ and $\hat{G}^{(b'b)}_{jl}$ followed by the parity symmetry and the properties of the Clebsch-Gordan coefficients, which can be organized as i) $0 \leq j \leq 2 \max_{a}(s_{a}), 0 \leq l \leq 2 \max_{a}(s_{a})$ ii) $\Pi_{a} \Pi_{b} (-)^{j}$ is positive, iii) $\Pi_{a} \Pi_{a'} (-)^{j} \Pi_{b} \Pi_{b'} (-)^{j}$ is positive. One see that the conditions are in fact the same with the branching decay $H \rightarrow a(\rightarrow 1\beta)[b(\rightarrow 34)]$. Consequently, SRs for the presence of different kinds of terms in $\hat{\Gamma}_{jl}$ will also be exactly the same. The non-interfering terms ($a = a'$ and $b = b'$) will show up in $\hat{\Gamma}_{jl}$ if $0 \leq j \leq 2s_{a}$ and $0 \leq l \leq 2s_{b}$, and $j$ and $l$ even. The term $a \leftrightarrow b$ interfering term ($a \neq a'$ and $b \neq b'$) will show up in $\hat{\Gamma}_{jl}$ if $|s_{a} - s_{a'}| \leq j \leq s_{a} + s_{a'}, \Pi_{a} \Pi_{a'} (-)^{j}$ positive, $0 \leq l \leq 2s_{b}$, and $j$ and $l$ even. The $b \leftrightarrow b'$ interfering term ($b \neq b'$ and $a = a'$) will show up in $\hat{1}\Gamma_{jl}$ if $|s_{b} - s_{b'}| \leq l \leq s_{b} + s_{b'}$, $\Pi_{b} \Pi_{b'} (-)^{j}$ positive, $0 \leq j \leq 2s_{a}$, and $j$ even. The $a \leftrightarrow a'$ and $b \leftrightarrow b'$ interfering term ($a \neq a'$ and $b \neq b'$) will show up in $\hat{\Gamma}_{jl}$ if $|s_{a} - s_{a'}| \leq j \leq s_{a} + s_{a'}$ and $|s_{b} - s_{b'}| \leq l \leq s_{b} + s_{b'}$, and both $\Pi_{a} \Pi_{a'} (-)^{j}$ and $\Pi_{b} \Pi_{b'} (-)^{j}$ positive. Especially, if the parities of $a$ and $a'$ as well as $b$ and $b'$ are opposite, all the aforementioned terms will be again well separated in $\hat{\Gamma}_{jl}$.

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