BPS-like bound and thermodynamics of the charged BTZ black hole

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Abstract. The charged Bañados-Teitelboim-Zanelli (BTZ) black hole is plagued by several pathologies: a) Presence of divergent boundary terms in the action, hence of a divergent black hole mass; b) Once a finite, renormalized, mass $M$ is defined black hole states exist for arbitrarily negative values of $M$; c) There is no upper bound on the charge $Q$. We show that these pathological features are an artifact of the renormalization procedure. They can be completely removed by using an alternative renormalization scheme leading to a different definition $M_0$ of the black hole mass, which is the total energy inside the horizon. The new mass satisfies a BPS-like bound $M_0 \geq \frac{\pi}{2} Q^2$ and the heat capacity of the hole is positive. We also discuss the black hole thermodynamics that arises when $M_0$ is interpreted as the internal energy of the system. We show, using three independent approaches (black hole thermodynamics, Einstein equations, Euclidean action formulation) that $M_0$ satisfies the first law if a term describing the mechanical work done by the electrostatic pressure is introduced.

1 Introduction

The discovery of black hole solutions in three-dimensional (3D) anti de Sitter (AdS) spacetime by Bañados, Teitelboim and Zanelli (BTZ) [1, 2] (for a review see Ref. [3]) enhanced our understanding of black holes and also played a key role in recent developments in gravity, gauge and string theory. From the point of view of black hole physics, the main lesson is that black holes can be formed by a singularity of the causal structure and not necessarily by a curvature singularity. From the point of view of the AdS/CFT correspondence, the BTZ black hole is the simplest realization of a 3D bulk gravity configuration that can be described by dual thermal CFT states [4, 5]. One of the most striking successes of this correspondence is the exact computation of the Bekenstein-Hawking entropy of the BTZ black hole using the dual two-dimensional CFT [6].

It was immediately realized that the BTZ solution allows also for charged generalizations, i.e. charged black hole in 3D AdS spacetime [1, 7]. Differently from the uncharged...
case, these black holes have a power-law curvature singularity and share the causal structure with their higher-dimensional cousins (e.g. the charged Reissner-Nordstrom solution in 4D AdS space).

Naively, one could expect that the analogy between 3D and higher-dimensional charged black holes can be pushed forward to cover the main physical features of the black hole, such as the mass spectrum, thermodynamics etc. In particular, this analogy could be very useful for investigating in a simplified context peculiar features of the charged black holes, such as the existence of extremal black holes states of zero temperature and non-vanishing entropy.

However, this seems not to be the case, at least at first sight. The charged BTZ black hole is plagued by several pathologies, which make it rather different from its higher-dimensional counterparts. The origin of these pathologies is well-understood; it can be traced back to the logarithmic behavior of harmonic functions in two dimensions, which implies that the electrostatic potential of the charged BTZ black hole diverges asymptotically as $\ln r$. The consequences are: a) When we vary the action we get divergent boundary terms, i.e. we have a divergent black hole mass; b) Using a suitable renormalization procedure, we can define a finite mass $M$ for the solution, but we find that black hole state exist for arbitrarily negative values of $M$; c) At fixed $M$ there is no upper bound on the charge $Q$. A further, recently discovered, manifestation of this problematic behavior, is the fact that the entropy function approach do not work for the extremal charged BTZ black hole [8].

These features make the charged BTZ black hole drastically different from the charged solutions in four and higher dimensions. In the latter case the black hole mass satisfies a BPS bound $M \geq a^2Q^2$, which guarantees that the mass is positive definite and that for a given mass the charge is bounded from above. The existence of this bound is usually a consequence of the supersymmetry of the extremal black hole.

Recently, an alternative renormalization procedure leading to a finite value $M_0$ for the mass of the charged BTZ black hole, has been proposed [9, 10, 11]. Physically, $M_0$ is the total energy (gravitational and electromagnetic) inside the black hole outer horizon. Moreover, the identification of $M_0$ with the conserved charge associated with time-translations is very natural from the point of view of the AdS/CFT correspondence [9]. It allows to reproduce, using the dual CFT, the Bekenstein-Hawking entropy of the hole and to consider the charged BTZ geometry as a bridge between two AdS$_2$ geometries, a near horizon one (AdS$_2 \times S^1$) and an asymptotic one (linear dilaton AdS$_2$) [9, 10].

In view of these results, one is led to ask if the use of this alternative renormalization procedure for the mass, allows also to cure the pathologies of the charged BTZ black hole. In this paper we investigate this issue. We will show that all the problematic features of the charged BTZ black hole can be removed if one uses $M_0$ as black hole mass. We will demonstrate that $M_0$ satisfies a BPS-like bound $M_0 \geq \frac{\pi}{2}Q^2$ and that for a black hole above extremality the heat capacity is positive and becomes zero in the extremal case. We also discuss the formulation of black hole thermodynamics when $M_0$ is interpreted as the internal energy of the thermodynamical system. We show, using three independent methods (black hole thermodynamics, Einstein equations, Euclidean action formulation) that $M_0$ satisfies the first law if a term describing the mechanical work done by the electrostatic pressure is introduced.

The structure of the paper is as follows. In Sect. 2 we review briefly the main features and pathologies of the charged BTZ black hole. In Sect. 3 we discuss in detail the two
renormalization schemes that can be used to get a finite black hole mass and show that the mass \( M_0 \) satisfies a BPS-like bound. In Sect. 4 we discuss the thermodynamics of the charged BTZ black hole in the two cases, when the internal energy of the system is identified with either \( M \) or \( M_0 \). In Sect. 5 we present our conclusions.

2 The charged BTZ black hole

The charged BTZ black hole solution is a U(1) generalization of the uncharged BTZ black hole [1]. In this paper the solution with a non zero electric charge \( Q \) and zero angular momentum will be considered.

The action is

\[
I = \int d^3x \sqrt{-g} \left( \frac{R + 2\Lambda}{2\pi} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right),
\]

where \( F_{\mu\nu} \) is Maxwell tensor, \( \Lambda = 1/l^2 \) is the cosmological constant (\( l \) is the AdS length) and we are using units such that 3D Newton constant \( G \) is dimensionless, \( G = \frac{1}{8} \).

The solution for the electrically charged, non rotating case is given by [7, 10]

\[
ds^2 = -f(r) dt^2 + f^{-1} dr^2 + r^2 d\phi^2,
\]

with metric function and Maxwell field

\[
f = \frac{r^2}{l^2} - M - \pi Q^2 \ln \frac{r}{w}, \quad F_{tr} = \frac{Q}{r},
\]

where \( M, Q, w \) are integration constants. Although the solution depends only on two integration constants (\( w \) can be absorbed in a redefinition of \( M \)), it is convenient to keep the dependence of the metric on the arbitrary length scale \( w \) explicit. The above solution represents a 3D, asymptotically AdS, black hole, with a power-law singularity at \( r = 0 \), where \( R \sim \pi Q^2 / r^2 \) and, generically, with an inner (\( r_- \)) and outer (\( r_+ \)) horizon. \( M, Q \) could be naively seen as the black hole mass and charge, respectively. \( M \) can be easily expressed as a function of the charge and of the outer horizon radius,

\[
M(r_+, Q, w) = \frac{r_+^2}{l^2} - \pi Q^2 \ln \left( \frac{r_+}{w} \right).
\]

Whereas the interpretation of \( Q \) as the black hole electric charge is straightforward, the same is not true for \( M \). In fact by varying the action [1], one finds a surface term which diverges in the limit \( r \to \infty \) [7]:

\[
\left( -\delta M - \pi \delta Q^2 \ln r \right) N(r),
\]

where \( N \) is the lapse function. The presence of the logarithmic divergent boundary term makes the black hole mass a poorly defined concept.

A second unpleasant feature emerges when one imposes a cosmic censorship condition, i.e. the absence of naked singularities. The requirement that the singularity at \( r = 0 \) is shielded by an event horizon is equivalent to requiring that the metric function \( f(r) \) evaluated on its minimum value, is equal or less than zero (corresponding respectively to
Figure 1: Region in the $M - Q$ phase space where the black hole exists. The region of existence is the shaded region. Extremal black holes are in the boundary line between the shaded and the unshaded regions.

one or two horizons). Introducing a function $\Delta(M, Q)$ the condition for the existence of the horizon(s) can be cast in the form

$$\Delta(M, Q) = f\left(r = r_{\text{min}} = \sqrt{\frac{\pi}{2} Q l}\right) = -M + \frac{\pi Q^2}{2} \left(1 - \ln \frac{\pi Q^2}{2}\right) \leq 0,$$

Eq. (6) is not a BPS-like bound, it can be satisfied by negative values of the mass $M$. This can be immediately seen considering the $M - Q$ phase diagram shown in Fig (1).

For $Q$ above a critical value $Q_0$ there are black hole solutions with arbitrarily negative values of $M$. The presence of black hole states with mass values unbounded from below makes the system intrinsically unstable and the definition of thermodynamical ensembles problematic. At this point one can wonder if the presence of black hole states with arbitrary negative mass is a physical feature of the system or an artifact due to the divergence of the boundary term. This question can be answered only after the issue of mass renormalization is discussed in detail.

3 Mass renormalization

The problem of the presence of a divergent boundary term can be handled with a renormalization procedure. This procedure can be implemented in systematic way by enclosing our system in a circle of radius $r_0$. This allows to rewrite the metric function (3) in the form

$$f(r) = -M_0(r_0, w) + \frac{r^2}{l^2} - \pi Q^2 \ln \left(\frac{r}{r_0}\right).$$

(7)
and to define a regularized mass:

\[ M_0(r_0, w) = M + \pi Q^2 \ln \left( \frac{r_0}{w} \right). \]  

(8)

\( w \) is now considered a running scale and \( M_0(r_0) \) is the total energy (gravitational and electromagnetic) inside a circle of radius \( r_0 \); in the limit \( r \to \infty \) one takes also \( r_0 \to \infty \), keeping the ratio \( r/r_0 = 1 \) \[9\] \[10\]. One is left with two possible options: i) \( M \) is held fixed and the space-time metric is scale-dependent; ii) The metric is \( w \)-invariant and \( M \) runs with \( w \).

Option i) is the renormalization procedure proposed in \[7\]. Eq. (8) is used to identify \( M \) as the total mass of the solution, an infinite constant is absorbed in \( M_0 \) and the logarithmic divergent term in Eq. (5) is removed. Apart from the drawback of having the metric (hence the horizons position and the black hole entropy) running with \( w \), this procedure does not solve the stability problem, hence does not allow for a consistent interpretation of the charged BTZ black hole as a thermodynamical system.

In this paper we use the \( w \)-invariant renormalization prescription ii) first proposed in Ref. \[9\]. To keep the metric function \( f \) unchanged as \( w \) runs, we held \( M_0(r_0, w) \) fixed, whereas \( M \) changes with \( w \): \( w \to \lambda w, \quad M \to M + \pi Q^2 \ln \lambda \). The boundary term (5) becomes now

\[ \left( -\delta M_0 - \pi \delta Q^2 \ln \frac{r}{r_0} \right) N(r), \]  

(9)

and in the limit \( r, r_0 \to \infty \) the divergent part is removed.

As a consequence of the \( w \)-invariance of \( f \) and \( M_0 \), we can arbitrarily choose \( w \) and \( M_0 \). Following Ref. \[10\] we choose to fix them in terms of the AdS length \( (w = l) \) and of the horizon position \( (r_0 = r_+) \).

This renormalization procedure allows us to associate in a consistent way to every charged BTZ black hole solution (2), (3) a finite mass given by

\[ M_0(r_+) = M + \pi Q^2 \ln \left( \frac{r_+}{l} \right). \]  

(10)

The metric function becomes

\[ f(r, M_0) = -M_0 + \frac{r^2}{l^2} - \pi Q^2 \ln \frac{r}{r_+}. \]  

(11)

It is important to stress that setting \( r_0 = r_+ \) we are implicitly assuming that at least one horizon is always present, i.e. the validity of the cosmic censorship conjecture.

We can identify the \( w \)-invariant mass \( M_0(r_+) \) as the conserved charge associated with time-translation invariance, instead of the mass \( M \). The renormalization prescription ii) has further nice features. The renormalized mass \( M_0 \) depends only on the horizon position, is always positive-definite and shares with the uncharged BTZ black hole the mass/horizon-position dependence:

\[ M_0(r_+) = \frac{r_+^2}{l^2}. \]  

(12)

Moreover, the identification of \( M_0 \) with the conserved charge associated with time-translation allows to reproduce exactly the Bekenstein-Hawking entropy of the charged BTZ black hole using a Cardy formula for the 2D dual CFT \[9\].
3.1 BPS-like bound

Let us now show that the use of $M_0$ as the physical mass of the system, allows, at least in principle, to solve the instability problem. The new mass spectrum can be found using Eq. (12) and the trivial relation $r_+ \geq r_{\text{min}} = \sqrt{\frac{n}{2}} Q l$. We have

$$M_0 \geq \frac{\pi}{2} Q^2.$$  \hspace{1cm} (13)

Because by writing Eqs. (10) and (11) we have implicitly assumed that at least one horizon is present, we expect the inequality (6) to be identically satisfied. In fact, expressing $\Delta$ of Eq. (6) as a function of $M_0$,

$$\Delta(M_0, Q) = -M_0 + \frac{\pi Q^2}{2} \left( 1 - \ln \frac{\pi Q^2}{2M_0} \right),$$  \hspace{1cm} (14)

and setting $\alpha = 2M_0/\pi Q^2$ the condition for the existence of the horizons, $\Delta \leq 0$, takes the form $\alpha - 1 \geq \ln \alpha$, which is always true.

Eq. (13) represents a BPS-like bound for the black hole mass. It takes a form similar to the bound satisfied by charged black holes in higher dimensions. The bound is saturated when in Eq. (13) the equality holds. In this case we have an extremal black hole, which is a state of mass $M_0 = (\pi/2)Q^2$, zero temperature and nonvanishing entropy $S = 2\pi \sqrt{2\pi l} Q$ (see Eq. (15) below). Again, these are features shared by higher dimensional charged black holes. The quadratic form of the $M_0 - Q$ phase diagram, which results from Eq. (13), eliminates the negative, unbounded from below, tail present in Fig. 1 and sets an upper bound on the black hole charge $Q$.

This result implies that the presence of black hole states with arbitrary negative mass is a consequence of identifying the energy of the system with the mass $M$. Moreover, it gives a strong hint that the $M_0 = \pi Q^2/2$ extremal black hole could be a stable configuration. Obviously, in the context of our discussion stability is just a consequence of the validity of the cosmic censorship conjecture. A formal proof of the stability of the extremal configuration would require a detailed analysis of the perturbation spectrum around the extremal black hole solution. Alternatively, stability can be proved by showing that the extremal background is supersymmetric [12], i.e. it allows for the existence of Killing spinors. A detailed analysis of the stability of the extremal black hole is outside the aim of this paper. In the next sections we will show that using $M_0$ as internal energy of the system allows for a consistent formulation of the thermodynamics of the charged BTZ black hole.

4 The first law of thermodynamics for the charged BTZ black hole

It is well known that the laws of black hole mechanics mimic the laws of thermodynamics. Formally, a black hole can be considered a thermodynamical system. Obviously, the thermodynamical behavior of our charged BTZ black hole will depend on the identification of the black hole parameters in terms of thermodynamical variables. For the temperature $T$, the entropy $S$, the electric potential $\Phi$ (thought of as chemical potential) there is no
ambiguity. $T, S, \Phi$ are given as usual in terms of, respectively, surface gravity, horizon area and time component of the vector potential,

$$
T = \frac{1}{4\pi} \left( \frac{2r_+}{l^2} - \frac{\pi Q^2}{r_+} \right),
$$

$$
S = 4\pi r_+ = 4\pi l \sqrt{\pi Q^2 \ln \frac{r_+}{l}} + M,
$$

$$
\Phi = A_0(r_+) = -2\pi Q \ln \frac{r_+}{l}. \tag{15}
$$

On the other hand, for the internal energy $E$ of the thermodynamical system we have two possible choices: we can identify $E$ either with $M$ or with $M_0$. We will show now that both choices lead, at least at the formal level, to a consistent thermodynamical formulation.

For $E = M$ the internal energy of the system is the total energy (gravitational and electrostatic) of the black hole and we expect the first principle to take the usual form. In fact differentiating $M(r_+, Q)$ in Eq. (11) and making use of Eqs. (15) one easily obtains the first principle in the form

$$
dM = TdS + \Phi dQ. \tag{16}
$$

The exact form $M(S, Q)$ can be easily determined, we have

$$
M(S, Q) = \frac{S^2}{16\pi^2 l^2} - \frac{\pi Q^2}{4\pi l} \ln \left( \frac{S}{4\pi l} \right). \tag{17}
$$

Conversely, when $E = M_0$ the internal energy is identified with the energy of the black hole inside the radius $r_+$. In this case we expect that the presence of radial pressure gradients will give rise to additional terms in Eq. (16). The new form of the first principle can be obtained differentiating $M_0$ given in equation (10) and using Eq. (16). One has,

$$
dM_0 = TdS + \Phi dQ + dK, \tag{18}
$$

where $K = \pi Q^2 \ln \left( \frac{r_+}{l} \right)$ is minus the electrostatic energy outside the horizon. The variation of $K$ cannot change the black hole entropy but represents mechanical work done by electrostatic pressure. We can compute $dK$ keeping constant the electrostatic potential $\Phi$, this allows to express charge variation in terms of displacement of the horizon,

$$
2\pi \ln \frac{r_+}{l} dQ = -\frac{2\pi Q}{r_+} dr_+. \tag{19}
$$

Using the previous equation one finds

$$
dM_0 = TdS + \Phi dQ - \frac{\pi Q^2}{r_+} dr_+. \tag{20}
$$

The last term in the previous equation is the work done by the radial pressure

$$
P_r = T_{rr} \tag{21}
$$

generated by the electrostatic field ($T_{\mu\nu}$ is the stress-energy tensor for the Maxwell field). Explicit computation of the $T_{rr}$ gives,

$$
P_r(r_+) = -\frac{Q^2}{2r_+^2}. \tag{22}
$$
Using Eq. (22) in (20) one obtains the first principle in the final form

\[ dM_0 = TdS + \Phi dQ + P_r dA, \]  

(23)

where \( A = \pi r_+^2 \) is the area inside the radius \( r_+ \). Notice that when \( dA > 0 \) the mechanical work \( P_r dA \) in Eq. (23), is negative, i.e it is done by the thermodynamical system. The pressure \( P_r(r_+) \) goes to zero when \( r_+ \to \infty \), in this situation \( M = M_0 \) and the two thermodynamical descriptions are equivalent.

The internal energy \( M_0 \) appearing in the first principle (23) appears to be a function of three independent extensive thermodynamical parameters \( S, Q, A \); a simple calculation gives

\[ M_0(S, Q, A) = \frac{S^2}{16\pi^2l^2} - \pi Q^2 \ln \left( \frac{S}{4\pi l} \right) + \frac{\pi Q^2}{2} \ln \left( \frac{A}{\pi l^2} \right). \]  

(24)

However, there are only two independent parameters because of the presence of a constraint. This constraint takes a different form for thermodynamical transformations at constant \( \Phi \) or constant \( Q \). In the first case the constraint takes the form \( \Phi dQ = -2P_r dA. \) (25)

Conversely, keeping the charge \( Q \) constant the constraint takes the form \( Q d\Phi = -2A dP_r. \) (26)

It is also interesting to compute the thermal capacity of the black hole at constant charge as a function of \( M_0 \). We have

\[ C = T \frac{\partial S}{\partial T} \bigg|_{Q} = 4\pi l \sqrt{M_0 \left( \frac{2M_0 - \pi Q^2}{2M_0 + \pi Q^2} \right)}. \]  

(27)

The heat capacity is always positive when the black hole is above extremality, \( M_0 \geq \pi Q^2/2 \), and becomes zero in the extremal case.

### 4.1 Derivation of the first law from Einstein’s equations

Black hole thermodynamics can be derived from the laws of black hole mechanics, i.e it is codified in Einstein equations. The first principle of thermodynamics for the charged BTZ black hole (23) can be derived from the \( rr \) component of Einstein’s equation \[ G_{rr} - \Lambda g_{rr} = \pi T^r_r, \]  

(28)

where \( G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \) is the Einstein tensor. Evaluating the equations (28) on the horizon and using Eq. (21) one has

\[ \frac{f'(r_+)}{2r_+} - \frac{1}{l^2} = \pi P_r. \]  

(29)

Multiplying both terms of this equation by \( d(r_+^2) \) and using Eqs. (15) and (12) we obtain

\[ dM_0 = TdS - P_r dA. \]  

(30)

Using Eq. (25) we easily find that equation (30) is equivalent to the first law (23).

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1A first principle of this form for the charged BTZ black hole has been also derived in Ref. [13, 14]. However, in those papers a different definition for the internal energy \( E \) is used. \( E \) is not identified with the mass \( M_0 \), but is given by our Eq. (10) with opposite sign of the second term in the r.h.s.
4.2 Euclidean action formulation

In this section we will show that the thermodynamics of the charged BTZ black hole described in the previous sections can be also derived using the Euclidean action formalism. In the Euclidean action approach to black hole thermodynamics, one can get Gibbs free energy through analytic continuation of the action, with suitable boundary terms, in Euclidean space.

To compute the Euclidean action $I_{E}$ we will follow the method of Bañados, Teitelboim e Zanelli [1], which use the Hamiltonian version of the action (1). The bulk contribution is equal to zero and the Euclidean action is completely given by three surface terms. The first surface term must be added at infinity and is given by the mass of the solution times the periodicity of of Killing time $\beta = 1/T$. The other two surface terms make sure that the variational derivative of the action vanishes on the horizon.

If we use the regularization scheme i) of section 3 the boundary term at infinity is given by $M$ and all together one has, for the Euclidean action:

$$I_{E} = \beta M - 4\pi r_{+} - \beta A_{0}(r_{+})Q.$$  \hspace{1cm} (31)

Gibbs free energy, $G(T, \Phi)$ describing the system in the grand canonical ensemble, is given by $G = TI_{E}$ and using Eq. (31) it turns out to be, as expected, the Legendre transform of $M$ with respect to $S$ and $Q$:

$$G(T, \Phi) = M - TS - \Phi Q.$$  \hspace{1cm} (32)

The description of the thermodynamical system trough $G(T, \Phi)$ corresponds to the choice of $M$ as the internal energy of the system. One can easily reproduce the entropy $S$ and the charge $Q$ as $S = -(\partial G)/(\partial T)$ and $Q = -(\partial G)/(\partial \Phi)$.

If we use instead the renormalization scheme ii) of section 3, the Euclidean action (31), hence Gibbs free energy does not change. The mass of the solution is now $M_{0}$ but the boundary term at infinity is still given by $M$, the logarithmic term in Eq. (10) being an horizon contribution. From Eq. (10) it follows that $M_{0}$ can be written as

$$M_{0} = M - \frac{1}{2}Q\Phi(r_{+}).$$  \hspace{1cm} (33)

Thus, the term needed to cancel the variational derivatives of the action on the horizon is now given by $-4\pi r_{+} - (1/2)\beta Q\Phi$. All these contribution sum up to the same result given in Eq. (31).

Corresponding to the choice of $M_{0}$ as the internal energy of the system, Gibbs free energy can be now expressed as a function of $T, \Phi, P_{r}$. This can be done by first making use of Eq. (33) to write $G$ in equation (32) in terms of $M_{0}$: $G = M_{0} - TS - \Phi Q + \frac{1}{2}\Phi Q$, then differentiating and using the first law (23) and the constraints (25), (26). One obtains

$$dG(T, \Phi, P) = -SdT - Qd\Phi - A_{0}dP_{r}.$$  \hspace{1cm} (34)

5 Conclusions

In this paper we have shown that the problematic features of the charged BTZ black hole are an artifact of the usual renormalization procedure for the divergent bare mass of the
hole. An alternative renormalization scheme leads to a different definition of black hole mass, which physically is the total energy inside the horizon. When described in terms of this mass $M_0$, the charged BTZ black hole behaves like the 4D Reissner-Nordstrom black hole. It satisfies a BPS-like bound that guarantees both positivity of the mass and an upper bound for the charge of the hole. The extremal black hole is a state of zero temperature and nonvanishing entropy. The thermal capacity of the hole is always positive and becomes zero for the extremal black hole.

We have also shown, using three different approaches, that the charged BTZ black hole allows for a consistent thermodynamical description when $M_0$ is interpreted as the internal energy of the system. The only change with respect to usual black hole thermodynamics is the appearance in the first law of a term describing the mechanical work done by the electrostatic pressure.

These results improve our understanding of charged black hole solution and could be also very useful in the AdS/CFT correspondence context. Similarly to the higher-dimensional cases the BTZ black hole is a bridge between a near-horizon AdS$_2$ and an asymptotic AdS$_3$ geometry [10]. This feature could be very useful for understanding the nature of AdS$_2$ quantum gravity and in particular the microscopic entropy of extremal charged black holes [17, 18, 19, 20, 21, 22, 23, 24].

In this paper we have not addressed at a full level the issue of the stability of the extremal charged BTZ black hole. The presence of the BPS-like bound [13], the vanishing of the temperature and of the thermal capacity in the extremal configuration strongly indicates that the extremal charged BTZ black hole is stable. However, a proof of this statement will require detailed analysis of the perturbation spectrum or, alternatively, the demonstration that this extremal state is a true supersymmetric BPS state.

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