Nuclear dependence of $Q^2$ evolution in the structure function $F_2$

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ABSTRACT

$Q^2$ evolution of the structure functions $F_2$ in tin and carbon nuclei is investigated in order to understand recent NMC measurements. $F_2$ is evolved by using leading-order DGLAP, next-to-leading-order DGLAP, and parton-recombination equations. NMC experimental result $\partial[F_2^{Sn}/F_2^{C}]/\partial[\ln Q^2] \neq 0$ could be essentially understood by the difference of parton distributions in the tin and carbon nuclei. However, we find an interesting indication that large higher-twist effects on the $Q^2$ evolution could be ruled out. Nuclear dependence of the $Q^2$ evolution could be interesting for further detailed studies.

† Email: kumanos or 94sm10@cc.saga-u.ac.jp. Information on their research is available at [http://www.cc.saga-u.ac.jp/saga-u/riko/physics/quantum1/structure.html](http://www.cc.saga-u.ac.jp/saga-u/riko/physics/quantum1/structure.html)
or at [ftp://ftp.cc.saga-u.ac.jp/pub/paper/riko/quantum1](ftp://ftp.cc.saga-u.ac.jp/pub/paper/riko/quantum1).

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1. Introduction

Nuclear modification of the structure function $F_2$ has been an interesting topic since the discovery of the EMC (European Muon Collaboration) effect in 1983. Although most studies discuss $x$ dependence of the modification, $Q^2$ dependence becomes increasingly interesting. It is because the NMC (New Muon Collaboration) showed $Q^2$ variations of the ratio $F_2^A/F_2^D$ with reasonably good accuracy [1]. Structure functions $F_2$ themselves cannot be calculated without using nonperturbative methods; however, $Q^2$ evolution of $F_2$ can be evaluated perturbatively. The phenomenon is called scaling violation, which is considered to be a strong evidence to support perturbative QCD. An intuitive way of describing the scaling violation is to use the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [2]. $Q^2$ variations of $F_2^A/F_2^D$ are calculated by the DGLAP equations [3, 4], and they are consistent with existing NMC experimental data.

It is not, however, obvious whether DGLAP could be applied to the nuclear case. Complex nuclear interactions may give rise to extra $Q^2$ factors in the evolution equations. In particular, the longitudinal localization size of a parton with momentum fraction $x$ could exceed an average nucleon separation in a nucleus if $x$ is small ($x < 0.1$). In this case, partons in different nucleons could interact and the interactions are called parton recombinations. Their contributions to the evolution equations have $A^{1/3}$ nuclear dependence [3]. Until recently, no nuclear dependence of the $Q^2$ evolution had been found experimentally. In fact, previous measurements are consistent with no nuclear dependence $\partial[F_2^A/F_2^D]/\partial[\ln Q^2] = 0$ [1]. However, it is reported recently by NMC [5] that there exist significant differences between tin and carbon $Q^2$ evolutions, $\partial[F_2^{Sn}/F_2^{C}]/\partial[\ln Q^2] \neq 0$. It is the first indication of nuclear effects in the $Q^2$ evolution of $F_2$ and is worth investigating theoretically.

The purpose of our study is to calculate the nuclear dependence $\partial[F_2^{Sn}/F_2^{C}]/\partial[\ln Q^2]$ theoretically and to compare the results with the NMC data. There are two major sources for the difference in $Q^2$ evolution of $F_2$. First, parton distributions themselves are different in tin and carbon nuclei. Second, nuclear interactions could modify the evolution equations. For example, the parton recombinations produce extra $Q^2$ dependent effects on the $Q^2$ evolution equations. Numerical solutions of DGLAP equations and those with recombination effects are discussed in Ref. [4].

In section 2, we employ input distributions obtained in a hybrid parton model with rescaling and recombination effects [3], then the $Q^2$ evolution is calculated by using the computer code in Ref. [4]. Three types of evolutions are tested, and they are leading-order (LO) DGLAP equations, next-to-leading-order (NLO) DGLAP, and evolution equations with parton recombinations (PR).
2. $Q^2$ evolution of $F_2$ in tin and carbon nuclei

Scaling violation has been well studied, and it corresponds to the physics that minute quark-gluon clouds around a parton could be seen by increasing $Q^2$. Evolution equations are derived by calculating parton splitting processes into two partons or by calculating corresponding anomalous dimensions. Parton-recombination contributions can also be included in the equations. $Q^2$ evolution is then described by integrodifferential equations, and the DGLAP [2] and PR [6] evolution equations are given by

$$\frac{\partial}{\partial t} q_i(x,t) = \int_x^1 \frac{dy}{y} \left[ \sum_j P_{qi} \left( \frac{x}{y} \right) q_j(y,t) + P_{qg} \left( \frac{x}{y} \right) g(y,t) \right]$$

$$+ \left( \text{recombination terms } \propto \frac{\alpha_s A^{1/3}}{Q^2} \right), \quad (1a)$$

$$\frac{\partial}{\partial t} g(x,t) = \int_x^1 \frac{dy}{y} \left[ \sum_j P_{qg} \left( \frac{x}{y} \right) q_j(y,t) + P_{gg} \left( \frac{x}{y} \right) g(y,t) \right]$$

$$+ \left( \text{recombination terms } \propto \frac{\alpha_s A^{1/3}}{Q^2} \right), \quad (1b)$$

where the variable $t$ is defined by $t = -\frac{2}{\beta_0} \ln[\alpha_s(Q^2)/\alpha_s(Q^0)]$. $\alpha_s$ is the running coupling constant and $\beta_0$ is given by $\beta_0 = (11/3)C_G - (4/3)T_R N_f$ with $C_G = N_c$ (number of color), $T_R = 1/2$, and $N_f=$number of flavor. In the PR evolution case, there is an extra evolution equation for a higher-dimensional gluon distribution. Explicit expressions of recombination contributions are found in Ref. [4, 6]. The first two terms in Eqs. (1a) and (1b) describe the process that a parton $p_j$ with the nucleon’s momentum fraction $y$ splits into a parton $p_i$ with the momentum fraction $x$ and another parton. The splitting function $P_{p_ip_j}(z)$ determines the probability that such a splitting process occurs and the $p_j$-parton momentum is reduced by the fraction $z$.

Because the splitting functions $P_{p_ip_j}$ are independent of nuclear interactions, there are two possibilities for the nuclear dependence in Eqs. (1a) and (1b). First, parton $x$ distributions are modified in a nucleus. It is known that quark distributions in a nucleus are effectively reduced at small $x$ due to shadowing mechanisms and medium $x$ due to binding, rescaling, and other effects. They are enhanced at large $x$ due to nucleon Fermi motion effects and also at $x \approx 0.1$. Modification of the gluon distribution in a nucleus is not well known at this stage. In particular, there is little information from the experimental side. However, we expect that the gluon distribution is shadowed at small $x$ [9]. If the parton distributions are modified in Eqs. (1a) and (1b), they give rise to different $Q^2$ evolution through the splitting processes. Therefore, the nuclear dependence comes entirely from the parton-distribution differences in the DGLAP evolution case.
Second, the parton-recombination mechanism supplies additional $\alpha_s A^{1/3}/Q^2$ effects in Eqs. (1a) and (1b). As it is mentioned in the introduction, partons in different nucleons cannot be considered independent at small $x$ because the parton localization size becomes larger than the nucleon size. The recombination probability is proportional to the number of nucleons in the longitudinal direction, so that there exists the factor $A^{1/3}$. The factor $\alpha_s/Q^2$ arises from the fact that the parton-parton recombination cross section is proportional to $\alpha_s/Q^2$. Hence, nuclear dependence exists in the recombination terms as well as in the parton distributions if the PR evolution is used.

In order to calculate $Q^2$ evolution of nuclear $F_2$ structure functions, we need to have input parton distributions at certain $Q^2$. Because the distributions themselves cannot be calculated exactly, they depend on a used model. In the present investigation, we should employ a model which can at least explain measured ratios $F_2^A/F_2^D$. As such a parton-model candidate, we have a hybrid model with parton-recombination and $Q^2$-rescaling mechanisms [3]. According to this model, both parton recombinations and $Q^2$ rescaling are calculated at $Q^2_0=0.8$ GeV$^2$. Then, parton distributions are evolved to those at larger $Q^2$. The model can explain measured $x$ and $Q^2$ dependence of the ratio $F_2^A/F_2^D$ by NMC. For the details of the model, we refer the reader to Ref. [3].

Calculated results for $\partial[F_2^{Sn}/F_2^C]/\partial[\ln Q^2]$ are shown at $Q^2=5$ GeV$^2$ together with preliminary NMC data [8] in Fig. 1. The dotted, solid, and dashed curves correspond to LO-DGLAP, NLO-DGLAP, and PR evolution results respectively. The QCD scale parameter is $\Lambda=0.2$ GeV and the number of flavor is three. The DGLAP evolution curves agree roughly with the experimental tendency; however, they underestimate the $Q^2$ variation in the region $0.01 < x < 0.05$. In the PR evolution, NLO effects in the DGLAP part are included. It is interesting to find that the PR results disagree with experimental data even in the sign. The large discrepancy from the DGLAP results is caused partly by the evolution from small $Q^2$ (0.8 GeV$^2$) to 5 GeV$^2$. The recombination contributions are higher-twist effects, so that they are very large in the small $Q^2$ region. Because of the significant discrepancy from the data, large parton-recombination contributions could be ruled out. However, it does not mean that the PR evolution itself is in danger. There is an essential parameter $K_{HT}$, which determines how large the higher-dimensional gluon distribution is ($xG_{HT}(x,Q^2_0) = K_{HT}[xg(x,Q^2_0)]^2$). We chose $K_{HT}=1.68$ so that $G_{HT}$ contribution is 10% to the gluon distribution [6]. However, as it is discussed in Ref. [3], the magnitude of $K_{HT}$ is unknown at this stage. In order to discuss the validity of the PR evolution, the constant $K_{HT}$ must be evaluated theoretically.

Because average $Q^2$ values of the NMC data are different at each $x$ point, we
should take into account the $Q^2$ variation of the derivative. According to the NMC’s preliminary results, the average $Q^2$ in the $x=0.01$ region is about a few GeV$^2$ and it is about 10–20 GeV$^2$ in the $x=0.1$ region. In order to show theoretical $Q^2$ variations, the derivative is calculated at $Q^2=2, 5, 20$ GeV$^2$ in the NLO-DGLAP case, and the results are shown in Fig. 2. The dashed, solid, and dotted curves correspond to the results at 2, 5, 20 GeV$^2$ respectively. In the $x=0.01$ region, the theoretical results with $Q^2=2$ GeV$^2$ agree with the NMC data. The results with $Q^2=20$ GeV$^2$ also agree roughly with data in the $x=0.1$ region. However, we still underestimate the derivative between these regions.

From these analyses, we find that the essential part of the NMC results could be understood within the parton-model framework of Ref. [3] together with the usual $Q^2$ evolution equations [4]. However, there are still discrepancies between the theoretical evolution results and the NMC data. So we check the sensitivity of the theoretical results on sea-quark and gluon modifications in nuclei. If the parton distributions in the tin and carbon nuclei are identical, the $Q^2$ derivative $\partial [F_{2n}^T/F_{2c}^C]/\partial [\ln Q^2]$ has to vanish in the DGLAP evolution. Therefore, the finite values in the DGLAP cases reflect nuclear modification of quark and gluon distributions. If the sea-quark shadowing is increased significantly at small $x$ in the tin with keeping same distributions in the carbon, the theoretical results agree with the NMC data in the region $x \approx 0.03$. On the other hand, the gluon shadowing is increased in the similar way, the disagreement becomes slightly larger. The sea-quark modification has more significant effects on the $Q^2$ derivative than the gluon modification does. In order to compare theoretical results with the NMC data more seriously, detailed studies on nuclear parton distributions as well as on nuclear $Q^2$ evolution equations are necessary.
3. Conclusions

We find that the NMC finding of nuclear $Q^2$ dependence, $\partial[F_2^S/F_2^C]/\partial[\ln Q^2] \neq 0$, could be essentially understood by ordinary $Q^2$ evolution equations. The most important factor for the derivative is the nuclear modification of parton distributions (especially sea-quark distributions) and is not the $Q^2$ evolution due to the parton recombinations. Our parton model together with the evolution equations describes the NMC data fairly well, although it slightly underestimate the nuclear difference. In our analysis, “large” higher-twist effects from the parton recombinations could be ruled out. However, it is encouraging to study the details of the recombination mechanism in comparison with the NMC data. Furthermore, studies of $x$ dependence in nuclear parton distributions are essential for understanding the details of the NMC data.

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Figure Captions

**Fig. 1** Nuclear dependence of $Q^2$ evolution is calculated at $Q^2=5$ GeV$^2$. The dotted, solid, and dashed curves are the results in the LO-DGLAP, NLO-DGLAP, and PR evolutions respectively. $K_{HT}=1.68$ is taken in the PR evolution [6]. The theoretical results are compared with the preliminary NMC data [8].

**Fig. 2** The derivative is calculated at $Q^2=2$, 5, and 20 GeV$^2$ in order to show $Q^2$ variations. The evolution method is the NLO-DGLAP. The dashed, solid, and dotted curves correspond to the results at $Q^2=2$, 5, and 20 GeV$^2$ respectively. They are compared with the NMC data.
Figure 1
Figure 2