A high accuracy method for large moments and product of inertia spacecraft

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Abstract. When measuring product of inertia (POI), the “moment of inertia” method can be applied with big measurable range and low accuracy. Comparing with it, the using of “Dynamic balancing method” can get the result of high accuracy, but only with small range of measurable center of mass eccentricity and POI. For a passive control reentry spacecraft which has big angle between principle axis and rotational axis, each way can’t achieve the demand of large moments, large POI and high accuracy. This paper provides a creative way which combines both ways we mentioned above to measure the POI with high accuracy, large eccentricity and large POI. Meanwhile, the data processing software and specific MGSE are also innovatively integrated in this way to achieve the requirements.

1. Introduction
Mass property characters, which including mass, center of gravity (COG), moment of inertia (MOI) and POI, are important parameters for spacecraft orbit control and attitude control [1]. For reentry cabin of space station, POI values are key parameters to assure a safe, stable reentry stage, so a high accuracy measurement is required for spacecraft designer [2]. In some cases, the reentry stage is designed under passive control, which means the cabin fly back without any attitude control thrusters. Traditionally, this kind of cabin has large bias moment, so how to adjust POI characters to proper values under large moments and with high accuracy is becoming vital important.

But the function of spin balancing machine is to reduce the dynamic and static unbalance [3], which can’t meet the requirement of passive control reentry cabin. Meanwhile, large moments and inclined principle axis will produce large unbalance which may exceed the maximum capacity of spin balancing machine [4]. High accuracy under this restricted condition becomes more difficult.

2. Measurement requirements
Suppose the definition of reentry cabin coordinate system is shown in Fig. 1. The structure coordinate is named as OXYZ, which original point is O, located on center point of small end, X axis is parallel to central axis of cabin and Y axis towards to IV quarter line. The second coordinate system is marked as O₁X₁Y₁Z₁, which original point is located on center of gravity of cabin, X₁ axis lies in O₀X₀ plane and has an angle of 18º with OX axis (positive direction towards large end), Y₁ axis lies in O₀X₀ plane perpendicular to O₀X₀ axis (positive direction near small end). All of the coordinate system is right hand system.
Figure 1. Coordinate system definition of cabin.

The requirements of the cabin is to assure $O_1X_1$ axis of second coordinate system is the principle axis according to append balancing weight on cabin while the bias moment caused by COG is larger than 600kgm.

3. Measurement method

In order to assure $O_1X_1$ axis is principle axis, the POI values about this axis need to be adjust to zero\textsuperscript{[5]}. So how to measure the POI values about the axis becomes a critical step. POI can be measured by indirect method measuring MOI values, and the advantage is the equipment has no restrict on cabin unbalancing, but the disadvantage of the method is the accuracy is lower than the demand. The principle of “moment of inertia” method is shown in Fig. 2.

Figure 2. MOI measuring equipment.

MOI about OL axis can be wrote as Eq. 1\textsuperscript{[6]}. 

\textbf{[5]}... 

\textbf{[6]}...
\[ I_L = I_x \cos^2 \alpha + I_y \cos^2 \beta + I_z \cos^2 \gamma - 2I_{xy} \cos \alpha \cos \beta - 2I_{xz} \cos \alpha \cos \gamma - 2I_{yz} \cos \gamma \cos \beta \]  

(1)

In this equation:
- \( I_x \), \( I_y \) and \( I_z \) are MOI values about OX, OY and OZ axis respectively;
- \( I_{xy} \), \( I_{yz} \) and \( I_{xz} \) are POI values about XOY, YOZ and XOZ plane respectively;
- \( \alpha \), \( \beta \) and \( \gamma \) are angles between OL and OX, OY and OZ respectively.

So, if 6 attitudes MOI values are measured in proper condition which \( \alpha \), \( \beta \) and \( \gamma \) are known, then the POI values can be calculated.

Because the accuracy of MOI method is limited and can’t meet the requirement of cabin, another dynamic balancing method is used when the POI have been adjusted into proper range using MOI method.

Dynamic balancing method is a high accuracy POI measuring method using spin balancing machine, but have restricted unbalancing force and unbalancing moments range, otherwise the machine can’t achieve high accuracy or may cause damage. So, Eq. 2 is used to verify capacity of the machine.

\[
\begin{cases} 
M \omega^2 r < 4000 \\
\omega^2 k \left( I_{xy} i + I_{xz} j \right) < 4000 
\end{cases}
\]  

(2)

In this equation:
- \( M \) is mass of object, \( r \) is COG vector, so \( M \times r \) is bias moments;
- \( \omega \) is rotational speed;
- \( I_{xy} \), \( I_{xz} \) are the POI values about the equipment plane.

Fig. 3 shows the relationship between rotational speed and POI value under different unbalancing moments.

![Figure 3. Relationships of rotational speed and POI](image)

According to this figure, POI and rotational speed should be restricted to special range to avoid unbalancing moments exceeding the capacity.
For unbalancing force, the bias moments are constant and can be reduced by special adapter describe in next chapter.

After the POI values and bias moments have been adjusted to special range, the dynamic balancing method can be used to measure accuracy POI values.

The calculated method of dynamic balancing method is listed in Eq. 3.

\[ x_1 U_1 + x_2 U_2 = I_{xy} i + I_{xz} j \]  \hspace{1cm} (3)

In this equation:

- \( U_1, U_2 \) are the balancing weight on plane \( X_1, X_2 \) respectively;
- \( x_1, x_2 \) are the distance of plane \( X_1, X_2 \) to original point respectively;
- \( I_{xy}, I_{xz} \) are the POI values about the equipment plane.

### 4. MGSE design

Because large bias moments caused by COG, special kind of mechanical grounding support equipment (MGSE) should be designed in order to assure the unbalancing force less than the capacity of spin balancing machine. Fig.4 is an adapter model designed for the cabin, and the bias center hole is designed to reduce bias moments. The distance between the two holes is same as the calculated COG value provided by the designer.

![Figure 4. Model of MGSE.](image)

Fig. 5 is the stain and deformation after installation using finite element analysis.
5. Data processing

Though the POI values about equipment plane have been adjusted by “moment of inertia” method and accurately measured by “dynamic balancing method”, the required values are under second coordinate system. So a post calculated software has been developed to transfer the data to required coordinate system.

Eq. 4 is the transfer method.

\[
[D']_{3x3} = [A]_{3x3} [D]_{3x3} [A]^{-1}
\]  

In this equation:
- \([D']_{3x3}\) is the calculated values;
- \([A]_{3x3}\) is inverse of rotation matrix;
- \([A]_{3x3}\) is rotation matrix;

Fig. 6 is the developed software interface using labview.

After each POI measurement, the POI values under second coordinate system are calculated by the software to verify whether the O1X1 axis is the principle axis and to decide if the balancing weight is suitable.

Repeat it several times to achieve measurement requirement.
6. Conclusion
For passive control reentry cabin with large moments and large product of inertia, inclined principle axis can be adjusted and high accuracy can be achieved according to combine “moment of inertia” method and “dynamic balancing method”, special MGSE and data processing software are needed to finish the goal.

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