Electromagnetic transition form factors of the Roper resonance in baryon chiral perturbation theory

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Abstract

We consider the electromagnetic transition form factors of the Roper resonance in the framework of an effective field theory of pions, nucleons and delta and Roper resonances as explicit degrees of freedom. Due to the lack experimental data in the region of applicability of low energy effective field theory we fit available free coupling constants and compare obtained results compare to the predictions of a theoretical model.

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I. INTRODUCTION

Chiral perturbation theory [1, 2] - low-energy effective field theory (EFT) of QCD leads to a successful description of the Goldstone boson sector. It turned out that generalisation of ideas of ChPT to the EFTs with heavy degrees of freedom is a non-trivial problem. In baryon chiral perturbation theory dimensionally regulated loop diagrams violate the power counting [3]. This problem has been solved by using the heavy-baryon approach [4–6] and later by choosing a suitable renormalization scheme [7–10]. While the Δ resonance and (axial) vector mesons can also be consistently included in low-energy EFT (see e.g. Refs. [11–20]), the treatment of heavier baryons such as the Roper resonance is more complicated. Ref. [21] presented new ideas on the extension of the range of applicability of chiral EFT beyond the low-energy region.

The Roper resonance has been found in a partial wave analysis of pion-nucleon scattering data long time ago [22]. Since then it has attracted particular interest because of being the first nucleon resonance that exhibits a decay mode into a nucleon and two pions, besides the decay into a nucleon and a pion. Another interesting feature is that the Roper appears below the first negative parity nucleon resonance, the $S_{11}(1535)$. Despite of much effort a satisfactory theory of the Roper resonance is still missing.

First steps in direction of addressing the Roper resonance state in a chiral EFT have been made in Refs. [21, 23–26]. The pole mass and the width of the Roper resonance has been calculated at one-loop order in Refs. [23, 24] and at two-loop order in Ref. [27], the magnetic moment was studied in Ref. [25]. The impact of the explicit inclusion of the Roper resonance in chiral EFT on the $P_{11}$ pion-nucleon scattering phase shifts has been studied in Ref. [26]. Recently electromagnetic transition form factors of the Roper resonance have been analysed in and EFT of pions, nucleons, Roper resonance and vector mesons [28].

In this work we calculate the electromagnetic transition form factors of the Roper resonance in a systematic expansion in the framework of baryon chiral perturbation theory with pions, nucleons, the delta and Roper resonances as explicit degrees of freedom. Unlike the Ref. [28] we consider only small values of the virtual photon momentum square $q^2 \sim M_{\pi}^2$.

The paper is organized as follows: in Section II we specify the effective Lagrangian, in Section III the transition form factors are defined contributing Feynman diagrams are identified. Section IV contains the numerical results and we summarize in Section V.

II. EFFECTIVE LAGRANGIAN

Below we specify the terms of the chiral effective Lagrangian which are relevant for the calculation of the electromagnetic transition form factors of the Roper at next-to-leading order. We consider pions, nucleons and the delta and Roper resonances as dynamical degrees of freedom. The effective Lagrangian can be written as

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{\pi \Delta} + \mathcal{L}_{\pi R} + \mathcal{L}_{\pi N \Delta} + \mathcal{L}_{\pi N R} + \mathcal{L}_{\pi \Delta R}, \]

where the subscripts indicate the dynamical fields which contribute to a given term. From the purely mesonic sector we only need the leading order terms given by [2, 29]

\[ \mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4} \text{Tr} \left[ \partial_\mu U \partial^\mu U^\dagger \right] + \frac{F^2 M^2}{4} \left[ U^\dagger + U \right] + i \frac{F^2}{2} \left[ (\partial_\mu U U^\dagger + \partial^\mu U^\dagger U) v_\mu \right], \]

\[ \mathcal{L}_{\pi N}^{(2)} = \frac{F^2}{4} \text{Tr} \left[ \partial_\mu U N^{\dagger} \partial^\mu U N \right] + \frac{F^2 M^2}{4} \left[ N^{\dagger} + N \right] + i \frac{F^2}{2} \left[ (\partial_\mu U N^{\dagger} + \partial^\mu U^{\dagger} N) v_\mu \right], \]

\[ \mathcal{L}_{\pi \Delta}^{(2)} = \frac{F^2}{4} \text{Tr} \left[ \partial_\mu \Delta^{\dagger} \partial^\mu \Delta \right] + \frac{F^2 M^2}{4} \left[ \Delta^{\dagger} + \Delta \right] + i \frac{F^2}{2} \left[ (\partial_\mu \Delta^{\dagger} + \partial^\mu \Delta^{\dagger}) v_\mu \right], \]

\[ \mathcal{L}_{\pi R}^{(2)} = \frac{F^2}{4} \text{Tr} \left[ \partial_\mu R^{\dagger} \partial^\mu R \right] + \frac{F^2 M^2}{4} \left[ R^{\dagger} + R \right] + i \frac{F^2}{2} \left[ (\partial_\mu R^{\dagger} + \partial^\mu R^{\dagger}) v_\mu \right], \]

\[ \mathcal{L}_{\pi N \Delta}^{(2)} = \frac{F^2}{4} \text{Tr} \left[ \partial_\mu N \partial^\mu \Delta \right] + \frac{F^2 M^2}{4} \left[ N \partial^\mu \Delta + \partial^\mu N \Delta \right] + i \frac{F^2}{2} \left[ (\partial_\mu N \partial^\mu \Delta + \partial^\mu N \partial^\mu \Delta) v_\mu \right], \]

\[ \mathcal{L}_{\pi N R}^{(2)} = \frac{F^2}{4} \text{Tr} \left[ \partial_\mu N \partial^\mu R \right] + \frac{F^2 M^2}{4} \left[ N \partial^\mu R + \partial^\mu N R \right] + i \frac{F^2}{2} \left[ (\partial_\mu N \partial^\mu R + \partial^\mu N \partial^\mu R) v_\mu \right], \]

\[ \mathcal{L}_{\pi \Delta R}^{(2)} = \frac{F^2}{4} \text{Tr} \left[ \partial_\mu \Delta \partial^\mu R \right] + \frac{F^2 M^2}{4} \left[ \Delta \partial^\mu R + \partial^\mu \Delta R \right] + i \frac{F^2}{2} \left[ (\partial_\mu \Delta \partial^\mu R + \partial^\mu \Delta \partial^\mu R) v_\mu \right]. \]
where $F$ is the pion decay constant in the chiral limit and $M$ is the leading term in the quark mass expansion of the pion mass $\bar{2}$. The external electromagnetic field $A_\mu$ is represented in $v_\mu = -e A_\mu \tau_3 / 2$, where $e^2 / (4\pi) \approx 1 / 137 \ (e > 0)$. The pion field in contained in the unimodular unitary $2 \times 2$ matrix $U$.

The terms of the effective Lagrangian with pions and baryons needed for our calculation are given by:

\[
\begin{align*}
\mathcal{L}_{\pi N}^{(1)} &= \bar{\Psi}_N \left\{ i \partial_\mu - m + \frac{1}{2} g \not\! \! \partial \gamma^5 \right\} \Psi_N , \\
\mathcal{L}_{\pi R}^{(1)} &= \bar{\Psi}_R \left\{ i \partial_\mu - m_R + \frac{1}{2} g_R \not\! \! \partial \gamma^5 \right\} \Psi_R , \\
\mathcal{L}_{\pi R}^{(2)} &= \bar{\Psi}_R \left\{ \bar{c}^R (\chi^+) \right\} \Psi_R , \\
\mathcal{L}_{\pi NR}^{(1)} &= \bar{\Psi}_R \left\{ \frac{g_{\pi NR}}{2} \gamma_\mu \gamma_5 u_\mu \right\} \Psi_N + \text{h.c.} , \\
\mathcal{L}_{\pi NR}^{(2)} &= \bar{\Psi}_R \left\{ \frac{c^R_6}{2} f^\mu_+ + \frac{c^R_7}{2} v^{(s)}_\mu \right\} \sigma^{\mu \nu} \Psi_N + \text{h.c.} , \\
\mathcal{L}_{\pi NR}^{(3)} &= \frac{i}{2} d_6^\mu \bar{\Psi}_R [D^\mu , f^{(s)}_\mu ] D^\nu \Psi_N + 2i d_7^\mu \bar{\Psi}_R (\partial^\mu v^{(s)}_\mu ) D^\nu \Psi_N + \text{h.c.} , \\
\mathcal{L}_{\pi}^{(1)} &= -\bar{\Psi}_i \tilde{\xi}^i_{\mu \nu} \Theta^{\mu \nu} (z_1) \omega_\alpha^j \Psi_N + \text{h.c.} , \\
\mathcal{L}_{\pi}^{(2)} &= h R \bar{\Psi}_i \tilde{\xi}^i_{\mu \nu} \Theta^{\mu \nu} (z) \omega_\alpha^j \Psi_R + \text{h.c.} .
\end{align*}
\]

Here $\Psi_N$ and $\Psi_R$ are isospin doublet fields of the nucleon and the Roper resonance, respectively. The $\Delta$ resonance is represented by the vector-spinor isovector-isospinor Rarita-Schwinger field $\Psi_\nu$ $[34]$, $\tilde{\xi}^i$ is the isospin-3/2 projector, $\omega_\alpha^j = \frac{1}{2} \text{Tr} \ [\tau^i u_\alpha]$ and $\Theta^{\mu \alpha} (z) = g^{\mu \nu} + z \gamma^\mu \gamma^\nu$ with an off-shell parameter $z$. We fix the off-shell structure of the interactions involving the delta by taking $g_3 = g_3 = 0$ and $z_1 = z = 0$. It has been shown that these off-shell parameters can be absorbed in LECs $[31, 33]$. Further building blocks are given by

\[
\begin{align*}
\mu &= \sqrt{U} , \\
\chi^+ &= M^2 (U^\dagger + U) , \\
u_\mu &= i \left[ u^\dagger \partial_\mu u - u \partial_\mu u^\dagger - i \left( u^\dagger v_\mu u - u v_\mu u^\dagger \right) \right] , \\
D_\mu \Psi_{N/R} &= \left( \partial_\mu + \Gamma_\mu - i v^{(s)}_\mu \right) \Psi_{N/R} , \\
(D_\mu \Psi)_{\nu,i} &= \partial_\mu \Psi_{\nu,i} - 2i \epsilon_{i j k} \Gamma_{\mu,k} \Psi_{\nu,j} + \Gamma_{\mu} \Psi_{\nu,i} , \\
\Gamma_\mu &= \frac{1}{2} \left[ u^\dagger \partial_\mu u + u \partial_\mu u^\dagger - i \left( u^\dagger v_\mu u - u v_\mu u^\dagger \right) \right] = \tau_k \Gamma_{\mu,k} , \\
v^{(s)}_{\mu \nu} &= \partial_\mu v^{(s)}_\nu - \partial_\nu v^{(s)}_\mu , \\
f^\mu_+ &= u f^\mu_+ u^\dagger + u^\dagger f^\mu_+ u , \\
f^\mu_+ &= \partial_\mu v_\nu - \partial_\nu v_\mu - i [v_\mu , v_\nu] .
\end{align*}
\]

A mixing kinetic term of the form $i \lambda_1 \bar{\Psi}_R \gamma_\mu D^\mu \Psi_N - \lambda_2 \bar{\Psi}_R \Psi_N + \text{h.c.}$ is not included in the effective Lagrangian since, using field transformations it can be reduced to the form specified above $[23]$. 

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III. ELECTROMAGNETIC TRANSITION FORM FACTORS OF THE ROPER RESONANCE

Most general parametrisation of the $\gamma^*NR$ vertex function contains three independent Lorentz structures. However due to the current conservation the coefficients of these structures satisfy one identity. Therefore the renormalized vertex function of the $R \rightarrow \gamma^*N$ transition can be parameterized as

$$\sqrt{Z_R} \tilde{\omega}^i(p_f) \Gamma^\mu(p_f, p_i) u^j(p_i) \sqrt{Z_N}$$

$$= \tilde{\omega}^i(p_f) \left[ \left( \gamma^\mu - q^\mu q^2 \right) \tilde{F}_1^*(Q^2) + \frac{i \sigma^{\mu\nu} q^\nu}{M_R + m_N} \tilde{F}_2^*(Q^2) \right] u^j(p_i) \sqrt{Z_N},$$

where $Q^2 = -(p_f - p_i)^2$ and $Z_R$ and $Z_N$ are the residues of the dressed propagators of the Roper and the nucleon, respectively.

By counting the mass differences $m_R - m_N$, $m_\Delta - m_N$ and $m_R - m_\Delta$ as of the same order as the pion mass and the pion momenta, i.e. $\sim q^1$ the standard counting can be applied to all diagrams contributing to the considered transition form factors. According to the rules of this counting a four-dimensional loop integration is of order $q^4$, an interaction vertex obtained from an $\mathcal{O}(q^n)$ Lagrangian counts as of order $q^n$, a pion propagator as order $q^{-2}$, and a nucleon propagator as order $q^{-1}$. Further, order $q^{-1}$ is assigned to the $\Delta$ and the Roper resonance propagators for non-resonant kinematics. The propagators of the delta and the Roper resonance get enhanced for resonant kinematics when they appear as intermediate states outside the loop integration [12]. In this case order $q^{-3}$ is assigned to these propagators. Up to next-to-next-to leading order according to the above counting there are tree and one loop diagrams contributing to the electromagnetic transition form factors.
of the Roper resonance, shown in Fig. 1. However, due to the large mass difference $m_R - m_N \sim 400 \text{ MeV} \gg M_\pi = 135 \text{ MeV}$, the above mentioned power counting cannot be trusted. Assigning order $\delta^1$ to $m_R - m_N$ it is more appropriate to count $M_\pi \sim \delta^2$. While different contributions of the same order according the standard counting now become of unequal importance, still contributions to the transition form factors up to next-to-next-to leading order are generated by the same diagrams, shown in Fig. 1. Loop diagrams calculated using the standard dimensional regularisation do not satisfy the power counting, however all power counting violating pieces are polynomial in momenta and the squared pion mass and hence can be absorbed in the renormalization of the coupling constants. We have explicitly verified that this is the case.

Notice that loop contributions to the residues of the nucleon and the Roper resonance propagators multiplied with the tree order contributions to the transition form factors start the order higher than the accuracy of our calculation and therefore we do not take them into account.

IV. NUMERICAL RESULTS

To obtain numerical results for the transition form factors we use the following standard values of the parameters [34]

\[
M_\pi = 139 \text{ MeV}, \quad m_N = 939 \text{ MeV}, \quad m_\Delta = 1210 \pm 1 \text{ MeV}, \quad \Gamma_\Delta = 100 \pm 2 \text{ MeV}, \\
m_R = 1365 \pm 15 \text{ MeV}, \quad F_\pi = 92.2 \text{ MeV}, \quad g_A = 1.27.
\] (6)

Further we substitute $h = 1.42 \pm 0.02$, where the latter is the real part of this coupling taken from Ref. [35] (the given error takes into account only the statistical uncertainties). The imaginary part of $h$ only contributes to orders beyond the accuracy of our calculation and therefore we do not include it here. We use $g_{\pi NR} = \pm (0.47 \pm 0.05)$ [27] and in what follows we take both signs into account which contributes to the error budget. Further, we assume $g_R = g_A$ ($g_A$ is axial coupling of pion and nucleon, difference between $g$ and $g_A$ is in higher order and can be neglected) and $h_R = h$, which corresponds to the maximal mixing assumption of Ref. [36], where the $\Delta$ and Roper resonances and the nucleon are considered as chiral partners in a reducible representation of the full QCD chiral symmetry group.

As there is no experimental data available for low $q^2$, we used the numerical results obtained from the parameterisation of Ref. [37]. We fixed the unknown parameters contributing at tree order by reproducing the $F_2^*$ form factor of the proton at low $q^2$.

In Fig. 2 we plot our obtained results.

V. SUMMARY

In current work we have calculated the electromagnetic transition form factors of the Roper resonance up to next-to-leading order in a systematic expansion of baryon chiral perturbation theory with pions, nucleons, delta and Roper resonances as dynamical degrees of freedom.

So far experimental data is not available in the region of applicability of chiral effective field theory. Therefore we fitted available free parameters and compared our results to predictions of the theoretical parametrization of Ref. [37].
FIG. 2: Numerical results for the transition form factors. Solid (red) lines correspond to the parameterisation of Ref. [37] and dashed (blue) lines represent the results for the current work.

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