Capillary surfers:
wave-driven particles at a fluid interface

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Active systems have recently attracted considerable interest for the possibility of extending statistical mechanics to incorporate non-equilibrium phenomena [1,2]. Among active systems, self-propelled particles are natural or artificial objects that convert energy from the environment into directed motion [3]. Vibrating platforms are suitable sources of diffuse energy to observe macroscopic self-propelled particles [4, 5] and their collective behaviour [6–10] with table-top experiments. Here we introduce capillary surfers: solid particles propelled by their self-generated wave field on the surface of a vibrating liquid bath. The surfer speed and its interaction with the environment can be tuned through the particle shape, mass, fluid properties and the vibration parameters. The wave nature of the interactions among surfers allows for multistability of states and promises a number of novel collective behaviours. Capillary surfers benefit from the interplay between dissipation and inertia manifesting as a field of unsteady surface waves, and thus have the potential to fill the gap [11] between dissipation- [2, 12–15] and inertia-dominated active systems [16–18].

Self-propulsion of artificial particles has been achieved from nanometric to centimetric scale with a number of different strategies [3]. At the macroscopic scale, artificial self-propelled particles can be obtained on tabletop experiments via the rectification of mechanical vibrations. Solid asymmetric particles can self-propel with an internal source of energy [19] or by converting energy from the vibration of the platform on which they move [120]. On the other hand, fluid interfaces are convenient platforms for the self-propulsion of natural [21] and artificial bodies, including solid particles [22, 23], drops [5, 26, 27] and small-scale robots [28]. When the
Figure 1: A capillary surfer self-propels on a fluid interface due to its self-generated waves (Supplementary Video 1). (a) Surfer geometry. (b) Side view schematic of the experimental setup (not to scale). (c) Oblique wave field visualization, in which colours are obtained from the distorted reflection of a yellow and blue background on the fluid surface. (d) Dependence of the surfer speed $U$ on the forcing frequency $f$ and forcing acceleration $\gamma$, as collapsed by Eq. (1), with $L = 4.20 \pm 0.03$ mm, $w = 2.70 \pm 0.03$ mm, $a = L/2$, the parameters being the same in (c). Inset: dependence of $U$ on the asymmetry $a/L$ for $f = 100$ Hz, $\gamma = 3.3$ g, $L = 6.44 \pm 0.02$ mm, $w = 4.13 \pm 0.02$ mm. We use $h_+ = 1.22$ mm and $h_- = 0.82$ mm throughout all experiments.

interface is vibrating, millimetric droplets may self-propel by bouncing on the sloped surface wave they generate in certain parameter regimes [5], and floating drops may be deformed and driven by Faraday surface waves [26, 27]. For floating drops, propulsion mechanisms based on asymmetric vortex generation [27] and wave radiation pressure [26] have been proposed, but the drops are difficult to manipulate directly due to their self-adaptive nature. The capillary surfers we introduce here are highly tunable and propel as a result of the asymmetric radiation pressure of the waves they generate on the vibrating liquid surface.

In our experiments, capillary surfers had the rectangular shape represented in Fig. 1a. The centre of mass was offset with respect to their in-plane geometrical centre. Surfers were deposited on a bath of water-glycerol mixture and were supported at the liquid-air interface by virtue of the equilibrium between their weight, hydrostatic forces and surface tension. The contact line of the bath was pinned to the surfer’s base perimeter. As a result of their mass asymmetry, surfers were slightly tilted in equilibrium (Fig. 1b) and the deformation of the interface varied along their perimeter. The liquid bath was vertically driven by an electromagnetic shaker with acceleration $\Gamma(t) = \gamma \cos \omega t$, with $\omega = 2\pi f$ and $f$ the forcing frequency in the range 40–100 Hz. (See Methods for more details on experimental materials and procedures.)

As soon as the bath was set into vibration, a surfer generated propagating surface waves as a result of the
relative vertical motion between the surfer and bath, a consequence of the surfer’s inertia. Correspondingly, the surfer moved along its long axis in the direction of its thinner half (Fig. 1b–d). In the absence of external perturbations or manufacturing imperfections, the surfer moved with constant velocity $U$ along a rectilinear trajectory. Taking inspiration from marine terminology, in the following we refer to the front and back of the surfer as the “bow” and “stern”, respectively. We proceeded by analysing the motion of a single surfer as a function of the forcing parameters, the surfer asymmetry, and its size (Fig. 1 and Supplementary Figures S1 and S2). For a given surfer size and asymmetry and a fixed forcing frequency $f$, the surfer speed increases with the forcing acceleration $\gamma$. For a fixed acceleration amplitude, the speed decreases with frequency.

In order to rationalise this behaviour, we consider the wave radiation stress generated by the surfer. The radiation stress $S$ of surface waves may be defined as “the excess flow of momentum due to the presence of the waves” [29]. For capillary waves of amplitude $A$ and wavenumber $k$ in the deep-water limit $kH \gg 1$, $S = (3/4)\sigma A^2k^2$, where $H$ is the bath depth and $\sigma$ the surface tension. Experimental measurements of the dependence of the surfer speed on its asymmetry show that the speed is maximised when the displacement of the centre of mass is maximised, which corresponds to the highest difference in the equilibrium deformation of the liquid surface between bow and stern (inset of Fig. 1d). The radiation stress generated by the surfer thus exhibits a fore-aft asymmetry: waves of larger amplitude $A_+$ are generated at the stern, where the effective mass is larger, while waves of smaller amplitude $A_-$ are generated at the bow. As a result, the surfer experiences a net propulsive force $F_p = (3/4)\sigma k^2w(A_+^2 - A_-^2)$, where $w$ is the surfer width. This propulsive force is resisted by viscous shear beneath the surfer. By approximating this flow as a locally fully-developed Couette flow, the viscous force on a surfer with speed $U$ is $F_D \approx \eta wL/\gamma^2$. The surfer speed is ultimately prescribed by the balance of $F_p$ and $F_D$, which yields

$$U = \frac{3H\sigma k^2}{4\eta L} \left( A_+^2 - A_-^2 \right) = \frac{3H\sigma}{4\eta L} \left( k\frac{\gamma}{\omega^2} \right)^2 \left( q_+^2 - q_-^2 \right),$$

where we assume that the wave amplitudes $A_{\pm}$ are proportional to the bath forcing amplitude $\gamma/\omega^2$ with unknown constants of proportionality $q_{\pm}$. Fig. 1d shows that the experimental data may be collapsed using this scaling relationship (see raw data in Supplementary Figure S1), confirming the hypothesis of radiation stress as the mechanism underlying surfer propulsion. The discrepancy observed at large forcing amplitude may be ascribed to the influence of nonlinear effects on the wave profile that arise for large wave amplitudes, since the expression for the radiation stress we used is derived for linear waves [29].

Surfers may interact with each other through the propagating waves they generate on the fluid interface. We observed that two surfers of equal size and speed can arrange into up to seven qualitatively distinct modes of interaction (Fig. 2 and Supplementary Video 2): head-to-head (c), back-to-back (d), orbiting (e), promenading (f), tailgating (g), t-bone (h) and jackknife (i). In the head-to-head (back-to-back) mode the two surfer sterns (bows) face each other (Fig. 2c,d). While these modes are static, the remaining five modes are dynamic. In the orbiting mode, the two surfers orbit around the system’s centre of mass (Fig. 2e). In the promenade mode, they proceed side by side with constant speed along a rectilinear trajectory (Fig. 2f). In the tailgating mode, they are aligned along their major axis, with the stern of one surfer pointing toward
Figure 2: The seven interaction modes of surfer pairs (see also Supplementary Video 2). (a,b) Side view (a) and top view (b) schematics of the theoretical model, in which a surfer is represented by two unequal point masses $m_+$ and $m_-$. (c–i) Modes observed in experiment (top) and theory (bottom), for $f = 100$ Hz and $\gamma = 3.3$ g. (c) Head-to-head, (d) back-to-back, (e) orbit, (f) promenade, (g) tailgate, (h) t-bone, (i) jackknife. In experiments, $L = 4.20 \pm 0.03$ mm, $w = 2.70 \pm 0.03$ mm, $a = L/2$. Parameters used for the theoretical predictions are detailed in the Methods.
the bow of the other, and they move with constant speed along a rectilinear trajectory (Fig. 2c). In the t-bone mode, the two major axes are perpendicular to each other and the stern of one surfer points toward the bow of the other, while they both follow a circular trajectory (Fig. 2a). The jackknife mode has a similar configuration except the two bows face each other (Fig. 2). Note that should the vibration be eliminated, the surfers cease to propel and immediately collapse in towards each other under the influence of capillary attraction.

These interaction modes can be captured by a conceptually simple theoretical model for the surfers’ horizontal dynamics. We model a surfer of mass \( m \) as a pair of point masses \( m_+ \) and \( m_- \), where \( m_+ > m_- \) and \( m_+ + m_- = m \), connected by a massless rod of length \( l = L/2 \) and floating on the surface of an inviscid and incompressible fluid bath (Fig. 2a,b). We describe the trajectory of the \( i \)th surfer by its center of mass \( \mathbf{x}_i(t) \in \mathbb{R}^2 \) and orientation (unit) vector \( \mathbf{n}_i(t) \), which points from \( m_+ \) to \( m_- \). The masses \( m_\pm \) are chosen to represent the surfer’s asymmetric mass distribution in experiments (Fig. 1a), and are located at \( \mathbf{x}_{i,\pm} = \mathbf{x}_i \mp m_+/m \). The masses are assumed to oscillate vertically at the forcing frequency \( \omega \) and amplitude \( \gamma/\omega^2 \) of the bath, and thus generate capillary waves of wavelength \( \lambda = 2\pi/k \) given by the dispersion relation in the deep-water limit, \( \omega^2 = \sigma k^3/\rho \). Since the waves are of small amplitude, \( \gamma/\omega^2 \ll \lambda \), their form may be deduced by solving the linearized hydrodynamic problem of a periodically oscillating point force acting on a fluid interface [30] (see Methods). Each surfer is a pair of capillary wave sources, which mediate the interaction between surfers.

Each surfer moves in response to three forces: a propulsive force due to radiation pressure \( F_p \mathbf{n}_i \), a drag force proportional to its velocity \( \dot{\mathbf{x}}_i \) due to the viscous shear stress under the surfer and a lateral capillary wave force proportional to the local gradient of the interfacial deformation. Specifically, each mass on a given surfer experiences a force due to the linear superposition of capillary waves generated by all other surfers, as shown schematically in Fig. 2b. Both the surfers and capillary waves oscillate at the forcing frequency \( \omega \), and the capillary interaction force is nonzero when time-averaged over the forcing period [30] (see Methods). Crucially, this force has an oscillatory spatial dependence \( \Phi(kr) \), oscillating between attractive and repulsive on the capillary wavelength \( \lambda \). The equations of motion for a collection of identical interacting surfers are thus

\[
\begin{align*}
\mathbf{m}_+ \ddot{\mathbf{x}}_i + \frac{m}{\tau_v} \dot{\mathbf{x}}_i &= F_p \mathbf{n}_i + F_c \sum_{\alpha,\beta = \pm 1} \mu_\alpha \mu_\beta \sum_{j \neq i} \Phi(k|\mathbf{x}_{j,\beta} - \mathbf{x}_{i,\alpha}|) \frac{\mathbf{x}_{j,\beta} - \mathbf{x}_{i,\alpha}}{|\mathbf{x}_{j,\beta} - \mathbf{x}_{i,\alpha}|}, \\
I \dddot{\theta}_i + \frac{I}{\tau_v} \dot{\theta}_i &= -l F_c \sum_{\alpha,\beta = \pm 1} \mu_\alpha \mu_\beta \sum_{j \neq i} \alpha \mu_{-\alpha} \Phi(k|\mathbf{x}_{j,\beta} - \mathbf{x}_{i,\alpha}|) \mathbf{n}_i \times \frac{\mathbf{x}_{j,\beta} - \mathbf{x}_{i,\alpha}}{|\mathbf{x}_{j,\beta} - \mathbf{x}_{i,\alpha}|} \cdot \mathbf{z},
\end{align*}
\]

where \( \mathbf{x}_{i,\alpha} = \mathbf{x}_i - \alpha \mu_{-\alpha} \mathbf{n}_i \), \( I \) is the moment of inertia, \( F_c = (mg)^2 k/\sigma \) is the capillary force coefficient and \( \tau_v = mH/\eta w L \) is the viscous timescale. The first and second equations account for the lateral force and torque balances on each surfer, respectively, as detailed in the Methods. The trajectory equations contain a single unknown parameter \( F_p \), whose value \( F_p = mU/\tau_v \) is directly inferred from the experimentally measured free speed \( U \) of a single surfer in isolation. Numerical simulations of the model [2] successfully recover the interaction modes of surfer pairs (see Methods for details), and the theoretically predicted wave forms are
qualitatively similar to those observed in experiments (Fig. 2c–i, lower panels). In each mode the two surfers exhibit discrete equilibrium spacings \( d \approx n\lambda \) where \( n = 1, 2, \ldots \). The maximum order \( n_{\text{max}} \) achievable per mode depends on the properties of the liquid, vibration, and surfers. We focused on the promenade mode as in our experiments it exhibited the largest order observed: \( n_{\text{max}} = 4 \) (Fig. 3). First, we fixed the forcing frequency and measured the promenade spacing for increasing forcing acceleration. The spacing \( d \) increases very slightly with forcing acceleration \( \gamma \) (Fig. 3e), but is significantly more sensitive to the forcing frequency \( f \). The normalized spacing \( d/\lambda \) is approximately independent of \( f \), showing that the equilibrium spacings are quantized by the capillary wavelength in the range of forcing frequencies explored (Fig. 3f). Numerical simulations of the theoretical model (2) exhibit excellent quantitative agreement with the experimental results (Fig. 3e,f).

Many-body experiments and simulations show that capillary surfers have potential as constituents of a novel active system (Fig. 4 and Supplementary Video 3). For instance, a many-body promenade mode has been observed in experiments (Fig. 4a) and simulations (Fig. 4d). Experiments reveal a super-orbiting state of eight surfers (Fig. 4b), and simulations show that a collection of nine surfers may execute a flocking state in an orderly lattice formation (Fig. 4c). While there have been extensive studies on overdamped active systems (i.e. bacterial suspensions) \(^2\), mediated by viscous hydrodynamic forces that decay monotonically with distance, a collection of surfers has the peculiar feature of being characterised by wave-mediated interactions, which results in long-range spatially-oscillatory forces defined by alternating regions of attraction and repulsion. This feature is a consequence of fluid inertia, and responsible for the multistability of a discrete set of interaction states, as documented here for the promenade mode (Fig. 3). The influence of multistability is particularly evident in the exotic many-body mode shown in Fig. 4d: the spacing between surfers is approximately either one or two capillary wavelengths, so the mode may be interpreted as an aggregate of \( n = 1 \) (Fig. 3a) and \( n = 2 \) (Fig. 3b) promenade modes.

Novel collective behaviours at a fluid interface have been observed with camphor boats \(^{22}\) and walking droplets \(^{10}\) previously, but only within a relatively narrow parameter space defined by the physical constraints of these systems. Furthermore, the richness of the two-body interaction landscape (Fig. 2) documented here for the surfer system already far exceeds those documented for the aforementioned systems. Ultimately, the surfer system is a uniquely tunable and accessible experimental platform that has the potential to fill the gap between active systems at low and high Reynolds numbers: the surfers used in the present work self-propel at intermediate Reynolds number \( \text{Re} = \rho UL/\eta \approx 1–10 \), where both inertia and viscous forces are relevant. Recent reviews \(^3\) underline the need for novel experimental platforms and theoretical frameworks to explore systems in this intermediate regime, which has received far less attention than the purely viscous \(^2\) and inertial \(^{16–18}\) regimes. It is anticipated that such systems could exhibit novel classes of self-organising behaviours, such as soliton-like waves, shock-like phenomena, new flocking states (e.g. Fig. 4) and non-equilibrium phase transitions \(^3\). Our discovery of the surfer system lays the foundation for a novel branch of exploration within the context of experimental and theoretical active matter,
Figure 3: Promenade modes observed in experiments (theory), shown in the top (bottom) panels in (a)–(d) and points (curves) in (e, f). In (a)–(d), $\gamma = 3.3 \, g$ and $f = 100 \, Hz$. (e) Dependence of the dimensionless inter-surfer spacing $d/\lambda$ on the forcing acceleration $\gamma$ for fixed forcing frequency $f = 100 \, Hz$. (f) Dependence of $d/\lambda$ on $f$ just below the Faraday instability threshold. In experiments, $f$ ranges from 50–100 Hz in increments of 10 Hz, and the corresponding values of $\gamma/g$ are 1.1, 1.5, 2.0, 2.3, 3.0 and 3.5. In the model, $d$ is defined as the distance between the surfers' centres of mass, while in experiments the surfers' finite width is accounted for by subtracting $w$. In experiments, $L = 4.20 \pm 0.03 \, mm$, $w = 2.70 \pm 0.03 \, mm$ and $a = L/2$. Parameters used for the theoretical predictions are detailed in the Methods.
Figure 4: Examples of collective behaviour of surfers (Supplementary Video 3). (a) Promenade mode of four surfers observed in experiments. (b) Super-orbiting mode of eight surfers observed in experiments. (c) Numerical simulation of a flocking state of nine surfers in a lattice formation. (d) Exotic promenade mode of thirteen surfers obtained in simulations. (a, b) $f = 100$ Hz, $\gamma = 3.3$ g, $L = 2.78 \pm 0.01$ mm, $w = 1.75 \pm 0.01$ mm, $a = L/2$. (c, d) Parameters used for the theoretical predictions are detailed in the Methods.

and can serve to help address outstanding questions while inspiring previously unforeseen directions.

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IH, GP, and DMH designed, constructed, and performed the experiments, and analyzed the experimental data. AUO developed the theoretical model and numerical simulation, and analyzed the simulation data. GP, AUO, and DMH interpreted the experimental and theoretical results and wrote the paper.

The authors declare that they have no competing financial interests.

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Methods

Liquid bath

The bath was composed of a glycerol-water mixture and had diameter 10 cm. In order to precisely infer the volume fraction of glycerol, we measured the mixture density \( \rho = 1.1755 \pm 0.0001 \text{ g/cm}^3 \) at \( T = 21.7 \degree C \) with a densitometer (DM 35 Basic, Anton Paar). Using an empirical formula [M1] the glycerol volume fraction was determined to be 63.2%. The dynamic viscosity \( \eta = 0.0192 \pm 0.0002 \text{ Pa-s} \) was measured at \( T = 22.0 \degree C \) using a conical rheometer (ARES-G2, TA Instruments), a result that is in close agreement with the viscosity \( \eta = 0.0184 \text{ Pa-s} \) obtained at the same temperature from an empirical formula [M2].

All experiments were conducted at the temperature \( T = 21.5 \pm 0.5 \degree C \), at which \( \rho = 1.175 \pm 0.003 \text{ g/cm}^3 \) and \( \eta = 0.0197 \pm 0.0005 \text{ Pa-s} \), where uncertainties are due to temperature fluctuations and estimated from empirical formulae [M1,M2]. The value \( \eta = 0.0197 \pm 0.0005 \text{ Pa-s} \) was used for the scaling in equation (I) and Fig. [I]. The surface tension \( \sigma = 66.4 \pm 0.5 \text{ mN/m} \) was measured at \( T = 21.5 \degree C \) by using a pendant drop method [M3]. A graduated cylinder with resolution 1 mL was used to measure and pour 45 mL of liquid into the bath. The resulting depth of the bath was \( H = 5.73 \pm 0.06 \text{ mm} \). The Faraday instability threshold \( \gamma_F \) of the vibrated bath was measured before, in between, and after sets of experiments in order to ensure that the physical properties of the liquid remained unchanged, and ranged in the interval \( \gamma_F = 3.75-3.85 \text{ g} \).

Surfer manufacturing

Centimeter-sized surfers with width \( w = 1.75-7.09 \text{ mm} \) and length \( L = 2.78-10.85 \text{ mm} \) were manufactured out of white Polytetrafluoroethylene (PTFE) sheets with density \( 2.2 \text{ g/cm}^3 \), which are chemically hydrophobic. An adhesive PTFE sheet with thickness 0.40 mm was attached to the upper surface of a PTFE sheet with thickness 0.82 mm. A laser cutter (Universal Laser Systems, VLS 4.60) was then used to cut out rectangular profiles with rounded corners (with radius equal to 0.17L) to avoid sharp corners along the contact line, which were observed to reduce the reproducibility of the surfer motion. In order to introduce an asymmetry in the surfer distribution of mass, a laser engraved line was used as a guide to cut out a portion of the upper adhered layer using a fine razor. The height of the surfer’s stern and bow were thus \( h_+ = 1.22 \text{ mm} \) and \( h_- = 0.82 \text{ mm} \), respectively.

Vibration setup

The bath container was made of acrylic and directly mounted on an electrodynamic shaker (The Modal Shop, 2025E) connected to an amplifier (The Modal Shop, 2100E21). Two accelerometers (PCB Piezoelectronics, 352C65) were attached at diametrically opposed ends of the container and their signals were acquired by a computer through a data acquisition device (National Instruments, USE-6343). The reported acceleration was the mean of the measurements of the accelerometers. A closed feedback loop was used to maintain the mean acceleration at a specified target value. The acceleration difference between the accelerometers was
also recorded at each frequency. The shaker was mounted on a isolated optical bench (Thorlabs, SDA75120) to reduce the influence of external vibrations. A monochrome USB camera (Allied Vision, Mako) with a macro lens for video acquisition was mounted above the bath and normal to its surface. In order to increase the contrast of the video recordings, the bath’s base was constructed out of a black acrylic plate. The system was placed inside an acrylic box to isolate the bath from ambient air currents and contaminants.

Visualization

Tracking was performed with the surfers appearing as white objects on a dark bath on the recordings of a camera placed vertically above the bath. The wave field was visualized using a semi-transparent mirror placed at 45 degrees relative to the vertical, and directly above the fluid bath. A light source with diffuser was directed horizontally toward the mirror. A black backdrop was placed behind the mirror to minimize exterior sources of light, and the camera was placed vertically above the mirror. An in-sync strobing effect was obtained with the camera and triggered at a frequency that was an integer divisor of \( f \). Out-of-sync videos were recorded with the camera frame rate slightly detuned from \( f \). High speed videos were recorded with a Phantom Miro R311 color high-speed camera with oblique orientation with respect to the bath using a reflection technique to visualize the waves \[M4]\.

Preliminary tests

The speed of 15 surfers with \( L = 4.2 \text{ mm}, \ w = 2.7 \text{ mm} \) and half upper layer (Fig. 1a) was measured for \( \gamma = 3.5 \text{ g} \) and \( f = 100 \text{ Hz} \). The mean speed was \( U = 2.27 \text{ mm/s} \) with standard deviation 0.25 mm/s. Among the 15 surfers, 6 were selected for the experiments as those exhibiting the straightest trajectories. Imperfections in the pinning of the contact line were observed to lead to curved trajectories.

Experimental procedures

Prior to each experiment, the liquid container was cleaned with ethanol, rinsed with deionized water and dried with clean compressed air. In order to avoid the effects of wave reflection or waves due to the oscillating meniscus, data were only recorded when the surfers were within 26.5 mm of the bath centre, where the surfer velocity was measured to be statistically independent of its proximity to the border. During single-surfer speed measurements, the fluid bath was changed for every 3 surfers tested. A linear least-squares fit was used to calculate the time-averaged speed of the surfer from the distance traveled by the surfer as a function of time. At \( f = 100 \text{ Hz} \) the error on the time-averaged speed ranged from 0.0001–0.001 mm/s, with a 95% confidence bound for a single surfer. The standard deviation among 6 surfers ranged from 0.1–0.01 mm/s.

The interaction modes of surfer pairs (Fig. 2c–i, top panels) were explored with 6 pairs, chosen among the 6 surfers according to the best similarity in propulsion speed. Here the fluid bath was changed and refreshed for every new pair. For the promenade spacing measurements at \( f = 100 \text{ Hz} \) (Fig. 3e), the standard deviation
of the spacing among pairs of surfers ranged from 0.01–0.1 mm.

For Fig. 1d (inset), the position of the surfer’s centre of mass was varied by varying the length of the PTFE upper layer in a surfer with \( L = 6.44 \pm 0.02 \) mm and \( w = 4.13 \pm 0.02 \) mm. These surfers were slightly larger than those used elsewhere in the paper in order to ease the manufacturing process. Five different positions of the centre of mass were explored and six surfers were used per position.

Surfers with five different sizes were used to investigate the effect of magnification. Six surfers per size were used. Each surfer was a scaled-up version of a sample surfer (so that the width, length and corner radius of curvature were magnified by a constant) but with the overall thickness kept constant. Each surfer had asymmetry parameter \( a = L/2 \). Here the fluid bath was changed every time the experiments on two surfers were completed. These results are reported in the Supplementary Materials (Fig. S2).

Uncertainties and error propagation

The uncertainty on the scaling of the surfer speed was calculated by substituting the formula for the wavenumber of capillary waves in the deep-water limit, \( k = (\rho \omega^2/\sigma)^{1/3} \), and \( \omega = 2\pi f \) into Eq. (1) and computing the square root of the variance formula for independent variables

\[
\Delta U = \sqrt{\left( \frac{\partial U}{\partial H} \right)^2 \Delta H^2 + \left( \frac{\partial U}{\partial \sigma} \right)^2 \Delta \sigma^2 + \left( \frac{\partial U}{\partial \gamma} \right)^2 \Delta \gamma^2 + \left( \frac{\partial U}{\partial f} \right)^2 \Delta f^2 + \left( \frac{\partial U}{\partial \eta} \right)^2 \Delta \eta^2 + \left( \frac{\partial U}{\partial \rho} \right)^2 \Delta \rho^2 + \left( \frac{\partial U}{\partial L} \right)^2 \Delta L^2},
\]

where \( \Delta H = 0.06 \) mm, \( \Delta \sigma = 0.5 \) mN/m, \( \Delta f = 0.1 \) Hz, \( \Delta \eta = 0.0005 \) Pa-s, \( \Delta \rho = 0.0003 \) g/cm\(^3\) and \( \Delta L = 0.01–0.05 \) mm was taken as twice the standard deviation of measurements on six surfers per \( L \). The uncertainty in the forcing acceleration \( \Delta \gamma/\gamma = 0.01–0.06 \) was measured in the range of frequency 40–100 Hz as the largest discrepancy between the measurements of the two accelerometers. The horizontal error bars in Fig. 1d represent the resulting uncertainty \( \Delta U \).

Theoretical model

A single surfer of length \( L \), width \( w \), asymmetry \( a \), stern (bow) heights \( h_+ (h_-) \), mass density \( \rho_s \), and mass \( m \) (Fig. 1b) is modeled as a pair of point masses \( m_+ = \rho_s w h_+ \) and \( m_- = (L-a) \rho_s w h_- \) connected by a rigid massless rod of length \( l = L/2 \) (Fig. 2a,b). We begin by deriving the equations of motion for the center of mass \( \mathbf{x}_i,\pm \) and orientation (unit) vector \( \mathbf{n}_i \) of the \( i \)th surfer. The masses are located at \( \mathbf{x}_i,\pm = \mathbf{x}_i \pm \mu_\pm \mathbf{n}_i \), where \( \mu_\pm = m_\pm/m \). They move under the influence of wave forces \( \mathbf{F}_\pm \), time-averaged over the forcing period \( \omega = 2\pi f \) of the bath, and drag forces \( -D_\pm \dot{\mathbf{x}}_i,\pm \), yielding the equations of motion

\[
m_+ (\ddot{\mathbf{x}}_i - \mu_- \dot{\mathbf{n}}_i) + D_+ (\dot{\mathbf{x}}_i - \mu_- \dot{\mathbf{n}}_i) = \mathbf{F}_+, \quad m_- (\ddot{\mathbf{x}}_i + \mu_+ \dot{\mathbf{n}}_i) + D_- (\dot{\mathbf{x}}_i + \mu_+ \dot{\mathbf{n}}_i) = \mathbf{F}_-.
\]

We now assume that \( D_\pm = m_\pm/\tau_v \), where \( \tau_v = mH/\eta w L \) is the viscous timescale obtained by computing the shear stress due to a locally fully-developed Couette flow on the underside of the surfer. Adding the two
are the kinematic and dynamic free surface boundary conditions, respectively. We let 
\[\ddot{\mathbf{x}}_i + D \dot{\mathbf{x}}_i = \mathbf{F}_+ + \mathbf{F}_-, \tag{5}\]
where \(D = D_+ + D_-\). To model the rotational dynamics, we take the cross product of the first equation in Eq. (4) with \(-\mu_\perp \mathbf{n}_i\), the second equation with \(\mu_\perp \mathbf{n}_i\), and add the two resulting equations:

\[\left[ m_- (\mu_+ l)^2 + m_+ (\mu_- l)^2 \right] \mathbf{n}_i \times \dot{\mathbf{n}}_i + \frac{1}{\tau_v} \left[ m_- (\mu_+ l)^2 + m_+ (\mu_- l)^2 \right] \mathbf{n}_i \times \dot{\mathbf{n}}_i = l \left[ \mu_+ \mathbf{n}_i \times \mathbf{F}_- - \mu_- \mathbf{n}_i \times \mathbf{F}_+ \right]. \tag{6}\]

Writing \(\mathbf{n}_i = (\cos \theta_i, \sin \theta_i)\), Eq. (6) reduces to

\[I \ddot{\theta}_i + \frac{l}{\tau_v} \dot{\theta}_i = l \left( \mu_+ \mathbf{n}_i \times \mathbf{F}_- - \mu_- \mathbf{n}_i \times \mathbf{F}_+ \right) \cdot \mathbf{z}, \tag{7}\]

where \(\mathbf{z}\) is the unit vector in the (vertical) \(z\)-direction and \(I = m_+ (\mu_+ l)^2 + m_- (\mu_- l)^2 = \mu_+ \mu_- ml^2\) is the moment of inertia.

We now deduce an expression for the capillary interaction forces \(\mathbf{F}_\pm\). Consider a point mass \(m_j\) with position \((x_j, z_j)\) oscillating periodically with frequency \(\omega\) on the surface of an inviscid and incompressible fluid bath with density \(\rho\) and surface tension \(\sigma\). We neglect gravitational effects on the waves and thus the parametric forcing on them due to the bath vibration. We also assume the waves to be of small amplitude, so that the governing equations for the velocity potential \(\phi_j(x, z, t)\) and interfacial deformation \(h_j(x, t)\) may be linearized:

\[(\Delta + \partial_{zz}) \phi_j = 0, \quad z \leq 0, \quad \partial_t h_j = \partial_z \phi_j, \quad z = 0, \quad -\rho \partial_t \phi_j + \sigma \Delta h_j + m_j \ddot{z}_j \delta(x - x_j) = 0, \quad z = 0, \tag{8}\]

where \(\Delta = \partial_{xx} + \partial_{yy}\) is the 2D Laplacian, and \(z = 0\) denotes the position of the undisturbed fluid interface. The first equation in Eq. (8) enforces incompressibility of the fluid, while the second and third equations are the kinematic and dynamic free surface boundary conditions, respectively. We let \(\ddot{z}_j = -g - \zeta_j \omega^2 \cos \omega t\), where \(\zeta_j\) is the oscillation amplitude of particle \(j\). By generalizing the derivation presented by De Corato & Garbin [30] to account for the static (non-oscillatory) deformation of the interface, we find that the solution to Eq. (8) is

\[h_j(x, t) = \frac{m_j g}{2\pi\sigma} \log |x - x_j| - \frac{m_j \zeta_j \omega^2}{12\sigma} G(k|x - x_j|) \cos \omega t, \tag{9a}\]

where \(G(r) = -H_0(r) - Y_0(r) + 2 \text{Re} [H_0(\varsigma r) - Y_0(\varsigma r)]\), \(\varsigma = e^{i\pi/3}\), \(H_0\) is the zeroth-order Struve function and \(Y_0\) is the zeroth-order Bessel function of the second kind [M5]. The first term in Eq. (9a) is the static deformation of the interface induced by the particle’s weight, while the second is the dynamic deformation whose form was derived by De Corato & Garbin [30]. The wave fields presented in Fig. 2c–i (lower panels), Fig. 3a–d (lower panels) and Fig. 4c,d correspond to
the linear superposition of these wave profiles over all point masses. In these plots, the first term in Eq. (9a) is neglected for the sake of facilitating visualization of the wave field’s oscillations.

We proceed by deriving an expression for the lateral force $F_{ij}$ exerted by the mass $m_j$ on a mass $m_i$ with position $(x_i, z_i)$, where $\ddot{z}_i = -g - \zeta_\omega^2 \cos \omega t$. The mass $m_i$ exerts a force $m_i \ddot{z} d$ on the interface, where $d = (-\nabla h_j, 1)/\sqrt{1 + |\nabla h_j|^2}$ is the unit normal vector to the interface. The interface thus exerts an equal and opposite force $-m_i \ddot{z} d$ on the mass, the horizontal component of which is $m_i \ddot{z} \nabla h_j$ in the limit of small deformations, $|\nabla h| \ll 1$. We thus obtain the expression

$$F_{ij} = m_i \ddot{z} \nabla h_j (x_i, t) = -m_i (g + \zeta_\omega^2 \cos \omega t) \nabla h_j (x_i, t). \quad (10)$$

Using Eq. (9) and the facts that $H'_0 = H_{-1}$ and $Y'_0 = -Y_1$, we find that the time-average of Eq. (10) over the oscillation period $2\pi/\omega$ is [30]

$$\langle F_{ij} \rangle = -\frac{m_i m_j g^2 k}{\sigma} \left[ \frac{1}{2\pi k |x_i - x_j|} + \frac{\xi_i \xi_j \Phi(k |x_i - x_j|)}{24} \right] \frac{x_i - x_j}{|x_i - x_j|}, \quad (11)$$

where $\xi_i = \zeta_\omega^2 / g$ and

$$\phi(r) = -G(r) = H_{-1}(r) - Y_1(r) - 2 \text{Re} \left[ \zeta (H_{-1}(r) + Y_1(r)) \right]. \quad (12)$$

The function $\phi(r)$ changes sign as $r$ is progressively increased [30], so the capillary force between the masses oscillates between attractive and repulsive as the distance between them increases.

We may now state the equations of motion for a collection of surfers, assuming that all of the surfers and their associated point masses have the same oscillation amplitude, $\xi_i = \xi$. Combining Equations (5), (7) and (11) yields the trajectory equations (2), where

$$\phi(r) = \frac{1}{2\pi r} + \frac{\xi^2}{24} \phi(r) \quad \text{and} \quad F_c = \frac{(mg)^2 k}{\sigma}. \quad (13)$$

Numerical Simulations

We proceed by non-dimensionalizing the trajectory equations (2) using $x \rightarrow kx$ and $t \rightarrow tkF_p \tau_v / m$:

$$\ddot{m} \ddot{x}_i + \ddot{x}_i = \ddot{n}_i + \ddot{F}_c \sum_{\alpha, \beta = \pm 1} \mu_\alpha \mu_\beta \sum_{j \neq i} \Phi(|x_j, \beta - x_i, \alpha|) \frac{x_j, \beta - x_i, \alpha}{|x_j, \beta - x_i, \alpha|},$$

$$\ddot{m} \ddot{h}_i + \ddot{h}_i = -\frac{\ddot{F}_c}{1/4 - \mu_0^2} \sum_{\alpha, \beta = \pm 1} \mu_\alpha \mu_\beta \sum_{j \neq i} \alpha \mu_\alpha \phi(|x_j, \beta - x_i, \alpha|) \ddot{n}_i \times \frac{x_j, \beta - x_i, \alpha}{|x_j, \beta - x_i, \alpha|} \cdot z, \quad (14)$$

where $\mu_0$ is given by $\mu_\pm = 1/2 \pm \mu_0$, $\ddot{t} = kF_P \tau_v$, $x_i, \alpha = x_i - \alpha \mu_\alpha \ddot{n}_i$, and the dimensionless parameters

$$\ddot{m} = \frac{kF_P \tau_v^2}{m} = kU \tau_v \quad \text{and} \quad \ddot{F}_c = \frac{F_c}{F_p} = \frac{F_c \tau_v}{mU} \quad (15)$$

are defined through the free speed $U = F_p \tau_v / m$ of a single surfer in isolation. We further assume that the surfers’ oscillation amplitudes are equal to the forcing amplitude of the bath, $\xi = \gamma / g$, so the model (14) contains the single unknown parameter $U$ that is inferred from experimental measurements of a single surfer.
in isolation. Equation (14) is solved using a fourth-order explicit Runge-Kutta method in MATLAB, and the Struve functions in the expression for $\Phi$ are evaluated using the toolbox “Struve functions” developed by T. P. Theodoulidis.

In accordance with the experiments, the simulations were conducted assuming a surfer with length $L = 4.3$ mm, width $w = 2.7$ mm, asymmetry parameter $a = L/2$, stern (bow) heights $h_+ = 1.2$ mm ($h_- = 0.8$ mm), mass density $\rho_s = 2.2$ g/cm$^3$ and mass $m = 0.026$ g. The fluid is assumed to have dynamic viscosity $\eta = 0.018$ Pa·s, density $\rho = 1.175$ g/cm$^3$, surface tension $\sigma = 66$ mN/m and depth $H = 5$ mm, for which the viscous timescale is $\tau_v = 0.61$ s.

The interaction modes shown in Fig. 2c–i (and Supplementary Video 2) and Fig. 3k–d (lower panels) were conducted using a forcing frequency $f = 100$ Hz and forcing acceleration $\gamma/g = 3.3$, for which the surfer free speed is $U = 1.9$ mm/s (SI Fig. 1) and the capillary wave number is $k = (\rho \omega^2/\sigma)^{1/3} = 1.9$ mm$^{-1}$. The associated dimensionless parameters are $\tilde{m} = 2.2$, $\tilde{l} = 4.1$, $\mu_0 = 0.1$ and $\tilde{F}_c = 2.3 \times 10^4$. For Fig. 3k, the forcing frequency was fixed, $f = 100$ Hz, while the forcing acceleration $\gamma/g = \xi$ was varied. The surfer free speed $U$ was inferred from the experimental data in Supplementary Figure S1 using linear interpolation/extrapolation. For Fig. 3 both $f$ and $\gamma/g$ were varied in experiments, so linear interpolation/extrapolation was used to determine both $U$ and $\xi$ in the simulations. For Fig. 4c,d (and Supplementary Video 3), the forcing frequency was $f = 100$ Hz and the forcing acceleration was $\gamma/g = 6.6$. The corresponding values of $U$ were extrapolated from the experimental data in SI Fig. 1.

The different modes presented in Fig. 2 and Fig. 4 were obtained by solving Eq. (14) with different initial conditions. Each curve in Fig. 3 was obtained by conducting a numerical sweep in the parameter $\gamma$; specifically, we found a promenade mode of a given order $n$ at the largest value of the forcing acceleration considered, $\gamma = 3.5$ g, and then progressively decremented $\gamma$ in steps of 0.05 g, using the final state of the prior simulation as the initial condition for the next. The curves terminate when a promenade mode could not be found. The same was done for Fig. 3, except the sweep was done in the parameter $f$ in steps of 1 Hz.

References: Methods

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