ELECTROWEAK PHASE TRANSITION IN STRONG MAGNETIC FIELDS IN THE STANDARD MODEL OF ELEMENTARY PARTICLES

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ABSTRACT

The electroweak phase transition in the magnetic and hypermagnetic fields is studied in the Standard Model on the base of investigation of symmetry behaviour within the consistent effective potential of the scalar and magnetic fields at finite temperature. It includes the one-loop and daisy diagram contributions. All discovered fundamental fermions and bosons are taken into consideration with their actual masses. The Higgs boson mass is chosen to be in the energy interval $75 \text{ GeV} \leq m_H \leq 115 \text{ GeV}$. The effective potential calculated is real at sufficiently high temperatures due to mutual cancellation of the imaginary terms entering the one-loop and the daisy diagram parts. Symmetry behaviour shows that neither the magnetic nor the hypermagnetic field does not produce the sufficiently strong first order phase transition. For the field strengths $H, H_Y \geq 10^{23} \text{ G}$ the electroweak phase transition is of second order at all. Therefore, baryogenesis does not survive in the Standard Model in smooth magnetic fields. The problems on generation of the fields at high temperature and their stabilization are also discussed in a consistent way. In particular, it is determined that the nonabelian component of the magnetic field $(gH)^{1/2} \sim g^{4/3}T$ has to be produced spontaneously. To investigate the stability problem the $W$-boson mass operator in the magnetic field at high temperature is calculated in one-loop approximation. The comparison with results obtained in other approaches is done.

1. INTRODUCTION

Among interesting problems of nowadays high energy physics there are two ones which, at first glance, are not connected with each other. These are the value of the Higgs boson mass $m_H$ and the strengths of magnetic fields $H$ which could be present in the early universe (see surveys [1] - [4]). Both problems are of
paramount importance for particle physics and cosmology. For instance, as was shown in Refs. [6], [8], [9] a large scale homogeneous hypercharge magnetic field $H_Y$ must essentially influence the type of the electroweak (EW) phase transition making it strong first order. Interest to the effects of strong magnetic fields has considerably increased recently when it was realized that a standard baryogenesis scenario in the Standard Model (SM) of elementary particles could not be established without the fields [13]. Different mechanisms to produce the fields have been proposed [20], [17], [11], [21], [18], [22]. One of the purposes of the present paper is to discuss some recent results on these topics. At the same time, in the Monte Carlo simulations and by the nonperturbative methods of quantum field theory [15], [14] the magnetic mass of nonabelian gauge fields of order $m_{mag.} \sim g^2 T$ has been determined. It screens the nonabelian component of the magnetic field at long distances. This is one of the reasons why an interest to the hypermagnetic field $H_Y$ was excited. This field, owing to its abelian nature, is not screened at finite temperature. The influence of hypermagnetic fields on symmetry behaviour has been first investigated for many years ago in Ref. [16] where the similarity to superconductivity was emphasized. This important observation plays a decisive role in the description of the EW phase transition. The problem which requires further investigations is the generation of strong abelian fields at high temperature. Interesting mechanism connecting with an abilian anomaly was suggested in Ref. [7].

Much more involving is situation with the magnetic field, although it is studied for many years. Currently, there are not common opinions about either the ways of producing and stabilization the field or its role at high temperature (see papers [13], [20], [27] and references therein). This situation, probably, finds an explanation in the nonabelian nature of the field and lack of simple analogies in condensed matter physics (mainly superconductivity is discussed). As an example of peculiarities, let us mention the phenomenon of condensation of the $W$- and $Z$-boson fields which originates from the instability of the perturbative vacuum in strong magnetic fields (see Refs. [30] - [32]). In such a situation, to treat problems with the external fields correctly the consistent calculations have to be carried out. Recent investigations of the phase transitions in the magnetic fields at high temperature [33], [35], [27] have used as a qualitative picture of the phenomenon the description in Refs. [36], [20] derived at a classical level for the case $m_H = m_Z$ corresponding to second type superconductivity. However, in these papers the effects of fermions (light and heavy) as well as the radiation corrections in the fields at high temperature have not been included but play an important role. The second goal of the present paper is to elaborate the situation with magnetic fields at high temperatures. We shall consider all the problems mentioned within consistent calculations allowing for the one-loop effective potential (EP) and the daisy diagram contributions in the external fields at high temperature. To find the latter ones the one-loop polarization functions of gauge bosons under these external conditions will be computed. As it is occurred, the influence of the fields
crucially depends on the values of the particle masses. So, to have a reliable quantitative description of the EW phase transition we fix them to be equal to the present day experimental data. The Higgs boson mass will be taken in the energy interval $75 \text{ GeV} \leq m_H \leq 115 \text{ GeV}$. The low bound corresponds to the mass values when perturbative methods are reliable. The upper bound is chosen to fit the present experimental low limit $m_H \geq 90 \text{ GeV}$. In our calculations, the external fields will be taken into account exactly through the Green functions. This insures, the results to be obtained should reproduce correctly the effects of the fields for all the values $m_H$ considered. It is also important to notice that in strong fields at high temperatures light fermions dominate, as it follows from the term $H^2 \log T/m_f$ entering the one-loop effective potential, where $m_f$ is the fermion mass. Actually, at different temperatures the different fermions give the dominant contributions.

The concept of symmetry restoration at high temperature has been intensively used in studying of the evolution of the universe at its early stages. Nowadays it is a corner stone in investigations of various problems of cosmology and particle physics \cite{37}, \cite{38}. In particular, the type of the EW phase transition and, hence, a further evolution of the universe depends on the mass of Higgs boson. As we mentioned above, the idea that strong fields make the EW phase transition strong first order, that is necessary to retain the standard baryogenesis (see recent survey \cite{13}), requires further investigations with radiation corrections allowed for. Since all the masses of fundamental particles, except $m_H$, are known one is able to investigate in detail the phase transition as the function of this parameter and to determine the properties of the vacuum. That is the main goal of the present paper.

Various aspects of the phase transitions in magnetic fields at high temperature have been discussed in literature \cite{39} - \cite{42}, \cite{18}. In Refs. \cite{13}, \cite{14}, considering the boson part of the Salam-Weinberg model, the EW phase transition in the strong fields was investigated and the vacuum structures of the phases have also been described. In Ref. \cite{13} side by side with temperature and magnetic field a chemical potential was incorporated. But the role of fermions has not been studied in detail.

Another aspect of the EW phase transition, which also has not been elaborated, is the influence of so-called daisy (or ring) diagrams at high temperature and strong fields. At zero field it has been investigated in Refs. \cite{45}, \cite{55}, \cite{46} where the importance of these diagrams to correctly describe symmetry behaviour was emphasized. In Ref. \cite{46} the t-quark mass was chosen of order 110 GeV. So, to account of the experimental value $m_t = 175 \text{ GeV}$, it has to be revised. In the present paper the EW phase transition will be studied for the case of the constant fields $H_Y$ and $H$. This is an adequate approximation for strong fields in the cases of the second order phase transition and the initial stage of the first order one when the bubbles are not large \cite{8}.

The content is as follows. In sect. 2, for convenience of readers, the results of
the investigations announced are described in a qualitative manner. In sects. 3, 4 the contributions of bosons and fermions to the one-loop EP $V^{(1)}(T, H, \phi_c)$ of classical the scalar $\phi_c$ and the external magnetic fields are calculated in the form convenient for numeric investigations. In sect. 5 the correlation corrections due to daisy diagrams are computed and the vacuum stability condition at high temperature is discussed. Special attention is devoted to computation of the daisy diagrams with the unstable (tachyonic) mode presenting in the magnetic field in the $W$-boson spectrum. In sect. 6 the restored phase with external fields is described. In sect. 7 the high temperature expansion of the EP is present. Then, in sect. 8 the EW phase transition in the hypercharge magnetic field is investigated. The same for the magnetic field case is carried out in sect. 9. In these two sections the detailed analysis of the phase transition is done. The one-loop polarization functions of $W$-bosons at high temperatures and strong magnetic fields are calculated in sect. 10. This, in particular, gives possibility to study self-consistently the vacuum stability. The spontaneous vacuum magnetization at high temperature is investigated in sect. 11. It is found that the magnetic fields of order $(gH)^{1/2} \sim g^{4/3}T$ is generated. Such strong fields affect all the processes at high temperatures. Comparison of the results obtained with that of other approaches is done in sect. 12. Discussion and concluding remarks are given in sect. 13. Appendix contains necessary information on the Mellin transformation technique used in calculations of the high temperature asymptotics of the EP.

2. QUALITATIVE PICTURE OF THE EW PHASE TRANSITION IN MAGNETIC FIELDS

As it is belived nowadays, the presence of different kind strong magnetic fields in the early universe is rather resonable than exotic phenomenon. In literature on this topic a lot of dynamic mechanisms to generate the fields are proposed (see surveys [4], [3] and references therein). In the present paper, we are not going to consider all of them. Our goal is to describe a consistent picture of the influence of the fields on the EW phase transition. In this section, we consider in a qualitative manner the mechanisms of producing the external fields, the ways of their action on the vacuum at zero and finite temperature and present the main results of our investigation.

2.1 The generation of the primordial magnetic fields

The generation of the fields can be devided in two classes: 1) generation due to processes that had happened at the EW phase transition; 2) creation of the fields before the EW phase transition epoch (for instance, at a GUT scale). To the first class we refer: the rotation of bubbles at the first order phase transition [22]; fluctuation of gradients of the Higgs field [3]; bubble collisions [24], [25]. These
mechanisms produce microscopic fields which had to be amplified by magnetohydrodynamic processes up to the macroscopic values required to fit the astronomic observations, as it is discussed in Refs. [50], [51].

To the second class of the processes we refer the spontaneous vacuum magnetization at high temperature discovered first at zero temperature by Savvidy [52] and the fields generated by strings [48]. Ones had been created, these fields were frozen in a cosmic plasma, evolved with it during the expansion of the universe and present at the EW phase transition. In what follows, the former mechanism will be investigated in detail. We shall show that it does work at high temperature. So, it could serve to produce the seed magnetic field (in contrast to conclusions of Refs. [34], [35]).

Let us remind the results of Refs. [52], [17], [20], [19]. The spontaneous vacuum magnetization has been derived from the investigation of the EP of covariantly constant (sourceless) chromomagnetic field

\[ H^a = H_0 \delta^a_3 \]

which is a solution to the classical Yang-Mills field equations, where \( H = \text{const} \) and \( a \) is an isotopic index,

\[ V(H, T) = \frac{H^2}{2} + V^{(1)}(H, T). \]  

(1)

It includes the tree-level and the one-loop parts. The minimum of the EP at high temperature \( T \) corresponds to the nonzero magnetic field of order \((gH)^{(1/2)} \sim g^2T\), \( g \) is gauge coupling constant. In the EW theory, the \( a = 3 \) component of the weak isospin just corresponds to the nonabelian part of usual magnetic field which we observe in the broken phase.

Very important for our analysis is the value of the vacuum field strength \( \sim g^2T \). In fact, as it will be shown in sect. 11, this value is increased when the correlation corrections are taken into account. The field is screened at long distances \( l \geq 1/m_{mag} \sim 1/g^2T \) by the magnetic mass of gauge field. However, inside this space domain the strong fields may exist and affect all the processes. Really, typical particle masses are of order \( m_T \sim gT \), therefore, for small \( g \) the Compton wave length \( \lambda_{\text{Compt.}} \sim 1/m_T \), giving the particle size, and the Larmor radius, \( r_{\text{Larmor}} \sim 1/(gH)^{1/2} \sim 1/g^2T \), determining the space range where the charged particle spectrum is formed, are both located inside a domain which is filled up by the field. Hence it follows that the field strengths of order \((gH)^{1/2} \sim g^2T \) or stronger are of interest at high temperature and, in particular, the Savvidy mechanism gives rise such intense fields.

Interesting mechanism to generate the hypercharge magnetic field due to the abelian anomaly was proposed in Ref. [7]. At present time it is not investigated in detail. So, in what follows we will not consider a consistent picture with this field. We will just assume that the strong external field \( H_Y \) present when the transition had happened.

1 In paper [20] the important cancellation of logarithmic terms entering the zero and the finite temperature parts of the EP was missed. This has been resulted in the incorrect value of the vacuum magnetic field spontaneously created at finite temperature.
Below, we shall study the influence of constant the usual magnetic \[^{18}\] and the hypercharge magnetic \[^{6}, {8}, {9}, {12}\] fields. Such the approximation is not artificial for strong fields. More definitely, the gradients of fields are negligible if the relation \(| \nabla H/H | \ll H/m \) holds \[^{49}\]. Here, \( m \) is a characteristic mass of the problem under consideration. It means that in strong fields the dominant effects are due to intensity of the fields. At high temperature, particle masses are of order \( m \sim gT \). As it was pointed out in Ref. \[^{8}\] this approximation works well for the second order phase transitions and for the first stages of the first order ones when the bubbles are not large. On the other hand, in Refs. \[^{7}, {6}\] it was argued that strong stochastic hypermagnetic fields are able to produce the baryon asymmetry of the universe. These problems are left beyond the scope of the present paper.

### 2.2 Mechanisms of acting of the external fields on a vacuum

The ways in which the magnetic and the hypermagnetic fields affect the vacuum scalar condensate are quite different. In the latter case, it is completely similar to the case of superconductivity, as it was investigated first in Ref. \[^{16}\]. In broken phase, the gauge field (\( U(1) \) gauge field in the Higgs model and \( Z \)-boson field in the EW theory) is screened by its mass. This is the consequence of the interaction term \( \sim A^2 \phi^2 \) in the Lagrangian, \( \phi \) is the scalar field. The influence of the external field is reduced to the increase of the vacuum energy and it manifests itself at tree level. In sufficiently strong fields the symmetry restoration happens and the gauge field mass \( M_A = g \phi_c \) becomes equal zero. For the critical field strength one has an estimate, \( H_Y^2/2 \sim O(M_A^4) \), which shows that the restoration happens when the energy density of the external field equals to that of the scalar condensate \( \sim M_A^4 \). This mechanism works both at zero and finite temperature. In the latter case, the critical value of \( H_Y \) is decreasing when the temperature is increasing, as in superconductors. If the mass \( m_H \leq m_z \), the vacuum of the SM is the first type ”superconductor” with respect to hypermagnetic field otherwise it is the second type one. This picture is determined at a classical level \[^{16}\]. The external hypermagnetic field delays the first order phase transition making it stronger, that is necessary for baryogenesis. This conclusion has been obtained in three approximation in Ref. \[^{10}\]. As it was discussed in detail in Ref. \[^{13}\], to have a standard baryogenesis scenario the ratio \( R = \Delta \phi_c/T_c \) of the order parameter jump to the critical temperature must be of order \( \sim 1.2 - 1.5 \). At zero external fields, one has \( R \sim 0.6 \) for \( m_H \sim 70 - 80 \text{ GeV} \).

In the EW theory, the \( U(1) \) symmetry corresponding to electromagnetic field is retained in the broken phase. Therefore, there are no couplings of the scalar and the electromagnetic fields at tree level. Hence, one would expect that the external fields affect the vacuum condensate through the radiation corrections as it was discussed in Ref. \[^{53}\]. However, in the nonabelian case the influence of fields on the scalar condensate is more complicate and the phase transition in strong
external magnetic fields can happen at tree level, because of the non-linearity of the field equations. The vacuum properties are determined by not only the scalar field condensate \( \phi_c \) but also the other order parameter - so-called W-boson condensate \([3, 13, 36]\) (see also surveys \([33, 32]\)). Actually, here one faces the situation with interacting order parameters. In the EW Lagrangian the next two terms, \( \sim F_{\mu\nu}W^+_\mu W^-_\nu \) and \( \sim W^+_\mu W^-_\nu \phi_c^2 \), enter, where \( F_{\mu\nu} \) is an electromagnetic field strength tensor, \( W^\pm_\mu \) is the W-boson field. The former interaction is due to a gyromagnetic ratio \( \gamma = 2 \) inherent the nonabelian theories. It is crucial for exciting in strong constant magnetic fields \( H \geq H_0 = M_w^2/e \) in the W-boson spectrum of the tachyonic (unstable) mode \( p_0^2 = p_3^2 + M_w^2 - eH \), \( p_3 \) is a momentum along the field direction, which then is condensed owing to the self-interaction \( \sim (W^+_\mu W^-_\mu)^2 \). The W-boson condensate influences the scalar field at classical level (due to the second of the above written interaction terms). As a result, the scalar condensate is eliminated and the condensates of W- and Z-boson fields are formed \([3, 13, 36]\). The threshold of the phase transition is determined by the value of the Higgs boson mass. If \( m_H < M_w \) the vacuum is the first type “superconductor” with respect to the magnetic field. In this case the homogeneous W-condensate is formed \([3, 14]\). For \( m_H > M_w \) the vacuum behaves as a second type ”superconductor”, therefore the lattice structure of the Abrikosov type formed by the W- and Z-boson fields is produced. This picture has been derived for arbitrary mass \( m_H \) in Refs. \([43, 58]\) and for the special value \( m_H = m_z \) in Refs. \([36, 32]\).

These vacuum properties have been determined in tree approximation when fermions do not contribute. But they do contribute in one-loop order, and, moreover, play a decisive role in the vacuum dynamics. In strong fields at high temperatures the influence of light fermions is increased, as the consequence of the term \( \sim H^2\log(T/m_f) \) entering the one-loop EP. Besides, other peculiarities must be taken into consideration. Namely, the EP contains additional \( T \)-dependent terms, such as \( \sim \phi_c^2 T^2, \sim eHT\phi_c \), etc. (see more details in sects. 5, 7), which are generated in the one-loop and higher orders of perturbation theory. These terms make the picture of the field action involving. Therefore, to investigate symmetry behaviour numeric calculations must be carried out.

### 2.3 Symmetry behaviour in magnetic fields

As it is well known \([45, 46, 55]\), at finite temperature side by side with the one-loop EP the correlation corrections described by the daisy diagrams should be added in the consistent calculations. Series of these diagrams are responsible for the long range effects and contain imaginary terms which cancel the imaginary part of the one-loop EP. As a result, the total EP is real at sufficiently high temperature. This important property is fulfilled also in the external fields when the contribution of daisy diagrams with unstable mode is allowed for \([10, 11]\).

Now, we are going to describe symmetry behaviour at high temperature and
strong magnetic fields. In what follows we assume that only one type of the fields is applied. Remind that in the broken phase the component of $H_Y$ responsible for the magnetic field is unscreened and the form of the EP curve corresponds to both the magnetic and the hypemagnetic fields. In order to investigate the EW phase transition we shall consider the function $\mathcal{V}' = \mathcal{V}(h, B, \phi) - \mathcal{V}(h, B, 0)$ describing the symmetry restoration. Here, the dimensionless magnetic, $h = H/H_0$, $H_0 = M_w^2/e$, and the scalar, $\phi = \phi_c/\delta(0)$, fields and the inverse temperature, $B = \beta M_w$, are introduced, $\delta(0)$ is the EP minimum position at zero temperatures and fields (more details see in sects. 8, 9). The normalized order parameter $\phi$ is changed from unit to zero that is convenient in numeric calculations.

In fact, symmetry breaking (restoration) can be realized in two ways. By the first order phase transition with a nonzero jump of the order parameter $\Delta \phi_c \neq 0$ or by the second order one when the order parameter is changing smoothly.

In Fig. 1 we depict symmetry behaviour at high temperatures for the first order phase transition. Notice that we consider the metastable electroweak minimum which is separated from a global unbounded state disposed at large values of $\phi^2$ by a wide and high potential barrier. The temperatures $T_{c_1}$ and $T_{c_2}$ are called the spinodal temperatures, which correspond the situations when bubes of the broken ($T_{c_1}$) and the restored ($T_{c_2}$) phases can not exist in the vacuum.

In Fig. 2 we show symmetry behaviour for the second order phase transition. In this case baryogenesis can not be realized because of washing away the baryon-antibaryon asymmetry (see survey [13]).

As we mentioned before, at zero external fields the value of the parameter $R \sim 0.6$ and so baryogenesis is not possible in the SM. Our main task here is to
investigate the situation with strong external fields been taken into consideration.

The general picture of the phase transitions is independent of the specific external conditions considered. The external fields change the parameters (or the type) of the phase transitions. Besides, because of the different mechanisms of influence of the magnetic and hypermagnetic fields on the vacuum the conditions fixing the transition temperature are also different for these fields (for more details see sects. 8, 9).

Now, let us describe the restored phase. For the magnetic field case, because of the presence of the tachyonic mode in the W-boson spectrum, the state \( \phi = 0 \) is unstable at zero temperature. To better understand the situation at \( T \neq 0 \), let us first discuss the one-loop case. To verify whether the vacuum is stable or not one has to consider the effective mass squared of the unstable mode defined as the sum of the ground state energy squared taken at zero momentum and the one-loop W-boson mass operator averaged over the ground state. It must be considered for the value of \( H \) condensed at high temperature. In particular, we have for the field \( H^{(1)} \) condensed at the one-loop level \( (gH)^{1/2} = (g^2/2\pi)T \): \[ M^2(H_c, T) = \Pi(H_c, T, n = 0, \sigma = +1) - gH_c > 0. \] Thus, the vacuum stabilization is observed. The situation which takes place when the correlation corrections are included is discussed in sect. 11.

In the restored phase the \( W \)-bosons do not interact with the hypermagnetic field. So, no instabilities occur in this case.

By investigating symmetry behaviour in either the hypermagnetic or the magnetic field within the total EP including the contributions of all the SM particles we have determined by numeric computations that for all \( m_H \) values considered...
the increase in the field strength is resulted in the weaker (not stronger) first order phase transition. For $H, H_Y \geq 0.1 - 0.5 \ \ 10^{24}$ G it becomes of the second order at all. Thus, we come to the conclusion that baryogenesis does not survive at these external conditions.

To better understand the role of fermions in symmetry behaviour let us adduce two terms of the asymptotic expansion of the one-loop EP in the limit of $T \to \infty, H \to \infty$. The first one is the term $\sim H^2 \log \frac{T}{m_f}$. Due to this term the light fermions are dominant at high temperature. The second term can be derived from the expansion of the zero temperature part: $\sim -eH m_f^2$ This term acts to make "heavier" the Higgs particles in the field. As a result, the second order temperature phase transition is stimulated due to strong fields.

We would like to complete this section with the comparison of the described results with that of other approaches (for more details see sect. 8, 12). The EW phase transition in the hypermagnetic field has been investigated in one-loop approximation to the EP in Ref. [8] and by the method combining perturbative results and lattice simulations in Ref. [9]. These authors became of different reasons have skipped the fermion part of the EP and therefore had no possibility to determine the form of the EP curve in strong fields at high temperature. In this respect our investigation filled up the gap existed. In our investigation the external fields have been taken into account exactly through Green’s functions. Therefore, in particular, influence of fermions on symmetry behaviour was correctly reproduced.

3. BOSON FIELD CONTRIBUTIONS TO $V^{(1)}(T, H, \phi_c)$

The Lagrangian of the boson sector of the Salam-Weinberg model is (see, for example, [61])

$$L = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + (D_\mu \Phi)^+ (D^\mu \Phi)$$

$$+ \frac{m^2}{2} (\Phi^+ \Phi) - \frac{\lambda}{4} (\Phi^+ \Phi)^2,$$

where the standard notations are introduced

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon^{abc} A_\mu^b A_\nu^c,$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

$$D_\mu = \partial_\mu + \frac{1}{2} ig A_\mu^a \tau^a + \frac{1}{2} ig' B_\mu.$$

The vacuum expectation value of the field $\Phi$ is

$$< \Phi > = \left( \begin{array}{c} 0 \\ \phi_c \end{array} \right).$$
The fields corresponding to the $W^-$, $Z$-bosons and photons, respectively, are

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm iA^2_\mu),$$

$$Z_\mu = \frac{1}{\sqrt{g'^2 + g^2}}(gA^3_\mu - g'B_\mu),$$

$$A_\mu = \frac{1}{\sqrt{g'^2 + g^2}}(g'A^3_\mu + gB_\mu).$$

(5)

To incorporate interaction with an external hypermagnetic field we add the term $\frac{1}{2}\vec{H}\vec{H}$ to the Lagrangian. The value of the macroscopic magnetic field generated inside the system will be determined by minimization of free energy. Interaction with classical electromagnetic field is introduced as usually by splitting the potential in two parts: $A_\mu = \vec{A}_\mu + A^R_\mu$, where $A^R$ describes a radiation field and $\vec{A} = (0, 0, Hx^1, 0)$ corresponds to the constant magnetic field directed along the third axis. We make use of the gauge-fixing conditions

$$\partial_\mu W^\pm_\mu = ie\vec{A}_\mu W^\pm_\mu \mp i\frac{g\phi_c}{2\xi}\phi^\pm = C^\pm(x),$$

(6)

$$\partial_\mu Z_\mu - \frac{i}{\xi'}(g^2 + g'^2)^{1/2}\phi_c\phi_z = C_z(x),$$

(7)

where $e = g\sin\theta_w$, $\tan\theta_w = g'/g$, $\phi^\pm, \phi_z$ are the Goldstone fields, $\xi, \xi'$ are the gauge fixing parameters, $C^\pm, C_z$ are arbitrary functions and $\phi_c$ is a value of the scalar field condensate. In what follows, all calculations will be carried out in the general relativistic renormalizable gauge (6),(7) and after that we set $\xi, \xi' = 0$ choosing the unitary gauge.

To compute the EP $V^{(1)}$ in the background magnetic field let us introduce the proper time, s-representation, for the Green functions

$$G^{ab} = -i\int_0^\infty ds \exp(-isG^{-1ab})$$

(8)

and make use the method of Ref. [59], allowing in a natural way to incorporate the temperature into this formalism. A basic formula of Ref. [59] connecting the Matsubara Green functions with the Green functions at zero temperature is needed,

$$G^{ab}_k(x, x'; T) = \sum_{n=-\infty}^{\infty} (-1)^{n+[x]}\sigma_k G^{ab}_k(x - [x]\beta u, x' - n\beta u),$$

(9)

where $G^{ab}_k$ is the corresponding function at $T = 0, \beta = 1/T, u = (0, 0, 0, 1)$, the symbol $[x]$ denotes an integer part of $x_4/\beta$, $\sigma_k = 1$ in the case of physical fermions and $\sigma_k = 0$ for boson and ghost fields. The Green functions in the right-hand side
of formula (9) are the matrix elements of the operators $G_k$ computed in the states $| x', a \rangle$ at $T = 0$, and in the left-hand side the operators are averaged over the states with $T \neq 0$. The corresponding functional spaces $U^0$ and $U^T$ are different but in the limit of $T \to 0$ $U^T$ transforms into $U^0$.

The one-loop contribution into EP is given by the expression

$$V^{(1)} = -\frac{1}{2} Tr \log G^{ab},$$

(10)

where $G^{ab}$ stands for the propagators of all the quantum fields $W^\pm, \phi^\pm, ...$ in the background magnetic field $H$. In the s-representation the calculation of the trace can be done in accordance with formula (11)

$$Tr \log G^{ab} = -\int_0^\infty \frac{ds}{s} tr \exp(-isG^{-1}_{ab}).$$

(11)

Details of calculations based on the s-representation and the formula (9) can be found in Refs. [59], [44], [19]. The terms with $n = 0$ in Eqs.(9), (10) give zero temperature expressions for the Green functions and the effective potential $V^{(1)}$, respectively. They are the only terms possessing divergences. To eliminate them and uniquely fix the potential we use the following renormalization conditions at $H, T = 0$ [44]:

$$\frac{\partial^2 V(\phi, H)}{\partial H^2} \bigg|_{H=0, \phi=\delta(0)} = \frac{1}{2},$$

(12)

$$\frac{\partial V(\phi, H)}{\partial \phi} \bigg|_{H=0, \phi=\delta(0)} = 0,$$

(13)

$$\frac{\partial^2 V(\phi, H)}{\partial \phi^2} \bigg|_{H=0, \phi=\delta(0)} = | m^2 |,$$

(14)

where $V(\phi, H) = V^{(0)} + V^{(1)} + \cdots$ is the expansion in a number of loops and $\delta(0)$ is the vacuum value of the scalar field determined in tree approximation.

It is convenient for what follows to introduce the dimensionless quantities: $h = H/H_0$, $\phi = \phi_0/\delta(0)$, $K = m_H^2/M_w^2$, $B = \beta M_w$, $\tau = 1/B = T/M_w$, $V = V/H_0^2$ and $M_w = \frac{2}{3} \delta(0)$. After reparametrization the scalar field potential is explicitly expressed through the ratio $K$,

$$V^{(0)} = \frac{h^2}{2} + K \sin^2 \theta_w(-\phi^2 + \phi^4/2).$$

(15)

Notice that $h$ in the case of the external hypermagnetic field is the component of $h_Y$ which remains unscreened in the broken phase. In the restored phase, it will be convenient to work in terms of the initial fields and we will carry out the corresponding calculations later.
The renormalized one-loop EP is given by the sum of the functions
\[ \mathcal{V}_1 = \mathcal{V}^{(0)} + \mathcal{V}^{(1)}(\phi, h, K) + \omega^{(1)}(\phi, h, K, \tau), \]  
where \( \mathcal{V}^{(1)} \) is the one-loop EP at \( T = 0 \), which has been studied already in Ref. [33]. It has the form:
\[ \mathcal{V}^{(1)} = \mathcal{V}^{(1)}_{w,z} + \mathcal{V}^{(1)}_{\phi}, \]
where
\[ \mathcal{V}^{(1)}_{w,z} = \frac{3\alpha}{\pi} \left[ h^2 \log \Gamma \left( \frac{1}{2} + \frac{\phi^2}{2h} \right) + h^2 \zeta'(-1) + \frac{1}{16} \phi^4 - \frac{1}{8} \phi^4 \log \frac{\phi^2}{2h} + \frac{1}{24} h^2 \right] - \frac{1}{24} h^2 \log(2h) \]
\[ + \frac{\alpha}{2\pi} \left[ -2h^2 + (h^2 + h\phi^2) \log(h + \phi^2) \right] + (h^2 - h\phi^2) \log |h - \phi^2| + \frac{i}{2} \alpha h(\phi^2 - h) \theta(h - \phi^2), \]  
and \( \omega^{(1)} \) is the temperature dependent contribution to the EP given by the terms of formulae (9), (10) with \( n \neq 0 \).

We outline the used calculation procedure considering the \( W \)-boson contribution as an example [19].

\[ \omega^{(1)}_w = \frac{\alpha K^2}{32\pi} \left[ (\frac{9}{2} \phi^4 - 3\phi^2 + \frac{1}{2}) \log \frac{3\phi^2 - 1}{2} \right] - \frac{27}{4} \phi^4 + \frac{21}{2} \phi^2 \]

and calculates the imaginary part for \( h > \phi^2 \) appearing due to the tachyonic mode \( \varepsilon^2 = p_3^2 + M_w^2 - eH \) in the \( W \)-boson spectrum [33]. It can be explicitly calculated by means of the analytic continuation taking into account the shift \( s \to s - i0 \) in the \( s \)-plane. Fixing \( \phi^2/h > 1 \) one can rotate clockwise the integration contour in the \( s \)-plane and direct it along the negative imaginary axis. Then, using the identity
\[ \frac{1}{\sinh s} = 2 \sum_{p=0}^{\infty} e^{-s(2p+1)} \]
and integrating over \( s \) in accordance with the standard formula
\[ \int_0^{\infty} ds s^{n-1} \exp(-\frac{b}{s} - as) = 2(\frac{b}{a})^{n/2} K_n(2\sqrt{ab}), \]
a, b > 0, one can represent the expression (20) in the form
\[ \Re \omega^{(1)} = -4 \frac{\alpha h}{\pi B} (3\omega_0 + \omega_1 - \omega_2), \] (23)

where
\[ \omega_0 = \sum_{p=0}^{\infty} \sum_{n=1}^{\infty} \frac{x_p}{n} K_1(nBx_p); x_p = (\phi^2 + h + 2ph)^{1/2} \] (24)

\[ \omega_1 = \sum_{n=1}^{\infty} \frac{y}{n} K_1(nBy), y = (\phi^2 - h)^{1/2} \] (25)

and in the range of parameters \( \phi^2 < h \) after analytic continuation
\[ \omega_1 = -\pi \sum_{n=1}^{\infty} \frac{|y|}{n} Y_1(nB | y |), \] (26)

\[ \omega_2 = \sum_{n=1}^{\infty} \frac{z}{n} K_1(nBz), z = (\phi^2 + h)^{1/2}, \] (27)

\( K_a(x), Y_a(x) \) are the MacDonald and Bessel functions, respectively. The imaginary part of \( \omega^{(1)}_w \) is given by the expression
\[ \Im \omega_1 = -2\alpha \frac{h}{B} \sum_{n=1}^{\infty} \frac{|y|}{n} J_1(nB | y |), \] (28)

\( J_1(x) \) is Bessel function. As it is well known, the imaginary term of the EP is signaling the instability of a system. In what follows, we shall consider mainly symmetry behaviour described by the real part of the EP. As the imaginary part is concerned, it will be cancelled in a consistent calculation including the one-loop and daisy diagram contributions to the EP.

Carrying out similar calculations for the \( Z \)- and Higgs bosons, we obtain [44]:
\[ \omega_z = -6\alpha \sum_{n=1}^{\infty} \frac{\phi^2}{\cos^2 \theta_w n^2 B^2} K_2(\frac{nB\phi}{\cos \theta}), \] (29)

\[ \Re \omega_{\phi} = \left\{ -\frac{2\alpha}{\pi} \sum_{n=1}^{\infty} \frac{t^2}{B^2 n^2} K_2(nBt) + \frac{\alpha}{\pi} \sum_{n=1}^{\infty} \frac{|t|^2}{n^2 B^2} Y_2(nB | t |) \right\}. \] (30)

where the variable \( t = [K_w(\frac{3\phi^2-1}{2})]^{1/2} \) at \( 3\phi^2 > 1 \) and the series with the function \( Y_2(x) \) has to be calculated at \( 3\phi^2 < 1 \). The corresponding imaginary terms are also cancelled as it will be shown below.

The above expressions (17), (23), (24), (30) will be used in numerical studying of symmetry behaviour at different \( H, T \). Notice the cancellation of a number of terms entering the zero-temperature part given Eqs. (17) and \( T \)-depended one. This fact has a general character and was used in checking of the correctness of calculations.
4. Fermion contributions to $V^{(1)}(H, T, \phi_c)$

The fermion one-loop EP in magnetic fields is well studied \[28\], \[29\], \[63\]. To find this explicit form at finite temperature let us consider the standard unrenormalized expression written in the $s$-representation

$$V^{(1)}_f \equiv \frac{1}{8\pi^2} \sum_{n=-\infty}^{\infty} (-1)^n \int_0^{+\infty} ds \frac{1}{s^3} e^{-(m_f^2+q^2n^2/4s)(eHs)\coth(eHs)}, \quad (31)$$

$m_f$ is the fermion mass. Here, we have incorporated the equation \[9\] to include a temperature dependence. In what follows, we shall allow for the contributions of all fermions - leptons and quarks - with their present masses. Usually, considering symmetry behaviour without field one restricts himself by a $t$-quark contribution, only. But in the case of external fields applied this is not a good idea, since the dependence of $V^{(1)}$ on $H$ is a complex function of the ratio $m_f^2/eH$. So, at some fixed values of $H, T$ the different kind dependencies on $H$ will contribute for fermions with different masses. Hence, a very complex dependence on the field takes place in general. We shall include this in a total by carrying out numerical computations and summing up over all the fermions. Now, separating a zero temperature contribution by means of the relation \[16\]

$$\sum_{-\infty}^{+\infty} = 1 + 2 \sum_{1}^{\infty}$$

and introducing the parameter $K_f = m_f^2/M_w^2 = G^2_Y u kawa/g^2$, we obtain for the zero temperature fermion contribution to the dimensionless EP,

$$V_f(h, \phi) = \frac{\alpha}{4\pi} \sum_f K_f^2(-2\phi^2 + \frac{3}{2}\phi^4 - \phi^4\log\phi^2)$$

$$= \frac{\alpha}{\pi} \sum_f (q_f^2 h^2/6 \log \frac{2 | q_f | h}{K_f})$$

$$= \frac{\alpha}{\pi} \sum_f [2q_f^2 h^2 \log \Gamma_1(\frac{K_f \phi^2}{2 | q_f | h}) + (2\zeta'(\frac{1}{2}) - \frac{1}{6})q_f^2 h^2]$$

$$+ \frac{1}{8} K_f^2 \phi^4 + \left(\frac{1}{4} K_f^2 \phi^4 - \frac{1}{2} K_f | q_f | h \phi^2 \right) \log \frac{2 | q_f | h}{K_f \phi^2}, \quad (32)$$

where $q_f$ is a fermion electric charge, the sum $\sum_f = \sum_{f=1}^{3} (leptons) + 3 \sum_{f=1}^{3} (quarks)$ counts the contributions of leptons and quarks with their electric charges. The $\Gamma_1$ function is defined as follows (see Refs. \[29\], \[33\]):

$$\log \Gamma_1(x) = \int_0^x dy \log \Gamma(y) + \frac{1}{2} x (x - 1) - \frac{1}{2} x \log(2\pi). \quad (33)$$
Figure 3: The Higgs field daisy diagrams giving contribution to the effective potential. Blobs stand for the neutral scalar field polarization operator calculated at zero momentum.

The finite temperature part can be calculated in a way described in the previous section. In the dimensionless variables it looks as follows:

\[
\omega_f = \frac{4}{\pi} \sum \frac{\sum (-1)^n}{\sum_{p=0}^{\infty} \sum_{n=1}^{\infty}} \left[ \frac{(2ph + K_f \phi^2)^{1/2}h}{Bn} K_1((2ph + K_f \phi^2)^{1/2}Bn) + \frac{((2p + 2)h + K_f \phi^2)^{1/2}}{Bn} hK_1(((2p + 2)h + K_f \phi^2)^{1/2}Bn) \right]. \tag{34}
\]

Again, a number of terms in Eqs. (32) and (34) are cancelled in the total, as in the bosonic sector.

These two expressions will also be used in numeric investigations of symmetry behaviour.

5. CONTRIBUTION OF DAISY DIAGRAMS

It was shown by Carrington [46] that at \( T \neq 0 \) the consistent calculation of the EP based on the generalized propagators, which include the polarization operator insertions, requires that daisy (ring) diagrams have to be added simultaneously with the one-loop contributions. These diagrams essentially affect the phase transition at high temperature and zero field [45], [46], [55]. Their importance at \( T \) and \( H \neq 0 \) was also pointed out in Refs. [10], [11].

As it is known [15], [45], the sum of daisy diagrams describes a dominant contribution of long distances. It is important when massless states appear in a system. So, this type of diagrams has to be allowed for when a symmetry restoration is investigated. To find the correction \( V_{\text{ring}}(H, T) \) at high temperature and magnetic field the polarization operators of the Higgs particle, photon and Z-boson at the considered background should be calculated. Then, \( V_{\text{ring}}(H, T) \) is given by a series depicted in figures 3, 4. Here, a dashed line describes the Higgs particles, the wavy lines represent photons and Z-bosons, the blobs correspond to the polarization operators in the limit of zero momenta. As also it is known [44], [46], in order to calculate the contribution of daisy diagrams the limiting expressions of the polarization operators \( \Pi_{\mu\nu}(k, T, H) \) at zero momenta, \( \Pi_{00}(k = 0, T, H) \), are to be substituted. This limit, called the Debye mass, can be
computed from the EP of the special type. The latter fact considerably simplifies our task.

Now, let us turn to calculations of $V_{\text{ring}}(H, T)$. It is given by the standard expression [15], [15], [46], [17]:

$$V_{\text{ring}} = -\frac{1}{12\pi \beta} \text{Tr} \left\{ [M^2(\phi) + \Pi_{00}(0)]^{3/2} - M^3(\phi) \right\},$$

(35)

where trace means the summation over all the contributing states, $M(\phi)$ is the tree mass of the corresponding state. The functions $\Pi_{00}(0)$ are: $\Pi_{00}(0) = \Pi(k = 0, T, H)$ for the Higgs particle; $\Pi_{00}(0) = \Pi_{00}(k = 0, T, H)$ - the zero-zero components of the polarization functions of gauge fields in the magnetic field taken at zero momenta. The above contributions are of order $\sim g^3(\lambda^{3/2})$ in the coupling constants whereas the two-loop terms have order $\sim g^4(\lambda^2)$. For $\Pi_{00}(0)$ the high temperature limits of polarization functions have to be substituted which are of order $\sim T^2$ for leading terms and $\sim g\phi_c T, (gH)^{1/2} T(\phi_c/T << 1, (gH)^{1/2}/T << 1)$ for subleading ones.

### 5.1 The polarization function of scalar field

For the next step of calculation, we remind that the effective potential is the generating functional of the one-particle irreducible Green functions at zero external momenta. So, to have $\Pi(0)$ we can just calculate the second derivative with respect to $\phi$ of the potential $V^{(1)}(H, T, \phi)$ in the limit of high temperature, $T >> \phi, T >> (eH)^{1/2}$, and then set $\phi = 0$. This limit can be calculated by means of the Mellin transformation technique (see, for instance, [19]) and the result looks as follows:

$$V^{(1)}(H, \phi, T)_{T \to \infty} = \left[ \left( \frac{C_f}{6} \phi_c^2 + \frac{\alpha \pi}{2 \cos^2 \theta_w} \phi_c^2 + \frac{g^2}{16} \phi_c^2 \right) T^2 \right]$$

$$+ \left[ \frac{\alpha \pi}{6} (3\lambda \phi_c^2 - \delta^2(0)) T^2 - \frac{\alpha}{\cos^3 \theta} \phi^3 T - \frac{\alpha}{3} \left( \frac{3\lambda \phi_c^2 - \delta^2(0)}{2} \right)^{3/2} T \right]$$

$$- \frac{1}{2\pi} \left( \frac{1}{4} \phi_c^2 + gH \right)^{3/2} T + \frac{1}{4\pi} eHT \left( \frac{1}{4} \phi_c^2 + eH \right)^{1/2}$$
\[
+ \frac{1}{2\pi} eHT \left( \frac{1}{4} \phi_c^2 - eH \right)^{1/2}.
\]

The parameter \( C_f = \sum_{i=1}^{3} G_{ii}^2 + 3 \sum_{i=1}^{3} G_{iiq} \) determines the fermion contribution of order \( \sim T^2 \) having relevance to our problem. We also have omitted \( \sim T^4 \) contributions to the EP. The terms of the type \( \sim \log[T/f(\phi, H)] \) cancel the logarithmic terms in the temperature independent parts [30], [31]. Considering the high temperature limit we restrict ourselves to linear and quadratic in \( T \) terms, only.

One else important expression, which also should be taken into account, is the linear in \( H \) term of the zero temperature EP which looks as follows:

\[
V_{(1)}(H, \phi_c)/H_0^2 = -\frac{\alpha}{2\pi} \phi^2 \sum_f K_f |q_f H|.
\]

It significantly affects symmetry behaviour and contributes to the Debye mass in strong fields.

Now, differentiating these expressions twice with respect to \( \phi \) and setting \( \phi = 0 \), we obtain

\[
\Pi(0) = \frac{\partial^2 V^{(1)}(\phi, H, T)}{\partial \phi^2} |_{\phi=0} = \frac{1}{24\beta^2} \left( 6\lambda + \frac{6e^2}{\sin^2 2\theta_w} + \frac{3e^2}{\sin^2 \theta_w} \right) + \frac{2\alpha}{\pi} \sum_f \left[ \frac{\pi^2 K_f}{3\beta^2} - |q_f H| K_f \right] + \frac{(eH)^{1/2}}{8\pi \sin^2 \theta_w \beta} e^2 (3\sqrt{2} \zeta(-\frac{1}{2}, \frac{1}{2})).
\]

The terms \( \sim T^2 \) give standard contributions to temperature mass squared coming from the boson and fermion sectors. The \( H \)-dependent term is negative since the difference in the brackets is \( 3\sqrt{2} \zeta(-\frac{1}{2}, \frac{1}{2}) - 1 \simeq -0.39 \). Formally, this may result in the negativeness of \( \Pi(0) \phi \) for very strong fields \( (eH)^{1/2} > T \). But this happens in the range of parameters where asymptotic axpansion is not applicable. Substituting expression (38) into Eq. (35) we obtain (in the dimensionless variables)

\[
\psi_{ring}^\phi = -\frac{\alpha}{3B} \left\{ \left( \frac{3\phi^2}{2} - \frac{1}{2} K + \Pi(0) \right)^{3/2} + \frac{\alpha}{3B} K \left( \frac{3\phi^2}{2} - \frac{1}{2} \right)^{3/2} \right\}.
\]

As one can see, the last term of this expression cancels the fourth term in Eq. (36), which becomes imaginary at \( 3\phi^2 < 1 \). This is the important cancellation preventing the infrared instability at high temperature.

Notice that Eq. (35) contains other term (the last one) which becomes imaginary for strong magnetic fields or small \( \phi^2 \). It reflects the known instability in the \( W \)-boson spectrum [30], [31].
5.2 The Debye masses of photons and Z-bosons

To find the $H$-dependent Debye masses of photons and Z-bosons the following procedure will be used. We calculate the one-loop contributions to the EP due to the $W$-bosons and the fermions in a magnetic field and some "chemical potential", $\mu$, which plays the role of an auxiliary parameter. Then, by differentiating them twice with respect to $\mu$ and setting $\mu = 0$ the mass squared $m_D^2$ will be obtained. Let us outline that in more detail for the case of fermion contributions where the result is well known.

The temperature dependent part of the one-loop EP of constant magnetic field at a non-zero chemical potential induced by an electron-positron vacuum polarization is [63]:

$$V_{\mathrm{ferm.}}^{(1)} = \frac{1}{4\pi^2} \sum_{l=1}^{\infty} \left( -1 \right)^{l+1} \int_0^\infty \frac{ds}{s^3} \exp \left( -\frac{\beta^2 l^2}{4s} - m^2 s \right) \coth(eHs)\cosh(\beta l \mu), \quad (40)$$

where $m$ is the electron mass, $e = g \sin \theta_w$ is the electric charge and s-representation is used. Its second derivative with respect to $\mu$ taken at $\mu = 0$ can be written in the form

$$\frac{\partial^2 V_{\mathrm{ferm.}}^{(1)}}{\partial \mu^2} = \frac{eH}{\pi^2} \beta^2 \sum_{l=1}^{\infty} \left( -1 \right)^{l+1} \int_0^\infty \frac{ds}{s} \exp \left( -m^2 s - \beta^2 l^2 / 4s \right) \coth(eHs). \quad (41)$$

Expanding $\coth(eHs)$ in series and integrating over $s$ in accordance with formula (22) we obtain in the limit of $T >> m, T >> (eH)^{1/2}$:

$$\sum_{l=1}^{\infty} \left( -1 \right)^{l+1} \left[ \frac{8m}{\beta l} K_1(m\beta l) + \frac{2}{3} \frac{(eH)^2 l \beta}{m} K_1(m\beta l) + \cdots \right]. \quad (42)$$

Series in $l$ can easily be calculated by means of the Mellin transformation (see Refs. [64], [19], [18]). To have the Debye mass squared it is necessary to differentiate Eq. (41) with respect to $\beta^2$ and take into account the relation of the parameter $\mu$ with the zero component of the electromagnetic potential: $\mu \rightarrow ieA_0$ [17]. In this way we obtain finally,

$$m_D^2 = g^2 \sin^2 \theta_w \left( \frac{T^2}{3} - \frac{1}{2\pi^2} m^2 + O((m\beta)^2, (eH \beta^2)) \right). \quad (43)$$

This coincides with the known result calculated from the photon polarization operator [62]. As one can see, the dependence on $H$ appears in the order $\sim T^{-2}$. To find the total fermion contribution to $m_D^2$ one has to sum up the expression (43) over all fermions and substitute their electric charges.

To calculate $m_D^2$ for Z-bosons it is sufficient to account of the fermion coupling to Z-field. It can be done by substituting $\mu \rightarrow i(g/2\cos \theta_w + g \sin^2 \theta_w)$. The result
differs from Eq. (13) by the coefficient at the brackets in the right-hand side which has to be replaced, \( g^2 \sin^2 \theta_w \rightarrow \frac{g^2}{4 \cos^2 \theta_w} + \tan^2 \theta_w \). One also has to add the terms coming due to the neutral currents and the part of fermion-Z-boson interaction which is not reproduced by the above substitution:

\[
m_D' = \frac{g^2}{8 \cos^2 \theta_w} (1 + 4 \sin^4 \theta_w) T^2.
\] (44)

Now, let us apply the above procedure to obtain the \( W \)-boson contribution. The corresponding EP (temperature dependent part) calculated at non-zero \( T, \mu \) looks as follows,

\[
V_w^{(1)} = -\frac{e H}{8 \pi^2} \sum_{l=1}^{\infty} \int_0^\infty \frac{ds}{s^2} \exp\left(-m^2 s - \frac{l^2 \beta^2}{4 s}\right) \left[ \frac{3}{\sinh(e H s)} + 4 \sinh(e H s) \cosh(\beta l \mu) \right].
\] (45)

All the notations are obvious. The first term in the squared brackets gives the triple contribution of the charged scalar field and the second one is due to the interaction with a \( W \)-boson magnetic moment. Again, after differentiation twice with respect to \( \mu \) and setting \( \mu = 0 \) it can be written as

\[
\frac{\partial^2 V_w^{(1)}}{\partial \mu^2} \bigg|_{\mu=0} = \frac{e H}{2 \pi^2} \frac{\partial}{\partial \beta^2} \sum_{l=1}^{\infty} \int_0^\infty \frac{ds}{s} \exp\left(-\frac{m^2 s}{e H} - \frac{l^2 \beta^2 e H}{4 s}\right) \left[ \frac{3}{\sinh(s)} + 4 \sinh(s) \right].
\] (46)

Expanding \( \sinh^{-1}s \) in series over Bernoulli’s polynomials,

\[
\frac{1}{\sinh s} = \frac{e^{-s}}{s} \sum_{k=0}^{\infty} \frac{B_k}{k!} (-2s)^k,
\] (47)

and carrying out all the calculations described above, we obtain for the \( W \)-boson contribution to \( m_D^2 \) of the electromagnetic field

\[
m_D^2 = 3 g^2 \sin^2 \theta_w \left[ \frac{1}{3} T^2 - \frac{1}{2 \pi} T (m^2 + g \sin \theta_w H)^{1/2} - \frac{1}{8 \pi^2} (g \sin \theta_w H) \right] + O(m^2/T^2, (g \sin \theta_w H/T^2)^2) \].
\] (48)

Hence it follows that spin does not affect the Debye mass in leading order. Other interesting feature is that the next-to-leading terms are negative.

The contribution of the \( W \)-boson sector to the \( Z \)-boson mass \( m_Z^2 \) is given by expression (48) with the replacement \( g^2 \sin^2 \theta_w \rightarrow g^2 \cos^2 \theta_w \). Summing up the expressions (13) and (48) and substituting them in Eq. (35), we obtain the photon part \( V_{\gamma \text{ring}} \), where it is necessary to express masses in terms of the vacuum value of the scalar condensate \( \phi_c \). In the same way the daisy diagrams of \( Z \)-bosons
$V_{ring}^z$ can be calculated. The only difference is the mass term of $Z$-field and the additional term in the Debye mass due to the neutral current $\sim \bar{\nu}\gamma_\mu\nu Z_\mu$. These three fields $- \phi, \gamma, Z -$ which become massless in the restored phase, contribute into $V_{ring}(H, T)$ in the presence of the magnetic field. At zero field, there are also terms due to the $W$-boson loops in Figs. 3, 4. But when $H \neq 0$ the charged particles acquire masses $\sim eH$ and these daisies can be neglected.

5. 3 Daisy diagrams of the tachyonic mode

A separate consideration should be spared to the tachyonic (unstable) mode in the $W$-boson spectrum: $p_3^2 = p_3^2 + M_w^2 - eH$. First of all, we notice that this mode is excited due to a spin interaction and it does not influence the $G_{00}$ component of the $W$-boson propagator. Secondly, in the fields $eH \sim M_w^2$ the mode becomes the long range state. Therefore, it should be included in $V_{ring}(H, T)$ side by side with other considered neutral fields. But in this case it is impossible to take advantage of formula (33) and we return to the initial EP containing the generalized propagators.

For our purpose it will be convenient to make use of the generalized EP written as the sum over the modes in the external magnetic field [17], [18]:

$$V_{gen}^{(1)} = \frac{eH}{2\pi \beta} \sum_{l=\infty}^{+\infty} \sum_{n=0, \sigma=0, \pm 1} \log[\beta^2(\omega_l^2 + \epsilon_n^2, p_3, H, T)]$$

where $\omega_l = \frac{2nl}{\beta}$, $\epsilon_n^2 = p_3^2 + M_w^2 + (2n + 1 - 2\sigma)eH$ and $\Pi(H, T)$ is the radiation mass squared of $W$-bosons in magnetic field at finite temperature. Denoting as $D_0^{-1}(p_3, H, T)$ the sum $\omega_l^2 + \epsilon^2$, one can rewrite eq. (49) as follows:

$$V_{gen}^{(1)} = \frac{eH}{2\pi \beta} \sum_{l=\infty}^{+\infty} \sum_{n, \sigma} \log[\beta^2 D_0^{-1}(p_3, H, T)]$$

$$+ \frac{eH}{2\pi \beta} \sum_{l=\infty}^{+\infty} \sum_{n, \sigma} \log[1 + (\omega_l^2 + p_3^2 + M_w^2 - eH)^{-1} \Pi(H, T)]$$

$$+ \sum_{n \neq 0, \sigma \neq \pm 1} \log[1 + D_0(\epsilon_n^2, H, T) \Pi(H, T)].$$

(50)

Here, the first term is just the one-loop contribution of $W$-bosons, the second one gives the sum of daisy diagrams of the unstable mode (as it can easily be checked by expanding the logarithm in a series). The last term describes the sum of the short range modes in the magnetic field and should be omitted.

Thus, to find $V_{ring}^{unstable}$ one has to calculate the second term in Eq. (50). In the high temperature limit we obtain:

$$V_{ring}^{unstable} = \frac{eH}{2\pi \beta} \{(M_w^2 - eH + \Pi(H, T))^{1/2} - (M_w^2 - eH)^{1/2}\}.$$  (51)
By summing up the one-loop EP and all the terms $V_{ring}$, we arrive at the total consistent in leading order EP.

Let us mention the most important features of the above expression. It is seen that the last term in Eq. (51) exactly cancels the "dangerous" term in Eq. (36). So, the EP is real and no instabilities appear at sufficiently high temperatures when $\Pi(H, T) > M_w^2 - eH$. To make a quantitative estimate of the range of validity of the total EP it is necessary to calculate the $W$-boson mass operator in a magnetic field at finite temperature and hence to find $\Pi(H, T)$. This is a separate and enough cogent problem which is considered in detail in Ref. [66]. Here, we only adduce the result of $\Pi(H, T)$ calculations:

$$\Pi_{unstable}(H, T) = < n = 0, \sigma = 1 | \Pi^\text{charged}_{\mu
u} | n = 0, \sigma = 1 > = \alpha[12, 33(eH)^{1/2}T + i4(eH)^{1/2}T],$$

(52)

where the average value of the mass operator in the ground state of the $W$-boson spectrum $| n = 0, \sigma = +1 >$ was calculated. This expression has been obtained in the limit $eH/T^2 << 1, B = M_w/T << 1$. Side by side with the real part responsible for the radiation mass squared the expression (52) contains the imaginary one describing the decay of the state. The latter term is small as the former one is compared and of order of the usual damping constants at high temperature. So, $Im\Pi(H, T)$ can be ignored in our problem. The radiation mass squared is positive. It acts to stabilize the spectrum. At $H = 0$ no screening is produced in one-loop order, as it should be at finite temperatures for transversal modes [13]. Thus, we come to conclusion that at sufficiently high temperature the effective $W$-boson mass squared $M_{w, eff.}^2 = M_w^2 - eH + \Pi(H, T)$ is positive and no conditions for $W$-boson condensation discussed in Refs. [43], [32] are realized.

6. RESTORED PHASE IN THE EXTERNAL MAGNETIC FIELDS

Having calculated the EP at $\phi \neq 0$ we are able to determine the kind of the EW phase transition for different $m_H, h$. That will be done in the next sections. Here, we are going to describe in more detail the restored phase with the hypermagnetic and magnetic fields. Let us consider first the former case.

To describe more precise the restored phase one has to calculate radiation corrections to the external field $H_Y$ at high temperature. Before doing that let

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2 Express (52) disagrees with the corresponding one of Ref. [34] where the average value of the gluon polarization operator in an abelian chromomagnetic field was calculated in weak field approximation and $\Pi(H, T)$ has been found to be zero. Most probably, the discrepancy is the consequence of the calculation procedure adopted by these authors when the gluon polarization operator was calculated at zero external field and then its average value has been calculated in the state $| n = 0, \sigma = +1 >$. Our expression is the high temperature limit of the mass operator which takes into account the external field exactly.
us remind that at $\phi = 0$ this field is completely unscreened whereas the non-
abelian constituents of the electromagnetic and $Z$-fields are screened at scales
$l \geq (g^2 T)^{-1}$ by the magnetic mass. Remind also that we are investigating the
separate influence of the external fields. This means that in the covariant deriva-
tive describing interaction with the external field $H_Y$ in the restored phase one
should maintain the $U(1)_Y$ term only: $D_\mu = \partial_\mu + \frac{1}{2} g B^\mu_{ext}$. We set the potential
as before, $B^\mu_{ext} = (0, 0, H_Y x^1, 0)$.

In the restored phase $W$-bosons do not interact with $H_Y$. The field dependent
part of the EP $V(\phi = 0, H_Y, T)$ is non-zero due to the contributions of fermions
and scalars. However, the fermion part depends logarithmically on temperature
($\sim (g'/2)^2 H_Y^2 \ln(T/T_0)$) and can be neglected as compared to the tree level term
$\frac{1}{2} H_Y^2$. This is not the case for the scalar field whose contribution to the on
one-loop EP is

$$V_{sc}^{(1)}(H_Y, T) = - \frac{(g'/2)^2 H_Y^2}{24\pi^2} \ln(T/T_0) + \frac{((g'/2)H_Y)^{3/2}T}{6\pi} + O(1/T).$$

The term logarithmically dependent on $T$ can again be neglected but the linear
in $T$ part should be retained. Since “hyperphotons” are massless in the restored
phase we also include the contribution of the corresponding daisy diagrams:

$$V^{ring}_{restored}(H_Y, T) = - \frac{T}{12\pi} \left[ \frac{2}{3} (g'/2)^2 T^2 + m^2_D \right] - \frac{((g'/2)H_Y)^{1/2}T}{2\pi} - \frac{1}{8\pi^2} (g'/2) H_Y^{3/2},$$

where $m^2_D = \frac{1}{4\pi} g'^2 T^2 \sum_{f(R,L)} Y_f^2$ is the sum over the fermion contributions to the
Debye mass of the “hyperphotons”, $Y_f$ are the hypercharges of $R$- and $L$- lep-
tons and quarks. Both these expressions have been calculated in a way described
in previous sections.

For convenience of numerical investigations we express Eqs. (53) and (54) in
terms of the dimensionless variables $h, B$: $V(H_Y, T)_{restored} = (H_0)^2 v_{restored}(h, B)$,

$$v_{restored}(h, B) = \frac{1}{2} \frac{h^2}{\cos^2 \theta} + \frac{\alpha}{3\sqrt{2\cos^3 \theta}} \frac{h^{3/2}}{B} - \frac{1}{3} \frac{\alpha}{B} \frac{7}{6} \frac{4\pi\alpha}{\cos^2 \theta B^2} - \frac{h^{1/2}}{2\sqrt{2\pi B \cos \theta}} - \frac{h}{16\pi^2 \cos^2 \theta} \frac{1}{3/2};$$

where $\alpha = e^2/4\pi$ and $h_Y = h/\cos \theta$.

Now, let us turn to the magnetic field case. In accordance with our approach
we set $H_Y = 0$. It will be important for what follows to remember recent results on
observation of the gluon magnetic mass in lattice simulations that was found to be of order $m_{mag} \sim g^2 T$ (as it has been expected from nonperturbative calculatios
in quantum field theory [15], [14]). The mass screens the nonabelian component
of the magnetic field at distances $l > l_m \sim (g^2 T)^{-1}$ but inside the space domain $l < l_m$ it may exist and affect all the processes at high temperatures, as it was discussed in sect. 2.1. This fact has not been taken into consideration in a number of investigations of the EW phase transition. In particular, in Ref. [24] (as in Ref. [20]) the field strength generated at finite temperature was erroneously estimated as coinciding with that at zero temperature.

The magnetic field in the restored phase may be homogeneous or not dependently on the stability of the charged boson spectrum in the field calculated with radiation corrections included. That can be checked in accordance with formulae (51) and (52) at $M_w = 0$. If the effective mass squared $M_{\text{eff}}^2(H, T)$ is positive, the perturbative vacuum is stable and the external field is homogeneous otherwise it is unstable and a lattice structure has to be generated due to the evolution of the instability. We shall see below, for the first order phase transition (which is the main topic in the present paper) $M_{\text{eff}}^2(H, T, \phi_c)$ is positive in the minimum of the EP when the symmetry restoration happens. Therefore, we will not investigate in detail the structure of the restored phase for different $H$ restricting ourselves by the case of the constant field, as in the broken phase.

To make a link between studies of symmetry behaviour in the external hypermagnetic field and the previous results for the usual magnetic field [44], [17] we notice that in the broken phase $H_Y$ and $H$ are connected by the relation $H = H_Y \cos \theta$. So, all investigations, dealing with symmetry behaviour in a magnetic field at high temperature, are relevant in the case of $H_Y$ in the respect of the form of the EP curve at different $T, H_Y$. The hypercharge field influences the scalar field condensate at tree level and acts to restore symmetry. That was the reason why it has been taken into account in the lower order in Refs. [4], [5]. But, as it will be shown below, for strong fields and heavy $m_H$ the form of the EP curve in the broken phase is very sensitive to the change of the parameters. Moreover, it is strongly depended on the correlation correction contributions. So, to have an adequate picture of the EW phase transition symmetry behaviour with the daisy diagrams included has to be investigated.

7. HIGH TEMPERATURE EXPANSION OF THE EFFECTIVE POTENTIAL

The most of investigations dealing with symmetry behaviour at high temperature used the limiting form of the EP at $T \to \infty$. It will be of interest to compare the results obtained in two cases - for the complete EP and for the asymptotic one.

First, let us adduce the high temperature limit of the sum of terms describing the contributions of the scalar field at zero and finite temperature, the $H$-independent contribution of $W$-bosons, $Z$-bosons as well as the contributions of
the second terms of corresponding daisy diagrams, Eq. (39):

\[
\omega = \frac{3\alpha}{8\pi \cos^4\theta}(\phi^2 + \phi^4(\log(B) - C)) + \frac{3\alpha}{4\pi}(\phi^4\log\phi - \frac{3}{4}\phi^4 + \phi^2) + \frac{\alpha K^2}{32\pi}\left[\frac{1}{2}(3\phi^2 - 1)^2\log\left(\frac{4\pi}{B^2K}\right) + 6\phi^2 - C(3\phi^2 - 1)^2\right] + \frac{\alpha}{\pi}\left[\frac{\pi}{12B^2}(3\phi^2 - 1) + \frac{\pi^2\phi^2}{2\cos^2\theta B^2} - \frac{2\pi\phi^3}{3\cos^3\theta B}\right],
\]

\(C = 0.5772\) is Euler’s constant. The first terms of daisies dependent on the Debye masses should be added separately.

The high temperature asymptotic of the fermion sector \(\omega_f\) looks as follows:

\[
\omega_f^{*} = -\frac{\alpha}{\pi}\sum_{f}\left[-\frac{\pi^2}{3}\frac{K_f\phi^2}{B^2} + \frac{1}{6}\phi\frac{h^2}{B^2}(\log(\frac{\pi}{4\alpha\phi + K_f\phi^2 B^2}) - 2C)\right] + \frac{K_f^2\phi^4}{4}\left[\frac{\pi}{4\alpha + K_f\phi^2 B^2} - 2C\right] + \frac{3}{4}].
\]

The term \(4\pi\alpha\) in Eq. (58) is introduced in order to correct the infrared infinity in the field. It effectively accounts of the fermion temperature mass. Just due to this infrared singularity the light fermions dominate in strong fields.

Now, let us write down the high temperature limits of the W-boson sector (expressions \(\omega_0, \omega_1, \omega_2\)) Eq. (23). Instead \(\Re\omega^{(1)}\) one has to substitute the sum \(\omega_{\text{spin}} + \omega_0 = \omega_w^{(1)}:\

\[
\omega_{\text{spin}} = -\frac{\alpha h}{2\pi}\left[(\phi^2 + h)\log(\phi^2 + h) - (\phi^2 - h)\log |\phi^2 - h|\right] - 2h + 8Ch - 2h\log\left(\frac{\pi}{B^2\mu^2}\right) + \frac{4\pi}{B}(\phi^2 + h)^{1/2}],
\]

\[
\omega_0 = -\frac{3\alpha}{\pi}\left[\frac{h^2}{12}(\log(\frac{\pi}{(h + \phi^2)^{1/2}B}) - C) - \frac{\phi^4}{4}(\log(\frac{4\pi}{(h + \phi^2)^{1/2}B}) - C)\right] - \frac{\pi^2\phi^2}{3B^2} + \frac{2\pi}{3B}(\phi^2 + h)^{3/2} - \frac{\pi h}{B}(h + \phi^2)^{1/2} + \frac{1}{16}\phi^2h - \frac{3}{16}\phi^4\right].
\]

Here, the term due to the unstable mode is omitted since it is cancelled by the second term of daisy diagrams generating by the unstable mode. As it is seen, a lot of terms from expression \(V_{\phi,\phi}^{*}\) at zero temperature are cancelled in the total. The term with \(\log(B\mu)\) is \(\phi\)-independent and therefore inessential when symmetry behaviour is investigated, \(\mu\) marks the normalization point.

Now, for completeness, let us write down explicitly the contributions of the \(\gamma\)- and \(Z\)- daisy diagrams:

\[
V_{\text{ring}}^{\gamma} = -\frac{\alpha}{3B}[\Pi_{\gamma}(h, B)]^{3/2},
\]

25
\[
\Pi_\gamma(h, B) = \sum_f \left( \frac{e^2 q_f^2}{3B^2} - \frac{e^2 q_f^2 K_f \phi^2}{2\pi^2} \right) + e^2 \left( \frac{1}{B^2} - \frac{3}{2\pi B} (\phi^2 + h)^{1/2} - \frac{3}{8\pi^2} h \right).
\]

As the contribution of the Z boson daisy diagram we get (taking into account the substitution \(e^2 \rightarrow g^2(\frac{1}{4\cos^2 \theta} + \tan^2 \theta)\) and addind other necessary terms):

\[
V_{\text{ring}}^z = -\frac{\alpha^3 B}{3B^3} [\Pi_z(h, B) + \frac{\phi^2}{\cos^2 \theta}]^{3/2} + \frac{\alpha^3 B}{3B} \frac{\phi^3}{\cos^2 \theta},
\]

where

\[
\Pi_z(h, B) = \sum_f \left( g^2 \left( \frac{1}{4\cos^2 \theta} + \tan^2 \theta \right) \left( \frac{q_f^2}{3B^2} - \frac{q_f^2 K_f \phi^2}{2\pi^2} \right) \right) + \frac{g^2}{8\cos^2 \theta} \left( \frac{1}{6B^2} - \frac{\sqrt{K} \phi}{2\pi B} \right) + \frac{g^2}{8B^2 \cos^2 \theta} \left( 1 + 4 \sin^4 \theta \right).
\]

The term, containing \(\sqrt{K}\) comes from the one-loop diagram of the Higgs field. Remind, for the case of the asymptotic EP the last term of Eq. (63) has already been included in Eq. (56).

8. SYMMETRY BEHAVIOUR IN STRONG HYPER-MAGNETIC FIELD

Let us investigate the EW phase transition in the hypermagnetic field for different values of \(m_H\). It can be done by considering the Gibbs free energy in the broken, \(G_{\text{broken}}(H_{\text{ext}}, \phi, T)\), and the restored, \(G_{\text{restored}}(H_{\text{ext}}, T)\), phases:

\[
G_{\text{broken}} = V(\phi, H, T) - \vec{H} \vec{H}_{\text{ext}} \cos \theta, \quad G_{\text{restored}} = V(0, H_Y, T) - \vec{H}_Y \vec{H}_{\text{ext}}.
\]

By minimization of these equations the fields \(H\) and \(H_Y\) generated in the vacuum have to be expressed through \(H_{\text{ext}}\). The first order phase transition can be determined within two equations:

\[
G_{\text{restored}}(H_{\text{ext}}^c, T, 0) = G_{\text{broken}}(H_{\text{ext}}^c, T, \phi(H_{\text{ext}}^c)),
\]
describing the advantage of the broken phase creation, where $\phi(H)_c$ is a scalar field vacuum expectation value at given $H, T$, which has to be found as the minimum position of the total EP,

$$\frac{\partial V(H, T, \phi_c)^{\text{total}}}{\partial \phi_c} = 0.$$  \hfill (67)

Hence the critical field strength can be calculated. In this expression and below we use for brevity $H$ instead of $H^{\text{ext}}$.

Having obtained the EP in the restored phase, the one-loop EP described by formulae (17), (23), (29)-(34) and the daisy diagram contributions $V_{\text{ring}}$ we are going to investigate symmetry behaviour. We shall present the results in two stages. First, we shall consider the total EP as the function of $\phi^2$ at various fixed $H, T, K$ and determine the form of the EP curves in the broken phase. In this way it will be possible to select the range of the parameters when the first order phase transition is realized. After that the temperature $T_c$ at given field strength $H_Y$ will be found.

As usually [33], to investigate symmetry behaviour we consider the difference $V' = Re[V(h, \phi, K, B) - V(h, \phi = 0, B)]$ which gives information about the symmetry restoration. We will present the results for two cases: 1) for the total EP in the broken phase; 2) for the high temperature limits of it. Then we will make a comparison.

In what follows, it will be also convenient to express the conditions of the phase transition in terms of the dimensionless variables $h, B, \phi$, taking into account the relation $h_Y = h/cos\theta$. Then, the Gibbs free energy

$$G_{\text{broken}}(h^{\text{ext}}, \phi, B) = \frac{h^2}{2} + v'(h, \phi, B) - hh^{\text{ext}},$$  \hfill (68)

has to be expressed in terms of $h^{\text{ext}}$ through the equation

$$h^{\text{ext}} = h + \frac{\partial v'(h, \phi, B)}{\partial h},$$  \hfill (69)

where $v'$ describes the one-loop and daisy diagram contributions to the EP. The phase transition happens when the condition

$$\frac{h^2}{2}tan^2\theta = v'_\text{restored}(h, B_c) - v'_\text{broken}(h, \phi_c, B_c)$$  \hfill (70)

holds. The function $v'_\text{restored}$ is given by Eq. (55). We also have substituted the field $h^{\text{ext}}$ by $h$.

The results on the phase transition determined by numeric investigation of the total EP are summarized in Table 1.
Table 1.

In the first column we show the hypermagnetic field strength in the broken phase (in dimensionless units). In the second and third ones the mass parameter $K = m^2_H/M_w^2$ and the critical temperature of the first order phase transition are adduced. Next two columns give the local minimum position $\phi_c(H, T_c)$ and its squared value at the transition temperature. The last two columns fix the ratio $R = 246GeV\phi_c(h, T_c)/T_c$, determining the advantage of baryogenesis, and the $W$-boson effective mass calculated in the local minimum of the EP at the corresponding field strength and the transition temperature. The parameter $M_w(h, T_c) = [(\frac{2}{3}\phi_c(h, T_c))^2 - eH + \Pi(h, T_c, \phi_c)]^{1/2}/M_w$ is the dimensionless $W$-boson mass with the radiation corrections included.

As it is seen, the increase in $h$ makes the phase transition weaker (not stronger as it was expected in Refs. [6], [8] by analogy to superconductivity in the external magnetic field). The ratio $R$ is less then unit for all the field strengths, whereas the baryogenesis condition is $R > 1.2 - 1.5$ [3]. Thus, we come to the conclusion that external hypermagnetic field does not make the EW phase transition strong enough to produce baryogenesis. Moreover, for strong fields the phase transition is of second order for all the values of $K$ considered.

Let us continue the analysis of data in the Table 1. For the field strengths $H > 0.1 - 0.5H_0(H_0 = M_w^2/c)$ the phase transition is of second or weak first-order. The $W$-boson effective mass squared (in dimensionless units) $M_w^2(\phi_c, h, B_c) = \phi_c^2(h, B_c) - h + \Pi(h, B_c)$ is positive for $h = 0.01$ and $h = 0.1$. Therefore, the local minimum is the stable state at the first order phase transition. For stronger fields, when the second order phase transition happens, the effective $W$-boson mass becomes imaginary. This reflects the known instability in the external magnetic field which exhibits itself even when the radiation mass of the tachyonic mode is included. But it does not matter for the problem of searching for the strong first order phase transition in the external field investigated in the present paper. The instability has to result in the condensation of $W$- and $Z$-boson fields at high temperature. However, this does not change the type of the phase transition.
In Refs. [6], [9], [8], [67] it has been concluded that the strong hypermagnetic field increases the strength of the first order phase transition and in this case baryogenesis survives in the SM. Our results are in obvious contradiction with this conclusion. To explain the origin of the discrepancies let us first consider Refs. [6], [8] where a perturbative method of computations has been applied. These authors, in studying of the EW phase transition, have allowed for the influence of the external field at tree level, only. That corresponds to the usual case of superconductors in the external magnetic field, and, as a consequence, they observed the strong first order phase transition. In fact, the type of the phase transition was just assumed, since no investigations of the EP curve with all the particles included for different $H_Y, T$ have been carried out. In the former paper there was the qualitative estimate of the field effect, whereas in the latter one the quantitative analysis in one-loop approximation for the temperature dependent part of the EP has been done. Actually, in both these investigations the influence of the external field was reduced to the consideration of the condition (24) fixing the transition temperature in the hypermagnetic field. The role of fermions and $W$-bosons in the field was not investigated at all. However, as we have seen, the fermions (heavy and light) are of paramount importance in the phase transition dynamics. Just due to them the EW phase transition becomes of second order in strong fields (for the values of $K$ when it is of first order in weak fields).

In Refs. [9], [67] the phase transition was investigated by the method combining the perturbation theory and the lattice simulations. As the first step in this approach the static modes only are maintained in the high temperature Lagrangian. The fermions are nonstatic modes and decoupled. The only fermion remainder is the t-quark mass entering the effective universal theory [13], [9], [67]. So, no fermion features in the external fields and hence no information about the form of the EP curve could be derived in this way. In our analysis, it has been observed that not only heavy but also light fermions are important in strong external fields, as one, in particular, can see from the term $H^2 \log T/m_f$ of the one-loop EP. At high temperature it influences symmetry behaviour considerably. Actually, for various field strengths the fermions with different masses are dominant and we have allowed for all of them. Besides, we have taken into consideration all the daisy correlation corrections in the external field that also affects symmetry behaviour.

We would like to stress that our perturbative results for the values of $K \sim 0.8 - 0.9$ are reliable. They are in agreement with nonperturbative analysis at zero field. The external field is taken into consideration exactly. For these mass $m_H \sim 75 - 80$ GeV we observed the change of the first order phase transition into the second order one with increase in the field strength. The same behaviour takes also place for $K > 1$ when the perturbative analysis may be not trusty. But, as we have determined, the picture of the symmetry behaviour is only quantitatively changed for heavy scalar particles: the first order phase transition in weak fields becomes the second order one for strong fields. These circumstances convince us
that the assumption of Ref. [3] that the hypermagnetic field makes the weak first-order phase transition strong enough is not proved by the detailed calculations.

In Table 2 we adduce the results for the phase transition obtained within the asymptotic EP.

| h   | K     | $T_c$ (GeV) | $\phi_c^2(h, T_c)$ | $\phi_c(h, T_c)$ | R    | $M_{w}(h, T_c)$ |
|-----|-------|-------------|---------------------|------------------|------|-----------------|
| 0.01| 0.85  | 104.678     | 0.097               | 0.3114           | 0.7319| 0.3358          |
| 0.01| 1.25  | 119.569     | 0.098               | 0.3146           | 0.6473| 0.3441          |
| 0.01| 2     | 142.01      | 0.102               | 0.3194           | 0.5532| 0.3563          |
| 0.05| 0.85  | 105.449     | 0.085               | 0.2915           | 0.6801| 0.3050          |
| 0.05| 1.25  | 120.241     | 0.094               | 0.3066           | 0.6765| 0.3319          |
| 0.05| 2     | 142.559     | 0.102               | 0.3194           | 0.5511| 0.3611          |
| 0.1  | 0.85  | second      | order               | phase transition |       |                 |
| 0.1  | 1.25  | 121.616     | 0.062               | 0.2490           | 0.5037| 0.2809          |
| 0.1  | 2     | 143.539     | 0.088               | 0.2966           | 0.5084| 0.3420          |

Table 2.

These data, as previous ones, show that the ratio $R$ is less than unit and the baryogenesis condition does not hold.

Now, let us compare the results in Tables one and two. As one can see, even for $h = 0.01$ they differ considerably for all the parameters except the critical temperatures. The first order phase transition for $K = 0.85$ determined by both of the potentials possesses the same characteristics. But this is not the case for $K = 1.25$, when the phase transition described by the exact EP is weaker first order. For the field strength $h = 0.1$ the exact EP predicts the weak first-order phase transition for all $K$ whereas the asymptotic one fixes the second order phase transition for $K = 0.85$. For $K = 1.25$, the jump of the order parameter determined by the asymptotic EP is twice larger than for the exact EP case. If the fields are stronger than $0.2 \cdot 10^24$ G both EP predict the second order phase transition. Thus, we conclude that asymptotic EP predicts conversion of the first order phase transition to the second order one for weaker fields whereas the first order phase transition described by it is stronger as compare to the transition derived within the exact EP.

9. SYMMETRY BEHAVIOUR IN STRONG MAGNETIC FIELD

As we have described qualitatively in sect. 2, the case of external magnetic field somehow differs from the hypermagnetic one. Nevertheless, a number of previous results dealing mainly with the form of the EP curve in the broken phase are relevant, since in this phase the unscreened constituent of the external
hypermagnetic field coincides with the magnetic field. The condition fixing the transition temperature is

\[ V_{\text{restored}}(H, T_c, 0) = V_{\text{broken}}(H, T_c, \phi_c(H, T_c)). \]  

(71)

The transition happens when the depth of the minima located at the beginning, \( \phi_c = 0 \), and at \( \phi_c \neq 0 \) is the same.

Below, as before, we present the results of numeric investigations of the phase transition obtained for the exact EP and for the high temperature limit of it. In Table 3 we show the characteristics of the first order phase transition determined within the exact EP.

| h    | K    | \( T_c \) (GeV) | \( \phi_c^2(h, T_c) \) | \( \phi_c(h, T_c) \) | R    | \( M_w(h, T_c) \) |
|------|------|----------------|------------------------|------------------------|------|------------------|
| 0.01 | 0.85 | 105.18         | 0.107                  | 0.3271                 | 0.76506 | 0.3504          |
| 0.01 | 1.25 | 120.77         | 0.045                  | 0.2121                 | 0.4321 | 0.2541          |
| 0.01 | 2    | 143.96         | 0.016                  | 0.1265                 | 0.2162 | 0.2031          |
| 0.1  | 0.85 | 106.35         | 0.098                  | 0.3130                 | 0.7241 | 0.2835          |
| 0.1  | 1.25 | 121.92         | 0.021                  | 0.1449                 | 0.292  | 0.1243          |
| 0.1  | 2    | 146.14         | 0.003                  | 0.0545                 | 0.0917 | 0.1272          |
| 0.5  | 0.85 | 108.23         | 0.092                  | 0.3033                 | 0.6894 | 0.4696 i        |
| 0.5  | 1.25 | second phase  |                       |                        |       |                  |
| 0.5  | 2    | second phase   |                       |                        |       |                  |

Table 3.

The second column shows the values of the parameter \( K = \frac{m_H^2}{M_w^2} \), corresponding to the Higgs boson masses 75 GeV, 90 GeV and 115 GeV, respectively. All the parameters are as in section 8.

It is seen, an increase in \( h \) makes the first order phase transition weaker, as in the case of hypermagnetic field. For strong fields \( H \geq 10^{22} - 10^{23} \) G the baryogenesis condition \( R \geq 1.2 - 1.5 \) is not satisfied.

Now, let us show the results for the asymptotic EP. They are gathered in Table 4.
First of all notice, the baryogenesis condition $R \geq 1.2$ is not satisfied for neither the exact EP nor the asymptotic one. The $R$ values are small for for all $K$ considered. Since the external field is accounted of exactly through the Green functions the effects of the field influence are also exactly reproduced. Thus, our analysis has shown that in the case of external magnetic field baryogenesis can not be generated in the SM. The comparison of the results for the exact EP (Table 3) and the asymptotic EP (Table 4) shows that in the latter case the second order phase transition is predicted for more weak external fields.

The main results of this and previous section can be summarized as follows: the external either the magnetic or the hypermagnetic field can not produce the strong first order EW phase transition. Baryogenesis does not survive in the SM due to these external conditions.

In the next two sections we shall investigate the problems dealing with the spontaneous magnetization of the vacuum at high temperature and stabilization of this state due to radiation corrections. We shall show that strong magnetic fields $H \sim g^{4/3}T^2$ have to be spontaneously created.

10. RADIATION SPECTRA OF $W$-BOSON IN STRONG MAGNETIC FIELD AT HIGH TEMPERATURE

$W$-boson mass operator in a constant magnetic field $H$ at zero temperature has been calculated and investigated in Refs. [69], [70]. In particular, it gave possibility to clarify the role of the radiation corrections in the problem of stabilization of the $W$-boson spectrum (see survey [70]). The temperature dependent radiation corrections to the $W$-boson spectrum has been studied in Ref. [66]. The longitudinal components of gauge fields acquire the temperature masses $\sim gT$ [13]. The tachyonic mode is the transversal state excited by a spin interaction. Its temperature mass can be calculated as the average value of the mass operator in the ground state of the spectrum.

To incorporate temperature the imaginary time formalism will be used. As in the case of zero temperature [69], the Schwinger operator method and $s$-representation will be applied. In general, after summation over discrete imagi-
nary frequencies this method becomes not applicable. However, it remains prac-
tically unchanged in the limit of high temperature when only the static modes
\( l = 0 \) contribute \[15\]. This approximation is sufficient to investigate the role of
the daisy diagrams and will be used in what follows.

We calculate the average value of the \( W \)-boson mass operator in the states
of \( W \)-boson spectrum in a magnetic field \( | n, \sigma > \), where \( n, \sigma \) are the Landau
level number and the spin projection variable, respectively. These functions,
\(< M(H, T, n, \sigma) > \), give the radiation temperature dependent masses of the states.
The effective mass squared is
\[ M^2(H, T) = M^2 - eH + Re < M(H, T, n = 0, \sigma = 1) >. \]
If this value is positive, the spectrum is stabilized by radiation corrections.

In the present section we consider a simplified model of electroweak interac-
tions (the boson part of it) based on the spontaneous breaking of
\( SU(2) \rightarrow U(1) \) gauge group. The Lagrangian is

\[
L = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \phi)^2 + \frac{m_0^2}{2} \phi^2 - \frac{\lambda}{4} \phi^4, \tag{72}
\]

where \( x^2 = x^a x^a, a = 1, 2, 3, \)
\( F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g e^{abc} A_\mu^b A_\nu^c, \)
\( D_\mu \phi = \partial_\mu \phi + e A_\mu \). This model is described, for example, in Ref. \[70\]. After the sponta-
neous symmetry breaking the charged components \( W^\pm_\mu = \sqrt{2} (A_1^1 \pm i A_2^1) \) asquir
the masses \( M = g \phi_c \), where \( \phi_c = | m_0 | / \sqrt{\lambda} \) is the vacuum value of the scalar
field, and the component \( A_\mu = A_3^\mu \) remains massless and is identified with the
electromagnetic potential. We also identify the gauge coupling constant \( g \) with
electric charge: \( g = e \). In fact, this model is the mass regularization of the Yang-
Mills theory. In the limit \( M \rightarrow 0 \) all the corresponding results are reproduced.
The results for Salam-Weinberg theory can also be obtained by the simple substi-
tutions, \( \gamma \)-contributions \( \rightarrow Z \)-contributions with the corresponding vertex and
propagator factors (see for details Ref. \[70\]).

To investigate the problem we are interested in, let us, as before, direct the
external magnetic field along the third axis \( H = H_3 \). The corresponding potential
is chosen in the form \( A_\mu^{ext} = (0, 0, 0, H x_1), H = const. \) To quantize the fields the
following gauge fixing condition is used:

\[
\partial_\mu W^{\pm\mu} - ie A^{ext}_\mu W^{\pm\mu} - M \phi^{\pm} = 0, \tag{73}
\]

where \( \phi^{\pm} = \frac{1}{\sqrt{2}} (\phi^1 \pm i \phi^2) \) are the charged Goldstone fields.

The \( W \)-boson mass operator in one-loop order is given by a standard series
of diagrams \[70, 83\]. It can be written in the form

\[
M_{\mu\nu} = \frac{e^2}{\beta} \sum_{k_4} \int \frac{d^3 k}{(2\pi)^3} [M^\phi_{\mu\nu}(k, P) + M^{vec}_{\mu\nu}(k, P)], \tag{74}
\]

where \( \beta = 1/T, k_4 = 2\pi l/\beta, l = 0, \pm 1, \pm 2, ..., \) operator \( P_\mu = i \partial_\mu + e A^{ext}_\mu \). The
first term in the Eq. (74) describes the contribution of virtual neutral scalar
particles and the second one gives that of virtual gauge fields (photons, W- and Goldstone bosons).

We restrict ourselves by investigation of the high temperature limit, that corresponds to the static mode \( l = 0 \). In this case the standard calculation procedure developed at zero temperature can be straightforwardly applied. Details of these calculations can be found in Refs. [69], [70], [66]. Here, we note the points specific for \( k_4 = 0 \) case. For brevity, in what follows we will consider the part \( M^+(k, P) \).

To integrate over \( d^3k \) in Eq. (74) one has to introduce the s-representation for each propagator entering and present the product of propagators as follows

\[
D_\phi(k)G_w(P - k) = -\int_0^1 du \int_0^\infty ds se^{i s H},
\]

where "Hamiltonian" is,

\[
H = (1 - u)\vec{k}^2 + u(\vec{P} - \vec{k})^2 - u(M^2 + 2ieF) - (1 - u)m^2,
\]

and \( m \) is the mass of the neutral scalar field. The \( k_4 \)-dependent part of the Hamiltonian is zero. Then, the integration is carried out by means of the procedure developed in Ref. [61]. The difference between the finite and zero temperature cases consists in the dimension of the corresponding integrals. This is reflected in the power of \( s \) appeared after integration. Three dimensional integral (at \( T \neq 0 \)) gives the factor \( s^{-3/2} \) whereas at \( T = 0 \) one obtains \( s^{-2} \) [61].

To find the energy of the states owing to the radiation corrections one has to define the \( W \)-boson mass-shell in the field at high temperature. In the static case it is described by the equations

\[
\begin{align*}
\left( \vec{P}^2 + M^2 \right)\delta_{ij} + 2ieF_{ij}^{ext} \right) W_j^- &= 0, \\
P_i W_i^- &= 0, \quad \phi^- = 0,
\end{align*}
\]

where \( j = 1, 2, 3 \) and the product \( P_4 W_4 = 0 \) because for static modes \( p_4 = 0 \). The states are normalized by the condition

\[
<n, \sigma \mid n', \sigma'> = \delta_{n,n'} \delta_{\sigma,\sigma'},
\]

\( n, n' = 0, 1, \ldots \) and \( \sigma, \sigma' = 0, \pm 1 \).

The average value of the mass operator in these states can be written in the form

\[
< M > = \frac{\alpha}{2\sqrt{\pi} \beta} \int_0^1 du \int_0^\infty \frac{dx}{\sqrt{x}} [e u H \Delta]^{-1/2} e^{-x u M^2 / e H}
\]

\[
\exp \left[ -(2n + 1)(\rho - x(1 - u) - 2u(1 - u)) M(x, u) \right],
\]

\( \alpha, \beta \))
where $M(x, u) = < n, \sigma | M_{ij} | n, \sigma >$ and

$$
\tanh\rho = \frac{(1-u)shx}{(1-u)chx + uhx/x}
$$

$$
\Delta = (1-u)^2 + 2u(1-u)\frac{sh2x}{2x} + u^2(\frac{shx}{x})^2.
$$

This very complicate expression can be investigated for different limits of interest. In particular, for high temperatures and strong fields, $\frac{eH}{M^2} \gg 1$, $\frac{eH}{T} \ll 1$ we obtain (in the reference frame $p_3 = 0$):

$$
< n, \sigma = +1 | M^{gauge} | n, \sigma = +1 > = \frac{e^2}{4\pi}(eH)^{1/2}T[12, 33 + 4n + i(3 + 6n)],
$$

$$
< n, \sigma = +1 | M^\phi | n, \sigma = +1 > = \frac{e^2}{4\pi}(eH)^{1/2}T[14, 63 + 4n + i(7 + 6n)],
$$

$$
Re < n, \sigma = -1 | M^{gauge} | n, \sigma = -1 > = \frac{e^2}{4\pi}(eH)^{1/2}T(11, 44 + 4n),
$$

$$
Re < n, \sigma = 0 | M^{gauge} | n, \sigma = 0 > = \frac{e^2}{4\pi}(eH)^{1/2}T(15, 44 + 4n).
$$

Above formulae give a general picture on the behaviour of the radiation energies for different spin states. As it is seen, the real part of $< M >$ is positive in the ground and excited states. It acts to stabilize the tree spectrum. The imaginary part describes the decay of the states due to transitions to the ones having lower energies.

Now, let us consider the effective $W$-boson mass squared:

$$
M^2(H, T) = M^2 - eH + \Re < n = 0, \sigma = +1 | M^{gauge} + M^\phi | n = 0, \sigma = +1 >
$$

$$
= M^2 - eH + 26, 96 \frac{e^2}{4\pi}(eH)^{1/2}T.
$$

This value is positive for sufficiently high temperatures. Hence, one can conclude that radiation corrections in the field stabilize the vacuum at high temperatures. The temperature mass of the transversal modes depends on the field and equals to zero when $H = 0$, as it should be [15].

Obtained results are of interest for cosmology. Namely, if at the EW phase transition epoch the magnetic field was present, the radiation mass of $W$-bosons could serve as the dynamic mechanism of the vacuum stabilization in both the broken and the restored phases. It was discussed in detail in the previous sections. To derive a consistent picture one has to consider a vacuum magnetization at high temperature with the correlation corrections taken into consideration.
11. THE SPONTANEOUS VACUUM MAGNETIZATION AT HIGH TEMPERATURE

The generation of magnetic fields in nonabelian gauge theories at finite temperature is of great importance for particle physics and cosmology. Its positive solution, in particular, will give a theoretical basis for investigations of the QCD vacuum at high temperature and the primordial magnetic fields in the early universe [20], [36]. In literature different mechanisms of producing the fields are discussed (see recent papers [6], [13] and references therein). In this section we investigate in more detail one of possibilities - the spontaneous magnetization of the vacuum of nonabelian gauge fields at finite temperature. This problem has been studied in one-loop order in Refs. [20], [17], [19] where the creation of the vacuum field was derived. In Ref. [17] a number of correlation corrections has been taken into account, but the polarization functions of gauge fields were not calculated and therefore a trusty conclusion about the phenomenon was not obtained. In Ref. [11] that has been considered with all necessary daisy diagrams included. Below, we shall follow this paper.

For simplicity, we investigate the vacuum magnetization at finite temperature within $SU(2)$ gluodynamics. Considering the abelian covariantly constant chromomagnetic field $H^a = \delta^{a3}H = \text{const}$ and finite temperature as a background, we calculate the EP containing the one-loop and daisy diagram contributions of the neutral and charged gluon fields. The field dependent Debye mass of neutral gluons can be computed from a special type EP. To find the one of the charged gluons the high temperature limits of the gluon polarization functions at the background are also calculated. It will be shown that in the adopted approximation the Savvidy level with the field strength $(gH)^{1/2} \sim g^{4/3}T$ is generated. This is strong field. Although it is screened at distances $l > (g^2T)^{-1}$ by the gluon magnetic mass, the spectrum of charged particles, being formed at Larmor’s radius $r \sim (gH)^{-1/2} \sim (g^{4/3}T)^{-1}$, is located inside this domain, $r << l_m$, for small $g$.

Most of results for the Yang-Mills (YM) theory can be obtained without actual calculations by making use of the results of the previous chapters. Really, the model considered in sect. 9 is a mass regularization of the YM theory. The external magnetic field just coincides with the abelian chromomagnetic field of interest. Therefore, the results for the latter one can be obtained by setting the mass of the charged gauge fields to zero and subtracting the contributions of the longitudinal spin projection of the massive charged gauge fields and the Goldstone fields. These transformations can be made freely. For instance, to get the contribution of charged gluons to the one-loop EP it is sufficient to substitute the factor 3 by the factor 2 in the part of Eq. (20) describing the contribution of massless ($M = 0$) charged spin zero particles. The same has to be done to find either the Debye mass of neutral gluons (in Eq. (48)) or the temperature masses of charged gluons (sect. 9), etc. Below, for convenience of further account, we
The high temperature limit of the one-loop EP is

\[
V^{(1)}(H, T) = \frac{H^2}{2} + \frac{11 g^2}{48 \pi^2} H^2 \log \frac{T^2}{\mu^2} - \frac{1}{3} \frac{(gH)^{3/2} T}{\pi} - i \frac{(gH)^{3/2} T}{2\pi} + O(g^2 H^2).
\] (83)

Notice the cancellation of \(H\)-dependent logarithms entering the vacuum and the statistical parts, \(\mu\) is a normalization point.

11.1 Daisy diagrams of the unstable mode and the neutral gluons

The contribution of daisy diagrams with the unstable mode is given by the expression

\[
V_{\text{unstable}} = gHT \frac{\Pi(H, T, n = 0, \sigma = +1)}{2\pi} - gH^{1/2} + i \frac{(gH)^{3/2} T}{2\pi}. \] (84)

From Eqs. (83) and (84) it is seen that the imaginary terms are cancelled out in the total. The final EP is real if the condition \(\Pi_{\text{unstable}}(H, T) > gH\) holds.

This expression must be supplemented by the term describing the contribution of the neutral gluon fields. In one-loop order it gives a trivial \(H\)-independent constant which can be omitted. However, these fields are long-range states and they do give \(H\)-dependent EP through the correlation corrections including the mass term \(\Pi^0(H, T)\). Corresponding part of the EP is described by the expression which can be recognized from the case \(H = 0\) \([46], [15]\):

\[
V_{\text{ring}} = \frac{1}{24} \Pi^0(H, T) T^2 - \frac{1}{12\pi C} (\Pi^0(H, T))^{3/2} + \frac{\Pi^0(H, T)}{32\pi^2} \left[ \log \left( \frac{4\pi T}{\Pi^0} \right)^{1/2} + \frac{3}{4} - C \right],
\] (85)

where \(\Pi^0(H, T) = \Pi^0_{00}(k = 0, H, T)\) is the zero-zero component of the neutral gluon polarization operator calculated in the external field at finite temperature and taken at zero momentum, \(C\) is Euler’s constant. The first term in Eq. (85) has order \(\sim g^2\) in coupling constant, the second term is of order \(\sim g^3\) and the last one \(\sim g^4\). Restricting ourselves by order \(\sim g^3\), it will be omitted in what follows. As usually, for \(\Pi(H, T)\) the high temperature limit of the function has to be used. The expression (85) needs in one additional comment. If one compares it with Eq. (83), the difference will be in the extra term \(\sim \Pi(T, H) T^2\). We have maintained this next-to-leading term because it is of great importance for problems under consideration. In studying of symmetry behaviour only the leading \((\phi\text{-independent})\) terms of the polarization functions were taking into account.
The mass squared $m_{neutral}^2 = \Pi_{00}(H, T, p_0 = 0)$, that is the Debye mass of neutral gluons, reads

$$m_D^2 = \frac{2}{3} g^2 T^2 - \frac{(gH)^{1/2}}{\pi} T - \frac{1}{4\pi^2} (gH) + O((gH)^2 / T^2). \quad (86)$$

Here, the first term is the well known temperature mass squared and other ones give the field-dependent contributions. They have negative signs that is important for what follows. Substituting expression (86) into equation (85) we obtain the correlation corrections due to neutral gluons.

The calculations, as we have described in sect 10., result in the high temperature limits of $\Pi_{unstable}(H, T)$

$$\Pi_{unstable}(H, T) = \langle n = 0, \sigma = +1 \mid \Pi_{\mu\nu}^{charged} \mid n = 0, \sigma = +1 \rangle = 12.33 \frac{g^2}{4\pi} (gH)^{1/2} T, \quad (87)$$

and of excited states $\Pi(n \neq 0, \sigma)$,

$$Re\Pi(p_4 = 0, n, p_3 = 0, H, T, \sigma = +1) = \frac{g^2}{4\pi} (gH)^{1/2} T (12.33 + 4n), \quad (88)$$

$$Re\Pi(p_4 = 0, n, p_3 = 0, H, T, \sigma = -1) = \frac{g^2}{4\pi} (gH)^{1/2} T (11.44 + 4n),$$

where the average values of the polarization operator in the corresponding states of the spectrum (91) are computed. These formulae have been obtained in the high temperature limit $gH / T^2 << 1$. The operator contains also an imaginary part which describes the decay of the states. But for the problem under consideration only the real part is needed because it is responsible for the radiation masses of particles.

Let us note the most important features of the expressions (87), (88). It is seen, at $H = 0$ no screening magnetic mass is produced in one-loop order. Second, the mass squared of the modes are positive and act to stabilize the spectrum of charged gluons at high temperatures. Therefore, in the nonzero chromomagnetic field at finite temperature the charged transversal gluons become massive.

### 11.2 Contribution of the daisy diagrams of charged gluons

To obtain the correlation corrections due to the longitudinal charged gluons the zero-zero component of the polarization operator has to be calculated. The Debye mass of charged gluons is found to be [66]

$$\Pi_{00}(k_4 = 0, k_3 = 0, H, T) = \frac{2}{3} g^2 T^2 + \frac{g^2}{4\pi} (gH)^{1/2} T (6 + 4n), \quad (89)$$

where again only the real part is adduced. Hence, the next-to-leading terms are the growing positive functions of $n$. 38
Now, let us turn to the generalized EP

$$V_{\text{gen}}^{(1)} = \frac{gH}{2\pi} \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \sum_{n=0,\sigma=\pm 1}^{\infty} \log[\beta^2 (\omega_l^2 + \epsilon_{n,\sigma,p_3}^2 + \Pi(n, T, H, \sigma))] \quad (90)$$

written as the sum of energies of the charged gluon field modes in the external chromomagnetic field (\(\omega_l = \frac{2\pi l}{\beta}\) - discrete imaginary energies)

$$\epsilon_{n,\sigma}^2 = p_3^2 + (2n + 1 - 2\sigma)gH + \Pi(H, n, \sigma, T) \quad (91)$$

and including the temperature masses \(\Pi(H, n, \sigma, T)\) of the modes which are also dependent on \(H\), the level number, \(n\), and the spin projection \(\sigma\). In general, the incorporation of the polarization functions into EP may result in wrong combinatoric factors for the two-loop diagrams. In our case, to be sure in the obtaining results the following procedure is applied. We subtract the term

$$V_s = \frac{gH}{2\pi} \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \sum_{n=0,\sigma=\pm 1}^{\infty} D_0(p_3, H, T)\Pi(H, n, T, \sigma) \quad (92)$$

from Eq. (90), that separates the contributions of the two-loop diagrams of charged gluons. One freely can check that the two-loop diagrams containing the one of loops with the neutral gluon lines can be accounted of within the generalized EP including the insertion of the neutral gluon polarization operator. After that the only two-loop diagram (two touch circles) allowing for the self-interaction of charged gluon fields in the vacuum remains to be computed separately.

Then, by substituting the expressions (87), (88), (89) into Eq. (90) and integrating over momentum and calculating the sums in \(n\), we obtain the daisy diagram contribution of charged gluon fields. The result can be expressed in terms of the generalized \(\zeta\)-function and looks as follows,

$$V_{\text{ring}}^{\text{ch}} = \frac{gHT}{2\pi} \left\{ \sqrt{2gHD} \left[ \zeta(-\frac{1}{2}, a_+) + \zeta(-\frac{1}{2}, a_-) + 2\zeta(-\frac{1}{2}, a_D) \right] - \sqrt{2gH} \left[ 3\zeta(-\frac{1}{2}, \frac{1}{2}) + \zeta(-\frac{1}{2}, \frac{3}{2}) \right] + (\Pi(H, T, n = 0, \sigma = +1)^{1/2} \right\}, \quad (93)$$

where the first term in the first squared brackets corresponds to the spin projection \(\sigma = +1\), the second term - \(\sigma = -1\) and the last one describes the part due to longitudinal charged gluons. The terms in the second squared brackets give the independent of \(\Pi(H, T)\) part of eq. (90). The last term in the curly brackets is due to radiation mass of the unstable mode. The notations are introduced: \(gHD = gH + \frac{g^2}{2\pi} (gH)^{1/2} T\), \(a_- = \frac{1}{2} + \frac{g^2}{4\pi} \frac{11.44(gH)^{1/2} T}{2gHD}\), \(a_+ = \frac{1}{2} + \frac{g^2}{4\pi} \frac{19.62(gH)^{1/2} T}{2gHD}\) and \(a_D = (\frac{2}{7}g^2 T^2 + \frac{3g^2}{2\pi}(gH)^{1/2} T + gH)/2gHD\). This expression is real for sufficiently high temperatures.
Substituting the expression (86) in eq. (85) and gathering all other contributions, we obtain the consistent EP.

To calculate the two-loop vacuum diagram describing the self-interaction of charged gluons the standard procedure can be applied. The computation details are given in Ref. [11]. Referring the readers to this paper, we write down here the resulting expression for the high temperature limit

\[
V_{ch}^{(2)}(H, T)_{|T \to \infty} = -\frac{g^2}{4\pi}(gH)^{1/2}T^3 + O(gHT^2).
\] (94)

The high temperature limit of the term subtracted from the generalized EP (92) is

\[
V_s(H, T)_{|T \to \infty} = \frac{g^2}{6\sqrt{2\pi}}\zeta\left(\frac{1}{2}, -\frac{1}{2}\right)(gH)^{1/2}T^3 + O(gHT^2).
\] (95)

It also gives the negative contribution to the leading terms of the asymptotic expansion since \(\zeta\left(\frac{1}{2}, -\frac{1}{2}\right) = 0.8093\). It worth to mention that these are the longitudinal modes that determine the high temperature behaviour of \(V_{ch}^{(2)}(H, T)\).

Having obtained the two-loop corrections to the EP, one can investigate the spontaneous vacuum magnetization at high temperature.

### 11.3 Vacuum magnetization and the stability problem

The derived EP is expressed through the well studied special functions. Therefore, it can easily be investigated numerically for any range of parameters entering. As usually, it is convenient to introduce the dimensionless variables: the field \(\phi = (gH)^{1/2}/T\) and the EP \(v(\phi, g) = V(H, T)/T^4\). The vacuum magnetization at high temperatures, \(T >> (gH)^{1/2}, \phi \to 0\), will be investigated within the following limiting form of the EP,

\[
v^{\text{total}}(\phi, g)_{|\phi \to 0} = \frac{\phi^4}{2g^2} + \frac{11\phi^4}{48\pi^2}\log\left(\frac{T^2}{\mu^2}\right) - \frac{1}{3}\phi^3\left(\frac{1}{\pi}\frac{g^2}{48\pi} - \frac{g^2}{\phi}\right) \nonumber
\]

\[
- \frac{1}{3}\left(\frac{2}{3}\right)^{3/2}\frac{27\phi^2}{\pi} - \frac{g^2}{\phi} - \frac{g^2}{6\sqrt{2\pi}} \cdot 0.8093\phi,
\] (96)

where other \(\sim g^3\) terms are omitted. The logarithmic term is signalling the asymptotic freedom of \(g^2(T)\) at high temperatures [19]. It includes explicitly the dependence on the scale parameter \(\mu\). Other terms present, respectively, the high temperature limits of the one-loop EP, the neutral gluon and the charged gluon daisies and the two-loop diagram of charged gluons. To obtain the term due to \(V_{ring}^{ch}\) the asymptotic expression for Zeta-function [72],

\[
\zeta\left(-\frac{1}{2}, a_D\right)_{|a_D \to \infty} = -\frac{2}{3}a_D^{3/2} + \frac{1}{2}a_D^{1/2} - \frac{1}{48}a_D^{-1/2} + O(a_D^{-3/2}).
\] (97)
was used. Zeta-functions with $a_+, a_-$ do not contribute in leading order. Since we are searching for the fields $\phi$ of order larger then $g^2$, we can omit the term $g^3$ and obtain for the condensed field

$$ (gH)^{1/2}_c = \frac{0.6}{\pi^{1/3}} g^{4/3}T. $$  \hspace{1cm} (98)

Thus, we come to the result that the ferromagnetic vacuum state indeed exists at high temperatures. The correlation corrections increase the field strength as compare to the one-loop value $(gH)^{1/2}_c \sim g^2T$.

Let us discuss the stability of the condensed field. The second derivative of the EP is positive for $H_c$ that means we have the minimum. The field is not changing in the direction $a = 3$ of the isotopic space. To check that this is indeed the case for the perpendicular directions $1, 2$ or $a = a^{\pm}$ responsible for excitation of charged fields $W^{\pm}$, one has to calculate the effective mass squared $M^2(H_c, T)$. First we consider the one-loop case. Substituting the value $(gH^{(1)}_c)^{1/2} = (g^2/2\pi)T$ in the one-loop polarization function, we find that the effective mass squared, $M^2(H^{(1)}_c, T) = \Pi^{(1)}(H^{(1)}_c, n = 0, \sigma = +1) - gH^{(1)}_c \geq 0$, is positive. Thus, the vacuum stabilization is observed in this consisten calculation. However, if one checks whether the one-loop gluon radiation mass stabilize the true vacuum magnetic field and substitutes the value $(gH^{(1)}_c)^{1/2} \sim g^{4/3}T$, the negative value of $M^2(H_c, T)$ will be obtained. The one-loop mass does not stabilize the spectrum and, hence, vacuum. Nevertheless, the one-loop result makes hopeful the idea to have the stable vacuum due to radiation corrections to the charged gluon spectrum. Naturally, to investigate this possibility the gluon polarization operator with the correlation corrections included should be calculated. This problem requires an additional investigation. Other interesting possibility is the formation at high temperatures of the gluon electrostatic potential, so-called $A_0$ condensate (see survey [68]), which also acts as a stabilizing factor [17]. To realize the latter scenario consistently the simultaneous spontaneous generation of both the $A_0$ condensate and the magnetic field should be investigated. If again the homogeneous vacuum field will be found to be unstable with these improvements made, the inhomogeneous fields of the lattice type discussed in Refs. [36], [43], [58] can be created. Really, since the condensed magnetic field is strong at high temperatures, the lattice structures having the cells of order $\sim 1/(g^{4/3}T) << 1/(g^2T)$ are located inside the domain where the fields are not screened by the gluon magnetic mass.

The above calculations have unambiguously determined the possibility of the vacuum magnetization at high temperature, although a number of questions concerning the vacuum stabilization and structure has to be investigated in order to derive a final picture. This could result in the presence of strong magnetic fields in the hot universe.
12. COMPARISON WITH OTHER APPROACHES

In this section we are going to compare our results with that of other investigations. That refer to either the phase transition or the generation of the magnetic field at high temperature. We begin with discussion of the EW phase transition in strong hypermagnetic fields.

First let us discuss the results of Refs. [6], [8] where it was concluded that strong external hypermagnetic field generates the sufficiently strong first order phase transition and baryogenesis survives in the SM. In these papers the influence of the field on the vacuum has been allowed for in tree approximation that gives the qualitative estimate of the effect. In Ref. [8] the temperature dependent part of the EP has been included in one-loop order whereas the field dependence was skipped. Naturally, further studying of the phenomenon should include the field dependent radiation and correlation corrections due to fermions and bosons. This is the problem that we have addressed to in the present investigation. The main idea was to determine the form of the EP curve in the broken phase and find the range of the parameters \( H, K \) when the EW phase transition of first order is happened. To elaborate that the consistent EP including the one-loop and daisy diagrams of all the fundamental particles has been calculated. As we have discovered, the role of fermions is crucial in a vacuum dynamics in strong fields at high temperature. They essentially affect the structure of the broken phase making the EW phase transition weaker as compare to the tree level results. The external field has been accounted of exactly through Green’s functions. The minimum of the EP was found to be real at sufficently high temperatures when the first order phase transition happens. This important property was established when the daisy diagrams of the tachyonic mode have been included. As a result, no conditions for \( W^{-} \) and \( Z \)-boson condensates are observed at high temperatures and the external field strengths corresponding to the first order phase transition. The condensates could be generated for stronger fields when the phase transition becomes of second order. But, this is not of interest here since we are looking for conditions when baryogenesis can survive.

In Refs. [9], [67] the EW phase transition in the hypermagnetic field has been investigated by means of a general method developed in Ref. [71] (see also survey [13]) which combines the perturbative computations and the lattice simulations. The results obtained therein have supported the main conclusions of Refs. [6], [8] discussed above. In more detail, it was found for the fields \( H_{Y} < 0.3T^{2} \) the first order phase transition becomes stronger, but it still turns into a crossover for masses \( m_{H} \geq 80 \text{ GeV} \). For stronger fields, a mixed phase analogous to a first type superconductor with a single macroscopic tube of symmetric phase, parallel to \( H_{Y} \), penetrating through the broken phase, has been observed.

As this aproach is concerned, we note that because of peculiarities of the calculation procedure adopted in Ref. [8] all fermions except t-quarks are decoupled as nonstatic modes and therefore the field dependent fermion contributions
as well as the correlation corrections could not be accounted of. That is why these calculations also do not reproduce correctly symmetry behaviour for strong external fields.

In the present approach, the value of the mass $m_H = 75 \text{ GeV} (= 0.85)$ is close to $m_H = 80 \text{ GeV}$ discussed in Ref. [4]. This is important for us because the first order phase transition is controlled by perturbative method used. But we could not observe the mixed phase. We have seen the crossover (or second order phase transition) for $H_Y = 10^{23} \text{ G}$ and $K = 0.85$ determined within the asymptotic EP. The exact EP in this case predicts the weak first-order phase transition. Since the EP is real in the minimum, no conditions for the vortex-like phase exist.

The Higgs boson mass values considered correspond to the cases when perturbative results are reliable ($K = 0.85$) and may be not trusty ($K = 1.25 , 2$). However, since the external field is taken into account exactly its effects are correctly reproduced. As we have seen, an increase in $H_Y$ makes the EW phase transition of second order for field strengths $H_Y \sim 0.5 \cdot 10^{24} \text{ G}$ for all the values of $K$ investigated. For weaker fields the phase transition is of first order but the ratio $R = \phi_c (H, T_c) / T_c$ is less then unit, that is unsufficient to generate baryogenesis.

Let us remind the situation with the magnetic field when the magnetic mass of order $m_{mag} \sim g^2 T$ is taken into consideration. This mass screens the nonabelian component $H \delta^{a3}$ at distances $l > l_m \sim (g^2 T)^{-1}$ but inside the space domain $l < l_m$ it may exist and affect all the processes at high temperatures. The latter fact has not been realized in a number of investigations [34], [35] where (as in Ref. [20]) the field strength generated at finite temperature has been erroneously estimated as coinciding with that at zero temperature. Our estimate of the field strength at high temperature makes the investigation of the EW phase transition in strong magnetic fields reasonable. But, as we have seen in sect. 9, in this case it also is impossible to generate the strong first order phase transition.

Now, let us consider in more detail the problems of the magnetic field generation and stabilization at high temperature. In the present survey we have investigated in detail the Savvidy mechanism. The ways of the vacuum stabilization have also been discussed. In this scenario, usual magnetic field should be treated as the projection of the chromomagnetic field created in a nonabelian gauge theory. Results of our investigation disagree with that of Refs. [34], [35] where it is claimed that spontaneous magnetization does not hold at high temperature. To clarify the origin of the discrepancy let us repeat the main statements of Refs. [34], [35]: 1) The field strength generated at finite temperature $gH \sim \Lambda^2$, where $\Lambda^2 = \mu^2 \exp(-\frac{48\pi^2}{11Ng^2(\mu)})$, coincides with that at zero temperature. This is much less than the magnetic mass squared $\sim (g^2 T)^2$. 2) Since the spectrum of charged particles in the magnetic field is formed at the Larmor radius scale, the weak long range fields are not produced being screened by the mass at distances $l \geq 1/g^2 T$. The error is the assumption that the field strength generated in the
vaccum is not changed when the temperature is switched on. As we have seen in sect. 11, strong abelian colour magnetic fields of order \((gH)^{1/2} \sim g^2 T\) (in one-loop approximation \([17]\), \([19]\)) or \((gH)^{1/2} \sim g^{4/3} T\) \([11]\) (when higher order corrections are included) is spontaneously generated at high temperature.

We also have observed in the consistent calculation that the Savvidy state is stable at high temperature when the one-loop EP and the one-loop gluon polarization operator are taken into account.

13. DISCUSSION

As it was realized recently, in the SM the usual scenario of baryogenesis can not be established without external fields \([13]\). By analogy to superconductivity it was assumed that strong hypermagnetic field is able to generate the strong first order phase transition for the values of mass \(m_H\) permitted by experiment \([6]\). Different mechanisms to produce magnetic fields have been proposed in literature. They can be divided in two groups: 1) generation of fields at EW phase transition \([22]\), \([23]\), \([24]\), \([25]\), \([26]\); 2) generation of fields beyond the SM scale (for instance, GUT theories) \([1]\), \([29]\). A special interest was in strong hypermagnetic field which due to its abalian nature is not screened at high temperature whereas the nonabelian component of the usual magnetic field is screened at distances \(l > 1/g^2 T\) by the gauge field magnetic mass. The influence of the magnetic fields generated at the EW scale on the first order phase transition was investigated in Ref. \([27]\). It has been shown that if the field strength is stronger then \(H_0 = 10^{24}\) G the phase transition is delayed. Of cause, it is relevant if the first order phase transition is realized. But this is not the case for the SM, where without external fields for the mass \(m_H \geq 60 - 70\) GeV the second order phase transition is predicted. Naturally, internal forces are not able to change the kind of the phase transition.

The investigations carried out in the present paper are refered to the second possibility assuming that the fields have been generated before the EW phase transition at a GUT scale. The dynamics of the hypermagnetic field generation has not been discussed here, although the corresponding mechanism was proposed in Ref. \([7]\). We just assumed that the field present in the early universe. As the magnetic field is concerned, in sects. 10, 11 we have investigated in detail its generation and stabilization at high temperature.

The results on the hypermagnetic field influence can be summarized as follows. In contrast to the conclusions of Refs. \([8]\), \([9]\) claiming that strong hypermagnetic field generates the strong first order EW phase transition, we observed an opposite effect. As we have seen in our numerical computations, the weak first-order EW phase transition becomes of second order in strong fields. The origin of the discrepancy is the following. These authors have not determined correctly the form of the EP curve at the transition temperature since the contributions of fermions and the correlation corrections in the fields were skipped. In fact, our analysis has completed the investigations not only for the hypermagnetic but
also for the magnetic field. We have discovered that the role of fermion radiation corrections is of great importance in the phenomenon investigated. To better understand the role of fermions in symmetry behaviour let us adduce two terms of the asymptotic expansion of the EP in the limit of $T \rightarrow \infty, H \rightarrow \infty$. The first one is the term $\sim H^2 \log \frac{T}{m_f}$. Due to this term the light fermions are dominating at high temperature. The second term can be derived from the expansion of the zero temperature part Eq. (32). This expression side by side with the leading term $\sim H^2 \log \frac{T}{m_f}$, which due to a “dimension parameter trading” is replaced by the above written term, contains the subleading one $\sim -eHm_f^2 \log \frac{eH}{m_f}$ (for details see Ref. [29]). This term acts to make "heavier" the Higgs particles in the field. As a result, the second order temperature phase transition is stimulated due to strong fields. We also have investigated the influence of different parts of the EP on symmetry behaviour. It was discovered that the change of the kind of the EW phase transition with increase in $H$ is due to the fermion temperature part of the EP.

The stability problem of the vacuum at high temperature and strong fields has also been investigated. Our EP (its minima) is real at the first order phase transition that is important for the reliability of the results obtained. The latter property is insured by the imaginary terms of daisy diagrams cancelling the corresponding ones of the one-loop EP. Thus, the total EP is suitable to correctly describe the phase transition. As we have found, the effective W-boson mass $M_w(H,T)$ is real in the local minimum, therefore no conditions for the formation of the vortex-like structure due to W- and Z-boson condensates exist in the broken phase. Probably, this is the reason why the condensates have not been observed in Ref. [3], although it is difficult to check that straightforwardly.

Furthermore, we have also shown that spontaneous vacuum magnetization indeed happens at high temperature giving the reliable mechanism of the magnetic field generation at a GUT scale. In principle, the field could be spontaneously created at the EW scale. However, in this case it is incorrect to treat the EW phase transition as the external field problem. It is necessary to apply consideration as in Ref. [27].

On the base of our analysis one has to conclude that baryogenesis can not survive in the SM if smooth external magnetic fields generated beyond the EW scale are included as environment.

As a final remark, we would like to stress once again that light fermions act to turn the first order phase transition into the second order one with increase in the field strength, independently of the values of the Higgs boson mass considered. This fact may be one of the reasons why it is of interest to carry out a similar investigation for the minimal supersymmetric SM or other supersymmetric extensions of it, because the fermion and the boson sectors enter such models on an equal footing.

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APPENDIX

General method of calculation of high temperature asymptotics has been developed in Ref. [64]. To outline the procedure used in the present paper let us consider the fermion contribution as an example:

\[ f^{(1)}_r = \sum_{n=1}^{\infty} (-1)^n K_{|r|}(n\omega). \]  

(A.1)

At first, let us make Mellin’s transformation of \( K_{|r|}(n\omega) \) with respect to parameter \( n \) [64]:

\[ K_{|r|}(s) = \int_0^\infty K_{|r|}(\omega t) t^{s-1} dt = \omega^{-s} 2^{-s-2} \Gamma\left(\frac{s}{2} - \frac{|r|}{2}\right) \Gamma\left(\frac{s}{2} + \frac{|r|}{2}\right), \]

where \( \Gamma(x) \) is \( \Gamma \)-function, and substitute it in Eq. (A.1). Then we find

\[ f^{(1)}_r = \frac{1}{8\pi i} \sum_{n=1}^{\infty} (-1)^n \int_{C_{r+i\infty}}^{C_{r+i\infty}} ds^n \zeta(s) (2^{1-s} - 1) \zeta(s), \]

(A.2)

where \( C_r > |r| \) and the integral is calculated along the straight line parallel to the imaginary axis. If one shifts the integration variable \( s \to s - r = s' \), the integration contour moves to the right, \( C_{2r} \to C'_{2r} \), for negative \( r \), and to the left, \( C_r \to C'_0 \), in the case of positive \( r \): \( 2r < C'_{2r} < 2r - 1; 0 < C'_0 < 1 \). Changing the sequence of the summation and the integration in Eq. (A.2) and taking into account the definition of Riemann’s \( \zeta \)-function [75]

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^s} = (2^{1-s} - 1) \zeta(s), \]

one obtains

\[ f^{(1)}_r = \frac{1}{8\pi i} \int_{C_{r-i\infty}}^{C_{r-i\infty}} ds' \zeta(s') (2^{1-s'} - 1) \frac{2}{\omega} s' + r \Gamma\left(\frac{s'}{2} - \frac{|r|}{2}\right) \Gamma\left(\frac{s'}{2} + \frac{|r|}{2}\right). \]

(A.3)

It is important to note that the contours \( C'_{2r}, C'_0 \) are the same as for \( K_{|r|}(x) \) after shifting of integration variable \( s \to s' \). The integral (A.3) is calculated by closing
the contour $C'$ to the left and summing up the residua of the integrand. For the contour $C'_{2r}$ one should include all the poles of $\Gamma$ and $\zeta$-functions. But for positive $r$, $0 < C_0 < 1$, the pole $s' = 1$ of $\zeta$-function is out of the contour and should be excluded.

In the case of fermion contributions the following integrals have to be calculated:

$$f_0^{(1)} = \frac{1}{2}(C + \log(\frac{\omega}{4\pi})) - \frac{2}{\pi} \sum_{n=1}^{\infty} (-\frac{\omega^2}{\pi^2})^n (1 - 2^{-2n-1}) \frac{\Gamma(n + 1/2)}{\Gamma(n)} \zeta(2n + 1), \quad (A.4)$$

$$f_r^{(1)} = -\frac{2^n}{\sqrt{\pi}} (\frac{\omega}{2})^{r-2} \sum_{n=1}^{\infty} (-\frac{\omega^2}{\pi^2})^n (1 - 2^{-2n-2r+1}) \frac{\Gamma(n + r - 1/2)}{\Gamma(n)} \zeta(2n+2r-1), \quad (A.5)$$

$r = 1, 2, ..., C - \text{Euler constant}$. As it is seen, in the limit $\omega \to 0 \ (T \to \infty)$ the first term in Eq. (A.4) is dominating. Other terms give the corrections of order $\sim \frac{1}{T^2}, \frac{1}{T^4}, ....$

The same procedure has been applied for contributions of boson fields and for particular values of $r$ the results of summations are as follows:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} K_2(n\omega) = \frac{2\zeta(4)}{\omega^2} - \frac{\zeta(2)}{2} \frac{\pi}{\omega} + \frac{\omega}{6} - \frac{\omega^2}{16} (\log \frac{4\pi}{\omega} - C + 3/4) + O(\omega^2), \quad (A.6)$$

$$\sum_{n=1}^{\infty} \frac{1}{n} K_1(n\omega) = \frac{\zeta(2)}{\omega} - \frac{\pi}{2} + \frac{\omega}{4} (\log \frac{4\pi}{\omega} - C + 1/2) + O(\omega^2), \quad (A.7)$$

$$\sum_{n=1}^{\infty} \frac{1}{n} Y_1(n\omega) = \frac{4\zeta(2)}{\omega} + \omega (\log \frac{4\pi}{\omega} - C + 1/2) + O(\omega^2), \quad (A.8)$$

$$\sum_{n=1}^{\infty} K_0(n\omega) = \frac{1}{2} (C - \log \frac{4\pi}{\omega}) + O(\omega). \quad (A.9)$$

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