Numerical study of the effective modulus of elasticity of three-dimensional mechanical metamaterial

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Abstract. Metamaterials are of great interest due to their unusual properties and promising practical application. The paper studies a mechanical metamaterial bar composed of 81 unit cells. The unit cell is shaped to a cube the faces of which consist of two-dimensional tetrachiral elements. The elastic deformation of a metamaterial specimen is numerically modeled in uniaxial loading. How the twist angle and Young modulus of the metamaterial specimen depend on the parameters of the chiral structure is demonstrated. Value ranges of the parameters in which the twist angle is largely affected are found. Cell parameters that have the greatest and least influence on the metamaterial twist angle and Young's modulus are determined. The effective Young's modulus decreases by a factor of approx. 20 when changing the inner radius of the ring. An almost constant effective Young's modulus is found.

1. Introduction

Mechanical metamaterials are media with a special structure that provides unusual effective properties whose values are not known for natural analogs or surpass them [1]. Mechanical metamaterials with unusual mechanical properties hold promise in aerospace and automotive engineering as well as in biomedical applications. The development of mechanical metamaterials is facilitated by the success of modern additive technologies. Additive manufacturing technologies are promising and competitive compared to traditional ones due to high productivity, economic efficiency, and the ability to create parts with complex geometry [2]. The 3D printing technology allows for the design of materials with previously inaccessible properties, for example, with the transformation of a longitudinal elastic wave into a transverse one [3].

In recent years, metamaterials with negative Poisson's ratio (auxetics) have gained popularity [4]. This property makes them ideal candidates for use in flexible structures such as transformable aircraft structures [5] and as analogues of spokes in nonpneumatic tires [6].

Numerous types of auxetic materials with negative Poisson's ratio [7, 8] are known. One of them is a metamaterial with chiral elements in its unit cells. A simple chiral element has a central ring and ribs tangential to it. Chiral structures take a name associated with the number of ribs.

Another interesting feature of chiral mechanical metamaterials is the elastic counterpart of optical activity in three-dimensional chiral structures [9], which converts a transverse linearly polarized elastic wave into the orthogonal transverse one. In the static case, such “mechanical activity” is associated with the appearance of an additional degree of freedom. It is a common knowledge that an
ordinary elastic solid cannot twist in tension or compression. This is demonstrated both experimentally and in the theoretical description of elastic deformation within Cauchy continuum mechanics.

Two-dimensional chiral structures have been very extensively studied. The study of three-dimensional chiral metamaterials is now actively engaged in [10], but a lesser extent is given to behavior under mechanical stress.

This work aim is to analyze the influence of cell parameters on the behavior of a tetrachiral mechanical metamaterial in uniaxial loading using numerical simulation.

2. Special features of numerical modeling
Numerical modeling is performed by the finite-element method using ANSYS software. A unit cell is taken as a system of ribs (beams) and as a set of three-dimensional solid elements in the finite-element calculation.

The mathematical statement of the problem of metamaterial deformation is made in terms of linear elasticity. We restricted ourselves to the case of displacements causing no contact interaction of cell elements. Elastic properties of a material are assumed to be \( E = 200 \text{ GPa} \) for Young’s modulus and \( \nu = 0.3 \) for Poisson’s ratio.

The present investigation is concerned with a metamaterial specimen composed of the unit cells that consist of chiral elements. A chiral element is a ring and ribs. In this paper the tetrachiral (4 ribs) structure (Fig. 1) is considered. To design a three-dimensional metamaterial, two-dimensional structures should be connected in such a way as to form cube faces. A unit cell of the metamaterial is thus obtained. The basic parameters values are presented in Table 1. The number of unit cells in the specimen is thought to be constant and equals \( n = 81 \). The specimen has three cells along axes \( x \) and \( z \) and nine cells along axis \( y \).

![Figure 1. Geometry of the tetrachiral structure of the unit cell.](image)

| Table 1. Initial parameters of a metamaterial structure. |
|-----------------|----------------|----------------|----------------|----------------|
| \( l \) (mm)    | \( t \) (mm)   | \( h \) (mm)   | \( r_1 \) (mm) | \( r_2 \) (mm) |
| 50              | 5              | 5              | 12.5           | 17.5           |
| \( \theta \) (°) | 20             |                |                |                |

The metamaterial behavior in uniaxial loading along the specimen length is analyzed. The specimen is subjected to the following boundary conditions: \( U_x^{\text{fix}} = U_y^{\text{fix}} = U_z^{\text{fix}} = 0 \) for the fixed surface (face) of the specimen and \( U_y^{\text{dis}} = 15 \text{ mm} \) for the opposite surface (face) of the specimen. Displacement should be set positive or negative, causing tension or compression, respectively. At the specified structural parameters, the 15 mm displacement corresponds to a 3% uniaxial strain of the metamaterial specimen.

The main responses used to study deformation features in the given material are (i) the twist angle \( \alpha \) of the displaced face of the metamaterial specimen and (ii) Young’s modulus \( E \).

Twist angle \( \alpha \) is evaluated by the change in coordinates of the end nodes of the displaced face of the specimen. Young’s modulus \( E \) is determined through a response to uniaxial loading at the fixed face.
The main dimension of a unit cell that is taken as constant is cell length $l$. The rest parameters are variable: $t$, $h$, $r_1$, $r_2$, $\theta$. The aim of the investigation is to determine how $\alpha$ and $E$ depend on the cell parameters $t$, $h$, $r_1$ and $\theta$. All the parameters are chosen to be independent.

3. Results and discussion

The results of the influence of the cell parameter variation on the twist angle of a three-dimensional metamaterial are given in Fig. 2. Values of $r_1$ vary from 12.5 mm to 0 mm. At a smaller inner radius of the metamaterial ring, the twist angle decreases. Evidently, the specific volume of the metamaterial increases in this case and more space is filled with continuous material. From Fig. 2a it is seen that the dependence is almost linear in the portion $r_1 = 12.5, \ldots, 4.5$ mm; a further decrease in the radius exerts no effect on $\alpha$.

If the rib thickness $h$ decreases from 5 to 1 mm with the equal step 1 mm, the earlier tendency to reduce the twist angle with decreasing specific volume does not hold. A decrease in $h$ increases the twist angle insignificantly (Fig. 2a), and the difference between the final and initial values is 0.22 °. This variation (about 3 %) allows a conclusion that $h$ has little effect on the metamaterial twist. As with parameter $r_1$, the angle $\alpha$ remains unchanged after $h = 2$ mm.

A decrease in the rib width $t$ causes a different behavior of the metamaterial. Values of $t$ vary from 5 to 1 mm with the step 1 mm (Fig. 2a). This variation gives the largest difference between the final and initial twist angles of the metamaterial, being equal to 2.83 °.

The initial angle $\theta$ equals 25 °; the value is chosen from the geometrical relation between the ring radius and rib length at the same thicknesses of the ring and rib provided that the rib is tangential to the ring. The value of $\theta$ grows to 40 °. Figure 2b indicates that an increase in the inclination angle of the rib forming the cell vertex causes the twist angle of the loaded metamaterial to decrease. The tendency holds up to the angle 45 ° until the metamaterial torsion stops. At an angle larger than 45 °, the material continues to twist but in the other direction.

![Figure 2. Twist angle of the metamaterial as a function of the variable parameters.](image)

The dependences of Young’s modulus on the variable parameters were also constructed (Fig. 3). Consider the influence of parameter $r_1$ (inner radius of the ring) on the above-mentioned value. In the metamaterial sample, the value of $E$ was 6.5 GPa at $r_1 = 12.5$ mm. With a decrease in the inner radius $r_1$ more efforts are required to deform the metamaterial, consequently, the higher is the value of Young’s modulus.

At a minimum value of $r_1 = 0$ mm, the ring space is filled with a solid body, therefore Young’s modulus increases to $E = 9.5$ GPa.
A study with variable parameters width $t$, thickness $h$, and inclination angle of the rib $\theta$ are similar. The slope of the line characterizes the influence of a variable parameter on Young's modulus. A tendency is noted: a decrease in the parameter value entails a decrease in the force required for a similar deformation of the metamaterial sample. This is due to a decrease in the specific volume of the metamaterial.

With a decrease in the variable parameters $h$ and $t$, the effective Young's modulus sharply decreases. The minimum value of $E = 1.3$ GPa. An almost constant effective Young’s modulus (Fig. 3b) upon varying $\theta$ is found.

![Graphs showing Young's modulus as a function of variable parameters](image)

**Figure 3.** Young's modulus as a function of the variable parameters.

As can be seen from Figure 3, the dependence $E (h)$ is seen to be linear, the dependence $E (t)$ is almost linear, and $E (\theta)$ is seen to be linear and almost constant.

It is worth noting that the effective elastic modulus of the metamaterial was found to be almost forty times less than the elastic modulus of the base material. That is quite justifiable if we take into account rather high effective porosity of the metamaterial and the well-known effect of porosity on the elastic properties of porous materials [11, 12].

4. Conclusion
Elastic deformation of a metamaterial specimen is numerically modeled in uniaxial compression and tension. It is demonstrated how the twist angle and Young’s modulus of the metamaterial specimen depend on the structural parameters of the unit cell. Cell parameters that have the greatest and least influence on the metamaterial twist angle and support force are determined. Value ranges of the parameters in which the twist angle is largely affected are found.

The greatest influence on the twist angle of the metamaterial is exerted by the variation in the width of the rib of the chiral element. The thickness of the rib (and of the entire unit cell) hardly affects the twist angle.

As the value of the inner radius of the ring element $r_1$ decreases, the specific volume of the metamaterial sample increases.

The effective Young’s modulus decreases by a factor of approx. 20 when changing parameter $r_1$. An almost constant effective Young’s modulus (Fig. 3b) upon varying $\theta$ is found.

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References
[1] Fisanov V V 2018 Russian Physics Journal 61(6) 1129–34
[2] Kolubaev A V, Tarasov S Y, Filippov A V, Denisova Y A, Kolubaev E A and Potekaev A I 2018 Russian Physics Journal 61(8) 1491–1498
[3] Kweun J M, Lee H J, Oh J H, Seung H M and Kim Y Y 2017 Phys. Rev. Lett. 118 205901-1–205901-6
[4] Evans K E, Nkansah M A, Hutchinson I J and Rogers S C 1991 Nature 353(6340) 124–125
[5] Lesieutre G, Browne J A and Frecker M 2011 Journal of Intelligent Material Systems and Structures 22(10) 979–986
[6] Ju J, Kim D-M and Kim K 2012 Composite Structures 94(8) 2285–95
[7] Goldstein R V, Gorodtsov V A, Lisovenko DS and Volkov M A 2019 Phys. Mesomech. 22(4) 261–268
[8] Tuch E V and Strebkova E A 2019 Russian Physics Journal 62(10) 705–709
[9] Gansel J K, Thiel M, Rill M S, Decker M, Bade K, Saile V and Freymann G 2009 Science 325(5947) 1513–15
[10] Chen Y, Frenzel T, Guenneau S, Kadic M, Wegener M 2020 Journal of the Mechanics and Physics of Solids 137 103877
[11] Smolin I Yu, Makarov P V, Eremin M O, and Matyko K S 2016 Proc. Struct. Integr. 2, 3353–60
[12] Savchenko N L, Sevostyanova I N, Sablina T Yu, Gomze L and Kulkov S N 2014 AIP Conf. Proc. 1623 547–550