Research on Electromagnetic Compatibility of High Frequency Switching Power Supply Based on Harmonic Balance Finite Element Method

Chunning Bu1*, Zhihong Zhang2, Tiantian Hou1, Xin Bai1 and Hong He1

1 Department of electronic and electrical engineering, Cangzhou Jiaotong College, Cangzhou, Hebei, 061110, China
2 Tianjin Product Quality Inspection Technology Research Institute Electrical Engineering Technology Research Center, Tianjin, 300074, China
*Corresponding author’s e-mail: buchunning@czjtu.edu.cn

Abstract. With the rapid development of electronic technology, high frequency switching power supply has been widely used in various electrical equipment, and its electromagnetic interference has attracted more and more attention. In this paper, the harmonic balance finite element method is used to solve the electromagnetic compatibility of high frequency switching power supply. In this method, the coefficients of sine wave are applied to represent the waveform, and the solution is generated by linear synthesis. We propose to combine it with the finite element method to solve the steady-state response of electromagnetic field in frequency domain. It is proved the method we proposed is superior to the traditional time domain method.

1. Introduction

High-frequency switching power supply has the characteristics of light weight, high efficiency, small size and low power consumption, and has been widely used in computer and peripheral equipment, medical equipment, household appliances and other fields. But these characteristics will bring some problems, such as leakage inductance, electromagnetic radiation and proximity effect of wire, which will cause a series of effects on electromagnetic compatibility (EMC). In addition, the waveform distortion may be caused by the nonlinear characteristics. This kind of waveform distortion will produce harmonics, which will not only increase the power loss in the core and winding, but also bring electromagnetic interference (EMI) to the electronic system [1-2]. There are a wide frequency range and strong amplitude in the EMI signal, which will pollute the electromagnetic environment and interfere with communication equipment and electronic instruments through conduction and radiation. Therefore, people attach more and more importance to the electromagnetic compatibility of electronic products. How to suppress the EMI of high-frequency switching power supply and make it conform to the relevant EMC standards has become an increasingly concerned problem for electronic product designers. This paper first introduces the harmonic balance finite element method (HBFEM), and applies it to high frequency switching power supply to analyze the electromagnetic interference problems caused by power loss leakage inductance and eddy current in high frequency switching power supply transformer.
2. The Relevant knowledge of harmonic balance analysis applied to magnetic circuit

The method of harmonic balance originates from the analysis and treatment of nonlinear circuits. In the 1960s, harmonic balance method is an effective method to prove the problem of steady-state nonlinear circuits. This method divides the nonlinear system into linear and nonlinear parts. The excitation source is attributed to the linear part, while the nonlinear part only includes the nonlinear term. In any case, the state variables connecting the linear part and the nonlinear part should be equal, and each state variable is represented by a Fourier series, so the harmonic balance matrix equation can be formed and the nonlinear problem can be analyzed.

In 1988, S. Yamada first combined the harmonic balance method with the finite element method [3] to solve the steady-state electromagnetic problems. Although the simulation of magnetization curve in this paper is relatively simple, the derivation of the magnetic susceptibility matrix reflecting the nonlinear characteristics is relatively complex, which limits the application of this method. After that, S. Yamada proposed that the method of Fourier series can be used to approximate the reluctance [4], and the reluctance coefficient of each element can be used to express the reluctance matrix in HBFEM. So the reluctance matrix has a simple form. J. Lu has made a further contribution to the development of this method. A field circuit coupling relationship based on harmonic balance finite element method is proposed and applied to the design of switching power supply. J. Gyselinck introduced the differential reluctance tensor into the HBFEM and solved the equation by Newton Raphson method [5]. F. Bachinger used the method to study the three-dimensional eddy current problem [6]; Wang Xifan proposed the steady-state analytical method of the circuit with saturated ferromagnetic coil based on the harmonic balance method, and used the HBFEM to study the saturated core loss and the electromagnetic characteristics of the third harmonic transformer.

In conclusion, the difference from the traditional time domain finite element method is that the coefficients of sine wave are applied to represent the waveform, and the solution is generated by linear synthesis. It combines the finite element method to solve the steady-state response of electromagnetic field directly in frequency domain. Therefore, the HBFEM is usually more superior than the traditional time domain method when it is independent of time constant and slightly nonlinear.

3. HBFEM for magnetic field generated by current source

In switching power supply, the magnetic core of some direct current-direct current converters, such as push-pull current converter and zero voltage switching resonant converter, can be considered to be excited by a current source with a current density of Js.

3.1. HBFEM matrix equation of magnetic field excited by current source

In two-dimensional space, assuming $\nabla \phi = 0$, the vector magnetic potential in the diffusion equation is discretized by Galerkin finite element method:

$$
G = \iint_S \left[ \frac{\partial N_i}{\partial x} \nabla A + \frac{\partial N_i}{\partial y} \nabla A \right] dxdy - \iint_S \left( J_s - \sigma \frac{\partial A}{\partial t} \right) N_i dxdy = 0
$$

(1)

Considering the orthogonality of trigonometric function, the magnetic resistivity $\nu$, vector magnetic potential $A$, shape function $N_i$ and $J_s$ are substituted into equation (1), and the harmonic balance method is used. Only the odd harmonics of a single element are considered, we can obtain the equation of harmonic balance finite element matrix:
\[ G = \frac{1}{4\Delta} \begin{bmatrix} (a_1a_1 + b_1b_1)D & (a_1a_2 + b_1b_2)D & (a_1a_3 + b_1b_3)D \\ (a_2a_1 + b_2b_1)D & (a_2a_2 + b_2b_2)D & (a_2a_3 + b_2b_3)D \\ (a_3a_1 + b_3b_1)D & (a_3a_2 + b_3b_2)D & (a_3a_3 + b_3b_3)D \end{bmatrix} \]

\[ \times \begin{bmatrix} A_1^c \\ A_2^c \\ A_3^c \end{bmatrix} + \frac{\sigma \omega \Delta}{12} \begin{bmatrix} 2N & N & N \\ N & 2N & N \\ N & N & 2N \end{bmatrix} \begin{bmatrix} A_1^c \\ A_2^c \\ A_3^c \end{bmatrix} = K_1^c + N^c A^c - K^c \] (2)

Where the vector magnetic \( A^e \) potential is a frequency domain expression, the \( \omega \) is fundamental frequency and \( \Delta \) represents the area of the triangle element. The constants \( a \) and \( b \) can be obtained from \( X \) and \( Y \) coordinates. The matrix of \( D \) and \( N \) are

\[ D = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} = \frac{1}{2} \]

\[ N = \begin{bmatrix} 2v_0 - v_{2e} & v_{2s} & v_{2e} - v_{4e} & -v_{2s} + v_{4s} \\ 2v_0 + v_{2e} & v_{2s} + v_{4s} & v_{2c} + v_{4c} \\ 2v_0 + v_{6c} & v_{6s} \\ \text{Symmetry} & 2v_0 - v_{10c} \end{bmatrix} \] (3)

The \( K_i^c \) can be expressed as

\[ K_i^c = \frac{\Delta^c}{3} \begin{bmatrix} J_{1s}, J_{1c}, J_{3s}, J_{3c}, \ldots \end{bmatrix}^T \] (5)

Therefore, the matrix equation of the system excited by the current source can be expressed as

\[ SA + MA - K = 0 \] (6)

By solving this matrix equation, all the harmonic components of \( a \) can be obtained. The order of harmonic component can be determined according to the actual situation.

3.2. Study on magnetic field of switching power supply excited by current source

The working frequency of switching power supply is usually 20kHz - 400kHz (pulse width modulation tuning switch) and 500kHz - 10mHz (zero current switching or zero voltage switching
oscillation switch). If the excitation function is a sine wave signal, the harmonic will appear in the magnetic field and current due to the saturation of the magnetic core. These harmonics will increase electromagnetic interference and power loss[7].

As shown in Figure 1 and Figure 2, there is usually an inductor at the input of the switch mode push-pull converter and the ZVS resonant converter, and the excitation source can be considered as the current source.

![Figure 1 Switch-mode push-pull converter](image)

![Figure 2 ZVS resonant converter](image)

4. **HBFEM for nonlinear magnetic field system excited by voltage source**

And for the most part, zero current switching resonant converters and pulse width modulation can be considered as magnetic systems excited by voltage sources. This system is often coupled with external circuits, as shown in Figure 3. Due to the nonlinear characteristics of the magnetic core, although the current of the input circuit is unknown, the current waveform presents saturation.

![Figure 3 Zero Current Switching resonant converter](image)

![Figure 4 Switch-mode positive converters](image)
According to Kirchhoff’s law and Faraday’s law, the relationship between magnetic field and voltage can be obtained for a single finite element subdivision element:

\[ u_k = C_k \begin{bmatrix} A^1 \\ A^2 \\ A^3 \end{bmatrix} \]  

(7)

Where matrix \( [C_k] \) is the coefficient related to voltage, and its calculation formula is

\[ C_k = \frac{\omega d_{i_k} \Delta^e}{3 S_{ck}^e} [N, N, N] \]  

(8)

\( S_{ck} \) is the cross-sectional area and the depth of \( z \) direction is \( d_0 \). \( N \) is the harmonic matrix, which is defined by equation (4). The voltage can be expressed as

\[ u_{ink} = U_k + S_{ck} Z_k J_k \]  

(9)

In the equation, matrix \( [Z_k] \) contains the coil impedance of the corresponding harmonic

\[ Z_k = \begin{bmatrix} Z_{0k} & 0 & \cdots & 0 \\ 0 & Z_{1k} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & Z_{nk} \end{bmatrix} \]  

(10)

For a series circuit, \( [Z_{nk}] \) in equation (10) can be expressed as

\[ Z_{nk} = Z_{fok} + Z_{Cok} + Z_{ток} \]  

(11)

The voltage \( [u_{nk}] \) including all unknown harmonic components is expressed as

\[ u_{nk} = [u_{0k}, u_{1k}, u_{1k}, u_{2k}, u_{2k}, \cdots]^T \]  

(12)

Equation (12) takes into account the zero, odd, and even harmonic components that are often contained in zero current switching (ZCS) resonant converters at the same time, so the current-related system matrix equation can be rewritten as

\[ SA + MA - G_k J_k = 0 \]  

(13)

In the equation, \( [G_k] \) can be obtained from a single finite element subdivision element

\[ G^e = \frac{\Delta^e}{3} \]  

(14)

By combining Equations (10) and (13), the total system equation with multiple input and output circuits can be obtained. Therefore, the HBFE equation of voltage source excitation is expressed as

\[ \begin{bmatrix} H & -G_1 & -G_2 & L & -G_k & \cdots \\ C_1 & S_{ck} Z_1 & 0 & 0 & 0 & \cdots \\ C_2 & 0 & S_{ck} Z_2 & 0 & 0 & \cdots \\ \vdots & 0 & 0 & \ddots & \vdots & \vdots \\ C_k & 0 & 0 & 0 & S_{ck} Z_k & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix} \begin{bmatrix} A \\ J_1 \\ J_2 \\ \vdots \\ J_k \end{bmatrix} = \begin{bmatrix} 0 \\ u_{in1} \\ u_{in2} \\ \vdots \\ u_{ink} \end{bmatrix} \]  

(15)
The A phase \( J_k \) in the above equation can be obtained from the above matrix equation. The matrix H can be given by \( S + M \).

5. Conclusion
This paper theoretically gives the HBFEM to solve the electromagnetic compatibility of high-frequency switching power supplies. Compared with other traditional methods, the method we proposed has many advantages. It uses the linear synthesis of sinusoidal functions to generate solutions, and uses the coefficient of the sine function to express the waveform, which can directly solve the steady-state response of the electromagnetic field in the frequency domain. It is proved that the HBFEM is usually advanced than the traditional time-domain method.

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