Anomalous magnetic and electric moments of $\tau$ and lepton flavor mixing matrix in effective lagrangian approach

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Abstract

In an effective lagrangian approach \cite{1} to new physics, the authors in ref. \cite{2} pushed tau anomalous magnetic and electric dipole moments (AMDM and EDM) down to $10^{-11}$ and $10^{-25} \ e\ cm$ by using a Fritzsch-Xing lepton mass matrix ansatz. In this note, we find that, in this approach, there exists the connection between $\tau$ AMDM and EDM and the lepton flavor mixing matrix. By using the current neutrino oscillation experimental results, we investigate the parameter space of lepton mixing angles to $\tau$ AMDM and EDM. We can obtain the same or smaller bounds of $\delta \alpha_{\tau}$ and $d_{\tau}$ acquired in ref. \cite{3} and constrain $\theta_l$ (the mixing angle obtained by long-baseline neutrino oscillation experiments) from $\tau$ AMDM and EDM.
1 Introduction

Some neutrino oscillation experiments in recent years indicate that neutrinos are massive and oscillate in flavor. That is to say, there exists lepton flavor mixing. Lepton flavor mixing is very important for solving some topics related to particle physics and cosmology, such as the lepton anomalous magnetic dipole moments (AMDM) and electric dipole moments (EDM) and lepton flavor violation (LFV) decays.

Theoretically, the standard model predicts $\delta\alpha_\tau = 1.1769(4) \times 10^{-3}$ and a very tiny $d_\tau$ from CP violation in the quark sector. Experimental analysis of the $e^+e^- \rightarrow \tau^+\tau^-\gamma$ process from L3 and OPAL collaborations gives that $\delta\alpha_\tau = 0.004 \pm 0.027 \pm 0.023$ and $d_\tau = (0.0 \pm 1.5 \pm 1.3) \times 10^{-16} \text{ e cm}$ [4]. We also note that the authors in ref. [5, 6] investigated $g - 2$ and radiative lepton decays with the effective lagrangian approach. Moreover, in ref. [2], T. Huang, Z.H. Lin and X.M. Zhang firstly introduced the non-universal effective operators and leaded the lepton flavor violation to the effective approach. The authors of ref. [4] presented an effective lagrangian to describe the effect of new physics and obtained the bounds of AMDM and EDM. By using a special lepton mass matrix ansatz [7], ref. [2] pushed the anomalous magnetic and electric dipole moments of tau lepton down to

$$|\delta\alpha_\tau| < 3.9 \times 10^{-11}, \quad |d_\tau| < 2.2 \times 10^{-25} \text{ e cm}. \quad (1)$$

However these results were obtained only in one Fritzsch-Xing ansatz which induces the nearly bi-maximal mixing pattern for atmospheric and solar neutrino oscillations.

In fact, there are a lot of lepton mass matrices ansatz [8] which are compatible with the current neutrino oscillation experimental data. Similar to the CKM matrix, the lepton mixing matrix can be also measured by experiments. However, at the present neutrino oscillation experiment level, it is difficult to put the precise values of lepton mixing matrix elements. From the several famous oscillation experiments, we can just obtain the range of these matrix elements. In the effective lagrangian approach, AMDM, EDM and lepton flavor violation are mainly related to the lepton mixing matrix. And thus we can find a connection between AMDM and EDM of tau lepton and the lepton flavor mixing matrix.
As a result, either can we investigate the parameter space of lepton mixing angles from experimental data of $\tau$ AMDM and EDM, or can study the bounds of $\delta a_\tau$ and $d_\tau$ imposed by the mixing angles obtained from the current neutrino oscillation experiments.

In this note, we briefly talk about AMDM, EDM and LFV in the effective lagrangian approach introduced in Sec. 2. In the sequent Sec., the lepton mixing matrix and the numerically results are presented. And we summarize the subject in the last Sec.

2 Effective lagrangian with magnetic and electric dipole moments operators

To study magnetic and electric moments beyond Standard Model (SM), the authors in refs. \cite{1, 2} considered an effective lagrangian approach to new physics:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum_i C_i \mathcal{O}_i, \quad (2)$$

where $\mathcal{L}_{\text{SM}}$ is the SM lagrangian, $\Lambda$ is the new physics scale, $\mathcal{O}_i$ are $SU_C(3) \times SU_L(2) \times U_Y(1)$ invariant operators and $C_i$ are constants which represent the coupling strengths of $\mathcal{O}_i$. A complete list of the operators can be found in Ref. \cite{3}. Related to the anomalous magnetic moment of tau lepton, there are two dimension-six operators

$$\mathcal{O}_{\tau B} = \bar{L_\tau} \sigma^{\mu\nu} \tau_R \Phi B_{\mu\nu} \quad (3)$$

$$\mathcal{O}_{\tau W} = \bar{L_\tau} \sigma^{\mu\nu} \frac{\sigma_i}{2} \tau_R \Phi W_{i\mu\nu} \quad (4)$$

where $\sigma^i$ is Pauli matrices $(i=1, 2, 3)$, $L_\tau = (\nu_\tau, \tau_L)^T$ the $\tau$ left-handed isodoublet, $\tau_R$ the right-handed singlet, $\Phi$ the Higgs scalar doublet, and $B_{\mu\nu}$ and $W_{i\mu\nu}$ are strengths of $U_Y(1)$ and $SU_L(2)$ gauge fields. Similarly, operators below are introduced to induce the electric dipole moments of $\tau$ \cite{4}.

$$\bar{O}_{\tau B} = \bar{L_\tau} \sigma^{\mu\nu} i \gamma_5 \tau_R \Phi B_{\mu\nu} \quad (5)$$

$$\bar{O}_{\tau W} = \bar{L_\tau} \sigma^{\mu\nu} i \gamma_5 \frac{\sigma^i}{2} \tau_R \Phi W_{i\mu\nu}. \quad (6)$$
When $\Phi$ gets vacuum expectation value, operators $\mathcal{O}_B$ and $\mathcal{O}_W$ can give rise to the tau anomalous magnetic moment. After the broken of electroweak symmetry and diagonalization of the mass matrices of the leptons and the bosons, the effective neutral current couplings of the leptons to the photon $\gamma$ is

$$
\mathcal{L}_{\text{Eff}}^\gamma = \mathcal{L}_{\text{SM}}^\gamma + e \frac{1}{2m_\tau} (-ik_\nu \sigma^{\mu\nu})S^\gamma \begin{pmatrix} \bar{e} \\ \bar{\mu} \\ \bar{\tau} \end{pmatrix}^T U^\dagger_I \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} U_I \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix},
$$

where

$$
S^\gamma = \frac{2m_\tau}{e} \sqrt{2} \sqrt{\frac{v}{\Lambda^2}} \left[ C_{\tau W} S_W - \frac{1}{2} C_{\tau B} C_W \right],
$$

where $S_W \equiv \sin \theta_W$, $C_W \equiv \cos \theta_W$, and $\theta_W$ is Weinberg angle. Matrix $U_I$ is the unitary matrix which diagonalizes the mass matrix of the charged leptons. It is not measurable and dependent on the basis choice. For simplicity, we can choose the basis where neutrino is diagonal and work on neutrino mass basis. So, the unitary matrix $U_I^\dagger$ here is just the lepton mixing matrix $V_I$,

$$
V_I = U_I^\dagger U_\nu,
$$

where $U_\nu$ is the unitary transformation matrix that diagonalize the neutrino mass matrices $M_\nu$. By representing

$$
\bar{V} = V_I \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} V_I^\dagger,
$$

the decay width of $l \to l' + \gamma$ is given by

$$
\Gamma(l \to l'\gamma) = \frac{m_\mu}{32\pi} \left( \bar{V}_{ll'} e S^\gamma \frac{m_\mu}{m_\tau} \right)^2,
$$

where $\bar{V}_{ll'}(l \neq l')$ are the non-diagonal elements of matrix $\bar{V}$ defined in Eq. (10). The new physics contribution to the tau anomalous magnetic moment is given by

$$
|\delta\alpha_\tau| = |\bar{V}_{\tau\tau} S^\gamma|.
$$

A similar formula holds for the tau electric dipole moment. So we can find $\tau$ AMDM, EDM and LFV decays are all related to lepton mixing through the matrix $V$. 

4
3 Lepton mixing matrix and \( \tau \) AMDM and EDM

The general form of the lepton flavor mixing matrix is [10]

\[
V_l = \begin{pmatrix}
C_{12}C_{13} & -S_{12}C_{13} & S_{13} \\
S_{13}S_{23}C_{12} + C_{23}C_{12}e^{i\delta} & -S_{13}S_{23}S_{12} + C_{23}C_{12}e^{i\delta} & -C_{13}S_{12} \\
S_{13}C_{23}C_{12} + S_{23}C_{12}e^{i\delta} & S_{13}C_{23}S_{12} + C_{23}C_{12}e^{i\delta} & C_{13}S_{23}
\end{pmatrix},
\]

(13)

where \( S_{ij} \equiv \sin \theta_{ij}, \) \( C_{ij} \equiv \cos \theta_{ij}, \) \( \theta_{ij} \) is the mixing angles between different flavors; and \( \delta \) is CP phase. Putting aside the LSND experiments [11], the mixing angles measured by atmospheric [12], solar [13] and long-baseline [14] experiments actually correspond to the \( \theta_{23}, \theta_{12}, \theta_{13} \) respectively. If neglecting the possible CP-violating phase, and substituting \( \theta_{13}, \theta_{23}, \theta_{12} \) with \( \theta_1, \theta_a, \theta_s, \) we can present the lepton mixing matrix as [8, 15]

\[
V_l = \begin{pmatrix}
C_lC_s & -S_aC_l & S_l \\
S_lS_aC_s + C_aC_s & -S_lS_aS_s + C_aC_s & -S_aC_l \\
-S_lC_aC_s + S_aS_s & S_lS_aC_a + S_aC_s & S_aC_l
\end{pmatrix},
\]

(14)

where \( S_a \equiv \sin \theta_a, \) \( C_a \equiv \cos \theta_a, \) and so on. The present experimental data favor \( \sin^2 2\theta_a > 0.8 \) or \( \theta_a \sim 32^\circ - 45^\circ, \) \( \sin^2 2\theta_1 < 0.1 \) or \( \theta_1 \sim 0^\circ - 9.2^\circ. \) There are two different possible cases to the solar neutrino oscillation: the long wave-length vacuum oscillation with \( \sin^2 2\theta_s \approx 1, \) or \( \theta_s \sim 45^\circ; \) and the matter-enhanced oscillation (MSW mechanism [16]) with \( \sin^2 2\theta_s \sim 10^{-3} - 10^{-2} \) (small-angle solution) and \( \theta_s \sim 1^\circ - 3^\circ \) or with \( \sin^2 2\theta_s \sim 0.65 - 1 \) (large-angle solution) and \( \theta_s \sim 27^\circ - 45^\circ. \)

From Eqs. (10)-(12) and (14), we get the explicit form of the anomalous magnetic moment expressed in \( \theta_l, \theta_a \) and \( \theta_s, \)

\[
|\delta \alpha_\tau| = \frac{m_\tau}{e m_\mu} \left| \frac{C_l S_a^2}{S_l S_s} \right| \sqrt{\frac{32\pi \Gamma(\mu \rightarrow e\gamma)}{m_\mu}}.
\]

(15)

By using the current experimental upper limits, \( BR(\mu^- \rightarrow e^-\gamma) < 4.9 \times 10^{-11} \) [17], we investigate the relation between \( \delta \alpha_\tau \) and \( d_\tau \) and the mixing angle \( \theta_l \) which are shown on Figs. 1, 2, 3 and 4. In the numerical calculation, we take \( \theta_a = 32^\circ \) and \( 45^\circ \) respectively, as well as \( \theta_s = 1^\circ, 27^\circ, 45^\circ. \)
In the Figs, such as Fig. 1, all three curves, which correspond to $\theta_s = 1^\circ, 27^\circ$ and $45^\circ$ respectively, as well as $\theta_a = 32^\circ$, are all decrease to the lepton mixing angle which constrained by the long-baseline neutrino oscillation experiments. On the one hand, Considering the bound of $\delta a_\tau$ in Eq. (1), we find that the curve A can be excluded. That is to saw, $\theta_s$ will be not in the region $1^\circ - 3^\circ$. This result is suited to large mixing angle MSW solution ($\theta_s \sim 27^\circ - 45^\circ$) which is favored by the recent data of the Super-K and SNO. For curve B (or C), the regions in which $\theta_l < 1.8^\circ$ (or $\theta_l < 3^\circ$) can be excluded. The similar analysis can be used to the other three Figs. On the other hand, from the lepton flavor mixing angles constrained by the experiments directly, we can obtain the bounds on $\delta a_\tau$ and $d_\tau$ same as or smaller than these from the ansatz in Eq. (1), if $\theta_l$ lies in the scope about from $1.8^\circ$ to $9.2^\circ$. In other words, by using the effective lagrangian approach and the lepton mixing matrix, we can push $\tau$ AMDM and EDM down to very tiny values.

4 Summary

We extend the work in ref. [2] to bound the AMDM and EDM of $\tau$ in the effective lagrangian by considering the current experimental lepton mixing data. The experimental limits on $\mu \rightarrow e\gamma$ can put strong limits on $\tau$ AMDM and EDM same as obtained in ref. [2]. Our results are compatible with the large mixing angle MSW solution which is favored by the recent data of the Super-K and SNO. The AMDM and EDM of $\tau$ can also give some constraints on the lepton mixing angles $\theta_s, \theta_a$ and $\theta_l$ by solar, atmospheric and long-baseline neutrino experiment respectively.

Acknowledgments

This work is supported in part by National Natural Science Foundation of China and Doctoral Programme Foundation of Institute of High Education of China. One of the authors(W.J.H) acknowledges supports from the Chinese Postdoctoral Science Foundation.
and CAS K.C. Wong Postdoctoral Research Award Fund. We grateful to Prof. T. Huang, X.M Zhang and Z.Z Xing for useful discussions.

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Figure 1: Fig of $|\delta \alpha_r|$ to $\theta_l$ when $\theta_a = 32^\circ$, as well as $\theta_s = 1^\circ$, $27^\circ$, $45^\circ$ respectively.
Figure 2: Same as Fig. 1 but $\theta_a = 45^\circ$.

Figure 3: Fig of $|d_\tau|$ to $\theta_1$ when $\theta_a = 32^\circ$, as well as $\theta_s = 1^\circ$, $27^\circ$, $45^\circ$ respectively.
Electric dipole moment $|d\tau|$ (e cm)

$|d\tau| = 2.2 \times 10^{-25}$

$|d\tau| = 1.1 \times 10^{-17}$

For all: $\theta_a = 45^\circ$

A $\theta_s = 1^\circ$

B $\theta_s = 27^\circ$

C $\theta_s = 45^\circ$

Figure 4: Same as Fig. 3 but $\theta_a = 45^\circ$. 