Differentiable Neural Architecture Learning for Efficient Neural Network Design

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Abstract—Automated neural network design has received ever-increasing attention with the evolution of deep convolutional neural networks (CNNs), especially involving their deployment on embedded and mobile platforms. One of the biggest problems that neural architecture search (NAS) confronts is that a large number of candidate neural architectures are required to train, using, for instance, reinforcement learning and evolutionary optimisation algorithms, at a vast computation cost. Even recent differentiable neural architecture search (DNAS) samples a small number of candidate neural architectures based on the probability distribution of learned architecture parameters to select the final neural architecture. To address this computational complexity issue, we introduce a novel architecture parameterisation based on scaled sigmoid function, and propose a general Differentiable Neural Architecture Learning (DNAL) method to optimize the neural architecture without the need to evaluate candidate neural networks. Specifically, for stochastic supernets as well as conventional CNNs, we build a new channel-wise module layer with the architecture components controlled by a scaled sigmoid function. We train these neural network models from scratch. The network optimization is decoupled into the weight optimization and the architecture optimization, which avoids the interaction between the two types of parameters and alleviates the vanishing gradient problem. We address the non-convex optimization problem of neural architecture by the continuous scaled sigmoid method with convergence guarantees. Extensive experiments demonstrate our DNAL method delivers superior performance in terms of neural architecture search cost, and adapts to conventional CNNs (e.g., VGG16 and ResNet50), lightweight CNNs (e.g., MobileNetV2) and stochastic supernets (e.g., ProxylessNAS). The optimal networks learned by DNAS surpass those produced by the state-of-the-art methods on the benchmark CIFAR-10 and ImageNet-1K dataset in accuracy, model size and computational complexity. Our source code is available at https://github.com/QingbeiGuo/DNAL.git

Index Terms—Deep Neural Network, Convolutional Neural Network, Neural Architecture Search, Automated Machine Learning

I. INTRODUCTION

Although convolutional neural networks have made great progress in various computer vision tasks, such as image classification [1]–[4], object detection [5]–[7] and semantic segmentation [8]–[11], their deployment into many embedded applications, including robotics, self-driving cars, mobile apps and surveillance cameras, is hindered by the constraints of model size, latency and energy budget. A lot of approaches have been proposed to improve the efficiency of neural networks to handle these hardware constraints. These approaches can be divided into three categories: conventional model compression [12]–[14], lightweight network design [15]–[17] and automatic neural architecture search [18]–[20]. Thanks to the over-parameterisation of deep neural networks, the conventional methods compress neural network models by different compression techniques, such as pruning [21], [22], network quantization [23], [24], tensor factorization [25], [26], and knowledge distilling [27]. The lightweight network is heuristically constructed by designing efficient modules, including group convolutions, depthwise separable convolutions, shuffle operations, etc. Recently, in order to automatically explore the large design space, the NAS methods leverage reinforcement learning [18], [28], [29], evolutionary optimisation algorithm [19], [30], [31] and gradient-based method [20], [32], [33] for efficient neural network search, achieving the state-of-the-art recognition performance.

However, these existing methods suffer from three problems. (1) Both the heuristic compression policy and lightweight module design require domain expertise to explore the architecture space. However, the space is so large that such hand-crafted methods cannot afford the architecture search cost. Due to the limitations imposed on the search space, the resulting neural networks are usually sub-optimal. Moreover, these methods have to take the constraint of hardware resources into account. Unfortunately, the computational complexity makes it prohibitive to produce application and hardware specific models. (2) Previous NAS methods exploit reinforcement learning and evolutionary optimisation algorithms to automatically explore the discrete search space, thus achieving the state-of-the-art recognition performance. However, such methods generate a large number of candidate neural architectures, more than 20,000 candidate neural networks across 500GPUs over 4 days in [34]. It is time-consuming to train and evaluate them so as to guide the neural net architecture search. (3) The existing DNAS methods relax the problem to search discrete neural architectures to optimize the probability of stochastic supernets, and allow us to explore continuous search spaces by using gradient-based methods. However, some DNAS methods still
require a few candidate neural architectures to identify the best candidate by sampling based on the probability distribution of learned architecture parameters [20].

To address these problems, we introduce a novel approach which converts the discrete optimisation problem into a continuous one. This is achieved by proposing a differentiable neural architecture learning method to automatically search for the optimal neural network parameterised in terms of a continuous scaled sigmoid function. This is the first work to apply the scaled sigmoid function to facilitate the search for efficient neural networks to the best of our knowledge. Specifically, for both conventional CNNs and stochastic supernets, we build a new channel-wise module layer controlled by the scaled sigmoid function, which can be inserted into any existing neural architectures without any special design. This module relaxes the discrete space of neural architecture search by continuous architecture representation. By progressively reducing the smoothness of the scaled sigmoid function, the continuous optimization problem is gradually turned into the original architecture optimization problem. Thus, the optimal neural architecture can be learned by using gradient-based methods with few epochs, while guaranteeing the convergence. No additional candidate neural networks are produced, significantly improving the efficiency of neural architecture search. In order to avoid the interaction between the weight optimization and the architecture optimization, the network optimization is decoupled into the weight optimization and the architecture optimization. This also alleviates the vanishing gradient problem. After optimizing the neural architecture, we achieve its potential representation ability by finetuning. Extensive experiments demonstrate that our DNAL method is applicable to conventional CNNs (e.g., VGG16 [2] and ResNet50 [3]), lightweight CNNs (e.g., MobileNetV2 [35]) and stochastic supernets (e.g., ProxylessNAS [36]), and achieves the state-of-the-art performance on the classification task on CIFAR-10 [37] and ImageNet-1K [38] in terms of model size, FLOPs, accuracy, and more importantly, search cost.

Our contributions can be summarized as follows.

- We build a new standalone control module based on the scaled sigmoid function to enrich the neural network module family to enable the neural architecture optimization.
- We relax the discrete architecture optimization problem into a continuous one and learn the optimal neural architecture by using gradient-based methods.
- Our DNAL method produces no candidate neural architectures but one, thus drastically improving the efficiency of neural architecture search.
- It is applicable to conventional CNNs, lightweight CNNs, and stochastic supernets for automated neural architecture learning.
- Extensive experiments confirm that our DNAL method achieves the state-of-the-art performance on various CNN architectures, including VGG16, ResNet50, MobileNetV2, and ProxylessNAS, over the task of CIFAR-10 and ImageNet-1K classification.

The rest of this paper is organized as follows: We first investigate the related work in Section II. We then present the differentiable neural architecture learning method in Section III. Subsequently, we demonstrate that our proposed DNAL method delivers superior performance through extensive experiments on various popular network models and datasets in Section V. We present an ablation study, which enhances the understanding of DNAL in Section VI. Finally, we draw the paper to a conclusion in Section VII.

II. RELATED WORK

In this section, we review various methods to yield efficient neural networks from three different perspectives.

Conventional Model Compression. The conventional compression method includes pruning [21], [22], network quantization [23], [24], tensor factorization [25], [26] and knowledge distillation [27]. Pruning is one of the most promising compression methods, which removes the redundant parts, including weights, channels and even layers, to compress neural networks based on heuristic rules. It is orthogonal to other methods to design more efficient neural networks. As the structured pruning is more efficient in reducing the parameters and computations, the channel-wise pruning methods have attracted more attention. Huang et al. introduced a scaling factor to scale the output of specific structures, such as channels, groups, and residual blocks, and added sparsity regularizations on the scaling parameters to force them to zero [12]. The structures corresponding to zero scaling factors will be pruned as the dispensable parts to promote the design of compact CNNs. Similarly, the scaling parameters in Batch Normalization (BN) are used to control the output of the corresponding channels without introducing any extra parameters [13]. In this work, we adopt a similar approach and propose a module scaling based on the scaled sigmoid function. The difference between the proposed approach and these methods is analyzed in detail in the supplementary material.

Most of these pruning methods require extra retraining to compensate the performance loss in accuracy due to pruning. This inevitably increases the computational cost, especially in the case of the iterative pruning and retraining scheme [39], [40]. Unfortunately, these methods achieve only sub-optimal solutions, because it is prohibitive to explore the whole search space using their human-based heuristics.

Lightweight Network Design. This kind of method aims to directly construct efficient neural networks by designing cheap but effective modules, rather than pruning redundant structures. Such modules have few parameters and low computation complexity with a reasonable representation capacity. They including group convolution [15], [41]–[43], depthwise separable convolution [16], [53] and shuffle operation [17].

These human-designed modules explore only a very small portion of the design space, thus discovering only sub-optimal solutions. Moreover, it is infeasible to design specific neural networks for every individual hardware platform. In practice, we usually reuse the neural networks for different devices to save the computation time, and adjust them to achieve a trade-off between accuracy and efficiency.

Neural Architecture Search. Recently, the methodology of neural architecture search has made a significant progress. The
NAS methods automatically explore the search space to find the optimal neural architectures by different optimization methods, such as reinforcement learning, evolutionary optimization algorithms and gradient-based methods.

The early works automatically searched the optimal neural architecture based on reinforcement learning [18], [28], [29] and evolutionary optimization algorithms [19], [30], [31] in a discrete search space. These methods generate thousands of candidate neural network architectures, and their validation set performance is treated as the reward or fitness to guide the search process. However, it is time-consuming to train and evaluate those candidate architectures. Various proxy techniques have been adopted to reduce the search cost, including the performance evaluation on a small dataset, training for few epochs and searching few blocks [36]. However, they do not fundamentally solve the problem of search cost.

To solve the problem effectively, the idea of differentiable neural architecture search was proposed in [20], [32], [33] to optimize the network architecture by gradient-based methods. These DNAS methods utilize the softmax function over parallel operation blocks to convert the discrete space into a continuous space, and formulate the neural architecture learning in a differentiable manner. The optimal neural architecture problem is then solved based on gradient search methods, which avoids enumerating individual network architectures and training/evaluating them separately. Nevertheless, some DNAS methods still sample multiple candidate architectures and training/evaluating them separately. For instance for the case of VGG16, as the total number of its channels is 4224, it contains $2^{4224} \approx 10^{1272}$ possible architectures. For deeper ResNet50, the number of possible architectures exceeds $10^{7992}$. The search space is so large that it is not feasible to enumerate them, let alone search for the optimal neural architecture. Moreover, it is a challenging to solve the non-convex optimization problem to find the optimal architecture in such a large search space.

C. Differentiable Neural Architecture Learning by Continuation

**Scaled Sigmoid Function.** We first introduce channel-wise architecture parameter vector $s$, which serves as an indicator to represent the architecture of a neural network. Here, $s = [s^1, ..., s^L]$, and for the $l$-th layer, $s^l = [s^l_{11}, ..., s^l_{1N_l}; ...; s^l_{M_lN_l}]$. DNAL learns an efficient neural architecture by converting the architecture parameters $s$ to a binary vector $b$. The binarization process can be implemented by taking the binary function $b = \text{binary}(s)$ as activation functions,

$$b = \text{binary}(s) = \begin{cases} 1, & \text{if } s > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $s \in s$ and $b \in b$.

However, it is infeasible to train the deep neural network with the standard back-propagation (e.g., SGD), as the binary function is non-smooth and non-convex. The binary function is ill-posed at zero, which is non-differentiable, and its gradient is zero for all nonzero values, which causes the vanishing gradient problem in the neural network optimization.

It is an open problem to optimize neural networks with non-smooth activation functions. Motivated by the continuation methods [45], we convert the optimization problem with ill-posed gradients to a manageable problem by smoothing the binary activation function. We find that there is a relationship between the binary function and the scaled sigmoid function which becomes binary when scale factor $\delta$ tends to infinity, as follows,

$$\lim_{\delta \to +\infty} \text{sigmoid}(\delta s) = \text{binary}(s),$$

where $p = \text{sigmoid}(\delta s) = 1/(1 + e^{-\delta s})$ is the scaled sigmoid function with hyper-parameter $\delta$ to control its transition from
Architecture Optimization.

We relax the problem of optimizing the neural architecture to
progressively sharpening the scaled sigmoid function transition
region. Specifically, we begin to optimize the neural architecture
with the smoothed sigmoid activation function, where $\delta_0 = 1$. By progressively increasing the scale factor $\delta$, the neural
architecture will gradually converge to a solution corresponding
to an optimal architecture defined by the resulting binary
function. In this paper, $\max(\delta) = 10^4$, which is sufficient
to guarantee the convergence. Therefore, our DNAL method
can optimize the neural architecture by using gradient-based
methods, while producing no additional candidate architectures.

Architecture Optimization. We build a new channel-wise
module layer to incorporate the Scaled Sigmoid activation
function, named by SS, and add the SS transformation after
the batch normalization layer, as shown in Fig. 2. The order
of SS layer is analyzed in the ablation study presented later
in the paper.

To achieve an efficient neural architecture, we define the
following loss function,

$$L = L_0 + \lambda_s \sum_{l=1}^{L} \sum_{i=1}^{M^l} \sum_{j=1}^{N^l} \text{sigmoid}(\delta s_{ij}).$$  \hspace{1cm} (4)

Here, the first term is the cross-entropy loss function. The
second term drives the scaled sigmoid activation of each
channel to zero, which tends to remove the less important
channels. The hyper-parameter $\lambda_s$ is a coefficient to achieve
an appropriate balance between accuracy-efficiency.

In the forward step, we calculate the output $\hat{x}^l$ of the $l$-th
layer as follows,

$$\hat{x}^l = \sum_{i=1}^{M^l} \text{ReLU}( \begin{pmatrix} x_{i1}^l \\ \vdots \\ x_{iN^l}^l \end{pmatrix} \odot \begin{pmatrix} \text{sigmoid}(\delta s_{i1}) \\ \vdots \\ \text{sigmoid}(\delta s_{iN^l}) \end{pmatrix}),$$  \hspace{1cm} (5)

where $\odot$ is the symbol of Hadamard product, and $x_{ij}^l$ is the
original output of the $j$-th channel of the $l$-th block in the $l$-th
layer. The sigmoid function serves as a weight coefficient to scale
the output of the corresponding channel. The outputs of
the blocks at each layer are aggregated as the layer output. The scale factor $\delta$ exponentially increases as the training process
progresses.

In the backward step, the gradient w.r.t. the architecture
parameters can be calculated as follows,

$$\frac{\partial L}{\partial s_{ij}} = \frac{\partial L}{\partial p_{ij}} \frac{\partial p_{ij}}{\partial s_{ij}} = \frac{\partial L}{\partial p_{ij}} \delta^2 s_{ij} (1 - \delta^l s_{ij}).$$  \hspace{1cm} (6)

Our approach has a number of significant advantages. We
update the architecture parameters $s$ by directly using a gradient
descent method. Due to the relative small number of architecture
parameters, compared to the number of weights, our
DNAL method exhibits a fast convergence for the architecture
optimization. Moreover, as we optimize the neural architecture
directly on the target tasks, we require no proxy tasks, such as
a smaller dataset for training. This ensures that the neural
architecture obtained by DNAL is optimal on the target
tasks. In this work, the architecture optimization consumes
about one-tenth of the resource needed for the parameter
optimization, specifically 20 epochs for CIFAR-10 and 10
epochs for ImageNet, as described in Section VI in detail.

Once the DNAL optimization process has converged, if
$\text{sigmoid}(\delta s_{ij}) = 1$, then the corresponding channel is signifi-
cantly retained. On the contrary, if $\text{sigmoid}(\delta s_{ij}) = 0$, the
corresponding channel is decisively pruned out as its output
has no contribution to the subsequent computation. However,
as this pruning process can result in channel dimension
incompatibility, we follow the technique in [33], and zero-
pad the missing channels for the sake of channel dimension
alignment. Moreover, after pruning, we disable all the SS layer,
so no special operations or structures are introduced. Finally,
we fine-tune the optimal sub-network to restore its representative
ability.

Sequential Optimization Strategy. In the approach described
so far, there are two challenging problems. With the increasing
value of the hyper-parameter $\delta$, the scaled sigmoid function
has larger and larger saturation zone where the gradient is zero
and this leads to the vanishing gradient problem. As a result,
back-propagation becomes inefficient, and this is harmful to the
optimization process. The other issues is the interaction between
the parameter optimization and the architecture optimization.
A channel is simultaneously affected by both its architecture
parameter and weights. For example, a channel has a near-
zero value of scaled sigmoid function but large weights. It is
arbitrary to take it as a less important channel, because the
channel still may contribute considerably to the next layer.

To tackle these two issues, we decouple the network
optimization into the weight optimization and the architecture
optimization. We found empirically that if the architectures are
optimized from the start without a suitable initialisation, the
architecture search will tend to fall into bad local optima.
Therefore, the weights are optimized first. At the weight
optimization stage, we disable the SS layer, which means that
the SS layer does not change the original channel output, i.e.,
$\text{sigmoid}(\delta s) = 1$ for each channel, and we use SGD to learn
only the weights. When optimizing the neural architecture, we
freeze the trainable layers, including convolutional layers and
BN layers, and focus only on the architecture parameters also by
SGD. Using the sequential optimization strategy alleviates the
vanishing gradient problem, as well as reducing the interaction
weights and architecture parameters. We will compare the sequential optimization strategy with the joint optimization in the following ablation study.

**DNAL Algorithm.** We depict the optimization process of our DNAL method in Alg. 1. The whole process consists of three stages: (1) Weight optimization stage. We learn only the weights by SGD with weight decay, while disabling the SS layer. (2) Architecture optimization stage. When learning architecture parameters, the weights are frozen. The scaled sigmoid technique is used to optimize the architecture parameters by SGD. (3) Finetuning stage. After pruning the channels with $\text{sigmoid}(\delta s) = 0$, we finetune the derived neural network to achieve a better accuracy.

**Algorithm 1** Differentiable neural architecture learning procedure.

**Require:**

- The stochastic supernet $\mathcal{N}$ with SS layers, a sequence $1 = \delta_1 < \cdots < \delta_n = +\infty$.

**Ensure:**

- the efficient neural network $\hat{\mathcal{N}}$.

1: randomly initializing the weights $W$, and disabling the SS layers
2: for each epoch $i = 1$ to $m$
3: optimize $W$ by SGD with respect to $\mathcal{L} = \mathcal{L}_0 + \lambda_w \|W\|_2$
4: end for
5: enabling the SS layers, initializing $s = 0$, and freezing the weights $W$
6: for each epoch $i = 1$ to $n$
7: optimizing $S$ by SGD with respect to $\mathcal{L} = \mathcal{L}_0 + \lambda_o \sum \sum \text{sigmoid}(\delta, s)$
8: end for
9: pruning the channels with $\text{sigmoid}(\delta s) = 0$
10: disabling the SS layers
11: finetuning the searched network $\hat{\mathcal{N}}$ by SGD with respect to $\mathcal{L} = \mathcal{L}_0 + \lambda_w \|W\|_2$

### IV. Analysis

In this section, we investigate the differences between DNAL and other related methods by comparison.

**Comparison with other DNAS methods.** Although our DNAL method adopts the same gradient-based approach as the existing DNAS methods to optimize the neural architecture, there are major differences between them in following three respects.

First, the existing DNAS methods applied the softmax function to learn the relative probability of each operation block, and then retain the block with the maximum probability to construct the optimal architecture, while abandoning the other components [20], [32], [33]. DNAL utilizes the scaled sigmoid function to learn the absolute probability of each channel. After converging to the original binary problem, we preserve the channels with probability 1, while removing the channels with probability 0.

Second, the existing DNAS methods choose the most likely operation block from multiple candidate operation blocks for each layer to construct the optimal neural architecture, which means each layer contains only one operation block [20], [32], [33]. However, DNAL learns more general structure, where each layer may contain different operation blocks with different channels. Thus, DNAL increases the search space size by orders of magnitude. This helps to improves the accuracy of neural networks, which is experimentally confirmed in the following section.

Third, after finishing the architecture search, some DNAS methods still sample several candidate architectures based on the distribution of architecture parameters, and select the best one by training them from scratch [20]. Our DNAL method yields directly the optimal architecture by the proposed method, producing no additional candidate architectures. This significantly reduces the computational cost.

**Comparison with other scaling methods.** In DNAL, we introduce the scaled sigmoid function as a mechanism to scale the output of each channel. The proposed method is significantly different from other scaling methods.

First, the existing scaling methods consider the scale factor as a coefficient to scale the output of some specific structures [12], including channels, groups and blocks. Some methods leveraged the learnable scale factor of BN without introducing extra parameters [13], [14]. We define an architecture parameter $s \in \mathbb{R}$, and use its scaled sigmoid function $\text{sigmoid}(\delta s) \in [0, 1]$, as the scale factor in a probabilistic way.

Second, these scaling methods impose a sparsity constraint on the scaling parameters to push them infinitely close to zero, and then prune the structures corresponding to zero or near-zero out. However, such pruning may degrade the performance. By contrast, our DNAL method forces the sparsity constraint on the scaled sigmoid function rather than the architecture parameters, driving them into the negative saturation zone, i.e., $\text{sigmoid}(\delta s) = 0$. Removing the channels with $\text{sigmoid}(\delta s) = 0$ causes no accuracy loss due to the consistency of the binary expression with the pruned structure.
V. Experiments

In this section, we empirically evaluate the proposed DNAL method on CIFAR-10 \[37\] and ImageNet-1K \[38\] for classification by using state-of-the-art CNN architectures, which include conventional CNNs (e.g., VGG \[2\] and ResNet \[3\]), lightweight CNNs (e.g., MobileNetV2 \[35\]) and stochastic supernets (e.g., ProxylessNAS \[36\]). We use PyTorch \[46\] to implement the proposed DNAL method.

A. Datasets

CIFAR-10. CIFAR-10 is a popular dataset of tiny images with 10 classes, and contains 50,000 images for training and 10,000 images for testing.

ImageNet-1K. ImageNet-1K is a subset of the ImageNet Large Scale Visual Recognition Challenge dataset. It contains 1.2M training images and 50K validation images as testing images, and is categorized into 1000 classes.

B. Classification on CIFAR-10

We evaluate the recognition performance on CIFAR-10, comparing against several popular convolutional neural networks, such as VGG16, ResNet56 and MobileNetV2.

Implementation. We use a variation of VGG16, as in \[47\]. In the first weight optimization stage, the initial model is trained for 100 epochs, and the learning rate is fixed to 0.1. In the architecture optimization stage, we learn the optimal neural architecture for 20 epochs with a constant learning rate 0.1. The scale factor $\delta$ grows from 1 to $10^4$ exponentially. In the last finetuning stage, the learning rate schedule is 0.1 for the first 30 epochs, 0.01 until the 80th epoch, and 0.001 to the 130th epoch. The batch-size is set to 128, weight decay 1e-4 and momentum 0.9. For ResNet56, we follow the same setting as VGG16, except for reducing the batch-size to 64. For MobileNetV2, the setting is different from VGG16. The weight optimization stage lasts for 150 epochs. The learned neural network is finetuned for 180 epochs. The learning rate is divided by 10 after 30 and 105 epochs. The weight decay is 4e-5.

VGG16. Tab. \[I\] shows the performance of different compression methods. Compared with Variational-pruning, DNAL achieves better accuracy (93.53\% vs. 93.18\%) with similar compression and acceleration rates. Compared with GAL-0.1, DNAL provides larger reductions in FLOPs and Params, while achieving better accuracy (93.75\% vs. 93.42\%). Compared with HRank, DNAL is significantly better in all respects (61.23 vs. 73.70 in FLOPs, 0.60 vs. 1.78 in Params and 92.33\% vs. 91.23\% in top-1 accuracy). In addition, we tried for a higher compression and acceleration rate of up to about 90× and 20×, respectively, for mikronet, achieving 89.27% top-1 accuracy and 99.51% top-5 accuracy. It demonstrates the ability of our proposed DNAL to find a more efficient neural networks.

ResNet56. We show the results for ResNet56 in Tab. \[III\]. The proposed DNAL outperforms HRank in all respects (83.11 vs. 88.72 in FLOPs, 0.59 vs. 0.71 in Params and 93.75\% vs. 93.52\% in top-1 accuracy). With a similar model size and computation complexity, we achieve better accuracy than NISP (93.75\% vs. 93.01\%). DNAL yields 1.3% higher top-1 accuracy and about 2× faster speedup than AMC, and also yields 0.32\% and 1.62\% higher top-1 accuracy than KSE and GAL-0.8 with a smaller model size and faster speedup, respectively. To explore more efficient neural models, we further compress the neural network, up to more than 70× for model size and more than 120× for computation complexity, achieving 83.48\% and 99.19\% in top-1 and top-5 accuracy, respectively.

MobileNetV2. The results for MobileNetV2 are shown in Tab. \[III\]. Since the network is already very computationally efficient, it is interesting to see whether it can be compressed further. Compared with FLGC, DNAL again demonstrates its outstanding ability to find efficient neural networks. When the acceleration rate is less than 2×, our DNAL achieves the best top-1 accuracy (94.30\%), close to the baseline. When the acceleration rate exceeds 2×, our DNAL achieves 94.01% top-1 accuracy, which is still better than FLGC. Although it is much harder to further compress the lightweight model, DNAL still manages to obtain 87.85% top-1 accuracy and 99.62% top-5 accuracy with an acceleration rate of about 20× and compression rate of roughly 30×.

C. Classification on ImageNet-1K

We further conduct the experiments for several popular CNNs, i.e., VGG16, ResNet50, MobileNetV2 and Proxyless-
NAS, to evaluate the recognition performance on the large-scale ImageNet-1K.

**Implementation.** Both VGG16 and ResNet50 are initially trained for 30 epochs with a fixed learning rate 0.1. Then, we optimize the architecture parameters for 10 epochs with a constant learning rate 0.01. The optimal neural architectures are learned by the scaled sigmoid function method with the exponential scale factor $\delta$ ranging from 1 to $10^4$. The architecture learning process is the same for other models, i.e., MobileNetV2 and ProxylessNAS. Finally, the derived neural networks are finetuned for 70 epochs. We use SGD with a mini-batch size of 256, a weight decay of 0.0001 and a momentum of 0.9. The learning rate is divided by 10 at 10 and 40 epochs. We train MobileNetV2 from scratch for 80 epochs. The learning rate starts from 0.1, and is tuned with a cosine decaying schedule. The derived neural networks are finetuned for 90 epochs. The weight decay is set to 4e-5. The other hyper-parameters are the same for both VGG16 and ResNet50. We remove the dropout from the last classifier layer of MobileNetV2. We use the same search space as ProxylessNAS but without the zero operation. We train the stochastic super-net for 100 epochs from scratch. The learning rate is set to 0.05 initially, and is tuned with the same decaying schedule as used for MobileNetV2. We finetune the derived neural networks for 110 epochs by SGD with a mini-batch size of 96. The other hyper-parameters are the same to MobileNetV2.

**VGG16.** Tab. IV shows the performance of different methods. Compared with GDP, our DNAL method achieves a faster acceleration rate ($2.59 \times$ vs. $2.42 \times$) and 1% higher top-1 accuracy (69.80% vs. 68.80%). Compared with both ThiNet and SSR, DNAL provides significantly better parameter reduction (77.05 vs. 131.44 and 77.05 vs. 126.7), while maintaining a comparable performance in top-1 accuracy.

**ResNet50.** For ResNet50, we summarize the performance comparison with various methods in Tab. V. We observe that DNAL outperforms SSR by a significant margin in all respects. Similarly, DNAL achieves better performance than both GDP and GAL. Compared with ThiNet-50, DNAL achieves 1.62% higher top-1 accuracy with similar FLOPs and parameter reductions (1.75 vs. 1.71 in FLOPs and 12.75 vs. 12.38 in Params). Similar observations can be found when comparing with HRank. DNAL yields 0.88% and 1.07% higher top-1 accuracy than HRank, respectively, while maintaining lower computation complexity. Compared with ABCPruner, DNAL is a slightly better in accuracy (73.65% vs. 73.52%) with fewer FLOPs (1.75 vs. 1.79).

**MobileNetV2.** To further demonstrate the effectiveness of our DNAL method, we also test it on modern efficient neural networks, i.e., MobileNetV2. The results are presented in Tab. VII. Here we compare DNAL with the state-of-the-art autoML model compression method, i.e. AMC. DNAL outperforms AMC by more than 0.2% with approximate FLOPs (217.24 vs. 211), and even beats it by 0.11% at smaller computation complexity (207.25 vs. 211). Its ability to compress the lightweight neural networks further is surprising.

**ProxylessNAS.** We show the compression performance of different NAS methods in Tab. VII. These searched models are divided into three categories according to the underlying NAS methods used. Our DNAL achieves 75.0% top-1 and 92.5% top-5 accuracy on ImageNet with only 3.6M parameters, which is a new state-of-the-art accuracy among different NAS methods. Compared with the EA-based NAS methods, DNAL is more than 2% and about 1% higher than CARS in top-1 and top-5 accuracy, respectively, with slightly fewer parameters. It even
approximate to the CARS-I model in accuracy with about 1.5× fewer parameters. Compared to the based-RL NAS methods, DNAL model attains a significantly better performance than both NASNet and MnasNet, while requiring fewer parameters. Our DNAL also surpasses DARTS by 1.7% and 1.2% in top-1 and top-5 accuracy, respectively, with significantly fewer parameters, and achieves almost the same top-1 accuracy as ProxylessNAS, but with 2× fewer parameters. And it achieves higher accuracy and fewer parameters than FBNet. Therefore, it fully demonstrates that our DNAL owns a stronger representability than those state-of-the-art NAS methods.

**Efficiency.** In this part, we further analyze the efficiency of the proposed DNAL method. To demonstrate its efficiency, we choose the number of training epochs, which is hardware-independent, as a metric of learning efficiency, for fair comparison. The experimental results are reported in Tab. IV, V, VI and VII. We easily observe that DNAL typically features high efficiency. For both VGG16 and ResNet50, to reach optimality, DNAL requires the minimum number of training iterations, with the exception of ABCPruner. DNAL is more than 2.2× more efficient than ThiNet. For the other two neural networks, i.e., MobileNetV2 and ProxylessNAS, our gradient-based DNAL method is empirically more efficient than the NAS methods based on reinforcement learning (e.g., AMC) and evolutionary optimisation algorithm (e.g., CARS), which in any case are much expensive in terms of the search cost due to the evaluation of lots of candidate neural architectures. Among the gradient-based NAS methods, DNAL is still superior in efficiency. The cost of searching for the optimal neural architectures of our DNAL method is particularly low, i.e., 20 epochs for CIFAR-10 and 10 epochs for ImageNet. We argue that the DNAL’s high efficiency derives from its differentiability of the objective function thanks to the scaled sigmoid method.

**VI. Ablation Study**

In this section, we report the results of an ablation study set up to investigate the impact of different factors on both VGG16 and MobileNetV2 using the classification task of CIFAR-10 as a vehicle.

**Effect of the scale factor $\delta$.** As the scale factor increases, the scaled sigmoid function has a larger saturation zone, which helps to binarize the neural architectures. Fig. 3 shows the distribution of the scaled sigmoid activations obtained with different scale factors. We initialize the scale factor to $\delta = 1$ and the architecture parameter to $s = 0$ at the beginning of the architecture optimization step, i.e., $\text{sigmoid}(\delta s) = 0.5$. We can see that the many scaled sigmoid activations are induced to zero under the influence of the scaled sigmoid regularization as the scale factor increases. When $\delta = 10^4$, all the channels are binarized. The number of channels with $\text{sigmoid}(\delta s) = 1$ defines the efficiency of the resulting neural networks.

**Effect of the order of SS layer.** We explore the effect of different placement of the SS layer in conjunction with three network configurations, i.e., Conv-SS-BN-ReLU, Conv-BN-SS-ReLU and Conv-BN-ReLU-SS configurations. Fig. 4 shows the test accuracy achieved with different network configurations. We observe that both the Conv-BN-SS-ReLU and Conv-BN-ReLU-SS configurations are close in the recognition performance, which is significantly better than Conv-SS-BN-ReLU. In the weight optimization stage, these three networks are identical because the SS layer is disabled. Thus, their behaviours are also consistent. In the architecture optimization stage, their network configurations become different due to the enabling SS layers. For both the Conv-BN-SS-ReLU and Conv-BN-ReLU-SS configurations, the optimized architectures improve the accuracy at the beginning of the architecture optimization. However, as the number of pruned channels increases, their performance gradually degrades. By contrast, the Conv-SS-BN-ReLU’s performance reduces dramatically. After finetuning, all exhibit improved performance, but both the Conv-BN-SS-ReLU and Conv-BN-ReLU-SS configurations are significantly better than Conv-SS-BN-ReLU.

**Table VI**

| Model         | FLOPs (M) | Params (M) | Top-1 (%) | Top-5 (%) | Search Cost (Epochs) |
|---------------|-----------|------------|-----------|-----------|----------------------|
| Baseline      | 300.79 (1.10 ×) | 3.50 (1.00 ×) | 71.52 | 90.15 | 120 |
| DNAL($\lambda_{56c-5}$) | 217.22 (1.58 ×) | 2.87 (1.22 ×) | 71.02 | 89.96 | 80+10=90 |
| AMC [49]      | 211 (1.43 ×) | 70.8 | 80+10=90 |
| DNAL($\lambda_{57e-5}$) | 207.25 (1.45 ×) | 2.78 (1.26 ×) | 70.91 | 89.79 | 80+10=90 |

**Table VII**

| Model         | Params (M) | Top-1 (%) | Top-5 (%) | Search Cost (Epochs) |
|---------------|------------|-----------|-----------|----------------------|
| Baseline      | 16.0       | 75.7      | 92.5 Alternating | 100+10+110 |
| DNAL($\lambda_{56c-5}$) | 3.6       | 75.0      | 92.5 gradient | 100+10+110 |
| CARS-A [19]   | 5.1        | 75.2      | 92.5 EA     | - |
| CARS-B [19]   | 3.7        | 72.8      | 91.6 EA     | - |
| NASNet-A [34] | 5.3        | 74.0      | 91.6 RL     | - |
| NASNet-B [34] | 5.3        | 72.8      | 91.3 RL     | - |
| NASNet-C [34] | 4.9        | 72.5      | 91.0 RL     | - |
| MnasNet-927 [53] | 4.4   | 74.8      | - RL     | - |
| MnasNet-657 [57] | 3.6      | 73.0      | - RL     | - |
| DARTS [52]    | 4.7        | 73.3      | 91.3 gradient | 600×250 |
| ProxylessNAS [56] | 7.1      | 75.1      | 92.5 gradient | 200+150 |
| FBNet-A [50]  | 4.3        | 73.0      | - gradient | 90+360 |
| FBNet-B [20]  | 4.5        | 74.1      | - gradient | 90+360 |
| FBNetC [20]   | 3.5        | 74.9      | - gradient | 90+360 |

Figure 3. Architecture binarization results with different scale factors (%). (a) VGG16 on CIFAR-10. (b) MobileNetV2 on CIFAR-10.
As we can see, setting the SS layer behind the BN layer helps to improve the recognition accuracy. We argue that the BN layer normalizes the distribution of feature maps for each layer, which enables the SS layer to operate under the same distribution of feature maps.

**Effect of the joint optimization.** In this paper, we optimize the weights and architecture parameters in a sequential manner. However, they can be optimized jointly. We compare these two optimization strategies in Fig. 5. For a fair comparison, the searched models are similar in the computational efficiency. At the beginning of the network optimization, the joint strategy has a faster convergence, and exhibits better accuracy. However, as the network continues to be optimised, the performance gradually degrades in accuracy. For the sequential strategy, after the architecture optimization, the resulting network rapidly recovers its performance by finetuning, surpassing the joint strategy optimization in accuracy.

This confirms that the joint optimization is hindered by the vanishing gradient problem. With the increasing scale factors $\delta$, more channels enter the saturation zone where the gradients of the architecture parameters are zero. This results in inefficient network optimization and in consequence a loss of accuracy. After the architecture optimization, DNAL disables the SS layer and optimizes only the weights, which avoids the vanishing gradient problem caused.

**VII. Conclusion**

We have presented a differentiable neural architecture learning method (DNAL). DNAL utilizes the scaled sigmoid function to relax the discrete architecture space into a continuous architecture space, and gradually converts the continuous optimization problem into the binary optimization problem. The optimal neural architecture is learned by gradient-based methods without the need to evaluate candidate architectures individually, thus significantly improving the search efficiency. We introduced a new SS module layer to implement the scaled sigmoid activation function, enriching the module family of neural networks for the optimization of neural architectures. The proposed DNAL method was applied to conventional CNNs, lightweight CNNs and stochastic supernets. Extensive experiments on CIFAR-10 and ImageNet-1K demonstrated that DNAL delivers state-of-the-art performance in terms of accuracy, model size and computational complexity, especially search cost.

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