Spin fluctuations in Sr$_2$RuO$_4$ from polarized neutron scattering: implications for superconductivity

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Triplet pairing in Sr$_2$RuO$_4$ was initially suggested based on the hypothesis of strong ferromagnetic spin fluctuations. Using polarized inelastic neutron scattering, we accurately determine the full spectrum of spin fluctuations in Sr$_2$RuO$_4$. Besides the well-studied incommensurate magnetic fluctuations we do find a sizeable quasi-ferromagnetic signal, quantitatively consistent with all macroscopic and microscopic probes. We use this result to address the possibility of magnetically-driven triplet superconductivity in Sr$_2$RuO$_4$. We conclude that, even though the quasi-ferromagnetic signal is stronger and sharper than previously anticipated, spin fluctuations alone are not enough to generate a triplet state strengthening the need for additional interactions or an alternative pairing scenario.

Superconducting Sr$_2$RuO$_4$ was proposed to be a solid-state analogue of He$_3$, i.e., a triplet superconductor, based on its proximity to SrRuO$_3$, a ferromagnetic (FM) metal. A simple model derived from the density-functional theory for SrRuO$_3$, CaRuO$_3$ and Sr$_2$YRu$_2$O$_8$ ascribed the mass and spin susceptibility renormalization to FM fluctuations, and predicted a triplet pairing. Experimental evidence pointing toward a particular (chiral-p) triplet was obtained, such as temperature-independent uniform susceptibility for the in-plane fields and time-reversal symmetry breaking. However, the dominant spin fluctuations in Sr$_2$RuO$_4$ are not FM, but incommensurate (IC) antiferromagnetic (AFM), and several experiments are inconsistent with either triplet states, or time-reversal breaking, or both. Various theories were proposed to explain triplet pairing by incorporating higher-order vertex corrections, the interplay of incommensurate charge and spin fluctuations, or orbital fluctuations, arriving at different superconducting states. Even the question about which bands drive pairing remains controversial.

The Fermi surface of Sr$_2$RuO$_4$ is known to tiny details. It has two quasi-one-dimensional (q1D) sheets, derived from $d_{xz, yz}$ and $d_{xy}$ orbitals, respectively. Sr$_2$RuO$_4$ exhibits an almost temperature independent normal-state susceptibility, which is enhanced by a factor around $q = 0$ compared to the DFT value. The enhancement factor of the IC fluctuations is even larger, since the bare susceptibility is larger. Also the electronic specific heat coefficient of about 38 mJ/mol-K$^2$ is enhanced by a factor of 3, yielding a Wilson ratio of 2. Similarly, quantum oscillations show strong and band-dependent mass renormalizations, which can be explained by quasi-ferromagnetic (qFM) fluctuations, in the spirit of He$_3$, but also in terms of local Hund’s rule fluctuations.

Inelastic neutron scattering (INS) experiments detect strong IC spin fluctuations at $q_{IC} = (0.3, 0.3, 0)$ arising from nesting in the q1D bands. Upon minor substitution with Ti or Ca this instability condenses into a static spin-density wave with the same Q. INS also assesses the anisotropy of magnetic excitations, which is known to favor triplet pairing, and find it to be non-negligible, but still small. Finally, recent high-resolution INS reveals that the nesting fluctuations do not change between the normal and superconducting states even for energies well below the superconducting gap. The NMR relaxation rate, $1/T_1T$, probes the spin susceptibility $\chi''(q, \omega)/\omega$ integrated over the entire Brillouin zone, and exhibits the same temperature dependence as the INS nesting signal, indicating that it is dominated by the latter. However, $1/T_1T$ also shows a weaker, temperature-independent offset, pointing to another contribution tentatively attributed to the FM response. This tendency towards ferromagnetism can be enhanced by Co or Ca substitution.

To this end, we have used polarized INS to search for the missing FM fluctuations in Sr$_2$RuO$_4$. The magnetic response consists of two components: a broad maximum around $q = 0$, which we will call qFM, and an IC, and much stronger, AFM component. We entered this full magnetic susceptibility into the BCS equations describing spin-fluctuation-induced superconducting pairing.

Because neutron polarization analysis suffers from a reduced intensity, we used a large sample of ten aligned crystals grown at Kyoto University with a total volume of 2.2 cm$^3$ and a mosaic spread of 1.9(2) degrees. Experiments were performed on the spectrometer IN20 at the Institut Laue Langevin. In general, neutron scat-
tering only senses magnetic components that are polarized perpendicular to the scattering vector \( \mathbf{Q} \). The polarization analysis distinguishes spin-flip (SF, with \( i = x, y, \) and \( z \) the direction of neutron polarization) and non-spin-flip (nSF) processes and adds further selection rules. Phonon scattering and nuclear Bragg peaks only contribute to the nSF channels, but magnetic scattering contributes to the SF channel when the magnetic component is perpendicular to the direction of neutron polarization, and to the nSF channel otherwise. We use the conventional coordinate system with \( x \) parallel to \( \mathbf{Q} \), \( z \) perpendicular to the scattering plane, and \( y = z \times x \).

Even with our large sample it was impossible to quantitatively analyze the qFM response by unpolarized INS, because it is too little structured in \( q \) space impeding a background (BG) determination, see supplemental material. In contrast, the polarization analysis permits a direct BG subtraction at each point in \( \mathbf{Q} \) and energy. For instance, \( 2\text{I}_{\text{SF}}(\mathbf{Q})-\text{I}_{\text{nSF}}(\mathbf{Q}) \) yields a BG-free total magnetic signal (up to a correction for the finite flipping ratio). Fig. 1(b-c) shows a representative scan through both the IC and the FM \( \mathbf{Q} \) positions. The full polarization analysis is shown for the SF (b) and the nSF (c) channels. The SF signals have been counted with better statistics, because the SF count rates always contain the magnetic signal and have a lower BG. Only the nSF\(_y\) and nSF\(_z\) channels contain a single magnetic component superposed with the larger nSF scattering, which contains all the phonon contributions. The appearance of the nesting signal in various channels is well confirmed; Fig. 1(b) clearly shows the anisotropy of the IC nesting signal at \((-0.3,0.7,0)\) discussed in Ref. 30.

The sharp enhancement at \((0,1,0)\) is present only in the nSF channel, which proves its non-magnetic character (the longitudinal zone-boundary phonon) 48 49. The finite flipping ratio was determined on several phonon modes, which integrates the signal of all individual crystals, yielding values between 8 and 10. The final analysis only used the SF data, corrected by the average flipping ratio, because of their higher signal to BG ratio 50.

Polarized INS results displaying the sum of the two magnetic components (in-plane plus out-of-plane) are shown in Fig. 1 for \( T = 1.6 \) K and in the supplemental material for \( T = 150 \) K. In order to compare scans taken at different but equivalent scattering vectors, a correction for the magnetic form factor has been applied. The observation of magnetic fluctuations in so many different scans unambiguously documents the existence of sizeable qFM fluctuations. The analysis furthermore yields the absolute scale of the magnetic response throughout the entire Brillouin zone, which allows us to construct a model for the full susceptibility \( \chi''(\mathbf{q}, E) \). The calibration into absolute susceptibility units has been performed by the comparison with the scattering intensity arising from an acoustic phonon, similar to the procedure described in Ref. 51. This calibration can be performed with high precision in the case of Sr\(_2\)RuO\(_4\), because the phonon dispersion is well known and a lattice dynamical model exists that was used to calculate the phonon signal strength at finite propagation vectors 48 49, while in most cases the \( q \rightarrow 0 \) limit is used as an approximation.

The quantitative model fitted to the data consists of two parts: the IC peaks centered at \( Q_{\text{IC}} \) and the broad and weakly \( q \)-dependent qFM part at the zone center. We write \( \chi''(\mathbf{q}, E) = \chi''_{\text{IC}}(\mathbf{q}, E) + \chi''_{\text{FM}}(\mathbf{q}, E) \), where

\[
\chi''_{\text{IC}}(\mathbf{q}, E) = \chi''_{\text{IC}} \frac{\Gamma_{\text{IC}} \cdot E}{E^2 + \Gamma_{\text{IC}}^2 [1 + \frac{\xi_{\text{IC}}^2}{\xi_{\text{IC}}^2} (\frac{\Delta q}{\xi_{\text{IC}}} )^2 ]^2}
\]  

(1)
is the single-relaxor formula with both $(\Gamma_q)^{-1}$ and $\chi'(q, 0)$ decaying with the same correlation length $\xi_{\text{IC}}$. Here $\Delta q = |q - q_{\text{IC}}|$, and is measured in the reciprocal lattice units, (r.l.u.), equal to $2\pi/\alpha$.

Equation (1) describes a typical magnetic response near an AFM instability [52]. The qFM term was described by a broad Gaussian, and its energy dependence in the single-relaxor form with the constant parameter $\Gamma_{\text{FM}}$:

$$\chi''_{\text{FM}}(q, E) = \chi'_{\text{FM}} \cdot \Gamma_{\text{FM}} \cdot E \cdot \frac{E^2}{E^2 + \Gamma_{\text{FM}}^2} \cdot \exp \left(-\frac{q^2}{W^2} \cdot 4 \ln(2)\right)$$  \hspace{1cm} (2)

and $q$ is the distance to the nearest 2D Bragg point. The parameters resulting from a global fit to the whole data set are given in Table I [53]. The model susceptibility was convoluted with the spectrometer resolution using the relsim program package [54] and scaled through phonon scattering [49] yielding the lines in Fig. 1 (d-h).

The corresponding real part of the susceptibility at zero energy $\chi'(q, E = 0)$, the amplitudes of the spectra at fixed $q$, is displayed in Fig. 2. The qFM signal shows no significant anisotropy and corresponds to the macroscopic susceptibility, which also exhibits only weak anisotropy [2, 3]. For the IC peak, the model describes the average of the in-plane and out-of-plane susceptibilities [30], with $\chi'_{\text{IC}}$ ($\chi'_{\text{ab}}$) slightly larger (smaller) than this value. The model was obtained by refining the only 6 parameters with the total set of 120 independent data points at 1.6 K and 76 at 150 K. Thus obtained $\chi'_{\text{IC}}$ and $\Gamma_{\text{IC}}$ are somewhat higher than those extracted from unpolarized INS [11, 20]. The correlation length $\xi_{\text{IC}}$ is less accurate but the qualitative decrease at higher temperature is unambiguous. In principle, one should consider the in-plane and out-of-plane components of the IC peak separately and then take their superposition, but the limited statistics does not allow for that. In contrast to the IC signal, the qFM one is basically temperature-independent, in agreement with the macroscopic measurement [22]. Thus, the qFM response becomes more visible at high temperatures. Note that, due to the simplicity of the model [53], the macroscopic susceptibility of $\sim 28 \mu_B^2/\text{eV} \cdot \text{(f.u.)}$ is smaller than in the model, $\sim 41 \mu_B^2/\text{eV} \cdot \text{(f.u.)}$.

The model $\chi''(q, E)$ can also be successfully verified against $1/T_1 T$ in NMR [41, 42, 55–57] and with specific heat data [22, 58, 59], see supplemental material [47]. The impact of the qFM fluctuations must not be underestimated because of the larger phase space, they yield about 85% of the specific-heat enhancement.

The qFM response does not correspond to the paramagnon scattering expected close to a FM transition [52]; instead it can be viewed as an AFM instability with a small but finite propagation vector near the Brillouin-zone center and a width that largely exceeds the length of the propagation vector. The superposition of several low-$q$ contributions can result in the observed broad feature centered at $q = (0, 0)$.

The raw data and the parameters used for the model are presented in Table I.

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| $T$ (K) | $\chi_{\text{FM}}$ | $W$ (r.l.u.) | $\Gamma_{\text{FM}}$ | $\chi'_{\text{IC}}$ | $\xi_{\text{IC}}$ | $\Gamma_{\text{IC}}$ |
|---------|-----------------|-------------|-----------------|-----------------|-----------------|-----------------|
| 1.6     | $22 \pm 1$      | $0.53 \pm 0.04$ | $15.5 \pm 1.4$ | $213 \pm 10$   | $9.7 \pm 0.5$   | $11.1 \pm 0.8$  |
| 150     | $22 \pm 2$      | $0.47 \pm 0.06$ | $19.0 \pm 3.5$ | $89 \pm 7$     | $6.1 \pm 0.5$   | $17.8 \pm 2.9$  |

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**TABLE I:** (Upper part) Parameters of the $\chi''(q, E)$ model for Sr$_2$RuO$_4$ refined with the polarized INS data for $T = 1.6$ and 150 K. (Lower part) The largest triplet, T, and singlet, S, eigenvalues (in arbitrary units) of the interaction matrices $V_s$ and $V_t$, respectively (Eq. 3), obtained for the isotropic susceptibility, $\chi'(q, 0)$ or for the anisotropic components $\chi'_{zz}$ and $\chi'_{\text{ab}}$; the largest eigenvalues for qFM or IC fluctuations only are shown together with those for the total susceptibility.

FIG. 2: The real part of the static susceptibility $\chi'(q, E = 0)$ as described by eqs. (1,2) along the zone diagonal (a) and for the entire zone (b) at 1.6 K and (c) at 150 K.
Table I. As expected, for the IC fluctuations alone singlet pairing is most stable, and the qFM ones give triplets. With the total susceptibility, the IC fluctuations significantly contribute to the triplet solution as well, but the ground state is still a singlet: the ratio of the largest singlet to the largest triplet eigenvalue is rather high, $R_{s/t} = 4.8$ [53]. Even a five times larger qFM part (clearly incompatible with the experiment) only reduces the ratio to $R_{s/t} = 1.4$. Sharpening the parameter $I(q)$ significantly helps the triplet case, but not enough; tripling $b$ to 1.32 only reduces $R_{s/t}$ to 2.2. Fig. 3 (c) and (d) present the SOPs for the most stable singlet and triplet solutions with the experimental set of parameters. The triplet solution is degenerate with the one rotated by $90^\circ$, so that a chiral state can be constructed. Note that both solutions have strong angular anisotropies (even vertical line nodes), not imposed by the $p$ or $d$ symmetries.

We have also estimated a potential effect of matrix elements in Eq. (3) by retaining only the intra-orbital pairing, as suggested in Ref. [1], or only interactions within the q1D and the q2D bands, but in either case the singlet solution remains much more stable than the triplet one. Applying the total susceptibility to the $\gamma$ band only results in a largely favored singlet state, $R_{s/t} = 3.5$. Magnetic anisotropy favors the triplet state [38-40]. The macroscopic susceptibility tells us that the qFM part has an easy-axis anisotropy of 20%. For the IC part, INS finds a larger anisotropy, of about a factor of two [30]. NMR places an upper limit at a factor of three [12]. Using the latter, we find the numbers shown in Table I. A chiral state with $d||z$ would have triplet-pair spins aligned in the $xy$ plane, and thus be disadvantaged compared to a spin-isotropic singlet state; the ratio rises to $R_{s/t} = 2.9$. A planar state with spins perpendicular to the planes will benefit from the easy-axis anisotropy, but not enough: the $R_{s/t}$ is still 2.9. Within simple spin-fluctuation theory it seems almost impossible to obtain a stable triplet solution even though the qFM signal is much sharper than previously thought.

In conclusion, we have identified the long-sought qFM fluctuations in Sr$_2$RuO$_4$, and, by comparing with the phonon scattering, quantitatively determined their amplitude. Combining this qFM signal and the nesting-driven IC response we have constructed the total magnetic susceptibility $\chi''(q,E)$ at all $q$, which is consistent with the macroscopic susceptibility, with the specific heat coefficient in the normal state and with the $1/T_1 T$ NMR results. Even though the experimentally determined qFM response is stronger and sharper than thought before, the IC component still dominates the spin-fluctuation spectrum in Sr$_2$RuO$_4$, so that the total susceptibility favors a singlet order parameter for a spin-fluctuation mediated pairing. Thus, if the superconductivity in Sr$_2$RuO$_4$ is triplet, interactions beyond simple spin-fluctuation exchange would be required for the pairing mechanism.

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[1] Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J. G. Bednorz and F. Lichtenberg, Nature **372**, 532 (1994).
[2] A. P. Mackenzie and Y. Maeno, Rev. Mod. Phys. **75**, 657 (2003).
[3] Y. Maeno, S. Kittaka, T. Nomura, S. Yonezawa and K. Ishida, J. of Phys. Soc. Jpn. **81**, 011009 (2012).
[4] T. M. Rice and M. Sigrist, J. Phys. Condens. Matter **7**, L643 (1995).
[5] I. Mazin and D. Singh, Phys. Rev. Lett. **79**, 733 (1997).
[6] I. I. Mazin and D. J. Singh, Phys. Rev. **B56**, 2556 (1997).
[7] C. Kallin, Rep. Prog. Phys. **75**, 042501 (2012).
[8] C. Kallin and A. Berlinsky, Rep. Prog. Phys. **79**, 054502 (2016).
[9] A. P. Mackenzie, T. Scaffidi, C. W. Hicks and Y. Maeno, npj Quantum Materials **2**, 40 (2017).
[10] I. Mazin and D. Singh, Phys. Rev. Lett. **82**, 4324 (1999).
[11] Y. Sidis, M. Braden, P. Bourges, B. Hennion, S. Nishizaki, Y. Maeno, and Y. Mori, Phys. Rev. Lett. **83**, 3320 (1999).
[12] T. Nomura and K. Yamada, J. Phys. Soc. Jpn. **69**, 3678 (2000).
[13] T. Nomura and K. Yamada, J. Phys. Soc. Jpn. **71**, 404 (2002).
[14] S. Raghu, A. Kapitulnik and S. A. Kivelson, Phys. Rev. Lett. **105**, 164601 (2010).
[15] T. Takimoto, Phys. Rev. B **62**, 14641(R) (2000).
[16] M. Tsuchiizu, Y. Yamakawa, S. Onari, Y. Ohno, and H. Kontani, Phys. Rev. B **91**, 155103 (2015).
[17] J.W. Huo, T.M. Rice and F.-C. Zhang, Phys. Rev. Lett. **110**, 167003 (2013).
[18] T. Scaffidi, and S. H. Simon, Phys. Rev. Lett. **115**, 087003 (2015).
[19] C. Bergemann, A. Mackenzie, S. Julian, D. Forsythe, and E. Ohmichi, Adv. in Phys. **52**, 639 (2003).
[20] C. N. Veenstra, Z.-H. Zhu, M. Raichle, B. M. Ludbrook, A. Nicolaou, B. Slomski, G. Landolt, S. Matsuda, S. Kittaka, Y. Maeno, J. H. Dil, I. S. Ellimov, M. W. Haverkort and A. Damascelli, Phys. Rev. Lett. **112**, 127002 (2014).
[21] M. Kim, J. Mravlje, M. Ferrero, O. Parcollet, and A. Georges, Phys. Rev. Lett. **120**, 126401 (2018).
[22] Y. Maeno, K. Yoshida, H. Hashimoto, S. Nishizaki, S.-I. Ikeeda, M. Nohara, T. Fujita, A. P. Mackenzie, N. E. Hussey, J. G. Bednorz, et al., J. Phys. Soc. Japan **66**, 1405 (1997).
[23] T. Oguchi, Phys. Rev. B **51**, 1385 (1995).
[24] D. Singh, Phys. Rev. B **52**, 1358 (1995).
[25] I. Hase and Y. Nishihara, J. Phys. Soc. Japan **65**, 3957 (1996).
[26] M. Braden, Y. Sidis, P. Bourges, P. Pfeuty, J. Kulda, Z. Mao, and Y. Maeno, Phys. Rev. B **66**, 064522 (2002).
[27] F. Servant, B. Fak, S. Raymond, J. P. Brison, P. Lejay, and J. Flouquet, Phys. Rev. B **65**, 184511 (2002).
[28] A. Liebsch, A.I. Lichtenstein, Phys Rev Lett. **84**, 1591 (2000).
contains an essential part that depends only on the scattering angle. By analyzing many $Q$ scans, a BG function depending on the scattering angle could be fitted. The variation of the BG also reflects the variation of the nSF count rates indicating that this part of the BG arises from the finite flipping efficiency. Although in the analysis of the magnetic signals, the size of the magnetic scattering has almost exclusively been obtained from the PA, these considerations show that the BG is well understood and properly mastered.

[51] N. Qureshi, P. Steffens, D. Lamago, Y. Sidis, O. Sobolev, R. A. Ewings, L. Harnagea, S. Wurmehl, B. Böchner, and M. Braden, Phys. Rev. B 90, 144503 (2014).

[52] T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism* (Springer-Verlag Berlin Heidelberg, 1985).

[53] The overestimation of $\chi'(0,0)$ seems to arise from the IC Lorentzian tails. The data can be equally well described assuming a Gaussian decay also for the IC amplitude, with the parameters $\chi_{IC}' = 214 \mu_B/eV^2$, $W=0.12$ r.l.u. and a constant $\Gamma_{IC}=11$ meV. With this description there is no contribution of IC parts to $q=0$, but the results for the BCS eigenvalue calculation are nearly unchanged. In particular the singlet is still favoured with an eigenvalue (in units of Table 1) of 78 compared to 20 for the triplet solution: $R_{s/t}=3.9$.

[54] A. Zheludev, reslib 3.4c, Oak Ridge National Laboratory, Oak Ridge, TN, 2006.

[55] T. Imai, A. Hunt, K. Thurber, and F. Chou, Phys. Rev. Lett. 81, 3006 (1998).

[56] C. Berthier, M. Julien, M. Horvati, and Y. Berthier, J. Phys. I France 6, 2205 (1996).

[57] H. Mukuda, K. Ishida, Y. Kitaoka, K. Asayama, Z. Mao, Y. Mori, and Y. Maeno, J. Phys. Soc. Japan 67, 3945 (1998).

[58] D. M. Edwards and G. G. Lonzarich, Philosophical Magazine 65, 1185 (1992).

[59] M. Hatatani and T. Moriya, J. Phys. Soc. Japan 64, 3434 (1995).

[60] O. Friedt, P. Steffens, M. Braden, Y. Sidis, S. Nakatsuji, and Y. Maeno, Phys. Rev. Lett. 93, 147404 (2004).

[61] P. Steffens, Y. Sidis, P. Link, K. Schmalzli, S. Nakatsuji, Y. Maeno, and M. Braden, Phys. Rev. Lett. 99, 217402 (2007).

[62] P. Steffens, O. Friedt, Y. Sidis, P. Link, J. Kulda, K. Schmalzli, S. Nakatsuji, and M. Braden, Phys. Rev. B 83, 054429 (2011).

[63] I. Eremin, D. Manske, S. G. Ovchinnikov, and J. F. Annett, Ann. Phys. 13, 149 (2004).

[64] S. Okamoto and A. J. Millis, Phys. Rev. B 70, 195120 (2004).

[65] I. Mazin and D. Singh, J. Phys. Chem. Solids 59, 2185 (1998).