Topological antiferromagnetic spintronics: Part of a collection of reviews on antiferromagnetic spintronics

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The recent demonstrations of electrical manipulation and detection of antiferromagnetic spins have opened up a chapter in the spintronics story. In this article, we review the emerging research field that is exploring synergies between antiferromagnetic spintronics and topological structures in real and momentum space. Active topics include proposals to realize Majorana fermions in an antiferromagnetic topological superconductors, to control topological protection of Dirac points by manipulating antiferromagnetic order parameters, and to exploit the anomalous and topological Hall effects of zero net moment antiferromagnets. We explain the basic physics concepts behind these proposals, and discuss potential applications of topological antiferromagnetic spintronics.
Topologically protected states of matter are unusually robust because they cannot be destroyed by small changes in system parameters. This feature of topological states has suggested an appealing strategy to achieve useful quantum computation. In spintronics, topological states provide for strong spin-momentum locking, high charge-current to spin-current conversion efficiency, large electron mobilities and long spin diffusion lengths, strong magnetoresistance, and efficient spin-filtering. Materials exhibiting topologically protected Dirac or Weyl quasiparticles in their momentum-space bands, and those exhibiting topologically non-trivial real-space spin textures, have both inspired new energy-efficient spintronics concepts.

In a topological insulator (TI), time reversal symmetry enforces Dirac quasiparticle surface states with spin-momentum locking (see Fig. 1(a)) and protection against backscattering. The much higher efficiency of magnetization switching by a current induced spin-orbit torque (SOT) in a TI/magnetically doped TI (MTI) heterostructure, than in a heavy-metal/ferromagnet (FM) bilayer is thought to be associated with spin-momentum locking, and is a paradigmatic example of the potential seen for topological materials in spintronics. A full microscopic understanding of the underlying current-spin conversion mechanism is however still absent. Progress in understanding and exploiting topological insulators in spintronics has so far been limited by unintentional bulk doping in TIs, and by the decreased stability of TI surface states at elevated temperatures. The practical utility of the topologically enhanced SOT has also been limited by the cryogenic temperatures at which known MTIs order.

A substantial rise in the critical temperature of a magnetic topological insulator (by a factor of 3 to 90 K) due to proximity coupling to adjacent antiferromagnet (AF) has recently been demonstrated in a heterostructure consisting of the metallic antiferromagnet CrSb sandwiched between two MTIs. Increased SOT efficiency at heterojunctions between TIs and ferrimagnetic CoTb alloys containing antiferromagnetically coupled Co and Tb sublattices has also been reported. The later effect persists to room temperature, but with decreased efficiency enhancement at higher temperatures. Research on using antiferromagnetism to achieve a role for topological materials in spintronics is however still at an early stage and many ideas have so far only been addressed theoretically. The practical advantages of TIs over heavy-metal systems for spin-orbit torques, for example, are not yet established. The forms of magnetism so far incorporated in magnetic TIs remain fragile because they are of interface or dilute-moment character.

Other new ideas, beyond simply making topological insulators magnetic, are emerging at a rapid pace. In this article we review topological antiferromagnetic spintronics, the emerging field that is exploring the interplay between transport, topological properties in either momentum space or real space, and antiferromagnetic order.

I. TOPOLOGICAL INSULATORS IN ANTIFERROMAGNETS AND MAJORANA FERMIONS

The roots of topological antiferromagnetic spintronics can be traced to studies of layered AFs of the SrMnBi2 type, which were thought to feature quasi 2D massive Dirac quasiparticles around the Fermi level. These were associated with the observation of enhanced mobilities, similarly to those in graphene. Masuda et al. have demonstrated manipulation of the Dirac quasiparticle current and the quantum Hall effect in a EuMnBi2 AF by an applied strong magnetic field, with the effect of the field mediated by Eu sublattices.

As pointed out by Mong et al., TI phases are possible in antiferromagnets even though time-reversal symmetry is broken and are protected instead by Taji where Taji is a half magnetic unit cell translation operation, as we illustrate in Fig. 1(a). The proposed low-temperature AF candidate, GdPtBi, has not yet been confirmed as a TI by angle resolved photoemission spectroscopy (ARPES), presumably due imperfect crystal momentum resolution of the measurement. A path of research related to topological superconductivity has demonstrated signatures of the coexistence of a 2D TI, i.e. the quantum spin Hall effect (QSHE), and a superconducting state in hole-doped and electron-doped antiferromagnetic monolayers of FeSe. FeSe is the metallic building block of the iron-based high-Tc superconductors. Surprisingly, the combined effect of substrate strain, spin-orbit coupling, and electronic correlations was shown to induce band inversion and QSHE edge states. These can in turn lead to the realization of Majorana zero modes, superconducting quasiparticle states with real wavefunctions that prevents decoherence, providing one route for the realization of quantum computing with topological qubits. The antiferromagnetic TI in combination with superconductivity can allow for an alternative realization of the Fu-Kane Majorana fermion proposal, possibly at higher temperatures. Separately, QSHE states in an AF have also been predicted in honeycomb lattice systems.

II. TOPOLOGICAL SEMIMETAL ANTIFERROMAGNETS

Topological semimetal states arise when conduction and valence bands touch at discrete points, lines, or planes in a bulk Brillouin zone at energies close to the Fermi level. The low energy physics of topological semimetals is governed by effective Dirac or Weyl equations. 3D Dirac and Weyl quasiparticles in non-magnetic bulk systems have attracted attention because of reports of suppressed backscattering, measurements of exotic topological surface states,
FIG. 1. Overview of the materials palate of topological antiferromagnetic spintronics: All panels show energy vs. crystal momentum band diagrams and magnetic structures. (a) Schematics of the spin-momentum locked surface-state Dirac quasiparticles in a TI. In the inset, we show the magnetic Gd sublattice of GdPtBi, an AF TI candidate. The magenta arrow marks the $T\frac{1}{2}$ symmetry that protects a TI state in an AF. (b) Schematics of the bulk Dirac quasiparticles of an AF Dirac semimetal (DSM) that must be located along a high symmetry line in the BZ, and the magnetic structure of the AF DSM candidate CuMnAs. The $\mathcal{PT}$ symmetry connects the magnetic Mn sublattices in pairs, and together with additional crystalline symmetry protects the DSM state as we explain in Fig. 2(b). (c) Massive Dirac quasiparticles and QAHE edge states (red dispersion). A quantized Hall conductivity can be produced by the spin-chirality, $\chi_{ijk} = \hat{S}_i \cdot (\hat{S}_j \times \hat{S}_k)$, of a non-coplanar spin texture. The quantized topological Hall effect (QTHE) in a non-coplanar insulating AF gives rise to quantized edge states as in the QAHE, as we explain in the text. For the sake of simplicity, the non-coplanar spins of the antiferromagnetic QTHE candidate K$_{0.5}$RhO$_2$ are translated to a common origin. (d) Weyl points act as sources and drains of Berry curvature (blue and red arrows). In a centrosymmetric WSM, the topological charges are distributed antisymmetrically around the $\Gamma$ point in the BZ. The noncollinear magnetic sublattice of antiferromagnetic Mn$_3$Ge forms kagome planes and has inversion symmetry $\mathcal{P}$. The pseudorelativistic linear band crossings in the left panels are found in the edge/surface states, whereas those in the right panels are realized in the bulk. The TI and DSM states (the two upper panels) cannot be realized in systems with ferromagnetic order, but are possible in antiferromagnets. In contrast, the QAHE/QTHE and WSM states (the two lower panels) are expected in both FMs and AFs, and can be formally obtained by breaking symmetries of the phases depicted in the two upper panels.

and interest in unique topological responses such as dissipationless axial currents. These properties are thought to be responsible for experimental observations of chiral magnetotransport and strong magnetoresistance, although the topological origin of these phenomena is not yet firmly established. For instance, the strong magnetoresistance in WTe$_2$ semimetals was originally explained on the basis on the carrier compensation in the tiny electron-hole pockets at the Fermi level, and only later linked to the presence of Weyl fermions.

A. Topological metal-insulator transitions in 3D Dirac semimetal antiferromagnets

In a system with time reversal $\mathcal{T}$ and spatial inversion $\mathcal{P}$ symmetries, the electronic bands are doubly degenerate resulting in a low energy Dirac Hamiltonian, $\mathcal{H}_D(k)$. In its simplest form:

$$\mathcal{H}_D(k) = \begin{pmatrix} \hbar v_F k \cdot \sigma & m \\ m & -\hbar v_F k \cdot \sigma \end{pmatrix}$$

(1)

where $\sigma$ is the vector of Pauli matrices, $v_F$ is the Fermi velocity, $k = q - q_0$ is the crystal momentum measured from the Dirac point at $q_0$, and $m$ is the mass (in units of energy). The corresponding energy dispersion reads $E_k = \pm \hbar v_F \sqrt{k_x^2 + k_y^2 + k_z^2 + \left(\frac{m}{\hbar v_F}\right)^2}$. The mass can vanish only when turned-off by a crystalline symmetry, and in
this case $\mathcal{H}_{\text{3D}}(k)$ describes the four-fold degenerate band touching of a 3D Dirac semimetal (DSM) illustrated schematically in Fig. 1(b). In a 3D DSM, the topological invariants and nontrivial surface states can be linked to the crystalline symmetry protecting the degeneracy.

The 3D DSM state is not possible in FMs because $T$-symmetry breaking prevents double band degeneracy. On the other hand, a topological crystalline 3D Dirac semimetal was predicted in an AF, namely in the orthorhombic phase of CuMnAs. Here $\mathcal{P}$ and $\mathcal{T}$ symmetries are absent separately, but the combined effective $\mathcal{PT}$ symmetry ensures double band degeneracy over the whole Brillouin zone. The DSM is in this case protected by $\mathcal{PT}$ symmetry together with an additional crystalline non-symmorphic symmetry, as we explain in Fig. 2. The orthorhombic CuMnAs AF provides an attractive hydrogen atom for magnetic DSMs induced by the band inversion, since only a single pair of Dirac points appears near the Fermi level of the ab initio band structure. Electron filling enforced semimetals with a single Dirac cone are also a possibility as indicated theoretically in two dimensional model AFs.

![Figure 2](image)

**FIG. 2.** (BOX:) Topological metal-insulator transition in the antiferromagnetic DSM CuMnAs. (a) The crystal and magnetic structure of the orthorhombic CuMnAs breaks time reversal and spatial inversion symmetry but preserves the combination $\mathcal{PT}$ which connects the $A-B$ and $C-D$ Mn sublattices (Cu and As atoms are omitted for brevity). In the presence of an electrical current, owing to the $\mathcal{PT}$ symmetry, a non-equilibrium staggered spin-polarization, $\delta s$, is generated that can reorient the antiferromagnetic moments. (b) For a Néel vector along the [001] axis, the crystal has a screw rotation symmetry $S_z$ which transforms the atom $A$ into the atom $C$ by a $\pi$-rotation along the $z$-axis followed by a $\left(\frac{1}{2}, 0, \frac{1}{2}\right)$-unit cell translation. $S_z$ prevents hybridization of doubly degenerate bands that have opposite eigenvalues of $S_z$ distinguished by “+” and “−” labels, and protects the DSM phase, as seen in the ab initio band structure. (c) For the [100] orientation of the Néel vector, the $S_z$ symmetry is broken and the Dirac fermions acquire a mass giving rise to a semiconducting phase. This Néel vector reorientation controlled transition is referred to as the topological metal-insulator transition and can lead to extremely large anisotropic magnetoresistance.

Novel effects have been predicted in topological DSM AFs that are based on the possibility of controlling topological states by controlling only Néel vector orientation, not the presence or absence of antiferromagnetic order, and this can be accomplished using current-induced spin-orbit torques. The latter effect, discussed in detail by Železny et al. in this focused issue, has been experimentally demonstrated in CuMnAs. The coexistence of Dirac fermions and spin-orbit torques in CuMnAs implies a new phase transition mechanism, referred to as the topological metal-insulator transition (MIT). The origin of the effect is in Fermi surface topology that is sensitive to the changes in the magnetic symmetry upon reorienting the Néel vector, as explained in Fig. 2. The transport counterpart of the topological MIT is topological anisotropic magnetoresistance (AMR), which in principle can reach extremely large values. The topological AMR can be understood as a limiting case of the crystalline AMR. The effect is different in origin and presumably more favorable for spintronics than the MIT observed in the pyrochlore iridate family which is driven by combined correlation and external field effect, or the extreme magnetoresistance observed in the AF topological metal candidate NdSb.

An antiferromagnetic Dirac nodal line semimetal has also been proposed. Here the band touching lines are protected by off-centered mirror plane symmetries and lead to distinct physical properties such as drum head surface states. Since the nodal-lines were observed several eV deep in the ab initio Fermi sea of tetragonal CuMnAs, the search is still on for more favorable candidate AF materials featuring nodal lines closer to the Fermi level.

### B. Weyl fermions in antiferromagnets

When $\mathcal{P}$ or $\mathcal{T}$ symmetry, or both, is broken and the double band degeneracy is lifted, the touching points of two non-degenerate bands can form a 3D Weyl semimetal (WSM), see Fig. 1(d). Fermi states in a WSM are described by...
the Weyl Hamiltonian \[ \mathcal{H}(\mathbf{k}) = \pm \hbar v_F \mathbf{k} \cdot \mathbf{\sigma}. \] (2)

Weyl points act as monopoles sources of Berry curvature flux and generate a topological charge defined by:

\[ C = \frac{1}{2\pi} \int_{\delta S} d^2k \mathbf{n} \cdot (\partial_k \mathbf{u}_k \times |\partial_k \mathbf{u}_k|) = \pm 1. \] (3)

Here \( \delta S \) is a small sphere surrounding the Weyl point with the surface normal vector \( \mathbf{n} \) and \( \Omega_k = (\partial_k \mathbf{u}_k \times |\partial_k \mathbf{u}_k|) \) is the momentum space Berry curvature, which can be viewed as a fictitious magnetic field. In the vicinity of the Weyl point the Berry curvature takes the monopole form, \( \Omega_k = \pm \mathbf{k}/(2k^3) \). Weyl points always come in pairs with opposite topological charges and do not rely on any specific symmetry protection. The only way to remove them is to annihilate two Weyl points with opposite topological charges. This is in contrast to the DSM case in which gaps can open also due to the hybridization among the degenerate band partners when symmetries are weakly broken, as we explained in the previous section. The 3D nature of the Weyl point is crucial here since the corresponding Weyl equation uses all three Pauli matrices. Consequently, any small perturbation, that is in general expressed as a linear combination of Pauli matrices, can be also be realized in the paramagnetic and AF phase of GdPtB by applying a magnetic field and, in contrast to DSMs, WSMs can in principle exist also in FMs.

The initial prediction of the WSM by Wan et al. was for pyrochlore AFs. Subsequently, a Weyl metal state was predicted in the chiral non-collinear centrosymmetric AF Mn₃Ge (see Fig.1(c)), a metallic system in which trivial Fermi surface pockets are also present. Figs.3(a,b) illustrates the density of states weighted by the surface contribution which exhibits the typical Fermi arc features (see also Fig.1(d)), open surface state constant energy surfaces that connect the bulk projections of Weyl points, as found in \textit{ab initio} calculations. We note that Weyl semimetal states can be also be realized in the paramagnetic and AF phase of GdPtB by applying a magnetic field and, in contrast to DSMs, WSMs can in principle exist also in FMs.

The presence of Weyl points in the pyrochlore AF Eu₂Ir₂O₇ was inferred from optical experiments. However, a direct spectroscopic identification (e.g., by ARPES) of these Weyl AFs has yet to be made. The YbMnB₃ AF was originally proposed to be WSM based on the ARPES identification of the bandcrossings consistent with the \textit{ab initio} bandstructure calculated for the canted AF moment, later supported by an optical study. However, recent magnetotransport and optical conductivity measurements in the YbMnB₃ AF were shown to be rather consistent with the Dirac quasiparticles. The Mn₃Ge AF was shown to host a large anomalous Hall effect (AHE) whose origin is discussed in the next section.

### III. HALL EFFECTS IN NONCOLLINEAR TOPOLOGICAL ANTIFERROMAGNETS

Until recently the AHE was viewed as a combined consequence of the time reversal symmetry breaking in a ferromagnet and spin-orbit coupling. In the case of collinear AFs, either \( T_{\downarrow} \mathcal{T} \) symmetry or \( \mathcal{P} \mathcal{T} \) symmetry forces the Hall conductivity to vanish. Recent \textit{ab initio} calculations inspired by earlier theoretical work have however shown that time-reversal symmetry breaking by AF order can yield a finite Hall response in some non-collinear AFs, even those with zero net magnetization and even in the absence of spin-orbit coupling. The time reversal symmetry breaking is manifested by a nonzero Berry curvature, as we show in Fig.3(d)). The intrinsic contribution to the Hall conductivity depends only on the band structure of the perfect crystal and can be calculated from linear response theory.

\[ \sigma_{xx} = \frac{e^2}{h} \sum_n \int_{\text{BZ}} \frac{d^3k}{(2\pi)^3} f(k) \Omega_y^n(k), \] (4)

where \( \Omega_y^n(k) \) is the \( y \)-component of the fictitious magnetic field, or the Berry curvature (cf. Eq.3), and \( n \) is the band index.

#### A. The Anomalous Hall effect in antiferromagnets

In the simplest toy model of a WSM (with a two Weyl points in the vicinity of the Fermi level), the AHE conductivity can be calculated by integrating the quantized two-dimensional Hall conductivities of momentum-planes that are
perpendicular to the line connecting Weyl points to obtain,
\[
\sigma_{xy} = \frac{e^2}{h} \frac{\Delta k_W}{2\pi},
\]
where \(\Delta k_W\) is the distance between Weyl points.\(^{61}\) The AHE was recently observed in the hexagonal noncollinear AFs Mn₃Sn and Mn₃Ge\(^{54,62,63}\), which have Weyl points close to the Fermi level. However, \textit{ab initio} calculations of the intrinsic AHE in Mn₃Ge, which predict a magnitude consistent with experiment (see Fig.3(e)), reveal that the dominant contribution to the AHE comes instead from localized avoided crossings in the band structure\(^{53}\). This property is illustrated in Fig.3(c) by calculating crystal momentum projected contributions of the Berry curvature/conductivity
\[
\Omega_y(k_x, k_y) \cdot (\partial_{k_x} \hat{d} \times \partial_{k_y} \hat{d}),
\]
where \(\hat{d}(k) = d(k)/|d(k)|\). The integral is quantized and relates directly to the Chern number (c.f. Eq.(3)).

While the AHE arises from the Berry curvature in the momentum space, other important spintronic phenomena can be associated with Berry curvatures in different parameter spaces. For instance, the spin-orbit torque tensor is defined by the linear response relation
\[
\mathbf{T} = \tau \mathbf{E},
\]
where \(\mathbf{T} = \frac{dn}{dt}\) is the SOT exerted on the magnetization \(\mathbf{m}\) in a magnet subject to an applied electric field \(\mathbf{E}\). The intrinsic part of the SOT can be rewritten in terms of a mixed Berry curvature, \(\Omega_{ij}^\text{int} = \hat{e}_i \cdot 2\mathbf{m} \sum_n \langle \partial_{\mathbf{m}^n} \hat{d}_i | \partial_{k_j} \hat{d}_n \rangle\), where \(\hat{e}_i\) denotes the \(i\)th Cartesian unit vector and \(\hat{d}\) is a
unit vector in the direction of magnetization. A large SOT in a topologically nontrivial insulating FM has been associated with the existence of the monopoles of the mixed Berry curvature. These are termed mixed Weyl points as they correspond formally to a Weyl Hamiltonian \( H(k, \mathbf{m}) = \hbar v_F (k_x \sigma_x + k_y \sigma_y) + v_\theta \theta \sigma_z \) in the mixed momentum-magnetization space. (Here \( \theta \) is the azimuthal angle of the magnetization.) The recent discovery of the SOT and the prediction of the DSM in antiferromagnetic CuMnAs motivates a search for analogous dissipationless (pronounced) SOTs in insulating AFs. Finally, we note that spintronics based on insulating AFs may utilize the concept of Weyl magnons (a boson analog of Weyl fermions), that has been proposed in pyrochlore AFs.

B. Topological Hall effect in antiferromagnets

Real-space order parameter textures can be induced in AFs, and their presence can be detected by the so-called topological Hall effect. In this phenomenon, the role of the spin-orbit coupling is substituted by the chirality of the spin texture (see inset in Fig.(1c)), \( \chi_{ijk} = \hat{S}_i \cdot \left( \hat{S}_j \times \hat{S}_k \right) \). (Note that the spin-chirality vanishes in coplanar AFs like Mn\(_3\)Ge.) The effect of the corresponding fictitious magnetic field, \( \mathbf{m} \cdot (\partial_j \mathbf{m} \times \partial_i \mathbf{m}) \), on the Bloch electrons generates a Hall response. The topological Hall effect can be experimentally distinguished from the AHE by, e.g., analyzing the disorder dependence. However, the distinction at surfaces might be difficult as was pointed out in studies of monolayer Fe deposited on an Ir(001) surface.

The Hall effect associated with the spin-chirality was initially reported in antiferromagnetic pyrochlore iridates, and later in MnSi chiral antiferromagnetic alloy. We note that the term "topological" used to label the effect does not imply in this case a correspondence to a topological invariant. This is in contrast to skyrmions discussed in more detail in the following section. A skyrmion spin texture carries an integer topological charge which is accompanied with a topological Hall effect. In this case, the term topological refers to the association of the Hall response to a topological invariant. Another example of such a correspondence is the quantum topological Hall effect which was proposed for the non-coplanar AF K\(_{0.5}\)RhO\(_2\), as we show schematically in Fig.(1d). Unlike skyrmions, here the topological charge occurs in the momentum space, i.e., is obtained from Eq.(1), giving an integer value.

Finally, topological systems were predicted also as promising generators of the spin Hall effect (SHE). The WSM TaAs was predicted to host a large spin Hall angle arising mainly from the nodal line anticrossing features. A large SOT in a topologically nontrivial insulating FM has been associated with the existence of the monopoles of the mixed Berry curvature. These are termed mixed Weyl points as they correspond formally to a Weyl Hamiltonian \( H(k, \mathbf{m}) = \hbar v_F (k_x \sigma_x + k_y \sigma_y) + v_\theta \theta \sigma_z \) in the mixed momentum-magnetization space. (Here \( \theta \) is the azimuthal angle of the magnetization.) The recent discovery of the SOT and the prediction of the DSM in antiferromagnetic CuMnAs motivates a search for analogous dissipationless (pronounced) SOTs in insulating AFs. Finally, we note that spintronics based on insulating AFs may utilize the concept of Weyl magnons (a boson analog of Weyl fermions), that has been proposed in pyrochlore AFs.

IV. ANTIFERROMAGNETIC SKYRMIONS

As noted above, position-dependent magnetization textures can also be topologically non-trivial. Skyrmions are noncollinear magnetization textures in which the spin quantization axis changes continuously over length scales that vary from a few nm to a few \( \mu m \). For two-dimensional systems, the winding number,

\[
Q^{(j)} = \frac{1}{4\pi} \int dx dy \, \mathbf{m}^{(j)} \cdot \left( \partial_x \mathbf{m}^{(j)} \times \partial_y \mathbf{m}^{(j)} \right),
\]

of a magnetization texture measures the number of times the sphere of magnetization directions is covered upon integrating over space and must take on integer values. Here \( \mathbf{m} = \mathbf{m}(x, y, z) \) is the normalised magnetization field in the real space and \( \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m}) \) is the fictitious emergent magnetic field. The antiferromagnetic skyrmion can be visualized as two interpenetrating ferromagnetic skyrmions, where the index \( (j) = (1, 2) \) labels the two antiferromagnetic sublattices, as shown in Fig.(2a). Microscopically, the skyrmionic magnetization modulation is caused by the Dzyaloshinskii-Moriya interaction (DMI) of non-centrosymmetric crystals. Whereas ferromagnetic skyrmions are often generated by interfacial DMIs, antiferromagnetic skyrmions are expected to be more abundant in crystals with bulk DMIs.

By comparing to Eq.(5), and (6) we see that the winding number \( Q \) topologically protects skyrmion textures in real space, just as Weyl points and the QAHE state are protected in momentum space. The observed energy barrier
FIG. 4. Antiferromagnetic skyrmions. (a) An antiferromagnetic skyrmion can be viewed as consisting of two antiferromagnetically coupled ferromagnetic skyrmions. For the sake of clarity, we draw the two opposite magnetic moments in the antiferromagnetic unit cell as coinciding and their moments as perfectly compensated. Note that the structure of the skyrmion is analogous to the momentum-space Berry curvature shown in the inset of Fig. 3(a). (b) A synthetic antiferromagnetic skyrmion in a Fe-Cu-Fe trilayer. (c) Micromagnetic simulation of ferromagnetic (upper panel) and antiferromagnetic (lower panel) skyrmion motion driven by a SOT. The ferromagnetic skyrmion is deflected by the Magnus force, while the antiferromagnetic skyrmion can move in a straight line due to mutual compensation between Magnus forces from the two magnetic sublattices, as schematically shown in panel (a). Panels (b)-(c) are adapted from Refs. 80 and 81.

for skyrmion annihilation in discrete magnetic skyrmions is of the order of ∼0.1 eV. Because this barrier is finite, skyrmion stability in experimental systems relies in part on other physical limitations, (e.g., a combined effect of spin rotation and skyrmion diameter shrinking) rather than from topological protection itself.

Skyrmions in non-centrosymmetric AFs were considered already some time ago in various contexts, including in connection with high-temperature superconductivity. However, spintronics aspects of antiferromagnetic skyrmions, namely their manipulation by an electrical current, were discussed only recently. Micromagnetic simulations show that antiferromagnetic skyrmions move faster than ferromagnetic skyrmions, can be driven with lower current densities and, most importantly, move in straight lines, as illustrated in Fig. 4(c). This important difference arises because the Magnus force which deflects ferromagnetic skyrmions has opposite sign for the two magnetic sublattices of an antiferromagnetic skyrmion, owing to the opposite topological numbers illustrated in Fig. 4(a). AF skyrmions were recently studied in detail also in synthetic AFs (e.g. in a Fe|Cu|Fe trilayer) in which skyrmions in the two ferromagnetic layers are coupled antiferromagnetically, as shown in Fig. 4(b). The topological SHE was suggested as a probe to monitor the AF skyrmions, as well as to generate a spin current.

V. PERSPECTIVES

The past year has seen important progress in coupling topological phases of matter with AF order. The fortunate lattice constant match between the Cr-doped TI (Bi,Sb)₂Te₃, and the high temperature AF CrSb (see Fig. 5(a)) has been exploited to grow epitaxial interfaces between these materials. The resulting heterostructures exhibit strengthened order in the MT, enhancing topological effects in its electronic properties. CrSb/(Bi,Sb)₂Te₃/CrSb trilayers exhibit cusps in the magnetoresistance, that presumably correspond to a topological phase transition of Dirac quasiparticles at the interface. Separately, in studies of Bi₂Se₃/CoTb heterostructures, it was also shown that a SOT from a TI can manipulate the AF-coupled sublattices in an adjacent ferrimagnet. Further progress in which perfectly compensated antiferromagnetic materials are employed can enable the advantages of AF spintronics, discussed throughout this issue, to be realized more fully. AF FeSe monolayers will likely be tested for the realization of Majorana-based topological quantum qubits. Making a p-n junction in a single layer of FeSe by gating can generate two regions - one superconducting and one with a QSHE. Coupling this system to two FM electrodes from both sides would lead to localization of Majorana modes at the interface as illustrated in Fig. 5(b). This two level state can function as a quantum bit which can nonlocally encode information and is robust against decoherence due to the real wavefunction of the Majorana modes.

In this brief review, we have focused on direct synergies between antiferromagnetic and topological properties in crystal momentum and real spaces. Novel magnetic topological phases of matter were predicted only very recently and in many cases remain experimentally elusive, e.g. AF TIs, DSMs, and WSMs, QAHE states, and skyrmions. When realized, these topological antiferromagnet states can lead not only to more stable topological nanospintronics devices, but also to unprecedented functionalities relying on the unique AF symmetries and the possibility of tuning them by
external means by coupling to the AF order. Fast topological memories, in which states are written by the topological SOT in an AF TI, or an AF DSM, and read out via the large magnetoresistance effects associated with band gap tuning, are among anticipated applications. Another possibility is opened by exploiting topological phase transitions, as we have explained for the CuMnAs Dirac AF in the Fig. 2. Here one can foresee the possibility of a topological transistor operating at high frequencies and low current densities (see Fig. 5(c))\textsuperscript{96}. In contrast to the related proposal in crystalline topological insulators\textsuperscript{97}, one can achieve highly mobile bulk Dirac quasiparticle current (present in the "ON" state) controllable by gating or by ultrafast SOT. We note that many of the novel effects we have discussed follow directly from antiferromagnetic symmetries and cannot be realized in FMs, for instance (i) magnetism combined with the QSHE, and superconductivity, and (ii) magnetism combined with Dirac semimetal phase. The conditions for a good Dirac quasiparticle in AF spintronics were discussed recently\textsuperscript{13}.

The sign and magnitude of the AHE in Mn\textsubscript{3}Ge depends on the noncollinear spin texture orientation. This together with the demonstration of the possibility of manipulating the noncollinear spin texture by a spin-torque\textsuperscript{98} can allow for memory devices in noncollinear AFs, with the electrical readout via the AHE, as illustrated in Fig. 3(d). Moreover, optical counterparts of the dc AHE should be present in non-collinear AF,\textsuperscript{99} opening the prospect for optical detection of topological effects and of antiferromagnetic opto-spintronic devices.

The nontrivial topologies of magnetization texture might also find applications. The skyrmion might represent the smallest micromagnetic configuration for storing information before going into truly quantum mechanical single spin qubits.\textsuperscript{10} For instance, in the skyrmionic racetrack memory, as shown in Fig. 5(d), the magnetic information is stored in skyrmions instead of magnetic domains separated by domains walls.\textsuperscript{101} Skyrmions can be driven by the SOT at low current densities and have advantages over domain walls especially in the curved parts of the race track thanks to their stability.

After almost a half century of research we have discovered many manifestations of topology playing a role in materials physics, from spin liquids, to Quantum Hall effects, to the theory of dislocations, and beyond. In this article, we have highlighted a newly emerging field, topological antiferromagnetic spintronics. Beyond providing an interesting new context in which to identify and understand the physical consequences of topological properties in momentum-space bands or real-space textures, topological antiferromagnetic spintronics suggests the tantalizing possibility of converting important fundamental advances to truly valuable new applications of quantum materials.

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