The convolution formula for a decay rate

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Abstract

The convolution formula is derived within the framework of the decay-chain method for decay channels with three and four particles in a final state. To get this formula exactly for unstable particles of any type one must modify the propagators of vector and spinor fields. In this work we suggest proper modifications and get the convolution formula by direct calculations. It was noted that this approach naturally arises in the model of unstable particles with random mass.

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1. Introduction

The unstable particles (UP) manifest themselves as resonances (intermediate states) in the scattering experiments and as decaying (evolving) states in the decay or oscillation experiments. The former case is treated with $S$-matrix or with renormalized propagator approach. In the second case we must take into account the instability or finite-width effects (FWE) in a special way. There are some different methods proposed in literature to describe the UP as quasi-stable state. The first method was formulated by Matthews and Salam in Ref. [1], where an uncertainty principle was taken into consideration. Bohm et al. [2] suggest the time asymmetric quantum theory of UP with relativistic Gamov vector, which describes UP in initial or final states. The model of UP with random (smeared, fuzzy) mass was suggested in Ref. [3]. This model is based on the uncertainty relation for the mass of UP and has a
close analogy to the elaboration [1]. An effective theory of UP is discussed in [4] where the authors have applied the method of correction factorization.

At last decade the so called convolution method (CM) has been used to evaluate FWE [5, 6]. This method consists in a semiphenomenological description of invariant mass distribution by Breit-Wigner-like density function. The convolution formula was derived within the framework of the model [3] as a direct consequence of the model approach. This formula was applied in a phenomenological way to calculate the hadron decay rates [7]. The FWE (or "mass smearing" effects) in hadron decays are very significant due to large hadron widths. The FWE in the near-threshold decays $t \to WZb$, $t \to cWW$, $cZZ$ and $A^0(h^0) \to tbW$ were calculated in Refs. [5, 6] where some analysis of CM applicability is fulfilled.

In this paper we systematically analyze the processes of type $\Phi \to \phi_1\phi \to \phi_1\phi_2\phi_3$, where $\phi$ is an UP with a large width and $\Phi, \phi_1, \phi_2$ are the stable or long-lived particles of any kind. The prescriptions are suggested for propagators and for polarization matrices of unstable vector and spinor fields, which exactly lead to the convolution formula for a decay rate. It was noted that these prescriptions naturally follow from the model [3]. The convolution formula for above mentioned decays was got by direct calculations without any approximations for all type of UP and for all possible type of $\Phi$ and $\phi_\alpha$.

2. Derivation of the convolution formula for a decay rate

The convolution formula (CF) can be obtained on the example of processes $\Phi \to \phi_1\phi \to \sum_{i,k} \phi_1\phi_i\phi_k$, where $\phi$ is an UP with a large width. In the general case:

$$\Phi \to \phi_1\phi \to \phi_1 \sum_n X_n,$$

(1)

where the sum runs over all decay channels of the unstable particle $\phi$. Some particles of the $X_n = (x_1, x_2, ..., x_n)$ can be unstable too. By means of Eq. (1) we represent the connection between the CM and decay-chain method (DCM). When the sum runs over $(\phi_i, \phi_k)$, which are the stable or long-lived particles, we have:

$$\Gamma(\Phi \to \phi_1\phi) = \sum_{i,k} \Gamma(\Phi \to \phi_1\phi_i\phi_k).$$

(2)

The CM gives the expression [5]:

$$\Gamma(\Phi \to \phi_1\phi) = \int_{q_i^2}^{q_f^2} \Gamma(\Phi \to \phi_1\phi(q)) \rho_\phi(q) dq^2.$$

(3)
In Eq. (3) the value \( \Gamma(\Phi \to \phi_1(q)) \) is the width, calculated in a stable particle approximation when \( m_\phi^2 = q^2 \) and \( \rho_\phi(q) \) is some invariant mass distribution function. From Eq. (1) it is clear that \( \rho_\phi(q) \) depends on all decay-chain of \( \phi \), that is, CF can be derived in principle from the DCM.

When the intermediate state \( \phi \) is scalar UP then for any type of initial \( \Phi \) and final \( \phi_k \) states a decay width can be represented in the factored form (see Appendix):

\[
\Gamma(\Phi \to \phi_1 \phi_2 \phi_3) = \int_{q_1^2} q_2^2 \Gamma(\Phi \to \phi_1 \phi(q)) \frac{q \Gamma(\phi(q) \to \phi_2 \phi_3)}{\pi |P_\phi(q)|^2} dq^2. \tag{4}
\]

In Eq. (4) \( q_1 = m_2 + m_3 \), \( q_2 = m_\phi - m_1 \) and \( P_\phi(q) \) is the propagator’s denominator of the scalar field \( \phi \). It is noteworthy that the derivation of the convolution formulae (3) or (4) don’t depends on the choice of \( P_\phi(q) \). This fact is important for a model definition of \( \rho(q) \).

For convenience and completeness we give in Appendix a full list of the expressions \( \Gamma(\Phi \to \phi_1 \phi) \) and \( \Gamma(\Phi \to \phi_1 \phi_2 \phi_3) \) for all possible types of \( \Phi \) and \( \phi_k \). From Eqs. (2) and (4) it follows that \( \rho_\phi(q) \) in Eq. (3) can be expressed in the form:

\[
\rho_\phi(q) = \frac{q}{\pi |P_\phi(q)|^2} \sum_{i,k} \Gamma(\phi(q) \to \phi_i \phi_k) \equiv \sum_{i,k} \rho_{ik} \phi(q). \tag{5}
\]

A generalization of Eq. (5) for the cases when three or more particles are in a final state is straightforward (see Appendix). Thus, we can always represent \( \Gamma(\Phi \to \phi_1 \phi) \), where \( \phi \) is scalar UP with a large width, by the convolution formula (3). If one uses the parametrization \( q \Gamma(q) = Im \Sigma(q) \) and Dyson-resummed propagator

\[
P_\phi(q) = q^2 - m_\phi^2(q) - i Im \Sigma_\phi(q), \quad m_\phi^2(q) = m_{\phi 0}^2 + Re \Sigma_\phi(q), \tag{6}
\]

then the \( \rho_\phi(q) \) can be written in the Lorentzian (Breit-Wigner type) form:

\[
\rho_\phi(q) = \frac{1}{\pi} \frac{Im \Sigma_\phi(q)}{[q^2 - m_\phi^2(q)]^2 + [Im \Sigma_\phi(q)]^2}. \tag{7}
\]

The expression (7) have been used in the previous papers [5, 6, 7].

The situation drastically changes when UP is vector or spinor field. Traditional propagators \( P_{\mu \nu}(q) = -i(g_{\mu \nu} - q_\mu q_\nu/m_\gamma^2)/P_\gamma(q) \) for the vector field \( V \) and \( \hat{P}(q) = i(\hat{q} + m_\Psi)/P_\Psi(q) \) for the spinor field \( \Psi \) do not make it possible to represent \( \Gamma(\Phi \to \phi_1 \phi_2 \phi_3) \) in the factored form [4]. As it was noted in Ref. [6] for the vector field case the numerator \( \eta_{\mu \nu} = -i(g_{\mu \nu} - q_\mu q_\nu/m_\gamma^2) \) destroys the factorization and we have an approximate CF. The same disfactorization takes place for the fermion propagator \( i(\hat{q} + m_\Psi) \) too. In this situation one should analyze the modification of propagators for vector and spinor field after Dyson summation. Such an analysis is complicated due to tensor and operator structure of \( \eta_{\mu \nu} \) and
\(\hat{q} + m\). There are no unit and strict definitions of these structures in literature. For example, the \(\eta_{\mu\nu}\) has the structure \(g_{\mu\nu} - q_{\mu}q_{\nu}/(m_{V}^{2} - im_{V}\Gamma_{V}^{0})\) in Ref. [4], \(g_{\mu\nu} - q_{\mu}q_{\nu}/(m_{V} - i\Gamma_{V}/2)^{2}\) in Ref. [5], \(g_{\mu\nu} - q_{\mu}q_{\nu}/q^{2}\) in Ref. [8], and \((g_{\mu\nu} - q_{\mu}q_{\nu}/q^{2})P_{T}(q^{2}) + (q_{\mu}q_{\nu}/q^{2})P_{L}(q^{2})\) in Ref. [9]. By direct calculation one can check that \(\eta_{\mu\nu} = -i(g_{\mu\nu} - q_{\mu}q_{\nu}/q^{2})/P_{V}(q)\), that is to CF.

The model of UP with random mass [3] contains the general designation how to modify propagators \(P_{\mu\nu}(q)\) and \(\hat{P}(q)\). The "smearing" of UP’s mass in according to uncertainty principle leads to the modified dispersion relation \(q^{2} = \mu^{2}\), where \(\mu\) is nonfixed random mass of UP with some distribution function \(\rho(\mu)\). In another way we have the "smeared" or "fuzzy" mass-shell and the general prescription \(m^{2} \rightarrow \mu^{2} = q^{2}\), where \(q\) is timelike arbitrary momentum. Then the modified propagators for vector and spinor fields are:

\[
P_{\mu\nu}(q) = -i(g_{\mu\nu} - q_{\mu}q_{\nu}/q^{2})/P_{V}(q), \tag{8}
\]

and

\[
\hat{P}(q) = i(\hat{q} + q)/P_{\Psi}(q), \quad q = \sqrt{(qq)}. \tag{9}
\]

In Eqs. (8) and (9) the \(P_{V}(q)\) and \(P_{\Psi}(q)\) are some functions, which don’t influence on convolution structure of the formulae (3) and (4). The prescription (8) coincides with the definition in Ref. [8]. It should be noted that the expressions (8) and (9) approximately coincide with standard ones at peak vicinity \(q^{2} \approx m^{2}(m)\).

Direct calculations with help of Eqs. (8) and (9) (see Appendix) lead to the factored form (4) for \(\Gamma(\Phi \rightarrow \phi_{1}\phi_{2}\phi_{3})\) when the UP in an intermediate state is vector or spinor field. The calculations were fulfilled for all possible type of particles \(\Phi\) and \(\phi_{k}\). For completeness we give a full list of the expressions for \(\Gamma(\Phi \rightarrow \phi_{1}\phi_{2}\phi_{3})\) in Appendix. Thus, the prescriptions (8) and (9) always lead to CF (1) or (3) and (4). The expression (7) runs out from Eq. (5) and parametrization \(q\Gamma(q) = Im\Sigma(q)\), when we use the expression (6). It should be noted that applicability of this approach in the context of gauge theories is limited to low orders because of the appearance of gauge dependence [8, 9, 10, 11, 12]. Moreover, we should redefine \(P_{\Psi}(q)\) for spinor UP with account of \(m_{\Psi}(q) = m_{o\Psi} + Re\Sigma_{\Psi}(q)\) and \(\Gamma_{\Psi}(q) = Im\Sigma_{\Psi}(q)\). However, as was noted early, the convolution structure is not subject to definition of \(P(q)\).

To get the expression for \(\Gamma(\Phi \rightarrow \phi_{1}\phi_{k})\), where \(\phi(q)\) is \(V\) or \(\Psi\), one need the definitions of polarization matrixes for vector and spinor fields. With prescription \(m \rightarrow q\) these definitions are:

\[
\sum_{e} e_{\mu}e_{\nu}^{*} = -(g_{\mu\nu} - q_{\mu}q_{\nu}/q^{2}), \tag{10}
\]
and
\[ \sum_{\nu} \nu^{\nu} (q) u_{\nu}^{\nu} (q) = \frac{q + q_{\alpha \beta}}{2q^{0}}. \]  

(11)

The expressions (8) - (11) are the continuation of standard ones to the "fuzzy" (smeared) mass-shell. The factorization of \( \Gamma(\Phi \to \phi_1 \phi_2 \phi_3) \) on "fuzzy" mass-shell results due to the right sides of Eqs. (10) and (11) equal to the numerators of Eqs. (8) and (9). Such a factorization takes place on the usual mass-shell.

Direct calculations do not lead to the CF in the cases when there are two or more UP in the intermediate or final states. In the framework of the model [3] for \( \Phi \to \phi_1 \phi_2 \), where \( \phi_1 \) and \( \phi_2 \) are the UP with large width, we get the expression for the width, which is usually applied in a phenomenological way:

\[ \Gamma(\Phi \to \phi_1 \phi_2) = \int dq_{1}^{2} \rho_{1}(q_{1}) \int dq_{2}^{2} \rho_{2}(q_{2}) \Gamma(\Phi \to \phi_1(q_{1})\phi_2(q_{2})). \]  

(12)

Representation of the width \( \Gamma(\Phi \to \phi_1 \phi_2 \phi_3) \) in the form (4) is actually the transition from DCM to CM. Such transitions are very complicated in the cases when there are many unstable and stable particles and there are many sections in the decay-chain. Instead of transition from DCM to CM we can derive the convolution formula in the framework of the model [3] for general case. In Ref. [3] such a derivation was done in the case of scalar UP, therefore the discussion of this problem for the vector and spinor UP is actual.

3. Summary

In this paper we have demonstrated in detail the connection between DCM and CM for the decay channels with three-particle final states. The convolution formula was derived with help of the prescription \( m \to q \) by direct calculations for all types of UP. It was noted that this prescription naturally arises in the model of UP with random mass [3]. Transition from DCM to CM is very complicated in the case of many-particle decay-chain. Therefore an alternative proof of CF in the general case is actual.

4. Appendix

In this section we use the \( \tilde{\lambda} \)-function, which describes the kinematics of the process \( \phi(q) \to \phi_1(k_1)\phi_2(k_2) \). In the \( \hat{q} = 0 \) frame of reference:

\[ \tilde{k}_{\alpha} = \frac{1}{2} q \tilde{\lambda}(k_1, k_2; q), \quad \tilde{k}_{\alpha} \equiv |\tilde{k}_{\alpha}|, \quad \alpha = 1, 2, \]  

(13)
where the $\tilde{\lambda}(k_1, k_2; q)$ is defined in analogy with the known Kallen function $\lambda(k_1, k_2; q) = q^2 \tilde{\lambda}(k_1, k_2; q)$:

$$\tilde{\lambda}(k_1, k_2; q) = \left[1 - 2 \frac{k_1^2 + k_2^2}{q^2} + \frac{(k_1^2 - k_2^2)^2}{q^4}\right]^{1/2}. \quad (14)$$

The expressions $\Gamma_i(\phi \rightarrow \phi_1 \phi_2)$ for all types of particles can be represented in the form:

$$\Gamma_i(\phi \rightarrow \phi_1 \phi_2) = \frac{g^2}{8\pi} \tilde{\lambda}(m_1, m_2; m) f_i(m_1, m_2; m), \quad (15)$$

where $f_i(m_1, m_2; m)$ depends on the interaction Lagrangian. To illustrate CM we use the simplest Lagrangians:

$$L_k = g \phi \phi_1 \phi_2; \ g \phi V_\mu V^\mu; \ g \phi \bar{\psi}_1 \psi_2; \ g V_\mu (\phi^\mu \phi_1 - \phi_1^\mu \phi); \ g V_\mu \bar{\psi}_1 \gamma^\mu (c_V + c_A \gamma_5) \psi_2; \ g \phi V_\mu V^\mu. \quad (16)$$

In Eq. (16) $\phi, V_\mu$ and $\psi$ are the scalar, vector and spinor fields. Then the $f_i(m_1, m_2; m)$ in Eq. (15) are defined by the following expressions:

$$\phi \rightarrow \phi_1 \phi_2, \ f_1 = \frac{1}{2m}. \quad (17)$$

$$\phi \rightarrow V_1 V_2, \ f_2 = \frac{1}{m} \left[1 + \frac{(m^2 - m_1^2 - m_2^2)^2}{8m_1^2 m_2^2}\right]. \quad (18)$$

$$\phi \rightarrow \psi_1 \psi_2, \ f_3 = m \left[1 - \frac{(m_1 + m_2)^2}{m^2}\right]. \quad (19)$$

$$\phi \rightarrow \phi_1 V, \ f_4 = \frac{1}{2} \frac{m^2}{m_2} \tilde{\lambda}^2(m_1, m_2; m), \ m_2 = m_V. \quad (20)$$

$$V \rightarrow \phi_1 \phi_2, \ f_5 = \frac{1}{6} m \tilde{\lambda}^2(m_1, m_2; m), \ m = m_V. \quad (21)$$

$$V \rightarrow \psi_1 \psi_2, \ f_6 = \frac{2}{3} m \left\{ (c_V^2 + c_A^2) \left[1 - \frac{m_1^2 + m_2^2}{2m^2} - \frac{(m_1^2 + m_2^2)^2}{2m^4}\right] \right\} + 3(c_V^2 - c_A^2) \frac{m_1 m_2}{m^2}. \quad (22)$$

$$V \rightarrow V_1 \phi, \ f_7 = \frac{1}{3m} \left[1 + \frac{(m^2 + m_1^2 - m_2^2)^2}{8m^2 m_1^2}\right], \ m_2 = m_\phi. \quad (23)$$

$$V \rightarrow V_1 V_2, \ f_8 = \frac{1}{24} \frac{m^5}{m_1^2 m_2} \left\{ 1 + 8(\mu_1 + \mu_2) - 2(\mu_1^2 + 16\mu_1 \mu_2 + 9\mu_2^2) + 8(\mu_1^3 - 4\mu_1^2 \mu_2 \right. \right.$$

$$- 4\mu_1 \mu_2^2 + \mu_2^3) + \mu_1^4 + 8\mu_1^2 \mu_2^2 + 8\mu_1 \mu_2^3 + \mu_2^4\}, \ \mu_a = \frac{m_a}{m_2}. \quad (24)$$

$$\psi \rightarrow \phi \psi_1, \ f_9 = \frac{1}{2} m \left(1 + \frac{2m_1}{m} + \frac{m_1^2 - m_2^2}{m^2}\right), \ m = m_\psi, \ m_2 = m_\phi. \quad (25)$$

$$\psi \rightarrow \psi_1 V, \ f_{10} = (c_V^2 + c_A^2) \left\{ \frac{(m_1^2 - m_2^2)^2}{2m^2} + \frac{m_2^2 (m_2^2 + m_1^2 - 2m_2^2)}{2m_2^2} \right\} - 3m_1 (c_V^2 - c_A^2), \quad (26)$$

\[ m = m_\psi, \ m_2 = m_V. \]

Using the expressions (17)-(26) we can represent $\Gamma(\Phi \rightarrow \phi_1 \phi_2 \phi_3)$ in a compact and universal form for all types of decay channels. Here we shortly describe the method of $\Gamma(\Phi \rightarrow \phi_1 \phi_2 \phi_3)$ calculation. This value can be always written as:

$$\Gamma = \frac{k}{p^0} \int J(|M(k_i, m_i)|^2) d\frac{k_1}{k_i^0}, \quad (27)$$
where $M(k_i, m_i)$ is an amplitude, $p$ and $k_i$ are momentum of $\Phi$ and $\phi_i$, $k$ is some numerical factor, and

$$J(|M|^2) = \int |M|^2 \delta(p - k_1 - k_2 - k_3) \frac{d\bar{k}_2 d\bar{k}_3}{k_2^0 k_3^0}. \tag{28}$$

The integral $J(|M|^2)$ is calculated in $\bar{q} = 0$ frame of reference and as a result we have the noncovariant expression

$$J(|M|^2) \rightarrow f(q^0, q^0 p^0, q^0 k^0_3, \bar{p}^2, ...). \tag{29}$$

This expression can be always reconstructed to covariant form using $\bar{q} = 0$:

$$q^0 \rightarrow q = \sqrt{(qq)}, \quad q^0 p^0 \rightarrow (qp), \quad q^0 k^0_1 \rightarrow (qk_1), \quad \bar{p}^2 = (p^0)^2 - m^2 \rightarrow (pq)^2/q^2 - m^2, ... \tag{30}$$

Then we pass to the $\bar{p} = 0$ frame of reference and change the variable in Eq. (27) according to

$$\frac{d\bar{k}_1}{k_1^0} = -\frac{1}{2m} \bar{k}_1 dq^2 d\Omega = -\frac{1}{4} \bar{\lambda}(q, m_1; m) dq^2 d\Omega. \tag{31}$$

Using this simple method and prescriptions (8), (9) we have got by tedious but straightforward calculations the general expression for $\Gamma(\Phi \rightarrow \phi_1 \phi_2 \phi_3)$, when $\Phi, \phi$ and $\phi_k$ are all of possible type particles:

$$\Gamma_{\alpha\beta}(\Phi \rightarrow \phi_1 \phi_2 \phi_k) = \frac{g_1^2 g_2^2}{26 \pi^3} \int_{q_1^2}^{q_2^2} \lambda(q, m_1; m_1 f_\alpha(q, m_1; m) \bar{\lambda}(m_i, m_k; q) f_\beta(m_i, m_k; q) q dq^2 |P_\phi(q)|^2, \tag{32}$$

where $q_1 = m_i + m_k$ and $q_2 = m - m_1$. From Eqs. (32) and (13) it follows:

$$\Gamma_{\alpha\beta}(\Phi \rightarrow \phi_1 \phi_2 \phi_k) = \int_{q_1^2}^{q_2^2} dq^2 \Gamma_{\alpha}(\Phi \rightarrow \phi_1 \phi(q)) \frac{q \Gamma(\phi(q) \rightarrow \phi_1 \phi_k)}{\pi |P_\phi(q)|^2}. \tag{33}$$

In the approximation

$$\Gamma(\Phi \rightarrow \phi_1 \phi) = \sum_{i,k} \Gamma(\Phi \rightarrow \phi_1 \phi_i \phi_k) \tag{34}$$

we get the known convolution formula

$$\Gamma(\Phi \rightarrow \phi_1 \phi) = \int_{q_1^2}^{q_2^2} \Gamma(\Phi \rightarrow \phi_1 \phi(q)) \rho_\phi(q) dq^2, \tag{35}$$

where

$$\rho_\phi(q) = \frac{q}{\pi |P_\phi(q)|^2} \sum_{i,k} \Gamma(\phi(q) \rightarrow \phi_i \phi_k). \tag{36}$$

The same result can be received for many-particle decay channels of UP $\phi \rightarrow \phi_1 \phi_2 \phi_3 ...$. For example, let us consider the decay chain $\Phi \rightarrow \phi_1 \phi \rightarrow \phi_1 \phi_2 \phi_3 \phi_4$, where $\phi_k$ are the scalar fields. Then for the simplest contact interaction we have:

$$\Gamma_{\phi} = \frac{g_1^2 g_2^2}{26 \pi^8 p^0} \int \frac{d\bar{k}_1}{k_1^0 |P_\phi(q)|^2} \int \int \delta(q - k_2 - k_3 - k_4) \frac{d\bar{k}_2 d\bar{k}_3 d\bar{k}_4}{k_2^0 k_3^0 k_4^0}, \tag{37}$$
where $q = p - k_1$ and

$$
\Gamma_{\phi}(q) \equiv \Gamma(\phi(q) \to \phi_1\phi_2\phi_3) = \frac{g_2^2}{2\pi^5q^0} \int \int \int \delta(q - k_2 - k_3 - k_4) \frac{d\vec{k}_2 d\vec{k}_3 d\vec{k}_4}{k_2^0 k_3^0 k_4^0}.
$$

(38)

From Eqs. (37), (38) and (??) it follows:

$$
\Gamma_{\phi} = \int_{q_1^2}^{q_2^2} dq^2 \Gamma_{\phi}(q) \frac{q \Gamma_{\phi}(q)}{\pi|P_{\phi}(q)|^2},
$$

(39)

where $\Gamma_{\phi}(q) \equiv \Gamma(\Phi \to \phi_1\phi(q))$. Using the factorizable $|M|^2$ we can get the result (39) by direct calculations for others types of particles $\phi_k$. It should be noted that the facoted (33) and convolution (35) structures take place for any choice of $P_{\phi}(q)$.

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