The Universal Specific Merger Rate of Dark Matter Halos

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Abstract

We employ a set of high resolution N-body simulations to study the merger rate of dark matter halos. We define a specific merger rate by normalizing the average number of mergers per halo with the logarithmic mass growth change of the hosts at the time of accretion. Based on the simulation results, we find that this specific merger rate, \( dN_{\text{merge}}(\xi, M, z) / d\xi / d \log M(z) \), has a universal form, which is only a function of the mass ratio of merging halo pairs, \( \xi \), and does not depend on the host halo mass, \( M \), or redshift, \( z \), over a wide range of masses \( 10^{12} \lesssim M \lesssim 10^{14} M_\odot h^{-1} \) and merger ratios \( \xi \gtrsim 10^{-2} \). We further test with simulations of different \( \Omega_m \) and \( \sigma_8 \), and get the same specific merger rate. The universality of the specific merger rate shows that halos in the universe are built up similarly, with a universal composition in the mass contributions and an absolute merger rate that grows in proportion to the halo mass growth. As a result, the absolute merger rate relates with redshift and cosmology only through the halo mass variable, whose evolution can be readily obtained from the universal mass accretion history (MAH) model of Zhao et al. Last, we show that this universal specific merger rate immediately predicts an universal unevolved subhalo mass function that is independent on the redshift, MAH or the final halo mass, and vice versa.

Unified Astronomy Thesaurus concepts: Cold dark matter (265)

1. Introduction

Subhalo accretion is one of the key ingredients of the hierarchical structure formation theory. In the \( \Lambda \) Cold Dark Matter (\( \Lambda \)CDM) framework, dark matter halos form from the small density perturbations in the early universe, and then grow in size by accreting surrounding smaller halos under gravity. These mergers are not only responsible for the growth of halos, but also lead to the formation of subhalos within halos. As galaxies form and evolve in the centers of halos, the merger history of a halo also determines the property and distribution of its galaxy population. It is thus important to investigate the composition and rate of halo mergers across cosmic time in order to better understand structure formation and galaxy evolution (e.g., Lacey & Cole 1993; Rodriguez-Gomez et al. 2015).

During the past, great efforts have been made to characterize halo merger in a CDM cosmology, either through (semi-) analytical models (Bond et al. 1991; Lacey & Cole 1993; Sheth & Lemson 1999; Sheth et al. 2001; Sheth and Tormen 2002; Neistein & Dekel 2008; Zhang et al. 2008a; Yang et al. 2011; Salvador-Solé & Manrique 2021; Salvador-Solé et al. 2022) or numerical simulations. Due to the complexity of the structure formation process, the latter allows for a more accurate and detailed study (Governato et al. 1999; Gottlöber et al. 2001; Berrier et al. 2006), especially in recent years with the rapid advances in computer technology. For example, Fakhouri & Ma (2008, FM08 hereafter) has proposed a fitting formula of the mean merger rate per halo \( dN_{\text{merge}}/N_{\text{halo}}/dz / d\xi \) based on FoF halos in the Millennium Simulation, where \( z \) is redshift, \( \xi \) the mass ratio of merging halo pairs. Their follow-up works can be found in Fakhouri & Ma (2009) and Fakhouri et al. (2010, hereafter FM10). They found that the mean merger rate per halo has low dependence on the halo mass \( M / 10^{13} \) and redshift \((1+z)^{0.099}\).

Although simulation is more reliable in capturing the detailed process of gravitational collapse in principle, the measurement of merger rate could be subject to a lot of tricky issues such as the construction of the merger tree (e.g., Srisawat et al. 2013; Han et al. 2018) and the way to identify halos and subhalos (e.g., Muldrew et al. 2011; Han et al. 2012; Onions et al. 2012; Knebe et al. 2013). As a result, some discrepancies still remain among the simulation measurements of the merger rates (e.g., Genel et al. 2009, hereafter G09; Genel et al. 2010, hereafter G10; Stewart et al. 2009; Wetzel et al. 2009, hereafter S09; Poole et al. 2017). Among these studies, G09 and S09 have found a more rapid increase in the merger rate with redshift at \( z < 1 \) compared to FM08 and FM10, while the discrepancies at higher redshift become reasonably small. Consequently, the fitting formulas of merger rate proposed in different works still lead to differing dependencies upon the halo mass, redshift, and cosmological parameters.

In this work, we propose a model that describes the halo merger rate in a simpler way without involving parameters related with the halo itself or to cosmology. This is done by normalizing the merger rate per halo with the logarithmic mass
growth rate of the host halo:

\[
dN_{\text{merge}}(\xi, M)/d\xi/d \log M = \frac{dN_{\text{merge}}/d\xi/d\xi}{d \log M/d\xi},
\]

(1)

where \(d \log M\) represents the mass growth ratio of the host, \(\log((M + dM)/M)\) \(\sim dM/M\), between redshift \(z\) and \(z + dz\). It describes the transient populations of infall subhalos relative to the host halo. We will show that this specific merger rate has a universal form for halos of different masses and redshifts and is only a function of the merger mass ratio \(\xi\). Moreover, we look at simulations with different matter density parameter \(\Omega_m\) and \(\sigma_8\), and find the form of the specific merger rate remains the same.

In addition to the above instantaneous merger rate at different redshifts, there are also amount of studies discussing the merger histories of present-day halos. For example, van den Bosch et al. (2005) has found that the accumulated unevolved subhalo mass function is approximately universal for halos with different masses, which has been later confirmed by other studies (Giocoli et al. 2008; Stewart et al. 2008; Yang et al. 2011; Jiang & van den Bosch 2014, 2016; Han et al. 2018). We will show that we can naturally derive this universality from our specific merger rate.

Contrary to a common consensus that halo growth can be divided into an early and a late phase that are contributed by statistically different types of mergers, our results suggest that these two phases are actually composed of the same types of mergers. As the density profile of a halo is shaped by its growth history, this means that the types of merger should not be responsible for the different slopes of the inner and outer profiles of the halo. This is consistent with an early claim that the density profile of a halo is not determined by the types of mergers it experienced (i.e., aggregation history Salvador-Solé et al. 2005).

In this work, we use a set of dark matter only simulations to study the halo merger rate. Based on the simulation results, we show that the Mass Accretion Histories (MAHs) of halos of different masses and cosmologies are composed of mergers of the same distribution in mass ratio. Compared to previous studies, our finding is simpler in the description and closer to the physics behind. This paper is organized as follows: In Section 2, we describe the simulation data and introduce the specific merger rate. Our main results are given in Section 3. We discuss the connections of our result to other merger statistics in Section 4 and compare with theoretical expectations from the excursion set model in Section 5. We conclude in Section 6. Throughout this work, we use log for base-10 logarithm and \(\ln\) for natural logarithm.

2. Data and Methodology

2.1. Simulation

Our work is based on a set of high resolution \(\Lambda\)CDM \(N\)-body simulations (Jing et al. 2007) (Table 1), which are run with the same number of particles \(N_p = 1024^3\). We label them as \(L_x\) \((x = 1, 2, 3, 4, 5, 6\) for \(\Lambda\)CDM). Among these simulations, \(L_1\) and \(L_2\) have the same cosmology but different box sizes: 150 and 300 Mpc \(h^{-1}\) \(\text{Mpc}\). \(L_3\) has the highest mass resolution and lower \(\Omega_m\) and \(\sigma_8\) compared to \(L_1\). \(L_4\) has the same mass resolution with \(L_2\) but a higher \(\sigma_8\) (0.95), and \(L_5\) has the same box size with \(L_2\) but smaller \(\Omega_m\). The baryon density parameter \(\Omega_b\) of \(L_x\) simulation is equal to 0.045. We also consider an Einstein–de Sitter (EDS) simulation for which \(\Omega_m = 1\) and \(\sigma_8\) and the shape of power spectrum are the same as \(L_3\). These simulations will allow us to study the possible cosmology dependence and analyze resolution issues. For each simulation, there are \(\sim 100\) snapshot outputs between redshift 17 and zero.

In all simulations, halos are identified using the standard Friends-of-Friends (FoF, Davis et al. 1985) algorithm with a linking length equal to 0.2 times the mean particle separation. Based on the FoF halos, subhalos are then identified using the Hierarchical Bound-Tracking (Han et al. 2012) algorithm. The mass of a self-bound subhalo in HBT is defined as its number of bound particles multiplied by the particle mass.

2.2. Merging Halo Pairs

The virial mass of a halo is defined at the radius where its inner matter density is equal to the critical density predicted from the spherical collapse model, \(\rho_{\text{vir}} = \Delta_{\text{vir}}/\rho_{\text{crit}}\), where \(\Delta_{\text{vir}} = 18\pi^2 + 82[\Omega_m(z) - 1] - 39[\Omega_m(z) - 1]^2\) according to Bryan & Norman (1998). If a satellite subhalo is located within the virial radius of its host halo at any given moment but is out of the virial radius at its previous snapshot, we define it as an “infall” event. On the contrary, if a satellite subhalo is in the virial radius at the previous snapshot but outside the virial radius at the given moment, we regard it as a “splashout” event. For an “infall” event, we define \(\xi\) as the mass ratio of the pair of merging halo progenitors. And for a “splashout” event, \(\xi\) is defined as the mass ratio of the halo pair right after the splashout.

We use the virial masses of the halo pair when calculating the merger mass ratio. However, it can be non-trivial to define virial masses for halos that are merging. Even though the centers of the progenitors are not contained in the virial radius of each other, their boundaries can still be overlapping. At this point, we have defined two types of halos: “independent halos” and “intersecting halos.” A “independent halo” does not intersect with any other larger halo in virial radius, while the “intersecting halo” partially overlaps with at least one other

![Table 1: Simulation Parameters](image-url)

| Simulation | Box Size \((h^{-1}\text{Mpc})\) | Particle Mass \((h^{-1}\text{Mpc})\) | \(\Omega_m\) | \(\Omega_b\) | \(h\) | \(\sigma_8\) | \(n_c\) |
|------------|-----------------|-----------------|-------------|-------------|-----|---------|-----|
| L1         | 150             | \(2.34 \times 10^8\) | 0.732       | 0.268       | 0.045 | 0.71    | 0.85 | 1   |
| L2         | 300             | \(1.87 \times 10^8\) | 0.732       | 0.268       | 0.045 | 0.71    | 0.85 | 1   |
| L3         | 100             | \(6.67 \times 10^7\) | 0.742       | 0.258       | 0.044 | 0.719   | 0.796| 0.963|
| L4         | 300             | \(1.87 \times 10^8\) | 0.742       | 0.268       | 0.045 | 0.71    | 0.95 | 1   |
| L5         | 300             | \(1.39 \times 10^9\) | 0.8         | 0.2         | 0.045 | 0.71    | 0.85 | 1   |
| EDS        | 100             | \(2.59 \times 10^8\) | 0.0         | 1.0         | 0.0   | 0.796   | 0.963|   |
larger halo in space. In the latter case, we calculate the virial mass of the larger halo in the normal way according to the spherical overdensity definition, while the virial mass of the smaller halo is computed excluding particles within the virial radius of the larger halo, similar to the treatment of Giocoli et al. (2008). The redefined halos are always located at the centers of HBT identified self-bound subhalos. In this work, we only consider the independent halo sample when selecting merged halos for analysis of the merger rate. Clearly, the mass of the accreted subhalo will be underestimated in our treatment if it is a crossing halo in the last snapshot. In this case, we use the virial mass of its progenitor in the last–last snapshot before merger\(^8\) to get the mass ratio \(\xi\). Besides, we only consider the merging between the first-order subhalos labeled by HBT and the host halo and have not accounted for mergers between higher level subhalos and the host, as higher level mergers can be analytically modeled subsequently once first-order merger rates are modeled.

2.3. The Specific Merger Rate

During the merger, the infalling halos might fall into and run out of the virial radius of host halos more than once. So for a halo with mass \(M_h\) at redshift \(z_1\), we define its number of “merger” events between two neighboring snapshots (\(z_1\) and \(z_2\)) as the number of “infall” events subtracting the “splashout” events: \(\Delta N_{\text{merge}}(\xi|M_h,z) = \Delta N_{\text{infall}} - \Delta N_{\text{splash}}\). Here the “splashout” has an opposite definition of “infall,” referring to the events that the subhalo flies out of the main halo. Meanwhile, we define the mass change ratio of the main branch halo as \(\Delta \log M_h = \log(M_h(z_1)/M_h(z_2))\), where \(z_1 < z_2\). By averaging the results of all the hosts with masses \(M_h\), we then statistically obtain the “specific merger rate” from the simulation: \(\langle \Delta N_{\text{merge}}(\xi|M_h,z) \rangle = \langle \Delta \log M_h \rangle / \xi\). By further integrating this expression on \(\xi\) over the range of \(\xi > \xi_{\text{min}}\), we could obtain the cumulative form: \(\langle \Delta N_{\text{merge}}(>\xi_{\text{min}}|M_h,z) \rangle / \langle \Delta \log M_h \rangle\).

The “specific merger rate” is the main quantity we study in this paper. In the following, we will use the simulation data to show that the (cumulative) specific merger rate is universal for halos with different masses, redshifts, and cosmologies.

3. Result

In this section, we mainly study the instantaneous specific merger rate for halos of different masses at \(z \geq 0\) using ΛCDM simulations. We also show that studying the merger rate along the evolution history of a specific population of halos produces the same result.

3.1. The Universality of the Specific Merger Rate

In the simulation, we bin halos at each snapshot into four mass ranges for statistics: \(10^{11} M_h h^{-1}\), \(10^{12} M_h h^{-1}\), \(10^{13} M_h h^{-1}\) and \(10^{14} M_h h^{-1}\). For each halo at a given snapshot, we find all the new mergers that occurred after the last snapshot. Meanwhile, we calculate the average mass change ratio \(\Delta \log M_h\) of halos of the same mass. The instantaneous specific merger rate is obtained by normalizing the average number of mergers with the mean mass change ratio \(\Delta \log M_h\). In Figure 1, we show one of such measurements with simulation L1. The left panel shows the results at redshift zero. From the figure we see that the specific merger rates for different mass halos are well consistent with each other, and a double Schechter (Han et al. 2018) provides a good description:

\[
dN_{\text{merge}} / d\xi / d \log M_h = (a_1 \xi^{b_1} + a_2 \xi^{b_2}) \exp(c \xi^d),
\]

where \(a_1, a_2, b_1, b_2, c, \) and \(d\) are free parameters. The fitting curve is shown in the red solid line with the best-fit parameters to be \((a_1, a_2, b_1, b_2, c, d) = (0.467, 17.362, -1.717, 0.212, -3.682, 0.937)\). For the numerical results in the figure, we make a lower cut-off in the mass ratio considering the mass resolution effect.

Next, we measure the specific merger rates at different redshifts. In order to show the redshift dependence more clearly, we use the cumulative form of the specific merger rate in the rest of the paper, which is obtained by integrating \(dN_{\text{merge}} / d\xi / d \log M_h\) over \(\xi > \xi_{\text{min}}\). Here we make five choices of \(\xi_{\text{min}}\) of integration for analysis: 0.6, 1/3, 0.1, 0.01, and 0.001. The results are shown in the right panel of Figure 1, in which the gray solid lines are achieved by directly integrating the fitting formula of Equation (2). The colored lines are simulation results, which are smoothed by averaging both \(\langle \Delta N_{\text{merge}}(\xi > \xi_{\text{min}}, z) \rangle\) and \(\langle \Delta \log M(z) \rangle\) over three adjacent data points whenever the fluctuation of the curve exceeds 20% of the mean. For a given \(\xi_{\text{min}}\), we find that the cumulative merger rate is almost a constant over redshift. This discovery is of great significance, implying that the normalized form of the merger rate likely has no dependence on cosmology, as the cosmological parameters vary significantly with redshift.

As a further illustration, we repeat the above analysis on the other four LCDM simulations which have different \(\Omega_m\) and \(\sigma_8\) values from L1. Besides, we also consider an EDS simulation (\(\Omega_m = 1\)) for comparison. These results are presented in Figure 2, in which all the simulations show good consistency with each other and with our model, and therefore can be seen as a robust test of the universality of the specific merger rate.

Among these simulations, L3 has the best mass resolution and thus is able to show merger rate down to \(\xi = 0.01\) for \(M = 10^{12} M_h h^{-1}\) halos. With the L3 simulation, we also find that the merger rate for \(M_h = 10^{12} M_h h^{-1}\) and \(\xi = 0.1\) overlaps with other lines, indicating that our conclusion might be valid for very low mass halos. Note that in our cumulative merger rate statistics above, the minimum number of particles of a host halo is about 320. In addition, we only consider pairs in which the less massive progenitor has more than \(\sim 30\) particles.

Overall, we see general remarkable agreement between our model and simulation results. All the clues above show that the specific merger rate is independent of halo mass, redshift, and cosmology at least over the range of halo mass \([10^{12}, 10^{14}] M_h h^{-1}\), mass ratio \(\xi \geq 0.01\) and redshift \([0, 5]\). In Section 5, we will discuss how this can be approximately understood in the framework of the Extended Press–Schechter (EPS) theory.

3.2. The Specific Merger Rate Along the Evolution History of a Given Halo Population

In previous sections, we considered halos of a given mass at different redshifts. Now we trace present-day halos of the same masses along their formation histories. This is particularly interesting for semi-analytical models.
Assuming $M_0$ is the mass of halos at redshift zero ($z_0$), and $M_d(z_d|M_0, z_0)$ is the main branch halo mass at redshift $z_d$, then the specific merger rate of $\xi$ at redshift $z_a$ can be obtained by dividing the number of merger events in $[z_a, z_a + dz]$ with the main branch mass change ratio: 

$$\langle \Delta N_{\text{merge}}(\xi, z_d|M_0, z_0) \rangle / \langle \Delta \log M_d \rangle,$$

where $\langle \Delta \log M_d \rangle = \langle \log[M_d(z, z_d|M_0, z_0)/M_d(z_a + dz|M_0, z_0)] \rangle$, 
\n$\xi = M_s/M_d$, and $M_s$ is the mass of the accreted subhalo. The specific merger rate as a function of redshift is shown in Figure 3.

As this statistic is more susceptible to the simulation resolution effect compared to the instantaneous merger rate, we only show the results for redshift less than 3. It is reassuring to see that we get exactly the same result as in Section 3.1, consistent with universal form of the instantaneous merger rates of halos at any redshift and halo mass.

### 4. Connections to other Merger Statistics

In this section, we discuss how our model is connected to the unevolved subhalo mass function and the absolute halo merger rate studied in other works. We also compare the predicted rate of major mergers to other studies in the literature.

#### 4.1. Equivalence to the Universal Un-evolved Subhalo Mass Function

An immediate application of the specific merger rate is to predict the unevolved subhalo mass function (USMF) $g(\xi_0) = dN(\xi_0)/d\xi_0$, where $\xi_0$ is the ratio between the mass of the subhalo at the time of accretion and the present-day host halo mass, such that $\xi_0 = \xi(z) M(z)/M_0$. Rewriting the specific merger rate as

$$f(\xi) \equiv \frac{dN(\xi|M, z)}{d\xi d \ln M},$$

$$= \frac{dN(\xi_0|M_0, z)}{d\xi_0} \frac{dM}{d \ln M} = \xi f(\xi),$$

we can integrate it over the mass increment to obtain the final USMF

$$g(\xi_0) = \int_0^{\xi_0} f(\xi)/\bar{M}^2 d\bar{M},$$

where $\bar{M} = M(z)/M_0$, with $M(z)$ being the host halo mass at redshift $z$ and $M_0$ the present-day mass. Alternatively, one can differentiate the USMF with respect to $M$ to derive the specific merger rate, obtaining

$$f(\xi) = \frac{dG(\xi)}{d\xi},$$

where $G(\xi) \equiv dN/d \ln \xi = \xi g(\xi)$. To get the universal specific merger rate, the universality of the USMF over different time is required. Note that Equation (5) is the mathematical inversion of Equation (4).

Because the specific merger rate is independent on the host halo mass and redshift, Equation (4) immediately leads to an important conclusion that the USMF is universal, which only depends on the mass ratio $\xi_0$ and is independent on the mass, redshift, or MAH of the host halo. This strong universality is a direct consequence of our finding that the halo accretion closely traces the mass accretion, so that the accumulated population of accreted halos does not depend on the path of the mass growth, in line with the “unbiased accretion” picture discussed in Han et al. (2016). In fact, multiplying our specific merger rate by $\xi$, we get

$$\frac{mdN}{d\xi dM} = \xi f(\xi),$$

which means the mass accreted through mergers at a fixed $\xi$ is exactly proportional to the mass increment $dM$ of the halo.

In Figure 4, we compare our model prediction with some other fitting functions of the USMF. The result of Giocoli et al. (2008) shows an analytical fit of their simulation measurements, for which virial definition is adopted for the halo mass and merger. The model of Yang et al. (2011) is a semi-analytical model which evolves progenitor halos from an empirically modified extended PS formula and the mass assembly model of Zhao et al. (2009). The result of Jiang & van den Bosch (2014) is a fitting function for the simulation
measurements of Li & Mo (2009), which also adopts the virial definitions of quantities. For the mass ratio larger than $10^{-2}$, our model prediction locates between the results of Giocoli

Figure 2. The cumulative specific merger rates measured for different simulations. The six panels from left to right and top to bottom are respectively for simulation L1, L2, L3, L4, L5 and EDS. The line styles and colors in this figure are the same as in Figure 1.

Figure 3. The cumulative distribution of merger ratios across the histories of present-day halos measured with simulation L1. Results for different present-day masses $M_0$ are shown by different line styles. The color in this figure has the same meaning as in Figure 1.

Figure 4. Unevolved subhalo mass function. Our model prediction is shown in the blue solid line. The dashed-dotted green line, dashed red line and dotted yellow line respectively show the model prediction from Yang et al. (2011), the fitting formula of Jiang & van den Bosch (2014) and the fitting formula of Giocoli et al. (2008).
et al. (2008) and Yang et al. (2011). Among these three curves, our result shows an overall excellent agreement with Jiang & van den Bosch 2014, but is higher at the large mass end ($\xi_0 > 0.3$), which indicates the massive subhalos are more sensitive to different treatments of the halo merger than the low mass subhalos. At the very low mass end $\xi_0 < 10^{-3}$, our prediction is slightly lower than all three models, which might be due to the resolution effect in our sample. It is worth recalibrating our model with simulation results in a larger dynamical range in the future. Han et al. (2018) provided universal fittings of the unevolved subhalo mass functions including contributions from all levels of subhalos. However, as our merger rate only accounts for first level subhalos, we do not compare with it here.

Even though the universality of the USMF has been proposed for a long time, the universality of the merger rate has not been known before. So we believe it is important that we explicitly point out this equivalence both analytically and experimentally, thus unifying our understanding of the two.

### 4.2. Estimating the Absolute Merger Rate

The specific merger rate shows us that the halo is built up self-similarly, indicating the merger rate per unit redshift depends on redshift only through the halo mass. Thus, one can decompose the merger rate of a host halo at $(M_{\text{obs}}, z_{\text{obs}})$ into two independent terms:

$$\frac{dN_{\text{merge}}(>\xi|M_{\text{obs}}, z_{\text{obs}})}{dz_{\text{obs}}} = \frac{dN_{\text{merge}}}{d \log M_{\text{obs}}} \times \frac{d \log M_{\text{obs}}}{dz_{\text{obs}}}.$$  \hfill (7)

The first term $dN_{\text{merge}}(>\xi|M_{\text{obs}}, z_{\text{obs}})/d \log M_{\text{obs}}$ is a constant over time and can be obtained with Equation (2). To predict the merger rate, we use a widely adopted model for the MAHs of dark matter halos, for which we refer the readers to Zhao et al. (2009) for more details. This MAH model is accurate and universal over large dynamical ranges: the same set of model parameters work well for different cosmological models and for halos of different masses at different redshifts. To get the instantaneous mass grow rate $dM/dz$, we only need to trace the MAH with one-step backward. We set the shift parameter to zero in the MAH model.

In Figure 5, we show our model predictions of $dN(>\xi)/dz$. In the first panel, we compare our results with the simulation results of L1(&2). We remove lines suffering from resolution effect. By adding a multiplicative factor of 0.82, our model of $dN(>\xi)/dz$ gives a good description of the simulation results. The multiplicative factor here is mainly due to the difference between our MAH and the model of Zhao et al. (2009), which may be attributed to our different halo samples, the different...
ways in getting the average mass growth ratio $\Delta \log M^{10}$ as well as the ways in computing the halo mass. In the upper right panel, we compare our model for halos with $M = 10^{13} M_\odot h^{-1}$ to three other studies. The dashed-dotted lines show the fitting formula given by FM10:

$$dN_{\text{merge}} / dz / d\xi = A \left( M / M_\ast \right) \alpha \exp \left[ \left( \frac{\xi}{\xi_0} \right)^{\beta} \left( 1 + z \right)^{\gamma} \right] (8)$$

where $\alpha, \beta, \gamma, \eta, A, \xi = (0.133, -1.995, 0.263, 0.0993, 0.0104, 9.72e-3)$ and $M_\ast = 10^{12} M_\odot$. In addition, we also show comparisons with G09 and S09. G09 is also based on the EPS formalism with redshift at $z = 0$, 9.72e-3 and another factor of 0.82 is adopted. The model is still able to describe the simulation results very well, while Zhao et al. (2009) has adopted a cosmology of $\Omega_m = 0.3$ and $\sigma_8 = 0.9$, and a slightly different fitting formula $dN / dz (\xi) = 0.27 \left( d\xi / dz \right) M_{12}^{1.15} \xi^{-0.5} \left( 1 - \xi \right)^{1.3}$, with $M_{12}$ being the mass in units of $10^{12} M_\odot h^{-1}$. These two results are shown by dashed lines and dotted lines, respectively, in the figure. To make a fair comparison, we set the cosmologies of these models the same as L1. On the whole, our model is broadly consistent with all three of these works. In more details, our model shows the best consistency with G09 in both the shape and amplitude. Compared with FM10, our results are consistent with theirs at $z > 1$, but show a more rapid increase with redshift at $z < 1$. The S09 results show a lower merger rate for $\xi \leq 0.01$ and a more rapid increase with redshift for $z < 1$ than other models. Overall, the consistency between our results and these models confirms the accuracy of our model of $dN / dz (\xi) / dz$.

As a further test, we show the results of the EDS cosmology in the lower panels. From the third panel, we can find that our model is still able to describe the simulation results very well, for which the same multiplicative factor of 0.82 is adopted. While all the other formulas in the fourth panel show obvious discrepancies in amplitude from our model.

To sum up, instead of providing a fitting formula to the simulation results of the merger rate $dN(\xi) / dz$, we propose a model by combining the specific merger rate with the MAH model of Zhao et al. (2009), which is more clear in physics. More importantly, our model is universal and enables us to estimate the merger rate in different cosmologies.

4.3. Estimating the History of Major Mergers

By applying Equation (7) along the MAH of a given halo, one can immediately predict its merger rate history. For a halo with mass $M_0$ and $z_0$, its MAH can be specified by the (Zhao et al. 2009) model, so that its merger rate history is given by:

$$dN_\text{d}(\xi, z_0 | M_0, 0) / dz_0 = dN_\text{d}(\xi, z_0 | M_0, 0) / d \log M_\text{d}(\xi, z_0 | M_0, 0) \times d \log M_\text{d}(\xi, z_0 | M_0, 0) / dz_0.$$  (9)

where $z_0$ the accretion time, and $M_0$ the main branch mass at $z_0$.

According to our conclusion, $dN / d \log M_\text{d}$ is a constant over redshift for a given $\xi$.

In Figure 6, we compare our model predictions of the major merger rates ($\xi \geq 1/3$) with the analytical model for the accretion of subhalos developed in Yang et al. (2011, hereafter Y11), by adopting the same MAH model of $M_\text{d}(\xi, z_0 M_0)$ (Zhao et al. 2009). Here we still set the shift parameter to zero in the model. We find that, by considering a multiplicative factor 0.89, we can get a consistent result with Y11. The difference in amplitude may be due to our different treatments in measuring the halo mass and merger rate.

5. Comparison with the EPS Prediction

In this section, we compare our results with predictions from the EPS formalism. Denoting $M_0$ as the descendant mass of a halo at $z_0$, $M_1$ the mass of its progenitor at $z_1 > z_0$ ($M_1 < M_0$), the number weighted conditional mass function of $M_1$ is given as (Lacey & Cole 1993; Cole et al. 2000):

$$\phi(M_1, z_1 | M_0, z_0) = \frac{M_0}{M_1 \sqrt{2 \pi} \Delta z} \exp \left( - \frac{(M_1 - M_0)^2}{2 \Delta z} \right) dM_1,$$  (10)

where $\Delta z = w(z_1) - w(z_0)$, $\Delta S = S(M_1) - S(M_0)$, $w(z_1)$ the critical overdensity at $z_1$. For binary mergers, the descendant mass $M_0$ can be written as the sum of two progenitors $M_0 = M_1 + M_2$. Consequently, the average merger rate per descendant halo can be related to the conditional mass function as (Sheth & Pitman 1997; Zhang et al. 2008a, 2008b)

$$\frac{\Delta N(\xi)}{\Delta z} = \frac{\phi(M_1, z_1 | M_0, z_0) M_0}{1 + \xi^2 \Delta z},$$  (11)

where $M_0$ can be either of $M_1$ or $M_2$, $\Delta z = z_1 - z_0$ is the time step and $M_0 = M_1 / M_2$ is the progenitor mass ratio. Due to the asymmetry of Equation (10) of the EPS model (Zhang et al. 2008a), here we assign $M_1$ as the less massive progenitor $M_2$. Meanwhile, the mass change of the host $\Delta \log M$ between $z_1$ and $z_0$ can be obtained as $\log(1 + \sum_j M_j / \Delta z)$$\xi(\Delta N(\xi))$, where $\Delta N(\xi)$ is the relative mass change of the host halo, $\Delta M(\xi) / M_1$, induced by mergers of $\xi$. For a small time step ($\Delta z \rightarrow 0$), $\Delta \log M$ is proportional to $M_1 / \Delta s(\xi) \log(1 + \sum_j M_j / \Delta z)$. In Figure 7, we show the EPS estimations of $f(\xi, M_0)$ for descendant halos at $z_0 = 0$ with $\Delta z = 0.02$. We can find that the specific merger rate estimated for $M_0 = 10^{10} M_\odot h^{-1}$ is almost the same as that for $M_0 = 10^{13} M_\odot h^{-1}$. Although slightly more minor mergers and fewer major mergers are found for the former, the difference is tiny at $\xi > 10^{-4}$. For a small time step, the exponential term in the conditional function diminishes. The mass related term $dS / dM_1 / \Delta z^{3/2}$ is found to have a rough power law relation with the progenitor mass $\xi$, which thus helps to remove the dependence of $f(\xi)$ on the descendant mass $M_0$ and leads to a quite universal form. The same conclusion holds for $z_0 > 0$.

It is encouraging to see that our simulation result is consistent with the EPS predictions for $10^{-4} < \xi < 10^{-1}$. While the EPS results are derived numerically without a simple expression, our empirical law from the simulation provides a much more direct and convenient way to predict the merger rate. As our result summarizes the simulation concisely, it can also serve as an important benchmark for calibrating
reveals a strong self-similarity in the merger-driven growth of halos, such that the merger rate scales with the mass growth rate, with the same mass ratio distribution in the progenitors at each step.

2. Consequently, the absolute merger rate \( dN(\xi|z, M)/d\xi/dzc \) depends on redshift only through the halo mass variable, which can be easily predicted from the specific merger rate combined with the universal MAH model of Zhao et al. (2009).

3. The same conclusion holds for present-day halos through their mass accretion histories.

4. The universal specific merger rate naturally results in a universal USMF that only depends on the mass ratio but not on the host halo mass, redshift, or MAH. This is a direct consequence of our finding that the halo accretion closely traces the mass accretion, resulting in a final progenitor distribution that is independent on the path of the mass growth, in line with the “unbiased accretion” picture discussed in Han et al. (2016).

The above conclusions are valid at least for halos over the range of mass \([10^{12}, 10^{14}] M_\odot h^{-1}\) and merger ratio \( \xi > 0.01 \). Compared with previous works, the specific merger rate defined in this work is simpler in the description and much easier to interpret physically, with no dependence on the cosmology. In particular, our formalism has the appealing advantage that it is mathematically equivalent to the universal USMF. As a result, our findings on the universality of the merger rate also substantially extend our understanding of the universality of the USMF over time and cosmology. Furthermore, our finding suggests that the structure evolution of a halo mainly depends on its mass growth rate rather than the composition of mergers.

Together with the universal USMF, we expect our findings can find wide applications in many problems. As our specific merger rate describes the simulation result in a concise and convenient way, it can serve as an important benchmark for developing and calibrating theoretical models of halo growth. For example, we have demonstrated that the EPS model can produce consistent results with ours, but in a much more involved way with subtle differences. Our model can also be used in conjunction with the universal distribution of infall orbits (Li et al. 2020) to generate initial conditions for halo mergers in semi-analytical or numerical studies of halo and galaxy formation.

In this work, we only focus on the mass distribution of mergers without discussing the spatial distribution of the merger remnants, i.e., subhalos. Despite this, our findings may be used to provide insights on the universal spatial distribution of unevolved subhalos (Han et al. 2016) as well. In fact, if approximating the progenitor distribution (Equation (2)) at the low mass end with a single power law of \( \sim a\xi^{-2} \), we can get \( dN/dm/dM \sim am^{-2} \). In this case, the number of subhalos accreted is exactly proportional to the mass accreted irrespective of accretion time, so that the accretion of subhalos is “unbiased” relative to mass accretion. This is exactly the “unbiased” accretion picture described in Han et al. (2016), which would result in the distribution of subhaloes tracing dark matter distribution dynamically. Alternatively, if we assume halos are built up essentially shell by shell from the inside out as in the CUSP formalism (Salvador-Solé & Manrique 2021; Salvador-Solé et al. 2021),...
Salvador-Solé et al. 2022, the number of mergers over time can be translated to the spatial distribution of accreted subhalos over radius. Under the same approximation of a $\sim \xi^{-2}$ distribution at the low mass end, the derived spatial distribution can become separable from the mass distribution, reproducing the Han et al. (2016) model. We note that the CUSP formalism itself does not rely on the assumption of inside-out growth, and can be applied to both purely accreting halos and systems experiencing major mergers.

Finally, our work can be further improved in several aspects. For example, to test our conclusion for halos over a larger dynamical range, we need simulations with larger box size and better resolution in the next step. In addition, the infall rate and splashout rate themselves are also important quantities that can be studied separately. In fact, we find that, although the normalized infall rate declines with redshift, it has no dependence on the halo mass. The same trend is found for the splashout rate. But the measurement of splash rates are very easily influenced by the simulation resolution, as the masses of splashed halos are usually low, and thus introduce resolution effects to our statistics. It also remains interesting to generalize our analysis to study the galaxy merger rate which are found to be qualitatively similar (Rodriguez-Gomez et al. 2015). Last but not least, it could be interesting to study the merger rate under alternative physical definitions of the halo boundary (e.g., depletion radius, Fong & Han 2021) which may be able to provide more consistent and natural definitions of the “merger” among haloes. We hope to study these issues in more details in the future.

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**References**

Berrier, J. C., Bullock, J. S., Barton, E. J., et al. 2006, ApJ, 652, 56
Bond, J. R., Cole, S., Efstathiou, G., & Kaiser, N. 1991, ApJ, 379, 440
Bryan, G. L., & Norman, M. L. 1998, ApJ, 495, 80
Cole, S., Lacey, C. G., Baugh, C. M., & Frenk, C. S. 2000, MNRAS, 319, 168
Davis, M., Efstathiou, G., Frenk, C. S., & White, S. D. M. 1985, ApJ, 292, 371
Fakhouri, O., & Ma, C.-P. 2008, MNRAS, 386, 577
Fakhouri, O., & Ma, C.-P. 2009, MNRAS, 394, 1825
Fakhouri, O., Ma, C.-P., & Boylan-Kolchin, M. 2010, MNRAS, 406, 2267
Fong, M., & Han, J. 2021, MNRAS, 503, 4250
Genel, S., Bouché, N., Naab, T., Stemberg, A., & Genzel, R. 2010, ApJ, 719, 229
Genel, S., Genzel, R., Bouché, N., Naab, T., & Stemberg, A. 2009, ApJ, 701, 2002
Giocoli, C., Tormen, G., & van den Bosch, F. C. 2008, MNRAS, 386, 2135
Gottlöber, S., Klypin, A., & Kravtsov, A. V. 2001, ApJ, 546, 223
Governato, F., Gardner, J. P., Stadel, J., Quinn, T., &Lake, G. 1999, AJ, 117, 1651
Han, J., Cole, S., Frenk, C. S., Benitez-Llambay, A., & Helly, J. 2018, MNRAS, 474, 604
Han, J., Cole, S., Frenk, C. S., & Jing, Y. 2016, MNRAS, 457, 1208
Han, J., Jing, Y. P., Wang, H., & Wang, W. 2012, MNRAS, 427, 2437
Jiang, F., & van den Bosch, F. C. 2014, MNRAS, 440, 193
Jiang, F., & van den Bosch, F. C. 2016, MNRAS, 458, 2848
Jing, Y. P., Suto, Y., & Mo, H. J. 2007, ApJ, 657, 664
Knebe, A., Pearce, F. R., Lux, H., et al. 2013, MNRAS, 435, 1618
Lacey, C., & Cole, S. 1993, MNRAS, 262, 627
Li, Y., & Mo, H. 2009, arXiv:0908.0301
Li, Z.-Z., Zhao, D.-H., Jing, Y. P., Han, J., & Dong, F.-Y. 2020, ApJ, 905, 177
Muldrew, S. L., Pearce, F. R., & Power, C. 2011, MNRAS, 410, 2617
Neistein, E., & Dekel, A. 2008, MNRAS, 388, 1792
Onions, J., Knebe, A., Pearce, F. R., et al. 2012, MNRAS, 423, 1200
Poole, G. B., Mutch, S. J., Croton, D. J., & Wyithe, S. 2017, MNRAS, 472, 3659
Rodriguez-Gomez, V., Genel, S., Vogelsberger, M., et al. 2015, MNRAS, 449, 49
Salvador-Solé, E., & Manrique, A. 2021, ApJ, 914, 141
Salvador-Solé, E., Manrique, A., & Botella, I. 2022, MNRAS, 509, 5305
Salvador-Solé, E., Manrique, A., & Solanes, J. M. 2005, MNRAS, 358, 901
Sheth, R. K., & Lemson, G. 1999, MNRAS, 305, 946
Sheth, R. K., Mo, H. J., & Tormen, G. 2001, MNRAS, 323, 1
Sheth, R. K., & Pinna, J. 1997, MNRAS, 285, 66
Sheth, R. K., & Tormen, G. 2002, MNRAS, 329, 61
Srisawat, C., Knebe, A., Pearce, F. R., et al. 2013, MNRAS, 436, 150
Stewart, K. R., Bullock, J. S., Barton, E. J., & Wechsler, R. H. 2009, ApJ, 702, 1005
Stewart, K. R., Bullock, J. S., Wechsler, R. H., Maller, A. H., & Zentner, A. R. 2008, ApJ, 683, 597
van den Bosch, F. C., Tormen, G., & Giocoli, C. 2005, MNRAS, 359, 1029
Wetzel, A. R., Cohn, J. D., & White, M. 2009, MNRAS, 395, 1376
Yang, X., Mo, H. J., Zhang, Y., & van den Bosch, F. C. 2011, ApJ, 741, 13
Zhang, J., Fakhouri, O., & Ma, C.-P. 2008a, MNRAS, 389, 1521
Zhang, J., Ma, C.-P., & Fakhouri, O. 2008b, MNRAS, 387, L13
Zhao, D.-H, Jing, Y. P., Mo, H. J., & Börner, G. 2009, ApJ, 707, 354

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11 Note the CUSP formalism itself does not rely on the assumption of inside-out growth, and can be applied to both purely accreting halos and systems experiencing major mergers.