CURRENT OPERATORS IN QUANTUM FIELD THEORY AND SUM RULES IN DEEP INELASTIC SCATTERING

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Abstract

It is shown that in the general case the canonical construction of the current operators in quantum field theory does not render a bona fide vector field since Lorentz invariance is violated by Schwinger terms. We argue that the nonexistence of the canonical current operators for spinor fields follows from a very simple algebraic consideration. As a result, the well-known sum rules in deep inelastic scattering are not substantiated.

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1 Relativistic invariance of the current operators

In any relativistic quantum theory the system under consideration is described by some (pseudo)unitary representation of the Poincare group. The electromagnetic or weak current operator $\hat{J}^\mu(x)$ for this system (where $\mu = 0, 1, 2, 3$ and $x$ is a point in Minkowski space) should satisfy the following necessary conditions.

Let $\hat{U}(a) = \exp(i\hat{P}_\mu a^\mu)$ be the representation operator corresponding to the displacement of the origin in spacetime translation of Minkowski space by the four-vector $a$. Here $\hat{P} = (\hat{P}^0, \hat{\mathbf{P}})$ is the operator of the four-momentum, $\hat{P}^0$ is the Hamiltonian, and $\hat{\mathbf{P}}$ is the operator of ordinary momentum. Let also $\hat{U}(l)$ be the representation operator corresponding to $l \in SL(2, C)$. Then

$$\hat{U}(a)^{-1}\hat{J}^\mu(x)\hat{U}(a) = \hat{J}^\mu(x-a)$$

(1)
\[ 
\hat{U}(l)^{-1} \hat{J}^\mu(x) \hat{U}(l) = L(l)^\mu_\nu \hat{J}^\nu(L(l)^{-1} x) \quad (2) 
\]

where \( L(l) \) is the element of the Lorentz group corresponding to \( l \) and a sum over repeated indices \( \mu, \nu = 0, 1, 2, 3 \) is assumed.

Let \( \hat{M}^{\mu\nu} (\hat{M}^{\mu\nu} = -\hat{M}^{\nu\mu}) \) be the representation generators of the Lorentz group. Then, as follows from Eq. (2), Lorentz invariance of the current operator implies

\[ 
[\hat{M}^{\mu\nu}, \hat{J}^\rho(x)] = -i \{ (x^\mu \partial^\nu - x^\nu \partial^\mu) \hat{J}^\rho(x) + g^{\mu\rho} \hat{J}^\nu(x) - g^{\nu\rho} \hat{J}^\mu(x) \} \quad (3) 
\]

where \( g^{\mu\nu} \) is the metric tensor in Minkowski space.

The operators \( \hat{P}^\mu, \hat{M}^{\mu\nu} \) act in the scattering space of the system under consideration. In QED the electrons, positrons and photons are the fundamental particles, and the scattering space is the space of these almost free particles ("in" or "out" space). Therefore it is sufficient to deal only with \( \hat{P}^\mu_{\text{ex}}, \hat{M}^{\mu\nu}_{\text{ex}} \) where "ex" stands either for "in" or "out". However in QCD the scattering space by no means can be considered as a space of almost free fundamental particles — quarks and gluons. For example, even if the scattering space consists of one particle (say the nucleon), this particle is the bound state of quarks and gluons, and the operators \( \hat{P}^\mu, \hat{M}^{\mu\nu} \) considerably differ from the corresponding free operators \( \hat{P}^\mu, \hat{M}^{\mu\nu} \). It is well-known that perturbation theory does not apply to bound states and therefore \( \hat{P}^\mu \) and \( \hat{M}^{\mu\nu} \) cannot be determined in the framework of perturbation theory. For these reasons we will be interested in cases when the representation operators in Eqs. (1) and (2) correspond to the full generators \( \hat{P}^\mu, \hat{M}^{\mu\nu} \).

Strictly speaking, the notion of current is not necessary if the theory is complete. For example, in QED there exist unambiguous prescriptions for calculating the elements of the S-matrix to any desired order of perturbation theory and this is all we need. It is believed that this notion is useful for describing the electromagnetic or weak properties of strongly interacted systems. It is sufficient to know the matrix elements \( \langle \beta | \hat{J}^\mu(x) | \alpha \rangle \) of the operator \( \hat{J}^\mu(x) \) between the (generalized) eigenstates of the operator \( \hat{P}^\mu \) such that \( \hat{P}^\mu | \alpha \rangle = P^\mu_{\alpha} | \alpha \rangle, \hat{P}^\mu | \beta \rangle = P^\mu_{\beta} | \beta \rangle \). It is usually assumed that, as a consequence of Eq. (1),

\[ 
\langle \beta | \hat{J}^\mu(x) | \alpha \rangle = \exp[i(P^\nu_{\beta} - P^\nu_{\alpha}) x_\nu] \langle \beta | \hat{J}^\mu | \alpha \rangle \quad (4) 
\]

where formally \( \hat{J}^\mu \equiv \hat{J}^\mu(0) \). Therefore in the absence of a complete theory we can consider the less fundamental problem of investigating the properties
of the operator $\hat{J}^\mu$. From the mathematical point of view this implies that we treat $\hat{J}^\mu(x)$ not as a four-dimensional operator distribution, but as a "nonlocal" operator satisfying the condition

$$\hat{J}^\mu(x) = \exp(i\hat{P}x)\hat{J}^\mu\exp(-i\hat{P}x) \quad (5)$$

The standpoint that the current operator should not be treated on the same footing as the fundamental local fields is advocated by several authors in their investigations on current algebra (see, for example, Ref. [1]). One of the arguments is that, for example, the canonical current operator in QED is given by [2]

$$\hat{J}^\mu(x) = \mathcal{N}\{\tilde{\psi}(x)\gamma^\mu\hat{\psi}(x)\} = \frac{1}{2}[\tilde{\psi}(x), \gamma^\mu\hat{\psi}(x)] \quad (6)$$

(where $\mathcal{N}$ stands for the normal product and $\hat{\psi}(x)$ is the Heisenberg operator of the Dirac field), but this expression is not a well-definition of a local operator. Indeed, Eq. (6) involves the product of two local field operators at coinciding points, i.e. $\hat{J}^\mu(x)$ is a composite operator. The problem of the correct definition of composite operators is the difficult problem of quantum field theory which so far has been solved only for a few models in the framework of renormalized perturbation theory (see, for example, Refs. [3, 4]) while the unambiguous way of calculating such operators beyond perturbation theory is not known.

It is well-known (see, for example, Ref. [5]) that it is possible to add to the current operator the term $\partial_\nu X^{\mu\nu}(x)$ where $X^{\mu\nu}(x)$ is some operator antisymmetric in $\mu$ and $\nu$. However it is usually believed [6] that the electromagnetic and weak current operators of a strongly interacted system are given by the canonical quark currents the form of which is similar to that in Eq. (6).

We will not insist on the interpretation of the current operator according to Eq. (5) and will not use this expression in the derivation of the formulas, but in some cases the notion of $\hat{J}^\mu$ makes it possible to explain the essence of the situation clearly. A useful heuristic expressions which follows from Eqs. (3) and (5) is

$$[\hat{M}^{\mu\nu}, \hat{J}^\rho] = -i(g^{\mu\rho}\hat{J}^\nu - g^{\nu\rho}\hat{J}^\mu) \quad (7)$$
2 Canonical quantization and the forms of relativistic dynamics

In the standard formulation of quantum field theory the operators $\hat{P}_\mu, \hat{M}_{\mu\nu}$ are given by

$$\hat{P}_\mu = \int \hat{T}_\mu^\nu(x)d\sigma_\nu(x), \quad \hat{M}_{\mu\nu} = \int \hat{M}_{\mu\nu}^\rho(x)d\sigma_\rho(x)$$

where $\hat{T}_\mu^\nu(x)$ and $\hat{M}_{\mu\nu}^\rho(x)$ are the energy-momentum and angular momentum tensors and $d\sigma_\mu(x) = \lambda_\mu \delta(\lambda x - \tau)d^4x$ is the volume element of the space-like hypersurface defined by the time-like vector $\lambda$ ($\lambda^2 = 1$) and the evolution parameter $\tau$. In turn, these tensors are fully defined by the classical Lagrangian and the canonical commutation relations on the hypersurface $\sigma_\mu(x)$.

In this connection we note that in the canonical formalism the quantum fields are supposed to be distributions only relative the three-dimensional variable characterizing the points of $\sigma_\mu(x)$ while the dependence on the variable describing the distance from $\sigma_\mu(x)$ is usual.

In spinor QED we define $V(x) = -L_{int}(x) = e\hat{J}_\mu(x)\hat{A}_\mu(x)$, where $L_{int}(x)$ is the quantum interaction Lagrangian, $e$ is the (bare) electron charge and $\hat{A}_\mu(x)$ is the operator of the Maxwell field (let us note that if $\hat{J}_\mu(x)$ is treated as a composite operator then the product of the operators entering into $V(x)$ should be correctly defined).

At this stage it is not necessary to require that $\hat{J}_\mu(x)$ is given by Eq. (3), but the key assumption in the canonical formulation of QED is that $\hat{J}_\mu(x)$ is constructed only from $\hat{\psi}(x)$ (i.e. there is no dependence on $\hat{A}_\mu(x)$ and the derivatives of the fields $\hat{A}_\mu(x)$ and $\hat{\psi}(x)$). Then the canonical result derived in several well-known textbooks and monographs (see, for example, Refs. [2]) is

$$\hat{P}_\mu = P_\mu + \lambda_\mu \int V(x)\delta(\lambda x - \tau)d^4x$$

$$\hat{M}_{\mu\nu}^\rho = M_{\mu\nu}^\rho + \int V(x)(x^\rho \lambda_\mu - x_\mu \lambda^\rho)\delta(\lambda x - \tau)d^4x$$

It is important to note that if $A_\mu(x), J_\mu(x)$ and $\psi(x)$ are the corresponding free operators then $\hat{A}_\mu(x) = A_\mu(x), \hat{J}_\mu(x) = J_\mu(x)$ and $\hat{\psi}(x) = \psi(x)$ if $x \in \sigma_\mu(x)$.

As pointed out by Dirac, any physical system can be described in different forms of relativistic dynamics. By definition, the description in the
point form implies that the operators \( \hat{U}(l) \) are the same as for noninteracting particles, i.e. \( \hat{U}(l) = U(l) \) and \( \hat{M}^{\mu\nu} = M^{\mu\nu} \), and thus interaction terms can be present only in the four-momentum operators \( \hat{P} \) (i.e. in the general case \( \hat{P}^\mu \neq P^\mu \) for all \( \mu \)). The description in the instant form implies that the operators of ordinary momentum and angular momentum do not depend on interactions, i.e. \( \hat{P} = P \), \( \hat{M} = M \) (\( \hat{M} = (\hat{M}^{23}, \hat{M}^{31}, \hat{M}^{12}) \)), and therefore interaction terms may be present only in the operators \( \hat{P}_0 \) and the generators of the Lorentz boosts \( \hat{N} = (\hat{M}^{01}, \hat{M}^{02}, \hat{M}^{03}) \). In the front form with the marked \( z \) axis we introduce the + and - components of the four-vectors as \( x^+ = (x^0 + x^z) / \sqrt{2} \), \( x^- = (x^0 - x^z) / \sqrt{2} \). Then we require that the operators \( \hat{P}_+^\mu \), \( \hat{P}_j^\mu \), \( \hat{M}_{12}^\mu \), \( \hat{M}_{-j}^{\mu+} \), \( \hat{M}_{-j}^{\mu-} \) (\( j = 1, 2 \)) are the same as the corresponding free operators, and therefore interaction terms may be present only in the operators \( \hat{M}_{-j}^\mu \) and \( \hat{P}^- \).

In quantum field theory the form of dynamics depends on the choice of the hypersurface \( \sigma_\mu(x) \). The representation generators of the subgroup which leaves this hypersurface invariant are free since the transformations from this subgroup do not involve dynamics. Therefore it is reasonable to expect that Eqs. (9) and (10) give the most general form of the Poincaré group representation generators in quantum field theory if the fields are quantized on the hypersurface \( \sigma_\mu(x) \), but in the general case the relation between \( V(x) \) and \( L_{int}(x) \) is not so simple as in QED. The fact that the operators \( V(x) \) in Eqs. (9) and (10) are the same follows from the commutation relations between \( \hat{P}^\mu \) and \( \hat{M}^{\mu\nu} \).

The most often considered case is \( \tau = 0 \), \( \lambda = (1, 0, 0, 0) \). Then \( \delta(\lambda x - \tau) d^4x = d^3x \) and the integration in Eqs. (9) and (10) is taken over the hyperplane \( x^0 = 0 \). Therefore, as follows from these expressions, \( \hat{P} = P \) and \( \hat{M} = M \). Hence such a choice of \( \sigma_\mu(x) \) leads to the instant form [7].

The front form can be formally obtained from Eqs. (9) and (10) as follows. Consider the vector \( \lambda \) with the components

\[
\lambda^0 = \frac{1}{(1 - v^2)^{1/2}}, \quad \lambda^j = 0, \quad \lambda^3 = -\frac{v}{(1 - v^2)^{1/2}} \quad (j = 1, 2)
\] (11)

Then taking the limit \( v \to 1 \) in Eqs. (9) and (10) we get

\[
\hat{P}^\mu = P^\mu + \omega^\mu \int V(x)\delta(x^+)d^4x,
\]

\[
\hat{M}^{\mu\nu} = M^{\mu\nu} + \int V(x)(x^\nu\omega^\mu - x^\mu\omega^\nu)\delta(x^+)d^4x
\] (12)
where the vector $\omega$ has the components $\omega^- = 1, \omega^+ = \omega^j = 0$. It is obvious that the generators (12) are given in the front form and that’s why Dirac [7] related this form to the choice of the light cone $x^+ = 0$.

In Ref. [7] the point form was related to the hypersurface $t^2 - x^2 > 0, t > 0$, but as argued by Sokolov [8], the point form should be related to the hyperplane orthogonal to the four-velocity of the system under consideration. We shall discuss this question below.

### 3 Incompatibility of canonical formalism with Lorentz invariance for spinor fields

A possible objection against the derivation of Eqs. (11) and (12) is that the product of local operators at one and the same value of $x$ is not a well-defined object. For example, if $x^0 = 0$ then following Schwinger [9], instead of Eq. (11), one can define $J^\mu(x)$ as the limit of the operator

$$J^\mu(x) = \frac{1}{2}\bar{\psi}(x + \frac{1}{2})\gamma^\mu \exp(i\epsilon \int_{x - \frac{l}{2}}^{x + \frac{l}{2}} A(x')dx')\psi(x - \frac{l}{2})$$

when $l \to 0$, the limit should be taken only at the final stage of calculations and in the general case the time components of the arguments of $\bar{\psi}$ and $\psi$ also differ each other (the contour integral in this expression is needed to conserve gauge invariance). Therefore there is a "hidden" dependence of $\hat{J}^\mu(x)$ on $\hat{A}^\mu(x)$ and hence Eqs. (11) and (12) are incorrect.

However, any attempt to move apart the arguments of the $\hat{\psi}$ operators in $\hat{J}^\mu(x)$ immediately results in breaking of locality. In particular, at any $l \neq 0$ in Eq. (13) the Lagrangian is nonlocal. We do not think that locality is a primary physical condition, but once the Lagrangian is nonlocal, the whole edifice of local quantum field theory (including the canonical Noether formalism) becomes useless. For these reason we first consider the results which follow from the canonical formalism.

In addition to the properties discussed above, the current operator should also satisfy the continuity equation $\partial J^\mu(x)/\partial x^\mu = 0$. As follows from this equation and Eq. (11), $[\hat{J}^\mu(x), \hat{P}_\mu] = 0$. The canonical formalism in the instant form implies that if $x^0 = 0$ then $\hat{J}^\mu(x) = J^\mu(x)$. Since $J^\mu(x)$ satisfies the condition $[J^\mu(x), P_\mu] = 0$, it follows from Eq. (12) that if $\hat{P}_\mu = P_\mu + V_\mu$ then
the continuity equation is satisfied only if

$$[V^0, J^0(x)] = 0$$  \hspace{1cm} (14)

where

$$V^0 = \int V(x) d^3x, \quad V(x) = -eA(x)J(x)$$ \hspace{1cm} (15)

We take into account the fact that the canonical quantization on the hypersurface $x^0 = 0$ implies that $A^0(x) = 0$.

As follows from Eqs. (1) and (3), the commutation relation between the operators $\hat{M}^0_i \ (i = 1, 2, 3)$ and $J^0(x) = 0$ should have the form

$$[\hat{M}^0_i, J^0(x)] = -x^i [\hat{P}^0, J^0(x)] - iJ^i(x)$$ \hspace{1cm} (16)

Since

$$[M^0_i, J^0(x)] = -x^i [P^0, J^0(x)] - iJ^i(x)$$ \hspace{1cm} (17)

it follows from Eqs. (10), (14) and (15) that Eq. (16) is satisfied if

$$\int y^i A(y) [J(y), J^0(x)] d^3y = 0$$ \hspace{1cm} (18)

It is well-known that if the standard equal-time commutation relations are used naively then the commutator in Eq. (18) vanishes and therefore this equation is satisfied. However when $x \to y$ this commutator involves the product of four Dirac fields at $x = y$. The famous Schwinger result [9] is that if the current operators in question are given by Eq. (13) then

$$[J^i(y), J^0(x)] = C \frac{\partial}{\partial x^i} \delta(x - y)$$ \hspace{1cm} (19)

where $C$ is some (infinite) constant. Therefore Eq. (18) is not satisfied and the current operator $\hat{J}^\mu(x)$ does not satisfy Lorentz invariance.

At the same time, Eq. (19) is compatible with Eqs. (13) and (13) since $\text{div}(A(x)) = 0$. One can also expect that the commutator $[\hat{M}^0_i, J^k(x)]$ is compatible with Eq. (5). This follows from the fact [11] that if Eq. (14) is satisfied then the commutator $[J^i(x), J^k(y)]$ does not contain derivatives of the delta function.

While the arguments given in Ref. [9] prove that the commutator in Eq. (19) cannot vanish, one might doubt whether the singularity of the commutator is indeed given by the right hand side of this expression. Of course, at
present any method of calculating such a commutator is model dependent, but the result that Eq. (16) is incompatible with Lorentz invariance follows in fact only from algebraic considerations. Indeed, Eqs. (14), (16) and (17) imply that if $M^{\mu\nu} = M^{\mu\nu} + V^{\mu\nu}$ then

$$[V^{0i}, J^0(x)] = 0$$  \hspace{1cm} (20)

Since $V^{0i}$ in the instant form is a nontrivial interaction dependent operator, there is no reason to expect that it commutes with the free operator $J^0(x)$. Moreover for the analogous reason Eq. (14) will not be satisfied in the general case. In terms of $\hat{J}^\mu$ one can say that the condition $\hat{J}^\mu = J^\mu$ is incompatible with Eq. (7) in the instant form.

To better understand the situation in spinor QED it is useful to consider scalar QED [11]. The formulation of this theory can be found, for example, in Ref. [12]. In contrast with spinor QED, the Schwinger term in scalar QED emerges canonically [9, 5]. We use $\varphi(x)$ to denote the operator of the scalar complex field at $x^0 = 0$. The canonical calculation yields

$$\hat{J}^0(x) = J^0(x) = i[\varphi^*(x)\pi^*(x) - \pi(x)\varphi(x)], \hspace{1cm} \hat{J}^i(x) = J^i(x) - 2eA^i(x)\varphi^*(x)\varphi(x), \hspace{1cm} J^i(x) = i[\varphi^*(x) \cdot \partial^i \varphi(x) - \partial^i \varphi^*(x) \cdot \varphi(x)]$$  \hspace{1cm} (21)

where $\pi(x)$ and $\pi^*(x)$ are the operators canonically conjugated with $\varphi(x)$ and $\varphi^*(x)$ respectively. In contrast with Eq. (13), the operator $V(x)$ in scalar QED is given by

$$V(x) = -eA(x)\mathbf{J}(x) + e^2A(x)^2\varphi^*(x)\varphi(x)$$  \hspace{1cm} (22)

However the last term in this expression does not contribute to the commutator (16). It is easy to demonstrate that as pointed out in Ref. [11], the commutation relations (3) in scalar QED are satisfied in the framework of the canonical formalism. Therefore the naive treatment of the product of local operators at coinciding points in this theory is not in conflict with the canonical commutation relations. The key difference between spinor QED and scalar QED is that in contrast with spinor QED, the spatial component of the current operator is not free if $x^0 = 0$ (see Eq. (21)). Just for this reason the commutator $[\hat{M}^{0i}, J^0(x)]$ in scalar QED agrees with Eq. (3), since the Schwinger term in this commutator gives the interaction term in $\hat{J}^i(x)$.

Now let us return to spinor QED. As noted above, the canonical formalism cannot be used if the current operator is considered as a limit of the
expression similar to that in Eq. (13). In addition, the problem exists what is the correct definition of $V(x)$ as a composite operator. One might expect that the correct definition of $J^\mu(x)$ and $V(x)$ will result in appearance of some additional terms in $V(x)$ (and hence in $V^0$ and $V^{0i}$). However it is unlikely that this is the main reason of the violation of Lorentz invariance. Indeed, as noted above, for only algebraic reasons it is unlikely that both conditions (14) and (20) can be simultaneously satisfied. Therefore, taking into account the situation in scalar QED, it is natural to conclude that the main reason of the failure of canonical formalism is that either the limit of $\hat{J}^\mu(x,0)$ when $x^0 \to 0$ does not exist or this limit is not equal to $J^\mu(x)$ (i.e. the relation $\hat{J}^\mu(x) = J^\mu(x)$ is incorrect).

Our conclusion implies that it is insufficient to consider $\hat{J}^\mu(x)$ as a limit of the expression in Eq. (13) when $l \to 0$. In turn this implies that the description of the commutator in Eq. (19) only by the Schwinger term (which is a consequence of Eq. (13)) may be insufficient. The results of several authors (see the discussion in Ref. [5]) show that if the time and space components of the arguments $x$ and $x'$ of the fields $\hat{\psi}(x)$ and $\hat{\psi}(x')$ defining the current operator are different then the equal time commutators of the components of this operator depend on the order in which the limits $x^0 - x^0' \to 0$ and $x - x' \to 0$ are calculated.

Let us mention the mathematical fact that the commutation relations (3) can be formally satisfied if we assume that $V(x)$ is given by Eq. (13), $\hat{J}^0(x) = J^0(x)$ but $\hat{J}^i(x) = J^i(x) - eCA^i(x)$ where $C$ is the same constant as in Eq. (20). However such a form of $\hat{J}^i(x)$ is not gauge invariant and therefore it cannot be obtained from the gauge invariant expressions similar to that in Eq. (13). Let us also recall that in perturbative QED the Schwinger terms do not play a role if we consider only the processes described by connected Feynman diagrams (see, for example, Refs. [13, 2, 12]). In addition, as noted in Sec. 1, in QED it is sufficient to consider only commutators involving $\hat{P}_{\mu x}$ and $M_{\mu x}$. Therefore there is no problem with Lorentz invariance of the $S$-matrix in QED. However the above considerations are important for the problem of constructing the current operators for strongly interacting particles (see Sec. 4).

By analogy with Ref. [9] it is easy to show that if $x^+ = 0$ then the canonical current operator in the front form $J^+(x^-, x_\perp)$ (we use the subscript $\perp$ to denote the projection of the three-dimensional vector onto the plane
12) cannot commute with all the operators \( J^i(x^-, x_\perp) \) \((i = -1, 2)\). As easily follows from the continuity equation and Lorentz invariance, the canonical operator \( J^+(x^-, x_\perp) \) should satisfy the relations

\[
[V^-, J^+(x^-, x_\perp)] = [V^{-j}, J^+(x^-, x_\perp)] = 0 \quad (j = 1, 2)
\]  

(23)

By analogy with the above consideration we conclude that these relations cannot be simultaneously satisfied and therefore either the limit of \( J^\mu(x^+, x^-, x_\perp) \) when \( x^+ \to 0 \) does not exist or this limit is not equal to \( J^\mu(x^-, x_\perp) \). Therefore the canonical light cone quantization is incompatible with the existence of the canonical current operator.

Let us also note that if the theory should be invariant under the space reflection or time reversal, the corresponding representation operators in the front form \( \hat{U}_P \) and \( \hat{U}_T \) are necessarily interaction dependent (this is clear, for example, from the relations \( \hat{U}_P P^+ \hat{U}_P^{-1} = \hat{U}_T P^+ \hat{U}_T^{-1} = \hat{P}^- \)). In terms of the operator \( \hat{J}^\mu \) one can say that this operator should satisfy the conditions

\[
\hat{U}_P (\hat{J}^0, \hat{J}) \hat{U}_P^{-1} = \hat{U}_T (\hat{J}^0, \hat{J}) \hat{U}_T^{-1} = (\hat{J}^0, -\hat{J})
\]  

(24)

Therefore it is not clear whether these conditions are compatible with the relation \( \hat{J}^\mu = J^\mu \). However this difficulty is a consequence of the difficulty with Eq. (2) since, as noted by Coester [14], the interaction dependence of the operators \( \hat{U}_P \) and \( \hat{U}_T \) in the front form does not mean that there are discrete dynamical symmetries in addition to the rotations about transverse axes. Indeed, the discrete transformation \( P_2 \) such that \( P_2 x := \{x^0, x_1, -x_2, x_3\} \) leaves the light front \( x^+ = 0 \) invariant. The full space reflection \( P \) is the product of \( P_2 \) and a rotation about the 2-axis by \( \pi \). Thus it is not an independent dynamical transformation in addition to the rotations about transverse axes. Similarly the transformation \( TP \) leaves \( x^+ = 0 \) invariant and \( T = (TP)P_2R_2(\pi) \).

4 Discussion

We have shown that if the representation generators of the Poincare group and the current operators are constructed in the framework of the canonical formalism then in the general case the current operators do not satisfy Lorentz invariance. One might think that owing to some correction of the canonical formalism the current operator will remain unchanged but the operators \( V^\mu \) and \( V^{\mu\nu} \) will change in such a way that the conditions (14) and
in the instant form or (23) in the front one will be simultaneously satisfied. However, as argued in the preceding section, there is no reason to believe that this may occur. Therefore we conclude that in the general case the canonical current operator does not exist and the Schwinger terms are insufficient to describe the most singular part of the equal time commutation relations. Let us discuss the consequences of these conclusions.

One of the problems considered in the literature (see, for example, Ref. [15]) is the problem of constructing the covariant $T$-product $T^*(\hat{J}^\mu(x),\hat{J}^\nu(y))$. It has been shown that in the presence of Schwinger terms the standard $T$-product $T(\hat{J}^\mu(x),\hat{J}^\nu(y))$ is not covariant but it is possible to add to $T(\hat{J}^\mu(x),\hat{J}^\nu(y))$ a contact term (which is not equal to zero only if $x^0 = y^0$) such that the resulting $T^*$-product will be covariant. In these investigations it was always assumed that the Lorentz invariance condition (3) for $\hat{J}^\mu(x)$ is compatible with Schwinger terms. However, in view of the above discussion the existence of the equal time commutators cannot be guaranteed in the general case, and if the commutators exist it is not clear whether the main singularities are exhausted by the Schwinger terms.

Let us now briefly consider the application of current algebra to deep inelastic scattering (DIS). The hadronic tensor in DIS is usually written as

$$ W^{\mu\nu} = \frac{1}{4\pi} \int e^{ixq} \langle N| [\hat{J}^{\mu}(\frac{x}{2}), \hat{J}^{\nu}(-\frac{x}{2})]|N\rangle d^4x \tag{25} $$

where $|N\rangle$ is the state of the initial nucleon and $q$ is the 4-momentum of the virtual photon, $W$ or $Z$ boson.

The argument of many authors is that since the quantity $q$ in DIS is very large then only the region of small $x$ contributes to the integral (25), and due to asymptotic freedom the current operators in Eq. (25) can be approximately taken free while the corrections to such obtained expressions are of order $\alpha_s(q^2)$ (where $\alpha_s(q^2)$ is the QCD running coupling constant). In particular, the well-known sum rules derived from current algebra (see, for example, Refs. [16]) are valid. However, as noted in Sec. 1, the operators $V^\mu$ and $V^{\mu\nu}$ in Eqs. (1-3) by no means can be considered as small perturbations of the free operators $P^\mu$ and $M^{\mu\nu}$ respectively (see Ref. [17] for more details).

The difficulties with equal time commutation relations were one of the reasons for the development of the operator product expansion (OPE) [18]. In the framework of the OPE one can consider $[\hat{J}^{\mu}(\frac{x}{2}), \hat{J}^{\nu}(-\frac{x}{2})]$ not only when
\( x^0 = 0 \) but also when \( x^2 \to 0 \). Symbolically this commutator is written as

\[
[\hat{J}(x^2), \hat{J}(-x^2)] = \sum_{i,n} C^i_n(x^2) x_{\mu_1} \cdots x_{\mu_n} \hat{O}^{\mu_1 \cdots \mu_n}
\]

where \( C^i_n(x^2) \) are the Wilson coefficient \( c \)-number functions and the \( \hat{O}^{\mu_1 \cdots \mu_n} \) are limits of some regular operators \( \hat{O}^{\mu_1 \cdots \mu_n}(x/2, -x/2) \) when \( x \to 0 \).

The expansion (26) has been proved only for a few models in the framework of renormalized perturbation theory \([13, 4]\). Meanwhile this expansion is usually used when perturbation theory does not apply, for example in DIS. Since QCD beyond perturbation theory is very complicated, the investigations whether the OPE is valid beyond this theory were carried out mainly in two-dimensional models (see Ref. \([20]\) and references therein) and there is no agreement between different authors whether the validity of the OPE beyond perturbation theory can be reliably established. Besides, the authors of Ref. \([20]\) did not consider the restrictions imposed on the current operator by its commutation relations with the representation operators of the Poincare group. It is important to note that the Poincare group in 1+3 spacetime is much more complicated than in 1+1 one (for example, the Lorentz group in 1+1 spacetime is one-dimensional), and, as noted in the preceding section, even in 1+3 spacetime the case of particles with spin is much more complicated than the case of spinless particles.

Let us consider DIS in the frame of reference where the \( z \) component of the momentum of the initial nucleon is positive and very large. This frame is usually called the infinite momentum frame (IMF). It is clear that the IMF can be obtained by choosing the four-vector \( \lambda \) as in Eq. (11) and taking the limit \( v \to 1 \). Therefore one can expect that the generators of the Poincare group in the IMF are given in the front form. This also follows from the following facts. The “minus” component of the four-vector \( q \) in the IMF is very large and much greater than the other components. Since \( qx = q^- x^+ + q^+ x^- - q_\perp x_\perp \), the integration in Eq. (25) is carried out over the region of very small \( x^+ \): \( x^+ \sim 1/q^- \) (strictly speaking such a conclusion is valid only if we consider only Fourier transforms of smooth functions while the Wilson coefficients are singular). Therefore the integration in Eq. (25) involves a small vicinity of the light cone \( x^+ = 0 \), and, as noted in Sec. 2, the light cone quantization leads to the front form.

Since the generators \( M^{-j} \) \( (j = 1, 2) \) in the front form are interaction dependent and the operators \( \hat{O}^{\mu_1 \cdots \mu_n}_i \) should transform as the tensors of the \( n \)-th
rank, these operators should properly commute with the $\hat{M}^{-j}$. Therefore, as follows from Eq. (26), each $\hat{O}$-operator should be interaction dependent, i.e. $\hat{O}^{\mu_1\cdots\mu_n}_{i} \neq O^{\mu_1\cdots\mu_n}_{i}$ for all $\mu_1 \cdots \mu_n$ and $i$. The equality $\hat{O}^{\mu_1\cdots\mu_n}_{i} = O^{\mu_1\cdots\mu_n}_{i}$ for all $\mu_1 \cdots \mu_n$ and $i$ takes place only if the current operator is free, i.e. in the parton model. Indeed, as shown by many authors (see, for example, Ref. [21] and references therein), the parton model is a consequence of the impulse approximation ($\hat{J}^\mu(x) = J^\mu(x)$) in the front form of dynamics. We see that the parton model is incompatible with Lorentz invariance (see Ref. [17] for details). The fact that $\hat{O}^{\mu_1\cdots\mu_n}_{i} \neq O^{\mu_1\cdots\mu_n}_{i}$ also follows from the factorization property of the OPE, according to which the Wilson coefficients are responsible for the hard part of the hadronic tensor and the operators $\hat{O}^{\mu_1\cdots\mu_n}_{i}$ are responsible for the soft part which cannot be determined in the framework of perturbation theory (see, for example, Ref. [22] and references therein).

If Eq. (26) is valid and nothing is known about the operators $\hat{O}^{\mu_1\cdots\mu_n}_{i}$ then the theory makes it possible to determine only the $q^2$ evolution of the moments of the structure functions. However there exist a very few cases when the theory claims that the values of the moments also can be calculated. In particular, to lowest order in $\alpha_s(q^2)$ the theory reproduces the well-known sum rules [16] and then the OPE makes it possible to determine their $q^2$ evolution.

Let us consider the OPE to lowest order in $\alpha_s(q^2)$. Then the conventional basis for twist two operators (see, for example, Ref. [23]) is

$$\hat{O}^{\mu_1\cdots\mu_n}_V = SN\{\hat{\psi}(0)\gamma^\mu D^\mu_1 \cdots D^\mu_n \hat{\psi}(0)\},$$

$$\hat{O}^{\mu_1\cdots\mu_n}_A = SN\{\hat{\psi}(x)\gamma^5 \gamma^\mu D^\mu_1 \cdots D^\mu_n \hat{\psi}(x)\}$$

(27)

where $D^\mu$ is the covariant derivative, the operator $S$ makes the tensor traceless and for simplicity we do not write flavor operators and color and flavor indices. In particular, it is claimed that the operator $\hat{O}^{\mu}_V$ is equal to the electromagnetic current operator $\hat{J}^\mu(0)$ (see Eq. (3)).

The conclusion that to lowest order in $\alpha_s(q^2)$ the basis for twist two operators is given by Eq. (27) follows from the assumption that since the operators $\hat{O}(x,y)$ are the basis of the expansion of the product of two local operators depending on $x$ and $y$ then the $\hat{O}(x,y)$ are not only regular when $x,y \to 0$ but also bilocal in $x$ and $y$, i.e. they depend only on fields and their derivatives at the points $x$ and $y$. However it is obvious that gauge invariance does not allow the operators in (27) to be limits of some bilocal
operators (this is clear, for example, from Eq. (13)). When we expand the product of two local operators over some basis, we connect the points $x$ and $y$ by fields propagating from $x$ to $y$ and therefore in the general case the interaction dependent operator $\hat{O}(x, y)$ is nonlocal. It is also important to note that since some of the coefficients $C_n^i((x - y)^2)$ are singular they cannot be expanded over the products of functions depending on $x$ and $y$.

The operators (27) indeed form the basis of the expansion (26) in the approximation when the current operator is free, i.e. in the parton model. However in the general case the operators $\hat{O}_i^{\mu_1 \cdots \mu_n}$ describe the contribution of the soft part of the interaction between quark and gluons to the expansion (26), and the lowest order in $\alpha_s(q^2)$ by no means implies that these operators have the same functional form as the free operators.

As shown in the preceding section, the canonical operator $N\{\hat{\bar{\psi}}(0)\gamma^\mu \hat{\psi}(0)\}$ does not transform as a vector operator and therefore if the operators in Eq. (27) are canonical then these operators do not transform as the corresponding tensor operators. This implies that the limit of $\hat{O}_i^{\mu_1 \cdots \mu_n}(x/2, -x/2)$ cannot be equal to the corresponding canonical operator in Eq. (27).

Let us suppose that the operator $\hat{O}_V^\mu$ depends on the interaction between quarks and gluons at distances of order $1/\Lambda$. The QCD running coupling constant at these distances $\alpha_s(\Lambda^2)$ is by no means small and the perturbative expansion over $\alpha_s(\Lambda^2)$ is meaningless. During their propagation from $x/2$ to $-x/2$ the quark and gluon fields can produce many virtual particles at distances $\sim 1/\Lambda$. Therefore in the general case the limit of $\hat{O}_V^\mu(x/2, -x/2)$ will depend not only on the field operators at $x = 0$ but also on the integrals of these operators over a region where these particles can be produced. As argued in the preceding section, it is not clear whether the limit of $\hat{J}_V^\mu(x)$ when $x \to 0$ exists, and, if it exists, it does not coincide with the canonical operator $J^\mu(0)$. In addition, the analogous considerations applied to $\hat{J}_V^\mu(0)$ show that it is not clear whether it is possible to construct the local operator $\hat{J}^\mu(x)$ beyond perturbation theory. As a result, neither $\hat{O}_V^\mu$ nor $\hat{J}_V^\mu(0)$ can be equal to the corresponding expression in Eq. (27). However there is no rule which prescribes the equality of the operators $O_V^\mu$ and $\hat{J}^\mu(0)$. Meanwhile the well-known sum rules (16) (which originally were derived from the equal time commutation relations) are based on the equalities of some $\hat{O}$-operators and the corresponding current operators at $x = 0$. Generally speaking such equalities take place only in the parton model and therefore we conclude that
the sum rules are not substantiated.

In Ref. [17] a model in which the current operator explicitly satisfies Eqs. (1-3) has been considered. It has been shown that the sum rules [16] are not satisfied in this model. Let us stress that our considerations do not exclude a possibility that for some reasons there may exist sum rules which are satisfied with a good accuracy. However the statement that the sum rules [16] unambiguously follow from QCD is not substantiated (it is also worth noting that these sum rules were derived when QCD did not exist).

In the present paper we considered the problem of constructing the current operators in the framework of the canonical quantization of field operators on some hypersurface. However in this case, in addition to the difficulties discussed above, there exists the difficulty which is ignored by many authors: once we assume that the field operators on this hypersurface are free we immediately are in conflict with the Haag theorem [24, 6].

Suppose we have constructed the operators \( \hat{P}^\mu, \hat{M}^{\mu\nu} \) in the point form not using the quantization on some hypersurface. As noted in Sec. 1, the notion of current is not necessary, and, as follows from above considerations, the possibility of constructing the local operator \( \hat{J}^\mu(x) \) beyond perturbation theory is problematic. For these reasons let us also suppose that the current operator can be treated only according to Eq. (5). Then the conditions (1-3) are automatically satisfied if \( \hat{J}^\mu = J^\mu \). The calculations of DIS on the nucleon and deuteron assuming that this relation is valid have been carried out in Refs. [17, 25]. The fact that in the general case the relation \( \hat{J}^\mu = J^\mu \) is incompatible with Lorentz invariance was pointed out in connection with the investigation of relativistic effects in few-quark and few-nucleon systems (see, for example, Ref. [26]).

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