Multipartite electronic entanglement purification with charge detection

Yu-Bo Sheng¹, Fu-Guo Deng², and Gui-Lu Long¹,³,⁴

¹ Department of Physics, Tsinghua University, Beijing 100084, China
² Department of Physics, Beijing Normal University, Beijing 100875, China
³ Key Laboratory for Atomic and Molecular NanoSciences, Tsinghua University, Beijing 100084, China
⁴ Tsinghua National Laboratory for Information Science and Technology, Beijing 100084, China

(Dated: April 29, 2013)

We present a multipartite entanglement purification scheme in a Greenberger-Horne-Zeilinger state for electrons based on their spins and their charges. This scheme works for purification with two steps, i.e., bit-flipping error correction and phase-error flip error correction. By repeating these two steps, the parties in quantum communication can get some high-fidelity multipartite entangled electronic systems.

Keywords: Quantum physics, Entanglement purification, Multipartite, Electrons, Charge detection, Decoherence

PACS numbers: 03.67.Pp Quantum error correction and other methods for protection against decoherence - 03.67.Hk Quantum communication

I. INTRODUCTION

Quantum entanglement is of vital importance in achieving tasks of quantum information processing and transmission [1]. Most of the practical quantum computation and quantum communication tasks require that the separated parties in distant locations share the maximally entangled state $\rho_{\text{max}}$ [2–10]. Especially, multipartite entangled states have many important applications in quantum computation [1] and quantum communication, such as controlled teleportation [11, 12], quantum secret sharing [13–15], quantum state sharing [16–20], and so on. All these tasks require multipartite entangled states to set up the quantum channel between legitimate participants in quantum communication. However, with local operations and classical communication, the two users in quantum communication can not create entanglement. If they want to share the entanglement separately, they have to create the entangled states locally and transmit them in a quantum channel. In a practical transmission, the channel noise cannot be avoided, which will make a maximally entangled state become a mixed one. This will decrease the fidelity of quantum teleportation or make quantum communication insecure.

Entanglement purification provides us a powerful tool to distil high-fidelity entangled states from less entangled ones [21–33]. The first entanglement purification protocol, which is based on the quantum controlled-not (CNOT) logic operations, was proposed by Bennett et al. [21] in 1996 for purifying a two-qubit Werner state. So far, most of entanglement purification protocols [21–29] are focused on bipartite entangled quantum systems and there are only several multipartite entanglement protocols, including high-dimension entanglement protocols [30–33] as the latter is more complicated than the former. In 1998, Murao et al. [30] proposed the first multipartite entanglement purification protocol with CNOT logic operations for quantum systems originally in a Greenberger-Horne-Zeilinger (GHZ) state. This protocol was extended to high-dimensional multipartite quantum systems in 2007 [32]. They use some generalized XOR gates to substitute the CNOT gates to fulfill their protocol.

Until now, most of the purification protocols are based on the photons as they are manipulated easily. On the other hand, conduction electrons can also be used to achieve quantum communication and computation processes since Beenakker et al. [34] broke through the obstacle of the no-going theorem [35] in 2004. An electron acts as a qubit in both charge degree of freedom and spin degree of freedom. If one measures the charge degree of freedom of an electron quantum system, it will leave its spin degree of freedom unaffected. By means of charge detections [36], Beenakker et al. [34] designed a protocol for a CONT gate between electronic qubits. Moreover, people have constructed entangled spins [37], achieved the entanglement concentration [38], prepared cluster states and designed a multipartite entanglement analyzer [39] with charge detections of electron quantum systems. A bipartite entanglement purification protocol was also presented in 2005 [40], although their protocol is only used to purify the Werner state, similar to the original entanglement purification protocol by Bennett et al. for photon pairs. After each purification step, they have to add another bilateral $\pi/2$ operations to recover the mixed state to Werner state in order to perform the next purification step, which make its efficiency low [22].

In this paper, we present a multipartite entanglement purification for electrons in a Greenberger-Horne-Zeilinger state based on their spins and their charges, resorting to charge detections and electron polarizing beam...
splitters. In this scheme, charge detections play the role of a parity check. The whole purification scheme can be divided into two steps, i.e., bit-flip error purification and phase-flip error purification. It does not need to add another operation to recover the mixed state to Werner state after each purifying step. That is, it is easier than the only one entanglement purification scheme for two-electron system [10], and it will have a practical application in solid quantum computation and communication.

II. MULTIPARTITE ELECTRONIC ENTANGLEMENT PURIFICATION WITH CHARGE DETECTIONS

Before we start to explain our purification scheme, we first introduce a basic element for entanglement purification scheme, i.e., a parity-check gate. Parity-check gates can be used to construct a Bell-state analyzer and a CNOT gate [14]. In Fig.1, the polarizing beam splitter (PBS) is used to transmit an electron in the spin-up state $|\uparrow\rangle$ and reflect an electron in the spin-down state $|\downarrow\rangle$. The charge detector (C) can distinguish the occupation number one from the occupation number 0 and 2, but it cannot distinguish the electron numbers between 0 and 2. That is, it can distinguish the occupation number even or odd. Let us suppose that two polarization qubits $a$ and $b$ initially in the states

$$|\Psi_a\rangle = \alpha_1|\uparrow\rangle + \alpha_2|\downarrow\rangle$$

and

$$|\Psi_b\rangle = \beta_1|\uparrow\rangle + \beta_2|\downarrow\rangle.$$  

Here

$$|\alpha_1|^2 + |\alpha_2|^2 = 1,$$

$$|\beta_1|^2 + |\beta_2|^2 = 1.$$  

These two qubits are transmitted into the spatial modes $a_1$ and $b_1$, respectively, and they interact with each other on the PBS. The whole state of the two electrons will evolve to

$$|\Psi_T\rangle_{ab} = \alpha_1\beta_1|\uparrow\uparrow\rangle_{ab} + \alpha_1\beta_2|\uparrow\downarrow\rangle_{ab} + \alpha_2\beta_1|\downarrow\uparrow\rangle_{ab} + \alpha_2\beta_2|\downarrow\downarrow\rangle_{ab}. $$

One observes immediately that the states $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ will lead the charge detection to have the charge occupation number $C = 1$ as each electron passes through a different path after the first PBS. The states $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ will lead the charge detection to $C = 0$ and $C = 2$, respectively. The charge detection cannot distinguish 0 and 2, and it will show the same result, i.e., $C = 0$ for simplicity. The states $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ can be distinguished from $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ by the different outcomes of the charge detection. So this device can be used to accomplish a parity check on a two-electron system. That is, one can get $\alpha_1\beta_1|\uparrow\uparrow\rangle + \beta_2|\uparrow\downarrow\rangle$ from the output $a_2b_2$ if $C = 1$ and get $\alpha_1\beta_2|\uparrow\downarrow\rangle + \alpha_2\beta_1|\uparrow\uparrow\rangle$ if $C = 0$.

Now, let us detail how this entanglement purification scheme works for multipartite electron systems. A multipartite Greenberger-Horne-Zeilinger (GHZ) state for spin 1/2 systems can be described as

$$|\phi^+\rangle_a = \frac{1}{\sqrt{2}}(|\uparrow\cdots\uparrow\rangle + |\downarrow\cdots\downarrow\rangle).$$

We first take three-particle GHZ states as an example to show the principle of this multipartite entanglement purification scheme and then extend to the case of N-particle systems. There are eight three-particle GHZ states, i.e.,

$$|\Phi^\pm\rangle_{ABC} = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\rangle \pm |\downarrow\downarrow\downarrow\rangle)_{ABC},$$

$$|\Phi^+_1\rangle_{ABC} = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\uparrow\rangle \pm |\uparrow\downarrow\uparrow\rangle)_{ABC},$$

$$|\Phi^+_2\rangle_{ABC} = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\uparrow\rangle \pm |\uparrow\uparrow\downarrow\rangle)_{ABC},$$

$$|\Phi^+_3\rangle_{ABC} = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\downarrow\rangle \pm |\uparrow\downarrow\downarrow\rangle)_{ABC}. $$

Here the subscripts A, B, and C represent the three electrons belonging to the three parities, say Alice, Bob, and Charlie, respectively. Initially, we suppose that the original GHZ state transmitted is $|\Phi^+\rangle_{ABC}$. The noisy channel will degrade the state and make the initial state be a mixed one. For example, the state $|\Phi^+\rangle_{ABC}$ may become $|\Phi^+\rangle_{ABC}$, say a bit-flip error, or become $|\Phi^-\rangle_{ABC}$, say a phase-flip error. Sometimes both a bit-flip error and a phase-flip error will take place such as $|\Phi^2\rangle_{ABC}$. So the task of purifying three-electron entangled systems can be divided into two step, i.e., purifying the bit-flip error and the phase-flip error. The principle of this multipartite entanglement purification for electron systems is shown in Fig.2.
with a probability of \( \frac{1}{2} F^2 \) and

\[
|\phi_1\rangle = \frac{1}{\sqrt{2}} (|\downarrow\downarrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\downarrow\downarrow\rangle) A_1 B_1 C_1 A_2 B_2 C_2
\]

with a probability of \( \frac{1}{2}(1 - F)^2 \). The cross-combinations \( |\Phi^+\rangle \otimes |\Phi^+_1\rangle \) and \( |\Phi^+_1\rangle \otimes |\Phi^+\rangle \) never lead all the three parties to have the same parity and can be eliminated automatically. In the spatial modes \( b_1 b_2 b_3 \), Hadamard \((H)\) operations are performed on the electrons \( A_2 B_2 C_2 \), which will lead to the transformation

\[
|\uparrow\rangle \rightarrow \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle),
\]

\[
|\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}} (|\uparrow\rangle - |\downarrow\rangle).
\]

After performing an \( H \) operation on each of the electrons \( A_2 B_2 C_2 \), Eq.(8) and Eq.(9) become

\[
|\phi\rangle' = \frac{1}{4} [(|\uparrow\uparrow\uparrow\rangle(|\uparrow\rangle + |\downarrow\rangle)^{\otimes 3} + |\downarrow\downarrow\downarrow\rangle(|\uparrow\rangle - |\downarrow\rangle)^{\otimes 3}]
\]

\[
= \frac{1}{4} [(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle) + (|\uparrow\uparrow\uparrow\rangle + |\downarrow\down\down\rangle)(|\uparrow\rangle - |\down\rangle) + (|\up\rangle + |\down\rangle)(|\up\rangle + |\down\rangle)]
\]

\[
|\phi_1\rangle' = \frac{1}{4} [(|\down\rangle - |\up\rangle)(|\up\rangle + |\down\rangle)(|\up\rangle + |\down\rangle)^{\otimes 3} + (|\up\rangle + |\down\rangle)(|\up\rangle + |\down\rangle)^{\otimes 3}]
\]

\[
= \frac{1}{4} [(|\down\rangle - |\up\rangle)(|\up\rangle + |\down\rangle)(|\up\rangle + |\down\rangle) + (|\up\rangle + |\down\rangle)(|\up\rangle - |\down\rangle) + (|\up\rangle + |\down\rangle)(|\up\rangle + |\down\rangle) - (|\up\rangle - |\down\rangle)(|\up\rangle - |\down\rangle)]
\]

Finally, Alice, Bob, and Charlie measure the polarization states of their electrons \( A_2 B_2 C_2 \) in the modes \( b_1 b_2 b_3 \). If they obtain the outcomes \(|\up\down\up\rangle A_2 B_2 C_2 \) or \(|\down\up\up\rangle A_2 B_2 C_2 \), they will obtain the GHZ state \( \sqrt{2}\phi_1 \) with a probability of \( \frac{1}{2} F^2 \) and the GHZ state \( \frac{1}{\sqrt{2}} (|\up\up\up\rangle + |\down\down\down\rangle) A_1 B_1 C_1 \) with a probability of \( \frac{1}{2}(1 - F)^2 \). That is, Alice, Bob, and Charlie will get a new ensemble \( \rho' \) with the fidelity of \( F' = \frac{F^2}{F^2 + (1 - F)^2} > F \) when \( F > 1/2 \) by keeping the electron systems \( A_1 B_1 C_1 \) if they only obtain one of the four outcomes \( \{|\up\down\up\rangle A_2 B_2 C_2 \}, \{|\down\up\up\rangle A_2 B_2 C_2 \}, \{|\down\down\up\rangle A_2 B_2 C_2 \}, \{|\up\up\down\rangle A_2 B_2 C_2 \} \). Certainly, they will get the outcomes \( \{|\up\down\up\rangle, \up\down\up\rangle, \down\up\up\rangle \} \) or \( \{|\down\down\down\rangle, \down\down\down\rangle \} \). In this time, Alice, Bob, and Charlie need only flip the relative phase of the electron system \( A_1 B_1 C_1 \) and will obtain the ensemble \( \rho' \).

On the other hand, if Alice, Bob, and Charlie all obtain an odd parity, they will get the state

\[
|\phi\rangle^o = \frac{1}{\sqrt{2}} (|\down\down\down\down\down\rangle + |\up\up\up\up\up\rangle) A_1 B_1 C_1 A_2 B_2 C_2
\]

with a probability of \( \frac{1}{2} F^2 \) and the state

\[
|\phi_1\rangle^o = \frac{1}{\sqrt{2}} (|\down\down\down\down\down\rangle + |\up\up\up\up\up\rangle) A_1 B_1 C_1 A_2 B_2 C_2
\]
with a probability of $\frac{1}{2}(1-F)^2$. Compared with Eq.(8) and (9), Alice, Bob, and Charlie only need to add a bit-flip operation on each of the three qubits $A_1B_2C_2$ and they can get the same result as that with the even parity.

By far, we have discussed the principle of the purification of the bit-flip errors for three-electron systems. This method can also be extended to purify the bit-flip errors in multipartite entangled systems. For example, the initial state of an $N$-electron quantum system can be described as:

$$|\Phi^+\rangle_N = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\cdots\uparrow\rangle + |\downarrow\downarrow\cdots\downarrow\rangle).$$

Suppose a bit-flip error may take place in the first electron. After transmission, each party makes a parity check on his two electrons coming from two entangled quantum systems and they all choose the same parity by classical communication, shown in Fig.2. Now, let us suppose they all choose the even parity case and then the original mixed state system becomes

$$|\phi^+\rangle_{2N} = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\cdots\uparrow\rangle + |\downarrow\downarrow\cdots\downarrow\rangle)$$

with a probability of $\frac{1}{2}F^2$ and

$$|\phi_1^+\rangle_{2N} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\cdots\uparrow\downarrow\cdots\uparrow\rangle + |\downarrow\uparrow\cdots\downarrow\uparrow\cdots\downarrow\rangle)$$

with a probability of $\frac{1}{2}(1-F)^2$. After the H operations on the $b_1b_2\cdots b_N$ modes, Eq.(10) and Eq.(17) become

$$|\phi^+_1\rangle_{2N} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\cdots\uparrow\downarrow\cdots\uparrow\rangle + |\downarrow\uparrow\cdots\downarrow\uparrow\cdots\downarrow\rangle)$$

$$|\phi_1^+_1\rangle_{2N} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\cdots\uparrow\downarrow\cdots\uparrow\rangle + |\downarrow\uparrow\cdots\downarrow\uparrow\cdots\downarrow\rangle)$$

After the measurements on the electrons in the modes $b_1b_2\cdots b_N$ with the basis $Z=\{|\uparrow\rangle, |\downarrow\rangle\}$, the parties will obtain a new ensemble $\rho''$ in which the fidelity of the state $|\Phi^+\rangle_N$ is $\frac{F^2}{F^2+(1-F)^2}$ if the number of $|\downarrow\rangle$ is even. They will get the same result with a phase-flip operation on each $N$-electron system kept if the number of the outcomes $|\downarrow\rangle$ is odd.

We have fully described the principle of bit-flip error purification on the first electron. If the bit-flip error takes place on other electrons, we can purify these errors in the same way and will get the same result like those discussed above.

B. Purification of phase-flip errors

Now we start to explain the principle of the phase-flip error purification in the present scheme. Usually, during the transmission, the relative phase between several entangled electrons is sensitive to path length instabilities, which have to be kept constant. This problem is analogous to the optical system for quantum communications [11] [12]. Phase-flip errors cannot be purified directly, but it can be converted into bit-flip errors. For example, the eight GHZ states shown in Eq.(6) can be transformed into the following eight states by adding an $H$ operation on each electron, i.e.,

$$|\Psi^+\rangle = \frac{1}{2}(|\uparrow\uparrow\cdots\uparrow\rangle + |\downarrow\downarrow\cdots\downarrow\rangle + |\downarrow\downarrow\cdots\downarrow\rangle + |\uparrow\uparrow\cdots\uparrow\rangle),$$

$$|\Psi^-\rangle = \frac{1}{2}(|\uparrow\uparrow\cdots\uparrow\rangle + |\downarrow\downarrow\cdots\downarrow\rangle + |\downarrow\downarrow\cdots\downarrow\rangle + |\uparrow\uparrow\cdots\uparrow\rangle),$$

$$|\Psi^+_1\rangle = \frac{1}{2}(|\uparrow\uparrow\cdots\uparrow\rangle + |\uparrow\down\cdots\down\rangle - |\down\up\cdots\up\rangle - |\down\down\cdots\down\rangle),$$

$$|\Psi^-_1\rangle = \frac{1}{2}(|\uparrow\up\cdots\up\rangle + |\down\down\cdots\down\rangle - |\down\down\cdots\down\rangle - |\up\up\cdots\up\rangle),$$

$$|\Psi^+_2\rangle = \frac{1}{2}(|\up\up\cdots\up\rangle - |\down\down\cdots\down\rangle + |\down\down\cdots\down\rangle + |\up\up\cdots\up\rangle),$$

$$|\Psi^-_2\rangle = \frac{1}{2}(|\up\down\cdots\down\rangle - |\down\up\cdots\up\rangle - |\down\down\cdots\down\rangle + |\up\up\cdots\up\rangle),$$

$$|\Psi^+_3\rangle = \frac{1}{2}(|\up\down\cdots\down\rangle - |\down\up\cdots\up\rangle - |\down\down\cdots\down\rangle + |\up\up\cdots\up\rangle),$$

$$|\Psi^-_3\rangle = \frac{1}{2}(|\up\down\cdots\down\rangle - |\down\up\cdots\up\rangle - |\down\down\cdots\down\rangle + |\up\up\cdots\up\rangle).$$

Suppose a phase-flip error may occur in the first electron and the initial state after the electron systems are transmitted over a noisy channel becomes a mixed state as follows

$$\rho_p = F|\Phi^+\rangle\langle\Phi^+| + (1-F)|\Phi^-\rangle\langle\Phi^-|.$$ (21)

After the transformation by adding an $H$ operation on each electron, Eq.(21) becomes

$$\rho_p = F|\Psi^+\rangle\langle\Psi^+| + (1-F)|\Psi^-\rangle\langle\Psi^-|.$$ (22)

It is interesting to find that in the state $|\Psi^+\rangle$, the number of $|\down\rangle$ in each items is even, but it is odd in the state $|\Psi^-\rangle$. We also find that all the GHZ states with the superscript + in each item is even but have the odd number of $|\down\rangle$ for -.

Now we detail the principle of the phase-flip error purification, shown in Fig.2. For two pairs $A_1B_1C_1$ and $A_2B_2C_2$ picked out from the ensemble $\rho_p$, their state can be viewed as the mixture of four pure states: $|\Psi^+\rangle\otimes|\Psi^+\rangle$, $|\Psi^+\rangle\otimes|\Psi^-\rangle$, $|\Psi^-\rangle\otimes|\Psi^+\rangle$, and $|\Psi^-\rangle\otimes|\Psi^-\rangle$ with the probabilities of $F^2$, $F(1-F)$, $F(1-F)$, and $(1-F)^2$, respectively. Each party makes a parity-check measurement on his two electrons with charge detection and then all parties check their results by classical communication. They only choose the case that all of them get the even parity and they discard the other cases. In this way, the cross-combinations $|\Psi^+\rangle\otimes|\Psi^-\rangle$ and $|\Psi^-\rangle\otimes|\Psi^+\rangle$ are eliminated automatically and the remaining items are

$$|\varphi\rangle = \frac{1}{2}(|\up\up\up\up\up\rangle + |\down\up\down\up\up\rangle + |\down\down\up\up\up\rangle + |\down\down\down\down\rangle)_{A_1B_1C_1A_2B_2C_2}.$$ (23)
and
\[
|\varphi\rangle = \frac{1}{2}(|↑↑↑↑↓⟩ + |↑↑↑↑↑⟩ + |↑↑↑↑↑⟩ + |↓↓↓↓↓⟩)A_{1}B_{1}C_{1}A_{2}B_{2}C_{2}.
\]

with the probabilities of \(F^2\) and \((1 - F)^2\), respectively. In order to get the three-electron entangled state \(|\Psi^+\rangle\), the parties first perform an \(H\) operation on each of the three electrons \(A_{2}B_{2}C_{2}\), which will make \(|\varphi\rangle\) and \(|\varphi\rangle\) evolve as
\[
|\varphi\rangle \rightarrow \frac{1}{\sqrt{2}}(|↑↑↑⟩A_{2}B_{2}C_{2}(|↑↑↑⟩ + |↑↑↓⟩ + |↑↓↑⟩ + |↓↑↑⟩)A_{1}B_{1}C_{1}C_{1}A_{2}B_{2}C_{2}.
\]

From Eq. (25) and Eq. (26), if the number of the outcome \(\downarrow\) in the measurements on \(A_{2}B_{2}C_{2}\) is even, i.e., \(|↑↑↑⟩, |↑↑↓⟩, |↑↓↑⟩,\) or \(|↓↑↑⟩\), the three parties can get \(|\psi^+\rangle\) with the fidelity of \(\sqrt{\frac{F^2}{2+1-F^2}}\). Otherwise, if it is odd, the three parties will get the state \(|\psi^-\rangle\) with the fidelity of \(\frac{1}{\sqrt{2+1-F^2}}\). The remaining state of \(|\psi^+\rangle\) or \(|\psi^-\rangle\) can be transformed into \(|\Phi^+\rangle\) or \(|\Phi^-\rangle\) by adding another \(H\) operation on each electron.

III. DISCUSSION AND SUMMARY

So far, we have fully described our purification scheme for multiparticle electronic entangled states. We have explained the entanglement purification principle for multiparticle electron systems with a special density matrices. That is, we have discussed the principle of our entanglement purification protocol for purifying the bit-flip error on the first particle and the phase-flip error, shown in Eqs. (17) and (21). In fact, in a practical environment, the mixed state may be the Werner-type state, or more complicated. For instance, a general three-particle mixed state can be written as:
\[
\rho_9 = F|\Phi^+\rangle\langle\Phi^+| + F_1|\Phi^-\rangle\langle\Phi^-| + \cdots + F_7|\Phi_5\rangle\langle\Phi_5| (27)
\]
where \(F + F_1 + \cdots + F_7 = 1\). In order to increase the fidelity \(F\), we should purify each unwanted item like \(|\Phi^-\rangle\), \(|\Phi_5\rangle\), and \(|\Phi_5\rangle\). The state \(\rho_9\) contains both bit-flip errors and phase-flip errors. Alice, Bob, and Charlie can purify the bit-flip error on the first particle and then the bit-flip error on the second particle, whose principle is similar to the case with only the bit-flip error on the first particle. In this way, Alice, Bob, and Charlie can purify the bit-flip errors on an arbitrary position, as discussed in Ref. [30]. By an \(H\) operation on each particle, the phase-flip errors in Eq. (27) can be transferred into the bit-flip errors, which can be purified with the similar way to the latter. That is to say, a general mixed state of multiparticle entangled electron systems can be purified by repeating both the bit-flip-error purification and the phase-flip-error purification, similar to the original polarization entanglement purification scheme for multiparticle Boson systems [30].

Charge detection has played a prominent role in constructing the parity-check gate, and also it is the key element of the present purification protocol. It has been realized by means of point contacts in a two-dimensional electron gas. For instance, Ref. [33] used the effect of the electric field of the charge on the conductance of an adjacent point contact to realize the charge detection. Ref. [34] proposed a realization of a charge parity meter which is based on two double quantum dots alongside a quantum point contact. The realization of such a device can be seen as a specific example of the general class of mesoscopic quadratic quantum measurement detectors which is investigated by Mao et al. [45].

In summary, we have proposed a multipartite entanglement purification protocol for electron systems with the help of parity-check gates. We first use the electronic polarizing beam splitters and charge detections to
construct the parity check gate and then detail the multipartite entanglement purification for electron systems in GHZ state. The present scheme does not require the controlled-not gate and it works for purification with two steps, i.e., bit-flipping error correction and phase-error flip error correction. By repeating these two steps, the parties in quantum communication can get some high-fidelity multipartite entangled electronic systems. These features will make this scheme have a practical application in solid quantum computation and communication in the future.

ACKNOWLEDGEMENTS

Y.B.S. is supported by China Postdoctoral Science Foundation under Grant No. 20090460365 and China Postdoctoral Special Science Foundation under Grant No. 201003132. G.L.L. is supported by the National Natural Science Foundation of China under Grant No. 10874098 and the National Basic Research Program of China under Grant No. 2009CB929402. F.G.D. is supported by the National Natural Science Foundation of China under Grant No. 10974020 and A Foundation for the Author of National Excellent Doctoral Dissertation of PR China under Grant No. 200723.

[1] M.A. Nielsen, I.L. Chuang Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, UK, 2000).
[2] A.K. Ekert, Phys. Rev. Lett. 67 (1991) 661.
[3] C.H. Bennett, G. Brassard, N.D. Mermin, Phys. Rev. Lett. 68 (1992) 557.
[4] N. Gisin, G. Ribordy, W. Tittel, H. Zbinden, Rev. Mod. Phys. 74 (2002) 145.
[5] G.L. Long, X.S. Liu, Phys. Rev. A 65 (2002) 032302.
[6] F.G. Deng, G.L. Long, Phys. Rev. A 68 (2003) 042315.
[7] X.H. Li, F.G. Deng, H.Y. Zhou, Phys. Rev. A 78 (2008) 022321.
[8] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W.K. Wootters, Phys. Rev. Lett. 70 (1993) 1895.
[9] C.H. Bennett, S.J. Wiesner, Phys. Rev. Lett. 69 (1992) 2881.
[10] X.S. Liu, G.L. Long, D.M. Tong, L. Feng, Phys. Rev. A 65 (2002) 022304.
[11] A. Karlsson, M. Bourennane, Phys. Rev. A 58 (1998) 4394.
[12] F.G. Deng, C.Y. Li, Y.S. Li, H.Y. Zhou, Y. Wang, Phys. Rev. A 72 (2005) 022338.
[13] M. Hillery, V. Bužek, A. Berthiaume, Phys. Rev. A 59 (1999) 1829.
[14] A. Karlsson, M. Koashi, N. Imoto, Phys. Rev. A 59 (1999) 162.
[15] L. Xiao, G.L. Long, F.G. Deng, J.W. Pan, Phys. Rev. A 69 (2004) 052307.
[16] R. Cleve, D. Gottesman, H.K. Lo, Phys. Rev. Lett. 83 (1999) 648.
[17] A.M. Lance, T. Symul, W.P. Bowen, B.C. Sanders, P.K. Lam, Phys. Rev. Lett. 92 (2004) 177903.
[18] F.G. Deng, X.H. Li, C.Y. Li, P. Zhou, H.Y. Zhou, Phys. Rev. A 72 (2005) 044301.
[19] F.G. Deng, X.H. Li, C.Y. Li, P. Zhou, H.Y. Zhou, Europ. Phys. J. D 39 (2006) 459.
[20] X.H. Li, P. Zhou, C.Y. Li, H.Y. Zhou, F.G. Deng, J. Phys. B 39 (2006) 1975.
[21] C.H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J.A. Smolin, W.K. Wootters, Phys. Rev. Lett. 76 (1996) 722.
Lett. 84 (2004) 4617.

[44] B. Trauzettel, A.N. Jordan, C.W.J. Beenakker, M. Büttiker, Phys. Rev. B 73 (2006) 235331.

[45] W. Mao, D.V. Averin, R. Ruskov, A.N. Korotkov, Phys. Rev. Lett. 93 (2004) 056803.