Effects which will not blur the message of the \( ^1\text{H} \left( ^{11}\text{Li}, ^9\text{Li} \right) ^3\text{H} \) reaction: observation of phonon–exchange pairing correlations in nuclei

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Abstract. The results of the \( ^1\text{H} \left( ^{11}\text{Li}, ^9\text{Li} \right) ^3\text{H} \) experiment carried out recently at TRIUMF have given detailed information concerning the structure and the reaction mechanism at work in the case of halo, exotic nuclei, providing important stepping stones for an eventual quantitative description of superfluidity in finite nuclei. In fact, the central role played by renormalization processes as testified by the excitation of the \( ^1/2^- \) (2.69 MeV) member of the \( ^9\text{Li} \left( 2^+ \otimes p_{3/2}(\pi)^- \right) \) multiplet (nuclear structure) together with the post–prior symmetry (reaction), testify to the fact that a detailed knowledge of the “bare” quantities, like e.g. the NN–interaction is likely to be less important, in a unified Nuclear Field Theory description of the structure and the reaction aspects of the experiment, than previously thought.

1. Introduction
Understanding a many–body physical phenomenon in terms of symmetries and of conservation laws is of relevance because it makes, as a rule, calculations simpler. Furthermore, because it allows to find out which processes are allowed and which are not.

This applies equally well to structure as well as to reaction states, in keeping with the fact that the two fields can be viewed as the bound and the continuum expression of the same physics. A simple example of this statement being the single-particle motion of a bound neutron in the last bound orbit of \( ^{207}\text{Pb} \), and the elastic scattering of a neutron off the nucleus \( ^{206}\text{Pb} \).

A particular relevant embodiment of these concepts is provided by the two-particle transfer reaction \( ^1\text{H} \left( ^{11}\text{Li}, ^9\text{Li} \right) ^3\text{H} \) exciting the \( 3/2^- \) (ground state) and the first excited \( 1/2^- \), 2.69 MeV state of \( ^9\text{Li} \) [1]. In particular, when viewed in terms of the Nuclear Field Theory (NFT) formalism of structure [2, 3, 4] and reactions [5] used in the analysis of the experiment of Tanihata et al (see [6]). Within this context in the present contribution we refer to [6] and [7] for all technical details of the analysis.

2. Nuclear structure
Starting from the nuclear structure aspect of this reaction, it is well established that the ground state of \( ^{11}\text{Li} \) can be viewed as the nuclear realization of Cooper’s model [8] where the role of
the Fermi sea is played by the $^9\text{Li}$ core, and the two fermions at the Fermi surface are the halo neutrons. They interact through the bare as well as the induced interactions, arising from the exchange of dipole vibration of the halo with respect to the charged core (pigmy resonance), as well as of the quadrupole vibrations of this core [9], and both dipole and quadrupole vibrations are of importance for the single-particle sequence and binding energy of the weakly bound neutrons. Within this context, $^9\text{Li}$ does not only act on the Cooper pair in terms of the Pauli principle as in the original model of metal superconductivity, but also by providing, through quadrupole collective vibrations, part of the glue which binds the two neutrons. In other words, the ground state of $^{11}\text{Li}$ can be written as a combination of monopole and quadrupole pair addition modes (see refs. [10, 11] and refs. therein). A simple consequence of angular momentum conservation would imply in principle, and within this context, that also a $1^+$ pair addition component will be present in the $^{11}\text{Li}$ ground state wavefunction. However, such modes have not been found to display collective properties (see e.g. ref. [12]), thus making such a possibility physically non operative. In keeping with the above argument one can write,

$$|^{11}\text{Li}(gs; \frac{3}{2}^-) = \alpha \left[ \Gamma^+_{\nu} (\nu) a^+_{p\frac{1}{2}}(\pi) \right]_{\frac{3}{2}^-} |0\rangle + \beta \left[ \Gamma^+_{2^+} (\nu) a^+_{p\frac{1}{2}}(\nu) \right]_{\frac{3}{2}^-} |0\rangle ,$$

where the creation operators of the neutron monopole and quadrupole pair addition (RPA) modes are

$$\Gamma^+_{0^+}(\nu) = \sum_j \left[ X_j \Gamma^+_{j^+}(\nu) - Y_j \Gamma^+_{j^+}(\nu) \right] , \quad \left( \Gamma^+_{j^+} = [a^+_{j^+}(\nu)a^+_{j^+}(\nu)]_{0^+} \right) ,$$

and

$$\Gamma^+_{2^+}(\nu) = \sum_{j1,j2} \left[ X_{j1,j2} \Gamma^+_{j1,j2}(\nu) - Y_{j1,j2} \Gamma^+_{j1,j2}(\nu) \right] , \quad \left( \Gamma^+_{j1,j2} = [a^+_{j1}(\nu)a^+_{j2}(\nu)]_{2^+} \right) ,$$

respectively. The “vacuum” state $|0\rangle$ is the correlated $^8\text{He}$ ground state. Of notice that the $p_{\frac{1}{2}}(\pi)$ proton state leading to the $^9\text{Li}$ ground state is assumed to act, throughout, as a spectator.

The two final states populated in the two neutron pick–up reaction are

$$|^{9}\text{Li}(gs; \frac{3}{2}^-) = a^+_{p\frac{1}{2}}(\pi)|0\rangle ,$$

and

$$|^{9}\text{Li}(2.69 \text{ MeV}; \frac{1}{2}^-) = \left[ a^+_{p\frac{1}{2}}(\pi) \Gamma^+_{2^+}(ph) \right]_{\frac{1}{2}^-} |0\rangle ,$$

where

$$|^{8}\text{He}(3.6 \text{ MeV}; 2^+) = \Gamma^+_{2^+}(ph)|0\rangle ,$$

the (RPA) phonon creation operator being

$$\Gamma^+_{2^+}(ph) = \sum_{ph} X_{ph} \Gamma^+_{ph} - Y_{ph} \Gamma^+_{ph} ,$$

with

$$\Gamma^+_{ph} = \left[ a^+_{p} a^+_{h} \right]_{2^+} ,$$

$p$ labeling a state above the Fermi energy of $|0\rangle$ and $h$ one below. Both proton and neutron excitations are to be considered [9] (see also ref. [11], Ch. 11).

A detailed and consistent calculation of the corresponding modes needs as input a mean field, which could also be a simple Saxon–Woods parametrization with a sensible $k$–mass, and the particle–vibration coupling strengths and associated transition probabilities. These quantities
emerge from the self–consistent relation that must exist between single–particle potential and nuclear density (and between its fluctuations) as described by the equations

\[
U_{nA} = \int d^3 r' \rho_A(\vec{r}') V_{nn} \left( |\vec{r} - \vec{r}'| \right), \quad \delta U_{nA} = \int d^3 r' \delta \rho_A(\vec{r}') V_{nn} \left( |\vec{r} - \vec{r}'| \right).
\] (9)

Once the mean field single–particle degrees of freedom and collective vibrations are determined, NFT provides the rigorous physical framework to describe their interweaving in terms of the particle–vibration coupling mechanism. In this way one obtains real (dressed, effective) particles, and vibrations. In other words, effective masses, charges, lifetimes, induced interactions as well as inelastic, and transfer form factors. In particular, in dealing with superfluidity in nuclei arising from the combined effect of bare and induced pairing interaction [11], even in the case in which the bare interaction \( V_{nn} \) is not well known. This is because one is confronted with a particular case of asymptotic free theories, where something important happens at the infrared limit, a phenomenon which does not depend on the details of the interaction above a given cut–off. Such theories are not only renormalizable but furthermore they can be accurately renormalized in terms of simple Hamiltonians (see e.g. [13] and refs. therein). Within this context one can posit that renormalization processes average out many of the uncertainties associated with the poor knowledge one may have of \( V_{nn} \), as testified by the recent confirmation [1, 6], of the associated predictions of ref. [9] (see also ref. [11] Ch.11) concerning the glue binding the two halo neutrons in \(^{11}\text{Li}\). Within this context NFT provides the elements to renormalize single–particle [14] as well as collective motion [15], and the associated particle–vibration coupling (vertex renormalization).

3. Reaction

To the ground state \( \rightarrow \) ground state transition (\( 3/2^- \rightarrow 3/2^- \)), processes with angular momenta and parity \( 0^+, 1^+ \) and \( 2^+ \) can in principle contribute (angular momentum conservation), while in connection with the ground state \( \rightarrow \) first excited state (\( 3/2^- \rightarrow 1/2^- \)) \( 1^+ \) and \( 2^+ \) are allowed (see also [1]). In the analysis of these processes carried out in ref. [6], the most important channels populating the two final states observed, were considered. One of the arguments used in this choice was based on the \( Q \)–values associated with each of the different reactions paths.

3.1. \( Q \)–values

The different reaction channels which can populate the two states observed in the \(^1\text{H} (^{11}\text{Li}, ^9\text{Li}) ^3\text{H} \) reaction directly, or in terms of multistep processes are:

(i) two–particle transfer, which receives contributions from successive, simultaneous and non–orthogonality channels [5, 16, 17], processes which are associated with the following \( Q \)–values

(a) \(^{11}\text{Li}(p,t)^9\text{Li}(gs)\) ; \( Q = 8.2 \text{ MeV} \),
(b) \(^{11}\text{Li}(p,t)^9\text{Li}^*(2.69 \text{ MeV})\) ; \( Q = 5.5 \text{ MeV} \),

(ii) Breakup channels

(a) two particle breakup, both neutrons in the \( s_{1/2} \) resonance of \(^{10}\text{Li} \) (most probable event) \(^{11}\text{Li}(p,p')^{10}\text{Li} + 2n(\text{cont.})\) ; \( Q \approx -0.5 \text{ MeV} \),
(b) one particle breakup \(^{11}\text{Li}(p,p')^{10}\text{Li} + n(\text{halo}) + n(\text{cont.})\) ; \( Q \approx -0.5 \text{ MeV} \),

(iii) One–particle transfer

(a) \(^{11}\text{Li}(p,d)^{10}\text{Li}\) ; \( Q \approx 1.9 \text{ MeV} \)

Such a process, considered as an on–the–energy–shell process, can populate the observed final states in keeping with the fact that once a neutron is picked–up from \(^{11}\text{Li}\), the other leaves the system almost at once. It can also populate the final states in an off–the–energy shell process, i.e. successive transfer.
(b) $^{10}$Li($d,t)^9$Li(gs) ; \[ Q = 6.3 \text{ MeV} , \]
and
$^{10}$Li($d,t)^9$Li(2.69 MeV) ; \[ Q = 3.6 \text{ MeV} . \]

(iv) Inelastic scattering

(a) entrance channel
$^{11}$Li($p,p')^{11}$Li* (1$^-$) ; \[ E \approx 0.5 \text{ MeV} ; \]
\[ Q \approx -0.5 \text{ MeV} . \]
Of course, this process is in competition with the break up process, in keeping with the fact that $^{11}$Li, being so weakly bound ($S_{2n} \approx 380 \text{ keV}$), displays no bound excited state

(b) exit channel
$^9$Li($t,t')^9$Li* (2.69 MeV) ; \[ Q = -2.69 \text{ MeV} . \]
This process can populate the final excited state, as a two–step process involving transfer to the ground state of $^9$Li (i.e. process (i)(a) above, $Q = 8.2 \text{ MeV}$) and then exit channel inelastic scattering.

All the above $Q$–values are to be reported to the bombarding conditions, that is,

Center of mass energy/nucleon = 2.75 MeV ,

Coulomb barrier $\approx 0.6 \text{ MeV}$ .

Of notice that negative $Q$–values have to be compensated by an input energy from the relative motion reservoir. In on–the–energy shell processes energy has to be conserved. In virtual processes, like the successive pick–up of two nucleons (motion reservoir. In on–the–energy shell processes energy has to be conserved. In virtual channels of processes (ii) display a finite overlap with the final states the two-halo neutrons have very small probability (cf. Table 1 ref. [6]). In particular the final

all processes but those associated with the successive and simultaneous transfer (pick-up) of

the time “contraction” associated with a very negative $Q$–value (larger $\Delta E$, smaller $\Delta t$), and thus of the chance of the process to occur. From a semiclassical point of view (see e.g. [5]), little applic able in the present case but anyhow much illuminating, a large negative $Q$–value implies an exponential function with a large exponent ($\exp \left( -i (Qt/h) \right)$) and thus a rapidly oscillating function which essentially zeroes the gentle behaved form factor contribution. Of notice that all processes but those associated with the successive and simultaneous transfer (pick-up) of the two-halo neutrons have very small probability (cf. Table 1 ref. [6]). In particular the final channels of processes (ii) display a finite overlap with the final states $^9$Li(3/2$^-$) + t and $^9$Li(1/2$^-$) + t (see Figs. 1(g) and 1(f) of ref. [6]).

3.2. Overlaps

While two–particle transfer is the specific tool to probe pair correlations in nuclei [10, 18], as testified by the fact that superfluidity is tantamount to a finite value of the pair operator $P^+ = \sum_{\nu>0} a^+_{\nu} a^0_{\nu}$ in the ground state ($\alpha_0 = \langle 0| P^+ |0 \rangle$) (see e.g. refs. [11, 18], and refs. therein), not all two–particle transfer processes are equally suited for the job. Let us give a couple of examples, starting from very general considerations. The perfect probe is one in which the Cooper pair to be transferred (or picked–up) displays the same spatial correlations in the target as in the projectile. Thus, if one would like to study pairing correlations in $^{120}$Sn and in $^{11}$Li, the specific probe are the reactions $^{120}$Sn + $^{118}$Sn → $^{118}$Sn + $^{120}$Sn, $^{120}$Sn + $^{122}$Sn → $^{122}$Sn + $^{120}$Sn and similarly $^{11}$Li + $^9$Li → $^9$Li + $^{11}$Li.

Of course in these cases the Cooper (2n) pair displays the same correlations in both target and projectile, and consequently, the corresponding overlaps of the Cooper pair in the initial (i) and final (f) channels $\Omega_n = \langle 2n(f)|2n(i) \rangle \approx 1$ (cf. Eq. (2.11) in ref. [10] and subsequent discussion). The main drawback of heavy ion reactions, in particular in the case of Sn, is the rather large Coulomb and hadronic fields involved. This implies, as a rule, large inelastic probabilities making it necessary $\gamma$–coincidence experiments or the like, to disentangle the gs → gs transition from all other multistep processes.
This is the reason why light ion two–particle transfer reactions have provided essentially most of the detailed nuclear structure information available on nuclear pair correlations. In this case, the correlations displayed by the two transferred neutrons in the heavy (target or residual nucleus) and in the light ion (projectile or outgoing particle) can be quite different and, consequently, $\Omega$ rather small. It is then important to be able to calculate the overlap $\Omega$ in an accurate way, making eventually use of detailed many–body wavefunctions for the light ion, e.g. the triton $t$. While the detailed properties of the $t$ described by such wavefunctions are rather different from those associated with simple Gaussian–like $t$–wavefunctions used as a rule to calculate the absolute $(p, t)$ or $(t, p)$ cross sections (see e.g. [16]), one does not expect the associated $\Omega$ to be much different. This is because the overlap depends essentially only on the mean square radius of the associated density, quantity which in the simplified $t$–wavefunctions is parametrically adjusted. In other words, it is quite unlikely that the detailed NN, NNN, etc. “bare” forces acting among nucleons, which are essential to obtain the right binding energies, deep inelastic form factors, etc. of e.g. the triton, could be of much relevance in determining the value of $\Omega$. In other words, the overlaps $\Omega$ calculated in the analysis of $^{11}\text{Li}(p, t)^{9}\text{Li}$ in terms of gaussian-like $t$–wavefunctions (see e.g. ref. [16]) are expected not to differ much from those obtained using so-called realistic $t$-wavefunctions.

3.3. The interaction responsible for transfer

Making use of post–prior symmetries (cf. e.g. [5]), that is, the fact that it is equivalent to consider that the transfer–field acts on the initial or on the final (intermediate) channel, in keeping with energy conservation (see Eq. (12)), it is possible to formulate a multistep two–particle transfer reaction solely in terms of the mean–field potential. It is worth noticing that, due to the relation

$$\langle \Psi_A | (V_{nn} - U_{nA}) | \Psi_A \rangle = 0 ,$$  \hspace{1cm} (10)

the corresponding results should, to a large extent, be independent of the choice of $V_{nn}$.

![Figure 1](image_url). Absolute differential cross section associated with the successive two–particle transfer contribution to the $^{11}\text{Li}(p, t)^{9}\text{Li}$ cross section. Prior labels the prior–prior representation in which $V_{np}$ is responsible for the transfer process. Post labels the post–post representation. In this case it is the nucleon nucleus potential (9) which controls the reaction process.

A clear example of the above reasoning is provided by Fig. 1. In it, the angular distribution associated with the successive transfer of two neutrons in the reaction $^{11}\text{Li}(gs)(p, t)^{9}\text{Li}(gs)$ [6] calculated in the post–post representation and in the prior-prior one are displayed. In the first
case the simple Wood–Saxon parametrization of Bohr and Mottelson [19] was used to describe the interaction responsible for transfer. In the second case a simple parametrization of $V_{np}$, sensible, for the purpose, but not adjusted to fulfill condition (10) was used [20]. Although one observes differences, the order of magnitude of the absolute cross section is the same in both calculations. Again, conservation arguments apply to a generic two–step reaction

$$a + A \rightarrow f + F \rightarrow b + B ,$$

(11)

which reads

$$H_a + H_A = H_f + H_F = H_b + H_B ,$$

(12)

and insures that nothing is gained to choose the post–post, post–prior or prior–prior representations of any of the reaction channel contributing to the process. Thus, $V_{nn}$ does not need to enter the reaction calculations.

4. Conclusions

While it is true, as stated above, that understanding a many–body system like the atomic nucleus in terms of symmetries and conservation laws implies a certain level of understanding, it is only when one can describe the behaviour of the system under study and the reaction channels used to probe it in terms of the motion of the individual nucleons and of the collective modes of the system as a whole, as well as of their interweaving, that one has obtained a true understanding of it. This is of course the goal achieved by Quantum Electrodynamics (QED) (see [21, 22] and refs. therein) to describe daily life phenomena, in particular in Feynman’s setup, a goal which within the low–energy nuclear physics scenario is also that of NFT [2, 3, 23], tailored after QED. In this theory as in NFT nothing is free, and what one measures, e.g. the single–particle energy (mass) is not the “bare” mass, but something else which includes the effect of renormalization processes [21].

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