Identifying Influential Brokers on Social Media from Social Network Structure

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Abstract

Identifying influencers in a given social network has become an important research problem for various applications, including accelerating the spread of information in viral marketing and preventing the spread of fake news and rumors. The literature contains a rich body of studies on identifying influential source spreaders who can spread their own messages to many other nodes. In contrast, the identification of influential brokers who can spread other nodes’ messages to many nodes has not been fully explored. Theoretical and empirical studies suggest that involvement of both influential source spreaders and brokers is important for successful information dissemination and viral marketing campaigns. However, the overlap between central nodes and influential brokers is small (less than 15%) in Twitter datasets. We also tackle the problem of identifying influential brokers from a given social network. By using three social media datasets, we investigate the characteristics of influential brokers by comparing them with influential source spreaders and central nodes obtained from centrality measures. Our results show that (i) most of the influential source spreaders are not influential brokers (and vice versa) and (ii) the overlap between central nodes and influential brokers is small (less than 15%) in Twitter datasets. We also tackle the problem of identifying influential brokers from centrality measures and node embeddings, and we examine the effectiveness of social network features in the broker identification task. Our results show that (iii) although a single centrality measure cannot characterize influential brokers well, prediction models using node embedding features achieve F₁ scores of 0.35–0.68, suggesting the effectiveness of social network features for identifying influential brokers.

Introduction

Identifying influencers from a social network has been a fundamental research task in the web and network science research communities (Lü et al. 2016; Li et al. 2018; Morone and Makse 2015; Al-Garadi et al. 2018; Banerjee, Jenamani, and Pratihar 2020). It has been shown that a few individuals called influencers play an important role in triggering a large-scale cascade of information diffusion (Pei et al. 2014; Katz and Lazarsfeld 1955). Thus, identifying influencers is considered to be crucial for conducting effective viral marketing campaigns (Richardson and Domingos 2002; Domingos and Richardson 2001; Kempe, Kleinberg, and Tardos 2003) and preventing the spread of unwanted information (e.g., fake news and rumors) (Budak, Agrawal, and El Abbadi 2011).

Several algorithms for identifying influencers have been proposed (Lü et al. 2016; Li et al. 2018; Al-Garadi et al. 2018; Banerjee, Jenamani, and Pratihar 2020). A common approach is to calculate centrality measures of nodes in a social network and extract the nodes with high centrality as influencers (Chen et al. 2012; Lü et al. 2016; Morone and Makse 2015). Traditional centrality measures include degree (Freeman 1979), closeness (Freeman 1979), betweenness (Freeman 1979), PageRank (Brin and Page 1998), and k-core index (Seidman 1983; Dorogovtsev, Goltsev, and Mendes 2006). New measures have been also used for identifying influencers, including VoteRank (Zhang et al. 2016) and Collective Influence (CI) (Morone and Makse 2015). While previous studies assumed either explicitly or implicitly that influencers are nodes who can spread their own messages to many other nodes (Kempe, Kleinberg, and Tardos 2003; Li et al. 2018; Zhang et al. 2016; Lü et al. 2016; Chen et al. 2012; Banerjee, Jenamani, and Pratihar 2020), another type of influencer on social media who can spread other users’ messages to many users has also been shown to play an important role in large-scale information diffusion (Bakshy et al. 2012; Weng, Menczer, and Ahn 2013; Liu-Thompkins and Rogerson 2012; Araujo, Neijens, and Vliegenthart 2017; Meng et al. 2018; Tsugawa 2019). We refer to the former as source spreaders and the latter as brokers (Burt 2000; Araujo, Neijens, and Vliegenthart 2017; Meng et al. 2018) (Fig. 1). Bakshy et al. (2012), and Weng et al. (2013) showed that information diffusion by brokers who bridge different communities affects the final sizes of information diffusion cascades. Araujo et al. (2017) showed that information brokers facilitate content diffusion for global brands. Meng et al. (2018) showed that the diffusion size of health-related information posted by the Centers for Disease Control and Prevention is significantly affected by broker involvement. Therefore, identifying influential brokers, as well as source spreaders, is important for successful information dissemination.
Influential source spreaders are users who can spread their own messages to many other users, and influential brokers are users who can spread other users’ messages to many users.

Because the characteristics of influential brokers have been unclear to date, we begin by examining their characteristics by comparison with influential source spreaders (RQ1) and central nodes obtained from centrality measures (RQ2). We then conduct experiments to identify influential brokers. We focus on node embeddings (Rossi, Zhou, and Ahmed 2018a,b; Goyal and Ferrara 2018; Grover and Leskovec 2016; Cui et al. 2018; Qiu et al. 2018, 2019), which are low-dimensional vector representations of nodes, and traditional centrality measures (Freeman 1979; Brin and Page 1998; Seidman 1983; Dorogovtsev, Goltsev, and Mendes 2006) as features for identifying influencers, and we examine their effectiveness (RQ3).

Our main contributions are summarized as follows.

- **RQ1** How different are influential source spreaders and influential brokers?

- **RQ2** Are influential brokers located at central positions in a social network?

- **RQ3** How accurately can we predict the influential brokers from a social network by using node embeddings, which incorporate complex structural information about the social network?

Because there are multiple definitions of influencers, most algorithms for identifying influencers aim either explicitly or implicitly to identify influential source spreaders (Riquelme and González-Cantergiani 2016). For instance, Cha et al. (2010) examined the influence of Twitter users using their numbers of followers, retweets, and mentions, and Bakshy et al. (2011) considered users who initiate large-scale retweet cascades as being influencers. Influencers are commonly referred to by other terms such as opinion leaders (Hu et al. 2012) and authorities (Bouguessa and Romdhane 2015). These studies regard influencers as users who can spread their own information or posts to many other users, and we refer to this type of influencer as an influential source spreader. Another line of research (Araujo, Neijens, and Vliegenthart 2017; Liu-Thompkins and Rogerson 2012; Li et al. 2014a; Burt 2000) is focused on influential brokers who can spread other users’ information or posts to many users. Information disseminated by influential brokers who bridge different communities is suggested to spread widely in several domains such as YouTube videos (Liu-Thompkins and Rogerson 2012), brand content (Araujo, Neijens, and Vliegenthart 2017), and health-related information (Meng et al. 2018). Araujo et al. (2017) and Li et al. (2014a) showed that both influential source spreaders and brokers play important roles in facilitating large-scale information diffusion cascades.

Although there are multiple definitions of influencers, most algorithms for identifying influencers aim either explicitly or implicitly to identify influential source spreaders (Kempe, Kleinberg, and Tardos 2003; Li et al. 2018; Banerjee, Jenamani, and Pratihar 2020; Zhang et al. 2016; Pei et al. 2014; Chen et al. 2012). A common way to evaluate the effectiveness of influencer identification algorithms is to use synthetic information diffusion models such as the susceptible–infected–removed (SIR) model (Zhang et al. 2016; Li et al. 2014b; Chen et al. 2012), the independent cascade model (Kempe, Kleinberg, and Tardos 2003), and the linear threshold model (Kempe, Kleinberg, and Tardos 2003). As a metric for evaluating the effectiveness of the algorithms, the number of users who receive information when the identified influencers are selected as seed nodes in the information diffusion models is used (Kempe, Kleinberg, and Tardos 2003; Zhang et al. 2016; Li et al. 2014b). In other words, many existing algorithms are shown to be effective for identifying users who can spread information to many others.
other users under synthetic information diffusion models. Some studies (Pei et al. 2014; Panagopoulos, Malliaros, and Vazirgianis 2020; Tsugawa and Kimura 2018) used real information diffusion trace data rather than synthetic models, but those studies also evaluated the users’ power as influential source spreaders.

Algorithms for identifying influential source spreaders fall roughly into three categories: (i) algorithms based on network topology (Morone and Makse 2015; Zhang et al. 2016; Lü et al. 2011; Li et al. 2014b; Chen et al. 2012), (ii) algorithms based on information diffusion models (Kempe, Kleinberg, and Tardos 2003; Li et al. 2018; Banerjee, Jena-man, and Pratihar 2020; Tang, Shi, and Xiao 2015; Tang, Xiao, and Shi 2014), and (iii) algorithms based on the records of users’ activities (Weng et al. 2010; Yamaguchi et al. 2010; Panagopoulos, Malliaros, and Vazirgianis 2020). Algorithms based on network topology estimate the influence of each user from the topological structure of a social network using network metrics of nodes. The metrics include traditional centrality measures (Freeman 1979; Brin and Page 1998), LeaderRank (Lü et al. 2011), VoteRank (Zhang et al. 2016), and CI (Morone and Makse 2015). The second category of algorithms, namely, those that use information diffusion models, are referred to as influence maximization algorithms (Kempe, Kleinberg, and Tardos 2003; Li et al. 2018; Banerjee, Jenanami, and Pratihar 2020); these identify a set of influential seed nodes that can spread information to many nodes under the given information cascade model (Kempe, Kleinberg, and Tardos 2003; Li et al. 2018; Banerjee, Jena-manami, and Pratihar 2020). In the seminal work by Kempe et al. (2003), the influence maximization problem was formulated as a combinatorial optimization problem, and greedy algorithms were proposed. Since then, several efficient influence maximization algorithms have been proposed, including TIM (Tang, Xiao, and Shi 2014) and IMM (Tang, Shi, and Xiao 2015) that can find influencers in huge networks based on influence cascade models. While the first two categories of algorithms use only social network structure and synthetic diffusion models, the third category of algorithms uses the records of users’ activities such as tweets and retweets in addition to the network structure. Such algorithms that are widely used include TwitterRank (Weng et al. 2010), TURank (Yamaguchi et al. 2010), and CELFIE (Panagopoulos, Malliaros, and Vazirgianis 2020). These algorithms have been shown to be effective for identifying influential source spreaders, but their effectiveness for identifying influential brokers remains unclear. As explained above, many algorithms for identifying influential source spreaders use network topology, and so we expect the latter also to be a promising source for identifying influential brokers.

Several algorithms for identifying structural hole spanners (Lou and Tang 2013; Lin et al. 2021; Xu et al. 2017) that bridge different communities have also been proposed. Structural hole spanners are defined as nodes whose removal from a network causes communities to become disconnected (Xu et al. 2017; Lou and Tang 2013). Empirical studies of tweet diffusion on social media have shown that inter-community diffusion of a tweet increases its final cascade size (Bakshy et al. 2012; Weng, Menczer, and Ahn 2013; Tsugawa et al. 2019), which suggests that structural hole spanners may be related to influential brokers. However, the relationship between structural hole spanners and influential brokers has not been investigated. A simple way to find structural hole spanners is to extract nodes with high betweenness as used in (Goyal and Vega-Redondo 2007). We compare influential brokers and central nodes based on betweenness, which are expected to be structural hole spanners, and we examine the relationship between the concepts of structural hole spanners and brokers.

Few studies have quantified a user’s influence as a broker, but a notable exception is that by Bhowmick et al. (2019), who proposed an algorithm called SmartInf for identifying influencers from past retweet cascades and evaluated the identified influencers’ power as brokers; that study suggests that past retweet cascades are useful for identifying influential brokers. In contrast, the effectiveness of network topology for identifying brokers has not been explored. We follow Bhowmick et al. (2019) and examine the usefulness of network embeddings and traditional centrality measures obtained from network topology for identifying brokers.

**Preliminaries**

**Notation**

A social network is represented as a directed graph $G = (V; E)$, where $V$ is a set of nodes representing social media users and $E$ is a set of links representing the relationships among those users. Link $(u, v) \in E$ represents the fact that user $u$ follows user $v$.

A sequence of retweets (reposts) and the original tweet (post) is referred to as an information diffusion cascade. Furthermore, $U_c = \{u_1^c, u_2^c, \ldots, u_{n^c}\}$ is the set of users who retweet the original tweet of cascade $c$, and $T_c = \{t_{u_1^c}, t_{u_2^c}, \ldots, t_{u_{n^c}}\}$ is the set of timestamps of the retweets in cascade $c$. Here, $u_1^c$ is the user who posts the $s$-th retweet in cascade $c$, $t_{u_i^c}$ is the timestamp of the retweet posted by user $u_i$ in cascade $c$, and $n^c$ is the cascade size of $c$. The user who posts the original tweet of $c$ is denoted as $u_0^c$, and the timestamp of the original tweet of $c$ is denoted as $t_0^c$. A set of diffusion cascades among the social media users $V$ is denoted as $D = \{c\}$, and a set of cascades initiated by node $v$ is denoted as $C_v = \{c \mid c \in D, u_0^c = v\}$. The set of users who retweet in cascade $c$ after user $v$ retweets in cascade $c$ is denoted as $R_v^c = \{u_i^c \mid t_{u_i^c} > t_0^c\}$.

**Definitions of Influence**

We define influencer scores of each user as both a source spreader and a broker.

**Source spreader score:** An influential source spreader is a user who can spread their own tweets to many other users. Thus, the source spreader score of user $u$ is defined as the number of users who retweet user $u$’s tweets and is given by

$$S_u = \bigcup_{c \in C_u} U_c.$$
This metric measures the popularity of user \( u \)'s tweets and was used in previous studies to evaluate the influence of nodes in social networks (Pei et al. 2014; Zhang et al. 2016) and for influence maximization problems (Panagopoulos, Malliaros, and Vazirgianis 2020; Kempe, Kleinberg, and Tardos 2003).

**Broker score:** An influential broker is a user who can spread other users’ tweets to many other users. Thus, following (Bhowmick et al. 2019) and analogously to the source spreader score, the broker score of user \( u \) is defined as the number of users who post retweets after user \( u \)'s retweets and is given by

\[
B_u = \left| \bigcup_{c \in D} R^c_u \right|.
\]

(2)

This metric is intended to measure the impact of user \( u \)'s retweets on the popularity of the retweeted tweets (Bhowmick et al. 2019). Since it is difficult to directly measure the impact of each retweet on the future popularity of the retweeted tweet, we assume that the users who participate in many large-scale cascades at their early stages are influential brokers.

**Methodology**

**Datasets**

We use three social media datasets that we refer to as the Twitter Japan, Twitter Nepal (Bhowmick et al. 2019), and Digg (Hogg and Lerman 2012) datasets, which contain both a social network of social media users and information diffusion cascades among them. The basic statistics of these datasets are given in Table 1.

- **Twitter Japan:** The Twitter Japan dataset contains a who-follows-whom network of Japanese Twitter users and their tweets and retweets posted during January 2014. This dataset was collected in our previous study (Tsugawa 2019; Tsugawa and Kimura 2018; Tsugawa and Kito 2017).

- **Twitter Nepal:** The Twitter Nepal dataset (Bhowmick et al. 2019) contains tweets and retweets about the 2015 Nepal earthquake, as well as a who-follows-whom network among the users involved in those tweets and retweets.

- **Digg:** The Digg dataset (Hogg and Lerman 2012) contains posts, their votes, and a who-follows-whom network among the users. In the Digg social media, users can make posts (called stories in Digg) and also vote on other users’ posts. A voted post is shown in the timelines of followers of the voted user. Thus, the sequence of a story and votes to it is regarded as a cascade (Lerman and Ghosh 2010). As given in Table 1, only 0.15% of users made any posts, with most users only voting, and so source spreader scores can be calculated for only a few users. Therefore, we used the Digg dataset not for identifying influential source spreaders but only for identifying influential brokers.

| Datasets   | Twitter Japan | Twitter Nepal | Digg  |
|-----------|---------------|---------------|-------|
| Num. users | 351,759       | 273,222       | 279,631 |
| Num. cascades | 8,419,432   | 26,424        | 3,553  |
| Num. retweets | 29,587,972   | 521,938       | 3,018,197 |
| Num. source users | 256,549     | 18,983        | 431    |

Table 1: Basic statistics of the datasets.

Our three datasets come from different domains, which is expected to be useful for examining how the characteristics of influencers differ across social media services, languages, and topics. Our datasets cover two social media services (i.e., Twitter and Digg) and three different languages (i.e., English, Nepali, and Japanese). Moreover, the Twitter Nepal dataset contains topic-specific cascades, whereas the Digg and Twitter Japan datasets contain non-topic-specific cascades. By using these three different types of datasets, we examine the characteristics of influencers in different domains.

**Centrality**

We extract influential brokers as well as influential source spreaders from the three datasets, and we examine the characteristics of the brokers and source spreaders using centrality measures. We use traditional popular centrality measures including degree centrality (Freeman 1979), closeness centrality (Freeman 1979), betweenness centrality (Freeman 1979), \( k \)-core index (Seidman 1983; Dorogovtsev, Goltsev, and Mendes 2006), and PageRank (Brin and Page 1998).

From social network \( G \) representing who-follows-whom relationships among social media users, we extract the top \( p \% \) of central nodes based on the centrality measures. We also extract the top \( p \% \) of users based on the source spreader and broker scores as influential source spreaders and brokers, respectively. We then examine the overlap among the extracted influencers (i.e., source spreaders and brokers) and central nodes. More specifically, we calculate \( p \% \) overlap scores (Pei et al. 2014; Tsugawa and Kimura 2018; Borgatti, Carley, and Krackhardt 2006) between influencers and central nodes as

\[
\text{Overlap}_p = \frac{|T^\text{inf}_p \cap T^\text{cent}_p|}{|T^\text{inf}_p|},
\]

(3)

where \( T^\text{inf}_p \) and \( T^\text{cent}_p \) are the sets of top-\( p \% \) influencers and central nodes, respectively. Previous studies (Chen et al. 2012; Lü et al. 2016) have shown that influential source spreaders tend to have high centrality; thus, overlap between influential source spreaders and central nodes is expected to be high. In contrast, whether influential brokers have high centrality is not yet clear.

**Node Embedding**

We also examine how influential brokers can be characterized by node embeddings (Rossi, Zhou, and Ahmed 2018b,a; Goyal and Ferrara 2018; Cui et al. 2018). A node embedding is a latent low-dimensional vector representation of a node in a network, and a node embedding technique
learns such vector representations of nodes from only the topological structure of a given network (Goyal and Ferrara 2018; Cui et al. 2018). While traditional centrality measures are human-crafted features that are defined explicitly using human knowledge about central nodes, node embeddings are features learned from the network data alone without explicitly using human knowledge about the node characteristics. Node embeddings have been shown to be effective for several tasks (Goyal and Ferrara 2018; Cui et al. 2018), which motivates us to examine the effectiveness of learned embeddings of nodes for characterizing influencers.

Among several options for node embedding techniques (Goyal and Ferrara 2018; Cui et al. 2018), we use DeepGL (Rossi, Zhou, and Ahmed 2018b,a). While with most node embeddings it is difficult to interpret the meaning of each dimension of the obtained vectors, DeepGL produces interpretable vector representations of nodes (Rossi, Zhou, and Ahmed 2018b,a; Fujiwara et al. 2020). Using interpretable embeddings, we try to understand the influencers’ characteristics that are not captured by traditional centrality measures.

In what follows, we briefly introduce DeepGL; see (Rossi, Zhou, and Ahmed 2018a) for the details. DeepGL uses the base features $x$ of each node. The base features of a node are represented as a vector, and each dimension can be any feature, such as a traditional centrality measure of the node or a node attribute. DeepGL learns the vector representation of each node from its base features $x$ and those of its neighbors by using a relational function $f$. A relation function is a combination of relational feature operators that can be applied to a base feature. A relational feature operator obtains a value from base feature values of one-hop neighbors of a target node. Examples of the relational feature operator include mean, sum, and max. In a directed network, in-neighbor, out-neighbor, and total-neighbor can be defined, and the operators for in, out, and total neighbors are denoted as $\Phi_{\text{in}}, \Phi_{\text{out}},$ and $\Phi_{\text{total}}$, respectively. $S$ is a summary function that returns a single value from multiple values, such as sum, mean, and max. For instance, $\Phi_{\text{mean}}(x)$ is a relational operator that calculates the mean $x$ of the in-neighbors of a node. In DeepGL, each dimension of the obtained representation vector of a node is defined as the combination of base features $x$ and relational operators. For instance, each dimension could be $\Phi_{\text{mean}}(\text{degree})$, which is the mean of the degree of neighboring nodes. More-complex features can be obtained, such as $\Phi_{\text{mean}} \circ \Phi_{\text{max}}(\text{degree})$, which for node $v$ is calculated as follows. First, for each neighbor $u$ of node $v$, the maximum degree among $u$’s neighbors is obtained. Then, the mean of the maximum values for node $v$’s neighbors is obtained. DeepGL can learn such complex features of nodes from a network in an unsupervised way by combining base features and relational functions.

### Predicting Influencers

We examine the effectiveness of the node embeddings obtained with DeepGL for identifying influential brokers. We conducted experiments on identifying influential brokers from the node embeddings using a supervised machine-learning framework. For comparison purposes, we also conducted experiments on identifying influential source spreaders. The task here is to identify the top-$p\%$ influential brokers (or source spreaders) from the DeepGL embeddings. We obtained the DeepGL embeddings from each social network $G$. We also obtained broker scores (or source spreader scores) for all nodes. Then, we annotated nodes with top-$p\%$ scores as influencers, and others as non-influencers. A fraction $q$ of nodes in each network $G$ are used as training data, and the others are used as test data. For the training data, class labels (i.e., influencers or non-influencers) are available for model training. We used LightGBM (Ke et al. 2017) as a classifier, and for each network we trained models of identifying influential brokers. LightGBM was chosen because it can produce feature importance of the obtained model and is known to be effective for machine learning tasks using tabular data (Ke et al. 2017). To cope with highly imbalanced class labels (i.e., there are far more non-influencers than influencers), we subjected the training data to downsampling and obtained balanced training data, where the numbers of non-influencers and influencers are the same. Moreover, 80% of the downsampled training data were used for model training, and 20% of the training data were used for hyperparameter tuning using Optuna (Akiba et al. 2019). The loss function was binary logarithmic loss. Note that the test data were still unbalanced. The parameter configurations for DeepGL are summarized in Table 2. We used the DeepGL implementation in (Fujiwara et al. 2020). By applying DeepGL to each social network, we obtained embedding vectors of nodes in an unsupervised way. For each setting, we performed 10 experiments and obtained average scores for classification accuracy.

### Results

#### Overlap Among Influential Brokers, Source Spreaders, and Central Nodes

We first examine the characteristics of influential brokers by comparing them with influential source spreaders and central nodes (RQ1 and RQ2). Figure 2 shows confusion matrices of 10% overlap scores among influential brokers, source spreaders, and central nodes. Note that influential source spreaders were not extracted from the Digg data because the number of source users was too small (see Table 1).

Figure 2 shows that overlaps between source spreaders and brokers are low. For the Twitter Japan dataset, the overlap score is 0.18, and for the Twitter Nepal dataset the score is less than 0.01. These results indicate that influential source
betweenness, PageRank, and closeness scores are relatively high. The overlap scores for brokers with source spreaders are low for all datasets, but the overlaps with source spreaders are relatively high. The overlaps between central nodes and brokers are also small, whereas overlaps with source spreaders are relatively large, which suggests that using a single centrality measure is effective for identifying source spreaders but not brokers.

Summary of answers to RQ1 and RQ2: The overlap between brokers and source spreaders is generally small (0.18 for the Twitter Japan dataset and less than 0.01 for the Twitter Nepal dataset), which suggests that brokers have different characteristics from source spreaders. The overlaps between central nodes and brokers are also small, whereas overlaps with source spreaders are relatively large, which suggests that using a single centrality measure is effective for identifying source spreaders but not brokers.

Difference between Influential Brokers and Source Spreaders

Furthermore, we investigate the difference between influential brokers and influential source spreaders. To characterize a social media user, we investigate the impact of their retweet on the subsequent retweets of other users. To quantify the impact of a retweet posted by user $u$ on the subsequent retweets, we define the average broker score per retweet of user $u$ as $B_u/r_u$, where $r_u$ is the number of retweets posted by user $u$. Note that $B_u$ is the broker score of user $u$ defined as Eq. (2). We compare the average broker score per retweet among influential brokers, influential source spreaders, and all users. We also compare the average number of retweets posted by influential brokers, influential source spreaders, and all users. Figure 3 shows the results.

These results clearly show the difference between influential brokers and influential source spreaders. Figure 3 shows that the average broker score per retweet of influential brokers is considerably higher than that of influential source spreaders. These results show that a tweet retweeted by an influential broker tends to spread more widely than does a tweet retweeted by an influential source spreader. Figure 3 also shows that the numbers of retweets posted by influential source spreaders tend to be either more than or comparable to the numbers of retweets posted by ordinary users in the Twitter datasets. In contrast, the average broker score per retweet of influential source spreaders is lower than that of ordinary users. This confirms that although influential source spreaders’ original tweets tend to spread widely,
DeepGL embedding features achieve higher accuracy than the models using only centrality features. Particularly for the Twitter Japan and Twitter Nepal datasets, the F1 scores of the embedding model are respectively 0.11 and 0.17 higher than those of the centrality model for the task of predicting the top-5% brokers, which indicates the effectiveness of the DeepGL embeddings for identifying influential brokers. For the Digg dataset, the embedding features are less effective than for the Twitter datasets, but the embedding model achieves higher accuracy than the centrality model.

For comparison, next we examine the effectiveness of the DeepGL embeddings for identifying influential source spreaders (Table 4). The training data were 20% of the nodes, and the other 80% were used as test data. In contrast to the previous results, these results show that using the DeepGL embeddings has little effect on identifying influential source spreaders. There is little difference in the prediction accuracy of the models using only traditional centrality measures and the models using the DeepGL embeddings. This indicates that the complex features obtained with DeepGL are not effective for identifying source spreaders. Moreover, combining Tables 3 and 4, we find that the accuracy of predicting influential brokers is comparable or even higher than that of predicting influential source spreaders.

Note that the obtained accuracy scores are not sufficiently high for practical use and should be improved in future research. For instance, a recent study (Ye, Liu, and Pan 2021) identified influential users in Sina Weibo with an F1 score of approximately 0.8 using state-of-the-art machine learning techniques such as node embeddings and graph neural networks. Although our definition of influential users differs from that used by Ye, Liu, and Pan (2021), their approaches can still be applied to the task of broker identification. Our results are useful as the first step toward identifying influential brokers, but it will be necessary to construct prediction models with better accuracy and practicality.

**Summary of answer to RQ3:** The accuracy of predicting the top-10% influential brokers using the DeepGL embedding features is 0.35–0.68, which is comparable with the accuracy of predicting influential source spreaders. Using node embeddings as features is suggested to be a more effective approach than using traditional centrality measures.

**Feature Importance**

Next, we investigate the important features that contribute to the influencer prediction in the embedding models, and we examine the characteristics of brokers and source spreaders. We obtained feature importance scores [known as the Gini importance (Louppe et al. 2013)] from the constructed models. Tables 5 and 6 give the top-five important features based on the importance score in the embedding models for the tasks of predicting the top-10% influential brokers and source spreaders, respectively. Here, we show the importance of a model selected randomly from the 10 constructed ones, but we confirmed that the important features were consistent across the 10 models.

Table 5 shows that for the Twitter datasets, traditional centrality measures are not included in the top-five features.
|       | Twitter Japan | Twitter Nepal | Digg |
|-------|---------------|---------------|------|
|       | precision     | recall        | F1   | precision     | recall        | F1   | precision     | recall        | F1   |
| centrality | 0.20          | 0.72          | 0.51 | 0.33          | 0.89          | 0.48 | 0.20          | 0.75          | 0.32 |
| embedding | 0.26          | 0.80          | 0.40 | 0.53          | 0.96          | 0.68 | 0.22          | 0.81          | 0.35 |

|       | Twitter Japan | Twitter Nepal | Digg |
|-------|---------------|---------------|------|
|       | precision     | recall        | F1   | precision     | recall        | F1   | precision     | recall        | F1   |
| centrality | 0.08          | 0.69          | 0.14 | 0.15          | 0.85          | 0.25 | 0.11          | 0.72          | 0.19 |
| embedding | 0.15          | 0.81          | 0.25 | 0.26          | 0.97          | 0.42 | 0.13          | 0.80          | 0.23 |

Table 3: Prediction accuracy for influential brokers. Models using DeepGL features achieve considerably higher accuracy than do models using only centrality measures.

|       | Twitter Japan | Twitter Nepal |
|-------|---------------|---------------|
|       | precision     | recall        | F1   | precision     | recall        | F1   |
| centrality | 0.17          | 0.80          | 0.28 | 0.11          | 0.60          | 0.19 |
| embedding  | 0.30          | 0.82          | 0.44 | 0.12          | 0.69          | 0.21 |

|       | Twitter Japan | Twitter Nepal |
|-------|---------------|---------------|
|       | precision     | recall        | F1   | precision     | recall        | F1   |
| centrality | 0.19          | 0.83          | 0.31 | 0.09          | 0.71          | 0.16 |
| embedding  | 0.26          | 0.97          | 0.42 | 0.13          | 0.80          | 0.23 |

Table 4: Prediction accuracy for influential source spreaders. The models using DeepGL features and those using only centrality measures have similar accuracy levels.

|       | Twitter Japan | Twitter Nepal | Digg |
|-------|---------------|---------------|------|
|       | precision     | recall        | F1   | precision     | recall        | F1   | precision     | recall        | F1   |
| centrality | 0.28          | 0.80          | 0.42 | 0.11          | 0.60          | 0.19 |
| embedding  | 0.30          | 0.82          | 0.44 | 0.12          | 0.69          | 0.21 |

|       | Twitter Nepal |
|-------|---------------|
|       | precision     | recall        | F1   |
| centrality | 0.08          | 0.69          | 0.14 |
| embedding  | 0.15          | 0.81          | 0.25 |

Table 5: Top-five importance scores of features in the embedding models for broker prediction. Complex features using multiple relational feature operators have higher importance. As base features, betweenness, k-core, and PageRank are used in the important features.

Table 6: Top-five importance scores of features in the embedding models for source spreader prediction. In contrast to the broker prediction models, centrality measures have high importance in the source spreader prediction models.

Complex embedding features that use three or four feature operators are shown to be effective features for identifying influential brokers. Features obtained from l-feature operators incorporate features of nodes in l-hop neighbors. Thus, features with a large number of feature operators incorporate complex higher-order structural features of nodes. These results suggest that such complex features are useful for identifying influential brokers in Twitter networks. For the Digg dataset, although the DeepGL embeddings are included in the top-five features, traditional centrality measures are also included. Moreover, the numbers of feature operators used in the DeepGL features are smaller than those for the Twitter datasets. For the Digg dataset, compared with the Twitter datasets, the effectiveness of the DeepGL features is suggested to be limited, which is consistent with the results of the prediction experiments (Table 3).

In contrast to the broker prediction, Table 6 shows that traditional centrality measures are included as top-five important features for the source spreader prediction tasks. Particularly for the Twitter Japan dataset, PageRank has the highest importance score. The DeepGL embedding features are also included in the top-five important features, but the
importance scores are comparable or lower than traditional centrality measures. These results confirm that for the source spreader prediction tasks, the DeepGL embeddings are not so effective, and traditional centrality measures are sufficient.

These results also show that many features based on betweenness centrality, PageRank, and $k$-core are included in the top-ranked lists. However, effective features are different for the tasks and datasets. From these results, we cannot find universal characteristics of influential brokers and source spreaders across different topics and user sets.

**Transferability**

The results in the previous subsection raise a new question about the transferability of the constructed models. We obtained quite different results for each different dataset, which suggests that a prediction model learned from one dataset may not be effective on other datasets. Therefore, we evaluate the prediction accuracy of a constructed model on one dataset when applying it to other datasets. We train a model using a source-domain dataset, then we evaluate its accuracy on a target-domain dataset; the training procedure is the same as that in the previous subsection. Since each dimension of the feature vector must be consistent between the training (i.e., source-domain) data and the test (i.e., target-domain) data, we transfer the DeepGL node embeddings from the source domain to the target domain when using the DeepGL embedding model. DeepGL can produce consistent node embedding vectors across different networks by using its inductive learning framework (Rossi, Zhou, and Ahmed 2018b,a).

Table 7 compares the $F_1$ scores of the task of identifying the top-10% influential brokers for different combinations of source- and target-domain datasets, and Table 8 does the same for the task of identifying the top-10% influential source spreaders. These results show that when the source and target domains are different, the $F_1$ scores are considerably lower than when the source and target domains are the same, suggesting that transferring a trained influencer identification model to different domains is difficult. Our results also suggest that it is more difficult to transfer a broker identification model than a source-spreader one. The models trained on different domains achieve only poor accuracy, particularly in the broker identification task. As suggested by the results in the previous subsection, traditional centrality measures are more useful for influential source-spreader identification than for influential broker identification, which might be the cause of the difference in transferability. Overall, the results suggest that models for identifying influencers should be trained for each domain to achieve high prediction accuracy.

**Effects of Amount of Training Data**

Finally, we investigate how the amount of training data affects the prediction accuracy of the constructed models. Figure 4 shows $F_1$ scores of the embedding models for each dataset while changing the fraction of training data. The task here was predicting the top-10% influencers, and the source and target domains were the same. Figure 4 shows that for all the datasets, the $F_1$ scores with 5% training data are almost the same as those with 50% training data. This suggests that 5% training data are enough to learn the model of identifying influencers. The results suggest that a small amount of training data is enough for obtaining influencer prediction models. While results in the previous subsection suggest that influencer identification models should be trained for each domain, the results in this subsection suggest that the training cost for each dataset is not large, which is preferable in practice.

**Discussion**

**Implications**

Our results show that the characteristics of influential brokers are different from those of influential source spreaders, which suggests that algorithms for identifying source spreaders cannot be used directly to identify brokers. Although designing centrality measures has been effective for identifying influential source spreaders (Lü et al. 2016; Li et al. 2014b; Zhang et al. 2016), our results suggest that such an approach is not effective for identifying brokers.

Our results show that the unique characteristics of influential brokers differ from those of influential source spreaders. Tweets retweeted by influential brokers tend to spread to many users, whereas tweets retweeted by influential source spreaders do not. Note that the causal relationship between a broker’s involvement in a retweet cascade and its future cascade size is unclear. One possible explanation is that a retweet posted by an influential broker facilitates other users’ retweets, which affects the future popularity of the tweet. The other explanation is that influential brokers retweet tweets that will be popular in the future in their early stages of retweet diffusion. In both cases, we reason that identifying influential brokers is useful. In the former case, influential brokers would be useful in viral marketing: a company could ask influential brokers to spread the tweets posted by its accounts so that those tweets spread widely. In the latter case, influential brokers would be useful for knowing which content will be popular in the future; also, identifying such users could help limit the spread of unwanted information by asking them not to do so.

Our results also show that while a single centrality measure has poor predictive power in identifying influential bro-
Limitations

This study has some limitations, which we discuss below along with future research directions. First, the effectiveness of other social network features for identifying brokers should be investigated. There are many options for node embedding techniques (Goyal and Ferrara 2018; Cui et al. 2018), and ones that are suitable for identifying brokers should be explored in future research. While using betweenness centrality alone is shown to be ineffective, we are interested in investigating the effectiveness of other techniques for identifying structural hole spanners (Lou and Tang 2013; Lin et al. 2021; Xu et al. 2017) for identifying influential brokers.

Second, the prediction accuracy should be improved for practical use. Although finding top influencers is a difficult task, we expect that there is room for improvement in the prediction accuracy of the models. As suggested in (Bhowmick et al. 2019), past log data of diffusion cascades are expected to be a useful source. Therefore, a model that incorporates both topological structure and diffusion cascades is expected to achieve higher accuracy. Moreover, using features of tweet contents such as linguistic features obtained from language models and like counts is also expected to be useful, and analyzing specific cases in which influential brokers play an important role (e.g., information diffusion regarding state-sponsored troll accounts (Zannettou et al. 2019)) may also provide useful insights.

Third, the applicability of the trained model to other users should be further investigated. Our results show that the trained models in this paper cannot be applied to different domains; this is not surprising because the three datasets differ considerably in terms of languages, cultures, topics, and social media platforms, but we should investigate further the transferability of our approach. For instance, a model trained on the Twitter Japan dataset might be applicable to different Japanese user sets. Also, we are interested in methods for transferring a pre-trained model to other domains (Qiu et al. 2020) for influencer identification, and clarifying a method for constructing a transferable influencer identification model is important future work.

Fourth, as discussed already, whether retweets posted by influential brokers do actually influence the retweeting behaviors of other users remains unclear. Clarifying a causal relation between one user’s retweet and other users’ retweets is challenging but important future work, and a possible way to tackle this problem would be to conduct a field experiment to ask social media users to retweet some specific tweets, and then compare their future diffusion patterns.

Conclusion

In this paper, we tackled the problem of identifying influential brokers who can spread other users’ messages to many users on social media. Using three social media datasets, we investigated the characteristics of influential brokers by comparing them with influential source spreaders and central nodes. Our results showed that most of the influential source spreaders are not influential brokers (and vice versa). We also showed that overlap between central nodes and influential brokers was less than 15% on the two Twitter datasets, which suggests that a heuristic that extracts highly central nodes as brokers is not a good approach. We conducted experiments of identifying influential brokers by using node embedding features obtained with DeepGL. Our results showed that models using DeepGL embeddings achieved \( F_1 \) scores of 0.35–0.68, which is a similar level of accuracy to that of identifying influential source spreaders. Moreover, models using DeepGL embeddings achieved higher accuracy than did those using only centrality measures, which indicates the effectiveness of DeepGL embeddings for identifying brokers. Our results showed the effectiveness of using network topology for identifying influential brokers, as well as the limitations of using traditional centrality measures.

| Source | Target | Twitter Japan | Twitter Nepal | Twitter Nepal |
|--------|--------|----------------|---------------|---------------|
|        |        | Japan Nepal Digg | Japan Nepal Digg | Japan Nepal Digg |
| centrality | 0.31 0.03 0.18 | 0.06 0.48 0.18 | 0.10 0.02 0.32 |
| embedding | 0.40 0.05 0.17 | 0.11 0.68 0.14 | 0.17 0.05 0.35 |

Table 8: Comparison of \( F_1 \) scores of the models among different combinations of source and target domain datasets when predicting top-10% brokers.
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