Dirac neutrinos and anomaly-free
discrete gauge symmetries

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Abstract

Relying on Dirac neutrinos allows an infinity of anomaly-free discrete gauge symmetries to be imposed on the Supersymmetric Standard Model, some of which are GUT-compatible.

A few introductory words

Dirac neutrinos have not (yet) been ruled out by experiment, see e.g. Ref. [1]. Provided a satisfactory explanation for the smallness of their masses, see e.g. Refs. [2, 3, 4, 5, 6, 7, 8, 9, 10, 11], they are an interesting alternative to the standard Majorana picture, in particular concerning cosmological aspects, see e.g. Refs. [12, 13, 14, 15, 16, 17]. For Dirac neutrinos, renormalization group effects have been studied in Ref. [18], and the possibility of having zero textures in the mass matrix of Dirac neutrinos has been considered in Ref. [19].

It is the purpose of this letter to draw the attention to yet another of their features: We will show that the assumption of Dirac rather than Majorana neutrino masses is consistent with infinitely many mutually non-equivalent discrete gauge symmetries (DGSs). They are so-called anomaly-free, and can be imposed on the Supersymmetric Standard Model (SSM) to forbid unwanted, e.g. proton endangering, operators. The possibilities thus far, namely baryon triality ($B_3$), recently introduced proton hexality ($P_6$) and well-known matter parity ($M_p$), cf. Refs. [20, 21, 22], are based on Majorana neutrino masses.

The content is as follows. In Sect. 1 and Appendix A we review Abelian discrete symmetries; in Sect. 2 and Apps. B C D E we describe how DGSs emerge from a high energy theory. Sect. 3 constitutes the main part of this text, determining the anomaly-free DGSs which rely on Dirac neutrinos. Sect. 4 discusses the physical implications of the various newly-found DGSs; Sect. 5 together with Appendix F analyzes whether the corresponding discrete charges are compatible with unification. In Sect. 6 as an example, we present toy models of how $\mathbb{Z}_4$-DGSs might arise. Sect. 7 concludes.

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1 Discrete symmetries

Thus far no processes which are lepton and/or baryon number violating have been observed, which is why the corresponding renormalizable and non-renormalizable operators in the Lagrangian density $\mathcal{L}$ have to be either strongly suppressed or absent altogether. The latter can be readily obtained by relying on a discrete symmetry (DS): One demands $\mathcal{L}$ to be invariant under a discrete transformation of the fields, $\varphi \rightarrow O^{DS}_z \varphi$. In case the DS is a $Z_N$-symmetry, with $N \in \{2, 3, \ldots \}$, this reads $O^{DS}_z = e^{2\pi i z} z$, with $z \in \{0, \ldots, N-1\}$. $Z_2$-symmetries are commonly labelled “parities”, Grossman and Haber [27] coined the word “triality” for a $Z_3$-symmetry, and in Ref. [22] $Z_6$-symmetries were called “hexalities”. We will encounter further “$N$-alities” later on. Of course, one can also have DSs which are not $Z_N$-symmetries (or direct products thereof), which means that they are non-Abelian. In the following, we will not concern ourselves with such non-Abelian DSs.

Assuming the existence of all Standard Model (SM) gauge invariant operators in the renormalizable and matter parity conserving superpotential (for notation see below)

$$\mathcal{W}_{\text{MSSM}} = h_d Q H_d D + h_u Q H_u U + h_u L H_d E + \mu H_d H_u ,$$ (1.1)

generation-independent $Z_N$-symmetries can be classified [21] according to which renormalizable lepton number ($n_{\text{lepton}}$) and/or quark number ($n_{\text{quark}}$) violating operators are forbidden: Allowing for the $n_{\text{lepton}}$-violating operator $QLD$ automatically allows the existence of $LLE$ and $LH_u$ from the DS point of view, see Eq. (1.1). Similarly, forbidding one of these three terms automatically forbids all; such a statement applies to non-renormalizable operators as well. So the (non-)existence of one operator is accompanied by the (non-)existence of a whole set of operators. Hence, concerning all lepton and/or baryon number violating operators up to dimension five [21, 22, 23, 24, 25] we find schematically, see Appendix A

$$QLD \iff \{QLD, LLE, LH_u, QULH_d, LH_u H_d H_u, QUL L^{|}, EH_d H_u^{|}, UDE\},$$

$$UDD \iff \{UDD, QQQH_d, QQD^{|}\},$$

$$QQQL \iff \{QQQL, UUD E\},$$

$$LH_u LH_u \iff \{LH_u LH_u\} .$$ (1.2)

The terms on the left-hand side should be viewed as representatives for the complete set on the right-hand side. [Lines 1, 3 and 4 will be extended in Eq. (1.1).] The representatives of the first two lines were used by Ibáñez and Ross, see Ref. [21], to classify the DSs in terms of the operators $QLD$ and $UDD$, see also Table 1

- A symmetry forbidding both operators is called a matter $N$-ality or generalized matter parity.
- If only $UDD$ is forbidden we speak of a baryon $N$-ality (generalized baryon parity).
- Forbidding only $QLD$ yields a lepton $N$-ality (generalized lepton parity).
- Allowing both operators is compatible only with a $Z_N$-symmetry where the discrete charges are proportional to the hypercharge $\hat{z}_i = Y_i \mod N$. Such a DS is trivial.

One can already see that no lepton or baryon $N$-ality is compatible with a Georgi-Glashow $SU(5)$, in which $QLD$ and $UDD$ both originate from a $10 5 5$. Likewise proton hexality $P_6$ [22] is incompatible with $SU(5)$ as it allows $QUQD$ but forbids $QQQL$, whereas both these operators come from a $10 10 5$.

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1 For instance, the renormalizable superpotential operators $QLD$ and $UDD$ together cause rapid proton decay if neither of them is very strongly suppressed. But also e.g. $QQQL$, though non-renormalizable and thus suppressed by presumably the gravitational scale, may cause havoc to the proton. So one needs help from e.g. family symmetries, see for instance Refs. [23, 24, 25, 26].

2 $Q, D, U, L, E, H_d$ and $H_u$ are the left-chiral superfields of the left-handed quark doublets, $d$-type and $u$-type antiquark singlets, left-handed lepton doublets, antilepton singlets and the two Higgs doublets, respectively; the $h^\dagger$ denote Yukawa matrices. $SU(3)_C$, $SU(2)_W$ and generational indices are suppressed.

3 Solving $z_Q + z_{H_d} + z_{\tau} = 0$, $z_Q + z_{H_u} + z_{\tau} = 0$, $z_L + z_{H_d} + z_{\tau} = 0$, $z_{H_d} + z_{H_u} = 0$, $z_Q + z_L + z_{\bar{\tau}} = 0$ and $z_{\bar{\tau}} + 2 z_{\tau} = 0$ gives that $z_i \propto Y_i \mod N$. 

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Table 1: The classification of different $\mathbb{Z}_N$-symmetries à la Ibañez and Ross, cf. Ref. [21]. For notations like $M_p$, $P_6$, etc. see Table 2.

| $\exists QLD$ | matter $N$-alities, e.g. $M_p$, $P_6$ | lepton $N$-alities, e.g. $L_p$ |
| $\exists QLD$ | baryon $N$-alities, e.g. $B_p, B_3$ | no DS at all |

The best-known example for a $\mathbb{Z}_2$-symmetry is $R$-parity ($R_p$) [31]. The discrete charge $z^{R_p}_\varphi$ for a field $\varphi$ is given by $z^{R_p}_\varphi = n_{\text{quark}}(\varphi) + n_{\text{lepton}}(\varphi) + 2 \cdot s(\varphi)$, $s$ being the spin. $R_p$ is defined for fields rather than superfields, providing a useful tool to classify whether a particle is part of the (2Higgs-)SM or whether it is a superpartner of one of these, i.e. whether it has a supersymmetric motivation. If one demands invariance of the Lagrangian density $\mathcal{L}$ under $R_p$ ($\mathcal{L}_{R_p}$), all lepton and baryon number violating renormalizable operators are forbidden.

$R_p$ can be modified to $R^{\text{uny}}_p$ [32, 33] which acts on whole superfields $\Phi$ (rather than fields): $z^{R^{\text{uny}}_p}_\varphi = n_{\text{quark}}(\Phi) + n_{\text{lepton}}(\Phi)$, constraining the super- and Kähler potential such that the result is $\mathcal{L}_{R^{\text{uny}}_p}$. Other examples of $\mathbb{Z}_2$-symmetries are baryon parity, $B_p$, $z^{B_p}_\varphi = n_{\text{quark}}(\Phi)$, and lepton parity, $L_p$, $z^{L_p}_\varphi = n_{\text{lepton}}(\Phi)$. Table 2 summarizes these common DSs together with the ones found in [21] and [22]. The primed DSs of Table 2 cannot originate from an anomaly-free $\mathcal{L}_{R_p}$ whereas $M_p$ is only Pati-Salam compatible, see Section 3. {\{B_3, B_3', B_3''\}} and {\{P_6, P_6', ..., P_6''''\}}. Examining the consequences of the $P_6$-symmetries, we find that they are very restrictive; as was proposed in Ref. [22]: "$P_6$ is the DS of the MSSM" if one relies on Majorana neutrinos.

## 2 Discrete gauge symmetries

It can be argued that global DSs are violated by quantum gravitational effects [34], which at first sight renders the use of DSs impractical. There is however a loop-hole: If the DS is a so-called DgaugeS (DGS), i.e. if it is the remnant/residual/left-over of a spontaneously broken local gauge symmetry, then no wormholes etc. screw up its performance [35, 36]. The underlying “mother symmetry” of course must not cause trouble with anomalies, from which follows that not every DS is automatically feasible; for instance, as we will see in Sect. 3 $B_p$, $B_p'$, $L_p$ and $L_p'$ of Table 2 cannot originate from an anomaly-free high-energy $U(1)$ symmetry.

In what follows we shall in a top-down fashion describe how DSs arise from a local gauge symmetry at high energies, listing step-by-step which transformations are performed and/or which assumptions are made, to finally arrive at the discrete anomaly equations which are the starting point of Sect. 3. We try to stay as general as possible as long as possible.

Though the local gauge symmetry could in principle be Abelian or non-Abelian, $R$- or non-$R$, for the rest of this paper, we shall consider a single non-$R$ local $U(1)$ gauge group. Hence we restrict our DGSs to be $\mathbb{Z}_N$-symmetries.

Hasty readers who are familiar with this subject may want to jump ahead directly to the next section, assuming a non-anomalous “mother” $U(1)_X$, no SM-singlets except the $U(1)_X$-breaking superfield $\mathfrak{A}$ with $X$-charge $N$ and three generations of right-handed neutrinos, all $X$-charges being integer numbers, the discrete charges of the MSSM superfields and the neutrinos being generation-independent, all SM-charged matter which is beyond the MSSM being heavy.

We start with an $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_X$-invariant quantum field theory supposedly coming from a string; $U(1)_Y$ is not yet to be identified with the SM-hypercharge, because for the sake of generality we take both $U(1)$-factors to be possibly anomalous at first. Of course, if instead one starts
Table 2: Common DSs. The first line gives the hypercharges of the superfields \( Q, \overline{D} \) etc.; the second and the third lines list the corresponding quark and lepton number; the other lines show the discrete charges of the superfields under various DSs.

| \( N \) | \( Q \) | \( \overline{D} \) | \( U \) | \( L \) | \( \overline{E} \) | \( H_d \) | \( H_u \) | comments |
|---|---|---|---|---|---|---|---|---|
| \( Y/Y_Q \) | -1 | 2 | -4 | -3 | 6 | -3 | 3 |  |
| \( n_{\text{quark}} \) | -1 | -1 | 0 | 0 | 0 | 0 | 0 |  |
| \( n_{\text{lepton}} \) | 0 | 0 | 0 | 1 | -1 | 0 | 0 |  |
| \( B_p \) | 2 | 1 | 1 | 0 | 0 | 0 | 0 | baryon \( N \)-ality, anomalous |
| \( B'_p \) | 2 | 0 | 1 | 1 | 0 | 1 | 1 | baryon \( N \)-ality, anomalous |
| \( L_p \) | 2 | 0 | 0 | 0 | 1 | 1 | 0 | lepton \( N \)-ality, anomalous |
| \( L'_p \) | 2 | 1 | 0 | 0 | 0 | 1 | 1 | lepton \( N \)-ality, anomalous |
| \( M_p \equiv R_{\text{susy}}^p \) | 2 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | matter \( N \)-ality, Pati-Salam compatible |
| \( M'_p \equiv R_{\text{susy}}^p' \) | 2 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | matter \( N \)-ality, \( SO(10) \) compatible |
| \( B_3 \) | 3 | 0 | 1 | 2 | 2 | 2 | 1 | baryon \( N \)-ality |
| \( B'_3 \) | 3 | 1 | 0 | 1 | 2 | 2 | 1 | baryon \( N \)-ality |
| \( B''_3 \) | 3 | 2 | 2 | 0 | 2 | 2 | 1 | baryon \( N \)-ality |
| \( P_6 \) | 6 | 0 | 5 | 1 | 4 | 1 | 1 | 5 | matter \( N \)-ality, same as \( M_p \times B_3 \) |
| \( P'_6 \) | 6 | 1 | 1 | 3 | 1 | 1 | 4 | 2 | matter \( N \)-ality, same as \( M_p' \times B''_3 \) |
| \( P''_6 \) | 6 | 2 | 3 | 5 | 4 | 1 | 1 | 5 | matter \( N \)-ality, same as \( M_p \times B'_3 \) |
| \( P'''_6 \) | 6 | 3 | 5 | 1 | 1 | 1 | 4 | 2 | matter \( N \)-ality, same as \( M_p \times B''_3 \) |
| \( P''''_6 \) | 6 | 4 | 1 | 3 | 4 | 1 | 1 | 5 | matter \( N \)-ality, same as \( M_p' \times B_3 \) |
| \( P'''''_6 \) | 6 | 5 | 3 | 5 | 1 | 1 | 4 | 2 | matter \( N \)-ality, same as \( M_p' \times B'_3 \) |

\( \text{e.g. with an } SU(5) \times U(1)_X - \text{invariant theory, some of the following points are obviously rendered moot, and other blatant steps [like the breaking of } SU(5) \text{] have to be introduced at obvious places. Up to short summaries of the corresponding points, we have relegated the first seven steps in which the K"ahler potential is canonicalized, the dilaton acquires a VEV and the anomaly is rotated into the } U(1)_X \text{ alone to Appendix B.} \)

1. The Kač-Moody matrix of the two \( U(1) \) factors is taken to be positive-definite.
2. The Kač-Moody matrix is diagonalized.
3. The effects of the \( U(1)_X \times U(1)_Y \) transformations are discussed: anomalies and the dilaton-originated Green-Schwarz shift.
4. The two above-mentioned effects mutually cancel.
5. The dilaton acquires a VEV, generating Fayet-Iliopoulos as well as kinetic gauge terms.
6. The kinetic gauge terms are canonicalized.
7. We rotate such that all Fayet-Iliopoulos terms are condensed in just one of the $U(1)$ factors.

8. One demands that some left-chiral superfields $A_i$ (not to be confused with the anomaly coefficients $A_{abc} = \text{Tr}c_i \{ T^a \}, T^b \} \cdot T^c$, the $T$’s being the gauge group generators and $\Omega_j$ are SM-uncharged but $X$-charged. The scalar components of the $A_i$ shall later acquire vacuum expectation values (VEVs) and thus play the role of Higgs fields for the $U(1)_X$; if $U(1)_X$ is generation-dependent, the $A_i$ are sometimes called flavons. The $\Omega_j$ on the other hand denote all other SM-singlets like e.g. a right-handed neutrino $\nu$.

9. Next, one requires for the $Y$-charges that $Y_Q + Y_{H_d} + Y_{\bar{D}} = Y_Q + Y_{H_u} + Y_{\bar{U}} = Y_L + Y_{H_d} + Y_{\bar{T}} = 0$. This, together with the vanishing of the anomaly coefficients $A_{CCY}$, $A_{WWY}$, $A_{GGY}$ and the assumption that all SM-charged matter beyond the MSSM is vectorlike, allows one to identify $U(1)_Y$ with the hypercharge, its values given in Table 2. Note that if a set of $X$-charges $X_i$ gives a certain value for the overall $X$-charge of a $Y$-invariant operator, then the set $X_i + \alpha Y_i$, with $\alpha \in \mathbb{R}$, constitutes the same value. The replacement $X_i \to X_i + \alpha Y_i$ is the so-called $Y$- or hypercharge-shift, parameterized by $\alpha$. Evidently, terms which are $U(1)_Y$-allowed and $U(1)_X$-forbidden/allowed are also forbidden/allowed after a $Y$-shift of the $X$-charges. Also the four linear anomalies $A_{CCX}$, $A_{WWX}$, $A_{YYX}$, $A_{GGX}$ [cf. Eq. (B.5)] remain invariant under this shift, whereas

\[
A_{XX} \to A_{XX} + 6\pi^2 \alpha^2 k_Y X_S, \quad A_{YXX} \to A_{YXX} + 4\pi^2 \alpha k_Y X_S.
\]

Here, $k_Y$ is the Kač-Moody level of $U(1)_Y$, and $X_S$ is a real parameter introduced in Eq. (B.4). Only if $X_S = 0$, all anomaly coefficients involving $U(1)_X$ are invariant under $Y$-shifts. In this case, due to the Green-Schwarz anomaly cancellation condition of Eq. (B.8), $U(1)_X$ is non-anomalous. Therefore, starting with an anomaly-free $U(1)_X$, the equations which constrain the remnant $\mathbb{Z}_N$-symmetry in Sect. 3 are not changed by $Y$-shifts. For an example of models related by a $Y$-shift see Sect. 6.

10. One postulates: With two sets of integers $n_i$, $n'_j$ fulfilling $\sum_i n_i Y_i = \sum_j n'_j Y_j = 0$, all $\sum_i n_i X_i$, i.e. all ratios of $X$-charges of terms which are $Y$-invariant, are rational numbers. Moreover, instead of making this operator-wise requirement, we demand in a field-wise fashion that the $X$-charges are such that all $X_i/X_j$ are rational numbers (charge quantization). If $X_S = 0$, this more restrictive requirement could be weakened to demanding that there is a $Y$-shift relating the original set of $X$-charges to another set for which all $X_i/X_j$ are rational numbers; for simplicity we shall not stick to this option.

11. The previous Item allows to rescale the $X$-charges such that they all take their smallest possible integer values.

12. With $\Phi$ now denoting any superfield which is not an $A_i$ (see Item 6), any super- or Kähler potential term $T$ composed of $k_{\text{max}}$ different species of superfields which is $SU(3)_C \times SU(2)_W \times U(1)_Y$-invariant can be written in the form $T = \Phi_1^{\phi_1} \cdot \Phi_2^{\phi_2} \cdot \ldots \cdot \Phi_{k_{\text{max}}}^{\phi_{k_{\text{max}}}}$, the $n_i$ being integer numbers, denoting how often the corresponding superfield appears in the term; note that $\phi^{-1}$ means $\phi$\textsuperscript{-1}. However, it is by far not guaranteed that $n_\phi_1 \cdot X_{\phi_1} + n_\phi_2 \cdot X_{\phi_2} + \ldots + n_{\phi_{k_{\text{max}}}} \cdot X_{\phi_{k_{\text{max}}}} = 0$. But suppose that the excess $X$-charge can be compensated by several powers of the superfield $A_1$. In this case $T = A_1^{-(n_\phi_1 \cdot X_{\phi_1} + n_\phi_2 \cdot X_{\phi_2} + \ldots + n_{\phi_{k_{\text{max}}}} \cdot X_{\phi_{k_{\text{max}}}})/X_{A_1}$. $T$ is $U(1)_X$-invariant. If there are several $A_j$ with different $X$-charges, it is for the purposes in this paper useful to work with an “effective $A$” or “reduced $A$” which we will label $\mathfrak{A}$. Taking into account the Giudice-Masiero/Kim-Nilles mechanism, its $X$-charge is the greatest common divisor of the $X$-charges of all the $A_j$, see Appendix C. $T$ then generalizes to

\[
\mathfrak{A}^{-(n_\phi_1 \cdot X_{\phi_1} + n_\phi_2 \cdot X_{\phi_2} + \ldots + n_{\phi_{k_{\text{max}}}} \cdot X_{\phi_{k_{\text{max}}}})/X_{\mathfrak{A}}, \quad \Phi_1^{\phi_1} \cdot \Phi_2^{\phi_2} \cdot \ldots \cdot \Phi_{k_{\text{max}}}^{\phi_{k_{\text{max}}}}.
\]
As an example, consider $X_{A_1} = \sqrt{13}$, $X_{A_2} = \frac{2}{3}\sqrt{13}$, $X_{A_3} = \frac{5}{3}\sqrt{13}$ and $X_{Q_i} = \frac{1}{3}\sqrt{13}$, $X_{H_d} = \frac{1}{15}\sqrt{13}$, $X_{T_{\tau \tau}} = \frac{2}{5}\sqrt{13}$ as a starting point. Then rescale the X-charges such that all fields have integer charges, thus multiply by $2 \cdot 3 \cdot 5/\sqrt{13}$, arriving at $X_{A_1} = 2 \cdot 3 \cdot 5 = 30$, $X_{A_2} = 3 \cdot 5 \cdot 7 = 105$, $X_{A_3} = 2 \cdot 5 \cdot 7 = 70$ and $X_{Q_i} = 6$, $X_{H_d} = 2$, $X_{T_{\tau \tau}} = 12$. The greatest common divisor of the $A_{1,2,3}$ is thus 5, so $|X_{A}| = 5$ (so e.g. $A = A_2A_1^tA_3^t$). Therefore, as we will argue later on, one arrives at a $Z_5$-symmetry with $z_{Q_i} = 1$, $z_{T_{\tau \tau}} = z_{H_d} = 2$. For another example see Appendix E.

13. There is however an important caveat to Eq. (2.2). As the Hamiltonian density necessarily is a polynomial of fields \cite{40, 41}, in order to satisfy the cluster decomposition principle (CDP) \cite{42}, i.e. distant experiments have uncorrelated results, one may only have integer exponents of the fields. This then translates to the requirement that every super- and Kähler potential term may contain only integer powers of the superfields, dictating that $n_{\phi_1} \cdot X_{\phi_1} + n_{\phi_2} \cdot X_{\phi_2} + ... + n_{\phi_{\text{max}}} \cdot X_{\phi_{\text{max}}}$ is an integer multiple of $X_\mathfrak{A}$, otherwise the whole term is forbidden.

14. The $A_i$ and thus also $\mathfrak{A}$ acquire VEVs, so $U(1)_X$ is broken. It must be ensured at all costs that those terms which are (phenomenologically) desired have $X$-charges which are integer multiples of $X_\mathfrak{A}$: In such a case, the operator in Eq. (2.2) produces $\langle \mathfrak{A} \rangle^{-\langle n_{\phi_1} \cdot X_{\phi_1} + n_{\phi_2} \cdot X_{\phi_2} + ... + n_{\phi_{\text{max}}} \cdot X_{\phi_{\text{max}}} \rangle / X_\mathfrak{A} \cdot T}$. On the other hand, terms which are undesired (like e.g. baryon number violating operators) might be assigned an overall $X$-charge which is not an integer multiple of $X_\mathfrak{A}$ so that the exponent of $\mathfrak{A}$ is fractional and the whole term thus forbidden. Therefore not all SM-invariant terms are necessarily generated, because the corresponding “mother terms” might be forbidden due to the CDP’s persistent constraints. These omissions are what one calls “forbidden due to a DGS”, the DGS being the remnant/residual/left-over of a spontaneously broken local gauge symmetry. If a super- or Kähler potential term is forbidden, then the $|X_\mathfrak{A}|^{th}$ power of this term is allowed for. This reasoning is precisely the same as the one which we reviewed in the beginning of Sect. I see also Eq. (2.5).

To parameterize the possible deviation of $n_{\phi_1} \cdot X_{\phi_1} + n_{\phi_2} \cdot X_{\phi_2} + ... + n_{\phi_{\text{max}}} \cdot X_{\phi_{\text{max}}}$ from being an integer multiple of $|X_\mathfrak{A}|$, one introduces the following decomposition of the $X$-charges

$$X_{\phi_j} = m_{\phi_j} \cdot |X_\mathfrak{A}| + z_{\phi_j}.$$  

$m_{\phi_j}$ and the discrete charge $z_{\phi_j}$ are both integer, the latter being restricted to $\{0, 1, ..., |X_\mathfrak{A}| - 1\}$. So if the sum of the $z_{...}$ of several superfields does not produce an integer multiple of $X_\mathfrak{A}$, the corresponding term is not allowed; we have a $Z_{|X_\mathfrak{A}|}$-symmetry. In the following we are going to work with the standard notation:

$$|X_\mathfrak{A}| \equiv N.$$  

The $N$ above however might not yet be the one showing up in “$Z_N$”: Suppose that $N = 24$, then the superfields suggest a $Z_{24}$-symmetry. But it might well be that for all SM gauge invariant operators the overall discrete charges are even, so that rescaling at the operator level effectively yields a $Z_{12}$-symmetry.

15. We demand that the $X$-charges of the superpotential terms $Q^i H_d \bar{D}^j$ and $Q^i H_u \bar{U}^j$ ($i, j \in \{1, 2, 3\}$) are integer multiples of $N$. Otherwise the corresponding Yukawa coupling constants would contain zero-entries due to the CDP, which would translate to unobserved zero-entries in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. So we find that the discrete charges of the quarks have to be generation-independent, although the original $X$-charges might well be generation-dependent: $m_{Q_i} \neq m_{Q_j}$ but $z_{Q_i} = z_{Q_j} \equiv z_Q$, see Eq. (2.3). In other words, discrete quark charges are family-universal.

16. For simplicity, we demand the same for the leptons. With only generation-independent discrete charges and the requirement that the three SSM Yukawa couplings are allowed by the discrete

\footnote{If one relies on Dirac neutrinos or a see-saw, the same arguments as in Item 14 apply, with the Maki-Nakagawa-Sakata matrix \cite{44} replacing the CKM matrix. One way to avoid this conclusion is the generation of (Majorana) neutrino masses from loop-effects, see Ref. \cite{44}.}
symmetry, i.e.
\[
\begin{align*}
z_Q + z_{\bar{H}_d} + z_{\bar{D}} & = 0 \mod N, \\
z_Q + z_{\bar{H}_u} + z_{\bar{U}} & = 0 \mod N, \\
z_L + z_{\bar{H}_d} + z_{\bar{E}} & = 0 \mod N,
\end{align*}
\]
the total discrete charge of any gauge-invariant term in the \(\mathbb{Z}\)+SSM sector can be expressed as, see also Refs. [22, 45, 46],
\[
z_{\text{total}} = (z_{\bar{U}} + z_D + z_D) \cdot Z + (z_Q + z_L + z_{\bar{D}}) \cdot Z + (z_{\bar{H}_d} + z_{\bar{H}_u}) \cdot Z + N \cdot Z,
\]
(2.4)
with \(Z\) representing an integer number. This result motivates the classification of the \(\mathbb{Z}_N\)-symmetries in Table 1.

17. We now add the three right-handed neutrinos \(\bar{N}\) to the theory, additionally requiring that the Yukawa terms \(LH_u\bar{N}\) are allowed by the DGS,
\[
z_L + z_{\bar{H}_u} + z_{\bar{N}} = 0 \mod N.
\]
(2.6)
Solving the four equations of Eqs. (2.4,2.6) with eight unknowns, we can express the \(z\) in terms of the four parameters \(m, n, p, r \in \{0, 1, ..., N-1\}\) (so \(QH_d\bar{D}, QH_u\bar{U}, LH_d\bar{E},\) and \(LH_u\bar{N}\) are required, but not \(H_d\bar{H}_u\)):
\[
\begin{align*}
z_Q & = r, & z_{\bar{U}} &= m - n + 2r, & z_{\bar{D}} &= -m - 4r, \\
z_L & = -n - p - 3r, & z_{\bar{E}} &= m + p + 6r, & z_{\bar{N}} &= -m + n + p, \\
z_{\bar{H}_d} & = -m + n - 3r, & z_{\bar{H}_u} &= m + 3r.
\end{align*}
\]
(2.7)
The coefficient of \(r\) is proportional to the hypercharge of the corresponding particle (see Table 2); hence \(r\) is the discrete version of the \(Y\)-shift-parameter \(\alpha\) in Item 9. Choosing \(r = 0\), we recover the same parameterization of discrete symmetries as in Ref. [21], here generalized to include the right-handed neutrinos.

18. The \(X\)-charges decompose according to Eq. (2.3). Using Eq. (2.7), we can rewrite the Green-Schwarz anomaly cancellation conditions in terms of the discrete parameters \(m, n, p, r\). For instance, for the anomaly \(A_{\text{CCX}}\) we obtain, see Eqs. (B.6,B.8),
\[
\frac{1}{2k_C} \left[ -N_f \cdot n + \sum_{i=1}^{N_f} (2m_{Q_i} + m_{D_i} + m_{U_i}) \cdot N + 2 \cdot A_{\text{CCX}}^{\text{beyond MSSM}} \right] = 2\pi^2 X_S,
\]
(2.8)
with \(N_f\) denoting the number of generations. This, however, does not specify everything, since we have not yet dealt with beyond-MSSM matter.

19. In the following, we list our assumptions about SM-charged matter which is not part of the MSSM:
- **C**-charged matter: There may be no massless colored particles, as these would have been seen already by experiment. What can in principle occur is colored matter in vectorlike pairs which is too heavy to have been detected so far. After \(U(1)_X\) breaking, the corresponding mass terms must therefore be \(\mathbb{Z}_N\)-invariant.
- **W**-charged matter: As for colored particles.
- **Y**-charged matter: We distinguish the following mutually independent cases, elucidated below

| Beyond-SM matter is heavy. | \(Y\)-charge is normal or large compared to SM \(Y\)-charges. | \(Y\)-charge is tiny compared to SM \(Y\)-charges. |
|---------------------------|----------------------------------------------------------|--------------------------------------------------|
| (a) o.k.                  | (b) renders \(A_{YYX}\) and \(A_{YXX}\) useless          |
| (c) not observed by experiment | (d) renders \(A_{YYX}\) and \(A_{YXX}\) useless          |
| Beyond-SM matter is light but not massless. | (e) not observed by experiment | (f) renders \(A_{YYX}\), \(A_{YXX}\) and \(A_{GGX}\) useless |
| Beyond-SM matter is massless. |                                             |                                                  |
(a) Just like before, heavy particles with reasonable $Y$-charges are acceptable.

(b,d) Heavy or light-but-not-massless particles with tiny $Y$-charges cannot be ruled out. The presence of such particles spoils the predictability of $A_{YYX}$ and $A_{YXX}$, which is why we shall not use these two constraints; for more details see Sect. 4 of Ref. [22].

(c,e) There may be no light or even massless particles with a reasonable, i.e. not too small, hypercharge, as these would have been seen already by experiment.

(f) In principle, one could also have massless particles with tiny (experimentally yet undetectable) hypercharges. Then, however, a systematic analysis of the discrete anomaly condition would not be possible. Hence, we demand such particles to be absent.

With these assumptions Eq. (2.8) reads

$$-N_f \cdot n + N \cdot Z = 4 \pi^2 X_S k_C ,$$

(2.9)

$Z$ symbolizing an integer number. A similar relation is obtained for $A_{WWX}$.

20. We demand unification of the three MSSM gauge coupling constants. That is, adopting the hypercharge normalization $Y_L = \frac{1}{2}$, we require that the Kač-Moody levels are related by $k_C = k_W = \frac{4}{5} \delta_Y$, see Eq. (2.10).

21. We demand the $U(1)_X$ to be anomaly-free, i.e. $X_S = 0$. (This makes Item 20 superfluous.)

22. The only massless $\Omega$-type particles (see Item 8) we shall admit are right-handed neutrino superfields $N$, i.e. particles whose discrete charge is such that their trilinear coupling to $LH_u$ is allowed, cf. Eq. (2.6). Massive $\Omega$s can be assumed as well without spoiling the analysis in Sect. 3. Other types of particles are classified in Appendix D. In the language of Appendix D we shall deal with “Case 3”, which has the term $LH_u LH_u$ not allowed, thus we will not have to deal with pseudo-Dirac neutrinos. Having constrained the SM-singlet particle content, the calculation of the gravitational anomaly $A_{GGX}$ becomes feasible, as well. Now, Eq. (2.9) and Item 21 together with the equivalent relations for $A_{WWX}$ and $A_{GGX}$ lead to the starting point of our investigation in the next section:

$$-N_f \cdot n + N \cdot Z = 0 ,$$

(2.10)

$$-N_f \cdot (n + p) + N_H \cdot n + N \cdot Z = 0 ,$$

(2.11)

$$-N_f \cdot (5n + p - m - \zeta_N) + 2N_H \cdot n + N \cdot Z + \eta \frac{N}{2} \cdot Z = 0 .$$

(2.12)

$N_H$ is the number of pairs of Higgs doublets. $\eta = 0, 1$ for $N$ is odd, even; furthermore, $\zeta_N = 0$ in a theory without light right-handed neutrinos and $\zeta_N = -m + n + p$ if there are $N_f$ generations of $N$. Note that the $r$-dependence drops out since the linear anomalies are invariants under $Y$-shifts, see Item 9.

23. Finally we integrate out heavy degrees of freedom, including the heavy $U(1)_X$ gauge boson. This might cause a rescaling of the discrete charges, for the MSSM sector could have a discrete symmetry which is a subgroup of the overall $\mathbb{Z}_N$-symmetry. Consider again the example of a $\mathbb{Z}_{24}$. Suppose that all MSSM superfields have even discrete charges, but some heavy particles have $z = 1$. Then, the $\mathbb{Z}_{24}$ cannot be rescaled to a $\mathbb{Z}_{12}$ like in the example at the end of Item 14. However, after the energies have dropped below the masses of the $z = 1$ heavy matter, one can integrate it out, and a rescaling (now only within the MSSM sector) becomes possible.

3 Anomaly-free Dirac-DGSs

Compared with Refs. [21, 22], we have added three right-handed neutrinos $\overline{N}$ to the light particle content. Analogously to [22] we now discuss the resulting discrete anomaly conditions, i.e. Eqs. (2.10,2.12). Note that in a scenario with light right-handed neutrinos the parameter $m$ remains unconstrained; so regardless of what the values for $N_f$ and $N_H$ are, it can take all $N$ values $m = 0, 1, ..., N - 1$. Restricting ourselves
to $N_f = 3$ and $N_H = 1$, we get

\begin{align}
-3n &= N\cdot \mathbb{Z}, \quad (3.1) \\
-2n - 3p &= N\cdot \mathbb{Z}, \quad (3.2) \\
3m - 13n - 3p + 3\zeta_N &= N\cdot \mathbb{Z} + \eta \frac{N}{2} \cdot \mathbb{Z}, \quad (3.3)
\end{align}

which can be linearly combined to give $3n = N\cdot \mathbb{Z}$, $3p - n = N\cdot \mathbb{Z}$ and \[ (3.3) - 2 \times (3.2) - 3 \times (3.1) \]

$3(m + p + \zeta_N) = N\cdot \mathbb{Z} + \eta \frac{N}{2} \cdot \mathbb{Z}$. If $\zeta_N = 0$ we recover Eqs. (2.21–2.23) of Ref. [22]; plugging in $\zeta_N = -m + n + p$, i.e. considering the case with three $\mathbb{N}$ (which from the viewpoint of the discrete anomaly conditions could have Majorana mass terms if $\zeta_N = 0, N/2$), yields

\begin{align}
3n &= N\cdot \mathbb{Z}, \quad (3.4) \\
3p - n &= N\cdot \mathbb{Z}, \quad (3.5) \\
6p + 3n &= N\cdot \mathbb{Z} + \eta \frac{N}{2} \cdot \mathbb{Z}. \quad (3.6)
\end{align}

The calculation of $3 \times (3.3) + 4 \times (3.5) - 2 \times (3.6)$ leads to the condition $-n = N\cdot \mathbb{Z}$ which reveals that $n = 0$, thus rendering Eq. (3.5) trivial. Interestingly enough, this is exactly the condition for having the bilinear term $H_uH_u$ allowed by the discrete symmetry, since $z_{H_uH_u} = n$, see Eq. (2.7). So without demanding it, the $\mu$-term emerges automatically due to anomaly considerations, unlike in Refs. [21, 22]. From Eq. (3.5) we now obtain

\begin{equation}
3p = N\cdot \mathbb{Z}. \quad (3.7)
\end{equation}

Only in those cases where $N$ is a multiple of three, $p$ can take a non-trivial value. However, there exist non-trivial DGSs also with $p = 0$, taking e.g. $m = N/2$ gives $M_p$.

With right-handed neutrinos, all anomaly-free DGSs can now be classified by the set of integers $(N; m, n, p) = (N; m, 0, p)$, with the constraint of Eq. (3.7). In contrast to Ref. [22], a Majorana mass term $\mathbb{N}\mathbb{N}$ is not imposed here. As this term has discrete charge $2(p - m)$, it is allowed only if either $(p = 0 \land m = N/2)$, $(p = N/2 \land m = N/2)$ or $(p = 2N/3 \land m = 2N/3)$; of course, $N$ must be divisible by 2 and/or 3. The classification of the DGSs in terms of the values of $N$ is shown in Table 3. The cases allowing for the Majorana mass term $\mathbb{N}\mathbb{N}$ are listed explicitly and correspond to $M_p$, $B_3$ and $P_6$ only. In order to comply with our requirement of having pure Dirac neutrinos, see Item [22] we discard these solutions of the anomaly conditions. All other cases, however, yield new anomaly-free DGSs, which we will call Dirac-DGSs. The $\mathbb{Z}_N$-symmetries up to $N = 6$ are given in Table 4 just to list a few. Thus, excluding 1) \{ $M_p$, $B_3$, $P_6$, 2 \} rescalings like $(6; 0, 0, 2) = (3; 0, 0, 1)$ as well as 3) double counting like $(3; 0, 0, 1) = (3; 0, 0, 2)$ \{ $(N; m, 0, p)$, $(N; M - m, 0, N - p)$ and $(a \cdot N; a \cdot m, 0, a \cdot p)$, with $a$ being a positive integer, give the same DGS, for the latter see Item 22 \}, we have many Dirac-DGSs\footnote{Having taken rescaling already into account, why is the number of DGSs in the case without Dirac neutrinos three and in the case with Dirac neutrinos \infty? With Majorana neutrinos, the possibility of rescaling the discrete charges leads to a finite number of distinct DGSs [22]. But allowing for Dirac neutrinos, the parameter $m$ is not constrained at all, therefore the choice of $N$ being an arbitrary prime number always leads to non-trivial Dirac-DGSs.} also with $N \leq 6$\footnote{Like the numerical syllables in trityality and hexality, we shall stick to Greek rather than Latin. Otherwise we would have e.g. tertiality, quartality, quintality, sextality and septyality.}.

- three trialties: $(3; 0, 0, 1)$, $(3; 1, 0, 0)$, $(3; 1, 0, 2)$,
- one tetrality: $(4; 1, 0, 0)$,
- two pentalities: $(5; 1, 0, 0)$, $(5; 2, 0, 0)$,
- three hexalities: $(6; 1, 0, 0)$, $(6; 1, 0, 2)$, $(6; 3, 0, 2)$.

Beyond Table 4 we easily also find

- three heptalties: $(7; 1, 0, 0)$, $(7; 2, 0, 0)$, $(7; 3, 0, 0)$,
- two octalities: $(8; 1, 0, 0)$, $(8; 3, 0, 0)$,
Table 3: Classifying the anomaly-free DGSs with right-handed neutrinos in terms of the value for \( N \). \( \sim (2|N) \) and \( \sim (3|N) \) denotes that \( N \) is not an integer multiple of 2 and 3, respectively. Note that the treatment in this section is more general than the one in Ref. [22]; “without Dirac neutrinos” is so-to-speak a special case of “with Dirac neutrinos”, see also Table 6 (no Dirac neutrinos means “not Case 3”, so Cases 1 and 2 remain, both with and without right-handed neutrinos \( \mathcal{N}_{\text{Maj}} \)).

- continuing to, say, \( N = 14 \), there are (all distinct) nine 9-alities, two 10-alities, five 11-alities, eight 12-alities, six 13-alities, three 14-alities, see also the Table in Appendix [4]

Before the discussion of the physical implications of the Dirac-DGSs, some comments concerning the purely Abelian anomaly conditions are in order.

- As observed in Ref. [22] and Item 19 the anomaly coefficients \( A_{YY} \) and \( A_{YXX} \) do not pose useful constraints on the DGSs because the hypercharges of heavy Dirac particles could be fractional; this statement holds true for Dirac-DGSs as well.

- On the other hand, in Ref. [22] \( A_{YXX} \) contained information about whether or not fractionally \( X \)-charged exotic matter has to be assumed for a given DGS. This is not the case for Dirac-DGSs as we will sketch in the following. It was shown in Ref. [22] that the cubic anomaly condition \( A_{YXX} = 0 \) can be written as \( \sum_{i} z_{i}^{3} = \text{RHS} \), with the \( z_{i} \) denoting the discrete charges of the particles in the \( \mathcal{N}+\text{SSM} \) sector. The RHS can take on only certain values depending on \( N \), cf. [22]’s Eqs. (A.3,A.4):

| RHS | \( \sim (2|N) \) | \( (2|N) \land \sim (4|N) \) | \( (4|N) \) |
|-----|----------------|-----------------|----------------|
| \( \sim (3|N) \) | \( N \cdot Z \) | \( \frac{N}{2} \cdot Z \) | \( N \cdot Z \) |
| \( (3|N) \) | \( 3N \cdot Z \) | \( 3\frac{N}{2} \cdot Z \) | \( 3N \cdot Z \) |

These possible values for the RHS must be compared to the sum over \( z_{i}^{3} \), which we can express in terms of the parameters \((m, n, p)\). In contrast to [22], we now have to include the three right-handed neutrinos with discrete charge \((p + n - m)\); this simplifies the resulting expression, see Eq. (A.1) of [22], considerably. Inserting \( n = 0 \), a necessity for all Dirac-DGSs, we get \( \sum_{i} z_{i}^{3} = 18 m^{2} p \),
Table 4: The easiest $\mathbb{Z}_N$-symmetries. The comment “same as $x$th” means that the symmetry is equivalent to the one in the $x$th line of the symmetries with identical $N$.

which, due to Eq. (3.7), is always an integer multiple of $6m^2N$. The RHS can match this value for all possible values of $N$. One therefore does not have to rely on fractionally $X$-charged heavy particles in order to meet the cubic anomaly condition. In this respect, $A_{XX}$ does not constrain the Dirac-DGSs.

4 The physics of Dirac-DGSs

In order to discuss the physical implications of the Dirac-DGSs, we investigate which lepton and/or baryon number violating operators are allowed for these new symmetries. As mentioned in Section 1, many of these operators come together with other operators if one assumes the presence of the MSSM superpotential terms, see Eq. (1.2). Even though the $\mu$-term is initially not required, it arises automatically for Dirac-DGSs due to anomaly considerations. Therefore, the classification of the lepton and/or baryon number violating operators up to dimension five given in Eq. (1.2) applies to the Dirac-DGSs as well.

However, in the Dirac case, there is a new particle, the right-handed neutrino $\nu$ with $n_{\text{lepton}} = -1$, which leads to additional SM invariant terms. We have to determine these new operators and group them together depending on their discrete charges: If, under a specific DGS $(N; m, 0, p)$, one term has for example discrete charge $p$ and another has charge $-p$, then both operators are simultaneously forbidden.
This cubic operator carries charge 3. Under the general Dirac-DGS, the discrete charge for this set of operators is given by Eq. (1.2), we obtain six sets of $n_{\text{lepton}}$- and/or $n_{\text{quark}}$-violating operators, which can be represented by the terms:

$$QLD, \quad LH_uLH_u, \quad UDD, \quad QQQL, \quad \overline{NNN}, \quad \overline{NNNN}. \quad (4.1)$$

Since our focus is to classify the Dirac-DGSs, i.e., those which forbid the Majorana mass term $\overline{NN}$, the $QLD$-set ($\ni \overline{N}$) and the $LH_uLH_u$-set ($\ni \overline{NNN}$) are never allowed by Dirac-DGSs. Comparing with Table 1 shows that Dirac-DGSs can never be baryon N-alities, but only matter or lepton N-alities. Let us therefore discuss the remaining four sets in turn.

- **$UDD$:** Under the general Dirac-DGS ($N; m, 0, 0$), the discrete charge of these operators is $\pm m$. They are thus present in theories where the DGS has $m = 0$. With Eq. (3.7), these are ($N; 0, 0, N \cdot Z/3$), leading to lepton triality ($3; 0, 0, 1$) as the only possibility after rescaling. All other Dirac-DGSs forbid $UDD$ and its accompanying operators, cf. Eq. (1.2); they are therefore all matter N-alities.

- **$QQQL$:** The discrete charge for this set of operators is given by $\mp p$. They are therefore present in all $Z_N$-symmetries with $N \neq 3 \cdot Z$ [see Eq. (3.7)] as well as in those symmetries with $N = 3 \cdot Z$ and $p = 0$.

- **$\overline{NNN}$:** This cubic operator carries charge $3(p - m)$. Due to Eq. (3.7), this is equivalent to the discrete (mod $N$) charge $3m$. Hence, $\overline{NNN}$ is allowed only if $m = \frac{3}{N} \cdot Z$. This together with $p = \frac{3}{N} \cdot Z$ shows that the cubic term arises only for $Z_3$-symmetries.

- **$\overline{NNNN}$:** Here we obtain the discrete charge $4(p - m)$. To find the Dirac-DGSs that allow this quartic term, we multiply the corresponding condition $4(p - m) = N \cdot Z$ by three and apply Eq. (3.7):

$$4 \cdot 3p - 12m = 3N \cdot Z, \quad \Rightarrow \quad 12m = N \cdot Z.$$  

Depending on whether $N$ is divisible by 2, 3 and/or 4, we get the conditions

$$p \in \left\{ 0, \frac{N \cdot Z}{3} \right\}, \quad m \in \left\{ 0, \frac{N \cdot Z}{2}, \frac{N \cdot Z}{3}, \frac{N \cdot Z}{4}, \frac{N \cdot Z}{6}, \frac{N \cdot Z}{12} \right\}.$$

After rescaling, the only $Z_N$-symmetries that have the potential to allow the term $\overline{NNNN}$ are those with $N = 2, 3, 4, 6, 12$. However, excluding $M_3$, $B_3$ and $P_6$, one can show explicitly that, of the remaining 0 + 3 + 1 + 3 + 8 possible Dirac-DGSs, only three symmetries allow this quartic term: (4; 1, 0, 0), (12; 1, 0, 4) and (12; 5, 0, 8), see also Appendix.

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It is easy to check which type of N-ality one obtains from a given parameter set ($N; m, n, p$). Using Eq. (2.7), the discrete charges of $QLD$ and $UDD$ are given as $z_{QLD} = m - 2n - p$ and $z_{UDD} = m - 2n$, respectively. With Table 1 we find that baryon N-alities require $[z_{UDD} \neq 0 \land z_{QLD} = 0]$, lepton N-alities must have $[z_{UDD} = 0 \land z_{QLD} \neq 0]$, and matter N-alities need $[z_{UDD} \neq 0 \land z_{QLD} \neq 0]$. 

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$^8$It is easy to check which type of N-ality one obtains from a given parameter set ($N; m, n, p$). Using Eq. (2.7), the discrete charges of $QLD$ and $UDD$ are given as $z_{QLD} = m - 2n - p$ and $z_{UDD} = m - 2n$, respectively. With Table 1 we find that baryon N-alities require $[z_{UDD} \neq 0 \land z_{QLD} = 0]$, lepton N-alities must have $[z_{UDD} = 0 \land z_{QLD} \neq 0]$, and matter N-alities need $[z_{UDD} \neq 0 \land z_{QLD} \neq 0]$. 

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Table 5: The lepton and/or baryon number violating operators occurring with Dirac-DGSs. The discrete charges of the operators are given as well as the symmetries that allow these terms.

In this section we analyze the compatibility of the GUTs with various grand unified theories (GUTs). Our starting assumption is that the gauge structure of the theory includes \( U(1)_X \times G_{\text{GUT}} \) where \( G_{\text{GUT}} \) is the gauge group of the chosen GUT, and the \( U(1)_X \) factor generates the low-energy discrete symmetry. We therefore get \( Z_N \times G_{\text{GUT}} \). This structure constrains the possible \( Z_N \)-symmetries because it requires all the fields of one \( G_{\text{GUT}} \) multiplet to have the same discrete charge. Note however that it is well possible to have a GUT-compatible DGS arising from a GUT-incompatible \( U(1)_X \); for an example see Section 6. From the low-energy point of view, the discrete charges are not uniquely fixed for a specific DGS given in terms of \((N; m, n, p)\). This ambiguity is parameterized by the integer \( r = 0, \ldots, N - 1 \) in Eq. (2.7) and can be exploited to find GUT compatible DGSs.

In the following, we discuss the constraints on the discrete charges for various GUT(-like) scenarios and their implication for the (non-)existence of the lepton and baryon number violating operators in the set \( QQQL \), see Table 5.

- SO(10): The 16 of \( SO(10) \) contains all quarks and leptons. Therefore this GUT group requires \( z_Q = z_U = z_D = z_L = z_{\bar{L}} = z_{\bar{E}} = z_N \). Imposing these relations on Eq. (2.7) and setting \( n = 0 \), we arrive at the necessary conditions for a GUT compatible Dirac-DGS:

\[
p = 0 \ , \quad m + r = 0 \mod N \ , \quad 4r = 0 \mod N \ . \quad (5.1)
\]

\footnote{A similar analysis in which the \( Z_{12} \)-symmetry is required to have \( M_p \) as a subgroup can be found in Ref. 17.}

\footnote{We point out that this is a simplifying assumption, since additional \( U(1) \) factors can arise when \( G_{\text{GUT}} \) breaks down to the Standard Model gauge group \( G_{\text{SM}} \), e.g. \( SO(10) \to SU(5) \times U(1)' \to SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)' \). In such a scenario, a combination of this \( U(1)' \) with the \( U(1)_X \) could be responsible for the emergence of the discrete symmetry. Here we shall however assume that the origin of the DGS is independent of the GUT gauge group.}
Since $p = 0$, the operators in the set $QQQL$ are always allowed. Note that this statement does not depend on the constraints of the anomaly conditions but only on the presence of the $\mu$-term ($n = 0$). After rescaling, there are actually only two $\mathbb{Z}_N$-symmetries ($N; m, n, p; r$) which are $SO(10)$ compatible: $(2; 1, 0, 0; 1) = M'_p$ (not a Dirac-DGS), cf. Table 3 as well as $(4; 1, 0, 0; 3)$, cf. Sect. 6. Note that $SO(10)$ compatible $\mathbb{Z}_N$-symmetries are, of course, also compatible with the following GUTs.

- **Georgi-Glashow $SU(5)$** [with a possible extra $U(1)'$-factor with charges $-1, 3, -1, 3, -1, -5, -2, 2$ for the superfields $Q, \bar{Q}, U, L, \bar{E}, \bar{N}, H_d, H_u$]: Here, the $10$ decomposes into $Q, \bar{U}, \bar{E}$, and the $\mathbb{5}$ into $\mathbb{D}, L$; the right-handed neutrino $\bar{N}$ lives in a singlet of $SU(5)$ 49]. So $z_Q = z_{\bar{E}} = z_{\bar{E}}$, $z_{\bar{D}} = z_L$, leads to the conditions

$$p = 0, \quad m + 5r = 0 \mod N \ . \tag{5.2}$$

Again, the $QQQL$-set is always allowed in this case. The $SU(5)$ compatibility of all $\mathbb{Z}_N$-symmetries up to $N = 14$ is shown in Appendix E stating explicitly the required values for $r$. It is easy to see that there exist infinitely many such DGSs: Consider for instance $(N; N - 5, 0, 0; 1)$ and $N$ being prime; then no rescaling is possible, so that there are at least as many $SU(5)$ DGSs as there are prime numbers.

- **Flipped $SU(5) \times U(1)'$** [the $U(1)'$ factor with charges $-1, -1, 3, 3, -5, -1, -2, 2$]: The embedding of the particles into the multiplets of flipped $SU(5)$ is similar to Georgi-Glashow $SU(5)$: One simply switches “up” and “down” for the $SU(2)_W$ singlets. Thus we have $10 \rightarrow Q, \bar{D}, \bar{N}$ and $\mathbb{5} \rightarrow U, L$; the right-handed electron $\bar{E}$ is in the singlet representation of $SU(5)$ and is charged only under $U(1)'$, see Ref. 51 and also Refs. 51 52. This yields $z_Q = z_{\bar{E}} = z_{\bar{E}}$, $z_{\bar{D}} = z_L$ leading to

$$p = 0, \quad m + r = 0 \mod N \ . \tag{5.3}$$

Also in this case, $QQQL$ cannot be forbidden by a DGS. As with $SU(5)$, there is an infinite number of flipped $SU(5)$ compatible DGSs.

- **Pati-Salam $[SU(4) \times SU(2)_W \times SU(2)_R]$** [the $(4, 2, 1)$ representation contains the fields $Q, L$, while the $(4, 1, 2)$ decomposes into $\mathbb{D}, U, \bar{E}, \bar{N}$ 53]. $[SU(3)_C$ comes from $SU(4)$, while $U(1)_Y$ stems from $SU(4) \times SU(2)_R$. $]$ Hence $z_Q = z_L$, $z_{\bar{E}} = z_{\bar{E}} = z_{\bar{N}}$, and then

$$p + 4r = 0 \mod N , \quad 2m + 6r = 0 \mod N \ . \tag{5.4}$$

As $p$ is not automatically zero, the operators in the set $QQQL$ can be forbidden by Pati-Salam compatible $\mathbb{Z}_N$-symmetries. Actually, there are only four DGSs which allow $QQQL$, namely $(2; 1, 0, 0; 0) = M_p$, $(2; 1, 0, 0; 1) = M'_p$ (both not Dirac-DGSs), and $(4; 1, 0, 0; 1)$, $(4; 1, 0, 0; 3)$: all other Pati-Salam $\mathbb{Z}_N$-symmetries forbid the operators of the set $QQQL$.

Interestingly, the number of such DGSs is finite. We have just stated that with $p = 0$, i.e. allowing $QQQL$, there are only two Dirac-DGSs. Let us therefore consider $p = \frac{N}{4}$. Multiplying the first condition of Eq. (5.4) by three and the second by two, we get $N + 12r = 3aN$, $4m + 12r = 2bN$, with unspecified integers $a, b$. Subtracting the first equation from the second and solving for $m$ yields

$$m = (2b - 3a + 1) \cdot \frac{N}{4} \ . \tag{5.5}$$

In the case of $N$ not being a multiple of 4, the parameters $a$ and $b$ have to be chosen such that $m$ is an integer. $p = \frac{N}{4}$ and Eq. (5.5) give rise to DGSs of the form (we neglect the value for $r$)

$$\begin{align*}
(N ; (2b - 3a + 1) \cdot \frac{N}{4} , 0 , \frac{N}{3}) & \iff (12 ; 3 (2b - 3a + 1) \cdot \frac{N}{12} , 0 , 4 \cdot \frac{N}{12}) \\
\end{align*}$$
In the last step we have rescaled all parameters with the common factor $\frac{N}{12}$, which, in general, need not be an integer. Further rescaling might be possible, depending on the values of $a$ and $b$. For $p = \frac{2n}{11}$ we obtain a similar result. This shows that Pati-Salam compatible $\mathbb{Z}_N$-symmetries are only possible for $N \leq 12$. Explicit counting yields 9 such Dirac-DGSs, of which 9 − 2 = 7 forbid $QQQL$.

Summarizing the above results, we have one $SO(10)$, an infinite number of (flipped) $SU(5)$ and nine Pati-Salam compatible Dirac-DGSs. The GUT compatibility of all $\mathbb{Z}_N$-symmetries with $N \leq 14$ is given in Appendix [F]. Almost all of them allow the operators of the set $QQQL$; in order to have proton-decay at an experimentally acceptable rate it is thus necessary to suppress the term $QQQL$ in these scenarios. Only seven Pati-Salam compatible DGSs forbid the set $QQQL$. In Appendix [F] we also give the other allowed sets of lepton and/or baryon number violating operators discussed in Section [I].

6 An example

To illustrate how a Dirac-DGS arises from a $U(1)_X$ gauge symmetry, and how distinct theories can be related by a hypercharge shift, and how these related theories give rise to different GUT-compatibilities, we consider three different sets of $U(1)_X$-charges to begin with:

| Model/Charges | $X_Q$ | $X_T$ | $X_T'$ | $X_L$ | $X_F$ | $X_{H_u}$ | $X_{H_d}$ |
|---------------|-------|-------|--------|-------|-------|----------|----------|
| 1             | 0     | -3    | 3      | 0     | -3    | 3        | 3        |
| 2             | 3     | 3     | -9     | -9    | 15    | 3        | -6       |
| 3             | 1     | -1    | -1     | -3    | 3     | 3        | 0        |

These three sets are all free of anomalies and mutually related by $Y$-shifts. We assume a vectorlike pair of $A$-fields: $X_{A_1} = -4$, $X_{A_2} = 4$. Then, after $U(1)_X$-symmetry breaking, we get a $\mathbb{Z}_4$-symmetry which might be called matter tetrality $M_4 = (4, 1, 0, 0)$:

| Model/Charges | $z_Q$ | $z_T$ | $z_T'$ | $z_L$ | $z_F$ | $z_{H_u}$ | $z_{H_d}$ |
|---------------|-------|-------|--------|-------|-------|----------|----------|
| 1             | 0     | 1     | 3      | 0     | 1     | 3        | 3        |
| 2             | 3     | 3     | 3      | 3     | 3     | 2        | 2        |
| 3             | 1     | 3     | 3      | 1     | 3     | 3        | 0        |

Therefore, the second model ($r = 3$) is compatible with $SO(10)$ and the last one ($r = 1$) is compatible with Pati-Salam, at least on the discrete level [but not on the $U(1)_X$-level]. See also Appendix [F].

7 Conclusion

When supersymmetrizing the SM, the introduction of a DS to avoid exotic processes is highly desirable. Such a DS is supposedly the remnant of a $U(1)$ broken at high energies. Assuming that the experimentally observed neutrinos are Majorana-type, only three $\mathbb{Z}_N$-symmetries for the MSSM sector are possible: $M_p$, $B_3$ and $P_6$. Allowing, however, for purely Dirac-type neutrinos (experimentally still possible), an infinite number of non-equivalent discrete anomaly-free $\mathbb{Z}_N$-symmetries is conceivable for the MSSM+$\mathbb{N}$. The existence of the $\mu$-term is a consequence, not an input, unlike for the three above-mentioned DGSs. Up to $N = 14$, we have listed all possible DGSs in Appendix [F] "decodable" with Eq. (2.7). Some of them are compatible with a GUT-scenario in the sense that the discrete charges are consistent with the direct product $\mathbb{Z}_N \times G_{GUT}$. Those DGSs going along with $SO(10)$, $SU(5)$ and flipped $SU(5)$ automatically allow for the $QQQL$-set superpotential operators.

\[11\] In fact, they can be calculated from Eq. (8.5) of Ref. [22] with $C_1 = 1$ and $C_2 = -1, 2, 0$. 

15
Analogously to Table 2 and in addition to the three Dirac-DGSs in Sect. 6, we have collected here five especially interesting Dirac-DGSs out of the many in Appendix F: We show the explicit charge assignments for all $\mathbb{Z}_{N \leq 6}$-symmetries ($N; m, n, p; r$) which forbid $QQQL$. The two trialities can be named unambiguously according to the classification in Table 1: When Pati-Salam compatible for a specific value for $r$, we give the discrete charges for these cases. Within the MSSM-sector, all but the first are as powerful as the aggressive $P_6$ in Table 2. Regarding the three Majorana-DGSs, i.e. $M_p, B_3$, and $P_6$, GUT compatibility and the absence of $QQQL$ mutually exclude each other.

| Dirac-DGS       | $N$ | $z_Q$ | $z_\tau$ | $z_\nu^c$ | $z_L$ | $z_\nu$ | $z_{H_d}$ | $z_{H_u}$ | comment                                                                 |
|-----------------|-----|-------|-----------|-----------|-------|---------|-----------|-----------|--------------------------------------------------------------------------|
| lepton triality $L_3''$ (3,0,0,1;2) | 3   | 2     | 1         | 1         | 2     | 1       | 0         | 0         | the only lepton $N$-ality, allows $UDD$ and $NNN$, Pati-Salam compatible due to $r = 2$ |
| matter triality $M_3$ (3,1,0,2; e.g. 0) | 3   | 0     | 1         | 2         | 1     | 0       | 1         | 2         | allows $NNN$ no $B$ up to dim-5                                          |
| $L_3$           | 6   | 0     | 0         | 0         | 0     | 0       | 0         | 0         | no $E$ and $B$ up to dim-5                                               |
| $(6; 1, 0, 2; e.g. 0)$ | 6   | 1     | 5         | 5         | 1     | 5       | 5         | 0         | no $E$ and $B$ up to dim-5, Pati-Salam compatible due to $r = 1$           |
| $(6; 3, 0, 2; 1)$ | 6   | 4     | 5         | 4         | 5     | 5       | 3         | 3         | no $E$ and $B$ up to dim-5, Pati-Salam compatible due to $r = 4$           |

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Appendix

A Sets of operators

Requiring the existence of the superpotential terms in Eq. (1.1) one finds that the lepton and/or baryon number violating operators are allowed/forbidden in sets. This can be shown by rearranging products of superfields. In the following, operators in parentheses are allowed by definition, see Eq. (1.1). Then the remaining terms always come in pairs, proving that both are simultaneously either present or absent. Occasionally, we artificially insert the product of a field and its complex conjugate, which is trivially invariant under a DGS. So e.g. the existence of $QL\bar{D}$ and necessarily $LH_{d}\bar{E}$ then requires the existence of $LL\bar{E}$:

$$QL\bar{D} \cdot (LH_d\bar{E}) \sim LL\bar{E} \cdot (QH_d\bar{D}).$$  \hspace{1cm} (A.1)

Among the four anomaly-free trialities in the table of Appendix F, three forbid $QQQL$ and are hence particularly interesting: The Majorana-DGS baryon triality $B_3 = (3; 1, 0, 1)$ as well as the two Dirac-DGSs lepton triality $L_3 = (3; 0, 0, 1)$ and matter triality $M_3 = (3; 1, 0, 2)$. Since all four anomaly-free $\mathbb{Z}_6$-symmetries are matter $N$-alities, we refrain from naming the three Dirac-DGSs; the remaining Majorana-DGS is already called proton hexality.
In the same manner one finds
\[
QL\bar{D} \cdot (H_d H_u) \sim L H_u \cdot (Q H_d \bar{D}),
\]
\[
(Q H_d \bar{D}) \cdot (Q H_u \bar{U}) \cdot (L H_d E) \sim Q \bar{U} \bar{E} H_d \cdot Q L \bar{D} \cdot (H_d H_u),
\]
\[
QL\bar{D} \cdot (H_d H_u)^2 \sim L H_u H_d H_u \cdot (Q H_d \bar{D}),
\]
\[
(Q H_d \bar{D}) \cdot (Q H_u \bar{U}) \cdot (L^1 L) \sim Q \bar{U} L^1 \cdot Q L \bar{D} \cdot (H_d H_u),
\]
\[
(Q H_d \bar{D}) \cdot (L H_d E) \cdot (H_u^\dagger H_u) \sim \bar{E} H_d H_u \bar{H} \cdot Q L \bar{D} \cdot (H_d H_u),
\]
\[
(Q H_u \bar{U}) \cdot (L H_d E) \cdot (\bar{D}^I \bar{D}) \sim UED^I \cdot Q L \bar{D} \cdot (H_d H_u),
\]
justifying the first “\(\iff\)” in Eq. (1.2). Likewise
\[
(Q H_d \bar{D})^2 \cdot (Q H_u \bar{U}) \sim Q QQ H_d \cdot UDD \cdot (H_d H_u),
\]
\[
(Q H_d \bar{D}) \cdot (Q H_u \bar{U}) \cdot (\bar{D}^I \bar{D}) \sim QQ \bar{D}^I \cdot UDD \cdot (H_d H_u); \tag{A.3}
\]
\[
(Q H_d \bar{D}) \cdot (Q H_u \bar{U})^2 \cdot (L H_d E) \sim UU DD \cdot QQ \bar{L} \cdot (H_d H_u)^2. \tag{A.4}
\]

Introducing the right-handed neutrino \(\bar{N}\) and demanding the interaction \(LH_u \bar{N}\), one can similarly prove the groups of operators given in Eq. (1.1).
\[
L H_u \bar{N} \cdot (L H_d E) \sim L L E \bar{N} \cdot (H_d H_u),
\]
\[
L H_u \bar{N} \cdot (Q H_d \bar{D}) \sim Q L \bar{D} \bar{N} \cdot (H_d H_u); \tag{A.5}
\]
\[
(Q H_d \bar{D}) \cdot (L H_u \bar{N}) \sim \bar{N} \cdot Q L \bar{D} \cdot (H_d H_u); \tag{A.6}
\]
\[
(L H_u \bar{N}) \cdot (L H_u \bar{N}) \sim L H_u L H_u \cdot \bar{N} \bar{N}; \tag{A.7}
\]
\[
(Q H_d \bar{D})^2 \cdot (Q H_u \bar{U}) \cdot (L H_u \bar{N}) \sim UDD \bar{N} \cdot QQ \bar{L} \cdot (H_d H_u)^2. \tag{A.8}
\]

B The first seven steps of the top-down list in Sect. 2

Our starting point is an \(SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_{X'}\)-invariant four-dimensional theory in which the dilaton \(S\) has not yet acquired a vacuum expectation value (VEV). Among others, there is the \(F\)-term
\[
\frac{1}{2} S \left( \begin{array}{cc} W_X & W_{Y'} \\ W_{X'} & W_Y \end{array} \right) \cdot K' \left( \begin{array}{cc} W_{X'} \\ W_Y \end{array} \right),
\]
with \(K'\) being a \(2 \times 2\) matrix, and the \(D\)-terms \(\bar{H} e^{2(V_{X'} X' \phi + V_{Y'} Y' \phi)} \) \(\phi\), and \(-\frac{1}{2} \ln(S + \bar{S} - X'SV_{X'} - Y'SV_{Y'})\).

1. \(K'\) has to be positive-definite, and it may be taken symmetric. Thus
\[
\frac{1}{4} S \left( \begin{array}{cc} W_X & W_{Y'} \\ W_{X'} & W_Y \end{array} \right) \cdot \left( \begin{array}{cc} k'_{11} & k'_{12} \\ k'_{12} & k'_{22} \end{array} \right) \cdot \left( \begin{array}{c} W_{X'} \\ W_Y \end{array} \right).
\]
\[
\tag{B.1}
\]

2. Next we perform an orthogonal transformation to diagonalize \(K'\). This mixes \(W_X\) and \(W_{Y'}\) (and equivalently \(V_X\) and \(V_{Y'}\)) as well as, for a given field \(\phi\), its charges \(X' \phi\) and \(Y' \phi\).
\[
K' \rightarrow K = \left( \begin{array}{cc} k_X & 0 \\ 0 & k_Y \end{array} \right).
\]
\[
\tag{B.2}
\]
Thus there is no kinetic mixing anymore between \(U(1)_X\) and \(U(1)_Y\). This diagonalization is spoiled by the renormalization group evolution; however, the resulting effects are small. \(k_X, k_Y\) are called the pseudo Kač-Moody levels of \(U(1)_X\) and \(U(1)_Y\).
At this point, one might ask the question: What are the conditions on the original $X'$- and $Y'$-charges such that after a rotation (like in Items 2,7) the $X$-charges may be generation-dependent whereas the $Y$-charges are generation-independent? We have

$$
\begin{pmatrix} V_{X'} \\ V_{Y'} \end{pmatrix} \rightarrow \begin{pmatrix} V_X \\ V_Y \end{pmatrix} = \begin{pmatrix} \cos \gamma & -\sin \gamma \\ \sin \gamma & \cos \gamma \end{pmatrix} \cdot \begin{pmatrix} V_{X'} \\ V_{Y'} \end{pmatrix};
$$

$\gamma$ is of course determined by demanding the $K'$ matrix to be diagonalized (or, in Item 7 that $Y_S$ is rotated away). This rotation gives

$$
\Phi e^{2(X' \cdot V_X + Y' \cdot V_Y) \cdot \Phi} \rightarrow \Phi e^{\gamma \cdot \begin{pmatrix} \cos \gamma \cdot X' \cdot \Phi - \sin \gamma \cdot Y' \cdot \Phi \end{pmatrix} V_X + \begin{pmatrix} \sin \gamma \cdot X' \cdot \Phi + \cos \gamma \cdot Y' \cdot \Phi \end{pmatrix} V_Y} \cdot \Phi.
$$

Now demand the resulting $Y$-charges to be generation-independent. Then, e.g. $Y_{Q_i} = Y'_{Q_i}$, leads to

$$
sin \gamma \cdot X'_{Q_i} + \cos \gamma \cdot Y'_{Q_i} = sin \gamma \cdot X_{Q_i} + \cos \gamma \cdot Y_{Q_i},
$$

so that the original $Y$-charges have to fulfill $Y'_{Q_i} = Y_{Q_i} + (X'_{Q_i} - X_{Q_i}) \cdot \tan \gamma$.

3. Having diagonalized $K$, we investigate the effects of a combined $U(1)_X \times U(1)_Y$ gauge-transformation (performed e.g. to prevent Goldstone bosons), i.e. the effects of

$$
\Phi \rightarrow e^{i (\Lambda_X X + \Lambda_Y Y_S) \cdot \Phi},
$$

$$
V_{X,Y} \rightarrow V_{X,Y} - i/2 (\Lambda_X Y - \Lambda_Y X),
$$

$$
S \rightarrow S - i/2 X_S \Lambda_X - i/2 Y_S \Lambda_Y.
$$

Here, the gauge transformation is parameterized by $\Lambda_{X,Y}$. The real-valued quantity denoted as $X_S$ is usually written as $\delta_{GS}^{X_S}$.

(a) Eq. (B.3) causes anomalies (as for the vanishing kinetic mixing terms, there are no mixed terms like $W_X W_Y$)

$$
\frac{1}{32 \pi^2} \left[ \lambda_X \left( A_{CCX} F_C \bar{F}_C + A_{WWX} F_W \bar{F}_W + A_{YYX} F_Y \bar{F}_Y + A_{XXX} F_X \bar{F}_X \right) \right],
$$

and the same with the replacements $\lambda_X \rightarrow \lambda_Y$, $A_{X} \rightarrow A_{Y}$, plus the anomalies with gravitation, with e.g. $A_{GGA} = \text{Trace} T_X = \frac{1}{6} \sum_i X_i$. The $\lambda_{X,Y}$ are the scalar components of $\Lambda_{X,Y}$, and the $A_{abc} = \text{Trace} [\{T^a, T^b\} \cdot T^c]$ are the anomaly coefficients. The gauge group generators $T^a$ are assumed to be according to the standard GUT-convention, so that e.g.

$$
A_{CCX} = \frac{1}{2} \sum_{i=1}^{N_f} (2X_{Q_i} + X_{D_i} + X_{Q_i}) + A^{beyond \text{ MSSM}},
$$

where $N_f$ is the number of families.

(b) Eq. (B.4) together with Eqs. (B.1,B.2) gives

$$
-i/8 (X_S \Lambda_X + Y_S \Lambda_Y) \cdot \begin{pmatrix} W_X & W_Y \end{pmatrix} \cdot \begin{pmatrix} k_X & 0 \\ 0 & k_Y \end{pmatrix} \cdot \begin{pmatrix} W_X \\ W_Y \end{pmatrix},
$$

which produces

$$
-\frac{1}{16} \left[ \lambda_X X_S (k_C F_C \bar{F}_C + k_W F_W \bar{F}_W + k_Y F_Y \bar{F}_Y + k_X F_X \bar{F}_X) \right],
$$

and the same with the replacements $\lambda_X \rightarrow \lambda_Y$, $X_S \rightarrow Y_S$ plus the shifts with gravitation.
4. The anomalies are required to be canceled by the dilaton-shifts, *i.e.* Items 3a) and 3b) mutually eliminate each other; this is the four-dimensional version of the Green-Schwarz mechanism \[54\], see also \[55\]. Thus it is ensured that the theory is gauge-invariant, *i.e.* one demands the following anomaly conditions, see *e.g.* Refs. \[56\] \[57\] \[58\], (and the same for $X \leftrightarrow Y$)

$$2\pi^2 X_S = \frac{A_{XX}}{k_X} = \frac{A_{YY}}{k_Y} = \frac{A_{CC}}{k_C} = \frac{A_{WW}}{k_W} = \frac{A_{GG}}{12}.$$ (B.8)

5. We let the dilaton acquire a VEV, $S \to S + \langle S \rangle$. So:

- $-\frac{1}{2}\ln(S + \overline{S} - X_S V_X - Y_S V_Y)$ gives with $\frac{S + \overline{S} - X_S V_X - Y_S V_Y}{2\Re\langle S \rangle}$ being small, $-\frac{1}{2}\ln(2\Re\langle S \rangle) - (S + \overline{S} - X_S V_X - Y_S V_Y)/(4\Re\langle S \rangle)$, producing an effective $D$-term $2\xi_X V_X + 2\xi_Y V_Y$ with

$$\xi_X = \frac{X_S}{8\Re\langle S \rangle}, \quad \xi_Y = \frac{Y_S}{8\Re\langle S \rangle}.$$ (B.9)

This is the Dine-Seiberg-Wen-Witten-mechanism \[59\] \[60\] \[61\] \[62\].

- From, *e.g.* $\frac{1}{4}k_CSW_CW_C$, we obtain the gauge kinetic terms and thus the gauge coupling constants. Using standard GUT-conventions and identifying $2k_C\Re\langle S \rangle = 2/g_C^2$, we find, with $g_{\text{string}} \equiv 1/\sqrt{2\Re\langle S \rangle}$,

$$g_C^2 k_C = g_W^2 k_W = g_Y^2 k_Y = g_X^2 k_X = 2g_{\text{string}}^2.$$ (B.10)

From Eqs. (B.8, B.9) and the relation above one finds that *e.g.* $\xi_X = \frac{g_{\text{string}}^2}{192\pi^2} \sum_X X_i$.

6. Now that $S$ has undergone the gauge shift (Item 3) and having acquired a VEV (Item 5), we soak up the constant coefficient of the $W_i \ldots W_i$, so that, *e.g.* $W_C W_C$ produces the kinetic term $\frac{1}{16\pi^2} F_C F_C$ rather than $\frac{1}{16\pi^2} F_C F_C$. Item 2 and Item 6 together are called the ‘canonicalization of the kinetic terms of $V_C$’.

7. Next, we perform yet another orthogonal transformation which leaves the freshly canonicalized kinetic terms invariant. This transformation rotates away $Y_S$, thus rendering $U(1)_Y$ non-anomalous; so now we have $A_{XX} = 0$, also written as $A_{YX} = 0$, and $A_{CC} = A_{WW} = A_{YY} = A_{GG} = 0$.

### C The $X$-charge of the “effective 2l”

In the following, we discuss the scenario with two $A$-type particles $A_i$ ($i = 1, 2$). For simplicity, we assume that their charges $X_{A_i}$ are positive integers; the generalization to negative $X$-charges is straightforward. After the breakdown of $U(1)_X$ the effective operators in the Lagrangian can only have an overall $X$-charge of the form

$$X_{\text{total}} = -a_1 \cdot X_{A_1} - a_2 \cdot X_{A_2},$$

with $a_i \in \mathbb{N}$ for superpotential terms and $a_i \in \mathbb{Z}$ for Kähler potential terms. Notice that, in principle, operators in the Kähler potential can be converted to effective operators in the superpotential via the Giudice-Masiero/Kim-Nilles mechanism \[38\] \[39\]. If the two $X_{A_i}$ have a greatest common divisor $d$, we can define new integers $x_{A_i} = X_{A_i}/d$. With this, Eq. (C.1) can be rewritten as

$$X_{\text{total}} = -d \cdot [a_1 \cdot x_{A_1} + a_2 \cdot x_{A_2}].$$ (C.2)

Evidently, $X_{\text{total}}$ is a multiple of $d$. If the square bracket is not restricted to any subset of $\mathbb{Z}$, we will end up with a $\mathbb{Z}_d$-symmetry after $U(1)_X$-breaking.

The question however remains whether the square bracket can actually take any integer value. To answer this, we first decompose $x_{A_i}$ into prime factors $\xi^{(i)}_{\alpha}$:

$$x_{A_i} = \prod_{\alpha} \xi^{(i)}_{\alpha}.$$ 

Since $x_{A_1}$ and $x_{A_2}$ do not have a common divisor, one necessarily has that $\xi^{(1)}_{\alpha} \neq \xi^{(2)}_{\beta}$, for all $\alpha, \beta$. Thus the least common multiple of both $x_{A_i}$ is just their product $x_{A_1} \cdot x_{A_2}$. If one can obtain any integer
within the interval \([0, x_{A_1} \cdot x_{A_2}]\) with an appropriate integer-valued linear combination of the \(x_{A_i}\), then the square bracket in Eq. (C.2) can take any integer value whatsoever. To check this, we consider the two linear combinations

\[
0 \leq a_1 \cdot x_{A_1} + a_2 \cdot x_{A_2} < x_{A_1} \cdot x_{A_2},
\]
\[
0 \leq b_1 \cdot x_{A_1} + b_2 \cdot x_{A_2} < x_{A_1} \cdot x_{A_2},
\]

with \(a_2, b_2 \in \{0, 1, \ldots, x_{A_i} - 1\}\) and \(a_1, b_1 \in \mathbb{Z}\) such that the linear combinations of \(x_{A_1}\) and \(x_{A_2}\) lie within the given interval. Assuming \(a_2 \neq b_2\), we can show that the two linear combinations can never be matched within the interval \([0, x_{A_1} \cdot x_{A_2}]\), since \(a_1 \cdot x_{A_1} + a_2 \cdot x_{A_2} = b_1 \cdot x_{A_1} + b_2 \cdot x_{A_2}\) can be rewritten as

\[
(a_2 - b_2) \cdot x_{A_2} = (b_1 - a_1) \cdot x_{A_1}.
\]

The factor \((a_2 - b_2)\) must therefore be a multiple of \(x_{A_1}\), which however is not the case for \(a_2 \neq b_2\) and \(a_2, b_2 \in \{0, 1, \ldots, x_{A_i} - 1\}\). Hence, two linear combinations of the form \(0 \leq a_1 \cdot x_{A_1} + a_2 \cdot x_{A_2} < x_{A_1} \cdot x_{A_2}\) always yield different values for different \(a_2\). Now there are \(x_{A_1}\) different \(a_2\). For each \(a_2\) one finds \(x_{A_2}\) different possible values for \(a_1\) such that the linear combination lies within the interval \([0, x_{A_1} \cdot x_{A_2}]\). Thus we can obtain \(x_{A_1} \cdot x_{A_2}\) different values within the interval \([0, x_{A_1} \cdot x_{A_2}]\) by integer-valued linear combinations of \(x_{A_i}\). This finally shows that the square bracket in Eq. (C.2) can take any integer value.

Likewise, this argumentation can be applied to cases with any number of \(U(1)_{X_i}\)-breaking fields \(A_i\). The remnant discrete symmetry is a \(\mathbb{Z}_{|X_{A_i}|}\) with \(|X_{A_i}| \equiv d\), the greatest common divisor of all \(X_{A_i}\).

### D Classification of SM-singlets

In Refs. [20][21][22] it was assumed that all non-MSSM particles, including the singlets \(\Omega\) (see Item 8), are heavy, i.e. two fields must pair up to allow a \(\mathbb{Z}_N\)-invariant mass term after \(U(1)_{X}\)-breaking. From this, one could find some simplifications of the anomaly conditions. If a massive \(\Omega\) has a trilinear coupling with \(LH_u\), i.e. the operator \(LH_u\Omega\) is allowed, it is called a Majorana neutrino \(\overline{N}_{\text{Maj}}\). Of course this does not exclude other \(\Omega\)s with discrete charges for which \(LH_u\Omega\) is forbidden – these \(\Omega\)s then do not carry lepton number and are hence not to be called “neutrinos”. They can have \(X\)-charges which are half-odd-integer or integer multiples of \(N\); other charges are not possible since they have to add up to an integer multiple of \(N\) in order to be heavy.\(^{13}\) Depending on the \(X\)-charge of the forbidden term \(LH_u\Omega\), there are three mutually exclusive types of non-neutrino \(\Omega\)s: Case 1 has a DGS such that \(LH_u\) and \(LH_u LH_u\) are both allowed, Case 2 has a DGS such that \(LH_u\) is not but \(LH_u LH_u\) is allowed, Case 3 has a DGS such that \(LH_u\) and \(LH_u LH_u\) are both not allowed; see the first two lines of Columns 2 and 3 of Table 6. So in Refs. [20][21][22] the following cases were treated: a) no heavy singlets, b) \(\overline{N}_{\text{Maj}}\), c) \(\Phi\), d) \(\overline{N}_{\text{Maj}} + \Phi\), e) \(\overline{N}_{\text{Maj}} + \Phi\), f) \(\Phi\), g) \(\overline{N}_{\text{Maj}} + \Phi\), h) see also Table 7.

The situation becomes even more complex once we admit massless \(\Omega\)s (as we necessarily have to do in order to deal with Dirac rather than Majorana neutrinos), see Table 6. There could in principle be exotic particles which are massless and do not get a mass at least after \(U(1)_{X}\)-breaking. One would have no or only little systematics in solving the discrete gravitation-anomaly condition if \(\Psi\) and/or \(\Gamma\) and/or \(\Theta\) and/or \(\Theta'\) existed (see Lines 2, 5 and 8 in Table 7) – of course there are solutions to the equations, but they are quite arbitrary, depending on which \(X\)-charges one has chosen. Similar to Item 19 (f), the existence of massless SM-neutral particle spoils the predictability of \(A_{\text{GGO}}\). For that reason we shall not admit these particles in our treatment here. [In Ref. 63, the discrete gravitation-anomaly condition is not solved and the singlet particle content is not specify, so that they effectively work with a theory with \(\Theta'\) and \(\Phi'\). See also Appendix 5.] On the other hand, the analysis of a theory containing Dirac neutrinos as well as heavy singlets does not differ from the analysis of Dirac neutrinos alone, so its results can be taken over wholesale.

---

\(^{13}\)For simplicity we shall exclude cases like two \(X\)-charges being 3/7 \(\cdot\) \(N\) and 4/7 \(\cdot\) \(N\). We assume that all particles within one “\(\Omega\)-category” have to have the same discrete charge.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$X_L/N = \text{int.}$ & $X_L/N = \text{int.} + \frac{1}{2}$ & $X_L/N \neq \text{int.}, \text{int.} + \frac{1}{2}$ \\
\hline
$\Rightarrow \exists \Omega \Omega$ & Case 1: $\Omega \equiv \mathcal{N}_{\text{Maj}}$ & Case 2: $\Omega \equiv \Phi'$ & Case 3: $\Omega \equiv \Xi$
\hline
$\Rightarrow \exists \Omega \Omega$ & Case 2: $\Omega \equiv \mathcal{N}_{\text{Maj}}'$ & Case 1: $\Omega \equiv \Phi$ & Case 3: $\Omega \equiv \Xi$
\hline
$\Rightarrow \exists \Omega \Omega$ & Case 3: $\Omega \equiv \mathcal{N}_{\text{Dirac}}$ & Case 3: $\Omega \equiv \Psi$ & Case 1: $\Omega \equiv \Theta$
\hline
\end{tabular}
\caption{Classification of different $\Omega$s, with Case 1 ($\exists LH_u, \exists LH_u LH_u$), Case 2 ($\not\exists LH_u, \exists LH_u LH_u$), Case 3 ($\not\exists LH_u, \not\exists LH_u LH_u$).}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Case & SM-singlet content & reference \\
\hline
1 & $\mathcal{N}_{\text{Maj}}, \Phi, \mathcal{N}_{\text{Maj}} + \Phi$ & treated in Refs. [20, 21, 22] \\
& $\Theta, \Phi + \Theta, \mathcal{N}_{\text{Maj}} + \Theta, \mathcal{N}_{\text{Maj}} + \Theta + \Phi$ & \\
\hline
2 & $\mathcal{N}_{\text{Maj}}', \Phi', \mathcal{N}_{\text{Maj}}' + \Phi'$ & treated in Refs. [20, 21, 22] \\
& $\Theta', \Phi' + \Theta'$ & examples given in [37, 63] \\
& $\mathcal{N}_{\text{Maj}}' + \Theta', \mathcal{N}_{\text{Maj}}' + \Theta' + \Phi'$ & \\
\hline
3 & $\mathcal{N}_{\text{Dirac}}, \mathcal{N}_{\text{Dirac}} + \Xi$ & treated here \\
& $\Xi$ & treated in Refs. [20, 21, 22] \\
\hline
& $\Psi, \Gamma, \Xi + \Psi, \Xi + \Gamma, \mathcal{N}_{\text{Dirac}} + \Psi, \mathcal{N}_{\text{Dirac}} + \Gamma, \Psi + \Gamma,$ & \\
& $\Xi + \mathcal{N}_{\text{Dirac}} + \Gamma, \Xi + \mathcal{N}_{\text{Dirac}} + \Psi, \Xi + \Psi + \Gamma,$ & \\
& $\mathcal{N}_{\text{Dirac}} + \Psi + \Gamma, \Xi + \mathcal{N}_{\text{Dirac}} + \Psi + \Gamma$ & \\
\end{tabular}
\caption{Mutually different theories and which of these are treated here.}
\end{table}

E Case study: a model by Jack, Jones and Wild

In Ref. [63] one is given a model with a non-anomalous $U(1)_X$ (only the mixed anomalies are imposed) and four $A$-superfields. Explicitly no right-handed neutrinos are assumed, so tacitly the existence of fields like $\Theta'$ and/or $\Phi'$ (see Appendix D) must be assumed to cancel $A_{GGX}$ and $A_{XXH}$. The model is of Case 2, i.e. $LH_u LH_u$ is allowed but not so $LH_u$. Their considerations lead to a set of $X$-charges (note that their $X_{\text{ET}} = e_2$ should read $3143/300$ and not $3143/100$) with a free parameter $X_{H_u} = h_2$; if we set $h_2 = 3\alpha$, then $\alpha$ is the parameter of a $Y$-shift. We are now going to extract which discrete symmetry is hidden in these $X$-charges. First we rescale all charges by a factor of 2700 so that they are all integers. Now, the $A$s have charges $-2700, -2700, -720, -234$. The greatest common divisor is 18, hence we have a $\mathbb{Z}_{18}$. Then we pick $h_2 = 2309/900$. Examining the resulting charges mod 18 gives 0, 15, 3, 12, 3; 3, 15 for the fields $Q^i, D^i, U^i, L^i, E^i; H_d, H_u$. Finally we re-rescale by a factor of three, giving the discrete charges of $P_0$, see Table 2.

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F  Table of all $Z_N$-symmetries with $N \leq 14$

The following table shows all $Z_N$-symmetries $(N; m, n, p)$ up to $N = 14$, which can be converted into the corresponding discrete charges with the help of Eq. (2.7). In addition to the Dirac-DGSs, i.e. those which forbid the Majorana mass term $\mathcal{NN}$, we list also the standard DGSs $M_p$, $B_3$ and $P_6$ for completeness. The latter three symmetries allow $\mathcal{NN}$ and thus $LH_uLH_d$, resulting in Majorana-type light neutrinos. Except for the only baryon $N$-ality $B_3$ (which is not a Dirac-DGS), all symmetries forbid the operators of the set $QLD$. Furthermore, except for the only lepton $N$-ality $L_3 = (3; 0, 0, 1)$, all Dirac-DGSs forbid $UDD$ and are matter $N$-alities. Not all DGSs are compatible with a GUT scenario. However, if they are, the parameter $r$ in Eq. (2.7) has to take specific values which are given in the table. Furthermore, the sets of lepton and/or baryon number violating operators allowed by the DGSs are marked with the symbol $\checkmark$.

| $(N; m, n, p)$ | QQL | NNL | NNM | GUT-compatibility |
|---------------|-----|-----|-----|-------------------|
| $M_p = (2; 1, 0, 0)$ | ✓   | ✓   | ✓   | $r = 0$: Pati-Salam |
| $M_p = (2; 1, 0, 1)$ | ✓   | ✓   | ✓   | $r = 1$: SO(10) |
| $M_p = (2; 1, 0, 2)$ | ✓   | ✓   | ✓   | $r = 2$: flipped SU(5) |
| $M_p = (4; 1, 0, 0)$ | ✓   | ✓   | ✓   | $r = 1$: Pati-Salam |
| $M_p = (4; 1, 0, 1)$ | ✓   | ✓   | ✓   | $r = 1$: SO(10) |
| $M_p = (4; 1, 0, 2)$ | ✓   | ✓   | ✓   | $r = 2$: flipped SU(5) |
| $M_p = (4; 1, 0, 3)$ | ✓   | ✓   | ✓   | $r = 3$: flipped SU(5) |
| $M_p = (4; 1, 0, 4)$ | ✓   | ✓   | ✓   | $r = 4$: flipped SU(5) |
| $M_p = (4; 1, 0, 5)$ | ✓   | ✓   | ✓   | $r = 5$: flipped SU(5) |
| $M_p = (4; 1, 0, 6)$ | ✓   | ✓   | ✓   | $r = 6$: flipped SU(5) |
| $M_p = (4; 1, 0, 7)$ | ✓   | ✓   | ✓   | $r = 7$: flipped SU(5) |
| $M_p = (4; 1, 0, 8)$ | ✓   | ✓   | ✓   | $r = 8$: flipped SU(5) |
| $M_p = (4; 1, 0, 9)$ | ✓   | ✓   | ✓   | $r = 9$: flipped SU(5) |
| $M_p = (4; 1, 0, 10)$ | ✓   | ✓   | ✓   | $r = 10$: flipped SU(5) |
| $M_p = (4; 1, 0, 11)$ | ✓   | ✓   | ✓   | $r = 11$: flipped SU(5) |
| $M_p = (4; 1, 0, 12)$ | ✓   | ✓   | ✓   | $r = 12$: flipped SU(5) |
| $M_p = (4; 1, 0, 13)$ | ✓   | ✓   | ✓   | $r = 13$: flipped SU(5) |
| $M_p = (4; 1, 0, 14)$ | ✓   | ✓   | ✓   | $r = 14$: flipped SU(5) |
| $M_p = (4; 1, 0, 15)$ | ✓   | ✓   | ✓   | $r = 15$: flipped SU(5) |
| $M_p = (4; 1, 0, 16)$ | ✓   | ✓   | ✓   | $r = 16$: flipped SU(5) |
| $M_p = (4; 1, 0, 17)$ | ✓   | ✓   | ✓   | $r = 17$: flipped SU(5) |
| $M_p = (4; 1, 0, 18)$ | ✓   | ✓   | ✓   | $r = 18$: flipped SU(5) |
| $M_p = (4; 1, 0, 19)$ | ✓   | ✓   | ✓   | $r = 19$: flipped SU(5) |
| $M_p = (4; 1, 0, 20)$ | ✓   | ✓   | ✓   | $r = 20$: flipped SU(5) |

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