COOLING FLOW BULK MOTION CORRECTIONS TO THE SUNYAEV-ZEL’DOVICH EFFECT

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Abstract

We study the influence of converging cooling flow bulk motions on the Sunyaev-Zel’dovich (SZ) effect. To that purpose we derive a modified Kompaneets equation which takes into account the contribution of the accelerated electron media of the cooling flow inside the cluster frame. The additional term is different from the usual kinematic SZ-effect, which depends linearly on the velocity, whereas the contribution described here is quadratic in the macroscopic electron fluid velocity, as measured in the cluster frame. For clusters with a large cooling flow mass deposition rate and/or a small central electron density, it turns out that this effect becomes relevant.

Key words: Cosmology; Galaxy clusters: cooling flows; Background radiations
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1 Introduction

The SZ-effect is becoming more and more an important astrophysical tool thanks to the rapid progress of the observational techniques, which allow increasingly precise measurements. It has thus become relevant to study further corrections to it, such as relativistic effects (Rephaeli, 1995), the shape of the galaxy cluster and its finite extension or a polytropic temperature profile (see e.g. Puy et al. (2000)), the presence of cooling flows (Schlickeiser, 1991; Majumdar and Nath, 2000), corrections induced by halo rotation (Cooray and Chen, 2001) or even by Brillouin scattering (Sandoval-Villalbazo and Maartens, 2002) and the influence of early galactic winds (Majumdar et al., 2001). These

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additional effects are of different relevance and often depend on the specific values of the parameters which describe a given cluster. Generally, they range from the percent level up to even 20-30% and accordingly in the subsequent determination of the Hubble constant. Taking into account such corrections is on one side relevant when determining the Hubble constant via the SZ-effect, and on the other hand they could be interesting as a tool to study the detailed structure of the cluster itself.

Following these lines we study here another effect, which has not yet been taken into account: The possible influence on the SZ-effect of the cooling flow bulk motion of the electron media inside the cluster frame. Indeed, one expects such a motion to be present in cooling flows, for which in some clusters there is evidence in the central regions (Allen et al. 1993; Fabian et al. 1991; Fabian 1994, 1997). Recently, Dupke and Bregman (2002) investigated the prospects to detect gas bulk velocities in clusters of galaxies through the kinematic SZ-effect, which depends linearly on the velocity component along the line of sight. Bulk motions along a given line of sight will contribute to the kinematic SZ term as long as their averaged velocity, in the observer frame, does not vanish. In the case of cooling flow bulk motion the averaged velocity, in the cluster frame, along a given line of sight vanishes in good approximation, since we assume spherical symmetric infall. Thus, the cooling flow bulk motion will not contribute as such to the kinematic SZ-effect. Indeed, the effect we consider depends quadratically on the velocity and clearly, the averaged quadratic velocity does not vanish along a line of sight in the cluster frame. As we will see, the considered effect usually turns out to be small, since the cooling flow bulk motion velocities are rather small, unless in the very central regions of a cooling flow clusters, where the cooling flow approaches a sonic radius and changes from the subsonic to the supersonic regime. Nonetheless, in some favourable cases it might be of the order of some percent of the thermal SZ-effect.

The aim of this paper is to examine the influence on the thermal SZ-effect of the moving electron media inside a cooling flow region of a galaxy cluster. As it will be seen in more detail later, this effect has clearly to be distinguished from the kinematic SZ-effect, which takes into account the motion of the whole cluster or a fraction of it along a given line of sight. In fact, the standard thermal SZ-effect describes the frequency dependent intensity change of the Cosmic Microwave Background (CMB) photons by inverse Compton scattering off the hot intracluster plasma (electrons), where the electrons are supposed to be static scatterers.

The paper is organised as follows: In section 2 we briefly outline the dynamics
of a homogeneous steady-state cooling flow model in order to get the velocity profile of the bulk motion as well as the corresponding electron density distribution. In section 3 we derive the modification to the standard Kompaneets equation due to the inclusion of bulk motion of the scatterers inside the cluster frame and in section 4 we present and discuss our results. In section 5 we give a summary and an outlook.

2 Velocity profile for a homogeneous cooling flow model

As an example to get a velocity profile which describes the bulk motion, we consider a homogeneous steady-state cooling flow model where the mass deposition rate \( \dot{m} \) is constant, negative and enters as a parameter in the model. No mass drops out of the flow. We neglect the possible influence of magnetic fields, rotation and viscosity. In this context, the cluster is expected to be in a relaxed state, so that hydrostatic equilibrium allows us to use an isothermal \( \beta \)-model (Sarazin, 1988). For spherical symmetry, the cooling flow can thus be described by a set of Euler equations. Mass, momentum and energy conservation read (Mathews and Bregman, 1978; Fabian et al., 1984; White and Sarazin, 1988; Sarazin, 1988):

\[
\begin{align*}
4\pi r^2 \rho_{CF} v &= \dot{m} \\
v \frac{dv}{dr} + \frac{1}{\rho_{CF}} \frac{dP}{dr} + \frac{GM(r)}{r^2} &= 0 \\
v \frac{dE}{dr} - \frac{P v}{\rho_{CF}} \frac{d\rho_{CF}}{dr} &= -\Lambda \rho_{CF},
\end{align*}
\]

(1)

where \( \rho_{CF}(r) \) and \( P(r) \) are the gas density and pressure, respectively, in the cooling flow and \( v(r) < 0 \) is the velocity of the inward directed cooling flow. \( M(r) \) is the gravitating cluster mass inside the radius \( r \), discussed earlier in the literature by the above authors, with the dark matter (\( DM \)) density profile as follows:

\[
\rho_{DM}(r) = \frac{\rho_0}{1 + (r/r_c)^2},
\]

(2)

where the central cluster density \( \rho_0 = 1.8 \cdot 10^{-25} g cm^{-3} \) and the core radius \( r_c = 250 \, kpc \) describe the profile of the cluster mass. As usual, we assume that the cooling flow makes no significant contribution to the cluster mass density and, that the gas self-gravity can be neglected. The internal energy is \( E(r) = \frac{3}{2} \theta(r) \), with the temperature parameter \( \theta \) which defines the square of
the isothermal sound speed $c_s$:

$$\theta(r) := c_s^2(r) = \frac{kT_{e,CF}(r)}{\mu m_p}, \hspace{1cm} (3)$$

where $\mu \approx 0.63$ is the mean molecular weight, $m_p$ the proton mass and $T_{e,CF}$ the electron temperature in the cooling flow. $\Lambda(\theta)$ defines the radiative optically thin cooling function for a low density plasma, which is defined so that $\Lambda \rho_{CF}^2$ is the cooling rate per unit volume in the gas. We take an analytical fit which has been used in earlier works (Sarazin and White, 1987; Majumdar and Nath, 2000):

$$\Lambda(\theta) = \frac{4.7 \cdot \exp \left[- \left(\frac{T}{3.5 \cdot 10^5 K}\right)^{4.5}\right]}{10^{-22} \text{erg cm}^{-3} \text{s}^{-1}} + 0.313 \cdot T^{0.08} \cdot \exp \left[- \left(\frac{T}{3.0 \cdot 10^6 K}\right)^{4.1}\right] + 6.42 \cdot T^{-0.2} \cdot \exp \left[- \left(\frac{T}{2.1 \cdot 10^7 K}\right)^{4.0}\right] + 0.000439 \cdot T^{0.35}. \hspace{1cm} (4)$$

Eliminating the density $\rho_{CF}$ with the mass conservation equation, the Euler system in (1) leads then to a system of two ordinary coupled differential equations for the cooling flow velocity $v(r)$ and the square of the isothermal sound speed $\theta(r)$ (Mathews and Bregman, 1978):

$$\frac{dv}{dr} = v \left(3GM - 10r\theta + \frac{m}{2\pi} \frac{\Lambda(\theta)}{v^2}\right) / \left(r^2(5\theta - 3v^2)\right)$$

$$\frac{d\theta}{dr} = 2 \left(\theta(2v^2r - GM) - (v^2 - \theta) \frac{m}{4\pi} \frac{\Lambda(\theta)}{v^2}\right) / \left(r^2(5\theta - 3v^2)\right) \hspace{1cm} (5)$$

To integrate this system we have to take into account that both equations have singularities at the sonic radius $r_s$, where $5\theta(r_s) = 3v^2(r_s)$ and where the cooling flow undergoes a transsonic transition. The cooling flow region is limited by the cooling radius $r_{cool} \approx 100 - 150 \text{kpc}$.

We stress that our goal is not to develop a sophisticated cooling flow model, but to get an idea of how the cooling flow bulk motion contributes to the $\text{SZ}$-effect. We, therefore, do not attempt to find solutions to the system of Eqs.(5) for the innermost supersonic region, $r < r_s$, where we expect shocks to be important. We then avoid the search of critical values\footnote{A transition from subsonic to supersonic occurs, if the numerators and denominators in the system of Eqs.(5) vanish at $r_s$. Since the expressions are indeterminate, they have to be replaced by nonsingular expressions by the Bernoulli-de l’Hôpital’s} at $r_s$ which would have to be matched to hydrostatic equilibrium at $r_{cool}$. We thus start our integration from

$$1$$
$r_{cool}$ towards $r_s$ and we stop when the Mach number $M = v/c_s$ is close to unity, $M \approx 0.9$. Reasonable initial values at $r_{cool}$ are: $v(r_{cool}) \approx v_T \approx 10 - 20 \text{ km/s}$, where $v_T$ is the turbulent velocity and $\theta(r_{cool})$ such that $t_{cool} = \frac{5}{2} \frac{\theta}{pc_{vA}} \leq t_{Hubble}$. In Fig.1 we have plotted the velocity $|v(r)|$ and the isothermal sound speed $c_s(r)$ as a function of the radius $r$. This figure illustrates clearly that the electrons are strongly accelerated in the cluster center. (For an extended discussion: see [Koch (1999)].) Fig.2 shows the corresponding electron density profile $n_{CF}(r)$ in the cooling flow region.

3 SZ contribution due to cooling flow bulk motion

It is well known that the Kompaneets equation ([Kompaneets, 1957]), resulting from a Boltzmann transport equation, describes the spectral shift of the CMB photons due to the inverse Compton process on the hot electrons of the intracluster medium: the SZ-effect - for a review, see [Birkinshaw (1999)]. The frequency dependent intensity change of the CMB photon field $\Delta I_K(x)$ after integration along the line of sight over the cluster (cl) dimension can be expressed as follows ([Sunyaev and Zeldovich, 1972; Rephaeli, 1995]):

$$\Delta I_K(x) = i_0 g(x) \int_{cl} \frac{kT_{e,cl}}{m_e c^2} \sigma_T n_{cl} dl_{cl},$$

where $x = \frac{h \nu}{kT}$ is the dimensionless frequency with $T$ the CMB temperature and $i_0 = \frac{2(kT)^3}{(hc)^2}$. The integral is the Comptonization parameter $y_\kappa$ describing the cluster properties with $T_{e,cl}, m_e$ the electron cluster temperature and mass, respectively. $n_{cl}$ is the electron number density in the cluster and $\sigma_T$ the Thomson cross section. ($k$, $h$, $c$ are the Boltzmann constant, the Planck constant and the speed of light, respectively.) The function $g(x)$ defines the spectral shape of the thermal SZ-effect where, for the plasma in clusters, we have $T_{e,cl} \gg T$:

$$g(x) = \frac{x^4 \exp x}{(\exp x - 1)^2} \left( \frac{x(\exp x + 1)}{\exp x - 1} - 4 \right).$$

It is crucial to notice that the standard Kompaneets equation describes a static scatterer, assuming that in the average the electrons are macroscopically at rest. This is no longer true for the electrons in an accelerated cooling flow, as it is described by the system of Eqs.(5). The Kompaneets equation has thus to rule, which will give initial values for Eq.(5) at $r_s$ if one assumes a temperature at the sonic radius.
be modified in such a way, that the (macroscopic) bulk velocity of the moving electron media is explicitly taken into account. 

Psaltis and Lamb (1997) gave a very detailed analysis of Compton scattering by static and moving media. They made a careful distinction between the electron rest frame, the fluid frame (comoving with the fluid) and the system frame. Starting from the Boltzmann equation in the system frame, introducing the proper Lorentz transformations, expanding to the appropriate orders and assuming that the velocity distribution in the fluid frame is a relativistic Maxwellian, they end up with the zeroth moment of the radiative transfer equation with emission and absorption included.

Under the condition that the radiation field (CMB) is isotropic in the system frame, and introducing the macroscopic electron bulk velocity \( v(r) \), the extended Kompaneets equation becomes (Psaltis and Lamb, 1997):

\[
\frac{1}{n_e \sigma T_c} \frac{\partial I}{\partial t} = \frac{\epsilon}{m_e c^2} \frac{\partial}{\partial \epsilon} (\epsilon I) + \left(kT_{e,cl} \frac{m_e c^2}{3\epsilon^2} + \frac{v^2}{3c^2}\right) \left[-4\epsilon \frac{\partial I}{\partial \epsilon} + \epsilon \frac{\partial^2}{\partial \epsilon^2} (\epsilon I)\right], \quad (8)
\]

where we neglected absorption, emission and induced scattering. \( \epsilon \) is the photon energy and \( I \) the corresponding intensity.

For CMB photons we have \( \epsilon \ll kT_{e,cl} + \frac{1}{3}m_e v^2 \). Thus, the above Eq.(8) becomes:

\[
\frac{1}{n_e \sigma T_c} \frac{\partial I}{\partial t} = \left(kT_{e,cl} \frac{m_e c^2}{3\epsilon^2} + \frac{v^2}{3c^2}\right) \left[-2\epsilon \frac{\partial I}{\partial \epsilon} + \epsilon^2 \frac{\partial^2 I}{\partial \epsilon^2}\right]. \quad (9)
\]

Eq.(9) shows that if the radiation field is isotropic in the system frame, Comptonization of the bulk motion of the electrons inside the cluster is described entirely by second order terms in \( v \), since all first order terms in \( v \) vanish identically. The effect is clearly different from the kinematic SZ-effect, where the cluster as a whole moves through the CMB radiation. This equation reduces, of course, to the standard Kompaneets equation for \( v = 0 \). Thus, the important point is that the effect of a non-zero velocity field gives an additive contribution to the standard thermal SZ-effect.

This way we can express the bulk motion contribution due to the cooling flow (CF) to the SZ-effect as follows:

\[
\frac{\partial I}{\partial t} = \left(\frac{\partial I}{\partial t}\right)_K + \left(\frac{\partial I}{\partial t}\right)_{CF}, \quad (10)
\]

where the first term on the right hand side is given by the standard Kompaneets equation and the second term is due to the moving electron media in the cooling flow.
Relating the CMB photon intensity field $I$ to the occupation number $\eta$ through $I = i_0 x^3 \eta$, we have for the cooling flow contribution from Eq.(9):

$$\left( \frac{\partial I}{\partial t} \right)_{CF} = \left[ -4x \frac{\partial}{\partial x} (x^3 \eta) + x \frac{\partial^2}{\partial x^2} (x^4 \eta) \right] i_0 \frac{\sigma_T n_{CF} v^2}{3},$$  \hspace{1cm} (11)

where $n_{CF}$ is the electron number density in the cooling flow region. Integrating over the cluster cooling flow region and assuming a Planckian photon field for $\eta$, we find:

$$\Delta I_{CF}(x) = i_0 g(x) \frac{1}{c^2} \int_{CF} n_{CF}(r) \sigma_T \frac{v^2(r)}{3} dl_{CF},$$  \hspace{1cm} (12)

which is the contribution due to the cooling flow bulk motion inside the cluster frame to the SZ-effect. From Eq.(9) we see that the spectral shape $g(x)$ of this additional term is the same as for the usual thermal SZ-effect. This clearly makes it more difficult to distinguish this contribution from the usual SZ-effect.

4 Results

In the following we get an estimate of this additional SZ contribution. In order to compare bulk motion contribution in the cooling flow region to the classical thermal SZ-effect, we introduce the following frequency independent ratio:

$$\alpha = \frac{\Delta I_{CF}}{\Delta I_K} = \frac{y_{CF}}{y_K},$$  \hspace{1cm} (13)

with $y_{CF} = \int_{CF} \frac{n_{CF}(r) v^2(r)}{3x^2} \sigma_T dl_{CF}$ and $y_K = \int_{cl} \frac{kT_{e,cl}}{m_e c^2} \sigma_T n_{cl}(r) dl_{cl}$, the Comptonization parameter.

We consider a typical spherical cluster with an isothermal $\beta$-model for the electron cluster density $n_{cl}$. When the line of sight goes through the center of a spherical cluster, we get for a large cluster radius $r_{cl}$, $r_{cl} \gg r_c$ - see e.g. (Puy et al., 2000):

$$y_K = \frac{kT_{e,cl}}{m_e c^2} \sigma_T r_c n_{0,cl} B(\frac{3}{2} \beta - \frac{1}{2}, \frac{1}{2}),$$  \hspace{1cm} (14)
where \( r_c \) is the core radius, \( n_{0,cl} \) the central electron density of the cluster and \( B \) the Beta function. This way we find:

\[
\alpha = \frac{2 m_e}{3 k T_{e,cl} r_c n_{0,cl} B(\frac{3}{2}, \beta - \frac{1}{2})} \int_{r_s}^{r_{cool}} n_{CF}(r) v^2(r) \, dl_{CF}.
\]

The integral is limited to the cooling flow region from \( r_{cool} \) to the sonic radius \( r_s \), as explained in section 2, along the line of sight through the cluster center. The integration boundary \( r_s \) depends on the mass deposition rate and the initial value conditions\(^\text{2}\).

As an example we take as typical values for clusters of galaxies the numbers found for A426, A478, A1795, A2029 and A2142 \( (\text{White, Jones and Formann, 1997}) \). All these clusters have a cooling flow with a mass deposition rate of \( \dot{m} \approx -300 M_\odot/\text{yr} \). The values for \( \beta \) and the central electron cluster density \( n_{0,cl} \) are from \( \text{[Mohr et al. 1999]} \). With these parameters we can estimate the ratio \( \alpha \) between the cooling flow bulk motion contribution and the classical thermal SZ-effect, see Eq.(15), which turns out to be of the order \( 10^{-5} \). As we cut out the most inner part of the cooling flow region, from the center to the sonic radius \( r_s \), this value must be interpreted as a lower limit to this effect. Thus at first glance the cooling flow bulk motion contribution is not very relevant for SZ observations and for the subsequent determination of the Hubble constant.

Nevertheless, as \( \alpha = \alpha(T_{e,cl}, n_{0,cl}, \beta, \dot{m}, r_c, r_{cool}) \) depends on several observationally inferred parameters which can substantially change, we performed also a numerical investigation by letting vary the various parameters entering into the determination of \( \alpha \). We find that, for instance:

\begin{itemize}
  \item For big mass deposition rates \( \dot{m} (\approx -3000 M_\odot/\text{yr}) \) and small cooling and core radii \( (\approx 100 - 150 \text{kpc}) \) \( \alpha \) increases to \( 10^{-4} \), by assuming that the other parameters have still average cluster values such as: \( T_{e,cl} = 7.5 \text{keV} \), \( \beta = \frac{2}{3} \), \( n_{0,cl} = 2.2 \cdot 10^{-2} \text{cm}^{-3} \).
  \item Whereas we expect \( \beta \approx \frac{2}{3} \) and \( T_{e,cl} \approx 7.5 \text{keV} \) not to vary substantially, the value for the central electron cluster density could easily be smaller by two orders of magnitude, thus reaching values as low as \( n_{0,cl} \approx 10^{-4} \text{cm}^{-3} \). If this is the case then \( \alpha \) can be of the order \( 10^{-3} - 10^{-2} \). Notice that \( n_{CF} \), since it depends on the total gravitating mass as can be seen from the system of Eqs.(1), will not change much by varying \( n_{0,cl} \).
  \item When taking into account the finite extension of a cluster or an aspherical gas distribution it turns out that \( y_K \) gets smaller by as much as 30% in some cases, see \( \text{[Puy et al. 2000]} \). Accordingly, \( \alpha \) increases by \( \approx 40\% \).
\end{itemize}

\(^2\) The sonic radius \( r_s \) is typically between 1 and 15 kpc.
Thus, the cooling flow bulk motion contribution to the SZ-effect can reach for some clusters the percent level and become relevant for a correct analysis of the SZ observations. Especially in view of the rapid progress in the observational techniques - which will provide in the near future much more accurate measurements - one might then be capable of observing also small deviations from the standard thermal SZ-effect. This will then enable observers either to obtain better determinations of the Hubble constant or more details on the state of the cluster.

5 Summary and Outlook

We showed how the Kompaneets equation has to be modified to include the effect of the bulk motion due to a homogeneous steady-state cooling flow. This effect is quadratic in the cooling flow velocity, as it is measured in the cluster frame, and thus different from the usual kinematic SZ-effect, which is linear in the velocity. The contribution strongly depends on the specific dynamics of the cooling flow and the cluster properties. Generally, it turns out to be rather small. However, for some clusters which have more extreme parameters, though not unrealistic, one finds a contribution at the percent level. One might speculate that this effect could be relevant when observing young clusters which are not yet virialized, still in their formation phase and for which one might expect the presence of rather large and extended regions with bulk motions, or perhaps in superclusters which are still in the formation phase. Such young clusters might be observable in future with the Planck satellite. Another interesting situation might arise from shocks in merging processes of clusters of galaxies, in situations where the average velocity along a line of sight approximately vanishes. If so, the merging process will not be detected through the kinematic SZ-effect which is linear in the velocity, but could be instead observed through the above discussed effect which is quadratic in the velocity.

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Fig. 1. Cooling flow velocity $|v(r)|$ and isothermal sound speed $c_s(r)$ as a function of radius. The mass deposition rate is $\dot{m} = -300 M_\odot \text{ yr}^{-1}$, $r_{cool} = 100 \text{ kpc}$. The initial values for the integration are $v(r_{cool}) = 15 \text{ km/s}$ and $c_s(r_{cool}) = 1600 \text{ km/s}$.

Fig. 2. Cooling flow electron number density $n_{CF}(r)$ as a function of radius. Cooling flow parameters as adopted in Fig.1.