Nonzero $\theta_{13}$ for Neutrino Mixing in a Supersymmetric $B-L$ Gauge Model with $T_7$ Lepton Flavor Symmetry

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Abstract

We discuss how $\theta_{13} \neq 0$ is accommodated in a recently proposed renormalizable model of neutrino mixing using the non-Abelian discrete symmetry $T_7$ in the context of a supersymmetric extension of the Standard Model with gauged $U(1)_{B-L}$. We predict a correlation between $\theta_{13}$ and $\theta_{23}$, as well as the effective neutrino mass $m_{ee}$ in neutrinoless double beta decay.
In a recent paper [1], a supersymmetric $B - L$ gauge model with $T_7$ lepton flavor symmetry is proposed with the following desirable features. (1) Neutrino tribimaximal mixing is achieved in a renormalizable theory, without the addition of auxiliary symmetries and particles. (2) The resulting neutrino mass matrix depends on only two complex parameters, and is of the same form already considered some time ago [2], using the discrete symmetry $A_4$ [3,4]. (3) The charged-lepton Yukawa sector exhibits a residual discrete $Z_3$ symmetry, i.e. lepton flavor triality [5, 6], under which $e, \mu, \tau \sim 1, \omega^2, \omega$, where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. (4) There are physical scalar doublets transforming as $\omega, \omega^2$ which will decay exclusively into leptons such that lepton flavor triality is conserved. (5) If the new gauge boson $Z'$ corresponding to the spontaneous symmetry breaking of $B - L$ [7] has a mass around 1 TeV, its production and decay into these exotic scalars may be observable at the Large Hadron Collider (LHC).

Recently, the T2K Collaboration has announced that a new measurement [8] has yielded a nonzero $\theta_{13}$ for neutrino mixing, i.e.

$$0.03 (0.04) \leq \sin^2 2\theta_{13} \leq 0.28 (0.34) \quad (1)$$

for $\delta_{CP} = 0$ and normal (inverted) hierarchy of neutrino masses. This indicates a possibly significant deviation from tribimaximal mixing [9] where $\theta_{13} = 0$, $\tan^2 \theta_{12} = 1/2$, and $\sin^2 2\theta_{23} = 1$ are predicted. Whereas the tribimaximal pattern has an elegant theoretical interpretation [4] in terms of the simplest application of $A_4$ [3], deviations from it are expected [4, 10]. In this paper, we present a variation of our previous $T_7$ proposal [1] and show how a different choice of the residual symmetry of the soft terms of this model will lead to a four-parameter neutrino mass matrix with nonzero $\theta_{13}$ and predicts a strong correlation between $\theta_{13}$ and $\theta_{23}$ as well as the effective neutrino mass $m_{ee}$ in neutrinoless double beta decay.

The tetrahedral group $A_4$ (12 elements) is the smallest group with a real 3 representation.
The Frobenius group $T_7$ (21 elements) is the smallest group with a pair of complex $3$ and $3^*$ representations. It is generated by

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

where $\rho = \exp(2\pi i/7)$, so that $a^7 = 1$, $b^3 = 1$, and $ab = ba^4$. The character table of $T_7$ (with $\xi = -1/2 + i\sqrt{7}/2$) is given by

| class | $n$ | $h$ | $\chi_1$ | $\chi_1'$ | $\chi_1''$ | $\chi_3$ | $\chi_3^*$ |
|-------|----|----|--------|--------|--------|--------|--------|
| $C_1$ | 1  | 1  | 1      | 1      | 1      | 3      | 3      |
| $C_2$ | 7  | 3  | 1      | $\omega$ | $\omega^2$ | 0      | 0      |
| $C_3$ | 7  | 3  | 1      | $\omega^2$ | $\omega$ | 0      | 0      |
| $C_4$ | 3  | 7  | 1      | 1      | 1      | $\xi$ | $\xi^*$ |
| $C_5$ | 3  | 7  | 1      | 1      | 1      | $\xi^*$ | $\xi$ |

Table 1: Character table of $T_7$.

The group multiplication rules of $T_7$ include

$$3 \times 3 = 3^*(23, 31, 12) + 3^*(32, 13, 21) + 3(33, 11, 22),$$

$$3 \times 3^* = 3(21^*, 32^*, 13^*) + 3^*(12^*, 23^*, 31^*) + 1(11^* + 22^* + 33^*)$$

$$+ 1'(11^* + \omega 22^* + \omega^2 33^*) + 1''(11^* + \omega^2 22^* + \omega 33^*).$$

Note that $3 \times 3 \times 3$ has two invariants and $3 \times 3 \times 3^*$ has one invariant. These serve to distinguish $T_7$ from $A_4$ and $\Delta(27)$. We note that $T_7$ was first considered for quark and lepton masses some time ago [11].

Under $T_7$, let $L_i = (\nu, l)_i \sim 3$, $l_i^c \sim 1, 1', 1''$, $i = 1, 2, 3$, $\Phi_i = (\phi^+, \phi^0)_i \sim 3$, and $\Phi'_i = (\phi'^+, -\phi'^-)_i \sim 3^*$. The Yukawa couplings $L_i l_j^c \Phi_k'$ generate the charged-lepton mass matrix

$$M_l = \begin{pmatrix} f_1 v_1' & f_2 v_1' & f_3 v_1' \\ f_1 v_2' & \omega^2 f_2 v_2' & \omega f_3 v_2' \\ f_1 v_3' & \omega f_2 v_3' & \omega^2 f_3 v_3' \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} v,$$
if \( v'_1 = v'_2 = v'_3 = v'/\sqrt{3} \), as in the original \( A_4 \) proposal \[3\].

Let \( \nu_i' \sim 3^* \), then the Yukawa couplings \( L_i \nu_j^c\Phi_k \) are allowed, with

\[
M_D = f_D \begin{pmatrix} 0 & v_1 & 0 \\ 0 & 0 & v_2 \\ v_3 & 0 & 0 \end{pmatrix} = f_D \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},
\]

(6)

for \( v_1 = v_2 = v_3 = v/\sqrt{3} \) which is necessary for consistency since \( v'_1 = v'_2 = v'_3 = v'/\sqrt{3} \) has already been assumed for \( M_l \). Note that \( \Phi \) and \( \Phi' \) have \( B - L = 0 \), and both are necessary because of supersymmetry.

Now add the neutral Higgs singlets \( \chi_i \sim 3 \) and \( \eta_i \sim 3^* \), both with \( B - L = -2 \). Then there are two Yukawa invariants: \( \nu_i'^c \nu_j^c \chi_k \) and \( \nu_i'^c \nu_j^c \eta_k \) (which has to be symmetric in \( i, j \)). Note that \( \chi_i^* \sim 3^* \) is not the same as \( \eta_i \sim 3^* \) because they have different \( B - L \). This means that both \( B - L \) and the complexity of the \( 3 \) and \( 3^* \) representations in \( T_7 \) are required for this scenario. The heavy Majorana mass matrix for \( \nu^c \) is then

\[
M_{\nu^c} = h \begin{pmatrix} u_2 & 0 & 0 \\ u_3 & 0 & 0 \\ 0 & u_1 \end{pmatrix} + h' \begin{pmatrix} 0 & u'_3 & u'_2 \\ u'_3 & 0 & u'_1 \\ u'_2 & u'_1 & 0 \end{pmatrix} = \begin{pmatrix} A & C & B \\ C & D & C \\ B & C & D \end{pmatrix},
\]

(7)

where \( A = hu_2 \), \( B = h' u'_2 \), \( C = h' u'_1 = h' u'_3 \), and \( D = hu_1 = hu_3 \) have been assumed. This means that the residual symmetry in the singlet Higgs sector is \( Z_2 \), whereas that in the doublet Higgs sector is \( Z_3 \). This misalignment is different from that assumed previously \[1\], but is nevertheless achievable with suitably chosen soft terms, i.e. \( \chi_2^* \chi_2 \), \( \chi_2'^* \chi_2' \), \( \chi_2 \chi_2' \) + H.c., \( \eta_2^* \eta_2 \), \( \eta_2'^* \eta_2' \), \( \eta_2 \eta_2' \) + H.c., \( \chi_1^* \chi_1 \) + \( \chi_3^* \chi_3 \), \( \chi_1^* \chi_1' \) + \( \chi_3^* \chi_3' \), \( \chi_1 \chi_1' \) + \( \chi_3 \chi_3' \) + H.c., \( \eta_1^* \eta_1 \), \( \eta_1^* \eta_1' \), \( \eta_1 \eta_1' \) + \( \eta_3 \eta_3' \) + H.c., \( \chi_2 \eta_2 \) + H.c., \( \chi_2 \eta_2' \) + H.c., \( \chi_2 \eta_2' \) + H.c., \( \chi_2 (\eta_1 + \eta_3) + H.c., \chi_2 (\eta_1 + \eta_3) \) + H.c., \( (\chi_1 + \chi_3) \eta_2 \) + H.c., \( (\chi_1 + \chi_3) \eta_2 \) + H.c., \( (\chi_1 + \chi_3) (\eta_1 + \eta_3) \) + H.c., \( (\chi_1 + \chi_3) (\eta_1 + \eta_3) \) + H.c.

The seesaw neutrino mass matrix is now

\[
M_{\nu} = -M_D M_{\nu^c}^{-1} M_D^T = - \frac{f_D^2 \nu^2}{3 \det(M_{\nu^c})} \begin{pmatrix} AD - B^2 & C(B - A) & C(B - D) \\ C(B - A) & AD - C^2 & C^2 - BD \\ C(B - D) & C^2 - BD & D^2 - C^2 \end{pmatrix},
\]

(8)
where \( \text{det}(M_{\nu c}) = A(D^2 - C^2) + 2BC^2 - D(B^2 + C^2) \). Redefining the parameters \( A, B, C, D \) to absorb the overall constant, we obtain the following neutrino mass matrix in the tribimaximal basis:

\[
M^{(1,2,3)}_{\nu} = \begin{pmatrix}
\frac{D(A + D - 2B)}{2} & \frac{C(2B - A - D)}{\sqrt{2}} & \frac{D(A - D)}{2} \\
\frac{C(2B - A - D)}{\sqrt{2}} & AD - B^2 & C(D - A)/\sqrt{2} \\
\frac{D(A - D)}{2} & \frac{C(D - A)}{\sqrt{2}} & (AD + D^2 + 2BD - 4C^2)/2
\end{pmatrix}.
\]

This is achieved by first rotating with the \( 3 \times 3 \) unitary matrix of Eq. (5), which converts it to the \((e, \mu, \tau)\) basis, then by Eq. (10) below. Note that for \( D = A \) and \( C = 0 \), this matrix becomes diagonal: \( m_1 = A(A - B), m_2 = A^2 - B^2, m_3 = A(A + B) \), which is the tribimaximal limit. Normal hierarchy of neutrino masses is obtained if \( B \simeq A \) and inverted hierarchy is obtained if \( B \simeq -2A \).

The neutrino mixing matrix \( U \) has 4 parameters: \( s_{12}, s_{23}, s_{13} \) and \( \delta_{CP} \). We choose the convention \( U_{\tau 1}, U_{\tau 2}, U_{e3}, U_{\mu 3} \to -U_{\tau 1}, -U_{\tau 2}, -U_{e3}, -U_{\mu 3} \) to conform with that of the tribimaximal mixing matrix

\[
U_{TB} = \begin{pmatrix}
\sqrt{2/3} & 1/\sqrt{3} & 0 \\
-1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\
-1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2}
\end{pmatrix}.
\]

then

\[
M^{(1,2,3)}_{\nu} = U_{TB}^T m' \begin{pmatrix}
m_1' & 0 & 0 \\
m_2' & m_3' & 0 \\
m_3' & 0 & m_3'
\end{pmatrix} U_{TB}^T U_{TB},
\]

where \( m'_{1,2,3} \) are the physical neutrino masses, with

\[
m_2' = \pm \sqrt{m_1'^2 + \Delta m^2_{21}}, \tag{12}
\]

\[
m_3' = \pm \sqrt{m_1'^2 + \Delta m^2_{21}/2 + \Delta m^2_{32}} \quad \text{(normal hierarchy)}, \tag{13}
\]

\[
m_3' = \pm \sqrt{m_1'^2 + \Delta m^2_{21}/2 - \Delta m^2_{32}} \quad \text{(inverted hierarchy)}. \tag{14}
\]

If \( U \) is known, then all \( m_{1,2,3,4,5,6} \) are functions only of \( m_1' \).
In our model, the neutrino mass matrix has only 4 parameters \( A, B, C, D \), so there are 2 conditions on \( m_{1,2,3,4,5,6} \). They are given by

\[
A = D + \frac{2m_4}{D}, \quad B = D + \frac{m_4 - m_1}{D}, \quad C = \frac{-m_6}{m_1 \sqrt{2}} = \frac{-m_5}{m_4 \sqrt{2}}, 
\]

\[
D^2 = \frac{(m_1 - m_4)^2}{2m_1 - m_2} = \frac{m_1^2(m_3 + m_1 - 2m_4)}{2m_1^2 - m_6^2}. \tag{15}
\]

We now input the allowed ranges of values for \( \Delta m_{31}^2, \Delta m_{32}^2, s_{12}, s_{23}, s_{13} \) assuming \( \delta_{CP} = 0 \). In that case, \( A, B, C, D \) can be chosen real. We then obtain \( m_{1,2,3,4,5,6} \) as a function of \( m'_1 \).

We now solve for \( m'_1 \) using the condition \( m_1 m_5 = m_4 m_6 \) from Eq.(15). Using this value of \( m'_1 \), we check Eq.(16) to see if the input values are allowed. In this way, we are able to find a strong correlation between \( s_{13} \) and \( s_{23} \) as shown in Fig. 1.

It is very well approximated by

\[
\sin^2 2\theta_{23} \simeq 1 - \frac{1}{2} \sin^2 2\theta_{13}, \tag{17}
\]

for all solutions. Using [12] \( \sin^2 2\theta_{13} < 0.135 \), this implies \( \sin^2 2\theta_{23} > 0.93 \).
The effective neutrino mass $m_{ee}$ in neutrinoless double beta decay is given by

$$m_{ee} = \frac{1}{3} |2m_1 + m_2 + 2\sqrt{2}m_6| = \frac{1}{3} |2AD + D^2 - 2BD - B^2 + 2C(2B - A - D)|,$$

and the kinematic $\nu_e$ mass in nuclear beta decay is $m_{\nu_e} = \sum_i |U_{ei}m^i|$. We find solutions for both normal and inverted hierarchies, using the central values of $\Delta m^2_{32} = 2.40 \times 10^{-3}$ eV$^2$ and $\Delta m^2_{21} = 7.65 \times 10^{-5}$ eV$^2$. We take 3 representative values of $\sin^2 2\theta_{12}$, i.e. 0.84, 0.87, 0.90. In Figures [2] to [4] we show the solutions for the physical neutrino masses as well as $m_{ee}$ and $m_{\nu_e}$ as functions of $\sin^2 2\theta_{13}$ in the case of normal hierarchy. In Figures [5] to [7] we show these in the case of inverted hierarchy. For $\sin^2 2\theta_{12} = 0.87$ (corresponding to $\tan^2 \theta_{12} = 0.47$), we plot in Figures [8] and [9] the $T_7$ parameters $(A + 2D)/3$, $B$, $C$, and $(A - D)/2$ in the case of normal and inverted hierarchies. It is clear that $C$ and $(A - D)/2$ are small, showing that these solutions deviate only slightly from the tribimaximal limit. In particular, $C = 0$ exactly works for normal hierarchy, but it implies $\sin^2 2\theta_{12} > 8/9$, i.e. $\tan^2 \theta_{12} > 1/2$ [3].

In conclusion, we have shown that a previously proposed [1] $T_7/B - L$ model of neutrino masses has a variation (supported by a $Z_2$ residual symmetry) which allows a nonzero $\theta_{13}$ and predicts the strong correlation $\sin^2 2\theta_{23} \simeq 1 - \sin^2 2\theta_{13}/2$ which is consistent with all data, including the recent T2K measurement [8]. We also predict the effective neutrino mass $m_{ee}$ in neutrinoless double beta decay.

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Figure 2: Normal hierarchy solution in case of $\sin^2 \theta_{12} = 0.84$.

Figure 3: Normal hierarchy solution in case of $\sin^2 \theta_{12} = 0.87$. 
Figure 4: Normal hierarchy solution in case of $\sin^2 2\theta_{12} = 0.90$.

Figure 5: Inverted hierarchy solution in case of $\sin^2 2\theta_{12} = 0.84$. 
Figure 6: Inverted hierarchy solution in case of $\sin^2 2\theta_{12}=0.87$. 

Figure 7: Inverted hierarchy solution in case of $\sin^2 2\theta_{12}=0.90$. 
Figure 8: $T_7$ parameters for normal hierarchy in case of $\sin^2 2\theta_{12}=0.87$.

Figure 9: $T_7$ parameters for inverted hierarchy in case of $\sin^2 2\theta_{12}=0.87$. 