The Structure of the Pion and Nucleon, and Leading Neutron Production at HERA

Garry Levman

Department of Physics, University of Toronto, Toronto ON M5S 1A7, Canada

Abstract

Attention is paid to recent results from the ZEUS Collaboration on the photo- and electro- production of leading neutrons in $e^+p$ collisions at HERA. Some implications for the structure of the pion and the nucleon are discussed.

Key words: nucleon, pion, meson exchange, structure
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1 Introduction

The ZEUS Collaboration has published data on leading neutron production in neutral current $e^+p$ collisions at HERA[1]. The data have been anticipated because of the light they cast on the structure of the pion and nucleon.

In [1] the ZEUS Collaboration uses an effective flux method[2] to determine the photon-pion total cross section at high $W$, and the deep inelastic structure function of the pion, $F_2^\pi(x, Q^2)$. In this note, the method is described; its assumptions and inherent difficulties are discussed; and the problem of error is confronted. Implications of the ZEUS data are examined critically for the role of meson exchange in the production of leading neutrons and for the structure of the pion.
Leading baryons are those produced with small transverse momentum, $p_T$, and carrying a large fraction $x_L$, of the incoming proton’s energy. They are thought to represent the conserved baryon current because the squared 4-momentum transfer, $t$, between the incoming proton and outgoing baryon is small([1], Fig. 1).

The production data for leading baryons can be understood in terms of meson exchanges. Consider the specific case $\gamma p \to Xn$. The cross section for the production of a leading neutron is given by a sum over all possible meson exchanges between the photon and proton which lead to an outgoing neutron at the proton vertex([1], Fig. 1):

$$\sigma_{\gamma p \to Xn}(x_L, t, W^2) \sim a_{\gamma p}(t) \sum_i f_{i/p}(x_L, t) \sigma_{\gamma i} \left( (1 - x_L)W^2, t \right)$$

where $i = \pi, \rho$, or $a_2$ meson; $\sigma_{\gamma i}$ is the total $\gamma i$ cross section; $f_{i/p}$ is the splitting function for $p \to n$ via $i$ exchange; and $a_{\gamma p}$ is a form factor which accounts for rescattering of the neutron (absorption) due to the finite size of the projectile. Projectiles other than the photon, and produced baryons other than the neutron are treated similarly. For example, in the reaction $pp \to Xp$ the $\omega$ and $f_2$ mesons, and the pomeron $\mathbb{P}$ also contribute. There are complications. The direct production of baryons which decay, such as $\Delta \to n\pi$, leads to the indirect production of nucleons; there can be two-meson exchange, and different exchanges can interfere; the form factor, $a_{\gamma p}$, can depend on $x_L$ and perhaps on the exchanged meson $i$. The simple factorization assumed in Eqn. 1 is only an approximation.

3 Leading Neutron Production and Pion Exchange

In hadro-production experiments (projectile=$p, \pi$) leading neutron production is believed to be dominated by pion exchange. The spectrum of leading neutrons in $pp$ collisions at Fermilab and the ISR is fairly well represented by a single term (one-meson-exchange) of the form

$$\sigma_{pp \to Xn} = \frac{1}{4\pi} \frac{g^2}{4\pi} \frac{-2t}{(t - m_\pi^2)^2} (1 - x_L)^{1 - 2t} \sigma_{\pi p} \left( (1 - x_L)W^2, t = 0 \right)$$
where $m_\pi$ is the mass of the pion, $g^2/4\pi = 14.5$, and $\sigma_{\pi p}$ is the measured, on-mass-shell, $\pi p$ total cross section. The splitting function

$$f_{\text{eff}}(x_L, t) = \frac{1}{4\pi} \frac{g^2}{4\pi} \frac{-2t}{(t - m_\pi^2)^2} (1 - x_L)^{1-2t}$$

is very close to that expected from pure one-pion exchange. The result is simple, but surprising. Note in Eqn. 3 the absence of

- an absorptive factor $a(t)$;
- an off-mass-shell correction, that is, a $t$ dependence to $\sigma(\pi p)$;
- contributions from $\rho$ and $a_2$ exchange, and $\Delta$ production.

The self-consistency of the meson exchange picture requires the presence of such ‘backgrounds’. In addition, the value, 14.5, of the $p\pi^0p$ coupling constant is larger than recent estimates[3,4] which suggest that $g^2/4\pi$ lies in the range (13.5–14.1).

The ZEUS Collaboration has reviewed (see [1], §10.1) the expected backgrounds to one-pion exchange which are known both from experimental measurements (eg. $\Delta$ production) and from theoretical calculations (eg. absorption). The effects are large, roughly 20-30%, apparently confounding the experimental observation embodied in Eqns. 2 and 3; however,

- $\rho$, $a_2$ exchange and $\Delta$ production increase neutron production, while absorptive effects decrease neutron production
- absorption preferentially removes $\rho$ and $a_2$ compared to $\pi$ exchange because $a(t)$ decreases with increasing $t$ and the contribution of the higher mass mesons increases relative to the pion at high $t$.

The conclusion to draw is that simple one-pion exchange agrees well with the hadro-production data because of a fortuitous near cancellation of absorptive effects and background contributions. The large value of the effective coupling constant (14.5) implies that the cancellation is not perfect, and that there is a residual $\lesssim 5\%$ contribution to the normalization. $f_{\text{eff}}$ is not the true pion flux, but rather an effective pion flux in the presence of competing and ‘compensating’ processes.

4 The Effective Flux

The effective flux of pions in the proton (effective splitting function) can be defined by experiment as the deconvolution of the measured semi-inclusive
differential cross section for $pp \rightarrow Xn$ and the measured $\pi p$ total cross section,

$$f_{eff}(x_L, t) \equiv \frac{d\sigma_{pp \rightarrow Xn}}{dx_L dt} / \sigma_{\pi p}$$

(4)

where deconvolution is written as division. By construction and definition $f_{eff}$ corrects one-pion exchange in $pp$ collisions for $\rho$ and $a_2$ exchange, $\Delta$ production, rescattering and off-mass-shell effects, etc. From Eqn. 1 it is seen that a ‘model’ for $f_{eff}$ is

$$f_{eff} \sim a_{p,p}(t) \sum_i f_{i/p}(x_L, t) \sigma_{ip}\left((1 - x_L)W^2, t\right) / \sigma_{\pi p}\left((1 - x_L)W^2, t = 0\right)$$

It is important to emphasize: experiments find that a convenient parameterization of $f_{eff}$ is given by Eqn. 3. Because of the universal behavior of hadronic cross sections $f_{eff}$ is also expected to work for $\pi p$ and $\gamma p$ collisions.

5 The Structure of the Pion and the Effective Flux

It has been proposed[8,9] to measure the structure function of the pion $F^\pi_2$ at HERA using virtual $\gamma^*\pi$ collisions assuming pion exchange dominates the interaction $\gamma^* p \rightarrow Xn$, just as it does in hadro-production.

A vital consistency check for any extraction of $F^\pi_2$ using the electro-production of leading neutrons is the demonstration that the photo-production measurements is in accord with the hadro-production measurements. The importance of the photo-production data arises because:

- $W$ is the single relevant leptonic variable ($Q^2 = 0$).
- the $W$ dependence of measured hadronic cross sections closely follows a simple universal behavior; at high $W$ the hadronic total cross sections behave as a power law $W^{2(\alpha_p - 1)}$, where $\alpha_p$ is a constant independent of the projectile and target.
- the photo-production ($Q^2 < 0.02$ GeV$^2$) and transition region ($0.1 < Q^2 < 0.74$ GeV$^2$) data from HERA are in good agreement with vector dominance in which the photon behaves like an exchanged vector meson and the collision is hadronic[7,10].

In contrast to photo-production, the situation in electro-production is less well understood. The additional degree of freedom quantified by $Q^2$ adds complication. Rescattering, background composition and levels, and factorization properties can vary with $Q^2$. Moreover, in deep inelastic scattering ($Q^2 \gtrsim 4$ GeV$^2$) the chief interest lies in measuring the $W$ (that is $x, x \approx Q^2/W^2$ at low
dependence of the cross section. In summary, a prerequisite for a believable determination of $F_2^\pi$ using high $Q^2$ electro-production data is a consistent determination of $\sigma_\gamma^\pi$ using photo-production data.

A standard technique for determining $\sigma_\gamma^\pi$ involves fitting the observed differential cross section for $\gamma p \rightarrow Xn$ while simultaneously correcting for background exchanges, Delta production, absorption, and off-mass-shell effects[11]. This procedure has the important advantage of providing an estimate of the size of the contributing sub-processes and giving a transparent estimate of the error. It has the disadvantage that there is a large number of effects to be accounted for, with a corresponding large degree of freedom. The parameters must be guessed, fit, or fixed by experiment.

The effective flux is used instead to directly determine $\sigma_\gamma^\pi$ without determining the individual sub-processes. This has the advantages of simplicity and directness, but the disadvantage that no information is obtained about the relative importance of the various processes which contribute. The error is difficult to estimate (see §6) because the amount of pion exchange present is not unambiguously determined. If pion exchange does not contribute, the result, although well defined, is meaningless.

The arguments which follow (in §5.1 and §5.2) give the essence of what obtains from the ZEUS measurement.

5.1 $\sigma_\gamma^\pi$ at high $W$

The $\gamma\pi$ total cross section is obtained from the photo-production data using

$$\sigma_\gamma^\pi = \frac{d\sigma(\gamma p \rightarrow Xn)/dx_L}{f_{eff}}$$

where the variable $t$ has been integrated out in the numerator. Substitution for $f_{eff}$ from its definition in Eqn. 4 shows that $\sigma_\gamma^\pi$ can be determined from (double) ratios

$$\frac{\sigma(\gamma_\pi)}{\sigma(\gamma p)} = \frac{\frac{d\sigma(\gamma p \rightarrow Xn)/dx_L}{\sigma(\pi p)\sigma(pp)}}{\frac{d\sigma(pp \rightarrow Xn)/dx_L}{\sigma(pp)}}$$

of measured quantities.
The ZEUS Collaboration has observed that the relative rate of neutron production in photo-production at HERA is half that of pp collisions. It follows from Eqn. 5 that $\sigma(\gamma\pi)/\sigma(\gamma p)$ is half $\sigma(\pi p)/\sigma(pp)$. Therefore, as ZEUS deduces directly,

$$\sigma(\gamma\pi) \simeq \sigma(\gamma p)/3$$

rather than two-thirds as expected from Regge factorization or the counting of valence quarks (the Additive Quark Model).

An important consistency check, which the ZEUS Collaboration has performed ([1], Fig. 6 & 8), is that the neutron energy and angular distributions are described by $f_{eff}$.

### 5.2 $F_2^\pi$ at low $x$

The double ratio (Eqn. 5) is robust for hadro- and photo-production; however, for electro-production, absorptive rescattering decreases as $Q^2$ increases, as discussed in [5] and [6] (see also [1], Fig. 9). At low $Q^2$ ($Q^2 \lesssim 1 \text{ GeV}^2$), $F_2^\pi$ and $\sigma(\gamma^*\pi)$ are related by

$$F_2^\pi \simeq \frac{Q^2}{4\pi^2\alpha} \sigma(\gamma^*\pi)$$  \hspace{1cm} (6)

which, in analogy with $\rho$ dominance in $\gamma p$ interactions[7], can be written as

$$F_2^\pi \simeq \frac{Q^2}{4\pi^2\alpha} \frac{m_\rho^2}{m_\rho^2 + Q^2} \sigma(\gamma\pi)$$  \hspace{1cm} (7)

where $m_\rho$ is the mass of the $\rho$ meson.

Equations 6 and 7 connect the photo-production $\gamma\pi$ cross section to the transition region $\gamma^*\pi$ cross section; together with the corresponding equations for $\gamma(\ast)p$ interactions,

$$\frac{\sigma(\gamma^*\pi)}{\sigma(\gamma\pi)} \simeq \frac{\sigma(\gamma^*p)}{\sigma(\gamma p)}$$  \hspace{1cm} (8)

obtains. In the kinematic region $0 < Q^2 < 4 \text{ GeV}^2$ the ratio of neutron production (i.e. tagged divided by all) increases. To maintain Eqn. 8 an absorptive correction needs to be applied. Rescattering decreases, and the effective flux must increase correspondingly in order to maintain consistency between
photo-production, the transition region, and deep inelastic scattering. The ZEUS data require for deep inelastic scattering (DIS) \( f_{\text{eff}} \rightarrow f_{\text{DIS}}^{\text{eff}} = 1.3 f_{\text{eff}} \).

The measurements show that 2.5\% of deep inelastic scattering events, independent of \( x \) and \( Q^2 \), have a leading neutron with \( 0.64 < x_L < 0.82 \) and \( p_T < 0.66 x_L \) GeV ([1], Fig. 10). The constancy of the ratio of neutron production in DIS as a function of \( x \) and \( Q^2 \) implies that \( F_2^\pi(x, Q^2) = k F_2^p((1 - x_L)x, Q^2) = k F_2^p(0.27x, Q^2) \) with a constant of proportionality \( k \) given by

\[
k = \frac{0.025}{\int \int f_{\text{eff}}^{\text{DIS}}(x_L, t)dx_Ldt} = 0.25
\]

where the integration is over the \( x_L \) and \( t(p_T) \) range covered by the ZEUS measurement. Because \( F_2^\pi(x) \propto (1/x)^\lambda \) with \( \lambda \approx 0.2[7] \)

\[
F_2^\pi(x, Q^2) \approx F_2^p(x, Q^2)/3
\]

6 Discussion

For the determination of \( F_2^\pi \) the effective flux method requires little theoretical input. Only the general concepts of meson exchange theory are needed. The data, hadro-production, photo-production and electro-production, determine everything. Note especially that

- one-pion-exchange is not assumed. It is only assumed that pion exchange dominates neutron production.
- the contribution of backgrounds is not neglected. The method accounts for \( \rho \) and \( a_2 \) exchange and for indirect neutron production through \( \Delta \) production and decay.
- the exact composition and relative contribution of the backgrounds is unimportant.
- absorptive effects are measured.
- the consistency of the photo-production \( (Q^2 < 0.02 \text{ GeV}^2) \), transition region \( (0.1 < Q^2 < 0.74 \text{ GeV}^2) \), and DIS \( (Q^2 > 4 \text{ GeV}^2) \) analyses requires and defines an absorptive correction.

If absorption approximately compensates for background exchanges in hadro- and photo-production, as the data suggest, then the 30\% rise in neutron ratio between photo-production and DIS is a measure of the relative contribution of the backgrounds.
The effective flux gives a normalization of $\sigma(\gamma\pi)$ which differs by a factor of two from that expected by Regge factorization or valence quark counting. The disagreement follows directly from the experimental observation that the relative rate of neutrons in photo-production is half that observed in hadro-production. Can $f_{\text{eff}}$, which is determined by $pp$ collisions, be wrong by a factor of two when used for photo-production?

Background exchanges and absorptive effects are of the same magnitude, but contribute with opposite signs. The ZEUS leading neutron data show that the relative size of absorptive effects is approximately 20-30%. Background processes must contribute at approximately this level, in agreement with estimates from phenomenological studies (reviewed in [1], §10.1). With this in mind, a conservative estimate for the error on using $f_{\text{eff}}$ for photo-production is obtained by assuming that pion exchange contributes $\gtrsim 50\%$ and that the backgrounds change by $\lesssim 50\%$ on moving from $pp$ to $\gamma p$ collisions. The change in $f_{\text{eff}}$ is then $\lesssim 25\%$. Suppose that the low neutron rate is not due to a small photon-pion cross section, but rather due to changes in absorption or backgrounds. Then absorption must increase or backgrounds decrease strongly in the photon-proton system compared to the proton-proton system.

If the meson exchange picture underestimates the normalization of $\sigma(\gamma\pi)$ by a factor of two, the theory is wrong, misapplied, or there is a significant missing ingredient. Then one cannot extract with any confidence the $x$ and $Q^2$ behavior of $F_2^\pi$.

If the photo-production result is disbelieved, the ZEUS data can be taken as evidence that meson exchange plays little role in the production of leading nucleons at HERA. One then argues that the ZEUS measurement is merely a reflection of $F_2^p$, arising because of factorization (limiting fragmentation) at the proton vertex and baryon conservation.

On the other hand, the experimental evidence for the meson exchange picture is good (see references in [1]). One can accept the picture and the normalization of $\sigma(\gamma\pi)$ and $F_2^\pi$ implied by the ZEUS data. In this case some conjectures can be make:

- the $x$ dependence of $F_2$ for all hadrons is similar at low $x$ and is determined mainly by the QCD evolution equations, only weakly by the valence structure of the hadron.
- the number of partons at low $x$ in the pion is 1/3 that of the proton; since the charged radius of the pion is 2/3 the proton’s, the volume density of partons in the pion is approximately the same as in the proton.
- there is a large probability for the proton to be found in a meson-nucleon or meson-Delta Fock state (the nucleon within the nucleon); the proton is a loosely bound meson-nucleon system composed of infinitely bound partons.
the quark-antiquark sea of a hadron is generated mainly by valence-valence interactions (three for the proton and one for the pion), and not by self interactions.

Moreover, there is a significant violation of the quark counting rules and Regge factorization.

7 Conclusions

In the context of the meson exchange picture of leading baryon production, the leading neutron data from ZEUS, taken together with the hadro-production data from Fermilab and the ISR, make a statement about the normalization of $\sigma(\gamma\pi)$. With the assumption that the $x$ and $Q^2$ dependence of the difference between pion exchange and the backgrounds is weak, $F_2^\pi$ can be extracted from the data. The normalization is the firmer of the determinations. If the normalization fixed by the photo-production data is incorrect, then an extraction of $F_2^\pi$ is doubtful since the self-consistency of the picture is questionable. In particular, the $x$ and $Q^2$ dependence extracted from the data is questionable because the high $Q^2$ region is less well understood than photo-production, both experimentally and phenomenologically.

If the determination of $F_2^\pi$ using the effective flux method is correct, then the ZEUS data have important implications for the structure of the nucleon and the pion.

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