The highest frequency kHz QPOs in neutron star low-mass X-ray binaries

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ABSTRACT
We investigate the detections with RXTE of the highest frequency kHz Quasi-periodic oscillations (QPOs) previously reported in six neutron star (NS) low-mass X-ray binaries. We find that the highest frequency kHz QPO detected in 4U 0614+09 has a 1267 Hz 3σ confidence lower limit on its centroid frequency. This is the highest such limit reported to date and of direct physical interest as it can be used to constrain QPO models and the supranuclear density equation of state (EoS). We compare our measured frequencies to maximum orbital frequencies predicted in full GR using models of rotating NSs with a number of different modern EoS and show that these can accommodate the observed QPO frequencies. Orbital motion constrained by NS and ISCO radii is therefore a viable explanation of these QPOs. In the most constraining case of 4U 0614+09, we find the NS mass must be $M < 2.1 \, M_\odot$. From our measured QPO frequencies, we can constrain the NS radii for five of the six sources we studied to narrow ranges ($\pm 0.1$–0.7 km) different for each source and each EoS.

Key words: accretion, accretion discs – equation of state – stars: neutron – X-rays: binaries.

1 INTRODUCTION
Quasi-periodic oscillations (QPOs) in the X-ray emission from accreting neutron star low-mass X-ray binaries (NS-LMXBs) have been observed with best-fitting centroid frequencies up to $\nu_0 \sim 1250$ Hz (references are given next).

As orbital frequencies of $\sim$kHz are expected in the close vicinity of NSs, QPOs at such frequencies have often been associated with orbital motion in the inner accretion disc (see for example Lamb & Psaltis 1998a; Stella & Vietri 1998; Miller, Titarchuk, Osherovich & Kuznetsov 1999; Kluzniak & Abramowicz 2001; Török et al. 2016). Additional evidence for this interpretation was presented by Bult & van der Klis (2015) in the form of a dependence of the pulse amplitude in the accreting millisecond pulsar SAX J1808.4–3658 on the kHz QPO frequency, indicating an azimuthal asymmetry is at the base of the QPO mechanism. Alternative physical explanations of kHz QPOs invoke for instance disc oscillation modes, see de Avellar et al. (2018) and Rezzolla et al. (2003).

The accretion disc is truncated at or above the NS surface, and for this reason, if the QPO frequency is the general relativistic orbital frequency at the inner disc edge ($\nu_{\text{orb}}$), the maximum QPO frequency puts limits on the mass (ISCO radius smaller than orbital radius) and radius (smaller than orbital radius) of the NS to first order in $j$ (Kluzniak, Michelson, Wagoner 1990):

$$M \lesssim 2.2 (\nu_{\text{orb}}/1000 \text{ Hz})^{-1} (1+0.75j) \, M_\odot$$

and

$$R \lesssim 19.5 (\nu_{\text{orb}}/1000 \text{ Hz})^{-1} (1+0.2j) \, \text{km.}$$

Here, $j \equiv cJ/GM^2$ is the dimensionless angular momentum (spin parameter) of the NS, with $J$ the angular momentum and $M$ the mass (e.g. Miller et al. 1999b). Clearly, these constraints tighten when $\nu_{\text{orb}}$ increases. For this reason, the question what is the highest reliably detected QPO frequency is of direct physical interest.

The highest kHz QPO frequency reported in the literature is $\nu_u = 1329 \pm 4$ Hz (3.5σ, $Q = 51$, single kHz QPO) in 4U 0614+09 (van Straaten et al. 2000). In that work, a 1273 ± 15 Hz (no significance quoted, $Q = 14$, single kHz QPO) QPO is also reported from another observation of the same source. Subsequent analysis by Boutelier, Barret & Miller (2009) (using an automated detection method with a lower limit on the detection significance of 6σ) of an extended data set on 4U 0614+09 including the observations in which the high-frequency kHz QPOs were reported by van Straaten et al. (2000) failed to confirm the presence of a significant kHz QPO > 1224 Hz (the highest frequency $\nu_u$ were reported by van Straaten et al. 2000) measured the 1329 ± 4 Hz QPO the authors reported a $\sim 2.6\sigma$ 1328 ± 27 Hz QPO ($Q = 29$, single kHz QPO). We list these detections in Table 1 along with the other highest frequency kHz QPOs previously reported for other sources.

All sources in the table are atoll sources at low luminosity. In all cases, the high-frequency kHz QPO was detected when the source...
was in the banana state (Markwardt, Strohmayer & Swank 1999; van Straaten et al. 2000; Di Salvo et al. 2001; Kaaret et al. 2002; Jonker et al. 2007; Altamirano et al. 2008). Burst oscillations were detected in other observations of 4U 0614+09, SAX J1750.8–2900, 4U 1636–53, 4U 1702–43, and 4U 1728–34; the NS spin frequency is known for these sources. SAX J1750–2900 and 4U 1636–53 and 4U 1636–53 were included in the study by van Doesburgh & van der Klis (2017) but no higher frequencies than those reported in the literature were found. In Jonker et al. (2001), it was suggested that the fractional root mean square (rms) of the kHz QPO is anticorrelated with source luminosity. All sources mentioned here seem to follow this trend, indicating that the high-frequency QPO states of these varied sources might form a homogeneous group.

In van Doesburgh & van der Klis (2017), we recently reported a highest kHz QPO frequency in RXTE archival data on 4U 1728–34 of 1276 ± 38 Hz, on 4U 0614+09 of 1369 ± 51 Hz, and on 4U 1702–43 of 1198 ± 15 Hz. In that work, a large data set was uniformly processed for a different purpose than identifying the highest QPO frequencies. In view of the relevance of the issue of the highest frequency reliably detected for a kHz QPO, in this paper we present a detailed study of the QPO frequencies reported in van Doesburgh & van der Klis (2017), compare these to the literature (reanalysing the data when necessary), and discuss implications for the supranuclear density equation of state (EoS) and kHz QPO models.

2 DATA ANALYSIS

2.1 Energy spectral analysis

We use the Crab-normalized soft and hard colours averaged per RXTE observation (single pointing of ~1.5–2.5 ks in length, archived under a single ObsID) as previously obtained by van Doesburgh & van der Klis (2017) following the method of van Straaten et al. (2000). The hard colour is defined as the ratio between the counts in the 9.7–16.0 keV and 6.0–9.7 keV energy bands, and the soft colour as that between the 3.5–6.0 keV and 2.0–3.5 keV bands.

2.2 Timing analysis

To calculate the power spectra, we use GoodXenon and Event mode data with a time resolution of 1/8192 s (~122 μs) or better. We rebin if necessary to 1/8192 s and divide the data into segments of 16 s. This results in power spectra with a Nyquist frequency of 4096 Hz and a lowest frequency and frequency resolution of 0.0625 Hz. We include channels corresponding to photon energies of 2–18 keV for reasons outlined in Section 2.2.1. No background or dead-time corrections were performed before calculating the power spectra; instead, we correct for these effects after averaging the Leahey-normalized power spectra. To do so, we subtract a counting noise model power spectrum incorporating dead-time effects (Zhang et al. 1995) following the method of Klein Wolt (2004). We renormalize the power spectra such that the square root of the integrated power in the spectrum equals the fractional rms amplitude of the variability in the signal (van der Klis 1989).

2.2.1 Energy channel selection

To determine the optimal energy band for the detection of the upper kHz QPO with νu > 1200 Hz, we obtain the detection significance of a selection of upper kHz QPOs reported in van Doesburgh & van der Klis (2017) at slightly lower frequency (1000 < νu < 1200 Hz) in three different energy ranges: 2–18 keV, 2–31 keV (we correct for detector gain changes overtime by selecting the appropriate channels), and all available energy channels (here, the maximum energy can vary between epochs). To increase signal to noise, we average the 16 s power spectra within an observation. We fitted the power spectra above 400 Hz with a constant (to account for the Poisson noise level, fixed to zero as we subtracted it in previous steps), plus two Lorentzians (all Lorentzian parameters are allowed to vary): the lower and upper twin kHz QPOs (designated Lq and Uq, respectively). We find that this frequency selection does not affect the frequency or the detection significance of the kHz QPOs compared to fitting the entire frequency range with a more complex model. Also, alternative handling of the Poisson noise level by using a floating constant did not affect the frequency or detection significance of the kHz QPOs. In Fig. 1, we plot the detection significance of Lq, approximated by the ratio of the best-fitting
integrated power in the power spectral component to its negative 1σ error, for different energy ranges. We find that, overall, a 2–18 keV energy selection optimizes the detection significance of \( L_u \) in the 1000 < \( \nu_u \) < 1200 Hz frequency range.

We apply the 2–18 keV energy selection to construct power spectra for the observations in which the \( \nu_u > 1200 \) Hz QPOs were reported. A further cut in energy suggested by the slightly lower hard colour of these observations does not improve the detection significance further.

### 2.3 Data selection

We plot hard colour versus intensity, per observation, for 4U 1728–34 and 4U 0614+09 in Figs 2 and 3. We indicate the observations for which twin kHz QPOs and single kHz QPOs were reported with \( \nu_u > 1200 \) Hz in van Doesburgh & van der Klis (2017). The observations with \( \nu_u > 1200 \) Hz (pink) all have single kHz QPOs and low hard colour. The frequency of the upper kHz QPO when it appears in a pair of twin QPOs evolves smoothly to the frequencies of the QPO detected as a single peak towards lower hard colour. We therefore identify it as the upper kHz QPO.

In an attempt to detect other high-frequency kHz QPOs, we fitted observations, both single and averaged, with high-frequency QPOs. For 4U 0614+09, we focused on observations with hard colour <0.5 or intensity >0.08 Crab and for 4U 1728–34 on observations with hard colour <0.76. We detect a > 3σ 1269 ± 12 Hz QPO in 4U 0614+09 (in 96404-01-10-01) not reported in previous works. No other kHz QPOs were found.

### 3 RESULTS

In Table 2, we list the best-fitting parameters of fits to power spectra obtained when including all energy channels and fitting all frequencies (the method we used in van Doesburgh & van der Klis 2017) as well as those to power spectra obtained in the 2–18keV energy range and only fitting frequencies >400 Hz. These results confirm the expectation based on the <1200 Hz QPO fits that the signal to noise generally improves when selecting the 2–18keV energy band (this is the case in 13 of 17 cases). None of the three fit parameters (centroid frequency, FWHM, and integral power) significantly change for the repeated fits reported in the table. We detect single upper kHz QPOs (\( L_u \)) in 15 of 16 cases. Next, we discuss Table 2 in further detail.
3.1 4U 0614+09

The highest frequency QPO detected at better than 3\(\sigma\) significance, a > 5\(\sigma\) QPO in 30056-01-03-03, has \(v_Q = 1288 \pm 8\) Hz. This is within 1\(\sigma\) of the 1273 \pm 15 Hz QPO reported in van Straaten et al. (2000), which was measured in an average of 30056-01-03-03 with two other observations close in time.

A number of other observations merit mentioning. In 40030-01-04-00, the observation previously suspected to feature the highest frequency QPO, we detect a 1311_{-29}^{+34} Hz QPO (2.5\(\sigma\)) with \(Q \sim 9\). This is a broader QPO than the \(Q \sim 29\), 1328 \pm 27 Hz one (\(\sim 2.6\sigma\)) reported by Boutelier et al. (2009). We do not reproduce the \(Q \sim 50\), 1329 \pm 4 Hz (3.5\(\sigma\)) QPO reported by van Straaten et al. (2000).

In van Doesburgh & van der Klis (2017), we reported a 1369 \pm 51 Hz QPO for 40030-01-06-00. In our current analysis, we detect a 4.5\(\sigma\) QPO with a centroid frequency of 1283_{-39}^{+43} Hz. The frequency difference between the two results is not significant (1.4\(\sigma\)). To further improve signal to noise, we average 40030-01-04-00 and 40030-01-06-00, as these observations are close in time, have similar hard and soft colour (to within 10 per cent) and show similar power spectra. We obtain a 1285\pm 31 Hz QPO at 4.9 \(\sigma\). In 40030-01-05-00, twin kHz QPOs are detected with the upper kHz QPO at 1333 \pm 25 Hz (2.3\(\sigma\)). The two QPOs (the lower peak is at 1121 \pm 16 Hz, 3\(\sigma\)) have a peak separation of 212 \pm 40 Hz, which is slightly less than (but still within 2\(\sigma\)) of the 320 \pm 40 Hz seen for \(v_Q < 1170\) Hz (Boutelier et al. 2009), so the two peaks may move together when the frequencies increase, as expected.

3.2 4U 1728–34

For 4U 1728–34, we exceed the previously reported highest frequencies of 1161 \pm 16 Hz (detected in an ensemble of observations with similar energy spectra) and 1276 \pm 59 (detected in a combination of 20083-01-02-000 and 20083-01-02-01, see Table 1). We detect a highest QPO frequency of 1302_{-8}\^{+13}_{-10} Hz (3.3\(\sigma\)) in observation 20083-01-02-000 alone. The best-constrained frequency, however, corresponds not to this 1302_{-8}\^{+13}_{-10} Hz peak but to the 1278_{-14}^{+15} Hz one detected in the combination of observations 50030-03-09-00 and 50030-03-09-01; see Section 3.7.

3.3 4U 1636–53

Because of its high-spin frequency (581 Hz), which makes this source particularly important for constraining the EoS (see Section 4.1), we reanalysed the high- frequency kHz QPO reported for 4U 1636-53 by Altamirano et al. (2008) for the ensemble of 31 observations in ‘Group N’ listed in table 6.1 of that work. We detect a 1255_{-29}^{+34} Hz QPO at 4.4\(\sigma\).

3.4 SAX J1750.8–2900

The highest kHz QPOs in SAX J1750.8–2900, another source with high spin (601 Hz), were reported by Kaaret et al. (2002). Following the analysis in that work, we use all data of observation 60035-01-01-00. We detect a 1250_{-7}^{+13} Hz QPO at 4.4\(\sigma\) that is consistent with the earlier report.

3.5 4U 1246–59

For 4U 1246–59 (spin frequency unknown), Jonker et al. (2007) reported a 1258 \pm 2 Hz QPO using an average of three observations. For the same data, we find a 1261_{-5}^{+9} Hz QPO when using the 2–18keV energy selection. This is consistent with the result reported by Jonker et al. (2007). Because the QPO is so narrow (\(Q > 50\)), a high-bin resolution is required to resolve it. This can be seen in Table 2, where the parameters in the fit with 75 bins per decade are badly constrained. We therefore deviate from our standard of 100 bins per decade for this QPO and use 200 instead.

3.6 4U 1702–43

Recently, Nättilä et al. (2017) obtained radius measurements by modelling the X-ray burst cooling tail spectra of 4U 1702–43. The authors fit atmosphere models directly to the data and find that the radius is constrained to be \(R = 12.4_{-6}^{+4} \pm 1\) at 97.7 per cent confidence. As it is interesting to compare the radius constraint obtained with spectral modelling to the constraint of the maximum QPO frequency on radius, we include 4U 1702–43 in our source selection. van Doesburgh & van der Klis (2017) report a highest kHz QPO of 1198_{-16}^{+59} Hz in a combination of 11 observations when including all energy channels. In our current analysis, we detect a 1213_{-15}^{+16} Hz QPO.

3.7 Lower limits

For the purpose of using the highest frequency QPOs to constrain the EoS, it is useful to obtain the highest observed lower limits on the QPO frequencies. The probability distributions of the QPO frequencies deduced from our multiparameter fits are not generally
Table 2. Fit parameters of the upper kHz QPOs with the highest frequencies. We provide an estimate of the significance from the rms-normalized integral power divided by its negative error (IP/σ~). Errors quoted here use Δχ² = 1. In the last column, we list the 3σ (99.73 per cent) lower limit evaluated using Δχ² = 7.74. Throughout this work, we use the bold-faced parameters. Regular type-faced parameters are given for comparison only.

| Source | ObsID | v₀ (Hz) | FWHM (Hz) | IP (× 10⁻⁴) | σ (IP/σ~) | Log. bin. factor | Energy channel selection | 3σ lower limit (Hz) |
|--------|-------|---------|-----------|-------------|-----------|----------------|-----------------------|------------------|
| 4U 0614+09 | 40429-01-06-00 | 1220±19 | 35±19 | 4.8±1.6 | 3.2 | −100 | All | – |
| | | 1220±13 | 71±9 | 5.7±1.7 | 3.4 | −75 (400 Hz) | 0–43 | – |
| | | 1200±18 | 104±19 | 6.2±1 | 3.3 | −100 (400 Hz) | 0–43 | 818 |
| 30056-01-05-00 | 122±4 | 50±9 | 6.7±1 | 6.9 | −100 | All | – |
| | | 1233±6 | 49±12 | 6.6±1.0 | 7.3 | −75 (400 Hz) | 0–49 | – |
| | | 1221±3 | 45±14 | 6.6±0.9 | 8.0 | −100 (400 Hz) | 0–49 | 1214 |
| 30056-01-03-04 | 1285±27 | 120±9 | 2.8±1.1 | 2.9 | −100 | All | – |
| | | 1285±30 | 159±12 | 2.9±1.2 | 2.9 | −75 (400 Hz) | 0–49 | – |
| | | 1292±28 | 133±10 | 2.8±0.9 | 2.9 | −100 (400 Hz) | 0–49 | 0 |
| 30056-01-03-03 | 1286±28 | 50±15 | 2.6±0.6 | 4.6 | −100 | All | – |
| | | 1284±5 | 44±7 | 3.0±0.8 | 6.8 | −75 (400 Hz) | 0–49 | – |
| | | 1288±8 | 61±16 | 2.9±0.6 | 5.3 | −100 (400 Hz) | 0–49 | 1267 |
| 40030-01-05-00 | 1328±15 | 71±19 | 2.6±1.2 | 2.7 | −100 | All | – |
| | | 1321±13 | 128±19 | 2.8±0.9 | 2.1 | −75 (400 Hz) | 0–49 | – |
| | | 1333±24 | 93±26 | 2.4±1.0 | 2.3 | −100 (400 Hz) | 0–49 | 0 |
| | | 1112±21 | 71±31 | 2.4±1.1 | 2.6 | −100 | All | – |
| | | 1111±11 | 72±39 | 2.4±1.1 | 2.3 | −75 (400 Hz) | 0–49 | – |
| | | 1121±16 | 72±35 | 2.7±0.9 | 3.0 | −100 (400 Hz) | 0–49 | n.a. |
| 40030-01-04-00 | 1318±24 | 108±14 | 1.8±0.7 | 2.1 | −100 | All | – |
| | | 1284±29 | 125±34 | 2.1±0.7 | 2.8 | −75 (400 Hz) | 0–49 | – |
| | | 1311±24 | 150±35 | 2.0±0.8 | 2.5 | −100 (400 Hz) | 0–49 | 917 |
| 40030-01-06-00 | 1369±51 | 321±38 | 6.5±1.9 | 3.9 | −100 | All | – |
| | | 1287±39 | 331±38 | 6.2±1.3 | 4.5 | −75 (400 Hz) | 0–49 | – |
| | | 1283±39 | 318±38 | 6.2±1.4 | 4.5 | −100 (400 Hz) | 0–49 | 1145 |
| 40030-01-04-06-00 | 1310±31 | 208±19 | 3.4±0.9 | 4.2 | −100 | All | – |
| | | 1282±27 | 223±19 | 3.7±0.8 | 4.9 | −75 (400 Hz) | 0–49 | – |
| | | 1285±31 | 249±36 | 3.8±0.8 | 4.9 | −100 (400 Hz) | 0–49 | 1185 |
| 96404-01-10-01 | 1266±10 | 103±20 | 2.5±0.6 | 3.6 | −100 | All | – |
| | | 1258±7 | 126±100 | 2.8±0.8 | 3.4 | −80 (400 Hz) | 0–43 | – |
| | | 1269±8 | 51±39 | 2.6±0.8 | 3.1 | −100 (400 Hz) | 0–43 | 1228 |
| 4U 1728–34 | 50030-03-09-00/01 | 1271±15 | 75±19 | 1.4±0.4 | 3.7 | −100 | All | – |
| | | 1283±18 | 51±48 | 1.1±0.4 | 4.1 | −75 (400 Hz) | 0–43 | – |
| | | 1278±14 | 76±32 | 1.2±0.3 | 3.3 | −100 (400 Hz) | 0–43 | 1221 |
| 20083-01-02-000/001 | 1277±35 | 238±102 | 1.3±0.3 | 4.3 | −100 | All | – |
| | | 1276±32 | 245±48 | 1.4±0.3 | 4.3 | −75 (400 Hz) | 0–49 | – |
| | | 1280±31 | 254±100 | 1.4±0.3 | 4.8 | −100 (400 Hz) | 0–49 | 1156 |
| 20083-01-02-000 | 1293±22 | 127±99 | 1.7±0.7 | 2.9 | −100 | All | – |
| | | 1300±15 | 93±59 | 1.6±0.5 | 3.4 | −75 (400 Hz) | 0–49 | – |
| | | 1302±13 | 69±54 | 1.4±0.4 | 3.3 | −100 (400 Hz) | 0–49 | 1213 |
| 4U 1636–53 | 60032-01-03-01+(30) | 1268±5 | 53±35 | 0.24±0.06 | 4.0 | −100 | All | – |
| | | 1259±9 | 93±120 | 0.36±0.18 | 4.1 | −75 (400 Hz) | 0–43 | – |
| | | 1255±11 | 134±109 | 0.43±0.10 | 4.4 | −100 (400 Hz) | 0–43 | 1119 |
| SAX J1750.8–2900 | 60035-01-01-00 | 1245±4 | 35±35 | 1.55±0.7 | 3.5 | −100 | All | – |
| | | 1250±15 | 63±31 | 1.63±0.58 | 3.6 | −75 (400 Hz) | 0–43 | – |
| | | 1250±9 | 54±31 | 1.68±0.53 | 3.8 | −100 (400 Hz) | 0–43 | 1219 |
| 4U 1246–59 | 90042-02-01/08-09-00 | 1258±3 | 10±10 | 26±3 | 7.7 | −150 | All | – |
| | | 1255±7 | 6±31 | 40±34 | 1.1 | −75 (400 Hz) | 0–43 | – |
| | | 1261±9 | 16±9 | 26±3 | 9.3 | −200 (400 Hz) | 0–43 | 1256 |
strictly Gaussian. In the last column of Table 2, we list the 3σ confidence lower limits on the frequencies obtained by evaluating the frequency at which $\chi^2$ exceeds the best-fitting value by $\Delta \chi^2 = 7.74$, leaving all other fit parameters free. We find that in several cases the $\chi^2$ hypersurface is less curved, and hence the constraint on frequency is weaker than we would have expected from the 1σ-error in the Gaussian approximation. In Fig. 4, we plot the confidence contours for the best-constrained upper kHz QPO frequencies detected in 4U 1728–34, 4U 0614+09, 4U 1636–53, SAX J1750–2900, 4U 1246–59, and 4U 1702–43 listed in Table 2. The kHz QPO that is best constrained to have a high frequency is the 1288 ± 8 Hz one in observation 30056-01-03-03 of 4U 0614+09, which is detected at >5σ. The 3σ lower limit on its centroid frequency is 1267 Hz.

4 DISCUSSION

Our highest 3σ lower limit on a kHz QPO frequency, in 4U 0614+09, is 1267 Hz. For 4U 1728–34, 4U 1636–53, SAX J1750–2900, 4U 1246–59, and 4U 1702–43, these values are 1221, 1119, 1219, 1256, and 1150 Hz, respectively.

4.1 EoS constraints

The limits on mass and radius given in equations (1) and (2) are of first order in $j$. They are accurate for NSs with slow spin and have been recommended for use up to $\nu_{\text{spin}} \approx 400$ Hz (Miller et al. 1998a), although this spin limit depends on the EoS. For faster spinning stars, the dependence of oblateness and internal structure of the NS on its rotation rate are not negligible, and their effect on the interior and exterior space–time must be taken into account. Numerical methods are necessary to obtain accurate limits on mass and radius in these cases, as the test-particle geodesic equations need to be solved in the appropriate space–time for a NS with a particular EoS (Morsink & Stella 1999).

We compute the highest allowed orbital frequency for a star with a given EoS and spin frequency using the RNS code by Stergioulas & Friedman (1995) that follows the method by Cook, Shapiro & Teukolsky (1992). We choose five representative EoS that span a wide range of stiffness but also accommodate the 2.01 ± 0.04 M⊙ mass of millisecond pulsar J0348+0432 (Antoniadis et al. 2013). HLPS1, HLPS2, and HLPS3 are soft, intermediate, and stiff versions of the realistic HLPS EoS (Hebeler et al. 2013) that describe the widest range of stiffness consistent with nuclear experiments. APR (Akmal, Pandharipande & Ravenhall 1998) includes relativistic effects in the three-potential formalism, and L (reviewed in Arnett & Bowers 1977) is based on mean-field theory Pandharipande & Smith (1975). The observation of the binary NS merger (Abbott et al. 2017), most likely rules out EoS L, but it is still useful to include an unrealistically stiff EoS to demonstrate the effect of stiffness on the constraints found in this paper.

Given an EoS and a spin frequency, a range of stellar models can be computed, each with a different mass, radius, and largest orbital frequency. As an example, Fig. 5 shows the largest orbital frequency for stars with different EoS spinning with a frequency of 415 Hz. In this figure, the largest orbital frequency for a particular EoS is represented by a curve with two branches. The low-mass branch for each EoS, labelled ‘NS Radius’, corresponds to stars that are larger than the ISCO radius, so that a test particle orbiting at the star’s surface has the largest possible orbital frequency. The high-mass branch for each EoS labelled ‘ISCO’ corresponds to stars that are smaller than the radius of the ISCO. The maximum orbital frequency for a given EoS and spin corresponds to the frequency at the cusp, where the two branches meet. In Fig. 5, we show a black horizontal line at 1267 Hz, the 3σ lower limit on the frequency of the most constraining QPO, 1267 Hz, for the NS 4U 0614+09 that spins at 415 Hz. If this QPO frequency corresponds to orbital motion, then it is clear that it is possible for all five representative EoS to explain the observed frequency. For a fixed value of spin, Fig. 5 also shows that the ISCO-frequency-branch curves for the different EoS follow approximately the same curve. All of the ISCO frequency curves intersect the 1267 Hz line at a value close to 2.1 M⊙, indicating an EoS-independent maximum possible mass for 4U 0614+09 if the orbital frequency interpretation of the upper frequency QPO is adopted. Since the QPO could correspond to orbital motion at distances farther from the star than the ISCO, smaller masses are allowed. In the case of the two stiffest EoS, the QPO frequency also provides a lower mass limit of 1.3 M⊙ for EoS HLPS3 and 1.8 M⊙ for EoS L. We assume that it is unphysical for NSs with masses smaller than 0.8 solar mass to exist, so there is no lower mass limit on the softer EoS given by this QPO interpretation. The observation of a higher frequency QPO from this NS has the potential to rule out the stiffer EoS in our sample.

Similar upper mass limits on the other NSs can be found by making similar plots of largest orbital frequency as a function of mass for the appropriate spin frequency. The resulting maximum masses are obtained by assuming the 3σ limit on the observed upper frequency QPO, which is orbital in nature. We find a mass constraint for 4U 1702–43 (<2.2 M⊙), SAX J1750.8–2900 (<2.3 M⊙), 4U 1636–53 (<2.4 M⊙), and 4U 1728–34 (<2.1 M⊙) using this interpretation. Since this limit is set by the ISCO radius, it is robust in the sense that its dependence on the EoS is weak.

A stronger assumption is that the highest frequency QPO corresponds to motion of a test particle at the ISCO radius. In the case of 4U 0614+09, the NS mass would be required to be 2.0 ± 0.1 M⊙ with the uncertainty coming from the unknown EoS.

The highest allowed orbital frequencies for the five representative EoS are shown as a function of spin frequency in Fig. 6. The largest 3σ lower limits on the upper kHz QPO frequency are shown in Fig. 6 for the five NSs with known values of spin. As can be seen from the figure, all these EoS allow orbital frequencies consistent

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1A code based on RNS designed to compute the ISCO is available at https://github.com/rns-alberta/isco

Table 2 – continued

| Source    | ObsID          | $\nu_0$ (Hz) | FWHM (Hz) | IP ($\times 10^{-3}$) | $\sigma$ (IP/$\sigma_\nu$) | Log. bin. factor | Energy channel selection | 3σ lower limit (Hz) |
|-----------|----------------|-------------|-----------|-----------------------|---------------------------|-----------------|--------------------------|-------------------|
| 4U 1702–43 | 80033-01-10-00(+11) | 1198±1.6    | 92±1.25   | 2.7±0.7               | 3.8                       | −100            | All                      | 0–43              | 1150               |
|           |                | 1121±0.5    | 154±0.10  | 3.2±0.1               | 3.9                       | −750 (>400 Hz)  | 0–43                     | 150               |

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Figure 4. Confidence contours for the best-fitting ν_u and integral power of the most confidently detected highest kHz QPOs in 4U 0614+09, 4U 1728–34, 4U 1636–53, SAX J1750–2900, 4U 1702–43, and 4U 1246–59. We plot the contours corresponding to $\Delta \chi^2 = 4.00, 6.00, \text{and } 7.74$. The 3σ (99.73 per cent confidence) lower limit corresponds to the lowest frequency covered by the outer contour.

Figure 5. Highest possible circular orbital frequency versus mass, around a NS with $v_{\text{spin}} = 415$ Hz for five EoS (HLPS1, HLPS2, HLPS3, APR, and L). The highest 3σ lower limit (1267 Hz) on the kHz QPO frequency for 4U 0614+09 ($v_{\text{spin}} = 415$ Hz) is plotted in black.

Figure 6. Highest possible circular orbital frequency versus spin frequency for stable NSs of any mass, for five EoS (HLPS1, HLPS2, HLPS3, APR, and L, see text). The highest 3σ lower limits on the kHz QPO frequency for 4U 1728–34 ($v_{\text{spin}} = 363$ Hz), 4U 0614+09 ($v_{\text{spin}} = 415$ Hz), 4U 1636–53 ($v_{\text{spin}} = 581$ Hz), SAX J1750.8–2900 ($v_{\text{spin}} = 601$ Hz), and 4U 1702–43 ($v_{\text{spin}} = 329$ Hz) are overplotted. Orbital motion is a viable explanation for the kHz QPO for each of these EoS.

with the 3σ lower limits on kHz QPO centroid frequencies that we have obtained, which supports the hypothesis that kHz QPO frequencies are orbital and constrained by NS and ISCO radii.

In Fig. 7, we plot the highest possible orbital frequency versus radius for these EoS for an assumed $v_{\text{spin}}$ of 329 Hz, the spin frequency of 4U 1702–34.

The lower limit to the highest observed frequency QPO, 1150 Hz, is plotted as a black horizontal line. If this QPO represents orbital motion, then for a given EoS the allowed properties of 4U 1702–34 must be given by a value that lies above the horizontal line on the appropriate curve. This restricts the possible range of radii for any particular EoS for this star to fairly narrow ranges ($\pm 0.1$–$\pm 0.7$ km depending on the EoS). A radius constraint of $12.4^{+0.6}_{-2.6}$ (3σ) was recently obtained by Nättilä et al. (2017). This range of radii would only allow the softer EoS: APR, HLPS1, and HLPS2. For these EoS, if the kHz QPO is generated at the ISCO and under the reasonable assumption that NS masses are >0.8 $M_\odot$, the NS mass in 4U 0614+09 would be 2.0 ± 0.1. However, we caution that the constraints found by Nättilä et al. (2017) assumes spherical, non-rotating stars, so it is not clear how to compare their radius constraint with our graph.

We also checked if our kHz QPO frequencies, if orbital, provide further constraints on the specific moment of inertia, a quantity that is important, for instance, in the relativistic precession model (Stella & Vietri 1998). We estimated $I_{45} m^{-1}$ (where $M = m \cdot M_\odot$ and $I = 10^{45} I_{45}$ g cm$^2$ are the NS mass and moment of inertia,
the predicted possible circular orbital frequency around a NS calculated in full GR taking into account the effect of their known spin on stellar structure and space–time for five relevant EoS. In the most constraining case of the 1267 Hz 3σ lower limit, we find an upper limit on the NS mass of $M < 2.1 \, M_\odot$. For 4U 1702–43, we find that soft to intermediate-stiff EoS (HLPS1-2, APR) is able to explain the recent radius constraint obtained by modelling the spectra of X-ray burst cooling tails (Näätäla et al. 2017). In combination with the highest 3σ lower limit on the kHz QPO in that source, the radius is constrained to a narrow range different for each EoS. Assuming the kHz QPO is generated at or just outside the ISCO, the mass of the NS can be obtained, which for 4U 0614+09 is $2.0 \pm 0.1 \, M_\odot$. We conclude that orbital motion around the NS constrained by stellar and ISCO radii is a viable explanation for the kHz QPO for each of the EoS included in this work.

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2For the 1219 Hz lower limit in SAX J1750.8–2900 ($v_{\text{spin}} = 601$ Hz) we find ranges for L (1.9–2.0), for HLPS3 (1.4–1.9), for HLPS2 (1.0–1.4), for HLPS1 (0.46–2), and for APR (0.8–1.1).
