Constraints on extra dimensions from atomic spectroscopy

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Abstract

We consider a hydrogen atom confined in a thick brane embedded in a higher-dimensional space. Due to effects of the extra dimensions, the gravitational potential is amplified in distances smaller than the size of the supplementary space, in comparison with the Newtonian potential. Studying the influence of the gravitational interaction modified by the extra dimensions on the energy levels of the hydrogen atom, we find independent constraints for the higher-dimensional Planck mass in terms of the thickness of the brane by using accurate measurements of atomic transition frequencies. The constraints are very stringent for narrow branes.

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I. INTRODUCTION

Since the beginning of the past century, theories of higher-dimensional spacetime have attracted attention of physicists mainly motivated by the Kaluza-Klein theory, which postulated the existence of a compact extra dimension with the size of the Planck length in order to formulate a unification theory of electromagnetism and gravitation. More recently, braneworld models, originally conceived as a framework to solve the hierarchy problem, have vigorously renewed the interest in this subject [1–4]. In the braneworld scenario, our observable universe is a submanifold isometrically embedded in a higher-dimensional space and, unlike the Kaluza-Klein model, the size of the extra dimensions can be much larger than Planck length or even infinite [4] without introducing any irremediable conflict with experimental data.

The main feature of the braneworld scenario is that the standard model fields are confined in a submanifold (3-brane), if they are not excited above a certain energy scale, which should be greater than TeV order [5]. On the other hand gravity has access to extra dimensions. Despite this freedom, due to the existence of a bound zero-mode, the gravitational force recovers its four-dimensional behavior for distances much larger than the characteristic length scale, $\ell$, of the supplementary space. In models with compact dimensions, $\ell$ corresponds to the size of the extra dimension, while in models of non-compactified extra dimension, $\ell$ is related to the curvature radius of the ambient space. Below this length scale, the influence of extra dimensions on the gravitational force become significant. It happens that $\ell$ can be much greater than the scale, $L_m$, in which the confined fields feel the effects of the extra dimension directly. In the original ADD model, $L_m < 10^{-19} \text{ m}$ [1], while, for branes inspired in string theories, $L_m$ is much smaller [2].

In sight of this, the search for traces of extra dimensions in the gravitational interaction arises as a paramount issue which can be empirically investigated. Direct tests of the gravitational force in microscopic domains, based on torsion balance experiments, found no deviation from the inverse square law, constraining, in this way, the radius $R (= \ell/2\pi)$ of the extra dimension. Considering the original ADD model [1], it is found that $R < 44 \mu\text{m}$, which is the tightest bound for models with only one extra dimension [7–11]. For greater codimensions, the most stringent constraints come from astrophysics [12, 13] and high-energy particle collisions [5, 6].
Here we study the effects of the extra dimensions in a bound system, in order to find independent constraints for the number and the size of the supplementary space. It is reasonable to expect that the extra dimensions modify significantly the behavior of gravity in distances shorter than $\ell$. If the gravitational field in the weak-field limit obeys Gauss law, then the gravitational potential of a point-like mass $m$ has the following behavior:

$$V(r) = \begin{cases} -\frac{Gm}{r}, & \text{for } r \gg \ell \\ -\frac{Gm}{r^{n+1}}, & \text{for } r \ll \ell \end{cases} \tag{1}$$

where $G_n$ is the gravitational constant in the higher-dimensional space with $n$ extra dimensions. The exact relation between $G_n$ and the Newtonian constant $G$ depends on the extra-dimensional volume, which, on its turn, depends on the topology and curvature of the supplementary space. However, in magnitude order, it is expected that $G_n \sim G\ell^n$.

In atomic systems the usual gravitational interaction is negligible. The average potential gravitational energy of a hydrogen atom in the ground state is of the order of $Gm_h m_e / a_0 \sim 10^{-38}$ eV, where $a_0$ is the Bohr radius. However, due to effects of the extra dimensions, the potential energy increases by a factor of the order of $(\ell/r)^n$ for short distances ($r < \ell$). Thus, depending on the values of $\ell$ and $n$, the gravitational energy could in principle modify the energy spectrum of the hydrogen significantly. Therefore, the highly accurate measurements of atomic transitions can be used to put empirical constraints on models of extra dimensions.

Treating the gravitational potential as a perturbation of the Hamiltonian of the hydrogen atom, we find that the energy shift caused by extra dimensions in a certain quantum state $\psi$ is proportional to $\langle r^{-(n+1)} \rangle$, i.e., the average of $r^{-(n+1)}$ in that state. It happens that, for $n \geq 2$, this average diverges for $S$-levels. This problem is connected with the fact that the short-distance behavior of the brane-to-brane graviton propagator is not computable in the original ADD model, even in the tree level, suggesting that it may depend on the ultraviolet details of a fundamental quantum theory of gravitation, as it was pointed out in Ref [6], where the effects of extra dimensions on high-energy particle collision via the gravitational interaction were investigated.

The first attempts to constrain extra-dimensional parameters by using atomic spectroscopy have tried to circumvent the divergence problem by establishing a cut-off radius $[14-18]$. However, proceeding in that way, the constraints become dependent on this arbitrary parameter. Another way to avoid the divergence problem is by considering the effects
of the modified potential on molecular spectroscopy instead of atomic spectroscopy \[19\]. However, this approach gives much weaker constraints.

In the classical level, the divergence problem arises because the matter distribution confined in the brane is singular from the perspective of the ambient space. It happens that such delta-like confinement is just an idealization that hides the internal structure of the brane. The relevance of the brane substructure to regulate divergences of a thin brane effective theory has been observed and studied before \[20, 21\]. In Ref \[20\], for instance, these divergences are treated by introducing a renormalization prescription. Here we follow a different procedure which relies on the fact that, in the thick brane scenario, the wave function of localized particles has a certain width \(\sigma\) in the extra-dimensional space. As we shall see, when the width is taken into account, the average \(\langle r^{-(n+1)} \rangle\) is finite and depends on \(\sigma\). As the value of width is bounded by the thickness of the brane, then our analysis provides a joint constraint of the higher-dimensional Planck mass and brane thickness.

II. AN ATOM IN A THICK BRANE

In the context of string theories, the brane may be considered as infinitely thin. However, in the framework of the field theories, it may have a thickness. For instance, if we consider an ambient space of (4+1)-dimensions, a 3-brane may be represented as a domain wall solution of some scalar field \[22\]. A Yukawa-type interaction between Dirac spinor fields and the scalar field can localize matter in the core of the domain wall. Due to this interaction, the state of zero-mode of the spinor field is given by the following wave function:

\[
\Psi (\mathbf{x}, z) = \exp \left[-h \int_0^z \phi_0 (y) \, dy\right] \psi (\mathbf{x}),
\]

where \(h\) is the coupling constant, \(\psi (\mathbf{x})\) represents a free spinor in the (3 + 1)-dimension, \(\phi_0 = \eta \tanh (z/\varepsilon)\) is the scalar in a domain wall configuration interpolating between two vacua \(\pm \eta\) of the scalar field. With respect to the transversal direction \(z\), the wave function has a peak at \(z = 0\) and exponentially falls as we move away from the center of the domain wall. The quantity \(\varepsilon\) defines the thickness of the brane and must be smaller than \(10^{-19}\) m, according to current experimental constraints \[1, 5\].

Following a similar reasoning, it is possible to devise a confinement mechanism for matter in topological defects of greater codimensions. By generalizing the above result for a space
with $n$ extra dimensions, we can consider that, from the phenomenological point of view, the wave function of localized particles can be written as $\Psi (x, z) = \chi (z) \psi (x)$. For the sake of simplicity, we will assume that the extra-dimensional part $\chi$ is a Gaussian function around the center of the brane with a standard deviation $\sigma$:

$$\chi (z) = \left( \frac{2}{\pi \sigma^2} \right)^{n/4} \exp \left( -\frac{z^2}{\sigma^2} \right).$$

(3)

Of course the width of the wave function in the transverse directions should satisfy $\sigma < \varepsilon < \ell$. At this point it is important to emphasize that, although we have chosen a Gaussian profile in the transverse direction, $\chi$ can be any normalizable function of $z$, as we shall see later.

Now we can describe the gravitational interaction between the proton and the electron in this scenario. In the linear regime of gravity, there exists a coordinate system (a gauge) in which the gravitational potential satisfies the Laplace equation in vacuum [23]. Thus, in the first order of $G_n$, the exact potential produce by a point-like mass in a higher-dimensional space can be obtained, at least formally, for any compact topology [24]. For instance, if the supplementary space has a topology of a $n-$dimensional torus with size $\ell$, then the gravitational potential of a mass $m$ at the point $R = (x, z)$ of the ambient space is given by

$$V (R) = -\frac{G_n m}{R^{n+1}} - \sum_i \frac{G_n m}{|R - R'_i|^{n+1}},$$

(4)

where $R'_i = (0, 0, 0, k_1, \ldots, k_n)$ and each $k_i$ is an integer number. The potential (4) is a solution of the $(3 + n)$-dimensional Laplace equation with the appropriate boundary conditions. Indeed, the form of vector $R_i$ ensures that $V (R)$ is periodic with respect to the extra-dimensional directions as required by the topology of the supplementary space. It is interesting to note that, from the perspective of the space $\mathbb{R}^{3+n}$, $V (R)$ can be viewed as a superposition of potentials generated by the real mass $m$ at the origin and by copies (mirrors images) of the mass localized by the vectors $R'_i$.

In the far zone, i.e., for $|x| >> \ell$, (4) reproduces the Newtonian potential. Moreover the corrections due to the extra dimensions can be written as $1/r \left( 1 + \alpha e^{-r/\lambda} \right) [24]$. This Yukawa-law form is vastly employed to constrain parameters of higher-dimensional models, by using data from the torsion balance experiments [11]. On the other hand, in a bound system, such as the atom, the major contribution comes from the first term of (4), at least if the atom is at the lowest states. The first corrections come from the mirror images closest to $m$. The potential of each one of these $2n$ first neighbors is less than $G_n m/\ell^{n+1} \sim Gm/\ell$, ...
which is negligible in comparison to the first term, as we can verify by calculating this term with the extra-dimensional estimated size, $\ell$, that we obtain in Figure 2. For a detailed estimate of the contribution of the mirror images see, for instance, the appendix of Ref. [25].

Thus, for our purpose, we can approximate the gravitational potential of a point-like source by the function $-G n m / R^{n+1}$, which is proportional to the Green function of the Laplace operator in the flat higher-dimensional space $\mathbb{R}^{n+3}$. Therefore, in this approximation, the potential produced by the proton, assuming that its mass $m_p$ is distributed on the spatial extension of the nucleus, is given by

$$\phi (R) = -G n \int \cdots \int \frac{\rho (R')}{|R - R'|^{n+1}} d^{3+n}R', \quad (5)$$

where the mass density is $\rho = |\Psi_p|^{2} m_p$ and $\Psi_p (x, z) = \chi (z) \psi_p (x)$ is the higher-dimensional wave function of the proton. We assume that the mass is uniformly distributed inside the nucleus, so the 3-dimensional part $\psi_p (x)$ is constant in the spatial extension of the nucleus and zero outside.

In the hydrogen the average kinetic energy of the electron is a small fraction of its rest energy. Thus, in a first approach, we are going to consider the non-relativistic picture of the atomic system. In the classical framework, the gravitational interaction between electron and proton is governed by the Hamiltonian $H_G = m_e \phi$. Now assuming that $H_G$ is a small term of the hydrogen Hamiltonian, it follows, from the perturbation method, that the energy shift corresponding to a certain atomic state, in the first order, is given by

$$\delta E_{\psi} = \int \cdots \int |\Psi_e|^{2} m_e \phi (R) d^{3+n}R, \quad (6)$$

where $\Psi_e (x, z)$ is the higher-dimensional wave function of the electron (more precisely, the reduced particle) which comprises the extra-dimensional part $\chi (z)$ and the usual solutions $\psi_e (x)$ of the Schrödinger equation for the hydrogen atom.

Therefore, the extra dimensions modify the energy spectrum of the hydrogen by means of the gravitational interaction between electron and proton. Considering the wave functions of $1S$ and $2S$ states ($\psi_{1S} = 1/\sqrt{\pi a_0^3} \exp (-r/a_0)$ and $\psi_{2S} = 1/\sqrt{8\pi a_0^5} (1 - r/2a_0) \exp (-r/2a_0)$, respectively), the shift in the frequency of the $2S - 1S$ transition, $\Delta \nu_G = |\delta E_{2S} - \delta E_{1S}| / h$, can be calculated from (6) and compared with the experimental value.
III. RESULTS AND DISCUSSION

The empirical value of the $2S - 1S$ transition frequency in the hydrogen atom is $f_{\text{exp}} = 2466061413187035 \text{ Hz}$ with an experimental error $\delta_{\text{exp}} = 10 \text{ Hz}$ \cite{26}. The theoretical prediction (based on well-established four-dimensional physics) agrees with the experimental value up to the order of the theoretical error ($\delta_{\text{th}}$), which corresponds to 32 kHz and is related to uncertainties in measurements of the proton radius \cite{27}. Thus, to be consistent, any effect of extra dimensions should be less than the combined errors. The constraints on the parameters of the extra dimensional theory are obtained by imposing that $\Delta \nu_{G} < \sqrt{\delta_{\text{th}}^2 + \delta_{\text{exp}}^2}$. The numerical analysis of this condition is summarized in Figure 1. It establishes a lower bound for the fundamental Planck mass of the higher dimensional space $M_D$ in terms of $\sigma$.

According to Refs. \cite{6, 23}, the fundamental Planck mass is related to $G_n$ by the formula $M_D^{2+n} = \Omega_n \left( \frac{\hbar}{c} \right)^n \hbar c/G_n$, where $\Omega_n = (2\pi)^n \Gamma \left( \frac{n+3}{2} \right) / [(n+2)2n^{(n+3)/2}]$. In Figure 1, the regions below the curves are excluded. It is worth mentioning that the width of the particle wave function in the transverse directions, $\sigma$, is smaller than the thickness of the brane ($\varepsilon$). Therefore, the analysis shows that the constraints are tighter for thinner branes.

The cases $n = 1$ and $n = 2$ are not shown because the corresponding constraints are very weak. For $n \geq 3$, the major contribution of the gravitational energy comes from the integral in the interior of the nucleus, when we consider realistic branes with $\varepsilon < 10^{-19} \text{ m}$. The leading term is of the order of $G_n m_p m_e / a_0^3 \sigma^{n-2}$. This dependence on $\sigma$ explains why narrow branes provide very stringent limits on the fundamental Planck mass.

![Figure 1. Lower bound for the higher-dimensional Planck mass $M_D$ (expressed in natural units)](image)
in terms of $\sigma$ (the wave function width in the transverse directions). The areas below the lines are excluded.

In the original ADD model, the supplementary space is a flat $n$-torus with radius $R$. The relation between $R$ and the higher-dimensional Planck mass $M_D$ (given by $G^{-1} = 8\pi R^n M_{D}^{2+n}$, where $M_{D}^{2+n} = M_{D}^{2+n}/[\hbar/c]^n \hbar c$, see [6]) can be used to constraint the radius of the extra dimension. Figure 2 shows upper limits on the extra dimension radius for $n = 3, 4, 5$ and 6. The regions above the curves are excluded.

![Figure 2. Upper limit for the radius of the extra dimensions in terms of $\sigma$ for $n=3, 4, 5$ and 6. The regions above the lines are excluded.](image)

As mentioned before, we have admitted that the wave function in the supplementary space has a Gaussian profile (3) for the sake of simplicity. Nevertheless, it is important to emphasize that this assumption does not play an essential role in our analysis. Actually, our results do not change, even if we consider other profiles, provided that the parameter $\sigma$ is defined by:

$$\frac{1}{\sigma^m} = \frac{\Gamma(n/2)}{\Gamma((n-m)/2)} \int \left| \chi_p(z_1) \right|^2 \left| \chi_e(z_2) \right|^2 \frac{d^m z_1 d^n z_2}{|z_1 - z_2|^m},$$

where $m$ is a positive integer that should satisfy the condition $m \leq (n - 1)$ and $\Gamma$ stands for gamma function. When the profile is Gaussian, this parameter $\sigma$ corresponds to the width of the Gaussian function.

At this point, we should stress that the above constraints were derived based on the classical regime of the gravitational interaction. Nevertheless, as it was emphasized in Ref. [6], quantum-gravity effects may become relevant in a length scale of the order of $l_D$ (the higher
dimensional Planck length, given by $l_D = (\hbar/c) M_D^{-1}$ or even in a greater scale, depending on the quantum-gravity theory, not yet known. If this is the case, then unexpected effects may emerge and could distort or even overshadow the classical results we have investigated here.

In the lack of a quantum-gravity theory, let us explore some possibilities. According to [28], quantum corrections to the gravitational potential energy may be estimated by treating General Relativity theory as an effective theory. The classical potential energy, in three-dimensional space, is given by the Newtonian term $GMm/d$, where $d$ is the distance between particles with mass $M$ and $m$. The quantum contributions are smaller by a factor of the order of $(l_p/d)^2$, where $l_p$ is the Planck length of the ordinary three-dimensional space [28]. In the higher-dimensional case, the classical potential is $G_n M m / d^{n+1}$ and the quantum corrections would be of the order of $(l_D/d)^{n+2}$, based on dimensional analysis.

Now, considering the hydrogen atom, it is instructive to define an effective extra-dimensional distance, $d_{\text{eff}}$, between proton and electron, though they cannot be considered as point-like particles in this bound system. By writing the atomic gravitational energy as $G_n m_p m_e / d_{\text{eff}}^{n+1}$, it follows that $d_{\text{eff}}^{n+1} \sim \sigma^{n-2} a_0^3$ in the ground state. Notice that $d_{\text{eff}}$ is a kind of geometric average of the wave packet widths in the transversal and parallel direction of the brane. Now, from the lower constraints on $M_D$, we can estimate upper bounds for $l_D$ as a function of $\sigma$. By a direct calculation, we can verify that the ratio $(l_D/d_{\text{eff}})^{n+2}$ depends on $n$ and $\sigma$, but, for any dimension and for any value of $\sigma$ investigated here, it is smaller than $10^{-8}$. Thus, if $d_{\text{eff}}$ is the relevant characteristic length scale that regulates the gravity behavior in this system, then we may expect that the classical contribution will be the leading gravitational influence in this context. However, as the fundamental quantum-gravity theory is not known, only experiments can decide on this question.

IV. COMPARISON WITH OTHER CONSTRAINTS

Experimental bounds for the fundamental Planck mass are determined from many areas of Physics. As we have mentioned before, when the codimension is greater than 2, some of the most stringent constraints of $M_D$ are established by high-energy particles collisions. Data analysis from monojet events in proton-proton collisions at the LHC gives the following lower bounds for $M_D$ in TeV/c²: $4.38$ ($n = 3$), $3.86$ ($n = 4$), $3.55$ ($n = 5$) and $3.26$
Examining Figure 1, we may say that, in this thick brane scenario, unless unpredictable quantum-gravity effects suppress the classical result, spectroscopy could give stronger constraints in the case the confinement parameter, \( \sigma \), is small.

However, as \( \sigma \) has an unknown value, it is interesting to investigate the inverse problem, i.e., to estimate the maximum influence of the gravitational energy on the hydrogen spectrum, considering the current constraints on \( M_D \) given by the LHC. In Figure 3, we show the gravitational contribution to the hydrogen \((2S - 1S)\) transition, \( \Delta E_G \), as a function of the confinement parameter, \( \sigma \).

Figure 3. The energy gap between the \( 2S - 1S \) states in the hydrogen atom due to higher-dimensional gravitational interaction, assuming LHC constraints for \( M_D \). The horizontal lines, labeled with \( \delta E_{\text{exp}} \) and \( \delta E_{\text{th}} \), are the current experimental precision and theoretical uncertainty, respectively, in the \( 2S - 1S \) transition.

Here the gravitational energy \( \Delta E_G \) is confronted with the current theoretical uncertainty \( \delta E_{\text{th}} \) and current experimental error \( \delta E_{\text{exp}} \). For each value of \( n \) considered, there is an interval in the \( \sigma \)-axis in which \( \Delta E_G \) is less than the theoretical uncertainty. However, even in this range, extra dimensions induce a huge amplification of the atomic gravitational energy in comparison with the ordinary three-dimensional case, which is of the order of \( 10^{-38} \) eV. Within these specific ranges of \( \sigma \), Figure 3 indicates how much the experimental precision should be improved in order to the hydrogen spectroscopy become capable of revealing traces of the supposed extra dimensions. For shorter \( \sigma \), the limit on \( \Delta E_G \) is still given by \( \delta E_{\text{th}} \). Once again, it is important to emphasize that this prediction is valid only if no quantum-gravity effects suppress the classical result. But, notice that, if quantum-gravity effects
reinforce the classical contribution, then signals of extra dimensions, though unpredictable, would be viewed earlier.

New forms of constraints can be obtained by considering relations between $\sigma$ and $l_D$ in advance. Let us admit that $l_D$ is smaller than $\sigma$ by a factor $x$, i.e., $l_D = x\sigma$, with $x < 1$. In this case, the condition that the energy shift between $2S$ and $1S$ states, due to the gravitational interaction, be smaller than the uncertainty $\delta E$ yields the following constraint:

$$c^2 M_D > \left[ \alpha_n \left( \frac{\hbar c}{a_0} \right)^3 \frac{m_p c^2 m_e c^2}{\delta E} \right]^{1/4} x^{(n-2)/4},$$  

(8)

where $\alpha_n = 7\gamma_n\Omega_n/8\pi$ and coefficients $\gamma_n$ have the following values: $\gamma_3 = 2\pi^{3/2}$, $\gamma_4 = 4\pi/3$, $\gamma_5 = \pi^{3/2}/3$ and $\gamma_6 = 4\pi/15$. Thus, for instance, if $\sigma = 10l_D$, then the lower bounds for $M_D$, assuming $\delta E$ of about $10^{-10}$ eV, would be of GeV order, therefore, $10^3$ weaker than the constraints from LHC. Equation (8) also shows a weak sensitivity of $M_D$ with respect to the experimental precision. Indeed, if $\delta E$ was reduced by a factor of $10^4$, then, $M_D$ would be improved 10 times, only. It is also interesting to compare these constraints with indirect limits obtained in lepton colliders, which seem to have a similar origin. Ref. [6] determined the maximum $M_D$ sensitivity which can be reached by studying the final state with photons and missing energy at an electron-positron collider. Considering $\sqrt{s} = 1$ TeV and integrated luminosity $L = 200$ fb$^{-1}$, the predicted bounds for $M_Dc^2$ in TeV are: 4.0 ($n = 3$), 3.0 ($n = 4$), 2.4 ($n = 5$), when the beam polarization is 90%. These limits are much greater than those from equation (8).

Another way to find new constraints is, instead of fixing a direct relation between $l_D$ and $\sigma$, to require that the proton energy density in the extra dimensional space, $\rho$, be smaller than the critical density, $\rho_D = M_Dc^2/l_D^{n+3}$, by some factor $y$. Admitting a Gaussian profile in the transverse direction, we can estimate the proton density, $\sim m_p c^2/\sigma^3 R^3_p$, in the center of the thick brane. From the condition $\rho/\rho_D = y$, we can write $\sigma$ in terms of $y$. By using this relation, we find constraints for the fundamental Planck mass in terms of $y$:

$$c^2 M_D > \beta_n \left( \frac{m_p c^2}{R^3_p / \hbar^3 c^3} \right)^{1/4} \left( \frac{R^3_p m_e c^2}{a_0^3 \delta E} \right)^{n/8} y^{(n-2)/8},$$  

(9)

where $\beta_n = \alpha_n^{n/8} \left[ (4\pi/3) (\pi/2)^{n/2} \right]^{(n-2)/8}$. Thus, taking $y = 0.1$, to make some estimation, we find $M_D > 1.6$ GeV/c$^2$ ($n = 3$) to $M_D > 76$ GeV/c$^2$ ($n = 6$), considering the current theoretical uncertainty $\delta E_{th}$. In comparison with the previous case, the constraints are much
more sensitive with respect the precision $\delta E$. Indeed, if the condition (9) is calculated with the current experimental error, $\delta E_{\text{exp}}$, the constraints will be at least $10^{3n/8}$ times stronger. Specifically, for $n = 6$, we would have $M_D > 32 \text{ TeV/c}^2$, which is more stringent than the collider constraints.

V. FINAL REMARKS

In the thick brane scenario, it is expected that the matter and the standard model fields do not feel directly the effects of the extra dimension in a length scale less than the thickness of the brane $\varepsilon$, while the modification in the gravitational field may arise in a scale $\ell$ much greater than $\varepsilon$. Taking this into account, we have seen that the corrections of the gravitational potential due to extra dimensions may give significant contribution in a bound system. Thus, by using precision measurements of $2S - 1S$ transition frequency of the hydrogen, we have obtained new constraints for the higher-dimensional Planck mass in terms of a confinement parameter, $\sigma$, of the matter inside the brane, which should be smaller than the brane thickness.

The constraints we find here are stronger for thinner branes. In fact, comparing our results with the current constraints from LHC data, which go from $M_D > 4.38 \text{ TeV/c}^2$ ($n = 3$) to $M_D > 3.26 \text{ TeV/c}^2$ ($n = 6$) [29, 30], we may conclude that, for some narrow branes, atomic spectroscopy could impose very stringent constraints for the fundamental Planck mass in higher-dimensional spaces.

Finally, it is important to emphasize that, as the higher-dimensional Planck mass can be much smaller than the four-dimensional Planck scale, it is expected that quantum-gravity effects may arise much sooner in comparison with the traditional picture without extra dimensions [6]. However, as the fundamental theory is not known, the supposed quantum effects are unpredictable. On the other hand, the constraints from the hydrogen spectroscopy we find here relies on the classical behavior of gravity, thus, it is important to highlight that the present results are validity only if the classical contributions are not suppressed by quantum-gravity effects.
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