Linear and nonlinear two-photon bunching on an atomic beam splitter

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Optical emitters strongly coupled to photons propagating in one-dimensional waveguides are a promising platform for building an optical quantum computer. Here, we present a theoretical study of the scattering of two indistinguishable photons on a single two-level atom in a Hong-Ou-Mandel set-up. By computing the dynamics, we can describe the system at any time of the scattering event. This allows us to highlight the one-to-one correspondence between the saturation of the atom and the effective interaction induced between the photons. Furthermore, we discuss potential applications for on-chip quantum computing.

Introduction.— The beam splitter (BS) is an elementary unit of quantum linear optics [1] and has applications in various fields, such as quantum computation [2, 3] based on the Hong-Ou-Mandel effect [4] or optical network [5]. Recent experimental progress has brought the linear BS beyond the conventional optical realization made of glass, for instance using Landau-Zener transitions for electronic spin states in a double quantum dot [6] or for an artificial atom in a superconducting circuit [7], modulating SQUIDs in a superconducting cavity [8] or even using electromagnetically induced transparency for slow light in an atomic vapor cell [9].

In this Letter, we investigate the BS transformation realized by a two-level system (referred to as an “atom” in the rest of the paper) on photons propagating in a one-dimensional (1D) waveguide, as illustrated in Fig. 1. Strong light-matter interaction makes the 1D waveguide a promising candidate for quantum information processing. Indeed, while photons are the preferred choice for communicating quantum information, their lack of interaction is a drawback for the implementation of two-qubit gates. In view of building an optical quantum computer, one way of introducing an effective interaction between photons at low light power is to use the atom as a nonlinear medium. Indeed, because it cannot absorb or emit more than one photon at a time, a pair of incident photons will not interact with the atom in the same way as a single photon does. To date, several practical devices based on this nonlinearity have been proposed, such as single-photon transistors and routers [15–17], on-chip amplifiers [18] or quantum non-demolition photodetectors [19].

In the present work, we study analytically how the saturation of the atom effectively affects the linear behavior of the atomic BS. Our transparent approach allows to compute the dynamics and thus to intuitively understand our results by monitoring the atomic excitation during the scattering event. This differs from previous models limited to post-scattering descriptions based on a real-space formalism [20–22] or input-output theory [23].

Model.— We consider the system illustrated in Fig. 1b, consisting of a 1D waveguide strongly coupled to an atom with resonance frequency $\omega_A$ between ground state $|g\rangle$ and excited state $|e\rangle$. The dipole Hamiltonian describing the interaction between the atom and the propagating photons, under rotating wave approximation, is given by [24]

$$\hat{H}_{\text{dipole}} = -i\hbar \int_0^\infty d\omega g_\omega \left[ |e\rangle \langle g| (\hat{a}_\omega + \hat{b}_\omega) - \text{H.C.} \right],$$

(1)

where $g_\omega$ is the coupling constant, and $\hat{a}_\omega$ ($\hat{b}_\omega$) is the annihilation operator of the forward- (backward-) propagating photon mode. Any relevant physical quantity is then readily obtained — at any time $t$ — by deriving the Heisenberg equation of the corresponding observable and solving a closed set of first-order differential equations (see Appendix A for further details on the derivations).

In analogy to a Hong-Ou-Mandel experiment with a conventional beam splitter (see Fig. 1a), we study the scattering of two indistinguishable photons propagating in opposite directions (see Fig. 1b). Specifically, the figure of merit is the average coincidence after the scattering event

$$\mathcal{C} \equiv \lim_{t \to \infty} \langle \psi_{in} | \hat{N}_a(t) \hat{N}_b(t) | \psi_{in} \rangle,$$

(2)

with $\hat{N}_a = \int_0^\infty d\omega \hat{a}_\omega \hat{a}_\omega^\dagger$ and $\hat{N}_b = \int_0^\infty d\omega \hat{b}_\omega^\dagger \hat{b}_\omega$ respectively the photon-number operators of forward and backward propagating modes, and $|\psi_{in}\rangle = |1_a, 1_b, g\rangle$ the initial state of the system. The average coincidence $\mathcal{C}$ corresponds to the probability of finding the two photons propagating in opposite directions, i.e. in different output modes, after encountering the atomic BS. Hence, a vanishing coincidence $\mathcal{C} = 0$ corresponds to perfect bunching.
of the photons into the same output modes of the BS, which is a signature of their bosonic nature universally known as a Hong-Ou-Mandel dip [4].

To provide further understanding of our results, we also derive the probability of excitation of the atom as a function of time \( t \)

\[
P_c(t) = \frac{\langle \psi_{in} | \hat{\sigma}_z(t) | \psi_{in} \rangle + 1}{2},
\]

where \( \hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g| \). \( P_c(t) \) varies between 0, corresponding to the atom being in the ground state \( |g\rangle \), and 1 when the atom is in the excited state \( |e\rangle \). Therefore, by tracking the atomic excitation during the scattering event, one can visually identify the correspondence between the saturation of the atom and the nonlinearity induced.

**Linear regime.**— We first investigate the limit where the two photon pulses, centered around frequency \( \omega_0 \), have a bandwidth \( \Omega \) much smaller than the interaction strength \( \gamma = 2\pi N g_0^2 A \). In this regime, widely popular among theoretical studies [20, 23], the input photons can be seen as being effectively monochromatic with frequency \( \omega_0 \). We can then derive the average coincidence after the scattering event for such input photons (see Appendix A)

\[
C^o = 1 - \frac{4(\Delta/\gamma)^2}{[1 + (\Delta/\gamma)^2]^2} = 1 - 4R^o T^o,
\]

with \( \Delta \equiv \omega_0 - \omega_A \) the detuning and \( R^o = 1 - T^o = 1/[1 + (\Delta/\gamma)^2] \) the single-photon reflection coefficient in the monochromatic limit [25]. Remarkably, the average coincidence \( C^o \) observed in the case of two photons is fully determined by the atomic response to individual single photons and follows the behavior of a linear BS (see Fig. 1a) with reflection coefficient \( R^o \). In other words, no interaction between the photons is mediated by the atom.

Fig. 2 shows the average coincidence (4) as a function of the normalized detuning \( \Delta/\gamma \). The single-photon reflection coefficient \( R^o \) is plotted in dashed for reference, and we recognize the well-known destructive interference between the forward-scattered and the incident waves that leads to full reflection at resonance, as observed in recent experiments [11, 17]. As discussed above, perfect bunching of the two photons coincides with a 50-50 BS situation for the single-photon atomic response, that is when \( |\Delta| = \gamma \). On the other hand, the photons always exit the atomic BS in opposite directions when they are at resonance \( \Delta = 0 \). This is expected since the atomic BS acts as a fully reflective mirror at the single-photon level.

The absence of nonlinearity, which might seem initially surprising given the two-level structure of the atom, can be understood by computing the atomic excitation in the monochromatic regime (see Appendix A)

\[
P^o_c(t) \approx 0 \quad \forall t.
\]

This vanishing probability physically means that the atom is mostly in the ground state during the scattering event. This so-called weak-excitation limit arises naturally due to the narrow bandwidth in frequency of the incoming photons \( \Omega \ll \gamma \), which implies a long pulse in the time domain compared to the atomic lifetime. Hence, the atomic response to each photon is not influenced by the other because the atom is never significantly excited.

In order to fully characterize our linear BS, we now focus on the balanced BS situation, \( R^o = T^o = 1/2 \). In particular, one would expect to observe an imperfect Hong-Ou-Mandel dip when the two input photons are not arriving simultaneously at the BS. Indeed, delaying the arrival of one photon with respect to the other reduces their indistinguishability, so that coincidences begin to appear. When the delay is large enough, the separation between the two pulses is sufficiently large to identify each photon. Consequently, they do not in-
terfere anymore and, from the addition of probabilities $R^\alpha R^\alpha + T^\alpha T^\alpha = 1/2$, half the recorded events become coincidences. This is illustrated in Fig. 3. We clearly observe the Hong-Ou-Mandel dip going from 0 coincidence, when the two photons arrive simultaneously at the BS, to 1/2 when the photons do not overlap anymore. The analogy between a linear BS and the atomic BS is thus complete in the monochromatic regime.

We have shown that when the light frequency bandwidth $\Omega$ is negligible compared to the interaction strength $\gamma$, the atomic BS behaves as a linear BS. Specifically, when tuned as a balanced BS, i.e. when $|\Delta| = \gamma$, two indistinguishable photons incoming in opposite directions will bunch into the same output mode. The output state thus reads $|\psi_{\text{out}}\rangle = \frac{1}{\sqrt{2}}(|a_+\rangle + |a_-\rangle) \otimes |g\rangle$, corresponding to a superposition of finding the two photons co-propagating to the left or to the right in Fig 1b. This creation of entanglement between separated photons impinging on a BS [26], also called Hong-Ou-Mandel effect, arises from the interference between probability amplitudes and is a direct manifestation of the bosonic nature of photons [27]. It has been shown that it can be used to probabilistically realize quantum logic operations with only linear quantum optics and postselection on detection events [2, 28]. However, the lack of photon-photon interaction precludes the realization of deterministic quantum computing [29]. To overcome this limitation, one exciting technique under extensive research consists in using the atom as a nonlinear medium in order to effectively implement an interaction between photons [30]. Following our results, it is now clear that this requires pulses of finite bandwidth $\Omega$ non negligible with respect to the interaction strength $\gamma$.

**Nonlinear regime.**— Motivated by these considerations, we now study how the atomic BS responds to square photon pulses with finite bandwidth $\Omega$. Indeed, if we were to find again a linear response, there would be no chance of using the atom as a mediator of photon-photon interaction. In the following, we focus on the resonant situation $\Delta = 0$, for which we derive the following average coincidence (see Appendix A)

$$C_\Omega = 1 - 3\Omega/\gamma \left[ 1 - \Omega/\gamma + e^{-\Omega/\gamma^2} (1 + \Omega/\gamma) \right]$$

$$\neq 1 - 4R^\Omega T^\Omega \quad \forall \Omega > 0,$$

where $R^\Omega = 1 - T^\Omega = 1 + (-1 + e^{-\Omega/\gamma^2})\Omega/\gamma$ is the single-photon reflection coefficient for a square pulse of bandwidth $\Omega$. As illustrated in Fig. 4, the coincidence (6) significantly differs from the linear response of a conventional BS. This is in direct contrast to the monochromatic regime. Furthermore, the deviation seems to be maximal when the bandwidth becomes comparable to the interaction strength $\Omega \approx \gamma$. As a striking example, let us consider a balanced BS at the single-photon level $R^\Omega = T^\Omega \approx 1/2$, which is realized for a normalized bandwidth $\Omega/\gamma \approx 1.25$. While a linear response would entail a HOM effect for indistinguishable photons and thus a vanishing coincidence, the coincidence at the output of the atomic BS (6) is $C_\Omega \approx 0.23$ (see thick line Fig. 4). To further test our analytical model, we have also redrewn recent results obtained by A. Nysteen et al. [31], where the authors study the scattering of Gaussian pulses — as a function of time — by means of numerical simulations. The agreement is remarkable and we observe again that the coincidence does not reach $0$.

To better understand the physical reason underlying the nonlinear response (6), we evaluate the atomic excitation during the scattering event. In particular, we derive the probability of excitation for the time interval corresponding to the duration of the input pulses. Indeed, it is during this period of time that the saturation of the atom by one of the photon will affect the scattering
of the other. We then find (see Appendix A)
\begin{equation}
P_e^\sigma(\tau) = \frac{\Omega}{\gamma} \left[ 1 - 2\Omega/\gamma + \right. \\
e^{-\frac{4\gamma}{\Delta^2}(1 + 8\tau + 10\Omega/\gamma + 2e^{\frac{2\Omega}{\gamma}}(-1 + 8\tau - 4\Omega/\gamma))},
\end{equation}
where \(\tau \in [0,1]\) represents time in units of pulse duration. As shown in Fig. 5, the atom is notably excited during most of the pulse duration when \(\Omega \approx \gamma\). This observation deepens our understanding of the balanced BS situation mentioned in the previous paragraph. Indeed, because of the atomic saturation, the atomic BS does not act as a perfect 50-50 BS at the two-photon level. Specifically, when the atom is being excited via the interaction with the two photons, the remaining part of the pulses have a higher chance of going through without interacting, yielding a larger probability of transmission. It is this contribution that leads to a non-vanishing coincidence, as observed in Fig. 4.

On the other hand, we observe in Fig. 5 that when we move away from the regime \(\Omega \approx \gamma\), the nonlinearity induced by the atom is decreasing. This is in agreement with the monochromatic limit (3) for \(\Omega \ll \gamma\), while in the case \(\Omega \gg \gamma\), most of the pulse is off-resonant, reducing the probability of exciting the atom. The regime \(\Omega \approx \gamma\) thus combines the advantages of pulses concentrated in time — for a higher intensity — and in frequency — for a resonant coupling — leading to significant nonlinearity induced by the atom.

Discussions.— We now discuss some of the potential applications for this atomic BS. As mentioned previously, the nonlinearity induced by the atom allows the realization of novel practical devices [15–19]. Our results thus highlight that such devices based on the interaction between coincident photons may have a significantly increased efficiency when operated with pulses of appropriate bandwidth \(\Omega \approx \gamma\).

For practical purposes, it is usually necessary to have a four-port BS where the input and output are in different ports, as illustrated in Fig. 1a. For the waveguide-atom system this can be done by connecting the waveguide to two circulators [17] as in Fig. 6. The outgoing photons are then rerouted to two different ports. In the linear regime, such an atomic implementation of a BS has advantage over conventional BS because it is easily tunable. Indeed, we see in Eq. (4) that the two photons are totally reflected at resonance \(\Delta = 0\), and totally transmitted when far detuned \(\Delta \gg \gamma\); in both cases there is no Hong-Ou-Mandel effect. By controlling the energy spacing of the two-level system and hence the detuning, it is possible to turn the Hong-Ou-Mandel effect ON or OFF. This can be done by adjusting the gate voltage and biased flux in the case of superconducting qubit [32], or an external field for the case of atom and quantum dot [33]. This tunability of an atomic BS may be useful in many quantum information processing applications that utilize the Hong-Ou-Mandel effect.

To see an example where the tunable atomic BS can be useful, we look at the two-qubit controlled-phase gate proposed in the Knill-Laflame-Milburn scheme for optical quantum computing [2]. The module illustrated in Fig. 6c implements such a gate on two dual-rays photonic qubits, where the 50-50 BS in the original proposal have been replaced by tunable atomic BS. When the atomic BS is in the mode ON the controlled phase-gate is implemented, but when it is in mode OFF there is no Hong-Ou-Mandel effect and one can verify that the output is the same as the input, that is, the module implements the identity operator. Such an ability to turn ON and OFF the controlled-phase gate allows the realization of a configurable integrated optical chip that is capable of running different computational tasks, each of the task usually requiring a different number of controlled-phase gates to be applied. Quantum dots and superconducting qubits are natural choices for the two-level system in an implementation on an integrated chip.

Conclusions.— We have presented a time-dependent study of the scattering of two photons on a quantum emitter. We have shown that in the case of quasi-monochromatic photons, a linear regime naturally arises where the atom behaves as a conventional BS, leading to a Hong-Ou-Mandel effect for the right parameters. We have also discussed a potential application of such a tunable BS, enabling to switch ON and OFF a two-qubit gate in an integrated optical chip. In addition, the nonlinearity induced by the atom has been investigated by monitoring the atomic excitation during the scattering event. The investigation of BS [34], mirrors [35] and interferometers [36] operating in the quantum regime opens the way to new exciting experiments, such as the quantum delayed-choice experiment [37, 38], where controlling devices usually considered classical now behave according to quantum mechanics.
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Appendix A: Derivation of the time-dependent averages

We give here a detailed derivation of the average coincidence (2) and probability of excitation of the atom (3) as given in the Main Text.

To describe the evolution of the system during the scattering event, it is convenient to work in the interaction picture with respect to the free Hamiltonian $\hat{H}_0 = \hbar \omega_A |\epsilon\rangle \langle \epsilon| + \int_0^\infty d\omega \hbar \omega \left( \hat{a}_\omega^\dagger \hat{a}_\omega + \hat{b}_\omega^\dagger \hat{b}_\omega \right)$. The total Hamiltonian then reads

$$\hat{H}_{\text{int}} = -i\hbar \int_0^\infty d\omega \, g_{\omega} \left[ \hat{\sigma}_+ \left( \hat{a}_\omega + \hat{b}_\omega \right) e^{-i(\omega - \omega_A)t} - \text{H.c.} \right],$$

(A1)

where $\hat{\sigma}_+ = |e\rangle \langle g|$ is the atomic raising ladder operator.

In the following we will use the Weisskopf-Wigner approximation [39, p. 207], where the coupling constant is evaluated at the transition frequency $g_\omega = g_{\omega_A}$.

The average coincidence (2) after the scattering event is $C = 1 - P_{aa}(t \to \infty) - P_{bb}(t \to \infty)$, where $P_{jj}$ is the probability of having two photons in mode $j$. We show below how to obtain $P_{aa}$; the probability $P_{bb}$ can be computed in a similar manner. We have $P_{aa} = \langle \psi_{in} | \hat{C}_{aa} | \psi_{in} \rangle$, where

$$\hat{C}_{aa} = \frac{1}{2} \int_0^\infty d\omega \int_0^\infty d\omega' \hat{a}_\omega^\dagger \hat{a}_{\omega'}^\dagger \hat{a}_\omega \hat{a}_{\omega'} = \frac{1}{2}(\hat{N}_a^2 - \hat{N}_a),$$

(A2)

with $\hat{N}_a = \int_0^\infty d\omega \, \hat{a}_\omega^\dagger \hat{a}_\omega$ the photon-number operator in mode $a$. Note that we omit the time dependence of the field and atom operators for clarity. The Heisenberg equation of motion yields the following closed set of first-order differential equations for the operators $\hat{C}_{aa}, \hat{N}_a, \hat{\sigma}_+, \hat{\sigma}_z, \hat{N}_a \hat{\sigma}_+, \hat{N}_a \hat{\sigma}_z$ [24]

$$\dot{\hat{C}}_{aa} = \frac{\gamma}{2} (\hat{N}_a + \hat{N}_a \hat{\sigma}_z + \sqrt{\gamma}(\hat{N}_a \hat{\sigma}_+ \hat{a}_0 + \text{H.c.}),$$

(A3)

where $\gamma = 2\pi g_{\omega_A}^2$ and $\hat{a}_0 = \frac{1}{\sqrt{2\pi}} \int_0^\infty d\omega \, \hat{a}_\omega (t_0) e^{-i(\omega - \omega_A)t}$.

To find the expectation value $\langle \psi_{in} | \hat{C}_{aa} | \psi_{in} \rangle$ we need to know the action of the free-pulse operator $\hat{a}_0$ on the state of the system $|\psi_{in}\rangle$ at the initial time $t_0$. For this, we first express the latter in terms of creation operators as

$$|\psi_{in}\rangle = \langle 1_a, 1_b, g | a_0, 1_b, g \rangle = \int_0^\infty d\omega \, f_a(\omega) \hat{a}_\omega^\dagger \int_0^\infty d\omega' \, f_b(\omega') \hat{b}_\omega^\dagger |\varnothing\rangle,$$

(A4)

where $|\varnothing\rangle \equiv |0_a, 0_b, g\rangle$ corresponds to the forward and backward propagating modes being in vacuum state while the atom is in the ground state and $f_a(\omega)$ ($f_b(\omega')$) is the shape of the photon pulse incoming in mode $a$ ($b$). Specifically, we have $f_a(\omega) = f_b(\omega)$ when the two photons are indistinguishable. We then obtain

$$\hat{a}_0 |\psi_{in}\rangle = e^{-i\Delta t} \xi_a(t) |a_0, 1_b, g\rangle,$$

(A5)

where $\xi_a(t) \equiv 1/\sqrt{2\pi} \int_0^\infty d\omega \, f_a(\omega) e^{-i(\omega - \omega_0)t}$ with $\omega_0$ the central frequency of the pulse, and $\Delta = \omega_0 - \omega_A$. The free-pulse operator $\hat{a}_0$ thus decreases the number of photons when applied on $|\psi_{in}\rangle$.

Using the first equation in Eq. (A3), we can now derive a differential equation for $\langle \psi_{in} | \hat{C}_{aa} | \psi_{in} \rangle$

$$\frac{d}{dt} \langle \psi_{in} | \hat{C}_{aa} | \psi_{in} \rangle = \frac{\gamma}{2} \left( \langle \psi_{in} | \hat{N}_a | \psi_{in} \rangle + \langle \psi_{in} | \hat{N}_a \hat{\sigma}_+ | \psi_{in} \rangle \right) + \sqrt{\gamma} \left( \langle \psi_{in} | \hat{N}_a \hat{\sigma}_z | 0_a, 1_b, g \rangle e^{-i\Delta t} \xi_a(t) + \text{c.c.} \right).$$

(A6)
We then continue to use Eq. (A3) to find the differential equations for the expectation values that appear on the RHS of the above equation.

Iterating this procedure gives a system of 19 first-order differential equations, in which the only time dependence on the RHS is given by the input pulses \( \xi_a(t) \) and \( \xi_b(t) \). This happens because the operators \( \hat{a}_0, \hat{b}_0 \) are placed on the right, and \( \hat{a}^\dagger_0, \hat{b}^\dagger_0 \) on the left of every terms. Therefore, the number of photons is always decreased and one eventually ends up with averages in the vacuum state \( \langle \varnothing | O | \varnothing \rangle \) with \( \hat{O} \) one of the operators whose derivative is given in the LHS of Eq. (A3). These quantities are easily known since the system does not evolve if the state is in the vacuum \( | \varnothing \rangle \). We observe that the final system of first-order differential equations can be solved one by one, which greatly simplifies the computation.

In the process described above, one is led to solve the differential equation

\[
\frac{d}{dt} \langle \psi_{in} | \hat{\sigma}_z | \psi_{in} \rangle = -2\gamma \left( 1 + \langle \psi_{in} | \hat{\sigma}_z | \psi_{in} \rangle \right) - 2\sqrt{\gamma} \langle \psi_{in} | \hat{\sigma}_+ | 1_0, g \rangle e^{-i\Delta t} \xi_a(t) + \text{c.c.} - 2\sqrt{\gamma} \langle \psi_{in} | \hat{\sigma}_- | 1_a, 0_0, b \rangle e^{-i\Delta t} \xi_b(t) + \text{c.c.},
\]

which gives the probability of excitation (3).

We now have all the necessary information to study the problem of two-photon bunching on the atom for square pulses of the form

\[
\xi_a(t) = \xi_b(t + T) = \begin{cases} \sqrt{\frac{\Omega}{2}} & \text{for } t_0 \leq t \leq \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}
\]

where \( T \) represents the delay between the two pulses incoming on the BS. The monochromatic regime then corresponds to the limit \( \Omega \ll \gamma \) and is independent of the exact pulse shape.

Specifically, the probability of excitation during the time interval \( t_0 \leq t \leq \frac{\pi}{2} \) is found to be

\[
\frac{\langle \psi_{in} | \hat{\sigma}_z | \psi_{in} \rangle + 1}{2} = \frac{\sigma e^{-2t'}}{\delta (\delta^2 + 1)^2} \left[ \delta \left( 2\sigma (2t' - 3)\delta^2 + 2t' + 5 + (\delta^2 + 1)^2 \right) + e^{2t'} (\delta^2 + \delta) (-2\sigma + \delta^2 + 1) - 2e^{2t'} \delta \left( (\delta^2 + 1)^2 - 2\sigma ((t' + 2)\delta^2 + t' - 2) \right) \cos(t'\delta) + 4\sigma e^{t'} (t'\delta^2 + (t' - 3)\delta^2 + 1) \sin(t'\delta) \right],
\]

where \( t' = \gamma(t - t_0) \), \( \delta = \Delta / \gamma \) and \( \sigma = \Omega / \gamma \) are respectively the normalized time, detuning and bandwidth. Eq. (5) is then readily obtained by considering the monochromatic regime \( \sigma \ll 1 \) while Eq. (7) corresponds to the resonant case \( \delta = 0 \).

The solution for the average coincidence \( \mathcal{C} \) is rather lengthy, but is greatly simplified in the monochromatic limit (Eq. (4)) or at resonance (Eq. (6)).
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