Optimal Siting and Sizing of Energy Storage Systems in Active Distribution Networks to achieve their Dispatchability

Ji Hyun Yi, Student Member, IEEE, Rachid Cherkauoui, Senior Member, IEEE, and Mario Paolone, Senior Member, IEEE

Abstract—This paper presents a method for the optimal siting and sizing of energy storage systems (ESSs) to be installed into active distribution networks (ADNs) to achieve their dispatchability. The problem formulation accounts for the uncertainty inherent to the stochastic nature of distributed energy sources and loads. Thanks to the operation of ESSs, the main optimization objective is to minimize the dispatch error, which accounts for the mismatch between the realization and prediction of the power profile at the ADN connecting point to the upper layer grid, while respecting the grid voltages and ampacity constraints. The proposed formulation relies on the so-called Augmented Relaxed Optimal Power Flow (AR-OPF) method. It expresses a convex full AC optimal power flow problem that is further decomposed into two levels: the first is a Benders decomposition technique. Consequently, the optimal size and site of the ESSs are adjusted. To validate the proposed method, extensive simulations are conducted on a real Swiss ADN of 55 nodes hosting a large amount of stochastic PV generation.

Index Terms—Energy storage systems, optimal power flow, active distribution networks, resource planning, dispatchability

NOMENCLATURE

Sets and Indices

\begin{itemize}
  \item \( l \in L \) Set of buses excluding the slack bus and lines whose downstream bus is bus \( l \)
  \item \( d \in D \) Set of days
  \item \( t \in T \) Set of time steps
  \item \( \phi \in \Phi_d \) Set of scenarios for day \( d \)
\end{itemize}

Variables

\begin{itemize}
  \item \( U_l \in \{0,1\} \) Installation status of the ESS at bus \( l \)
  \item \( C_l \) Energy reservoir of the ESS at bus \( l \)
  \item \( R_l \) Power rating of the ESS at bus \( l \)
  \item \( \tilde{p}_l(t,d) \) Average of the active load over all scenarios at bus \( l \) for time \( t \) and day \( d \)
  \item \( \Delta p_l(\phi,t) \) Deviation of prosumption for scenario \( \phi \) and time \( t \) from \( \tilde{p}_l(t,d) \)
  \item \( \tilde{f}_l(t,d) \) Average of squared internal current causing losses over all scenarios in line \( l \) for time \( t \) and day \( d \)
  \item \( \Delta f_l(\phi,t) \) Deviation of squared internal current for scenario \( \phi \) and time \( t \) from the average \( f_l(t,d) \)
  \item \( D P_d(t) \) Dispatch plan associated with time \( t \) and day \( d \) at the grid connecting point (GCP)
\end{itemize}

Parameters

\begin{itemize}
  \item \( b_l \) Half of the total shunt susceptance of line \( l \)
  \item \( z_l=r_l+jx_l \) Total longitudinal impedance of line \( l \)
  \item \( P_{l}^{\text{max}}, Q_{l}^{\text{max}} \) Upper limits of active and reactive power flows for line \( l \), respectively
  \item \( u^{\text{max}}_{E}, u^{\text{min}}_{E} \) Upper bounds\ Lower bounds of the squared nodal voltage magnitude
  \item \( E_{l}^{\max}, E_{l}^{\min} \) Maximum\ Minimum allowed SoE levels
  \item \( \lambda_{\phi,d} \) Probability of scenario \( \phi \) on day \( d \)
  \item \( B \) Total budget for installing ESS units
  \item \( C_{l}^{\max}, C_{l}^{\min} \) Maximum\ minimum possible ESS energy reservoir capacity at bus \( l \)
  \item \( R_{l}^{\max}, R_{l}^{\min} \) Maximum\ minimum possible ESS power
\end{itemize}

\( \omega_l(\phi,t) \) Compensated error by the ESSs at bus \( l \) for scenario \( \phi \) and time \( t \)

\( \epsilon_l(\phi,t) \) Uncovered error at bus \( l \) for scenario \( \phi \) and time \( t \)

\( \theta_d(\phi,t) \) Daily error reduction rate for scenario \( \phi \), time \( t \) and day \( d \)

\( f_l(\phi,t) \) Square of current magnitude causing losses in line \( l \)

\( v_l(\phi,t) \) Square of voltage magnitude at bus \( l \)

\( s_l=p_l+jq_l \) Aggregated prosumption at bus \( l \)

\( S_l^f=P_l^f+jQ_l^f \) Upstream complex power flow to line \( l \)

\( S_l^{j} = P_l^{j} + jQ_l^{j} \) Auxiliary variable of upstream complex power flow to line \( l \) (upper bound of \( S_l^f \))

\( S_l^{\text{b}} = P_l^{\text{b}} + jQ_l^{\text{b}} \) Downstream complex power flow to bus \( l \) from line \( l \)

\( S_l^{b} = P_l^{b} + jQ_l^{b} \) Auxiliary variable of complex power flow to bus \( l \) from line \( l \) (upper bound of \( S_l^{\text{b}} \))

\( s_l^E_{l}=p_l^E+jq_l^E \) State-of-energy of ESS installed at bus \( l \)

\( u_{l}^{j}(\phi,t) \) Positive unserved load at bus \( l \) for scenario \( \phi \) and time \( t \)

\( u_{l}^{b}(\phi,t) \) Negative unserved load at bus \( l \) for scenario \( \phi \) and time \( t \)

\( \gamma^m(\phi,t) \) Slack variable associated with the realized losses deviation at \( m^{th} \) iteration for scenario \( \phi \) and time \( t \)

\( \zeta(\phi,t) \) Slack variable associated with the additional coverage of losses deviation for scenario \( \phi \) and time \( t \)
rating capacity at bus $l$

$CR_{\text{min}}$  Minimum power ramping rate of ESS

$\mathcal{L}_e, \mathcal{L}_u, \mathcal{L}_p$  ESS investment costs (fixed installation, energy reservoir, power rating)

$w_l, w_u, w_e$  Weight coefficient associated to the grid losses, unserved load and error between the dispatch plan and the active slack power in each scenario, respectively

$N_d$  Number of days in each day-type in a year

$Y$  Planning horizon

I. INTRODUCTION

The volatility in power system introduced by the high penetration of non-dispatchable and stochastic generation necessitates enough flexible resources to support the controllability of the system. In particular, the power generation from distributed renewable resources incurs unexpected fluctuations in the prosumption\(^1\) within the distribution grid. The deviation of the network infeed from the prescheduled power, called dispatch error, should be compensated by the system operating reserve of the grid. As a result, with a massive capacity of non-dispatchable DG, distribution network operators (DNO) could be exposed to high imbalance cost due to the large capacity of operating reserves that could be required. Meanwhile, there has been increasing interest in utilizing ESSs as a buffer to compensate for the imbalance in order to control the network infeed. It consists of following an anticipated power schedule with high accuracy minimizing consequently the imbalance cost\(^1\). The optimal control strategies of ESS for a dispatchable feeder has been addressed through several recent publications\(^2\)–\(^4\). In this respect, the optimal sizing and siting of ESS considering both of the investment cost and the associated grid operational advantages still deserve to be more investigated in view of an efficient utilization of ESS.

The problem of optimal siting and sizing of ESS for active distribution networks (ADNs) has been extensively investigated through numerous researches thanks to the versatile services that ESS can offer such as:

1) Technical purposes : minimizing network losses\(^5\)–\(^7\), voltage control\(^7\)–\(^9\), mitigating line congestion\(^7\), improving the quality of power supply\(^5\),\(^10\)

2) Economical purposes : minimizing system operation costs\(^11\), providing ancillary service to TSO\(^12\), mitigating the risk in energy market extra-costs via load shifting and power arbitrage\(^13\)

Several works mentioned above put a special accent on handling operational risks triggered by large forecasting uncertainties of renewable resources\(^7\),\(^8\). However, none of them considered the dispatch error brought by the stochasticity of prosumption. To the best of the authors knowledge, the optimal allocation of ESS considering its control to support the dispatchability of ADN is still a challenge to be addressed, and it represents the focus of this paper.

The capacity and location of ESS should strongly comply with the characteristics of the chosen control strategy and the subsequent operational conditions of the system. In this regard, the optimal power flow (OPF) that accurately models operation and control of distribution networks should be embedded in the planning tools for ADNs in order to guarantee their performances. Various approaches have been introduced to tackle the non-linearity and non-convexity of the OPF problem. They can be classified according to the following categories:

1) Modification of the power flow for convexification\(^9\);

2) Meta-heuristic methods\(^5\),\(^6\),\(^10\),\(^13\);

3) Relaxation approaches\(^8\),\(^11\),\(^12\),\(^14\).

One of the widely used approaches is convexification of AC-OPF applying semi-definite programming (SDP)\(^14\) or second-order cone programming (SOCP) relaxation\(^8\),\(^11\),\(^12\). The SOCP relaxation method proposed in\(^15\) has been more often implemented for the optimal ESS allocation for radial grids due to its superiority over the SDP relaxation in terms of computational efficiency. It is worthwhile noting that each of the works employing the SOCP relaxation defined the objective function to be strictly increasing with the grid losses to hold the exactness of the solution. Meanwhile, the author of\(^16\) and\(^16\) addressed the limitation of the SOCP relaxation method investigating the necessary conditions to guarantee the exactness of the solution. The drawbacks of the method were explicitly underlined by the fact that the exactness of the solution cannot be guaranteed especially in the cases of reverse line power flows and the cases where the upper bound of nodal voltage and the line ampacity constraints are binding. Actually, it brings significant limitations on the applicability of the method to ADNs hosting DG units with large capacities. Moreover, the model neglects the transverse elements of the lines, which can bring infeasibility of the solution especially when ADNs are composed of coaxial underground cables. In this context, the authors in\(^17\) proposed the Augmented Relaxed OPF (AR-OPF) to convexify the AC-OPF for a radial grid. Their contribution demonstrates that the conditions for the exactness of the solution are mild and hold for realistic distribution networks. The AR-OPF was implemented as the core of their subsequent work on the optimal ESS planning problem while embedding grid reconfiguration\(^17\).

In this paper, we propose an operation-driven planning strategy of ESS to achieve the dispatchability of the distribution feeder while including the AR-OPF model. Furthermore, to comply with the condition that ensures the exactness of the relaxed OPF, we formulated the sizing problem into two blocks by modifying the objective term related to the dispatch error and introducing associated constraints and variables. Meanwhile, we apply the Benders decomposition to handle the inherent multi-layered decisions with numerous scenarios\(^18\). The contribution of the paper are:

1) The optimal allocation of ESS is determined to address on achieving dispatchability of the ADN in the presence of prosumption uncertainty.

2) An exact convex model of OPF is employed to accurately reflect the operational condition of the ADN.

3) The structure of the planning problem and the objective function are formulated accounting for the correlation to be necessarily observed between the optimality criteria and the exactness of the OPF relaxation.

The structure of the paper is as follows: In Section II, we introduce the structure of the optimization problem and explain the key parts in detail. In Section III, the proposed problem formulation and solution approach are described, followed by some simulation results in Section IV. Finally, Section V concludes the paper.

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\(^1\) Prosumption is defined as the load consumption minus the generated power from distributed generation.
II. Problem Structure

In this section, the key elements composing the proposed method are explained. To begin with, we introduce the formulation for achieving the dispatchability of ADN, embedded in a daily OPF problem. Then, we describe the AR-OPF model that accurately models the power flow and grid security constraints.

A. System description

In this paper, the capability of dispatchability account only for active power. The active power through the grid connecting point (GCP) \( P_{G} \) to an upper layer grid is expected to follow a day-ahead schedule or a pre-determined daily dispatch plan \( D P_{d} \) derived from a forecasting tool. The scheduling horizon of the dispatch plan is composed of discrete intervals with the index \( t \in T \). At each node of the ADN, a non-dispatchable aggregated prosumption \( s_{l} \) is located. Then, at each node where an ESS is allocated (i.e., \( U_{l} = 1 \)), it is dispatched according to active power \( p_{l}^{E} \) and reactive power \( q_{l}^{E} \). The dispatched active power compensates for the gap between the dispatch plan and the realized active power indeed at GCP. This is defined as dispatch error. In summary, the ESS allocation problem can be described using a two-stage decision process: the first stage deals with the decision binary variables on the location of the ESS \( U_{l} \) and the decision continuous variables on the capacity of the ESS energy reservoir \( C_{l} \) and its power rating \( R_{l} \); the second stage deals with the decision variables on the ESS active \( p_{l}^{E} \) and reactive power \( q_{l}^{E} \) outputs.

B. Dispatch plan of the distribution feeder

The operational benefit of ESS allocation for the ADN dispatchability is achieved with a set of dispatch plans, each referring to a typical day. It is understood in this section that the variables with subscript \( l, \phi, t \) are defined for \( \phi \in \Phi_{d}, t \in T \), respectively. The daily dispatch plan for day \( d \) follows the predicted point of the total prosumption considering the losses as shown in (1). The prosumption scenarios for each day are generated with the assumption that the prosumption profile follows normal distribution. Therefore, the prosumption and the losses predictions are calculated by averaging the prosumption and the losses over scenarios. As shown in (2) and (3), we can define the prosumption at each bus \( l \) and the line losses of the line \( l \) with two terms respectively: the predicted point and the deviated power from the predicted point (we would refer to it as error hereafter). In this context, the dispatch error is formally defined as the total error of prosumption plus the line losses over the buses/lines:

\[
DP_{d}(t) = \sum_{l \in L} (\hat{p}_{l}(t) + r_{F_{l}}(t, d)) = \forall l, \forall t \tag{1}
\]

\[
p_{l}(\phi, t) = \hat{p}_{l}(t) - \Delta n_{l}(\phi, t), \quad \forall l, \forall \phi, \forall t \tag{2}
\]

\[
r_{F_{l}}(\phi, t) = r_{F_{l}}(t, d) - r_{l}\Delta F_{l}(\phi, t), \quad \forall l, \forall \phi, \forall t \tag{3}
\]

The installed ESSs can compensate for the dispatch error as much as their capacities can allow. To quantify the error covered, or not covered, by the ESSs, respectively, we express the total error at each time interval and scenario as follows.

\[
\sum_{l \in L} (\Delta p_{l}(\phi, t) + r_{l}\Delta F_{l}(\phi, t)) = \sum_{l \in L} (\epsilon_{l}(\phi, t) + \omega_{l}(\phi, t)), \quad \forall \phi, \forall t \tag{4}
\]

where \( \epsilon_{l} \) represents the leftover error that cannot be covered by ESS at each bus \( l \) and \( \omega_{l} \) indicates the compensated error at each bus \( l \) by the ESSs located at each bus \( l \). Finally, the objective term to minimize the power deviation from the daily dispatch plan for all scenarios and time intervals is expressed as follows.

\[
\min_{\phi, \forall t} \sum_{l \in L} \lambda_{\phi, l} \sum_{l \in L} \epsilon_{l}(\phi, t) \tag{5}
\]

C. Augmented Relaxed Optimal power flow

The dispatch plan is computed solving the OPF problem. For a radial power network where the lines are modeled as a two-terminal equivalent circuit, the Kirchhoff’s law to Fig. 1 are given in (8)-(11). The variables with subscript \( l \) are defined for \( \ell \in L \). The upstream bus of bus \( l \) is denoted as up\( l \). \( G \) is the adjacency matrix of the network, where \( G_{k,l} \) is defined for \( k, l \in L \) and \( G_{k,l} = 1 \) if \( k = up\ l \) or if 0. The internal current flowing through the longitudinal impedance is expressed with inequality constraint as shown in (11) applying the SCOP relaxation. It is noteworthy that the equality of (11) holds only when the terms in the objective function are strictly increasing with respect to the internal current \( f_{l} \) of the II model (see Fig. 1).

\[
S_{l} f_{l} = S_{l} f_{l} + z_{l} f_{l} - (v_{up}(l) + v_{l})b_{l}, \quad \forall l \tag{8}
\]

\[
S_{l} f_{l} = S_{l} f_{l} + \sum_{m \in L} G_{l,m} S_{l} f_{l}, \quad \forall l \tag{9}
\]

\[
v_{l} = v_{up}(l) + 229 \left( \sum_{m \in L} G_{l,m} S_{l} f_{l} + z_{l} f_{l} \right), \quad \forall l \tag{10}
\]

\[
f_{l} \geq \left| S_{l} f_{l} + z_{l} f_{l} + v_{up}(l) \right|, \quad \forall l \tag{11}
\]

In order to avoid any inexact solution (i.e., any solution that makes the lefthand side of (11) strictly greater than the righthand side, and thus without physical meaning). Authors of [17] introduced auxiliary variables to formulate Augmented Relaxed OPF (AR-OPF) model. These are \( f, S, \hat{v} \) and \( S \).
which stand for the upper bounds for internal current, apparent power, voltage and lower bounds for the apparent power respectively. The power flow equations are defined as well with that set of auxiliary variables as shown in (12)-(18), and the voltage constraint, the ampacity constraint from the sending end and the receiving end are modeled as indicated in (19)-(21). Finally, (22) are added to obtain the exactness conditions.

\[
\begin{align*}
S^l_I &= s_I + \sum_{m \in \mathcal{L}} G_{l,m} S^m_I - j(v_{up(l)} + v_l)b_I, \quad \forall l \\
\hat{S}^l_I &= s_I + \sum_{m \in \mathcal{L}} G_{l,m} \hat{S}^m_I, \quad \forall l \\
v_l &= v_{up(l)} - 2 \mathcal{R}_l \left( z_l^I (\hat{S}^l_I + jv_{up(l)}b_I) \right), \quad \forall l \\
S^l_I &= s_I + \sum_{m \in \mathcal{L}} G_{l,m} S^m_I, \quad \forall l \\
\hat{S}^l_I &= s_I + \sum_{m \in \mathcal{L}} G_{l,m} \hat{S}^m_I, \quad \forall l \\
\hat{f}_{l,v_l} &\geq \max \left\{ \left| \hat{P}^b_l \right|, \left| \hat{P}^b_l \right|^2 \right\} + \max \left\{ \left| \hat{Q}^b_l - jv_l b_l \right|, \left| \hat{Q}^b_l - jv_l b_l \right|^2 \right\}, \quad \forall l \\
\hat{f}_{l,v_{up(l)}} &\geq \max \left\{ \left| \hat{P}^b_l \right|, \left| \hat{P}^b_l \right|^2 \right\} + \max \left\{ \left| \hat{Q}^b_l + jv_{up(l)} b_l \right|, \left| \hat{Q}^b_l + jv_{up(l)} b_l \right|^2 \right\}, \quad \forall l \\
\hat{v}_m &\leq v_{max}, \quad \forall l \\
\hat{v}_m &\leq v_{max}, \quad \forall l \\
\hat{I}_{l,v_{up(l)}} &\geq \max \left\{ \left| \hat{P}^b_l \right|, \left| \hat{P}^b_l \right|^2 \right\} + \max \left\{ \left| \hat{Q}^b_l, \left| \hat{Q}^b_l \right|^2 \right\}, \quad \forall l \\
\hat{I}_{l,v_l} &\geq \max \left\{ \left| \hat{P}^b_l \right|, \left| \hat{P}^b_l \right|^2 \right\} + \max \left\{ \left| \hat{Q}^b_l, \left| \hat{Q}^b_l \right|^2 \right\}, \quad \forall l \\
\hat{P}^b_l &\leq \hat{I}_{l,v_{up(l)}}, \quad \hat{Q}^b_l \leq \hat{Q}_{l,v_{up(l)}}, \quad \forall l
\end{align*}
\]

For the sake of readability, the equations mentioned above are grouped and represented as follows:

\[
\Theta(\varphi, \kappa) \geq 0
\]

(23)

where \(\varphi = \{S^I, v, f, \hat{S}^I, \hat{v}, \hat{f}, S^b, s\}\) is the set of variables and \(\kappa = \{z, h, v_{max}, v_{min}, I_{max}\}\) is the set of parameters. The notation without subscript corresponds to the vector of variable and parameter for all buses/lines.

The set of grid constraints employing the auxiliary variables slightly shrinks the original feasible solution space, removing solution space related to undesirable or extreme operation points of the network. Then the optimal solution obtained by applying the SOCP relaxation is guaranteed to correspond to the optimal solution of the original OPF under mild conditions.

D. Energy storage systems

1) ESS allocation: The variables with subscript \(l\) are defined for \(l \in \mathcal{L}\). As shown in (24), the investment cost of ESS, which consists of fixed installation cost, energy reservoir cost, and power rating cost, should not exceed a budget. In reality, available power ratings and energy capacities are often restrained as seen in (25) and (26) due to various physical constraints involving for instance manufactural or geographical factors. Typically, commercial ESS has a defined \(C\)-rate, which is a measure of the rate at which ESS is discharged relatively to its maximum energy capacity, as shown in (27).

\[
\sum_{l \in \mathcal{L}} (I_c U_l + I_p R_l + I_c C_l) \leq B, \quad \forall l
\]

(24)

\[
R_{min} U_l \leq R_l \leq R_{max} U_l, \quad \forall l
\]

(25)

\[
C_{l, min} U_l \leq C_l \leq C_{l, max} U_l, \quad \forall l
\]

(26)

\[
R_l \leq \frac{C_l}{CR_{min}}, \quad \forall l
\]

(27)

2) Modeling of ESSs operation: The operational characteristic of ideal ESS at time interval \(t\) is modeled with equations (28)-(30). The variables with subscript \(l\) are defined for \(l \in \mathcal{L}\). The feasibility capability of a given ESS is linearized by constructing an inscribed square within the original circular capability curve defined by the maximum apparent power of the ESS (see (28)). The state-of-energy (SoE) level of ESS changes with the charge/discharge of the ESS at every time interval as described in (29). Also, (30) indicates that the SoE of the ESS should be within the maximum and minimum allowed SoE levels.

\[
\frac{-R_l}{\sqrt{2}} \leq \hat{P}^E_l (t) \leq \frac{R_l}{\sqrt{2}}, \quad \forall l
\]

(28)

\[
E^E_l (t+1) = E^E_l (t) + \Delta t * \hat{P}^E_l(t), \quad \forall l
\]

(29)

\[
E_{l, min} \leq E^E_l (t) \leq E_{l, max}, \quad \forall l
\]

(30)

In the interest of brevity, (23)-(27) are indicated as follows:

\[
\Xi(\eta, \xi) \geq 0
\]

(31)

where \(\eta = \{P, Q, E^E, U, R, C\}\) is the set of variables and \(\xi = \{\Delta t, E_{min}, E_{max}\}\) is the set of parameters. The notation without subscript corresponds to the vectors of variables and parameters for all buses/lines.

III. PROBLEM FORMULATION

The objective of the problem is to determine the optimal sizes and sites of ESSs so that the active power through the GCP (connecting point to the higher level grid) follows the dispatch plan with minimal deviation. However, it is clear that the dispatch error described as (6) in Sec. II-B does not increase while the total grid losses increase. Therefore, the exactness of the solution cannot be guaranteed if (6) is included in the objective function of the optimization problem with the AR-OPF model.

Therefore, we propose to decompose the problem into 2 blocks so that we can separate (6) from the AR-OPF problem and convey it to the first block where an approximated OPF formulation is considered. The whole algorithm of the proposed approach is illustrated in Fig. 3.

In the 1st block problem, the optimal ESSs allocation, the daily dispatch plans, and the corresponding ERR are calculated employing linear approximated OPF ignoring the grid losses. Only the nodal voltage constraints are considered regarding the security constraints, ignoring the ampacity limits to reduce computational burden. Instead, the ampacity limits are included within the 2nd block problem. Afterwards, the outputs of the 1st block are used as inputs for the 2nd block.

In the 2nd block problem, the objective is to refine the optimal allocation of the ESSs considering several operation scenarios and achieving the same level of ERR calculated in the 1st block problem. In this respect, ERR is implemented as an additional constraint to an AR-OPF model and the size and site of the ESSs are iteratively adjusted thanks to a Benders decomposition technique. This iterative process starts initially with a feasibility check of the ESS allocation resulting from the 1st block.
A. 1st block problem

In the 1st block problem, we minimize the investment and total penalty costs over the planning horizon to find out the optimal size and site of the ESSs and the optimal dispatch plan. To compute daily dispatch plans, we embed approximated OPF constraints for all scenarios into a two-stage mixed-integer linear programming problem. The OPF is formulated by the linear Distflow model in which shunt elements are considered, whereas the grid losses are neglected. In this way, the reactive power generated by the shunt impedance of the lines is accounted for in the nodal voltage constraints. Meanwhile, neglecting the formulation of the internal current ($f_l$) is less likely to affect the feasible solutions in this stage since the ampacity constraint is ignored. The equations related to the dispatch plan are included as $(24)-(27)$, and $(32)$-$(38)$. We substitute $(32)$ into the active power balance equation, yielding $(33)$. The lossless Distflow power flow including the ESS power and grid characteristics are expressed via $(34)$-$(37)$ and the security constraint is shown with $(38)$. All the variables within $(33)-(38)$ are defined for every lines ($\ell \in \mathcal{E}$), scenarios ($\phi \in \Phi_d$), time intervals ($t \in T$), and days ($d \in D$); however, the corresponding indices are omitted for the sake of brevity.

$$DP_d(t) = \sum_{\ell \in \mathcal{E}} \bar{p}_\ell(t, d), \forall t, \forall d$$

$$\sum_{\ell \in \mathcal{L}} \Delta p_{\ell} = \sum_{\ell \in \mathcal{L}} (e_{\ell} + \omega_{\ell}), \forall \phi, \forall t, \forall d$$

$$P_{\ell}^I = P_{\ell}^F = \bar{p}_\ell - \Delta p_{\ell} + P_{\ell}^F + \sum_{m \in \mathcal{E}} g_{\ell,m} P_{\ell,m}^I, \forall t, \forall \phi, \forall t, \forall d$$

$$Q_{\ell}^I = Q_{\ell}^F + \sum_{m \in \mathcal{E}} g_{\ell,m} Q_{\ell,m}^I - (v_{\text{up}}(t) + v_{\text{up}}(t) )b_{\ell}, \forall t, \forall \phi, \forall t, \forall d$$

$$Q_{\ell}^I = Q_{\ell}^F + \sum_{m \in \mathcal{E}} g_{\ell,m} Q_{\ell,m}^I, \forall t, \forall \phi, \forall t, \forall d$$

$$v_l = v_{\text{up}}(t) - 2\Re(z_l^T (S_l^T + j v_{\text{up}}(t) b_l)), \forall t, \forall \phi, \forall t, \forall d$$

$$v_{\text{min}} \leq v_l \leq v_{\text{max}}, \forall t, \forall \phi, \forall t, \forall d$$

The objective function is defined to minimize the investment cost ($IC$) of ESS and the annual penalty cost ($\sum_{d \in D} N_d PC_d$) regarding the uncovered dispatch error over the planning horizon $Y$, as shown in $(39)$. $\Omega_1$ and $\Omega_2$ represents the set of control variables in the first and second stage decision process, respectively. The constraints regarding the ESS allocation and operation explained in Sec. [1]-[2] are included (i.e., $(24)-(27)$, $(32)-(38)$). By solving the problem, the optimal allocation of ESSs along with the dispatch plan is obtained, and the daily ERRs are calculated by $(7)$ with respect to days with the index $d \in D$.

$$\begin{align*}
\text{minimize} & \quad IC + Y \sum_{d \in D} N_d PC_d \\
\text{subject to:} & \quad (5), (24)-(27), (32)-(38) \\
& \quad \Xi(\eta(\phi, t, d), \xi) \geq 0, \forall \phi, \forall t, \forall d \\
& \quad IC = \sum_{l \in \mathcal{L}} (T_{l} U_{l} + T_{p} R_{l} + T_{c} C_{l}) \quad \forall t \in T, \forall \ell \in \mathcal{E} \\
& \quad PC_d = \sum_{l \in \mathcal{L}} \sum_{\ell \in \mathcal{E}} \lambda_{\ell, \phi, d} \Xi(\ell(\phi, t), t), \forall \phi, \forall t, \forall d
\end{align*}$$

B. 2nd block problem

The objective of the 2nd block problem is to adjust the optimal size of ESS considering grid losses and to determine the optimal site that can minimize the grid losses. We formulate the targeted optimization problem using the Benders decomposition technique. In this respect, the 2nd block problem is decomposed into a master problem and several parallel subproblems that each represents a daily OPF problem. The master problem determines the sites and sizes of the ESSs. Then the fitness of the determined allocations is evaluated in the subproblems in terms of grid losses and unserved load. The unserved load is introduced to ensure the feasibility of subproblem regardless of the ESS allocation. In other words, it takes values only when the security constraints or the ERR constraints are binding.

The first step of the 2nd block is to solve parallel subproblems, checking the operational condition of ADN under the ESS allocation and the corresponding ERR calculated from the 1st block. The objective of the subproblem and the dual values of ESS allocation are computed and sent to the master problem to construct the first Bender’s cut. Through multiple iterations between the master problems and subproblems, the convergence is reached when the gap between the Lower Bound LB (see $(46)$) and the Upper Bound UB (see $(58)$) becomes less than a given tolerance (see Fig. 2).

1) Master problem: The formulation of the master problem is given in $(43)$. The master problem computes the lower bound of the planning problem by summing the investment cost and the lower approximation of the subsequent expected subproblem costs. Each $\alpha_d$ represents the subproblem cost for days classified into each day type. It is initially bounded by $\alpha$, the parameter given as the lower bound for the subproblem cost. $n \in \mathcal{N}$ is the index of benders iterations. In every $n$th iteration, Benders cuts represented by $Y^2(n)$ (see $(59)$) are added.
as additional constraints for days $d \in D$, as shown in (43), 46
represents that the Lower Bound LB is determined by the optimal solution of the master problem.

\[
\min_{U.C.R.\alpha} IC + \sum_{d \in D} \alpha_d \tag{43}
\]
subjected to : (24)–(27),

\[
\alpha_d \geq \alpha_d, \quad \forall d \tag{44}
\]

\[
\alpha_d \geq \gamma_l^{(m)}, \quad \forall d, \forall n \tag{45}
\]

\[
LB=IC^* + \sum_{d \in D} \alpha_d \tag{46}
\]

2) Subproblem: In the subproblem associated with day $d$, a daily AR-OPF model with the time-step discretization of 15 minutes evaluates the operational advantages of ESSs while considering real operational conditions. The variables with subscript $l$, $\phi$, $t$ are defined for $l \in L$, $\phi \in \mathcal{D}_d$, $t \in T$, respectively. The sufficiency of the ESS size is assessed by checking if the uncovered dispatch error (see (3)) satisfies the ERR for day $d$ as shown in (47). As indicated in (48), we introduce positive and negative unserved load terms to represent upward and downward deviation of a prosumption scenario from the prosumption prediction, respectively. They correspond to the amount that should be curtailed to primarily comply with the ERR constraint along with other security constraints even in the case of insufficient capacity of ESSs.

\[
| \sum_{l \in L} \epsilon_l (\phi, t) | \leq \theta_{d} (\phi, t), \quad \forall \phi, \forall t \tag{47}
\]

\[
p_l ( \phi, t) = p_l ( \phi, t) + ul_l^+ ( \phi, t) - ul_l^- ( \phi, t), \tag{48}
\]

\[
\forall ul_l^+, ul_l^- \in \mathbb{R}^+, \quad \forall l, \forall \phi, \forall t \tag{49}
\]

\[
\sum_{l \in L} (\Delta p_l ( \phi, t) + r_l \Delta f_l ( \phi, t) ) = \sum_{l \in L} (\epsilon_l ( \phi, t) + \omega_l ( \phi, t)), \quad \forall \phi, \forall t \tag{50}
\]

The AR-OPF problem embedding the dispatchability for the subproblem is formulated by replacing the prosumption $p_l$ with $p_l'$ and substituting $49$ and $3$ into $4$, expressing the error as shown in (50). We also switch the prosumption $p_l$ with $p_l'$ in $1$, the active power balance equations formulated with the original state variables (i.e., $8$, $9$) and the auxiliary variables (i.e., $12$, $13$, $15$, $16$).

In short, the equations of AR-OPF model including ESSs are re-defined as follows:

\[
\Theta^\prime ( \phi^\prime ( \phi, t), \kappa ) \geq 0, \quad \forall \phi, \forall t \tag{51}
\]

where $\phi^\prime = (S_l^t, v, f, S_l^t, v, f, S_l, s_l = (p_l + p_{E}) + j (q_l - q_{E}))$ is the set of variables and $\kappa = \{ z, b, b_{max}, b_{min}, I_{min} \}$ is the set of parameters. It should be noted that the ESS power is governed by the set of operational constraints as follows:

\[
\Xi (\eta ( \phi, t), \xi ) \geq 0, \quad \forall \phi, \forall t \tag{52}
\]

However, we can intuitively expect that having $50$ in the AR-OPF model could not be compliant with the mathematical formulation of the power flow equations. For instance, it is physically impossible to satisfy (50) in the case of an insufficient capacity of ESS to satisfy the given ERR (i.e., $\omega_l$ is too limited to make $\epsilon_l$ comply with the ERR constraints). The equation can only be satisfied by reducing the total error (the left-hand side of (50)) to be as the same as the right hand side. Since the prosumption error is fixed, the magnitude of the grid losses error should take an unrealistically big value to cancel out the prosumption error. It would induce the increase of the internal current $f_l$ such that the left-hand side of (11) becomes strictly greater than the right-hand side, which leads to the inexactness of the solution. Therefore, we introduce two slack variables, $\gamma_l$ and $\zeta$, to replace $50$ by (53) and (54) such that the value of the internal current $f_l$ would never deviate away from the real value (i.e., the exactness of the solution is guaranteed). Through the iterative process introduced in Algorithm 1, the error of grid losses is realized in obtaining the dispatch plan when the absolute value of the updated slack variable $\zeta$ becomes below a tolerance.

\[
\sum_{l \in L} (\Delta p_l ( \phi, t) + r_l \Delta f_l ( \phi, t) ) = \sum_{l \in L} (\epsilon_l ( \phi, t) + \omega_l ( \phi, t)), \quad \forall \phi, \forall t \tag{53}
\]

\[
\sum_{l \in L} (\Delta p_l ( \phi, t) + \gamma_l^m ( \phi, t) ) = \sum_{l \in L} (\epsilon_l ( \phi, t) + \omega_l ( \phi, t)), \quad \forall \phi, \forall t \tag{54}
\]

Algorithm 1 Iterative realization of grid losses deviation

Input : $\theta, s=p+jq$, $R^*$ (see (56)), $C^*$ (see (57))

1: Initialization : $m=1, \gamma^1=0, \zeta=1$;  
2: while $|\zeta| \geq \text{tolerance}$ do
3: Solve a sub problem including $53$ and $54$  
4: $\gamma^{m+1} \leftarrow \gamma^m + \zeta$  
5: $m \leftarrow m+1$
6: end while

Finally, the subproblem is described with the objective defined as to minimize the total grid losses and unserved load to satisfy the ERR constraint, and the operation period corresponds to all days grouped into each day type over the planning horizon.

\[
\min_{\varphi^\prime, \eta, \bar{u}^+, \bar{u}^-} : SC_d = Y N_d \sum_{l \in T} \lambda_{l, d} (w_l) \sum_{l \in L} r_l f_l ( \phi, t) \tag{55}
\]

subject to:

$R_l = R_l^* + \mu_{l, d}, \quad \forall l$  
$C_l = C_l^* + \theta_{l, d}, \quad \forall l$  

where $w_l$ and $w_u$ are the weight coefficients associated with the grid losses minimization and unserved load, respectively.  

$56$ and $57$ describe that the ESS power ratings and the energy reservoirs are fixed to the optimal solution values of the master problem, $\mu_{l, d}$ and $\theta_{l, d}$ are the dual of constraints related to the fixed ESS capacities, and are used to form the benders cuts for the master problems as shown in (59). The Upper Bound UB is calculated summing the optimal investment cost and the optimal subproblem costs as shown in (58).

\[
UB = IC^* + \sum_{d \in D} SC_d^* \tag{58}
\]

\[
\Gamma_d^{(n)} = \left[ SC_d^* - \sum_{l \in L} (\mu_{l, d} (R_l - R_l^*) - \theta_{l, d} (C_l - C_l^*))) \right], \quad \forall d, \forall n \tag{59}
\]
IV. SIMULATION

In this section, we validate the performance of the proposed methods with an existing Swiss distribution network with 55 bus and large capacity of renewable generation, as shown in Fig. 3. The base voltage is 21kV and the base 3 phase power is 6MVA. Within the distribution grid, 2.7MWp of PV generation capacity and 805kVA of hydropower generation capacity is installed. It should be noted that the stochasticity of renewable generation was considered for only the load consumption and PV generation. The planning horizon is set as 10 years, and we assume that the load consumption does not grow over the planning horizon, and the interest rate is zero. According to the indications of the operator of this grid, the number of candidate nodes for ESS installation is set as 5 out of 55 nodes (see Table I).

### TABLE I

| Parameter | Value |
|-----------|-------|
| Maximum power rating per site | 3MW |
| Maximum energy reservoir per site | 4MWh |
| Installation cost for energy reservoir | $300/kWh |
| Installation cost for power rating | $200/kVA |
| Capital investment cost per site | $0.1M |

![Considered 55 bus distribution feeder.](image)

We assumed that prosumption forecasts and scenarios for the simulation have been generated from a reliable forecasting and scenario generation methodology. To reduce the computational burden, we have considered 8 typical days to cover the seasonal variation of the prosumption over the year. 1000 scenarios with equal probabilities were reduced to 10 scenarios by applying K-medoids clustering algorithm [19]. To make sure to achieve the dispatchability, the penalty cost for the dispatch error is assumed as $700/MWh, which is significantly higher than a typical price settled in energy markets.

A. Planning with 1 day and 5 scenarios under hourly dispatch

In the purpose of illustrating the role of the 1st and 2nd block of the problem, we demonstrate the result of the simplified simulation considering an hourly dispatch for 1 representative day in two cases: case 1 with the original ampacity of the lines specified for the given grid and case 2 with the ampacity of one line reduced to 1/5 of the original value. Table II illustrates the optimal allocation result of ESS. The optimal result from the 1st block specifies the capacity in size of power rating and energy reservoir of ESS. However, the determined allocation of ESS cannot be guaranteed to be feasible and optimal to satisfy the ERR for the real operation of the grid, since the grid losses and the ampacity constraint were neglected in the 1st block problem. After the 2nd block of the problem, as shown in the result of case 1 in Table II, the optimal site of the ESS considering the objective of minimization of the grid losses is determined as Node 4, resulting in the reduction of the grid losses compared to the result of the 1st block problem.

In case 2, the ampacity of the line between Node 20 and Node 4 was reduced to 1/5 of the original value. We can observe that a part of the prosumption was curtailed to satisfy the ERR in the condition of restrained ampacity limit with the determined ESS allocation from the 1st block. In this regard, the result of the 2nd block shows the change in the allocation of the ESS due to the bottleneck of the line. The size of the ESS on Node 4 is reduced, and another ESS is allocated after re-allocating the ESS.

![ESS allocation result](image)

![Comparison between the result of 1st block and 2nd block](image)

B. Planning with the full scenarios under 15 min dispatch

The effectiveness of the proposed planning procedure is verified with the full set of scenarios with 8 typical days under 15 min interval dispatch. Table IV shows the optimal obtained ESS locations and sizes. We exhibit the results for two cases corresponding to case 1 with no ESS installed and case 2 with the optimal allocation of ESS. Fig. 4 illustrates the operation result for 1 typical day, showing the prosumption prediction considering 10 scenarios of the prosumption profiles, the dispatch plan, and the active power infeed through GCP corresponding to each scenario in the case of no ESS (see Fig. 4(a)) and the optimal ESS allocation (see Fig. 4(b)). The dispatch result without ESS shows that the dispatch error...
is significant, especially in the time where the production from PV is high. On the other hand, in the case with the optimal ESS allocation, the active power infed of every prosumption scenario follows the dispatch plan with small error. The cost analysis between the case with ESS and without ESS in Table V demonstrates quantitatively the capability of ESS to handle uncertainties within the grid. The annual dispatch error of the case without ESS is about 9 times of that in the case with ESS. The difference in the dispatch error is translated into the significant gap in the total cost for 10 years of operation: $6.72 Million with the default system configuration, and $1.73 Million with the optimal ESS allocation. Consequently, this result demonstrates the advantages for the DSO to invest on ESS in view of their technical and economical profit.

### TABLE IV

ESS ALLOCATION RESULT

| Problem | Location | Power rating | Energy reservoir |
|---------|----------|--------------|------------------|
| 1st Block | 45       | 986.10kVA    | 2.323MWh         |
| 2nd Block | 4        | 990.68kVA    | 2.327MWh         |

### TABLE V

COST AND OPERATIONAL ADVANTAGE COMPARISON

| Horizon | Without ESS | With ESS |
|---------|-------------|----------|
| 10 years | Total cost: $6.72M | $1.73M |
|          | Investment cost: - | $1.00M |
|          | Penalty cost: $6.72M | $0.73M |
| 1 year   | Uncovered error: 927.11MWh | 104.74MWh |
|          | Grid losses: 50.754MWh | 50.513MWh |
|          | Total energy consumed: 7.281GWh | 7.3140GWh |

![Fig. 4. Prosumption prediction, dispatch plan and active power through GCP in each scenario: (a) Day 1(No ESS), (b) Day 1(With ESS).](image)

### V. CONCLUSION

In this study, we have presented a tool for the optimal planning of ESSs within a distribution network to achieve its dispatchability. We have shown that the uncertainty of the prosumption can be compensated sufficiently with the allocation and exploitation of ESS. The non-approximated and convex OPF model, or the AR-OPF model is implemented to account for the operational conditions of the distribution network accurately. The planning problem is decomposed into two blocks to satisfy the condition for the exactness of the solution via the AR-OPF model. In the 1st block, the allocation of ESS is determined along with the corresponding dispatch ERR by implementing the linearly approximated OPF model. The AR-OPF is used in the 2nd block of the problem to check whether the allocated capacity is compatible with real operation of the grid to satisfy the ERR and to determine the optimal location of the ESS to minimize the grid losses. We validated the effectiveness of the proposed method for the real Swiss ADN of 55 nodes by demonstrating that the allocation of ESS successfully reduced the dispatch error.

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