Photon Production from Nonequilibrium Disoriented Chiral Condensates in a Spherical Expansion

Yeo-Yie Charng and Kin-Wang Ng
Institute of Physics, Academia Sinica, Taipei, Taiwan, R.O.C.

Chi-Yong Lin and Da-Shin Lee
Department of Physics, National Dong Hwa University, Hua-Lien, Taiwan, R.O.C.

(Dated: August 2002)

We study the production of photons through the non-equilibrium relaxation of a disoriented chiral condensate formed in the expanding hot central region in ultra-relativistic heavy-ion collisions. It is found that the expansion smooths out the resonances in the process of parametric amplification such that the non-equilibrium photons are dominant to the thermal photons over the range 0.2-2 GeV. We propose that to search for non-equilibrium photons in the direct photon measurements of heavy-ion collisions can be a potential test of the formation of disoriented chiral condensates.

PACS numbers: 25.75-q, 11.30.Rd, 11.30.Qc, 12.38.Mh

In relativistic heavy ion collisions, the highly Lorentz contracted nuclei essentially pass through each other, leaving behind a hot plasma in the central rapidity region with large energy density corresponding to temperature above 200 MeV where the chiral symmetry is restored. This plasma then cools down via rapid hydrodynamic expansion through the chiral phase transition during which the long-wavelength fluctuations become unstable and grow due to the spinoidal instabilities [1, 2, 3]. The growth of these unstable modes results in the formation of disoriented chiral condensates (DCCs) (see also Refs. [4, 5]). The DCCs are the correlated regions of space-time where the chiral order parameter of QCD is chirally rotated from its usual orientation in isospin space. Subsequent relaxation of the DCCs to the true QCD vacuum is expected to radiate copious soft pions which could be a potential experimental signature of the chiral phase transition observable in the ultra-relativistic heavy ion collisions at RHIC and LHC although such a signature has been looked for and not seen in the lower energy heavy ion collisions at the CERN SPS [6]. However, since these emitted pions will undergo the strong final interaction, the signal may be severely masked and become indistinguishable from the background. It then becomes important to study other possible signatures of DCCs that would be less affected by the final state interaction. Electromagnetic probe such as photon and lepton with longer mean free path in the medium serves as a good candidate and can reveal more detailed non-equilibrium information on the DCCs with minimal distortion [7, 8, 9].

Minakawa and Muller [10] have suggested that the presence of strong electromagnetic fields in relativistic heavy ion collisions induces a quasi-instantaneous “kick” to the field configuration along the $\pi^0$ direction such that it is plausible that the chiral order parameter in the DCC domains, if formed, will acquire a component in the direction of the neutral pion. The production of photons through the non-equilibrium relaxation of a DCC within which the chiral order parameter initially has a non-vanishing expectation value along the $\pi^0$ direction and subsequently oscillates around the minimum of the effective potential has been considered. In Ref. [9], Boyanovsky et al. have extensively studied the photon production from the low energy coupling of the neutral pion to photon via the $U_A(1)$ anomalous vertex. They have found that for large initial amplitudes of the $\pi^0$ field photon production is enhanced by parametric amplification. These processes are non-perturbative with a large contribution during the non-equilibrium stages of the evolution and result in a distinct distribution of the produced photons. Later, the authors in Ref. [11] have taken into account another dominant contribution that also involves the dynamics of $\pi^0$ due to the decay of the vector meson through the electromagnetic vertex. Although the corresponding dimensionless effective coupling involving the vector meson is quite perturbatively small, for the large amplitude oscillations of the $\pi^0$ mean field, the contribution to the photon production is of the same order of magnitude as the anomalous interaction. However, they have ignored the hydrodynamical expansion and adopted the simple “quench” phase transition from an initial thermodynamic equilibrium state at a temperature ($T_i$) higher than the critical temperature ($T_c$) for the chiral phase transition cooled instantaneously to zero temperature, which has been widely used in the study of non-equilibrium phenomena of DCCs [3, 4, 5, 7].
In this paper, we will consider the effect of the hydrodynamical expansion of the plasma to the production of photons from the non-equilibrium relaxation of a DCC. Although the “quench” scenario can capture the qualitative features of this non-equilibrium problem, we will see that the dynamics of parametric amplification as well as spinoidal instabilities is considerably modified by the plasma expansion. In fact, our study of the photon production based upon this strongly out-of-equilibrium scenario bears analogy to the recent studies in the context of the off-shell photon production from the out-of-equilibrium expanding quark-gluon plasma with a finite life time \[12\]. To make a comparison, we also compute the thermal equilibrium photon emission from the hot quark-gluon plasma as well as the hadronic matter following an adiabatic quark-gluon phase transition by convoluting the equilibrium photon production rates at finite temperature with the expansion dynamics.

In ultra-relativistic heavy-ion collisions, it is expected that the rapidity density of the particles produced in the hot central region has a plateau. This implies an approximate Lorentz boost invariance along the longitudinal direction in the evolution of the hot plasma in the central region \[13\]. However, at late times following the heavy nuclei collisions, a transverse flow can be generated due to the multi-scattering between the produced particles, as such the expansion becomes three dimensional \[14\]. Here we will simply assume that the hydrodynamical flow is spherically symmetric, and that the boost is along the radial direction. We will see that this assumption greatly simplifies the treatment of the photon field in the expanding space-time. We consider this as the first simplest attempt to tackle the problem.

The natural coordinates for spherical boost invariant hydrodynamical flow are the proper time \(\tau\) and the space-time rapidity \(\eta\) defined as

\[
\tau \equiv (t^2 - r^2)^{\frac{1}{2}}, \quad \eta \equiv \frac{1}{2} \ln \left(\frac{t + r}{t - r}\right),
\]

where \((t, r)\) are the coordinates in the laboratory,

\[
t = \tau \cosh \eta, \quad r = \tau \sinh \eta.
\]

The ranges of these coordinates are set to be \(0 \leq \tau < \infty\) and \(0 \leq \eta < \infty\), restricted to the forward light cone. In terms of spherical coordinates, the Minkowski line element is given by

\[
ds^2 = dt^2 - d\vec{r}^2 = d\tau^2 - \tau^2(d\eta^2 + \sinh^2 \eta d\theta^2 + \sin^2 \eta \sin^2 \theta d\phi^2).
\]

Eq. (4) is the Robertson-Walker metric for an open expanding universe with the scale factor \(R(\tau) = \tau\) \[13\]. For \(\eta < 1\), the open metric \[4\] can be approximated by the flat Robertson-Walker metric,

\[
ds^2 = d\tau^2 - \tau^2(d\eta^2 + \eta^2 d\theta^2 + \eta^2 \sin^2 \theta d\phi^2),
\]

\[
= d\tau^2 - \tau^2 d\eta^2.
\]

This flat metric will serve as the background comoving frame under which to study non-equilibrium photon production from DCC domains. This is a good first-order approximation since the boost invariance is in anyway applied only for small values of \(\eta\). Furthermore, \(\eta < 1\) corresponds to the space-time with \(r/t < 0.76\) which has covered a major portion of the forward light cone.

The relevant phenomenological effective action in a general expanding space-time is given by

\[
S = \int d^4x \sqrt{g} (L_\sigma + L_A + L_{\pi^0A}),
\]

where

\[
L_\sigma = -\frac{1}{2\sqrt{g}} g^{\mu\nu} \partial_\mu \Phi \cdot \partial_\nu \Phi + \frac{1}{2} M_\Phi^2 \Phi \cdot \Phi - \lambda \left(\Phi \cdot \Phi\right)^2 + h\sigma,
\]

\[
L_A = -\frac{1}{4} g^{\alpha\beta} g^{\gamma\delta} F_{\alpha\beta} F_{\gamma\delta},
\]

\[
L_{\pi^0A} = \frac{1}{\sqrt{g}} \frac{e^2}{32\pi^2 f_\pi} \epsilon^{\alpha\beta\mu} F_{\alpha\beta} F_{\mu\nu} + \frac{1}{(\sqrt{g})^2 8m_\pi^2 m_\pi^2} e^{2\nu2} \epsilon^{\alpha\beta\gamma\delta} g_{\epsilon\delta} \partial_\epsilon \pi^0 \partial_\gamma \pi^0 F_{\mu\nu} F_{\alpha\beta},
\]

where \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\) is the photon field, and \(\Phi = (\sigma, \pi^0, \vec{\pi})\) is an \(O(N + 1)\) vector of scalar fields with \(\vec{\pi} = (\pi^1, \pi^2, ..., \pi^{N-1})\) representing the \(N - 1\) pions. Note that the signature is \((-+++)\), and we have added \(1/\sqrt{g}\)
to each $\epsilon^{\alpha\beta\mu\nu}$ because it is a tensor density of weight $-1$ \[14\]. The phenomenological parameters in the effective Lagrangian above can be determined by the low-energy pion physics as follows:

$$m_\pi \approx 600\text{ MeV}, \quad f_\pi \approx 93\text{ MeV}, \quad \lambda \approx 4.5, \quad T_c \approx 200\text{ MeV},$$

$$h \approx (120\text{ MeV})^3, \quad m_V \approx 782\text{ MeV}, \quad \lambda_V \approx 0.36,$$  \hspace{1cm} (10)

where $V$ is identified as the $\omega$ meson, and the coupling $\lambda_V$ is obtained from the $\omega \rightarrow \pi^0\gamma$ decay width \[10\]. This effective action has been obtained by two of us \[11\] in the study of out-of-equilibrium photon production from DCC domains by taking the rapid "quench" phase transition scenario. Here we will consider the similar non-equilibrium phenomena taking account of the hydrodynamical expansion. It must be noticed that this is an effective field theory with an ultraviolet momentum cutoff of the order of $\Lambda \approx 1\text{ GeV}$. The effective vertices obtained are in terms of the perturbative theory without involving in-medium modifications. In-medium effects will enter only through the non-equilibrium Green’s functions for the meson fields as we will see below. To obtain these effective vertices including the strongly out of equilibrium effects, one should integrate out the quark fields and the vector meson in the context of the fully non-equilibrium formalism. It deserves to tackle in the near future.

It is well known that the minimal coupling of photons to the metric background is conformally invariant. Thus, in the conformally flat metric \[3\], it is convenient to work with the conformal time defined by

$$du \equiv \tau_i \frac{d\tau}{\tau},$$  \hspace{1cm} (11)

where $\tau_i$ is the initial proper time after which we expect that the quark-gluon plasma is formed and thermalized. Integration gives

$$u = \tau_i \ln \left( \frac{\tau}{\tau_i} \right).$$  \hspace{1cm} (12)

Rewrite the flat metric \[3\] as

$$ds^2 = -g_{\mu\nu}dx^\mu dx^\nu = a^2(u)(du^2 - d\vec{x}^2),$$  \hspace{1cm} (13)

where $a(u) = \tau/\tau_i = e^{u/\tau_i}$ and $d\vec{x} = \tau_d d\vec{y}$. Writing explicitly in terms of the comoving coordinates and defining $\Phi = \Phi_\alpha/a$, the action \[3\] becomes

$$S = \int du \, d^3\vec{x} \, \mathcal{L} = \int du \, d^3\vec{x} \, \left( \mathcal{L}_\sigma + \mathcal{L}_A + \mathcal{L}_{\pi^0A} \right),$$  \hspace{1cm} (14)

where

$$\mathcal{L}_\sigma = -\frac{1}{2} \eta^{\mu\nu} \partial_\mu \Phi_\alpha \cdot \partial_\nu \Phi_\alpha + \frac{1}{2} \left[ M_\sigma^2 + \frac{1}{a} \frac{d^2 a}{du^2} \right] \Phi_\alpha \cdot \Phi_\alpha - \lambda \left( \Phi_\alpha \cdot \Phi_\alpha \right)^2 + a^3 h \sigma_\alpha,$$  \hspace{1cm} (15)

$$\mathcal{L}_A = \frac{1}{4} \eta^{\alpha\mu} \eta^{\beta\nu} F_{\alpha\beta} F_{\mu\nu},$$  \hspace{1cm} (16)

$$\mathcal{L}_{\pi^0A} = \frac{e^2 \pi_0^0}{32\pi^2 a f_\pi} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} + \frac{1}{a^2} \frac{e^2 \lambda_0}{8 M_\sigma^2 m_V^2} \epsilon^{\alpha\beta\gamma\delta} \eta_{\alpha\beta} \eta_{\gamma\delta} \partial_\lambda \left( \frac{\pi_0^0}{a} \right) \partial_\gamma \left( \frac{\pi_0^0}{a} \right) F_{\mu\nu} F_{\alpha\beta},$$  \hspace{1cm} (17)

where $\Phi_\alpha = (\sigma_\alpha, \pi_0^0, \pi^\alpha)$ and $\eta^{\mu\nu}$ is the Minkowski metric. In terms of the conformal time, the effective action now has analogy with the effective action in Minkowski spacetime with the time dependent mass term and interactions.

The dynamics of the non-equilibrium expectation values as well as correlation functions of quantum fields can be obtained by implementing the Schwinger-Keldysh closed-time-path formulation of non-equilibrium quantum field theory. This formulation is used to describe the evolution of an initially prepared density matrix, and requires a path integral defined along a closed time contour. This techniques have successfully been employed elsewhere within many different contexts and we refer the readers to the literature for details \[17\]. Here we first shift $\sigma_\alpha$ and $\pi_\alpha^0$ by their expectation values with respect to an initial non-equilibrium states:

$$\sigma_\alpha(\vec{x}, u) = \phi_\alpha(u) + \chi_\alpha(\vec{x}, u), \quad \langle \sigma_\alpha(\vec{x}, u) \rangle = \phi_\alpha(u),$$

$$\pi_\alpha^0(\vec{x}, u) = \zeta_\alpha(u) + \psi_\alpha(\vec{x}, u), \quad \langle \pi_\alpha^0(\vec{x}, u) \rangle = \zeta_\alpha(u),$$  \hspace{1cm} (18) (19)

where
with the tadpole conditions:

\[ \langle \chi_a(\vec{x}, u) \rangle = 0, \quad \langle \psi_a(\vec{x}, u) \rangle = 0, \quad \langle \bar{\pi}_a(\vec{x}, u) \rangle = 0. \] (20)

One can impose these tadpole conditions to all orders in the corresponding expansion to derive the non-equilibrium evolution equations of field expectation values. In addition, the large-\( N \) approximation will be implemented below (Eqs. (2.7)-(2.9) of Ref. [9]) to obtain the factorized Lagrangian which provides a non-perturbative framework to self-consistently incorporate quantum fluctuation effects from the strong \( \sigma - \pi \) interactions. As for the issue of the gauge choice, we adopt here the Coulomb gauge which involves only the physical degrees of freedom without any redundant fields to study non-equilibrium photon production from the oscillating mean fields within the DCC domains in a spherical expansion.

Using the tadpole conditions (20), the large-\( N \) equations of motion for the mean fields involving the perturbatively electromagnetic corrections can be obtained as follows:

\[
\begin{align*}
&\left[ \frac{d^2}{du^2} + m_a^2(u) + 4\lambda \phi_a^2(u) + 4\lambda \zeta_a^2(u) + 4\lambda \langle \bar{\pi}_a^2 \rangle(u) \right] \phi_a - a^3 h = 0, \\
&\left[ \frac{d^2}{du^2} + m_a^2(u) + 4\lambda \phi_a^2(u) + 4\lambda \zeta_a^2(u) + 4\lambda \langle \bar{\pi}_a^2 \rangle(u) \right] \zeta_a - \frac{e^2}{32\pi^2 f_\pi a} (F \tilde{F}) \\
&+ \frac{e^2 \lambda^2_p}{4m^2_\pi^2 m^2_v} \frac{1}{a \, du} \frac{1}{a^2 \, du} \left( \frac{\zeta_a}{a} \right) \varepsilon^{\mu \nu \sigma} \varepsilon^{\alpha \beta \delta} \eta_{\sigma \delta} \langle F_{\mu \nu} F_{\alpha \beta} \rangle \\
&+ \frac{e^2 \lambda^2_p}{4m^2_\pi^2 m^2_v} \frac{1}{a^3 \, du} \left( \frac{\zeta_a}{a} \right) \varepsilon^{\mu \nu \sigma} \varepsilon^{\alpha \beta \gamma} \eta_{\sigma \delta} \langle \partial_\gamma F_{\mu \nu} F_{\alpha \beta} \rangle = 0.
\end{align*}
\] (21)

We then decompose the fields \( \bar{\pi}_a \) and \( \tilde{A}_T \) into their Fourier mode functions \( U_{ak}(u) \) and \( V_{Ak}(u) \) respectively,

\[ \bar{\pi}_a(\vec{x}, u) = \int \frac{d^3k}{(2\pi)^3 \omega_{\bar{\pi}_a,k,i}} \bar{\alpha}_k U_{ak}(u) e^{i\vec{k} \cdot \vec{x}} + \text{h.c.}, \] (22)

\[ \tilde{A}_T(\vec{x}, u) = \int \frac{d^3k}{(2\pi)^3 \omega_{\tilde{A}_k,i}} \tilde{A}_T(k, u) \\
= \int \frac{d^3k}{(2\pi)^3 \omega_{\tilde{A}_k,i}} \left\{ b^*_{+k} V_{+k}(u) e_{+k} + b^*_{-k} V_{-k}(u) e_{-k} \right\} e^{i\vec{k} \cdot \vec{x}} + \text{h.c.}, \] (23)

where \( \bar{\alpha}_k \) and \( b_{\pm k} \) are destruction operators, and \( e_{\pm k} \) are circular polarization unit vectors. The frequencies \( \omega_{\bar{\pi}_a,k,i} \) and \( \omega_{\tilde{A}_k,i} \) can be determined from the initial states and will be specified below. The mode equations for the photons as well as the pions can be read off from the quadratic part of the effective factorized Lagrangian as

\[ \frac{d^2}{du^2} + k^2 + m_a^2(u) + 4\lambda \phi_a^2(u) + 4\lambda \zeta_a^2(u) + 4\lambda \langle \bar{\pi}_a^2 \rangle(u) \right] U_{ak} = 0, \] (24)

\[ \frac{d^2}{du^2} + \left\{ 1 - \frac{e^2 \lambda^2_p}{m^2_\pi m^2_v} \frac{\zeta^2}{a^2} \right\} k^2 + \frac{e^2}{2\pi^2 f_\pi} \zeta_k \right\} V_{1k} = 0, \] (25)

\[ \frac{d^2}{du^2} + \left\{ 1 - \frac{e^2 \lambda^2_p}{m^2_\pi m^2_v} \frac{\zeta^2}{a^2} \right\} k^2 + \frac{e^2}{2\pi^2 f_\pi} \zeta_k \right\} V_{2k} = 0, \] (26)

where \( m_a^2(u) = -(1/2) a^2 M^2 - \bar{a}/a, \zeta(u) = \zeta_a(u)/a, k = [\vec{k}] \), and the dot means \( d/du \). The expectation values with respective to the initial states are given by

\[ \langle \bar{\pi}_a^2 \rangle(u) = (N - 1) \int \frac{d^3\vec{k}}{(2\pi)^3 \omega_{\bar{\pi}_a,k}} \left| U_{ak}(u) \right|^2 \coth \left[ \frac{\omega_{\bar{\pi}_a,k}}{2T_i} \right], \]

\[ \varepsilon^{\alpha \beta \mu \nu} \langle F_{\alpha \beta} F_{\mu \nu} \rangle(u) = \int \frac{d^3\vec{k}}{(2\pi)^3 \omega_{\tilde{A}_k}} \frac{\frac{d}{du}}{d u} \left( \frac{4k}{u} \right) \left| V_{1k}(u) \right|^2 - \left| V_{1k}(u) \right|^2, \]

\[ \varepsilon^{\mu \nu \sigma} \varepsilon^{\alpha \beta \delta} \eta_{\sigma \delta} \langle F_{\mu \nu} F_{\alpha \beta} \rangle(u) = \int \frac{d^3\vec{k}}{(2\pi)^3 \omega_{\tilde{A}_k}} \frac{\frac{d}{du}}{d u} \left( \frac{4k^2}{u} \right) \left| V_{1k}(u) \right|^2 + \left| V_{2k}(u) \right|^2, \]
\[ e^{\mu \nu \alpha} \epsilon_{\alpha \beta} \langle \partial_\gamma F_{\mu \nu} F_{\gamma \beta} \rangle = \int \frac{d^3 \vec{k}}{(2\pi)^3 \omega_{Ak}} (4k^2) \frac{d}{du} \left[ |V_{1k}(u)|^2 + |V_{2k}(u)|^2 \right], \tag{27} \]

where \( \langle \pi_n^2 \rangle(u) \) is self-consistently determined by Eq. (24). In particular, when \( u = 0 \), it becomes the gap equation,

\[ \langle \pi_n^2 \rangle(0) = (N - 1) \int^\Lambda \frac{d^3 \vec{k}}{(2\pi)^3 \omega_{Ak,i}} \coth \left[ \frac{\omega_{\pi_n,k,i}}{2T_i} \right]. \tag{28} \]

Notice that the photonic medium effects have been ignored in the above expressions of the photon field expectation values due to the fact that the photons interact electromagnetically so that their mean free paths are expected to be longer than the estimated size of the quark-gluon plasma fireball, and the produced photons will escape freely toward the particle detector having no enough time to build up their population in the plasma. In this conformal frame, the momentum cutoff we choose depends linearly on \( a \) so as to keep the physical momentum cutoff \( \Lambda \) fixed in the laboratory frame. As a result, the initial conditions for the mode functions become more subtle here. Here we choose

\[ U_{ak < \Lambda}(0) = 1, \quad \dot{U}_{ak < \Lambda}(0) = -i \omega \pi_n, \omega_{Ak,i} = k; \]

\[ \omega_{\pi_n,k,i} = k^2 + m_a^2(0) + 4\lambda\phi_a^2(0) + 4\lambda\pi_n^2(0) + 4\lambda\pi_n^2(0) \equiv k; \]

\[ V_{1,2k < \Lambda}(0) = 1, \quad \dot{V}_{1,2k < \Lambda}(0) = -i \omega \pi_n, \omega_{Ak,i} = k; \]

\[ \omega_{\pi_n,k,i} = k^2 + m_a^2(0) + 4\lambda\phi_a^2(0) + 4\lambda\pi_n^2(0) + 4\lambda\pi_n^2(0) \equiv k; \]

\[ V_{1,2k > \Lambda}(u_k) = 0, \quad \dot{V}_{1,2k > \Lambda}(u_k) = -i \omega \pi_n, \omega_{Ak,i} = k. \tag{29} \]

\[ U_{ak > \Lambda}(0) = 1, \quad \dot{U}_{ak > \Lambda}(0) = -i \omega \pi_n, \omega_{Ak,i} = k; \]

\[ \omega_{\pi_n,k,i} = k^2 + m_a^2(0) + 4\lambda\phi_a^2(0) + 4\lambda\pi_n^2(0) + 4\lambda\pi_n^2(0) \equiv k; \]

\[ V_{1,2k > \Lambda}(u_k) = 0, \quad \dot{V}_{1,2k > \Lambda}(u_k) = -i \omega \pi_n, \omega_{Ak,i} = k. \tag{30} \]

Therefore, for the momentum lying below the momentum cutoff \( \Lambda \) at the initial conformal time, the initial condition for the mode function can be set at \( u = 0 \) as shown in Eq. (23). However, for the momentum above the cutoff momentum \( \Lambda \) at \( u = 0 \), it will become dynamical when \( k \leq \Lambda (u) \) after \( u = u_k = \tau, \ln(k/\Lambda) \), and the initial condition for this mode function must be set at that conformal time \( u_k \) as in Eq. (30). The photon spectral number density at time \( u \) is given by the expectation value of the number operator for the asymptotic photons [3],

\[ \frac{dN}{d^3 \vec{x} d^3 \vec{k}} = \frac{dN_+}{d^3 \vec{x} d^3 \vec{k}} + \frac{dN_-}{d^3 \vec{x} d^3 \vec{k}} = \left\langle N_k(u) \right\rangle \]

\[ = \frac{1}{2k} \left[ \dot{A}_T(\vec{k}, u) \cdot \dot{A}_T(-\vec{k}, u) + k^2 \dot{A}_T(\vec{k}, u) \cdot \dot{A}_T(-\vec{k}, u) \right] - 1 \]

\[ = \frac{1}{4k^2} \left[ \dot{V}_{1k}(u)^2 + k^2 |V_{1k}(u)|^2 \right] + \frac{1}{4k^2} \left[ \dot{V}_{2k}(u)^2 + k^2 |V_{2k}(u)|^2 \right] - 1, \tag{31} \]

and the invariant photon spectral production rate is given by \[ \frac{kdN}{dud^3 \vec{x} d^3 \vec{k}} = \frac{1}{4} \left\{ \frac{e^2}{m_e^2 m_e^2} \frac{\dot{q}^2}{k^2} \frac{d}{du} \left[ |V_{1k}|^2 + |V_{2k}|^2 \right] + \frac{e^2}{2\pi^2 f_\pi} \frac{\dot{q}}{\omega} \frac{d}{du} \left[ |V_{1k}|^2 - |V_{2k}|^2 \right] \right\}. \tag{32} \]

We now need to relate this spectral production rate to the invariant production rate measured in the metric [3] in order to obtain the photon spectrum in the laboratory frame. From the coordinate transformations [3], we find that

\[ q = p \cosh \eta + p_r \sinh \eta, \]

\[ q_\eta = \rho \sinh \eta + p_r \cosh \eta, \]

\[ q_\theta = p_\theta, \]

\[ q_\phi = p_\phi, \tag{33} \]

and the zero-mass condition of the photon, i.e., \( g^{\mu \nu} p_\mu p_\nu = 0 \),

\[ q = \frac{|\vec{q}|}{\tau} = \frac{1}{\tau} \left( q^2 + \frac{1}{\sin^2 \eta} q^2 \right)^\frac{1}{2}, \]

\[ p = \frac{|\vec{p}|}{\rho} = \left( \rho^2 + \frac{1}{\rho^2} q^2 \right)^\frac{1}{2}, \tag{34} \]
where \( q_\mu = (q, \vec{q}) \) is the photon four-momentum in the metric (4) and \( p_\mu = (p, \vec{p}) \) is that measured in the laboratory frame. Because of the spherical symmetry of the problem, for convenience, we can choose \( \vec{p} \) to be along the \( z \)-axis, which is also the polar axis. This gives \( p_\tau = p \cos \theta \), and then from Eq. (33),
\[
q = p (\cosh \eta + \cos \theta \sinh \eta).
\]
Hence, from the momentum transformation laws (33), one can change the momentum variables from \( q_\mu \) into \( p_\mu \) in the invariant production rate making use of the fact that the spectral particle number density \( dN \) is invariant under such transformations. We then find that
\[
\frac{dN}{d\tau} = \frac{q}{d\eta d\vec{q} d\tau d\eta d\theta d\phi} = \frac{p}{d^3 \vec{p}} \tau^3 \sinh^2 \eta \sin \theta d\tau d\eta d\theta d\phi ,
\]
where \( d\Gamma \) stands for the invariant phase space element in the comoving frame. Therefore, the photon spectrum measured in the laboratory at the final time \( \tau_f \) can be obtained by integrating the invariant spectral production rate over the space-time history of the hydrodynamical evolution in the comoving frame as
\[
\frac{p dN}{d^3 \vec{p}} = \int_{\tau_1}^{\tau_f} d\tau \int_0^{\eta_{\text{max}}} d\eta \int_0^\pi d\theta \int_0^{2\pi} d\phi \tau^3 \sinh^2 \eta \sin \theta \frac{dN}{d\tau} (\tau, q) ,
\]
where \( q = p (\cosh \eta + \cos \theta \sinh \eta) \). Now, in the small \( \eta \) approximation, one can link the invariant rate in the comoving frame to that of the conformal frame as given in Eq. (33), where \( k = q \tau / \tau_i \) and \( \vec{k} = \vec{q} / \tau_i \), in the following way:
\[
\frac{dN}{d\tau} \sim \frac{q}{d\tau d^3 \vec{q} \bar{q}} \sim \left( \frac{\tau_i}{\tau} \right)^2 \frac{k}{dud\bar{u}d\vec{q}d\vec{k}} \frac{dN}{d\tau} (u, k),
\]
where \( u = \tau_i \ln (\tau_i / \tau_i) \). Then, we can approximate
\[
\frac{p dN}{d^3 \vec{p}} \sim \int_{\tau_1}^{\tau_f} d\tau \int_0^{\eta_{\text{max}}} d\eta \int_0^\pi d\theta \int_0^{2\pi} d\phi \tau^2 \tau_i \sinh^2 \eta \sin \theta \frac{k}{dud\bar{u}d\vec{q}d\vec{k}} (u, k)
\]
\[
\sim \frac{2\pi \tau_i^2}{p} \int_{\tau_1}^{\tau_f} d\tau \int_{\eta_{\text{max}}/\tau_i}^{\eta_{\text{max}}/\tau_i} d\eta \int_{\bar{u} = \eta_{\text{max}}/\tau_i}^{\bar{u} = \eta_{\text{max}}/\tau_i} d\bar{u} \int_{\vec{k} = \eta_{\text{max}}/\tau_i}^{\vec{k} = \eta_{\text{max}}/\tau_i} d\vec{k} \left[ \cosh (\eta_{\text{max}}) - \frac{1}{2} \left( \frac{k}{p \tau_i} + \frac{p \tau_i}{k} \right) \right] \frac{k}{dud\bar{u}d\vec{q}d\vec{k}} (u, k).
\]
The final expression is obtained after changing the integration variable from \( \theta \) to \( k \) as well as carrying out the \( \phi \)- and \( \eta \)-integrals.

In contrast with the non-equilibrium photon production from the DCC domains, the equilibrium photon emission arising from an expanding quark-gluon plasma (QGP) as well as hadronic matter can be obtained by convoluting the equilibrium photon production rates at finite temperature with the expansion dynamics. We currently believe that after the ultra-relativistic heavy collisions, in the central rapidity regime, parton-parton multi-scatterings cause the produced quarks and gluons to reach local thermal equilibrium at the time scale of \( \tau_i \), and then the so-called QGP with high energy/entropy density is expected to be formed. After the formation of a QGP, the dynamics of the thermal plasma is dominated by the hydrodynamical expansion. The hydrodynamical approach is appropriate since in this stage quarks and gluons interact in such a dense regime that their mean free paths are much smaller than typical wavelengths of various collective phenomena. Subsequently, the plasma cools adiabatically according to the equation of state of a hot QGP down to the quark-hadron phase transition when the QGP starts to hadronize. Assuming that the phase transition is a first order one, quarks, gluons, and hadrons will coexist in a mixed phase in which the released latent heat keeps the temperature of the plasma unchanged at the critical temperature even though the plasma continues to expand. This mixed phase will exist until all the matter has converted into the hadrons. The system in the hadronic phase will continuously expand with the dynamics governed by the equation of state of a hot hadron gas. As a result, the temperature continuously drops till the so-called freeze-out temperature is reached where the mean free path for all hadrons is of the order of the size of the plasma, and after that all hadrons cease further interactions. We would like to emphasize at this stage that according to such a hydrodynamical picture, the strongly out of equilibrium scenario of either supercooling or superheating has been ignored in the sense that the entropy of the system is assumed to be locally conserved for the whole evolving history of the system. Therefore, photon emission under this adiabatic expansion is in striking contrast with that from the DCC domains associated with a strongly out of equilibrium scenario. In the Bjorken’s hydrodynamical model, the QGP as well as hadronic matter can be described as a fluid in local thermal equilibrium. The energy density, pressure, entropy etc. will depend on the proper time only, and will not depend on the space-time rapidity under the assumption of a boost invariance in the central rapidity region. The dynamics of the fluid is determined by the conservation laws of the energy-momentum,
baryon number, and entropy. In addition, an equation of state of the fluid in the various phases can serve as an input to close this set of dynamical equations to be solved self-consistently. Here, to be consistent with the calculation of the non-equilibrium photon production from DCC domains, we also assume that the underlying hydrodynamical expansion is of a spherical expansion with the coordinates of the proper time \( \tau \) and the space-time rapidity \( \eta \) that results from free-streaming particles with a collective velocity \( v = r/t \) along the radial direction, where the fluid rapidity is equal to the space-time rapidity. In the central rapidity region which is baryon free, the conservation law of entropy for such an adiabatic expansion reads

\[
s(T)\tau^3 = s(T_i)\tau_i^3. \tag{40}
\]

This conservation law will play an important role in determining the cooling law as well as the duration time scales of various phases of the evolving system from the initial QGP phase, to the quark-hadron mixed phase, and to the final hadronic phase.

In the QGP phase, the entropy density is dominated by the relativistic massless gas composed of quarks and gluons given by

\[
s_Q = \frac{2\pi^2}{45} g_G T^3, \tag{41}
\]

where the degeneracy \( g_G = (2 \cdot 8 + 2 \cdot 2 \cdot 3 \cdot 2 \cdot \frac{3}{2}) \) for the two-flavor quarks and the eight SU(3) gluons. On the other hand, the entropy density of hadronic matter is taken as

\[
s_H = \frac{2\pi^2}{45} g_H(T) T^3, \tag{42}
\]

where \( g_H(T) \) is an effective degeneracy for a hadron gas. We introduce a temperature dependence in this effective degeneracy to account for the medium effects from the coupled hadron gas in the temperature regime of our interest. The variation of the effective degeneracy can be parametrized as \( g_H(T) \sim 13(T/{\text{fm}^{-1}})^3 \) where the generally acceptable value of \( \delta \) is about \( \delta \approx 3.4 \) of slight model-dependence (here we adopt the Walecka model) \([13]\). We are now in a position to determine the time scale in the QGP phase which starts from the initial formation time \( \tau_i \) until \( \tau_Q \) when the temperature drops to the critical temperature \( T_c \) of the quark-hadron phase transition and a phase transition to hadronic matter starts. Following the conservation law of entropy together with an expression of \( s_Q \) in Eq. (41), the cooling law in the QGP phase can be obtained as \( T = (\tau_i/\tau) T_i \), and thus \( \tau_Q = (T_i/T_c) \tau_i \). However, during the quark-hadron mixed phase, the relative proportion of QGP to hadronic matter must vary with time. Since the temperature of the quark-hadron mixed state remains constant at \( T = T_c \) even though the plasma continuously expands, the entropy density of the mixed state is equal to the sum of the entropy densities of QGP and hadronic matter at \( T = T_c \), \( s_Q^c \) and \( s_H^c \), respectively weighted by their fractions, \( f_Q(\tau) \) and \( f_H(\tau) \):

\[
s_{\text{mix}}(\tau) = f_Q(\tau) s_Q^c + f_H(\tau) s_H^c, \tag{43}
\]

where \( f_Q(\tau) + f_H(\tau) = 1 \). Initially at \( \tau = \tau_Q \), the phase consists entirely of QGP, i.e. \( f_Q(\tau_Q) = 1 \), and at the end, entirely of hadronic matter at \( \tau_H \), i.e. \( f_H(\tau_H) = 1 \). Thus, the life time of the mixed state is \( \tau_H - \tau_Q \). \( f_Q(\tau), f_H(\tau) \), and \( \tau_H \) are to be determined from the entropy conservation law \([10]\) which presumably also holds in the mixed state. Then, from Eqs. (43) and (44), we have

\[
(f_Q(\tau) s_Q^c + [1 - f_Q(\tau)] s_H^c) \tau^3 = s_Q(\tau_Q)^3. \tag{44}
\]

Hence, we obtain

\[
f_Q(\tau) = \frac{s_Q^c (\tau_Q)^3}{s_Q^c - s_H^c}, \quad f_H(\tau) = 1 - f_Q(\tau). \tag{45}
\]

Therefore, the mixed phase ends at \( \tau_H = (s_Q^c/s_H^c)^{1/3}\tau_Q \) determined from \( f_Q(\tau_H) = 0 \) above. As for the hadronic phase, substituting the associated entropy density of a hot hadron gas \([12]\) into the entropy conservation law \([11]\) can lead to the cooling law in this phase given by \( T = T_c(\tau_H/\tau)^{3/(\delta+3)} \) from which the freeze-out time can be obtained as \( \tau_f = \tau_H (T_c/T_f)^{(\delta+3)/3} \).

After having obtained all relevant duration time scales in various phases from the QGP phase to the hadronic phase, we now turn into the discussions of the photon production processes from the QGP and the hadronic phase. The calculation of the thermal photon production rates in the high energy regime \( E >> T \) from the QGP at one-loop order in the hot thermal loop (HTL) approximation has been obtained by Kapusta et al. to account for
the photon production from the annihilation of a quark-antiquark pair into a photon and a gluon \((q\bar{q} \rightarrow g\gamma)\) as well as the absorption of a gluon by a quark (antiquark) emitting a photon \((g(q)g \rightarrow q(\bar{q})\gamma)\), similar to Compton scattering in QED [13]. However, the recent study by Aurel et al. has shown that the two-loop contributions to the photon production under the HTL approximation from quark (antiquark) bremsstrahlung \((qqg \rightarrow q(q)\gamma)\), and quark-antiquark annihilation with scattering \((q\bar{q}g(q) \rightarrow q(q)\gamma)\) are of the same order of magnitude as that in one-loop order [20]. Their contributions to the photon production rates prove to be dominant at high photon energies. Later, we will adopt this two-loop result (Eqs. (1.2,3) of Ref. [21]) to calculate the equilibrium photon emission during the phases that involve the QGP. As for the photon production processes from the hadronic phase, it is known that the dominant processes in the energy regime of our interest are those of the reactions \(\pi\pi \rightarrow \rho\gamma, \pi\rho \rightarrow \pi\pi\gamma\) [23]. The reaction for producing photons that involves the intermediary axial vector meson \(a_1\) is also important, for example, \(\pi\rho \rightarrow a_1 \rightarrow \pi\pi\gamma\) [24]. All of these hadronic processes will be taken into account in the calculation of the equilibrium photon emission from the thermal hadrons. A point to be stressed is that we consider the photon production processes from the low-mass hadrons only while the higher mass resonances would be Boltzmann suppressed. We then use the invariant photon production rates either from the QGP or from the hadrons combining with the relevant cooling laws as well as their expansion dynamics to numerically obtain the total equilibrium photon emission for the whole evolution of the system. Notice that the equilibrium photon production invariant rates obtained in the literature [13,23] in the rest frame of the emitting matter can be expressed in terms of the comoving frame of a thermal bath with temperature being a function of the proper time \(\tau\) under the adiabatic expansion. We can denote those invariant rates from the QGP and the hadron gas in the comoving frame respectively by \(dN/d\tau \big|_Q\) and \(dN/d\tau \big|_H\) that both depend on the photon four-momentum \(q_\mu\) as well as the temperature \(T(\tau)\) with \(\tau\) dependence following their respective cooling laws. Finally, the equilibrium photon emission starting from the formation time \(\tau_i\) down to the freeze-out time \(\tau_f\) in the laboratory frame can be obtained by a Lorentz boost from the comoving frame to the center of mass frame. Thus it becomes with Eq. (47)

\[
\frac{dN}{d\vec{p}} = \int_{\tau_i}^{\tau_f} d\tau \int_{0}^{\tau_{max}} d\eta \int_{0}^{\pi} d\theta \int_{0}^{2\pi} d\phi \, \frac{\tau^3\sin^2\eta\sin\theta}{\sin^2\gamma} \frac{dN}{d\tau} \bigg|_H \left( q, T = T_e \frac{\tau_1}{\tau} \right)
\]

\[= \int_{\tau_i}^{\tau_f} d\tau \int_{0}^{\tau_{max}} d\eta \int_{0}^{\pi} d\theta \int_{0}^{2\pi} d\phi \, \frac{\tau^3\sin^2\eta\sin\theta}{\sin^2\gamma} \frac{dN}{d\tau} \bigg|_H \left( q, T = T_e \left( \frac{\tau}{\tau_1} \right)^{\frac{\tau}{\tau_1}} \right)
\]

where again \(q = p(\cosh\eta + \cos\theta\sinh\eta)\) in Eq. (35).

We now perform the numerical study. We first solve the coupled differential equations Eq. (21), Eqs. (24-26), and the field expectation values Eq. (27) self-consistently with the initial conditions \(\zeta(0) = 0.3\) fm\(^{-1}\) and \(\dot{\zeta}(0) = \dot{\phi}(0) = 0\) in the conformal time frame. The initial temperature \(T_i\) at the QGP formation time \(\tau_i = 1.0\) fm\(^{-1}\) that corresponds to the initial conformal time \(u = 0\) is chosen to be \(T_i = 1.0\) fm\(^{-1}\). We set \(N = 3\) for 3 pions. In Figs. 1(a) and 1(b), we show that \(\zeta(u)\) evolves with strong damping, while the amplitude of the field derivative \(\dot{\zeta}(u)\) grows. The strong damping is due to the back-reaction effects from the creation of excitations of the quantum fluctuations via parametric amplification. This translates into photon production as we will discuss later. In fact, the growing \(\dot{\zeta}(u)\) makes photon production become more effective. The spectra of emitted photons in the conformal time frame are depicted for \(u = 1, 1.5,\) and \(2\) fm\(^{-1}\) respectively in Fig. 1(c). They feature prominent peaks. Notice that the peaks are located at \(k \approx \omega_\zeta/2\) and \(k \approx \omega_\dot{\zeta}\), where \(\omega_\zeta\) is the time-averaged oscillation frequency of \(\dot{\zeta}\) which can be read off from Fig. 1(b). Since \(\omega_\dot{\zeta}\) increases with time, both of the peaks move towards the higher momenta as we can see from Fig. 1(c). Therefore, it is evident that the photon production is due to parametric amplification. Figs. 2(a), 2(b), and 2(c) are shown for a large initial amplitude \(\zeta(0) = 1.0\) fm\(^{-1}\) and \(\dot{\zeta}(0) = \dot{\phi}(0) = 0\). This is clearly the non-perturbative effects driven by the parametric amplification which exhibits the features of unstable bands leading to the growth of the fluctuation modes. The growth of the fluctuations turns into the profuse photon production in the modes within the unstable bands. It is not surprising that with a large initial field expectation value, the peaks in the photon spectrum (Fig.2(c)) are about an order of magnitude larger than the previous case.

The photon spectrum in the laboratory frame emitted from the non-equilibrium DCC domains can be obtained by integrating the rates [23] over the spacetime history [24]. To make the comparison, we calculate the thermal photon emission from the adiabatic quark-hadron phase transition in Eq. (40) with the photon production rates for the processes discussed above. To do so, for the non-equilibrium DCC case, we assume that the QGP is formed with temperature \(T_i = 1.0\) fm\(^{-1}\) at the formation time \(\tau_i = 1.0\) fm. In addition, the cooling law in the QGP phase, i.e. \(\tau = (T/T_i)\tau_i\), gives the time scale \(\tau_Q = 1.1\) fm when the QGP begins hadronizing and the subsequent quark-hadron

The photon spectrum in the laboratory frame emitted from the non-equilibrium DCC domains can be obtained by integrating the rates [23] over the spacetime history [24]. To make the comparison, we calculate the thermal photon emission from the adiabatic quark-hadron phase transition in Eq. (40) with the photon production rates for the processes discussed above. To do so, for the non-equilibrium DCC case, we assume that the QGP is formed with temperature \(T_i = 1.0\) fm\(^{-1}\) at the formation time \(\tau_i = 1.0\) fm. In addition, the cooling law in the QGP phase, i.e. \(\tau = (T/T_i)\tau_i\), gives the time scale \(\tau_Q = 1.1\) fm when the QGP begins hadronizing and the subsequent quark-hadron
FIG. 1: (a) Temporal evolution of the $\pi^0$ mean field $\zeta(u)$. (b) Evolution of the derivative $\dot{\zeta}(u)$. (c) Photon spectral number density $dN/dx^3 dk$ for $\zeta(0) = 0.3$ fm$^{-1}$ at different times, where $u$ is in units of fm.
FIG. 2: As in Fig. 1 for $\zeta(0) = 1.0 \, \text{fm}^{-1}$. 
mixed state is formed at the critical temperature assumed to be $T_c = 0.9 \text{ fm}^{-1}$. One can also estimate the time when the mixed state ends, $\tau_H = (s_0^Q/s_0^H)^{1/3} = 1.77 \text{ fm}$, where the entropy densities are evaluated at $T = T_c$. In the final stage, the system turns into the hadronic phase starting at $\tau_H$ till the freeze-out time $\tau_f = 5.6 \text{ fm}$ estimated from the cooling law of the hadronic phase, $\tau_f = \tau_H (T_c/T_f)^{(\delta+3)/3}$, where $\delta \simeq 3.4$ [18] and the freeze-out temperature is chosen as $T_f = 0.5 \text{ fm}^{-1}$.

Figs. 3(a), 3(b), and 3(c) display the photon emission in the laboratory frame from the non-equilibrium DCC domains (solid line for $\zeta(0) = 1.0 \text{ fm}^{-1}$ and dashed line for $\zeta(0) = 0.3 \text{ fm}^{-1}$) and the adiabatic evolution of the QGP phase down to the hadronic phase (dotted line) for the spacetime rapidity $\eta = 0.65$, 0.8, and 1.0 respectively. In all three cases, our results indicate that the non-equilibrium photon emission from DCC domains for an initial value of the $\pi^0$ component of an order parameter about $1.0 \text{ fm}^{-1}$ that undergoes large amplitude oscillations with respect to the minimum value of the effective action is an order of magnitude larger than the photon emission from the QGP to the hadronic phase through an adiabatic first order phase transition. On the other hand, for an initial value of the $\pi^0$ below $0.3 \text{ fm}^{-1}$, due to small amplitude oscillations, the non-equilibrium DCC photon emission is not so effective.
as compared to the thermal photon emission. Therefore, we believe that if DCC domains are indeed formed, the nonlinear dynamics of the $\pi^0$ oscillations can create an enormous amount of non-equilibrium photons from the DCC domains. These photon emission enhancement effects could potentially provide a distinct experimental signature. In addition, the emitted non-equilibrium photon spectra from DCC domains in a hydrodynamical expansion shown in Fig. 3 reveal the dramatically different features as compared with the results in Refs. [9, 11], where the authors have considered the similar non-equilibrium phenomena in the temperature quenching scenario without convoluting the photon production rate with the plasma expansion, and shown that prominent peaks appear in the photon spectrum locating at resonant momenta of the parametric amplification determined by the frequency of the $\pi^0$ oscillations. Since the measured emitted photons are chosen with the momentum along the $z$ direction and the plasma expansion is along the radial direction, the Doppler effect would cause the emitted photons either blue-shifted if the photon comes from the place where the direction of its momentum is along with that of the expansion at that place, or red-shifted if the direction of its momentum is opposite to that of the expansion. As a result, as we can see in Fig. 3, the Doppler effect leads to smoothing out the resonant peaks found in Refs. [9, 11]. In addition, one can also estimate the corresponding boost factor arising from the background expansion effect. From Eq. (33) and $k = q\tau/\tau_i$, one can relate the momentum $k$ in the conformal frame to the momentum $p$ in the laboratory frame by $p \simeq e^{\eta}k$, where $\eta = 0.65$, 0.8 and 1.0 in Fig. 3. This background expansion may boost the photon energy, say below the ultraviolet momentum cutoff of 1 GeV produced from the low-energy effective interactions in Eqs. (17, 18), to the energy as high as the order of 2 GeV.

In conclusion, we have studied the production of photons through the non-equilibrium relaxation of a disoriented chiral condensate within which the chiral order parameter initially has a non-vanishing expectation value along the $\pi^0$ direction, taking into account the hydrodynamical expansion of the central rapidity region. The production of non-equilibrium photons driven by the oscillation of the $\pi^0$ field due to parametric amplification is modified by the expansion which smoothes out the resonant photon production in the "quench" scenario. However, the resulting photon spectrum exceeds that for thermal photons from quark-gluon plasma and hadronic matter for photon energies around 0.2 – 2 GeV. These non-thermal photons can be a potential test of the formation of disoriented chiral condensates in relativistic heavy-ion-collision experiments.

We would like to thank D. Boyanovsky, Hector de Vega, and S.-Y. Wang for their useful discussions. The work of Y.-Y. Charng, K.-W. Ng, and D.-S. Lee were supported in part by the National Science Council, ROC under the grants NSC90-2811-M-001-071, NSC90-2112-M-001-028, and NSC90-2112-M-259-011 respectively.

[1] K. Rajagopal and F. Wilczek, Nucl. Phys. B 399, 395 (1993); K. Rajagopal and F. Wilczek, Nucl. Phys. B 404, 577 (1993); for a review, see K. Rajagopal in “Quark-Gluon Plasma”, Ed. by R. C. Hwa (World Scientific, Singapore, 1995).
[2] F. Cooper, Y. Kluger, E. Mottola, and J. P. Paz, Phys. Rev. D 51, 2377 (1995); Y. Kluger, F. Cooper, E. Mottola, J. P. Paz, and A. Kovner, Nucl. Phys. A 590, 581 (1995); F. Cooper, Y. Kluger, and E. Mottola, Phys. Rev. C 54, 3298 (1996); M. A. Lampert, J. F. Dawson, and F. Cooper, Phys. Rev. D 54, 2213 (1996).
[3] D. Boyanovsky, H. J. de Vega, and R. Holman, Phys. Rev. D 51, 734 (1995).
[4] J. D. Bjorken, Int. J. Mod. Phys. A7, 4189 (1992); Acta Phys. Polon. B 23, 561 (1992); A. Anselm, Phys. Lett. B 217, 169 (1989); A. Anselm and M. Ryskin, Phys. Lett. B 226, 482 (1991); J. P. Blaizot and A. Krzywicki, Phys. Rev. D 46, 246 (1992); K. L. Kowalski and C. C. Taylor, hep-ph/9211252 (unpublished); J. D. Bjorken, K. L. Kowalski, and C. C. Taylor, Proceedings of Les Rencontres de Physique del Valle d’Aoste, La Thuile (SLAC PUB 6109) (1993); G. Amelino-Camelia, J. D. Bjorken, and S. E. Larsson, Phys. Rev. D 56, 6942 (1997); J. D. Bjorken, Acta Phys. Polon. B 28, 2773 (1997).
[5] S. Gavin, A. Gocksch, and R. D. Pisarski, Phys. Rev. Lett. 72, 2143 (1994); S. Gavin and B. Muller, Phys. Lett. B 329, 486 (1994); Z. Huang and X.-N. Wang, Phys. Rev. D 49, 4335 (1994); J. Randrup, Phys. Rev. Lett 77, 1226 (1996); Phys. Rev. D 55, 1188 (1997); Nucl. Phys. A 616, 531 (1997).
[6] M. M. Aggarwal et al. [WA98 Collaboration], Phys. Lett. B 420, 169 (1998); M. M. Aggarwal et al. [WA98 Collaboration], Phys. Rev. C 64, 011901 (2000).
[7] Z. Huang and X.-N. Wang, Phys. Lett. B 383, 457 (1996); Y. Kluger, V. Koch, J. Randrup, and X.-N. Wang, Phys. Rev. C 57, 280 (1998).
[8] D. Boyanovsky, H. J. de Vega, R. Holman, and S. Prem Kumar, Phys. Rev. D 56, 5233 (1997).
[9] D. Boyanovsky, H. J. de Vega, R. Holman, and S. Prem Kumar, Phys. Rev. D 56, 3929 (1997).
[10] H. Minakata and B. Muller, Phys. Lett. B 377, 135 (1996).
[11] D.-S. Lee and K.-W. Ng, Phys. Lett. B 492, 303 (2000).
[12] S.-Y. Wang and D. Boyanovsky, Phys. Rev. D 63, 051702 (2001); S.-Y. Wang, D. Boyanovsky, and K.-W. Ng, Nucl. Phys. A 699, 819 (2002).
[13] J. D. Bjorken, Phys. Rev. D 27, 140 (1983).
[14] J. Sollfrank et al., Phys. Rev. C 55, 392 (1997).
[15] S. Weinberg, Gravitation and Cosmology, (John Wiley & Sons, New York, 1972).
[16] R. Davidson, N. C. Mukhopadhyay, and R. Wittman, Phys. Rev. D 43, 71 (1991).
[17] D. Boyanovsky and H. J. de Vega, Phys. Rev. D 47, 2343 (1993); D. Boyanovsky, D.-S. Lee, and A. Singh, Phys. Rev. D 48, 800 (1993); D. Boyanovsky, H. J. de Vega, and R. Holman, Phys. Rev. D 49, 2769 (1994); D. Boyanovsky, H. J. de Vega, R. Holman, D.-S. Lee, and A. Singh, Phys. Rev. D 51, 4419 (1995); D. Boyanovsky, M. D’Attanasio, H. J. de Vega, R. Holman, and D.-S. Lee, Phys. Rev. D 52, 6805 (1995); D. Boyanovsky, H. J. de Vega, D.-S. Lee, Y. J. Ng, and S.-Y. Wang, Phys. Rev. D 59, 125009 (1999); S.-Y. Wang, D. Boyanovsky, H. J. de Vega, D.-S. Lee, and Y. J. Ng, Phys. Rev. D 61, 065004 (2000).
[18] J.-e Alam, S. Sarkar, P. Roy, T. Hatsuda, and B. Sinha, Ann. of Phys. 286, 159 (2001).
[19] J. I. Kapusta, P. Lichard, and D. Seibert, Phys. Rev. D 44, 2774 (1991); ibid 47, 4171 (1993).
[20] P. Aurenche, F. Gelis, R. Kobes, and H. Zaraket, Phys. Rev. D 58, 085003 (1998).
[21] F. D. Steffen and M. H. Thoma, Phys. Lett. B 510, 98 (2001).
[22] H. Nadeau, J. Kapusta, and P. Lichard, Phys. Rev. C 45, 3034 (1992); 47, 2426 (1993).
[23] L. Xiong, E. Shuryak, and G. E. Brown, Phys. Rev. D 46, 3798 (1992).
[24] T. Peitzmann and M. H. Thoma, Phys. Rep. 364, 175 (2002).