We discuss transverse polarization distribution and fragmentation functions, in particular, T-odd functions with transverse momentum dependence, which might be relevant for the description of single transverse spin asymmetries. The role of intrinsic transverse momentum in the expansion in inverse powers of the hard scale is elaborated upon. The sin $\phi$ single spin asymmetry in the process $e^+ p \rightarrow e' \pi^+ X$ as recently reported by the HERMES Collaboration is investigated, in particular, by using the bag model.

§1. Introduction

Transverse polarization distribution and fragmentation functions parameterize transverse spin effects in hard scattering processes. Although in general little is known experimentally on most of these functions, some transverse spin experiments have been performed. For instance, large single transverse spin asymmetries have been observed in the process $p p^\uparrow \rightarrow \pi X$. Therefore, the question arises which transverse polarization distribution and/or fragmentation functions are relevant for their description. This might include so-called T-odd fragmentation functions, which are expected to arise due to final state interactions. The main observation is that the description of this specific process in terms of transverse spin functions will lead to power suppressed single spin asymmetries, unless one takes into account the intrinsic transverse momentum of the partons, see e.g. Ref.\cite{2}. We will discuss how the transverse momentum dependence of asymmetries can contain information (in terms of distribution and fragmentation functions) at leading order, that would be power suppressed if integrated over the transverse momentum.

Especially the T-odd functions with transverse momentum dependence might be relevant for the description of single transverse spin asymmetries, since these functions link the transverse momentum and transverse spin (of either quarks or hadrons) with a specific handedness. The different functions will lead to different angular dependences. Hence, studying the angular dependences of asymmetries (and their transverse momentum dependence) is a most promising way to unravel the origin(s) of transverse spin asymmetries. This is for instance demonstrated by a recent result by the HERMES Collaboration\cite{3}, as will be discussed.
§2. Distribution and fragmentation functions

Transverse spin asymmetries in hadron-hadron collisions require an explanation that involves quarks and gluons. A large scale allows for a factorization of such processes into parts describing the soft physics convoluted with a hard subprocess cross section. We will first focus on the Drell-Yan (DY) process, i.e. lepton pair production. In lowest order –the parton model approximation– this process consists of two soft parts, the correlation functions called $\Phi$ and $\overline{\Phi}$. In Fig. 1 the leading order diagram is depicted. One can decompose the quark momenta $p$ and $k$ into parts that are along the direction of the parent hadron, the so-called lightcone momentum fractions, and deviations from that direction. In case one integrates over the transverse momentum of the lepton pair one only has to consider the correlation functions as functions of the lightcone momentum fractions, for instance $\Phi(x)$. Its most general parameterization, that is in accordance with the required symmetries (hermiticity, parity, time reversal), is given by:

$$\Phi(x) = \frac{1}{2} \left[ f_1(x) P + g_1(x) \lambda \gamma_5 P + h_1(x) \gamma_5 S_T P \right]. \quad (2.1)$$

Other common notation is $q$ for $f_1$, $\Delta q$ for $g_1$ and $\delta q$ or $\Delta_T q$ for $h_1$.

At the parton model level one finds the well-known double transverse spin asymmetry $A_{TT} \propto |S_{1T}| |S_{2T}| \cos(\phi_{S_1} + \phi_{S_2}) h_1(x_1) \overline{h}_1(x_2)$, which is one possibility to obtain information on the transversity distribution function $h_1$, for instance at RHIC. At this order there are no single transverse spin asymmetries. However, these might arise from corrections to this lowest order diagram: the perturbative and/or the higher twist corrections. The former will typically yield single transverse spin asymmetries of the order $\alpha_s m_q/\sqrt{s}$ which are expected to be small. The higher twist corrections require parameterizing the correlation function $\Phi(x)$ to include contributions proportional to the hadronic scale (say the hadron mass),

$$\Phi(x) = \text{Eq. (2.1)} + \frac{M}{2} \left[ e(x) 1 + g_T(x) \gamma_5 S_T + h_L(x) \frac{\lambda}{2} \gamma_5 \left[ \not{n} + \not{n}_- \right] \right]. \quad (2.2)$$

The distribution functions $e, g_T, h_L$ are so-called twist-3 functions; these will show up in the cross section suppressed by $M/Q$, where $Q$ is a hard scale. At leading order in

Fig. 1. The leading order contribution to the Drell-Yan process.

Fig. 2. A contribution to the process $p p \rightarrow \pi X$. 
Transverse polarization distribution and fragmentation functions

\(\alpha_s\), i.e. \((\alpha_s)^0\), but at the order \(1/Q\), one finds no single or double transverse spin asymmetries in the DY cross section. The functions \(g_T\) and \(h_L\) will only appear in the asymmetry \(A_{LT}\).

Therefore, in order to produce a large single transverse spin asymmetry in the DY process (no experimental data exists however), one needs some conceptually non-trivial mechanism, like soft gluon poles, since regular perturbative and higher twist contributions appear to be either small or absent. For the case of pion production in \(p p\) scattering there exist a more conventional explanation of the large observed single spin asymmetries. It involves a T-odd fragmentation function. Such functions are expected to be present even if time reversal invariance is applied, because of the final state interactions between the outgoing hadron and the other fragments. For the description of a quark fragmenting into a pion plus anything, one needs the fragmentation correlation function \(\Delta(z)\), which one parameterizes in accordance with the required symmetries (including time reversal)

\[
\Delta(z) = T\text{-even part} + \frac{M}{2} \left[ D_T(z) \epsilon_{\mu\nu} \gamma_{T\mu} S_{T\nu} - E_L(z) \lambda_i \gamma_5 + H(z) \frac{i}{2} [\not{n}_-, \not{n}_+] \right].
\]

The twist-3 fragmentation functions \(D_T, E_L, H\) are T-odd. The T-odd fragmentation function \(H\) can be responsible for the single spin asymmetries, e.g. via a diagram as depicted in Fig. 2. This yields power suppressed asymmetries, which can be investigated experimentally by changing the energy scale over a wide range (again RHIC can provide this information). The question we like to address here is: how to obtain a single spin asymmetry that is not suppressed by powers of the hard scale? A possible solution is to include intrinsic transverse momentum, i.e. replace \(\Phi(x) \rightarrow \Phi(x, p_T)\) and \(\Delta(z) \rightarrow \Delta(z, k_T)\). For the T-even distribution functions in DY this replacement leads to double spin asymmetries. But T-odd functions with transverse momentum dependence can lead to single spin asymmetries at leading order. The transverse momentum dependent distribution functions are defined as

\[
\Phi(x, p_T) = \frac{1}{2} \left[ f_1 P - g_1 s P \gamma_5 - h_1 T i \sigma_{\mu\nu} \gamma_5 S_T^\mu P^\nu - h_{1s}^T i \sigma_{\mu\nu} \gamma_5 P_T^\mu P^\nu \right],
\]

where \(f_1 = f_1(x, p_T)\), etc. and we use the shorthand notation

\[
(\ldots)_{1s}(x, p_T) \equiv \lambda (\ldots)_{1L}(x, p_T^2) + \frac{(p_T \cdot S_T)}{M} (\ldots)_{1T}(x, p_T^2).
\]

The correlation function \(\Delta(z, k_T)\) is given by

\[
\Delta(z, k_T) = T\text{-even part} + \frac{1}{2} \left[ D_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma} P^\nu k_T^\rho S_T^\sigma}{M} + H_1^\perp \frac{\sigma_{\mu\nu} k_T^\mu P^\nu}{M} \right].
\]

The fragmentation functions \(D_{1T}^\perp\) and \(H_1^\perp\) are T-odd functions and as can be seen in Figs. 3 and 4 such T-odd effects link transverse momentum and transverse spin (orthogonal to the transverse momentum) with a specific orientation (handedness). The chiral-even function \(D_{1T}^\perp\) is expected to be relevant for transversely polarized leptons off transversely polarized hadrons at SLAC and by SMC.

\(^{a)}\) The chiral-even, T-even function \(g_T = g_1 + g_2\) has been studied by deep inelastic scattering of leptons off transversely polarized hadrons at SLAC and by SMC.
Fig. 3. The chiral-even, T-odd function $D_{1T}^\perp$ signals different probabilities for $q \rightarrow \Lambda(k_T, \pm S_T) + X$.

A production\textsuperscript{10}, for instance in $pp \rightarrow \Lambda^1 X$ (also measurable at RHIC). The chiral-odd function\textsuperscript{11} $H_{1T}^\perp$, also called the "Collins effect" function, might be relevant for $pp \rightarrow \pi X$ asymmetry via\textsuperscript{12}: $A_T \sim h_1(x_1) \otimes f_1(x_2) \otimes H_{1T}^\perp(z, k_T)$. Like the transverse momentum of the lepton pair in the DY process, the transverse momentum of the pion now originates from the intrinsic transverse momentum of the initial partons in addition to transverse momentum generated perturbatively by radiating off some additional parton(s) in the final state (hence the transverse momentum of the pion need not be small).

There is an experimental indication\textsuperscript{13} from analyzing a particular angular dependence (a $\cos 2\phi$ dependence\textsuperscript{14}) in the unpolarized process $e^+ e^- \rightarrow Z^0 \rightarrow \pi \pi X$, where the pions belong to opposite jets, that the Collins effect is in fact a few percent of the magnitude of the ordinary unpolarized fragmentation function. Comparison to the magnitude at lower energies of course requires evolution.

The two T-odd fragmentation functions satisfy the following sum rules\textsuperscript{15,17}:

$$\sum_h \int dz \ z \ H_{1T}^{(1)}(z) = 0, \quad \sum_h \int dz \ z \ D_{1T}^{\perp(1)}(z) = 0, \quad \text{(2.6)}$$

where $F^{(1)}(z) = z^2 \int d^2 k_T \ k_T^2/(2M^2) \ F(z, z^2 k_T^2)$.

The function $D_{1T}^\perp$ can also be probed in charged current exchange processes, since it is chiral-even, as opposed to the chiral-odd functions like $h_1, H_{1T}^\perp$.

So far we have not commented on the fact that the T-odd fragmentation functions appearing in the parameterization of $\Delta(z)$ will lead to power suppressed contributions, whereas the two T-odd functions of $\Delta(z, k_T)$ can contribute at leading order in the expansion in inverse powers of the hard scale. This is the subject of the next section.

§3. Transverse momentum at leading "twist"

A process like deep inelastic scattering has only two scales, the hadronic scale (say the hadron mass $M$) and the large scale of the virtual photon ($Q$). The explicit
mass term in front of the function \( g_T \) in \( \Phi(x) \) Eq. (2.2) can therefore only lead to a term \( M/Q \) in the cross section.

In the case of a less inclusive experiment, for instance \( e p \rightarrow e' \pi X \), one can also observe the transverse momentum \( P_{\pi\perp} \) of the pion. This semi-inclusive cross section will depend on three dimensionful quantities: \( M, |P_{\pi\perp}| \) and \( Q \). The function \( g_T \) will again lead to a contribution \( \sim M/Q \), since it is not sensitive to the transverse momentum of the pion; it will not average to zero if one averages over \( P_{\pi\perp} \). On the other hand, one might be sensitive to functions that will disappear upon averaging. For these functions the appropriate scales are \( M \) and \( P_{\pi\perp} \). Terms proportional to \( M/|P_{\pi\perp}| \) will appear without expanding in \( M/|P_{\pi\perp}| \). We will see an explicit example below.

In order to arrive at a single transverse spin asymmetry that is not suppressed by inverse powers of the hard scale, one can consider cross sections differential in the transverse momentum of the pion. In that case one is sensitive to the transverse momentum of the quarks directly and in case this concerns intrinsic transverse momentum of the quarks inside a hadron, the effects need not be suppressed by \( 1/Q \). The point is that if the transverse momentum of the pion is (solely) produced by perturbative QCD corrections, each factor of transverse momentum has to be accompanied by the inverse of the scale in the elementary hard scattering subprocess: \( 1/Q \). But in case of intrinsic transverse momentum of the quarks (or gluons) the relevant scale is not \( Q \), but the hadronic scale \( M \). In other words, one is not allowed to make an expansion of \( \Phi(x, p_T) \) in terms of \( p_T/Q \), since there could be terms of order \( p_T/M \), which are not small in general.

In processes with two (or more) soft parts, like semi-inclusive leptoproduction, the intrinsic transverse momentum of one soft part is linked to that of the other soft part resulting in effects, e.g. azimuthal asymmetries, not suppressed by \( 1/Q \). These effects will show up at relatively low (including nonperturbative) values of \( |P_{\pi\perp}| \). By radiating off hard partons the fragmenting quark and hence the pion might achieve a higher transverse momentum. Of course, if this produces a very high transverse momentum (\( \sim Q \)), then this will lead to suppression. But the point is: in case one observes the transverse momentum of the pion, one can probe \( \Phi(x, p_T) \) and \( \Delta(z, k_T) \), without suppression by \( 1/Q \).

Let us investigate the example of \( e\bar{p} \rightarrow e\pi X \) and consider cross sections integrated, but weighted with a function of the transverse momentum of the pion:

\[
\langle W \rangle_{P_e P_p} = \int d\phi^e d^2P_{\pi\perp} W \frac{d\sigma_{[e\bar{p} \rightarrow e\pi X]}_{P_e P_p}}{dx dy dz d\phi^e d^2P_{\pi\perp}},
\]

where \( W = W(|P_{\pi\perp}|, \phi^e, \phi_S) \). \( P_e \) and \( P_p \) are the polarizations of the electron and proton with respect to the virtual photon. We use \( O \) for unpolarized, \( L \) for longitudinally polarized (\( \lambda \neq 0 \)) and \( T \) for transversely polarized (\( |S_T| \neq 0 \)) particles.

For pion production only expressions with the fragmentation functions \( D_1, H^+_1, E \) and \( H \) contribute to the order we consider. The asymmetries involving a fragmentation function \( D_1 \) can also be obtained by looking at the asymmetry in jet production. The jet is just a means of observing a transverse momentum. For simplicity we will
now restrict to jet production (i.e., keeping only $D_1(z) = \delta(1-z)$) and consider pion production in the next section.

For the case of $P_{e}P_{p} = LT$ we find the following power suppressed azimuthal spin asymmetry

$$\frac{\langle 1 \rangle_{LT}}{[4\pi \alpha^2 s/Q^4]} = -\cos \phi_{\Lambda e} |S_T| y \sqrt{1-y} \frac{M}{Q} \sum_{a,\bar{a}} c_{a}^2 x^2 g_{TT}^a(x). \quad (3.2)$$

On the other hand, if one weights with powers of the observed transverse momentum (hence $\langle |P_{\text{jet} \perp}| \rangle$ is a scale in the problem) one obtains for instance the following leading order expression

$$\frac{\langle |P_{\text{jet} \perp}|/M \cos \Phi \rangle_{LT}}{[4\pi \alpha^2 s/Q^4]} = \cos \phi_{\Lambda e} |S_T| y (1 - \frac{1}{2} y) \sum_{a,\bar{a}} c_{a}^2 x g_{TT}^{(1)a}(x). \quad (3.3)$$

The function $g_{TT}^{(1)a}(x) = \int d^2 P_T p_T^2/(2M^2) g_{TT}^a(x, p_T^2)$ appears in the quantity

$$\Phi_{\Lambda e}^{a}(x) \equiv \int d^2 P_T p_T^2 \Phi(x, p_T) = \frac{M}{2} \left[ g_{TT}^{(1)}(x) S_T^{\alpha} P - h_{1L}^{(1)}(x) \lambda_{\gamma 5} \gamma^\alpha P \right]. \quad (3.4)$$

Clearly the relevant scale $M$ has to be compensated in the cross section and as is seen from Eq. (3.3) the factor $\Phi_{\Lambda e}^{a}(x)$ is related to a quark-gluon-quark matrix element that will always show up $M/Q$ suppressed in DIS. And in fact, the function $g_{TT}^{(1)}$ is a well-known quantity in the Wandzura-Wilczek approximation: it equals (upon neglecting quark masses) $x g_{WW}(x)$, where $g_{WW} = g_1 + g_2^{WW}$. The existing data on $g_T$ are consistent with $g_T = g_1^{WW}$. This shows that part of the "twist-3" information (the dominant contribution in fact) can be obtained without suppression factors of $1/Q$, by doing a less inclusive experiment, namely by observing a transverse momentum. Of course the average transverse momentum is also a function of $Q$ and experimentally the observation of transverse momenta is more difficult, but the point remains that the same information (e.g. $g_{WW}^{WW}$) can enter with different scales in different quantities.

### §4. Single spin asymmetry in $e \bar{p} \rightarrow e' \pi^+ X$

Recently, the HERMES Collaboration reported a sinus $\Phi'$ asymmetry in the process $e \bar{p} \rightarrow e' \pi^+ X$, where the target has a polarization along the electron beam direction and $\phi' (= \phi_{\Lambda e}')$ is the angle of the transverse momentum of the pion with respect to the lepton scattering plane, cf. Fig. 4. The measured asymmetry has an analyzing power of $0.022 \pm 0.005 \pm 0.003$. The fact that the pion prefers to be out-of-plane in the asymmetry (a sinus $\Phi'$ distribution) is indicative of a T-odd effect. The asymmetry can indeed be expressed in terms of a chiral-odd, T-odd fragmentation function with transverse momentum dependence, the Collins effect function $H_{1+}$, if a factorized picture like in Ref. 4 is assumed:

$$W^{\mu\nu} = \int d^2 P_T d^2 k_T \delta^2 (p_T + q_T - k_T) \text{Tr} \left[ \Phi(x, p_T) \gamma^\mu \Delta(z,k_T) \gamma^\nu \right]. \quad (4.1)$$
One can easily project out the $\sin \phi$ dependence from the cross section by weighted integration. We define

$$A_{OL} = \frac{\langle W \rangle_{P_{\perp} P_{\parallel}}}{\langle 1 \rangle_{OO}}$$

where

$$W = \frac{|P_{\pi \perp}|}{2 M_{\pi}} \sin \phi_{\pi}.$$  

We will assume the asymmetry arises mainly from the dominant flavor, i.e. we take into account only the contribution from $u \rightarrow \pi^+$. Furthermore, we will neglect transverse momentum effects inside the proton. We find the following expressions for the relevant asymmetries:

$$A_{OL} \propto \lambda (2 - y) \sqrt{1 - y} \frac{M}{Q} x h_1(x) H_1^{(1)}(z),$$

$$A_{OT} \propto |S_T| (1 - y) h_1(x) H_1^{(1)}(z),$$

where $H_1^{(1)}(z) = z^2 \int d^2 k_T k_T^2 / (2 M_{\pi}^2) H_1(z, z^2 k_T^2)$.

The polarization of the target $P_{\perp}$ is in the lepton scattering plane and in fact along the electron beam direction, hence, it is a combination of $L$ and $T$, depending on $y = (P \cdot q) / (P \cdot l)$. We find

$$\frac{|S_T|}{\lambda} = \sqrt{1 - y} \frac{2 M x}{Q} \implies \frac{A_{OL}}{A_{OT}} = \frac{2 - y}{1 - y} \frac{h_L(x)}{h_1(x)}.$$  

A striking feature is that the contribution from the target spin transverse to the virtual photon momentum also enters at subleading order in $M/Q$, even though the relevant functions ($h_1$ and $H_1^{(1)}$) are leading twist functions. For $0.2 < y < 0.8$ one then finds

$$2 \frac{h_L}{h_1} \lesssim \frac{A_{OL}}{A_{OT}} \lesssim 6 \frac{h_L}{h_1}.$$  

Using a bag model calculation⁶, we confirm the observation of Kotzinian et al.¹⁷, that $A_{OL}$ is the dominant contribution to the asymmetry (for $0 < x < 0.3$), see Figs. 6 and 7. In Ref. ¹⁷ they consider the Wandzura-Wilczek approximation; the bag model calculation of $h_L^{WW} = -2 h_1^{(1)WW} / x$ deviates considerably from $h_L$ in the small $x$ region though. We also observe that for larger $x$ (above 0.5), but smaller values of $y$, $A_{OT} \gg A_{OL}$ (we restrict to $x < 0.8$ due to model artifacts).

Measuring the same asymmetry for a target polarized transversely to the electron beam, really probes the unsuppressed asymmetry $A_{OT}$, since in this case $A_{OL}$ will
contribute $M^2/Q^2$ suppressed as one can see by interchanging the role of $|S_T|$ and $\lambda$ in Eq. (4.4).

The asymmetry $A_{LO}$ has also been measured and was found to be consistent with zero, coinciding with the Wandzura-Wilczek expectation:

$$A_{LO} \propto \lambda e \sqrt{1 - y} \left( \frac{M}{Q} x e(x) H_1^{(1)}(z) \right)^{WW} m_u \approx 0.$$  (4.6)

We conclude that the HERMES Collaboration might have obtained the first experimental information on the functions $h_1$ and $h_L$.

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