Research on Weak Signal Line Spectrum Enhancement Algorithm Based on Correlation Detection

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Abstract. The correlation detection and adaptive line spectrum enhancement(ALE) algorithm in weak signal detection is presented. Aiming at the problem that the traditional ALE has no obvious effect of line spectrum enhancement at low signal-to-noise ratio(SNR), we propose a modified ALE algorithm based on coherent integrator. Combining this ALE and sliding window correlation, the effect of ALE is clearly improved. The simulation results show that the weak signal receiving ability can be improved by 8 dB under the same line spectrum condition.

1. Introduction
Line spectrum enhancement is a common problem in weak signal detection in the fields of communication, sonar, radar and geophysics. That is, a sinusoidal or narrowband signal is detected in a wideband interference background[1-3]. At present, the mature weak transient signal detection techniques include wavelet denoising, adaptive digital filtering, Fourier transform in frequency domain and so on. The lowest detectable signal-to-noise ratio (SNR) is about -10 dB[4]. Chaotic sequence prediction algorithm, duffing chaotic oscillator synchronous differential detection method[5-6], wavelet singular value decomposition detection method[7-8] is a relatively new weak signal detection algorithm developed in recent years. The signal to noise ratio (SNR) of target signal with known spectrum is lower than -25 dB. At the same time, cross-correlation detection is a very effective wideband weak signal detection technique[9-13].

This technique improves signal-to-noise ratio (SNR) of target signal by cross-correlation calculation of two signals. The power of target signal can be enhanced by cross-correlation operation, and the line spectrum of narrowband signal can be improved. At the same time, cross correlation operation can eliminate the uncorrelated part of the noise in the receiving circuit[9]. In addition, cross-correlation detection does not need to obtain prior information of target signal and has low computational complexity, so it is widely used in weak signal(SNR < 0)[14] detection.

Adaptive Line-spectrum Enhancement (ALE) algorithm is widely used to improve the linear spectrum of target signal in background noise according to the principle of cross-correlation detection. However, the traditional ALE algorithm requires a high signal to noise ratio(SNR). When the SNR is low, the effect of line spectrum proposed by this traditional ALE algorithm is poor[13]. Therefore, an improved ALE algorithm based on coherent accumulation technique is proposed in this paper. Combined with the sliding window cross-correlation method and its post-processing technique, the
iterative noise in the operation can be reduced, the system gain can be enhanced, and the line spectrum detection ability under the condition of low signal-to-noise ratio can be improved.

2. Correlation Detection

Let the observed mixed signal sequence be \( x(k) = s(k) + n(k) \), \( s(k) \) is a useful signal (or the target signal), \( n(k) \) is noise signal.

2.1. Prior Knowledge is Available

If prior knowledge \( s(k) \) is available, \( s(k) \) and \( n(k) \) is independent of each other. Then the useful signal can be detected by summing the cross correlation function, whose cross-correlation function is

\[
R_{sx}(m) = E\{s(k) \cdot x(k + m)\} = R_{sx}(m) + R_{sn}(m) = R_{sn}(m) \tag{1}
\]

It can be seen from formula (1) that if the cross-correlation between the observed sequence \( x(k) \) and the useful signal equal to the auto-correlation of the useful signal, the useful signal is included in the observation sequence.

2.2. Prior Knowledge is not Available

If prior waveform of \( s(k) \) is not available, but knowing that it is a periodic sequence, then the self-correlation function of \( x(k) \) can be used to detect the existence of \( s(k) \) and its approximate periodic value, the auto-correlation function is

\[
R_{xx}(m) = E\{x(k) \cdot x(k + m)\} = R_{xx}(m) + R_{nn}(m) + R_{nm}(m) \tag{2}
\]

Knowing \( s(k) \) to be a periodic sequence, then autocorrelation should also be a periodic sequence, whose period is equal to the period of the useful signal. Although \( R_{s}(m) \) contains the auto-correlation of the noise \( R_{nn}(m) \), but \( R_{nn}(m) \) usually attenuates to zero as \( m \) increases. For example, white noise has a large value only at \( m = 0 \), the value of white noise is zero if \( m \neq 0 \). Therefore, as long as the value of the time interval \( m \) is large enough, the desired result can be correctly detected. Reference document\(^{[15]}\) describes in detail the methods and steps of removing cross-correlation in a correlation detection receiver.

3. ALE Algorithm and Its Improvement

Adaptive Line Enhancement (ALE) algorithm was first developed by Widrow\(^{[16]}\). At present, it has been widely used in the fields of spectrum estimation, spectral line estimation and narrowband detection. When wideband signal is added to narrow band signal, the mixing signal is delayed and without independent reference signal. The effect of delay is to de-correlate wideband noise, while sinusoidal or narrowband signals do not. The least mean square (LMS) criterion is used to adaptively match the correlated sinusoidal signal or narrowband signal, then the signal can be separated.

Because the ALE with the least mean square (LMS) adaptive algorithm has iterative noise, its processing gain is poor when the input signal-to-noise ratio is low. Therefore, a method to improve the adaptive line spectrum intensifier using coherent accumulation technique is proposed\(^{[16-19]}\). In the process of accumulation, the signal components can be added in-phase, while the interference components can only be added with energy, so a certain processing gain can be obtained, and the effect of iterative noise can be further eliminated. On the basis of the coherent accumulation algorithm, an improved adaptive line spectrum enhancement (ALE) algorithm is proposed in this paper, as shown in Figure1.
The above improved ALE algorithm is as follows:

\[
\begin{align*}
    y(k) &= \sum_{i=0}^{M-1} \omega_i(k)x(k - \Delta - i) \\
    y_x(k) &= y(k) + \sum_{i=0}^{M-1} \beta y_x(k - \Delta - i)\omega_i(k) \\
    e(k) &= x(k) - y(k) \\
    \omega_i(0) &= 0, \quad 1 \leq i \leq M \\
    \omega_i(k + 1) &= \omega_i(k) + 2\mu(e(k) + a_i e(k - 1) + a_2 e(k - 2) + a_3 e(k - 3)x(k - i)) \\
    0 &< a_3 < a_2 < a_i < 1
\end{align*}
\]

(3)

\( \beta \) is a constant in the above formula, and \( 0 < \beta < 1 \) to make the system converge. \( \mu \) is the adaptive learning step. When the \( \mu \) value is small, the autocorrelation radius of the effective signal cannot be learned adaptively, and the signal cannot get the corresponding peak value after the adaptive linear spectrum intensifier. When the \( \mu \) is large, it causes the adaptive abnormal, which makes the SNR (signal-to-noise ratio) increase. The simulation results show that the \( \mu \) is usually between 0.0003 and 0.01. \( M \) is the order of adaptive filter. If we need to increase the gain of the system, we only need to increase the number of \( M \). After \( M \) increases, the bandwidth of the filter becomes narrow, which removes some noise and increases the SNR. The \( M \) value cannot be too large, otherwise the adaptive linear spectrum enhancer may be limited. In the subsequent simulation of the paper, we take \( M=64 \). The selection of delay quantity \( \Delta \) in ALE is very important \cite{13}, the value \( \Delta \) should be larger than the time correlation radius of background noise, so that the interference components in \( x(k - \Delta) \) and \( x(k) \) are independent of each other. At the same time, the value of \( \Delta \) should be less than the time correlation radius of the target signal, so that the target signal \( s(k - \Delta) \) is still related to the \( s(k) \), so the output of the ALE is the best estimate of the target signal.

According to formula (3), the system function \( H_x(z) \) of the improved ALE can be obtained as follows:

\[
H_x(z) = \frac{Y_x(z)}{X(z)} = \frac{H(z)}{1 - \beta \cdot H(z)}
\]

(4)

In the formula, \( H(z) \) is the Z transformation of the transverse filter and the delay \( \Delta \).

As can be seen from Equation (4), the improved ALE has the characteristics of the ARMA filter, its bandwidth is narrower than that of the transverse filter and has stronger filtering noise capability,
thus processing gain of the system is improved. In addition, because the improved ALE not only uses the current error information, but also the previous error information, the linear combination of several output errors greatly eliminates the effect of iterative noise. As a result, the processing gain of the system is further improved.

4. Sliding window cross correlation and mean period processing

The improved ALE algorithm is superior to the traditional ALE algorithm in noise filtering ability and processing gain, but when the environmental noise is too high ($SNR < 0 \ dB$), the ALE processing is not enough to provide the system gain. There will still be some "false peaks" of signals. To provide further processing gain, sliding window cross-correlation and average period processing can be performed after ALE.

Sliding window cross-correlation means the proceeding of cross-correlation of a set of signals, then sliding fixed points to do the next cross-correlation, and then the two sets of result data and signals are compared and analyzed to find out the similarities between the two cross-correlation. Because the statistical characteristics of target signal and environmental noise signal are different, noise signals are generally random, and without obvious statistical characteristics. The target signals have some cross-correlation and obvious statistical characteristics. By using the difference of the statistical characteristics of the two signals, the detection reliability of the target signal can be improved by using the sliding window cross-correlation method.

If the sliding length of the signal is $L$ and the window length of the correlation integral is $H$, the correlation between the two signals is as follows:

$$R_{x_1x_2}(k,n) = \frac{1}{H} \sum_{l=0}^{H-1} x_1(l + L \cdot k)x_2(l - n + L \cdot k)$$

(5)

In Equation (5), $k$ denotes the correlation group number, and $n$ denotes the displacement value of the two received signal sequences.

Under the condition of practical weak signal detection, because of the complexity of environmental conditions, after the signal is processed by sliding window cross-correlation, the mixed signal must be post-processed, such as periodic average processing. The arithmetic mean period of sequence $x(k)$ is as follows

$$\bar{x} = \frac{1}{N} \sum_{k=1}^{N} x(k)$$

(6)

$\bar{x}$ represents the arithmetic average in the window. The arithmetic average is obtained by using the arithmetic average post-processing method to analyze the correlation coefficients of each group.

The sliding length $L$ must be greater than the time-dependent radius of the noise signal, so that the environmental noise signal of any two cross-correlation signals is not correlated. However, the peak position of the two cross-correlation signals of the target signal has almost no change, even if there is a very small change, there is no effect on the actual data. When the target signal is superimposed, the corresponding waveform peak of the same frequency will also be superimposed, and the correlation peak will be further sharpened. However, since the environmental noise signal is random, the peaks will cancel each other after the cross-correlation overlap, and thus the noise effect can be reduced.

$$R_{y}(n) = \frac{1}{N} \sum_{k=0}^{N} R_{x_1x_2}(k,n)$$

(7)

5. Simulation analysis

The paper simulates the signal line spectrum after ALE algorithm under different signal to noise ratio, and studies the effect of different SNR about the effect of ALE. At the same time, under the condition of worse environment noise ($SNR<0$), the improved effect of sliding window cross-correlation method and post-processing method on line spectrum after ALE is analyzed by simulation.
Simulation Condition: In line spectrum analysis, the frequency components of the target signal are respectively: 14 Hz, 28 Hz, 42 Hz, 65 Hz, 80 Hz. The carrier signal is Gaussian white noise signal and the input signal-to-noise ratio is 0 dB.

After the signal is processed by ALE, the signal length of average period processing is 30, the sliding window length is 3s, the sliding distance is 0.3 s.

When simulating the cross correlation of the sliding window, the received continuous spectrum frequency is between 200 Hz and 800 Hz. The carrier is Gaussian broadband white noise, the input signal to noise ratio(SNR) is -8 dB, the sampling rate is $F_s = 10 kHz$, and the sliding window length is 1024 points.

Simulation Conclusion: In all simulation Figures, the ordinate and the abscissa represent the size and frequency of the signal line spectrum, respectively. Comparing the effect of signal line spectrum in simulation diagram with the relative size of noise spectrum, it can be found that when the SNR is low (SNR=0 dB), if only ALE processing is carried out, the effect of line spectrum has hardly been improved, see Figure 2 (a)(b). The ordinate of Fig.2-Fig.5 represents the magnitude of the signal, the abscissa of Fig.2-Fig.4 represents the frequency, and the abscissa of Fig.5 represents the time delay. From the Fig.5, If the signal-to-noise ratio (SNR) is increased (SNR=5 dB), ALE can significantly improve the line spectrum, as shown in Fig.3. If the ALE processing is combined with the sliding window cross-correlation processing and the average period processing, even if the signal-to-noise ratio is reduced to -8 dB, the received signal will have a distinct power line spectrum, as shown in Fig.4. Under the two conditions of Fig.5 (a) (b), the correlation of signals is simulated. we can see that the correlation between the sliding window cross-correlation and post processing technology has significantly improved, which is consistent with the spectrum improvement results observed in Fig. 4.

It can be seen from the simulation Fig.5(b) that the result of the signal after the sliding window cross-correlation and post-processing is very obvious, which can effectively reduce the influence of the environmental noise signal. B denote the bandwidth of received signal, T denote the pulse interval of received signal, H denote the sliding length of the sliding window. G indicates the processing gain of system. The simulation results show that the processing gain $G_1$ before the sliding window cross-correlation processing is as follows:

$$G_1 = 5 \log BT$$

The processing gain after the sliding window cross-correlation is $G_2$

$$G_2 = 5 \log BT + 5 \log H$$

By comparing the above formulas (8) and (9), we can see that the system processing gain $G_2$ with sliding windows is significantly larger than that of the $G_1$ without sliding windows.Moreover, with the increase of sliding window length H, the processing gain $G_2$ increases further.
6. Conclusion

Aiming at the problem that the traditional adaptive linear spectrum intensifier is not effective in low SNR, an improved linear spectrum intensifier is proposed in this paper. It also proposes adding the sliding window cross-correlation and post-processing technology after the line spectrum enhancer, which can further reduce the iterative noise and improve the effect of line spectrum enhancement. The simulation results show that the detection ability of weak signals can be improved by 8 dB under the same linear spectrum condition. This paper provides a new approach for weak signal detection under complex ambient noise.

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References

[1] ZHANG Y., LIU S.H., LIU W.D. (2015) Detection of Weak Transient Electromagnetic Signals Based on Generalized Cross Correlation and Chaotic Time Series Prediction Integrate Algorithm. Journal of Electronics & Information Technology, 11:2769-2775.

[2] YANG J., CHENG N.P., NI S.Y. (2020) Performance Analysis of Welch Algorithm in Weak
Signal Detection. Computer Simulation, 5:235-240.

[3] YUAN Y.Y., LI Z., ZHANG Y.F. (2011) Studies of adaptive line enhancement algorithm for high-speed moving targets. Applied Acoustics, 4:306-313.

[4] WU Y.F., HUANG S.P., JIN G.B. (2013) Study on partial discharge by coupled Duffing oscillators. Acta Physica Sinica, 13:1305051-1305058.

[5] LI N., LI X.K., LIU C.H. (2016) Detection method of a short-time Duffing oscillator array with variable amplitude coefficients. Journal of Harbin Engineering University, 12:1645-1652.

[6] LU M., MA S.S., DING J.F. (2017) Weak signal detection system based on Duffing oscillator with linear driven by to-be-detected signal. Journal of Central South University (Science and Technology), 3:721-728.

[7] XU Y.K., SHUANG K. (2014) Detection of transient signal based on adaptive singular value decomposition. Journal of Electronics & Information Technology, 3:583-588.

[8] XU Y.K., SHU A.K. (2014) Detection of transient weak signal based on wavelet transform and singular value decomposition. Journal of China University of Petroleum (Edition of Natural Science), 3:181-185.

[9] ZHANG T.T., ZHANG W.Y. (2013) Suppression of narrowband interference using frequency domain cross-correlation. Journal of University of Chinese Academy of Sciences, 5:671-681.

[10] Alink O, Smeenge A R, and Kokkeler A B J. (2011) Exploring the use of two antennas for cross-correlation spectrum sensing. IEEE VTC Fall, San Francisco, United States: IEEE Press, pp. 1-5.

[11] Ho W, Gharpurey R. (2011) A cross-correlation based signal detector with two step down-converter robust to finite image rejection. IEEE ISC-AS. Rio de Janeiro, Brazil: IEEE Press, pp. 2197-2200.

[12] Alink O, Kokkeler A B J, Klumperink E A. (2011) Lowering the SNR wall for energy detection using cross-correlation. IEEE Transactions on Vehicular Technology, 8:3748-3757.

[13] PAN Z.M., DING H., CHENG M. (2013) Application of adaptive LMS algorithm and correlation in signal detection for GMI magnetic sensor. Journal of National University of Defense Technology, 1:142-146.

[14] LIU H.T., CONG W.H., PAN X. (2007) Line spectral detection of tone weak signal -- an adaptive line enhancement technique using coherent addition and frequency domain batch. Journal of Zhejiang University (Engineering Science), 12:2048-2051.

[15] ZHAO H.H., HUANG H.S., ZHANG W. (2015) Cross-correlation rejection algorithm based on Beidou receiver. Journal of Nanjing University of Posts and Telecommunication (Natural Science Edition), 6:39-43.

[16] HU P., GONG S.G., CAI X.D. (2012) Improvement of adaptive line enhancement and its application in detection of ship shaft-rate electric field signal. Journal of Wuhan University of Technology (Transportation Science & Engineering), 6:1217-1220.

[17] LI D.H. (2015) Faint signal detection technology based on correlation accumulation. Shipboard Electronic Countermeasure, 5:43-51.

[18] LUO B., WANG M.F., WANG S.C. (2017) A highly efficient weak target line-spectrum detection algorithm. Technical Acoustics, 36:171-176.

[19] JI C.P., XIU S., Ji W.J. (2019) Improved Weak Signal Detection for Inverse Phase Transition of Duffing Oscillator, Journal of Data Acquisition and Processing, 34:223-233.