Application of full waveform inversion method in underground pipeline detection

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Abstract. Urban underground pipelines are an important part of today's urban infrastructure. A comprehensive and systematic grasp of the current status of urban underground pipelines can provide people with basic information on urban planning, construction and management, and help people better develop and use cities and their underground space. Full waveform inversion is a well-known method that could reconstruct the underground velocity structures with high resolution. In this paper, we test the ability of full waveform inversion method in underground pipeline detection. The numerical test shows that we could use full waveform inversion to detect the regular urban underground pipelines in different sizes and depths.

1. Introduction

Underground pipelines appear and develop with the development of cities. It is an important part of today's urban infrastructure. The accurate description of urban underground pipelines through different detection methods can guide urban planning, design, construction, and management. It can avoid blind construction during work, damage underground pipelines, and even cause disasters. Seismic exploration is a geophysical exploration method that uses the response of artificially induced seismic waves to image underground structures. It could provide an accurate image of the underground structures at different depths. Nowadays, seismic methods have been widely used in urban underground space exploration. The resolution of conventional seismic exploration methods is between 3-10 meters, and its resolution is limited relative to the scale of urban underground pipelines. Seismic full waveform inversion is a powerful and advanced tool that uses seismic data to reconstruct high-resolution underground velocity fields directly. The main goal was to find a physical model that could explain real seismic data. Seismic full waveform inversion can be used for fine imaging of underground structures. It is the seismic data inversion method with the highest resolution at present. Its highest resolution can reach below 0.5 meters at a depth of 10 meters, so it has the ability to identify underground pipelines. There already some application of full waveform inversion in urban seismic exploration[1-3]. In this paper, we apply the full waveform inversion method to a synthetic urban underground pipeline data set. The results show that we could restructure the velocity model, which represents the underground pipeline in the target depth. The method has great potential in the urban underground pipeline detection.
2. Theory
Seismic full waveform inversion is a powerful and advanced tool to reconstruct the underground velocity field in high resolution using seismic data. The main goal is to find a model (generally the P wave velocity field) that can explain the real seismic data. It was first introduced about 35 years ago [4,5]. The method has been developed by many researchers[6-11].

Full waveform inversion drives the update of model parameters by minimizing the residuals of simulation data and observation data. The seismic full waveform inversion workflow could be explained in the following figure.

![Fig.1 The full waveform inversion workflow.](image)

The objective function of conventional full waveform inversion could be described as following

\[ E(m) = \frac{1}{2} \sum_{x_s} \sum_{x_r} \sum_t [u(t, x_r, x_s, m) - d(t, x_r, x_s)] \]

where \( E \) is the objective function, \( u \) is the synthetic seismic data, \( d \) is the observed seismic data, \( m \) is the modeling parameters, \( x_r \) is receiver location, \( x_s \) is source location, \( t \) is time. Solving this problem is the same as solving the following nonlinear inversion problem,

\[ \min \{ E(m) \} = \frac{1}{2} \| u(m) - d \|^2. \]

The derivative of the objective function with respect to the model parameters is

\[ \frac{\partial E}{\partial m} = \left( \frac{\partial u}{\partial m}, u - d \right). \]

\( \frac{\partial u}{\partial m} \) is the Frechet derivative (sensitive kernel function, Jacobian matrix).

To calculate this function by using a global optimization algorithm is enormous. Most FWI algorithms now use a local optimization algorithm to solve it[8]. We could use the gradient of the objective function

\[ g(m) = \frac{\partial E(m)}{\partial m} = \frac{2}{m^3} \sum_t \left( \sum_x \frac{\partial^2 u}{\partial t^2} R \right) \]

where \( R \) is the back-propagated residual wavefield, which is calculated by setting residual data as a virtual source and conducting forward modeling from the maximum time point to minimum time point. The gradient of the objective function can be calculated by the cross-correlation of the back-propagated residual wavefield and the second derivative of the forward-propagated source wavefield. After obtaining the gradient, we can update velocity by the following iteration formula \( m_{k+1} = m_k + \alpha \cdot d_k \).

\( \alpha \) is the step length and \( d_k \) is update direction. \( d_k = -H_k^{-1} \cdot g_k \), where \( H_k \) is the Hessian matrix. We use the conjugate gradient method to calculate the descent direction. It solves the current descent direction by weighting the current gradient and the last step descent direction.

3. Synthetic data sets
In order to test the method, we generate a synthetic data set to represent the seismic data set which acquired in the urban area with underground pipelines. The model size is 241*241 and the grid size is 0.05m. We put three pipelines in different depths (2m, 6m, and 10m). Each pipe is 9m long, and its width is gradually increasing from 0.15m to 0.25m (0-3m with diameter 0.15m, 3-6m with diameter...
0.2m, and 6-9m with diameter 0.25m). We use 1000m/s as the background velocity and 2000m/s for the pipe velocity in the model. Figure 2 shows the velocity model which is used in the modeling. In the urban engineering seismic exploration, we usually could use a Portable High-frequency Vibrator and MEMS receivers. In theory, we could get broadband seismic data sets. Therefore, we use ricker wavelets with 300Hz and 1000Hz pick frequency in the forward modeling. The source and receiver locations are place at zero depth on the top of the model with different intervals. PML boundary condition is used to reduce the boundary reflections. Table 1 shows some major modeling parameters. To test the validity of the full waveform inversion in the pipelines detection, we generated five different data sets based on the forward modeling as shown in Table 2.

![Fig.2 The true velocity model.](image)

Table 1. The modeling parameters

| dx  | dz  | nx  | nz  | Source pick frequency |
|-----|-----|-----|-----|------------------------|
| 0.05| 0.05| 241 | 241 | 300Hz/1000Hz           |

Table 2. The synthetic data sets

1. 1000Hz source wavelet, 240 sources and receivers with 0.05m interval.
2. 300Hz source wavelet, 240 sources and receivers with 0.05m interval.
3. 1000Hz source wavelet, 240 sources with 0.05m interval and 24 receivers with 0.5m interval.
4. 1000Hz source wavelet, 12 sources with 2m interval and 240 receivers with 0.05m interval.
5. 300Hz source wavelet, 240 sources and 24 receivers with 0.5m interval.

4. Inversion results

We use conventional time-domain full waveform inversion in the synthetic data test. The inversion parameters are the same for each data set. The starting model is the background velocity model with 1000m/s. Total iteration is 150 times for each inversion. Figure 3 shows the results for data set 1 and 3. The inversion results show that in the ideal condition full waveform inversion could reconstruct the pipeline velocity in different depth. The resolution is reducing with the increasing depth, but we could still see the pipelines very clear at 10 m depth. The results are very similar when we reduce the receiver number to 1/10 of the full data set. Figure 4 shows the results when we reduce the shot number to 1/20 of the full data set. The results are still very good. We could recognize the pipelines very clear at different depths. As we know in the realistic case, usually, we could not obtain such high frequency data in the surface monitoring condition. Therefore, we also test the method in a more realistic condition. Figure 5 shows another inversion results of the 300Hz data sets. When the data set has a lower frequency range, the resolution is reduced compared with high frequency range one. The width of the pipeline is not recovered very accurate. But we could still find the pipeline in the right location in the inversion results.
5. Conclusions
We test the ability of full waveform inversion method in the urban underground pipeline detection. In the ideal case we could image the underground pipelines with correct size and locations. In a more realistic case with lower data frequency range, when reducing the source and receiver number to a certain level, we could still locate the pipeline. The results show that the full waveform inversion method could be a potential tool in detecting the underground pipelines in a certain condition.

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