Application of the Wave and Finite Element Method to Calculate Sound Transmission Through Cylindrical Structures

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Abstract. This paper concerns the prediction of sound transmission through a cylindrical structure. The problem considered is that of sound generated by a line source located exterior to a two-dimensional circular cylinder which produces sound waves which transmit through the cylinder to an internal medium. An analytical solution is presented for the case of sound transmission through a thin cylindrical shell, by modelling the shell response using the Flugge-Byrne-Lur’ye equations. This solution is then compared to calculations where the response of the cylinder is calculated using the Wave and Finite Element (WFE) method. The WFE method involves modelling a small segment of a structure using traditional finite element (FE) methods. The mass and stiffness matrices of the segment are then used to calculate the response of the structure to excitation by an acoustic field. The WFE approach for calculating sound transmission is validated by comparison with the analytic solution. Formulating analytic solutions for more complicated structures can be cumbersome whereas using a numerical technique, such as the WFE method, is relatively straightforward.

1. Introduction

It is relatively straightforward to develop analytical models to predict the transmission of sound through a hollow cylinder made from a thin, homogeneous, isotropic material. However, for hollow cylinders constructed of more complicated materials e.g. a sandwich panel consisting of a foam core with skins made of an orthotropic material, analytic methods become difficult to formulate and thus numerical approaches are often preferable. This paper describes a numerical method for predicting sound transmission through a cylindrical structure using the wave and finite element (WFE) method. This method involves meshing a small segment of the structure with the resulting mass and stiffness matrices being post-processed to determine the structural and acoustic response.

The WFE method is a well-established technique for determining the characteristics of wave propagation in structures, including cylinders e.g. [3] (Ch 5). Renno and Mace [4] have also used the WFE method to calculate the forced response of a cylinder. In this paper we extend the WFE method in order to predict 2D sound transmission through a cylindrical structure (it is assumed that there is no variation of any property (structural or acoustic) along the axis of the cylinder). Note that the authors have recently published a paper [5] and are presenting a paper at this conference [6] describing the application of the WFE method to calculate sound transmission through flat plane structures. In §2 an analytical method is presented for calculating the acoustic field produced by a line source located...
exterior to a circular cylindrical shell made from a homogeneous, isotropic material. The response of the cylinder to the acoustic field is calculated using the Flugge-Byrne-Lur’ye shell equations [1]. In §3 the WFE method for calculating the structural response of a cylinder to an acoustic field is presented and in §4 the results of calculations made using both the analytical and numerical method are compared and shown to be in excellent agreement.

2. Analytical formulation

The problem considered is shown in figure 1 below. A line source is located exterior to a cylinder at radius $r = r_s$ and azimuthal angle $\phi_s$. The mean location of the inner and outer surfaces of the cylinder are respectively located at $r = a_i$ and $r = a_o$ and the cylinder has thickness $h$ and mean surface radius $R$.

![Figure 1. Schematic showing a cylinder with an exterior line source](image)

2.1. Acoustic field produced by a line source exterior to the cylinder

The acoustic pressure $p(x, t)$ at location $x$, and time $t$, within the fluid exterior or interior to the cylinder is assumed to satisfy the acoustic wave equation

$$\nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = -\rho_0 \frac{\partial q}{\partial t},$$

where $c_0$ and $\rho_0$ are, respectively, the ambient speed of sound and density of the fluid and $q(x, t)$ is the strength of the volume flow sources within the fluid.

A line-source of strength $Q_0(t)$ produces a volume source strength

$$q = Q_0 \frac{\delta(r - r_s)}{r} \delta(\phi - \phi_s).$$

The acoustic particle velocity in the radial direction, $u(x, t)$, is related to the acoustic pressure by eq. (3) below.

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial r}$$

It will also be useful to define the following Fourier transform pair
\[
\bar{p}_n = \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{\infty} p \exp(i\omega t - in\phi) \, dt \, d\phi,
\]

(4)

\[
p = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{p}_n \exp(-i\omega t + in\phi) \, d\omega.
\]

(5)

Taking the Fourier transform of eqs. (1) and (3) yields

\[
\bar{p}_n = \frac{i\omega_0 \bar{Q}_0}{2\pi} \exp(-in\phi_s) W(r),
\]

(6)

and

\[
\bar{u}_n = -\frac{i}{\omega_0} \frac{\partial \bar{p}_n}{\partial r}.
\]

(7)

where \( k = \omega/c_0 \) and \( W(r) \) must satisfy the following inhomogeneous ordinary differential equation:

\[
\frac{d}{dr} \left( r \frac{dW}{dr} \right) + \left( k^2 r - \frac{n^2}{r} \right) W = \delta(r - r_s).
\]

(8)

The solution to eq. (8) is given by (see, for example, Appendix 6.B of ref. [2])

\[
W = -\frac{w_+(r_s)w_-(r_<)}{r[w_+(r)w_-'(r) - w_-(r)w_+'(r)]},
\]

(9)

where \( r_> \) is the greater of \( r \) and \( r_s \), \( r_< \) is the lesser of \( r \) and \( r_s \) and \( w_+ \) and \( w_- \) are two linearly independent solutions of the homogeneous form of eq. (6) with \( w_- \) satisfying the boundary condition at \( r = a_o \) and \( w_+ \) satisfying the Sommerfeld radiation condition as \( r \to \infty \).

For this problem we choose

\[
w_-(r) = J_n(kr) + A_n H_n^{(1)}(kr), \quad w_+(r) = H_n^{(1)}(kr),
\]

(10)

where \( A_n \) is an unknown coefficient which is yet to be determined. Substituting eq. (10) into eq. (9) and then inserting the result into eq. (8) gives the following expression for the Fourier transform of the pressure exterior to the cylinder

\[
\bar{p}_n = \frac{\omega_0 \bar{Q}_0}{4} H_n^{(1)}(kr_>)[J_n(kr_<) + A_n H_n^{(1)}(kr_<)] \exp(-in\phi_s), \quad r \geq a_o.
\]

(11)

The radial velocity on the inner and outer surfaces of the vibrating cylinder is denoted \( v^i \) and \( v^o \) respectively. Note that the surface displacements are assumed to be small enough such that boundary conditions can be applied at the mean radii of the inner and outer surfaces of the cylinder which are respectively denoted \( a_i \) and \( a_o \).
The boundary condition on the exterior surface (at $r = a_o$) requires that the acoustic particle velocity in the radial direction at $r = a_o$ is equal to the radial velocity of the outer surface of the cylinder i.e.

$$\vec{v}_n^o = -\frac{i}{\omega \rho_0} \frac{\partial \vec{p}_n}{\partial r} = -\frac{ikQ_0}{4} H_n^{(1)} (kr) \left[ J'_n (ka_o) + A_n H_n^{(1)} (ka_o) \right] \exp (-in \phi_s). \quad (12)$$

Assuming $\vec{v}_n^o$ is known then eq. (14) can be rearranged to give the following expression for $A_n$

$$A_n = \frac{4i\vec{v}_n^o \exp (in \phi_s)}{kQ_0 H_n^{(1)} (kr) H_n^{(1)'} (ka_o)} - \frac{J'_n (ka_o)}{H_n^{(1)'} (ka_o)} \quad (13)$$

We therefore have the following expression for the Fourier transform of pressure on the outer surface of the cylinder

$$\vec{p}_{o,n} = \frac{\omega \rho_0 Q_0}{4} H_n^{(1)} (kr) \left[ J_n (ka_o) \frac{J'_n (ka_o)}{H_n^{(1)} (ka_o)} H_n^{(1)'} (ka_o) \right] \exp (-in \phi_s)$$

$$+ i\rho_0 c_0 \vec{v}_n^o H_n^{(1)} (ka_o) \frac{H_n^{(1)'} (ka_o)}{H_n^{(1)} (ka_o)}. \quad (14)$$

Inside the cylinder, $\vec{p}_n$ must satisfy

$$\frac{d^2 \vec{p}_n}{dr^2} + \frac{1}{r} \frac{d \vec{p}_n}{dr} + \left( k^2 - \frac{n^2}{r^2} \right) \vec{p}_n = 0, \quad (15)$$

subject to the boundary condition

$$\vec{p}_n^i = -\frac{i}{\omega \rho_0} \frac{\partial \vec{p}_n}{\partial r}, \quad (16)$$

at $r = a_i$.

The solution to eq. (15), which is finite along the cylinder axis, is of the form

$$\vec{p}_n = B_n J_n (kr), \quad r < a_i, \quad (17)$$

where $B_n$ is a coefficient which is determined from the boundary condition and which evaluates as

$$B_n = \frac{i \rho_0 c_0 \vec{p}_n^i}{J'_n (ka_i)}. \quad (18)$$

The Fourier transform of the pressure on the inner surface of the cylinder is thus

$$\vec{p}_{i,n} = i\rho_0 c_0 \frac{J_n (ka_i)}{J'_n (ka_i)} \vec{p}_n^i. \quad (19)$$
2.2. Analytical model for the structural response of a thin cylindrical shell

The equations of motion for a thin cylindrical shell of thickness \( h \) with mean-surface radius \( R \) are given by the 2-D form of the Flugge-Byrne-Lur’ye equations [1],

\[
\begin{align*}
- \frac{Eh}{R^2(1 - \nu^2)} \frac{\partial^2 w_\phi}{\partial \phi^2} - \frac{Eh}{R^2(1 - \nu^2)} \frac{\partial w_r}{\partial \phi} + \rho_s h \frac{\partial^2 w_\phi}{\partial t^2} &= 0, \\
\frac{Eh}{R^2(1 - \nu^2)} \frac{\partial w_\phi}{\partial \phi} + \left[ \frac{Eh}{R^2(1 - \nu^2)} + \frac{Eh^3}{12R^4(1 - \nu^2)} \right] w_r + \frac{Eh}{R^2(1 - \nu^2)} \frac{\partial^2 w_r}{\partial \phi^2} &+ \frac{Eh^3}{R^4(1 - \nu^2)} \frac{\partial^2 w_r}{\partial \phi^2} + \rho_s h \frac{\partial^2 w_r}{\partial t^2} = p_i - p_o, 
\end{align*}
\]

(20)

(21)

where \( p_i - p_o \) is the force per unit area acting on the shell mean-surface (which is assumed to act in the radial direction), \( E \) is Young’s modulus, \( \nu \) is Poisson’s ratio, \( w_\phi \) is the displacement of the shell mean-surface in the \( \phi \)-direction and \( w_r \) is the displacement of the shell mean-surface in the \( r \)-direction.

Applying the Fourier transform to the Flugge-Byrne-Lur’ye equations yields

\[
\begin{align*}
\frac{Ehn^2}{R^2(1 - \nu^2)} \bar{w}_\phi,n - \frac{iEhn}{R^2(1 - \nu^2)} \bar{w}_r,n - \rho_s h \omega^2 \bar{w}_\phi,n &= 0, \\
\frac{iEhn}{R^2(1 - \nu^2)} \bar{w}_\phi,n &+ \left[ \frac{Ehn^2}{12R^4(1 - \nu^2)} - \frac{Ehn^2}{6R^4(1 - \nu^2)} + \frac{Eh^3}{12R^4(1 - \nu^2)} + \frac{Eh}{R^2(1 - \nu^2)} \right] \bar{w}_r,n = \bar{p}_{i,n} - \bar{p}_{o,n}. 
\end{align*}
\]

(22)

(23)

It is assumed that the velocity of the inner and outer surfaces of the cylinder are equal to the radial velocity of the shell mean-surface i.e. \( \bar{v}_\phi_i = \bar{v}_\phi_o = -i\omega \bar{w}_r,n \).

Substituting eqs. (14) and (19) into eq. (23) yields the following matrix equation which describes the Fourier transform of the response of the thin cylindrical shell to the acoustic field produced by the line source

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \bar{w}_\phi,n \\ \bar{w}_r,n \end{bmatrix} = \begin{bmatrix} 0 \\ E \end{bmatrix},
\]

(24)

where

\[
\begin{align*}
A &= \frac{Ehn^2}{R^2(1 - \nu^2)} - \rho_s h \omega^2, & B &= - \frac{iEhn}{R^2(1 - \nu^2)}, & C &= \frac{iEhn}{R^2(1 - \nu^2)}' \\
D &= \frac{Ehn^2}{12R^4(1 - \nu^2)} - \frac{Ehn^2}{6R^4(1 - \nu^2)} + \frac{Eh^3}{12R^4(1 - \nu^2)} + \frac{Eh}{R^2(1 - \nu^2)} - \rho_s h \omega^2 \\
&\quad - \rho_0 c_0 \omega \begin{bmatrix} \frac{H_n^{(1)}(ka_i)}{H_n^{(1)}(ka_0)} - \frac{H_n^{(3)}(ka_i)}{H_n^{(3)}(ka_0)} \\
\frac{H_n^{(1)}(ka_0)}{H_n^{(1)}(ka_i)} \end{bmatrix},
\end{align*}
\]

(25a-e)
\[ E = -\frac{\omega \rho_0 \bar{Q}_0}{4} H_n^{(1)}(kr_3) \begin{bmatrix} J_n(ka_0) & J_n'(ka_0) \\ \frac{H_n^{(1)}(ka_0)}{H_n^{(1)}(ka_0)} & H_n^{(1)}(ka_0) \end{bmatrix} \exp\{-in\phi_s\}. \]

The matrix equation can be solved to give \( \bar{\omega}_{r,n} \) and \( \bar{\omega}_{\phi,n} \) which can then be used in eqs. (11) and (17) to calculate the acoustic pressure field exterior and interior to the cylinder.

Note that in many cases (usually when the acoustic medium has a low density relative to the density of the cylindrical shell) it is expected that the loading on the structure due to acoustic pressure produced by its vibration may be neglected. This can be done by setting the term in square brackets in eq. (25d) (which defines the matrix component \( D \)) equal to zero.

3. WFE method

In this section the WFE method is applied to calculate the response of a cylinder to acoustic excitation. Note that the analysis presented considers each Fourier component separately.

A curved rectangular segment is used to model a small segment of the structure from which the cylinder can be constructed by tessellation. The segment subtends an angle \( \Delta \phi \) which is assumed to be small enough such that the curved segment can be approximated using a flat rectangular segment. The flat segment has dimensions \( \Delta x \times \Delta y \) where \( \Delta x = R\Delta \phi \) and \( \Delta y \) is the length of the segment along the cylinder axis. According to ref [4], the dimensions of the segment should be chosen as in conventional finite element (FE) analysis e.g. six (or more) elements per wavelength are needed to accurately represent the motion. In all cases the vector of degrees of freedom (dofs), \( \mathbf{q} \), of the segment (illustrated in fig. 2 below) are partitioned as

\[ \mathbf{q} = [\mathbf{q}_{lb}^T \quad \mathbf{q}_{rb}^T \quad \mathbf{q}_{lt}^T \quad \mathbf{q}_{rt}^T]^T, \]

where the superscript \( T \) denotes transposition and the subscripts \( l, r, b, t \), correspond to left, right, bottom, top and internal nodes. The vector of internal nodal forces, \( \mathbf{f} \), and external nodal forces, \( \mathbf{e} \), are partitioned in the same manner.

\textbf{Figure 2.} Rectangular element used to model a segment of the cylinder

To include the effect of the curvature of the cylinder, the local coordinates (from where the FE model was obtained) are rotated around the \( y \)-axis by an angle \( \Delta \phi \). Thus, FE matrices of the “curved” segment become

\[ \mathbf{M} = \overline{R}^T \mathbf{M}_{LOC} \overline{R}, \quad \mathbf{K} = \overline{R}^T \mathbf{K}_{LOC} \overline{R}, \]

(27)
where $K_{LOC}$ and $M_{LOC}$ are the segment’s stiffness and mass matrices in the local coordinate system. The rotation matrix $R$ is given by

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & R & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & R \end{bmatrix},$$

where $R$ allows the rotation of the $(rb)$ and $(rt)$ nodes of the segment. Note that in the above equation, $I$, $0$ and $R$ are matrices of order equal to the number of nodal dofs in the FE model. $I$ is an identity matrix, $0$ is a matrix of zeros and $R$ is defined below

$$R = \begin{bmatrix} \cos(\Delta \phi) & 0 & -\sin(\Delta \phi) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin(\Delta \phi) & 0 & \cos(\Delta \phi) & 0 & -\sin(\Delta \phi) & 0 \\ 0 & 0 & 0 & \cos(\Delta \phi) & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sin(\Delta \phi) & 0 & \cos(\Delta \phi) \end{bmatrix}.$$

The equation of motion of the segment is

$$[K - \omega^2M]q = f + e.$$

Because the method considers a single Fourier component, all parameters vary in the tangential direction by the factor $\exp\{in\phi\} = \exp\{i\gamma_x x\}$, where $\gamma_x = n/R$. Thus each parameter will vary across the element in the tangential direction by a factor $\lambda_x = \exp\{i\gamma_x \Delta x\}$, where $\Delta x = R \Delta \phi$.

The dofs are thus related by

$$q = \Lambda_q q_{lb}, \quad \Lambda_q = \begin{bmatrix} I \\ \lambda_x I \\ I \end{bmatrix},$$

where $I$ is an identity matrix of order equal to the number of dofs at each node.

Substituting eq. (31) into eq. (30) and pre-multiplying the result by $\Lambda_q^H$, where the superscript $H$ denotes the complex transposition operator, yields

$$\Lambda_q^H [K - \omega^2M] \Lambda_q q_{lb} = \Lambda_q^H f + \Lambda_q^H e,$$

The terms on the right-hand side of eq. (32) can be written as

$$\Lambda_q^H f = \begin{bmatrix} I & \lambda_x^{-1}I & \lambda_x^{-1}I \\ \lambda_x^{-1}I & I & \lambda_x^{-1}I \\ \lambda_x^{-1}I & \lambda_x^{-1}I & I \end{bmatrix} \begin{bmatrix} f_{lb} \\ f_{rb} \\ f_{rt} \end{bmatrix} = f_{lb} + \lambda_x^{-1} f_{rb} + f_{lt} + \lambda_x^{-1} f_{rt},$$

and
\[ A^H_q e = \begin{bmatrix} I & \lambda^{-1}I & \lambda^{-1}I \end{bmatrix} \begin{bmatrix} e_{ib} \\ e_{rb} \\ e_{lt} \\ e_{rt} \end{bmatrix} = e_{ib} + \lambda^{-1}e_{rb} + e_{lt} + \lambda^{-1}e_{rt}. \]  

(34)

The equilibrium conditions at the right edge of the segment imply that \( f_{ib} + \lambda^{-1}f_{rb} + f_{lt} + \lambda^{-1}f_{rt} = 0 \). Because the problem is two-dimensional it is also required that \( e_{ib} + \lambda^{-1}e_{rb} = e_{lt} + \lambda^{-1}e_{rt} \). This gives

\[ \frac{1}{2} D q_{ib} = e_{ib} + \lambda^{-1}e_{rb}, \]

(35)

where

\[ D = A^H_q [K - \omega^2 M] A_q. \]

(36)

It will be assumed that the velocities of the inner and outer surfaces of the cylinder are equal i.e. \( \ddot{\theta}_n = \dot{\ddot{\theta}}_n = -i \omega \ddot{w}_{r,n} \). A single Fourier component of the net force per unit area in the radial direction exerted on the inner and outer surfaces of the cylinder, \( (\ddot{p}_n - \ddot{p}_n^0) \exp\{i n \phi\} \), can be determined using eq. (14) and (19) to give

\[ (\ddot{p}_n - \ddot{p}_n^0) \exp\{i n \phi\} = g \exp\{i n \phi\} + h \ddot{w}_{r,n} \exp\{i n \phi\}, \]

(37)

where

\[ g = -\frac{\omega \rho_0 Q_0}{4} H_n^{(1)}(kr) \left[ J_n(ka_0) - \frac{J_n'(ka_0)}{H_n^{(1)}(ka_0)} H_n^{(1)}(ka_0) \right] \exp\{-i n \phi_s\}, \]

(38)

and

\[ h = \omega \rho_0 c_0 \left( \frac{J_n(ka_i)}{J_n'(ka_i)} - \frac{H_n^{(1)}(ka_0)}{H_n^{(1)}(ka_0)} \right). \]

(39)

For cases where the effect of cylinder vibration on fluid loading can be neglected, the equations above can be simplified by setting \( \ddot{w}_{r,n} = 0 \) in eq. (37).

Consistent nodal forces can be obtained using the method described in [4], or alternatively and less accurately, the external excitation can be lumped at the nodes which is the approach adopted here. The radial component of the external nodal force at the \((lb)\) and \((rb)\) nodes are respectively given by

\[ e_{ib} (w_z) \approx \int_0^{L_y} \int_0^{L_x} \left( \ddot{p}_n - \ddot{p}_n^0 \right) \exp\{i n \phi\} Rd \phi dy = \frac{i}{2n} L_y R \left( \ddot{p}_n - \ddot{p}_n^0 \right) \left[ \exp\left\{ \frac{i n L_x}{2R} \right\} - 1 \right], \]

(40)

and
The nodal force vector can also be decomposed into two components, one of which is proportional to the dof vector \( q_{ib} \) giving

\[
e_{ib} = g_{ib} + H_{ib} q_{ib}, \quad e_{rb} = g_{rb} + H_{rb} q_{rb} = g_{rb} + \lambda_s H_{rb} q_{ib}
\]

where \( g_{ib} \) and \( g_{rb} \) are vectors of zeros except for the radial components which are

\[
g_{ib}(w_z) = \frac{i}{2n} \frac{L_y R}{n} \left[ 1 - \exp\left\{ \frac{i n L_x}{2R} \right\} \right]
\]

\[
g_{rb}(w_z) = \frac{i}{2n} \frac{L_y R}{n} \left[ \exp\left\{ \frac{i n L_x}{2R} \right\} - \exp\left\{ \frac{i n L_x}{R} \right\} \right]
\]

and \( H_{ib} \) and \( H_{rb} \) are matrices of zeros except for the diagonal component associated with the nodal displacement in the radial direction

\[
H_{ib}(w_z, w_z) = \frac{i}{2n} \frac{L_y R}{n} \left[ 1 - \exp\left\{ \frac{i n L_x}{2R} \right\} \right],
\]

\[
H_{rb}(w_z, w_z) = \frac{i}{2n} \frac{L_y R}{n} \left[ \exp\left\{ \frac{i n L_x}{2R} \right\} - \exp\left\{ \frac{i n L_x}{R} \right\} \right].
\]

The final equation of motion for the cylinder being excited by the acoustic field is thus

\[
\frac{1}{2} D - H_{ib} - H_{rb} \right] q_{ib} = g_{ib} + \lambda_s^{-1} g_{rb},
\]

which can be inverted to give \( q_{ib} \).

4. Results

In this section the WFE method is used to calculate the structural response and acoustic radiation from a cylinder excited by sound from an acoustic line source. The results of the calculation are compared with the results of calculations made using the analytic method. Note that the WFE method utilises the SHELL181 element from ANSYS to obtain the mass and stiffness matrices. This element is rectangular with 4 nodes and six degrees of freedom at each node: translations in the nodal \( x \), \( y \) and \( z \) directions and rotations about the nodal \( x \), \( y \) and \( z \) axes. The cylinder has density 7800kg.m\(^{-3}\), Young’s modulus 200×10\(^9\) Pa, Poisson ratio, 0.3, mean-surface radius 0.3m and a thickness of 1.8mm. The fluid interior and exterior to the cylinder has density 1000kg.m\(^{-3}\). The line source is located at a radius of 0.4m and has a strength of 1m\(^2\). Figure 3 plots the magnitude of the radial displacement of the shell \( \bar{w}_{r,n} \) against frequency, \( \omega \), for four different values of \( n \). Note that the agreement between
the WFE method and the analytical solution is generally good. Figure 4 plots the magnitude of the acoustic pressure against radius using the structural response calculated using the WFE method.

Figure 3. Plot of $|\tilde{\omega}_{r,n}|$ versus $\omega$ comparing the accuracy of the WFE calculation to the analytical solution.
5. Conclusions and future work

A WFE method for predicting two-dimensional sound transmission through a cylindrical structure has been developed and validated against an analytic solution. Future work will include: the calculation of consistent nodal forces, extension of the method to 3D problems and the application of the method to more complicated structures.

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