Measurement and modelling of forced convective heat transfer coefficient and pressure drop of Al₂O₃- and SiO₂-water nanofluids

J E Julia¹, L Hernández¹, R Martínez-Cuenca¹, T. Hibiki², R Mondragón³, C Segarra³, J C Jarque³

¹ Departamento de Ingeniería Mecánica y Construcción, Universitat Jaume I, Campus de Riu Sec, 12071-Castellón de la Plana, Spain
² Nuclear Engineering Department. Purdue University. West Lafayette. Indiana, USA
³ Instituto de Tecnología Cerámica, Universitat Jaume I, Campus de Riu Sec, 12071-Castellón de la Plana, Spain
E-mail: bolivar@emc.uji.es

Abstract. Forc enective heat transfer coefficient and pressure drop of SiO₂- and Al₂O₃-water nanofluids were characterized. The experimental facility was composed of thermal-hydraulic loop with a tank with an immersed heater, a centrifugal pump, a bypass with a globe valve, an electromagnetic flow-meter, a 18 kW in-line pre-heater, a test section with band heaters, a differential pressure transducer and a heat exchanger. The test section consists of a 1000 mm long aluminium pipe with an inner diameter of 31.2 mm. Eighteen band heaters were placed all along the test section in order to provide a uniform heat flux. Heat transfer coefficient was calculated measuring fluid temperature using immersed thermocouples (Pt100) placed at both ends of the test section and surface thermocouples in 10 axial locations along the test section (Pt1000). The measurements have been performed for different nanoparticles (Al₂O₃ and SiO₂ with primary size of 11 nm and 12 nm, respectively), volume concentrations (1% v., 5% v.), and flow rates (3 10³Re<10⁵). Maximum heat transfer coefficient enhancement (300%) and pressure drop penalty (1000%) is obtained with 5% v. SiO₂ nanofluid. Existing correlations can predict, at least in a first approximation, the heat transfer coefficient and pressure drop of nanofluids if thermal conductivity, viscosity and specific heat were properly modelled.

1. Introduction

Low thermal conductivity of fluids constitutes one of the most important limitation factors in heat transfer applications. In order to improve the thermal conductivity of fluids S.U.S Choi in 1995 proposed the use of nanofluids [1]. Nanofluids are defined as dilute suspensions with solid (nano)particles smaller than 100 nm. Nanofluids present some important advantages over the conventional colloidal suspensions such as high stability, reduced particle clogging and high heat transfer capabilities. However, the applicability of nanofluids in heat transfer systems is limited by the increase of the viscosity of the fluid (which increases the pumping power).

Most of the published works have been focused on the thermal conductivity enhancement measurement and modelling in nanofluids. These studies strongly suggest that nanoparticle clustering holds the key for the thermal conductivity enhancement [2-3]. However, since the heat transfer
performance is a better indicator than the effective thermal conductivity for nanofluids, the modelling of nanofluid heat transfer coefficients is gaining attention from researchers. Experimental data available shows a significant enhancement of convective heat transfer in parallel channels [4], tubes [5-7] and annular flow channels [8]. Despite the fact that the theoretical models for nanofluid heat transfer coefficients are quite limited, there is a strong implication that particle size and shape could play an important role in the level of heat transfer enhancement. Under similar conditions, the enhancement of heat transfer coefficient is much higher for carbon nanotubes [9] than that for disc-shaped TiO\textsubscript{2} nanofluids [7]. In addition, a number of other mechanisms have been proposed by considering the two-phase nature of nanofluids, which includes the thermal dispersion effect [10] and particle migration effect [11], but none of them is conclusive.

Most of the heat transfer coefficient equations obtained for nanofluids are modifications from traditional equations such as Dittus-Boelter [12] or Gnielinski equations [13] with some empirical parameters related with the nanofluid properties [14]. Therefore, these equations are only valid for certain nanofluids over small parameter ranges and proper models involving thermal conductivity, viscosity and specific heat are needed [15].

In this work, forced convective heat transfer coefficient and pressure drop of water based nanofluids of SiO\textsubscript{2} and Al\textsubscript{2}O\textsubscript{3} have been characterized. The experimental facility was composed of a thermal-hydraulic loop with aluminium horizontal test section composed of a tube with a length of 1000 mm and an inner diameter of 31.2 mm. The measurements have been performed for different nanoparticles (Al\textsubscript{2}O\textsubscript{3} and SiO\textsubscript{2}), volume concentrations (1\% v., 5\% v.), and flow rates (3 \times 10^{3}<Re<10^{5}).

2. Experimental procedure

2.1. Experimental set-up

Figure 1 shows a sketch of the thermal-hydraulic loop. The experimental facility is composed of a tank (inner volume of 0.118 m\textsuperscript{3}) with an 6 kW immersed heater (Watlow Ref. L14JX8B), a centrifugal pump (Espa Ref. XVM8 03F15T), a bypass with a globe valve, an electromagnetic flow-meter (Badger Meter Magnetoflow M1000, ±0,50\%), a 18 kW in-line pre-heater (Watlow Ref. CBLC737C00S-20), a test section with band heaters, a differential pressure transducer (Inversys IDP10, ±0,060\%) and a heat exchanger.

![Figure 1. Layout of the forced convective thermal loop](image)

Figure 2 shows a schematic representation of the test section and instrumentation. The test section consists of a 1000 mm long aluminium pipe with an inner diameter of 31.2 mm and a wall thickness of 3.6 mm. Eighteen band heaters are placed all along the test section in order to provide a uniform heat...
flux. The maximum heat flux available is 107 k/m². Heat transfer coefficient was calculated measuring fluid temperature using immersed thermocouples (Pt100, ±0.05°C) placed at both ends of the test section and surface thermocouples in 10 axial locations along the test section (Pt1000, ±0.1°C) [16]. Surface thermocouples were inserted 2 mm in the tube wall in order to avoid direct heating from the band heaters. Finite element simulation of the test section, heating system and thermocouples provided uniform heating with temperatures differences below 0.2 °C.

Figure 2. Schematic representation of the test section for single phase heat transfer experiments.

The measured parameters in the test section were the heat transfer coefficient ($h$) and the friction loss ($\Delta P$).

2.2. Forced convective heat transfer coefficient calculation, $h$

Forced convective heat transfer coefficient calculation was based on the work of Yu et al. (2009) [16]. The heat transfer coefficient is defined as:

$$h(x) = \frac{q}{T_{w,in}(x) - T_f(x)}$$

where $q$ is the heat flux, $x$ is the axial location and $T_{w,in}$ and $T_f$ are the test section inner wall temperature and fluid temperature, respectively.

The heat flux was determined through an energy balance:

$$Q = \frac{\rho c_p Q}{\pi D L} \left(T_{f, out} - T_{f, in}\right) = \Delta T_f$$

where $D$ is the inner diameter of the test section, $L$ is the length of the test section, $\rho$, $c_p$ and $Q$ the fluid density, heat capacity and volumetric flow rate, respectively. Both temperatures $T_{f, in}$ at $x=0$ and $T_{f, out}$ at $x=L$ can be measured with the immersed thermocouples placed at both ends of the test section. Hence, the heat flux ($q$) can be determined using the above equation. The test section is thermal insulated using Superwool 607 (10 mm thickness, $k=0.06$ W/m K). However, no complete insulation is possible due to the test section configuration (thermocouples, power supply cables, etc) and a heat loss of 10%-25% is produced depending on the test conditions.

It should be noted that axial heat flux is not considered in the $h$ calculation since experimental measurements show that it is, at least, 20 times smaller than the radial heat flux contribution. In addition, radial dependence of $T_{f, in}$ and $T_{f, out}$ has been checked measuring fluid temperature at different radial locations using a 0.25 mm K-type thermocouple. Radial dependence of $T_{f, in}$ is negligible. Radial dependence of $T_{f, out}$ depends on the fluid velocity and heat flux. In order to take the radial dependence into account in the $h$ calculation and averaged measurement of $T_{f, out}$ is performed using a Pt100 probe with a sensing length of 30 mm crossing the pipe diameter.

The fluid local temperature, $T_f(x)$, was calculated from a linear relation between test section inlet and outlet temperatures ($T_{f, in}$ and $T_{f, out}$).

$$T_f(x) = \frac{x}{L} T_{f, out} + \left(1 - \frac{x}{L}\right) T_{f, in}$$

(3)
The inner wall temperature, $T_{w, in}$, was determined from a radial heat conduction calculation by using the thermo-resistors placed in the outer wall of the test section ($T_{w, out}(x)$) and the local heat generated in the test section wall per unit length $q'(x)$:

$$T_{w, in}(x) = T_{w, out}(x) + \frac{q'}{4\pi k_{tube}} \frac{1 + \ln \left( R_d \right)}{1 - (R_d)}$$

where $k_{tube}$ is the thermal conductivity of the test section and $R_d$ is defined as,

$$R_d = \left( \frac{D}{D_{ext}} \right)^2$$

where $D_{ext}$ takes into account the thermocouple position inside the test section wall.

3. Results and discussion

3.1. Forced convective heat transfer coefficient, $h$

In order to check the accuracy of the experimental system, the forced convective heat transfer coefficient of a model fluid (distilled water) was measured and compared with standard correlations. Figure 3 shows the comparison of the local Nusselt number provided by the experimental system (10 measurements at different axial locations shown in Figure 2) and the Dittus-Boelter correlation. The Nusselt number is defined as:

$$Nu = \frac{hD}{k_f}$$

and the Dittus-Boelter correlation is given by,

$$Nu = 0.023 Re^{0.8} Pr^{0.4}$$

where $Re$ and $Pr$ are the conventional Reynolds and Prandtl numbers given by,

$$Re = \frac{\mu D}{\mu}$$

$$Pr = \frac{c_p \mu}{k_f}$$

![Figure 3. Comparison of local Nusselt numbers measured by the system and predicted by Dittus-Boelter correlation (distilled water).](image.png)

Distilled water $\mu$, $c_p$, and $k_f$ values have been obtained from standard correlations. In all the experiments the inlet fluid temperature was 60°C and the wall maximum temperature was 95°C. From the figure 3, it is possible to observe that the experimental Nusselt number is within the ±15% range of the value provided by the Dittus-Boelter correlation. The main uncertainty source in the experimental Nusselt number measurement is the temperature difference $\Delta T_f$ of the fluid. The temperature
difference value decrements with $Re$ value, since the maximum heat flux available was 107 kW/m$^2$ and the $Pr$ value is almost constant in all the experimental conditions. Consequently, the uncertainty in the Nusselt number measurement increases with the Nusselt number value.

Forced convective heat transfer coefficient of water based nanofluids of SiO$_2$ and Al$_2$O$_3$ have been characterized. The SiO$_2$ nanofluid was produced by the two-step method using commercial powder Aerosil 200 (Degussa). The powder is composed of amorphous hydrophilic silica nanoparticles with primary units of 12 nm and a specific surface of 200 m$^2$/g. The particle size of the real agglomerates present in the suspension was found to be 201 nm. The 90% of the particles had a diameter lower than 345 nm. The nanofluid was dispersed using an ultrasonic probe and stabilized by $pH$ value adjustment ($pH=9$).

The Al$_2$O$_3$ nanofluid is commercially available in suspension (Aerodisp W925, Degussa). The primary particles have a diameter of 11 nm. The particle size of the real agglomerates present in the suspension was 127 nm. The 90% of the particles have a diameter lower than 218 nm. The nanofluid was stabilized by $pH$ value adjustment ($pH=4$). More technical details about the nanofluid production and the complete $\mu$, $cp$ and $k$ characterization can be found in [15].

Figure 4 a) shows the forced convective heat transfer coefficient of distilled water and SiO$_2$ and Al$_2$O$_3$ – water nanofluids with 1% and 5% volume concentrations (SilX00 and AluX00 in the figure). Error analysis can be found in Appendix A. All the experiments were performed with a nanofluid inlet temperature at the test section of 60ºC and a maximum wall temperature of 95ºC. $Re$ number varied along the measurements since the nanofluid viscosity is higher than distilled water and its value depends on nanoparticle type and volume concentration [15]. Nanofluid values of $\mu$, $cp$ and $k$ values have been obtained from a previous work [15]. It has to be mentioned that heat transfer experiments with nanofluids at $Re$=3000 were not performed in order to prevent nanoparticle deposition. Heat transfer coefficient values of 1% v. nanofluids (Sil100 and Alu100 in the figure) are quite similar to those measured for distilled water. However, heat transfer coefficient values of 5% v. nanofluids (Sil500 and Alu500) show an important increment compared to those found for distilled water (180% for Al$_2$O$_3$ and 300% for Si$_2$O$_3$ at $Re$~3x10$^4$). Heat transfer for 3000<$Re$<5000 values is quite similar in all nanofluids and distilled water. Heat transfer in 5% v. nanofluids is similar up to $Re$~12x10$^3$, and from then, the increment in SiO$_2$ nanofluid is higher than in the case of Al$_2$O$_3$.

The differences found in the heat transfer coefficients in Figure 4 a) can be explained using standard correlations. Figure 4 b) shows the comparison of the averaged Nusselt number provided by the experimental system (length weighted averaged values over 10 measurements at different axial
locations) and the Dittus-Boelter correlation. All the nanofluid Nusselt number values are within the ±40% range of the value predicted by the correlation for \(Re>10^4\). If the 5% v. Al\(_2\)O\(_3\) values are not considered, the range is reduced to ±10%. Therefore, standard correlations like Dittus-Boelter can be used, at least in a first approximation, to predict heat transfer coefficient in turbulent flow if thermal conductivity, viscosity and specific heat are properly modelled. More experimental and modelling work is needed in order to predict the heat transfer behaviour of nanofluids in the transition flow region (3000<\(Re<10000\))

3.2. Pressure drop, \(\Delta P\)

Pressure drop in the test section was a direct measurement in the experimental setup. Since the test section is horizontal, the measurement of the differential pressure transducer provides the pressure drop measurement. Figure 6 a) shows the pressure drop measurements for all the fluids and \(Re\) number values. The pressure drop value for a fixed \(Re\) number increases with the nanofluid viscosity. This fact can be explained since a higher flow rate is needed to reach the \(Re\) value.

![Figure 6](image)

**Figure 6.** Comparison of pressure loss of distilled water and SiO\(_2\) and Al\(_2\)O\(_3\) nanofluids, a) pressure drop measurements and b) friction factor calculation and measurement.

Friction factor can be obtained from the pressure drop experimental data,

\[
\Delta P = f \frac{L \rho v^2}{D} 
\]

(10)

where \(f\) is the friction factor and it can be compared with the results given by the Colebrook-White equation [17],

\[
\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}}\right) 
\]

(11)

where \(\varepsilon\) is the roughness of the tube. Error analysis can be found in Appendix B.

Figure 6 b) shows the comparison between the measured friction factor and the predictions given by the Colebrook-White equation. It is possible to observe that all the measured values are within a ±20% range of the value predicted by a standard correlation.

4. Conclusions

Forced convective heat transfer coefficient and pressure drop of SiO\(_2\)- and Al\(_2\)O\(_3\)- water nanofluids with 1% v., 5% v. have been measured in thermal-hydraulic loop. The experiments have been
performed at a fixed inlet fluid temperature of 60ºC and a maximum wall temperature of 95ºC. The \( Re \) number value was varied between \( 3 \times 10^3 \) and \( 10^5 \). \( Pr \) number was kept almost constant in the experiments.

Forced convective heat transfer coefficient was increased using 5% v. nanofluid compared to distilled water values (180% for \( \text{Al}_2\text{O}_3 \) and 300% for \( \text{Si}_2\text{O}_3 \) at \( Re \sim 3 \times 10^4 \)). Dittus-Boelter correlation could predict, at least in a first approximation, the heat transfer coefficient of nanofluids in turbulent flows when thermal conductivity, viscosity and specific heat were properly modelled.

Pressure drop was increased using nanofluids and its increment was related to fluid viscosity. Standard friction factor correlation could predict the pressure drop of nanofluids when viscosity was properly modelled.

Acknowledgments

The authors gratefully acknowledge the financial support from the Spanish Ministry of Science and Innovation (projects CTQ2010-21321-C02-01 and CTQ2010-21321-C02-02).

References

[1] Choi S U S 1995 ASME Int. Mech. Eng. Congress and Exp San Francisco, CA, USA.
[2] Hong, K. S., Hong, T. and Yang, H. 2006. Appl. Phys. Lett. 88 Art. No 031901.
[3] Gao, J. W., Zheng, R. T., Ohtani, H., Zhu, D. S. and Chen, G. 2009. Nano Lett., 9, 4128 – 4132.
[4] Lee, S. and Choi, S. U. S 1996 Int. Mech. Eng. Congress and Exp. Atlanta, USA.
[5] Xuan, Y. and Li, Q. 2003. J. Heat Transf., 125, 151–155.
[6] Wen, D. S., and Ding, Y. L. 2004. Int. J. Heat and Mass Transf., 47, 5181– 5188.
[7] Yang, Y., Zhang, Z., Grulke, E., Anderson, W., & Wu, G. 2005. Int. J. Heat and Mass Transf. 48, 1107– 1116.
[8] Heris, S., Etemad, S. G., and Esfahany, M. 2006. Int. Comm. Heat and Mass Transf., 33, 529– 535.
[9] Ding, Y. L., Alias, H., Wen, D. S., & Williams, R. A. 2006. Int. J. Heat and Mass Transf. 49, 240–250.
[10] Xuan, Y. M., & Roetzel, W. 2000. Int. J. Heat and Mass Transf., 43, 3701– 3707.
[11] Wen, D. S., and Ding,Y. L. 2005. J. Nanoparticle Research, 7, 265–274.
[12] Dittus, F.W. and Boelter, L.M.K. 1930. Univ. Calif. Publ. Eng. 2, 443–461.
[13] Gneilinski, V. 1976. Int. Chem. Eng. 16, 359–368.
[14] Duangthongsuk, W. and Wongwises, S. 2010. Int. J. Heat and Mass Transf. 53, 334–344.
[15] Mondragon, R., Segarra, C., Jarque, J. C., Julia, J. E., Hernandez, L., Martinez-Cuenca, R. 2012. 6th European Thermal Sciences Conference. Poitiers-Futuroscope. France.
[16] Yu, W., France D. M., Smith D. S., Timofeeva, E. V., Routbort J L 2009 Int. J. Heat Mass Transfer 52 3606–3612.
[17] Colebrook, C.F., 1939. J. Inst. Civ. Eng. Lond, 11, 133-156.
Appendix

A. Heat transfer coefficient error analysis

Heat transfer coefficient error, $\delta h$, can be calculated from equation (1) by,

$$
\delta h = \delta q \left[ \frac{\partial}{\partial T} \left( \frac{\partial h}{\partial (T_{w,\text{in}} - T_f) \delta (T_{w,\text{in}} - T_f) \right) \right]^{\frac{1}{2}}
$$

(A.1)

where $\delta$ denotes the accuracy in the measured value.

From equation (2) it is possible to obtain $\delta q$ as:

$$
\delta q = \delta \rho \left[ \frac{\partial}{\partial \rho} \delta q \right]^2 + \delta c_p \left[ \frac{\partial}{\partial c_p} \delta q \right]^2 + \delta L \left[ \frac{\partial}{\partial L} \delta L \right]^2 + \delta D \left[ \frac{\partial}{\partial D} \delta D \right]^2 + \delta T \left[ \frac{\partial}{\partial T} \delta T \right]^2
$$

(A.2)

In the last equation, $\delta \rho=0.1\%$ and $\delta c_p=4.5\%$ are given by the experimental technique. The tube manufacturer provides $\delta L=10^{-3}$ m and $\delta D=2 \times 10^{-5}$ m. The value of $\delta Q$ is given by the flow meter accuracy and its value is 0.50%. From equation (3) it is possible to obtain $\delta \Delta T_f$ as:

$$
\delta \Delta T_f = \left( \frac{\partial}{\partial \lambda} \delta \lambda \right)^2 + \left( \frac{\partial}{\partial \Delta T \delta \Delta T_f} \right)^2
$$

(A.3)

where $\delta \lambda=10^{-3}$ m and $\delta T_{f,\text{in}}, \delta T_{f,\text{out}}$ are provided by the thermocouples manufacturer as 0.05 ºC.

From equation (4) it is possible to obtain $\delta \Delta T_{w,\text{in}}$ as:

$$
\delta \Delta T_{w,\text{in}} = \delta T_{w,\text{in}} \left( \frac{\partial}{\partial \rho} \delta T_{w,\text{in}} \right)^2 + \left( \frac{\partial}{\partial D} \delta T_{w,\text{in}} \right)^2 + \left( \frac{\partial}{\partial \lambda} \delta T_{w,\text{in}} \right)^2 + \left( \frac{\partial}{\partial \rho} \delta \lambda \right)^2
$$

(A.4)

where $\delta T_{w,\text{in}}$ is provided by the thermocouple manufacturer as 0.1 ºC and $\delta \lambda$ is obtained from literature as 0.01%. The value of $\delta R_d$ is obtained by the following expression,

$$
\delta R_d = \left( \frac{\partial}{\partial D} \delta D \right)^2 + \left( \frac{\partial}{\partial \rho} \delta \rho \right)^2
$$

(A.5)

where $\delta D$ is measured as $10^{-4}$ m

B. Friction factor error analysis

Friction factor error, $\delta f$, can be calculated from equation (10) by,

$$
\delta f = \left( \frac{\partial f}{\partial \Delta P} \delta \Delta P \right)^2 + \left( \frac{\partial f}{\partial \rho} \delta \rho \right)^2 + \left( \frac{\partial f}{\partial D} \delta D \right)^2 + \left( \frac{\partial f}{\partial \lambda} \delta \lambda \right)^2
$$

(B.1)

where $\delta \Delta P$ is provided by the pressure transducer manufacturer as 0.06%.