Interference effects in an Aharonov–Bohm ring with random quantum dots

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Abstract. We investigate electron transport through quantum dots embedded in an Aharonov–Bohm ring. One or both of the dots are chaotic and are treated by random matrix theory. The conductance and conductance fluctuations are calculated as a function of the ratio of the level broadening to the average level spacing in the random dots using random scattering matrix theory with a Poisson kernel. Several interference mechanisms in the ring are examined. When one random dot and a single resonance level coexist and the multiple reflection between them is taken into account, the conductance is suppressed as the Fermi level of the leads approaches resonance in the large level broadening regime. For coupled random dot systems, characterized by two different ratios, the conductance is sensitive to the difference between the ratios when the multiple scattering inside the ring is disregarded, while it does not depend on the difference when the scattering is effective.
1. Introduction

Quantum dot systems [1]–[3] have been studied, with the aim of understanding the phase coherence of quantum mechanical wavefunctions. Elastic scattering processes by an irregular boundary of the dot and impurities inside the dot induce the interference of electron trajectories. This interference results in quantum fluctuations. The statistical properties of the fluctuations can be analysed by random matrix theory (RMT) [4]–[7]. The system is categorized by several universality classes depending on the symmetry of the system. We can monitor this intrinsic property in electron transport by attaching leads to the dot. However, when the dot–leads coupling becomes stronger, we must treat the whole system quantum mechanically. In fact, a recent numerical calculation of a chaotic dot shows that there are trajectories which connect between the leads [8].

The coexistence of strong and weak coupling modes between the dot and leads is a common feature for open quantum dot systems. It can induce nontrivial phenomena which do not exist in weak coupling systems. The Fano effect is a prominent example, where the system consists of a direct transport path between the leads and discrete levels [9]. As a result of the interference between these two contributions, the transmission spectrum is generally asymmetric while it is symmetric for discrete level systems. The asymmetric peak is characterized by a single complex parameter, called the Fano parameter. The Fano effect itself is a ubiquitous phenomenon which appears in a wide variety of resonant scattering experiments in atomic physics and condensed matter physics, including a microwave cavity [10] and optical absorption [11]. In recent experiments, the Fano effect has been demonstrated in quantum dot systems [12]–[15]. In these experiments, the Fano parameter is controlled by external gate voltages and magnetic fields.

Electron transport through the Aharonov–Bohm (AB) ring with quantum dots has been investigated, with the aim of understanding electron coherence [1, 16, 17], and thus it is an ideal system for studying the Fano effect as well. Theoretical works have been investigated...
to understand the peculiarity of the Fano effect in the AB ring systems [18]–[23]. The role of randomness has also been discussed. For example, when a chaotic dot is embedded in an AB ring, electron transport under the AB flux is governed by the Altshuler–Aronov–Spivak mechanism [24]–[28]. Recently, Clerk et al [29] investigated the random Fano system, where the asymmetric resonance levels are distributed randomly and RMT was used to analyse the statistical distribution of the Fano parameter.

In this paper, we discuss the interplay between the randomness and the interference effect due to the AB ring. We focus on the following two points: (i) the strong dot–lead coupling regime in random dots and (ii) the mechanism by which electrons interfere inside the ring, which have not been emphasized in previous studies.

We discuss the system with strong dot–lead coupling. For a random quantum dot, as we will discuss, it is natural to introduce the ratio of the level broadening to the average level spacing, which characterizes the energy scale of the dot. In conventional weak quantum dot systems, this ratio is less than unity. In contrast, when the dot is widely opened to external leads, many resonances of the system can overlap considerably. In nuclear physics, a similar situation is realized; Ericson discussed the interference among these overlapped resonances, and showed that this interference induces nontrivial systematic fluctuations of the stochastic nuclear reaction cross-section [30, 31]. In quantum dot systems, the corresponding phenomena were discussed [32]–[34]. We analyse such systems using the notion of quantum dots. We systematically change the dot–lead coupling from small to large values to demonstrate how this parameter affects electron transport.

An open quantum dot system consists of various kinds of transport channels such as direct coupling channels between the leads, weak coupling channels, and randomly distributed levels. As a simple and ideal example of such a situation, we discuss the AB ring consisting of a random dot, and a regular quantum dot or direct path between the leads; we call it the regular–random system. In addition, we discuss the situation where both dots are random, calling it the random–random system. We also discuss how different interference mechanisms inside the ring give qualitatively different results in electron transport through the ring. In general, an electron goes around the ring many times until it escapes from the ring. On the other hand, if the ring is relatively opened to external leads, multiple scattering in the ring can be disregarded, and the transmission amplitude through the ring is simply the sum of the wavefunctions passing through the arms. In addition, motivated by the work of [29], we take into account a direct nonresonant path in a single dot. We discuss these three different interference mechanisms in the presence of random levels.

We discuss electron transport through the ring with random dots using the scattering matrix formulation. We define the random Hamiltonian model based on RMT [35]–[40]. This model is equivalent to the random S-matrix formulation [41]–[45] which disregards the detail of the system configuration, and thus it is a rather generic approach to the interference effect due to randomness. We should mention the relation between our model and the random Fano model discussed in [29] for regular–random systems. In [29], the statistical properties of many Fano resonances are discussed. On the other hand, we assume only the single resonant level in the regular dot, and it is affected by the random levels. Thus our attention is fixed on the single regular resonance. To discuss the statistical properties of the transport we must prepare different realizations of the random dot.

The S-matrix approach to the AB ring model is summarized in section 2. The relation between the S-matrix and the random Hamiltonian is summarized in section 3 while the random
S matrix approach is discussed in section 4. We discuss electron transport through a single random dot in section 5. In section 6, we discuss the regular–random systems while in section 7, we discuss the random–random systems. The conclusions follow afterwards. Note that we have studied related problems in a preliminary report [46], and in an extended version of the report [47].

2. S-matrix formulation of AB ring

We first summarize the S-matrix formulation of electron transport through an AB ring with two dots as depicted in figure 1. Electron transport through each dot and the ring are characterized by a scattering matrix $S_i$, and $S$, respectively

$$S_i = \begin{pmatrix} r_i & t_i \\ t'_i & r'_i \end{pmatrix} \quad (i = 1, 2), \quad S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}. \quad (1)$$

The coupling between the dot and lead is described by a $3 \times 3$ matrix. The method which we follow below was originally investigated in [16], and developed in [48, 49]. (The detail is shown in appendix A.) In the following, we assume the symmetric dot–lead coupling between the left and right leads. The total transmission coefficient $t$ is

$$t = \frac{t_1(1 + s_2 - r_2 - r'_2) + t_2(1 + s_1 - r_1 - r'_1)}{Z}, \quad (2)$$

where

$$Z = 1 + s_1s_2 - (r_1r_2 + r'_1r'_2) - (t_1t'_2 + t'_1t_2) + \frac{1}{4}(1 + s_1 - r_1 - r'_1)(1 + s_2 - r_2 - r'_2) + \frac{1}{4}(1 - r_1)(1 - r_2) - \frac{1}{2}(s_1 - r_1)(s_2 - r_2) + \frac{1}{4}(1 - r'_1)(1 - r'_2) - \frac{1}{2}(s_1 - r'_1)(s_2 - r'_2), \quad (3)$$

with $s_i = r_ir'_i - t_it'_i$. Note that this complicated formula for $t$ is obtained by the following simple relation [47]

$$\frac{1 + S}{1 - S} = \frac{1 + S_1}{1 - S_1} + \frac{1 + S_2}{1 - S_2}. \quad (4)$$

(See also equation (12) below.)
When the ring is widely opened to external leads, the number of channels going outside will increase. Such a situation has been realized experimentally [50]. In this case, the multiple scattering inside the ring can be disregarded, and as a result $t$ will be simply given by $t = t_1 + t_2$ although the normalization factor will be necessary in general. Note that this limit cannot be described by the model depicted in figure 1. It has been discussed that attaching extra leads to the ring can reduce the multiple scattering in the ring, which imitates an open geometry [19]. In this paper, we assume the condition a priori rather than deriving it, and we discuss how the two situations, with and without multiple scattering, bring different results.

3. S-matrix and Hamiltonian

According to the scattering theory (see, e.g. [2, 3, 35]), the S-matrix is

$$S = \frac{1 - i\pi w^\dagger(1/E^+ - H)w}{1 + i\pi w^\dagger(1/E^+ - H)w},$$

where $H$ denotes the dot Hamiltonian, the size of which is $N$, and $w$ describes the dot–lead tunnelling coupling matrix

$$w = \begin{pmatrix}
    w^{(L)}_1 & w^{(R)}_1 \\
    \vdots & \vdots \\
    w^{(L)}_N & w^{(R)}_N
\end{pmatrix},$$

where $L$ (R) refers to the coupling to the left (right) lead.

3.1. Regular quantum dot

As a simple example, the S-matrix of a quantum dot with a single energy level $E_0$ ($N = 1$) is summarized

$$S = \frac{1 - i\pi(w^\dagger w/E^+ - E_0)}{1 + i\pi(w^\dagger w/E^+ - E_0)} = \frac{1 - i\tilde{\gamma}}{1 + i\tilde{\gamma}}$$

where we have introduced the $2 \times 2$ matrix $\tilde{\gamma} = \gamma/(E^+ - E_0)$ with $\gamma = \pi w^\dagger w$, explicitly given by

$$\gamma = \frac{1}{2} \begin{pmatrix}
    \Gamma_L & \sqrt{\Gamma_L \Gamma_R} \\
    \sqrt{\Gamma_L \Gamma_R} & \Gamma_R
\end{pmatrix},$$

where $\Gamma_{L/R}$ is the level broadening. For simplicity we assume the symmetric coupling $\Gamma_L = \Gamma_R = \Gamma$ in the following. The S-matrix is then

$$S = \frac{1}{\epsilon + i} \begin{pmatrix}
    \epsilon & -i \\
    -i & \epsilon
\end{pmatrix},$$
with

\[ \epsilon = \frac{E - E_0}{\Gamma}. \]  

(10)

The conductance measured in units of \(2e^2/h\) is \(g = |S_{21}|^2\). We obtain the well-known formula

\[ g = \frac{\Gamma^2}{(E - E_0)^2 + \Gamma^2}. \]  

(11)

When two dots are embedded in the AB ring as discussed in section 2, \(H\) is a \(2 \times 2\) diagonal matrix with energies \(E_1\) and \(E_2\), and then

\[ \tilde{\gamma} = \tilde{\gamma}_1 + \tilde{\gamma}_2. \]  

(12)

If \(S_i = -(1 - i\tilde{\gamma}_i)/(1 + i\tilde{\gamma}_i)\) and \(S = -(1 - i\tilde{\gamma})/(1 + i\tilde{\gamma})\), then we obtain equation (4). Note that there are extra minus signs in the above expressions compared to equation (7), which do not change the final results.

3.2. Random quantum dot

We next discuss a random quantum dot described by RMT. We consider the Gaussian ensemble, where the probability density of the Hamiltonian \(H\) is given by

\[ P(H) = C \exp \left( -\frac{\pi^2}{2N\Delta^2} \text{tr} H^2 \right), \]  

(13)

where \(N\) is the size of the matrix \(H\), \(\Delta\) is the mean level spacing, and \(C\) is a normalization constant. The matrix elements of \(H\) that belong to the Gaussian unitary ensemble are given by

\[ H_{ij} = \begin{cases} \frac{\Delta}{\sqrt{N}} R_{ii} & (i = j), \\ \frac{\Delta}{\sqrt{2N}} (R_{ij} + iR'_{ij}) & (j < i), \end{cases} \]  

(14)

and \(H_{ij} = H_{ji}^*\), where \(R\) and \(R'\) are the random numbers which obey the standard normal (Gaussian) distribution. The dot–lead coupling matrix \(w\) can be given either by fixed or random numbers. As long as \(w^\dagger w\) is fixed, as we see below, the result is the same. Note that we do not discuss the fluctuations of \(w^\dagger w\) here. As an example which satisfies this condition, we discuss the case where the matrix elements of \(w\) are random real numbers given by

\[ w_j^{(l/R)} = \sqrt{\frac{\Gamma}{2}} R''_j, \]  

(15)

where \(R''\) is a uniform random distribution in the range \((0, 1)\) with the constraint of \(\sum_{i=1}^N (R''_i)^2 = 1\). From equations (14) and (15), we can calculate the \(S\)-matrix for the random quantum dot via equation (5). The conductance is obtained after taking the ensemble average.
4. Random S-matrix approach

Although the random Hamiltonian method is a microscopic approach, electron transport does not depend on the detail of the system configuration after taking an average over the samples. From the viewpoint of numerical calculations, the random Hamiltonian approach requires one to deal with the large size of the matrix, and it consumes computational resources. Thus we use an alternative effective approach; the randomness is directly imposed on the S-matrix, simulated by the circular ensembles [6]. The probability distribution of the S-matrix is given by the Poisson kernel [41, 42, 45]

$$P_\beta(S) d\mu_\beta(S) \propto \frac{1}{|\det(1 - S(S)\dagger)|^{\beta+2}} d\mu_\beta(S),$$  \hspace{1cm} (16)

$P_\beta(S) d\mu_\beta(S)$ represents the probability measure and $\langle S \rangle$ is the average value of the S-matrix. $d\mu_\beta(S)$ denotes the measure for the circular ensemble. $\beta = 1, 2$ and $4$ for the orthogonal, unitary and symplectic ensembles, respectively. Note that we have assumed single channel wires in the leads.

4.1. S-matrix and random Hamiltonian

The microscopic information of the model appears in $\langle S \rangle$ in addition to $\beta$. A random dot is characterized by the average level spacing $\Delta$, and the level broadening. The averaged Green function of the random dot is given by

$$\left\langle \frac{1}{E^+ - H} \right\rangle = \frac{\pi}{N\Delta} e^{-i\nu},$$  \hspace{1cm} (17)

with $\cos z = \pi E/2N\Delta$. The dot-lead coupling matrix $\gamma$ has the form

$$\gamma = \frac{N\Gamma}{2} \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix},$$  \hspace{1cm} (18)

where we have introduced a phenomenological parameter $a$ ($0 \leq a \leq 1$). It is zero when the direct tunnelling between the left and right leads is negligible.

Using these results we obtain

$$\langle S \rangle = \frac{1 - i\tilde{\gamma}}{1 + i\tilde{\gamma}},$$  \hspace{1cm} (19)

with

$$\tilde{\gamma} = \gamma \frac{\pi}{N\Delta} = \frac{X}{2} \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix},$$  \hspace{1cm} (20)

where

$$X = \frac{\pi\Gamma}{\Delta}. \hspace{1cm} (21)$$
4.2. Random S-matrix elements

The choice of random parameters in the random S-matrix depends on the universality class. For example, for the unitary ensemble ($\beta = 2$),

$$S_2 = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\alpha'} \end{pmatrix} \left( -\sqrt{1 - \tau} \quad \sqrt{1 - \tau} \right) \begin{pmatrix} e^{i\gamma} & 0 \\ 0 & e^{i\gamma'} \end{pmatrix} = \begin{pmatrix} -\sqrt{1 - \tau} e^{i(\alpha + \gamma)} & \sqrt{\tau} e^{i(\alpha' + \gamma')} \\ \sqrt{\tau} e^{i(\alpha + \gamma')} & \sqrt{1 - \tau} e^{i(\alpha' + \gamma)} \end{pmatrix}$$

(23)

with $0 < \tau < 1$, $0 < \alpha, \alpha', \gamma, \gamma' < 2\pi$, and the integration measure

$$d\mu_{\beta=2}(S) = C \, d\tau \, d\alpha \, d\alpha' \, dy \, dy'$$

(24)

where $C$ is a normalization constant.

It is known that the random S-matrix approach is equivalent to the random Hamiltonian approach with the Poisson kernel [40]. We numerically confirmed this point. In figure 2, we show the conductance through the ring with one regular dot and one random dot for several parameters of $\epsilon$ and $X$ for the unitary ensemble. The plots with symbols are calculated by the random Hamiltonian method while the plots with lines are calculated by the random S-matrix method. Since both methods give good agreement, we use the random S-matrix method in the following.
Figure 3. Electron transport through a single random dot with the unitary (GUE), orthogonal (GOE), and symplectic (GSE) ensembles. (a) The definition of $X$. (b) $g$ versus $X$. (c) $(g^2) - \langle g \rangle^2$ versus $X$.

5. Random dot

We first discuss transport properties through a single random dot. In figure 3, the definition of $X$ is schematically shown, and the averaged conductance $g$ and the conductance fluctuations $(g^2) - \langle g \rangle^2$ are plotted as a function of $X$ for the unitary, orthogonal, and symplectic ensembles. When $X < 2$, $g$ increases as $X$ increases. This simply means the stronger dot–lead coupling leads to larger transmission through the dot. When $X > 2$, in contrast, $g$ decreases as $X$ increases. In this regime, many levels overlap each other within the level broadening. The interference among the levels rather suppresses the conductance. Note that when $X = 2$, $\langle S \rangle = 0$, and thus the Poisson kernel is independent of the random matrix elements. On the conductance fluctuations, $(g^2) \sim \langle g \rangle^2$, which indicates the quantum fluctuation effect due to randomness is strong in electron transport.

The analytical expression of $g$ for the unitary ensemble is obtained by the supersymmetry method [51]

$$g = \frac{T}{3} + \frac{T^2}{6},$$

where $T = T_1 = T_2$ with the eigenvalues $T_1$ and $T_2$ of $1 - \langle S \rangle \langle S \rangle^\dagger$, which characterizes the transmission of the system, and in fact it is nothing but the conductance for regular systems. Note that $g$ is expressed only by $T$. $T$ depends on $X$ [46, 47]

$$T(X) = \frac{2X}{(1 + X/2)^2}.$$

$T(X)$ is the maximum of $T = 1$ when $X = 2$. Equation (26) means two values of $X$, $X_1 (X_1 < 2)$ and $X_2 (X_2 > 2)$ give the same value of $T$. Similar results are obtained for the orthogonal and symplectic ensembles; the transport properties are certain functions of $T(X)$, which are checked numerically.
6. Regular–random system

In this section, we discuss the following three models: a random dot characterized by $X$ couples with

**model 1**: the non-resonant direct path $a$, where the second dot is detached,

**model 2**: a regular dot (the energy level is $\epsilon$) with $t = t_1 + t_2$,

**model 3**: a regular dot with $t$ given by equation (2).

For model 1, as $a$ increases, the coupling between the leads becomes stronger while for models 2 and 3, as $\epsilon$ decreases it becomes stronger. These two parameters, $a$ and $\epsilon$, play similar roles to control the direct transmission through the ring. We discuss model 2 to compare with model 3, which includes multiple scattering inside the ring.

In figure 4, the conductance and conductance fluctuations are plotted as a function of $X$ for these three cases. The solid lines correspond to weak direct coupling while the dotted lines to strong direct coupling. For model 1, the enhancement of $g$ as a function of $a$ is prominent when $X$ is large, while it is not when $X$ is small. For model 2, the direct channel just shifts $g$ of the single random dot because of the transmission through the regular dot. This also means that $g$ is always written in terms of $T(X)$ except for the trivial constant. On the other hand, model 3 gives the following results; when $X < 2$, $g$ is enhanced as $\epsilon$ decreases, while when $X > 2$, $g$ is suppressed as $\epsilon$ decreases.

At the strongest direct coupling limit, $a = 1$ or $\epsilon = 0$, the differences among the models become clear. For model 1, $g = X^2/(1 + X)^2 + T'(X)/4$ with $T'(X) = 4X/(1 + X)^2$ [46], which monotonically increases as $X$ increases. For model 3, $g = 1/(1 + X/2)^2 + T(X)/4$ with $T(X)$ given by equation (26) [46], which monotonically decreases as $X$ increases. A peak structure at $X = 0$ is obtained only for model 3. This result means the resonant level in the regular dot characterizes electron transport. In fact, the conductance and its distribution function $P(g) = \langle \delta(g - |S_{12}|^2) \rangle$ do not depend on the universality class in the random dot [47].

For models 1 and 3, $\langle g^2 \rangle \sim \langle g \rangle^2$ while for model 2, $\langle g^2 \rangle > \langle g \rangle^2$. The results of models 1 and 3 are more or less similar to the one in a single random dot. This indicates the fluctuations are dominated by the random dot in contrast to the result of $g$. This is clearly seen when $\epsilon = 0$ in model 3. The fluctuations show a peak around $X = 2$ instead of the peak at $X = 0$. For model 2, $\langle g^2 \rangle - \langle g \rangle^2$ increases monotonically as $\epsilon$ decreases. This result comes from the fluctuations due to the interference terms between the two dots, which is enhanced as $\epsilon$ decreases.

In an open geometry, different types of channels will be involved in electron transport; in particular, strong coupling channels between the leads and the quantum fluctuations due to randomness naturally appear. Electron transport is influenced by the interference between them while the mechanism is not clear a priori. The above results can suggest the mechanism when electron transport is measured as a function of the dot–lead coupling. For instance, the presence of multiple scattering can be checked by whether the conductance can be enhanced in the relatively weak coupling regime. In contrast, the absence of multiple scattering can be seen in large conductance fluctuations. If the direct channel is imposed on the random channels as in model 1, stronger dot–lead coupling simply enhances the conductance since both $a$ and $X$ increase. In this way, the dot–lead coupling of the random dot will be able to uncover the difference between the interference mechanisms.
7. Random–random system

In this section, we discuss electron transport through coupled random dots that belong to the unitary ensemble with two parameters of $X_1$ and $X_2$

$$X_1 = X(1 + p), \quad X_2 = X(1 - p),$$

(27)
Figure 5. The conductance $g$ versus $p$ for several values of $X$ for the random and random dots system with $X_1 = X(1 + p)$ and $X_2 = X(1 - p)$ ($0 < p < 1$); (a) model 2 and (b) model 3.

with $0 \leq p \leq 1$. We discuss the following two models as in the regular–random systems: the transmission amplitude through the system is

- **model 2’**: $t = t_1 + t_2$,
- **model 3’**: $t$ given by equation (2).

In figure 5, $g$ is plotted as a function of $p$ for several values of $X$. For model 2’, $g$ is sensitive to $p$. When $X$ is small, $g$ decreases monotonically as $p$ increases. When $X$ is large, $g$ increases as $p$ increases. The results are explained by

$$g = g(X_1) + g(X_2),$$

(28)

where $g(X)$ is given by equations (25) and (26). For model 3’, $g$ is insensitive to $p$ in contrast to the results of model 2’. This indicates that the asymmetry in the dot–lead coupling between the dots is smeared out by multiple reflection in the ring. This result is explained by equations (12), (20) and (21), because from these equations the effective dot–lead coupling $X'$ of the system for model 3’ is

$$X' = X_1 + X_2,$$

(29)

and $g = g(X')$, which is independent of $p$. Equations (28) and (29) demonstrate the difference between the two models, the AB ring with and without multiple reflection. This shows that the interference mechanism inside the ring can have an influence even when both dots are random. In other words, the random dot system can be utilized to detect multiple scattering inside the ring.

The difference between the models can be seen when $g$ is plotted as a function of $X$ as well, which is shown in figure 6. When $p = 0$, for model 2’ it shows a peak at $X = 2$ as the one for the single random dot shown in figure 3. For model 3’, the peak is located at $X = 1$. This is explained by equations (29): the effective dot–lead coupling $X'$ is

$$X' = 2X_1.$$

(30)
We can extend this result to the case where $M$ dots are connected in parallel between the dots; $X' = \sum_{i=1}^{M} X_i$ and the peak position is located at $X' = 2/M$. On the other hand, the peak is still at $X' = 2$ for model 2'. As $p$ increases, $g$ depends on $p$ for model 2' while it does not for model 3'. For model 2', the double peak structure can be obtained as a function of $X$ because of two distinct values of the dot–lead coupling. We also calculate the conductance fluctuations; $\langle g^2 \rangle \sim \langle g \rangle^2$ for model 2' while $\langle g^2 \rangle > \langle g \rangle^2$ for model 3' as in the regular–random systems.

8. Conclusions

We have discussed an AB ring system with random quantum dots. Electron transport depends on the ratio of the level broadening to the average level spacing in the random dot. For regular–random hybrid systems, the multiple scattering in the ring induces a distinct feature; in the strong dot–lead coupling regime, the conductance is suppressed as the direct coupling between the leads becomes stronger while the conductance is enhanced in the weak dot–lead coupling regime. The difference remains even when both of the dots are random; the conductance is insensitive to the difference in the dot–lead coupling between the dots when there is multiple scattering inside the ring, while it is sensitive to the difference when the scattering is disregarded.

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Appendix A. S-matrix formulation through the AB ring

The S-matrix for the each quantum dot is defined by

\[
\begin{pmatrix}
a_2 \\
a_4 \\
\end{pmatrix} = S_1 \begin{pmatrix}
b_2 \\
b_4 \\
\end{pmatrix} = \begin{pmatrix}
r_1 & t'_1 \\
t_1 & r'_1 \\
\end{pmatrix} \begin{pmatrix}
b_2 \\
b_4 \\
\end{pmatrix}, \tag{A.1}
\]

\[
\begin{pmatrix}
a_3 \\
a_5 \\
\end{pmatrix} = S_2 \begin{pmatrix}
b_3 \\
b_5 \\
\end{pmatrix} = \begin{pmatrix}
r_2 & t'_2 \\
t_2 & r'_2 \\
\end{pmatrix} \begin{pmatrix}
b_3 \\
b_5 \\
\end{pmatrix}. \tag{A.2}
\]
The connection between a lead and the ring is given by a Folk model S-matrix \[48, 49\]

\[
\begin{pmatrix}
  b_1 \\
  b_2 \\
  b_3
\end{pmatrix} = \begin{pmatrix}
  \frac{1 - \eta_1}{2} & \frac{1 + \eta_1}{2} & \sqrt{\frac{1 - \eta_1^2}{2}} \\
  -\frac{1}{2} & \frac{1 - \eta_1}{2} & \sqrt{\frac{1 - \eta_1^2}{2}} \\
  \frac{1 - \eta_1^2}{2} & \frac{1 - \eta_1^2}{2} & \eta_1
\end{pmatrix}
\begin{pmatrix}
  a_1 \\
  a_2 \\
  a_3
\end{pmatrix},
\]

(A.3)

\[
\begin{pmatrix}
  b_6 \\
  b_4 \\
  b_5
\end{pmatrix} = \begin{pmatrix}
  \frac{1 - \eta_2}{2} & \frac{1 - \eta_2}{2} & \sqrt{\frac{1 - \eta_2^2}{2}} \\
  -\frac{1}{2} & \frac{1 - \eta_2}{2} & \sqrt{\frac{1 - \eta_2^2}{2}} \\
  \frac{1 - \eta_2^2}{2} & \frac{1 - \eta_2^2}{2} & \eta_2
\end{pmatrix}
\begin{pmatrix}
  a_6 \\
  a_4 \\
  a_5
\end{pmatrix}.
\]

(A.4)

We are interested in the total S-matrix

\[
\begin{pmatrix}
  b_1 \\
  b_6
\end{pmatrix} = S
\begin{pmatrix}
  a_1 \\
  a_6
\end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix}
  a_1 \\
  a_6
\end{pmatrix}.
\]

(A.5)

The transmission \(t\) is expressed as

\[
t = \left[ \frac{1 + \eta_1}{2} \frac{1 + \eta_2}{2} t_1 (1 + s_2 - r_2 - r_2') + \sqrt{\frac{1 - \eta_1^2}{2}} \sqrt{\frac{1 - \eta_2^2}{2}} t_2 (1 + s_1 - r_1 - r_1') \right] \frac{1}{Z_0},
\]

(A.6)

where

\[
Z_0 = \frac{1 + \eta_1}{2} \frac{1 + \eta_2}{2} (1 + s_2 - r_2 - r_2') - \sqrt{\frac{1 - \eta_1^2}{2}} \sqrt{\frac{1 - \eta_2^2}{2}} (t_1 t_2' + t_1' t_2) + \frac{1 - \eta_1}{2} \frac{1 - \eta_2}{2} \times (1 + s_1 - r_1 - r_1') (1 + s_2 + r_2 + r_2') + \frac{1 + \eta_1}{2} \frac{1 - \eta_2}{2} (1 - s_2 - r_2 + r_2') (1 - r_1')
\]

\[
+ \frac{1 - \eta_1}{2} \frac{1 + \eta_2}{2} (1 - s_2 + r_2 - r_2') (1 - r_1)
\]

(A.7)

with \(s_i = r_i r_i' - t_i t_i' (i = 1, 2)\). If \(\eta_1 = \eta_2 = 1/3\), we obtain equations (2) and (3).

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