ML Supported Predictions for SAT Solvers Performance

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Abstract

In order to classify the indeterministic termination behavior of the open source SAT solver CryptoMiniSat in multi-threading mode while processing hard to solve boolean satisfiability problem instances, internal solver runtime parameters have been collected and analyzed. A subset of these parameters has been selected and employed as features vector to successfully create a machine learning model for the binary classification of the solver’s termination behavior with any single new solving run of a not yet solved instance. The model can be used for the early estimation of a solving attempt as belonging or not belonging to the class of candidates with good chances for a fast termination. In this context a combination of active profiles of runtime characteristics appear to mirror the influence of the solver’s momentary heuristics on the immediate quality of the solver’s resolution process. Because runtime parameters of already the first two solving iterations are enough to forecast termination of the attempt with good success scores, the results of the present work deliver a promising basis which can be further developed in order to enrich CryptoMiniSat or generally any modern SAT solver with AI abilities.

Keywords: AI, artificial intelligence, ML, machine learning, SAT solver, security

1 Introduction

The significance of SAT\textsuperscript{1} solvers as a core technology for the analysis, synthesis, verification and testing of security properties of hardware and software products is established and well known. Automated Reasoning techniques for finding bugs and flaws examine nowadays billion lines of computer code with the use of Boolean and Constraint Solvers on domains of interest. However due to the continuous improvement of SAT solver efficiency during the last decades and the dramatic scalability of these solvers against large real-world formulas, many new application cases arise in which SAT solvers get deployed for tackling hard problems, which were believed to be in general intractable but yet get solved [5]. SAT solvers are also known to be used for security protocol analysis [21, 1] as well as for tasks like the automated verification of access control policies, automatic

\textsuperscript{1}SAT – satisfiability
Anomaly Detection in network configuration policies [15] and verification of general Access Control Systems where access rules are first encoded in SAT representation [6, 16]. Furthermore SAT-based cryptanalysis methods are in advance and report increasing successes, like for example cryptographic key recovery by solving instances which encode diverse cipher attacks [9, 19, 17, 10] etc.

Also the solution of constraint optimization problems for real-world applications on the basis of already very competitive MaxSAT solvers, the great majority of which are core-guided, heavily relying on the power of SAT solvers, is the best way of proving unsatisfiability of subsets of soft constraints, or unsat cores, in an iterative way towards an optimal solution [8]. New algorithmic solutions instantiating innovative approaches to solve various data analysis problems in ML, like correlation clustering, causal discovery, inference etc. are based on SAT and Boolean optimization solvers. Decision and optimization problems in artificial intelligence profit by the application of SAT solvers [18] but also the opposite direction is pursued, namely the improvement of SAT solving using ML [4, 12, 13]. The intrinsic connection of the two subjects, SAT solvers and AI, seems to be growing especially also under the scope of global trends intending to incorporate AI and ML technologies in the majority of the next generation cybersecurity solutions. The improvement of SAT solver performance by means of AI is a topic of intense and general interest and motivated this work.

2 Organization of this paper and contributions

First, a brief account is given here of previously gained experience with CryptoMiniSat [20], gathered while studying the solver’s behavior when engaged for the solution of hard CNF instances representing KPA in cryptanalysis. Similar instances whose solution is far from being trivial, are used also for the results produced in this work [11]. However these instances are here taken as an example of especially hard instances to solve while the achieved results should be relevant to the solution of arbitrary hard instances having similar features to those of the here employed ones. In what follows the motivation of the present work is substantiated and then the procedure followed to produce our results as well as the results themselves will be presented. In the last part we summarize about this work and discuss about further investigations planned. In the past the CMS solver’s performance has been studied by carrying out runtime tests both with the solver in default configuration and with various solver switches set. The tests showed that the solver runtime until the solution is found, in case the job doesn’t previously stop because of some time-limit setting, is subjected to distinct statistical variations. This is due to the indeterministic behavior of the solver in multi-thread operation mode. The complexity of the here discussed problems though excludes one-thread operation from being an option. The performed runtime tests were in average highly time consuming both when a termination was reached and when the job had to be interrupted because it reached some previously defined upper run time limit. The job interruption practice was motivated by an amplitude of experience showing that if some long

\(^2\)ML – machine learning

\(^3\)CNF – conjunctive normal form; KPA – known-plaintext attacks

\(^4\)CMS – CryptoMiniSat
runtime limit has been surpassed without a solution found, the majority of test runs do not terminate at all. As a matter of fact, even under identical solver parameter configuration when running several tests to solve one and the same instance, one cannot avoid diverging solving times or finding no solution at all for instances which are solvable by construction. All cryptanalytic instances used in experiments possess by construction one single solution, which was in this case the sought-for cryptographic key. The performance of a large number of tests had allowed a statistical analysis of the non-deterministic solver runtimes to empirically define command line parameter combinations for CMS which yield best runtimes medians for the type of instances taken under examination. The application of an AAC tool which followed [14], allowed a systematic exploration of the configuration parameter space reaching beyond the empirical tests to discover even better configuration parameter combinations. The results of those efforts have been quite encouraging, demonstrating a lowering of the median of runtimes by 30% to finally 90% with the application of the AAC [11]. The limitation we see however in further pursuing this approach is that when a semi-automatic tuning of the solver’s configuration parameters is to be performed, this has to be carried out on the basis of previously cumulated values of best achieved runtimes. Those best runtimes are of course obtainable at a high computational and time cost which should probably have to be repeatedly afforded, in case the expensively discovered effective configuration parameter settings are not globally valid but rather problem specific.

2.1 Contributions of the present work

In the present work we show how to create a prognosis concerning the successful termination of a solving process not by properly adjusting some solver configuration parameters but by using internal runtime parameters of the beginning of the process during the process. Instead of analyzing effectiveness of the solver’s configuration in a post-hoc manner, that is following the event of numerous successful terminations, we have here chosen to observe and analyze the joint evolution of solver internal parameters that are dynamically changing during the automatic state transformation of the instance while the solver is searching for a solution. These parameters which are issued by the solver when running in verbose mode, have been at first separately investigated in order to see if they demonstrate any correlation to the duration of job-runtimes and final successful termination. This question could not get uniquely answered. This circumstance motivated the tryout of a subset of the solver’s dynamically changing runtime parameters as model features for building an ML model with the purpose to detect if the solver’s active state evolution follows a direction with good chances to terminate timely or not. Taking into consideration the fact that the joint observation of parameters that belong to the two or three first iterations of the solving process showed to be sufficient to construct the ML model so as to get some decent classification results, this approach can be seen as a workable basis to later devise and incorporate an internal mechanism in the solver to trigger early changes of search strategy on the basis of internal short-termed collective parameter changes at a minimal time cost. Given the fact that the time length of the solver’s iterations normally essentially grows with the iteration’s num-

[AAC – automatic algorithm configuration]
3 Solver runtime parameters: description, analysis, and illustration

We have selected runtime parameters which we consider to be in general significant for characterizing the state of the solving process, and this not only in relation to the here regarded CNF instances. The parameter charts plotted in the graphics below are representative for the processing of instances of similar features. As features of the here employed CNF instances we observe the number of variables $L$, the number of CNF clauses $N$, the length of clauses (minimum, lower quartile, median, upper quartile, and maximum length), the sum of all occurrences of all variables, the fraction of clauses by length in the CNF instance, the occurrence of literals in clauses, and their mean value [7].

The three instances whose runtime parameters are plotted here belong to three different variations of the same mathematical problem (known-plaintext attack on the round-reduced AES-64 model cipher) with instance densities $N/L$ assuming the values 303.4 (18-vs), 304.2 (20-vs) and 306.6 (30-vs). We could prove that similar parameter graphics are produced for instances created on the basis of the same mathematical problem but with different problem parameters (different number of plaintexts, for our instances 18, 20, and 30, respectively and/or different key).

The parameter names are almost self-explanatory in the context of the functionality of a CDCL solver. We have experimented with two slightly different sets of parameters for the creation of the ML model and these sets are displayed in the first and second columns of table 1, where the abbreviation `props` stands for propagations.

| Set 1            | Set 2            |
|------------------|------------------|
| all-threads      | all-threads      |
| conflicts/second | conflicts/second |
| blocked-restarts | blocked-restarts |
| restarts         | restarts         |
| props/decision   | props/decision   |
| props/conflict   | literals/conflict|

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*CDCL – conflict driven clause learning*
The parameter **all-threads** expresses the total time consumed by a solver iteration calculated by adding the solving times contributed by the various threads during this iteration. All here discussed parameters are regularly delivered by the solver at the end of each and every iteration, with the solver running in verbose mode. In Figure 1 the **conflicts/second** parameters for six different runs of the same instance that terminated at different times with the solver identically configured for all runs, is exemplarily depicted. There exist differences in the structure, especially the heights and in the positions of the peaks of the curves which seem to associate to the length of the runtime until a successful termination of the corresponding job (see legends in Figure 1) has been reached.

![Figure 1: parameter evolutions for the instance 20-vs](image)

It seems plausible to attribute to the steep rising number of conflicts at the beginning of the solving-process (that means during the first and/or the second iterations) an early generation of a lot of additional useful informations for the solver. This event accelerates the learning effect which would justify the assumption of a direct association of a shorter solution runtime to the occurrence
of many conflicts at the beginning of the solving process. Many conflicts per second along the search path naturally results in a shorter iteration time, as there is a limit of conflicts per iteration, allowed by the solver, which in this case gets earlier reached. Advantageous are therefore short iteration times, arising out of the occurrence of high conflict rates during short iterations.

The markers on the curves correspond to the begin/end of an iteration. A short first iteration time sets the begin of the corresponding plot closer to the y-axis and it is notable that curves starting far to the left describe evolutions of solutions with mostly shorter therefore better runtimes.

Curves which despite their starting far to the left do evolve to describe long and therefore bad termination runtimes, usually demonstrate also early negative or very flat gradients as regards the conflicts/second parameter. To this, one can compare the two relatively flat curves with termination times 215,108 and 123,359 seconds, respectively (see legend of Figure 1). These two curves could also be categorized as not successfully terminating runs, if an upper time-limit had been set for the allowed runtime of the corresponding jobs.

Comparable results are depicted in Figure 2 (Figure 3) where the evolution of nine (seven) different runs of the instance 18-vs (30-vs) are shown. The instance in Figure 2 is the most difficult to solve in comparison to the other two instances of Figure 1 and Figure 3 respectively. In this more difficult case one observes several intensive or less intensive learning phases in a sequence, represented by more than one distinct peaks in the corresponding curve plots. Fine differences in the start values seem to play also a considerable role for the consequent runtime evolution. Not only the duration of the first iteration but also the initial value of the parameter conflicts/second is in this respect significant. Striking is again the appearance of the late starting and thoroughly flat curve corresponding to the very late termination time of 352,324 seconds (see legends of Figure 2).

Considering the fact that no continuous correlation connecting any of the runtime parameters with the corresponding solver termination time is to be found, the question arises if one can use statistical traits of the joint evolution of parameters in a combination, not in order to discover conditions for some optimal solution time but in order to distinguish between a solver path leading to a termination and one which does not.

The second runtime parameter here checked is that of the restarts. Restarts is a critically important heuristic according to many experts in the field of CDCL SAT-solver research. Restart schemes have been evaluated in detail and a particular benefit could be identified in frequent restarts when combined with phase saving [2]. Restarts are considered to compact the assignment stack and frequent restarts enhance the quality of the learnt clauses thus shorten the solution time. In a recent work an ML-based restart policy has been introduced to trigger a restart every time an unfavorable forecast regarding the quality of the expected new to be created learned clauses arises[13].

In Figures 4, 5 and 6 there are depicted the courses of the corresponding restarts parameters for the same runs whose conflicts/second parameters have been plotted above.

A comparison between these new three plots reveals as a common indicator of unpromising (not timely expected to terminate) runs, the corresponding poorly structured, and at parts continuously evolving course of a curve. This adverse curve-shape feature, if observed alone, becomes especially obvious only
Figure 2: parameter evolutions for the instance 18-vs

when watched over a time period which is longer than two or three iterations. In combination with the iteration length, given by the all-threads parameter though, the restarts parameter can indeed count as a criterium to forecast a timely termination during the first two iterations alone.

Similar remarks apply well for blocked-restarts, the complementary parameter, and the rest of the parameters taken for the ML model building, whose plots are not displayed here out of space considerations. In conclusion, one can ascertain that a missing agility in the timely evolution of runtime parameters and especially in combination with a low conflict rate at the beginning of the solving process, plausibly signalize little chance for a timely termination of a solving process.

Figure 7 shows pairwise scatterplots of the six (top) and seven (bottom) runtime parameters (compare first and second columns respectively of table 1. Also different CNF instances were used for each plot). These parameters correspond to the first of the two iterations taken for building the ML model. The plots help to reveal pairwise correlations between the model features. They indicate
also the intricacy implied in finding an analytical function for the classification. Points with different colors symbolize different classification tags.

4 ML models with CMS-runtime parameters as model features

In order to investigate the feasibility of correctly and fast predicting the finding of a solution with the CMS solver on the basis of an ML model, we used CMS-runtime parameters of previous test-run cases to construct NN\textsuperscript{7} classifiers. For the practical implementation we have employed the high-level Keras framework [3]. Keras is an open source neural-network library written in Python. It is capable of running on top of TensorFlow, Cognitive Toolkit (CNTK), or Theano. It was developed with a focus on enabling fast experimentation.

The training datasets for the models were compiled by extraction of the

\textsuperscript{7}NN – neural network
Figure 4: restarts for the instance 20-vs

runtime parameters out of log-data originating in a set of test run cases, incorporating two equal numbers of terminating and not terminating processes (150 all together for each CNF instance) and which were then accordingly tagged with 1 or 0. We explored three different NN models for binary classification, each of them has been trained with the same training datasets. The test datasets had in each case half the magnitude of the corresponding training dataset (75 test cases).

The first NN model we tried was also the simplest one consisting of mainly two layers of neurons. For the first layer, the input layer, there was taken a number of neurons equal to the runtime parameters of the CMS, planned to serve as features for the model. See table 1. One and the same runtime parameter was considered as a different model feature if belonged to a different iteration, so that we had 12 (14) input parameters for the model. The second layer (output layer) had just one neuron and delivered the binary classification.

The second NN model had an intermediate layer added with half as much neurons as the first layer, here six. The additional, hidden layer helps an NN-
model to better approximate the classification of non-linearly separable data.

The third NN model was derived from the second model by dropping a percentage of the input values for the second and the third layer which should help prevent overfitting. This model is called a multilayer perceptron for binary classification. In all NN-models non-linear activation functions were used.

Independent of model, when employing the 6 features of the first column of table 1 we had for all three instance-cases an equally good hit ratio near 90% while the formal model accuracy was 100%. With the seven parameters as model features of the second column of table 1 these simple models deliver less success with a hit score of about 70%. This shows that parameter (feature) selection is very important. Also, a more developed NN model might be necessary for a reliable classification prediction. These first results encourage the building of further more sophisticated models in the future.

Figure 5: restarts for the instance 18-vs
Figure 6: restarts for the instance 30-vs

5 Conclusions and future work

The SAT solver CryptoMiniSat reenacting produces a series of runtime parameter values while processing a CNF instance until solving or failing to solve it within a time limit. We showed that it is possible to select and employ subsets of these runtime parameters, the same for all runs, and use them as features to build an ML model for solver runtime classification. For any future attempt to solve a new CNF instance of features similar to those of the instance used to create the training data for the model, this model enables the early classification of the attempt as a fast or a late terminating run. By early is meant that the parameter profiles released during the first two solver iterations are already sufficient for a classification of this solving attempt.

In a future work the model should be extended so as to become capable to generate forecasts for attempts to solve problems bigger than the ones which it has been trained for. Also the possibility to extend the model so as to generate forecasts for CNF instances of any features and especially for hard random instances of any parametrization lies in the focus of our plans.
Figure 7: Pairwise scatterplots of six (top) and of seven (bottom) CMS runtime parameters. Each point indicates a CMS run. Colors indicate the classification of the test run.
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