A Comparative Study of Various Artificial Intelligence Based Agents for the Game of Angry Birds With and Without Splitting

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Abstract. In a game of angry birds, birds are fired from a slingshot and are targeted towards stationary pigs located at different fixed distances from the slingshot. The angry birds have to be fired in such a way that it lands as close as possible to the pigs' location. The goal is to develop an artificial intelligence-based model that would play the angry birds game based on the past human experience. In this game, the user will give the initial velocity and the angle of projection. Based on these parameters, the shot will be played, and the outcome is stored as a tuple consisting of the initial velocity, the angle of projection, and the location of pigs that have not been destroyed in a database. The machine learning-based agent reads the data from the database, trains itself based on the outcome of previous shots stored in the database, and plays the best possible shot according to the data retrieved from the database. Two machine learning models have been proposed, which are the K Nearest Neighbours model and the Naive Bayes model. The third model is the stochastic gradient descent model, which plays a shot based on the minimization of the distance between the angry bird and the pig using an objective function in terms of the initial velocity and splitting angle. The performance of both these agents has been compared with the human agent’s performance in terms of the average number of wins per 100 games.

1. Introduction

Angry birds are supposed to be fired using the slingshot and are to be targeted towards the pigs. However, there are a limited number of angry birds available. Therefore it is necessary to develop a mathematical model that helps destroy the maximum number of pigs using a minimum number of angry birds.

Adil Paul et al. [23] discussed a case based reasoning approach to the angry birds game by gaining experience from a random agent that constantly explores the solution space and improves the quality of current solutions for cases where a single angry bird does not split into two smaller angry birds. Francesco Calimeri et al. [6] presented an artificially intelligent agent that tried to maximize damage to the pigs for the case where the angry bird did not split into two smaller angry birds. Deborah Moore-Russo et al. [20] explained the game of angry birds using mathematics concepts. Sebastian Ruder [28] gave an overview of various optimization algorithms such as the stochastic gradient descent method used in our problem. G Almasi [1] discussed the techniques for the design and implementation of the Message Passing Interface. Sean M Stewart [30] discussed an approach to projectile motion when the projectile is subject
to resistive forces. However, this model does not explain the projectile splitting when traveling through mid-air.

Recently, a lot of work has been done on how to win the game of angry birds when the angry bird does not split into two smaller angry birds. Various optimization algorithms have also been developed to optimize various machine learning objective functions. The Message Passing Interface (MPI) [3] is a library in C programming language that enables parallel programming with communication between different processes using MPI_Send() and MPI_Recv() function. To the best of our knowledge, limited work has been done on the angry birds model where a large angry bird splits in mid-air into two smaller angry birds and lands at two different points. In this paper, a parallel stochastic gradient descent method using MPI, and machine learning-based agents using the K Nearest Neighbours classifier and the Naive Bayes classifier have been implemented to solve the angry birds’ problem with and without splitting.

2. Derivation of objective functions for the angry birds game

2.1. Angry Birds Without Splitting

Let us consider an angry bird of mass $m$ with an initial velocity $u$ and an angle of projection $\theta$. In the $y$ direction, the acceleration due to gravity $g$ is constant [7].

![Trajectory of an angry bird without splitting](image)

**Figure 1.** This figure represents the two dimensional notion of an angry bird when projected from the origin with a certain angle of projection and a certain initial velocity.

Horizontal component of velocity = $u_x = u \cos \theta$

Vertical component of velocity = $u_y = u \sin \theta$

The horizontal range of the projectile or the angry bird is given by:

$$R = u \cos \theta t,$$

where $t$ is the total time of flight of the angry bird [5].

When the angry bird begins its projectile motion and when it ends its projectile motion, then the vertical displacement or the height of the angry bird from the ground is considered to be zero. So there are two timings for which the vertical displacement or height from the ground is zero [2]. The time can be derived by solving the following equations.

$$u \sin \theta t - \frac{gt^2}{2} = 0$$

(2)
\[ \Rightarrow t(\sin \theta - \frac{gt}{2}) = 0 \]  

Hence,

\[ \Rightarrow t = 0 \text{ or } t = \frac{2usin\theta}{g} \]  

Substituting (4) in (1) we get:

\[ R = u\cos \theta \times \frac{2usin\theta}{g} \]  

\[ \Rightarrow R = \frac{2u^2 \cos \theta \sin \theta}{g} \]  

\[ \Rightarrow R = \frac{u^2 \sin 2\theta}{g} \]  

So we can say that the angry bird lands at a point which is at a horizontal distance \( R \) from the origin.

Now let us consider a fixed point \( a \) at a distance from the origin which is the point where the maximum damage to the pigs occurs. Currently, since \( d \) can be either positive or negative depending on \( R \), we assume that there is only one such point of maximum damage to the pigs.

The distance between the two points is given by:

\[ d = a - R \]  

Now \( d \) can be either positive or negative depending on \( R \). So the square of the distance between the two points is to be taken into consideration to remove the effect of the negative sign [11].

Placing value of (7) in the below equation, we obtain our objective function:

\[ d^2 = (a - R)^2 \]  

\[ \Rightarrow f(u, \theta) = d^2 = (a - R)^2 \]  

\[ \Rightarrow f(u, \theta) = (a - \frac{u^2 \sin 2\theta}{g})^2. \]  

It is necessary for the angry bird to land as close as possible to the point \( a \). So \( f \) is the objective function which is to be minimized.
2.2. Optimal strategy for game of angry birds without splitting

The first step is to find the stationary points or the points where the value of the first order partial derivative with respect to both $u$ as well as $\theta$ is zero \[22\].

Thus the conditions that have to be satisfied are:

\[
\frac{\partial f}{\partial u} = 0 \quad (12)
\]
\[
\frac{\partial f}{\partial \theta} = 0 \quad (13)
\]
\[
\frac{\partial f}{\partial u} = 2(a - u^2 \sin^2 \theta g)(-\frac{2usin2\theta}{g}) = 0 \quad (14)
\]

By solving this equation, we can figure out that for this value to be zero, either $usin2\theta = 0$ or $(a - \frac{u^2 \sin^2 \theta}{g}) = 0$.

So the possible solutions for this equation are:

(i) $u = 0$

(ii) $\sin 2\theta = 0$

(iii) $a - \frac{u^2 \sin^2 \theta}{g} = 0 \Rightarrow \frac{u^2 \sin 2\theta}{g} = a \Rightarrow u^2 \sin 2\theta = ag \Rightarrow \sin 2\theta = \frac{ag}{u^2}$

Differentiating with respect to $\theta$, we get:

\[
\frac{\partial f}{\partial \theta} = 2(a - \frac{u^2 \sin^2 \theta}{g})(-\frac{2u^2 \cos 2\theta}{g}) = 0 \quad (15)
\]

So the possible solutions for this equation are:

(i) $u^2 = 0$

(ii) $\cos 2\theta = 0 \Rightarrow \theta = \frac{\pi}{4}$

(iii) $\sin 2\theta = \frac{ag}{u^2}$

For any point to be a stationary point, any single point where $\frac{\partial f}{\partial u} = 0$ and $\frac{\partial f}{\partial \theta} = 0$ has to be taken since it is necessary for both the first order derivatives to be zero \[13\]. For this purpose, we consider take into consideration a point where $\theta = \frac{\pi}{4}$ and $u^2 = ag$ as one possible solution.

Substituting these values in the objective function (11), we get:

\[
\Rightarrow f(u, \theta) = \left( a - \frac{agsin2\pi}{g} \right)^2 \quad (16)
\]
\[
\Rightarrow f = (a - a)^2 \quad (17)
\]
\[
\Rightarrow f = 0. \quad (18)
\]

Since the objective function is a squared value, the value can never be negative, the stationary points substituted will give local minima?.

Now we consider all possible stationary points where $\sin 2\theta = \frac{ag}{u^2}$ and substitute them in the objective function (11).
\[ f(u, \theta) = (a - \frac{u^2ag}{u^2g})^2 \]  

(19) 

\[ f = (a - a)^2 \]  

(20) 

\[ f = 0. \]  

(21) 

Since the squared distance cannot be negative, zero is the minimum possible value of the objective function. So \( u = \sqrt{ag} \) and \( \theta = \frac{\pi}{4} \) are the optimal solutions.

### 2.3. Game of angry birds with splitting

Let us considered an angry bird which has been fired from a slingshot at an initial velocity \( u \) at an angle of projection \( \theta \) with respect to the \( x \) axis or the horizontal which which is the point \((0,0)\) and the height \( h \) at which the angry bird splits into two smaller angry birds.

![Figure 2. This figure shows the trajectory of angry birds splitting into two smaller angry birds when there are no horizontal forces acting on the angry birds](image)

The initial velocity of the angry bird with respect to the \( x \) axis and the \( y \) axis is given by:

\[ u_x = u \cos \theta \]  

(22) 

\[ u_y = u \sin \theta \]  

(23) 

The \( x \) and \( y \) component of the velocities of the angry bird at a vertical height \( h \) from the ground with a velocity \( v \) at that point is given by:

\[ v_x = v \cos \alpha \]  

(24) 

\[ v_y = v \sin \alpha \]  

(25)
Due to gravity, the vertical component of velocity decreases and the horizontal component of velocity remains the same [26]. The angle of the trajectory of the angry bird with the trajectory is $\alpha$. Therefore the equations obtained are:

\[ v^2 \sin^2 \alpha = u^2 \sin^2 \theta - 2gh \]  
\[ v^2 \cos^2 \alpha = u^2 \cos^2 \theta \]  

Now, the time $t$ which is taken for the angry bird to reach a height $h$ is given in:

\[ vsin\alpha = ucos\theta - gt \]  
\[ \Rightarrow t = \frac{ucos\theta - vsin\alpha}{g} \]  
\[ \Rightarrow t = \frac{ucos\theta - \sqrt{u^2sin^2\theta - 2gh}}{g} \]  

After that, the angle $\alpha$ is calculated.

\[ tan^2 \alpha = \frac{u^2sin^2\theta - 2gh}{u^2cos^2\theta} \]  
\[ \Rightarrow tan\alpha = \sqrt{\frac{u^2sin^2\theta - 2gh}{u^2cos^2\theta}} \]  
\[ \Rightarrow \alpha = tan^{-1}\left(\sqrt{\frac{u^2sin^2\theta - 2gh}{u^2cos^2\theta}}\right) \]  

From the above equations, we derive an equation for the total velocity $v$.

\[ v^2 = v^2 \cos^2 \alpha + v^2 \sin^2 \alpha \]  
\[ \Rightarrow v^2 = u^2 \cos \theta + u^2 \sin^2 \theta - 2gh \]  
\[ \Rightarrow v = \sqrt{u^2 - 2gh} \]
Now we use the law of conservation of energy [29] for deriving the equations of the splitting of the angry birds.

Thus $v' = \frac{v}{\sqrt{2}}$

Let the distance travelled by the two angry birds be $d'_1$ and $d'_2$ and the time taken for the angry bird to complete these two trajectories be $t_1$ and $t_2$. For simplicity, we assume $\beta = \frac{\pi}{4}$.

$$d'_1 = \frac{v}{\sqrt{2}} \cos(\beta - \alpha)t_1$$

(37)

$$d'_2 = \frac{v}{\sqrt{2}} \cos(\beta + \alpha)t_2$$

(38)

Now we will consider the smaller angry bird which will move in the downward direction.

The time taken to travel on these trajectories is given by:

$$t_1 = \frac{v}{\sqrt{2}} \sin(\alpha + \beta) + \frac{1}{g} \sqrt{\frac{v^2 \sin^2(\alpha + \beta)}{2} + 2gh}$$

(39)

$$t_2 = -\frac{v}{\sqrt{2}} \cos(\alpha + \beta) + \frac{1}{g} \sqrt{\frac{v^2 \cos^2(\alpha + \beta)}{2} + 2gh}$$

(40)

Let us now consider the landing points of the two trajectories to be $d_1$ and $d_2$.

$$d_1 = u \cos \theta t + d'_1$$

(41)

$$d_2 = u \cos \theta t + d'_2$$

(42)

Since the fixed points where the damage done to the pigs are at a distance of $a$ and $b$ respectively from the origin, the objective function which we obtain is the sum of square of both the distances. Thus the objective function is given by:

$$f(u, \theta, h) = (a - d_1)^2 + (b - d_2)^2.$$ 

(43)

2.4. Game of angry birds with splitting in the presence of resistive electric field

Let us consider a particle of a mass $m$ and a charge $q$ which is subject to an electric field $E$ in the negative $x$ direction. Before the splitting of the charged angry bird, the horizontal distance travelled by the angry bird is $p$. The height at which the angry bird splits is $h$. The initial velocity of the angry bird is $u$ and the angle of projection of the angry bird is $\theta$. The velocity of the angry bird at a height $h$ from the ground is $v$ and the angle with the horizontal at that point is $\alpha$. In the $y$ direction, the acceleration due to gravity is constant.

The acceleration of the angry bird in the negative $x$ direction is given by:

$$A = \frac{qE}{m}$$

(44)
Figure 3. This figure shows the trajectory of an angry bird splitting into two smaller angry birds in presence of a resistive horizontal electric field.

The $x$ and $y$ components if the velocity $v$ are $v_x = v \cos \alpha$ and $v_y = v \sin \alpha$.

\[ v^2 \sin^2 \alpha = u^2 \sin^2 \theta - 2gh \quad (45) \]

\[ v^2 \cos^2 \alpha = u^2 \cos^2 \theta - 2ap \quad (46) \]

Now,

\[ v \sin \alpha = u \sin \theta - gt \quad (47) \]

\[ \Rightarrow t = u \sin \theta - gt \quad (48) \]

\[ \Rightarrow t = \frac{u \sin \theta - \sqrt{u^2 \sin^2 \theta - 2gh}}{g} \quad (49) \]

Now we will also find out the value of $p$ which is the horizontal distance travelled by the charged angry bird before splitting.

\[ p = u \cos \theta t - \frac{At^2}{2} \quad (50) \]

\[ p = u \cos \theta \left[ \frac{u \sin \theta - \sqrt{u^2 \sin^2 \theta - 2gh}}{g} \right] - \frac{A}{2} \left( \frac{u \sin \theta - \sqrt{u^2 \sin^2 \theta - 2gh}}{g} \right)^2 \quad (51) \]
After this, we will find out the value of the angle $\alpha$.

$$\tan^2 \alpha = \frac{u^2 \sin^2 \theta - 2gh}{u^2 \cos^2 \theta - 2ap}$$  \hspace{1cm} (52)

$$\Rightarrow \alpha = \tan^{-1} \left( \frac{u^2 \sin^2 \theta - 2gh}{u^2 \cos^2 \theta - 2ap} \right)$$  \hspace{1cm} (53)

Let the time of flight after the two smaller angry birds be given by $t_1$ and $t_2$.

$$t_1 = \frac{v}{\sqrt{2g}} \sin(\alpha + \beta) + \frac{1}{g} \sqrt{\frac{v^2}{2} \sin^2(\alpha + \beta) + 2gh}$$  \hspace{1cm} (54)

$$t_2 = \frac{v}{\sqrt{2g}} \cos(\alpha + \beta) + \frac{1}{g} \sqrt{\frac{v^2}{2} \cos^2(\alpha + \beta) + 2gh}$$  \hspace{1cm} (55)

Now it would be possible to figure out the distances $d_1$ and $d_2$.

$$d_1 = p + \frac{v}{\sqrt{2}} t_1 - \frac{at_1^2}{2}$$  \hspace{1cm} (56)

$$d_2 = p + \frac{v}{\sqrt{2}} t_2 - \frac{at_2^2}{2}$$  \hspace{1cm} (57)

Similarly, the objective function in this case will be:

$$f(u, \theta, h) = (a - d_1)^2 + (b - d_2)^2.$$  \hspace{1cm} (58)

3. Solution to optimize the angry birds game and the stochastic gradient descent method

3.1. Optimal strategy for game of angry birds without splitting

The first step is to find the stationary points or the points where the value of the first order partial derivative with respect to both $u$ as well as $\theta$ is zero.

Thus the conditions that have to be satisfied are:

$$\frac{\partial f}{\partial u} = 0$$  \hspace{1cm} (59)

$$\frac{\partial f}{\partial \theta} = 0$$  \hspace{1cm} (60)
\[
\frac{\partial f}{\partial u} = 2(a - \frac{u^2 \sin 2\theta}{g})(-\frac{2u \sin 2\theta}{g}) = 0 \tag{61}
\]

By solving this equation, we can figure out that for this value to be zero, either \(usin2\theta = 0\) or \(a - \frac{u^2 \sin 2\theta}{g} = 0\).

So the possible solutions for this equation are:

(i) \(u = 0\)
(ii) \(\sin 2\theta = 0\)
(iii) \(a - \frac{u^2 \sin 2\theta}{g} = 0 \Rightarrow \frac{u^2 \sin 2\theta}{g} = a \Rightarrow u^2 \sin 2\theta = ag \Rightarrow \sin 2\theta = \frac{ag}{u^2}\)

Differentiating with respect to \(\theta\), we get:

\[
\frac{\partial f}{\partial \theta} = 2(a - \frac{u^2 \sin 2\theta}{g})(-\frac{2u^2 \cos 2\theta}{g}) = 0 \tag{62}
\]

So the possible solutions for this equation are:

(i) \(u^2 = 0\)
(ii) \(\cos 2\theta = 0 \Rightarrow \theta = \frac{\pi}{4}\)
(iii) \(\sin 2\theta = \frac{ag}{u^2}\)

For any point to be a stationary point, any single point where \(\frac{\partial f}{\partial u} = 0\) and \(\frac{\partial f}{\partial \theta} = 0\) has to be taken since it is necessary for both the first order derivatives to be zero. For this purpose, we consider take into consideration a point where \(\theta = \frac{\pi}{4}\) and \(u^2 = ag\) as one possible solution.

Substituting these values in the objective function (11), we get:

\[
\Rightarrow f(u, \theta) = (a - \frac{agsin\frac{2\pi}{4}}{g})^2 \tag{63}
\]
\[
\Rightarrow f = (a - a)^2 \tag{64}
\]
\[
\Rightarrow f = 0. \tag{65}
\]

Since the objective function is a squared value, the value can never be negative, the stationary points substituted will give local minima.

Now we consider all possible stationary points where \(\sin 2\theta = \frac{ag}{u^2}\) and substitute them in the objective function (11).

\[
\Rightarrow f(u, \theta) = (a - \frac{u^2ag}{u^2g})^2 \tag{66}
\]
\[
\Rightarrow f = (a - a)^2 \tag{67}
\]
\[
\Rightarrow f = 0. \tag{68}
\]

Since the squared distance cannot be negative, zero is the minimum possible value of the objective function. So \(u = \sqrt{ag}\) and \(\theta = \frac{\pi}{4}\) are the optimal solutions.
3.2. Optimal strategy for game of angry birds with splitting

Let us consider an objective function \( f(u, \theta, h) \) which is given in (43). The stochastic gradient descent has a learning rate \( \eta \) which generally lies between 0 and 1. The vector \( w \) consists of the variable parameters in the functions which are \( u, \theta \) and \( h \).

Therefore,

\[
w^T = [u, \theta, h]
\] (69)

These variables are randomly initialized and an iterative process is used to calculate the new value of the vector \( w \). The iterative method used to calculate the value of \( w \) is given by:

\[
w(k + 1) = w(k) - \eta \nabla f,
\] (70)

where

\[
(\nabla f)^T = \left[ \frac{\partial f}{\partial u}, \frac{\partial f}{\partial \theta}, \frac{\partial f}{\partial h} \right].
\] (71)

Now if the value of \( w(k + 1)^T w(k) \) is below a certain threshold \( \epsilon \ll 1 \) which is positive, then we conclude that convergence has been achieved and thus we have reached the solution. For our model \( \eta = 0.0000000001 \) and \( \epsilon = 0.00000000000001 \).

It would not be possible to find the absolute minimum value of the function using the stochastic gradient descent method since there are a large number of local minima [14]. For this purpose, the parallel stochastic gradient descent method has been implemented using the Message Passing Interface (MPI) to obtain a more optimal solution to this problem. In this method, random initialization is done for each process, and the local minimum is calculated for each process. Using the \texttt{MPI\_Send()} function, the local minimum is sent from all processes to the main process with \texttt{id} = 0. In the process with \texttt{id} = 0, the \texttt{MPI\_Recv()} function receives the minima from all the processes and the minimum of all minima is considered to be the absolute minimum [15].

3.3. Optimal Strategy of a game of angry birds with splitting in the presence of resistive electric field

The stochastic gradient descent method explained in subsection B can be used to find the minimum value of the objective function [4]. The values of \( \eta \) and \( \epsilon \) are the same for the third model and the second model.

4. Machine Learning Based Agent Playing Angry Birds Game

Machine learning-based agents play the angry birds game based on past human experience [9]. The agent reads data consisting of the initial velocity, the angle of projection, the height of splitting, and the points at which the angry birds are located, and the number of pigs destroyed in that particular single shot. Based on the experience gained from playing previous shots, the agent randomly chooses an initial velocity, an angle of projection, and a height of splitting and checks whether the shot played increases the probability of winning or not [21]. If the shot does not increase the probability of winning the game, initial velocity, the angle of projection, and splitting height are chosen once again. This process is repeated until a certain number of trials are performed. If it is not possible for the machine to obtain values of initial velocity, angle of projection, and splitting height, then the last chosen values are taken to play the shot for the game.
4.1. KNN based agent
In our feature space, we have a large number of data points. These data points are labeled [10, 25]. The label represents the number of pigs destroyed based on the initial velocity, the angle of projection and the height of the splitting of the angry birds, and the pigs’ location. For the purpose of classifying a testing point, which means predicting the number of pigs destroyed when a shot is taken, five nearest neighbors are taken into consideration [16]. For all the neighbors, we check how many pigs are killed by a shot. A counter is used to count the number of neighbors for a particular number of pigs that have been destroyed. Once the number of pigs that would be destroyed is calculated, the decision is made by the agent whether to choose a new shot or play the existing shot.

4.2. Naive Bayes based agent
In our model, the naive Bayes classifier classifies the probability of a certain number of pigs being destroyed by a certain point, given that there are a large number of points indicating the number of pigs destroyed corresponding to a particular initial velocity, angle of projection, and a height of splitting. It is given that there are certain points located within the testing data point in a circle of a certain radius where the testing data point or the testing angry bird shot (data point) is the center of the circle [27, 17, 19, 12]. Let $X$ be the probability that the data point corresponding to the point lies within the circle, $Y_c$ be the probability that a certain shot destroys $c$ pigs. The probability of that a shot will destroy $c$ pigs is given as follows [18, 24]:

$$P(Y_c|X) = \frac{P(X|Y_c) * P(Y_c)}{P(X)} \quad (72)$$

The value of $c$ for which $P(Y_c|X)$ is maximized is selected. If $c \geq 1$, then the shot is played, otherwise the shot is not played.

5. Results
The results are to be discussed in terms of the average number of times an agent would win when the agent would play the game 100 times. This means that we will perform multiple experiments and put the results in a table. Table 1 shows the results when the angry birds game is played without splitting; table 2 shows the results when the angry birds game is played with splitting, and table 3 shows the results when the angry birds game is played with splitting in the presence of a resistive electric field.

5.1. Results when angry birds is played without splitting

| Agent                  | Number of wins per 100 |
|------------------------|------------------------|
| Human                  | 75                     |
| Stochastic gradient descent | 93                     |
| KNN                    | 39                     |
| Naive Bayes            | 34                     |

5.2. Results when angry birds is played with splitting

| Agent                  | Number of wins per 100 |
|------------------------|------------------------|
| Human                  | 61                     |
| Stochastic gradient descent | 66                     |
| KNN                    | 31                     |
| Naive Bayes            | 28                     |
5.3. Results when angry birds is played with splitting in the presence of a resistive electric field

| Agent             | Number of wins per 100 |
|-------------------|------------------------|
| Human             | 59                     |
| Stochastic gradient descent | 67                 |
| KNN               | 29                     |
| Naive Bayes       | 30                     |

6. Conclusions

From the experiments that we have conducted, it can be concluded that the stochastic gradient descent method plays the best in terms of the number of games won per 100 games. The human agent also performs fairly well when it plays the game. However, the two machine learning-based agents cannot play the angry birds game so well. In a competition where the time available for a particular shot is limited, the machine learning-based agent would perform much better compared to the stochastic gradient descent method since the time taken for the objective function to converge the local or global minimum is extremely large. However, for competitions that are not time-bound, the stochastic gradient descent based agent would outperform the machine learning-based agent since it is very accurate.

References

1. Almasi, G., Archer, C., Castanos, J.G., Gunnels, J.A., Erway, C.C., Heidelberger, P., Martorell, X., Moreira, J.E., Pinnow, K., Ratterman, J., Steinmacher-Burrow, B.D., Gropp, W., Toonen, B.: Design and implementation of message-passing services for the blue gene/l supercomputer. IBM Journal of Research and Development 49(2.3), 393–406 (2005). DOI 10.1147/rd.492.0393
2. Bace, M., Ilijic, S., Narancic, Z., Bistricic, L.: The envelope of projectile trajectories. European journal of physics 23(6), 637 (2002)
3. Blas, J.G., Carretero, J.: Recent advances in the message passing interface (2014)
4. Bottou, L.: Large-scale machine learning with stochastic gradient descent. In: Proceedings of COMPSTAT’2010, pp. 177–186. Springer (2010)
5. Brown, R.A.: Maximizing the range of a projectile. The Physics Teacher 30(6), 344–347 (1992)
6. Calimeri, F., Fink, M., Germano, S., Humenberger, A., Ianni, G., Redl, C., Stepanova, D., Tucci, A., Wimmer, A.: Angry-hex: an artificial player for angry birds based on declarative knowledge bases. IEEE Transactions on Computational Intelligence and AI in Games 8(2), 128–139 (2015)
7. Chudinov, P.: Extension of application field of analytical formulas for the computation of projectile motion in midair. Revista Brasileira de Ensino de FAsica ^ 35, 1 – 5 (2013). URL http://www.scielo.br/scielo.php?script=sciarttext&pid=S1806-11172013000100010&nrm=iso
8. Deb, K.: Multi-objective optimization. In: Search methodologies, pp. 403–449. Springer (2014)
9. Devillers, L., Vidrascu, L., Lamel, L.: Challenges in real-life emotion annotation and machine learning based detection. Neural Networks 18(4), 407–422 (2005)
10. Dudani, S.A.: The distance-weighted k-nearest-neighbor rule. IEEE Transactions on Systems, Man, and Cybernetics (4), 325–327 (1976)
11. Fowler, J.F.: The linear-quadratic formula and progress in fractionated radiotherapy. The British journal of radiology 62(740), 679–694 (1989)
12. Frank, E., Hall, M., Pfahringer, B.: Locally weighted naive bayes. In: Proceedings of the Nineteenth conference on Uncertainty in Artificial Intelligence, pp. 249–256. Morgan Kaufmann Publishers Inc. (2002)
13 Gill, P.E., Murray, W., Wright, M.H.: Practical optimization. SIAM (2019)
14 Gropp, W., Gropp, W.D., Lusk, E., Skjellum, A., Lusk, A.D.F.E.E.: Using MPI: portable parallel programming with the message-passing interface, vol. 1. MIT press (1999)
15 Gropp, W., Lusk, E., Doss, N., Skjellum, A.: A high-performance, portable implementation of the mpi message passing interface standard. Parallel computing 22(6), 789–828 (1996)
16 Keller, J.M., Gray, M.R., Givens, J.A.: A fuzzy k-nearest neighbor algorithm. IEEE transactions on systems, man, and cybernetics (4), 580–585 (1985)
17 Lewis, D.D.: Naive (bayes) at forty: The independence assumption in information retrieval. In: European conference on machine learning, pp. 4–15. Springer (1998)
18 Lindley, D.V.: Fiducial distributions and bayes’ theorem. Journal of the Royal Statistical Society. Series B (Methodological) pp. 102–107 (1958)
19 McCallum, A., Nigam, K., et al.: A comparison of event models for naive bayes text classification. In: AAAI-98 workshop on learning for text categorization, vol. 752, pp. 41–48. Citeseer (1998)
20 Moore-Russo, D., Diletti, J., Strzalec, J., Reeb, C., Schillace, J., Martin, A., Arabeyyat, T., Prabucki, K., Scanlon, S.: A study of how angry birds has been used in mathematics education. Digital Experiences in Mathematics Education 1(2-3), 107–132 (2015)
21 Narayan-Chen, A., Xu, L., Shavlik, J.: An empirical evaluation of machine learning approaches for angry birds. In: IJCAI Symposium on AI in Angry Birds, pp. 1–7 (2013)
22 Nocedal, J., Wright, S.: Numerical optimization. Springer Science Business Media (2006)
23 Paul, A., Hullermeier, E.: A cbr approach to the angry birds game. (2015)
24 Pawlak, Z.: Rough sets, decision algorithms and bayes’ theorem. European Journal of Operational Research 136(1), 181–189 (2002)
25 Peterson, L.E.: K-nearest neighbor. Scholarpedia 4(2), 1883 (2009)
26 Pound, R.V., Snider, J.L.: Effect of gravity on gamma radiation. Physical Review 140(3B), B788 (1965)
27 Rish, I., et al.: An empirical study of the naive bayes classifier. In: IJCAI 2001 workshop on empirical methods in artificial intelligence, vol. 3, pp. 41–46 (2001)
28 Ruder, S.: An overview of gradient descent optimization algorithms. arXiv preprint arXiv:1609.04747 (2016) Short form of title 17
29 Sarton, G., Mayer, J., Joule, J., Carnot, S.: The discovery of the law of conservation of energy. Isis 13(1), 18–44 (1929)
30 Stewart, S.M.: An analytic approach to projectile motion in a linear resisting medium. International Journal of Mathematical Education in Science and Technology 37(4), 411–431 (2006). DOI 10.1080/00207390600594911. URL https://doi.org/10.1080/00207390600594911