Viability of the cluster mass function formalism in parametrised modified gravity

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Model-independent parametrisations for examining departures from General Relativity have been increasingly studied over the past few years. Various observables have been used to constrain the parameters and forecasts for future surveys have been carried out. In one such forecast, galaxy cluster counts were used to constrain the parameters. Here, we carry out a limited set of $N$-body simulations, with a modified Poisson equation, to examine the accuracy of existing mass functions for modified gravity cosmologies. As well as altering the gravitational calculation, we include the effect of a screening scale to ensure consistency of the theory with solar system tests. Our results suggest that if a screening scale exists its effect can be taken into account in the cluster count calculation through its effect on the linear matter power spectrum. If this is done, the accuracy of the standard mass function formalism in modified gravity theories with reasonably small departures from General Relativity, as tested in this work, is comparable to the standard case.

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I. INTRODUCTION

Several explanations have been put forward to explain the apparent accelerating cosmological expansion. The most popular idea is that the acceleration is due to a distinct form of energy in the universe, generically known as dark energy. The simplest example of what could constitute dark energy is a cosmological constant $\Lambda$, a non-dynamical form of energy that may be interpreted as arising as an unconstrained integration constant in the equations of motion or from the vacuum energy generated by the quantised fields that make up the matter content of the universe. Dynamical forms of dark energy involving scalar fields that are evolving with time can also lead to late-time acceleration. Some have also argued that the observation that the expansion is accelerating results from the misuse of the Cosmological Principle in interpreting the expansion history from the inhomogeneous distribution of matter around us. An alternative perspective to the above both lines of thought above is to explain the departure from the expected late-time behaviour by introducing modifications to the theory of General Relativity (GR) itself.

There has been an increased interest in modified gravity theories in cosmology over the past decade or so. The large number of proposed alternatives to General Relativity (GR) has led to interest in examining possible deviations from GR in a more model independent way. Much work has gone into creating these parametrisations, examining their consistency, constraining them with current data and forecasting future experiments (see [2] for a comprehensive review). Many different observations and combinations of observations have been considered for constraining these parametrisations. The key point with the combinations is the issue of distinct degeneracies in different sets of observations. In particular, observations such as the integrated Sachs-Wolfe effect in the Cosmic Microwave Background (CMB) and cosmic shear surveys, both being effects on the energies and trajectories of photons that are relativistic, constrain the sum of the two scalar potentials in the metric $\Psi + \Phi$ and thus constrain a particular combination of the Modified Gravity Parameters (MGPs) as will be introduced later. This degeneracy is best broken with an observation that depends on a different linear combination of the scalar potentials. Observations relating to the growth of structure in non-relativistic matter, such as redshift space distortions or galaxy surveys, are a natural choice for breaking the degeneracy. Recently [1], we argued that combining future cluster counts with CMB and cosmic shear observations could yield high precision constraints on the simplest form of MGPs particularly in the case where the modifications are constant at low redshift.

Galaxy cluster counts can be predicted for a particular combination of cosmological parameters by starting from the linear matter power spectrum and using mass functions to calculate how many bound structure will form at different mass scales. The mass functions are semi-analytic formulas whose form derives from the original Press-Schechter ideas [3] and its extensions [4,7]. The functions are now calibrated using detailed and extensive $N$-body simulations. In this paper, we present the results of a limited set of of $N$-body simulations aimed at investigating the accuracy of existing mass functions for the MGP formalism. Due to the computational demand of running the required $N$-body simulations we have restricted our investigation to a single point in the phase space of MGPs i.e. a single choice in the modification of the strength of gravity that is constant at low redshift. As such we are not attempting to accurately calibrate an extension of the mass function formalism in the phase space of MGPs. We are instead addressing the question of whether the existing mass function formalism can be used as is in forecasts involving cluster counts such as

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that carried out in [11].

This paper is organised as follows. In section II we discuss the mass function formalism and some of the results in the literature regarding modified gravity models. The modifications required to run N-body simulation in such models of parametrised modified gravity are summarised in section III and the results of running our suite of simulations are described in section IV. We conclude with a discussion of the results in section V.

II. MASS FUNCTIONS

Galaxy clusters are some of the largest collapsed structures in the universe. According to the standard scenario of hierarchical structure formation within the ΛCDM cosmological model, clusters typically consist of hot gas bound in a large cold dark matter halo. Clusters are a useful cosmological probe as their size corresponds to scales near the linear to non–linear transition in the underlying dark matter power spectrum. This has several consequences: they probe the tail of the matter perturbation spectrum and are therefore a sensitive probe of growth. In addition, galaxy cluster counts can be predicted from linear theory, using semi-analytic formulae or formulae calibrated from N-body simulations. Cluster counts and the properties of clusters have been used to constrain cosmological parameters and in particular they have been used to constrain dark energy, which may be responsible for late–time acceleration of the cosmological expansion (see for example [8, 9]), and some studies looking at constraining the growth factor γ (see for example [10, 11]).

The number of clusters observable over a fraction of the sky $f_{\text{sky}}$ and with a redshift dependent mass resolution limit $M_{\text{lim}}(z)$ in a redshift bin spanning the interval $z$ to $z + \Delta z$ can be calculated by integrating the comoving number density $dn/dM$ of objects with mass $M$

$$N_{\Delta z} = 4\pi f_{\text{sky}} \int_{z}^{z+\Delta z} dz' \int_{M_{\text{lim}}(z')}^{\infty} dM \frac{dn}{dM} \frac{dV}{dz'd\Omega},$$

where $dV/dzd\Omega = r^2(z)/H(z)$ is the comoving volume at redshift $z$ in a flat universe, with $H(z)$ the Hubble rate and $r(z) = \int_{0}^{z} dz'/H(z')$ the comoving distance to that redshift.

The comoving number density of objects in a given mass range $dn/dM$ is known as the mass function and much work has gone into predicting its shape for a given linear power spectrum. Early, semi-analytical estimates of the mass function resulted in the Press-Schechter formalism [3], where the mass function is given by

$$\frac{dn}{dM} = -\sqrt{2 \frac{\rho}{\pi}} \frac{d\sigma_M}{dM} \delta_c \exp \left[ -\frac{1}{2} \frac{\delta_c^2}{\sigma_M^2} \right],$$

where $\rho$ is the background density of dark matter today, $\delta_c = 1.686$ is the critical density contrast for collapse of a spherical perturbation and $\sigma_M^2$ is the variance of the dark matter fluctuations in spheres of radius $R = (3M/4\pi\rho)^{1/3}$ defined by the integral of the linear matter power spectrum $P(k)$ over wavenumber $k$

$$\sigma_M^2 = \frac{1}{2\pi^2} \int_0^\infty W^2(kR) P(k) k^2 dk,$$

with top-hat filter function

$$W(kR) = 3 \left( \frac{\sin(kR)}{(kR)^2} - \frac{\cos(kR)}{(kR)^2} \right).$$

Successive studies have refined the formalism resulting in more complex expressions with increased accuracy [4–7]. The functional forms of these mass functions have been used to fit the results of N-body simulations for ΛCDM cosmologies and agreement is at the 5-10% level for ΛCDM cosmologies [12, 13]. The inaccuracy increases to 10% or more for more general cosmologies [14, 15]. In the what follows we select the following form from [14], henceforth W05:

$$\frac{dn}{dM} = -0.7234 \frac{\rho}{M} \frac{d\sigma_M}{dM} \frac{1}{\sigma_M} \exp \left[ -\frac{1.1982}{\sigma_M^2} \right] \times$$

$$\left( 0.2538 + \sigma_M^{1.1625} \right).$$

The mass function formalism has been used extensively in constraining extensions to the standard ΛCDM cosmology. Some examples are redshift dependent dark energy equation of state models $w$CDM [12, 13] and some models of modified gravity including $f(R)$ [15, 16], DGP [17, 18] and coupled scalar field cosmologies [19, 20]. However, in some cases it is unclear how well the mass function formalism will reproduce the correct cluster counts if the modifications alter significantly the process of structure formation.

There are three reasons why the expressions commonly used to predict cluster counts may not be valid in modified models. Firstly the linear matter power spectrum the formalism is based on will, in general, differ from the standard cosmology for the same matter content. Secondly the expressions are calibrated with respect to standard ΛCDM N-body simulations or semi–analytical arguments and the coefficients obtained in this way may not be the correct one in modified cosmologies. As such, even if two models may give the same linear matter power spectrum, they could differ in their cluster counts predictions because, for example, the critical density $\delta_c$ may be different in the modified model. Thirdly, the expressions themselves may not be a corrected parametrisation of the transformation of a linear power spectrum into a description of the mass distribution of clusters.

Typically only the first possibility is addressed in works where cluster counts are used to constrain modified cosmologies. This is achieved by inputing into the formalism a modified linear matter power spectrum that has been obtained by solving the linear growth equations for the dark matter. This is by far the simplest way to account
for the modifications since the second and third concerns require the use of N-body runs to be addressed. Here we will focus on how well the mass formalism does in reproducing parameterised modified gravity cluster counts if the expressions are unchanged and the modifications are solely taken into account by modifications of the linear matter power spectrum. The results will inform us on whether using the standard formalism can be an informative tool in investigating the constraining power of cluster counts in modified gravity models. In particular we will be able to determine whether forecasts made in [1] may be biased because of the way the mass function formalism was used in the prediction of cluster counts in modified gravity models.

III. N-BODY SIMULATIONS AND MODIFIED GRAVITY PARAMETRISATION

Numerous sets of phenomenological "Modified Gravity Parameters" (MGPs) have been suggested in the literature, see e.g. [21] for a partial translation table and [22] for a discussion of the differences with some of the parametrisations. Most of the parametrisations are phenomenological modifications to the Einstein equations and typically involve a parameter relating to the strength of gravity and a parameter relating the two scalar potentials in the metric. Our potentials are defined as scalar perturbations of a flat, FRW metric

\[ g_{00} = -\left[1 + 2\Psi(\vec{x}, t)\right], \]
\[ g_{ij} = a^2(t)\left[1 - 2\Phi(\vec{x}, t)\right]\delta_{ij}, \]

(6)

where we have made the conformal Newtonian gauge choice to fix the remaining two scalar degrees of freedom in the perturbed metric. In [6], \( \Psi \) is the Newtonian potential and is responsible for the acceleration of massive particles whereas \( \Phi \) is the curvature potential, which also contributes to the acceleration of relativistic particles. For this work, since N-body simulations are essentially Newtonian plus an expanding background, we have only used one MGP, \( \mu \). This is the parameter in the Poisson equation that controls the strength of gravity. In fourier space,

\[ k^2(\Psi(a, k) = -4\pi G a^2 \mu(a, k)\rho(a)\Delta(a, k). \]

(7)

where, \( a \) is the FRW scale factor, \( G \) is Newton’s constant, \( \rho \) is the background density of cold dark matter and \( \Delta \) is the gauge invariant density contrast given by

\[ \Delta = \delta + \frac{3aHv}{k}, \]

(8)

where the cold dark matter density contrast is defined as \( \delta = \delta\rho(a, k)/\rho(a) \), \( v \) is velocity of the dark matter and \( H \) is the Hubble parameter. In principle, \( \mu \) could be any function of time and space, however here we have kept a simple treatment of it. In particular, we are considering the situation where, at a set redshift of \( z_{mg} \), the parameter transitions from its GR value of 1 to some other value. Except for the inclusion of a screening scale, there is no scale dependence inserted into the phenomenology. Screening mechanisms are an important component of modified gravity models as they allow models that differ from GR to reproduce solar system tests. There are several mechanisms in the literature, including the chameleon mechanism [23] that operates in \( f(R) \) gravity and the Vainshtein mechanism [24] that operates in DGP gravity. To further our phenomenological approach to modified gravity, we use a simpler treatment where the modifications to gravity cease below a set length scale, \( \ell_{sc} \).

In order to investigate the use of mass functions when forecasting cluster counts in models of parameterised modified gravity, we have made some simple modifications to the publicly available N-body, Smooth Particle Hydrodynamics (SPH) code Gadget-2 [25]. Gadget-2 is a parallel N-body code that calculates the gravitational acceleration with a TreePM approach [26]. For this investigation we are only interested in the dynamics of the dominant dark matter and we do not include any component with non-zero pressure. Since the N-body simulation probes the low-velocity, Newtonian regime on sub-horizon scales, the only modification of gravity that affects the simulations is any modification to the Poisson equation that encompasses all gravitational aspects of the interaction in such codes. In Gadget-2 the Poisson equation is solved differently on large and small scales. On large scales, a regular grid (the “mesh”) is placed in the simulation volume. The mass of each particle is then interpolated onto the mesh. This discrete density field is then Fourier transformed and the Poisson equation is solved in Fourier space. This yields the gravitational acceleration of particles in the simulation due to distant mass distribution in the simulation volume. On smaller scales, the force on each particle is obtained by calculating the Newtonian potential directly. A hierarchy of increasingly higher resolution cells (the “tree”) is constructed. Far from the point where the potential is being calculated, it is sufficient to use cells of coarse resolution containing multiple particles treated together at their centre of mass. Nearer to the point where the potential is being calculated, increased resolution is required until, at sufficiently short distances, the effect of each particle is included separately. The two components of the potential are combined via a matching filter to obtain the potential governing the overall force acting on each particle at every time step. Thus modifications to gravity must be encoded in both contributions to the total potential i.e. in both Fourier domain solution and direct force calculation. Our modifications to Gadget-2 are as follows: Both the potential calculated by the particle mesh and the force calculated by the tree structure are modified by a factor of \( \mu \) for distances above a screening scale of proper length \( \ell_{sc} \). This screening scale can be switched off such that the only modification acting is the change in the
strength of gravity. We ran the modified gravity simulations both with and without the screening scale for a series of simulation boxes each with the same volume and mass resolution (see table I for the parameter values used in the simulations). The expansion history in the simulations is that of a standard ΛCDM cosmology for both GR and MGP runs. This is in order to isolate the effect of the MGP on the growth of structure from any change due to the expansion history. Each simulation is started at a redshift $z_i = 50$ and average the result of each simulation over a number of runs with independent random seeds to reduce the effect of sample variance in our final cluster counts.

The linear matter power spectra today used for the mass function prediction for both the GR+ΛCDM runs and the modified gravity runs were computed using the modified version of CAMB [27] MGCAMB [28]. We have modified MGCAMB to incorporate the same modifications to gravity as the N-body simulation, namely a constant value for $\mu$, different to the fiducial GR value of 1, that switches on after redshift $z_{\mu}$ and with the option to recover GR below a screening scale $\ell_{sc}$. For all simulations in this work we have chosen a fiducial value of $\mu = 1.05$ for the modification. The second MGP, $\eta = \Phi/\Psi$, has been kept at its GR value in the MGCAMB code and does not come in to the $N$-body simulations. We consider a flat cosmology with the following parameters. The dimensionless Hubble rate in units of 100 Km s$^{-1}$ Mpc$^{-1}$, $h = 0.71$, the density of matter (baryons + dark matter) and dark energy in units of the critical energy density, $\Omega_m = 0.265$, and $\Omega_\Lambda = 0.735$ respectively, the optical depth to recombination $\tau = 0.088$, the amplitude of primordial, super horizon curvature perturbations $A_s = 2.157 \times 10^{-9}$ at $k = 0.05$ h Mpc$^{-1}$ and their spectral index $n_s = 0.963$. These parameters correspond to the WMAP 7-year best-fit parameters [29]. This model yields a large scale structure normalisation of $\sigma_8 = 0.804$ for the standard deviations of fluctuations on scales of 8 $h^{-1}$ Mpc.

The initial condition for each of the $N$-body simulations were calculated using the transfer functions obtained from the MGCAMB run. The transfer functions are used to generate a grid of particles with velocities and displacements consistent with the power spectrum of the matter expected at the starting redshift $z_i$. For this work we have used the 2LPT [30] [31] package to obtain the initial velocities and displacements. The package uses a second order Lagrange perturbation scheme to evolve the displacements and velocities of particles to the desired redshift. This is more accurate than using the first order Zeldovich approximation.

We obtained independent initial conditions for each of the several realisations run for each box and particle number. In the simulations used here, all of the particles are cold dark matter particles. For each box size and particle number, we also specified the smoothing scale $\epsilon$.

FIG. 1: The number of clusters as a function of mass of the cluster found in the our simulations for the GR case ($\mu = 1$) compared to the modified gravity case, with and without screening scale. In the modified gravity case the 5% increase in $\mu$ leads to an increase in clusters on all scales when a screening scale is not included. The result is more complicated when a screening scale is included with less cluster being produced at the low mass scales as expected.

| Gravity | $z_i$ | #runs | $L_{box}$ ($h^{-1}$Mpc) | $N_{part}$ | $\epsilon$ ($h^{-1}$kpc) | $\mu$ | $z_{\mu}$ | $\ell_{sc}$ ($h^{-1}$Mpc) | $\sigma$ |
|---------|------|-------|------------------------|-----------|------------------------|------|----------|------------------------|--------|
| GR      | 50   | 10    | 400                    | 256$^3$   | 45                     | 1.0  | n/a      | n/a                    | 400    |
| MGP's   | 50   | 10    | 400                    | 256$^3$   | 45                     | 1.05| 50       | 50                     | n/a    |
| MGP's   | 50   | 10    | 400                    | 256$^3$   | 45                     | 1.05| 50       | 50                     | n/a    |

TABLE I: Parameters for simulations.
We used a friends-of-friends halo finder\textsuperscript{1} with the standard linking length equal to 20\% of the inter-particle separation, $L_{\text{box}}/\sqrt{N_{\text{part}}}$. In principle, this linking length should be changed to take into account how virialisation is different for different cosmologies and gravity theories, however it is not clear how this should be done for parametrised modified gravity and we have made the standard choice in all of our calculations. It is now known that a systematic bias arises when determining the mass of halos with the friends-of-friends algorithm [14]. As shown in [14] this can be corrected for by modifying the number of particles in each halo according to the prescription $N_{\text{corr}} = N \times (1 - N^{-0.6})$, where $N$ is the original number of clusters found by the friends-of-friends algorithm in each mass interval and $N_{\text{corr}}$ is the unbiased value. We have applied this correction to the halos found in our simulations and also imposed a minimum number of (uncorrected) particles per halo of 40. The halos were then binned into logarithmic mass bins of width 0.2, between $10^{13} h^{-1} M_{\odot}$ and $10^{15} h^{-1} M_{\odot}$ where $M_{\odot}$ represent a solar mass.

\textsuperscript{1} [http://www-hpcc.astro.washington.edu/tools/fof.html](http://www-hpcc.astro.washington.edu/tools/fof.html)

In this section we examine the output of the simulations at a $z = 0$. Firstly, we examine the change to cluster counts due to the modification of gravity. Figure \ref{fig:GR} shows the total number of clusters found in the GR simulation, as well as the modified gravity cases with and without a screening scale. We plot the average value of the 10 independent realisations. The error bars show the standard deviation estimated from the 10 samples. As expected, the number of clusters increases due to the gravity being stronger in our fiducial modified model with $\mu > 1$. The increase is larger towards the higher masses. The screening scale reduces the increase in clusters in modified gravity as we would naively expect. In addition, there is a slight decrease in the number of cluster at the lowest masses. It is not clear if this decrement is significant or physically relevant but it may be consistent with the fact that at some mass scale the effect of screening will mean that objects that would have formed clusters in the GR case end up bound together with larger clusters which grow faster on scales beyond the screening scale.

In figure \ref{fig:mass_function} we compare the numbers of clusters in our simulations to the predictions in each case. The linear matter power spectra used as inputs to the W05 mass function are computed using the MGCAMB code, with $\mu = 1$ for the GR case, and $\mu = 1.05$ for the modified
V. DISCUSSION

We have shown that the mass function formalism can work for simple phenomenological models of modified gravity without a screening scale. To model modified gravity with a screening scale, its effect needs to be included in the calculation of the linear matter power spectrum in order to retain the accuracy of the mass function formalism. Of course, the mass function formalism is likely to break down for more extreme departures from GR, however it isn’t obvious that large deviations are allowed by current data. In addition, more complicated modelling of the $\mu$ parameter, such as including scale dependence may reduce the accuracy of the mass function predictions. However, as long as the scale dependence manifests in the linear matter power spectrum and the deviations from GR are relatively small then the mass function prediction may still be accurate. This will require further work. The work presented here needs to be extended in other ways as well, to more precisely determine the range of validity of the mass function formalism. Varying the cosmological parameters, linking length and parametrisation of $\mu$ will further test the validity of the mass functions. In addition, running bigger boxes and boxes with improved resolution will test the mass function over a larger range of cluster masses. The mass function should also be tested over a range of redshifts and with baryons included, rather than solely dark matter particles. In addition, an examination of spherical collapse in model independent modified gravity and/or an examination of virialisation in these theories may shed further light on finding halos and how mass functions work in modified gravity simulations.
[32] A. Jenkins, C. S. Frenk, S. D. M. White, J. M. Colberg, S. Cole, A. E. Evrard, H. M. P. Couchman, and N. Yoshida, MNRAS 321, 372 (2001), arXiv:astro-ph/0005260.

[33] K. Heitmann, Z. Lukić, S. Habib, and P. M. Ricker, apjl 642, L85 (2006), arXiv:astro-ph/0601233.

[34] C. Lacey and S. Cole, MNRAS 271, 676 (1994), arXiv:astro-ph/9402069.

[35] J. Einasto, A. A. Klypin, E. Saar, and S. F. Shandarin, MNRAS 206, 529 (1984).

[36] M. Davis, G. Efstathiou, C. S. Frenk, and S. D. M. White, Astrophys. J. 292, 371 (1985).