Photon-neutrino interactions in magnetic fields

R. SHAISULTANOV*

Budker Institute of Nuclear Physics
630090, Novosibirsk 90, Russia

Abstract

The low-energy two neutrino-two photon interactions in the presence of homogeneous magnetic field are studied. The cross sections in external magnetic field are shown to be larger than in vacuum by factor $\sim (m_W/m_e)^4 (B/B_c)^2$. The energy-loss rate due to the process $\gamma\gamma \rightarrow \nu\bar{\nu}$ in magnetic field is obtained.

*Email:shaisultanov@inp.nsk.su
Low-energy neutrino-photon interactions may be of interest in astrophysics and cosmology. A well-known example is the neutrino pair production via $\gamma \gamma \rightarrow \nu \bar{\nu}$, which provides an energy loss mechanism for stellar processes, and related processes, such as the neutrino photon scattering $\gamma \nu \rightarrow \gamma \nu$, that also may be important in the study of stellar evolution. Unfortunately the amplitude in any channel is known to be highly suppressed. In Gell-Mann showed that in the four-Fermi limit of the standard model the amplitude is exactly zero to order $G_F$ because by Yang theorem two photons cannot couple to $J = 1$ state. Therefore the amplitude is suppressed by additional factors of $\omega/m_W$, where $\omega$ is the photon energy and $m_W$ is the $W$ mass. For example, in the case of massless neutrinos, the amplitude for $\gamma \nu \rightarrow \gamma \nu$ in the Standard model is suppressed by factor $1/m_W^2$. As a result the cross section is exceedingly small. Also recently Dicus and Repko considered the processes $\gamma \nu \rightarrow \gamma \gamma \nu$, $\gamma \gamma \rightarrow \gamma \nu \bar{\nu}$ and $\nu \bar{\nu} \rightarrow \gamma \gamma \gamma$, the cross sections are shown to be much larger than the elastic cross section $\sigma(\gamma \nu \rightarrow \gamma \nu)$ for photon energies $\omega > 1$ keV. By now it is well known that in many astrophysical environments the absorption, emission, or scattering of neutrinos and photons occurs in the presence of strong magnetic fields with strength of order $B_c = m_e^2/e = 4.41 \times 10^{13}$ Gauss. Many processes were studied in the presence magnetic fields. Among them, for example, photon splitting $\gamma \rightarrow \gamma \gamma$ and neutrino Cherenkov process $\nu \rightarrow \nu \gamma$.

In this paper we will consider low-energy two neutrino-two photon interactions in the presence of homogeneous magnetic field. At $B/B_c \leq 0.1$ it is enough to use following effective action:

$$\mathcal{L}_{\text{eff}} = 4G_F a \frac{\alpha^{3/2}}{\sqrt{2} \sqrt{4\pi} m_e^4} \left[ \frac{5}{180} (N_{\mu\nu} F_{\mu\nu}) (F_{\lambda\rho} F_{\lambda\rho}) - \frac{14}{180} N_{\mu\nu} F_{\nu\lambda} F_{\lambda\rho} F_{\rho\mu} \right]$$

where $N_{\mu\nu}$ is

$$N_{\mu\nu} = \partial_{\mu} \left( \bar{\psi} \gamma_{\nu}(1 + \gamma_5)\psi \right) - \partial_{\nu} \left( \bar{\psi} \gamma_{\mu}(1 + \gamma_5)\psi \right),$$

and $a = 1 - \frac{1}{2} \left( 1 - 4 \sin^2 \theta_W \right)$ for $\nu_e$, where the first term in $a$ is the contribution from the $W$ exchange diagram and second one from the $Z$ exchange diagram. For $\nu_\mu$ and $\nu_\tau$ the
W contribution is multiplied by very small factor \((m_e/m_\mu)^4\) or \((m_e/m_\tau)^4\) and thus we can ignore it. Now taking into account only the \(Z\) exchange contribution we will get numerically small \(a \simeq -\frac{1}{2} + 2 \sin^2 \theta_W = -0.04\) and hence in the subsequent discussion we will deal only with electron neutrinos. Notice also that the effective Lagrangian \((\ref{eq:lagrangian})\) provides an adequate description of processes with \(\omega < m_e\) \([\ref{eq:effective-lagrangian}]\). In this action we now substitute \(F_{\mu\nu} \rightarrow F_{\mu\nu} + f_{\mu\nu}\), where \(f_{\mu\nu}\) is quantized electromagnetic field and \(F_{\mu\nu}\) from now on denotes external field.

Retaining terms bilinear in \(f_{\mu\nu}\) we obtain final effective action

\[
\mathcal{L}_{\text{eff}} = 4 G_F a \frac{\alpha^{3/2}}{\sqrt{2}} \frac{1}{\sqrt{4\pi} 180 m_e^4} N_{\mu\nu} \left[ 5 \left( F_{\mu\nu} f_{\lambda\rho} f^{\lambda\rho} + 2 f_{\mu\nu} f^{\lambda\rho} F_{\lambda\rho}\right) - 14 \left( F_{\nu\lambda} f^{\lambda\rho} f_{\mu\nu} + F^{\lambda\rho} f_{\nu\lambda} f_{\mu\nu} + F_{\mu\rho} f^{\lambda\rho} f_{\nu\lambda}\right) \right]
\]

Using \((\ref{eq:lagrangian})\) we can find the amplitudes for \(\gamma\gamma \rightarrow \nu\bar{\nu}\) and cross related reactions. The amplitude for \(\gamma\gamma \rightarrow \nu\bar{\nu}\) can be represented in the form

\[
\mathcal{M} = 8 G_F a \frac{\alpha^{3/2}}{\sqrt{2}} \frac{1}{\sqrt{4\pi} 180 m_e^4} \pi(p_1) \gamma_\mu (1 + \gamma_5) \nu(p_2) \mathcal{J}^\mu
\]

where

\[
\mathcal{J}^\mu = 6 (F k_1)^\mu \left[ (k_1 \varepsilon_2) (k_2 \varepsilon_1) - (k_2 k_1) (\varepsilon_2 \varepsilon_1) \right] - 6 k_2^\mu (k_1 F \varepsilon_1) (k_1 \varepsilon_2) - 14 k_2^\mu \left[ (\varepsilon_2 F k_1) (k_2 \varepsilon_1) - (k_2 F k_1) (\varepsilon_2 \varepsilon_1) + (\varepsilon_1 F k_2) (k_1 \varepsilon_2) - (\varepsilon_1 F \varepsilon_2) (k_2 k_1) \right] + 6 \varepsilon_2^\mu (k_1 F \varepsilon_1) (k_1 k_2) + 28 \varepsilon_1^\mu \left[(\varepsilon_2 F k_1) (k_2 k_1) - (k_2 F k_1) (\varepsilon_2 k_1) \right] + (k_1 \leftrightarrow k_2, \varepsilon_1 \leftrightarrow \varepsilon_2)
\]

The amplitudes for \(\gamma\nu \rightarrow \gamma\nu\) and \(\gamma\bar{\nu} \rightarrow \gamma\bar{\bar{\nu}}\) can be easily obtained from \((\ref{eq:lagrangian},\ref{eq:effective-lagrangian})\) with the use of cross symmetry. Then the cross section of process \(\gamma\gamma \rightarrow \nu\bar{\nu}\), averaged over polarizations of incoming photons, is given by

\[
\sigma_B(\gamma\gamma \rightarrow \nu\bar{\nu}) = \frac{1}{3(180)^2} \frac{G_F^2 a^2}{\pi^2 m_e^8} \left[ 1112 (k_1 k_2)^2 \left[ (k_1 F^2 k_1) + (k_2 F^2 k_2) \right] + 1480 (k_1 k_2)^2 \left( k_2 F^2 k_1 \right) - 440 (k_1 k_2) \left( k_2 F k_1 \right)^2 + 392 (k_1 k_2)^3 F_{\mu\nu} F^{\mu\nu} \right]
\]

From \((\ref{eq:effective-lagrangian})\) the cross sections of neutrino-photon interactions in magnetic field \(\sigma_B\) can be easily estimated. If we take, for example, \(k_1 = \omega (1, 1, 0, 0)\) and \(k_2 = \omega (1, -1, 0, 0)\) with \(\overrightarrow{B} = (0, 0, B)\) we will get
\[ \sigma_B = 6.6 \cdot 10^{-51} \left( \frac{\omega}{m_e} \right)^6 \left( \frac{B}{B_c} \right)^2 \text{cm}^2 \]  

(7)

A comparison between (7) and results of [3–6] shows that in external magnetic field neutrino-photon cross sections are enhanced by factor \( \sim (m_W / m_e)^4 (B/B_c)^2 \).

Let us now study how the process \( \gamma\gamma \rightarrow \nu\bar{\nu} \) contribute to stellar energy-loss of a star with strong magnetic field \( B = 10^{12} - 10^{13} \) Gauss (neutron star). The energy loss rate (in ergs/sec cm\(^3\)) in our case is equal to

\[
Q = \frac{1}{(2\pi)^6} \int \frac{2d^3k_1}{e^{\omega_1/T} - 1} \int \frac{2d^3k_2}{e^{\omega_2/T} - 1} \frac{(k_1k_2)}{\omega_1\omega_2} (\omega_1 + \omega_2) \sigma_B (\gamma \gamma \rightarrow \nu \bar{\nu})
\]

(8)

After integration we obtain

\[
Q = \frac{256}{637875} \left[ 973\pi^2 \zeta(5) + 12610 \zeta(7) \right] G_F^2 \alpha^2 a^2 m_e^4 \left( \frac{B}{B_c} \right)^2 T^{13} = 0.4 \cdot 10^{10} T_9^{13} \left( \frac{B}{B_c} \right)^2 \frac{\text{erg}}{\text{s cm}^3}
\]

(9)

where \( T_9 \) is temperature in units of \( 10^9 \) °K. A comparison between Q and energy-loss rates due to other processes, such as pair neutrino and photoneutrino processes (see e.g. [11–13]), shows that, at \( T \geq 10^9 \) °K, Q cannot be neglected in astrophysical considerations.

In addition taking into account the statement of [14] that the family of processes \( \gamma\nu \rightarrow \gamma\gamma\nu \), \( \gamma\gamma \rightarrow \gamma\nu\bar{\nu} \) and \( \nu\bar{\nu} \rightarrow \gamma\gamma\gamma \), whose cross sections are of order \( \sim 10^{-55} (\omega/m_e)^{10} \text{cm}^2 \) [7], could be quite important in astrophysics, we may conclude that processes discussed above may also be of some importance in astrophysics.


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