Laser propagation in a highly magnetized over-dense plasma

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Propagation of right-hand circularly polarized laser into highly magnetized over-dense collisional plasma is studied from basic equations. Plasma with density up to $10^{23}$ cm$^{-3}$ is heated from room temperature to 1 keV in one nanosecond within depth of 1 µm by Nd:YAG laser with moderate irradiance of around $10^{14}$ W cm$^{-2}$.

1 INTRODUCTION

The research on inertial confinement fusion started as early as 1960s. In this process, the energy of incident laser is mainly deposited in the corona region of laser-produced plasma and energy transfer to compressed over-dense deuterium-tritium fuel is achieved through shock waves or secondary particles such as energetic electrons or ions [1][2], which intrinsically decreases the conversion efficiency from laser to compressed fuel. This propagation of electromagnetic field in magnetized plasma has been studied since 1960s [4][5], and mainly in the region of weakly magnetic field and low frequency of radio wave. When magnetic field is extremely high, it is found that laser can propagate into over-dense plasma which was studied in paper [3] where process of collisional heating was not taken into account and the related basic physics is described briefly below.

The propagation of circularly polarized (CP) laser into a highly magnetized plasma with $B > 1$ demonstrates some special features. For left-hand circularly polarized (LHCP) laser, the relative permittivity is written as

$$\varepsilon_{L} = 1 - n/(1 + B)$$ (1)

It is clear that the incident laser will be reflected where $n = 1 + B$ in both weakly ($B < 1$) or highly ($B > 1$) magnetized plasma. On the other hand, for right-hand circularly polarized (RHCP) laser, the relative permittivity

$$\varepsilon_{R} = 1 - n/(1 - B) = 1 + n/(B - 1)$$ (2)

is no longer confined to the regime of $0 < \varepsilon < 1$, it enters a completely new regime of $\varepsilon_{B} > 1$. Here, $n = n_{c}/n_{e}$ is dimensionless plasma density and $B = B_{0}/B_{c}$ is dimensionless magnetic field, with $n_{c} = m_{e}c\omega_{L}^{2}/4\pi\varepsilon_{0}^{2} \approx 1.12 \times 10^{21}\lambda_{L}^{-2}$ in unit of per cubic centimeters, $B_{c} = m_{e}c\omega_{L}/e \approx 1.07 \times 10^{4}\lambda_{L}^{-1}$ in unit of tesla, $\lambda_{L}$ is laser wavelength in unit of micrometer, and $\omega_{L}$ is angular frequency of laser in unit of radian per second. For fundamental wavelength of Nd:YAG laser, $B_{c}$ is around $10^{4}$ tesla.

In recent years, various techniques have been developed to increase the amplitude of magnetic field greatly [6][7][8], such as through superconductor, laser-driven capacity coil, flux compression and et al. For example, microsecond magnetic field of 1200 T is generated by electromagnetic flux-compression [9] and picosecond magnetic field as high as $7 \times 10^{4}$ tesla is generated by podemotive force of intense laser during its interaction with solid target [10]. As a result, laser interaction with magnetized plasma in the region of $B > 1$ might become reality in the not-too-far future. Based on the basic equations of laser plasma interaction, the propagation of RHCP laser in magnetized over-dense collisional plasma will be analytically invesitaged with moderate laser irradiance thus process of collisional absorption is dominant and effect of plasma heating is taken in account, which is a step forward of previous work [3].
2 LASER PROPAGATION IN HIGHLY MAGNETIZED PLASMA

Consider a step-like density profile with vacuum \((z < 0)\) below and over-dense plasma \((z \geq 0)\) above the density step. External magnetic field is applied along \(z+\) direction, propagation direction of CP laser. CP laser is normally incident on plasma from the left at moderate irradiance, which is slightly above the ionization threshold and excludes both pondermotive and relativistic effects. Thus, the governing equations of the laser interactions with magnetized plasma is written as

\[
\begin{align*}
\partial_t \mathbf{u} &= -\mathbf{E} - \mathbf{u} \times \mathbf{B}_z - \mathbf{v}_u - n^{-1} \nabla p \\
\nabla \times \nabla \times \mathbf{E} + \partial_t \mathbf{E} &= \partial_t (n \mathbf{u}) \\
\nabla \cdot \mathbf{E} &= Z n_l - n \\
\partial_t n &= -\nabla \cdot (n \mathbf{u})
\end{align*}
\]

(3)

where \(p = nT\) is dimensionless plasma pressure, \(T = T_{eV}/(m_e c^2)\) is dimensionless plasma temperature with \(T_{eV}\) being plasma temperature in unit of electron volt, \(E = E_L/(m_e c \omega_L/e)\) is dimensionless electric field of laser, \(u = u_e/c\) is dimensionless velocity with \(c\) being velocity of light in vacuum, \(x = k_L x_0\) is dimensionless space and \(t = \omega_L t_0\) is dimensionless time, respectively. The dimensionless electron-ion collisional rate is

\[
\nu = \nu_{ei} / \omega_L \approx 1.72 \ln Z n \lambda_L^{-1} T_{eV}^{3/2}
\]

(4)

where \(\nu_{ei} = 2.91 \times 10^{-6} Z n_{eV} \ln \Lambda T_{eV}^{3/2} s^{-1}\) is electron-ion collisional rate, \(Z\) is number of free electron per atom set to 1 and \(\ln \Lambda\) is Coulomb logarithm set to be 1 here.

As a vector, the electric field in plasma can be written in the form \(\mathbf{E} = \mathbf{E}_t + \mathbf{E}_l\), with transverse components \(\mathbf{E}_t = -\nabla \phi\) and longitudinal component \(\mathbf{E}_l = -\mathbf{E} / \nu\), satisfying \(\nabla \cdot \mathbf{E}_l = 0\) and \(\nabla \times \mathbf{E}_l = 0\), respectively. Consider laser normally incident on the solid target, the transverse mode \(\mathbf{E}_t\) is perpendicular to laser direction, while the longitudinal component \(\mathbf{E}_l\) along laser direction. It is known that the conversion between the transverse and longitudinal modes occurs as soon as the laser frequency is close to the local plasma frequency. But in the present case, the plasma density under consideration is much higher than the critical density, thus the mode conversion (or the resonance) between the transverse and longitudinal modes is not likely to occur. Thus, in the absence of mode conversion, the starting equations can be written as

\[
\begin{align*}
\partial_t (\mathbf{u}_t - \mathbf{a}) + \mathbf{v}_t \cdot \mathbf{u}_t + \mathbf{u}_t \times \mathbf{B}_z &= -\partial_t \mathbf{u}_t - \mathbf{E}_L - \mathbf{v}_t - \nabla p / n = 0 \\
\nabla^2 \mathbf{a} - \partial_t \mathbf{a} - n \mathbf{u}_t &= n \mathbf{u}_t - \partial_t \mathbf{E}_L = 0
\end{align*}
\]

(5)

3 TRANVERSE COMPONENT

Because fusion laser usually has flat-top both spatially and temporally, the dimensionless potential vector of laser can be simplified as below within the temporal and spatial range of laser pulse

\[
\mathbf{a} = a_0 e^{i(kx - \sigma t)} (\tilde{x} - i \sigma \tilde{y})
\]

(6)

where \(k_\sigma = \sqrt{\varepsilon_\sigma} = k_\sigma + i \kappa_\sigma\) is the dimensionless complex wave vector, \(\varepsilon_\sigma = \varepsilon_\sigma + i \kappa_\sigma\) is relative permittivity of plasma, \(\sigma = -1\) for RHCP and \(\sigma = 1\) for LHCP laser. With \(\mathbf{u}_t = u_t (\tilde{x} - i \sigma \tilde{y})\), the force equation for transverse movement of electron fluid and transverse velocity of electron in CP laser become

\[
\begin{align*}
\partial_t \mathbf{u}_t &= \partial_t \mathbf{a} - \mathbf{u}_t \times \mathbf{B}_z - \mathbf{v}_t \\
\mathbf{u}_t &= \frac{\mathbf{a}}{1 + \sigma B_z + i \nu}
\end{align*}
\]

(7)

(8)

The relative permittivity of magnetized plasma is obtained as below, same as in paper [3].

\[
\varepsilon_\sigma = 1 - \frac{n}{1 + \sigma B_z + i \nu}
\]

(9)
As laser is incident into vacuum-plasma interface, the reflection and transmission coefficients of laser electric field are given by Fresnel equations [12]

\[
r_\sigma = \frac{(1 - k_\sigma)}{(1 + k_\sigma)} \quad t_\sigma = 2/(1 + k_\sigma)
\]

As laser propagates inside plasma, it deposits energy and obeys equation of energy conservation [13]

\[
\partial_t(w_P + w_L) = -\nabla \cdot S_t - Im(J_t \cdot E_t)
\]

Here, \( w_P = \frac{3}{2} \cdot nT \) is dimensionless energy density of plasma, assuming that plasma behaves like ideal gas [14]. \( w_L = (\varepsilon_r E^2 + B^2)/2 = (1 + \varepsilon_r)a^2/2 \) is dimensionless energy density of laser in plasma. \( \nabla \cdot S_t = \partial_z(a_1^* a_1) \) is laser absorption rate in plasma where \( a_1 = t_\sigma \cdot a \) is dimensionless potential vector of transmitted laser. Imaginary part of \( -J_t \cdot E_t = -nu_t \cdot a_1 \) is the plasma heating rate due to transverse electron current. After required parameters are inserted from above other equations, equ. 11 is the core equation to solve the problem of our interest, RHCP laser propagation into highly magnetized over-dense collisional plasma. First, the plasma temperature at vacuum-plasma interface, \( T(0, t) \), is obtained and the transmission coefficients of laser into plasma is calculated. Then equ.11 is used again inside plasma to obtain the temperature distribution of plasma, \( T(z, t) \). Thus the problem is fully solved.

4 RESULTS AND DISCUSSIONS

The conditions of interest are as below: the irradiance of incident RHCP laser is around \( 10^{14} \text{ Wcm}^{-2} \) (a=0.01) with wavelength of \( \lambda_L = 1.064 \mu m \), the plasma density is around \( 10^{23} \text{ cm}^{-3} \) (n = 100), the initial temperature of plasma is \( T_0 = 300 \text{ K} \) and the amplitude of magnetic field is around \( 10^5 \text{ Tesla} \) (B=10). Thus, the propagation of laser and temperature distribution of plasma are shown in fig. 1a and 1b, respectively, at t = 1 ps and 1 ns after the incidence of RHCP laser onto vacuum-plasma interface. Poynting vector of laser inside plasma normalized by that of incident laser and dimensionless energy density of plasma are shown in fig. 2a and 2b at t = 1 ps, 10 ps, 100 ps and 1ns, respectively. It is found that there exist a steep temperature jump (named heating front here) moving slowly into plasma. In front of the heating front, both the amplitude of laser and temperature of plasma are very small and plasma temperature decays slowly to that of background plasma. Behind the front, there exists a "uniform" temperature platform where plasma temperature is very high and laser absorption rate is very small. At t = 1 ns, the energy density of plasma is around \( 3.4 \times 10^7 \text{ J/cm}^3 \) (or \( 1.4 \times 10^7 \text{ J/g} \) for specific energy) and the conversion efficiency from laser to plasma is around 3\%, which is
Figure 2: (a) Poynting vector of RHCP laser in plasma normalized to that of incident laser v.s position and (b) Energy density of plasma v.s position at t = 1 ps, 10 ps, 100 ps and 1 ns (from left to right). The other parameter are the same as in fig.1.

calculated based on the the areal energy density of incident laser of around $10^5$ J/cm$^2$ and the areal energy density of plasma of around $3 \times 10^3$ J/cm$^2$.

To explain the phenomena of steep heating front and uniform temperature platform, the relationship between imaginary part of dimensionless wave vector of RHCP laser in plasma with plasma temperature is studied for both non-magnetized and longitudinally magnetized collisional plasma based on equ. 9 and shown in fig. 3a. In magnetized plasma, $k_i$ is peaked at certain temperature of $T_c \approx 10$ eV and very small at both low and high temperature. Thus, when RHCP laser is incident into plasma with $T \ll T_c$, laser propagates with small absorption rate, as shown at t = 1 ps in fig.1. Then as the plasma temperature increases, laser is greatly absorbed which increases plasma temperature very fast and forms a steep heating front. At last, as the temperature of plasma is far above $T_c$, the absorption rate becomes very small again and laser propagates ”freely” inside the uniform temperature platform.

Figure 3: (a) Imaginary part of wave vector $k_i$ of RHCP laser in plasma v.s plasma temperature $T(eV)$ with n = 100 for B=0 (black) and B = 10 (red). (b) Averaged temperature in plasma platform (black) and position of heating front (red) v.s plasma density at t = 1 ns with B = 10, a = 0.01, $\lambda_L = 1.064 \mu m$ and $T_0 = 300$ K.

The above phenomenon is investigated by 2D-3V (two dimensional in space and three dimensional in velocity) particle-in-cell (PIC) simulation with LAPINS code to generate warm dense matter [15], where processes of dynamic ionization and conduction are included. In this paper, plasma is aluminum-based with initial temperature of 0.01 eV, laser irradiance is $10^{15} Wcm^{-2}$ at wavelength of 1 $\mu m$, and dimensionless magnetic field is B = 1.2. Our model is also applied to the same condition, where the
ionization level is set to be Z =10. Compared to fig. 2(f) obtained by PIC simulation [15], similar temperature level is obtained but the extension is smaller and the heating front in PIC simulation is not as steep as that in our model. Some other processes are also neglected in our model and need to be taken into account in more "realistic" situations, for example, plasma hydrodynamics and pressure of magnetic field.

With our model, the relationship between temporal evolution of temperature and extension of platform with parameters such as laser irradiance and wavelength, plasma density and initial temperature, magnetic field can be easily obtained. For example in case of fig.1 and fig.2, it is found out that $T_f(keV) \approx 1.42 \cdot t_0^{0.4}$ and $z_f(\mu m) \approx 0.96 + 0.19 \log_{10}(t_\text{ns})$. The velocity of heating front is calculated to be $v_f(\mu m/\text{ns}) \approx 0.087 \cdot t_0^{-1}$ and the energy absorption efficiency from laser to plasma is around 3% at 1 ns. When only one of the parameters varies, the change of temperature and extension of platform is described below.

- At higher plasma density, temperature of platform increases while extension decreases, as shown in fig.3b at $t = 1$ ns.
- At higher laser irradiance, both temperature and extension of platform increase.
- At stronger magnetic field or larger wavelength, temperature of platform decreases while extension increases. Especially, when wavelength is $\lambda_L = 10.6 \mu m$ for CO$_2$ laser, it is found that $T_f(keV) \approx 0.6 \cdot t_0^{0.4}$, $z_f(\mu m) \approx 2.2 + 0.6 \log_{10}(t_\text{ns})$ and absorption efficiency is around 3% at 1 ns.
- At higher initial temperature of plasma, temperature of platform does not change much but extension of platform increases. Especially when initial plasma temperature is much higher than $T_c$, $T_0 \gg T_c$, the extension increases a lot. For example, when $T_0 = 100$ eV, it is found that $T_f(keV) \approx 1.45 \cdot t_0^{0.4}$, $z_f(\mu m) \approx 22 + 7.4 \log_{10}(t_\text{ns})$ and energy absorption efficiency is as high as 60% at 1 ns.

Thus, the temperature of plasma is in range of around 100 – 1000 eV with extension of a couple of micrometers within duration of 0.01 – 1 ns. Enlargement of the extension could be succeeded with pre-heated plasma (i.e. pre-compressed target shown in fig.1 in paper [16]) or infra-red laser (i.e. CO$_2$ laser in BESTIA project [17]). By the way, the required amplitude of magnetic field is as "low" as 1000 tesla when CO$_2$ laser is used. The specific energy of plasma is around $1.4 \cdot 10^7 J/g$ at 1 ns in fig.2b, which is comparable to specific energy of $6.4 \cdot 10^6 J/g$ generated by accelerated heavy ions in Facility for Antiprotons and Ion Research [18]. Thus, this process might be used in the generation of warm dense matter [15] or high energy density plasma. In the field of inertial confinement fusion, energy gain is one of the critical issues. The energy density of magnetic field in this paper is around $4 \cdot 10^9 J/cm^3$, thus the total energy is around 17 kJ in a compressed fuel with radius of 0.1 millimeter or around 17 MJ in sphere with radius of 1 millimeter. However, if only a narrow plasma channel is magnetized to guide RHCP laser into compressed fuel, the required magnetic energy will be decreased greatly, i.e. around 5 kJ in volume with radius of 20 micrometer and length of 1 millimeter. Such field might be generated by compression of initially much lower magnetic field. And this approach is, in some way, similar to that of fast ignition [2] or Magnetized Liner Inertial Fusion [19].

5 CONCLUSION

From the basic equations, the process of heating and propogation of RHCP into highly magnetized overdense collisional plasma is investigated, and phenomena of steep heating front and uniform temperature platform are found and explained. This result is a step forward of previous work [3] and roughly mimicks the result by PIC simulation [15]. Scaling relationship is also obtained, which might be able to guide the related potential applications in the future.

6 Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.
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