Towards Robust DNNs: An Taylor Expansion-Based Method for Generating Powerful Adversarial Examples

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Abstract: Although deep neural networks (DNNs) have achieved successful applications in many fields, they are vulnerable to adversarial examples. Adversarial training is one of the most effective methods to improve the robustness of DNNs, and it is generally considered as a minimax point problem that minimizes the loss function and maximizes the perturbation. Therefore, powerful adversarial examples can effectively simulate perturbation maximization to solve the minimax problem. In paper, a novel method was proposed to generate more powerful adversarial examples for robust adversarial training. The main idea is to approximates the output of DNNs in the input neighborhood by using the Taylor expansion, and then optimizes it by using the Lagrange multiplier method to generate adversarial examples. The experiment results show it can effectively improve the robust of DNNs trained with these powerful adversarial examples.

Keywords: DNNs; Adversarial examples; Taylor expansion; Lagrange multiplier method; dual problem; Gauss-Newton Method

1 Introduction

DNNs have achieved successful applications in many fields such as bioinformatics [1,2], speech recognition [3,4], and computer vision [5,6]. However, recent work has found that DNNs are vulnerable to adversarial examples. Szeig et al. [7] first noticed the existence of adversarial examples of DNNs in the image classification domain: a natural image added with a tiny perturbation can deceive the DNN to produce an incorrect prediction. These adversarial examples often have good concealment and harmfulness, as it is hard for human vision to detect them. The existence of adversarial examples caused widespread concern. Researchers have explored DNN properties and proposed many typical attack methods such as FGSM [8], JSMA [9], Deepfool[10], C&W [11], PGD [12], M-DI2-FGSM[13] etc. to generate adversarial examples.

The generation of adversarial examples can usually be modeled as the optimization problem of the loss function with respect to the input. Due to the high nonlinearity of DNNs, it is very difficult to solve this optimization problem. In this paper, a method is proposed using Taylor
expansion to approximate the complicated input-output mapping relationship of DNNs in a small neighborhood of the input example (using $L_p$ norms constraints) to replace the nonlinear part of DNNs. After that, we construct a dual problem with Lagrange multiplier method. Since the output of the quadratic Taylor expansion can be approximated to the predicted output of DNNs in a tiny neighbourhood of the input, obtaining adversarial examples directly on DNNs is equivalent to obtain it on a similar quadratic function from the view of classification. The advantages of our method are: (1) It can effectively avoid falling into local optimizations; (2) The dual problem based on the Lagrange multiplier method is capable of exactly calculate the extreme value; (3) It can be used to estimate the robust upper bound of DNNs accurately. Therefore, compared with other multi-step attack methods, our method has advantages on both calculation accuracy and speed.

Since our method can avoid falling into local optimizations, compared with one-step attack methods, it is closer to the optimal solution and the generated adversarial examples are more powerful. The experimental results show that adversarial examples generated by our method have high transferability and good concealment. As a result, adversarial training with our adversarial examples can effectively improve the robustness of DNNs.

The main innovations and contributions of our work can be summarized as follows:

(1) We apply Taylor theorem in small field ($L_p$ norms constraints) of input, generating a quadratic Taylor expansion which has similar output to DNNs. Compared with all previous methods, this method can avoid being trapped by local optimizations, and get more effective adversarial examples.

(2) We systematically evaluated the selection of objective functions, and the experiments show that our method can generate powerful adversarial examples regardless of the defense of gradient mask for DNNs.

(3) Experimental evaluation was conducted on MNIST and cifar-10 dataset. It is proved by the reliable experiment that our adversarial examples have high transferability and are capable of improving the robustness of DNNs through adversarial training. To make it easier for its researchers to use our work to evaluate the robustness of other defense systems, The complete code is available at https://github.com/zhangximin2019/zhangximin

2 Background

This section provides background on our work, covering the fundamentals of DNNs and the most representative attack methods available nowadays.

2.1 Deep Neural Networks and Notation

DNNs can generally be represented as a multidimensional function $F: \mathcal{X} \mapsto \mathcal{Y}$, $X \in \mathcal{X}$ stands for $d$-dimensional input variable, $Y \in \mathcal{Y}$ is a $m$-dimensional probability vector which stands for Confidence of $m$ classes. An $N$-layer DNNs receives an input $X$ and produces the corresponding output as follows:
\[ F(X) = F^{(N)}(\ldots F^{(2)}(F^{(1)}(X))) \]

\( F^{(i)} \) represents the computational output of layer \( i \) of DNNs. These layers can be convolutional, pooled, or other forms of neural network layers. The last layer of DNNs is usually the Softmax layer, defined as \( F^{(N)}(Z) = \text{Softmax}(Z) = \exp(z_i) / \sum_{i=1}^{m} \exp(z_i) \), where \( Z = F^{(N-1)}(\cdot) \) is the output vector of the previous layer (also known as the last hidden layer). The final prediction label is obtained by \( y = \arg \max_{i=1..m} F(X)_i \), where \( F(X) = \text{Softmax}(Z) \).

2.2 Adversarial Examples of DNNs

Szegedy et al. [7] first found the existence of adversarial examples in DNNs. More formally, in the space \( \mathbb{R}^{m \times n} \), we think of an image \( X \) which is \( m \times n \) as a point. Our goal is to find a point \( X' = X + \delta, \|\delta\|_p < C \) which is close enough to \( X \). Such point \( X' \) is called the adversarial example. This \( X' \) and \( X \) belong to the same category from the perspective of human eye, but the subtle perturbation \( \delta \) deceives DNNs into judging it as a different class from \( X \), i.e.

\[ F(X') = F(X + \delta) = y' \quad \text{s.t.} \quad y' \neq y \]

In general, the perturbation \( \delta \) is constrained by the \( L_p \) norm \( (p \in 0, 2, \infty) \), \( \|X' - X\|_p \leq C \).

2.3 Threat Models in Deep Learning

There are a number of methods to generate adversarial examples, but they are all have constraints. Since the capability of adversarial examples or the robustness of the attack is based upon what adversary is allowed to do. Without such limitation, adversary can replace the given image with any image, violating the definition of adversarial examples. To this end, we define these assumptions as threat models, which typically include attack targets and attack capabilities.

(1) Adversarial Goals

Adversarial goals in threat models can be defined as a specific formula that needs to be detected and defended. In DNNs, the classification of adversarial Goals is helpful for us to define this specific formula. Therefore, in threat models, the classification of adversarial goals is very important. In this paper, adversarial goals are divided into two categories: (a) Untargeted attack: misclassify adversarial examples to any incorrect class; (b) Targeted attack: misclassify adversarial examples to specified incorrect class.

In this paper, both untargeted attack and targeted attack are based on the change of confidence, and the method in this paper is the first attack method to reduce confidence through optimization method from the mathematical perspective to generate adversarial examples.

(2) Adversarial Capabilities

Adversarial examples can also be divided into white box attack and black box attack according to how much information adversary has about the target DNNs. The so-called white box
attack means that adversary knows everything about DNNs, including training data, activation function, topology structure, weight coefficient and so on. The black box attack assumes that adversary cannot obtain the internal information of the target DNNs and can only obtain the output of the model, including labels and confidence.

Because the gradient information of target DNNs needs to be mastered, the method in this paper belongs to white box attack. However, since adversarial examples generated by the method in this paper is highly transferable, it is easy to build the agent DNNs locally through the method in the paper [18] to successfully realize the black box attack.

2.4 Attack algorithms

We selected some typical gradient-based methods to compare with the methods proposed in this paper. The existing typical methods include FGSM, JSMA, Deepfool, C&W, PGD, M-DI$^2$-FGSM, etc.

(1) FGSM

Goodfellow et al.[8] presents a method for rapidly generating adversarial examples under $L_\infty$ distance called FGSM (Fast Gradient Sign Method): 

$$X' = X + \varepsilon \cdot \text{sign}(\nabla_{X} J(F(X; \theta), y))$$  \hspace{1cm} (3)

where $J$ is the loss function, $\varepsilon$ is the perturbation limit on the sign gradient direction $\text{sign}(\cdot)$. FGSM has the advantages of low computational complexity and the ability to generate a large number of adversarial examples in a short time.

(2) JSMA

Papernot et al.[9] proposed a targeted attack method under $L_0$ distance. According to the adversarial saliency maps, the input components were ranked in descending order, and the components with strong adversarial saliency were selected to add perturbation $\delta^t$. For the target class $t$, adversarial saliency map of component $S(X, t)[i]$ is defined as :

$$S(X, t)[i] = \begin{cases} 
0, & \text{if } \frac{\partial F_i(X)}{\partial x_i} < 0 \text{or } \sum_{j \neq i} \frac{\partial F_j(X)}{\partial x_i} > 0 \\
\left( \frac{\partial F_i(X)}{\partial x_i} \right) \sum_{j \neq i} \frac{\partial F_j(X)}{\partial x_i}, & \text{otherwise}
\end{cases}$$ \hspace{1cm} (4)

where $J_F = \left[ \frac{\partial F_j(X)}{\partial x_i} \right]_i$ represents DNNs Jacobian matrix.

(3) Deepfool

Moosav-dezfooli et al.[10] proposed the Deepfool method, which employs linear approximation for gradient iterative attack. For a binary classifier, the following iterative process can be used to describe:
\[ \delta_i = -\frac{f(X'_i)}{\|\nabla f(X'_i)\|_2} \nabla f(X'_i) \]
\[ X'_{i+1} = X'_i + \delta_i \]  
\[ \delta = \sum_i \delta_i \]  

Here, \( X'_i \) is the adversarial examples for the \( i \)-th iteration, \( X'_0 = X \). \( f(X'_i)/\|\nabla f(X'_i)\|_2 \) is the estimated distance between \( X'_i \) and the decision boundary \( f(X'_i) \). \( -f(X'_i)/\|\nabla f(X'_i)\|_2 \) is the gradient direction of \( X'_i \) toward the decision boundary.

**4) C&W**

Carlini and Wagner proposed a targeted iterative attack method based on gradient descent. Based on their further studies [11,19,20], C&W attacks are effective against most existing defenses. They modeled the process of generating the adversarial examples as the following optimization problem, minimizing the disturbance while maximizing the model classification error:

\[ \min_{\delta} \|\delta\|_p + c \cdot g(X + \delta) \]  

If and only if \( F(X') = y' \), \( g(X') \geq 0 \). Through experimental evaluation, they found that the most effective function \( g \) was:

\[ g(X') = \max(\max_{\omega \neq y'}(\text{Softmax}(X')_\omega) - \text{Softmax}(X')_y, -k) \]  

Where \( k \) is the constant that controls confidence.

**5) PGD**

Aleksander et al. [12] explained the process of generating adversarial examples as a simple one-step solution to solve the internal maximization problem of saddle point problem. Based on this, they proposed a derivative method of FGSM, called PGD. The essence of this method is to project gradient descent on the loss function:

\[ X' = \prod_{X} (X + \epsilon \cdot \text{sign}(\nabla_X J(f(X; \theta), y))) \]  

**6) M-DI²-FGSM**

Momentum and diverse inputs are two completely different ways to alleviate the overfitting phenomenon. Xie et al. [13] combined them naturally to form a much stronger attack, i.e., Momentum Diverse Inputs Iterative Fast Gradient Sign Method (M-DI²-FGSM):

\[ g_{n+1} = \mu g_n + \frac{\nabla_X L(T(X_n^{\text{adv}}; p), y^{\text{true}}; \theta)}{\|\nabla_X L(T(X_n^{\text{adv}}; p), y^{\text{true}}; \theta)\|_1} \]  

\[ X_{n+1} = \text{Clip}_X \{X_n + \alpha \cdot \text{sign}(\nabla_X L(X_n; y^{\text{true}}; \theta))\} \]
3 Our approach

We present an algorithm to find appropriate perturbation $\delta$ under $L_p$ distance. So that the confidence of adversarial examples $X'$ being classified as a correct class is minimized in untargeted attack, or the confidence of adversarial examples $X'$ being classified as a target class is maximized in a targeted attack.

For untargeted attack, it is to reduce the confidence of the correct output class; for targeted attack, it is to improve the confidence of the target class. Therefore, the core of the proposed method is constructing a quadratic Taylor expansion to approximate the complicated input output projection of DNNs in a small neighbourhood of the input example by using gradient information. Then, the Lagrange multiplier method is used to construct the dual function and calculate the extreme value to obtain the perturbation $\delta$.

The first step of our method is to calculate the gradient information of DNNs. Considering that the output of the last hidden layer and the Softmax layer of DNNs can both provide gradient information of DNNs, while the output of the Softmax layer is the output of the last hidden layer after normalization. The normalized Softmax layer will smooth the gradient, and the author of [9] insist on using the last hidden layer for the calculation. This is justified by the extreme variations introduced by the logistic regression computed between these two layers to ensure probabilities sum up to 1, leading to extreme derivative values. They believe this reduces the quality of information on how the neurons are activated by different inputs. In this paper, the gradient was calculated and tested on the last hidden layer and Softmax layer respectively to analyze the difference. The output of the last hidden layer is called Logist vector, and the output of the Softmax layer is called confidence vector in our paper.

Considering that adversarial examples are obtained in a little neighbourhood, where the output of DNNs may still be highly nonlinear, it is prone to fall into the local optimal value even if the multi-step iteration is adopted. For this reason, the output of DNNs at the last hidden layer or Softmax layer is derived by making use of Taylor series expression. The nonconvex and nonlinear part of DNNs is replaced by a simple quadratic function. Then, the Lagrange multiplier method is used to construct the dual function and add the constraint to form an optimization formula that is easy to calculate. In this way, effective perturbation values can be obtained to generate adversarial example $X'$.

3.1 Problem Formulation

This section will choose the objective function and establish the optimization model according to the attack target in 2.2. Assume that adversarial examples are generated on the last
hidden layer. The output of each neuron in the last hidden layer of DNNs is the logist value assigned to the class that the neuron represents, and the predict label of \( X \) is \( y = \arg \max_{i=1,...,m} (z_i) \). That is, the greater the value of \( z_i \), the more likely it is to be classified as the \( i \)-th class, and vice versa. Since the purpose of adversarial examples \( X' \) is to add a small amount of perturbation leading to the misclassification of DNNs. Therefore, assuming \((X, y_j)\), our goal is to find a small perturbation \( \delta \), so that the logist value \( z_j(X + \delta) \) is as small as possible, which means that the lower the probability of \( X + \delta \) being classified as the \( j \)-th class, the higher the probability that DNNs produces misclassification.

Assume that the output of the Softmax layer is used as a reference to generate adversarial examples. \( Y \) coming out from the Softmax layer represents the normalized confidence of different classes. The larger the value of \( Y_i \), the more likely it is to be classified as the \( i \)-th class, and vice versa. The difference from the last hidden layer is that the output of the Softmax layer is normalized, which means that the drop in \( f_j(X) \) corresponds to the rise in \( \sum_{i \neq j} f_i(X) \). Intuitively, reducing the confidence of the correct class at the Softmax layer is more helpful in generating adversarial examples. Our results show that normalization does not make a big difference to our approach. At this point, our method is more adaptable than the first-order gradient information based attack represented by JSMA.

We modeled the process of calculating optimal perturbation \( \delta \) as an optimization problem with confidence as the objective function. According to different layers of DNNs and different attack targets, three different objective functions are defined as follows:

- \( T_1 = z_j(X + \delta) \) and \( T_2 = f_j(X + \delta) \) represent the output of the correct class of \( X + \delta \) in the last hidden layer and the Softmax layer, respectively. If either of these two expressions is reduced to the minimum, the adversarial example can be obtained.
- \( T_2 = z_i(X + \delta) \) and \( T_5 = f_i(X + \delta) \) respectively represent the output of target class of \( X + \delta \) in the last hidden layer and the Softmax layer. If either of these two expressions is raised to the highest level, targeted adversarial example will be generated.
- \( T_3 = z_i(X + \delta) - \max_{i \neq j} (z_i(X + \delta)) \) and \( T_6 = f_i(X + \delta) - \max_{i \neq j} (f_i(X + \delta)) \) represent the difference values of target class output and maximum class output on the last hidden layer and Softmax layer respectively. When the difference value reaches the maximum, the probability of generating targeted adversarial example is the maximum.

(1) Untargeted Attack

Our goal is to find \( \delta \) in the constraint that minimizes \( f_j(X + \delta) \). Therefore, according to the expression of adversarial examples in [7], we describe the problem of finding perturbation \( \delta \)
for $X$ and constructing adversarial examples as:

$$\begin{align*}
\text{minimize} & \ T_i \text{ or } T_d \\
\text{s.t.} & \ F(X+\delta) \neq j, \ X + \delta \in [0,1]^n, \ \|\delta\|_p < C
\end{align*}$$

(11)

(2) Targeted Attack

Suppose $t$ is the target class, then the goal is to find the $\delta$ that maximizes $f_i(X+\delta)$ within the constraint. We model this process as:

$$\begin{align*}
\text{maximize} & \ T_2 \text{ or } T_3 \text{ or } T_5 \text{ or } T_6 \\
\text{s.t.} & \ F(X+\delta) = t, \ X + \delta \in [0,1]^n, \ \|\delta\|_p < C
\end{align*}$$

(12)

This paper studies a special input case, that is, the benign sample $X$ is not a meaningful natural picture, it may be a pure black or white picture, or it may be a meaningless messy code. Our adversarial examples can trick DNNs and classify adversarial examples into our target class $t$. We add a condition $F(X) \neq t$ to equation (12).

3.2 Generate adversarial examples based on the Taylor expansion

For DNNs, we propose a novel adversarial example generation algorithm, and prove its effectiveness in later experiments. Note that both the objective function $z_i$ and $f_i$ in section 3.1 can use the same method to solve the gradient information. Therefore, for the convenience of narration, $z_i$ and $f_i$ are uniformly denoted as $F$ in this section.

We use quadratic Taylor expansion to approximate the nonlinear part of $F$ in the neighbourhood of $X$, transforming the constrained nonlinear optimization problem into a constrained linear optimization problem, and then use Lagrange multiplier method to construct the dual problem to find the optimal solution within the $L_p$ norms constraints. This process not only reduces the difficulty of solving the optimization problem, but also improves the solution accuracy.

(1) Compute the gradient matrix $\nabla F(Y)$ and Hessian matrix $\nabla^2 F(Y)$ for the given sample $X$.

$$\begin{align*}
\nabla F_j(X) &= \left[ \frac{\partial F_j(X)}{\partial X_i} \right]_{n \times 1} \\
\nabla^2 F_j(X) &= \left[ \frac{\partial^2 F_j(X)}{\partial X_i \partial X_j} \right]_{n \times m}
\end{align*}$$

(13) (14)

(2) Use Taylor expansion to approximate $F_j$ in the neighbourhood $U(X,\delta)$ of $X$.

$$
F_j(X') = F_j(X + \delta) \doteq T(\delta) = F_j(X) + \nabla F_j(X) \delta + \frac{1}{2} \delta^T \nabla^2 F_j(X) \delta
$$

(15)

$X'$ is the moving point in the neighbourhood. $F_j(X')$ is the logist or Softmax value of $X'$
that is classified as $Y$. The lower the value, the lower the probability of being judged as the $i$-th class, the higher the probability of being misclassified.

(3) Calculate $\delta$ by using the Lagrange multiplier method: In equation (3), $\delta$ is the only unknown. Therefore, we transform the problem of generating adversarial examples into a nonlinear optimization problem under inequality constraints:

$$\min T(\delta) \quad s.t. \|\delta\|^2_p \leq C$$

We construct the Lagrange function:

$$L(\delta, \lambda) = T(\delta) + \lambda(\|\delta\|^2_p - C)$$

$$= F(X) + \nabla F(X)^T \delta + \frac{1}{2} \delta^T \nabla^2 F(X) \delta + \lambda(\|\delta\|^2_p - C)$$  \hspace{1cm} (17)

The nonlinear optimization problem with inequality constraints is transformed into an unconstrained optimization problem. For better calculation, the original problem (16) is transformed into a dual problem:

$$\min -g(\lambda) \quad s.t. \lambda \geq 0$$  \hspace{1cm} (18)

Where the dual function $g(\lambda) = \inf L(\delta, \lambda)$. According to the weak duality property, the optimal value $d^*$ of equation (18) is the optimal lower bound of the original problem (16), that is, the convex optimization problem approximates the original problem. The optimal solution of equation (18) must satisfy the KKT condition as follows:

$$\begin{cases}
\nabla \lambda - g(\lambda) = 0 \\
\lambda \geq 0
\end{cases}$$  \hspace{1cm} (19)

Assume that when $\lambda = \lambda^*$, $-g(\lambda^*)$ can get the minimum; when $\delta = \delta^*$, $T(\delta^*)$ be able to get the minimum. According to the principle of weak duality property, the optimal value of the original problem is not less than the optimal value of the dual problem, i.e. $g(\lambda^*) \leq T(\delta^*)$. If the original function is a convex function and satisfies the Slater condition, then $g(\lambda^*) = T(\delta^*)$. However, due to the high nonlinearity of DNNs, $\nabla^2 F(X)$ is difficult to be proved as a positive definite matrix. So we can think of $(\delta^*, \lambda^*)$ as an approximately optimal solution to $L(\delta, \lambda)$.

Figure 1 shows the relationship between DNNs and dual functions.
Figure 1: The relationship between DNNs and dual functions. This picture plots the process of generating adversarial example on MNIST and CIFAR10 dataset. For the target function $T_1$, with the constant $C$ increasing, the loss value of DNNs and the dual function changes. We can see that the changes of DNNs and dual functions tend to be consistent.

If the obtained $\delta$ satisfies the condition $\exists t \in I, F_j(X + \delta) > F_j(X) + \delta^T \nabla F_j(X)$, then $\delta$ is the best perturbation to generate adversarial example. In this way, we obtain the optimal solution $\delta$ for the optimization problem with inequality constraints through the Lagrange multiplier method, thus generating the adversarial example $X'$. The above method can be easily extended to all non-cyclic DNNs. The only requirement is that the activation function is differentiable, which has been satisfied by the characteristics of BP algorithm. The whole process is shown in Algorithm 1.

Algorithm 1 Generate adversarial examples based on Taylor expansion

1. $X'$ is a benign example

**Input:** $X, C$

**Output:** $X'$

1: $F_y(X) \leftarrow \left( \frac{\partial F_y(X)}{\partial X_i} \right)_{i=1}^m$, $\nabla^2 F_y(X) \leftarrow \left( \frac{\partial^2 F_y(X)}{\partial X_i \partial X_j} \right)_{i,j}$

2: while $y_{X+\delta} = y_X$ and $\|\delta\|_p \leq C$

3: $T(\delta) = F_y(X) + \nabla F_y(X)^T \delta + \frac{1}{2} \delta^T \nabla^2 F_y(X) \delta$

   //Use Taylor expansion to approximate $F_y$ in the neighbourhood $U(X, \delta)$ of $X$

4: $L(\delta, \lambda) = F_y(X) + \nabla F_y(X)^T \delta + \frac{1}{2} \delta^T \nabla^2 F_y(X) \delta + \lambda(\|\delta\|_p^q - C)$

   //Construct the Lagrange function

5: $\min -\inf L(\delta, \lambda) \quad s.t. \quad \lambda \geq 0$

   //Construct the dual problem

6: $(\delta^*, \lambda^*) \leftarrow \nabla_y L(\delta, \lambda) = 0, \lambda(\|\delta\|_p^q - C) = 0, \|\delta\|_p^q - C \leq 0, \lambda \geq 0$

   //The KKT condition is used to find the optimal solution

7: $y_{X+\delta^*} \leftarrow \arg \max_{i=1...m} F_y(X + \delta^*)_i$

8: $C \leftarrow C + 0.01$

9: end while

10: return $X + \delta^*$

The selection of $C$ is also involved in generating adversarial examples through equation (18), which is also a very important step. $C$ is used to constrain $\delta$. If the value of $C$ is too
large, the success rate of generating an adversarial example is higher, but the invisibility of the adversarial example will be weakened, and vice versa. Therefore, the choice of $C$ is crucial. Empirically, the most suitable $C$ is the minimum one which satisfies $\exists t \in I, F_i(X + \delta) > F_i(X + \delta)$ after solving equation (14). We verify this by running our $T_i$ from $C = 0$ on the MNIST and CIFAR10 dataset separately. We plot lines in Figure 2.

![Figure 2: Change in constant $C$. We plot the relationship between constant $C$ and the change of cross entropy loss value of DNNs, when the objective function $T_i$ generates adversarial examples on MNIST and CIFAR10 dataset respectively.](image)

3.3 Generate adversarial examples based on the Gauss-Newton Method

Because of the use of Taylor expansion formula, the method in 3.2 can generate effective and highly transferable adversarial examples, but the shortcomings of this method are also very obvious. Since the Taylor expansion formula involves the calculation of the Hessian matrix, and the second-order term in the Hessian matrix is usually difficult to calculate or requires a large amount of calculation, it is also not advisable to use the secant line approximation of the whole Hessian. Therefore, we can also use gauss-newton method to simplify the calculation.

Gauss-Newton Method is a specialized method for minimizing the least squares cost $(1/2)\|F_i(X + \delta)\|^2$. Given a point $X$, the pure form of the Gauss-Newton Method is based on linearizing $F_i(X + \delta)$ to obtain

$$R(X + \delta, X + \delta^k) = F_i(X) + \nabla F_i(X)^T(\delta - \delta^k)$$

(20)

and then minimizing the norm of the linearized function $R$:

$$\delta^{k+1} = \arg \min_{\delta \in \mathbb{R}^n} (1/2)\|R(X + \delta, X + \delta^k)\|^2$$

$$= \arg \min_{\delta \in \mathbb{R}^n} (1/2)\|F_i(X)\|^2 + 2(\delta - \delta^k)^T\nabla F_i(X)F_i(X)$$

$$+ (\delta - \delta^k)^T\nabla F_i(X)\nabla F_i(X)^T(\delta - \delta^k)$$

(21)

Assuming that the matrix $\nabla F_i(X)\nabla F_i(X)^T$ is invertible, the above quadratic minimization
\( \delta^{k+1} = \delta^k - (\nabla F_j(X)\nabla F_j(X)^\top)^{-1}\nabla F_j(X)F_j(X) \) \hspace{1cm} (22)

Notice that because of the high nonlinearity of DNNs, we can't prove the matrix \( \nabla F_j(X)\nabla F_j(X)^\top \) is invertible. To ensure descent, and also to deal with the case where the matrix \( \nabla F_j(X)\nabla F_j(X)^\top \) is singular (as well as enhance convergence when this matrix is nearly singular), the equation (22) is rewritten as follows:

\[ \delta^{k+1} = \delta^k - \alpha^k (\nabla F_j(X)\nabla F_j(X)^\top + \Delta^k)^{-1}\nabla F_j(X)F_j(X) \] \hspace{1cm} (23)

where \( \alpha^k \) is a stepsize chosen by one of the stepsize rules. The matrix \( \nabla F_j(X)\nabla F_j(X)^\top \) is a symmetric matrix certainly, so there is a matrix \( \Delta^k = -\lambda_{\min}(\nabla F_j(X)\nabla F_j(X)^\top)I \) which is a diagonal matrix that makes \( \nabla F_j(X)\nabla F_j(X)^\top + \Delta^k \) positive definite. The algorithm is as follows:

**Algorithm 2** Generate adversarial examples based on the Gauss-Newton Method

\[ X \] is a benign example

**Input:** \( X, \ y, \ C, \ \delta = 0 \)

**Output:** \( \delta \)

1: while \( y_{X+\delta} = y \) and \( \|\delta\|_p \leq C \) do

2: \( \nabla F_j(X) \left[ \frac{\partial F_j(X)}{\partial X_i} \right]_{i=1}^{m} \),

\( H \leftarrow \nabla F_j(X) \cdot \nabla F_j(X)^\top \),

\( \Delta^k \leftarrow -\lambda_{\min}(\nabla F_j(X)\nabla F_j(X)^\top)I \)

3: \( \delta^{k+1} = \delta^k - \alpha^k (H + \Delta^k)^{-1}\nabla F_j(X)F_j(X) \)

4: \( X \leftarrow X + \delta \)

5: return \( \delta \)

This is an algorithm that can generate adversarial examples quickly, but the accuracy is not high due to the addition of the identity matrix to ensure positive definite. In other words, the concealment of the adversarial examples generated by this algorithm is not as good as we expected. However, we use meaningless images to generate adversarial examples, then we do not have to worry about the problem of concealment. We'll test that experimentally. In addition, ‘our method’ we mentioned in this paper is the method in Section 3.2.
4 Construct robust DNNs through adversarial training

For the purpose of training a robust DNNs reliably, this paper does not use the method that directly focuses on improving the robustness against specific attacks, but first proposes specific requirements that a robust DNNs should be satisfied. Ruitong Huang et al.[12,15,16] described this specific requirement as a min-max optimization problem. On the one hand, we have to find an adversarial version of a given data point $X$ that achieves a high loss. On the other hand, we have to train a model and find the model parameters which minimize the loss of DNNs to adversarial examples. This is exactly the problem to construct robust DNNs through adversarial training.

Szegedy et al.[15] first proposed to use both adversarial examples and benign examples as training data, and the experiment proved that it is an effective method to defense adversarial examples. Aleksander et al.[12], from the perspective of optimization, believed that adversarial training is an optimization problem about saddle points, and they extended traditional ERM training to robust training.

Adversarial training: Suppose $(X,y_{true}) \in D$ is the original training data. Adversarial examples can be obtained under constraint $\varepsilon$ and $J(\cdot)$ is the loss function. Described as follows:

$$\theta^* = \arg \min_{\theta} \mathbb{E}_{(X,y_{true}) \sim D} \left[ \max_{\|X_{adv} - X\|_\infty \leq \varepsilon} J(f(X_{adv};\theta), y_{true}) \right]$$

(24)

It can be found that (24) is a saddle point problem, a combination of an inner maximization problem and an outer minimization problem. The inner maximization problem is to find adversarial examples that achieves a high loss. The outer minimization problem is to find the model parameters that can minimize the adversarial loss under some kind of adversarial attack. Current work on adversarial examples usually focuses on specific defensive mechanisms, or on attacks against such defenses[12].An important feature of min-max optimization problem is that attaining small adversarial loss gives a guarantee that targeted attack cannot fool DNNs. By definition, it is possible to construct a robust DNNs which can defense all kinds of attacks. Hence, adversarial training is an optimal balance between model accuracy and robustness.

Equation (24) defines the goal that an ideal DNNs should achieve, and quantifies its robustness. When $\mathbb{E}$ approaching infinity, the corresponding DNNs has perfect robustness against specified attack. This section studies the structure of adversarial training under the background of DNNs. These studies will lead us to use DNNs training to produce DNNs that are highly resistant to a wide range of adversarial attacks. Hence, we now focus our attention on
obtaining a good solution to Equation (24).

4.1 Inner maximization problem

The inner maximization problem corresponds to constructing valid adversarial examples, which is a non-concave internal maximum problem. Since this problem requires us to maximize a non-concave function, this is difficult to deal with. Our method approximates the output of the non-concave function in the input neighbourhood through the Taylor expansion function, and then we turn this second order function into a convex optimization problem by using the dual problem. Our method is more conducive to finding the extreme value within the constraint range and avoiding falling into the local optimal solution, which is exactly the defect of the existing typical attack methods.

In order to explain that our method can solve the inner maximization problem effectively, we take MNIST and CIFAR10 dataset as examples, and randomly pick up the pictures that can be correctly classified by DNNs for testing.

Experiment in Section 5 showed that, as we had expected, our method which uses the second-order Taylor expansion function to approximate the output of DNNs, and then uses the dual function to transform not only avoid falling into the local optimal value, and find the global optimal solution within the constraint range, but also make sure that the point $X$ found by the second-order function can also be input into DNNs to get the extreme value.

4.2 Outer minimization problem

The previous discussion shows that the inner maximization problem can be solved successfully by using our method. In order to train the adversarial network, we also need to solve the outer minimization problem of equation (6), that is, to find the model parameters to minimize the adversarial loss.

The main method to minimize the loss function is stochastic gradient descent (SGD) when training DNNs. An effective way to calculate the gradient of the outer optimization problem is to calculate the gradient of the loss function at the maximum value of the inner problem. This corresponds to adding the adversarial examples to the original training data set in the adversarial training. Of course, it is not clear that this is a valid descent direction for the min-max optimization problem. However, for continuously differentiable functions, the Danskin theorem - a classical optimization theorem - states that this is indeed true[12], and that the gradient of the inner maximization problem corresponds to descent directions for the min-max optimization problem. In 5.2, we will prove the effectiveness of the our method for adversarial training through experiments.
5 Evaluation

We now use our experimental setup to answer the following questions:(1) Vertical comparison between all objective functions on different dataset;(2) Horizontal comparison between FGSM, JSMA, Deepfool, C&W, PGD, M-DI2-FGSM and our method on different dataset;(3) Can our method improve the robustness of DNNs through adversarial training?(4) Whether Our adversarial examples are sufficiently transferable or not.

5.1 Experimental Setup

Our experiment will be conducted on MNIST and CIFAR10 dataset to verify the effectiveness of our method. MNIST is a popular handwritten dataset widely used in the machine learning community. It consists of ten classes from digit 0 to 9, containing a total of 70,000 handwritten digit images. We select 60,000 images as training data and 10,000 images as test data. Each image is in the size of 28×28 pixels.

The CIFAR-10 dataset consists of 60000 32×32 color images in 10 classes, with 6000 images per class. There are 50000 training images and 10000 test images. The dataset is divided into five training batches and one test batch, each with 10000 images. The test batch contains exactly 1000 randomly-selected images from each class.

We use the standard model for each dataset. For MNIST, we use the standard 3-layer convolutional neural network which achieves 99.2% accuracy. For cifar-10, we trained a standard 4-layer convolutional neural network which achieves 95.3% accuracy.

5.2 Evaluating Measure

We use $L_p$, PSNR and ASR to measure the effectiveness of our method. The value of $L_p$ is generally used to measure the global or local added perturbation, which is a measure of the concealment of adversarial examples. In order to better evaluate the concealment of adversarial examples, we listed "Peak Signal to Noise Ratio" (PSNR) as one of the indicators. PSNR, as the most common and widely used objective measurement of image quality, can effectively evaluate the concealment of adversarial examples. ASR stands for the probability of success in generating adversarial examples. When ASR is not 100%, $L_p$ and PSNR are for successful attacks only.

A. Compare the objective functions

|       | MNIST |       |       |       | CIFAR10 |       |       |
|-------|-------|-------|-------|-------|---------|-------|-------|
|       | $L_2$ | PSNR  | ASR   |       | $L_2$  | PSNR  | ASR   |
| $T_1$ | 0.91  | 73.03 | 100%  |       | 0.29   | 87.41 | 100%  |
| $T_4$ | 1.40  | 71.46 | 100%  |       | 0.30   | 87.45 | 100%  |

Table 1. Evaluation of untargeted attack by different objective functions on MNIST and CIFAR10
B. Compare existing classic methods with our method

To verify the effectiveness of our adversarial examples, we used JSMA, C&W, FGSM,
Deepfool and M-DI²-FGSM for comparison, where codes of JSMA, C&W, FGSM and Deepfool come from Cleverhans[22] and the codes for PGD and M-DI²-FGSM come from the link given by the author in the original text [12,13]. In addition, to ensure the rigor of the evaluation, we use the same model and the same batch of test data to verify the above methods.

For FGSM, we take $\varepsilon = 0.01$. If the target class adversarial examples can be generated within the specified step size, the adversarial examples will be returned for evaluation, otherwise it will be regarded as a failure. As a derivative method of FGSM, PGD has an upper limit $\varepsilon = 8.0$ for each pixel on the pixel scale of "0-255". For JSMA, we aim to generate adversarial examples. We extend the constraint on perturbation, and modify the iteration termination condition to classify the target class successfully. That is, no matter how much perturbation is required, we report success if the attack produce adversarial examples with the correct target label. But JSMA is unable to run on CIFAR10 due to an inherent significant computational cost for searching saliency map[11]. If we remove the search process, JSMA’s ability to generate adversarial examples is greatly reduced. Therefore, we did not use JSMA in the CIFAR10 experiment. Note that CW is a bit different from the above gradient-based methods in that it is an optimization-based attack. In this experiment, $L_2$ norm attack in C&W is adopted, and we set $\varepsilon = 1$, learning rate $= 0.1$. For Deepfool, note that in our implementation, the noise calculated as $f/\|w\| w$ instead of $f/\|w\| w/\|w\|$, where $\|w\|$ is the $L_2$ norm.

In our experiment, 500 pictures that could be correctly judged by the initial model were randomly selected from MNIST and CIFAR10 for testing. After all, if an image can be misclassified without a perturbation, then the meaning of generate adversarial examples is lost. In addition, in the case of target attack, we also divided adversarial examples into the best case and the worst case according to the image quality of the adversarial examples. In other words, it is to compare the perturbation strength superimposed by different methods to generate adversarial examples. The results are shown in the following table.

|          | MNIST | CIFAR10 |
|----------|-------|---------|
|          | PSNR  | ASR     | PSNR  | ASR     |
|          |       |         |       |         |
| **Best** |       |         |       |         |
| **Case** |       |         |       |         |
| $Our L_0$| 239.1 | 71.21%  | -     | -       |
| JSMA     | 183.5 | 58.54%  | -     | -       |
| $Our L_2$| 1.70  | 67.21%  | 0.39  | 84.83%  | 100%   |
| C&W      | 1.04  | 61.66%  | 0.28  | 63.26%  | 100%   |
| $Our L_\infty$ | 1.34  | 70.86%  | 1.32  | 70.56%  | 100%   |
| FGSM     | 4.12  | 74.47%  | 1.72  | 64.72%  | 100%   |
| **Average** |       |         |       |         |
| $Our L_0$| 243.4 | 74.04%  | -     | -       |
| JSMA     | 199.6 | 58.66%  | -     | -       |
|                | \(L_2\) | PSNR   | ASR  | \(L_2\) | PSNR   | ASR  |
|----------------|---------|--------|------|---------|--------|------|
| **Our** \(L_2\)  | 1.84    | 67.21  | 100% | 0.50    | 82.86  | 100% |
| C&W            | 2.21    | 60.68  | 96.2%| 0.62    | 65.52  | 100% |
| **Our** \(L_{\infty}\) | 2.32    | 70.90  | 100% | 2.10    | 69.74  | 100% |
| FGSM          | 5.09    | 73.10  | 45.3%| 1.78    | 63.75  | 89.5%|
| **Our** \(L_0\)  | 332.4   | 72.13  | 100% | -       | -      | -    |
| JSMA          | 265.4   | 57.67  | 100% | -       | -      | -    |
| **Worst Case** |         |        |      |         |        |      |
| **Our** \(L_2\)  | 2.24    | 69.75  | 100% | 0.54    | 82.13  | 100% |
| C&W            | 3.30    | 58.80  | 100% | 0.35    | 59.54  | 100% |
| **Our** \(L_{\infty}\) | 2.20    | 61.34  | 100% | 3.2     | 65.12  | 100% |
| FGSM          | 5.21    | 65.24  | 38.2%| 1.92    | 61.80  | 76.3%|

Table 3. Comparison of three targeted attack algorithms on the MNIST and CIFAR10 dataset.

|                | MNIST | CIFAR10 |
|----------------|-------|---------|
| \(L_p\)       | PSNR  | ASR     | \(L_p\) | PSNR  | ASR     |
| \(T_1\)       | 0.91  | 73.03   | 100%    | 0.29  | 87.41   | 100% |
| \(T_4\)       | 1.40  | 71.46   | 100%    | 0.30  | 87.45   | 100% |
| PGD           | 5.17  | 67.72   | 100%    | 1.63  | 78.76   | 86.49%|
| Deepfool      | 1.66  | 73.80   | 88.12% | 0.16  | 85.20   | 81.44%|
| M-DI\(^2\)-FGSM | 3.14  | 65.12   | 56.6%  | 1.85  | 75.12   | 48.1% |

Table 4. Comparison of three untargeted attack algorithms on the MNIST and CIFAR10 dataset.

We use the \(L_p\) norm and PSNR value to measure the concealment after adding perturbation. Experiments show that on different dataset, compared with the existing classical attack methods, the adversarial examples generated by our method have better imperceptibility, and our method can produce target class adversarial examples for any picture.

In [9], JSMA uses the last hidden layer instead of the Softmax layer to calculate the adversarial saliency map. The essence of this approach is to iteratively modify the pixels with the maximum derivative value until adversarial examples is generated or the number of pixels modified exceeds the limit. The authors gave a simple example to show how small input perturbations found using the forward derivative can induce large variations of the neural network output, but they didn’t explain the mathematical derivation. We believe that the mathematical basis of this method comes from the fact that in the neighbourhood of a fixed value \(X\), DNNs satisfy: For small \(\| \delta \|\), there is \( F(X + \delta) \approx F(X) + \nabla F(X) \cdot \delta \). Therefore, JSMA can generate adversarial examples by searching the adversarial saliency map.

However, authors believe that the extreme variations introduced by the Softmax layer lead to extreme derivative values. This reduces the quality of information on how the neurons are activated by different inputs and causes the forward derivative to loose accuracy when generating saliency maps. Therefore they compute the forward derivative of the network using the last hidden
layer instead of the Softmax layer. According to equation (3), the author selected the pixel with the maximum value of $S(X,t)$, but essentially wanted to find the pixel with the maximum value of $\frac{\partial F_i(X)}{\partial X_i}$ as the pixel most conducive to classification as the target class $t$ after adding perturbation. However, without the normalization of the Softmax layer, the author cannot guarantee that the increase of $\frac{\partial F_i(X)}{\partial X_i}$ will bring the decrease of $\left| \sum_{j \neq i} \frac{\partial F_i(X)}{\partial X_j} \right|$. So the greater the value of $S(X,t)$ doesn't mean the greater the value of $\frac{\partial F_i(X)}{\partial X_i}$. This means that the pixels selected in the above equation are not technically the key pixels in the targeted attack. We have reason to believe that this is why the capability of JSMA is not as good as we expected. However, our method overcomes this disadvantage. Whether it is in the last hidden layer or in the Softmax layer, the our method can effectively modify pixel points and effectively generating adversarial examples.

In [11], C&W is committed to solving $\min \| \delta \|_p + c \cdot f(X + \delta)$ s.t. $X + \delta \in [0,1]^n$. The authors use binary search to determine the value of constant $c$, which is a mechanical search method that is far less accurate and flexible than the Lagrange multiplier method we use. At the same time, C&W uses $\delta_i = \frac{1}{2} (\tanh(w_i) + 1) - x_i$ to expand the search space. Admittedly, this method is very conducive to searching more powerful adversarial examples, but it will also bring a very large search cost. While our method can be successful without having to pay such a high price.

In [12], PGD transforms constrained optimization problem into unconstrained optimization problem, which is easy to implement and suitable for solving large-scale optimization problems to generate effective adversarial examples. However, in order to ensure the effectiveness of iteration, it takes a long time to calculate the projection of iteration points and the convergence rate is slow. In addition, when PGD iterates the iteration point from outside to inside of the constraint through Equation (8), part of the iteration information is inevitably lost. In our paper, after approximating the input output mapping relationship of DNNs by using quadratic Taylor expansion, the dual function is constructed by using Lagrange multiplier method to optimize and generate adversarial examples. The Lagrange multiplier method can also transform unconstrained optimization problems into constrained optimization problems. However, unlike the projected gradient descent algorithm, the Lagrange multiplier method does not lose iteration information due to iteration, so the optimization results are more accurate.

Both Deepfool and FGSM often get trapped in local optimum due to their algorithmic characteristics, so the global optimal solution cannot be obtained. In this paper, quadratic Taylor expansion is used to approximate the input output mapping relationship of DNNs, which can
effectively skip the local optimum. M-DI²-FGSM combines momentum and diverse inputs naturally to form an attack with stronger transferability, but experiments have proved that its ASR is not ideal.

C. Generate adversarial examples for adversarial training based on Taylor Expansion

Based on the discussion in section 4, we can now apply our proposed method to train robust classifiers. Through previous experiments, it can be found that our method can better solve the inner maximization problem in section 4 than the existing classical attack methods. When conducting adversarial training, we can observe that the cross entropy loss decreases continuously to minimize outer problems, as shown in Figure 3. This result shows that our training is successful, and our method helps to enhance the robustness of DNNs.

![Figure 3: The change process of cross entropy loss in the process of adversarial training](image)

(a)MNIST (b)CIFAR10

Figure 3: The change process of cross entropy loss in the process of adversarial training

In order to verify the effectiveness of our adversarial examples for adversarial training, We used natural examples, our adversarial examples, JSMA adversarial examples, C&W adversarial examples, FGSM adversarial examples, PGD adversarial examples, Deepfool adversarial examples and M-DI²-FGSM adversarial examples to verify on the DNNs adversarial trained by above adversarial examples respectively.

The experimental classification accuracy is shown in the form of heat map. As shown in figure 4, it is easy to find that the DNNs obtained by our method through adversarial training is more robust than other models, and the attack ability of our adversarial examples are stronger.

The transferability of adversarial examples allows it to act on another model, even though this model is very different from the original one[7,14]. Therefore, a robust defense must guard against the transferability of adversarial examples, and the strength of adversarial examples is also reflected in its transferability.
Figure 4: Evaluate the attack capability of the adversarial examples generated by different methods on the DNNs adversarial trained by different adversarial examples on different dataset.

The attacks based on gradient calculation is divided into single-step attacks[14] and iterative attacks[7,23]. In general, iterative attacks tend to overfit specific network parameters, although they perform well in white box attacks, but they weaken the transferability of the adversarial examples. However, single-step attacks usually underfit to the network parameters thus producing adversarial examples with slightly better transferability, but it cannot achieve satisfactory results in the white box attack.

Our method obtain the gradient matrix and Hessian matrix for DNNs simultaneously. Then a quadratic Taylor expansion is proposed to approximate the output of DNNs in a small neighbourhood of the input example and it doesn't overfit. Therefore, our method can achieve a good balance between overfitting and underfitting for the network parameters of DNNs, and it performs noticeably well under both white-box and black-box settings.

Figure 4 also shows the success rate of the adversarial examples generated by our method under both white-box and black-box settings compared to other attack methods. It is clear that our adversarial examples are more transferable than others.

D. Generate synthetic digits

Based on the above experiments, we found that we can make any picture into adversarial examples of target class, this theory is also applicable to meaningless pictures. [9] and [11] have both done such experiments. They use all-black image and all-white image to generate adversarial examples that make no sense to humans but misclassified by DNNs. Here is the result.
Target Classification

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Totally Black** | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) | ![Image](image4.png) | ![Image](image5.png) | ![Image](image6.png) | ![Image](image7.png) | ![Image](image8.png) | ![Image](image9.png) | ![Image](image10.png) |
| **Totally White**  | ![Image](image11.png) | ![Image](image12.png) | ![Image](image13.png) | ![Image](image14.png) | ![Image](image15.png) | ![Image](image16.png) | ![Image](image17.png) | ![Image](image18.png) | ![Image](image19.png) | ![Image](image20.png) |

Figure 5: Targeted attack for the MNIST where the starting image is totally black or white. For random synthetic digits in [9], one can clearly recognize the target digit, but method in Section 3.3 does not have this flaw. For random synthetic digits in [11], although the perturbation is very small, its calculation cost is very large, which is greatly inferior to the method in Section 3.3.

6 Related work

Since Szeigy et al.[7] discovered adversarial examples, the safety of DNNs[24] has become an active research topic. The researchers classified the attack and discussed the adversarial capabilities[25,26]. Szegedy et al.[7] proposed a box-constrained LBFGS method to generate adversarial examples. Goodfellow et al.[14] proposed FGSM to generate adversarial examples efficiently by performing a single gradient step. Kurakin et al.[23] extended it to an iterative version, and found that the adversarial examples also exist in the physical world. Dong et al.[27] proposed a broad class of momentum-based iterative algorithms to boost the transferability of adversarial examples.

The above work calculates the gradient of DNNs to generate adversarial examples[7,9,14,28,29]. These work calculates the gradient not to update the weight of DNNs to improve the network, but to update the input and then make DNNs misclassify. They first define the cost function for the output of DNNs and then optimize the cost function to generate adversarial examples. However, these cost functions are often difficult to calculate or have to bear a large computational cost to optimize. Unlike these methods, our method uses a quadratic Taylor expansion to approximate the complicated input output mapping relationship of DNNs in a small neighbourhood of the input example directly. This is an equation that can directly find the extremum in the neighborhood, which is more accurate and requires less computational cost. Therefore, our method can generate adversarial examples which are very effective under both white-box and black-box settings.

The existence of adversarial examples reveals the vulnerability of DNNs. In order to improve the robustness of DNNs, researchers have proposed many methods of square defense adversarial...
examples,[14,30] proposed to use both adversarial examples and benign examples as training data to increase the robustness of DNNs. Tram`er et al.[31] proposed ensemble adversarial training, which augments training data with perturbations transferred from other models, in order to improve the robustness of DNNs. We recommend using counter training as a defense. Adversarial training requires a large number of effective adversarial examples to be generated at low cost, which is the main purpose of this paper. Min-max optimization problem is considered in confrontational training[32,33]. However, the results mentioned in [32,30] are different from those in our paper. Firstly, the authors believes that the inner maximization problem is difficult to solve, and the innovation of the method in this paper overcomes this problem. We approximate the output of DNNs, then the Lagrange multiplier method is used to construct the dual function for optimization. It is proved theoretically and experimentally that our method can obtain the optimal solution for the inner maximization problem. Secondly, they only considered first-order adversarial, and we also experimented with multi-step iterative methods. Furthermore, although the experiments in [33] produced promising results, they were evaluated only on the basis of FGSM. However, the assessment limited to FGSM is not entirely reliable. Therefore, our method is compared with many methods to obtain a more reliable experimental result.

7 Conclusion

In this paper, we propose a novel and more powerful attack method. We use a quadratic Taylor expansion to approximate the input output mapping relationship of DNNs in a small neighbourhood of the input example (using $L_p$ norms constraints), replacing the nonlinear part of DNNs. After that, the Lagrange multiplier method is used to construct the dual function for optimization and calculate the extreme value to generate adversarial examples. This method can efficiently generate a large number of effective adversarial examples at a small cost to solve the inner maximization problem in the min-max optimization problem.

Experimental results on MNIST and CIFAR10 show that compared with the existing classical attack methods, our method has better concealment, transferability, and can significantly improve the robustness of DNNs through adversarial training. Compared with single-step attacks, our method has higher transferability while maintaining the concealment of adversarial examples. Compared with iterative attacks, our method can solve the internal maximization problem more effectively and accurately. Therefore, our proposed method can be used as a benchmark to evaluate the robustness of DNNs and an effective defense in the future.
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