GAMMA-RAY EMISSION OF ACCELERATED PARTICLES ESCAPING A SUPERNOVA REMNANT IN A MOLECULAR CLOUD

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ABSTRACT

We present a model of gamma-ray emission from core-collapse supernovae (SNe) originating from the explosions of massive young stars. The fast forward shock of the supernova remnant (SNR) can accelerate particles by diffusive shock acceleration (DSA) in a cavern blown by a strong, pre-SN stellar wind. As a fundamental part of nonlinear DSA, some fraction of the accelerated particles escape the shock and interact with a surrounding massive dense shell producing hard photon emission. To calculate this emission, we have developed a new Monte Carlo technique for propagating the cosmic rays (CRs) produced by the forward shock of the SNR, into the dense, external medium. This technique is incorporated in a hydrodynamic model of an evolving SNR which includes the nonlinear feedback of CRs on the SNR evolution, the production of escaping CRs along with those that remain trapped within the remnant, and the broadband emission of radiation from trapped and escaping CRs. While our combined CR-hydroescape model is quite general and applies to both core collapse and thermonuclear SNe, the parameters we choose for our discussion here are more typical of SNRs from very massive stars whose emission spectra differ somewhat from those produced by lower mass progenitors directly interacting with a molecular cloud.

Key words: acceleration of particles – cosmic rays – ISM: supernova remnants – shock waves

Online-only material: color figures

1. INTRODUCTION

Many core-collapse supernovae (SNe) are expected to explode within their parent molecular clouds. Because of the influence of the surrounding material, the manifestation of the supernova remnant (SNR) can differ substantially depending on the progenitor star type. For a relatively low progenitor mass below \( \sim 12–14 M_\odot \), the stellar wind and photoionizing radiation are not sufficient to substantially clear out the surrounding cloud and already at a radius of about 6 pc the remnant can enter a radiative phase with a shock directly interacting with the molecular cloud (e.g., Chevalier 1999). The radiative shock with a typical velocity below \( \sim 150 \text{ km s}^{-1} \) can accelerate and compress cosmic rays (CRs) and produce non-thermal radiation (Bykov et al. 2000; Uchiyama et al. 2010). Recently, the Large Area Telescope on board the Fermi Gamma-ray Space Telescope detected GeV emission from SNRs IC 443, W44, and 3C 391 known to be directly interacting with molecular clouds (see, e.g., Abdo et al. 2010a, 2010b; Castro & Slane 2010).

Higher mass young stars with masses above \( \sim 16 M_\odot \) (of B0 V type and earlier) are likely to create low-density bubbles and H II regions with radii \( \sim 10 \text{ pc} \) surrounded by a massive shell of matter swept up from the molecular cloud by the wind and the ionizing radiation of the star over its lifetime. In this case, a strong SN shock propagates for a few thousand years in tenuous circumstellar matter with a velocity well above \( 10^3 \text{ km s}^{-1} \) before reaching the dense massive shell where it decelerates rapidly.

Regardless of the SN type, the blast wave of the SNR is expected to accelerate ambient material and generate relativistic electrons and ions, i.e., CRs, which produce strong non-thermal radiation. A preponderance of evidence suggests that the particle acceleration mechanism most likely responsible is diffusive shock acceleration (DSA; e.g., Blandford & Eichler 1987; Jones & Ellison 1991; Malkov & Drury 2001).

We note that despite the common acceleration mechanism, the appearance of the two classes of SNRs we mention can differ very substantially. For early progenitor stars, one can expect that a sizeable fraction of the \( \gamma \)-ray emission is produced by the CR ions that escape the forward shock (FS) and interact with the dense surrounding shell, while for lower mass progenitors, the bulk of the non-thermal radiation is likely to come from trapped CRs.

While the CRs produced by the SNR generate non-thermal emission across the spectrum from radio to TeV \( \gamma \)-rays, the \( \gamma \)-rays are of particular interest because they may be produced in proton–proton (or heavier ion) collisions of ultra-relativistic particles. In fact, there are three populations of shock-accelerated CRs that are important for producing \( \gamma \)-rays: relativistic electrons producing \( \gamma \)-rays through inverse Compton and non-thermal bremsstrahlung, CR ions that remain trapped within the FS precursor, and CR ions that are accelerated by the FS but escape upstream. These three populations are produced simultaneously by DSA but they have very different properties and will have very different \( \gamma \)-ray signatures.3

As has been known for some time (e.g., Ellison et al. 1981; Eichler 1984; Ellison & Eichler 1984; Berezhko & Krymskii 1988), a large fraction of the energy in particles accelerated at strong shocks can escape at an upstream boundary. In fact, the fraction of all galactic CRs that originate as escaping particles is likely to be significant and escaping CRs may even provide the bulk of CRs at the “knee” and above. The importance of modeling escaping CRs was discussed before the advent of DSA (e.g., Schwartz & Skilling 1978) and is attracting considerable attention currently within the DSA paradigm (see, e.g., Ptuskin & Zirakashvili 2005; Caprioli et al.

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3 A fourth-particle population that we do not consider here are secondary electron–positron pairs produced by proton–proton interactions (see, for example, Gabici et al. 2009). These leptons will produce inverse Compton emission and may be important depending on the external mass concentration.
In principle, particle-in-cell (PIC) simulations can solve this problem exactly. However, it must be noted that PIC simulations cannot yet solve the full NL shock problem for SNRs. Most current efforts with PIC simulations are needed (see Vladimirov et al. 2008 for a full discussion).

A number of stationary, nonlinear (NL) models of DSA can provide the integrated escaping CR energy flux as a fraction of the parameterized overall acceleration efficiency, but no model is yet able to determine the spectral shape of escaping CRs taking into account the self-consistent production of magnetic instabilities produced by both the trapped CRs in the shock precursor and the escaping CRs. Such a treatment is not yet feasible, creating an important problem since the interpretation of the $\gamma$-ray emission from young SNRs depends critically on the uncertain spectral shape of both the trapped and escaping CRs. Therefore, a suitable parameterization of the shape of the escaping CR flux is needed to allow comparisons with $\gamma$-ray observations of young SNRs in the hope of constraining NL DSA models.

It is important to note, of course, that SNRs are not stationary and the dynamics of an expanding remnant, even in the simplest spherical case, adds an additional factor to the issue of CR escape. As the remnant expands, the precursor region beyond the FS that is filled with CRs expands producing a “dilution” of CR energy density. This effect has been studied in detail by Berezhko and coworkers (e.g., Berezhko et al. 1996a, 1996b; see also Drury 2010). In a real shock, the dilution effect is coupled to escape since the lowering of the CR energy density results in less efficient generation of magnetic turbulence and this will change the escape of CRs. Both the flow of energy out of the shock by escape and the dilution of the CR energy density influence the Rankine–Hugoniot conservation relations in similar ways. Both act as energy sinks and both result in an increase in the shock compression and other NL effects. In the stationary, plane-shock approximation for DSA used here, we ignore dilution and only include the escape of CRs at an upstream free escape boundary. It has been shown, however, that this plane-shock approximation gives essentially the same results for the shock structure as in a spherical, expanding shock if the specific mode of escape is unimportant (see Berezhko & Ellison 1999). The mode of escape becomes important, however, if the escaping CR flux is used to calculate $\gamma$-ray emission in material external to the shock. Since we neglect dilution, our escaping CR fluxes, and the $\gamma$-ray emission we calculate from these CRs, are overestimated.

In this paper, we present a new Monte Carlo technique for propagating escaping CRs and calculating their $\gamma$-ray production via pion-decay, in the circumstellar medium (CSM) surrounding the outer SN blast wave. This treatment of escaping CRs is added to our CR-hydro simulation (e.g., Ellison et al. 2004, 2010, and references therein) producing a coherent model where a number of related elements of the SNR are treated more or less self-consistently. The hydrodynamic simulation couples the efficient production of CRs to the SNR evolution, including the production of escaping CRs as an intrinsic part of the DSA process. The escaping CRs are emitted from the FS as the SNR evolves, their propagation is followed as they diffuse in the CSM, and the $\gamma$-ray emission of these CRs is calculated consistently with that from the CRs (protons and electrons) that remain trapped within the remnant. While a number of important approximations are required, including the neglect of precursor CR dilution, this model represents a fairly complete and internally self-consistent description of an SNR interacting with a non-homogeneous CSM.

The importance of freshly produced, CRs interacting with their local environment to produce $\gamma$-rays has been recognized for some time and an extensive literature exists in this field. In a generalization of the model of Gabici & Aharonian (2007) (and previous work, e.g., Aharonian & Atoyan 1996), Gabici et al. (2009) calculate the broadband emission, from radio to TeV $\gamma$-rays, from CRs produced by an SNR interacting with a nearby molecular cloud. They emphasize that, depending on the parameters, the $\gamma$-ray emission can exceed other bands by a large factor, suggesting that some unidentified TeV sources might be associated with clouds illuminated by nearby SNRs. Gabici et al. (2009) also note the importance of the shape of the $\gamma$-ray spectrum for identifying GeV–TeV sources.

The model used by Gabici et al. (2009) is based on that of Ptuskin & Zirakashvili (2005) and includes a description of the evolution of the SNR and the spectrum of escaping CRs. In Gabici et al. (2009), the important parameter $p_{\text{max}}$, the maximum cutoff momentum for the CR spectrum, is parameterized as $p_{\text{max}}(t) \propto t^{-\delta}$, where $t$ is the age of the SNR and $\delta$ is taken to be $\sim 2.48$ to match the CR data below the knee, as measured at Earth. Our parameterization of $p_{\text{max}}$ differs considerably from this as we discuss below. An important result of Ptuskin & Zirakashvili (2005; see Berezhko & Krymskiı 1988 for an earlier derivation) that is incorporated in the Gabici et al. (2009) model and is not modeled here is that, when integrated over the whole Sedov phase, the total CR spectrum is a power law of the form $F_{\text{esc}} \propto p^{-4}$. Recently, Casanova et al. (2010) have applied the Gabici et al. (2009) model to SNR RX J1713.7–3946 taking into account the details of the ambient gas distribution.

The model presented here is similar to that of Lee et al. (2008). In both cases, the evolving SNR is modeled with a spherically symmetric hydrodynamic simulation where the efficient production of CRs via DSA is coupled to the remnant dynamics. The main difference is in the treatment of the diffusion of escaping CRs in the region beyond the SNR FS. The work of Lee et al. (2008) uses a “boxel” technique whereby, at each time step and each spatial grid in the three-dimensional simulation box, particles are exchanged between the adjacent boxels according to the particle momentum, location, and density gradient. In the model presented here, we use a Monte Carlo technique to propagate escaping CRs in the region beyond the FS. These two methods of propagation have distinct advantages and disadvantages, and both differ importantly from more analytic models based on a direct solution of a diffusion equation. In any case, we feel the problem of CRs produced by relatively young SNRs interacting with dense, local...
material is important enough to be considered with a variety of complementary techniques.

Other differences between the boxel model of Lee et al. (2008) and our new Monte Carlo model are all based on recent refinements of the CR-hydro model (see Ellison et al. 2010 and references therein) and on how we parameterize the escaping CR distribution, described below. While these refinements are important for modeling specific remnants, they do not significantly change the results given in Lee et al. (2008).

2. MODEL

The model we present here consists of two main parts. The CR-hydro part is used to calculate the evolution of an SNR and is essentially the same as that described in Ellison et al. (2007, 2010), Patnaude et al. (2009), and references therein. The evolution of the spherically symmetric remnant is coupled to the efficient production of CRs and the production of thermal and non-thermal emission is calculated (see Ellison et al. 2010 for recent work modeling the broadband emission from SNR RX J1713.7−3946). The DSA is determined in the CR-hydro model using the semi-analytic model of Blasi and coworkers (e.g., Blasi 2002; Amato & Blasi 2005; Blasi et al. 2005). The injection scheme for this model has been discussed in detail in a number of previous papers (see Caprioli et al. 2010 for recent extensions of the model) but we note that we use a slightly different injection method than typically used by Blasi and coworkers. Since the diffusion approximation upon which the semi-analytic model is based does not apply to thermal particles, a parameter, $\eta_{inj}$, must be defined that specifies what fraction of thermal particles obtain a superthermal energy and are injected into the DSA mechanism. Given this parameter, the NL DSA mechanism determines the fraction of shock ram kinetic energy that goes into superthermal particles, i.e., the acceleration efficiency $\epsilon_{DSA}$.

The only difference in our implementation of this injection model and that of Blasi and coworkers is that we specify $\epsilon_{DSA}$ and then set $\eta_{inj}$ accordingly. Both schemes are approximations since, in an evolving SNR, both $\eta_{inj}$ and $\epsilon_{DSA}$ are likely to be functions of age. For simplicity, we hold $\epsilon_{DSA}$ constant.

The Blasi et al. model that we use also implicitly assumes that the shock is planar and stationary. Apart from the neglect of CR dilution,\(^5\) this approximation will be reasonably accurate as long as the diffusion length of the highest energy CRs is a small fraction of the shock radius. The sharp X-ray synchrotron edges often seen in SNRs (e.g., Warren et al. 2005; Eriksen et al. 2011) imply the presence of amplified magnetic fields which will result in short diffusion lengths. In our models here we assume that the diffusion length of protons with maximum momentum $p_{max}$ is $1/10$ of the shock radius, a small enough value to validate the planar approximation yet allow $p_{max} \sim 10^{2−10^3} m_p c$, consistent with most models of CR production in SNRs.

Accounting for escaping CRs is essential in efficient DSA and escaping CRs are implicitly included in Blasi’s semi-analytic description. However, until now we have not included them in the production of radiation in the remnant environment in our CR-hydro model. The neglect of radiation produced by escaping CRs is justified if the SNR is in a uniform CSM with no external density enhancements. In this case, the emission from trapped CRs interacting with the shocked material is always much greater than that produced by escaping CRs in the less dense, unshocked external medium (see Model B in Figure 5).

The second and new part of our model is a calculation of the escaping CR distribution that emerges from the SNR FS and the propagation and interaction of these escaping CRs in a dense, spherically symmetric shell external to the SNR. Depending on the density of the external material, $\gamma$-rays produced by the escaping CRs can overwhelm those produced by trapped CRs, as emphasized by Gabici et al. (2009). We note that while here we restrict ourselves to spherical symmetry for the external mass distribution, it is straightforward to generalize the Monte Carlo technique to arbitrary mass distributions.

2.1. Escaping CR Distribution

As we make clear in describing our parameterized escaping CR model, both the fraction of energy in escaping CRs and their spectral shape are uncertain. However, while controversial for some years, the idea that some fraction of the most energetic particles in a shock undergoing DSA must escape, regardless of whether the shock is stationary or not, is now generally accepted although certain qualifications are still made (see Drury 2010).

We believe that energetic particle escape is a fundamental and unavoidable part of DSA that must occur in all supercritical collisionless shocks regardless of $p_{max}$ or time evolution because (1) observations and modeling of the Earth bow shock (e.g., Scholer et al. 1980; Mitchell et al. 1983; Ellison et al. 1990) support escape, (2) particle escape is an intrinsic part of many particle-in-cell (PIC) simulations (e.g., Giacalone et al. 1997; Giacalone & Ellison 2000), and (3) DSA requires self-generated turbulence to work over any reasonable dynamic range. Since CRs must interact with self-generated turbulence to be further accelerated, the highest energy CRs far upstream in the shock precursor will always lack sufficient turbulence to remain nearly isotropic and some fraction will escape. These escaping CRs will generate turbulence for the next generation of CRs, creating a bootstrap effect. As mentioned above, the dilution of CR energy density that occurs in spherical, expanding shocks will be coupled to CR escape through the magnetic turbulence generation.

Given the assumptions and approximations of the model, the semi-analytic description of Blasi et al. (2005) determines the energy in escaping CRs, $\mathcal{Q}_{esc}$, but does not determine the shape of the distribution. While other work does determine the shape (e.g., Vladimirov et al. 2006; Zirakashvili & Ptuskin 2008; Caprioli et al. 2010), the shape that results in these models depends importantly on arbitrary parameters and the assumptions made for the diffusion of the highest energy escaping CRs.

Since the shapes of the trapped and escaping CR distributions, at the highest accelerated energies, are critical for modeling both X-ray synchrotron emission and GeV–TeV $\gamma$-ray emission, we feel it is important to have a flexible, i.e., parameterized, model that can be compared to observations to provide information on the uncertain plasma processes until an adequate theory of self-generated turbulence in the presence of escaping particles is developed (see Bykov et al. 2011 for recent work on long-wavelength instabilities that may influence the maximum momentum CRs can obtain in a given shock).

As an example of the complexities that may exist, the amplified long-wavelength fluctuations discussed in Bykov et al. (2011) may result in particle acceleration by the resonant second-order Fermi mechanism. The stochastic acceleration rate, $\tau_{acc}^{-1}$, for particles with spatial diffusion coefficient $\kappa(p)$...
in the shock precursor is $v_{ph}^2/\kappa(p)$, where $v_{ph}$ is the phase velocity. While this rate may be below the first-order acceleration rate, it may still be high enough to influence the spectra shape at the highest particle energies achieved by first-order DSA. The spectral index of particles accelerated by the second-order Fermi mechanism depends on the parameter $\tau_{ac}/T_{esc}$, where $T_{esc}$ is the escape time (e.g., Petrov & Bykov 2008). In the case of resonant stochastic particle acceleration by long-wavelength fluctuations, $\tau_{ac}/T_{esc} \propto M_f^2/[k_1 r_g(p_{max})]$, where the characteristic wave number of the CR instability (cf. Bell 2004; Bykov et al. 2011) is

$$k_1 = \frac{4\pi \overline{v}^2}{c B}. \quad (1)$$

Here, $\overline{v}$ is the mean CR current, $r_g(p_{max})$ is the CR gyroradius at $p_{max}$ in the magnetic field $B$, and $M_f$ is the FS Alfvénic Mach number.

Therefore, for a large enough precursor CR current, $\overline{v}$, as expected for efficient DSA, the parameter $\tau_{ac}/T_{esc}$ may influence the shape of the CR distribution in the spectral break region. For instance, if a shock of velocity $V_{sk}$ produces a power-law spectrum of accelerated particles up to some maximum momentum and transfers a fraction $\eta$ of the shock ram pressure to CRs, then $\tau_{ac}/T_{esc} \propto \eta^{-1} (c/V_{sk})$ with a weak dependence on the particle momentum. The smaller $\tau_{ac}/T_{esc}$, the larger is the second-order Fermi effect and preliminary work (A. Bykov 2011, in preparation) suggests that $\tau_{ac}/T_{esc} \lesssim 100$ is needed to see a significant modification of the spectral shape. While more exact estimates are difficult, we might expect $\eta \gtrsim 0.5$ and $V_{sk} \gtrsim 5000$ km s$^{-1}$ to produce a noticeable effect.

When the shock accelerated particles approach $p_{max}$, they begin leaving the upstream region of the shock and the approximate power-law distribution of particles that remain in the shock turns over in a fashion that will depend on the diffusion coefficient of the highest energy particles. Whatever the plasma processes are for escaping particles, the shape of the escaping distribution, $F_{esc}(p)$, is determined by how CRs leave the shock and is, therefore, coupled to the trapped distribution. Since no current model of self-generated turbulence adequately describes the diffusion of escaping CRs, the diffusion coefficient is generally assumed to be Bohm-like right up to $p_{max}$ and independent of position relative to the shock.

Here, we parameterize the escaping CR phase-space distribution, $F_{esc}(p)$, as a modified parabola centered at $p_{max}$, where $p_{max}$ is the maximum momentum CRs would obtain if the acceleration cut off sharply when the upstream diffusion length $\kappa(p_{max})/V_{sk} = L_{FEF}$, where $V_{sk}$ is the speed of the FS and $L_{FEF}$ is a free escape boundary. In our SNR model, the maximum momentum is determined primarily by an arbitrary parameter, $f_{sk}$, which is the fraction of the shock radius equal to the diffusion length of protons with momentum $p_{max}$; i.e., $L_{FEF} = f_{sk} R_{sk}(t)$, where $R_{sk}(t)$ is the radius of the FS at time, $t$. For all of the examples shown here, we set $f_{sk} = 0.1$; a factor small enough to be consistent with the planar shock approximation in the Blasi et al. DSA calculation.

Our scheme for determining $p_{max}$ gives a very different result from the parameterization used in Gabici et al. (2009), as indicated in the bottom panel of Figure 1. Gabici et al. (2009) argue that magnetic field amplification (MFA) may contribute to a strong decrease in $p_{max}$ as a function of time since it might be expected that MFA is strongest at early times, yielding a large magnetic field and a higher $p_{max}$. As the remnant ages, MFA might decrease, producing a stronger time dependence than the standard SNR evolution would suggest. MFA is not included in the examples we show here. We caution, however, that NL feedback may reduce the full effects of MFA (e.g., Vladimirov et al. 2008; Caprioli et al. 2008, 2009) and we feel it is unlikely that a time dependence as strong as assumed by Gabici et al. (2009) will be obtained. In any case, our main purpose here is to introduce a new propagation tool for escaping CRs and not be overly concerned with details that are still subject to active research.

An approximate expression for the CRs that remain trapped within the SNR is (e.g., Ellison et al. 2004)

$$F_{tr} = f_{SA}(p) \frac{v_{esc}}{v_{ph}} \left( \frac{p}{p_{max}} \right)^{v_{esc}} \exp \left( -\left( \frac{p}{p_{max}} \right)^{v_{esc}} \right). \quad (2)$$
where \( f_{SA} \sim (p/p_{\text{max}})^{-4} \) is the quasi-power-law DSA distribution obtained by the standard semi-analytic model, \( \alpha_{\text{cut}} \) is an arbitrary parameter that determines the turnover around \( p_{\text{max}} \), and \( p_{\text{max}} \), as mentioned, is determined by the SNR dynamics and \( f_{sk} \). The distribution of escaping CRs, \( F_{\text{esc}}(p) \), is parameterized by assuming that it is a parabola in \( \log (p^4 F_{\text{esc}}) \) space, that is,

\[
\log [(p')^4 F_{\text{esc}}(p)] = -a[\log (p') - \log (1)]^2 + b,
\]

where \( p' = p/p_{\text{max}} \). Initially, we determine \( b \) such that

\[
f_{\text{trap}}(p_{\text{max}}) = F_{\text{esc}}(p_{\text{max}}),
\]

which yields \( b = \log(e^{-1}) = -0.434 \).

The width of the parabola, \( a \), is matched to \( f_{\text{trap}}(p) \) as follows. We determine the momentum \( p_c > p_{\text{max}} \) where the trapped CR distribution drops by some factor, \( 1/N_c \), below its value at \( p_{\text{max}} \), i.e.,

\[
f_{\text{trap}}(p_c) = 1/N_c.
\]

Specifying \( N_c \) uniquely determines \( p_c \). We then obtain \( a \) by setting

\[
F_{\text{esc}}(p_c)/F_{\text{esc}}(p_{\text{max}}) = N_c,
\]

that is,

\[
a(\alpha_{\text{cut}}) = -\log [(p')^4 N_c]/[(\log p')^2],
\]

where \( p' = p/p_{\text{max}} \) and we have written \( a(\alpha_{\text{cut}}) \) to emphasize that the width of the escaping distribution depends on the cutoff parameter in the trapped CR distribution. The final normalization for \( F_{\text{esc}} \) is set by the total energy in the escaping distribution, \( Q_{\text{esc}} \), which is an output of the semi-analytic DSA model.

In the top two panels of Figure 2, we show examples where \( \alpha_{\text{cut}} \) is varied between 0.5 and 2 with \( N_c = 3 \) and 10. All other parameters of the CR-hydro model are the same for these examples. In all panels, the heavy-weight (black) curves are the CRs that remain trapped in the SNR, \( f(p) \), and these distributions, unlike the escaping CRs, have undergone adiabatic losses during the \( t_{SNR} = 1000 \) yr age of the remnant.\(^7\) The parameters, \( \alpha_{\text{cut}} \) and \( N_c \), allow a fairly wide range of shapes in the critical region around \( p_{\text{max}} \), although it is important to note that, in our model, for all reasonable values of \( \alpha_{\text{cut}} \) and \( N_c \), the escaping CR distribution is expected to be narrow compared to the trapped CRs. This differs from the work of Gabici et al. (2009), as mentioned above, and of Ohira et al. (2010, 2011) who assume a power-law form for the escaping CR distribution.

In the bottom panel of Figure 2, we compare our parameterization (red dotted curve) using \( \alpha_{\text{cut}} = 1 \) and \( N_c = 30 \) to the form presented in Zirakashvili & Ptuskin (2008, blue dashed curve). Other than a slight offset of the peak, this choice of \( \alpha_{\text{cut}} \) and \( N_c \) matches the Zirakashvili & Ptuskin (2008) result quite well. We could have obtained an equally good match with a different combination of \( \alpha_{\text{cut}} \) and \( N_c \). The quality of this match with \( \alpha_{\text{cut}} = 1 \) leads us to fix \( N_c = 30 \) and leave \( \alpha_{\text{cut}} \) as a single free parameter for the coupled shapes of the cutoff in the trapped CRs and the escaping distribution.

\(^7\) We note the distinction that the trapped distributions, \( f(p) \), in these plots determine \( f_{5A0} \), exactly from the CR-hydro model and the semi-analytic DSA calculation, as opposed to the approximate expression for \( f_{\text{trap}} \) used in Equation (2) to fit the modified parabola.

Figure 2. Top two panels show trapped (black, heavy-weight curves) and escaping CR (red, light-weight curves) proton distributions for simulations where \( \alpha_{\text{cut}} \) and \( N_c \) have been varied. The distributions are summed at the end of the simulation at \( t_{SNR} = 1000 \) yr and the escaping CR distributions are those leaving the FS before any propagation occurs. In the bottom panel, we compare our parabola fit with \( \alpha_{\text{cut}} = 1 \) and \( N_c = 30 \) (red dotted curve) to the result from Zirakashvili & Ptuskin (2008, blue dashed curve). Both curves have been normalized to the total energy in escaping CRs.

(A color version of this figure is available in the online journal.)

2.2. Monte Carlo Model of Cosmic Ray Propagation

Given the form for the escaping distribution, we propagate the escaping CRs using a Monte Carlo technique.\(^8\) As the CR-hydro simulation evolves, \( F_{\text{esc}}(p) \) is calculated for spherical shells at time steps, \( \Delta t \), as the FS overtakes fresh circumstellar material. As the outer-most shell is formed, escaping CRs leave the shell and diffuse into the CSM with a momentum and density-dependent mean free path given by

\[
\lambda_{\text{CSM}} = \lambda_{\text{CSM,0}}(r_g/r_{\text{g,0}})^{\beta_n}(n_{\text{CSM}}/n_0)^{-\beta_n}.
\]

Here, \( r_g = pc/(\epsilon B) \) is the gyroradius, \( n_{\text{CSM}} \) is the CSM proton number density, and \( \alpha_{\text{cut}} \) and \( \beta_n \) are parameters. For scaling, we use \( n_0 = 1 \) cm\(^{-3} \), \( r_{g,0} = 10 \) GeV/(\( \epsilon B_{\text{CSM,0}} \)), and \( B_{\text{CSM,0}} = 3 \) \( \mu \)G. The normalization of the CSM diffusion coefficient, \( D_{\text{CSM,0}} = \lambda_{\text{CSM,0}}/c/3 \), can be estimated from CR propagation studies (see, for example, Ptuskin et al. 2006;\(^9\)

\(^8\) Many of the elements of our Monte Carlo propagation model are similar to that used to model nonlinear DSA and are described in detail in Jones & Ellison (1991) and Ellison et al. (1996) and references therein.
and the escaping CRs quickly fill the CSM out to the end of the simulation box.

The process continues until $t_{\text{SNR}}$ is reached during which time some numbers of CR filled shells have been formed within the SNR. The escaping CRs fill the CSM region with a distribution that depends on Equation (8) and the properties assigned to the CSM.

2.3. Circumstellar Medium Properties

We model the spherically symmetric CSM with a dense shell sitting on a low-density, uniform background of density $n_{\text{uni}}$. The shell has a maximum density $n_{\text{shell}}$ and an inner radius $R_{\text{shell}}$ which, for the examples in this paper, is greater than the outer radius of the SNR at $t_{\text{SNR}}$, that is, the blast wave of the SNR has not yet reached the dense shell at the end of the simulation. An additional parameter is the total mass in the shell, $M_{\text{shell}}$.

The dashed (red) curves in the top two panels of Figure 3 show the density and mass distribution for a CSM with $n_{\text{uni}} = 0.1$ cm$^{-3}$, $n_{\text{shell}} = 100$ cm$^{-3}$, $R_{\text{shell}} \sim 10$ pc, and $M_{\text{shell}} = 10^4 M_\odot$. Note that the dense shell smoothly rises from $n_{\text{uni}} = 0.1$ cm$^{-3}$ and the rise is centered on $R_{\text{shell}}$. The solid (black) curves show the CSM with no shell. The total extent of the simulation box for these examples is $\sim 20$ pc. Also shown in the top panel (dotted blue curve) is the density profile of the SNR at the end of the simulation, i.e., at $t_{\text{SNR}} = 1000$ yr. The FS, contact discontinuity, and reverse shock (RS) can easily be discerned from the figure.

The addition of the escaping CR distribution requires additional parameters and Table 1 gives the parameters for the CSM diffusion and the parameters for the external medium. As mentioned above, we restrict ourselves to a spherically symmetric CSM in this first presentation of the Monte Carlo propagation model.

3. RESULTS

We use the following environmental parameters for all of our examples: (1) the SN explosion energy, $E_{\text{SN}} = 10^{51}$ erg; (2) the ejecta mass, $M_{\text{ej}} = 1.4 M_\odot$; (3) the distance to the SNR, $d_{\text{SNR}} = 1$ kpc; and (4) the ambient magnetic field throughout the CSM, $B_{\text{CSM}} = 3 \mu$G.

For the DSA of trapped and escaping CRs, we fix the following: (1) the fraction of FS radius used to determine $p_{\text{max}}$, $f_{\text{fs}} = 0.1$; (2) the MFA factor, $B_{\text{amp}} = 1$, i.e., no MFA is used; (3) the matching factor defined in Equation (5), $N_c = 30$; and (4) the DSA efficiency, $\varepsilon_{\text{DSA}} = 50\%$.
The parameters for DSA and the CSM propagation that are varied for our examples are given in Table 1. Again we note that we are not attempting a detailed fit to any particular remnant and that our model is not restricted to the particular values for parameters we use here. Any of the environmental or DSA parameters can be modified to match a specific object.

In Figure 3, we show results for Models A (with a dense external shell) and B (no external shell), as listed in Table 1. The top two panels were discussed in Section 2.3. In the bottom panel of Figure 3, we show the escaping CR densities at \( t_{\text{SNR}} = 1000 \) yr. The escaping densities shown are for a single momentum near the peak of \( F_{\text{esc}} \) and the parameters assumed for the CSM propagation are noted in the figure. The escaping CRs are emitted from the SNR as it evolves so the escaping CRs that were produced earliest have been diffusing for approximately \( t_{\text{SNR}} = 1000 \) yr and many have left the simulation box.

For the case where the CSM is uniform (solid black curves), the escaping CRs diffuse outward and uniformly fill the region beyond the SNR FS with a density that decreases uniformly with radius as expected. With the dense shell, the escaping CR density drops rapidly as the CRs enter the shell. With this \( M_{\text{shell}} = 10^4 \, M_\odot \), the shell is about 2 \( \text{pc} \) thick. CRs that propagate beyond \( R_{\text{max}} \approx 20 \, \text{pc} \) are removed from the simulation.

In the top panel of Figure 4, we show the CR distributions for both the CRs that remain trapped in the SNR (black solid and dotted curves) and escaping CRs.\(^9\) In all cases, the integrated distributions are determined at \( t_{\text{SNR}} = 1000 \) yr. The dot-dashed (red) curve in the top panel is the summed escaping distribution as the CRs leave the FS, i.e., before they propagate into the external CSM. The solid and dashed blue curves are the escaping CRs, at \( t_{\text{SNR}} = 1000 \) yr, after propagation and we remind the reader that our escaping CR fluxes are overestimates since we do not model dilution which, in fact, occurs simultaneously with escape. The bottom panel shows just the escaping CRs with an expanded scale. Note that throughout this paper we include only escaping protons and ignore escaping heavier ions and trapping electrons. Trapped electrons are considered for inverse Compton emission. The distributions for the escaping CRs after propagation are lower than the distribution as CRs leave the FS for two reasons. The first is that some CRs escape from the simulation box at \( R_{\text{max}} \). The second is that some escaping CRs diffuse back into the SNR and these CRs are ignored and not included in the escaping (blue) distributions in Figure 4. The CRs that remain trapped in the shock are summed from the contact discontinuity to the FS.

In Figure 5, we show the various photon components for the models with \( \alpha_{\text{fs}} = \beta_n = 0.5 \) and \( D_{\text{CSM,0}} = 1 \times 10^{27} \, \text{cm}^2 \, \text{s}^{-1} \). The results for the two models are identical except for the pion-decay emission from the escaping CRs. As expected, escaping CRs interacting with the dense external shell produce substantially more emission than those interacting with the uniform CSM. In both cases, however, the emission from escaping CRs is much more strongly peaked than the pion-decay emission from the trapped CR protons. For the trapped CRs, the relative intensity of the pion-decay emission and the inverse Compton emission depends on the various parameters chosen, most particularly

\(^9\) In all of the examples in this paper, we only calculate CRs accelerated at the forward shock and ignore those accelerated by the reverse shock.
in the CSM and this produces a fairly strong effect on the pion-decay emission from the escaping CRs. While the emission from escaping CRs shown in Figures 5 and 7 is summed over the entire region from the outer radius of the SNR at $R_{\text{SNR}}$ to $R_{\text{max}} \sim 20$ pc, when a dense shell is present, most of the emission originates in the shell, as expected.

In Figure 8, we compare escaping CR distributions for different power-law dependences of the gyroradius, i.e., $\alpha_{rg} = 1/3$ (Model E), $\alpha_{rg} = 1/2$ (Model F), and $\alpha_{rg} = 1$ (Model G). For variety, the models in Figure 8, along with those in Figure 9 below, use a different set of CSM parameters than the models discussed thus far, as shown in Table 1. For the three examples shown, the CSM parameters are identical and the mean free paths differ only in the value of $\alpha_{rg}$; the normalization of the diffusion coefficient $D_{\text{CSM},0}$ is the same in the three models. In Figure 8, the solid (black) curve is Model C, the dotted (red) curve is Model A, and the dashed (blue) curve is Model D.

(A color version of this figure is available in the online journal.)

Figure 7. Gamma-ray emission for the three cases shown in Figure 6. Since only the interaction of escaping CRs with the external CSM is varied, the pion-decay emission from the trapped CRs within the SNR is the same for the three cases. Referring to Table 1, the solid (black) curve is Model C, the dotted (red) curve is Model A, and the dashed (blue) curve is Model D.

(A color version of this figure is available in the online journal.)

In Figure 7, we compare the pion-decay emission for the three examples given in Figure 6, all with the same ambient density distribution. The CRs trapped in the SNR are the same for these cases so the pion-decay emission from the trapped CRs (dot-dashed curve) is the same in the three models. Also identical for the three cases is the escaping CR flux as it emerges from the FS. The sole difference is the scattering strength, $D_{\text{CSM},0}$.
Figure 8. Escaping CR distributions, $F_{\text{esc}}$, (top panel), and density profiles (bottom panel) for Models E, F, and G, as listed in Table 1. The index $\alpha_{\text{rg}}$ is varied as shown and $\beta_n = 0.5$ in all cases. For these examples, and those shown in Figure 9, $n_{\text{uni}} = 1 \text{ cm}^{-3}$, $n_{\text{shell}} = 10 \text{ cm}^{-3}$, $R_{\text{shell}} = 7 \text{ pc}$, and the densities in the bottom panel are for a particular momentum near the peak of the escaping CR distribution. The simulation box extends to 12 pc.

Figure 9. Escaping CR distributions, $F_{\text{esc}}$, (top panel), and density profiles (bottom panel) for Models H, F, and I, as listed in Table 1. Note that the solid (green) curves (Model F) are identical in Figures 8 and 9. The index $\beta_n$ is varied as shown and $\alpha_{\text{rg}} = 0.5$ in all cases.

4. DISCUSSION AND CONCLUSIONS

As part of a comprehensive model of an evolving SNR undergoing efficient CR production, we have presented a Monte Carlo technique that describes the diffusion of CRs that escape from the FS of the remnant and propagate into a dense, external shell. While a number of calculations of escaping CRs and their $\gamma$-ray production have been performed (see, for example, Lee et al. 2008; Ohira et al. 2011; Drury 2010; and references therein), there remain many unresolved issues for this important problem. Our Monte Carlo method makes different assumptions than analytic calculations based on solving a diffusion equation and in some ways is less restrictive, particularly if energy losses are included during propagation.

The important features of our model include (1) the energy content of the escaping CR distribution is determined with the shock-accelerated CRs that remain trapped within the SNR using a planar, stationary, NL model of efficient DSA that neglects dilution (i.e., Blasi et al. 2005); (2) the acceleration of CRs produces changes in the hydrodynamics that modifies the evolution of the SNR; (3) the shape of the trapped CR distribution at the highest energies, which is uncertain due to a lack of a well-developed theory of turbulence generation for anisotropic particles, is parameterized consistently with the shape of the escaping CR distribution; (4) the broadband continuum photon emission from escaping and trapped CRs is determined with a single set of environmental and model parameters; and (5) although not emphasized or shown in the plots here, the thermal X-ray emission is included consistently with the broadband continuum emission (e.g., Ellison et al. 2010 and references therein).

The examples we show indicate the complexity and importance of including escaping CRs in a consistent fashion with CRs that remain trapped within the SNR. The shape of the GeV–TeV emission, particularly the low-energy kinematic cutoff, is important as one of the main ways of determining whether this emission is pion-decay or inverse Compton. If other features are discernable, they may provide clues to the importance of the escaping CRs and external density enhancements. We note that all of the spectra shown here are integrated over the region between the contact discontinuity and the FS and are not line-of-sight projections. It should be clear from Figure 6 that line-of-sight projections might show additional strong effects as escaping CRs interact with nearby dense material. Line-of-sight projections will be included in future work.
An important parameter that we have not varied here is the efficiency of DSA. In all of our examples, we set $E_{\text{DSA}} = 50\%$, i.e., 50% of the FS ram kinetic energy flux goes into CRs (trapped and escaping) at any instant. In fitting an actual SNR, $E_{\text{DSA}}$ is a parameter that may or may not be constrained by the observations. A considerable amount of work has led to the conclusion that $E_{\text{DSA}} \sim 50\%$ is a likely figure for young SNRs but this efficiency will definitely vary between remnants, may vary during the remnant lifetime, and may even vary at different locations in a single SNR (see, for example, Völk et al. 2003). We note that Monte Carlo shock simulations that include MFA and have parameters typical of young SNRs (e.g., Vladimirov et al. 2006; Vladimirov et al. 2008) show total acceleration efficiencies $E_{\text{DSA}} \gtrsim 50\%$ with a sizable fraction of total shock ram kinetic energy ($\gtrsim 30\%$) placed in escaping CRs.

Since we set $E_{\text{DSA}} = 50\%$ for all of our examples, and the other SNR parameters that determine what fraction of explosion energy ends up in CRs are kept constant, the third panel in Figure 1 gives the results for all of our models. After 1000 yr, $\sim 30\%$ of the SN explosion energy has gone into all CRs with $\sim 10\%$ going into escaping CRs. At 10,000 yr, $\sim 50\%$ has gone into all CRs with $\sim 20\%$ going into escaping CRs.

In this initial presentation of our Monte Carlo technique, we have exploded the SN in a uniform CSM with an external, spherically symmetric shell of dense material. This simple scenario shows how important escaping CRs can be for modeling non-thermal emission of young SNRs. It is not meant to match any particular object. The Monte Carlo propagation part of the CR-hydro model can be easily generalized to include asymmetric external mass distributions, such as those expected when remnants interact with a dense molecular cloud (e.g., SNR RX J1713.7$-$3946). Future work will also model $\gamma$-rays produced when escaping CRs interact with the complex structure of a dense surrounding shell as expected from a progenitor stellar wind.

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