Deterministic SWAP gate using shortcuts to adiabatic passage

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Abstract

We theoretically propose an alternative method to realize a deterministic SWAP gate using shortcuts to adiabatic passage based on the approach of Lewis-Riesenfeld invariants in cavity quantum electronic dynamics (QED). By combining Lewis-Riesenfeld invariants with quantum Zeno dynamics, the SWAP gate can be achieved deterministically. The numerical results show that our scheme presents a fast and robust approach to achieve SWAP gates.

Keywords: SWAP gate, shortcuts to adiabatic passage, quantum Zeno dynamics

(Some figures may appear in colour only in the online journal)

1. Introduction

In quantum computing, the quantum logic gate is a basic operation. Recently, a number of schemes have been proposed to perform quantum logic gates by using optical devices [1], quantum dots [2, 3], QED systems [4], ion traps and superconducting devices [5–9]. A universal set of quantum operations can be constructed by a series of single- and two-qubit gates. An efficient method of quantum information processing requires that all the operations are robust against decoherence, easily prepared and measured. The adiabatic passage technique provides the robustness of the method against small smooth variations of field parameters, and the decoherence caused by spontaneous emission can be avoided if the dynamics follows dark states, i.e. states without components on lossy excited states. The adiabatic passage technique usually needs a long process [10]. If the required evolution time is too long, the speed of the system evolution will be slowed down, and the dissipation caused by decoherence, noise, and losses would destroy the expected dynamics finally.

Shortcuts to adiabatic passage is a promising technique for quantum information processing which actually fights against the decoherence, noise, or losses that are accumulated during a long operation time. Thus, a variety of schemes have been proposed to construct shortcuts to adiabatic passage in both theory and experiment [11–25]. Shortcuts to adiabatic passage of logical gates operation have been presented, too. Chen et al [26] proposed a scheme of shortcuts to adiabatic passage for performing a \( \pi \) phase gate, and we have proposed a scheme of shortcut to adiabatic passage for constructing the multiqubit controlled phase gate [27].

In this paper, we effectively combine the advantages of shortcuts to adiabatic passage and quantum Zeno dynamic (QZD) [28, 29] to implement a SWAP gate. It does not need the composition of element gates, but directly implements the deterministic SWAP gate through designing resonant laser pulses by the invariant-based inverse engineering. The logical SWAP gate in our scheme can be performed in a much shorter time than that based on adiabatic passage technique, and it is very robust to the decoherence caused by atomic spontaneous emission and cavity decay.

This paper is structured as follows: in section 2, we give a brief description of the preliminary theory about Lewis-Riesenfeld invariants and QZD. In section 3, we effectively combine the shortcuts to adiabatic passage and QZD to implement the deterministic SWAP gate. Section 4 shows the numerical simulation results and feasibility analysis. The conclusion appears in section 5.

2. Preliminary theory

2.1. Lewis-Riesenfeld invariants

We first make a brief introduction of the Lewis-Riesenfeld invariants theory [30, 31]. Considering a system which is governed by a time-dependent Hamiltonian \( H(t) \), we can seek the
time-dependent Hermitian invariants $I(t)$ related to the original Hamiltonian $H(t)$ to satisfy

$$i\hbar \frac{\partial I(t)}{\partial t} = [H(t), I(t)].$$

Obviously, its expectation value remains constant all the time, and may drive the system state along an initial eigenstate of $I(t)$. For the time-dependent Schrödinger equation $i\hbar \frac{\partial \Psi(t)}{\partial t} = H(t)|\Psi(t)\rangle$, the solution can be expressed by the superposition of dynamical modes $|\Phi_n(t)\rangle$ of the invariants $I(t)$

$$|\Psi(t)\rangle = \sum_n C_n e^{i\alpha_n(t)}|\Phi_n(t)\rangle,$$

where $n = 0, 1, ..., \text{and } C_n$ is one of the time-independent amplitudes, $\alpha_n$ is the Lewis-Riesenfeld phase. $|\Phi_n(t)\rangle$ is one of the orthonormal eigenvectors of the invariant $I(t)$ with the corresponding real eigenvalue $\lambda_n$, satisfying $I(t)|\Phi_n(t)\rangle = \lambda_n|\Phi_n(t)\rangle$. And the Lewis-Riesenfeld phase satisfies

$$\alpha_n(t) = \frac{1}{\hbar} \int_0^t dt' \langle \Phi_n(t') | i\hbar \frac{\partial}{\partial t'} - H(t') | \Phi_n(t') \rangle.$$

### 2.2. Quantum Zeno dynamics

The quantum Zeno effect was first proposed by Misra and Sudarshan, and it can be used to reduce the influence of the decoherence and dissipation by inhibiting the transition between quantum states through the frequent measurements [32]. We consider a system which is governed by the Hamiltonian

$$H_C = H_{\text{obs}} + KH_{\text{meas}},$$

where $H_{\text{obs}}$ is the Hamiltonian of the investigated quantum system and $H_{\text{meas}}$ is the interaction Hamiltonian performing the measurement. $K$ is a coupling constant, and when $K \to \infty$, the whole system is governed by the evolution operator

$$U(t) = \exp \left[-it \sum_n (K\lambda_n P_n + P_n H_{\text{meas}} P_n) \right].$$

where $P_n$ is one of the eigenprojections of $H_{\text{meas}}$ with eigenvalues $\lambda_n$.

### 3. Shortcuts to adiabatic passage for the deterministic SWAP gate

We consider a system, in which the atoms are fixed inside an optical cavity, as shown in figure 1. Each atom possesses three ground states $|0\rangle$, $|1\rangle$, $|a\rangle$ and two excited states $|e\rangle$, $|u\rangle$. The transitions $|0\rangle \leftrightarrow |e\rangle$ and $|a\rangle \leftrightarrow |e\rangle$ are coupled to the laser pulses, with the corresponding Rabi frequencies $\Omega_a(t)$ and $\Omega_e(t)$, respectively. The transition $|1\rangle \leftrightarrow |e\rangle$ is strongly coupled to the single mode cavity field with the coupling constant $g$. And the auxiliary excited state $|u\rangle$ is only used to implement the one-qubit operation.

Now we show the scheme to realize a deterministic SWAP gate. The initial state $|\Psi_0\rangle$ of the two atoms and the cavity field is defined as

$$|\Psi_0\rangle = a_{00}|00\rangle_\alpha|0\rangle_C + a_{01}|01\rangle_\alpha|0\rangle_C + a_{10}|10\rangle_\alpha|1\rangle_C + a_{11}|11\rangle_\alpha|1\rangle_C,$$

where $|nm\rangle_\alpha|0\rangle_C$ denotes atoms $A$ and $B$ are in the state $|nm\rangle_\alpha|0\rangle_C$ and the cavity is in vacuum state $|0\rangle_C$. $\alpha_{nm}$ denotes the amplitude of the state $|nm\rangle_\alpha|0\rangle_C$, and satisfies the normalization condition. The SWAP gate makes the values of the two qubits exchange and then the output state becomes

$$|\Psi\rangle = a_{00}|00\rangle_\alpha|0\rangle_C + a_{01}|01\rangle_\alpha|1\rangle_C + a_{10}|10\rangle_\alpha|1\rangle_C + a_{11}|11\rangle_\alpha|0\rangle_C.$$
The Hamiltonian of the system reads

$$H = \alpha_{00}|00\rangle_\Lambda|00\rangle_\Lambda + \alpha_{01}|01\rangle_\Lambda|00\rangle_\Lambda + \alpha_{10}|01\rangle_\Lambda|00\rangle_\Lambda + \alpha_{11}|11\rangle_\Lambda|00\rangle_\Lambda,$$

thus, the SWAP gate is obtained.

In the following, we show the details of how to realize the SWAP gate by combining the shortcuts to adiabatic passage and QZD. For the first step, the input laser pulses are resonant with A atomic transition $|0\rangle_a \rightarrow |e\rangle_a$ and B atomic transition $|a\rangle_B \rightarrow |e\rangle_B$ with the corresponding Rabi frequencies $\Omega_{\Lambda AB}(t)$ and $\Omega_{\Lambda AB}(t)$, respectively. As a result, the state of the system turns into

$$|\Psi\rangle = \alpha_{00}|00\rangle_\Lambda|00\rangle_\Lambda + \alpha_{01}|01\rangle_\Lambda|00\rangle_\Lambda + \alpha_{10}|01\rangle_\Lambda|00\rangle_\Lambda + \alpha_{11}|11\rangle_\Lambda|00\rangle_\Lambda,$$

$$H_t = H_l + H_c,$$

$$H_l = \Omega_{\Lambda AB}(t)|e\rangle_\Lambda \langle 0| + \Omega_{\Lambda AB}(t)|e\rangle_\Lambda \langle a| + \text{H.c.},$$

$$H_c = g_{\Lambda A}|e\rangle_\Lambda \langle 1| + g_{\Lambda B}|e\rangle_\Lambda \langle 1| + \text{H.c.},$$

where $a$ is the annihilation operator, and $g_{\Lambda B}$ is the coupling strength between cavity mode and the trapped atom. In the following, we will set $g_{\Lambda A} = g_{\Lambda B}$ for simplicity. Since the lasers do not couple with the atomic state $|1\rangle$, the states $|1\rangle_\Lambda|00\rangle_\Lambda$ and $|01\rangle_\Lambda|00\rangle_\Lambda$ are decoupled from the other states in this step, and will not participate the evolution any more. For the state $|00\rangle_\Lambda|00\rangle_\Lambda$, the evolution subspace can be spanned by the basis vectors $|\varphi_1\rangle = |00\rangle_\Lambda|00\rangle_\Lambda$ and $|\varphi_2\rangle = |01\rangle_\Lambda|00\rangle_\Lambda$, and $|\varphi_3\rangle = |10\rangle_\Lambda|00\rangle_\Lambda$. The Hamiltonian of the system reads

$$H_t = \Omega_{\Lambda AB}(t)|e\rangle_\Lambda \langle 0| + g_{\Lambda A}|e\rangle_\Lambda \langle 1| + \text{H.c.},$$

For the condition $g_{\Lambda A} \gg \Omega_{\Lambda AB}(t)$, the Hilbert space is split into three invariant subspaces $I_1 = \{|\varphi_1\rangle\}$, $I_2 = \{c \varphi_2\rangle - |\varphi_2\rangle\}$ and $I_3 = \{c \varphi_3\rangle + |\varphi_3\rangle\}$ with the eigenvalues $\lambda_1 = 0$, $\lambda_2 = -g_{\Lambda A}$, and $\lambda_3 = g_{\Lambda A}$. The corresponding projections are $P_{\alpha} = |\alpha\rangle \langle \alpha|$. For simplicity, since the lasers do not couple with the atomic state $|1\rangle$, the states $|1\rangle_\Lambda|00\rangle_\Lambda$ and $|01\rangle_\Lambda|00\rangle_\Lambda$ are decoupled from the other states in this step, and will not participate the evolution any more. For the state $|00\rangle_\Lambda|00\rangle_\Lambda$, the evolution subspace can be spanned by the basis vectors $|\varphi_1\rangle = |00\rangle_\Lambda|00\rangle_\Lambda$ and $|\varphi_2\rangle = |01\rangle_\Lambda|00\rangle_\Lambda$, and $|\varphi_3\rangle = |10\rangle_\Lambda|00\rangle_\Lambda$. The Hamiltonian of the system reads

$$H_{\text{eff}} = g_{\Lambda A}|e\rangle_\Lambda \langle 1| + g_{\Lambda B}|e\rangle_\Lambda \langle 1|,$$

which indicates that the transition between $|00\rangle_\Lambda|00\rangle_\Lambda$ and $|01\rangle_\Lambda|00\rangle_\Lambda$ is eliminated. For another state $|01\rangle_\Lambda|00\rangle_\Lambda$, the system evolves in the subspace spanned by the basis vectors $|\varphi_4\rangle = |01\rangle_\Lambda|00\rangle_\Lambda$, $|\varphi_5\rangle = |e1\rangle_\Lambda|00\rangle_\Lambda$, $|\varphi_6\rangle = |11\rangle_\Lambda|00\rangle_\Lambda$, $|\varphi_7\rangle = |1e\rangle_\Lambda|00\rangle_\Lambda$, and $|\varphi_8\rangle = |1a\rangle_\Lambda|00\rangle_\Lambda$. The evolution system is governed by the Hamiltonian

$$H_2 = \Omega_{\Lambda AB}(t)|e\rangle_\Lambda \langle 0| + g_{\Lambda A}|e\rangle_\Lambda \langle 1| + \text{H.c.},$$

By using the similar way to the above processes, the effective Hamiltonian of this system reads

$$H_{\text{eff}} = \frac{1}{\sqrt{2}} (\mu |e\rangle \langle 0| \Omega_{\Lambda AB}(t) |\phi_1\rangle + \Omega_{\Lambda AB}(t) |\phi_3\rangle + H.c.),$$

where $|\mu\rangle = \frac{1}{\sqrt{2}} (-|\phi_2\rangle + |\phi_4\rangle)$. To speed up the transition from $|\phi_1\rangle$ to $-|\phi_3\rangle$ by the invariant-based inverse engineering, we must find out the invariant Hermitian operator $I(t)$ which satisfies (1). Since the effective Hamiltonian $H_{\text{eff}}$ possesses the SU(2) dynamical symmetry, the invariant $I(t)$ can be given by

$$I(t) = \frac{1}{\sqrt{2}} \chi (\cos \nu \sin \beta |\mu\rangle \langle \phi_1| + \cos \nu \cos \beta |\mu\rangle \langle \phi_3| + i \nu |\phi_1\rangle \langle \phi_3| + H.c.),$$

where $\chi$ is an arbitrary constant with units of frequency to keep $I(t)$ with dimensions of energy. The time-dependent auxiliary parameters $\nu$ and $\beta$ satisfy the equations

$$\nu = \frac{1}{\sqrt{2}} (\Omega_{\Lambda AB}(t) \cos \beta - \Omega_{\Lambda AB}(t) \sin \beta),$$

$$\beta = \frac{1}{\sqrt{2}} \tan \nu (\Omega_{\Lambda AB}(t) \cos \beta + \Omega_{\Lambda AB}(t) \sin \beta).$$

From (19) the expressions of $\Omega_{\Lambda AB}(t)$ and $\Omega_{\Lambda AB}(t)$ can be derived as follows:

$$\Omega_{\Lambda AB}(t) = \sqrt{2} (\beta \cot \nu \sin \beta + \nu \cos \beta),$$

$$\Omega_{\Lambda AB}(t) = \sqrt{2} (\beta \cot \nu \cos \beta - \nu \sin \beta),$$

where the dot denotes a time derivative. The eigenstates of the invariant $I(t)$ are

$$|\Phi_0(t)\rangle = \cos \nu \cos \beta |\phi_1\rangle - \sin \nu |\mu\rangle - \cos \nu \sin \beta |\phi_3\rangle,$n

$$|\Phi_\beta(t)\rangle = \frac{1}{\sqrt{2}} \left( \sin \nu \cos \beta \pm \sin \beta |\phi_1\rangle + i \cos \nu |\mu\rangle \right) - \sin \nu \sin \beta - i \cos \beta |\phi_3\rangle,$n

with the eigenvalues $\epsilon_0 = 0$ and $\epsilon_\pm = \pm 1$, respectively. The solution of the Schrödinger equation $i \hbar \frac{\partial |\Psi(t)\rangle}{\partial t} = H(t)|\Psi(t)\rangle$ can be written as the superposition of the eigenstates of $I(t)$

$$|\Psi(t)\rangle = \sum_{n = 0, \pm} C_n e^{i\alpha_n |\Phi_n(t)\rangle},$$

where $\alpha(t)$ is the Lewis-Riesenfeld phase in (3), and $C_n$ is a time-independent amplitude. In order to get the final state $-|\phi_3\rangle$, we choose the parameters appropriately

$$\nu(t) = \epsilon, \quad \beta(t) = \frac{\pi t}{2 \nu},$$

where $\epsilon$ is a time-independent small value. From a detailed calculation, we can obtain

$$\Omega_{\Lambda AB}(t) = \frac{\pi}{\sqrt{2} \nu} \cot \epsilon \sin \frac{\pi t}{2 \nu},$$

$$\Omega_{\Lambda AB}(t) = \frac{\pi}{\sqrt{2} \nu} \cot \epsilon \cos \frac{\pi t}{2 \nu}.$$

When $t = t_f$, it
\[ |\Psi(t_f)\rangle = -\sin \epsilon \sin \alpha |\phi_1\rangle \\
+ (-i \sin \epsilon \cos \epsilon + i \sin \epsilon \cos \epsilon \cos \alpha |\mu\rangle \\
+ (-\cos^2 \epsilon - \sin^2 \epsilon \cos \alpha |\phi_2\rangle, \]  
where \( \alpha = \pi/2 \sin \epsilon \) and \( \alpha_1 \) and \( \alpha_2 \) are the Rabi frequencies. As long as \( \alpha \) satisfies the condition \( \alpha = 2\pi n (N = 1, 2, 3, \ldots) \), \( |\Psi(t_f)\rangle = -|\phi_2\rangle = -|1\rangle_{AB}|0\rangle_C \) can be achieved. Then the first step is realized successfully, and \( |\Psi(t_f)\rangle \) can be obtained.

The second step is just similar to the first step. In this step, the laser pulses are resonant with \( B \) atomic transition \( |0\rangle_B \leftrightarrow |e\rangle_B \) and \( A \) atomic transition \( |a\rangle_A \leftrightarrow |e\rangle_A \) with corresponding Rabi frequencies \( \Omega_{\text{AB}}(t) = \frac{\pi}{2\sqrt{2}} \cot \epsilon \sin \frac{\pi}{2\sqrt{2}} \) and \( \Omega_{\text{ad}}(t) = \frac{\pi}{2\sqrt{2}} \cot \epsilon \cos \frac{\pi}{2\sqrt{2}} \), respectively. In this step, the states \( |0\rangle_{AB}|0\rangle_C \), \( -|1\rangle_{AB}|0\rangle_C \) and \( |1\rangle_{AB}|0\rangle_C \) will be remained the same, while the population of the state \( |1\rangle_{AB}|0\rangle_C \) is completely transferred to the state \( -|1\rangle_{AB}|0\rangle_C \), then \( |\Psi(t)\rangle = -|0\rangle_{AB}|0\rangle_C - \alpha_0|1\rangle_{AB}|0\rangle_C - \alpha_0|0\rangle_{AB}|1\rangle_C + \alpha_1|1\rangle_{AB}|0\rangle_C \) can be obtained.

The third step is just a one-qubit operation with the help of the auxiliary state \( |a\rangle_{AB} \). The input laser pulses are resonant with \( A(B) \) atomic transition \( |0\rangle_{AB} \leftrightarrow |a\rangle_{AB} \) and \( |a\rangle_{AB} \leftrightarrow |a\rangle_{AB} \) with the corresponding Rabi frequencies \( \Omega_{\text{ad}}(t)' = \frac{\pi}{2\sqrt{2}} \cot \epsilon \sin \frac{\pi}{2\sqrt{2}} \) and \( \Omega_{\text{AB}}(t)' = \frac{\pi}{2\sqrt{2}} \cot \epsilon \cos \frac{\pi}{2\sqrt{2}} \), respectively. The Hamiltonian in this step is given as
\[
H_3 = \sum_{i=A,B} \Omega_{\text{ad}}(t)'|i\rangle_u\langle i| + \Omega_{\text{AB}}(t)'|a\rangle_u\langle a| + \text{H.c.} \tag{26}
\]
For the initial atomic state \( |a\rangle_{AB} \), the population will finally transfer to the state \( -|0\rangle_{AB} \) by using the similar way in the first step. Thus, we can obtain \( |\Psi(t)\rangle = \alpha_0|0\rangle_{AB}|0\rangle_C + \alpha_0|0\rangle_{AB}|1\rangle_C + \alpha_0|0\rangle_{AB}|0\rangle_C + \alpha_0|1\rangle_{AB}|0\rangle_C \) that is the result of the SWAP gate. Figure 2 represents these three steps for constructing the SWAP gate.

4. Numerical simulations and feasibility analysis

In the following, we make the numerical simulations to verify the validity of the SWAP gate. Figure 3(a) shows the time-dependent laser pulse \( \Omega_{\text{ad}}(t)g(t) \) as a function of \( g(t) \) for a fixed value \( \epsilon = 0.25 \), \( g_A = g_B = 10g_0 \) and \( t_f = 20g_0 \). With these parameters, the Zeno condition can be met well. The populations of the states \( |01\rangle_{AB}|0\rangle_C \) and \( |10\rangle_{AB}|0\rangle_C \) swap perfectly, as shown in figures 3(a) and (b). Whether a scheme is available largely depends on the robustness to the loss and decoherence, so in the following, we consider the effects of loss and decoherence on implementing the SWAP gate. The corresponding master equation for the whole system density matrix \( \rho(t) \) has the following form:

\[
\rho(t) = -i[H_{\text{total}}, \rho(t)] - \frac{\gamma_0}{2}[a_d a(t) - 2a(t)a^d - \rho(t)a^d a(t)] \\
- \frac{\gamma_1}{2} \sum_{l=e,u} \sum_{k=0,1,\ldots} [\sigma^A_{lk}(t) - 2\sigma^A_{lk}(t)\sigma^A_{lk} + \rho(t)\sigma^A_{lk}] \\
- \frac{\gamma_2}{2} \sum_{l=e,u} \sum_{k=0,1,\ldots} [\sigma^B_{lk}(t) - 2\sigma^B_{lk}(t)\sigma^B_{lk} + \rho(t)\sigma^B_{lk}], \tag{27}
\]
where \( H_{\text{total}} = H_1 + H_2 + H_3 \). \( \gamma \) is the cavity decay rate, \( \gamma_{AB} \) is \( A(B) \) atomic spontaneous emission rate from the excited state \( |l\rangle_{AB}(l = e, u) \) to the ground state \( |k\rangle_{AB}(k = 0, 1, a) \), respectively. \( \sigma_{lk} = |l\rangle\langle k| \). For simplicity, we assume \( \gamma_e = \gamma_u = \gamma \) and the initial condition \( \rho(0) = |\Psi_0\rangle\langle \Psi_0| \). In figure 4, the fidelity of the SWAP gate is plotted versus the dimensionless parameter \( \gamma/g \) with different values of \( e/g_0 \) by numerically solving the master (27). From figure 4 we can see that the fidelity of our SWAP gate is higher than 97.6% even when \( \gamma = 0.1g_{AB} = g_0 \) and \( \kappa = g_{AB} = 10g_0 \). It shows that the SWAP gate in our scheme is robust against decoherence due to cavity decay and atomic spontaneous emission.

Now we give a brief analysis of the feasibility in experiment for this scheme. The scheme can be realized with trapped ions and nitrogen-vacancy color center in diamond [34], cavity QED systems [35–37] or with impurity levels in a solid, such as...
In this case, $\gamma \approx 0.0034g_{AB0} = 0.034g_{g0}$, $\kappa \approx 0.0046g_{AB0} = 0.040g_{g0}$, and with these parameters the fidelity of the SWAP gate can reach 99%. So our scheme is robust against both the cavity decay and atomic spontaneous emission and could be very promising and useful for quantum information processing.

5. Conclusion

In summary, we have proposed a promising scheme for implementing the SWAP gate through the shortcut to adiabatic passage and QZD instead of relying on the compositions of a large number of elementary gates. We also study the influences of atomic spontaneous emission and cavity decay on the fidelity through numerical simulation. The numerical simulation results show that our scheme is very robust against the decoherence caused by atomic spontaneous emission and cavity decay. We believe that our scheme will be useful to realize quantum algorithms, such as Shor’s algorithm for prime factoring or Grover’s algorithm for database search.

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Pr$^{3+}$ ions in Y$_2$SiO$_5$ crystal [38]. In recent experiments, the fabrication of various high-Q microcavities including whispering-gallery-mode cavities [39, 40], micropost cavities, and one- or two-dimensional photonic-crystal microcavities [41, 42] also have been well designed. And the suitable parameters of toroidal microcavity system for strong-coupling cavity QED have been investigated and the parameters can be chosen as $(g, \kappa, \gamma)/2\pi = (750, 3.5, 2.62)$ MHz [40].

Figure 3. (a) Temporal profile of the time dependence Rabi frequencies $\Omega(t)/g_0$ versus $t/g_0$ with $\Omega(t) = \Omega_A(t)$ (dash blue line), $\Omega_B(t)$ (solid blue line), $\Omega_{AB}(t)$ (dash red line), $\Omega_{AB}(t)$ (solid red line), $\Omega'_{AB}(t)$ (solid green line), $\Omega_{AB}(t)$ (dash green line). (b) Time evolutions of the populations of the corresponding system states with the initial state $|01\rangle$. (c) Time evolutions of the populations of the corresponding system states with the initial state $|10\rangle$. The system parameters are set to be $\varepsilon = 0.25$, $g_A = g_B = 10g_0$ and $\gamma = 20g_0$.

Figure 4. The effect of atomic spontaneous emission $\gamma$ on the fidelity of the SWAP gate with different values of the cavity decay $\kappa$. In this case, $\gamma \approx 0.0034g_{AB0} = 0.034g_{g0}$, $\kappa \approx 0.0046g_{AB0} = 0.040g_{g0}$, and with these parameters the fidelity of the SWAP gate can reach 99%. So our scheme is robust against both the cavity decay and atomic spontaneous emission and could be very promising and useful for quantum information processing.
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