Instant preheating in a scale invariant two measures theory

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Abstract

The instant preheating mechanism in the framework of a scale invariant two measures theory is studied. We introduce this mechanism into a non oscillating inflationary model as another possible solution to the reheating of the universe in this theory. In this framework, we consider that the model includes two scalar matter fields, the first a dilaton field, that transforms under scale transformations and it will be considered also as the field that drives inflation and the second, a scalar field which will interact with the inflaton through an effective potential. By assuming this interaction term, we obtain a scenario of instant radiation or decay of particles according to the domain the effective mass of the field that interacts with the inflaton. Also, we consider a scale invariant Yukawa interaction and then after performing the transition to the physical Einstein frame we obtain an expression for the decay rate from our scalar field going into two fermions. Besides, from specific decay rates, different constraints and bounds for the coupling parameters associated with our model are found.

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I. INTRODUCTION

It is well known that the inflationary universe models have solved some problems present in the hot big bang model, such as the horizon, flatness, monopole problem etc. However, the biggest feature of the inflationary stage is that it provides a causal interpretation to explicate the observed anisotropy of the cosmic microwave background (CMB) radiation and moreover this framework gives account of the distribution of large scale structures.

In order to study the inflationary stage, the scalar field or inflaton plays a fundamental role in the evolution of the early universe. In this context, the inflationary epoch can be described as a regime with a rapid accelerated expansion occurred during the early universe produced by the inflaton.

To describe the inflationary epoch, we have different gravitation theories and models that give account of the evolution of the early universe. In particular we can distinguish the scale invariant two measures theory that produces an accelerated expansion of the universe by means of the evolution of a single scalar field or inflaton field with an effective potential.

In relation to the two measures theories models, these utilize a non Riemannian measure of integration in the frame of the action. In particular in the situation of a scale invariant theory, the scale invariance was spontaneously broken from the equations of motion related with the degrees of freedom on the non Riemannian measure of integration in the framework of the action. In this sense, we can mention that the degrees of freedom that determine a non Riemannian measure of integration in four dimensions can be represented by scalar fields. In this sense, utilizing the measure of integration and also in the frame of the action different models with several scalar fields in four dimensions have been studied in the literature. The application of this scale invariant two measures theory to an emergent universe scenario was developed in ref. It corresponds to a non singular cosmological type of stage previous to inflation (emergent scenario) in which the universe begins as a static universe to later connect with the inflationary epoch. In order to use the two measures theory to describe the dark energy in the present universe, in ref. was considered that the two measures of integration leading to two independent integration constants and these constants break scale invariance, and characterize the strength of the dark energy density. Additionally, in ref. the curvaton reheating mechanism in a scale invariant two measures
theory defined in terms of two independent non-Riemannian volume forms was studied. In this context, the model has two scalar matter fields, a dilaton and it transforms under scale transformations and it corresponds to the inflaton field of the inflationary model while the other scalar field does not transform under scale transformations played the role of a curvaton field\[15\]. The introduction of the curvaton field in this scenario occurs due to the problematic of connecting the inflationary epoch in the framework of the scale invariant two measures theory, with the reheating of the universe and its subsequent connection with the radiation era \[16, 17\].

In relation to the reheating of the universe, we have that at the end of inflationary epoch the energy density of the universe can be interpreted as a combination of kinetic and potential energies of the inflaton to late dominate the kinetic energy \[18\]. In the process of reheating of the universe, the matter and radiation of the universe are produced generally through the decay of the scalar field or another field (decay parameter), while the temperature increases in many orders of magnitude and then the universe connects with the radiation regime of the standard big-bang model \[19, 20\].

In order to study the reheating of the universe the scenario of oscillations of the inflaton field (at the minimum of the potential) is an important part for the standard mechanism of reheating. Nevertheless, it is possible to find some inflationary models where the effective potential associated to the inflaton does not have a minimum and then the scalar field does not oscillate and then the standard mechanism of reheating does not work \[21\]. Thus, these kinds of models with these effective potentials are known in the literature as non-oscillating models, or simply NO models \[22\]. Interesting examples of these are the Quintessential inflation models which connect an early inflation with a late slowly accelerated phase, as the models considered by Peebles and Vilenkin \[23\] as well as \[8–10, 13, 24, 25\].

Originally, in order to solve this problematic for these NO models was the introduction of a mechanism that incorporates the gravitational particle production \[26\]. Nevertheless, this mechanism of reheating of the universe becomes inefficient and it presents several problems associated with the observational data, see ref. \[27\].

The introduction of the curvaton field as other mechanism of reheating after inflation in these NO models was considered in refs. \[16, 17, 20, 22\]. In this context, the decay rate of the scalar curvaton field into conventional matter gives account a mechanism of reheating of the universe. In this sense, introducing an effective potential associated to the curvaton field
is possible to reheat the universe \cite{20, 28}. This model of reheating does not need to introduce an interaction between the scalar field that drives inflation and other scalar field \cite{17, 20, 29}.

Another mechanism of reheating known in the literature is called instant preheating \cite{29}. In this scenario, after of the inflationary regime the inflaton field moves quickly producing particles which can be bosons and/or fermions. This mechanism corresponds to a non-perturbative process and it happens almost instantly \cite{29} and also this scenario does not need oscillations or parametric resonance of the inflaton field. In this sense, because the production of particles can happen immediately after the end of inflationary regime, within less than one oscillation of the field that drives inflation, the reheating of the universe can occurs efficiently. In order to study the instant reheating is indispensable to consider the interaction between the scalar field that drives inflation and another scalar field $\sigma$. Depending of the interaction between the inflaton field and the field $\sigma$ (via an effective potential) the effective masses of the particles $\sigma$ can be small or large at the moment when the particles are produced for later increase or decrease when the inflaton field moves to large values. In this mechanism the production of particles $\sigma$ begins nearly instantaneously assuming the nonadiabatically condition given by the ratio between the evolution of the effective masses of the particles-$\sigma$ and the square of these \cite{29}. For a review of reheating see refs. \cite{29, 31} and for instant preheating, see \cite{24, 32}.

The goal of this investigation is to analyze the instant preheating in a scale invariant two independent non-Riemannian volume-forms. In this sense, we investigate how the interaction term between the inflaton field that drives inflation and other scalar field (from the effective potential) in this theory modifies the results on the produced particles in this scenario and preheating of the universe. In this form, we will analyze the instant preheating in our model and in particular the energy density of produced particles and the decay rates in order to in account of the temperature and constraints on the parameters given by the observations.

For the application of the developed formalism, we will analyze some examples assuming two decay parameters. From these decay rates, we will study the different conditions of time, in order to obtain the bounds on the coupling parameters associated to these decay rates.

The outline of the paper goes as follow: in Sect. II we give a brief description of two independent non-Riemannian volume-forms. In Sect. III the instant preheating scenario is analyzed. The Sect. IV describes the instant radiation in which the energy density of the
field-σ decays as radiation. The Sect. V explains the radiation from the decay. In Sect. VI we obtain the decay rate for the particles σ going into two fermions. The Sect. VII analyzes the decay rate and constraints on the parameters of our model, and in Sect. VIII includes our conclusions.

II. TWO INDEPENDENT NON-RIEMANNIAN VOLUME-FORMS

In this section, we discuss a brief description of the two independent non-Riemannian volume-forms. We follow the general structure of the references [9, 10], but now we will enrich the field content of the theory with a new field σ which will not transform under scale transformations, so we write,

$$S = \int d^4x \Phi_1(A)[R + L^{(1)}] + \int d^4x \Phi_2(B) \left[ L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} \right],$$

where Φ1(A) and Φ2(B) are two independent non-Riemannian volume-forms and defined as

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda}, \quad \text{and} \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda},$$

respectively.

The quantities L(1,2) correspond to two different Lagrangians of two scalar fields, the dilaton ϕ, which will play the role of an inflaton and an additional scalar field σ. In this form, the Lagrangians can be written as

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi - \frac{1}{2}g^{\mu\nu}\partial_\mu \sigma \partial_\nu \sigma - \frac{\mu^2 \sigma^2}{2} \exp\{-\alpha \phi\} - V(\phi),$$

where $$V(\phi) = f_1 \exp\{-\alpha \phi\}$$ and the Lagrangian

$$L^{(2)} = -\frac{b}{2}e^{-\alpha \phi}g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi + U(\phi),$$

in which $$U(\phi) = f_2 \exp\{-2\alpha \phi\}$$. Here the quantities α, f1, f2 are dimension full positive parameters and the parameter b is a dimensionless one. Also, the quantity Φ(H) denotes the dual field strength of a third auxiliary 3-index antisymmetric tensor gauge field defined as

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda}. \quad (3)$$

We mention that the scalar potentials have been chosen such that the action given by eq.(1) is invariant under global Weyl-scale transformations with which

$$g_{\mu\nu} \to \lambda g_{\mu\nu}, \quad \Gamma^\mu_{\nu\lambda} \to \Gamma^\mu_{\nu\lambda}, \quad \phi \to \phi + \frac{1}{\alpha} \ln \lambda, \quad \sigma \to \sigma,$$
Analogously, from the invariant under we have multiplied by an exponential factor the scalar kinetic term in $L^{(2)}$ and also by the scalar curvature $R$ and $R^2$ couple to the two different modified measures. The equations of motions of the measure fields lead to several simple relations. First, the variation of the tensor field $H$ implies that the ratio between the measure $\Phi_2$ and $\sqrt{-g}$ is a constant:

$$\frac{\Phi_2}{\sqrt{-g}} = \chi_2 = \text{constant}. \quad (4)$$

Likewise, the variation with respect to $\Phi_1$ and $\Phi_2$ leads to the the Lagrangians coupling to $\Phi_1$ and $\Phi_2$ being constants that we may call $M_1$ and $M_2$:

$$R + L^{(1)} = -M_1 = \text{constant}, \quad (5)$$

and

$$L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{constant}, \quad (6)$$

while equation (11) does not break scale invariance, since the two measures $\Phi_2$ and $\sqrt{-g}$ transform identically under scale transformations. The same cannot be said however concerning (5) and (6) while the left hand side in these equations transforms, the right hand side ($M_1$ and $M_2$) are constants and does not transform. We get then spontaneous breaking of scale invariance.

We proceed in the so called first order formalism, where the connection is at the action level independent of the metric, in this case we can vary with respect to the metric and the consistency with the equations (5) and (6) allows us to solve for $\frac{\Phi_2}{\sqrt{-g}} = \chi_1$, which is given by,

$$\frac{1}{\chi_1} = \frac{V + \frac{\mu^2 a^2}{2} e^{-\alpha \varphi} - M_1}{2 \chi_2 (U + M_2)}. \quad (7)$$

Here we have considered the case in which $\epsilon = 0$ and $b = 0$, respectively.

Defining Einstein frame by a conformal transformation, we obtain an effective action, which in the case of $\epsilon = 0$ and $b = 0$ is governed by a canonical minimally coupled scalar field with the following effective Lagrangian given by

$$L_{eff} = -\frac{1}{2} \tilde{g}^{\mu\nu} \partial \varphi \mu \partial \varphi \nu - \frac{1}{2} \tilde{g}^{\mu\nu} \partial \sigma \mu \partial \sigma \nu - U(\varphi, \sigma), \quad (8)$$

where the Weyl-rescaled metric $\tilde{g}_{\mu\nu}$ is defined as

$$\tilde{g}_{\mu\nu} = \chi_1 g_{\mu\nu}. \quad (9)$$
and the effective potential is given by

\[ U_{\text{eff}}(\phi, \sigma) = \left( V + \frac{\mu^2 \sigma^2}{2} e^{-\alpha \phi} - M_1 \right)^2 = \left( \frac{f_1 e^{-\alpha \phi} + \mu^2 \sigma^2}{2} e^{-\alpha \phi} - M_1 \right)^2. \]  

(10)

III. INSTANT PREHEATING

In order to explain the instant preheating scenario in our model we will consider that the effective potential given by Eq. (10) presents the interaction term given by

\[ U_{\text{eff}}(\phi, \sigma) \approx \mu^2 \beta^2 \sigma^2 e^{-2 \alpha \phi} = m_1^2 \sigma^2 e^{-2 \alpha \phi}, \]

(11)

where the constant \( \beta \) is defined as \( \beta^2 = \frac{f_1}{2 \chi_2 M_2} \) and \( m_1 = \mu \beta \). Here we have assumed that \( M_2 \gg f_2 e^{-2 \alpha \phi} \) and \( f_1 e^{-\alpha \phi} \gg M_1 \), since during the inflationary scenario we have used the values \( M_1 \sim 10^{-60} \), \( f_1 \simeq f_2 \sim 10^{-8} \) and \( M_2 \sim 1 \), from observational data, see ref. [9]. Thus, the effective mass of the scalar field \( \sigma \) becomes

\[ m_{\sigma} = \mu \beta e^{-\alpha \phi} = m_1 e^{-\alpha \phi}, \]

(12)

since the effective mass of \( \sigma \) is defined as \( m_{\sigma}^2 = \partial^2 U_{\text{eff}}(\varphi, \sigma) / \partial \sigma^2 \).

Following refs. [36, 37] we will consider that the production of particles \( \sigma \) starts to change nonadiabatically under the condition \( |\dot{m}_{\sigma}| \geq m_{\sigma}^2 \) with which the scalar field \( \varphi \) can be written as

\[ \varphi \sim -\frac{1}{\alpha} \ln \left( \frac{\alpha |\dot{\varphi}_0|}{\mu \beta} \right), \]

(13)

where \( \dot{\varphi}_0 \) denotes the value of the velocity of the scalar field when this field rolls on the asymptotically flat potential after of the inflationary epoch. During this stage the mechanism of particle production starts nearly instantaneously in the time interval given by

\[ \Delta t \sim \frac{|\varphi|}{|\dot{\varphi}_0|} \sim \frac{1}{\alpha |\dot{\varphi}_0|} \left| \ln \left( \frac{\alpha |\dot{\varphi}_0|}{\mu \beta} \right) \right| > 0. \]

(14)

Also, we mention that during this time all effects associated to the expansion of the universe can be ignored in the process of particle production.

Now, in order to determine the velocity of the scalar field \( \dot{\varphi}_0 \), we can consider the break down approximation in which

\[ \ddot{\varphi} \simeq -\frac{\partial V(\varphi)}{\partial \varphi} = \frac{\alpha f_1^2}{2 \chi_2 M_2} e^{-2 \alpha \phi}, \]

(15)
where we have considered that the potential $V(\varphi) = \frac{f_1^2}{4x_2M_2^2}e^{-2\alpha \varphi}$ from eq. (10), see ref. [9].

Under this approximation, we find that the solution of the eq. (15) for the scalar field $\varphi(t)$ results

$$\varphi(t) = \frac{1}{\alpha} \ln \left[ \frac{e^{-\alpha \sqrt{C_1(t+C_2)}}}{2} \left( k_1^2 + \frac{e^{2\alpha \sqrt{C_1(t+C_2)}}}{\alpha C_1} \right) \right],$$  \hspace{1cm} (16)

where $C_1$ and $C_2$ are two integration constants and $k_1$ is defined as $k_1^2 = \frac{\alpha f_1^2}{2x_2M_2^2}$.

From this solution we can find that the velocity of the scalar field $\dot{\varphi}$ is given by

$$\dot{\varphi}(t) = \sqrt{C_1} \left( \frac{e^{2\alpha \sqrt{C_1(t+C_2)}} - \alpha C_1 k_1^2}{e^{2\alpha \sqrt{C_1(t+C_2)}} + \alpha C_1 k_1^2} \right),$$  \hspace{1cm} (17)

which for $\alpha$ big the above expression big quickly approaches the asymptotic value $\sqrt{C_1}$ i.e., $\dot{\varphi}_0 \sim \sqrt{C_1}$.

Thus, in order to obtain the value of the $\dot{\varphi}_0$, we can consider that the initial conditions for the scalar field and its velocity can be fixed at the end of inflationary epoch. In this way, we assume the slow roll approximation in which at the end of inflation we have $\varphi_{\text{end}} = -\alpha^{-1} \ln(2\alpha M_1/f_1)$ and $\dot{\varphi}_{\text{end}} = \frac{2M_1\alpha^2}{\sqrt{3}x_2M_2^2}$, see ref. [9]. From these initial conditions and considering the eqs. (16) and (17), we find that the asymptotic velocity of the scalar field becomes

$$\dot{\varphi}_0 \simeq \sqrt{C_1} \simeq \dot{\varphi}_{\text{end}} \sqrt{1 + \frac{3}{2\alpha^2}}.$$  \hspace{1cm} (18)

Here we note that for large-$\alpha$ the velocity $\dot{\varphi}_0 \simeq \dot{\varphi}_{\text{end}}$. Thus, the time interval given by eq. (14) can be approximated to $\Delta t \sim (\alpha |\dot{\varphi}_{\text{end}}|)^{-1} \ln \left( \frac{\alpha |\dot{\varphi}_{\text{end}}|}{\mu \beta} \right)$.

On the other hand, the occupation number $n_k$ of the particles $\sigma$ with momentum $k$ in the time interval $\Delta t$ is defined as [36–38]

$$n_k = \exp \left[ -\pi (k \Delta t)^2 \right],$$  \hspace{1cm} (19)

and then considering eq. (14) we obtain that the occupation number can be written as

$$n_k = \exp \left[ -\frac{\pi}{\alpha^2 \dot{\varphi}_0^2} \left( \ln \left[ \frac{\alpha |\dot{\varphi}_0|}{\mu \beta} \right] \right)^2 \right].$$  \hspace{1cm} (20)

In fact, we can assume that the definition of the occupation number given by eq. (19) still is valid for massive particles of the scalar field $\sigma$ of effective mass $m_\sigma$ under replacement of the momentum $k^2$ by $k^2 + m_\sigma^2$ [37]. Thus, eq. (19) can be modified as $n_k = \exp \left[ -\pi (k^2 + m_\sigma^2) \Delta t^2 \right]$ with which the occupation number becomes

$$n_k = \exp \left[ -\frac{\pi}{\alpha^2 \dot{\varphi}_0^2} \left( \ln \left[ \frac{\alpha |\dot{\varphi}_0|}{\mu \beta} \right] \right)^2 \right].$$  \hspace{1cm} (21)
Now, this quantity can be integrated to establish the density of $\sigma$ particles denotes by $n_\sigma$ and defined as $n_\sigma = \frac{1}{2\pi^2} \int_0^\infty dk k^2 n_k$. In this way, we find that the density of $\sigma$ particles $n_\sigma$ results

$$n_\sigma = \frac{1}{8\pi^3} \left[ \frac{\alpha |\dot{\phi}_0|}{\ln \left( \frac{\alpha |\dot{\phi}_0|}{\mu \beta} \right)} \right]^3 \exp \left[ -\frac{\pi m_\sigma^2}{\alpha^2 |\dot{\phi}_0|^2} \left( \ln \left( \frac{\alpha |\dot{\phi}_0|}{\mu \beta} \right) \right)^2 \right]. \quad (22)$$

We note that naturally in our model the number of produced particles is not exponentially suppressed, since the mass of the scalar field $\sigma$ decreases for large-$\varphi$ ($m_\sigma \propto e^{-\alpha \varphi}$). Thus, the number density of particles during their creation results $n_\sigma = \frac{1}{8\pi^3} \left[ \frac{\alpha |\dot{\phi}_0|}{\ln \left( \frac{\alpha |\dot{\phi}_0|}{\mu \beta} \right)} \right]^3$, however it decreases as $a^{-3}(t)$ with which the number of produced particles in terms of the time can be written as

$$n_\sigma = \frac{1}{8\pi^3} \left[ \frac{\alpha |\dot{\phi}_0| a_0}{a(t) \ln \left( \frac{\alpha |\dot{\phi}_0|}{\mu \beta} \right)} \right]^3. \quad (23)$$

Here we have used that at the moment of particle production the scale factor is given by $a_0$.

Additionally, we have that the energy density of produced particles $\rho_\sigma$ is defined as \[33, 34\]

$$\rho_\sigma = \frac{1}{(2\pi a)^3} \int_0^\infty n_k \sqrt{\frac{k^2}{a^2} + m_\sigma^2} \, (4\pi k^2) dk. \quad (24)$$

Here, we can note that interestingly there are two limit cases given by $m_\sigma \gg k/a$ and $m_\sigma \ll k/a$, because the effective mass of the $\sigma$-field $m_\sigma$ depends of the time. Thus, initially after of the inflationary stage, we can consider that the dominant term becomes the mass $m_\sigma$ over the physical momentum $k/a$. Later, product of the decrease in the time of the mass $m_\sigma$, the dominant term corresponds to the momentum i.e., $k/a \gg m_\sigma$. In the following, we will analyze these two limits separately.

### IV. INSTANT RADIATION

In this section we will study the process in which the energy density of produced particles of the field $\sigma$ decays as radiation. We call this process as instant radiation and it occurs for large-time when the mass of the $\sigma$-field decreases and then the effective mass tends to zero with which $m_\sigma \ll k/a$. In this situation we find that energy density of the $\sigma$-field from Eq.\((24)\) becomes

$$\rho_\sigma = B a^{-4}, \quad (25)$$
where the constant $B$ is defined as

$$B = \left[ \frac{\alpha \dot{\varphi}_0}{\left(2^{1/2} \pi \ln \left[ \frac{\alpha |\varphi_0|}{\mu^3} \right] \right)} \right]^4.$$ 

In this context, the equation of motion for the inflation field $\varphi$ including backreaction of produced $\sigma$ particles on the field $\varphi$ can be written as

$$\ddot{\varphi} + 3H \dot{\varphi} = \alpha \mu^2 \beta^2 e^{-2\alpha \varphi} \langle \sigma^2 \rangle,$$

where the expectation value $\langle \sigma^2 \rangle$ is defined as

$$\langle \sigma^2 \rangle \approx \frac{1}{2\pi^2} \int \frac{n_k k^2 dk}{\sqrt{(k/a)^2 + m^2_\sigma}}.$$

Thus, for the case of the instant radiation ($m_\sigma \ll k/a$) we find that $\langle \sigma^2 \rangle$ becomes

$$\langle \sigma^2 \rangle \approx \frac{1}{2\pi^2} \int n_k k dk = \frac{B_1}{a^2(t)},$$

where the constant $B_1$ is given by $B_1 = \sqrt{B}/(2\pi)$.

In this way, the equation of motion for the inflaton field in the situation in which the effective mass $m_\sigma \ll k/a$ including the backreaction term becomes

$$\ddot{\varphi} + 3H \dot{\varphi} = \alpha \mu^2 \beta^2 B_1 \frac{e^{-2\alpha \varphi(t)}}{a^2(t)}.$$

We observe that the backreaction effect decreases very quickly due to exponential decay product of evolution of $\varphi(t)$ that appears in the right hand side of this equation. Thus, the backreaction of produced $\sigma$ particles on $\varphi$ disappears naturally from the effective potential given by eq.(11). Also, from the condition $m_\sigma \ll k/a$ and considering that the scale factor $a(t) \propto t^{1/3}$ together with neglecting the backreaction of eq.(29), we find that the constraint for the $\alpha$ parameter becomes $\alpha > (2\sqrt{3})^{-1}$, if we want the condition $m_\sigma \ll k/a$ to be maintained during the time.

Additionally, we note that in the scenario of instant radiation, if nothing else happens, meaning non-decay of the $\sigma$ particles, and since the mass of these particles approaches to zero, then we obtain that these particles will asymptotically behave as radiation, as can be seen from eq.(26). However, we mention that this spectrum is not thermal becomes the distribution in the occupation number is not Boltzmann distribution (see eq.(19)), since the spectrum is not thermal in order to obtain a real thermal spectrum a thermalization process is required. The thermalization should bring all particle species [39, 40].
V. RADIATION FROM DECAY

In this section, we can analyze the case where the mass $m_\sigma \gg k/a$, for the dominant range of integration of the momentum. In this limit we have

$$\rho_\sigma = m_\sigma n_\sigma = \frac{\mu \beta}{8\pi^3} \frac{\alpha |\dot{\varphi}_0| a_0}{\ln \left( \frac{\alpha |\dot{\varphi}_0|}{\mu \beta} \right)} \left[ \frac{\alpha |\dot{\varphi}_0|}{\ln \left( \frac{\alpha |\dot{\varphi}_0|}{\mu \beta} \right)} \right]^3 \frac{e^{-\alpha \varphi(t)}}{a^3(t)} \propto \frac{e^{-\alpha \varphi(t)}}{a^3(t)}, \quad (30)$$

here, we have called to this stage as radiation from decay.

On the other hand, the equation of motion for the inflation field $\varphi$ after of the particles production can be written as

$$\ddot{\varphi} + 3H \dot{\varphi} = \alpha \mu^2 \beta^2 e^{-2\alpha \varphi} \langle \sigma^2 \rangle, \quad (31)$$

where the expectation value $\langle \sigma^2 \rangle$ from eq.(27) and assuming $m_\sigma \gg k/a$ can be written as

$$\langle \sigma^2 \rangle \approx \frac{1}{2\pi^2} \int \frac{n_k k^2 dk}{\sqrt{(k/a)^2 + m_\sigma^2}} \approx \frac{n_\sigma}{m_\sigma} \approx A e^{\alpha \varphi(t)} a^3(t), \quad (32)$$

in which the constant $A$ is defined as

$$A = \frac{1}{8\mu^2 \pi^3} \left[ \frac{\alpha |\dot{\varphi}_0|}{\ln \left( \frac{\alpha |\dot{\varphi}_0|}{\mu \beta} \right)} \right]^3.$$

In this form, using eq.(32) we find that eq.(31) can be rewritten as

$$\ddot{\varphi} + 3H \dot{\varphi} = \alpha m_\sigma n_\sigma = \frac{\alpha \mu \beta}{8\pi^3} \left[ \frac{\alpha |\dot{\varphi}_0|}{\ln \left( \frac{\alpha |\dot{\varphi}_0|}{\mu \beta} \right)} \right]^3 \frac{e^{-\alpha \varphi(t)}}{a^3(t)}. \quad (33)$$

In order to analyze the behavior of the field $\varphi(t)$ from eq.(33), we will consider as a first approximation neglects backreaction of produced particles. In this context, we can consider that the energy density of the $\sigma-$particles ($\rho_\sigma = m_\sigma n_\sigma$) becomes subdominant and then the right hand side of eq.(33) can be negligible. In fact, from eq.(33) (or analogously of eq.(31)), we note that naturally our effective potential gives rise to a force which produces that the inflaton field continues its movement to infinity.

Under this approximation the scale factor is given by $a(t) \sim t^{1/3}$ and then the Hubble parameter becomes $H = (3t)^{-1}$. Thus, if one neglects backreaction, the solution for the scalar field as a function of the time can be written as

$$\varphi(t) = \frac{2}{\sqrt{3}} \ln \left( \frac{t}{t_0} \right), \quad (34)$$
where the constant $t_0$ is defined as $t_0 = (3H_0)^{-1}$ and it corresponds to the initial time during the phase transition time between inflation and kination regime.

In this way, replacing eq. (34) in the equation for the energy density of produced particles given by eq. (30) we have

$$\rho_\sigma = \alpha_1 \left( \frac{t}{t_0} \right)^{-2\alpha/\sqrt{3}} \left( \frac{a_0}{a} \right)^3,$$

(35)

where the constant $\alpha_1$ is defined as

$$\alpha_1 = \frac{\mu \beta}{8\pi^3} \left[ \frac{\alpha |\dot{\phi}_0|}{\ln \left( \frac{\alpha |\dot{\phi}_0|}{\mu^3} \right)} \right]^3.$$

Additionally, during the kination regime the energy density of the background decreases as $\rho(t) \sim \dot{\phi}^2 \sim a^{-6}$ or $\rho(t) = 6H_0^2 (a_0/a)^6$ and we can consider that both densities achieve equilibrium i.e., $\rho \sim \rho_\sigma$. In this sense, if the densities $\rho$ and $\rho_\sigma$ are of the same order, we can assume that this situation occurs at the equilibrium time $t_{eq}$ given by

$$t_{eq} = \left[ \frac{2}{3\alpha_1 t_0^6} \right]^{1/\delta_2},$$

(36)

where the constants $\delta_1$ and $\delta_2$ are defined as

$$\delta_1 = -(1 + 2\alpha/\sqrt{3}), \quad \text{and} \quad \delta_2 = (1 - 2\alpha/\sqrt{3}),$$

respectively. In this way, the value of the scalar field at the time $t_{eq}$ becomes

$$\varphi(t = t_{eq}) = \varphi_{eq} = \frac{2}{\sqrt{3} \delta_2} \left[ \ln \left( \frac{2}{3\alpha_1} \right) + (\delta_1 - \delta_2) \ln t_0 \right].$$

(37)

We note that in particular for values of $\alpha \gg 1$, we have

$$\varphi_{eq} \approx - \frac{1}{\alpha} \ln \left[ \frac{2}{3\alpha_1 t_0^6} \right] = - \frac{1}{\alpha} \ln \left[ \frac{\dot{\phi}_0^2}{2\alpha_1} \right].$$

(38)

Here, we have used that $t_0 = 2/(\sqrt{3} \dot{\phi}_0)$, in which $\dot{\phi}_0 \approx \sqrt{C}$, see eq. (18).

VI. SCALE INVARIANT COUPLING OF $\sigma$ AND $\varphi$ FIELDS: DECAY RATE OF THE $\sigma$ PARTICLES TO FERMIONS

In this section, we want to analyze now a coupling of the field $\sigma$ to a fermionic spin 1/2 field $\Psi$. We will consider possible couplings while respecting scale invariance. Let us consider first the $\Psi$ kinetic term coupled to the measure $\Phi_1$

$$S = \int \Phi_1 L_{kin},$$

(39)
where \( L_{\text{kin}} \) is given by,

\[
L_{\text{kin}} = \frac{i}{2} \bar{\Psi} \left( \gamma^a e^\mu_a \overset{\rightarrow}{\nabla}_\mu \Psi - \bar{\Psi} \overset{\leftarrow}{\nabla}_\mu \gamma^a e^\mu_a \Psi \right),
\]

where

\[
\overset{\rightarrow}{\nabla}_\mu \Psi = \partial_\mu \Psi + \frac{1}{2} \omega^{ab}_\mu \sigma_{ab} \Psi,
\]

and

\[
\overset{\leftarrow}{\nabla}_\mu \Psi = \partial_\mu \bar{\Psi} - \bar{\Psi} \frac{1}{2} \omega^{ab}_\mu \sigma_{ab}.
\]

The \( \gamma^a \) matrices are metric independent (m.i.) while \( \bar{\Psi} = \Psi^\dagger \gamma^0 \) is as well m.i. Since under a scale transformation we have \( \Phi_1 \to e^\theta \Phi_1 \), then \( S_{\text{kin}} \) is invariant under

\[
\omega^{ab}_\mu \to \omega^{ab}_\mu,
\]

\[
\Psi \to e^{-\theta} \Psi,
\]

\[
\bar{\Psi} \to e^{-\theta} \bar{\Psi},
\]

and

\[
g_{\mu\nu} \to e^\theta g_{\mu\nu},
\]

which is equivalent also to \( e^a_\mu \to e^{-\theta} e^a_\mu \) and \( e^a_\mu \to e^\theta e^a_\mu \). Thus, the bilinear quantity \( \Psi \bar{\Psi} \) transforms as \( \Psi \bar{\Psi} \to e^{-2\theta} \Psi \bar{\Psi} \).

So, since \( \sigma \) is invariant under scale transformations, we see that a coupling to the measure \( \Phi_1 \) must also require a factor \( e^{\alpha \varphi} \)

\[
\sigma \Phi_1 e^{\alpha \varphi} \Psi \bar{\Psi}.
\]

Like wise, the coupling to the measure \( \Phi_2 \) (or \( \sqrt{-g} \) which transforms the same way and which is proportional to \( \Phi_2 \)), must contain a factor of \( e^{3\alpha \varphi} \) leading to an invariant term

\[
\sigma \Phi_2 e^{3\alpha \varphi} \bar{\Psi} \Psi.
\]

Thus, the "scale invariant Yukawa type interaction" between the field \( \sigma \) and the fermions must include \( \varphi \) in the following way,

\[
\int \sigma (g_1 \Phi_1 e^{\alpha \varphi} \bar{\Psi} \Psi + g_2 \Phi_2 e^{3\alpha \varphi} \bar{\Psi} \Psi) d^4 x.
\]

To properly use this interaction, we must transform to the Einstein Frame, use the Einstein Frame metric \( \bar{g}_{\mu\nu} \) and the Einstein Frame fermion field \( \Psi_{e.f.} \). For the case \( \epsilon = b = 0 \),
we have that \( \bar{g}_{\mu\nu} = \chi_1 g_{\mu\nu} \), or equivalently, \( \bar{e}_\mu = \chi_1^{-\frac{1}{2}} e_\mu \), and \( \bar{e}_\mu = \chi_1^{-\frac{1}{2}} e_\mu \). Additionally, the Einstein Frame fermion field satisfies the normal Dirac equation in the curved space \( \bar{g}_{\mu\nu} \) and must be defined as

\[
\Psi_{e.f.} = \chi_1^{-\frac{i}{2}} \Psi,
\]

and one can also check that \( \Psi_{e.f.} \) is scale invariant.

We now will look at the interaction terms after transformation to Einstein Frame, in different phases of the theory. For this we look \( \Psi \) field as a test field which is produced in a \( \sigma \) and \( \phi \) background. That means that we will not consider the effects of the \( \Psi \) field in the equation for \( \chi_1 \). In this way, we can mention that there are two interesting cases:

1. Let us consider the limit \( \phi \to -\infty \), which corresponds to the inflationary period and in this case, the constants \( M_1 \) and \( M_2 \) can be ignored. Therefore, we can obtain that the quantity \( \chi_1 = \frac{2\chi_2 f_2}{f_1} e^{\alpha \phi} \), under such conditions, we can look at the \( g_2 \) coupling: \( g_2 \sigma \Phi_2 e^{i \alpha \phi} \bar{\Psi} \Psi \) becomes in E.F. \( g_2 \chi_2^{-\frac{1}{2}} \sigma \sqrt{\bar{g}} \bar{\Psi}_{e.f.} \Psi_{e.f.} \), which is therefore \( \phi \) independent.

Similar effect takes place for the \( g_1 \) coupling in the inflationary limit \( \phi \to -\infty \) in which the quantity \( g_1 \sigma \Phi_1 e^{i \alpha \phi} \bar{\Psi} \Psi \) transforms to E.F. as \( g_1 \left( \frac{f_1}{2\chi_2 f_2} \right)^{\frac{1}{2}} \sigma \sqrt{\bar{g}} \bar{\Psi}_{e.f.} \Psi_{e.f.} \), which is therefore again \( \phi \) independent.

2. Now let us do the same calculation in the inflationary regime in which we study particle creation. In this case, the quantity \( 1/\chi_1 \) becomes

\[
\frac{1}{\chi_1} = \frac{1}{2\chi_2} \frac{V - M_1}{U + M_2},
\]

and we can neglect \( M_1 \) in the numerator and \( U \) in the denominator, obtaining therefore

\[
\frac{1}{\chi_1} = \frac{1}{2\chi_2} \frac{V}{M_2},
\]

which implies that \( \chi_1 \) results

\[
\chi_1 = \frac{2M_2 \chi_2}{f_1} e^{-\alpha \phi}.
\]

Here we see that this dependence is inverse to that of the one in the inflationary phase (where one can ignore the constants of integration \( M_1 \) and \( M_2 \)) and as a result we will get a strong \( \phi \) dependence of the \( g_1 \) and \( g_2 \) couplings.

Let us start with the \( g_2 \) coupling: \( g_2 \sigma \Phi_2 e^{\frac{i}{2} \alpha \phi} \bar{\Psi} \Psi \) becomes in E.F.

\[
g_2 \chi_2 \left( \frac{f_1}{2M_2 \chi_2} \right)^{\frac{1}{2}} \sigma \sqrt{\bar{g}} \bar{\Psi}_{e.f.} \Psi_{e.f.}
\]
We see a very strong growth of the coupling as $\varphi$ increases in this regime. Now for the $g_1$ coupling: $g_1 \sigma \Phi \frac{\alpha \varphi}{2} \Psi \Psi$ becomes in E.F. $g_1 \left( \frac{f_1}{2\chi_2 M_2} \right) \frac{1}{2} e^{\alpha \varphi} \sigma \sqrt{E F} \Psi_{e.f.} \Psi_{e.f.}$, which again grows as $\varphi$ grows.

In this form, we can define that the decay rate for $\sigma$ going into two fermions becomes

$$\Gamma(\sigma \rightarrow \Psi \Psi) = \frac{g^2 m_\sigma}{8\pi},$$

(54)

where the $g$-coupling is given by

$$g = g_1 \left( \frac{f_1}{2\chi_2 M_2} \right) \frac{1}{2} e^{\alpha \varphi} + g_2 \chi_2 \left( \frac{f_1}{2M_2 \chi_2} \right) \frac{3}{2} e^{3\alpha \varphi}.$$  

(55)

Here we note that the decay parameter $\Gamma$ given by eq.(54) increases with the growth of the scalar field $\varphi$, (see eq.(55)) and then the $\sigma$-particles tend to decay at large values of $\varphi$.

**VII. DECAY RATES AND CONSTRAINTS**

In this section we can study two decay rates in order to obtain different constraints on the parameters of our model. In the following, we will analyze the decay rate for the specific cases in which the coupling parameters $g_1 = 0$ and the other $g_2 \neq 0$ and vice versa.

In this context, we consider the special cases in which the coupling parameter $g_1 = 0$ and the other coupling parameter $g_2 \neq 0$ with which the $\Gamma$-coefficient is reduced to

$$\Gamma(\sigma \rightarrow \Psi \Psi) = \Gamma_2(\sigma \rightarrow \Psi \Psi) = c_2 g_2^2 e^{5\alpha \varphi},$$

(56)

where the constant $c_2$ is defined as $c_2 = \frac{\mu^3 \chi_2^2}{8\pi} \left( \frac{f_1}{2M_2 \chi_2} \right)^3$. Here, we have tagged the decay rate in this situation as $\Gamma_2$.

For the other instance in which the coupling parameter $g_2 = 0$, we have that the decay rate results

$$\Gamma(\sigma \rightarrow \Psi \Psi) = \Gamma_1(\sigma \rightarrow \Psi \Psi) = c_1 g_1^2 e^{\alpha \varphi},$$

(57)

in which $c_1 = \frac{\mu^3}{8\pi} \left( \frac{f_1}{2\chi_2 M_2} \right)$.

Now, as we discussed earlier we have considered that both densities become equivalent and this occurs at the equilibrium time given by eq.(36). At least during this time, we can consider that the inflaton field $\varphi$ spends most of the time previous to the equilibrium time $t_{eq}$ in which the inflaton takes the value $\varphi_{eq}$. In fact, the backreaction is unimportant for times shorter than $t_{eq}$ and then we can assume that the decay rate at that time limit $t_{eq}$
denotes by $\Gamma(\varphi = \varphi_{eq}) = \Gamma_{eq}$ satisfies the condition in which the particles $\sigma$ will decays to fermions $\Psi$.

At the equilibrium time, we find that the decay rate for the special case in which $g_1 = 0$ from eqs.\[35\] and \[56\] can be written as

$$
\Gamma_2(\sigma \rightarrow \Psi \Psi)\bigg|_{\varphi = \varphi_{eq}} \simeq c_2 g_2^2 \left( \frac{3 \alpha_1 t_0^2}{2} \right)^5.
$$

Analogously, we obtain that the decay rate $\Gamma_1$ at the equilibrium time for the special case $g_2 = 0$ results

$$
\Gamma_1(\sigma \rightarrow \Psi \Psi)\bigg|_{\varphi = \varphi_{eq}} \simeq c_1 g_1^2 \left( \frac{3 \alpha_1 t_0^2}{2} \right)^5.
$$

Additionally, we will assume that during the kinetic stage the Hubble factor decreases so that its value is similar to the decay rate $\Gamma$. Thus, we can consider that the scalar field $\sigma$ decayed under the condition $H(t_{dec}) = \frac{1}{3t_{dec}} \simeq \Gamma$, where $t_{dec}$ corresponds to the time when the scalar field $\sigma$ decayed and as the field $\varphi$ spends most of the time previous to the equilibrium time $t_{eq}$ we note that this time satisfies the condition $t_{dec} < t_{eq}$.

In particular for the case in which $g_1 = 0$ and considering the decay rate $\Gamma_2$ given by eq.\[55\], we find that the time when the scalar field $\sigma$ decayed $t_{dec}$ results

$$
t_{dec} \simeq \frac{1}{3c_2 g_2^2} \left( \frac{2}{3 \alpha_1 t_0^2} \right)^5.
$$

Similarly, for the situation in which $g_2 = 0$, we obtain that the time $t_{dec} \simeq \frac{1}{c_1 g_1^2} \left( \frac{2}{9 \alpha_1 t_0^2} \right)^5$, when we considered the decay rate $\Gamma_1$. Thus, under the condition $t_{dec} < t_{eq}$ we find that for the case $g_1 = 0$ we have

$$
g_2^2 > \frac{\sqrt{3}}{6 c_2} \left( \frac{2}{3 \alpha_1 t_0^2} \right)^5 |\dot{\varphi}_0|,
$$

and for the case in which $g_2 = 0$, we obtain that the lower limit for the coupling $g_1$ becomes

$$
g_1^2 > \frac{\sqrt{3}|\dot{\varphi}_0|}{9 c_1 \alpha_1 t_0^2},
$$

here we have considered eq.\[37\] for the equilibrium time. Note that in both cases the lower bounds for the coupling parameters are proportional to the velocity at the end of inflation, since for large $\alpha$ we have $\dot{\varphi}_0 \simeq \dot{\varphi}_{end}$.

On the other hand, in order to obtain the temperature at the equilibrium time $T(t = t^*_eq) = T_{eq}(t^*_eq)$, we can consider that previous to the equilibrium time, the scalar field $\sigma$ has totally decayed. This situation occurs when the densities satisfy the condition $\rho(t^*_eq) \sim \rho_{\sigma}(t^*_eq)$. Here we have used the notation $t^*_eq$ for the time when the scalar field $\sigma$ has completely decayed and thus differentiate it from the equilibrium time $t_{eq}$ since this time is different according on whether $\sigma$ field decays or not.
As the energy density of the background $\rho(a)$ decays during the kinetic epoch as $\rho \propto a^{-6}$ and the energy density $\rho_\sigma(a)$ as radiation i.e., $\rho_\sigma \propto a^{-4}$, we have

$$\rho(t_{eq}) = \rho(t_{dec}) \left( \frac{a(t_{dec})}{a(t_{eq})} \right)^6, \quad \text{and} \quad \rho_\sigma(t_{eq}) = \rho_\sigma(t_{dec}) \left( \frac{a(t_{dec})}{a(t_{eq})} \right)^4,$$

with which from condition $\rho(t_{eq}) \approx \rho_\sigma(t_{eq})$, we find that the temperature at the equilibrium $T_{eq} \approx \rho_\sigma^{1/4}(t_{eq})$ can be written as

$$T_{eq} \approx \rho_\sigma^{1/4}(t_{eq}) = \rho_\sigma^{1/4}(t_{dec}) \sqrt{\frac{\rho_\sigma(t_{dec})}{\rho(t_{dec})}}.$$

On the other hand, as we have that the scalar field $\sigma$ decayed under the condition in which $H(t_{dec}) \approx \Gamma$, then we assume that the energy density of the background $\rho(t_{dec}) = 6H^2 = 6\Gamma^2$ and for the energy density $\rho_\sigma(t_{dec})$ we have $\rho_\sigma(t_{dec}) \approx \sqrt{\frac{3}{\alpha}} \Gamma |\dot{\varphi}_0|$. In this way, by using Eq. (63) we find that the temperature at the equilibrium can be written as

$$T_{eq} \approx 10^{-1} |\dot{\varphi}_0|^{3/4} \Gamma^{-1/4} \approx 10^{-1} |\dot{\varphi}_{end}|^{3/4} \Gamma^{-1/4},$$

where we have used that the velocity $\dot{\varphi}_0 \approx \dot{\varphi}_{end}$ for values of $\alpha \gg 1$.

In this form, from eq. (64) we can analyze the temperature at the equilibrium for the specific cases of the coupling parameters $g_1$ and $g_2$. Thus, in particular for the case in which coupling parameter $g_1 = 0$, we find that the temperature $T_{eq}$ becomes

$$T_{eq} \approx 10^{-2} \frac{|\dot{\varphi}_{end}|^{13/4}}{c_2^{1/4} g_2^{1/2} \alpha_1^{5/4}},$$

and combining with eq. (60), we obtain a lower bound for the velocity at the end of inflation $\dot{\varphi}_{end}$ in terms of the temperature at the equilibrium $T_{eq}$ given by

$$|\dot{\varphi}_{end}| > 10^3 T_{eq}^2.$$
From eq. (68) we can obtain different constraints on the parameter \( \alpha \) depending on the temperature \( T_{eq} \) considered, since the lower bound for \( \alpha \) is given by \( \alpha > 10^{31} T_{eq} \).

As example, by assuming that temperature at the equilibrium corresponds to the big bang nucleosynthesis (BBN) temperature \( T_{eq} \sim T_{BBN} \) in which \( T_{BBN} \sim 10^{-22} \) (in units of \( M_{Pl} \)), we obtain that the lower bound for the parameter \( \alpha \) results \( \alpha > 10^9 \). Now if we assume that the temperature \( T_{eq} \) corresponds to the electroweak temperature \( T_{ew} \sim 10^{-17} \), we obtain that the lower limit for the parameter \( \alpha \) results \( \alpha > 10^{14} \). Note that these constraints for the parameter \( \alpha \) are consistent with considering large values of \( \alpha \) i.e., \( \alpha \gg 1 \).

Now for the special case in which the coupling term \( g_2 = 0 \), we get that the temperature at the equilibrium becomes

\[
T_{eq} \sim 10^{-1} \frac{|\dot{\varphi}_{end}|^{5/4}}{c_1^{1/4} g_1^{1/2} \alpha^{1/4}},
\]

and combining with eq. (61), we obtain a lower limit for \( \dot{\varphi}_{end} \) as a function of the temperature \( T_{eq} \) given by

\[
|\dot{\varphi}_{end}| > 10^{3/2} T_{eq}^2.
\]

As before, this expression gives a lower bound on the ratio \( \alpha^2 / \chi_2^{1/2} \) results

\[
\frac{\alpha^2}{\chi_2^{1/2}} > 10^{3/2} \frac{M_2^{1/2}}{M_1} T_{eq}^2,
\]

and then we have

\[
\alpha > 10^{3/4} \frac{1}{\sqrt{2} U^{1/4}_\text{(+)}} T_{eq}.
\]

As in the previous case, by considering that the temperature \( T_{eq} \sim T_{BBN} \), we obtain that the parameter \( \alpha > 10^8 \) and for the case in which the temperature \( T_{eq} \) corresponds to the \( T_{ew} \), we find that the constraint for \( \alpha > 10^{14} \). Again, we note that these results are consistent with assuming values of \( \alpha \gg 1 \).

On the other hand, we will obtain other constraints on the parameters of our model, by considering at least another conditions during the decay of the \( \sigma \) particles.

In fact, we can consider the condition in which the time when the field-\( \sigma \) decayed \( t_{dec} \) is such that \( t_{dec} > t_0 \), where the time \( t_0 \approx H(t_0)^{-1} \sim H(t_{end})^{-1} = H_{end}^{-1} \). As at the end of inflationary epoch the Hubble parameter \( H_{end} = (V(\varphi_{end})/6)^{1/2} = (V_{end}/6)^{1/2} \), in which the effective potential at the end of inflation is \( V_{end} = f_1 e^{-2\alpha \varphi_{end}}/(2\sqrt{\chi_2 M_2}) \), with which we find that the time \( t_0 \) is given by \( t_0 \approx (6/V_{end})^{1/2} = 2e^{\alpha \varphi_{end}} (3\sqrt{\chi_2 M_2}/f_1)^{1/2} \).
In this way, considering the condition in which \( t_{\text{dec}} > t_0 \), we find an upper bound for the coupling parameter \( g_2 \) associated to the decay rate \( \Gamma_2 \) for the case in which the coupling parameter \( g_1 = 0 \) given by

\[
g_2 < 8 \times 10^3 \left[ \frac{M_1}{\chi_2^{1/3} f_1 U_{(+)}^{27/12} \alpha^{17/3} \mu} \right]^3. \tag{73}
\]

Here, we have used that the time when the field-\( \sigma \) decayed for the case in which the parameter \( g_1 = 0 \) becomes

\[
t_{\text{dec}} \simeq \frac{1}{3c_2 g_2^2} \left( \frac{2}{3\alpha_1 t_0^2} \right)^5.
\]

In order to evaluate an upper limit for the parameter \( g_2 \), we consider the special case in which \( \alpha = 10^{10} \) results \( g_2 < 10^{32}/\mu^2 \). For the particular case in which \( \alpha = 10^{15} \) the upper bound for \( g_2 \) corresponds to \( g_2 < 10^{45}/\mu^3 \). Note that by increasing the value of the parameter \( \alpha \) decreases the upper bound for the coupling parameter \( g_2 \). Here, as before we have considered the values \( M_1 = 4 \times 10^{-60} \) (in units of \( M_4^4 \)), \( U_{(+)} = 10^{-120} \) (in units of \( M_4^4 \)), \( \chi_2 = 10^{-3} \) and \( f_1 = 2 \times 10^{-8} \), respectively [9].

Now, for the special case in which \( g_2 = 0 \) and considering that the time \( t_{\text{dec}} \) is given by

\[
t_{\text{dec}} \simeq \frac{1}{c_2 g_2^2} \left( \frac{2}{9\alpha_1 t_0^2} \right),
\]

we find that the upper bound on the coupling parameter \( g_1 \) associated to \( \Gamma_1 \) becomes

\[
g_1 < 15 \left[ \frac{M_1^2}{2^{1/2} f_1 U_{(+)} \alpha^2 \mu} \right]. \tag{74}
\]

In particular, for the specific value \( \alpha = 10^{10} \) we obtain that the upper bound for \( g_1 \) and it to \( g_1 < 2 \times 10^{-10}/\mu \) and for the value \( \alpha = 10^{15} \) we get the bound \( g_1 < 10^{-20}/\mu \). Again we have used the values of ref. [9], for \( M_1 \), \( f_1 \) and \( U_{(+)} \).

In this context, we will obtain a range for the coupling parameters \( g_1 \) and \( g_2 \) associated to the decay rates \( \Gamma_1 \) and \( \Gamma_2 \), by using the condition in which the \( \sigma \) field decays (at the time \( t_{\text{dec}} \)) before reaching equilibrium (at the time \( t_{\text{eq}} \)) wherethat \( t_{\text{dec}} < t_{\text{eq}} \) and from the time condition when the field-\( \sigma \) decayed at the time \( t_{\text{dec}} \) is greater than the time \( t_0 \sim H^{-1}_{\text{end}} \sim V^{-1/2}_{\text{end}} \) i.e., \( t_{\text{dec}} > t_0 \).

In this form, unifying both time conditions, we find that the range for the parameter \( g_2 \) associated to decay parameter \( \Gamma_2 \) for the specific case in which the parameter \( g_1 = 0 \) is given by

\[
8 \times 10^3 \left[ \frac{M_1}{\chi_2^{1/3} f_1 U_{(+)}^{27/12} \alpha^{17/3} \mu} \right]^3 > g_2 > \frac{3^{1/4}}{\sqrt{6} c_2} \left( \frac{2}{3\alpha_1 t_0^2} \right)^{5/2} \left| \dot{\varphi}_{\text{end}} \right|^{1/2}. \tag{75}
\]

Here we have used eqs. (60) and (73) together with the fact that \( \dot{\varphi}_0 \simeq \dot{\varphi}_{\text{end}} \) for large \( \alpha \).
In order to find a numerical range for the parameter $g_2$, we consider the special case in which $\alpha = 10^{10}$ results $(10^7/\mu^3)(\ln[10^{-26}/\mu])^{15/2} < g_2 < 10^{132}/\mu^3$. For the particular case in which $\alpha = 10^{15}$ the range for the coupling parameter $g_2$ corresponds to $(10^{-75}/\mu^3)(\ln[10^{-11}/\mu])^{15/2} < g_2 < 10^{45}/\mu^3$. We note that the range for the coupling parameter $g_2$ is very large. Here, as before we have considered that the time $t_0 = (6/V_{end})^{1/2}$ together with the values $M_1 = 4 \times 10^{-60}$ (in units of $M_{Pl}$), $U(+) = 10^{-120}$ (in units of $M_{Pl}$), $\chi_2 = 10^{-3}$ and $f_1 = 2 \times 10^{-8}$, respectively [9].

Analogously, for the specific case in which the parameter $g_2 = 0$, we find that the range for the coupling parameter $g_1$ can be written as

$$15 \left[ \frac{M_1^2}{2^{1/2} f_1 U(+)} \frac{\alpha^2 \mu}{\alpha_1} \right] > g_1 > \frac{3^{1/4} |\dot{\varphi}_{end}|^{1/2}}{3 t_0 \sqrt{c_1 \alpha_1}}.$$

Here we have considered that $\dot{\varphi}_0 \simeq \dot{\varphi}_{end}$ together with limits given by eqs. (61) and (74), respectively.

As before, in order to obtain a range for the parameter $g_1$, we assume the special case in which the parameter $\alpha = 10^{10}$ results $(10^{-12}/\mu)(\ln[10^{-26}/\mu])^{3/2} < g_1 < 2 \times 10^{-10}/\mu$, where the quantity $(\ln[10^{-26}/\mu])^{3/2} < 10^2$ or $\mu > 4 \times 10^{-36}$ in order to satisfy the range for the parameter $g_1$. For the particular case in which $\alpha = 10^{15}$ the range for the coupling parameter $g_1$ corresponds to $(10^{-24}/\mu)(\ln[10^{-11}/\mu])^{3/2} < g_1 < 10^{-20}/\mu$, with $\mu > 10^{-213} \simeq 0$. We note that the range for the parameter $g_1$ is very narrow in relation to $g_2$. Here, as before we have used that the time $t_0 = (6/V_{end})^{1/2}$ together with the values $M_1 = 4 \times 10^{-60}$ (in units of $M_{Pl}$), $U(+) = 10^{-120}$ (in units of $M_{Pl}$), $\chi_2 = 10^{-3}$ and $f_1 = 2 \times 10^{-8}$, respectively [9].

VIII. CONCLUSION

In this paper we have analyzed in detail the instant preheating mechanism in a scale invariant two measures theory. In this frame we have studied the instant preheating for a NO model where the potential associated to inflaton field does not have a minimum. Moreover, we have assumed that this preheating mechanism is applied to an effective potential that presents an interaction between the inflaton field $\varphi$ and other scalar field $\sigma$ given by eq. (11).

In our analysis, we have noted that the instant preheating and in particular the particles production $\sigma$ strongly depends on the interaction between the the fields $\varphi$ and $\sigma$.

From the energy density of the produced particles of the field-$\sigma$, we have obtained two
limit decays that depend on the effective mass $m_\sigma$ in relation to the physical momentum. In the first situation, we have found that the energy density of produced particles-$\sigma$ decays as radiation, in a process that we called instant radiation and it occurs when the effective mass satisfied the condition $m_\sigma \ll k/a$. In this stage we have observed that the backreaction of produced particles-$\sigma$ on the equation of motion associated to the inflaton field $\varphi$ disappears naturally product of the evolution of the inflaton and exponential decay of the backreaction term.

For the situation in which the effective mass of the field $\sigma$ satisfies the reverse situation in which $m_\sigma \gg k/a$, we have analyzed the possibility that the energy density of produced particles $\sigma$ is of the same order as the energy density of the background defining an equilibrium time.

Further, we have studied the decay rate in the framework of the scale invariant coupling of the scalar fields $\varphi$ and $\sigma$ and as this last field decays to fermions. Here, after performing transition to the physical Einstein frame we have considered a Yukawa interaction and then we have found an expression for the decay rate from our scalar field going into two fermions, see eq. (54). From these results we have analyzed two decay rates separately assuming the values of the coupling parameters associated to the decay parameters. In this analysis, we have found different constraints on the coupling parameters of the decay $\Gamma$, considering the imposed conditions from the time when the scalar field decayed, the equilibrium time and the initial time of the kinetic epoch.

Additionally, we have determined the temperature at the equilibrium for the different cases of the coupling parameters of $\Gamma$ and as example we have compared our results with the nucleosynthesis and electroweak temperatures, respectively.

Finally in this article, we have not addressed the process of the particles production considering other reheating mechanisms such as gravitational particle production from massless or heavy particles [26, 41]. In this sense, we hope to return to this point in the near future.
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