Electroweak production of hybrid mesons in a Flux-Tube simulation of Lattice QCD

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We make the first calculation of the electroweak couplings of hybrid mesons to conventional mesons appropriate to photoproduction and to the decays of $B$ or $D$ mesons. $E1$ amplitudes are found to be large and may contribute in charge exchange $\gamma p \rightarrow nH^+$ allowing production of (amongst others) the charged $1^{-+}$ exotic hybrid off $a_2$ exchange. Axial hybrid meson photoproduction is predicted to be large courtesy of $\pi$ exchange, and its strange hybrid counterpart is predicted in $B \rightarrow \psi K\Lambda(1^+)$ with $b.r. \sim 10^{-4}$. Higher multipoles, and some implications for hybrid charmonium are briefly discussed.

An outstanding problem in the Standard Model is how the non-Abelian, gluon, degrees of freedom behave in the limit of strong QCD. Lattice QCD predicts a spectroscopy of glueballs \[1\] and hybrid mesons \[2\], but there are no unambiguous signals against which these predictions can be tested.

A major stumbling block in the case of hybrids is that while predictions for their masses \[2,3\], hadronic widths \[4,5\] and decay channels \[4,5,6\] are rather well agreed upon, the literature contains no discussion of their production rates in electroweak interactions (beyond VMD in one exotic channel \[7\]). Meanwhile a significant plank in the proposed upgrade of Jefferson Laboratory is its assumed ability to expose the predicted hy-

... not fully visible...
It has been argued that this dependence $\vec{r}_Q = f(\vec{r}, \vec{y})$ gives significant contributions to static properties of hadrons, such as charge radii, $(r^2)_c$ and to the slope of the Isgur-Wise function $\rho(v \cdot v')$. Specifically for $QQ$

$$r^2_Q = \frac{1}{4} \left[ 1 + \frac{8b}{\pi^3 m^4} \sum_{i=1}^{\infty} (1/p^3) \right] (r^2)$$  \hspace{1cm} (1)$$

where the $\sum_i^{\infty} (1/p^3) \sim 1.2$ arises from the sum over all modes contributing to zero-point oscillations of the flux-tube. Isgur and Wise showed that these “transverse excursions” give huge ~ 51% corrections in light quark systems where $m_Q = m_d$, and ~ 13% corrections in heavy-light $Q\bar{Q}$ systems. Furthermore the $\sum_i^{\infty} (1/p^3)$ is ~ 80% saturated by its $p = 1$ term. Together, these suggest that the transition amplitudes to the lowest hybrids ($p = 1$ phonon modes) could be substantial. We shall now demonstrate that this can be so, at least for certain quantum numbers.

The respective amplitudes for conventional $E1$ transitions and the hybrid excitation come from expanding the incoming plane wave to leading order in the momentum transfer, thereby enabling the linear terms in $\vec{q} \cdot \vec{r}_Q$ to break the orthogonality of initial and final wavefunctions and cause the transition.

By combining with the tensor decomposition of the current-quark interaction, we may calculate excitation amplitudes to hybrids, and compare with those for conventional mesons in various multipoles. We will give extensive details elsewhere. In this first note, we illustrate the principle in electromagnetic interactions and in what promises to be a prominent heavy flavour decay channel.

A general feature of operators required to excite the lowest hybrid states (the first flux-tube mode) is the presence of the transverse position vector $\vec{y}$ to break the orthogonality between the lowest $Q\bar{Q}$ state and the “$\vec{y}$ excited” hybrid states. Hence in photoproduction one accesses $E1$ or (orbitally excited) $M1$ transitions in leading order. These are $\Delta S = 0$, e.g. $0^+_{Q^-} \to 0^+_{H^\pm}$ or $1_{Q^-}^0 \to (0,1,2)_{H}^{\pm}$. (Note that states with the “wrong” charge conjugation will only be accessible for flavoured mesons, e.g. in $\gamma p \to H^+ n$, and hence will have no analogue for $c\bar{c}$ and other $I = 0$ states).

Transitions involving spin-flip, $\Delta S = 1$, will need a $\vec{\sigma}$ spin operator as well as the above. Such terms arise as finite size corrections to the $\vec{\sigma} \cdot \vec{B}$ magnetic interaction and also in the spin-orbit interaction $\vec{\sigma} \cdot \vec{r}_Q \times \vec{E}$, in $J_{em}$. These are normally non-leading effects at $O(v/c)^2$ in amplitude and hence much suppressed for heavy flavours. They are known to give non-negligible contributions to some light flavour transitions. However, unlike the leading $\Delta S = 0$ terms, their effects are less well defined (e.g. binding effects and other relativistic corrections can play a role at this order). It is results for the $\Delta S = 0$ $E1$ transitions that are most reliable and on which we primarily focus in this first evaluation.

### E1 excitations

The familiar $E1$ amplitude between $Q\bar{Q}$ conventional states (e.g. $\gamma\pi \leftrightarrow b_1$) is

$$M(\gamma\pi \leftrightarrow b_1) = \left( \frac{e_1}{m_1} - \frac{e_2}{m_2} \right) b(r_\pi) |q|^2 \mu$$

where $b(r_\pi)$ is the radial wavefunction moment $\int_0^\infty r^2 dr b(r_\pi) r R_\pi(r)$, and $\mu$ is the reduced mass of the $Q\bar{Q}$. In line with ref. we use constituent masses which subsume contributions from the string.

The analogous amplitude for exciting the $\vec{y}$ oscillator between spin singlet states leads to $M \equiv M (\delta_+ - \delta_-)$ where

$$M(\gamma\pi \to a_{1H}) = \left( \frac{e_1}{m_1} + \frac{e_2}{m_2} \right) H(r_\pi) |q|^2 \sqrt{\frac{b}{3\pi^3}} \delta_m$$

(3) where the factors $\delta_\pm$ refer to the flux tube $p = 1$ phonon polarisation transverse to the body vector $\vec{r}$, while the $\delta_{m,\pm}$ refers to the hybrid polarisation in the fixed axes $x, y, z$. The transition $\gamma\pi \leftrightarrow a_{1H}$ is seen to vanish when $m_1 \gg m_2$ and $e_1 = -e_2$ in accord with the constraints of $C$ conjugation. The above formula can be immediately taken over to flavoured states where $m_1 \neq m_2$.

The parity eigenstates in the flux tube are given in ref. Following that reference we denote the number of positive or negative helicity phonon modes by $\{n_+, n_-\}$, which for our present purposes will be $\{1,0\}$ or $\{0,1\}$. Parity eigenstates $\pm$ are then the linear superpositions of $\{\{1,0\} \mp \{0,1\}\}$ such that for $\pi\gamma E1$ transitions we have

$$\langle P = -|\pi\gamma\rangle = 0; \quad \langle P = +|\pi\gamma\rangle = \sqrt{2} M;$$

This applies immediately to the excitation of the hybrid $a_{1H}^\pm$ in $\gamma\pi^\pm \to a_{1H}^\pm$ where there is no spin flip between the spin singlet $\pi$ and $a_{1H}$. In general we can write the radiative width $\Gamma(A \to B\gamma)$ as

$$\frac{4}{m_A} \frac{E_B}{2J_A + 1} \sum_{m_J}^2 |\sqrt{2} M(m_A^J, m_B^J) = m_A^J + 1)|^2$$

where the sum is over all possible helicities of the initial meson. The ratio of widths $\frac{\Gamma_{E1}(a_{1H}^\pm \to \pi^\pm\gamma)}{\Gamma_{E1}(b_1 \to \pi^\mp\gamma)}$ is then

$$\frac{72}{\pi^3 m_n^2} \frac{b(r_\pi)}{b(r_\bar{\pi})} \frac{|H(r_\pi)|^2}{|H(r_\bar{\pi})|^2} \left[ |q_H|^2 \exp\left(\frac{-|q_H|^2}{8\beta_H^2}\right) \right]$$

(4)
where the factor in square brackets includes the $q^3$ phase-space and a “typical” form-factor taken from the case of harmonic-oscillator binding [13].

Compare the form of this ratio driven by eqs. [23, 33] with the transverse contribution to the elastic charge radius, eq. [3]. In the approximation used here, the $E1$ transitions to the leading states saturate the dipole sum rule. This suggests the possibility of generalising some of our specific results into sum rules relating the elastic properties of hadrons to the excitation of their hybrid states [10].

In the Isgur-Paton adiabatic model [33] with a variational harmonic-oscillator solution we obtain $|H(r)|^2 / |r|^4 \approx 1.0$, so the radial moments do not suppress hybrids [14]. We follow ref [3] and use the standard parameters $b = 0.18\text{GeV}^2, m_n = 0.33\text{GeV}$ so that the prefactor $\sqrt{\frac{2}{3}} \frac{b}{m_n} \approx 3.8$ and hence there is no hybrid suppression from the flux-tube dynamics.

Within our variational solution $\beta_H = 255\text{MeV}, \beta_x = 281\text{MeV}, \beta_\rho = 335\text{MeV}$, so we see the $p = 1$ hybrid state being of roughly the same size as the $L = 1$ conventional state. The main uncertainty is the computed size of the $E1$ transition matrix elements. Assuming that this hybrid has mass $\sim 1.9\text{GeV}$ [2, 3, 11], and using the measured width $\Gamma(b_0^+ \rightarrow \pi^+\gamma) = 230 \pm 60\text{keV}$ [17] we predict that

$$\Gamma(a_1^+H \rightarrow \pi^+\gamma) = 2.1 \pm 0.9\text{MeV},$$

where the error allows for the uncertainty in $\beta_x$ [12, 15, 17].

The equivalent $E1$ process for spin triplet $QQ$ states is $(0,1,2)_H^+ \leftrightarrow \rho\gamma$, where the only difference from the $S = 0$ case is the addition of $L, S$ Clebsch-Gordan factors coupling the $QQ$ spin and flux-tube angular momentum to the total $J$ of the hybrid meson in question. The matrix element is analogous to eq. [3] multiplied by the Clebsch-Gordan $(1 + 1; m_\rho)|Jm_j\rangle$. We find (for $J = 0, 1, 2$ in this $E1$ limit),

$$\Gamma(b_1^+H \rightarrow \rho^+\gamma) = 2.3 \pm 0.8\text{MeV},$$

where the error reflects the uncertainties in the conventional $E1$ strength and $\beta_{b_1}$ and where we have taken $m_H = 1.9\text{GeV}$.

### Heavy Flavor Decays

As discussed after eq. [10], the $|M|^2$ for the weak transition $B \rightarrow \psi K_H(1^{+})$ is expected to have strength $\sim 13\%$ relative to its “counterpart” $B \rightarrow \psi K(1^{+})$. Empirically $B^+ \rightarrow \psi K(1^{+})(1230)$ is the single largest branching mode in $B^+ \rightarrow \psi X$ with $\text{b.r.} = (1.8 \pm 0.5) \times 10^{-3}$ while $B^+ \rightarrow \psi K(1^{+})(1400) \lesssim 0.5 \times 10^{-3}$. These rates involve both parity conserving (vector) and violating (axial) contributions and their relative strengths depend on the mixing between the $^3P_1$ and $^1P_1$ basis states.

| state | $\bar{u}d$ | $u\bar{s}$ |
|-------|---------|--------|
| $1S_0$ | $\pi\bar{c}c$ | $\bar{u}d$ |
| $3S_1$ | $(11; 1m_1|J_Hm_H)$ | $\rho\bar{c}c$ |
| $3P_0$ | $\sqrt{3}(11; 1m_1|J_Hm_H)$ | $\bar{u}d$ |
| $3P_1$ | $(1m_L+1; 1m_S|J_Hm_H)^*$ | $\bar{u}d$ |

**TABLE I:** Photon-Meson-Hybrid matrix elements: $\sqrt{2}M = \left(\frac{\alpha}{\pi m_H^2} + \frac{\alpha^2}{\pi m_H^4}\right) \sqrt{2|q|} \sqrt{\frac{b}{m_H^4}} |\langle r\rangle|$, should be multiplied by the Clebsch-Gordan factor in the second column to give the overall matrix element for a positive helicity photon. The numbers quoted in columns three and four are $|\bar{q}|/m_H$ $(10^{-3}\text{GeV}^{-1})$, evaluated using the results of [3], except those in brackets which use the $\beta$-values of [17].

These rates would lead one to expect an order of magnitude $b.r.$ for $B^+ \rightarrow \psi K_H(1^{+}) \gtrsim 10^{-4}$.

Explicit calculation confirms this. (For technical reasons our analysis of heavy-light dynamics is not identical to the original formulation of [10]. Details are in [18]). The transition matrix element has the structure

$$M \sim \langle K_H|V_\mu - A_\mu|B\rangle f_\psi m_\psi e^\mu_\psi$$

where $f_\psi = 0.4\text{GeV}$ [19]. A non-relativistic expansion of the vector and axial operators is made for both longitudinal and transverse components and terms linear in $\bar{q}$ or $\bar{p}$ identified. This is algebraically tedious but in essence parallels the approach illustrated earlier. The expectation values of these linear terms in $\bar{q}$ space generate the transitions to hybrid $K_H(1^{+})$; the analogous terms in $\bar{r}$ space lead to the familiar $K(1^{+})$ states. For $\Delta S = 0$ transitions $B \rightarrow \psi K_H(1^{+})$,

$$V_\mu \approx \frac{p_\rho^\mu m_\rho + m_\psi}{m_\rho m_\psi}; A_\mu \rightarrow A_T \sim |\bar{q}| q_\rho^\mu / m_\rho m_\psi.$$

Hence the transition to $K_H(1^{+})$ is large because the dominant $\langle V_\mu \rangle$ contributes in $S$-wave; by contrast $K_H(1^{-})$ receives its $S$-wave from the $|\bar{q}|/m_\rho$ suppressed $\langle A_\mu \rangle$ while the vector current contributes to $P$-waves. Explicit calculation confirms this where as a function of $m_H = (1.8; 2.0; 2.2)\text{GeV}$ we find

$\text{b.r.}[B \rightarrow \psi K_H(1^{-})] = (1.2; 0.5; 0.2) \times 10^{-5}$;

$\text{b.r.}[B \rightarrow \psi K_H(1^{+})] = (4.5; 2.3; 1.0) \times 10^{-4}$. Furthermore we find that $K_H(1^{+})$ is dominantly produced with longitudinal polarisation.

While fine details of the model may be questioned, the $O(10^{-4})$ branching ratio to this hybrid appears robust
and accessible to experiment. It is intriguing therefore that there is an unexplained enhancement at low $q_\psi$, corresponding to high mass $K$ systems, of this magnitude. While suggestive, it would be premature to claim this as evidence for hybrid production. Radial excitations of the $K(1^{+})$ are expected in this region, and in the ISGW model, extended to exclusive hadronic decays and assuming standard factorisation arguments, we find these to have $b.r. \sim 10^{-4}$, though slightly less than the hybrid. Other strange mesons in this mass range are likely to be suppressed due to their high angular momentum which give powerful orthogonality suppressions at small $q$. It is the S-wave character of the hybrid and axial production that drives their significant production rates.

To test these predictions experimentally, first identify the $\psi$ vertex and reconstruct the $B$ from the decay hadrons and thereby the invariant mass distribution of the strange system. Observation of significant axial strength around 2 GeV, produced by parity conserving S-wave amplitudes at $b.r. > 10^{-4}$ would prove strong evidence for the presence of the hybrid meson and warrant further studies of how to quantify the relative production and mixing of these axial mesons. In turn it would underpin our predictions of significant $E1$ transitions to such states in photoproduction.

There is also the possibility of hybrid Charmonium in $B \to \psi H X$. Predicting this involves knowledge of flux-tube formation dynamics which goes beyond the present work.

**Conclusions**

We confirm Isgur’s conjecture that electromagnetic transitions to hybrids may be significant. We find this to be true for certain $E1$ transitions for light flavours in charge exchange.

Within this model we also anticipate that $gg$ interactions initiate significant cascades such as $\psi H \to \psi \eta (\eta')$ and the diffractive transition $\gamma N \to 2_H^{+}^{-} N$; these currents will disturb the flux tube by direct analogy with the electromagnetic transitions discussed here.

These results promise an active programme of future research at an upgraded Jefferson Laboratory and at CLEO-c. They also encourage mining existing data on $B$ decays and inclusion in future plans for heavy flavor decays. In particular there is the intriguing observation of an as yet unexplained enhancement in $B \to \psi X$ in the kinematic region where $K_H$ is expected, and with a strength compatible with that predicted for $K_H = 1^+$. We urge further investigation of this, and the other channels identified in this note.

We shall give a detailed discussion elsewhere.

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