Steepest Descent Multimodulus Algorithm for Blind Signal Retrieval in QAM Systems

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Abstract
We present steepest descent (SD) implementation of multimodulus algorithm (MMA2-2) for blind signal retrieval in digital communication systems. In comparison to stochastic approximate (gradient descent) realization, the proposed SD implementation of MMA2-2 equalizer mitigates inter-symbol interference with relatively smooth convergence and superior steady-state performance.

Keywords: Blind equalization; multimodulus algorithm (MMA2-2); steepest descent; adaptive filter; channel equalization

1. Introduction
The multimodulus algorithm (MMA2-2) [1, 2] is given as

\[ w_{n+1} = w_n + \mu \left( R_m - y_{R,n}^2 \right) y_{R,n} x_n - j \mu \left( R_m - y_{I,n}^2 \right) y_{I,n} x_n, \quad (1) \]

where \( j = \sqrt{-1} \), \( R_m \) is a positive statistical constant, \( x_n \) is channel observation vector, \( w_n \) is equalizer vector, and \( y_n = w_n^H x_n = y_{R,n} + j y_{I,n} \) is equalizer output. The update (1) is probably the most popular and widely studied multimodulus algorithm capable of equalizing multi-path transmission channel blindly and recovering carrier phase jointly in quadrature amplitude modulation based wireless, wired and optical communication systems. The update, however, is stochastic approximate in nature, works on symbol-by-symbol basis, and is relatively slower in convergence when compared to its batch counterparts. Moreover, even in successfully converged state, the error function in update expression is non-zero except for instances when \( |y_n| = \sqrt{R} \); these fluctuations (as quantified in [3]) cause delay in switching to decision-directed mode and lead to decision errors causing loss of information.

In order to exploit full potential of MMA2-2, there is a new practice in literature to realize it in batch mode. In this context, Han et al. discussed a number of methods including steepest descent implementation for constant modulus algorithms (CMA) and relaxed convex optimization for MMA2-2 in [3] and [4], respectively. In [5], Shah et al. discussed batch MMA2-2 by exploiting iterative blind source separation framework and came up with Givens and hyperbolic rotations based batch MMA2-2. Also in [6], authors transformed MMA2-2 cost into an analytical problem and solved that for both batch and adaptive processing using subspace tracking methods. The most rigorous treatment appeared in [7], where a batch MMA2-2 is obtained which included an analytical transformation to a set of coupled canonical polyadic decompositions by using subspace methods. Recently, Han and Ding [9] suggested a steepest descent batch implementation of a class of CM algorithms where the update process did not require equalizer outputs (no feedback) and rather relied directly on statistics obtained from the received signal. Motivated by that approach, in this correspondence, we present a steepest descent implementation of MMA2-2 by estimating required batch statistics iteratively while maintaining simplicity of its adaptive structure. To the best of our knowledge, a steepest descent implementation of MMA2-2 has not been realized in literature.

2. Feedforward Steepest Descent Algorithms
In order to realize a steepest descent implementation of (1), we need to estimate expected value of its error function.

\[ w_{n+1} = w_n + \mu E \left[ \left( R_m - y_{R,n}^2 \right) y_{R,n} x_n - j \left( R_m - y_{I,n}^2 \right) y_{I,n} x_n \right] \quad (2) \]

We evaluate this expectation in forward driving manner as advocated in [9]. According to which, we replace \( y_n \) with \( \bar{w}_n^H x_n \), and evaluate statistical average of matrix quantities involving \( x_n \) conditioned on \( w_n \). Exploiting the facts that

\[ y_{R,n} = \frac{1}{2} \left( \bar{w}_n^H x_n + x_n^H \bar{w}_n \right) \quad (3a) \]
\[ y_{I,n} = \frac{1}{2} \left( \bar{w}_n^H x_n - x_n^H \bar{w}_n \right) \quad (3b) \]

and after some manipulations, we obtain

\[ E \left[ \left( R_m - y_{R,n}^2 \right) y_{R,n} x_n - j \left( R_m - y_{I,n}^2 \right) y_{I,n} x_n \right] = E \left[ \left( R_m \bar{w}_n^H x_n - \frac{1}{2} \left( x_n^H \bar{w}_n \right)^2 \bar{w}_n^H x_n - \frac{1}{2} \left( \bar{w}_n^H x_n \right)^2 x_n \right) \right] \]
\[ = R_m E \left[ x_n \bar{w}_n^H x_n \right] - \frac{1}{2} \bar{w}_n E \left[ x_n \bar{w}_n^H x_n \bar{w}_n^H x_n^H \bar{w}_n \right] - \frac{1}{2} \bar{w}_n E \left[ \left( \bar{w}_n^H x_n \right)^3 \right] x_n \quad (4) \]
We can show that:

$$E[x_nx_n^Hw_nw_n^Hx_nx_n^Hw_nw_n^H] = E[x_nx_n^Hw_nw_n^Hx_nx_n^Hw_nw_n^H]$$

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Further, we obtain:

$$E[(w_n^Hx_n)_3x_n] = E[w_n^Hx_n]$$

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Finally, we can estimate required statistics either by taking ensemble average over a batch of data or iteratively updating the estimate at each time index. At index $n$, an iterative estimate of expectation $E[f_n]$ where $f_n$ is some matrix with random variable’s entries, may be obtained as $S_n = (1 - \lambda)S_{n-1} + \lambda f_n$, $0 < \lambda < 1$. Next, using $S_n^I$, $S_n^II$, and $S_n^III$ denote iterative estimates of $E[x_n] = E[x_nx_n^H]$, $E[x_nx_n^H]$, and $E[x_n]$ respectively, we obtain feed-forward steepest descent MMA-2 (SD-MMA-2) as given by:

$$SD-MMA-2$$

$w_{n+1} = w_n + \mu R_n S_n w_n$

$$- \frac{2}{\mu} S_n^I \text{vec}[w_n \text{vec}[w_nw_n^H]^T]$$

$$- \frac{2}{\mu} S_n^II \text{vec}[w_n \text{vec}[w_nw_n^H]^T]^T$$

$$S_n = (1 - \lambda)S_{n-1} + \lambda x_n x_n^H$$

$$S_n^II = (1 - \lambda)S_{n-1} + \lambda x_nx_n^H$$

$$S_n^III = (1 - \lambda)S_{n-1} + \lambda x_nx_n^H$$

Considering a fixed channel, assume that the (steady-state) estimates of statistics $S_n^I$, $S_n^II$, and $S_n^III$ are available, say from the received large batch of data. Now, solving $\partial J/\partial w = 0$ and exploiting these available statistics, we obtain the following offline fixed-point steepest descent algorithm:

$$w \leftarrow \frac{S_n^{-1}}{2 \lambda} \left(3 S_n^{II} \text{vec}[w \text{vec}[ww^H]^T] + S_n^{III} \text{vec}[w \text{vec}[ww^H]^T]^T\right)$$

where $S_n^I$, $S_n^{II}$ and $S_n^{III}$ are offline estimates of $S_n^I$, $S_n^{II}$ and $S_n^{III}$ respectively. However, note that the iteration (10) is found to be diverging which is a common problem in fixed-point procedure when matrix inverse is involved; see [11], eq. (21) and details therein. To improve this situation, we add a step-size in (10), obtaining a stabilized (offline) fixed-point algorithm:

$$FP-MMA-2$$

$w \leftarrow w + \mu R_n S_n w - \frac{2}{\mu} S_n^{II} \text{vec}[w \text{vec}[ww^H]^T] - \frac{2}{\mu} S_n^{III} \text{vec}[w \text{vec}[ww^H]^T]^T$
given as \( \mathbf{h}_n = [-0.023 - 0.0345j, 0.0804 - 0.0804j, 0.2068 - 0.1149j, 0.678 + 0.1378j, 0.1277 + 0.0345j, -0.1232 - 0.1103j, -0.023 - 0.021j, 0.0176 + 0.1196j, 0.0115 + 0.0118j] \). The signal-to-noise-ratio is 30 dB. The equalizer length is 15, initialized with a unit spike at center tap, and all algorithms use step-size of \( 10^{-4} \).

The ISI measure in dB at \( n \)th time index is

\[
\text{ISI}_n = 10 \log_{10} \left( \frac{1}{N_{\text{runs}}} \sum_{k=1}^{N_{\text{runs}}} \frac{\max\{|t_{n,k}(i)|^2|}{\max\{|t_{n,k}|^2|} \right) \tag{12}
\]

where \( t_{n,k} \) is the overall channel-equalizer impulse response vector at index \( n \) in the \( k \)th run of simulation. \( t_{n,k}(i) \) represents the \( i \)th entity of \( t_{n,k} \), and \( \max\{|t_{n,k}|^2| \) represents the largest squared amplitude in \( t_{n,k} \).

For fixed channels, we choose \( \lambda = 1/n \) (\( n \) is time index) so that the required statistics are estimated over all received data. Fig. 1(a) demonstrates convergence behaviors of MMA2-2 and SD-MMA2-2, averaged over 400 and 50 independent runs \( (N_{\text{runs}}) \), respectively. We notice that the ISI mitigation achieved by SD-MMA2-2 is far better in steady-state when allowed to converge at the same rate as that of MMA2-2. In Fig. 1(b), single trajectory of ISI convergence of each MMA2-2 and SD-MMA2-2 is shown. We can note that the SD-MMA2-2 exhibits far smoother and more stable convergence than MMA2-2 (for fixed channel scenario), and this is the reason why we used fewer independent runs for the ensemble averaging of ISI trajectories in SD-MMA2-2 than MMA2-2.

4. Conclusions

A steepest descent implementation of MMA2-2 for blind signal recovery has been proposed and demonstrated to mitigate ISI. The proposed equalizer has been found to yield better steady-state performance than stochastic approximate gradient descent MMA2-2. Thus, the proposed approach seems to be quite a promising substitute for traditional counterpart on fixed channels. Future work includes: (a) application to time-varying channels, (b) evaluation of optimal step-sizes, and (c) application to MIMO systems.

References

[1] Shafayat Abrar and Asoke Kumar Nandi. Blind equalization of square-QAM signals: a multimodulus approach. IEEE Transactions on Communications, 58(6), 2010.
[2] Shafayat Abrar and Syed Ismail Shah. New multimodulus blind equalization algorithm with relaxation. IEEE Signal Processing Letters, 13(7):425–428, 2006.
[3] Ali Waqar Azim, Shafayat Abrar, Azzedine Zerguine, and Asoke Kumar Nandi. Steady-state performance of multimodulus blind equalizers. Signal Processing, 108:509–520, 2015.
[4] Huy-Dung Han. Batch algorithms for blind channel equalization and blind channel shortening using convex optimization. University of California, Davis, 2012.
[5] Huy-Dung Han, Zhi Ding, and Muhammad Zia. A convex relaxation approach to higher-order statistical approaches to signal recovery. IEEE Transactions on Vehicular Technology, 66(1):188–201, 2017.
[6] Syed Awaiz Wahab Shah, Karim Abed-Meraim, and Tareq Yousef Al-Naffouri. Multi-modulus algorithms using hyperbolic and given rotations for blind deconvolution of mimo systems. In IEEE International Conference on Acoustics, Speech and Signal Processing, pages 2155–2159, IEEE, 2015.
[7] Steredenn Daumont and Daniel Le Guennec. An analytical multimodulus algorithm for blind demodulation in a time-varying MIMO channel context. International Journal of Digital Multimedia Broadcasting, 2010.
[8] Otto Debals, Muhammad Sohail, and Lieven De Lathauwer. Analytical multi-modulus algorithms based on coupled canonical polyadic decompositions. Technical report, Technical Report 16-150, ESA-T-STADUS, KE Leuven, Leuven, Belgium, 2016.
[9] Huy-Dung Han and Zhi Ding. Steepest descent algorithm implementation for multichannel blind signal recovery. IET Communications, 6(18):3196–3203, 2012.
[10] David A Harville. Matrix algebra from a statistician’s perspective, volume 1. Springer, 1997.
[11] Aapo Hyvarinen. Fast and robust fixed-point algorithms for independent component analysis. IEEE Transactions on Neural Networks, 10(3):626–634, 1999.
[12] Vicente Zarzoso and Pierre Comon. Blind and semi-blind equalization based on the constant power criterion. IEEE Transactions on Signal Processing, 53(11):4363–4375, 2005.
[13] Vicente Zarzoso et Pierre Comon. Optimal step-size constant modulus algorithm. IEEE Transactions on communications, 56(1), 2008.
[14] Vicente Zarzoso and Pierre Comon. Robust independent component analysis by iterative maximization of the kurtosis contrast with algebraic optimal step size. IEEE Transactions on Neural Networks, 21(2):248–261, 2010.
[15] Giorgio Picchi and Giancarlo Prati. Blind equalization and carrier recovery using a “stop-and-go” decision-directed algorithm. IEEE Transactions on Communications, 35(9):877–887, 1987.

Figure 1: ISI convergence: (a) averaged trajectories. (b) Randomly selected single trajectory for channel-2.