Measure and collapse of participatory democracy in a two-party system

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Abstract. ‘Measure what is measurable, and make measurable what is not so’ (Galileo Galilei). According to this quotation we do not ask why we need to measure democracy, but if it is possible to measure something that is not unequivocally defined. Although, a final agreement on the definition of democracy is unlikely, the idea that it is a form of governance based on collective decision-making seems to be uncontested. On the premise that in a high-quality democracy citizens (agents) not only must have equal participation rights but must want to participate in shaping decisions, we propose, as an effective measure of democracy in a two-party political system, the percentage of the total population that actually voted in a given election for one of two major parties. Thus, we disregard not only nonvoters but also those who vote for smaller parties, whose votes will not have a substantial impact on the election and consequently they will not be ‘in the loop’, even theoretically. To describe such a system a sociophysics model based on the $S=1$ Ising model (Blume–Capel) is proposed. The measure of democracy, index $V_D$, is analyzed as a function of interparty conflict.

Keywords: renormalisation group, interacting agent models
1. Introduction

More than half of the world’s countries can be considered democracies. However, the quality of democracy in particular countries may be quite different. The question is whether there is a way to distinguish the quality of democracy or, in other words, whether there is a sensible measure of democracy. There are a number of measures available—indices such as the Democracy Index, Freedom House, Polity, Democracy Barometer or Vanhanen Index, which try to take into account various aspects of democracy and consequently are based on many indicators and subjective assessments.

The Democracy Index is based on 60 indicators grouped into five categories: electoral process, civil liberties, political participation and culture, and ranks countries as one of four types: ‘full democracies’, ‘flawed democracies’, ‘hybrid regimes’ and ‘authoritarian regimes’ (incidentally, from a physicist’s point of view these categories resemble the 18th century definitions of the Fahrenheit scale points: aestus intolerabilis (blinding heat), calor ingens (great heat), aer temperatus (moderate), aer frígidos (cold), ...). Freedom House assesses the current state of political rights and civil liberties in each state on a scale from 1 (most free) to 7 (least free) and states are then classified as ‘free’, ‘partly free’ or ‘not free’. Polity’s conclusions are based on an evaluation of an election, the nature of political participation and the extent of checks on executive authority. The Polity scale ranges from −10 to 10, from ‘autocracies’ (−10 to −6), through anocracies (−5 to 5) to democracies (6 to 10). The Democracy Barometer (DB) [1] is based on the idea that one can measure the degree of fulfillment of the nine ‘functions’ deduced from three principles: Freedom (functions: Individual liberties, Rule of law, Public sphere), Control (Competition, Mutual constraints, Governmental capability) and Equality (Transparency, Participation, Representation). The DB consists of a total of 100 indicators. The Vanhanen Index [2] is based on two clearly defined quantitative indicators corresponding to the two theoretical dimensions of democratization.
called ‘competition’ and ‘participation’. According to the Vanhanen idea the ‘degree of competition’ in a given political system is indicated by the electoral success of the smaller parties, and the ‘degree of electoral participation’ is measured by the percentage of the total population that actually voted in a given election. These two variables are taken with the same weight to construct an index of democratization (ID), the Vanhanen Index.

None of the above mentioned indices has received common acceptance, and except for the Vanhanen Index their construction is rather complicated and linked to a certain extent with policy. So, it seems to be also helpful to analyze the problem of the measure of democracy by using statistical physics models or a sociophysics approach [3–5].

2. The model

The starting point is the premise that in a high-quality democracy citizens (agents) not only must have equal participation rights, which is obvious, but also must want to participate in shaping decisions. In this paper we confine ourselves to considering the two-party system, i.e. a political system in which the electorate votes mostly for one of two major parties. So, one or other party can win a majority in the legislature. In consequence, votes given to smaller parties have only a formal meaning with no real impact on shaping decisions. The classical example of a state with a two-party system is of course the USA, where in fact all members of the parliament belong to one of the two major parties. However, more common is the two-party system where two major parties dominate elections but there are third parties that have some seats in the legislature. Examples are the United Kingdom, or Poland for the last eight years.

As an effective measure of democracy in the two-party political system we propose the percentage of the total population that actually voted for one of the two major parties in a given election. Thus we divide the whole population entitled to vote into three groups: the electorate of the first party, called \( L \), the electorate of the second party, called \( C \), and the others, called \( F \). The last group comprise those who vote for the smaller (third) parties, who in the main vote ‘against’ and are fully aware that their voices will not have a major impact on the practical outcome of the election, floating voters and indifferent citizens. So, the effective democracy measure \( V_D \) is given by

\[
V_D = \frac{N_L + N_C}{N_L + N_C + N_F},
\]

where \( N_L, N_C, N_F \) denote the numbers of voters for the particular parties.

According to the idea of sociophysics social behavior can be modeled in the same way that physics models natural phenomena [3]. The most popular and useful physics model applied to describe social behavior is undoubtedly the Ising model [3–5, 7, 8]. So, in the language of sociophysics we consider a group of \( n \) agents (citizens), where \( n = n_L + n_C + n_F \), and \( n_L, n_C, n_F \) denote initial numbers of voters for the parties ‘\( L \)’, ‘\( C \)’ and ‘\( F \)’, respectively. Each agent has attached an Ising variable (spin) \( S_i^\alpha \), where
\[ \alpha = L, C, F \text{ and } i = 1, 2, ..., n_{\alpha}. \] In this case the Ising variable has three possible values: we take \( S_i = 1 \) when agent \( i \) is a voter for the ‘\( L \)’ party, \( S_i = -1 \) when the agent is a ‘\( C \)’-voter, and \( S_i = 0 \) for an ‘\( F \)’-voter. Analogously, as in the physics case we introduce coupling between two agents from the same group \( K_{\alpha} (\alpha = L, C, F) \), which is a measure of the unity of views or satisfaction of being a member of the same group. In a stable situation the coupling \( K_{\alpha} \) is negligible because usually the members of the ‘\( F \)’ party have no common views. To distinguish creeds of the electorates of the ‘\( L \)’ and ‘\( C \)’ parties we introduce an external field \( H_{\beta} (\beta = L, C) \) coupled linearly with each agent of the ‘\( L \)’ and ‘\( C \)’ groups. The members of the ‘\( F \)’ group are not able to distinguish between ‘\( L \)’ and ‘\( C \)’ parties, so their ‘creed’ has to be independent of the sign \( +, - \), i.e. a ‘field’ \( D \) should be coupled to \((S_i^F)^2\). Finally, confining ourselves to a one-dimensional arrangement of particular subgroup members, one has three decoupled chains described by the Hamiltonian:

\[ \tilde{H}_0 = - \sum_{\alpha=L,C,F} n_{\alpha} K_{\alpha} (S_i^\alpha S_{i+1}^\alpha) - \sum_{\beta=L,C} H_{\beta} (S_i^\beta) - D_F \sum_{\alpha=L,C} (S_i^\alpha)^2. \]

Postulating a principle of maximum satisfaction [3] one can find the equilibrium state of the model described by the Hamiltonian (2). And if

\[ \text{sgn}(H_L) = -\text{sgn}(H_C), \quad K_{\alpha} > 0, \quad \text{and} \quad D_F < -K_{F}, \]

then, for an isolated system that is the counterpart of the physical ground state (zero temperature) all agents of the ‘\( L \)’ group have spin +1, all agents of the ‘\( C \)’ group have spin −1, and all members of the ‘\( F \)’ group have spin 0. In this paper we consider only equilibrium properties of the system, which in physics depend on temperature. In principle such a quantity does not exist in social systems. However, there is a social meaning of temperature \( T \) in sociophysics as an overall approximation for all random events that influence decisions but are not included in the model [6]. Accordingly, one can assume that social systems have a ‘temperature’ in their steady state that validates an application of finite-temperature statistical physics methods to study social systems. At \( T = 0 \) all agents from ‘\( L \)’ group have spin +1, from ‘\( C \)’ group −1 and from ‘\( F \)’ group 0, but at finite temperature only \( N_{\alpha}^+ \) members of ‘\( \alpha \)’ group still have spin +1, \( N_{\alpha}^- \) spin −1 and \( N_{\alpha}^0 \) spin 0. Consequently, at finite temperature the ‘\( \alpha \)’ party has \( N_{\alpha} \) voters (\( \alpha = L, C, F \)):

\[ N_L = \sum_{\alpha=L,C,F} N_{\alpha}^+, \quad N_C = \sum_{\alpha=L,C,F} N_{\alpha}^-, \quad N_F = \sum_{\alpha=L,C,F} N_{\alpha}^0 \]

and the quantities \( N_{\alpha}^\chi \) (\( \chi = +, -, 0 \)) are expressed by the spin averages in the following way:

\[ N_{\alpha}^+ = \frac{1}{2} (\langle S_{\alpha}^2 \rangle + \langle S_{\alpha} \rangle), \quad N_{\alpha}^- = \frac{1}{2} (\langle S_{\alpha}^2 \rangle - \langle S_{\alpha} \rangle), \quad N_{\alpha}^0 = 1 - N_{\alpha}^+ - N_{\alpha}^- \]

As a measure of political strife between electorates of the two major parties ‘\( L \)’ and ‘\( C \)’ we introduce a coupling \( Q \),

\[ -Q \sum_i (S_i^L)^2 (S_i^C)^2. \]
The choice of such a coupling prefers an exchange of voters between the ‘L’ (or ‘C’) and ‘F’ groups rather than between ‘L’ and ‘C’, which is possible but less probable from the ideological point of view.

3. The method

The obvious way to analyze flows of voters between the parties is by computer simulations. However, in this paper we concentrate on the equilibrium properties by using the linear renormalization group transformation to study the Hamiltonian (2), (6). We start with the three decoupled chains (2), assuming that initially the number of the voters in each group (L, C, F) is the same, \( n = \alpha n \), and

\[
H_0 = -\beta H_0 = \sum_{\alpha=L,C,F} H_0^\alpha, \quad H_0^\alpha = k_\alpha \sum n S_i^\alpha S_{i+1}^\alpha + h_\alpha \sum n S_i^\alpha + d_\alpha \sum (S_i^\alpha)^2, \tag{7}
\]

where \( k_\alpha = -K_\alpha / T \), \( h_\alpha = -H_\alpha / T \), \( d_\alpha = -D_\alpha / T \). The minimal set of parameters to describe our model consists of three intrachain couplings \( k_L = k_C, h = h_L = -h_C \) and \( d = d_F \) and yields

\[
H_0^L = k \sum n S_i^L S_{i+1}^L + h \sum n S_i^L, \quad H_0^C = k \sum n S_i^C S_{i+1}^C - h \sum n S_i^C, \quad H_0^F = d \sum (S_i^F)^2. \tag{8}
\]

The renormalization group (RG) transformation for the Hamiltonian (7) is defined by

\[
\text{exp}[H'_0(\sigma)] = \text{Tr}_S P(\sigma, S)\text{exp}[H_0(S)], \tag{9}
\]

and the weight operator \( P(\sigma, S) \) that couples the original \( S \) and effective \( \sigma \) spins is chosen in the linear form [9]

\[
P(\sigma, S) = \prod_i p_i = \prod_i \left( 1 - S_{2i+1}^2 - \sigma_{i+1}^2 + \frac{1}{2} S_{2i+1} \sigma_{i+1} + \frac{3}{2} S_{2i+1}^2 \sigma_{i+1}^2 \right). \tag{10}
\]

For the decoupled chains the transformation (9) and (10) is a decimation transformation where in each step of the procedure every other spin is killed and the renormalized Hamiltonian can be written in the form

\[
H'(\sigma^\alpha) = \sum_{\alpha=L,C,F} \ln \text{Tr}_S \exp H_0(\sigma). \tag{11}
\]

Unlike the case of the two-state model (\( S = 1/2 \) Ising model), the decimation transformation for a three-state (\( S = 1 \)) model generates new interactions

\[
\xi_0(S_i^\alpha)^2 S_{i+1}^\alpha, \quad q_0(S_i^\alpha)^3 (S_{i+1}^\alpha)^2, \quad h_F S_i^F, \quad d_L(S_i^C)^2, \quad d_C(S_i^C)^2, \tag{12}
\]

and finally

\[
\ln \text{Tr}_S \exp H_0(\sigma) = \ln \left[ f_0 + f_1 \sigma_i^\alpha + f_2 \sigma_i^\alpha \sigma_{i+1}^\alpha + f_3 (\sigma_i^\alpha)^2 + f_4 (\sigma_i^\alpha)^2 \sigma_{i+1}^\alpha + f_5 (\sigma_i^\alpha \sigma_{i+1}^\alpha)^2 \right]
\]

\[
= z_0 + h' \sigma_i^\alpha + k' \sigma_i^\alpha \sigma_{i+1}^\alpha + d'_0 (\sigma_i^\alpha)^2 + f'_0 (\sigma_i^\alpha)^2 \sigma_{i+1}^\alpha + q'_0 (\sigma_i^\alpha \sigma_{i+1}^\alpha)^2. \tag{13}
\]
The renormalized parameters $\alpha_h'$, $\alpha_k'$, $\alpha_d'$, $\alpha_j'$, $\alpha_q'$ and $\alpha_z$ as functions of the original interactions are presented in appendix A. The constant term $\alpha_z$ (independent of effective spins $\sigma$) can be used to calculate the ‘free energy’ per site

$$f = \frac{1}{3} \sum_{n=1}^{\infty} \frac{z_L^{(n)} + z_C^{(n)} + z_F^{(n)}}{2^n},$$

where $n$ numbers the RG steps, and hence the spin averages $\langle S^{(n)} \rangle$ and $\langle (S^{(n)})^2 \rangle$. In figure 1 the temperature dependences of the spin averages found from the RG procedure for infinite chains (solid lines), and exact results for three three-site chains (dashed lines), are presented for the model with

$$D_F = -1.1, \quad K_L = K_C = 0.5, \quad H_L = -H_C = 0.01.$$

As seen, the results for the infinite and three-site chains converge at $T = 0$ and for high temperatures.

In order to consider the interchain (intergroup) coupling (6) we apply a cluster approximation. In this approximation one considers a finite number of isolated cells (cluster), disregarding the remaining cells of the system [10]. Outwardly, in our case the simplest cluster possible is made of two three-site cells from ‘$L$’ and ‘$C$’ subsystems, and the contribution to the renormalized energy of this cluster is

$$\ln \langle e_i^{\sigma_i} e_j^{\sigma_j} \rangle_0, \quad q = q_{LC} = -Q/T,$$

where

$$\langle A \rangle_0 = \frac{\text{Tr}_S A P(\sigma, S) e_0^H}{\text{Tr}_S P(\sigma, S) e_0^H}. $$

However, as usual the RG procedure generates new couplings, whose original values are equal to zero, and one has to consider general interactions of the isolated set of the three three-site cells from ‘$L$’, ‘$C$’ and ‘$F$’ subsystems.

Figure 1. Temperature dependence of the magnetization $\langle S_i \rangle$ (a) and $\langle S_i^2 \rangle$ (b) for three noninteracting chains. Solid lines denote infinite chains and dashed lines three-spin chains.
\[
\ln(e^{H_f})
\]

with

\[
H_f = \sum_{\alpha=\beta=L,C,F} k_{\alpha\beta} \sum_{i=1}^{3} S_i^\alpha S_i^\beta + \sum_{\alpha=\beta=L,C,F} q_{\alpha\beta} \sum_{i=1}^{3} (S_i^\alpha S_i^\beta)^2 + \sum_{\alpha=\beta=L,C,F} j_{\alpha\beta} \sum_{i=1}^{3} (S_i^\alpha S_i^\beta)^2 \\
+ \sum_{\alpha=\beta=L,C,F} k_{\alpha\beta}^d \sum_{i=1}^{3} S_i^\alpha S_{i+1}^\beta + \sum_{\alpha=\beta=L,C,F} q_{\alpha\beta}^d \sum_{i=1}^{3} (S_i^\alpha S_{i+1}^\beta)^2 + \sum_{\alpha=\beta=L,C,F} j_{\alpha\beta}^d \sum_{i=1}^{3} (S_i^\alpha S_{i+1}^\beta)^2 \\
+ \sum_{\alpha=\beta=L,C,F} \tilde{j}_{\alpha\beta} \sum_{i=1}^{3} (S_i^\beta S_i^\alpha) + \sum_{\alpha=\beta=L,C,F} \tilde{j}_{\beta\alpha} \sum_{i=1}^{3} (S_i^\beta S_{i+1}^\alpha)
\]

Altogether, one has to consider, formally, 39 coupling parameters: 15 single-chain (3 chains times 5 parameters), of which, in the minimum set, three are originals, \(k, h, d\) (equation (8)), and 24 interchain couplings (equation 19), of which one is original, \(q\) (equation 16). However, it is quite easy to find analytical forms of the renormalized couplings and perform the RG iterations.

4. The \(V_D\) index

As mentioned in the Introduction, random events but also information noise can play a similar role in social systems to temperature in physical systems. So, one can find the temperature dependences of the number of particular party voters and \(V_D\). We start with the noninteracting subsystem models: (i) defined by the parameters (15) and, to check the possible role of \(K_F\) coupling, (ii) additionally \(K_F = 1\). Because we assume that the numbers in each group of voters are the same, \(n_L = n_C = n_F\), initially (at the ground state) the numbers in each group of ‘L’, ‘C’ and ‘F’ voters are the same, \(N_L = N_C = N_F = 1\) per agent.

The results are presented in figure 2. As seen for model (i) the voter numbers start with \(N_i = 1\), and then as the temperature increases, \(N_L\) and \(N_C\) decrease, reach a minimum and then increase to 1 as \(T \to \infty\). Similarly, the \(V_D\) index starts with \(\frac{2}{3}\) at \(T = 0\) and beyond the minimum it reaches the same value \(\frac{2}{3}\) as \(T \to \infty\). For model (ii) both \(N_L(N_C)\) and \(V_D\) first increase, reach a maximum, pass through a minimum and then increase again to 1 and \(\frac{2}{3}\) as \(T \to \infty\), respectively.

We are now in a position to evaluate the dependence of the particular parties’ voter numbers on the intergroup coupling \(Q\) (equations (6) and (16)). Let us first consider the finite system of three chains with three agents in each, with the coupling parameters as in (15) and \(Q > 0\) at \(T = 0\) (ground state), \(T = 0.05\) and \(T = 0.2\). As is seen from the left plots of figure 3, at the ground state the initial agent arrangement \(N_L = N_C = N_F = 1\) is conserved until \(Q = \frac{1}{3}\), then there is a jump of \(N_L = N_C = 1\) to \(\frac{1}{2}\), and \(N_F\) to 2. The jump is gradually smeared by rising temperature (middle and right plots of figure 3). For a nonsymmetric case \(K_L \neq K_C\) (figure 4), the ground state configuration is essentially different, and if for example \(K_L > K_C\) then \(N_L\) does not depend on \(Q\) whereas
NC drops to 0 and $N_F$ jumps to 2 at $Q = \frac{1}{3}$. At higher temperature the behavior of $N_i$ in the nonsymmetric case is similar to that in the symmetric one. In the bottom plots of figures 3 and 4 the $Q$-dependence of the $V_D$ index is shown. For low temperature $V_D$ drops sharply at $Q = \frac{1}{3}$, and for higher temperature it decreases rapidly from $\frac{2}{3}$ to $\frac{1}{3}$.
Now we proceed to the RG analysis of the infinite chains. To calculate the average (18) we use the identity

\[
\exp\left[ k_{\alpha\beta} S_\alpha S_\beta + q_{\alpha\beta} (S_\alpha S_\beta)^2 + j_{\alpha\beta} (S_\alpha^2 S_\beta^2 + J_{\beta\alpha} S_\alpha S_\beta^2) \right] = 1 + K_{\alpha\beta} S_\alpha S_\beta + Q_{\alpha\beta} (S_\alpha^2 S_\beta^2) + J_{\alpha\beta} (S_\alpha^2 S_\beta^2 + J_{\beta\alpha} S_\alpha S_\beta^2),
\]

where

\[
K_{\alpha\beta} = \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta}^2 - j_{\beta\alpha}^2} \left( e^{2k_{\alpha\beta}} - e^{2j_{\alpha\beta}} + 2e^{k_{\alpha\beta} + 2j_{\alpha\beta}} \right),
\]

\[
Q_{\alpha\beta} = -1 + \frac{1}{4} e^{q_{\alpha\beta} + k_{\alpha\beta} - j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} + j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} + j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} + k_{\alpha\beta} + j_{\beta\alpha}}.
\]

\[
J_{\alpha\beta} = -1 + \frac{1}{4} e^{q_{\alpha\beta} + k_{\alpha\beta} - j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} + j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} + j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} + k_{\alpha\beta} + j_{\beta\alpha}}.
\]

To evaluate the RG transformation one has to know the chain averages \( \langle S_\alpha^0 \rangle \), \( \langle (S_\alpha^0)^2 \rangle \), \( \langle S_\alpha^0 S_\beta^0 \rangle \), \( \langle (S_\alpha^0 S_\beta^0)^2 \rangle \) and \( \langle (S_\alpha^0)^2 S_\beta^0 \rangle \). It is quite easy to find their closed expressions, for example:

\[
\langle S_\alpha^0 \rangle = \sigma_\alpha^0, \quad \langle S_\beta^0 \rangle = \sigma_\beta^0, \quad \alpha = L, C, F
\]

\[
\langle S_\alpha^2 \rangle = G_\alpha^0 + G_\alpha^1 (\sigma_\alpha^0 + \sigma_\alpha^0) + q_\alpha^0 (\sigma_\alpha^0)^2 + G_\alpha^2 ((\sigma_\alpha^0)^2 + (\sigma_\alpha^0)^2)
\]

\[
+ G_\alpha^3 (\sigma_\alpha^0)^2 (\sigma_\alpha^0)^2 + (\sigma_\alpha^0)^2 (\sigma_\alpha^0)^2)
\]

The coefficients \( G_\alpha^i \) are presented in appendix B.

In the cluster approximation with three three-site (-agent) blocks ‘L’, ‘C’, ‘F’ taking into account only two-site coupling, the RG transformation has the form of 39 recursion relations. By iterating these relations and collecting the constant terms generated in each step of the iteration process one can calculate numerically the ‘free energy’ and then the averages \( \langle S_\alpha^0 \rangle \) and \( \langle (S_\alpha^0)^2 \rangle \). In figure 5 these averages are presented.
as functions of interblock coupling $Q$ for the model with $D_F = -1.1$, $H_L = -H_C = 0.01$ in two cases: (i) symmetric $K_L = K_C = 0.5$ and (ii) nonsymmetric $K_L = 0.5$ and $K_C = 0.48$ at $T = 0.25$. Knowing the averages $\langle S_i^a \rangle$ and $\langle (S_i^a)^2 \rangle$, one can find the number of particular parties’ voters (5) and $V_D$ index (1). The results for symmetric and nonsymmetric cases are presented in figures 6 and 7, respectively. It is seen that the dependences of the voter numbers on $Q$ for an infinite system differ significantly from those for three-site blocks. However, in both symmetric and nonsymmetric cases, as for the finite system at low temperature, the $V_D$ index changes slowly for sufficiently small $Q$ and then drop sharply to a constant value.

In physical systems the coupling parameters $K_a$, $D_a$, $H_a$ and $Q_a$ have a plausible interpretation even if they have an effective character. Such an interpretation, of course, is not so obvious for social systems. However, one can assume that there is a positive coupling between the members of the same political environment, the measure of which is the parameter $K_a$, and some parameter that separates the creeds of the particular party voters, $H_a$. Analogously, a negative $D_F$ can be considered as a measure of discouragement to take part in public life; on the other hand, a positive $D_{L(C)}$ is a measure of citizen participation. In figure 8 the $Q$-dependences of index $V_D$ of the symmetric ($K_L = K_C = 0.5$, $D_F = -1.1$, $D_{L(C)} = 0$) and nonsymmetric ($K_L = 0.5$, $K_C = 0.4$, $D_F = -1.1$, $D_{L(C)} = 0$) models considered above are compared with the results for the model with $K_F = 1$ and positive $D_{L(C)} = 0.5$. As one would expect, in the latter case the range of $Q$ in which $V_D$ changes ever so slightly is much broader.

5. Summary

It is unlikely that a simple statistical physics model could be used to predict a social event, although certain sociophysicists believe that it is possible in some cases and, for example, Serge Galam [3] claims ‘I do not think history could be predicted even in principle, given our current tools of research and perception of the world’; however, at the same time he expresses a hope that ‘sociophysics in the future may yield real predictive tools’. Anyway, it seems that sociophysics models can be successfully used to describe, explain and point out general features of social behavior.
In this paper we propose the three-state Ising-like statistical physics model to describe the influence of the social interplay between electorates of the two major parties, embodied by the coupling $Q$, on the quality of democracy. The minimal number of parameters that define the model is three: the measure of the unity of views of voters of the two major parties ($L$, $C$), $k = -K_L/T = -K_C/T$, the field that differentiates the creeds of the particular party voters, $h = -H_L/T = -H_C/T$, and the measure of a discouragement to take part in public life of the $F$-group citizens, $d = -D_F/T$. The measure of the quality of democracy is $V_D$, an index defined as the percentage of the total population that actually voted for the two major parties in a given election. This index reflects not only the rights but also the inclination of citizens to participate in decision-making, even if theoretically, which can be treated as the essence of democracy. To check the universality of the results we have applied three sets of original parameters: (i) a symmetric model ($K_L = K_C = 0.5$, $K_F = 0$, $D_F = -1.1$, $D_L(C) = 0$), (ii) a nonsymmetric model ($K_L = 0.5$, $K_C = 0.4$, $K_F = 1$, $D_L(C) = 0.5$), and (iii) a second nonsymmetric model ($K_L = 0.5$, $K_C = 0.4$, $K_F = 1$, $D_L(C) = 0.5$, others as above). In all cases, there is a range of $Q$ in which the index $V_D$ changes slightly: it first increases with $Q$, passes a maximum, then at some characteristic point $Q_f$ starts to fall rapidly, and at $Q_c$ reaches a constant value (figure 8). At the same time for the symmetric model
(i) the numbers of voters of both major parties first slightly increase with increasing $Q$ and then sharply decrease (figure 6). For the nonsymmetric case ($K_L > K_C$), only the number of $L$-party voters increases, reaches a maximum and then drops to some constant value, whereas the number of $C$-party voters decreases immediately with increasing $Q$ (figure 7). When the values of both $Q_f$ and $Q_c$ and the location of the maximum of $V_D$ depend on the model parameters, a collapse of $V_D$ seems to be a general feature of the present model.

We conclude from the model that in the two-party political system a reasonable level of conflict between the electorates of the two major parties can be beneficial for both parties and moreover for the quality of democracy measured by the index $V_D$. However, for a higher level of conflict (higher degree of polarization), citizen participation decreases rapidly. For $Q > Q_c$, only so-called hard or fixed electorates of the major parties want to take part in public life. A high percentage—and in the extreme case most of society—decline to vote for a party that can win a majority in the legislature and therefore decline to participate in real decision-making, which in fact means the collapse of high-quality democracy.

Appendix A. Decimation transformation parameters

The renormalized parameters (13) as functions of the original interactions $k_{\alpha}, h_{\alpha}, d_{\alpha}$ (RG recursion relations) are as follows:

$$
\begin{align}
  z_\alpha &= \lambda_0^\alpha, \quad h'_\alpha = \lambda_1^\alpha - \lambda_2^\alpha, \quad k'_\alpha = \frac{1}{4}(-2\lambda_3^\alpha + \lambda_4^\alpha + \lambda_5^\alpha), \quad d'_\alpha = -2\lambda_0^\alpha + \lambda_1^\alpha + \lambda_2^\alpha), \\
  j'_\alpha &= \frac{1}{4}(-2\lambda_1^\alpha + 2\lambda_2^\alpha + \lambda_1^\alpha + \lambda_5^\alpha), \quad q'_\alpha = \frac{1}{4}(4\lambda_0^\alpha - 4\lambda_1^\alpha - 4\lambda_2^\alpha + 2\lambda_3^\alpha + \lambda_4^\alpha + \lambda_5^\alpha). \\
  \lambda_i^\alpha &= \ln f_i^\alpha, \quad \omega_i^\alpha = \frac{1}{f_i^\alpha}, \quad i = 0, 1, \ldots, 5, \quad \alpha = L, C, F.
\end{align}
$$

Figure 8. $V_D$ index as a function of $Q$ for three models: symmetric $K_L = K_C = 0.5$, $K_F = 0$ (solid line), nonsymmetric with $K_L = 0.5$, $K_C = 0.4$, $K_F = 1$ (dotted) and nonsymmetric with $K_L = 0.5$, $K_C = 0.4$, $K_F = 1$, $D_L = D_C = 0.5$ (dashed) at $T = 0.2$. 

\begin{thebibliography}{1}
\bibitem{1} doi:10.1088/1742-5468/2015/10/P10006
\end{thebibliography}
$f_0^\alpha = 1 + e^{-d_\alpha h_\alpha} + e^{d_\alpha h_\alpha}$,

$f_1^\alpha = \frac{1}{2}(e^{-d_\alpha h_\alpha} + e^{-2d_\alpha h_\alpha})e^{d_\alpha h_\alpha} + e^{d_\alpha h_\alpha}$,

$f_2^\alpha = \frac{1}{2}(e^{-d_\alpha h_\alpha} + e^{-2d_\alpha h_\alpha})e^{d_\alpha h_\alpha} + e^{d_\alpha h_\alpha}$,

$f_3^\alpha = e^{d_\alpha h_\alpha} + e^{d_\alpha h_\alpha}$,

$f_4^\alpha = e^{d_\alpha h_\alpha} + e^{d_\alpha h_\alpha}$,

$f_5^\alpha = e^{d_\alpha h_\alpha} + e^{d_\alpha h_\alpha}$.

(A.3)

Appendix B. Single-chain averages

$G_0^\alpha = c_p^\alpha g_0^\alpha,$

$G_1^\alpha = (c_p^\alpha + c_q^\alpha)g_1^\alpha + c_p^\alpha(g_0^\alpha + g_2^\alpha)$,

$G_2^\alpha = c_p^\alpha g_0^\alpha + c_p^\alpha g_2^\alpha + c_q^\alpha(g_0^\alpha + g_2^\alpha),

G_3^\alpha = 2(c_p^\alpha + c_q^\alpha)(g_1^\alpha + g_2^\alpha) + c_p^\alpha(g_0^\alpha + 2g_2^\alpha + g_4^\alpha) + (2c_p^\alpha + c_q^\alpha)g_0^\alpha$,

$G_4^\alpha = (c_p^\alpha + c_q^\alpha)g_0^\alpha + c_p^\alpha(g_1^\alpha + g_4^\alpha) + c_q^\alpha(g_1^\alpha + g_3^\alpha) + c_2^\alpha g_1^\alpha + 2g_2^\alpha + g_4^\alpha$,

$G_5^\alpha = 2c_p^\alpha(g_2^\alpha + g_4^\alpha) + 2c_p^\alpha g_4^\alpha + 2c_q^\alpha(g_1^\alpha + g_3^\alpha) + c_q^\alpha g_1^\alpha + c_2^\alpha g_1^\alpha + 2g_2^\alpha + g_4^\alpha + c_q^\alpha g_4^\alpha$.

(B.1)

where

$c_p^\alpha = \frac{1}{2}(\omega_0^\alpha - \omega_2^\alpha),

\quad c_q^\alpha = \frac{1}{4}(\omega_0^\alpha + \omega_4^\alpha - 2\omega_2^\alpha),

\quad c_q^\alpha = \frac{1}{2}(\omega_0^\alpha + \omega_4^\alpha) - \omega_0^\alpha.$

$\frac{1}{2}(\omega_1^\alpha - \omega_3^\alpha)$,

$\frac{1}{2}(\omega_2^\alpha - \omega_1^\alpha + \omega_3^\alpha - \omega_0^\alpha),

\quad c_j^\alpha = \omega_0^\alpha - \omega_1^\alpha - \omega_2^\alpha + \frac{1}{2}(\omega_3^\alpha + \omega_4^\alpha + \omega_5^\alpha).$ (B.2)

and

$g_0^\alpha = \text{Tr}_S S_2^\alpha [1 - (S_1^\alpha)^2](1 - (S_3^\alpha)^2)e^{H_0^\alpha},

\quad g_1^\alpha = \frac{1}{2}\text{Tr}_S S_2^\alpha S_1^\alpha [1 - (S_3^\alpha)^2]e^{H_0^\alpha},

\quad g_2^\alpha = \frac{1}{4}\text{Tr}_S S_2^\alpha S_0^\alpha e^{H_0^\alpha},

\quad g_3^\alpha = \text{Tr}_S S_2^\alpha S_0^\alpha S_1^\alpha [1 - (S_3^\alpha)^2]e^{H_0^\alpha},

\quad g_4^\alpha = \frac{1}{2}\text{Tr}_S S_2^\alpha [1 - \frac{3}{2}(S_1^\alpha)^2]e^{H_0^\alpha},

\quad g_5^\alpha = \text{Tr}_S S_2^\alpha S_3^\alpha [1 - \frac{3}{2}(S_1^\alpha)^2]e^{H_0^\alpha}.$ (B.3)

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