Chapter 8

Vibration Simulation of Electric Machines

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Abstract

This chapter deals with the basics of vibration calculations in electrical machines. It includes a brief introduction to the sources of vibration in electrical machines. In addition, the construction of electric machines is briefly summarized. It also describes the influence of individual parts of electric machines on vibration generation. The chapter also deals with the important steps that need to be taken when calculating vibration signal waveform using finite element method (Ansys). The individual sections summarize the most important requirements for setting the vibration calculation and it also deals with minimizing the calculation errors.

Keywords: vibration, finite element method, electric machines, vibration simulation, vibration sources, Ansys, Maxwell

1. Introduction

The vibration of electric machines is one of the causes of the function of electric machines. This is an undesirable phenomenon that cannot be completely separated. The vibrations of electric machines are very depending on the operating state of the electric machine. In the diagnostics of electrical machines, vibrations are used to identify failures of both electrical machines as well as mechanical connections to other machines and power failures. In practice, electrical machines can experience many types of malfunctions that can affect the function of the electrical machine itself or even destroy it. In some cases, vibrations generated by electrical machine failures can also damage other machinery near this machine. For this reason, prevention and early detection of malfunctions is important. In electrical machines, most of the progressive failures begin gradually to appear on the level or frequency spectrum of vibrations. Each failure has other symptoms (other vibration frequencies, direction, size, etc.). This makes it possible to determine the type of failure based on long-term tracking. In order to be able to identify in a timely manner, the type of malfunction that arises, it is necessary to know the manifestations of the individual faults. In
earlier times, the only possibility of tracking and measuring the development of malfunctions was done on real machines. On the basis of this experience, the same foreseeability disturbances could be assumed in other machines. This process has been simplified with the onset of computing and finite element utilization. It is now possible to simulate the physical models in electrical machines in full extent. It is possible to interface individual types of models (mechanical, electromagnetic, thermal) and then achieve very accurate results. The Ansys program is a program that allows to solve physical phenomena in electrical machines. Thanks to the individual modules, it is possible to make electromagnetic, thermal and mechanical design of any electrical machine and then simulate its behavior in different operating states. Especially today, when using many types of inverters, it is a great advantage to connect a model to an electrical circuit. It allows to solve the influence of different methods of power supply on electric machine. For example, what effect the vibrations of the electric machine will have on higher harmonic generated inverter.

The problem of calculating vibrations in electrical machines is very complex due to the number of physical phenomena, and it is necessary to handle a large amount of information from many areas (mechanics, magnetism, etc.). For this reason, this chapter focuses on a basic approach to solution issues. In solving a particular problem, it is necessary to take into account the time requirement of individual calculations and to perform a sufficient amount of calculation simplifications which are based on the results requirements analysis. Simplification may involve adjustments to a particular model that is used for the calculations. Another simplification may be the neglect of some of the vibration sources that operate in the electric machine, and so on. The main requirement is that the simplification of the model does not cause the error to be calculated. Therefore, it is necessary to familiarize themselves with the construction of the simulated machine, the sources of vibration, the functions of individual parts and their effect on the propagation of vibrations [1, 2].

2. Vibration fundamentals

Vibrations are a mechanical phenomenon. It can be said that this is the movement of a flexible body or environment whose individual body vibrates around the equilibrium position.

The forces acting on the vibrating body define the motion equation:

\[ m \frac{d^2 x}{dt^2} = F(t) - kx - b \frac{dx}{dt} \]  

(1)

where \( m \) is body mass, \( x \) is deviation from the steady state of the body, \( F(t) \) is force dependent on time, \( k \) is stiffness of the spring, and \( b \) is coefficient of damping.

The forces acting on any system create the oscillation itself. In a simple case, the oscillation has a harmonic character. This occurs when system is exposed to a single source with a constant exciting force. For the description of harmonic oscillation, the relationship is used:

\[ x(t) = x_{\text{max}} \sin(2\pi ft) \]  

(2)

where \( x(t) \) is displacement value, \( x_{\text{max}} \) is maximum displacement value and \( f \) is vibration frequency.
This relationship applies to very simple oscillations. There are a number of sources and influences in electric machines that affect the vibration generation. The actual course of vibration displacement is, therefore, the sum of forces that change over time with different frequencies. The vibration displacement of the electric machine is shown in Figure 1. Fast Fourier Transformation is used to convert the signal into the frequency area and the result of this transformation is shown in Figure 1. Figure 1 also shows the proportion of individual harmonics on the vibration signal [2–4].

3. Electric machine construction

For the calculation of vibrations in electrical machines, it is necessary to get basic information about their basic construction. The electric machine consists of a magnetic circuit. The magnetic circuit focuses most of the magnetic field into a defined area. The magnetic circuit itself is made of steel plates connected to the stator, respectively into the rotor. There are grooves cut on the internal circumference of the stator, into which the winding is inserted. The winding itself is one of the most important parts of electric machines. Copper with good electric conductivity and with 99% purity is used as a material of winding. In some applications, aluminum alloy of similar purity is used as a material. All electric motors have many other mechanical parts. These include a shaft on which the rotor plates are mounted. Although the shaft is, in most cases, a simple component that is made of a machined steel rod, it can have a great effect on the vibration of the machine. The main parameter that can affect the vibration is the quality of the processing and the quality of the whole rotor balancing. Due to the possible inhomogeneity of the material, the so-called mass unbalance can occur, causing the unwanted vibrations generated by the machine. The vibration level and frequency depend on the rotation speed of the rotor itself. Rotors are balancing in production to reduce this phenomenon.

Another important part of electrical machines is bearings. Many types of bearings are used in electrical machines. Ball bearing or roller bearings are commonly used. Nowadays, electromagnetic bearings are also used in special applications. From the vibration point of view, two separate phenomena occur in the bearings. The first is generating vibrations. This is in
trouble-free condition caused, for example, by skipping the balls. The fault condition is the result of a missing lubricant. Failures can also be caused by poor quality lubricants. The second factor that affects the vibration of the electric machine is transmission between the rotor and the stator. Depending on the design of the bearings, they are partially damped.

The bearings are located in a bearing shield that is attached to the frame of the electric machine. A stator of the electric machine is also placed in the frame. There are several types of frames. The most commonly used are foot frame or flanges frame. Based on the type of frame, the machine is mechanically connected to the device. Again, the method of attachment of the machine affects the propagation or, respectively, vibration damping in the electrical machine. A terminal box is also attached to the frame and it serves to connect the power supply. The power supply method of the electric machine is another important factor. Electric machines are divided according to the following types of power supply:

- **Asynchronous**: Electric machines powered by AC voltage, either in one or in three phases. These are the most commonly used machines in the industry. They have different rotations of the magnetic field in the stator and the rotor.

- **Synchronous**: Electrical machine whose rotating speed is proportional to the frequency of the alternating current supply and independent of the load. Synchronous machines are very often used as a generator.

- **DC machines**: Electrical machine powered by direct current. In most cases, it works with a static field in static machine parts. Permanent magnets may appear in their construction.

As can be seen from the brief description of the construction of the electric machine, it is a mechanically and intricately complex device with many variations. Various factors can affect the formation or propagation of vibrations. For this reason, it is always necessary to determine which parameters and structural elements are inserted into the calculation process and which are neglected [5–7, 10, 11].

### 3.1. Electrical machine failures

The generation of vibrations in an electric machine influences several design parameters. The influence on the generation of vibrations is mainly due to components design, mainly their shape and quality of production. With the time of use of the electrical machine in operation, vibration and wear of individual components increase.

Vibration sources are identified in vibration spectrum. Each vibration source takes effect of specific frequency in the spectrum. The amplitude is proportional to the degree of damage. For each source that causes the peaks at corresponding frequencies with increasing deviation, the value of peaks increases [2, 8].

Examples of electrical machine faults that can be modeled with finite element method are:

- **Unbalance of the rotor**: The unbalance depends on the distribution of the center of gravity of the rotor relative to its axis of rotation. Because of the uneven distribution of matter, the imbalance causes centrifugal force, noise and rotor vibration. With higher speeds, vibration is increasing.
• Eccentric rotor: Eccentricity occurs especially when the rotary axis is shifted relative to the geometric axis. Because of the eccentric rotor, there is a variable air gap between the stator and the rotor that generates pulsating vibrations. The greatest vibration reaches the first harmonic component. The rotor eccentricity contributes to vibration and noise. It causes an unbalanced pull of magnetic force in the rotor and bending the shaft.

There are two types of eccentricity:

Static eccentricity: Situation, when the rotor is deflected from the center of the engine and still rotates around its own axis of rotation. This is because of the static eccentricity. The size of the air gap is not constant over its entire circumference, resulting in stronger interactions between the stator and rotor magnetic fields in places with a smaller air gap.

Dynamic eccentricity: In the latter case, dynamic eccentricity occurs when the rotor rotates in the geometric center of the engine but does not rotate around its own axis of rotation. The air gap is a function of both position and time. The variable air gap rotates at a frequency equivalent to the rotational speed of the rotor.

• Bent shaft: The cause of the shaft deformation is the difference between the geometric axis and the axis of rotation. The geometric axis of the bent shaft has the shape of a curve. If the axis of rotation is not a straight line, it is a bend shaft. If the center of gravity does not lie on the rotary axis, the rotor is unbalanced [2, 8].

4. Vibration sources

For any calculation of the vibration level, it is necessary to become familiar with the various sources that generate these vibrations in electric machines. According to the physical principle, sources of vibration can be divided into several groups:

• Electromagnetic sources
• Mechanical sources
• Aerodynamic sources

Vibrations of electromagnetic and mechanical origin occur in all rotating electric machines. As a source of aerodynamic origin, it is usually a fan. Fan is often not a part of the construction of electric machines. For this reason, this chapter does not deal with the problem of calculating the vibrations thus generated [1].

4.1. Electromagnetic sources

Part of the vibration of electric machines is of electromagnetic origin. Their cause is the oscillation of the machine frame and its parts caused by electromagnetic forces. These forces are due to higher harmonics of the supply current, magnetic saturation, phase asymmetry, magnetostriction or disturbances in the magnetic circuit or electrical component of the machine.
The frequency spectrum of these vibrations has discrete character. Vibrations caused by the electrical causes occur mainly in the radial direction. Vibrations are occurred in the case of more varied air gap size in the axial direction, for example, due to non-symmetrical rotor mounting [1].

4.2. Mechanical sources

Mechanical vibrations are mainly caused by bearings, rotor balancing, machining of rotating parts and rotor mounting. Mechanical vibrations produced into electrical machines are also caused by connected devices. These external vibration sources include clutch misalignment or gearing, wedge gears or vibrations caused by connected loads [1].

4.3. Aerodynamic sources

The basic aerodynamic source of vibration is the fan in electric machine. Any obstruction that is exposed to air flow can generate vibration. The main cause of fan noise is the formation of turbulent airflow around the fan blades [1, 2, 4].

5. Vibration simulation using finite element method (FEM)

As already indicated in the previous sections of the chapter, the vibration of electric machines is a phenomenon that interferes with several physical areas (mechanics, electromagnetism, etc.) [8–10]. Therefore, the entire calculation process needs to be divided into several parts:

a. Determining Vibration Sources: at first, it is necessary to decide which resources to count. In the case of this chapter, the calculation is simplified only for the occurrence of vibrations by the effect of a time-dependent electromagnetic field. For this reason, it is possible to use Maxwell 2D or Maxwell 3D. This module allows to calculate the time-varying effect of the force in the magnetic circuit depending on the change of the electric current. The use of this program also allows to connect to a simulator of electrical circuits (program Simulor). After connecting the supply current to the Maxwell model on the simpler electric circuit, it is possible to calculate the changes in the magnetic circuit caused by the control logic, that is, speed control.

b. Model creation: creating a model of an electric machine is one of the most important parts of the calculation itself. The user must choose between 2D and 3D model. The 2D model is much simpler, and therefore, the calculation itself takes a very short time. On the other hand, this is a great simplification of the calculation. The 3D model will allow for a more accurate calculation and consideration of the more influences affecting the calculation. However, the calculation of 3D models is considerably more demanding for computational power, and therefore, the calculation time is considerably longer.

c. Calculation of forces caused by selected sources: the next step is to determine the forces that act on the electrical machine. In this chapter, this is defined as a force of electromagnetic...
origin. The Maxwell program is used to determine them. These calculations can be supplemented by the calculation of the actual frequencies of the electric machine and also by external influences (such as asymmetry, etc.). An important factor is determining the right time step.

d. Determining vibration on the model: determination of vibration on a particular model is the result of a mechanical analysis [8–10] (Figure 2).

6. Model creation

The model of an electric machine can be made in several ways. One of them is the use of modern CAD systems to create geometry and its subsequent import into the computing environment. Another option is to create a model directly in the Ansys (using DesignModeler).

Another choice is to use the RMxprt environment. This module is primarily designed for rapid calculations of electrical machines. It features an environment for fast input of electrical machine dimensions. At the beginning of the job, the user selects a template that matches the specific machine type. The user then enters the main machine dimensions, slot size and slot type, and other parameters using simple tables.

RMxprt contains the following electrical machine templates:

- Synchronous machine
- Permanent magnet DC motor/DC machine
- Claw-pole alternator
- DC machine
- Single-phase/three-phase induction machine
- Universal motor

Figure 2. The Ansys workbench modules.
User needs to know a lot of information to create a model in RMXprt. One of the items to know is the basic design dimensions. Furthermore, it is necessary to know how to place the windings in the grooves and also the dimensions of the individual wires. All of these parameters affect the end result [10] (Figure 3).

7. Determining the time step

When calculating the vibration of an electrical machine, it is important to note, as in the case of its measurement, the need to take into account the sampling frequency of the search signal. In the case of finite element calculation, this sampling frequency is represented by the time step of the individual calculations.

Results of transient analysis are linear approximate in the Ansys program. This can cause data loss. Example of choosing a time step or sampling frequency is shown in Figure 4 on the simple signal. The used signal shows the sinus function with frequency 1 Hz. When selecting a large time step (specifically 3 Hz), this function is approximated by a straight line. There is a complete loss of function. When there is use 5 values on signal sampling and there is use linear interleaving function for his reconstruction, then constructed signal has triangle waveform. The calculated signal is the same as the original function only when using a sampling frequency of 36 Hz (and higher). This example shows how the time step can affect the results of the time waveform

Figure 3. Models of induction machine generated by RMxprt: a) 2D model and b) 3D model.

Figure 4. Linear approximation of sin signal.
calculations of primitive function. Even greater influence can be experienced on the course of the calculated waveforms of the vibrational signals, which are much more complicated.

For the sampling of the general signal, the so-called Nyquist condition is mentioned in the literature. This indicates that each frequency-limited time function can be replaced by samples taken with the \( T_{sam} \) period, which is equal to half of the overturned value of the \( f_{\text{max}} \), the highest frequency contained in the sampling signal. Therefore:

\[
f_{sam} = \frac{1}{T_{sam}} \geq 2f_{\text{max}} \tag{3}
\]

where \( f_{sam} \) is sampling frequency and \( f_{\text{max}} \) is maximal frequency included in the signal.

As shown in Figure 4, when this state is used for a complex signal (such as the vibration are complex), there has been a strong deformation of the entire calculated waveform. Determining the time step requires some experience and knowledge that leads to a compromise between the quality of the result and the time consumption of the calculation [4].

### 8. Electromagnetic simulation

Electromagnetic simulation is a possible solution in the Maxwell program, which is one of the Ansys software package modules. Maxwell module is used to calculate the magnetic field on 2D and 3D models. The calculation itself is based on Maxwell’s equations. These equations can be written in a differential form:

\[
\nabla \times E = -\frac{\partial B}{\partial t} \tag{4}
\]

\[
\nabla . B = 0 \tag{5}
\]

\[
\nabla \times H = J + \frac{\partial D}{\partial t} \tag{6}
\]

\[
\nabla . D = \rho \tag{7}
\]

where \( E \) is electric field intensity, \( B \) is magnetic flux density, \( H \) is magnetic field intensity, \( J \) is current density on surface, \( D \) is electric flux density, and \( \rho \) is volume charge density.

Some of these parameters depend on the properties of the used material:

\[
B = \mu_0 \mu_r H \tag{8}
\]

where \( \mu_0 \) is permeability of vacuum and \( \mu_r \) is relative permeability of material.

\[
D = \varepsilon_0 \varepsilon_r E \tag{9}
\]
where \( \varepsilon_0 \) is permittivity of vacuum and \( \varepsilon_r \) is permittivity of magnetic material.

\[
J = \sigma . E
\]  

(10)

where \( \sigma \) is electric conductivity.

To determine the vibrations in the electric machine model, the force acting between the rotor and the stator must be determined from the magnetic field in the air gap. For the calculation of these forces, it is possible to use the Maxwell stress tensor. Based on this, it is possible to write for two components electromagnetic forces on a 2D model equation:

- For radial direction

\[
F_{\text{rad}} = \frac{L_{\text{stk}}}{2 \mu_0} \int (B_n^2 - B_t^2) dl
\]  

(11)

- For tangential direction

\[
F_{\text{tan}} = \frac{L_{\text{stk}}}{2 \mu_0} \int B_n B_t dl
\]  

(12)

where \( B_n \) is normal component of flux density, \( B_t \) is tangential component of flux density, \( l \) is length of stator edge, and \( L_{\text{stk}} \) is stack length of the motor [2, 8, 9].

In the Maxwell environment, the following relationships to determine the individual power components [3] can be used:

\[
F_{\text{rad}} = F_x \cos \Theta_{\text{tip}} + F_y \cos \Theta_{\text{tip}}
\]  

(13)

\[
F_{\text{tan}} = -F_x \cos \Theta_{\text{tip}} + F_y \cos \Theta_{\text{tip}}
\]  

(14)

The radial force component acts perpendicularly to each tip, and the teeth cause radial deformation and vibration. Meanwhile, the tangential force acts on the rotor and produces rotational torque and also causes torsional strains.

When determining the behavior of an electrical machine, the time course of the supply voltage is decisive. This chapter deals with the calculation of vibrations on an asynchronous motor. This type of machine is powered by three-phase alternating voltage. For the simplest case, the time course is harmonious, containing one harmonic. In many real cases, the supply voltage is not smooth. The effect on these waveforms may be the power supply or the function of the inverter connected to the electric machine.

For calculation, the voltage was given in the following phases:

- Phase A: \( U_{\text{max}} \sin(2\pi \times 50 \times \text{time}) \)
- Phase B: \( U_{\text{max}} \sin(2\pi \times 50 \times \text{time} - 2\pi/3) \)
- Phase C: \( U_{\text{max}} \sin(2\pi \times 50 \times \text{time} - 4\pi/3) \)
From the point of view of the electromagnetic calculation of electrical machines, the magnetic magnet induction is the most important variable in the air gap. This magnitude depends on the design of the particular electric machine (number of slots, etc.). The figure of this magnitude at half the length of the air gap is shown in Figure 5. It is seen that magnetic flux density waveform is not as smooth as the supply voltage waveform. This waveform is displayed on a line representing the half of the air gap. It is clear that the quality of this process is very dependent on the quality of the mesh. The number of elements can affect the resulting signal waveform and also its frequency spectrum. Generally, it is recommended to use at least four elements representing the width of the air gap. Since the width of the air gap of the electric machine can range from tens to millimeters, it is a factor that can greatly influence the solving time and calculation difficulty.

The total time course of the absolute value of the forces in the electric machine is shown in Figure 6. As shown in Figure 6, the influence of linear approximation between the individual time steps is evident [2, 8, 9].

9. Mechanical simulation

The mechanical analysis itself in the Ansys program serves to determine the deformations based on the forces applied to the model. The main variable, which in this case describes the vibration, is the displacement of the individual components of the model. It also serves to determine a deformation and displacement of the body at each point in the mesh when finite element method is used. The calculation is based on the following equation:

\[
[M][a] + [C][v] + [K][x(t)] = [F(t)]
\]  

(15)

Figure 5. Magnetic field density in the air gap.

Figure 6. The time course of forces in the electric machine.
where $F(t)$ is load vector, $M$ is mass, $C$ is damping, $K$ is stiffness, $x$ is displacement vector, $a$ is acceleration vector ($a = \ddot{x}(t)$), and $v$ is velocity vector ($v = \dot{x}(t)$).

On the basis of this equation, the individual displacements of the bodies are then determined. Several other parameters have to be taken into account to set the calculation correctly. One of the things that need to be set is the material properties of the individual parts of the simulated machine. The material properties are the material density, the Poisson’s ratio and the Young’s modulus for mechanical analysis. These material properties can be found on the Internet or requested from the material supplier.

Another parameter that affects the quality of the result is the final set-up of the mesh. There is a need to focus on where forces work or where there is a small cross-section on model. At these points of the model, it is necessary to manually adjust the strength of the mesh to avoid any unwanted effect on the result.

Another parameter is to determine the properties of the contact surfaces. Particularly, for more complex models, results are poor where there is a poor contact surface setting. The Ansys mechanical analysis module uses five types of contact surfaces such as bounded, without separation, without friction, friction and pulping. Their choice depends on the knowledge of the construction of the electrical machine and on the way of the mechanical connection of its individual parts [4, 8] (Figure 7).

10. Conclusions

Since the vibration of electric machines is a carrier of information about its state, the issue of vibration calculation is very topical. When designing any electrical machine, the designer must reach to a certain standard of vibration which is given by the norm. In many cases, when designing a new machine, there were problems with vibration. This problem occurs only after the prototype has been created. By using finite element methods in vibration determination, designer can achieve very interesting results that can help minimize the vibration of electrical machines at the design stage. It can mean considerable savings for the manufacturing company.

The calculation of vibrations using the Ansys program is a very complex issue. The correct setup of electromagnetic computation requires experience not only with calculating electrical
machines but also with program Maxwell’s control and adjustment. The actual modeling of the machine itself is also quite demanding. Whether the user uses a classical CAD system or RMxpert, it needs a lot of information that can have a detrimental effect on the calculation. One of the most important information is the dimensions of the electric machine. This information can be obtained from the technical documentation. Another factor that can influence the calculation is the knowledge of the materials used and especially their properties. Much information about these materials can be found on the Internet. In many cases, the values of material properties are measured in specific conditions (e.g., at temperature 22°C) and then these values do not have to match the calculation conditions. The ideal source of information on material properties is the supplier of construction materials.

Once a model has been created, it is necessary to set-up individual analyses. It should be noted that, for example, electromagnetic analysis has different mesh quality requirements than mechanical analysis. There are automatic features for mesh creation in Ansys Workbench and Maxwell environment. For vibration calculations, it is necessary to edit the quality mesh on certain parts of the model manually in many cases. However, this requires knowledge not only of the procedures for both types of calculations but also of some experience with different types of analyses. Another non-negligible part of the calculation is the time step selection for transient analysis. If the time step is too long, there is a loss of data that could be critical to the calculation. Conversely, when setting a small time step, an unreasonable load on the computing hardware will occur and an increase in the computational time will be necessary. Given the number of calculations that need to be made to achieve the relevant results, a minor change may mean an extension of day calculations.

It can be said that modern programs using finite element methods allow the calculation of a wide range of physical problems. As far as the calculations of vibrations of electric machines are concerned, it is a very complex issue, which affects many areas (electricity, magnetism, mechanics, thermal). The calculation of the complete vibrational spectrum of the electric machine with all vibration sources is possible, but it is very time consuming. Therefore, it is always necessary to focus on solving a certain part of this issue. Based on the analysis of the results, requirements and the analysis of the input parameters, it is possible to simplify the solution considerably, which allows to achieve reasonable results in a sufficiently short time. An ideal solution to the vibration problem of electric tools using finite element method is to build a team of workers with knowledge and experience from different industries who will cooperate.

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