Debris Flows in Direct Dark Matter Searches-The modulation effect

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The effect of some possible non standard WIMP velocity distributions, like the Debris Flows recently proposed, on the direct dark matter detection rates is investigated. We find that such distributions may be deciphered from the data, especially if the time variation of the event rates due to the annual motion of the Earth is observed.

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I. INTRODUCTION

The combined MAXIMA-1 [1], BOOMERANG [2], DASI [3] and COBE/DMR Cosmic Microwave Background (CMB) observations [4] imply that the Universe is flat [3] and that most of the matter in the Universe is Dark [6, 7], i.e. exotic. Combining the data of these quite precise experiments on e finds:

$$\Omega_b = 0.0456 \pm 0.0015, \Omega_{\text{CDM}} = 0.228 \pm 0.013, \Omega_{\Lambda} = 0.726 \pm 0.015.$$ 

Since any "invisible" non exotic component cannot possibly exceed 40% of the above $\Omega_{\text{CDM}}$, exotic (non baryonic) matter is required and there is room for cold dark matter candidates or WIMPs (Weakly Interacting Massive Particles).

Even though there exists firm indirect evidence for a halo of dark matter in galaxies from the observed rotational curves, see e.g the review [9], it is essential to directly detect such matter. The possibility of such detection, however, depends on the nature of the dark matter constituents and their interactions.

Since the WIMP’s are expected to be extremely non relativistic, with average kinetic energy $\langle T \rangle \approx 50 \text{ keV}(m_{\text{WIMP}}/100 \text{ GeV})$, they are not likely to excite the nucleus. So they can be directly detected mainly via the recoiling of a nucleus $(A,Z)$ in elastic scattering. The event rate for such a process can be computed from the following ingredients[10]: i) The elementary nucleon cross section. This most important parameter will not, however, be the subject of the present work. We will adopt the view that it can be extracted from the data of event rates, if and when such data become available. From limits on the event rates, one can obtain exclusion plots on the nucleon cross sections as functions of the WIMP mass. ii) knowledge of the relevant nuclear matrix elements obtained with as reliable as possible many body nuclear wave functions, iii) knowledge of the WIMP density in our vicinity and its velocity distribution.

In the standard nuclear recoil experiments, first proposed more than 30 years ago [11], one has to face the problem that the reaction of interest does not have a characteristic feature to distinguish it from the background. So for the expected low counting rates the background is a formidable problem. Some special features of the WIMP-nuclear interaction can be exploited to reduce the background problems. Such are:

i) the modulation effect: this yields a periodic signal due to the motion of the earth around the sun. Unfortunately this effect, also proposed a long time ago [12] and subsequently studied by many authors [10, 13–21], is small and becomes even smaller than 2% due to cancelations arising from nuclear physics effects,

ii) backward-forward asymmetry expected in directional experiments, i.e. experiments in which the direction of the recoiling nucleus is also observed. Such an asymmetry has also been predicted a long time ago [22], but it has not been exploited, since such experiments have been considered very difficult to perform. Some progress has, however, has recently been made in this direction and they now appear feasible [22, 34]. In such experiments the event rate depends on the direction of observation. In the most favorable direction, opposite to the sun’s direction of motion, is comparable to the standard event rate. The sensitivity of these experiments for various halo models has also been discussed [27,28]. Furthermore we should mention that in such experiments [23,26,34] all events are counted. If some interesting events can be found, they can be established by further analyzing them by the direction of the observed recoils.

An essential ingredient in direct WIMP detection is the WIMP density in our vicinity and, especially, the WIMP velocity distribution. The dark matter in the solar neighborhood is commonly assumed to be smoothly distributed in space and to have a Maxwellian velocity distribution. Some of the calculations have considered various forms of phenomenological non symmetric velocity distributions [35–38] [19, 26, 27] and some of them even more exotic dark matter flows like the late infall of dark matter into the galaxy, i.e caustic rings [39–43] and Sagittarius dark matter [44].
In addition to the above models very recently it was found that the velocity distributions measured in high resolution numerical simulations exhibit deviations from the standard Maxwell-Boltzmann assumption, especially at large velocities [45, 46]. Furthermore a distinction was between a velocity structure that is spatially localized, such as streams [47, 48], and that which is spatially homogenized, which was designated as “debris flow” [49]. Both streams and debris flows arise from the disruption of satellites that fall into the Milky Way, but differ in the relative amount of phase-mixing that they have undergone. Implications of streams [50] and the debris flows in direct dark matter searches have also been considered [51].

In the present paper we will discuss in some detail the effect of these debris flows on the event rates of direct dark matter experiments for a variety of targets such as those employed in XENON10 [52], CoGENT [53], DAMA [54, 55] and PICASSO [56, 57]. We will also study the effect of these flows on the time variation of these rates due to the annual motion of the Earth [58] (modulation effect) as a function of the energy transfer and the WIMP mass and compare them with the standard M-B distribution. The effects of debris flows in directional experiments will be studied elsewhere.

\section{II. The Formalism for the WIMP-Nucleus Differential Event Rate}

The most interesting quantity which depends on the velocity distribution is the quantity \( g(v_{\text{min}}) \). For the M-B distribution in the local frame it is defined as follows:

\[
g(v_{\text{min}}, v_E(\alpha)) = \frac{1}{\sqrt{\pi} v_0^3} \int_{v_{\text{min}}}^{v_{\text{max}}} e^{-\left(\frac{v^2}{v_0^2} + 2v \cdot v_E(\alpha) + v_E^2(\alpha)/v_0^2\right)}dv d\Omega
\]  

For isotropic debris flows [51] it is given by:

\[
g(v_{\text{min}}, v_E(\alpha)) = \int_{v_{\text{min}}}^{v_{\text{max}}} \frac{f(v)}{v} dv, \quad f(v) = \begin{cases} \frac{v}{v_{\text{flow}}v_E(\alpha)}, & v_{\text{flow}} - v_E(\alpha) < v < v_{\text{flow}} + v_E(\alpha) \\ 0, & \text{otherwise} \end{cases}
\]

These functions are shown in Fig. [II]

In what follows we will find it useful to useful to expand \( g(v_{\text{min}}, v_E(\alpha)) \) in in powers of \( \delta \), the ratio of the Earth’s velocity around the sun divided by the velocity \( v_0 \) of the sun around the galaxy (220 km/s). Keeping terms up to linear in \( \delta \approx 0.135 \) and expressing everything in dimensionless variables we obtain:

\[
v_0 g(v_{\text{min}}, v_E(\alpha)) = \Psi_0(x) + \Psi_1(x) \cos \alpha, \quad x = \frac{v_{\text{min}}}{v_0}
\]
where $\Psi_0(x)$ represents the quantity relevant for the average rate, $\Psi_1(x)$, which is proportional to $\delta$, represents the modulation and $\alpha$ is the phase of the Earth ($\alpha = 0$ around June 3rd). In the case of the flows they were derived from the semi-analytic approximations of simulations as discussed by Spergel and co-workers [51]. In the case of a M-B distribution take the following form:

$$\Psi_0(x) = \frac{1}{2}(\text{erf}(1-x) + \text{erf}(x+1) + \text{erfc}(1-y_{esc}) + \text{erfc}(y_{esc}+1) - 2)$$

(4)

$$\Psi_1(x) = \frac{1}{2\delta} \left( \frac{-\text{erf}(1-x) - \text{erf}(x+1) - \text{erfc}(1-y_{esc}) - \text{erfc}(y_{esc}+1)}{2} + \frac{e^{-(x+1)^2}}{\sqrt{\pi}} - \frac{e^{-(y_{esc}+1)^2}}{\sqrt{\pi}} + 1 \right)$$

(5)

where erf($x$) and erfc($x$) are the error function and its complement respectively $y_{esc} \approx 2.8v_0$ is the escape velocity. For isotropic debris flows one finds:

$$\Psi_0(x) = \begin{cases} \frac{1}{2} & 0 < x < y_f - 1 \\ \frac{1+y_f-x}{2y_f} & y_f - 1 < x < 1 + y_f \\ 0 & x > 1 + y_f \end{cases}, \quad y_f = \frac{v_{flow}}{v_0}$$

(6)

$$\Psi_1(x) = \delta \begin{cases} 0 & 0 < x < y_f - 1 \\ \frac{x-y_f}{y_f} & y_f - 1 < x < 1 + y_f \\ 0 & x > 1 + y_f \end{cases}, \quad y_f = \frac{v_{flow}}{v_0}$$

(7)

We note that the variable $x$ depends on the nuclear recoil energy $E_R$ as well as the WIMP-nucleus reduced mass. As we shall see below there is an additional dependence of the rates on $E_R$ coming from the nuclear form factor.

At Earth-frame velocities greater than 450 km/s, debris flow comprises more than half of the dark matter at the Sun’s location, and up to 80% at even higher velocities [51]. In the VL2 simulation, the combination of debris flows and standard M-B is very well fit by the function

$$\epsilon(x) = 0.22 + 0.34 \left( \text{erf} \left( \frac{x}{220} \frac{185}{185} \right) + 1 \right)$$

(8)

This function is exhibited in Fig. 2. In this case we find:

$$\Psi_i(x) \rightarrow (1 - \epsilon(x)) \Psi_i^{MB}(x) + \epsilon(x)\Psi_i^f(x), \quad i = 0, 1$$

(9)

The functions $\Psi_0(x)$ and $\Psi_1(x)$ are exhibited in Fig. 3. As expected in the case of the flows $\Psi_0(x)$ falls off linearly for large values of $x$. Note that in all cases $\Psi_1(x)$ takes both positive and negative values, which affects the location of the maximum of the modulated rate as a function of $\alpha$, depending on the target and the WIMP mass. We will explore this effect of the different distributions in direct experiments searching any time dependence of the rates.

Once these functions are known the formalism to obtain the direct detection rates is fairly well known (see e.g. the recent reviews [59, 60]). So we will briefly discuss its essential elements here. The differential event rate can be cast in the form:

$$\frac{dR}{dE_R} \big|_A = \frac{dR_0}{dE_R} \big|_A + \frac{d\dot{H}}{dE_R} \big|_A \cos \alpha$$

(10)

where the first term represents the time averaged (non modulated) differential event rate, while the second gives the time dependent (modulated) one due to the motion of the Earth (see below). Furthermore

$$\frac{dR_0}{dE_R} \big|_A = \frac{\rho_x}{m_X} Am_p \sigma_n \left( \frac{\mu_r}{\mu_p} \right)^2 \sqrt{<v^2>_A} \frac{1}{Q_0(A)} \frac{dt}{du}$$

$$\frac{d\dot{H}}{dE_R} \big|_A = \frac{\rho_x}{m_X} Am_p \sigma_n \left( \frac{\mu_r}{\mu_p} \right)^2 \sqrt{<v^2>_A} \frac{1}{Q_0(A)} \frac{dh}{du}$$

(11)
FIG. 2: The function $\epsilon(x)$, $x = \frac{\nu_{\text{min}}}{\nu_0}$ as a function of $x$, which gives a possible combination of a M-B distribution and debris flows\[51\].

FIG. 3: The functions $\Psi_0(x)$ and $\Psi_1(x)$ as a function of $x = \frac{\nu_{\text{min}}}{\nu_0}$. Note that these functions have been computed at $\alpha = 0$, i.e. the average local velocity. Note also that the variable $x$ depends on the nuclear recoil energy $E_R$ as well as the WIMP-nucleus reduced mass. Otherwise the labeling of the curves is the same as that of Fig. \[1\].

with with $\mu_r$ ($\mu_p$) the WIMP-nucleus (nucleon) reduced mass, $A$ is the nuclear mass number and $\sigma_n$ is the elementary WIMP-nucleon cross section. $m_\chi$ is the WIMP mass and $m_t$ the mass of the target. Furthermore one can show that

\[
\frac{dt}{du} = \sqrt{\frac{2}{3}} a^2 F^2(u)\Psi_0(a\sqrt{u}) , \quad \frac{dh}{du} = \sqrt{\frac{2}{3}} a^2 F^2(u)\Psi_1(a\sqrt{u}) \tag{12}\]

with $a = (\sqrt{2} \mu_r b \nu_0)^{-1}$, $\nu_0$ the velocity of the sun around the center of the galaxy and $b$ the nuclear harmonic oscillator size parameter characterizing the nuclear wave function. $u$ is the energy transfer $Q$ in dimensionless units given by

\[
u = \frac{E_R}{Q_0(A)} , \quad Q_0(A) = [m_p A b^2]^{-1} = 40 A^{-4/3} \text{ MeV} \tag{13}\]

and $F(u)$ is the nuclear form factor. In the present calculation they were obtained in context of the nuclear shell model in the spirit of\[61\] (for the spin induced process see,e.g. [61, 62]). The form factor is important in the case of
FIG. 4: The square of the nuclear form factor for a heavy target, e.g. $^{127}$I (a) and an intermediate target, e.g. $^{73}$Ge (b). For light targets the effect of the form factor is small.

a heavy target and large WIMP mass, i.e. for large recoil energies (see Fig. 4).

Note that the parameter $a$ depends both on the WIMP, the target and the velocity distribution. Note also that for a given energy transfer $E_R$ the quantity $u$ depends on $A$.

Sometimes one writes the differential rate as:

$$
\frac{dR}{dE_R}|_A \propto \rho_\chi \frac{m_1}{m_\chi} A m_p \sigma_n \left( \frac{\mu_r}{\mu_p} \right)^2 \sqrt{<v^2>^2} A^2 \frac{1}{Q_0(A)} \left( \frac{dt}{du} (1 + H(a\sqrt{E_R/Q_0(A)}) \cos \alpha) \right)
$$

In this formulation $H(a\sqrt{E_R/Q_0(A)})$, the ratio of the modulated to the non modulated differential rate, gives the relative differential modulation amplitude. It coincides with the ratio $\Psi_1(a\sqrt{E_R/Q_0(A)})/\Psi_0(a\sqrt{E_R/Q_0(A)})$, i.e. it is independent of the nuclear form factor and depends only on the reduced mass and the velocity distribution. It is thus the same for both the coherent and the spin mode. Note that it can take both positive and negative values, which affects the location of the maximum of the modulated rate as a function of $\alpha$. For the convenience of the analysis of experiments, however, we will present our results in the form of Eq. 11.

III. SOME RESULTS ON DIFFERENTIAL RATES

We will apply the above formalism in the case of I and Na, which are components of the target NAI used in the DAMA experiment [54, 55] and Ge employed, e.g. by the CoGeNT experiment [53]. The results for the Xe target [52] are similar to those for I and for the $^{19}$F target [54, 57] are similar to those for Na. The differential rates $\frac{dE_R}{dQ}|_A$ and $\frac{dE_R}{dQ}|_A$, for each component ($A = 127$ and $A = 23$) and for $A = 73$ are exhibited in Fig. 5-10. The nuclear form factor has been included (for a heavy target, like $^{127}$I or $^{131}$Xe, its effect is sizable even for an energy transfer of 10 keV, see Fig. 4).

The introduction of debris flows makes a small contribution at low energy transfers. As expected [51] it tends to increase the differential rate at high energy transfers. This is particularly true for light small WIMP-nucleus reduced mass (see Figs 7 and 9). One, however, does not see any particular signature in the shape of the resulting curve. Furthermore the event rate in this region is about five times smaller than the maximum. One, however, observes an interesting pattern concerning the time varying (modulated) part of the rate (see Figs 8 and 10). For a heavy target, like $^{127}$I or $^{131}$Xe, it is not surprising that, for WIMPs with relatively large mass, the modulation becomes negative, i.e. the rate becomes minimum in June 3rd, for all models considered here. For low WIMP masses, however, the sign of the modulation due to the flows is opposite to that of the M-B distribution. Thus the use of the light target nucleus $^{19}$F, combined with the low detection threshold of 1.7 keV for recoil nuclei, makes PICASSO particularly sensitive to low mass dark matter particles and gives it also some leverage in the low mass region of the spin independent sector. The present stage of the experiment [63] is approaching the sensitivity to challenge or confirm the claims of seasonal modulations by the DAMA [55] and CoGeNT [53] experiments. A similar situation arises in the case of an intermediate target, like $^{73}$Ge. Here the M-B distribution yields a negative value only for very low energy transfers. The situation becomes most interesting in the case of a light target, see Fig. 8. Here, with the possible exception of quite low energy transfers, which perhaps are below or very near threshold, the M-B distribution yields a positive modulation
FIG. 5: The differential rate \( \frac{dR}{dQ} \), as a function of the recoil energy for a heavy target, e.g. \(^{127}\text{I}\) assuming a nucleon cross section of \(10^{-7}\text{pb}\). Panels (a) (b), (c) and (d) correspond to 5, 20, 50 and 100 GeV WIMP masses. Otherwise the notation is the same as that of Fig. [1].

amplitude, i.e. a maximum on June 3rd, while the result of debris flows is to cause a change in sign as one moves to high energy transfers. Also in this case the modulation amplitude tends to increase as the energy transfer increase, while the corresponding contribution due to the M-B distribution tends to decrease. We should remark though that the total rate (average+modulated) tends to decrease at high momentum transfers. We should also stress that we have presented here the absolute modulate rate (events per kg target per year). The relative modulated amplitude (the ratio of the time varying rate divided by the time averaged) maybe larger.

The above results, as we will see in the next section, have important implications in the total event rates.

Sometimes, as is the case for the DAMA experiment, the target has many components. In such cases the above formalism can be applied as follows:

\[
\frac{dR}{dQ} \bigg|_{A} = \sum_{i} X_{i} \frac{dR}{dQ} \bigg|_{A_{i}}, \quad u \rightarrow u_{i}, \quad X_{i} = \text{the fraction of the component } A_{i} \text{ in the target}
\]  \hspace{1cm} (15)

We will not, however, pursue such an analysis.

### IV. SOME RESULTS ON TOTAL RATES

For completeness and comparison we will briefly present our results on the total rates. Integrating the differential rates discussed in the previous section we obtain the total rate \( R \), adding the corresponding time averaged rate \( R_{0} \) and the total modulated rate \( \tilde{H} \), given by:

\[
R = R_{0} + \tilde{H} = \frac{\rho_{\text{e}}}{m_{\chi}} \frac{m_{t}}{A_{\text{mp}}} \left( \frac{\mu_{r}}{\mu_{p}} \right)^{2} \sqrt{\langle v^{2} \rangle} A^{2} \sigma_{n} t \left( 1 + h \cos \alpha \right),
\]  \hspace{1cm} (16)

with

\[
t = \int_{Q_{\text{th}}/Q_{0}(A)}^{(y_{\text{max}}/\alpha)^{2}} dt \, du, \quad h = \frac{1}{t} \int_{Q_{\text{th}}/Q_{0}(A)}^{(y_{\text{max}}/\alpha)^{2}} dh \, du.
\]  \hspace{1cm} (17)
FIG. 6: The differential rate \( \frac{d\bar{H}}{dQ} \), as a function of the recoil energy for a heavy target, e.g. \(^{127}\text{I}\) assuming a nucleon cross section of \(10^{-7}\)pb. Panels (a), (b), (c) and (d) correspond to to 5, 20, 50 and 100 GeV WIMP masses. Otherwise the notation is the same as that of Fig. 1.

\( y_{\text{max}} \) is the maximum velocity allowed by the distribution and \( Q_{\text{th}}(A) \) is the energy cut off imposed by the detector.

The obtained results for quantities \( R_0 \) and \( h \) are exhibited in Figs 11-13 assuming a nucleon cross section of \(10^{-7}\)pb.

In the case of a heavy target the average event rate attains the maximum value of 30 events per kg of target per year at a WIMP mass of 25 GeV, while for heavy WIMPS it eventually falls to about 5 kg/y at 500 GeV (to a good approximation it falls inversely proportional to the WIMP mass above the 200 GeV). For an intermediate target we get 15 kg/y at 25 GeV, with an asymptotic value of 4 kg/y. For a light target the maximum becomes 2.5 kg/y at 20 GeV. Again the asymptotic value at 500 GeV is about 1/5 of the maximum.

It is clear that, as far as the time average rates \( R_0 \) are concerned, the debris flows do not exhibit any characteristic signature to differentiate them from the standard M-B distribution. The relative modulation amplitude \( h \), however, exhibits a very interesting feature, namely, if caused by the flows, it is negative for all targets, even for the light ones, and in the entire WIMP mass range (minimum in June). On the other hand if it is caused by the M-B distribution it is positive in the case of light targets regardless of the WIMP mass. It is also positive for intermediate/heavy targets, if the WIMPs are relatively light. Then the maximum occurs on June 3rd as expected. It becomes negative only for relatively heavy WIMPs. Thus it is an experimental challenge to measure the small time dependence of the event rate with a relative difference between the maximum and the minimum of \(2h \approx 4\%\). From such data on both light and heavy targets, if and when they become available, one may be able: i) to get a hint about the size of the WIMP mass and ii) infer the existence of flows.

V. DISCUSSION

In the present paper we first obtained results on the differential event rates, both modulated and time averaged, focusing our attention on the effects of debris flows. We found that:

- The flows indeed enhance the time averaged rates at relatively high energy transfers compared to the M-B distribution. All rates, however, fall as the energy transfer increases. This fall is only partially due to the velocity distribution. It is also caused by the nuclear form factor, in particular in the case of heavy targets.
FIG. 7: The differential rate \( \frac{dR}{dQ} \), as a function of the recoil energy for a light target, e.g. \(^{23}\text{Na}\) assuming a nucleon cross section of \(10^{-7}\) pb. Panels (a) (b), (c) and (d) correspond to 5, 20, 50 and 100 GeV WIMP masses. Otherwise the notation is the same as that of Fig. 1.

- In view of the dependence of above rates on the unknown WIMP mass, one does not have a clear signature to differentiate the debris flows from the standard distribution.

- The differential modulated rates provide such a signature, the sign of the modulation amplitude, which determines the position of the maximum. The debris flows tend to favor a negative sign (minimum on June 3rd), while the standard WIMPs favor a positive sign when the target is light or even when the target is heavy but the WIMP is light (maximum on June 3rd).

We then proceeded and calculated the total event rates as functions of the WIMP mass. We presented results obtained with a zero threshold energy. For higher threshold energies we expect the debris flows to be suppressed a little less than the standard WIMPs\[^{58}\], since the differential event rates associated with the former attain smaller values at low energy transfers. The time averaged rates are affected by the debris flows, but one does not find a characteristic feature to differentiate the debris flows from the standard WIMPs. Such a feature is provided by the sign of the modulation amplitude. The debris flows tend to favor minimum in June 3nd (opposite to that expected), while for the standard WIMPs it is positive (maximum on June 3nd) in the case of light targets, regardless of the WIMP mass, or for heavy targets for relatively light WIMPs.

In conclusion a measurement of the modulation amplitude expected at the scale of \(2 \pi \times 4\%\) may shed light on the scale of the WIMP mass and determine the presence of any non standard velocity distributions. These issues maybe settled better, if data on directional experiments become available. Such theoretical explorations are currently under study.

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FIG. 8: The differential rate $\frac{dH}{dQ}$, as a function of the recoil energy for a light target, e.g. $^{23}$Na assuming a nucleon cross section of $10^{-7}$ pb. Panels (a), (b), (c) and (d) correspond to 5, 20, 50 and 100 GeV WIMP masses. Otherwise the notation is the same as that of Fig. 1.

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FIG. 9: The differential rate $dR/dQ$, as a function of the recoil energy for an intermediate target, e.g. $^{73}$Ge assuming a nucleon cross section of $10^{-7}$pb. Panels (a) (b), (c) and (d) correspond to 5, 20, 50 and 100 GeV WIMP masses. Otherwise the notation is the same as that of Fig. [1].

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FIG. 10: The differential rate $\frac{d\tilde{H}}{dQ}$, as a function of the recoil energy for an intermediate target, e.g. $^{73}$Ge assuming a nucleon cross section of $10^{-7}$pb. Panels (a) (b), (c) and (d) correspond to to 5, 20, 50 and 100 GeV WIMP masses. Otherwise the notation is the same as that of Fig. 1.
FIG. 11: The total rate $R_0$ (top panels) and the relative modulation amplitude $h$ (bottom panels) as a function of the WIMP mass in GeV in the case of a heavy target $^{127}$I at zero threshold. Otherwise the notation is the same as that of Fig. 1.

FIG. 12: The same as in Fig. 11 for a light target, e.g. $^{23}$Na.
FIG. 13: The same as in Fig. 11 for an intermediate target, e.g. $^{73}$Ge.