Flat-band dark polaritons in two-dimensional chiral-waveguide quantum electrodynamics

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Abstract. We consider a two-dimensional extension of the one-dimensional waveguide quantum electrodynamics and investigate the nature of linear excitations in two-dimensional arrays of qubits (particularly, semiconductor quantum dots) coupled to networks of chiral waveguides. We show that the combined effects of chirality and long-range photon mediated qubit-qubit interactions lead to the emergence of the two-dimensional flat bands in the polaritonic spectrum, corresponding to slow strongly correlated light.

1. Introduction
Arrays of quantum emitters, placed in the vicinity of photonic nanostructures are now the focus of intensive research and constitute the field of waveguide quantum electrodynamics (WQED) [1, 2]. While specific material platforms of WQED span from artificial arrays of cold atoms [3] to radio-frequency circuits with superconducting qubits [4, 5], all of them possess a common feature, namely the long range inter-qubit interaction, mediated by the exchange of the propagating photons. The onset of long-range correlations enables to access the plethora of intriguing physical phenomena such as emergence of unconventional topological phases [6, 7] and collective super-radiance and sub-radiance [8, 9, 10], and it paves the way towards the design and engineering of prospective quantum networks [11].

The study of the dispersion properties of the polaritons in WQED is thus an crucial task for understanding the nature of unconventional multiphotonic states in the considered systems. In this sense, the present work address the nature of linear excitations systems consisting of one- or two-dimensional arrays of atoms or artificial atoms (semiconductor quantum dots) coupled to a single chiral waveguide or a network of chiral waveguides.

2. One-dimensional array
We start from the consideration of the 1D case. The corresponding geometry is schematically illustrated in Fig. 1. The effective real-space Hamiltonian of the system, obtained via the Schrieffer-Wolff transformation [12, 13], reads

$$\mathcal{H}_{\text{eff}} = \sum_{n=1}^{N} \delta(b_{n,R}^\dagger b_{n,L} + \text{h.c.}) - i \frac{\Gamma_0}{2} \sum_{m,n=1}^{N} \Theta(m-n) e^{iqd|m-n|} (b_{m,R}^\dagger b_{n,R} + b_{n,L}^\dagger b_{m,L}), \quad (1)$$
where the individual decay rate of a single atom $\Gamma_0 = q^2/v$ defines the characteristic energy scale of the system, $q = \omega/v$ is the wave vector of the photon, mediating the interaction between qubits equidistantly spaced by $d$, and $\Theta$ is the Heaviside step function defined within the half-maximum convention. Consequently, the corresponding $k$-space Hamiltonian for the infinite lattice is given by

$$
H_{\text{eff}} = \frac{\Gamma_0}{4} \sum_k \left[ \cot \left( \frac{qd - kd}{2} \right) b_{k,R}^\dagger b_{k,R} + \cot \left( \frac{qd + kd}{2} \right) b_{k,L}^\dagger b_{k,L} \right] + \sum_k \delta(\epsilon_{k,R}^0) + \text{h.c.}.
$$

The shape of the dispersion curves $\epsilon_k$, shown in Fig. 2, is determined by the phase $qd$ and detuning $\delta$. For the case of $\delta = 0$ illustrated by the upper panels, $R$ and $L$ modes are completely decoupled. The anti-Bragg structure dispersion with $qd = \pi/2$ for finite $\delta$ presents a middle slow polariton branch gapped from the upper and lower fast polariton bands [see Fig. 2 (d)]. For Bragg structures, $qd = \pi$ and $qd = 2\pi$, $\delta$ is responsible for opening a gap around $\epsilon_k = 0$ as can be seen in Figs. 2 (e,f). The eigenmodes dispersion equation for the considered setup $\cos(kd) = \frac{1}{2} \text{Tr}(T)$ obtained in terms of transfer matrix ($T$) over the single period of the structure composed by a layer of thickness $d$ intersected by a scatterer is given by

$$
\cos(kd) = \cos(qd) \left[ \frac{(\omega - \epsilon_k)^2 - \delta^2 - \Gamma_0^2/4}{(\omega - \epsilon_k)^2 - \delta^2 + \Gamma_0^2/4} \right] - \sin(qd) \left[ \frac{(\omega - \epsilon_k)\Gamma_0}{(\omega - \epsilon_k)^2 - \delta^2 + \Gamma_0^2/4} \right]
$$

with its eigenvalues shown in Fig. 2 for different phases $qd$. One can see that the exact solution can diverge significantly from those obtained within the Markov approximation. In particular, it does not contain nonphysical regions with infinite group velocity around $k = 0$ for $qd = 2\pi$, where an additional gap is opened instead and substantially modifies the band structure in the vicinity of the band edges $k = \pm \pi/d$ for the case $qd = \pi$.

For finite structures, the effective Hamiltonian (1) is characterized by complex eigenenergies $\epsilon$, with the real part corresponding to a frequency shift relative to $\omega$, and the imaginary part corresponding to the radiative decay rate, $\Gamma_{1D} = -\text{Im}(\epsilon)$ [14]. It is noteworthy that the dependence of the darkest state decay rate on the number of atoms for the considered setup, presented in Fig. 2 (g), reveals a scaling of $\Gamma_{1D} \sim N^{-1}$ in contrast to the $\Gamma_{1D} \sim N^{-3}$ scaling, characteristic for 1D waveguides hosting linear polarized modes.
3. Two-dimensional lattice model

In the two-dimensional (2D) model, we consider an \(N \times N\) square lattice of atoms at the nodes of a quadratic waveguide composed of a set of horizontal and vertical chiral waveguides in the \(xy\) plane as depicted in Fig. 1 (b). The effective 2D \(k\)-space Hamiltonian is given by

\[
\mathcal{H}_{\text{eff}}^{2D} = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger (\mathcal{H}^{x}_{\text{eff}} + \mathcal{H}^{y}_{\text{eff}}) \Psi_{\mathbf{k}}, \quad \text{with} \quad \Psi_{\mathbf{k}} = (b_{k_x R_y}, b_{k_x L_y}, b_{k_x x})^T.
\]

Its expression in the vicinity of \(k_x, k_y = 0\) for \(qd = \pi\) reads

\[
\mathcal{H}_{\text{eff}}^{2D} = \frac{\Gamma_0}{8} \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger \begin{pmatrix}
  k_x d + \delta_y & 2\delta_x - \delta_y & -\frac{1}{\sqrt{2}}k_y d \\
  2\delta_x - \delta_y & -k_x d + \delta_y & \frac{1}{\sqrt{2}}k_y d \\
  -\frac{1}{\sqrt{2}}k_y d & \frac{1}{\sqrt{2}}k_y d & 2\delta_y
\end{pmatrix} \Psi_{\mathbf{k}}.
\]  

(4)
with corresponding dispersion shown in Fig. 3 (a-c). For the case of degenerate frequencies, \( \delta_x, \delta_y = 0 \), the low-energy spectrum reveals a profile identical to spectra characteristic of Lieb lattices, with a pair of Dirac cones intersected by a flat band (Fig. 3 (a)). With a finite detuning in the waveguides parallel to the \( y \) axis, \( \delta_y = 0.02\Gamma_0 \), the spectrum shows anisotropy along \( k_y \) as well as the emergence of a gap, as it can be seen in Fig. 3 (b). Naturally, if a detuning is introduced in the waveguides parallel to the \( x \) axis the anisotropy will appear along \( k_x \) instead of \( k_y \). In Fig. 3 (c) both detunings are finite, so that the anisotropy is present along both the \( x \) and \( y \) directions, respectively.

The diagonalization of the effective real-space Hamiltonian for \( qd = \pi \) within the single occupation limit leads to the radiative decay rate and the photonic distribution shown in Fig. 3(d-g). The imaginary part of the darkest state displayed in Fig. 3 (d) reveals the collective radiative decay rate scaling \( \Gamma_{2D} \sim N^{-3} \). Although the 1D chain presents a purely chiral propagation for \( \delta = 0 \), the 2D lattice within the condition \( \delta_x, \delta_y = 0 \) does not reveal purely chiral propagation due to the admixture of excitations stemming from perpendicular waveguides. This admixture can be perceived by the off-diagonal terms proportional to \( k_y \), such as \( b_k^{R_x} b_k^{R_y} \), in Hamiltonian (4). As a consequence, the radiative decay rate scaling \( \Gamma_{2D} \sim N^{-3} \) is observed instead of a scaling \( \sim N^{-1} \) exclusively observed for purely chiral propagation.

4. Conclusions

We analyzed dispersions and decay rates of the polariton modes emerging in 1D and 2D arrays of multilevel atoms coupled to chiral waveguides. In particular, it was demonstrated that in the 2D case the low-energy effective Hamiltonian is similar to Hamiltonians characteristic of the Lieb lattice model and thus generates polariton flat bands, which due to the large density of states may host a rich family of strongly interacting multiphotonic states.

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