Amplitude, period and orientation of the moiré patterns in barrier 3D displays

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ABSTRACT

The experimental measurements of the amplitude, period, and orientation of the moiré patterns in a digital autostereoscopic barrier-type 3D display are presented as functions of the angle across a wide angular range with a small increment. The slope and width of the functions are estimated. Simultaneous branches are observed and analyzed. The theoretical interpretation is based on the wavevectors and the transfer function. The results can help prevent or minimize the moiré effect in displays.

ARTICLE HISTORY
Received 11 December 2017
Accepted 4 March 2018

KEYWORDS
Moiré effect; moiré minimization; moiré amplitude; moiré period; moiré orientation

1. Introduction

The moiré effect is caused by the interaction of the rays passing through two or more periodic layers [1,2]. In digital three-dimensional (3D) autostereoscopic displays based on LCD panels, the layers are highly regular (periodic). The periodicity of both layers (the LCD pixel grid and the barrier plate) is one reason for the appearance of the periodic moiré patterns in such displays as a result of the interference between the pixels and the barrier.

In this paper, the moiré effect in three-dimensional (3D) displays with the horizontal parallax (HPO) (i.e. in displays with a one-dimensional parallax barrier plate consisting of repeated parallel lines) is considered [3,4]. To improve the image quality, the moiré patterns must be eliminated [5–7]. In the current research, the parameters of the moiré patterns of two parallel layers without separation were measured so that the distance from the camera would have no effect on the visual appearance of the patterns.

Plain coplanar layers can be turned around from the normal to their surface. Then all the characteristics of the moiré patterns become functions of the turnaround angle and may have some local extrema across the angular range. The maxima of the amplitude correspond to the strong patterns; the minima, to the weak patterns or to their absence. The period of the moiré patterns vs. the rotation angle will be referred to as the ‘period function’; similarly, two other characteristics will be called ‘orientation function’ and ‘amplitude function.’

In the current content, the ‘moiré angles’ are the angles of the local maxima of the period function. Typically, these coincide with the maxima of the amplitude function. It was previously shown [8] that in the digital displays with square pixels, the moiré angles are rational angles (the tangent [9] of a rational angle is a rational number). Therefore, with respect to the moiré effect in digital displays, the rational angles have a special significance, and here, they will sometimes be mentioned by fractions (meaning the tangents).

Amplitude is an important characteristic of any physical phenomenon. One of the factors affecting amplitude is the modulation transfer function (MTF). MTF is an improved concept of the visibility circle, which models the human visual system through a binary function. The influence of MTF becomes essential when the moiré patterns are observed from afar, which does not allow the human eyes to resolve the individual screen pixels. Such screens and viewing distances (when the individual pixels are not recognizable) are presently typical for displays.

At some angles, the moiré patterns have a very small amplitude and are unrecognizable. At many other angles, only one pattern is detected. In a few cases, however, several patterns can be observed together. These are called ‘branches’. The independent branches have different orientations whereas the harmonics depend on one another; i.e. they have multiple periods but are oriented at the same angle. Switched branches were observed in Ref. [10]. Simultaneous branches are reported in this paper.
Typically, one of the branches visually prevails and suppresses others so only one pattern can be seen; but whenever possible, an attempt was made to keep two or three strongest branches with the highest amplitude and the longest period.

The measurements of the period and orientation of the moiré patterns in 3D displays were made previously [11], although across a limited range and without details near the moiré angles. In the research [12], a relatively large increment (5°) was used. One of the few attempts to experimentally measure the amplitude of the moiré patterns is Ref. [13]. At the same time, there are many papers about the moiré period. This may mean that the moiré patterns’ amplitude is more difficult to measure than the period or orientation, and therefore, a detailed picture of the behavior of the moiré patterns is yet uncertain. As a result, it is still problematic to draw a qualitative comparison of the visual quality of displays in relation to the moiré effect. As far as the authors know, direct measurements were not performed before. The current study was an attempt to compensate for the lack of information about the moiré amplitude in digital autostereoscopic displays.

The paper is organized as follows. In Section 2, a theory of the period and orientation based on the wavevectors of the gratings is reviewed. Also, the amplitude theory based on the MTF of the human eye, which, in this particular case, can be approximated by polynomials, is proposed. In Section 3, the measurement technique is described, and the experiment results of the measurements of three main parameters of patterns (period, amplitude, and orientation) across a wide angular range between $-20°$ and $+100°$ with an angular increment of about 1° (and even less near the rational angles) are presented. Section 4 finalizes the paper.

2. Theory

In the layout herein, the gratings are coplanar; a grid (2D screen of LCD pixels) is static while a grating (1D linear barrier) can be rotated around the normal to the screen (see Figure 1). In such configuration, the moiré patterns are periodic parallel bands (stripes).

In theory, the square pixels are assumed, although the practical design of some LCD devices may be based on rectangular pixels.

2.1. Wavevector of moiré patterns

Generally, the wavevector of a moiré pattern (briefly, the moiré wavevector) is the difference between the wavevectors of the gratings [1]. Several (up to 3) harmonics of gratings are considered.

When the plate rotates, its wavevector turns around each spectral component of the grid together with all its spectral components. Among all the combinational spectral components, the components whose trajectories are close to the origin of the spectral domain (within the visibility circle) were chosen. In the current paper, the radius of the visibility circle was approximately 0.8 of the fundamental wavenumbers; such value ensures the visibility of the moiré patterns while both the pixel grid and the grating remain invisible. The extremely important concept of the visibility circle is presented in Ref. [1].

This moiré factor is defined as the ratio of the moiré period to the period of the grating. For the concept of the moiré magnifier, refer to Ref. [14].

Consider the 2D spectrum of the square grid. The spectral components of the grid are located at the nodes with integer coordinates $mk_0$ and $nk_0$, where $k_0$ is the fundamental wavenumber (i.e. the first spectral component of the spectrum) and $m, n$ are the integer numbers of harmonics. Each node can be a center of a trajectory; in the complex plane, $z_c = (m + jn)k_0$, where $j$ is the imaginary unit; as in Refs. [15,16], and in Ref. [17], where the complex numbers are also used. Figure 2 shows an example of a trajectory of a grating with wavevector $k_1$ rotated around the spectral component with arbitrary values $m = 5$ and $n = 2$.

The moiré wavevector is the ray $OD$ from the origin to the spectral component of the rotated plate whose center of rotation is point $C$. The equation of the trajectory with

![Figure 1. Layout of coplanar layers.](image)

![Figure 2. An example trajectory in the complex plane. The location of the center and the radius are chosen arbitrarily.](image)
radius $k_1$ is shown as:

$$z = z_c - k_1 \cdot e^{i\alpha}$$  \hspace{1cm} (1)

The complex number $z$ can be rewritten in the polar form $z = a \exp(j\varphi)$, where modulus $a$ and argument $\varphi$ are the wavenumber and orientation, respectively. Then the wavenumber of the moiré pattern is as follows (see also Equation (12) in Ref. [17]):

$$k_m = k_1 \sqrt{1 + \rho^2 - 2 \rho \cos(\alpha - \alpha_M)},$$  \hspace{1cm} (2)

where $\rho = |z_c|/k_1 = (m^2 + n^2)^{1/2}k_0/k_1$ is the ratio of the periods of the layers and $\alpha_{\text{MAX}}$ is the moiré angle.

$$\alpha_M = \arctan \frac{n}{m}.$$  \hspace{1cm} (3)

The period of the moiré patterns appears in the literature (see, e.g. Equation (9) in Ref. [18]); it is the inverse wavevector Equation (2).

$$\lambda = \frac{\lambda_1}{\sqrt{1 + \rho^2 - 2 \rho \cos(\alpha - \alpha_M)}}.$$  \hspace{1cm} (4)

The corresponding magnification factor is

$$\mu = \frac{1}{\sqrt{1 + \rho^2 - 2 \rho \cos(\alpha - \alpha_M)}}.$$  \hspace{1cm} (5)

The orientation of the pattern is given by

$$\varphi = \arctan \frac{nk_0 - k_1 \sin \alpha}{mk_0 - k_1 \cos \alpha} = \arctan \frac{\rho \sin \alpha_M - \sin \alpha}{\rho \cos \alpha_M - \cos \alpha}.$$  \hspace{1cm} (6)

 Particularly, for identical gratings with the horizontal wavevector ($n = 0$, $m = 1$, and $k_1 = 1$), the orientation is given by $\tan \varphi = 1/\tan(\alpha/2)$, as known from book [1].

The theoretical functions defined by Equations (5) and (6) are shown in Figure 3 for several values of $\rho$. The graphs shown in Figure 3 depict the behavior of the patterns near the moiré angle. At that angle, the orientation of the pattern coincides with the orientation of the plate (up to a possible difference in $\pi$ radians).

$$\varphi_M = \alpha_M.$$  \hspace{1cm} (7)

At the moiré angle, the period reaches a local maximum. Equation (7) shows that the orientation of the moiré patterns (the direction of the moiré wavevector) coincides with the direction of the wavevector of the plate [11], in which case the lines of the patterns are parallel to the lines of the barrier plate.

The slope of the orientation curve Equation (6) and Figure 3(b) (measured in degrees of the moiré angle per degree of the plate orientation) at the moiré angle is

$$\varphi'_M = \frac{1}{\rho - 1}.$$  \hspace{1cm} (8)

The moiré factor at the moiré angle is as follows (see also Equations (2)–(11) in Ref. [1]):

$$\mu_{\text{MAX}} = \frac{1}{\rho - 1}.$$  \hspace{1cm} (9)

Equation (9) shows that the closer $\rho$ is to 1 (or, equivalently, the closer the modulus of the current spectral component of grid $|z_c| = (m^2 + n^2)^{1/2}k_0$ is to the radius of the trajectory of grating $k_1$), the longer the period (see Figure 3(a)); and in the case of $\rho = 1$, it is infinite. In both Equations (8) and (9), the closer $\rho$ is to 1 (or, equivalently, the closer $k_1$ is to $z_c$), the higher the slope;
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Figure 4. MTF for \( d = 4 \text{ mm}, \; \lambda = 555 \text{ nm} \) (solid line). The dashed line shows the profile of the visibility circle.

and when \( \rho = 1 \), it is infinite (see Figure 3(a)). Also, the width of the bell-shaped period function characterizes the value of \( \rho \). That is, the half width of the period curve at level \( h < 1 \) is

\[
\Delta \alpha = \frac{1 - \rho}{h} \sqrt{1 - \frac{h^2}{\rho}}. \tag{10}
\]

When \( \rho = 1 \), the half width is zero. This means that for the exact match, the period function looks like a delta function, but for a larger (or smaller) \( \rho \), it is wider.

2.2. Amplitude of the moiré patterns

There are many factors affecting the amplitude of the moiré patterns, such as the light radiation conditions, the background illumination, and the polarizers in LCDs, and MTF is one of them. In a straightforward approach, the amplitude of the spectral component outside the visibility circle is zero while within the visibility circle, it does not depend on the angle of rotation of the plate because the relations between the amplitudes of the spectral components are not changed due to the rotation.

The MTF of the human eye (or camera) depends on the spatial frequency. The famous and fruitful concept of the visibility circle is a binary approximation of MTF (see Figure 4).

The visibility circle and the MTF can be qualified as the first- and second-order approximations. Although the visibility circle works perfectly for flat (coplanar) layers, it needs an updating in the non-coplanar case. Therefore, in the layout herein, the visible amplitude depends on the distance from the origin of the spectral domain along the spectral trajectory (i.e., on the rotation angle of the plate by Equation (2)).

According to Ref. [19], the general expression for the radial MTF of humans depends on the diameter of the pupil (2 mm ≤ \( d \) ≤ 6 mm), the spatial frequency of patterns \( u \), and the wavelength of light \( \lambda \). The spatial frequency of the moiré patterns observed from distance \( L \) is

\[
u = \frac{L}{\lambda_m \mu}, \tag{11}
\]

where \( \mu \) is the magnification factor by Equation (5) and \( \lambda_m \) is the period of moiré fringes on an original surface (i.e., the inverse spatial frequency at zero distance from the surface).

In this research, it is sufficient to consider the typical parameters, such as \( d = 4 \text{ mm} \) and \( \lambda = 555 \text{ nm} \). The MTF for these parameters is shown in Figure 4. Moreover, for the above typical parameters, the exact MTF [19] can be approximated by polynomials (e.g. the third-order approximation).

\[
M(u, 4) = -2 \cdot 10^{-6} u^3 + 4 \cdot 10^{-4} u^2 - 3 \cdot 10^{-2} u + 1
\]

has a less than 2% interpolation error.

The amplitude calculated using Equation (12) is shown in Figure 5. The amplitude function near its maximum is close to the triangular function.

3. Experiments

In the experiments in this study, the barrier plate was directly placed (without a gap) on the surface of an LCD screen with a changeable angle between the axis of the plate and the axis of the screen (see Figure 1). A uniform white field (the pixels of identical brightness across the whole screen) was constantly displayed.

In overlapped plain physical layers touching each other, a gap is practically unavoidable at least due to the non-zero thickness of the physical objects. In a barrier display, a gap between the barrier and the screen is essential for 3D imaging. In either case (touching layers or layers intentionally installed with a gap), the amplitude of the moiré patterns in barrier displays is not affected by the gap; with that, the variation of the other parameters...
is small. This is valid for the barrier case, where the characteristics of the moiré patterns can be experimentally evaluated based on direct touch. This seems opposite the lenticular case, where the amplitude has a strong dependence on the gap while the other parameters seem to be similar to the barrier case. The lenticular case, however, is outside the scope of the current paper.

Experiments were performed with barrier plates of different periods and opening ratios. An experiment included many incremental steps. At each step, the plate was turned around by a small angular increment (normally about 1°, but in the neighborhood of the moiré angles, it was smaller), and was photographed. The moiré patterns appeared in the neighbourhood of only some specific angles.

The experiments were performed with the barrier samples 50, 75, and 150 lines per inch (lpi); these values are often referred to as 'pitches'. The corresponding periods were 0.508, 0.339, and 0.170 mm, respectively. The size of each sample was 8.5 \times 8.5 cm. The period of the screen pixels of the LCD screen was 0.266 mm (the corresponding ratios of the periods of layers $\rho$ were 1.910, 1.273, and 0.634).

To provide equal photographic conditions for different plates, several samples were assembled together on the common base (up to six samples). A photograph of the plate with multiple samples is shown in Figure 6.

For the calibration, the longest-period sample was used, in which case the intensities in the opened and closed areas of the plate could be measured separately. Thus, the amplitudes in different plates could be directly compared. (Figure 7)

As a result, hundreds of experiment photographs of the moiré patterns were taken and processed.

3.1. Image processing

To retrieve the numerical values from numerous photographs, a semi-automatic software system that measures the period, orientation, and amplitude of the photographed moiré patterns (technically, the plain waves) was designed.

The image processing was based on the windowed 2D Fourier transform. In the angular measurements, the Hann window was applied; in the amplitude measurements, the flat-top window was applied. There are...
numerous spectral peaks in the 2D Fourier transform of each photograph. As such, the lower-amplitude peaks were eliminated, and among the remaining ones, the peaks with lower spatial frequencies were chosen. The peak's location in the spectral domain determines the orientation and wavenumber of the moiré patterns; hence, the period. Technically, the amplitude of the first spectral component in the spectrum of the moiré patterns was measured. Typically, there were from one to six peaks at each rotation angle. Among these, the branches had to be recognized.

The branches were classified manually based on the similarity of the measured parameters (period, angle, and amplitude) along the branch (i.e. a relatively smooth change of the measured parameter). The maximum amplitude and the maximum period were calculated individually for each branch. Some weak low-amplitude and short-period branches were eliminated especially when these coexisted with the high-amplitude or long-period branches, which used to suppress the weak branches visually.

Note that except for the neighborhood of the rational angles, the moiré patterns were not detected experimentally. Also, note that Figures 8–10 were obtained in the experiments and thus contain some noise.

### 3.2. Measurement of the moiré parameters

The three parameters of the moiré patterns that were measured at each angle \( \alpha \) were the period, orientation, and amplitude. In an example shown in Figure 8, the patterns were detected near the 0°, 27°, 45°, and 90° angles, which are close to the rational angles 0, 1/2, 1, and \( \infty \). The set of angles where the moiré patterns appeared was distinct for each plate.

As described in Section 2.1., at the moiré angle, the orientation of the moiré wave coincides with angle \( \alpha \). The values of the orientation function lie between \(-90^\circ\) and \(180^\circ\) (see Figure 8(a)). Instead of such, the modified (‘wrapped’) function \( \psi - \alpha \), which has a symmetrical range of values from \(-90^\circ\) to \(+90^\circ\), was considered; at the moiré angles, the wrapped function crosses the abscissa (see Figure 8(b)).

Generally speaking, several harmonics can be recognized in a non-sinusoidal moiré wave. The harmonics are characterized by the same orientation, multiple periods, and smaller amplitudes. Therefore, the expected branches with the coinciding angle functions and the double or triple periods are harmonics but not the branches (see Figure 9). The higher harmonics were excluded from the measurements in this study.

### 3.3. Characteristics of the moiré patterns in the barrier display

Examples of the three measured parameters are shown in Figure 10. The amplitude is given in arbitrary units (after the calibration by pure black and pure white areas). Figure 10 shows that the behaviors of the three functions at the 0° and 90° angles are very similar.

In the left column of Figure 10, the moiré waves were detected near the 0°, 45°, and 90° angles (the rational angles 0, 1, and \( \infty \)). There was only one wave at the 45° angle, but two branches were detected at the 0° and 90° angles. In the right column, the pattern at angle 0° has three branches while the 90° angle has a single branch. There are a few extra branches, however, with lower amplitudes at other angles. Namely, there are four additional waves at \( \pm 18.4^\circ\), \(26.7^\circ\), \(33^\circ\), and \(56.3^\circ\) in the right column of Figure 10. These low-amplitude branches include fewer points and may thus look somehow incomplete but still recognizable.

Sometimes, especially near angle 0, two or three branches are detected. Also, it can be observed in Figure 10 that the experimental amplitude function is similar to the period function. Also, compare the experiment period and the amplitude in Figure 10 with the theoretical Figures 3(a) and 5. This confirms these authors’ assumption [20] of the better visibility of the moiré waves of the longer period. Roughly speaking, the longer is the period of the moiré patterns while the higher is the amplitude on a branch.
Figure 9. Two harmonics of a non-sinusoidal moiré wave observed in the experiment: (a) orientation (the same for both harmonics); and (b) period (differs twice).

In the aforementioned experiment example, the branches of the orientation function with different slopes can be recognized at the moiré angle 0. They also differ in terms of the width of the period curves. The shorter the amplitude is, the lower the slope. For the other \( \rho \) values, the picture of the branches may look completely different.

Across the experiments, eight moiré angles (0°, ±18.4°, 26.7°, 33°, 45°, 56.3°, 63.4°, and 90°; tangents...
0, 1/3, ½, 2/3, 1, 3/2, 2, and ∞) were detected by the image processing software. Here, no distinction is made between the positive, negative, and symmetric angles; thus, the −18°, +18°, and 108° angles are considered the same 1/3 rational angle. One expected 71.6° (tangent 3) angle was missed, however, probably because of the small amplitude of the moiré patterns at that angle.

Among the detected moiré angles, the highest amplitudes of the moiré patterns were observed at the 0°, 45°, and 90° angles (see Figure 11). The maximum amplitude at these three angles is approximately five times higher than that at the other moiré angles. As can be seen in Figure 11, the 0°, 45°, and 90° angles definitely prevail based on their amplitude.

Among the different barrier periods, the period of the moiré patterns is the longest: 0.508 mm (50 lpi), where it is approximately 2.5 times higher than that at any other period. The highest amplitude of the moiré patterns was observed at 0.339 mm (75 lpi), where it is approximately three times longer. Example graphs for the 45° moiré angle are shown in Figure 12. The same or very similar scenarios take place at the 0° and 90° angles (there are insufficient data to verify this observation at the other moiré angles).

Thus, the experiments in this study showed that the relatively medium periods of the barrier plates (0.3–0.5 mm) exhibited the highest amplitudes and the longest periods of the moiré patterns.

The experiment results on the amplitude as a function of the opening ratio at three angles (0°, 27°, and 45°) in two displays with a close period of pixels (0.266 and 0.282 mm) are shown in Figure 13. The corresponding experiment photographs are given in Figure 14.

Figure 13 confirms that the amplitude of the moiré patterns increased when the opening ratio increased. Previously, a similar behavior was experimentally confirmed [13], but only at the 0° angle.

4. Discussion

The experimental orientation functions shown in Figure 10(a,d) are clearly S-shaped, as predicted by Equation (6); similarly, the experimental period functions shown in Figure 10(b,e) are bell-shaped, in accordance with Equation (4) (see also Figure 3). For the triangular shape of the amplitude function, refer to Equation (12) and Figure 5. Among the three measured functions, the orientation and period functions were clear and smooth while the amplitude function sometimes looked unclear. Therefore, the accuracy of the amplitude measurements should be improved.
The characteristic property that the moiré patterns are parallel to the lines of the barrier plate can be a practical indicator of the exact moiré angle.

The basic mathematical model of a camera commonly accepted and very widely used in computer graphics and image processing is the perspective (central) projection transformation (see, e.g. [21] and others). The central projection is characterized by the location of the center of projection and the location and orientation of the principal plane (the screen). This model represents a focused camera with no distortions, although some features of a lens can be added. In this study, this model was completely relied upon, with a pinhole camera actually simulated.

The aforementioned camera model is free from aberrations. Nevertheless, the distortions like barrel or pincushion could happen in a physical camera, especially at large angles. The view angles, however, are less than 5°, and there is no need to consider any of the optical aberrations or distortions at such small angles.

To eliminate or reduce the effect of the other parameters on the amplitude, in the experiments in this study, the camera axis was always orthogonal to the screen; each photograph was individually calibrated, etc. The measurement software devised by these authors recognizes the moiré patterns at the rational moiré angles up to $m, n = 3$.

As in the layout herein, the camera axis is always orthogonal to the screen, and the planes of the object and of the image are parallel to each other. In this case of two parallel lines, ‘the ratio of the distance between points, and the distance between their images, are constant’, as stated in Ref. [22]. In essence, this statement, together with many similar ones from other sources, means a complete absence of projective distortions in the case of the object parallel to the screen. Therefore, in particular, a square is mapped onto a square (not a general quadrilateral); moreover, the projection of a periodic object onto the image plane is a periodic function.

The width of the period function may explain the fact that in an experimental display exhibited at Creative Korea Expo 2015, moiré patterns were not observed even at the small angle 6° despite the fact that this angle is very close to 0, where the probability is highest [11]. In this case, no moiré patterns were observed because the mentioned display had the exact match $\rho = 4.0$, and according to Equation (10), its period curve had to be very narrow.

Interestingly, the experiment graph Figure 11 is somewhat similar to the moiré probability graph in Ref. [11]. This may mean that the probability essentially depends on the amplitude. The angles between the rational angles with $m, n \leq 3$ are good candidates for moiré-free angles in the displays with square pixels combined with the linear (1D) barrier. In practice, however, other rational angles should not be excluded a priori.

5. Conclusion

The amplitude of the moiré patterns in three-dimensional (3D) displays was measured directly. The period and orientation of the moiré patterns were also measured as functions of the angle. All the measurements were made with a small angular increment across a wide angular region. Simultaneous branches were observed and analyzed.

The theoretical interpretation is given based on the wavevectors and the MTF. The experimental functions followed the theoretical predictions.

The measurement of the moiré parameters, including the amplitude, can be useful, such as in the comparison of the characteristics of displays based on the photographs
of the screens with the moiré patterns and calibration markers. It can be one of the standard numerical characteristics of the image quality. The results can be effectively applied to the practical method of selection of a moiré-free angle of the barrier or lenticular plate, and furthermore, to the multi-component (and thus more reliable) minimization of the moiré effect in digital displays as well as in other devices where the moiré effect should be reduced or eliminated.

**Disclosure statement**

No potential conflict of interest was reported by the authors.

**Funding**

This work was supported by the Cross-Ministry Giga KOREA Project through the Korean Government, Ministry of Science and ICT (MSIT) [GK17D0200].

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