Modal expansion of optical far-field quantities using quasinormal modes

Felix Binkowski1, Fridtjof Betz1, Rémi Colom1, Martin Hammerschmidt2, Lin Zschiedrich2, and Sven Burger1,2

1Zuse Institute Berlin, Takustraße 7, 14195 Berlin, Germany
2JCMwave GmbH, Bolivarallee 22, 14050 Berlin, Germany

Abstract. We discuss an approach for modal expansion of optical far-field quantities based on quasinormal modes (QNMs). The issue of the exponential divergence of QNMs is circumvented by contour integration of the far-field quantities involving resonance poles with negative and positive imaginary parts. A numerical realization of the approach is demonstrated by convergence studies for a nanophotonic system.

Introduction

For the study of physical phenomena in nano-optical systems, a modal description is the most instructive approach. Modal expansion techniques using QNMs [1–3] have been proposed to analyze light-matter interaction in nanoresonators [4–7]. As the QNMs are the solutions to open systems, they decay in time and are characterized by complex eigenfrequencies. State-of-the-art approaches use the electromagnetic fields of the QNMs in the near-field region of the resonant systems to expand near-field quantities of interest. However, far-field properties of optical systems are important for many applications because typical experiments perform measurements in the far-field region. QNMs diverge exponentially in the far-field region [2, 3], which is a key issue for modal expansion techniques. Approaches using model approximations with real-valued frequencies have been proposed to overcome the divergence problem [8–10].

Here, we discuss an approach for modal expansion of optical far-field quantities [11]. The approach is based on the complex eigenfrequencies of QNMs. The divergence issue in the far-field region is circumvented by introducing contour-integral-based expressions of the far-field quantities involving resonance poles with negative and positive imaginary parts. In this way, one can derive nondiverging expansions of the far-field quantities while the model with complex-valued frequencies of the resonant systems can be retained. We demonstrate the approach by convergence studies for a nanophotonic system.

Modal expansion of far-field quantities

In nano-optics, QNMs, $\hat{E}(\omega_0) \in \mathbb{C}^3$, are solutions to the time-harmonic Maxwell’s equations in second-order form,

$$\nabla \times \mu_0^{-1} \nabla \times \hat{E}(\omega_0) - \omega_0^2 \varepsilon(\omega_0) \hat{E}(\omega_0) = 0,$$

(1)

where $\omega_0 \in \mathbb{R}$ is the angular frequency, $\mu_0$ is the vacuum permeability, and $\varepsilon(\omega_0)$ is the permittivity tensor. For simplification of the notation, we omit the spatial dependence of the quantities. The eigenfrequencies $\tilde{\omega}_k \in \mathbb{C}$ corresponding to the QNMs have negative imaginary parts as the QNMs have to satisfy outgoing radiation conditions.

The approach proposed in [11] is demonstrated by decomposing the energy flux density,

$$s(\hat{E}(\omega_0), \hat{E}^*(\omega_0)) = \frac{1}{2} \text{Re} \left( \hat{E}^*(\omega_0) \times \frac{1}{i\omega_0 \mu_0} \nabla \times \hat{E}(\omega_0) \right) \cdot \mathbf{n},$$

Figure 1. One-dimensional resonator defined by different refractive indices, where $n_2 > n_1$. Solving the Helmholtz equation with a source term corresponding to incoming plane waves yields solutions for the electric field, $\mathbf{E}(x, \omega)$ and $\mathbf{E}^*(x, \omega)$. For simplification, only the real parts of the scattered fields (a.u.) outside the resonator are shown. (a) Diverging field $\mathbf{E}(x, \tilde{\omega}_{k\Delta}) = A e^{i\omega_{k\Delta}(x-x_0)}$, where $\tilde{\omega}_{k\Delta} = \omega_{k\Delta} + \Delta \tilde{\omega}_k$ is a frequency close to $\omega_{k\Delta}$.

(b) Nondiverging field $\mathbf{E}^*(x, \tilde{\omega}_{k\Delta}) = Re e^{i\omega_{k\Delta}(x-x_0)}$.
where the field $E'(\omega_0)$ is the complex conjugate of the electric field $E(\omega_0)$ and $n$ is the normal vector on the associated far-field sphere. The Riesz projection expansion (RPE) is used to expand $s(E(\omega_0), E'(\omega_0))$ into modal contributions \([7, 12]\). The RPE is based on complex contour integration, which means that $s(E(\omega_0), E'(\omega_0))$ has to be evaluated for complex frequencies. This is not straightforward as $s(E(\omega_0), E'(\omega_0))$ is nonholomorphic. This challenge has been addressed by exploiting the relation $E'(\omega_0) = E(-\omega_0)$ for $\omega_0 \in \mathbb{R}$, which is also a solution to Eq. (1). The field $E(-\omega_0)$ has an analytical continuation into the complex plane $\omega \in \mathbb{C}$, denoted by $E'(\omega)$. This field yields the required analytical continuation given by $s(E(\omega), E'(\omega))$. With this, Cauchy’s integral formula,

$$s(E(\omega_0), E'(\omega_0)) = \frac{1}{2\pi i} \oint_{C_0} \frac{s(E(\omega), E'(\omega))}{\omega - \omega_0} d\omega,$$

is exploited for the closed integration path $C_0$ around $\omega_0$, where $s(E(\omega), E'(\omega))$ is holomorphic inside of $C_0$. Cauchy’s residue theorem leads to

$$s(E(\omega_0), E'(\omega_0)) = -\sum_{k=1}^{K} \left[ \frac{1}{2\pi i} \oint_{C_k} \frac{s(E(\omega), E'(\omega))}{\omega - \omega_0} d\omega \right] - \sum_{k=1}^{K} \left[ \frac{1}{2\pi i} \oint_{\tilde{C}_k} \frac{s(E(\omega), E'(\omega))}{\omega - \omega_0} d\omega \right] + \frac{1}{2\pi i} \oint_{\tilde{C}_0} \frac{s(E(\omega), E'(\omega))}{\omega - \omega_0} d\omega, \tag{2}$$

where $\tilde{C}_1, \ldots, \tilde{C}_K$ are contours around the resonance poles of $E(\omega)$, given by $\tilde{\omega}_1, \ldots, \tilde{\omega}_K$, and $C_1, \ldots, C_K$ are contours around the resonance poles of $E'(\omega)$, given by $\tilde{\omega}'_1, \ldots, \tilde{\omega}'_K$. The contour $C_0$ comprises $\omega_0$, the resonance poles $\tilde{\omega}_0, \ldots, \tilde{\omega}_K$ and $\tilde{\omega}'_0, \ldots, \tilde{\omega}'_K$, and no further poles. The Riesz projections

$$\tilde{s}_k(E(\omega_0), E'(\omega_0)) = -\frac{1}{2\pi i} \oint_{C_k} \frac{s(E(\omega), E'(\omega))}{\omega - \omega_0} d\omega$$

are modal contributions for the energy flux density. The contribution

$$s_t(E(\omega_0), E'(\omega_0)) = \frac{1}{2\pi i} \oint_{\tilde{C}_0} \frac{s(E(\omega), E'(\omega))}{\omega - \omega_0} d\omega$$

is the remainder containing nonresonant components as well as contributions corresponding to eigenfrequencies outside of the contour $C_0$.

The presented approach is based on computing the quantity $s(E(\omega), E'(\omega))$ by solving Eq. (1) for $\omega$ and for $-\omega$. Due to the compensation of the factors $e^{i\omega_0 r/\epsilon}$ and $e^{-i\omega_0 r/\epsilon}$ of the fields in the far-field region, this yields a nondiverging quadratic form $s(E(\omega), E'(\omega))$, where a product of $E(\omega)$ and $E'(\omega)$ is involved. In this way, modal expansions of far-field quantities can be computed. To illustrate this, a one-dimensional resonator with the fields $E(x, \omega)$ and $E'(x, \omega)$ fulfilling the corresponding Helmholtz equation is considered. Figure 1(a) sketches the diverging field $E(x, \tilde{\omega}_0)$, which relates to a QNM of the problem as $\tilde{\omega}_0 = \tilde{\omega}_0 + i \Delta \tilde{\omega}$ is a frequency close to the eigenfrequency $\tilde{\omega}_0$. Figure 1(b) shows the nondiverging field $E'(x, \tilde{\omega}_0)$ outside of the resonator. Note that the frequency $\tilde{\omega}_0$ represents a point on an integration contour $C_0$ from Eq. (2). The product $E(x, \tilde{\omega}_0) \cdot E'(x, \tilde{\omega}_0)$ shows a nondiverging behavior and relates to the energy flux density. The approach also applies to arbitrary three-dimensional problems, where, in the far-field region, $E(r, \omega) \sim e^{i\omega_0 r/\epsilon} (1/\epsilon)$ and $E'(r, \omega) \sim e^{-i\omega_0 r/\epsilon} (1/\epsilon)$.

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