The Variation of the Forces acting in a Helical Cylindrical Gear according to its operating Modes

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Abstract. The main objective of this paper is a five operating mode comparison case study that reveals the values of the main three force components acting in the contact point where two gears are meshed together. For the determination of this force components, the initial part of the paper will consist in the calculation of the moment of torsion for the output pinion of the gearbox, with the help of the total power of the power turbine that drives the main shaft. This will also conclude that of the total power generated by the 18802,242 [kW] turbine power plant, the power turbine that drives the main shaft through the main gearbox will return the power of 5000 [kW].

Keywords: Helical cylindrical gear, type 22 frigate, helix angle.

1. Introduction
In this paper, the research object will be the main gearbox configuration of a type 22 frigate. The gearing is made up of a main wheel and shaft, the forward end being coupled to the Oil Transfer Box whilst the after end is coupled to the forward intermediate propeller shaft. The main shaft also carries the main thrust collar.

Meshing with the main wheel are two secondary pinions, upper and lower. The secondary pinions are connected to the primary wheels by a quill shaft. The forward end of each quill shaft is connected to the pinion by a flanged coupling whilst the after end is connected to the primary wheel by a fine tooth coupling. Fitted to the forward end of the upper quill shaft, via an extension tube, is the shaft disc brake. The turning gear engages via a sliding dog clutch, into the after end of the upper quill shaft.

The primary pinion meshes with both primary wheels and forms the input member of the main gearing. Forward of the pinion is the main Gas Turbine clutch, aft of the pinion is the Cruising Turbine clutch. A schematic of the above mentioned is listed in Figure 1.
2. The main forces acting in a Helical Cylindrical Gear

At the contact point of two opposed teeth (from the pinion and from the wheel) there is a main (normal) force $F_n$. This force can be decomposed in three components on three orthogonal directions: the Tangential Component ($F_t$), the Axial Component ($F_a$) and the Radial Component ($F_r$) such as in Figure 2.

![Figure 2](image-url)  

Each of the three components can be found on both wheels that mesh together and they are equal as value but they have opposite directions (they cancel each other out according to Newton’s third law) and they are defined according to the definition of the moment of torsion:

The Tangential Component (such as displayed in figures 3 and 4):
The Axial Component (such as in figure 3):
\[ F_a = F_t \cdot \tan \beta \text{ [KN]} \]  

Where \( \beta = m \left( \frac{F_n}{F_t} \right) \)

The Radial Component (such as displayed in figures 3 and 5):
\[ F_r = F_t \cdot \frac{\tan \alpha_n}{\cos \beta} = F_t \cdot \tan \alpha_t \text{ [KN]} \]  

Where \( \alpha_n = m \left( \frac{F_n}{F_t} \right) \) and \( \alpha_t = m \left( \frac{F_t}{F_nt} \right) \)

Therefore the main force will be the result of the three components:
\[ F_n = \sqrt{F_t^2 + F_a^2 + F_r^2} = \frac{F_t}{\cos \alpha_n \cos \beta} \text{ [KN]} \]
3. **Determination of the power from the gas turbine installation**

In order to compute the values of the three components acting at the contact point of two opposite teeth, it is needed the moment of torsion for the output pinion. The output moment will be computed taking into account five different operating modes of the gas turbine plant. Each operating mode will be a percentage of the total power of the power turbine that drives the main shaft through the main gearbox. The next part of the paper will compute this total power.

The most common used naval Diesel fuel has the following concentration:
- \( c = 85.6\% \) (carbon)
- \( h = 13\% \) (hydrogen)
- \( s = 1.08\% \) (sulfur)
- \( o = 0.01\% \) (oxygen)
- \( n = 0.29\% \) (nitrogen)
- \( a = 0.02\% \) (other compounds)

The amount of air required to burn one kilogram of fuel will be:

\[
m_{\text{air}} = \frac{\frac{g_c}{8} + \frac{g_h}{8} + \frac{g_s}{8} - \frac{g_o}{8} - \frac{g_n}{8} - \frac{g_a}{8}}{23.2} \quad \text{Kg}_{\text{air}} \quad \text{Kg}_{\text{fuel}}
\]

\[
m_{\text{air}} = 14.35 \quad \text{Kg}_{\text{air}} / \text{Kg}_{\text{fuel}}
\]

As shown in figure 6, the input functional parameters for temperature and pressure of the gas turbine installation are \( T_1 = 318 \) [K] (the temperature within the machinery compartment) and \( p_1 = 17 \) [bar] (the input air pressure). Also, for this specific installation (Rolls Royce Olympus TM3B) it is known the hourly fuel consumption as being 5000 \( \text{Kg}_{\text{fuel}} / \text{h} \) and the fact that the energy flow is the one indicated by the arrows.

![Figure 6 Diagram of the type 22 frigate main gas turbine plant](image)

where **K** is the air compressor, **CA** is the combustion chamber, **TG** is the gas generator and **TP** is the power turbine.

The second purpose of this paragraph is to prove that from the total power of one given gas turbine plant only one fraction of it is actually used for training the propeller shaft (via the gearbox).

The output temperature from the **K** compressor will be:

\[
T = 318 \cdot 17^{0.26} = 664,268 \quad [\text{K}]
\]
It is considered that the value of the coefficient of excess air $\alpha$ (between 1.5 and 1.8) is 1.7. Taking into account the (6) result and the hourly fuel consumption will be:

$$m_{\text{air}} = c_h \cdot \frac{1}{3600} \cdot \alpha \cdot m_{\text{air}}$$  \hspace{1cm} (8)

$$m_{\text{air}} = 33,881 \left[ \frac{\text{Kgair}}{s} \right]$$  \hspace{1cm} (9)

The airflow received by the compressor or the training power of the compressor will be:

$$Q_{\text{airK}} = m_{\text{air}} \cdot c_{\text{air}} \cdot (T - T_0)$$  \hspace{1cm} (10)

Where $c_{\text{air}}$ is the specific heat of air. The (10) formulae gets:

$$Q_{\text{airK}} = 33,881 \cdot 1 \cdot (664,268 - 318) = 11731,906 \text{ [kW]}$$  \hspace{1cm} (11)

Taking into account the frictional losses and considering the air compressor’s efficiency as 0.85 the power from the gas turbine plant will be:

$$Q_{\text{TG}} = \frac{Q_{\text{airK}}}{\eta_{\text{K}}} = \frac{11731,906}{0.85} = 13802,242 \text{ [kW]}$$  \hspace{1cm} (12)

So, of the total power generated by the 18802,242 [kW] turbine power plant, the power turbine that drives the main shaft through the main gearbox will return the power of 5000 [kW].

4. Helical Cylindrical Gear Geometric Model Parameters

For the next part of the paper, the research object will be a simple helical cylindrical gear. The three components of the main force acting at the contact point of the wheel will be further analyzed. For this, the input shaft will be from the main Gas Turbine.

In Table 1 are described five different operating modes for the main Gas Turbine that leads to different values for the components of the main force.

| Operating Mode | PCL [\%] | Input power [kW] | Main shaft speed [rpm] | Power Turbine Speed [rpm] |
|----------------|---------|------------------|------------------------|--------------------------|
| 1              | 82      | 4100             | 180                    | 3700                     |
| 2              | 75      | 3750             | 164                    | 3400                     |
| 3              | 61      | 3050             | 134                    | 2700                     |
| 4              | 40      | 2000             | 87                     | 1800                     |
| 5              | 16      | 800              | 52                     | 1000                     |

The PCL (Power Control Lever) establishes the amount of power provided to the shaft related to the maximum output power of the power turbine.

The moments of torsion for the input, respectively the output pinion are:

$$M_{t1} = \frac{P_1}{\omega_1} \text{ [kNm]}$$  \hspace{1cm} (13)

$$M_{t2} = \frac{P_2}{\omega_2} \text{ [kNm]}$$  \hspace{1cm} (14)

$P_1$ and $P_2$ are the output power of the main Gas Turbine, respectively the output power for the gearbox.

$$P_2 = \eta_{\text{gearbox}} P_1 \text{ [kW]}$$  \hspace{1cm} (15)

$$\eta_{\text{gearbox}} = 0.94 ... 0.98 \text{ [-]}$$  \hspace{1cm} (16)

$\eta_{\text{gearbox}}$ represents the output power loss due to the moving elements of the main gearbox. $\omega_1$ and $\omega_2$ are the angular speeds for the input respectively the output pinion.
\[
\omega_{1,2} = \frac{2n_1,2}{60} \text{ [rpm]}
\]

\(n_1\) and \(n_2\) are the speeds of the input, respectively the output shaft.

For simplicity of the calculus, it is taken into account a single stage reduction.

It is known that

\[
M_{t2} = \frac{1}{\eta_{\text{gearbox}}} \frac{n_2}{n_1} \quad \text{[kNm]}
\]

Therefore

\[
M_{t2} = \eta_{\text{gearbox}} M_{t1} \frac{n_1}{n_2} \quad \text{[kNm]}
\]

Applying the (1) and (19) formulas for \(d=1.8\text{m}\), the results for the five operating modes are shown in Table 2.

**Table 2.** Calculation of the Tangential Component related to each operating mode.

| Operating Mode | Input moment of torsion \(M_{t1}\) [kNm] | Output moment of torsion \(M_{t2}\) [kNm] | Tangential Component [kN] |
|----------------|-----------------------------------------|------------------------------------------|--------------------------|
| 1              | 10.587                                  | 204.565…213.270                         | 227.294…236.966          |
| 2              | 10.538                                  | 205.356…214.094                         | 228.173…237.883          |
| 3              | 10.793                                  | 204.416…213.114                         | 227.129…236.794          |
| 4              | 10.616                                  | 206.457…215.243                         | 229.397…239.159          |
| 5              | 7.643                                   | 138.168…144.047                         | 153.520…160.052          |

The next part of the paper will compare the values for the main components (1), (2) and (3) related to three different helix angles: 15°, 30° and 45°. For each operating mode there will be analyzed all the possible values for \(\alpha_n\), between 0 and 90°.

**Case 1: \(\beta=15^0\)**

Applying the (2), (3) and (4) formulas, the results are obtained in Table 3.

**Table 3.** Calculation of the Axial and Radial Components and the main Forces related to each operating mode (for \(\alpha_n=20^0, 40^0, 60^0\) and \(80^0\)).

| Operating Mode | Tangential Component [kN] | Axial Component [kN] | Radial Component [kN] | Main Force [kN] |
|----------------|----------------------------|----------------------|-----------------------|-----------------|
| 1              | 227,294…236,966           | 60,903…63,494        | 85,466…89,291         | 250,413…261,069 |
| 2              | 227,294…236,966           | 60,903…63,494        | 197,450…205,852       | 307,178…320,249 |
| 3              | 227,294…236,966           | 60,903…63,494        | 407,572…424,915       | 470,624…490,650 |
| 4              | 227,294…236,966           | 60,903…63,494        | 1334,521…1391,308     | 1355,108…1412,771 |
| 5              | 227,294…236,966           | 60,903…63,494        | 199,277…207,757       | 310,020…323,213 |
| 6              | 227,294…236,966           | 60,903…63,494        | 409,148…426,560       | 472,444…492,549 |
| 7              | 227,294…236,966           | 60,903…63,494        | 1339,681…1396,692     | 1360,348…1418,238 |
| 8              | 227,294…236,966           | 60,903…63,494        | 1339,681…1396,692     | 1360,348…1418,238 |
| 9              | 227,294…236,966           | 60,903…63,494        | 1339,681…1396,692     | 1360,348…1418,238 |
Case 2: $\beta=30^0$
Applying the (2), (3) and (4) formulas, the results are obtained in Table 4.

| Operating Mode | Tangential Component [kN] | Axial Component [kN] | Radial Component [kN] | Main Force [kN] |
|----------------|---------------------------|---------------------|---------------------|----------------|
| 1              | 227,294…236,966           | 131,228…136,812    | 95,526…99,591      | 279,300…291,185 |
| 2              | 227,294…236,966           | 131,228…136,812    | 220,227…229,598    | 342,612…357,191 |
| 3              | 227,294…236,966           | 131,228…136,812    | 454,588…473,932    | 524,913…547,249 |
| 4              | 227,294…236,966           | 131,228…136,812    | 95,895…99,976      | 280,380…292,312 |
| 5              | 227,294…236,966           | 131,228…136,812    | 221,078…230,486    | 343,937…358,574 |
| 6              | 227,294…236,966           | 131,228…136,812    | 456,346…475,766    | 526,942…549,367 |
| 7              | 227,294…236,966           | 131,228…136,812    | 1488,464…1551,803  | 1517,271…1581,839 |

Case 3: $\beta=45^0$
Applying the (2), (3) and (4) formulas, the results are obtained in Table 5.

| Operating Mode | Tangential Component [kN] | Axial Component [kN] | Radial Component [kN] | Main Force [kN] |
|----------------|---------------------------|---------------------|---------------------|----------------|
| 1              | 227,294…236,966           | 131,228…136,812    | 95,526…99,591      | 279,300…291,185 |
| 2              | 227,294…236,966           | 131,228…136,812    | 220,227…229,598    | 342,612…357,191 |
| 3              | 227,294…236,966           | 131,228…136,812    | 454,588…473,932    | 524,913…547,249 |
| 4              | 227,294…236,966           | 131,228…136,812    | 95,895…99,976      | 280,380…292,312 |
| 5              | 227,294…236,966           | 131,228…136,812    | 221,078…230,486    | 343,937…358,574 |
| 6              | 227,294…236,966           | 131,228…136,812    | 456,346…475,766    | 526,942…549,367 |
| 7              | 227,294…236,966           | 131,228…136,812    | 1488,464…1551,803  | 1517,271…1581,839 |
5. Conclusions, results and discussions

For the simple helical cylindrical gear, the five operating modes, the three different values for the helix angle (15\textdegree, 30\textdegree and 45\textdegree), the following results were obtained:

**Case 1: \( \beta = 15\textdegree \)**
Figure 7 Variation of the Tangential, Axial and Radial Components and the main Forces related to each operating mode

Case 2: $\beta=30^0$
Figure 8.3 – OM3

Figure 8.4 – OM4

Figure 8.5 – OM5

**Figure 8** Variation of the Tangential, Axial and Radial Components and the main Forces related to each operating mode
Case 3: $\beta=45^0$

Figure 9.1 – OM1  
Figure 9.2 – OM2  
Figure 9.3 – OM3  
Figure 9.4 – OM4
Figure 9.5 – OM5

Figure 9 Variation of the Tangential, Axial and Radial Components and the main Forces related to each operating mode

For simplicity, the displayed results were only for the minimum values of the three components. It is obvious that the variation pattern is identical for all three helix angle values and for each operating mode. The steepness of the Radial Component and the Main Force curves becomes more obvious for $\alpha_n$ greater than 50°.

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