SUSY Ward identities in 1-loop perturbation theory

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We present preliminary results of a study of the supersymmetric (SUSY) Ward identities (WIs) for the $N=1$ SU(2) SUSY Yang-Mills theory in the context of one-loop lattice perturbation theory. The supersymmetry on the lattice is explicitly broken by the gluino mass and the lattice artifacts. However, the renormalization of the supercurrent can be carried out in a scheme that restores the nominal continuum WIs. The perturbative calculation of the renormalization constants and mixing coefficients for the local supercurrent is presented.

1. INTRODUCTION

SUSY gauge theories present different non-perturbative aspects which are the object of current research, for example the possible mechanisms for dynamical supersymmetry breaking. The simplest SUSY gauge theory is the $N=1$ SUSY Yang-Mills theory. For SU($N_c$) it has $(N_c^2 - 1)$ gluons and the same number of massless Majorana fermions (gluinos) in the adjoint representation of the color group.

To formulate supersymmetry on the lattice we follow the ideas of Curci and Veneziano [1]. They adopt the Wilson formulation for the $N=1$ SUSY Yang-Mills theory. Supersymmetry is broken explicitly by the Wilson term and the finite lattice spacing. In addition, a soft breaking due to the introduction of the gluino mass is present. It is proposed that supersymmetry can be recovered in the continuum limit by tuning the bare gauge coupling $g$ and the gluino mass $m_g$ to the SUSY point, at $m_g = 0$, which also coincides with the chiral point. In previous publications [2], and references therein, we have investigated these issues.

Another independent way to study the SUSY (chiral) limit is by means of the SUSY WIs. On the lattice they contain explicit SUSY breaking terms. In this framework, the SUSY limit is defined to be the point in parameter space where these breaking terms vanish and the SUSY WIs take their continuum form.

Our collaboration is currently focussing on the study of the SUSY WIs on the lattice, either with Monte Carlo methods [3] or by a perturbative calculation of the renormalization constants and mixing coefficients of the lattice supercurrent. For a different perturbative approach to SUSY WIs see [4].

2. SUSY WARD IDENTITIES ON THE LATTICE

In the Wilson formulation of the $N=1$ SUSY Yang-Mills theory [1] the gluonic part of the action is the standard plaquette action while the fermionic part reads...
\[ S_f = \text{Tr} \left\{ \frac{1}{2a} \left( \bar{\lambda}(x)(\gamma_{\mu} - r)U_{\mu}^\dagger(x)\lambda(x + a\hat{\mu})U_{\mu}(x) \right) + \left( m_0 + \frac{4\epsilon}{a} \right) \bar{\lambda}(x)\lambda(x) \right\}. \]

Supersymmetry is not realized on the lattice. One can still define transformations that reduce to the continuum SUSY ones in the limit \( a \to 0 \). A possible choice is

\[
\delta U_{\mu}(x) =\begin{cases} \epsilon \bar{\gamma}_{\mu} \lambda(x) + \epsilon \bar{\lambda}(x + a\hat{\mu})U_{\mu}(x) & \text{for the gauge field} \\ agf \epsilon \bar{\gamma}_{\mu} \lambda(x + a\hat{\mu})U_{\mu}(x) & \text{for the fermion field} \end{cases}
\]

\[
\delta \lambda(x) = -\frac{i}{g} \sigma_{\rho\tau} G_{\rho\tau}(x) \epsilon
\]

where \( G_{\rho\tau} \) is the clover plaquette operator.

### 2.1. Ward identities and operator mixing

Compared to the numerical simulations \( \bar{3} \) a lattice perturbative calculation of the SUSY WIs introduces new aspects. In order to do perturbation theory we have to fix the gauge, which implies that new terms appear in the SUSY WIs: the gauge fixing term (GF) and the Faddeev-Popov term (FP) while contact terms (CT) appear off-shell \( \bar{3} \). Taking into account all contributions coming from the action, the bare SUSY WIs read

\[
\left\langle O \Delta_{\mu} S_{\mu}(x) - 2m_0 O \chi(x) + \frac{\delta O}{\delta \bar{\epsilon}(x)} \right\rangle_{\epsilon = 0}
\]

\[
- \delta S_{\text{GF}} \frac{\delta S_{\text{FP}}}{\delta \bar{\epsilon}(x)} \frac{\delta S_{\text{FP}}}{\delta \bar{\epsilon}(x)} \right\rangle = \left\langle O X_S(x) \right\rangle.
\]

\( X_S \) is the symmetry breaking term, whose specific form depends on the choice of the lattice supercurrent. We define the lattice supercurrent to be

\[
S_{\mu}(x) = \frac{2i}{g} \text{Tr} \left\{ G_{\rho\tau}(x) \sigma_{\rho\tau} \gamma_{\mu} \lambda(x) \right\}.
\]

We choose a non-gauge invariant operator insertion \( O := A_\mu^a(y) \lambda^b(z) \). Gauge dependence implies that operator mixing with non-gauge invariant terms has to be taken into account for the operator renormalization. \( X_S \) mixes with operators of equal or lower dimension \( \bar{3} \)

\[
X_S(x) = \bar{X}_S(x) - (Z_S - 1) \Delta_{\mu} S_{\mu}(x) - 2\tilde{m}\chi(x) - Z_T \Delta_{\mu} T_{\mu}(x) - \sum_i Z_{A_i} A_i.
\]

The additional operators \( A_i \) do not appear in the on-shell gauge invariant numerical approach of \( \bar{3} \). They are either BRS-exact, \( A = \delta_{\text{BRS}} \tilde{A} \), or vanishing using the equation of motion. Moreover, because the \( A_i \) do not appear in the SUSY WIs at tree level, the \( Z_{A_i} \) are \( O(g^2) \).

We are forced to go off-shell, contrary to the numerical simulations that are in the on-shell regime, otherwise it is not possible to separate the contributions of \( T_{\mu} \) and \( S_{\mu} \). Finally, infrared divergences are treated using the Kawai procedure \( \bar{5} \), which gives the general recipe to renormalize a Feynman diagram at one-loop order.

### 2.2. Perturbative calculation

We calculate the matrix elements of the terms in the SUSY WIs for general external momenta \( p \) (for the gauge field) and \( q \) (for the fermion field) and the projections over \( \gamma_{\mu} \) and \( \gamma_{\mu}\gamma_5 \) matrices.

The lattice SUSY transformations of the gauge field \( A_{\mu} \) are not identical to the continuum ones: the transformation of the gauge link \( U_{\mu} \) determines the transformation properties of \( A_{\mu} \). Up to \( O(g^2) \),

\[
\delta A_{\mu}^a = i(\bar{\epsilon}(x) \gamma_{\mu} \lambda^b(x) + \bar{\epsilon}(x + a\hat{\mu}) \gamma_{\mu} \lambda^b(x + a\hat{\mu}))+ \frac{i}{2} agf_{abc}(\bar{\epsilon}(x) \gamma_{\mu} \lambda^c(x) - \bar{\epsilon}(x + a\hat{\mu}) \gamma_{\mu} \lambda^c(x + a\hat{\mu})) A_{\mu}^a
\]

\[
- \frac{i}{24} a^2 g^2 (2\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}) A_{\mu}^a A_{\mu}^d \left\{ \left( \bar{\epsilon}(x) \gamma_{\mu} \lambda^a(x) + \bar{\epsilon}(x + a\hat{\mu}) \gamma_{\mu} \lambda^a(x + a\hat{\mu}) \right) \right\},
\]

which reduces to the continuum SUSY transformation, \( \delta A_{\mu}^a = 2i\bar{\epsilon}(\gamma_{\mu} \lambda^a - \gamma_5 \lambda^a \gamma_5) \), in the limit \( a \to 0 \).

At one-loop order, three propagator integrals on the lattice are tabulated in \( \bar{3} \) in terms of lattice constants plus the continuum counterparts:

\[
C_{0,p,\mu_\nu\rho}(p, q) = \frac{1}{\pi^2} \int d^4 k \frac{k_\mu k_\nu k_\rho}{k^2(k + p)^2(k + q)^2}.
\]

With the help of \( \bar{3} \) one can express \( C_0(p, q) \), which for arbitrary external momenta \( p \) and \( q \) is a complicated expression in terms of Spence functions, as

\[
C_0(p, q) = \frac{1}{\Delta} \left( Li_2 \left( \frac{p \cdot q - \Delta}{q^2} \right) - Li_2 \left( \frac{p \cdot q + \Delta}{q^2} \right) \right)
\]
\[ + \frac{1}{2} \log \left( \frac{p \cdot q - \Delta}{p \cdot q + \Delta} \right) \log \left( \frac{(q - p)^2}{q^2} \right), \]

where \( \Delta^2 = (p \cdot q)^2 - p^2 q^2 \). The \( C_{\mu}(p, q) \), \( C_{\mu\nu}(p, q) \), \( C_{\mu\nu\rho}(p, q) \) can be written recursively in terms of scalar functions \( p^2, q^2, p \cdot q \) and \( C_0(p, q) \) multiplying Lorentz components of the external momenta \( p \) and \( q \). The general results for arbitrary external momenta \( p \) and \( q \) are very long (sometimes up to 1000 terms). Therefore a small momenta expansion is required.

### 2.3. Renormalization Constants

One can write the matrix elements in the form

\[ \langle O_{\Delta\mu S_{\mu}} \rangle = S_F(q) \cdot \Lambda_{SS} \cdot D(p) \cdot \delta_{ab}, \]

where \( S_F(q) \) and \( D(p) \) are the full gluino and gluon propagators, \( \delta_{ab} \) is the color factor and \( \Lambda_{SS} \) is the matrix element with amputated external propagators. For small momenta, \( \Lambda_{SS} \) yields at one-loop order

\[ \Lambda_{SS} = \left( 2(p - q)_\mu p_\nu \sigma_{\nu\alpha} \gamma_\mu (1 + T^S_S) + i(p - q)_\mu (\gamma_\mu \delta_{\alpha\alpha} - p_{\alpha\gamma} \gamma_\alpha) T^T_T + \cdots \right), \]

where \( T^S_S \) is the coefficient of the one-loop contribution which is proportional to the tree level expression of \( \Lambda_{SS} \), \( T^T_T \) is the coefficient of the contribution proportional to the tree level \( \Lambda_{TT} \). The tree level matrix element of \( \langle O_{\Delta\mu T_{\mu}} \rangle \) reads

\[ \Lambda_{TT} = i(p - q)_\mu (\gamma_\mu \delta_{\alpha\alpha} - p_{\alpha\gamma} \gamma_\alpha). \]

Collecting the various contributions in the WIs and expanding them in terms of a basis of Lorentz-Dirac structures, the renormalization and mixing constants can be obtained from the coefficients as

\[ -(Z_S - 1) = T^S_S + T^CT_S + T^G_F + T^G_F + T^F_F, \]

\[ -Z_T = T^T_T + T^CT_T + T^G_T + T^G_T + T^F_T. \]

We consider an appropriate choice of projections, which for small momenta and on tree level read,

\[ \text{Tr}(\gamma_5 \Lambda_{AT}) = 4i(p_\mu p_\nu - p^2 \delta_{\mu\nu} - p_\alpha q_\alpha) \]

\[ + p \cdot q \delta_{\mu\alpha}, \]

\[ \text{Tr}(\gamma_5 \Lambda_{TS}) = 0, \]

\[ \text{Tr}(\gamma_5 \Lambda_{SS}) = 8i(p_\mu p_\nu - p^2 \delta_{\mu\nu} - p_\alpha q_\alpha) \]

\[ + p \cdot q \delta_{\mu\alpha}, \]

in order to extract the coefficients. Typically the \( T_T^{(1)}, T_S^{(1)} \) are constants plus logarithms depending on the external momenta. Our preliminary results for \( Z_S \) and \( Z_T \) show a good agreement with the numerical data of [3].

### 3. OUTLOOK

It is possible to study the SUSY WIs by means of lattice perturbation theory and to determine the coefficients \( Z_T \) and \( Z_S \) in the off-shell regime.

The contributions for all diagrams have been written down explicitly for small external momenta. Preliminary results are in accordance with Monte Carlo data, but still we have to increase the precision of the numerical integrations and to perform several checks before presenting the final results in a forthcoming publication.

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