Spin–orbit interaction in non-paraxial Gaussian beams and the spin-only measurement of optical torque

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Abstract

A circularly polarized focussed Gaussian beam carries total angular momentum of $\hbar$ per photon about the beam axis, but less than $\hbar$ spin per photon, due to the focussing of the beam. The remainder of the angular momentum is carried as orbital angular momentum. When such beams are used to rotate microscopic birefringent particles in optical tweezers, the change in angular momentum can be optically measured. However, this measurement is made using the collimated transmitted beam, rather than the focussed beam. Therefore, the conversion of spin to orbital angular momentum by focussing or collimating the beam is expected to affect the measurement. We show that for the typical cases where rotating optical tweezers are used for such measurements, the error due to spin–orbit conversion is unimportant, but there exist cases where a spin-only torque measurement would be completely erroneous.

Keywords: spin, orbit, optical tweezers, optical angular momentum, spin–orbit interaction

(Some figures may appear in colour only in the online journal)

1. Introduction

The development of optical tweezers [1] saw its prompt deployment in biophysics, with the quantitative non-contact force measurements they made available proving transformative [2, 3]. The application of optical forces for the trapping and manipulation of microparticles was soon followed by the application of optical torques [4–7]. The microscopic replication of the classic almost-simultaneous experiments of Beth and Holbourn [8, 9] in optical traps [4, 5] opened the path to absorption-free transfer of optical angular momentum and straightforward optical measurement of the torque. Since this beginning, the use of optical torque in optical tweezers has reached the point of practical application in, e.g. microrheology with rotating probes allowing excellent spatial resolution [10].

The little hero of much of this work has been the vaterite microsphere, a polycrystalline assembly of crystals of vaterite, one of the crystal forms of calcium carbonate which exhibits very strong birefringence [11]. While vaterite microspheres are much more optically-complex than a simple waveplate, they still perform the basic function of altering the degree of circular polarisation of the trapping beam, and therefore the spin angular momentum flux, with minimal incidental absorption. They can be produced (almost) spherical in shape, simplifying the fluid mechanics required for, for example, quantitative microrheology.
The basic process of measuring the transfer of spin angular momentum to a trapped vaterite particle (or, indeed, any other birefringent probe) takes place outside the optical trap. The trapping beam begins as a collimated beam. The polarisation state of this beam is adjusted as desired, and the beam is focussed to a diffraction-limited spot by a high numerical aperture microscope objective. The polarisation of this beam is modified by the trapped birefringent probe particle, and the transmitted beam (i.e. the forward-scattered light) is collected and collimated, and its polarisation state is measured to determine the optical torque.

However, the focussing of the beam by the high numerical aperture lens converts spin angular momentum to orbital angular momentum, and it has not previously been clear whether this should be a significant problem for measurements of this type. We presented a simple outline of the theory of this spin–orbital coupling in our previous paper on this topic [12]. The essential principle is that in a focussed beam, only the component of the spin angular momentum parallel to the beam axis contributes to the overall spin angular momentum of the beam about the beam axis. This is the same geometry that results in the focussed beam carrying less linear momentum than a collimated beam, which is what allows three-dimensional trapping by a single beam [1]. There is a further complication: focussing the beam with a non-absorbing rotationally-symmetric lens (e.g. an ideal lens) will not change the angular momentum flux of the beam, since no torque can be exerted on that lens [13]. Since the spin angular momentum flux of the beam must decrease, there must be a corresponding increase in the orbital angular momentum flux. This orbital angular momentum is carried by an optical vortex structure of the axial component of the electric field of the beam, which is produced by the focussing. In [12], we calculate the axial component of the electric field using a multipole representation of the beam, but any method yielding the correct fields will suffice. For example, the Richards–Wolf representation of the focussed field [14] gives the same vortex structure. Richards and Wolf did not consider the circularly polarised case in [14] but the result is apparent from their linear polarisation case and the phase relations they give on page 365 of [14].

The question can be asked whether this effect might have been observable in our earlier experiment on the trapping and rotation of calcite fragments [4]. Clearly, this question can be extended to asking whether this spin–orbit coupling by focussing can influence quantitative measurements of torque using vaterite microspheres.

1.1. Outline

Following this introduction, we present the essentials of the theory of optical tweezers [1, 15, 16] and rotating optical tweezers [10], including the quantitative measurement of torque via spin angular momentum [3, 17, 18]. We continue to an overview of spin and orbital angular momenta, including their distinction and coupling during focussing of a beam [12]. Next, we outline our computational methods.

From there, we proceed to our results. We consider a number of specific cases and questions:

(a) Interaction of a focussed beam with a calcite sheet. How do torque measurements based on the spin of the collimated transmitted beam compare with the actual torque?
(b) Is there a simple example to show that a spin-only torque measurement can be completely incorrect? A rotationally-symmetric radially-birefringent structure is a natural example to check, since (a) the torque will be zero due to the symmetry [13] and (b) the polarisation of the transmitted light will change.
(c) Noting that much of the current work on rotating optical tweezers, especially when quantitative optical measurements of the torque are made, uses vaterite microspheres [11, 19, 20], how does the accuracy of such optical torque measurements depend on the focussing of the beam?
(d) How are optical torque measurements on vaterite microspheres affected by motion within an optical trap, including Brownian motion and/or optical torques that can tilt the optic axis relative to the horizontal plane?

2. Theory

2.1. Optical tweezers

It is well-known that the optical forces in optical tweezers result from the transfer of linear momentum from the trapping beam to the particle in the trap. The key that allows three-dimensional trapping with a single beam is the dependence of the axial linear momentum flux on how tightly focussed the beam is. A collimated beam carries $n \pi k$ momentum per photon in the direction of propagation, where $n$ is the refractive index of the medium, $\pi$ is the reduced Planck constant, and $k$ is the free-space wavenumber. If the beam is focussed, the magnitude of the momentum flux will still be $n \pi k$, locally, but only the vector component parallel to the beam axis will contribute to the axial momentum flux. Thus, if a focussed beam is made more collimated by a particle in the trap, the axial momentum flux increases, and the reaction force on the particle will be in the opposite direction, ‘upstream’ along the beam. This ‘reverse’ force allows three-dimensional trapping [1].

Light can also carry angular momentum, and therefore the trapping beam can also exert an optical torque on a particle in the trap. Generally, such torques will occur when the particle in the trap is anisotropic, whether this is due to material anisotropy (e.g. birefringence) or non-spherical shape (shape birefringence [21] or other shape effects), even when the beam carries zero angular momentum per photon. Such torques are often transient, acting to turn the particle into an equilibrium orientation within the trap. For example, elongated particles will often align with the long axis along the beam axis. The commonly-seen torques act to (a) align particles along the beam axis, (b) align particles with the direction of polarisation, if the beam is linearly polarised, and (c) spin particles about the beam axis, if the beam is circularly polarised. More complicated motions are possible for complex shapes. The first two of these torques, (a) and (b), often compete, with alignment along the beam axis dominating for large elongated particles and alignment along the direction of polarisation dominating...
for small particles when the long axis comfortably fits within the focal spot of the beam. The optical effect of such small particles can typically be described in terms of shape birefringence [21]. For particles with material birefringence, negative uniaxial particles will align along the beam axis and positive uniaxial particles will align along the polarisation direction, unless shape effects result in a different alignment [22]. For flattened particles, both (a) and (b) can occur together, rather than being in competition, since the particle has two long axes. The last two of these torques, (b) and (c), will compete in the case of an elliptically polarised beam, and either alignment or spinning can result, depending on the details of the particle and the ellipticity of the beam.

The orientation of the particle within the trap is important for the existence of torques about the beam axis, namely torques (b) and (c). If the particle is rotationally symmetric about the beam axis, the particle will not change the angular momentum per photon about the beam axis. Particles such as elongated spheroids or cylinders are rotationally symmetric about their long axis, and if they align along the beam axis, will not experience any torques about this axis. Similarly, if a negative uniaxial sphere is trapped, it will align with its optic axis along the beam axis, and despite being birefringence, will not experience either of torques (b) or (c). If one wishes to trap a spherical birefringent particle in order to exert torques about the beam axis, the particle should be positive uniaxial (like a vaterite microsphere). For non-spherical particles, the shape can result in trapping in an orientation where torques (b) and (c) can still be exerted on the particle [23, 24].

2.2. Spin and orbital angular momenta

Since the electromagnetic field is a vector field, we can divide the angular momentum carried by the beam into spin or intrinsic angular momentum and orbital angular momentum. The spin density is independent of our choice of origin about which to take moments, while the orbital angular momentum density does depend on that choice. Note that the distinction applies to the densities. In the (common) special case where the total linear momentum in some plane is zero, the integral of the orbital angular momentum density is invariant of the choice of point in the plane about which moments are taken; this has been termed intrinsic orbital angular momentum, by analogy with spin angular momentum [25].

The separation of the total angular momentum into spin and orbital angular momenta is not without difficulty in the general case. Notably, it is not generally possible to make this division in a way that is simultaneously gauge invariant and Lorentz invariant. However, here we are interested in the special case of a time-harmonic beam rather than the general case. For a time-harmonic beam, a gauge-independent separation of the two types of angular momenta is straightforward [26], and we can write the time-averaged spin density $s_i$ in terms of the electric field amplitude vector components $E_i$ and the Levi–Civita symbol as

$$s_i = i\epsilon_0\epsilon_{ijk}E_jE_k^*/(2\omega),$$  

and similar expressions can be written in any locally orthonormal coordinate system. This allows us to calculate the spin density for any known time-harmonic electromagnetic field.

Further, in the far field of the beam (i.e. sufficiently far from the focus so that the field is locally a plane wave), only the vector component of the spin density in the local direction of propagation can be non-zero, and that component is proportional to the local degree of circular polarisation and the Poynting vector. Thus, the spin angular momentum flux can be simply measured [27], unlike the orbital angular momentum flux, the quantitative measurement of which is a formidable problem, especially when the light passes through an optical tweezers system [28].

2.3. Coupling between spin and orbital angular momenta during focussing

As we described in [12], spin angular momentum can be converted to orbital angular momentum when we focus a circularly polarised beam. First, the axial spin angular momentum flux must be reduced by the focussing of the beam. This results from the same geometric considerations that give a reduced linear momentum flux for a focussed beam. Second, if this focussing is performed by a rotationally-symmetric lens (i.e. a normal lens), the angular momentum per photon about the symmetry axis of the lens cannot change.
The spin flux about the beam axis must be reduced by focusing, but the total angular momentum flux about the beam axis must remain the same. These two facts are simply reconciled by the conversion of the lost spin to orbital angular momentum. This conversion is illustrated in figure 1.

2.4. Rotating optical tweezers

Combining the previous few points, we have (a) a recipe for the quantitative measurement of optical torques using a suitable probe particle, such as a positive uniaxial birefringent particle, and (b) a potential difficulty resulting from the spin–orbit conversion through focussing.

To measure the spin angular momentum flux, we can collect and collimate the light that has passed through the optical trap, and measure its degree of circular polarisation. It is not necessary to measure the complete Stokes vector of the light—we only require the degree of circular polarisation. For example, we can construct a circular polarisation analyser using a quarter-wave plate, a polarising beamsplitter, and two photodetectors. This yields the powers \( P_{\text{left}} \) and \( P_{\text{right}} \) of the left- and right-circularly-polarised components of the beam.

The optical torque is then simply found from the change in circular polarisation due to the particle [17, 29]. We can measure the circular polarisation in the absence of the particle, or if we know that the trapping beam is either circularly or linearly polarised, simply assume appropriate values for \( P_{\text{left}} \) and \( P_{\text{right}} \).

We can also perform this measurement in a linearly-polarised basis, which can give a more accurate result for a circularly-polarised trapping beam and small torques. In this case, we will measure the powers of two linearly polarised components, and the torque will be given by the difference between them:

\[
\tau_{\text{CP}} = \frac{P_{\text{high}} + P_{\text{low}} - 2(P_{\text{high}}P_{\text{low}})^{1/2}}{\omega} \quad (3)
\]

or

\[
\tau_{\text{CP}} = \frac{P - (P^2 - \Delta P_{\text{HL}}^2)^{1/2}}{\omega} \quad (4)
\]

where \( P = P_{\text{high}} + P_{\text{low}} \) is the total power and \( \Delta P_{\text{HL}} = P_{\text{high}} - P_{\text{low}} \) is the difference between the signals, and \( P_{\text{high}} \) and \( P_{\text{low}} \) are the powers of the two linearly polarised components, with the higher and lower power respectively. Here, we assume that the handedness of the circular polarisation does not change—this will be the case when the torque is small.

Just as focussing a circularly polarised beam will convert some spin angular momentum to orbital angular momentum, the collimation of the collected light is also expected to couple the spin and orbital angular momenta fluxes of the beam. Therefore, the spin torque as measured by the measurement described above is expected to differ from the spin torque exerted on the particle. Judging by past quantitative measurements, such as those referred to in our introduction above, and others showing excellent quantitative agreement between optical tweezers based microrheometry and conventional macrorheometry [30], this is not a problem with the method, but rather a great advantage as it appears to provide a sufficiently reliable measurement of the total torque, including both the spin and orbital parts. That is, the spin–orbit coupling due to collimating the collected light converts the change in the orbital angular momentum flux of the focussed beam to a change in the spin angular momentum flux of the collimated beam.

However, we do not in general collect all of the outgoing light. The light that can be collected is limited by the numerical aperture of the condenser or objective that collects the transmitted light. Light that is reflected by the particle (i.e. backscattered light) will not be collected, nor will light that is scattered at large angles (i.e. sidescattered light).

In general, an assumption must be made about the spin angular momentum of the uncollected light. For a birefringent probe particle, we can expect backscattered light to have spin similar to that of the transmitted light, since light reflected from the first surface will have its spin unchanged, and light reflected from the far surface will cross the particle twice and experience double the change in spin compared to the transmitted light, resulting in an average change equal to that of the transmitted light. For any particle, we can expect sidescattered light to have approximately zero spin about the beam axis, since only the vector component in the direction of the beam axis, which must be small for sidescattered light, will contribute to the axial spin. These considerations give rise to two simple assumptions about the spin angular momentum per photon of the uncollected light is (a) zero, or (b) equal to that of the collected light. We will compare these two assumptions below.

Clearly, the conversion of spin to orbital angular momentum can potentially affect such measurements. If the original conversion of spin to orbital angular momentum is reversed by the collimation when the transmitted light is collected, there will be no effect on the final measurement. However, when a trapped particle alters the polarisation state of the beam, we cannot, in general, guarantee that this will be the case.

From our experience with quantitative measurement of optical torques based on determination of the circular polarisation, we expect that, for birefringent probe particles we typically use (e.g. vaterite microspheres), any error due to the spin–orbit conversion is relatively small, compared to the uncertainties from the measurement of the particle radius and nonsphericity of the particles. Confirming that this is indeed the case will be one of our main results below.

It seems unlikely that such measurements will, in general, always be correct, and we will search for a simple example showing that a spin-only measurement of the orbital torque can indeed be very wrong.

2.5. Computational methods

Computational modelling of optical trapping allows us to calculate both the spin and orbital contributions to the optical torque in the focal region of the beam, and the spin–orbital coupling to due the focussing of the trapping beam as well as the collection/collimation of the transmitted beam. We carry out these calculations using the T-matrix method [31–33] as implemented in our optical tweezers toolbox [34, 35].
We use the discrete dipole approximation to calculate the T-matrices of our birefringent objects [36]. This code is included in the newest version of our toolbox [35].

The spin that would be measured by the spin-only torquemeter is calculated by integrating the outward vector component of the spin of the transmitted light over a spherical surface in the far field, over the acceptance angle of the condenser collecting the light. We assume that the numerical aperture of the condenser is equal to that of the objective focussing the trapping beam. This outward vector component would become the axial component when the beam is collimated by the condenser. We perform this calculation in spherical coordinates, with $r^2 s_r = c_0 \text{Im}(E_y E_y^*) / \omega$, numerically integrating this quantity over a spherical surface in the far field.

To calculate the actual torque, and its spin and orbital components, we integrate the angular momentum flux and spin flux over a spherical surface. This integration can be performed analytically by taking the far-field limit, reducing the calculation to the summation of products of the amplitudes of the vector spherical wavefunctions used to represent the beam in the T-matrix method [27, 34, 37]. The orbital part of the torque is simply the difference between the total torque and the spin torque [37, 38]. These spin and orbital components of the torque are those exerted on the particle in the trap by the focussed beam.

The ‘Torquemeter’ results in the remaining figures are equivalent to the ‘Collimated’ result in figure 1, while the ‘Torque’ results are equivalent to the total angular momentum flux (spin plus orbital). We consider our birefringent objects located at the beam focus rather than at a trapping location as a stable equilibrium is dependent on the numerical aperture of the objective, the object properties, and may not necessarily exist. We include code as supplementary material with this paper that is compatible with our toolbox to perform this integration and output the spin per photon and the proportion of total power for both the forward and backward scattered light.

We give the torques in terms of the torque efficiency $Q_T$, which is related to the torque $\tau$ by

$$\tau = Q_T P / \omega, \quad (5)$$

where $P$ is the beam power.

3. Results

3.1. Calcite sheet

First, we consider the simple case of a calcite sheet as an example of a birefringent uniaxial crystal. Here, we calculate the effect of a calcite sheet. Rather than choose a special thickness such as a half-wave plate, we choose an essentially arbitrary thickness, in this case a thickness of 4.75 $\mu$m, which is somewhat thicker than a zero-order half-wave plate, which would be 3.24 $\mu$m thick. Thus, we expect the torque for a collimated beam to be well above 1$\hbar$ per photon, but less than 2$\hbar$ per photon.

Calculations showing the optical torque and simulated measurements of the torque using the spin-only torquemeter are shown in figure 2. We can see that for the lower beam convergence angles shown, there is a significant ($\approx$7%) contribution to the torque by orbital angular momentum, but the torque-meter measurement agrees closely with the optical torque. Therefore, it appears that the orbital angular momentum is converted back to spin angular momentum when the transmitted light is collected and collimated.

For larger beam convergence angles, there is a small but significant discrepancy between the optical torque and the torquemeter measurement. This is probably the result of light being reflected by the calcite sheet, as the reflection coefficient will increase with the increasing angles of incidence of the plane wave components of the beam as the beams become increasingly tightly focussed.

The change in the spin angular momentum here is quite large (and much larger than the spin torque acting on the vaterite particles we will consider below). Despite this large change, the spin-only torque measurement is still accurate. This is an encouraging sign as far as the reliability of spin-only torquemeter applications in practice.

We also see that it appears to be important to consider the uncollected light. We will examine this further for vaterite microspheres below.

3.2. An example of failure

The theoretical considerations above suggest a simple example to test for the failure of spin-only torque measurement. A rotationally-symmetric birefringent object aligned with its optic axis along the beam axis will experience zero torque, due to the rotational symmetry of both its shape and material properties about the beam axis.

One example of such an object is a $q$-plate, or radial waveplate. A half-wave $q$-plate, for example, will reverse the circular polarisation (and therefore the spin angular momentum) of a circularly polarised beam. However, since the change in total angular momentum must be zero (since the torque must
be zero), the $\pm 2\hbar$ change in spin per photon is matched by a $\pm 2\hbar$ change in orbital angular momentum per photon [39]. (We could place a conventional half-wave plate after the half-wave $q$-plate and restore the spin angular momentum to its original state. This device will simply add $2\hbar$ angular momentum per photon. A sequence of $m$ such devices can be used to obtain a total change in orbital angular momentum of $2m\hbar$ per photon.)

Since a vaterite microsphere has a radial structure in its outer layers, it will appear as a $q$-plate to the beam if its optic axis is aligned along the beam axis. This is not a stable equilibrium orientation for a vaterite, but still provides us with a simple example to test for the failure of spin-only torque measurement. The spin and orbital torques, and the spin-only torquemeter measurement for a vaterite particle as a function of the angle between the beam axis and its optic axis are shown in figure 3. The vaterite is aligned with its optic axis along the beam axis at the left-hand side of this graph. The total torque is zero, with equal and opposite spin and orbital components.

There is a large change in the spin angular momentum, which results in a large torque according to the spin-only torquemeter. Thus, this shows that in at least some cases, spin-only torque measurement can be very wrong. We can also see from this figure that as the optic axis approaches a horizontal orientation, the torquemeter measurement becomes increasingly accurate as the vaterite particle appears to the beam as a conventional waveplate rather than a $q$-plate. Further, the slope of the torque and torquemeter curves becomes small toward the right-hand side of the graph, indicating that small fluctuations of the orientation (e.g. due to rotational Brownian motion) will result in very little change in either the torque or the torquemeter measurement.

3.3. Vaterite microspheres

Next, we can investigate the accuracy of the spin-only torquemeter in its typical application: measuring the torque exerted on vaterite microspheres. Simulated torque measurements are shown in figure 4, showing the effect of the radius of the vaterite microsphere and the numerical aperture of the trap. The torquemeter measurement and the actual optical torque are in close agreement. Note that the computational error in these results is less than the thickness of the line on the graphs.

3.4. Assumptions for the uncollected light

Since the torquemeter employed in experiments collects only the forward scattered light, an assumption must be made about the contribution from the uncollected light. The two simplest assumptions are that the spin angular momentum per photon of the uncollected light is (a) zero; or (b) equivalent to the spin angular momentum of the collected light. Figure 5 shows the error between the measured torque from the torquemeter and the actual torque acting on a vaterite microsphere at the beam focus. A range from 0.5 $\mu$m to 2.5 $\mu$m radius vaterites are shown as these sizes are commonly used in microrheology experiments.

From figure 5, the contribution of the uncollected light does not agree with either assumption over the range of vaterite radii, predominantly due to the amount of back scattered light being dependent on the vaterite size. This gives rise to a third assumption, where we can determine a radius-dependent model to scale the spin per photon of the collected light. A simple case would be to use a quadratic as this scales with the area of the vaterite and is assumption (3), shown in figure 5.

However, an alternative scaling model could be determined that considered the destructive and constructive interference of the back scattered light that gives rise to the pattern in figure 6. From an experimental point of view, the radius cannot be determined with enough precision to account for these interference patterns.

Overall, the assumption that the uncollected light has zero spin gives reasonably accurate results for most of the vaterite particles considered. The quadratic assumption (3) can be used to improve the accurate for mid-sized vaterites.
Figure 4. Left: measurement of torque for vaterites of different radii with a trapping beam of NA = 1.2. Right: measurement of torque for a 1.5 μm radius vaterite in an optical trap and varying numerical aperture.

Figure 5. The percent error between the measured torque and the actual torque for two assumptions of the uncollected light: its spin angular momentum per photon (1) is zero; (2) is equal to the collected light; or (3) is scaled by an empirical quadratic function dependent on the radius.

The assumption that the uncollected light has spin equal to the collected light is poor for small vaterite particles. In this case, much of the uncollected light is scattered at large angles, and as discussed earlier, is expected to have approximately zero spin about the beam axis.

3.5. Vaterite microspheres moving in an optical trap

Finally, we can consider two cases of a vaterite particle moving in an optical trap. First, we can consider a vaterite particle trapped and spinning in a circularly polarised beam, and include the effect of translational and rotational Brownian motion. The rotational Brownian motion will change the orientation of the optic axis, including tipping it away from the equilibrium horizontal position. As discussed above, we do not expect this to result in a large error, but it is still prudent to explicitly check the effect.

Figure 7(left) shows the torque and torque measurement for such a spinning vaterite. We see that the rotational Brownian motion does result in a small variation in both the measured torque and the spin and orbital components of the torque. However, there is very little variation in the total torque. The fluctuation in the torque meter measurement is small compared to typical uncertainties in the torque measurement, and negligible compared to the effect of uncertainties in the vaterite radius and shape.

The simulation in figure 7(left) included 10 ms prior to the results shown allowing the vaterite to move into its stable equilibrium position as this reflects experimental measurements. The shift in axial position from the beam focus to this trapping location effects both the actual torque and the measured torque meter values, however, this error is also negligible compared to existing experimental uncertainties.

The second case we can consider is the alignment of a vaterite particle with the plane of polarisation in a linearly
polarised trap. This motion is used, for example, in active microrheology where the orientation of the particle is switched between two states \[10\]. The simulated measurement is shown in figure 7(right). Again, it matches well suggesting the method works well in linearly polarised light.

4. Discussion and conclusion

We have shown that when the spin-only torquemeter is used to measure the optical torque on vaterite microspheres, the conversion of spin to orbital angular momentum by the focussing of the trapping beam has little effect. In practice, we can assume that the change in orbital angular momentum in the focal region is adequately measured through the reverse conversion of orbital angular momentum to spin when the transmitted light is collimated.

Further, we have shown that assuming that the uncollected light has the same spin as the collected light can result in significant errors for small vaterite microspheres. It is better to assume that the uncollected light has zero spin, or we can fit a quadratic transition between the assumptions of zero spin (which is expected to be the better assumption for small vaterites, where the majority of the uncollected light is sidescattered) and spin equal to the collected light (which
should be a better assumption for large vaterites, where the majority of the uncollected light is backscattered).

Finally, we also demonstrated that spin-only torquemeter measurements can be very wrong, with the counterexample of a vaterite microsphere aligned with its optic axis along the beam axis, acting as a $q$-plate. In this orientation, it alters the spin of the beam, which is measured by the torquemeter as a torque, while the actual torque is zero, due to the rotational symmetry about the beam axis.

Our results above provide a general picture of the most common case of optical torque measurements using trapped vaterite particles. Noting that vaterite microspheres have been trapped in vacuum rather than the more usual liquid medium [40, 41], it is worth noting that while the effects of spin–orbit coupling by focussing will be similar to when vaterites are trapped in liquid, the much larger refractive index contrast will make the error in measurement due to the uncollected light much larger. We can expect this error due to uncollected light to be the dominant error in an optical torque measurement.

Data availability statement

The data generated and/or analysed during the current study are not publicly available for legal/ethical reasons but are available from the corresponding author on reasonable request.

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