Controlling the temperature of bones using pulsed CO\textsubscript{2} lasers: observations and mathematical modeling

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Abstract: Temperature of porcine bone specimens are investigated by aiming a pulsed CO\textsubscript{2} laser beam at the bone-air surface. This method of controlling temperature is believed to be flexible in medical applications as it avoids the uses of thermal devices, which are often cumbersome and generate rather larger temperature variations with time. The control of temperature using this method is modeled by the heat-conduction equation. In this investigation, it is assumed that the energy delivered by the CO\textsubscript{2} laser is confined within a very thin surface layer of roughly 9 \(\mu\)m. It is shown that temperature can be maintained at a steady temperature using a CO\textsubscript{2} laser and we demonstrate that the method can be adapted to be used in tandem with another laser beam. This method to control the temperature is believed to be useful in de-contamination of bone during the implantation treatment, in bone augmentation when using natural or synthetic materials and in low-level laser therapy.

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1. Introduction

Bone tissue ablation [1–4] is now considered an alternative method for various type of medical surgery. In some cases, bone tissue ablation shows no discernible evidence of thermal damage as opposed to using a standard drill, which can cause fragmentation into numerous small bone chips [4, 5] during a surgical procedure. Apart from bone ablation, laser treatment of animal
bones involving moderate radiant exposure of He-Ne laser light in low-level laser therapy (LLLT) has shown to be successful on osteogenesis after controlled surgical fracture [6, 7]. In these controlled surgical fractures, trabecular regions are being exposed. As trabecular regions are being exposed, we are investigating how well the temperature of cancellous bone can be controlled by a CO\textsubscript{2} laser beam working in tandem with a visible laser. Other uses of Argon laser light involving moderate radiant exposure has shown to give excellent results in the treatment of primary angle closure glaucoma [8]. In tissue engineering, temperature control of the bone near the implant surface is very important as devitalization of the overlying tissue may occur if the temperature exceeds 47\degree C [9–11]. Although no major thermal damages were noticed in both cortical and cancellous bone de-contamination above 50\degree C [12], it is still very desirable to control temperature in medical sciences. As studies appear to be done separately for cortical and cancellous bone in the literature, only results for cancellous porcine bones are reported in our investigations.

In all of the aforementioned applications related to orthopaedic surgery, a procedure that is well temperature-controlled is desirable to prepare the bone before it is drilled or cut. The method used to heat a given bone specimen by CO\textsubscript{2} laser can be done in tandem with a visible laser that is exciting natural intracellular chromophores such as porphyrins or cytochromes [4] found in blood supplying bones. In this tandem method, the CO\textsubscript{2} laser would be controlling the bone temperature, which is absorbing in the far-infrared and visible light such as He-Ne taking the same path as the CO\textsubscript{2} laser would be useful to excite natural chromophores.

In this investigation, we show that the temperature of the cancellous part of porcine bones can be controlled using a constant flux of CO\textsubscript{2} radiation. A similar method was proposed to control the hard cortical part of the bone under a constant CO\textsubscript{2} laser flux [13], a complete mathematical treatment supported by experimental results for cancellous porcine bone is presented. A complete mathematical model would take into account the blood supply in the bone cancellous structure. For the dry bones that are currently available, it was deemed that a simplified mathematical model that treats the bone as a uniform section was realistic. The current mathematical model would provide a good basis for more complete models where the complexities of including the effects of blood/water could be studied more rigorously.

2. Methodology

The absorption coefficient $\alpha$ of bone is very large [2, 3] at the CO\textsubscript{2} laser wavelength $\lambda$. As the melting point of mineral bones is quite high [3] ($T_{\text{Melt}} \sim 1280\degree C$), the material can be heated to at least 300\degree C without significantly affecting its physical properties and burning the material.

A CO\textsubscript{2} laser, Firestar 60t from Synrad, was used at $\lambda = 10.6\mu m$ to investigate all the bone samples in this study. The beam from the CO\textsubscript{2} laser aperture was reflected by two mirrors mounted into the periscope assembly shown in Fig. 1. The pair of mirrors were adjusted so that the beam would propagate in a direction parallel to the plane of the optic table. The CO\textsubscript{2} laser beam was collimated by a pair of lenses of focal lengths $f_1$ and $f_2$, respectively. Using the pair of collimation lenses, the beam was magnified to fill about one third of the lens surface of a third lens having a focal length of $f_3$. This final lens focuses the CO\textsubscript{2} beam a distance of 50 mm behind it (as shown in Fig. 1). The bone sample is placed further away, in the diverging field, at about 25 cm from the third lens' focal point. The 2D optical scanner mirrors sweeps the beam along the plane of the figure at a frequency of typically 100 Hz and in a direction perpendicular to the plane of the figure at a very low frequency of typically 0.5 Hz. The surface scanned by the 2D mirror system is roughly three times that of the cross sectional area of the bone specimen.

Porcine bones were chosen because laser ablation resulting in these animal bones is similar to the findings in hard biological tissues such as human teeth [3]. Furthermore, the cortical and
cancellous parts of the bone mineral density of porcine bones are comparable to those of human bones [14]. The bone samples were cut out of cylindrical rib bones about 60 mm long. Their cross sections were nearly circular with diameters ranging from 7 to 10 mm and their lengths were about 10 mm. The bones were placed in boiling water for approximately 30 minutes then they were placed in a 3% hydrogen peroxide solution for approximately four hours. Following this procedure, the bones were soaked for a few minutes in distilled water, then wiped with low abrasive tissue and were finally left to dry in a well ventilated area for a 24 hour period.

During each trial, the bone specimen was placed with their cross-section side facing towards the incident beam. The samples were placed on a thick glass substrate roughly 2 cm thick. The glass substrate (not shown in Fig. 1) lying directly on the metallic optical table absorbs the CO$_2$ radiation during the heating procedure and prevented too much heat from being transferred through the bone samples into the table.

At the location where the bone samples were placed, the CO$_2$ beam exposing each sample had a broad Gaussian profile with the beam spot size (at $1/e^2$) of 20 mm. The spot size was much larger than the average bone cross-sectional diameter. The CO$_2$ beam was also steered with the optical scanner mirrors along two independent directions at the aforementioned frequencies to uniformly heat the bone’s cross-sectional area. Each bone sample were centered within the CO$_2$ laser beam prior to scanning. An infra-red temperature sensor (IR-USB from Omega Engineering) with an accuracy of 1°C was aimed at the cross-sectional surface of the
bone and was adjusted in order to measure the temperature of any object placed at that position. When the operator’s finger was positioned at the working location, the thermometer measured a temperature of 30°C. A red laser aligned along the CO\textsubscript{2} beam was used to track down the invisible far-infrared radiation exposing the bone samples during the heating procedure in our investigations. Another laser with a different wavelength can also be used in tandem with the CO\textsubscript{2} beam if we want to generate fluorescence on a substance that is overlaying the bone under a well-controlled temperature in future work.

Since bones absorb much of the radiation at the CO\textsubscript{2} wavelength, only a very thin layer confined near the surface of the bone will be heated. The thickness $\tau$ of this layer attenuates the laser beam irradiation by direct absorption according to the Lambert-Beer law \cite{15, 16}.

Therefore, in the case of a cylindrical disk the laser irradiation will decay exponentially from the surface as

$$I(z) = I_0 e^{-\alpha z} \quad (1)$$

where $\alpha$ is the absorption coefficients, $I_0$ is the laser irradiation at the material surface and $z$ is the distance from the surface into the material.

At a depth of $z = 3/\alpha$ from the surface of the material, Eq. (1) predicts that the laser irradiation is approximately 5% of the initial value. As a result, for our investigation, we estimate that the thickness of the layer heated by the CO\textsubscript{2} laser is $\tau = 3/\alpha$.

According to values reported in the literature \cite{2, 3}, the absorption coefficient $\alpha$ for bone is roughly 3300 cm\textsuperscript{-1}, which means $\tau \sim 9 \mu$m at $\lambda = 10.6 \mu$m.

The scattering coefficient reported in the literature for hard bones in the mid-infrared from 2 $\mu$m to 10.6 $\mu$m is on the order of 10 cm\textsuperscript{-1} \cite{21, 22} or is too small to be measured \cite{23, 24} at $\lambda = 10.6 \mu$m. As the scattering coefficients reported for hard tissues is very small compared to the absorption coefficient $\alpha$, the scattering has been neglected in Eq. (1) and is not taken into account in the mathematical model.

3. Experimental results

The bones were placed as shown in the experimental set-up illustrated in Fig. 1. The cancellous cross-section of the bone is facing towards the incident CO\textsubscript{2} laser beam as shown in Fig. 2. A computer-controlled procedure was used to heat our bone samples at constant temperature. To maintain a bone sample at $T \sim 50^\circ$C in this procedure, a series of CO\textsubscript{2} laser pulses at a modulation frequency of 5 kHz were delivered to the sample at a power of about 7.4 W during 15 seconds. The power was set to a value between 7.35 W and 12.6 W so the sample temperature could ramp-up between 55$^\circ$C and 85$^\circ$C at a modulation frequency of 5 kHz. This phase is shown as step A in Fig. 3 for a sample maintained near 52$^\circ$C.

Immediately following the first series of pulses at 5 kHz, a second series of pulses were delivered to the samples at double the modulation frequency (10 kHz) at the same power or at a slightly different power. During the short period of time while the modulation frequency was changed from 5 kHz to 10 kHz, the laser was turned off. This is seen as a small dip in temperature in Fig. 3 at the beginning of step B. The series of pulses at 10 kHz was applied so that the bone would recover its temperature loss during the very short period of time it was turned off seen in the later section of step B in Fig. 3.

The frequency of the pulses was changed to 20 kHz in order to keep the temperature reasonably uniform in time. This is shown in step C in Fig. 3. A similar method was also shown to be successful in the heating a liquid such as water \cite{17}.

Immediately following the series of pulses at 20 kHz, the laser was turned off and the bone samples were left to cool (this is shown in step D in Fig. 3.)

In Fig. 4 we present the results of heating porcine bones using the method that we outlined in Fig. 3. We note that the bone specimens were kept at given temperature during step C.
Fig. 2. Cylindrical porcine bone samples. a) Bone sample with finite length $L$ used during the experimental investigations. b) Semi-infinite bone assumed in the mathematical model.

Fig. 4 we note that temperature can be controlled at fairly fixed temperature (step C) using different condition during the heating procedure. Note in Fig. 4 that the temperature of the bone cross-sectional area is increasing quickly to reach a constant value within $55^\circ C - 85^\circ C$ range depending on the power delivered by the laser during step A. The laser is turned off for a short period of time, typically 10ms immediately after step A to change the modulation frequency from 5 kHz to 10 kHz. During this short period of time, the bone cools off and the temperature drops just before the laser is turned on at its higher frequency of 10 kHz. Some decreases in temperature can be seen in Figs. 4(c), 4(d) and 4(f) after 15s (end of step A), 40s (end of step B) and 25s (end of step B) respectively, when the laser frequency was changed. The interval between each data point shown in all graphs in Fig. 4 is one second.

A shorter pause could be implemented by the routine when the modulation frequency is changed from 10 kHz to 20 kHz. The modulation frequency was changed to 20 kHz and the same laser power was maintained, we note that the temperature of the bone specimens is nearly constant during step C. In addition, we note that step C lasts for ∼60 s in Figs. 4(a) and 4(b), for ∼90 s in Figs. 4(c) and 4(d) and for more than 100 s in Figs. 4(d) and 4(e). We also note that the decrease in temperature is rather modest in Figs. 4(a) to 4(e) when the laser frequency is changed. However, when the frequency of the pulses is changed from 10 kHz to 20 kHz in Fig. 4(f), we observe a significant decrease in the temperature. This large decrease in temperature in Fig. 4(f) may be due to the temperature gradient between the bone sample surface (∼85°C) and the ambient air (∼22°C).
Fig. 3. Four step procedure to heat the spongy part of the bone. In step A of this process, the bone specimen is rising its temperature rapidly within 15s. The specimen temperature is recorded for a few seconds just before the laser is turned on. During step A, the laser power is typically around 8 W and its modulation frequency is 5 kHz. A small drop in temperature is observed after step A due to a short pause period in the routine to switch the laser modulation frequency. In step B, the laser is pulsed at a modulation frequency of 10 kHz for a period of 10 seconds at a laser power slightly smaller than in step A. In step C, the laser is pulsed at a modulation frequency of 20 kHz for a period varying from 60 to 150 s. When the modulation frequency changes from 10 kHz to 20 kHz another sudden drop in temperature is observed at the coding outset of step C. Then the laser is turned off and a few points are being recorded during this period (step D).

4. Mathematical model

The porcine bone specimens were modeled using a short cylinder with a height of $L$ and a diameter $d$, as shown in Fig. 2. Each specimen is exposed to the laser beam along the cylindrical face facing up as shown in Fig. 2(a). Since the length $L$ of each cylindrical specimen is within 7-10 mm, it is much greater than $\tau$ and as a result the specimen can be modeled as a semi-infinite cylinder as shown in Fig. 2(b). The cylinder is assumed to be axisymmetric and homogeneous with constant physical properties. The standard 1-D heat conduction equation can be written as [18]

$$\frac{1}{\kappa} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2}$$

(2)

where $\kappa = k/ (\rho c)$ is the thermal diffusivity (m$^2$/s), $\rho$ is the mass density (kg/m$^3$), $c$ is the specific heat (J/kg/K) and $k$ is the thermal conductivity (W/m/K).

A finite difference solution [19] of the heat conduction equation was derived. The boundary conditions at the surface of the specimen are determined using the surface energy balance [19].
Fig. 4. The same as in Fig. 3 except that the power of the laser and the CO\textsubscript{2} sweeping frequency were (a) 7.7 W (step A for 20 s) and 7.4 W (steps B for 10 s and C for 30 s). Optical scanner operating at 50 Hz during all steps, (b) 8.8 W (steps A for 15 s and B for 95 s, no step C). Optical scanner operating at 80 Hz for all steps, (c) 8.1 W (steps A for 10 s and B for 20 s) and 7.7 W (step C for 120 s). Optical scanner operating at 100 Hz during all steps, (d) same as (c) except that step C lasted for 100 s, (e) 8.8 W (steps A and B for 20 s and step C for 90 s). Optical scanner operating at 100 Hz and (f) 13.7 W (step A for 20 s), 12.6 W (step B for 10 s) and 12.3 W (step C for 20 s). Optical scanner operating at 50 Hz for steps A and B while it is operating at 100 Hz for step C.
For the surface illuminated by the laser \((z = 0)\) we have a mixed boundary condition that can be written as

\[-kdT \frac{dz}{dz} - h_c(T - T_\infty) - \varepsilon\sigma(T^4 - T_\infty^4) - q_{\text{laser}} = 0\]  

(3)

where the first term is the heat transferred through the surface by conduction into the material, the second term is the heat transferred by convection, \(h_c\) is the convective heat transfer coefficient (W/m\(^2\)/K), the third term is the heat transferred by radiation (\(\varepsilon\) is the emissivity of bone, \(\sigma\) is the Stefan-Boltzmann constant, \(T_\infty\) is the temperature of the ambient air) and finally, \(q_{\text{laser}}\) is the heating source due to the laser. We note from Eq. (3), that the radiation term (3rd term), makes Eq. (2) non-linear in \(T\). The mathematical model considers a solid bone and as a result, scattering from the bone was neglected in Eqs. 1 and 3.

The bone is in contact with the glass plate at \(z = L\) and we have the following boundary conditions

\[T_{\text{bone}} = T_{\text{glass}}\]  

(4)

and,

\[-k_{\text{bone}} \frac{dT}{dz} = -k_{\text{glass}} \frac{dT}{dz}\]  

(5)

The temperatures of the bone and of the glass are identical at the contact surface and the heat flux through both media are equal.

4.1. Basic assumptions

Before we can proceed to compute the temperature as a function of time by solving Eq. (2) subject to the boundary conditions described in Eqs. 3, 4 and 5, we must make a few assumptions. For our numerical study, we assumed that the laser beam had a ‘top-hat’ beam pattern and that the beam size was twice as much as the cross-sectional area of the bone. In other words, we assumed that the intensity of the beam was uniform over the entire surface of the bone.

The initial temperature at \(t = 0\) of the specimen was assumed to be uniform and equal to the temperature of the ambient air namely, \(T_\infty = 25^\circ\text{C}\). We assumed that the bone was homogeneous and that it’s thermal properties were constant throughout the material. We also assumed that the emissivity of the bone was \(\varepsilon = 1\) [20, 25, 26].

The thermal properties for cancellous bone were taken from [16] and were used in the numerical simulations that are presented in Figs. 5 and 6. According to [16], a thermal conductivity of \(k = 0.31\text{ W/m/K}\), a mass density of \(\rho = 1178\text{ kg/m}^3\) and a heat capacity of \(c = 2274\text{ J/kg/K}\) results in a value of \(\kappa = 1.1572 \times 10^{-7}\text{ m}^2/\text{s}\) for the thermal diffusivity of cancellous bone.

4.2. Numerical results

In Fig. 5, we present the temporal variation of the surface temperature obtained from our numerical simulations for the first 3 laser heating pulses. We find that the temperature rises rapidly at the outset of the heating pulse and when the laser is turned off the temperature profile decreases slightly. This can be seen as a "ramped step function". For larger values of the duty cycle we note that the temperature increases slightly more than for the cases when the duty cycle is smaller. Nevertheless, in all cases, the cooling is not sufficient to return the surface to its original temperature and as a result the surface temperature continues to increase as time progresses. These results are consistent with those found in [27, 28].

In Fig. 6(a), we present the temporal variation of the surface temperature obtained from Eq. (2) for 100 seconds for 4 different values of the convective heat transfer coefficient, the specific heat was assigned a value of \(c = 2274\text{ J/kg/K}\) [16]. In this figure we simulated a laser that was turned off for 1 s while the laser frequency was being changed from 5 kHz to 10 kHz at \(t = 20\text{s}\) and from 10 kHz to 20 kHz at \(t = 40\text{s}\). This can be seen as deep minimums.
in the surface temperatures at these transition times. We observe that as \( h_c \) is reduced, the temperature increases rapidly as a function of time. This would be consistent with the notion that as less heat is removed from the surface due to convection, the hotter the surface becomes as it absorbs energy from the laser. For the largest value of \( h_c = 200 \text{W/m}^2/\text{K} \), we note that as time progresses the temperature begins to flatten out. This can also be seen in Fig. 6(b), in which we plot the time derivative of the surface temperature as a function of time. As time progresses we note that the time derivative of the surface temperature for all values of \( h_c \) approaches 0 indicating that the surface of the bone specimen is approaching a constant temperature.

One of the main difficulties that we encountered while modeling the bone sample is that there does not seem to be any consensus in the literature on the value of the specific heat for either cancellous or cortical bones. Values ranging from \( 1.15 \times 10^3 \text{J/kg/°C} \) have been reported by [29–33] up to values as high as \( 2.274 \times 10^3 \text{J/kg/°C} \) [16]. In a study by [34], they showed that the specific heat of bovine femurs increases linearly as a function of temperature and depends on its mineral composition as well as its water content.

We present Fig. 7 which depicts the temporal variations of the surface temperature obtained using Eq. (2) for the experimental parameters shown in Fig. 4(a) namely, 7.7 W for \( 20s < t < 40s \) (step A) and 7.4 W for \( 40s < t < 80s \) (for steps B and C) and turned off when \( t > 80s \) for different values of specific heat. In the first two cases, the specific heat was set to a constant value of \( c = 2.274 \times 10^3 \text{J/kg/°C} \) (solid black trace) [16] and \( c = 1.3 \times 10^3 \text{J/kg/°C} \) (solid blue trace) [31].
Fig. 6. Top panel shows the numerical results for temporal variation of the average temperature within $\tau$ using Eq. (2) for $h_c = 5$ W/m$^2$/K (black trace), $h_c = 50$ W/m$^2$/K (blue trace), $h_c = 100$ W/m$^2$/K (red trace) and $h_c = 200$ W/m$^2$/K (green line) for a laser power of 70 W for a duty cycle (DC) of 22% for $0 < t < 20s$, 15% for $20s < t < 40s$ and 15% for $40s < t < 100s$. Bottom panel shows temporal variation of the time derivative of the surface temperatures for the same values of $h_c$ as the top panel.

For the other two cases we assumed that the specific heat increases linearly as a function of temperature: $c = 200 + 10T$ (blue dash-dot trace) and $c = 100 + 10T$ (black dash-dot trace). Many other linear models for the heat capacity were tried but these seemed to give the best results for our simplified model.

We remind the reader that for all of the cases that are presented in Fig. 7, the convective coefficient $h_c$, and thermal conductivity $k$, were held to constant values of 100 W/m$^2$/K and 0.31 W/m/K, respectively. The initial temperature that was used for all of these cases was obtained from the observed values. As a result, the initial temperature in our model was set at $T_\infty = 29^\circ$C which is the same initial temperature as in Fig. 4(a). Lastly, in order to facilitate the comparison between the computed temperatures and those that were measured using the IR sensor, we superimposed the measured temperatures shown as red triangles in Fig. 7.

We observe from Fig. 7 that for all of the 4 cases presented, the surface temperature increases as a function of time. In the first case, where $c = 2.274 \times 10^3$ J/kg/°C (black solid line), the temperature rises to $T \sim 47^\circ$C at 80s. We also note that the rate at which the temperature increases at the outset of the laser heating, $20s < t < 30s$, does not agree very well with the rate with which the observed values are seen to increase (red triangles). The observed temperatures

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show a significant jump from the initial temperature of \( T = 29^\circ C \) to \( T \sim 47^\circ C \) in approximately 1 s. Our simplified model is not able to replicate this abrupt change in temperature for this particular value of heat capacity.

In the second case when \( c = 1.3 \times 10^3 \text{J/kg/}^\circ \text{C} \) (solid blue trace) [31] we notice that the temperature increases in a similar fashion as in the first case but it reaches a slightly higher value of \( T \sim 50^\circ C \) at 80 s. We further note that the agreement between the computed and the observed rate of change in the temperature at the outset of the laser heating improves somewhat when a smaller value for the specific heat is used in our model however it still does not quite capture the measured trend.

In the next two cases, we assumed that the specific heat vary as linear functions of temperature. In the third case we assumed that the heat capacity was described as \( c = 100 + 10T \) (black dash-dot trace) while in the fourth case the heat capacity was described as \( c = 200 + 10T \) (blue dash-dot trace). In both these cases we observe that the rate at which the computed temperature increase at the outset heating agrees much better with those that were measured. We further note that in both of the last two cases, the computed temperatures converge towards the measured temperature. The differences between the computed temperature and the measured temperature are less than \( \sim 10\% \) for 30 s < \( t < 80 \) s.
From these 4 cases we can see that the numerical results rely significantly on the value of the specific heat that we assign to the bone in the model. It appears that a better agreement with the observed temperatures is obtained if we assume that the specific heat varies as a linear function of temperature.

In all of the cases that are presented in Fig. 7, we note that the simulated surface temperature decreases slightly when the laser beam modulation frequency is changed from 5 kHz to 10 kHz and from 10 kHz to 20 kHz. To model the effect of changing the modulation frequency in our simulations, we assumed that the laser was turned off for 0.1 s while the laser modulation frequency was changed. This can be seen as sharp decreases in the temperatures at the time when the frequencies were changed namely at \( t = 40 \) s and \( t = 50 \) s. Although one could easily argue that our model is much too simplified, Fig. 7 shows that our model gives reasonable results in comparison to the measured temperatures. This suggests that our model could be a useful tool to obtain a first order prediction of the average temperature within \( \tau \) of bones.

5. Conclusion

It was shown experimentally that the surface temperature of porcine bones can be controlled to a constant temperature within 1 or 2 degrees within the 50–85°C range. This small temperature range of less than 2°C would allow a good enough resolution for bone treatment during exposure for at least one minute.

A very simplified model of the bone structure was used in this study. We assumed that the bone was a uniform axisymmetric cylinder possessing the thermal parameters equivalent to those of cancellous bone. The composition of the bone, its porosity, the fact that bones are not uniform as well as the effect of the beam scanning across the bone sample could affect the computed temperatures however these were not considered in the model. Nevertheless, using our very simplified mathematical description which included heat transfer by conduction, convection and radiation to model the temporal variation of the temperature obtained results that were within \( \sim 10\% \) of the observed values.

The model predicts that the temperature reaches a near constant value \( \sim 50\text{°C} \) for \( h_c = 100\text{W/K} \) when the specific heat of the bone is permitted to vary linearly with temperature in the model(see Fig. 7). Furthermore, the agreement between the observed and the computed rates of change in temperature at the outset of the laser heating improves significantly when the specific heat is allowed to vary as a function of temperature in the model.

A more sophisticated model that takes into account the change of \( h_c \) and \( k \) as functions of the bone surface temperature as suggested in [13] as well as changes in the composition, the porosity and the sweeping of the CO\(_2\) laser beam may improve the mathematical predictions of the average temperature.