Josephson junction parametric amplifiers are playing a crucial role in the readout chain in superconducting quantum information experiments. However, their integration with current 3D cavity implementations poses the problem of transitioning between waveguide, coax cables and planar circuits. Moreover, Josephson amplifiers require auxiliary microwave components, like directional couplers and/or hybrids, that are sources of spurious losses and impedance mismatches that limit measurement efficiency and amplifier tunability. We have developed a wireless architecture for these parametric amplifiers that eliminates superfluous microwave components and interconnects. This greatly simplifies their assembly and integration into experiments. We present an experimental realization of such a device operating in the 9 – 11 GHz band with about 100 MHz of amplitude gain-bandwidth product, on par with devices mounted in conventional sample holders. The simpler impedance environment presented to the amplifier also results in increased amplifier tunability.

Josephson junction parametric amplifiers are playing a crucial role in the readout chain in superconducting quantum information experiments. However, their integration with current 3D cavity implementations poses the problem of transitioning between waveguide, coax cables and planar circuits. Moreover, Josephson amplifiers require auxiliary microwave components, like directional couplers and/or hybrids, that are sources of spurious losses and impedance mismatches that limit measurement efficiency and amplifier tunability. We have developed a wireless architecture for these parametric amplifiers that eliminates superfluous microwave components and interconnects. This greatly simplifies their assembly and integration into experiments. We present an experimental realization of such a device operating in the 9 – 11 GHz band with about 100 MHz of amplitude gain-bandwidth product, on par with devices mounted in conventional sample holders. The simpler impedance environment presented to the amplifier also results in increased amplifier tunability.

Josephson junction parametric amplifiers are playing a crucial role in the readout chain in superconducting quantum information experiments. However, their integration with current 3D cavity implementations poses the problem of transitioning between waveguide, coax cables and planar circuits. Moreover, Josephson amplifiers require auxiliary microwave components, like directional couplers and/or hybrids, that are sources of spurious losses and impedance mismatches that limit measurement efficiency and amplifier tunability. We have developed a wireless architecture for these parametric amplifiers that eliminates superfluous microwave components and interconnects. This greatly simplifies their assembly and integration into experiments. We present an experimental realization of such a device operating in the 9 – 11 GHz band with about 100 MHz of amplitude gain-bandwidth product, on par with devices mounted in conventional sample holders. The simpler impedance environment presented to the amplifier also results in increased amplifier tunability.

Josephson junction parametric amplifiers are playing a crucial role in the readout chain in superconducting quantum information experiments. However, their integration with current 3D cavity implementations poses the problem of transitioning between waveguide, coax cables and planar circuits. Moreover, Josephson amplifiers require auxiliary microwave components, like directional couplers and/or hybrids, that are sources of spurious losses and impedance mismatches that limit measurement efficiency and amplifier tunability. We have developed a wireless architecture for these parametric amplifiers that eliminates superfluous microwave components and interconnects. This greatly simplifies their assembly and integration into experiments. We present an experimental realization of such a device operating in the 9 – 11 GHz band with about 100 MHz of amplitude gain-bandwidth product, on par with devices mounted in conventional sample holders. The simpler impedance environment presented to the amplifier also results in increased amplifier tunability.

Josephson junction parametric amplifiers are playing a crucial role in the readout chain in superconducting quantum information experiments. However, their integration with current 3D cavity implementations poses the problem of transitioning between waveguide, coax cables and planar circuits. Moreover, Josephson amplifiers require auxiliary microwave components, like directional couplers and/or hybrids, that are sources of spurious losses and impedance mismatches that limit measurement efficiency and amplifier tunability. We have developed a wireless architecture for these parametric amplifiers that eliminates superfluous microwave components and interconnects. This greatly simplifies their assembly and integration into experiments. We present an experimental realization of such a device operating in the 9 – 11 GHz band with about 100 MHz of amplitude gain-bandwidth product, on par with devices mounted in conventional sample holders. The simpler impedance environment presented to the amplifier also results in increased amplifier tunability.
The tunable inductance of the SQUID is \( L \). The WJA circuit consists of a SQUID shunted by a split parallel-plate capacitor (blue box) fabricated on a sapphire chip. The pads that were each \( 2 \times 5 \) mm long and \( 0.25 \) mm wide were chosen. Moreover, because the coupling is a flat function of \( d \) (to first order) around \( d = (2n + 1)\lambda_g/4 \), the design is robust against errors in fabrication or machining that affect the resonator frequency or its position from the wall.

Although \( Q = 100 \) was chosen for this particular device, as shown in Fig. 2(a), the \( Q \) can be tuned by over three orders of magnitude by choosing \( l \) between \( 0.5 \) mm and \( 5 \) mm. While the \( Q(l) \) dependence can be calculated analytically, a finite element electromagnetic solver, such as HFSS[29], was used for a more detailed analysis. These simulations confirmed that the \( Q \propto l^{-2} \) behavior predicted by Eqn. 1 for short antenna lengths breaks down for longer antenna lengths when the size of the dipole antenna becomes comparable to the size of the waveguide. The behavior of the coupling as a function of the chip distance from the wall, \( d \), was also simulated; the coupling should be maximum when the chip is at an antinode of the electric field in the waveguide and minimum when it is at a node. As shown in Fig. 2(b), the simulations confirm the expected divergence in the \( Q \) when \( d = n\lambda_g/2 \) where \( n \) is an integer and \( \lambda_g \) is the guide wavelength[30]; the \( Q \) diverges at these points because \( Z_L(d = n\lambda_g/2) = 0 \). Moreover, the coupling is a flat function of \( d \) (to first order) around \( d = (2n + 1)\lambda_g/4 \), the design is robust against errors in fabrication or machining that affect the resonator frequency or its position from the wall.

Fabricating the lumped-element circuit for the WJA required making both parallel-plate capacitors and a Josephson junction on chip. The same three-layer fabrication process developed for conventional JBAs[7, 11, 31] was used to avoid the complication of making vias. First, a 150 nm niobium layer was deposited on the wafer and patterned using optical lithography and reactive-ion etching to define the shared ground plane of the split parallel-plate capacitors. Next, 200 nm silicon nitride was deposited conformally over the entire wafer using plasma-enhanced chemical vapor deposition to form the dielectric layer of the capacitors; the thickness and dielectric constant \( (\epsilon_r = 7) \) of this layer are used to calculate the size of the capacitors needed for a 3.5 pF capacitance. Finally, the circuit parameters were chosen to be \( I_0 = 4 \) \( \mu \)A and \( C = 3.5 \) pF for a maximum linear resonance frequency around 9.5 GHz; in addition, to make the amplifier bandwidth larger than the typical qubit cavity bandwidth, we chose \( Q = 100 \) which required two antenna pads that were each \( 2.5 \) mm long and \( 0.25 \) mm wide separated by \( 150 \) \( \mu \)m gaps, sizes that can be readily fabricated using either optical or electron-beam lithography.

\[
Z_L(d) = \sqrt{\frac{L + L_{stray}}{C}}
\]

where \( Z_L(d) \) is the impedance seen by the resonator which is a function of the chip distance from the wall, \( d \), \( Z_c = \sqrt{L + L_{stray}}/C \) is the characteristic impedance of the resonator, where \( L_{stray} \) is the stray inductance in the circuit. Thus, the resonator \( Q \) can be tuned controllably by changing \( C_c \), which in turn is proportional to the length of the dipole antenna, \( l \), in the limit of short lengths. To realize an amplifier designed for qubit read-out, the circuit parameters were chosen to be \( I_0 = 4 \) \( \mu \)A and \( C = 3.5 \) pF for a maximum linear resonance frequency around 9.5 GHz; in addition, to make the amplifier bandwidth larger than the typical qubit cavity bandwidth, we chose \( Q = 100 \) which required two antenna pads that were each \( 2.5 \) mm long and \( 0.25 \) mm wide separated by \( 150 \) \( \mu \)m gaps, sizes that can be readily fabricated using either optical or electron-beam lithography.
a bi-layer of electron-beam resist was spun on the wafer and patterned using electron-beam lithography. Double angle evaporation of aluminum was used to define the Josephson junctions in the 8 µm × 2 µm SQUID loop as well as the top plates of the capacitors and the coupling antenna.

The WJAs were cooled down in a cryogen-free dilution refrigerator to about 50 mK. First, the amplifier’s linear resonance frequency and bandwidth were measured by looking at the reflected phase using a vector network analyzer (VNA). The reflected phase (Fig. 3(a) inset) showed a 360° phase roll characteristic of an overcoupled resonator. From this, the coupling was found to be Q = 100 as desired and agreeing with simulation results. Next, by applying current to an external superconducting coil magnet, the linear frequency of the WJA was tuned with flux. As shown in Fig. 3(a), the linear resonance frequency tuned from its maximum value of 11.4 GHz down to 9 GHz. By fitting the resonance frequency as a function of flux to the equation:

\[
f(\Phi) = \frac{1}{2\pi \sqrt{C L_J \left( \frac{L_{stray}}{L_J} + \frac{1}{\cos \frac{\pi \Phi}{\Phi_0}} \right)}}
\]

the circuit parameters were found to be \( C = 1 \) pF, \( L_{stray} = 120 \) pH and \( I_0 = 4.6 \) µA with error bars of about 10%. The discrepancy in the measured capacitance from the desired value is a result of operating the circuit near the self-resonance frequency of the capacitor; in future iterations of the WJA, it would be advantageous to use thinner dielectric layers and/or larger dielectric constants to avoid this problem. Nevertheless, the device still operated in the desired frequency range. More importantly, as shown in Fig. 3(b), the Q varied by only 10% between 9 GHz and 11 GHz. Above 11 GHz, the Q changed rapidly because at these higher frequencies, the chip distance from the wall is significantly different from \( \lambda_g/4 \). The almost constant Q as a function of frequency not only agrees well with simulation results but also suggests that the environment’s impedance is well matched over the whole frequency range. Because the Q of the WJA can be varied by changing the antenna length independent of the device frequency, this architecture offers a valuable control knob over the amplifier bandwidth that is unavailable in conventional JBAs.

Another important figure of merit for single effective-junction parametric amplifiers is the inductance participation ratio, \( p = L_{stray}/(L_J + L_{stray}) \). Using the the extracted values of \( L_{stray} \) and \( L_J \), the \( p \) was found to vary between about 0.6 and 0.4 over the entire frequency range (see blue trace in Fig. 3(b)). Together the Q and \( p \) are important because their product determines both whether the resonator will amplify and the maximum signal power that can be amplified without saturation[19]. To realize an amplifier with sufficient gain for qubit readout, it is necessary that \( Qp \gtrsim 5 \); however, to maximize the amplifier saturation power, the \( Qp \)-product needs to be as low as possible with \( Qp \approx 5 \) to 10 being optimal[19]. In particular, for the present device, the \( Qp \) product was around 50 by design. Since \( Qp \) varies by only about 20%
over the whole frequency range (see Fig. 3(c)), this device is expected to show tunability and constant amplifier performance over the operation band. Combined with the increased control over the Q and p in this architecture, this will allow for fuller optimization of the amplifier’s properties in future iterations.

The WJA was characterized as a phase-sensitive amplifier by applying two pump tones, \( \omega_{p1} \) and \( \omega_{p2} \), symmetrically detuned by 500 MHz from the signal tone at \( \omega_s \) through the weakly coupled pump port. Different gains at the same frequency were achieved by keeping the flux through the SQUID constant while changing the two pump powers. As shown in Fig. 4(a), the amplifier achieved gains between unity and over 25 dB. In particular at 20 dB of gain, the instantaneous amplifier bandwidth is 13 MHz (much larger than typical cavity bandwidths used in qubit experiments).

The amplifier’s dynamic range was characterized by measuring its maximum gain as a function of input signal power from the VNA and finding the power at which the gain fell by 1 dB (the \( P_{-1dB} \) power); at 20 dB of gain, we found that the saturation power was \(-132\) dBm which corresponded to approximately 0.7 photons in the 13 MHz amplifier bandwidth (see Fig. 4(b)). Additionally, the slope of \( P_{-1dB} \) power versus gain (orange line in Fig. 4(b)) was \(-1.2\) dB/db, close to the ideal slope of \(-1\) dB/db expected for an ideal linear parametric amplifier[16]. The dynamic range of this device, while comparable to conventional JBAs, could be further increased by reducing the \( Qp \)-product by a factor of ten to \( Qp \approx 5 \); while this could be done either by reducing the \( Q \) or the \( p \), the current \( Q \approx 100 \) is optimal for qubit readout and so reducing the participation ratio is desirable. This can be achieved, for example, by using multiple SQUIDs with larger \( I_0 \) and hence a smaller \( L_j \).

A final figure of merit, especially for parametric amplifiers, is the amplifier’s noise temperature, \( T_N \). According to Caves[32], when the WJA is operated as a phase-sensitive parametric amplifier, the added noise is zero for the amplified quadrature (\( T_N = 0 \)) while \( T_N = \infty \) for the de-amplified quadrature. The noise temperature for this amplifier was estimated from the increase in the noise measured by a room temperature spectrum analyzer when the WJA was turned on. When the WJA was off, i.e \( G = 0 \), the noise measured at room temperature is entirely dominated by noise added by the high electron mobility transistor (HEMT) amplifier. On the other hand, when the WJA is turned on, the measured noise increases because the spectrum analyzer now also receives amplified quantum noise from the parametric amplifier. We call the noise visibility ratio the amount by which the combined noise of the WJA and the HEMT is greater than the HEMT noise alone.

As shown in Fig. 4(c), the noise visibility increased with WJA gain; when \( G = 25 \) dB (blue trace), the ratio was about 8.5 dB indicating that less than 15% of

---

**FIG. 3.** Figure 3 — Q and Participation Ratio versus Resonator Frequency. (a) Resonance Frequency versus Flux: The reflected phase (inset) is plotted as a function of frequency revealing a 360° degree phase roll characteristic of an over-coupled resonator. Changing the flux allows this resonance frequency to be tuned over 2 GHz. By fitting the resonance frequency as a function of the flux through the SQUID, the circuit parameters of the device are extracted to be \( I_0 = 4.6 \mu A, C = 1 \) pF and \( L_{stray} = 120 \) pH. B) Q and p versus Frequency: A plot of the resonator Q as a function of the resonance frequency shows that \( Q \approx 100 \), varying by around 10% over most of the frequency range except above 11 GHz where it rapidly increases by around 40%; this is because at these higher frequencies, the chip location is significantly different from the electric field anti-node. On the other hand, the participation ratio varies smoothly with resonance frequency; at the highest resonator frequency, the participation ratio is the smallest since \( L_j \) is at its maximal value. (b) Qp Product versus Frequency: Over the range of operation, the Qp product is relatively flat varying by about 20% above and below its average value of \( Qp = 50 \).
The power at room temperature was added HEMT noise. The dips in the traces at the center frequency are a consequence of the WJA behaving as a phase-sensitive amplifier at \( \omega_s \). Since only one of the two quadratures is amplified, the measured noise should be 3 dB lower. However, the finite resolution bandwidth of the spectrum analyzer smears out the dip and consequently, the measured dips were smaller than the expected 3 dB (see inset of Fig. 4(c)) and have a width of 2.5 MHz equal to the spectrum analyzer’s resolution bandwidth. At \( G = 20 \) dB, the difference in noise power of 4.5 dB was comparable to other parametric amplifiers, JBAs or JPCs, measured in this experimental setup indicating that the noise performance of the WJA is comparable to standard paramps. For this measurement setup, the system noise temperature, \( T_{sys}^H \), which is the effective noise temperature of the HEMT and the losses between the plane of the WJA and the HEMT, is assumed to be 35 K. From the measured increase of 4.5 dB at \( G = 20 \) dB, we estimate an upper bound on the noise added by the WJA of \( T_N \leq 2.5T_Q \), where \( T_Q = \frac{\hbar \omega_s}{2k_B} \).

In conclusion, we have designed and operated a wireless implementation of a Josephson parametric amplifier for readout of a superconducting qubit in a 3D cavity. Our design replaces the usual connections between the chip of the amplifier and the qubit port by a simple on-chip antenna placed in the center of a waveguide. This leads to a much simplified assembly of critical components. The WJA exhibits the same gain, bandwidth, dynamic range and noise properties as a conventional JBA while also offering improved tunability and increased control over both the quality factor \( Q \) and participation ratio \( p \). With this better control, it is possible to increase the amplifier dynamic range with a WJA consisting of multiple SQUIDs in series with correspondingly large critical current junctions. The simplification of the microwave environment seen by the Josephson element should also eliminate sources of loss that currently limit the measurement efficiency of circuit QED experiments. The incorporation of a WJA in a qubit readout chain is currently under way. Finally, this architecture could be easily extended to realize a wireless JPC by adding dipole antenna to a lumped JPC circuit and placing it at the intersection of two waveguides with perpendicular polarizations.

Fabrication assistance from M. Rooks, as well as the Yale Institute for Nanoscience and Quantum Engineering facilities are gratefully acknowledged. This research was supported by the National Science Foundation under Grants No. DMR-1066060 and No. DMR-0653377, the US Army Research Office Grants No. W911NF-09-1-0514 and No. W911NF-14-1-0011.
[1] S. Wahlsten, S. Rudner, and T. Claeson, Journal of Applied Physics 49, 4248 (1978).
[2] J. Mygind, N. F. Pedersen, O. H. Soerensen, B. Dueholm, and M. T. Leivinsen, Applied Physics Letters 35, 91 (1979).
[3] B. Yurke, L. R. Corruccini, P. G. Kaminsky, L. W. Rupp, A. D. Smith, A. H. Silver, R. W. Simons, and E. A. Whittaker, Phys. A 39, 2519 (1989).
[4] I. Siddiqi, R. Vijay, F. Pierre, C. M. Wilson, M. Metcalfe, C. Rigetti, L. Frunzio, and M. H. Devoret, Phys. Rev. Lett. 93, 207002 (2004).
[5] I. Siddiqi, R. Vijay, F. Pierre, C. M. Wilson, L. Frunzio, M. Metcalfe, C. Rigetti, R. J. Schoelkopf, M. H. Devoret, D. Vion, and D. Esteve, Phys. Rev. Lett. 94, 027005 (2005).
[6] M. A. Castellanos-Beltran and K. W. Lehnert, Applied Physics Letters 91, 083509 (2007).
[7] R. Vijay, Josephson Bifurcation Amplifier, Ph.D. thesis, Yale University (2008).
[8] T. Yamamoto, K. Inomata, M. Watanabe, K. Matsuba, T. Miyazaki, W. D. Oliver, Y. Nakamura, and J. S. Tsai, Applied Physics Letters 93, 042510 (2008).
[9] M. A. Castellanos-Beltran, K. D. Irwin, G. C. Hilton, L. R. Vale, and K. W. Lehnert, Nat. Phys. 4, 929 (2008).
[10] A. Kamal, A. Marblestone, and M. Devoret, Phys. Rev. B 79, 184301 (2009).
[11] M. Hatridge, R. Vijay, D. H. Slichter, J. Clarke, and I. Siddiqi, Phys. Rev. B 83, 134506 (2011).
[12] J. Y. Mutus, T. C. White, E. Jeffrey, D. Sank, R. Barends, J. Bochmann, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, J. Kelly, A. Megrant, C. Neill, P. J. O’Malley, P. Roushan, A. Vainsencher, J. Wenner, I. Siddiqi, R. Vijay, A. N. Cleland, and J. M. Martinis, Applied Physics Letters 103, 122602 (2013).
[13] L. Zhong, E. Menzel, R. Di Candia, P. Eder, M. Ihmig, A. Baust, M. Haeberlein, E. Hoffmann, K. Inomata, T. Yamamoto, Y. Nakamura, E. Solano, F. Deppe, A. Marx, and R. Gross, New Journal of Physics 15, 125013 (2013).
[14] N. Bergeal, R. Vijay, V. E. Manucharyan, I. Siddiqi, R. J. Schoelkopf, S. M. Girvin, and M. H. Devoret, Nat. Phys. 6, 296 (2010).
[15] N. Bergeal, F. Schackert, M. Metcalfe, R. Vijay, V. E. Manucharyan, L. Frunzio, D. E. Prober, R. J. Schoelkopf, S. M. Girvin, and M. H. Devoret, Nature 465, 64 (2010).
[16] B. Abdo, F. Schackert, M. Hatridge, C. Rigetti, and M. Devoret, Appl. Phys. Lett. 99, 162506 (2011).
[17] N. Roch, E. Flurin, F. Nguyen, P. Morfin, P. Campagne-Ibarcq, M. H. Devoret, and B. Huard, Phys. Rev. Lett. 108, 147701 (2012).
[18] B. Abdo, A. Kamal, and M. H. Devoret, Phys. Rev. B 87, 014508 (2013).
[19] F. Schackert, A Practical Quantum-Limit Parametric Amplifier Based on the Josephson Ring Modulator, Ph.D. thesis, Yale University (2013).
[20] R. Vijay, D. H. Slichter, and I. Siddiqi, Phys. Rev. Lett. 106, 110502 (2011).
[21] M. Hatridge, S. Shankar, M. Mirrahimi, F. Schackert, K. Geerlings, T. Brecht, K. M. Sliwa, B. Abdo, L. Frunzio, S. M. Girvin, R. J. Schoelkopf, and M. H. Devoret, Science 339, 178 (2013).
[22] K. W. Murch, S. J. Weber, C. Macklin, and I. Siddiqi, Nature 502, 211 (2013).
[23] R. Vijay, C. Macklin, D. H. Slichter, S. J. Weber, K. W. Murch, R. Naik, A. N. Korotkov, and I. Siddiqi, Nature 490, 77 (2012).
[24] G. de Lange, D. Ristè, M. J. Tiggelman, C. Eichler, L. Tornberg, G. Johansson, A. Wallraff, R. N. Schouten, and L. DiCarlo, arXiv:1311.5472 (2013).
[25] L. Sun, A. Petrenko, Z. Leghtas, B. Vlastakis, G. Kirchmair, K. M. Sliwa, A. Narla, M. Hatridge, S. Shankar, J. Blumoff, L. Frunzio, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, arXiv:1311.2534 (2013).
[26] M. H. Devoret and R. J. Schoelkopf, Science 339, 1169 (2013).
[27] H. Paik, D. I. Schuster, L. S. Bishop, G. Kirchmair, G. Catelani, A. P. Sears, B. R. Johnson, M. J. Reagor, L. Frunzio, L. I. Glazman, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Phys. Rev. Lett. 107, 240501 (2011).
[28] C. Rigetti, J. M. Gambetta, S. Poletto, B. L. T. Plourde, J. M. Chow, A. D. Córcoles, J. A. Smolin, S. T. Merkel, J. R. Rozen, G. A. Keefe, M. B. Rothwell, M. B. Ketchen, and M. Steffen, Phys. Rev. B 86, 100506 (2012).
[29] ANSYS HFSS version 15.1.
[30] D. M. Pozar, Microwave Engineering (John Wiley & Sons Inc., 2012).
[31] L. Frunzio, Design and fabrication of superconducting circuit for amplification and processing of quantum signals, Ph.D. thesis, Université Paris-sud (2006).
[32] C. M. Caves, Phys. Rev. D 26, 1817 (1982).