Bright soliton dynamics in Spin Orbit-Rabi coupled Bose-Einstein condensates

P. S. Vinayagam\textsuperscript{a,b}, R. Radha\textsuperscript{a,*}, S. Bhuvaneswari\textsuperscript{c}, R. Ravisankar\textsuperscript{c}, P. Muruganandam\textsuperscript{c}

\textsuperscript{a}Centre for Nonlinear Science (CeNSc), PG and Research Department of Physics, Government College for Women (Autonomous), Kumbakonam 612001, India
\textsuperscript{b}Department of Physics, United Arab Emirates University, P.O.Box 15551, Al-Ain, United Arab Emirates
\textsuperscript{c}Department of Physics, Bharathidasan University, Palkaliperur Campus, Tiruchirapalli 620024, India

Abstract

We investigate the dynamics of a spin-orbit (SO) coupled BECs in a time dependent harmonic trap and show the dynamical system to be completely integrable by constructing the Lax pair. We then employ gauge transformation approach to witness the rapid oscillations of the condensates for a relatively smaller value of SO coupling in a time independent harmonic trap compared to their counterparts in a transient trap. Keeping track of the evolution of the condensates in a transient trap during its transition from confining to expulsive trap, we notice that they collapse in the expulsive trap. We further show that one can manipulate the scattering length through Feshbach resonance to stretch the lifetime of the confining trap and revive the condensate. Considering a SO coupled state as the initial state, the numerical simulation indicates that the reinforcement of Rabi coupling on SO coupled BECs generates the striped phase of the bright solitons and does not impact the stability of the condensates despite destroying the integrability of the dynamical system.

Keywords: Coupled nonlinear Schrödinger system, Bright Soliton, Gauge transformation, Lax pair

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1. Introduction

The advent of Bose-Einstein condensates (BECs) in rubidium \cite{1} and the subsequent experimental identification of bright \cite{2,3} and dark solitons \cite{2,3} for attractive and repulsive binary interaction, respectively contributed to a resurgence in the investigation of ultra cold matter. At ultralow temperatures, the macroscopic wave function of BECs can be described by the mean field Gross-Pitaevskii (GP) equation, which is essentially a variant of the celebrated nonlinear Schrödinger (NLS) equation \cite{7}. It is worth pointing out at this juncture that the behavior of single (scalar) component BECs is influenced by the external trapping potential and binary interatomic interaction. Experimental realization of vector (or two component) BECs in which two (or more) internal states or different atoms can be populated has given a fillip to the investigation of multi-component BECs. In contrast to the single component BECs, the multi-component BECs exhibit rich dynamics by virtue of inter-species and intra species binary interaction which can be either attractive or repulsive. This extra freedom associated with multi-component BECs enables them to display novel and rich phenomenon like multidomain walls \cite{8,10}, spin switching soliton pairs (either bright-bright \cite{11,13}, dark-dark \cite{14} or bright-dark, etc) which can never be witnessed in single component BECs.

Recently, in a landmark experiment, Spielman group at NIST have engineered a synthetic spin-orbit (SO) coupling for a BEC \cite{15}. In the experiment, two Raman laser beams were used to couple a two component BEC consisting of (predominantly) two hyperfine states of \textsuperscript{87}Rb. The momentum transfer between laser beams and atoms contributes to the rich possibility of creating synthetic electric and magnetic fields. Recent investigations have explored the possibility of identifying tunable spin orbit coupled BECs \cite{16,21} with various trapping potentials and stable regimes of condensates have been observed. The identification of stripe phase in the investigation of spin orbit coupled BECs...
has only rekindled the enthusiasm in this domain of interest as the striped phase is believed to be an indication of the phase transition taking place in a spin orbit coupled BEC. Also, the influence of spin-orbit coupling in a one-dimensional BEC, particularly the interplay of the SOC, Raman coupling, and nonlinearity induced precession of the soliton’s spin has been studied recently [23]. The existence of “Striped phase”, which essentially consists of a linear combination of plane waves and this has contributed to the idea of using ultra cold atoms for the implementation of a quantum simulator [24].

At this juncture, it should be mentioned that even though nonlinear excitations like vortices [25], Skymions [26], and bright solitons [27] have been generated in a SO coupled BEC, the dynamics of SO coupled BECs in a time dependent harmonic trap governed by a two coupled GP equation has not been exactly solved analytically. In this paper, we construct the linear eigenvalue problem of SO coupled GP equation in a transient harmonic trap and show that it is completely integrable. We then generate bright solitons solutions and track their evolution in a transient harmonic trap. We observe that the addition of SO coupling contributes to the rapid oscillations of real and imaginary parts of the order parameter and this occurs at a relatively lower value of SO coupling parameter in a time independent harmonic trap compared to their counterparts in a transient trap. Tracing the evolution of the bright solitons (or the condensates) during its transition from confining to expulsive trap, we notice that the condensates collapse suddenly in the expulsive trap. By employing Feshbach resonance management, we show that one can retrieve the condensates by stretching the lifespan of confining trap. Then considering a SO coupled state as the initial state, we numerically study the impact of Rabi coupling on the condensates. The results of our investigation indicate that the reinforcement of Rabi coupling on SO coupled BECs generates striped solitons and leaves no impact on the stability of BECs. We also emphasize that all the above occurs despite the transition of the dynamical system to the nonintegrable regime.

2. The model and Lax pair

We consider a spin-orbit coupled quasi-one dimensional BEC in a parabolic trap with longitudinal and transverse frequencies $\omega_x \ll \omega_z$. Assuming equal contributions of Rashba [28] and Dresselhaus [29] SO coupling (as in the experiment of Ref. [15]), which can be described at sufficiently low temperatures by a set of coupled GP equation of the form [30, 32]:

$$i \frac{\partial \psi_1}{\partial t} = \left[ -\frac{1}{2} \frac{\partial^2}{\partial x^2} + V(x,t) - \gamma(t) \left( |\psi_1|^2 + |\psi_2|^2 \right) \right] \psi_1 + \Omega \psi_2,$$

$$i \frac{\partial \psi_2}{\partial t} = \left[ -\frac{1}{2} \frac{\partial^2}{\partial x^2} + V(x,t) - \gamma(t) \left( |\psi_1|^2 + |\psi_2|^2 \right) \right] \psi_2 + \Omega \psi_1. \quad (1)$$

In the above equation, the term $\pm i k_L \partial / \partial x$ represents the momentum transfer between the laser beams and atoms due to SO coupling, $\gamma(t)$ represents binary attractive interaction, and the linear cross coupling parameter $\Omega$ denotes Rabi coupling and $V(x,t) = \Lambda(t)^2 x^2 / 2$, where $\Lambda(t) = \omega_x / \omega_z$, is the time dependent trap frequency.

Switching off the Rabi coupling ($\Omega = 0$), and employing the following transformation

$$\psi_1(x,t) = q_1(x,t) \exp \left[ \frac{i}{2} k_L (k_L t - 2x) \right], \quad (2a)$$

$$\psi_2(x,t) = q_2(x,t) \exp \left[ \frac{i}{2} k_L (k_L t + 2x) \right]. \quad (2b)$$

Equation (1) can be written in a simpler form by eliminating the SO coupling term as

$$i \frac{\partial q_1}{\partial t} = \left[ -\frac{1}{2} \frac{\partial^2}{\partial x^2} + V(x,t) - \gamma(t) \left( |q_1|^2 + |q_2|^2 \right) \right] q_1, \quad (3a)$$

$$i \frac{\partial q_2}{\partial t} = \left[ -\frac{1}{2} \frac{\partial^2}{\partial x^2} + V(x,t) - \gamma(t) \left( |q_1|^2 + |q_2|^2 \right) \right] q_2. \quad (3b)$$

We emphasize that the model governed by equations (3) is exactly integrable if either Rabi coupling (Zeeman splitting) ($\Omega$) or SO coupling ($ik_L$) is taken into account, but not both of them [33].
The above coupled equations (3a) and (3b) admit the following Lax-pair
\[
\Phi_t + \mathcal{U}\Phi = 0,
\]
\[
\Phi_x + \mathcal{V}\Phi = 0,
\]
where \( \Phi = (\phi_1, \phi_2, \phi_3)^T \) is a three-component Jost function,
\[
\mathcal{U} = \begin{pmatrix} i\zeta(t) & U_{12} & U_{13} \\ U_{21} & -i\zeta(t) & 0 \\ 0 & 0 & -i\zeta(t) \end{pmatrix}, \quad \mathcal{V} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix}.
\]
with
\[
U_{12} = \sqrt{\gamma(t)}q_1(x, t) \exp [i\phi(x, t)],
\]
\[
U_{13} = \sqrt{\gamma(t)}q_2(x, t) \exp [i\phi(x, t)],
\]
\[
U_{21} = -\sqrt{\gamma(t)}q_1'(x, t) \exp [-i\phi(x, t)],
\]
\[
U_{31} = -\sqrt{\gamma(t)}q_2'(x, t) \exp [-i\phi(x, t)],
\]
\[
V_{11} = i\zeta(t) [c(t)x - \zeta(t)] + \frac{i}{2} \gamma(t) \left( |q_1(x, t)|^2 + |q_2(x, t)|^2 \right),
\]
\[
V_{12} = \sqrt{\gamma(t)} [c(t)x - \zeta(t)] q_1(x, t) \exp [i\phi(x, t)] + \frac{i}{2} \sqrt{\gamma(t)} |q_1(x, t)| \exp [i\phi(x, t)],
\]
\[
V_{13} = \sqrt{\gamma(t)} [c(t)x - \zeta(t)] q_2(x, t) \exp [i\phi(x, t)] + \frac{i}{2} \sqrt{\gamma(t)} |q_2(x, t)| \exp [i\phi(x, t)],
\]
\[
V_{21} = -\sqrt{\gamma(t)} [c(t)x - \zeta(t)] q_1'(x, t) \exp [-i\phi(x, t)] + \frac{i}{2} \sqrt{\gamma(t)} |q_1'(x, t)| \exp [-i\phi(x, t)],
\]
\[
V_{22} = -i\zeta(t) [c(t)x - \zeta(t)] - \frac{i}{2} \gamma(t)|q_1(x, t)|^2,
\]
\[
V_{23} = -\frac{i}{2} \gamma(t)q_1'(x, t)q_2(x, t),
\]
\[
V_{31} = -[c(t)x - \zeta(t)] \sqrt{\gamma(t)}q_2(x, t) \exp [i\phi(x, t)] + \frac{i}{2} \sqrt{\gamma(t)} |q_2(x, t)| \exp [i\phi(x, t)],
\]
\[
V_{32} = -\frac{i}{2} \gamma(t)q_1(x, t)q_2'(x, t) \exp [2i\phi(x, t)],
\]
\[
V_{33} = -i\zeta(t) [c(t)x - \zeta(t)] - \frac{i}{2} \gamma(t)|q_2(x, t)|^2.
\]
In the above, \( \phi(x, t) \equiv c(t)x^2/2 \), and \([\ldots]\) denotes differentiation with respect to \( x \). The compatibility condition \( \mathcal{U}_x - \mathcal{V}_t + [\mathcal{U}, \mathcal{V}] = 0 \) generates the SO coupled GP equation (1) without Rabi coupling (\( \Omega = 0 \)), while the spectral parameter \( \zeta(t) \) obeys the following equation:
\[
\frac{d}{dt} \zeta(t) = c(t)\zeta(t),
\]
with
\[
\lambda(t)^2 = \frac{d}{dt} c(t) - c(t)^2,
\]
and
\[
c(t) = \frac{d}{dt} \ln \gamma(t).
\]
It may be noted that a similar Riccati equation (9) has been employed to solve GP-type equations \([34–38]\). In fact, the identification of the Riccati-type equation (9) gives the first signature of complete integrability of Equation (1).
with \((\Omega = 0)\). Equation (9), which determines the parabolic potential strength, \(\lambda^2(t)\), demonstrates that it is related to the interaction strength, \(\gamma(t)\), through the integrability condition, which can be derived by simply substituting Equation (10) in Equation (9):

\[
\gamma(t) \frac{d^2}{dt^2} \gamma(t) - 2 \left( \frac{d}{dt} \gamma(t) \right)^2 + \lambda(t)^2 \gamma^2(t) = 0. \tag{11}
\]

Thus, the system of coupled GP equations (3) is completely integrable for suitable choices of \(\lambda(t)\) and \(\gamma(t)\), which are consistent with equation (11). For condensates in a time independent harmonic trap, \(\lambda(t) = \lambda_0\) (a constant), Equation (11) yields \(\gamma(t) = \gamma_0\exp(\lambda_0 t)\), where \(\gamma_0\) is an arbitrary constant.

Considering a vacuum seed solution, i.e., \(q_1(x, t) = q_2(x, t) = 0\), and employing gauge transformation approach [39,42], we obtain

\[
q_1(x, t) = 2e_1 \frac{\beta(t)}{\sqrt{\gamma(t)}} \exp\left[i\xi(x, t) - i\phi(x, t)\right] \text{sech} \theta(x, t), \tag{12a}
\]

\[
q_2(x, t) = 2e_2 \frac{\beta(t)}{\sqrt{\gamma(t)}} \exp\left[i\xi(x, t) - i\phi(x, t)\right] \text{sech} \theta(x, t), \tag{12b}
\]

where

\[
\theta(x, t) = 2\beta(t)x - 4 \int \alpha(t)\beta(t) dt + 2\delta, \tag{13a}
\]

\[
\xi(x, t) = 2\alpha(t)x - 2 \int \left[\alpha(t)^2 - \beta(t)^2\right] dt - 2\chi, \tag{13b}
\]

\[
\alpha(t) = a_0 \exp \int c(t) dt, \quad \beta(t) = b_0 \exp \int c(t) dt, \tag{13c}
\]

\[
\gamma(t) = \gamma_0 \exp \int c(t) dt, \quad \phi(x, t) = \frac{1}{2}c(t)x^2. \tag{13d}
\]

\(\delta\) and \(\chi\) are arbitrary real parameters, and \(e_1\) and \(e_2\) are coupling constants subject to the constraint \(|e_1|^2 + |e_2|^2 = 1\). A formal solution to the coupled GP equation (1), in the absence of Rabi term, can be straightforwardly written using the transformation (5). The solutions given by Equations (2) illustrate the momentum transfer between laser and atoms when a vector BEC with order parameters \(q_1\) and \(q_2\) driven by bright solitons given by Equations (12) is irradiated with a laser beam.

3. Soliton dynamics of spin-orbit-Rabi coupled BEC in a transient and time independent trap

Choosing a transient trap shown in figure (1) we show the real, imaginary and absolute value of the macroscopic order parameters \(\psi_1\) and \(\psi_2\) without and with spin orbit coupling in figures (2a) and (2b). Comparison of figures (2a) and (2b) indicates that the addition of SO coupling contributes to the rapid oscillations of the real and imaginary parts of order parameters \(\psi_1\) and \(\psi_2\). The fact that the phase of \(\psi_1\) and \(\psi_2\) oscillates with space and time as it is evident from (2) contributes to oscillating nature of spin orbit coupled real and imaginary parts of order parameter. Keeping track of the evolution of the condensates in the transient trap, we observe that the condensate collapses during time evaluation as shown in figure (3b) for \(t = 10\). In other words, the oscillating nature of real and imaginary parts of order parameter is sustained as long as the trap remains confining in nature \((\lambda(t)^2 > 0)\) and the condensates collapse as soon as the trap becomes expulsive \((\lambda(t)^2 < 0)\). In addition to the expulsive trap, the fact that the scattering length \(\gamma(t)\) is driven by \(\exp(0.02t^2)\) [by virtue of (11)] contributes to the collapse of BECs in quasi one dimension. However, by employing Feshbach resonance and manipulating the trap frequency appropriately, one can increase the longevity of the confining trap and recover the condensates as shown in figure (4). For a quasi one dimensional attractive BEC confined in a harmonic trap with frequencies \(\omega_x = 2\pi \times 20\text{Hz}\) and \(\omega_z = 2\pi \times 1000\text{Hz}\), the trap frequency becomes \((\omega_x/\omega_z)^2 = 0.0004\). The fact that the condensates are recovered for \(c(t) = 0.0004\) (see figure (4)) is consistent with the results observed recently in [27]. The stabilization of the condensates by manipulating the trapping frequency
Figure 1: Evolution of the time dependent trap for $c(t) = 0.04t$

Figure 2: Real, Imaginary parts and absolute value of the order parameters $\psi_1$ and $\psi_2$ from (2) and (12) at time $t = 0$ with $c(t) = 0.04t$: (a) without spin-orbit coupling, $k_L = 0$ and (b) with spin-orbit coupling $k_L = 8$. The other parameters are $\alpha_0 = 0.31, \beta_0 = 0.5, \gamma_0 = 2, \chi_1 = 0.5, \delta_1 = 0.2, \epsilon_1 = 0.33$ and $\epsilon_2 = \sqrt{1-\epsilon_1^2}$.

through Feshbach resonance is also numerically verified and the corresponding density profiles are shown in figure 5. It should also be emphasized that the present integrable model offers the luxury of retrieving the condensates by tuning the time dependent trap frequency through Feshbach resonance. Switching off the time dependence of the trap, one observes the same oscillatory behavior for a relatively small value of SO coupling and larger trap frequency as shown in figure 6. However, the subsequent time evolution leads to collapse of the condensates which can once again be
stabilized employing Feshbach resonance.

We then study the impact of Rabi coupling on bright solitons numerically by considering the spin orbit coupled profile in (2) at \( t = 0 \) as the initial condition. The numerical simulations are employed using a split-step Crank-Nicolson method [43–46]. The initial profile is refined to be a stationary state using imaginary time propagation, which is then evolved with real time propagation by adding the correct phase \( \xi(x, 0) \) and \( \theta(x, 0) \) from (13). Figure 7 depicts the initial profile (\( \Omega = 0 \)) from (2) at \( t = 0 \) and final profile of the stationary state as obtained from imaginary time propagation with \( \Omega = 10 \). Figures 8 and 9 show the numerically simulated density profile of the SO coupled condensates in the presence of Rabi coupling in a transient and time independent harmonic trap respectively. The presence of stripes in figures 8 and 9 is a clear signature of SO coupling consistent with the results of [27]. The observation of striped phase which is an indication of the phase transition taking place in spin orbit coupled BECs...
Figure 5: Contours plots of densities $|\psi_1|^2$ and $|\psi_2|^2$ as a function of time from (2) and (12) for (a) $c(t) = 0.04t$ and (b) $c(t) = 0.0004t$ with $k_L = 8$ and the other parameters being the same as in figure 2.

Figure 6: Oscillating real and imaginary parts of the order parameter in the time independent trap for a relatively small choice of SO coupling $k_L = 4$ and larger trap frequency $c(t) = 0.4$. The other parameters are the same as in figure 4.

is also consistent with the results of spin orbit coupled spin-1 BECs [22]. Eventhough we have shown the system governed by (1) to be completely integrable only in the absence of Rabi coupling ($\Omega = 0$), we observe that the addition of Rabi coupling does not impact the stability of the condensates as it is evident from figures 8 and 9. In other words, eventhough the addition of Rabi coupling to SO coupled condensates takes the dynamical system to the nonintegrable regime, one does not obviously observe its signature on the condensates and they dwell in the stable
regime. The addition of different trap strengths (or different interaction strengths) merely changes the trajectories of bright solitons without impacting the stability of the condensates either as shown in figures 9(a)-(c). The tunability of transient harmonic trap to produce stable condensates is reminiscent of the tunability of stable SOC-BECs in a double well potential with the adjustable Raman frequency [21].

4. Conclusion

In this paper, we have unearthed the Lax-pair of the SO coupled BECs in a transient harmonic trap and shown them to be completely integrable. We observe that the addition of SO coupling contributes to the rapid oscillations in the real and imaginary parts of order parameter. Tracing their evolution in a transient harmonic trap, we observe that the condensates collapse in the expulsive trap. However, by employing Feshbach resonance management and manipulating trap frequency appropriately, we are able to retrieve the condensates. The oscillatory behavior in the condensates occurs at a relatively small SO coupling parameter in a time independent harmonic trap compared to its counterpart in a transient trap. Then, considering a SO coupled state as the ground state, we study numerically the impact of Rabi coupling on BECs. We observe that the reinforcement of Rabi coupling in a SO coupled BEC gives
Figure 9: Contour plots of densities $|\psi_1|^2$ and $|\psi_2|^2$ as a function of time in the presence of Rabi coupling ($\Omega = 10$) with $k_L = 8$ from the numerical solution of (1) for different trap strengths (a) $c_0 = 0.0004$, (b) $c_0 = 0.04$ and (c) $c_0 = 1$. The other parameters are the same as in figure 2.

rise to striped solitons and one does not see any signature of instability in the condensates. It would be interesting to study the impact of SO coupling by considering a Rabi coupled BEC as the initial state and the results will be published later.
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