A Promised Value Approach to Optimal Monetary Policy*

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Abstract

This paper characterizes optimal commitment policy in the New Keynesian model using a recursive formulation of the central bank’s infinite-horizon optimization problem in which promised inflation and output gap – as opposed to lagged Lagrange multipliers – act as pseudo-state variables. Our recursive formulation is motivated by (Kydland, F. and Prescott, E. C. (1980). Journal of Economic Dynamics and Control Vol. 2, pp. 79–91). Using three well-known variants of the model – one featuring inflation bias, one featuring stabilization bias and one featuring a lower bound constraint on nominal interest rates – we show that the proposed formulation sheds new light on the nature of the intertemporal trade-off facing the central bank.

I. Introduction

Optimal commitment policy is a widely adopted approach among economists and policymakers to studying the question of how to best conduct monetary policy. For example, at the Federal Reserve, the results of optimal commitment policy analysis from the FRB/US model have for some time been regularly presented to the Federal Open Market Committee to help inform its policy decisions (Brayton, Laubach and Reifschneider, 2014). Most recently, in many advanced economies where the policy rate was constrained at the effective lower bound (ELB), the insights from the optimal commitment policy in a stylized New Keynesian model have played a key role in the inquiry on how long the policy rate should be kept at the ELB (Bullard (2013), Evans (2013), Kocherlakota (2011), Plosser (2013), Woodford (2012)). Accordingly, a deep understanding of optimal commitment policy is as relevant as ever.

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In this paper, we contribute to a better understanding of optimal commitment policy in the New Keynesian model – a workhorse model for analysing monetary policy – by characterizing it using a novel recursive method. Our method uses promised values of inflation and output as pseudo-state variables in the spirit of Kydland and Prescott (1980) instead of lagged Lagrange multipliers as in the standard method of Marcet and Marimon (2019). We describe our recursive approach – which we will refer to as the promised value approach – in three variants of the New Keynesian model that have been widely studied in the literature: the model with inflation bias, the model with stabilization bias and the model with an ELB constraint. In each model, we define the infinite-horizon problem of the Ramsey planner, provide the recursive formulations of the Ramsey planner’s problem via the promised value approach, and describe the trade-off facing the central bank in determining the optimal commitment policy.1

The idea of using promised values as pseudo-state variables to recursify the infinite-horizon problem of the Ramsey planner was first suggested by Kydland and Prescott (1980) in the context of an optimal capital taxation problem. Later, Chang (1998) and Phelan and Stacchetti (2001) formally described, as an intermediate step towards characterizing sustainable policies, the recursive formulation of the Ramsey planner’s problem using promised marginal utility in models with money and with fiscal policy respectively. However, because their focus was on characterizing sustainable policies, they did not solve for the Ramsey policy. To our knowledge, we are the first to formulate and solve the Ramsey policy using the promised value approach.2

Our aim is not to argue that readers should use the promised value approach instead of the Lagrange multiplier approach. Rather, our aim is to show that the promised value approach can be a useful analytical tool to supplement the analysis based on the standard Lagrange multiplier approach. Both approaches should be able to find the same allocation; we indeed find that both approaches reliably compute the optimal commitment policies in the New Keynesian model. However, the Ramsey policies are often history dependent in complex ways, and it is not always straightforward for researchers to understand the trade-off facing the central bank. Accordingly, it is useful for researchers to have an alternative way to analyse the Ramsey policy, as it may provide new insights on the optimal commitment policy.3

One difficulty associated with the promised value approach is that it requires researchers to compute the set of feasible promised values (see discussion in Marcet and Marimon (2019)). We find that the extent to which this computation poses a challenge depends on the model. For the model with inflation bias and the model with stabilization bias, the promised rate of inflation is the only pseudo-state variable, and we analytically show that the set of feasible promised inflation rates are identical to the set of feasible actual inflation

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1 In the online appendix, we also analyse the model with the ELB and quantitative easing.

2 The only exception is a recent lecture note by Sargent and Stachurski (2018) which characterizes the Ramsey policy in the linear-quadratic version of the model of Cagan (1956) using the promised value approach. Note that, while the Lagrange multiplier approach is almost always used in solving the Ramsey problems in business cycle models, the promised value approach is extensively used in the literature of dynamic contract.

3 In simple models such as the model with inflation bias considered in the next section, one can easily reverse-engineer policy functions associated with the promised value approach from those associated with the Lagrange multiplier approach. However, in more complicated setup, such reverse engineering is either very complicated or impossible without some numerical methods.
rates – which is a primitive of the models – under non-restrictive conditions. For the model with the ELB constraint, the set of feasible promised inflation-output pairs cannot be found analytically, and one needs a computationally non-trivial method described in Chang (1998) and Phelan and Stacchetti (2001), among others, to find the set. In our numerical example, we find that the set is large and does not represent a binding constraint for the control variables in the Bellman equation. Thus, if one wants to casually use the promised value approach, abstracting from the task of characterizing the set of feasible promises is unlikely to be harmful.

Part of our contribution is pedagogical. As discussed above, the idea of solving the Ramsey policy using the promised value approach has been around for a few decades. Yet, researchers almost always use the Lagrange multiplier approach to solve the Ramsey policy. Our detailed description of how to adopt the promised value approach to a well-known optimal policy problem will be useful to other researchers who would like to adopt this approach to other interesting optimal policy problems.

In addition to Chang (1998) and Phelan and Stacchetti (2001) who recursified the Ramsey planner’s problem using promised values, our paper is closely related to the large literature on optimal policy in New Keynesian models. Optimal commitment policies in the model with inflation bias and in the model with stabilization bias have been studied by many, including Clarida, Gali and Gertler (1999), Gali (2015), and Woodford (2003). Optimal commitment policy in the model with the ELB constraint has been studied by Eggertsson and Woodford (2003), Jung, Teranishi and Watanabe (2005), Adam and Billi (2006), and Nakov (2008), among others. All of these papers – often implicitly but sometimes explicitly – rely on the method of Marcet and Marimon (2019) and use the lagged Lagrange multipliers as pseudo-state variables to characterize optimal commitment policies; our contribution is to provide an alternative method to characterize them.

Our paper is also related to Waki, Dennis and Fujiwara (2018) who study a mechanism-design problem under private information in a New Keynesian model. The recursive characterization of their mechanism-design problem features promised inflation as a pseudo-state variable, as in our paper, and the limiting full-information version of their model corresponds to the standard New Keynesian model with stabilization bias we consider in section IV. Our paper is different from their work because we (i) examine the intertemporal trade-off facing the central bank under the promised-value approach, (ii) contrast it with the trade-off under the Lagrange multiplier approach, and (iii) consider two other versions of the New Keynesian model – one with inflation bias and the other with ELB – that are commonly used in the literature.

The rest of the paper is organized as follows. Sections II, III and IV study the model with inflation bias, the model with stabilization bias, and the model with the ELB constraint, respectively. In each section, we first present the infinite-horizon problem of the Ramsey planner and describe how the infinite-horizon problem is made recursive under the promised value approach. We then discuss the dynamics of the Ramsey equilibrium, describe the

4While we do not consider open economy models in this paper, there is a growing literature analysing optimal policy in open economy models. See, for example, Groll and Monacelli (2020) and Soffritti and Zanetti (2008), among many others. Our promised value approach may also be fruitfully applied to the analysis of optimal policy in open economy models.
key trade-off the central bank faces, and contrast the promised value approach with the standard Lagrange multiplier approach. Section V concludes.

II. Model with inflation bias

Our first model is the one with inflation bias, which is a version of the standard New Keynesian model in which the inefficiency associated with monopolistic competition in the product market is not offset by a production subsidy. As the model is standard, we refer interested readers to Woodford (2003) and Galí (2015) for more detailed descriptions. The economy starts at time one. The model is log linearized around its deterministic steady state. Its private sector equilibrium conditions at time $t$ are given by

$$y_t = \sigma y_{t+1} + \pi_{t+1} - r_t + r^*, \quad (1)$$

$$\pi_t = \kappa y_t + \beta \pi_{t+1}, \quad (2)$$

where $y_t$, $\pi_t$, and $r_t$ are the output gap, inflation and the policy rate respectively. $\sigma$, $\kappa$, $\beta$ are the inverse intertemporal elasticity of substitution, the slope of the Phillips curve, and the time discount rate respectively. $r^*$ is the long-run natural rate of interest. Equations (1) and (2) are referred to as the Euler equation and the Phillips curve respectively. In the model with inflation bias, we abstract from the ELB constraint on $r_t$. With this abstraction, the Euler equation does not constrain the allocations the central bank can choose; it merely pins down the policy rate given the sequence of inflation and output. This abstraction is a common practice in the literature.

We assume that $y_t \in \mathbb{K}_Y$ and $\pi_t \in \mathbb{K}_\Pi$ where $\mathbb{K}_Y$ and $\mathbb{K}_\Pi$ are closed intervals on the real line, $\mathbb{R}$. For any variable $x$, let us denote $\{x_t\}_{t=1}^\infty$ by a bold font $\mathbf{x}$. We say $(y, \pi)$ (i.e. $\{y_t, \pi_t\}_{t=1}^\infty$) is a competitive outcome if equation (2) is satisfied for all $t \geq 1$, and use $\mathbf{CE}$ to denote the set of all competitive outcomes.

The sequence of values, $\{V_t\}_{t=1}^\infty$, associated with a competitive outcome, $\{y_t, \pi_t\}_{t=1}^\infty$, is given by

$$V_t = \sum_{k=t}^\infty \beta^{k-t} u(y_k, \pi_k),$$

where $u(\cdot, \cdot)$, the payoff function, is given by

$$u(y, \pi) = -\frac{1}{2} [\pi^2 + \lambda (y - y^*)^2]. \quad (3)$$

This quadratic payoff function can be derived as the second-order approximation to the household welfare.\(^5\) The presence of $y^*$ in this objective function captures the inefficiency associated with monopolistic competition in the product market. The problem of the Ramsey planner is to choose a competitive outcome that maximizes the time-one value as follows:

$$V_{ram,1} = \max_{(y, \pi) \in \mathbf{CE}} V_1. \quad (4)$$

\(^5\) See, for example, Galí (2015) for the derivation.
The Ramsey outcome is defined as the solution to this optimization problem and is denoted by \( \{ y_{\text{ram},t}, \pi_{\text{ram},t} \}_{t=1}^\infty \). The value sequence associated with the Ramsey outcome is denoted by \( \{ V_{\text{ram},t} \}_{t=1}^\infty \).

Promised value approach

Under the promised value approach, the infinite-horizon optimization problem of the Ramsey planner given by equation (4) is divided into two steps. In the first step, the following constrained infinite-horizon Ramsey problem is formulated:

\[
 w^*(\eta) = \max_{(y, \pi) \in \Gamma(\eta)} - \frac{1}{2} \sum_{t=1}^\infty \beta^{t-1} [\pi_t^2 + \lambda (y_t - y^*)^2],
\]

where \( \Gamma(\eta) \) is the set of competitive outcomes in which the initial inflation, \( \pi_1 \), is \( \eta \). This set is formally defined in the online appendix. In the second step, the Ramsey planner chooses the initial inflation promise, \( \eta \), that maximizes \( w^*(\eta) \). That is,

\[
 V_{\text{ram},1} = \max_{\eta \in \Omega} w^*(\eta),
\]

where \( \Omega \) is the set of time-one inflation rates consistent with the existence of a competitive outcome. This set is formally defined and computed analytically in online appendix.

By the standard dynamic programming argument, it can be shown that \( w^*(\eta) \) satisfies the following functional equation:

\[
 w(\eta) = \max_{y \in \mathcal{Y}, \pi \in \mathcal{P}, \eta' \in \Omega} u(y, \pi) + \beta w(\eta'),
\]

subject to

\[
 \pi = \eta \quad \text{and} \quad \pi = \kappa y + \beta \eta',
\]

where \( \eta \) is the promised rate of inflation for the current period from the previous period, and \( \eta' \) is the promised rate of inflation for tomorrow. Conversely, if a bounded function, \( w : \Omega \to \mathbb{R} \), satisfies this functional equation, then \( w = w^* \).

Let \( \{ w_{\text{PV}}(\cdot), y_{\text{PV}}(\cdot), \pi_{\text{PV}}(\cdot), \eta_{\text{PV}}(\cdot) \} \) be the value and policy functions associated with this Bellman equation. The Ramsey value sequence and the Ramsey outcome are obtained by iterating over these functions with the time-one inflation rate set to the argmax of \( w^*(\eta) \) in equation (5).

Lagrange multiplier approach

It is useful to contrast the recursive formulation of the promised value approach with that of the more standard Lagrange multiplier approach of Marcet and Marimon (2019). In the Lagrange multiplier approach, a saddle-point functional equation is used to recursify the

\[\text{(5)}\]

\[\text{(6)}\]

\[\text{subject to}\]

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6The proof is closely related to the proof of the Bellman optimality principle. See Chang (1998).
A promised value approach

In the model with inflation bias, it is given by

$$W(\phi) = \min_{\phi'} \max_{y \in K_Y, \pi \in K_{\Pi}} f(y, \pi, \phi') + \beta W(\phi'),$$

where $f(\cdot)$, the modified payoff function, is given by

$$f(y, \pi, \phi') = u(y, \pi) + \phi'(\pi - \kappa y) - \phi \pi.$$

Let $(y_{LM}(\cdot), \pi_{LM}(\cdot), \phi'_{LM}(\cdot))$ be the policy functions associated with this saddle-point functional equation. One can find the Ramsey outcome by iterating over these policy functions with the initial Lagrange multiplier set to zero.

### Analysis of optimal policy

For both the Bellman equation of the promised value approach and the saddle-point functional equation of the Lagrange multiplier approach, the payoff function is quadratic and the constraints are linear. This linear-quadratic structure allows us to solve the model analytically for both approaches. However, to describe how the promised value approach works in a transparent way, we use a numerical example in the main text and relegate the closed-form solutions to online appendix. The parameter values used in the numerical example are from Woodford (2003) and are shown in Table 1.

Figure 1 shows the policy functions for the promised rate of inflation in the next period and the output gap in the current period as well as the value function associated with the Bellman equation.

In the promised value approach, the initial inflation rate is given by the argmax of the value function associated with the Bellman equation – shown in the right panel of Figure 1. According to the panel, the initial inflation rate – indicated by the dashed vertical line – is slightly below 0.2%. Once the initial inflation rate is determined, the dynamics of the economy are sequentially pinned down by the policy functions linking the promised rate of inflation in the current period ($\eta$) to the promised rate of inflation in the next period ($\eta'$) and output in the current period ($y$), shown in the left and middle panels respectively. For example, the time-two inflation rate is determined by the policy function for the promised rate of inflation evaluated at the initial inflation rate and is shown by the pentagram in the

### Table 1

| Parameter | Value |
|-----------|-------|
| $\beta$   | 0.9925|
| $y^*$     | 0.01  |
| $\lambda$ | 0.003 |
| $\kappa$  | 0.024 |
left panel. The time-one output is determined by the policy function for output evaluated at the initial rate of inflation and is shown by the pentagram in the middle panel. The black dots in the policy functions trace the dynamics of the economy afterward.

The implied dynamics of the economy are shown in Figure 2. The central bank has an incentive to generate a positive inflation rate at time one, which is associated with a level of output gap that is above zero but below \( y^* \). Inflation and output converge eventually to zero, a well-known feature of the optimal commitment policy in this model.\(^9\)

To understand the trade-off associated with the Bellman equation (equation (6)), we show in Figure 3 the objective function to be maximized and its two subcomponents – today’s payoff, \( u(\cdot, \cdot) \), and the discounted continuation value, \( \beta w(\cdot) \) – at \( t = 20 \) when the economy has essentially converged to its steady state of zero inflation so that \( \eta = 0 \). Note that two arguments for the payoff function, inflation and output, are functions of \( \eta \) and \( \eta' \) (recall that \( \pi = \eta \) and \( \pi = \kappa y + \beta \eta' \)). Thus, the payoff function \( u(\pi, y) \) can be transformed to an indirect payoff function, \( u^*(\eta' | \eta) \). To be more explicit, we will examine

\[
\tilde{w}(\eta' | \eta) := u^*(\eta' | \eta) + \beta w(\eta')
\]

conditional on \( \eta = 0 \), and see how each of \( \tilde{w}(\eta' | \eta = 0) \), \( u^*(\eta' | \eta = 0) \), and \( \beta w(\eta') \) depends on \( \eta' \).\(^{10}\)

The fact that inflation is zero at the steady state is captured by the fact that the objective function evaluated at \( \eta = 0 \), shown by the left panel, is maximized at \( \eta' = 0 \). The optimality of promising zero inflation in the next period when the promised inflation rate for the current period is zero reflects two competing forces. The first force is how the promised inflation rate affects today’s payoff. Given that the central bank needs to deliver zero inflation today,

\(^9\) In the online appendix, we contrast the Ramsey equilibrium to the Markov perfect equilibrium and the value-maximizing pair of inflation and output.

\(^{10}\) \( w(\cdot) \) is the value function associated with the Bellman equation for the promised value approach, and is constructed analytically, as shown in the online appendix.
the lower the promised inflation rate is for next period, the higher the output today has to be in order to satisfy the Phillips curve.\textsuperscript{11} Because of the presence of $y^*$ in the payoff function, a higher output (a lower inflation) means a higher payoff as long as output is below $y^*$. Thus, the central bank has an incentive to promise some deflation next period, as captured by the middle panel of Figure 3 which shows that today’s utility is maximized at $\eta' < 0$.

The second force is how the promised inflation affects the discounted continuation value. As shown in the right panel of Figure 3, a higher promised inflation rate is associated with a higher continuation value up to a certain point, as a higher future inflation is associated with a higher future level of output that is closer to $y^*$. The optimality of promising a zero inflation rate reflects these two competing effects of adjusting the inflation promise on the today’s payoff and on the discounted continuation value.

\textsuperscript{11}To see this, set $\eta = 0$ in the two constraints in the Bellman equation (6).
Figure 4. Policy functions from the Lagrange multiplier approach – model with inflation bias –

Notes: $\phi_{-1}$ is the lagged Lagrange multiplier, whereas $\phi$ is the Lagrange multiplier in the current period. The rate of inflation is expressed in annualized percent. The output gap is expressed in percent. The blue line on the right panel is the 45 degree line.

We will close the section by examining the policy functions from the standard Lagrange multiplier approach. Figure 4 shows the policy functions for inflation, output and the Lagrange multiplier associated with the saddle-point functional equation (7). Unlike in the promised value approach, these functions are functions of the lagged Lagrange multiplier, $\phi_{-1}$. Time-one allocations are given by the policy functions evaluated at the initial lagged Lagrange multiplier of zero and are indicated by the pentagram. The black dots trace the dynamics of inflation, output and the Lagrange multiplier after the first period. According to the right panel, the Lagrange multiplier eventually converges to a positive value. As the Lagrange multiplier converges, inflation and output also converge to zero, as shown in the left and middle panels respectively. The dynamics of inflation and output derived from the Lagrange multiplier approach are of course identical to those implied by the promised value approach shown in Figure 2. In the online appendix, we provide analytical proof for their equivalence.

III. Model with stabilization bias

Our second model is the model with stabilization bias. The private sector equilibrium conditions in this model at time $t$ are given by

$$\begin{align*}
\sigma y_i(s') &= \sigma E_t y_{t+1}(s'^{t+1}) + E_t \pi_{t+1}(s'^{t+1}) - r_t(s') + r^*, \\
\pi_i(s') &= \kappa y_i(s') + \beta E_t \pi_{t+1}(s'^{t+1}) + s_t.
\end{align*}$$

The key difference between this model and the model in the previous section is that in this model, there is a cost-push shock, denoted by $s_t$, that additively enters into the Phillips curve. The cost-push shock follows an N-state Markov process and its possible values are given by the set, $\mathbb{S} := \{e_1, e_2, \ldots, e_N\}$. The probability of moving from state $i$ to state $j$ is denoted by $p(e_j|e_i)$. $s'$ denotes the history of shocks up to time $t$. That is, $s' := \{s_h\}_{h=1}^t$. Because there is uncertainty, the allocations are state-contingent and depend on $s'$. © 2020 The Authors. Oxford Bulletin of Economics and Statistics published by Oxford University and John Wiley & Sons Ltd.
As in the model with inflation bias and consistent with common practice in the literature on stabilization bias, we abstract from the ELB constraint on the policy rate, which in turn allows us to abstract from the Euler equation. We assume that $y_t \in \mathbb{K}_Y$ and $\pi_t \in \mathbb{K}_\Pi$, where $\mathbb{K}_Y$ and $\mathbb{K}_\Pi$ are closed intervals on the real line, $\mathbb{R}$. For any variable $x$, let us denote its state-contingent sequence $\{x_t(s_t)\}_{t=1}^{\infty}$ by $x(s)$ (bold font) and its state-contingent sequence with the time-one state $s_1 = s$ by $x(s)$. We say $(y, \pi)$ is a competitive outcome if the Phillips curve is satisfied for all $t \geq 1$. We use $\text{CE}$ to denote the set of all competitive outcomes and use $\text{CE}(s)$ to denote the set of competitive outcomes in which the initial state $s_1$ is $s$.

The sequence of values $\{V_t(s_t)\}_{t=1}^{\infty}$ associated with a competitive outcome is given by

$$V_t(s') = \sum_{k=t}^{\infty} \beta^{k-t} \sum_{s' \mid s'} \mu(s' \mid s') u(y_k(s'), \pi_k(s')),$$

where $\mu(s' \mid s)$ is the conditional probability of observing $s'$ after observing $s$. The payoff function, $u(\cdot, \cdot)$, is given by

$$u(y, \pi) = -\frac{1}{2} \left[ \pi^2 + \lambda y^2 \right].$$

(8)

The Ramsey problem is to choose the state-contingent sequences of inflation and output to maximize the time-one value for each $s \in \mathbb{S}$. That is,

$$V_{\text{ram}, 1}(s) = \max_{(y, \pi) \in \text{CE}(s)} V_1(s).$$

The Ramsey outcome is defined as the state-contingent sequences of inflation and output that solve this optimization problem and is denoted by $\{y_{\text{ram}, t}(s'), \pi_{\text{ram}, t}(s')\}_{t=1}^{\infty}$. The value sequence associated with the Ramsey outcome is denoted by $\{V_{\text{ram}, t}(s')\}_{t=1}^{\infty}$.

Promised value approach

As in the model with inflation bias, the infinite-horizon optimization problem of the Ramsey planner is divided into two stages. In the first stage, the constrained Ramsey problem is formulated as follows:

$$w^*(\eta, s) = \max_{(y(s), \pi(s)) \in \Gamma(\eta, s)} \left[ -\frac{1}{2} \sum_{t=1}^{\infty} \beta^{t-1} \sum_{s' \mid s_1 = s} \mu(s' \mid s) \left[ \pi_t^2 + \lambda y_t^2 \right] \right],$$

where $\Gamma(\eta, s)$ is the set of competitive outcomes with the initial state $s_1 = s$ in which the initial inflation is $\eta$. This set is formally defined in the online appendix. In the second stage, the Ramsey planner chooses the initial inflation to maximize $w^*(\eta, s)$:

$$V_{\text{ram}, 1}(s) = \max_{\eta \in \Omega(s)} w^*(\eta, s),$$

(9)

where $\Omega(s)$ is the set of time-one inflation rates consistent with the existence of a competitive outcome with the initial state $s_1 = s$. This set is formally defined and computed analytically in the online appendix. The Bellman equation associated with the first-stage constrained Ramsey problem is given by

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\[
\begin{align*}
w(\eta_i, e_i) &= \max_{y \in K \cap \Pi} u(y, \pi) + \beta \sum_{j=1}^{N} p(e_j | e_i) w'_{j}(\eta_j', e_j) \\
\text{subject to} \\
\pi &= \eta_i \\
\pi &= \kappa y + \beta \sum_{j=1}^{N} p(e_j | e_i) \eta_j' + e_i,
\end{align*}
\]  

where we are now explicit about the specifics of the shock (recall that \(S := \{e_1, e_2, \ldots, e_N\}\))

Note that the control variables include \(\eta_j'\) for each \(j \in \{1, 2, \ldots, N\}\).

Let \(\{w_{PV}(\cdot), y_{PV}(\cdot), \pi_{PV}(\cdot), \{\eta_{PV,j}'(\cdot)\}_{j=1}^{N}\}\) be the value and policy function associated with the Bellman equation. Note that there are \(N\) promised inflation rates that have to be chosen. The Ramsey value sequence and the Ramsey outcome can be obtained by iterating over these functions with the time-one inflation set to the argmax of \(w^*(\eta, s)\) in equation (9).

**Lagrange multiplier approach**

The saddle-point functional equation associated with the Ramsey planner’s problem above is given by

\[
W(\phi, e_i) = \min_{\phi'} \max_{y \in K, \pi \in K\Pi} f(y, \pi, \phi, \phi', e_i) + \beta \sum_{j=1}^{N} p(e_j | e_i) W'(\phi', e_j)
\]

where \(f(\cdot)\), the modified payoff function, is given by

\[
f(y, \pi, \phi, \phi', e_i) = u(y, \pi) + \phi' (\pi - \kappa y - e_i) - \phi \pi.
\]

Let \(\{V_{LM}(\cdot), \pi_{LM}(\cdot), \phi_{LM}(\cdot)\}\) be the policy functions associated with this saddle-point functional equation. One can find the Ramsey outcome by iterating over these policy functions with the initial Lagrange multiplier set to zero.

**Analysis of optimal policy**

Given the linear-quadratic structure of the model, the solutions to the Bellman equation from the promised value approach and the saddle-point functional equation can be obtained analytically. However, we will again use a numerical example to illustrate the mechanics of the promised value approach in a transparent way; the analytical results are provided in the online appendix. To make the exposition as transparent as possible, we will assume that (i) there are only two states (high and normal), (ii) the economy starts in the high state, (iii) the economy will move to the normal state with certainty in period 2 and (iv) the normal state is absorbing. In the remainder of this section, we will use the notation \(e_N\) and

12 As a result, the larger the number of exogenous states is, the larger the number of policy functions to solve for is. However, because an increase in the number of exogenous states does not affect the state space, it does not necessarily lead to increased computational burden.

13 As in the model with inflation bias, we confirm that the upper and lower bounds implied by the closed intervals on choice variables are not binding in equilibrium.
### TABLE 2

Parameters and transition probabilities – model with stabilization bias –

| $\beta$ | $\lambda$ | $\kappa$ | $e_h$ | $e_n$ | $p(e_h|e_h)$ | $p(e_n|e_h)$ | $p(e_h|e_n)$ | $p(e_n|e_n)$ |
|-------|-------|-------|------|------|-------------|-------------|-------------|-------------|
| 0.9925 | 0.003 | 0.024 | 0.001 | 0.001 | 1           | 0           | 1           | 1           |

Figure 5. Policy functions from the promised value approach – model with stabilization bias –

Notes: $\eta$ is the rate of inflation that was promised in the previous period and needs to be delivered in the current period. $\eta'$ is the promised rate of inflation for the next period. These rates are expressed in annualized percent. $w$ is the value associated with the Bellman equation (equation (10)). The blue lines on the top-left and bottom-left panels are the 45 degree lines.

$e_n$, instead of $e_1$ and $e_2$, to refer to the high and normal states respectively. Parameter values are shown in Table 2. The values for the parameter governing the private sector behaviour are the same as in the previous section.

Figure 5 shows the policy functions for the promised inflation rate in the next period and output in the current period as well as the value function associated with the Bellman equation of the promised value approach. The top and bottom panels are for the high state and the normal state respectively.

The initial inflation rate is given by the argmax of the value function from the high state, shown by the top-right panel of the figure. The initial inflation – indicated by the dashed
vertical line – is about 0.25%. Once the time-one inflation rate is determined, the time-two inflation rate ($\pi_2$) and the time-one output ($y_1$) are determined by the high-state policy functions shown in the top-left and top-middle panels respectively. Subsequent sequences of inflation and output – shown by the black dots – are determined by the normal-state policy functions shown in the bottom panels.

Figure 6 shows the implied dynamics of inflation, output and the value. A well-known feature of the optimal commitment policy in the model with stabilization bias is that, in the initial period, the central bank promises to undershoot its inflation target once the shock disappears. Relative to the equilibrium under the Markov perfect policy – shown by the dashed lines – in which the central bank does not have a commitment technology, such promise of undershooting improves the trade-off between inflation and output stabilization at $t=1$ through expectations when the economy is buffeted by the cost-push shock, allowing the central bank to achieve a higher period-one value. The undershooting of inflation and output will fade gradually, and inflation and output will eventually converge to zero.

To understand the trade-off the central bank faces in choosing to create deflation in the second period, we show in Figure 7 the objective function associated with the Bellman equation – shown in the left panel – and its two subcomponents – shown in the middle and right panels – at time one when the cost-push shock is present. Specifically, we will examine

$$\bar{w}(\eta'|\eta, e = e_h) := u^*(\eta'|\eta, e = e_h) + \beta w(\eta', e = e_n)$$

conditional on $\eta = \eta_1$, and how each of $\bar{w}(\eta'|\eta = \eta_1, e = e_h)$, $u^*(\eta'|\eta = \eta_1, e = e_h)$, and $w(\eta', e = e_n)$ depends on $\eta$. Note that, as in the model with inflation bias, today’s payoff function can be written as a function of $\eta$ and $\eta'$, because today’s output is a function of today’s inflation and inflation in the next period.

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14 The online appendix formulates the optimization of the discretionary central bank and solves for the Markov perfect policy.

15 In defining $\bar{w}$, we embed the shock structure that $e = e_h$ at time one and $e = e_n$ thereafter. Note that, as shown in the online appendix, $w(\cdot)$ – the value function in the Bellman equation for the promised value approach – is computed analytically together with the policy functions.
Consistent with Figures 5 and 6, the value of the objective function, $w(\cdot)$, is maximized at $\eta' < 0$. To understand why some deflation is optimal, we need to examine how $\eta'$ affects today’s payoff as well as the discounted continuation value, shown in the middle and right panels of Figure 7 respectively.

On the one hand, because the inflation rate in the current period has been chosen in the previous period, today’s payoff is maximized when today’s output is zero. Conditional on the initial promised inflation rate of $\eta = \eta_1 = 0.26/400$ and the cost-push shock of $e = e_h = 0.1/100$, the Phillips curve implies that zero output today is achieved only by promising some deflation for the next period, as indicated by the dashed vertical line in the middle panel of Figure 7. On the other hand, the discounted continuation value is maximized if the central bank chooses to promise zero inflation for the next period ($\eta' = 0$), as shown by the dashed vertical line in the right panel of Figure 7. The promised inflation rate of zero maximizes the discounted continuation value because it is associated with fully stabilized paths of inflation and output in the future and thus with the highest possible continuation value of zero. The optimal rate of promised inflation – indicated by the solid vertical line – balances these two forces. All told, the overall objective function – shown in the left panel of Figure 7 – is maximized at $\eta' < 0$.

Finally, Figure 8 shows the policy functions for inflation, output and the Lagrange multiplier associated with the saddle-point functional equation (7) of the Lagrange multiplier approach. The top and bottom panels are for the high state and the normal state respectively. The time-one inflation, output, and Lagrange multiplier are indicated by the pentagrams in the top panels, which are the high-state policy functions evaluated at the initial lagged Lagrange multiplier of zero. Thereafter, the dynamics of the economy are governed by the normal-state policy functions shown in the bottom panels – because the cost-push shock is assumed to disappear after the first period – and are traced by the black dots. In the online appendix, we analytically verify the equivalence of the dynamics of the economy obtained from the promised value and Lagrange multiplier approaches.
Figure 8. Policy functions from the Lagrange multiplier approach – model with stabilization bias –
Notes: $\phi_{-1}$ is the lagged Lagrange multiplier, whereas $\phi$ is the Lagrange multiplier in the current period. The rate of inflation is expressed in annualized percent. The output gap is expressed in percent. The blue lines on the top-right and bottom-right panels are the 45 degree lines.

IV. Model with the ELB

Our final model features the ELB constraint on nominal interest rates and a natural rate shock. The private sector equilibrium conditions at time $t$ are given by

$$\sigma y_t(s') = \sigma E_t y_{t+1}(s^{t+1}) + E_t \pi_{t+1}(s^{t+1}) - r_t(s') + r^* + s_t$$  \hspace{1cm} (11)

$$\pi_t(s') = \kappa y_t(s') + \beta E_t \pi_{t+1}(s^{t+1})$$  \hspace{1cm} (12)

where $s_t$ is a natural rate shock following a N-state Markov process. We assume that $y_t \in K_Y$ and $\pi_t \in K_{\Pi}$ where $K_Y$ and $K_{\Pi}$ are closed intervals on the real line, $\mathbb{R}$. We introduce the ELB constraint on the policy rate by imposing that $r_t \in [r_{ELB}, r_{max}]$, where $r_{ELB}$ is the ELB constraint on the policy rate.\(^{16}\)

\(^{16}\)In the online appendix, we will also study a variant of the model in this section that includes a role for quantitative easing.
Possible values of the natural rate shock are given by the set, $\mathbb{S} := \{\delta_1, \delta_2, \ldots, \delta_n\}$. The probability of moving from state $i$ to state $j$ is denoted by $p(\delta_j|\delta_i)$. $s'$ denotes the history of shocks up to time $t$. That is, $s' := [s_h]_{h=1}^t$. Because there is uncertainty, the allocations are state-contingent and depend on $s'$.

For any variable $x$, let us denote its state-contingent sequence $\{x_t(s')\}_{t=1}^\infty$ by a bold font $\mathbf{x}$. The future outcome of the natural rate shock is denoted by $\mathbf{s}$. We say $\mathbf{(v, \pi, r)}$ is a competitive outcome if equations (11) and (12) are satisfied for all $t \geq 1$. We use $\text{CE}$ to denote the set of all competitive outcomes and use $\text{CE}(s)$ to denote the set of competitive outcomes in which the initial state $s_1$ is $s$.

The value sequence, $\{V_t(s')\}_{t=1}^\infty$, associated with a competitive outcome is given by

$$V_t(s') = \sum_{k=t}^\infty \beta^{k-t} \sum_{s\in\mathbb{S}} \mu(s^k|s') v(y_k(s^k), \pi_k(s^k)),$$

where $\mu(s^k|s')$ is the conditional probability of observing $s^k$ after observing $s'$. The payoff function, $v(\cdot, \cdot)$, is given by equation (8) from the previous section. The Ramsey planner’s problem is to choose the state-contingent sequences of inflation and output to maximize the time-one value for each $s \in \mathbb{S}$. That is,

$$V_{\text{ram},1}(s) = \max_{(v(s), \pi(s), r(s)) \in \text{CE}(s)} V_1(s). \quad (13)$$

The Ramsey outcome is defined by the solution to this problem and is denoted by $\{v_{\text{ram},t}(s'), \pi_{\text{ram},t}(s'), r_{\text{ram},t}(s')\}_{t=1}^\infty$. The value sequence associated with the Ramsey outcome is denoted by $\{V_{\text{ram},t}(s')\}_{t=1}^\infty$.

Promised value approach

As in the previous two models, the infinite-horizon optimization problem of the Ramsey planner is divided into two stages. In the first stage, the constrained Ramsey problem is formulated as follows:

$$w^*(\eta_1, \eta_2, s) = \max_{(v(s), \pi(s), r(s)) \in \Gamma(\eta_1, \eta_2, s)} \left[ \frac{1}{2} \sum_{t=1}^\infty \beta^{t-1} \sum_{s'|s_1=s} \mu(s'|s) \left( \pi_k(s')^2 + \lambda y_1(s')^2 \right) \right],$$

where $\Gamma(\eta_1, \eta_2, s)$ is the set of competitive outcomes with the initial state $s_1 = s$ in which the initial output and inflation are $\eta_1$ and $\eta_2$ respectively. This set is more formally defined in the online appendix. In the second stage, the Ramsey planner chooses the initial inflation and output promises that maximize $w^*(\eta_1, \eta_2, s)$:

$$w^*(s) = \max_{(\eta_1, \eta_2) \in \Omega(s)} w^*(\eta_1, \eta_2, s), \quad (14)$$

where $\Omega(s)$ is the set of pairs of time-one inflation rates and output gaps consistent with the existence of a competitive outcome with the initial state $s_1 = s$. This set is formally defined and computed numerically in the online appendix. The Bellman equation associated with the first-stage constrained Ramsey problem is given by.
\[ w(\eta_{1,i}, \eta_{2,i}, \delta_i) = \max_{y \in \mathbb{S}, \pi \in \mathbb{K}, r \in \mathbb{K}, (\eta_{1,j}', \eta_{2,j}') \in \Omega_j, j = 1}^{N} u(y, \pi) + \beta \sum_{j=1}^{N} p(\delta_j | \delta_i) w(\eta_{1,j}', \eta_{2,j}', \delta_j) \]  

subject to
\[
\begin{align*}
    y &= \eta_{1,i} \\
    \pi &= \eta_{2,i} \\
    \sigma y &= \sum_{j=1}^{N} p(\delta_j | \delta_i) [\sigma \eta_{1,j}' + \eta_{2,j}'] - r + r^* + \delta_i \\
    \pi &= ky + \beta \sum_{j=1}^{N} p(\delta_j | \delta_i) \eta_{2,j}',
\end{align*}
\]

where we are now explicit about the specifics of the shock (recall that \( \mathbb{S} := \{\delta_1, \delta_2, \ldots, \delta_n\} \)). Let \( \{w_{PV}(\cdot), y_{PV}(\cdot), \pi_{PV}(\cdot), \eta_{1,j}^{PV}(\cdot), \eta_{2,j}^{PV}(\cdot)\}_{j=1}^{N} \) be the value and policy functions associated with this Bellman equation. Note that there are \( N \) promises for both inflation and output that have to be chosen. The Ramsey value sequence and the Ramsey outcome can be obtained by iterating over the policy functions for inflation, output, and the policy rate found in the first step with time-one output and inflation set to the argmax of \( w^*(\eta_1, \eta_2, s) \) in equation (14).

**Lagrange multiplier approach**

The saddle-point functional equation associated with the infinite-horizon optimization problem of the Ramsey planner above (equation (13)) is given by
\[ W(\phi_1, \phi_2, \delta) = \min_{\phi_1', \phi_2'} \max_{y', \pi', r, \phi_1', \phi_2', \delta_1} f(y', \pi', r, \phi_1', \phi_2', \phi_2', \delta_1) + \beta \sum_{j=1}^{N} p(\delta_j | \delta_i) W(\phi_1', \phi_2', \delta_j), \]  

where \( f(\cdot), \) the modified payoff function, is given by
\[
\begin{align*}
    f(y, \pi, r, \phi_1, \phi_1', \phi_2, \phi_2', \delta_i) &= u(y, \pi) + \phi_1'(r - r^* + \sigma y - \delta_i) - \frac{\phi_1}{\beta} (\sigma y + \pi) + \phi_2' (\pi - ky) - \phi_2 \pi.
\end{align*}
\]

Let \( \{y_{LM}(\cdot), \pi_{LM}(\cdot), r_{LM}(\cdot), \phi_{1,LM}(\cdot), \phi_{2,LM}(\cdot)\} \) be the policy functions associated with this saddle-point functional equation. As in the previous two models, one can find the Ramsey value and outcome by iterating over these policy functions with the initial Lagrange multiplier set to zero.

**Analysis of optimal policy**

Unlike the first two models, the model with the ELB constraint cannot be solved analytically under either approach. Thus, we solve the model numerically. The solution methods are standard and their details are described in the online appendix. As in the model with
A promised value approach

Table 3

| Parameters and transition probabilities – model with effective lower bound (ELB) – |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| β               | λ               | κ               | r_{ELB}         | δ_c             | δ_n             | p(δ_c|δ_c)        | p(δ_n|δ_c)        | p(δ_c|δ_n)        | p(δ_n|δ_n)        |
| 0.9925          | 0.003           | 0.024           | 0              | -0.02           | 0               | 1              | 0               | 1               | 1               |

Figure 9. Policy functions from the promised value approach – model with effective lower bound –

Notes: η_1 and η_2 are the output gap and the rate of inflation, respectively, that were promised in the previous period and need to be delivered in the current period. \( \eta'_1 \) and \( \eta'_2 \) are the promised output gap and the promised rate of inflation, respectively, for the next period. \( w \) is the value associated with the Bellman equation (equation (15)). The rate of inflation is expressed in annualized percent. The output gap is expressed in percent.

To simplify the shock structure in order to describe the mechanics of the promised value approach in a transparent way, we will assume that (i) there are only two states (crisis and normal), (ii) the economy starts in the crisis state, (iii) the economy will move to the normal state with certainty in the second period and (iv) the normal state is absorbing. In the remainder of the section, we will use the notation \( \delta_c \) and \( \delta_n \), instead of \( \delta_1 \) and \( \delta_2 \), to refer to the crisis and normal states respectively. Parameter values are shown in Table 3. The values for the parameters governing the private sector behaviour are the same as those in the previous sections.
Figure 10. Dynamics – model with effective lower bound –

Notes: The rate of inflation and the policy rate are expressed in annualized percent. The output gap is expressed in percent.

Figure 9 shows the policy functions for the promised inflation and output in the next period as well as the value function associated with the Bellman equation from the promised value approach, while Figure 10 shows the dynamics of inflation, output, the policy rate, and the value implied by these functions.

The pair of the initial inflation rate and output is given by the argmax of the crisis-state value function – shown in the top-right panel of Figure 9 – and is indicated by the solid vertical line. The initial inflation rate and output are about minus 0.01% and minus 0.6% respectively. Once the initial inflation rate and output are determined, the dynamics of the economy are governed by the normal-state policy functions linking the promised inflation rate and output today to the promised inflation rate and output next period, shown by the bottom panels. The dots in the policy functions trace the dynamics of the economy. The economy’s dynamics are shown in Figure 10.

The key feature of the optimal commitment policy in the model with the ELB constraint is that in the initial period, the central bank promises to overshooting inflation and output once the crisis shock disappears in the second period – a feature well-known in the literature (Eggertsson and Woodford (2003); Jung et al. (2005); Adam and Billi (2006)). The
overshooting commitment mitigates the declines in inflation and output at the ELB via expectations. After the second period, inflation and output gradually approach to their steady state values of zero. Note that, under the optimal discretionary policy – shown by the dashed lines – there is no overshooting in the aftermath of the crisis shock, and the declines in inflation and output are larger during the crisis than under the optimal commitment policy.17

Figure 11 shows the trade-off associated with the Bellman equation, given by equation (15), in the first period when the economy is in the crisis state today but is expected to return to the normal state in the next period. Given the initial inflation rate and output the central bank has to deliver today ($\eta_1$ and $\eta_2$), the Phillips curve pins down the promised inflation in the next period. Specifically,

$$\eta'_2 := \pi' = \frac{1}{\beta}(\pi - \kappa y)$$

$$= \frac{1}{\beta}(\eta_2 - \kappa \eta_1)$$

Thus, the only control variable available for the central bank to adjust is the promised output for the next period, $\eta'_1$. Thus, the objective function to be maximized can be written as

$$\tilde{w}(\eta'_1 | \eta_1, \eta_2, \delta = \delta_c) := u^*(\eta'_1 | \eta_1, \eta_2, \delta = \delta_c) + \beta\tilde{w}(\eta'_2 | \eta_2', \delta = \delta_n)$$

conditional on $(\eta_1, \eta_2) = (\eta_{1,1}, \eta_{2,1})$ and how each of $\tilde{w}(\eta'_1 | \eta_1, \eta_2, \delta = \delta_c), u^*(\eta'_1 | \eta_1, \eta_2, \delta = \delta_1)$, and $w(\eta'_1 | \eta'_2, \delta = \delta_n)$ depends on $\eta'$.18 Note that the second subcomponent – the next period’s

17 We formulate the optimization problem of the discretionary central bank and solve for the Markov perfect equilibrium in the online appendix.

18 In defining $\tilde{w}$, we embed the shock structure that $\delta = \delta_c$ at time one and $\delta = \delta_n$ thereafter. Note that, as shown in the online appendix, $w(\cdot)$ – the value function in the Bellman equation for the promised value approach – is computed together with the policy functions.
value function – depends on both $\eta_1'$ and $\eta_2'$. However, because the knowledge of $(\eta_1, \eta_2)$ at time-one pins down $\eta_2'$ at time two, we write it as $w(\eta_1' | \eta_2', \delta = \delta_n)$ to emphasize that the dependence of the next period’s value function on $\eta_1'$.

The left panel shows how the overall objective function varies with the promised output, whereas the middle and right panels show how the two subcomponents of the overall objective function, today’s payoff and the discounted continuation value, vary with the promised output.

As shown in the middle panel, today’s payoff is constant, as it depends only on the current-period inflation rate and output that were promised in the previous period and thus does not depend on the promised output for the next period. Thus, what maximizes the discounted continuation value – shown in the right panel – also maximizes the overall objective function – shown in the left panel. As indicated by the solid vertical line in that panel, the discounted continuation value is maximized at around $\eta_1' = 0.2$, meaning that it is optimal to promise an output overshoot.

Why does a positive time-two output maximize the continuation value? There are two counteracting forces. On the one hand, because the time-two inflation is given – it is implied by the time-one output and inflation $\eta_{1,1}$ and $\eta_{2,1}$, as discussed above – promising the time-two output of zero maximizes the time-two payoff. On the other hand, given that the time-two inflation implied by $\eta_{1,1}$ and $\eta_{2,1}$ is positive, delivering zero time-two output implies that the time-three inflation is positive and even slightly higher than the time-two inflation rate because of the time-two Phillips curve constraint. The Phillips curve constraint means that, by promising a higher output for time two, the central bank ensures that the time-three inflation is closer to zero. And, the closer the promised inflation rate is to zero, the higher the continuation value is, as can be seen in the bottom-right panel of Figure 9. The continuation value is maximized at a positive time-two output because of these two counteracting forces.

While $\eta_1' = 0.2$ maximizes the time-one value, this is not the level of output the central bank ends up promising for $t = 2$ because of the ELB constraint on the policy rate. The ELB constraint on the policy rate puts a lower bound on the promised output the central bank can choose due to the Euler equation; given today’s output and the rate of inflation in the next period, the policy rate needs to be sufficiently low in order to support a low level of output in the next period. In Figure 11, any promised output below the dashed vertical line is associated with a negative policy rate in the current period. The maximum is attained when the promised output is at its lower bound and the policy rate is zero.

Turning to the Lagrange multiplier approach, we show in Figure 12 the policy functions and value function associated with the saddle-point functional equation (16). The time-one inflation, output and Lagrange multipliers are given by the crisis-state policy functions – shown in the top panels – evaluated at $(\phi_{1,-1}, \phi_{2,-1}) = (0, 0)$ and are indicated by the pentagrams. Thereafter, the dynamics of the economy are determined by the normal-state policy functions shown in the bottom panels and are traced by the black dots. The dynamics of inflation and output derived from the Lagrange multiplier approach are identical to those implied by the promised value approach, up to the accuracy of the numerical methods used. In the online appendix, we contrast the dynamics obtained from the promised value and

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19 With $y_2 = 0$, the time-two Phillips curve implies that $\pi_3 = \pi_2 / \beta$. © 2020 The Authors. *Oxford Bulletin of Economics and Statistics* published by Oxford University and John Wiley & Sons Ltd.
Figure 12. Policy functions from the Lagrange multiplier approach – model with effective lower bound –
Notes: $\phi_{1,-1}$ and $\phi_{2,-1}$ are the lagged Lagrange multipliers, whereas $\phi_1$ and $\phi_2$ are the Lagrange multipliers in the current period. The rate of inflation is expressed in annualized percent. The output gap is expressed in percent.

Lagrange multiplier approaches and show that the differences are of a magnitude in line with the numerical errors associated with the global solution methods.

V. Conclusion

In this paper, we characterized optimal commitment policies in three well-known versions of the New Keynesian model using a novel recursive approach – which we called the promised value approach – inspired by Kydland and Prescott (1980). Under the promised value approach, promised inflation and output act as pseudo-state variables, as opposed to the lagged Lagrange multipliers under the standard approach of Marcet and Marimon (2019). The Bellman equation from the promised value approach sheds new light on the trade-off facing the central bank and provides fresh perspectives on optimal commitment policies. The promised value approach can serve as a useful analytical tool for those economists interested in analysing optimal monetary policy.

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Supporting Information

Additional supporting information may be found in the online version of this article:

Technical Appendix for Online Publication: A Promised Value Approach to Optimal Monetary Policy.