Pion electromagnetic form-factor with domain wall fermions

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Motivated by recent measurements at J-Lab, the pion electromagnetic form-factor is investigated with quenched domain wall fermions and a renormalization group improved gauge action called DBW2. We see that quark mass dependence of the form-factor with finite momentum transfer $s$ is rather small.

1. Introduction

The pion electromagnetic (EM) or charge form-factor is one of the simplest quantity in QCD. Some new experiments on this quantity at relatively large momentum transfers are under way\textsuperscript{[1]}. While there exist a few lattice computations \textsuperscript{[2]}, none of them respect the chiral symmetry sufficiently well. The new experimental data with small errors might not only impose strong restrictions to some phenomenological models, but give stringent test of lattice QCD. We calculate the pion EM form-factor in the quenched lattice QCD with domain wall fermions (DWF) with several momentum transfers which partially cover the above experiments. The DWF enables us to access lighter quark masses than ever, because it has good chiral properties which are quite important for the Nambu-Goldstone bosons like a pion.

2. Simulation details

We employ the DWF action for the quark action. The number of sites in the fifth direction is $L_s = 12$ and the domain wall height $M_5 = 1.8$. Quark masses are taken to be $m_q a = 0.08, 0.06, 0.04$ and 0.02 whose pion masses in the physical unit are about 760, 660, 540 and 390 MeV, respectively.

The gauge action is a renormalization group improved gauge action called DBW2 with the lattice size $16^3 \times 32$,

$$S_G[U] = -\frac{\beta}{3} \left( 1 - 8c_1 \right) \sum_{x, \mu < \nu} P[U]_{\mu\nu} + c_1 \sum_{x, \mu \neq \nu} R[U]_{\mu\nu} \right)$$

where $P[U]_{\mu\nu}$ is the real part of the trace of the usual $1 \times 1$ plaquette in the $\mu, \nu$ plane at a point $x$ and $R[U]_{\mu\nu}$ is that of the $1 \times 2$ rectangles in the $\mu, \nu$ plane. $c_1 = -1.4069$ is computed non-perturbatively using the Swendsen’s blocking and the Schwinger-Dyson method. The gauge coupling is taken to be $\beta = 0.87$ which gives the lattice spacing $a^{-1} \sim 1.3$ GeV and a sufficient spatial lattice volume $V = (2.4 \text{fm})^3$ for a pion. The number of configuration is 100. Some fundamental chiral properties on the DWF and DBW2 actions are elaborated in ref.\textsuperscript{[3]}. Explicit chiral symmetry breaking due to the finiteness of the fifth direction $L_s$ is characterized by the residual mass $m_{\text{res}}$. In fact it is shown in ref.\textsuperscript{[3]} that the DBW2 gauge action gives smaller $m_{\text{res}}$ than those of other (Wilson, Symanzik and Iwasaki) gauge actions. Our present results of $m_{\text{res}}$ and the linearly extrapolated value are shown in Fig.\textsuperscript{[4]}. They are much smaller than the quark masses $m_q$.

Now we turn to the pion EM form-factor $F_\pi(Q^2)$ defined by

$$F_\pi(Q^2)(p_\mu + p'_\mu) = \langle \pi^+(p) | j_\mu | \pi^+(p') \rangle.$$  \hspace{1cm} (2)

where $q^2 = -Q^2 = (p - p')^2$ and $j_\mu$ is the EM current. We note $F_\pi(0)$ is the electric charge of a pion and we employ this relation as a numerical

\textsuperscript{*}We thank RIKEN, Brookhaven National Laboratory and the U.S. Department of Energy for providing the facilities essential for the completion of this work.
check. The structure of the 3-point function in eq.(2) in our calculation is schematically shown in Fig. 2. We use an appropriate smeared source and a local sink. The separation between the source and the sink is 24 in the lattice unit. A momentum is injected at the sink and flows into the current insertion, i.e., only one quark propagator has a non-zero momentum. This method is statistically efficient for small momentum transfers. The presently calculated momentum transfers of the form-factor are $\vec{p} = (0, 0, 0), (1, 0, 0)$ and $(1, 1, 0)$ in the unit of $2\pi/La$ with the spatial lattice size $L$. They correspond to physical values of $Q^2 = 0, 0.26, 0.53$ GeV$^2$, respectively. Larger momentum transfers will be investigated in near future. We employ the sequential source technique\[4\], when we solve the quark propagator with the finite momentum.

The EM current taken here is the local current, $j_\mu = \bar{q}_\mu q$, which is not conserved on the lattice and requires operator renormalization to compare with the matrix element in the continuum,

$$\langle \pi | j_\mu | \pi \rangle_{\text{cont}} = Z_V \langle \pi | j_\mu | \pi \rangle_{\text{lat}}$$ \hspace{1cm} (3)

where $Z_V$ is the vector current renormalization factor. Instead of $Z_V$, however, we use an axial-vector current renormalization factor $Z_A$, because this quantity is so far well investigated in the derivation of the pion decay constant and the relation $Z_A = Z_V$ is satisfied in DWF up to $O(a^2)$ errors. This equality is confirmed numerically in the DWF and DBW2 action within a few percent\[5\]. We show the present results of $Z_A$ in Fig. 3 in the chiral limit ($m_l = -m_{\text{res}}$), $Z_A = 0.7798(5)$.

### 3. Results

The computations were carried out by QCDSP in BNL and the Columbia university. We have calculated the pion EM form-factor from both the electric ($\langle \pi | j_\mu=4 | \pi \rangle$) and magnetic ($\langle \pi | j_\mu=1 | \pi \rangle$) sectors of the 3-point function and checked the consistency within the errors. We show in Figs. 4 and 5 as typical examples of calculated pion form-factors with the non-zero momentum transfers deduced from the matrix elements in the electric sector ($\mu = 4$). We see clear plateaus to give...
the renormalized pion form-factors in the figures.

All the fitted results after the renormalization are summarized in Table. We see excellent agreement with unity for $Q^2 = 0$, which means that our codes and the operator renormalization $Z_A$ are quite successful. Quark mass dependence of the form-factors with the finite momenta seems to be rather small, although they have relatively large statistical errors.

Table 1
Fitted results of the pion EM form-factor for each $Q^2[GeV^2]$ and quark mass (100 configurations).

| $m_f$ | $Q^2 = 0$ | $Q^2 = 0.26$ | $Q^2 = 0.53$ |
|-------|-----------|--------------|--------------|
| 0.08  | 1.003(19) | 0.783(24)    | 0.603(48)    |
| 0.06  | 1.010(22) | 0.779(31)    | 0.583(73)    |
| 0.04  | 1.013(27) | 0.761(44)    | 0.564(138)   |
| 0.02  | 1.011(34) | 0.689(77)    | n/a          |

4. Summary and outlook

We have calculated the pion electromagnetic form-factor using the quenched DWF and DBW2 gauge action. Because of the good chiral and scaling properties of the action, we have investigated lighter quark masses than ever. The present results with small momentum transfers show rather small quark mass dependence.

Calculations at larger momentum transfers are necessary to obtain a functional form of the form factor to be compared with experiments, or models such as vector meson dominance. Such calculations are now in progress.

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