TIDAL DISRUPTION OF A STAR BY A BLACK HOLE: OBSERVATIONAL SIGNATURE

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ABSTRACT

We have modeled the time-variable profiles of the Hα emission line from the nonaxisymmetric disk and debris tail created in the tidal disruption of a solar-type star by a $10^6 M_\odot$ black hole. Two tidal disruption events were simulated using a three-dimensional relativistic smooth particle hydrodynamics code to describe the early evolution of the debris during the first 50–90 days. We have calculated the physical conditions and radiative processes in the debris using the photoionization code CLOUDY. We model the emission-line profiles in the period immediately after the accretion rate onto the black hole became significant. We find that the line profiles at these very early stages of the evolution of the postdisruption debris do not resemble the double-peaked profiles expected from a rotating disk, since the debris has not yet settled into such a stable structure. As a result of the uneven distribution of the debris and the existence of a “tidal tail” (the stream of returning debris), the line profiles depend sensitively on the orientation of the tail relative to the line of sight. Moreover, the predicted line profiles vary on fairly short timescales (of the order of hours to days). Given the accretion rate onto the black hole, we also model the Hα light curve from the debris and the evolution of the Hα line profiles in time.

Subject headings: black hole physics — galaxies: nuclei — hydrodynamics — line: profiles

1. INTRODUCTION

1.1. Tidal Disruption of a Star by a Black Hole and Related Issues

A star in an orbit around a massive black hole can get tidally disrupted during its close passage by the black hole. After several orbital periods, the debris from the disrupted star settles into an accretion disk and gradually falls into the black hole (Rees 1988; Cannizzo et al. 1990; Syer & Clarke 1992; Loeb & Ulmer 1997). As material gets swallowed by the black hole, intense UV or soft X-ray radiation is expected to emerge from the innermost rings of the accretion disk (Rees & Frates 1976; Lightman & Shapiro 1977; Frank 1978; Phinney 1989; Sembay & West 1993; Magorrian & Tremaine 1999; Syer & Ulmer 1999). For black hole masses $M_{bh} < 10^6 M_\odot$, tidal disruption theory predicts flares with luminosities of the order of the Eddington luminosity with durations of the order of months and with spectra that peak in the UV/X-ray domain band (Rees 1988; Evans & Kochanek 1989; Ulmer 1999; Kim et al. 1999; Gezari et al. 2003). High-energy flares from the central source illuminate the debris, the photons get absorbed, and some are reemitted in the optical part of the spectrum (i.e., the light is “reprocessed”). One of the spectral lines in which this phenomenon can be observed is the Balmer series Hα line ($\lambda_{rest} = 6563$ Å).

The disruption of a star begins when the star approaches the tidal radius, $r_t \approx r_\rm{tidal}(M_{bh}/M_\star)^{1/3}$, the point where the surface gravity of the star equals the tidal acceleration from the black hole across the diameter of the star ($r_\star$ and $M_\star$ are the radius and mass of the star, and $M_{bh}$ is the mass of the black hole). A $10^6 M_\odot$ black hole is often used as a prototypical example in tidal disruption calculations. This choice is motivated by the criterion for a solar-type star to be disrupted before it crosses the black hole horizon (i.e., the Schwarzschild radius, $r_S$) in order for emission to be observable. For supermassive black holes with $M_{bh} > 10^8 M_\odot$, $r_S > r_t$, and the star falls into the black hole before it gets disrupted.

The tidal disruption process has been the subject of many simulations (Carter & Luminet 1982, 1983; Bicknell & Gingold 1983; Evans & Kochanek 1989; Khokhlov et al. 1993; Laguna et al. 1993b; Flogolov et al. 1994; March et al. 1996; Diener et al. 1997; Ayala et al. 2000; Ivanov & Novikov 2001; Ivanov et al. 2003). It has been shown that tidal processes in the vicinity of a massive black hole could lead to tidal capture, tidal heating, and tidal spin-up of a star (Novikov et al. 1992; Alexander & Kumar 2001; Alexander & Hopman 2003; Alexander & Morris 2003) and, in some cases, ultimately to the explosion of the star. Such explosions, as well as accretion of postdisruption debris, should manifest themselves as luminous flares from the centers of galaxies (Carter & Luminet 1982; Rees 1988). Stars close to a black hole may experience mixing or may ejection some of their mass (Alexander 2004). As a consequence, stellar populations in nuclear clusters are expected to be somewhat unusual in comparison with populations whose evolution was not affected by a massive black hole (Alexander & Livio 2001; Di Stefano et al. 2001). This has important implications for observations of the stellar cluster in the center of our Galaxy (Ghez et al. 2000; Schödel et al. 2002; Eckart et al. 1999; Figer et al. 2000; Gezari et al. 2002), where a high concentration of otherwise rare blue He supergiants has been observed (Krabbe et al. 1991; Najarro et al. 1994).

The tidal disruption and accretion of stars can fuel black holes in the centers of galaxies (Hills 1975; Duncan & Shapiro 1982; David et al. 1987a, 1987b; Murphy et al. 1991; Freitag & Benz 2002; Yu 2003), and its contribution to nuclear activity in galaxies and the growth of the black hole mass depends on the rate of disruption events in a galaxy. The predicted tidal disruption rate in a typical inactive galaxy is $10^{-4}–10^{-5}$ yr$^{-1}$ (Magorrian & Tremaine 1999; Alexander 2004). This value is consistent with the rate of UV/X-ray outbursts observed with ROSAT from inactive nuclei selected as tidal disruption

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candidates (Donley et al. 2002). The rate of tidal disruptions in active and more luminous nuclei is estimated to be lower, with the lowest value of $10^{-9}$ yr$^{-1}$ for galaxies with the most massive black holes. This may occur partly because massive central black holes ($M_{bh} > 10^8 M_\odot$) swallow stars promptly, without disruption, and partly because stars are less centrally concentrated in these galaxies (Magorrian & Tremaine 1999). The observed UV/X-ray flaring rate in these galaxies is about $9 \times 10^{-4}$ galaxy$^{-1}$ yr$^{-1}$ (Donley et al. 2002), suggesting that in such nuclei, outbursts may be due to another mechanism, such as accretion disk instabilities (Siemiginowska et al. 1996; Burderi et al. 1998).

The tidal encounter of a compact star with a black hole can also result in emission of gravitational waves, which may be observable with upcoming instruments. More specifically, compact stars (helium stars, white dwarfs, neutron stars, and stellar-mass black holes), which can withstand large tidal forces without being disrupted, may get captured in relativistic orbits around a supermassive black hole. Because of the inspiral and decay of the orbit, those objects are expected to emit the peak of their gravitational wave power in the LISA frequency band (Hils & Bender 1995; Sigurdsson & Rees 1997; Freitag 2001). It has been recently suggested by Freitag (2003) that very low mass main-sequence stars (MSSs; $M \ll 1 M_\odot$) may contribute to events detected by LISA, which was not previously expected for capture of these objects by a supermassive black hole. Although MSSs produce a relatively weak gravitational signal during the in-spiral, compared to compact objects, their detection in the Galactic center is more likely because such stars have a predicted close-encounter rate that is 1 order of magnitude higher than that of white dwarfs (WDs) and about 2 or more orders of magnitude higher than that of neutron stars (NSs) and stellar-mass black holes. These compact MSSs are expected to produce a strong enough signal to allow for 0.5–2 detections from our Galactic center, with a signal-to-noise ratio of 10 or higher, for a LISA mission duration of 1 yr (Freitag 2003). Moreover, MSSs are expected to be disrupted relatively early during the in-spiral, giving rise to possibly detectable electromagnetic flares. The sudden appearance of an electromagnetic counterpart to a transient gravitational wave source, expected in the case of MSSs and helium stars, could allow identification of the progenitor. In the case of a tidal disruption event in another galaxy, the coincidence of an electromagnetic flare and a gravitational wave signal would provide an indication that the event occurred at the very nucleus of the galaxy and possibly allow the measurement of the redshift. More compact objects that can spiral in without being disrupted (such as WDs, NSs, and black holes) are expected to produce stronger gravitational wave signatures and weaker or no electromagnetic waves, with the exception of WDs, in which tidal interaction may trigger thermonuclear explosion (García-Senz et al. 1999).

1.2. Observational Motivation: Transient Emission Lines in Inactive Galaxies and LINERs

In view of the above theoretical considerations, it is necessary to make predictions of the likely observational signatures of a tidal disruption event. The prompt UV/soft X-ray flash that is expected to accompany the disruption does provide strong evidence for such an event and has in fact been detected in a number of galaxies with ROSAT and the Hubble Space Telescope (Brandt et al. 1995; Grupe et al. 1995a, 1995b, 1999; Donley et al. 2002; Bade et al. 1996; Komossa & Greiner 1999; Greiner et al. 2000; Renzini et al. 1995; Li et al. 2002). However, the duration of this flash is short enough that it can easily be missed. Aftereffects with a longer duration, such as line emission from the debris, have a better chance of being detected. If the appearance of emission lines just after an X-ray flare could be detected from the same object, it could be used to identify the tidal disruption in the early phase and would provide strong support for the overall picture, but such cases are rare (see Cappellari et al. 1999; Gezari et al. 2003).

A set of tantalizing observations in the past decade show that several LINERs (low-ionization nuclear emission regions; Heckman 1980) have transient Balmer emission lines, which are often double-peaked; examples include NGC 1097 (Storchi-Bergmann et al. 1993), M81 (Bower et al. 1996), NGC 4450 (Ho et al. 2000), NGC 4203 (Shields et al. 2000), and NGC 3065 (Eroceles & Halpern 2001). Such line profiles are characteristic of rotating disks and resemble the persistent double-peaked Balmer lines found in about 10%–20% of broad-line radio galaxies and about 3% of all active galaxies (e.g., Eroceles & Halpern 1994, 2003; Strateva et al. 2003). Their abrupt appearance in LINERs led to suggestions that this transient event was related to the tidal disruption of a star by a supermassive, nuclear black hole (Eroceles et al. 1995b; Storchi-Bergmann et al. 1995) or a change in the structure of the accretion disk associated with a change in accretion rate (Storchi-Bergmann et al. 1997).

To investigate the possibility of line emission from the postdisruption debris and to evaluate the suggestion that the transient double-peaked lines of LINERs are related to tidal disruption events, we have undertaken a calculation of the strength and profile of the Hα line emitted from the debris. In §2, we describe two smoothed particle hydrodynamics (SPH) simulations of tidal disruption on which we base our further calculation of the line properties. Our line profile calculation follows the method used for line profiles emitted by relativistic, Keplerian disks and is described in §3. In §4 we present the resulting line profiles, and in §5 we discuss the physical conditions in the debris and the approximations used. In §6 we summarize our conclusions and consider future prospects.

2. QUALITATIVE DESCRIPTION OF TWO SPH SIMULATIONS

Tidal disruption simulations were carried out with a three-dimensional relativistic SPH code in order to study the dynamical evolution of the postdisruption debris. The SPH code used provides a description of relativistic fluid flows in a static curved spacetime geometry (Laguna et al. 1993a, 1993b). We use it to simulate the tidal disruption of a star in the potential of a Schwarzschild black hole.

The MSS is modeled as a polytrope with index $\Gamma = 5/3$. The density profile of the predisruption star is determined by the Lane-Emden equations. The star is initially placed on a parabolic orbit at a distance of $700r_g$ from the black hole, where $r_g = r_s/2 = GM/c^2 = M$ is the gravitational radius and $M$ is the mass of the black hole. Hereafter, we use units in which $G = c = 1$, where $G$ is the gravitational constant and $c$ is the speed of light, and we adopt $r_g = M$ as a natural unit of length. The gravitational radius of a $10^6 M_\odot$ black hole is $r_g = 1.48 \times 10^{11}$ cm = 4.92 lt-s, and the dynamical time at a given radius is $t_\text{dyn} \sim (r^3/GM)^{1/2} = 4.92(r/r_g)^{3/2}$ s.

The strength of the tidal encounter is given by the ratio of tidal radius to pericentric distance, $\eta = r_p/r_g$. For the case of a $1 M_\odot$ star and a $10^6 M_\odot$ black hole, $r_p \approx 47r_g$. The two SPH simulations describe the tidal disruption for the case of a
mildly relativistic encounter, $\eta = 1.2$. This value has been selected as a likely scenario for tidal disruption of an MSS. We do not investigate the strongly relativistic cases in which tidal compression could lead to the explosion of a star because such an explosion could introduce an uncertainty in the distribution of the debris mass over binding energy and consequently in the spatial distribution and kinematics of the debris.

The self-gravity of the star is accounted for initially. Once the star gets disrupted, the self-gravity becomes unimportant and the debris particles follow nearly Keplerian orbits. It can be shown for the case of the Keplerian potential that the rate of return of bound debris to the pericenter follows $dM/dt \propto r^{-5/3}$ (Rees 1988; Phinney 1989). This behavior of the debris return rate has been observed in our SPH simulations. Once bound debris starts to rain down on the black hole, it is expected to cause the initial rapid rise in the emitted UV/X-ray light curve and steady decay with the power-law index of $-5/3$ later on.

The two different simulations have 5000 and 20,000 particles (hereafter 5k and 20k, respectively) contributing equally to the mass of a 1 $M_\odot$ star. The 5k SPH calculation follows the debris for 94 days in total. After 34 days significant accretion onto the black hole begins. Our investigation follows the evolution of the line profiles in the last 60 days. The 20k SPH simulation spans 53 days, during which the evolution of the line profiles was followed for the last 6 days (Table 1). Using both the 5k and 20k SPH simulations in the line profile modeling, we take advantage of the longer time span in the former and better resolution achieved with the larger number of particles in the latter.

Figure 1 shows particle distribution maps after the second pericentric passage, at the beginning of the accretion phase and at the end of the 5k simulation. At the early stages of the tidal event most of the particles were located in the pronounced tidal tail. Sixty days later, about 20% of the particles are scattered from the tidal tail and form a quasi-spherical distribution, with most of its mass concentrated in the equatorial plane (Cannizzo et al. 1990; Loeb & Ulmer 1997; Ulmer et al. 1998; Ulmer 1999; Menou & Quataert 2001). This is a consequence of the intersection of the leading part of the tidal tail with itself (Kochanek 1994; Lee et al. 1996; Kim et al. 1999; Ayal et al. 2000). We refer to the spheroidal part of the debris as the halo and to its planar component as the disk. The remaining 75% of particles are still confined to the tail, and 1% are accreted onto the black hole. There is a concern that some fraction of particles $\sim (\tau_{\text{run}}/\tau_{\text{run}})\sqrt{N}$, where $\tau_{\text{run}}$ and $N$ are the total duration and total number of particles in the run, respectively, of the halo are an artifact of the SPH simulation. This can be due to the tendency of the SPH numerical method to preserve the constant number of neighbor particles for each particle during the calculation. In the regions with a small density of particles this leads to a “smoothing” over a large spatial range, and it may introduce the scatter of particles from the debris plane to the halo. These particles cannot be distinguished from the population of particles scattered out of the debris plane by the intersection of the tidal tail with itself. We further discuss the implications of the spheroidal halo for the emission-line profiles and total Hα luminosity in § 5.2.

The velocity distribution in the tail is “bimodal” where the central part of the tail exhibits very low radial velocities; particles on the near (right-hand) side of the tail “flow” toward the black hole, while particles on the far (left-hand) side are moving in the opposite direction. This is a consequence of the energy distribution throughout the debris in the disruption process: after the initial disruption event 50% of the debris stays bound to the system and the other 50% is unbound. This effect has been predicted by theory in the case of stellar disruption after a single flyby of the star (Rees 1988) and has been observed in tidal disruption simulations. The symmetry in the distribution of the debris over binding energy is a consequence of the spin-up of the star at the expense of orbital kinetic energy. The spin-up initially causes the development of a quadrupolar deformation. As the tidal interaction gets stronger, the star starts to shed its mass, since the material in the stellar bulge has reached the escape velocity at the star’s surface. One portion of the stellar debris ends up deeper in the potential well of the black hole, which causes further spread in binding energy of the debris. This effect determines which portion of the debris stays bound to the black hole (Rees 1988). Following the second passage of the debris through pericenter, approximately 66% of the mass is unbound, 33% remains bound, and only about 1% is accreted by the black hole. The maximal approaching and receding velocities in the debris, with respect to the stationary observer positioned at infinity, are of order $10^3$ km/s. The dynamical evolution in the 20k run is the same; it is just followed over a shorter evolutionary timescale. Since the tidal tail includes a large fraction of bound and unbound particles in both simulations, its morphology and velocity field greatly influence the observed line profiles.

3. LINE PROFILE CALCULATION AND TIME DELAY OF REPROCESSED LIGHT

3.1. Calculation of Line Profiles

We follow the line profile calculations carried out by Chen & Halpern (1989) and Eracleous et al. (1995b) to obtain the observed profile from a Keplerian, relativistic, thin disk in the weak-field approximation. The description of the debris as a flat, thin structure is justified by the fact that the height of the debris is 3 orders of magnitude less than its dimensions in the orbital plane. The main objective of the calculation is to obtain the final expression for the flux density in the observer’s frame as a function of parameters defined in the reference frame of the debris. Figure 2 shows the coordinate system and the geometry of the debris. The observer is located on the positive $z$-axis, at a distance $d = +\infty$, and above the orbital plane at $i = 30^\circ$ to the $z'$-axis. Since the calculation is presented in cited papers, we just introduce its main steps and comment on its application to the case of tidal disruption debris.
The total emission-line flux received from the debris by an observer at infinity is given by an integral over the plane of the image produced at infinity, namely,

\[ F = \int \frac{d\nu}{C_{23}} \int \int d\Sigma I_{\nu}/C_{23} \tag{1} \]

where \( \nu, I_{\nu}, \) and \( d\Sigma \) are the frequency, specific intensity, and solid angle element measured in the frame of the image (i.e., of the observer). Using the impact parameter of rays at infinity, \( b \), as a coordinate in the image plane and exploiting the Lorentz invariance of the quantity \( I_{\nu}/\nu^3 \) and the fact that the debris is confined to a plane, equation (1) can be transformed into an expression for the flux density (i.e., the line profile) in terms of coordinates and physical quantities in the source frame (see detailed derivation in Chen et al. 1989; Chen & Halpern 1989; Eracleous et al. 1995b),

\[ f_{\nu} = \int \int d\Sigma I_{\nu} \]

\[ = \frac{M^2 \nu_0 \cos i}{d^2} \int_{\xi_{\text{in}}}^{\xi_{\text{out}}} d\xi \int_{0}^{2\pi} d\phi' I_{\nu}(\xi, \phi') \psi(\xi, \phi'), \tag{2} \]

where the new polar coordinates in the debris plane are the dimensionless radius \( \xi \equiv r/r_g \) and the azimuthal angle \( \phi' \). In practice, the integration is performed by summing over particles, assigning to each particle an emissivity according to its position, as derived from calculation with the code CLOUDY (Ferland 1996; see § 4.1 in this paper). The limits of integration describe the portion of the debris that emits the H\( \alpha \) line, between radii \( \xi_{\text{in}} \) and \( \xi_{\text{out}} \). The function \( D(\xi, \phi') \), the “Doppler factor,” is determined by the phase-space distribution of the emitting particles and the metric and describes the effects of gravity and the motion of the emitting particles on the energies of the emitted photons. The function \( \psi(\xi, \phi') \) is
determined by the geometrical distribution of the emitting particles and the metric and describes the effects of curved trajectories of light rays. In the special case in which the debris is confined to a plane, these functions are given by

\[
D = \frac{(1 - 2/\xi)^{1/2}}{\gamma} \left\{ 1 - \beta_r \left( 1 - (b/r)^2 (1 - 2/\xi)^{1/2} \right) \right\}^{-1} + \frac{\beta_r (b/r) \sin \phi \sin \phi'}{(1 - \sin^2 \phi \cos^2 \phi')^{1/2}},
\]

with \(\gamma\) the Lorentz factor, \(\beta_r\) and \(\beta_r'\) the radial and azimuthal velocities of debris particles in the source frame, and

\[
\psi(\xi, \phi') = 1 + \frac{1}{\xi} \left( 1 - \sin i \cos \phi' \right).
\]

The above analytic expression for \(\psi(\xi, \phi')\) has been derived in the weak-field approximation and is accurate to order \(\xi^{-1}\). This approximation is appropriate in our case because the portions of the debris that experience a strong gravitational field are also highly ionized and make a negligible contribution to the \(H_\alpha\) flux (see discussion in § 5.1). The ratio \(b/r\) describes how rays emitted from the debris are mapped to points in the image at infinity and is given by \(b/r = (1 - \sin^2 \phi \cos^2 \phi')^{1/2} \psi(\xi, \phi')\). Finally, the Lorentz factor is given by \(\gamma = [1 - \beta_r^2 (1 - 2/\xi)^{2} - \beta_r'^2 (1 - 2/\xi)^{1/2} - \beta_r^2 (1 - 2/\xi)^{1/2} - \beta_r'^2 (1 - 2/\xi)^{-1/2}]^{-1}\).

The emission properties of the debris are described by the local specific intensity, \(I_{\nu c}\). We take the local line profile to be a Gaussian corresponding to a velocity dispersion \(\sigma\) (in units of the speed of light). The width of the local line profile represents not only internal motions of the line-emitting gas (not captured by the SPH simulation) but also the velocity range between the discretized debris points we use in our numerical integration. We further assume that the emissivity of the line is a power law with radius (see discussion in §§ 4.1 and 5.1). Therefore, we write the specific intensity as

\[
I_{\nu c} = \frac{c_0 \xi^{-\eta}}{2(2\pi)^{1/2}\sigma} \exp \left[ -\frac{(1 + X - D)^2}{2D^2\sigma^2} \right].
\]

where \(X\) is defined by \(1 + X \equiv \nu/\nu_0\), where \(\nu\) and \(\nu_0\) are observed and rest-frame frequency, respectively, and \(c_0\) is a constant.

In the final model, the line profiles are described by the following parameters: the inner and outer radius of the line-emitting portion of the debris, \(\xi_{in}\) and \(\xi_{out}\), the particle emissivity power-law index \(\eta\) and \(\eta_{disk}\), the inclination of the debris plane \(i\), the local velocity dispersion \(\sigma\) in units of \(c\), and the radius of the central continuum source \(\xi_0\). The last parameter is relevant to the calculation of the travel time across the debris, which we describe in § 3.2, below.

3.2. Time Delay of the Reprocessed Light Emitted by the Debris

In the emission model we adopt, a central source of finite dimensions illuminates the debris. The luminosity of this source is proportional to the accretion rate onto the black hole. Because of the finite velocity of light, at any given time (in the observer’s frame), different portions of the debris are seen to respond to a different level of illumination. It is also noteworthy that the length scales and timescales in this problem span a very large dynamic range. As a consequence, the light crossing time of the outer portions of the debris is longer than the dynamical time of the inner portions, which makes it necessary for us to follow the redistribution of the debris in phase space and the variations of the X-ray source carefully.

It is simple to show that travel time delay for light rays can be written as

\[
\Delta t_{\text{travel}} = (\xi - \xi_0) (1 - \sin i \cos \phi'),
\]

where \(\xi\) and \(\phi'\) are the coordinates of a particle in the orbital plane, \(\xi_0\) is the radius of the central source, and \(i\) is the inclination. Since light rays travel in the gravitational potential of the black hole, they suffer an additional, relativistic time delay, which can be calculated from the equation of geodesics for photons (Weinberg 1972, p. 202). The assumptions are that photons travel in an isotropic gravitational field and that their trajectories can be considered coplanar with the observer and the black hole. We consider only the gravitational delay caused by the black hole’s potential and assume that the debris does not have any significant gravitational influence on a light ray. This is a reasonable assumption, since \(\beta_{deb}/\beta_{bh} \sim 10^{-12}\).

In our notation, the gravitational time delay for a nonrotating black hole can be expressed as

\[
\Delta t_{\text{gr}} \approx \frac{1}{(\xi - \xi_0)} + 2 \ln \left[ \frac{\xi + (\xi^2 - \xi_0)^{1/2}}{\xi \cos i} \right] - \cos \phi' \left[ \frac{1 - \cos i}{1 + \cos i} \right]^{1/2} + 2 \ln \left[ \frac{1 + \sin i}{\cos i} \right].
\]

The overall delay for a particle caused by light travel and general relativistic effects is then

\[
\Delta t = \Delta t_{\text{travel}} + \Delta t_{\text{gr}}.
\]

We only need to account for relative time delays, which we calculate relative to a ray coming from the origin of the coordinate system of the debris. We find that the general relativistic time delay in our calculations never exceeds 10% of the travel time delay, and it is typically of order a few percent.

4. RESULTS AND IMPLICATIONS

4.1. Light Curves and Emissivity of the Debris

The output of the 5k run comprises 351 frames describing the evolution of the debris morphology over 60 days of accretion, with a time step of approximately 4 hr (see Table 1). This is a fine enough temporal resolution to trace the redistribution of particles in the debris. For comparison, a particle at \(r = 200r_g\) orbits the black hole with a period of approximately 24.5 hr. During the accretion phase of the simulation the number of particles decreases because of infall in the black hole. Tracking this number allows us to follow the accretion rate and construct the X-ray light curve. The total amount of mass accreted in this run is about \(10^{-2} M_{\odot}\). The X-rays resulting from accretion illuminate the debris out to large distances from the black hole and power the emission of \(H_\alpha\) photons.

The output of the 20k run consists of 204 temporal frames spanning 6 days of accretion with a time step of about 45 minutes (the coverage is not even; it includes gaps, since the
behavior of the debris can be captured even with sparse sampling. Because of the short time span of accretion, the number of accreted particles is small (less than 0.1% of the total mass), and consequently, the illuminating, X-ray light curve is not smooth. To address this issue we compute the Hα light curve for two more, fiducial, illumination light curves: one consistent with the debris return rate predicted by theory (≈10−50; Rees 1988; Phinney 1989) and the other constant in time. This means that the simulation would lead to a Hα luminosity of the debris component as calculated for the time-averaged value of incident luminosity $L_{\text{acc}} = 1.5 \times 10^{44} \text{ ergs s}^{-1}$.

On the basis of the particle distribution in the simulation, we find that the density in the tidal tail decreases with time because the tail gets stretched as the debris evolves (cf. Fig. 1). We find that for the typical SPH time frame the density distribution in the tail can be approximated as $n_{\text{H}2} \propto \xi$ and in the range $n_{\text{H}2} = 10^{14} - 10^{15} \text{ cm}^{-3}$. With path lengths of $9 \times 10^{13}$ and $\sim 5 \times 10^{15} \text{ cm}$, the corresponding tail column densities are in the range $N_{\text{H}2} = 10^{27} - 10^{31} \text{ cm}^{-2}$. The low and high values of the column density correspond to lines of sight along the short and long axes of the tail, respectively. (See Table 2 for values of physical parameters in the debris.) The density of particles scattered from the tail in the spheroidal halo around the black hole is fairly low in comparison. The halo density reaches a maximum in the plane of the debris ($\geq 3$ species overdensity), where particles orbiting the black hole form a disk of radius $\sim 2500 \text{yr}$.

The number density in the disk decreases with radius as $n_{\text{H}2} = 1 \times 10^{12} \text{ cm}^{-2}(\xi/500)^{-2.1}$. The corresponding column densities in the directions orthogonally and radially through the disk are $N_{\text{H}2} = 7 \times 10^{20}$ and $3 \times 10^{25} \text{ cm}^{-2}$, respectively. The spheroidal part of the halo, formed from particles scattered out from the plane of the debris, has an estimated number density of $n_{\text{H}2} = 3 \times 10^{11} \text{ cm}^{-3}(\xi/500)^{-1.4}$ and corresponding column density of $N_{\text{H}2} = 1 \times 10^{25} \text{ cm}^{-2}$.

The CLOUDY calculations show that the Hα line power emitted by the mostly neutral tidal tail in response to X-ray illumination decays with distance from the center as $Q_{\text{tail}} \propto r^{-6.6}$, as a consequence of the density distribution in the tail and the geometry of the debris (i.e., illumination incidence). The corresponding particle emissivity for the tail derived from the surface emissivity decays with the distance as $\epsilon_{\text{tail}} \propto r^{-2.4}$ (Fig. 3). The disk and halo components are almost completely ionized, and only $1$ in $10^{37}$ hydrogen atoms is neutral, on average. The emissivity of the disk decreases with distance from the black hole as $\epsilon_{\text{disk}} \propto r^{-2.2}$, approximately. The equivalent particle emissivity distribution for the disk is $Q_{\text{disk}} \propto r^{-0.06}$. Therefore, in our description of the surface emissivity of the debris (eq. [5]) we use power-law indices of $\beta_{\text{tail}} = 2.4$ for the tail and $\beta_{\text{disk}} = 0.06$ for the disk. The results of the CLOUDY calculations and the power-law prescriptions derived from them are summarized in Figure 3. The total Hα luminosities contributed by the tail and the disk for the time-averaged value of the illumination $L_{\text{acc}} = 1.5 \times 10^{44} \text{ ergs s}^{-1}$, as calculated by CLOUDY, are $L_{H\alpha} \approx 1 \times 10^{36} \text{ ergs s}^{-1}$ and $L_{H\alpha} \approx 1 \times 10^{37} \text{ ergs s}^{-1}$, respectively. The calculated value of

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**Table 2**

| Debris Region | $N_{\text{H}2}$ (cm$^{-2}$) | $n_{\text{H}2}$ (cm$^{-3}$) | $m_{\text{H}2}/n_{\text{H}2}$ | $m_{\text{H}2}/n_{\text{H}2}$ | $T$ (K) | $U$ (erg) | $L_{H\alpha}$ (ergs s$^{-1}$) |
|---------------|-----------------|-----------------|------------------|------------------|-------|--------|-----------------|
| Tail...........| $10^{27} - 10^{31}$ | $10^{14} - 10^{15}$ | $\sim 1$ | $\sim 10^{-3}$ | $5 \times 10^{4}$ | $10^{-5} - 0.1$ | $1 \times 10^{36}$ |
| Disk...........| $10^{27} - 10^{25}$ | $10^{10} - 10^{12}$ | $10^{-8} - 0.3$ | $0.7 - 1$ | $1 \times 10^{5}$ | $20$ | $1 \times 10^{37}$ |
| Halo............| $10^{25}$ | $10^{7} - 10^{12}$ | $10^{-7}$ | $1$ | $1 \times 10^{4}$ | $27$ | $6 \times 10^{38}$ |

* The column density: low and high values correspond to directions orthogonally and radially through the debris component.
* The average value of temperature over radius.
* Ionization parameter: range of values in the tail and average in the disk and halo.
* The Hα luminosities from the debris components as calculated for the time-averaged value of incident luminosity $L_{\text{acc}} = 1.5 \times 10^{44} \text{ ergs s}^{-1}$. 

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2 We have verified that the energy distribution of the debris particles in our simulation would lead to a $\propto r^{-5.3}$ return rate of particles to the pericenter, based on the Keplerian orbits.
the luminosity for the spheroidal component is $L_{\text{halo}} \approx 6 \times 10^{38}$ erg s$^{-1}$ (also listed in Table 2).

We discuss the emission properties of the debris in § 5.1 and their implications for the observability in § 5.3. Finally, with the above emissivity prescriptions we calculate the observed H$\alpha$ luminosity curve of the debris at a particular time step by computing the time at which the light was emitted from the debris and by finding the ionizing flux that was illuminating that location at the time the emission occurred, according to the light-travel time from the black hole to that particular region of the debris.

Figure 4 shows three different H$\alpha$ light curves from the debris confined to a plane (assuming $\zeta_\text{in} = 500$, $\zeta_\text{out} = 40,000$) during the 60 day accretion phase of the 5k simulation. Figure 4a shows the accretion luminosity on a logarithmic scale (solid curve), calculated from the accretion rate of the debris in the SPH simulation. The UV/X-ray luminosity curve is arbitrarily scaled and overplotted on the top of the H$\alpha$ curve for comparison. It is noticeable that the H$\alpha$ light curve departs from the accretion light curve at late times, though the departure appears small in the logarithmic plot (used here because of the large dynamic range of the light curves). The same effect is more noticeable in Figure 4b, where the accretion luminosity is proportional to $r^{-3/2}$ and the H$\alpha$ light curve is plotted on a linear scale. The H$\alpha$ light curve roughly follows the shape of the incident UV/X-ray light curve at early times but decays faster at late times. The faster decay in the H$\alpha$ light curve reflects the debris evolution in time: as the tail becomes more elongated, the incident photons travel a longer way to illuminate the debris. Consequently, the intensity of the illuminating light gets lower in the later stages of the tidal disruption event. The relative decay rate of the H$\alpha$ light curve with respect to the UV/X-ray light curve diminishes about 80 days after the accretion started. We find that this late relative rise in the H$\alpha$ luminosity is due to the increase in the number of particles in the disk component. As particles diffuse from the high-density tail to the lower density disk, in later stages of the simulation, their emission efficiency increases and they contribute a significant amount of H$\alpha$ light to the light curve. To isolate the effect of the debris evolution in time from the evolution of the illuminating light curve, we calculate the H$\alpha$ light curve in the case of constant illumination (Fig. 4c). Here the relative departure of the H$\alpha$ light curve from the UV/X-ray light curve...
can be interpreted as a consequence of the expansion and redistribution of the debris. The Hα luminosity appears to level off at late times because the debris disk begins to settle into a quasi-steady configuration.

In summary, the observed Hα flux depends sensitively on the UV/X-ray light curve, on the distribution of matter that makes up the inner portion of the debris, and on how quickly particles redistribute themselves in phase space. The main features of the Hα light curve are an initial rise followed by a decline, with superposed fluctuations. The initial rise is a consequence of the propagation of the initial illumination front through the debris at the speed of light. The fluctuations are a result of the fluctuations in the accretion rate, which are caused, in turn, by the finite number of particles employed in the simulation. The decay rate of the Hα light curve is determined by the decay rate of the UV/X-ray light curve and the debris expansion and redistribution rate.

4.2. Line Profiles from the Debris and Their Variability

We have computed sample line profiles emerging from the debris for the following choices of model parameters:

1. Inclination angle of the plane of the debris, i.—We assumed that the observer is located on the positive z-axis, at a distance d → ∞ at i = 30° to the z-axis. Changing the inclination changes the values of the projected velocities (i.e., the overall width of the line profile) but has very little effect on the line profile shape otherwise.

2. Inner and outer radius, \( \xi_{\text{in}} \) and \( \xi_{\text{out}} \)—The adopted inner radius of the debris is the inner boundary of the region from which Hα emission is expected to emerge. The choice of the inner radius depends on physical conditions in the debris, as we explain in § 5.1. Here we explore several cases with \( \xi_{\text{in}} \) between 200 and 10,000. The outer radius \( \xi_{\text{out}} = 40,000 \) is naturally set by the dimensions of the system.

3. Particle emissivity power-law index, \( \beta \)—As noted in § 4.1 above, we find that \( \beta_{\text{disk}} = 2.4 \) in the debris tail and \( \beta_{\text{disk}} = 0.06 \) in the disk. These indices describe the emissivity per particle as a power law and correspond to indices \( \beta_{\text{ss}} \) and \( \beta_{\text{sd}} \) in the description of the surface emissivity. The adopted values for emissivity indices significantly influence the profile shapes. Higher values of \( \beta \) weight the emissivity toward smaller radii where the projected velocity is higher.

4. Velocity dispersion, \( \sigma \), and central source radius \( \xi_0 \)—The adopted value of the velocity dispersion for the profiles presented was \( \sigma = 100 \text{ km s}^{-1} \). A lower limit on the velocity dispersion is set by the velocity difference measured for pairs of adjacent particles. This value represents the dispersion due to the finite number of resolving elements in the simulation and equals 20 km s\(^{-1}\) for 90% of particles close to the debris plane. The constraint on the upper limit of the velocity dispersion comes from the velocity dispersion due to small-scale turbulence, \( \sigma_{\text{turb}} \), which is the distance from the massive black hole, and \( \xi_0 \) is the smallest dimension of the fluid (i.e., width or thickness of the tail). Since the real turbulence could be substantially smaller than the upper limit because of dissipation by internal shocks, we adopt a value of 100 km s\(^{-1}\). Larger values of \( \sigma \) produce wider profiles and smooth out sharp features. The central source radius is arbitrarily chosen to be \( \xi_0 = 200 \). It implies a central source of finite dimensions such as a corona of ionized plasma, or a vertically extended accretion flow in the innermost parts of an accretion disk. Its effect on the line profiles is rather small.

In Figures 5–9 we show sample line profiles to illustrate how they evolve in time and how they are affected by the choice of model parameters and by the orientation of the observer. Figure 5 is a “trailed spectrogram” summarizing the temporal evolution of the line profiles from the two different SPH runs; it is a two-dimensional map of the Hα emission as a function of projected velocity and time. Figure 6 shows a different representation of the evolution of the line profile with time, which effectively comprises selected time slices from the trailed spectrogram. Figures 7 and 8 show how the inner radius of the line-emitting region and the azimuthal orientation of the observer affect the observed line profiles. Figure 9 shows the effect of the different values of velocity dispersion on the shape of the line profiles. The main properties and features of our results are as follows:

Profile variability with time.—A property that is immediately obvious in the line sequence is the change of the profile shape with time (Figs. 5 and 6). It is noticeable that the adopted low value of the velocity dispersion allows us to resolve individual particles in the trailed spectrograms, orbiting around the black hole. The evolution of the line intensities in time roughly follows the behavior of the UV/X-ray luminosity but decays somewhat faster in time (see § 4.1). The multi-peaked line profile is a consequence of the velocity field of the inner debris, which consists of the inner portion of the tidal tail that is falling toward the black hole (toward the observer) and debris that is rotating around the black hole after being scattered. The line profiles and their variability could be observationally important features of the debris just formed from tidal disruption. The variable line profiles might be observed and recognized on the relatively short timescale of hours to days.

Effect of the inner radius, \( \xi_{\text{in}} \)—The profiles become broader as the inner radius of the line-emitting regions decreases, since higher velocity gas resides at smaller radii (see Fig. 7). The approximate full width at zero intensity of the profiles ranges from \( 4500 \text{ km s}^{-1} \) for \( \xi_{\text{in}} = 10,000 \) to \( 18,000 \text{ km s}^{-1} \) for \( \xi_{\text{in}} = 200 \). We find that line profiles change from the profiles dominated by the emission redward from the rest wavelength for \( \xi_{\text{in}} < 1000 \) to narrower profiles dominated by the blueward emission from the tail for \( \xi_{\text{in}} > 1000 \), since for large values of \( \xi_{\text{in}} \), the high-velocity rotating gas in the vicinity of the black hole is excluded and the dominant contributions to the line profile come from the tidal tail. The intensity of the line also decreases with increasing inner radius, making the outer regions of the debris harder to observe.

Effect of observer orientation, \( \phi_0 \)—Because of the non-axisymmetric geometry and velocity field, the line profiles emitted by the debris depend on the orientation of the tidal tail relative to the observer. In Figure 8 we show the effect of the azimuthal orientation \( \phi_0 \) of the debris, with respect to the observer. The values of \( \phi_0 \) are \( 45°, 90°, 120°, 180°, 220°, \) and 270°, as measured in a counterclockwise direction from the positive \( x \)-axis to the observer’s line of sight. These can be compared with the profile corresponding to the same time in Figure 6 for \( \phi_0 = 0° \). The position of the peaks in Figure 8 varies relative to the rest wavelength, since the relative direction of bulk motion of the material depends on the observer’s orientation. For example, it is possible to distinguish the emission from the tail for the range of azimuthal orientations \( 90°–220° \). The tail emission in these profiles appears as the most blueshifted peak, since these are the orientations for which different portions of the tail flow toward the observer.
Effect of velocity dispersion, $\sigma$.—In Figure 9 we show the effect of four different values of the velocity dispersion in calculation of the emission-line profiles. A value of $20\,\text{km s}^{-1}$ is the lower limit of velocity dispersion set by the discrete nature of the SPH simulation, while the upper limit of $800\,\text{km s}^{-1}$ is determined by the small-scale turbulence in the debris. The velocity dispersion of particles in the halo (about 1500 particles in the 20k run) is significantly higher and reaches $6000\,\text{km s}^{-1}$. As the value of the velocity dispersion increases, the profile features get smoothed out, until only a smooth, double-peaked profile is observed.

Effect of illuminating light curve.—We have computed model profiles for several different X-ray illumination light curves, keeping all the other parameters fixed. We used (a) the light curve obtained from the accretion rate in the 5k simulation, (b) the light curve from the accretion rate as predicted by theory, i.e., $\propto t^{-5/3}$ (Rees 1988; Phinney 1989), and (c) a light curve that is constant in time (Fig. 4). We find that the line profile shapes do not depend sensitively on the shape of the light curve. This is a consequence of the centrally "weighted" emissivity profile of the debris, which causes the innermost emission region to be the dominant contributor of the H$\alpha$ light. In the innermost region of the debris the dynamic range in light-travel times is not large; therefore, the illumination of the innermost emitting region is almost instantaneous. Over the very short light crossing time of the central emitting region, the gradient in the UV/X-ray light curve is small and the illumination is nearly constant over this region. The fast fluctuations in the illuminating light curve, on the other hand, are smoothed out during reprocessing in the debris and cannot be identified in the H$\alpha$ light curve.

In summary, we find that profiles are not significantly influenced by the shape of the illuminating light curve. The profile shapes are affected, however, by the inner radius of the line-emitting region and redistribution of the debris. The inner radius can change with the advance or recession of the ionization front into the debris, which is controlled primarily by the density of the debris. The most notable effect of the inner radius is on the width of the profile. Since the physical conditions can change very rapidly during flares, this mechanism causes the line profiles to change on the light crossing time-scale (minutes to hours) and evolve from wide and multi-peaked to narrow and vice versa (the recombination time is negligible in comparison to the light crossing time of the debris; therefore, particles in the debris respond effectively instantaneously to changes in the incident flux). The redistribution of the debris in phase space, on the other hand, takes more time: $\sim24\,\text{hr}$ for particles in the innermost part of the emitting region. This redistribution of the debris also may cause a transition from narrower to wider profiles; however, it takes place gradually, on the timescale of days.

Fig. 5.—Trailed spectrogram of the simulated H$\alpha$ emission-line profiles from the 5k simulation spanning 60 days (left) and from the 20k simulation spanning 6 days (right). This is effectively a two-dimensional intensity map vs. projected velocity of the emitting material and time. Darker shades correspond to higher intensity. The scale on the right represents time since the tidal disruption event.
5. DISCUSSION

5.1. Physical Conditions in the Debris and Radiative Processes

We have calculated the physical conditions and radiative processes in the debris using the photoionization code CLOUDY, version 94 (Ferland 1996). It is necessary to know the physical conditions in order to assess the validity of the assumptions made in our line profile calculations.

Since the physical conditions in the tidal tail, disk, and halo differ noticeably, these three regions naturally emerge as separate components of the tidal debris. The spheroidal halo is an oblate structure of particles scattered out of the plane of the debris. The disk is produced by the flow of particles from the tail that turn around the black hole and form a higher density circular component concentrated close to the plane of the debris. The disk shows a smooth transition to the halo in terms of density and physical parameters (see Table 2). The physical differences among the components arise as a consequence of the number density, which is several orders of magnitude higher in the tail. The particle density of the halo is uncertain, because of the numerical scatter, and consequently its luminosity contribution is also subject to uncertainty. We nevertheless have calculated and presented physical properties of the spheroidal component, as given by the SPH simulation, and we discuss the implications of its presence for the line profiles and the observability in the next section.

The temperature of the debris tail reaches $3 \times 10^4$ K in the hottest parts of the tail (i.e., the illuminated face of the tail) and equals 5000 K on average in the partially ionized and neutral parts of the tail. The temperature in the disk ranges from 8000 K in the inner region to $1 \times 10^5$ K at the outer rim of the disk, with the average $\sim 10^5$ K. The mean temperature of the halo is $1 \times 10^4$ K and ranges from $3 \times 10^7$ K in the central parts of halo to $1 \times 10^4$ K in the outer halo. The ionization parameter is calculated for all components as $U = \Phi_H / n_H c$, where $\Phi_H$ is the flux of ionizing photons, $n_H$ is the total hydrogen density, and $c$ is the speed of light. The ionization parameter in the tail ranges from 0.1 in the parts of the tail closest to the source of ionization to $10^{-5}$ on the far side of the tail. The ionization parameter in the disk is almost constant throughout the disk, with a value of about 20. The ionization parameter in the halo is higher, with an average value of about 27. As a consequence, the disk and the halo are in a much higher state of ionization relative to the tail. The disk shows a wide range of hydrogen ionization fractions over its radius, with the strongest ionization at the outer rim of the disk where density is lowest. In the halo the fraction of neutral hydrogen atoms ranges from $10^{-10}$ to $10^{-7}$, with an average value close to $10^{-7}$. In contrast, the tail is mostly neutral, with only one hydrogen ion per $10^3$ hydrogen nuclei. The properties of the tidal disruption regions are summarized in Table 2.

Fig. 6.—Sequence of H$\alpha$ profiles emitted from the region $\xi \in 500 - 40000$ over a period of 6 days (20k run). The relative time from the beginning of the accretion phase onto the black hole is marked next to each profile. The accretion phase begins 47 days after the tidal disruption. The inclination of the debris plane and the velocity shear are as marked on the figure.
The ionization state of the debris, as well as its abundance and density, determines the radiative processes dominant in the debris. The implication of the assumed solar metallicity is that emission lines from metals play an important role in the cooling of the debris. We focus our attention on processes relevant to the final H/CO luminosity. Two processes contribute to the H/CO luminosity of the debris: recombination and collisional excitation. The dominant recombination channel is the recombination of a photoelectron to the \( n = 3 \) level followed by a decay to the \( n = 2 \) level via emission of an H\(\alpha\) photon. Emission by collisional excitation occurs when hydrogen atoms in the \( n = 1 \) and 2 levels are promoted to the \( n = 3 \) level by collisions with energetic photoelectrons and then deexcite radiatively. The relative contribution of H\(\alpha\) emission through recombination relative to collisions is

\[
\frac{L_{H\alpha,\text{rec}}}{L_{H\alpha,\text{coll}}} = \frac{\alpha_{\text{rec}} n_{\text{H II}} V_{\text{rec}}}{\alpha_{\text{coll}} n_{\text{H I}} V_{\text{coll}}},
\]

where \( \alpha_{\text{rec}} \) and \( \alpha_{\text{coll}} \) are the respective coefficients for recombination and collisional processes that lead to emission of an H\(\alpha\) photon, \( n_{\text{H II}} \) and \( n_{\text{H I}} \) are the number density of hydrogen ions and atoms, and \( V_{\text{rec}} \) and \( V_{\text{coll}} \) are the parts of the debris volume that give rise to emission through recombination and collisional deexcitation, respectively.

The recombination coefficient for an electron to recombine from the continuum to the \( n = 3 \) level can be written as \( \alpha_c = \sum_{n=3}^\infty \alpha_n \) for a given temperature. The adopted recombination coefficients for the tail, disk, and halo are \( \alpha_{\text{tail}} = 3.4 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1} \), \( \alpha_{\text{disk}} = 1.8 \times 10^{-14} \text{ cm}^3 \text{ s}^{-1} \), and \( \alpha_{\text{halo}} = 1.8 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1} \) (Osterbrock 1989). Similarly, it is possible to estimate the rate of collisions that will promote an electron to the \( n = 3 \) state. The high optical depths in the Lyman series cause a significant electron population in the \( n = 2 \) level. The photoelectron energy distribution for low-energy levels of hydrogen differs from the Boltzmann distribution, and the number density is not sufficiently high for collisions to lead to thermal equilibrium (eq. [10.57] of Krolik 1999). In particular, thermalized energy levels will be \( n > 1 \) for the tail, \( n > 2 \) for the halo, and \( n > 3 \) for the disk, and the relative populations of these levels will be in agreement with their statistical weights. It is possible then to estimate the collisional excitation coefficients for transitions from the \( n = 1 \) and 2 levels to the thermalized \( n = 3 \) level (eq. [3.21] of Osterbrock 1989). This approximately yields collisional excitation rates for the tail, disk, and halo of \( \alpha_{\text{tail}} = 1.2 \times 10^{-11} \text{ cm}^3 \text{ s}^{-1} \), \( \alpha_{\text{disk}} = 1.1 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1} \), and \( \alpha_{\text{halo}} = 9.0 \times 10^{-11} \text{ cm}^3 \text{ s}^{-1} \).
In the disk marked on the top of the figure. 6 days, 6 hr, 0 minutes. Size of the emitting region and inclination are as instrumental resolution of 2 affect the total output in H/C28 depths recombination and collisions from equation (9) to be about calculate the ratio of respective relative contributions from ratio of emitting volumes for two processes, it is possible to fractioned in the thin ionized and partially ionized layers of the tail (we refer to them as the tail “skin”). From here a certain fraction of H/C25 illuminating photons, and typically has a value of order 107. photons traveling across the tail, along the lines of incidence of which leads to the ejection of the debris with high velocities, perpendicular to the plane. However, none of the above-mentioned structures were resolved beyond doubt in the simulations. A halo-like structure is expected to be in hydrostatic equilibrium, and its size should be determined by the radiation pressure from the central source. Ulmer et al. (1998) and Loeb & Ulmer (1997) argue that the luminosity incident upon the halo can rarely be exactly tuned for the structure to rest in equilibrium. They further calculate that for a super-Eddington luminosity the halo will expand and cool down on a timescale of months to years, until it becomes gravitationally unbound and is blown away. For a sub-Eddington luminosity, on the other hand, the halo is expected to collapse because of insufficient support from radiation pressure.

On the basis of its physical conditions, it is obvious that, if real, the halo observed in our simulations would make the

5.2. Spheroidal Halo: Implications for Line Profiles and Luminosity

In our predictions of the observational signature of the postdisruption debris, we have not included any contributions from the spheroidal halo because we doubt that this structure, as predicted by our SPH simulation, is real. In this section we discuss this issue further and justify our approach. We also describe qualitatively the effect of such a halo on the observational appearance of the debris.

The halo is very likely produced artificially because a number of particles are scattered from the disk every time the flow of particles turns around the black hole and intersects with the inflowing stream of particles. The number of particles contributed to the halo by numerical scatter is proportional to $\sqrt{N}$, where $N$ is the total number of particles in the run, and multiplied by a factor $\tau_{\text{halo}}/\tau_{\text{dyn}}$ as described in § 2. The dynamical timescale of the disk is about 7 days for the outermost particles, which implies that the number of scattered particles is of the order of 1000. This is an upper limit of the uncertainty, where the total number of particles in the halo, in the 20k run, reaches the number of 1500 particles in the later stages of the debris evolution.

It has been pointed out by several authors that a spheroidal halo (Ulmer et al. 1998; Loeb & Ulmer 1997; Cannizzo et al. 1990; Alexander & Livio 2001) with a central radiatively supported torus (Rees 1988; Loeb & Ulmer 1997; Cannizzo et al. 1990; Evans & Kochanek 1989) may form around the black hole as a consequence of a self-compression of the flow, which leads to the ejection of the debris with high velocities, perpendicular to the plane. However, none of the above-mentioned structures were resolved beyond doubt in the simulations. A halo-like structure is expected to be in hydrostatic equilibrium, and its size should be determined by the radiation pressure from the central source. Ulmer et al. (1998) and Loeb & Ulmer (1997) argue that the luminosity incident upon the halo can rarely be exactly tuned for the structure to rest in equilibrium. They further calculate that for a super-Eddington luminosity the halo will expand and cool down on a timescale of months to years, until it becomes gravitationally unbound and is blown away. For a sub-Eddington luminosity, on the other hand, the halo is expected to collapse because of insufficient support from radiation pressure.

On the basis of its physical conditions, it is obvious that, if real, the halo observed in our simulations would make the

halo is optically thin to H/C11 photons ($N_{H/C11} \approx 2 \times 10^{-2}$) and the electron scattering optical depth is about 5. In the late stages of the debris evolution $N_{H/C11} \approx 1 \times 10^4$ and $\tau_e$ stays approximately the same in the halo. The high values of $\tau_e$ of the halo material destroy the fraction of the H/C11 photons emitted by the part of the debris embedded in the halo. The evolution of the spherical halo would further help the process of fading of H/C11 emission with time along with the decreasing accretion luminosity. For the purposes of radiative transfer calculations, we have modeled the tail, the disk, and the halo as three separate components, and therefore we do not account for the secondary absorption and electron scattering by the halo of H/C11 photons created in the tail. Both opacity mechanisms cause the destruction of H/C11 photons if the spherical halo is fully evolved and possibly wipe out the H/C11 emission line. If the same process operates in the centers of galaxies that are tidal disruption candidates, it should be possible to observe the disappearance of the broad H/C11 emission line on the scale of months.

![Figure 9. – H/C11 emission line profiles simulated for four different values of velocity dispersion. A velocity dispersion of 100 km s$^{-1}$ is the equivalent of an instrumental resolution of 2 Å. The relative time for the profile frames is 6 days, 6 hr, 0 minutes. Size of the emitting region and inclination are as marked on the top of the figure.](image)

With the values of the rate coefficients, the ionization state of the tail, disk, and halo (n/H$_{II}$/n$_{II}$ $\approx 10^{-3}$ in the tail and 10$^7$ in the halo and disk on average), and taking into account the ratio of emitting volumes for two processes, it is possible to calculate the ratio of respective relative contributions from recombination and collisions from equation (9) to be about $1 \times 10^{-6}$, $6 \times 10^3$, and $2 \times 10^6$. These values are not unexpected: collisional excitation is the dominant mechanism for production of H/C11 photons in the tail, where the density is highest, while recombination is dominant in the disk and halo.

Some of the H/C11 photons created by the two processes are destroyed on the way out of the debris. The mechanisms that affect the total output in H/C11 luminosity are the absorption in the H/C11 transition and electron scattering, with respective optical depths $\tau_{H/C11}$ and $\tau_e$. The optical depth $\tau_{H/C11}$ is very high for the H/C11 photons traveling across the tail, along the lines of incidence of illuminating photons, and typically has a value of order 10$^7$. Electron scattering in the tail has an average optical depth of $\tau_e \approx 10$. Consequently, the majority of H/C11 photons are created in the thin ionized and partially ionized layers of the tail (we refer to them as the tail “skin”). From here a certain fraction of H/C11 photons escapes the debris and reaches the observer. In the disk $\tau_{H/C11} \approx 4 \times 10^6$ and $\tau_{H/C11} \approx 8$. In the early stages of accretion, while the luminosity is still super-Eddington, the
dominant contribution to the line emission from the debris. In the late stages of the debris evolution, our simulations show the halo becoming optically thick to Hα photons. Moreover, the long diffusion time of photons in the halo \( t_{\text{diff}} \approx \tau r/c \approx 3.5 \text{ hr} \), where \( r \) is the inner radius of the halo) may smear out the time variability of the emission-line profiles, which is one of the main signatures of the disruption event. The high value of the velocity dispersion of the particles in the halo would smear out the line profiles and very likely make them unobservable. It is therefore necessary to address in future studies what fraction of the halo (if any) is really present and what fraction is contributed by the numerical scatter.

If a halo manages to form, survives the super-Eddington phase, and achieves hydrostatic equilibrium, it will be transparent to Hα photons only during the super-Eddington phase. At later times, we expect that it will become optically thick and will be the primary source of illumination of the outer debris. Moreover, it will fade on a timescale of months to years. In this picture, which is consistent with the predictions of Loeb & Ulmer (1997), the shape of the observed line profiles should be similar to the profiles computed here for large values of the inner radius (see Fig. 7). Whether the profile (noticeable here for \( \xi = 10, 000 \) at the wavelength \( \lambda = 6,500 \text{ Å} \)) will appear as blueshifted or redshifted would depend on the orientation of the debris tail with respect to the observer.

5.3. Observability of Emission Lines and Their Uniqueness as a Tidal Disruption Signature

The CLOUDY calculation predicts a time-averaged Hα luminosity from the tidal debris of about \( 10^{36}, 10^{37} \), and \( 6 \times 10^{38} \) ergs s\(^{-1}\) for the tail, disk, and halo, respectively. In the earlier stages of the disruption event when the UV/X-ray luminosity is super-Eddington, the Hα luminosity is expected to be up to 80 times higher than its average value and comparable to that of tidal disruption candidates observed in the local universe (see Fig. 4). The examples are NGC 4450 (at 16.8 Mpc) with an Hα luminosity of \( L_{\text{Hα}} = 1.8 \times 10^{39} \) ergs s\(^{-1}\) (Ho et al. 2000) and NGC 1097 (at 22 Mpc) with \( L_{\text{Hα}} = 7.7 \times 10^{39} \) ergs s\(^{-1}\) (Storchi-Bergmann et al. 1995). Thus, the emission-line signature of a tidal disruption event should be detectable at least out to the distance of the Virgo Cluster. In practice, however, the detection of such emission lines from low-luminosity sources may be complicated by the weak contrast relative to the underlying stellar continuum.

On the basis of observational constraints from known tidal disruption candidates, it should be possible to detect some variable properties of the line profiles and light curves predicted here. One of the first observable effects of tidal disruption should be UV/X-ray flash accompanied by a decaying light curve, mirrored in the delayed response of the Hα light curve of the debris, with some scatter. The line profile intensities are expected to decay accordingly in time. Other effects to look for are the change in the number of peaks in the line profile and relative fluctuations in the intensity of the peaks, as well as their shift in wavelength.

The temporal variability of the Hα emission line profiles from the postdisruption debris is one of the important indicators of a tidal disruption event. In order to capture the rapid profile variability, because of the variable illumination, the exposure time should be comparable to the light crossing time of the innermost regions of the line-emitting debris, which has the fastest and strongest response to the ionizing radiation. Longer exposures are expected to capture the average shape of the rapidly varying line profiles. The light crossing time of the innermost regions of the debris is about \( 8 \xi / M_6 \) minutes (where \( \xi = \xi / 100 \)), while the exposure times are typically about 30–60 minutes (for galaxies at the distance of the Virgo Cluster, for example). Thus, if an event is caught early in its evolution and the light crossing time is relatively long (i.e., \( M \geq 10^6 \text{ M}_\odot \)), there is a chance of detecting variability caused by changing illumination over the course of one to a few nights. On longer timescales, variability is caused by changes in the structure of the debris. In the presence of the spheroidal halo, the variability of the lines may be modified by the long diffusion timescale of photons through the halo. The component of the tidal tail outside the halo will then still respond to the variability but on the timescale set by the light reprocessed by the halo.

In view of the predictions from our profile calculation, an important question is whether variable multipeaked line profiles can originate in some other physical scenario or can be regarded as the unique signature of tidal disruption. Multi-peaked emission lines are likely to be the signature of an inhomogeneity in the phase-space distribution of the emitting material. Because of the asymmetry of the emitting region, the direction of the observer has a large influence on the observed line profile, making it difficult, if not impossible, to infer the exact emission geometry from a particular multipeaked profile. Nevertheless, the variability pattern of the line profiles can serve as a general indicator of a tidal disruption event. Additional observational indicators can be used in conjunction to diagnose a tidal disruption event, for example, a sharp X-ray/UV flash preceding the appearance of the emission lines, an emission-line spectrum indicative of a hard ionizing continuum (i.e., the presence of ionic species with high ionization potentials), and the characteristic decay of the emission-line flux on timescales of weeks to months after the event (the flux of different emission lines is expected to decay at different rates; see, e.g., Eracleous et al. 1995a).

5.4. Approximations in the Calculation

The finite resolution of the simulation could introduce uncertainties in the accretion light curve caused by the discretized accretion of the stellar material onto the black hole. Another possible effect is that the finite number of particles does not completely reveal the morphology of the debris and some of its components may stay hidden. For example, when the majority of the particles are confined to the tail it is hard to say whether there is an accretion disk forming around the black hole out of a small number of particles. Consequently, there is a concern that profiles calculated for low values of the inner radius, i.e., \( \xi_{\text{in}} = 200 \), may represent the contribution from a small number of particles in the central region and therefore be dominated by small-number noise.

A possible source of error is the assumption of a thin, flat structure (i.e., confined to a plane). In the early stages of the evolution of the debris, the majority of the particles are still in the tidal tail, located in a plane, which makes the assumption valid. During the evolution of the debris the number of particles that orbit the black hole in an almost spherical distribution increases (Cannizzo et al. 1990; Loeb & Ulmer 1997; Ulmer et al. 1998; Ulmer 1999; Menou & Quataert 2001). At that point it is possible to distinguish three structural components of the debris: the relatively planar tidal tail and the disk and the spheroidal halo. The halo is made up of particles scattered from the tail by shocks during the pericentric approach of the debris or during the intersection of the tail with
itself (Kochanek 1994; Lee et al. 1996; Kim et al. 1999; Ayal et al. 2000) and some fraction of particles contributed by the numerical noise. Since the tidal tail includes the majority of the particles and most of the halo mass is concentrated close to the equatorial plane, even in our last frame, the assumption of a thin disk is still reasonable. If, however, the mass in the spherical halo increases at very late times, the assumption of a planar geometry needs to be reconsidered.

We also adopt the weak-field approximation in our calculations, which is a fairly small source of error (of order 1% and less). This is valid, since we adopt $\xi_{\text{in}} = 200$ as the innermost radius of the line-emitting debris. The few particles interior to this radius would not contribute to the $H_\alpha$ emission because their close proximity to the source of ionizing radiation would make them fully ionized. In the case of a physical scenario in which emission of $H_\alpha$ is not possible because of a highly ionized debris, the same profile formalism can be used to calculate the emission from other lines emitted under these conditions.

6. CONCLUSIONS

We modeled the emission-line luminosity and profile from the debris released by the tidal disruption of a star by a black hole in the early phase of evolution. Our model predicts prompt optical evolution of postdisruption debris and profile shapes different from circular and elliptical disk model profiles. Since line profiles observed so far in LINERs look more disklike and evolve slowly, the observations are likely to have caught the event at late times ($\geq$6 months after the initial disruption), after the debris has settled into a quasi-stable configuration.

The line profiles can take a variety of shapes for different orientations of the debris tail relative to the observer. Because of the very diverse morphology of the debris, it is almost impossible to uniquely match the multi-peaked profile with the exact emission geometry. Nevertheless, the profile widths and shifts are strongly indicative of the velocity distribution and the location of matter emitting the bulk of the $H_\alpha$ light. Profile shapes do not depend sensitively on the shape of the light curve of the X-rays illuminating the debris. They strongly depend on the distance of the emitting material from the central ionizing source, which is a consequence of the finite propagation time of the ionization front and the redistribution of the debris in phase space. It may be possible to distinguish between the two effects observationally on the basis of their different characteristic timescales. The onset of the optically thick spheroidal halo should cause the disappearance of the broad $H_\alpha$ emission line on the timescale of months and give rise to a narrower, strong, blueshifted or redshifted emission line, arising from the portion of the tidal tail unobscured by the halo.

If X-ray flares and the predicted variable profiles could be observed from the same object, they could be used to identify the tidal disruption event in its early phase. The X-ray flares can be promptly detected by all-sky synoptic X-ray surveys and high-energy burst alert missions such as Swift. The evolution of the tidal event may then be followed with optical telescopes from the ground on longer timescales and give insight into the next stage of development of the debris. Thus, simulations of the tidal disruption process on longer timescales (of order several months to a few years) are sorely needed. Calculations of the long-term evolution of a tidal disruption event can predict the type of structure that the debris finally settles into and whether its emission-line signature resembles the transient double-peaked lines observed in LINERs. This study would provide an important insight into the evolution of LINERs.

Finally, the observed rate of tidally disrupted solar-type stars can constrain the rate of captured compact objects (which are important gravitational wave sources) and the capture rate of main-sequence stars in our Galaxy, which are expected to emit the peak of the gravitational radiation in the LISA frequency band and can be detected in the local universe.

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