Beamforming Design for Large-Scale Antenna Arrays Using Deep Learning

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Abstract—Beamforming (BF) design for large-scale antenna arrays with limited radio frequency chains and the phase-shifter-based analog BF architecture, has been recognized as a key issue in millimeter wave communication systems. It becomes more challenging with imperfect channel state information (CSI). In this letter, we propose a deep learning based BF design approach and develop a BF neural network (BFNN) which can be trained to learn how to optimize the beamformer for maximizing the spectral efficiency with hardware limitation and imperfect CSI. Simulation results show that the proposed BFNN achieves significant performance improvement and strong robustness to imperfect CSI over the conventional BF algorithms.

Index Terms—Deep learning (DL), millimeter wave (mmWave), beamforming (BF) design, large-scale antenna arrays, neural network (NN), beamforming neural network (BFNN).

I. INTRODUCTION

A. Background and Motivations

Recently beamforming (BF) design, especially, hybrid analog and digital beamforming (HBF) design, for millimeter wave (mmWave) communication systems with large-scale antenna arrays has been receiving much attention for its main advantage of providing enough BF gains to compensate for the severe path loss at affordable hardware cost and power consumption [1], [2]. It has been recognized that among all the difficulties in this complicated BF optimization problem the most difficult one is the constant modulus constraint on analog BF due to its phase-shifter-based architecture [3], [4].

In the conventional model-based design approach to handle this difficulty, there have been some algorithms that use mature mathematical methods, such as orthogonal matching pursuit (OMP) [1], generalized eigenvalue decomposition (GEVD) [3], and manifold optimization [4], etc. However, these algorithms either require some approximations to simplify the original objective function, or require a lot of serial time-consuming iterations to obtain a solution. Moreover, most of them assume perfect channel state information (CSI).

In another aspect, recently works on intelligent communications with the help of data-based deep learning (DL) technology have shown its great potential in dealing with traditional complicated and challenging problems [5], [6], [7], [8]. Inspired by these works, in this letter, we devote to applying the DL technology to solve the complicated BF design problem for mmWave systems with hardware limitation and imperfect CSI. Our motivations can be explained from the following three aspects:

- First, it is well known that as the BF design is a quite complicated non-convex problem due to the joint optimization of multiple variables and the constant modulus constraint, it is unlikely to find a closed-form optimal solution [1]. As DL has been regarded as an efficient technology to deal with intractable problems [8], [9], it would be interesting to see what could be obtained if using DL to solve the BF optimization problem.
- Second, through a large number of training iterations with a lot of samples, the DL-based schemes have been shown to have the ability to understand the complicated characteristics of the wireless channels [5]. Compared with the conventional works assuming perfect CSI [1], [3], [4], [10], the DL based approach is expected to possess strong robustness to imperfect CSI.
- Third, most efficient traditional BF algorithms require a number of optimization iterations to achieve good performance which consume a lot of time [3], [4], [10]. However, thanks to the acceleration of the graphics processing unit on a neural network (NN), the DL-based schemes can operate fast by parallel computation and thus be more applicable for high speed communications.

B. Novelty and Contributions

In this letter, we propose a DL-based BF design approach and develop a BF neural network (BFNN) which can be trained to learn how to optimize the beamformer for maximizing the spectral efficiency (SE) with hardware limitation and imperfect CSI. The contributions can be summarized as follows:

- New design approach: As the analog beamformer has a specific architecture consisting of a number of phase shifters and it outputs analog signals directly to the transmit antenna array, we cannot apply the traditional DL design approach [5], [6] by replacing the beamformer by a black box (a multi-layer NN) and training it in the whole communication link. Instead, we propose a novel DL design approach by developing the BFNN which directly outputs the optimized beamformer based on the input of the estimated CSI.
- Loss function: Different from traditional NN for physical layer optimization, where the loss function is defined as the mean square error (MSE) between the true signals (labels) and the recovered ones [5], [6], [11], in the developed BFNN, we propose a new self-defined loss function which is closely relevant to a certain performance metrics (e.g. SE) and does not need explicit labels.
- Handling the constant modulus constraint: We propose to add a novel Lambda layer as the final layer of the BFNN,
which can guarantee that the output analog beamformer satisfies the constant modulus constraint.

- Robustness to imperfect CSI: As the proposed BFNN can be trained from the input of imperfect CSI to learn how to approach the ideal SE, we show from simulation results that the BFNN achieves significant performance improvement and strong robustness to imperfect CSI over the conventional BF algorithms.

Since in the mmWave HBF design the optimal digital beamformer normally has a closed-form solution \[3, 4\], as an initial work and for the ease of presentation, in this letter we focus on the analog BF design, which is the most difficult part in the HBF design, and consider the scenario of a large-scale antenna array with only one radio frequency (RF) chain. However, after introducing the detailed design approach and showing the effectiveness through simulations, we will finally discuss the generality of the proposed DL-based BF design approach to more complicated scenarios such as HBF with multiple RF chains.

II. SYSTEM MODEL

Consider in Fig. 1 the downlink of a narrowband multiple-input and single-output (MISO) mmWave system with the analog BF (precoding) architecture, where a base station (BS) with one RF chain and \(N_t\) antennas transmits one data stream to a user with only one receive antenna.\(^1\) Let \(s\) denote the transmitted symbol with normalized average symbol energy, i.e., \(E(|s|^2) = 1\). The symbol is first multiplied by a scalar digital precoder \(v_{\text{D}}\), and then by an \(N_t \times 1\) analog precoder \(v_{\text{RF}}\), which is implemented using phase shifters. The finally precoded signal is then given by \(x = v_{\text{RF}}v_{\text{D}}s\).

In general, as shown in \[2\], a multiple-input and multiple-output (MIMO) mmWave channel \(H\) can be characterized by a geometry-based channel model as follows

\[
H = \sqrt{\frac{N_t N_r}{L}} \sum_{l=1}^{L} \alpha_l \phi_l^R \phi_l^T,
\]

where \(\alpha_l\) denotes the complex gain of the \(l\)th path, and \(\phi_l^R\) and \(\phi_l^T\) denote the antenna array response vectors at the BS and the user, respectively, with \(\phi_l^R\) and \(\phi_l^T\) denoting the azimuth angles of departure and arrival associated with the \(l\)th path. In the considered system, as \(N_t = 1\), we simply set \(\alpha_l \phi_l^T = 1\) and denote the resulting channel vector as \(h^R\).

Then, the received signal at the user can be represented by

\[
r = h^R v_{\text{RF}}v_{\text{D}}s + n, \quad n \text{ is the additive noise satisfying the circularly symmetric complex Gaussian distribution with zero mean and covariance } \sigma^2.\]

In this letter, the SE, which has been widely used in existing BF designs \[4, 10\], is chosen as the optimization objective, which is given as follows for the considered system

\[
R = \log_2 \left( 1 + \frac{1}{\sigma^2} ||h^R v_{\text{RF}}||^2 \right),
\]

Considering the constant modulus constraint, \(||v_{\text{RF}}||^2 = 1\), for \(i = 1, \ldots, N_t\), and the maximum transmit power constraint \(||v_{\text{RF}}v_{\text{D}}||^2 \leq P\), it can be proved that the optimal \(v_{\text{D}}\) for maximizing \(R\) is given by \(\sqrt{P/N_t}\). Then, the BF optimization problem for \(v_{\text{RF}}\) is given by

\[
\text{minimize} \quad \log_2(1 + \frac{1}{\sigma^2} ||h^R v_{\text{RF}}||^2)
\]

subject to \(||v_{\text{RF}}||^2 = 1\), for \(i = 1, \ldots, N_t\), (3)

where \(\gamma = \frac{P}{\sigma^2}\) denotes the signal to noise ratio (SNR).

III. DL MODEL AND THE DESIGN OF BFNN

In this section, we first introduce the new challenges when applying the DL technology to solve \[5\] with imperfect CSI. Then, we describe the BFNN architecture and elaborate three key characteristics of the BFNN in response to the challenges.

A. Challenges

As mentioned in Section \[13\], since the analog beamformer has a specific architecture containing phase shifters and its analog output cannot be realized by a digital NN, we cannot follow the traditional approach to replace the beamformer by a multi-layer NN and train it in the BS-user communication link. Here, we propose a different DL design approach by designing a BFNN that directly outputs \(v_{\text{RF}}\) to solve \[3\]. However, this design is not trivial due to the following three challenges.

- What should the input of the BFNN be? Most existing DL-based works \[5, 11\] took the received baseband digital signal as the input. However, this approach cannot be applied here as the received signal itself is a function of the analog precoder to be optimized. Furthermore, the number of dimensions of the received signal (one receive antenna) is much lower than that of the precoder (\(N_t\)) to be optimized.

- How to guarantee that the output \(v_{\text{RF}}\) satisfies the constraint? As it is well known that complex output is not well supported by most DL frameworks (e.g., Tensorflow, Pytorch), it would be more difficult to further impose the constant modulus constraint on the output.

- What should the label be when training the BFNN? In almost all of the conventional intelligent communication designs \[5, 7, 11\], the label is set exactly as the transmitted bits or perfect CSI. However, it is difficult to find a proper label for this BF design problem. For example, if we take an optimized analog beamformer based on a traditional algorithm as the label, it is obvious that the resulting BFNN would not perform better than the traditional algorithm.

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\(^1\) Although this letter focuses on the narrowband analog BF design with the aid of DL, the design approach can be generalized to a broadband MIMO mmWave system with HBF, as will be discussed in Section \[4\].
The architecture of the proposed BFNN is shown in Fig. 2(a). It consists of a number of dense units and a self-defined Lambda layer at the end. Each dense unit consists of a batch normalization layer, a dense layer, a rectified linear unit (Relu) activation function and a dropout layer [9], as shown in Fig. 2(b). For each unit or layer, its input and output tensors along with their dimensions are depicted accordingly. In response to the above three challenges, we make the following specific consideration when designing the BFNN.

1) Input of the BFNN: Inspired by the traditional model-based RF designs [3], [4], [10] with the assumption of perfect CSI at the transmitter, we assume that there is a channel estimator at the BS and set the estimated CSI, \( h_{\text{est}} \), as the input of the BFNN. Note that as the input is not necessary to be the perfect CSI, the BFNN is expected to learn how to optimize the beamformer with imperfect CSI via the training process. Besides, in order to enhance the generalization ability for different SNRs, the SNR value \( \gamma \) associated with each sample is also input to the BFNN.

2) Lambda Layer: To ensure that the output of the BFNN, i.e., \( v_{RF} \), is a complex vector satisfying the constant modulus constraint, an innovative self-defined Lambda layer is added at the end of the BFNN, as shown in Fig. 2(a). Specifically, letting \( \theta \) denote its real-valued input (the output of the last dense unit), its complex-valued output is given by

\[
v_{RF} = \exp(j \cdot \theta) = \cos(\theta) + j \cdot \sin(\theta),
\]

where \( j = \sqrt{-1} \). It can be seen from (4) that \( \theta \) has a clear physical meaning that each element of \( \theta \) corresponds to the phase of each analog BF coefficient in \( v_{RF} \).

3) Loss Function and Label: Different from conventional supervised learning designs [5], [6], [7], where specific labels are necessary and the loss function is usually defined as the MSE or cross-entropy between the true signals (labels) and recovered ones, in our design, there is no need of labels and the BFNN is trained with the following new loss function directly related to the objective in (3)

\[
\text{LOSS} = -\frac{1}{N} \sum_{k=1}^{N} \log_2(1 + \frac{\gamma_k}{N_l} ||h_{RF,k}||^2),
\]

where \( N \) denotes the total number of training samples, and \( \gamma_k \), \( h_{RF,k} \) and \( v_{RF,k} \) represent the SNR, CSI and output analog beamformer associated with the \( k \)th sample. It is worth noting that during the training process, \( \gamma_k \) and \( h_{RF,k} \) directly come from the randomly generated samples while \( v_{RF,k} \) is the output of the BFNN. Also note that here the loss function in (5) does not require any labels and is similar to a reward function in reinforcement learning. In another view, since the DL technology is essentially a gradient descend method [9], the reduction of loss value with training iterations leads to the increase of the average SE.

From the above consideration, it can be seen that during the offline training process, by taking \( h_{\text{est}} \) as the input and using

\[2\text{Note that as the complex-valued computation is not well supported by most DL frameworks, } h_{\text{est}} \text{ should be first converted to a } 2N_t \times 1 \text{ real-valued vector by concatenating its real and imaginary parts.}\]

the true channel response \( h \) in the loss function, the BFNN can be trained to approach the ideal SE with perfect CSI as much as possible, and thus has certain robust performance to channel estimation errors when deployed online.

IV. SIMULATION RESULTS

Throughout the simulations, a half-wave spaced uniform linear array with \( N_t = 64 \) is deployed at the BS. We apply the same mmWave channel model with exactly the same parameters as those in [2]. That is, the mmWave channel samples are generated according to [1], where \( L \) is set to 3, \( \alpha_l \) satisfies independently and circularly symmetric Gaussian distribution with zero mean and unit variance, and \( \phi_l \) satisfies independently uniform distribution in \([0, 2\pi]\). Two conventional HBF algorithms in the special case of one RF chain are considered for comparison, i.e., the OMP algorithm [11] and the alternating minimization algorithm using phase extraction (PE-Alt) in [4]. Besides, the performance of fully-digital (FD) BF is provided as a benchmark. All the source codes and data sets of the following simulations can be found in [12].

A. Results with Perfect CSI

In the ideal case of perfect CSI, we set four dense units in the proposed BFNN, with 2048, 1024, 256, 64 neurons in their corresponding dense layers, respectively. The perfect CSI is fed into the BFNN to generate \( v_{RF} \) and also used to compute the loss function (5). It is found that a total of \( 2 \times 10^5 \) samples is enough to train the BFNN and the dropout layer is not necessary in this case. Batch size and epoch are set to 512 and 1000, respectively. Fig. 3 shows the SE as a function of SNR for different BF algorithms. It can be seen that the OMP algorithm performs worst because of its limitation on a predefined beamformer set. However, the proposed BFNN performs even a little bit better than the PE-Alt algorithm and close to the FD BF, and thus possesses very competitive potential to the state-of-the-art algorithms.

B. Results with Imperfect CSI

In this more practical case, we adopt the channel estimation algorithm in [2] for obtaining \( h_{\text{est}} \). In the conventional PE-Alt and OMP algorithms, \( h \) is directly replaced by \( h_{\text{est}} \) when computing the BF coefficients. In the proposed BFNN, we
set five dense units with 4096, 2048, 1024, 256, 64 neurons in their corresponding dense layers, and set a drop rate of 0.45, 0.35, 0.3, 0.2, and 0 for each dropout layer, respectively. A total of $4 \times 10^5$ samples is used to train the BFNN with batch size and epoch set to 4096 and 1000, respectively.

Fig. 4(a) shows the SE versus SNR performance under two channel estimation levels, which are characterized by two pilot-to-noise power ratios (PNRs), i.e., $-10\,\text{dB}$ and $20\,\text{dB}$. It is assumed here that the estimation of the number of channel paths is correct, i.e., $L_{\text{est}} = 3$. It can be seen from Fig. 4(a) that the proposed BFNN outperforms the conventional algorithms, with the improvement becoming larger for smaller PNRs.

In practical systems, there also exist estimation errors in estimating the number of total paths $L$. Due to the sparsity of the mmWave channels and considering the estimation complexity, $L_{\text{est}}$ is preset to a small value $L_{\text{est}} = 1$ and only picks up the strongest path [13]. Fig. 4(b) shows the SE performance for different $L_{\text{est}}$ values. It can be seen from this figure that the proposed BFNN significantly outperforms the conventional algorithms with the improvement becoming larger for less accurate $L_{\text{est}}$.

In summary, it can be concluded that the proposed BFNN exhibits much stronger robustness to imperfect CSI than the conventional algorithms. The less accurate the channel estimate is, the larger the performance gap is. This is because through many iterations with large training sets, the BFNN has been trained to learn the characteristics of the mmWave propagation channels, as well as the relationship between the imperfect CSI and the ideal SE with perfect CSI.

V. DISCUSSION OF THE GENERALITY OF BFNN

Although the BFNN is designed specifically for the analog BF, it has good generality for other more complicated BF problems. For example, in the broadband scenario, by concatenating the multi-tap channel vectors, $h_n$ for $n = 0, \ldots, D-1$ with $D$ denoting the maximum channel delay normalized by the symbol period, as the input and redefining the broadband SE as the loss function, the BFNN can be extended to optimize the analog beamformer. For another example, considering the HBF problem with multiple RF chains, according to conventional HBF works [3], [10], normally the optimal digital beamformer can be solved with a closed-form solution and the difficulty is in the optimization of the analog one. Thus, a simple extension of the current BFNN is to increase the output dimensions from $N_t$ to $N_{\text{RF}}N_t$ for the $N_t \times N_{\text{RF}}$ analog BF matrix and use a new loss function for HBF. Following the similar idea, the BFNN can also be considered in the joint transmit and receive BF design or the multi-user BF design.

VI. CONCLUSION

We have proposed a DL-based BF design approach for mmWave systems with large-scale antenna arrays. With some special designs on the self-defined Lambda layer and the loss function, the proposed BFNN can well handle the challenges of hardware limitation and imperfect CSI in mmWave systems. Although without extensive hyperparameter tuning, simulations have shown the competitive performance of the BFNN and provided valuable insights for future BF designs. Due to the generality of BFNN, it is of great interest to extend the BFNN to more complicated BF problems in future work.

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