Fast magnetosonic waves in pulsar winds

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Abstract

Fast magnetosonic waves in a magnetically-dominated plasma are investigated. In the pulsar wind, these waves may transport a significant fraction of the energy. It is shown that the nonlinear steepening and subsequent formation of multiple shocks is a viable mechanism for the wave dissipation in the pulsar wind. The wave dissipation both in the free pulsar wind and beyond the wind termination shock is considered.

Key words: pulsars general { supernova remnants { MHD { waves

1 Introduction

The mechanisms of the energy transfer from the pulsar to the pulsar wind nebula still remain obscure. It is widely accepted that pulsars emit an electron-positron plasma, which forms an ultrarelativistic magnetized wind. The rotational energy of the neutron star is carried mostly by electromagnetic fields as Poynting flux. In the MHD wind and should be eventually transferred to the radiating particles. The wind jets at a shock front located in the case of the Crab Nebula, where the wind is an electron-positron plasma, and changes in the MHD frame of reference.

The existence of magnetosonic waves in rarefied magnetized plasmas is well established (e.g., Aksnes et al. 1975). However, we assume that only true MHD waves (those satisfying the condition $\nabla \times B = 0$) may be generated by the rotating magnetosphere. The reason is that according to the conventional view, the plasma density in the pulsar wind is sufficiently large such that low-frequency electromagnetic waves are heavily damped (e.g., Aksnes et al. 1978; Matos & Mendes 1996). There are four types of MHD waves but only the entropy and the fast magnetosonic (FM) waves may propagate far beyond the light cylinder, where the magnetic field is nearly co-axial to the pulsar rotation period. These oscillations propagate outwards as MHD waves. In the equatorial region, the magnetic field line at a given radius alternates in direction with the frequency of rotation, being connected to a different magnetic pole every half-period. Such an alternating field is transported by the ow in the form of the striped wind (Michel 1971, 1982; Bogovalov 1999), which may be considered as an entropy wave. At high latitudes, where the magnetic field does not change sign, the magnetic oscillations are transferred away in the form of FM waves (generation of FM waves by the rotating, slightly nonaxisymmetric magnetic field was considered by Bogovalov 2001a).

The entropy wave decays because of the current starvation in current sheets separating strips with opposite magnetic fields (Nov 1975; Michel 1982, 1994; Coroniti 1990). Lyubarsky & Kirk (2001) showed that the ow sign, cantly accelerates in the course of reconnection and this dilates the distance over which the wave decays. At typical conditions, the dissipation radius exceeds the radius of the termination shock, therefore one should conclude that the Poynting flux in the striped wind is not dissipated until the wind enters the termination shock. All the energy should release within the shock where the ow decelerates.

In this article, the fate of FM waves is considered. It will be shown that these waves may be described within the MHD framework throughout the pulsar wind up to the termination shock and beyond. The reason is that in a magnetically-dominated plasma, the FM waves excite an all-conductivity current, the oscillations of the magnetic field being nearly compensated by the displacement current. In MHD regime, FM waves may decay due to the nonlinear steepening and subsequent formation of multiple shocks. It will be shown that for typical pulsar parameters, the waves may decay only beyond the termination shock. However in rapidly spinning pulsars, like the Crab, FM waves may de-
where the \( \omega \) reaches the term in a shock provided and density in the wind is high enough.

The article is organized as follows. In Sect. 2, properties of FM S waves in a magnetically dominated plasma are outlined qualitatively in Sect. 2. These estimates are applied to the pulsar wind in Sect. 3. The wave decay after the multiple shocks from action is considered in Sect. 4. Conclusions are summarized in Sect. 5. In Appendix 1, exact solutions for nonlinear FM S waves in the magnetically dominated plasma are presented. In Appendix 2, the energy and momentum fluxes transferred by FM S waves are derived.

2 FM S WAVES IN A MAGNETICALLY DOMINATED PLASMA

In this section, I consider FM S waves in the wind frame. Let us consider waves propagating perpendicularly to the magnetic field because far enough from the pulsar, the magnetic field is nearly toroidal whereas the waves propagate radially. Only qualitative statements are outlined here; exact solutions are presented in Appendix 1.

When the FM S wave propagates perpendicularly to the magnetic field, plasma oscillates along the wave direction together with the magnetic field. The \( \mathbf{u} \) freezing condition ties the plasma density and the magnetic field together:

\[
\mathbf{B}^0 = \mathbf{n} \mathbf{b} = \text{const.}
\]

where \( \mathbf{B}^0 \) and \( \mathbf{n} \) are the magnetic field and the plasma density in the proper plasma frame, the subscript * stands for quantities averaged over the wave period. The quantities measured in the proper plasma frame differ from those measured in the wind frame because the plasma oscillates. In this section, I neglect this difference assuming the oscillation velocity to be non-relativistic. More general consideration is presented in Appendix 1. With the aid of Eq. (1), one can express the magnetic energy and the pressure via the plasma density as follows:

\[
P = \mathbf{p} + \frac{\mathbf{B}^2 n^2}{8} ; \quad E = s^* + \frac{\mathbf{B}^2 n^2}{8} ;
\]

where \( \mathbf{p} \) and \( s^* \) are the plasma pressure and energy density, correspondingly. The velocity of the wave may be now found immediately as the sound velocity in such a medium (throughout the paper all velocities are expressed in units of the speed of light, \( c = 1 \)):

\[
s^*_t = \frac{\partial P}{\partial E} = \frac{w s^* + \mathbf{B}^2 n^2 = 4}{w + \mathbf{B}^2 n^2 = 4} ; \quad (2)
\]

where \( w = \mathbf{p} + s^* \) is the enthalpy of the plasma, \( s = \mathbf{p}/\partial \mathbf{p} \) is the thermodynamical sound velocity.

In the pulsar wind, the magnetic energy density significantly exceeds the plasma energy density. One can conveniently express all the values via the parameter

\[
\mathbf{B}^2 n^2 / 4 w ; \quad (3)
\]

which is twice the ratio of the magnetic to the plasma energy density in the wind frame and the ratio of the Poynting \( \mathbf{u} \) to the plasma energy \( \mathbf{u} \) in the laboratory frame. One can see that in the FM S velocity is close to the speed of light. The corresponding Lorentz factor, \( \mathbf{r} = (1 - \mathbf{s})^{1-2} \), may be written as

\[
\mathbf{s} = \frac{1}{1 - \mathbf{s}} ;
\]

In the small amplitude, linear wave, the electric field may be written as \( \mathbf{E}^0 = 2 \mathbf{B}^0 \), whereas variations in the magnetic field are \( \mathbf{B}^0 = B^0 (n = \mathbf{m}) \). Taking into account that variations in the density and in the velocity are related by the standard expression \( n \mathbf{m} = n = 2 \mathbf{m} \), one can see that

\[
\mathbf{E}^0 = \frac{n}{n} = B^0 \quad (4)
\]
to within a factor \( 1 \). Because the electric field may not exceed the magnetic field, the wave amplitude should be limited by

\[
B^0 < B^0 = 2 ;
\]

It is shown in the Appendix 1 that when the wave amplitude approaches the limiting value, the oscillation velocity becomes ultrarelativistic; the plasma density goes to zero at some phase of the wave period and moreover the characteristic scale for the nonlinear steepening of the wave decreases to zero.

Now let us find what plasma a density is necessary to allow the MHD solution. The plasma density should satisfy the evident condition \( s = j \), where \( j \) is the conductivity current excitide in the wave. Substituting into the Maxwell equations

\[
r \mathbf{B}^0 = 4 j + \mathbf{E}^0 / \mathbf{c} \quad \rightarrow \quad r \mathbf{E}^0 = \mathbf{E}^0 = \mathbf{E}^0 / \mathbf{c} ;
\]

a sinusoidal wave \( \exp ( i \omega t + k \cdot x ) \) with the dispersion law \( \omega = k x \), one can see that the cold plasma may provide the necessary conductivity current if

\[
\mathbf{f}_s > \mathbf{B}^0 / \mathbf{B}^0 ; \quad (6)
\]

where \( \mathbf{f}_s = \mathbf{E}^0 \mathbf{m} \) is the Larmor frequency. We consider the electron-positron plasma therefore throughout the paper \( m \) is the electron mass.

At the conditions (5, 6), FM S waves in a magnetically dominated plasma may be considered in MHD approximation. In the MHD regime, the wave may decay as a result of nonlinear steepening and subsequent formation of multiple shocks. The wave steepening occurs because the wave velocity depends on the plasma density and therefore the compressive part of the wave moves faster than the expansive one. In the strongly magnetized plasma, even a large amplitude wave is only weakly nonlinear because the wave velocity varies with the density only by a factor of about \( 1 \). The reason is that, as it was shown above, the conductivity current, which only introduces nonlinearity into the MHD equations, is small in the magnetically dominated plasma.
The difference in the velocities between the compressive and expansive parts of the wave may be estimated from Eq. (2) as

$$\alpha_{\text{k}} = \frac{n}{\alpha_{\text{n}}}$$

The shock form is when the leading edge of the wavefront becomes vertical (e.g., Landau & Lifshitz 1959). This occurs within a characteristic nonlinear time it takes for the compressive part to shift relative the expansive one by 1 = !. Therefore the shock from after the wave travels a distance

$$x_{\text{shock}} = l = \alpha_{n}$$

This estimate is confirmed by a rigorous derivation in Appendix A. After the shock formation, the wave continues to distort at the characteristic scale $x_{\text{n}}$ such that eventually all the wave energy dissipates in the shock. However, both and $m$ may change because the plasma heats up and accelerates (or decelerates) considerably in the course of the wave dissipation. Therefore, numerically the dissipation scale may differ from the shock formation scale even though the same expression (7) is valid for both scales.

### 3 FM S Waves in the Pulsar Wind

Far beyond the light cylinder, the wind may be considered purely radial, whereas the magnetic field is purely toroidal. The wind is assumed to be super-FAST, $V_{\text{out}} > c_{\text{s}}$. Transforming the electron magnetic fields from the wind frame into the pulsar frame, one can see that the condition (5) reduces to the condition $B < B_{0}$ (the factor of 2 appears because in the proper plasma frame $E = 0$ whereas in the pulsar frame $E = v B - B_{0}$). So FM S waves may be generated by the rotating oblique domain magnetosphere at high latitudes where the magnetic field does not change the sign. In the equatorial belt (its width depends on the angle between the magnetic and the rotation axes of the pulsar) an entropy wave is generated in the form of the stripped wind. Generally, a superposition of an entropy and FM S waves is generated here (one can talk about the superposition because even a large amplitude FM S wave is weakly nonlinear in the magnetically dominated plasma) however, an important point is that an entropy wave arises here inevitably because there is no other MHD wave that may transfer alternating magnetic field in a high-plasma.

Now let us consider validity of MHD approximation (Eq. (6)) for the FM S wave. The magnetic field in the pulsar wind is predominantly toroidal and may be presented as (from now and hereafter only averaged quantities will be considered therefore the subscript * will be omitted)

$$B = B_{*} \frac{R_{L}}{R}$$

where $R_{L} = c_{\text{s}} = 5 \times 10^{8} \text{ cm}$ is the light cylinder radius, $P$ the pulsar period. The magnetic field at the light cylinder may be estimated as

$$B_{L} = \frac{R_{L}}{R_{*}} \approx \frac{30}{P} \text{ G}$$

where $= 10^{30} \text{ G cm}^{-3}$ is the magnetic moment of the star. The wave frequency in the pulsar frame is the pulsar rotation frequency; in the wind frame the frequency is $= \omega \times 2$). Noting that the magnetic field in the wind frame is

$$B_{*} = B_{L} = \frac{R_{*}}{R} \approx \frac{30}{P} \text{ G}$$

This radius should be compared with the radius of the standing shock where the wind term vanishes. One can place the upper limit on this radius by equating the magnetic pressure in the wind to the ram pressure of the medium, $B^{2}/8 \pi = V^{2}$, where $= 1 \text{ g cm}^{-3}$ is the density of the interstellar gas, $V = 100V_{\text{c}} \text{ km/s}$ the velocity of pulsar through it. Substituting Eq. (8), one obtains

$$R_{\text{shock}} = 7 \times 10^{4} \frac{30}{P} \text{ cm}$$

The radius of the term in the shock may be less than (10) if the pulsar is surrounded by a dense medium. Comparing Eq. (10) with Eq. (9) one can see that FM S waves in the pulsar wind may be considered in MHD approximation.

The wave may decay in multiple shocks that arise as a result of nonlinear steepening. Transforming the characteristic nonlinear scale (7) to the pulsar frame, one gets

$$R_{\text{shock}} = 4 \frac{2}{5} R_{L} \text{ cm}$$

The magnetic parameter of the wind depends on the plasma density. In the pulsar frame, one can conveniently express the density, $N = n$, via the dimensionless multiplicity factor, $\zeta$, as

$$N = \frac{B_{L}}{2 \epsilon} \frac{R_{L}}{R}$$

Before the multiple shocks are formed, the pulsar wind is cold, $W = m n$, and propagates with a constant Lorentz factor $\gamma$. The magnetic parameter remains constant,

$$\omega = \frac{B^{2}}{4 \pi m n} = \frac{1}{2 \epsilon} \zeta$$

where $\gamma = B_{L} = m n$ is the gyrofrequency at the light cylinder. Now one can write the shock formation distance as

$$R_{0} = 4 \frac{2}{5} \frac{R_{L}}{2} \text{ cm}$$

For typical pulsar parameters, $\epsilon = 100$, $\gamma = 10$ (H. B. K. A. M. 2001), this radius satisfies the condition (9) therefore MHD description of the wave is just correct. For the typical parameters, this radius is one order the ionization shock radius (10) for all pulsars with the exception of the millisecond ones. However, there is strong evidence to suggest that the multiplicity parameter in the Crab pulsar signifi cantly exceeds the typical one and may be as high as $10^{4}$ (Shklovsky 1970; Rees & Gunn 1974). Extra pairs may be produced together with the observed gamma radiation in the upper magnetosphere (Cheng, Ho, and Rudak 1988; Lyubarsky 1996). At such a plasma density, the multiple shocks arise before the wave reaches the term ionization shock (observed in the Crab at $10^{7} \text{ cm}$ from the pulsar). So formation of the shocks in the FM S waves may occur both in the free pulsar wind within the term ionization shock and in the pulsar wind nebula beyond the term ionization shock.

If the multiple shocks are not formed until the wave reaches the term ionization shock (i.e., if the radius (13) exceeds the ionization shock radius), the...
waves enter the nebula through the shock. In case the upstream ow is not Pointing dom inated, the
downstream ow is non-relativistic; the non-linear scale (11) is extrem ely sm all in this case and there-fore the waves should decay im mediately, in fact al-ready w ithin the shock. But if the upstream ow is still Pointing dom inated, the shock is weak and the
downstream ow is relativistic (Kundt & Krot schek 1982, Kennel & Coroniti 1984). The waves pass freely the shock from upstream because reaction is im-possible. Param eters of the waves are nearly not a-ected by weak shocks (e.g., Anderson 1963) how-ever the non-linear scale decreases because the ow deca-terates (the statement by Kennel & Coroniti that the downstream Lorentz factor remains large, $\beta_{\text{down}}$, is based on the assump tion that the ow is spherical ly sym metric: one can see that even slight deviation of the ow lines from radial may result in signi cant deceleration of the ow). Eventually the multiple shocks are formed. Investigation of the ow in the nebula and search for the multiple shocks forma- tion zone is out of the scope of this article. I sim-ply assume that the multiple shocks do form at some point and consider evolution of the ow par-ameters in the course of the wave dissipation. It will be shown in the next section that in the sub-FMS ow (beyond the term iation shock) the wave energy dis-sipates im mediately after the multiple shocks ar-rise, dissipation being accom panied by the ow deceler-ation down to 1. Hence although the Pointing dom inated w ind could not be brought to rest in the term iation shock, the wave dissipation provides an effective mechanism of the ow braking.

4 WAVE DECAY

4.1 Basic equations

After the multiple shocks arise, the wave energy dissipates. This occurs at the non-linear scale (11), which now may change because the released energy heats the plasma ow. One can consider evolution of the ow par-ameters in the course of the wave decay applying the conservation laws to the system. Let us separate the average w uk of conserving values into the wave and the ow parts. Such a separa-tion may be performed in general terms for all am be-plu-oid waves (see Appendix 2). The average plasma density, $n$, and velocit y, $v$, (and the corresponding Lorentz factor) are de-ned such that the wave does not contribute to the m atter ow (in Appendix 2 the averaged quantities are asked by tilde; here we work only with averaged quantities and tilde is om mitted). The continuity equation may be written as (the ow is considered as locally spherically sym metric)

$$n v R^2 = n_0 v_0 R_0^2; \quad (14)$$

where the subscript "0" is referred to the quantities at the shock forma-tion radius (13). The energy equation may be written as

$$\frac{1}{R^2} \frac{d}{dR} \frac{v R^2}{2} + \frac{B^2}{4} v R^2 = Q; \quad (15)$$

where $Q$ is the energy transferred from the wave to unit plas ma volume per unit time. The magnetic eld strength may be expressed via the density making use of the frozen ow condition (see Eq. A.11.6)

$$\frac{B}{R n} = \frac{B_0}{R_0 n_0}; \quad (16)$$

The entropy equation describes the entropy growth as a re-sult of the wave decay:

$$\frac{d S}{d} = \frac{d}{d n} + \frac{d T}{d d}; \quad (17)$$

where $d = dR$ is the proper time. We assume that just after the shock forma-tion, the plasma is heated to relativistic tem peratures, $w = 4nT$.

Because the wave propagates in the proper plasma a frame with the velocity $v_{\text{rel}} = 1 \ (l = 1)$, its energy and m om entum are equal, in this frame $e_k$ to within $1 = (\text{see Appendix 2})$. Therefore if the energy $d^2$ per unit mass is absorbed in the proper plasma a frame $e_k$ then the m om entum $d^2$ per unit mass is also transferred from the wave to the plasma. Then in the pulsar frame, the absorbed energy is $dE = 2d^2$. Taking into account that $dV dt$ is invariant, one can write the energy release rate as

$$Q \frac{dE}{dV dt} = 2 n d^2; \quad (18)$$

Substituting $d^2 = T dS$, one writes the entropy equation in the form

$$2^3 3n \frac{dT}{dR} = \frac{T}{dn} = Q; \quad (19)$$

Let us now turn to the energy equation (15). Making use of Eqs. (14, 16), one can see that the Pointing ow (the second term in the brackets in the left-hand side) is nearly independent of $R$ in the ultrarelativistic case. Expanding this term in $1 = \frac{T}{T_0}$ to the next order and retaining in the f rst term only leading terms in $1 = \frac{T}{T_0}$ because this term is already sm all as $1 = \frac{T}{T_0}$, one can reduce Eq. (15) to

$$\frac{d}{dR} \frac{v R^2}{2} + \frac{B^2}{8} v R^2 = Q^2; \quad (20)$$

Substituting $n$ from Eq. (14) into Eqs. (18, 19), one gets nally

$$\frac{d}{dR} \frac{T}{T_0} + \frac{a^2}{3} = \frac{Q^2}{4T_0 \frac{\delta n}{\frac{\delta R}^2}}; \quad (20)$$

$$3 \frac{d}{dR} \left( \frac{T}{T_0} \right) + \frac{T}{T_0} \frac{d}{dR} \left( \frac{T}{T_0} \right) + \frac{2T}{T_0} \frac{\delta R}{\delta T} = \frac{Q^2}{2T_0 \frac{\delta n}{\frac{\delta R}^2}}; \quad (21)$$

where the factor

$$a = \frac{3B_0^2}{32 n T_0}; \quad (22)$$

is less than unity in super-FMS ow s and exceeds unity in the opposite case. It was found above that at all event w ind param eters, the multiple shocks may arise in FMS waves both before the wind reaches the term iation shock and be-yond the shock in the pulsar wind nebula. Therefore one should consider the wave decay both in the super-FMS (free w ind) and in the sub-FMS (beyond the term iation shock) ow s. The ow behavior in these cases is just opposite there-fore let us consider them separately.
4.2 Wave decay in the super-FMS ow

The ow in the wind is super-FMS, in this case a 1 and the plasma is cold at \( R_0 \) so one can take \( T_0 \) m. Eliminating \( Q \) from Eqs.(20, 21) and neglecting the term with \( a \), one gets

\[
\frac{d(T-T_0)}{dR} = \frac{T}{T_0} \frac{d(\alpha)}{dR} + 2\frac{T}{T_0} = 0;
\]

which yields

\[
\frac{\alpha R^2}{16 \, \text{nT}} = \frac{\alpha R^2}{R_0^2};
\]

so in the super-FMS ow heating of the medium is accompanied by acceleration.

The energy release is caused by the nonlinear wave distortion. Therefore the fraction of the wave energy dissipated at the distance \( R \) may be roughly estimated as \( R = R_{n1} \). One should substitute in the expression for the nonlinear scale (11) parameters varying in the course of the energy dissipation. Variation of \( m \) may be found making use of Eqs.(14, 16, 23):

\[
\frac{B^2}{16 \, \text{nT}} = \left( \frac{\alpha R^2}{R_0^2} \right).\]

Now the dissipated fraction of the wave energy may be estimated as \( R = \left( \frac{R_{n1}}{\alpha R} \right) \) after the wave dissipates. This implies, with the aid of Eq.(23), that \( R = R_{n1} \).

So the wave dissipation at \( R = R_0 \) heats the plasma a till the temperature \( T = T_0 \) and accelerates it to the Lorentz factor \( \frac{1}{\sqrt{1-x}} \). Then the heated plasma cools and accelerates in the Bernoulli regime, \( T' = T_0 \).

At the distance \( R_1 = \frac{R_0}{R_0} \), all the released energy transforms into the kinetic energy and the plasma reaches the Lorentz factor

\[
1 + \frac{\alpha R^2}{R_0^2} = 1 = 13 \times 10^7 \times \frac{30}{p^2};
\]

4.3 Wave decay in the sub-FMS ow

If the multiple shocks formation radius (13) exceeds the termination shock radius the waves pass the termination shock and enter the sub-FMS ow. In the sub-FMS case, the ow decelerates in the course of energy release (see below) and the nonlinear scale sharply decreases just after the multiple shocks formation. Therefore the wave decay scale turns out to be small as compared with the radius and one can consider the ow in the plane geometry. In this case Eqs.(20, 21) reduce to

\[
\frac{d}{dx} \left( \frac{T}{T_0} \right) = \frac{a_0^2}{\alpha^2} \frac{x^2}{3} = \frac{1}{2} ;\]

\[
\frac{d(T-T_0)}{dx} - \frac{T}{T_0} \frac{d(\alpha)}{dx} = 2;\]

where

\[
x = \frac{Z}{Q} \frac{dR}{dx} \]

is the ratio of the transferred energy to the initial plasma energy. When the wave decays, \( x \) becomes very large and reaches the initial ratio of the wave energy to the plasma energy; for large amplitudes waves this ratio is about \( \alpha^2 \).

Integrating Eq.(24), one yields

\[
T = T_0 \left( 1 + \frac{a}{3} x \right) + \frac{a_0^2}{\alpha^2} \left( \frac{x}{3} \right)^2;\]

Substituting \( T \) from this relation into Eq.(25), one gets the equation

\[
\frac{2}{3} \left( \frac{T}{T_0} \right) = \frac{2 a^2}{3 x} + \frac{4 a_0^2}{\alpha^2} \left( \frac{x}{3} \right)^2 = 1 ;\]

which is linear with respect to \( (x) \). The solution is

\[
\frac{2 a^2}{3 x} + \frac{4 a_0^2}{\alpha^2} \left( \frac{x}{3} \right)^2 = 1 ;\]

where the sign of the square root should be positive for \( a < 1 \) and, correspondingly, negative for \( a > 1 \) to satisfy the initial conditions.

One should see that the super-FMS ow \( a < 1 \) accelerates whereas the sub-FMS ow \( a > 1 \) decelerates in the course of energy release. For \( a > 1 \) one finds, with the aid of Eq.(22),

\[
\frac{R}{T_0} = \frac{2 a}{3 x} + \frac{q}{x} ; \quad T_0 = \frac{3 x}{8 a};\]

It follows from Eqs.(14, 16) that in the narrow decay zone, \( R = R_0 \), the magnetic field remains constant while the ow remains relativistic, \( x \). The absorbed energy is transformed into the internal plasma energy and the magnetization parameter decreases:

\[
\frac{B^2}{4 \, \text{nT}} = \frac{2 a}{x};\]

The wave energy dissipates completely when \( x = 1 \). The nonlinear scale (11) decreases already when \( x \) exceeds unity i.e., when the fraction of the dissipated energy is small ( \( 1 = 0 \)) therefore the total dissipation scale turns out to be small, \( R_0 = 0 \).

When the wave decays completely, \( x \) and one gets

\[
1 = 1 ;\]

However this solution becomes invalid when, in the proper plasma frame, the particle Larmor radius, \( r = (eB)^2 \), exceeds the wavelength, \( \lambda = 2 R \). Making use of Eqs.(8, 12, 22, 27), one can write the condition \( r > 2 R \) as

\[
\frac{2 x^2}{3} = \left( \frac{R_0}{R_0} \right);\]

The right-hand side of this expression is small therefore only a small fraction of the energy is transferred to the plasma in MHD regime; most of the energy is dissipated when the Larmor radius exceeds the wavelength. This may result from formation of a high energy tail in the particle energy distribution.
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5 CONCLUSIONS

In the pulsar wind, a significant fraction of the Poynting flux may be transported by FM waves. The plasma in the pulsar wind is magnetically dominated in the sense that in the proper plasma frame, the magnetic energy density exceeds the plasma energy density. This condition is equivalent to the condition that in the plasma frame, the Poynting flux dominates the plasma energy flux. In such a plasma, FM waves excite a small conductivity current, oscillations of the magnetic field being nearly compensated by the displacement current. Thus a very small plasma conductivity is sufficient to keep this current and MHD description of these waves is justifiably through the pulsar wind till the term ionization shock and beyond.

These waves may decay in multiple shocks that arise through nonlinear steepening of the waves. Because the conductivity current is small at the pulsar wind conditions, even a large amplitude FM wave is nearly linear and therefore shocks are formed at large distances from the pulsar. Depending on the plasma density in the wind, this may occur either before or after the shock reaches the termionation shock and beyond the shock. In any case the wave energy is dissipated in the multiple shocks. Energy release in the super-FM S ow is accompanied by the plasma heating and acceleration. The plasma accelerates further on. Therefore if the waves dissipate in the free, super-FM S pulsar wind, all the wave energy eventually transforms into the kinetic energy of the ow. If the waves do not dissipate in the wind, the energy remains Poynting-dominated. In this case the ionization shock should be weak and the downstream ow remains ultra-relativistic. Such a ow does not match the slow expansion of the nebula; this was considered as an evidence for a conversion of a significant fraction of the Poynting flux into the particle energy flux before the pulsar wind reaches the termionation shock (Rees & Gunn 1974; Kenneel & Coroniti 1984). However it was shown above that the wave dissipation in the sub-FM S downstream ow is accompanied by the ow deceleration to sub-relativistic velocities. Hence even if the ow remains Poynting-dominated at the termionation shock, the wave dissipation downstream the shock may provide the necessary deceleration.

Only a fraction of the total Poynting flux is transported by the FM waves; therefore the proposed mechanism does not solve the problem but should be considered as an element of the future complete theory. The mechanism considered may also play an essential role in gas-ray burst models involving Poynting energy dissipation from compact objects (Usov 1992, 1994; Meszaros & Rees 1997; Kuznietz & Rudge 1999; Blackman & Y. I., 1998; Spruit 1999; Lyutikov & Blackman 2001; Denkhahn 2002; Denkhahn & Spruit 2002).

APPENDIX 1. NONLINEAR FM WAVES IN A MAGNETICALLY DOMINATED PLASMA

Relativistic nonlinear MHD waves are considered in the general case by Akhiezer et al. (1975). Here we consider only the waves propagating perpendicular to the ambient magnetic field in the case 1.

Let us not consider the waves in the plasma geometry. It is followed from the continuity equation

\[ \frac{\partial \theta}{\partial t} + \frac{\theta}{\partial x} n v = 0 \]

(A11)

and the frozen-in condition

\[ \frac{\partial \theta}{\partial t} + \frac{\theta}{\partial \xi} v B = 0 \]

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that the magnetic field may be presented in the form

\[ B = \frac{b}{n} B \quad \text{const}; \]  

(A12)

where the subscript * stands for quantities at some fixed point in such a way that \( B^* \) is equal to \( B \) averaged over the wave period. The dynamical equation may be presented in the form of the energy equation

\[ \frac{\partial T_{01}}{\partial t} + \frac{\partial T_{01}}{\partial x} = 0; \]  

(A13)

where the components of the energy-momentum tensor are

\[ T_{01} = w^* v + \frac{B^2}{4} 2 v; \]  

(A14)

\[ T_{00} = w^* p + \frac{1 + v^2}{8} B^2 2 v; \]  

(A15)

Note that this set of equations is reduced, with the aid of Eq. (A12), to the standard hydrodynamic equations with

\[ E = w^* + \frac{B^2}{8}; \quad P = p + \frac{B^2}{8} = K + \frac{B^2 n^2}{8}; \]

\[ W = E + P = w^* + p + \frac{B^2}{4}; \]  

(A16)

The nonlinear simple wave may be found from the condition that all dependent variables are functions of one of them, e.g., \( n \) (Landau & Lifshitz 1959). This means that Eq. (A1.3) should be equivalent to Eq. (A1.1), i.e.

\[ \frac{dT_{01}}{dt} = \frac{d(n v)}{dt}; \]  

(A17)

The last equation reduces to

\[ \frac{dv}{dn} = \frac{s_h}{n} \]  

(A18)

where the FM s velocity \( s_h \) is given by Eq. (2). For the small amplitude wave, \( n \), one obtains the linear FM s wave propagating with the phase velocity \( s_h = \text{const} \). For a strong wave, \( s_h \) depends on the local density, the wave becomes nonlinear. However, in the strongly magnetized case, \( s_h \) goes to unity and therefore even a strong wave is nearly linear.

Substituting \( s_h = 1 \) into Eq. (A18), one obtains

\[ \frac{n}{n} = \frac{1 + v}{1 + v} \]  

(A19)

In the frame moving, in average, with the plasma \( v = 0 \), one gets for the small amplitude wave

\[ v = n = \text{const}; \]  

In the case \( n \), Eq. (A1.9) reduces to

\[ n = \text{const}; \]  

The waveform moves along the characteristic of Eq. (A1.1):

\[ \frac{dx}{dt} = \frac{d(n v)}{dn} = \frac{v + s_h}{1 + v + 1} = \frac{2 w n^2}{b n^3} \]  

(A110)

Fast magnetosonic waves in pulsar winds

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considering specific waveforms. Let us define the average plasma density, \( n \), and velocity, \( v \), by the relations

\[
<n \nu> = n \nu \quad ; \tag{A 2.1}
\]

\[
<n > = n \quad ; \tag{A 2.2}
\]

where the angular brackets denote averaging over the wave period. With such a definition, there is no mass flux associated with the wave.

The total energy and momentum densities are (see Eq. (A 1. 4 - A 1. 6))

\[
T_{00} = W^2 \quad ; \tag{A 2.3}
\]

\[
T_{01} = W v^2 \quad ; \tag{A 2.4}
\]

Averaging this values, one can present them as a superposition of a wave part dependent only on average plasma parameters and a wave part dependent on the wave amplitude. Calculations are simplified in the frame where plasma is at rest in average, \( v = 0 \).

In a small amplitude wave, the energy and momentum are of the second order in the wave amplitude, therefore linear relations, like (see Eq. (A 1.8))

\[
\frac{n}{n} = \frac{v}{\alpha_0} \quad ;
\]

may be used only in the second order terms. The average of the first order terms may be expressed via the second order terms expanding Eqs. (A 2.1, A 2.2) in \( v \) and \( n \) to the second order. The result is

\[
<n > = \frac{1}{2} n < (v)^2 > ;
\]

\[
<v > = \frac{1}{\alpha_0} < (v)^2 > ;
\]

Now expanding Eqs. (A 2.3, A 2.4) and making use of the thermodynamical expression \( dE = dn = W = n \); because the wave is isentropic, all thermodynamical values may be considered as functions of \( n \), one yields

\[
<T_{00} > = E + W < (v)^2 > ;
\]

\[
<T_{01} > = \frac{1}{\alpha_0} W < (v)^2 > ;
\]

where \( E (n) \) etc. So the energy density is separated into a part which depends only on the average medium density and the wave part which is proportional to the wave amplitude squared. The momentum density of the medium is zero in the proper plasma frame therefore only the wave momentum contributes to \( < T_{01} > \). The energy and momentum of the wave coincide in the proper plasma frame, to within \( 1 \alpha_0 \leq 1 \). Making the Lorentz transformation, one can get components of the energy-momentum tensor in an arbitrary frame of reference.