DIAMAGNETISM OF 2D-FERMIONS IN THE STRONG NONHOMOGENEOUS
STATIC MAGNETIC FIELD $B = B(0, 0, 1/cosh^2(\frac{x-n}{\delta}))$: gas magnetization, static
magnetic susceptibility, chemical potential and gas compressibility.

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We study diamagnetism of a gas of fermions moving in a nonhomogeneous magnetic field $B = B(0, 0, 1/cosh^2(\frac{x-n}{\delta}))$. The gas magnetization, the static magnetic susceptibility, the chemical potential and the gas compressibility are discussed and compared with the uniform field case. General need to study dynamics of electrons in different types of magnetic fields follows from a large number of experimental situations in which its understanding enables physicists to obtain new information.

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Recently, using results of our exact description of the spinless fermion motion in a nonhomogeneous magnetic field $B = B(0, 0, 1/cosh^2(\frac{x-n}{\delta}))$, we studied ground state energy properties of a gas of spinless fermions moving in this field. For densities lower than some critical value, $\nu < \nu_c(B, \delta)$, the corresponding total energy is lower than that of the uniform field state.\[1\]. However, physical properties of a gas of spinless two-dimensional (2d) -fermions moving in this field were not studied in \[1\]. It is the aim of this paper to fill in this gap.

Energy of a gas of free spinless fermions moving in a plane in a uniform field perpendicular to this plane is larger or equal to its total energy $E_T(0, \nu) = 2\pi t N \nu^2$ in the zero field:

$$\Delta E_h(n) \equiv E_T(B, \nu) - E_T(0, \nu) = 2\pi t N (\nu_{n+1} - \nu) (\nu - \nu_n),$$

(1)

where $\nu_{n+1} \geq \nu > \nu_n, \nu_n \equiv n \frac{\Phi}{\Phi_0}, n = 0, 1, \ldots; \Phi_0$ is a unit of the magnetic flux, $\Phi \equiv Ba^2, t \equiv \frac{\hbar^2}{2ma^2}$; $\nu$ is the number density of the gas, $\nu \equiv N_j / L^2, N_j$ is the total number of fermions, $Na^2$ is the area of the square with the side length $L = a\sqrt{N}$, to which the motion is bounded, $m$ is the fermion mass. Here $a$ is a characteristic length of the system, its value is of the order of a lattice constant value. The energy degeneracy occurs in \[1\] only if a number of Landau levels is completely filled (e.g. if for some $n \nu_n = \nu$). A nonhomogeneity of the field introduced by a local field intensity decrease leads to competition of two tendencies: a decrease of the single fermion energy level due to decreased value of the field intensity and a decrease of the every energy level degeneracy due to larger spacing between centers of neighboring orbits within the region of smaller fields. Spectrum of 2d Bloch electrons in a periodic magnetic field was studied in \[2\], where the magnetic unit cell is assumed to be commensurate with the lattice unit cell. Their work is extension of previous studies of free electrons in periodic magnetic field \[3\] to the lattice case. In our work we concentrate our attention on the nonperiodic modulated field.

Let us consider motion of a spinless fermion gas bounded to the square $LxL$ in a nonhomogeneous static magnetic field perpendicular to this plane. We neglect the lattice periodic potential influence on the gas energy spectrum in this paper. We consider motion in more details the limit in which nonhomogeneity disappears and a uniform field appears. In difference to \[2\] and \[3\] we do not consider a periodic magnetic field. Recently, \[4\], an exact description of motion of the quantum spinless fermion in a nonhomogeneous magnetic field described by the vector potential $A = (0, B\tanh(\frac{x-n}{\delta}), 0)$ was found. Let us now describe those results from \[4\] which are relevant for our further calculations in this paper. In the case of motion of a quantum spinless fermion in a nonhomogeneous magnetic field described by the vector potential $A = (0, B\tanh(\frac{x-n}{\delta}), 0)$ the energy spectrum of the motion in the $x$-direction is splitted, see in \[4\], into a discrete and a continuous parts for general values of the field $B$ and of the nonhomogeneity.
parameter $\delta$. We take $x_0 = 0$ in the following, thus the field has its maximum intensity at $x = 0$. Let us consider the limit of strong fields ($F \equiv 2\pi a^2 \Phi' >\!> 1/2$, where $\Phi' \equiv B\delta^2$). In this limit it is sufficient to take into account the lowest energy levels of the spectrum. The eigenvalues of the energy corresponding to this part of the spectrum are given by, see in [4]:

$$E_n(p) = \frac{p_y^2}{2m}(1 - \frac{F^2}{((\frac{1}{4} + F^2)^{\frac{1}{2}} - ((1/2) + n)^2)^2} + (\frac{\hbar^2}{2m\delta^2})(F^2 - ((\frac{1}{4} + F^2)^{\frac{1}{2}} - (1/2 + n))^2),$$

where $n = 0, 1, \ldots \lceil n_{max} \rceil$, here $\lceil n \rceil$ denotes an integer part of a real number $n$, $p_y$ is the $y$-momentum. Let us define $P \equiv \frac{|p_y|\delta}{\hbar}$. The number $n_{max}$ is defined by:

$$n_{max} = (\frac{1}{4} + F^2)^{\frac{1}{2}} - (1/2) - (|P| \cdot F)^{\frac{1}{2}},$$

for given values of $P$ and $F$.

For $F^2 \gg \frac{1}{4}$ and for small quantum numbers $n$, $n = 0, 1, 2, \ldots$, the energy $E_n(p_y)$ expanded into series of $1/F$ powers takes the form:

$$E_n(p_y) \approx \hbar\omega(n + \frac{1}{2}) - (\frac{\hbar^2}{2m\delta^2})(n + \frac{1}{2})^2 + \frac{1}{4} - \frac{p_y^2}{mF}(n + \frac{1}{2}) + (\frac{\hbar^2}{8mF^2})(n + \frac{1}{2}) + O(\frac{1}{F^3}).$$

where $\omega \equiv \frac{\hbar c}{e a}$ is the cyclotron frequency. The energy levels become degenerate in the limit of strong but modulated fields if the energy expansion above is restricted to the first two terms, which are of the $F^1$ and $F^0$ orders respectively. The largest value of the third term in this expansion is negligible with respect to the second term:

$$\max\left(\frac{p_y^2}{mF}(n + \frac{1}{2})\right) << (\frac{\hbar^2}{2mF^2})(n + \frac{1}{2}).$$

if we take into account that there exists a natural cut-off for $p_y$ momenta, $\max(|p_y |) = \frac{\pi \hbar}{a}$, due to the underlying crystal and if we assume that the field intensity $B$ satisfies the inequality:

$$\frac{\Phi}{\Phi_0} >\!> 8\pi^2,$$

where $\Phi \equiv B.a^2$. The degeneracy of the n-th level appears due to the lost of the energy dependence on $p_y$ momentum. These levels are, [4], modified Landau levels with energies in the form

$$E_n = \hbar\omega(n + \frac{1}{2}) - \frac{\hbar^2}{2m\delta^2}(n + \frac{1}{2})^2 + \frac{1}{4} + O(1/F), \quad (2)$$

where

$$n = 0, 1, \ldots << n_m; n_m \approx F.$$

Note that

$$\frac{\hbar^2}{2m\delta^2} = 4t(L\frac{25}{28})^2/N.$$ 

Every energy level $E_n$ is degenerated within considered approximation, its degeneracy $D_n$ is found to be:

$$D_n = D_L \frac{\tanh(\frac{\hbar}{2})}{\frac{25}{28}}, \quad (3)$$
Here the characteristic length $L$ and the nonhomogeneity parameter $\delta$ satisfy the inequality:

$$\tanh(L/2\delta) < (1 - \frac{2}{F}(n + \frac{1}{2})).$$

The Landau level degeneracy is $D_L \equiv \frac{2\pi e^2}{hc} N$ as in the case of the uniform field. The form of the degeneracy $D_n$ given above holds for all orders of $F$. The large $F$ expansion in (2) limits its validity to the region of system parameters given by the inequality below (3) which follows from the usual, [5], boundary conditions: periodicity in the y-direction perpendicular to the x-axis and limits on the position of the orbit center in the x-direction to the region $< -L/2, +L/2 >$. The orbit center x-coordinate $x_c$ is given, [4], by

$$\tanh(x_c/\delta) = \left(-\frac{p_y \delta}{\hbar}\right)/F.$$ 

The ground state energy $E_T(B, \delta, \nu)$ for spinless fermion gas with density $\nu$ in the limit of strong but nonhomogeneous fields specified by $B, \delta$ given as difference between the nonhomogeneous field state and the zero field state energy is found to be:

$$\Delta E_{nh}(n) \equiv E_T(B, \delta, \nu) - E_T(0, \nu) = 2\pi tN[(\nu - \nu_n)\frac{\tanh(L)}{25}](\nu_{n+1}\frac{\tanh(L)}{25} - \nu) +$$

$$+ \left(1 - \frac{\tanh(L)}{25}\right)[\nu(2\nu_n + \nu_1) - \frac{\tanh(L)}{25}(\nu_{n+1}\nu_n)] -$$

$$- \frac{ta^2}{\delta^2} N[\nu(n^2 + n + 1) - \nu_n(\frac{2n^2}{3} + n + \frac{1}{3})].$$

The total energy difference (4) is found assuming that there are $n$ levels $0, 1, ..., n - 1$ filled and that the $n$-th level is filled partially. The gas density $\nu$ in (4) is limited by the following inequalities:

$$\nu_{n+1}\frac{\tanh(L)}{25} \geq \nu > \nu_n\frac{\tanh(L)}{25},$$

$$\nu_n \equiv n\Phi/\Phi_0.$$ 

The uniform field result (1) follows from (4) and (5) in the limit $\delta \rightarrow \infty$ keeping values of all the other system parameters constant.

Let us now calculate some of physical characteristics of our gas of spinless fermions in the considered nonhomogeneous magnetic field at zero temperature. Then let us compare these characteristics (magnetization, static magnetic susceptibility, chemical potential and compressibility) with the corresponding characteristics in the uniform field case. The comparison enable us to describe influence of nonhomogeneity of the magnetic field in the large intensity and nonhomogeneity limit on a gas of fermions.

The magnetization is found to be given by

$$m_z(B, \delta) \equiv \frac{1}{N} \frac{\partial E_T(B, \delta, \nu)}{\partial B} = \left(a^2\frac{\Phi_0}{\Phi}\right)[-\frac{ta^2}{\delta^2}N\frac{2n^2}{3} + n + \frac{1}{3}] +$$

$$+ \frac{ta^2}{\delta^2}N\frac{2n^2}{3} + n + \frac{1}{3}].$$

We see from (6) that when the nonhomogeneity parameter $\delta$ increases its value to infinity we obtain the uniform field result. When this parameter is finite then the magnetization increases its value (its absolute value decreases).

The static magnetic susceptibility $\chi_{zz}(B, \delta)$ is found to be given by:

$$\chi_{zz}(B, \delta) \equiv \frac{\partial m_z}{\partial B} = \left(a^2\frac{\Phi_0}{\Phi}\right)^2[-4\pi tN\frac{2n^2}{3} + \frac{ta^2}{\delta^2}N\frac{2n^2}{3} + n + \frac{1}{3}].$$

$$+ \frac{ta^2}{\delta^2}N\frac{2n^2}{3} + n + \frac{1}{3}].$$
We see that the susceptibility becomes less diamagnetic than in the uniform field case due to presence of the field nonhomogeneity where the field intensity is decreased, and thus also where the density of states is lower.

The chemical potential difference between the nonhomogeneous field state and the zero field state $\Delta \mu$ is given by

$$\Delta \mu = \frac{1}{N} \left( \frac{\partial \Delta E_{nh}(n)}{\partial \nu} \right) = 2\pi t\left[ (\nu_1(2n+1)-2
\nu) - \frac{ta^2}{2^2} (n^2 + n + 1) \right].$$

(8)

The single fermion energy shift down due to presence of nonhomogeneity decreases the chemical potential and is described in (8) by the second term. The first term has the same form as in the uniform field case. Note, however, that the density of particles, $\nu$, is in the nonhomogeneous field case bounded from above and below by limiting values which are dependent on the nonhomogeneity of the field, see (5). In the uniform field case there exists symmetry when filling a given energy level ($\nu \rightarrow \nu_{n+1}^-$) and when emptying the same level ($\nu \rightarrow \nu_{n+1}^+$):

$$\mu(\nu \rightarrow \nu_{n+1}^-) = -\mu(\nu \rightarrow \nu_{n+1}^+).$$

This symmetry is lost whenever the nonhomogeneity parameter is finite.

The difference of the inverse compressibility $\Delta \frac{1}{\kappa}$ of the gas in the nonhomogeneous field state and the zero field state is given by

$$\Delta \frac{1}{\kappa} \equiv -\frac{\partial \mu}{\partial \nu} = 4\pi t$$

(9)

From (9) it follows that the inverse compressibility is not affected by the presence of the nonhomogeneity. Thus decrease of the chemical potential with increase of the gas density is the same in the uniform field case and in the nonhomogeneous field case.

Results of this paper describe some of the physical properties of 2d-fermion gas moving in our specific type of the nonhomogeneous field. We expect that qualitatively these results describe modification of the uniform field case due to presence of a static nonhomogeneity of general type. General need to study dynamics of electrons in different types of magnetic fields follows from a large number of experimental situations in which its understanding enables physicists to obtain new information. In the solid state physics these situations occur studying such effects as: the Hall effect; magnetoresistivity; anomalous skin effects in a magnetic field; cyclotron resonances in metals; magnetoplasma waves in metals and semiconductors; quantum oscillatory effects like: the de Haas - van Alsen effect, oscillations of the entropy, of the volume, of the specific heat, of the thermopower and of other thermodynamic characteristics, the Shubnikov-de Haas effect, oscillations of the surface impedance, of the Hall coefficients, of the sound absorption coefficients and of the sound velocity, magnetoacoustic oscillations (Pippard’s geometric resonance oscillations), giant quantum oscillations of absorption in metals, quasiclassical size effects (radiofrequency size effects, cut off effects of cyclotron resonance and of quantum oscillations, oscillations on cut off orbits, the Sondheimer effect) and quantum size effects (mesoscopic phenomena).

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