Conductance and shot noise in the helimagnet tunnel junction

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Abstract

As a result of sinusoidal spatial modulation, helimagnet induces spin-dependent diffracted transmission. In this work, we propose a scattering matrix treatment to the general transport problem characterized by sinusoidal diffraction. The conductance and shot noise properties of the normal metal/helimagnet/normal metal heterostructure are investigated. It is found that the shot noise is suppressed and demonstrates rise-and-fall variations as a result of interference between different diffracted channels. Sharp change of the shot noise occurs when one and both diffracted beams disappear into evanescent modes at the helimagnet spiral wave vector equal to one and two times the electron Fermi wave vector $q = k_F$ and $2k_F$.

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I. INTRODUCTION

Diffraction is centuries-old understanding of the single-beam-in multi-beam-out phenomenon in optics, acoustics, quantum mechanics, and all wave equation governed scattering processes, when the middle media has some property periodically varying in space. Naturally we should know that as a simple case transmission of the electron through a sinusoidal-height quantum potential barrier or sinusoidal-depth well demonstrates diffraction effect. Although in vast numerical treatments such as the plane-wave expansion method for metamaterials, this problem is just a building block within, we think it is important to particularly treat the quantum grating effect by developing a general scattering method and provide the detailed physical picture \textit{ab initio} from the Schrödinger equation, which to our knowledge is not covered in literature. Also, the sinusoidal-height quantum potential barrier itself can be the model of a real material and a real transport device based on it. The recent widely-focused material, helimagnet (HM), lends a very appropriate platform.

The HM is a kind of magnetic state\textsuperscript{1} with its spin spiraling in two or three dimensions characterized by a single spiral wavevector $Q$, which is different from conventional spatially uniform ferromagnet and antiferromagnet. When a single electron passes through the HM structure, spin-dependent diffraction occurs. Recently the transport properties of the HM-embedded devices are targeted from different view angles in literature. Manchon \textit{et al.}\textsuperscript{2} and us\textsuperscript{3} observed the spin-dependent diffraction effect in the transmission at the ferromagnet/HM interface and through the thin-layer HM junction, respectively. Some functional devices were proposed based on the HM such as the persistent spin currents\textsuperscript{4}, spin-field-effect transistor\textsuperscript{5}, tunneling anisotropic magnetoresistance\textsuperscript{6}, and spin resonance\textsuperscript{1}. Conductance characteristic of the HM spin configuration and spiral period was found in the Fano resonance spectrum of a quasi-one-dimensional WG containing a thin conducting HM layer as a donor impurity\textsuperscript{7}. Ac gate potential driven Quantum pumping behavior was also investigated recently\textsuperscript{8}. Although some works were done to investigate the HM-related transport properties, there lacks an overall description of the diffraction scenario.

As far as a good transport approach can go, beyond the conductance, we also considered the diffraction governed shot noise properties. As a consequence of the quantization of charge and defined by quantum contribution in the current fluctuations, shot noise is useful to obtain information on a system which is not available through conductance measurements\textsuperscript{9}. Two
of the most significant shot noise experiments are carrier charge confirmations of the Cooper pair\textsuperscript{10} and Laughlin quasiparticle\textsuperscript{11}. In most cases, the properties of quantum correlation are reflected in the Fano factor $F$, which is defined by proportion of the real shot noise $S$ to Poisson noise $2eI$ ($I$ is the average current), the latter of which corresponds to single quasiparticle transmission without correlation. Therefore, some levels of the Fano factor have typical physical meaning. $F = 1$ characterizes Poisson noise. Besides the ideal case, the Fano factor approaches 1 when the transmission is extremely small corresponding to uncorrelated transport and closed channel in ballistic tunneling. $F = 0$ characterizes full correlation and maximal quantum coherence. In real conductors, the Fano factor approaches 0 when the transmission reaches 1 corresponding to open channels in ballistic transport. In some cases with strong electron-electron interaction involved\textsuperscript{12}, the shot noise can be enhanced beyond 1. $F = 1/2$ characterizes the effect of Pauli exclusion and $F = 1/3$ characterizes diffusive transport when open and closed channels distributes in disorder such as diffusive metals\textsuperscript{13} and graphene\textsuperscript{14}. With understanding of the physics underlying different Fano factor levels, we could suppose that diffraction enhances transmission, different diffraction channels have strong coherence, and the shot noise should be thus suppressed. Our theory would confirm this supposition in detail.

Also, the transport properties are governed by diffraction. For the HM spiral wave vector $q$ larger than two times the electron Fermi wave vector $2k_F$, both diffraction beams degrade into evanescent surface modes and do not contribute to the transmission, in which case the transmission is identical to that of a plain barrier. As a result, sharp change in the conductance, shot noise, and Fano factor occurs at $q = 2k_F$, lending a potential transport measurement of $q$.

II. THEORETICAL FORMULATION

Our model is sketched in Fig. 1 (a). An HM interlayer is put between two semiinfinite free regions extending in the $x$-$y$ plane. For electrons, those free regions can be normal metal leads. Transport direction is along the $z$ coordinate. The HM spin varies in space with the vector field \begin{equation} \mathbf{n}_r = \left[ \sin \left( qx \right), 0, \cos \left( qx \right) \right] . \end{equation}
$q$ is the spin wave vector and the helix is two dimensional modulating in the $x$ direction. The spin exchange between the free electron and the HM magnetization giving rise to a space-dependent Zeeman term in the Hamiltonian, which is

$$H = -\frac{\hbar^2}{2m^*} \nabla^2 + (Jn_\epsilon \cdot \sigma + V_0) \delta (z).$$

(2)

Here, $m^*$ is the HM electron effective mass. $J$ refers to space and momentum averages of the exchange coupling strength. $\sigma$ is the Pauli matrix. We assume an ultrathin HM layer located at the $z = 0$ plane, so its effect in the Hamiltonian can be approximated by a $\delta$-function. $V_0$ is the electrostatic potential of the HM. For insulating HM, $V_0 > 0$ is a barrier potential; for conducting HM, $V_0 < 0$ is a well potential. We consider the former case.

Scattered by the HM interlayer, the electron wave function with incidence, reflection, and transmission beams in the two free regions can be written as

$$\begin{align*}
\psi_I(x, z) &= \sum_{n=\infty}^{+\infty} (A_{n\sigma}^i e^{ik_{xn}x} e^{ik_{zn}z} \chi_{\sigma} + A_{n\sigma}^o e^{-ik_{zn}z} e^{-ik_{zn}z} \chi_{\sigma}), \quad z < 0, \\
\psi_{II}(x, z) &= \sum_{n=\infty}^{+\infty} (B_{n\sigma}^i e^{ik_{zn}z} e^{-ik_{zn}z} \chi_{\sigma} + B_{n\sigma}^o e^{ik_{zn}z} e^{ik_{zn}z} \chi_{\sigma}), \quad z > 0,
\end{align*}$$

(3)

where $k_{xn} = k_x + nq$ and $k_{zn} = \sqrt{k_F^2 - k_y^2 - k_{zn}^2}$ with the Fermi wave vector $k_F = \sqrt{2m_e E_F / \hbar}$, $E_F$ the electron Fermi energy and $m_e$ the free electron mass. $A_{n\sigma}^i$ and $B_{n\sigma}^i$ are the probability amplitudes of the incoming waves from the lower and upper leads, respectively, while $A_{n\sigma}^o$ and $B_{n\sigma}^o$ are those of the outgoing waves. Diffraction occurs in the $x$ direction. Translation symmetry protects the plane wave component in the $y$-direction $e^{ik_y y}$ unchanged during transmission.

By continuity equation at the HM interface

$$\psi_I (x, 0^-) = \psi_{II} (x, 0^+),$$

(4)

and

$$\frac{\hbar^2}{2m_e} \left. \frac{\partial \psi_I}{\partial z} \right|_{z=0^-} + (V_0 + w) \psi_I (x, 0^-) = \frac{\hbar^2}{2m_e} \left. \frac{\partial \psi_{II}}{\partial z} \right|_{z=0^+},$$

(5)

with

$$w = \begin{bmatrix} \cos (qx) & \sin (qx) \\ \sin (qx) & -\cos (qx) \end{bmatrix},$$

(6)

the scattering matrix relation

$$\begin{pmatrix} A_{n\sigma}^o \\ B_{n\sigma}^o \end{pmatrix} = \begin{pmatrix} t^{\sigma\tau}_{nm} & t'^{\sigma\tau}_{nm} \\ t^{\sigma\tau}_{nm} & t'^{\sigma\tau}_{nm} \end{pmatrix} \begin{pmatrix} A_{m\tau}^i \\ B_{m\tau}^i \end{pmatrix} = \mathbf{s}_{\sigma\tau} (k_n, k_m) \begin{pmatrix} A_{m\tau}^i \\ B_{m\tau}^i \end{pmatrix}$$

(7)
can be obtained. \( r_{\sigma\tau}^{nm} \) and \( t_{\sigma\tau}^{nm} \) represent respectively the reflection and transmission amplitudes from the spin-\( \tau \), \( m \)th-order diffraction channel to the spin-\( \sigma \), \( n \)th-order diffraction channel. \( r_{\sigma\tau}^{\sigma\tau} \) and \( t_{\sigma\tau}^{\sigma\tau} \) are the corresponding backward amplitudes. \( \mathbf{k}_n = (k_{xn}, k_{yn}, k_{zn}) \) is the three-dimensional wavevector labeling diffraction channels. Considering the real current flux, the scattering matrix

\[
\hat{S}_{\sigma\tau}^{RL}(\mathbf{k}_n, \mathbf{k}_m) = \frac{\text{Re} (k_{zn})}{\text{Re} (k_{zm})} \hat{S}_{\sigma\tau}^{RL}(\mathbf{k}_n, \mathbf{k}_m).
\]

(8)

With the scattering matrix \( \hat{S}_{\sigma\tau}^{RL}(\mathbf{k}_n, \mathbf{k}_m) \), the conductance and shot noise at low temperatures can be calculated as follows\(^9\).

\[
G = \frac{e^2}{h} \int_0^{2\pi} \int_0^{\pi/2} \sum_{\sigma,\tau, n} \left| \hat{S}_{\sigma\tau}^{RL}(\mathbf{k}_n, \mathbf{k}_0) \right|^2 k_F^2 \sin \theta \theta_{in} d\theta_{in} d\phi_{in}.
\]

(9)

\[
S = 2\frac{e^3}{h} \int_0^{2\pi} \int_0^{\pi/2} \sum_{\sigma,\tau, n} \left\{ \left| \hat{S}_{\sigma\tau}^{RL}(\mathbf{k}_n, \mathbf{k}_0) \right|^2 \left[ 1 - \left| \hat{S}_{\sigma\tau}^{RL}(\mathbf{k}_n, \mathbf{k}_0) \right|^2 \right] \right\} k_F^2 \sin \theta \theta_{in} d\theta_{in} d\phi_{in}.
\]

(10)

Here, \( \theta_{in} \) and \( \phi_{in} \) are the incident polar and azimuthal angles, respectively. The Fano factor can be defined by \( F = S/(2eG) \).

III. NUMERICAL RESULTS AND INTERPRETATIONS

Numerical results of the diffracted transmission for \( q = 0.5k_F \) are shown in Fig. 1. \( T_{0,\pm1} \) is defined as \( \sum_{\sigma,\tau} \left| \hat{S}_{\sigma\tau}^{RL}(\mathbf{k}_{0,\pm1}, \mathbf{k}_0) \right|^2 \). The zero-order transmission \( T_0 \) has spherical symmetry in the incident angle space, namely, in the electron wave vector space. It approaches maximum at normal incidence, which is natural for standard barrier tunneling as the wave vector in the propagating direction and hence the current flux approaches maximum. The 1 and \(-1\) order transmission is at minimum in normal incidence due to the \( x-z \) plane symmetry of the HM spiral. When the incident angle increases, \( T_1 \) and \( T_{-1} \) increases, approaches maximum and abruptly disappears into evanescent modes for the incident azimuthal angle \( \phi_{in} = 0 \) and polar angle \( \theta_{in} = \pi/6 \) and \( \theta_{in} = -\pi/6 \), respectively. The angle can be analytically obtained by the equation of \( k_{zn,\pm1} = \sqrt{k_F^2 - (k_F \sin \theta_{in} \sin \phi_{in})^2 - (k_F \sin \theta_{in} \cos \phi_{in} \pm 0.5k_F)^2} = 0 \). To the other side of the incident sphere, \( T_1 \) and \( T_{-1} \) disappears at the glazing angle.

To investigate the conductance and shot noise properties of the HM tunnel junction, angle averaged quantities of Eqs. (9) and (10) are shown in Fig. 2. From panel (a), it can be seen that the conductance monotonously increases with the Fermi wave vector \( k_F \).
It weakly depends on the HM spiral wave vector $q$ and is not visible in the figure. The conductance increase in $k_F$ is due to larger current flux and more contributing channels. The weak dependence on $q$ is due to diffusion of the diffraction effect by angle average. The angle-averaged shot noise as functions of $q$ and $k_F$ is shown in panel (b). Prominent diffraction effect can be seen in the noise spectrum. As a result of diffraction, different diffracted channels interact and give rise to the multiple peaks in the shot noise. The pink and red dotted lines correspond to $q = k_F$ and $q = 2k_F$, respectively. For $q$ larger than $2k_F$, all diffracted waves degrade into evanescent modes and variation of the shot noise became smooth without abrupt rises and falls. For $q$ larger than $k_F$ and smaller than $2k_F$, $T_1$ disappears for all positive incident angles and $T_{-1}$ disappears for all negative incident angles. For $q$ smaller than $k_F$, diffracted transmission from part of the incident semi-sphere becomes evanescent and does not contribute to the transport as can be seen in Fig. 1. At the two sides of the pink dotted line in Fig. 2 (b) when $q$ is close to $k_F$, strength of the two diffracted waves $T_1$ and $T_{-1}$ matches each other, giving rise to maximal channel-coherence. Therefore, peaks of the shot noise are dramatically enhanced at the two side of the pink dotted line labeling $q = k_F$. When the absolute value of $k_F$ is small relative to $q$, almost no diffraction channel exists and the shot noise is extremely small. There is an abrupt increase in the shot noise when diffracted channels begin to contribute to the transport.

The relative strength of the shot noise in comparison with the Poisson noise is measured by the Fano factor. In Fig. 2 (c) and (d), we show numerical results of the Fano factor. From panel (c), it can be seen that the absolute value of the Fano factor is smaller than 1/3 throughout the considered parameter space. As summarized in the Introduction, this regime is between the complete ballistic tunneling regime of $F = 0$ and the diffusive tunneling regime of $F = 1/3$. Our considered HM tunnel junction is a low $\delta$-barrier with the spiral modulating in the spin space. The $\delta$-barrier strength $V_0 = 50 \text{ meV} \cdot \text{Å}$ and the HM exchange coupling strength $J = 20 \text{ meV} \cdot \text{Å}$. Therefore, even without the spiral modulation, the transmission is very large and close unity. The spiral modulation trifurcated the incident electron plane wave into the 0 and $\pm 1$ order diffracted waves. The main 0-order transmission still governs with $T_0$ approaching 1 and $T_{\pm 1}$ three to four orders smaller than it. These transmission properties can be clearly seen in Fig. 1. As an effect, the system approximates an open conductor and the Fano factor is very small. However, coherence between different diffraction channels significantly influences the shot noise. There are two prominent phenomena. One is that
the shot noise is dramatically suppressed relative to the poisson value. The other is that the shot noise demonstrates rise-and-fall variations. Therefore, the Fano factor is smaller for larger $k_F$ for stronger transmission and more contributing channels. Also oscillations can be seen in the Fano factor specified in the panel (d). The Fano factor oscillates as a function of $q$ for $q < 2k_F$. The oscillation is small in comparison with its decrease as a function of $k_F$ and is not prominently seen in the panel (c). When $q > 2k_F$ diffraction disappears, the shot noise properties resemble a plain barrier. For small Fermi energies, there is a dividing line in the shot noise between the diffraction affected transport for $q < 2k_F$ and the ordinary barrier scattering governed transport for $q > 2k_F$, which is already seen in the panel (b). In the panel (d) the black and red star symbols label the dividing position of $q = 2k_F$ for different Fermi energies. The shot noise properties also lend a potential detection of the HM spiral period and spin wave vector.

IV. CONCLUSIONS

In conclusion, we theoretically investigated the conductance and shot noise properties in the HM tunnel junction. With the developed scattering matrix scheme, the general procedure and formulas to calculate the diffracted transmission probabilities, the conductance, and the shot noise are given. At a certain incident angle, one of the diffracted waves $T_1$ or $T_{-1}$ disappears into an evanescent mode and that transmission coefficient abruptly falls into zero giving rise to prominent rise-and-fall oscillations in the angle-averaged shot noise. Two dividing lines of $q = k_F$ and $q = 2k_F$ characterize the regimes that one or both of the diffracted waves complete disappear into evanescent modes for all incident angles. When $q > 2k_F$, the shot noise properties resemble that of a plain $\delta$-barrier. At the two sides of $q = k_F$, strength of the two diffracted channel matches each other giving rise to strong oscillation in the shot noise. Due to correlation among different diffracted channels, the shot noise are additionally suppressed relative to the Poisson value. The diffraction-affected transport properties are prominently demonstrated in the shot noise and Fano factor.
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FIG. 1: (a) Schematic illustration of the diffraction effect in the helimagnet tunnel junction. The helimagnet spin spirals in the $x$-$z$ plane. An incident plane wave can be diffracted into sidebands with the $x$-component wave vector adding or subtracting a spiral wave vector $q$. Higher order diffraction decays exponentially justifying the $\pm 1$ order cutoff. For $q$ larger than one Fermi wave vector $k_F$, one diffraction beam degrades into evanescent surface mode and does not contribute to the transmission. For $q$ larger than $2k_F$, both diffraction beams degrade into evanescent surface modes and do not contribute to the transmission, in which case the transmission resembles that of a plain $\delta$-barrier. (b) Order-0, (c) order-1, (d) order-$-1$ transmission $T_0$, $T_1$, $T_{-1}$ in the incident angle space. For panels (b), (c), and (d), $E_F = 100$ meV, $V_0 = 50$ meV·Å, $J = 20$ meV·Å, $q = 0.5k_F$. Order-0 transmission has spherical symmetry. Order-1 and $+1$ transmission disappears at certain positive and negative incident angles, respectively. The two surfaces in panels (c) and (d) correspond to the two peaks at which the diffracted beam disappears. One is for grazing incidence, the other is at a certain positive and negative incident angle for $T_1$ and $T_{-1}$, respectively.
FIG. 2: Conductance $G$ (a), shot noise $S$ (b), and the Fano factor $F$ (c) as functions of the HM spiral wave vector $q$ and the electron Fermi wave vector $k_F$. $G$ is in unit of $e^2/(h\AA^2)$. $S$ is in unit of $2e^3/(h\AA^2)$. In panel (b), the red and pink dotted line corresponds to $q = 2k_F$ and $q = k_F$, respectively. (d) Fano factor as a function of $q$ for two different Fermi energies. The star symbol is at the Fermi energy with $q = 2k_F$. 
