Influence of Phonon dimensionality on Electron Energy Relaxation

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We studied experimentally the role of phonon dimensionality on electron-phonon (e-p) interaction in thin copper wires evaporated either on suspended silicon nitride membranes or on bulk substrates, at sub-Kelvin temperatures. The power emitted from electrons to phonons was measured using sensitive normal metal-insulator-superconductor (NIS) tunnel junction thermometers. Membrane thicknesses ranging from 30 nm to 750 nm were used to clearly see the onset of the effects of two-dimensional (2D) phonon system. We observed for the first time that a 2D phonon spectrum clearly changes the temperature dependence and strength of the e-p scattering rate, with the interaction becoming stronger at the lowest temperatures below ~ 0.5 K for the 30 nm membranes.

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It is an established fact that at sub-Kelvin temperatures the thermal coupling between conduction electrons and the lattice becomes very weak. This has significant implications for the operation of low-temperature detectors and coolers, or for any solid-state systems where dissipation and cooling are relevant. Low-temperature electron-phonon (e-p) interaction has been studied widely during the past decades, but mostly only for the case in which the phonons are fully three-dimensional (3D). However, due to significant advances in fabrication of thin suspended structures, many practical devices and detectors exist in which the phonons are expected to move freely only within the plane of a membrane, forming a quasi-2D system.

The question how the two-dimensionality of the phonon modes influences e-p interaction has been addressed theoretically for certain cases, but no clear experimental observation of the effect has been reported to date, although several attempts have been made.

In this paper, we show for the first time experimentally that the electron-phonon interaction clearly changes depending on the dimensionality of the phonons, as expected from theory. E-p coupling was measured with the help of sensitive NIS tunnel junction thermometry, for thin Cu wires on suspended silicon nitride (SiN) membranes with thickness varying from 30 nm to 750 nm, which spans the transition from 2D to 3D phonons. In addition, samples with identical Cu wires on bulk substrates were also measured for comparison. For the thinnest membranes, the e-p interaction was strengthened in comparison with the bulk samples, and its temperature dependence changed significantly, as is predicted by the theory.

In the presence of stress-free boundaries, the bulk transversal and longitudinal phonon modes (with sound velocities $c_t$ and $c_l$, respectively) couple to each other and form a new set of eigenmodes, which in the case of a suspended membrane are known as the horizontal shear modes ($h$), and symmetric ($s$) and antisymmetric ($a$) Lamb modes. The frequencies $\omega$ for the $h$ modes are simply $\omega = c_t \sqrt{k_t^2 + (m\pi/d)^2}$, where $k_t$ is the wave vector component parallel to the membrane surfaces, $d$ is the membrane thickness and the integer $m$ is the branch number. However, the dispersion relations of the $s$ and $a$ Lamb modes cannot be given in a closed analytical form, but have to be calculated numerically. The lowest three branches, dominant for thin membranes at low temperatures, have low frequency analytical expressions: $\omega_h = c_t k_t$, $\omega_s = c_s k_t$, and $\omega_a = \frac{\hbar}{m} k_t^2$, where $c_s = 2c_t \sqrt{(c_t^2 - c_l^2)/c_l^2}$ is the effective sound velocity of the $s$ mode, and $m^* = \hbar \left[ 2c_t d \sqrt{(c_t^2 - c_l^2)/3c_l^2} \right]^{-1}$ is an effective mass for the $a$-mode “particle.” This lowest $a$-mode with its quadratic dispersion is mostly responsible for the non-trivial behavior of the e-p interaction.

Note that already a single free surface affects the modes, and the e-p interaction, as the bulk modes couple and form another new set of eigenstates, including the surface localized Rayleigh-mode. Thus, the widely observed result for e-p power flow $P = 2V(T_e - T_p)$ from a metal volume $V$ with $T_e$ the electron and $T_p$ the phonon temperature, is not expected to hold even for thin enough films on bulk substrates.

A schematic of the Cu wire samples on suspended silicon nitride membranes and the measured circuit is shown in Fig. 1. 17 samples were made on either suspended membranes or bulk substrates, where nitridized (100) Si wafers with 30, 200 and 750 nm thick low-stress SiN top layers were used as the substrate for both cases. The suspension of the SiN$_x$ membranes (size 600×300 $\mu$m$^2$) was achieved by anisotropic backside wet etching of the silicon substrate in KOH, and the metallic structures were fabricated using standard e-beam lithography and multi-angle shadow mask evacuation techniques. As the e-p interaction strength is sensitive to the thickness and disorder level of the metal, we minimized its effect by evaporating the Cu wires of a specific thickness on all the different substrates simultaneously. Ultrathin Cu layers ($t$=14-30 nm) were used to strengthen the effect of the thin membranes. The oxide layer forming the tunnel junction barriers was produced by thermal oxidation of Al. Table 1 presents the essential dimensions of the
samples discussed in this paper, measured by scanning electron (SEM) and atomic force (AFM) microscopies. The electron mean free path \( l \) was determined from the resistance of the wire at base temperature \( 60 \) mK, using the accurately measured dimensions of the wire.

**Table I:** Parameters for samples. M= suspended Si\(_x\) membrane and B= bulk substrate. B6 had an oxidized Si substrate.

| Sample | Si\(_x\) d | Cu t | V | l | \( \tau \)(0.2K) | \( \tau \)(0.8K) |
|--------|------------|------|---|---|---------------|---------------|
| M1     | 30 14      | 2.71 | 5.7 | 2.6 | 0.16          |               |
| B1     | 30 14      | 2.46 | 4.9 | 7.1 | 0.030         |               |
| M2     | 200 14     | 2.44 | 4.6 | 15.0 | 0.11         |               |
| B2     | 200 18     | 3.67 | 4.1 | 6.4 | 0.045         |               |
| M3     | 30 19      | 5.50 | 11.2 | 2.2 | 0.30          |               |
| B3     | 30 19      | 4.62 | 9.8 | 4.3 | 0.034         |               |
| M4     | 750 22     | 6.09 | 10.3 | 3.1 | 0.030         |               |
| B4     | 750 22     | 5.87 | 8.7 | 3.9 | 0.013         |               |
| M5     | 30 32      | 6.09 | 22  | 1.8 | 0.31          |               |
| B5     | 30 32      | 5.09 | 19  | 2.7 | 0.038         |               |
| B6     | - 32      | 7.10 | 22  | 1.6 | 0.031         |               |

**FIG. 1:** (Color online) A Schematic of the suspended samples and the measuring circuit. Red lines are the normal metal Cu, light gray Al for SINIS-junctions and dark gray Al or Nb for SN-junctions.

We used the hot-electron technique \cite{2} to measure the e-p interaction by overheating the electrons by Joule heat power \( P \) and measuring the resulting electron temperature \( T_e \). All the samples had two electrically isolated Cu normal metal wires next to each other (Fig. 1). The longer wire \( (L = 500 \mu m) \) was heated by applying a slowly ramping voltage across the pair of superconducting Nb (or Al) leads in direct metallic contact to Cu, forming SN junctions. These junctions provide excellent electrical, but very poor thermal conductance due to Andreev reflection, as the junctions are biased within the superconducting gap \( \Delta \). Thus, due to the lack of outdiffusion of electrons and the long length of the wire, input heat is distributed uniformly in the interior of the wire and the electron gas cools dominantly by phonons, instead of diffusively \cite{18} or by thermal photons \cite{12}. Since \( L \gg L_{e-c} \), the electron-electron scattering length, electron temperature is also well defined without complications from non-equilibrium \cite{20}. In our sample geometry the electron temperature is measured with two additional Al leads forming a NIS tunnel junctions pair (SINIS) in the middle of heated wire, as a function of input power \( P = IV \) measured in a four probe configuration. The purpose of the short Cu wire, with additional SINIS thermometer on it, is to give an estimate of the local phonon temperature \( T_p \), as the e-p power flow depends on both \( T_e \) and \( T_p \).

The current-biased Al SINIS thermometer is ideally suited to measure temperature below a few Kelvins, \cite{2} due to its high sensitivity (in our DC measurement \( \sim 0.1 \) mK at 0.1 K) and low power dissipation. In addition, for all the data here, the SINIS voltage vs. temperature response follows the BCS theory without fitting parameters very accurately at least down to \( \sim 0.2 \) K, where typically saturation sets in. This saturation depends on the strength of the e-p interaction (size of thermometer and type of substrate) and the amount of filtering, and thus we conclude that it is most likely caused by external noise heating. For this reason we take the most conservative approach and assume that all saturation is caused by it, in which case we can use BCS theory to convert the measured voltage data for all temperatures.

Even if the electrons lose their energy overwhelmingly to the phonons in our sample geometry, it is still possible that the measured temperature is not only determined by the e-p interaction. This is because the emitted phonons could be removed so ineffectively from the membrane that the phonon transmission becomes a bottleneck for the energy flow. Bulk scattering of phonons at low temperatures is very weak \cite{7}, even for thin disordered membranes \cite{21}, as is boundary resistance for thin films on bulk substrates \cite{22,23}. In contrast, almost nothing quantitative is known about the boundary resistance between a thin metal film and a thin 2D membrane, or between a thin 2D membrane and a bulk substrate. However, it seems clear that if the combined metal film and membrane thickness is below the thermal wavelength of the phonons, the phonon modes in the two materials are strongly coupled, leading to an effectively non-existent boundary resistance. Hence, if we check that the membrane temperature \( T_p \) is not too high compared to \( T_e \) (effective enough hot phonon removal), we can be confident that the measured \( T_e \) reflects the e-p interaction.

Figure 2 shows the main result of the measurements, with \( T_e \) and \( T_p \) plotted vs. the heating power density \( p = P/V \) for all membrane thicknesses (30 nm, 200 nm and 750 nm). In addition, data from a few representative bulk samples are shown. Compared to the corresponding bulk substrate sample (B4), \( T_e \) of the 750 nm membrane (M4) shows no difference at all, and it effectively behaves as bulk. This is reasonable, because for the 750 nm membrane the estimated dimensionality cross-over temperature \cite{24,25} \( T_{cr} = \hbar c_t/(2k_B d) \) is \( \sim 30 \) mK, with \( c_t = 6200 \) m/s for SiN. The phonon temperatures \( T_p \), however, show a big difference: The
bulk samples show almost no response from the saturation value of the thermometer \( T_e \approx 190 \text{ mK} \), whereas the membrane phonons heat up measurably, most likely due to the boundary resistance between the membrane and the bulk. Nevertheless, this increase in \( T_p \) for all samples is small enough not to influence the e-p interaction. For the 200 nm thick membrane (M2) \( (T_{cr} \sim 110 \text{ mK}) \), at low heating power densities \( [p < 40 \text{ pW}/(\mu \text{m})^3] \) the temperature dependence follows the behavior of the bulk sample (B2), although with a difference in the absolute value. This shows that the strength of the e-p coupling weakens compared to the bulk. At higher powers and temperatures \( (p > 40 \text{ pW}/(\mu \text{m})^3) \), where \( T_e > 0.6 \text{ K} \), \( T_e \) starts to increase more rapidly in the membrane sample, most likely due to the boundary resistance effects. The phonons in the 30 nm thick membrane sample (M1) are expected to be in the 2D limit at low temperatures \( (T_{cr} \sim 0.5 \text{ K}) \), and a clear sign of this can be seen in Fig. 2 as a strongly different behavior of the measured \( T_e \) vs. \( p \) curve with respect to all other samples. Below \( \sim 6 \text{ pW}/(\mu \text{m})^3 \) the e-p coupling is notably stronger \( (T_e \text{ lower}) \) than in the corresponding bulk (B1) or any other sample, but again at highest temperatures the influence of other effects starts to dominate over the e-p coupling.

To study the temperature dependence of the data in Fig. 2 more accurately, we plot the logarithmic derivatives \( d(\log p)/d(\log T_e) \) in Fig. 3 (a) for different membrane samples. For low heating powers \( (T_e^n >> T_p^n) \), \( \rho_{e-p} \approx T_e^n \), where \( n \) is the power law of the e-p interaction, thus in that regime \( d(\log p)/d(\log T_e) = n \). Typically this exponent is \( n \approx 5 \) for thicker \( (t > 30 \text{ nm}) \) metal films on bulk substrates \[4, 17\], if the disorder in the film is not too strong \[20, 21, 28\]. From Fig. 3 (a) we first of all see that for the 30 nm membrane sample M1, the difference to the bulk sample B1 is very clear. The M1 data has a plateau of \( n \approx 4.5 \) between \( p = 0.1 - 6 \text{ pW}/(\mu \text{m})^3 \), while for B1, \( n \approx 5 \). Typically this exponent is \( n \approx 5 \), which, for comparison, is observed for a more typical \( t = 32 \text{ nm} \) Cu wire on oxidized Si (B6). This behavior is qualita-

![FIG. 2: (Color online) Measured electron and phonon temperatures \( T_e \) and \( T_p \) versus the applied heating power density in log-log-scale.](image1)

![FIG. 3: (Color online) Numerical logarithmic derivatives of the measured data in Fig. 2 (a) \( T_e \) data for M1 and B1, (b) \( T_e \) data for M2 and B2, (c) \( T_e \) data for M4 and B4.](image2)
The membrane close to the transition region (T < 0.5 K) the thinnest membranes can have a factor 2-3 strengthening effect, whereas at higher temperatures the thermal relaxation from membranes can be an order of magnitude weaker compared to bulk samples. The membrane close to transition region (d = 200 nm) was shown to have a weaker (~ factor of two) e-p interaction strength than the bulk samples. Thinning the metal film on bulk substrates also leads to a sizeable weakening of the e-p interaction. The observed power law exponent for the 2D limit is consistent with n ≈ 4.5, and is much smaller than the corresponding bulk exponent n = 6...7. A reduction by more than a factor one gives indirect evidence of the importance of the flexural, dispersive Lamb-modes for the membrane electron-phonon interaction, in agreement with theory [9, 10].

FIG. 4: (Color online) (a) Tc versus p = P/V for 30 nm membrane samples M1,M3,M5. (b) Tc versus p for bulk samples, from top to bottom B1 (top), B3, B5 and B6 (bottom). (c) d(log p)/d(log T) of the data in (a). (d) d(log p)/d(log T) of the data in (b). From top to bottom: green line B1 (top), magenta B3, blue B5, Red B6 (bottom). In (d) noise has been filtered to help the eye.

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