Spin multistability in dissipative polariton channels

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We present a model for theoretical description of the dynamics of a system of spinor cavity polaritons in real space and time accounting for all relevant types of the interactions and effective magnetic fields. We apply our general formalism for the consideration of the polarization dynamics of the coherently driven one dimensional polariton channel. We investigate effect of the temperature and the longitudinal-transverse splitting on the spin (polarization) multistability and hysteresis arising from the polarization-dependent polariton-polariton interaction. We show that the effect of the phase of the driving laser pump is as important as its strength and demonstrate that the multistability behavior can survive up to high temperatures in presence of longitudinal-transverse splitting.

I. INTRODUCTION

Cavity polaritons are composite particles, arising from strong coupling between photonic mode of a planar semiconductor microcavity and exciton transition in Quantum Well (QW), embedded in the cavity at the point where the electric field of the confined electromagnetic state reaches its maximum. Having a hybrid half-light - half-matter nature cavity polaritons demonstrate a set of peculiar properties which make them different from other quasiparticles in solid state systems. Extremely low effective mass of the cavity polaritons (about $10^{-4} - 10^{-5}$ of the free electron mass) together with strong polariton-polariton interactions makes polaritonic system an ideal candidate for observation of the variety of quantum collective phenomena at surprisingly high temperatures. Achievement of polariton Bose-Einstein condensation was first reported at $T = 20K$1 and later on even at room temperature2. Later on polariton superfluidity3, Josephson effect4 and formation of topological excitations5 were experimentally observed. Other theoretically predicted effects such as polariton self-trapping42, polariton-mediated superconductivity7 still wait for their experimental confirmation.

Besides fundamental interest, quantum microcavities in the strong coupling regime can be used for optoelectronic applications8. For more then a decade, the only object of the study in this context was polariton laser9, a novel type of the coherent emitter which explores the possibility of the polariton BEC. In the last years, however, the emphasis became to shift on other types of the devices based on transport properties of cavity polaritons in real space. It was noticed that the peculiar spin structure of polaritons opens a way for creation of optical analogs of spintronic components (so-called spinoptronic devices10). With respect to optics, spin-optronics has the advantage of being able to use particle-particle interactions occurring in nanostructures and resulting in strong nonlinearities. With respect to spintronics, it has the advantage of strongly reducing the dramatic impact of carrier spin relaxation or decoherence, which has severely limited the achievement or the functionality of any working semiconductor-based spintronic devices.

In this context, the analysis of one-dimensional (1D) polariton transport is of particular importance11, as 1D polariton channels are fundamental building blocks of such future spinoptronic devices as polariton neurons12 and polariton integrated circuits13. It should be noted, that current state of growth technology offers a large variety of methods of the lateral confinement of cavity polaritons14 and polariton quantum wires (1D polariton...
channels) can be routinely produced.

Currently, the theoretical study of transport of spinor cavity polaritons in the real space is based on the assumption of full coherence of the polaritonic system. Polariton-polariton interactions are either neglected\cite{15-17}, either treated within frameworks of spinor Gross-Pitaevskii equations (GPe)\cite{18}. The non-coherent processes coming from the interaction of the polaritonic system with a phonon bath in most cases are not accounted for or treated within simple phenomenological models lacking microscopic justification\cite{19-21}. On the other hand, it is clear that polariton-phonon interaction is of crucial importance, as it provides a thermalization mechanism for a polaritonic ensemble and drastically affects such experimentally observable quantities as first and second order coherencies\cite{22,23}.

It should be noted, that for spatially homogeneous polariton system polariton-phonon interactions can be accounted for using a system of the semiclassical Boltzmann equations\cite{24-27}. This method, however, has several serious drawbacks. First, it is based on the assumption that the system is fully incoherent, and thus variety of intriguing nonlinear phenomena such as bistability and multistability can not be described. Second, it provides the information about occupation numbers in the reciprocal space only, and thus can not be used for description of the dynamics of the spatially inhomogeneous system. This makes this formalism inappropriate for modeling of spinoptronic devices based on polariton transport in real space.

In the present paper we present formalism suitable for the description of the dynamics of an inhomogeneous spinor polariton system in real space and time accounting for all relevant types of the processes. Namely, we take into account polariton-polariton and polariton-phonon interactions and effective longitudinal-transverse (TE-TM) magnetic field acting on polariton spin. Our consideration is based on the Lindblad approach for density matrix dynamics and represents a generalization of our previous work where spinless case was considered\cite{28,29}. We use our results for modeling of the spin dynamics of the polaritons in 1D channels, investigating the role played by decoherence at different temperatures.

II. FORMALISM

We describe the state of the system (polaritons plus phonons) by its density matrix $\chi$, for which we apply the Born approximation factorizing it into the phonon part which is supposed to be time-independent and corresponds to the thermal distribution of acoustic phonons $\chi_{ph} = \exp\{-\frac{H_{ph}}{k_B T}\}$ and the polariton part $\chi_{pol}$ whose time dependence should be determined, $\chi = \chi_{ph} \otimes \chi_{pol}$. Our aim is to find dynamic equations for the time evolution of the single-particle polariton density matrix in real space and time

$$\rho_{\sigma,\sigma'}(\mathbf{r},\mathbf{r}',t) = Tr\left\{ \hat{\psi}_{\sigma}^\dagger(\mathbf{r},t)\hat{\psi}_{\sigma'}(\mathbf{r}',t)\rho \right\}$$

$$= \langle \hat{\psi}_{\sigma}^\dagger(\mathbf{r},t)\hat{\psi}_{\sigma'}(\mathbf{r}',t) \rangle$$

where $\hat{\psi}_{\sigma}(\mathbf{r},t)$ are operators of the spinor polariton field, the subscripts $\sigma,\sigma' = \pm 1$ denote the $z$-projection of the spin of cavity polaritons and correspond to right- and left-circular polarized states and the trace is performed by all the degrees of freedom of the system. The particularly interesting quantities are matrix elements with $\mathbf{r} = \mathbf{r}'$ which give the density and polarization of the polariton field in real space and time

$$n(\mathbf{r},t) = \sum_{\sigma = \pm 1} \rho_{\sigma,\sigma}(\mathbf{r},t),$$

$$s_z(\mathbf{r},t) = \frac{1}{2} [\rho_{+,+}(\mathbf{r},t) - \rho_{-,+}(\mathbf{r},t)],$$

$$s_x(\mathbf{r},t) + is_y(\mathbf{r},t) = \rho_{+,+}(\mathbf{r},t) - \rho_{-,+}(\mathbf{r},t).$$

The off-diagonal matrix elements with $\mathbf{r} \neq \mathbf{r}'$ also have physical meaning and describe spatial coherence in the system.

To obtain expressions for temporal dynamics of the components of single particle density matrix it is convenient to go to the reciprocal space, making a Fourier transform of the one-particle density matrix,

$$\rho_{\sigma,\sigma'}(\mathbf{k},\mathbf{k}',t) = (2\pi)^d/L^d \int e^{i(\mathbf{k}\mathbf{r} - \mathbf{k}'\mathbf{r}')} \rho_{\sigma,\sigma'}(\mathbf{r},\mathbf{r}',t) d\mathbf{r}d\mathbf{r}'$$

$$= Tr\left\{ a_{\sigma,k}^+ a_{\sigma',k'} \chi \right\} \equiv \langle a_{\sigma,k}^+ a_{\sigma',k'} \rangle,$$

where $d$ is the dimensionality of the system ($d = 2$ for non-confined polaritons, $d = 1$ for the polariton channel), $L$ is its linear size, $a_{\sigma,k}$, $a_{\sigma,k}^+$ creation and annihilation operators of the polaritons with circular polarization $\sigma$ and momentum $\mathbf{k}$. Note, that we have chosen the prefactor in a Fourier transform in such a way, that the values of $\rho(\mathbf{k},\mathbf{k}',t)$ are dimensionless, and diagonal matrix elements give occupation numbers of the states in discretized reciprocal space. Knowing the density matrix in reciprocal space, we can find the density matrix in real space straightforwardly applying the inverse Fourier transform.

The total Hamiltonian of the system can be represented as a sum of two parts,

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2,$$

where the first term $\mathcal{H}_1$ describes the "coherent" part of the evolution, corresponding to free polariton propagation, polariton-polariton interactions and the effect of TE-TM splitting, and the second term $\mathcal{H}_2$ corresponds to the dissipative interaction with acoustic phonons. The two terms affect the polariton density matrix in a qualitatively different way.
A. Polariton-polariton interactions

The part of the evolution corresponding to $\mathcal{H}_1$ is given by the following expression

$$
\mathcal{H}_1 = \sum_{k\sigma} E_k a_{k,\sigma}^+ a_{k,\sigma} + \sum_{k,\sigma} \Omega(k) a_{k,\sigma}^+ a_{k,-\sigma} + U_1 \sum_{k_1,k_2,p,\sigma} a_{k_1,\sigma}^+ a_{k_2,\sigma}^+ a_{k_1+p,p,\sigma} a_{k_2-p,p,\sigma} + U_2 \sum_{k_1,k_2,p,\sigma} a_{k_1,\sigma}^+ a_{k_2,-\sigma}^+ a_{k_1+p,\sigma} a_{k_2-p,-\sigma},
$$

where $E_k$ gives the dispersion of the polaritons, $\Omega(k)$ is the TE-TM splitting corresponding to the in-plane effective magnetic field leading to the rotation of the pseudospin of cavity polaritons, $U_1$ is the matrix element of the interaction between polaritons of the same circular polarization, $U_2$-matrix element of the interaction between polaritons of opposite circular polarizations. In the current paper we neglect the $p$-dependence of the polariton-polariton interaction constant coming from Hopfield coefficients for simplicity. As well, we will suppose $\Omega(k) = \text{const}$ which corresponds well to the situation of the polariton channel (but not for 2D polariton system).

The effect of $\mathcal{H}_1$ on the evolution of the density matrix is described by the Liouville-von Neumann equation,

$$
i \hbar (\partial_t \chi)^{(1)} = [\mathcal{H}_1; \chi],
$$

which after the use of the mean field approximation leads to the following dynamic equations for the elements of the single-particle density matrix in the reciprocal space (the derivation is completely analogous to those presented in Ref. [28]):

$$
-i\hbar \{\partial_t \rho_{\sigma,\sigma'}(k,k')\}^{(1)} = (E_k - E_{k'}) \rho_{\sigma,\sigma'}(k,k') + \Omega [\rho_{-\sigma,-\sigma'}(k,k') - \rho_{\sigma,-\sigma'}(k,k')] + U_1 \sum_{k_1,p} [\rho_{\sigma,\sigma'}(k_1,k_1-p) \rho_{\sigma,\sigma'}(k-p,k_1') - \rho_{\sigma,-\sigma'}(k_1,k_1-p) \rho_{\sigma,-\sigma'}(k,k')]
$$

$$
+ U_2 \sum_{k_1,p} [\rho_{-\sigma,-\sigma'}(k_1,k_1-p) \rho_{\sigma,\sigma'}(k-p,k') - \rho_{-\sigma',-\sigma'}(k_1,k_1-p) \rho_{\sigma,-\sigma'}(k,k')].
$$

B. Scattering with acoustic phonons

Polariton-phonon scattering corresponds to the interaction of the quantum polariton system with the classical phonon reservoir. It is of dissipative nature, and thus straightforward application of the Liouville-von Neumann equation is impossible. One should rather use the approach based on the Lindblad formalism, which is standard in quantum optics and results in the master equation for the full density matrix of the system[30]. For the convenience of the reader, we give the main steps of the derivation of the dissipative part of dynamic equations for spinor polariton system, omitting however all technical details which can be found elsewhere[30].

The Hamiltonian of the interaction of polaritons with acoustic phonons in Dirac picture can be represented as

$$
\mathcal{H}_2(t) = \mathcal{H}^{-}(t) + \mathcal{H}^{+}(t) = \sum_{\sigma,k,q} D(q) e^{i(E_{k+q}-E_k) t} a_{k,\sigma}^+ a_{k,q,\sigma} b_{q} e^{-i\omega_q t} + b_{-q}^+ e^{i\omega_q t},
$$

where $a_{k,\sigma}$ are operators for spinor polaritons, $b_{q}$ operators for spinless phonons, $E_k$ and $\omega_q$ are dispersion relations for polaritons and acoustic phonons respectively, $D(q)$ is the polariton-phonon coupling constant. In the last equality we separated the terms $\mathcal{H}^{\pm}$ where a phonon is created, containing the operators $b^{\pm}$, from the terms $\mathcal{H}^{-}$ in which it is destroyed, containing operators $b$. Now, one can consider a hypothetical situation when polariton-polariton interactions are absent, and the redistribution of the polaritons in reciprocal space is due to the scattering with acoustic phonons only. One can rewrite the Liouville-von Neumann equation in an integro-differential form and apply the so called Markovian approximation, corresponding to the situation of fast phase memory loss (see Ref. [30] for the details and discussion of limits of validity of the approximation)

$$
(\partial_t \chi)^{(2)} = -\frac{1}{\hbar^2} \int_{-\infty}^{t} dt' [\mathcal{H}_2(t); [\mathcal{H}_2(t'); \chi(t)]] = \delta_{\Delta E} \left[2 (\mathcal{H}^{+} \chi \mathcal{H}^{-} + \mathcal{H}^{-} \chi \mathcal{H}^{+}) - (\mathcal{H}^{+} \mathcal{H}^{+} + \mathcal{H}^{-} \mathcal{H}^{-}) \chi - \chi (\mathcal{H}^{+} \mathcal{H}^{-} + \mathcal{H}^{-} \mathcal{H}^{+}) \right],
$$

where the coefficient $\delta_{\Delta E}$ denotes energy conservation and has dimensionality of inverse energy and in the calculation taken to be equal to the broadening of the polariton state[31]. For time evolution of the mean value of any arbitrary operator $\langle \hat{A} \rangle = Tr(\chi \hat{A})$ due to scattering with phonons one thus has (derivation of this formula is represented in Ref. [28]):

$$
\left\{ \partial_t \langle \hat{A} \rangle \right\}^{(2)} = \delta_{\Delta E} \left( \langle \mathcal{H}^{-}; [\hat{A}; \mathcal{H}^{+}] \rangle + \langle \mathcal{H}^{+}; [\hat{A}; \mathcal{H}^{-}] \rangle \right). \tag{12}
$$
Putting \( \hat{A} = a_{\sigma,\mathbf{k}'} \) in this equation we get the contributions to the dynamic equations for the elements of the single-particle density matrix coming from polariton-phonon interaction:

\[
\{ \partial_t n_{\mathbf{k},\sigma} \}^{(2)} = \sum_{\mathbf{q},E_k < E_{k+q}} 2W(\mathbf{q}) \left[ (n_{\mathbf{k},\sigma} + 1)n_{\mathbf{k+q},\sigma} n_{\mathbf{q},\sigma}^{ph} + 1 \right] n_{\mathbf{q},\sigma}^{ph} + \frac{1}{2} (\rho_{\sigma,-\sigma}(\mathbf{k},\mathbf{k}) \rho_{\sigma,\sigma}(\mathbf{k} + \mathbf{q}, \mathbf{k} + \mathbf{q}) + \rho_{-\sigma,\sigma}(\mathbf{k},\mathbf{k}) \rho_{\sigma,-\sigma}(\mathbf{k} + \mathbf{q}, \mathbf{k} + \mathbf{q})) \\
+ \sum_{\mathbf{q},E_k > E_{k+q}} 2W(\mathbf{q}) \left[ n_{\mathbf{k},\sigma} (n_{\mathbf{k+q},\sigma} n_{\mathbf{q},\sigma}^{ph} + 1) - n_{\mathbf{k},\sigma} (n_{\mathbf{k+q},\sigma} + 1) n_{\mathbf{q},\sigma}^{ph} + 1 \right] , \tag{13}
\]

\[
\{ \partial_t \rho_{\sigma,-\sigma}(\mathbf{k},\mathbf{k}') \}^{(2)} = \sum_{\mathbf{q},E_k < E_{k+q}} W(\mathbf{q}) \left[ \rho_{\sigma,\sigma}(\mathbf{k},\mathbf{k}') (n_{\mathbf{k+q},\sigma} n_{\mathbf{q},\sigma}^{ph} + 1) + \rho_{-\sigma,-\sigma}(\mathbf{k},\mathbf{k}') \rho_{\sigma,-\sigma}(\mathbf{k} + \mathbf{q}, \mathbf{k} + \mathbf{q}) \right] \\
- \sum_{\mathbf{q},E_k > E_{k+q}} W(\mathbf{q}) \left[ \rho_{\sigma,\sigma}(\mathbf{k},\mathbf{k}') (n_{\mathbf{k+q},\sigma} n_{\mathbf{q},\sigma}^{ph} + 1) + \rho_{-\sigma,-\sigma}(\mathbf{k},\mathbf{k}') \rho_{\sigma,-\sigma}(\mathbf{k} + \mathbf{q}, \mathbf{k} + \mathbf{q}) \right] \tag{14}
\]

\[
\{ \partial_t \rho_{\sigma,-\sigma}(\mathbf{k},\mathbf{k}') \}^{(2)} = \sum_{\mathbf{q},E_k < E_{k+q}} W(\mathbf{q}) \left[ \rho_{\sigma,\sigma}(\mathbf{k},\mathbf{k}') (n_{\mathbf{k+q},\sigma} n_{\mathbf{q},\sigma}^{ph} + 1) + \rho_{-\sigma,-\sigma}(\mathbf{k},\mathbf{k}') \rho_{\sigma,-\sigma}(\mathbf{k} + \mathbf{q}, \mathbf{k} + \mathbf{q}) \right] \\
- \sum_{\mathbf{q},E_k > E_{k+q}} W(\mathbf{q}) \left[ \rho_{\sigma,\sigma}(\mathbf{k},\mathbf{k}') (n_{\mathbf{k+q},\sigma} n_{\mathbf{q},\sigma}^{ph} + 1) + \rho_{-\sigma,-\sigma}(\mathbf{k},\mathbf{k}') \rho_{\sigma,-\sigma}(\mathbf{k} + \mathbf{q}, \mathbf{k} + \mathbf{q}) \right] \tag{15}
\]

\[
\{ \partial_t \rho_{\sigma,-\sigma}(\mathbf{k},\mathbf{k}') \}^{(2)} = \sum_{\mathbf{q},E_k < E_{k+q}} W(\mathbf{q}) \left[ \rho_{\sigma,\sigma}(\mathbf{k},\mathbf{k}') (n_{\mathbf{k+q},\sigma} n_{\mathbf{q},\sigma}^{ph} + 1) + \rho_{-\sigma,-\sigma}(\mathbf{k},\mathbf{k}') \rho_{\sigma,-\sigma}(\mathbf{k} + \mathbf{q}, \mathbf{k} + \mathbf{q}) \right] \\
- \sum_{\mathbf{q},E_k > E_{k+q}} W(\mathbf{q}) \left[ \rho_{\sigma,\sigma}(\mathbf{k},\mathbf{k}') (n_{\mathbf{k+q},\sigma} n_{\mathbf{q},\sigma}^{ph} + 1) + \rho_{-\sigma,-\sigma}(\mathbf{k},\mathbf{k}') \rho_{\sigma,-\sigma}(\mathbf{k} + \mathbf{q}, \mathbf{k} + \mathbf{q}) \right] \tag{16}
\]

where, \( n_{\mathbf{k},\sigma} = \rho_{\sigma,\sigma}(\mathbf{k},\mathbf{k}) \), \( \rho_{-\sigma,-\sigma}(\mathbf{k},\mathbf{k}) = s_<(\mathbf{k}) + is_>(\mathbf{k}) \) and \( W(\mathbf{q}) \) denote spin-independent scattering rates with acoustic phonons (see Ref. 28 for the details). The first two equations corresponding to \( \mathbf{k} = \mathbf{k}' \) are nothing but the spinor Boltzmann equations for polariton-phonon scattering describing the redistribution of the polaritons in the reciprocal space which were obtained earlier using another technique. The equations for off-diagonal matrix elements with \( \mathbf{k} \neq \mathbf{k}' \) describe their decay which physically corresponds to the decay of the coherence in the system coming from polariton-phonon interactions.
C. Pumping terms

In this paper we concentrate on the case when the system is pumped by external coherent laser beam. The corresponding Hamiltonian can be introduced as

$$\mathcal{H}_{cp} = \sum_{\mathbf{k},\sigma} p_{\mathbf{k},\sigma}(t) a_{\mathbf{k},\sigma}^+ + \text{h.c.}$$

(17)

Here $p_{\mathbf{k}}$ is the Fourier transform of the pumping amplitude in real space

$$p_{\sigma}(\mathbf{x},t) = P_{\sigma}(\mathbf{x}) e^{i\mathbf{k}_p \cdot \mathbf{x}} e^{-i\omega_p t},$$

(18)

where $P_{\sigma}(\mathbf{x})$ is the pumping spot profile in real space, $\mathbf{k}_p$ is an in-plane pumping vector resulting from the inclination of the laser beam as respect to the vertical, and $\omega_p$ is the pumping frequency of the single-mode laser. Time evolution of the arbitrary element of density matrix is given by

$$\{\partial_t \rho_{\sigma,\sigma'}(\mathbf{k},\mathbf{k}')\}^{(cp)} = \frac{i}{\hbar} \{ p_{\mathbf{k},\sigma}^*(t) \langle a_{\mathbf{k},\sigma'} \rangle - p_{\mathbf{k}',\sigma'}(t) \langle a_{\mathbf{k},\sigma}^* \rangle \},$$

(19)

where the time evolution of the mean values of the annihilation operator reads

$$\partial_t \langle a_{\mathbf{k},\sigma} \rangle = -\frac{i}{\hbar} p_{\mathbf{k},\sigma}(t) - \frac{i}{\hbar} \mathcal{E}_k \langle a_{\mathbf{k},\sigma} \rangle - \frac{i}{\hbar} \Omega_k \langle a_{\mathbf{k},-\sigma} \rangle$$

$$- \frac{i}{\hbar} \sum_{\mathbf{k}_2,p} \left( U_1 \rho_{\sigma,\sigma}(\mathbf{k}_2,\mathbf{k}_2 - \mathbf{p}) + U_2 \rho_{-\sigma,-\sigma}(\mathbf{k}_2,\mathbf{k}_2 - \mathbf{p}) \right) \langle a_{\mathbf{k}+\mathbf{p},\sigma} \rangle$$

$$+ \sum_{\mathbf{q},E_k < E_{k+q}} W(\mathbf{q}) \left[ (n_{\mathbf{k}+\mathbf{q},\sigma} - n_{\mathbf{q}}^{ph}) \langle a_{\mathbf{k},\sigma} \rangle + \rho_{-\sigma,\sigma}(\mathbf{k} + \mathbf{q},\mathbf{q} + \mathbf{q}) \langle a_{\mathbf{k},-\sigma} \rangle \right]$$

$$- \sum_{\mathbf{q},E_k > E_{k+q}} W(\mathbf{q}) \left[ (n_{\mathbf{k}+\mathbf{q},\sigma} + n_{\mathbf{q}}^{ph} + 1) \langle a_{\mathbf{k},\sigma} \rangle + \rho_{-\sigma,\sigma}(\mathbf{k} + \mathbf{q},\mathbf{q} + \mathbf{q}) \langle a_{\mathbf{k},-\sigma} \rangle \right].$$

(20)

D. Dynamics of the polarization

The dynamics of the circular polarization degree $\varphi_c$ of the light emission from the ground $k = 0$ state can be defined as

$$\varphi_c = \frac{n_{k=0}^+ - n_{k=0}^-}{n_{k=0}^+ + n_{k=0}^-},$$

(21)

where $n_{k=0}^+(t)$ and $n_{k=0}^-(t)$ stand for the populations of polaritons with pseudospin $\pm 1$ correspondingly in the ground state of the dispersion.

One should mention the effect of longitudinal-transverse splitting $\Omega$ on the polarization degree dynamics, since it couples $\sigma^+$ and $\sigma^-$ modes together. Its role becomes more evident if one switches to the pseudospin formalism which is described in the introductory part of the manuscript. From the formal point of view, TE-TM splitting is equivalent to the effective magnetic field in +x direction $\mathbf{\Omega} = \mathbf{e}_x \Omega$ (along the quantum wire). In the same time, the polariton-polariton interaction gives rise to another effective magnetic field oriented in z direction (structure growth axes) $\Omega_{p-p} = \mathbf{e}_z (U_1 - U_2)(n^+ - n^-)$ (see Ref. [33]). Therefore, the total effective magnetic field represents superposition $\Omega_{tot} = \Omega_{p-p} + \Omega$. Accordingly, it is possible to rewrite the kinetic equations as a coupled equations for occupation number $n_\sigma$ and in-plane pseudospin $\mathbf{S}_\perp$ [38]. Considering only the effect of effective magnetic fields (assuming infinite lifetime and absence of interaction with phonons), coupled equations are given as

$$\partial_t n_{k=0}^+ \propto \mathbf{e}_x \cdot (\mathbf{S}_\perp \times \mathbf{\Omega}),$$

(22)

$$\partial_t \mathbf{S}_\perp \propto (\mathbf{S}_\perp \times \mathbf{\Omega}_{p-p}) + \frac{1}{2} (n_{k=0}^+ - n_{k=0}^-) \mathbf{\Omega}.$$ (23)

This corresponds to the precession of the pseudospin along time dependent magnetic field, which leads to its non-trivial dynamics.

III. RESULTS AND DISCUSSION

We consider a microcavity based on AlGaAs family of alloys and use the following parameters. The Rabi splitting was taken equal to 15 meV, polariton effective mass $3 \times 10^{-4}$ of the free electron mass and detuning between the pure photonic and excitonic modes 3 meV. The polariton quantum wire is 50 $\mu$m long and 2 $\mu$m wide. Further, we use typical polariton lifetime in a medium Q-factor microcavity, $\tau = 2$ ps. The polariton-polariton and polariton-phonon scattering rates have been taken independent on the wavevector for simplicity. The matrix element of polariton-polariton interaction was estimated using expression $U \approx 3 E_0 a_0^2/\hbar$, where $E_0$ is the exciton binding energy, $a_0$ is its Bohr radius and $S$ is the area of the wire, which gives $U \approx 20$ neV. The polariton-phonon scattering rate $W = 10^3$ s$^{-1}$. Pump laser is detuned
above the energy of the lower polariton branch by $\delta = 1$ meV. We consider the case of spatially homogeneous cw pump of different polarizations.

The bistable (for spinless condensate) and multistable (if one accounts for the spin) behavior of a polariton system in 1D and 2D quantum systems has already been investigated theoretically in a number of works (see, for instance, Ref. [34]) and was reported experimentally [35, 36]. Most of the theoretical approaches are based on solution of the GPe. Unfortunately, this technique does not allow to account for the dissipation dynamics of polaritons due to interaction with the crystal lattice (phonon-mediated processes). The density-matrix approach, which is being developed in current manuscript, does. In the limiting case of zero temperature we immediately reproduce the results obtained by the GPe, as expected.

The multistability (with multi-hysteresis) characteristic is shown in Fig. 2 for different temperatures in range 1-100 K, in the absence of TE-TM splitting. In this case, since there is no mechanism of the transition between $\sigma^+$ and $\sigma^-$ modes, this effect can be understood in terms of independent bistable dynamics of $\sigma^+$ and $\sigma^-$ modes. Accordingly, in inset we present the population hysteresis curve of a spinless polaritons to clarify the forthcomming discussion. Let’s begin with inset. In the certain range of pumps the polariton population can take two different values depending on the history of the pumping process. If we slowly increase the intensity of pump, at some threshold value $P_{th}^{\omega\pi}$ the population of the ground state jumps up abruptly due to the resonance of the blue shifted polariton energy with the energy of the laser mode. The system keeps staying at this high-populated state with further increase of the pump intensity. In the backward direction, when we decrease the intensity of pump, the bistable transition to the low-populated state appears at the lower pump intensity ($P_{th}^{\omega\pi} < P_{th}^{\sigma\pi}$) and hysteresis curve appears.

With account for spin, polariton-polariton interaction becomes polarization-dependent, which leads to multi-stability of the polariton circular polarization (see Ref. [34] for the detailed discussion of the situation at $T = 0$). This effect is illustrated in main plot of Fig. 2 for different temperatures, where the pump intensity is fixed and its circular polarization degree $\varphi_p$ is being changed. Let us explain this phenomenon with the help of above discussion for spinless case. Keeping the total pump intensity, lets change its circular polarization from $\sigma^- (\phi = \pi$, and $\varphi_p = -1)$ to $\sigma^+ (\phi = 0$, and $\varphi_p = 1)$, see Fig. [1]. Initially there exists only $\sigma^-$ polaritons in the ground state, thus $\varphi_c = -1$. As $\varphi_p$ is increased, $\sigma^+$ component starts to become more populated, and at certain threshold value of $\varphi_p$ the first bistable jump up in $\varphi_c$ occurs that implies the abrupt increase of the $\sigma^+$ component. Further increase of $\varphi_p$ leads to the second jump up of the polarization degree $\varphi_c$ due to the bistability jump down of $\sigma^-$ component from high population state to low population state. Finally the system reaches the state with only $\sigma^+$ component and $\varphi_c = 1$. In the backward direction (decrease pump polarization degree from +1 to -1) the first jump down is due to the abrupt increase of $\sigma^-$ component, while the second jump down is explained by the abrupt decrease of $\sigma^+$ component occupancy.

With increasing temperature, the multistability loops start to shrink and become totally destroyed at about $T \approx 100$ K. It occurs due to the dissipation processes coming from interaction with acoustic phonons. At higher temperature the spin-independent polariton-phonon interaction makes the dependence $\varphi_c(\varphi_p)$ quasi-linear, as it should be expected, indeed, in the case when coherent nonlinearities play no role and there is no transition between circular polarized components the polarization degree of the system should coincide with those of the pump.

Now let us introduce the TE-TM splitting to see its effect on polarization multistability. The corresponding term removes the isotropy in the xy plane since it acts as an effective magnetic field in +x direction. Consequently, the population of each component $n_k^\omega$ becomes dependent not only on circular polarization of the pump, but also on its in-plane component as can be seen from Eq. [22]. In Fig. 3, the dependence of the internal circular polarization degree of the system $\varphi_c$ is plotted as a function of the circular polarization degree of the pump $\varphi_p$ for three different in-plane angles $\theta$ between the in-plane pseudospin of the pump and direction along the wire: axis Ox (see Fig. [1]). Azimuthal angle $\theta$ comes in the pump-
that relative phase drastically modifies the profile of the pump pseudospin. The pump pseudospin (different azimuthal angles \( \theta \)) for different xy-plane projections of the pseudospin of the pump can be understood from the first term of the kinetic equation for \( S_1 \), where the two cases (\( \theta = 0 \) and \( \theta = \pi \)) give contributions with opposite signs. Therefore, the internal circular polarization degree becomes highly sensitive to the choice of meridian on the surface of the Poincaré sphere along which the pump laser evolves from \( \sigma^- \) to \( \sigma^+ \) state becomes crucial for the polarization dynamics of the polariton system. It was shown before in Fig. 2, in the absence of TE-TM splitting due to the dissipative nature of polariton-phonon interactions, the hysteresis behavior is washed out at 100 K. On the other hand, if \( \Omega \neq 0 \), as in Fig. 4, the bistable behavior can be recovered and bistability phenomena can survive up to higher temperatures as compared to the case \( \Omega = 0 \). However, instead of two-stepped hysteresis loop shown in Fig. 3 we observe only one-stepped behavior. This result suggests the transition from two-independent modes dynamics (the two modes are the mode with \( \sigma^+ \) and the mode with \( \sigma^- \) polarizations) to a single collective mode dynamics. In fact, at some critical value of the TE-TM splitting (around 0.1 meV in our parameter regime), transition from high population state to low population state of the one mode is always accompanied by the simultaneous transition from low population state to high population state of the other mode and crossover from the multistable behavior to the bistable occurs.

Let us finally analyze the combined effect of the TE-TM splitting and scattering on phonons (in the rest of the calculations the in-plane component of the pump pseudospin is taken along the +x direction, i.e., \( \theta = 0 \)). As it was shown in Eq. 20, this term is responsible for this behavior. The particles align their pseudospin parallel to the strong effective magnetic field in the +x direction. The last term in Eq. 20 is responsible for this behavior. The particles align their pseudospin parallel to the strong effective magnetic field in the +x direction.
magnetic field to minimize the total energy in the system. Meanwhile, the hysteresis behavior vanishes, and the difference between the backward and forward swapings disappears.

IV. CONCLUSIONS

In conclusion, we developed a formalism for the description of the dissipative dynamics of an inhomogeneous spinor polariton system in real space and time accounting for polariton-polariton interactions, polariton-phonon scattering and effect of the TE-TM effective magnetic field. We applied our formalism to one-dimensional polariton condensate at different temperatures to investigate the dynamics of the circular polarization of the system when it is driven by the external homogeneous laser pump. We showed that the polarization of the condensate is highly sensitive not only to the history of the strength of the pump, but also to the phase of the elliptical polarization degree of this pump. In the presence of TE-TM field we observe the survival of this phenomena up to very high temperatures.

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31 A. Kavokin and G. Malpuech, Cavity Polaritons (Elsevier Academic Press, Amsterdam, 2003).
32 K.V. Kavokin, I. A. Shelykh, A. V. Kavokin, G. Malpuech, P. Bigenwald, Phys. Rev. Lett. 92, 017401 (2004).
33 I.A. Shelykh, A.V. Kavokin, Yu. Rubo, T. C. H. Liew and G. Malpuech, Semicond. Sci. Technol. 25, 013001 (2010).
34 N. A. Gippius, I. A. Shelykh, D. D. Solnyshkov, S. S. Gavrilov, Yuri G. Rubo, A. V. Kavokin, S. G. Tikhodeev, and G. Malpuech, Phys. Rev. Lett. 98, 236401 (2007).
35 A. Baas, J.-Ph. Karr, M. Romanelli, A. Bramati, and E. Giacobino, Phys. Rev. B 70, 161307 (2004).
36 T. K. Paraiso, M. Wouters, Y. Leger, F. Morier-Genoud, and B. Deveaud-Pledran, Nat. Mater. 9, 655 (2010).
37 A. Amo, T. C. H. Liew, C. Adrados, R. Houdre, E. Giacobino, A. V. Kavokin and A. Bramati, Nat. Phot. 4, 361 (2010).
38 Shelykh, I. A, A. V Kavokin, and G. Malpuech. Phys. Status Solidi (b) 242, 2271 (2005).
39 A. Smerzi, S. Fantoni, S. Giovanazzi, and S. R. Shenoy, Phys. Rev. Lett. 79, 4950 (1997).
40 G.J. Milburn, J. Corney, E. M. Wright, D. F. Walls, Phys. Rev. A 55, 4318 (1997).
41 S. Raghavan, A. Smerzi, S. Fantoni, and S. R. Shenoy, Phys. Rev A 59, 620 (1999).
42 I. A. Shelykh, D. D. Solnyshkov, G. Pavlovic, and G. Malpuech Phys. Rev. B, 78, 041302 (2008).