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H. Kagan,54 R. Kass,54 J. P. Morris,54 A. M. Rahimi,54 J. J. Regensburger,54 Q. K. Wong,54 N. L. Blount,55 J. Brau,55 R. Frey,55 O. Igonkina,55 J. A. Kolb,55 M. Lu,55 R. Rahmat,55 N. B. Sinev,55 D. Strom,55 J. Strube,55 E. Torrence,55 N. Gagliardi,56 A. Gaz,56 M. Margoni,56 M. Morandin,56 A. Pompili,56 M. Posocco,56 M. Rotondo,56 F. Simonetto,56 R. Stroili,56 C. Voci,56 E. Ben-Haim,57 H. Briand,57 G. Calderini,57 J. Chauveau,57 P. David,57 L. Del Buono,57 Ch. de la Vaissière,57 O. Hamon,57 Ph. Leruste,57 J. Malèlès,57 J. Ocaz,57 A. Perez,57 J. Prendi,57 L. Gladney,58 M. Biasini,59 R. Cavoureri,59 E. Manoni,59 C. Angelini,60 G. Batignani,60 S. Bettarini,60 M. Carpinelli,60 R. Cenci,60 A. Cervelli,60 F. Forti,60 M. A. Giorgi,60 A. Lusiani,60 G. Marchiori,60 M. A. Mazur,60 M. Morganti,60 N. Neri,60 E. Paoloni,60 G. Rizzo,60 J. J. Walsh,60 M. Haire,61 J. Biesiad,62 P. Elmer,62 Y. P. Lau,62 C. Lu,62 J. Olsen,62 A. J. S. Smith,62 A. V. Telnov,62 E. Baracchini,63 F. Bellini,63 G. Cavoto,63 D. del Re,63 E. Di Marco,63 R. Faccini,63 F. Ferrarotto,63 F. Ferroni,63 M. Gaspero,63 P. D. Jackson,63 L. Li Gioi,63 M. A. Mazzoni,63 S. Morganti,63 G. Piredda,63 F. Polci,63 F. Renga,63 C. Voena,63 M. Ebert,64 T. Hartmann,64 H. Schröder,64 R. Waldi,64 T. Adye,65 G. Castelli,65 B. Franke,65 E. O. Olaiya,65 S. Ricciardi,65 W. Roethel,65 F. F. Wilson,65 S. Emery,66 M. Escalier,66 A. Gaidot,66 S. F. Ganzhur,66 H. Gamel de Monchenault,66 W. Kozanecki,66 G. Vasseur,66 Ch. Yèche,66 M. Zito,66 X. R. Chen,67 H. Liu,67 W. Park,67 M. V. Purohit,67 J. R. Wilson,67 M. T. Allen,68 D. Aston,68 R. Bartoldus,68 P. Bechtle,68 N. Berger,68 R. Claus,68 J. P. Coleman,68 M. R. Convery,68 J. C. Dingfelder,68 J. Dorfan,68 G. P. Dubois-Felsmann,68 W. Dunwoodie,68 R. C. Field,68 T. Glatzman,68 S. J. Gowdy,68 M. T. Graham,68 P. Grenier,68 C. Hast,68 T. Hryn’ova,68 W. R. Innes,68 J. Kaminski,68 M. H. Kelsey,68 H. Kim,68 P. Kim,68 M. L. Kocian,68 D. W. G. S. Leith,68 S. Li,68 S. Luitz,68 V. Luth,68 H. L. Lynch,68 D. B. MacFarlane,68 H. Marsiske,68 R. Messner,68 D. R. Muller,68 C. P. O’Grady,68 I. Ofte,68 A. Perazzo,68 M. Perl,68 T. Pulliam,68 B. N. Ratcliff,68 A. Roodman,68 A. A. Salnikov,68 R. H. Schindler,68 J. Schwiening,68 A. Snyder,68 J. Stelzer,68 D. Su,68 M. K. Sullivan,68 K. Suzuki,68 S. K. Swain,68 J. M. Thompson,68 J. Va’vra,68 N. van Bakel,68 A. P. Wagner,68 M. Weaver,68 W. J. Wisniewski,68 M. Wittgen,68 D. H. Wright,68 A. K. Yarritu,68 K. Yi,68 C. C. Young,68 P. R. Burchat,69 A. J. Edwards,69 S. A. Majewski,69 B. A. Petersen,69 L. Wilden,69 S. Ahmed,70 M. S. Alam,70 R. Bula,70 J. A. Ernst,70 V. Jain,70 B. Pan,70 M. A. Saeed,70 F. R. Wappler,70 S. B. Zain,70 M. Krishnamurthy,71 S. M. Spanier,71 R. Eckmann,72 J. L. Ritchie,72 A. M. Ruland,72 C. J. Schilling,72 R. F. Schiwitters,72 J. M. Izen,73 X. C. Lou,73 S. Ye,73 F. Bianchi,74 F. Gallo,74 D. Gamba,74 M. Pelliccioni,74 M. Bomben,75 L. Bosiso,75 C. Cartaro,75 F. Cossutti,75 G. Della Ricca,75 L. Lanceri,75 L. Vitale,75 V. Azzolini,76 N. Lopez-March,76 F. Martinez-Vidal,76 D. A. Milanes,76 A. Oyanguren,76 J. Albert,77 J. L. Ritchie,77 B. Bhuyan,77 K. Hamano,77 R. Kowalewski,77 I. M. Nugent,77 J. M. Roney,77 R. J. Sobie,77 P. F. Harrison,78 J. Ilic,78 T. E. Latham,78 G. B. Mohanty,78 H. R. Band,79 X. Chen,79 S. Danu,79 K. T. Flood,79 J. J. Hollar,79 P. E. Kutter,79 Y. Pan,79 M. Pierini,79 R. Prepost,79 S. L. Wu,79 and H. Neal80
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We present a search for $B$ decays to a charged scalar meson $a_0^+$ and a $\pi^0$ where the $a_0^+$ decays to an $\eta$ meson and a $\pi^+$. The analysis was performed on a data sample consisting of $383 \times 10^6 B\bar{B}$ pairs collected with the BABAR detector at the PEP-II asymmetric-energy $B$ Factory at SLAC.
find no significant signal and set an upper limit on the product branching fraction $B(B^+ \rightarrow a_{10}^+ \pi^0) \times B(a_{0}^+ \rightarrow \eta \pi^+)$ of $1.4 \times 10^{-6}$ at the 90% confidence level.

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The structure of scalar mesons is a subject of some debate [1, 2]. Proposed models include two-quark or four-quark states with potential contributions from glue-balls or a molecular admixture of $K\bar{K}$ meson pairs. Measurement of the branching fraction for the mode $B^+ \rightarrow a_{10}^+ \pi^0$ [3] is expected to provide an effective test of the two- and four-quark models [3]. The Feynman diagrams for the decay in the two-quark case are shown in Figure 1. Those for the four-quark case are similar except for an $s\bar{s}$ pair produced from the vacuum internal to the $a_{10}^+$ meson. The color-allowed electroweak tree diagram shown in Figure 1(a) is suppressed for all $a_{10}^+$ models since the $W^+$ is constrained to decay to states of even $G$-parity (a generalization of $C$ symmetry to cover particle multiplets) within the Standard Model, whereas the $a_{10}^+$ has odd $G$-parity [4].

FIG. 1: The Feynman diagrams contributing to the process $B^+ \rightarrow a_{10}^+ \pi^0$ in the two-quark model. (a) is the external (color-allowed) tree, (b) the internal (color-suppressed) tree, (c) the annihilation process and (d) the gluonic penguin process.

The amplitudes for the above diagrams depend on the $a_{10}^+$ model used; in particular the annihilation diagram is heavily suppressed in a four-quark model. Hence measurement of the branching fraction provides the potential for model discrimination. In the two-quark case, the predicted branching fractions go as high as $2 \times 10^{-7}$ [3]. However, in the four-quark case the prediction for the branching fraction is an order of magnitude lower.

The branching fraction for the result quoted below will be given in terms of the product $B(B^+ \rightarrow a_{10}^+ \pi^0) \times B(a_{0}^+ \rightarrow \eta \pi^+)$ since the branching fraction $B(a_{0}^+ \rightarrow \eta \pi^+)$ is not well measured, although it is thought to be approximately 85% [1].

The analysis presented in this paper is based on 347 fb$^{-1}$ of data collected at the $\Upsilon(4S)$ resonance with the $BaBar$ detector at the PEP-II asymmetric-energy $e^+e^-$ collider located at the Stanford Linear Accelerator Center. This corresponds to $(383\pm 4) \times 10^6 BB$ pairs.

The $BaBar$ detector has been described in detail previously [2]. Track parameters of charged particles are measured by a combination of a 5-layer double-sided silicon vertex tracker and a 40-layer drift chamber (DCH), both operating in the 1.5 T magnetic field of a superconducting solenoid. Photons and electrons are identified using a CsI(Tl) electromagnetic calorimeter. Further charged particle identification (PID) is provided by measurements of the average energy loss ($dE/dx$) in the tracking devices and by an internally-reflecting, ring-imaging Čerenkov detector (DIRC) covering the central region.

The analysis focuses on $a_{10}^+$ mesons produced from the decay $B^+ \rightarrow a_{10}^+ \pi^0$, followed by $a_{0}^+ \rightarrow \eta \pi^+$, where the $\eta$ meson subsequently decays to $\gamma \gamma$ or $\pi^+ \pi^- \pi^0$ final states. The $\pi^0$ mesons used are reconstructed via the decay $\pi^0 \rightarrow \gamma \gamma$. The selections used for the analysis are the result of an optimization procedure based on ensemble Monte Carlo (MC) studies. In these studies, a sample of MC candidates is produced for given selection criteria by generating randomly from probability density function (PDF) distributions defined with the selection applied. By re-fitting to the datasets for each set of selection criteria it is possible to select the set that yields the maximum sensitivity to signal. This is done independently for each decay mode considered. In both cases $a_{10}^+$ candidates are required to satisfy $0.8 < m_{\eta \pi} < 1.2$ GeV/$c^2$ with the $\eta$ candidates satisfying $0.51 < m_{\pi \gamma} < 0.57$ GeV/$c^2$ or $0.540 < m_{3\pi} < 0.555$ GeV/$c^2$. The $\pi^0$ produced from the $\eta \rightarrow \pi^+ \pi^- \pi^0$ decay is required to satisfy $0.10 < m_{\pi^0} < 0.16$ GeV/$c^2$. The $\pi^0$ daughter of the $B$ candidate is required to satisfy $0.115 < m_{\pi^0} < 0.150$ GeV/$c^2$. This selection is tighter than for the $\pi^0$ produced from the $\eta$ meson since it is of significantly higher energy and therefore has a better resolution. The charged track from the $a_{10}^+$ candidate decay is required not to satisfy DIRC- and
DCH-based PID criteria consistent with a kaon hypothesis. This PID selection has been measured to be more than 80% efficient for tracks with momenta up to 4 GeV/c with a pion mis-identification rate lower than 10% over the same range.

A B meson candidate is characterized kinematically by the energy difference \( \Delta E \equiv E_B - \frac{1}{\sqrt{s}} \) and energy-substituted mass \( m_{ES} \equiv (\frac{1}{s} - p_B^2)^{1/2} \), where \( s \) is the square of the centre-of-mass energy of the colliding beams, \((E_B, p_B)\) is the candidate B meson 4-momentum and all values are expressed in the \( \Upsilon(4S) \) frame. Signal events peak around zero for \( \Delta E \), and at the B meson mass for \( m_{ES} \). The resolutions for \( \Delta E \) and \( m_{ES} \) are approximately 30 MeV and 3 MeV/c², respectively. We require \(|\Delta E| \leq 0.35 \text{ GeV} \) and \( 5.20 \leq m_{ES} \leq 5.29 \text{ GeV}/c^2 \) as an input for the fit used to extract signal and background parameters (described below) in order to maximize the available statistics.

The principal source of background in the analysis arises from random combinations in continuum \( e^+e^- \to q\bar{q} \) (\( q = u, d, s, c \)) events. These contributions are reduced in part by placing a selection on the variable \(|\cos(\theta_{TB})|\), where \( \theta_{TB} \) is the angle between the thrust axis of the B candidate and the thrust axis of the rest of the event calculated in the \( \Upsilon(4S) \) frame. Candidates formed in jet-like \( q\bar{q} \) events will peak at \(|\cos(\theta_{TB})|\) values approaching 1, whereas signal B decays will follow an almost flat distribution as they are isotropic in this angle. We require \(|\cos(\theta_{TB})| < 0.7 \) for both \( \eta \) channels. The final variable used in the analysis is a linear Fisher discriminant \( F \) that consists of the angles of the B momentum and B thrust axis (in the \( \Upsilon(4S) \) frame) with respect to the beam axis, and the zeroth and second Legendre moments of the energy flow computed with respect to the B thrust axis \([8]\). The reconstruction efficiencies after selection are presented in Table I.

The analysis uses an extended unbinned maximum-likelihood fit to extract yields for the modes under study. The input variables to the fit are \( \Delta E \), \( m_{ES} \), \( F \) and the \( a_0^\pm \) candidate resonance mass \( m_{\eta\pi} \). The extended likelihood function for the fit is defined as:

\[
\mathcal{L} = \frac{e^{-(\sum n_j)}}{N!} \prod_{i=1}^{N} \left[ \sum_{j=1}^{M} n_j P_j \right] ,
\]

where \( P_j \) is the normalized PDF for a given fit component \( j \). For each candidate \( i \) the PDF is evaluated using the fit variables of that candidate. The \( M \) fit components are the signal and all background contributions. The total number of candidates is given by \( N \) with the yield associated with each fit component given by \( n_j \). The fit for each \( \eta \) channel consists of 16 components modeling signal and continuum candidates separately as well as charged and neutral charmed B meson decays. There are then 12 components modeling individual charmedless modes which were found to contribute a background to the signal. The yields for all \( B \) background components are held fixed in the final fit using values calculated from the latest branching fraction estimates \([9]\), whereas the signal and continuum background yields are allowed to vary.

The fit model is constructed in order to extract signal candidates effectively from a sample where multiple reconstruction hypotheses exist for each event. The signal MC events have an average candidate multiplicity of 1.4 for both \( \eta \) decay modes.

In this analysis separate PDFs were used to discriminate between correctly and incorrectly reconstructed signal candidates in MC. This was achieved by using MC information to separate the signal MC candidates into an almost pure sample of correctly reconstructed candidates and a sample consisting mainly of incorrectly reconstructed candidates. By iteratively fitting the separate PDFs to each sample in turn, a consistent set of PDFs for the two cases was obtained. The component for correctly reconstructed candidates was then taken to model signal candidates in the final fit to data. The fraction of events in the MC that were identified as correctly reconstructed by the fit was approximately 62% for both \( \eta \) channels. The signal candidate yield resulting from the fit to MC was verified to be consistent with that expected.

The shapes of the distributions for incorrectly reconstructed signal were found to be similar to continuum background and thus any such candidates are assumed to be absorbed into the yield associated with the continuum PDF. Modeling signal candidates in this way was shown using ensemble MC studies to provide better sensitivity to signal than other methods. As a final test, the method was validated using ensemble MC studies to show that it introduced no bias into the final fit result.

Any continuum and \( B \) backgrounds that remain after the event selection criteria have been applied are identified and modeled using Monte Carlo simulation based on the full physics and detector models \([10]\). Charmless \( B \) decays providing a background to the signal are identified by analyzing the MC candidates passing selection from a large mixed sample of Standard Model \( B \) decays. Charged and neutral charmed \( B \) decays are modeled separately and individual components are included for each charmedless \( B \) decay mode found to contribute. The PDF parameters for each \( B \) background component are obtained from MC samples and held fixed in the final fit to data. Those for the continuum background shape are left free in the final fit. The contributions from two charmedless backgrounds with the same final state as signal, those for \( B^+ \to a_0(1450)^+\pi^0 \) and non-resonant \( B^+ \to \eta\pi^+\pi^0 \), are estimated using fits to the relevant regions of the Dalitz plane. Any potential interference effects were neglected since the fits gave no significant yields for these modes.

The total PDFs are modeled as products of the PDFs for each of the four fit variables. The signal shapes in \( \Delta E \), \( m_{ES} \), \( m_{\eta\pi} \) and \( F \) are modeled with a Novo-
TABLE I: The results of the fit to the full data set, and other values required for calculating the branching fraction. All B background yields were held fixed. The upper limit is shown first with only the statistical error and then with the total error.

| Required Quantity/Result | \( \eta \rightarrow \gamma \gamma \) | \( \eta \rightarrow \pi^+ \pi^- \pi^0 \) |
|--------------------------|---------------------------------|---------------------------------|
| Candidates to fit        | 103054                          | 31626                           |
| Fixed B Background (candidates) | 1640 | 942 |
| Signal Yield (candidates) | -8 ± 19                         | 13±13                           |
| Continuum Yield (candidates) | 101400±300 | 30700±200 |
| ML Fit Bias (candidates)  | 5.2±3.0                         | -2.0±1.3                        |
| Efficiencies and BF\(\text{s}\) |                                |                                |
| Efficiency (%) \(B(\eta \rightarrow X)\) (%) | 16.3±0.1 | 10.2±0.1 |
| Branching Fraction \(\times 10^{-6}\) | -0.6\(^{+0.8}_{-0.7}\) \((\text{stat})\) +0.4 \((\text{syst})\) | 1.7\(^{+1.6}_{-1.4}\) \((\text{stat})\) +0.3 \((\text{syst})\) |
| Combined Mode Results |                                |                                |
| Branching Fraction \(\times 10^{-6}\) | 0.1\(^{+0.7}_{-0.6}\) \((\text{stat})\) +0.3 \((\text{syst})\) | < 1.4 \((\text{statistical error only})\) |
| Significance | 0.1\(\sigma\) \((\text{stat + syst})\) | |
| Upper Limit 90% C.L. \((\times 10^{-6})\) | < 1.3 \((\text{statistical error only})\) | < 1.4 \((\text{total error})\) |

The results of the analysis are presented in Table I. The statistical errors on the signal yields are defined using the change in the central value when the quantity \(-2\ln L\) increases by one unit from the minimum. The significance is taken as the square root of the difference between the value of \(-2\ln L\) for zero signal and the value at the minimum (including additive systematics).

For the purposes of the branching fraction calculation we assume that the \(T(4S)\) decays with an equal rate to both \(B^+B^-\) and \(B^0\bar{B}^0\). The fit bias is measured using an ensemble MC study based on a parameterization taken from the fit to data with all yield values taken from data. Where a negative yield is found a value of zero is used for the study. The branching fraction results from the two \(\eta\) decay modes are combined by forming the product of the likelihood functions, after their maxima have been shifted to account for fit bias. The functions themselves are defined by computing the likelihood values for signal yields around the maximum. Systematic errors are included at the required stages in the calculation depending on correlations between the two \(\eta\) channels.

We find no significant signal in either \(\eta\) decay mode and thus quote upper limits on the branching fraction at the 90% confidence level (C.L.), taken to be the branching fraction below which lies 90% of the total of the likelihood integral in the positive branching fraction region.

In Figure 2 we show projections of each of the four fit variables for both the \(\eta \rightarrow \gamma \gamma\) and \(\eta \rightarrow \pi^+ \pi^- \pi^0\) decay modes. To enhance the visibility of a potential signal, the candidates in these figures have been required to satisfy the condition that the likelihood ratio \(L_{\text{sig}}/[L_{\text{sig}} + \Sigma L_{\text{bkg}}]\) for any candidate be greater than 0.6. Here \(L_X\) is the likelihood for a given event being described by either the signal or background model. The likelihoods are calculated for each figure separately, excluding the variable being plotted. As can be seen there is no significant signal peak for either mode.

The largest sources of systematic uncertainty in the analysis arise from poor knowledge of the \(a_0^+\) lineshape and from the error in the estimated background contributions. By varying the width of the \(a_0^+\) Breit-Wigner between 50 and 100 MeV/c\(^2\) we predict an uncertainty of approximately +5 and −4 candidates for \(\eta \rightarrow \gamma \gamma\) and +0.5 and −1 candidate for \(\eta \rightarrow \pi^+ \pi^- \pi^0\). Varying the charmed \(B\) yields within their branching fraction errors (or ±100% where a limit is used), and the charmed \(B\) yields by ±10%, gives an estimated uncertainty of ±4 candidates in \(\eta \rightarrow \gamma \gamma\) and ±1 candidate in \(\eta \rightarrow \pi^+ \pi^- \pi^0\). The error due to the uncertainty in the fit bias was calculated as the sum in quadrature of 50% of the measured bias and its statistical error, as taken from the ensemble MC.
study described above. This value was calculated to be approximately ±3 candidates in the η → γγ channel and ±1 candidate for η → π⁺π⁻π⁰.

Further sources of systematic uncertainty, which are multiplicative rather than additive, affect the efficiency and thus enter into the branching fraction calculation. Limited signal MC statistics account for 0.4% in both η decay modes. Auxiliary studies on inclusive control samples [8], predict errors of 0.5% per charged track and 3% per reconstructed η or π⁰ decaying to two photons. The estimate of the number of produced B̅B̅ events is uncertain by 1.1%. The uncertainties in B daughter product branching fractions are taken to be 2% for η → γγ and 3% for η → π⁺π⁻π⁰ [9]. A summary of all systematic error contributions is presented in Table III.

In conclusion, we do not find a significant signal for the mode B⁺ → a₀⁺π⁰. We set an upper limit at 90% C.L. on the branching fraction B(B⁺ → a₀⁺π⁰) × B(a₀⁺ → ηπ⁺) of 1.4×10⁻⁶, suggesting that there is insufficient sensitivity with the current dataset to probe the predicted theoretical parameter space, with the largest predicted branching fraction being 2×10⁻⁷ [3]. We are therefore unable to comment on the validity of any of the current models of the a₀⁺.

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[1] Particle Data Group, Y.-M. Yao et al., J. Phys. G33, 1 (2006), “Note on Scalar Mesons”, p. 546.
[2] V. Baru et al., Phys. Lett. B 586, 53 (2004).
[3] Throughout this paper, charged-conjugate decays are also implied.
[4] S. Laplace and V. Shelkov, Eur. Phys. Jour. C 22, 431 (2001).
[5] D. Delepine, et al., Eur. Phys. Jour. C 45, 693 (2006).
[6] H.-Y. Cheng, et al., Phys. Rev. D 73, 014017 (2006).
[7] BABAR Collaboration, B. Aubert et al., Nucl. Instrum. Methods Phys. Res., Sect. A 479, 1 (2002).
[8] BABAR Collaboration, B. Aubert et al., Phys. Rev. D 70, 032006 (2004).
[9] Particle Data Group, Y.-M. Yao et al., J. Phys. G33, 1 (2006); Heavy Flavour Averaging Group (HFAG), E. Barberio, et al., hep-ex/0603003 (2006).

[10] The BABAR detector Monte Carlo simulation is based on GEANT4: S. Agostinelli et al., Nucl. Instrum. Methods Phys. Res., Sect. A 506, 250 (2003).
[11] The Novosibirsk function is defined as
\[
f(x) = A_s \exp(-0.5(\ln^2(1+\Lambda \tau(x-x_0))/\tau^2 + \tau))
\]
where \(A = \sinh(\tau \sqrt{\ln 4})/(\sigma \tau \sqrt{\ln 4})\), the peak is \(x_0\), \(\tau\) is the tail parameter and \(A_s\) is a normalization factor.
[12] E852 Collaboration, S. Teige et al., Phys. Rev. D 59, 012001 (1998).
[13] ARGUS Collaboration, H. Albrecht, et al., Phys. Lett. B 241, 278 (1990); Function defined as
\[
f(x) = N x [(1 - x/E_{\text{beam}})^2 \exp [p(1 - (x/E_{\text{beam}}))^2]]^{1/2},
\]
where \(N\) is a normalization factor, \(p\) a shape parameter and \(E_{\text{beam}}\) is 50% of the centre-of-mass energy of the colliding beams.
[14] K. S. Cranmer, Comput. Phys. Commun. 136, 198 (2001).
[15] See for instance BABAR Collaboration, B. Aubert et al., Phys. Rev. D 69, 071101 (2004), and references therein.