Asymmetries of anti-triplet charmed baryon decays

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Abstract

We analyze the decay processes of $B_c \rightarrow B_nM$ with the $SU(3)_F$ flavor symmetry and spin-dependent amplitudes, where $B_c(B_n)$ and $M$ are the anti-triplet charmed (octet) baryon and nonet meson states, respectively. In the $SU(3)_F$ approach, it is the first time that the decay rates and asymmetries are fully and systematically studied without neglecting the $\mathcal{O}(15)$ contributions of the color anti-symmetric parts in the effective Hamiltonian. Our results of the up-down asymmetries based on $SU(3)_F$ are quite different from the previous theoretical values in the literature. In particular, we find that $\alpha(\Lambda^+_c \rightarrow \Xi^0 K^+)_ {SU(3)} = 0.94^{+0.06}_{-0.11}$, which is consistent with the recent experimental data of $0.77 \pm 0.78$ by the BESIII Collaboration, but predicted to be zero in the literature. We also examine the $K^0_S - K^0_L$ asymmetries between the decays of $B_c \rightarrow B_n K^0_S$ and $B_c \rightarrow B_n K^0_L$ with both Cabibbo-allowed and doubly Cabibbo-suppressed transitions.
I. INTRODUCTION

Recently, the Belle collaboration has measured the absolute branching ratio of $\Lambda_c^+ \rightarrow pK^-\pi^+$ with high precision \cite{1}, resulting in the world average value of $\mathcal{B}(\Lambda_c^+ \rightarrow pK^-\pi^+) = (6.23 \pm 0.33)\%$, given by the Particle Data Group (PDG) \cite{2}. This decay mode is the so-called golden channel as most of $\Lambda_c^+$ decay branching fractions are presented relative to it. Subsequently, this golden mode, along with many other $\Lambda_c^+$ ones, has also been observed by the BESIII Collaboration \cite{3–12} with $\Lambda_c^+\bar{\Lambda}_c^-$ pairs, produced by $e^+e^-$ collisions at a center-of-mass energy of $\sqrt{s} = 4.6$ GeV, having a uniquely clean background to study the anti-triplet charmed baryon state of $\Lambda_c^+$. In particular, the decay of $\Lambda_c^+ \rightarrow \Sigma^+\eta'$ has been seen for the first time with $\eta'$ in the final states for the charmed baryon decays \cite{12}. In addition, the absolute decay branching fraction of $\Xi_c^0 \rightarrow \Xi^-\pi^+$, which involves the anti-triplet charmed baryon state of $\Xi_c^0$, has also been measured by the Belle collaboration \cite{13}. Clearly, a new experimental physics era for charmed baryons has started.

On the other hand, the theoretical study of the charmed baryon decays has faced several difficulties. The most serious one is that the factorization approach in the non-leptonic decays of charmed baryons is not working. For example, the Cabibbo-allowed decays of $\Lambda_c^+ \rightarrow \Sigma^0\pi^+$ and $\Lambda_c^+ \rightarrow \Sigma^+\pi^0$ do not receive any factorizable contributions, whereas the experimental data show that their branching fractions are all close to $O(10^{-2})$ \cite{2}, indicating the failure of the factorization method. In addition, the complication of the charmed baryon structure makes us impossible to directly evaluate the decay amplitude in a model-independent way. It is known that the most reliable and simple way to examine the charmed baryon processes is to use the flavor symmetry of $SU(3)_F$ \cite{14–25}. Indeed, it has been recently demonstrated that the results for the charmed baryon decays based on the $SU(3)_F$ approach \cite{17–25} are consistent with the current experimental data. Nevertheless, the charmed baryon decays have been extensively studied in various dynamical models \cite{26–36}, particularly, the recent dynamical calculations of the singly Cabibbo-suppressed $\Lambda_c^+$ decays by Cheng, Kang and Xu (CKX) \cite{36} based on current algebra.

For the two-body charmed baryon decay of $B_c \rightarrow B_nM$, with $B_c,B_n$ and $M$ the anti-triplet charmed (octet) baryon and nonet meson states, respectively, beside its decay branching fraction, there exits another interesting physical observable, the up-down asymmetry $\alpha$, which is related to the longitudinal polarization of $B_n$. Currently, there are three experi-
mental measurements of the up-down asymmetries in the charmed baryon decays \[2\], along with the recent one by BESIII \[11\], given by

\[ \alpha(\Lambda_c^+ \to \Xi^0 K^+)_{\text{exp}} = 0.77 \pm 0.78, \] (1)

which has been suggested to be approximately zero in the previous theoretical studies with the dynamical models \[26, 31, 33, 34\] as well as the \(SU(3)_F\) approach \[16\]. However, the up-down asymmetries in \(B_c \to B_n M\) were not discussed in the previous studies with \(SU(3)_F\) in Refs. \[17–25\].

In addition, it has been noticed that the physical Cabibb-allowed dominated decay processes of \(B_c \to B_n K_L^0\) and \(B_c \to B_n K_S^0\) are the same when the doubly Cabibbo-suppressed contributions are taking to be zero \[19\]. However, in some of these processes, the doubly Cabibbo-suppressed transitions are not negligible, which can be examined by defining the \(K_L^0 - K_S^0\) asymmetries between the \(K_L^0\) and \(K_S^0\) modes \[19\] to track the interferences.

In this work, we will systematically analyze the decay processes of \(B_c \to B_n M\) with the \(SU(3)_F\) symmetry with all operators under \(SU(3)_F\). We will also include the effect of the \(\eta - \eta'\) mixing. There are two different ways to link the amplitudes among the processes by \(SU(3)_F\). The first one is a purely mathematical consideration. By imposing the \(SU(3)\) group, we are able to write down the amplitude by tensor contractions. The second one is the diagrammatic approach, in which one draws down all the possible diagrams for the decay process with ascertaining that the amplitude from each diagram shall be the same by interchanging up, down and strange quarks. Both ways have their own advantages. The tensor method is easier to cooperate with the other symmetry and it allows us to estimate the order of the contribution from the amplitude with the Wilson coefficients. Explicitly, it could cooperate with the \(SU(3)\) color symmetry and take account of the strange quark mass as the source of the \(SU(3)_F\) symmetry breaking \[15, 22\]. On the other hand, the diagrammatic approach can distinguish the factorizable and non-factorizable amplitudes \[32\]. The close relations between the two methods have been examined in Ref. \[37\]. In Ref. \[23\], it has been proved to be useful if one combines both methods.

This paper is organized as follows. In Sec. II, we give the formalism for the two-body charmed baryon decays of \(B_c \to B_n M\), in which we first write the decay amplitudes in terms of parity conserved and violated parts under the \(SU(3)_F\) flavor symmetry, and then display the decay rates and asymmetries. In Sec. III, we show our numerical results and present
discussions. We conclude in Sec. IV. In Appendix A, we list the all decay amplitudes of the anti triplet baryon states in terms of the SU(3)$_F$ parameters. We give the definitions of the up-down and longitudinal polarization asymmetries in Appendix B.

II. FORMALISM

To study the two-body decays of the anti-triplet charmed baryon ($B_c$) to octet baryon ($B_n$) and nonet pseudoscalar meson ($M$) states, we write the hadronic state representations under the SU(3)$_F$ flavor symmetry to be

\[
B_c = (\Xi^0_c, -\Xi^+_c, \Lambda^+_c),
\]

\[
B_n = \begin{pmatrix}
\frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ \\
\frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\
\Xi^- & \Xi^0 - \sqrt{\frac{2}{3}}\Lambda
\end{pmatrix},
\]

\[
M = \begin{pmatrix}
\frac{1}{\sqrt{2}}(\pi^0 + c_\phi\eta + s_\phi\eta') & \pi^+ & K^+ \\
\pi^- & \frac{1}{\sqrt{2}}(-\pi^0 + c_\phi\eta + s_\phi\eta') & K^0 \\
K^- & \bar{K}^0 & -s_\phi\eta + c_\phi\eta'
\end{pmatrix},
\]

respectively, where $(c_\phi, s_\phi) = (\cos \phi, \sin \phi)$ and $\phi = 39.3^\circ$ [38] to describe the mixing between $\eta_8$ and $\eta_0$ of the octet and nonet sates for $\eta$.

From $c \rightarrow u\bar{d}s$, $c \rightarrow u$ and $c \rightarrow u\bar{s}d$ transitions at tree level, the effective Hamiltonian is given by [39]

\[
\mathcal{H}_{\text{eff}} = \sum_{i=+,-} \frac{G_F}{\sqrt{2}} c_i \left( V_{cs} V_{ud} O^{ds}_i + V_{cd} V_{ud} O^{qq}_i + V_{cd} V_{us} O^{su}_i \right),
\]

with

\[
O^{qq\pm}_\pm = \frac{1}{2} [(\bar{u}q_1) V_{-A}(\bar{q}_2 c) V_{-A} \pm (\bar{q}_2 q_1) V_{-A}(\bar{u}c) V_{-A}],
\]

where $(V_{cs}, V_{cd}, V_{us}) \simeq (1, -s_c, -s_c^2)$ with $s_c \equiv \sin \theta_c \approx 0.225$ [2] and $\theta_c$ the Cabibbo angle, $c_i$ (i=+,-) represent the Wilson coefficients, $G_F$ is the Fermi constant, $O^{qq\pm}_\pm$ and $O^{qq}_\pm \equiv O^{dd}_\pm - O^{ss}_\pm$ are the four-quark operators, and $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$. In Eq. (3), the decays associated with $O^{ds}_\pm$, $O^{qq}_\pm$ and $O^{sd}_\pm$ are the so-called Cabibbo-allowed (favored), singly Cabibbo-suppressed and doubly Cabibbo-suppressed processes, respectively.

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Note that $O_{+(-)}$, corresponding to the $O(T5(6))$ representation, is (anti)symmetric in flavor and color indices. The tensor forms of $H(T5)$ and $H(6)$ under $SU(3)_F$ are given by

$$H(T5)^{ij}_k = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad \begin{pmatrix}
0 & s_c & 1 \\
s_c & 0 & 0 \\
1 & 0 & 0
\end{pmatrix}, \quad \begin{pmatrix}
0 & -s_c^2 & -s_c \\
-s_c^2 & 0 & 0 \\
-s_c & 0 & 0
\end{pmatrix},$$

$$H(6)^{ij} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 2 & -2s_c \\
0 & -2s_c & 2s_c^2
\end{pmatrix},$$

respectively. In general, we write the spin-dependent amplitude of $B_c \to B_n M$ as

$$\mathcal{M}(B_c \to B_n M) = i\bar{u}_{B_n} (A - B\gamma_5) u_{B_c},$$

where $A$ and $B$ are the $s$-wave and $p$-wave amplitudes, corresponding to the parity violating and conserving ones, and $u_{B_{n,c}}$ are the baryon Dirac spinors, respectively. From Eqs. (3) and (6), we can decompose $A$ and $B$ in terms of the tensor forms under $SU(3)_F$ as

$$A = a_0 H(6)^{ij} (B'_c)^{ik} (B_n)^{jk}(M)_l^i + a_1 H(6)^{ij} T^{ik} (B_n)^{jk}(M)_l^i + a_2 H(6)^{ij} T^{ik} (B_n)^{jk}(M)_l^i + a_3 H(6)^{ij} (B_n)^{jk}(M)_l^i + a_4 H(6)^{ij} (B_n)^{jk}(M)_l^i + a_5 (B_n)^{jk}(M)_l^i + H(T5)^{ij}_k (B_c)_k,$$

$$B = b_0 H(6)^{ij} (B'_c)^{ik} (B_n)^{jk}(M)_l^i + b_1 H(6)^{ij} T^{ik} (B_n)^{jk}(M)_l^i + b_2 H(6)^{ij} T^{ik} (B_n)^{jk}(M)_l^i + b_3 H(6)^{ij} (B_n)^{jk}(M)_l^i + b_4 H(6)^{ij} (B_n)^{jk}(M)_l^i + b_5 (B_n)^{jk}(M)_l^i + H(T5)^{ij}_k (B_c)_k,$$

respectively, where $(B'_c)^{ij} = \epsilon^{ijk} (B_c)_k$. Here, we have assumed that the mass dependences of $A$ and $B$ are negligible, while the Wilson coefficients of $c_i$ have been absorbed into the $SU(3)_F$ parameters $a_i$ and $b_i$. The full expansions of $A(B_c \to B_n M)$ are listed in Appendix A, while those of $B(B_c \to B_n M)$ can be derived by replacing $a_i$ in $A(B_c \to B_n M)$ with $b_i$.

Since the operator $O(T5) \sim (\bar{u}q_1)(\bar{q}_2c) + (\bar{q}_2q_1)(\bar{u}c)$ is symmetric in color index, whereas the baryon states are antisymmetric, the contributions of $O(T5)$ from the nonfactorizable part to the amplitude vanish, so that we only need to consider the factorizable amplitude from $O(T5)$ [23]. By combining the topological diagram in Fig. [1] one concludes that only $a_6$ and $b_6$ terms in Eq. (6) contain the factorizable contributions in $O(T5)$, in which the octet

$$a_6 = \ldots$$

and

$$b_6 = \ldots$$

are given by

$$a_6 = \ldots$$

and

$$b_6 = \ldots$$

respectively.
meson state $M$ is directly given by the weak interaction alone as demonstrated in Ref. \[23\]. As a result, in our calculations we will neglect the terms associated with $a_0$, $a_4$, $a_5$ and $a_7$ and $b_0$, $b_4$, $b_5$ and $b_7$ in Eq. (6).

The decay angular distribution of the direction $\hat{\mathbf{p}}_{B_n} = \mathbf{p}_{B_n}/p_{B_n}$ ($p_{B_n} \equiv |\mathbf{p}_{B_n}|$) of $B_n$ in the rest frame of $B_c$ is found to be

$$d\Gamma/d\theta \propto 1 + \alpha \hat{\mathbf{P}}_{B_n} \cdot \hat{\mathbf{p}}_{B_n} = 1 + \alpha \cos \theta,$$

(8)

where $\mathbf{P}_{B_n}$ is the polarization vector of $B_n$ with the longitudinal component being $P_{B_n} = \alpha$, $\theta$ is the angle between $\mathbf{P}_{B_n}$ and $\hat{\mathbf{p}}_{B_n}$ and $\alpha$ is the so-called up-down asymmetry parameter, given by

$$\alpha = \frac{2\kappa \text{Re}(A^*B)}{|A|^2 + \kappa^2|B|^2}, \quad \kappa = \frac{p_{B_n}}{E_{B_n} + m_{B_n}}$$

(9)

with $E_{B_n}$ and $p_{B_n}$ the energy and three momentum of $B_n$. The definitions of the up-down and longitudinal asymmetries can be found in Appendix B. Consequently, we obtain the decay rate as

$$\Gamma = \frac{p_{B_n}}{8\pi} \left( \frac{(m_{B_c} + m_{B_n})^2 - m_M^2}{m_{B_c}^2} |A|^2 + \frac{(m_{B_c} - m_{B_n})^2 - m_M^2}{m_{B_c}^2} |B|^2 \right)$$

(10)

To extract the doubly Cabibbo-suppressed contributions in the Cabibbo-allowed dominating decays of $B_c \to B_n K^0_L/K^0_S$, we also define the $K^0_S - K^0_L$ asymmetry parameter as

$$R_{K^0_{S,L}}(B_c \to B_n) = \frac{\Gamma(B_c \to B_n K^0_S) - \Gamma(B_c \to B_n K^0_L)}{\Gamma(B_c \to B_n K^0_S) + \Gamma(B_c \to B_n K^0_L)}.$$  

(11)

### III. NUMERICAL RESULTS AND DISCUSSIONS

We now determine the $SU(3)_F$ parameters through the experimental data \[2, 11-13, 40\], listed in Table III where we have also shown the reproduced values for the observables. Note
TABLE I. Comparisons of the decay branching ratios and asymmetries between the experimental data \[2, 11, 13, 40\] and theoretical reproductions with $SU(3)_F$.

| Channel          | $\mathcal{B}_{\text{exp}}$ | $\alpha_{\text{exp}}$ | $\mathcal{B}_{SU(3)_F}$ | $\alpha_{SU(3)_F}$ |
|------------------|-----------------------------|------------------------|--------------------------|---------------------|
| $\Lambda_c^+ \to \Lambda^0\pi^+$ | $(13.0 \pm 0.7) \times 10^{-3}$ | $-0.91 \pm 0.15$ | $(13.0 \pm 0.7) \times 10^{-3}$ | $-0.87 \pm 0.10$ |
| $\Lambda_c^+ \to pK_S^0$      | $(15.8 \pm 0.8) \times 10^{-3}$ |                           | $(15.7 \pm 0.8) \times 10^{-3}$ | $-0.89^{+0.26}_{-0.11}$ |
| $\Lambda_c^+ \to \Sigma^0\pi^+$ | $(12.9 \pm 0.7) \times 10^{-3}$ |                           | $(12.7 \pm 0.6) \times 10^{-3}$ | $-0.35 \pm 0.27$ |
| $\Lambda_c^+ \to \Sigma^+\pi^0$ | $(12.4 \pm 1.0) \times 10^{-3}$ | $-0.45 \pm 0.32$ | $(12.7 \pm 0.6) \times 10^{-3}$ | $-0.35 \pm 0.27$ |
| $\Lambda_c^+ \to \Sigma^+\eta$      | $(4.1 \pm 2.0) \times 10^{-3}$ |                           | $(3.2 \pm 1.3) \times 10^{-3}$ | $-0.40 \pm 0.47$ |
| $\Lambda_c^+ \to \Sigma^+\eta'$    | $(13.4 \pm 5.7) \times 10^{-3}$ |                           | $(14.4 \pm 5.6) \times 10^{-3}$ | $1.00^{+0.00}_{-0.17}$ |
| $\Lambda_c^+ \to \Xi^0K^+$      | $(5.9 \pm 1.0) \times 10^{-3}$ | *0.77 ± 0.78 | $(5.6 \pm 0.9) \times 10^{-3}$ | $0.94^{+0.06}_{-0.11}$ |

**$\mathcal{R}_{\Xi^0}$** | 0.210 ± 0.028

*This value is not included in the data input. **$\mathcal{R}_{\Xi^0} \equiv B(\Xi^0 \to \Lambda K_S^0)/B(\Xi^0 \to \Xi^-\pi^+)$.*

That in our fit, we have used the original data point of $B(\Lambda_c^+ \to p\pi^0) = (0.8 \pm 1.3) \times 10^{-4}$ from the BESIII Collaboration \[40\], but the result of $\alpha(\Lambda_c^+ \to \Xi^0K^+) = 0.77 \pm 0.78$ \[11\] is not included. Consequently, there are 16 experimental data inputs to fit with 10 $SU(3)_F$ parameters in Eq. (7), given by

$$ (a_1, a_2, a_3, a_6, \tilde{a}, b_1, b_2, b_3, b_6, \tilde{b}) , $$

resulting in the degree of freedom (d.o.f) to be 6, where $\tilde{a}$ and $\tilde{b}$ are defined by

$$ \tilde{a} \equiv a_0 + \frac{1}{3} (a_1 + a_2 - a_3) , \quad \tilde{b} \equiv b_0 + \frac{1}{3} (b_1 + b_2 - b_3) . $$
respectively. Note that the $\eta_0$ amplitude depends only on $\bar{a}$ and $\bar{b}$. By performing the minimal $\chi^2$ fitting as shown in Ref. [21], we obtain

$$(a_1, a_2, a_3, a_6, \bar{a}) = (4.34 \pm 0.50, -1.33 \pm 0.32, 1.25 \pm 0.36, -0.26 \pm 0.64, 1.77 \pm 0.83) \times 10^{-2}\text{GeV}^2,$$

$$(b_1, b_2, b_3, b_6, \bar{b}) = (-9.20 \pm 2.09, -8.03 \pm 1.19, 1.42 \pm 1.61, -4.05 \pm 2.48, 13.15 \pm 5.56) \times 10^{-2}\text{GeV}^2.$$  (14)

The correlation coefficients between i-th and j-th $SU(3)_F$ parameters in Eq. (12) are given by

$$R = \begin{pmatrix}
1 & 0.64 & 0.59 & -0.58 & -0.30 & 0.96 & -0.47 & 0.67 & -0.66 & 0.25 \\
0.64 & 1 & -0.17 & -0.07 & -0.38 & 0.61 & -0.59 & 0.38 & -0.12 & 0.36 \\
0.59 & -0.17 & 1 & -0.67 & 0.01 & 0.55 & 0.03 & 0.57 & -0.75 & -0.05 \\
-0.58 & -0.07 & -0.67 & 1 & 0.11 & -0.65 & 0.21 & -0.59 & 0.93 & -0.05 \\
-0.30 & -0.38 & 0.01 & 0.11 & 1 & -0.31 & 0.34 & -0.29 & 0.15 & -0.35 \\
0.96 & 0.61 & 0.55 & -0.65 & -0.31 & 1 & -0.51 & 0.63 & -0.70 & 0.27 \\
-0.47 & -0.59 & 0.03 & 0.21 & 0.34 & -0.51 & 1 & -0.59 & 0.15 & -0.29 \\
0.67 & 0.38 & 0.57 & -0.59 & -0.29 & 0.63 & -0.59 & 1 & -0.69 & 0.22 \\
-0.66 & -0.12 & -0.75 & 0.93 & 0.15 & -0.70 & 0.15 & -0.69 & 1 & -0.10 \\
0.25 & 0.36 & -0.05 & -0.05 & -0.35 & 0.27 & -0.29 & 0.22 & -0.10 & 1
\end{pmatrix}.  \quad (15)$$

In our fit, we find that $\chi^2/d.o.f = 0.5$, which indicates that our results with the $SU(3)_F$ symmetry can well explain all current existing experimental data for the decay branching ratios and up-down asymmetries. Indeed, as seen in Table II our reproductions based on $SU(3)_F$ are all consistent with the corresponding experimental measurements. However, it is important to pointed out that our values of $B(\Xi^0 \to \Xi^- \pi^+) = (2.21 \pm 0.14) \times 10^{-2}$ and $|\alpha(\Xi^0 \to \Xi^- \pi^+)| = 0.98^{+0.02}_{-0.07}$ are consistent with, but higher than, the corresponding data of $(1.80 \pm 0.52) \times 10^{-2}$ [13] and $0.6 \pm 0.4$ [2], respectively.

It is worth to take a closer look on the parameters in Eq. (14). As mentioned early, $H(15)$ only contributes to the factorization amplitudes, which can be parametrized only in terms of $a_6$ and $b_6$ terms, corresponding to the vector and axial-vector currents in the baryonic matrix elements, respectively. Our result of $b_6 \gg a_6$ in Eq. (14) suggests that the axial-vector part of the factorization contribution is much larger than the vector one. This can be understood as follows. In the decay of $B_c \to B_n M$, the pseudoscalar meson part of the factorization approach is given by

$$\langle 0| j_5^p | M \rangle = i f_M q^\mu,$$  (16)
where $f_M$ is the meson decay constant, while $q^\mu$ is the four-momentum of $M$, which is also equal to the four-momentum difference between the initial and final baryons of $B_c$ and $B_n$. Consequently, we get that

$$q^\mu \langle B_n|\bar{q}\gamma_\mu c|B_c\rangle \gg q^\mu \langle B_n|\bar{q}\gamma_\mu c|B_c\rangle = i \langle B_n|\partial^\mu (\bar{q}\gamma_\mu c)|B_c\rangle ,$$

(17)

where $q$ stands for the light quarks. In the case of the $SU(4)$ flavor symmetry, in which the charm quark is also treated as $q$, Eq. (17) is automatically satisfied as the right-handed part is zero. It is clear that the inequality in Eq. (17) depends on the parameters $a_6$ and $b_6$, which are not quite determined yet, particularly $a_6$. In fact, from Table IX in Appendix A, we have that

$$A(\Lambda_c^+ \rightarrow p\pi^0) = \sqrt{2} \left( a_2 + a_3 - \frac{a_6}{2} \right),$$

(18)

in which $a_2$ and $a_3$ get almost canceled out each other, resulting in that it could be dominated by the $a_6$ terms. In this case, the experimental search for the up-down asymmetry as well as the future measurement on the branching ration of $\Lambda_c^+ \rightarrow p\pi^0$ will be helpful to obtain the precise value of $a_6$.

In Tables II, III and IV, we list our predictions of the branching ratios and up-down asymmetries for the Cabibbo-allowed, singly Cabibbo-suppressed and doubly Cabibbo-suppressed decays, respectively. In the tables, we have also presented the values of $A$ and $B$, which are useful to understand the up-down asymmetries as well as the comparisons with those given by specific theoretical models. We note that some of our results for the up-down asymmetries have been discussed for the first time in the literature, while the decay branching ratios are almost the same as those in Refs. [17,23]. In particular, we find that $B(\Lambda_c^+ \rightarrow p\pi^0) = (1.2 \pm 1.2) \times 10^{-4}$, which is consistent with our previous value of $(1.3 \pm 0.7) \times 10^{-4}$ in Ref. [23] and $0.8 \times 10^{-4}$ calculated by the pole model with current algebra in Ref. [36] as well as the current experimental upper limit of $2.7 \times 10^{-4}$ [2]. In addition, the decay branching ratio for the related Cabibbo-suppressed mode of $\Lambda_c^+ \rightarrow n\pi^+$ is predicted to be $(8.5 \pm 1.9) \times 10^{-4}$, in comparison with $(6.1 \pm 2.0) \times 10^{-4}$ in Ref. [23] and $2.7 \times 10^{-4}$ in Ref. [36]. We remark that most of the predictions in the present work with the spin-dependent amplitudes have small uncertainties comparing to those of our previous study with $SU(3)_F$ in Ref. [23] except the decay of $\Lambda_c^+ \rightarrow p\pi^0$ due to the cancellation effect as well as the correlations in Eq. (15).

To compare our predictions of the up-down asymmetries with those in the literature, we summarize the values of $\alpha$ for the Cabibbo-allowed and singly Cabibbo-suppressed decays
TABLE II. Predictions of the branching ratios and up-down asymmetries for the Cabibbo-allowed decays, where we have also listed the values of $A$ and $B$ in the unit of $10^{-1} G_F GeV^2$.

| channel          | $A$     | $B$     | $10^3 B$ | $\alpha$ |
|------------------|---------|---------|----------|-----------|
| $\Lambda_c^+ \to \Lambda^0 \pi^+$ | $-0.33 \pm 0.06$ | $1.62 \pm 0.12$ | $13.0 \pm 0.70$ | $-0.87 \pm 0.10$ |
| $\Lambda_c^+ \to pK_S$     | $0.64 \pm 0.11$  | $-0.99 \pm 0.45$ | $15.7 \pm 0.8$  | $-0.89^{+0.26}_{-0.11}$ |
| $\Lambda_c^+ \to pK_L$      | $-0.62 \pm 0.10$ | $1.04 \pm 0.42$ | $15.5 \pm 0.8$  | $-0.92^{+0.21}_{-0.08}$ |
| $\Lambda_c^+ \to \Sigma^0 \pi^+$ | $-0.63 \pm 0.02$ | $0.37 \pm 0.29$ | $12.7 \pm 0.6$  | $-0.35 \pm 0.27$ |
| $\Lambda_c^+ \to \Sigma^+ \pi^0$ | $0.63 \pm 0.02$  | $-0.37 \pm 0.29$ | $12.7 \pm 0.6$  | $-0.35 \pm 0.27$ |
| $\Lambda_c^+ \to \Sigma^+ \pi^0$ | $-0.34 \pm 0.07$ | $0.26 \pm 0.44$ | $3.2 \pm 1.3$   | $-0.40 \pm 0.47$ |
| $\Lambda_c^+ \to \Sigma^+ \eta$  | $-0.69 \pm 0.26$ | $-4.80 \pm 1.54$ | $14.4 \pm 5.6$  | $1.00^{+0.00}_{-0.17}$ |
| $\Lambda_c^+ \to \Xi^0 K^+$    | $0.27 \pm 0.06$  | $1.61 \pm 0.24$ | $5.6 \pm 0.9$   | $0.94^{+0.06}_{-0.11}$ |
| $\Xi_c^+ \to \Sigma^+ K_S$   | $-0.23 \pm 0.09$ | $-0.43 \pm 0.39$ | $4.9^{+5.9}_{-4.2}$ | $0.89^{+0.11}_{-0.46}$ |
| $\Xi_c^+ \to \Sigma^+ K_L$   | $0.16 \pm 0.08$  | $0.54 \pm 0.35$ | $3.8^{+5.1}_{-3.5}$ | $1.00^{+0.00}_{-0.18}$ |
| $\Xi_c^+ \to \Xi^0 \pi^+$    | $-0.22 \pm 0.06$ | $0.12 \pm 0.23$ | $3.8 \pm 2.0$   | $-0.32 \pm 0.52$ |
| $\Xi_c^0 \to \Xi^- \pi^+$    | $0.84 \pm 0.08$  | $-2.25 \pm 0.3$ | $22.1 \pm 1.4$  | $-0.98^{+0.07}_{-0.02}$ |
| $\Xi_c^0 \to \Lambda^0 K_S$  | $0.50 \pm 0.05$  | $-0.57 \pm 0.27$ | $5.0 \pm 0.3$   | $-0.70 \pm 0.28$ |
| $\Xi_c^0 \to \Lambda^0 K_L$  | $-0.53 \pm 0.05$ | $0.56 \pm 0.27$ | $5.5 \pm 0.3$   | $-0.66 \pm 0.28$ |
| $\Xi_c^0 \to \Sigma^0 K_S$   | $0.03 \pm 0.07$  | $-0.50 \pm 0.25$ | $0.5 \pm 0.4$   | $-0.34 \pm 0.95$ |
| $\Xi_c^0 \to \Sigma^0 K_L$   | $0.02 \pm 0.06$  | $0.42 \pm 0.22$ | $0.3^{+0.5}_{-0.3}$ | $0.28 \pm 0.71$ |
| $\Xi_c^0 \to \Sigma^+ K^-$   | $-0.27 \pm 0.06$ | $-1.61 \pm 0.24$ | $5.9 \pm 1.1$   | $0.81 \pm 0.16$ |
| $\Xi_c^0 \to \Xi^0 \pi^0$   | $-0.44 \pm 0.06$ | $1.50 \pm 0.23$ | $7.6 \pm 1.0$   | $-1.00^{+0.07}_{-0.00}$ |
| $\Xi_c^0 \to \Xi^0 \eta$    | $0.65 \pm 0.08$  | $1.64 \pm 0.55$ | $10.3 \pm 2.0$  | $0.93^{+0.07}_{-0.19}$ |
| $\Xi_c^0 \to \Xi^0 \eta'$   | $0.61 \pm 0.25$  | $4.27 \pm 1.51$ | $9.1 \pm 4.1$   | $0.98^{+0.02}_{-0.27}$ |
TABLE III. Legend is the same as Table II but for the singly Cabibbo-suppressed decays with an overall factor of $\sin \theta_c$ for $A$ and $B$ omitted.

| channel                  | $A$         | $B$         | $10^4 B$     | $\alpha$     |
|--------------------------|-------------|-------------|--------------|--------------|
| $\Lambda^+_c \rightarrow p\pi^0$ | $0.01 \pm 0.22$ | $-1.50 \pm 0.77$ | $1.2 \pm 1.2$ | $-0.05 \pm 0.72$ |
| $\Lambda^+_c \rightarrow p\eta$ | $-1.65 \pm 0.41$ | $3.32 \pm 1.65$ | $11.5 \pm 2.7$ | $-0.96^{+0.30}_{-0.04}$ |
| $\Lambda^+_c \rightarrow p\eta'$ | $2.27 \pm 0.57$ | $9.99 \pm 3.7$ | $27.1 \pm 12.3$ | $0.96^{+0.04}_{-0.19}$ |
| $\Lambda^+_c \rightarrow n\pi^+$ | $-0.10 \pm 0.16$ | $-3.99 \pm 0.45$ | $8.5 \pm 1.9$ | $0.12 \pm 0.19$ |
| $\Lambda^+_c \rightarrow \Lambda^0 K^+$ | $1.50 \pm 0.13$ | $0.80 \pm 0.80$ | $6.5 \pm 1.0$ | $0.32 \pm 0.30$ |
| $\Lambda^+_c \rightarrow \Sigma^0 K^+$ | $1.01 \pm 0.13$ | $-3.46 \pm 0.53$ | $5.4 \pm 0.7$ | $-1.00^{+0.06}_{-0.00}$ |
| $\Lambda^+_c \rightarrow \Sigma^+ K_S$ | $1.01 \pm 0.13$ | $-3.46 \pm 0.53$ | $5.4 \pm 0.7$ | $-1.00^{+0.06}_{-0.00}$ |
| $\Lambda^+_c \rightarrow \Sigma^+ K_L$ | $1.01 \pm 0.13$ | $-3.46 \pm 0.53$ | $5.4 \pm 0.7$ | $-1.00^{+0.06}_{-0.00}$ |
| $\Xi^+_c \rightarrow \Lambda^0 \pi^+$ | $0.12 \pm 0.16$ | $-3.40 \pm 0.56$ | $12.3 \pm 4.0$ | $-0.19 \pm 0.24$ |
| $\Xi^+_c \rightarrow pK_S$ | $-1.01 \pm 0.13$ | $3.46 \pm 0.53$ | $21.7 \pm 3.8$ | $-0.93^{+0.10}_{-0.07}$ |
| $\Xi^+_c \rightarrow pK_L$ | $1.01 \pm 0.13$ | $-3.46 \pm 0.53$ | $21.7 \pm 3.8$ | $-0.93^{+0.10}_{-0.07}$ |
| $\Xi^+_c \rightarrow \Sigma^0 \pi^+$ | $1.81 \pm 0.09$ | $-1.04 \pm 0.78$ | $25.5 \pm 2.6$ | $-0.38 \pm 0.26$ |
| $\Xi^+_c \rightarrow \Sigma^+ \pi^0$ | $-1.89 \pm 0.21$ | $-0.28 \pm 1.22$ | $26.9 \pm 6.6$ | $0.10 \pm 0.43$ |
| $\Xi^+_c \rightarrow \Sigma^+ \eta$ | $1.53 \pm 0.36$ | $1.29 \pm 1.85$ | $17.2 \pm 10.7$ | $0.55 \pm 0.53$ |
| $\Xi^+_c \rightarrow \Sigma^+ \eta'$ | $1.39 \pm 0.61$ | $10.55 \pm 3.48$ | $38.3 \pm 18.1$ | $0.84^{+0.16}_{-0.34}$ |
| $\Xi^+_c \rightarrow \Xi^0 K^+$ | $-0.10 \pm 0.16$ | $-3.99 \pm 0.45$ | $8.2 \pm 1.9$ | $0.18 \pm 0.28$ |
| $\Xi^0_c \rightarrow \Lambda^0 \pi^0$ | $-0.05 \pm 0.13$ | $2.94 \pm 0.49$ | $2.3 \pm 0.8$ | $-0.09 \pm 0.23$ |
| $\Xi^0_c \rightarrow \Lambda^0 \eta$ | $-0.13 \pm 0.33$ | $6.81 \pm 1.27$ | $6.2 \pm 1.9$ | $-0.44 \pm 0.28$ |
| $\Xi^0_c \rightarrow \Lambda^0 \eta'$ | $2.26 \pm 0.69$ | $11.48 \pm 4.36$ | $18.4 \pm 8.8$ | $0.94^{+0.06}_{-0.24}$ |
| $\Xi^0_c \rightarrow pK^-$ | $0.61 \pm 0.15$ | $3.71 \pm 0.55$ | $5.0 \pm 1.1$ | $0.67 \pm 0.17$ |
| $\Xi^0_c \rightarrow nK_S$ | $-1.44 \pm 0.05$ | $0.84 \pm 0.68$ | $3.8 \pm 0.2$ | $-0.47 \pm 0.34$ |
| $\Xi^0_c \rightarrow nK_L$ | $1.44 \pm 0.05$ | $-0.84 \pm 0.68$ | $3.8 \pm 0.2$ | $-0.47 \pm 0.34$ |
| $\Xi^0_c \rightarrow \Sigma^0 \pi^0$ | $0.72 \pm 0.22$ | $-3.51 \pm 0.65$ | $3.8 \pm 0.7$ | $-0.88^{+0.21}_{-0.12}$ |
| $\Xi^0_c \rightarrow \Sigma^0 \eta$ | $-1.09 \pm 0.26$ | $-0.91 \pm 1.31$ | $1.4 \pm 0.7$ | $0.05 \pm 0.74$ |
| $\Xi^0_c \rightarrow \Sigma^0 \eta'$ | $-0.99 \pm 0.43$ | $-7.46 \pm 2.46$ | $3.5 \pm 1.8$ | $0.80^{+0.20}_{-0.37}$ |
| $\Xi^0_c \rightarrow \Sigma^+ \pi^-$ | $-0.61 \pm 0.15$ | $-3.71 \pm 0.55$ | $3.9 \pm 0.8$ | $0.78 \pm 0.17$ |
| $\Xi^0_c \rightarrow \Sigma^- \pi^+$ | $1.94 \pm 0.19$ | $-5.18 \pm 0.7$ | $13.3 \pm 0.9$ | $-1.00^{+0.04}_{-0.00}$ |
| $\Xi^0_c \rightarrow \Xi^0 K_S$ | $-1.44 \pm 0.05$ | $0.84 \pm 0.68$ | $3.6 \pm 0.2$ | $-0.32 \pm 0.25$ |
| $\Xi^0_c \rightarrow \Xi^0 K_L$ | $-1.44 \pm 0.05$ | $0.84 \pm 0.68$ | $3.6 \pm 0.2$ | $-0.32 \pm 0.25$ |
| $\Xi^0_c \rightarrow \Xi^- K^+$ | $-1.94 \pm 0.19$ | $5.18 \pm 0.70$ | $9.8 \pm 0.7$ | $-0.96^{+0.08}_{-0.04}$ |
TABLE IV. Legend is the same as Table II but for the doubly Cabibbo-suppressed decays with an overall factor of $\sin^2 \theta_c$ for $A$ and $B$ omitted.

| channel | $A$       | $B$       | $10^3 B$ | $\alpha$ |
|---------|-----------|-----------|----------|-----------|
| $\Lambda_c^+ \to nK^+$ | $-0.12 \pm 0.03$ | $0.06 \pm 0.12$ | $0.4 \pm 0.2$ | $-0.41 \pm 0.61$ |
| $\Xi_c^+ \to \Lambda^0 K^+$ | $-0.20 \pm 0.02$ | $-0.26 \pm 0.13$ | $3.4 \pm 0.8$ | $0.76 \pm 0.24$ |
| $\Xi_c^+ \to p\pi^0$ | $0.10 \pm 0.02$ | $0.60 \pm 0.09$ | $6.0 \pm 1.4$ | $0.65 \pm 0.17$ |
| $\Xi_c^+ \to p\eta$ | $0.21 \pm 0.05$ | $-1.20 \pm 0.24$ | $19.8 \pm 7.0$ | $-0.70 \pm 0.15$ |
| $\Xi_c^+ \to p\eta'$ | $-0.48 \pm 0.13$ | $-2.19 \pm 0.83$ | $43.5 \pm 22.9$ | $0.88^{+0.12}_{-0.25}$ |
| $\Xi_c^+ \to n\pi^+$ | $0.14 \pm 0.03$ | $0.85 \pm 0.13$ | $12.1 \pm 2.8$ | $0.65 \pm 0.17$ |
| $\Xi_c^+ \to \Sigma^0 K^+$ | $-0.32 \pm 0.03$ | $0.85 \pm 0.11$ | $11.9 \pm 0.7$ | $-0.99^{+0.05}_{-0.01}$ |
| $\Xi_c^0 \to p\pi^-$ | $0.14 \pm 0.03$ | $0.85 \pm 0.13$ | $3.1 \pm 0.7$ | $0.65 \pm 0.17$ |
| $\Xi_c^0 \to n\pi^0$ | $-0.10 \pm 0.02$ | $-0.60 \pm 0.09$ | $1.5 \pm 0.4$ | $0.65 \pm 0.17$ |
| $\Xi_c^0 \to n\eta$ | $0.21 \pm 0.05$ | $-1.20 \pm 0.24$ | $5.0 \pm 1.8$ | $-0.70 \pm 0.15$ |
| $\Xi_c^0 \to n\eta'$ | $-0.48 \pm 0.13$ | $-2.19 \pm 0.83$ | $11.0 \pm 5.8$ | $0.88^{+0.12}_{-0.25}$ |
| $\Xi_c^0 \to \Sigma^- K^+$ | $-0.45 \pm 0.04$ | $1.20 \pm 0.16$ | $6.0 \pm 0.4$ | $-0.99^{+0.05}_{-0.01}$ |

of $B_c \to B_n M$ in Tables V and VI respectively. In the tables, the data are taken from the experimental values in Ref. [2], KK and Iva correspond to the calculations with the covariant quark models by Korner and Kramer (KK) [26] and Ivanov et al. (Iva) [33], XK, CT and Zen are based on the pole models by Xu and Kamal (XK) [27], Cheng and Tseng (CT) [29] and Zenczykowski (Zen) [31], SV1, CT', UVK($l$) and CKX are related to the considerations of current algebra by Sharma and Verma (SV1) [34], Cheng and Tseng (CT) [29], Uppal, Verma and Khanna (UVK) without (with) the baryon wave function scale variation [30] and Cheng, Kang and Xu (CKX) [36], and SV2($l$) represent the results with $SU(3)_F$ by Sharma and Verma with two different signs of $B(\Lambda_c^+ \to \Xi^0 K^+)$ [16], respectively. As seen in Table V, our results of the up-down asymmetries are quite different from those in the literature [16, 26, 31, 33, 34]. In particular, it is interesting to see that we predict that

$$\alpha(\Lambda_c^+ \to \Xi^0 K^+)_F = 0.94^{+0.06}_{-0.11}$$

which is consistent with the current experimental data of $0.77 \pm 0.78$ in Eq. (11), but
TABLE V. Summary of our results with $SU(3)_F$ and those in the literature for the up-down asymmetries of the Cabibno-allowed charmed baryon decays, where the data, KK, XK, CT, UVK, Zen, Iva, SV1, and SV2 are from the PDG [2], Korner and Kramer [26], Xu and Kamal [27], Cheng and Tseng [29], Uppal, Verma and Khanna [30], Zencyzykowski [31], Ivanov *et al.* [32], Sharma and Verma [34], and Sharma and Verma [16], respectively.

| channel | our result | data | KK | XK | CT | UVK | Zen | Iva | SV1 | SV2 |
|---------|------------|------|----|----|----|-----|-----|-----|-----|-----|
| $\Lambda_c^+ \rightarrow \Lambda^0 \pi^+$ | $-0.89_{-0.11}^{+0.26}$ | $-0.91 \pm 0.15$ | $-0.70$ | $-0.67$ | $-0.99$ | $-0.87$ | $-0.99$ | $-0.95$ | $-0.99$ | input |
| $\Lambda_c^+ \rightarrow pK^0$ | $-0.90_{-0.10}^{+0.22}$ | $-1.0$ | $0.51$ | $-0.90$ | $-0.99$ | $-0.66$ | $-0.97$ | $-0.99$ | $-0.99 \pm 0.39$ |
| $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$ | $-0.35 \pm 0.27$ | $0.70$ | $0.92$ | $-0.49$ | $-0.32$ | $0.39$ | $0.43$ | $-0.31$ | $-0.45 \pm 0.32$ |
| $\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$ | $-0.35 \pm 0.27$ | $-0.45 \pm 0.32$ | $0.70$ | $0.92$ | $-0.49$ | $-0.32$ | $0.39$ | $0.43$ | $-0.31$ | input |
| $\Lambda_c^+ \rightarrow \Sigma^+ \eta$ | $-0.40 \pm 0.47$ | $0.33$ | $-0.94$ | $0$ | $0.55$ | $-0.99$ | $0.92 \pm 0.47$ |
| $\Lambda_c^+ \rightarrow \Sigma^+ \eta'$ | $1.00_{-0.17}^{+0.00}$ | $-0.45$ | $0.68$ | $-0.91$ | $-0.05$ | $0.44$ | $-0.75 \pm 0.38$ |
| $\Lambda_c^+ \rightarrow \Xi^0 K^+$ | $0.94_{-0.11}^{+0.06}$ | $0.77 \pm 0.78$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $\Xi_c^+ \rightarrow \Sigma^+ \bar{K}^0$ | $0.98_{-0.16}^{+0.02}$ | $-1.0$ | $0.24$ | $-0.09$ | $1.0$ | $-0.99$ | $-0.38$ | $0.03 \pm 0.31$ |
| $\Xi_c^+ \rightarrow \Xi^0 \pi^+$ | $-0.32 \pm 0.52$ | $-0.78$ | $-0.81$ | $-0.77$ | $1.0$ | $-1.0$ | $-0.74$ | $0.03 \pm 0.29$ |

$\Xi_c^+ \rightarrow \Xi^0 K^0$ | $0.68 \pm 0.28$ | $-0.68 \pm 0.4$ | $-0.38 \pm 0.38$ | $-0.99$ | $-0.79$ | $-0.84$ | $-0.99$ | $-0.96 \pm 0.38$ |
| $\Xi_c^+ \rightarrow \Xi^0 \bar{K}^0$ | $0.07 \pm 0.90$ | $-0.96 \pm 0.99$ | $-0.96 \pm 0.99$ | $-0.50 \pm 0.55$ | $-0.15$ | $0.07 \pm 0.67$ |
| $\Xi_c^+ \rightarrow \Sigma^+ K^-$ | $0.81 \pm 0.16$ | $0.92 \pm 0.92$ | $-0.92$ | $0.21$ | $0.94$ | $-0.80$ | $-0.99 \pm 0.37$ |
| $\Xi_c^+ \rightarrow \Xi^0 \pi^0$ | $-1.00_{-0.27}^{+0.00}$ | $0.94_{-0.19}^{+0.07}$ | $-0.92$ | $-0.92$ | $-0.92$ | $0.14 \pm 0.34$ |
| $\Xi_c^+ \rightarrow \Xi^0 \eta$ | $0.94_{-0.27}^{+0.00}$ | $-0.38$ | $-1.0$ | $-0.32$ | $0.65$ | $-0.63 \pm 0.40$ | $-0.99 \pm 0.42$ |
TABLE VI. Summary of our results with SU(3)$_F$ and those in the literature for the up-down asymmetries of the singly Cabibbo-suppressed charmed baryon decays, where UVK, SV2 and CKX are from Refs. 30, 16 and 36, respectively.

| channel                  | our result   | UVK$^{(i)}$ | SV2$^{(i)}$ | CKX  |
|--------------------------|--------------|-------------|-------------|------|
| $\Lambda^+_c \rightarrow p\pi^0$ | $-0.05 \pm 0.72$ | 0.82 (0.85) | 0.05 (0.05) | $-0.95$ |
| $\Lambda^+_c \rightarrow p\eta$  | $-0.96^{+0.30}_{-0.04}$ | $-1.00 \ (-0.79)$ | $-0.74 \ (-0.45)$ | $-0.56$ |
| $\Lambda^+_c \rightarrow p\eta'$ | $0.96^{+0.04}_{-0.19}$  | 0.87 (0.87) | $-0.97 \ (-0.99)$ |
| $\Lambda^+_c \rightarrow n\pi^+$ | $0.12 \pm 0.19$  | $-0.13 \ (+0.68)$ | 0.05 (0.05) | $-0.90$ |
| $\Lambda^+_c \rightarrow \Lambda^0 K^+$ | $0.32 \pm 0.30$ | $-0.99 \ (-0.99)$ | $-0.54 \ (0.97)$ | $-0.96$ |
| $\Lambda^+_c \rightarrow \Sigma^0 K^+$ | $-1.00^{+0.06}_{-0.00}$ | $-0.80 \ (-0.80)$ | 0.68 (-0.98) | $-0.73$ |
| $\Lambda^+_c \rightarrow \Sigma^+ K^0$ | $-1.00^{+0.06}_{-0.00}$ | $-0.80 \ (-0.80)$ | 0.68 (-0.98) | $-0.74$ |

different from all theoretical predictions in the literature. For example, it has been suggested that this asymmetry is approximately zero in dynamical models 26, 31, 33, 34, while the authors in Ref. 16 have also taken it to be zero as a data input when the SU(3)$_F$ symmetry is imposed. In our fit, the value in Eq. (1) has not been included as an input in order to see its value based on the SU(3)$_F$ approach. Since the error of our predicted result in Eq. (19) is small, we are confident that $\alpha(\Lambda^+_c \rightarrow \Sigma^0 K^+)$ should be much larger than zero and close to one. To clarify this issue, a further precision measurement on this asymmetry is highly recommended.

In addition, due to the vanishing contributions to the decays from the $a_4$, $a_5$, $a_7$ and $a'_0$ terms of $\mathcal{O}(\Lambda_{15})$, we get

$$A(\Lambda^+_c \rightarrow \Sigma^0 K^+) = A(\Lambda^+_c \rightarrow \Sigma^+ K^0_S, \Sigma^+ K^0_L) = \sqrt{2}(a_1 - a_3)s_c,$$

$$B(\Lambda^+_c \rightarrow \Sigma^0 K^+) = B(\Lambda^+_c \rightarrow \Sigma^+ K^0_S, \Sigma^+ K^0_L) = \sqrt{2}(b_1 - b_3)s_c,$$

leading to the fitted values of

$$B(\Lambda^+_c \rightarrow \Sigma^0 K^+, \Sigma^+ K^0_S, \Sigma^+ K^0_L) = (5.4 \pm 0.7) \times 10^{-4},$$

$$\alpha(\Lambda^+_c \rightarrow \Sigma^0 K^+, \Sigma^+ K^0_S, \Sigma^+ K^0_L) = -1.00^{+0.06}_{-0.00},$$

as given in Table III. Note that the decay branching ratio of $\Lambda^+_c \rightarrow \Sigma^0 K^+$ has been measured to be $(5.2 \pm 0.8) \times 10^{-4}$ 2, which agrees with with that in Eq. (21). Future measurements
TABLE VII. Irreducible amplitudes, decay branching ratios and up-down and $K^0_S-K^0_L$ asymmetries of $B_c \to B_n K^0_L/K^0_S$ with both Cabibbo-allowed and doubly Cabibble-suppressed transitions, where the $B$ amplitudes can be obtained directly by substituting $a_i$ with $b_i$.

| channel                        | Irreducible amplitude for $A$                                                                 | $10^3\mathcal{B}_{SU(3)}F$ | $\alpha_{SU(3)}F$ | $R_{K^0_{S\leftrightarrow L}}$ |
|--------------------------------|------------------------------------------------------------------------------------------------|-----------------------------|-------------------|-------------------------------|
| $\Lambda_c^+ \to pK^0_S$      | $\sqrt{2}((a_1 - \frac{a_6}{2}) + (a_3 - \frac{a_6}{2})s_c^2)$                               | 15.7 ± 0.8                 | -0.89^{+0.26}_{-0.11} | 0.009 ± 0.011                 |
| $\Lambda_c^+ \to pK^0_L$      | $-\sqrt{2}((a_1 - \frac{a_6}{2}) - (a_3 - \frac{a_6}{2})s_c^2)$                               | 15.5 ± 0.9                 | -0.92^{+0.21}_{-0.08}  |                               |
| $\Xi_c^+ \to \Sigma^0 K^0_S$  | $\sqrt{2}((a_3 - \frac{a_6}{2}) + (a_1 - \frac{a_6}{2})s_c^2)$                               | 4.9^{+5.9}_{-4.2}          | 0.89^{+0.11}_{-0.46}  | 0.118 ± 0.078                 |
| $\Xi_c^+ \to \Sigma^0 K^0_L$  | $\sqrt{2}((a_3 - \frac{a_6}{2}) - (a_1 - \frac{a_6}{2})s_c^2)$                               | 3.8^{+5.1}_{-3.5}          | 1.00^{+0.00}_{-0.18}  |                               |
| $\Xi_c^0 \to \Sigma^0 K^0_S$  | $(a_2 + a_3 - \frac{a_6}{2}) + (a_1 - \frac{a_6}{2})s_c^2$                                    | 0.5 ± 0.4                  | -0.34 ± 0.95         | 0.170 ± 0.146                 |
| $\Xi_c^0 \to \Sigma^0 K^0_L$  | $-(a_2 + a_3 - \frac{a_6}{2}) + (a_1 - \frac{a_6}{2})s_c^2$                                    | 0.3^{+0.5}_{-0.3}          | 0.28 ± 0.71          |                               |
| $\Xi_c^0 \to \Lambda^0 K^0_S$ | $\frac{1}{\sqrt{3}}((2a_1 - 2a_2 - a_3 - \frac{a_6}{2}) - (a_1 - 2a_2 - 2a_3 + \frac{a_6}{2})s_c^2)$ | 5.0 ± 0.3                  | -0.66 ± 0.28         | -0.043 ± 0.003                |
| $\Xi_c^0 \to \Lambda^0 K^0_L$ | $\frac{1}{\sqrt{3}}((2a_1 - 2a_2 - a_3 - \frac{a_6}{2}) + (a_1 - 2a_2 - 2a_3 + \frac{a_6}{2})s_c^2)$ | 5.5 ± 0.4                  | -0.70 ± 0.28         |                               |

on $\Lambda_c^+ \to \Sigma^+ K^0_S$ and $\Lambda_c^+ \to \Sigma^+ K^0_L$ are important as they can tell us if Eqs. (20) and (21), which can also be derived through the isospin symmetry, are right or wrong.

We now concentrate on the decay processes of $B_c \to B_n K^0_L$ and $B_c \to B_n K^0_S$, which involve both Cabibbo-allowed and doubly suppressed transitions, as shown in Table VII. If we ignore the later contributions associated with $\sin^2\theta_c$, $B(B_c \to B_n K^0_F) = B(B_c \to B_n K^0_F)$. Clearly, the $K^0_S - K^0_L$ asymmetry depends on the doubly Cabibbo-suppressed parts of the decays. As shown in Table VII the central values for the first three asymmetries are predicted to be around 10% or more, which are consistent with those in Ref. [19]. For $\Xi_c^0 \to \Sigma^0 K^0_S/K^0_L$, the up-down asymmetry of $R_{K^0_{S\leftrightarrow L}}(\Xi_c^0 \to \Sigma^0)$ has different sign, indicating that the effect of the doublyCabibbo-suppressed transition is not ignorable in these decay processes. Explicitly, we find out that $B(\Xi_c^0 \to \Lambda^0 K^0_F)$ can be a little larger than $B(\Xi_c^0 \to \Lambda^0 K^0_S)$, in which the $K^0_S - K^0_L$ asymmetry is predicted to be $-(4.3 \pm 0.3)\%$ with a tiny uncertainty, which agrees well with $-(3.7 \pm 0.4)\%$ in Ref. [19].
IV. CONCLUSIONS

We have studied the two-body decays of $B_c \to B_n M$ with the $SU(3)_F$ flavor symmetry based on the spin-dependent $s$ and $p$-wave amplitudes of $A$ and $B$, respectively. These amplitudes, which have been decomposed in terms of the $SU(3)_F$ parameters $a_i^{(t)}$ and $b_i^{(t)}$, allow us to examine the longitudinal polarization of $P_{B_n}$, which is related to the up-down asymmetry of $\alpha$. We have obtained a good $\chi^2$ fit for the ten $SU(3)_F$ parameters in Eq. (14) from the all possible contributions of $O(6)$ and $O(15)$ with 16 data points in Table I in the $SU(3)_F$ approach, in which all experimental data for the decay branching ratios and up-down asymmetries can be explained. Consequently, we have systematically predicted all decay branching ratios and up-down asymmetries of the Cabibbo-allowed, singly Cabibbo-suppressed and doubly Cabibbo-suppressed charmed baryon decays. In particular, our results of $\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = (2.21 \pm 0.14) \times 10^{-2}$ and $\alpha(\Xi_c^0 \to \Xi^- \pi^+) = -0.98^{+0.07}_{-0.02}$ are consistent with the data of $(1.80 \pm 0.52) \times 10^{-2}$ and $-0.6 \pm 0.4$, respectively. We have also found that $\mathcal{B}(\Lambda_c^+ \to p \pi^0) = (1.2 \pm 1.2) \times 10^{-4}$, which is consistent with the current experimental upper limit of $2.7 \times 10^{-4}$. In addition, we have gotten that $\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+, \Sigma^+ K^0_S, \Sigma^+ K^0_L) = (5.4 \pm 0.7) \times 10^{-4}$ and $\alpha(\Lambda_c^+ \to \Sigma^0 K^+, \Sigma^+ K^0_S, \Sigma^+ K^0_L) = -1.00^{+0.06}_{-0.00}$, which are also guaranteed by the isospin symmetry.

We have shown in Table V that our predictions of the up-down asymmetries are quite different from the theoretical values in the literature for most of the decay modes. In particular, we have found that $\alpha(\Lambda_c^+ \to \Sigma^0 K^+, SU(3)) = 0.94^{+0.06}_{-0.11}$ in Eq. (19), which is consistent with the current experimental data of $0.77 \pm 0.78$ in Eq. (11), but much larger than zero predicted in the literature. A future precision measurement on this asymmetry is clearly very important as our prediction based on $SU(3)_F$ is close to one with a small uncertainty, which can be viewed as a benchmark for the $SU(3)_F$ approach.

We have also explored the $K^0_S - K^0_L$ asymmetries in the decays of $B_c \to B_n K^0_L/K^0_S$ with both Cabibbo-allowed and doubly Cabibbo-suppressed transitions. The asymmetries depend strongly on the contributions from the doubly Cabibbo-suppressed contributions. Clearly, the measurements of these asymmetries are good tests for the doubly Cabibbo-suppressed transitions.

In conclusion, we give a systematic consideration of the up-down asymmetries in the two-body charmed baryon decays of $B_c \to B_n M$ as well as the $K^0_S - K^0_L$ asymmetries in
the decays of $B_c \to B_n K^0_L / K^0_S$ in the $SU(3)_F$ approach. Some of our predictions based on $SU(3)_F$ are different from those in the dynamical models, can be tested by the experiments at BESIII and Belle.

**Appendix A: Irreducible Amplitudes**

In this Appendix, we provide the irreducible amplitudes $A_{B_c \to B_n M}$ from Eq. (7) based on the flavor $SU(3)_F$ symmetry, while those of $B_{B_c \to B_n M}$ can be obtained by substituting $b_i$ with $a_i$ in $A_{B_c \to B_n M}$. Note that in the limits of $\eta = \eta_8$ and $\eta' = \eta_0$, one has that $s_\phi = \sqrt{2} c_\phi$, resulting in the $\eta' = \eta_0$ modes only contain $\tilde{a}$. In Tables VIII IX and X we show the Cabibbo-allowed, singly Cabibbo-suppressed and doubly Cabibbo-suppressed amplitudes of $A_{B_c \to B_n M}$, respectively. Here, we have only considered the factorizable amplitudes from $O(15)$, so that the terms associated with $a_{0,4,5,7}$ and $b_{0,4,5,7}$ are set to be zero.

**Appendix B: Up-down and Longitudinal Polarization Asymmetries**

From Eq. (8), the up-down is defined by

$$\alpha = \frac{d\Gamma(\vec{P}_{B_n} \cdot \hat{p}_{B_n} = +1) - d\Gamma(\vec{P}_{B_n} \cdot \hat{p}_{B_n} = -1)}{d\Gamma(\vec{P}_{B_n} \cdot \hat{p}_{B_n} = +1) + d\Gamma(\vec{P}_{B_n} \cdot \hat{p}_{B_n} = -1)},$$

which is equal to the longitudinal polarization asymmetry, i.e. $P_{B_n} = \alpha$.

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| Channel | \( A \) |
|---------|---------|
| \( \Lambda_c^+ \to \Lambda^0 \pi^+ \) | \( \frac{\sqrt{6}}{3} (-a_1 - a_2 - a_3 - a_6) \) |
| \( \to pK^0 \) | \(-2a_1 + a_6\) |
| \( \to \Sigma^0 \pi^+ \) | \( \sqrt{2}(-a_1 + a_2 + a_3) \) |
| \( \to \Sigma^+ \pi^0 \) | \( \sqrt{2}(a_1 - a_2 - a_3) \) |
| \( \to \Sigma^+ \eta \) | \( \sqrt{2} \right|_{\phi} (a_1 - a_2 + a_3 + 6a) + \frac{2}{3} s_{\phi} (-a_1 - a_2 + a_3) \) |
| \( \to \Sigma^+ \eta' \) | \( \frac{2}{3} c_{\phi} (a_1 + a_2 - a_3 + 3a) + \sqrt{2} s_{\phi} (-a_1 - a_2 + a_3 + 3a) \) |
| \( \to \Xi^0 K^0 \) | \(-2a_2\) |
| \( \Xi^0 \to \Sigma^+ K^0 \) | \( 2a_3 - a_6\) |
| \( \Xi^+ \to \Sigma^+ \bar{K}^0 \) | \(-2a_3 - a_6\) |
| \( \Xi^+ \to \Sigma^0 \bar{K}^0 \) | \( \frac{\sqrt{6}}{3} (-2a_1 + a_2 + a_3 + \frac{a_6}{2}) \) |
| \( \to \Sigma^0 \bar{K}^0 \) | \( \sqrt{2}(-a_2 - a_3 + \frac{a_6}{2}) \) |
| \( \to \Sigma^+ K^- \) | \( 2a_2\) |
| \( \to \Xi^0 \bar{K}^0 \) | \( \sqrt{2}(a_1 - a_3) \) |
| \( \to \Xi^0 \eta \) | \( \sqrt{2} c_{\phi} (-a_1 - 2a_2 - a_3 - 6a) + \frac{2}{3} s_{\phi} (a_1 - 2a_2 - a_3 - 3a) \) |
| \( \to \Xi^0 \eta' \) | \( \frac{2}{3} c_{\phi} (-a_1 + 2a_2 + a_3 + 3a) + \sqrt{2} s_{\phi} (a_1 - 2a_2 - a_3 + 6a) \) |
| \( \to \Xi^- \pi^+ \) | \( 2a_1 + a_6\) |

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| Channel | \( \sin^{-1} \theta_c A \) |
|---------|------------------|
| \( \Lambda^+_c \to \Lambda^0 K^+ \) | \( \frac{\sqrt{3}}{3}(a_1 - 2a_2 + a_3 + a_6) \) |
| \( \to p\pi^0 \) | \( \sqrt{2}(a_2 + a_3 - \frac{a_6}{2}) \) |
| \( \to p\eta \) | \( \sqrt{2} c_\phi(-\frac{2a_1}{3} + \frac{a_2}{3} + \frac{a_3}{3} + \frac{a_6}{2} + 2\tilde{a}) + \frac{1}{3} s_\phi(-4a_1 + 2a_2 + 2a_3 + 3a_6 - 6\tilde{a}) \) |
| \( \to p\eta' \) | \( \frac{1}{3} c_\phi(4a_1 - 2a_2 - 2a_3 - 3a_6 + 6\tilde{a}) + \sqrt{2} s_\phi(-\frac{2a_1}{3} + \frac{a_2}{3} + \frac{a_3}{3} + \frac{a_6}{2} + \tilde{a}) \) |
| \( \to n\pi^+ \) | \( 2a_2 + 2a_3 + a_6 \) |
| \( \to \Sigma^0 K^+ \) | \( \sqrt{2}(a_1 - a_3) \) |
| \( \to \Sigma^+ K_S \) | \( \sqrt{2}(a_1 - a_3) \) |
| \( \to \Sigma^+ K_L \) | \( \sqrt{2}(a_1 - a_3) \) |
| \( \Xi^+_c \to \Lambda^0 \pi^+ \) | \( \frac{\sqrt{3}}{3}(a_1 + a_2 - 2a_3 - \frac{a_6}{2}) \) |
| \( \to pK_S \) | \( \sqrt{2}(-a_1 + a_3) \) |
| \( \to pK_L \) | \( \sqrt{2}(a_1 - a_3) \) |
| \( \to \Sigma^0 \pi^+ \) | \( \sqrt{2}(a_1 - a_2 + \frac{a_6}{3}) \) |
| \( \to \Sigma^+ \pi^0 \) | \( \sqrt{2}(-a_1 + a_2 + \frac{a_6}{3}) \) |
| \( \to \Sigma^+ \eta \) | \( \sqrt{2} c_\phi(\frac{a_1}{3} + \frac{a_2}{3} + \frac{2a_3}{3} - \frac{a_6}{2} + 2\tilde{a}) + \frac{1}{3} s_\phi(2a_1 + 2a_2 + 4a_3 - 3a_6 - 6\tilde{a}) \) |
| \( \to \Sigma^+ \eta' \) | \( \frac{1}{3} c_\phi(-2a_1 - 2a_2 - 4a_3 + 3a_6 + 6\tilde{a}) + \sqrt{2} s_\phi(\frac{a_1}{3} + \frac{a_2}{3} + \frac{2a_3}{3} - \frac{a_6}{2} + 2\tilde{a}) \) |
| \( \to \Xi^0 K^+ \) | \( 2a_2 + 2a_3 + a_6 \) |
| \( \Xi^0_c \to \Lambda^0 \nu^0 \) | \( \frac{\sqrt{3}}{3}(-a_1 - a_2 + 2a_3 - \frac{a_6}{2}) \) |
| \( \to \Lambda^0 \eta \) | \( \frac{\sqrt{3}}{3} c_\phi(-a_1 - a_2 + \frac{a_6}{2} + 6\tilde{a}) + \frac{\sqrt{3}}{3} s_\phi(-a_1 - a_2 + \frac{a_6}{2} - 3\tilde{a}) \) |
| \( \to \Lambda^0 \eta' \) | \( \frac{\sqrt{3}}{3} c_\phi(a_1 + a_2 - \frac{a_6}{2} + 3\tilde{a}) + \frac{\sqrt{3}}{3} s_\phi(-a_1 - a_2 + \frac{a_6}{2} + 6\tilde{a}) \) |
| \( \to pK^- \) | \( -2a_2 \) |
| \( \to nK_S \) | \( \sqrt{2}(-a_1 + a_2 + a_3) \) |
| \( \to nK_L \) | \( \sqrt{2}(a_1 - a_2 - a_3) \) |
| \( \to \Sigma^0 \pi^0 \) | \( a_1 + a_2 - \frac{a_6}{2} \) |
| \( \to \Sigma^0 \eta \) | \( c_\phi(-\frac{a_1}{3} + \frac{a_2}{3} + \frac{2a_3}{3} - \frac{a_6}{2} - 2\tilde{a}) + \sqrt{2} s_\phi(-\frac{a_1}{3} - \frac{a_2}{3} - \frac{2a_3}{3} + \frac{a_6}{2} + \tilde{a}) \) |
| \( \to \Sigma^0 \eta' \) | \( \sqrt{2} c_\phi(\frac{a_1}{3} + \frac{a_2}{3} + \frac{2a_3}{3} - \frac{a_6}{2} - \tilde{a}) + s_\phi(-\frac{a_1}{3} - \frac{a_2}{3} - \frac{2a_3}{3} + \frac{a_6}{2} - 2\tilde{a}) \) |
| \( \to \Sigma^+ \pi^- \) | \( 2a_2 \) |
| \( \to \Sigma^- \pi^+ \) | \( 2a_1 + a_6 \) |
| \( \to \Xi^0 K_S \) | \( \sqrt{2}(-a_1 + a_2 + a_3) \) |
| \( \to \Xi^0 K_L \) | \( \sqrt{2}(-a_1 + a_2 + a_3) \) |
| \( \to \Xi^- K^+ \) | \( -2a_1 - a_6 \) |
TABLE X. Doubly Cabibbo-suppressed amplitudes for $A_{B_c \to B_n M}$

| Channel | $\sin^{-2}\theta_c A$ |
|---------|------------------------|
| $\Lambda_c^+ \to pK^0$ | $2a_3 - a_6$ |
| $\to nK^+$ | $-2a_3 - a_6$ |
| $\Xi_c^+ \to \Lambda^0 K^+$ | $\frac{\sqrt{6}}{3}(-a_1 + 2a_2 + 2a_3 + \frac{a_6}{2})$ |
| $\to \Sigma^+ K^0$ | $-2a_1 + a_6$ |
| $\to p\pi^0$ | $-\sqrt{2}a_2$ |
| $\to p\eta$ | $\frac{\sqrt{2}}{3}c_\phi(2a_1 - a_2 - 2a_3 - 6\tilde{a}) + \frac{2}{3} s_\phi(2a_1 - a_2 - 2a_3 + 3\tilde{a})$ |
| $\to p\eta'$ | $\frac{2}{3}c_\phi(-2a_1 + a_2 + 2a_3 - 3\tilde{a}) + \frac{\sqrt{2}}{3} s_\phi(2a_1 - a_2 - 2a_3 - 6\tilde{a})$ |
| $\to n\pi^+$ | $-2a_2$ |
| $\to \Sigma^0 K^+$ | $\sqrt{2}(-a_1 - \frac{a_6}{2})$ |
| $\Xi_c^0 \to \Lambda^0 K^0$ | $\frac{\sqrt{6}}{3}(-a_1 + 2a_2 + 2a_3 - \frac{a_6}{2})$ |
| $\to p\pi^-$ | $-2a_2$ |
| $\to n\pi^0$ | $\sqrt{2}a_2$ |
| $\to n\eta$ | $\frac{\sqrt{2}}{3}c_\phi(-6\tilde{a} + 2a_1 - a_2 - 2a_3) + \frac{2}{3} s_\phi(3\tilde{a} + 2a_1 - a_2 - 2a_3)$ |
| $\to n\eta'$ | $\frac{2}{3}c_\phi(-3\tilde{a} - 2a_1 + a_2 + 2a_3) + \frac{\sqrt{2}}{3} s_\phi(-6\tilde{a} + 2a_1 - a_2 - 2a_3)$ |
| $\to \Sigma^0 K^0$ | $\sqrt{2}(a_1 - \frac{a_6}{2})$ |
| $\to \Sigma^- K^+$ | $-2a_1 - a_6$ |
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