NLO CALCULATIONS OF THE EXCLUSIVE PROCESSES IN PQCD

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We present a generally applicable reduction formalism which makes it possible to express an arbitrary tensor and scalar one-loop Feynman integral, with N external lines and massless propagators, in terms of a basic set of eight fundamental scalar Feynman integrals with 2, 3, and 4 external lines, for arbitrary external kinematics. The formalism is particularly suitable for the NLO calculations of exclusive processes at large momentum transfer in pQCD, where all previously developed reduction methods fail due to the presence of collinear external on-shell lines.

1 Motivation

In testing various aspects of QCD the hadronic exclusive processes (EP) at large momentum transfer in which the total number of particles (partons) in the initial and final states is \( N \geq 6 \) are becoming increasingly important.

Leading order (LO) predictions have been obtained for many EP processes. Owing to the fact that the LO predictions in perturbative QCD (pQCD) do not have much predictive power, the inclusion of higher-order corrections is essential because they have a stabilizing effect, reducing the dependence of the LO predictions on the renormalization and factorization scales and the renormalization scheme. However, only a few EP have been analyzed at the next-to-leading order (NLO).

Obtaining radiative corrections in pQCD requires the evaluation of one-loop Feynman integrals with massless propagators (quarks masses can be neglected in high-energy processes). These integrals contain IR divergences (both soft and collinear) and need to be regularized. The most suitable regularization method for pQCD calculations is dimensional regularization.

As it is well known, in calculating Feynman diagrams mainly three difficulties arise: reduction of tensor integrals to scalar integrals, reduction of scalar integrals to a set of basic scalar integrals and the evaluation of the basic scalar integrals.
Considerable progress has recently been made in developing efficient approaches for calculating one-loop Feynman integrals with a large number of external lines. As far as the calculation of one-loop $N$-point massless integrals is concerned, the most complete and systematic method has been developed by Binoth at al.\[1\] It does not, however, apply to all cases of practical interest. Namely, being obtained for the non-exceptional external momenta it cannot be applied to the integrals in which the set of external momenta contains subsets comprised of two or three collinear on-shell momenta. Integrals of this type arise when performing the leading-twist NLO analysis of hadronic EP at large momentum transfer in pQCD.

With no restrictions regarding the external kinematics, in this paper we describe an efficient, systematic and completely general method for reducing an arbitrary one-loop $N$-point massless integral to a set of basic integrals.

2 Reduction of tensor integrals

In order to obtain one-loop radiative corrections to physical processes in massless gauge theory, the integrals of the following type are required:

$$I_{\mu_1\cdots\mu_P}^N(D;\{\nu_i\}) = \frac{(\mu^2)^{2-D/2}}{(2\pi)^D} \int \frac{d^Dl}{A_1^{\nu_1} A_2^{\nu_2} \cdots A_N^{\nu_N}} I(l_{\mu_1} \cdots l_{\mu_P}), \quad A_i = (l + r_i)^2 + i\epsilon. \quad (1)$$

This is a rank $P$ tensor one-loop $N$-point Feynman integral with massless internal lines in $D$ dimensions, where $l$ is the loop momentum, $\mu$ is the usual dimensional regularization scale and $\nu_i \in \mathbb{N}$ are arbitrary powers of the propagators. The corresponding scalar integral, we denote by $I_0^N$. These integrals represent generalizations of the usual integrals in practical calculations, where $\nu_i = 1$. However, the most natural presentation of the reduction method discussed here is in terms of these generalized integrals.

The corresponding expressions of the above integrals in Feynman parameter space\[2\] reads

$$I_0^N(D;\{\nu_i\}) = \frac{1}{(4\pi)^2} (4\pi \mu^2)^{2-D/2} \Gamma\left(\sum_{i=1}^N \nu_i - D/2\right) \prod_{i=1}^N \Gamma(\nu_i) \int_0^1 \left(\prod_{i=1}^N \frac{dy_i}{y_i^{\nu_i-1}}\right) \delta\left(\sum_{i=1}^N y_i - 1\right) \left[-\sum_{i,j=1}^N y_i y_j (r_i - r_j)^2 - i\epsilon\right]^{D/2 - \sum_{i=1}^N \nu_i}, \quad (2)$$

$$I_{\mu_1\cdots\mu_P}^N(D;\{\nu_i\}) = \sum_{k,j_1,\cdots,j_N \geq 0 \atop 2k + \sum_{i=1}^N j_i = P} \left\{[g]^k [r_1]^{j_1} \cdots [r_N]^{j_N}\right\}_{\mu_1\cdots\mu_P} \frac{(4\pi \mu^2)^{P-k}}{(-2)^k} \left(\prod_{i=1}^N \frac{\Gamma(\nu_i + j_i)}{\Gamma(\nu_i)}\right) \times I_0^N(D + 2(P - k);\{\nu_i + j_i\}), \quad (3)$$

where $\left\{[g]^k [r_1]^{j_1} \cdots [r_N]^{j_N}\right\}_{\mu_1\cdots\mu_P}$ represents a symmetric (with respect to $\mu_1 \cdots \mu_P$) combination of tensors, each term of which is composed of $k$ metric tensors and $j_i$ external momenta $r_i$ (for example, $\{g r_1\}_{\mu_1\mu_2\mu_3} = g_{\mu_1\mu_2} r_{\mu_3} + g_{\mu_1\mu_3} r_{\mu_2} + g_{\mu_2\mu_3} r_{\mu_1}$). The general results \[2\] and \[3\] represent massless versions of the results that have originally been derived by Davydychev\[3\] for the case of massive Feynman integrals. With the decomposition \[3\], the problem of calculating the tensor integrals has been reduced to the calculation of the general scalar integrals.

3 Reduction of scalar integrals

As is well known, the direct evaluation of the general scalar integral \[2\] represents a non-trivial problem. However, with the help of recursion relations, the problem can be significantly
some algebraic manipulations the identity (4) takes the form

$$\sum_{i,j=1}^{N} (r_j - r_i)^2 z_i \nu_j I_0^N(D; \{\nu_k + \delta_{kj}\}) = \sum_{i,j=1}^{N} z_i \nu_j I_0^N(D; \{\nu_k + \delta_{kj} - \delta_{ki}\}) - (D - \sum_{j=1}^{N} \nu_j) z_0 I_0^N(D; \{\nu_k\}).$$

In arriving at (5), it has been understood that $I_0^N(D;\ldots,\nu_l,0,\nu_{l+1},\ldots) \equiv I_0^{N-1}(D;\ldots,\nu_l,\nu_{l+1},\ldots)$.

The relation (5) represents the starting point for the derivation of the recursion relations for scalar integrals. We have obtained the fundamental set of recursion relations by choosing the arbitrary constants $z_i (i = 1 \cdots N)$ so as to satisfy the following system of linear equations:

$$\sum_{i=1}^{N} r_{ij} z_i = C, \quad j = 1, \ldots, N; \quad r_{ij} = (r_i - r_j)^2,$$

(6)

where $C$ is an arbitrary constant. If (6) is taken into account, the relation (5), after a few manipulations, reduces to recursion relation

$$C I_0^N(D - 2; \{\nu_k\}) = \sum_{i=1}^{N} z_i I_0^N(D - 2; \{\nu_k - \delta_{ki}\}) + (4\pi \mu^2)(D - 1 - \sum_{j=1}^{N} \nu_j) z_0 I_0^N(D; \{\nu_k\}),$$

(7)

where $z_i$ are given by the solution of the system (6). This is a generalized form of the recursion relation which connects the scalar integrals in different number of dimensions.

By directly choosing the constants $z_i$ in (5) in a such a way that $z_i = \delta_{ik}$, for $k = 1, \cdots, N$, we arrive at a system of $N$ equations which is always valid:

$$\sum_{j=1}^{N} (r_k - r_j)^2 \nu_j I_0^N(D; \{\nu_i + \delta_{ij}\}) = \sum_{j=1}^{N} \nu_j I_0^N(D; \{\nu_i + \delta_{ij} - \delta_{ik}\}) - (D - \sum_{j=1}^{N} \nu_j) I_0^N(D; \{\nu_i\}).$$

(8)

If the system (8) can be solved with respect to $I_0^N(D; \{\nu_i + \delta_{ij}\}, j = 1, \cdots, N$, the solutions represent the recursion relations.

The use of the relations (7) and (8) in practical calculations depends on whether the kinematic determinants

$$\det(R_N) = \begin{pmatrix} 0 & r_{12} & \cdots & r_{1N} \\ r_{12} & 0 & \cdots & r_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ r_{1N} & r_{2N} & \cdots & 0 \end{pmatrix}, \quad \det(S_N) = \begin{pmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & r_{12} & \cdots & r_{1N} \\ 1 & r_{12} & 0 & \cdots & r_{2N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & r_{1N} & r_{2N} & \cdots & 0 \end{pmatrix}$$

are equal to zero or not. We distinguish the following four different types of recursion:

- **Case I:** $\det(S_N) \neq 0, \det(R_N) \neq 0; \quad (C \neq 0 \text{ and } z_0 \neq 0)$
- **Case II:** $\det(S_N) \neq 0, \det(R_N) = 0; \quad (C = 0 \text{ and } z_0 \neq 0)$
- **Case III:** $\det(S_N) = 0, \det(R_N) \neq 0; \quad (C \neq 0 \text{ and } z_0 = 0)$
• Case IV: \( \det(S_N) = 0, \det(R_N) = 0; \) \((C \neq 0 \text{ and } z_0 = 0)\) or \((C = 0 \text{ and } z_0 = 0)\),

where we indicated the necessary choice for the constants \(C\) and \(z_0\) in a way that the system [9] has solution and the most useful recursion relations emerge.

Making use of the relations (17) and (18), each scalar integral \(I_N^0(D; \{\nu_k\})\) can be represented as a linear combination:

\[
I_N^0(D; \{\nu_k\}) = \sum_i c_i(D, r_{ij}) I_{N-1}^0(D^{(i)}; \{\nu_k^{(i)}\}) + \lambda I_N^0(D'; \{1\}),
\]

(10)

where for the dimension \(D'\) one usually chooses \(4 + 2\varepsilon\) or \(6 + 2\varepsilon\). Infinitesimal parameter \(\varepsilon\) is regulating the divergences. Parameter \(\lambda\) equals 0 for Cases II, III and IV[2]. It follows that in all the above cases, with the exception of the Case I, the integrals with \(N\) external lines can be represented in terms of the integrals with smaller number of external lines. Consequently, then, there exists a fundamental set of integrals of the form \(I_N^0(4 + 2\varepsilon; \{1\})\) in terms of which all integrals can be represented as a linear combination.

Therefore[2], any dimensionally regulated one-loop \(N\)-point Feynman integral can be represented in terms of six types of box integrals \((N = 4)\), one type of triangle integral \((N = 3)\) and the general (arbitrary \(D, \nu_1\) and \(\nu_2\)) two-point integral \((N = 2)\). The types of the integrals are determined by number of vanishing kinematic variables. Five of the six basic box integrals are IR divergent in 4 dimensions, while the basic triangle integral is finite. However, all the basic box integrals are finite in 6 dimensions. Thus, an alternative fundamental set of integrals is comprised of five box integrals in 6 dimensions, one box and triangle integral in 4 dimensions and the general two-point integral. This fundamental set of integrals is particularly interesting because the integral \(I_3^0\) is the only divergent one. In the final result, all the divergences, IR as well as UV, are contained in the general two-point integrals and associated coefficients. The expressions for all relevant basic integrals can be found in the literature[2,6,7].

4 Conclusion

Through the tensor decomposition and scalar reduction procedure presented, any massless one-loop Feynman integral with generic 4-dimensional momenta can be expressed as a linear combination of a fundamental set of scalar integrals: six box integrals in \(D = 6\), a triangle integral in \(D = 4\), and a general two-point integral. All the divergences present in the original integral are contained in the general two-point integral and associated coefficients.

Acknowledgments

I would like to thank the organizers of the XXXVIIIth Rencontres de Moriond conference for the invitation and for an exciting conference.

References

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