Current Algebra Approach to Heavy Top Effects in $Z \rightarrow b + \bar{b}$

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ABSTRACT

The $O(\alpha m_t^2/m_W^2)$ and $O(\alpha\alpha_s m_t^2/m_W^2)$ corrections to the partial decay width $\Gamma(Z \rightarrow b + \bar{b})$ are computed using the current algebra formulation of radiative corrections. This framework allows one to easily enforce the relevant Ward identity that greatly simplifies the calculations. As a result, the one-loop $O(\alpha m_t^2/m_W^2)$ contribution is computed through the investigation of only two convergent diagrams. The computation of the QCD corrections to the one-loop $O(\alpha m_t^2/m_W^2)$ term involves fewer diagrams than in the standard approach. In particular, the number of infrared divergent contribution is reduced. The calculation is performed in the dimensional regularization scheme and no term more divergent than $1/(n-4)$ is found. Our result confirms the screening of the one-loop top mass effect recently found by Fleischer et al.
1 Introduction

It is well known that the electroweak radiative parameters $\delta \rho$ \[1\] and $\Delta r$ \[2\] are affected by virtual top exchanges in the self-energies of the vector bosons through terms that depend quadratically on $m_t$, the mass of the top quark. Moreover, due to the isodoublet nature of the top, $m_t^2$ corrections are also found in the $Z\overline{b}b$-vertex.

The status of the theoretical calculations of these corrections is quite advanced. At the moment we have available for the vacuum polarization functions, beside the one-loop calculation \[3\], the complete perturbative $O(\alpha\alpha_s)$ QCD contribution \[4, 5\], studies of the $t\overline{t}$ threshold effects \[6\] and the leading $O((\alpha m_t^2/m_W^2)^2)$ term \[7, 8\]. Concerning the $Z\overline{b}b$-vertex, the one-loop calculation was performed by several groups a few years ago \[9–11\]. Recently, the leading $O(\alpha\alpha_s m_t^2/m_W^2)$ \[12\] and the $O((\alpha m_t^2/m_W^2)^2)$ terms \[8\] have been computed.

The calculation of the two-loop corrections has proven to be a difficult task. Indeed, apart from the perturbative QCD contribution to the vacuum polarization, the two-loop contributions have been evaluated in some approximation, either neglecting all the masses and momenta but the top mass \[7, 12\] or retaining besides the top only the higgs mass \[8\]. However, these approximations are sufficient to derive the most interesting part of the corrections, namely the leading $m_t^2$ contribution.

It is the aim of this paper to apply techniques used in the current algebra formulation of radiative corrections \[13\] to derive the leading $O(\alpha m_t^2/m_W^2)$ and $O(\alpha\alpha_s m_t^2/m_W^2)$ corrections in the partial decay width $\Gamma(Z \rightarrow b + \overline{b})$. As will be seen in the following, these techniques greatly simplify the calculations. In the most interesting case, namely the two-loop $O(\alpha\alpha_s m_t^2/m_W^2)$ correction, in addition to facing fewer contributions than in the standard calculation \[12\] one deals with less divergent diagrams. Furthermore, the treatment of the infrared divergent (IR) contribution is greatly simplified. Our result shows that the QCD corrections have opposite sign to that of the one-loop contribution, leading then an increase in the $m_t$ upper bound.

\[1\]For an updated analysis of the QCD effects in the electroweak corrections see Ref. \[6\] and references therein.
We find agreement with the recent calculation of the $\alpha s G_\mu m_t^2$ term by Fleischer et al..

The paper is organized as follows. In Section 2 the current algebra derivation of the $O(\alpha^2/m^2_W)$ one-loop term is presented. Section 3 is devoted to the calculation of the QCD corrections to this term. In Section 4 we discuss the results. The Appendices give some details about the QCD calculation. In Appendix A we show that the formulae used for the four and five-point correlation functions (Cfr. Eqs. (18a,16)) contain correctly the two-loop wave function renormalization of the external legs. Appendix B lists the individual contributions of the various diagrams.

2 One-loop $O(\alpha^2/m^2_W)$ correction

In this section we derive the leading one-loop $O(\alpha^2/m^2_W)$ term in the $Z b \bar{b}$-vertex using two and three-point correlation functions. This allows us to set the framework for our subsequent discussion of the QCD corrections and to derive the basic Ward identity that enters into the calculation.

In order to fix our notation we write the part of the Standard Model (SM) Lagrangian density that describes the interaction of the $W$, $Z$ and unphysical scalars with fermions as

$$L_{int} = -\frac{g}{\sqrt{2}} (W_\mu^+ J_\mu^W + h.c.) - \frac{g}{c} Z_\mu J_\mu^Z - \frac{g}{2 m_W} \left[ \Phi_2 S_2 + \sqrt{2} (\Phi^S + h.c.) \right] \tag{1}$$

where $g$ is the $SU(2)$ coupling, $m_W$ stands for the mass of the $W$ boson, $c$ is an abbreviation for $\cos \theta_W$, $J_\mu^Z$ and $J_\mu^W$ are the fermionic currents coupled to $Z$ and $W$ respectively, $W^+$ is the field that creates a $W^+$ meson, $\Phi_2$ and $\Phi$ the unphysical counterparts associated with the $Z$ and $W$ and

$$S_2 = 2 \partial_\mu J_\mu^Z = -i \bar{\psi} m^0 C_3 \gamma_5 \psi \tag{2a}$$

$$S = -i \partial_\mu J_\mu^W = \bar{\psi} \Gamma \psi. \tag{2b}$$
In Eqs.(2a,2b) \( \psi \) represents the column vector \( \psi \equiv (t, b)^T \), \( m^0, C_3 \) and \( \Gamma \) are the \( 2 \times 2 \) matrices

\[
m^0 = \begin{pmatrix} m_t^0 & 0 \\
0 & 0 \end{pmatrix}
\]

(2c)

\[
C_3 = \begin{pmatrix} 1 & 0 \\
0 & -1 \end{pmatrix}
\]

(2d)

\[
\Gamma = \begin{pmatrix} 0 & 0 \\
-m_t^0 a_+ & 0 \end{pmatrix},
\]

(2e)

\( a_+ \equiv \frac{1+\gamma_5}{2} \) and the superscript 0 on \( m_t \) refers to the bare mass. As it is evident from Eqs.(2c,2d) we are considering only the third generation and taking the bottom quark as massless.

The one-loop vertex diagrams contributing to the \( O(\alpha m_t^2/m_W^2) \) correction are depicted symbolically in Fig.1. The circles represent the sum of diagrams in which the ends of the \( \Phi^\pm \) propagators are attached in all possible ways to the external \( b\bar{b} \) quarks.

The amplitude corresponding to Fig.1(a) can be expressed as

\[
\gamma_\mu^\nu = -i \frac{g}{c} \lim_{q \to q} T_\mu^\nu(\bar{q}, p, p')
\]

(3a)

with

\[
T_\mu^\nu = \frac{-i}{2} \frac{g^2}{2m_W^2} \int \frac{d^n k}{(2\pi)^n} \frac{1}{\mu^{n-4}} \frac{1}{k^2 - m_W^2} \int d^n y e^{-i\vec{q}\cdot\vec{y}}
\]

\[
\times \int d^n x e^{i\vec{k}\cdot\vec{x}} < p' p | T^* \left[ J_\mu^\nu_Z(y) (S^\dagger(x) S(0) + h.c.) \right] | 0 >,
\]

(3b)

where \( n \) is the dimension of space-time, \( \mu \) is the 't Hooft mass scale, \( T^* \) is the covariant time–ordered product, \( p \) and \( p' \) the momenta of the \( \bar{b} \) and \( b \) quark respectively and \( q = p + p' \) the momentum carried by the Z. As the \( O(\alpha m_t^2/m_W^2) \) term should be gauge invariant by itself.
we find it convenient to carry out the calculation in the 't Hooft–Feynman gauge in which the propagator of the $\Phi$ field has the form $i(k^2 - m_w^2)^{-1}$.

As shown in Ref. [13,14], the limiting procedure in Eq. (3a) affects the insertion of the $\Phi$ fields on the external lines in such a way that $V_{\Phi}^{\mu}$ contains not only the proper vertex correction but also the contributions from the wave function renormalization of the $b$ quarks.

To trigger a Ward identity we contract $T_\Phi^\alpha$ with $\bar{q}_\alpha$ obtaining

$$\bar{q}_\alpha T_\Phi^\alpha = D_\Phi + \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - m_w^2} [V_\Phi(k - \bar{q}) - V_\Phi(-k - q)]$$  (4)

where $D_\Phi$ is an expression analogous to $T_\Phi^\mu$ with the replacement $J Z^\mu \rightarrow -i\partial_\mu J Z^\mu$ and

$$V_\Phi(k) = \frac{-g^2}{4m_w^2} \left( c^2 - \frac{s^2}{2} \right) \int d^n x e^{ik \cdot x} \langle p' p | T^* (S^\dagger(x) S(0) - S^\dagger(0) S(x)) | 0 \rangle,$$  (5)

with $s^2 = 1 - c^2$. In deriving Eq. (5) we have used the commutation relation

$$\delta(x_0 - y_0) [J_Z^0(x), S(y)] = -\frac{c^2 - s^2}{2} S(x) \delta^n(x - y).$$  (6)

Differentiating Eq. (4) with respect to $\bar{q}_\mu$ one obtains

$$T_\Phi^{\mu} = -\bar{q}_\alpha \frac{\partial}{\partial \bar{q}_\mu} T_\Phi^\alpha + \frac{\partial}{\partial \bar{q}_\mu} D_\Phi - \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - m_w^2} \frac{\partial}{\partial k_\mu} V_\Phi(k - \bar{q}).$$  (7)

It is easy to show, following, for example, the discussion in Section VII of Ref. [13] that the first term in the r.h.s. of Eq. (7) is $O(\alpha)$ instead of $O(\alpha m_w^2)$. Terms of this kind will be considered throughout all our calculations as subleading and therefore neglected. After a partial integration in the last term in Eq. (7) we can write for $T_\Phi^{\mu}$

$$T_\Phi^{\mu} = \frac{\partial}{\partial \bar{q}_\mu} D_\Phi + \frac{g^2}{4m_w^2} (c^2 - \frac{s^2}{2}) \int \frac{d^n k}{(2\pi)^n} \frac{k^\mu}{(k^2 - m_w^2)^2} \int d^n x e^{i(k - \bar{q}) \cdot x} \langle p' p | T^* (S^\dagger(x) S(0) - S^\dagger(0) S(x)) | 0 \rangle.$$  (8)
As we are interested in the leading \(m_t\) dependent contribution, we can set \(\bar{q} = q = 0\) in the second term of Eq.(8) and obtain

\[
(T_{\Phi}^\mu)_{L.T.} = \frac{\partial}{\partial \bar{q}_\mu} D_\Phi + \frac{g^2}{2m_W^2} (c^2 - s^2) \int \frac{d^n k}{(2\pi)^n \mu^{n-4}} \frac{k^\mu}{(k^2 - m_t^2)^2} \times \int d^n x e^{ik \cdot x} \left< \frac{p'p}{2} | T^* (S^\dagger(x) S(0)) | 0 > \right.,
\]

where the subscript \(L.T.\) reminds us that we are considering only the leading term.

Turning our attention to Fig.1(b) we write the amplitude as

\[
U_{\Phi}^\mu = -i g \tilde{U}_{\Phi}^\mu \quad (10a)
\]

\[
\tilde{U}_{\Phi}^\mu = -\frac{g^2}{2m_W^2} (c^2 - s^2) \int \frac{d^n k}{(2\pi)^n \mu^{n-4}} \frac{k^\mu}{(k^2 - m_t^2)(k^2 + q^2 - m_t^2)} \times \int d^n x e^{ik \cdot x} \left< \frac{p'p}{2} | T^* (S^\dagger(x) S(0)) | 0 > \right..
\]

It is immediate to see that the leading term in \(\tilde{U}_{\Phi}^\mu\), obtained by putting \(q = 0\) in Eq.(10b), cancels exactly the second term in the r.h.s. of Eq.(9). We conclude that the leading \(O(\alpha m_t^2)\) correction in the \(Zb\bar{b}\)-vertex is given by \(-i \frac{2}{c} \frac{\partial}{\partial \bar{q}_\mu} D_\Phi\bigg|_{\bar{q}=q}\) or, recalling Eq.(2a),

\[
(V_{\Phi}^\mu + U_{\Phi}^\mu)_{L.T.} = i g^3 \frac{3c}{8cm_W^2} \frac{\partial}{\partial \bar{q}_\mu} \left\{ \int \frac{d^n k}{(2\pi)^n \mu^{n-4}} \frac{1}{k^2 - m_t^2} \int d^n y e^{-i\bar{q} \cdot y} \times \int d^n x e^{ik \cdot x} \left< \frac{p'p}{2} | T^* \left[ S_2(y) (S^\dagger(x) S(0) + h.c.) \right] | 0 > \right> \right\}_{\bar{q}=q}.
\]

To express the r.h.s. of Eq.(11) in terms of Feynman diagrams we use Wick’s theorem. We now observe that a non-zero contribution is obtained only when \(S_2\) is acting on a top line. Fig.2 represents schematically the fermion lines in Eq.(11) for \(\bar{q} \neq q\) before the \(k\) integration is performed. The dotted line stands for the \(\bar{q}\) momentum absorbed by the \(S_2\) operator while the dashed lines indicate the momenta emitted or absorbed by the \(S\) and \(S^\dagger\) operators. For example, in Fig.2(a) \(\bar{q}\) is absorbed by \(S_2(y)\) while \(k\) is emitted by \(S^\dagger(x)\). Similarly in Fig.2(b)
\( k \) is emitted by \( S(x) \) and \( \bar{q} \) is absorbed by \( S_2(y) \).

Using an anticommuting \( \gamma_5 \) the contributions of Fig.2(a) and Fig.2(b) to Eq.(11) are

\[
2(a) = \frac{g^2 m_t^4}{8c m_W^2} \frac{\partial}{\partial \bar{q}_\mu} \left\{ \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p')\bar{q}_a v(p)}{[k^2 - m_W^2][(k - p + \bar{q})^2 - m_t^2][(k - p)^2 - m_t^2]} \right\}_{\bar{q} = q} \tag{12a}
\]

\[
2(b) = \frac{g^2 m_t^4}{8c m_W^2} \frac{\partial}{\partial \bar{q}_\mu} \left\{ \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p')\bar{q}_a v(p)}{[k^2 - m_W^2][(k + p' - \bar{q})^2 - m_t^2][(k + p')^2 - m_t^2]} \right\}_{\bar{q} = q} \tag{12b}
\]

where the l.h.s. in the above equations indicates the appropriate diagram in Fig.2 and \( a_- = 1 - \gamma_5 \). It is easy to see that the differentiation with respect to \( \bar{q}_\mu \) of the denominators in Eqs.(12) gives a zero contribution after the limit \( \bar{q} \to q \) is taken. Differentiating the numerators and then putting \( p' = p = m_W = 0 \) one obtains

\[
(V_{\phi}^\mu + U_{\phi}^\mu)_{L.T.} = -i \frac{g}{2c} \frac{g^2}{16\pi^2} \frac{m_t^2}{2m_W^2} \bar{u}(p')\gamma^\mu a_- v(p).
\tag{13}
\]

Eq.(13) gives the contribution of the leading \( O(\alpha m_t^2 / m_W^2) \) vertex correction. In the on–shell scheme where \( s^2 = \sin^2 \theta_W \) is defined according to [3]

\[
\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2}
\]

and the partial width \( \Gamma(Z \to f + \bar{f}) \), where \( f \) is a massless fermion, can be written as [15]

\[
\Gamma(Z \to f + \bar{f}) = N_c \frac{G_F m_Z^3}{\sqrt{2}} |\rho_{ff}(m_Z^2)| \left| 1 - 4I_3^f Q_f s^2 \text{Re} k_f + 8Q_f^2 s^4 |k_f|^2 \right| \tag{14}
\]

we see that the term (13) introduces a modification in the electroweak form factors \( \rho_{bb} \) and \( k_b \) equal to \(-4x_t \) and \(+2x_t \) respectively, where \( x_t = (G_\mu m_t^2) / (\sqrt{2} 8\pi^2) \). In Eq.(14) \( N_c \) refers to the color factor, \( I_3^f \) and \( Q_f \) stand for the \( I_3 \) and charge quantum number of the fermion and \( \delta \rho_{irr} = 3x_t \). The counterpart of \( \rho_{bb} \) and \( k_b \) in the \( \overline{MS} \) scheme, namely the quantity \( \mathcal{P}_{bb} \) and \( \hat{k}_b \) [15], will be accordingly modified by the factors \(-\hat{\alpha} / (4\pi \hat{s}^2)(m_t^2 / m_W^2) \) and
\[ \hat{\alpha} = \frac{e^2(m_Z)}{(4\pi)} \] and \[ \hat{s}^2 = \sin^2 \hat{\theta}_W(m_Z) \] are the \( \overline{\text{MS}} \) coupling and weak angle evaluated at \( \mu = m_Z \).

3 Two-loop \( O(\alpha\alpha_s \frac{m_t^2}{m_W^2}) \) correction

We proceed to compute the QCD corrections to the leading \( O(\alpha\frac{m_t^2}{m_W^2}) \) term, i.e. the \( O(\alpha\alpha_s \frac{m_t^2}{m_W^2}) \) contribution. We begin by noticing that the wave function renormalization of the external \( Z \) has no contribution \( O(\alpha\alpha_s \frac{m_t^2}{m_W^2}) \), therefore all the QCD corrections are obtained by adding a gluon, in all possible ways, to the diagrams of Fig.1. The result is shown in Fig.3. The circles represent now the sum of diagrams of various kind. i) Diagrams where both the ends of the \( \Phi \) as well as the gluon propagator are attached to the external \( b, \bar{b} \) lines. These are reducible diagrams contributing to the wave function renormalization of the external fermions. ii) Diagrams where either the \( \Phi \) or the gluon propagator has both ends attached to an external line while the other propagator is acting internally. iii) Diagrams where each propagator has one end attached to an external fermion while the other end is attached internally.

Fig.3(a) can be expressed as a five-point correlation function or

\[ V_{\Phi g}^\mu = -i \frac{g}{c} \lim_{\bar{q} \rightarrow q} T_{\Phi g}^\mu(\bar{q}, p, p') \] (15a)

\[ T_{\Phi g}^\mu = -\frac{1}{2} \frac{g^2}{m_W^2} \frac{g_s^2}{g_s^2} \int \frac{d^n k_1}{(2\pi)^n} \frac{1}{k_1^2 - m_W^2} \int \frac{d^n k_2}{(2\pi)^n} \frac{1}{k_2^2 - m_W^2} \int d^n y e^{-i \bar{q} \cdot y} \int d^n x_1 e^{ik_1 \cdot x_1} \times \int d^n x_2 e^{ik_2 \cdot x_2} \int d^n x_3 e^{-ik_2 \cdot x_3} < p' p | T^* [J_{s \lambda}^\mu(y)J_s^\lambda(x_2)J_{s \lambda}(x_3) (S^\dagger(x_1) S(0) + h.c.)] | 0 >, \] (15b)

where \( g_s \) and \( J_s^\lambda \) are the \( SU(3)_c \) coupling and current respectively and we have suppressed the color indices. Analogously to the one-loop vertex function \( V_{\Phi}^\mu, V_{\Phi g}^\mu \) contains not only the one-particle irreducible (1PI) two-loop corrections to the proper vertex but also the contribution of the wave function renormalization of the external lines. Concerning the latter, as shown in
Appendix A, the limiting procedure in Eq. (15a) does correctly take into account the two-loop 1PI contribution of it but there is a mismatch in the numerical coefficient of the reducible part. To cure it we replace Eq. (15a) by

\[ \tilde{V}_{\mu \Phi}^\alpha = -i \frac{g}{c} \lim_{\bar{q} \to q} (T_{\Phi g}^\mu - \Pi^{(1)}_g T_{\Phi}^\mu) \]  

where

\[ \Pi^{(1)}_g = \frac{g_s^2}{16\pi^2} \Gamma(2 - \frac{n}{2}) (2 - n) \int_0^1 d\alpha (1 - \alpha) \left[ \lambda^2 (1 - \alpha) \right]^{-2+n/2} \]  

is the QCD one-loop wave function renormalization constant in \( n \) dimensions and \( \lambda \) is a fictitious gluon mass introduced to regularize the IR divergences.

The four-point correlation function depicted in Fig.3(b), \( U_{\mu \Phi}^\mu \), presents an analogous over-counting in the field renormalization of the external legs. Analogously to Eq. (16) the correct factor can be obtained by subtracting a term proportional to the one-loop two-point correlation function \( U_{\mu \Phi}^\mu \). We write the corrected amplitude as

\[ \tilde{U}_{\mu \Phi}^\mu = -i \frac{g}{c} (U_{\mu \Phi}^\mu - \Pi^{(1)}_g \tilde{U}_{\mu \Phi}^\mu) \]  

\[ \tilde{U}_{\mu \Phi}^\mu = \frac{i g^2}{2m_W^2 g_s^2 (c^2 - s^2)} \int \frac{dk_1}{(2\pi)^n} \frac{k_1^\mu}{m_W^2 - m_W^2} \frac{k_2^2}{(k_1 + q)^2 - m_W^2} \int \frac{dk_2}{(2\pi)^n} \frac{1}{m_W^2} \times \int d^\mu x_1 e^{ik_1 \cdot x_1} \int d^\mu x_2 e^{ik_2 \cdot x_2} \int d^\mu x_3 e^{-ik_2 \cdot x_3} < p' p \mid T^{\ast} \left[ J_{s \lambda}^\lambda(x_2) J_{s \lambda}^\lambda(x_3) S^\dagger(x_1) S(0) \right] \mid 0 >. \]  

The terms \( \Pi^{(1)}_g T_{\Phi}^\mu \) in Eq. (16) and \( \Pi^{(1)}_g \tilde{U}_{\mu \Phi}^\mu \) in Eq. (18a) can be understood as follows. In the one-loop vertex functions \( T_{\Phi}^\mu \) and \( \tilde{U}_{\mu \Phi}^\mu \) we can consider the bra \( < p' p \mid \) as dressed with respect to the strong interactions. Expansion up to \( O(\alpha_s) \) gives the above two contributions.

In order to reduce Eq. (15b) to a more tractable expression we apply the same procedure developed in Section 2, namely we contract \( T_{\Phi}^\alpha - \Pi^{(1)}_g T_{\Phi}^\alpha \) with \( \bar{q} \alpha \) and then we differentiate with respect to \( \bar{q} \mu \). The important point to notice is that \( J_{s \lambda}^\lambda \) commutes with \( J_{Z \lambda}^{\mu} \). Therefore,
going through the same steps as in Section 2, we reach the two-loop counterpart of Eq. (11) or

\[
\left( T_{\Phi g}^\mu \right)_{L.T.} = \frac{\partial}{\partial q_\mu} \mathcal{D}_{\Phi g} - \frac{ig^2}{2m_t^2} g_s^2 (c^2 - s^2) \int \frac{d^n k_1}{(2\pi)^n} \frac{k^\mu_1}{\mu^{n-4} (k_1^2 - m_t^2)^2} \int \frac{d^n k_2}{(2\pi)^n} \frac{1}{k_2^\mu} \times \int d^m x_1 e^{i k_1 \cdot x_1} \int d^m x_2 e^{i k_2 \cdot x_2} \int d^m x_3 e^{-i k_2 \cdot x_3} \langle p' p | T^* \left[ J^\lambda_s (x_2) J^\lambda_s (x_3) S^\dagger (x_1) S(0) \right] | 0 \rangle, \tag{19}
\]

where \( \mathcal{D}_{\Phi g} \) is again an expression obtained by \( T_{\Phi g}^\mu \) with the replacement \( J_\mu^g \rightarrow -i \partial_\mu J_\mu^g \). The second term in the r.h.s. of Eq. (19) cancels the leading contribution of Eq. (18b) and we are left with

\[
\left( \tilde{V}_{\Phi g}^\mu + \tilde{U}_{\Phi g}^\mu \right)_{L.T.} = -\frac{ig}{c} \left( \frac{\partial}{\partial q_\mu} \mathcal{D}_{\Phi g} - \Pi_\mu^{(1)} \frac{\partial}{\partial q_\mu} \mathcal{D}_\Phi \right) \bigg|_{\bar{q} = q} \tag{20}
\]

The diagrams contributing to \( \mathcal{D}_{\Phi g} \) are obtained by adding a virtual gluon, in all possible ways, to the graphs of Fig. 2. The types of diagrams are shown in Fig. 4. To every graph in the figure there corresponds two diagrams. In the first one the momentum \( k_1 \) is emitted by \( S^\dagger (x) \) while in the second it is emitted by \( S(x) \).

To get an ultraviolet (UV) finite answer we have to add to Eq. (20) the counterterm contributions, that we indicate as c.t.. The only counterterm at our disposal comes from the bare top mass. Taking as renormalized quantity the zero of the real part of the inverse propagator, the so-called “on-shell” (OS) mass, we have for the counterterm \( \delta m_t \)

\[
\delta m_t = \frac{g_s^2}{16\pi^2} m_t \Gamma (2 - \frac{n}{2}) \int_0^1 d\alpha \left[ 2 (1 - \alpha) + n \right] (m_t^2 \alpha^2)^{-2 + n/2}. \tag{21}
\]

In Fig. 5 the types of diagrams contributing to c.t. are depicted. The cross represents the insertion of \( \delta m_t \). Figs. 5(a, b, c) are due to the presence of \( m_t^0 \) in the operators \( S_2 \) and \( S \) (Cfr. Eqs. (2c, 2e)). Similarly to Fig. 4, every graph in Fig. 5 represents two contributions with different momentum insertions.
Evaluation of the contributions of Fig.4 and Fig.5 (Cfr. Appendix B) gives

$$\left( \bar{\Phi}^\mu \bar{\phi} g + c.t. \right)_{L.R.} = -i \frac{g}{2 c} \frac{g^2}{16 \pi^2} \frac{m_t^2}{2 m_W^2} \frac{g_5^2}{16 \pi} (c_1 + c_2 \xi(2)) C_F \bar{u}(p') \gamma^\mu a_- v(p)$$  \hspace{1cm} (22)

where $C_F$ is the eigenvalue of the quadratic casimir operator for the fundamental representation of $SU(N)$, namely $(N^2 - 1)/(2N)$, $(C_F = 4/3$ for $SU(3)_c$),

$$c_1 = -\frac{1}{2} + B_r \hspace{1cm} (23a)$$

$$c_2 = -6 \hspace{1cm} (23b)$$

and

$$\xi(2) = - \int_0^1 dx \frac{\ln(1 - x)}{x} = \frac{\pi^2}{6}. \hspace{1cm} (23c)$$

In Eq.\hspace{1cm} (23a) $B_r$ represents the contribution of the IR divergent diagrams 4(a), 4(b), 4(c) and of $\frac{\partial \Pi^{(1)}}{\partial q^\mu} D_{\Phi} \bigg|_{q^2} \bigg|$ once the UV pole has been subtracted. Explicitly

$$B_r = - \ln^2 \frac{\lambda^2}{q^2} - 3 \ln \frac{\lambda^2}{q^2} - 3 + \frac{\pi^2}{3}. \hspace{1cm} (24)$$

Comparing Eq.\hspace{1cm} (22) with Eq.\hspace{1cm} (13) we see that the QCD corrections modify the leading $O(\frac{\alpha m_t^2}{m_W^2})$ term in the $Zbb$-vertex by

$$\frac{\alpha_s}{4\pi} (c_1 + c_2 \xi(2)) C_F.$$  

The inclusion of the $O(\alpha\alpha_s \frac{m_t^2}{m_W^2})$ corrections in the decay width $\Gamma(Z \rightarrow b + \bar{b})$ can be performed in the following way. We write the $Zbb$-vertex, $V^\mu$, as

$$V^\mu = -i \frac{g}{c} \gamma^\mu a + b \gamma_5 \frac{2}{2} \hspace{1cm} (25)$$
where

\[ a = a_0 + a_1 e + a_1 s + a_2 \]  \hspace{1cm} (26a)

\[ b = b_0 + b_1 e + b_1 s + b_2 \]  \hspace{1cm} (26b)

with

\[ a_0 = -\frac{1}{2} + \frac{2}{3}s^2; \quad b_0 = \frac{1}{2} \]  \hspace{1cm} (26c)

\[ a_1 e = \frac{g^2}{16\pi^2} \frac{m^2_r}{4m^2_W}; \quad b_1 e = -a_1 e \]  \hspace{1cm} (26d)

\[ a_1 s = \frac{\alpha_s}{4\pi} a_0 c_1 C_F; \quad b_1 s = \frac{\alpha_s}{4\pi} b_0 c_1 C_F \]  \hspace{1cm} (26e)

\[ a_2 = \frac{\alpha_s}{4\pi} a_1 e (c_1 + c_2 \xi(2)) C_F; \quad b_2 = -a_2. \]  \hspace{1cm} (26f)

The decay width can be written as

\[ \Gamma (Z \to b + \bar{b}) = \frac{g^2}{48\pi} m_Z (a^2 + b^2). \]  \hspace{1cm} (27)

Expanding the squares in Eq.(27), keeping terms up to the order we are considering, one obtains

\[ \Gamma (Z \to b + \bar{b}) = \frac{g^2}{48\pi} m_Z \left[ (a_0^2 + b_0^2)(1 + \frac{\alpha_s}{2\pi} c_1 C_F) + 2(a_0 - b_0)a_1 e \left( 1 + \frac{\alpha_s}{4\pi} (2c_1 + c_2 \xi(2)) C_F \right) \right]. \]  \hspace{1cm} (28)

It is clearly understood that in Eq.(28) the factor \( B_r \) present in \( c_1 \) is evaluated at \( q^2 = m^2_Z \).

A few remarks about the above result are now in order. i) All the calculation has been performed analytically. The integration over the Feynman parameters has been checked, always analytically, with the algebraic manipulation program MAPLE [16]. ii) In every diagram in Fig.4 and Fig.5 but 4(a) we have neglected all the momenta and masses compared to the top mass. In 4(a) we were forced to keep in addition the momentum transfer \( q \) due
to the presence of IR divergent terms. iii) We found it advantageous to regularize the IR divergencies by giving a small mass to the gluon instead of using dimensional regularization. In this way we did not introduce any $1/\epsilon^2$ term ($\epsilon = n - 4$) in the calculation keeping the n-dimensional algebra simpler.

Eq.(28) is IR divergent in the $c_1$ coefficient (Cfr. Eqs.(23a) and (24)). As is well known, to eliminate the IR divergencies it is necessary to take into account the QCD bremsstrahlung. In particular, to be consistent with the order of our calculation, we need to consider the bremsstrahlung with the inclusion of the leading $O(\alpha_s m_W^2)$ term. The discussion of this correction can be performed on the same footing as the $O(\alpha_s m_W^2)$ term. In fact, the cancellation between Eq.(18b) and the second term in Eq.(19) will work even in the presence of a single $J^\lambda_s$ operator. The result is that the $O(\alpha_s m_W^2)$ term in the QCD bremsstrahlung is obtained by adding a real gluon in all possible ways to the diagrams in Fig.2. We note that a gluon emitted by a virtual fermion line gives rise to a correction that does not contribute to the leading term. It is then easy to show that the $O(\alpha_s m_W^2)$ correction can be absorbed in a redefinition of the vector and axial coupling of the $Z$. Therefore we can write for the rate $\Gamma(Z \rightarrow b + \bar{b} + g)$ [17]

$$\Gamma(Z \rightarrow b + \bar{b} + g) = \frac{g^2}{48\pi} m_Z (\tilde{a}^2 + \tilde{b}^2) \frac{\alpha_s}{2\pi} \left[ -c_1 + \frac{3}{2} \right] C_F \tag{29}$$

with

$$\tilde{a} = a_0 + a_1 e \tag{30a}$$

$$\tilde{b} = b_0 + b_1 e. \tag{30b}$$

Expanding the squares in Eq.(29), retaining terms up to the order we are interested, we have

$$\Gamma(Z \rightarrow b + \bar{b} + g) = \frac{g^2}{48\pi} m_Z \left( a_0^2 + b_0^2 + 2(a_0 - b_0)a_1 e \right) \frac{\alpha_s}{2\pi} \left[ -c_1 + \frac{3}{2} \right] C_F. \tag{31}$$
Finally, summing Eq. (28) and Eq. (31) one obtains

\[ \Gamma (Z \rightarrow b + \bar{b}) + \Gamma (Z \rightarrow b + \bar{b} + g) = \frac{g^2}{48\pi} m_Z \left( (a_0^2 + b_0^2) \left( 1 + \frac{3\alpha_s}{4\pi} C_F \right) + 2(a_0 - b_0)a_{1e} \left( 1 + \frac{\alpha_s}{4\pi} (3 + c_2 \xi(2)) C_F \right) \right). \]  (32)

Eq. (32) is the final result. The \( O(\frac{m_t^2}{m_W^2}) \) correction, represented by the term \( a_{1e} \) in Eq. (32), gets modified into \( (C_F = 4/3) \)

\[ a_{1e} \rightarrow a_{1e} \left( 1 - \frac{\alpha_s}{\pi} \frac{\pi^2 - 3}{3} \right) \].  (33)

Remembering that \( a_{1e} \) is written as \( (G_\mu m_t^2)/(\sqrt{2} 8\pi^2) \) in the OS and \( \alpha_s/(4\pi s^2) m_t^2/(4 m_W^2) \) in the \( \overline{MS} \) formulation, we see that one can take into account the leading QCD effect in \( \rho_{bb} \) and \( k_b \), or correspondingly \( \overline{\rho}_{bb} \) and \( \hat{k}_b \), just by replacing in the one-loop \( O(\frac{m_t^2}{m_W^2}) \) term \( m_t^2 \) by

\[ m_t^2 \rightarrow m_t^2 \left( 1 - \frac{\alpha_s}{\pi} \frac{\pi^2 - 3}{3} \right) \].  (34)

Eq. (34) coincides with the result found by Fleischer et al [12].

4 Conclusions

In the previous Sections we have shown that the use of current correlation functions and their associated current algebra provides a very powerful and compact framework to discuss the \( O(\frac{m_t^2}{m_W^2}) \) corrections in the \( Zb\bar{b} \)-vertex.

It is well known that the \( O(\frac{m_t^2}{m_W^2}) \) term should be finite by itself at the one-loop level. In fact there is no counterterm available to cancel any divergent contribution proportional to \( m_t^2 \). The formalism of current correlation functions allows one to combine several Feynman diagrams and easily enforce the Ward identity that guarantee the finiteness of this term. The actual calculation of the leading \( O(\frac{m_t^2}{m_W^2}) \) correction becomes then trivial.
Furthermore the current algebra framework is very suitable to discuss the QCD corrections to the one-loop $O(\alpha \frac{m_t^2}{m_W^2})$ term. In fact, because the strong and weak currents commute, the one-loop Ward identity in the electroweak sector is preserved. This is crucial for the structure of the divergent terms. Indeed no poles more divergent than $1/\epsilon$ are found (Cfr. Appendix B) because all the divergent terms are due to the QCD substructure. In comparison the standard two-loop calculation presents $O(1/\epsilon^2)$, or even $O(1/\epsilon^3)$ contributions if the IR divergencies are regularized using dimensional regularization [12]. From a practical point of view this fact greatly helps in performing the calculation. Especially the structure of the IR divergent terms is very simplified. We find only two different IR contributions (Fig.4(a) and 4(b)) and their evaluation in the IR part is very similar to the computation of the one-loop QCD corrections to the $Z\bar{b}b$-vertex.

Concerning the physical significance of the $O(\alpha \alpha_s \frac{m_t^2}{m_W^2})$ calculation, we find that the QCD corrections to $O(\frac{m_t^2}{m_W^2})$ term in $\Gamma(Z \rightarrow b + \bar{b})$ have opposite sign to that of the one-loop contribution. A similar situation happens in the electroweak parameters $\delta \rho$ and $\Delta r$. A detailed analysis [6] shows that the inclusion of the perturbative QCD higher order effects in the vacuum polarization functions increases the prediction for $m_t$ derived from current measurements. The values obtained for $m_t$ including the $O(\alpha \alpha_s)$ corrections are larger than those obtained using only the electroweak calculation by an amount between 5 and 10 GeV for $90 \leq m_t \leq 200$ GeV. Using $\alpha_s = 0.118$ [18], Eq.(34) gives as correction factor for $m_t 4.4\%$, that is consistent with the result of Ref. [9]. In fact, a comparison between the correction found for $\delta \rho$, i.e. $-\frac{2\pi^2+6}{9}\frac{\alpha_s}{\pi} \sim -2.860\frac{\alpha_s}{\pi}$ [4], and the value we obtained, $-\frac{\pi^2-3}{3}\frac{\alpha_s}{\pi} \sim -2.290\frac{\alpha_s}{\pi}$, shows that the two results are numerically comparable. Both corrections are quite large, however, it should be pointed out that they are scheme dependent. In our calculation, as well as in Ref. [4], the OS mass definition for the renormalized top mass has been used. If we employ instead a $\overline{MS}$ definition for the top mass, $\hat{m}_t(\mu = \hat{m}_t)$, we have that the counterterm $\delta m_t$ has no finite part, and therefore in the evaluation of the diagrams in Fig.5 only the terms proportional to $\delta$ should be retained (Cfr. Appendix B). This changes the coefficient.
multiplying $\frac{\alpha_s}{\pi}$ from $-\frac{\pi^2-3}{3}$ to $-\frac{\pi^2-11}{3}$, i.e. from $\sim -2.290$ to $\sim +0.377$, a much smaller number. Similarly the use of $\tilde{m}_t$ as renormalized mass in $\delta \rho$ changes the numerical coefficient in front of $\frac{\alpha_s}{\pi}$ from $-\frac{2\pi^2+6}{9}$ to $-\frac{2\pi^2-18}{9}$, i.e. from $\sim -2.860$ to $\sim +0.193$, again a much smaller number.

Finally we want to stress that our calculation of $O(\alpha\alpha_s\frac{m^2_t}{m^2_W})$ terms and the one presented in Ref. [12] are completely independent. Because in Ref. [12] the IR divergencies were regularized using dimensional regularization it is not possible to make any intermediate check, not even at the level of $\Gamma(Z \to b + \bar{b})$. It is a welcome fact that our result coincides with the one obtained in Ref. [12].

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Appendix A

We wish to show that Eqs.(16) and (18a) contain correctly the field renormalization of the external quarks. The wave function renormalization constant $Z_b$ for a massless $b$ quark is equal to

$$Z_b = \frac{1}{1 - \Pi_b} \quad (A1)$$

where $\Pi_b = \Sigma_b(\not{p})/\not{p}|_{\not{p}=0}$ with $\Sigma_b$ equal to $-i$ times the self-energy of the $b$. Expanding $Z_b$ up to $O(\alpha\alpha_s)$ considering only the contributions due to the $\Phi$ and the gluon we have

$$Z_b = 1 + \Pi_b + \Pi_b^2 + \cdots \simeq 1 + \Pi^{(1)}_\Phi + \Pi^{(1)}_g + \Pi^{(2)} + 2 \Pi^{(1)}_\Phi \Pi^{(1)}_g + \cdots \quad (A2)$$
where $\Pi_\Phi^{(1)}$ is the one-loop contribution due to the $\Phi$, $\Pi_g^{(1)}$ the corresponding one for the gluon and $\Pi^{(2)}$ the two-loop 1PI mixed $\Phi$–gluon term. Eq. (A2) tells us that any 1PI self-energy diagram inserted in an external leg should be multiplied by a factor $1/2$ while the total counting of the product of one-loop objects should be equal to 2.

We consider first the vertex function $V_{\Phi g}^{\mu}$. Among the various contributions of the $T^\ast$–product in Eq. (15b) there are diagrams in which neither the strong currents nor the operators $S$ and $S^\dagger$ enclose the vertex of the $J_Z^{\mu}$ current. We begin by discussing these diagrams for the case of the $\bar{b}$ leg. They are depicted in Fig. 6(a–g). The wiggly line in the figure stands for the $\bar{q}$ momentum absorbed by the $J_Z^\mu$ operator. Again, as in Fig. 4 and Fig. 5, every graph represents two contributions with different momentum insertions.

Indicating with $\Pi^{(2)}_{\alpha(a)}$ ($\alpha = a, b, c, d$) the self-energy graph, divided by the relevant momentum, inserted in the external leg of the diagram obtained by Fig. 6(a) attaching together the dashed lines, we can write the sum of the two diagrams represented by 6(a) as

$$6(a) = \lim_{\bar{q} \to q} -i g c \bar{u}(p') \gamma^\mu \frac{a_0 + b_0 \gamma_5}{2} \frac{1}{p' - q} \frac{1}{2} \left[ \Pi^{(2)}_{\alpha(a)} p + \Pi^{(2)}_{\alpha(a)} (p' - \bar{q}) \right] v(p)$$

(A3)

Remembering that $\Pi^{(2)}$ has the form $\Pi^{(2)} = z a_+$ and observing that the first term in the square bracket in Eq. (A3) is zero we have

$$6(a) = -i g c \frac{1}{2} z_{6(a)} \bar{u}(p') \gamma^\mu \frac{a_0 + b_0 \gamma_5}{2} a_- v(p).$$

(A4)

The factor $1/2 z_{6(a)}$ is the correct contribution of the self-energy diagram associated with Fig. 6(a) to the field renormalization of the $\bar{b}$.

Let’s consider now the graph 6(b). Each of the two diagrams of this kind, called 6(b1) and 6(b2), can be written as

$$6(b_i) = \lim_{\bar{q} \to q} i g c \bar{u}(p') \gamma^\mu \frac{a_0 + b_0 \gamma_5}{2} \frac{1}{p' - q} \frac{1}{2} \left[ \prod_{i(b_i)} (p, p', \bar{q}) p + \prod_{i(b_i)} (p, p', \bar{q}) (p' - \bar{q}) \right] v(p)$$

(A5)
where \( i = 1, 2 \) and the \( \Pi_{6(b_i)} \)'s are related to \( \Pi^{(2)}_{6(b)} \) through

\[
\Pi^{(2)}_{6(b)} = \lim_{\bar{q} \rightarrow q} \left( \Pi_{6(b_1)}(p, p', \bar{q}) + \Pi_{6(b_2)}(p, p', \bar{q}) \right). \tag{A6}
\]

It is clear that only the \( \Pi_{6(b_1)}(p, p', \bar{q}) \) term contributes in Eq. (A5). However, an explicit calculation shows that there is a relation between the \( \Pi_{6(b_i)} \)'s, i.e.

\[
\Pi_{6(b_1)} = \Pi_{6(b_2)}; \quad \Pi_{6(b_2)} = \Pi_{6(b_1)}'. \tag{A7}
\]

Therefore when we sum \( 6(b_1) \) and \( 6(b_2) \) and we use the relations (A7) and Eq. (A6) we get

\[
6(b) = -ig \frac{1}{2} z_{6(b)} \bar{u}(p') \gamma^\mu \frac{a_0 + b_0 \gamma_5}{2} a_0 v(p) \quad \tag{A8}
\]

that gives again the correct contribution to the field renormalization.

The sum of the diagrams \( 6(c) \) and \( 6(d) \) can be discussed in a similar fashion as \( 6(b) \) obtaining

\[
6(c) + 6(d) = -ig \frac{1}{c} \left( \frac{1}{2} z_{6(c)} + \frac{1}{2} z_{6(d)} \right) \bar{u}(p') \gamma^\mu \frac{a_0 + b_0 \gamma_5}{2} a_0 v(p). \tag{A9}
\]

Therefore we conclude that \( \mathcal{V}_{\Phi g}^\mu \) takes correctly into account the two-loop 1PI contribution to the wave function renormalization.

Beside the 1PI contribution Eq. (A2) contains the product of one-loop terms. Diagrams \( 6(e\text{--}g) \) represent the corresponding contributions in \( \mathcal{V}_{\Phi g}^\mu \) for the \( \bar{b} \) leg. Due to the limiting procedure we have that each diagram carries a factor \( 1/2 \). Therefore the sum \( (6e\text{--}g) \) plus the corresponding contribution of the \( b \) leg gives 3 times \( \Pi^{(1)}_{\Phi} \Pi^{(1)}_{g} \). There are other reducible contributions to \( \mathcal{V}_{\Phi g}^\mu \) that do not give the correct renormalization factor. In particular diagrams \( 6(h) \) is counted 1 in \( \mathcal{V}_{\Phi g}^\mu \) because the limiting procedure does not affect it. We know, however, that it should be counted 1/2 because of the wave function renormalization of the external \( \bar{b} \) leg. The same happens for the symmetric diagram where the gluon is on the
external b line. Therefore we have that in $V_{\Phi g}^\mu$ the product of one-loop $\Phi$ diagrams times $\Pi_g^{(1)}$ is counted one time more than the correct contribution. The second term in Eq. (16) exactly cures this problem. The vertex function $U_{\Phi g}^\mu$ can be discussed similarly to diagram 6(h). The subtraction indicated in Eq.(18a) gives again the correct counting.

Appendix B

In this Appendix we list the contributions of Fig.4 to $D_{\Phi g}$ and of Fig.5 to c.t.. Defining

$$\delta = \frac{2}{\epsilon} - 2\gamma + 2 \ln 4\pi - 2 \ln \frac{m_\tau^2}{\mu^2}$$

and omitting the common factor

$$\frac{g^2 m_\tau^2}{4} \frac{g_s^2}{m_w^2 (16\pi^2)^2} C_F \bar{u}(p') \gamma q / a - v(p)$$

we have

4(a) = $\frac{-5}{2} + \frac{\pi^2}{3} - \ln \frac{q^2}{m_\tau^2} - \ln^2 \frac{\lambda^2}{q^2} - 4 \ln \frac{\lambda^2}{q^2}$

4(b) = 4(c) = $-\delta - \frac{1}{2} + \ln \frac{\lambda^2}{m_\tau^2}$

4(d) = 4 $\delta + 13 - 6 \xi(2)$

4(e) = 4(f) = $-\frac{5}{2} \delta - \frac{43}{4} + 3 \xi(2)$

4(g) = 4(h) = $4 \delta + \frac{9}{2} - \frac{3}{2} \xi(2)$

4(i) = 4(l) = $+\frac{7}{2} - \frac{3}{2} \xi(2)$

5(a) = 5(b) = 5(c) = $-3\delta - 7$

5(d) = 5(e) = $\frac{3}{2} \delta + \frac{13}{2}$
The left-hand-sides in the above equations indicate the appropriate diagrams in Fig.4 and Fig.5. In the same normalization the contribution of $-\Pi^{(1)}_g \frac{\partial}{\partial q_\mu} \! D_\delta \bigg|_{q=q}$ is equal to $-4(\beta)$.

References

[1] M. Veltman, Nucl. Phys. B123 (1977) 89.

[2] A. Sirlin, Phys. Rev. D22 (1980) 971.

[3] W.J. Marciano and A. Sirlin, Phys. Rev. D22 (1980) 2695.

[4] A. Djouadi and C. Verzegnassi, Phys. Lett. B195 (1987) 265; A. Djouadi, Nuovo Cimento 100A (1988) 357.

[5] B.A. Kniehl, Nucl. Phys. B347 (1990) 86.

[6] S. Fanchiotti, B.A. Kniehl and A. Sirlin, “Incorporation of QCD Effects in Basic Corrections of the Electroweak Theory”, preprint CERN-TH-6749/92 and NYU-TH-92/12/05.

[7] J.J. van der Bij and F. Hoogeveen, Nucl. Phys. B283 (1987) 477.

[8] R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci and A. Viceré, Phys. Lett. B288 (1992) 95.

[9] A. Akhundov, D. Bardin and T. Riemann, Nucl. Phys. B276 (1986) 1.

[10] W. Beenakker and W. Hollik, Z. Phys. C40 (1988) 141.

[11] J. Bernabeu, A. Pich and A. Santamaria, Phys. Lett. B200 (1988) 569.

[12] J. Fleischer, O.V. Tarasov, F. Jegerlehner and P. Rączka, Phys. Lett. B293 (1992) 437.

[13] A. Sirlin, Rev. Mod. Phys. 50, (1978) 573.

[14] G. Preparata and W.I. Weisberger, Phys. Rev. 175 (1968) 1965; L.S. Brown, Phys. Rev. 187 (1969) 2260.
Figure Caption

Fig.1 One-loop vertex diagrams contributing to the $O(\alpha \frac{m^2}{m_W^2})$ corrections to the $Z \rightarrow b + \bar{b}$ process. The figures schematically represent the sum of Feynman diagrams in which the propagators are attached in all possible ways to the external lines.

Fig.2 Momentum insertions associated with $D_\Phi$. The dotted line indicates a $\bar{q}$ momentum absorbed by the $S_2$ operator. The dashed lines stand for the momenta absorbed or emitted by the $S^\dagger$ and $S$ operators.

Fig.3 Two-loop vertex diagrams contributing to the $O(\alpha \frac{m^2}{m_W^2})$ corrections to the $Z \rightarrow b + \bar{b}$ process. The meaning of the circles is explained in the text at the beginning of Section 3.

Fig.4 Types of graphs contributing to $D_{\Phi g}$. Every graph represents two diagrams with different momentum insertions (see text). The meaning of the lines is as in Fig.2.

Fig.5 Types of diagrams belonging to the counterterm contributions ($c.t.$). Every graph has the same meaning as in Fig.4.

Fig.6 Subset of graphs contributing to $V_{\Phi g}^{\mu}$. The meaning of the graphs is the same as in Fig.4 and Fig.5. The wiggly line stands for the $\bar{q}$ momentum absorbed by the $J_{Z}^{\mu}$ operator.