I. INTRODUCTION

The helicon antenna, consisting of two rings separated and joined by two curving limbs, is a well-known device in its application as an ion source for electric propulsion \cite{1,2}. Driven by a radio-frequency amplifier, its energy couples to the plasma medium to drive the dissociation of propellant material, such as argon gas. Here we investigate the vector potential produced by such a device in vacuum, with an eye towards its adaptation as a source of RF energy for injection into a burning plasma.

Our methodology is based on the numerical evaluation of solutions to the inhomogeneous field equations

$$\nabla A^\mu = -\mu_0 J^\mu$$

in Lorenz gauge, where \(A^\mu = (a/c_0, A)\) is the four-vector for the potential, \(J^\mu = (c_0 J, J)\) is the four-vector for the current source, and \(c_0^2 = \mu_0 \varepsilon_0\) gives the speed of light in vacuum. The d'Alembertian operator is \(\Box^2 \equiv \nabla^\mu \nabla_\mu = -c_0^{-2} \partial_t^2 + \nabla^2\) in a frame with metric signature (−, +, +, +). We subscribe to the reductionist viewpoint \cite{5,6} that the potential is a more fundamental description than the field formulation, and that the Lorenz gauge makes explicit the relation between the continuity of the potential and the continuity of the source \(\partial_t J^\mu = 0\).

The solution \(A^\mu\) for a given \(J^\mu\) is found by directly imaging the current source throughout the region, by which we mean that the contribution from each source element \(J^\mu(t', r')\) is calculated at a point \((t, r)\) with regard to the propagation delay, or retarded time \(t' = t - r/c_0\), as well as the spatial distance to the element \(\Delta r = r - r'\). The antenna is assumed to be driven by a feed current arranged so that its contribution is negligible, leaving to calculate only the potential produced by the antenna. We neglect any charge accumulation along the conductor, so that the static charge density vanishes, thus \(j = 0\) and \(\nabla \cdot J = 0\). The solution to the wave equation for a line source is then written

$$A_C(t, r) = \frac{\mu_0}{4\pi} \int_{r'} \frac{I_C(t', r')}{\Delta r} dl,$$

where the subscript \(C\) reminds us that these vectors must be expressed in Cartesian components \(A = (A_X, A_Y, A_Z)\), neglecting any self-interaction between different parts of the antenna.

The antenna is modeled as a collection of discrete line current elements \(I_k \equiv I(r_k')\), so that the integral becomes a sum \(\int_{r'} dl \rightarrow \sum_k \Delta k\), where \(\Delta k\) is the length of the \(k\)th element. The antenna is assigned typical size parameters of half-width \(w = 0.1 \text{ m}\) and half-height \(h = 0.2 \text{ m}\) (or 1 and 2 dm respectively), and the feed current is normalized to \(\mu_0 I_0/4\pi = 1\). We assume that the driving current is provided along a twisted pair cable so that the opposing phases produce canceling contributions to the overall potential. The Cartesian \((X, Y, Z)\) and cylindrical \((R, \phi, Z)\) coordinates are related by \(X = R\cos \phi\) and \(Y = R\sin \phi\). The static analysis calculates the potential in the \(XY\) plane at various \(Z\) along the cylindrical axis. The dynamic analysis calculates the potential in the \(ZR\) plane at various angles \(\phi\) around the cylindrical axis. The propagation analysis computes \(A\) in the \(ZR\) plane at \(\phi = \pi/4\) and various times \(t\). Our methodology is a straightforward implementation of classical electrodynamics in the potential formulation, such as found in Ref. \cite{3}.

II. STATIC ANALYSIS

For the static, time independent analysis, the Lorenz gauge is equivalent to the Coulomb gauge, \(\nabla \cdot A = 0\). Position \(r'(\lambda, \phi')\) along the helicon antenna may be parametrized by the azimuth \(\phi'\) and the line distance along the conductor \(\lambda\) to the location where the driving current is attached, here taken to be at \((X', Y', Z') = (w, 0, -h)\). The ring at \(Z' = h\) will be called the “head”, and the one at \(Z' = -h\) will be the “tail”. The vertical limbs we call “arms”, and the geometry of the antenna is depicted in Fig. 1 where the “legs” carrying the driving current are not shown. It takes two revolutions around the circle to cover the entire antenna, thus the positive half is identified with \(\phi'_+ \in [0, 2\pi]\) and the negative with \(\phi'_- \in [0, -2\pi]\). The azimuth is parametrized over \(N_\phi\) values, so that \(\Delta \phi = 2\pi/N_\phi\). The line elements for the rings have a length \(\Delta k = w\Delta \phi\), and those for the arms have a
The helicon antenna is modeled as a collection of discrete line current elements as described in the text. The geometry is shown in units of 1 dm = 0.1 m throughout.

By Kirchhoff’s point rule, the magnitude of the current along either branch of each ring is \( I_r = I_0 \), and along the arms it is \( I_a = I_0 \). The arms must cover a height of 2\( h \) over an azimuthal range of \( \pi \), thus the ratios \( I_2/I_\phi = 2h/\pi w \) and \( (I_a/I_\phi)^2 = 1 + (I_2/I_\phi)^2 \) determine \( I_X = -I_\phi \sin \phi' \) and \( I_Y = I_\phi \cos \phi' \) for \( I_\phi = I_0[1 + (2h/\pi w)^2]^{-1/2} \) so that \( I_\phi = I_0(-\sin \phi', \cos \phi', 2h/\pi w) \), and similarly for \( I_i \) with sign changes as appropriate for the two branches of each ring. The contribution from each element \( \mathbf{I}(r'_k) \) to the solution \( \mathbf{A}(r) \) is calculated by multiplying the current \( \mathbf{I}_k \) by \( \Delta_k/\Delta_r \) for each component in \((X,Y,Z)\).

On the midplane of the antenna \( Z = 0 \) dm, the vector potential \( \mathbf{A} \) has its greatest magnitude at the locations of the arm elements \((0, \pm w, 0)\), as seen in Fig. 2 and is dominated by the \( X \) and \( Z \) components as expected; the \( Y \) component is about a tenth of the magnitude of the others. At three quarters of the antenna height, \( Z = 1 \) dm shown in Fig. 3 the locations of the arms are clearly discerned in the vector potential. In the plane of the head ring, \( Z = 2 \) dm shown in Fig. 4 the vector potential is mostly in the azimuthal direction, with a strong vertical component only at the junctions of the head ring with the arms. Away from the antenna, at \( Z = 3 \) dm shown in Fig. 5 a significant vector potential still exists, with both vertical and azimuthal components.
FIG. 2. Static vector potential at $Z = 0$.

FIG. 3. Static vector potential at $Z = 1$.

FIG. 4. Static vector potential at $Z = 2$.

FIG. 5. Static vector potential at $Z = 3$. 
For a driving frequency of $\nu = 13.54$ MHz, the phase difference at the maximum distance $\lambda_{\text{max}}$ from the location of the current feed legs is 3.7%. The colorbar indicates the phase in units of degrees, and the length of the arrow represents the magnitude of the real current at $t/\tau = 0$.

B. High frequency

Now let us suppose that the antenna is driven at a higher frequency, say $\nu = 541.6$ MHz, whose vacuum wavelength is considerably shorter $\lambda_0 \sim 0.55$ m so that $\lambda_{\text{max}}/\lambda_0 = 149\%$. The phase structure of the current along the antenna is displayed in Fig. 11, where we see that the magnitude of the real current is zero where $\theta_k + \theta_t = \pm \pi/2$. Repeating the azimuthal scan at the same angles as above for $t/\tau = 0$, the results are displayed in Fig. 12 through Fig. 15. The vector potential has a complicated structure, the most interesting feature of which is the strong magnitude of $A$ which develops at azimuths $\phi = \pi/4$ and $\phi = 3\pi/4$.

The complicated phase structure along the antenna indicates that we are in a regime in which interference effects can lead to the propagation of an electromagnetic pulse. Restricting attention to the ZR plane at azimuth $\phi = \pi/4$, let us now investigate how the pulse evolves through time. Considering the set of times given by $t/\tau \in [1,8]/8$, we display the results in Figs. 16 through 19 respectively. Times separated by one half period have the same magnitude for $A$ and vector components of opposite sign. At the end of one cycle, the potential is back
FIG. 9. Vector potential for $\nu = 13.54$ MHz at $\phi = \pi/2$ and $t/\tau = 0$.

$\phi/\pi = 2/4, t/\tau = 0/8$

FIG. 10. Vector potential for $\nu = 13.54$ MHz at $\phi = 3\pi/4$ and $t/\tau = 0$.

$\phi/\pi = 3/4, t/\tau = 0/8$

FIG. 11. For a driving frequency of $\nu = 541.6$ MHz, the phase difference at the maximum distance $\lambda_{\text{max}}$ from the location of the current feed legs is 149%. The colorbar indicates the phase in units of degrees, and the length of the arrow represents the magnitude of the real current at $t/\tau = 0$.

IV. DISCUSSION

The original motivation for this study was simply to develop an interesting exercise in classical electrodynamic field theory, with possible utility to investigators in electric propulsion. However, by playing around with the driving frequency, we have found evidence that a significant pulse may be generated which propagates away from the antenna. We expect interesting results when the ratio $\lambda_{\text{max}}/\lambda_0$ is at half-integer multiples, so that constructive interference from the head and tail rings leads to a prop-
FIG. 12. Vector potential for $\nu = 541.6$ MHz at $\phi = 0$ and $t/\tau = 0$.

FIG. 13. Vector potential for $\nu = 541.6$ MHz at $\phi = \pi/4$ and $t/\tau = 0$.

FIG. 14. Vector potential for $\nu = 541.6$ MHz at $\phi = \pi/2$ and $t/\tau = 0$.

FIG. 15. Vector potential for $\nu = 541.6$ MHz at $\phi = 3\pi/4$ and $t/\tau = 0$. 
FIG. 16. Vector potential for $\nu = 541.6$ MHz at $\phi = \pi/4$ and $t/\tau = 1/8$.

FIG. 17. Vector potential for $\nu = 541.6$ MHz at $\phi = \pi/4$ and $t/\tau = 1/4$.

FIG. 18. Vector potential for $\nu = 541.6$ MHz at $\phi = \pi/4$ and $t/\tau = 3/8$.

FIG. 19. Vector potential for $\nu = 541.6$ MHz at $\phi = \pi/4$ and $t/\tau = 1/2$. 

$\phi/\pi = 1/4$, $t/\tau = 1/8$  
$\phi/\pi = 1/4$, $t/\tau = 2/8$  
$\phi/\pi = 1/4$, $t/\tau = 3/8$  
$\phi/\pi = 1/4$, $t/\tau = 4/8$
FIG. 20. Vector potential for $\nu = 541.6$ MHz at $\phi = \pi/4$ and $t/\tau = 5/8$.

FIG. 21. Vector potential for $\nu = 541.6$ MHz at $\phi = \pi/4$ and $t/\tau = 3/4$.

FIG. 22. Vector potential for $\nu = 541.6$ MHz at $\phi = \pi/4$ and $t/\tau = 7/8$.

FIG. 23. Vector potential for $\nu = 541.6$ MHz at $\phi = \pi/4$ and $t/\tau = 8/8$. Online this figure is linked to a short video showing the temporal evolution of the vector potential.
agating pulse. Noticing the similarity between the higher frequency here and the range spanned by existing RF injection plasma heating technologies based on resonance with the ion cyclotron and the lower hybrid frequencies, together spanning a range of about 0.05 to 10 GHz \[3\], we wonder if the helicon antenna might be adapted for use in fusion engineering. For that purpose, the inclusion of a ground plane to achieve unidirectional propagation might be necessary to allow for mounting on the outside of the vacuum vessel, or perhaps the antenna unaltered can be placed towards the interior of the central solenoid in a tokamak so that both propagation pulses can be utilized.

The model here has made several simplifying approximations which need to be more fully developed. As the material of the conductor is unspecified, we have used the vacuum values of \(\varepsilon_0\) and \(\mu_0\) in the determination of the phase along the antenna \(\theta_k\). For greater veracity, the values of \(\varepsilon\) and \(\mu\) appropriate for the chosen material should be used instead. A volume or surface current density could be used for the antenna rather than the line current used here, with straightforward modifications to the evaluation of the integrals. More interesting but more difficult is the incorporation of self interactions between different parts of the antenna—the current \(I_k\) cannot be immune to the effects of the current at other locations \(I_k'\), which are beyond the scope of this article. Finally, one is interested in how the propagating electromagnetic pulse would interact with a medium of ionized gas located in the region beyond the antenna rings. By properly tuning the antenna dimensions and driving frequency, one hopes to drive resonances inside the plasma medium.

V. CONCLUSION

In this article we have presented a numerical evaluation of the vector potential produced by the helicon antenna in vacuum for both the static case and the dynamic case of nonzero driving frequency. We have considered both a typical driving frequency and one somewhat greater, where the higher frequency produces a significant electromagnetic pulse which propagates along the cylindrical axis. The frequency of the propagating pulse for typical size parameters lies within the range spanned by ion cyclotron and lower hybrid resonances, suggesting that an adaptation of the helicon antenna design might be useful for RF injection plasma heating. Extensions of the model for a more realistic description of an experimental apparatus are outlined.

[1] F. F. Chen, Plasma Phys. Control. Fusion 33, 339 (1991).
[2] M. Yano and M. L. R. Walker, Physics of Plasmas 13, 063501 (2006).
[3] D. D. Palmer and M. L. R. Walker, Journal of Propulsion and Power 25, 1013 (2009).
[4] G. Chen, A. V. Arefiev, R. D. Bengtson, B. N. Breizman, C. A. Lee, and L. L. Raja, Physics of Plasmas 13, 123507 (2006).
[5] A. V. Arefiev and B. N. Breizman, Physics of Plasmas 13, 062107 (2006).
[6] G. Rousseaux, Annales de la Fondation Louis de Broglie 30, 387 (2005).
[7] G. Rousseaux, Annales de la Fondation Louis de Broglie 28, 261 (2003).
[8] D. Griffiths, Introduction to Electrodynamics, 2nd ed. (Prentice-Hall, Inc., Englewood Cliffs, NJ, USA, 1989).
[9] R. Dendy, Plasma Physics: an Introductory Course (Cambridge University Press, Cambridge, UK, 1993).