Calculation of the $^3\text{He}$(in-flight $K^-,n$) reaction for searching the deeply-bound $K^-pp$ state

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Abstract

The formation of a deeply-bound $K^-pp$ state by the $^3\text{He}$(in-flight $K^-,n$) reaction is investigated theoretically in the distorted-wave impulse approximation using the Green’s function method. The expected inclusive and semi-exclusive spectra at $p_{K^-} = 1.0$ GeV/c and $\theta_n = 0^\circ$ are calculated for the forthcoming J-PARC E15 experiment. We employ optical potentials between the $K^-$ and "pp" core-nucleus, and demonstrate systematically the dependence of the spectral shape on $V_0$ and $W_0$, which are the real and imaginary parts of the strength for the optical potential, respectively. The necessary condition to observe a distinct peak of the $K^-pp$ bound state with $I = 1/2$, $J^\pi = 0^-$ in the spectrum turns out to be that the value of $V_0$ is deeper than $\sim -100$ MeV and $W_0$ shallower than $\sim -100$ MeV, of which the strength parameters come up to recent theoretical predictions.

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INTRODUCTION

Over the last several years, many theoretical and experimental efforts have been devoted for investigations of deeply-bound kaonic nuclei \([1]\), but there is no conclusive proof on their existence at the present stage. In order to study properties of antikaon \(K^-/\bar{K}^0\) in nuclear medium, it is an important subject to verify the reliable evidence of deeply-bound kaonic nuclei because antikaons might condense in the interior of compact star \([2]\).

Among various antikaon nuclear systems, the three-body \(\bar{K}NN\) bound state with a configuration of \([\bar{K} \otimes \{NN\}_{I=1}]_{I=1/2}\) is expected to be the lightest and the most fundamental kaonic nucleus. This state is symbolically called “\(K^-pp\)” in this paper. In 1963, Nogami \([3]\) has firstly suggested a possible existence of the \(K^-pp\) bound state by a rather crude calculation. About 40 years later, Yamazaki and Akaishi \([4]\) restarted to study the structure of the \(K^-pp\) bound state on the basis of a quantitative few-body calculation with the Brueckner theory, and predicted that the binding energy and width for \(K^-\) are \(B.E. = 48\text{ MeV}\) and \(\Gamma = 61\text{ MeV}\), respectively. Shevchenko, Gal and Mares \([5]\) performed the \(\bar{K}NN-\pi\Sigma N\) coupled channel calculation for the \(K^-pp\) bound state by the Faddeev approach, and obtained \(B.E. = 55-70\text{ MeV}\) and \(\Gamma = 95-110\text{ MeV}\). Several theoretical works \([6, 7, 8]\) also supported the existence of the \(K^-pp\) bound state, but the calculated binding energy and width are not converged theoretically.

On the other hand, FINUDA collaboration at DAΦNE \([9]\) reported experimentally evidence of a deeply-bound \(K^-pp\) state in invariant mass spectroscopy by \(K^-pp \rightarrow \Lambda + p\) decay processes from \(K^-\) absorption on \(^6\text{Li}, ^7\text{Li}\) and \(^{12}\text{C}\) at rest. The measured energy and width are \(B.E. = 115\text{ MeV}\) and \(\Gamma = 67\text{ MeV}\), respectively. However, Magas et al. \([10]\) claimed that it is not necessary to postulate a \(K^-pp\) bound state in order to explain a bump structure in the FINUDA data, because of the \(Y+N\) pair interaction with the residual nucleus, following the \(K^-\) absorption on two nucleons in these nuclei. Although this data analysis is continued by the FINUDA group, the situation is still unsettled. More experimental data are required in order to confirm whether the \(K^-pp\) system has a deeply-bound state or not.

Recently, Iwasaki et al. \([11]\) proposed a new experiment searching the deeply-bound \(K^-pp\) state at J-PARC, by the missing-mass spectrum of the \(^9\text{He}\text{(in-flight }K^-,n)\) reaction, together with the invariant-mass spectra detecting all particles via decay processes from the \(K^-pp\) bound state (J-PARC E15). Our purpose is to clarify theoretically the expected inclusive
and semi-exclusive spectra of the $^3$He(in-flight $K^-,n$) reaction for preparing the forthcoming J-PARC E15 experiment.

As for (in-flight $K^-,N$) reactions on $^{12}$C and $^{16}$O targets, Kishimoto et al. \[12\] performed measurements for searching deeply-bound kaonic nuclei, but no clear evidence was observed. Yamagata et al. \[13\] calculated the corresponding inclusive spectra by $^{12}$C, $^{16}$O(in-flight $K^-,p$) reactions, using $K^-$ optical potentials based on Chiral unitary and phenomenological approaches; they suggested that any distinct peaks of deeply-bound kaonic states will not be observed even if the bound states exist. The predicted states cannot be separated due to their broad width. It would be naively expected that if we choose a lighter nucleus as a target, a signal for formation of kaonic nuclei appears more clearly, whereas no theoretical calculation has been done for the $^3$He target so far. Yamazaki and Akaishi \[14\] also proposed the $K^-pp$ formation via $p+p \rightarrow K^++\Lambda(1405)p^+ \rightarrow K^-+K^-pp$, assuming a $\Lambda(1405)p$ doorway. Such a $K^-pp$ experiment with a $\sim 4$ GeV proton beam on a deuterium target is planned by FOPI collaboration at GSI \[15\].

In this Letter, we demonstrate firstly the inclusive and semi-exclusive spectra of the $^3$He(in-flight $K^-,n$) reaction at $p_{K^-} = 1.0 \text{ GeV/c}$ and $\theta_n = 0^\circ$ within the distorted wave impulse approximation (DWIA), in order to predict the production cross section of the $K^-pp$ bound state and to examine the experimental feasibility. As studied in Ref. \[13\], we employ the Green’s function method by Morimatsu and Yazaki \[16\] in the DWIA framework, which describes well unstable hadron nuclear systems, comparing theoretical spectra with experimental ones \[17, 18\].

FRAMEWORK

**DWIA with the Green’s function method**

In the DWIA framework \[19\], the inclusive double-differential cross section of the $^3$He(in-flight $K^-,n$) reaction at the forward direction $\theta_n = 0^\circ$ in the lab system is written \[17\] as

$$\frac{d^2\sigma}{dE_n d\Omega_n} = \beta \left[ \frac{d\sigma(0^\circ)}{d\Omega_n} \right]^{\text{(elem)}} S(E),$$

(1)

where $[d\sigma(0^\circ)/d\Omega_n]^{\text{(elem)}}$ is a Fermi-averaged cross section for the elementary $K^- + N \rightarrow N + K^-$ forward scattering which is equivalent to a backward $K^-N$ elastic scattering \[12\] in
the lab system. Here we consider only the non-spin-flip $K^- + n \to n + K^-$ reaction channel. The kinematical factor $\beta$ expresses the translation from the two-body $K^-\cdot N$ lab system to the $K^-\cdot ^3\text{He}$ lab system, and it is defined as

$$\beta = \left(1 - \frac{E_{\alpha}^{(2)} - p_n^{(2)}}{E_{K_{\text{res.}}}^{(2)} - p_n^{(2)}} \right) \frac{p_n E_n}{p_n^{(2)} E_n},$$

where $p_{K^-}$ is the incident $K^-$ momentum, and $p_n^{(2)}$ and $p_n (E_n^{(2)}$ and $E_n)$ denote the outgoing neutron momenta (energies) for a nucleon target and $^3\text{He}$ target, respectively, in the lab frame; $E_{K_{\text{res.}}}^{(2)}$ is the residual $K^-$ energy in the final state. $S(E)$ denotes the strength function of the $K^-pp$ system, which is expressed in the Green’s function method [16], as

$$S(E) = -\frac{1}{\pi} \text{Im} \left[ \sum_{\alpha,\alpha'} \int d\mathbf{r} d\mathbf{r'} f_{\alpha}(\mathbf{r}) G_{\alpha,\alpha'}(E; r, r') f_{\alpha'}(\mathbf{r'}) \right],$$

with

$$f_{\alpha}(\mathbf{r}) = \chi^{(-)*} \left( p_n, \frac{M_C}{M_{K^-pp}} \mathbf{r} \right) \chi^{(+)} \left( p_{K^-}, \frac{M_C}{M_A} \mathbf{r} \right) \langle \alpha \mid \psi_n(\mathbf{r}) \mid i \rangle,$$

where $\chi^{(+)} (\chi^{(-)})$ is distorted waves of the incoming $K^-$ (outgoing neutron) with the momentum $p_{K^-} (p_n)$; $M_A$, $M_C$ and $M_{K^-pp}$ are the masses of the $^3\text{He}$ target, the core-nucleus and the $K^-pp$ system, respectively. The factor of $M_C/M_A$ and $M_C/M_{K^-pp}$ takes into account the recoil effects. Here we assumed to use the recoil factor of $M_C/M_{K^-pp}$ in not only $\chi^{(-)}$ but also $\chi^{(+)}$, for simplicity. $\langle \alpha \mid \psi_n(\mathbf{r}) \mid i \rangle$ is a hole-state wave function for a struck neutron in the target, where $\alpha$ denotes the complete set of eigenstates for the system. We employ a simple $(1s)^3$ harmonic oscillator model for the $^3\text{He}$ wave function. Then, the relative $2N-N$ wave function has the form of $\phi_{2N-N}(r) \propto \exp(-r^2/2a^2)$, $a = b_N \sqrt{3/2}$. The size parameter $b_N$ is taken to be 1.30 fm, which reproduces the recent experimental r.m.s charge radius of $^3\text{He}$, $\sqrt{<r^2>} = 1.94$ fm [21]. In this calculation, we assumed the “$pp$” pair as a rigid core with $S_N = \ell_N = 0$, for simplicity. This assumption would be suitable for describing the considerable structure of the deeply-bound $K^-pp$ state, as mentioned below. Therefore, Green’s function for the $K^-pp$ system, $G_{\alpha,\alpha'}$, in Eq.[3] is obtained by solving numerically the following Klein-Gordon equation with a $K^-pp$ optical potential $U^\text{opt}$.

$$\{(E - V_{\text{Coul}}(\mathbf{r}))^2 + \nabla^2 - \mu^2 - 2 \mu U^\text{opt}(\mathbf{r}) \} G(E; \mathbf{r}, \mathbf{r'}) = \delta^3(\mathbf{r} - \mathbf{r'}),$$

where $\mu$ is the reduced mass between the $K^-$ and the “$pp$” core-nucleus, and $V_{\text{Coul}}$ the Coulomb potential with the finite nuclear size effect.
Moreover, the strength function $S(E)$ can be decomposed into two parts \[16\] as

$$S(E) = S^{\text{con}}(E) + S^{\text{esc}}(E),$$

where $S^{\text{con}}(E)$ describes the $K^-$ conversion processes including the decay modes of $K^-pp \rightarrow \pi^+ + \Sigma/\Lambda + N$ (mesonic) from the one-body absorption and $K^-pp \rightarrow \Sigma/\Lambda + N$ (nonmesonic) from the two-body absorption, and $S^{\text{esc}}(E)$ the $K^-$ escaping processes leaving the core-nucleus in the ground state:

$$S^{\text{con}}(E) \equiv -\frac{1}{\pi} f^\dagger G^\dagger (\text{Im} U^{\text{opt}}) G f,$$

$$S^{\text{esc}}(E) \equiv -\frac{1}{\pi} f^\dagger (1 + G^\dagger U^{\text{opt}}) (\text{Im} G_0) (1 + U^{\text{opt}} G) f,$$

where $G_0$ is the free Green’s function. Here we used an abbreviated notation for $G, f$ and $U^{\text{opt}}$. We evaluate the distorted waves within the eikonal approximation as

$$\chi^{(-)*}(p_n, r) = \exp \left[ -i p_n \cdot r - \frac{i}{v_n} \int_z^{+\infty} U_n(b, z') dz' \right],$$

$$\chi^{(+)}(p_{K^-}, r) = \exp \left[ +i p_{K^-} \cdot r - \frac{i}{v_{K^-}} \int_{-\infty}^z U_{K^-}(b, z') dz' \right]$$

with an impact parameter coordinate $b$ and the optical potential for $\lambda = K^-$ or $n$,

$$U_{\lambda}(r) = -i \frac{v_{\lambda}}{2} \tilde{\sigma}_{\lambda N}^{\text{tot}} (1 - i\alpha_{\lambda N}) \rho(r),$$

where $\rho(r)$ is a nuclear density distribution, and $\tilde{\sigma}_{\lambda N}^{\text{tot}}$ and $\alpha_{\lambda N}$ are the isospin-averaged total cross section and the ratio of the real to imaginary parts of the forward amplitude for the $\lambda + N$ scattering, respectively. These parameters are chosen to be $\tilde{\sigma}_{K^-N}^{\text{tot}} = \tilde{\sigma}_{nN}^{\text{tot}} = 40 \text{ mb}$ and $\alpha_{K^-N} = \alpha_{nN} = 0 \[12, 22\].

**Optical potential for $K^-pp$ system**

Yamazaki and Akaishi \[4\] obtained the $K^-pp$ optical potential between the $K^-$ and the “$pp$” pair, performing a variational three-body calculation. This YA optical potential is parametrized in a Gaussian form with a range-parameter $\beta = 1.09 \text{ fm}$, as

$$U^{\text{opt}}_{K^-pp}(r) = (V_0 + i W_0) \exp \left[ -(r/\beta)^2 \right] ,$$

where $V_0 = -300 \text{ MeV}$ and $W_0 = -70 \text{ MeV}$. Note that these parameters are determined within a non-relativistic framework. Solving the Klein-Gordon equation with this potential,
we find that the $K^-$ binding energy and width account for $B.E. = 51$ MeV and $\Gamma = 68$ MeV, respectively. Although these values are slightly changed from $B.E. = 48$ MeV and $\Gamma = 61$ MeV given in Ref. [4], we employ the YA potential having $(V_0, W_0) = (-300$ MeV, $-70$ MeV) as a standard one, for a first step toward our quantitative investigation. Shevchenko, Gal and Mareš [5] also obtained $B.E. \sim 70$ MeV and $\Gamma \sim 110$ MeV as the maximum values for the $K^-pp$ bound state by the Faddeev calculation. We can simulate $B.E. = 72$ MeV and $\Gamma = 115$ MeV as a complex Klein-Gordon energy, adjusting the parameters to $(V_0, W_0) = (-350$ MeV, $-100$ MeV) within the potential form of Eq.(12). We adopt these values of $(V_0, W_0)$ as an upper limit for the reliable parameters.

RESULTS AND DISCUSSION

Let us consider the $^3$He(in-flight $K^-,n$) reaction at the incident $K^-$ momentum $p_{K^-} = 1.0$ GeV/c and the forward direction $\theta_n = 0^\circ$, where the lab cross section of the elementary reaction process is maximal [12, 23]. The lab cross section for the $K^-+n \rightarrow n+K^-$ forward scattering is $\sim 27$ mb/sr [23] in free space, and is reduced to $\sim 15$ mb/sr with Fermi-averaging [24]. These values are about three times larger than those for the $K^-+p \rightarrow p+K^-$ forward scattering for the $(K^-,p)$ reaction at $p_{K^-} = 1.0$ GeV/c, e.g., 8.8 mb/sr [13] in free space and 5.2 mb/sr [22] with Fermi-averaging. We adopt 15 mb/sr as a value of $[d\sigma(0^\circ)/d\Omega_n]^{(elem)}$ in Eq.(1), and we notice that the momentum transfer $Q(0^\circ) = p_{K^-} - p_n$ is negative ($Q(0^\circ) < 0$) which means that the residual $K^-$ recoils backwards relative to the incident $K^-$ in the lab frame, therefore, the kinematical factor $\beta$ in Eq.(2) enhances the spectrum by a factor 1-2 depending on $p_n = 1.0$-1.4 GeV/c.

For missing-mass spectroscopy, Fig. 1 shows the calculated inclusive spectrum of the $^3$He(in-flight $K^-,n$) reaction as a function of the ejected neutron momentum $p_n$, and the components of the partial-wave angular momentum states with $L$. Here we used the YA potential with $(V_0, W_0) = (-300$ MeV, $-70$ MeV). We find that a distinct peak of the $K^-pp$ bound state with $I = 1/2$, $J^\pi = 0^-$ ($L = S = 0$) appears around $p_n = 1280$ MeV/c, which has $B.E. \sim 50$ MeV and $\Gamma \sim 70$ MeV. The absolute value of the integrated formation cross section amounts to 3.5 mb/sr. Note that the effective neutron number with the distortion for the $K^-pp$ bound state is $N_{1s_n \rightarrow 1s_{K^-}}^{DW} = 0.23$ with $D_{\text{dis}} = 0.47$, of which value is more than ten times as large as the effective proton number $P_{1s_p \rightarrow 1s_{K^-}}^{DW} = 0.013$ with $D_{\text{dis}} = 0.095$ for
the $K^-$ nuclear $1s$ bound state via the $1p_p \rightarrow 1s_{K^-}$ transition in $^{12}$C. This is a major advantage of the use of the $s$-shell target nucleus such as $^3$He. We stress that the calculated spectrum consists of mainly the states with $L = 0$ below the $K^-$ emitted threshold ($p_n = 1224$ MeV/c), whereas the components of $L \geq 1$ dominates continuum states for the quasi-free (QF) region. This $L = 0$ dominance in the bound region is caused by a small size of the $s$-shell nucleus, contrast to the case of $p$-shell nuclei, $^{12}$C and $^{16}$O. Moreover, the recoil effects for the light $s$-shell nuclear target play an important role in increasing the cross section of the $K^-pp$ bound state; if we replace the recoil factors of both $M_C/M_{K^-pp}$ and $M_C/M_A$ by 1 in Eq. (4), the cross section of the peak in the bound region is reduced by half, whereas the yields in the QF region grow up, and the QF peak is shifted to the lower $p_n$ side and its width is broader. When we use a reasonable factor of $M_C/\bar{M}_{AK}$ where $\bar{M}_{AK}$ is the mean mass of $M_A$ and $M_{K^-pp}$, instead of $M_C/M_{K^-pp}$, we find that the cross section around the peak in the bound region is enhanced by about 10%.

— Fig.1 —

In Fig. 2, we display contributions of the $K^-$ escape and conversion components into the inclusive spectrum, using the YA potential. Since the “$pp$” rigid-core nucleus is assumed in this calculation, the $\bar{K}NN$ continuum spectrum is described as $K^-+$“$pp$” continuum states. Thus the $K^-$ escape spectrum in Eq. (8) might not be fully explained in the QF region where the $^3$He break-up processes of $K^- + ^3$He $\rightarrow n + p + p + K^-$ occur. On the other hand, this assumption seems to work well in the bound region; decay processes of $K^-pp \rightarrow \pi + \Sigma/\Lambda + N$ and $K^-pp \rightarrow \Sigma/\Lambda + N$ are effectively described by the imaginary potential, $\text{Im}U_{\text{opt}}$, if we choose an appropriate one. Such a conversion spectrum in Eq.(7) expresses semi-exclusive $K^- + ^3$He $\rightarrow n + \Sigma/\Lambda + X$ processes detecting the selected particles in invariant-mass spectroscopy at the J-PARC E15 experiment. Therefore, the treatment of the “$pp$” core-nucleus would be justified in not only the bound region but also the continuum region, as long as we concern the conversion spectrum. In Fig. 3, we show also the escape and conversion spectra using $(V_0, W_0)=(-350 \text{ MeV}, -100 \text{ MeV})$, in order to simulate the results by Shevchenko, Gal and Mareš. We confirm that a peak of the $K^-pp$ bound state is broadened because a value of $\Gamma$ is so larger than that obtained by the YA potential. The corresponding bump is barely recognized in the spectrum.

— Fig.2 —

— Fig.3 —
Mareš, Friedman and Gal [25] introduced a phase space suppression factor \( f(E) \) multiplying the imaginary part of the \( K^- \) optical potential. When a pole of the \( K^-pp \) bound state is located below the \( \pi \Sigma N \) threshold in the complex energy plane, its width would be narrower by the phase space suppression because the \( K^N \rightarrow \pi \Sigma \) decay channel is closed there. Several predictions for the \( K^-pp \) bound state [4, 5, 7, 8] suggest that its pole resides above the \( \pi \Sigma N \) threshold, except the results in Ref. [6]. In order to see the effects, we replace \( W_0 \) by \( W_0 \times f(E) \) in Eq. (12). Here we assumed that the \( K^- \) is absorbed from 80\% one-nucleon and 20\% two-nucleons, as shown in Refs. [13, 25]. Indeed, the helium bubble chamber experiment [26] suggested that the ratio of \( K^- \) absorption on two nucleons to all \( K^- \) absorption process in \(^4\)He at rest amounts to 16\%. Taking into account the phase space factor, we adjust a value of \( W_0 \) to be \(-93\) MeV so as to reproduce the same \( \Gamma \) of the \( K^-pp \) bound state. Since \( S^{on}(E) \) can be further decomposed into the corresponding spectra from the one- and two-nucleon absorptions, we can estimate each component for these decay modes. Fig. 4 shows the effects of the phase space factor in the calculated spectrum. We find that the shape of the inclusive spectrum is not so changed when the phase space factor is turned on; the spectrum of the \( K^-pp \) bound state, however, is sizably reduced near the \( \pi \Sigma N \) threshold \((p_n \sim 1330\) MeV/c\) because the \( K^-pp \) below the threshold can decay into only the \( K^-pp \rightarrow Y + N \) process from the two-nucleon absorption, which contributes slightly to the spectrum on the whole.

— Fig.4 —

It is worth studying shape behaviors of the strength function \( S(E) \) for the \( K^-pp \) bound state on the parameters of \( V_0 \) and \( W_0 \), systematically. As shown in Fig. 3 we change the value of \( V_0 \) from \(-50\) MeV to \(-350\) MeV, keeping \( W_0 = -70\) MeV. We see that a peak of the bound state gives rise to a cusp-like structure at \( V_0 = (-100)-(-50)\) MeV (shallow). In Fig. 6 we show the shape behavior of \( S(E) \) when we change the value of \( W_0 \) from \(-40\) MeV to \(-150\) MeV, keeping \( V_0 = -300\) MeV. We find that choosing \( W_0 = -150\) MeV (deep), the peak is completely invisible even if the bound state exists. Consequently, in order to identify a peak of the \( K^-pp \) bound state in the inclusive spectrum, we realize that the value of \( V_0 \) must be deeper than \( \sim -100\) MeV and also that of \( W_0 \) shallower than \( \sim -100\) MeV. The parameter set for the YA potential [4] fulfills this condition, while that simulating the results by Shevchenko, Gal and Mareš [5] stands on the edge of the signal observation in terms of the width. We believe that it is possible to extract variable information on the \( K^-\)
optical potential from the shape behavior of the inclusive and semi-exclusive spectra.

— Fig.5 —
— Fig.6 —

Doté et al. [27] predicted the possibility of extreme nuclear shrinkage in the $K^-NNN$ system (strange tribaryon) by anti-symmetrized molecular dynamics (AMD) calculations. Such nuclear contraction is strongly related to the antikaon condensation in compact star [2]. It is interesting to evaluate the nuclear contraction in the $K^-pp$ bound state by the $(K^-, n)$ reaction. When a harmonic oscillator model is applied to $^3\text{He}$, the r.m.s. distance of the “$pp$” pair is 1.90 fm in the $K^-pp$ bound state, whose value is reduced from that of 2.25 fm in $^3\text{He}$, as discussed in Ref. [4]. However, this effect is not expected to be observed because the cross section of the $K^-pp$ bound state is only slightly suppressed by about 4%.

Our simplified calculation in this paper would bring some uncertainties for the resultant spectrum: Only the $K^- + n \rightarrow n + K^-$ forward scattering has been considered, omitting the $K^- + p \rightarrow n + \bar{K}^0$ charge exchange process which can also contribute to the $K^-pp$ formation through the coupling between $K^-pp$ and $\bar{K}^0pn$ channels. This contribution is roughly estimated by replacing the Fermi-averaged transition amplitude $f_{K^-n\rightarrow nK^-}$ for the elementary $K^- + n \rightarrow n + K^-$ process by the isoscalar transition amplitude $f_{\Delta I=0} = -\sqrt{2/3}(f_{K^-n\rightarrow nK^-}+\frac{1}{2}f_{K^-p\rightarrow n\bar{K}^0})$ which is equivalent to a $u$-channel isospin-0 one for $(K^-,\bar{K})$ reactions, including a spectroscopic factor for the $K^-pp I=1/2$ state formed on $^3\text{He}$. As a result, the cross section of the $K^-pp$ bound state is enhanced by about 18% with Fermi-averaging. It is noticed that the $K^-pp$ state with $I=1/2$ dominates the $(K^-,n)$ reaction at $p_{K^-} = 1.0 \text{ GeV}/c$ because $|f_{\Delta I=0}|^2/|f_{\Delta I=1}|^2 \simeq 14$, where $f_{\Delta I=1}$ is the isovector transition amplitude [28]. This nature justifies our assumption that we treat $[\bar{K} \otimes \{NN\}]_{I=1}I=1/2$ as $K^-pp$ restrictedly in this calculation. In order to get more quantitative results, a full coupled-channel calculation would be needed. Moreover, the choice of the parameters in the eikonal distorted waves also changes the absolute value of the cross section, but the distortion effect is not significant for $^3\text{He}$. For the in-flight $(K^-,N)$ reaction, it would be not appropriate to use the decay rate measured by $K^-$ absorption at rest, considering that its value depends on atomic orbits where $K^-$ is absorbed through atomic cascade processes [29, 30]. More theoretical and experimental considerations are needed.
SUMMARY AND CONCLUSIONS

We have examined the $^3$He(in-flight $K^-,n$) reaction at $p_{K^-} = 1.0$ GeV/c and $\theta_n = 0^\circ$ for the forthcoming J-PARC E15 experiment. We have demonstrated how the spectral shape of the inclusive spectrum is changed by the strength parameters of $V_0$ and $W_0$ in the $K^-\text{-"pp"}$ optical potential, and also obtained the semi-exclusive $K^-$ conversion spectrum which has the ability to be compared with the data by the experiment measuring the missing- and invariant-mass simultaneously. We believe that our considerable assumptions are adequate for the first step toward our purpose. In conclusion, the necessary condition to observe a distinct peak of the $K^-pp$ bound state turns out to be that $V_0$ is deeper than $\sim -100$ MeV and $W_0$ shallower than $\sim -100$ MeV. The $K^-pp$ bound state predicted by Yamazaki and Akaishi [4] fulfills this condition, while that by Shevchenko, Gal and Mareš [5] stands on the edge of the signal observation in terms of the width. To analyze in detail the forthcoming experimental data at J-PARC, it will be necessary to improve our theoretical treatment. This investigation now is in progress [28].

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FIG. 1: The calculated inclusive spectra of the $^3$He(in-flight $K^-,n$) reaction at $p_{K^-} = 1.0$ GeV/c and $\theta_n = 0^\circ$ as a function of the neutron momentum, using the YA optical potential with $(V_0, W_0) = (-300$ MeV, $-70$ MeV). The vertical dashed line indicates the corresponding neutron momentum of $p_n = 1224$ MeV/c at the $K^-$ emitted threshold. The contributions of partial-wave angular momentum states with $L = 0, 1$ and $2$ are also drawn.
FIG. 2: Same as Fig. 1 but the inclusive spectrum is decomposed into the conversion processes of $K^- + ^3\text{He} \rightarrow n + \Sigma/\Lambda + X$ (dashed line), and the escape processes of $K^- + ^3\text{He} \rightarrow n + K^- + \sim pp$ (dotted line). Note that the conversion spectrum is identical to the inclusive one in the bound region.
FIG. 3: Same as Fig. 2 but the case of \((V_0, W_0)=(-350 \text{ MeV}, -100 \text{ MeV})\).
FIG. 4: The effects of the phase space suppression factor in the $^3$He(in-flight $K^-,n$) reaction at $p_{K^-} = 1.0$ GeV/c and $\theta_n = 0^\circ$. The YA optical potential is used. The solid and dashed lines denote the inclusive spectra with and without taking into account the phase space factor, respectively. The dot-dot-dashed line indicates the contribution of the one-nucleon absorption process, $K^-+^3$He $\rightarrow n+\pi+Y+N$, and the dotted line the two-nucleon one, $K^-+^3$He $\rightarrow n+Y+N$. 
FIG. 5: The behavior of the strength function for the $^3\text{He}(\text{in-flight } K^-, n)$ reaction at $p_{K^-} = 1.0$ GeV/c and $\theta_n = 0^\circ$ on the strength parameter of $V_0$. $W_0$ is fixed to be $-70$ MeV. The horizontal axis denotes the energy of the $K^- pp$ system measured from the $K^-$ emitted threshold; $E < 0$ means a bound state of the $K^- pp$ system.
FIG. 6: The behavior of the strength function for the $^3$He(in-flight $K^-,n$) reaction at $p_{K^-} = 1.0$ GeV/c and $\theta_n = 0^\circ$ on the strength parameter of $W_0$. $V_0$ is fixed to be $-300$ MeV. See also the caption in Fig. 5.