Determination of the Higgs-boson couplings and $H - A$ mixing in the generalized SM-like Two Higgs Doublet Model

P.Nieżurawski, A.F.Żarnecki
Institute of Experimental Physics, Warsaw University,
M.Krawczyk
Institute of Theoretical Physics, Warsaw University

March 26, 2022

Abstract

The feasibility of measuring the Higgs-boson properties at the Photon Collider at TESLA has been studied in detail for masses between 200 and 350 GeV, using realistic luminosity spectra and detector simulation. We consider the Two Higgs Doublet Model (II) with SM-like Yukawa couplings for $h$, parametrized by only one parameter ($\tan \beta$). The combined measurement of the invariant-mass distributions in the $ZZ$ and $W^+W^-$ decay-channels is sensitive to both the two-photon width $\Gamma_{\gamma\gamma}$ and phase $\phi_{\gamma\gamma}$. From the analysis including systematic uncertainties we found out that after one year of Photon Collider running with nominal luminosity the expected precision in the measurement of $\tan \beta$ is of the order of 10%, for both light ($h$) and heavy ($H$) scalar Higgs bosons. The $H - A$ mixing angle $\Phi_{HA}$, characterizing a weak CP violation in the model with two Higgs doublets, can be determined to about 100 mrad, for low $\tan \beta$. 
1 Introduction

A photon collider has been proposed as a natural extension of the $e^+e^-$ linear collider [1]. The physics potential of a photon collider is very rich and complementary to the physics program of the $e^+e^-$ and hadron-hadron colliders. It is an ideal place to study the mechanism of the electroweak symmetry breaking (EWSB) and the properties of the Higgs-boson. In paper [2] we have performed realistic simulation of the production at the TESLA Photon Collider [3] of the SM Higgs boson with masses above 150 GeV for $W^+W^-$ and $ZZ$ decay channels. We found that precise measurements of both the $higgs \rightarrow \gamma\gamma$ partial width, $\Gamma_{\gamma\gamma}$ and the phase, $\phi_{\gamma\gamma}$ are needed for determination of the Higgs-boson couplings, see also [4, 5, 6, 7, 8]. Therefore, as we have found in [2], it is extremely important to combine both $W^+W^-$ and $ZZ$ channels, as the first one, due to a large background leading to large interference effects is very sensitive to a phase, while the second one - to a partial width. From combined analysis of $W^+W^-$ and $ZZ$ decay channels the $\gamma\gamma$ partial width $\Gamma_{\gamma\gamma}$ can be measured with an accuracy of 3 to 8% and the phase of the $\gamma\gamma h$ amplitude $\phi_{\gamma\gamma}$ with an accuracy between 30 and 100 mrad.

Once we found how useful these channels are for the SM Higgs study, the natural is to investigate models with the extended Higgs sector. In this paper we continue the analysis from [2] by extending it to the Two Higgs Doublet Model 2HDM (II). We consider one of the SM-like versions of this model, with Yukawa couplings as in SM (up to a sign). For both CP-conserving as well as CP-violating scenarios we perform the combined analysis of $W^+W^-$ and $ZZ$ invariant-mass distributions and extract the corresponding Higgs-boson coupling to gauge bosons, which is governed by only one parameter ($\tan \beta$). For 2HDM model with a CP violation we estimate the precision of the measurement of $H - A$ mixing angle. The systematic uncertainties of the coupling and mixing angle measurements are estimated. Results given in this paper supersede preliminary results presented in the first part of our earlier paper [9].

2 Event simulation

The Compton back-scattering of a laser light off high-energy electrons is considered as a source of high energy, highly polarized photon beams at Photon Collider [1]. According to the current design [3], the energy of the laser photons is assumed to be fixed for all considered electron-beam energies. With 100% circular polarization of laser photons and 85% longitudinal polarization of the electron beam the luminosity spectra peaked at high $\gamma\gamma$ invariant masses is expected.

Our analysis uses the CompAZ parametrization [10] of the realistic luminosity spectra for a Photon Collider at TESLA [11]. We assume that the centre-of-mass energy of colliding electron beams, $\sqrt{s_{\text{ee}}}$, is optimized for the production of a Higgs boson with a given mass. All results presented in this paper were obtained for an integrated luminosity corresponding to one year of the photon collider running, as given by [11]. The total photon-photon luminosity increases from about 600 fb$^{-1}$ for $\sqrt{s_{\text{ee}}} = 305$ GeV (optimal beam energy choice for $M = 200$ GeV) to about 1000 fb$^{-1}$ for $\sqrt{s_{\text{ee}}} = 500$ GeV ($M = 350$ GeV).

The analysis described in this work was performed in two steps. In the first step, we use samples of events generated with PYTHIA 6.152 [12] to study selection efficiency and invariant-
mass resolution in reconstruction of the events $\gamma\gamma \rightarrow W^+W^-/ZZ$, as a function of the $\gamma\gamma$ centre-of-mass energy, $W_{\gamma\gamma}$. We consider the direct vector-boson production in $\gamma\gamma$ interactions as well as Standard Model Higgs-boson decays into the vector bosons and contribution from the interference terms. To take into account effects which are not implemented in PYTHIA (photon beam polarization, interference term contribution, direct $\gamma\gamma \rightarrow ZZ$ production) we use the standard method used in various experimental analyses called a reweighting procedure. Each generated event is attributed a weight given by the ratio of the differential cross-section for a vector-boson production in the polarized photon interactions [4, 5, 13] to the PYTHIA differential cross section for given event. The fast simulation program SIMDET version 3.01 [14] is used to model the TESLA detector performance.

A good invariant-mass resolution is essential for the considered measurement. For the $W^+W^-$ events only $W^+W^- \rightarrow q\bar{q}q\bar{q}$ decay channel is considered, as without knowing the exact beam-photon energies, which is always a case for the Photon Collider, the semileptonic $W^\pm$ decays worsen the mass resolution. The final selection efficiency for $\gamma\gamma \rightarrow W^+W^-$ events is found to lay between 20% for $W_{\gamma\gamma} \sim 200 \text{ GeV}$ and 16% for $W_{\gamma\gamma} \sim 400 \text{ GeV}$ (including 47% probability for hadronic decays of both $W^\pm$). The resolution in the reconstructed $\gamma\gamma$ invariant mass for these events, described by the parameter $\Gamma$ (from the Breit–Wigner type fit), changes from about 6.5 GeV at $W_{\gamma\gamma} = 200 \text{ GeV}$ to about 13 GeV at $W_{\gamma\gamma} = 400 \text{ GeV}$.

For $ZZ$ events, only $ZZ \rightarrow l\bar{l}q\bar{q}$ decay channel is considered, where one $Z$ decays into $e^+e^-$ or $\mu^+\mu^-$. Lepton tagging and the invariant-mass reconstruction for both the lepton pair and the two hadronic jets is crucial for a suppression of the background from the direct $\gamma\gamma \rightarrow W^+W^-$ events. After all cuts, the selection efficiency for $ZZ$ events is only about 5%, mainly due to a small branching ratio for the considered channel (9.4% for $ZZ \rightarrow l\bar{l}q\bar{q}$, $l = e, \mu$), however, the final sample is very clean. For the $l\bar{l}q\bar{q}$ final state we get the invariant-mass resolution $\Gamma$ changing from about 5.5 GeV at $W_{\gamma\gamma} = 200 \text{ GeV}$ to about 7.5 GeV at $W_{\gamma\gamma} = 400 \text{ GeV}$. Details of event selection and the description of the invariant mass resolution are given in [2].

The invariant-mass resolutions obtained from a full simulation of $W^+W^-$ and $ZZ$ events (based on the PYTHIA and SIMDET programs), have been parametrized as a function of the $\gamma\gamma$ centre-of-mass energy, $W_{\gamma\gamma}$. They can be used to obtain the parametric description of the expected invariant mass distributions, for $\gamma\gamma \rightarrow W^+W^-$ and $\gamma\gamma \rightarrow ZZ$ events, avoiding time consuming event generation procedure. For arbitrary model, and for arbitrary values of model parameters, expected mass distributions can be calculated by numerical convolution of the parametrized resolutions with the CompAZ spectra and the cross section formula. This approach, developed in [2], was used to obtain results described in the remaining part of this paper.

3 Measurement of $\Gamma_{\gamma\gamma}$ and $\phi_{\gamma\gamma}$ from the $WW/ZZ$ invariant-mass distributions in SM

In this section we summarize results of [2], where the feasibility of measuring the Standard Model Higgs-boson $W^+W^-$ and $ZZ$ decay channels at the $\gamma\gamma$ option of TESLA has been
studied for a Higgs-boson mass above 150 GeV. The signal, i.e. the Higgs-boson decays into the vector bosons, and the background from direct vector-bosons production are used to extract the width and the phase of the loop coupling $\text{higgs} \rightarrow \gamma\gamma$. For the $ZZ$ final-state a direct, i.e. non-resonant, process is rare as it occurs via loop only. On contrary, the non-resonant $W^+W^-$ production is a tree-level process, and is expected to be large. Therefore, also an interference between the signal of $W^+W^-$ production via the Higgs resonance and the background from the direct $W^+W^-$ production may be large. This effect can be used to access an information about the phase $\phi_{\gamma\gamma}$. For the Higgs-boson masses around 350 GeV we found in [2] that the phase $\phi_{\gamma\gamma}$ is more sensitive to the loop contributions of new, heavy charged particles than the width $\Gamma_{\gamma\gamma}$ itself.

The parametric description of the expected invariant mass distributions for $\gamma\gamma \rightarrow W^+W^-$ and $\gamma\gamma \rightarrow ZZ$ events was obtained by the numerical convolution of the cross-sections formula (including $\Gamma_{\gamma\gamma}$ and $\phi_{\gamma\gamma}$ as model parameters) with the CompAZ spectra and the parametrized resolution. Based on this description, many experiments were simulated, each corresponding to one year of a Photon Collider running at TESLA at a nominal luminosity. The “theoretical” distributions were then fitted, simultaneously to the “observed” $W^+W^-$ and $ZZ$ mass spectra, with the width $\Gamma_{\gamma\gamma}$ and phase $\phi_{\gamma\gamma}$ considered as the only free parameters. Assuming the Standard Model Higgs-boson branching ratios, and with a proper choice of the electron-beam energy, the $\gamma\gamma$ partial width can be measured with an accuracy of 3 to 8%, while the phase of the amplitude with an accuracy between 35 and 100 mrad [2].

The $\phi_{\gamma\gamma}$ measurement opens a new window to a precise determination of the Higgs-boson couplings and to search for a “new physics”. It turns out that the phase is constrained predominantly by the $W^+W^-$ invariant-mass distribution, thanks to the large interference between Higgs-boson decay into $W^+W^-$ and non-resonant (SM) $W^+W^-$ production. However, the two-photon width of the Higgs-boson is much better constrained by the measurement of the $ZZ$ mass spectra, as the non-resonant background is much smaller here. A precise determination of both parameters is only possible when both measurements are combined.

The promising results obtained for the SM Higgs boson encourage to the evolution of the analysis towards the models with more Higgs doublets. Presented analysis extends the approach developed for SM Higgs boson to the selected Two Higgs Doublet Model (II) scenarios. For all considered sets of parameter values the expected invariant mass distributions were calculated, taking into account model predictions for both the production cross-section (including interference term contributions) and the Higgs-boson branching ratios. Parametrized distributions were used to simulate many experiments, each corresponding to one year of a Photon Collider running. Errors (and correlations) in $\Gamma_{\gamma\gamma}$ and $\phi_{\gamma\gamma}$ measurement, expected from the simultaneous fit to the observed $W^+W^-$ and $ZZ$ mass spectra, where then used to determine expected uncertainties of model parameters.
4 Determination of the Higgs-boson couplings in the CP-conserving model $B_h$

In our previous analysis [2] we indicated possible deviations from the Standard Model predictions resulting from the loop contributions to the $h\gamma\gamma$ vertex of new heavy charged particles. However, deviations in the two-photon width and phase can also appear if the couplings of the Higgs-boson to the other particles are different than those predicted by the Standard Model. Both such effects are taken into account in this analysis, based on a simple extension of SM, namely 2HDM (II), see also [15].

In this section we consider the Two Higgs Doublet Model (II) (2HDM (II)) with CP conservation. The Higgs sector of such model contains $h$, $H$, $A$ and $H^{\pm}$ bosons, and is characterized (in the $Z_2$ symmetric case) by mixing angles $\alpha$ and $\beta$.

4.1 The Model and its tests at future colliders

The simplest version of the 2HDM is the case where couplings are parametrized by one mixing angle. Often one considers 2HDM when $h$ couples to gauge bosons as in SM - however then also all Yukawa couplings are as in SM. Here we assume instead that the Yukawa couplings of $h$ are equal (up to sign) to the corresponding SM Higgs-boson couplings. Then the coupling of $h$ to gauge bosons is governed by $\tan\beta$, as it is shown in table 1. The corresponding Yukawa and gauge boson couplings of $H$ and $A$ bosons are also uniquely determined by $\tan\beta$, as can be seen in the table. Note that here $H$ and $A$ have similar Yukawa couplings (up to $i\gamma_5$ factor).\footnote{In our analysis the loop $h\gamma\gamma$ and $H\gamma\gamma$ vertexes appear. The couplings of $h$ and $H$ to the charged higgs boson $H^{\pm}$, contributing to such vertexes, are calculated according to the 2HDM II potential [15] assuming $\mu=0$.}

This scenario, called by us a model $B_h$, can be treated as a generalized Standard Model-like 2HDM (II) scenario $B$ for $h$, introduced in [15]. Solutions $B_{h+u}$ and $B_{h-d}$ of [15] correspond to our model $B_h$ in the limit of $\tan\beta \ll 1$ and $\tan\beta \gg 1$, respectively.

In 2HDM there is a possibility that one of the Higgs boson couplings vanishes, for example the coupling to the EW gauge bosons. The corresponding sum rule

$$\left(\chi_X^h\right)^2 + \left(\chi_X^H\right)^2 + \left(\chi_X^A\right)^2 = 1,$$

where $X$ denotes a fermion or a vector boson, $X = u, d, V$, ensures only that at least one of the other Higgs bosons has a nonzero coupling.

In the solution $B_h$ the lightest Higgs-boson $h$ couplings to fermions are equal to the Standard Model, except that for up-type fermions the sign is opposite.\footnote{This is equivalent to the following condition imposed on the mixing angles $\alpha$ and $\beta$ of 2HDM (II): $\alpha = -\frac{\pi}{2} - \beta$.} In such a case, the two-gluon partial width, $\Gamma_{gg}$, dominated by the top-quark loop contribution, is very close to the SM predictions.

We observe that couplings of $h$ to the EW gauge-bosons $V, V = W^\pm, Z^0$, may significantly differ from the corresponding SM couplings. However, since the decays into $WW$ and $ZZ$ are...
the dominating ones in the considered Higgs boson mass range\(^3\) this potentially large effect cancels out in the branching ratios (as it affects in similar strength both the partial width and the total width). Therefore, to be sensitive to deviations from the SM couplings, one has to study processes where the coupling \(hVV\) is involved in the production of the Higgs boson and not only in its decays. Taking this into account we expect that, i.e. \(gg \rightarrow h \rightarrow VV\) at LHC could indicate no deviations from SM,\(^4\) and at the same time sizable effect can appear at LC (eg. in Higgsstrahlung process \(e^+e^- \rightarrow Z h\)). Also one can expect large deviations from the SM prediction in the two photon width \(\Gamma_{\gamma\gamma}\) and phase \(\phi_{\gamma\gamma}\) which can be measured at the Photon Collider, as described below.

Finally we note that when the Higgs boson coupling to \(V\) vanishes, so does the corresponding branching ratio and the sensitivity of the considered measurements is lost. This is a reason why the determination of \(\tan \beta\) from measurement of \(h\) production and decays to \(W^+W^-\) and \(ZZ\) is not possible for \(\tan \beta \approx 1\).

### 4.2 \(\gamma\gamma \rightarrow h/H\) in the model \(B_h\)

In the Standard Model, the dominant contributions to the \(h\gamma\gamma\) coupling are due to the \(W^\pm\) and top-quark loops. Therefore, the process of resonant Higgs boson production at the Photon Collider is sensitive to the Higgs-boson couplings to the gauge-bosons. Moreover, as the phase of the \(W^\pm\) contributions to the \(\gamma\gamma \rightarrow h\) amplitude differs from that of the top loop contribution, not only the two-photon partial width \(\Gamma_{\gamma\gamma}\) is sensitive to the the Higgs boson coupling, but also the phase \(\phi_{\gamma\gamma}\). Both parameters can be precisely measured at the Photon Collider (see Section 3), allowing us to constrain the values of \(\chi_V\) and \(\tan \beta\) from the combined measurement of \(W^+W^-\) and \(ZZ\) invariant-mass distributions. An important observation is that the ratio of \(h\) and \(H\) branching ratios to the vector bosons, \(\text{BR}(higgs \rightarrow ZZ)/\text{BR}(higgs \rightarrow W^+W^-)\) does not depend on the \(\tan \beta\) value and for a given Higgs-boson mass it is expected to be the same as in the Standard Model.

For \(\tan \beta \ll 1\) one gets \(\chi_V^h \approx 1\). Nevertheless, one expects significant deviations from the Standard Model predictions for a light Higgs-boson \(h\), both for the two-photon width and phase,

\(^3\)For the SM Higgs boson with mass of 200 to 350 GeV contribution of other decay channels is about 0.4%.
\(^4\)In solution \(B_h\), deviations of more than 10% from SM predictions (on the number of \(WW\) and \(ZZ\) decays) are only expected for \(|\chi_V| < 0.2\), i.e. for \(\tan \beta \approx 0.8 - 1.2\).

| \(\chi_u\) | \(\cos(2\beta)\) | \(-\frac{1}{\tan \beta}\) | \(-\tan \beta\) | \(-\sin(2\beta)\) | \(0\) |
|---|---|---|---|---|---|
| \(h\) | \(H\) | \(A\) |

Table 1: Couplings of the neutral Higgs-bosons to up- and down-type fermions, and to vector bosons, relative to the Standard Model couplings, for the considered solution \(B_h\) of the SM-like 2HDM (II).
since as compared to the SM there is a change of a relative sign of the top-quark and the $W$ contributions. In 2HDM II (sol. $B_h$) the two-photon width is significantly larger than in the Standard Model, where these two contributions partly cancel each other. For $\tan \beta \sim 1$ the two-photon width decreases, due to the suppressed $W$-loop contribution (here $\chi_V = \cos(2\beta) \approx 0$). Finally, for large values of $\tan \beta$ ($\cos(2\beta) \approx -1$), the two-photon width of the light Higgs-boson $h$ tends to be close to the expectations of the Standard Model, since the only difference is due to the presence of the heavy charged Higgs-boson in the loop. The opposite, as compared to the SM one, sign of the down-type fermion contributions gives negligible effect.

The two-photon width and phase can be investigated also for the heavy scalar Higgs-boson $H$ for model $B_h$, with couplings as given in table 1.

### 4.3 Determination of $\tan \beta$

**Results for $h$ production** Results for the light Higgs-boson $h$ with mass $M_h = 300$ GeV, from the measurement of the two-photon width (times the vector-boson branching ratio) and phase are presented in Fig. 1 for various $\tan \beta$ values and the charged Higgs-boson mass of 800 GeV. Statistical error contours (1σ) on the expected deviation from the Standard-Model predictions are obtained from the combined fit to the invariant-mass distributions for $W^+W^-$ and $ZZ$ events. They correspond to $L_{\gamma\gamma} \approx 840$ fb$^{-1}$. These contours show that the measurement of the two-photon width and phase for the light Higgs-boson $h$ decaying into $W^+W^-$ and $ZZ$ would allow a precise determination of the $\tan \beta$ value. The possible ambiguity in the measurement of the two-photon width, observed for low value of $\Gamma_{\gamma\gamma} \cdot BR(h \rightarrow VV)$, can be resolved by the phase measurement, which clearly distinguishes between low $\tan \beta$ and large $\tan \beta$ solutions.

The statistical error on the extracted $\tan \beta$ value is shown in Fig. 2 for different values of the light Higgs-boson mass $M_h$. The expected error in the $\tan \beta$ determination is smallest (from about 1.5% for $M_h = 200$ GeV to about 4% for $M_h = 350$ GeV) for $\tan \beta \approx 0.7$, i.e. close to 1. Although the resonant production cross section is small in this region the Higgs-boson coupling to the vector bosons is most sensitive to $\tan \beta$. For very high and very low $\tan \beta$, when the relative Higgs-boson coupling to the vector bosons is close to ±1 (table 1), the precise measurement of $\tan \beta$ is not possible. For $\tan \beta \approx 1$ no direct measurement and no error estimate is possible, as the coupling to the vector bosons vanishes. In such a case only limits on the $\tan \beta$ value can be set within the considered model.

**Results for $H$ production** The measurement of the two-photon width and phase has been investigated also for the heavy scalar Higgs-boson $H$ of the SM-like Two Higgs Doublet Model (II) sol. $B_h$ (as before), with couplings as given in table 1. Statistical error contours (1σ) on the expected deviations from the Standard Model predictions are presented in Fig. 3, for the heavy scalar $H$ with mass $M_H = 300$ GeV, while a light Higgs-boson mass is set to 120 GeV and that of the charged Higgs-boson to 800 GeV. For $\tan \beta \sim 1$ both the two-photon width and phase of $H$ are close to the expectations of the Standard Model (for a given $M_H$). For $\tan \beta > 1$ both the top-quark and $W$ contributions to the two-photon width are strongly suppressed and the precision of the measurement deteriorates fast with increasing $\tan \beta$. For
The deviation from the SM for the light Higgs-boson $h$ with mass 300 GeV in the SM-like 2HDM II (sol. $B_h$), with charged Higgs-boson mass of 800 GeV for different values of tan\(\beta\). Statistical error contours (1\(\sigma\)) on the measured deviation from the Standard Model predictions for the phase $\phi_{\gamma\gamma}$ and for $\Gamma_{\gamma\gamma} \times BR(h \to VV)$, correspond to $L_{\gamma\gamma} \approx 840$ fb\(^{-1}\). Contour labeled 'SM' indicates the expected precision for the Standard Model.

For $\tan\beta < 1$ the $W$-loop contribution decreases with decreasing $\tan\beta$, however the top-quark contribution to the two-photon width increases. As a result, the two-photon width decreases slightly for $\tan\beta \sim 0.5$ and then starts to increase with decreasing $\tan\beta$. For $\tan\beta \sim 0.1$ the Higgs-boson decay to $c\bar{c}$ starts to dominate. The expected number of events with the $W^+W^-$ and $ZZ$ decays drops rapidly and the measurement becomes problematic again.

The statistical error on the extracted $\tan\beta$ value is shown in Fig. 4. Results are given for four values of heavy scalar Higgs-boson mass $M_H$, from 200 to 350 GeV. The expected error in the $\tan\beta$ determination is smallest (1–2\%) for $\tan\beta \approx 0.2$. For larger values of $\tan\beta$, $0.3 \leq \tan\beta \leq 0.8$, the precision depends strongly on the Higgs-boson mass. For mass of 200 GeV it changes between 2 and 4\%, whereas for mass of 350 GeV it is between 2 and 10\%. We checked that the precise measurement is also possible for $1.5 \leq \tan\beta \leq 5$, with statistical errors from 3–4\% for $M_H = 200$ GeV to 10–20\% for $M_H = 350$ GeV.
2HDM(II) Sol.B

\[ \tan(\beta) = \frac{\text{tg}(\beta)}{\text{tg}(\beta)} \times \% \]

200 GeV

250 GeV

300 GeV

350 GeV

Figure 2: Statistical error in the determination of $\tan \beta$, for four values of the light Higgs-boson mass $M_h$. The simultaneous fit to the observed $W^+W^-$ and $ZZ$ mass spectra is considered for the SM-like 2HDM II (sol. $B_h$), with charged Higgs-boson mass of 800 GeV. Centre-of-mass energy of colliding electron beams $\sqrt{s_{ee}}$ is optimized for each mass $M_h$.

5 Determination of $H - A$ mixing for model $B_h$ with a weak CP violation

In the general Two Higgs Doublet Model [16], the mass eigenstates of the neutral Higgs-bosons $h_1$, $h_2$ and $h_3$ do not match CP eigenstates $h$, $H$ and $A$. We consider here the CP-violating Two Higgs Doublet Model based on solution $B_h$, with a weak CP violation through a small mixing between $H$ and $A$ states. We study a simple option, where the couplings of the lightest mass-eigenstate $h_1$ (with mass 120 GeV) are expected to correspond to the couplings of $h$ boson, whereas couplings of $h_2$ and $h_3$ states can be described as the superposition of $H$ and $A$ couplings, as follows from table 1. For the relative basic couplings we have:

\[
\begin{align*}
\chi_{h_1}^X & \approx \chi_h^X, \\
\chi_{h_2}^X & \approx \chi_X^H \cdot \cos \Phi_{HA} + \chi_X^A \cdot \sin \Phi_{HA}, \\
\chi_{h_3}^X & \approx \chi_X^A \cdot \cos \Phi_{HA} - \chi_X^H \cdot \sin \Phi_{HA},
\end{align*}
\]  

where $X$ denotes a fermion or a vector boson, $X = u, d, V$. We study the feasibility of the determination of the mixing angle $\Phi_{HA}$ from the combined measurement of the two-photon width and phase for the Higgs-boson mass-eigenstate $h_2$. 

9
Figure 3: As in Fig. 1 for the heavy Higgs-boson $H$ with mass 300 GeV. A light Higgs-boson mass is assumed to be $M_h = 120$ GeV.

Figure 4: As in Fig. 2 for a heavy Higgs boson $H$, with a light Higgs-boson mass of $M_h = 120$ GeV.
It should be stressed that, in the considered case of CP violation via $H - A$ mixing, only the invariant mass distributions are sensitive to the mixing angle $\Phi_{HA}$. In contrast, the angular correlations in Higgs-boson decays $higgs \rightarrow WW/ZZ \rightarrow 4f$ can be used to establish an evidence for direct CP-violation in Higgs-boson couplings [17].

We perform the combined analysis of $W^+W^-$ and $ZZ$ decay channels, of the two-photon width (times vector-boson branching ratio) and phase measurement for the scalar Higgs-boson $h_2$ with mass $M_{h_2} = 300$ GeV. Results are presented in Fig. 5, for the light Higgs-boson mass $M_{h_1} = 120$ GeV and $M_{H^{\pm}} = 800$ GeV. Error contours (1$\sigma$) on the measured deviation from the Standard Model predictions are shown for $\Phi_{HA} = 0$, i.e. when CP is conserved, and for CP violation with $\Phi_{HA} = \pm 0.3$ rad. Even a small CP-violation can significantly influence the measured two-photon width and two-photon phase, and therefore it is possible to determine precisely both the CP-violating mixing angle $\Phi_{HA}$ and the parameter $\tan \beta$.

Next we address a question: how well can one establish conservation of CP-symmetry in the considered model? The first estimate can be read out from Fig. 6 where the statistical error in the determination of the angle $\Phi_{HA}$, around $\Phi_{HA} = 0$ value, is shown. The results are presented as a function of $\tan \beta$ for four values of Higgs-boson mass $M_{h_2}$, from 200 to 350 GeV. As above, we assume a light Higgs-boson mass is equal to 120 GeV and the charged Higgs-boson mass of 800 GeV. Here $\Phi_{HA}$ is considered as the only free parameter in the fit. Influence of error correlations between $\Phi_{HA}$ and $\tan \beta$, which have to be taken into account when both parameters are determined simultaneously from the fit, will be discussed in section 6.

The expected statistical error in the determination of $\Phi_{HA}$ is smallest ($\sim 20$ mrad) for $\tan \beta \approx 0.3$. For $\tan \beta \sim 1$ the error changes from about 30 mrad for mass of 350 GeV to 80 mrad for mass of 200 GeV. For larger values of $\tan \beta$ the precision of the measurement worsens fast with increasing $\tan \beta$ value. In the considered range of $\tan \beta$ the precision of phase measurement improves when Higgs-boson mass increases.

6 Systematic uncertainties

In sections 4 and 5 we considered the statistical errors of the extracted model parameter ($\tan \beta$ or $\Phi_{HA}$) expected from the one-parameter fit of the theoretical expectations (cross section convoluted with luminosity spectra and detector resolution) to the measured invariant-mass distributions of $W^+W^-$ and $ZZ$ events. However, as a large sample of events is expected, especially in the $\gamma\gamma \rightarrow W^+W^-$ channel, systematic uncertainties can significantly influence the final precision and they have to be taken into account. In case of 2HDM with CP violation, also the possible correlations between $\Phi_{HA}$ and $\tan \beta$ have to be considered if both parameters are used in the fit.

The following sources of the experimental systematic uncertainties were considered in the presented analysis:

- uncertainty in the integrated $\gamma\gamma$ luminosity
- uncertainty in the shape of the luminosity spectra
Figure 5: As in Fig.1 for the SM-like 2HDM II (sol. B_h) with CP-violation for the heavy Higgs-boson $h_2$ with mass 300 GeV and couplings from Eq. 1. A light Higgs-boson has mass $M_{h_1} = 120$ GeV. Three values of $H - A$ mixing angle $\Phi_{HA} = -0.3, 0, 0.3$ are considered.

Figure 6: Verifying a CP-conservation for the SM-like 2HDM II (sol. B_h). Statistical error in the determination of the $H - A$ mixing angle $\Phi_{HA}$ for four values of heavy Higgs-boson mass $M_{h_2}$, as a function of $\tan \beta$ value. One parameter fit to the observed $W^+W^-$ and $ZZ$ mass spectra, is considered for the SM-like 2HDM II (sol. B_h) with weak CP-violation, for light Higgs-boson mass of 120 GeV, charged Higgs-boson mass of 800 GeV, and $\Phi_{HA} = 0$, Eq. (1).
• uncertainty in the Higgs-boson mass (from other measurements)
• uncertainty in the total Higgs-boson width
• energy and mass scale uncertainty of the detector
• uncertainty in the reconstructed mass resolution

In order to take these uncertainties into account we allow additional parameters to vary in the fit. Three model parameters, which were fixed in the previous approach, are now considered as free parameters: the integrated $\gamma\gamma$ luminosity, the Higgs-boson mass and the total Higgs-boson width. To describe uncertainty in the shape of the luminosity spectra, two new parameters $A$ and $B$ were introduced, modifying the CompAZ spectra according to the formula:

$$\frac{dL}{dW_{\gamma\gamma}} = \frac{dL^{\text{CompAZ}}}{dW_{\gamma\gamma}}(1 + A \cdot \sin \pi x + B \cdot \sin 2\pi x)$$

where $x = \frac{W_{\gamma\gamma} - W_{\text{min}}}{W_{\text{max}} - W_{\text{min}}}$. This accounts for possible smooth variations of the luminosity spectra in the invariant-mass range from $W_{\text{min}}$ to $W_{\text{max}}$ considered in the fit.\(^5\) If the detector energy scale and mass resolution are also considered as free parameters in the fit, very large correlations between fitted parameters are observed. This is because, in the limited mass range used for the fit, deviation of the invariant mass spectra due to the energy scale shift is similar to the one resulting from the shift in the Higgs-boson mass. Similar is true for the mass resolution and the Higgs-boson width. Therefore, energy scale and mass resolution were fixed in the fit as the variations of the Higgs-boson mass and total width already account to large extent for possible uncertainties of these parameters.

Variations of the five parameters listed above allow us to account for possible deviations of the invariant-mass distributions from the nominal model expectation due to the experimental systematic uncertainties. We do not impose any additional constraints on these parameters, which could arise from the independent measurements (e.g. luminosity measurement in other process or Higgs-boson mass measurement at LC). Therefore our estimate of systematic effects should be considered as the conservative one.

In addition to all experimental uncertainties also theoretical uncertainties should be considered. Unfortunately they are difficult to estimate, partly because some of corrections have not yet been calculated. Therefore our study is $K$-factor = 1 type of analysis and should be extended by including higher order corrections in the future.

In Fig. 7 we present the influence of systematic effects on $\tan \beta$ (upper plot) and $\Phi_{HA}$ (lower plot) measurement, for a heavy Higgs-boson with mass of 300 GeV. We assume here the SM-like 2HDM II (sol. $B_h$), with the light Higgs-boson mass of 120 GeV, charged Higgs-boson mass of 800 GeV, and no $H - A$ mixing (i.e. $\Phi_{HA} = 0$). Errors expected without (dashed lines) and with (solid lines) systematic uncertainties are compared. Also shown is the comparison of the errors expected from the simultaneous fit of both $\tan \beta$ and $\Phi_{HA}$ (thick lines) and from separate fits (thin lines). Systematic uncertainties significantly influence the precision of the

---

\(^5\)The width of the invariant-mass window in which the fit was performed changes from 60 GeV for Higgs-boson mass of 200 GeV to 100 GeV for mass of 350 GeV.
measurement, both for \( \tan \beta \) and \( \Phi_{HA} \). The effect depends strongly on \( \tan \beta \) value. Systematic effects increase the expected error by up to factor of 5 for \( \tan \beta \) measurement at the highest \( \tan \beta \) values. Also the correlations between \( \tan \beta \) and \( \Phi_{HA} \), in the simultaneous fit of both parameters, increase the expected errors, for some cases by factor of 2 or more. It should also be noted that the effect of the parameter correlations is significantly larger when systematic uncertainties are taken into account.

7 Final results including systematic uncertainties

Results presented in sections 4 and 5 have been corrected for the systematic effects and the possible parameter correlations, as described in section 6. Final results of the analysis, for sol. \( B_h \) without CP violation, are presented in Fig. 8. Total error in the determination of \( \tan \beta \), as expected from the combined fit to the observed \( W^+W^- \) and \( ZZ \) mass spectra, is presented for the light (upper plot) and the heavy (lower plot) Higgs boson. In the wide range of the considered \( \tan \beta \) and Higgs-boson mass values, \( \tan \beta \) can be measured with precision better than 10%. Although the systematic uncertainties significantly influence the measurement, the total error of the order of 2% is still expected for most favorable choice of model parameters.

Total errors in the determination of \( \tan \beta \) and the angle \( \Phi_{HA} \), for sol. \( B_h \) with a possible weak CP violation through a small mixing between \( H \) and \( A \) states, are presented in Fig. 9, for four values of heavy Higgs-boson mass. Errors are evaluated for light Higgs-boson mass of 120 GeV, charged Higgs-boson mass of 800 GeV, and \( \Phi_{HA} = 0 \). The error on \( \tan \beta \) increases significantly when \( \Phi_{HA} \) is included in the fit (compare Fig. 8). In most of the considered parameter space it is between 5 and 20%. The error on \( \Phi_{HA} \) is below \( \sim 100 \) mrad for \( \tan \beta \leq 1 \) and increases rapidly for high \( \tan \beta \) values.

We conclude that in the low \( \tan \beta \) region the assumption that CP-symmetry is conserved in SM-like 2HDM (II) can be precisely verified at the Photon Collider. However, the fact that the CP-conserving solution \( B_h \) fails to describe the data, does not necessarily prove the violation of CP within this model. Observed discrepancies could also point to the more general solution of 2HDM (II). In such a case combined analysis of LHC, Linear Collider and Photon Collider data is needed to establish a possible evidence for CP-violation [18, 19].

8 Conclusions

The feasibility of measuring the Higgs-boson properties at the the Photon Collider at TESLA has been studied in detail for masses between 200 and 350 GeV, using realistic luminosity spectra and detector simulation. We consider the so called solution \( B_h \) of the Standard Model-like Two Higgs Doublet Model, with and without CP-conservation. For the CP conserving case, Yukawa couplings of the lightest Higgs-boson \( h \) have the same absolute values as in the Standard Model, and the coupling to the EW gauge-bosons is governed by only one parameter - \( \tan \beta \). We consider this simple model a generalized SM-like solution \( B \) for \( h \), as the LHC measurement of \( h \) production in the gluon-gluon fusion process would indicate no deviations
Figure 7: Influence of the systematic uncertainties and parameter correlations on the expected precision in determination of $\tan \beta$ (upper plot), and the mixing angle $\Phi_{HA}$ (lower plot) for heavy Higgs-boson mass of 300 GeV. Errors obtained from the simultaneous fit of both parameters (thick lines) and from separate fits (thin lines) are compared. The fit to the observed $W^+W^-$ and $ZZ$ mass spectra, is considered for the SM-like 2HDM II (sol. $B_h$), with the light Higgs-boson mass of 120 GeV, charged Higgs-boson mass of 800 GeV, and no $H - A$ mixing ($\Phi_{HA} = 0$), Eq. 1.
Figure 8: Total error in the determination of $\tan \beta$, for four values of heavy Higgs-boson mass. The simultaneous fit to the observed $W^+W^-$ and $ZZ$ mass spectra, is considered for the light Higgs boson (upper plot) and the heavy Higgs boson (lower plot) of the SM-like 2HDM II (sol. $B_h$), with charged Higgs-boson mass of 800 GeV. For measurement of a heavy Higgs boson $H$, a light Higgs-boson mass is set to $M_h = 120$ GeV. Systematic uncertainties related to the luminosity spectra, Higgs boson mass and total width, energy scale and mass resolution are taken into account.
Figure 9: Total error in the determination of tan $\beta$ (upper plot) and the $H - A$ mixing angle $\Phi_{HA}$ (lower plot), as a function of tan $\beta$ value, for four values of heavy Higgs-boson mass $M_{h2}$. The simultaneous fit of both parameters to the observed $W^+W^-$ and $ZZ$ mass spectra, is considered for the SM-like 2HDM II (sol. $B_h$), with light Higgs-boson mass of 120 GeV, charged Higgs-boson mass of 800 GeV, and no $H - A$ mixing ($\Phi_{HA} = 0$), Eq. 1. Systematic uncertainties related to the luminosity spectra, Higgs boson mass and total with, energy scale and mass resolution are taken into account.
from SM. From the combined measurement of the invariant-mass distributions in the $ZZ$ and $W^+W^-$ decay-channels, the parameter of the model can be precisely determined. After taking into account possible systematic uncertainties of the measurement, we found out that after one year of Photon Collider running the expected precision in the measurement of the Higgs-boson coupling ($\tan \beta$) is of the order of 10%, for both light and heavy scalar Higgs boson. In case of the Two Higgs Doublet Model solution $B_h$ with a weak CP violation, the $H - A$ mixing angle can be constrained. For low $\tan \beta$ values precision of about 100 mrad can be obtained (in a small-mixing approximation).

Acknowledgments

We would like to thank our colleagues from the ECFA/DESY study groups for useful comments and suggestions. This work was partially supported by the Polish Committee for Scientific Research, grant no. 1 P03B 040 26 and project no. 115/E-343/SPB/DESY/P-03/DWM517/2003-2005. P.N. acknowledges a partial support by Polish Committee for Scientific Research, grant no. 2 P03B 128 25. M.K. acknowledges a partial support by the European Community’s Human Potential Programme under contract HPRN-CT-2000-00149 Physics at Colliders.

References

[1] I. F. Ginzburg, G. L. Kotkin, V. G. Serbo and V. I. Telnov, JETP Lett. 34 (1981) 491 [Pisma Zh. Eksp. Teor. Fiz. 34 (1981) 514];
   I. F. Ginzburg, G. L. Kotkin, V. G. Serbo and V. I. Telnov, Nucl. Instrum. Meth. 205 (1983) 47;
   I. F. Ginzburg, G. L. Kotkin, S. L. Panfil, V. G. Serbo and V. I. Telnov, Nucl. Instrum. Meth. A219 (1984) 5;
   V. I. Telnov, Nucl. Instrum. Meth. A294 (1990) 72.

[2] P. Nieżurawski, A. F. Żarnecki, M. Krawczyk, J. High Energy Phys. 11 (2002) 034.

[3] TESLA Technical Design Report, Part 6, Chapter 1: B. Badelek et al., Photon Collider at TESLA, Int. J. Mod. Phys. A19 (2004) 5097.

[4] I. F. Ginzburg and I. P. Ivanov, Phys. Lett. B408 (1997) 325.

[5] G.J. Gounaris, J. Layssac, P.I. Porfyriadis, F.M. Renard, Eur.Phys.J. C13 (2000) 79.

[6] G.J. Gounaris, P.I. Porfyriadis and F.M. Renard, Eur. Phys. J. C19 (2001) 57.

[7] E. Asakawa, J. i. Kamoshita, A. Sugamoto and I. Watanabe, Eur. Phys. J. C14 (2000) 335; E. Asakawa, S. Y. Choi, K. Hagiwara and J. S. Lee, Phys. Rev. D62 (2000) 115005.

[8] E. Asakawa and K. Hagiwara, Eur. Phys. J. C31 (2003) 351;

[9] P. Nieżurawski, A. F. Żarnecki, M. Krawczyk, submitted to EPS’2003, July 17-23 2003, Aachen, Germany, abstract # 605; hep-ph/0307175.
[10] A.F. Żarnecki, Acta Phys.Polon. B34 (2003) 2741, 
http://info.fuw.edu.pl/~zarnecki/compaz/compaz.html

[11] V. I. Telnov Nucl. Instrum. Meth. A 355 (1995) 3; V. I. Telnov, A Code for the simulation of luminosities and QED backgrounds at photon colliders, talk presented at Second Workshop of ECFA-DESY study, Saint Malo, France, April 2002.

[12] T. Sjostrand, P. Eden, C. Friberg, L. Lonnblad, G. Miu, S. Mrenna and E. Norrbin, Comp. Phys. Comm. 135 (2001) 238.

[13] G.Belanger, F.Boudjema, Phys. Lett. B288 (1992) 210; D.A.Morris, et al., Phys. Lett. B323 (1994) 421.

[14] M. Pohl, H. J. Schreiber, DESY-99-030.

[15] I. F. Ginzburg, M. Krawczyk and P. Osland, Nucl. Instrum. Meth. A472:149, 2001; hep-ph/0101331; hep-ph/0101208.

[16] I. F. Ginzburg, M. Krawczyk and P. Osland, hep-ph/0211371; I. F. Ginzburg and M. Krawczyk, hep-ph/0408011.

[17] D.J. Miller, S.Y. Choi, B. Eberle, M.M. Mühlleitner and P.M. Zerwas, Phys. Lett. B505 (2001) 149; S.Y.Choi, D.J.Miller, M.M.Mühlleitner and P.M.Zerwas, Phys. Lett. B553 (2003) 61; C.P.Buszello, I.Fleck, P.Marquard, J.J. van der Bij, Eur. Phys. J. C32 (2004) 209; C. P. Buszello, P. Marquard and J. J. van der Bij, hep-ph/0406181; M.T.Dova, S.Ferrari, Phys. Lett. B605 (2005) 376; P.Nieżurawski, A.F.Żarnecki, M.Krawczyk, Acta Phys. Polon. B36 (2005) 833.

[18] P.Nieżurawski, A.F.Żarnecki, M.Krawczyk, Determination of the basic Higgs-boson couplings from combined analysis of WW/ZZ decays at LHC, LC and Photon Collider, submitted to ICHEP’2004, abstract #12-0740.

[19] R.M.Godbole et al., CP Studies of the Higgs Sector: A contribution to the LHC / LC Study Group document, hep-ph/0404024; LHC/LC Study Group: G. Weiglein et al., Physics Interplay of the LHC and the ILC, hep-ph/0410364.