Interpreting and Extending the Guided Filter via Cyclic Coordinate Descent

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Abstract—The guided filter (GF) is a widely used smoothing tool in computer vision and image processing. However, to the best of our knowledge, few papers investigate the mathematical connection between this filter and the least-squares optimization. In this paper, we first interpret the guided filter as the cyclic coordinate descent (CCD) solver of a least-squares objective function. This discovery implies an extension approach to generalize the guided filter since we can change the least-squares objective function and define new filters as the first pass iteration of the CCD solver of modified objective functions. In addition, referring to the iterative minimizing procedure of the CCD, we can derive new rolling filtering schemes. So, we are reasonable to say that our discovery not only reveals an approach to design new GF-like filters adapting to specific requirements of applications but also offers thorough explanations for two rolling filtering schemes of the guided filter as well as the method to extend them. Experiments prove our new proposed filters and rolling filtering schemes could produce state-of-the-art results.

Index Terms—Guided filter (GF), cyclic coordinate descent (CCD), rolling filtering schemes.

I. INTRODUCTION

The most fundamental smoothing tools in image processing should be edge-aware filters. We can roughly divide them into two categories: the explicit filter and the implicit filter. The first kind of filters takes a mapping operator to transform inputs to outputs. The well-known Gaussian, bilateral and guided filters [1]–[3] are part of this type because we can formulate them as the convolution operators explicitly. In contrast, the mapping operator of the implicit filter is not given. Instead, the filtering output is considered as the minimizer of an objective function. Xu et al. [4] provide an instance of this kind of filters.

Each kind of filters has its merits and demerits. The explicit filter is easy to implement and often has low computational cost. But, it is not an easy task to design a new explicit filter and analyze its rolling filtering behavior as people usually define the mapping operator according to their intuition and this encumbers the mathematical analysis of the explicit filter. In order to achieve state-of-the-art results, the implicit filter spend considerable computational time. This is because the final results are yielded by the iterative solvers such as gradient descent [5]. Disregarding the above shortcoming, the implicit filter brings more convenience: designing a new implicit filter can be reduced to proposing an objective function and finding its solver, which has been well studied and owns a solid theoretical foundation.

It is possible to draw upon the strong points of one kind of filters to overcome the shortcomings of another kind of filters. Specifically, if we establish the connection between the mapping operator of the explicit filter and the iterative solver of the objective function of the implicit filter, the problem cannot be addressed from the explicit/implicit perspective alone would be solved by unifying two filtering viewpoints. The benefits of this joint perspective are twofold:

• the filtering behavior of the explicit filter as well as its rolling filtering usage is described by the iterative solver. We thus not only can define new filters by modifying objective functions but also are able to disclose their rolling filtering usages from the minimizing procedure of their iterative solver.

• the explicit filter deepens our understanding of the implicit filter as it entitles each minimizing pass a filtering connotation other than its original optimization interpretation. This connection facilitates the intuitive understanding of each iteration as well as its functions in optimization.

Establishing the connection between the explicit filter and the implicit filter is not new. Let \( q, p, L \) and \( \Lambda \) be an \( N \times 1 \) output vector, constraint, \( N \times N \) Laplacian matrix and diagonal...
matrix, respectively. He et al. [3] proved the output of the guided filter approximates to one Jacobi iteration in optimization (1). But He’s discovery leaves much to be desired because the guided filter and the iterative solver of optimization (1) are not strictly equal and therefore the Jacobi algorithm cannot describe the behavior of guided image filtering with multiple times.

$$\min_{q} (q - p)^{T} \Lambda (q - p) + q^{T} L q$$  \hspace{1cm} (1)$$

Considering the potential benefits of the joint perspective for the guided filter [3], we will disclose the equivalence between the guided filter and the cyclic coordinate descent solver of the least squares optimization. Further, the connection is exploited to extend the guided filter as well as its rolling filtering scheme. Our main contributions are threefold:

- We unveil that the guided filter equals to the cyclic coordinate descent solver of a least squares objective function and point out the rolling filtering usage of the guided filter can be viewed as the minimizing procedure for the objective function.
- We find a general framework to define new GF-like filters and develop novel instances of GF-like filters in this framework.
- We offer a mathematical foundation for two rolling filtering schemes of the guided filter and the approach to extend them.

II. RELATED WORK

In the literature, a lot of efforts are devoted to disclosing the connection between the explicit filter and the implicit filter. Li and Osher [6] reveal the median filter corresponds to the analytic form of the minimizer of the sum of the weighted absolute error, thus it has fully explainable connections to global energy minimization. However, this discovery between the mapping operator and the closed-form solution is not as useful as the relationship that connects the mapping operator of an explicit filter to the iterative solver of the implicit filter since the latter makes the two different filters be complementary.

Different from the way of [6], Elad [7] shows the bilateral filter (BF) [2] emerges from a Bayesian optimization, as a single iteration of the well-known Jacobi algorithm. But he did not prove the bilateral filter is the solver of the Bayesian optimization. Dong and Acton [8] make progress by showing the bilateral filter equals to the iterative reweighting solver, which is a good approximation to the Newton’s method [5]. Later, Caraffa et al. [9] introduce guidance information to the robust optimization and define the guided bilateral filter as the solver of the robust optimization. Imitating the iteratively minimizing procedure, they invent a rolling filtering scheme for the guided bilateral filter.

Since 2010, the guided filter has attracted much attention. In order to address its defect caused by the box window, Lu et al. [10] design a cross window for the cross-based local multipoint filter (CLMF). However, their geometric-adaptive window is still problematic. To solve the problem, Tan et al. [11] design a symmetrical window for their multipoint filter with local polynomial approximation and range guidance (MLPA). In addition, Tan et al. [11] introduce a spatial regularization term to the guided filter and make their MLPA spatial-aware. But the ability is at the cost of increasing runtime drastically. This shortcoming is conquered by Dai et al. [12] who assemble the fully connected guided filter (FCGF) by introducing tree distance to the guided filter. Similarly, incorporating edge-aware weights into the guided filter, Li et al. [13] propose the weighted guided filter (WGF) to address the halo artifacts of the guided filter. Unlike Li et al. [13], Qiu et al. [14] propose the guided image filtering as a nonconvex optimization, which is solved by the majorize-minimization algorithm.

Although the guided filter is designed as one pass, non-iterative filter, its rolling filtering scheme arouses extensive concerns recently. Seo and Milanfar [16] propose an iterative guided filtering method, which is also adopted by Yelameli et al. [17], for robust flash denoising/deblurring. Unlike previous work, Zhang et al. [18] propose a novel Rolling Guidance Filtering (RGF) scheme with the complete control of detail smoothing under a scale measure. However, none of above work establishes the connection between the guided filter and some iterative solvers and thus is not able to benefit from unifying two filtering schemes.

The remainder of this paper is organized as follows: in section III, we establish the connection between the guided filter and the cyclic coordinate descent solver of the least squares optimization. Employing this connection, we propose several new GF-like filters and rolling filtering schemes in section IV. In the last section, we conduct extensive experiments to disclose the smoothing ability of newly proposed filters and rolling filtering schemes. Final results prove that our filters and rolling filtering schemes are superior to other methods. At last we note that our paper involves lots of abbreviations. For easy reference, we list all of them in Table I.

III. THE EQUIVALENCE

The guided filter has a close connection with the cyclic coordinate descent solver of the least squares optimization. As outlined in Fig 1, we will devote this section to exhibit the equivalence between the mapping operator of the guided filter and the cyclic coordinate descent solver of the least squares optimization.

A. The Definition of the Guided Filter

Initially, the guided filter is defined in the two steps local multipoint filtering framework [19]:

1) Multipoint Estimation: calculating multipoint estimates \(q_{i}^{k}\) for each pixel \(i\) in the image domain \(\Omega\) according to the linear transform \(q_{i}^{k} = a_{k} I_{i} + b_{k}, \forall k \in \omega_{i}\) in a window \(\omega_{i}\) centered at \(i\), where \(I\) is the guidance image, \((a_{k}, b_{k})\) is the minimizer (3) of (2), \(p\) is the filtering input, \(\varepsilon\) is a constant,
Table I. Lists Of Abbreviations

| Abbreviation | Description               |
|--------------|---------------------------|
| CCD          | Cyclic Coordinate Descent |
| BF           | Bilateral Filter          |
| CLMF         | Cross-based Local Multipoint Filter |
| MLPA         | Multipoint filter with Local Polynomial Approximation and range guidance |
| FCGF         | Fully Connected Guided Filter |
| WGF          | Weighted Guided Filter    |
| LLSURE       | Local Linear SURE-based Edge-preserving filter |
| GF           | Guided Filter             |
| IGF          | Inverse Guided Filter     |
| SGF          | Spatial-aware Guided Filter |
| TVGF         | Total Variation Guided Filter |
| CGF          | Conservative Guided Filter |
| ICGF         | Inverse Conservative Guided Filter |
| RGF          | Rolling Guidance Filtering |
| RMSF         | Rolling Mutual Structure Filtering |
| GF-RMSF      | GF based RMSF             |
| CGF-RMSF     | CGF based RMSF            |
| RFN filtering| Rolling Flash/No-Flash Filtering |
| SIMD         | Single Instruction Multiple Data |

Mathematically, the cyclic coordinate descent is based on the idea that the minimizer of a multivariable function $F$ can be obtained by minimizing it along one direction at a time. That is, in each iteration, for each index $i$, we can verify that $\theta(3)$ $\theta(4)$ are the closed-form solutions of $\theta(6)$.

By cyclically minimizing $\theta(3)$ $\theta(4)$, we can obtain a sequence $x^n_0$ and get a sequence $x^n_0$ $\ldots$ that has $F(x^n_0) \geq F(x^n_1) \geq F(x^n_2) \geq \ldots$.

Applying the cyclic coordinate descent algorithm to optimize the objective function $\theta(6)$, we can verify that $\theta(3)$ $\theta(4)$ are the closed-form solutions of $\theta(6)$, by cyclically minimizing $\theta(a,b)$. In the first step, let $q^n_0 = p$ and $\theta_0(q^n, I, \varepsilon) = \sum_{i\in\Omega}(a_k I_i + b_k - q^n_0)^2 + \varepsilon d_k^2$, the cyclic coordinate descent minimizes optimization $\theta(7)$.

$$\min_{a_k,b_k} \sum_{k\in\Omega}(a_k I_i + b_k - q^n_0)^2 + \varepsilon a_k^2$$

We can formulate the closed-form solutions of $a^n_0, b^n_0$ of $\theta(7)$ as $\theta(8)$.

$$a^n_{k+1} = \arg \min_{a_k,b_k} \theta_0(q^n, I, \varepsilon)$$

$$b^n_{k+1} = \theta_0(q^n, I, \varepsilon) - a^n_{k+1} E_{\theta_0}(I)$$

In the second step with fixed $a^n_{k+1}, b^n_{k+1}$, the cyclic coordinate descent algorithm computes the minimizer of $\theta(9)$.

$$q^{n+1}_i = \arg \min_{q_i} \theta_1(a^{n+1}, b^{n+1}, I)$$

where $\theta_1(a^{n+1}, b^{n+1}, I) = \sum_{k\in\Omega}(a^{n+1} I_i + b^{n+1} - q_i)^2$ and $q^{n+1}_i$ can be formulated as $\theta(10)$.

$$q^{n+1}_i = E_{\theta_0}(a^{n+1}) I_i + E_{\theta_0}(b^{n+1})$$
We note that (8) (10) are same to (3) (4) except for an extra iteration index n. This implies the guided filter equals to the first cyclic coordinate descent iteration of (6) with an initial guess $q^0 = p$. Moreover, let GF($q^n, I, \varepsilon$) denote the filtering outputs of the guided filter with respect to the input $q^n$, the rolling filtering scheme (11)

$$q^{n+1} = \text{GF}(q^n, I, \varepsilon)$$

(11)
can be interpreted as the cyclic coordinate descent minimizing procedure of (6). The reason is that the filtering result of the guided filter with the input $q^n$ obtained from the $n^\text{th}$ cyclic coordinate descent iteration equals to $q^{n+1}$ i.e. $n+1^\text{th}$ cyclic coordinate descent iteration. To further broaden the understanding for above two equivalence, we outline this discovery and main idea in Fig 1. Specifically, by exploiting the equivalence,

- we can modify the objective function and exploit the cyclic coordinate descent to define new GF-like filters due to the equivalence between the guided filter and one cyclic coordinate descent iteration.
- we can derive rolling filtering schemes from the cyclic coordinate descent algorithm because the iteratively minimizing procedure of the cyclic coordinate descent indicates new rolling filtering schemes.

IV. THE METHOD FOR EXTENSION

The equivalence between the guided filter and the cyclic coordinate descent algorithm exposes possible ways to extend the filter. In this section, we are going to develop new GF-like filters and their rolling filtering schemes according to the technical roadmap illustrated in Fig 1.

A. New GF-Like Filters

New filters can be derived by modifying the objective function (6) and defining them as the first CCD pass of new objective functions. Following this roadmap, we propose five GF-like filters (i.e, SFG, TVFG, CGF, IGF, ICGF) and two rolling filtering schemes (i.e, CGF-RMSF and RFNF). Refer to Table I for details of these abbreviations.

1) The Spatial-Aware Guided Filter (SGF): Tan et al. [11] and Dai et al. [12] complain that the guided filter only considers the color similarity. To address the problem, Tan introduces a spatial regularization term into his multipoint filter and Dai appends a spatial tree weight into the fully connected guided filter. However, both methods will increase the runtime significantly. In addition, the fully connected guided filter is apt to yield artifacts in the rolling guidance filtering [18] as the minimal spanning tree used by the fully connected guided filter often fails to represent the geometric structure of the iteratively refined guidance. In this regard, we derive the spatial-aware guided filter, a GF-like counterpart for the bilateral filter used in the rolling guidance filtering, from the cyclic coordinate descent solver of (12),

$$\min_{a_k, b_k} \sum_{k \in \Omega} \sum_{i \in \omega_k} (w_{ki} (a_k I_i + b_k - q_i)^2 + \varepsilon a_k^2)$$

(12)

where $\sigma$ is a constant controlling the spatial similarity. Our spatial-aware guided filter not only takes the Gaussian spatial similarity $w_{ki} = \exp(-\|k-i\|^2/\sigma)$ into account but also can be integrated with the rolling guidance filtering scheme [18] satisfactorily in terms of computational efficiency and smoothing quality.

The minimizer of (12) can be found by iteratively computing (13) (14) with an initial guess $q^0 = p$, where $E_{\omega_k} (x) = \sum_{j \in \omega_k} w_{kj} x_j / \sum_{j \in \omega_k} w_{kj}$ and $D_{\omega_k} (x) = E_{\omega_k} (x^2) - (E_{\omega_k} (x))^2$ denote weighted average/variance in the window $\omega_k$ with respect to the gaussian weight $w_{kj}$. Following the cyclic coordinate descent iteration for the guided filter, we define the spatial-aware guided filter as (13) (14) with $n = 0$ and $q^0 = p$.

$$a_k^{n+1}, b_k^{n+1} = \text{SGF}^n(q^n, I, \varepsilon)_k$$

$$= \arg \min_{a_k, b_k} \sum_{i \in \omega_k} (w_{ki} (a_k I_i + b_k - q_i)^2 + \varepsilon a_k^2)$$

$$a_k^{n+1} = \frac{E_{\omega_k} (I q^n) - E_{\omega_k} (I) E_{\omega_k} (q^n)}{D_{\omega_k} (I) + \varepsilon}$$

$$b_k^{n+1} = E_{\omega_k} (q^n) - a_k^{n+1} E_{\omega_k} (I)$$

$$q_i^{n+1} = \text{SGF}(q^n, I, \varepsilon)_i$$

$$= \arg \min_{q_i} \sum_{i \in \omega_k} w_{ki} (a_k^{n+1} I_i + b_k^{n+1} - q_i)^2$$

$$= \frac{E_{\omega_k} (a_k^{n+1} I_i + b_k^{n+1} - q_i)^2}{\varepsilon}$$

(13)

(14)

2) The Total Variation Guided Filter (TVGF): The guided filter has no idea about what kind of the output is preferred because the cost function (6) of the guided filter only considers the constraint between the guidance I and output q. In order to produce the most favorite noise free result, we assemble a new cost function (15)

$$\min_{q, a, b} \sum_{k \in \Omega} (\lambda TV^2 (q_k) + \sum_{i \in \omega_k} ((a_k I_i + b_k - q_i)^2 + \varepsilon a_k^2))$$

(15)

by appending a Total Variation (TV) regularization term $TV(g) = \sqrt{\sum_{i \in \omega_k} |\partial_i g|^2 + \varepsilon |\partial_i g|^2}$ to the cost function (6). Putting $P_2(a_k^{n+1}, b_k^{n+1}, I) = \sum_{k \in \Omega} (\lambda TV^2 (q_k) + \sum_{i \in \omega_k} ((a_k^{n+1} I_i + b_k^{n+1} - q_i)^2)$ and $q^0 = p$, we can achieve the minimizer by iteratively calculating (16) (17)

$$a_k^{n+1}, b_k^{n+1} = \arg \min_{a_k, b_k} P_2(q^n, I, \varepsilon)$$

$$q_i^{n+1} = \arg \min_{q_i} P_2(a_k^{n+1}, b_k^{n+1}, I)$$

(16)

(17)

according to the cyclic coordinate descent algorithm. The optimal solutions of $a_k^{n+1}, b_k^{n+1}$ are expressed as (18) which is same to the definition (7) of the guided filter.

$$a_k^{n+1} = \frac{E_{\omega_k} (I q^n) - E_{\omega_k} (I) E_{\omega_k} (q^n)}{D_{\omega_k} (I^n) + \varepsilon}$$

$$b_k^{n+1} = E_{\omega_k} (q^n) - a_k^{n+1} E_{\omega_k} (I)$$

(18)

Different from (9), the minimizer $q_i^{n+1}$ of (17) equals to (19),

$$q_i^{n+1} = \mathcal{F}^{-1} \left( \frac{\mathcal{F}(f^{n+1})}{|\omega_k|} \right)$$

(19)
where

\[ f_{i}^{n+1} = \sum_{k \in \Omega} q_{k}^{n+1} I_{i} + b_{k}^{n+1}, \]

\[ D = \mathcal{F}^{\ast}(\hat{c}_{\epsilon}) \mathcal{F}(c_{\epsilon}) + \mathcal{F}^{\ast}(\hat{c}_{\epsilon}) \mathcal{F}(\hat{c}_{\epsilon}), \]

\( \mathcal{F} \) is the Fast Fourier Transform (FFT) operator and \( \mathcal{F}^{\ast} \) denotes the complex conjugate. \( \mathcal{F}(1) \) is the Fourier Transform of the delta function and \( |o_{k}| \) denotes the pixel number in the window \( o_{k} \). Moreover, the plus, multiplication and division in (19) are all component-wise operators.

We define the total variation guided filter as the first cyclic coordinate descent pass of (18) (19). Above cyclic coordinate descent minimization can be written in the rolling filtering form (20), where \( q = TVGF(p, I, \epsilon, \lambda) \) denotes the filtering output \( q \) of the total variation guided filter.

\[ q^{n+1} = TVGF(q^n, I, \epsilon, \lambda) \quad (20) \]

3) The Conservative Guided Filter (CGF): We found that the minimizers of objective functions (6) (15) are trivial zeros. This discovery indicates that both guided filter and total variation guided filter will consume the “energy” of images at each iteration and therefore are dissipative. Note that previous GF-like filters such as the cross-based local multipoint filter [10], the multi-point filter with local polynomial approximation and range guidance [11], the weighted guided filter [13], the fully connected guided filter [12] and the local linear SURE-based edge-preserving filter [14] are also dissipative filters. Readers can verify this from their trivial results of multiple time filtering. We consider that an ideal filter should be conservative, i.e. the rolling filtering result must converge to a nontrivial solution. To achieve this goal, we build the cost function (21) and exploit its cyclic coordinate descent iteration to derive the conservative guided filter. Here \( g \) denotes a reference image used to constrain the derivation between the output \( q \) and \( g \) via the data term \((q_{i} - g_{i})^{2}\). In practice, we usually set it as the input \( p \). So, the solution of (21) must be nontrivial.

\[ \min_{q,a,b} \sum_{k \in \Omega} \sum_{i \in o_{k}} ((a_{i} I_{i} + b_{k} - q_{i})^{2} + \epsilon a_{i}^{2}) + \lambda (q_{i} - g_{i})^{2} \quad (21) \]

According to the cyclic coordinate descent algorithm, we can achieve the minimal point of the objective function (21) by iteratively computing (22) (23)

\[ a_{k}^{n+1}, b_{k}^{n+1} = \arg \min_{a_{k}, b_{k}} \mathcal{P}_{0}(q^n, I, \epsilon) \quad (22) \]

\[ q_{i}^{n+1} = \arg \min_{q_{i}} \mathcal{P}_{3}(a^{n+1}, b^{n+1}, I, g, \lambda) \quad (23) \]

with an initial guess \( q^{0} = p \), where

\[ \mathcal{P}_{3}(a^{n+1}, b^{n+1}, I, g, \lambda) = \sum_{k \in \Omega} (a_{k} I_{i} + b_{k} - q_{i})^{2} + \lambda (q_{i} - g_{i})^{2} \]

Note that the solution of (22) is same to (18). Putting \( \alpha = \frac{\lambda}{\sum_{i \in o_{k}} 1} \), we can reformulate the solution of (23) as

\[ q_{i}^{n+1} = (1 - \alpha)GF(q^n, I, \epsilon, \lambda) + \alpha g_{i} \quad (24) \]

Similar to the definition of the total variation guided filter, the conservative guided filter is defined as the first pass iteration of (22) (23). We thus have \( q = CGF(p, I, g, \epsilon, \lambda) = (1 - \alpha)GF(p, I, \epsilon) + \alpha g \). Further, the cyclic coordinate descent minimizing procedure for the objective function (21) can be reformulated in the rolling filtering form (25).

\[ q^{n+1} = CGF(q^n, I, g, \epsilon, \lambda) \quad (25) \]

4) The Inverse Guided Filters: In the filtering scheme of the guided filter, the guidance image is used to compute the smoothing result. People may raise the following question naturally: can we inverse the filtering procedure by estimating the guidance \( G \) from a smoothing result \( q \)? Luckily, the answer is positive. We employ the objective function (26) with the initial guess \( G^{0} = I \) to formulate the Inverse Guided Filter (IGF).

\[ \min_{G,a,b} \sum_{k \in \Omega} \sum_{i \in o_{k}} ((a_{k} G_{i} + b_{k} - q_{i})^{2} + \epsilon a_{i}^{2}) \quad (26) \]

Applying the cyclic coordinate descent algorithm to the cost function (26), we iteratively calculate following two subproblems (27) (28),

\[ a_{k}^{n+1}, b_{k}^{n+1} = \arg \min_{a_{k}, b_{k}} \mathcal{P}_{0}(q^n, G^n, \epsilon) \quad (27) \]

\[ q_{i}^{n+1} = \arg \min_{q_{i}} \mathcal{P}_{4}(a^{n+1}, b^{n+1}, G) \quad (28) \]

where \( \mathcal{P}_{0}(p, G^n, \epsilon) = \sum_{k \in \Omega} ((a_{k} G_{i} + b_{k} - q_{i})^{2} + \epsilon a_{i}^{2}) \) and

\[ \mathcal{P}_{4}(a^{n+1}, b^{n+1}, G) = \sum_{k \in \Omega} (a_{k} G_{i} + b_{k} - q_{i})^{2} \quad (29) \]

Minimizing \( \mathcal{P}_{0}(q^n, G^n, \epsilon) \) in (27), we can formulate the closed-form solution of \( a^{n+1}, b^{n+1} \) as (29).

\[ a_{k}^{n+1} = \frac{E_{0}(G^n q) - E_{0}(G^n)E_{0}(q)}{D_{0}(G^n) + \epsilon} \]

\[ b_{k}^{n+1} = E_{0}(q) - a_{k}^{n+1} E_{0}(G^n) \quad (29) \]

Solving the least squares optimization (28), we have (30)

\[ G^{n+1} = \frac{E_{0}(a^{n+1} q) - E_{0}(a^{n+1}q_{i}^{n+1})}{E_{0}(a^{n+1}q_{i}^{n+1})} \quad (30) \]

and define the inverse guided filter as the first cyclic coordinate descent pass of (29) (31) with \( G^{0} = I \) and denote its output \( G \) as (31).

\[ G = IGF(q, I, \epsilon) \quad (31) \]

Similarly, the Inverse Conservative Guided Filter (ICGF) \( G = ICFG(q, I, \epsilon, \lambda) \) is defined as the first cyclic coordinate descent pass of (32). Here \( g \) controls the derivation between the output \( G \) and \( g \). In practice, we usually set it as the input guidance \( I \). In the first step,

\[ \min_{G,a,b} \sum_{k \in \Omega} \sum_{i \in o_{k}} ((a_{k} G_{i} + b_{k} - q_{i})^{2} + \epsilon a_{i}^{2}) + \lambda (G_{k} - g_{k})^{2} \quad (32) \]

we solve \( a_{k}^{n+1}, b_{k}^{n+1} = \arg \min_{a_{k}, b_{k}} \mathcal{P}_{0}(q^n, G^n, \epsilon) \) and obtain (33).

\[ a_{k}^{n+1} = \frac{E_{0}(G^n q) - E_{0}(G^n)E_{0}(q)}{D_{0}(G^n) + \epsilon} \]

\[ b_{k}^{n+1} = E_{0}(q) - a_{k}^{n+1} E_{0}(G^n) \quad (33) \]

In the second step, we optimize \( \min_{G} \sum_{k \in \Omega} \sum_{i \in o_{k}} ((a_{k} G_{i} + b_{k} - q_{i})^{2} + \epsilon a_{i}^{2}) + \lambda (G_{k} - g_{k})^{2} \) and thus have (34).

\[ G^{n+1} = \frac{\sum_{k \in \Omega} (a_{k}^{n+1} q_{i} - a_{k}^{n+1} b_{k}^{n+1}) + \lambda g_{i}}{\sum_{k \in \Omega} (a_{k}^{n+1} b_{k}^{n+1})^{2} + \lambda} \quad (34) \]

Note that it is unwise to use the inverse guided filter or the inverse conservative guided filter alone because they usually do
are not equal. The reason is that in the rolling minimizing should be same to the input. In fact, the output and input will be itself. Intuitively, we may consider that the output are equal, the mutual structure for the same image improvement for these rolling filtering schemes.

**B. Rolling Filtering Schemes**

Although the guided filter is designed as a non-iterative filter, its rolling filtering usages still have important applications. We devote this section to the theoretical explanation and improvement for these rolling filtering schemes.

1) Rolling Mutual Structure Filtering (RMSF): The guided filter assumes the geometric structure of both guidance and input coincides with each other. But it is a strong assumption and will cause that the guided filter yields texture mapping artifacts. One way to deal with the inconsistent structure between the guidance and the input is to estimate their mutual structures. Based on the discovery of Shen et al. [20], we minimize (35) to obtain textureless results, where \( E(q, a, b, G, c, d) = \sum_{i}E_{i}\sum_{\varepsilon}(a_{i}G_{i} + b_{i} - q_{i})^{2} + \varepsilon a_{i}^{2} + ((c_{i}q_{i} + d_{i} - G_{i})^{2} + \varepsilon c_{i}^{2}) \).

\[
\min_{q,a,b,G,c,d} E(q, a, b, G, c, d) \quad (35)
\]

The optimization can be solved by iteratively computing four subproblems (36)-(39) with \( q^{0} = p \) and \( G^{0} = I \), where \( a_{i}(x) = \frac{1}{1 + k_{m}(x^{2})} \). (36) (37) are used to estimate the linear coefficients used in the guided filter and the inverse guided filter. (38) (39) calculate the linear combination of the guided filter and the inverse guided filter. So we say that we disclose the filtering explanation for the rolling mutual structure filtering successfully and thus we call above procedure the guided filter based rolling mutual structure filtering. For clarity, we list the guided filter based rolling mutual structure filtering algorithm in Algorithm 2.

\[
\begin{align*}
    a_{k}^{n+1}, b_{k}^{n+1} &= \arg\min_{a_{k}, b_{k}} P_{0}(q_{k}^{n}, G_{k}^{n}, \varepsilon) \quad (36) \\
    c_{k}^{n+1}, d_{k}^{n+1} &= \arg\min_{c_{k}, d_{k}} P_{0}(G_{k}^{n}, q_{k}^{n}, \varepsilon) \quad (37) \\
    q_{i}^{n+1} &= \arg\min_{q_{i}} P_{1}(a_{k}^{n+1}, b_{k}^{n+1}, G_{i}^{n}) \\
    &\quad + P_{2}(c_{k}^{n+1}, d_{k}^{n+1}, q_{i}^{n}) \\
    q_{i}^{n+1} &= a_{i}(c_{i}^{n+1})GF^{n}(G_{i}^{n}, q_{i}^{n}, \varepsilon) + (1 - a_{i}(c_{i}^{n+1}))IGF^{n}(G_{i}^{n}, q_{i}^{n}, \varepsilon) \quad (38) \\
    G_{i}^{n+1} &= \arg\min_{G_{i}} P_{1}(c_{k}^{n+1}, q_{i}^{n+1}, q_{i}^{n}) \\
    &\quad + P_{2}(a_{k}^{n+1}, b_{k}^{n+1}, q_{i}^{n}) \\
    G_{i}^{n+1} &= a_{i}(a_{i}(c_{i}^{n+1}))GF^{n}(G_{i}^{n}, q_{i}^{n}, \varepsilon) + (1 - a_{i}(a_{i}(c_{i}^{n+1})))IGF^{n}(G_{i}^{n}, q_{i}^{n}, \varepsilon) \quad (39)
\end{align*}
\]

Note that the filtering pair, the guided filter and the inverse guided filter, play important roles in the guided filter based rolling mutual structure filtering algorithm according to (38) (39). Specifically, when the guidance \( G^{0} \) and input \( q^{0} \) are equal, the mutual structure for the same image \( G^{0} = q^{0} \) will be itself. Intuitively, we may consider that the output should be same to the input. In fact, the output and input are not equal. The reason is that in the rolling minimizing procedure (38) (39), the guided filter smooths out details but the inverse guided filter plays the role of preserving the major structure from a smoothed input. Thanks to the two antagonistic terms, the rolling mutual structure filtering can preserve the major structure and suppress details/textures in final results. Due to the same reason, the output cannot be same to the input.

The objective function proposed by Shen et al. [20] can be reduced to (35) if Shen’s parameters \( \lambda = \beta = 0 \). However, the equivalence does not imply our filtering interpretation for the rolling mutual structure filtering is trivial. One major contribution of our work is that we first disclose the filtering explanation for each iteration step. In addition, we can disclose things that are not revealed by Shen. For instance, Shen reports the rolling filtering scheme (40) cannot produce mutual structure filtering results. However, he does not provide an explanation for its filtering behavior. Here we employ the guided filter based rolling mutual structure filtering stated above to illustrate the reason. Comparing (38) (39) with (40), we can find that (38) (39) have two extra inverse guided filter terms which learn guidance from input. So, the mutual structure filtering result is the linear combination of the guided filter and the inverse guided filter. In contrast, (40) only considers the guided filter part which will wipe out details without the help of the inverse guided filter.

\[
\begin{align*}
    \bar{q}_{i}^{n+1} &= GF(q_{i}^{n}, G_{i}^{n}, \varepsilon)_{i} \\
    \bar{G}_{i}^{n+1} &= GF(G_{i}^{n}, q_{i}^{n+1}, \varepsilon)_{i} \quad (40)
\end{align*}
\]

Another contribution of our filtering interpretation is that we can employ the rolling mutual structure filtering interpretation to define the inverse conservative guided filter based rolling mutual structure filtering. This is because we can substitute the filter pair \( GF(p, I, \varepsilon), IGF(p, I, \varepsilon) \) in the guided filter based rolling mutual structure filtering with \( CGF(p, I, \varepsilon, \lambda), ICGF(p, I, \varepsilon, \lambda) \) to assemble the inverse conservative guided filter based rolling mutual structure filtering which is illustrated in Algorithm 3 and has better filtering results.
Algorithm 3 The CGF-Based RMSF Algorithm (The Conservative Guided Filter Based Rolling Mutual Structure Filtering Algorithm)

1: procedure CGF-RMSF
2: \( p, I, \varepsilon, \lambda, \beta, N \)
3: \( q^0 = p, G^0 = I \)
4: for \( n = 0 \) to \( N \) do
5: \( a_k^{n+1}, b_k^{n+1} = \arg \min_{a_k, b_k} P_0(q^n, G^n, \varepsilon) \)
6: \( c_k^{n+1}, d_k^{n+1} = \arg \min_{c_k, d_k} P_0(G^n, q^n, \varepsilon) \)
7: \( \alpha_i(a^{n+1}) = \frac{1}{1 + E_{a_i}(a_i^{n+1})} \alpha_i(a^{n+1}) \)
8: \( q_i^{n+1} = \alpha_i(c^n_i)(\text{CGF}(q^n_i, G^n, p, \varepsilon, \lambda_i) + (1 - \alpha_i(c^n_i))(\text{ICGF}(G_i^n, q^n_i, I, \varepsilon, \beta_i)) \)
9: \( G_i^{n+1} = \alpha_i(a^n_i)(\text{CGF}(G_i^n, q_i^{n+1}, I, \varepsilon, \beta_i) + (1 - \alpha_i(a^n_i))(\text{ICGF}(q_i^{n+1}, G_i^n, p, \varepsilon, \lambda_i)) \)
10: end for
11: end procedure

C. Rolling Flash/No-Flash Filtering (RFNF)

To enhance the quality of flash/no-flash image pairs, Seo and Milanfar [16] take the guided filter to synthesize a new image which composes a base image \( B \) and Milanfar [16] take the guided filter to synthesize a new image which composes a base image \( B \) and a detail image \( D \) computed from the flash/no-flash image pairs \((I^f, I^n)\) and offer a spectral analysis to illustrate why the rolling usage (41) can yield better result. Differently, we will interpret (41) as an approximation for the cyclic coordinate descent solver of the objective function (42),

\[
q^{k+1} = \text{GF}(q^k, I^f, \varepsilon) + \lambda (I^f - \text{GF}(I^f, I^f, \varepsilon))
\]

(41)

with \( q^0 = I^n \) can yield better result. Differently, we will interpret (41) as an approximation for the cyclic coordinate descent solver of the objective function (42),

\[
\min_{q, a, I} \sum_{(k, \omega) \in \Omega} ((a_k I_i + b_k - q) - \omega a_k^2) + \lambda (q_k - I_k^2)
\]

(42)

where \( I^f = \text{GF}(I^f, I^f, \varepsilon) + \tau (I^f - \text{GF}(I^f, I^f, \varepsilon)) \). Let \( g \) be an alias of \( I^f \), the objective function is same to (21). Hence we can achieve the minimizer by iteratively computing (43) with \( \alpha = \frac{\lambda}{|I^f - \text{GF}(I^f, I^f, \varepsilon)|} \).

\[
q_i^{n+1} = \left(1 - \alpha\right) \text{GF}(q^n_i, I^f, \varepsilon) + \alpha I_i^f
\]

(43)

If \( \alpha \approx 0 \) and \( \tau = \frac{\lambda}{\alpha} \), we have \((1 - \alpha) \approx 1\) and \( \alpha I^f \approx \lambda (I^f - \text{GF}(I^f, I^f, \varepsilon)) \). In addition, \( q_i^{n+1} \) in (43) reduces to \( \text{GF}(q^n_i, I^f, \varepsilon) + \lambda (I^f - \text{GF}(I^f, I^f, \varepsilon)) \) which has the same form with (41). This discovery convinces that the rolling filtering scheme of Seo and Milanfar [16] is just an approximation for a special case of (43). We therefore can generalize (41) to (43). More importantly, the generalization produces much better results in motion deblurring.

V. Comparison and Experiments

We now demonstrate that our new filters and rolling filtering schemes are capable to generate state-of-the-art results for different applications, where all filters are implemented in C++ without SIMD (single instruction multiple data) optimization on an i7 CPU with 4GB memory and five GF-like filters, including the cross-based local multipoint filter (CLMF) [10], the multipoint filter with local polynomial approximation and range guidance (MLPA) [11], the weighted guided filter (WGF) [13], the fully connected guided filter (FCGF) [12] and the local linear SURE-based edge-preserving filter (LLSURE) [14], are used to perform comparison.

A. Computational Complexity

Similar to the guided filter, the spatial-aware guided filter can be computed in linear time as it only substitutes the average operator \( E_0(x) \) used in the guided filter for the weighted average operator \( E^n_0(x) \). The computational complexity of the spatial-aware guided filter therefore is determined by \( E^n_0(x) \) of which the computational complexity is same to the Gaussian convolution \( \sum_{j \in q_0} w_{ij} x_j \). As is known to us, \( \sum_{j \in q_0} w_{ij} x_j \) can be computed in linear time via the acceleration algorithms [1]. Table II reports the runtime of the spatial-aware guided filter to filter one-megapixel image. The speed of the spatial-aware guided filter is almost same to the guided filter and significantly faster than the multipoint filter of Tan et al. [11] and the fully connected guided filter of Dai et al. [12].

As for the total variation guided filter, it no longer can be computed in linear time as the computational complexity of FFT operator \( F \) is \( O(n \log n) \). However, it does not mean that the total variation guided filter cannot be computed efficiently because the implementation of FFT is highly optimized on modern hardware [21]. We can verify this in Table II as it does not increase the runtime very much.

The computational complexity of the conservative guided filter, the inverse guided filter and the inverse conservative guided filter is same to the guided filter which is linear computational complexity because all of them only involve pointwise arithmetic calculations and the average operator \( E(x) \). Table II reports the runtime of eleven filters to filter one-megapixel image. The speed of our conservative guided filter, inverse guided filter and inverse conservative guided filter is almost same to the guided filter and significantly faster than
the cross-based local multipoint filter, the multipoint filter with local polynomial approximation and range guidance and the fully connected guided filter.

### B. Texture Smoothing and Enhancement

Fig 2 plots the texture smoothing and enhancement results produced by the bilateral filter (BF) [2] and GF-like filters: the guided filter (GF) [3], the cross-based local multipoint filter (CLMF) [10], the multipoint filter with local polynomial approximation and range guidance (MLPA) [11], the weighted guided filter (WGF) [13], the fully connected guided filter (FCGF) [12], the spatial-aware guided filter (SGF) in the Rolling Guidance Filtering (RGF) [18] scheme. From the texture smoothing results and their close-ups in the first two rows, we can observe that the result of SGF is similar to BF. Contrastively, GF, CLMF and WGF fail to remove all textures because they do not consider the spatial similarity which is the key to perform textures-aware smoothing in the RGF scheme.

Similar to SGF, MLPA and FCGF take the spatial similarity into account too and therefore produce acceptable results. However, the biggest problem of them is the running cost: MLPA and FCGF spend more time than SGF. Although BF is comparable with SGF in texture smoothing, it suffers from gradient reversal artifacts in image enhancement as illustrated in the third row and the close-up in the fourth row. In contrast, all GF-like filters including our SGF do not have the problem according to the experiment results.

### C. Noise and Haze Removal

Benefiting from the total variation regularization [23], [24], the total variation guided filter (TVGF) is able to reduce noise without structure degradation. For qualitative comparison, we demonstrate the denoising results of seven filtering methods in the first two rows of Fig 3. Visually, only the results of FCGF and LLSURE are comparable with our TVGF. Table III adopts three indices including PSNR [25], MSE [26] and SSIM [27] to estimate the denoising quality. Our TVGF ranks first on all three indices.

The total variation regularization also empowers TVGF the halo artifacts suppression ability, which is the selling point of WGF. The second row in Fig 3 and its close-ups show an instance from the single image haze removal experiment. From this figure, it is not difficult to find that only the results of TVGF and WGF do not suffer from halo artifacts.
D. Multiple Time Filtering for Stereo Matching

In the stereo matching framework [28], the guided filter is employed to smooth each slice of the cost volume one pass. However “Is it optimal to filter each slice one pass?” We find that the answer is negative. The reason is that the PBP performance curve of the guided filter, illustrated in the first row of Fig 4, is a parabola-like line. We owe this to the dissipation property of the guided filter. Specifically speaking, the PBP performance increases with the iteration number on an interval \([0, \tilde{N})\) since the noise is removed gradually without degrading image edges very much. Here \(\tilde{N}\) denotes the optimal filtering number and usually is greater than one. With the filtering time increasing (i.e. \(N \in (\tilde{N}, \infty)\)), the structure information of inputs will be depleted by dissipative filters, so the PBP performance will decrease on the interval \((\tilde{N}, \infty)\). But it is very hard to decide the optimal filtering pass \(\tilde{N}\) in advance. Unlike the dissipative guided filter, the PBP curve of the conservative guided filter is monotonically decreasing because the conservative guided filter is conservative and can preserve the structure information of input no matter how many times filtering are applied. The property implies an easy way to choose \(N\) for the conservative guided filter (CGF): within an admissible computational burden, we make \(N\) as large as possible. The ability of the conservative guided filter brings us much convenience as we no longer need to tweak \(N\) carefully as we do for the guided filter.

E. Major Structure Extraction

We compare the ability of the guided filter based rolling mutual structure filtering (40) and the methods of Xu et al. [4], Zhang et al. [18], Shen et al. [20] in extracting the major structure and illustrate results in Fig 5. The rolling filtering scheme (40) does not yield satisfactory result as it only considers the smoothing part of Rolling Mutual Structure Filtering (RMSF) [20]. The results of Xu et al. [4] and Zhang et al. [18] are much better than (40). This is because Xu et al. [4] design a relative total variation term to extract main structures and suppress textures. Different from Xu et al. [4], Zhang et al. [18] design a new filter and
employing it to filter images with the complete control of detail smoothing under a scale measure. However, taking a close look at their results, we can easily find that the two methods could not distinguish major structures from textures very well and thus not only leave details in the major structures but also blur some important major structures. To solve this problem, Shen proposes mutual-structure for joint filtering. Although the method of Shen can produce the same results as the GF based RMSF, our CGF based RMSF derived from the GF based RMSF are able to remove textures more clearly. More importantly, we provide a more elegant and general way to deal with this task.

**F. Flash/No-Flash Deblurring**

Motion blur due to camera shake is an annoying problem while taking pictures. Our generalized Rolling Flash/No-Flash Filtering (RFNF) can be applied to flash/no-flash deblurring. The method of Zhou et al. [29] as well as the method (41) of Seo and Milanfar [16] are used to perform comparison. Fig 6 illustrates the results of three methods, where no-flash images suffer from mild noise and strong motion blur. As shown in the close-ups, our method outperforms the method of Zhou et al. [29] by obtaining much finer details with better color contrast even though our method does not estimate a blur kernel at all. In addition, compared with our method, Zhou’s method also suffers from over-saturated regions in the blurred image. Unlike Zhou et al. [29], the results of Seo and Milanfar [16] are rather satisfactory. However, their edges are not as sharp as ours.

**VI. CONCLUSION AND FUTURE WORK**

The major contribution of our work is to disclose the equivalence between the guided filter and the cyclic coordinate descent solver of a least squares optimization. The equivalence provides us new insight on how to extend the guided filter as well as the rolling filtering scheme. Specifically, employing the equivalence, we define the guided filter as the first pass iteration of the cyclic coordinate descent solver and derive
new rolling filtering scheme from the minimizing procedure. In addition, we modify the objective function of the guided filter and obtain several GF-like filters and two rolling filtering usages from new objective functions.

In this paper, totally five GF-like filters (i.e., SGF, TVGF, CGF, IGF, ICGF) and two rolling filtering schemes (i.e., CGF-RMSF and RFNF) are proposed. We conduct extensive experiments to disclose/verify the smoothing property/ability of these filters and rolling filtering schemes. Experimental results prove their advantages.

However, we think that these achievements should not be the limit of the power of the connection between the guided filter and the cyclic coordinate descent. Our future work will be devoted to deriving more GF-like filters as well as their rolling schemes according to new objective functions adapted to various tasks in computer vision and graphics. Other than the filtering quality, the smoothing efficiency is another critical problem. We will incorporate with the acceleration technique [30]–[34] to speed up proposed GF-like filters and rolling filtering schemes.

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