Yu. P. Goncharov · F. F. Pavlov

Estimates for parameters and characteristics of the confining SU(3)-gluonic field in \(\phi\)-meson from leptonic widths

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Abstract The paper is devoted to applying the confinement mechanism proposed earlier by one of the authors to estimate the possible parameters of the confining SU(3)-gluonic field in vector \(\phi\)-meson. The estimates obtained are consistent with the leptonic widths of the given meson. The corresponding estimates of the gluon concentrations, electric and magnetic colour field strengths are also adduced for the mentioned field at the scales of the meson under consideration.

Keywords Quantum chromodynamics · Confinement · Mesons · Nuclear forces

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1 Introduction and Preliminary Remarks

According to the point of view of quantum chromodynamics (QCD) the conventional nuclear forces between two nucleons should be just a residual interaction among quarks composing nucleons. On the other hand, for a long time (see, e.g., Refs. [1; 2]) one considers the so-called repulsive core of nuclear forces at small distances to be obligatory to the exchange by neutral vector mesons \(\rho, \omega, \phi\). So, a description of vector mesons in terms of quark and gluonic degrees of freedom could to a certain extent be useful for the problem of nuclear forces.

As is known, at present no generally accepted quark confinement mechanism exists that would be capable to calculate a number of nonperturbative parameters characterizing mesons (masses, radii, decay constants and so on) appealing directly to quark and gluon degrees of freedom related to QCD-Lagrangian. At best there are a few scenarios (directly not connected to QCD-Lagrangian) of confinement that restrict themselves mainly to qualitative considerations with small possibilities of concrete

Yu. P. Goncharov
Theoretical Group, Experimental Physics Department, State Polytechnical University, Sankt-Petersburg 195251, Russia
E-mail: ygonch77@yandex.ru

F. F. Pavlov
Theoretical Group, Experimental Physics Department, State Polytechnical University, Sankt-Petersburg 195251, Russia
E-mail: pavlovfedor@mail.ru
calculation. In view of it in [3, 4, 5] a confinement mechanism has been proposed which was based on the unique family of compatible nonperturbative solutions for the Dirac-Yang-Mills system directly derived from QCD-Lagrangian. The word unique should be understood in the strict mathematical sense. Let us write down arbitrary SU(3)-Yang-Mills field in the form \( A = A_\mu dx^\mu = A_\mu^a \lambda_a dx^\mu \) (\( \lambda_a \) are the known Gell-Mann matrices, \( \mu = t, r, \vartheta, \varphi \); \( a = 1, \ldots, 8 \) and we use the ordinary set of local spherical coordinates \( r, \vartheta, \varphi \) for spatial part of the flat Minkowski spacetime).

In fact in [3, 4, 5] (see also Appendix C in Ref. [19]) the following theorem was proved:

The unique exact spherically symmetric (nonperturbative) solutions (depending only on \( r \) and \( r^{-1} \)) of SU(3)-Yang-Mills equations in Minkowski spacetime consist of the family of form

\[
A_{1t} \equiv A_t^3 + \frac{1}{\sqrt{3}} A_8^t = -\frac{a_1}{r} + A_1, \quad A_{2t} \equiv -A_1^3 + \frac{1}{\sqrt{3}} A_8^t = -\frac{a_2}{r} + A_2,
\]

\[
A_3t \equiv -\frac{2}{\sqrt{3}} A_8^t = \frac{a_1 + a_2}{r} - (A_1 + A_2),
\]

\[
A_{1\varphi} \equiv A_3^\varphi + \frac{1}{\sqrt{3}} A_8^\varphi = b_1 r + B_1, \quad A_{2\varphi} \equiv -A_3^\varphi + \frac{1}{\sqrt{3}} A_8^\varphi = b_2 r + B_2,
\]

\[
A_{3\varphi} \equiv -\frac{2}{\sqrt{3}} A_8^\varphi = -(b_1 + b_2)r - (B_1 + B_2)
\]

(1)

with the real constants \( a_1, A_3, b_1, B_1 \) parametrizing the family. Besides in [4, 5] (see also [8]) it was shown that the above unique confining solutions (1) satisfy the so-called Wilson confinement criterion [6, 7]. Up to now nobody contested this result so if we want to describe interaction between quarks by spherically symmetric SU(3)-fields then they can be only the ones from the above theorem. On the other hand, the desirability of spherically symmetric (colour) interaction between quarks at all distances naturally follows from analysing the \( p\bar{p} \)-collisions (see, e.g., [9]) where one observes a Coulomb-like potential in events which can be identified with scattering quarks on each other, i.e., actually at small distances one observes the Coulomb-like part of solution (1). Under this situation, a natural assumption will be that the quark interaction remains spherically symmetric at large distances too but then, if trying to extend the Coulomb-like part to large distances in a spherically symmetric way, we shall inevitably come to the solution (1) in virtue of the above theorem.

The applications of the family (1) to the description of both the heavy quarkonia spectra [10, 11, 12, 13] and a number of properties of pions, kaons, \( \eta \)- and \( \eta' \)-mesons [14, 15, 16, 17, 18, 12, 20] showed that the confinement mechanism is qualitatively the same for both light mesons and heavy quarkonia. At this moment it can be described in the following way.

The next main physical reasons underlie linear confinement in the mechanism under discussion. The first one is that gluon exchange between quarks is realized with the propagator different from the photon-like one, and existence and form of such a propagator is a direct consequence of the unique confining nonperturbative solutions of the Yang-Mills equations [3, 4, 5]. The second reason is that, owing to the structure of the mentioned propagator, quarks mainly emit and interchange the soft gluons so the gluon condensate (a classical gluon field) between quarks basically consists of soft gluons (for more details see [3, 4, 5]) but, because of the fact that any gluon also emits gluons (still softer), the corresponding gluon concentrations rapidly become huge and form a linear confining magnetic colour field of enormous strengths, which leads to confinement of quarks. This is by virtue of the fact that just the magnetic part of the mentioned propagator is responsible for a larger portion of gluon concentrations at large distances since the magnetic part has stronger infrared Singularities than the electric one. In the circumstances physically nonlinearity of the Yang-Mills equations effectively vanishes so the latter possess the unique nonperturbative confining solutions of the Abelian-like form (1) (with the values in Cartan subalgebra of SU(3)-Lie algebra) which describe the gluon condensate under consideration. Moreover, since the overwhelming majority of gluons is soft they cannot leave the hadron (meson) until some gluons obtain additional energy (due to an external reason) to rush out. So we also deal with the confinement of gluons.

Finally, one should say that the unique confining solutions similar to (1) exist for all semisimple and non-semisimple compact Lie groups, in particular, for SU(N) with \( N \geq 2 \) and U(N) with \( N \geq 1 \) [3]. Explicit form of solutions, e.g., for SU(N) with \( N = 2, 4 \) can be found in [5] but it should be emphasized that components linear in \( r \) always represent the magnetic (colour) field in all the mentioned solutions.
Especially the case U(1)-group is interesting which corresponds to usual electrodynamics. Under this situation, as was pointed out in Refs. 3, 11, 14, parameters $A_{1,2}$ of solution (1) are inessential for physics in question and we can consider $A_1 = A_2 = 0$. Obviously we have $\sum_{j=1}^3 A_{j\ell} = \sum_{j=1}^3 A_{j\nu} = 0$ which reflects the fact that for any matrix $T$ from SU(3)-Lie algebra we have $Tr T = 0$. Also, as has been repeatedly discussed by us earlier (see, e. g., Refs. 3, 11), from the above form it is clear that the solution (1) is a configuration describing the electric Coulomb-like colour field (components $A_{\nu}^a$) and the magnetic colour field linear in $r$ (components $A_{\nu}^a g_{\nu\rho}$) and we wrote down the solution (1) in the combinations that are just needed further to insert into the corresponding Dirac equation.

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The aim of the present paper is to continue obtaining estimates for $A_{j\ell}, b_j, B_j$ for concrete mesons starting from experimental data on spectroscopy of one or another meson. We here consider vector $\phi$-meson [22].

Of course, when conducting our considerations we shall rely on the standard quark model (SQM) based on SU(3)-flavor symmetry (see, e. g., Ref. [22]) so in accordance with SQM $\phi = ss$.

Section 2 contains a specification of main relations derived from the confinement mechanism in question. Section 3 gives the independent estimates for the mean radius $r$ of $\phi$-meson from its leptonic widths which are used in Section 4 for obtaining estimates for parameters of the confining SU(3)-gluonic field in the meson under consideration. Section 5 employs the obtained parameters of SU(3)-gluonic field to get the corresponding estimates for such characteristics of the mentioned field as gluon concentrations, electric and magnetic colour field strengths at the scales of vector meson in question while Section 6 is devoted to discussion and concluding remarks.

Further we shall deal with the metric of the flat Minkowski spacetime $M$ that we write down (using the ordinary set of local spherical coordinates $r, \vartheta, \varphi$ for the spatial part) in the form

$$ds^2 = g_{\mu\nu} dx^\mu \otimes dx^\nu \equiv dt^2 - dr^2 - r^2(d\vartheta^2 + \sin^2 \vartheta d\varphi^2).$$

Besides, we have $|\delta| = |\det(g_{\mu\nu})| = (r^2 \sin \vartheta)^2$ and $0 \leq r < \infty, 0 \leq \vartheta < \pi, 0 \leq \varphi < 2\pi.$

Throughout the paper we employ the Heaviside-Lorentz system of units with $h = c = 1$, unless explicitly stated otherwise, so the gauge coupling constant $g$ and the strong coupling constant $\alpha_s$ are connected by relation $g^2/(4\pi) = \alpha_s$. Further we shall denote $L_2(F)$ the set of the modulo square integrable complex functions on any manifold $F$ furnished with an integration measure, then $L_2^a(F)$ will be the $a$-fold direct product of $L_2(F)$ endowed with the obvious scalar product while $\dagger$ and $*$ stand, respectively, for Hermitian and complex conjugation.

When calculating we apply the relations $1 \text{ GeV}^{-1} \approx 0.197 \text{ fm}$, $1 \text{ s}^{-1} \approx 0.658 \times 10^{-24} \text{ GeV}^3$, $1 \text{ V/m} \approx 0.231 \times 10^{-21} \text{ GeV}^2$, $1 \text{ T} \approx 4\pi \times 10^{-7}\text{H/m} \times 1 \text{ A/m} \approx 0.693 \times 10^{-15} \text{ GeV}^2$.

Finally, for the necessary estimates we shall employ the $T_{\mu\nu}$-component (volumetric energy density $\delta E$) of the energy-momentum tensor for a SU(3)-Yang-Mills field which should be written in the chosen system of units in the form

$$T_{\mu\nu} = - F^a_{\mu\alpha} F^a_{\nu\beta} g^{\alpha\beta} + \frac{1}{4} F^a_{\mu\rho} F^a_{\nu\sigma} g^{\rho\sigma} g_{\mu\nu}.$$
2 Specification of Main Relations

2.1 Meson wave functions

The meson wave functions are given by the unique nonperturbative modulo square integrable solutions of the mentioned Dirac equation in the confining SU(3)-field of (1) $\Psi = (\Psi_1, \Psi_2, \Psi_3)$ with the four-dimensional Dirac spinors $\Psi_j$ representing the jth colour component of the meson, so $\Psi$ may describe the relative motion (relativistic bound states) of two quarks in mesons and is at $j = 1, 2, 3$ (with Pauli matrix $\sigma_1$)

$$\Psi_j = e^{-i\omega_j t} \psi_j \equiv e^{-i\omega_j t} \left( \frac{F_{j1}(r)\Phi_j(\theta, \varphi)}{F_{j2}(r)\sigma_1 \Phi_j(\theta, \varphi)} \right)_j,$$

with the 2D eigenspinor $\Phi_j = (\Phi_{j1}, \Phi_{j2})$ of the Euclidean Dirac operator $D_0$ on the unit sphere $S^2$, while the coordinate $r$ stands for the distance between quarks.

In this situation, if a meson is composed of quarks $q_{1,2}$ with different flavours then the energy spectrum of the meson will be given by $\epsilon = m_{q_1} + m_{q_2} + \omega$ with the current quark masses $m_{q_k}$ (rest energies) of the corresponding quarks and an interaction energy $\omega$. On the other hand at $j = 1, 2, 3$

$$\omega_j = \omega_j(n_j, l_j, \lambda_j) =$$

$$A_j g^2 a_j b_j \pm (n_j + \alpha_j) \sqrt{(n_j^2 + 2n_j \alpha_j + A_j^2) \mu_0^2 + g^2 b_j^2 (n_j^2 + 2n_j \alpha_j)}$$

$$n_j^2 + 2n_j \alpha_j + A_j^2$$

with the gauge coupling constant $g$ while $\mu_0$ is a mass parameter and one should consider it to be the reduced mass which is equal to $m_{q_1} m_{q_2}/(m_{q_1} + m_{q_2})$ with the current quark masses $m_{q_k}$ (rest energies) of the corresponding quarks forming a meson (quarkonium), $a_3 = -(a_1 + a_2)$, $b_3 = -(b_1 + b_2)$, $B_3 = -(B_1 + B_2)$, $A_j = \lambda_j - gB_j$, $\alpha_j = \sqrt{A_j^2 - g^2 a_j^2}$, $n_j = 0, 1, 2, ..., \lambda_j = \pm(l_j + 1)$ are the eigenvalues of Euclidean Dirac operator $D_0$ on a unit sphere with $l_j = 0, 1, 2, ...,\n
In line with the above we should have $\omega = \omega_1 = \omega_2 = \omega_3$ in energy spectrum $\epsilon = m_{q_1} + m_{q_2} + \omega$ for any meson (quarkonium) and this at once imposes two conditions on parameters $a_j, b_j, B_j$ when choosing some experimental value for $\epsilon$ at the given current quark masses $m_{q_1}, m_{q_2}$.

The general form of the radial parts of (4) can be found, e.g., in Refs. [17, 18, 19] and within the given paper we need only the radial parts of (4) at $n_j = 0$ (the ground state) that are

$$F_{j1} = C_j P_j e^{a_j} e^{-\beta_j r} \left( 1 - \frac{gb_j}{\beta_j} \right), \quad F_{j2} = gb_j + \beta_j,$$

$$F_{j2} = iC_j Q_j e^{a_j} e^{-\beta_j r} \left( 1 + \frac{gb_j}{\beta_j} \right), \quad Q_j = \mu_0 - \omega_j$$

with $\beta_j = \sqrt{\mu_0^2 - \omega_j^2 + g^2 b_j^2}$, while $C_j$ is determined from the normalization condition $\int_0^\infty (|F_{j1}|^2 + |F_{j2}|^2) dr = \frac{1}{4}$. The corresponding eigenspinors of (4) with $\lambda = \pm 1$ ($l = 0$) are

$$\lambda = -1: \Phi = C \left\{ \frac{e^{i\varphi}}{e^{-i\varphi}} \right\} e^{i\varphi/2}, \quad \text{or} \quad \Phi = C \left\{ \frac{e^{i\varphi}}{-e^{-i\varphi}} \right\} e^{-i\varphi/2},$$

$$\lambda = 1: \Phi = C \left\{ \frac{e^{-i\varphi}}{e^{i\varphi}} \right\} e^{i\varphi/2}, \quad \text{or} \quad \Phi = C \left\{ \frac{-e^{-i\varphi}}{e^{i\varphi}} \right\} e^{-i\varphi/2}$$

with the coefficient $C = 1/\sqrt{2\pi}$ (for more details, see Refs. [17, 18, 19]).
2.2 Choice of quark masses and the gauge coupling constant

Obviously, we should choose a few quantities that are the most important from the physical point of view to characterize meson under consideration and then we should evaluate the given quantities within the framework of our approach. In the circumstances let us settle on the ground state energy (mass), the root-mean-square radius and the magnetic moment. All three magnitudes are essentially nonperturbative ones, and can be calculated only by nonperturbative techniques.

Within the present paper we shall use relations (5) at \( n_j = 0 = \ell_j \) so energy (mass) of meson under consideration is given by

\[
\mu = 2m_s + \omega
\]

with \( \omega = \omega_j(0, 0, \lambda_j) \) for any \( j = 1, 2, 3 \) whereas

\[
\omega = g^2a_1b_1 + \alpha_1 \mu_0 = \frac{g^2 a_2 b_2}{|A_2|} + \alpha_2 \mu_0 = \frac{g^2 a_3 b_3}{|A_3|} + \alpha_3 \mu_0 = \mu - 2m_s
\]

(8)

with \( \mu = 1019.455 \text{ MeV} \). As a consequence, the corresponding meson wave functions of (4) are represented by (6) and (7). It is evident for employing the above relations we have to assign some values to quark mass and gauge coupling constant \( g \). We take the current quark mass used in [15; 16; 17; 13; 18] and it is \( m_s = 107.5 \text{ MeV} \). Under the circumstances, the reduced mass \( \mu_0 \) of (5) will be equal to \( m_s/2 \).

As to the gauge coupling constant \( g = \sqrt{4\pi \alpha_s} \), it should be noted that recently some attempts have been made to generalize the standard formula for \( \alpha_s = \alpha_s(Q^2) = 12\pi/[(33 - 2n_f) \ln(Q^2/A^2)] \) (\( n_f \) is number of quark flavours) holding true at the momentum transfer \( \sqrt{Q^2} \to \infty \) to the whole interval \( 0 \leq \sqrt{Q^2} \leq \infty \). If employing one such a generalization used in Refs. [23; 24] which we have already discussed elsewhere (for more details see [15; 16; 17; 13; 18]) then (when fixing \( A = 0.234 \text{ GeV}, n_f = 3 \)) we obtain \( g \approx 3.771 \) necessary for our further computations at the mass scale of \( \phi \)-meson.

2.3 Electric form factor and the root-mean-square radius

The relations (4), (6) and (7) allow us to compute an electric form factor of a meson as a function of the square of momentum transfer \( Q^2 \) in the form (for more details see [15; 16; 17; 13; 18])

\[
f(Q^2) = \sum_{j=1}^{3} f_j(Q^2) = 
\sum_{j=1}^{3} \frac{(2\beta_j)^{2\alpha_j+1}}{6\alpha_j} \cdot \sin[2\alpha_j \arctan(\sqrt{|Q^2|/(2\beta_j))]} \n \frac{\sqrt{|Q^2|}(4\beta_j^2 - Q^2)^{\alpha_j}}{(4\beta_j^2 - Q^2)^{\alpha_j}} \]

(9)

which also entails the root-mean-square radius of the meson (quarkonium) in the form

\[
< r > = \sqrt{\sum_{j=1}^{3} \frac{2\alpha_j^2 + 3\alpha_j + 1}{6\beta_j^2}}
\]

(10)

that is in essence a radius of confinement.

2.4 Magnetic moment

Also it is not complicated to show with the help (4), (6) and (7) that the magnetic moments of mesons (quarkonia) with the wave functions of (4) (at \( l_j = 0 \)) are equal to zero [15; 16; 17; 13; 18], as should be according to experimental data [22].

Though we can also evaluate the magnetic form factor \( F(Q^2) \) of meson (quarkonium) which is also a function of \( Q^2 \) (see Refs. [13; 16]) the latter will not be used in the given paper so we shall not dwell upon it.
2.5 Remarks about the relativistic two-body problem

It is well known (see any textbook on quantum mechanics, e.g., Ref. [27]) that nonrelativistic two-body problem, when the particles interact with potential \( V(\mathbf{r}_1, \mathbf{r}_2) \) depending only on \( |\mathbf{r}_2 - \mathbf{r}_1| = r \), reduces to the motion of one particle with reduced mass \( m = m_1 m_2 / (m_1 + m_2) \) in potential \( V(r) \), where \( r \) becomes the ordinary spherical coordinate from triplet \((r, \theta, \varphi)\). I.e., the bound states of two particles are the bound states of the particle with mass \( m \) in potential \( V(r) \) and they are the modulo square integrable solutions of the corresponding Schrödinger equation. Another matter is relativistic two-body problem. As we emphasized in Refs. [4; 5], up to now it has no single-valued statement. But if noting that the most fundamental results of nonrelativistic quantum mechanics (hydrogen atom and so on) are connected with potentials \( V(r) \) which are a part of the electromagnetic field \( A = (A_0, \mathbf{A}) \), i.e. \( V(r) = A_0, \mathbf{A} = 0 \) then one may propose some formulation of the relativistic two-body problem. Really, now \( V(r) \) additionally obeys the Maxwell equations and if we want to generalize the corresponding Schrödinger equation to include an interaction with arbitrary electromagnetic field for \( \mathbf{A} \neq 0 \) then the answer is known: this is the Dirac equation with the replacement \( \partial_\mu \to \partial_\mu - ig A_\mu \), \( g \) is a gauge coupling constant. Indeed, when going back in nonrelativistic limit at the light velocity \( c \to \infty \) the magnetic field \( \mathbf{A} \) vanishes because, as is well known, in the world with \( c = \infty \) there exist no magnetic fields (see any elementary textbook on physics). At the same time \( V(r) = A_0 \) does not vanish and remains the same as in nonrelativistic case and the Dirac equation turns into the Schrödinger equation (see Ref. [27]). But then we can see that \( m \) in the above Dirac equation should consider the same reduced mass as before since in nonrelativistic limit we again should come to the standard formulation of two-body problem through effective particle with reduced mass \( m \). So we can draw the conclusion that if an electromagnetic field is a combination of electric \( V(r) = A_0 \) field between two charged elementary particles and some magnetic field \( \mathbf{A} = \mathbf{A}(r) \) (which may be generated by the particles themselves and also depends only on \( r \), distance between particles) then there are certain grounds to consider the given (quantum) relativistic two-body problem to be equivalent to the one of motion for one particle with usual reduced mass in the mentioned electromagnetic field. As a result, we can use the Dirac equation for finding possible relativistic bound states for such a particle implying that this is really some description of the corresponding two-body problem. Under this situation we should remark the following.

1. Although for simplicity we talked about electromagnetic field but everything holds true for any Yang-Mills field, in particular, for SU(3)-gluonic field while the Maxwell equations are replaced by the Yang-Mills ones.

2. There arises the question: whether the Maxwell or Yang-Mills equations possess solutions with spherically symmetric \( A_0(r), \mathbf{A}(r) \)? The answer is given by the uniqueness theorem (see Section 1).

3. The Dirac equation in such a field has the nonperturbative spectrum \( (5) \) and the latter should be treated as the nonperturbative interaction energy of two quarks and \( r \) as the distance between quarks and so on. So eq. \( (5) \) should be understood just in this manner. In line with the above we should have \( \omega = \omega_1 = \omega_2 = \omega_3 \) (with \( \omega_j \) of \( (5) \)) in energy spectrum \( \epsilon = m_{q_1} + m_{q_2} + \omega \) for any meson (quarkonium) and this at once imposes two conditions on parameters \( a_j, b_j, B_j \) of solution \( (1) \) when choosing some experimental value for \( \epsilon \) at the given current quark masses \( m_{q_1}, m_{q_2} \). At last, the Dirac equation in question is obtained from QCD lagrangian (with one flavor) if mass parameter in the latter is taken to be equal to the above reduced mass. Therefore, we can say that meson wave functions \( (4) \) are the nonperturbative solutions of the Dirac-Yang-Mills system directly derived from QCD-lagrangian.

4. To summarize, there exist good physical and mathematical grounds for formulation of the above relativistic two-body problem that has a correct nonrelativistic limit and is Lorentz and gauge invariant. It is clear that all the above considerations can be justified only by comparison with experimental data but now we obtain some intelligible programme of further activity which has been partly realized in many our papers cited above.

Much of the above was discussed in Refs. [4; 5] but perhaps in other words.

3 An estimate of \( \langle r \rangle \) from leptonic widths

The question now is how to estimate \( \langle r \rangle \) independently to then calculate it according to \( (10) \) within framework of our approach. For this aim we shall employ the widths of leptonic decays \( \phi \to e^+ e^- \).
and $\phi \to \mu^+\mu^-$ which are approximately equal to $I_0 \approx 1.27$ keV and $I_{10} \approx 1.22262$ keV, respectively, according to Ref. [22]. Under this situation one can use a variant of formulas often employed in the heavy quarkonia physics (see, e.g., Ref. [13]). In their turn such formulas are actually based on the standard expression from the elementary kinetic theory of gases (see, e.g., Ref. [23]) for the number $\nu$ of collisions of a molecule per unit time

$$\nu = \sqrt{2} \sigma < v > n,$$  

(11)

where $\sigma$ is an effective cross section for molecules, $< v >$ is a mean molecular velocity, $n$ is the concentration of molecules. If replacing $\nu \to I_{9,10}$ we may fit (11) to estimate the leptonic widths $I_{9,10}$ when interpreting $\sigma$ as the cross section of creation of $e^+e^-$ or $\mu^+\mu^-$ from the pair $\bar{s}s$ due to electromagnetic interaction, $< v >$ and $n$ as, respectively, a mean quark velocity and concentration of quarks (antiquarks) in $\phi$-meson. To obtain $\sigma$ in the explicit form one may take the corresponding formula for the cross section of creation of $e^+e^-$ from the muon pair $\mu^+\mu^-$ (see, e.g., Ref. [26]) and, after replacing $\alpha_{em} \to Q\alpha_{em}$, $m_\mu \to m_s$ with electromagnetic coupling constant $\alpha_{em}=1/137.0359895$ and muon mass $m_\mu$, obtain

$$\sigma = \frac{4\pi N Q^2 \alpha_{em}^2}{3s} \left( 1 + \frac{2m_s^2}{s} \right) \sqrt{1 - \frac{4m_s^2}{s}},$$  

(12)

where leptonic masses $m_l$ are $m_e = 0.510998918$ MeV for electron and $m_\mu = 105.658389$ MeV for muon, accordingly, the Mandelstam invariant $s = 2m_\mu^2$ with $\mu = 1019.455$ MeV, $N$ is the number of colours and $Q = 1/3$ for $\phi$-meson. To get $< v >$ one may use the standard relativistic relation $v = \sqrt{T^2 + 2E_0}(T + E_0)$ with kinetic $T$ and rest energies $E_0$ for velocity $v$ of a point-like particle. Putting $T = \mu - 2m_s$, $E_0 = m_s$ we shall gain

$$< v > = \sqrt{1 - 2m_s/\mu}. $$

(13)

At last, obviously, $n = 1/V$, while $V$ is the volume of a region where the process of annihilation $\bar{s}s \to e^+e^-$ or $\mu^+\mu^-$ occurs, so $V = 4\pi < r_{an} >^3/3$ with some $< r_{an} >$. Then the relations (11)–(13) entail the independent estimate for $< r_{an} >$

$$< r_{an} > = \left( \frac{3\sigma \sqrt{2} \sqrt{1 - 2m_s/\mu}}{4\pi \Gamma/(1 - m_s/\mu)} \right)^{1/3}$$

(14)

with $\sigma$ of (12) and $\Gamma = I_{9,10}$. When inserting $N = 3$, $\mu = 1019.455$ MeV, $m_s = 107.5$ MeV into (14) we shall have $< r_{an} > \approx 0.883$ fm for $\Gamma = I_0$ and $< r_{an} > \approx 0.889$ fm for $\Gamma = I_{10}$. It should be noted that no experimental values exist for the radius $< r >$ in the case of $\phi$-meson[22], but if taking into account that in the case of proton with mass 938 MeV we have $< r > \approx 0.875$ fm [22], then if considering the size of $\phi$-meson to be approximately the same as for proton we should put, for example, $< r > \approx < r_{an} >$ to make computation more sensible.

In further considerations we can use this independent estimate of $< r >$ while calculating $< r >$ according to (10) which will impose certain restrictions on parameters of the confining SU(3)-gluonic field in $\phi$-meson.

At last, we should note that there exists the formula of Van Royen–Weisskopf for the widths of the leptonic decays of neutral vector mesons (see, e.g., Ref. [24])

$$\Gamma = \frac{16\pi \alpha_{em}^2 Q_V^2}{M_V} |\psi(0)|^2$$

(14')

with the meson mass $M_V$ and the effective charge $Q_V$ of quarks in the meson so $Q_V = 1/2, 1/18, 1/9$, respectively, for $\rho, \omega, \phi$-mesons.

There arises, however, the question: as should be understood $|\psi(0)|^2$ in (14') within the framework of our approach because our wave functions (4) consist of three components. Probably the most natural prescription would be $|\psi(0)|^2 = \sum_{j=1}^3 |\psi_j(0)|^2$ with $\psi_j$ of (4). But then many combinations are possible.
Table 1  Gauge coupling constant, mass parameter $\mu_0$ and parameters of the confining SU(3)-gluonic field for $\phi$-meson

| Particle | $g$  | $\mu_0$ (MeV) | $a_1$ | $a_2$ | $b_1$ (GeV) | $b_2$ (GeV) | $B_1$ | $B_2$ |
|----------|-----|---------------|-------|-------|-------------|-------------|-------|-------|
| $\phi-\bar{s}s$ | 3.771 | 53.750 | 0.668 | -0.263 | 0.223 | -0.536 | -0.450 | -0.440 |

E.g., $|\psi_1(0)| \neq 0$, while $|\psi_2(0)| = |\psi_2(0)| = 0$, or $|\psi_1(0)| = |\psi_2(0)| = 0$ while $|\psi_3(0)| \neq 0$ and so on. Every such a choice has its physical interpretation and entails its own estimates for the gluonic field parameters. The analysis of all the possibilities is worth writing the separate paper. Under this situation we decided in the given paper to restrict ourselves to a simpler estimate for $<r>$ adduced above in this Section.

4 Estimates for parameters of SU(3)-gluonic field in $\phi$-meson

4.1 Basic equations

Now we should consider the equations (8) and (10) as a system which should be solved compatibly when $\mu = 1019.455$ MeV, $m_s = 107.5$ MeV, $g \approx 3.771$ while the possible value of $<r>$ has been estimated in previous section. While computing for distinctness we take all eigenvalues $\lambda_j$ of the Euclidean Dirac operator $D_0$ on the unit 2-sphere $S^2$ equal to 1.

4.2 Chiral limit and numerical results

As was remarked in Refs. [18; 19], the Dirac equation in the field (1) possesses a nontrivial spectrum of bound states even for massless fermions. As a result, mass of any meson remains nonzero in chiral limit when masses of quarks $m_q \to 0$ and meson masses will only be expressed through the parameters of the confining SU(3)-gluonic field of (1). This purely gluonic residual mass of meson should be interpreted as a gluonic contribution to the meson mass.

I.e., the confinement mechanism under consideration gives us a possible approach to the problem of chiral symmetry breaking in QCD [18; 19]; in chiral symmetric world masses of mesons are fully determined by the confining SU(3)-gluonic field between (massless) quarks and are not equal to zero. Accordingly chiral symmetry is a sufficiently rough approximation holding true only when neglecting the mentioned SU(3)-gluonic field between quarks and no additional mechanism of the spontaneous chiral symmetry breaking connected to the so-called Goldstone bosons is required. Referring for more details to [18; 19], we can here only say that, e.g., mass of $\phi$-meson has also a purely gluonic contribution and we may be interested in what part of the $\phi$-meson mass is obligatory to that contribution. Indeed, in chiral limit $m_{q_1}, m_{q_2} \to 0$ we obtain from (8)

$$\langle \mu \rangle_{\text{chiral}} \approx \frac{g^2 a_1 b_1}{\lambda_1 - g B_1} \approx \frac{g^2 a_2 b_2}{\lambda_2 - g B_2} \approx \frac{g^2 (a_1 + a_2)(b_1 + b_2)}{\lambda_3 + g (B_1 + B_2)} \neq 0.$$  

and we can see that in chiral limit the meson masses are completely determined only by the parameters $a_1, b_1, B_1, B_2$ of SU(3)-gluonic field between quarks, i.e. by interaction between quarks, and those masses have the purely gluonic nature. So one can use the parameters $g, a_1, b_1, B_1$ to compute $\langle \mu \rangle_{\text{chiral}}$ which in fact represents the sought gluonic contribution to the meson masses. The results of the numerical compatible solving of equations (8) and (10) are gathered in table 1 and 2.
5 Estimates of gluon concentrations, electric and magnetic colour field strengths

5.1 Estimates

Now let us recall that, according to Refs. [5; 12], one can confront the field (1) with $T_{00}$-component (volumetric energy density of the SU(3)-gluonic field) of the energy-momentum tensor (3) so that

$$T_{00} \equiv T_{tt} = \frac{E^2 + H^2}{2} = \frac{1}{2} \left( \frac{a_1^2 + a_1 a_2 + a_2^2}{r^4} + \frac{b_1^2 + b_1 b_2 + b_2^2}{r^2 \sin^2 \vartheta} \right) \equiv \frac{A}{r^4} + \frac{B}{r^2 \sin^2 \vartheta} \tag{16}$$

with electric $E$ and magnetic $H$ colour field strengths and real $A > 0, B > 0$. One can also introduce magnetic colour induction $B = (4\pi \times 10^{-7} \text{H/m}) H$, where $H$ in A/m.

To estimate the gluon concentrations we can employ (16) and, taking the quantity $\omega = \Gamma$, the full decay width of a meson, for the characteristic frequency of gluons we obtain the sought characteristic concentration $n$ in the form

$$n = \frac{T_{00}}{\Gamma} \tag{17}$$

so we can rewrite (16) in the form $T_{00} = T_{\text{coul}} + T_{\text{lin}}^0$ conforming to the contributions from the Coulomb and linear parts of the solution (1). This entails the corresponding split of $n$ from (17) as $n = n_{\text{coul}} + n_{\text{lin}}$.

The parameters of Table 1 were employed when computing and for simplicity we put $\sin \vartheta = 1$ in (16). Also there was used the following present-day full decay width of $\phi$-meson $\Gamma = 4.26$ MeV, whereas the Bohr radius $a_0 = 0.529 \cdot 10^5 \text{ fm}$ [22].

Table 3 contains the numerical results for $n_{\text{coul}}, n_{\text{lin}}, n, E, H, B$ for the meson under discussion.

5.2 Concluding Remarks

As is seen from Table 3, at the characteristic scales of $\phi$-meson the gluon concentrations are huge and the corresponding fields (electric and magnetic colour ones) can be considered to be the classical ones with enormous strengths. The part $n_{\text{coul}}$ of gluon concentration $n$ connected with the Coulomb electric colour field is decreasing faster than $n_{\text{lin}}$, the part of $n$ related to the linear magnetic colour field, and at large distances $n_{\text{lin}}$ becomes dominant. It should be emphasized that in fact the gluon concentrations are much greater than the estimates given in Table 3 because the latter are the estimates for maximal possible gluon frequencies, i.e. for maximal possible gluon impulses (under the concrete situation of $\phi$-meson). The given picture is in concordance with the one obtained in Refs. [15; 16; 17; 18; 19]. As a result, the confinement mechanism developed in Refs. [3; 4; 5] is also confirmed by the considerations of the present paper.

It should be noted, however, that our results are of a preliminary character which is readily apparent, for example, from that the current quark masses (as well as the gauge coupling constant $g$) used in computation are known only within the certain limits and we can expect similar limits for the
magnitudes discussed in the paper so it is necessary further specification of the parameters for the confining SU(3)-gluonic field in \( \phi \)-meson which can be obtained, for instance, by calculating widths of radiative decays of type \( \phi \rightarrow \pi^0 + \gamma \), \( \phi \rightarrow \eta + \gamma \) and so on [22]. We hope to continue analysing the given problems elsewhere.

Finally, one can emphasize that though there exists a number of papers devoted to miscellaneous aspects of the vector meson physics (see e.g. Refs. [28] and references therein) but all of them do not directly appeal to the quark and gluonic degrees of freedom as should be from the first principles of QCD. Really, they use the so-called potential approach where interaction between quarks is described by potential of form \( V(r) = a/r + br \) with some constants \( a \) and \( b \). We cannot, however, speak about the above potential \( V(r) \) as describing some gluon configuration between quarks. It would be possible if the mentioned potential were a solution of Yang-Mills equations directly derived from QCD-Lagrangian since, from the QCD-point of view, any gluonic field should be a solution of Yang-Mills equations (as well as any electromagnetic field is by definition always a solution of Maxwell equations). But in virtue of the uniqueness theorem of Section 1 it is impossible: Coulomb and linear parts belong to different parts of the YM-potentials, accordingly, to the colour electric and colour magnetic parts so they cannot be united in one component of form \( a/r + br \).

On the contrary, within the framework of our approach the words quark and gluonic degrees of freedom make exact sense: gluons come forward in the form of bosonic condensate described by parameters \( a_j, b_j, B_j \) from the unique exact solution (1) of the Yang-Mills equations while quarks are represented by their current masses \( m_q \).

6 Conclusion

The main idea of quark confinement may be borrowed from classical electrodynamics. Indeed, let us recall the well-known case of motion of a charged particle in the homogeneous magnetic field (see, e.g., Ref. [31]). In the latter case the particle moves along helical curve with lead of helix \( h = 2\pi m v \cos \alpha/(qH \sqrt{1 - v^2}) \) and radius \( R = m v \sin \alpha/(qB \sqrt{1 - v^2}) \), where \( \alpha \) is an angle between vectors of the particle velocity \( v \) and magnetic induction \( B \). \( q \) is a particle charge, \( m \) is a particle mass. As a consequence, the homogeneous magnetic field does not give rise to the full confinement of the particle since the latter may go to infinity along the helical curve. The situation is not changed at quantum level as well: there exist no bound states in the homogeneous magnetic field [32]. But if estimating module of \( B \) at \( R \sim 10^{-13} \) in \( = 1 \) fm for electron then \( B \) will be of order \( 10^{23} \) T. I.e., if considering that quarks are confined by a colour magnetic field that they themselves create then one needs a colour magnetic field between quarks with \( B \) of such an order and that field should not allow any quark to go to infinity. It is clear this field should be a solution of the Yang-Mills equations. Really, if speaking about Minkowski spacetime then searching for classical solutions of the Yang-Mills equations makes sense because at large distances between quarks the latter are surrounded with a huge number of gluons that are emitted by both quarks and gluons themselves. Under this situation it is quite plausible that confinement of quarks arises due to certain properties of such gluonic clouds while the latter should be described just by classical solutions of the Yang-Mills equations.

As was shown in Refs. [33] (see also Appendix C in Ref. [19]), the necessary solution has the form (1) for group SU(3) and is unique in a certain sense. This result allowed us to propose a quark confinement mechanism which was successfully applied to meson spectroscopy. Indeed, as follows from (10) at \( |b_j| \rightarrow \infty \) we have \( < r > \sim \sqrt{\sum_{j=1}^{3} \frac{1}{|g(b_j)|^2}} \), so in the strong magnetic colour field when \( |b_j| \rightarrow \infty \), \( < r > \rightarrow 0 \), while the meson wave functions of (4) and (6) behave as \( \Psi_j \sim e^{-g |b_j| r} \), i.e., just the magnetic colour field of (1) provides two quarks with confinement. This situation also holds true at classical level [21].

The given paper extends this mechanism over vector mesons and further study of that approach will be connected with analysis of other concrete mesons and baryons with its help.
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