Equilibrium thermodynamic properties of two-component and two phase mixtures

S. Benjelloun · R. Boukharfane

Received: date / Accepted: date

Abstract We present general calculations allowing to express the thermodynamical coefficients and thermophysical properties (compressibility, thermal coefficients and heat capacities) of a material composed of a mixture of two constituents or phases, regardless of the equations of state considered for each of the constituent or for the mixture. We consider mixtures under complete thermodynamical equilibrium, either with mass exchange between the two constituents (phase change), such as an oil and gas mixture (black-oil) and water-vapor system or with two immiscible phases, such as air and water mixtures (foam).

Keywords Thermodynamic coefficient · Thermophysical properties · thermal coefficients · speed of sound · two phase systems · liquid vapor equilibrium

1 Introduction

The thermodynamic state of a system can be described using a number of state quantities, called state functions. For a closed system at thermodynamic equilibrium, only two of these quantities are independent. Therefore, the state functions can be linked either by functional relations (e.g. an equations of state), which specifies relationships between these quantities, or by thermodynamical identities linking the infinitesimal variations (or derivatives) of a triplet of these state quantities. If we limit ourselves to the 5 fundamental state functions : \( \varrho \), \( p \), \( T \), \( s \), and \( e \), being the specific density (the inverse of the specific volume), pressure, temperature, specific entropy and specific internal energy, respectively. Then, \( (5) = 10 \) thermodynamical identities can be expressed for a closed system. Other state functions, such as enthalpy \( h \) and other thermodynamic potentials are obtained as simple functions of these fundamental quantities, e.g.

\[
h = e + \frac{p}{\varrho}
\]
First order derivatives of these state functions, with respect to, and as functions of, a given pair of variables are generally called thermodynamic coefficients. Examples are compressibility coefficients such as the speed of sound, the bulk modulus, and the calorific coefficients such as heat capacities, Grüneissen coefficients; see [Benjelloun(2021)] for a detailed presentation of the thermodynamic identities and the relations between thermodynamic coefficients.

2 Thermodynamic properties of a two immiscible phases mixture or mixtures without mass exchange

We consider a system formed of two immiscible components at complete thermodynamic equilibrium (pressure and temperature). An example of system that can be modeled by this is any mixture of non reacting gases (air), water and air mixtures (foam), oil and water mixture, etc. We suppose that the phases are immiscible and that there is no phase change or mass transfer between the two constituents. The system is supposed closed, and we set $x_1 = x$ to be the mass fraction of constituent 1, $x_2 = 1 - x$ the mass fraction of component 2. Therefore, considering an elementary homogeneous part of the system, $x$ is a constant. We note $\alpha_1 = \alpha$ and $\alpha_2 = 1 - \alpha$ the volumetric fractions. $\alpha$ and $x$ are related by means of the following relations

\[
\begin{align*}
x &= \frac{x_1 \alpha_1}{\alpha_1 + (1-x)\alpha_2}, \\
\alpha &= \frac{x}{x_1 + (1-x)x_2}.
\end{align*}
\]

(2)

We consider the mixture equilibrium quantities defined from the specific extensive quantities. Namely, the specific volume for the mixture

\[
\begin{align*}
v &= x v_1 + (1-x)v_2, \\
v &= \frac{x}{v_1} + \frac{(1-x)}{v_2},
\end{align*}
\]

(3)

the specific internal energy

\[
e = xe_1 + (1-x)e_2
\]

(4)

and the specific entropy

\[
s = xs_1 + (1-x)s_2
\]

(5)

At equilibrium, this closed bi-constituent system is bi-variant. Indeed, if we choose a pair of independent state functions to describe the state of each component, the equality of temperature and pressure between the two phases gives two constraints on the four state function. The two expressions for the first law of thermodynamics for each component can be expressed as follows

\[
\begin{align*}
de_1 &= Tds_1 - pdv_1, \\
de_2 &= Tds_2 - pdv_2.
\end{align*}
\]

(6)

and can be combined to get the first law for the mixture quantities

\[
de = Tds - pdv.
\]

(7)
Hence, and as for pure phases, we can define the equilibrium speed of sound in the mixture $c^m$ and its equilibrium Grüneissen coefficient $k$ as follows

$$dp = c^2 dg + gkT ds,$$

and the isobaric specific heat coefficient $C_v$ by

$$dT = \frac{T}{C_v} ds + \frac{kT}{\varrho} dg.$$

All thermodynamic identities and relations for pure phases (as Reech and Mayer relations) are valid for the equilibrium mixture quantities. The mixture thermodynamic coefficients that can be easily derived, as function of the pure phases quantities, are the derivatives of $v$, $e$ and $s$, with respect to $p$ at constant temperature $T$, or with respect to $T$ at constant pressure $P$. For instance, deriving $v = x v_1 + (1-x)v_2$ with respect to $P$ at constant $T$, we have

$$-\frac{\partial v}{\partial P} = \frac{e^2}{(c^T)^2} = \frac{v_1^2}{(c_1^T)^2} + (1-x)\frac{v_2^2}{(c_2^T)^2},$$

where $c_T$ is the isothermal speed of sound, and the isothermal compressibility is given by $\varrho (c_T)^2 = \chi_T$. Now deriving the volume $T$ at constant $P$ to get

$$\frac{\partial v}{\partial T} \bigg|_P = v \alpha_P = x v_1 \alpha_P^1 + (1-x)v_2 \alpha_P^2$$

From the specific entropy we get

$$\frac{\partial s}{\partial T} \bigg|_P = C_P = \frac{C_P^1}{T} + (1-x)\frac{C_P^2}{T}$$

and

$$\frac{\partial s}{\partial P} = \frac{1}{\varrho kC_v} = \frac{1}{\varrho_1 k_1 C_1} + \frac{1}{\varrho_2 k_2 C_2} + (1-x)\frac{1}{\varrho_2 k_2 C_2}$$

We have then

$$\left\{ \begin{array}{l}
\frac{1}{c_T^2} = x \frac{1}{c_1^2} + (1-x) \frac{1}{c_2^2} \\
\alpha_P = \varrho \left( x \frac{\alpha_P^1}{c_1^2} + (1-x) \frac{\alpha_P^2}{c_2^2} \right) \\
C_P = x C_1 + (1-x)C_2 \\
C_v = C_P - T \alpha_P x c_T^2 \\
\gamma = \frac{C_P}{C_{v,P}} \\
p_c = \frac{C_P - T \alpha_P c_T^2}{C_P - T \alpha_P c_T^2} \\
k = \frac{\alpha_P c_T^2}{C_P - T \alpha_P c_T^2} \\
\alpha_v = \varrho k C_v = \varrho \alpha_P c_T = \frac{x \alpha_P^1}{c_1^2} + (1-x) \frac{\alpha_P^2}{c_2^2} \end{array} \right.$$
We see in particular that \( c^2 \) does not depend only on \( c_1 \) and \( c_2 \) but on \( k \) and \( C_v \) as well

\[
\frac{1}{\varrho^2 c^2} = \frac{x}{\varrho_1^2 (c_1^2)} + \frac{1-x}{\varrho_2^2 (c_2^2)} - \frac{T \left( \frac{\varrho_1 \alpha_1}{\varrho_1^2} + \frac{(1-x)\alpha_2}{\varrho_2^2} \right)}{xC_1^2 + (1-x)C_2^2}
\]

or equivalently

\[
\frac{1}{\varrho^2 c^2} = \frac{1}{\varrho_1^2 c_1^2} + (1-x) \frac{1}{\varrho_2^2 c_2^2} - x \frac{T \alpha_1^2}{\varrho_1^2 C_1^p} - (1-x) \frac{T \alpha_2^2}{\varrho_2^2 C_2^p} - \frac{T \left( \frac{\varrho_1 \alpha_1}{\varrho_1^2} + \frac{(1-x)\alpha_2}{\varrho_2^2} \right)}{xC_1^2 + (1-x)C_2^2}.
\]

The above formula is equivalent to formulae given in [Landau and Lifshitz(1989), Temkin(1992)]. Here we have developed formulae for all the other thermodynamic quantities, i.e., \( \alpha_v, \alpha_p, C_v, C_p, \gamma, \) etc.

### 2.1 Application to ideal gases

We recall that the ideal gases EOS is given by

\[
p = r \varrho T, \quad \gamma = \gamma(T), \quad C_v = C_v(T) = \frac{r}{\gamma(T) - 1}.
\]

and that we have the following expressions for the thermodynamic coefficients

\[
C_p = \frac{\gamma \varrho}{\gamma - 1}, \quad \alpha_p = \frac{1}{T}, \quad c^2 = \frac{\gamma P}{\varrho} = \gamma r T, \quad \gamma T, \quad \gamma T, \quad k = \gamma - 1.
\]

A mixture of ideal gases with EOS \( p = r_1 \varrho_1 T \) is an ideal gas with \( p = r_\varrho T = \frac{\varrho}{\varrho_1} \varrho_1 T \).

Where we have \( (M \) is the molar mass, and we recall that \( x \) is the mass fraction)

\[
\begin{align*}
p &= \varrho r T, \quad r = x_1 r_1 + x_2 r_2, \\
\alpha_p &= \alpha_p^1 = \alpha_p^2 = \frac{1}{T}, \\
C_p &= x_1 C_p^1 + x_2 C_p^2 = \frac{\gamma T}{\gamma - 1}, \\
\gamma(T) &= \frac{\frac{\varrho_1 \gamma + \varrho_2 \gamma}{\varrho_1 + \varrho_2}}{\varrho_1 + \varrho_2}, \\
C_v &= x_1 \frac{2}{\gamma - 1} C_v^1 + x_2 \frac{2}{\gamma - 1} C_v^2 = \frac{\gamma T}{\gamma - 1}, \\
c_T^2 &= \frac{\gamma T}{\gamma - 1} c_T^1 + \frac{\gamma T}{\gamma - 1} c_T^2, \\
\gamma &= \frac{x_1 C_p^1 + x_2 C_p^2}{x_1 C_{v,1} + x_2 C_{v,2}}.
\end{align*}
\]

From Mayer relation for ideal gases \( C_p - C_v = r \), we obtain

\[
C_v = x_1 C_{p,1} + x_2 C_{p,2} - r = x_1 C_{v,1} + x_2 C_{v,2},
\]

Note that the last relation is only correct for the perfect gases EOS. Hence

\[
\gamma = \frac{x_1 C_{p,1} + x_2 C_{p,2}}{x_1 C_{v,1} + x_2 C_{v,2}}.
\]

We see in particular that if \( \gamma_1(T) = \gamma_2(T) \) (but not necessary \( r_1 = r_2 \)) then \( \gamma(T) = \gamma_1(T) = \gamma_2(T) \).
3 Thermodynamic properties of two-phase mixture with mass exchange

We consider a system, in thermodynamic equilibrium, made up of two phases of the same constituent, with the possibility of mass exchange between the two phases (phase change). An example is a liquid-vapor system. We consider the evolutions and the properties of this system along the equilibrium curve. Each phase taken on its own is an open system that can be described using three variables \((X_i, Y_i, n_i)\), which gives a total of six variables. The equilibrium between the two phases results in 4 variables of the system. However, if they are expressed in the variables \((T, v, s, e, x)\) or \((v, s, P, e, x)\), the state variables \((T, v, s, e, x)\) of state variables among \((X, Y)\) and state variables of the mixture \((v, s, e, x)\) are naturally defined as

\[
\begin{align*}
  s &= xs_1 + (1 - x)s_2, \\
  e &= xe_1 + (1 - x)e_2, \\
  v &= xv_1 + (1 - x)v_2.
\end{align*}
\]

The monophasic state variables \(s_1, s_2, v_1, v_2, e_1, e_2, \mu = \mu_1 = \mu_2, \text{etc.} \), are also state variables of the system. However, if they are expressed in the variables \((P, T)\), we note that for the mixture these quantities are related to \(T\) (or to \(P\)). Therefore, we cannot use them together with \(T\) or \(P\) as state variable. However, we note that this implies a particular dependence for the quantities of the mixture in \(x\) and \(P\) (or in \(x\) and \(T\)). For example, we have

\[
s(P, x) = xs_1(P) + (1 - x)s_2(P)
\]

To establish the relations between the mixture state variables, we first remark that the state variables \((v, s, e)\) verify the first thermodynamic identity \(^1\):

\[
de = Tds - pdv
\]

We can therefore define for the two-phase mixing system the functions \(c^2, k, C^n, \alpha, \text{etc.} \). All thermodynamic identities and relations apply to the equilibrium quantities and state variables of the mixture. We recall however that the identities and coefficients using \(dT\) and \(dP\) (or \(dT\) and \(dP\) ) simultaneously, have no meaning and are to be eliminated, \(i.e.,\) the thermodynamic identities that we will keep will imply a

\(^1\) One way to prove it is to use \(g = e + Pv - TS = xg_1 + (1 - x)g_2\) and notice that \(dg = xdg_1 + (1 - x)dg_2 + (g_2 - g_1)dx = xdg_1 + (1 - x)dg_2\), because at equilibrium \(g_1 = g_2 = \frac{m_1}{m_{tot}}\). So \(dg = vdp - sdT\) and the first thermodynamic identity follows.
triplet among \((dv, ds, dT, de, dx)\) or \((dv, ds, dP, de, dx)\). As previously stated, it suffices to express two of the thermodynamic properties for the mixture to deduce the others by the different relations. On the other hand, the fact that \(\mu_1 = \mu_2\) yields to \(h_1 - h_2 = T(s_1 - s_2) = L\), and this quantity is called Latent heat of phase change.

The Claparon-Clausis relation is obtained by differentiating \(\mu_1 = \mu_2\) with respect to \(T\)

\[
\frac{dP}{dT} = \frac{s_2 - s_1}{v_2 - v_1} = \frac{L}{T(v_2 - v_1)}.
\]

(27)

We note also the formula for \(\frac{d^2P}{dT^2}\) from [Krafcik and Sánchez Velasco(2014)].

\[
\frac{d^2P}{dT^2} = \frac{c_p^2 - c_v^2}{T(v_2 - v_1)} - 2 \frac{c_p^2 v_2 - c_v^2 v_1}{v_2 - v_1} \frac{s_2 - s_1}{v_2 - v_1} + \frac{v_2 \chi^2 - v_1 \chi_1^2}{v_2 - v_1} \left(\frac{s_2 - s_1}{v_2 - v_1}\right)^2.
\]

(28)

From \(P = P(T)\) we note that \(c^2 = k^2 T C^v\) and \(K_T = 0\), \(C^v = \infty\), \(\alpha_p = \infty\), \(\gamma = \infty\). Moreover,

\[
\frac{dP}{dT} = \frac{gc^2}{kT} = \frac{gC^v(gc^2 - pk)}{gkC^v - p} = \frac{gkC^v}{(31)}
\]

(29)

We therefore have two relations for the three variables \(k\), \(C^v\) and \(c^2\), which are expressed as follows

\[
c^2 = k^2 T C^v
\]

(30)

and

\[
\frac{c^2}{vkT} = \frac{s_2 - s_1}{v_2 - v_1}
\]

(31)

We note that [Bennett et al.(1964)]Bennett, Kahl, and Wedemeyer show the following relation

\[
c^2 = v^2 \frac{T}{C^v} \left(\frac{dP}{dT}\right)^2 = v^2 \frac{T}{C^v} \frac{(s_2 - s_1)^2}{(v_2 - v_1)^2} = \frac{v^2 L^2}{C^v T(v_2 - v_1)^2}.
\]

(32)

This relation can also be obtained from the two relations (30) and (31). Therefore, we can write

\[
k = \frac{c^2}{v^2 \chi^2 - \chi_1^2}.
\]

(33)

and

\[
C^v = \frac{v^2 T (s_2 - s_1)^2}{c^2 (v_2 - v_1)^2}
\]

(34)

Now, it remains to find \(c^2\) which we will do in the remaining of this section, following a method similar to the one in [Landau and Lifshitz(1989)]. In the following, we will note the phase 1 as \(G\) (gaz), and phase 2 as \(L\) (liquid). Since we have

\[
v(x, p) = xv_G(P) + (1 - x) v_L(P), \quad s(x, p) = xs_G(P) + (1 - x) s_L(P).
\]

(35)

if we choose the thermodynamical variables \(P\) and \(s\) to describe the system on the saturation curve, we can write

\[
\frac{\partial v}{\partial P}_{\text{sat}, x} = \frac{\partial v}{\partial P}_{\text{sat}, x} + \frac{\partial v}{\partial s}_{\text{sat}, x} - \frac{\partial s}{\partial P}_{\text{sat}, x}.
\]

(36)

\(\text{We write } v(x, p) = \delta(s(x, p), p)\) and we derive with respect to \(x\) on both sides
Thus
\[ \left. \frac{\partial v}{\partial P} \right|_{s,t, \text{sat}} = \left. \frac{\partial v}{\partial P} \right|_{s,t, \text{sat}} - \left. \frac{\partial s}{\partial P} \right|_{s,t, \text{sat}} \]  
(37)
and hence using (35)
\[ \left. \frac{\partial v}{\partial P} \right|_{s,t, \text{sat}} = \left. \frac{\partial v}{\partial P} \right|_{s,t, \text{sat}} - \frac{v_G - v_L}{s_G - s_L} \left. \frac{\partial s}{\partial P} \right|_{s,t, \text{sat}} \]  
(38)
which yields to
\[ \left. \frac{\partial v}{\partial P} \right|_{s,t, \text{sat}} = x \left( \frac{\partial v_G}{\partial P} \right|_{s,t, \text{sat}} - \frac{v_G - v_L}{s_G - s_L} \left. \frac{\partial s}{\partial P} \right|_{s,t, \text{sat}} \right) + (1 - x) \left( \frac{\partial v_L}{\partial P} \right|_{s,t, \text{sat}} - \frac{v_G - v_L}{s_G - s_L} \left. \frac{\partial s}{\partial P} \right|_{s,t, \text{sat}} \right) \]  
(39)
Noting \( L = h_G - h_L = T(s_G - s_L) \) the latent heat of phase change. The first term of the right hand side of Eq. (39) can be expressed as follows
\[ \frac{\partial v_G}{\partial P} \bigg|_{s,t, \text{sat}} - \frac{v_G - v_L}{s_G - s_L} \frac{\partial s}{\partial P} \bigg|_{s,t, \text{sat}} = \frac{\partial v_G}{\partial P} \bigg|_{s,t, \text{sat}, T} + \frac{\partial v_G}{\partial T} \bigg|_{s,t, \text{sat}, P} \frac{\partial s}{\partial P} \bigg|_{s,t, \text{sat}} \]  
(40)
Clapeyron’s relation gives \( \left. \frac{\partial P}{\partial s} \right|_{s,t, \text{sat}} = \frac{L}{T(v_G - v_L)} = \frac{s_G - s_L}{v_G - v_L} \), and therefore the right hand side of Eq. (40) becomes
\[ \frac{\partial v_G}{\partial P} \bigg|_{s,t, \text{sat}, T} + \frac{\partial v_G}{\partial T} \bigg|_{s,t, \text{sat}, P} \frac{T(v_G - v_L)}{L} \]  
(41)
Combining the fourth relation of Maxwell \( \left. \frac{\partial s}{\partial P} \right|_{s,t, \text{sat}} = \left. \frac{\partial v}{\partial T} \right|_{s,t, \text{sat}, P} \) and the definition of \( C^p \)
as \( C^p = T \left. \frac{\partial s}{\partial P} \right|_{s,t, \text{sat}, P} \), we obtain
\[ \frac{\partial v_G}{\partial P} \bigg|_{s,t, \text{sat}, T} + \frac{\partial v_G}{\partial T} \bigg|_{s,t, \text{sat}, P} \frac{2T(v_G - v_L)}{L} - \frac{T(v_G - v_L)^2 C^p_G}{L^2} \]  
(42)
The second term of Eq. (39) is transformed in the same way and we get
\[ -\frac{v^2}{c^2} = x \left( \frac{\partial v_G}{\partial T} \bigg|_{s,t, \text{sat}, P} + \frac{\partial v_L}{\partial T} \bigg|_{s,t, \text{sat}, P} \frac{2T(v_G - v_L)}{L} - \frac{T(v_G - v_L)^2 C^p_L}{L^2} \right) + (1 - x) \left( \frac{\partial v_L}{\partial T} \bigg|_{s,t, \text{sat}, P} + \frac{\partial v_L}{\partial T} \bigg|_{s,t, \text{sat}, P} \frac{2T(v_G - v_L)}{L} - \frac{T(v_G - v_L)^2 C^p_L}{L^2} \right) \]  
(43)
and
\[-\frac{v^2}{c^2} = x \left( \frac{\gamma_G^2 v_G^2}{c_G^2} + \alpha_G v_G^2 \frac{2T(v_G - v_L)}{L} - \frac{T(v_G - v_L)^2 C_G^p}{L^2} \right) + (1 - x) \left( \frac{\gamma_L^2 v_L^2}{c_L^2} + \alpha_L v_L^2 \frac{2T(v_G - v_L)}{L} - \frac{T(v_G - v_L)^2 C_L^p}{L^2} \right), \tag{44}\]
which allows to write
\[c^2 = \frac{v^2}{xK_1 + (1 - x)K_2}, \tag{45}\]
where
\[
\begin{align*}
K_1 &= \frac{\gamma_G^2 v_G^2}{c_G^2} - \alpha_G v_G^2 \frac{2T(v_G - v_L)}{L} + \frac{T(v_G - v_L)^2 C_G^p}{L^2}, \\
K_2 &= \frac{\gamma_L^2 v_L^2}{c_L^2} - \alpha_L v_L^2 \frac{2T(v_G - v_L)}{L} + \frac{T(v_G - v_L)^2 C_L^p}{L^2}. \tag{46}\end{align*}
\]
From the equations (45), (33) and (34) we can deduce:
\[k = \frac{v}{v_G - v_L} \left( xK_1 + (1 - x)K_2 \right), \tag{47}\]
and
\[C_v = \frac{L^2}{T(v_G - v_L)^2} \left( xK_1 + (1 - x)K_2 \right) \tag{48}\]
All the other thermodynamic coefficients can be expressed from the three above \(c^2\), \(C_v\) and \(k\).

3.1 Special cases
If we assume \(v_G \gg v_L\), such us in the case of a liquid-vapour system, the speed of sound formula become
\[c^2 = \frac{(x + (1 - x) \frac{v_L}{v_G})^2}{x \left( \frac{\gamma_G^2}{c_G^2} - \frac{2\alpha_G T v_G^2}{L} + \frac{T C_G^p}{L^2} \right) + \mathcal{O} \left( x \frac{1}{v_L} \right) + (1 - x) \left( \frac{T C_G^p}{L^2} \right) + \mathcal{O} \left( (1 - x) \frac{v_L}{v_G} \right)}, \tag{49}\]
For the case \(x \ll 1\), and still assuming that \(v_G \gg v_L\) we end up with
\[c^2 = \frac{L^2}{TC_G^p} \left( x + \frac{v_L}{v_G} \right)^2 + \mathcal{O} \left( \frac{v_L^2}{v_G^2} \right) + \mathcal{O} \left( x \frac{v_L^2}{v_G^2} \right). \tag{50}\]
Thus,
\[c \simeq \left( x + \frac{v_L}{v_G} \right) \frac{L}{\sqrt{TC_G^p}}, \tag{51}\]
which is a very low value. This is consistent with experimental values where a significant drop in the speed of sound is observed in a liquid with cavitation [Kieffer(1977)]. Within this limit, we obtain
\[k = \frac{c^2}{xv_2 + (1 - x)v_1} \simeq \left( x + \frac{v_L}{v_G} \right) \frac{L}{TC_G^p} \tag{52}\]
and

\[ C^v = \frac{(xv_2 + (1 - x)v_1)^2}{Tc^2} \frac{L^2}{(v_2 - v_1)^2} \simeq C^p_L \]

The last identity \( C^v \simeq C^p_L \) may be surprising but one should keep in mind that the mixture coefficients, such as \( C^v \), are derivatives along the saturation line. The identity \( C^v \simeq C^p_L \) is also not at all intuitive, and we note that in the paper [Kieffer(1977)], the author uses the intuitive hypothesis \( C^v \simeq C^v_L \) (for \( x \ll 1 \)) which we proved above to not be generally correct. The case \((1 - x) \ll 1 \) yields to

\[
\begin{align*}
  c^2 &= \frac{c^2_G - c^2}{\gamma^2_G - c^2_G} \left( a_G \frac{T}{L^2} + \frac{2\alpha^2_G}{L^2} \right) + O \left( \frac{x}{\sqrt{v_G}} \right) + O(1 - x) \\
  k &= \frac{k_G - c^2_G}{\gamma^2_G - c^2_G} \left( 2\alpha_G + \frac{Tc^2}{L^2} \right) + O \left( \frac{x}{\sqrt{v_G}} \right) + O(1 - x) \simeq \frac{c^2}{L^2} \\
  C^v &\simeq \frac{\gamma^2_G L^2}{T^2} - 2\alpha_G L - C^p_G
\end{align*}
\]

Numerical applications for ideal gas give a slightly reduced speed compared to \( c_G \).

4 Conclusion

We presented general expressions for the thermodynamic properties of a mixture of two components, as functions of the component properties. Generally, these properties are obtained by developing an equation of state for the mixture, constructed from a pre-chosen equation of state for the component, such as Peng-Robinson EOS. The proposed approach is more general as it may relax the hypothesis underlying the choice of an equation of state. It may also be applied directly to tabulated data and experimental measurements for thermo-physical properties. In other hand, some authors or engineers may be attempted to use simple arithmetic averaging values for mixtures quantities, and we hope that our formulae provide a ‘ready-to-use’ alternative to adopt for correctness.

References

Benjelloun(2021). S. Benjelloun. Thermodynamic identities and thermodynamic consistency of Equation of States. arXiv preprint arXiv:2105.04845, 2021.

Bennett et al.(1964) Bennett, Kahl, and Wedemeyer. F. D. Bennett, G. D. Kahl, and E. H. Wedemeyer. Resistance changes caused by vaporization waves in exploding wires. Technical report, Army Ballistic Research Lab Aberdeen Proving Ground MD, 1964.

Kieffer(1977). S. Kieffer. Sound speed in liquid-gas mixtures: Water–air and water–steam. J. Geophys. Res., (82):2895–2904, 1977.

Kračík and Sánchez Velasco(2014). M. Kračík and E. Sánchez Velasco. Beyond Clausius–Clapeyron: Determining the second derivative of a first-order phase transition line. American Journal of Physics, 82(4):301–305, 2014.

Landau and Lifshitz(1989). L. D. Landau and E. M. Lifshitz. Course of Theoretical Physics Volume 6-Fluid Mechanics. Butterworth-Heinemann, 1989.

Temkin(1992). S. Temkin. Sound speeds in suspensions in thermodynamic equilibrium. Physics of Fluids A: Fluid Dynamics, 4(11):2399–2409, 1992.