Compressive Sensing Magnetic Resonance Image Reconstruction and Denoising using Convolutional Neural Network

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Abstract: Restoration of high-quality brain Magnetic Resonance Image (MRI) from the sparse under-sampled complex $k$-space signal is a widely studied ill-posed inverse transform problem. A deep learning-based data-adaptive and data-driven convolutional technique has been proposed for high-quality MRI recovery from its under-sampled complex domain $k$-space signal. The uniform subsampling process is very slow in phase-encoding to generate high-resolution images. The longer scan times degrade the perceptual image quality. Various factors contribute to image degradation during data acquisition such as the inception of body motion artifacts, the thermal energy effects of the body, and random noise artifacts due to voltage fluctuations. Keeping in view the patient’s critical condition and comfort, longer scan times are not preferred in practice. To reduce the image acquisition time, noise levels, and motion artifacts in the MR images, Compressive Sensing (CS) provides an accelerated way to reconstructs the high-quality MR image from very limited signal measurements acquired much below the Nyquist rate. However, such data acquisition strategies require advanced computer algorithms for the reconstruction of high-quality MRI from the undersampled MRI data. An improved CNN-based MRI reconstructed algorithm has been presented in this paper which shows better performance to reconstruct high-quality MRI than similar other MR image reconstruction algorithms. The performance of the proposed algorithm is measured by image quality checking tools such as normalized-MSE, PSNR, and SSIM.

Keywords: Magnetic resonance image, compressive sensing, sparse-encoding, $k$-space, convolution neural network, image denoising, image reconstruction.

1. Introduction
MRI is a major medical image acquisition technique but very slow in scanning the MR signal. MRI is used to acquire medical images of internal human anatomy, soft-tissue organs, and other structural parts of the human body. These medical images assist the medical practitioners in diagnosis and disease detection in clinical practices. Medical MR imaging technique is a popular non-invasive and safe imaging technique among other existing medical imaging modalities such as X-rays, Computerised Tomography, ultrasound, positron-emitting transmission (PET), etc. because it does not use any harmful radio-ionized material like x-rays and CT images to acquire MR images. However, it requires a longer scan time which is practically not preferred considering the patient’s condition and comfort in real situations.

According to the Shannon-Nyquist theory [1], an MRI signal can be regenerated from its under-sampled $k$-space counterpart if it has sampled at twice the number of samples of its peak frequency. The slow MRI data acquisition makes it ineffective in critical cases such as in high-priority emergency conditions when...
Immediate scans are required. Further, the longer scan time can cause discomfort and claustrophobic feeling to the patients. Critical and pediatric patients can't remain static without movements and in the breath-holding state during long scan times [2]. Compressive sensed (CS) imaging acquisition techniques accelerate the low data acquisition process by reducing long image scan time which is usually taken by present days MRI scanners. This task is accomplished by reconstructing high-resolution MRIs from the compressively sensed complex domain k-space signal using some non-linear reconstruction methods from even 10% of the Nyquist data sampling rate [2]. The under-sampled MRI signal is acquired in the complex frequency domain and is used to reconstruct the MRI [3], [4]. CS can provide high-quality fast image reconstruction from sparsely represented MR data by adopting a non-linear image reconstruction strategy. However, to accomplish the said target, different issues related to it are required to be addressed first such as long scanning time, high image recovery time, under-sampling optimization, sparse representation of subsampled data, etc, to attain the fast reconstruction of quality images. Denoising sparsely represented data can enhance the reconstruction image quality by suppressing the artifacts and noise contents [5].

CS has attracted a wider interest of the research community for greater potential to undertake and explore the new research strategies in under-sampled MRI reconstruction. It is one of the fast data collection methodologies which exploit the data sparsity to reduce the MRI scanning time. For high-quality MR image reconstructions from sparsely represented data also exist such as partially separable reconstruction techniques such as k-t methods and other model-based reconstruction approaches. The central idea of sparse data representation is that MRI data is compressible which contains redundant information. Because these techniques are solely based on sparse characteristics of data in place of hard parallelism which is the potential for high-speed MRI data acquisition exceeds that of parallel imaging techniques [6]. In the following figure, the relationship of fully-sampled and compressive sensed undersampled MRI data acquisition scheme has been presented.

![Figure 1](image)

**Figure 1.** In the first row from (a) is reference image (b) is with 50% scanned factor-2 (e) and (f) are with fully sampled with factor-2 & 4 respectively and (e) is with additive Gaussian noise of 10 dB SNR. The second row from (f) to (i) is indicating from fully to under-sampled k-space data.

The motivation for this paper is to present a robust CNN-based data-driven computer algorithm for high-resolution MRI reconstruction. The purpose of this method is to accelerate the MR image acquisition and reconstruction process by acquiring under-sampled compressively sensed under-sampled k-space data. The complex domain MRI signal is transformed by Fourier transform converting it into sparse image plain
coefficients. Subsequently, artifact removal and denoising operation is also performed simultaneously to produce high-quality image reconstruction. The rest of the paper is organized such as in Section-2 given the similar existing works, Section-3 contains the proposed methods. Section-4 contains experimental evaluation, Section-5 demonstrates the experiment simulation, results, and comparisons with other existing methods. Section-6 presents the conclusion and future scope of the method.

2. Related Work

The compressive sensing data acquisition represents the MRI data in less number of radiofrequency impulses acquired in an undersampled linear combination of MR signal values in the k-space. Compressive sensed data is used to recover the high-quality MR image from a set of undersampled MRI data by alleviating the long MRI scanning time. Therefore, compressive sensing provides under-sampled sparse k-space data, which is usually contaminated with noise and aliasing artifacts due to incoherence in the transform domain.

The parallel imaging techniques received data signals from multiple scanner coils and combine them to reconstruct the images [7]. In the parallel hardware-level data acquisition, compressive sensing utilizes the multi radio-frequency pulses receiving from multi-coils for aliasing effect removal. The parallel acquisition methods have their limitations and drawbacks as these introduce artifacts and noise effects in the reconstructed images which further required more preprocessing. The recent works show the ability of compressive sensing techniques to represent the sparse data for rapid MRI reconstruction [4], [8]. To avoid the reconstruction artifacts that occurred due to geometric regularities, Bandlet wavelet basis functions were used to remove the noise effects [9]. The conventional compressive sensing image reconstruction techniques use uniform sparse transformation methods such Fourier transform, Total Variational (TV), and wavelet transformation methods [4],[10], [11]. The TV regularization utilizes the gradients sparsity including in the basis functions. The CS has also been applied in functional MRI which is also reconstructed from sparsely acquired data both in spatial and time domain [12]–[16]. The major issue with uniform data sparsifying transform is that it does not image-specific which may result in highly undersampled due to inadequate sparseness [3]. This drives to the use of small size local sparse-patches that can generate data-specific dictionaries which contain local image features and represents sparsity at a much higher undersampling rate [3], [17]–[20]. The transform domain sparse k-space data such as wavelets and contourlets is represented as a dictionary which plays a key role in high-quality MRI reconstruction in compressive sensing MR data acquisition.

Other sparse representation-based MR image reconstruction methods are in use such as dual-tree wavelet transform [21] and an overcomplete contourlet [22]. Wavelets are used to recover minute pixel-based image features and contourlets recover curve-like image details [23]. In addition, adding the Total Variation (TV) method can improve the recovered MR image quality by 1.3 dB than wavelet + Total variation scheme. An extended version of the wavelet-based sparse transform method is the Gaussian Scale Mixture (GSM) method [24], [25], which exploits the dependencies between wavelet coefficient and compressive data.

The data-adaptive compressive sensing dictionary learning approach is more capable to represent the sensed data in more sparse coefficients. Only a few non-zero data elements are sufficient to represent the entire k-space grid data. For example, an MRI $x \in C^{p}$ where $x_{(i,j)} \in C^{n}$ is a n dimensional vector representing the 2D MR image patch of size $\sqrt{n} \times \sqrt{n}$ pixels, starting pixels indexing locations from the left-top corner $(i,j)$ pixel grid. The dictionary $D \in \ell_{1}$ represents the image patches. Each dictionary patch $D$ with $K$ vectors, terms as atoms and each $n^{th}$ vector represents a squared size patch of image data coefficients. Each patch vector $x(i,j)$ in $D$ is said to be sparse. When $K$ to be equal n, the total number of dictionary atoms will equal to the size of the $\sqrt{n} \times \sqrt{n}$ and $D$ to be read as one complete basis. Else if $K > n$, the dictionary $D$ will be called as overcomplete.

In [17], CNN based MRI reconstruction method was proposed for highly undersampled MRI data by a 3D sparse convolution encoding scheme. This method accelerates the Fourier transform-based 3D convolution
by a filtering updations with iterative learning from the sparse coefficients. Small square patch-based non-local filters were applied in [26]–[31]. The non-local methods have been demonstrated robust image denoising performance while preserving the maximum image details than the previous compressed sensing reconstruction methods as given in [32], [33]. In [34], a unified approach is followed for the reconstruction of MR images using bi-directional recurrent-neural network (RNN) and CNN which directly undersampled in the k-space by acquiring compressive sensing data acquisition. An adaptive CNN method has been presented in [35] which exploits the k-space data by interpolation (ACNN-k-Space) and utilizes the residual encoding-decoding technique to recover the signal values to reconstructs the images by interpolating sparse k-space coefficients. Each image slice is interpolated by integrating within its nearest neighboring sequences. The CNN-based model is being widely used in different medical image processing applications for diagnosis and classification purposes CT, MRI, and chest x-ray images [36]. But the major issue and challenge in medical image processing for reconstruction and denoising is that a single subject voluminous data set is not always available in each case. Up to a few years back, the non-local means filters were popular and have been used in MRI denoising and reconstruction, however, sparsity property is not explored much in these filters [37]. In [5], a rotation-invariant non-local means (RI-NLM3D) filter is applied to sparse DCT transformed coefficients combining with self-similar regions in the MR image. This method shows enhanced denoising and sparse reconstruction images. It has been observed from the published research articles in scientific journals that not much attention has been paid towards deep learning CNN-based medical image denoising and reconstruction techniques.

3. Methods

The initial MRI reconstruction starts with the under-sampled k-space coefficients simply by taking the inverse of Fourier transformed coefficients. The unknown zero-filled k-space coefficients are filled by replacing with simply by the inverse transformed coefficient, hence called zero-filled reconstruction. MRI y is to be reconstructed from N number of radio-frequency pulse samples acquired in the k-space by MR scanner and the image x ∈ CN is formulated from this data is written as:

\[ y = F_0 x + v \]  

where \( y \in C^M \) and \( (M < N) \) is the k-space complex transformation data \( x \) which is corrupted with random Gaussian white noise \( v \) of variance \( \sigma^2 \), and \( y \in C^N \) and \( F_0 \in C^{N \times N} \) is the undersampled sparse Fourier transform data matrix \( F \in C^{M \times N} \) and the undersampled operator \( U \in C^{M \times N} \) which is fully extracted using different sub-sampling schemes to calculate the partial Fourier domain matrix of data coefficients. The graphical procedural strategy of this method is represented in the following figures.

**Figure 2.** The graphical pipeline of MRI data acquisition from (a) fully-sampled k-space (b) 1-D data (c) fully sampled with factor-2 (d) with added noise and (e) fully sampled with factor-2 k-space MRI reconstruction.

The compressive sensing MRI acquisition aims to acquire fewer data than the Nyquist rate and reconstruct the high-resolution MRI from compressive sensed sparse k-space MRI data.
Figure 3. Conventional fully sampled MRI acquisition and image reconstruction

The given set of image patches \( Y = (y_j)_{j=1}^m \in \mathbb{R}^n \) of \( m \) signals where \( y_j \in \mathbb{R}^n \). The dictionary-learning approach aims to extract the best dictionary \( D = (d_i)_{i=1}^p \) of \( P \) atoms of \( d_i \in \mathbb{R}^p \) sparse encodes data to recover an approximated image for each \( y_j \in \mathbb{R}^n \) patch \( n \) of size \( w \times w \) from the given sparse image.

The sparse encoding of a single data patch \( y = y_j \) for some \( j = 1...m \) is obtained by minimizing the \( \ell^1 \) constrained optimization with norm-minimization and is written as:

\[
\min_{\|x\|_0 \leq k} \frac{1}{2} \|y - Dx\|^2
\]  

where the \( \ell^1 \) is the pseudo-norm of \( x \in \mathbb{R}^n \) and is given as:

\[
\|x\|_0 \{i, x(i) \neq 0\} = \sum_{i=1}^n \mathbb{1}_{x(i) \neq 0}
\]

The parameter \( k > 0 \) is used to control the amount of sparsity of data.

The optimization is performed on both the data dictionary \( D \) and the set of coefficients \( X = (x_j)_{j=1}^m \in \mathbb{R}^n \), where \( j = 1,...,m \) and \( x_j \) is the set of coefficients of \( y_j \) the data patch. The optimization on both sets \( D \) and \( X \) can be written as:

\[
\min_{D \in \mathbb{R}^n \times \Phi_x \forall x} E(X, D) = \frac{1}{2} \|Y - DX\|^2 = \frac{1}{2} \sum_{j=1}^m \|y_j - Dx_j\|^2
\]

The constraint on the dictionary \( D \) is set as:

\[
D = \{D \in \mathbb{R}^n \mid D \text{ is sparse, } D, ||D|| \leq 1\}
\]
The dictionary columns are unit-normalized and the sparsity constraints on \( X \) is set as:

\[
\Phi_k = \{ X \in \mathbb{R}^{n \times p} \mid X_{.,j} \leq k \}
\]  

(6)

Let \( P(x) \) is a set of all image patches in which each pixel of each patch \( y \) appears with the same frequency in different overlapping patches. This overlapping patch generation scheme will allow a faster image reconstruction process [29].

The dictionary \( D \in \mathbb{R}^{n \times p} \) where \( p \geq n \) atoms in \( \mathbb{R}^n \) are the initial dictionary extracted by a selection of random patches normalization to a unit-norm. A dictionary of an input image is created and display as:

![Input Image](a) and [Dictionary Patches](b) created from input (a) of one slice of brain MRI.

**Figure 4.** Dictionary (b) is created from input (a) of one slice of brain MRI

\( P(y) \) can be considered a matrix of \( d \times n \) the size of patches of the image. Each column of \( P(y) \) the image matrix represents a patch vector and can be seen as a shifted vector of the image. When the dictionary \( D \in \mathbb{R}^{n \times p} \) when \( p \geq n \) atoms in \( \mathbb{R}^n \) then the initial dictionary \( D \) is extracted by random selection of MRI patches and normalizing them to norm one.

### 3.1. Updating the \( X \) coefficients:

The optimization of \( X \) coefficients requires for each vector \( y_j = Y_{.,j} \) to \( x_j = X_{.,j} \) computation that provides the solution:

\[
\min \frac{1}{2} \| y - Dx_j \|^2
\]

(7)

which is non-smooth and non-convex minimization. A heuristic to solve it is to compute a stationary point of the energy using the forward and backward iterative scheme given as:

\[
x_j \leftarrow \text{Pr}_{\Phi_k} (x_j - \tau D^*(Dx_j - y))
\]

(8)

where \( \tau < \frac{2}{\| DD^* \|} \). The dictionary atoms \( |\bar{x}(1)| \leq ... \leq |\bar{x}(n)| \) are arranged in an ordered magnitude of the vector \( x \in \mathbb{R}^n \) and the orthogonal projection on \( \Phi_k \) denotes to \( z = \text{Pr}_{\Phi_k} (x) \) for all \( ith \) vectors as:

\[
\forall i = 1, ..., n, \quad z(i) = \begin{cases} x(i) & \text{if } x(i) \geq |\bar{x}(k)|, \text{and} \\ 0 & \text{otherwise} \end{cases}
\]

(9)
3.2. Updating the $D$ coefficients

After computing the coefficients of $X$, the dictionary atoms are also updated by the minimizing error term in the image reconstruction action and is written as:

$$
\min_{D \in D} \frac{1}{2} \| Y - DX \|^2 \tag{10}
$$

The minimization can also be performed with the projected gradient descent optimizer as defined below:

$$
D \leftarrow \text{Pr}(D - \tau(DX - Y)X^*) \tag{11}
$$

where $\tau < 2/\|XX^*\|$. 

3.3. Convolution Neural Networks (CNN)

The CNN models learn the useful image details and restore the same in the reconstructed images. A CNN model in this work consists of nine convolution layers those convolved with batch-normalization and ReLU function to recover the image details with convolutional weight filters. A max-pooling layer down-sampled the extracted data to retain the low-frequency high energy data values and the last fully-connected output layer produces the final output image values. A piecewise rectified linear activation function (ReLU) is used in each convolution output layer to provide coefficient values between 0 and non-negative numbers. A CNN model is trained to generate small image patches of 32×32 sizes and a filter network of size 3×3 is used to convolve the image patches to restore the useful image details in the reconstructed images.

The first layer convolve with 192×3×3×1 channel filters for gray-scale input MR image slice and extracts 192 feature maps from a noisy image. The ReLU activation function is applied to the convolution layer for non-linearity operation and the 3×3 kernels used to convolve with each slice of the MRI voxels. Next, except final layer all the hidden layers are of identical type convolution of 3×3 of 192 filters to extract 192 features. Batch-normalization alongwith ReLU is applied on a two subsequent convolution layers to extract useful image features. The data-flow diagram of the scheme presented in this paper is given in the following figure.

![Figure 5](image_url) 

**Figure 5.** MRI reconstruction data-flow diagram from the under-sampled compressed MRI signal using CNN model

The architecture of the CNN model will be presented in the concerned section. The compressively sensed raw MRI signal is accumulated in the k-space grid. The fully sampling of MRI data at the Nyquist rate is requires a longer scanning time normally between 30-60 minutes for one complete scan. Compressive sensing data acquisition is used to accelerate the MRI generation by halving the Nyquist rate. This is used to accelerate the reconstruction of the high-quality MRI by sparse-encoded reconstruction for timely diagnosis. The high-resolution MRI is recovered from the sparse k-space by alleviating the long...
computational time. The compressive sensing is achieved by convolving the k-space sampling mask with the fully-sampled MRI signal.

The complete procedure of reconstruction and denoising can be explained in the following steps:

1. MRI volume was first transformed by forward-Fourier transform in k-space coefficients.
2. K-space sub-sampled with 10%-25% of the frequency domain coefficients.
3. A sparse dictionary $D$ is trained by extracting useful image details from the full k-space data
4. A fully sampled image is reconstructed using the CNN model from sparse slice-wise dictionary weights by convolution operations.
5. The denoising operation is performed on the recovered image by the CNN-based noise residue learning method by convolution and batch normalization, iteratively optimizing data error loss.
6. Stop when the denoising reconstruction error rate is minimum.

3.4. CNN-based optimization by rank minimization

The sparse MRI reconstruction problem can be efficiently resolved by the matric rank minimization approach. Taking the logarithm of the matrix determinant [38] is minimized iteratively by computing the local minimum. CNN model can be applied iteratively to reduce the value of the logarithm which can also be used in matrix rank minimization. The CNN model kernel filters of size 3×3 are trained with MRI patches generated from a fully sampled noise-free image. These patches are termed learned dictionaries which are used to suppress the noise levels in the recovered MR images from the undersampled sparse noisy image patches with random white Gaussian noise. A prior known noise variance effectively removes the noise contents and the denoising process becomes fast and efficient. The first input layer takes a 32×32 image patch which is convoluted with 3×3×192 filters of different weights with stride 1 with zero paddings. The subsequent convolution network layers second, third, and fourth identically perform the same process at each layer and followed by an activation function ReLU is applied. An outlier zero is padded with each patch to produce the same output patch size as the input patch size. The max-pooling operation is applied to downsample the extracted image features with low frequency high valued edge details, and object boundaries.

Figure 6. Architecture model for patch-wise image feature learning from the noisy input MRI for network model training.
CNN is applied to the square small image patches for reconstructed image approximation from the noisy sparse k-space measurement. The network of CNN kernel weights is applied on extracted image patches by a convolution operation. Several patches of the image contain redundant image information in image regions. Different clusters are formed by grouping small equal-sized with similar characteristic patches extracted from the input images. This approach reduces the optimization problem. The input set of all square patches is considered inherently sparse and similarity of patches is generated for each patch of size $\sqrt{n} \times \sqrt{n}$ from the center of each patch. A group $G$ is formed considering all image patches similarity with the formula written as:

$$G = \{(f_k \| \hat{g}_f - g_{fk} \| < T)\}$$ \hspace{1cm} (12)

where $\hat{g}_f$ is a noise-free image patch and $g_{fk}$ is the group of corresponding noisy patches. $f_i$ is the minimization function less than a predefined threshold value $T$. After grouping all similar image patches, subsequent coefficients matrix $b$ is evaluated from all patch matrixes as written below:

$$b = [b_1, b_2, ..., b_{n-1}]$$ \hspace{1cm} (13)

A group of patches that belongs to $G^{\text{move}}$ and every vector in $f_k$ is a $k^{\text{th}}$ patch. When the matrix $b_i$ is contaminated with Rician probability of noise distribution, the reconstruction results changed and the MRI is reconstructed from the sparse k-space by matrix rank minimization. The reconstruction matrix $B$’s rank can be recovered from the following optimized problem solver:

$$B = \arg \min B, s.t. \| b - B \|_2^2 \leq \lambda_{a_k}^2$$ \hspace{1cm} (14)

where $\| . \|_2^2$ is $L_2$ a normalized matrix and $\lambda_{a_k}^2$ is indicating the variance of Rician distributed noise in MRI and this problem can be solved by minimizing the rank of the system $B$. An efficient non-convex optimization solution can be provided by the matrix rank minimization for efficient image restoration.

3.5. CNN Training

The proposed CNN model is trained with a fully sampled freely available BrainWeb MRI simulated database of T1-weighted MRI. The CNN model is trained with fully sampled training data to predict the missing data values from the sparse signal components. Image patches of $32 \times 32$ are generated by sliding windows from the input images. The stride =1 is used to slide the filter masks to avoid overlapping due to zero padding outliers. The network trained with setting the network model learning-rate value at 0.001 and the batch-size one by adopting the gradient optimization in the learning parameters. Norm-2 data loss function applied to minimize the reconstruction information loss. The rate of information loss in the reconstructed images has been computed estimating the amount of noise parameter and image patches reconstruction loss parameters. The choice of parameter selection for estimation in the convolution architecture is arbitrary.

The extracted k-space image patches are used as input to the convolution layer of CNN to convolve the target kernel weights by adding random Gaussian noise with varying noise levels. Next these k-space patch convolved with the undersampling mask to extract the sparse image patches. The noise corrupted k-space sparsely represented data is used to generate spatial 2D MRI by the inverse transform of complex domain coefficients. The square $32 \times 32$ image patches extracted from spatial domain images were used for training the CNN model for feature learning in the given images. The convolution filter weight parameters are adopted to optimize the information loss between the target reconstructed noise-free image patches and noisy input image patches. The output of the convolution operation is the weight parameter matrixes and denoised image patches. Each patch has been convolved with the weight matrixes iteratively to reconstruct
the noise-free high-quality image. The proposed model trained with the 256×256 MRI voxels by increasing the training dataset many folds.

This paper contains only the noise-free high-quality MRI reconstruction estimating from the undersampled k-space sensitivity maps. For an application like brain tumor localization, segmentation and edge tracing requires inter-patch correlation for high accuracy. To avoid over-fitting training and speed up the training, the dropout connection technique can be applied. The training phase requires a huge amount of training data and long training times whereas testing can be performed with very little data in a very short time. The non-availability of large medical imaging data is a major issue and a big challenge for the training of deep-CNN-based models for MR image reconstruction and denoising applications. The architecture of CNN for compressive sensing estimation and training the weighted dictionary to learn the important feature details during the network training phased is given in the following image.

3.6. Loss function

The loss function \( L \) estimates the information loss by calculating the data difference between the original input and reconstructed output images. The smaller information loss indicates the high quality of the reconstructed image and higher information loss indicates the low recovered image quality and poor is the performance of the reconstruction algorithm. By fine-tuning the filter weights we can control the convergence of filter weight values and significantly increase the performance of the reconstruction quality of the output image patches. The MR image reconstruction quality is monitored by minimizing the mean square error (MSE) between the input and recovered images as defined below:

\[
L = ||i - f(i)||_2
\]  

(15)

The overlapping of stride \( s \) for each image patch has adjusted the null values of outlier pixels values due to zero padding and minimized by:

\[
\min ||i - f(i)||_2 \quad \text{such that} \quad -1 \leq s \leq 1
\]

(16)

and the local mean of the current denoised patch \( y \) and its noise component \( k \) is estimated as:

\[
i = f\{y\} - f\{k\}
\]

(17)
Here, \( f \) is the function used to estimate the mean value. \( f\{y\} \) estimates the image patches from sparse k-space coefficients and \( f\{k\} \) is the estimation of the noise from the training process. The estimation of the noise-free patch from the noise corrupted image patch, the above equation can be re-written as:

\[
\log_q(y|i) = \frac{1}{2\sigma^2} \sum (y_{r,n} - i_{r,n})^2
\]

(18)

where \( r \) and \( n \) are representing the indexing of image \( y \) and \( \sigma \) representing the noise standard deviation in the noise degraded image patches. The normalized mean-squared error (nMSE) is optimized during the training phase as:

\[
\min_\beta p = f\{(i - \bar{i} - \beta(y - \bar{y}))^2\}
\]

(19)

3.7. Testing

The MRI patches generated from undersampled noisy k-space data are given as inputs to the CNN model and processed by the hidden network layers by convolving with convolutional kernel weight parameters. The dictionary of atoms weight parameters is used to reconstruct the noise-free patches by recovering the mission k-space data in the undersampled image patches. As the denoising is performed by patch-wise classification, the testing is very fast in comparison to the training phase. The dictionary generated during the training phase is used to recover the denoised image patches in the testing phase. The final reconstructed image is obtained by aggregating from the \( i^{th} \) image-patches of undersampled sparse k-space data which is defined and written as:

\[
\hat{i} = \bar{i} + \frac{f\{(y_i - \bar{y})^2\) - \sigma^2(y_i - \bar{y})\)}{f\{(y_i - \bar{y})^2\}}
\]

(20)

The CNN architecture for the testing phase is given below. The sparse k-space patches are input to the CNN model. There are four convolution layers. Input image patch is given to the first convolutional layer. From the second to the fourth layer, a linear activation function ReLU is applied followed by a max-pooling is applied to retain the low-frequency high-value edge details. The final layer is the fully connected output layer which provides the fully sampled reconstructed and denoised image.

![Figure 8. CNN architecture for the testing phase in image reconstruction from learned dictionary patches](image)

3.8. MRI denoising

CNN MRI denoising model is based on a two-way approach called feature encoder and decoder. This feature encoder-decoder network comprises four hidden convolutional layers, four activation layers followed by max-pooling layer, and finally a fully-connected output layer. The receptor of data neurons is enhanced by down-sampling in the max-pooling operation. The overall architecture of the CNN model is present here in figure-7 above. The encoder convolution layer uses 3×3 size convolution kernels with a
A linear activation ReLU function (Rectified Linear Unit) is applied to shift the convolution results. The max-pooling layer is used to down-sample the spatial resolution of the output image features by convolving a $2 \times 2$ filter convolution operation with $2 \times 2$ weighted kernels.

A $181 \times 217$ image is encoded with a $3 \times 3 \times 192 \times 1$ channel feature map followed by a convolution layer. The lowest convolution layer generates a $192$ channel feature map. Mainly encoder during the encoding phase doubles the feature maps and simultaneously reduces the size of the encoder at every stage similar to the U-Net model given in [39].

The information decoding process involves the up-sampling of the image feature maps by the deconvolution operation. However, in the image reconstruction process, some of the features-map information is lost which resulted in degradation of the perceptual image quality up to a large extent and the final reconstructed image may lose vital information. This will be very crucial to eliminate this problem. This problem is resolved by fusing the original image features by encoding features from the corresponding feature maps in the upsampling layers to updated the feature maps in the recovered images.

The decoder process involves the deconvolution process using $3 \times 3$ weight filter kernels and followed by a non-linear ReLU operation is applied to the encoding process. The $192$-channel input image feature maps are extracted from the image during encoding and are deconvoluted to $128$-channel feature maps during decoding and upsampling operation. The max-pooling layer downsampling the feature map by reducing the output feature map dimensions after every convolution operation. The feature encoding-decoding process maps image feature details at the same level in the images in the convolution layers. The final fully connected output layer produces the expected number of feature classes to regenerate the final noise-free MRI.

4. Experimental Evaluation

The proposed CNN-based method has been evaluated with BrainWeb [40] simulated T1-weighted and T2-weighted simulated MRI database. The image denoising reconstruction performance has been evaluated by experimenting with $181 \times 217 \times 181$ dimensions MRI volume of BrainWeb simulated MRI dataset with unknown and known varying noise levels. We have used 40 MRI slices of $181 \times 217$ out of which 30 were used to train the CNN model and 10 have been used for validation. In addition, a similar type of strategy is adopted by adding noise of different noise levels ranging from 3% to 11% in noise-free MRI volumes. The experimental results have also been compared with various existing techniques such as SparseMRI reconstruction, Compressive Sensing with Non-local Total Variation reconstruction, and low-rank modeling of local k-space neighborhood methods.

The implementation of the proposed algorithm has been tested online using cloud platform https://colab.research.google.com/ on 64-bit Windows-10 on Intel i5 CPU with 16-GB RAM. The model has been trained online using online cloud GPU. The experiment results show the comparative reconstruction and denoised performance with other existing algorithms as shown in the results.

Some of the methods given here were implemented and evaluated using the publically available codes provided by the concerned authors for evaluation and reproducible research purposes. Several quantitative measures such as MSE, PSNR, and SSIM have been utilized as defined and given below here.

4.1. Qualitative and Statistical Evaluation

The quantitative quality performance of the denoised and reconstruction images is evaluated by using the normalized mean-square-error (n-MSE), peak signal-to-noise ratio (PSNR), and structural-similarity-index metrics. The n-MSE between the reference image and the reconstructed image is calculated with the following system:
\[ \text{nMSE} = \frac{\sum_{i=1}^{M} \sum_{j=1}^{M} [(f(i,j) - \hat{f}(i,j))^2]}{\sum_{i=1}^{M} \sum_{j=1}^{M} (f(i,j))^2} \]  

whereas \( f(i,j) \) is the ground truth reference image and \( \hat{f}(i,j) \) is the reconstructed image. For reconstruction image quality quantification, the PSNR parameter is calculated as the ratio between the peak-intensity value with the squared root of mean square error which is written as:

\[ \text{PSNR} = \frac{\text{MAX}(f^2)}{\text{nMSE}} \]

For denoising image quality, in addition to PSNR, SSIM is also calculated by the following formula as defined below:

\[ \text{SSIM} = \frac{(2\mu_f\mu_f + C_1) \times (2\sigma_{f \times \hat{f}} + C_2)}{(\mu_f^2 + \mu_f^2 + C_1) \times (\sigma_f^2 + \sigma_f^2 + C_2)} \]

In this equation \( \mu_f \) and \( \mu_f \) are means, \( \sigma_f^2 \) and \( \sigma_f^2 \) are variances and \( \sigma_{f \hat{f}} \) are the covariances of the ground-truth referenced image \( f(i,j) \) and reconstructed image \( \hat{f}(i,j) \) respectively.

### 4.2. Results and Comparative Evaluation

The freely available to download and use BrainWeb [40] simulated MRI dataset has been used for experimental work and to evaluate the quantitative and qualitative evaluation of this method. Only the T1-weighted gray-scale 3D dataset is considered for testing purposes. MRI slices from this imaging modality are selected to convert into k-space by complex Fourier transformation and then an undersampled mask is applied to obtain the compressed data containing only 25% to 50% k-space data. In the network training, fully sampled noisy images were used to train the model by extracting the useful image features and edge details.

To generate noisy data, Fourier transforms image space is used by additive white Gaussian noise in the complex frequency domain k-space data. Further image patches of 32×32 were generated from the full-sized image slices. All the generated patches were indexed and fed to the CNN in sequence to estimate the missing sensitivity.

The three quantitative MSE, PSNR, and SSIM have been calculated on BrainWeb simulated T1-W MRI dataset and have been given in table 1 below.

| Sr. No. | Noise Levels | MSE   | PSNR  | SSIM  |
|---------|--------------|-------|-------|-------|
| 1       | 3            | 3.449 | 37.38 | 0.970 |
| 2       | 5            | 4.705 | 34.68 | 0.949 |
| 3       | 7            | 5.777 | 32.90 | 0.927 |
| 4       | 9            | 6.751 | 31.54 | 0.905 |
| 5       | 11           | 7.701 | 30.40 | 0.882 |
| Average | 7            | 5.676 | 33.38 | 0.927 |

The qualitative performance compared with few other existing reconstruction techniques and given in Table-2 below.

| Technique(s) | PSNR | SSIM |
|--------------|------|------|

Table 2: The reconstruction performance comparison of the proposed method with other existing methods
According to the performance evaluation and comparison is given in the table-2 above, the proposed method shows better performance. The results obtained in terms such as MSE, PSNR, and SSIM have been plotted in the following figures.

**Table 2**

| Method                  | MSE  | PSNR |
|-------------------------|------|------|
| Zero-filled             | 29.84| 0.887|
| SparseMRI [9]           | 30.29| 0.860|
| CS+NLTV [28]           | 30.33| 0.888|
| G-LORAKS [8], [[13], [[16] | 27.78| 0.837|
| **Proposed CNN-based**  | **33.38** | **0.927** |

According to the performance evaluation and comparison is given in the table-2 above, the proposed method shows better performance. The results obtained in terms such as MSE, PSNR, and SSIM have been plotted in the following figures.

**Figure 9.** The left-side figure gives a comparison of noise levels vs MSE vs PSNR and the right-side shows noise levels vs SSIM score.
Figure 10. The first row contains (a) reference image (b) noise corrupted image. Second-row (c) to (f) contains denoised images results and in the third row from (g) to (j) depicts noise residual images.

5. Conclusion
This paper represents the fast MRI k-space data acquisition and reconstruction of the high-quality MRI by denoising from the under-sampled compressive sensed complex domain MRI data. The convolution-neural-network-based techniques have been widely used in machine learning and deep learning computation models to predict the expected outputs by training the neural network filter to restore the image data. A patch-based strategy is adopted to extract the useful image feature details by training a layered architecture to process the image patches. The trained dictionaries are used to recover the missing image patch information. The random white complex Gaussian-noise is removed by the noise filtering process to recover the high-quality images. The information loss in the recovered images and reconstructed image quality is evaluated using the image quality metrics. The performance of the proposed method is comparable with other exiting image denoising and reconstruction methods. The presented technique still has the potential to improve its performance by testing with more realistic medical imaging data. In the future, this method will go through more testing phases with different assumptions and a variety of MRI data to make it a more robust and reliable reconstruction and image denoising method.

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