Review of Methods for Solving Inverse Problems of Identification of the Magnetization of Permanent Magnets in Electrical Devices

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Abstract. The article shows the need for identification of permanent magnets by studying the distribution of magnetization over their volume by solving inverse problems. The main approaches to solving inverse problems are considered: minimization of some functional and multiple solution of the direct problem of calculating the magnetic field; solution of an ill-posed problem and determination, using regularization methods, of a pseudosolution that is stable to small perturbations. An overview of the work performed aimed at improving the theoretical and practical approaches to solving these problems is given. New mathematical models and methods for calculating the magnetic field have been developed, which provide a decrease in the number of unknown parameters, an improvement in stability and a decrease in time consumption in the problems of identifying the magnetization of permanent magnets in electrical devices. The influence of the experimental data error on the solution result is estimated. Examples of solving test problems and experimental studies are given, which have shown the effectiveness of the proposed methods and approaches. The results obtained can also be used to solve direct and inverse problems of electrical engineering.

1. Introduction
Permanent magnets (PM) are increasingly used in actuators (electric motors, electromagnetic drives, electromechanical devices - relays, contactors, etc.) of dynamic systems in various fields of technology due to their size-saving and energy-saving properties. At the same time, devices with PM have a drawback: under the influence of overheating, mechanical shocks, current surges in the windings, the PM is demagnetized, which can lead to malfunction and failure of devices. In this regard, it becomes necessary to identify PM, that is, to assess their state by studying the distribution of magnetization over the PM volume by solving inverse problems. This is because the magnetization cannot be measured directly. However, it can be estimated indirectly by measuring the values of the magnetic induction in accessible places. For example, in the air gap of the device or in the space surrounding the magnet, and then, solving the inverse problem, determine the magnetization. Identification tasks are subdivided into diagnostics, macromodeling and flaw detection tasks. Note that inverse problems are also used for optimal design: for structural and parametric synthesis of objects [1]. Inverse problems of identification and synthesis differ in the type of additional information. Additional information during identification is a set of measured values of a certain quantity, for example, magnetic induction. In the case of synthesis, these are the values of mainly integral characteristics, for example, the force of attraction of the armature to the core of the electromagnet. Synthesis algorithms, in contrast to identification, can
contain a procedure for optimizing any characteristic of the research object, for example, minimizing the mass of the device.

Inverse problems are referred to as ill-posed, that is, the solution to the problem may not be unique and not stable. It is proved that the problems under consideration have the uniqueness of their solutions [2–6], and the stability of the solution will be ensured by restricting the class of the required functions [7], that is, by passing to conditionally correct (correct by AN Tikhonov) problems [8]. A review of methods for solving inverse problems, carried out in [9], showed that two approaches are applied to solving inverse problems: multiple solutions of the direct problem of calculating the field by one of the numerical methods and minimizing the functional corresponding to the problem; the formulation and solution of an ill-posed problem and the definition of a pseudo-solution stable to small perturbations using the regularization method. The first approach is more versatile and is used both for identification and for optimal design of new devices. The second approach is used more often, but only when identifying parameters.

Direct problems are solved by both analytical and numerical methods. Numerical methods meet the requirements of optimal design and identification to a greater extent. For example, these are methods of reducing field calculation problems to the calculation of equivalent equivalent circuits (magnetic, electrical, thermal circuits) [10–15], the finite difference method [16–18], the finite element method [19–21], the boundary element method [22–24], the method of boundary integral equations [25–28], the method of spatial integral equations [29–32], as well as combinations of these methods [33–36]. The methods listed above are grid-based and therefore lead to systems of algebraic equations of large dimension. This increases the time for solving the inverse problem, since, as a rule, a multiple numerical solution of the direct problem of calculating the field is performed. More economical meshless methods: the method of fundamental solutions and its modifications [37–40], as well as approaches combining mesh and meshless methods are practically not used at present [41–45]. The deterministic and stochastic approaches to the estimation of the error in solving inverse identification problems have been applied. Not enough attention is paid to the application of the hierarchy of mathematical models. All of the above indicates the need for further improvement of methods for solving inverse problems of PM identification.

2. Methods for solving inverse identification problems

As a result of the research carried out in the field of improving methods for solving inverse problems of identifying the magnetization of PM in electrical devices, the following results were obtained. A new method for identifying the magnetic state of a solitary PM using only a scalar magnetic potential is investigated. The issue of measuring the values of the magnetic potential based on the integration of the experimental values of the magnetic induction is considered. Two methods for identifying the magnetic state of PM are considered on the basis of the integral equation of magnetostatics from the data of measurements of induction in the surrounding space. A novelty is the study of various variants of the inhomogeneous distribution of magnetization in the volume of the PM [46].

The solution to the inverse problem depends on the choice of points for measuring the magnetic induction, which make it possible to obtain the most accurate estimate of the distribution of the PM magnetization. A study was made of the influence of the placement of the Hall sensor on the assessment of the distribution of magnetization over the PM volume. The research technique is based on conducting a series of numerical experiments by sequentially solving pairs of forward-reverse problems and modeling noise using a pseudo-random sequence generator. The solution of the direct problem of the analysis of the magnetic field, which makes it possible to obtain the distribution of the magnetic induction from the given values of the magnetization of permanent magnets and the properties of the medium, made it possible to dispense with physical experiments. The error specified by the pseudo-random sequence generator was introduced to simulate the measurement process in the calculated values. Such a scheme has such an important advantage over a physical experiment as the possibility of comparing the calculated values of the PM magnetization with the initial distribution, which makes it possible to estimate the accuracy of solving the inverse problem.
In the course of numerical experiments, we considered systems composed of PM and isotropic ferromagnets with a linear characteristic. The method of boundary integral equations, based on the use of scalar potentials of a simple and double layer, was used to solve direct and inverse problems [47]. This method allows us to reduce the inverse problem of identifying the PM magnetization to a system of integral equations of the first and second kind relative to the PM magnetization and the density of fictitious magnetic charges at the boundaries of the frame elements of the device under consideration. Numerical implementation of the method leads to an ill-conditioned system of linear algebraic equations (SLAE), which is solved by the Tikhonov regularization method.

The following conclusions and practical recommendations were formulated as a result of the study: the considered method of identifying magnetization is inapplicable for electromagnetic systems with incorporated PM, which do not allow measuring the induction directly next to the pole; for electrical devices with the placement of PM on the surface, allowing the measurement of magnetic induction in the air gap adjacent to the magnet pole, the considered method is applicable, but the result essentially depends on the choice of points for measuring the magnetic induction; placement points of the Hall sensor are recommended to be located as close as possible above the poles of the PM; In the case of mirror symmetry, which is common in most devices, it is desirable to use dot placement that has the same symmetry; the asymmetric placement of the points, as well as their location behind the magnetic screen, do not allow obtaining a reliable estimate of the magnetization of the PM; in conditions when at all points for all measured components of the magnetic induction vector the same value of the relative error is characteristic, it is recommended to place some of the points close to the edges of the magnets; when using a decreasing sequence as a source of values of the regularization parameter, it is recommended to select a value one million times less than the maximum modulus of the coefficient matrix element as the initial value.

A modification of the method for assessing the magnetization of a PM from the known distribution of the magnetic field in the surrounding space, taking into account the presence of soft magnetic ferromagnetic materials with known characteristics, has been developed [48]. The novelty of the developed modification lies in the possibility of taking into account additional regularizing information in the form of the direction of the easy magnetization axis for each PM, if it is known. In comparison with the original method [38], the modification is proposed to reduce the dimension of the system of equations, and, consequently, to save RAM and reduce the computation time, to use a priori information about the direction of the easy magnetization axis, which makes it possible to go from finding the magnetization vector to finding a scalar the magnitude of the modulus of magnetization. The idea of the modification is as follows: with anisotropy, the PM has one axis of easy magnetization and two axes of hard magnetization, and if the initial direction of magnetization is known in advance (coinciding with the direction of the axis of easy magnetization), then the other two components can be zeroed, reducing the number of unknowns by three times, attributable to this magnet. A transformation matrix is introduced that stores information about the direction of the easy magnetization axis for each PM of the system, or information that the direction is not known in advance.

In order to verify the proposed modification, a program has been developed that allows to simulate the measurement process: to calculate the direct problem and find the magnetic induction at the points of the air gap, then introduce an error into the "measurement results" and solve the inverse problem. The numerical experiments carried out showed that the proposed approach, in addition to speeding up the calculations, leads to a decrease in the error on the average calculated on the basis of the Euclidean norm.

In the mathematical model of the magnetic field, the influence of the hysteresis of the material of the frame of an electrical device on the result of solving the inverse problem of identifying the magnetization of a PM is considered. An algorithm has been developed for estimating the permissible thickness of the magnetic hysteresis loop of the electrical device frame material, for which the behavior of the material can be described by an unambiguous characteristic $B(H)$, without distorting the result of solving the inverse problem of PM magnetization identification. The effect of the width of the hysteresis loop of the frame material on the accuracy of the result of solving the inverse problem of identifying the PM
magnetization is investigated. The main magnetization curve is calculated using the Langevin formula. The hysteresis loop is calculated over the selected range of magnetic intensity values according to the Giles-Atherton model. The identification of the magnetization of a rectangular PM on a ferromagnetic plate is considered as a model problem. A method for optimizing the location of induction measurement points has been developed and implemented. The optimization criterion is the standard deviation of the calculated magnetization from the exact solution. Optimization of the placement of measurement points leads to an improvement in the solution by more than 2 times.

The direct and inverse problems of calculating the fields of inclusions or cavities of toroidal shapes are considered [49]. The solution in the form of a series is very economical in terms of computational costs, since a few first harmonics are usually sufficient to represent the solution with satisfactory accuracy. The inverse problem is to find the magnetic permeability in some part of the volume. Residual magnetization is taken into account by piecewise linear approximation of the hysteresis loop.

A study of methods for non-destructive testing of PM magnetization was carried out on the basis of solving the inverse problem of magnetostatics. New are the results related to the optimization of the location of the induction measurement points, taking into account the differences in the measurement error at different measurement points. According to the practice of magnetic flaw detection, the influence of the field of external currents was not taken into account. The presence of a ferromagnetic layer was taken into account. An approach based on the integral equation of magnetostatics is used to identify the magnetization. The scalar magnetic potential is introduced for a stationary field. The corresponding differential method (cells) is used to solve the integral equation of magnetostatics. The volume of the magnetic material is divided into cells (elementary parallelepipeds). The magnetization is considered constant within the cell. Integrals over the area of cells are calculated analytically, and the stress for points outside the volume of the magnetic material is calculated using the corresponding analytical relations. Tikhonov's method, based on minimizing a functional with a certain regularization parameter, which leads to a SLAE already with a square matrix, is used to regularize the SLAE. Different measurement points are characterized by different information content due to the presence of measurement errors. The weighted amount method was applied to account for this. The weighting factor is assumed to be inversely proportional to the measurement error. The minimized functional in the regularization method takes the form of a weighted sum in connection with the noted circumstance. Moreover, the weighting factors are inversely proportional to the measurement errors. Identification of magnetization in the region of a rectangular permanent magnet located on a ferromagnetic base is considered as a model problem. Optimization of the location of measurement points is considered. The weighted sum method was used to improve the model's adequacy. The weight of the term was taken to be inversely proportional to the measurement error. The use of optimization and the method of weighting factors significantly increases the efficiency of the method under consideration [50].

Methods for identifying the magnetic state of a PM based on the measurement data of the induction in the surrounding space are considered. The methods are based on the application of the corresponding integral equation of magnetostatics. The integrals in the matrix of coefficients of the SLAE and their first derivatives are found exactly analytically, and their new expression is obtained without discontinuous functions. The presence of singular points in derivative functions and their influence on the computational process is noted. Comparison with the method based on numerical differentiation is made. It is assumed that the source of the error is the measurement and calculation of the induction and (or) the scalar magnetic potential in the surrounding space. The influence of the number of cells in the division of the PM region and measurement points is investigated. Various methods of regularization (Lavrent'ev, Tikhonov, and others) are considered. The influence of the factor of finiteness of the discharge grid and the asymptotic character of the convergence of the method are noted. The results obtained can also be used to solve the inverse problem for a system of ferromagnetic bodies and in test problems using other methods [51].

Two-sided methods for calculating the characteristics of the magnetic field of electrical systems containing ferromagnets and PM, based on the application of the Lagrange multiplier method to the equations of the electromagnetic field in terms of the scalar potential, are considered. The Lagrange
multiplier approach has the advantage of being it is applicable for solving both dynamic and stationary problems with distributed parameters. The corresponding adjoint partial differential equations are obtained for different optimality criteria (both for uniform and mean square metrics). The solution of the problem of calculating two-sided estimates of the solution in calculating the static magnetic field in a ferromagnet placed in a third-party uniform magnetic field is considered. This method is also applicable for calculating PM fields, for which it is required to take into account the residual magnetization and the finite width of the hysteresis loop. Corresponding ratios are given for this purpose. It is assumed that the main source of error in the calculation is the approximate values of the magnetic permeability of the medium. The results obtained can also be used to solve direct and inverse problems for a system of ferromagnetic bodies and in test problems using other methods [52].

Two-sided methods for calculating the characteristics of the magnetic field of electrical systems containing ferromagnets and PM are considered. The methods are based on the application of the Pontryagin maximum principle to the equations of the electromagnetic field in terms of scalar and vector potentials. The transition is used from the differential formulation of boundary value problems of the magnetic field to the corresponding discrete-continuous formulation in the form of a system of ordinary differential equations, to which the classical theory of the maximum principle is applicable. After finding the equations of the boundary value problem of the maximum principle, the inverse passage to the limit is carried out to the differential form by tending the grid step to zero. The corresponding adjoint partial differential equations are obtained for different optimality criteria. The solution of the problem of calculating two-sided estimates of the solution when calculating the magnetic field in a ferromagnet placed in a uniform external magnetic field is considered. This method is also applicable for calculating the fields of permanent magnets, for which it is required to take into account the residual magnetization, the finite width of the hysteresis loop. Corresponding ratios are given for this purpose. It is assumed that the main source of error is the approximate values of the magnetic permeability of the medium in the calculation. The results obtained can also be used to solve direct and inverse problems for a system of ferromagnetic bodies and in test problems using other methods [53].

To ensure stable operation of devices with PM, it is necessary that the maximum magnetic field strength in a permanent magnet be less with a certain margin of coercive force. It is proposed to estimate the coercive force by the known dependences of the magnetic induction on the magnetic field strength and temperature, as well as the value of the magnetization. However, the features of these characteristics are such that, with this approach, the coercive force is determined with an error exceeding by more than an order of magnitude the error in determining the magnetization. In this regard, a new method for PM identification is proposed [54]. According to it, the temperature is measured in accessible places. Next, the inverse problem of heat transfer is solved using a mathematical model of the temperature field of the device. In this case, the stator-gap heat transfer coefficients can be adjusted. The minimization of the objective function, the sum of the squares of the differences between the measured and calculated temperatures at the selected points, is carried out by the gradient method. A magnetic field simulation must be performed to estimate the maximum magnetic field. The method makes it possible to take into account both the magnetic and the thermal state of the PM installed in an electric machine at relatively low computational costs. The information obtained is necessary to select the type of PM when constructing the pole of the corresponding machine. The method can be used to identify and design other electrical devices.

A method for solving inverse problems of optimal design of electrical devices [55] is proposed. The effectiveness of the method is due to the use of a hierarchy of mathematical models: at the first stage, the designed device is represented by an equivalent magnetic circuit with a magnetic permeability of ferromagnets equal to infinity, at the second stage, by a chain with a finite magnetic permeability, at the third stage, the equations of a stationary magnetic field are used. Inverse problems are solved analytically at the first and second stages, at the third stage numerically. This approach makes it possible to determine the initial approximations of the required parameters with a sufficiently high accuracy and reduce the time of their numerical refinement at the third stage, as well as reduce the total time for solving the problem.
The finite element method is used at the third stage to solve direct problems of calculating the magnetic field and forces; the gradient descent method is used to solve optimization problems. Reducing the time for solving the problem is achieved here by transforming the constraints into objective functions and sequentially minimizing these functions over a limited number of variables. One of the objective functions (mass) is minimized analytically, the rest – numerically. The problem of optimal design of energy-saving electromagnets, in the design of which a PM is built in, providing energy savings, was solved by the proposed method. Comparison of the proposed method with the known ones showed its acceptable accuracy (the error does not exceed 1%) and a reduction in the solution time by about half.

A method for mathematical modeling of three-dimensional magnetic fields in unbounded regions containing subregions with nonlinear dependences \(B(H)\) is proposed. Its application is considered by the example of calculating the magnetic field and determining the magnetomotive force (MMF) of a push-pull actuator, the active element of which is made of a shape memory ferromagnet. The computational algorithm is based on the combined finite element method (calculation of the field in nonlinear subdomains) and fundamental solutions (calculation of the field in the space surrounding the ferromagnet). The field in a linear subdomain is decomposed into two fields: from the coils with current and the magnetization of the ferromagnet. This allows you to go from vector quantities to scalar variables. For the first time in the method of fundamental solutions, point vector magnetic moments are used, which make it possible not only to increase the accuracy of the method, which is proved by solving the test problem, but also to eliminate the numerical instability characteristic of magnetic dipoles. The results obtained make it possible to effectively solve direct and inverse problems of three-dimensional magnetic fields in the design and identification of electrical devices [56].

A variant of the method of point sources for modeling plane-parallel stationary magnetic fields is proposed, in which new sources (vector magnetic moments) are used [57]. The simulation uses the decomposition of the field in the surrounding space into two: the field of currents of the coils in the absence of actuators and the magnetization field of the actuators, which is replaced by the field of magnetic moments located in the ferromagnetic element of the actuator in the absence of coils. A test problem with a known analytical solution is considered as an example: it is necessary to perform mathematical modeling of the magnetic field formed after placing a ferromagnetic cylinder with radius \(R\) and magnetic permeability \(\mu^+\) in a uniform field. A system of equations based on boundary conditions is designed to determine the moments. A more complex case is presented: the section of the object under consideration is a rectangle. In this case, several field sources are used, located outside and inside the rectangle. The application of the proposed method for solving the problem of the synthesis of an actuator (determination of the magnetomotive force from a given magnetic field strength) is described. The dependence of the simulation error on the number of field sources is investigated.

The new PM model is constructed for the analysis of magnetic fields and allows, like the known models (current and charge), to reduce the amount of calculations and, thereby, to increase the efficiency of solving the inverse problem of PM identification [58]. The originality and novelty of the new model lies in the use of point field sources - vector magnetic moments. The moments make it possible to reduce to a simple summation of the contributions of individual sources the calculation of the fields of a number of devices, for example, electric machines with PM [59]. When identifying PM of various shapes, it is proposed to use the moment model. Modern research methodology based on solving inverse problems is used to solve identification problems. The application of the moment model can significantly reduce the computed costs with high accuracy [60]. These sources can also be used in the analysis and synthesis of fields of electromagnetic devices containing ferromagnets.

It is proposed to use the gridless method of point magnetic moments to calculate the energy of devices with PM, which allows one to exclude the operations of integration over the volume and surfaces, replacing them by summing the contributions of the moments, and significantly reducing the total number of unknowns. The method allows the use of well-known sources (charges). The stages of the method implementation are given and examples of its implementation are considered. The calculation results coincide with the known analytical solutions for bodies in the form of a ball and a cylinder. Numerical analysis showed that the total field of the PM, including the field of the PM itself and the
field of the surrounding space, consisting of piecewise homogeneous subdomains, is equal to zero. An example of the placement of point magnetic moments in a PM in the form of a parallelepiped is shown. The calculation of its field showed the effectiveness of the method in comparison with the finite element method. The results obtained allow us to recommend the use of the method of point magnetic moments for calculating the PM energy.

3. Conclusion
A review of the proposed by the authors improved methods for solving inverse problems of PM identification in electrical devices is performed.

Modifications of the method for solving the inverse problem of estimating the magnetization distribution over the PM volume are developed on the basis of boundary integral equations for scalar potentials and are aimed at improving stability and reducing time costs. Despite some acceleration, methods for solving the inverse problem are still time consuming. The developed methods allow performing computations in parallel, which is especially important given the current level of development of computing technology. There are two levels suitable for parallel computations: compilation and solution of SLAE using several streams (the proposed methods ultimately reduce to solving SLAE); solution of several SLAE simultaneously (in optimization methods, parallel calculations for several points from the domain of the minimized functional; in methods based on Tikhonov regularization, these are simultaneous calculations for different values of the regularization parameters).

A method for PM identification with allowance for temperature, which significantly increases the accuracy of solutions, is proposed. A new field source (a point magnetic moment) has been introduced into the calculations. The field modeling method is developed on its basis. The third PM model is built. It differs from the known two (current and charge) by reducing the number of computational operations. Replacing the dipoles with moments improves the accuracy of solutions and eliminates numerical instability.

Using numerical analysis, it is shown that the total energy of the PM, including the energies of the PM itself and the surrounding piecewise homogeneous space (air, ferromagnet, air), is zero. The results obtained can be used for identification, analysis, and optimal synthesis of PM in various electrical devices.

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