Conference Paper

Mathematical Model of Forest Fire Soil-thrower Movement

Olga Dornyak, Ivan Bartenev, Mikhail Drapalyuk, Dmitry Stupnikov, Sergey Malyukov, and Evgeny Pozdnyakov

Voronezh State University of Forestry and Technologies named after G.F. Morozov, Voronezh, Russian Federation

Abstract

The design of a forest fire soil-thrower made to prevent and eliminate ground forest fires is presented. A mathematical model of machine movement has been developed, which enables to study the laws of the interaction process of the design with the soil. It is accepted that the machine has two degrees of freedom. The mathematical model has been obtained using the Lagrange equations of the second kind. The design and technological parameters of the forest fire soil-throwing machine, affecting the efficiency of its work, including mass and width of the grip of the ripper casing, mass, radius and frequency of rotation of the milling tool, the number and geometric parameters of the blades are taken into account. Mathematical model enables to determine the effect of these parameters on the characteristics of the movement of ripper casing and milling working body. A mathematical model is needed to synchronize the translational motion of the unit and the rotational motion of the rotor. Formulas have been obtained for the steady motion of the forest fire soil-thrower, that determine the hauling power of tractor and torque that ensures the operation of milling tools.

Keywords: forest fires, forest fire soil-thrower, ripper casing, milling tool, blades, mathematical model

1. Introduction

Forests are important part of the ecosystem and they play a significant role to preserve and maintain the environment. The main hazard is forest fires because its consequences are terrible in nature. Therefore, there is a need to detect and extinguish fire before it spreads to destroy the resources [1]. Forest fire spreading is a complex process affected by multi-factors. Understanding the relationships between these multi-factors and forest fire spreading trend is vital to predicting the fire spreading promptly and accurately to make the strategy in forest fire extinguishing [2–5].
Millions of hectares of forest are damaged and die each year as a result of forest fires, millions of tons of combustion products are released into the atmosphere; fauna dies, protective forest functions are reduced, threat to human lives is created [6–10].

Forest fire - uncontrolled spontaneous spread of fire on forest land, covered and not covered with forest vegetation. Ground, crown and peat forest fires are distinguished [11–13].

Among all the registered fires, ground fires, the fire of which spreads through the forest floor and the lower tier of forest vegetation, make most of the cases (97 - 98%) and covered areas (87 - 89%).

Modern technical means used for carrying out preventive and forest fire activities have various drawbacks: they create narrow mineralized strip, do not form ground ripraps of necessary volumes, have low productivity, have insufficient constructive reliability of milling tools, which restrains their use on heavy forest soils [14, 15].

2. Methods and Equipment

Therefore, a forest fire soil throwing machine has been developed for the prevention of ground forest fires in conditions of heavy soils and soils saturated with roots of trees and shrubs (Figure 1). It consists of a frame with a hinged mechanism 1, a milling working body (rotor) 2, which has blades for throwing soil 3, a protective ripper 4 equipped with windows 5 through which the soil is thrown out, safety knives 6, hydraulic systems and control elements.

![Figure 1: General view of a forest fire soil thrower.](image-url)
Before starting work, the forest fire soil thrower is hung on the rear hitch of the tractor, the hydraulic drive of the machine is connected to its hydraulic system, and the required deflection angle of the blades is set on a milling tool.

The design chart of the developed machine is shown in Figure 2. After starting the tractor, hydraulic motor begins to spin the milling tool. During the forward movement of the unit, the ripper with safety knives fixed on it is sunk into the soil, loosens it and cuts small roots. After that, the loosened soil goes to the milling working body and is thrown by the blades through one of the two windows provided in the ripper casing. The distance of soil throw is adjusted by changing the rotation frequency of milling tool and deflection angle of blades that run the soil.

![Figure 2: Design scheme of forest fire soil thrower: $B$ -- width of mineralized strip created by the ripper; $B_1$ -- width of mineralized strip created by milling tool; $H_m$ -- mineralized strip depth.](image)

The reliability of the machine is increased in comparison with existing counterparts due to the protection of milling working body with ripper casing, which completely covers cutter from strikes against stumps and roots. The efficiency of the machine on heavy soils is achieved due to the preliminary loosening of soil with ripper casing, equipped with three knives, which cut small roots and prevent milling tool from large roots and stumps.

### 3. Results and Discussion

A mathematical model is proposed to study the laws of the interaction process of forest fire soil thrower design with soil.

It is accepted that deformation of all the elements of the machine can be neglected, and the machine itself moves along a straight horizontal surface.

The studied design has two degrees of freedom. $X$ - the coordinate of the point $O_1$ of the base stand of ripper casing (body 1) and $\varphi$ - the angle of rotation of the
rotor (body 2) relative to the axis of rotation are chosen as generalized coordinates. We introduce a fixed coordinate system OXYZ, associated with the ground (OX axis is horizontal and parallel to the movement trajectory of any point of ripper casing) (Figure 3, a). We also introduce moving coordinate systems O1X1Y1Z1, rigidly connected with ripper casing, and O2X2Y2Z2 which is rigidly connected with the blade. Coordinate system O2X2Y2Z2 is shown for blade 2 (body 22) in Figure 3, a.

![Figure 3: Design scheme of ripper casing of a forest fire soil thrower.](image)

Lagrange equations of the second kind in general form are written as follows [16]:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial x} \right) - \frac{\partial T}{\partial x} = Q_x
\]

(1)

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \phi} \right) - \frac{\partial T}{\partial \phi} = Q_\phi
\]

(2)

where \( T \) -- kinetic energy, J; \( Q_x, Q_\phi \) -- generalized forces with dimensions H and H. m respectively.

Under the made assumptions, the kinetic energy of the mechanical system is determined by the expression

\[
T = \frac{1}{2} \cdot (m_1 + m_2) \cdot \dot{x}^2 + \frac{1}{2} \cdot J_{x2} \dot{\phi}^2
\]

(3)

where \( m_1, m_2 \) -- the masses of ripper casing and milling tool, kg; \( J_{x2} \) -- the moment of rotor inertia relative to the axis of rotation, kg \( \cdot \) m².

External forces applied to the mechanical system under study (active forces and reaction forces of bonds) are used to calculate the generalized forces.

The group of active forces includes casing weight \( \overrightarrow{G}_1 \), rotor weight \( \overrightarrow{G}_2 \), traction force \( \overrightarrow{F}_{\text{traction}} \) forces with the moment \( M_{\text{rot.mom}} \) applied to the rotor (Figure 3, a). The system of external reactions of relations due to the interaction of the working body with the ground has been considered.
The forces of resistance to the movement of ripper casing base stand, distributed over its lower and upper surfaces, are represented by two components directed along the normal and tangential to the surface of the base. The lower surface of ripper casing base stand slides along the surface of dense soil, loosened elements of the soil move along the upper surface of the base of the ripper casing. Thus, the element $ds_1$ of the base stand with the center at some point $M(x, y, z)$ (Figure 3, b) is acted upon by forces which are perpendicular to the plane of the base stand $\vec{N}^{(d)}_1(x, y, z)ds_1, \vec{N}^{(a)}_1(x, y, z)ds_1$, and sliding friction forces directed opposite to the speed of ripper casing base stand relative to the ground $\int_{slid.frict.force}^{(d)}(x, y, z)ds_1, \int_{slid.frict.force}^{(a)}(x, y, z)ds_1$.

Indicating the sliding friction coefficients of the steel base on a dense and loosened soil as $f^{(d)}_1$ and $f^{(a)}_1$, let’s take according to Coulomb’s law

$$\int_{slid.frict.force}^{(d)} = f^{(d)}_1 N^{(d)}_1 \int_{slid.frict.force}^{(a)} = f^{(a)}_1 N^{(a)}_1$$ (4)

It’s obvious that $|N^{(a)}_1| = |N^{(d)}_1|$. Assuming that the trajectories of soil particles are almost parallel to the base, we get

$$N^{(a)}_1 = N^{(d)}_1 = |\vec{q}(x, y, z)| \cos \alpha$$ (5)

where $\vec{q}ds$ -- weight of the loosened soil column with the base $ds$ in the vicinity of the selected point $M(x, y, z)$, Pa; $\alpha$ -- cutting angle, rad.

Function $q(x, y, \text{and } z)$ depends on the density and shape of the soil prism, formed on the surface of the ripper casing base stand when moving the machine. The definition of this function is possible by solving a conjugate problem for a solid-state structure interacting with a half-space of a non-uniform continuous medium. Such a task is not considered in this work, and the density and shape of displaced soil volume are considered to be given values.

The contribution of resistance forces to the movement of loosened soil from the side walls of the ripper casing is small compared with the resistance forces to the movement of the ripper casing base stand, therefore these forces are not taken into account.

The resistance forces to the blades movement depend on the mass of the soil taken by the blade. The magnitude of the resistance force becomes zero as soon as a portion of the soil is thrown into the window. The design involves the installation of three or six blades. The implementation of the operating modes of the forest fire soil thrower is carried out under the condition that the height of the ground "cushion" at the rear wall must be less than or equal to $R/2$, where $R$ -- length of the blade, m. The geometric evaluation of the machine design shows that at any time moment, in the case of three blades, one of them is in contact with soil, and in the case of six blades - two of them.
Distribution density of the forces of soil normal pressure on the blade number \( i \), Pa (Figure 4, a) is shown like \( \vec{N}_{2i} = \vec{N}_{2i}(\phi, x_2, y_2) \). \( \vec{N}_{2i} \) dependence on \( \phi \) is connected with changes in the conditions of blade contact with soil during its rotation. \( \vec{N}_{2i} \) dependence on \( x_2, y_2 \) coordinates is determined by the shape of taken soil volume. Friction forces \( \vec{f}_{\text{slid.frict.force}_{2i}} \) distributed over the blade in accordance with Coulomb’s law and characterized by the function are also necessary to be taken into account. We note that \( \vec{N}_{2i} \) and \( \vec{f}_{\text{slid.frict.force}_{2i}} \) vectors should also be determined from the solution of the adjoin problem.

**Figure 4:** Design scheme of the milling working body of the forest fire soil thrower.

It should be noted that the surface elements of the blade with number \( i \) are in contact with soil if

\[
\frac{\pi}{2} + \gamma_{\text{rip.cas.}} < \phi_i - \left[ \frac{\phi_i}{2\pi} \right] \cdot 2\pi < \frac{3\pi}{2} - \gamma_{\text{rip.cas.}}.
\]

Here square brackets are used to denote the function “integer part of a number”. \( \gamma_{\text{rip.cas.}} \) angle depends on the height of the prism on the basis of ripper casing, while \( \sin (\gamma_{\text{rip.cas.}}) = (R - H)/R \) (Figure 4, b).

If \( \phi_i \) angular coordinate of the \( i \)-th blade satisfies the condition

\[
\frac{3\pi}{2} - \gamma_{\text{rip.cas.}} < \phi_i - \left[ \frac{\phi_i}{2\pi} \right] \cdot 2\pi < \frac{5\pi}{2} + \gamma_{\text{rip.cas.}}.
\]

then this blade is not in contact with the soil, and the resistance force is \( \vec{N}_{2i} = 0 \).

Let us take the formula for calculating the blade motion resistance force which is proportional to the square of the linear velocity of the blade element, as for the case of body motion in an unlimited viscous medium:

\[
N_{2i} = \mu^{**}V^2 = \mu^{**}(\dot{\phi}y_2)^2.
\]

where \( \mu^{**} \) -- drag coefficient, \( Pa \cdot s^2/m^2 \); \( V \) -- the speed of the blade element with \( y_2 \) coordinate.
In this case, if the blade with the number \(i-1\) (the blade located behind the blade \(i\) in the direction of rotation) has an angular coordinate \(\phi_{i-1} > \gamma_1\), where \(\gamma_1\) -- angle, determining the position of the window, then the force of resistance to the soil movement is significantly reduced for the \(i\) blade elements. This is due to the fact that these elements shift the ground into the air space of an open window, and the \(i-1\) adjacent rigid blade does not impede such movement.

Let us assume that the resistance force to the blade movement for \(\phi_{i-1} > \gamma_1\) is defined as in a linearly viscous medium:

\[
N_{2i} = \mu^*V = \mu^*(\phi y_2)
\]

(9)

where \(\mu^*\) -- drag coefficient, \(Pa \cdot s/m\).

The forces of soil resistance to cutting depend on the cutting angle, the angle of sharpening and the speed of working body movement. In this paper, the simplest relation for the horizontal component of the cutting resistance force \(F_c\) of the following form is used

\[
F_c = bh\sigma_r
\]

(10)

where \(b\) -- cutting element width, \(m\); \(h\) -- thickness of cut soil chip, \(m\); \(\sigma_r\) -- soil resistivity to cut, \(Pa\).

After calculating the generalized forces, the Lagrange equations of the second kind for the system under study take the form

\[
\begin{cases}
(m_1 + m_2) \ddot{x} = F_{traction} - (f^{(d)} + f^{(u)}) \cos^2\alpha \int_{S_1} q(x_1, y_1) \, ds - bh\sigma_r \\
J_x \ddot{\phi} = M_{rot.mom.} - \sum_{i=1}^{N} \int_{S_{2i}} N_{2i}(y_2, \phi_i) \cdot y_2 \, ds
\end{cases}
\]

(11)

where \(S_1\) -- base surface area of the ripper casing, \(m^2\); \(S_{2i}\) -- the area of the blade with the number \(i\) which is in contact with the ground, \(m^2\).

The equations (11) describe the dynamic behavior of the mechanical design of a forest fire soil throwing machine. These equations can be used to develop algorithms for controlling the speed of the machine, as well as to determine the influence of design parameters on the displacement velocity of ripper casing and the angular velocity of milling tool rotation.

If the motion of the mechanical system is steady, that is \(\ddot{x} = 0, \ddot{\phi} = 0\), then we can calculate the necessary traction force and torque value in this mode from equations (11):

\[
\begin{cases}
F_{traction} = (f^{(d)} + f^{(u)}) \cos^2\alpha \int_{S_1} q(x_1, y_1) \, ds + bh\sigma_r \\
M_{rot.mom.} = \sum_{i=1}^{N} \int_{S_{2i}} N_{2i}(y_2, \phi_i) \cdot y_2 \, ds
\end{cases}
\]

(12)
4. Conclusion

An improved design of forest fire soil thrower is presented, providing the possibility of its use on heavy soils and soils saturated with roots of tree and shrub vegetation.

A mathematical model of the movement of forest fire soil thrower (11) has been obtained, which makes it possible to determine the laws of ripper casing and milling working body motion and use them for effective control over the machine. The model is necessary to synchronize the non-stationary modes of translational tractor motion and rotational rotor motion with given input parameters.

The design formulas (12) have been obtained for the case of steady motion of soil throwing machine, which determine the values of the traction force from the tractor and the torque which ensures the operation of the milling tool.

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Conflict of Interest

The authors have no conflict of interest to declare.

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