A numerical model of Trichel pulses in air; the effect of pressure

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Abstract. In this paper a three-species two-dimensional numerical model is used for the simulation of the Trichel pulse regime of corona discharge in air in a point-plane configuration. The effect of air pressure on Trichel pulse characteristics is investigated and the numerical results are compared with the available experimental data. The effect of corona electrode radius of curvature on charge per pulse and the relation between charge per pulse and the average charge per pulse is also studied, and compared with the experimental data reported by Atten et al.

1. Introduction
If a negative DC voltage above the corona onset level is applied to a sharp electrode in an electronegative gas, a pulsating current will appear in the circuit. These pulses are called Trichel pulses [1] after Trichel, who first discovered them and did extensive research on their physics. At voltages close to the corona onset level, Trichel pulses are irregular; however, with a small increase in voltage they become very regular. Due to the many ionic species and reactions involved in this process, Trichel pulse regime of corona discharge is an interesting yet a complicated phenomenon.

Most of the models presented so far for modelling the corona discharge are either single-species (only one charge carrier is assumed in the air gap and other species are neglected), steady-state or one-dimensional (1D). A single-species model might be useful for some engineering applications where an approximate estimate of average current and space charge density suffices. However, it does not help with understanding the detailed distributions of charge carriers during corona discharge. Moreover, all of these models fail if a good estimate of corona discharge characteristics is required. Therefore, presenting a model, which can accurately predict the Trichel pulse behavior during corona discharge has still been missing. In addition, most of the experimental and theoretical studies on corona discharge are performed at normal pressure (1 atm). Investigating the effect of pressure on the Trichel pulses is of interest and has not been widely discussed.

In this paper, a two-dimensional (2D) dynamic model, which includes three ionic species (positive and negative oxygen ions, and electrons), is used for modelling Trichel pulses in air. The effect of pressure on Trichel pulse characteristics is studied and compared with the available experimental data [2]. The ionization layer profiles in the air gap during one Trichel pulse are shown and the ionization layer thickness along the axis versus time is presented. The effect of tip radius on the charge per pulse is also investigated and the relation between charge per pulse and the average charge per pulse is studied. The numerical results are compared with the available experimental data wherever possible [2].
2. Mathematical model and numerical algorithm

The corona discharge is modelled in a point-plane configuration (figure 1). A hyperboloidal conducting needle with a tip radius curvature of \( r_p \) and length \( L \) is placed at a distance \( d \) from the infinitely large grounded conducting plate. A negative DC potential \( V \) is applied to the needle. An external (ballast) resistance \( R_{\text{ext}} \) is connected in series between the needle and the voltage source. The ambient gas is air. This system is assumed to be axisymmetric, which means that the equations should be solved in 2D using variables \( x \) and \( r \), where \( x \) is the axial distance and \( r \) indicates the radial distance.

For the simulation of the corona discharge, both the electric field and charge density distributions need to be calculated. The electric field is governed by the Poisson equation (1), while the charge density distribution can be determined by solving charge transport equations for all three ionic species considered in the model [3]. A generic form of the charge transport equation is shown in Equation (2). These four equations are combined with the Kirchhoff voltage equation for the electric circuit, where the total current in the circuit is obtained using Sato’s formula [4]. Details of the equations, reaction coefficients and the boundary conditions can be found in [3].

\[
\nabla \cdot \varepsilon \mathbf{E} = N_{\text{total}} \quad (1)
\]

\[
\frac{\partial N}{\partial t} = S - \nabla \cdot (N \mathbf{W}) + \nabla \cdot (D \nabla N) \quad (2)
\]

where \( \varepsilon \) is the gas permittivity, \( \mathbf{E} \) is the electric field, \( N \) is the charge density of the ionic species, \( \mathbf{W} \) is the drift velocity, \( D \) is the diffusion coefficient and \( S \) is the source term.

To investigate the effect of pressure, it should be noted that the number of oxygen molecules \( (NO_2) \) and mobility of species are affected by pressure such that \( NO_2 \) is proportional to \( P \) and \( \mu \) is inversely proportional to \( P \).

The Finite Element Method (FEM) was used to solve the Poisson equation. For the charge continuity equation, the Flux Corrected Transport (FCT) technique was added to FEM to prevent the FEM results from diverging [3, 5].

3. Results

The numerical model presented in [5] was first used to produce a series of Trichel pulses for the applied voltage of \( V = -9 \) kV, assuming an external resistance of \( R_{\text{ext}} = 100 \) k\( \Omega \) and normal pressure of 1 atm (figure 2). In this case, the ionization layer thickness along the axis of symmetry versus time and 2D profiles of this layer in the air gap were shown during one Trichel pulse. The effect of pressure on the Trichel pulse frequency was investigated for a constant applied voltage. The ratio between charge per pulse and average charge per pulse as calculated by Atten et al. [2] is also calculated and discussed in detail.

Ionization layer

The ionization layer thickness along the axis is defined as the distance from the corona electrode to the point at which the ionization coefficient is equal to the attachment coefficient. Figures 3 and 4 show the thickness of this layer along the symmetry axis and electric field on the corona electrode tip versus time, respectively. The ionization coefficient is related to the electric field, therefore, the variation of the electric field and ionization layer thickness has similar profiles as these figures show.

Figure 5 shows the ionization layer profile at different instants of one Trichel pulse: half pulse rising point (1), maximum current point (2), and two points in the falling section of the pulse (points 3 and 4). At points 1 and 2, the ionization layer thickness is the largest. However at point 2, the ionization stops on the tip of the corona electrode. The diminishing of this layer gradually continues with time along the electrode surface until ionization completely stops in the entire air gap.
Effect of pressure on Trichel pulse characteristics

As reported in [6], for a given needle voltage the Trichel pulse frequency has an inverse relation with pressure. At higher pressures, there is a larger number of oxygen molecules per cubic meter and this produces an inverse relation between mobility and pressure, since ions have more frequent collisions with neutral molecules. Therefore, in these gases the Trichel pulse period increases and the frequency decreases. On the other hand, experiments show that pulse duration has an inverse relation with pressure [2], which is confirmed by our numerical results. Table 1 shows the variation of Trichel pulse period and pulse duration versus gas pressure.

Comparison and discussion

Comparing numerical results with the experimental data [2] results in the following conclusions:

1) The mean charge per pulse increases with corona electrode radius. As Table 2 shows, for the same applied voltage \( V = -7 \text{kV} \), reducing the radius of the corona electrode decreases the generated charge per pulse. It can be explained by the fact that increasing the radius of the corona electrode enlarges the region involved in the ionization. Therefore, the generated charge per pulse increases. As the \( Q \) increases, the Trichel pulse period decreases, because larger space charge needs more time to travel. Therefore, it takes more time for the electric field to increase to a value at which a new Trichel pulse can be generated.

2) The ratio \( Q/Q_c \) (charge per pulse divided by mean charge per pulse) is always larger than 0.5. Charge per pulse, \( Q_o \), was calculated by integrating the corona current in time intervals at which the corona current is above 10% of the maximum current. In Atten’s paper [2], it is reported that in high pressure air and with no external resistance, \( Q/Q_c \) is always smaller than 0.5. Since our numerical results were for normal pressure and non-zero external resistances this was not confirmed by our model, but from Figure 6 it is obvious that decreasing the external resistance from 100 k\( \Omega \) to 10 k\( \Omega \) decreases this ratio significantly. Therefore, it is reasonable to extrapolate and expect that for zero external resistance, this ratio can be below 0.5 as reported in [2].

Figure 1. Circuit model of the system

Figure 2. Trichel pulses for \( V = -9 \text{kV} \)

4. Conclusions

In this paper, a 2D numerical model, which incorporates three ionic species (electrons, positive and negative oxygen ions) was used for simulating the negative corona discharge in point-plane configuration in air and Trichel pulses were obtained at different air pressures. Moreover, the variation of the ionization layer in the air gap during one Trichel pulse and the ionization layer thickness along the axis versus time were shown. The effects of pressure on Trichel pulse characteristics were investigated and it was confirmed that for a given needle voltage, the Trichel pulse frequency, the average corona current and the pulse duration are inversely proportional with pressure. It has been shown that the mean charge per pulse has a direct relation with \( r_p \). Contrary to the experimental data presented in [2], the numerical simulation predicts the ratio \( Q/Q_c \) is larger than 0.5. It is suggested that this discrepancy is due to the presence of the external surge resistor.
Figure 3. The thickness of the ionization layer

Figure 4. Electric field on the corona electrode

Figure 5. Ionization profile at different time instants

Figure 6. $Q_i/Q$ for $V = -8kV$

Table 1. Trichel pulse period and pulse duration versus gas pressure ($V=-9kV$, $R_{ext}=100\mu\Omega$)

| Air pressure (atm) | Trichel pulse period (µs) | Pulse duration (µs) |
|-------------------|--------------------------|---------------------|
| 0.8               | 4.88                     | 0.30                |
| 1                 | 6.61                     | 0.28                |
| 1.2               | 11.07                    | 0.19                |

Table 2. Charge per pulse (in pico coulombs) for two configurations with different radii

| Pulse number | 2      | 3      | 4      |
|--------------|--------|--------|--------|
| Radius = 100 µm | 115.61 | 114.17 | 111.49 |
| Radius = 10 µm    | 23.781 | 21.906 | 21.415 |

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