Hyperbolic Orbits in the Solar System: Interstellar Origin or Perturbed Oort Cloud Comets?

Arika Higuchi,1⋆ and Eiichiro Kokubo2

1RISE Project, National Astronomical Observatory of Japan, Osawa, Mitaka, Tokyo 181-8588, Japan
2Division of science, National Astronomical Observatory of Japan, Osawa, Mitaka, Tokyo 181-8588, Japan

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ABSTRACT

We study the dynamical properties of objects in hyperbolic orbits passing through the inner Solar system in the context of two different potential sources: interstellar space and the Oort cloud. We analytically derive the probability distributions of eccentricity, $e$, and perihelion distance, $q$, for each source and estimate the numbers of objects produced per unit of time as a function of these quantities. By comparing the numbers from the two sources, we assess which origin is more likely for a hyperbolic object having a given eccentricity and perihelion distance. We find that the likelihood that a given hyperbolic object is of interstellar origin increases with decreasing eccentricity and perihelion. Conversely, the likelihood that a hyperbolic object has been scattered from the Oort cloud by a passing star increases with decreasing eccentricity and increasing perihelion. By carefully considering their orbital elements, we conclude that both 1I/2017 U1 ’Oumuamua ($e \approx 1.2$ and $q \approx 0.26$ au) and 2I/2019 Q4 Borisov ($e \approx 3.3$ and $q \approx 2$ au) are most likely of interstellar origin, not scattered from the Oort cloud. However, we also find that Oort cloud objects can be scattered into hyperbolic orbits like those of the two known examples, by sub-stellar and even sub-Jovian mass perturbers. This highlights the need for better characterization of the low mass end of the free-floating brown dwarf and planet population.

Key words: comets: general – Oort cloud

1 INTRODUCTION

The standard formation scenario of planetary systems naturally suggests that interstellar space is filled with many planetesimals because exo-giant planets ejection planetesimals during planet formation, as the planets in the Solar system did (e.g., Dones et al. 2004). Planetesimals that are almost but not completely ejected from the planetary system survive as Oort cloud comets in the planetary system. Oort cloud comets become observable from Earth when their perihelion distances become small due to external forces. For example, when a star penetrates the Oort cloud, the star drills a narrow tunnel through the Oort cloud by ejecting the comets within some distance from the star as described in Figure 1. Some of the ejected comets make a last perihelion passage as their farewells to the Solar system before becoming fully interstellar objects. In other words, both interstellar space and the Oort cloud are possible as sources of objects moving along hyperbolic orbits.

1I/2017 U1 ’Oumuamua, (hereafter U1) is the first highly eccentric ($e \approx 1.2$) object identified in the solar system, with an effective velocity at infinity $V \approx 26$ km s$^{-1}$ (e.g., Williams 2017). This velocity cannot be explained by planetary perturbations because U1 did not encounter any of the planets (Meech et al. 2017). Many observations of U1’s shape, thermal properties, colours, absence of cometary activity, tumbling rotational state, and non-gravitational acceleration have been reported (e.g., Jewitt et al. 2017; Meech et al. 2017; Ye et al. 2017; Bannister et al. 2017; Knight et al. 2017; Micheli et al. 2018; Fraser et al. 2018; Bolin et al. 2018) and are summarized in Bannister et al. (2019). Peculiar physical properties of U1 include its extremely elongated or oblate (Maslchenko 2019) shape and its lack of cometary activity. Together, these properties are unlike those found in other small Solar system objects. However, physical peculiarities alone are not enough to exclude the possibility that U1 might be a Solar system body deflected from the Oort cloud. We examine this possibility here. A second hyperbolic object, the comet C/2019 Q4 (2I/Borisov, hereafter Q4), was discovered by G. Borisov on August 30, 2019, observing from MARGO, Nauchnij, in

* E-mail: higuchi.rika@nao.ac.jp

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the Crimean peninsula. Soon after that the Q4s interstellar nature was confirmed. Q4 has a very high eccentricity of $e = 3.3$, a comet-like appearance and spectrum similar to those of D-type asteroids (de León et al. 2019; Fitzsimmons et al. 2019; Jewitt & Luu 2019).

While most long-period comets have $e < 1$, some are known with $e \gtrsim 1$. Królikowska & Dybczyński (2017) calculated the orbits of long-period comets carefully taking into account the perturbations from planets and the non-gravitational forces to infer their original elements, defined as the orbital elements at 250 au from the Sun before the perihelion passage (e.g., Królikowska 2014; Królikowska & Dybczyński 2017). Królikowska & Dybczyński (2019) collected data for a full sample of long-period comets discovered over the 1801-2017 period and calculated their original orbital elements. They used the JPL Small Body Database Search Engine to construct a complete list of long-period comets discovered since 1801, omitting sungrazing comets. They found that, in most cases, the comets followed elliptical (bound) orbits prior to their last perihelion. Figure 2(a) shows the original eccentricities $e_{\text{orig}}$ and perihelion distances $q_{\text{orig}}$ of 11 comets in original, marginally hyperbolic orbits from Table 1 in Królikowska & Dybczyński (2019). While these comets could also come from the interstellar space, it is more likely that their eccentricities exceed unity only because of uncertainties in the astrometry. In that case, comets in Figure 2(a) are dynamically the same as other long-period comets, but different from U1 and Q4 shown in 2(b).

Here, we derive analytically the probability distributions of eccentricity, $e$, and perihelion distance, $q$, for hyperbolic orbits derived from either interstellar space or the Oort cloud. We estimate the ratio of numbers of objects from the two sources and the dependence of this ratio on various parameters of the Oort cloud and the interstellar objects.

In Section 2, we describe the derivation of the likelihood that interstellar objects have a given value of $b$, the impact parameter to the Sun and $V$, the velocity at infinity. Section 3 follows the methodology applied in Section 2 but for the production of comets scattered from the Oort cloud on hyperbolic orbits. In Section 4, we plot the probabilities derived in sections 2 and 3 on the $e$ vs. $q$ plane and make comparison between interstellar objects and hyperbolic Oort cloud comets. In Section 5, we compare the expected numbers of interstellar objects and hyperbolic Oort cloud comets with an assumption that the Solar system recently had an encounter with a passing object. The properties of a passing object implied by the orbits of U1 and Q4 are discussed in section 6. Section 7 gives a summary and discussion.

### 2 INTERSTELLAR OBJECTS (ISO)

Assuming a uniform spatial distribution and a Maxwellian velocity distribution, the number of interstellar objects (hereafter ISOs) encountering the Sun with the velocity at infinity between $V$ and $V + \delta V$ and the impact parameter between $b$ and $b + \delta b$ per time is given by

$$\delta N_{\text{ISO}}(V, b) = 2\pi b \delta b \rho_{\text{ISO}}(V) \delta V,$$

where $\rho_{\text{ISO}}$ is the total number density of ISOs and $\rho(V)$ is a Maxwellian distribution,

$$\rho(V) = \frac{2}{\pi} \frac{V^2}{a^3} \exp \left( -\frac{V^2}{2a^2} \right).$$

Here, $a = \sqrt{\pi/8}(V)$ and $(V)$ is the mean velocity. We assume

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1. MPEC 2019-R106 : COMET C/2019 Q4 (Borisov) https://minorplanetcenter.net/mpec/K19/K19RA6.html
2. MPEC 2019-S72 : 2I/Borisov = C/2019 Q4 (Borisov) https://minorplanetcenter.net/mpec/K19/K19S72.html
3. https://ssd.jpl.nasa.gov/query.cgi
that ISOs are planetesimals ejected from planetary systems by scattering from giant planet(s). Other fragments might be generated by tidal disruption of planets (Cuk 2018; Rafikov 2018) but their expected contribution is small and neglected here. We estimate the number density of ISOs generated by stars of spectral type ‘i’ as

\[ \rho_{\text{ISO}} = \rho_{\text{str}} \rho_{\text{gp}} n_{\text{OC}} \delta_{\text{ISO}} \]

(3)

where \( \rho_{\text{str}} \) is the number density of the stars, \( \rho_{\text{gp}} \) is the probability that the stars have one of more giant planets, \( n_{\text{OC}} \) is the number of comets in the Oort cloud around each star, and \( \delta_{\text{ISO}} \) is set so that \( n_{\text{OC}} \delta_{\text{ISO}} \) gives the number of ISOs generated by a star of type ‘i’. We use \( \rho_{\text{str}} \) in García-Sánchez et al. (2001) and for simplicity set \( \rho_{\text{gp}} = 0.015, 0.1, \) and 0 for MK, GFA, and other type stars, respectively (Moro-Martín et al. 2009). Assuming that the number of Oort cloud comets is proportional to the mass of the parent star \( m_i \), we set \( n_{\text{OC}} = \rho_{\text{SS}} n_{\text{SS}} (m_i/m_\odot) \), where \( n_{\text{SS}} \) is the number of Oort cloud comets in the Solar system. We use \( m_i \) summarized by Rickman et al. (2008), substituting all the values assumed above and summing over all the stellar types, to obtain the total number density of ISOs as

\[ \rho_{\text{ISO}} = \rho_{\text{SS}} n_{\text{SS}} \sum_{i=0}^{13} \rho_{\text{str}} \rho_{\text{gp}} \left( \frac{m_i}{m_\odot} \right) \approx \Gamma n_{\text{SS}} \delta_{\text{ISO}} \]

(4)

where \( \Gamma \approx 10^{-3} \) [pc\(^{-3}\)]. Table 1 lists our adopted values. We assume that the velocity distribution of ISOs is similar to that of their parent stars, which is \( (V) \approx 50 \) km s\(^{-1}\) as summarized in Table 1 (Rickman et al. 2008). Substituting Equations (2) and (4) into Equation (1), we obtain

\[ \delta N_{\text{ISO}}(V, b) = C_{\text{ISO}} V V \delta V \delta b, \]

(5)

where \( C_{\text{ISO}} \) is the number density of ISOs with a given \( V \) and \( b \), written as

\[ C_{\text{ISO}} = 2 \pi \Gamma n_{\text{SS}} \delta_{\text{ISO}} V \rho(V) b. \]

(6)

### 3 HYPERBOLIC OORT CLOUD COMETS (HOC)

We first derive the velocity and impact parameter of a hyperbolic Oort cloud comet (hereafter HOC) against the Sun after an encounter with a passing object by using the two-body scattering formula. Then, we derive the expected number of HOCs for given \( V \) and \( b \) by taking into account the number density of comets in the Oort cloud.

#### 3.1 VELOCITY AND IMPACT PARAMETER GIVEN BY A PASSING OBJECT

We assume that an object that approaches the Sun passes on a straight trajectory. We describe each encounter of the object with a comet using the following parameters: \( m_\star \) and \( V_\star \), the mass and velocity of the object, \( b_{\text{Sun}} \), the impact parameter of the object to the Sun, \( \mathbf{b}_{\text{HOC}} \), the impact parameter vector from the comet to the object, and \( \mathbf{r}_\star \), the position vector of the object from the Sun at the moment when the object has the closest approach to the comet. We assume that the comet is not moving relative to the Sun and \( V \) and \( b \) of scattered comets are determined only by the perturber. Also considering \( b_{\text{Sun}} \gg b_{\text{HOC}} \), we approximate the position vector of the comet from the Sun with \( r = \mathbf{r}_\star \).

The angle between the velocity vectors of the comet to the object before and after the encounter \( \theta \) is given as a function of only \( V \) and \( V_\star \). The angle \( \theta \) determines the position of the object at the encounter so that the comet has an orbit with \( b \) after the encounter (appendix A1). We find \( r_\star \) that gives \( V \) and \( b \) as (appendix A2),

\[ r_\star = b_{\text{Sun}} \left( 1 - \frac{V^2}{4V_\star^2} \right)^{-\frac{1}{2}}. \]

#### 3.2 EXPECTED NUMBER OF HOCs PER UNIT OF TIME

We estimate the number of HOCs encountering the Sun with a velocity between \( V \) and \( V + \delta V \) and an impact parameter between \( b \) and \( b + \delta b \) per unit of time as

\[ \delta N_{\text{HOC}} = p_{\text{ene}} \delta \rho_{\text{OC}}(r), \]

(8)

where \( p_{\text{ene}} \) is a probability of having an encounter with an object, \( \delta V \) is an element of volume per unit of time (dimensions of \( L^3 \) m\(^{-1}\)) placed at distance \( b_{\text{HOC}} \) from the passing object,

\[ \delta g = \frac{8 (G m_\text{s})^2}{V V_{\text{Sun}}^2} V^{-3} \delta V \delta b, \]

(9)

and \( \rho_{\text{OC}}(r) \) is the number density of comets in the Oort cloud at \( r \). The probability \( p_{\text{ene}} \) is 1 if the Solar system just had an

| type | \( m_\text{s} \) [M\(_\odot\)] | \( \rho_{\text{OC}} \) [10\(^{-3}\) pc\(^{-3}\)] | \( \rho_{\text{gp}} \) | \( V_\star \) [km s\(^{-1}\)] |
|------|-----------------|-----------------|-------|----------------|
| B0   | 9               | 0.06            | 0     | 24.6           |
| A0   | 3.2             | 0.27            | 0     | 27.5           |
| A5   | 2.1             | 0.44            | 0.1   | 29.3           |
| F0   | 1.7             | 1.42            | 0.1   | 36.5           |
| F5   | 1.3             | 0.64            | 0.1   | 43.6           |
| G0   | 1.1             | 1.52            | 0.1   | 49.8           |
| G5   | 0.93            | 2.34            | 0.1   | 49.6           |
| K0   | 0.78            | 2.68            | 0.015 | 42.6           |
| K5   | 0.69            | 5.26            | 0.015 | 54.3           |
| M0   | 0.47            | 8.72            | 0.015 | 50.0           |
| M5   | 0.21            | 41.55           | 0.015 | 51.8           |
| wd   | 0.9             | 3.0             | 0     | 80.2           |
| gi   | 4               | 0.43            | 0     | 49.7           |

Table 1. Stellar parameters used in this paper. ‘wd’ and ‘gi’ indicate white dwarfs and giant stars, respectively. The last column gives the mean heliocentric velocity. The values are taken from García-Sánchez et al. (2001), Moro-Martín et al. (2009), and Rickman et al. (2008).
encounter with an object, and if not, \( p_{\text{enc}} = 0 \). The value of \( p_{\text{enc}} \) averaged over the age of the Solar system is discussed in section 6. The element of volume per unit time, \( \delta g \), is defined so that comets contained within \( \delta g \) have a velocity between \( V \) and \( V + \delta V \) and an impact parameter between \( b \) and \( b + \delta b \) (appendix A3). We model the distribution of comets in the Oort cloud as \( \rho_{\text{OCC}}(r) = \rho_{\text{OCC}}^0 r^{-\gamma} \). Numerical studies show \( \gamma \approx 3 \) (e.g., Dones et al. 2004). Assuming that the Oort cloud has inner and outer edges at \( r_{\text{min}} \) and \( r_{\text{max}} \), respectively, we have

\[
\hat{\rho}_0 = \begin{cases} 
\frac{(y - 3)\gamma - 3}{4\pi r_{\text{min}}^{y-3}} & \text{for } \gamma > 3 \\
\frac{\pi \log(r_{\text{max}}/r_{\text{min}})^{-1}}{\gamma} & \text{for } \gamma = 3
\end{cases},
\]

where we assume \( r_{\text{min}} \ll r_{\text{max}} \) for \( \gamma > 3 \). Substituting Equations (9) and (10) into Equation (8), we obtain

\[
\delta N_{\text{HOC}}(V, b) = C_{\text{HOC}} \delta V \delta b,
\]

where \( C_{\text{HOC}} \) is the number density of HOCs at a given \( V \) and \( b \) and using Equation (7) written as

\[
C_{\text{HOC}} = \frac{8(Gm_*)^2}{V_*} \rho_{\text{OCC}}^0 \sum_{\text{Sun}} \left(1 - \frac{V_*^2}{4V_*^2}ight)^{y/2} V^{-3}.
\]

4 DISTRIBUTIONS OF ECCENTRICITY AND PERIHElON DISTANCE

We convert the distributions of \( V \) and \( b \) into those of \( e \) and \( q \) assuming that all objects move on hyperbolic orbits whose focus is at the Sun (appendix A4). The numbers of ISOs and HOCs encountering the Sun with eccentricity between \( e \) and \( e + \delta e \) and the perihelion distance between \( q \) and \( q + \delta q \) per time is given by

\[
\delta n(e, q) = C J \delta e \delta q = \delta n(V, b) J,
\]

where \( \delta n(V, b) \) and \( C \) represent \( \delta N_{\text{ISO}}(V, b) \) or \( \delta N_{\text{HOC}}(V, b) \) and \( C_{\text{ISO}} \) or \( C_{\text{HOC}} \), respectively, and \( J \) is the determinant of the Jacobian between the \((V, b)\) and \((e, q)\) frames.

Panels (a), (b), and (c) in Figure 3 show the contours of the two-dimensional probability distributions for ISOs obtained from Equations (5) and (13) on the \( e \) vs. \( q \) plane. The values of the contours are normalized at U1: we call this normalized probability \( p_{\text{ISO}}(e, q) \). We adopt \( \langle V \rangle = 20 \text{ km s}^{-1} \), 50 km s\(^{-1}\), and 100 km s\(^{-1}\).

For any value of \( \langle V \rangle \) shown in Figure 3, the probability increases with decreasing \( e \) and \( q \). The ridge roughly following the equi-velocity curve given by

\[
V = \sqrt{\frac{Gm_*(e - 1)}{q}} \quad (\text{for each mode } V_{\text{eq}}(V) = \sqrt{\frac{\sqrt{7}/2}{2}} V \text{ is seen}),
\]

however, the distribution is rather flat. The probability at the same eccentricity as U1’s but at \( q = 1 \) au, is given by \( p_{\text{ISO}}(e = 1.2, q = 1) \approx 1.3, 0.34, 0.28, \) for \( \langle V \rangle = 20 \text{ km s}^{-1}, 50 \text{ km s}^{-1}, \) and 100 km s\(^{-1}\), respectively. The probability at Q4’s \( e \) and \( q \) is given by \( p_{\text{ISO}}(e = 3.3, q = 2) \approx 0.12, 0.30, 0.34 \) for \( \langle V \rangle = 20 \text{ km s}^{-1}, 50 \text{ km s}^{-1}, \) and 100 km s\(^{-1}\), respectively. This implies that U1’s orbit is more typical of ISOs than Q4’s.

For HOCs, we examine the probability distribution of \( e \) and \( q \) per unit of time but over an encounter with an object because it varies with time during the encounter. We weight Equation (12) by \( 2b/sin \alpha \), the path length of the object where it can generate comets with given \( V \) and \( b \) (see Figure 4). Figure 3(d) shows the probability distribution obtained from Equations (11) and (13) on the \( e \) vs. \( q \) plane for HOCs integrated over an encounter with an object with \( V = 50 \text{ km s}^{-1} \). The probability diverges at \( e = 1 \) and \( q \to \infty \). We obtain \( p_{\text{HOC}}(e = 1.2, q = 1) \approx 0.10 \). The distribution is steep compared to that of ISOs where \( e \) is small. This result barely changes with \( V \). The black dotted lines in Figure 3(d) show the equi-velocity curves for \( V = 10, 26, \) and 60 km s\(^{-1}\). Comets on equi-velocity curves arrive at the Sun almost at the same time because \( b \ll b_{\text{Sun}} \). At \( q = 1 \) au on the \( V = 26 \text{ km s}^{-1} \) curve, \( p(=1.76, q = 1) \approx 0.4 \). This implies that, among the HOCs \( V = 26 \text{ km s}^{-1} \) that arrive at the Sun around the same time, U1’s and \( q \) are as likely for an origin as ISOs as such as ISOs. The arrival time is calculated as \( t_{\text{arr}}(V) = (w/V - (w/tan \alpha)/V) = b_{\text{Sun}}[1 - (3/8)V^2/V_{\text{eq}}^2]V/V_{\text{eq}} < 1, \) where \( w \) is the path length of the HOC (see Figure 4). This means that the HOCs with larger \( V \) arrive at the Sun earlier than those with smaller \( V \). In other words, the advance members of a comet shower are more consistent with U1 than other comets coming after them. Note that \( p_{\text{HOC}}(e, q) \) is independent of \( m_* \) and \( b_{\text{Sun}} \).

5 RATIO OF ISO TO HOC

Integration of Equations (5) and (11) over ranges of given eccentricity and perihelion distance gives the absolute numbers of ISOs and HOCs per time. However, we prefer to discuss ratio of ISOs to HOCs, because their absolute numbers strongly depend on the uncertain size distributions. In what follows, we have implicitly assumed that ISOs and HOCs have the same size-frequency distributions, allowing \( n_{\text{OCC}} \) to be canceled out.

We define the ratio of the number of HOCs to that of ISOs for given \( e \) and \( q \) as

\[
\mathcal{H} = \frac{\delta N_{\text{HOC}}}{\delta N_{\text{ISO}}} = \frac{C_{\text{HOC}}}{C_{\text{ISO}}},
\]

which tells us which source is more likely given a particular \( e \) and \( q \) pair. We assume that an encounter of the Solar system with an object HOC occurs and set \( p_{\text{enc}} = 1 \).

Figure 5 shows contours of \( \mathcal{H} \) on the \( e \) vs. \( q \) plane for \( \langle V \rangle = 50 \text{ km s}^{-1} \), \( b_{\text{Sun}} = 10^3 \text{ au} \), \( m_* \), \( m_0 = 10^2 \text{, and } V = 50 \text{ km s}^{-1} \). Other parameters are fixed at \( G = 10^{-3}, \gamma = 10, \) and \( \gamma = 3. \) At the \( e \) and \( q \) of U1 and Q4 in Figure 5, \( \mathcal{H} \approx 10^{-3} \) and \( 10^{-4} \), respectively. This means that both U1 and Q4 would be less likely to be HOCs, even if the Solar system had a recent encounter with a passing object as assumed above. One can easily calculate \( \mathcal{H} \) for any \( p_{\text{Sun}}, m_*, \gamma, \) and \( \gamma = 10^3 \), \( m_0 = 10^2 \), \( \gamma = 3 \), and \( \gamma = 10^{-3} \), respectively. This means that both U1 and Q4 would be less likely to be HOCs, even if the Solar system had a recent encounter with a passing object as assumed above.
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\[ v = V_m (20) \]

\( U_1 \)’s siblings with \( v_2 < v_1 < v_{U1} \)

\[ t = t_{\text{obs}} \]

\[ t = t_{\text{obs}}(v_1) \]

\[ t = t_{\text{obs}}(v_2) \]

\[ \text{trajectory of star D} \]

\[ b_{\text{sun}} = 10^4 \text{ au} \]

\[ m_* = 10^{-2} m_\odot \]

\[ V_\text{sun} = 50 \text{ km s}^{-1} \]

\[ V = 50 \text{ km s}^{-1} \]

\[ V = 10, 26, 60 \text{ km s}^{-1} \]

Figure 3. Panels (a), (b), and (c): scaled contours of the two-dimensional probability distributions plotted on the \( e \) vs. \( q \) plane for ISOs for \( V = 20 \text{ km s}^{-1} \) (a), \( 50 \text{ km s}^{-1} \) (b), and \( 100 \text{ km s}^{-1} \) (c) and an equi-\( V \) curve for each \( V_m(\langle V \rangle) \) (thin black curve). Panel (d): scaled contours of the two-dimensional probability distributions integrated over an encounter with an object with \( V_* = 50 \text{ km s}^{-1} \) plotted on the \( e \) vs. \( q \) plane and equi-\( V \) curves for \( V = 10, 26, \text{ and } 60 \text{ km s}^{-1} \) (thin black curves). Black squares in each panel indicate \( U_1 \) and Q4.

Figure 4. Geometry among the Sun, Star D (a passing object), \( U_1 \), and \( U_1 \)’s siblings plotted on the plane that contains the Sun and the trajectory of Star D, as an example of the HOC production. Star D passes \( b_{\text{sun}} \) at \( t = 0 \). \( U_1 \) arrives at the Sun at \( t = t_{\text{obs}} \).

Figure 5. Contours of ratios of the number of HOCs to that of ISOs on the \( e \) vs. \( q \) plane, obtained from Equation (15) for \( \gamma = 3 \), \( b_{\text{sun}} = 10^4 \text{ au} \), \( m_* = 10^{-2} m_\odot \), \( V_\text{sun} = 50 \text{ km s}^{-1} \), and \( V = 50 \text{ km s}^{-1} \). Thin black curve shows eq. (16). Black squares indicate \( U_1 \) and Q4.
and (c) in Figure 6 are for \((e, q) = (1.2, 0.26)\), \((e, q) = (3.3, 2)\), and \((e, q) = (1.2, 10)\), respectively. Closed areas between the curves and the y-axis in Figure 6 show the range for passing objects to have \(H = 1\). The curve is roughly defined by a horizontal line at the lower limit of \(V_v\) and a diagonal line for constant \(b_{\text{Sun}}^{(e+1)}V_v\). Figures 6(a) and (b) clearly show that a close encounter with \(b_{\text{Sun}} \sim 10^7\) au is required for \(H > 1\) if \(m_\ast\) is as small as \(< 0.01\) \(m_\odot\). The range for \(H > 1\) becomes larger for larger \(q\). If we can observe objects with \(q\) up to 10 au, an encounter with an object with \(m_\ast = 0.01\) \(m_\odot\) and \(b_{\text{Sun}} \sim 10^6\) au is enough for \(H > 1\) (Figure 6(c)).

\[
\text{PROPERTIES OF A HYPOTHETICAL PERTURBER}
\]

Suppose a hypothetical object, which we will call “Star D”, scattered an Oort cloud comet onto a hyperbolic object with the velocity at infinity, \(V_v\). What can we say about the current position and the mass-range of Star D and about the averaged encounter frequency of the Solar system with similar objects?

We assume that Star D is moving along a straight trajectory shown in Figure 4. The distance traveled since the instant of time that corresponds to the encounter with Star D until now is estimated from \(l = b_\odot/\sin\alpha\) and, for Star D, \(l_\ast = lV_v/V\). Then the distance to the Sun from the current position of Star D is approximated by Equation (A8). In 3-dimensional space, the geometry of the trajectory of Star D and \(r_\ast\) is axisymmetric about the trajectory of U1. Therefore, Equation (A8) defines a torus-like volume with a cross section given by the uncertainties of \(b_\odot\) and \(V_v\). Star D has \(r_\ast \sim 2b_\odot\) for \(V_v = 50\) km s\(^{-1}\), where \(b_\odot \leq r_{\text{max}} \sim 10^6\) au to penetrate the Oort cloud.

A lower limit to the mass of Star D, \(m_{\text{D}}^{\text{min}}\), is set by the requirement to avoid a collision, which occurs when impact parameter required to give \(V\) (Equation (A13)) becomes smaller than the physical radius of Star D. This leads to

\[
m_{\text{D}}^{\text{min}} = \left(\frac{3}{4\pi G_\odot}\right) V_v^3 \rho_\ast^{-1} \left(\frac{4V_v^2}{V^2} - 1\right)^{-1/2}, \tag{16}
\]

where \(V_v\) and \(\rho_\ast\) are the velocity and density of Star D. Figure 7 shows the contours of \(m_{\text{D}}^{\text{min}}\) derived from Equations (14) and (16) on the \(e\) vs. \(q\) plane for \(V_v = 50\) km s\(^{-1}\) and \(\rho_\ast = 10^3\) kg m\(^{-3}\). We have \(m_{\text{D}}^{\text{min}} \approx 2 \times 10^{-4}\) \(m_\odot\) for the production of both U1 (\(V \approx 26\) km s\(^{-1}\)) and Q4 (\(V \approx 32\) km s\(^{-1}\)). This corresponds to \(\sim 0.2\) Jupiter masses. For the other (\(e \sim 1\)) comets in Figure 2(a), we have \(m_{\text{D}}^{\text{min}} \lesssim 10^{-5}\) \(m_\odot\) (a few Earth masses).

An upper limit to the mass of Star D, \(m_{\text{D}}^{\text{max}}\), can be set by the fact that Star D has not been found by the widefield infrared survey explorer (WISE; Wright et al. (2010)). The free-floating planetary-mass object closest to the Sun is WISE J085510.83-071442.5 (Luhman 2014). Its distance and mass are estimated, respectively, as 2.23 ± 0.04 pc (Luhman & Esplin 2016) and 3-10 jovian masses, assuming an age of 1-10 Gyr (Luhman 2014). Taking this as a measure of the sensitivity of WISE to nearby sub-stellar objects, any jovian mass object with the same brightness as the closest one would have been detected within 1-1.5 pc.

![Figure 6](image-url) Curves for \(H_{\text{BCS}0} = 1\) for given \(e\) and \(q\) on the \(b_\odot - V_v\) plane obtained by solving Equation (15)=1 for \(V_v = 20\) km s\(^{-1}\) (orange), 50 km s\(^{-1}\) (black), 100 km s\(^{-1}\) (blue), and \(m_\ast/m_\odot = 10^{-2}\) (dashed) and 10\(^{-1}\) (solid). Panels (a), (b), and (c) are for \((e, q) = (1.2, 0.26), (3.3, 2),\) and \((1.2, 10)\), respectively.
We analytically derive the expected distributions of eccentricity, $e$, and perihelion distance, $q$, for objects belonging to two distinct populations. First, we consider initially unbound objects entering the Solar system from interstellar space (ISOs). Second, we consider initially bound objects from the Oort cloud (HOCs) scattered onto hyperbolic trajectories by gravitational interaction with a passing star (see Figure 3). We estimate the numbers of ISOs and HOCs and evaluate them by using their ratio, $\mathcal{H}$, on the $e$ vs. $q$ plane (Figure 5).

1. We find that hyperbolic objects with small $e$ and small $q$ are the most likely to have an interstellar origin. Conversely, hyperbolic objects with small $e$ but large $q$ have a higher likelihood of having been scattered from the Oort cloud.

2. Both 1I/‘Oumuamua (2017 U1) and 2I/Borisov (2019 Q4) have orbits most consistent with an interstellar origin. While an origin by scattering from the Oort cloud cannot be rejected, this possibility has a very low probability of occurrence in the absence of a recent and very close stellar encounter, for which we have no evidence.

3. We find that passing bodies of sub-stellar mass (down to $\sim 0.2 M_J$) are capable of deflecting Oort cloud comets into hyperbolic orbits like those of 1I/‘Oumuamua (2017 U1) and 2I/Borisov (2019 Q4).

Future observations of two kinds are needed to provide an improved understanding of the dynamics and origin of hyperbolic objects in the Solar system. First, the distribution of orbital elements of such bodies, especially in the eccentricity vs. perihelion distance plane, will help determine the ratio of interstellar to scattered Oort cloud sources. Second, measurements of the abundance and distribution of sub-stellar (even sub-Jupiter) mass perturbers near the Sun are needed to quantify the role of scattering from the Oort cloud.

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Figure 7. Contours of $m_{D}^{\text{max}}$ derived from Equations (14) and (16) on the $e$ vs. $q$ plane for $V_e = 50 \text{ km s}^{-1}$ and $\rho_e = 10^3 \text{ kg m}^{-3}$. Black dotted curve shows $V = 2V_e$. (No solution below this curve). Black squares indicate U1 and Q4.
APPENDIX A: DERIVATION

A1 Scattering Angle

We assume a passing object that approaches the Sun on a hyperbolic orbit. Using non-rotational coordinates centered on the object having a given hyperbolic orbit defined by \( v_{\infty} \) and \( b_0 \), a comet encounters the object with the velocity and impact parameter \( v \) and \( b_{\text{HOC}} \), respectively. The velocity of the object at the moment of the closest approach is given by

\[
v_s = \sqrt{v_{\infty}^2 + \frac{2GM}{r_s}}.
\]  

where \( M = m_\odot + m_* \). The angle between the velocity vectors of the comet before and after the encounter \( \theta \) is given by

\[
\tan \frac{\theta}{2} = \frac{GM_s}{v_{\infty}^2 b_{\text{HOC}}}.
\]

Then, the velocity of the comet to the Sun after the encounter is expressed as

\[
V = \sqrt{V_s^2 + 2V_s v \cos \theta} = 2V_s \sqrt{\frac{\tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}}.
\]

For \( \theta \ll 1 \),

\[
V \approx \frac{2GM_s}{V_s b_{\text{HOC}}},
\]

which is the velocity change given by the impulse approximation. Scattering that gives the velocity as large as the U1’s, which is \( V_s \), cannot be dealt with using the impulse approximation. Equation (A3) gives

\[
\tan \frac{\theta}{2} = \left(\frac{4V_s^2}{V^2 - 1}\right)^{1/2}.
\]

Next, we choose the non-rotating Cartesian coordinates centered on the Sun such that the \( x \) axis is anti-parallel to \( V_s \), the \( z \) axis is anti-parallel to the angular momentum vector of the object, and the \( y \) axis is perpendicular to the \( x \) and \( z \) axes. The velocity vector of the comet after the encounter is expressed as \( V = V(\cos \alpha \cos \beta, \sin \alpha \cos \beta, -\sin \beta) \), where \( \alpha = (\pi + \theta)/2 \) is the angle between the \( x \) axis and \( V \) and \( \beta \) is the angle between \( b_{\text{HOC}} \) and the reference plane. Using Equation (A5), we have

\[
\sin \alpha = \left(1 - \frac{V^2}{4V_s^2}\right)^{1/2}.
\]

A2 Object Position

For the comet to have a trajectory with \( b \) after the encounter, the position of the object during the encounter must be determined. Let the angle between \( r_s \) and the \( x \) axis be \( \alpha_\ast \). From the conservation of angular momentum,

\[
\sin \alpha_\ast = \frac{b_0 V_{\infty}}{r_s v_s}.
\]

By combining Equations (A6) and (A7) and using Equation (A1), we find \( r_s \) that gives \( V \) and \( b \) as

\[
r_s = b_0 \left(1 - \frac{V^2}{4V_{\infty}^2}\right)^{-1/2} S,
\]

\[
S = \left[1 + \left( \frac{GM}{V_{\infty}^2 b_0}\right)^2 \left(1 - \frac{V^2}{4V_{\infty}^2}\right)^{-1}\right]^{-1} \frac{GM}{V_{\infty}^2 b_0} \left(1 - \frac{V^2}{4V_{\infty}^2}\right)^{-1} \approx 1.
\]
where \( V \neq 0 \). The assumption of \( S = 1 \) corresponds to the approximation that the trajectory of the object is not hyperbolic but a straight line. We give \( S = 1 \) and \( V_{\infty} = V \) since this is true in almost all cases in this paper.

A3 Derivation of \( \delta g \)

The tiny volume \( \delta g \) is defined with the following equation so that comets contained within \( \delta g \) have \( V \) and \( b \):

\[
\delta g = [2\pi b_{\text{HOC}} \delta b_{\text{HOC}} \times V \times \delta \beta], \tag{A10}
\]

where the ring-area with the radius of \( b_{\text{HOC}} \) decides \( V \) and \( \delta \beta \) gives the direction of \( V \) to meet the Sun with \( b \). Using \( b_{\odot} \gg b_{\text{HOC}} \), the relation between \( \beta \) and \( b \) is

\[
\beta \simeq \sin \beta = \frac{b}{r_{E} \sin \alpha}. \tag{A11}
\]

Substituting Equation (7) into Equation (A11) and carrying out the differentiation, we obtain

\[
\delta \beta = \frac{\delta b}{r_{E} \sin \alpha} = \frac{\delta b}{b_{\odot}}. \tag{A12}
\]

The explicit expression of \( b_{\text{HOC}} \) is given from Equations (A2) and (A5) as

\[
b_{\text{HOC}} = \frac{Gm_{\odot}}{V_{*}^{2}} \sqrt{\frac{4V_{*}^{2}}{V^{2}} - 1}. \tag{A13}
\]

By carrying out the differentiation of Equation (A13), we obtain

\[
\delta b_{\text{HOC}} = -4 \left( \frac{Gm_{\odot}}{V_{*}^{2}} \right)^{2} V^{-3} b_{\text{HOC}} \delta V. \tag{A14}
\]

Substituting Equations (A12) and (A14) into Equation (A10), we obtain \( \delta g \) as a function of \( V \) and \( b \) (Equation (9)).

A4 Coordinate Transformation from Impact Parameters to Orbital Elements

From the relation of \( V = \sqrt{Gm_{\odot}(e - 1)}/q \) and \( b = q\sqrt{(e + 1)/(e - 1)} \), the determinant of the Jacobian between the \( (V, b) \) and \( (e, q) \) frames is calculated as

\[
J = \begin{vmatrix} \frac{\partial V}{\partial e} & \frac{\partial V}{\partial q} \\ \frac{\partial b}{\partial e} & \frac{\partial b}{\partial q} \end{vmatrix} = \frac{1}{2} \sqrt{\frac{Gm_{\odot}}{q(e + 1)}} \frac{e}{e - 1}. \tag{A15}
\]

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