Deep inelastic $J/\psi$ production at HERA in the $k_T$-factorization approach
and its consequences on the nonrelativistic QCD

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Abstract. In the framework of the $k_T$-factorization approach, we analyse the inclusive and inelastic production of $J/\psi$ particles in deep inelastic ep scattering. We take into account both colour-singlet and colour-octet production channels. We inspect the sensitivity of theoretical predictions to the choice of model parameters. Our theoretical results agree reasonably well with recent experimental data collected by the collaboration H1 at HERA.

1 Introduction

Over the last decade, investigation of the $J/\psi$ production mechanisms in hadron-hadron and lepton-hadron collisions continues to attract significant attention from both theoretical and experimental sides. The puzzling history of $J/\psi$ traces back to the early 1990s, when the measurements [1]-[4] of the $J/\psi$ and Υ hadroproduction cross sections revealed a more than one order-of-magnitude discrepancy with theoretical expectations [5]-[7]. This fact has induced extensive theoretical activity. In particular, it led to the introduction of a new production mechanism, the so called colour-octet model [8]-[16]. Since then, the colour-octet model has been believed to give the most likely explanation of the quarkonium production phenomena, although there are also some indications that it is not working well [17]. One of the problems is connected with the photoproduction data [18], [19] where the contribution from the colour-octet mechanism is unnecessary or even unwanted [20]-[22] as the experimental results can be described within the colour-singlet model alone (if the next-to-leading-order contributions are taken into account) [23]. Another difficulty refers to the $J/\psi$ spin alignment. If, as expected, the dominant contribution comes from the gluon fragmentation into an octet $c\bar{c}$ pair, the mesons must have strong transverse polarization [22]-[26]. This is in disagreement with the data [27], [28], [29], [30] which point to unpolarized or even longitudinally polarized mesons.

A different strategy is represented by the $k_T$-factorization approach [31]-[34] (see also recent review [35] and references therein). In this approach, one focuses on the resummation of “small $x$” logarithms (i.e., the terms $[\ln(\mu^2/\Lambda^2) \alpha_s]^n$, $[\ln(\mu^2/\Lambda^2) \ln(1/x) \alpha_s]^n$, and $[\ln(1/x) \alpha_s]^n$ to all orders in $n$ and takes into account the effects of finite transverse momenta of partons. The resummation results in the “unintegrated” parton distributions which generalize the QCD factorization beyond the collinear approximation.

The effects of the initial gluon transverse momentum and gluon off-shellness were
shown to have an important impact on the $J/\psi$ production properties in the photon-gluon fusion colour-singlet model [36], [37]. Several attempts have been made in the literature to incorporate the colour-octet model within the $k_T$-factorization scheme [38], [39]. An extensive analysis of the production of $J/\psi$, $\chi_c$, and $\Upsilon$ mesons in $p\bar{p}$ collisions at the Fermilab Tevatron has been recently presented in Ref. [40]. Our paper is a logical continuation of this line. Here, we concentrate on the inclusive and inelastic production of $J/\psi$ particles by virtual photons in deep inelastic $ep$ scattering at HERA. The calculations are based on the $k_T$-factorization approach and on the nonrelativistic QCD formalism where we take into account both colour-singlet and colour-octet channels.

The outline of the article is as follows. First, in Sec. 2, we briefly recall the basic principles of the colour-singlet and colour-octet models and explain their extension to the $k_T$-factorization approach. The necessary technical details are described in Sec. 3. The numerical results followed by a discussion are displayed in Sec. 4. Finally, the concluding remarks are collected in Sec. 5.

2 Theoretical framework

In the framework of the colour-singlet approach [5]-[7], the production of any heavy meson is described as the perturbative production of a colour-singlet $QQ$ pair in a state with properly arranged the quantum numbers, according to the quarkonium state under consideration. For the production of $J/\psi$ particles, the relevant partonic subprocess is

$$\gamma + g \to ^3S_1[1] + g,$$

where we follow the standard spectroscopic notations, and the number in the brackets stands for the colour representation of the $cc$ pair. The corresponding Feynman diagrams are depicted in Fig. 1(a) (assuming also five possible permutations of the photon and gluon lines). The formation of a meson from the quark pair is a nonperturbative process. Within the nonrelativistic approximation which we are using, this probability reduces to a single parameter related to the meson wave function at the origin $|\mathcal{R}(0)|^2$, which is known for the $J/\psi$ and $\Upsilon$ families from the measured leptonic decay widths.

In addition to the above, we consider the colour-octet production scheme [8]-[11], which implies that the heavy $QQ$ pair is perturbatively created in a hard subprocess as an octet colour state and subsequently evolves into a physical quarkonium state via emitting soft (nonperturbative) gluons, which may be interpreted as a series of “classical” colour-dipole transitions. Although these transition probabilities can, in principle, be expressed in terms of field operators and therefore calculated, no such calculation exists to date. Thus, the transition probabilities remain free parameters, which are assumed to obey a definite hierarchy in powers of $v$, the relative velocity of the quarks in the bound system under study. This freedom is commonly used to estimate the colour-octet parameters by adjusting them to experimental data.

In the case when the colour-octet $Q\bar{Q}$ state is allowed, there appear additional contributions from the diagram of Fig. 1(a) and another set of diagrams shown in Figs. 1(b) and 1(c) (including all possible permutations). The graphs shown in Figs. 1(a) and 1(b) correspond to the following partonic subprocesses

$$\gamma + g \to ^1S_0[8] + g,$$  
$$\gamma + g \to ^3S_1[8] + g,$$  
$$\gamma + g \to ^3P_J[8] + g,$$
while the ones shown in Fig. 1(c) refer to the subprocesses

\[ \gamma + g \rightarrow {}^1S_0[8], \]
\[ \gamma + g \rightarrow {}^3P_J[8]. \]

(The production of \(^3S_1[8]\) state in a \(2 \rightarrow 1\) photon-gluon fusion process is forbidden by the colour and charge parity conservation.) Although the \(2 \rightarrow 2\) subprocesses \((2)-(4)\) are of formally subleading order in \(\alpha_s\) in comparison with \((5)-(6)\), their role cannot be regarded as small. Since they contribute to very different regions of the phase space, they even can dominate over the \(2 \rightarrow 1\) subprocesses under the experimental conditions. We will discuss this in more detail in Sec. 4. Note also that the \(2 \rightarrow 2\) subprocesses are indispensable for the inelastic events (i.e., the events with large final state hadron mass).

The colour-octet matrix elements (usually denoted in the literature as \(\langle 0 | O^8 | 0 \rangle\)) responsible for the nonperturbative transitions in \((2)-(6)\) are related to the fictitious colour-octet wave functions, that are used in calculations in place of the ordinary colour-singlet wave functions:

\[ \langle 0 | O^8 | 0 \rangle = \frac{9}{2\pi} \pi \langle R^8(0) \rangle^2 = \frac{9}{2\pi} \pi \langle \Psi^8(0) \rangle^2. \]

This equation applies to all \(S\)-wave colour-octet states, and a similar relation holds for \(R^8(0)\) and \(\Psi^8(0)\) if the \(P\)-wave colour-octet states are involved. For the sake of uniformity, we will be consistently using the notation in terms of \(R(0)\) and \(R'(0)\) for both colour-singlet and colour-octet contributions.

A generalization of the above formalism to the \(k_T\)-factorization approach implies two important steps. These are the introduction of noncollinear gluon distributions (which we show here) and the modification of the gluon spin density matrix in the parton level matrix elements (which we explain in the next section).

In the numerical analysis, we have tried two different sets of \(k_T\)-dependent gluon densities. In the approach of Ref. [41] based on a leading-order perturbative solution of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [42] equation, the unintegrated gluon density \(F_g(x,k_T^2,\mu^2)\) is calculated as a convolution of the ordinary (collinear) gluon density \(G(x,\mu^2)\) with universal weight factors:

\[
F_g(x,k_T^2,\mu^2) = \int_x^1 G(\eta,k_T^2,\mu^2) \frac{x}{\eta} G(\frac{x}{\eta},\mu^2) \frac{d\eta}{\eta},
\]

\[
G(\eta,k_T^2,\mu^2) = J_0(2\sqrt{\frac{\alpha_s}{\eta k_T^2}} \ln(1/\eta) \ln(\mu^2/k_T^2)), \quad k_T^2 < \mu^2,
\]

\[
G(\eta,k_T^2,\mu^2) = I_0(2\sqrt{\frac{\alpha_s}{\eta k_T^2}} \ln(1/\eta) \ln(k_T^2/\mu^2)), \quad k_T^2 > \mu^2,
\]

where \(\mu\) is the factorization scale, \(J_0\) and \(I_0\) stand for Bessel functions (of real and imaginary arguments, respectively), and \(\bar{\alpha}_s = 3\alpha_s/\pi\). In the leading-order approximation, the parameter \(\bar{\alpha}_s\) is connected to the Pomeron intercept \(\alpha(0) = 1 + \Delta\), with \(\Delta = \bar{\alpha}_s 4 \ln 2\). We use the value of \(\Delta = 0.35\) as it is accepted in many other our papers ([43] and references therein).

Another parametrization is based on the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [44] evolution equation. This approach was originally proposed in [31],
[42] and is now frequently discussed in the literature [45], [46]. It recalls the kinematic relation between the virtuality $q^2$ and the transverse momentum $k_T$ of a parton: $q^2 = k_T^2/(1-x)$. Consequently, the ordinary gluon density $G(x, q^2)$ may be considered as giving the $k_T^2$ distribution also. In this approach, the unintegrated gluon density is derived from the “collinear” density by simply differentiating it with respect to $q^2$:

$$\mathcal{F}_g(x, k_T^2, \mu^2 = k_T^2) = \frac{d}{dq^2} G(x, q^2)|_{q^2=k_T^2}. \quad (10)$$

As the BFKL and DGLAP equations are known to collect different logarithms, we find it worth exploring the numerical consequences of this difference. For consistency, the same leading-order (LO) GRV set [47] was used in both cases as the input collinear density.

3 Matrix elements and differential cross section

At first, we consider the relevant $2 \to 2$ partonic subprocesses as they are given by the photon-gluon fusion mechanism. Let $k_1$, $k_2$, $k_3$, $p_c$, and $p_e$ be the momenta of the incoming virtual photon and gluon, the outgoing gluon and the outgoing heavy (charmed) quark and antiquark, respectively; $\epsilon_1$, $\epsilon_2$, and $\epsilon_3$ the photon and gluon polarization vectors; $m_c$ the quark mass, $m_\psi = 2m_c$, $k = k_2 - k_3$, and $a$, $b$, and $c$ the eight-fold colour indices of the incoming gluon, the outgoing gluon, and the (coloured) $c\bar{c}$ state. We also introduce the projection operator $J(S, L)$, which guarantees the proper spin and orbital angular momentum of the $c\bar{c}$ state under consideration. Then, the photon-gluon fusion matrix elements read

$$\mathcal{M}(\gamma g \to \psi g) = \text{tr}\{\mathcal{K}_1 (\vec{p}_c - \vec{k}_1 + m_c, \mu_1) \mathcal{K}_2 (\vec{p}_c - \vec{k}_3 + m_c, \mu_2) \mathcal{K}_3 J(S, L)\} \times C_\psi \text{tr}\{T^a T^b T^c\} \left[\frac{1}{2} - 2(p, k_1)\right]^{-1} \left[\frac{1}{2} + 2(p, k_3)\right]^{-1} + 5\text{ permutations}, \quad (11)$$

$$\mathcal{M}(\gamma g \to \psi g) = \text{tr}\{\mathcal{K}_1 (\vec{p}_c - \vec{k}_1 + m_c) \gamma_\mu J(S, L)\} iG^{(3)}(k_2, \epsilon_2, -k_3, \epsilon_3, -k, \mu) \times C_\psi f^{abd} \text{tr}\{T^c T^d\} \left[\frac{1}{2} - 2(p, k_1)\right]^{-1} \left[\frac{1}{2} + 1\right] \text{ permutation}. \quad (12)$$

In the above expression, $G^{(3)}$ is related to the standard QCD three-gluon coupling

$$G^{(3)}(p, \lambda, q, \mu, k, \nu) = (q-p)^\nu g^{\lambda\mu} + (k-q)^\lambda g^{\mu\nu} + (p-k)^\mu g^{\nu\lambda}. \quad (13)$$

The factor represented by the $SU(3)$ generator matrix $T^c$ has to be replaced by the unity matrix if the outgoing $c\bar{c}$ state is a colour singlet. The coefficient $C_\psi$ stands for the normalization of the $c\bar{c}$ colour wave function and is equal to $1/\sqrt{3}$ and $1/2$ for the singlet and octet states, respectively.

The projection operator $J(S, L)$ reads for the different spin and orbital angular momentum states [5]-[7]:

$$J^{(1)S_0} \equiv J(S=0, L=0) = \gamma_5 (\vec{p}_c + m_c)/m_\psi^{1/2}, \quad (14)$$

$$J^{(3)S_1} \equiv J(S=1, L=0) = \gamma_5 (\vec{p}_c + m_c)/m_\psi^{1/2}, \quad (15)$$

$$J^{(3)P_1} \equiv J(S=1, L=1) = (\vec{p}_c - m_c) \gamma^\mu (\vec{p}_c + m_c)/m_\psi^{3/2}. \quad (16)$$

States with various projections of the spin momentum onto the $z$ axis are represented by the polarization vector $\epsilon(S_z)$.
In the nonrelativistic approximation which we are using, the relative momentum \( q \) of the quarks in the bound state is treated as a small quantity. So, it is useful to represent the quark momenta as follows:

\[
p_c = \frac{1}{2} p_\psi + q, \quad \bar{p}_c = \frac{1}{2} p_\psi - q,
\]

where \( p_\psi \) is the momentum of the final state quarkonium. The probability for the two quarks to form a meson depends on the bound state wave function \( \Psi(q) \). Therefore, we multiply the matrix elements (11)–(12) by \( \Psi(q) \) and perform integration with respect to \( q \). The integration is performed after expanding the integrand around \( q = 0 \):

\[
M(q) = M|_{q=0} + q^\alpha (\partial M/\partial q^\alpha)|_{q=0} + \ldots
\]

Since the expressions for \( M|_{q=0}, (\partial M/\partial q^\alpha)|_{q=0} \), etc., are no longer dependent on \( q \), they may be factored outside the integral sign. A term-by-term integration of this series then yields [7]:

\[
\int \frac{d^3q}{(2\pi)^3} \Psi(q) = \frac{1}{\sqrt{4\pi}} R(x = 0),
\]

\[
\int \frac{d^3q}{(2\pi)^3} q^\alpha \Psi(q) = -i \epsilon^\alpha(L_z) \sqrt{3} \frac{1}{\sqrt{4\pi}} R'(x = 0),
\]

etc., where \( R(x) \) is the spatial component of the wave function in the coordinate representation (the Fourier transform of \( \Psi(q) \)). The first term contributes only to \( S \)-waves, but vanishes for \( P \)-waves because \( R_P(0) = 0 \). On the contrary, the second term contributes only to \( P \)-waves, but vanishes for \( S \)-waves because \( R'_S(0) = 0 \). States with various projections of the orbital angular momentum onto the \( z \) axis are represented by the polarization vector \( \epsilon(L_z) \).

The polarization vectors \( \epsilon(S_z) \) and \( \epsilon(L_z) \) are defined as explicit four-vectors. In the frame where the \( z \) axis is oriented along the quarkonium momentum vector, \( p_\psi = (0, 0, |p_\psi|, E_\psi) \), these polarization vectors read:

\[
\epsilon(\pm 1) = (1, \pm i, 0, 0)/\sqrt{2}, \quad \epsilon(0) = (0, 0, E_\psi, |p_\psi|)/m_\psi.
\]

States with definite \( S_z \) and \( L_z \) can be translated into states with definite total momentum \( J \) and its projection \( J_z \) using the Clebsch-Gordan coefficients:

\[
\epsilon^{\mu\nu}(J, J_z) = \sum_{L_z, S_z} \langle 1, L_z; 1, S_z | J, J_z \rangle \epsilon^\mu(L_z) \epsilon^\nu(S_z).
\]

As far as the gluon spin density matrix is concerned, one should take into account that gluons generated in the parton evolution cascade do carry non-negligible transverse momentum and are off mass shell. One can trace a clear analogy between the \( k_T \)-factorization approach and the equivalent photon approximation in QED showing that the polarization properties of the off-shell incoming gluon are similar to those of the incoming virtual photon. The off-shell photon spin density matrix is given by the full lepton tensor

\[
\epsilon^\mu \epsilon^{\nu*} \sim L^{\mu\nu} = 8 p^\mu p^\nu - 4(pk) g^{\mu\nu},
\]

where \( p \) is the momentum of the beam particle, and \( k \) is the momentum of the emitted photon. A similar anzatz (Eq. (24), see below) is used in the \( k_T \)-factorization approach.
Neglecting the second term in the right hand side of (23) in the small \( x \) limit, \( p \gg k \), one arrives at the spin structure \( \overline{e^\mu e^{\nu\tau}} \sim \rho^\mu \rho^\nu \). The latter can be rewritten in the form

\[
\overline{e^\mu e^{\nu\tau}} = k^\mu k^{\nu\tau}/|k_T|^2,
\]

where we have represented the 4-momentum \( k \) as \( k = xp + k_T \) and applied a gauge shift \( e^\mu \to e^\mu - k^\mu/x \). This formula converges to the usual \( \sum e^\mu \varepsilon^\nu = -g^\mu\nu \) when \( k_T \to 0 \). In the present calculations, we use Equ. (23) for virtual photons, and Equ. (24) for off-shell gluons. The expressions (23) and (24) merge with each other in the ultrahigh energy limit. The effect of the second term in (23) at HERA energies is found to be about 5 to 10 percent [43]. As we will see in Section 4, the presence of longitudinal components in the off-shell gluon spin density matrix has important impact on the quarkonium polarization. The final state gluon in (11)-(12) is assumed on-shell, \( \sum \varepsilon^\mu \varepsilon^{\nu\tau} = -g^\mu\nu \). The evaluation of the traces in Eqs. (11)-(12) is straightforward and is done using the algebraic manipulation system FORM [48].

To calculate the cross section of a physical process we have to multiply the matrix elements squared by the gluon distribution functions and perform integration over the final state phase space. The multiparticle phase space \( \prod d^3p_i/(2E_i) \delta^4(\sum p_{in} - \sum p_{out}) \) is parametrized in terms of transverse momenta, rapidities, and azimuthal angles: \( d^3p_i/(2E_i) = (\pi/2)dp^2_T dy_i d\phi_i/(2\pi) \). Let \( \phi_1 \) and \( \phi_2 \) be the azimuthal angles of the scattered electron and the initial gluon, and \( y_\psi, y_3, \phi_\psi, \phi_3 \) the rapidities and the azimuthal angles of the \( J/\psi \) particle and the coproduced gluon. Then, the fully differential cross section reads:

\[
d\sigma(ep \to e'\psi X) = \frac{\alpha^2}{16\pi} \frac{e^2}{s^2} \frac{|R(0)|^2}{4} \sum_{\text{spins}} \frac{1}{8} \sum_{\text{colours}} |\mathcal{M}(\gamma g \to \psi g)|^2 \\
\times \mathcal{F}_g(x_2, k^2_{2T}, \mu^2) \ d\epsilon^2_{1T} \ d\epsilon^2_{2T} \ dp^2_T \ dy_3 \ dy_\psi \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{d\phi_\psi}{2\pi}.
\]

The phase space physical boundary is determined by the inequality [49]

\[
G(\hat{s}, \hat{t}, k^2_3, k^2_1, k^2_2, m^2_\psi) \leq 0,
\]

where \( \hat{s} = (k_1 + k_2)^2, \hat{t} = (k_1 - p_\psi)^2 \), and \( G \) is the standard kinematic function [49].

The initial gluon momentum fractions \( x_1 \) and \( x_2 \) are calculated from the energy-momentum conservation laws in the light cone projections:

\[
\begin{align*}
(k_1 + k_2)_E + p_{||} &= x_1 \sqrt{s} = m_{\psi T} \exp(y_\psi) + |k_{3T}| \exp(y_3), \\
(k_1 + k_2)_E - p_{||} &= x_2 \sqrt{s} = m_{\psi T} \exp(-y_\psi) + |k_{3T}| \exp(-y_3),
\end{align*}
\]

\( m_{\psi T} = (m^2_\psi + |p_{\psi T}|^2)^{1/2} \). Here, we preserve exact kinematics and do not neglect the “small” light-cone component of the gluon momentum. The multidimensional integration in Eq. (25) has been performed by means of the Monte-Carlo technique, using the routine VEGAS [50].

4 Numerical results and discussion

We start the discussion by presenting a comparison between our theoretical calculations and experimental data collected by the H1 collaboration at HERA [30]. The collaboration reports on the measurement of a number of differential cross sections:
\[ \frac{d\sigma}{dp_T^2, \psi}, \frac{d\sigma}{dp_T^2, \psi}, \frac{d\sigma}{d\omega^2}, \frac{d\sigma}{d\phi}, \frac{d\sigma}{dy}, \frac{d\sigma}{dy^*}, \text{and } \frac{d\sigma}{dW}, \text{where } p_{T,\psi} \text{ and } p_{T,\psi}^* \text{ are the } J/\psi \text{ transverse momenta in the laboratory and } \gamma^*p \text{ center-of-mass systems, respectively, } z \text{ is the } J/\psi \text{ inelasticity variable defined as } z = (p_\psi p_p)/(k_1 p_p), Q^2 = -k_1^2 \text{ is the photon virtuality, } y \text{ and } y^* \text{ are the } J/\psi \text{ rapidities in the laboratory and } \gamma^*p \text{ systems, and } W \text{ is the } \gamma^*p \text{ invariant energy. The data collected in the kinematic range } 2 \text{ GeV}^2 < Q^2 < 100 \text{ GeV}^2, 50 \text{ GeV} < W < 225 \text{ GeV}, 0.3 < z < 0.9, p_{T,\psi}^* > 1 \text{ GeV}^2 \text{ will be referred to as "sample 1" (see Fig. 2), while the data collected in the kinematic range } 12 \text{ GeV}^2 < Q^2 < 100 \text{ GeV}^2, 50 \text{ GeV} < W < 225 \text{ GeV}, 0.3 < z < 0.9, p_{T,\psi}^* > 1 \text{ GeV}^2, p_{T,\psi}^* > 6.4 \text{ GeV}^2 \text{ will be referred to as "sample 2" (see Fig. 3).}

On the theoretical side, we have examined the sensitivity of model predictions to the choice of the gluon distribution functions, the renormalization scale in the strong coupling constant, the value of the quark mass, and the values of the nonperturbative colour-octet matrix elements.

The effect of the different equations (BFKL versus DGLAP) which govern the evolution of gluon densities is found to be as large as a factor of 2 in the production cross section. This is illustrated by a comparison of dash-dotted and dashed histograms in Figs. 2 and 3, respectively. Note, however, that the parametrizations which we have used here represent two extreme cases, so that most of the other parametrizations available in the literature may be expected to lie in between our curves (see [35], [43]).

A similar effect is connected with the renormalization scale \( \mu^2 \) in the running coupling constant \( \alpha_s(\mu^2) \). The calculations made with \( \mu^2 = k_{2T}^2 \) and \( \mu^2 = m_{\psi T}^2 \) are represented by the dash-dotted and dotted histograms in Figs. 2 and 3. Note that the setting \( \mu^2 = k_{2T}^2 \) is only possible in the \( k_T \)-factorization approach, while it is meaningless in the collinear calculations where the parton transverse momentum is neglected: \( k_{2T} \equiv 0 \).

The quark mass plays in the calculations two essentially different roles. The “current” mass \( m_c \) is present in the expressions for the perturbative matrix elements (11)-(12). The “constituent” mass determines the phase space of the reaction via its connection to the physical mass of the final state, \( m_\psi = 2m_c \). However, it is worth pointing out that this connection is not strict. In fact, it may be violated by the effects of binding energy and internal quark motion [51]. The sensitivity of model predictions to the quark mass setting was examined in a toy calculation, where the “current” mass \( m_c \) and the “constituent” mass \( m_\psi/2 \) were treated as independent parameters. We found that the production cross section is only sensitive to the “constituent” mass but remains rather stable against variations in the “current” mass. In the rest of the paper we will be always using \( m_c = m_\psi/2 = 1.55 \text{ GeV} \).

The contributions from the \( 2 \to 1 \) colour-octet subprocesses are cut away by the experimental restriction \( z < 0.9 \). Turning to the \( 2 \to 2 \) colour-octet contributions, one has to take care about the infrared instability of the relevant matrix elements. In a rigorous approach, one has to consider the corresponding \( 2 \to 1 \) subprocesses at next-to-leading order. Then, the interference between the LO and NLO contributions must cancel the divergent parts of the \( 2 \to 2 \) subprocesses. Such calculations have been performed in the collinear factorization in [52]. Since the corresponding results are not yet available in the \( k_T \)-factorization, we use an approximate phenomenological approach. In order to restrict the \( 2 \to 2 \) subprocesses to the perturbative domain, we introduce the regularization parameter \( q_{\text{reg}}^2 \), so that all propagators are kept away from their poles by a distance not less than \( q_{\text{reg}} \). It may be argued that the nonperturbative parts of the \( 2 \to 2 \) subprocesses can be absorbed into \( 2 \to 1 \) subprocesses; that is, when the emitted gluon is soft, one can consider the final state as represented by a single
particle rather than by two. This our suggestion is similar to the one made in papers [8]-[10], [53] in the usual factorization scheme. In this approach, the regularization parameter $q_{\text{reg}}^2$ in the $2 \to 2$ processes and the nonperturbative colour-octet matrix elements in the $2 \to 1$ processes must be correlated [53] to avoid double counting between the hard and soft gluons in the final state (and so, to avoid sensitivity of the results to the choice of $q_{\text{reg}}^2$). The numerical results shown in Figs. 2 and 3 are obtained with setting $q_{\text{reg}}^2 = 1 \text{ GeV}^2$ (which may be regarded as the lower limit of the perturbative domain).

The results shown in Figs. 2 and 3 are obtained with the nonperturbative colour-octet matrix elements of Ref. [11]. If the values extracted from the recent analysis [40] were used instead, the contribution from the colour-octet states would be a factor of 5 lower\(^1\). Irrespective to the particular choice of the nonperturbative matrix elements, the production of $J/\psi$ particles at the conditions under study is not dominated by the colour-octet mechanism. This conclusion is also supported by an independent analysis [37] of recent ZEUS photoproduction data [54]. In general, the data are consistent with the predictions of the colour-singlet model and lie within the theoretical band provided by reasonable variations in model parameters. Although no need is seen in the colour-octet contributions, their presence does not lead to serious discrepancies in the visible kinematic area.

We find it instructive to proceed the discussion by presenting a comparison with another data displayed by the H1 collaboration in their earlier publication [55]. The data collected in the range $2 \text{ GeV}^2 < Q^2 < 80 \text{ GeV}^2$, $40 \text{ GeV} < W < 180 \text{ GeV}$, $z > 0.2$ constitute two correlated samples. In the “inelastic” sample, an additional cut on the final state hadron mass was applied, $M_X > 10 \text{ GeV}$, while the sample without any additional cuts is referred to as “inclusive”.

Although these data are, probably, less convincing in statistics, they are more useful and interesting from the theoretical point of view. Firstly, in the inclusive production sample, the high-$z$ kinematic range $z \simeq 1$ provides an access to the $2 \to 1$ colour-octet contributions. Secondly, the cut on the final state hadron mass $M_X$ in the inelastic sample prevents the $2 \to 2$ colour-octet matrix elements from infrared divergence.

In Fig. 4, we present a comparison between the data and our predictions for the inelastic production. The final state hadron mass $M_X$ is calculated as the invariant mass of the proton remnant and the final state gluon produced in the $2 \to 2$ subprocesses (1)-(4). The requirement that the final state mass $M_X$ be large means that the final state gluon must be hard. Thus, the theoretical estimates of the colour octet contributions are no longer dependent on the artificial regularization parameter $q_{\text{reg}}^2$. One can see that the data are reasonably described within the colour-singlet production mechanism, and the colour-octet contributions are not needed. Moreover, the colour-octet contributions to the high-$z$ region look even superfluous. The overall situation seems to favour the low values of the nonperturbative matrix elements proposed in [40].

In the inclusive production sample (see Fig. 5), a significant deficit is seen in the $z$-distribution at $z \simeq 1$, although the intermediate values of $z$ are described quite well. In the $J/\psi$ transverse momentum spectrum, the deficit is only seen at low

\(^1\)The nonperturbative matrix elements given in Ref. [11] are as follows: $|R_{3S_1}(0)|^2 = 8 \times 10^{-1} \text{ GeV}^3$, $|R_{3S_0}(0)|^2 = 8 \times 10^{-3} \text{ GeV}^3$, $|R_{3S_1}(0)|^2 = 8 \times 10^{-3} \text{ GeV}^3$, $|R_{3P_0}(0)|^2 = 7 \times 10^{-3} \text{ GeV}^5$. According to [40], the $\langle 3S_0 \rangle$ and $\langle 3P_0 \rangle$ matrix elements should be reduced by a factor of 5, while the $\langle 3S_1 \rangle$ matrix element should be reduced by at least a factor of 50 or even set to 0. This violates the naive nonrelativistic QCD scaling rules, but is consistent with estimates obtained within the $k_T$-factorization approach by other authors [38], [39].
While there is no discrepancy at higher \( p_{T,\psi} \) values. These properties may be taken as an indication that the inclusive sample contains large contributions from diffractive processes. As a consequence, our calculations underestimate the absolute \( J/\psi \) production rate by a factor of 3.

We conclude the discussion by presenting our results on the \( J/\psi \) spin alignment in Fig. 6. The data are expressed in terms of the leptonic decay angular parameter \( \alpha \), which characterizes the azimuthal angle distribution measured in the \( J/\psi \) rest frame with respect to a given reference axis: \( d\Gamma_l \sim 1 + \alpha \cos(\theta) \). The cases \( \alpha = 1 \) and \( \alpha = -1 \) correspond to transverse and longitudinal polarizations of the \( J/\psi \) meson, respectively. Our calculations show that the fraction of longitudinally polarized mesons increases with increasing \( Q^2 \), which is a consequence of the enhancement of the longitudinal component in the polarization vector of a virtual photon. Unfortunately, the experimental data are rather indefinite. In this case, we can derive no conclusions on the agreement or disagreement between the theory and experiment.

In the situation which we consider, the effects of the gluon off-shellness are shadowed by the effects of highly virtual photons and are not clearly visible. At a different situation, when either the initial photons are real or the process is mediated by gluons only, the \( J/\psi \) polarization properties are found to be perfectly compatible with the predictions of the \( k_T \)-factorization approach, as it was demonstrated earlier in Refs. [36], [37] and [40] devoted to the analysis of \( J/\psi \) photoproduction and hadroproduction data.

5 Conclusion

Here we have addressed the issue of performing a global analysis of quarkonium electroproduction within the \( k_T \)-factorization approach. The state of the art has not yet reached the precise quantitative level. There are uncertainties connected with the choice of unintegrated gluon densities, the renormalization scale in the strong coupling constant, the inclusion of next-to-leading-order subprocesses, and the nonperturbative colour-octet transitions.

At the same time, the \( k_T \)-factorization approach shows a number of important achievements. As a general feature, the model behavior is found to be perfectly compatible with the available data on the production of various quarkonium states at modern colliders. The model succeeds in describing the \( p_T \)-spectra of \( J/\psi \), \( \chi_c \), and \( \Upsilon \) mesons at the Fermilab Tevatron and provides a consistent picture of the production of \( J/\psi \) mesons by real and virtual photons at HERA (as it was demonstrated earlier in Refs. [36], [37] and [40]). The model even succeeds in describing the polarization phenomena observed in both \( pp \) and \( ep \) interactions, thus providing an important insight for solving a long-term puzzle.

As a result of the complex interplay of all theoretical uncertainties, the numerical analysis becomes rather ambiguous. The electroproduction data can be successfully described within the leading-order colour-singlet mechanism alone. No need is seen in the colour-octet contributions. At the same time, there are no serious contradictions with the data if the colour-octet contributions are included. The effects of the different unintegrated gluon distributions can hardly be separated from the effects connected with the running coupling constant.
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Figure 1: Feynman graphs representing the photon-gluon fusion mechanism in the colour-singlet and colour-octet models. Only the perturbative skeleton is presented; the soft gluons corresponding to the nonperturbative colour-octet transitions are not shown.

Figure 2: A comparison between the theoretical predictions and experimental data [30] in the kinematic range $2 < Q^2 < 100 \text{ GeV}^2$, $50 < W < 225 \text{ GeV}$, $0.3 < z < 0.9$, $p_{T,\psi}^2 > 1 \text{ GeV}^2$. Dash-dotted histogram, the colour-singlet contribution with BFKL gluon density and $\alpha_s(k^2_{2T})$; dashed histogram, the colour-singlet contribution with DGLAP gluon density and $\alpha_s(k^2_{2T})$; dotted histogram, the colour-singlet contribution with BFKL gluon density and $\alpha_s(m^2_{T,\psi})$; solid histogram, the sum of the colour-singlet and colour-octet contributions, with BFKL gluon density, $\alpha_s(k^2_{2T})$, colour-octet matrix elements as in [11], and $q^2_{\text{reg}} = 1 \text{ GeV}^2$.

Figure 3: A comparison between the theoretical predictions and experimental data [30] in the kinematic range $12 < Q^2 < 100 \text{ GeV}^2$, $50 < W < 225 \text{ GeV}$, $0.3 < z < 0.9$, $p_{T,\psi}^2 > 1 \text{ GeV}^2$, $p_{T,\psi}^2 > 6.4 \text{ GeV}^2$. Dash-dotted histogram, the colour-singlet contribution with BFKL gluon density and $\alpha_s(k^2_{2T})$; dashed histogram, the colour-singlet contribution with DGLAP gluon density and $\alpha_s(k^2_{2T})$; dotted histogram, the colour-singlet contribution with BFKL gluon density and $\alpha_s(m^2_{T,\psi})$; solid histogram, the sum of the colour-singlet and colour-octet contributions, with BFKL gluon density, $\alpha_s(k^2_{2T})$, colour-octet matrix elements as in [11], and $q^2_{\text{reg}} = 1 \text{ GeV}^2$.

Figure 4: A comparison between the theoretical predictions and experimental data [55] on the inelastic $J/\psi$ production in the range $2 < Q^2 < 80 \text{ GeV}^2$, $40 < W < 180 \text{ GeV}$, $z > 0.2$, $M_X > 10 \text{ GeV}$. Dash-dotted histogram, the colour-singlet contribution with BFKL gluon density and $\alpha_s(k^2_{2T})$; dashed histogram, the colour-singlet contribution with DGLAP gluon density and $\alpha_s(k^2_{2T})$; dotted histogram, the colour-octet contribution with BFKL gluon density, $\alpha_s(k^2_{2T})$ and colour-octet matrix elements as in [11]; solid histogram, the sum of the colour-singlet and colour-octet contributions.

Figure 5: A comparison between the theoretical predictions and experimental data [55] on the inclusive $J/\psi$ production in the range $2 < Q^2 < 80 \text{ GeV}^2$, $40 < W < 180 \text{ GeV}$, $z > 0.2$. Dash-dotted histogram, the colour-singlet contribution with BFKL gluon density and $\alpha_s(k^2_{2T})$; dashed histogram, the colour-singlet contribution with DGLAP gluon density and $\alpha_s(k^2_{2T})$; dotted histogram, the colour-octet contribution with BFKL gluon density, $\alpha_s(k^2_{2T})$, colour-octet matrix elements as in [11], and $q^2_{\text{reg}} = 1 \text{ GeV}^2$; solid histogram, the sum of the colour-singlet and colour-octet contributions.

Figure 6: A comparison between the theoretical predictions and experimental data on the $J/\psi$ spin alignment represented in terms of the decay lepton angular distributions. Left panel, kinematic range 1 (as in Fig. 2) [30], $Q^2 < 6.5 \text{ GeV}^2$; right panel, kinematic range 1 (as in Fig. 2) [30], $Q^2 > 6.5 \text{ GeV}^2$. 
