The role of boundary conditions on the free surface of a liquid metal in an electrovortex flow

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Abstract. An electro-vortex flow between two hemispherical electrodes is considered. The influence of the type of boundary condition on the surface of a conducting liquid medium on the velocity field in the volume is studied numerically. The dependences of the velocity on the axis of the vessel on the radius of the small electrode and the parameter of the electric vortex flow are obtained for various types of boundary conditions on the surface.

1. Introduction

For carrying out magnetohydrodynamic experiments, metals that remain liquid at room temperature are most convenient - mercury and various alloys based on indium and gallium. At the same time, on the one hand, gallium alloys have an advantage over mercury due to their non-toxicity, on the other hand, the open liquid metal surface is oxidized in air.

This oxide film behaves like a flexible film, i. e. the adhesion condition is realized on it, and the surface deformation under certain conditions can be neglected. Observations of an open surface (using appropriate experimental setups) can provide information on the flow structure if oxides on the surface are chemically eliminated, and the type of boundary condition changes. We considered the effect of the type of boundary condition on the surface on the electrovortex flow (EVF) in a hemispherical container.

EVF is formed as a result of interaction of the non-uniform electric current passing through the liquid metal with own magnetic field of this current [1]. Such flows significantly affect many processes in mechanical engineering (electro-welding) and electrometallurgy (electroslag remelting, various electric melting furnaces) [2].

We consider the system where electric current $I_0$ propagates from the small hemispherical electrode with the radius $R_1$ through the conductive media (liquid metal) to the big hemispherical electrode with the radius $R_2$ (Figure 1). Electromagnetic force $F=J\times B$ (where $J$ is the current density and $B$ is the total magnetic field consisting of the self-magnetic field and the external magnetic fields) causes the liquid to move. In such an axisymmetric system (without the influence of external magnetic fields), EVF has the form of a one toroidal vortex. Various aspects of this problem were considered, for example in [3-5].
2. Numerical model

We considered steady, two-dimensional, axisymmetric ($\partial / \partial \phi = 0$) system of equations. To numerically solve the Navier - Stokes equation, we used the finite volume method. Area calculation area (figure 1) was a quarter circle with internal radius $R_1$ (small electrode) and external radius $R_2=1$ (large electrode) and consisted of 2500-20000 quadrangular cells for different grids. The wall boundary conditions were set on both electrode and we considered two types of the boundary condition on the upper surface. Wall boundary condition when $U_{\theta}|_{\theta=\pi/2}=0$ and slip boundary condition when $U_{\theta}|_{\theta=\pi/2}=0$. 

Calculations were carried out with the electrodynamical approximation, when the influence of the magnetic field on the velocity field was considered only by adding the source of the electromagnetic force to the motion equation $F = J \times B$, excluding electric currents induced by the moving fluid. Possibility of using this approach is due to the small value of magnetic Reynolds number: $Re_m=\mu_0 \sigma U R^2 << 1$. For typical value of velocity $U \sim 0.1 \text{ m/s}$, $Re_m \sim 0.04$. Here $\mu_0$ – magnetic constant, $\sigma$ – electrical conductivity.

Let’s enter the following scales: for length - $R_2$, electric current density - $I_0 = I_0 / R_2^2$, magnetic field induction $B_0 = \mu_0 I_0 / R_2$, electromagnetic force - $f_0 = \mu_0 I_0^2 / R_2^3$, velocity - $U_0 = \sqrt{f_0 R_2 / \rho}$, pressure $p_0 = \rho U_0^2$.

Applying these scales to the Navier-Stokes equation written in spherical coordinates (3) as in [2] we can get specific dimensionless parameters: so-called the parameter of the electrovortex flow $S$. Here $e$ - unit vector.

\[
(U \cdot \nabla)U = -\nabla p + \frac{1}{\sqrt{S}} \Delta U + \frac{\cos \theta - 1}{4 \pi^2 r^3 \sin \theta} e_\theta. 
\]  

(1)

3. Results

In Figure 2, the dependence of the axial velocity $U_z$ on $z$ at several numbers $S$ at radius $R_1=0.001$ are presented. These data are quite correlated with the results obtained in [6]. We can see that the maximum speed is located approximately in the same place on the $z$-axis for different types of boundary condition on the surface.
Figure 2. The dependence of the axial velocity on z coordinate. 1 – S=1; 2 – 1e3, 3 – 1e6. Index “a” – wall boundary condition, no index – slip boundary condition.

Figure 3 shows the dependences of the maximum axial velocity on the parameter S for different values of the small electrode radius $R_1$ for different types of boundary conditions. We limited ourselves to calculations for $R_1$ < 0.1, since at large $R_1$, the flow significantly changes the geometry in the nonlinear case, and a flow in a large volume turns into a flow in a slot, which is already noticeable for $R_1 = 0.1$. Up to $S \sim 1000$, the flow is in a linear mode. In this mode, the shape of the velocity profile $U_z(z)$ does not depend on the parameter S, which in this case is only a scale factor. It can be seen from the graphs that with increasing current, the influence of the type of boundary conditions on the velocity field decreases. The degree of influence can be characterized by the value of $\gamma_{BC}$.

$$\gamma_{BC} = \frac{U_{z,\text{max}}^{\text{wall}} - U_{z,\text{max}}^{\text{slip}}}{U_{z,\text{max}}^{\text{slip}}}.$$  \hspace{1cm} (2)

Figure 4. shows the dependence of the value of $\gamma_{BC}$ on the parameter S for different small electrode radii $R_1$. Then $\gamma_{BC} = 0$ means the absence of the influence of the type of the boundary condition, and the maximum value, which means the maximum effect of the type of the boundary condition, is achieved at S between 0 and 1000. The radius of the small electrode $R_1$ affects the maximum speed, with decreasing $R_1$, the maximum speed increases, thus the dependence of the limiting values of $\gamma_{BC}$ (influence parameter of the type of boundary condition).

In Figure 5. shows the dependence of the maximum value of $\gamma_{BC}$ on the size of the small electrode is presented. An increase in the velocity due to a decrease in the radius of the small electrode leads to a decrease in the influence of the boundary condition at the same parameter S.
4. Conclusions
Calculations of the axial velocity of an electric vortex flow in a hemisphere were carried out for different currents at different values of the radius of the small electrode for different types of boundary conditions on the surface. It is shown that the degree of influence of the type of boundary condition decreases with increasing current and decreasing the radius of the small electrode. The maximum limiting values of the parameter of the influence of the boundary conditions on the axial velocity are obtained. Further, these results will be verified by comparison with the analytical solutions presented in [1], [7], [8].

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