Systematic study of unfavored $\alpha$ decay half-lives of closed shell nuclei related to ground and isomeric states

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In present work, the unfavored $\alpha$ decay half-lives and $\alpha$ preformation probabilities of closed shell nuclei related to ground and isomeric states around $Z = 82$, $N = 82$ and 126 shell closures are investigated by adopting two-potential approach from the perspective of valence proton (hole) and isospin asymmetry of the parent nucleus. The results indicate that $\alpha$ preformation probability is linear dependence on $N_pN_n$ or $N_pN_nI$, the same as the case of favored $\alpha$ decay in our previous work [X.-D. Sun et al., Phys. Rev. C 94, 024338 (2016)]. $N_p$, $N_n$, and $I$ represent the number of valence protons (holes), the number of valence neutrons (holes), and the isospin of the parent nucleus, respectively. Fitting the $\alpha$ preformation probabilities data extracted from the differences between experimental data and calculated half-lives without a shell correction, we give two linear formulas of the $\alpha$ preformation probabilities and the values of corresponding parameters. Based on the formulas and corresponding parameters, we calculate the $\alpha$ decay half-lives for those nuclei. The calculated results can well reproduce the experimental data.

I. INTRODUCTION

$\alpha$ decay was defined in 1899 by Rutherford, and the quantum tunnel theory was independently put forward to estimate the probability of an $\alpha$ particle tunneling through the Coulomb barrier in 1928 by Gurney and Condon [1] and Gamow [2]. Since then, $\alpha$ decay, as one of the most important tools to study unstable nuclei, neutron-deficient nuclei and superheavy nuclei, has been a hot area of research in nuclear physics [3–8]. Theoretically, $\alpha$ decay shares the similar theory of barrier penetration with different kinds of charged particles’ radioactivity, for instance, heavy ion emission, single proton emission, spontaneous fission [9–14], and so on. Experimentally, $\alpha$ decay is the main decay mode for most of the new synthesis of superheavy nuclei and sometimes it is the unique.

Meanwhile, for some very unstable new synthetic nuclei, $\alpha$ decay is the effective way to determine their identities (the number of protons and neutrons).

Usually, $\alpha$ decay is described as a process of a preformed $\alpha$ particle tunneling through the potential barrier between $\alpha$ cluster and the daughter nucleus, and the preformed probabilities of $\alpha$ cluster are different for various nuclei. Therefore, the calculated $\alpha$ decay constant should be multiplied by a preformation factor of $\alpha$ particle. Nevertheless, we rarely know formation and movement of $\alpha$ particle inside the parent nuclei, as a result of the complicated structure of the quantum many-body systems. Therefore, there are a few works [15–18] studying $\alpha$ preformation probabilities from the viewpoint of microscopic theory. Phenomenologically, $\alpha$ preformation probabilities are obtained by the ratios of theoretical calculations without considering the preformation factors to experimental half-lives [19–22]. Recent research has shown that, the pairing effect, the shell effect, and different spin-parity states of daughter and parent nuclei are the major factors determining $\alpha$ preformation probabilities [17]. Seif et al. have proposed that the $\alpha$ preformation probability, considering isospin, shell effect and valence proton-neutron interaction, is proportional to $N_pN_n$ for even-even nuclei around proton $Z = 82$, neutron $N = 82$ and 126 shell closures [7], where $N_p$, $N_n$ denote valence protons (holes) and valence neutrons (holes) of parent nucleus, respectively. Furthermore, in our previous work [23], the $\alpha$ preformation probabilities of odd- $A$ and doubly-odd nuclei favored $\alpha$ decay also satisfy this relationship. Therefore, it is interesting to validate whether this linear relationship still exist in unfavored $\alpha$ decay of closed shell nuclei. In this work, we investigate the $\alpha$ decay half-lives and $\alpha$ preformation probabilities for unfavored $\alpha$ decay of closed shell nuclei around $Z = 82$, $N = 82$ and 126 closed shells, respectively. Our results indicate that in unfavored $\alpha$ decay of closed shell nuclei, the $\alpha$ preformation probabilities are still linear related with $N_pN_n$, i.e. valence proton-neutron interaction plays an important role on $\alpha$ preformation probabilities. The calculated results can well reproduce the experimental data from NUBASE2012 [24].

This article is organized as follows. In next section, the theoretical framework of the $\alpha$ decay half-life is briefly presented. The detailed calculations and discussions are given in Sec. III. In this section, we investigate the $\alpha$ preformation probabilities from the viewpoint of the valence proton-neutron interaction and isospin effect, respectively. Sec. IV is a brief summary.

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II. THEORETICAL FRAMEWORK

$\alpha$ decay half-life $T_{1/2}$, as an important indicator of nuclear stability, can be calculated by the $\alpha$ decay width $\Gamma$ and written as

$$T_{1/2} = \frac{\hbar n}{\Gamma}. \quad (1)$$

In general, $\alpha$ decay can be approximatively treated as a stationary state problem, on account of the $\Gamma$ is much smaller than the $\alpha$ decay energy $Q_\alpha$. Hence, we can adopt two-potential approach (TPA), which has been successfully applied to deal with metastable states, to calculate $\alpha$ decay half-life [25–30]. In the framework of TPA, the $\alpha$ decay width is calculated as below

$$\Gamma = P_\alpha \frac{\hbar^2 FP}{4\mu}, \quad (2)$$

where $\mu$ is the reduced mass of daughter nucleus and $\alpha$ particle, and $\mu = \frac{m_\alpha m_d}{m_\alpha + m_d}$ with $m_d$ and $m_\alpha$ being mass of daughter nucleus and $\alpha$ particle, respectively.

$P$ is the semiclassical Wentzel-Kramers-Brillouin (WKB) barrier penetration probability, namely, Gamow factor. It can be expressed as

$$P = \exp(-2\int_{r_2}^{r_3} k(r) dr), \quad (3)$$

where $k(r) = \sqrt{\frac{2\mu}{\hbar^2}}(Q_\alpha - V(r))$ is the wave number of the $\alpha$ particle, and $r$ is the center of mass distance between the daughter nucleus and the preformed $\alpha$ particle. $V(r)$ is the $\alpha$-core potential.

The normalized factor $F$, denoting the assault frequency of $\alpha$ particle, is approximatively calculated by

$$F \int_{r_1}^{r_2} \frac{1}{2k(r)} dr = 1, \quad (4)$$

where $r_1$, $r_2$ and the above $r_3$ are the classical turning points, they can be obtained through solving $V(r_1) = V(r_2) = V(r_3) = Q_\alpha$.

$P_\alpha$ means the $\alpha$ preformation probability, recent researches indicate that it rapidly decline in near closed shell and mildly change in an open shell region [19–21]. $P_\alpha$ increases with the increase of valence nucleons up to next closed shell, and decreases with the decrease of valence holes. Usually, the value of $P_\alpha$ is defined as

$$P_\alpha = P_{0 \alpha} \frac{T_{\text{calc}}}{T_{\text{expt}}^{1/2}}, \quad (5)$$

where $T_{\text{expt}}^{1/2}$ denotes experimental half-life, $T_{\text{calc}}^{1/2}$ represents the calculated $\alpha$ decay half-life based on an assumption that $\alpha$ preformation probability is a different constant for different kinds of nuclei. In accordance with the calculations by adopting the density-dependent cluster model (DDCM) [31], $P_{0 \alpha}$ is 0.43 for even-even nuclei, 0.35 for odd-A nuclei, and 0.18 for doubly-odd nuclei.

The total interaction potential $V(r)$, which is composed of nuclear potential $V_N(r)$, Coulomb potential $V_C(r)$, and centrifugal potential $V_l(r)$, can be written as

$$V(r) = V_N(r) + V_C(r) + V_l(r). \quad (6)$$

In this work, we choose a cosh parametrized form for the nuclear potential $V_N(r)$, obtained by analyzing experimental data of $\alpha$ decay [32], which can be expressed as

$$V_N(r) = -V_0 \frac{1 + \cosh(R/r_0)}{\cosh(R/r_0) + \cosh(R/a_0)}, \quad (7)$$

where $V_0$ and $a_0$ mean the depth and diffuseness of the nuclear potential, respectively. In our previous work, we have obtained a set of parameters by analyzing the 164 even-even nuclei experimental $\alpha$ decay half-lives data, which is $a_0 = 0.5958$ fm and $V_0 = 192.42 + 31.059 \frac{N_d - Z_d}{A_d}$ MeV [33] with the $N_d$, $Z_d$, and $A_d$ being the number of neutrons, protons and mass number of the daughter nucleus, respectively. In this work, we also adopt these parameters to calculate the nuclear potential.

$V_C(r)$, the Coulomb potential, is regarded as the potential of a uniformly charged sphere with sharp radius $R$, which can be expressed as

$$V_C(r) = \left\{ \begin{array}{ll}
\frac{Z_a e^2}{2 R} [3 - \left(\frac{r}{R}\right)^2], & r < R, \\
\frac{Z_a e^2}{2 R} e^{\frac{R - r}{a}}, & r > R,
\end{array} \right. \quad (8)$$

where $Z_a = 2$ is the number of protons for preformed $\alpha$ particle. The centrifugal potential $V_l(r)$ can be estimated by

$$V_l(r) = \frac{\hbar^2 (l + 1/2)^2}{2 \mu r^2}, \quad (9)$$

where $l$ is the orbital angular momentum taken away by the $\alpha$ particle. $l = 0$ for the favored $\alpha$ decays, while $l \neq 0$ for the unfavored decays. Adopting the Langer modified centrifugal barrier, for one-dimensional problems, $l(l + 1) \rightarrow (l + 1/2)^2$ is a necessary corrections [34]. According to the conservation law of angular momentum [35], the minimum angular momentum $l_{\text{min}}$ taken away by the $\alpha$ particle can be obtained by

$$l_{\text{min}} = \left\{ \begin{array}{ll}
\Delta_j, & \text{for even} \Delta_j \text{ and } \pi_p = \pi_d, \\
\Delta_j + 1, & \text{for even} \Delta_j \text{ and } \pi_p \neq \pi_d, \\
\Delta_j, & \text{for odd} \Delta_j \text{ and } \pi_p \neq \pi_d, \\
\Delta_j + 1, & \text{for odd} \Delta_j \text{ and } \pi_p = \pi_d,
\end{array} \right. \quad (10)$$

where $\Delta_j = |j_p - j_d|$, $j_p$, $\pi_p$, $j_d$, $\pi_d$ are spin and parity values of the parent and daughter nuclei, respectively.

The sharp radius $R$ is calculated by

$$R = 1.28 A^{1/3} - 0.76 + 0.8 A^{-1/3}. \quad (11)$$

The above empirical formula is derived from the nuclear droplet model and proximity energy [36]. $A$ is the mass number of parent nucleus.
III. RESULTS AND DISCUSSIONS

The aim of present work is to study the effect of valence nucleons (holes) and isospin on $\alpha$ preformation probabilities and half-lives of unfavored $\alpha$ decay belong to nuclei around the $Z = 82, N = 82$ and 126 shell closures. For the odd-$A$ and doubly-odd nuclei, excitation of single nucleon causes the high-spin isomers. Our previous works [23, 37] indicate that both ground and isomeric states can be treated in a unified way for $\alpha$ decay parent and daughter nuclei.

Many researches show that the more smaller valence nucleons (holes) nuclei have, the more smaller $\alpha$ preformation probabilities should be [19-21]. In addition, valence proton-neutron interaction approximately remain unchanged in the same shell region [38]. Recently, it is found that $\alpha$ preformation probabilities are linear with product of valence protons (holes) and valence neutrons (holes) for even-even nuclei around $Z = 82, N = 82$ and 126 closed shells. Furthermore, Self et al. propose that isospin asymmetry also makes an important contribution to $\alpha$ preformation probabilities [7]. Moreover, our previous work [23] indicates that the linear relationship between $P_\alpha$ and $N_pN_n$ still exist for the favored $\alpha$ decay of odd-$A$ and doubly-odd nuclei in the same regions. However, for unfavored $\alpha$ decay, it is more difficult than its counterparts because of the unpaired nucleons and different spin-parity state of parent and daughter nucleus. So it is necessary to study whether $\alpha$ preformation probabilities of unfavored $\alpha$ decay of closed shell nuclei still exist the liner relationship with $N_pN_n$.

For more intuitively study, we plot a nuclide distribution map in Fig. 1, and the area are marked as Region I to V in accordance with valence nucleons (holes), respectively. In this paper, we concentrate on Regions I, III, and IV, on account of a little number of nuclei with $\alpha$ radioactivity in Regions II and V. For purpose of a deeper insight into relationship between $\alpha$ preformation probabilities and $N_pN_n$, we study from standpoint of nuclear shell and isospin asymmetry, respectively within Eq. (11) and (12).

$$P_\alpha = a \frac{N_pN_n}{Z_0 + N_0} + b,$$

$$P_\alpha = cN_pN_nI + d,$$

where $Z_0, N_0$ are adjacent magic number of proton and neutron, respectively. $a, b, c$ and $d$ are adjustable parameters extracted from the fittings of $P_\alpha$ of Table I-III and listed in the Table IV (the upper part for the case of odd-$A$ and bottom half for doubly-odd nuclei). Based on Eq. (11), (12) and corresponding parameters, we calculate the $\alpha$ decay half-lives and express as $T_{1/2}^{\text{calc}1}, T_{1/2}^{\text{calc}2}, T_{1/2}^{\text{calc}3}$, respectively, which are listed in the last two columns of Table I-III. The first seven columns of Table I-III are $\alpha$ transition, $\alpha$ decay energy $Q_\alpha$, spin-parity transformation, the minimum orbital angular momentum $l_{\text{min}}$ taken away by $\alpha$ particle, the experimental half-life $T_{1/2}^{\exp}$, calculated half-life $T_{1/2}^{\text{calc}1}$ by TPA with $P_\alpha = P_0$, extracted $\alpha$ preformation probability $P_\alpha$, respectively.

In Region I, proton numbers are below $Z = 82$ shell closure, and neutron numbers above $N = 82$ closed shell. Therefore, the $N_pN_n$ are negative. The calculations of odd-$A$ nuclei in Region I are listed in Table I. In Region III, the proton numbers are above the $Z = 82$ closed shell,

![FIG. 1.](image1.png)

**FIG. 1.** (color online) Nuclide chart is divided into five regions. The cyan and magenta lines denote the $Z = 82, N = 126$ nuclear shell closures, respectively. The black dotted line represents $N = 104$.

![FIG. 2.](image2.png)

**FIG. 2.** (color online) The linear relationship between $\alpha$ preformation probabilities and $\frac{N_pN_n}{Z_0 + N_0}$. $N_p, N_n$ represent valence protons (holes) and neutrons (holes) of parent nucleus, respectively. $Z_0, N_0$ mean the magic numbers of proton and neutron, respectively. The blue solid and dash dot line denote the fittings of nuclei in Region I and, II, respectively. The red dot and dash line represent the fittings of nuclei in Region III, IV, respectively.
but the neutron numbers below the $N = 126$ closed shell, so the $N_pN_n$ are negative. Similarly, in the Region IV the $N_pN_n$ are positive. The detailed calculations of odd-A nuclei in Region III, IV are given in upper half and bottom half of Table II, respectively.

For the unfavored $\alpha$ decay of doubly-odd nuclei, the detailed calculations are listed in Table III, in this table, Regions I, III, and IV are involved. From Table I-III, we can find that last two columns $T_{\text{exp}}^{1/2}$, $T_{\text{calc}}^{1/2}$ are well conform because of Eq. (11) and (12) are two different perspectives for studying the liner relationships between $N_pN_n$ and $P_\alpha$. From Table IV, we can clearly see that the
TABLE III. Same as Table I, but for unfavored $\alpha$ decay of doubly-odd nuclei.

| $\alpha$ transition | $Q_\alpha$ (MeV) | $\beta^+ \to \beta^-$ | $I_{\text{min}}$ | $T_{\text{exp}}^{\text{calc}}$ (s) | $P_\alpha$ | $T_{1/2}^{\text{calc}}$ (s) | $T_{1/2}^{\text{calc}}$ (s) |
|---------------------|-----------------|----------------------|-----------------|-----------------|-----------|-------------------|-------------------|
| $^{154}\text{He}^{m} \to ^{150}\text{Th}^{m}$ | 3.823 | $8^+ \to 9^+$ | 2 | $1.86 \times 10^7$ | 4.23 $\times 10^7$ | 0.410 | 2.57 $\times 10^7$ | 3.34 $\times 10^7$ |
| $^{168}\text{Re} \to ^{164}\text{Ta}$ | 5.068 | $(7^+) \to (3^+)$ | 4 | $8.80 \times 10^4$ | 1.32 $\times 10^5$ | 0.270 | 1.27 $\times 10^5$ | 1.24 $\times 10^5$ |
| $^{170}\text{Ir}^{m} \to ^{166}\text{Re}$ | 6.265 | $(8^+) \to (7^+)$ | 2 | $2.25 \times 10^6$ | 1.01 $\times 10^6$ | 0.081 | 1.86 $\times 10^6$ | 1.36 $\times 10^6$ |
| $^{172}\text{Ir} \to ^{168}\text{Re}$ | 5.985 | $(3^+) \to (7^+)$ | 4 | $2.20 \times 10^2$ | 5.42 $\times 10^1$ | 0.044 | 2.43 $\times 10^2$ | 2.26 $\times 10^2$ |
| $^{180}\text{Ti} \to ^{176}\text{Au}$ | 6.715 | $(4^{-}) \to (5^{-})$ | 2 | $1.70 \times 10^4$ | 8.47 $\times 10^{-1}$ | 0.009 | $-3.92 \times 10^0$ | $-5.18 \times 10^0$ |

Nuclei in Region I around $Z = 82$, $N = 82$ shell closure

| $^{190}\text{Bi}^{m} \to ^{186}\text{Bi}$ | 6.836 | $(3^+) \to (7^+)$ | 4 | $8.16 \times 10^0$ | 5.72 $\times 10^0$ | 0.126 | 1.00 $\times 10^1$ | 8.70 $\times 10^0$ |
| $^{192}\text{Bi}^{m} \to ^{188}\text{Bi}$ | 6.343 | $(3^+) \to (7^+)$ | 4 | $2.77 \times 10^2$ | 4.43 $\times 10^2$ | 0.288 | 3.00 $\times 10^2$ | 2.80 $\times 10^2$ |
| $^{194}\text{Bi}^{m} \to ^{190}\text{Bi}$ | 6.207 | $(10^{-}) \to (9^{-})$ | 2 | $3.84 \times 10^2$ | 4.12 $\times 10^2$ | 0.193 | 4.17 $\times 10^2$ | 3.89 $\times 10^2$ |

Nuclei in Region III around $Z = 82$, $N = 126$ shell closure

| $^{210}\text{At} \to ^{206}\text{Bi}$ | 5.696 | $(5^+) \to (9^+)$ | 2 | $5.56 \times 10^4$ | 8.18 $\times 10^4$ | 0.265 | 5.22 $\times 10^4$ | 5.21 $\times 10^4$ |
| $^{210}\text{At} \to ^{206}\text{Bi}$ | 5.361 | $(5^+) \to (6^+)$ | 2 | $1.66 \times 10^7$ | 8.14 $\times 10^5$ | 0.009 | 1.07 $\times 10^6$ | 1.18 $\times 10^6$ |

Nuclei in Region IV around $Z = 82$, $N = 126$ shell closure

| $^{212}\text{Bi}^{m} \to ^{208}\text{Tl}$ | 6.455 | $(8^{-}, 9^{-}) \to 5^+$ | 3 | $2.24 \times 10^4$ | 2.40 $\times 10^4$ | 0.002 | 5.64 $\times 10^4$ | 6.83 $\times 10^4$ |
| $^{214}\text{Bi}^{m} \to ^{210}\text{Tl}$ | 5.621 | $(1^{-}) \to 5^+$ | 5 | $3.14 \times 10^{-1}$ | 1.57 $\times 10^{-2}$ | 0.009 | 7.92 $\times 10^{-3}$ | 8.20 $\times 10^{-3}$ |

values of adjustable parameters $b$ and $d$ in Eq. (11) and (12) are approximatively equal in each region because the isospin $I$ changes little.

For intuitively, the linear relationships of unfavored $\alpha$ decay for odd-$A$ nuclei between $P_\alpha$ and $\frac{N_p N_n}{Z_0 + N_0}$, $N_p N_n I$ as Eq. (11), Eq. (12) are plotted in Fig. 2 and 3, respectively. Similarly, for the unfavored $\alpha$ decay of doubly-odd nuclei, the corresponding linear relationships are depicted in Fig. 4 and 5, respectively. In Fig. 2-5, the blue circle and the red triangle represent the nuclei around at $Z = 82$, $N = 82$ and $Z = 82$, $N = 126$ closed shells, respectively. The lines are linear fittings between $P_\alpha$ and $\frac{N_p N_n}{Z_0 + N_0}$, $N_p N_n I$, respectively, and also are predictions of Eq. (11) and (12). The corresponding parameters $a$, $b$, $c$, and $d$ are given in the Table IV. From Fig. 2 and 3, we can find that valence proton-neutron interaction have an

![FIG. 3. (color online) Same as Fig. 2, but it depicts linear relationship between $\alpha$ preformation probabilities and product of valence protons (holes), neutrons (holes) and isospin asymmetry as $N_p N_n I$.](image)

![FIG. 4. (color online) Same as Fig. 2, but it depicts doubly-odd nuclei in accordance with $\frac{N_p N_n}{Z_0 + N_0}$](image)
obvious difference in different shell closures. From Fig. 4 and 5, we can intuitively find that the lines’ variation tendencies still satisfy above equations although the nuclei number of unfavored $\alpha$ decay is small.

Intuitively, we can find that the linear relationship in Regions III and IV are better than those in Region I from Fig. 2-5. It might because the doubly magic core at $Z = 82$, $N = 82$ is unbound, and the nucleons in the core play an essential role on $P_\alpha$ [23]. Meanwhile, $Z = 82$, $N = 126$ are stable doubly magic core.

### TABLE IV. The parameters of Eqs. (11) and (12) that show $\alpha$ preformation probabilities are linearly related to $N_pN_n$.

| Region | $a$  | $b$  | $c$  | $d$  |
|--------|-----|-----|-----|-----|
| odd-A Nuclei | -1.65948 | -0.11308 | -0.06898 | 0.02948 |
| III    | -0.8437  | 0.05854  | -0.03726  | 0.0402  |
| IV     | 0.51361  | 0.00585  | 0.01281   | 0.00585 |
| doubly-odd Nuclei | -0.82097  | -0.12653 | -0.04695 | -0.13455 |
| III    | -2.72853 | -0.02778 | -0.09794 | -0.04321 |
| IV     | 0.53443  | -0.00317 | 0.01402   | -0.00363 |

### IV. SUMMARY

In summary, we systematically study unfavored $\alpha$ decay of closed shell nuclei related to ground and isomeric states around $Z = 82$, $N = 82$ and 126 closed shells, respectively, within a two-potential approach. Our research indicates that, for unfavored $\alpha$ decay of closed shell nuclei, $P_\alpha$ are still linear to $N_pN_n$ or $N_pN_nI$, in addition, shell effect and valence proton-neutron interaction still plays an important role in $P_\alpha$. Our calculations are in good agreement with the experimental data.

![Graph showing $P_\alpha$ vs. $N_pN_nI$ for $Z=82$, $N=82$ and $N=126$]

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