Effect of magnetic field on Rayleigh-Taylor instability of two superposed fluids

P K Sharma, Anita Tiwari and R K Chhajlani

1,2 BUIT, Barkatullah University Bhopal (M.P.)-462026 India
3 School of Studies in Physics, Vikram University Ujjain (M.P.) India

E-mail : pks_buit30@yahoo.com

Abstract. The effect of two dimensional magnetic field on the Rayleigh-Taylor (R-T) instability in an incompressible plasma is investigated to include simultaneously the effects of suspended particles and the porosity of the medium. The relevant linearized perturbation equations have been solved. The explicit expression of the linear growth rate is obtained in the presence of fixed boundary conditions. A stability criterion for the medium is derived and discussed the Rayleigh Taylor instabilities in different configurations. It is found that the basic Rayleigh-Taylor instability condition is modified by the presence of magnetic field, suspended particles and porosity of the medium. In case of an unstable R-T configuration, the magnetic field has a stabilizing effect on the system. It is also found that the growth rate of an unstable R-T mode decreases with increasing relaxation frequency thereby showing a stabilizing influence on the R-T configuration.

1. Introduction

Currently, there is much interest in the evolution of the Rayleigh-Taylor (R-T) and hydrodynamic instabilities in both laboratory, inertial confinement, fusion and astrophysical plasmas. The Rayleigh Taylor instability occur when adverse density gradient exist, e.g., in the case of a heavy fluid supported by the lighter fluid in a gravitational field. Chandrasekhar [1] has discussed the R-T instability of two incompressible fluids taking into account different parameters, viz. magnetic field viscosity, surface tension and uniform rotation. Hans [2] has studied the effects of collisions with neutral atoms on the Rayleigh-Taylor in a composite medium. Many others e.g. Talwar [3], Bhatia [4], Sharma and Shrivastava [5]. Sharma and Chhajlani [6] have extended his work using different parameters and assumptions.

The effect of suspended particle on the stability of superposed fluids might be of importance to industrial and chemical engineering. Saffman [7] has studied in detail a dust gas in magnetohydrodynamics. Sharma et al. Chhajlani [8] have investigated effect of rotation and suspended dust particles on the R-T instability of two superposed magnetizing conducting plasma.

In the recent years, the wide applications of porosity in industrial, geophysical situation and chemical engineering created considerable interest, particularly among geophysical fluid dynamics. Porosity is of use for the study of physical properties of comets, meteorites and interplanetary dust in astrophysical context McDonnel [9]. Wooding [10] has considered the Rayleigh instability of a thermal boundary layer in flow through a porous medium. Recently, Prajapati et al. [11,12] have studied the effect of porosity on R-T instability with suspended particles.
The aim of this paper is to study the effect of two dimensional horizontal magnetic field on the Rayleigh Taylor instability of two superposed incompressible conducting fluids in porous medium.

2. Linearized equations of motion for the perturbation

The system under investigation is made of two semi infinite, non dissipative, incompressible, porous, magnetofluids under uniform horizontal magnetic field \( H(\text{Hx,Hy},0) \) with suspended dust particles. Consider a Cartesian system of axes with the z direction as vertical. Suppose that a plane interface of discontinuity exist at \( z=0 \). The static interface between the two fluids is initially flat and the magnetofluid with density \( \rho_2 \) in above the one with density \( \rho_1 \) where \( \rho_2 > \rho_1 \). If the system is perturbed the variables representing the difference from the hydrostatic state are \( u(u,v,w), h(hx,hy,hz), \delta p, \delta \rho \) the velocity, magnetic field, pressure and density.

It is supposed that suspended dust particles experience force \( KN(V-U) \) in the fluid, where \( K \) is a constant given by \( K = 6 \pi a \mu \) (Stokes drag formula), ‘\( a \)’ being the particle radius.

\[
\frac{\rho}{\varepsilon} \frac{\partial \bar{u}}{\partial t} = -\nabla \delta \rho + \frac{KN}{\varepsilon} (v-u) + g \delta \rho - \frac{\mu}{k_1} u + \frac{\mu_\alpha}{4\pi} \left[ (\nabla \times h) \times H + (\nabla \times H) \times h \right]
\]  
(1)

\[
\frac{\partial}{\partial t} \rho = (u\nabla)\rho = 0,
\]  
(2)

\[
\frac{\partial h}{\partial t} = (H\nabla) u - (u\nabla) H
\]  
(3)

\[
\nabla u = 0,
\]  
(4)

\[
\nabla h = 0
\]  
(5)

\[
\left[ \frac{\tau}{\varepsilon} + 1 \right] v = u,
\]  
(6)

Where \( \tau = m/6\pi a \mu \) denotes the relaxation time for the suspended dust particles and \( \varepsilon \) denotes the medium porosity. As a result of this Darcy’s law the usual viscous term in the equation of fluid motion is replaced by the resistance term \( (-\mu/k_1)u \) where \( \mu \) is the viscosity of the fluid, \( k_1 \) is the permeability of the medium.

A solution in normal modes is proposed to these equations:

\[
\text{exp}(ik_x x + ik_y y + \text{int})
\]  
(7)

where \( k_x \) and \( k_y \) are the components of the wave numbers \( (k^2 = k_x^2 + k_y^2) \) and \( n \) is the growth rate of the harmonic perturbations.

A combination of the above equations and use of Eq.(7) leads to an equation of the z-component of the velocity \( w \) alone:

\[
\left[ \frac{i}{\varepsilon} n + v / k_1 + a_0 \frac{(in)}{\varepsilon} (in)^2 \right] \left[ D(\rho Dw) - k^2 Dw \right] + \left[ \frac{(H_x k_x + k_x H_y)^2}{4\pi(a\varepsilon n)} \right] (D^2 - k^2)w = 0
\]  
(8)

Where \( a_0 = mN/\rho \) denotes the mass concentration of the particles and \( D = d/dz \). Equation (8) is the general dispersion relation incorporate the effect of the magnetic field and of the suspended dust particles in the porous medium. Here we have assumed the same density of suspended particles in both the regions \( z < 0 \) and \( z > 0 \).

3. Discussion

The solution of Eq.(8) by using the boundary conditions across the interface of two fluids can be written as

\[
2
\[
\sigma^2 + \sigma^2 \left[ f_s \left( 1 + \frac{2mN}{\rho_1 + \rho_2} \right) + \frac{g}{k_1} (\beta_1 v_1 + \beta_2 v_2) \right] + \\
\sigma \left\{ \frac{g}{k_1} (\beta_1 v_1 + \beta_2 v_2) + 2(k + V_A + k + V_B)^2 - gk(\beta_2 - \beta_1) \right\} + \\
f_s \left[ 2(k + V_A + k + V_B)^2 - gk(\beta_2 - \beta_1) \right] = 0. \tag{9}
\]

Here, \( f_s = 1/\tau \), \( \sigma = (\text{in}) \) and \( \beta_1 = \rho_1 / (\rho_1 + \rho_2) \), and \( \beta_2 = \rho_2 / (\rho_1 + \rho_2) \).

For simplicity, we considered the Alfvén velocities of two plasmas are the same, so that
\[
V_{A,\beta}^2 = \frac{\mu_e H^2}{4\pi(\rho_1 + \rho_2)} \tag{10}
\]

Eq.(9) gives the dispersion relation of R-T configuration for two superposed fluids of different densities including permeability, porosity, suspended dust particles with two dimensional uniform horizontal magnetic field.

### 3.1 Stable configuration (\( \beta_1 > \beta_2 \)) or (\( \rho_1 > \rho_2 \))

In this potentially stable arrangement when the lower fluid is heavier than the upper fluid, it clearly represent the stable configuration. If we applied necessary condition of the Hurwitz criterion on equation (9) we find that all the coefficients are positive and real implying the stability of the considered system.

### 3.2 Unstable configuration (\( \beta_2 > \beta_1 \)) or (\( \rho_2 > \rho_1 \))

The system is stable or unstable depends on the following conditions

#### 3.2.1. If  \( 2(k + V_A + k + V_B)^2 > gk(\beta_2 - \beta_1) \) the system is stable because there is no change in sign. So it has no positive root.

#### 3.2.2. If  \( 2(k + V_A + k + V_B)^2 < gk(\beta_2 - \beta_1) \) the system is unstable because constant term is negative therefore allows one change of sign and so has one positive root. The occurrence of a positive root implies that the system is unstable. Thus for the unstable case (\( \beta_2 > \beta_1 \)) the system is stable or unstable according as  \( 2(V_A \sin \theta + V_B \cos \theta)^2 \) is greater than or smaller than \( gk(\beta_2 - \beta_1) \). In the absence of magnetic field, equation (9) has one positive root, and so the system is unstable for (\( \beta_2 > \beta_1 \)). But the magnetic field has got a stabilizing effect and completely stabilizes the wave-number band \( k > k_c \),

Where \( k_c = \frac{g(\beta_2 - \beta_1)}{2[V_A \sin \theta + V_B \cos \theta]^2} \) \tag{11}

and \( \theta \) is the inclination of the wave vector \( k \) to the direction of the magnetic field \( H \) i.e. \( kx = k \sin \theta \) and \( ky = k \cos \theta \). By this configuration we can say that the condition of R-T instability and cut-off wave-number do not depend upon medium porosity and suspended dust particles but depend on magnetic field only. We also find that the condition of R-T instability remains the same but there will be a change in the growth rate of R-T instability due to multiplication of relaxation frequency of suspended dust particles in the constant term of Eq. (9).

In order to perform the numerical interpretation on the growth rate of Rayleigh Taylor instability of two superposed magnetized plasma with suspended dust particles, medium viscosity and porosity we rewrite the dispersion relation (9) in dimensionless form as

\[
\sigma^2 + \sigma^2 \left[ f_s \left( 1 + 2\alpha' \right) + \frac{2\alpha \nu}{k_1} \right] + \sigma \left\{ 2\frac{g}{k_1} \nu' + 2(V_A \sin \theta + V_B \cos \theta)^2 - (\beta_2 - \beta_1) \right\} + \\
f_s \left[ 2(V_A \sin \theta + V_B \cos \theta)^2 - (\beta_2 - \beta_1) \right] = 0.
\tag{12}
\]

where \( \nu' = \mu / (\rho_1 + \rho_2) \) and \( \alpha' = mN / (\rho_1 + \rho_2) \).
In Fig. (1-2), we have plot the growth rate ($\sigma^*$) vs relaxation frequency ($f_s^*$) for the different values of medium porosity ($\varepsilon$), and dust particle density ($\alpha$). The numerical values of all the parameters are taken as arbitrary to study their effect on the growth rate. From the curves we find that growth rate of the system decreases as we increase the values of medium porosity and dust density. According to discussion above, the Rayleigh-Taylor instability is suppressed by horizontal magnetic field, porosity and dust particle density.

4.Acknowledgement

The authors (P.K. Sharma and Anita Tiwari) are thankful to Prof. R.K. Pandey, Director, BUJT and Prof. Nisha Dubey, Hon’ble V.C. Barkatullah University, Bhopal for their constant encouragement in this work. The authors (P.K.S. and A.T.) are also expresses their sincere thanks to MPCST, Bhopal for providing Research Fellow and financial assistance in the research project.

5. References

[1] Chandrasekhar S 1991 Hydrodynamic and Hydromagnetic Stabilit (Clarendon Press Oxford).
[2] Hans H K 1968 Nucl. Fusion 8 89
[3] Talwar S P J. Geophysical Research 69 2707
[4] Bhatia P K 1970 Cosmic Electrodynamics 1 269
[5] Sharma R C and Shrivastava K M 1974 Austr. J. Phys. 27 53
[6] Sharma P K and Chhajlani R K 1998 Phys. Plasmas 5 2202
[7] Saffman P G 1962 J. Fluids Mech. 13 120
[8] Sharma P K, Prajapati R P and Chhajlani R K 2010 Acta Physica Polonica A 118 576
[9] McDonnel J A M 1978 Cosmic Dust (Toronto: John Wiley and Sons)
[10] Wooding R A 1960 J. Fluid Mech. 9 183
[11] Prajapati R P, Soni G D, Sanghvi R K and Chhajlani R K 2008 Z. Naturforsch 64a 455
[12] Prajapati R P and Chhajlani R K 2010 J. Porous Media 13(9) 765