Self-interaction and mass in quantum field theory

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Qualitative implications of electroweak theory are reconsidered on the assumption that the unique source of fermion rest mass is self-interaction via coupling to gauge fields. This implies small but nonzero mass for neutrinos, and suggests that successive fermion generations are distinct coupled-field eigenstates of a self-interaction mass operator. For a scalar Higgs field, this mechanism can account for the SU(2) symmetry breaking of electroweak theory without a biquadratic self-interaction. The implied Higgs particle mass could be very small, eluding any search limited to heavy particles.

I. IMPLICATIONS OF SELF-INTERACTION

In relativistic perturbation theory, the self-interaction mass of an electron is a sum over momentum transfer, logarithmically divergent unless the sum is somehow cut off. This divergence indicates that the theory is incomplete, but its logarithmic character implies that any physically valid cutoff can occur only for very large momentum transfer, or at very small distances. A convincing self-contained theory would require a cutoff mechanism that implies correct observed masses for all fermions. The implications of standard quantum field theory, modified only by such a cutoff, are reconsidered here, and appear to resolve several mysteries remaining in accepted current theory. This argues for a renewed effort to understand the cutoff mechanism.

A classical point-particle would interact with itself through the static electromagnetic field generated by its electric charge. This implies an infinite mass, inconsistent with observed reality. In quantum field theory the model is fundamentally different. Each physical particle is created from the physical vacuum, which must be postulated to have no net energy or current density, although polarizable. The static field of a charged fermion is compensated due to the net neutrality of all leptons and quarks. The radiative component remains, generated by virtual excitations of gauge fields coupled to each elementary fermion field. Any interacting fermion is necessarily dressed by the accompanying virtual radiation. Bare fermion mass can simply be omitted from the Lagrangian density, but self-interaction mass due to virtual radiation fields cannot be eliminated unless there is no mechanism for virtual excitation. An immediate implication is that neutrinos carry a small mass, due to virtual excitation of the weak gauge fields.

That a physical fermion field is a quasiparticle which acquires mass from its self-interaction is consistent with the structure of the Dirac equation: a mass parameter couples field components of opposite parity whose energy values have opposite sign. The field equation for a bare fermion coupled to gauge fields can be rewritten so that a mass parameter replaces a 4-vector transition operator linear in the self-interaction gauge fields. The mass parameter takes the form of an eigenvalue of the transition operator, identified as a self-interaction mass operator. Diagonalization in the Fock space of the interacting system defines a canonical transformation from bare fermions to dressed quasiparticles, identified as physical fermions. This transformation breaks chiral symmetry as it produces nonvanishing mass. Although this is implicit in standard field theory, an algebraic formulation of the theory will be developed here in which this transformation is explicit.

A significant implication of this analysis is that for fermions the self-interaction mass operator might very well have several eigenvalues that correspond to discrete states pushed down below successive overlapping continua. This could explain the existence of higher generations of fermions, such as heavy leptons and their corresponding neutrinos.

If local charge neutrality is postulated (cosmic jellium), or atomic nuclei are treated as point charges, standard renormalization theory defines a self-contained quantum electrodynamics, restricted to electrons and the Maxwell field. Extending Maxwell theory to non-abelian gauge symmetry, the unified electroweak theory of Weinberg and Salam incorporates neutrinos, quarks and SU(2) weak gauge fields. Fermion masses are given their observed values, justified by the quantitative success of renormalized QED. Renormalization also justifies absorbing vacuum polarization into the observed electric charge unit as a renormalized coupling constant.

Assuming that fermion masses are solely an expression of the self-interaction induced by coupling to gauge fields might appear to conflict with an essential element of electroweak theory. This theory postulates a biquadratic self-interaction from which an assumed scalar boson (Higgs) field acquires mass. This induces a canonical transformation that gives mass to the weak gauge fields but not to the transformed Maxwell field. Yukawa terms that couple the Higgs and fermion fields are independently postulated, and are assumed to account for fermion mass in general.

It is argued here that since radiative self-interaction is inherent in the formalism, these Yukawa terms are unnecessary. Neutrino mass requires at least a right-handed
chiral isospin singlet neutrino field, probably quantitatively negligible. Radiative self-interaction is shown here to be relevant to a scalar boson field. This can replace the biquadratic Higgs self-interaction while retaining the essential structure of electroweak theory. This argument implies a mechanism that forces the residual mass of the Maxwell field to zero. The Higgs mass could result solely from coupling to the weak gauge fields, and might be very small. Such a Higgs particle would be missed by current searches for a heavy scalar boson.

II. SELF-INTERACTION MASS IN QED

QED theory requires two distinct postulates. The first is the dynamical postulate that the action integral \( W = \int \mathcal{L} dx \) of the Lagrangian density \( \mathcal{L} \) over a specified space-time region is stationary with respect to variations of the independent fields \( A_\mu \) and \( \psi \), subject to fixed boundary values. The second postulate attributes algebraic commutation or anticommutation properties to these elementary fields.

Defining \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), the QED Lagrangian density is

\[
\mathcal{L} = -(1/16\pi)F^{\mu\nu}F_{\mu\nu} + i\hbar c \psi^\dagger \gamma^0 \gamma^\mu D_\mu \psi. \tag{1}
\]

Coulping to the electromagnetic 4-potential \( A_\mu \) occurs through the covariant derivative

\[
D_\mu = \partial_\mu + i(-e/\hbar c)A_\mu, \tag{2}
\]

where \(-e\) is the renormalized electronic charge. The notation used here defines covariant 4-vectors

\[
x_\mu = (ct, \mathbf{r}), \partial_\mu = (\partial/c\partial t, \mathbf{\nabla}),
A_\mu = (\phi, -\mathbf{A}), \ j_\mu = (c\rho, -j).
\]

Spatial components have reversed signs in the corresponding contravariant 4-vectors, indicated by \( x^\mu \), etc. The Dirac matrices are represented in a form appropriate to a 2-component fermion theory, in which chirality \( \gamma^5 \) is diagonal for mass-zero fermions,

\[
\gamma^\mu = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}. \tag{3}
\]

The dynamical postulate implies covariant field equations,

\[
\partial^\mu F_{\mu\nu} = (4\pi/e)j_\nu + (4\pi/e)(-ec\psi^\dagger \gamma^0 \gamma_\nu \psi), \tag{4}
\]

\[
i\hbar c \gamma^\mu D_\mu \psi = 0. \tag{5}
\]

In units such that \( \hbar = c = 1 \), decomposing \( A_\mu = A^m_{\mu\nu} + A^{ext}_{\mu\nu} \) into self-interaction and external subfields, and using notation \( \mathcal{A}(x) = \gamma^\mu A_\mu(x) \), the fermion field equation is

\[
\{i\gamma^\mu \partial_\mu + eA^{ext}\} \psi = -eA^{int}\psi = \hat{m}\psi, \tag{6}
\]

defining a mass operator \( \hat{m} = -eA^{int} \).

III. SELF-INTERACTION MASS AS AN EIGENVALUE

The self-interaction mass operator \( \hat{m} = -eA^{int} \) acts in a Fock space defined by products of creation and annihilation operators. The mass operator retains the triplet-odd character of the classical field \( A_\mu(x) \). What is proposed here is to rewrite Eq.\( (6) \) as a renormalized Dirac equation

\[
\{i\gamma^\mu \partial_\mu + eA^{ext}\} \psi = \{\hat{m} - m\} \psi = 0. \tag{7}
\]

Eq.\( (7) \) exhibits the algebraic structure implied by renormalization. A canonical transformation of field operators and the vacuum state diagonalizes the mass operator and determines a c-number mass. This transformation mixes field components of positive and negative energy, breaking chiral symmetry. The resulting Dirac equation combines chiral massless Pauli spinors into 4-component Dirac bispinors.

The relevant Fock space is parametrized by mass parameter\( m \) in the renormalized Dirac equation. Since \( m_0 = 0 \) for bare fermions, the computed eigenvalue can be identified with \( \delta m(m) \) in standard perturbation theory. A consistency condition \( \delta m(m) = m \) is imposed because the right-hand member of Eq.\( (7) \) must vanish.

The self-energy of a free electron can be evaluated to order \( e^2 \) using Feynman’s rules. The relevant Feynman diagram describes virtual emission and reabsorption of a photon of 4-momentum \( k \) by a free electron. The present analysis replaces perturbation theory for this virtual process by solution of the algebraic eigenvalue equation \( \{\hat{m} - m\} \psi = 0 \). For restricted \( k \), the electron mass \( m \) should be computable to relative accuracy \( a = e^2/kc \) using only single-photon virtual excitations. It will be assumed here that the electromagnetic field and the electronic charge have been renormalized.

The Fock space relevant here is characterized by a bare fermion field \( \psi(x) \) with parametric mass \( m \). Expanded in a complete set of solutions \( u_\mu(x) \) of the Dirac equation with mass \( m \), \( \psi(x) = \sum_\mu u_\mu(x) a_\mu \), where \( x = (x, t) \). The amplitude coefficients \( a_\mu \) are fermion destruction operators, with anticommutators

\[
\{a_\mu, a_\nu^\dagger\} = \delta_{\mu\nu}, \quad \{a_\mu, a_\nu\} = 0, \quad \{a_\mu^\dagger, a_\nu^\dagger\} = 0. \tag{8}
\]

A model vacuum state is defined such that \( a_\mu|\text{vac}\rangle = 0 \), where \( a_\mu^\dagger = a_\mu^\dagger(\epsilon_\alpha > 0) \), \( a_\mu = a_\mu^\dagger(\epsilon_\alpha \leq 0) \).

For the interacting system considered in QED, the renormalized Maxwell field is expanded in terms of bosonic amplitude operators as \( A_\mu(x) = \sum_k A_{\mu k}(x) b_k \), such that \( b_k|\text{vac}\rangle = 0 \). The physical electronic charge \(-e\) defines a renormalized coupling constant that incorporates the effects of vacuum polarization.

The mass operator is to be diagonalized in a basis of virtual excitations of the model vacuum state. This replaces the bare creation operator \( a_\mu^\dagger \) by a dressed operator whose leading terms are \( \eta_\mu = a_\mu^\dagger c_\mu + \sum_k b_k a_\mu^\dagger c_{\mu-k} - c_{\mu-k} a_{\mu-k}^\dagger \).
Here \( a^\mu_k \) creates a bare electron of 4-momentum \( p \) and \( b^\dagger_{\nu k} \) creates both a bare electron of 4-momentum \( p - k \) and a photon of 4-momentum \( k \). At this level of approximation, the implied mass \( m \) is a \( c \)-number sum over \( k \) if the sum converges. As in standard renormalized QED, logarithmic divergence is indicated, and a cutoff can be introduced consistent with the observed mass.

If only terms of order \( \alpha \) are retained in all matrix elements of \( m \), the mass eigenvalue here must agree with this order with the Feynman self-energy

\[
\delta m(m) = \frac{3\alpha}{4\pi} m \left( \ln \frac{\Lambda^2}{m^2} + \frac{1}{2} \right),
\]

introducing cutoff \( \Lambda \) to eliminate a logarithmic ultraviolet divergence. \( \delta m \) vanishes for \( m \to 0 \). The consistency condition for the physical electron mass requires \( \Lambda/m = \exp(286.7) \), far beyond the range of any feasible experiment.

Chirality considerations imply that \( \delta m(0) = 0 \), consistent with the perturbation formula. The algebra of \( \gamma \) matrices implies that chirality is conserved by transition matrix elements of the form \( \int \bar{\psi}_a \gamma^\mu W^a_\mu \psi_b \). When \( m = 0 \), bare states of opposite energy have definite but opposite chirality, and cannot be mixed by such virtual transitions. Hence virtual transitions cannot affect the mass operator.

### IV. U(2) LOCAL GAUGE SYMMETRY

Gauge symmetry of classical U(2) vector fields can be analyzed following the logic of Yang and Mills for SU(2). The derivation here follows Sect.10.3 of reference. Elementary 2x2 matrices are defined by the unit matrix \( \tau_0 \) and the Pauli matrices \( \tau = \{ \sigma_x, \sigma_y, \sigma_z \} \). Any element of U(2) takes the form \( \exp(i\tau_0 \bar{\gamma}^a \gamma^a) \) for real \( a(x) \), using the summation convention for repeated index \( k = 0, 1, 2, 3 \). The gauge field \( W^a_\mu = \{ B_\mu, W_\mu \} \) has U(1) and SU(2) component subfields as indicated. Metric tensor \( g_{\mu\nu} \) here has diagonal elements \( 1, -1, -1 \).

Covariant derivative \( D_\mu \) is defined as\( \partial_\mu + \frac{ig}{2} \tau^a \gamma_\mu W^a_\mu \). In detail, \( W^a_{\mu\nu} = D_\mu W^a_\nu - D_\nu W^a_\mu \). The field tensor \( W^a_{\mu\nu} = D_\mu W^a_\nu - D_\nu W^a_\mu \). In detail, \( W_{\mu\nu} = \{ \partial_\nu B_\mu - \partial_\mu B_\nu, \partial_\nu W_\mu - \partial_\mu W_\nu - \frac{i}{2} \epsilon^{\mu\nu\rho\sigma} W_{\rho\sigma} \times W_{\rho\sigma} \} \).

\[
U(x) = 1 - \frac{i}{2} g \tau^a \gamma^a (x) + \cdots \text{determines an infinitesimal local gauge transformation} \ W^a_\mu(x) \to U W^a_\mu U^{-1} + \frac{i}{2} \epsilon^{\mu\nu\rho\sigma} (\partial_\rho U(x)) U^{-1} \text{and implies} \ W^a_{\mu\nu} \to U W^a_{\mu\nu} U^{-1}. \]

For U(2) fields with no kinematic mass, the Lagrangian density

\[
\mathcal{L}_W = -\frac{1}{4} W^a_{\mu\nu} W^a_{\mu\nu} \text{is gauge invariant}. \]

Conventional field strength units here absorb a factor \( 4\pi \).

The Euler-Lagrange equations for the gauge fields follow from \( \partial_{\mu} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} W^a_\nu)} - \frac{\partial \mathcal{L}}{\partial W^a_\nu} = 0 \). The individual terms here are

\[
\frac{\partial \mathcal{L}}{\partial W^a_{\tau\sigma}} \frac{\partial W^a_{\tau\sigma}}{\partial (\partial_{\mu} W^a_\nu)} = \{ B^\mu_{\nu}, W^\mu_{\nu} \}
\]

implying gauge field equations \( \partial_\mu B^\mu_{\nu} = 0 \) and \( \partial_\mu W^\mu_{\nu} = \frac{ig}{2} W^a_\mu \times W^a_{\nu} = \frac{i}{2} W^a_{\mu\nu} \). This SU(2) self-interaction implies finite mass for \( W_\mu \), which breaks U(2) symmetry. \( B_\mu \) has no self-interaction, and hence no resulting mass. It could be identified with the Maxwell field \( A_\mu \), except that U(2) theory has no interaction between the U(1) and SU(2) subfields, not defining an electromagnetic current density for the fields \( W^a_\mu = (W^1_\mu \mp W^2_\mu) \sqrt{2} \), which are observed to carry unit charge.

### V. THE SU(2) SCALAR FIELD

Although U(2) theory does not require a scalar Higgs field, a 2-component scalar field \( \Phi \) can be introduced without violating any symmetry. The covariant derivative \( D_\mu \Phi \) defines a gauge-invariant Lagrangian density

\[
\mathcal{L}_\Phi = \frac{i}{2} (D_\mu \Phi)^\dagger (D^\mu \Phi), \text{ with no kinetic mass term. Because} \]

\( \mathcal{L}_\Phi \) is quadratic in the covariant derivatives, it also describes interaction among the gauge fields. The Euler-Lagrange equation for \( \Phi \) takes the form

\[
\frac{\partial}{\partial \mu} \frac{\partial \mathcal{L}_\Phi}{\partial (D_\mu \Phi)^\dagger} - \frac{\partial (D_\mu \Phi)^\dagger}{\partial \Phi} \frac{\partial \mathcal{L}_\Phi}{\partial (D_\mu \Phi)^\dagger} = D_\mu D^\mu \Phi = 0,
\]

which can be written as \( \{ \partial_\mu \partial^\mu + m^2 \} \Phi = 0 \).

The mass operator \( \bar{m}^2 \) is constructed from gauge fields generated by virtual excitations of \( \Phi \), which determine a source current density for the gauge fields. This implies a self-interaction mechanism analogous to that of leptons and quarks. The mass operator is diagonalized by a canonical transformation that associates a minimal mass eigenvalue with a global stationary state of the renormalized vacuum. This determines different masses for the renormalized component fields \( \phi_0 \) and \( \phi_+ \), breaking SU(2) symmetry without a biquadratic term in the Lagrangian density. The transformation minimizes self-interaction mass by decoupling \( A_\mu \), the most strongly interacting component of the transformed U(2) gauge field, from the transformed scalar field \( \phi_0 \). As a result, \( \phi_0 \) has zero electric charge, and its self-interaction mass is due only to the renormalized weak gauge fields. At the same time, the interaction strength of \( A_\mu \) is maximized by forcing its mass to zero. This eliminates the largest self-interaction mass term for both \( \phi_0 \) and \( A_\mu \). This transformation can be identified with the canonical transformation assumed in electroweak theory.

Assuming isospin \( t_3 = \frac{1}{2} \) and hypercharge \( y = 1 \), the self-interaction eigenstates define isospin \( t_3 = \mp \frac{1}{2} \) and electric charge factor \( Q = 0, 1 \), which vanishes for renormalized field \( \phi_0(x) \). A bosonic field of positive mass can vanish exactly, since there is nothing to exclude occupation numbers \( n = 0 \) in physical states. Only the component field of lowest mass would remain in a physical
ground state. The Higgs condition $\phi_+ = 0$ might simply be a dynamical implication of self-interaction, restated as $\langle \phi_+ / \phi_0 \rangle \rightarrow 0$ for physically accessible states.

For renormalized coupling constants $g_1 \neq g_2$, interactions with the gauge bosons arise from terms in the covariant derivative $D_\mu \Phi = \{(\partial_\mu + \frac{ig_2}{\sqrt{2\hbar c}} B_\mu + \frac{ig_1}{2\sqrt{2\hbar c}} \tau \cdot W_\mu )\Phi \}$. If $\langle \phi_+ / \phi_0 \rangle \rightarrow 0$, the residual covariant derivative is $\Phi = \Phi_\mu \phi_0$;

$$D_\mu \Phi = \left( \frac{ig_2}{\sqrt{2\hbar c}} \sqrt{W_\mu^+ \phi_0} \right),$$

where $Z_\mu = W_\mu^3 \cos \theta W - B_\mu \sin \theta W$ is a linear transform of the neutral gauge fields. The Weinberg angle $\theta_W$ is defined such that $\sin \theta_W = g_1 / \sqrt{g_1^2 + g_2^2}$. The decoupled orthogonal field $A_\mu = W_\mu^3 \sin \theta W + B_\mu \cos \theta W$ is the physical Maxwell field. The Lagrangian term $-\frac{1}{2} W_{\mu\nu}^+ W_{\mu\nu}^+$ is replaced by $-\frac{1}{2} \tilde{W}_{\mu\nu}^+ \tilde{W}_{\mu\nu}^+$, omitting interaction terms among the weak gauge fields, where

$$\tilde{W}_{\mu\nu}^+ = (\partial_\mu + \frac{ig_2}{\hbar c} \sin \theta W A_\mu) W_{\nu}^+ - (\partial_\nu + \frac{ig_2}{\hbar c} \sin \theta W A_\nu) W_{\mu}^+.$$ (13)

This transformed Lagrangian exhibits $U(1)$ gauge invariance for $Q = +1$. The implied electric charge unit is $e = g_2 \sin \theta_W = -g_1 \cos \theta_W$.

In the appropriate energy range, $e = \sqrt{4\pi/129} = 0.3121$ in units such that $\hbar = c = 1$. The Fermi constant $G_F = 1.16639 \times 10^{-5} \text{(GeV)}^{-2}$ determines the ratio $(g_2/M_W)^2 \approx 4 \sqrt{3} G_F$. Combining these values, the empirical mass $M_W = 80.33 \text{ GeV}$ implies coupling constants $g_1 = 0.3554, g_2 = 0.6525$, so that $\sin^2 \theta_W = 0.2288$ (accepted value $0.2315$).

Denoting SU(2) self-interaction mass, without the scalar field, by $M_0$, $\Delta M_W = M_W - M_0$ and $\Delta M_Z = M_Z - M_0$ such that $\cos \theta_W = \Delta M_W / \Delta M_Z$ for the symmetry-breaking mass induced by the scalar field. Empirical $M_W = 80.33 \text{ GeV}, M_Z = 91.19 \text{ GeV}$, and $\sin^2 \theta_W = 0.2315$ imply $M_0 = 3.15 \text{ GeV}$ and $\Delta M_W = 77.18 \text{ GeV}, \Delta M_Z = 88.04 \text{ GeV}$.

The transformed Lagrangian density for $\phi_0$ is

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi_0 \partial^\mu \phi_0 + \frac{1}{4} g_2^2 \frac{1}{(\hbar c)^2} W_\mu^+ W_\mu^\nu \phi_0^2 + \frac{1}{8} \frac{g_1^2 + g_2^2}{(\hbar c)^2} Z_\mu Z^\nu \phi_0^2.$$ (14)

Because $\phi_0$ interacts only with the weak gauge fields, it is electrically neutral. It has no electromagnetic self-interaction or resulting mass, while the Maxwell field remains massless. The interaction terms define dynamical mass in the gauge field equations.

These terms imply mass proportional to $|\phi_0|$ for both $W_\mu^\pm$ and $Z_\mu$. They augment the direct self-interaction in the SU(2) field equations and break SU(2) symmetry.

Any biquadratic term in the Lagrangian implies a nonlinear Poisson-like field equation for $\phi_0$. Yukawa coupling to fermions implies an inhomogeneous equation driven by fermion densities. The signs and magnitudes of these terms may have cosmological implications.

VI. CONCLUSIONS

If the unique source of mass in electroweak theory is direct or indirect self-interaction, traditional quantum field theory contains formal mechanisms that for fermions can explain the finite but small mass of neutrinos, while providing a rationale for the existence of fermion generations distinguished only by mass. Applied to a scalar (Higgs) boson in a nominal U(2) manifold, symmetry-breaking consistent with the electroweak model is driven by a canonical transformation that minimizes the scalar mass and forces the residual Maxwell field to be massless. The present analysis implies that the self-interaction mass of the Higgs boson may arise from weak interactions only, and might be very small, analogous to neutrino mass. This is the only physical implication contrary to the Standard Model, and is clearly subject to experimental test. It is inconsistent to postulate that neutrino mass is exactly zero, since self-interaction mediated by weak gauge fields is implied.

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