Scaling behavior in convection-driven Brazil-nut effect

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A striking experimental observation in vibrated granular materials is the Brazil-nut effect (BNE), where a large intruder particle immersed in a vertically shaken bed of smaller particles rises to the top, even when it is much denser\cite{1}. The usual practice, while describing these experiments, has been to use the dimensionless acceleration \( \Gamma = a \omega^2 / g \), where \( a \) and \( \omega \) are respectively the amplitude and the angular frequency of vibration and \( g \) is the acceleration due to gravity\cite{2,3,4}. Considering a vibrated quasi-two-dimensional bed of mustard seeds, we show here that the peak-to-peak velocity of shaking \( v = a \omega \), rather than \( \Gamma \), is the relevant parameter in the regime where boundary-driven granular convection is the main driving mechanism. We find that the rise-time \( \tau \) of an intruder is described by the scaling law \( \tau \sim (v - v_c)^{-2} \), where \( v_c \) is identified as the critical vibration velocity for the onset of convective motion of the mustard seeds. This scaling form holds over a wide range of \((a, \omega)\), diameter and density of the intruder.

Besides its obvious technological importance, granular matter is one of the most interesting examples of a driven dissipative system. Shaken granular matter exhibits various interesting phenomena such as convection, segregation and jamming. The BNE involves the motion of a single intruder in a vibrated bed of granular material. A related effect is the segregation obtained on vibrating a mixture of particles of different sizes and densities. While some progress has been made, a complete understanding of these observations is still lacking. The consensus now is that different segregation mechanisms such as convection, void filling, air-drag and kinetic-energy driven methods are responsible for the effect, each being dominant in different parameter regimes\cite{5,6,7}. In this Letter, we show that in a broad parameter regime, convection is responsible for BNE in quasi-two dimensions. It is well-known that granular convection can be either boundary-wall-driven or buoyancy-driven. Boundary-driven convection is unique to granular systems and refers to the case where the surface properties of the side walls and their shape are crucial in determining the formation and nature of the convection roll\cite{8,9,10}. Buoyancy-driven granular convection, on the other hand, is analogous to fluid convection and is seen under strong driving\cite{11,12}.

One of the simplest measurements in a granular convecting fluid is that of the rise-time \( \tau \) of a large intruder. This quantity effectively provides a single observable characterizing the complex velocity field in the system. The intruder rise-time has been used as a probe for understanding the effects of driving frequency and amplitude, interstitial air-flow, the size and density of the intruder, and wall effects on BNE. Duran et al.\cite{13} studied BNE in a quasi-two-dimensional bed of aluminium beads. They demonstrated two different segregation mechanisms: arching effects in the range \( \Gamma_c < \Gamma \lesssim 1.5 \) with \( \Gamma_c \approx 1 \), and a convective regime for \( \Gamma \gtrsim 1.5 \). In the arching regime, intruder rise was observed only when intruder-to-bead diameter ratio was large enough. In the convective regime, the rise-time was independent of intruder diameter. For a three-dimensional bed of glass beads, Knight et al.\cite{14} studied the rise time of tracer particles in the boundary-driven convection regime. For a fixed driving frequency they reported a \( \tau \sim (\Gamma - \Gamma_c)^{-2.5} \) dependence with \( \Gamma_c \approx 1.2 \), while for fixed \( \Gamma \) values, they obtained an exponential dependence on frequency. In a later work, Vanel et al.\cite{15} looked at the motion of an intruder and reported deviations from the above behaviour. However, none of these studies have obtained any scaling of the rise-time data. An understanding of the dependence of rise-time on driving amplitude and frequency therefore remains incomplete.

The present work reports experimental measurements of the rise-time of an intruder in a vertically vibrated quasi-two-dimensional bed of mustard seeds (Fig. 1). The measurements were made under a fairly wide range of driving parameters, obtained by varying frequencies and amplitudes \((2.5 \lesssim \Gamma \lesssim 25)\), intruder densities and diameters. The most remarkable finding of this work is the collapse of the rise-
time data when plotted as a function of the driving velocity \( v = a\omega \). Throughout the parameter regime where the collapse holds, we find that boundary driven convection is the dominant mechanism responsible for the rise of the intruder. Shaking the bed vertically results in a thin downward moving layer of mustard seeds adjacent to each side wall, which sets up bulk upward motion elsewhere in the system (Fig. 1b). This convective motion of the medium carries the intruder to the top of the bed. There is no dependence of the rise-time on the density of the intruder, with the intruder behaving approximately like a mass-less tracer particle tracking the convective flow of the background granular medium. This is also evident from the very similar flow patterns of the medium obtained with and without the intruder (Fig. 2). We note that \( v \) has been used as a scaling parameter in the vibro-fluidized regime\([15,17]\) that is typically realized at much higher driving (\( \Gamma \gtrsim 50 \)) and is not wall-driven. However, no scaling has been achieved so far in the boundary-driven convection regime. Given the non-homogeneous flow fields that are set up in our experiments, the dependence of \( \tau \) on shaking velocity is rather surprising.

In our experiments, we measure the rise-times \( \tau \) for several peak-to-peak amplitudes \( a \) and frequencies \( f = \omega/2\pi \), for various relative densities \( \rho_r = (\text{intruder density})/(\text{mustard seed density}) \). The rise-time decreases with increasing shaking frequencies and amplitudes, as seen in the upper inset of Fig. 3. It is observed that \( \tau \) is almost independent of density over the range used in these experiments (\( \rho_r = 0.30 - 2.34 \)). This is in contrast to the three-dimensional result\([3,18,19]\), where a non-monotonic dependence of \( \tau \) on \( \rho_r \) was found. There is no scaling of \( \tau \) with \( \Gamma \) (lower inset of Fig. 3). In contrast, a plot of \( \tau \) as a function of the shaking velocity \( v = a\omega \) (Fig. 3) shows an excellent collapse of the data. This collapse holds for the entire range of intruder densities and the two intruder sizes (diameter 1.7 cm and 1 cm) that were used. The collapsed \( \tau \)-vs-\( v \) data of Fig. 3 is well described by the scaling-law \( \tau = A(v - v_c)^{-2} \), where \( A \) and \( v_c \) are fitting parameters. For the range of \( \rho_r \) values investigated, \( v_c \) is found to be around 15 – 18 cm/sec and \( A \) is around 13,000 ± 500 cm²/sec.

The absence of any dependence of the rise-time on the size and density of the intruder implies that convection is the domi-
inherent driving mechanism. To confirm this premise, we make an independent measurement of the convection velocity of the mustard seeds without the intruder. Over a range of shaking frequencies and amplitudes, we measure the velocity of tagged mustard particles which are moving down along the two side walls. The velocity of the boundary layer particles is an order of magnitude larger than the mean velocity of the intruder. This, when plotted against $v = \omega r$ for shaking frequencies: i) 20 Hz (□), ii) 35 Hz (△) and iii) 50 Hz (○). Each linear fit (denoted by solid line) is extrapolated to estimate the peak-to-peak shaking velocity for the onset of convection. The $x$-intercepts lie in the range of 16 – 17 cm/sec. Inset: Plot of the convection velocities at the side-wall as a function of $\Gamma$.

![Graph showing convection velocities at the side walls.](image)

**FIG. 4: Convection velocities at the side walls.** The convection velocities, in the absence of the intruder, are plotted versus the peak-to-peak velocity of shaking $v = \omega r$ for shaking frequencies: i) 20 Hz (□), ii) 35 Hz (△) and iii) 50 Hz (○). Each linear fit (denoted by solid line) is extrapolated to estimate the peak-to-peak shaking velocity for the onset of convection. The $x$-intercepts lie in the range of 16 – 17 cm/sec. Inset: Plot of the convection velocities at the side-wall as a function of $\Gamma$.

Our understanding of the different parameter regimes for boundary driven convection is summarized in the phase-diagram in Fig. 5. We note that while $\Gamma > 1$ is necessary for bulk convection, it is not a sufficient condition. As seen in the phase diagram, there is a region of parameter values, with $\Gamma > 1$ and $v < v_c$, where no bulk convection occurs. In this regime, we observe some motion of the mustard seeds restricted entirely to the the upper corners of the bed. Such conditions cannot give rise to BNE for the intruder sizes studied here. We expect that for much larger intruder sizes, other mechanisms may come into play which can drive BNE. We also find that on varying the bed-height, the scaling of rise-time continues to hold for smaller heights (supplementary material) but breaks down for larger heights. The parameters $A$ and $v_c$ depend on the bed height. The microscopic origin of the parameters $A$ and $v_c$ is not clear. They depend on the bed-particle diameter and the bed-height, which are easily characterizable quantities, but also on details such as the wall, and bed-particle roughness, which are harder to characterize. Finally, we anticipate that a similar scaling should hold even in three-dimensional granular systems, in some parameter regime.

**Methods:** The experimental setup consists of a quasi-two-dimensional rectangular acrylic cell of height 25 cm, width 13 cm and gap 6 mm. The cell is filled up to a height $h$ with black mustard seeds of density 1.11 g/cc and sizes between 1.5 and 2 mm. The intruder is a large acrylic disk of diameter 1.7 cm, thickness 5 mm and density 1.07 g/cc. The thickness of the intruder is chosen such that mustard seeds are not able to enter the space between the intruder and the cell walls. The cell is mounted on a 50 kgf DESPL electrodynamic shaker executing vertical sinusoidal vibrations of the required amplitudes and frequencies. The narrow side walls of the cell are roughened by lining them with fine sandpaper (ISO grit designation P – 100). For these boundary conditions, the experiments were reproducible. However, for smooth walls it was found that the convection patterns were irreproducible and often the system would jam. The humidity of the lab was maintained at 40%.

In order to produce fixed initial conditions corresponding roughly to a fixed spatial distribution of mustard seeds, the cell was initially vibrated at a high $\Gamma$ and the intruder was pushed to a fixed depth $h_1$ (Fig. 1a). The driving was then removed and the top layer leveled. The cell was then vibrated at the required $a$ and $f$. The upward motion of the intruder was recorded using a Panasonic Lumix DMC-TZ3 digital camera at a rate of 33 frames/sec. High speed tracking of mustard-seed convection at the side walls was done using an IDT MotionPro high speed camera at a rate of 150 frames/sec. The tagged particles for this experiment were yel-
low mustard seeds of sizes and densities within the same range as the black mustard seeds. These were dispersed randomly in the bed. The tracking of the intruder (Fig. 1a) and the coloured particles near the side walls was done using a Labview based particle tracking algorithm. In order to change the density of the intruder, several acrylic intruders with central holes were made. By changing the size of the air holes or inlaying them with copper, a range of density ratios $\rho_r = 0.30 - 2.34$ (Fig. 1b) was achieved. $\rho_r = 0.3$ for acrylic with 1.4 cm air hole, 0.8 for acrylic with 1.2 cm air hole, 1.07 for solid acrylic disk and 2.34 for acrylic with 0.65 cm copper.

The rise-time of the particle was computed by estimating the time taken by the intruder to rise from a fixed starting position $\ell_3$ to a fixed final position $\ell_3$ (Fig. 1f). The mean rise-time $\tau$ was computed by averaging over 10 trials for every parameter set. For the convection velocity at the wall, the fall time of only those seeds which remained in the first boundary layer were considered and the mean velocity was obtained by averaging over many seeds.

The granular convection phenomenon was studied in greater detail by filling up the rectangular cell with alternating layers of black and yellow mustard seeds, with each layer being approximately 2 cm thick.

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FIG. 6: Intruder position vs. time plot. Position vs. time plots for an intruder of $\rho_r = 1.07$, for i) $f = 20$ Hz, $a = 4.8$ mm (green), ii) $f = 35$ Hz, $a = 2.0$ mm (red) and iii) $f = 50$ Hz, $a = 1.2$ mm (blue). The dashed horizontal lines (black) represent $h_2$ and $h_3$ (see Fig. 1a of main paper). The rise-time of the particle is computed by estimating the time it takes for the intruder to rise from the position $h_2$ to $h_3$. The vertical dashed lines in green, red and blue enclose the time windows required by the intruder to rise between $h_2$ and $h_3$ for the three shaking experiments (i), (ii) and (iii) respectively.
FIG. 7: Different realizations of intruder trajectories. Position vs. time plots for 10 different trials of the rise of a $\rho_r = 1.07$ intruder for $f = 50Hz$ and $\alpha = 1.4$ mm. The trajectories show roughly quadratic dependence on time and different repeats of the experiment for fixed control parameters lead to slightly different trajectories and correspondingly different rise-times.
FIG. 8: Plots of the rise-time for different bed-heights. Plot of $\tau$ versus $v = a\omega$ for two different bed heights $h = 8\text{cm}$ (green) and $h = 10\text{cm}$ (red), for three different frequencies $20\text{ Hz} (\square)$, $35\text{ Hz} (\triangle)$ and $50\text{ Hz} (\bigcirc)$. In all experiments, the relative density was $\rho_r = 1.07$ and the intruder size was $1.7\text{ cm}$. We see good collapse of data for both heights. The fits are to the scaling form $A/(v - v_c)^2$ with $A \approx 13000\text{ cm}^2/\text{sec}$, $v_c \approx 15\text{ cm/ sec}$ for the $10\text{ cm}$ bed and $A \approx 7000\text{ cm}^2/\text{sec}$, $v_c \approx 7\text{ cm/ sec}$ for the $8\text{ cm}$ bed.
FIG. 9: Tagged particle trajectories (y-vs-t) near wall. Each bright, slanted yellow trajectory represents the time-evolution of the y (vertical)-coordinate of a tagged particle (yellow mustard seed) adjacent to the side wall, for three different parameter sets. The slope of the yellow trajectories give the velocities of the tagged particles as (a) 4.2 cm/sec, (b) 5.0 cm/sec and (c) 4.3 cm/sec respectively. The white sinusoidal trace (horizontal) represents the time-evolution of a reference white mark on the acrylic cell.