Lepton collider probes for Majorana neutrino effective interactions

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Received: 10 January 2022 / Accepted: 17 May 2022 / Published online: 20 June 2022
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Abstract The extension of the standard model with new high-scale weakly coupled physics involving right-handed neutrinos in an effective field theory framework (SMNEFT) allows for a systematic study of heavy neutrinos phenomenology in current and future experiments. We exploit the outstanding angular resolution in future lepton colliders to study the sensitivity of forward–backward asymmetries to the effective signals. These are weakly coupled fields possibly at the electroweak (EW) scale, and their interactions with the SM particles, beyond the seesaw mixing with active neutrino states, could be described by an effective field theory (EFT) including them as low-energy degrees of freedom: the standard model effective field theory framework extended with right-handed neutrinos (SMNEFT),1 with operators known up to dimension $d = 9$ [6–10]. On the other hand, approaches like the so-called neutrino non-standard interactions (NSI) and general neutrino interactions (GNI) incorporate right-handed neutrinos within the standard model effective field theory (SMEFT) [11–14], considering their Majorana and/or Dirac nature.

The extensions of the type I seesaw renormalizable Lagrangian with effective interactions of higher dimension for the right-handed neutrinos are attracting attention, since the naive seesaw heavy $N$ production rates and decay widths can be challenged by the effective interactions, leading to a variety of signals that can be studied at the Large Hadron Collider (LHC) and other colliders [15–27]. We will focus here on a simplified scenario with only one heavy Majorana neutrino $N$, considering its interactions to be dominated by new physics at a higher energy scale and parameterized by the $d = 6$ SMNEFT operators, and we will neglect the effect of the renormalizable Yukawa Lagrangian term $N L \phi$ that gives place to the heavy-active neutrino mixings $U_{lN}$, as it is strongly constrained not only by the naive seesaw relation $U_{lN}^2 \sim m_\nu / M_N \sim 10^{-14} - 10^{-10}$ required to account for the light $\nu$ masses [28], but also by experimental constraints. Our previous works involved the Majorana $N$ decay [29,30] and surveyed its production signals in colliders for masses $m_N$ above the EW scale [31–35], focusing on lepton number-violating (LNV) processes, and in the order

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1 Introduction

The simplest standard model (SM) extensions that can account for light neutrino masses, and hence the neutrino oscillation phenomena, predict the existence of sterile right-handed neutrinos $N_R$ with Majorana mass terms, as the type I [1–5] and also the linear and inverse seesaw mechanisms. These are weakly coupled fields possibly at the electroweak (EW) scale, and their interactions with the SM particles, beyond the seesaw mixing with active neutrino states, could be described by an effective field theory (EFT) including them as low-energy degrees of freedom: the standard model effective field theory framework extended with right-handed neutrinos (SMNEFT),1 with operators known up to dimension $d = 9$ [6–10]. On the other hand, approaches like the so-called neutrino non-standard interactions (NSI) and general neutrino interactions (GNI) incorporate right-handed neutrinos within the standard model effective field theory (SMEFT) [11–14], considering their Majorana and/or Dirac nature.

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1 Also called $N_R$SMEFT and $\nu_R$SMEFT in the literature.
\[ m_N = 1 - 10 \text{ GeV scale}, \text{where its decay is dominated by the} \ N \rightarrow \nu_{\nu} \text{ channel} \ [36] \text{and can be produced in the decays of} B \text{ mesons} \ [37, \ 38]. \]

The search for EW-scale massive neutrino states \( N \) is among the goals of future \( e^+e^- \text{ colliders}, as they provide a clean environment with lower SM backgrounds compared with a hadronic machine. The literature regarding the use of lepton colliders in past, existing, and proposed experiments like the linear ILC \ [39] \text{or circular colliders like the FCC-ee} \ [40] \text{and the CEPC} \ [41] \text{to study the production of heavy neutrinos is very extensive} \ [16, \ 17, \ 42-54]. \]

In this article, we want to exploit the remarkable angular resolution that will be available for the detection of final leptons in future lepton colliders to study the sensitivity of forward-backward asymmetries to discover the possible \( N \) leptons in future lepton colliders to study the sensitivity of the dimension 5 operators, following the treatment presented in \ [5] \text{and shown in Table 1. The effective operators above can be classified by their Dirac-Lorentz structure into scalar, vectorial, and tensorial. The couplings of the tensorial operators are naturally suppressed by a loop factor } 1/(16\pi^2), \text{as they are generated at one-loop level in the ultraviolet (UV) complete theory} \ [6, \ 62]. \]

\[ \frac{1}{\Lambda d-4} \sum J \alpha J O J \]

where \( d \) is the mass dimension of the operator \( O_J \), \( \alpha_J \) are the effective couplings, and the sum in \( J \) goes over all independent interactions at a given dimension \( d \).

The authors of Ref. \ [7] \text{performed a detailed study of the phenomenology of dimension 5 SMNEFT operators. These include the Weinberg operator } \( O_W = (L\phi)(\phi^\dagger L') \ [60] \text{which contributes to the light neutrino masses, } \ O_{N=0} = (N N^c)(\phi^\dagger \phi) \text{ which gives Majorana masses and couplings of the heavy neutrinos to the Higgs (its LHC phenomenology has been studied in} \ [15, \ 17, \ 19, \ 26, \ 61]), \text{and the operator } O_{N=0} = N (\phi^\dagger) \text{ which gives magnetic moments for the heavy neutrinos, which is identically zero if we include just one sterile neutrino } N \text{ in the theory. As they do not contribute to the processes studied discarding the heavy-light neutrino mixings—we will only consider the contributions of the dimension 6 operators, following the treatment presented in} \ [6, \ 8] \text{and shown in Table 1. The effective operators above can be classified by their Dirac-Lorentz structure into scalar, vectorial, and tensorial. The couplings of the tensorial operators are naturally suppressed by a loop factor } 1/(16\pi^2), \text{as they are generated at one-loop level in the ultraviolet (UV) complete theory} \ [6, \ 62]. \]

\[ \frac{1}{\Lambda d-4} \sum J \alpha J O J \]

2 Effective interactions formalism

The SM Lagrangian is extended with only one right-handed neutrino \( \nu_R \) with a Majorana mass term, which gives a massive state \( N \) as an observable degree of freedom. The new physics effects are parameterized by a set of effective operators \( O_J \) constructed with the SM and the \( \nu_R \) fields and satisfying the \( SU(2)_L \otimes U(1)_Y \text{ gauge symmetry} \ [6, \ 8, \ 59] \text{. The effect of these operators is suppressed by inverse powers of the new physics scale } \Lambda \text{. The total Lagrangian} \text{is organized as follows:} \]

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_{d=5}^{\infty} \frac{1}{\Lambda d-4} \sum J \alpha J O_J^d \text{.} \]

\[ \text{Note that we do not include the type I seesaw Lagrangian terms giving the Majorana and Yukawa terms for the sterile neutrinos.} \]
involving tensorial interactions. However, the four-fermion operators in the third and fourth groups in Table 1 require a specific renormalizable SM extension for Madgraph5 to be able to handle them.

The operator \( O_{\mu NN1} \), leading to the second term in Eq. (2), is implemented with the mediation of an auxiliary charged vector boson \( V_0 \) with the Lagrangian terms \( \Phi^\dagger \gamma^\mu \gamma^5 \Phi N \) and \( \bar{N} \gamma^\mu \gamma^5 N \), which is integrated out numerically by setting its mass to a very high value\(^3\) and its decay width to zero. As we are interested in the possible family mixings between the lepton and quark sides, we define a 3 × 3 matrix coupling \( \alpha(i,j)_{\mu NN1} \) in flavor space, which allows us to choose between flavor-diagonal or non-diagonal interactions.

The scalar-mediated four-fermion operators \( O_{LNLI} \), \( O_{QuNL} \), and \( O_{LNQd} \) each lead to two Lagrangian terms in Eq. (2). These can be implemented using neutral (\( S_0 \), \( S_1 \), \( S_2 \)) and charged (\( S_{P0} \), \( S_{P1} \), \( S_{P2} \)) scalar auxiliary mediators, which carry a flavor index. For instance, in the case of \( O_{LNLI} \), we include the Lagrangian terms \( \gamma^\mu ( \gamma^\mu )^\dagger \) to generate the first term in the second row of Eq. (2), and the terms \( \gamma^\mu ( \gamma^\mu )^\dagger \) to generate the second. We also include for them 3 × 3 matrix couplings in flavor space \( \alpha(i,j)_{LNLI} \), \( \alpha(i,j)_{QuNL} \), and \( \alpha(i,j)_{LNQd} \) which let us choose flavor-diagonal or non-diagonal interactions.

The operator \( O_{QNld} \) explicitly mixes quarks and leptons in fermion lines. It can be implemented by the mediation of “leptoquark” fields we call \( S_{l1} \) and \( S_{l2} \), with the Lagrangian terms \( \tilde{u} \gamma^\mu \gamma^\nu \gamma^\mu \gamma^\nu \tilde{l} \) and \( \bar{c}(\gamma^\mu \gamma^\nu \gamma^\mu \gamma^\nu \bar{c} \gamma^\mu \gamma^\nu \gamma^\mu \gamma^\nu \bar{l}) \). These fields are electrically charged (\( Q(S_U) = 2/3 \), \( Q(S_D) = -1/3 \)) and carry SU(3) color indices. We make two versions for this operator: choosing to preserve the quark flavor on the four-fermion vertex, or the flavor of the fermion line. The former is the option written in Eq. (2).

3 In our numerical implementation, all the auxiliary mediators are fixed to have a mass \( M = 10^8 \) GeV and zero decay width.

The vectorial operators involving two \( N \) fields \( O_{fNN} \) are implemented with the mediation of auxiliary massive neutral vector fields \( V_f \) (this is done separately for each interaction with a fermion \( f = l, u, d, l, L \)). We write Lagrangian terms \( \tilde{N} \gamma^\mu \gamma^5 N \gamma^\mu \gamma^5 V_f \) and \( \bar{f} \gamma^\mu \gamma^5 P_R fV_f \), which are coupled to generate four-fermion vertices given by \( O_{fNN} \) in Table 1. These operators do not contribute to the processes studied in this work, or to the \( N \) decay, so we do not consider them further. For another implementation of the leptoquark and vector four-fermion interactions, see Ref. [24]. See also Ref. [22] for an implementation of the \( O_{N1\Phi} d = 6 \) operator together with seesaw mixings at next-to-leading order (NLO).

We have checked that our model implementation reproduces our previous results for the \( N \) decay width, in every channel [29,30], and also the cross sections for every 2-2 four-fermion scattering involving one \( N \) field.

2.2 Constraints on effective couplings

Recent works on right-handed effective neutrino interactions (SMNEFT), including dimension 6 operators, have derived constraints on the different effective couplings values \( \alpha_T \), or alternatively on the new physics scale \( \Lambda \). Most constraints are valid for Majorana \( m_N \) masses below the benchmark points we consider in this work: \( m_N = 50, 100, 150 \) GeV.

Reference [18] sets bounds on four-fermion effective couplings with one and two \( N \) fields, obtaining values below unity for \( \Lambda = 1 \) TeV and masses \( m_N \lesssim 1 \) GeV. Reference [20] studies the SMNEFT also including type I seesaw mixings for the \( N \) with SM light neutrinos. They study the mass regime in which the decay channel \( N \rightarrow \nu \gamma \) is dominant, for \( m_N \lesssim 10 \) GeV. More recently, in Ref. [25], the authors obtain exclusion limits for the sensitivity reach in the \( m_N - \Lambda \) plane, discarding the heavy-light mixings and using effective operators with one \( N \), and also for masses \( m_N \).
below 50 GeV, proposing a displaced-vertex search strategy for the high-luminosity LHC. Also, the authors in Ref. [24] study operators with two \( N \) fields and type I seesaw mixings, showing sensitivity reaches in the \( m_N - \Lambda \) plane for masses up to 1 TeV for fixed values of the heavy-light mixings (which drive the \( N \) decay for the considered scenarios), using displaced \( N \) decays in the LHC with ATLAS and several far detector experiments. As these operators do not contribute to the processes studied in this work, we do not restrict our effective couplings to these constraints. Our group has obtained bounds for the effective couplings, separating vector, scalar, and tensor contributions, considering \( N \)-mediated B meson decays in LHCb and Belle, which also apply for \( m_N < 5 \text{ GeV} \) [37].

In Ref. [22], the authors obtain a bound on the leptonic right-handed current \( \mathcal{O}_{\nu R}^{(2)} \) coupling for \( m_N = 100 \) (1000) GeV from CMS results [67], assuming the \( N \) only decays via seesaw mixings. It can be translated, when considering only one massive heavy neutrino state (almost coinciding with the \( \mu N \) neutrino state), into the bound \( \sigma_W^{(2)} \lesssim 2 \times 20 \) (20), for \( \Lambda = 1 \text{ TeV} \), considering the \( \mu N \) interaction. The bounds for the first fermion family \( (\alpha_W^{(1)}) \) are \( 1.5 - 2 \) times weaker. These bounds are well above the numerical values considered in our analysis.

For this work, we do take into account existing constraints [68] on the operators contributing to neutrinoless double beta decay (0\( \nu \beta \beta \) decay) following the treatment in Ref. [30], and we set the bound \( \omega_{0\nu\beta\beta} = 3.2 \times 10^{-2} \left( \frac{m_N}{100 \text{ GeV}} \right)^{1/2} \) for \( \Lambda = 1 \text{ TeV} \), for the couplings of the \( \mathcal{O}_{\nu R N\phi}^{(1)}, \mathcal{O}_{\nu N\phi}^{(1)}, \mathcal{O}_{\nu N\phi}^{(2)}, \mathcal{O}_{N\phi}^{(1)}, \mathcal{O}_{N\phi}^{(2)} \) operators. These appear as contributions to the \( \Gamma_N \) total width and to the \( N \) production vertex in the considered processes. We do not impose constraints on the other effective operators.

3 Collider analysis

We consider the single production of Majorana neutrinos \( N \) in the process \( e^+ e^- \rightarrow N \), followed by a purely leptonic decay \( N \rightarrow \mu^- \mu^+ \nu \) or a semi-leptonic decay \( N \rightarrow \mu^- j \). The single \( N \) production mode in an \( e^+ e^- \) collider in our simplified effective interactions model, depicted in Fig. 1a, is dominated by the \( \mathcal{O}_{N\phi}^{(1)} \) operator contribution, because of the strong constraint imposed on the alternative \( \mathcal{O}_{N\phi}^{(2)} \) interaction by the non-observation of the neutrinoless double beta decay, as discussed in Sect. 2.1.

We start by studying the pure leptonic process \( e^+ e^- \rightarrow \nu \mu^- \mu^+ \nu \), where the Majorana \( N \) is produced in the primary vertex together with a light neutrino, and subsequently decays into a di-muon pair and a light neutrino, as depicted in the upper diagrams in Fig. 1b. The decay of the \( N \) in vertex \((I I)\) to a di-muon pair and a light neutrino involves the contributions of the operators \( \mathcal{O}_{L_{N\phi}}^{(2)} \) and \( \mathcal{O}_{N\phi}^{(2)} \), now for the second family flavor.

We also aim to study the semi-leptonic process \( e^+ e^- \rightarrow \nu \mu^- j \), where the Majorana \( N \) decays into a muon and two jets. While the \( N \) production mechanism remains unchanged, the decay of the \( N \) in vertex \((I I)\) now involves the contributions of the vector four-fermion operator \( \mathcal{O}_{d_{N\phi}}^{(2)} \) together with the vector \( \mathcal{O}_{N\phi}^{(1)} \) and the scalars \( \mathcal{O}_{L_{N\phi}}^{(2)}, \mathcal{O}_{Q_{N\phi}}^{(2)}, \mathcal{O}_{Q_{N\phi}}^{(2)} \) and \( \mathcal{O}_{Q_{N\phi}}^{(2)} \) as shown in the lower diagrams in Fig. 1b.

We can thus explicitly take into account the following tree-level generated effective Lagrangian terms involved in those processes:

\[
\mathcal{L} = \frac{1}{\Lambda^2} \sum_{i,j} \left\{ -\alpha_W^{(i,j)} \frac{m_{\nu_i} m_{W}^2}{\sqrt{2}} \bar{\nu}_i \gamma^\mu P_R N W^- + \alpha_{V_0}^{(i,j)} \bar{\nu}_j \gamma^\mu P_R d_j \bar{\nu}_i \gamma_5 P_R N \\
+ \alpha_{S_0}^{(i,j)} \bar{\nu}_j P_R N \bar{\nu}_i P_R \{ j \bar{\nu}_i \gamma_5 P_R N \bar{\nu}_i P_R \{ j \bar{\nu}_i \gamma_5 P_R N \\
+ \alpha_{S_2}^{(i,j)} \bar{\nu}_i P_R d_j \bar{\nu}_i P_R N - \bar{\nu}_j P_R d_j \bar{\nu}_i P_R N \\
+ \alpha_{S_3}^{(i,j)} \bar{\nu}_i P_R d_i \bar{\nu}_i P_R N - \bar{\nu}_j P_R d_i \bar{\nu}_i P_R N \right\} + h.c. \right\}.
\]

(2)

The tree-level \( N \) production cross section (Fig. 1a) can be calculated as

\[
\sigma(e^+ e^- \rightarrow \nu \mu^- \mu^- \nu) = \frac{(m_N^2 - s)^2}{8\pi \Lambda^4 s} \left\{ \alpha_{S_0}^{(i,j)^2} \left( \frac{m_N^2 + 2s}{8s} \right) + \alpha_{W}^{(1)^2} \frac{m_W^2}{(s + m_W^2 - m_N^2)} \right\},
\]

(3)

while the pure leptonic and semi-leptonic tree-level \( N \) decay widths can be found in Appendix B in [30].

The irreducible SM backgrounds for both the pure leptonic \( e^+ e^- \rightarrow \nu \mu^- \mu^- \nu \) and the semi-leptonic \( e^+ e^- \rightarrow \nu \mu^- j \) processes involve diagrams with intermediate standard vector bosons (photons, \( Z \)) and Higgs bosons in \( s \) channels, which subsequently decay into muon pairs, light neutrino pairs, or quark pairs, and \( W \) bosons decaying leptonically or hadronically. The dominant SM backgrounds for both processes are events that come from \( e^+ e^- \rightarrow W^- W^+ \), with both \( W \) decaying leptonically in the first case [43, 53], and the \( W^- \) decaying hadronically in the second [47].

As we want to tackle the single \( N \) production, which brings at least one undetectable light neutrino in the final state, we cannot rely on explicitly observing lepton number violation as evidence for the Majorana \( N \) production. Given that our signals include the \( N \rightarrow \mu^- W^+ \) channel coming from the \( \mathcal{O}_{N\phi}^{(2)} \) contribution, we cannot simply rely on either the invariant mass distribution of the jet pair or the trans-
verse mass of the $\nu\mu^+$ pair (which peak at the $W$ mass for the dominant SM background) to impose cuts [47,50,53]. Rather, we will rely on the signal topology and kinematics, where depending on its energy and mass the boosted $N$ will produce its daughter particles in a collimated cone, and exploit the very good angular and energy resolution at future electron–positron colliders to construct forward–backward asymmetries between the leptons (and jet) in the final states to disentangle the $N$ contribution.

The proposed signals can be studied in future lepton colliders like the linear ILC [39] or circular colliders like the FCC-ee [69] and the CEPC [70]. For concreteness, throughout the paper we will consider an electron–positron collider with center of mass energy $\sqrt{s} = 500$ GeV and integrated luminosity $L = 500$ fb$^{-1}$ for estimating the numbers of events. These values correspond to one of the proposed ILC operation modes [71].

In order to study the prospects of measuring the single $N$ production with pure leptonic and semi-leptonic decays, we have implemented the effective interaction model in FeynRules 2.3 [63], as described in Sect. 2.1. The UFO output [64] was input into MadGraph5_aMC@NLO 2.5.5 [65,66], and we generate LHE events at the parton level, which are read by the embedded version of PYTHIA 8 [72] and then are interphased to Delphes 3.5.0 [73] with the DSiDi card [74] for a fast detector simulation. The analysis of the generated events at the reconstructed level is made with the expert mode in MadAnalysis5 1.8.58 [75].xx

For concreteness, we explore a few benchmark scenarios, and simplify the parameter space, setting all the effective couplings $\alpha, g$ in Eq. (1) and Table 1 to the same numerical value $\alpha$ (except for the operators with charged leptons of the first family constrained by the $0\nu\beta\beta$-decay bound, as explained in Sect. 2.1). We also fix the new physics scale $\Lambda = 1$ TeV and show our results for different $N$ mass values $m_N$.

We generate events by asking for the processes $e^+e^- \rightarrow \nu\mu^-\mu^+\nu$ (pure-leptonic) and $e^+e^- \rightarrow \nu\mu^-jj$ (semi-leptonic), with $\nu = \nu_e, \bar{\nu}_e, \ell = e, \mu, \tau$, and only light quark jets ($j = u, d, c, s$). These events include the signals with the production of an intermediate $N$ state (even in $t$-channels), as well as the pure-SM processes, allowing for interference (signal plus background events: $S + B$). We also generate signal-only events ($S$), by producing the heavy $N$ explicitly by $e^+e^- \rightarrow \nu N$ and asking for the leptonic $N \rightarrow \mu^-\mu^+\nu$ or semi-leptonic $N \rightarrow \mu^-jj$ decays.

We adopt the following basic acceptance cuts for both the pure-leptonic and the semi-leptonic process: we keep transverse momenta for jets $p_T^j > 20$ GeV and leptons $p_T^\ell > 10$ GeV, pseudorapidities $|\eta_j| < 5, |\eta_\ell| < 2.5$, and isolation between jets and leptons $\Delta R_{j,\ell} > 0.4$.

The number of events and cutflows for the signal-only ($S$), signal plus background ($S + B$, generated with interference), and SM background-only ($B$) datasets are shown in Table 2 for the pure-leptonic and Table 3 for the semi-leptonic processes.

In Fig. 2 we show the parton-level cross section values we obtain for both processes, generating signal plus background ($S + B$) event datasets with different values of the effective couplings $\alpha$, together with the SM background-only ($B$) values. For $m_N$ near 120 GeV, we find an enhancement of the signal cross section, due to the contribution of the $O^{ij}_{Nl\phi}$ operator to the $N$ production and decay when the $W$ is on-shell.

3.1 Leptonic channel

For the pure leptonic mode $e^+e^- \rightarrow \nu\mu^-\mu^+\nu$, with two light neutrinos in the final state, we exploit the precision

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We have checked that our SM value for the cross section $\sigma_{SM}(e^+e^- \rightarrow \nu\mu^-jj)$ agrees with [47] (using their cuts and the CEPC Delphes card).
in measurements of the muon pair momenta. As the $N$ is produced together with a light neutrino in a 2-2 process, the energy and boost of the $N$ are completely determined in the CM frame for each mass value $m_N$. Its production will be reflected in the dependence of the various observables on the summed energy of the di-muon pair $E_{\mu\mu} = E(\mu^- + \mu^+)$. A preliminary test to see the dependence of the di-muon pair separation in $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$ with $E_{\mu\mu}$ and the intermediate $N$ mass $m_N$ is shown in Fig. 3a. We calculate the $\Delta R_{\mu\mu}$ value averaged over events for each $E_{\mu\mu}$ bin. Here we generate signal-only (S) events producing the $N$ and setting its decay $N \rightarrow \mu^+\mu^- \nu$ explicitly, for some benchmark mass values and couplings $\alpha = 0.2$.

As expected, we find that for low $m_N = 50$ GeV, the muons come out roughly in the same direction, and with growing $m_N$ they are less boosted and start to separate. Also, for a fixed value of $m_N$, with increasing energy, the muons are more boosted, and $\Delta R_{\mu\mu}$ diminishes.

In order to more precisely analyze this behavior, we consider a forward–backward asymmetry $A_{\mu\mu}^{FB}$ between the flight directions of the outgoing muon and anti-muon, for different values of their summed energies $E_{\mu\mu}$. We thus define:

$$A_{\mu\mu}^{FB} = \frac{N_+ - N_-}{N_+ + N_-},$$

(4)

where $N_{\pm}$ is the number of events with a positive (negative) value of $\cos(\theta)$, the angle between the final muon and anti-muon flight directions in the Lab (or CM) frame in the $e^+e^-$ collision. We plot $A_{\mu\mu}^{FB}(E_{\mu\mu})$ for signal-only (S) events in Fig. 3b, for the same benchmark mass values and couplings, confirming the aforementioned behavior: for low $N$ mass, the muons are boosted and emerge in the same direction ($A_{\mu\mu}^{FB} = 1$) and tend to separate ($A_{\mu\mu}^{FB} < 1$) even coming out mostly in opposite directions ($A_{\mu\mu}^{FB} < 0$) when $m_N$ increases for low values of the summed energies of the outgoing muons. When their energy increases, the muon pairs tend to be very boosted, giving a positive asymmetry value.

With these hints on the signal-only kinematical behavior, we explore the possibility of using this forward–backward asymmetry to separate the events in the realistic dataset generated with the SMNeff6 UFO and allowing for effective-SM interference (S + B) from the SM-only (B) events for the pure leptonic process $e^+e^- \rightarrow \nu\mu^-\mu^+\nu$. We study different kinematical variables distributions for the process in order to separate the signals from the SM background. We apply a selection cut keeping events with a minimum value of missing transverse energy (MET > 25 GeV), accounting for neutrinos in the final state. The distribution of the signal-only (S) and background-only (B) number of events with $E_{\mu\mu}$ is shown in Fig. 4a. We find that a cut on $E_{\mu\mu} < 240$ GeV helps to separate the signals from the dominant SM background, which peaks at $E_{\mu\mu} = \sqrt{s}/2$, in a symmetric configuration where the muon–anti-muon pair shares half the energy with the unobservable light neutrinos. The corresponding number of events cutflow is shown in Table 2.

Although the signal-only (S) events seem to be able to be discovered with a sensitivity greater than 5$\sigma$ in an event-counting experiment with effective couplings $\alpha = 0.2$ applying the MET and $E_{\mu\mu}$ cuts, we want to test the possibility for discovering the signals in the full (S + B) dataset. So we take one step forward and use the asymmetry in Eq. (4) to perform a $\Delta\chi^2$ test to find the statistical significance of the separation between the signal plus background ($A_{\mu\mu}^{S+B}$) and background-only ($A_{\mu\mu}^{B}$) datasets for the asymmetry values.

We build a $\Delta\chi^2$ function as

$$\Delta\chi^2 = \frac{\sum_{E_i} \left( A_{\mu\mu}^{S+B}(E_i) - A_{\mu\mu}^{B}(E_i, m_N, \alpha) \right)^2}{\left( (\Delta A_{\mu\mu}^{S+B})^2 + (\Delta A_{\mu\mu}^{B})^2 \right)},$$

(5)
Fig. 3  Average $\Delta R_{\mu\mu}$ (a) and $A_{\mu\mu}^{FB}$ (b) distributions with $E_{\mu\mu}$, signal-only (S) no cuts

Fig. 4  Number of events (a) and $A_{\mu\mu}^{FB}$ (b) distributions with $E_{\mu\mu}$, for $\alpha = 0.2$, $\Lambda = 1$ TeV, $\sqrt{s} = 500$ GeV, $\mathcal{L} = 500$ fb$^{-1}$

Table 2 $e^+e^- \rightarrow \nu\mu^-\mu^+\nu$ (pure-leptonic). Number of events for signal-only (S), signal and SM background (S + B, generated with interference), and SM background (B) for $\sqrt{s} = 500$ GeV, $\mathcal{L} = 500$ fb$^{-1}$. Muons cut: one muon and anti-muon in final state. MET cut: select events with $\text{MET} > 25$ GeV. $E_{\mu\mu}$ cut: select events with $E_{\mu\mu} < 240$ GeV (see Fig. 4a). Signal significance: $S/\sqrt{(S + B)}$ for event-counting experiment

| $\alpha = 0.2$ | $m_N = 50$ GeV | $m_N = 100$ GeV | $m_N = 150$ GeV | SM |
|---------------|----------------|----------------|----------------|-----|
| Cuts:         | S  | S + B | S  | S + B | S  | S + B | B  |
| Muons         | 544| 33545| 1659| 34612| 901| 33860| 33060| |
| MET           | 539(99%)| 27098(81%)| 1617(97%)| 28035(81%)| 884(98%)| 27296(81%)| 26543(80%)| |
| $E_{\mu\mu}$  | 511(94%)| 7085(21%)| 1524(92%)| 8070(23%)| 734(81%)| 7351(22%)| 6771(20%)| |
| $S/\sqrt{(S + B)}$ | 6.1| 16.9| 8.6| |

where $E_i = E_{\mu\mu}$, the muon summed energy bins shown in Figs. 3 and 4. We sum in quadrature the error on the number of events for both asymmetry values in each summed energy bin, $(\Delta A_{\mu\mu}^B)^2 + (\Delta A_{\mu\mu}^{S+B})^2$, which are taken to be Poisson-distributed [34].

In Fig. 4b we show the values and error bars of the asymmetry $A_{\mu\mu}^{S+B}$ and $A_{\mu\mu}^B$ in $E_{\mu\mu}$ bins, for distinct $m_N$ values and for $\alpha = 0.2$. The label shows the value $Z$ of the number of standard deviations $Z_\sigma$ of statistical significance we obtain for each dataset with the $\Delta \chi^2$ test. The signal plus background (S + B) events dataset can be well separated from the SM background-only events (B) when $m_N = 100$ giving 12$\sigma$, and $m_N = 150$ GeV giving 7$\sigma$, but not for $m_N = 50$ GeV, due to the lower cross section of this signal,
as can be seen in Fig. 2a. We find that this lower mass value can be resolved with \( Z = 8 \) for couplings \( \alpha = 0.25 \) using the same MET and \( E_{\mu} \) cuts.

### 3.2 Semi-leptonic channel

For the semi-leptonic process \( e^+ e^- \rightarrow \nu \mu^- j j \), we aim to see the imprints of the intermediate \( N \) on its decay products by measuring the asymmetry \( A_{\mu}^{FB} \) between the flight directions of the final muon and the jet with the highest transverse momentum, defined equivalently to Eq. (4). We expect to see the dependence of the asymmetry with the energy of the muon \( E_{\mu} \) for distinct \( m_N \) and \( \alpha \) values.

As we did with the pure leptonic channel, in Fig. 5a we plot the distribution of the average value of the separation \( \Delta R_{\mu j} \) between the muon and the most energetic jet in each muon energy bin \( E_{\mu} \) for signal-only events, before applying any cuts. We find that the muon and jet separation depends mostly on the mass of the intermediate Majorana neutrino \( m_N \): for low mass, the products of the decay \( N \rightarrow \mu^- j j \) are more boosted and emerge in a narrower cone. This effect increases slightly with higher muon energy, and in fact we do not have events in the higher \( E_{\mu} \) bins for the \( m_N = 50 \text{ GeV} \) signal. In the \( m_N = 150 \text{ GeV} \) signal events, the muon and the jet are more separated. In Fig. 5b we plot the asymmetry \( A_{\mu}^{FB} \) for signal-only events before cuts. Consistently, the events with a lighter \( N \) produce a maximal value \( A_{\mu}^{FB} = 1 \), showing that the muon and jet emerge in the same direction. This changes for the higher \( m_N \) values. Here, the \( N \) is less boosted, and the muon and jet emerge in an increasingly open angle for higher muon energies. This can be seen by considering that the more energetic muons tend to emerge in the direction of the \( N \), and the jets go in the opposite direction (they mostly come from an on-shell \( W \); see the right-bottom diagram of Fig. 1b).

The behavior is different for the SM backgrounds. As we mentioned, the dominant SM contribution to the semi-leptonic process comes from events with pair production of \( W \) bosons \( e^+ e^- \rightarrow W^- W^+ \) with \( W^- \rightarrow \nu \mu^- \) and \( W^+ \rightarrow j j \). Here, the prompt \( W^- \) and \( W^+ \) are produced back to back. This gives us muons and jets emerging in opposite directions, and thus \( A_{\mu}^{FB} \sim -1 \) for the dominant SM background contribution, in events with highly energetic muons, as can be seen in the SM background-only (B) events in Fig. 6b.

We explored the distribution of the generated signal (S) and background (B) events with several kinematic variables, and select events with missing transverse energy (MET) greater than 25 GeV to account for the final-state neutrino. We also find that the transverse mass of the final muon-neutrino system, calculated in our reconstructed level data as the transverse mass (MT) of the missing transverse energy (MET)-muon system, MT-MET(\( \mu \))\(^5\), can help to separate the signal from the SM background, which peaks at the \( m_W \) value, reflecting the fact that the muon-neutrino pair comes from a \( W^- \). In Fig. 6a we show the number of events distribution with the MT-MET(\( \mu \)) variable for the signals (S) and the SM background (B). We find that a cut selecting events with MT-MET(\( \mu \)) > 85 GeV keeps more than 70% of the events for the three \( m_N \) signal datasets, while keeping 9% of the SM (B) events. The cutflow is shown in Table 3.

After applying these cuts, we plot the distribution of the asymmetry \( A_{\mu}^{FB} \) for muon energy bins \( E_{\mu} \) in Fig. 6b. The cut applied in MT-MET(\( \mu \)) reduces the number of the dominant SM background events in the lower \( E_{\mu} \) bins, giving a less negative value for \( A_{\mu}^{FB} \) for the background-only dataset. We

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\(^5\) The transverse mass variable of the muon-missing transverse energy system is defined as MT-MET(\( \mu \))= \( \sqrt{2p_T^\mu p_T^{miss}(1 - \cos(\Delta \phi(p_T^\mu, p_T^{miss}))}) \); we use the name given in MadAnalysis5.
Fig. 6 Number of events distribution with MT-MET(μ) variable (a) and $A_{FB}^{μ}$ distribution with $E_μ$ for signal plus background (S + B) and SM background-only (B) events for $α = 0.2$, $Λ = 1$ TeV, $\sqrt{s} = 500$ GeV, $L = 500$ fb$^{-1}$.

Table 3 $e^+ e^- \rightarrow νμ^- jj$ (semi-leptonic). Number of events for signal-only (S), signal and SM background (S + B, generated with interference), and SM background (B) for $\sqrt{s} = 500$ GeV, $L = 500$ fb$^{-1}$.

| Cuts:         | $m_N = 50$ GeV | $m_N = 100$ GeV | $m_N = 150$ GeV | $m_N = 50$ GeV | $m_N = 100$ GeV | $m_N = 150$ GeV |
|---------------|---------------|-----------------|-----------------|---------------|-----------------|-----------------|
| Muon          | 223           | 134944          | 2027            | 136584        | 2062            | 136799          |
| MET           | 223 (100%)    | 96046 (71%)     | 2005 (99%)      | 97615 (71%)   | 2048 (99%)      | 98189 (72%)     |
| MT-MET        | 161 (72%)     | 12140 (9%)      | 1627 (80%)      | 13779 (10%)   | 1755 (85%)      | 14058 (10%)     |
| $S/\sqrt{(S+B)}$ | 1.46         | 13.86           | 14.8            |               |                 |                 |

Also see that the datasets with signal and background events (S + B) for high $N$ mass values (with more signal-only events than the $m_N = 50$ GeV dataset) give even positive $A_{FB}^{μ}$ values for events with low-energy muons, as expected from Fig. 5b.

We find the generated signal plus background (S + B) datasets for $m_N = 100$ GeV and $m_N = 150$ GeV with couplings $α = 0.2$ can be separated from the SM background-only (B) dataset with a statistical significance of $6σ$ and $12σ$, respectively. The dataset with $m_N = 50$ GeV cannot be separated with the use of the asymmetry, again due to the lower cross section for this signal, as found in Fig. 2b.

4 Summary

The effective field theory extending the standard model with sterile right-handed neutrinos $NR$ (SMNEFT) parametrizes new high-scale weakly coupled physics in a model-independent manner, allowing for a systematic study of their phenomenology in current and future experiments. We consider massive Majorana neutrinos coupled to ordinary matter by dimension 6 effective operators, focusing on a simplified scenario with only one right-handed neutrino added, which provides us with a manageable parameter space to probe. We implement the effective interactions, which include four-fermion operators with the Majorana $N$ field in FeynRules, and make them suitable for simulation with MadGraph5, introducing renormalizable UV interactions with mediators which are integrated out numerically in order to reproduce the low-energy SMNEFT behavior.

We exploit the remarkable angular resolution in future lepton colliders to study the sensitivity of forward–backward asymmetries to discover the possible single production of heavy Majorana neutrinos via $e^+ e^- \rightarrow N ν$, followed by a purely leptonic decay $N \rightarrow μ^- μ^+ ν$ or a semi-leptonic decay $N \rightarrow μ^- jj$, for masses $m_N > 50$ GeV. In this regime, the $N$ production and decays are dominated by scalar and vectorial four-fermion single $NR$ operators $O_{LNLl}$, $O_{dLNl}$, $O_{QuNL}$, $O_{LNQd}$, and $O_{QNld}$, as well as the vectorial bosonic $O_{NΦ}$. This is an alternative analysis to searches using displaced vertices and fat jets, in a higher mass regime, where the $N$ is short-lived but can be found by the angular distribution of its decay products, which depending on their energy can be more or less collimated, leaving a clear imprint on forward–backward asymmetries between them.
By performing a dedicated Monte Carlo simulation with MadGraph5 for the ILC with center of mass energy $\sqrt{s} = 500$ GeV and integrated luminosity $L = 500$ fb$^{-1}$, and implementing a novel analysis with MadAnalysis5 using the expert mode, we have shown that a forward-backward asymmetry between the final muon separation in the pure leptonic decay mode allows us to detect the signal plus background (generated with interference) over the SM background-only events with a sensitivity up to 12σ for $m_N = 100$ GeV, for effective couplings $\alpha = 0.2$ and new physics scale $\Lambda = 1$ TeV. In the case of the semileptonic decay, we can compare the final muon and higher $p_T$ jet flight directions and find the highest sensitivity for the $m_N = 150$ GeV signal dataset, again with values around 12σ.

In summary, we conclude that the use of forward-backward asymmetries in future lepton colliders can probe heavy neutrino new physics in final states with light neutrinos which challenge simpler analyses, with a sensitivity that will eventually allow us to put bounds on four-fermion effective operators for a higher $m_N$ mass regime, complementary to proposed displaced vertices searches.

Acknowledgements We thank CONICET (Argentina) and PEDECIBA, CSIC, and ANII, under grant PR-FCE-3-2018-148577 (Uruguay), for their financial support.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The simulations that support the present findings are available from the corresponding author upon reasonable request.]

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Funded by SCOAP3.

References

1. P. Minkowski, $\mu \to e\gamma$ at a rate of one out of 1-billion muon decays? Phys. Lett. B 67, 421 (1977). https://doi.org/10.1016/0370-2693(77)90435-X

2. R.N. Mohapatra, G. Senjanovic, Neutrino mass and spontaneous parity violation. Phys. Rev. Lett. 44, 912 (1980). https://doi.org/10.1103/PhysRevLett.44.912

3. T. Yanagida, Horizontal symmetry and masses of neutrinos. Prog. Theor. Phys. 64, 1103 (1980). https://doi.org/10.1143/PTP.64.1103

4. M. Gell-Mann, P. Ramond, R. Slansky, Complex spinors and unified theories. Conf. Proc. C790927, 315 (1979). arXiv:1306.4669 [hep-th]

5. J. Schechter, J.W.F. Valle, Neutrino masses in SU(2) x U(1) theories. Phys. Rev. D 22, 2227 (1980). https://doi.org/10.1103/PhysRevD.22.2227

6. F. del Aguila, S. Bar-Shalom, A. Soni, J. Wudka, Heavy Majorana neutrinos in the effective Lagrangian description: application to hadron colliders. Phys. Lett. B 670, 399 (2009). https://doi.org/10.1016/j.physletb.2008.11.031. arXiv:0806.0876 [hep-ph]

7. A. Aparici, K. Kim, A. Santamaria, J. Wudka, Right-handed neutrino magnetic moments. Phys. Rev. D 80, 013010 (2009). https://doi.org/10.1103/PhysRevD.80.013010. arXiv:0904.3244 [hep-ph]

8. Y. Liao, X.-D. Ma, Operators up to dimension seven in standard model effective field theory extended with sterile neutrinos. Phys. Rev. D 96, 015012 (2017). https://doi.org/10.1103/PhysRevD.96.015012. arXiv:1612.04527 [hep-ph]

9. S. Bhattacharya, J. Wudka, Dimension-seven operators in the standard model with right handed neutrinos. Phys. Rev. D 94, 055022 (2016). https://doi.org/10.1103/PhysRevD.94.055022. arXiv:1505.05264 [hep-ph] [Erratum: Phys. Rev. D 95(3), 039904 (2017). https://doi.org/10.1103/PhysRevD.95.039904]

10. H.-L. Li, Z. Ren, M.-L. Xiao, J.-H. Yu, Y.-H. Zheng, Operator bases in effective field theories with sterile neutrinos: d ≤ 9. JHEP 11, 003 (2021). https://doi.org/10.1007/JHEP11(2021)003. arXiv:2105.09329 [hep-ph]

11. I. Bischer, W. Rodejohann, General neutrino interactions from an effective field theory perspective. Nucl. Phys. B 947, 114746 (2019). https://doi.org/10.1016/j.nuclphysb.2019.114746. arXiv:1905.08699 [hep-ph]

12. T. Han, J. Liao, H. Liu, D. Marfatia, Scalar and tensor neutrino interactions. JHEP 07, 027 (2020). https://doi.org/10.1007/JHEP07(2020)027. arXiv:2004.13869 [hep-ph]

13. J. Terol-Calvo, M. Törölt, A. Vicente, High-energy constraints from low-energy neutrino nonstandard interactions. Phys. Rev. D 101, 095010 (2020). https://doi.org/10.1103/PhysRevD.101.095010. arXiv:1912.09131 [hep-ph]

14. F.J. Escríhuela, L.J. Flores, O.G. Miranda, J. Rendón, Global constraints on neutral-current generalized neutrino interactions. JHEP 07, 061 (2021). https://doi.org/10.1007/JHEP07(2021)061. arXiv:2105.06484 [hep-ph]

15. A. Caputo, P. Hernandez, J. Lopez-Pavon, J. Salvado, The seesaw portal in testable models of neutrino masses. JHEP 06, 112 (2017). https://doi.org/10.1007/JHEP06(2017)112. arXiv:1704.08721 [hep-ph]

16. D. Barducci, E. Bertuzzo, The see-saw portal at future Higgs factories: the role of dimension six operators. J. High Energ. Phys. 2022, 77 (2022). https://doi.org/10.1007/JHEP06(2022)077. arXiv:2201.11754 [hep-ph]

17. D. Barducci, E. Bertuzzo, A. Caputo, P. Hernandez, B. Mele, The see-saw portal at future Higgs factories. JHEP 03, 117 (2021). https://doi.org/10.1007/JHEP03(2021)117. arXiv:2011.04725 [hep-ph]

18. J. Alcaide, S. Banerjee, M. Chala, A. Titov, Probes of the Standard Model effective field theory extended with a right-handed neutrino. JHEP 08, 031 (2019). https://doi.org/10.1007/JHEP08(2019)031. arXiv:1905.11375 [hep-ph]

19. J.M. Butterworth, M. Chala, C. Englert, M. Spannowsky, A. Titov, Higgs phenomenology as a probe of sterile neutrinos. Phys. Rev. D 100, 115019 (2019). https://doi.org/10.1103/PhysRevD.100.115019. arXiv:1909.04665 [hep-ph]

20. A. Biekötter, M. Chala, M. Spannowsky, The effective field theory of low scale see-saw at colliders. Eur. Phys. J. C 80, 743 (2020). https://doi.org/10.1140/s10052-020-8339-2. arXiv:2007.00673 [hep-ph]

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57. A. Hofer, L. Sehgal, Pair production and correlated decay of heavy Majorana neutrinos in e+ e− collisions. Phys. Rev. D 54, 1944 (1996). https://doi.org/10.1103PhysRevD.54.1944. arXiv:hep-ph/9603240
58. G. Cvetic, C. Kim, C. Kim, Heavy Majorana neutrinos at e+ e− colliders. Phys. Rev. Lett. 82, 4761 (1999). https://doi.org/10.1103PhysRevLett.82.4761. arXiv:hep-ph/9901255
59. J. Wudka, A short course in effective Lagrangians. AIP Conf. Proc. 531, 81 (2000). https://doi.org/10.1063/1.1315034. arXiv:hep-ph/0002180
60. S. Weinberg, Baryon and lepton nonconserving processes. Phys. Rev. Lett. 43, 1566 (1979). https://doi.org/10.1103PhysRevLett.43.1566
61. M.L. Graesser, Broadening the Higgs boson with right-handed neutrinos and a higher dimension operator at the electroweak scale. Phys. Rev. D 76, 075006 (2007). https://doi.org/10.1103PhysRevD.76.075006. arXiv:0704.0438 [hep-ph]
62. C. Arzt, M. Einhorn, J. Wudka, Patterns of deviation from the standard model. Nucl. Phys. B 433, 41 (1995). https://doi.org/10.1016S0550-3213(94)00336-D. arXiv:hep-ph/9405214
63. A. Alloul, N.D. Christensen, C. Degrande, C. Duhr, B. Fuks, FeynRules 2.0—a complete toolbox for tree-level phenomenology. Comput. Phys. Commun. 185, 2250 (2014). https://doi.org/10.1016S0010-4655(14)00012-0arXiv:1310.1921 [hep-ph]
64. C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer, T. Reiter, UFO—the universal FeynRules output. Comput. Phys. Commun. 183, 1201 (2012). https://doi.org/10.1016S0010-4655(12)00122-arXiv:1108.2040 [hep-ph]
65. J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H.S. Shao, T. Stelzer, P. Torrielli, M. Zaro, The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations. JHEP 07, 079 (2014). https://doi.org/10.1007JHEP07(2014)079. arXiv:1405.0301 [hep-ph]
66. J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer, T. Stelzer, MadGraph 5: going beyond. JHEP 06, 128 (2011). https://doi.org/10.1007JHEP06(2011)128. arXiv:1106.0522 [hep-ph]
67. A.M. Sirunyan et al. (CMS), Search for heavy neutral leptons in events with three charged leptons in proton–proton collisions at $\sqrt{s} = 13$ TeV. Phys. Rev. Lett. 120, 221801 (2018). https://doi.org/10.1103PhysRevLett.120.221801. arXiv:1802.02965 [hep-ex]