Proton decay and new contribution to $0\nu2\beta$ decay in SO(10) with low-mass $Z'$ boson, observable $n - \bar{n}$ oscillation, lepton flavor violation, and rare kaon decay

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**Abstract:** In the conventional approach to observable $n - \bar{n}$ oscillation through Pati-Salam intermediate gauge symmetry in $SO(10)$, the canonical seesaw mechanism is also constrained by the symmetry breaking scale $M_R \sim M_C \leq 10^6$ GeV which yields light neutrino masses several orders larger than the neutrino oscillation data. Recently this difficulty has been evaded through TeV scale gauged inverse seesaw mechanism while predicting experimentally verifiable $W_R^\pm Z_R$ bosons with a new dominant contribution to $W_L - W_L$ mediated neutrinoless double beta decay and other observable phenomena, but with proton lifetime far beyond the accessible limits in foreseeable future. In the present work, adopting the view that we may have only a nonstandard TeV scale $Z'$ gauge boson accessible to the Large Hadron Collider while $W_R^\pm$ may be heavy and currently inaccessible, we show how a class of non-supersymmetric $SO(10)$ models allow experimentally verifiable proton lifetime and the new contributions to neutrinoless double beta decay in the $W_L - W_L$ channel, lepton flavor violating branching ratios, observable $n - \bar{n}$ oscillation, and lepto-quark gauge boson mediated rare kaon decays. The occurrence of Pati-Salam gauge symmetry with unbroken D-parity and two gauge couplings at the highest intermediate scale guarantees precision unification in such models. This symmetry also ensures vanishing GUT threshold uncertainy on $\sin^2 \theta_W$ or on the highest intermediate scale. Although the proton lifetime prediction is brought closer to the ongoing search limits with GUT threshold effects in the minimal model, no such threshold effects are needed in a non-minimal model even for lifetimes close to the Super-Kamiokande limit. We derive a new analytic expression for the $0\nu\beta\beta$ decay- half-life and show how the existing experimental limits impose the lower bound on the lightest of the three heavy sterile neutrino masses, $M_{S1} \geq 14 \pm 4$ GeV, irrespective of the nature of hierarchy of light neutrino masses. We also derive a new lower bound on the lepto-quark gauge boson mass mediating rare kaon decay, $M_{\text{lepto}} \geq (1.53 \pm 0.06) \times 10^6$ GeV, which is easily accommodated in the model. The $n - \bar{n}$ mixing times are predicted in the range $\tau_{n-\bar{n}} \simeq 10^8 - 10^{13}$ sec covering those accessible to ongoing experiments.

**Keywords:** Grand Unified Theory, Proton decay, Neutron-antineutron oscillation, Rare kaon decays, Neutrinoless double beta decay and Lepton number violation
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1 INTRODUCTION

Although the standard model (SM) of strong, weak, and electromagnetic interactions has unraveled the gauge origin of fundamental forces and the structure of the universe while successfully confronting numerous experimental tests, it has a number of limitations. Experimental evidences of tiny neutrino masses compared to their charged lepton counterparts also raises the fundamental issue on the origin of these masses as well as the nature of neutrinos: whether Dirac or Majorana - a question whose answer rests on the detection and confirmation of $0\nu2\beta$ decay process on which there are a number of ongoing experiments. The SM predicts lepton flavor violating (LFV) decays many orders smaller than the current experimental limits which appear to be compatible via supersymmetric theories. Thus, in the absence of supersymmetry so far, it is important to explore non-supersymmetric (non-SUSY) extensions of the SM with sizable LFV decay branching ratios.

Several limitations of the SM are removed when it is embedded in a popular grand unified theory (GUT) like $SO(10)$ which has potentialities to achieve precision unification of the three forces, accommodate tiny neutrino masses through various seesaw mechanisms, provide spontaneous origins of Parity and CP-violations, and a host of other interesting physical phenomena. Even without any additional flavor symmetry, the model succeeds in representing all fermion masses of three generations while observable baryon number violating processes are generic among its predictions of new physics beyond the SM. Apart from proton decay, theoretical models have been proposed for experimentally observable signature of the other baryon number violating process such as $n - \bar{n}$ and $H - \bar{H}$ oscillations, and double proton decay through GUTs out of which $n - \bar{n}$ oscillation has attracted considerable interest. While in most of the conventional models, the intermediate breaking of Pati-Salam gauge symmetry $SU(2)_L \times SU(2)_R \times SU(4)_C(\equiv G_{224})$ has been exploited at $\mu = M_C \sim 10^6 \text{ GeV}$, in a very interesting recent development it has been proposed to realize the process even if the symmetry breaks near the GUT scale such that the SM gauge symmetry rules all the way down to the electroweak scale. In this model the diquark higgs mediating the oscillation process are tuned to have masses at the desired intermediate scale or lower. The model also explains the observed baryon asymmetry of the universe by a novel mechanism. Prediction of $|\Delta(B - L)| \neq 0$ proton decay mode is another special attractive feature of this model. The neutrino masses and mixings in these models are governed by high scale canonical or type-II seesaw mechanisms with $B - L$ gauge symmetry breaking occurring near the GUT-scale.

We consider worth while to pursue the conventional approach mainly because we are interested in associating all relevant physical mass scales with the spontaneous breakings of respective intermediate gauge symmetries. A non-standard extra $Z'$ boson which is under experimental investigation at LHC energies is possible by extension of the SM. If such an extension is through Pati-Salam symmetry at higher scale, the model has the interesting possibility of accommodating observable $n - \bar{n}$ oscillation and rare kaon decays. If Pati-Salam symmetry in turn emerges from a GUT scenario like $SO(10)$, it provides interesting possibilities of gauge coupling unification and GUT-scale representation
of all charged fermion masses with the prediction of Dirac neutrino mass matrix. If one fermion singlet per generation is added to the SO(10) framework, it has the interesting possibility of explaining light neutrino masses and mixings by experimentally verifiable gauged inverse seesaw mechanism. Whereas the non-supersymmetric SM as such predicts negligible contributions to charged lepton flavor violating (LFV) decays, the TeV scale inverse seesaw mechanism predicts LFV branching ratios only $4 - 5$ order smaller than the current experimental limits. Embedding such a mechanism through heavier right-handed neutrinos provides further interesting realisation of additional new dominant contributions to neutrino-less double-beta decay in the $W_L - W_L$ channel through the exchanges of sterile neutrinos which turn out to be Majorana fermions in the model.

In this work we attempt to revive the conventional approach [31–34] but by evading the light neutrino mass constraint through inverse seesaw formula gauged by the TeV scale symmetry $SU(2)_L \times U(1) \times U(1)_{B-L} \times SU(3)_C$ manifesting in an extra $Z'$ boson which might be detected by ongoing search experiments at the Large Hadron Collider, a strategy which has been adopted recently in non-supersymmetric (non-SUSY) SO(10) grand unified theory (GUT) [36, 55]. Low energy signature of lepto-quark gauge bosons is also predicted through rare kaon decay $K_L \rightarrow \mu \bar{\nu}$ with branching ratios close to the current experimental limit [35]. Once the experimentally testable gauged inverse seesaw mechanism is made operative, the model is found to predict a number of new physical quantities to be verified by ongoing search experiments at low and accelerator energies. They include (i) dominant contribution to $0\nu\beta\beta$ rate in the $W_L - W_L$ channel due to heavy steriele neutrino exchanges leading to the lower bound on the lightest sterile neutrino mass $M_{S1} \geq 14 \pm 4$ GeV, (ii) unitarity-violating contributions to branching ratios for lepton flavor violating (LFV) decays, (iii) leptonic CP-violation due to non-unitarity effects, (iv) experimentally verifiable $|\Delta(B-L)| = 0$ proton decay modes such as $p \rightarrow e^+\pi^0$ (v) lepto-quark gauge-boson mediated rare kaon decay with $Br.(K_L \rightarrow \mu \bar{\nu}) \approx 10^{-12}$, and (vi) observable $n - \bar{n}$-oscillation mixing time $10^8 - 10^{13}$ sec with the possibility of a diquark Higgs scalar at the TeV scale.

The quark-lepton symmetric origin of the Dirac neutrino mass matrix ($M_D$) is found to play a crucial role in enhancing the effective mass parameter for $0\nu\beta\beta$ decay. We also briefly discuss how a constrained (unconstrained) value of the RH neutrino mass matrix emerges from the SO(10) structure with one 126 (two 126’s) from the GUT-scale fit to charged fermion masses.

Although some of the results of the present work were also derived in a recent work [36], the model required the asymmetric left-right gauge symmetry at $\simeq 10$ TeV leading to the prediction of $W_R^\pm, Z_R$ gauge bosons at LHC energies. In the present SO(10) model, we show that even though only a TeV scale $Z'$ boson [37] is detected at the LHC, a number of these observable predictions are still applicable even if the $W_R^\pm$ boson masses are beyond the currently accessible LHC limit. In contrast to the earlier model, in the present work we predict proton lifetime to be accessible to ongoing search experiments. The symmetry breaking chain of the model is found to require $SU(2)_L \times SU(2)_R \times SU(4)_C \times D(g_{2L} = g_{2R}) = g_{224D}$ gauge symmetry at the highest intermediate scale ($M_P$) which eliminates the possible presence of triangular geometry of gauge couplings around the GUT scale. This in turn determines the unification mass precisely, at the meeting point of two gauge
coupling constant lines. The other advantage of this symmetry is that it pushes most of the larger-sized submultiplets down to the parity restoring intermediate scale reducing the size of GUT-threshold effects on the unification scale and proton lifetime while the GUT-threshold effects on $\sin^2 \theta_W$ or $M_P$ have exactly vanishing contribution [41, 42, 45].

We derive a new formula for half-life of $0\nu\beta\beta$ decay as function of three heavy sterile neutrino masses and provide a new plot of the half-life against the lightest of the three masses. From the existing experimental values of life-time [6–9], we derive the lower limit on the sterile neutrino mass of the first generation $M_{S1} > 14 \pm 4$ GeV which is also experimentally verifiable. This result is found to hold irrespective of the nature of hierarchy of light neutrino masses. We find that the saturation of half-life obtained by Heidelberg-Moscow experiment does not necessarily require light neutrinos to be quasi-degenerate. These results on the half-life prediction on $0\nu\beta\beta$ decay and the derived bound on $M_{S1}$ is also applicable to the model of ref.[36].

This paper is organized in the following manner. In Sec. 2 we discuss the specific $SO(10)$ symmetry breaking chain and study predictions of different physically relevant mass scales emerging as solutions to renormalization group equations. In Sec. 3 we discuss predictions of proton lifetime accessible to ongoing search experiments. Lower bound on the lepto-quark gauge boson mediating rare-kaon decay is derived in Sec.4 where mixing times for $n-\bar{n}$ oscillation are also predicted. In Sec. 5 we discuss the derivation of Dirac neutrino mass matrix from GUT-scale fit to the charged fermion masses where, in a minimal $SO(10)$ structure, we also show how the model predicts the RH neutrino mass eigenvalues which can be detected at the LHC. Fits to the neutrino oscillation data are discussed in Sec. 6. In Sec. 7 we discuss the model estimations of LFV decay branching ratios and CP-violating parameter due to non-unitarity effects. In Sec. 8 we obtain the model estimations on the dominant contributions to $0\nu2\beta$ process and study variation of half-life as a function of sterile neutrino masses. The paper is summarized with conclusions in Sec. 9. In the Appendix we derive analytic formulas for GUT threshold effects on $\ln(M_P/M_Z)$ and $\ln(M_U/M_Z)$.

2 PRECISION GAUGE COUPLING UNIFICATION AND MASS SCALES

In the conventional approach to investigation of gauge coupling unification, usually the semi simple gauge symmetry to which the GUT gauge theory breaks is a product of three or four individual groups. As a result the symmetry below the GUT-breaking scale involves three or four gauge couplings. The most popular of such examples is $SO(10) \to G_I$ where $G_I = SU(2)_L \times U(1)_Y \times SU(3)_C$ which has three different gauge couplings, $g_Y, g_{2L}$ and $g_{3C}$, whose renormalization group (RG) evolution creates a triangular region around the projected unification scale making the determination of the scale more or less uncertain. Even though the region of uncertainty is reduced in the presence of intermediate scales, it exists in principle when, for example, $G_I = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$, that included three or four gauge couplings. Only in the case when $G_I = SU(2)_L \times SU(2)_R \times SU(4)_C \times D$, the Pati-Salam symmetry with LR discrete symmetry [19] ($\equiv$ D-Parity) [31, 32], there are two gauge couplings $g_{2L} = g_{2R}$ and $g_{4C}$, and the meeting point of the two RG-evolved
coupling lines determines the unification point exactly. Several interesting consequences of this intermediate symmetry have been derived earlier including vanishing corrections to GUT-threshold effects on $\sin^2 \theta_W$ and the intermediate scale \cite{41, 42, 45, 46}. We find this symmetry to be essentially required at the highest intermediate scale in the present model to guarantee several observable phenomena as $SO(10)$ model predictions while safeguarding precision unification.

We consider the symmetry breaking chain of non-SUSY $SO(10)$ GUT which gives a rich structure of new physics beyond the SM provided the Pati-Salam symmetry occurs as an intermediate symmetry twice: once between the high parity breaking scale ($M_P$) and the GUT scale ($M_{GUT}$) and, for the second time, without parity between the $SU(4)_C$ breaking scale ($M_C$) and $M_P$

$$SO(10) \xrightarrow{M_T} SU(2)_L \times SU(2)_R \times SU(4)_C \times D \quad [G_{224D}, g_{2L} = g_{2R}]$$

$$\xrightarrow{M_T} SU(2)_L \times SU(2)_R \times SU(4)_C \quad [G_{224}, g_{2L} \neq g_{2R}]$$

$$\xrightarrow{M_C} SU(2)_L \times U(1)_R \times U(1)_{(B-L)} \times SU(3)_C \quad [G_{2113}]$$

$$\xrightarrow{m_W} SU(2)_L \times U(1)_Y \times SU(3)_C \quad [G_{SM}]$$

The first step of spontaneous symmetry breaking is implemented by giving GUT scale VEV to the D-Parity even Pati-Salam singlet contained in $54_H \subset SO(10)$ leading to the left-right symmetric gauge group $G_{224D}$ with the equality in the corresponding gauge couplings $g_{2L} = g_{2R}$. The second step of breaking occurs by assigning Parity breaking VEV to the D-Parity odd singlet $\eta(1,1,1) \subset 210_H$ \cite{31, 32} resulting in the LR asymmetric gauge theory $G_{224}(g_{2L} \neq g_{2R})$. The third step of breaking to gauge symmetry $G_{2113}$ is implemented by assigning VEV of order $M_C \sim 10^5 - 10^6 \text{ GeV}$ to the neutral component of the $G_{224}$ sub-multiplet $(1,3,15) \subset 210_H$. This technique of symmetry breaking to examine the feasibility of observable $n - \bar{n}$ through the type of intermediate breaking $G_{224} \rightarrow G_{2113}$ was proposed at a time when neither the neutrino oscillation data, nor the precision CERN-LEP data were available \cite{32–34}. The gauge symmetry $G_{2113}$ that is found to survive down to the TeV scale is broken to the SM by the sub-multiplet $\Delta_R(3,1,10) \subset 126_H$ leading to the low-mass extra $Z'$ boson accessible to LHC. At this stage RH Majorana mass matrix $M_N = f \langle \Delta_R^0 \rangle$ is generated through the Higgs Yukawa interaction. The last step of breaking occurs as usual through the VEV of the SM doublet contained in the sub-multiplet $\phi(2,2,1) \subset 10_H$. The VEV of the neutral component of RH Higgs doublet $\xi_R(1,2,4)$ under $G_{224}$ symmetry contained in $16_H$ ($SO(10)$) is used to generate the $N - S$ mixing mass term needed for extended seesaw mechanism. For the sake of fermion mass fit at the GUT, scale we utilize two Higgs doublets for $\mu \geq 5 \text{ TeV}$. Out of these two, the up type doublet $\phi_u \subset 10_{H_1}$ contributes to Dirac masses for up quarks and neutrinos, and the down type doublet $\phi_d \subset 10_{H_2}$ contributes to masses of down type quarks and charged leptons. We will see later in this work how the induced VEV of the sub-multiplet $\xi(2,2,15) \subset 126_H$ \cite{36, 67} naturally available in this model plays a crucial role in splitting quark and lepton masses at the GUT scale and determining the value of $M_D$. In one interesting scenario, the GUT
scale fit to fermion masses and mixings results in the diagonal structure of RH neutrino mass matrix near the TeV scale which is accessible for verification at LHC energies.

Using extended survival hypothesis [23, 25] the Higgs scalars responsible for spontaneous symmetry breaking and their contributions to $\beta-$ function coefficients up to two-loop order are given in Tab.5. One set of allowed solutions for mass scales and GUT-scale fine-structure constant is

$$
M_0^R = 5 \text{ TeV}, M_\Delta = M_C = 10^{5.5} - 10^{6.5} \text{ GeV}, M_P = 10^{13.45} \text{ GeV},
$$

where $M_\Delta$ represents the degenerate mass of diquark Higgs scalars contained in $\Delta_R(1, 3, \bar{10}) \subset 126_H$.

The renormalization group evolution of gauge couplings is shown in Fig.1 exhibiting precision unification.

![Figure 1](image)

**Figure 1.** gauge coupling unification including $\xi(2, 2, 15)$

We have noted that when $M_\Delta < M_C$, there is a small decrease in the unification scale that is capable of reducing the proton lifetime predictions by a factor $3 - 5$. One example of this solution is,

$$
M_0^R = 5 \text{ TeV}, M_\Delta = 10^4 \text{ GeV}, M_C = 10^6 \text{ GeV}, M_P = 10^{12.75} \text{ GeV},
$$

It is interesting to note that the present LHC bound on the diquark Higgs scalar mass [68] is

$$
(M_\Delta)_{\text{exp.}} \geq 3.75 \text{ TeV}.
$$

(2.3)
As discussed in the context of \( n - \bar{n} \) oscillation in Sec. 4, our model accommodates a TeV scale diquark with observable mixing time. But substantial decrease in the unification scale and the corresponding decrease in proton lifetime is possible when the bi-triplet Higgs scalar \( \Theta_H(3,3,1) \subset 54_H \) is lighter than the GUT scale by a factor ranging from \( \frac{1}{15} \) to \( \frac{1}{25} \). These solutions are discussed in the following section.

3 LOW MASS \( Z' \) AND PROTON DECAY

3.1 Low-mass \( Z' \) boson

In the solutions of RGEs with precision unification, we have found that \( g(B-L) = 0.72 - 0.75 \) and \( g_{1R} = 0.40 - 0.42 \) in the range of values \( M_R^0 = v_{B-L} = 3 - 10 \text{ TeV} \). This predicts the mass of the \( Z' \) boson in the range

\[
M_{Z'} = 1.75 - 6.1 \text{ TeV},
\]

whereas the current experimental bound from LHC is \( (M_{Z'})_{\text{expt}} \geq 2.5 \text{ TeV} \). Thus, if such a \( Z' \) boson in the predicted mass range of the present model exists, it is likely to be discovered by the ongoing searches at the LHC.

3.2 Proton Lifetime for \( p \to e^+\pi^0 \)

3.2.1 Predictions at two-loop level

The formula for the inverse of proton-decay width \([69–71]\) is

\[
\Gamma^{-1}(p \to e^+\pi^0) = \frac{64\pi f_\pi^2}{m_p} \left( \frac{M_U^4}{gg'} \right) \frac{1}{|A_L|^2|\alpha_H|^2(1 + D + F)^2 \times R}. \tag{3.2}
\]

where \( R = [(A_{SR}^2 + A_{SL}^2)(1 + |V_{ud}|^2)]^2 \) for \( SO(10) \), \( V_{ud} = 0.974 \) = the \((1,1)\) element of \( V_{CKM} \) for quark mixings, \( A_{SL}(A_{SR}) \) is the short-distance renormalization factor in the left (right) sectors and \( A_L = 1.25 \) = long distance renormalization factor. \( M_U \) = degenerate mass of 24 superheavy gauge bosons in \( SO(10) \), \( \alpha_H \) = hadronic matrix element, \( m_p \) = proton mass = 938 MeV, \( f_\pi \) = pion decay constant = 139 MeV, and the chiral Lagrangian parameters are \( D = 0.81 \) and \( F = 0.47 \). Here \( \alpha_H = \alpha_H(1 + D + F) = 0.012 \text{ GeV}^3 \) is obtained from lattice gauge theory computations. In our model, the product of the short distance with the long distance renormalization factor \( A_L = 1.25 \) turns out to be \( A_R \simeq A_L A_{SL} \simeq A_L A_{SR} \simeq 3.20 \). Then using the the two-loop value of the unification scale and the GUT coupling from eq.(2.1) gives

\[
\tau_p(p \to e^+\pi^0) \simeq 5.05 \times 10^{35} \text{ yrs} \tag{3.3}
\]

whereas the solution of RGEs corresponding to eq.(2.2) gives

\[
\tau_p(p \to e^+\pi^0) \simeq 1.05 \times 10^{35} \text{ yrs}. \tag{3.4}
\]

For comparison we note the current experimental search limit from Super-Kamiokande is \([72, 78–80]\)

\[
(\tau_p)_{\text{SuperK}} \geq 1.4 \times 10^{34} \text{ yrs}. \tag{3.5}
\]
A second generation underground water Cherenkov detector being planned at Hyper-Kamiokande in Japan is expected to probe higher limits through its 5.6 Megaton year exposure leading to the partial lifetime \( (\tau_p)_{\text{HyperK}} \geq 1.3 \times 10^{35}\text{yrs} \) if actual decay event is not observed within this limit. Thus our model prediction in eq. (3.4) barely within the planned Hyper-K limit although this the prediction in eq. (3.3) nearly 4 times larger than this limit.

If the proton decay is observed closer to the current or planned experimental limits, it would vindicate the long standing fundamental hypothesis of grand unification. On the other hand proton may be much more stable and its lifetime may not be accessible even to Hyper K experimental search programme. These possibilities are addressed below.

### 3.2.2 GUT scale and proton life-time reduction through bi-triplet scalar

We note that the present estimation of the GUT scale can be significantly lowered so as to bring the proton-lifetime prediction closer to the current Super-K. limit if the the Higgs scalar bi-triplet \( \Theta_H(3,3,1) \subset 54_Y \) of \( SO(10) \) is near the Parity violating intermediate scale. For example in Figure 2, we have shown how in this model only the unification scale is lowered while keeping the other physical mass scales unchanged as in eq. (2.1) for a value of \( M_{331} = 9 \times 10^{13} \text{ GeV} \).

![Figure 2](image_url)

**Figure 2.** Same as Figure 1 but with the Higgs scalar bi-triplet of mass \( 9 \times 10^{13} \text{ GeV} \).

In Table 1 we have presented various allowed values of the GUT scale and the proton life-time for different combinations of the diquark Higgs scalar masses \( M_\Delta \) contained in
The components have superheavy masses around the GUT scale. It was shown in refs.\cite{10,11} that there could be significant threshold effects on the unification scale arising out of heavy and super-heavy particle masses was pointed out especially in the context grand desert models \cite{38,39,40,44} and in intermediate scale SO(10) models \cite{41,42,43,45,46,47}.

Table 1. Predictions on lifetime for the decay $p \rightarrow e^+ \pi^0$ with lower values of masses of the bi-triplet and the diquark Higgs scalars.

\[
\begin{array}{cccccc}
M_{\Delta} \text{ (GeV)} & M_P \text{ (GeV)} & M_{(3,3,1)} \text{ (GeV)} & M_G \text{ (GeV)} & \alpha_2^{-1} & \tau_p \text{ (years)} \\
10^{4.0} & 10^{12.73} & 10^{14.00} & 10^{15.37} & 22.37 & 4.65 \times 10^{33} \\
10^{4.0} & 10^{12.73} & 10^{14.50} & 10^{15.66} & 22.08 & 1.03 \times 10^{34} \\
10^{4.0} & 10^{12.73} & 10^{15.00} & 10^{15.75} & 21.79 & 2.32 \times 10^{34} \\
10^{4.0} & 10^{12.73} & 10^{15.92} & 10^{15.92} & 21.22 & 1.05 \times 10^{35} \\
10^{4.5} & 10^{12.89} & 10^{14.00} & 10^{15.60} & 23.16 & 6.58 \times 10^{35} \\
10^{4.5} & 10^{12.89} & 10^{14.50} & 10^{15.69} & 22.88 & 1.47 \times 10^{34} \\
10^{4.5} & 10^{12.89} & 10^{15.50} & 10^{15.87} & 22.19 & 7.26 \times 10^{34} \\
10^{4.5} & 10^{12.89} & 10^{15.95} & 10^{15.95} & 22.01 & 1.49 \times 10^{35} \\
10^{5.0} & 10^{13.05} & 10^{14.00} & 10^{15.62} & 23.94 & 8.45 \times 10^{33} \\
10^{5.0} & 10^{13.05} & 10^{14.50} & 10^{15.71} & 23.66 & 1.89 \times 10^{34} \\
10^{5.0} & 10^{13.05} & 10^{15.50} & 10^{15.89} & 23.08 & 9.44 \times 10^{34} \\
10^{5.0} & 10^{13.05} & 10^{15.98} & 10^{15.98} & 22.79 & 2.11 \times 10^{35} \\
\end{array}
\]

\[\Delta_R(1,3,\overline{10}) \subset 126_H\] which mediate $n - \bar{n}$ oscillation process. Even for a the bi-triplet mass $M_{GUT}/\sqrt{15}$ we note a reduced value of the unification scale at $M_U = 10^{15.63}$ GeV and the corresponding proton lifetime at $\tau_P = 4.6 \times 10^{33}$ yrs when $M_\Delta \sim 10^4$ GeV. The estimated lifetimes without including the GUT-threshold effects is found to be in the range $\tau_P = 4.6 \times 10^{33}$ yrs to $2.1 \times 10^{35}$ yrs, most of which are between the Super-K and the Hyper-K limits.

An important source of uncertainty on $\tau_P$ in GUTs is known to be due to GUT-threshold effects as illustrated in the following sub-section.

### 3.2.3 Estimation of GUT-threshold effects

That there could be significant threshold effects on the unification scale arising out of heavy and super-heavy particle masses was pointed out especially in the context grand desert models \cite{38,39,40,44} and in intermediate scale SO(10) models \cite{41,42,43,45,46,47}.

In order to examine how closer to or farther from the current experimental bound our model predictions could be, we have estimated the major source of uncertainty on proton lifetime due to GUT threshold effects in SO(10) with intermediate scales \cite{46,47} taking into account the contributions of the superheavy (SH) components in $54_H$, $126_H$, $210_H$, $10_{H_1}$ and $10_{H_2}$ in the case of the minimal model

\[
210_H \supset \Sigma_1(2,2,10) + \Sigma_2(2,2,\overline{10}) + \Sigma_3(2,2,6) + \Sigma_4(1,1,15),
\]

\[
54_H \supset S_1(1,1,20) + S_2(3,3,1) + S_3(2,2,6),
\]

\[
126_H \supset \Delta_1(1,1,6), \quad 10_{H_1} \supset H_i(1,1,6), i = 1, 2,
\]

where the quantum numbers on the RHS are under the gauge group $G_{224}$ and the components have superheavy masses around the GUT scale. It was shown in refs.\cite{41,42,45}.
that when $G_{224D}$ occurs as intermediate symmetry, all loop corrections due to superheavy masses $m_{SH} \geq M_P$ cancels out from the predictions of $\sin^2 \theta_W$ and also from $M_P$ obtained as solutions of RGEs for gauge couplings while the GUT threshold effect on the unification scale due to the superheavy scalar masses assumes an analytically simple form. As outlined in the Appendix, even in the presence of two more intermediate symmetries below $M_P$, analogous formulas on the GUT-threshold effects are also valid

$$\Delta \ln \left( \frac{M_U}{M_Z} \right) = \frac{\lambda'^{U}_{2L} - \lambda'^{U}_{4C}}{\delta \left( a''_{4L} - a''_{4C} \right)}$$

(3.8)

where $a''_{ij}$ is one-loop beta function coefficients in the range $\mu = M_P - M_U$ for the gauge group $G_{224D}$. In eq. (3.8)

$$\lambda'^{U}_{2L} = b'_Y + \sum_{SH} b'^{SH}_i \ln \left( \frac{M^{SH}}{M_U} \right), \quad i = 2L, 2R, 4C$$

(3.9)

$b'_Y = tr(\theta'_Y)$ and $b'^{SH}_i = tr(\theta'^{SH}_i)$ where $\theta'_Y$ ($\theta'^{SH}_i$) are generators of the gauge group $G_{224D}$ in the representations of superheavy gauge bosons (Higgs scalars). The one-loop coefficients for various SH components in eq. (3.7) contributing to threshold effects are [46]

$$b'^{SH}_i = (0, 0, 4), \quad b'^{SH}_i = \{0, 0, 16\}, \quad b'^{SH}_i = \{10, 10, 12\}, \quad b'^{SH}_i = \{6, 6, 4\}, \quad b'^{SH}_i = \{0, 0, 16\}$$

(3.10)

where we have projected out the would-be Goldstone components from $S_3$ leading to

$$\lambda'^{U}_{2L} - \lambda'^{U}_{4C} = 2 - 6\eta_{210} - 2\eta_{54} - 2\eta_{126} - 4\eta_{10},$$

(3.11)

with $\eta_X = \ln(M_X/M_U)$, and we have made the plausible assumption that all SH scalars belonging to a particular $SO(10)$ representation have a common mass such as $M_{210} = M_{210}(i = 1 - 4)$ for $210_H$ and so on for other representations [47]. Utilising the model coefficients $a''_{2L} = 44/3$ and $a''_{4C} = 16/3$, and using eq. (3.11) in eq. (3.8) gives

$$M_U/M_U^0 = 10^{(0.25 \ln \eta)/2.3025}$$

(3.12)

where $\eta = 10(1/10)$ depending upon our assumption that SH components are $10(1/10)$ times heavier/lighter than the GUT scale. By applying these GUT-threshold effects to the solutions of RGE in eq. (2.2), we obtain

$$M_U = 10^{15.92 \pm 0.25} \text{GeV},$$

$$\tau_p(p \to e^+ \pi^0) \simeq 5.05 \times 10^{35} \pm 1.0 \pm 0.34 \text{ yrs}$$

(3.13)

where the first uncertainty is due to GUT threshold effects, and the second uncertainty derived in Appendix B, is due to the 1σ level uncertainties in the experimental values of $\sin^2 \theta_W(M_Z)$ and $\alpha_S(M_Z)$. It is clear from eq. (3.13) that our prediction covers wider range of values in proton lifetime prediction including value few times larger than the current Super-K. limit.

Similarly each of the numerical values in the last column of Table 1 is modified by this additional uncertainty factor of $10^{\pm 1 \pm 0.32}$ in the estimated lifetimes.
4 RARE KAON DECAY AND $n - \bar{n}$ OSCILLATION

In this section we discuss the model predictions on rare kaon decays mediated by lepto-quark gauge bosons of $SU(4)_C$ that occurs as a part of Pati-Salam intermediate gauge symmetry $SU(2)_L \times SU(2)_R \times SU(4)_C$ which breaks spontaneously at the mass scale $\mu = M_R^+ = M_C$. The lepto-quark Higgs scalar contribution to the rare decay process is suppressed in this model due to the natural values of their masses at $M_C = 10^6$ GeV and smaller Yukawa couplings.

4.1 Rare Kaon Decay $K_L \rightarrow \mu \bar{e}$

Earlier several attempts have been made to derive lower bound on the lepto-quark gauge boson mass [19, 73, 74]. In this subsection we update the existing latest lower bound on the $SU(4)_C$-leptoquark gauge boson mass [73] using the improved measurement on the branching ratio and improved renormalization group running of gauge couplings due to the running VEV’s and the additional presence of $G_{2113}$ gauge symmetry in between $G_{224}$ and the SM gauge symmetries. Experimental searches on rare kaon decays in the channel $K_L \rightarrow \mu^\pm e^\mp$ have limited its branching ratio with the upper bound [35]

$$\text{Br.} (K_L \rightarrow \mu \bar{e})_{\text{expt.}} = \frac{\Gamma (K_L \rightarrow \mu^e \bar{e})}{\Gamma (K_L \rightarrow \text{all})} < 4.7 \times 10^{-12}$$

(4.1)

Figure 3. Feynman diagram for rare kaon decays $K_L^0 \rightarrow \mu^\pm e^\mp$ mediated by a heavy lepto-quark gauge boson of $SU(4)_C$ gauge symmetry.

The leptoquark gauge bosons of $SU(4)_C$ in the adjoint representation $(1,1,15)$ under $G_{224}$ mediate rare kaon decay $K_L \rightarrow \mu^\pm e^\mp$ whose Feynman diagram is shown in the Figure 3. Analytic formulas for the corresponding branching ratio is [73, 74],

$$\text{Br.} (K_L \rightarrow \mu \bar{e}) = \frac{4\pi^2 \alpha^2 (M_C) m_{K_L}^4 R}{G_F^2 \sin^2 \theta_C m_{\mu}^2 (m_s + m_d)^2 M_C^4},$$

(4.2)

where the factor $R$ includes renormalization effects on the quark masses $m_d$ or $m_s$ from $\mu = M_C$ down to $\mu = \mu_0 = 1$ GeV through the $G_{2113}$, the SM and the $SU(3)_C$ gauge symmetries.
Noting that the down quark or the strange quark mass satisfies the following renormalization group equations,

\[ m_{d,s}(M_C) = \frac{m_{d,s}(\mu_0)}{\eta_{\text{em}}} R_{2113} R_{213}^{(6)} R_{213}^{(5)} R_{QCD}^{(5)} R_{QCD}^{(4)} R_{QCD}^{(3)} \]  \quad (4.3)

where

\[ R_{2113} = \Pi_i \left( \frac{\alpha_i(M_C)}{\alpha_i(M_R^0)} \right)^{-C_i/2a_i^{(1)}}, \quad i = 2L, 1R, B - L, 3C, \]

\[ R_{213}^{(6)} = \Pi_i \left[ \frac{\alpha_i(M_R^0)}{\alpha_i(m_t)} \right]^{-C_i/2a_i^{(2)}}, \quad R_{213}^{(5)} = \Pi_i \left[ \frac{\alpha_i(m_t)}{\alpha_i(M_Z)} \right]^{-C_i/2a_i^{(3)}}, \quad i = 2L, Y, 3C, \]

\[ R_{QCD}^{(5)} = \left[ \frac{\alpha_S(M_Z)}{\alpha_S(m_b)} \right]^{-4/a^{(4)}}, \quad R_{QCD}^{(4)} = \left[ \frac{\alpha_S(m_b)}{\alpha_i(m_c)} \right]^{-4/a^{(5)}}, \quad R_{QCD}^{(3)} = \left[ \frac{\alpha_S(m_c)}{\alpha_S(\mu^0)} \right]^{-4/a^{(6)}}, \]  \quad (4.4)

where the input parameters used in above eq. (4.4) are: \( C_1 = (0, 1/4, 8) \), \( C_2 = (0, -1/5, 8) \) and the one-loop beta-coefficients relevant for our present work are \( a_i^{(1)} = (-3.57/12, 37/8, -7) \), \( a_i^{(2)} = (-19/6, 41/10, -7) \), \( a_i^{(3)} = (-23/6, 103/30, -23/3) \), \( a^{(4)} = -23/3 \), \( a^{(5)} = -25/3 \), \( a^{(6)} = -9 \). Now we can obtain the renormalization factor in eq.(4.2)

\[ R = \left[ R_{2113} R_{213}^{(6)} R_{213}^{(5)} R_{QCD}^{(5)} R_{QCD}^{(4)} R_{QCD}^{(3)} \right]^{-2}. \]  \quad (4.5)

Using eq.(4.4) and eq.(4.5) and eq.(4.1), we derive the following inequality,

\[ F_L(M_C, M_R^0) > \left[ \frac{4\pi^2 m_K^2 R_p}{G_F^2 \sin^2 \theta_C m^2(\mu^2)(m_s + m_d)^2} \times 10^{11.318} \right]^{1/4}, \]  \quad (4.6)

where

\[ F_L(M_C, M_R^0) = M_C \alpha_S^{-3/14} (M_C) \alpha_Y^{-1/82} (M_R^0) \left[ \frac{\alpha_{B-L}(M_C)}{\alpha_{B-L}(M_R^0)} \right]^{-1/74} \alpha_Y^{1/82} (m_t) \alpha_C^{-2/7} (m_t), \]

\[ R_p = \left[ R_{2113} R_{213}^{(6)} R_{213}^{(5)} R_{QCD}^{(5)} R_{QCD}^{(4)} R_{QCD}^{(3)} \right]^{-2}. \]  \quad (4.7)

In Fig. 4 the function \( F_L(M_C, M_R^0) \) in the LHS of eq. (4.6) is plotted against \( M_C \) for a fixed value of \( M_R^0 = 5 \) TeV, where the Horizontal lines represent the RHS of the same equation including uncertainties in the parameters. Thus, for the purpose of this numerical estimation, keeping \( M_R^0 \) fixed at any value between 5 – 10 TeV, we vary \( M_C \) until the LHS of eq.(4.6) equals its RHS.

For our computation at \( \mu_0 = 1 \) GeV, we use the inputs \( m_K = 0.4976 \) GeV, \( m_d = 4.8^{+0.7}_{-0.3} \) MeV, \( m_s = 95 \pm 5 \) MeV, \( m_\mu = 105.658 \) MeV, \( G_F = 1.166 \times 10^{-5} \) GeV\(^{-2} \), and \( \sin^2 \theta_C = 0.2254 \pm 0.0007 \), \( m_t = 4.18 \pm 0.03 \) GeV, \( m_c = 1.275 \pm 0.025 \) GeV, \( m_t = 172 \) GeV. At \( \mu = M_Z \) we have used \( \sin^2 \theta_W = 0.23166 \pm 0.00012 \), \( \alpha_s = 0.1184 \pm 0.0007 \), \( \alpha^{-1} = 127.9 \) and utilized eq.(4.1)–eq.(4.7). With \( M_R^0 = 5 \) TeV and \( M_Z' \approx 1.2 \) TeV, the
existing experimental upper bound on $\text{Br.}(K_L \rightarrow \mu^\mp e^\pm)$ gives the lower bound on the $G_{224}$ symmetry breaking scale

$$M_C > (1.932^{+0.082}_{-0.074}) \times 10^6 \text{GeV}. \quad (4.8)$$

Noting from Fig.1 that in our model $\alpha_S(M_C) = 0.0505$, we get from eq.(4.8) as rare-kaon decay constraint on the $SU(4)_C$-leptoquark gauge boson mass

$$M_{\text{lepto}} > (1.539^{+0.065}_{-0.059}) \times 10^6 \text{GeV}. \quad (4.9)$$

where the uncertainty is due to the the existing uncertainties in the input parameters. From the derived solutions to RGEs for gauge couplings this lower bound on the leptoquark gauge boson mass is easily accommodated in our model.

### 4.2 Neutron-Antineutron Oscillation

Here we discuss the prospect of this model predictions for experimentally observable $n - \bar{n}$ oscillation while satisfying the rare-kaon decay constraint by fixing the $G_{224}$ symmetry breaking scale at $M_C \sim 2 \times 10^6$ GeV as derived in eq.(4.8). The Feynman diagrams for the $n - \bar{n}$ oscillation processes are shown in left- and right-panel of Fig.5 where $\Delta_{u^c d^c}, \Delta_{d^c d^c}$, and $\Delta_{u^c d^c}$ denote different diquark Higgs scalars contained in $\Delta_R(1,3,\overline{10}) \subset 126_R$. The amplitude for these two diagrams can be written as

$$\text{Amp}^{(a)}_{n \rightarrow \bar{n}} = \frac{f_{11}^3 \lambda_{UB-L}}{M_1^4 \Delta_{u^c d^c} \Delta_{d^c d^c}}, \quad \text{Amp}^{(b)}_{n \rightarrow \bar{n}} = \frac{f_{11}^3 \lambda_{UB-L}}{M_2^4 \Delta_{d^c d^c} \Delta_{u^c d^c}}. \quad (4.10)$$
where $f_{11} = (f_{\Delta_{u^c d^c}}) = (f_{\Delta_{d^c d^c}}) = (f_{\Delta_{u^c u^c}})$ from the $SO(10)$ invariance and the quartic coupling between different di-quark Higgs scalar has its natural value i.e, $\mathcal{O}(0.1) - \mathcal{O}(1)$.

The $n - \bar{n}$ mixing mass element $\delta m_{n\bar{n}}$ and the dibaryon number violating amplitude $W_{(B=2)} = \text{Amp}^{(a)} + \text{Amp}^{(b)}$ are related up to a factor depending upon combined effects of hadronic and nuclear matrix element effects

$$\delta m_{n\bar{n}} = (10^{-4} \text{GeV}) \cdot W_{(B=2)}. \quad (4.11)$$

The experimentally measurable mixing time $\tau_{n\bar{n}}$ is just the inverse of $\delta m_{n\bar{n}}$

$$\tau_{n\bar{n}} = \frac{1}{\delta m_{n\bar{n}}}. \quad (4.12)$$

With $v_{B-L} = 5$ TeV in the degenerate case, when all diquark Higgs scalars have identical masses $M_\Delta = 10^5$ GeV, the choice of the parameters $f_{11} \approx \lambda \sim \mathcal{O}(0.1)$ gives $\tau_{n\bar{n}} = 6.58 \times 10^9$ sec. As described below our $SO(10)$ model can fit all charged fermion masses and CKM mixings at the GUT scale with two kinds of structures: (i) only one $126_H$, and (ii) two Higgs representations $126_H$ and $126'_H$. In the minimal case the Yukawa coupling $f$ of $126_H$ to fermions has a diagonal structure,$\,$

$$f = \text{diag}(0.0236, -0.38, 1.5), \quad (4.13)$$

which gives through eq. (4.10), eq. (4.11), and eq. (4.12)

$$\tau_{n\bar{n}} = 10^8 - 10^{10}\text{secs}. \quad (4.14)$$

This model prediction is accessible to ongoing search experiments [76]. However, the GUT scale fit to the fermion masses can be successfully implemented without constraining the $f$ values when a second $126'_H$ is present at the GUT scale with all its component at $M_U$ except $\xi'(2, 2, 15)$ being around the $M_D$ scale. Then using $f_{11} = 0.1 - 0.01$, the estimated value turns out to be

$$\tau_{n\bar{n}} \sim 10^9 - 10^{13}\text{sec}. \quad (4.15)$$

Out of this the mixing time in the range $10^9 - 10^{10}$ sec can be probed by ongoing experiment [76].
$$
\begin{array}{cccc}
0.1 & 0.1 & 10^5 & 10^5 \\
0.0236 & 0.1 & 10^5 & 10^5 \\
0.0236 & 1.0 & 10^5 & 10^5 \\
0.1 & 0.1 & 10^4 & 10^5 \\
0.0236 & 1.0 & 10^4 & 10^5 \\
0.0236 & 1.0 & 10^5 & 10^4 \\
\end{array}
$$

Table 2. Predictions for $n-\bar{n}$ oscillation mixing time as a function of allowed couplings and masses of diquark Higgs scalars in the model described in the text.

\section{Determination of Dirac Neutrino Mass Matrix}

The Dirac neutrino mass near the TeV scale forms an essential ingredient in the estimations of inverse seesaw contribution to light neutrino masses and mixings as well as the LFV and LNV processes in this model in addition to predicting leptonic CP-violation through non-unitarity effects. Since the procedure for determination of $M_D$ has been discussed earlier \cite{36}, we mention it briefly here in the context of the present model. In order to obtain the Dirac neutrino mass matrix $M_D$ and the RH Majorana mass matrix $M_N$ near TeV scale, at first the PDG values \cite{49} of fermion masses at the electroweak scale are extrapolated to the GUT scale using the renormalization group equations (RGEs) for fermion masses in the presence of the SM for $\mu = M_Z - 5$ TeV, and from $\mu = 5 - 10$ TeV using the RGEs in the presence of $G_{2113}$ symmetry \cite{55, 82}. From $\mu = 5 - 100$ TeV, RGEs corresponding to two Higgs doublets in the presence of $G_{2113}$ symmetry are used \cite{55}. These two doublets which act like up-type and down type doublets are treated to have originated from separate representations $10_H$ and $10_H$ of $SO(10)$. For mass scale $\mu \geq 10^5$ GeV till the GUT scale the fermion mass RGEs derived in the presence of the $G_{224}$ and $G_{224D}$ symmetries \cite{56} are exploited. Then at the GUT scale $\mu = M_{GUT}$ we obtain the following values of mass eigenvalues and the CKM mixing matrix:

\begin{align}
\begin{pmatrix}
0.976 & 0.216 & -0.0017 - 0.0035i \\
-0.216 - 0.0001i & 0.976 - 0.0000i & 0.0310 \\
0.0083 - 0.0035i & -0.03 - 0.0007i & 0.999
\end{pmatrix},
\end{align}

Formulas for different fermion mass matrices at the GUT scale have been discussed in \cite{36, 55}

\begin{align}
M_u^0 = G_u + F, \quad M_D^0 = G_u - 3F, \\
M_d^0 = G_d + F, \quad M_t = G_d - 3F,
\end{align}
where \( G_u = Y_1 v_u, G_d = Y_2 v_d \), and in the absence of \( 126_H \) in those models, the diagonal structure of \( F \) was shown to originate from available non-renormalizable higher dimensional operators. The new interesting point here is that the present model permits \( F \) to be renormalizable using the ansatz \([67]\) \( F = f v_\xi \), and the induced VEV \( v_\xi \) of \( \xi(2, 2, 15) \subset 126_H \) is predicted within the allowed mass scales of the \( SO(10) \) while safeguarding precision gauge coupling unification.

Using the charged-lepton diagonal mass basis and eq. \((5.2)\) we have

\[
M_e(M_{\text{GUT}}) = \text{diag}(0.000216, 0.0388, 0.9620) \text{ GeV},
\]

\[
G_{d,ij} = 3 F_{ij}, \ (i \neq j).
\]

In the present model, type-II seesaw contribution being negligible and the neutrino oscillation data being adequately represented by inverse seesaw formula, there is no compelling reason for the Majorana coupling \( f \) to be non-diagonal. On the other hand diagonal texture of RH neutrino mass matrix has been widely used in the literature in a large class of \( SO(10) \) models. Moreover, as we see below, the diagonal structure of \( f \) which emerges in the minimal model exactly predicts the RH neutrino masses accessible to LHC and the neutrino oscillation data.\(^*\) We then find that diagonal texture of \( f \) gives the matrix \( G_d \) to be also diagonal leading to the relations

\[
G_{d,ii} + F_{ii} = m_i^0, \ (i=d,s,b),
\]

\[
G_{d,ij} - 3 F_{jj} = m_j^0, \ (j = e, \mu, \tau).
\]

\[
F = \text{diag} \frac{1}{4}(m_d^0 - m_e^0, m_s^0 - m_\mu^0, m_b^0 - m_\tau^0),
\]

\[
= \text{diag}(2.365 \times 10^{-4}, -0.0038, +0.015) \text{ GeV},
\]

\[
G_d = \text{diag} \frac{1}{4}(3 m_d^0 + m_e^0, 3 m_s^0 + m_\mu^0, 3 m_b^0 + m_\tau^0),
\]

\[
= \text{diag}(9.2645 \times 10^{-4}, 0.027224, 1.00975) \text{ GeV},
\]

where we have used the RG extrapolated values at the GUT scale. It is clear from the value of the mass matrix \( F \) in eq. \((5.5)\) that we need a small VEV \( v_\xi \sim 10 \text{ MeV} \) to fit the fermion mass fits at the GUT scale. To verify that this \( v_\xi \) is naturally obtained in this model, we note that the spontaneous symmetry breaking in this model \( G_{224} \to G_{2113} \) occurs through the VEV of \( (1, 3, 15) \subset 210_H \). Then the desired trilinear term in the scalar potential \( V \) gives the natural value of the VEV

\[
V = \lambda_3 M_{\text{GUT}} 210_H 126_H^1 10_H
\]

\[
= \lambda_3 M_{\text{GUT}}(1, 3, 15)_{210} (2, 2, 15)_{126} (2, 2, 1)_{10_{1,2}}
\]

\[
v_\xi \sim \lambda_3 M_{\text{GUT}} M_C v_{\text{ew}} / M_\xi^2 = 10 \text{MeV} - 100 \text{MeV},
\]

\(^*\) Alternatively the fermion masses at the GUT scale can be fitted by the diagonal coupling \( f' \) of a second \( 126_H \), whose \( \xi(2, 2, 15) \) component can be fine-tuned to have mass at the same intermediate scale to provide the desired VEV. In this case the \( f \) and RH Majorana neutrino mass matrix \( M_N \) is allowed to possess a general \( 3 \times 3 \) matrix structure without any a priori constraint.
for $M_\xi = 10^{12} - 10^{13}$ GeV.

Repeating the RG analysis of ref.[36] we have verified that the precision gauge coupling unification is unaffected when $\xi(2, 2, 15)$ occurs at such high intermediate scales except for an increase of the GUT scale by nearly 2 and the GUT fine structure constant by nearly three times. That the Parity violating scale and the GUT scale would be marginally affected is easy to understand because the contribution due to $\xi(2, 2, 15)$ to all the three one-loop beta-function coefficients are almost similar $\delta b_{2L} = \delta b_{2R} = 5, \delta b_{4C} = 5.333$. That the unification is bound to occur can be easily seen because there are only two gauge coupling constant lines for $\mu > M_P$.

Using the computed values of $M_\nu^0$ and the value of $F$ from eq. (5.5) in eq. (5.2), gives the the matrix $G_u$ at $\mu = M_{GUT}$. Another by product of this fermion mass fit at the GUT scale is that the matrix elements of $F$ now gives $f = \text{diag}(f_1, f_2, f_3)$ and consequently the RH neutrino mass hierarchy $M_{N_1} : M_{N_2} : M_{N_3} = 0.023 : -0.38 : 1.5$. This hierarchy is consistent with lepton-number and lepton flavor violations discussed in Sec.2, Sec.4, and Sec.5

$$G_u(M_{GUT}) = \begin{pmatrix} 0.0095 & 0.0379 - 0.0069i & 0.0635 - 0.1671i \\ 0.0379 + 0.0069i & 0.2637 & 2.117 + 0.0001i \\ 0.0635 + 0.1672i & 2.117 - 0.0001i & 51.444 \end{pmatrix} \text{GeV.} \quad (5.7)$$

Now using eq. (5.5) and eq. (5.7) in eq. (5.2) gives the Dirac neutrino mass matrix $M_D$ at the GUT scale

$$M_D^0(M_{GUT}) = \begin{pmatrix} 0.00876 & 0.0380 - 0.0069i & 0.0635 - 0.1672i \\ 0.0380 + 0.0069i & 0.3102 & 2.118 + 0.0001i \\ 0.0635 + 0.1672i & 2.118 - 0.0001i & 51.63 \end{pmatrix} \text{GeV.} \quad (5.8)$$

Noting that $F = f v_\xi = \text{diag}(f_1, f_2, f_3)v_\xi$ in eq.(5.5), $v_\xi = 10$ MeV gives $(f_1, f_2, f_3) = (0.0236, -0.38, 1.5)$. Then the allowed solution to RGES for gauge coupling unification with $M_R^0 = v_R = 5$ TeV gives $M_{N_1} = 115$ GeV, $M_{N_2} = 1.785$ TeV, and $M_{N_3} = 7.5$ TeV. While the first RH neutrino is lighter than the current experimental limit on $Z_R$ boson mass, the second one is in-between the $Z_R$ and $W_R$ boson mass limits, but the heaviest one is larger than the $W_R$ mass limit. These are expected to provide interesting collider signatures at LHC and future accelerators. This hierarchy of the RH neutrino masses has been found to be consistent with lepton-number and lepton flavor violations discussed in Sec.2, Sec.4, and Sec.5. Then following the top-down approach we obtain the value of $M_D$ at the TeV scale

$$M_D(M_R^0) = \begin{pmatrix} 0.02274 & 0.0989 - 0.0160i & 0.1462 - 0.3859i \\ 0.0989 + 0.0160i & 0.6319 & 4.884 + 0.0003i \\ 0.1462 + 0.3859i & 4.884 - 0.0003i & 117.8 \end{pmatrix} \text{GeV.} \quad (5.9)$$

We will use $M_N = (0.115, 1.7, 7.8)$ TeV and the $M_D$ matrix of eq.(5.9) to predict LFV and LNV decays in the next two sections.
6 FITTING THE NEUTRINO OSCILLATION DATA BY GAUGED INVERSE SEESAW FORMULA

In the presence of three singlet fermions $S_i$ ($i = 1, 2, 3$), the inverse seesaw mechanism [36, 50–52, 55] is implemented in the present model through the $SO(10)$ invariant Yukawa Lagrangian that gives rise to the $G_{2113}$ invariant interaction near the TeV scale [36, 55] where $\chi_R(1,1/2,-1,1) \subset 16_H$ generates the $N - S$ mixing term,

$$L_{\text{Yuk}} = Y^a 16.16.10^a H + f 16.16.126^f H + y_\chi 16.1.16^\dagger H + \mu_S 1.1$$

$$\supset Y^T \ell_L N_R \Phi_1 + f N_R^c N_R \Delta_R + F N_R S \chi_R + S^T \mu_S S + \text{h.c.}$$

This Lagrangian gives rise to the $9 \times 9$ neutral fermion mass matrix after electroweak symmetry breaking.

$$M = \begin{pmatrix}
0 & 0 & M_D \\
0 & \mu_S & M \\
M_D^T & M^T & M_N
\end{pmatrix}, \quad (6.1)$$

In contrast to the SM where all three matrices $M_N, M$, and $\mu_S$ have no dynamical origins, in this model the first two have dynamical interpretations $M_N = f v_R, M = y_\chi v_\chi$; only $\mu_S$ suffers from this difficulty.

In this model the RH neutrinos being heavier than the other two fermion mass scales in the theory with $M_N \gg M > M_D, \mu_S$, they are at first integrated out from the Lagrangian, which, in the $(\nu, S)$ basis, gives the $6 \times 6$ mass matrix

$$M_{\text{eff}} = -\begin{pmatrix}
M_D M_N^{-1} M_D^T & M_D M_N^{-1} M^T \\
M M_N^{-1} M_D^T & M M_N^{-1} M^T - \mu_S
\end{pmatrix}, \quad (6.2)$$

This is further block diagonalized to find that the would be dominant $type - I$ seesaw contribution completely cancels out leading to the gauged inverse mass formula for light neutrino mass matrix and also another formula for the sterile neutrinos(S)

$$m_\nu = M_D M_N^{-1} \mu_S (M_D M_N^{-1})^T \quad (6.3)$$

$$m_S = \mu_S - M M_N^{-1} M^T \quad (6.4)$$

The complete $6 \times 6$ unitary mixing matrix which diagonalizes the light-sterile neutrino effective mass matrix $M_{\text{eff}}$ is

$$V_{6\times6} = W \cdot U = \begin{pmatrix}
1 - \frac{1}{2} XX^\dagger & X \\
-X^\dagger & 1 - \frac{1}{2} X^\dagger X
\end{pmatrix} \cdot \begin{pmatrix}
U_\nu & 0 \\
0 & U_S
\end{pmatrix} \quad (6.5)$$

In this extended inverse seesaw scheme, the light neutrinos are actually diagonalized by a matrix which is a part of the full $6 \times 6$ mixing matrix $V_{6\times6}$

$$\mathcal{N} \simeq \begin{pmatrix}
1 - \frac{1}{2} XX^\dagger
\end{pmatrix} U_{\text{PMNS}} = (1 - \eta) U_{\text{PMNS}} \quad (6.6)$$
where \( \eta = \frac{1}{2} M_D M^{-1} (M_D M^{-1})^\dagger \) is a measure of non-unitarity contributions. In the \((\nu, S, N)\) basis, adding RH Majorana mass \( M_N \) to eq.(6.2), the complete mixing matrix \([36, 57]\) diagonalizing the resulting \(9 \times 9\) neutrino mass matrix turns out to be

\[
\mathcal{V} = \begin{pmatrix}
\nu_{\alpha i}^\nu & \nu_{\alpha i}^S & \nu_{\alpha i}^N \\
\nu_{\beta j}^S & \nu_{\beta j}^S & \nu_{\beta j}^N \\
\nu_{\gamma k}^N & \nu_{\gamma k}^N & \nu_{\gamma k}^N
\end{pmatrix}
\begin{pmatrix}
(1 - \frac{1}{2} XX^\dagger) U_\nu & (X - \frac{1}{2} Z Y^\dagger) U_S & Z U_N \\
-X^\dagger U_\nu & (1 - \frac{1}{2} (X^\dagger X + Y Y^\dagger) Y^\dagger) U_S & (Y - \frac{1}{2} X^\dagger Z) U_N \\
y^\dagger X^\dagger U_\nu & -Y^\dagger U_S & (1 - \frac{1}{2} Y^\dagger Y) U_N
\end{pmatrix},
\]

(6.7)
as shown in the appendix. In eqn.(6.7) \( X = M_D M^{-1}, \ Y = M M^{-1}, \ Z = M_D M_N^{-1}, \) and \( y = M^{-1} \mu_S. \)

Although the \(N - S\) mixing matrix \( M \) in general can be non diagonal, we have assumed it to be diagonal partly to reduce the unknown parameters and as we shall see the LFV effects constrain the diagonal elements. Noting that \( \eta_{\alpha \beta} = \frac{1}{2} \sum_{k=1}^{3} (M_{Dak} M_{Dak}^*)/M_k^2, \) the entries of the \( \eta \) matrix are constrained from various experimental inputs like e.g. rare lepton decays, invisible Z-boson width, neutrino oscillations etc. For illustration let us quote the bound on these elements of \( \eta \) on 90\% C.L. \(^1\) \( |\eta_{ee}| \leq 2.0 \times 10^{-3}, \ |\eta_{\mu \mu}| \leq 8.0 \times 10^{-4}, \ |\eta_{\tau \tau}| \leq 2.7 \times 10^{-3}, \ |\eta_{e \mu}| \leq 3.5 \times 10^{-5}, \ |\eta_{e \tau}| \leq 8.0 \times 10^{-3}, \) and \( |\eta_{\mu \tau}| \leq 5.1 \times 10^{-3}.\) Whereas the possible CP phases of the elements of \( \eta_{\alpha \beta} (= \phi_{\alpha \beta}) \) are not constrained, the knowledge of \( M_D \) matrix given in eq.(5.9) and saturation of the lower bound on \( |\eta_{\tau \tau}| = 2.7 \times 10^{-3} \) leads to a relation between diagonal elements of \( M, \)

\[
\frac{1}{2} \left[ \frac{0.17}{M_1^2} + 23.853 \sum \frac{13876.84}{M_2^3} \right] = 2.7 \times 10^{-3}
\]

(6.8)
The above relation can be satisfied by the partial degenerate values of \( M \) as \( M_1 = M_2 \geq 100 \) GeV and \( M_3 \geq 2.15 \) TeV while it also accommodates the complete non-degenerate values \( M_1 \geq 10 \) GeV, \( M_2 \geq 120 \) GeV, and \( M_3 \geq 2.6 \) TeV. For degenerate \( M, \) this gives \( M_1 = M_2 = M_3 = 1.6 \) TeV. The elements of \( \eta \) can be different for different values of \( M \) allowed in our model. We need to know the PMNS mixing matrix and \( \eta \) in order to estimate the non-unitarity lepton mixing matrix \( N_{3 \times 3}. \)

Our analysis carried out for a normal hierarchy (NH) of light neutrino masses can be repeated also for inverted hierarchical (IH) or for quasi-degenerate (QD) masses to give correspondingly different values of the \( \mu_S \) matrix. For example, using NH for which \( m_\nu^\text{diag} = \text{diag}(0.00127, 0.00885, 0.0495) \) eV consistent with the central values of a recent global analysis of the neutrino oscillation parameters \([58]\) \( \Delta m_{sol}^2 = 7.62 \times 10^{-5} \) eV\(^2, \ \Delta m_{atm}^2 = 2.55 \times 10^{-3} \) eV\(^2, \ \theta_{12} = 34.4^\circ, \ \theta_{23} = 40.8^\circ, \ \theta_{13} = 9.0^\circ, \delta = 0.8\pi \) and assuming vanishing Majorana phases \( \alpha_1 = \alpha_2 = 0, \) we use the non-unitarity mixing matrix \( N = (1 - \eta) U_{\text{PMNS}}, \) and the relation \( m_\nu = N m_\nu N^T, \) to derive the form of \( \mu_S \) matrix from the light neutrino

\(^{1}\)For related references on the 90\% C.L. of the bounds on the elements \( |\eta_{\alpha \beta}| \) see references cited in \([36, 55]\).
are found to be more dominant compared to RH neutrinos, the branching ratios due to exchanges of sterile neutrino (S) go over number of sterile neutrinos $S_j$ and for heavy right-handed Majorana neutrinos $N_k$ and the mixing matrices are $V^\nu S = \{X U_S\}_{\alpha j}$ and $V^\nu N = \{Z U_N\}_{\alpha k}$ with $X = \frac{M_S}{M_N}$ and $Z = \frac{M_N}{M_N}$. The allowed ranges of $M_i, (i = 1, 2, 3)$ from the LFV constraint eq.(6.8) and the predicted values of $M_{N_i}, (i = 1, 2, 3)$ now determine the mass eigen values of the sterile neutrinos leading to $M_{S_i} = [12.5, 49, 345.6] \text{ GeV}$ for $M = \text{diag}[40, 300, 1661] \text{ GeV}$, and $M_N = \text{diag}[115, 1785, 7500] \text{ GeV}$.

The neutrino mixing matrices are estimated numerically

$$\mathcal{N} \equiv V^\nu = \begin{pmatrix} 0.8143 - 0.0008i & 0.5588 + 0.0002i & 0.1270 + 0.0924i \\ -0.3587 - 0.0501i & 0.6699 - 0.0343i & -0.6472 - 0.0001i \\ 0.4489 - 0.0571i & -0.4849 - 0.0394 & -0.7438 - 0.0001i \end{pmatrix},$$

$$V^\nu S = \begin{pmatrix} 0.0542 & 0.0325 - 0.0052i & 0.0086 - 0.0227i \\ 0.2358 + 0.0380i & 0.2075 & 0.2869 \\ 0.3465 + 0.9155i & 1.597 & 6.920 \end{pmatrix} \times 10^{-2}i,$$

$$V^\nu N = \begin{pmatrix} 0.0170 & 0.0053 - 0.0009i & 0.0018 - 0.0048i \\ 0.0740 + 0.0119i & 0.0340 & 0.0608 \\ 0.1089 + 0.2865i & 0.2625 & 1.467 \end{pmatrix} \times 10^{-2}.$$

Compared to RH neutrinos, the branching ratios due to exchanges of sterile neutrino ($S_i$) are found to be more dominant

$$\text{Br} (\mu \to e + \gamma) = 3.5 \times 10^{-16}.$$
Similarly, other LFV decay amplitudes are estimated leading to \[63\]
\[
\begin{align*}
\text{Br} (\tau \rightarrow e + \gamma) &= 3.0 \times 10^{-14}, \\
\text{Br} (\tau \rightarrow \mu + \gamma) &= 4.1 \times 10^{-12}.
\end{align*}
\] (7.6)

These branching ratios are accessible to ongoing search experiments.

We have also noted here that the leptonic CP-violating parameter due to non-unitarity effects is \(J \simeq 10^{-5}\) which is similar to the model prediction of ref.[36].

8 NEW CONTRIBUTIONS TO NEUTRINO-LESS DOUBLE BETA DECAY IN THE \(W_L^− - W_L^−\) CHANNEL

In the generic inverse seesaw, there is only one small lepton number violating scale \(\mu_S\) and the lepton number is conserved in \(\mu_S = 0\) limit leading to vanishing nonstandard contribution to the 0\(\nu\)2\(\beta\) transition amplitude. On the contrary, in the extended seesaw under consideration, it has been shown for the first time that there can be a new dominant contributions from the exchanges of heavy sterile neutrinos [36]. The main thrust of our discussion will be new contribution arising from exchange of heavy sterile neutrinos \(S_i\) with Majorana mass \(M_S = \mu_S - M(1/M_N)MT\) as explained in Sec.6. Because of heavy mass of \(W_R\) boson in this theory, the RH current contributions are damped out.

\[
\text{Figure 6. } W_L^− - W_L^− \text{ mediated channel with light } \nu_i \text{ and sterile } S_i \text{ Majorana neutrino exchanges.}
\]

In the mass basis, we have \(\nu_\alpha = N_{\alpha i} \nu_{m_i} + V_{\nu_i}^{\nu S_i} S_{m_j} \). In addition to the well known standard contribution in the \(W_L^− - W_L^−\) channel shown in the left-panel of Fig.6, we note the new contribution shown in the right-panel of Fig.6 with the corresponding amplitudes

\[
\begin{align*}
A_L^{\nu} &\propto G_F^2 \left( \nu_{ei} \nu_{ei} \right)^2 \frac{m_{\nu_i}}{p^2}, \\
A_L^{S} &\propto G_F^2 \left( \nu_{ej} \nu_{ej} \right)^2 \frac{M_{S_j}}{M_T}.
\end{align*}
\] (8.1) (8.2)

where \(|p^2| \simeq (190 \text{ MeV})^2\) represents neutrino virtuality momentum and \(G_F = 1.2 \times 10^{-5} \text{ GeV}^{-2}\).
Noting from eq.(6.7) that $(\nu_{ei}^S)^2 = (M_D/M_{ei})^2$, the RHS of eq.(34) is expected to dominate because of three reasons: (i) Dirac neutrino mass origin from quark-lepton symmetry in $SO(10)$, (ii) Smaller values of diagonal elements of the $N-S$ mixing matrix $M$, (iii) smaller eigen values of the heavy sterile Majorana neutrino mass: $M_S = \mu_S - M(1/M_N)M^T$. The mixing matrix elements necessary for prediction of $0\nu\beta\beta$ amplitude can be represented as,

$$N_{e1} = 0.819, \quad N_{e2} = 0.552, \quad N_{e3} = 0.156,$$

$$\nu_{ei}^S = 0.00015, \quad \nu_{e2}^S = 0.00068, \quad \nu_{e3}^S = 0.00022.$$  \hspace{1cm} (8.3)

### 8.1 New formula for half-life and bound on sterile neutrino mass

We derive a new formula for half-life of $0\nu\beta\beta$ decay as a function of heavy sterile neutrino masses and other parameters in the theory. We then show how the current experimental bounds limit the lightest sterile neutrino mass to be $M_{S1} \geq 14 \pm 4 \text{ GeV}$.

Using results discussed in previous sections, the inverse half-life is presented in terms of $\eta-$ parameters and others including the nuclear matrix elements [36, 64–66]

$$\left[ T_{1/2}^{0\nu} \right]^{-1} = G_{01}^{0\nu}|\mathcal{M}_{0\nu}|^2|\eta_0 + \eta_S|^2.$$  \hspace{1cm} (8.4)

where the dimensionless particle physics parameters are

$$\eta_0 = \sum_i (\nu_{ei}^S)^2 m_i / m_e, \quad \eta_S = \sum_i (\nu_{ei}^S)^2 m_p / M_{S_i}.$$  \hspace{1cm} (8.5)

In eqn.(8.5), $m_e (m_i)=$ mass of electron (light neutrino), and $m_p =$ proton mass. In eqn.(8.4), $G_{01}^{0\nu}$ is the the phase space factor and besides different particle parameters,
it contains the nuclear matrix elements due to different chiralities of the hadronic weak currents such as \((M^0_{\nu})\) involving left-left chirality in the standard contribution. Explicit numerical values of these nuclear matrix elements discussed in ref. [36, 64–66] are given in Table. 3.

| Isotope | \(G_{01}^{0\nu}[10^{-14}\text{yrs}^{-1}]\) | \(M^0_{\nu}\) | \(M^0_N\) |
|---------|---------------------------------|-----------------|----------|
| \(^{76}\text{Ge}\)     | 0.686                          | 2.58–6.64       | 233–412  |
| \(^{82}\text{Se}\)     | 2.95                           | 2.42–5.92       | 226–408  |
| \(^{130}\text{Te}\)    | 4.13                           | 2.43–5.04       | 234–384  |
| \(^{136}\text{Xe}\)    | 4.24                           | 1.57–3.85       | 160–172  |

Table 3. Phase space factors and nuclear matrix elements with their allowed ranges as derived in Refs. [36, 64–66].

In terms of effective mass parameter, the inverse half-life for neutrinoless double beta decay is given as,

\[
\left[ T_{0\nu}^{1/2} \right]^{-1} = \frac{\Gamma_{0\nu\beta\beta}}{\ln 2} = G_{0\nu} \left| \frac{M_{\nu}}{m_e} \right|^2 \times \left| m_{ee}^{\text{eff}} \right|^2 ,
\]  

(8.6)

with \(m_{ee}^{\text{eff}} = m_{ee}^\nu + m_{ee}^S\),

where \(G_{0\nu}\) contains the phase space factors, \(m_e\) is the electron mass, and \(M_{\nu}\) is the nuclear matrix element and the effective mass parameters are

\[
m_{ee}^\nu = \mathcal{N}_{ei}^2 m_{\nu_i} , \quad m_{ee}^S = p^2 \left( \frac{\nu_e \delta_i}{M_{Si}} \right)^2 ,
\]  

(8.7)

where \(p^2 = -|p|^2\). With \(|\langle p^2 \rangle| = \left| m_e m_p M^0_N / M^0_{\nu} \right| \simeq (120 - 200) \text{MeV}^2\), \(M_{\nu R} \simeq 10^5 \text{GeV}\), we predict the effective mass for \(0\nu\beta\beta\) transition rate for light neutrino masses,

\[
|m_{ee}^\nu| = \mathcal{N}_{e1}^2 m_{\nu_1} + \mathcal{N}_{e2}^2 m_{\nu_2} + \mathcal{N}_{e3}^2 m_{\nu_3} \simeq \begin{cases} 0.004 \text{eV} & \text{NH}, \\ 0.048 \text{eV} & \text{IH}, \\ 0.23 \text{eV} & \text{QD}. \end{cases}
\]  

(8.8)

For direct prediction of half-life as a function of heavy sterile neutrino and its comparison with experimental data of ongoing search experiments, we derive the following analytic formula

\[
T_{0\nu}^{1/2} = K_{0\nu}^{-1} \times \frac{M^{2}_{\nu} M^{2}_{\delta}}{|\langle p^2 \rangle|^2 (M_{D,i})^4} \left[ 1 + a \frac{M^{2}_{\nu}}{M^{2}_{S2}} + b \frac{M^{2}_{\nu}}{M^{2}_{S3}} - \delta \right]^{-2},
\]  

(8.9)
Figure 8. Prediction of half life-time for neutrinoless double beta decay in this model in the $W_L^- - W_L^-$ channel as a function of lightest sterile neutrino mass $M_{S_1}$ for light NH neutrino masses (Yellow band), but due to sterile neutrino exchanges. The band of uncertainty is due to the uncertainty in the neutrino virtuality momentum $|p| = 120\text{MeV} - 200\text{MeV}$. The upper dashed-horizontal lines are predictions only due to light neutrino exchanges of NH and IH patterns of masses. The lower horizontal lines are lower bounds of three experimental groups [6–9].

\[ K_{0\nu} = 1.57 \times 10^{-25} \text{ yrs}^{-1} \text{ eV}^{-2} \] and
\[ a = \frac{M_{D_{12}}^2 M_{N_2}}{M_{D_{11}}^2 M_{N_1}}, \quad b = \frac{M_{D_{23}}^2 M_{N_3}}{M_{D_{11}}^2 M_{N_1}}, \quad \delta = \frac{m_{ee}^\nu M_{N_1}}{M_{D_{11}}^2 |p|^2}. \] (8.10)

This formula is different from the one obtained using type-II seesaw dominance in $SO(10)$ with TeV scale $Z^{\prime \prime}$ [85]. Using the predicted value of $M_D$ from eq.(5.9) and derived values of heavy RH Majorana neutrino mass matrix, $M_N = \text{diag}(115, 1750, 7500)$ GeV from the GUT-scale fit to the fermion masses we obtain from eq.(8.10)
\[ a = 1.666 - i 0.394, \quad b = -3.7815 - i 3.3456. \] (8.11)

For different values of the diagonal matrix $M = \text{diag}(M_1, M_2, M_3)$ consistent within the non-unitarity constraint eq.(6.8), and the $M_N = \text{diag}(115, 1785, 7500)$ GeV, we derive mass eigenvalues $\hat{M}_S = (M_{S_1}, M_{S_2}, M_{S_3})$ using the formula
\[ \hat{M}_S = -(M_1^2/M_{N_1}, M_2^2/M_{N_2}, M_3^2/M_{N_3}). \] (8.12)

The moduli of these eigenvalues and the corresponding elements of $M$ are given in Table 4. It is clear from eq.(8.9) that the half-life is a function of three mass eigenvalues $M_{S_1}$, $M_{S_2}$ and $M_{S_3}$ while all other parameters are known. For fixed values of $M_{S_2} = 50$ GeV and
Table 4. Eigenvalues of sterile neutrino mass matrix for different allowed $N - S$ mixing matrix elements.

$M_{S3} = 394$ GeV, we have plotted half-life against the lightest sterile neutrino mass $M_{S1}$, as shown in Fig.8 by neglecting the light neutrino exchange contribution in eq.(8.9) ($\delta = 0$).

It is evident from eq.(8.9) that for $M_{S3} \gg M_{S2} \gg M_{S1}$, a log($T_{1/2}$) vs log($M_{S1}$) would exhibit a linear behavior. The half-life for neutrino less double beta decay is presented by the blue-color band of Fig.8 which is due to existing uncertainty in the nuclear matrix elements as well as the resulting range of allowed values of $|\langle p \rangle| = 120$ MeV - 200 MeV. The two dashed horizontal lines represent half-life predictions for the standard hierarchical (NH) and inverted hierarchical (IH) cases by taking light neutrino exchange contribution only. The colored solid horizontal lines at the bottom of the figure represent recent experimental lower limits by three different groups [6–9]. It is quite clear from Fig.8 that the lightest of the three heavy sterile neutrino masses has the lower bound

$$|M_{S1}|_{\text{LNV}} \geq 14 \pm 4\text{GeV}. \quad (8.13)$$

Further eq.(8.9) also predicts that this lower bound would not be affected significantly as long as $M_{S3} \gg M_{S2} \gg M_{S1}$ which is easily satisfied by the type of solutions allowed by the non-unitarity constraint on $\eta_{\alpha\beta}$.

Including the light neutrino exchange contribution in the formula for NH, IH and QD cases ($\delta \neq 0$) and for fixed value of $|p| = 160$ MeV we have plotted the half-life as a function of $M_{S1}$ as shown in Fig.9 where the upper, lower, and the middle curves represent the QD, NH, and IH pattern of light neutrino masses, respectively. Because of the opposite sign of the light-neutrino and the sterile-neutrino exchange contributions, there is partial cancellation between the two effective mass parameters especially in the QD case resulting in a somewhat larger life-time compared to the NH case or the IH case. This figure gives the lower bound at $M_{S1} \gtrsim 14$ GeV for NH and IH cases as before, but $M_{S1} \gtrsim 12.5$ GeV
Figure 9. Same as Fig.8 with sterile neutrino exchanges but for NH, IH, and QD patterns of light neutrino masses. The value of neutrino virtuality momentum has been fixed at $|p| = 160$ MeV for all the three cases.

for the QD case. But when the whole range of uncertainty in the $|p^2|$ is included, this difference disappears.

We have checked that, although a naive extrapolation of the formula of eq. (8.9) for higher eigen values of $M_{S_i}$ ($i = 1, 2, 3$) resulting from larger values of $M_1, M_2, M_3$ gives a maximum in the half-life finally settling down at $T_{1/2} \sim 2 \times 10^{25}$ yrs., these solutions contradict the prediction of TeV scale $Z'$ boson accessible to LHC. They also correspond to LFV decay branching ratios substantially lower than those estimated here.

It is clear that even when the whole range of uncertainties in $|p|$ are included the three curves overlap although for a fixed value of $|p|$ the QD case gives a factor of nearly 7 times larger lifetime prediction compared to the other two cases especially when $M_{S_1} \simeq 20$ GeV. This suggests that even if the uncertainties in $|p|$, mainly emerging due to nuclear matrix elements is considerably reduced, detection of $0\nu\beta\beta$ decay life time in the QD case would require more accurate measurements, if the light neutrinos are quasidegenerate and $M_{S_1} = 15-20$ GeV. Because of the underlying gauged inverse nature of the inverse seesaw mechanism relying upon the validity of the constraints on the mass matrices $M_N \gg M \gg M_D, \mu_S$ and the non-unitarity constraint emerging from LFV decays, large values of $M_{S_1}$ beyond 20 GeV are disallowed.

Apart from these small and moderate cancellations between the two contributions, we do not find any large cancellation leading to large half-life inaccessible to ongoing $0\nu\beta\beta$ decay searches. We conclude that the sterile neutrino exchange contribution dominates the decay process in most of the regions of the parameter space especially for NH and IH cases for all allowed $M_{S_1}$ values. In the QD case this holds for $M_{S_1}$ values up to $\sim 20$ GeV.
allowed by the model.

9 SUMMARY AND DISCUSSIONS

We have implemented extended seesaw mechanism in a class of $SO(10)$ models containing one additional fermion singlet ($S$) per generation leading to TeV scale $Z'$ boson and heavy RH Majorana neutrino ($N$) masses via $U(1)_R \times U(1)_{B-L}$ gauge symmetry breaking generated through the Higgs representation $126_H$ while the $N - S$ mixing matrix $M$ is generated through the VEV of RH doublet Higgs contained in $16_H$. Inspite of the presence of the TeV scale RH neutrino mass matrix $M_N$, and naturally dominant Dirac neutrino mass matrix ($M_D$) in the model, the would-be large contribution due to type-I seesaw cancels out. The type-II seesaw contribution is damped out because of large parity violating scale and the TeV scale $B - L$ breaking. The formula for light left-handed neutrino masses and mixings are adequately well represented by the gauged inverse seesaw formula. The Dirac neutrino mass matrix $M_D$ that plays a crucial role in the inverse seesaw formula, non-unitarity effects and predictions of LFV decays and $0\nu\beta\beta$ decay is obtained by fitting the charged fermion masses and CKM mixings at the GUT scale for which the induced VEV of $\xi(2,2,15) \subset 126_H$ is utilized in addition to two separate Higgs doublets originating from $10_H^{(1,2)}$. The roles of two different types of $SO(10)$ structures corresponding to the presence of (i) a single representation $126_H$ leading to a diagonal structure of RH neutrino mass matrix, or (ii) two representations $126_H$ and $126'_H$ leading to a general structure of RH neutrino mass matrix, are discussed with their respective impact on the phenomenology of observable $n - \bar{n}$ oscillation. While the dominant new contribution to $0\nu2\beta$ decay in the $W_L - W_L$ channel due to sterile neutrino exchanges, saturates the current experimental limits arrived at various experimental groups, the branching ratios for LFV decays, and rare kaon decays are noted to be within the accessible ranges of ongoing search experiments. Using RG analysis, we have derived the lower bound on the lepto-quark gage boson mass mediating rare kaon decays to be $M_{\text{lepto}} \geq (1.53 \pm 0.06) \times 10^6$ GeV which is easily accommodated in the GUT scenario. The unification constraint on gauge couplings of the $SO(10)$ model is found to permit diquark Higgs scalar masses extending from $\sim (10-100)$ TeV leading to observable $n - \bar{n}$ oscillation while satisfying flavor physics constraints [86]saturating the lepto-quark gauge boson mass bound. These suggests that the model is also simultaneously consistent with observable rarekaon decay by ongoing search experiments.

Compared to the recent interesting proposal of ref.[26–28], although successful generation of baryon asymmetry of the universe has not been implemented so far in this model, we have one extra gauge boson accessible to LHC. Likewise, in our model the lepto-quark gauge boson mediated $K_L \rightarrow \mu\bar{e}$ is also accessible to ongoing search experiments. Whereas the new $B - L$ violating proton decay is predicted to be accessible in ref.[26–28], in our case it is $B - L$ conserving proton decay $p \rightarrow e^+\pi^0$. Whereas the type-I seesaw mechanism associated with high $B - L$ breaking scale is generally inaccessible to direct experimental tests, in our case the TeV-scale gauged inverse seesaw is directly verifiable. In the minimal
model the predicted values of the RH neutrino masses are also accessible for verification at LHC.

Even though the model is non-supersymmetric, it predicts similar branching ratios as in SUSY models for LFV processes like $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, and $\tau \rightarrow e\gamma$. Even for the Dirac phase $\delta = 0, \pi, 2\pi$ of the PMNS matrix, the model predicts the leptonic CP-violation parameter $J \approx 10^{-5}$ due to non-unitarity effects. We have explicitly derived a new formula for the half life of $0\nu\beta\beta$ decay as a function of the sterile neutrino masses in the model and derived the lower bound $M_{S_1} \geq 14 \pm 4$ GeV imposed by the current experimental limits on the half life. In this model as also in the model of ref.[36], the lifetime corresponding to Heidelberg Moscow experiment does not necessarily require the light neutrinos to be quasi-degenerate. We have checked that the analytic formula obtained for the half life for the dominant $0\nu\beta\beta$ in the $W_L-W_L$ channel of ref.[36], is analogous to the formula obtained in this work with almost the same lower bound on the lightest sterile neutrino mass when the decay amplitudes in the RH sector is neglected because of much heavier $W_R$ mass.

The predicted proton-lifetime in the minimal model is found to be $\tau_p(p \rightarrow e^+\pi^0) \simeq 5.05 \times 10^{35\pm1.0\pm0.34}$ yrs where the first(second) uncertainty is due to GUT-threshold effects(experimental errors). This lifetime is accessible to ongoing and planned experiments. We have noted significant reduction of the predicted lifetime, bringing the central value much closer to the current Super K. limit with $\tau_p(p \rightarrow e^+\pi^0) = 1.1 \times 10^{34}$ yrs$-5.05 \times 10^{35}$ yrs when the effect of a lighter bi-triplet Higgs contained in the representation $54_H \subset SO(10)$ is included. We conclude that even though the model does not have low-mass RH $W_{R}^\pm$ bosons in the accessible range of LHC, it is associated with interesting signatures on lepton flavor, lepton number and baryon number violations and rare kaon decays.
10 APPENDIX A

10.1 Estimation of experimental and GUT-threshold uncertainties on the unification scale

10.1.1 Analytic formulas

In contrast to other intermediate gauge symmetries, SO(10) model with $G_{224L}$ intermediate symmetry was noted to have the remarkable property that GUT threshold corrections arising out of superheavy masses or higher dimensional operators identically vanish on $\sin^2 \theta_W$ or the $G_{224L}$ breaking scale [41, 42, 45, 47]. We show how this property can be ensured in this model with precision gauge coupling unification while predicting vanishing GUT-threshold corrections on $M_P$, analytically, but with non-vanishing finite corrections on $M_{GUT}$. We derive the corresponding GUT threshold effects in $SO(10)$ model with three intermediate symmetry breaking steps, $G_{224L}$, $G_{224}$, and $G_{2113}$ between the GUT and the standard model whereas the uncertainties in the mass scales has been discussed in ref.[46] only with single intermediate breaking. The symmetry breaking chain under consideration is

\[
SO(10) \frac{\alpha''}{M_U} \xrightarrow{G_{224L}} \frac{\alpha''}{M_P} \xrightarrow{G_{224}} \frac{\alpha'}{M_C} \xrightarrow{G_{2113}} \frac{\alpha}{M_R^G} \xrightarrow{SO} G_{13},
\]

(10.1)

where $a''_i$, $a''$ and $a_i$ are, respectively, the one-loop beta coefficients for the gauge group $G_{2L2p}$, $G_{2L2p}$, $G_{2L1R1R}$, and $G_{SM} \equiv G_{2L1H}$. Following the formalism used in ref.[46, 47], one can write the expressions for two different contributions of $\sin^2 \theta_W (M_Z)$, and $\alpha_s (M_Z)$:

\[
16\pi \left( \alpha_s^{-1} - \frac{3}{8} \alpha_{em}^{-1} \right) = \mathcal{A}_P \ln \left( \frac{M_P}{M_Z} \right) + \mathcal{A}_U \ln \left( \frac{M_U}{M_Z} \right) + \mathcal{A}_C \ln \left( \frac{M_C}{M_Z} \right) + \mathcal{A}_0 \ln \left( \frac{M_0^R}{M_Z} \right) + f_M^U, \tag{10.2}
\]

where,

\[
\mathcal{A}_0 = (8a_{3C} - 3a_{2L} - 5a_Y) - (8a'_{3C} - 3a'_{2L} - 3a'_{1R} - 2a'_{B-L}),
\]

\[
\mathcal{A}_C = (8a''_{3C} - 3a''_{2L} - 3a''_{1R} - 2a''_{B-L}) - (6a''_{2C} - 3a''_{2L} - 3a''_{2R}),
\]

\[
\mathcal{A}_P = (6a''_{4C} - 3a''_{2L} - 3a''_{2R}) - (6a''_{4C} - 6a''_{2L}),
\]

\[
\mathcal{A}_U = (6a''_{4C} - 6a''_{2L}),
\]

\[
f_M^U = \lambda_M^U - \lambda_M^U.
\]

Similarly,

\[
16\pi \alpha_{em}^{-1} \sin^2 \theta_W \left( \frac{3}{8} \right) = \mathcal{B}_P \ln \left( \frac{M_P}{M_Z} \right) + \mathcal{B}_U \ln \left( \frac{M_U}{M_Z} \right) + \mathcal{B}_C \ln \left( \frac{M_C}{M_Z} \right) + \mathcal{B}_0 \ln \left( \frac{M_0^R}{M_Z} \right) + f_\theta^U, \tag{10.3}
\]

with

\[
\mathcal{B}_0 = (5a_{2L} - 5a_Y) - (5a'_{2L} - 3a'_{1R} - 2a'_{B-L}),
\]

\[
\mathcal{B}_C = (5a''_{2L} - 3a''_{1R} - 2a''_{B-L}) - (5a''_{2L} - 3a''_{2R} - 2a''_{4C}),
\]

\[
\mathcal{B}_P = (5a''_{2L} - 3a''_{2R} - 2a''_{4C}) - (2a''_{2L} - 2a''_{4C}),
\]

\[
\mathcal{B}_U = (2a''_{2L} - 2a''_{4C}),
\]

\[
f_\theta^U = \frac{1}{3} (\lambda_M^U - \lambda_M^U).
\]
It is well known that threshold effects at intermediate scales are likely to introduce discontinuities in the gauge couplings thereby destroying possibilities of precision unification. This fact has led us to restrict the model with vanishing intermediate scale threshold corrections by assuming relevant sub-multiplets to have masses exactly equal to their respective intermediate scales which is applicable to the intermediate scales $M_R^0, M_R^+$, and $M_C$ in the present work.

Denoting $C_0 = 16\pi \left(\alpha_s^{-1} - \frac{3}{8} \alpha_{em}^{-1}\right)$, and $C_1 = 16\pi \alpha_{em}^{-1} \left(\sin^2 \theta_W - \frac{3}{8}\right)$, one can rewrite the eq. (10.2), and eq. (10.3) for $M_P$ and $M_U$ as

$$A_P \ln \left( \frac{M_P}{M_Z} \right) + A_U \ln \left( \frac{M_U}{M_Z} \right) = D_0 = C_0 - A_C \ln \left( \frac{M_C}{M_Z} \right) - A_0 \ln \left( \frac{M_R^0}{M_Z} \right) - f'_U,$$  
(10.4)

$$B_P \ln \left( \frac{M_P}{M_Z} \right) + B_U \ln \left( \frac{M_U}{M_Z} \right) = D_1 = C_1 - B_C \ln \left( \frac{M_C}{M_Z} \right) - B_0 \ln \left( \frac{M_R^0}{M_Z} \right) - f'_U.$$

(10.5)

A formal solution for these two sets of eqns. (10.4), and (10.5),

$$\ln \left( \frac{M_U}{M_Z} \right) = \frac{D_1 A_P - D_0 B_P}{B_U A_P - A_U B_P},$$

(10.6)

$$\ln \left( \frac{M_P}{M_Z} \right) = \frac{D_0 B_U - D_1 A_U}{B_U A_P - A_U B_P}.$$  
(10.7)

In this present work, we derive two types of uncertainties in the mass scales of $SO(10)$ model; i.e, the first one comes from low energy parameters taken from their experimental errors and another one arising from the threshold corrections accounting the theoretical uncertainties in the mass scales due to heavy Higgs fields present at GUT scale. These two categories are presented below:

### 10.1.2 Uncertainties due to experimental errors in $\sin^2 \theta_W$ and $\alpha_s$

In eqns. (10.4) and (10.5) the low energy parameters are contained in $C_0$ and $C_1$. As a result, we have got further simplified relations relevant for experimental uncertainties, i.e, $\Delta (D_0) = \Delta (C_0)$ and $\Delta (D_1) = \Delta (C_1)$, and hence,

$$\Delta \ln \left( \frac{M_U}{M_Z} \right)_{\text{expt.}} = \frac{\Delta (C_1) A_P - \Delta (C_0) B_P}{B_U A_P - A_U B_P} = \left[ (16\pi) \alpha_{em}^{-1} (\delta \sin^2 \theta_W) \right] A_P - \left[ \frac{(16\pi) \alpha_s^{-1}}{\alpha_s^2} (\delta \alpha_s) \right] B_P,$$

(10.8)

$$\Delta \ln \left( \frac{M_P}{M_Z} \right)_{\text{expt.}} = \frac{\Delta (C_0) B_U - \Delta (C_1) A_U}{B_U A_P - A_U B_P} = \left[ \frac{(16\pi) \alpha_s^{-1}}{\alpha_s^2} (\delta \alpha_s) \right] B_U - \left[ (16\pi) \alpha_{em}^{-1} (\delta \sin^2 \theta_W) \right] A_U,$$

(10.9)

where, the errors in the experimental values on electroweak mixing angle $\sin^2 \theta_W$ and strong coupling constant $\alpha_s$ as $\sin^2 \theta_W = 0.23102 \pm 0.00005, \; \alpha_s = 0.118 \pm 0.003$ giving $\delta \alpha_s = \pm 0.003$ and $\delta \sin^2 \theta_W = \mp 0.00005$. 

- 30 -
10.1.3 Uncertainties in $M_U$ with vanishing correction on $M_P$

In the present work, we have considered minimal set of Higgs fields belonging to a larger $SO(10)$ Higgs representation implying other Higgs fields which do not take part in symmetry breaking will automatically present at GUT scale. Since we can not determine the masses of these heavy Higgs bosons and, hence, they introduce uncertainty in other mass scales $M_P$ and $M_U$ via renormalization group equations resulting source of GUT threshold uncertainty in our predictions for proton life time. For this particular model, the GUT threshold corrections to D-parity breaking scale and unification mass scale is presented below

\[
\Delta \ln \left( \frac{M_U}{M_Z} \right) \bigg|_{\text{GUT Th.}} = \frac{\Delta (D_1) A_P - \Delta (D_0) B_P}{B_U A_P - A_U B_P} - \frac{f_U^U}{6 \left( a^m_{2L} - a^m_{4C} \right)}, \quad (10.10)
\]

\[
\Delta \ln \left( \frac{M_P}{M_Z} \right) \bigg|_{\text{GUT Th.}} = \frac{\Delta (D_0) B_U - \Delta (D_1) A_U}{B_U A_P - A_U B_P} = \frac{B_U f_M^U - A_U f_g^U}{24 \left( a^m_{2L} - a^m_{4C} \right) \left( a^m_{2L} - a^m_{4C} \right)} = 0. \quad (10.11)
\]

The last step resulting in vanishing GUT-threshold correction analytically follows by using expressions for $f_M^U, f_g^U, B_U$ and $A_U$ derived in 10.1.1. This was proved in ref.[45].
| Group $G_I$          | Higgs content                                                                 | $a_i$ | $b_{ij}$               |
|---------------------|------------------------------------------------------------------------------|-------|------------------------|
| $G_{1_{2L3C}}$      | $\Phi(\frac{1}{2}, 2, 1)_{10}$                                              | 41/10 | (199/50 27/10 44/5)    |
|                     |                                                                               | 9/10  | 35/6 12                |
|                     |                                                                               | 11/10 | 9/2 12 -26             |
| $G_{1_{B-L1R2L3C}}$| $\Phi_1(0, \frac{1}{2}, 2, 1)_{10} \oplus \Phi_2(0, \frac{1}{2}, 2, 1)_{10}$ | 37/8  | (269/16 63/8 9/4 4)    |
|                     | $\Delta_R(-1, 1, 1, 1)_{126} \oplus \chi_R(\frac{1}{2}, \frac{1}{2}, 1, 1)_{16}$ | 57/12 | 63/8 33/4 12           |
|                     |                                                                               | 3/2   | 1 8 12                 |
|                     |                                                                               | 1/2   | 3/2 9/2 -26            |
| $G_{2_{2R4C}}$      | $\Phi_1(2, 2, 1)_{10} \oplus \Phi_2(2, 2, 1)_{10}$                          | 8/3   | 37/3 6 45/2            |
|                     | $\Delta_R(1, 3, \overline{10})_{126} \oplus \chi_R(1, 2, \overline{1})_{16}$| 29/3  | 6 1103/3 1275/2        |
|                     | $\sigma_R(1, 3, 15)_{210}$                                                   | 16/3  | 9/2 255/2 736/3        |
| $G_{2_{2R4CD}}$     | $\Phi_1(2, 2, 1)_{10} \oplus \Phi_2(2, 2, 1)_{10}$                          | 44/3  | (1298/3 51 1755/2)     |
|                     | $\Delta_R(3, 1, 10)_{126} \oplus \Delta_R(1, 3, \overline{10})_{126}$      | 44/3  | 51 1298/3 1755/2       |
|                     | $\chi_L(2, 1, 4)_{16} \oplus \chi_R(1, 2, \overline{1})_{16}$              | 16/3  | 351/2 351/2 1403/2     |

Table 5. One and two loop beta coefficients for different gauge coupling evolutions described in text taking the second Higgs doublet at $\mu \geq 5$ TeV.

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