$K$ induced formation of the $f_0(980)$ and $a_0(980)$ resonances on proton targets

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We perform a calculation of the cross section for nine reactions induced by $K$ scattering on protons. The reactions studied are $K^-p \rightarrow \Lambda\pi^+\pi^-$, $K^-p \rightarrow \Sigma^0\pi^+\pi^-$, $K^-p \rightarrow \Lambda\pi^+\pi^-$, $K^-p \rightarrow \Sigma^0\pi^+\pi^-$, $K^-p \rightarrow \Sigma^+\pi^-\eta$, $K^-p \rightarrow \Lambda\pi^+\pi^-$, $K^-p \rightarrow \Sigma^0\pi^+\pi^-$, $K^-p \rightarrow \Sigma^+\pi^-\eta$. We find that in the reactions producing $\pi^+\pi^-$ a clear peak for the $f_0(980)$ resonance is found, while no trace of $f_0(500)$ appears. Similarly, in the cases of $\pi\eta$ production a strong peak is found for the $a_0(980)$ resonance, with the characteristic strong cusp shape. Cross sections and invariant mass distributions are evaluated which should serve, comparing with future data, to test the dynamics of the chiral unitary approach used for the evaluations and the nature of these resonances.

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I. INTRODUCTION

Kaon beams are becoming a good source for new investigations in hadron physics. At intermediate energies J-PARC offers good intensity secondary kaon beams up to about 2 GeV/c [1, 2]. DAPHNE at Frascati provides low energy kaon beams [3, 4]. Very recently plans have been made for a secondary meson beam Facility at Jefferson Lab, which includes kaons, both charged and neutral [5]. One of the aims is to produce hyperons ($\equiv Y$), which are not as well studied as nucleons or deltas [6], and also cascade states, which are even less known [5, 7, 8]. In the present paper we address a different problem using kaon beams, which is the kaon induced production of the $f_0(980)$ and $a_0(980)$ resonances. The reactions proposed are $Kp \rightarrow \pi\pi Y$ and $Kp \rightarrow \pi\eta Y$, which produce the $f_0(980)$ and $a_0(980)$ resonances respectively. These two resonances are the most emblematic scalar resonances of low energy which have generated an intense debate as to their nature, as $q\bar{q}$, tetraquarks, meson molecules, glueballs, dynamically generated states, etc. [9]. By now it is commonly accepted that these mesons are not standard $q\bar{q}$ states but "extraordinary" states [10]. The coupling of some original $q\bar{q}$ state to meson-meson components demanding unitarity has as a consequence that the meson cloud eats up the original seed becoming the largest component [11-14]. The advent of chiral dynamics in its unitarized form in coupled channels, the chiral unitary approach, has brought new light into the subject and the resonances appear from the interaction of pseudoscalar mesons, usually taken into account by coupled Bethe Salpeter equations with a kernel, or potential [15-18] extracted from the chiral Lagrangians [19], or equivalent methods like the inverse amplitude method [20, 21]. A recent review on this issue makes a detailed comparative study of work done on these issues, strongly supporting this latter view [22].

The study of $B$ and $D$ decays [23, 24] has also offered a new valuable source of information on these states and has stimulated much theoretical work [25-32]. Yet, little is done in reactions involving baryons, with the exception of $f_0(980)$ photoproduction, done in Refs. [33, 34], for which predictions had been done in Ref. [35], which also had been addressed theoretically lately [36, 37]. With this scarce information, the use of proton targets to produce these states, now induced by kaons, is bound to be a new good source of information which should narrow our scope on the nature of these resonances.

One of the outcomes of the chiral unitary theories is that the $f_0(980)$ couples strongly to $KK$ although it decays into $\pi\pi$ which is an open channel. On the other hand, the $a_0(980)$ couples both to $KK$ and to $\pi\eta$, which becomes the decay channel. The use of kaon beams to produce these resonances offers one new way in which to test these ideas, since the original kaon, together with a virtual kaon that will act as a mediator of the process, will produce the resonances using the entrance channel to which they couple most strongly. We will study different processes, having $f_0(980)$ or $a_0(980)$ in the final state, together with a $\Lambda$ or a $\Sigma$ and we will use both $K^-$ or $\bar{K}^0$ to initiate the reaction. In total we study nine reactions for which we evaluate $d^2\sigma/dM_{\text{invcos}}(\theta)$ and make predictions for the dependence on the energy of the beam, the invariant mass of the final two mesons and the scattering angle, $\theta$.

The contents of the article are organized as follows. In Sec. [11] we revisit the chiral unitary approach for the $f_0(980)$ and $a_0(980)$ resonances. In Sec. [11] we present...
the formalism and main ingredients of the model. In Sec. IV we present our main results and, finally, in the last section we summarize our approach and main findings.

II. THE CHIRAL UNITARY APPROACH FOR THE \( f_0(980) \) AND \( a_0(980) \) RESONANCES

Following Refs. 13, 38, we start from the coupled channels, \( \pi^+\pi^-, \pi^0\pi^0, \pi^0\eta, K^+K^-, K^0\bar{K}^0 \), and evaluate the transition potentials from the lowest order chiral Lagrangians of Ref. 19. Explicit expressions for \( s \) wave, which we consider here, can be seen in Refs. 25, 26. Then, by using the on shell factorization of the Bethe-Salpeter equation in coupled channels 39, 40, one has in matrix form

\[
T = V + VGT; \quad T = \left[1 - VG\right]^{-1}V, \quad (1)
\]

where \( V \) is the transition potential and \( G \) the loop function for two intermediate meson propagators which must be regularized. Following Ref. 22 we take a cut off in three momenta of 600 MeV, demanded when the \( \eta\eta \) channel is considered explicitly. Eq. (1) provides the transition \( T \) matrix, \( t_{ij} \), from any one to the other channels, and we shall only need the \( t_{K^+K^-\to\pi^+\pi^-}, t_{K^0\bar{K}^0\to\pi^0\pi^0}, t_{K^+\bar{K}^0\to\pi^0\eta}, t_{K^0\bar{K}^0\to\pi^0\eta} \) matrix elements. The first two matrix elements contain a pole associated to the \( f_0(980) \), while the latter two contain the pole of the \( a_0(980) \), although this resonance is quite singular and appears as a big cusp around the \( K\bar{K} \) threshold, both in the theory as in experiments 41, 42. The \( f_0(980) \) couples strongly to \( K\bar{K} \) channel with \( \pi\pi \) the decay channel, and the \( a_0(980) \) couples strongly to \( K\bar{K} \) and \( \pi\eta \) channels.

III. FORMALISM

From the perspective that the \( f_0(980) \) and \( a_0(980) \) resonances are generated from the meson-meson interaction, the picture for \( f_0(980) \) and \( a_0(980) \) anki-kaon induced production proceeds via the creation of one \( K \) by the \( Kp \) initial state in a primary step and the interaction of the \( K \) and \( K \) generating the resonances. This is provided by the mechanism depicted in Fig. 1 by means of a Feynman diagram.

Let us study the \( K^-p \to \Lambda(\Sigma^0)\pi^+\pi^- (\pi^0\eta) \) as a reference. From this reaction we shall be able to construct the other five reactions with minimal changes. In this case, we want to couple the \( K^- \) with another \( K^+ \) to form the resonances. The first thing one observes is that one of the kaons (the \( K^+ \)) is necessarily off shell, since neither the \( \Lambda \) nor the \( \Sigma^0 \) can decay into \( Kp \). Then, in principle one needs the \( K^+K^- \to \pi^+\pi^- (\pi^0\eta) \) amplitude with the \( K^+ \) leg off shell, which can be evaluated from the chiral Lagrangians. Yet, the structure of these Lagrangians is such that the potential can be written as 13

![Figure 1: Feynman diagram for the \( Kp \to \pi\pi(\pi^0\eta)Y \) reaction.](image1)

![Figure 2: Contact term stemming from the Feynman diagram of Fig. 1 from the off shell part of the \( K^+K^- \to \pi^+\pi^- (\pi^0\eta) \) transition potential.](image2)

However, the chiral Lagrangian for meson baryon 43, 44, upon expanding on the number of pion fields, contains also contact terms with the same topology as the one generated from the off shell part of the amplitude 45 which cancel this latter term. The result is that one can take just the on shell \( K\bar{K} \to \pi\pi(\pi\eta) \) amplitude in the diagram of Fig. 1 and ignore the contact terms stemming from the meson baryon Lagrangian. These cancellations were observed before in Ref. 46 in the study of the \( \pi N \to \pi N \) reaction and in Ref. 47 for the study of the pion cloud contribution to the kaon nucleus optical potential.

The other ingredient that we need for the evaluation of the diagram of Fig. 1 is the structure of the
Yukawa meson-baryon-baryon vertex. Using chiral Lagrangians and keeping linear terms in the meson field, the Lagrangian can be written as

\[ \mathcal{L} = \frac{D}{2} \langle \bar{B} \gamma^\mu \gamma_5 |u_\mu, B \rangle + \frac{F}{2} \langle \bar{B} \gamma^\mu \gamma_5 |u_\mu, B \rangle \]

\[ = D + F \langle \bar{B} \gamma^\mu \gamma_5 u_\mu \rangle + \frac{D - F}{2} \langle \bar{B} \gamma^\mu \gamma_5 Bu_\mu \rangle, \tag{3} \]

where the symbol \( <> \) stands for the trace of SU(3). The term linear in meson field gives

\[ u_\mu \simeq -\sqrt{2} \frac{\partial_\mu \Phi}{f} \tag{4} \]

with \( f \) the pion decay constant, \( f = f_\pi = 93 \text{ MeV} \), and \( \Phi, B \) the meson and baryon SU(3) field matrices given by

\[ \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^+ \\ \frac{1}{\sqrt{2}} \pi^- & - \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 & - \frac{1}{\sqrt{2}} \pi^0 - \frac{1}{\sqrt{6}} \eta \\ \frac{1}{\sqrt{2}} \Sigma^0 & - \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\ \frac{1}{\sqrt{2}} \Sigma^- & - \frac{1}{\sqrt{2}} \Sigma^- & \Sigma^- & n \end{pmatrix}, \tag{5} \]

\[ B = \begin{pmatrix} \frac{1}{\sqrt{2}} \bar{\Sigma}^0 + \frac{1}{\sqrt{6}} \Lambda & \frac{1}{\sqrt{2}} \bar{\Sigma}^0 + \frac{1}{\sqrt{6}} \Lambda & 0 & 0 \\ \frac{1}{\sqrt{2}} \bar{\Sigma}^- & - \frac{1}{\sqrt{2}} \bar{\Sigma}^- & 0 & 0 \end{pmatrix}. \tag{6} \]

We take \( F = 0.795, D = 0.465 \) in this work at the tree level, consistent with the findings of Ref. \[48\]. The explicit expression of the SU(3) matrix elements of Eq. \[4\] leads to the following expression

\[ \mathcal{L} \to i \left( \alpha \frac{D + F}{2f} + \beta \frac{D - F}{2f} \right) \bar{u}(p', s'_B) \gamma_5 u(p, s_B), \tag{7} \]

where \( u(p, s_B) \) and \( \bar{u}(p', s'_B) \) are the ordinary Dirac spinors of the initial and final baryons, respectively, and \( p, s_B \) and \( p', s'_B \) are the four-momenta and spins of the baryons, while \( q = p - p' \) is the four momentum of the meson. The values of \( \alpha \) and \( \beta \) are tabulated in Table \[\text{I}\] of the paper.

**TABLE I: Coefficients for the \( \bar{K}N \) Y couplings of Eq. \[7\].**

|             | \( K^- p \to \Lambda \) | \( K^- p \to \Sigma^0 \) | \( K^- n \to \Sigma^- \) |
|-------------|--------------------------|---------------------------|--------------------------|
| \( \alpha \) | \( -\sqrt{2} \)          | 0                         | 1                        |
| \( \beta \)  | \( -\sqrt{3} \)          | 1                         | \( \sqrt{2} \)           |
| \( K^0 n \to \Lambda \) | \( -\sqrt{2} \)          | 0                         | 0                        |
| \( \beta \)  | \( -1 \)                  | \( \sqrt{3} \)            | \( \sqrt{2} \)           |

Altogether we can write the amplitude for the diagram of Fig. \[\text{I}\] as

\[ T = -it_{KK \to MM} \frac{1}{q^2 - m_K^2} \left( \alpha \frac{D + F}{2f} + \beta \frac{D - F}{2f} \right) \]

\[ \times \bar{u}(p', s'_B) \gamma_5 u(p, s_B) F(q^2), \tag{8} \]

where we have added the customary Yukawa form factor that we take of the form

\[ F(q^2) = \frac{\Lambda^2}{\Lambda^2 - q^2}, \tag{9} \]

with typical values of \( \Lambda \) of the order of 1 GeV.

The sum and average of \( |T|^2 \) over final and initial polarization of the baryons is easily written as

\[ \sum_{s_p} \sum_{s'_{\Lambda/\Sigma}} |T_{i\ell}|^2 = |T_{K\bar{K} \to MM}|^2 \left( \frac{1}{q^2 - m_K^2} \right)^2 \times \]

\[ \frac{(M_p + M')^2}{4M_pM'} \left[ (M_p - M')^2 - q^2 \right] \times \]

\[ \left( \alpha_i \frac{D + F}{2f} + \beta_i \frac{D - F}{2f} \right)^2 F^2(q^2), \tag{10} \]

where \( M_p, M' \) are the masses of the proton and the final baryon (\( \Lambda \) or \( \Sigma \)). The subindex \( i \) stands for different reactions.

We can write \( q^2 \) in terms of the variables of the external particles and have

\[ q^2 = M_p^2 + M'^2 - 2EE' + 2|\vec{p}| |\vec{p}'| \cos \theta, \tag{11} \]

where \( \vec{p}, \vec{p}' \) and \( E, E' \) are the momenta and energies of the proton and the final baryon, and \( \theta \) is the angle between the direction of the initial and final baryon, all of them in the global center of mass frame (CM). The \( \vec{p}, \vec{p}' \) and \( E, E' \) have the form as

\[ |\vec{p}| = \frac{\lambda^{\frac{1}{2}} |s, m_K^2, M_p^2|}{2\sqrt{s}}, \tag{12} \]

\[ |\vec{p}'| = \frac{\lambda^{\frac{1}{2}} |s, M_p^2, M'^2|}{2\sqrt{s}}, \tag{13} \]

\[ E = \sqrt{M_p^2 + |\vec{p}|^2}, \tag{14} \]

\[ E' = \sqrt{M'^2 + |\vec{p}'|^2}, \tag{15} \]

where \( s \) is the invariant mass square of the \( \bar{K}p \) system and \( \lambda \) is the Källen function with \( \lambda(x, y, z) = (x - y - z)^2 - 4yz \).

We can write the differential cross section as

\[ \frac{d^2\sigma}{dM_{\text{inv}}d\cos\theta} = \frac{M_pM'}{32\pi^3} \frac{|\vec{p}'|^2 |\vec{p}|}{s} \sum_{s_p} \sum_{s'_{\Lambda/\Sigma}} |T|^2, \tag{16} \]

with \( |\vec{p}'| \) the momentum of one of the mesons in the frame where the two final mesons are at rest,

\[ |\vec{p}'| = \frac{\lambda^{\frac{1}{2}} |M_{\text{inv}}^2, m_1^2, m_2^2|}{2M_{\text{inv}}}, \tag{17} \]

where \( M_{\text{inv}} \) is the invariant mass of the two mesons system, and \( m_1 \) and \( m_2 \) are the masses of the two mesons, respectively. Note that the \( KK \to MM \) scattering amplitudes \( t_{KK\to MM} \) depend on \( M_{\text{inv}} \) only.
We want to study nine reactions

\[ K^- p \rightarrow \Lambda \pi^+ \pi^- \], \( K^- p \rightarrow \Sigma^0 \pi^+ \pi^- \], \( K^- p \rightarrow \Lambda \pi^0 \eta \], \( K^- p \rightarrow \Sigma^0 \pi^0 \eta \], \( K^- p \rightarrow \Sigma^0 \pi^0 \eta \], \( K^0 p \rightarrow \Sigma^0 \pi^+ \pi^- \], \( K^0 p \rightarrow \Sigma^0 \pi^0 \eta \], \( K^0 p \rightarrow \Sigma^0 \pi^0 \eta \].

The Yukawa vertices for KBB are summarized in Table I. The \( K\bar{K} \rightarrow MM \) amplitudes are discussed above. However, only the \( I_3 = 0 \) components are studied there, corresponding to zero charge. We have three cases with \( \pi \eta \) where the charge is non zero, \( K^- p \rightarrow \Sigma^+ \pi^- \eta \], \( K^0 p \rightarrow \Lambda \pi^+ \eta \] and \( K^0 p \rightarrow \Sigma^0 \pi^+ \eta \]. We can easily relate the \( K\bar{K} \rightarrow \pi \eta \) amplitudes to the \( K^+ K^- \rightarrow \pi^0 \eta \) which is evaluated in the case of zero charge, using isospin symmetry. Indeed, recalling the phases \( |K^- \rangle = |1/2, -1/2 \rangle \), \( |\pi^\pm \rangle = -|1, 1 \rangle \), \( |\pi^- \rangle = |1, -1 \rangle \), we can write in terms of the total isospin

\[ \begin{align*}
|K^+ K^- \rangle &= -\frac{1}{\sqrt{2}} |1, 0 \rangle - \frac{1}{\sqrt{2}} |0, 0 \rangle, \\
|K^0 K^- \rangle &= -|1, -1 \rangle, \quad |K^+ K^0 \rangle = |1, 1 \rangle, \\
|\pi^+ \eta \rangle &= -|1, 1 \rangle, \quad |\pi^- \eta \rangle = |1, -1 \rangle,
\end{align*} \]

and then we find

\[ \begin{align*}
t_{K^+ K^- \rightarrow \pi^0 \eta} &= -\frac{1}{\sqrt{2}} t_{K^0 K^- \rightarrow \pi^0 \eta}, \\
t_{K^0 K^- \rightarrow \pi^- \eta} &= \sqrt{2} t_{K^+ K^- \rightarrow \pi^- \eta}, \\
t_{K^0 K^+ \rightarrow \pi^+ \eta} &= \sqrt{2} t_{K_1 K^- \rightarrow \pi^+ \eta}.
\end{align*} \]

With these ingredients we will use Eq. (16) to evaluate the cross section in each case, and all we must do is change the \( t_{K\bar{K},MM} \) in each case and the values of \( \alpha \) and \( \beta \). These magnitudes are summarized in Table I.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Reaction & \( t_{K\bar{K},MM} \) & \( \alpha \) & \( \beta \) & Resonance \\
\hline
\hline
\( K^- p \rightarrow \Lambda \pi^+ \pi^- \) & \( t_{K^- K^0 \rightarrow \pi^+ \pi^-} \) & \(-\frac{2}{\sqrt{3}}\) & \( \frac{1}{\sqrt{3}} \) & \( f_0(980) \) \\
\( K^- p \rightarrow \Sigma^0 \pi^+ \pi^- \) & \( t_{K^- K^0 \rightarrow \pi^+ \pi^-} \) & \( 0 \) & \( 1 \) & \( f_0(980) \) \\
\( K^- p \rightarrow \Lambda \pi^0 \eta \) & \( t_{K^- K^0 \rightarrow \pi^0 \eta} \) & \(-\frac{2}{\sqrt{3}}\) & \( \frac{1}{\sqrt{3}} \) & \( a_0(980) \) \\
\( K^- p \rightarrow \Sigma^0 \pi^0 \eta \) & \( t_{K^- K^0 \rightarrow \pi^0 \eta} \) & \( 0 \) & \( 1 \) & \( a_0(980) \) \\
\( K^- p \rightarrow \Sigma^+ \pi^- \eta \) & \( \sqrt{2} t_{K^+ K^- \rightarrow \pi^0 \eta} \) & \( 0 \) & \( \sqrt{2} \) & \( a_0(980) \) \\
\( K^0 p \rightarrow \Lambda \pi^+ \eta \) & \( \sqrt{2} t_{K^- K^0 \rightarrow \pi^0 \eta} \) & \(-\frac{2}{\sqrt{3}}\) & \( \frac{1}{\sqrt{3}} \) & \( a_0(980) \) \\
\( K^0 p \rightarrow \Sigma^0 \pi^+ \eta \) & \( \sqrt{2} t_{K^- K^0 \rightarrow \pi^0 \eta} \) & \( 0 \) & \( 1 \) & \( a_0(980) \) \\
\( K^0 p \rightarrow \Sigma^+ \pi^+ \pi^- \) & \( t_{K^0 K^0 \rightarrow \pi^+ \pi^-} \) & \( 0 \) & \( \sqrt{2} \) & \( f_0(980) \) \\
\( K^0 p \rightarrow \Sigma^+ \pi^0 \eta \) & \( t_{K^0 K^0 \rightarrow \pi^0 \eta} \) & \( 0 \) & \( 2 \) & \( f_0(980) \) \\
\hline
\end{tabular}
\end{table}

**IV. RESULTS**

We have a dependence of the cross section in the energy, \( M_{inv} \), and scattering angle \( \theta \) given by Eq. (11). We first evaluate the cross section for \( \theta = 0 \), in the forward direction. In Fig. 3 we show the numerical results of \( d\sigma/dM_{inv} \cos \theta \) for \( \cos(\theta) = 1 \) as a function of \( M_{inv} \) of the \( \pi^+ \pi^- \) for \( K^- p \rightarrow \Lambda (\Sigma^0) \pi^+ \pi^- \) reactions. We have chosen \( \sqrt{s} = 2.4 \) GeV, corresponding to the \( K^- \) momentum \( p_K = 2.42 \) GeV in the laboratory frame. 1 One can see that there is a clear peak around \( M_{inv} = 980 \) MeV which is the signal for the \( f_0(980) \) resonance that was produced by the initial \( K^+ K^- \) coupled channel interactions and decaying into \( \pi^+ \pi^- \) channel. On the other hand, the magnitude of the cross section for \( \Lambda \) production is of the order of 10 times larger than for \( \Sigma^0 \) production, because the coupling of \( K \Lambda \) is stronger than the \( K \Sigma \) coupling.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Theoretical predictions for \( S \) wave \( \pi^+ \pi^- \) mass distributions for \( K^- p \rightarrow \Lambda (\Sigma^0) \pi^+ \pi^- \) reactions at \( \sqrt{s} = 2.4 \) GeV and \( \cos(\theta) = 1 \).}
\end{figure}

In Fig. 4 we show the numerical results of \( d\sigma/dM_{inv} \cos \theta \) for \( \cos(\theta) = 1 \) as a function of \( M_{inv} \) of the \( \pi \eta \) for \( K^- p \rightarrow \Lambda (\Sigma^0) \pi^0 \eta \) and \( K^- p \rightarrow \Sigma^+ \pi^- \eta \) reactions. In this case we see also a clear peak/cusp around \( M_{inv} = 980 \) MeV which corresponds to the \( a_0(980) \) state.

Similarly, we show our results for \( K^0 p \) reactions in Fig. 5. One can see again the clear peaks for \( a_0(980) \) and \( f_0(980) \) resonances around \( M_{inv} = 980 \) MeV.

In all the reactions mentioned above, we observe clear peaks for the \( f_0(980) \) in the case of the \( \pi^+ \pi^- \) production or for the \( a_0(980) \) in the case of \( \pi\eta \) production. It is remarkable that in the case of the \( f_0(980) \) production there is no trace of the \( f_0(500) \) \( (\sigma) \) production. This is reminiscent of what happens in \( B^0 \rightarrow J/\psi \pi^+ \pi^- \), where a clear peak is seen for the \( f_0(980) \) but no trace is observed of the \( f_0(500) \). 23

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1 In the laboratory frame, \( s = m_K^2 + m_p^2 + 2m_p \sqrt{m_K^2 + p_K^2} \).
FIG. 4: Theoretical predictions for $S$ wave $\pi\eta$ mass distributions for $K^-p \to \Lambda(\Sigma^0)\pi^0\eta$ and $K^-p \to \Sigma^+\pi^-\eta$ reactions at $\sqrt{s} = 2.4$ GeV and $\cos(\theta) = 1$.

FIG. 5: Theoretical predictions for $S$ wave $\pi\eta$ and $\pi^+\pi^-$ mass distributions for $K^0 p \to \Lambda(\Sigma^0)\pi^+\eta$ and $K^- p \to \Sigma^+\pi^-\eta(\pi^+\pi^-)$ reactions at $\sqrt{s} = 2.4$ GeV and $\cos(\theta) = 1$.

It is much as a cusp around the $K\bar{K}$ threshold, but with a large strength. As we remarked earlier, this feature is common to all reactions where the $a_0(980)$ is produced with good statistics [41, 42].

Furthermore, in Figs. 6 to 8 we show the results for $d\sigma/dM_{inv}d\cos\theta$ for the $Kp$ reactions at the peak of the invariant mass, $f_0(980)$, $a_0(980)$ respectively, as a function of $\cos\theta$. Because we considered only the contributions from the $t$ channel $K$ exchange, the reactions peak forward and one can see a full down of about of a factor 10 in the cross section from forward to backward angles, where contributions from $s$ and $u$ channels could be dominant.

Finally, we now fix $M_{inv} = 980$ MeV at the peak of the resonance and $\cos\theta = 1$ and look at the dependence of the cross section with the energy of the $K$ beam. Because the $\Lambda$ production is larger than the $\Sigma$ production, we show only the results for the $\Lambda$ production in Fig. 9. We observe that the cross section grows fast from the reaction threshold and reaches a peak around $p_K = 2.5$ GeV.

V. CONCLUSIONS

In this work, we study the production of $f_0(980)$ and $a_0(980)$ resonances in the $Kp$ reaction with the picture that these two resonances are dynamically generated within the coupled pseudoscalar-pseudoscalar channels interaction in $I = 0$ and 1, respectively. This is the first evaluation of the cross section for these reactions. In the cases of $\pi^+\pi^-$ production we find a neat peak for the $f_0(980)$ production and no production of the $f_0(500)$. This feature is associated to the fact that the resonance is created from $K\bar{K}$ and the $f_0(980)$ has a strong coupling $K\bar{K}$ while the $f_0(500)$ has a very small production.

offered an explanation for this fact. Indeed, in this reaction at the quark level one produces $cc$, that makes the $J/\psi$, and a $s\bar{s}$ pair. This pair hadronizes into two mesons which are not $\pi\pi$, but mostly $K\bar{K}$ or $\eta\eta$. Then these particles undergo final state interaction producing the resonances. However, the $K\bar{K}$ couples strongly to the $f_0(980)$ resonance and very weakly to the $f_0(500)$, and this explains the observed features. In this case we have the $K\bar{K}$ producing the resonances and, similarly, we find a production of the $f_0(980)$ and not of the $f_0(500)$.

The reactions with $\pi\eta$ in the final state produce the $a_0(980)$ resonance. It is interesting to observe the shape.
that the coupling to this component. Thus, in spite of the fact that the $f_0(980)$ is observed in the $\pi^+\pi^-$, to which the $f_0(500)$ couples strongly, one finishes with a negligible signal for $f_0(500)$ in this reaction. This feature is also observed in the $B_s \to J/\psi\pi^+\pi^-$ reaction and we find a natural explanation of both reactions within the chiral unitary approach to the nature of these resonances. It would be good to have the reactions proposed implemented in actual experiments to narrow the scope on possible interpretations of the nature of these resonances. Some alternative explanations for the features observed in the $B_s \to J/\psi\pi^+\pi^-$ reaction are given for instance in Ref. [49] and would be good to see what these pictures would predict for the reactions studied here.

The reactions with the $\pi\eta$ production give rise also to a clear peak corresponding to the $a_0(980)$. This resonance appears as border line in the chiral unitary approach, corresponding to a state slightly unbound, or barely bound. The fact is that shows up clearly in a form of a strong cusp around the $K\bar{K}$ threshold, and this feature is observed in recent experiments with large statistics. It would be good to see what happens when the experiment is done. We should also note that our theoretical approach provides the absolute strength for both the $f_0(980)$ and $a_0(980)$ production and this is also a consequence of the theoretical framework that generates dynamically these two resonances.

We have assumed a $t$-channel dominance, based on the strong coupling of the resonances to $K\bar{K}$. This has as a consequence that the nine reactions that we have studied have a definite weight, the largest differences coming from the Yukawa MBB couplings which are well known. Comparison of the strength of these reactions could serve to assert the dominance of the production model that we have assumed.

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