Cost Optimization of Two-Dimensional Warranty Products under Preventive Maintenance

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1. Introduction

During the current warranty service, in order to reduce the cost of warranty and improve availability, it is usually necessary to carry out preventive maintenance after a certain period of product operation. At present, for many products, the warranty service has gradually changed from the corrective maintenance strategy to a preventive maintenance strategy. In many literature studies, the warranty strategies involving preventive maintenance have been studied [1–6]. For 2D warranty products which deteriorate with time and usage (for example, a new vehicle has a warranty period of 2 years or 50,000 km, and the warranty period is terminated regardless of which period is reached first), such products have the characteristics of high price and high requirement for product availability, so preventive maintenance is very necessary. Preventive maintenance plays an important role in restoring and maintaining product reliability. For 2D warranty products, proper preventive maintenance can affect the failure rate of products, thus changing the number of product failure during the warranty period and reducing maintenance cost. At present, for products used by large-scale industrial and mining enterprises, in order to meet the needs of product maintenance in the daily use process, the user usually has a certain maintenance force. However, during the warranty period, due to the influence of human resources, material resources, technology, and other factors, there is still a certain gap between the user and the manufacturer in terms of maintenance force. During the warranty period, only preventive maintenance performed by the user may lead to the problem of low reliability recovery and high maintenance cost. Therefore, preventive maintenance assisted by the manufacturer could be a useful method.

Profit-making is the fundamental goal of the manufacturer, and their first task is to complete the annual production plan. During the scheduled production plan period, it is difficult for the manufacturer to change the production plan temporarily and deploy personnel to carry out preventive maintenance of products for the user. This will cause the increase of product failure times and even affect the intact rate of products and other indicators. In order to solve this problem, under the existing warranty strategy, this paper proposes a two degrees preventive maintenance strategy performed by the manufacturer and the user during the warranty period. However, in the current literature on the warranty strategy model, preventive maintenance is mostly a single degree of maintenance. Huang assumed that preventive maintenance is incomplete
maintenance with constant maintenance degree during the warranty period. He established a cost model under preventive maintenance strategy and obtained the optimal maintenance scheme [7]. Under the cycle preventive maintenance strategy, Shafiee assumed that each preventive maintenance cost is proportional to the product usage time and establishes a warranty cost model [8]. Tao proposed preventive maintenance during the initial warranty period and extended warranty period, and the maintenance cost was borne by the manufacturer. Under the premise of controlling the total profit of the manufacturer, the impact of the number of preventive maintenance on the length of the extended warranty period was studied [9]. Alqahtani considered the optimization of warranty cost from the perspective of the manufacturer, adopts a warranty strategy for periodic preventive maintenance of recycled products, and establishes a warranty cost model [10]. Zhou proposed to divide the warranty period into multiple stages and perform different periods of preventive maintenance in different stages. Compared with the period preventive maintenance, it proved that this strategy can reduce the warranty cost under certain conditions [11]. Wang assumed the preventive maintenance cost for based warranty period and extended warranty period were shared by the manufacturer and the user in different proportions. Regarding the lowest warranty cost as goal, he found out the optimal preventive maintenance cycle from perspective of the user and the manufacturer [12]. Shang introduced maintenance cost threshold to decide whether to replace products after failure and determined preventive maintenance interval under minimum maintenance cost [13]. Considering the manufacturer’s point of view, Wang proposed to carry out periodic incomplete preventive maintenance and studied the optimal warranty strategy for products with different usage rates [14]. Cheng proposed a maintenance strategy under the condition of delaying the first preventive maintenance for products that deteriorate with time. Assuming the maintenance cost of each preventive maintenance was different, the corresponding maintenance cost model was established [15]. Tong assumed that the usage rate of product during the warranty period is changed. During warranty period, the product was subjected to several maintenance, and the cost model was established. By example, it was proved that the model can improve the accuracy of the calculation result of the warranty cost [16]. Nasrourm proposed two strategies of incomplete preventive maintenance performed separately by the user and the manufacturer during the warranty period and assuming that the user had certain maintenance capabilities, then established a cost model [17]. Wang proposed a cost rate model for periodic preventive maintenance and sequential preventive maintenance and compared the advantages and disadvantages of two models [18]. In the above literature, the warranty strategy provided a single degree of maintenance preventive maintenance. Only Liao proposed a warranty strategy with preventive maintenance including complete maintenance, incomplete maintenance, and predictive maintenance [19].

Regardless of the preventive maintenance strategy from the perspective of the user or the manufacturer, the maintenance cost is closely related to many factors such as the basic warranty period, extended warranty period, maintenance frequency, and maintenance cost threshold. Under a certain maintenance cycle, how to balance the various factors to allocate the proportion of user maintenance and manufacturer maintenance cost becomes a core issue restricting maintenance costs. The model [20, 21] proposed by Cheng provides a novel idea for solving the above problems, and the regression results of the model can be used as an effective basis for allocation decision. However, at present, no literature has proposed a combination of two different degrees of incomplete maintenance. In the warranty service of real engineering applications, such a combination will have great application space. Unfortunately, none of the current warranty cost models can be applied to this combination. Therefore, we establish the corresponding cost model in this paper. First of all, this model can provide a new strategy for product warranty. Second, this model can fill the gap in the current two degrees preventive maintenance model. Last but not the least, this model can provide a new solution for the manufacturer and the user to reduce the cost of warranty.

According to the degree of product recovery after maintenance, the degree of maintenance can be divided into three types. Suppose $\delta(\delta \in [0, 1])$ represents the repair factor. The first type is called minimum maintenance. In this type $\delta = 0$, each repair restores the product to its prefailure degree. The second one is called complete maintenance, and in this type $\delta = 1$, each repair restores the product to be like a new product. The third one is called incomplete maintenance [22], and the degree of maintenance is between the two, in this type $0 < \delta < 1$. In order to reduce the cost of warranty, preventive maintenance is usually incomplete maintenance or complete maintenance. Three different maintenance degrees have their own characteristics. Compared with the time and cost of maintenance, the minimum maintenance, incomplete maintenance, and complete maintenance increase in turn. The research shows through maintenance, the failure rate of the product at this moment can be reduced to the failure rate at a certain moment before the actual age, which is called the virtual age [23]. For incomplete preventive maintenance, it can be assumed that the virtual age just in the preceding preventive maintenance interval is reduced by maintenance activity, or the virtual age before the preventive maintenance can be reduced. This paper applies the second hypothesis, in which the Proportional Age Setback model [24] is used. The same hypothetical model is mentioned in other literature studies [25, 26]. In the process of incomplete preventive maintenance, different degree of maintenance corresponds to different repair factors, which reduces the virtual age of products at different degrees, and the maintenance cost of higher degree of maintenance will be higher. This rule is consistent with the maintenance situation in practical application.

The rest of this paper is structured as follows: a 2D warranty cost optimization model is proposed in the second part. The third part explains the calculation process of the model. In order to verify the validity of the model, an example is given in the fourth part. In the fifth part, the results
are discussed. And the last part gives the summary and development direction of the research content.

2. Model Establishment

2.1. Notations

\((T_w, U_w)\): 2D warranty period of products

\((T_0, U_0)\): 2D preventive maintenance interval during warranty period

\(m\): degree of preventive maintenance

\(\delta(m)\): repair factor after preventive maintenance

\(C_p\): preventive maintenance cost

\(C_f\): maintenance cost after product failure

\(r\): product usage rate

\(\lambda(t|r)\): initial failure rate of product (The failure rate of the product under usage rate of \(r\))

\(C(T_0, U_0, N)\): expected value of warranty cost

2.2. Model Description. As can be seen in Figure 1, \((T_w, U_w)\) is the 2D warranty period of the product: (1) the product is less utilized with the usage rate \(r_1\) and the warranty terminates at time \(T_w\); (2) the product is more utilized with the usage rate \(r_2\) and the warranty terminates at time \(U_w\).

For a given usage rate \(r\), product usage \(u\) is a linear function of time \(t\), with \(u = r \times t\). The failure rate of the product is expressed by \(\lambda(t|r)\) when its usage rate is \(r\), and expression [27] is

\[ \lambda(t|r) = \theta_0 + \theta_1 r + \theta_2 r^3 + \theta_3 r t^2, \]  

where the value of \(\theta_0, \theta_1, \theta_2, \theta_3\) can be estimated according to the historical failure rate of the product. \(r_w(r_w = U_w/T_w)\) is product design usage rate. Periodic preventive maintenance is carried out with \((T_0, U_0)\) as the interval during the warranty period. Among them, \(T_0\) is the time interval of periodic preventive maintenance and \(U_0\) is the usage interval of periodic preventive maintenance. No matter which deadline arrives first, preventive maintenance is carried out. After preventive maintenance, the failure rate of the product is reduced. When the product breaks down, the manufacturer is responsible for the maintenance, and the degree of maintenance is minimum maintenance, which means the failure rate of the product remains unchanged before and after this maintenance. And this hypothesis put forward by Cha and Finkelstein [28] is adopted here. At present, there are three main methods to express the 2D failure rate, namely, the bivariate method, the composite scale method, and the usage rate method. The usage rate method is widely used. Usually, it is assumed that the usage rate of a single product will remain unchanged during the warranty period. This assumption is also proved in the literature [29, 30]. However, due to the difference in product habits, the usage rate of each user is different. For batch products, usage rate is a random variable subject to a certain distribution. By investigating the use of products, we can know the distribution. The distribution functions of usage and probability density functions are expressed by \(G(r)\) and \(g(r)\), respectively. And the product usage rate obeys Weibull distribution

\[ g(r) = \left(\frac{\beta}{\alpha}\right) \left(\frac{r}{\alpha}\right)^{\beta - 1} e^{-\left(\frac{r}{\alpha}\right)^\beta}, \]  

where \(\alpha\) is the scale parameter and \(\beta\) is the shape parameter.

During the warranty period, the total number of preventive maintenance activities is \(N_w\). After preventive maintenance, the repair effect performed by the manufacturer is better than that performed by the user. In order to reduce total product failures during the warranty period, we assume the manufacturer performs several preventive maintenance activities, and the rest are done by the user. For the convenience of research, we specify that the manufacturer shall only perform preventive maintenance once during the warranty period. The degree of incomplete maintenance is expressed by \(m\). The repair factor is \(\delta(m)\), and the relationship [31] between \(m\) and \(\delta(m)\) is

\[ \delta(m) = (1 + m)e^{-m}. \]  

3. Usually, the maintenance degree can be divided into 6, namely, \(m = [0, 1, 2, 3, 4, 5]\). The degree of incomplete preventive maintenance by the user and the manufacturer is \(m_1\) and \(m_2\) (\(m_2 > m_1\)), respectively. Corresponding repair factors are \(\delta(m_1)\) and \(\delta(m_2)\), and cost of preventive maintenance is not equal under different maintenance degrees, which are \(C_{pl}\) and \(C_{p2}\), respectively. The virtual age of the product changes after incomplete maintenance. We assume incomplete maintenance is carried out at time \(T\), the virtual age before maintenance is indicated by \(T^-\), and the virtual age after maintenance is indicated by \(T^+\). The product failure rate after maintenance is

\[ \lambda(T^+) = \lambda(\delta(m)T^-) \]  

Assuming that among the preventive maintenance activities of \(N_w\) times, only the \(N\)th one is performed by the manufacturer and the remaining \((N_w - 1)\) times are performed by the user, then \(N \in [1, N_w]\) exists. When the manufacturer’s preventive maintenance time \(N\) changes, the failure rate of the product will also change, thus affecting the number of product failure and maintenance cost during the
warranty period. The product warranty cost during the warranty period consists of two parts: the cost of preventive maintenance and the cost of repairing maintenance.

When the interval \((T_0, U_0)\) of preventive maintenance is determined, \(N_w\) can be known for the products used at a certain usage rate. Although the manufacturer’s time \(N\) for preventive maintenance may be any of the \(N_w\) preventive maintenance, no matter how \(N\) changes, the total cost of preventive maintenance is the sum of the cost of one time preventive maintenance performed by the manufacturer and \((N_w - 1)\) times preventive maintenance performed by the user. Therefore, the main factor affecting the warranty cost is the repairing maintenance cost during the warranty period. Because when \(N\) changes, it will have a greater impact on the trend of product failure rate and then affect the total number of product failures during the warranty period. We take products with relatively low usage rate as an example. Under the same interval of preventive maintenance, \(N_1\) and \(N_2\) are used to indicate the different time of preventive maintenance performed by the manufacturer. The dotted line in this figure represents the changing trend of the initial failure rate of the product. When periodic preventive maintenance is carried out during the warranty period, the failure rate will decrease after preventive maintenance. After preventive maintenance performed by the manufacturer, the failure rate will change more obviously. However, due to the different time of \(N_1\) and \(N_2\), the impact on product failure rate during the whole warranty period is also distinct. Therefore, the number of product failures during the warranty period will vary, resulting in different repairing maintenance cost. This shows that different values of \(N\) can lead to different warranty cost.

When the time \(N\) of preventive maintenance by the manufacturer is determined, different intervals of preventive maintenance \((T_0, U_0)\) will also have different effects on product failure rate. Compared with Figures 2(b) and 2(c), preventive maintenance is carried out by the manufacturer for the \(N_2\)th time. Because the interval of preventive maintenance in Figure 2(c) is longer than that in Figure 2(b), the failure rate of the products during the warranty period is higher, which leads to more total failures and higher repair cost. From this, we can see when 2D preventive maintenance interval \((T_0, U_0)\) changes, the cost of warranty also changes.

From the above analysis, we know that whichever changes in \(T_0, U_0, \) and \(N\) has an impact on the total cost during the warranty period. So, to minimize warranty cost, we should select the appropriate \(T_0, U_0, \) and \(N\).

2.3. Model of Warranty Cost. During the warranty period, the failure rate of the product after the \(k\)th preventive maintenance is expressed by \(\lambda_k (t|r) (k \in [0, N_w])\). According to the relationship between \(r_w\) and \(r_0\), it is necessary to divide the product warranty cost into two cases: \(r_0 \leq r_w\) and \(r_0 > r_w\).

With the change of the usage rate \(r\), the preventive maintenance interval and the maintenance deadline will also change during the warranty period. It can be divided into six situations. As shown in Table 1.

When \(r_0 \leq r_w\), the relative relationship between \(r_0\) and \(r_w\) can be shown in Figure 3.

In situation 1, the product’s warranty deadline is \(T_{w}^r\), while the preventive maintenance interval is \(T_0\). And the total number of preventive maintenance is \(N_w = \lceil T_w / T_0 \rceil \) ("\(\lfloor \cdot \rfloor\)“ denotes downward rounding of "\(*\")

After the \(k\)th preventive maintenance, the failure rate \(\lambda_k (t|r)\) of the product changes as follows: if \(k = 0\),

\[
\lambda_k (t | r) = \lambda (t | r),
\]

if \(1 \leq k \leq N - 1\),

\[
\lambda_k (t|r) = \lambda \left( t - kT_0 + \delta(m_1)T_0 + \delta^2(m_1)T_0 + \cdots + \delta^k(m_1)T_0 \right) | r
\]

\[
= \lambda \left( t - kT_0 + \delta(m_2) \delta(m_1)T_0 + \cdots + \delta(m_2)\delta^{k-1}(m_1)T_0 \right) | r
\]

(6)

if \(k = N\),

\[\lambda_k (t|r) = \lambda \left( t - kT_0 + \delta(m_2) T_0 \right) | r
\]

\[= \lambda \left( t - kT_0 + \delta(m_2) \sum_{i=0}^{k-1} \delta^i(m_1)T_0 \right) | r
\]

(7)
Table 1: Preventive maintenance interval and warranty deadline under different usage rate.

| Situation | Scope of $r$ | Preventive maintenance interval | Warranty deadline |
|-----------|--------------|---------------------------------|-------------------|
| 1         | $r \leq r_0 \leq r_w$ | $T_0$ | $T_w$ |
| 2         | $r_0 < r \leq r_w$ | $U_0/r$ | $T_w$ |
| 3         | $r_0 \leq r_w < r$ | $U_0/r$ | $U_w/r$ |
| 4         | $r \leq r_w < r_0$ | $T_0$ | $T_w$ |
| 5         | $r_w < r \leq r_0$ | $T_0$ | $U_w/r$ |
| 6         | $r_w < r_0 < r$ | $U_0/r$ | $U_w/r$ |

Figure 2: Schematic diagram of product failure rate during the warranty period. (a) The $N_1$th preventive maintenance is performed by the manufacturer. (b) The $N_2$th preventive maintenance is performed by the manufacturer. (c) The $N_3$th preventive maintenance is performed by the manufacturer.

Figure 3: Relative relationship between $r_0$ and $r_w$. 
and if \( N + 1 \leq k \leq N_w \),

\[
\lambda_k (t|r) = \lambda \left( t - kT_0 + \left( \delta (m_1)T_0 + \delta^2 (m_1)T_0 + \cdots + \delta^{k-N} (m_1)T_0 \right) \\
+ \left( \delta (m_2)\delta^{k-N} (m_1)T_0 + \delta (m_2)\delta^{k-N-1} (m_1)T_0 + \cdots + \delta (m_2)\delta^{k-1} (m_1)T_0 \right) \right) |r
\]

\[
= \lambda \left( t - kT_0 + \sum_{i=1}^{k-N} \delta (m_i)T_0 + \delta (m_2) \sum_{j=k-N}^{k-1} \delta^j (m_1)T_0 |r \right).
\]

(8)

In the warranty strategy, the product failure in each preventive maintenance interval is regarded as a nonhomogeneous Poisson process (NHPP) [32], and the number of product minimum maintenance in a preventive maintenance interval is

\[
M = \int_{T_j}^{T_k} \lambda (t)dt,
\]

(9)

\[
C_{Tf} = C_f \left( \int_{0}^{T_j} \lambda_0 (t|r)dt + \int_{T_j}^{2T_j} \lambda_1 (t|r)dt + \cdots + \int_{N_wT_j}^{(N_w-1)T_j} \lambda_{N-1} (t|r)dt + \int_{N_wT_j}^{T_k} \lambda_{N_w} (t|r)dt \right)
\]

\[
= C_f \left( \sum_{i=0}^{N_w-1} \int_{iT_j}^{(i+1)T_j} \lambda_i (t|r)dt + \int_{N_wT_j}^{T_k \lambda_{N_w} (t|r)dt} \right).
\]

(10)

The total cost of preventive maintenance is

\[
C_{T_P} = C_{P_1} (N_k - 1) + C_{P_2}.
\]

(11)

In situation 1, the cost during the warranty period is as follows:

\[
C_1 (T_0, U_0, N) = C_{T_P} + C_{T_f} = C_{P_1} (N_k - 1) + C_{P_2} + C_f \left( \sum_{i=0}^{N_w-1} \int_{iT_j}^{(i+1)T_j} \lambda_i (t|r)dt + \int_{N_wT_j}^{T_k \lambda_{N_w} (t|r)dt} \right).
\]

(12)

In other situation of different usage rates, the calculation methods of product warranty cost are similar with situation 1, but there are differences only in preventive maintenance interval and maintenance deadline. Therefore, the calculation process is no longer redundant, and the calculation equation of maintenance cost in other cases is given directly:

\[
C_n (T_0, U_0, N) = C_{P_1} (N_k - 1) + C_{P_2} + C_f \left( \sum_{i=0}^{N_w-1} \int_{iP_i (n)}^{(i+1)P_i (n)} \lambda_i (t|r)dt + \int_{N_wP_i (n)}^{W_d (n)} \lambda_{N_w} (t|r)dt \right),
\]

(13)

where \( n(=1, 2, 3, 4, 5, 6) \) represents the situation of different usage rates, \( P_i (n) \) represents the preventive maintenance interval in situation \( n \), and \( W_d (n) \) represents the warranty deadline in situation \( n \).

\[
C (T_0, U_0, N) = \int_{0}^{T_0} C_1 (T_0, U_0, N)dG(r) + \int_{T_0}^{T_w} C_2 (T_0, U_0, N)dG(r) + \int_{T_w}^{\infty} C_3 (T_0, U_0, N)dG(r).
\]

(14)
According to formula (13), in case of \( r_0 > r_w \), the expected value of warranty cost during the warranty period is

\[
C(T_0, U_0, N) = \int_0^{T_w} C_4(T_0, U_0, N)dG(r) + \int_{r_w}^{\infty} C_5(T_0, U_0, N)dG(r) + \int_{r_w}^{\infty} C_6(T_0, U_0, N)dG(r).
\]

(15)

3. Model Analysis

From the expression of cost model, we can see that it is difficult to solve it directly by the method of functional analysis because of the complexity of this model. With the help of MATLAB software, the numerical algorithm is used to solve the model. The solving steps are as follows:

Step 1: according to the scope of the product warranty period, we select suitable preventive maintenance interval \((T_0, U_0)\). A limited number of \((T_0, U_0)\) is generated with a fixed step size \(\Delta T_0\) and \(\Delta U_0\).

Step 2: in each group \((T_0, U_0)\), it is easy to determine the total number \(N_w\) of preventive maintenance. Then, we let \(N\) traverses the value in \([1, N_w]\) and get different combinations of \((T_0, U_0, N)\).

Step 3: according to formula (14) and (15), the corresponding warranty cost \(C(T_0, U_0, N)\) of different combinations \((T_0, U_0, N)\) is calculated, and the minimum warranty cost is obtained as the optimal solution \(C^*(T_0, U_0, N)\), as well as the corresponding optimal \(T_0^*, U_0^*\) and \(N^*\).

4. Example Analysis

In order to reduce the cost of 2D warranty, preventive maintenance is performed for a new type of dump truck during the warranty period. Preventive maintenance is accomplished by two degrees of maintenance force, the manufacturer, and the user. The 2D warranty period is \(T_w = 5\) years and \(U_w = 16 \times 10^4\) km. According to the evaluation of relevant experts, the degree of preventive maintenance performed by the user is \(m_1 = 1\), while by the manufacturer is \(m_2 = 2\). The values of the remaining parameters are \(\delta(m_1) = 0.74, \delta(m_2) = 0.41, C_f = 60\) dollars, \(C_{p1} = 230\) dollars, and \(C_{p2} = 650\) dollars; \(\alpha = 3, \beta = 2, \theta_0 = 0.7, \theta_1 = 0.5, \theta_2 = 0.6\), and \(\theta_3 = 0.8\).

With the help of MATLAB software, the numerical algorithm is used to calculate the model. \(T_0\) takes the value in \([0, T_w]\) with step \(\Delta T = 0.02T_w\), and \(U_0\) takes the value in \([0, U_w]\) with step \(\Delta U = 0.02U_w\). A total of 2500 groups of \((T_0, U_0)\) are generated. Then, we calculate the warranty cost \(C(T_0, U_0, N)\) for each group \((T_0, U_0)\) when \(N\) takes different values and draw a three-dimensional diagram of the lowest warranty cost value \(C(T_0, U_0)\) under different \((T_0, U_0)\), as shown in Figure 4.

According to the calculation results, the minimum cost is 3,144 dollars when \((T_0, U_0, N) = (1.3, 4.16 \times 10^4, 2)\). By contrast, if only preventive maintenance is carried out by the user, the minimum maintenance cost is 3,211 dollars when \((T_0, U_0) = (0.9, 2.88 \times 10^4)\). So, the new warranty strategy can save 2.13% of the warranty cost.

Through the data analysis of warranty cost, we can see that under this new strategy, the warranty cost has the following rules:

1. If the value of \(U_0\) is smaller, the warranty cost varies greatly with the interval \(T_0\); if the value of \(U_0\) is larger, the cost of warranty varies little with the interval \(T_0\). When \(U_0 = 0.02U_w\) (preventive maintenance is performed at a lower usage interval) and \(U_0 = 0.9U_w\) (preventive maintenance is performed at a longer usage interval) separately, let \(T_0\) take uniformly valued in \([0, T_w]\), and we calculate the warranty cost and the variance. The results are shown in Figure 5. It is known that the warranty cost varies greatly with the change of \(T_0\) in \(U_0 = 0.02U_w\).

2. If the value of \(T_0\) is smaller, the change of maintenance cost with interval \(U_0\) is not obvious; if the value of \(T_0\) is larger, the change of maintenance cost with interval \(U_0\) is obvious. When \(T_0 = 0.02T_w\) and \(T_0 = 0.9T_w\) separately, let \(U_0\) take uniformly valued in \([0, U_w]\), and we calculate the warranty cost and the variance. The results are shown in Figure 6. It is known that the warranty cost varies greatly with the change of \(U_0\) in \(T_0 = 0.9T_w\).

3. With the increase of \(N\), warranty cost has different trends with a definite of \((T_0, U_0)\). From the results of the calculation, three groups of cost of different \((T_0, U_0)\) are selected to compare the trend of warranty cost with the change of \(N\). The corresponding warranty cost data are shown in Table 2, and the trend is shown in Figure 7. The result shows that when \((T_0, U_0) = (0.7, 2.24)\), with the increase of \(N\), the warranty cost first decreases and then increases. When \((T_0, U_0) = (0.9, 5.12)\), the warranty cost increases with the increase of \(N\), while when \((T_0, U_0) = (1.1, 1.92)\), the warranty cost decreases with the increase of \(N\). The units of \(T_0\), \(U_0\) and...
From the above data analysis, it is obvious that in this warranty strategy, whichever value of $T_0$, $U_0$, and $N$ changes will affect the warranty cost.

5. Sensitivity Analysis

$C(T_0, U_0, N)$ are, respectively, “years,” “$10^4$ km,” and “dollars.”

From the above data analysis, it is obvious that in this warranty strategy, whichever value of $T_0, U_0$, and $N$ changes will affect the warranty cost.

### Table 2: Maintenance cost under fixed $T_0, U_0$, and different $N$ conditions.

| $N$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-----|---|---|---|---|---|---|---|
| $(T_0, U_0)$ | (0.7, 2.24) | 3,621 | 3,537 | 3,481 | 3,464 | 3,487 | 3,554 | 3,776 |
| | (0.9, 5.12) | 3,420 | 3,435 | 3,933 | — | — | — | — |
| | (1.1, 1.92) | 4,738 | 4,610 | 4,497 | 4,330 | — | — | — |

### Table 3: Result of optimal warranty cost under different $C_f$.

| $C_f$ | $T_0^*$ | $U_0^*$ | $N^*$ | $C^*(T_0^*, U_0^*, N)$ | Cost increase |
|------|--------|--------|------|------------------------|---------------|
| 500  | 1.3    | 4.16   | 2    | 2,805                  | 0             |
| 600  | 1.3    | 4.16   | 2    | 3,144                  | 12.09         |
| 700  | 0.9    | 3.24   | 3    | 3,468                  | 23.64         |
| 800  | 0.9    | 2.88   | 3    | 3,739                  | 33.31         |
| 900  | 0.9    | 2.52   | 4    | 4,011                  | 42.97         |

### Table 4: Result of optimal warranty cost under different $C_{p1}$.

| $C_{p1}$ | $T_0^*$ | $U_0^*$ | $N^*$ | $C^*(T_0^*, U_0^*, N)$ | Cost increase |
|----------|--------|--------|------|------------------------|---------------|
| 1900     | 0.9    | 3.24   | 3    | 3,037                  | 0             |
| 2100     | 1.3    | 4.16   | 2    | 3,104                  | 2.22          |
| 2300     | 1.3    | 4.16   | 2    | 3,144                  | 3.53          |
| 2500     | 1.3    | 4.16   | 2    | 3,184                  | 4.85          |
| 2700     | 1.3    | 4.16   | 2    | 3,224                  | 6.17          |

From the results of Tables 3–5, we can see that $C_f$, $C_{p1}$, and $C_{p2}$ have different influence on the warranty cost. By comparison, the change of $C_f$ has the greatest impact on the warranty cost $C^*(T_0^*, U_0^*, N)$ in the 2D warranty period. This
result can provide a basis for the manufacturer and user to specify a warranty plan.

6. Conclusions

In this paper, taking warranty cost of 2D warranty product as the research object, the new warranty strategy which includes two degrees preventive maintenance is proposed. And we establish the corresponding cost model. The numerical example shows that this strategy can reduce the cost of warranty and provide a scientific basis for the manufacturer and the user to formulate reasonable warranty schemes. The cost model can provide a new solution for product warranty cost prediction in engineering applications. In addition, the model can also assist the manufacturer and the user in making warranty decisions. And it is easy to compare the impact of different preventive maintenance interval on warranty cost. However, this paper considers only one preventive maintenance performed by the manufacturer during the warranty period. In the next step, we can also consider multiple preventive maintenance by the manufacturer. In addition, we only study the cost of warranty product. In the next step, we should further study the availability of warranty product.

Data Availability

The data that support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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