Analysis on Vibration Characteristics of Flexible Rotating Manipulator

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Abstract. Aiming at the problem that the inertia effect of the tip mass causes the accuracy decrease of the manipulator's movement, the floating coordinate system was utilized to describe the manipulator's elastic deformation displacement, and the nonlinear coupling term caused by the transverse deformation was taken into account. According to Lagrange equation of the second type, the system dynamics equation expressed by the modal function was derived. Combined with numerical simulation, the influence of the tip mass on the vibration characteristics of the system was analyzed. Studies have shown that the tip load of the flexible manipulator increases the vibration response amplitude of the system and reduces the response frequency. Reducing the length of the manipulator is beneficial to improve its motion accuracy. Research results have application value for manipulator optimization design and reliability evaluation.

1. Introduction
Manipulators play an important role in the field of industrial manufacturing. The large-scale rigid motion of the rotary manipulator is coupled with its small displacement flexible motion. This coupling effect brings certain difficulties to the dynamic analysis and precise control of the robot system. Considering the influence of the rigid-flexible coupling effect, the traditional rigid-body system dynamics theory cannot accurately describe the dynamic characteristics of the manipulator. Based on the small deformation hypothesis, it has been a hot topic for scholars to study the effect of manipulator flexibility on the vibration characteristics of the system.

Fung[1] et al. established dynamic models of flexible double-link and single-link manipulator under nonlinear constraints based on the Hamilton principle, and concluded that the rigid body motion and flexible motion of the connecting rod are nonlinearly coupled. Bascetta [2] et al. studied the identification method of tip load inertia parameters. Sayahkarajy [3] et al. established a finite element model of a planar two-link manipulator and compared it with the calculation results of the assumed modal method, which demonstrated the correctness of the finite element method. In addition, Oguamanam [4] et al. investigated the natural vibration of flexible manipulators with tip mass. Al-Bedoor [5] et al. first paid attention to the influence of flexible joints on the dynamic characteristics of rotating manipulators. Wei [6] et al. developed a global modal method to solve the natural frequency of a flexible joint manipulator with tip mass. Furthermore, My [7] et al. proposed the recursive formula of the dynamic equation of the multi-link manipulator for the problems of joint motion and elastic deformation of the connecting rod. Literature [8-10] studied the "dynamic stiffening" effect of rotating manipulators.

This paper establishes a rigid-flexible coupling model of a rotating manipulator under the action of gravity field, and derives the dynamic equations of the coupling system. The assumed mode method is
used to discretize the equation, and numerical calculations is performed according to the fourth-order Runge-Kutta method to study the influence of the tip mass and the length of the manipulator on the vibration characteristics of the system.

2. Physical Model
The manipulator is simplified as a Euler-Bernoulli cantilever beam of equal section, without considering the influence of shear deformation. As is shown in Fig. 1, the inertial coordinate system $X_0O_Y$ is established to describe the large range rigid motion of the manipulator, the origin $O_0$ is at the center of the rotating shaft, and the $O_0X$-axis is vertically downward. The floating coordinate system $x_0y$ is used to describe the elastic deformation of the manipulator. The origin $o_1$ is at the beginning end of the manipulator. The $o_1x$-axis is the central axis of the manipulator when it is not deformed. Any point $P$ reaches the point $P'$ after elastic deformation, $u$ and $v$ are the axial and transverse deformation respectively, and $w_c$ is the axial shortening deformation caused by the transverse deformation, which is called the nonlinear coupling deformation. Note that $L$ is the arm length, $\rho$ is the density, $A$ is the cross-sectional area, $I$ is the moment of inertia of the section, $E$ is the elastic modulus, $m$ is the tip mass, $J_c$ is the moment of inertia, $a$ is the radius of the shaft, $J_h$ is the moment of inertia of the shaft, $\theta$ is the angular displacement, and $\tau$ is the driving torque.

As is shown in Fig. 1, the point position vector of $P$ in the inertial coordinate system is

$$R_p = a + A(\theta)(r_p + w_p)$$  \hspace{1cm} (1)

Where, $a$ is the position vector of the origin $o_1$ in the inertial coordinate system, $r_p$ is the position vector of point $P$ in the floating coordinate system, $A(\theta)$ is the conversion matrix from the floating coordinate system to the inertial coordinate system, $w_p$ is the deformation displacement vector, $i$ and $j$ are the unit direction vector of the floating coordinate system.

$$a = [a \cos \theta \quad a \sin \theta]^T$$  \hspace{1cm} (2)

$$r_p = [x_p \quad 0]^T$$  \hspace{1cm} (3)

$$A(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$  \hspace{1cm} (4)

$$w_p = (u + w_c)i + vj$$  \hspace{1cm} (5)

![Fig.1. Structure model of manipulator](image)

The axial shortening deformation of the manipulator can be expressed as

$$w_c(x,t) = -\frac{1}{2} \int_0^t \left( \frac{\partial v(\xi,t)}{\partial \xi} \right)^2 d\xi$$  \hspace{1cm} (6)

According to the separated variable method and the assumed mode method, the transverse
deformation displacement of the manipulator can be expressed as

\[ v(x,t) = \sum_{i=1}^{N} \Phi_i(x) q_i(t) = \Phi q \]  

(7)

In the formula, the \( i \)th modal function is

\[ \Phi_i(x) = A \left[ \cos \frac{\lambda_i}{L} x - \cosh \frac{\lambda_i}{L} x + \zeta_i \left( \sin \frac{\lambda_i}{L} x - \sinh \frac{\lambda_i}{L} x \right) \right] \]  

(8)

Where, \( \Phi_i(x) \) is the \( i \)th mode of the manipulator, \( q_i(t) \) is the corresponding \( i \)th modal coordinate, \( \Phi \) is the modal row vector, \( q \) is the modal coordinate column vector, and \( N \) is the modal truncation number.

By deriving Equ. (1), we can get

\[ \dot{R}_p = \dot{a} + \dot{A}(\theta)(r_p + \omega_p) + A(\theta)\dot{\omega}_p \]  

(9)

In the same way, the position displacement and velocity vector of the tip mass are as follows,

\[ R_m = a + A(\theta)(r_e + \omega_e) \]  

(10)

\[ \dot{R}_m = \dot{a} + \dot{A}(\theta)(r_e + \omega_e) + A(\theta)\dot{\omega}_e \]  

(11)

In the formula, \( r_L \) is the displacement of the concentrated mass when the manipulator is not deformed, and \( \omega_L \) is the deformation vector at the end of the manipulator.

3. Kinetic equations

According to Equs. (9) and (11), the kinetic energy of the manipulator system can be expressed as

\[ T = \frac{1}{2} J_h \dot{\theta}^2 + \frac{1}{2} \int_0^L \rho A \dot{R}_p^2 \dot{R}_p + \frac{1}{2} m R_m^2 \dot{R}_m \]  

(12)

Without considering the higher-order terms of \( \omega_i(x,t) \), Eq. (12) can be simplified as

\[ T = \frac{1}{2} \int_0^L \rho A \left[ (a + x)^2 + v^2 \right] \dot{x}^2 dx + \frac{1}{2} \int_0^L \rho A (a + x) w_e \dot{x} dx + \frac{1}{2} \int_0^L \rho A v^2 \dot{x} dx + \frac{1}{2} \int_0^L \rho A \dot{v}^2 \dot{x} dx + \frac{1}{2} m \left[ (a + L)^2 + v^2 \right] \dot{\theta}^2 + m \dot{\theta}^2 \left( a + L \right) w_e \bigg|_{x=L} + \frac{1}{2} mv^2 \bigg|_{x=L} \]  

(13)

The potential energy of elastic deformation, and the gravitational potential energy of the tip mass and manipulator constitute the potential energy of the system, which can be expressed as

\[ V = \frac{1}{2} \int_0^L E I \left( \frac{\partial^2 v}{\partial x^2} \right)^2 \dot{x} dx - \rho A g \int_0^L \left[ (a + x) \cos \theta - v \sin \theta \right] \dot{x} dx - mg \left[ (a + L) \cos \theta - v \sin \theta \right] \bigg|_{x=L} \]  

(14)

The generalized coordinate array of system is defined as

\[ r = [\dot{\theta}, q]^T \]

According to the Lagrange equation of the second type, the system motion control equation can be obtained

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{r}} \right) - \frac{\partial T}{\partial r} + \frac{\partial V}{\partial r} = \dot{F}_r \]  

(15)

In the formula, \( \dot{F}_r \) is the generalized force corresponding to the non-conservative force, which is only related to the rotational torque \( \tau \) at the cylindrical gear in this paper, \( \dot{F}_r = [r, \dot{\theta}]^T \).

Substituting Equs. (7), (13) and (14) into Equ. (15), the system dynamics equation can be obtained as
\[
\begin{align*}
\dot{\theta} \left[ J_h + \left( \int_0^L \rho A(a+x)^2 \, dx + m(a+L)^2 \right) \right] + q^T \left( \int_0^L \rho A \Phi^T \Phi \, dx + m \Phi(L)^T \Phi(L) \right) q +
\left( \int_0^L \rho A(a+x) \Phi \, dx + m(a+L) \Phi(L) \right) \dot{q} +
2 \dot{q}^T \left( \int_0^L \rho A \Phi^T \Phi \, dx + m \Phi(L)^T \Phi(L) \right) \dot{q} +
g \cos \theta \left( \int_0^L \rho A \Phi \, dx + m \Phi(L) \right) q - 2 \dot{q}^T \left( \int_0^L \rho A(a+x) \Phi^T \Phi \, dx + m \Phi(L)^T \Phi(L) \right) \dot{q} +
g \sin \theta \left( \int_0^L \rho A(a+x) \Phi \, dx + m(a+L) \right) - 2 m q^T \left( \int_0^L \Phi^T \Phi \, dx \right) \dot{q} = \tau
\end{align*}
\]

Eqns. (16) and (17) are expressed in matrix form as follows

\[
\begin{bmatrix}
M_{\theta\theta} & M_{\theta q} \\
M_{q\theta} & M_{qq}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\dot{q}
\end{bmatrix}
+ \begin{bmatrix} 0 \\
0 \end{bmatrix} \theta + \begin{bmatrix} 0 \\
0 \end{bmatrix} q = \begin{bmatrix} F_0 \\
F_q \end{bmatrix}
\]

(18)

Where,

\[
M_{\theta\theta} = J_h + \left( \int_0^L \rho A(a+x)^2 \, dx + m(a+L)^2 \right)
+ q^T \left( M_1 + m \Phi(L)^T \Phi(L) \right) q
\]

\[
M_{\theta q} = M_2 + m(a+L) \Phi(L)
\]

\[
M_{q\theta} = M_{q\theta}^T
\]

\[
M_{qq} = M_1 + m \Phi(L)^T \Phi(L)
\]

\[
K_{qq} = K_1 - \dot{\theta}^2 \left( M_3 - C_1 \right) - \dot{\theta}^2 \left( m \Phi(L)^T \Phi(L) - m(a+L) H(L) \right)
\]

\[
F_0 = \tau - 2 \dot{q}^T \left( M_3 - C_1 \right) q - 2 \dot{q}^T \left( m \Phi(L)^T \Phi(L) - m(a+L) H(L) \right) \dot{q}
\]

\[
- g \cos \theta \left( M_3 + m \Phi(L) \right) q - g \sin \theta \left( \rho A(2aL + L^2) / 2 + m(a+L) \right)
\]

\[
F_q = -g \sin \theta \left( M_3 + m \Phi(L) \right)
\]

\[
H(x) = \int_0^x \Phi(x)^T \Phi(x) \, \, dx
\]

\[
M_1 = \int_0^L \rho A \Phi^T \Phi \, dx
\]

\[
M_2 = \int_0^L \rho A(a+x) \Phi \, dx
\]

\[
M_3 = \int_0^L \rho A \Phi \, dx
\]

\[
K_1 = \int_0^L E I \Phi^T \Phi \, dx
\]

\[
C_1 = \int_0^L \rho A(a+x) H(x) \, dx
\]
4. Numerical simulation

Numerical calculations are carried out for the heavy lifting robot to study the vibration characteristics of the manipulator under the inertial effect of the tip load. The mode truncation number is 3 and the initial conditions for solving Equ. (18) are $\theta|_{t=0} = 0$, $q_1(0) = q_2(0) = q_3(0) = 0$. Other simulation parameters are shown in Table 1.

| Parameter | Value   |
|-----------|---------|
| $L$       | 0.7m    |
| $\rho$    | 7850kg·m$^{-3}$ |
| $EI$      | $4.27 \times 10^3$ N·m$^2$ |
| $m$       | 20kg    |
| $a$       | 0.05m   |
| $A$       | $1.6 \times 10^{-3}$ m$^2$ |

According to the data in Table 1, the transverse vibration response of the manipulator is calculated. Fig. 2 shows the relationship between the driving torque and the rotational angular displacement of the manipulator. The driving torque decreases as the angular displacement of the manipulator increases. Fig. 3 shows the coordinates of the first three modes. It can be seen that the coordinates of the 1st mode play a leading role in the vibration displacement analysis. The coordinates of the higher modes constitute the disturbance component of the vibration response. Therefore, the coordinates of the first three modes can achieve sufficient calculation accuracy. Next, the influence of tip load and manipulator length on the vibration characteristics of the system is analyzed.

Manipulator vibration affects the upward swing of the tip load. At the same time, the inertia effect of tip load causes disturbance to the vibration system of manipulator. Take the end load mass of 0, 10 kg and 20 kg for numerical calculation respectively. Fig. 4 shows the angular velocity of the system rotation. As the tip load increases, the angular velocity amplitude decreases and the oscillation frequency decreases. Fig. 5 shows the transverse vibration displacement of the manipulator. Under the action of gravity field, the static deformation of the manipulator is caused by the tip load, so the horizontal vibration balance position is in the negative direction of the y-axis.

Fig. 6 shows the change rule of the elastic potential energy of the manipulator. When the load mass increases from 0 to 20kg, the amplitude of the elastic potential energy of the manipulator increases from $1.52J$ to $2.37J$, an increase of $55.92\%$, which indicates that the increase of the tip load causes the increase of the overall elastic deformation of the system. According to the research results, in the material lifting mechanism, improving the bending stiffness of the manipulator can effectively control the amplitude of the bending deformation, which is beneficial to ensure the accuracy and reliability of the manipulator's movement.
The manipulator is the main body of the vibration system, and the influence of the length of the manipulator on the vibration characteristics of the system is further considered. The length of the manipulator is respectively 0.5m, 0.6m and 0.7m for numerical calculation. According to the driving control law given in Fig. 2, the angular velocity of the manipulator rotation is shown in Fig. 7. As the length of the manipulator increases, the system oscillation frequency decreases. Fig. 8 and Fig. 9 are the transverse deformation of the manipulator and the elastic deformation potential energy change law. When the length of manipulator increases from 0.5m to 0.7m, the maximum amplitude of tip bending deformation increases from 0.167mm to 0.342mm, and the maximum amplitude of elastic deformation potential energy of the system increases from 1.47J to 2.31J. The increase in the length of the manipulator leads to an increase in the inertia of the system. Under the conditions of a certain driving
control law, the angular velocity of the system decreases and the amount of elastic deformation increases. In addition, similar to the impact of the tip load on the vibration characteristics of the system, the impact of the increase in the length of the manipulator is also reflected in the increase in the flexibility of the system and the weakening of the system oscillation. In the analysis of dynamic characteristics of multi-link robots, the cumulative effect of motion errors caused by the elastic deformation of the manipulator should be considered.

5. Conclusion
In this paper, considering the influence of the high-order coupling deformation of the manipulator, the rigid-flexible coupling dynamic equation of the rotating manipulator is derived according to the Lagrange equation of the second type. The equation is discretized using the assumed mode method, and the tip load and the length of the manipulator are analyzed through the numerical calculation results. The effect of vibration characteristics is as follows: as the tip load increases, the deformation potential energy of the manipulator increases, and the vibration response amplitude of the system increases. In order to improve the motion accuracy of the manipulator, the stiffness of the manipulator should be appropriately increased; in the research of high-precision manipulators, By reducing the length of the manipulator, the vibration response amplitude of the system can be effectively controlled. This article does not consider the influence of the shape and size of the tip load. The next step is to study the dynamic characteristics of the system when the tip mass centroid deviates from the manipulator axis, and investigate the influence of the tip mass eccentricity on the manipulator's vibration shape and natural frequency, so as to provide a reference for the optimal design of industrial robots.

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