An adaptive inelastic magnetic mirror for Bose-Einstein condensates

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We report the reflection and focussing of a Bose-Einstein condensate by a new pulsed magnetic mirror. The mirror is adaptive, inelastic, and of extremely high optical quality. The deviations from specularity are less than 0.5 mrad rms, making this the best atomic mirror demonstrated to date. We have also used the mirror to realize the analog of a beam-expander, producing an ultra-cold collimated fountain of matter waves.

A major aim of the field of atom optics is to build analogs of mirrors, lenses, and waveguides for manipulating ultra-cold atoms in applications ranging from high-resolution lithography to ultra-sensitive atom interferometry. To this end, static magnetic fields and the optical dipole potential have already been used to both reflect and focus conventional cold atomic clouds, and laser-cooled neutral atoms have also been focused in both 1D [8] and 3D [9] using pulsed magnetic fields. Since a gaseous Bose-Einstein condensate (BEC) [10] is the ultimate source of coherent atoms, it is essential to develop high-quality atom-optical elements which can be applied to BECs. A first step in this direction was the recent demonstration of a flat mirror based on the optical dipole force [11].

In this Letter we show that a different kind of atomic mirror [12], based on pulsed magnetic fields [12, 13], can be used to manipulate Bose condensates. This new mirror has several attractive features: it can be inelastic, it permits adjustable three-dimensional focussing, and it has extremely high optical quality. In addition to demonstrations of simple reflection and focussing, these properties also allow us to realize a BEC “beam expander” which produces an extremely cold, collimated beam of matter waves. Finally, we show that the evolution of the condensate in these experiments is in good agreement with theory.

All magnetic atom-optical elements make use of the Stern-Gerlach potential $U = -\mu B$ experienced by an atom moving adiabatically in a magnetic field of magnitude $B$ (with $\mu$ being the component of the atomic magnetic moment parallel to the field). For weak-field seeking atoms, any field for which $B$ increases in the direction of the initial atomic velocity will act as a simple mirror. If the atoms are to be focussed as well as reflected, then $B$ must in addition have positive curvature in the appropriate directions. Other desirable properties of a condensate mirror are equipotential surfaces with sufficient smoothness to preserve coherence, and compatibility with the ultra-high vacuum environment of a typical BEC apparatus. Since these conditions must all be satisfied by the magnetic trap in which the condensate is formed, the trap field itself provides an ideal starting point for the construction of a mirror. Particularly important here is the fact that all magnetic elements in the trap are a relatively large distance $d \gtrsim 20$ mm from the atoms. Thus, because curvature in the magnetic field scales as $d^{-3}$, microscopic corrugations are drastically reduced compared to mirrors in which atoms make a close ($d < 100 \mu m$) approach to the surface. Such corrugations, and their optical analogs, presently limit the quality of existing atomic mirrors [7, 14] and so large-scale mirrors might provide an easier route to fully coherent atomic manipulation. Also, with this mirror the condensate is focussed in three dimensions with a single magnetic pulse, in contrast to previous work with cold atoms which has either been restricted to one dimension [8] or required the judicious application of two magnetic pulses [9].

In our experiment the magnetic mirror is formed by combining a horizontal Ioffe-Pritchard [15] magnetic trap with a uniform vertical magnetic field $B_c$. We use a co-ordinate system where the symmetry axis of the trap is in the $z$ direction, gravity acts in the $-y$ direction and the origin is at the center of the magnetic trap. The total magnetic mirror field to second-order is then

$$B(r) = \{0, B_c, B_0\} + B'_x \{x, -y, 0\} \quad + B''_y \{-x, y, 0\}$$

The parameters $B_0$, $B'_x$, and $B''_y$ are respectively the bias field, gradient, and curvature of the trap. One can show that the magnitude of this field (Fig. 1) is essentially parabolic in the axial ($z$) direction, but hyperbolic in the offset radial co-ordinate $r = \sqrt{x^2 + (y - y_c)^2}$ and thus effectively linear for radii $r > \sqrt{2B_0/B'_x}$. The potential therefore closely approximates that of a horizontal cylindrical mirror. The control field $B_c$ allows us to shift the minimum of the potential vertically from $y = 0$ to $y_c = B_c/B'_x$, moving the center of curvature of the mirror to $y_c$ and making the radius of curvature at a particular height $-y$ below the trap center equal to $y + y_c$. A condensate falling under gravity can be reflected by a brief pulse of the magnetic mirror field. During the pulse the atoms experience a potential that is weakly confining in the $z$ direction, strongly confining (with strength dependent on $B_c$) in the $x$ direction and almost linear in the $y$ coordinate. The linear variation of the potential with height results simply in a change in the center-of-mass motion (i.e. bouncing), whereas the
parabolic dependence in the two horizontal directions focuses the condensate. The field pulses can be repeated at appropriate intervals to produce multiple reflections.

The BECs used in our experiments are produced as follows. About $10^6$ $^{87}$Rb atoms are collected at the low-pressure end of a double magneto-optical trap (MOT) system. Three simple extended-cavity diode lasers provide the necessary 780 nm ($5\pi^2 S_{1/2} \rightarrow 5p^2 P_{3/2}$) MOT trapping and repumping light. For both MOTs the trap laser light is detuned $\Delta = -13$ MHz (i.e. red) of the $F = 2 \rightarrow F' = 3$ transition, whilst the repumping light is resonant with the $F = 1 \rightarrow F' = 2$ line. The atoms are cooled to 40 $\mu$K in optical molasses ($\Delta = -35$ MHz), optically pumped on the $F = 2 \rightarrow F' = 2$ transition into the $|F, m_F| = |2, 2\rangle$ state, and then transferred into the Ioffe-Pritchard magnetic trap. The trap coils are formed from water-cooled 3 mm o.d. copper tubing: a 9-turn cuboidal baseball coil with average side dimensions 45 $\times$ 45 $\times$ 55 mm$^3$, and two three-turn 64 $\times$ 101 mm$^2$ rectangular bias coils. With a typical bias field of 1 G, the axial and radial magnetic trapping frequencies were measured to be $\nu_z = 10$ Hz and $\nu_r = 223$ Hz respectively at a current of 220 A. The trap lifetime was typically 70 s. After a 32 s RF evaporative cooling ramp pure condensates are reproducibly obtained with more than $10^9$ atoms. The atoms are imaged in absorption using an 8 $\mu$s pulse of near-resonant light from a beam which propagates horizontally at an angle of 60° to the $z$ axis of the magnetic trap. We refer to the resulting horizontal axis in our images as $z'$. The imaging system has a magnification of 0.80. Comparison of the absorption images with a Thomas-Fermi model shows that distortion of the probe beam by the initially optically dense BEC becomes negligible after 10 ms of expansion.

Fig. 2 (a) and (b) are typical sequences of absorption images showing reflection and focussing of the condensate by mirrors of small and large radii of curvature respectively. The magnetic field, with a current of 155 A in the magnetic trap coils, was pulsed on for 5 ms every 35 ms. This pulse duration was chosen to give an elastic bounce for our magnetic acceleration $\mu B^2/m \approx 6g$. In these sequences we have suppressed a horizontal motion of the condensate, discussed below, to illustrate the bouncing more clearly and focus attention on the evolution of the condensate shape.

The rapid growth in the condensate width seen in Fig. 3 (a) is a consequence of the instability of the cavity formed by the magnetic mirror and gravity. A gravity-cavity is unstable, meaning that the atomic trajectories walk out of the cavity, when the the radius of curvature $R$ of the mirror is less than twice the release height $h$. This is the case for Fig. 3 (a), where the radius of curvature in the $xy$ plane $R_{xy} = h \approx 1.4$ mm. The corresponding classical motion is easy visualized in this case. Since the gravitational potential energy is much larger than the initial kinetic energy of the condensate, all atomic trajectories strike the mirror at an angle very close to vertical. The reflected trajectories then all cross the mirror axis at a focal point $\approx 0.5 R_{xy}$ above the mirror surface on their way to turning points at height $h$. The condensate will therefore be strongly focused through this focal point after each bounce, as can be clear seen after the second bounce in Fig. 3 (a), even though our viewpoint is at an angle of 60° to the $z$ axis. In contrast Fig. 3 (b) corresponds to the stable case $R_{xy} = 5h$. The evolution here is similar to that expected for bouncing on a flat mirror.

A complete theoretical analysis of our experiment would require a numerical integration of the Gross-Pitaevskii equation on a large mesh in three dimensions, a rather formidable task. We have therefore developed two simpler theoretical models of the bounce dynamics. The first extends the work of Castin and Dum who showed that a BEC, in the Thomas-Fermi regime and confined in a time-dependent parabolic potential $U(r, t) = \frac{\omega_0^2}{2} \sum_{j=1}^{3} \omega_j(t)^2 r_j^2$, has an atomic spatial distribution which obeys the simple scaling law:

$$n(r, t) = \max \left\{ \frac{n_0}{\lambda_1 \lambda_2 A_3} \left( 1 - \sum_{j=1}^{3} \frac{r_j^2}{A_j^2} \right), 0 \right\}$$

where $\lambda_j(t)$ is the solution of

$$\frac{d^2 \lambda_j(t)}{dt^2} + \omega_j(t)^2 \lambda_j(t) - \frac{\omega_j(0)^2}{\prod_{i=1}^{3} \lambda_i(t)^{1+\delta_{ij}}} = 0,$$

$\lambda_j(0) = 1$ and the initial Thomas-Fermi radii are $A_j$. It can be shown that if the potential is parabolic about the time-varying center $r_j(t)$ of the BEC, i.e. if $U(r, t) = \frac{\omega_0^2}{2} \sum_{j=1}^{3} \omega_j(t)^2 (r_j - r_{ej}(t))^2$, then the same scaling law applies for the BEC if one subtracts out the overall center-of-mass motion and uses the magnetic curvatures acting at the condensate center. This model is analytic, and therefore fast to compute, and it includes the effects of mean-field repulsion. In order to verify that this locally harmonic potential approximation is valid for our mirror, we developed a second model, based on a Monte Carlo simulation of the classical dynamics. The simulation computes the classical trajectories of $10^5$ atoms with initial positions
FIG. 2: Sequences of experimental absorption images taken at times \( t = 2, 4, 6, \ldots, 74 \) ms for control fields (a) \( B_c = 0 \) and, (b) \( B_c = 70 \) G. Each individual image in a sequence is 0.5 mm wide (the \( z' \) direction) and 2.0 mm high (the \( y \) direction). In the image sequence (c) the control field is \( B_c = 0 \) during the first bounce, and the control field and pulse duration during the second bounce are chosen to collimate and launch the BEC. In this sequence the individual images have dimensions \( 0.5 \times 2.7 \) mm\(^2\).

and velocities chosen at random so as to give the Thomas-Fermi result for the free expansion (Eq. 2). In this model the mirror potential can be treated exactly, but atomic interactions after the initial mean-field energy driven expansion are neglected. Both theoretical models make the reasonable approximation that diffraction is negligible, and both use values of the parameters characterizing our mirror \((B_0, B', B'', B_c)\) measured in other experiments.

Fig. 3 shows that the Monte Carlo simulation is in excellent agreement with the experimental data of Fig. 2(b). The horizontal motion seen here (and suppressed in Fig. 2) is partly due to a tilt in the magnetic trap with respect to gravity, and partly due to the fact that for our coil orientation the magnetic potentials have an increasingly large slope in the \( z \) direction as \( y \) decreases (see Fig. 1). This slope also causes the condensate to rotate in the \( zy \) plane during a bounce. The analytic Thomas-Fermi calculation yields very similar results, except that the rotation of the condensate is not reproduced because this model assumes a perfectly harmonic trap.

To make a more quantitative comparison of the experimental data with these models, we fitted both the Monte Carlo simulation and the experimental absorption images to a Thomas-Fermi column density to obtain the condensate radii (see inset in Fig. 4). These two quantities can also be obtained directly from our analytic Thomas-Fermi model. The comparison between experiment and the two models is shown in Fig. 4. The two models are in good agreement with each other, confirming the validity of their respective approximations, and both are in good agreement with the measured condensate radii.

The horizontal size of the bouncing condensate is a sensitive probe of the optical quality of the mirror because any corrugations or other aberration will increase the size of the condensate on each bounce. We therefore repeated the two simulations, adding the effects of an rms mirror roughness of 0.5 mrad to the condensate on each bounce. The results are shown as the dashed curves in Fig. 4. The discrepancy between these curves and the experimental data implies that any deviations from specularity in our mirror are less than 0.5 mrad. This performance is at least a factor of three better than the best atomic mirrors based on evanescent waves \([14]\) and magnetic media \([2]\), making our pulsed magnetic mirror the smoothest cold atom mirror yet demonstrated.

In addition to the extremely good optical quality, the mirror also has the property of being adaptive (changing \( B_c \) changes the focal length) and inelastic (changing the pulse duration changes the vertical impulse imparted to the BEC). A second experiment was performed to explicitly take advantage of these aspects of the mirror. We realized a BEC “beam expander”, by first expanding the BEC by reflection from a short radius \((R_{xy} = h)\) mirror, and then
on the second reflection launching $[23]$ and collimating it by a longer interaction with a weakly focusing mirror. The results are shown in Fig. 3 (c). This process produces a low density, ultracold, condensate fountain, which might find application, for example, in atomic clocks. It can also be regarded as a demonstration of delta-kick cooling $[24]$.

Fig. 4 shows that the evolution of a condensate reflected by the magnetic mirror is well described by classical physics. This is in contrast to the reflection of condensates by light sheets $[11]$, where the reflected condensate develops a complicated self-interference structure near the turning point of its trajectory. Fringes occur when the de Broglie wavelength of the condensate is large at positions where the classical atomic trajectories cross, a condition which is never satisfied for our mirror. Of course in many atom-optical applications the absence of interference fringes is an advantage.

Our two theoretical models do suggest that non-classical evolution should be visible in the $xy$ plane under certain conditions. In particular, for the case $R_{xy} = h$ (Fig. 2 (a)) the density and the “waist” size at the tight focus near the $R_{xy}/2$ point are respectively high enough and small enough that the effects of atomic interactions and diffraction might be visible. The condensate’s behavior near this tight waist provides a sensitive probe of the mirror quality, so it would therefore be interesting to image the condensate along the trap axis and compare the results with a full calculation using the time-dependent 3D Gross-Pitaevskii equation. Another topic for future investigation is the coherence of the mirror. Since the trap field is sufficiently well-controlled to produce a condensate in the first place, it seems very likely that the mirror will preserve coherence. Nevertheless it would be desirable to demonstrate this directly, e.g. by splitting a condensate coherently and then interfering the two reflections.

In conclusion, we have demonstrated adjustable focusing and reflection of a BEC from a pulsed magnetic mirror, and shown that the evolution of the reflected condensate is well-described by a simple theory. The mirror establishes a new limit for the optical quality of atom-optical elements. Finally, we have also used the adaptive and inelastic properties of the mirror to realize an atomic “beam expander” and used it to make a condensate fountain.

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