DOUBLE-MODE RR LYRAE VARIABLES: PULSATIONAL MASSES REVISITED

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ABSTRACT

Double-mode RR Lyrae variables (i.e., radial variables which are simultaneously pulsating in both fundamental and first-overtone modes) appear to be fundamental tools for investigating the mass of old Population II horizontal-branch (HB) stars. The most widespread method adopted for evaluating the masses of these objects is based on the Petersen approach, which relies only on pulsational periods ($P_1/P_0$ versus $P_0$), and therefore is independent of any preliminary evaluation of the reddening and/or the distance modulus of the stellar cluster.

In this paper we supply an overview of the mass estimates and underline the role played by opacities as well as by full-amplitude nonlinear models for removing the discrepancy between pulsational and evolutionary masses. On the basis of the comparison between the theoretical scenario and double-mode RR Lyrae stars belonging to selected Galactic globular clusters (IC 4499, M3, M15, M68, NGC 2419, NGC 6426), we show that the $P_1/P_0$ versus $P_0$ diagram can also provide valuable constraints on the luminosity of these variables.

Subject headings: globular clusters: individual (IC 4499, M3, M15, M68, NGC 2419, NGC 6426) — stars: horizontal-branch — stars: oscillations — stars: variables: other

1. INTRODUCTION

The evolution and the final fate of stellar structures are mainly governed by the amount of original mass. According to such plain evidence, the large number of present theoretical interpretations of the evolutionary status of stars and stellar systems provides tight constraints on the mass of the investigated objects. An independent estimation of stellar masses would of course be of paramount interest, since it would represent prima facie evidence for the physical reliability of the adopted evolutionary scenario. As is well known, radial pulsating structures offer such an opportunity for the simple reason that the pulsations are mainly governed by gravity. On this simple basis, the periods should depend, as they actually do, on pulsator masses and radii (i.e., on the stellar parameters $M$, $L$, and $T$). Jorgensen & Petersen (1967) originally suggested that the occurrence of double-mode pulsators could give the unique opportunity to provide a straightforward evaluation of pulsator masses taking into account only the ratio between the fundamental ($P_0$) and the first-overtone periods ($P_1$).

According to this scenario, Petersen (1973) introduced the Petersen diagram (PD) $P_1/P_0$ versus $P_0$ as a suitable tool for estimating the actual mass value of double-mode pulsators. The application of this procedure to RR Lyrae stars dates back to Cox, King, & Hodson (1980). On the basis of the PD they found a mass value of about $M/M_\odot = 0.65$ for the only double-mode RR Lyrae (RRd) star known at that time (AQ Leonis; Jerzykiewicz & Wenzel 1977). Since then the discovery of new RRd variables in several Galactic globular clusters (Clement et al. 1986, hereafter C86) and in dwarf spheroidal galaxies (Nemec 1985a; Kaluzny et al. 1995) has brought on an interesting discussion about the constraints on their pulsational and evolutionary characteristics. Cox, Hodson, & Clancy (1983, hereafter CHC) investigated the PD for RRd pulsators in the metal-poor Oosterhoff II (Oo II) cluster M15 and derived a pulsational mass of the order of $M/M_\odot = 0.65$. The same authors suggested a mass of the order of $M/M_\odot = 0.55$ for the two RRd pulsators belonging to M3, the prototype of intermediate metallicity Oosterhoff I (Oo I) clusters. These results were subsequently confirmed by Nemec (1985b) and by C86, who found similar mass values for RRd variables in the Oo I cluster IC 4499.

However, for the quoted HB pulsators current evolutionary theories foresee larger masses, namely $M/M_\odot \simeq 0.8$ and $\simeq 0.65$ for Oo II and Oo I clusters, respectively (see Bono et al. 1996a). Such a disturbing discrepancy between the masses of RRd variables determined from pulsational and from evolutionary theories, was settled as soon as Cox (1991) found that pulsational models incorporating new and updated opacity evaluations were able to reconcile pulsational and evolutionary predictions. The settling of this long-standing discrepancy was thus regarded as evidence for the reliability of the new opacity tables.

In this paper we present a new investigation of the Petersen approach, which discloses some unexpected results and sheds new light on the matter. We show that opacity, as originally suggested by Cox (1991, 1995), is the key physical ingredient that produces the disagreement between pulsational and evolutionary masses. However, we also find that this discrepancy,
even using old opacities, can be consistently removed either by adopting a much finer spatial resolution in linear computations or by relying on detailed nonlinear models. Nevertheless, an exhaustive solution to the problem of RR Lyrae masses can only be achieved if both new opacities and full-amplitude, detailed, nonlinear, nonlocal, and time-dependent convective models are taken into account.

2. MASSES AND LUMINOSITIES OF RRd VARIABLES

During the last few years we have carried out an extensive survey of limiting amplitude, nonlinear models of RR Lyrae variables (Bono & Stellingwerf 1994, hereafter BS). The main purpose of this project is to examine the dependence of modal stability and pulsation behavior on astrophysical parameters (for complete details see Bono et al. 1996a). As a by-product of this investigation, we revisited the problem of pulsator masses by investigating the dependence of the Petersen diagram on the various assumptions governing theoretical calculations.

The sequences of static envelope models were analyzed in the linear nonadiabatic approximation (Castor 1971), and each model was required to cover the outer 90% of the stellar photospheric radius. The outer boundary condition was typically fixed at an optical depth of the order of 0.001. The linear models were constructed by neglecting convection and by adopting the analytical approximation of old Los Alamos “King” opacity tables provided by Stellingwerf (1975a, 1975b). On the basis of these assumptions a typical coarse model is characterized by 100–150 zones and a few percent of the total stellar mass. Complete details of the mass ratio between consecutive zones and the method adopted for constraining the hydrogen ionization region are given in Stellingwerf (1975a) and BS.

As a starting point, Figure 1 shows the theoretical PD obtained for selected values of stellar masses and luminosities. The models plotted in this figure present a stable linear limit cycle in the first two modes. To understand the meaning of theoretical data displayed in this figure, we recall that linear models provide evaluations of periods independently of the actual limit cycle stability of a given mode. As a consequence, we have to bear in mind that a rather large amount of data in similar figures should be regarded as unphysical, since they supply the ratio $P_1/P_0$ even where either the fundamental or the first-overtone modes present an unstable nonlinear limit cycle.

The period ratios of the M15 RRd variables plotted in Figure 1 were evaluated taking into account different estimates (Nemec 1985b; Kovacs, Shlosman, & Buchler 1986; Clement & Walker 1990; Purdue et al. 1995). The error bar plotted in the lower right-hand corner refers to these measurements. The comparison between theoretical models and observational data, shown in the figure, clearly supports previous results given in the literature under similar theoretical assumptions and discloses the occurrence of the “mass discrepancy problem.” At the same time, Figure 1 shows that at a given fundamental period, the period ratio $P_1/P_0$ appears largely independent of the assumed luminosity level.

In order to investigate the dependence of linear periods on the spatial resolution previously adopted, a new set of linear detailed models have been computed by adopting the prescriptions suggested by BS. The number of zones for these new sequences of models is increased by roughly a factor of 2 with respect to the coarse ones and ranges from 200 to 300. Figure 2 shows the results of these new computations, disclosing that the “mass discrepancy problem” also appears affected by the method adopted to discretize the physical structure of the static envelope model. As a matter of fact, we find that periods provided by linear, nonadiabatic, radiative models constructed with a finer spatial resolution partially remove the degeneracy of the luminosity levels. Moreover, as a most relevant point, these calculations now suggest that the mass value of Oo II RRd variables should be of the order of $M/M_\odot = 0.8$, whereas Oo I RRd variables should increase to about $M/M_\odot = 0.70$, in much better agreement with evolutionary prescriptions (see Bono et al. 1996a).

However, BS have already shown that linear periods are only a first, though good, approximation of the pulsational periods obtained from a more appropriate nonlinear treatment of the pulsation. Thus the problem arises if linear predictions about RRd masses are preserved in the nonlinear approach. To properly address this fundamental theoretical question, Figure 3 displays the results of several sequences of nonlinear, nonlocal, and time-dependent convective models constructed by assuming the same equation of state and the same opacities adopted in the linear regime. According to the negligible influence of spatial resolution on nonlinear, limiting amplitude characteristics and modal stability (BS and refer-
ence therein), in order to speed up the calculations required by the nonlinear approach only coarse static envelope models were taken into account.

The dynamic behavior of the envelope models was examined for the first two modes, and the static structures were forced out of equilibrium by perturbing the linear radial eigenfunctions with a constant velocity amplitude of $20 \text{ km s}^{-1}$. The method adopted for initiating nonlinear models unavoidably introduces a spurious component of both periodic and nonperiodic fluctuations which are superimposed on the pure radial motions. As a consequence, before the dynamic behavior approaches the limit cycle stability it is necessary to carry out extensive calculations. The fundamental and first-overtone sequences have been integrated in time for at least 2000 periods. The models located close to the fundamental blue edge and to the first-overtone red edge were followed for a longer time interval (2000–6000 periods), since in these regions of the instability strip before the dynamic motions approach their asymptotic behavior, a switchover to a different mode could take place even after several thousand periods. The integration is generally stopped as soon as the nonlinear work term is vanishing and the pulsational amplitudes present a periodic similarity of the order of $10^{-7}$.

Therefore, it turns out that the decrease of theoretical points plotted in Figure 3 is tightly connected with the morphology of the “OR” region, since only envelope models that present stable nonlinear limit cycles both in the fundamental and in the first overtone modes were taken into account. Moreover, data in Figure 3 reveal that the nonlinear PD differs intrinsically from the canonical linear PD. In fact, in this new context the spurious theoretical points connected with models that present a unique stable limit cycle (fundamental or first overtone) have obviously disappeared. A direct, interesting consequence of this new theoretical scenario is that the comparison between nonlinear periods and observational data can now give useful information on both stellar masses and luminosities of the pulsators. The reader interested in a thorough analysis concerning the evaluation of these parameters on the basis of RRd variables belonging to both Oo I and Oo II clusters is also referred to Cox (1995) and Walker (1995). In the evaluation of masses we eventually find that nonlinear results do not fully support linear indications. In fact, on the basis of nonlinear periods we obtain a stellar mass of $M/M_J \approx 0.7$ for Oo II cluster pulsators, whereas for the RRd variables in IC 4499 we estimate a mass of the order of $M/M_\odot = 0.60$. As a consequence, the agreement found by relying on linear, detailed models has to be regarded as an artifact of the computational procedure. Moreover, for M15 and M68 pulsators we find a luminosity around $\log(L/L_\odot) \approx 1.8$, which appears somewhat larger than the currently accepted evolutionary predictions.

Bearing in mind the present scenario, we now take into account the effects of the new opacities provided by Rogers & Iglesias (1992) for temperatures higher than $10^4$ K and by Alexander & Ferguson (1994) for lower temperatures. The reader interested in the method adopted for handling the new opacity tables is referred to Bono, Incerpi, & Marconi (1996b). For the sake of conciseness, we briefly quote the mass evaluations obtained from linear computations: $M/M_\odot = 0.72$, 0.60 (coarse models) and $M/M_\odot = 0.78$, 0.65 (detailed models) for pulsators in Oo II and Oo I clusters, respectively. Figure 4 shows nonlinear periods based on updated radiative opacities. The comparison with observational data now points to a promising theoretical scenario, since it predicts a stellar mass slightly greater than $M/M_\odot = 0.8$ for RRd pulsators in Oo II clusters and a mass value around $M/M_\odot = 0.65$ for RRd variables in IC 4499. Both results are now in excellent agree-
ment, within the error bar, with canonical evolutionary predictions. Data plotted in Figure 4 also suggest a luminosity level of the order of $\log \left( \frac{L}{L_J} \right) \approx 1.7$ for Oo II RRd variables, whereas the corresponding luminosity level for RRd variables in IC 4499 falls between the computed luminosity levels at $\log \left( \frac{L}{L_J} \right) \approx 1.61$ and 1.72. The overall good agreement with evolutionary predictions, presented in Bono et al. (1996a), shows that, thanks to the updated physical input, both pulsational and evolutionary theories converge to form a homogeneous scenario concerning the long-debated question of RR Lyrae luminosity in globular clusters.

Finally, it is worth noting that the two RRd variables in M3 appear slightly more massive and more luminous than RRd variables in IC 4499. According to current metallicity estimates for these clusters ($[\text{Fe/H}]_{\text{M3}} = -1.7$, $[\text{Fe/H}]_{\text{IC 4499}} = -1.5$), even this finding appears in satisfactory agreement with the evolutionary prescriptions.

3. CONCLUSIONS

In this paper we have revisited the approach based on the PD for determining the masses of RRd variables. It is shown that the pulsator masses evaluated through the comparison between periods obtained in a linear, nonadiabatic, radiative regime and observational data might be affected by substantial systematic errors. On the other hand, the periods provided by the surveys of nonlinear, nonlocal, and time-dependent convective models point out that, even though the discrepancy between linear and nonlinear periods has often been considered negligible, it plays a key role in properly defining the location of double-mode pulsators inside the Petersen diagram ($P_1/P_0$ versus $P_0$).

As a most relevant point, we found that a nonlinear Petersen diagram, constructed taking into account both nonlinear models and new radiative opacities, simultaneously provides valuable constraints not only on the stellar masses but also on the luminosities of RRd variables. The pulsational masses and luminosities of double-mode pulsators obtained in this new theoretical framework confirm the results recently provided by Cox (1995). The comparison with observational data of RRd variables in both Oo I and Oo II Galactic globular clusters shows a satisfactory agreement with current evolutionary and pulsational predictions. At the same time, this agreement supplies a new piece of evidence against the suggested anomaly of HB star luminosities. Further applications of this new approach for constraining the physical parameters of RRd variables belonging to the Galactic field, to the central region of the LMC, and to dwarf spheroidal galaxies are under way.

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REFERENCES

Alexander, D. R., & Ferguson, J. W. 1994, ApJ, 437, 879
Bono, G., Caputo, F., Castellani, V., & Marconi, M. 1996a, A&A, in press
Bono, G., Incerpi, R., & Marconi, M. 1996b, ApJ, 462, L97
Bono, G., & Stellingwerf, R. F. 1994, ApJS, 93, 233 (BS)
Castor, J. I. 1971, ApJ, 166, 109
Clement, C. M., & Nemec, J. M. 1990, JRASC, 84, 434
Clement, C. M., Nemec, J. M., Robert, N., Wells, T., Dickens, R. J., & Bingham, E. A. 1986, AJ, 92, 825 (C86)
Clement, C. M., & Walker, I. 1990, AJ, 101, 1352
Cox, A. N. 1991, ApJ, 381, L71
———. 1995, in ASP Conf. Ser. 78, Astrophysical Applications of Powerful New Databases, ed. S. J. Adelman & W. L. Wiese (San Francisco: ASP), 243
Cox, A. N., Hodson, S. W., & Clancy, S. P. 1983, ApJ, 266, 94 (CHC)
Cox, A. N., King, S. P., & Hodson, S. W. 1980, ApJ, 236, 219
Jerzykiewicz, M., & Wenzel, W. 1977, Acta Astron., 27, 35
Jorgensen, H. E., & Petersen, J. O. 1967, Z. Astrophys., 67, 377
Kaluzny, J., Kubiak, M., Szymański, M., Udalski, A., Krzeminski, W., & Mateo, M. 1995, A&A, 112, 407
Kovacs, G., Shlosman, I., & Buchler, J. R. 1986, ApJ, 307, 593
Nemec, J. M. 1985a, AJ, 90, 204
———. 1985b, AJ, 90, 249
Petersen, J. O. 1973, A&A, 27, 89
Purdue, P., Silberman, N. A., Gay, P., & Smith, H. A. 1995, AJ, 110, 1712
Rogers, F. J., & Iglesias, C. A. 1992, ApJS, 79, 507
Stellingwerf, R. F. 1975a, ApJ, 195, 441
———. 1975b, ApJ, 199, 705
Walker, A. R. 1994, AJ, 108, 555
———. 1995, in ASP Conf. Ser. 83, Astrophysical Applications of Stellar Pulsation, ed. R. S. Stobie & P. A. Whitelock (San Francisco: ASP), 198