On genus-one-corrected extremal black holes and the correspondence principle

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Abstract

We discuss charged black-hole solutions to the equations of motion of the string-loop-corrected effective action. At the string-tree level, these solutions provide backgrounds for the "chiral null model". The effective action contains gravity, dilaton and moduli fields. Analytic solutions of the one-loop-corrected equations of motion are presented for the extremal magnetic and dyonic black holes. Using the fact that in magnetic solution the loop-corrected dilaton is non-singular at the origin, we apply the correspondence principle to show that the entropy of the loop-corrected magnetic black hole can be interpreted as the microscopic entropy of the D-brane system.

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1 Introduction

At present, string theory is considered the best candidate for a fundamental theory that would provide a consistent quantum theory of gravity unified with other interactions [1]. Thus, there is a problem of understanding how the intrinsically stringy effects modify Einstein gravity. In this paper we are going to focus on two of these effects: the presence of scalar fields such as the dilaton and the moduli; and higher-genus contributions modifying the tree-level effective action. We consider a class of black-hole solutions obtained in the "chiral null models" embedded into the (compactified) heterotic string theory [2] which includes the majority of all superstring black holes, in particular, electric, magnetic and dyonic black holes. Within the "chiral null models" it was possible to show that the Bekenstein-Hawking entropy can be interpreted as statistical entropy of string oscillating states [3] and [4] (for another class of models see review [5] and references therein).

In paper [6] a class of solutions of the model effective action for the abelian field interacting with the scalar moduli fields in flat 4D space-time was investigated. Moduli-dependence of gauge couplings mimicked higher-loop string effects. In paper [7], using a model string effective action which includes a small constant threshold correction to the gauge coupling, an electric black hole solution was considered. However, no modulus dependence of the model effective action was taken into account and the coupling of the dilaton to the electric field strength was assumed to be small.

In this paper we investigate a class of "realistic" charged black-hole solutions obtained by using a 4D effective action which includes dilaton and modulus fields. The gauge fields enter the action with gauge couplings that include threshold corrections modifications modeling string-loop effects.

In section 2 we review the structure of the 4D dyonic black hole in heterotic string theory provided by the chiral null model. In section 3 we consider the structure of the genus-one (torus topology) correction to the low-energy string effective action and discuss its dependence on the moduli.

Section 4 is devoted to calculation of modifications of geometric and thermodynamic properties of the black-hole solutions to the string-loop-corrected equations of motion.

As compared to previous results, the present investigation is, to our knowledge, the first one where string effects are considered for a realistic class of superstring black hole solutions.

Using the explicit loop-corrected solution for the extremal magnetic black hole, we show that dilaton is non-singular at the origin. This opens a possibility to apply to this system the correspondence principle and to show that the geometrical entropy of magnetic black hole is reproduced as the microscopic entropy of the weakly-interacting D-brane system.

2 Dyonic 4D black hole in toroidally compactified heterotic string theory

In this paper we discuss black hole solutions obtained in conformal chiral null models with curved transverse part. These can be considered embedded in heterotic string theories. The 4D low energy effective action is obtained by dimensional reduction of the 10D superstring effective action onto the transverse part.

The 4D effective action includes higher-genus contributions modifying the tree-level effective action. We consider a class of black-hole solutions obtained in the "chiral null models" embedded into the (compactified) heterotic string theory which includes the majority of all superstring black holes, in particular, electric, magnetic and dyonic black holes. Within the "chiral null models" it was possible to show that the Bekenstein-Hawking entropy can be interpreted as statistical entropy of string oscillating states (for another class of models see review and references therein).

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The 4D low energy effective action (in the Einstein frame) is
\[
S = \int d^4x \sqrt{-g} \left( R - \frac{1}{2} (\partial_\mu \phi)^2 - (\partial_\mu \sigma)^2 - (\partial_\mu \gamma)^2 \right.
- \frac{1}{4} e^{-\phi - 2\gamma}(F_{(1)})^2 - \frac{1}{4} e^{-\phi + 2\sigma}(F_{(2)})^2
- \frac{1}{4} e^{-\phi - 2\gamma}(F_{(3)})^2 - \frac{1}{4} e^{-\phi - 2\sigma}(F_{(4)})^2 \bigg),
\]
where \(\phi\) is the dilaton field, \(\sigma\) and \(\gamma\) are moduli related to the radii of the \(T^2\). Here \(F_{(1)}\) and \(F_{(3)}\) are magnetic fields strengths, and \(F_{(2)}\) and \(F_{(4)}\) are electric field strengths. The nontrivial 6D backgrounds of the chiral null model (the remaining part of the metric is flat) obtained as solutions of equations of motion of this action are
\[
ds^2 = -\lambda(r) dt^2 + \lambda^{-1}(dr^2 + r^2 d\Omega_2^2)
\]
\[
\lambda^2(r) = FK^{-1}k f^{-1}
\]
\[
2\phi = \ln FK^{-1}k f^{-1}
\]
\[
e^{2\sigma} = FK
\]
\[
e^{2\gamma} = fk
\]
where \(F^{-1}, K, f\) and \(k^{-1}\) are harmonic functions satisfying the equations \(\delta^{ij}\partial_i \partial_j K = 0\) for \(i, j = 1, 2, 3\), etc.. The gauge fields electric and magnetic fields potentials are \(A_{(2)} = (K^{-1}, 0)\), \(A_{(4)} = (F, 0)\), and \(A_{(1)} = (0, a_s)\), \(A_{(3)} = (0, b_s)\), \(s = 1, 2, 3\). The latter satisfy the relations
\[
2\partial_\mu b_\mu = -\epsilon_{pq\mu} \partial^\mu F^q
\]
\[
2\partial_\mu a_\mu = -\epsilon_{pq\mu} \partial^\mu k^{-1}.
\]
Electric and magnetic charges are obtained from the asymptotics of the field strengths calculated with the gauge potentials \(A_{(2)}, A_{(4)}\) and \(A_{(1)}, A_{(3)}\) respectively.

### 3 One-loop corrections

The tree level effective action in string theory receives string-loop corrections due to integration over world sheets of higher topologies [9]. Here we are going to consider the one-string-loop corrections to the gauge couplings [3]. The effective action is
\[
S = \int d^4x \sqrt{-g} \left( R - \frac{1}{2} (\partial_\mu \phi)^2 - (\partial_\mu \sigma)^2 - (\partial_\mu \gamma)^2 \right.
- \frac{1}{4} e^{-\phi + 2\gamma + \Delta}(F_{(1)})^2 - \frac{1}{4} e^{-\phi + 2\sigma + \Delta}(F_{(2)})^2
- \frac{1}{4} e^{-\phi - 2\gamma + \Delta}(F_{(3)})^2 - \frac{1}{4} e^{-\phi - 2\sigma + \Delta}(F_{(4)})^2 \bigg).
\]

\(^1\) We discuss a simplified case without the axion field which can be also included in the chiral null model.

\(^2\) A more detailed analysis of relevant threshold corrections to the effective action will be given elsewhere.
For toroidal compactification, the one-loop correction to the action (1) is
\[\Delta = -b \ln(T_2|\eta(T)|^4U_2|\eta(U)|^4),\]
where \(\eta\) is the Dedekind \(\eta\)-function and the moduli \(T\) and \(U\) are,
\[T = T_1 + iT_2 = B + i\sqrt{\det G},\]
\[U = U_1 + iU_2 = (G_{12} + i\sqrt{\det G})/G_{11}.\]
Here, \(G_{ij}\) are the components of the "internal" metric, which in our case are \(G_{12} = 0, G_{22} = fk = e^{2\gamma}\) and \(G_{11} = FK = e^{2\sigma}\); \(B\) is the axion field which in our case is zero and \(b\) is related to the one-loop \(\beta\)-function coefficients associated with the \(N = 2\) subsection [9]. Note that for the abelian gauge fields which we consider, the constant \(b\) is negative.

Using the modular property of the \(\eta\)-function \(\eta(-1/\tau) = (-i\tau)^{1/2}\eta(\tau)\) one can check that the choice \(G_{11} = fk\) and \(G_{22} = FK\) yields the same result for \(\Delta\);
\[\Delta = -b \ln(fk|\eta(i\sqrt{FK fk})|^4|\eta(i\sqrt{FK})|^4).\]

Some comments on the properties of the threshold corrections are in order. For the purely electric \((f = k^{-1} = 1)\) extremal \((F^{-1} = K)\) black hole as well as for the purely magnetic \((F^{-1} = K = 1)\) extremal \((f = k^{-1})\) black hole, the threshold correction is constant. The same is true if all the four charges are equal \((F^{-1} = K = f = k^{-1})\).

For the solutions we consider below the threshold correction tends to a finite nonzero value as the radius \(r\) goes to zero and infinity. For intermediate values of \(r\), the threshold correction is a slowly varying function of \(r\). This provides a motivation for considering constant threshold corrections to these black holes. For small \(r\) we are near the horizon of the tree-level solution and for asymptotically large \(r\) we obtain the mass and charges of the black hole. It seems natural to consider also near extremal black holes \((\Delta Q \ll Q)\) where the threshold corrections can be developed in a power series in \(\Delta Q/Q\) near the corresponding extremal values.

### 4 Black hole geometry with genus-one corrections

The equations of motion obtained from the action (4) are:
\[0 = g^{\nu\mu}D_\nu \partial_\mu \phi + \frac{1}{4}e^{-\phi + 2\gamma}(F_{(1)})^2 + \frac{1}{4}e^{-\phi + 2\sigma}(F_{(2)})^2 \]
\[+ \frac{1}{4}e^{-\phi - 2\gamma}(F_{(3)})^2 + \frac{1}{4}e^{-\phi - 2\sigma}(F_{(4)})^2,\]
\[0 = 2g^{\nu\mu}D_\nu \partial_\mu \sigma - \frac{1}{2}e^{-\phi + 2\sigma}(F_{(2)})^2 + \frac{1}{2}e^{-\phi - 2\sigma}(F_{(4)})^2 \]
\[= \frac{1}{4}\partial \Delta \frac{1}{4\partial \sigma}((F_{(2)})^2 + (F_{(4)})^2),\]
\[0 = 2g^{\nu\mu}D_\nu \partial_\mu \gamma - \frac{1}{2}e^{-\phi + 2\gamma}(F_{(1)})^2 + \frac{1}{2}e^{-\phi - 2\gamma}(F_{(3)})^2 \]
\[= \frac{1}{4}\partial \Delta \frac{1}{4\partial \gamma}((F_{(1)})^2 + (F_{(3)})^2).\]
\[0 = \partial_\mu \left( \sqrt{-g}(e^{-\phi+2\gamma} + \Delta)g^{\mu\nu} F^{(1)}_{\mu\nu} \right)\]  
\[0 = \partial_\mu \left( \sqrt{-g}(e^{-\phi+2\sigma} + \Delta)g^{\mu\nu} e^{\alpha} F^{(2)}_{\mu\nu} \right)\]  
\[0 = \partial_\mu \left( \sqrt{-g}(e^{-\phi-2\gamma} + \Delta)g^{\mu\nu} e^{\nu} F^{(3)}_{\mu\nu} \right)\]  
\[0 = \partial_\mu \left( \sqrt{-g}(e^{-\phi-2\sigma} + \Delta)g^{\mu\nu} e^{\nu} F^{(4)}_{\mu\nu} \right)\]

Note that the string-tree-level chiral null model provides a solution to the low energy effective action \([\bar{F}]\) which in a special renormalization scheme does not receive \(\alpha'\) corrections. Solutions of the one-loop-corrected effective action \([\bar{F}]\) modify the harmonic condition for the functions \(F^{-1}, K, f, k^{-1}\) which is substantial in the proof of the absence of \(\alpha'\) corrections. The \(\alpha'\) corrections become important near the horizon which for all the black holes we consider here is at \(r = 0\). If the modified functions \(F^{-1}, K, f, k^{-1}\) satisfy harmonic equation, we can expect that there will be no \(\alpha'\) corrections. Below we shall see that this can be the case for some black holes, for example, for the extremal dyonic solution.

### 4.1 Magnetic extremal black hole

In this subsection we consider purely magnetic \((F^{-1} = K = 1)\) extremal \((f = k^{-1})\) black holes. These are one of the few black holes for which the correspondence principle \([8]\) does not apply. The reason lies in the behavior of the dilaton near the horizon. This "pathology" of magnetic solutions was already noted in the flat space analysis of \([8]\). As mentioned in the previous section, the threshold correction for this black hole is independent of \(r\) and equal to \(\Delta = -\delta \ln |\eta(i)|^b\) which we will consider to be numerically small. We shall look for solutions to the equations (8)-(15) in the form:

\[g_{00} = -\tilde{\lambda}_1 = -\frac{1}{f_0} - \Delta \lambda_1\]
\[g_{ij} = \delta_{ij} \tilde{\lambda}_2^{-1} = \delta_{ij} \left( \frac{1}{f_0} + \Delta \lambda_2 \right)^{-1}\]
\[\phi = \ln f_0 + \Delta \phi_1\]
\[\gamma = 0\]
\[F_{ij}^{(1)} = F_{ij}^{(3)} = -\epsilon_{ijk} \partial^k (f_0 + \Delta f_1)\]

Here \(f_0\) is a solution of harmonic equation \(\delta_{ij} \partial_i \partial_j f_0 = 0\). In the first order in \(\Delta\) equations are

\[0 = \partial^2 \phi_1 + \frac{\lambda_1 + \lambda_2}{2f_0} (\partial f_0)^2 - \frac{\phi_1}{f_0^2} (\partial f_0)^2 - \frac{1}{2} \partial f_0 \partial (\lambda_2 - \lambda_1)\]
\[ 0 = \frac{1}{2} \partial^2 \phi_1 + f_0 \partial^2 \lambda_2 + \frac{9}{4} \partial f_0 \partial \lambda_2 + \frac{1}{4} \partial f_0 \partial \lambda_1 - \frac{1}{2f_0} \partial f_0 \partial \phi_1 \]
\[ + \frac{\lambda_1 + \lambda_2}{4f_0} (\partial f_0)^2 \]
\[ 0 = \frac{f_0}{2} \partial_i \partial_j (\lambda_2 - \lambda_1) + \frac{2}{f_0} \partial_i (f_0 \partial_j \phi_1) \]
\[ + \frac{1}{f_0} \partial_i (f_0 \partial_j \lambda_2) - \frac{1}{2f_0} \partial_i f_0 \partial_j f_0 - \frac{1}{2} \partial_i \partial_j f_0 \partial f_0 \partial \phi_1 \]
\[ + \frac{\lambda_1}{f_0} \partial_i f_0 \partial_j f_0 + \frac{1}{2f_0} \partial_i \partial_j f_0 + \frac{1}{2f_0} \partial_i f_0 \partial_j f_0 \]
\[ \lambda_2 (\partial_i f_0 \partial_j f_0 + \frac{1}{2} \partial_i \partial_j f_0) \]
\[ 0 = \frac{f_0}{2} \partial^2 (\lambda_2 - \lambda_1) + \frac{1}{2} \partial^2 \phi_1 + \frac{2}{f_0^2} \partial f_0 \partial f_1 - \frac{1}{2f_0} \partial f_0 \partial \phi_1 \]
\[ + \frac{3}{4} f_0 \partial f_0 \partial \lambda_2 - \frac{1}{4} \partial f_0 \partial \lambda_1 - \frac{\phi_1}{f_0^2} (\partial f_0)^2 + \frac{3(\lambda_1 + \lambda_2)}{4f_0} (\partial f_0)^2 + \frac{1}{f_0} \partial f_0 \partial \phi_1, \]
\[ \text{(21)} \]
\[ \text{here the first equation is the dilaton equation of motion (8), the second equation is the} \]
\[ (00) \text{ components of equation (15), and the last two equations are the transverse and} \]
\[ \text{longitudinal parts of the (ij) component of equation (15) The equations for gauge} \]
\[ \text{fields are satisfied if we assume that all fields depend on } r \text{ through } f_0. \text{ It is convenient} \]
\[ \text{to rewrite} \]
\[ \text{the equations in new variables} \]
\[ \psi = \frac{\lambda_1 + \lambda_2}{2f_0} - \phi_1 \]
\[ \tilde{\lambda} = \lambda_2 - \lambda_1 \]
\[ \text{(23)} \]
\[ \text{(24)} \]
\[ \text{Integrating the system in these new variables effectively requires solving first-order} \]
\[ \text{differential equations. Integration constants are defined so that for large values of } r \text{ the} \]
\[ \text{metric is asymptotic to the Lorentz metric. We obtain} \]
\[ \phi = \ln f_0 + \frac{\Delta}{2} f_0 (\ln f_0 - 1) \]
\[ f = f_0 - \frac{\Delta}{2} f_0^2 \]
\[ \tilde{\lambda}_1 = \frac{1}{f_0} + \frac{3 \Delta}{2} \ln f_0 \]
\[ \tilde{\lambda}_2 = \frac{1}{f_0} - \frac{\Delta}{2} \ln f_0. \]
\[ \text{(25)} \]
\[ \text{Similar logarithmic corrections to the geometry of black holes appeared in a completely} \]
\[ \text{different quantum gravity approach in [10]. The asymptotic charges are} \]
\[ M = \frac{P}{2} (1 - \frac{3\Delta}{2}) \]
\[ Q_m = P (1 - \Delta) \]
\[ D = P \]
\[ \text{(26)} \]
\[ \text{where } D \text{ is the dilaton charge. Although the tree-level solution was BPS saturated, the} \]
\[ \text{genus-one-corrected solution is no longer BPS saturated: } M \neq Q_m. \text{ The location of the} \]
\[ \text{horizon is found from the equation} \]
(note that $\Delta < 0$). From this transcendental equation we find that for small values of $\Delta$ the horizon is located at $r = 3|\Delta|P/2 \ln(2/3|\Delta|) \square$, which is close to zero. We see that that horizon is pushed outward and the mass is increased as in the case of the electric black hole considered in \cite{7}. It is interesting to note that if the black hole we consider were non-abelian $b > 0$ yielding positive $\Delta > 0$ (note that $\ln |\eta(iT)|^4$ is always negative), there would be no horizon.

\subsection*{4.2 Dyonic extremal black hole}

Here we consider the case where all the charges are equal ($F^{-1} = K = f = k^{-1}$), now all the moduli as well as the dilaton are zero (see definitions \cite{2}). Before beginning calculations similar to the those of the previous subsection, note that the one-loop-corrected effective action is now of the form

$$S = \int d^4 x \sqrt{-g} \left( R - \frac{1}{2} \Delta \left( F_1^2 + F_2^2 \right) \right). \quad (28)$$

From this expression it is clear that a solution can be found by redefining electric and magnetic fields and leaving the metric intact. Namely

$$F_{ij}^{(1)} = -\varepsilon_{ijk} \partial^k \frac{f_0}{\sqrt{1 + \Delta}}$$
$$F_{0i}^{(2)} = \partial_i (\sqrt{1 + \Delta f_0})^{-1} \quad (29)$$

After this redefinition the action \eqref{28} takes the form of the tree-level effective action. Decomposing the metric and gauge fields as in the previous subsection, we obtain other solutions to \eqref{28}, but only \eqref{29} is asymptotic to the solution of the tree-level effective action. The requirement that both solutions have the same asymptotics is natural from the physical point of view, because small perturbation of the couplings should not affect the spacetime far from the black hole. The electric and magnetic charges are modified in a different way: $Q_e = \sqrt{1 + \Delta} Q$ and $Q_m = Q/\sqrt{1 + \Delta}$. One can see that the BPS condition is violated (note, however, that in the first order in $\Delta M = Q = \frac{1}{2}(Q(1 + \Delta/2) + Q(1 - \Delta/2))$). Although the charges are modified, the entropy is unchanged. The position of the horizon is again at $r = 0$. Since the modified potentials are obtained by rescaling the tree-level ones, they satisfy harmonic equation and receive no $\alpha'$ corrections.

\subsection*{4.3 Electric extremal black hole}

For the purely electric ($f = k^{-1} = 1$) extremal ($F^{-1} = K$) black hole, using the following ansatz

$$F_{0i} = \partial_i K_0^{-1} + \Delta \partial_i K_1$$
$$g_{00} = -\left( \frac{1}{K_0} + \Delta \lambda_1 \right)$$
$$g_{ij} = \delta_{ij} \left( \frac{1}{K_0} + \Delta \lambda_2 \right)^{-1}$$

...
where all functions depend only on the \( r \), one finds that there is no solution to the one-loop effective action. The obstruction is connected with the fact that for this ansatz the equation for the \( g^{ij} \) components of the metric gives two equations since there are two different tensor structures (\( \partial_i \xi \partial_j \xi \) and \( \delta_{ij} (\partial K)^2 \)). In the magnetic case, because of the presence of the antisymmetric tensor in the definition of magnetic field, the system of equations of motion can be satisfied if all the fields depend on \( r \) through \( f_0 \). In the electric case the gauge field equation is not so easily satisfied and yields a system of equations that has no solutions for the spherically symmetric configuration. More exactly, for the ansatz with the special part of the metric proportional to \( \delta_{ij} \) we have five independent equations and four variables. The same is true for a general spherically-symmetric configuration in the magnetic case without the special assumption that \( r \)-dependence is through \( f_0 \). We conclude that for the electric extremal black hole the one-loop correction breaks the the spherical symmetry of the tree-level solution.

5 Genus-one-corrected magnetic black hole and the correspondence principle

The reason why the correspondence principle [3, 5, 8] cannot be applied to magnetic black hole is the behavior of the string coupling constant \( g_{st} = e^\phi \) as one approaches the horizon. At the tree level, we have

\[
g_{st} = f_0 = 1 + \frac{P}{r}
\]

and for \( r \to 0 \), \( g_{st} \to \infty \). Using the one-loop-corrected dilaton, we find that now the coupling constant is

\[
g_{st} = \exp(\phi) = f_0 \exp \left( -\frac{\Delta}{2} f_0 \left( \ln f_0 - 1 \right) \right).
\]

Now as one approaches the horizon which is close to \( r = 0 \), the coupling constant decreases. This is the behavior required for applicability of the correspondence principle. In this section we will show that using the correspondence principle one can find a microscopic description of the entropy of 4D magnetic black hole. The classical (macroscopic) entropy of the magnetic black hole, defined as one fourth of the area of the horizon is

\[
S = \frac{A}{4} = \pi (\lambda_2^{-1} r^2) |_{\text{hor}} = \frac{9}{8} \pi |\Delta| P^2 \ln(2/3|\Delta|)
\]

String tree-level black hole solution which is represented by the ”chiral null model” can be embedded both in NS-NS [12] and R-R sectors [13] of string theory. Let us consider the R-R embedding in type IIB theory. We use the one-loop-corrected magnetic black hole solution as background in bosonic part of the 2D world-sheet action of IIB superstring theory. Conformal invariance of this theory is preserved if we consider both tree-level and one-string-loop contributions to the effective action. The bosonic part of the latter is precisely the action [4] which equations of motion were solved to obtain the deformed, i.e. loop-corrected backgrounds.
modify the chiral null model in 5D so that under compactification to 4D it reproduces
the one-loop-corrected magnetic black hole. The 4D background is

$$
\begin{align*}
\tilde{\lambda}_1 &= \frac{1}{f_0} - \frac{3|\Delta|}{2} \ln f_0 \\
\tilde{\lambda}_2 &= \frac{1}{f_0} + \frac{|\Delta|}{2} \ln f_0 \\
\phi_{4D} &= \ln f_0 - \frac{|\Delta|}{2} f_0 (\ln f_0 - 1) \\
F_{ij} &= -\epsilon_{ijk} \partial^k (f_0 + \frac{|\Delta|}{2} f_0^2)
\end{align*}
$$

The 5D background which compactified to 4D provides the one-loop-corrected magnetic
black hole is

$$
\begin{align*}
ds_5^2 &= g_{00} dt^2 + dy^2 + 2a_i dy dx^i \nonumber \\
B_{yi} &= a_i \\
\phi &= \phi_{4D} \\
a_i dx^i &= P(1 + |\Delta| + \frac{|\Delta| P}{r})(1 - \cos \theta) d\varphi \\
g_{00} &= -(1 - \frac{3|\Delta|}{2} f_0 \ln f_0) \exp(-\frac{|\Delta|}{2} f_0 (\ln f_0 - 1)) \\
\tilde{f}^2 &= \frac{2f_0^2}{2 + |\Delta| f_0 \ln f_0} \exp(-\frac{|\Delta|}{2} f_0 (\ln f_0 - 1)).
\end{align*}
$$

Here we have considered the extremal case so that the dilaton in both dimensions is the
same. The obtained configuration is a solitonic brane with the RR gauge field $B$. The
mass, charge and entropy of this solution are (we omit the numerical factors)

$$
\begin{align*}
M &\sim \frac{|\Delta| P}{g^2 \alpha'^4} V \\
Q &\sim \frac{(1 + |\Delta|) P}{g \alpha'^{1/2}} \\
S &\sim \frac{|\Delta| \ln(2/3|\Delta|) P^2}{g^2 \alpha'^4} V
\end{align*}
$$

where $V$ is the volume of the six-dimensional torus formed by $y$ and other five compact
coordinates that in (35) we did not write explicitly. This is still a strong-coupling description; to find the weak-coupling description we apply the correspondence principle. At
weak coupling the solitonic brane, now with the required behavior of the dilaton, transforms into a state of D-branes. Following [8] we assume that at the transition point the square of the radius of the horizon is of order $\alpha'$ and the masses of the solitonic brane and D-brane configuration are equal.
Under these assumptions we consider a system of $Q$ D branes with a small number of excited long strings. In this case the entropy is

$$S_D \sim \alpha'^{1/2} \delta E,$$

$$\delta E \sim E - \frac{QV}{g\alpha'^{7/2}}.$$  \hfill (38)

Here $E$ is the energy of the D-brane system which at the transition point is equal to the mass of the solitonic brane, the charges of the solitonic brane and the D-brane system are also equal. Substituting the expression of the energy, we find the entropy of the excited D-brane system:

$$S_D \sim \frac{QV}{(1 + |\Delta|)g\alpha'^3} \sim \frac{V}{g^2(|\Delta| \ln(2/3|\Delta|))^{7/2}P^6}.$$  \hfill (39)

This is exactly the same expression for the entropy as that of the solitonic brane \[36\] since

$$S \sim \frac{|\Delta| \ln(2/3|\Delta|)P^2}{g^2\alpha'^4}V \sim \frac{V}{g^2(|\Delta| \ln(2/3|\Delta|))^{7/2}P^6}.$$  \hfill (40)

Exciting the D-brane system by a large number of massless open strings on the D-branes \[8\] gives an entropy that does not match that of the solitonic brane. Probably, this is connected to the fact that although the solitonic brane we discussed here behaves in many respects as a black p-brane, it is not a p-brane, but a kind of a hybrid \[2\].

Usually it is expected that unless $\alpha'$-corrections are absent because of special properties of solution (such as $N = 4$ supersymmetry of the world-sheet action \[1\]), these corrections become important near the singularity and modify the leading term of solution. Qualitative examples of such modifications for the fundamental string were considered in \[14\]. The effect of the $\alpha'$-corrections was argued to smear the singularity and to move the horizon from $r = 0$ to a distance of order $(\alpha')^{1/2}$, thus providing a realization of the ”stretched horizon” \[15\]. A remarkable property of the string-loop-corrected magnetic black hole is that now string-loop correction modifies solution in the same way: it moves the horizon from $r = 0$ and drastically changes the behavior of dilaton which becomes regular at $r = 0$\[3\]. The latter property was crucial for applicability of the correspondence principle.

## 6 Conclusions

We investigated charged extremal black-hole solutions of equations of motion of the one-string-loop-corrected effective action which provide backgrounds for the ”chiral null model” \[3\].

In the purely magnetic case we found an explicit analytical solution for the extremal black hole. It appears that geometry is modified; namely, background receives logarithmic corrections which are in agreement with corrections found in a different approach to quantum gravity \[11\]. The threshold corrections change the values of the mass and magnetic charge of the black hole and move the horizon from $r = 0$ to a distance proportional to the threshold correction and the charge.

\[3\]For magnetic black hole string-loop-corrected solution the function $\tilde{f}$ no longer satisfies the harmonic equation.
For the extremal dyonic black hole, we found solution which has the form of the tree-level solution with the rescaled electric and magnetic fields. It follows that since the modified backgrounds satisfy harmonic equation they receive no $\alpha'$ corrections \cite{1,2}. The entropy of the dyonic black hole remain the same as at the tree level, although the charges are modified.

For the purely electric black hole, we found that equations of motion with threshold corrections have no spherically symmetric solution. The same is true for generic spherically-symmetric magnetic solution, but in the latter case because of the special form of magnetic field it was possible to construct a particular solution.

Of special interest is application of the correspondence principle to the loop-corrected solution for the extremal magnetic black hole. In this case, loop corrections modify the tree-level solution so that the dilaton which, at the tree-level, is singular at the horizon, becomes a smooth function which vanishes at $r = 0$. This means that the string coupling is weak near the horizon and opens the possibility to apply the correspondence principle to semi-quantitative microscopic calculation of the entropy of the magnetic black hole. It was shown that in the picture in which the weak-coupling D-brane system is described by a small number of long strings the Bekenstein-Hawking entropy of magnetic black hole is reproduced by the D-brane system.

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References

[1] M. Green, J. Schwarz and E. Witten, Superstring Theory, Cambridge University Press, 1987.

[2] G.T. Horowitz and A.A. Tseytlin, Phy. Rev. D51, 2896 (1995), hep-th/9409021.
   A.A. Tseytlin, Mod. Phys. Lett. A11, 689 (1996), hep-th/9601177.
   M. Cvetic and A.A. Tseytlin, Phys. Rev. D53, 5619 (1996), hep-th/9512031.
   M. Cvetic and A.A. Tseytlin, Phys. Lett. B366, 95 (1996), hep-th/9510097.

[3] L. Susskind, Some Speculations about Black Hole Entropy in String Theory, hep-th/9309143.

[4] F. Larsen and F. Wilczek, Phys. Lett. B375, 37 (1996), hep-th/9511064.

[5] G.T. Horowitz, Quantum States of Black Holes, in Proceedings of the symposium on Black Holes and Relativistic Stars, in honor of S. Chandrasekhar, December 1996, gr-qc/9704072.

[6] M. Cvetic and A.A. Tseytlin, Nucl. Phys. B416, 137 (1994), hep-th/9307123.

[7] K. Chan, Mod. Phys. Lett. A12, 1597 (1997), hep-th/9610121.

[8] G.T. Horowitz and J. Polchinski, Phys. Rev. D55, 6189 (1997), hep-th/9612146; Self Graviting Fundamental Strings, hep-th/9707170.

[9] V.S. Kaplunovsky, Nucl. Phys. B307, 145 (1988), hep-th/9205070.
   L. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B355, 649 (1991).

[10] V.P.Frolov, W. Israel and S.N. Solodukhin, Phys. Rev. D54, 2732 (1996), hep-th/9602105.

[11] F.W.J. Olver, Asymptotics and Special Functions, N.Y., Academic Press, 1974.

[12] A.A. Tseytlin, Nucl. Phys. B477, 431 (1996), hep-th/9605091.

[13] A. Strominger and C. Vafa, Phys. Lett. B379, 99 (1996), hep-th/9601029.
    G. Horowitz and A. Strominger, Phys. Rev. Lett. 77, 2368 (1996), hep-th/9602051.
    C. Callan and J. Maldacena, Nucl. Phys. B472, 591 (1996), hep-th/9602043.

[14] A.A. Tseytlin, Phys. Lett. B363, 223 (1995), hep-th/9509050.

[15] L. Susskind and J. Uglum, Phys. Rev. D50, 2700 (1994), hep-th/9401070.