THE MYSTERY OF PARITY

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And should I not take pity on Nineveh, that great city, with more than a hundred and twenty thousand inhabitants who do not know their right hand from their left, and many beasts besides?

I Introduction

Our world does not exhibit left-right symmetry at the level of familiar objects: biological [1] and polar [2] molecules, organic chemicals [3], human anatomy, and much more. However, before 1956 it was widely [4] (though not universally [5]) assumed that the fundamental laws of physics exhibited that symmetry (parity invariance). When it was called into question for the weak interactions [6], experiments [7, 8, 9] quickly showed that in fact the weak interactions had a definite handedness, involving left-handed particles and right-handed antiparticles.

Could parity violation at the microscopic level be responsible for what we see in the macroscopic world? Despite calculations claiming this to be so (see, e.g., [10]), Tom Erber has pointed out that this asymmetry need not stem from the microscopic level, but can arise spontaneously in very simple systems. For $N$ point charges arranged on the surface of a unit sphere, the lowest-energy state is mirror-symmetric for $2 \leq N \leq 10$, but asymmetric for $N = 11$ [2, 11] and specific higher values of $N$. In like manner (see also [12]), although the Coriolis force tends to send water down a drain counterclockwise in the Northern Hemisphere and clockwise in the Southern, initial conditions play a far more important role.

In this article we shall be concerned with microscopic parity invariance and a mystery which it presents. This consists of a marriage of internal and space-time symmetries, forbidden when the space-time symmetry consists of the whole Poincaré group [13] but permitted in this case because of the discrete nature of the parity transformation. We will argue that this marriage could point to regularities underlying the nature of quarks and leptons, and to extensions of particle interactions beyond those known today.

In Section II we briefly review the observed pattern of quarks and leptons, noting the great difference between the masses of the light neutrinos and the remaining fermions. In Section III we express this difference in group-theoretic terms, relying on an oft-employed five-dimensional geometric construction based on the group $SO(10)$. The role of parity reversal in this language is extremely simple, consisting of reflection of one of the five coordinates. Possible consequences of this observation are given in Section IV, while Section V concludes.
II Quark and lepton patterns

The observed quarks and leptons fall into three families. One distinguishes left-handed from right-handed states. Each left-handed family consists of a quark electroweak doublet [transforming as a triplet of color SU(3)], a lepton doublet [transforming as a singlet of color SU(3)], and the corresponding antiparticles which are all electroweak singlets. In each right-handed family the roles of the particles and antiparticles are reversed. For Dirac particles (the quarks and charged leptons) the left-handed and right-handed states and the corresponding antiparticles are combined into one four-component object with a specific “Dirac” mass. The possibility that a neutrino can be its own antiparticle allows for left-handed neutrinos and right-handed antineutrinos (the “active” participants in weak interactions) to have one “Majorana” mass while the “sterile” left-handed antineutrinos and right-handed neutrinos have another. In Fig. I we show present information on quark and lepton masses, quoting Dirac masses for the charged fermions and direct upper limits on masses of “active” neutrinos which may or may not be of Majorana type.

Neutrinos are known to mix with one another, so that the states of definite mass (denoted $\nu_1$, $\nu_2$, and $\nu_3$) are linear combinations of the “flavor” eigenstates $\nu_e$, $\nu_\mu$, and $\nu_\tau$. Neutrino oscillation experiments find $\Delta m^2_{21} \equiv m_2^2 - m_1^2 = (7.59^{+0.19}_{-0.21}) \times 10^{-5}$ eV$^2$, $\Delta m^2_{32} \equiv m_3^2 - m_2^2 = (2.43 \pm 0.13) \times 10^{-3}$ eV$^2$ [14]. If $m_1^2 \ll m_2^2, m_3^2 \ll m_{2,3}^2$, then $m_1 \ll m_2 \simeq 9$ meV, $m_3 \simeq 50$ meV. However, all the neutrino masses could be larger and quasi-degenerate. In any case a cosmological bound implies that the sum of the (active) neutrino masses must not exceed 0.28 eV [15], far below the direct limits depicted in Fig. I.

Fig. I represents one of the great puzzles of today’s particle physics. Do the masses of the quarks and leptons (and their mixing under the weak interactions) represent some deep
underlying structure (as in the Periodic Table of the Elements), or the solution of some anarchic dynamics (as in the Titius-Bode law describing planetary orbits)? For the present we bypass this question and discuss the structure of a single family, which we shall denote

\[
F = \begin{pmatrix}
  u \\
  d \\
  \nu_e \\
  e^-
\end{pmatrix}.
\] (1)

### III Geometry of grand unified groups

The strong interactions are described by an \(SU(3)_C\) (C for color) Yang-Mills gauge theory, while the gauge symmetry of the electroweak interactions is \(SU(2)_L \otimes U(1)_{Y_W}\), where the subscript \(L\) indicates that the interaction applies to left-handed fermions (and right-handed antifermions), while the subscript \(Y_W\) denotes weak hypercharge. Georgi and Glashow [16] found an ingenious way to unify \(SU(3) \otimes SU(2) \otimes U(1)\) into an \(SU(5)\) group; the 15 observed left-handed quarks and leptons (excluding left-handed antineutrinos) of each family are apportioned into 5-dimensional and 10*-dimensional representations of \(SU(5)\). However, the pattern becomes much simpler when \(SU(5)\) is included into the group \(SO(10)\) [17, 18]. The 5- and 10*-dimensional representations of \(SU(5)\) combine with an \(SU(5)\) singlet, the right-handed neutrino, into a single 16*-dimensional spinor representation of \(SO(10)\), a group of rank 5 whose representation members may be identified by their coordinates in a 5-dimensional vector space. The spinor consists of next-to-nearest neighbors on the vertices of a 5-dimensional hypercube in this space. Its members may be identified by vectors of the form

\[
\left( \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2} \right),
\] (2)

with an odd number of + signs for 16* and an even number for its conjugate 16 representation [19]. Other representations of \(SO(10)\) have simple depictions in this language: For example, members of the vector 10-plet of \(SO(10)\) are denoted by

\[
(\pm 1, 0, 0, 0, 0) + \text{permutations}.
\] (3)

The group \(SO(10)\) has rank 5, so there are five mutually commuting observables which may be represented in it. As the color \(SU(3)\) subgroup of \(SO(10)\) has rank two, one may take color isospin \(I_{3C}\) and hypercharge \(Y_C\) as two of the observables. For \(SU(2)_L\) one takes its third component \(I_{3L}\), while weak hypercharge will be denoted by \(Y_W\). The electromagnetic charge is \(Q = I_{3L} + Y_W/2\). A fifth observable \(Q_\chi\), lying in \(SO(10)\) but outside \(SU(5)\), will be defined shortly.

One may now measure the value of any observable for an \(SO(10)\) representation member by taking its projection along a specific five-dimensional vector, e.g.:

\[
V(I_{3C}) = \left( +\frac{1}{2}, -\frac{1}{2}, 0, 0, 0 \right) ; \quad V(Y_C) = \left( +\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, 0, 0 \right) ,
\]

\[
V(I_{3L}) = \left( 0, 0, 0, +\frac{1}{2}, -\frac{1}{2} \right) ; \quad V(Y_W) = \left( -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, 1, 1 \right) ; \quad V(Q) = \left( -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0 \right) .
\]
An additional observable denoted can be represented by $V(Q_{\chi}) = (1, 1, 1, 1)/\sqrt{10}$, where we have chosen to normalize $Q_{\chi}$ in the same way as $I_{3C}$ or $I_{3L}$. In the 16-plet of SO(10), 5-plet SU(5) members have $Q_{\chi} = -3/\sqrt{40}$, 10*-plet members have $Q_{\chi} = 1/\sqrt{40}$, and the SU(5) singlet has $Q_{\chi} = 5/\sqrt{40}$.

The specific members of the left-handed family (1) may be denoted by the following spinors. A subscript 1, 2, 3 denotes the color label; we display only one color of each quark. We shall adopt the shorthand $\pm$ for coordinates $\pm 1/2$ [20]. We shall also put a vertical bar between the first three indices, denoting color SU(3), and the last two, denoting weak SU(2). We then have

$$u_{L1} = (+ - | + -) ; \quad d_{L1} = (+ - | - +) ; \quad \nu_{L} = (+ + | + -) ; \quad e_{L} = (+ + | - +) . \quad (4)$$

Each of these is a weak doublet with $I_{3L} = \pm 1/2$, as the last two indices are unequal. The corresponding antiparticles, obtained by reversing the signs of the first four indices, are

$$\bar{u}_{L1} = (- + | - -) ; \quad \bar{d}_{L1} = (- + | + +) ; \quad \bar{\nu}_{L} = (- - | - -) ; \quad \bar{e}_{L} = (- - | + +) . \quad (5)$$

The $\bar{\nu}_{L}$ has no charges within the Standard SU(3)$_C$⊗SU(2)$_L$⊗U(1)$_{YW}$ Model; it is sterile.

An interesting three-dimensional projection of the five-dimensional space may be obtained by defining the horizontal plane to be the two-dimensional vector space describing color SU(3) and the vertical axis to be electric charge. The members of an SO(10) 16-plet may then be represented as two cubes stacked corner-to-corner, as shown in Fig. 2.

![Figure 2: Projection of SO(10) 16-plet describing a quark-lepton family into the space of color (horizontal plane) ⊗ electric charge (vertical axis)](image)

So far we have not discussed the right-handed states. These, it turns out, are related to the corresponding left-handed states by a simple reversal of the fifth index. Thus, for right-handed particles we have

$$u_{R1} = (+ - | + +) ; \quad d_{R1} = (+ - | - -) ; \quad \nu_{R} = (+ + | + +) ; \quad e_{R} = (+ + | - -) , \quad (6)$$
while for right-handed antiparticles we have
\[\bar{u}_{R1} = (-+\bar{+}|-+\bar{+}) \; ; \; \bar{d}_{R1} = (-+\bar{+}|+\bar{+}) \; ; \; \bar{\nu}_R = (-++|---) \; ; \; e^+_R = (-++|+-) \; . \quad (7)\]

It is now the right-handed particles which are electroweak singlets, while the right-handed antiparticles are electroweak doublets. In particular, the right-handed neutrino $\nu_R$ is sterile with respect to Standard Model charges.

All this is familiar to practitioners of grand unified theories. Indeed, the unbroken SO(10) symmetry is left-right symmetric [18, 21]; it is a non-zero expectation value of the charge $Q_\chi$ which destroys this symmetry. This could arise at any mass scale from a Higgs mechanism. If the scale is several TeV or less, one might be able to observe the corresponding neutral gauge boson (a “$Z_\chi$” [22, 23]) at the CERN Large Hadron Collider (LHC). A very large mass scale, however, could be associated with a large Majorana mass of right-handed neutrinos.

One can also envision the breaking of SO(10) as proceeding first through its subgroup SO(6) $\otimes$ SO(4) (easily illustrated on the fingers of two hands). The SO(6) is isomorphic to an SU(4) group which may be thought of an extended color, regarding leptons as the fourth color [18]. Its subgroup containing color is SU(3)$_C$ $\otimes$ U(1)$_{B-L}$, where $B$ and $L$ are baryon and lepton number. The SO(4) is isomorphic to SU(2)$_L$ $\otimes$ SU(2)$_R$. The subsequent breaking of SU(2)$_R$ would be responsible for parity-noninvariance of the electroweak theory. A handy expression for electric charge, instead of the uninspiring relation involving weak hypercharge, is
\[Q = I_3^R + I_3^L + (B - L)/2.\]

The vectors projecting out $I_3^R$ and $B - L$ are
\[V(I_3^R) = \left( 0, 0, 0, +\frac{1}{2}, \frac{1}{2} \right) \; ; \; V(B - L) = \left( -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, 0, 0 \right) . \quad (8)\]

What is puzzling about the parity operation is that, although it is a transformation of the Poincaré group, it corresponds to a simple operation in the SO(10) five-dimensional vector space. Because it is a discrete transformation, it evades the Coleman-Mandula theorem [13] which forbids the combination of internal and Poincaré symmetries except as a direct product. Its violation is also deeply implicated in how the Standard Model arises from some higher symmetry. In the next section we argue that such a symmetry is likely to exist on the basis of our very incomplete knowledge about the nature of matter in the Universe.

\section*{IV Expanded symmetries}

Ordinary matter makes up a small fraction of the known energy density of the Universe; dark matter comprises about five times as much [24]. We have little clue as to its nature.

Imagine a TeV-scale effective symmetry SU(3) $\otimes$ SU(2) $\otimes$ U(1) $\otimes$ G, where the beyond-Standard-Model (BSM) group G could be any number of extensions currently on the market. One can classify the new types of matter very generally as shown in Table I [25]:

Grand unified theories well beyond SO(10) have been proposed. The $E'_8$ of $E_8 \otimes E'_8$ in the heterotic string [26] could play a role of “shadow matter” which communicates only weakly with our world. The fifth coordinate in the SO(10) description, whose reversal we have shown induces parity reflections, could play a wider role in an extended vector space of more than five dimensions.

The spinors of SO(2N) groups may be represented as alternate vertices of hypercubes in $N$ dimensions. These spinors and their conjugates each have $2^{N-1}$ members. If $N > 5$, one can
Table I: Possible types of matter classified according to SM and BSM (G) transformation.

| Type of matter | Std. Model | G     | Example(s) |
|----------------|------------|-------|------------|
| Ordinary       | Non-singlet| Singlet| Quarks, leptons |
| Mixed          | Non-singlet| Non-singlet| Superpartners |
| Shadow         | Singlet   | Non-singlet| $E'_8$ of $E_8 \otimes E'_8$ |

ask what fraction of those have the form $(+++-+-|a_{N-5} \ldots a_N)$ or $(-+-+-|a_{N-5} \ldots a_N)$, where the first five indices refer to the SO(10) subgroup of SO(2N), and hence would be “sterile” under charges of the Standard Model. The answer is the same as for $N = 5$, namely 1/16. One is seeking, rather, a scheme where most of the matter is sterile under Standard Model charges. The existence of a large amount of dark matter in our Universe could be a key to guessing the structure of a large Grand Unified Group, and perhaps incidentally helping to solve the mystery of parity violation.

V Conclusion

The advent of the CERN Large Hadron Collider will offer one possible window into extended grand unified theories, through the discovery of new forms of matter or new gauge bosons. One of the simplest such examples would be the gauge boson $Z_\chi$ coupled to the charge $Q_\chi$ mentioned above [22, 23]. It may turn out in retrospect that the role of parity and its violation in our current understanding of unified theories was just a foretaste of a much richer structure.

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