Abstract—This paper considers a device-to-device (D2D) underlaid cellular network where an uplink cellular user communicates with the base station while multiple direct D2D links share the uplink spectrum. This paper proposes a random network model based on stochastic geometry and develops centralized and distributed power control algorithms. The goal of the proposed power control algorithms is twofold: ensure the cellular users have sufficient coverage probability by limiting the interference created by underlaid D2D users, while also attempting to support as many D2D links as possible. For the distributed power control method, expressions for the coverage probabilities of cellular and D2D links are derived and a lower bound on the sum rate of the D2D links is provided. The analysis reveals the impact of key system parameters on the network performance. For example, the bottleneck of D2D underlaid cellular networks is the cross-tier interference between D2D links and the cellular user, not the D2D intra-tier interference. Numerical results show the gains of the proposed power control algorithms and accuracy of the analysis.

Index Terms—Power control, device-to-device communication, cellular networks, Poisson point process, stochastic geometry.

I. INTRODUCTION

Device-to-device (D2D) communication underlaid with cellular networks allow direct communication between mobile users [1]–[3]. D2D is an attractive approach for dealing with local traffic in cellular networks. The initial motivation for incorporating D2D communication in cellular networks is to support proximity-based services, e.g., social networking applications or media sharing [3]. Assuming there are proximate communication opportunities, D2D communication may also increase area spectral efficiency, improve cellular coverage, reduce end-to-end latency, or reduce handset power consumption [4], [2]. In spite of these potential gains, the coexistence problem between D2D and cellular communication in the same spectrum is challenging due to the difficulty of interference management [2]. Specifically, the underlaid D2D signal becomes a new source of interference. As a result, cellular links experience cross-tier interference from the D2D transmissions whereas the D2D links need to combat not only the inter-D2D interference but also the cross-tier interference from the cellular transmissions. Therefore, interference management is essential to ensure successful coexistence of cellular and D2D links.

Power control is an effective approach to mitigate interference in wireless networks; it is broadly used in current wireless systems. In this paper, we propose power control methods for interference coordination and analyze their performance in D2D underlaid cellular networks. In particular, we consider a hybrid random network model using stochastic geometry and develop two different power control algorithms for the proposed network model. With a carefully designed (centralized) power control technique, we show that multiple D2D links may communicate successfully while guaranteeing reliable communication for the existing cellular link. This shows that, with an appropriate power control technique, underlaid D2D links help to increase the network sum-throughput without causing unacceptable performance degradation to existing cellular links.

A. Related Work

There has been considerable interest in power control techniques for D2D underlaid cellular networks. A simple power control scheme was proposed in [5] for a single-cell scenario and deterministic network model, which regulates D2D transmit power to protect the existing cellular links. To maximize the sum rate of the network, a D2D transmit power allocation method was proposed in [6] for the deterministic network model. A dynamic power control mechanism for a single D2D link communication was proposed in [7], which targets improving the cellular system performance by mitigating the interference generated by D2D communication. The main idea was to adjust the D2D transmit power via base station (BS) to protect cellular users. In [8], a power minimization solution with joint subcarrier allocation, adaptive modulation, and mode selection was proposed to guarantee the quality-of-service demand of D2D and cellular users. In prior work [1], [2], [9]–[17], D2D power control strategies are developed and evaluated in a deterministic D2D link deployment scenario. For a random network model, spectrum sharing between ad hoc and cellular networks was studied in [18]–[20] but power control – an essential component of D2D underlaid cellular networks – has not been addressed. Power control has been studied in other random ad hoc networks without considering cellular networks (see e.g., [21]–[23]). In our paper, we propose power control algorithms and analyze their performance in a D2D underlaid cellular network.
B. Contributions

In this paper, we consider a D2D underlaid cellular network in which an uplink cellular user wants to communicate with the BS while multiple D2D links coexist in the common spectrum. In such a network, we model the D2D user’s locations using a spatial Poisson point process (PPP). This model is motivated by the fact that D2D users are distributed randomly over the network like ad hoc nodes.

In the D2D underlaid cellular network, we propose a centralized and a distributed power control algorithm. The main idea of the centralized algorithm is to design the transmit power of mobile users so as to maximize the signal-to-interference-plus-noise ratio (SINR) of the cellular link while satisfying the individual target SINR constraints for D2D links. Using the fact that the centralized power allocation problem is convex, we solve it with a feasibility set increment technique. One main observation is that the centralized power control approach can significantly improve the overall cellular network throughput due to the newly underlaid D2D links while guaranteeing the coverage probability of pre-existing cellular links.

We also propose a simple distributed on-off power control algorithm. Note that the centralized algorithm requires global channel state information (CSI) possibly at a centralized controller; this may incur high CSI feedback overhead. To resolve this issue, the proposed on-off power control method requires CSI knowledge about the direct link between the transmitter and its corresponding receiver only. In particular, for the distributed power control method, we derive expressions including the coverage probabilities of both cellular and D2D links and a lower bound on the sum rate of D2D links using stochastic geometry. One important insight obtained from the analysis is that on-off power control strategy for the uplink user is actually optimal in terms of the coverage probability of the cellular link, agreeing with the finding in ad hoc networks [23]. Further, we derive the optimal D2D transmission probability which maximizes the lower bound of the sum rate of D2D links when the distributed on-off power control algorithm is used. In contrast to the centralized power control method, the distributed power control algorithm is not sufficient to guarantee reliable cellular communication, though it does improve the cellular network throughput by additional D2D communication. We finally verify the results by simulating two different D2D link deployment scenarios.

The remainder of this paper is organized as follows. In Section II, the proposed model for D2D underlaid cellular networks is described. The proposed power control algorithms are presented in Section III. In Section IV, the analytical expressions for the cellular user coverage probability and the typical D2D link coverage probability are derived when the distributed power control method is applied. For the distributed power control, the sum rate lower bound of D2D links is derived in Section V. Simulation results are provided in Section VI to demonstrate the effectiveness of the proposed algorithms and to validate the analytical results, which are followed by our conclusions in Section VII.

II. PROBLEM FORMULATION

In this section, we present the system model and describe network metrics that will be used in this paper.

A. System Model

We consider a D2D underlaid cellular network, as shown in Fig. 1. In this model, let the circular disk \( C \) with radius \( R \) denotes the coverage region of BS centered at the origin. We assume that one cellular uplink user is uniformly located in this region. Further, we assume that the locations of the D2D transmitters are distributed in the whole \( \mathbb{R}^2 \) plane according to a homogeneous PPP \( \Phi \) with density \( \lambda \). For a given D2D transmitter, its associated receiver is located at a fixed distance away with isotropic direction. We assume all nodes have one antenna.

Under the given assumptions, the \( K \) D2D transmitters in \( C \) is a Poisson random variable with mean \( E[K] = \lambda \pi R^2 \). Given one realization of the PPP \( \Phi \), the received signals at D2D receiver \( k \) and the BS are given by

\[
y_k = h_{k,k}d_{k,k}^{-\alpha/2}s_k + h_{k,0}d_{k,0}^{-\alpha/2}s_0 + \sum_{l=1, l \neq k}^{K} h_{k,l}d_{k,l}^{-\alpha/2}s_l + n_k, \tag{1}
\]

\[
y_0 = h_{0,0}d_{0,0}^{-\alpha/2}s_0 + \sum_{k=1}^{K} h_{0,k}d_{0,k}^{-\alpha/2}s_k + n_0, \tag{2}
\]

where subscript 0 is used for the uplink signal to the BS and subscript \( k, k \neq 0 \), are used for D2D links; \( s_k \) and \( n_0 \) denote the signal sent by D2D transmitter \( k \) and the uplink user; \( y_k \) and \( y_0 \) represent the received signal at D2D receiver \( k \) and the BS; \( n_k \) and \( n_0 \) denote the additive noise at D2D receiver \( k \) and the BS distributed as \( \mathcal{CN}(0, \sigma^2) \); \( h_{k,k} \) and \( h_{0,k} \) represent the distance-independent fading from D2D transmitter \( k \) to receiver \( k \) and the channel from D2D transmitter \( k \) to the BS, and are independently distributed as \( \mathcal{CN}(0, 1) \). Here, we use a far-field assumption in the distance dependent path-loss model, i.e., \( d_{k,k}^{-\alpha} = \min(1, d_{k,k}^{-\alpha}) \) for all \( k, l \) where \( d_{k,j} \) denotes the distance from transmitter \( j \) to receiver \( k \) and \( \alpha \) is the path-loss exponent. The transmit power satisfies the
average transmit power constraint, i.e., $\mathbb{E} [ |s_k|^2 ] \leq P_{avg,c}$ and $\mathbb{E} [ |s_k|^2 ] \leq P_{avg,d}$ for $k \in \{1, \ldots, K\}$, respectively. Then the SINR at D2D receiver $k$ and the BS are given by

$$\text{SINR}_k(K, p) = \frac{|h_{k,k}|^2 d_{k,k}^{-\alpha} p_k}{|h_{k,0}|^2 d_{k,0}^{-\alpha} p_0 + \sum_{\ell \neq k} |h_{k,\ell}|^2 d_{k,\ell}^{-\alpha} p_\ell + \sigma^2},$$

$$\text{SINR}_0(K, p) = \frac{|h_{0,0}|^2 d_{0,0}^{-\alpha} p_0}{\sum_{k=1}^K |h_{0,k}|^2 d_{0,k}^{-\alpha} p_k + \sigma^2},$$

where $p = [p_0, p_1, \ldots, p_K]^T$ denotes transmit power profile vector with $p_i$ being the transmit power of transmitter $i$.

### B. Performance Metrics

We are interested in the coverage probability of the cellular link and D2D links. The cellular coverage probability is defined as

$$\hat{P}^{(C)}_{\text{cov}} = \mathbb{E}[P^{(C)}_{\text{cov}}(p)] = \mathbb{E}[\mathbb{P}(\text{SINR}_0(K, p) \geq \beta_0)],$$

where $\beta_0$ represents the minimum SINR value for reliable uplink connection. Similarly, the D2D coverage probability is defined as

$$\hat{P}^{(D)}_{\text{cov}} = \mathbb{E}[P^{(D)}_{\text{cov}}(p)] = \mathbb{E}[\mathbb{P}(\text{SINR}_k(K, p) \geq \beta_k)],$$

where $\beta_k$ represents the minimum SINR value for reliable D2D link connections. Further, the sum rate of all D2D links is defined as

$$R^{(D)} = \mathbb{E} \left[ \sum_{k=1}^{|S|} \log_2 \left( 1 + \text{SINR}_k(K, p) \right) \right],$$

where $|S|$ denotes the number of D2D links scheduled for transmission.

### III. Power Control Algorithms

When the global CSI is available at the central controller, a centralized power control algorithm is proposed, which maximizes the SINR of the cellular link while satisfying the SINR constraints for both the cellular link and D2D links. Further, when the transmitter has CSI of the direct link of the corresponding receiver only, a distributed on-off power control algorithm is proposed.

#### A. Centralized Power Control

A main difference between ad hoc networks and underlaid D2D cellular networks is that centralized power control is possible when the D2D links are managed by the BS. For other management strategies, centralized power control can provide an upper bound on what can be achieved with more decentralized algorithms.

Suppose that the BS has global channel state information (CSI). Under this assumption, the centralized power control problem is formulated as

$$\begin{align*}
\max_{\{p_0, p_1, \ldots, p_K\}} \quad & \frac{G_{0,0} p_0}{\sum_{k=1}^K G_{0,k} p_k + \sigma^2} \\
\text{subject to} \quad & \frac{G_{k,k} p_k}{\sum_{\ell \neq k} G_{k,\ell} p_\ell + \sigma^2} \geq \beta_k, \\
& 0 \leq p_0 \leq P_{avg,c}, \\
& 0 \leq p_k \leq P_{avg,d},
\end{align*}$$

where $k, \ell \in K = \{0, 1, 2, \ldots, K\}$ and $G_{i,j} = |h_{i,j}|^2 d_{i,j}^{-\alpha}, \forall i, j \in \{0, 1, \ldots, K\}$. This optimization problem can be compactly written in a vector form as

$$\begin{align*}
\max \quad & g_0^T p \\
\text{subject to} \quad & (I - F) p \geq b, \\
& 0 \leq p \leq p_{avg},
\end{align*}$$

where $g_0^T = [G_{0,0}, 0, \ldots, 0]$, $g_0^T = [0, G_{0,1}, G_{0,2}, \ldots, G_{0,K}]$, $p_{avg} = [P_{avg,c}, P_{avg,d}, \ldots, P_{avg,d}]^T$, and the normalized channel gain matrix $F$ and target SINR vector $b$ are defined as

$$F_{k,\ell} = \begin{cases} 0, & k = \ell, \\ \frac{\beta_k G_{k,k}}{G_{k,k}} , & k \neq \ell. \end{cases}$$

$$b = \left[ \frac{\beta_0 \sigma^2}{G_{0,0}}, \frac{\beta_1 \sigma^2}{G_{1,1}}, \ldots, \frac{\beta_K \sigma^2}{G_{K,K}} \right]^T.$$

Since the objective function (linear-fractional function) is quasi-convex and the constraint set is convex (a polytope in particular) with respect to power profile vector $p$, the optimal solution can be obtained by using standard convex programming tools, provided that the feasible set is nonempty. Note that the matrix $F = [f_0, f_1, \ldots, f_K]$ is comprised of nonnegative elements and is irreducible because all the active D2D links interfere each other. By the Perron-Frobenious theorem, the following well-known lemma proved in [24] gives a necessary and sufficient condition on the feasibility of the optimization problem [10].

**Lemma 1:** [24] The constraint set in the optimization problem [10] is nonempty if and only if the maximum modulus eigenvalue of $F$ is less than one, i.e., $\rho(F) < 1$, where $\rho(\cdot)$ denotes the spectral radius of a matrix.

We next describe our proposed centralized algorithm to solve the optimization problem [10]. First we assume that D2D receivers can feedback all the perfect normalized channel gains $G_{i,k}$ and target SINR information $\beta_i$ to the BS. Using this assumption, the BS then computes the transmit power used for both D2D transmitters and the uplink user. Note that the feasible set should be nonempty to obtain the optimal solution, i.e., $\rho(F) < 1$. Since the normalized channel gains $G_{i,k}$, however, are random variables (the locations of all the transmit nodes are random variables), there exists a non-zero probability that the power control solution is infeasible, i.e., $\mathbb{P}(\rho(F) \geq 1) \neq 0$, especially when the number of D2D links $K$ is large. When the solution is infeasible, an admission control method is needed in conjunction with the power control algorithm to provide a feasible solution to the power control problem by selecting a subset of D2D links. This D2D link selection problem may be solved by...
brute-force search, which requires \( \sum_{k=1}^{K} (\binom{K}{k}) \) computations. The computational complexity grows exponentially with \( K \). Instead of brute-force search, we propose an efficient D2D link selection algorithm with low computational complexity for this problem. The key idea is to drop D2D communication links successively that causes the maximum sum of the interference power in the network until the feasibility condition is satisfied. For \( K \) given D2D links, we first test feasibility condition of the optimization problem in (10). If \( \rho(\mathbf{F}) > 1 \), i.e., the feasibility set is empty, we select the \( k \)-th D2D transmitter such that it creates the maximum sum of interference power to all other receivers, i.e., \( k = \arg \max_{k \in K \setminus \{0\}} \| \mathbf{f}_k \|_2 \), and then accordingly we remove the \( k \)-th row and column to reduce the size of matrix \( \mathbf{F} \). We keep reducing the size of matrix \( \mathbf{F} \) until the feasibility condition is satisfied. Table I summarizes the proposed D2D link selection method in conjunction with power control.

\[ \text{TABLE I} \]

| Step | Algorithm |
|------|-----------|
| Initialization | Set initial \( \mathbf{F}^\ell \) for \( \ell = 0 \), assuming \( K \) D2D links are all active. |
| Step 1 | Test feasibility condition \( \rho(\mathbf{F}^\ell) < 1 \). If this condition is satisfied, go to Step 5. Otherwise, go to Step 2. |
| Step 2 | Pick the column of \( \mathbf{F}^\ell \) such that \( k = \arg \max_{k \in K \setminus \{0\}} \| \mathbf{f}_k \|_2 \). |
| Step 3 | Generate a reduced matrix \( \mathbf{F}^{\ell+1} \) by removing the \( k \)-th column and row vectors in \( \mathbf{F}^\ell \). |
| Step 4 | Update \( \mathbf{F}^{\ell+1} = \mathbf{F}^\ell \). Go to Step 1. |
| Step 5 | Solve the optimization problem in (10). |

In this subsection, we provide a distributed power control algorithm. In the absence of coordination, each D2D transmitter chooses its transmit power to maximize its own rate towards its intended receiver, disregarding the interference caused to the others. The proposed on-off method is to select the D2D transmit power from the decision set \( \{0, P_{\text{max, } d}\} \) solely based on knowledge of the direct link information and a threshold \( G_{\text{min}} \) that is fixed and known by all users. The power used by D2D pair \( k \) is given by

\[
p_k = \begin{cases} 
P_{\text{max, } d} & \text{if } G_{k,k} > G_{\text{min}}; \\ 0 & \text{otherwise.} \end{cases}
\]

Therefore, the transmit power of each D2D user is an independent Bernoulli random variable. Since \( |h_{k,k}|^2 \sim \text{Exp}(1) \) and thus \( P(|h_{k,k}|^2 \geq x) = e^{-x} \), for a given distance \( d_{k,k} \), the transmission probability of the proposed power control method is given by

\[
P[G_{k,k} > G_{\text{min}}] = P[|h_{k,k}|^2d_{k,k}^{-\alpha} > G_{\text{min}}] = \exp\left(-G_{\text{min}}d_{k,k}^{-\alpha}\right). \tag{11}
\]

Therefore, the peak power \( P_{\text{max, } d} \) is defined as

\[
P_{\text{max, } d} = \frac{P_{\text{avg, } d}}{\exp\left(-G_{\text{min}}d_{k,k}^{-\alpha}\right)}, \tag{12}
\]

which guarantees the average D2D transmit power of on-off power control is still \( P_{\text{avg, } d} \). Note that the proposed power control method is distributed as each D2D transmitter decides its transmit power by the own channel gain \( |h_{k,k}|^2 \) only, which is independent from all other network parameters.

**Remark 1:** The proposed power control algorithm may be useful in non-random networks because it can be applicable in any realization of the proposed random network. Therefore, the randomness in the network modeling is not a key part of the algorithm, rather it is a component of the analysis to show that it works.

**IV. COVERAGE PROBABILITY ANALYSIS FOR DISTRIBUTED POWER CONTROL**

In this section, we derive the cellular link coverage probability, propose an optimal power control strategy for the cellular link under the average transmit power constraint, and derive the D2D link coverage probability. The coverage probability is computed using stochastic geometry. To enable the stochastic geometric analysis, we assume that the transmit power of each D2D transmitter is i.i.d. with distribution function \( F_{p_k}(\cdot) \) and that the transmit power of the uplink user is independent and has distribution function \( F_{p_\theta}(\cdot) \). Note that the coverage probability analysis we provide in this section is valid for any distributed power control algorithms that select its own transmit power independently of the transmit power used at the other D2D transmitters.

**A. Cellular Link Coverage Probability**

Assume that the BS is located at the origin. The SINR of the typical uplink is given by

\[
\text{SINR}_0 = \frac{p_0|h_{0,0}|^2d_0^{-\alpha}}{\sum_{k \in \Phi} p_k|h_{0,k}|^2d_{0,k}^{-\alpha} + \sigma^2}. \tag{13}
\]

Further, since the cellular user’s location is distributed uniformly in the circle with radius \( R \), the distribution function of the distance \( d_{0,0} \) of the cellular link is given by

\[
F_{d_{0,0}}(r) = \begin{cases} 
0 & \text{if } r < 0; \\
\frac{r^2}{\pi R^2} & \text{if } 0 \leq r \leq R; \\
1 & \text{if } r \geq R.
\end{cases}
\]

The following theorem provides an analytical formula for the uplink coverage probability.

**Theorem 1:** The cellular link coverage probability is

\[
\hat{P}_{\text{cov}}(C) = \mathbb{P}_{X} \left[ e^{-a_1X - a_2X^{1/2}} \right], \tag{14}
\]

where \( a_1 = \sigma^2\beta_0, a_2 = \frac{\pi \lambda_0^2}{\sin(\frac{\pi}{2})}\mathbb{E}[P_{\theta}], X = p_0^{-1}d_0^{-\alpha} \) with cdf \( F_X(x) = \int F_{d_{0,0}}(x^{1/2}p^{1/2})dF_{p_\theta}(p) \).
Proof 1: See Appendix A.

Theorem 1 provides an intuition that how important network parameters affect the cellular link coverage probability. For example, we observe that $\Pi_{\text{cov}}^{(C)}$ depends on two D2D-related network parameters: $\lambda$ and $E[p_k^2]$. In particular, $\Pi_{\text{cov}}^{(C)}$ decreases as the density $\lambda$ of D2D transmitters increases, which is intuitive as higher D2D link density causes more interference to the cellular link. Further, the random D2D power control $p_k$ affects $\Pi_{\text{cov}}^{(C)}$ only through its $\frac{2}{\alpha}$-th moment. This implies that the system can control the impact of D2D links on the cellular link by constraining $E[p_k^2]$ and then find the optimal distribution of $p_k$ to maximize cellular link coverage probability.

We next provide a simple lower bound for $\Pi_{\text{cov}}^{(C)}$ which only depends on the certain moments of $p_k$ and $d_{0,0}$ (rather than the distributions). This lower bound is formalized in the following Corollary.

Corollary 1: Cellular link coverage probability $\Pi_{\text{cov}}^{(C)}$ can be lower bounded as

$$\Pi_{\text{cov}}^{(C)} \geq \Pi_{\text{cov},\text{lb}}^{(C)} = e^{d_{0,0} - a_2(\frac{4}{\alpha})^2} R^2 \cdot \mathbb{E}[p_k^{-\frac{2}{\alpha}}]^{\frac{2}{\alpha}}. \quad (15)$$

Proof 2: See Appendix B.

B. Optimal Cellular Link Power Control

As shown in Theorem 1, the coverage probability of cellular link is a function of the transmit power of the uplink user. Thus, it is interesting to optimize the uplink transmit power under an average power constraint to improve the coverage probability. Conditioning on the location of the uplink user, i.e., $d_{0,0} = d$, the cellular link coverage probability can be expressed as $\phi(p_0) = e^{-a_1d^\alpha p_0^{-1} - a_2d^2p_0^{-\frac{2}{\alpha}}}$. Then, the cellular link power control problem is formulated as

$$\begin{align*}
\text{maximize} & \quad \int \phi(p_0) dF_{p_0,0}(p_0) \\
\text{subject to} & \quad p_0 dF_{p_0,0}(p_0) = P_{\text{avg},c} \\
& \quad dF_{p_0,0}(p_0) = 1. \quad (16)
\end{align*}$$

Note that this conditional cellular link coverage probability maximization problem is of infinite dimension, which is difficult to solve in general. Nevertheless, for this specific optimization problem, the optimal solution has a remarkably simple structure, which is stated in the following theorem.

Theorem 2: Denote $\phi(p_0) = e^{-a_1d^\alpha p_0^{-1} - a_2d^2p_0^{-\frac{2}{\alpha}}}$. Then there exists a non-degenerate $p^*(d) \in (0, \infty)$ that maximizes $\frac{\phi(p_0)}{p_0}$. Further, conditional on $d_{0,0} = d$ the optimal power allocation strategy maximizing the cellular link coverage probability is on-off power control.

Proof 3: See Appendix C.

From Theorem 2 the on-off power control strategy provides the cellular user with the optimal coverage probability performance and the optimal transmission power $p_0^*(d)$ is a maximizer of the function $\frac{\phi(p_0)}{p_0}$. Although we are able to derive $p_0^*(d)$ explicitly, we are able to find a closed form expression in the interference limited regime, i.e., $\sigma^2 = 0$.

Corollary 2: For the interference limited regime ($\sigma^2 = 0$), the optimal transmit power of the cellular user is

$$p_0^*(d) = \max \left\{ \frac{\mathbb{E}[p_k^2]}{\text{sinc}(\frac{4}{\alpha})}, \mathbb{E}[K]^{\alpha/2} \beta_0 \left( \frac{d}{R} \right)^\alpha, P_{\text{avg},c} \right\}. \quad (17)$$

Proof 4: Let denote $x = \frac{d}{P_0}$. Then, for the interference limited regime, the objective function for the uplink power optimization problem becomes $\phi(x) = x \exp(-a_2d^2x^{2/\alpha})$. From the first order Karush-Kuhn-Tucker (KKT) condition, i.e., $\frac{\partial}{\partial x} \phi(x) = 0$, we obtain the maximizer $x^* = \left( \frac{1}{a_2d^2} \right)^{\frac{\alpha}{2}}$. Putting $a_2 = \frac{x\lambda^\alpha d^{2\alpha}}{\text{sinc}(\frac{4}{\alpha})} \mathbb{E}[p_k^2] = \frac{\mathbb{E}[K]^{\alpha/2} \beta_0}{R^2 \text{sinc}(\frac{4}{\alpha})} \mathbb{E}[p_k^2]$ and using $p_0^* = x^*$, we obtain the first term in (17). Note that since the cellular user uses binary power control, the cellular user’s transmit probability becomes $P_{\text{avg},c} / p_0^*(d) \leq 1$.

The optimal transmit power of the cellular user $p_0^*(d)$ behaves in two different ways according to the location of the user. When $\left( \frac{\mathbb{E}[p_k^2]}{\text{sinc}(\frac{4}{\alpha})} \right)^{\frac{\alpha}{2}} \mathbb{E}[K]^{\alpha/2} \beta_0 \left( \frac{d}{R} \right)^\alpha < P_{\text{avg},c}$ (in the cell center), the cellular user uses the constant transmit power $P_{\text{avg},c}$. In contrast, when $\left( \frac{\mathbb{E}[p_k^2]}{\text{sinc}(\frac{4}{\alpha})} \right)^{\frac{\alpha}{2}} \mathbb{E}[K]^{\alpha/2} \beta_0 \left( \frac{d}{R} \right)^\alpha > P_{\text{avg},c}$ (in the cell edge), the transmit power increases proportionally to $d^\alpha$ implying that the cellular user should increase the transmit power according to the inverse of path-loss, agreeing with intuition. Further, the uplink user is required to increase the transmit power linearly according to $\mathbb{E}[K]^{\frac{\alpha}{2}}$ where $K$ is the random number of D2D links in the coverage of the BS. Using the optimal cellular user transmit power obtained in Corollary 2 we have a simple bound on the cellular user coverage probability for the interference-limited regime.

Corollary 3: For the interference limited regime ($\sigma^2 = 0$), the cellular user coverage probability is lower bounded as

$$\Pi_{\text{cov}}^{(C)}(d) \geq \begin{cases} 
\frac{P_{\text{avg},c} \exp(-1)}{p_0^*(d)}, & \text{for } p_0^*(d) > P_{\text{avg},c} \\
\left( \frac{\mathbb{E}[p_k^2]}{\text{sinc}(\frac{4}{\alpha})} \right)^{\frac{\alpha}{2}} \mathbb{E}[K]^{\alpha/2} \beta_0 \left( \frac{d}{R} \right)^\alpha, & \text{for } p_0^*(d) \leq P_{\text{avg},c}.
\end{cases} \quad (18)$$

Proof 5: Recall that the lower bound of cellular user coverage probability is $\Pi_{\text{cov}}^{(C)}(d) \geq P_{\text{avg},c} / \int dF_{p_0,0}(p_0)$ as shown in (5). Since the optimal power control strategy of the uplink user is the binary power control, i.e., $p_0 = p_0^*(d)$ with probability $P_{\text{avg},c} / p_0^*(d)$ and $p_0 = 0$ with probability $1 - P_{\text{avg},c} / p_0^*(d)$, the coverage probability in the interference regime ($\sigma^2 = 0$) becomes

$$\Pi_{\text{cov}}^{(C)} \geq \frac{P_{\text{avg},c} \mathbb{E}[p_k^2]}{p_0^*(d) \left\{ \int dF_{p_0,0}(p_0) \exp(-a_2d^2p_0^*(d)^{-2/\alpha}) \right\}}. \quad (19)$$

Using the solution of $p_0^*(d)$ in (17) and $a_2 = \frac{\mathbb{E}[K]^{\alpha/2} \beta_0}{R^2 \text{sinc}(\frac{4}{\alpha})} \mathbb{E}[p_k^2]$, we obtain the desired lower bound.

Example 1 (On-Off Power Control): In this example, let us consider a set of typical parameters: the path-loss exponent...
\(\alpha = 4\), the cell radius \(R = 500m\), the target SINR \(\beta_0 = -3\) dB, the average number of D2D links \(\mathbb{E}[K] = \lambda \pi R^2 = 15\), the average transmit power of the D2D transmitters \(P_{\text{avg}, d} = 0.1\) mW, and the average transmit power of the cellular user \(P_{\text{avg}, c} = 100\) mW. We assume that the on-off power control method is used for the D2D links with success probability \(\Pr[G_{k,k} \geq G_{\text{min}}] = \exp(-G_{\text{min}} d_{k,k}^{-\alpha})\) with the parameter \(G_{\text{min}} = d_{k,k}^{-\alpha}\) which gives us \(\mathbb{E}[G_{k,k}^{1/2}] = \sqrt{\mathbb{E}[P_{\text{avg}, d}] \exp(-1)}\). In this set of parameters, if the cellular user is located in the half of cell radius \(d = 125\), the optimal uplink transmission power \(p_0^*(R/2) = \max \left\{ \frac{\mathbb{E}[p_k^2]}{\text{sinc}(\frac{\pi}{R})}, \mathbb{E}[K]^{\alpha/2} \beta_0 \left( \frac{d}{R} \right)^\alpha, P_{\text{avg}, c} \right\} = 100\) mW because of \(\frac{\mathbb{E}[p_k^2]}{\text{sinc}(\frac{\pi}{R})} \mathbb{E}[K]^{\alpha/2} \beta_0 \left( \frac{d}{R} \right)^\alpha \approx 0.6\) mW, implying that the average transmit power is used in this case.

Thus, the lower bound of the cellular user coverage probability is \(\hat{P}_{\text{cov}}(C)(R/2) \approx 0.92\). Alternatively, if we assume that the cellular user is located at the cell edge with \(d = 0.8R\), then the target SINR \(\beta_0 = 3\) dB, and the average number of D2D links \(\mathbb{E}[K] = \lambda \pi R^2 = 39\), the transmission power of the cellular user becomes \(p_0^*(0.9R) = 113\) mW, which means that the uplink user opportunistically sends its uplink signal using transmit power 113 mW with probability of \(\frac{113}{114}\). Therefore, it gives a coverage probability performance \(\hat{P}_{\text{cov}}(C)(0.8R) \approx 0.29\).

### C. D2D Link Coverage Probability

We derive an expression for the coverage probability for the typical D2D link. Consider an arbitrary communication D2D pair \(k\) and assume that the D2D receiver is located at the origin. Then, \(\text{SINR}_k = \frac{p_k|h_{k,k}|^2 d_{k,k}^{-\alpha}}{\sum_{x\in \Phi \setminus \{k\}} p_l|h_{k,l}|^2 ||x_i||^{-\alpha} + p_0|h_{k,0}|^2 d_{k,0}^{-\alpha} + \sigma^2}\), where \(||x_i|| = d_{k,i}\). Using the same approach we used to prove Theorem 1, we need to compute two Laplace transforms \(\mathbb{E}[e^{-sp_0|h_{k,0}|^2 d_{k,0}^{-\alpha}}]\) and \(\mathbb{E}[e^{-s\sum_{x\in \Phi \setminus \{k\}} p_l|h_{k,l}|^2 ||x_i||^{-\alpha}}]\) to derive the distribution of SINR_k.

First let us focus on \(\mathbb{E}[e^{-sp_0|h_{k,0}|^2 d_{k,0}^{-\alpha}}]\). As we assume the uplink user and the D2D receiver are randomly positioned in the disk with radius \(R\), the pdf \(f_{d_{k,0}}(r)\) is given by \(\hat{f}_{d_{k,0}}(r) = \frac{2r}{R^2} \left( \frac{2}{\pi} \cos^{-1} \left( \frac{r}{2R} \right) - \frac{r}{\pi R} \left( 1 - \frac{r^2}{4R^2} \right) \right)\), \(0 \leq r \leq 2R\). Besides, \(|h_{k,k}|^2\) is a random variable with the exponential distribution, i.e. \(|h_{k,0}|^2 \text{Exp}(1)\) and \(p_0\) has cdf \(F_{p_0}(p)\). Noting further that \(p_0, |h_{k,0}|^2\), and \(d_{k,0}\) are independent, we have \(\mathbb{E}[e^{-sp_0|h_{k,0}|^2 d_{k,0}^{-\alpha}}]\) and \(\hat{f}_{d_{k,0}}(r)\) do not exist. Instead, we use the Gamma approximation for the distribution of \(p_0|h_{k,0}|^2 d_{k,0}^{-\alpha}\) inspired by [26] as in the following lemma.

**Lemma 2:** Using the Gamma approximation, the Laplace transform of \(Y = p_0|h_{k,0}|^2 d_{k,0}^{-\alpha}\) is approximated as

\[
\hat{L}_Y(s) = (1 + \theta s)^{-\kappa},
\]

where

\[
\kappa = \frac{\mathbb{E}[p_k^2]|d_{k,0}^{\alpha}|}{\mathbb{E}[p_0^2]|d_{k,0}^{\alpha}| - \mathbb{E}[p_0^2]|d_{k,0}^{\alpha}|}, \quad \theta = \frac{2\mathbb{E}[p_0^2]|d_{k,0}^{\alpha}|}{\mathbb{E}[p_0^2]|d_{k,0}^{\alpha}|} - \mathbb{E}[p_0^2]|d_{k,0}^{\alpha}|.
\]

**Proof 6:** See Appendix D.

Using Lemma 2 and a similar approach as in the previous subsection, it is possible to derive the complementary cumulative distribution function (ccdf) of SINR_k, and the coverage probability for the typical D2D link is given in the following theorem.

**Theorem 3:** The coverage probability of the typical D2D link is given by

\[
\Pr(\text{SINR}_k \geq \beta) = \mathbb{E}_X \left[ e^{-b_1 Z - b_2 Z^2} \hat{L}_Y(\beta Z) \right],
\]

where \(b_1 = \sigma^2 \beta, b_2 = \frac{\sigma^2 \lambda_k \beta |\mathbb{E}[h_{k,0}^\beta]|}{|\text{sinc}(\frac{\pi}{R})|}, Z = p_0^{-1} d_{k,0}^{-\alpha}\) with cdf \(F_Z(z) = \int F_{d_{k,0}}(x^\frac{1}{\alpha} p_0^\frac{2}{\alpha}) dF_{p_0}(p)\), and \(\hat{L}_Y(s)\) is given in Lemma 2.

**Proof 7:** See Appendix E.

Note that the D2D coverage probability has a penalty term \(\hat{L}_Y(\beta Z)\) compared with that of the cellular user due to the interference coming from the cellular users. Intuitively, the coverage performance of the D2D links decreases as increasing the average transmit power used by the cellular user.

### V. A LOWER BOUND ON SUM RATE OF D2D LINKS

In this section, we derive a lower bound on the sum rate of D2D links when the proposed on-off power control is applied. Assuming Gaussian signal transmission from all the active links, the distribution of the interference will be Gaussian. Thus, the achievable sum rate of D2D links is

\[
R^{(D)} = \mathbb{E} \left[ \sum_{k=1}^{|S|} \log_2 \left( 1 + \frac{G_{k,k} \text{Max, } d}{\sum_{\ell \neq k} G_{k,\ell} \text{Max, } d + G_{k,0} P_{\text{avg, c}} + \sigma^2} \right) \right],
\]

where \(|S|\) denotes the number of active links selected by the proposed on-off power control algorithm, i.e., \(|S| = \lambda \pi R^2 \mathbb{P}(G_{k,k} \geq G_{\text{min}})\). By defining the inter-D2D link interference at the \(k\)-th D2D receiver as \(I_k = \sum_{\ell \neq k} G_{k,\ell}\) for \(\ell \in \{1, 2, \ldots, |S|\}\) and assuming \(P_{\text{avg, c}} = P_{\text{Max, d}}\), a lower
where SNR = $P_{\text{avg}} / d$. The first equality (a) comes from the fact that $G_{k,k} \geq G_{\text{min}}$ conditioned on the event that D2D link $k$ is active; (b) follows from the convexity of $\log(1 + \frac{x}{1+e})$ with respect to $x$; (c) results from applying Jensen’s inequality. Interestingly, the lower bound on the total ergodic sum rate of all active D2D links in (30) is determined by four factors: (1) the total number of active D2D links $|S|$; (2) the operating SNR; (3) the total average interference power created by all active links on the whole network, i.e., $I_{\text{avg}} = \frac{1}{|S|} \sum_{k \in S} I_k + G_{k,k} P_{\text{max},d}$, where recall that $I_k = \sum_{\ell \neq k} G_{k,\ell} \lambda |h_k,\ell|^2 d_{k,\ell}^{-\alpha}$; (4) the channel gain threshold $G_{\text{min}}$, which is a power control constant. Due to the PPP assumption on the locations of D2D transmitters and the assumption of far-field path loss model, i.e., $I_k = \sum_{\ell \neq k} |h_k,\ell|^2 \text{min}(1,d_{k,\ell}^{-\alpha})$, the mean of the interference power at the D2D receiver $k$ generated by transmitters in the disk area is given by (assume that $R > 1$)

$$E[I_{\text{avg}}] = E \left[ \frac{1}{|S|} \sum_{k \in S} I_k + G_{k,0} P_{\text{max},d}^{-1} \right]$$

$$= \frac{1}{|S|} \sum_{k \in S} E[I_k] + \frac{1}{|S|} \sum_{k \in S} E[G_{k,0}] P_{\text{max},d}^{-1}$$

$$\geq \frac{\alpha - 2 R^{2-\alpha}}{\alpha - 2} \lambda \pi + E[G_{k,0}] P_{\text{max},d}^{-1}$$

$$\geq \frac{\alpha - 2 R^{2-\alpha}}{\alpha - 2} \lambda \pi + \frac{1}{R^2} P_{\text{max},d}^{-1}$$

where (a) follows from the Campbell’s formula, (b) follows from the fact that $d_{k,0}$ and $h_{k,0}$ are independent and $E[|h_{k,0}|^2] = 1$, and (c) comes from the fact the inverse moment of Euclidian distance between two points that are randomly thrown in a circle with radius $R$ is $E[d_{0,k}^\alpha] \simeq \frac{1}{\alpha} (25)$ and $E[d_{0,k}^\alpha] \geq E[d_{0,k}^\alpha]$ for $\alpha \geq 3$. We can then express the lower bound as

$$R^{(D)} \geq |S| \log_2 \left( 1 + \frac{G_{\text{min}}}{I_k + G_{k,0} P_{\text{max},d}^{-1} + \frac{1}{\text{SNR}}} \right)$$

$$\geq \lambda \pi R^2 \exp \left( -\frac{\alpha - 2 R^{2-\alpha}}{\alpha - 2} \lambda \pi + \frac{1}{R^2} P_{\text{max},d}^{-1} + \frac{1}{\text{SNR}} \right)$$

$$\geq \lambda \pi R^2 \exp \left( -\frac{\alpha - 2 R^{2-\alpha}}{\alpha - 2} \lambda \pi + \frac{1}{R^2} P_{\text{max},d}^{-1} + \frac{1}{\text{SNR}} \right)$$

where $R = \log_2 \left( 1 + \frac{G_{\text{min}}}{I_k + G_{k,0} P_{\text{max},d}^{-1} + \frac{1}{\text{SNR}}} \right)$ is the (low bounded) rate-per-link. Applying the fact that $P(G_{k,k} \geq G_{\text{min}}) = \exp(-G_{\text{min}} d_{k,k}^\alpha)$, we have

$$R^{(D)} \geq \lambda \pi R^2 \exp \left( -G_{\text{min}} d_{k,k}^\alpha \right) R.$$  

(37)

We remark on the results as follows. On the one hand, choosing a large $G_{\text{min}}$ yields a higher rate-per-link $R$ which is intuitively true as only D2D links with very good channel can be active by choosing a large $G_{\text{min}}$. On the other hand, larger $G_{\text{min}}$ leads to smaller number of active D2D links within the disk. Hence, a good choice of $G_{\text{min}}$ balancing these two competing factors will lead to a high D2D sum rate $R^{(D)}(|S|)$. In general, it is difficult to obtain a closed form solution for $G_{\text{min}}$ that maximizes the sum rate lower bound (37). For the interference limited regime (e.g., a dense D2D network, i.e., $\lambda > G_{\text{min}}$, or high transmit power used by the cellular user, i.e., $P_{\text{max},d} > G_{\text{min}}$), it is possible to obtain a closed form solution of $G_{\text{min}}$. Using (log$(1 + x) \approx x$ for small $x$, the sum rate lower bound in (37) becomes

$$R^{(D)} \geq \lambda \pi R^2 \exp \left( -G_{\text{min}} d_{k,k}^\alpha \right) \frac{\alpha - 2 R^{2-\alpha}}{\alpha - 2} \lambda \pi + \frac{1}{R^2} P_{\text{max},d}^{-1} + \frac{1}{\text{SNR}}$$

$$= \delta(R, \lambda, P_{\text{max},d}^{-1}) \exp \left( -G_{\text{min}} d_{k,k}^\alpha \right) G_{\text{min}},$$

(38)

where $\delta(R, \lambda, P_{\text{max},d}^{-1}) = \frac{\alpha - 2 R^{2-\alpha}}{\alpha - 2} \lambda \pi + \frac{1}{R^2} P_{\text{max},d}^{-1} + \frac{1}{\text{SNR}}$. To find the best threshold $G_{\text{min}}$, the following optimization problem can be considered:

$$\max_{G_{\text{min}} \geq 0} \delta(R, \lambda, P_{\text{max},d}^{-1}) \exp \left( -G_{\text{min}} d_{k,k}^\alpha \right) G_{\text{min}}.$$  

(39)

Although the objective function is not concave, the optimal solution of $G_{\text{min}}$ can be obtained by finding the first order KKT condition since the objective function has an unique maximum value for $G_{\text{min}} \geq 0$. The first order KKT condition gives us

$$\exp \left( -G_{\text{min}} d_{k,k}^\alpha \right) \delta(R, \lambda, P_{\text{max},d}^{-1}) = \frac{d_{k,k}^\alpha \exp \left( -G_{\text{min}} d_{k,k}^\alpha \right) G_{\text{min}} \delta(R, \lambda, P_{\text{max},d}^{-1})}{d_{k,k}^\alpha}.$$  

(40)

From the first order KKT condition, the optimal D2D transmission parameter solution is $G_{\text{min}} = d_{k,k}^\alpha$. Using this solution, the sum rate lower bound in (38) can be simply expressed as

$$R^{(D)} \geq e^{-1} \frac{\delta(R, \lambda, P_{\text{max},d}^{-1})}{d_{k,k}^\alpha}.$$  

(41)

This lower bound result is notable in that when $\lambda$ goes to infinity, the sum rate of underlaid D2D links increases linearly with the spatial packing ratio $d_{k,k}^\alpha$ of D2D transmissions. In particular, when the cross-tier interference $P_{\text{avg},d}^{-1}$ is large enough, the sum rate bound can be expressed as $R^{(D)} \geq e^{-1} \frac{\delta(K)}{d_{k,k}^\alpha}$, implying that the cross-tier interference $P_{\text{avg},d}^{-1}$ degrades the D2D sum rate performance significantly.

VI. NUMERICAL RESULTS

In this section, we provide numerical results for the D2D underlaid cellular system. From our simulation results, we first show the performance gain of the proposed power control methods compared to the no power control case in terms of the
### TABLE II  
**SIMULATION PARAMETERS**

| Parameters                                      | Values                                      |
|-------------------------------------------------|---------------------------------------------|
| Cell radius ($R$)                               | 500 (m)                                     |
| The D2D link range ($d_{k,k}$)                  | 50 (m)                                      |
| D2D link density ($\lambda$)                    | 0.00002 and 0.00005                        |
| Average number D2D links ($K$)                   | $E[K] = \pi R^2 \lambda \in \{15, 39\}$   |
| Path-loss exponent ($\alpha$)                   | 4                                           |
| Target SINR threshold ($\beta$)                 | -3, 3, 9, 12, 15 (dB)                      |
| The average transmit power of the cellular user  | $P_{avg, c} = 100$ mW                      |
| The average transmit power of the D2D transmitters | $P_{avg, d} = 0.1$ mW                      |
| Noise variance ($\sigma^2$) for 1MHz bandwidth   | -143.97 (dBm)                               |
| The number of realizations                      | 1000 geometry drops                        |

---

Fig. 2. A snap shot of link geometry for a D2D underlaid cellular network when the dense D2D link deployment scenario, i.e., $\lambda=0.00005$.

Fig. 3. Coverage probability performance of both the cellular and D2D links according to different power control methods when the D2D links are sparse, i.e., $\lambda=0.00002$.

---

cellular user and the D2D user coverage probability. Further, to compare the accuracy of the analysis provided in Section IV, we compare the coverage probability results obtained from simulations with the theoretical bounds.

1) **Simulation Setup:** Fig. 2 shows one snap shot of the cell geometry. As illustrated, the BS is located at the center position $(0,0)$ in $\mathbb{R}^2$ plane and the cellular user is uniformly dropped within the range of $R = 500$ m. The D2D transmitters are dropped according to PPP with the density parameter $\lambda \in \{0.00002, 0.00005\}$ so that the average number of D2D links equals $E[K] = \pi R^2 \lambda \in \{15, 39\}$. Further, for a given D2D transmitter’s location, the corresponding D2D receiver is isotropically dropped at a fixed distance $d_{k,k}=50$ m away from the D2D transmitter. Since the D2D communication are supposed to be of short range compared to the cellular link, we assume that the average transmit power of the cellular user and D2D links are equal to $P_{avg, c} = 100$ mW and $P_{avg, d} = 0.1$ mW. Since the number of D2D links $K$ is a random variable and we evaluate the coverage probability and sum rate performance of the proposed algorithms by averaging 1000 independent realizations. Further, the optimal transmission scheduling parameter $G_{min}$ is obtained using the line search method, which finds a maximizer of the function in (37). The parameters used in the simulations are summarized in Table II.

2) **Coverage Probability Comparison in Sparse D2D Link Deployment:** Suppose the sparse D2D link deployment scenario where the average number of D2D links in the cell equals $E[K] = \pi R^2 \lambda = 15.7$. In this scenario, we compare the coverage probability of the cellular link and the D2D links under different power control algorithms. As shown in Fig. 3, we observe that the proposed power control methods improve the cellular user coverage probability. The proposed power control methods also provide increased D2D link coverage probability compared to the no power control case, especially in the low target SINR regime. This implies that the power control methods are efficient to mitigate both intra-D2D and cross-tier interference. In particular, one remarkable observation is that the centralized power control achieves nearly perfect cellular user coverage probability performance, i.e., (no outage) over different target SINR values, while successfully supporting a large number of active D2D links (83 %) when target SNIR $\beta = 3$ dB. Meanwhile, the on-off distributed power control method yields performance gains for both cellular and D2D links compared to that of no power control case when the target SINR is larger than 9 dB. For example, when the target SINR is 12 dB, the on-off power control method provides 10% cellular link and 5% D2D link coverage probability.
performance gains compared to the no power control case.

3) Coverage Probability Comparison in Dense D2D Link Deployment: Consider a dense D2D link deployment scenario where the average number of D2D links in the cell equals \( E[K] = \pi R^2 \lambda = 39 \). For the dense D2D link deployment, as shown in Fig. 4, we observe similar trends as in the sparse D2D link deployment case. One interesting point is that the cellular user’s coverage probability is not degraded as the number of D2D links increases when the centralized power allocation method is applied because the proposed admission control ensures that the uplink user is protected. This implies that the centralized power control method is able to support reliable uplink performance regardless of the density of D2D links. Meanwhile, the D2D user coverage probability performance becomes deteriorated because of the increased intra-D2D link interference. It is notable that the proposed on-off power control method improves the D2D link coverage probability performance compared to that of no power control case in a more broad range of the target SINR. Although the D2D user coverage probability performance decreases in the dense scenario, the total number of successful D2D transmissions is large than that of the sparse D2D link deployment scenario. For example, when the target SINR is 3 dB, the total numbers of successful D2D transmissions in both sparse and dense scenarios are about \(|S|_{\text{sparse}} = E[K P_{\text{cov}(D)}(p)] = 15 \times 0.83 \approx 12\) and \(|S|_{\text{dense}} = E[K P_{\text{cov}(D)}(p)] = 39 \times 0.68 \approx 27\), respectively. The performance gain from the centralized power control method is summarized in Table III.

4) D2D Link Sum Rate Performance: As shown in Fig. 5, the D2D link sum rate performance behaves differently than the D2D link coverage performance. In the case of no power control, the sum rate of D2D does not change in the target SINR as the D2D transmission power is independent of the target SINR. Meanwhile, when the proposed on-off power control is applied, the sum rate performance of D2D links increases with respect to the target SINR. This is because in the high target SINR regime, the uplink user opportunistically sends the signal to the BS using the uplink binary power control, which decreases the cross-tier interference to the D2D receivers. Another interesting point is that the centralized power control method provides the worst D2D sum rate. This is because it was designed to maximize the coverage probability, implying that many D2D links are inactive to guarantee the uplink user coverage. Therefore, the binary uplink user power control method is useful to increase the cellular user coverage probability performance as well as the sum rate of D2D links.

5) Validation of Theoretical Analysis: Let us compare the theoretical results derived in Section IV with the numerical results by simulations. By comparing them, as shown in Fig. 6, we observe that the theoretical bound on the coverage probability of the cellular user in (25) is well matched with the simulation results when the distributed on-off power control method is used. For the D2D user coverage probability, we can find the coverage probability performance gap between the analytical result in (18) and simulation results. This discrepancy comes from only taking into account the interference of D2D transmitters that are located in the cell circle in the simulations, but our analytical results are obtained by considering all D2D interfering links in \( \mathbb{R}^2 \) plane. Nevertheless, the analytical results provide the same D2D coverage probability trends. From these observations, our approximated coverage probability bounds for the D2D underlaid cellular network when all nodes use independent transmission power allocation strategies. The gap between the bounds and simulation results becomes large when we consider the centralized power control method, which designs the transmit power of all nodes by considering interaction between all wireless links. This centralized power control method creates correlation between transmit power statistics used by nodes. In our analysis, however, this correlation effect is ignored, which leads to increase the gap.

\[ \text{Fig. 4. Coverage probability performance of the cellular link according to different power control methods when the D2D links are dense, i.e., } \lambda = 0.00005. \]

\[ \text{Fig. 5. Sum rate performance of D2D links when the D2D links are dense, i.e., } \lambda = 0.00005. \]
where in the second last equality we use the fact that $|h_{0,0}|^2 \sim \text{Exp}(1)$ and thus $P(|h_{0,0}|^2 \geq x) = e^{-x}$. Conditioned on the transmit power of the typical uplink transmitter $p_0 = p$ and the distance $d_{0,0} = d$ from the cellular transmitter to BS, we next compute the second term (45). To this end, we need the Laplace transform $L_\phi(s) = E[e^{-s\sum_{k \in F} p_k|h_{0,k}|^2||x_k||^{-\alpha}}]$. The last step is to derive the probability distribution of $X = p_0^{-1}d_{0,0}^{\alpha}$.

$$F_X(x) = P(p_0^{-1}d_{0,0}^{\alpha} \leq x) = \int P(d_{0,0} \leq (xp)^{\frac{1}{\alpha}})dF_{p_0}(p) = \int F_{d_{0,0}}(x^{\frac{1}{\alpha}}p^{\frac{1}{\alpha}})dF_{p_0}(p).$$ (48)

**B. Proof of Theorem 2**

Let $\phi(x) = e^{-a_1x-a_2x^{\frac{2}{\alpha}}}$. We compute the first and second derivative of $\phi(x)$ as follows:

$$\phi'(x) = -e^{-a_1x-a_2x^{\frac{2}{\alpha}}}(a_1 + a_2\frac{2}{\alpha}x^{\frac{1}{\alpha}-1}),$$ (49)

$$\phi''(x) = -e^{-a_1x-a_2x^{\frac{2}{\alpha}}}(a_1 + a_2\frac{2}{\alpha}x^{\frac{1}{\alpha}-1})^2 + e^{-a_1x-a_2x^{\frac{2}{\alpha}}}(a_2\frac{2}{\alpha}(1 - \frac{2}{\alpha})x^{\frac{1}{\alpha}-2}).$$ (50)

As $\alpha > 2$, $\phi''(x) \geq 0$ for $x \geq 0$ and thus $\phi(x)$ is convex for $x \geq 0$. Applying Jensen’s inequality, we obtain

$$F \phi(C) = E_X\left[e^{-a_1X-a_2X^{\frac{2}{\alpha}}} \right] \geq e^{-a_1E[X]-a_2E[X]^{\frac{2}{\alpha}}},$$ (51)

where $E[X] = E[p_0^{-1}d_{0,0}^{\alpha}] = E[p_0^{-1}]E[d_{0,0}^{\alpha}]$ due to the independence of $P_C$ and $D_C$. Finally, $E[d_{0,0}^{\alpha}]$ can be computed explicitly.

$$E[d_{0,0}^{\alpha}] = \int r^\alpha dF_{d_{0,0}}(r) = \int_0^R r^\alpha \frac{2r}{R^2}dr = \frac{2}{2 + \alpha}R^\alpha.$$ (52)

**C. Proof of Corollary 2**

Note that $\phi(p)$ is positive-valued and continuous when $p > 0$. Also, $\lim_{p \to +0} \frac{\phi(p)}{p} \to 0$ and $\lim_{p \to +\infty} \frac{\phi(p)}{p} \to 0$. These facts imply that there exists a non-degenerate $p^*(d) \in (0, \infty)$ that achieves the maximum value of $\frac{\phi(p)}{p}$. If we ignore the

| Target SINR $\beta$ | -3 dB | 6 dB | 9 dB | 12 dB | 15 dB |
|---------------------|-------|------|------|-------|-------|
| Cellular link coverage probability | 1 | 1 | 1 | 0.99 | 0.98 |
| $|S^N_{[\text{off}]}| = E[KP_{C0}^N\{p\}]$ (Dense D2D link) | 31 | 24 | 20 | 16 | 13 |
| $|S^N_{[\text{on}]}| = E[KP_{C0}^N\{p\}]$ (Sparse D2D link) | 13 | 12 | 11 | 9 | 8 |
constraint $\int dF_{p_0}(p) = 1$ for the time being and consider the following relaxed conditionalcellular link coverage optimization problem:

$$\max \int \phi(p)dF_{p_0}(p)$$

subject to $$\int pdF_{p}(p) = P_{\text{avg}}, c.$$ (53)

Let $dG(p) = \frac{p}{P_{\text{avg}}}dF_{p_0}(p)$. Then the above optimization problem can be equivalently formulated as

$$\max P_{\text{avg}}\cdot c\int \frac{\phi(p)}{p}dG(p)$$

subject to $\int dG(p) = 1$. (54)

whose optimal solution is $G^*(p^*)(d) = G^*(p^{**}(d)) = 1$ and $G^*(p) = 0$ for $p \neq p^*(d)$. Therefore, we conclude that the binary power control strategy is optimal.

D. Proof of Lemma 2

A Gamma distributed random variable $Y$ has pdf

$$f_Y(y) = \frac{1}{\theta} \Gamma(\kappa) y^{\kappa-1}e^{-\frac{y}{\theta}}, y > 0,$$ (55)

where $\kappa > 0$ is shape parameter and $\theta > 0$ is scale parameter. Fitting the first and second moment yields

$$\mathbb{E}[p_{0}|h_{k,i}|^2]d_{k,i}^{-\alpha} = \mathbb{E}[p_{0}|h_{k,i}|^2]d_{k,i}^{-\alpha} = \kappa \theta$$ (56)

$$\text{Var}(p_{0}|h_{k,i}|^2)d_{k,i}^{-\alpha} = \mathbb{E}[p_{0}|h_{k,i}|^2]d_{k,i}^{-\alpha} - (\mathbb{E}[p_{0}|h_{k,i}|^2]d_{k,i}^{-\alpha})^2$$

$$= 2\mathbb{E}[p_{0}^2]|d_{k,i}^{-\alpha}| - \mathbb{E}[p_{0}^2]|d_{k,i}^{-\alpha}| = \kappa \theta^2,$$ (57)

from which we can solve for $\kappa$ and $\theta$, which are given in the proposition. Then the proof is then completed by invoking the Laplace transform of Gamma distribution $L(s) = (1 + \kappa s)^{-\kappa}$.

E. Proof of Theorem 3

To prove Theorem 3, we need to derive the cdf of SINR$_k$. To this end, using Slivnyaks theorem [27], it is easy to see that

$$L_{\Phi\backslash\{k\}}(s) = \mathbb{E}[e^{-s\sum_{x \in \Phi,\{k\}} p_{i}|h_{k,i}|^{2}|x_i|^{-\alpha}} | k \in \Phi]$$

$$= L_{\Phi}(s) = e^{-\mathbb{E}_s\sum_{x \in \Phi,\{k\}}|p_{i}|h_{k,i}|^{2}|x_i|^{-\alpha}d_{k,i}^{-\alpha}}.$$ (58)

It follows that

$$P(\text{SINR}_k \geq \beta) = P\left(\frac{p_{i}|h_{k,i}|^{2}d_{k,i}^{-\alpha}}{\sum_{x \in \Phi,\{k\}} p_{i}|h_{k,i}|^{2}|x_i|^{-\alpha} + p_{j}|h_{k,j}|^{2}d_{k,j}^{-\alpha} + \sigma^2} \geq \beta\right)$$

$$= P\left(|h_{k,i}|^{2} \geq \beta^{-1}d_{k,i}^{-1}d_{k,i}^{\alpha}C\right)$$ (59)

$$= P\left(\sum_{x \in \Phi,\{k\}} p_{i}|h_{k,i}|^{2}|x_i|^{-\alpha} + p_{j}|h_{k,j}|^{2}d_{k,j}^{-\alpha} + \sigma^2 \geq \beta^{-1}d_{k,i}^{-1}d_{k,i}^{\alpha}C\right)$$ (60)

$$= P\left(\sum_{x \in \Phi,\{k\}} p_{i}|h_{k,i}|^{2}|x_i|^{-\alpha} + p_{j}|h_{k,j}|^{2}d_{k,j}^{-\alpha} + \sigma^2 \geq \beta^{-1}d_{k,i}^{-1}d_{k,i}^{\alpha}C\right)$$ (61)

$$= P\left(\sum_{x \in \Phi,\{k\}} p_{i}|h_{k,i}|^{2}|x_i|^{-\alpha} + p_{j}|h_{k,j}|^{2}d_{k,j}^{-\alpha} + \sigma^2 \geq \beta^{-1}d_{k,i}^{-1}d_{k,i}^{\alpha}C\right)$$ (62)

where $C = \sum_{x \in \Phi,\{k\}} p_{i}|h_{k,i}|^{2}|x_i|^{-\alpha} + p_{j}|h_{k,j}|^{2}d_{k,j}^{-\alpha} + \sigma^2$. This completes the proof.
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