Using fuzzy logic to support maintenance decisions according to Resilience-Based Maintenance concept

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1. Introduction

The maintenance of technical systems is of particular importance in the era of growing competition and ever higher requirements in quality, reliability, and productivity of organizations’ functions and tasks. According to [5], maintenance for complex socio-technical systems can be defined as a combination of activities which ensures that physical assets continue to fulfill their intended tasks effectively (performing required functions), efficiently (at minimum use of resources), and safely (at a minimum human and environmental risk). Therefore, the main goals of the maintenance processes of technical systems are today considered to provide [42]: 1) an appropriate level of functionality of a technical facility, 2) declared durability of a facility, 3) security of a facility and its environment, and 4) effective use of available resources supporting basic processes. The achievement of these goals is possible thanks to an appropriately selected maintenance strategy.
for technical systems. Currently, the primary forms of maintenance can be [25]:
- **Pre-planned maintenance**: includes early maintenance tasks such as cleaning, greasing, lubricating, zero-setting, and recording key measurements. Often conducted by non-maintenance staff. Usually called First-line maintenance.
- **Planned maintenance**: also known as scheduled maintenance, and its timing and scope are both known in advance.
- **Shutdown maintenance**: planned maintenance but carried out when production or plant is shut down.
- **Breakdown maintenance**: carried out when equipment fails to meet its desired function. This may involve repairs, replacements, or adjustments as considered necessary.
- **Emergency maintenance**: carried out only when either inspection or breakdown maintenance has identified its necessity.

As all technical systems operate under conditions of uncertainty and variability resulting from, among others, the uncertainty of operating processes, environment, or a modeling process, the problem of appropriate selection of the maintenance strategy arises. This is especially visible for systems performing in deep uncertainty, where the disruptive events occur very rarely. For these systems, the classic probabilistic approach to maintenance modeling cannot be implemented due to a lack of operational data. The possible solution to this problem may be connected with the provision of resilient organizations to prevent or minimize the effects of high-level failures [23, 26].

Resilience theory is concerned with successfully responding to the unpredictability and uncertainty of change [4]. When referring to the resilience of industrial assets, several authors have highlighted the importance of maintenance to physical asset management and suggested ways to improve maintenance in relation to improved dependability of the assets (e.g. [29]). Moreover, the relations between maintenance, safety, risk, and resilience are especially highlighted in work [14]. Later, safety performance of organizations in relation to the decision-making processes is analyzed in [12]. The research findings constitute the base for the authors in [2], where resilience engineering issues are investigated in safety research and organizational practice. Based on the obtained survey results, in another work [20] the authors define maintenance as a safety barrier in process system operations. They analyze overall system performance in terms of, among others, maintenance costs, safety impact, environmental impact, and asset damage. Safety-II domain, defined as safety management through guided adaptability, is later investigated in [33]. The authors analyze the relations between resilience engineering and safety domains. In this new approach to safety, it is assumed that failures were the flip side of successes, or in other words, things that go right and things that go wrong happen basically the same way. Therefore, we may state that resilience is a key issue in ensuring the safe and reliable operation of systems and organizations’ effective management.

Following this, there is a necessity to investigate the relationship between resilience and maintenance performance. For this reason, this article aims to introduce a new approach to system maintenance based on resilience concept implementation, called Resilience-Based Maintenance (RBM). The proposed concept is based on Maintenance Support Potentials (MSP) introduction. The MSP constitutes the base for measuring an organization’s Maintenance Support Capacity (MSC). Moreover, based on the MSP definition, the authors develop a fuzzy-based organization’s maintenance support potential level assessment method. The proposed approach considers two main maintenance support potentials parameters – potential readiness level and process regency – and four main steps, including organization’s MSP and their assessment parameters identification/evaluation, MSP weights assessment, Maintenance Support Capacity in an organization assessment, and final reasoning in terms of maintenance recommendations proposition. The fuzzy theory is implemented in the MSP parameters estimation process.

The developed assessment method’s implementation possibilities are based on the example of a selected global manufacturer from the automotive sector.

The proposed concept was preliminarily introduced in the authors’ research work [5], where the simple investigation of maintenance potentials assessment possibilities based on scoring method was presented. In this study, the authors extend the previously done research by introducing a more systematic description of the approach, new assessment methodology, and the implementation possibilities of the RBM concept.

To sum up, the authors’ contribution in this study includes:
- introduction of a new Resilience-Based Maintenance concept that bases on Maintenance Support Potentials definition,
- a new concept of organization’s maintenance support potential level,
- development of a three-step assessment method to assess the organization’s maintenance support potential level ratio in order to define the organization’s Maintenance Support Capability, etc.

Therefore, the article structure includes, apart from the Introduction section, a detailed review of the literature in the area of classification of basic maintenance strategies for technical systems, based on which the concept of Resilience-Based maintenance is described. Next, a proposed new maintenance concept based on resilience theory is introduced. Moreover, the Maintenance Support Potentials are defined as a measure of an organization’s support capability in the area of maintenance management. Later, in Section 4, the authors introduce the proposed method for an organization’s maintenance support potential level assessment. The implementation possibilities of the developed method are presented in Section 5. Section 6 presents the obtained results and their discussion. Finally, Section 7 provides conclusions, limitations of the study, and suggestions for the authors’ future research works to optimize the maintenance strategy based on the concept of Resilience-Based Maintenance.

2. Related work

2.1. Defining maintenance process

Today, technical systems should be designed, operated, and maintained in a safe, reliable, robust, durable, sustainable, and resilient way [34, 42]. In order to satisfy such goals, organizations develop and implement effective maintenance management processes. Following the European Standard PN-EN 13306:2010 [31], maintenance management may be defined as all activities of the management that determine the maintenance objectives, strategies, and responsibilities and implement them by means such as maintenance planning, maintenance control and supervision, improvement of methods in the organization including economic aspects. The main challenge for the maintenance manager is to structure maintenance procedures and activities to be undertaken to achieve the strategic objectives associated with them [6, 11]. In addition, following the European Standard EN 17007:2017 [32], proper maintenance needs technical skills, techniques, methods to properly utilize assets like factories, power plants, vehicles, equipment, and machines (Figure 1). As a result, it is necessary to consider maintenance issues in an organization in a more holistic way, not only limited to such problems, like maintenance planning or selection of an appropriate maintenance strategy [43]. It also requires looking at the issues related to ensuring an effective maintenance system by considering issues related to safety, risk, and
2.2. Uncertainty Modelling

The consequence of knowledge imperfections is the uncertainty in the maintenance process. We understand the concept of uncertainty as a situation of having limited knowledge such as:

- the order, nature, or state of things is unknown, and
- the consequence, extent, or magnitude of circumstances, conditions, or events is unpredictable.

There are many forms of uncertainty, but the most common is its division into two categories: aleatory and epistemic uncertainty [8, 46]. The aleatory uncertainty is understood as an inherent variation associated with the engineered system or the environment under consideration. It can be observed in random experiments and described by probability distributions. Traditional reliability engineering and risk analysis applications tend to model only the aleatory uncertainties, leading to significant underestimations of the real risks and over-estimation of reliability [16]. However, the epistemic uncertainty is not an inherent property of the system or its environment, and it results from our inability to understand as well as describe and model reality. Thus, in this case, the standard probabilistic methods are not useful [41, 52].

In 2005 Lotfi A. Zadeh proposed a generalized uncertainty theory (GTU), which attempts to unify the approach to uncertainty [48]. The GTU theory was based on the concepts of granular structures and generalized constraints. The basic assumptions of these concepts are illustrated in Figure 2. Let $X$ be a variable taking values in a universe of discourse, $U$, then $a$ is a singular value of $X$ (e.g., a singleton), implying that there is no uncertainty about $X$’s value. If this is not the case, then a granular value of $X$, $A$, may be viewed as a representation of the state of knowledge about $X$’s value.

Informally, a granule of a variable $X$ is a clump of $X$ values drawn together by indistinguishability, equivalence, similarity, proximity, or functionality. For example, intervals (crisp or fuzzy) are granules and different probability distributions [1]. The concept of granularity underlies the concept of a linguistic variable - a concept introduced by L. A. Zadeh in the paper "Outline of A New Approach to the Analysis of Complex Systems and Decision Processes" [49]. A linguistic variable’s concept plays a pivotal role in many fuzzy logic applications [7, 10, 21, 30, 35, 45]. Four primary rationales underlie the granulation of attributes and the use of linguistic variables:

- the bounded ability of sensory organs to resolve detail and store information,
- when numerical information is not available,
- when an attribute is not quantifiable because we do not have a numerical scale for it,
- when there is an acceptance for imperfection (e.g., inaccuracy or imprecision), which can be exploited through granulation to achieve tractability and communication economy.

There is a close connection between granularity and uncertainty. Suppose $X$ is a variable, and we are looking for the value of this variable. If the answer is "X is a", where $a$ is a singleton, then there is no uncertainty in $X$’s information because the information is singular. Nevertheless, if the answer is "X is approximately a", in the abbreviation "X is *a*", there is some uncertainty in the information because information is described as granular. Therefore, the granularity may be equated to non-singularity. In the context of standard probability theory, $a$ would generally be interpreted as a probability distribution centered on $a$. In GTU, $X$’s information is viewed as a generalized constraint on $X$, or more specifically, as a granule characterized by a generalized constraint. A probability distribution can be seen as a particular case of a generalized constraint.

A generalized constraint, GC, is defined as an expression of the form [47]:

$$GC : X isr R$$

where: $X$ is the constrained variable; $R$ is a constraining relation which, in general, is non-bivalent; and $r$ is an indexing variable that identifies the modality of the constraint, that is, its semantics.

The principal modalities of generalized constraints are summarized in the following.

a) Probabilistic ($r = p$)

$$X isp R$$

with $R$ – the probability distribution of $X$. For example:

$$X isp N(m,\sigma)$$

means that $X$ is a normally distributed random variable with mean $m$ and variance $\sigma^2$.

If $X$ is a random variable that takes values in a finite set $\{x_1, ..., x_n\}$ with respective probabilities $p_1, ..., p_n$, then $X$ may be expressed as:
with the semantics:

\[ Prob(X = u_i) = p_i \quad i = 1, \ldots, n \]  

(5)

In GTU, a probabilistic constraint is viewed as an instance of a generalized constraint. When \( X \) is a generalized constraint, the expression \( X \) isp \( R \) is interpreted as a probability qualification of \( X \), with \( R \) as \( X \)'s probability [47]. For example:

\[ (X \text{ is big}) \text{isp likely} \]  

(6)

It means that the probability of the fuzzy event \( \{X \text{ is big}\} \) is likely, where "big" is a fuzzy subset of the real line.

b) Possibilistic \((r = \text{blank})\)

\[ X \text{ is } R \]  

(7)

with \( R \) playing the role of the possibility distribution of \( X \). For example:

\[ X \text{ is } [a,b] \]  

(8)

means that \([a, b]\) is the set of possible values of \( X \). Next example:

\[ X \text{ is small} \]  

(9)

In this case, the fuzzy set labeled small is the possibility distribution of \( X \), and \( \mu_{\text{small}} \) is the membership function of small, then the semantics of "\( X \text{ is small} \)" is defined by [48]:

\[ \text{Poss} \{X = u\} = \mu_{\text{small}}(u) \]  

(10)

where \( u \) is a generic value of \( X \).

c) Veristic \((r = v)\)

\[ X \text{ isv } R \]  

(11)

where \( R \) plays the role of a verity (truth) distribution of \( X \). In particular, if \( X \) takes values in a finite set \([u_1, \ldots, u_n]\) with respective verity (truth) values \( t_1, \ldots, t_n \), then \( X \) may be expressed as:

\[ X \text{ isv } (t_1|u_1 + \ldots + t_n|u_n) \]  

(12)

meaning that \( \text{Ver} \{X = u_i\} = t_i \), \( i = 1, \ldots, n \).

When \( X \) is a generalized constraint, the expression \( X \text{ isv } R \) is interpreted as verity (truth) qualification of \( X \). For example:

\[ (X \text{ is small}) \text{isv very true} \]  

(13)

should be interpreted as "it is very true that \( X \text{ is small}\)." The semantics of truth qualification is defined in [47]:

\[ \text{Ver} \{X = R\} \text{ is } t = X \text{ is } \mu_R^{-1}(1(t)) \]  

(14)

Where \( \mu_R^{-1} \) is the inverse of the membership function of \( R \), and \( t \) is a fuzzy truth value, which is a subset of \([0, 1]\).

Therefore, there are two classes of fuzzy sets: (b) possibilistic and (c) veristic. In the case of a possibilistic fuzzy set, the grade of membership is the degree of possibility. In the case of a veristic fuzzy set, the grade of membership is the degree of verity (truth).

L. A. Zadeh [50] introduced the concept of fuzzy sets as a generalization of the classical set theory. In fuzzy sets, each space \( X \) element can belong partially to a set \( A \) and partly to its complement \( \overline{A} \). Fuzzy sets are defined by the membership function corresponding to the functional characteristics of classical sets. Each set \( X \) element has the assigned value that defines the degree of membership to the fuzzy set. The standard fuzzy sets membership function belongs to a range \([\alpha, \beta]\) and if we deal with the normal fuzzy sets \( \alpha = 0 \) and \( \beta = 1 \). Thus, the membership function of the set \( X \) is:

\[ \mu_A : X \rightarrow [0,1] \]  

(15)

We can distinguish three cases here:

a) \( \mu_A(x) = 1 \) – means full membership in the fuzzy set \( A \)
b) \( \mu_A(x) = 0 \) – means the lack of membership in the fuzzy set \( A \)
c) \( 0 < \mu_A(x) < 1 \) – means a partial membership in the fuzzy set \( A \)

A fuzzy set \( A \) is contained in the fuzzy set \( B \) only when \( \mu_A(x) < \mu_B(x) \) for each \( x \in X \), and the fuzzy set \( A \) equals the fuzzy set \( B \) only when \( \mu_A(x) = \mu_B(x) \). The complement of set \( A \) is a fuzzy set \( \overline{A} \) with a membership function \( \mu_{\overline{A}} = 1 - \mu_A \).

Although the inference based on the fuzzy set theory and multi-valued logic is more complex and less intuitive, thanks to widely available computer tools supporting the fuzzy inference process, it is becoming more common [22].

Uncertainty assessment is particularly important for planned maintenance and is mainly based on probabilistic models \((r = p)\). However, these models’ effective use is only possible if the data on the damage processes are sufficiently numerous and stationary processes. These conditions are not fulfilled in high uncertainty situations, where rare events occur, and these events’ consequences are difficult to predict.

The authors propose to use possibility-based procedures to model the maintenance process under these conditions \((r = \text{blank})\) and fuzzy set theory. In the absence of statistical data, this approach allows objectifying expert knowledge, which is inherently subjective partially.

3. The concept of Resilience-Based Maintenance

The starting point for our considerations is the model of the maintenance process presented in Fig. 1. From this model, it follows that the prerequisite for the proper performance of the maintenance process in the organization is an extensive system implementing all support processes. We called it Maintenance Support System (MSS) and assumed that its fundamental characteristic is Maintenance Support Capability (MSC), which is defined as follows:

Maintenance Support Capability is the ability of an organization to ensure that physical assets continue to fulfill their intended tasks effectively, efficiently, and safely, under given expected as well as unexpected conditions of use and maintenance.

Following this, a measure of an organization’s Maintenance Support is its capacity to create and maintain specific potentials over time to resiliently respond to any foreseeable and unpredictable operating events.

We propose to name these potentials Maintenance Support Potentials (MSP) and the entire maintenance system based on this concept – Resilience-Based Maintenance. These potentials are as follows (based on [13]):

- **PR – The Potential to respond**: knowing what to do and being able to react correctly to any threats and hazards (e.g., changes, disturbances, and disruptions) by activating correctly planned and prepared actions, by adjusting the required mode of operation, or by introducing new activities, procedures or processes.

- **PM – The Potential to monitor**: being able to monitor all signals from the internal and external environment that may affect
an organization’s performance in the near-term or long-term future.

• **PL – The Potential to learn:** being able to draw conclusions from experience, in particular ‘to learn the right lessons from the right experiences’. It also includes changing values, criteria, and even the organization’s goals, depending on the type of change in the situation.

• **PA – The Potential to anticipate:** knowing what to be expected and predicting future developments considering particular potential disruptions, constraints, and changing operating conditions.

A functional diagram of the Maintenance Support System broken down into individual subsystems: monitoring, response, learning, and anticipation is shown in Figure 3. Thus, the general model of Maintenance Support Capability can be represented as follows:

\[
MSC = \{PM, PR, PL, PA\}
\]

Before beginning a response action, some special conditions must be fulfilled, such as requesting and receiving permission or authorization. When a response is started, the availability of specific resources should be required (e.g., information, staff, materials, and tools). While the response is being carried out, it may be necessary to maintain a given degree of normal functioning, even during an emergency action. Because responses are often complex and aggregated processes, the proper timing and synchronization of them can also be crucial in creating the Potential to respond.

Learning can be understood as the active and intended modification of processes and procedures describing the organization’s behavior in specific situations. The primary purpose of Potential to learn (PL) is to improve the organization’s ability to respond, monitor, and anticipate. Each organization should learn from both negative and positive examples. In general, negative situations are rare and irregular, so learning, in this case, is a reaction to some unusual event or situation (e.g., a disruption or an accident). A typical rule for starting the learning process is to state that an event or signal is significantly different from expectations. This type of learning is called reactive or event-driven.

An influential learning culture should meet four necessary conditions, namely:

- create favorable conditions and learning opportunities,
- establish main rules at which learning take place (e.g., limits and thresholds for monitored signals),
- define conditions of similarity between individual situations to enable the generalization of results obtained from monitoring,
- create objective conditions for verifying the learning process and confirming its effectiveness.

Usually, a high level of learning culture is achieved primarily by using a broad-perspective and focusing on exceptional but rare cases.

Creating Potential for anticipation (PA) in an organization is conducive to supporting anticipatory thinking technologies. Where monitoring is about observing and looking at something to see whether it is significantly changing, anticipation is more about thinking and imagining outside the event horizon. The primary purpose of anticipation is to imagine alternative scenarios and predict what can happen in the future. Therefore, anticipation depends on the assumptions made about the future and models used for prediction. Three basic types of modeling are applied in practice: deterministic, probabilistic, and realistic. The first one relies on the assumption that the future is a simple reflection of the past, both in terms of similarity in size and frequency. The basis of the probabilistic approach is the assumption that the unknown future is an extrapolation of the known past, taking into account randomness. The third method is based on the assumption that understanding past events and relationships between them makes it possible to predict the possible course of events in the future, taking into account the uncertainty that such a prediction is burdened with. Therefore, the anticipation can be seen as an art of art rather than a science and depends very much on the person or team’s imagination that deals with it. This process runs at variable speeds, with unpredictable timing.

Consequently, the question arises on how to assess the maintenance support capability for an organization. To answer this question and, indeed, define the organization’s maintenance support capability, there is introduced a new organization’s maintenance support potential level assessment ratio (MSPi). It can be evaluated based on the following formula:

\[
MSP_i = \sum_{i=1}^{n} P_i \cdot w_i, \quad i = 1, 2, \ldots, n
\]

where:
- \(MSP_i\) – organization’s maintenance support potential level;
- \(P_i\) – ith maintenance support potential;
- \(w_i\) – weight for ith maintenance support potential;
- \(n\) – number of analyzed maintenance support potentials.
As a result, in order to gain benefit from the maintenance support potential level assessment, maintenance managers should:
1. understand MSS and identify maintenance management priorities for the near, medium, and long term;
2. identify assets, human, and material resources, as well as define the possible maintenance strategies to follow;
3. define possible responding strategies appropriate to disruptions occurring;
4. balance maintenance costs vs. risks for disruptions.

Implementation of such defined main steps for MSS identification and improvement needs a methodological approach use. Following this, the proposed approach adopted in this study consists of three main phases. The first step bases on qualitative analysis implementation. During this phase, the identification of the problem and definition of maintenance support potentials is performed. Moreover, the main parameters for maintenance support potentials assessment are identified. The second phase includes quantitative analysis performance. The collection of experts’ opinions about the defined maintenance support potentials and their evaluation parameters is carried out at this stage. There are also defined weights for all maintenance support potentials to reflect a company’s maintenance management priorities. Due to the lack of possibility to use accurate statistical data, it was proposed in the described method to estimate both parameters by experts using the fuzzy logic concept. The analytical approach to determine these values is presented in the 4.2 Subsection. The last phase – an output phase provides the organization’s maintenance support potential level ratio assessment and reasoning on the level of \( MSP_O \) obtained.

Additionally, at this stage, the reasoning process on the maintenance recommendations proposition is performed. Figure 4 represents the graphical view of the proposed complete methodology followed.

![Fig. 4. Organization’s maintenance support potential level analysis procedure](image)

### 4. Fuzzy-based method for assessment of organization’s maintenance support potential level

Before companies can devise effective means of enhancing maintenance support capacity, managers must first understand the universe of maintenance potentials as well as the conditions that drive them. Then, after gaining specific knowledge about maintenance support potentials assessment, companies can proceed to select and tailor the most effective maintenance strategies. A detailed description of the main phases of the proposed assessment method is presented in the next subsections.

#### 4.1. Qualitative analysis of Maintenance Support Potentials

The first two steps of the analysis are used to identify the investigated maintenance support potentials and their evaluation parameters. In the developed method, the MSPs are based on resilience potentials introduced by Erik Hollnagel [13] and presented in detail in Section 3. According to [4], the MSP are usually analyzed following the six main evaluation areas. The characteristic of these areas is presented in Tables 1-4 for each MSP respectively.

**Table 1. Assessment of the Potential to respond – the main evaluation areas**

| Procedure | Result of the procedure |
|-----------|------------------------|
| P11 | Disruptive events (DE) identification |
| List of the DEs |
| P12 | Disruptive events (DE) relevance |
| Verified list of DEs |
| P13 | Respond to DEs planning |
| List of the responds to DEs |
| P14 | Respond to DEs adequacy |
| Verified list of the responds to DEs |
| P15 | Respond parameters defining: |
| • triggering and ending criteria; |
| • respond delay (activating speed); |
| • resources capability |
| Verified list of the responds parameters to DEs |
| P16 | Readiness to respond |
| Verification rules |

**Table 2. Assessment of the Potential to monitor – the main evaluation areas**

| Procedure | Result of the procedure |
|-----------|------------------------|
| P21 | Performance indicators (PI) identification |
| List of the PIs |
| P22 | Performance indicators (PI) relevance |
| Verified list of PIs |
| P23 | Timeliness of PIs determination |
| Time delay for individual PIs |
| P24 | Measurement accuracy of PIs defining |
| Sensitivity for individual PIs |
| P25 | Measurement frequency of PIs defining |
| Rules for taking measurements |
| P26 | Measurement results plausibility |
| Rules for checking results |

The presented tables contain a detailed specification of individual factors that should be considered when assessing MSP. Such a presentation of these factors is useful for performing a preliminary analysis of the investigated organization - at the data collection stage. However, at the stage of performing a detailed quantitative analysis, such an approach would generate a very high degree of model complication. Therefore, the authors propose to group the most critical evaluation factors of individual MSP into two main parameters – potential readi-
ness level and process regency (Fig. 5). These parameters correspond to the defined above main areas of assessment.

All the evaluation factors connected with time-frequency, timeliness or forecasting perspective refer to the regency parameter. The factors that influence organization respond capacity, measurement accuracy, learning process efficiency, or forecasting process effectiveness are attributed to the readiness parameter.

4.2. Quantitative analysis – assessment of Maintenance Support Potentials

During this phase, the main steps are to: collect expert opinions, weights assess, and fuzzy model implementation.

**Step 1. Expert opinions collection:**
First, the experts provide their opinions for the defined two MSP (P) assessment parameters. The experts’ opinions are collected using linguistic scales. The proper definition of linguistic variables is based on expert knowledge and depends on the industry type. However, the general description of the linguistic variables is proposed in Tables 5 – 7.

**Step 2. Assessment of weights for MSP:**
Later, there is a necessity to assess MSP weights based on the experts’ knowledge. Let \( W = \{w_1, w_2, \ldots, w_n\} \) be the vector for MSP weights. Based on the available literature, this vector may be evaluated based on one of the three main approaches implementation.

**Firsts approach.** The parameters weights are expressed precisely by real numbers (crisp data) when satisfying the following assumption:

\[
\sum_{j=1}^{n} w_j = 1
\]  

**Second approach.** A vector of linguistic values may also express the weights’ parameter. In this approach, there is defined the scale of linguistic terms. Thus, usually, there are used expressions to give the evaluation value of the chosen parameter by seven linguistic terms, from “Very big” to “Very small” concerning seven fuzzy scales (see, e.g. [51]). Following this, the larger weight is given to the parameter, the greater importance is given to that parameter of MSP evaluation.

**Third approach.** The last method of weights parameters estimation may be based on AHP method implementation. Due to the uncertainty in implementing the MSP assessment process, the fuzzy AHP method should be used to find fuzzy preference weights [36]. Saaty developed the AHP method in 1980 (according to [28]). Buckley’s fuzzy theory was incorporated into the AHP method in 1985 and presented in work [3]. The procedure for fuzzy AHP implementation into criteria weight evaluation is presented, e.g., in [28, 36]. According to their studies, the procedure bases on the two main steps:

- to construct fuzzy pairwise comparison matrices based on decision-makers opinion,
- to compute the fuzzy weights by normalization.

Selection of the appropriate approach for estimating the weighting factors will depend directly on the managers, their skills/expertise level, and the knowledge of possible evaluation tools. The most straightforward approach is based on the scoring method implementation but will produce a very subjective result depending on the evaluation team’s preferences. The application of the AHP method will allow balancing the results obtained by assigning weights according to the level of importance of each maintenance potential in relation to the others. In turn, the second approach can be used when the assessment of the importance of individual maintenance support potentials is carried out by many experts from different departments of the company. This will unify the assessment in relation to the different levels of experience of the experts.

**Step 3. Fuzzification of risk parameters:**
When the expert opinions are collected and weights assessed, the fuzzy set theory is used to model the MSP parameters and obtain their assessed value. The fuzzy set theory makes the comparison process between the experts’ opinions clearer and easier. A triangular FN is represented by a triplet \((a, b, c)\), and its membership function is given by:

\[
\mu(x) = \begin{cases} 
0 & \text{for } x < a \\
\frac{x - a}{b - a} & \text{for } a \leq x \leq b \\
\frac{c - x}{c - b} & \text{for } b \leq x \leq c \\
0 & \text{for } x > c 
\end{cases}
\]  

\[ (19) \]
The FN parameters meaning is straightforward: $a$ and $c$ are the lower and upper bounds of fuzzy number $A_z$, respectively, and $b$ denotes the modal value of fuzzy number $A_z$.

If there were collected opinions from different experts, there is a necessity to aggregate them to obtain the $P_i$ level. According to [9], the aggregation of experts’ opinion can be performed using the arithmetic mean aggregation operator. The mean aggregation operator, defined on fuzzy triangular numbers $(a_1, b_1, c_1), (a_2, b_2, c_2), \ldots, (a_m, b_m, c_m)$, delivers the result as $(x, y, z)$ according to the formula:

\[
x = \frac{1}{m} \sum_{k=0}^{m} a_k,
\]

\[
y = \frac{1}{m} \sum_{k=0}^{m} b_k
\]

\[
z = \frac{1}{m} \sum_{k=0}^{m} c_k
\]

For the transparency of the presented method, the authors do not consider experts weighting. However, when there is a need to differentiate the obtained opinions depending on an expert’s significance, the authors recommend introducing the experts’ normalized weights. Thus, the aggregated fuzzy number of the $i$th basic opinion may be estimated as [27]:

\[
M_{zl} = \sum_{l=1}^{m} A_{zi}, \text{and } i = 1, 2, \ldots, n_p
\]

where: $M_{zl}$ represents aggregated fuzzy number of the $i$th parameter; $W_l$ is an $l$th experts’ normalized weight; $A_{zi}$ is the fuzzy number of $i$th parameter given by $l$th expert judgment; $n_p$ is the number of parameters; $m$ is the number of experts.

According to the expert’s trait, the examples of weighting scores are presented, e.g., in [27, 44]. Moreover, the survey of known methods for fuzzy opinions aggregation is given in, e.g. [15, 39].

**Step 4. Fuzzy interference system:**
After carrying out experts’ opinions aggregation, the next steps of this phase are connected with \( P_i \) quantification. This process is based on a Mamdani fuzzy model use [24]. The Mamdani fuzzy interference mechanism is based on the compositional rule of inference proposed by Zadeh [50]. The Mamdani fuzzy model’s main components are Fuzzification, Knowledge base, Fuzzy Interference System, and Defuzzification [38]. A scheme of the \( P_i \) assessment process based on the Mamdani fuzzy model is shown in Figure 6.

![Fig. 6. Stages of the use of fuzzy sets according to Mamdani fuzzy model](image)

As it was mentioned, the fuzzification process is based on the use of TFN. A triangular FN converts the linguistic scales in the range of 0-1 using its membership function. The knowledgebase consists of the rule base and membership functions of inputs. The rule base includes a number of IF-THEN rules used to capture the imprecise modes of reasoning [40].

The fuzzy interference system (FIS) is designed to map the fuzzy inputs and rule the outputs using a fuzzy set theory. Due to the Mamdani model use, the FIS has based on MIN and MAX operators implementation. The MIN operator is used for combination and implication operations. The MAX operator is used to aggregate the fuzzy outputs. Finally, the defuzzification process is aimed at the conversion of the fuzzy output into crisp output.

**Step 5. Fuzzified output defuzzification:**
A survey of the most commonly known defuzzification methods is presented, e.g., in [37]. There are many sources of uncertainty in evaluating MSP parameters, so the authors propose using the centroid of area defuzzification method for defuzzification process performance. Thus, the crisp output is estimated as [38]:

\[
\text{Centroid of area, } z^* = \frac{\int \mu_A(z) \cdot zdz}{\int \mu_A(z) dz} \tag{22}
\]

where: \( z^* \) – the crisp value for the \( z \) output (defuzzified output); \( \mu_A(z) \) – the aggregated output membership function; \( z \) – universe of discourse.

This crisp output value is later implemented in the Output phase for \( MSP_o \) level estimation.

### 4.3. Organization’s maintenance support potential level assessment with the reasoning process

The organization’s maintenance support potential level is estimated based on the previous phases’ results in the last phase. According to the obtained \( MSP_o \) level, maintenance-related decisions can be made accordingly.

Based on the obtained level of estimated ratio, the decision-maker should first correctly interpret the obtained values. Table 8 describes the possible \( MSP_o \) levels. The authors propose a 5-grade scale for \( MSP_o \) ratio assessment.

According to the obtained overall ratio level, the decision-maker may take appropriate actions. When the overall ratio level is not acceptable, managers should take the following actions:
- first, the definition of maintenance management policies and procedures as a basis for MSP implementation,
- introduction of disruptive events identification processes and possible responding parameters definition; establishing the possible influence of adverse events occurrence on maintenance processes performed in an organization,
- maintenance measurement processes definition with a selection of possible performance indicators,
- analysis of possible to be implemented in organization forecast models, which provide the most efficient maintenance management process.

This means the manager must seek additional management actions for company maintenance support capability introduction and improvement or increase prevention and preparedness (connected with, e.g., maintenance policy definition) without reducing profits. Success at this task requires a good understanding of organization’s maintenance support system, both broad and tailored to the manager’s own company. Moreover, it constitutes the initial step of MSS creation in an organization.

For organizations where the overall ratio assumes values accepted by managers, decisions concerns maintenance recommendations. When the overall ratio is acceptable, the most common maintenance recommendations are the following maintenance policies defined by a producer. When the obtained level of an overall ratio is higher than the acceptable level, the organization maintenance capability is enough to introduce maintenance strategies that satisfy reliability or risk/safety assumptions. The appropriate recommendations will depend on the type of organization, its physical assets, and industry sector and should be compatible with ISO 5500x standards indications.

Following this, the general structure for MSC in organization development may be compatible with the one presented in Fig. 7. The most crucial improvement ways are indicated in every of the analyzed maintenance support potentials.

![Fig. 7. MSC in organization development – possible directions of company’s related tasks](image)

### 5. Application of the proposed approach in a company from the automotive sector

To illustrate the proposed fuzzy-based decision method’s implementation possibility, the authors analyzed a case company from the automotive sector. The investigated company is located in Poland in the Lower Silesia region and is a global manufacturer of compres-
Table 9. Parameters linguistic scores for all defined MSP based on experts' opinions

| MSP | Process regency level | Readiness level |
|-----|-----------------------|-----------------|
| P1 – Potential to respond | VH | H |
| P2 – Potential to monitor | H | H |
| P3 – Potential to learn | M | H |
| P4 – Potential to anticipate | M | M |

Table 8. Organization’s maintenance support potential levels – description and ratio levels

| Ranking category | Description | MSP, range |
|------------------|-------------|------------|
| EXCELLENT (E)    | Full development and implementation of maintenance support potentials in the organization; a system for collecting and using information (about adverse events as well as processes for responding to their occurrence) following the concept of a learning organization. Integration of a maintenance management system with an enterprise management strategy. Parameters of maintenance support potentials in an organization evaluated regularly. | 93-100 |
| VERY SATISFACTORY (VS) | The parameters of a given maintenance support potential in an organization are fully defined and implemented, the monitoring of the level of maintenance support potentials is based on a defined system of operational indicators, the evaluation process is still carried out on an irregular basis; however, information on potential adverse events is collected in a systematic manner. | 75-92 |
| SATISFACTORY (S) | Maintenance support potential parameters at a satisfactory level - potential implementation procedures are defined and verified, there are no rules and principles for potentials evaluation, a system for measuring maintenance support potentials is still not fully developed; standards for potential time parameters are not defined or just pre-defined. | 54-74 |
| ACCEPTABLE (A) | Maintenance support potential parameters at an acceptable level - procedures for implementation of potential are pre-defined; there are no rules and principles for evaluation of maintenance potentials, but possible undesirable events are preliminarily identified; there are no standards for time parameters of Potential, and a probability of their evaluation is estimated at a deficient level. | 31-53 |
| UNACCEPTABLE (UA) | No activities are carried out to implement and evaluate maintenance support potentials in an organization; no efforts (or very little) are made to identify adverse events and their impact on maintenance processes, no management policies and procedures in the maintenance area. | 0-30 |
In the next step, the proposed fuzzy model needs to be implemented. Following this, the input parameters are to be fuzzified. The obtained linguistic scores given by the experts are converted to corresponding fuzzy set numbers. The Triangular and Trapezoidal FNs used in the presented case study to represent the linguistic scales of input and output parameters are shown in Figure 8.

Next, there is a necessity to determine IF-THEN rules. Based on the experts’ knowledge, there were proposed 25 rules – presented in Table 10 and one additional, which defines the situation when there is no potential identified (rule 26). According to this, for example, rule 1 is defined as:

IF Process regency is Very Low and Readiness level is Very Low, THEN P1 level is Unacceptable.

Rule 26 is defined as:

If Process regency is Impossible and Readiness level is None, THEN P1 level is NO POTENTIAL.

Table 10. MSP level decision matrix

| Process regency | Readiness level |
|-----------------|-----------------|
| VH              | S               | S               | VS              | E               | E               |
| H               | A               | S               | VS              | VS              | E               |
| M               | A               | A               | S               | S               | VS              |
| L               | UA              | A               | A               | S               | S               |
| VL              | UA              | UA              | A               | S               | S               |
| VL              | L               | M               | H               | VH              |

With MATLAB software, there is possible to obtain the final MSP score from the constructed FIS. Figure 9 presents the adopted rules in the used MATLAB software for chosen P1 assessment. To obtain P1’s final score from the constructed FIS, Equation (22) is used for the defuzzification of the fuzzy set resulting from the Mamdani algorithm.

Table 11 presents the obtained organization’s maintenance support potential level for the analyzed company.

Table 11. Evaluated organization’s maintenance support potential level

| Potential to respond | P1 | P1*wi |
|----------------------|----|-------|
| 91.36                | 22.84 |
| Potential to monitor | 75  | 18.75 |
| Potential to learn   | 55  | 13.75 |
| Potential to anticipate | 55  | 13.75 |
| MSPo                 | 69.09 |

The results given in the Table 11 are obtained considering the assumptions that all decision rules have the same weights (the same importance). The discussion of obtained results is presented in the next section.

6. Results and discussion

The proposed case study gives the possibility to analyze how the developed fuzzy-based assessment method may be used to evaluate the maintenance support capability level in an organization. The proposed method allows the possibility to employ linguistically exert knowledge and engineering judgment to make a more realistic evaluation in the maintenance management capability area. The complete results of the proposed FIS for MSP assessment are presented in Figure 10.
allow for precise identification of disruptive events and their parameters/consequences (e.g., duration/removal from the system, costs, and delays). The frequency of PI’s defining and monitoring is also defined.

Despite implementation in the case company of systematic, periodic analysis of current business performance and ways to address risks, the Potential to learn still needs to be improved. This company’s maintenance support potential still needs to be supplemented with, among others, possible resources for the learning process and its time parameters.

Moreover, according to the expert opinions, the case company’s Potential to anticipate is at the medium level. This is mainly due to its focusing on the current state of the operational performance level. There is a lack of solutions focused on predicting future developments on particular potential disruptions, constraints, and changing operating conditions. The leading implemented solutions assess the analyses of the company’s current operational/strategic and tactical level made once a year (in the last month of the fiscal year).

Following this, according to the obtained results, there is still the necessity to define processes that would predict potential future adverse events with a significant impact on the implemented operational processes. There is a lack of solutions that would focus on predicting future development direction in relation to specific potential disruptions, constraints, and changing operating conditions.

Consequently, the main recommendations for the analyzed company regarding its maintenance processes are as follows:

- use the results obtained from risk management in the planning of effective maintenance processes (maintenance strategies) to help achieve the required availability, reliability, and safety levels dictated by the business;
- develop guidelines for the forecasting process, preventive procedures, and maintenance management scenarios for identified disruptive events;
- assess strategy elements aimed at achieving the required availability, reliability, and safety levels dictated by the business (e.g., critical spare management, operational controls, and failure response measures);
- provide transparently and verifiably costing in the area of maintenance management;
- embrace and develop approaches that seek to continually improve efficiency and effectiveness of company’s activities (e.g., connected with learning objectives description, learning process rules definition);
- be compliant with statutory and regulatory imperatives.

7. Conclusions

Maintenance management is one of the most important issues nowadays. The appropriate maintenance decisions can achieve significant financial benefits (reducing maintenance costs) and increasing the company’s operational indicators. However, when defining an organization’s maintenance support capability, the focus should be on eliminating possible hazards and preventing failures, and developing an organization’s potentials for resilient performance. Following this, the authors introduced a new concept on Resilience-Based Maintenance, which is based on implementing the main resilience potentials given by Erik Hollnagel [13]. Its proper use in a company is based on the necessity of its performance investigation in such areas as current state and knowledge about possible hazard events, the possibility of learning from the obtained experience, or the ability to anticipate unwanted events. Moreover, in this study, the authors proposed the organization’s maintenance support potential level assessment methodology, which considers two evaluation parameters – readiness level and process regency. The fuzzy logic structure allowed the experts to capture the experts’ opinions in linguistic terms for the defined two MSP parameters and evaluate the overall ratio level.

The analyzed case study shows the possibilities of using a given method in the decision-making process. Hence, the selected case’s assessment procedure allowed us to verify the complexity of the adopted procedure, the substantive scope of the developed assessment tool and allowed us to determine the intensity of implementation work.

Following the case study, we may state that the analyzed company is well prepared to respond to everyday hazards. The main problem is connected with developing such tools and skills that will give the possibility to predict the future.

At this stage of carried out research analyses, the authors may point out two main possible limitations of the proposed method. First, the method limitation may be connected with the managers’ correctness of performed assessment process. The managers (experts) may give incorrect answers during the internal audit performance to obtain higher ratings than the actual level of achieved maintenance management capability in an organization. The second possible limitation is the possibility of omission of specific steps during the proposed assessment procedure performance. Following this, to obtain reliable results, it is necessary to follow the procedure and appropriately evaluate the actual level of the maintenance support potentials being assessed in the model.

The results presented in the article are preliminary studies that the authors will develop in their future research. Further analysis will focus on the business continuity concept implementation and physical asset management concept use to extend the proposed Resilience-Based Maintenance approach.

To conclude, the proposed methodology is to be used for organization maintenance support capability level assessment and may be performed by maintenance management and safety officers. Moreover, it gives preliminary information that can be useful for the development of maintenance strategies as well as the selection of the most hazardous areas in the audited companies. Therefore, it provides essential information on the need to control disruptive events and implement safety improvements. Moreover, the proposed organization’s maintenance support potential level assessment method may be used in various industry sectors.

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