Supersymmetric Inflation and Large-scale Structure *

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*Work done in collaboration with J.A. Adams & G.G. Ross [1]

Abstract

In effective supergravity theories following from the superstring, a modulus field can quite naturally set the necessary initial conditions for successful cosmological inflation to be driven by a hidden sector scalar field. The leading term in the scalar potential is cubic hence the spectrum of scalar density perturbations necessarily deviates from scale-invariance, while the generation of gravitational waves is negligible. The growth of large-scale structure is then consistent with observational data assuming a critical density cold dark matter universe, with no need for a component of hot dark matter. The model can be tested thorough measurements of cosmic microwave background anisotropy on small angular scales.

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I. INTRODUCTION

It is well known that a sufficiently long period of accelerated, non-adiabatic expansion in the early universe, driven by the false vacuum energy of a scalar field, successfully solves the horizon and flatness problems of the standard Big Bang model, as well as the cosmological monopole problem of grand unified theories \cite{2}. In the ‘new’ inflationary model \cite{3}, a single bubble of the true vacuum expands sufficiently in the vacuum energy dominated De Sitter epoch, so as to contain the entire universe visible today and drive it to the critical density; the vacuum energy is then converted to radiation, ‘reheating’ the universe and starting off the standard Friedmann-Robertson-Walker evolution. Furthermore, the density perturbations generated by quantum fluctuations of the scalar field driving inflation have a (nearly) scale-invariant spectrum, as is required by observations \cite{3}.

However it was observed over a decade ago that in order to respect the observational limit on the perturbation amplitude deduced from the isotropy of the 2.73 K cosmic microwave background (CMB), the scalar potential has to be extremely flat and protected against radiative corrections. It became evident that the only plausible candidates for the ‘inflaton’ are gauge singlet fields in supersymmetric theories \cite{4}, which were recognized already as being the most probable extension of physics beyond the Standard Model $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ Model \cite{5}.

However all such models \cite{4} were found to be plagued with various phenomenological problems, in particular the production during reheating of massive unstable particles such as gravitinos whose late decays can disrupt the standard cosmology \cite{6} as well as the excitation during inflation of weakly coupled scalar fields associated with supersymmetry breaking, which too release their energy rather late generating an unacceptable amount of entropy \cite{7}. (The latter problem is particularly acute for the flat directions (or moduli) of string theories \cite{8,9}.) It was also established that thermal effects in the early universe cannot localize the inflaton field at the origin as is required to ensure a sufficiently long period of inflation \cite{1}. However given random initial conditions, as is appropriate for a weakly coupled field, successful inflation was shown to be possible if the inflaton has its global minimum at the origin and evolves towards it from an initial vacuum expectation value (vev) beyond the Planck scale. Such a ‘chaotic’ inflation model \cite{3} accommodates a wide variety of potentials (albeit with arbitrary fine tuning) hence attention drifted away from the specific problems encountered by supersymmetric inflationary models.

Subsequently, precision accelerator data \cite{10} have confirmed that the most likely solution to the ‘hierarchy’ problem posed by a fundamental Higgs boson in the Standard Model is indeed (broken) supersymmetry just above the electroweak scale. Moreover, this enables successful unification of the strong and electroweak gauge couplings at a scale of $\approx 2 \times 10^{16}$ GeV as well as providing an elegant mechanism for electroweak symmetry breaking and an understanding of the pattern of fermion masses \cite{11}. The superpartners of the known particles should have masses no higher than a few TeV so can be directly created at the forthcoming LHC or possibly even at LEP 2. The lightest supersymmetric particle is typically the neutralino, a neutral weakly interacting mixture of the superpartners of the gauge and Higgs bosons. It naturally has a relic abundance of order the critical density \cite{12} and is therefore an excellent candidate for the cold dark matter (CDM) \cite{13} which is required in all viable models of large-scale structure formation \cite{14}. This provides strong motivation to reexamine the problems connected with inflation in supersymmetric theories, specifically $N = 1$ supergravity, the phenomenologically successful effective field theory below the Planck scale \cite{1}. We focus on models where supersymmetry breaking occurs in a ‘hidden’ sector and is communicated to the visible sector through gravitational interactions.
Meanwhile on the cosmological front, the discovery by COBE of temperature fluctuations in the CMB on angular scales larger than the causal horizon at (re)combination has provided strong indirect support for inflation. The power spectrum of the fluctuations is consistent with a scale-invariant primordial perturbation, and the statistics with a random Gaussian field, both as predicted by inflation [15]. Thus the spectrum of scalar density perturbations can be normalized directly to COBE (taking into account any gravitational wave component which would also contribute to the CMB anisotropy). The primordial spectrum is modified on scales smaller than the horizon size at matter-radiation equality by a ‘transfer function’ characteristic of the matter content of the universe [14]. The power spectrum inferred from observations of the clustering and motions of galaxies [16] can then be compared with the theory. Of particular interest is whether the problem of excessive small-scale power in a CDM universe (assuming scale-invariant primordial fluctuations) [17] can be resolved by the spectral ‘tilt’ expected from supersymmetric inflation [9,18], rather than by invoking a component of hot dark matter. Ongoing and future observations of the CMB anisotropy on small angular scales [19] will provide an independent test of this possibility.

II. REQUIREMENTS OF THE INFLATIONARY POTENTIAL

The main theoretical problem in constructing an inflationary model based on supergravity is that the large cosmological constant during inflation breaks global supersymmetry, giving all scalar fields a ‘soft’ mass of order the Hubble parameter [20]. In the simplest models, the inflaton potential thus acquires a curvature too large to allow inflation to proceed for the required number of e-folds. The problem is characteristic of the scalar potential (along a F-flat direction) of a singlet field in the hidden sector having minimal kinetic terms, hence various solutions have been proposed which modify one or the other of these assumptions, e.g. introduction of non-minimal kinetic terms, or specific interactions of the inflaton with gauge fields, or identification of the inflaton as a D-(rather than F-) flat direction or even as a modulus field [21]. Here, I would like to discuss a new mechanism leading to successful inflation in the low-energy effective supergravity theory following from the superstring [1]. The interesting observation is that in a wide class of such theories, the equations of motion have an infra-red fixed point at which successful inflation can occur, even for minimal kinetic terms, along a F-flat direction.

First, let us briefly review the necessary ingredients for successful inflation with a scalar potential \( V(\phi) \). Essentially all model generating an exponential increase of the cosmological scale-factor \( a \) satisfy the ‘slow-roll’ conditions [22]

\[
\frac{\dot{\phi}}{V'} \approx -\frac{V'}{3H}, \quad \epsilon \equiv \frac{M^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \quad |\eta| \equiv \left| \frac{M^2 V''}{V} \right| \ll 1, \quad (1)
\]

where \( H \approx \sqrt{V/3M^2} \) is the Hubble parameter during inflation, and the normalized Planck mass \( M \equiv M_{Pl}/\sqrt{8\pi} \approx 2.44 \times 10^{18} \) GeV. Inflation ends (i.e. \( \dot{a} \) drops through zero) when \( \epsilon, |\eta| \approx 1 \). The spectrum of adiabatic scalar perturbations is [22]

\[
\delta^2_H(k) = \frac{1}{150\pi^2 M^4} \frac{V_*}{\epsilon_*}, \quad (2)
\]

where \( * \) denotes the epoch at which a scale of wavenumber \( k \) crosses the ‘horizon’ \( H^{-1} \) (more correctly, Hubble radius) during inflation, i.e. when \( aH = k \). The CMB anisotropy measured by
COBE [13] allows a determination of the fluctuation amplitude at the scale, $k_{\text{COBE}}^{-1} \sim H_0^{-1} \sim 3000 \, h^{-1} \, \text{Mpc}$, corresponding roughly to the size of the presently observable universe, where $h \equiv H_0/100 \, \text{km sec}^{-1} \, \text{Mpc}^{-1}$ is the present Hubble parameter. The number of e-folds before the end of inflation when this scale crosses the Hubble radius is

$$N_{\text{COBE}} \equiv N_*(k_{\text{COBE}}) \simeq 51 + \ln \left( \frac{k_{\text{COBE}}^{-1}}{3000 \, h^{-1} \, \text{Mpc}} \right) + \ln \left( \frac{V_\star}{3 \times 10^{14} \, \text{GeV}} \right) + \ln \left( \frac{V_\star}{V_{\text{end}}} \right) - \frac{1}{3} \ln \left( \frac{V_{\text{end}}}{3 \times 10^{14} \, \text{GeV}} \right) + \frac{1}{3} \ln \left( \frac{T_{\text{reheat}}}{10^5 \, \text{GeV}} \right),$$

(3)

where we have indicated the numerical values anticipated for the various energy scales in our model. (Note that $N_{\text{COBE}}$ is smaller than the usually quoted [22] value of 62 because the reheat temperature must be low enough to suppress the production of unstable gravitinos which can disrupt primordial nucleosynthesis [6].) The COBE observations sample CMB multipoles up to $k^{-1} \sim 20$, where the $l$th multipole probes scales around $k^{-1} \sim 6000h^{-1}\text{Mpc}$. The low multipoles, in particular the quadrupole, are entirely due to the Sachs-Wolfe effect on super-horizon scales ($k^{-1} > k_{\text{dec}}^{-1} \sim 180h^{-1}\text{Mpc}$) at CMB decoupling and thus a direct measure of the primordial perturbations. However the high multipoles are (increasingly) sensitive to the composition of the dark matter which determines how the primordial spectrum is modified through the growth of the perturbations on scales smaller than the horizon at the epoch of matter-radiation equality, i.e. for $k^{-1} < k_{\text{eq}}^{-1} \sim 80h^{-1}\text{Mpc}$. Thus the normalization of the spectrum [2] to the COBE data is sensitive to its $k$ dependence and also on whether there is a contribution from gravitational waves to the CMB anisotropy. The 4-year data is fitted by a scale-free spectrum, $\delta_H^2 \sim k^{n-1}$, $n = 1.2 \pm 0.3$, with $Q_{\text{rms}} = 15.3^{+3.8}_{-2.8} \mu\text{K}$ [15]. For a scale-invariant ($n = 1$) spectrum, $Q_{\text{rms}} = 18 \pm 1.6 \mu\text{K}$, so assuming that there are no gravitational waves, the amplitude for a $\Omega = 1$ CDM universe is $\delta_H = (1.94 \pm 0.14) \times 10^{-5}$ [23]. Using eq. (2), the vacuum energy at this epoch is then given by

$$V_{\text{COBE}} \simeq (6.7 \times 10^{16} \, \text{GeV})^4 \epsilon_{\text{COBE}},$$

(4)

showing that the inflationary scale is far below the Planck scale [22]. A similar limit obtains, viz. $V_{\text{COBE}} \lesssim (4.9 \times 10^{16} \, \text{GeV})^4$, if the observed anisotropy is instead ascribed entirely to gravitational waves, the amplitude of which, in ratio to the scalar perturbations, is just [22]

$$r = 12.4 \, \epsilon.$$

(5)

Thus it is legitimate to study inflation in the context of an effective field theory. The potential then has the generic form

$$V \sim \Lambda^4 \left[ 1 + c_n (\phi/M)^n \right].$$

(6)

In the usual model of chaotic inflation, one has $\phi/M \gg 1$ so the first term on the rhs is negligible and $\epsilon$ and $\eta$ are small because they are proportional to $(\phi/M)^{-2}$. Alternatively $\phi$ may start with a vev much smaller than the Planck scale during inflation, in which case the smallness of $V'$ and $V''$, and hence $\epsilon$ and $\eta$, results from the relative smallness of the second term on the rhs.

To take into account both cases, let us expand the (slowly varying) potential about the value $\phi^*$ in inflaton field space at which the observed density fluctuations are produced. Writing $\phi = \bar{\phi} + \phi^*$ (in units of $M$) we have

$$V(\phi) = \Lambda^4 \left[ 1 + c_1 \bar{\phi} + c_2 \bar{\phi}^2 + c_3 \bar{\phi}^3 + c_4 \bar{\phi}^4 + \ldots \right].$$

(7)
Here we have factored out the overall scale of inflation $\Lambda$, which we have seen must be small relative to the Planck scale $M$. The constraints on the parameters in the potential following from the slow-roll conditions (4) are therefore

$$c_1 \ll 1, \quad c_2 \ll 1, \quad c_3 \tilde{\phi} \ll 1, \quad c_4 \tilde{\phi}^2 \ll 1 \ldots$$

The first test for an inflationary model is whether these conditions are naturally satisfied. Many complicated models have been proposed which purport to do so [21], although this is not always evident on closer examination. We consider here the simplest possibility employing a single inflaton field in minimal supergravity.

### III. NATURAL SUPERGRAVITY INFLATION

In supersymmetric theories with a single supersymmetry generator ($N = 1$), complex scalar fields are the lowest components, $\phi^a$, of chiral superfields $\Phi^a$ which contain chiral fermions, $\psi^a$, as their other component. (We will take $\Phi^a$ to be left-handed chiral superfields so that $\psi^a$ are left-handed fermions.) Masses for fields will be generated by spontaneous symmetry breaking so that the only fundamental mass scale is the Planck scale, $M$. (This is aesthetically attractive and also what follows if the underlying theory generating the effective low-energy supergravity theory follows from the superstring.) The $N = 1$ supergravity theory describing the interaction of the chiral superfields is specified by the Kähler potential

$$G(\Phi, \Phi^\dagger) = d(\Phi, \Phi^\dagger) + \ln |f(\Phi)|^2,$$

which yields the scalar potential

$$V = e^{d/M^2} \left[ F_A^A (d_B^B)^{-1} F_B - 3 |f|^2 / M^2 \right] + D - \text{terms},$$

where

$$F_A^A \equiv \frac{\partial f}{\partial \Phi^A} + \left( \frac{\partial d}{\partial \Phi^A} \right) \frac{f}{M^2}, \quad (d_B^B)^{-1} \equiv \left( \frac{\partial^2 d}{\partial \Phi^A \partial \Phi^B} \right)^{-1}.$$

Here the function $d$ sets the form of the kinetic energy terms of the theory

$$L_{\text{kin}} = \frac{\partial^2 d}{\partial \phi^A \partial \phi^B} \partial_\mu \phi^A \partial^\mu \phi^B,$$

while the superpotential $f$ determines the non-gauge interactions of the theory. For canonical kinetic energy terms, $d = \sum_A \phi_A^\dagger \phi^A$, the potential takes the relatively simple form

$$V = \exp \left( \sum_A \phi_A^\dagger \phi^A \left[ \sum_B |\frac{\partial f}{\partial \phi^B}|^2 - 3 |f|^2 \right] \right).$$

In order for there to be a period of inflation, it is necessary for at least one of the terms $|\partial f / \partial \phi^B|$ to be non-zero. However, these are precisely the order parameters for supersymmetry so this corresponds to supersymmetry breaking during inflation. While there are several possible mechanisms for such breaking, it suffices for the purposes of this discussion to simply assume that one of the terms has
nonvanishing value $\Lambda^4$, where $\Lambda$ denotes the supersymmetry breaking scale. Now expansion of the exponential in eq. (13) shows that $c_2 = 1$ and $c_4 = 1$ in eq. (13), in conflict with the requirement (8) for successful inflation. We see that the problem can be traced to the presence of the overall factor involving the exponential in the scalar potential (13).

In ref. [9] we suggested that in theories with moduli the problem is easily avoided. Moduli are fields in superstring theories which, in the absence of supersymmetry breaking, have no potential. The moduli vevs serve to determine the fundamental couplings of the theory and for the moduli of interest here they appear in the superpotential only in combination with non-moduli fields, serving to determine the latter’s couplings in terms of their vevs. We argued [9] that the quadratic terms in the potential involving the non-moduli fields such as the inflaton would be absent for special values of these vevs and, since the resultant potential would drive inflation, just this desired configuration would come to dominate the final state of the universe. Subsequently we realized [1] that it is not even necessary to invoke such an ‘anthropic principle’ because there is a quasi-fixed point in the evolution of the moduli. As we discuss below, this ensures, for initial values in the basin of attraction of the fixed point, that the cancellation of the quadratic terms applies, ensuring that condition (8) is satisfied. Now although the moduli have a flat potential in the absence of supersymmetry breaking, once supersymmetry is broken they may acquire a potential through the moduli dependence of the $d$ function in the scalar potential (14). This is potentially disastrous because such a potential would drive the moduli vevs away from the value needed to cancel the quadratic inflaton term. However the kinetic term often has a larger symmetry than the full Lagrangian; for example the canonical form has an $SU(N)$ symmetry where $N$ is the total number of chiral fields. In this case there will be many moduli left massless even when supersymmetry is broken because they will be (pseudo) Goldstone modes associated with the spontaneous breaking of this symmetry. These moduli can play the role discussed above eliminating the quadratic term in the inflaton potential [1].

The mechanism proposed in ref. [1] applies to a large class of models, the only condition being that the kinetic term does indeed have a symmetry leading to pseudo-Goldstone modes. We have given [1] two specific examples to illustrate the idea in detail, one for the case where the kinetic term has a larger symmetry than the full Lagrangian, and another where the potential discussed above follows from a symmetry of the full theory. In both cases the field potential is of the general form

$$V(|\tilde{\phi}|, \varphi) = \Lambda^4 \left( 1 + \beta |\tilde{\phi}|^2 \varphi + \gamma |\tilde{\phi}|^3 + \delta |\tilde{\phi}|^4 + \ldots \right)$$

(14)

where further terms have been added in the expansion of $V$. The cubic term may arise from a cubic term in the superpotential [1]; this is allowed if the additional ($U(1)$) symmetry of the $\phi$ field in an $R$-symmetry. (Alternatively there may be another modulus with $U(1)$ charge such that a cubic term can appear in the kinetic function $d$.) If the cubic term is not present, then the quartic term, which is always allowed in the kinetic term by the $SU(2)$ and $U(1)$ symmetries of the model, will dominate. Note that the parameters $\beta$, $\gamma$ and $\delta$ are all naturally of order unity.

For successful ‘new’ inflation, we are interested in initial conditions which lead to $|\tilde{\phi}|$ being small but there is nothing which constrains the initial conditions of $\varphi$. However since the potential (14) has an infrared fixed point with $\tilde{\phi} = \varphi = 0$, any initial value of $\tilde{\phi}$ and $\varphi$ will be driven there if they are within the domain of attraction, given (for positive $\beta$) by

$$\varphi \geq \frac{3|\gamma|}{2\beta} \left[ 1 + \sqrt{1 + \frac{4}{9} \left( \frac{\beta}{|\gamma|} \right)^2} \right] |\tilde{\phi}| .$$

(15)
Thus, without any fine tuning of the initial conditions (beyond the condition that the fields lie in this domain of attraction), the fields are driven to fixed values and the potential becomes a constant, driving a period of inflation. (We have chosen $\beta$ to be positive, while $\gamma$ should be negative if it is to lead to an inflationary potential.) Moreover this fixed point corresponds to a point of inflection in the potential which is unstable with respect to small perturbations. Thus inflation is naturally terminated by a new mechanism as follows. The equations of motion for $\phi$ and $|\tilde{\phi}|$ are

$$\ddot{\phi} + 3H\dot{\phi} = -\beta |\tilde{\phi}|^2, \quad |\ddot{\tilde{\phi}}| + 3H|\dot{\tilde{\phi}}| = -\beta \phi |\tilde{\phi}| + 3|\gamma||\tilde{\phi}|^2,$$

so while $\phi$ is positive, the fields are driven to the fixed point and inflation begins. However $\phi$ has fluctuations of order the Hawking temperature of the De Sitter vacuum, $T_H = H/2\pi$, so should it fluctuate and become negative, the fields will be driven away from the fixed point thus ending inflation. (For $\beta$ negative, the reverse would be the case.) The initial conditions for this stage are $\phi, |\tilde{\phi}| \sim H$; thereafter, as we see from eq. (16), $|\tilde{\phi}|$ will grow more rapidly than $\phi$ and the cubic term in the potential will soon dominate.

There are two distinctive features of the potential (14) which ensure that, after the transition to positive $\phi$, there will be an inflationary period yielding density fluctuations of the magnitude observed. The first is that this potential has a very small gradient in the neighbourhood of the origin in field space so it generates a long period of slow-roll inflation during which quantum fluctuations are naturally small. The second feature is that the full potential, including higher order terms, is governed by an overall scale, $\Lambda$. The reason is that the potential arises from the $d$ term of eq. (9) which, in the absence of supersymmetry breaking, gives rise to the kinetic term and thus does not contribute to the potential, vanishing when derivatives are set to zero. Thus the potential is proportional to the (fourth power of the) overall supersymmetry breaking scale, $\Lambda$. This scale is plausibly of $\mathcal{O}(10^{14})$ GeV during inflation and, in conjunction with the small slope, correctly yields the required magnitude of density fluctuations.

**IV. IMPLICATIONS FOR LARGE-SCALE STRUCTURE AND CMB ANISOTROPY**

The inflationary period following from a potential of the form (14) with no quadratic term (and $\gamma = -4$) has been closely studied earlier. The field value when perturbations of a given scale cross the Hubble radius is obtained by integrating the equation of motion (16) back from the end of inflation, which occurs at $\tilde{\phi}_{\text{end}} \simeq M/6|\gamma|$ when $\epsilon = 1$. Thus $\tilde{\phi}_\star \simeq M/3|\gamma|[N_\star(k) + 2]$ and using eq.(3) we find a logarithmic (squared) deviation from scale invariance for the scalar perturbations,

$$\delta_H^2(k) = \frac{9\gamma^2}{75\pi^2} \frac{\Lambda^4}{M_4^4}[N_\star(k) + 2]^4.$$  

This corresponds to a ‘tilted’ spectrum, $\delta_H^2(k) \propto k^{n-1}$, with

$$n(k) = 1 + 2\eta - 6\epsilon \simeq \frac{N_\star(k) - 2}{N_\star(k) + 2},$$

i.e. $n \simeq 0.92$ for $N_\star = 51$ corresponding to the scales probed by COBE. We emphasize that a leading cubic term in the potential gives the maximal departure from scale-invariance. The slope of the potential is tiny, $\epsilon = 1/18\gamma^2(N_\star + 2)^4 \simeq 7.0 \times 10^{-9}\gamma^{-2}$, but its curvature is not:
\( \eta = -2/(N_* + 2) \simeq -0.038 \). Consequently, although the spectrum is tilted, the gravitational wave background \([8]\) is negligible. Furthermore the tilt would be greater if \( N_* \) is smaller, for example if there is a second epoch of ‘thermal inflation’ when the scale-factor inflates by \( \sim 20 \) e-folds \([24]\) so that the value of \( N_* \) appropriate to COBE is 31 rather than 51, and \( n \simeq 0.88 \). We normalize the spectrum \([17]\) to the CMB anisotropy using the expression for the (ensemble-averaged) quadrupole,

\[
\frac{\langle Q_{\text{rms}}^2 \rangle}{T_0^2} = \frac{5C_2}{4\pi} = \frac{5}{4} \int_0^\infty \frac{dk}{k} \, j_2^2 \left( \frac{2k}{H_0} \right) \delta_H^2(k), \tag{19}
\]

where \( j_2 \) is the second-order spherical Bessel function. According to the COBE data \([13,23]\), \( Q_{\text{rms}} \simeq 20 \pm 2 \mu K \) for \( n \simeq 0.9 \) which fixes the inflationary scale to be

\[
\frac{\Lambda}{M} \simeq 2.8 \pm 0.14 \times 10^{-4} \, |\gamma|^{-1/2}, \tag{20}
\]

consistent with general theoretical considerations of supersymmetry breaking \([1]\).

The spectrum of the (dimensionless) rms mass fluctuations after matter domination (per unit logarithmic interval of \( k \)) is given by \([14]\)

\[
\Delta^2(k) \equiv \frac{k^3 P(k)}{2\pi^2} = \delta_H^2(k) \left( \frac{k}{aH} \right)^4, \tag{21}
\]

where \( P(k) \) is the usual power spectrum and the ‘transfer function’ \( T(k) \) takes into account that linear perturbations grow at different rates depending on the relation between their wavelengths, the Jeans length and the Hubble radius. For CDM we use \([14]\),

\[
T(k) = \left[ 1 + \left\{ ak + (bk)^{3/2} + (ck)^{2} \right\}^\nu \right]^{-1/\nu}, \tag{22}
\]

with \( a = 6.4\Gamma^{-1}h^{-1}\text{Mpc}, b = 3\Gamma^{-1}h^{-1}\text{Mpc}, c = 1.7\Gamma^{-1}h^{-1}\text{Mpc} \) and \( \nu = 1.13 \), where the ‘shape parameter’ is \( \Gamma \simeq \Omega h e^{-20N} \[25\].

The cosmological parameters adopted for ‘standard’ CDM are \( h = 0.5 \) and \( \Omega_N = 0.05 \[14\]. However, observational uncertainties still permit the Hubble parameter to be as low as 0.4 \[26\]. Also the nucleon density parameter \( \Omega_N \) may be as high as 0.033\( h^{-2} \), taking into account the recent upward revision of the \( ^4\text{He} \) mass fraction \[27\]. We show \( P(k) \) for \( \Omega_N = 0.05, 0.1 \) and \( h = 0.4, 0.5 \) in figure \[6\], having taken account of non-linear gravitational effects at small scales using the prescriptions of ref. \[25\] (PD) and ref. \[28\] (BG). The tilt in the primordial spectrum which increases logarithmically with decreasing spatial scales allows a good fit on scales of \( \sim 1 – 100 \text{Mpc} \) to the data points obtained \[29\] from the angular correlation function of APM galaxies, if the Hubble parameter (nucleon density) are taken to be at the lower (upper) end of the allowed range. (Only 1\( \sigma \) statistical errors are shown; at \( k \lesssim 0.05h\text{Mpc}^{-1} \), there are also large systematic errors \[29\] so the apparent discrepancy here needs further investigation.) Note that the expected characteristic “shoulder” at small scales due to the non-linear evolution is clearly visible in the APM data. Other studies of tilted spectra \[8,11\] focussed on the linear evolution and/or used a compendium \[25\] of data from different surveys (having different systematic biases) rather than one set of high quality data. We conclude that the problem with the excess power on small scales in the COBE-normalized standard CDM model \[17\] is naturally alleviated in supergravity inflation as anticipated earlier \[8,18\], with no need for a component of hot dark matter.
FIGURES

(a) SUSY CDM (n = 0.92), \( \Omega_N = 0.05, \ h = 0.4 \)
(b) SUSY CDM (n = 0.88), \( \Omega_N = 0.1, \ h = 0.5 \)

FIG. 1. Predicted non-linear power spectra of density fluctuations in cold dark matter, normalized to COBE and compared with data inferred from the APM galaxy survey.

FIG. 2. Predicted variance of the density field smoothed over a sphere of radius \( 8h^{-1}\) Mpc, compared with observational limits (horizontal planes) inferred from rich clusters of galaxies.

FIG. 3. Predicted angular power spectra of CMB anisotropy, normalized to COBE and compared with data from current ground-based and balloon experiments.
We also calculate some averaged quantities of observational interest. A common measure of large-scale clustering is the variance, \( \sigma(R) \), of the density field smoothed over a sphere of radius \( R \), usually taken to be \( 8 h^{-1} \) Mpc, given in terms of the matter density spectrum by

\[
\sigma^2(R) = \frac{1}{H_0^2} \int_0^\infty W^2(k R) \delta_H^2(k) T^2(k) k^3 \, dk,
\]

where a ‘top hat’ smoothing function, \( W(k R) = 3 \left[ \sin(k R)/(k R)^3 - \cos(k R)/(k R)^2 \right] \), has been used. As seen from figure 2, the observational value of \( \sigma(8 h^{-1} \) Mpc) = 0.60\,^{+0.19}_{-0.15} \) (95% c.l.), inferred from the abundances of rich clusters of galaxies \([17,32]\) favours high tilt, high \( \Omega_N \) and low \( h \).

Another interesting quantity is the smoothed peculiar velocity field or ‘bulk flow’,

\[
\sigma_v^2(R) = \frac{1}{H_0^2} \int_0^\infty W^2(k R) e^{-(12 h^{-1} k)^2} \delta_H^2(k) T^2(k) k \, dk,
\]

where, for direct comparison with observations, we have applied an additional gaussian smoothing on 12\( h^{-1} \)Mpc. For the two models shown in figure 1 we find,

\[
\sigma_v(40 h^{-1} \)Mpc) = 383\pm38 \text{ km sec}^{-1} \ (N_{\text{COBE}} = 51, \ \Omega_N = 0.05, \ h = 0.4),
= 320\pm32 \text{ km sec}^{-1} \ (N_{\text{COBE}} = 31, \ \Omega_N = 0.1, \ h = 0.5).
\]

(25)

\[
\text{to be compared with the POTENT III measurement of } \sigma_v(40 h^{-1} \)Mpc) = 373 \pm 50 \text{ km sec}^{-1} \ [33].
\]

We do not consider constraints coming from the abundances of collapsed objects at high redshift such as Lyman-\( \alpha \) clouds and quasars \([31,35]\), as this involves many astrophysical uncertainties at present.

An unambiguous test of the model is the predicted CMB anisotropy. To compute this accurately requires numerical solution of the coupled linearized Boltzmann, Einstein and fluid equations for the perturbation in the photon phase space distribution. We use the COSMICS computer codes \([34]\) to calculate the angular power spectrum using the primordial scalar fluctuation spectrum \([17]\). The first 1000 multipoles are plotted in figure 3, taking \( \Omega_N = 0.05, 0.1 \), along with a compendium of recent observations \([19]\), and the prediction of standard CDM is shown for comparison. The height of the first ‘Doppler peak’ is preferentially boosted for the higher value of \( \Omega_N \) and this is favoured by the CMB observations in conjunction with the large-scale structure data, as has been noted independently \([35]\). For a given value of \( \Omega_N \) the effect of the spectral tilt is to suppress the heights of all the peaks. Although present ground-based observations are inconclusive, this prediction will be definitively tested by the forthcoming satellite-borne experiments, MAP and COBRAS/SAMBA.

In summary, inflationary model building has received a new impetus as a consequence of the impressive progress in observations of large-scale structure and CMB anisotropy which can discriminate between such models. It appears quite plausible that within the next decade such astronomical data will provide a direct window to physics at the unification scale.

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