Phase diagram and validity of one-parameter scaling near the two-dimensional metal-insulator transition

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We explore the scaling description for a two-dimensional metal-insulator transition (MIT) of electrons in silicon. Near the MIT, $\partial T/p = (-1/p)d(ln g)/d(ln T)$ is universal (with $p$, a sample dependent exponent, determined separately; $g$--conductance, $T$--temperature). We obtain the characteristic temperatures $T_0$ and $T_1$ demarking respectively the quantum critical region and the regime of validity of single parameter scaling in the metallic phase, and show that $T_1$ vanishes as the transition is approached. For $T < T_1$, the scaling of the data requires a second parameter. Moreover, all of the data can be described with two-parameter scaling at all densities – even far from the transition.

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Many recent experiments have provided strong evidence for a two-dimensional (2D) disordered metallic state in Si metal-oxide-semiconductor field-effect transistors (MOSFETs) and in other 2D systems [1, 2], but the microscopic nature of the metallic phase still awaits a solid understanding. Since there have been predictions for 2D metallic behavior for quite some time [3], it should not be too surprising to see a metal-insulator transition (MIT), but on the other hand it appears to violate rather general and widely accepted assumptions [4]. It is apparent from the experimental data [2, 3, 4, 5] that spin interactions are an important component of the metallic behavior [6, 7, 8, 9], and it is suspected that the dynamical Coulomb interactions are important [10, 11, 12] since the non-interacting (screened) carriers are known to be insulating in 2D [13].

The physics of the MIT can be reduced to certain scaling forms that describe the behavior of the conductivity $\sigma$ as a function of temperature $T$, sample length $L$, and carrier density $n$. A simple single-parameter scaling has proved to be a successful description of the critical behavior of $\sigma$ in most 2D systems. It fails, however, in the metallic phase at the lowest temperatures, where $\sigma(T)$ becomes very weak and, in some systems, it even appears to saturate as $T \to 0$. From the careful analysis of the experiments discussed below, we find that two-parameter scaling is more appropriate in that regime. In fact, we show that all of the data on both insulating and metallic sides of the transition can be described with two-parameter scaling at all densities – even far from the transition. Such scaling has been proposed theoretically near the MIT in the presence of dangerously irrelevant variables [3, 13], and it also provides an excellent description of the MIT in a different type of the 2D metal [14]. This form of the two-parameter scaling function is consistent with the apparent success of the conventional single-parameter scaling in the limited range of $(n, T)$ phase space.

One signature of the 2D MIT is a reversal of the sign of $d\sigma/dT$. (\(\sigma\) is written throughout dimensionlessly – so all numerical values are multiplied implicitly by $e^2/h$ to recover SI units). Measurements of the conductance $G = (w/L)\sigma$ ($w$ is device width) as a function of the carrier density $n$ at several values of $T$ reveal a crossing point at a critical density $n_c$. Near enough to the MIT, a one-parameter scaling scheme holds,

$$\sigma(n, T) \sim f(\delta_n) = f(\delta_n/\sigma_p),$$

where $\delta_n = (n - n_c)/n_c$, $\delta_n^* = \delta_n/\sigma_p$, and $p = 1/2\nu$ is the scaling exponent describing the critical behavior of correlations scales of temporal and spatial fluctuations. $f(\delta_n/\sigma_p) \sim \exp(\delta_n/\sigma_p)$ for $T > T_0 \sim |\delta_n^*|^{1/p}$ in the quantum critical region [13]. Even for $T < T_0$, the data still can be scaled according to Eq. (1) to form a single curve. The single-parameter scaling fails at $T = T_1 < T_0$, and we show that the scale $T_1 = T_1(\delta_n)$ vanishes as the transition is approached.

We examine the phase diagram of $\sigma$ near the 2D MIT in two samples with peak mobility $\mu_{\text{el}} \approx 0.5 \text{ m}^2/\text{Vs}$ at $T = 4.2$ K. To obtain $T < 4.2$ K, we used a $He^4$ dilution refrigerator with heavily shielded wiring. Small AC signals were measured with lock-in amplifiers. The samples were generic two-probe Si MOSFETs with a 50 nm thick oxide layer; sample 17 was $L \times w = 5 \times 11.5 \mu m$ and sample 27 was $254 \times 254 \mu m$. Both devices have a density of oxide interface states $N_{\text{Ox}} \leq 10^{14}/\text{m}^2$, which is measured with standard techniques [17].

Figure 1(a) contains $\sigma(\delta_n, T)$ from a 2D inversion layer (sample 17) below 4 K, and the conventional one-parameter scaling. Results appear in Fig. 1(b). The individual curves of $\sigma(\delta_n)$ at fixed $T$ all cross at a critical density $n_c$ as expected. As shown in Fig. 1(b) the raw data for $\sigma(\delta_n^*)$ collapse on to a single curve [$\text{master curve} \sigma(\delta_n^*) \sim f(\delta_n^*)$] for a given 2D system if a certain exponent $p$ is chosen in $\delta_n^* = \delta_n/\sigma_p$. This collapse fits the suggestion of a quantum critical point at $T = 0$ and $\delta_n = 0$. Critical conductivities in other systems fall in the range $0.3 < \sigma_c < 5$, and so do the present results. Sample...
FIG. 1. (a) $\sigma(\delta_n, T)$ for 0.4 K < $T$ < 4 K from sample 17 with a schematic phase diagram inset. Arrows in the main figure indicate the direction of flow for decreasing temperature. The phase diagram comprises two curves for $T_0 \propto |\delta_n|^{1/p}$ (solid line) and $T_1 \propto |\delta_n|^{1/q}$ (dotted line) that separate the $\delta_n - T$ phase space into three regions. The quantum critical regime (QCR) is indicated schematically by cross-hatching. For $T > T_1$ a single parameter can be used to scale the conductivity and for $T < T_1$ scaling with one parameter fails. (b) Scaling of data in $\ln \sigma$ vs $\delta_n/T^p$ for both samples (upper curve is sample 27 and lower curve is sample 17). The solid line emphasizes the linearity of $\ln \sigma$ in $\delta_n$ near the MIT. (c) In the regime of validity of one-parameter scaling ($T > T_1$), the universal behavior of $z\nu \beta_T \sim \nu \beta(g)$ is shown for six Si MOSFETs that span a wide range of parameters.

27 exhibits the same features as sample 17 with slightly different parameters: $\sigma_c$ of the two samples (0.32 for sample 17 and 0.50 for sample 27) differ from each other by 50% even though the values of $n_c$ (1.7 $\times$ 10$^{15}$/m$^2$ and 1.9 $\times$ 10$^{15}$/m$^2$, respectively) and the exponents $p$ (1/2.0 and 1/2.2, respectively) are within 10%. As predicted [13], the scaled data $\ln \sigma(\delta_n^*)$ in Fig. 1(b) are approximately linear near enough to the transition, for $T > T_0$ (emphasized by the solid line). This is tantamount to the symmetry between $\sigma(\delta_n)$ and $1/\sigma(-\delta_n)$, which has been predicted [13] and observed [13]. Beyond the linear range (marked in the figure as $T_0$), the data continue to scale on both the insulating and metallic sides. At some point, however, the scaling fails on the metallic side and the individual traces of $\sigma(\delta_n^*)$ at fixed $T$ diverge from $\sigma_1(\delta_n^*)$. These points in $\delta_n^*$ can be read as positions in the phase diagram for the MIT. A phase diagram like that proposed for quantum phase transitions [13] is inset in Fig. 1(a). The critical point is at $T = 0$ and $\delta_n = 0$ and crossover lines separate the quantum critical region from the insulating and metallic regions. We have added a second (dotted) curve $T_1(\delta_n)$ demarking the region ($T < T_1$) where the conventional one-parameter scaling breaks down. $T_1$ was obtained from individual traces of $\sigma(\delta_n^*)$ by locating their departure from $\sigma_1(\delta_n^*)$ as shown in Fig. 1(a). It is obvious that the difference between one-parameter scaling and the experiment can reach as much as $|\sigma(\delta_n^*) - \sigma_1(\delta_n^*)|/\sigma_c = 35\%$. The $T_1$ points are plotted as a function of $\delta_n$ in Fig. 1(b), which shows clearly that the higher temperature data can be scaled over a wider range of $\delta_n$. From Fig. 1(b), the characteristic temperature $T_1$ appears to have a power-law dependence on $\delta_n$, as might be expected near a critical point. The power-law dependence of $T_1$ obtains in both samples and the exponents are very similar: 0.54 ± 0.01 for sample 17 and 0.49 ± 0.03 for sample 27.

For $T > T_1$, one-parameter scaling holds. We show that, even though different parameters $p$, $n_c$, and $\sigma_c$ are required to obtain the master curves, there is a universal feature to the ultimate scaling function of the conductance as a function of length. In particular, we find that the function $\beta_T/p = (1/p)[-d(\ln g)/d(\ln T)]$ is universal. The results of this transformation are shown in Fig. 1(c) for the two samples studied here and compared with four other samples studied in the past [2,20]. Clearly the curve $\beta_T/p$ is universal in the vicinity of the MIT. These experiments span two orders of magnitude in sample dimension [21], an order of magnitude in critical exponent $p$ [22], about an order of magnitude in $\mu_4$-2, different surface electrostatics [3], and different manufacturing procedures. We expect the relation to hold in other 2D systems, and in fact a somewhat similar curve was obtained for higher peak-mobility Si MOSFETs [21]. With the assumption that the length over which quantum coherence obtains is growing algebraically as $T \to 0$ so that $L_\varphi \propto 1/T^{1/2}$ [22], we can write the scaling law [4]
that were used to accomplish single-parameter scaling at for finite size effects may play a role in the failure of one-parameter scaling. On the other hand, a larger value of \( p' \), i.e., a larger deviation from one-parameter scaling, is found in a larger sample, but it would be interesting to study if there is any systematic dependence of all the critical exponents on the sample length.

The data (Fig. 3) also show that \( \sigma_c\) depends very weakly on temperature: \( \sigma_c = \sigma(n=0,T) \propto T^{p'} \), resulting in \( \sigma_c = 0 \) at \( T = 0 \). It is important to note, however, that the direct observation of a significant reduction of \( \sigma_c \) as \( T \to 0 \) can not be achieved at experimentally accessible temperatures because of the very small exponent \( p' \). For this reason, our result is not inconsistent with the constant values of \( \sigma_c \) observed in other experiments. Of course, we also can not distinguish this temperature dependence from a logarithm, which is another form proposed for corrections to scaling for certain quantum critical points [24]. On the other hand, our result is in agreement with the recent work [10] on the transition between a different type of the 2D metal and an insulator, but the values of \( p' \) are considerably different.

\[ \beta(g) = d(\ln g)/d(\ln L) \sim d(\ln g)/d(\ln L_c) \text{ as } \beta(g) \sim z\beta_T. \]

Recent numerical work has suggested [23] that \( \nu\beta(g) \) is a universal function close to the MIT.

On the metallic side, for \( T < T_1 \), we find that each \( \sigma(n,T) \) trace deviates from the master curve in the same way, regardless of temperature, which suggests that it should be possible to scale all of the curves using two parameters. This, together with the predictions of two-parameter scaling for certain forms of MIT (e.g., with a strong triplet coupling, i.e., the spin-dependent part of the electron-electron interaction is large) [13], encouraged us to examine a new scaling scheme according to

\[ \sigma(n,T) \sim T^{p'} f(\delta_n/T^{p'}). \]  \hspace{1cm} (2)

This scaling form has been applied to a different form of the 2D metal with considerable success [10]. As can be seen in Fig. 3, with this form we can scale all of the data for \( 0.4 < T < 1.5 \) K from the insulating side through to the metallic side using the same values of \( p' \) that were used to accomplish single-parameter scaling at \( T > T_1 \). For each sample, the data collapse onto a sin-

FIG. 2. (a) Method of observing crossover from one-parameter to two-parameter regimes by measuring deviations from the \( \sigma_i \) at each \( T \). Dotted lines depict (arbitrarily chosen) 2% deviation. (b) Experimental phase diagram for both samples. Open and solid dots mark \( T_0 \), open and solid diamonds mark \( T_1 \). For each sample, the phase diagram is separated into the quantum critical regime (QCR) where \( \ln \sigma \propto \delta_n, T_1 < T < T_0 \) regime where the \( T \)-dependence of \( \sigma \) is weaker but the one-parameter scaling still works, and the \( T < T_1 \) regime where scaling can be achieved only with two parameters.

\[ \delta(g) = d(\ln g)/d(\ln L) \sim d(\ln g)/d(\ln L_c) \text{ as } \beta(g) \sim z\beta_T. \]

Fig. 3(c) thus shows the universality of \( z\nu\beta_T \sim \nu\beta(g) \).
presumably reflecting the different universality classes of the two situations.

In the metallic phase, as $T \to 0$, $\sigma(T)$ becomes a very weak function after an initial rapid increase. As a result of this “saturation” of $\sigma(T)$, single-parameter scaling fails at the lowest temperatures (see also Ref. [28]) and the scale $T_1$ must exist. We have identified explicitly $T_1 = T_1(\delta_n)$ and shown that it vanishes as the transition is approached. Furthermore, we have shown that all of the conductivity data in both metallic and insulating regimes can be scaled with two parameters according to Eq. (2). That scaling form is consistent with the observation that $\sigma(T)$ becomes very weak (“saturates”) in the metallic phase as $T \to 0$. We reiterate that the scaling form (2) has been predicted for certain models [15]. The predictions do not, however, describe our data in detail (e.g. the sign or magnitude of $p'$). Scaling form (4) has been also used successfully to describe a different type of a 2D MIT [13].

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