CP violation in $D^0 \to K^+ \pi^-$

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Abstract

In this paper we study the direct CP asymmetry of the doubly Cabibbo-suppressed decay mode $D^0 \to K^+ \pi^-$ within standard model and two Higgs doublet model with generic Yukawa structure. In the standard model we derive the corrections to the tree level amplitude, generated from the box and di-penguin diagrams, required for generating the weak CP violating phases. We show that these phases are so tiny leading to a direct CP asymmetry of order $10^{-9}$. Regarding the two Higgs doublet model with generic Yukawa structure we derive the Wilson coefficients relevant to $D^0 \to K^+ \pi^-$. After taking into account all constraints on the parameter space of the model we show that charged Higgs couplings to quarks can lead to a direct CP asymmetry of order $10^{-3}$ which is 6 orders of magnitude larger than the standard model prediction.
I. INTRODUCTION

Up to now, no signals for new particles beyond the standard model (SM) have been seen in colliders. New Physics (NP) can be also probed through indirect searches in colliders, for instances searching for signals of flavor violation in the quark sector forbidden in the SM, lepton number violation and CP violation beyond the one predicted by SM.

In the SM the origin of CP violation is the Cabibbo-Kobayashi-Maskawa (CKM) matrix describing the quark mixing \[1, 2\]. The presence of some complex elements in the CKM matrix allows CP violation that has been observed in kaon and B mesons \[3-6\]. Moreover, recently LHCb has reported the first measurement of CP violation in the baryon sector using the baryonic decay mode \(\Lambda_b \rightarrow p \pi^- \pi^+ \pi^-\) decays \[7, 8\]. On the other hand, in the D sector, great experimental progress has been achieved in the last decade. The \(D^0 - D^0\) mixing was discovered in 2007 after combining the results from BABAR \[9\], Belle \[10\] and CDF \[11\]. Later, at LHCb, the mixing has been firmly established after the first experimental observations of the slow mixing rate of the \(D^0 - \bar{D}^0\) oscillations \[12\]. Despite this progress there is still no experimental evidence for direct CP violation in charm. The first two full years data taking at LHCb are consistent with CP conservation in charm \[13\].

CP violating processes involving the up-type quark are expected to be seen only in the charmed mesons. In this sector CP violation within SM is expected to be small because the relevant combination of elements of the CKM matrix is of order \(10^{-3}\) \[14\]. Generally two body non-leptonic D decays can be classified into Cabibbo-Favored (CF), single Cabibbo-suppressed (SCS) and Double Cabibbo Suppressed (DCS). This classification is based on the power of the suppression factor \(\lambda \simeq |V_{us}| \simeq |V_{cd}|\) which appears in their amplitudes \[14\].

Within SM CP-asymmetry of order \(10^{-3}\) has been predicted in some SCS decay modes \[15\]. More SCS decay modes have been investigated in the framework of the SM. The results showed that a larger CP-asymmetry of order \(10^{-2}\) can be obtained for the decay modes \(D^0 \rightarrow K_s K_s\) \[16\]. In a recent study of the SCS decays \(D^0 \rightarrow K_s K^{*0}\) and \(D^0 \rightarrow K_s \bar{K}^{*0}\) the direct CP asymmetry have been estimated in the SM to be as large as \(3 \times 10^{-3}\) \[17\].

We turn now to CF two body non-leptonic D decays. In a previous study we showed that the direct CP asymmetry of \(D^0 \rightarrow K^- \pi^+\) can be less than or equal \(1.4 \times 10^{-10}\) in the framework of the SM \[18\]. This almost vanishing asymmetry favors this decay mode to be smoking gun for NP beyond SM. In fact one expects to have similar situation for the DCS decay.
mode $D^0 \rightarrow K^+\pi^-$. So the objectives of this study is to give a prediction of the direct CP asymmetry of this process in SM and to explore NP contribution to this asymmetry arising from two Higgs doublet model with generic Yukawa structure.

Simple extensions of the SM include the two Higgs doublet models (2HDM)\cite{19, 20}. These models keep the gauge structure of the SM untouched. They only extend the scalar sector by adding new scalars. 2HDM can be classified to several types according to their couplings to quarks and leptons. For instances, 2HDM type I, II or III (for a review see ref. \cite{21}). The 2HDM III has complex couplings to quarks. As a consequence these couplings are relevant for generating the desired CP violating weak phases. Other motivation for 2HDM III includes their ability to explain $B \rightarrow D\tau\nu$, $B \rightarrow D^\star\tau\nu$ and $B \rightarrow \tau\nu$ simultaneously while other types such as 2HDM I and 2HDM II cannot \cite{22}.

This paper is organized as follows. In Sec. II we study the SM contribution to the amplitude of the decay mode $D^0 \rightarrow K^+\pi^-$. At tree-level the amplitude has no source of the weak CP violating phases required for non-vanishing direct CP asymmetry. Accordingly, we consider the loop-level and derive the contributions generated from box and di-penguin diagrams. In addition, we calculate the SM prediction of the direct CP asymmetry. In Sec. III we derive the contributions relevant to the Wilson coefficients of $D^0 \rightarrow K^+\pi^-$ originated from a charged Higgs couplings to the quarks in a two Higgs doublet model with generic Yukawa structure. Finally, we give our conclusion in Sec. IV.

II. DIRECT CP ASYMMETRY OF $D^0 \rightarrow K^+\pi^-$ WITHIN SM

Within SM the weak effective Hamiltonian governing the decay process $D^0 \rightarrow K^+\pi^-$ can be written as

$$H^{SM}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V^\ast_{cd} V_{us} \left( a_1 \bar{d} \gamma_\mu c L \bar{u} \gamma^\mu s_L + a_2 \bar{u} \gamma_\mu c L \bar{d} \gamma^\mu s_L \right) + \text{h.c.} = \frac{G_F}{\sqrt{2}} V^\ast_{cd} V_{us} \left( a_1 O_1 + a_2 O_2 \right) + \text{h.c.} \quad (1)$$

where $a_1 \equiv c_1 + c_2 / N_c$ and $a_2 \equiv c_2 - c_1 / N_C$ and $N_C$ is the color number. In naive factorization approximation (NFA) the amplitude of $D^0 \rightarrow K^+\pi^-$ can be written as

$$A_{D^0 \rightarrow K^+\pi^-} = -i \frac{G_F}{\sqrt{2}} V^\ast_{cd} V_{us} \left[ a_1 X^{K^+}_{D^0\pi^-} + a_2 X^{D^0}_{K^+\pi^-} \right], \quad (2)$$
FIG. 1. Feynman diagram for DCS processes: Box contribution.

where $X^{P_1}_{P_2P_3}$ is given by

$$X^{P_1}_{P_2P_3} = i f_P \Delta^2_{P_2P_3} F^{P_2P_3}_0 (m^2_{P_1}), \quad \Delta^2_{P_2P_3} = m^2_{P_2} - m^2_{P_3}$$

(3)

here $f_P$ is the $P$ meson decay constant and $F^{P_2P_3}_0$ is the form factor.

In NFA, there is no source for the strong CP conserving phases required for having non vanishing direct CP asymmetries. Consequently this factorization approximation is irrelevant to the study of CP violation. On the other hand the mass of the charm quark is not heavy enough to allow for a sensible heavy quark expansion, such as in QCD factorization and soft collinear effective theory, and it is not light enough for the application of chiral perturbation theory $^{[23]}$. A possible approach to study charm decays in a model-independent way is the so called the diagrammatic approach $^{[23–28]}$. Within this approach, the amplitude is decomposed into parts corresponding to generic quark diagrams according to the topologies of weak interactions. For each one of these topological diagrams, the related magnitude and relative strong phase can be extracted from the data without making further assumptions, apart from flavor SU(3) symmetry $^{[23]}$.

In the diagrammatic approach the amplitude of the decay process $D^0 \to K^+\pi^-$ can be written as $^{[23]}

$$A_{D^0\to K^+\pi^-} = V_{cd}^* V_{us} (T'' + E'')$$

(4)

where $T''$ is the tree level color-allowed external W-emission quark diagram and $E''$ is the W-exchange quark diagram. Their magnitudes and their strong phases can be found in Ref.$^{[23]}$. Comparing Eqs.$^{[2]}$ and $^{[4]}$ we get

$$T'' = \frac{G_F}{\sqrt{2}} a_1 f_K (m_D^2 - m^2_{\pi}) F^{D\pi}_0 (m^2_K)$$

$$E'' = \frac{G_F}{\sqrt{2}} a_2 f_D (m^2_K - m^2_{\pi}) F^{K\pi}_0 (m^2_D)$$

(5)
FIG. 2. Feynman diagram for DCS processes: di-penguins contribution.

In the SM the Wilson coefficients $a_1$ and $a_2$ and the CKM elements $V_{cd}$ and $V_{us}$ are all real. Thus at this level there is no source for the CP violating weak phases required for non vanishing CP asymmetry. Possible CP violating weak phases can be generated through box and di-penguin diagrams in fig.1 and fig.2. The box contribution to the total SM weak effective Hamiltonian can be expressed as

$$\Delta H_{\text{box}}^{\text{SM}} = \frac{G_F^2 m_W^2}{2\pi^2} V_{cd}^{*} s_l c_L \bar{d} \gamma_{\mu} d + \bar{u} \gamma_{\mu} u_L V_{us} f(x_U, x_D)$$

$$= \frac{G_F^2 m_W^2}{2\pi^2} \lambda_{U, D}^{\text{SM}} f(x_U, x_D) O_2$$

where

$$B = \lambda_{U, D}^{\text{SM}} f(x_U, x_D)$$

and

$$f(x, y) = \frac{7xy - 4}{4(1-x)(1-y)} + \frac{1}{x-y} \left[ \frac{y^2 \log y}{(1-y)^2} \left( 1 - 2x + \frac{xy}{4} \right) - \frac{x^2 \log x}{(1-x)^2} \left( 1 - 2y + \frac{xy}{4} \right) \right]$$

Clearly $B_x$ will be a complex number due to the presence of the complex CKM elements. Thus the desired CP violating weak phases are generated through the box contribution to the weak effective Hamiltonian.
The other contribution to the total SM weak effective Hamiltonian is the di-penguin contribution generated via diagram in Fig. (2) and can be written as

$$\Delta H_{\text{dipeng.}} = - \frac{G_F^2 \alpha_S}{8 \pi^3} [\lambda_{ct}^D E_0(x_D)] [\lambda_{ds}^U E_0(x_U)] \bar{d} \gamma^\mu T_a s_L (g^{\mu\nu} \Box - \partial^\mu \partial^\nu) \bar{u} \gamma^\nu T^a c_L$$

$$= - \frac{G_F^2 \alpha_S}{8 \pi^3} P_g \bar{d} \gamma^\mu T^a s_L (g^{\mu\nu} \Box - \partial^\mu \partial^\nu) \bar{u} \gamma^\nu T^a c_L$$

$$\equiv \frac{G_F^2 \alpha_S}{16 \pi^3} P_g \mathcal{O}$$

(9)

where $T^a$ are the generator of $SU(3)_C$ and the Inami function is given by

$$E_0(x) = \frac{1}{12(1-x)^4} [x(1-x)(18-11x-x^2) - 2(4-16x+9x^2) \log(x)]$$

(10)

The quantity $P_g$ can be expressed in terms of CKM elements and Inami function as

$$P_g \equiv [\lambda_{ct}^D E_0(x_D)] [\lambda_{ds}^U E_0(x_U)] = [V_{cs}^* V_{us} (E_0(x_s) - E_0(x_d)) + V_{cb}^* V_{ub} (E_0(x_b) - E_0(x_d))]$$

$$[V_{cs} V_{cd} (E_0(x_c) - E_0(x_u)) + V_{ts} V_{td}^* (E_0(x_t) - E_0(x_u))]$$

(11)

As can be seen from Eq. (11), $P_g$ is a complex number due to the presence of the complex CKM elements. Thus di-penguin contribution generates CP violating weak phases. We now proceed to express the operator $\mathcal{O}$ and find that

$$\mathcal{O} = \bar{d} \gamma^\mu T^a s_L (g^{\mu\nu} \Box - \partial^\mu \partial^\nu) \bar{u} \gamma^\nu T^a c_L = \bar{d} \gamma^\mu T^a s_L \Box (\bar{u} \gamma^\nu T^a c_L) + \bar{d} \Box T^a s_L \bar{u} \gamma^\nu T^a c_L$$

$$= -q^2 \bar{d} \gamma^\mu T^a s_L \bar{u} \gamma^\nu T^a c_L - (m_d \bar{d} T^a s_{s-p} + m_s \bar{d} T^a s_{s+p}) \cdot (m_c \bar{u} T^a c_{s+p} + m_u \bar{u} T^a c_{s-p})$$

$$-q^2 \bar{d} \gamma^\mu T^a s_L \bar{u} \gamma^\nu T^a c_L - m_d m_c \bar{d} T^a s_L \bar{u} T^a c_R - m_s m_u \bar{d} T^a s_R \bar{u} T^a c_L$$

$$-m_d m_u \bar{d} T^a s_L \bar{u} T^a c_R - m_s m_c \bar{d} T^a s_R \bar{u} T^a c_R$$

(12)

This expression can be further simplified using

$$\bar{d} \gamma^\mu T^a s_L \bar{u} \gamma^\nu T^a c_L = \frac{1}{2} \left( O_1 - \frac{1}{N_C} O_2 \right)$$

$$\bar{d} T^a s_L \bar{u} T^a c_R = -\frac{1}{4} \bar{d} \gamma^\mu c_R \bar{u} \gamma^\nu s_L - \frac{1}{2N_C} \bar{d} s_L \bar{u} c_R$$

$$\bar{d} T^a s_R \bar{u} T^a c_L = -\frac{1}{4} \bar{d} \gamma^\mu c_L \bar{u} \gamma^\nu s_R - \frac{1}{2N_C} \bar{d} s_R \bar{u} c_L$$

$$\bar{d} T^a s_L \bar{u} T^a c_L = -\frac{1}{4} \bar{d} c_L \bar{u} s_L - \frac{1}{16} \bar{d} \sigma_{\mu\nu} c_L \bar{u} \sigma^\mu s_L - \frac{1}{2N_C} \bar{d} s_L \bar{u} c_L$$

$$\bar{d} T^a s_R \bar{u} T^a c_R = -\frac{1}{4} \bar{d} c_R \bar{u} s_R - \frac{1}{16} \bar{d} \sigma_{\mu\nu} c_R \bar{u} \sigma^\mu s_R - \frac{1}{2N_C} \bar{d} s_R \bar{u} c_R$$

(13)
Upon taking the expectation values, we obtain

\[
\langle O \rangle = -q^2 \langle \bar{d} \gamma_i T^a s_L \bar{u} \gamma^- T^a c_L \rangle - m_d m_c \langle \bar{d} T^a s_L \bar{u} T^a c_L \rangle - m_s m_u \langle \bar{d} T^a s_R \bar{u} T^a c_L \rangle \\
- m_d m_u \langle \bar{d} T^a s_L \bar{u} T^a c_L \rangle - m_s m_c \langle \bar{d} T^a s_R \bar{u} T^a c_R \rangle \\
\approx -\frac{q^2}{2} \left( 1 - \frac{1}{N^2} \right) X_{D^0 \pi^-}^{K^+} + \frac{m_d m_c}{4} \left( 1 - \frac{1}{N} \right) X_{D^0 \pi^0}^{K^+} + \frac{5m_s}{8Nm_d} m_D^2 X_{D^0}^{K^+} 
\]

(14)

where \( q^2 \) is the gluon momentum. For the decay \( D^0 \to K^+ \pi^- \), one can approximate \( q^2 = (p_c \mp p_u)^2 = (p_\pi - p_D/2)^2 = (m_D^2 + m_K^2)/2 + 3m_\pi^2/4 \), by assuming that \( p_c \simeq p_D \) and \( p_u \simeq p_\pi/2 \). Finally, including both box and di-penguin contributions leads to a modification of the Wilson coefficients \( a_1 \) and \( a_2 \) as \( a_i \to a_i + \Delta a_i \) for \( i = 1, 2 \) and \( \Delta a_i \) are given by

\[
\Delta a_1 = -\frac{G_F m_W^2}{\sqrt{2} \pi^2 V^*_{cd} V_{us} N} B_\pi - \frac{G_F \alpha_s}{4\sqrt{2} \pi^3 V^*_{cd} V^*_{us}} \left[ \frac{q^2}{2} \left( 1 - \frac{1}{N^2} \right) - \frac{m_d m_c}{4} \left( 1 - \frac{1}{N} \right) \right] P_g \\
\Delta a_2 = -\frac{G_F m_W^2}{\sqrt{2} \pi^2 V^*_{cd} V_{us} N} B_\pi - \frac{G_F \alpha_s}{4\sqrt{2} \pi^3 V^*_{cd} V^*_{us}} 5m_s m_d^2 P_g
\]

(15)

For the numerical analysis we need to specify the values of the CKM elements. The CKM matrix can be fully determined by the measurement of four independent parameters. A convenient determination is from tree-level charged current decays only, which can be used to find the mixing angles\[30\]. These are \( |V_{us}|, |V_{cb}| \) and \( |V_{ub}| \) in addition to the CP violating angle \( \gamma \) of the unitarity triangle. For the \( CKM \) matrix element \( V_{td} \) we follow Ref.\[31\] and evaluate it using

\[
V_{td} = |V_{us}| |V_{cb}| R_t e^{-i\beta},
\]

(16)

where

\[
R_t = \sqrt{1 + R_b^2 - 2R_b \cos \gamma}, \quad R_b = \left( 1 - \frac{\chi^2}{2} \right) \frac{1}{\lambda} |V_{cb}|, \quad \cot \beta = \frac{1 - R_b \cos \gamma}{R_b \sin \gamma}
\]

(17)

From kaon decays we have\[32\] \( |V_{us}| = 0.2248 \pm 0.0006 \). The accuracy of the current experimental determination of \( \gamma \) by the LHCb collaboration\[33\] \( \gamma = (72.2^{+6.8}_{-7.2})^\circ \). The situation for \( |V_{cb}| \) and \( |V_{ub}| \) is quite unsatisfactory. This is due to the significant discrepancies in the determined values using inclusive or exclusive decays. The uncertainties in extracting \( |V_{ub}| \) from inclusive and exclusive decays are different to a large extent. Exclusively, the most precise determination of \( |V_{ub}| \) \( |V_{cb}| \) is obtained from the decay \( B \to \pi \ell \nu \) \( B \to D^* \ell \nu \)\[30,34\]\[30,35\].
TABLE I. Predictions for $\Delta a_1$ and $\Delta a_2$ corresponding to the inclusive, the exclusive and the combined input values of the CKM elements $|V_{cb}|$ and $|V_{ub}|$.

|   | inclusive | exclusive | combined   |
|---|-----------|-----------|------------|
| $\Delta a_1$ | $2.2 \cdot 10^{-7} e^{179.68^\circ i}$ | $2.2 \cdot 10^{-7} e^{179.74^\circ i}$ | $2.2 \cdot 10^{-7} e^{179.71^\circ i}$ |
| $\Delta a_2$  | $2.2 \cdot 10^{-6} e^{179.88^\circ i}$ | $2.2 \cdot 10^{-6} e^{179.90^\circ i}$ | $2.2 \cdot 10^{-6} e^{179.89^\circ i}$ |

$|V_{ub}^{\text{incl.}}| = (3.72 \pm 0.16) \times 10^{-3}$, $|V_{cb}^{\text{incl.}}| = (39.04 \pm 0.75) \times 10^{-3}$ \hspace{1cm} (18)

On the other hand we have from the inclusive decay $B \rightarrow X_u \ell \nu$ ($B \rightarrow X_c \ell \nu$) \hspace{1cm} (32) \hspace{1cm} (36)

$|V_{ub}^{\text{incl.}}| = (4.41 \pm 0.15^{+0.15}_{-0.19}) \times 10^{-3}$, $|V_{cb}^{\text{incl.}}| = (42.00 \pm 0.65^{+0.15}_{-0.19}) \times 10^{-3}$ \hspace{1cm} (19)

A combination of the inclusive and exclusive of $|V_{ub}|$ and $|V_{cb}|$ determinations is quoted \hspace{1cm} (37)

$|V_{ub}| = (4.09 \pm 0.39) \times 10^{-3}$, $|V_{cb}| = (40.5 \pm 1.5) \times 10^{-3}$ \hspace{1cm} (20)

Having these input values we can now obtain the numerical values of $\Delta a_1$ and $\Delta a_2$ given in Eq. (15). We list their predictions in Table I. Clearly from Table I the predicted values of $\Delta a_1$ and $\Delta a_2$ corresponding to the inclusive, the exclusive and the combined input values of the CKM elements $|V_{cb}|$ and $|V_{ub}|$ are approximately equal. Thus in the following we give our predictions corresponding to the combined input values of $|V_{cb}|$ and $|V_{ub}|$.

The direct CP asymmetry of $D^0 \rightarrow K^+ \pi^-$ can be expressed as

$$A_{CP} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{2r \sin(\phi_2 - \phi_1) \sin(\alpha)}{|1 + r|^2} = \kappa \sin(\phi_2 - \phi_1)$$ \hspace{1cm} (21)

where we have defined

$$\kappa = \frac{2r \sin(\alpha)}{|1 + r|^2}$$ \hspace{1cm} (22)

with $r = |E''/T''|$ and $\alpha = \alpha_{E''} - \alpha_{T''}$. The phases $\alpha_{E''}$ and $\alpha_{T''}$ are the strong phase of the amplitudes $E''$ and $T''$ respectively. The weak phases $\phi$ are defined through

$$\phi_i = \tan^{-1} \left( \frac{|\Delta a_i| \sin \Delta \phi_i}{a_i + |\Delta a_i| \cos \Delta \phi_i} \right)$$ \hspace{1cm} (23)
where \( \Delta \phi_i \) is the phase of \( \Delta a_i \). With \( a_1 = 1.2 \pm 0.1 \) and \( a_2 = -0.5 \pm 0.1 \) and \( \Delta a_1 \) and \( \Delta a_2 \) given in the last coulomb of Table I we find that \( \sin(\phi_2 - \phi_1) \simeq -9 \times 10^{-9} \) and hence

\[
A_{CP} \simeq -9 \times 10^{-9} \kappa \tag{24}
\]

For \( T'' = (3.14 \pm 0.06) \times 10^{-6} \) and \( E'' = (1.53^{+0.07}_{-0.08}) \times 10^{-6} e^{i(122 \pm 2)} \) we find that \( \kappa \simeq 0.37 \) and thus \( A_{CP} \simeq -3.36 \times 10^{-9} \). Clearly the predicted direct CP asymmetry within SM is so tiny.

### III. MODELS WITH CHARGED HIGGS CONTRIBUTIONS

In 2HDM III the physical mass eigenstates are \( H_0 \) (heavy CP-even Higgs), \( h_0 \) (light CP-even Higgs) and \( A_0 \) (CP-odd Higgs) and \( H^\pm \). In this model both Higgs doublets can couple to up-type and down-type quarks. As a consequence the couplings of the neutral Higgs mass eigenstates can induce flavor violation in Neutral Currents at tree-level. In the down sector these flavor violating couplings are stringently constrained from flavor changing neutral current processes [22, 38]. Thus in the following we consider only charged Higgs couplings to quarks that can be expressed as [22, 39]:

\[
\mathcal{L}_{H^\pm}^{\text{eff}} = \bar{u}_f \Gamma^{H^\pm \text{LR}}_{u_f d_i} P_R d_i + \bar{u}_f \Gamma^{H^\pm \text{RL}}_{u_f d_i} P_L d_i, \tag{25}
\]

where

\[
\Gamma^{H^\pm \text{LR}}_{u_f d_i} = \sum_{j=1}^{3} \sin \beta V_{f j} \left( \frac{m_{d_j}}{v_d} \delta_{j i} - \epsilon^d_{j i} \tan \beta \right),
\]

\[
\Gamma^{H^\pm \text{RL}}_{u_f d_i} = \sum_{j=1}^{3} \cos \beta \left( \frac{m_{u_j}}{v_u} \delta_{j f} - \epsilon^u_{j f} \tan \beta \right) V_{ji} \tag{26}
\]

Here \( v_u \) and \( v_d \) denote the vacuum expectations values of the neutral component of the Higgs doublets, \( \tan \beta = v_u / v_d \) and \( V \) is the CKM matrix. Applying the Feynman-rules given in Eq.(25) allows us to derive the effective Hamiltonian, resulting from the tree level exchanging charged Higgs diagram, that governs the process under consideration. The effective Hamiltonian can be expressed as

\[
\mathcal{H}_{\text{eff}}^{H^\pm} = \frac{G_F}{\sqrt{2}} V_{cd} V_{us} \sum_{i=1}^{4} C_i^H(\mu) Q_i^H(\mu), \tag{27}
\]
The Wilson coefficients $C_i^H$ are obtained by perturbative QCD running from $M_{H^\pm}$ scale to the scale $\mu \simeq m_c$ relevant for hadronic decay and $Q_i^H$ are the relevant local operators at $\mu \simeq m_c$. The operators are given as

$$
Q_1^H = (\bar{d} P_R c)(\bar{u} P_L s),
$$

$$
Q_2^H = (\bar{d} P_L c)(\bar{u} P_R s),
$$

$$
Q_3^H = (\bar{d} P_L c)(\bar{u} P_L s),
$$

$$
Q_4^H = (\bar{d} P_R c)(\bar{u} P_R s),
$$

(28)

Their corresponding Wilson coefficients $C_i^H$, at $\mu = m_H$ scale, can be expressed as

$$
C_1^H = \frac{\sqrt{2} \cos^2 \beta}{G_F V_{cd}^* V_{us} m_H^2} \left( \frac{m_u V_{us}}{v_u} - \sum_{j=1}^3 V_{ij} \epsilon_{d_j}^u \tan \beta \right) \left( \frac{m_c V_{cd}^*}{v_u} - \sum_{k=1}^3 V_{k1} \epsilon_{d_k}^u \tan \beta \right),
$$

$$
C_2^H = \frac{\sqrt{2} \sin^2 \beta}{G_F V_{cd}^* V_{us} m_H^2} \left( \frac{m_s V_{us}}{v_d} - \sum_{j=1}^3 V_{ij} \epsilon_{d_j}^d \tan \beta \right) \left( \frac{m_d V_{cd}^*}{v_d} - \sum_{k=1}^3 V_{k1} \epsilon_{d_k}^d \tan \beta \right),
$$

$$
C_3^H = \frac{2 \beta}{G_F V_{cd}^* V_{us} m_H^2} \left( \frac{m_u V_{us}}{v_u} - \sum_{j=1}^3 V_{ij} \epsilon_{d_j}^u \tan \beta \right) \left( \frac{m_d V_{cd}^*}{v_d} - \sum_{k=1}^3 V_{k1} \epsilon_{d_k}^d \tan \beta \right),
$$

$$
C_4^H = \frac{2 \beta}{G_F V_{cd}^* V_{us} m_H^2} \left( \frac{m_s V_{us}}{v_d} - \sum_{j=1}^3 V_{ij} \epsilon_{d_j}^d \tan \beta \right) \left( \frac{m_c V_{cd}^*}{v_u} - \sum_{k=1}^3 V_{k1} \epsilon_{d_k}^u \tan \beta \right),
$$

(29)

Having deriving the effective Hamiltonian we proceed now to discuss the experimental constraints on the flavor-changing parameters $\epsilon_{ij}^q$, for $q = d, u$, appear in the Wilson coefficients. We start first by discussing the constraints on $\epsilon_{ij}^d$. For $i \neq j$ case we find that $\epsilon_{ij}^d$ are stringently constrained from FCNC processes because of the tree-level neutral Higgs exchange [22, 38]. Thus, we are left with only $\epsilon_{11}^d, \epsilon_{22}^d$. These couplings can be constrained upon applying the naturalness criterion of 't Hooft to the quark masses [22]. Based on this criterion, the smallness of a quantity is only natural if a symmetry is gained in the limit in which this quantity is zero [22]. As a result it is unnatural to have large accidental cancellations without a symmetry forcing these cancellations. Applying this criterion to the quark masses in the 2HDM III we get [22]

$$
|\epsilon_{ij}^d| \leq \frac{|V_{ij}| \max \left[ m_{d_i(u_i)}, m_{d_j(u_j)} \right]}{|v_{u(d)}|}.
$$

(30)
This bound shows that $\epsilon_{11}^d, \epsilon_{22}^d$ will be severely constrained by their small quark masses. This is also the case for the coupling $\epsilon_{11}^u$. Thus we conclude that all the couplings $\epsilon_{ij}^d$ and $\epsilon_{11}^u$ that appear in the Wilson coefficients in Eq. (29) will lead to negligible effects and hence can be safely dropped. Thus, to a good approximation, we can write

$$C_H = \sum_{i} V_{ij}^* \epsilon_{ij}^d \tan \beta,$$

for $i = 1, 2, 3, 4, 5$. This results in

$$C_H^1 \simeq \sqrt{2} \cos^2 \beta \left( \sum_{j=2}^{3} V_{j2}^* \epsilon_{j1}^d \tan \beta \right) \left( \sum_{k=1}^{2} V_{k1}^* \epsilon_{k1}^u \tan \beta \right),$$

$$C_H^4 \simeq -\frac{\sin 2\beta m_s}{\sqrt{2} G_F V_{cd}^* m_H^2 v_d} \left( \sum_{k=1}^{2} V_{k1}^* \epsilon_{k2}^u \tan \beta \right),$$

$$C_H^2 = C_H^3 \simeq 0.$$  \hfill (31)

It should be noted that in obtaining Eq. (31) several other approximations have been taken. First, we have dropped the terms suppressed by the small quark masses $m_u$ and $m_d$ and the terms suppressed by the CKM element $V_{ud}$. Second, in $C_H^4$ we dropped the term proportional to $m_c m_s$ as it is real and thus it will not be relevant for generating weak phases. Finally, in $C_H^H$ we dropped the terms proportional to $m_c$ as it will be numerically much smaller than the other terms due to the suppression factor $V_{cd}^* (v \sin \beta) \simeq V_{cd}^* / v \simeq -10^{-3}$ for large $\tan \beta$ case of our interest. It should be noted also that, for large $\tan \beta$ case $m_s / v_d = m_s / (v \cos \beta)$ becomes large and so $C_H^4$ becomes comparable with $C_H^H$.

The total amplitude of $D^0 \to K^+ \pi^-$, including Higgs contribution, can be written as

$$A^{SM+H} = \left( C_1^{SM} + \frac{1}{N} C_2^{SM} + \chi^K (C_1^H - C_4^H) \right) X_{D^0 K^+ \pi^-} - \left( C_2^{SM} + \frac{1}{N} C_1^{SM} + \frac{1}{2N} (C_1^H - \chi^D \chi^H C_4^H) \right) X_{D^0 K^+ \pi^-}$$

where

$$\chi^K = \frac{m_K^2}{(m_c - m_d)(m_u + m_s)},$$

$$\chi^D = \frac{m_D^2}{(m_c + m_u)(m_d - m_s)}.$$  \hfill (33)

Eq. (32) can be expressed in terms of the amplitudes $T''$ and $E''$ introduced before as:

$$A^{SM+H} = V_{cs}^* V_{ud} (T''^{SM+H} + E''^{SM+H})$$

where

$$T''^{SM+H} = \frac{G_F}{\sqrt{2}} a_4^{SM+H} f_K (m_D^2 - m_{D'}^2) F_{0}^{D'} (m_D^2),$$

$$E''^{SM+H} = \frac{G_F}{\sqrt{2}} a_2^{SM+H} f_D (m_{D'}^2 - m_D^2) F_{0}^{K'} (m_D^2).$$  \hfill (35)
where

\[
a_1^{SM+H} = (a_1 + \Delta a_1 + \Delta a_1^H)
\]

\[
a_2^{SM+H} = -(a_2 + \Delta a_2 + \Delta a_2^H)
\]

(36)

with

\[
\Delta a_1^H = \chi^K(C_1^H - C_4^H)
\]

\[
\Delta a_2^H = \frac{1}{2N}(C_1^H - \chi^{D^0}C_4^H)
\]

(37)

Clearly Higgs contributions affect only the short distance physics (Wilson coefficients) leaving the strong phases unaffected.

In a recent study a lower bound on the charged Higgs mass in 2HDM of Type II has been set after taking into account all relevant results from direct charged and neutral Higgs boson searches at LEP and the LHC, as well as the most recent constraints from flavour physics [40]. The bound reads \( m_{H^\pm} \gtrsim 600 \) GeV independent of \( \tan \beta \). This bound should be also respected in 2HDM III [22].

For \( \tan \beta = 50 \) and \( m_{H^\pm} = 600 \) GeV we find that

\[
\Delta a_1^H \approx -0.08 \epsilon_{12}^u + 0.02 \epsilon_{22}^u + 2.88 \epsilon_{22}^u \epsilon_{21}^u - 0.52 \epsilon_{12}^u \epsilon_{31}^u - 0.12 \epsilon_{22}^u \epsilon_{31}^u
\]

\[
\Delta a_2^H \approx 0.20 \epsilon_{12}^u - 0.05 \epsilon_{22}^u + 0.24 \epsilon_{22}^u \epsilon_{21}^u + 0.04 \epsilon_{12}^u \epsilon_{31}^u - 0.01 \epsilon_{22}^u \epsilon_{31}^u
\]

(38)

where we have neglected the terms that are proportional to \( \epsilon_{12}^u \epsilon_{21}^u \) due to the strong constraint imposed on \( \epsilon_{12}^u \epsilon_{21}^u \) from \( D - \bar{D} \) mixing [38]. So we are left only with \( \epsilon_{12}^u, \epsilon_{22}^u \) and \( \epsilon_{21}^u, \epsilon_{31}^* \). The electric dipole moment of the neutron and the observable \( B \to \tau \nu \) can be used to set constraints on the coupling \( \epsilon_{31}^u [38] \). These constraints indicate that the terms proportional to \( \epsilon_{22}^u \epsilon_{31}^* \) will be much smaller compared to the terms proportional to \( \epsilon_{22}^u \) only and so these terms can be safely neglected. For similar reason we can drop the terms proportional to \( \epsilon_{12}^u \epsilon_{31}^* \) in comparison to the terms proportional to \( \epsilon_{12}^u \) only. Thus we get

\[
\Delta a_1^H \approx -0.08 \epsilon_{12}^u + 0.02 \epsilon_{22}^u + 2.88 \epsilon_{22}^u \epsilon_{21}^u
\]

\[
\Delta a_2^H \approx 0.20 \epsilon_{12}^u - 0.05 \epsilon_{22}^u + 0.24 \epsilon_{22}^u \epsilon_{21}^u
\]

(39)

The couplings \( \epsilon_{12}^u \) and \( \epsilon_{21}^u \) can be constrained using the process \( \bar{D}^0 \to \mu^+ \mu^- \) [38]. The resulting bounds can be expressed in terms of \( m_{H^\pm} \) and \( \tan \beta \) as [38]
\[ |\epsilon_{12,21}^u| \leq 3.0 \times 10^{-2} \left( \frac{m_{H^{\pm}}/500\text{GeV}}{\tan \beta/50} \right)^2 \]  

(40)

for \( m_{H^{\pm}} = 600 \text{ GeV} \) and \( \tan \beta = 50 \) we get \( |\epsilon_{12,21}^u| \leq 4.32 \times 10^{-2} \). These bounds indicate that maximum values of the real and imaginary parts of \( \epsilon_{12,21}^u \) will be roughly of order \( 10^{-2} \). We proceed now to discuss the constraints imposed on the coupling \( \epsilon_{22}^u \). The processes \( D(s) \to \tau \nu, D(s) \to \mu \nu \) can constraint the real part of \( \epsilon_{22}^u \) while the constraints on the imaginary part of \( \epsilon_{22}^u \) are weak [38]. For \( m_{H^{\pm}} = 600 \text{ GeV} \), \( \tan \beta = 50 \) and assuming real \( \epsilon_{22}^u \) the strongest bound \(-0.3 \leq \epsilon_{22}^u \lesssim 0.3 \) has been obtained by combining the constraints from \( D \to \mu \nu \) and \( D \to \mu \nu \) [38]. Regarding the imaginary part of \( \epsilon_{22}^u \), for \( m_{H^{\pm}} = 600 \text{ GeV}, \tan \beta = 50 \), the constraints from the electric dipole moment of the neutron reads \(-0.16 \leq Im(\epsilon_{22}^u) \lesssim 0.16 \) [38]. Other processes such as \( D-\bar{D} \) mixing and \( K-\bar{K} \) mixing can be used to set bounds on \( \epsilon_{22}^u \). However these bounds are weaker than the bounds obtained from \( D(s) \to \tau \nu, D(s) \to \mu \nu \) and the electric dipole moment of the neutron [18, 38].

The real parts of \( \Delta a_1^H \) and \( \Delta a_2^H \) are expected to be much smaller than the SM contributions, \( a_1 \) and \( a_2 \), and hence we can be safely neglect them and keep only the imaginary parts required for generating the weak phases. Thus we get

\[
\Delta a_1^H \simeq -0.037 I \simeq 0.037 e^{-90^\circ i} \\
\Delta a_2^H \simeq 0.009 I \simeq 0.009 e^{90^\circ i} 
\]  

(41)

where we kept the dominant terms only after keeping in mind the bounds on \( \epsilon_{12}^u, \epsilon_{21}^u \) and \( \epsilon_{22}^u \). We consider now two scenarios. In the first scenario we assume that \( \epsilon_{22}^u \) is pure real and the other couplings \( \epsilon_{12}^u \) and \( \epsilon_{21}^u \) are pure complex. In the second scenario we assume that \( \epsilon_{22}^u \) is pure complex and the other couplings \( \epsilon_{12}^u \) and \( \epsilon_{21}^u \) are pure real. In each scenario we take the maximum value of \( \epsilon_{ij}^u \) for \( ij = 12, 21, 22 \) allowed from the constraints discussed before.

In the first scenario we find that

\[
\Delta a_1^H \simeq -0.037 I \simeq 0.037 e^{-90^\circ i} \\
\Delta a_2^H \simeq 0.009 I \simeq 0.009 e^{90^\circ i} 
\]  

(42)

while in the second scenario we find that
\[ \Delta a_1^H \simeq 0.023 I \simeq 0.023 e^{90^\circ i} \]
\[ \Delta a_2^H \simeq -0.006 I \simeq 0.006 e^{-90^\circ i} \] (43)

The direct CP asymmetry of \( D^0 \to K^+\pi^- \), including Higgs contributions, can be expressed as

\[
A_{CP} = \frac{|A^{SM+H}|^2 - |\bar{A}^{SM+H}|^2}{|A^{SM+H}|^2 + |\bar{A}^{SM+H}|^2} \simeq \kappa \sin(\phi_2^H - \phi_1^H) \] (44)

where \( \kappa \) is given as before and the weak phases \( \phi_1^H \) and \( \phi_2^H \) are defined through

\[
\phi_i^H = \tan^{-1} \left( \frac{|\Delta a_i^H| \sin \Delta \phi_i^H}{a_i} \right) \] (45)

where \( \Delta \phi_i^H \) is the phase of \( \Delta a_i^H \). Thus for the first scenario we find that

\[
A_{CP} \simeq 0.01 \kappa \simeq 5 \times 10^{-3} \] (46)

while for the second scenario the predicted CP asymmetry is

\[
A_{CP} \simeq -0.007 \kappa \simeq -3 \times 10^{-3} \] (47)

Clearly the predicted direct CP asymmetries in the two scenarios are 6 orders of magnitude larger than the SM predicted one.

**IV. CONCLUSION**

In this paper we have studied the direct CP asymmetry of \( D^0 \to K^+\pi^- \) within standard model and two Higgs doublet model with generic Yukawa structure. In the standard model the tree-level amplitude has no source of the weak phases required for generating direct CP asymmetry. As a result we derived the corrections to the tree level amplitude generated from the box and di-penguin diagrams. We found that these correction can generate weak CP violating phases. However these phases are so tiny leading to a direct CP asymmetry
of order $10^{-9}$. With this tiny CP asymmetry the decay mode $D^0 \to K^+\pi^-$ can serve as a probe of new sources of weak CP violating phases that can be generated in new physics beyond standard model.

As an example of new physics beyond standard model we considered the two Higgs doublet model with generic Yukawa structure. Within this model we have derived the Wilson coefficients corresponding to the decay process $D^0 \to K^+\pi^-$ of our interest. After discussing the relevant constraints on the parameter space of the model, relevant to the process $D^0 \to K^+\pi^-$, we have shown that charged Higgs couplings to quarks can lead to a direct CP asymmetry of order $10^{-3}$. This asymmetry is 6 orders of magnitude larger than the standard model prediction.

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