The Diquark Model for Exclusive Reactions

P. Kroll¹ ²
Fachbereich Physik, Universität Wuppertal
Gaußstrasse 20, D-42097 Wuppertal, Germany
E-mail: kroll@theorie.physik.uni-wuppertal.de

Abstract

The present status of the diquark model for exclusive reactions at moderately large momentum transfer is reviewed. That model is a variant of the Brodsky-Lepage approach in which diquarks are considered as quasi-elementary constituents of baryons. Recent applications of the diquark model, relevant to high energy physics with electromagnetic probes, are discussed.

1 Introduction

Exclusive processes at large momentum transfer are described in terms of hard scatterings among quarks and gluons [1]. In this so-called hard scattering approach (HSA) a hadronic amplitude is represented by a convolution of process independent distribution amplitudes (DA) with hard scattering amplitudes to be calculated within perturbative QCD. The DAs specify the distribution of the longitudinal momentum fractions the constituents carry. They represent Fock state wave functions integrated over transverse momenta. The convolution manifestly factorizes long (DAs) and short distance physics (hard scattering). It however turned out that most processes are not dominated by the perturbative contribution at experimentally accessible values of momentum transfer. Non-perturbative dynamics still plays a crucial role in that kinematical region and, hence, the HSA although likely the correct asymptotic picture for exclusive reactions, needs modifications.

In a series of papers [2]-[7] such a modification has been proposed in which baryons are viewed as being composed of quarks and diquarks. The latter are treated as quasi-elementary constituents which partly survive medium hard collisions. Diquarks are an effective description of correlations in the wave functions and constitute a particular model for non-perturbative effects. The diquark model may be viewed as a variant of the HSA appropriate for moderately large momentum transfer and it is designed in such a way that it evolves into the standard pure quark HSA asymptotically. In so far the standard HSA and the diquark model do not oppose each other, they are not alternatives but rather complements. The existence of diquarks is a hypothesis. However, from experimental and theoretical approaches there have been many indications suggesting the presence of diquarks. For instance, they were introduced in baryon spectroscopy, in nuclear physics, in astrophysics, in jet fragmentation and in weak interactions to explain the famous ∆I =
1/2 rule. Diquarks also provide a natural explanation of the equal slopes of meson and baryon Regge trajectories. For more details and for references, see [3]. It is important to note that QCD provides some attraction between two quarks in a colour \( \bar{3} \) state at short distances as is to be seen from the static reduction of the one-gluon exchange term.

Even more important for our aim, diquarks have also been found to play a role in inclusive hard scattering reactions. The most obvious place to signal their presence is deep inelastic lepton-nucleon scattering. Indeed the higher twist contributions, convincingly observed by the NMC [8], can be modelled as lepton-diquark elastic scattering. Baryon production in inclusive \( pp \) collisions also reveals the need for diquarks scattered elastically in the hard interaction [9]. For instance, kinematical dependences or the excess of the proton yield over the antiproton yield find simple explanations in the diquark model. No other explanation of these phenomena is known as yet.

2 The Diquark Model

As in the standard HSA a helicity amplitude for the reaction \( AB \to CD \) is expressed as a convolution of DAs and hard scattering amplitudes (\( s, -t, -u \gg m_i^2 \))

\[
M(s, t) = \int dx_C dx_D dx_A dx_B \Phi^*_C(x_C) \Phi^*_D(x_D) T_H(x_i, s, t) \Phi_A(x_A) \Phi_B(x_B) \tag{1}
\]

where helicity labels are omitted for convenience. Implicitly it is assumed in (1) that the valence Fock states consist of only two constituents, a quark and a diquark (antiquark) in the case of baryons (mesons). In so far the specification of the quark momentum fraction \( x_i \) suffices; the diquark (antiquark) carries the momentum fraction \( 1 - x_i \). If an external particle is point-like, e.g. a photon, the corresponding DA is to be replaced by \( \delta(1 - x_i) \). As in the standard HSA contributions from higher Fock states are neglected. This is justified by the fact that that such contributions are suppressed by powers of \( \alpha_s/t \) as compared to that from the valence Fock state (if only S-wave hadrons are involved).

In the diquark model spin 0 (\( S \)) and spin 1 (\( V \)) colour antitriplet diquarks are considered. Within flavour SU(3) the \( S \) diquark forms an antitriplet, the \( V \) diquark an sextet. Assuming zero relative orbital angular momentum between quark and diquark and taking advantage of the collinear approximation, the valence Fock state of a ground state octet baryon \( B \) with helicity \( \lambda \) and momentum \( p \) can be written in a covariant fashion (omitting colour indices)

\[
|B; p, \lambda \rangle = f_S \Phi^S_B(x) B_S u(p, \lambda) + f_V \Phi^V_B(x) B_V (\gamma^\alpha + p^\alpha/m_B) \gamma_5 u(p, \lambda)/\sqrt{3} \tag{2}
\]

where \( u \) is the baryon’s spinor. The two terms in (2) represent configurations consisting of a quark and either a scalar or a vector diquark, respectively. The couplings of the diquarks with the quarks in a baryon lead to flavour functions which e.g. for the proton read

\[
B_S = u S_{[u,d]} \quad B_V = [u V_{[u,d]} - \sqrt{2} d V_{[u,u]}]/\sqrt{3}. \tag{3}
\]

The DAs \( \Phi^S_B(\lambda) \) are conventionally normalized as \( \int dx \Phi = 1 \). The constants \( f_S \) and \( f_V \) play the role of the configuration space wave functions at the origin.

The DAs containing the complicated non-perturbative bound state physics, cannot be calculated from QCD at present. It is still necessary to parameterize the DAs and to fit
the eventual free parameters to experimental data. Hence, both the models, the standard HSA as well as the diquark model, only get a predictive power when a number of reactions involving the same hadrons is investigated. In the diquark model the following DAs have been proven to work satisfactorily well in many applications [3-7]:

\[
\Phi^B_S(x)N^B_S x(1-x)^3 \exp \left[ -b^2(m_q^2/x + m_s^2/(1-x)) \right] \\
\Phi^B_V(x)N^B_V x(1-x)^3(1+5.8x-12.5x^2) \exp \left[ -b^2(m_q^2/x + m_v^2/(1-x)) \right].
\]

The constants \(N^B_S\) and \(N^B_V\) are fixed through the normalization (e.g. for the proton \(N_S^p = 25.97\), \(N_V^p = 22.92\)). The DAs exhibit a mild flavour dependence via the exponential whose other purpose is to guarantee a strong suppression of the end-point regions. The parameters appearing in the exponentials are not considered as free parameters since the final results (form factors, amplitudes) depend on their actual values only mildly. The following values for the parameters are chosen: \(b = 0.498\,\text{GeV}^{-1}\), \(m_u = m_d = 350\,\text{MeV}\), \(m_S = m_V = 580\,\text{MeV}\). It is to be stressed that the quark and diquark masses only appear in the DAs (4); in the hard scattering kinematics they are neglected.

The hard scattering amplitudes \(T_H\), determined by short-distance physics, are calculated from a set of Feynman graphs relevant to a given process. Diquark-gluon and diquark-photon vertices appear in these graphs which, following standard prescriptions, are defined as

\[
\begin{align*}
SgS & : \quad ig_s t^a (p_1 + p_2)_\mu \\
VgV & : \quad -i g_s t^a \left\{ g_{\alpha\beta}(p_1 + p_2)_\mu - g_{\beta\mu} \left[ (1 + \kappa) p_2 - \kappa p_1 \right]_\alpha \\
& \quad - g_{\mu\alpha} \left[ (1 + \kappa) p_1 - \kappa p_2 \right]_\beta \right\}
\end{align*}
\]

where \(g_s = \sqrt{4\pi\alpha_s}\) is the QCD coupling constant. \(\kappa\) is the anomalous magnetic moment of the vector diquark and \(t^a = \lambda^a/2\) the Gell-Mann colour matrix. For the coupling of photons to diquarks one has to replace \(g_s t^a\) by \(-\sqrt{4\pi\alpha e_D}\) where \(\alpha\) is the fine structure constant and \(e_D\) is the electrical charge of the diquark in units of the elementary charge. The couplings \(DgD\) are supplemented by appropriate contact terms required by gauge invariance.

The composite nature of the diquarks is taken into account by phenomenological vertex functions. Advice for the parameterization of the 3-point functions (diquark form factors) is obtained from the requirement that asymptotically the diquark model evolves into the standard HSA. Interpolating smoothly between the required asymptotic behaviour and the conventional value of 1 at \(Q^2 = 0\), the diquark form factors are actually parametrized as

\[
F_S^{(3)}(Q^2) = \frac{Q^2}{Q^2_S + Q^2} , \quad F_V^{(3)}(Q^2) = \left( \frac{Q^2_V}{Q^2_V + Q^2} \right)^2.
\]

The asymptotic behaviour of the diquark form factors and the connection to the hard scattering model is discussed in more detail in Ref. [3-4]. In accordance with the required asymptotic behaviour the \(n\)-point functions for \(n \geq 4\) are parametrized as

\[
F_S^{(n)}(Q^2) = a_S F_S^{(3)}(Q^2) , \quad F_V^{(n)}(Q^2) = \left( a_V \frac{Q^2_V}{Q^2_V + Q^2} \right)^{n-3} F_V^{(3)}(Q^2).
\]

The constants \(a_{S,V}\) are strength parameters. Indeed, since the diquarks in intermediate states are rather far off-shell one has to consider the possibility of diquark excitation and
break-up. Both these possibilities would likely lead to inelastic reactions. Therefore, we have not to consider these possibilities explicitly in our approach but excitation and break-up lead to a certain amount of absorption which is taken into account by the strength parameters. Admittedly, that recipe is a rather crude approximation for \( n \geq 4 \). Since in most cases the contributions from the \( n \)-point functions for \( n \geq 4 \) only provide small corrections to the final results that recipe is sufficiently accurate.

The diquark hypothesis has striking consequences. It reduces the effective number of constituents inside baryons and, hence, alters the power laws. In elastic baryon-baryon scattering, for instance, the usual power \( s^{-10} \) becomes \( s^{-6}F(s) \) where \( F \) represents the net effect of diquark form factors. Asymptotically \( F \) provides the missing four powers of \( s \). In the kinematical region in which the diquark model can be applied \((-t, -u \geq 4 \text{GeV}^2)\), the diquark form factors are already active, i.e. they supply a substantial \( s \) dependence and, hence, the effective power of \( s \) lies somewhere between 6 and 10. The hadronic helicity is not conserved in the diquark model at finite momentum transfer since vector diquarks can flip their helicities when interacting with gluons. Thus, in contrast to the standard HSA, spin-flip dependent quantities like the Pauli form factor of the nucleon can be calculated.

3 Electromagnetic Nucleon Form Factors

This is the simplest application of the diquark model and the most obvious place to fix the various parameters of the model. The Dirac and Pauli form factors of the nucleon are evaluated from the convolution formula (1) with the DAs (4) and the parameters are determined from a best fit to the data in the space-like region. The following set of parameters

\[
\begin{align*}
 f_S &= 73.85 \text{ MeV}, \quad Q_S^2 = 3.22 \text{ GeV}^2, \quad a_S = 0.15, \\
 f_V &= 127.7 \text{ MeV}, \quad Q_V^2 = 1.50 \text{ GeV}^2, \quad a_V = 0.05, \quad \kappa = 1.39;
\end{align*}
\]

provides a good fit of the data [5]. \( \alpha_s \) is evaluated with \( \Lambda_{QCD} = 200 \text{ MeV} \) and restricted to be smaller than 0.5. The parameters \( Q_S \) and \( Q_V \), controlling the size of the diquarks, are in agreement with the higher-twist effects observed in the structure functions of deep inelastic lepton-hadron scattering [8] if these effects are modelled as lepton-diquark elastic scattering. The Dirac and the Pauli form factors of the proton are very well reproduced. The predictions for the two neutron form factors are also in agreement with the data. However, more accurate neutron data are needed in the \( Q^2 \) region of interest in order to determine the model parameters better. The nucleon’s axial form factor [9] and its electromagnetic form factors in the time-like regions [1] have also been evaluated. Both the results compare well with data. Even electroexcitation of nucleon resonances has been investigated [10, 11]. In the case of the \( N\Delta \) form factor the model results agree very well with the data presented in [12] while the model seems to provide to large values for the Coulomb form factor [13].

4 Real Compton Scattering (RCS)

\( \gamma p \to \gamma p \) is the next reaction to which the diquark model is applied to. Since the only hadrons involved are again protons RCS can be predicted in the diquark model without any adjustable parameter. The results of the diquark model for RCS are shown in Fig. [4] for
Figure 1: (left) The scaled cross section for RCS off protons vs. $\cos \theta$ for three different photon energies. The experimental data are taken from [14].

Figure 2: (right) The integrated $\gamma\gamma \rightarrow p\bar{p}$ cross section ($|\cos \theta| \geq 0.6$). The solid line represents the diquark model prediction [7]. Data are taken from CLEO [18].

three different photon energies [4, 7]. Note that in the very forward and backward regions the transverse momentum of the outgoing photon is small and, hence, the diquark model which is based on perturbative QCD, is not applicable. Despite the rather small energies at which data [14] are available, the diquark model is seen to work rather well. The predicted cross section does not strictly scale with $s^{-6}$. The results obtained within the standard HSA are of similar quality [15]. A purely soft, overlap-like contribution can also explain these data [16]. The diquark model also predicts interesting photon asymmetries and spin correlation parameters (see the discussion in [4]). Even a polarization of the proton, of the order of 10%, is obtained [4]. This comes about as a consequence of helicity flips generated by vector diquarks and of perturbative phases produced by propagator poles appearing within the domains of the momentum fraction integrations. The appearance of phases to leading order of $\alpha_s$ is a non-trivial prediction of perturbative QCD [17]; it is characteristic of the HSA and is not a consequence of the diquark hypothesis.

Two-photon annihilation into $p\bar{p}$ pairs is related to RCS by crossing. The only difference is that now the diquark form factors are needed in the time-like region. The continuation of the diquark form factors from the space-like to the time-like region is described in [6]. The diquark model predictions for the integrated $\gamma\gamma \rightarrow p\bar{p}$ cross section is compared to the CLEO data [18] in Fig. 2. At large energies the agreement between predictions and experiment is good. The predictions for the angular distributions are in agreement with the CLEO data too. The diquark model predictions are also in agreement with the recent VENUS data [19].

5 Virtual Compton scattering (VCS)

This process is accessible through $ep \rightarrow ep\gamma$. An interesting element in that reaction is that, besides VCS, there is also a contribution from the Bethe-Heitler (BH) process where the final state photon is emitted from the electron. Electroproduction of photons offers many possibilities to test details of the dynamics: One may measure the $s$, $t$ and $Q^2$ dependence as well as that on the angle $\phi$ between the hadronic and leptonic scattering planes. This allows to isolate cross sections for longitudinal and transverse virtual photons.
Asymmetry prediction for CEBAF $s=5\text{GeV}^2$, $Q^2=1\text{GeV}^2$, $k_oL=6\text{GeV}$.

$\cos \theta = -0.6$
$\cos \theta = -0.2$
$\cos \theta = +0.2$
$\cos \theta = +0.6$

Figure 3: Diquark model predictions for the electron asymmetry in $ep \rightarrow ep\gamma$ [7].

One may also use polarized beams and targets and last but not least one may measure the interference between the BH and the VC contributions. The interference is sensitive to phase differences.

At $s$, $-t$ and $-u \gg m_p^2$ (or small $|\cos \theta|$ where $\theta$ is the scattering angle of the outgoing photon in the photon-proton center of mass frame) the diquark model can also be applied to VCS [7]. Again there is no free parameter in that calculation. The model can safely be applied for $s \geq 10\text{GeV}^2$ and $|\cos \theta| \leq 0.6$. For the future CEBAF beam energy of 6 GeV the model is at its limits of applicability. However, since the diquark model predictions for real Compton scattering agree rather well with the data even at $s \geq 5\text{GeV}^2$ (see Fig. 1) one may expect similarly good agreement for VCS. Predictions for the VCS cross section are given in [7].

Of interest is also the electron asymmetry in $ep \rightarrow ep\gamma$: 

$$A_L = \frac{\sigma(+) - \sigma(-)}{\sigma(+) + \sigma(+)}$$

where $\pm$ indicates the helicity of the incoming electron. $A_L$ measures the imaginary part of the longitudinal – transverse interference. According to the model, $A_L$ is large in the region of strong BH contamination (see Fig. 3). In that region, $A_L$ measures the relative phase between the BH amplitudes and the VCS ones. The magnitude of the effect shown in Fig. 3 is sensitive to details of the model and, therefore, should not be taken literally. Despite of this our results may be taken as an example of what may happen. The measurement of $A_L$, e.g. at CEBAF, will elucidate the underlying dynamics of VCS strikingly.

6 Summary and outlook

The diquark model which represents a variant of the HSA, combines perturbative QCD with non-perturbative elements. The diquarks represent quark-quark correlations in baryon wave functions which are modelled as quasi-elementary constituents. This model has been applied to many photon induced exclusive processes at moderately large momentum transfer (typically $\simeq 4\text{GeV}^2$). From the analysis of the nucleon form factors the parameters specifying the diquark and the DAs, are fixed. Compton scattering and two-photon annihilations of $p\bar{p}$ can then be predicted. The comparison with existing data reveals that the diquark model works quite well and in fact much better then the pure quark HSA.

Predictions for the VCS cross section and for the $ep \rightarrow ep\gamma$ cross section have also been made for kinematical situations accessible at the upgraded CEBAF and perhaps at
future high energy accelerators like ELFE@HERA. According to the diquark model the BH contamination of the photon electroproduction becomes sizeable for small azimuthal angles. The BH contribution also offers the interesting possibility of measuring the relative phases between the VC and the BH amplitudes. The electron asymmetry $A_L$ is particularly sensitive to relative phases. In contrast to the standard HSA the diquark model allows to calculate helicity flip amplitudes, the helicity sum rule does not hold at finite $Q^2$. One example of an observable controlled by helicity flip contributions is the Pauli form factor of the proton. Also in this case the diquark model accounts for the data.

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