2+1-dimansional traversable wormholes supported by positive energy

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We revisit the shapes of the throats of wormholes, including thin-shell wormholes (TSWs) in 2 + 1 dimensions. In particular, in the case of TSWs this is done in a flat 2 + 1 -dimensional bulk spacetime by using the standard method of cut-and-paste. Upon departing from a pure time-dependent circular shape i.e., \( r = a(t) \) for the throat, we employ a \( \theta \)-dependent closed loop of the form \( r = R(t, \theta) \), and in terms of \( R(t, \theta) \) we find the surface energy density \( \sigma \) on the throat. For the specific shape we find that the total energy which supports the wormhole is positive and finite. In addition to that we analyze the general wormhole’s throat and by considering a specific equation of \( r = R(\theta) \) instead of \( r = r_0 \), and upon certain choices of functions for \( R(\theta) \) we find the total energy of the wormhole positive.

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I. INTRODUCTION

In the theory of wormholes the prime important issue concerns energy which turns out to be negative (i.e. exotic matter) to resist against gravitational collapse. This and stability related matters informed by Morris and Thorne [1] were restructured later on by Hochberg and Visser [2]. Nonexistence of negative energy in classical physics / Einstein’s general relativity persisted as a serious handicap. Given this fact, and without reference to quantum theory in which negative energy has rooms at smallest scales to resolve the problem of microscopic wormholes, how can then one tackle with the large scale wormholes?. In this study, first we restrict ourselves to thin-shell wormholes (TSWs) which are tailored by the cut-and-paste technique of spacetimes [3, 4]. In our view the theorems proved in [2] for general wormholes should be taken cautiously and mostly relaxed when the subject matter is TSWs [5]. One important point that we emphasize / exploit is that the throat need not have a circular topology. It may depend on the angular variable as well, for example. This is the case that we naturally confront in static, non-spherical spacetimes. One such example is the Zipoy-Voorhees (ZV)-geometry which deviates from spherical symmetry by a deformation / oblateness parameter [6]. We employed this to show that the overall / total energy can be made positive although locally, depending on angular location it may take negative values [7]. We construct the simplest possible TSW in 2 + 1 –dimensions whose bulk is made of flat Minkowski spacetime. Such a wormhole was constructed first by Visser [3], for the spherical throat case in 3 + 1 –dimensions. In our case of 2 + 1 –dimensions the only non-zero curvature is at the throat which consists of a ring, apt for the proper junction conditions. For the shape of the throat we assume an arbitrary angular dependence in order to attain ultimately a positive total energy (i.e. normal matter). In other words, the throat surface is chosen as \( F = r - R(t, \theta) = 0 \), for an appropriate function \( R(t, \theta) \). For \( R(t, \theta) = a(t) \), we recover the circular topology considered to date. Note that \( t \) is the coordinate time measured by an external observer. As a possible choice we employ \( R(0, \theta) = R_0 (\theta) = \sqrt{\cos \frac{\pi}{2} \theta + 1} \) which represents a starfish shape. The crucial point about the path of the throat is that it must be convex rather than concave in order to attain anything but exotic matter. A circle, which is concave yields the undesired negative energy. We present various alternatives for the starfish geometrical shapes to justify our argument. The limiting case of almost zero periodic dependence on the angle brings us to that of total energy zero (=a vacuum) on the throat, which amounts to making TSW from a vacuum.

With this much information about 2 + 1 –dimensional TSWs we extend our argument to the 2 + 1 –dimensional general traversable wormhole which is considered to be a brane in 3 + 1 –dimensional flat spacetime. We provide explicit examples to show that 2 + 1 –dimensional wormholes can be fueled by a total positive energy and the null energy condition is satisfied.

Organization of the paper is as follows. In Section II we study TSWs with general throat shapes. 2 + 1 –dimensional wormholes induced from 3 + 1 –dimensional flat spacetime is considered in Section III. The paper ends with Conclusion in Section IV.

II. THIN-SHELL WORMHOLES WITH GENERAL THROAT SHAPE

In this section we consider a model of TSW in 2 + 1 –dimensional flat spacetime. Hence, the bulk metric is
given by
\[ ds^2 = -dt^2 + dr^2 + r^2d\theta^2. \] (1)
Following \cite{[1]} we introduce \( M^\pm \) as two incomplete manifolds from the original bulk and then we paste them on an identical hypersurface with equation
\[ F(t, r, \theta) = r - R(t, \theta) = 0 \] (2)
to make upon them a complete manifold known as the TSW. The throat is located on the shell \( r = R(t, \theta) \) and therefore \( R(t, \theta) \) is a general function of \( \theta \) and \( r \) but not arbitrary. As, \( r = R(t, \theta) \) is going to be the throat which connects two different spacetimes, in \( 2 + 1 \)–dimensions it must be a closed loop. We choose \( x^\alpha = (t, r, \theta) \) for the bulk and \( \xi^t = (t, \theta) \) for the hypersurface. Therefore while the bulk metric is given by
\[ g_{\mu\nu} = \text{diag} \left(-1, 1, r^2 \right), \] (3)
the induced metric on the shell \( h_{ij} \) is obtained by using
\[ h_{ij} = \frac{\partial x^\alpha}{\partial \xi^i} \frac{\partial x^\beta}{\partial \xi^j} g_{\alpha\beta}. \] (4)
One finds
\[ ds_\Sigma^2 = - \left( 1 - \dot{R}^2 \right) dt^2 + (R^2 + R'^2) d\theta^2 + 2R'R'dtd\theta \] (5)
in which a prime and a dot stand for derivative with respect to \( \theta \) and \( t \), respectively. Next, we find the extrinsic curvature tensor defined as
\[ K^\pm_{ij} = -n^\pm_\gamma \left( \partial^2 x^\gamma_{\partial \xi^i \partial \xi^j} + \Gamma^\gamma_{\alpha \beta} \frac{\partial x^\alpha}{\partial \xi^i} \frac{\partial x^\beta}{\partial \xi^j} \right), \] (6)
in which
\[ n^\pm_\gamma = \pm \frac{1}{\sqrt{\Delta}} \frac{\partial F(t, r, \theta)}{\partial x^\gamma} \] (7)
with
\[ \Delta = \frac{\partial F(t, r, \theta)}{\partial x^\alpha} \frac{\partial F(t, r, \theta)}{\partial x^\beta} g^{\alpha\beta}. \] (8)
Using (2), we find
\[ \Delta = 1 + \left( \frac{R'}{R} \right)^2 - \dot{R}^2. \] (9)
The exact form of the normal vector is found to be
\[ n^\pm_t = \pm \frac{1}{\sqrt{\Delta}} \left(-\dot{R} \right), \] (10)
\[ n^\pm_r = \pm \frac{1}{\sqrt{\Delta}} \] (11)
and
\[ n^\pm_\theta = \pm \frac{1}{\sqrt{\Delta}} \left(-R' \right), \] (12)
such that \( |n| = 1 \). The bulk’s line element (1) admits \( \Gamma^\theta_{\theta \tau} = \Gamma^\theta_{\tau \theta} = \frac{1}{r} \), \( \Gamma^\theta_{\theta \theta} = -r \) while the rest of Christoffel symbols are zero. Therefore, the extrinsic curvature tensor elements become
\[ K^+_t\theta = -n^+_r R' - n^+_\theta \left( \frac{\dot{R}}{R} \right) \] (13)
\[ K^+_t\tau = -n^+_r \ddot{R} \] (14)
\[ K^+_\theta\theta = -n^+_r \left( R'' - R - 2n^+_\theta \frac{R'}{R} \right). \] (15)
Israel junction conditions \cite{[3]} read
\[ k^i_j - k\delta^i_j = -8\pi S^j_i \] (16)
in which
\[ S^j_i = \begin{pmatrix} -\sigma & q_1 \\ q_2 & p \end{pmatrix} \] (17)
is the energy-momentum tensor on the thin-shell, \( k^i_j = K^+_i_j - K^-_i_j \) and \( k = \text{trace} \left( k^i_j \right) \). Note that \( q_1 \) and \( q_2 \) are appropriate pressure terms. Combining the results found above we get
\[ k_{tt} = -\frac{2\ddot{R}}{\sqrt{\Delta}} \] (18)
\[ k_{\theta\theta} = -2 \frac{\Delta}{\sqrt{\Delta}} \left( R'' - R - 2R'^2 \frac{R'}{R} \right) \] (19)
and
\[ k_{t\theta} = -\frac{2}{\sqrt{\Delta}} \left( \dot{R}' - \frac{R'\ddot{R}}{R} \right). \] (20)
Furthermore, one finds
\[ k^{\pm}_t = h^{tt} k_{tt} + h^{t\theta} k_{t\theta} \] (21)
\[ k^{\pm}_\theta = h^{\theta\theta} k_{\theta\theta} + h^{t\theta} k_{tt} \] (22)
\[ k^{\pm}_t = h^{tt} k_{tt} + h^{t\theta} k_{t\theta} \] (23)
and
\[ k^{\pm}_\theta = h^{tt} k_{tt} + h^{t\theta} k_{t\theta}. \] (24)
We recall that
\[ h_{ij} = \begin{pmatrix} -B & H \\ H & A \end{pmatrix} \] (25)
which implies
\[ k^{ij} = \left( \frac{A}{\sqrt{B - H^2}} \frac{H}{\sqrt{A - B + H^2}} \right) \]

in which \( A = (R'^2 + R^2), \ B = 1 - \dot{R}^2 \) and \( H = R \dot{R}' \).

Considering (26) in (21-25) we find
\[ k_t = -2 \frac{[\dot{R}^2 (\dot{R} - \frac{R \ddot{R}}{R}) - (R'^2 + R^2) \dot{R}]}{[R'^2 + R^2]}; \]
\[ k^\theta = -2 \frac{[R'' - R - 2 \frac{R'^2}{R} + \ddot{R} (\ddot{R} - \frac{R \dddot{R}}{R})]}{[R'^2 + R^2]}; \]
\[ k^\phi = -2 \frac{[\dot{R} \dot{R}' \dddot{R} + (1 - \dot{R}^2) (\dot{R} - \frac{R \dddot{R}}{R})]}{[R'^2 + R^2]}; \]
\[ k^\rho = -2 \frac{[\dot{R} \dddot{R}' - (R'^2 + R^2) (\dot{R}' - \frac{R \dddot{R}}{R})]}{[R'^2 + R^2]}. \]

Therefore the Israel junction conditions imply
\[ \sigma = \frac{1}{8 \pi} k^\rho \]
and
\[ p = \frac{1}{8 \pi} k_t. \]

In static equilibrium, one may set \( R = R_0 (\theta) \) and \( \dot{R} = \ddot{R} = 0 \) which consequently yield
\[ \sigma_0 = \frac{1}{4 \pi} \frac{[R''_0 - R_0 - 2 \frac{R'^2}{R_0}]}{(R'^2 + R^2_0) \sqrt{1 + (\frac{R'_0}{R_0})}}, \]
and
\[ p_0 = q_10 = q_{20} = 0. \]

This is not surprising since the bulk spacetime is flat. Therefore in static equilibrium, the only nonzero component of the energy-momentum tensor on the throat is the energy density \( \sigma_0 \). We note that the total matter supporting the wormhole is given by
\[ \Omega = \int_0^{2\pi} \int_0^\infty \sqrt{-g} \sigma_0 \delta (r - R) \, dr \, d\theta. \]

In the sequel we consider the various possibilities of the shape of the throat including the circular one. The first case to be checked is the circular throat i.e., \( R_0 = 1 \). This leads to \( \sigma_0 = \frac{1}{4 \pi} \frac{1}{R_0} \) and clearly violates the null energy condition which states that \( \sigma_0 + p_0 \geq 0 \).

For a specific function of \( R_0 (\theta) \), there are four different possibilities: i) \( \sigma_0 < 0 \) on entire domain of \( \theta \in [0, 2\pi] \), ii) \( \sigma_0 \leq 0 \) or \( \sigma_0 \geq 0 \) but the total energy \( \Omega < 0 \), iii) \( \sigma_0 \leq 0 \) or \( \sigma_0 \geq 0 \) with the total energy \( \Omega > 0 \) and iv) \( \sigma \geq 0 \). Herein,
\[ \Omega = \int_0^{2\pi} R_0 \sigma_0 d\theta, \]
is the total energy on the throat. In what follows we present illustrative examples for all cases. We note that the specific cases given below can be easily replaced by other functions but we must keep in mind that although \( R_0 (\theta) \) is a general function, \( r = R_0 \) must present a closed path in \( 2 + 1\)-dimensions.

FIG. 1: The geometry of the throat for \( R_0 (\theta) = \frac{1}{0.5 \cos^2 \theta + 1} \) with its energy density distribution \( \sigma_0 \). We see that the signature of the curvature is positive everywhere and as a result the matter is exotic everywhere.

FIG. 2: The geometry of the throat when \( R_0 (\theta) = \frac{1}{0.5 \cos^2 \theta + 1} \) and its energy density distribution \( \sigma_0 \). This figure shows that \( \sigma_0 \) is positive when the curvature is negative and vice versa. The total energy which supports the wormhole, however, is negative.
FIG. 3: The geometry of the throat when $R_0(\theta) = \frac{1}{\cos(\frac{\theta}{2}) + 1}$ and $\sigma_0$ in terms of $\theta$. Note that $\sigma_0$ is overwhelmingly positive and so is the total energy.

FIG. 4: The geometry of the throat when $R_0(\theta) = \frac{1}{\sqrt{|\cos(\frac{\theta}{2})| + 1}}$ and $\sigma_0$ in terms of $\theta$. In this figure $\sigma_0$ is positive everywhere as the curvature is positive, and the total energy is positive.

A. $\sigma_0 < 0, \Omega < 0$

In the first example, the throat is deformed from a perfect circle to an oval shape given by

$$R_0(\theta) = \frac{1}{0.5 \cos^2 \theta + 1}. \quad (37)$$

The shape of the throat and $\sigma_0$ are shown in Fig. 1a and 1b, respectively. As we observe here in Fig. 1b, energy density is negative everywhere for $\theta \in [0, 2\pi]$. This is also seen from the shape of the throat whose curvature is positive on $\theta \in [0, 2\pi]$. Although the total exotic matter for throat of the form of a circle of radius one is $-0.5$, in the case of (37) the total exotic matter is $-0.48111$ in geometrical unit. This shows that a small deformation causes the total exotic matter to be less.

B. $\sigma_0 \leq 0, \Omega < 0$

As our second case we consider

$$R_0(\theta) = \frac{1}{0.5 \cos^2 (3\theta) + 1}. \quad (38)$$

The shape of the throat looks like a starfish as it is displayed in Fig. 3a. The behavior of the energy density $\sigma_0$ is depicted in Fig. 3b. As it is clear $\sigma_0$ is positive everywhere except at the neighborhood of the corners of the throat where the curvature is positive. The total energy, however, is positive i.e., $\Omega = 0.38888$ unit.

C. $\sigma_0 \leq 0, \Omega > 0$

For

$$R_0(\theta) = \frac{1}{\cos \left(\frac{\theta}{2}\right) + 1}. \quad (39)$$

the shape of the throat is given. The corresponding energy density $\sigma_0$ is shown in Fig. 3b. As one can see the energy density is positive wherever the curvature of the throat is negative and vise versa. The overall energy is negative given by $\Omega = -0.39339$ unit.

D. $\sigma_0 > 0, \Omega > 0$

For this case let's consider

$$R_0(\theta) = \frac{1}{\sqrt{|\cos \left(\frac{\theta}{2}\right)| + 1}}. \quad (40)$$

which admits a throat of the shape shown in Fig. 2a. The corresponding energy density $\sigma_0$ is shown in Fig. 2b. As one can see the energy density is positive wherever the curvature of the throat is negative and vise versa. The overall energy is negative given by $\Omega = -0.39339$ unit.
which is shown in Fig. 4a. In this case $\sigma_0$ is positive everywhere as it is shown in Fig. 4b and the total energy is positive i.e. $\Omega = 0.40561$ unit.

### E. Parametric ansatz for $R_0(\theta)$

To complete our analysis we look at the case given in Section B and generalize the form of $R_0(\theta)$ as given by

$$R_0(\theta) = \frac{1}{\epsilon \cos^2(\theta) + 1} \quad (41)$$

in which $\epsilon \in \mathbb{R}^+$ and $n = 2, 3, 4, \ldots$. In Fig. 5 we plot $R_0(\theta)$ in terms of different $n$ and $\epsilon = 0.1$. The total exotic matter of each case is also calculated. We observe that increasing $n$ decreases the magnitude of the exotic matter. When $n$ goes to infinity (i.e. an infinite oscillation) the total energy goes to zero while at each point the energy density shows a positive or negative fluctuation. Of course the assumption of infinite frequency takes us away from the domain of classical physics, probably to the quantum domain. In the latter, particle creation from a vacuum is well-known. Herein, instead of particles we have formation of wormholes. The ansatz (41) shows how we go to the vacuum case $\Omega \to 0$ with $n \to \infty$. Yet we wish to abide by the classical domain with $n \ll \infty$.

### III. 2+1-DIMENSIONAL WORMHOLE INDUCED BY 3+1-DIMENSIONAL FLAT SPACETIME

We consider the $3+1$-dimensional Minkowski spacetime in the cylindrical coordinates

$$ds^2 = -dt^2 + dr^2 + dz^2 + r^2 d\theta^2 \quad (42)$$

with the substitution $z = \xi(r, \theta)$. This gives the line element

$$ds^2 = -dt^2 + \left(1 + \xi_r(r, \theta)^2\right) dr^2 + \left(r^2 + \xi_\theta(r, \theta)^2\right) d\theta^2 + 2\xi_r(r, \theta) \xi_\theta(r, \theta) \, dr \, d\theta \quad (43)$$

in which $\xi_r(r, \theta) = \frac{\partial \xi(r, \theta)}{\partial r}$ and $\xi_\theta(r, \theta) = \frac{\partial \xi(r, \theta)}{\partial \theta}$ and $\xi(r, \theta)$ is a function of $r$ and $\theta$. Using this line element, the Einstein’s tensor is obtained with only one nonzero component i.e.,

$$G^\xi_\mu = -\rho \frac{r^3 \xi_\xi r^2 + \xi_\xi \xi_\xi \xi_\theta r^2 - \xi_\theta^2 + 2\xi_\xi \xi_\xi \xi_\theta r - \xi_\xi^2 (r^2 + \xi_\theta^2)^2}{(r^2 + \xi_\theta^2 + \xi_\xi^2 r^2)^2} \quad (44)$$

Einstein’s equation $(8\pi G = 1 = c)$ reads

$$G^\nu_\mu = T^\nu_\mu \quad (45)$$

in which $T^\nu_\mu$ is the energy momentum tensor. The latter implies that the only non-zero component of the energy-momentum tensor is $T^t_t = -\rho$ component and therefore

$$\rho = \frac{r^3 \xi_\xi r^2 + \xi_\xi \xi_\xi \xi_\theta r^2 - \xi_\theta^2 + 2\xi_\xi \xi_\xi \xi_\theta r - \xi_\xi^2 (r^2 + \xi_\theta^2)^2}{(r^2 + \xi_\theta^2 + \xi_\xi^2 r^2)^2} \quad (46)$$

The total energy which supports the wormhole is obtained by

$$\Omega = \int_0^{2\pi} \int_0^\infty \rho \sqrt{-g} dr d\theta. \quad (47)$$

### A. Flare-out conditions

To have a wormhole we observe that $z = \xi(r, \theta)$ must be chosen aptly for a general wormhole structure. For instance, one may consider in the first attempt $\xi(r, \theta) = \xi(r)$ and following that we find

$$ds^2 = -dt^2 + \left(1 + \xi'(r)^2\right) dr^2 + r^2 d\theta^2, \quad (48)$$

so that the form of energy density becomes

$$\rho = \frac{\xi''}{r (1 + \xi'^2)^2}. \quad (49)$$

The expression (48) is comparable with the Morris-Thorne’s static wormhole

$$ds^2 = -e^{2\Phi(r)} dt^2 + \frac{1}{1 - \frac{b(r)}{r}} dr^2 + r^2 d\theta^2, \quad (50)$$

in which $\Phi(r)$ and $b(r)$ are the red-shift and shape functions, respectively. In the specific case (48), one finds $\Phi(r) = 0$ and $b(r) = r \left(\frac{\xi'(r)^2}{1 + \xi'(r)^2}\right)$. The well-known flare-out condition introduced by Morris and Thorne implies

![FIG. 6: The geometry of the throat when $R_0(\theta) = 1$ for (a) and $R_0(\theta) = \frac{1}{0.5 \cos^2 \theta + 1}$ and for (b) with $r_0 = 1$.](image)
if \( r = r_0 \) is the location of the throat, i) \( b(r_0) = r_0 \) and ii) for \( r > r_0 \), \( b'(r) < b(r) \). In terms of the new setting, i) implies that at the throat \( \xi' = \pm \infty \) and ii) states that \( \xi'' < 0 \) for \( r > r_0 \). In addition to these conditions at the throat we have \( z = \xi(r_0) = 0 \).

Next, we introduce the location of the throat at \( z = 0 \) and \( r = R_0(\theta) \) in which \( \theta \) is a periodic function of \( \theta \). These mean that \( z = \xi(R_0, \theta) = 0 \). Now, for a general function for \( z = \xi(r, \theta) \) we impose the same conditions as the Morris-Thorne wormholes i.e., \( \xi, \xi_r, \xi_{rr} < 0 \) for \( r > R_0(\theta) \) and at the location of the throat (where \( z = 0 \) and \( r = R_0(\theta) \) ) \( \xi_r(R_0, \theta) = \pm \infty \).

### B. An Illustrative Example

Here we present an explicit example. Let’s consider

\[
\xi = \pm 2r_0 \sqrt{\left( \frac{r}{R_0(\theta)} - 1 \right)}
\]  

(51)

in which the first condition i.e., \( \xi = 0 \) at the location of the throat \( r = R_0(\theta) \) is fulfilled. Next, the expression,

\[
\xi_r \xi_{rr} = \frac{1}{2} \frac{r_0^2}{(1 - \frac{r}{R_0})^2} R_0^3 < 0
\]

(52)

imposes \( R_0(\theta) > 0 \) on the entire domain of \( \theta \) i.e. \( \theta \in [0, 2\pi] \). We note also that \( R_0(\theta) \) must be a periodic function of \( \theta \) to have \( r = r_0 R_0(\theta) \) a closed loop, which is going to be our throat. The forms of

\[
\xi_r = \frac{\pm r_0}{R_0 \sqrt{\frac{r}{R_0} - 1}}
\]

(53)

and

\[
\xi_{rr} = -\frac{\pm r_0}{2R_0^2 \left( \frac{r}{R_0} - 1 \right)^{3/2}}
\]

(54)

suggest that at the throat \( \xi_r \to \pm \infty \) and also \( \xi_{rr} \to -\infty \) as expected. The form of energy density in terms of \( R \), however, becomes

\[
\rho = \frac{R_0^3 r_0^2 \left( R_0'' R_0 - R_0'^2 - 2R_0'^2 \right)}{2r_0 R_0' \left( r_0^2 R_0'^2 + (r_0^2 + r R_0 - R_0^2) \right)^2}
\]

(55)

The latter implies that any periodic function of \( R_0(\theta) \) which keeps \( R_0'' R_0 - R_0'^2 - 2R_0'^2 > 0 \) can represent a traversable wormhole with positive energy. In the case of \( R_0 = r_0 \) or \( \xi = \pm 2r_0 \sqrt{\frac{r}{r_0} - 1} \), the wormhole is shown in Fig. 6 whose energy density is given by

\[
\rho = \frac{r_0}{2r_0^3}.
\]

(56)

In Figs. 6-8 we plot the wormholes with \( R_0(\theta) \) given in (37), (38), (39), (40) and (41), respectively. Also in Fig. 9, the energy density \( \rho \) corresponds to the individual cases of (a), (b), (c) and (d) for (37), (38), (39) and (40), respectively which are given in terms of \( r \) and \( \theta \) with \( r_0 = 1 \). We see, for instance, that in Fig. 9d the energy is positive everywhere.
FIG. 9: The energy densities of the wormholes given in (37) for (a), (38) for (b), (39) for (c) and (40) for (d). As we observe, in the cases (b) and (c) energy density gets positive value for some interval while for (d) \( \rho > 0 \) everywhere. In (a) the energy density is negative everywhere.

IV. CONCLUSION

Our principal aim in this study is to establish a traversable wormhole with normal (i.e. non-exotic) matter in 2 + 1−dimensions. For the TSWs the strategy is to assume a closed angular path of the form \( r = R(t, \theta) \), where for \( R(t, \theta) = a(t) \), we recover the circular throat topology. Since this leads to exotic matter it is not attractive by our assessment. Let us note that we use the coordinate time instead of the proper time. Our analysis shows that any concave-shaped \( R_0(\theta) = R(0, \theta) \) around the origin undergoes the same fate. However, a convex-shaped \( R_0(\theta) \) seems promising in obtaining a normal matter. This is shown by explicit ansatzes whose plots suggest starfish-shaped closed curves for the throat of a 2 + 1−dimensional wormhole. Locally, for specific angular range it may yield negative energy, but in total the energy accumulates on the positive side. This result supports our previous finding that for the non-spherical spacetimes, i.e. the ZV-metrics, the throats can be non-spherical and in turn one may obtain a TSW in Einstein’s theory with a positive total energy [7]. The conclusion drawn herein for 2+1−dimensions therefore can be generalized to higher dimensions without much effort. A similar construction method has been employed for the general wormholes. We have treated the 2 + 1−dimensional wormhole as a brane in 3 + 1−dimensional Minkowski space and show the possibility of physical traversable wormhole. It turns out, however, that the rabbit emerges from only very special hats, not from all hats.

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