Low-lying $2^+$ states in neutron-rich oxygen isotopes in quasiparticle random phase approximation

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Abstract

The properties of the low-lying, collective $2^+_1$ states in neutron-rich oxygen isotopes are investigated in the framework of self-consistent microscopic models with effective Skyrme interactions. In RPA the excitation energies $E_{2^+_1}$ can be well described but the transition probabilities are much too small as compared to experiment. Pairing correlations are then accounted for by performing quasiparticle RPA calculations. This improves considerably the predictions of $B(E2)$ values and it enables one to calculate more reliably the ratios $M_n/M_p$ of neutron-to-proton transition amplitudes. A satisfactory agreement with the existing experimental values of $M_n/M_p$ is obtained.

_PACS numbers:_ 23.20.Js, 21.60.Jz, 21.10.Re
The prospects of nuclear physics studies with nuclei far from stability open up a wide range of possibilities for refining our understanding of nuclear properties in terms of microscopic descriptions and effective nucleon-nucleon interactions. One of the important aspects is the ability to disentangle neutron and proton contributions to collective transitions between low-lying states and the ground state. Experimentally, this can be done in a phenomenological way \[1\] by combining the information obtained in measurements involving various hadronic probes and purely electromagnetic probes. For instance, numerous experimental studies have been performed on \(^{18}\text{O}\) using different hadronic probes like proton \[2\], neutron \[3\], or pion \[4,5\] scattering. More recently, proton scattering on \(^{20}\text{O}\) \[6\] yielded information on the first \(2^+\) state in this unstable neutron-rich oxygen isotope. On the other hand, studies involving only electromagnetic properties such as electron scattering \[7\], Coulomb excitation or lifetime measurement \[8\] have also been done for these nuclei. While the excitation processes of purely electromagnetic nature are sensitive only to the protons and give access to the proton transition amplitudes and transition densities, the hadronic processes are sensitive to both proton and neutron transition densities. Therefore, it is possible by a combined analysis of the data from electromagnetic and hadronic processes to determine experimentally for a given excited state the transition amplitudes \(M_p\) and \(M_n\) corresponding to protons and neutrons, respectively \[1\]. Proton scattering experiments yielding \(M_n/M_p\) values have been recently performed on neutron-rich sulfur and oxygen isotopes \[6,9,10\].

In this work, we present microscopic calculations of low-lying \(2^+\) states in neutron-rich oxygen isotopes. These calculations are based on effective Skyrme interactions and they are performed in the framework of the random phase approximation (RPA) and the quasiparticle random phase approximation (QRPA). In microscopic models the properties of the states depend on two main inputs, the single-particle spectrum and the residual two-body interaction. In the present approach these two features are linked since the same effective interaction determines the Hartree-Fock (HF) single-particle spectrum and the residual particle-hole interaction. This approach has proved to be an efficient mean to predict properties of collective excitations like giant resonances \[11\] and it has also been used for calculating low-lying collective states in closed-shell nuclei \[12\].

In unstable nuclei we usually don’t deal with closed-shell or closed-subshell systems and therefore, the HF and RPA calculations must be done with additional approximations. The HF calculations are carried out assuming spherical symmetry and using the standard filling approximation with equal occupation numbers for all \((jm)\)-substates of the partially filled \(j\)-subshell. The RPA calculations are then to be carried out taking into account these occupation numbers. However, pairing correlations can be important in such nuclei and they must be taken into account. Their effects will be described by HFBCS calculations for the ground states and by QRPA calculations for the excitation spectra.

The HF and HFBCS method in spherical nuclei with Skyrme interactions is well-known \[13,14\]. For the pairing interaction we simply choose a constant gap given by \[15\]:

\[
\Delta = 12A^{-\frac{1}{3}}\text{MeV}\ .
\]

In a more realistic treatment of the pairing, the gap would depend on the single-particle state considered and it would tend to zero when the subshell is far from the Fermi level. Thus, in the constant gap approximation it is necessary to introduce a cut-off in the single-particle space. Above this cutoff subshells don’t participate to the pairing effect. In the case
of oxygen isotopes, we choose the BCS subspace to include the $1s, 1p$ and $2s - 1d$ major shells.

The results of the HF and HFBCS calculations performed for the nuclei $^{18,20,22}\text{O}$ using typical Skyrme interactions are summarized in Table I. For all the interactions, the binding energies per particle decrease with increasing neutron number but SGII has a different behavior as compared to the other interactions and it predicts larger values of $B/A$. The pairing correlations decrease $B/A$ by about 4% in all cases. The neutron radii increase substantially from $^{18}\text{O}$ to $^{22}\text{O}$ and the proton radii also increase slightly as a result of neutron-proton attraction. The effect of neutron pairing correlations is to redistribute neutron densities to the tail region and therefore, this leads to a small increase in $r_{n}$ (and also in $r_{p}$ for the reason mentioned above).

The HF-RPA model with Skyrme effective forces is also well-known. We only mention that in this work we solve the RPA equations in configuration space, choosing the particle-hole space so as to exhaust the energy-weighted sum rules. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic oscillator basis [17]. To generalize the HF-RPA to the QRPA model we follow the standard procedure [18]. Denoting by $a_{\alpha}^{\dagger}, a_{\alpha}$ the creation and annihilation operators of a particle in a HF state $\alpha = (j_{\alpha}, m_{\alpha})$ and by $c_{\alpha}^{\dagger}, c_{\alpha}$ the corresponding operators for a quasiparticle state, we have:

$$c_{j_{\alpha}m_{\alpha}}^{\dagger} = u_{\alpha} a_{j_{\alpha}m_{\alpha}}^{\dagger} - v_{\alpha} (-1)^{j_{\alpha} + m_{\alpha}} a_{j_{\alpha} - m_{\alpha}},$$  

where the BCS amplitudes $u_{\alpha}, v_{\alpha}$ satisfy the normalization condition:

$$u_{\alpha}^2 + v_{\alpha}^2 = 1.$$  

These amplitudes are determined by solving the BCS equations. One can then build the two-quasiparticle creation operators in an angular momentum coupled scheme:

$$C_{\alpha\beta}^{\dagger}(JM) = (1 + \delta_{\alpha\beta})^{-1/2} \sum_{m_{\alpha}m_{\beta}} \langle j_{\alpha}j_{\beta}m_{\alpha}m_{\beta}|JM\rangle c_{\alpha}^{\dagger}c_{\beta}^{\dagger}. $$

In QRPA the nuclear excitations correspond to phonon operators which are linear combinations of two-quasiparticle creation and annihilation operators:

$$Q^{\nu\dagger}(JM) = \sum_{\alpha \geq \beta} X^{\nu}_{\alpha\beta}(J) C_{\alpha\beta}^{\dagger}(JM) + (-1)^M Y^{\nu}_{\alpha\beta}(J) C_{\alpha\beta}(J - M).$$

Making use of the condition that the QRPA ground state $|\tilde{0}\rangle$ is a vacuum of phonon:

$$Q^{\nu}(JM)|\tilde{0}\rangle = 0,$$

one can then derive the QRPA equations whose solutions yield the excitation energies $E_{\nu}$ and amplitudes $X^{\nu}_{\alpha\beta}, Y^{\nu}_{\alpha\beta}$ of the excited states.

An important quantity that characterizes a given state $\nu = (E_{\nu}, LJ)$ is its transition density:

$$\delta \rho^{\nu}(r) \equiv \langle \nu | \sum_{i} \delta(r - r_{i}) | \tilde{0}\rangle,$$

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and a similar definition of the neutron (proton) transition density $\delta \rho_\nu (\delta \rho_\nu)$ with the summation in eq. (7) restricted to neutrons (protons). In QRPA the radial part of the transition density is:

$$\delta \rho_\nu (r) = \sum_{\alpha \geq \beta} \varphi_\alpha (r) \varphi_\beta^* (r) < \beta | Y_{L0} | \alpha > \{ X_{\alpha \beta} (J) - Y_{\alpha \beta} (J) \} \{ u_\alpha v_\beta + (-1)^J v_\alpha u_\beta \}$$,  \hspace{1cm} (8)$$

where $\varphi_\alpha (r)$ is the radial part of the wavefunction of the quasiparticle state $\alpha$. As an example the QRPA neutron and proton transition densities of the first $2^+$ state in $^{20}\text{O}$ calculated with the interaction SGII are shown in Fig.1. The neutron transition density is shifted outwards as compared to the proton transition density due to the presence of a neutron skin. Clearly, the two transition densities do not scale like $N/Z$ as it is sometimes assumed and this has quantitative consequences as we shall see below.

The neutron and proton matrix elements $M = < \nu | r^L Y_{L0} | \tilde{0} >$ of a multipole operator are obtained by integrating the corresponding transition densities over $r$:

$$M_{n,p} = \int \delta \rho_\nu (r) r^{L+2} dr$$,  \hspace{1cm} (9)$$

and the reduced electric multipole transition probabilities are calculated as:

$$B(EL)_{n,p} = |M_{n,p}|^2$$,  \hspace{1cm} (10)$$

We have calculated the $J^\pi = 2^+$ states in $^{18,20,22}\text{O}$ using RPA and QRPA with SIII, SGII and SLy4 interactions. The energies and $B(E2)_p$ values of the first $2^+$ states are shown in Fig.2 together with the existing experimental values in $^{18}\text{O}$ and $^{20}\text{O}$. The data come from experiments involving electromagnetic processes such as Coulomb excitation or lifetime measurement [8]. For the $E2^+$ energies, standard RPA reproduces very well experimental values especially for SGII and SLy4. On the other hand, QRPA deteriorates this agreement and it predicts the first $2^+$ states at somewhat higher energies. The theoretical prediction of the energies of low-lying states in models based on a HF or HFBCS mean field is a delicate task because these energies are sensitive to the spin-orbit part of the mean field while the spin-orbit component of the two-body effective interaction is not so well determined. For the three interactions used here there is a clear difference in the QRPA $E2^+$ energies between SGII and the other interactions. In the case of $B(E2)_p$ values RPA predicts too small values with the three interactions. However, the agreement becomes very good for QRPA both in $^{18}\text{O}$ and $^{20}\text{O}$, especially for SGII and SLy4 interactions. This indicates that pairing effects are important for transition probabilities of the first $2^+$ state in these nuclei.

In Fig.3 are shown the ratios $M_{n,p}$ calculated with the three interactions within RPA and QRPA. On the same figure are displayed the experimental values taken from ref [6]. It can be seen that the RPA results are somewhat larger than those of QRPA due to the very small $B(E2)_p$ values obtained in RPA. In comparison with the data, the QRPA results are in good agreement in $^{20}\text{O}$ whereas they are slightly too large in $^{18}\text{O}$. The three interactions predict different $A$ dependence in these oxygen isotopes for the QRPA ratios. It would be interesting to obtain the experimental value of $M_{n,p}$ in $^{22}\text{O}$ as well as the corresponding $B(E2)$ transition probability. If one assumes the quadrupole excitation to be purely isoscalar the ratio $M_{n,p}$ would be equal to $N/Z$. Taking as a guideline the QRPA results calculated with SGII one sees that the ratio of neutron-to-proton transition
amplitudes is about \((1.8 - 2.0)N/Z\), thus indicating that the low-lying \(2^+\) state has an important isovector component. This is not surprising since in these neutron-rich nuclei there are neutron particle-hole configurations at low energy which have no counterpart on the proton side and therefore, these neutron configurations necessarily introduce both isoscalar and isovector type of excitations.

In summary, we have investigated the properties of the low-lying, collective \(2^+_1\) states in neutron-rich oxygen isotopes in the framework of self-consistent microscopic models. Within the RPA model the excitation energy \(E_{2^+_1}\) can be well described but the transition probabilities are much too small as compared to experiment. In these open subshell nuclei the pairing correlations can be important and therefore, we have extended the previous model to the HFBCS approximation at the mean field level and we have performed QRPA calculations for the excited states. The quasiparticle microscopic description improves considerably the predictions of \(B(E2)\) values and it enables us to calculate more reliably the ratios \(M_n/M_p\) of neutron-to-proton transition amplitudes. These calculated values differ noticeably from the naive \(N/Z\) estimate and they are in satisfactory agreement with experiment. However, the QRPA overestimates the \(E_{2^+_1}\) energies. The sensitivity of positive parity low-lying states to the effective interaction should give one a handle on some specific components of the force, for instance the two body spin-orbit part.

Further proton scattering results on \(^{18}\text{O}\) and \(^{20}\text{O}\) will be available soon \(^{[13]}\), yielding energies and \(M_n/M_p\) ratio for the first \(2^+\) and \(3^-\) states. It would also be useful to perform experiments on more neutron-rich oxygen isotopes to establish firmly the trend of \(M_n/M_p\) as a function of \(N/Z\).

We would like to thank G. Colò, T. Suomijärvi and C. Volpe for useful discussions.
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Figure captions

**Figure 1.** Neutron and proton transition densities of the first $2^+$ state in $^{20}$O, calculated in QRPA with interaction SGII.

**Figure 2.** Energies and B(E2)$_p$ values of the first $2^+$ states in oxygen isotopes. Open and black symbols correspond to RPA and QRPA calculations, respectively. Three effective interactions are used: SIII (circles), SGII (triangles), SLy4 (stars). Experimental values are shown as crosses, with error bars for B(E2)$_p$.

**Figure 3.** The $M_n/M_p$ ratios in oxygen isotopes. The notations are the same as in Fig. 2.
|     | \(^{18}\)O | \(^{20}\)O | \(^{22}\)O |
|-----|------------|------------|------------|
|     | \(r_n\) (fm) | \(r_p\) (fm) | B/A (MeV) | \(r_n\) (fm) | \(r_p\) (fm) | B/A (MeV) | \(r_n\) (fm) | \(r_p\) (fm) | B/A (MeV) |
| SIII |            |            |           |            |            |           |            |            |           |
| HF   | 2.77       | 2.65       | 7.74      | 2.89       | 2.67       | 7.58      | 2.97       | 2.69       | 7.48      |
| BCS  | 2.79       | 2.67       | 7.43      | 2.91       | 2.69       | 7.28      | 3.03       | 2.71       | 7.16      |
| SGII |            |            |           |            |            |           |            |            |           |
| HF   | 2.76       | 2.64       | 8.30      | 2.87       | 2.65       | 8.15      | 2.95       | 2.66       | 8.07      |
| BCS  | 2.80       | 2.66       | 7.95      | 2.92       | 2.68       | 7.79      | 3.02       | 2.69       | 7.69      |
| SLy4 |            |            |           |            |            |           |            |            |           |
| HF   | 2.83       | 2.69       | 7.74      | 2.94       | 2.70       | 7.57      | 3.02       | 2.71       | 7.46      |
| BCS  | 2.85       | 2.72       | 7.42      | 2.98       | 2.73       | 7.25      | 3.08       | 2.74       | 7.13      |

TABLE I.
Neutron and proton r.m.s. radii, and binding energy per particle in the nuclei \(^{18}\)O, \(^{20}\)O and \(^{22}\)O calculated with interactions SIII \([14]\), SGII \([11]\) and SLy4 \([16]\). The rows labeled HF and BCS show Hartree-Fock and Hartree-Fock-BCS results, respectively.
\[ \rho(r) = \rho_{\text{SGII}}(r) \]
\[ \delta \rho_p \]
\[ \delta \rho_n \]
