Heavy quark spin multiplet structure of $P(\ast)\Sigma_{Q}^{(\ast)}$ molecular states

Yuki Shimizu, 1 Yasuhiro Yamaguchi, 2 and Masayasu Harada 1

1 Department of Physics, Nagoya University, Nagoya 464-8602, Japan
2 Theoretical Research Division, Nishina Center, RIKEN, Hirosawa, Wako, Saitama 351-0198, Japan

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We study the structure of heavy quark spin (HQS) multiplets for heavy meson-baryon molecular states in a coupled system of $P(\ast)\Sigma_{Q}^{(\ast)}$, with constructing the one-pion exchange potential with S-wave orbital angular momentum. Using the light cloud spin basis, we find that there are four types of HQS multiplets classified by the structure of heavy quark spin and light cloud spin. The multiplets which have attractive potential are determined by the sign of the coupling constant for the heavy meson-pion interactions. Furthermore, the difference in the structure of light cloud spin gives the restrictions of the decay channel, which implies that the partial decay width has the information for the structure of HQS multiplets. This behavior is more likely to appear in hidden-bottom sector than in hidden-charm sector.

I. INTRODUCTION

The exotic hadrons are the very interesting research subjects in hadron and nuclear physics. In 2015, the Large Hadron Collider beauty experiment (LHCb) collaboration announced the observation of two hidden-charm pentaquarks, $P_{c}^{+}(4380)$ and $P_{c}^{+}(4450)$, in the decay of $\Lambda_{b} \to J/\psi K^{-}p$ [1][3]. Their masses are $M_{4380} = 4380 \pm 8 \pm 28$ MeV and $M_{4450} = 4449.8 \pm 1.7 \pm 2.5$ MeV, and decay widths are $\Gamma_{4380} = 205 \pm 18 \pm 86$ MeV and $\Gamma_{4450} = 39 \pm 5 \pm 19$ MeV. The spin and parity $J^{P}$ of them are not well determined. The one state is $J = 3/2$ and the other state is $J = 5/2$ and they have opposite parity.

Before the LHCb observation, some theoretical studies of hidden-charm pentaquarks were done [1][7]. The LHCb announcement, there are many analyses based on the hadronic molecular state [8][24], compact pentaquark state [25][34], quark-cluster model [34], baryocharmonium model [35], hadroquarkonia model [36], topological soliton model [37], and meson-baryon molecules coupled with five-quark states [38]. The kinematical rescattering effects are also discussed in Refs. [39][43].

There are many theoretical descriptions for $P_{c}^{+}$ pentaquarks. Among those pictures, the hadronic molecular one has been used for several other exotic hadrons, especially near the thresholds. For example, since the mass of $X(3872)$ is close to the $DD^{\ast}$ threshold, $X(3872)$ includes the $DD^{\ast}$ molecule structure [44]. The masses of $P_{c}^{+}(4380)$ and $P_{c}^{+}(4450)$ are slightly below the thresholds of $D\Sigma_{c}^{\ast}$ and $D^{\ast}\Sigma_{c}$, respectively. They can be considered as the loosely bound state of heavy meson and heavy baryon.

Charm quarks are included in $P_{c}^{+}$ pentaquarks. The masses of the heavy quarks, charm and bottom, are much larger than the typical scale of low energy QCD, $\Lambda_{QCD} \sim 200$MeV. For the heavy quark region, there is a characteristic property in the quark interaction. The spin dependent interaction of the heavy quark is suppressed by the inverse of the heavy quark mass, $1/m_{Q}$. By this suppression, heavy quark spin symmetry (HQS) is appeared in the heavy quark limit [45][49]. As a result, we can decompose the total spin $\vec{J}$ to heavy quark spin $\vec{S}$ and the other spin $\vec{j}$:

\[ \vec{J} = \vec{S} + \vec{j}. \]  

The total spin is conserved and heavy quark spin is also conserved in the heavy quark limit because of the suppression of the spin dependent force. Thus the other spin part is also conserved. This conservation leads to the mass degeneracy of heavy hadrons. Let us consider the heavy meson $q\bar{q}$ with a light quark $q$ and a heavy quark $Q$. For $j \geq 1/2$, there are two degenerate states with total spin $J_{\pm} = j \pm 1/2$.

These two states are called HQS doublet. There is only $J = 1/2$ state for $j = 0$, hence it is called HQS singlet.

Such HQS multiplet structure is seen in the charm and bottom hadron mass spectrum. For example, the small mass difference is obtained between the heavy-light pseudoscalar ($J = 0$) and vector ($J = 1$) mesons, 140 MeV between $D$ and $D^{\ast}$, and 45 MeV between $B$ and $B^{\ast}$. These mass splittings are much smaller than those in the light quark sectors, 600 MeV between $\pi$ and $\rho$, and 400 MeV between $K$ and $K^{*}$. This observation indicates that the approximate heavy quark spin symmetry is realized in the charm and bottom quark sectors, and these two mesons with $J = 0,1$ belong to the HQS doublet having the heavy spin $S = 1/2$ and the other spin $j = 1/2$.

The approximate mass degeneracy is also observed in the heavy-light baryons. The mass splitting between $\Sigma_{c}$ ($J = 1/2$) and $\Sigma_{b}^{\ast}$ ($J = 3/2$) ($\Lambda_{b}$ and $\Sigma_{b}^{0}$) is about 65 MeV (20 MeV). They are the HQS doublet state with the heavy spin $S = 1/2$ and the other spin $j = 1$. On the other hand, the heavy-light baryons $\Lambda_{c}$ and $\Lambda_{b}$ with the light diquark spin 0 are a HQS singlet state.
In this paper, we study the structure of HQS multiplets of $Q\bar{Q}qqq$-type pentaquarks regarding them as molecular states of $P^{(*)}\Sigma_Q^{(*)}$. Here, $P^{(*)}$ means a HQS doublet meson with an anti-heavy quark like $D(B)$ and $D^*(B^*)$ and $\Sigma_Q^{(*)}$ stands for a HQS doublet baryon with a heavy quark like $\Sigma_c(S_b)$ and $\Sigma_c^*(S_b^*)$.

The HQS doublet structures of $P^{(*)}$ meson and $\Sigma_Q^{(*)}$ baryon which have one heavy quark are well known. HQS multiplet structure of $P^{(*)}N$ molecular state with a single heavy quark is discussed in Refs. [20–22]. They showed that the degeneracy of $j \pm 1/2$ states can be expanded to multi-hadron system. In this paper, we study the HQS multiplet structure of $P_c$-like pentaquarks as a doubly heavy quark systems. The appearance of the HQS multiplet structures of $\bar{q}q$ baryons and pion interaction Lagrangian is given by [56, 58]

$$\mathcal{L}_{\text{BB}} = \frac{3}{2} g_P \bar{q} q e^{\mu \rho \sigma} \tau_\nu \left[ \bar{S}_\mu A_\nu S_\rho \right].$$

The superfield $S_\mu$ for $\Sigma_Q$ and $\Sigma_Q^*$ is represented as

$$S_\mu = \bar{\Sigma}^{(*)}_{Q_\mu} - \sqrt{\frac{1}{3}} (\gamma_\mu + v_\mu) \gamma_5 \Sigma_Q.$$

The heavy baryon fields $\Sigma^{(*)}_{Q_\mu}$ are defined by

$$\Sigma^{(*)}_{Q_\mu} = \begin{pmatrix} \Sigma_Q^{(*)+} \\ \frac{1}{\sqrt{2}} \Sigma_Q^{(*)+} \\ \frac{1}{\sqrt{2}} \Sigma_Q^{(*)0} \\ \Sigma_Q^{(*)0} \end{pmatrix}.$$

$\Sigma_Q$ and $\Sigma_Q^*$ are spin 1/2 and 3/2 baryon fields in the HQS doublet. For the coupling constant $g_P$, we use $g_P = (\sqrt{5}/3)g_4$ and $g_4 = 0.999$ estimated in Ref. [58]. The coupling $g_4$ is determined by the decay of $\Sigma_Q^* \rightarrow \Lambda_c \pi$ and its sign follows the quark model estimation.

We construct the one pion exchange potential using the above Lagrangians. At each vertex, we introduce a tensor force by the quark model estimation.

In this section, we construct the OPEP for $P^{(*)}\Sigma_Q^{(*)}$ molecular states based on the heavy quark symmetry and the chiral symmetry. The $P^{(*)}$ mesons and pion interaction Lagrangian is given by [53–57]

$$\mathcal{L}_{HH\pi} = g_{\pi} \text{Tr} \left[ H H \gamma_\mu \gamma_5 A_\mu \right].$$

The heavy meson doublet field $H$ is

$$H = \frac{1 + \hat{\gamma}_5}{2} \left[ P^{(*)}_\mu \gamma_\mu + i P \gamma_5 \right].$$

$P$ and $P^*$ are pseudoscalar meson and vector meson fields in the HQS doublet. The axial vector current for the pion is given by

$$A_\mu = i \frac{1}{2} (\xi^j \partial_\mu \xi_j - \xi \partial_\mu \xi^j),$$

where $\xi = \exp(i\pi/\sqrt{2} f_\pi)$. The pion decay constant is $f_\pi = 92.4$ MeV and the pion field $\hat{\pi}$ is defined by

$$\hat{\pi} = \begin{pmatrix} \pi^0/\sqrt{2} \\ \pi^- \pi^0/\sqrt{2} \end{pmatrix}.$$

The coupling constant $g_\pi$ is determined as $|g_\pi| = 0.59$ from the decay of $D^* \rightarrow D \pi$.

The $\Sigma_Q^{(*)}$ baryons and pion interaction Lagrangian is given by [56, 58]

$$\mathcal{L}_{BB\pi} = \frac{3}{2} g_\pi \bar{q} q e^{\mu \rho \sigma} \tau_\nu \left[ \bar{S}_\mu A_\nu S_\rho \right].$$

The superfield $S_\mu$ for $\Sigma_Q$ and $\Sigma_Q^*$ is represented as

$$S_\mu = \bar{\Sigma}^{(*)}_{Q_\mu} - \sqrt{\frac{1}{3}} (\gamma_\mu + v_\mu) \gamma_5 \Sigma_Q.$$

The heavy baryon fields $\Sigma^{(*)}_{Q_\mu}$ are defined by

$$\Sigma^{(*)}_{Q_\mu} = \begin{pmatrix} \Sigma_Q^{(*)+} \\ \frac{1}{\sqrt{2}} \Sigma_Q^{(*)+} \\ \frac{1}{\sqrt{2}} \Sigma_Q^{(*)0} \\ \Sigma_Q^{(*)0} \end{pmatrix}.$$

$\Sigma_Q$ and $\Sigma_Q^*$ are spin 1/2 and 3/2 baryon fields in the HQS doublet. For the coupling constant $g_\pi$, we use $g_\pi = (\sqrt{5}/3)g_4$ and $g_4 = 0.999$ estimated in Ref. [58]. The coupling $g_4$ is determined by the decay of $\Sigma_Q^* \rightarrow \Lambda_c \pi$ and its sign follows the quark model estimation.

We construct the one pion exchange potential using the above Lagrangians. At each vertex, we introduce a cutoff parameter $\Lambda$ via the monopole type form factor

$$F(q) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + |q|^2},$$

where $m_\pi$ is a mass of the exchanging pion, and $q$ is its momentum. We use the same cutoff for $P^{(*)}P^{(*)}\pi$ and $\Sigma_Q^{(*)}\Sigma_Q^{(*)}\pi$ vertices for simplicity, and fix the value of cutoff 1600 MeV and 1500 MeV.

In the present analysis, we concentrate on the S-wave $P^{(*)}\Sigma_Q^{(*)}$ molecular states to clarify their HQS multiplet structures. In the Hadronic Molecule (HM) basis, the spin structures of molecular states are described by the product of meson-baryon spins. Then, the possible spins of the $P^{(*)}\Sigma_Q^{(*)}$ states are

$$\tilde{P} \Sigma_Q = [Qq]_0 \otimes [Q|d|1/2] = \frac{1}{2},$$

$$\tilde{P} \Sigma_Q^* = [Qq]_0 \otimes [Q|d|3/2] = \frac{3}{2}.$$
\[ \bar{P}^* \Sigma_Q = [\bar{Q}q]_1 \otimes [Qd]_{1/2} = \frac{1}{2} \otimes \frac{3}{2} \] \quad (13)
\[ \bar{P}^* \Sigma_Q^* = [\bar{Q}q]_1 \otimes [Qd]_{3/2} = \frac{1}{2} \otimes \frac{3}{2} \oplus \frac{5}{2} \] \quad (14)

where \( Q, \bar{Q}, q \) and \( d \) stand for a heavy quark, heavy antiquark, light quark and diquark in \( \Sigma_Q^{(*)} \) baryon, respectively, and the index \( j \) of \([\alpha]_j\) means the spin of \( \alpha \).

The wavefunctions and OPEPs for each spin state are
\[ \psi_{1/2}^{\text{HM}} = \begin{pmatrix} \bar{P}^* \Sigma_Q \end{pmatrix}_{1/2} \]
\[ \psi_{3/2}^{\text{HM}} = \begin{pmatrix} P^* \Sigma_Q \end{pmatrix}_{3/2} \]
\[ V_{\pi,1/2}^{\text{HM}}(r) = \frac{g_{\pi} f_\pi}{f^2} \left( \begin{array}{c} 0 \\ -\frac{1}{\sqrt{3}} \frac{1}{3} \frac{1}{3} \frac{1}{6} \frac{1}{6} \end{array} \right) C_\pi(r) \]
\[ V_{\pi,3/2}^{\text{HM}}(r) = \frac{g_{\pi} f_\pi}{f^2} \left( \begin{array}{c} 0 \\ -\frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5} \end{array} \right) C_\pi(r) \]
\[ \psi_{5/2}^{\text{HM}} = \begin{pmatrix} \bar{P}^* \Sigma_Q^* \end{pmatrix}_{5/2} \]
\[ V_{\pi,5/2}^{\text{HM}}(r) = -\frac{g_{\pi} f_\pi}{f^2} C_\pi(r) \]

The function \( C_\pi(r) \) is defined as
\[ C_\pi(r) = \frac{m^2}{4 \pi} \left[ e^{-m \pi r} - e^{-\Lambda \pi r} - \frac{L^2 - m^2}{2 \Lambda} e^{-\Lambda \pi r} \right] \] \quad (21)

It should be noted that we subtract the contact terms from the potential.

III. HQS MULTIPLET STRUCTURE OF \( \bar{P}^*(\Sigma_Q^{(*)}) \)

We construct the OPEP for HM base in Sec. II. However, it is inconvenient to see the structure of HQS multiplet. In this section, we introduce the Light Cloud Spin (LCS) basis, where the spin structure of \( Q\bar{Q}qd \) states is divided into the heavy quark spin \([Q\bar{Q}]_S\) and light cloud spin \([q[d]_{1/2}]_L\). It is a natural spin description in the heavy hadron systems, because heavy quark spin and light cloud spin are separately conserved in heavy quark effective theory.

Here, we treat the HQS structure of doubly heavy system in the following manner: The pentaquark as a bound state of \( \bar{P}^{(*)} \) and \( \Sigma_Q^{(*)} \) is labeled by the velocity \( v \) of the pentaquark. It is natural to assume that both \( \bar{P}^{(*)} \) and \( \Sigma_Q^{(*)} \) have the same velocity \( v \).

The spin structures of \( \bar{P}^{(*)}\Sigma_Q^{(*)} \) molecular states in LCS basis are given by
\[ [Q\bar{Q}]_0 \otimes [q[d]_{1/2}]_1/2 = \frac{1}{2} \] (singlet), \quad (22)
\[ [Q\bar{Q}]_0 \otimes [q[d]_{3/2}]_1/2 = \frac{3}{2} \] (singlet), \quad (23)
\[ [Q\bar{Q}]_1 \otimes [q[d]_{1/2}]_1/2 = \frac{1}{2} \otimes \frac{3}{2} \oplus \frac{5}{2} \] (doublet), \quad (24)
\[ [Q\bar{Q}]_1 \otimes [q[d]_{3/2}]_1/2 = \frac{1}{2} \otimes \frac{3}{2} \oplus \frac{5}{2} \] (triplet). \quad (25)

There are four types of HQS multiplets, spin 1/2 singlet, spin 3/2 singlet, spin (1/2, 3/2) doublet and spin (1/2, 3/2, 5/2) triplet which are classified by the heavy quark spin \( S = 0, 1 \) and the light cloud spin \( j = 1/2, 3/2 \).

Using unitary transformation matrices, we translate the basis from HM basis to LCS basis. For spin 1/2,
\[ \psi_{1/2}^{\text{LCS}} = U_{1/2}^{-1} \psi_{1/2}^{\text{HM}} \]
\[ = \begin{pmatrix} [Q\bar{Q}]_0 \otimes [q[d]_{1/2}]_{1/2} \\ [Q\bar{Q}]_1 \otimes [q[d]_{1/2}]_{1/2} \end{pmatrix} \]
\[ \psi_{3/2}^{\text{LCS}} = U_{3/2}^{-1} \psi_{3/2}^{\text{HM}} \]
\[ = \begin{pmatrix} [Q\bar{Q}]_0 \otimes [q[d]_{3/2}]_{1/2} \\ [Q\bar{Q}]_1 \otimes [q[d]_{3/2}]_{1/2} \end{pmatrix} \]

For spin 3/2,
\[ \psi_{5/2}^{\text{LCS}} = U_{5/2}^{-1} \psi_{5/2}^{\text{HM}} \]
\[ = \begin{pmatrix} [Q\bar{Q}]_0 \otimes [q[d]_{3/2}]_{1/2} \\ [Q\bar{Q}]_1 \otimes [q[d]_{3/2}]_{1/2} \end{pmatrix} \]

For spin 5/2,
Here, we call the components labeled by (singlet, doublet, or triplet) of \( \psi_{j}^{LCS} \) as spin \( J \) (singlet, doublet, or triplet) state. For instance, the first component of \( \psi_{1/2}^{LCS} \) in Eq. (29), i.e. \( \left[ \bar{Q}Q \right]_{0} \otimes \left[ g[d] \right]_{1/2} \) singlet is given by

\[
\sum_{1/2} = \left( \frac{1}{2}, \frac{1}{2} \right) - \frac{1}{2\sqrt{3}} \frac{2}{3\sqrt{2}} \frac{2}{6}
\]

\[
\sum_{3/2} = \left( \frac{1}{2}, -\frac{1}{2} \right) \sqrt{\frac{15}{6}} \frac{\sqrt{3}}{3} \frac{1}{6}
\]

\[
U_{5/2} = 1 .
\]

The potential matrices in LCS basis are diagonalized corresponding to the HQS triplet components. We find the particular values of the matrix elements of the OPEP; +1 for \( \left[ \bar{Q}Q \right]_{0} \otimes \left[ g[d] \right]_{1/2} \) and \( \left[ \bar{Q}Q \right]_{1} \otimes \left[ g[d] \right]_{1/2} \), and -2 for \( \left[ \bar{Q}Q \right]_{0} \otimes \left[ g[d] \right]_{3/2} \) and \( \left[ \bar{Q}Q \right]_{1} \otimes \left[ g[d] \right]_{3/2} \). Hence, these components play a different role, either an attraction or a repulsion, depending on the whole sign of the potential.

IV. NUMERICAL RESULT

Before solving coupled channel Schrödinger equations under the LCS basis potential, let us discuss the sign assignment of a coupling constant of the heavy meson-baryon interaction, \( |g| = 0.59 \). In the usual case, its sign is taken as plus following quark models. However, only the absolute value is determined by the decay of \( D^{*} \rightarrow D^{\pi} \) [39], and the sign of \( g \) is not determined.

This sign assignment is important in the present study. For example, the coefficients of the HQS singlet and doublet component are +1 in the spin 1/2 potential of LCS basis in Eq. (27). Thus, these potentials play as a repulsive one when we assign \( g = +0.59 \), but they are the attractive potentials when we choose \( g = -0.59 \). On the other hand, the HQS triplet with the coefficient \(-1/2\) has the attractive potential for \( g = +0.59 \) and repulsive

\[ V_{x, 5/2}^{LCS}(r) = U_{5/2}^{-1} V_{x, 5/2}^{HM} U_{5/2} \]

\[ = - \frac{g_{1}}{2f_{\pi}^{2}} C_{\pi}(r) . \tag{31} \]

FIG. 1. Attractive potential \( V(r) = - \frac{g_{1}}{2f_{\pi}^{2}} C_{\pi} \) for \( \Lambda = 1000 \) (purple solid curve) and 1500 MeV (green dotted curve), where \( g = 0.59 \), \( g_{1} = 0.942 \) and \( f_{\pi} = 92.4 \text{MeV} \).

A. Result in case of \( g = +0.59 \)

When we assign as \( g = +0.59 \), the HQS multiplets which have attractive potential are \( J^{P} = 3/2^{-} \) singlet and \( J^{P} = (1/2^{-}, 3/2^{-}, 5/2^{-}) \) triplet. The potential is written as

\[ V(r) = - \frac{g_{1}}{2f_{\pi}^{2}} C_{\pi}(r) , \tag{35} \]

and we show it in Figure [1]

Firstly, we show the results obtained by solving the Schrödinger equation with preserving the heavy quark spin symmetry. We define the spin averaged mass for \( \bar{P}^{(*)} \) mesons and \( \Sigma_{Q}^{(*)} \) baryons as

\[ M_{P^{\text{ave}}} = \frac{M_{P} + 3M_{P^{*}}}{4} , \tag{36} \]

\[ M_{\Sigma_{Q}^{\text{ave}}} = \frac{2M_{\Sigma_{Q}} + 4M_{\Sigma_{Q}^{*}}}{6} , \tag{37} \]

to deal with the degeneracy of the HQS doublet meson and baryon, respectively. The masses of relevant charmed and bottomed hadrons are shown in Table [I]

The spin averaged reduced mass is defined as

\[ \mu_{\text{ave}} = \frac{M_{P^{\text{ave}}}M_{\Sigma_{Q}^{\text{ave}}}}{M_{P^{\text{ave}}} + M_{\Sigma_{Q}^{\text{ave}}}} . \tag{38} \]

When \( \mu_{\text{ave}} = 1.102, 1.474, 1.699 \) and 2.779 GeV, the spin averaged masses of \( \bar{D}^{(*)}\Sigma_{c}^{(*)} \), \( \bar{B}^{(*)}\Sigma_{b}^{(*)} \), \( B^{(*)}\Sigma_{c}^{(*)} \), and \( B^{(*)}\Sigma_{b}^{(*)} \) are reproduced, respectively. We solve the Schrödinger equation with keeping the heavy quark spin

\[ 1 \text{ It could be argued that the relative sign of } g \text{ and } g_{1} \text{ is not determined.} \]
symmetry by changing the mass parameter $\mu_{ave}$ from 1 GeV to 100 GeV. To obtain the bound state solutions, we use Gaussian expansion method [60]. The results of $\Lambda = 1000$ and 1500 MeV are shown in Figure 3. All four states, spin 3/2 singlet and spin (1/2, 3/2, 5/2) triplet, are degenerate because of the heavy quark spin symmetry and their bound state solutions are obtained for all range of $\mu_{ave}$.

Next, we show the results including the effect of the heavy quark spin symmetry breaking. The breaking is introduced by the nonzero mass difference between the HQS multiplets, namely $\bar{P}$ and $P^*$, and $\Sigma_Q$ and $\Sigma_Q^*$. To see the mass dependence of a binding energy, the heavy hadron masses are parametrized as follows:

\[
\begin{align*}
M_{\bar{P}} &= 2\mu + \frac{a}{2\mu} + \frac{w}{(2\mu)^2}, \\
M_{P^*} &= 2\mu + \frac{b}{2\mu} + \frac{x}{(2\mu)^2}, \\
M_{\Sigma_Q} &= 2\mu + \frac{c}{2\mu} + \frac{y}{(2\mu)^2}, \\
M_{\Sigma_Q^*} &= 2\mu + \frac{d}{2\mu} + \frac{z}{(2\mu)^2},
\end{align*}
\]

where $\mu$ is a parameter corresponding to the reduced mass of $\bar{P}^{(*)}\Sigma_Q^{(*)}$ state. The eight parameters of $a, b, c, d, w, x, y$ and $z$ are fixed to reproduce the eight hadron masses in Table I by taking $\mu = 1.102$ (2.779) GeV for charm (bottom) sector. The values of these parameters are shown in Table II. Note that the charm (bottom) hadron masses are reproduced when we take $\mu = 1.102$ (2.779) GeV, and the heavy quark spin symmetry is restored as the mass parameter $\mu$ increases. The energies obtained by solving the Schrödinger equations with the effect of heavy quark spin symmetry breaking are shown in Figure 4. The labels in Figure 3, e.g. Spin 1/2 triplet, are named as being at the heavy quark limit. For instance, the solid line named as Spin 1/2 triplet displays the energy of the state which becomes the spin 1/2 triplet state at the heavy quark limit. We note that the components belonging to the same $J^P$ state can be mixed in the finite hadron mass region as shown later, while they are not mixed at the heavy quark limit. The energies of the $\bar{D}^{(*)}\Sigma_Q^{(*)}, \bar{D}^{(*)}\Sigma_Q^{(*)}, B^{(*)}\Sigma_Q$ and $B^{(*)}\Sigma_Q^*$ states are correspond to the values at $\mu = 1.102, 1.474, 1.699$ and 2.779 GeV, respectively.

All four states are degenerate and the binding energy is −13.7 MeV for $\Lambda = 1000$ MeV and −22.3 MeV for $\Lambda = 1500$ MeV in heavy quark limit. As $\mu$ becomes smaller, the degeneracy is solved. At $\mu = 1.102$ GeV, only two (three) states can be bound for $\Lambda = 1000$ MeV (1500 MeV).

For the spin 1/2 and 3/2 states, each components is completely separated in the heavy quark limit as shown in Eqs. (27) and (29). In the finite heavy hadron mass region, however, the kinetic term with the nonzero mass splitting of the HQS multiplets gives a mixing of the HQS singlet, doublet and triplet components. The percentage of (singlet, doublet, triplet) components belonging to the same state at the heavy quark limit. We note that the components are named as being it at the heavy quark limit. For instance, the solid line named as Spin 1/2 triplet displays the energy of the state which becomes the spin 1/2 triplet state at the heavy quark limit. We can see that the effect of heavy quark spin symmetry breaking is small.

### B. Result in case of $g = −0.59$

In the case of $g = −0.59$ , the attractive multiplets are $J^P = 1/2^-$ singlet and $J^P = (1/2^-, 3/2^-)$ doublet. The potential is written as

\[
V(r) = \frac{g g_1}{f_\pi^2} C_\pi(r),
\]

and it is shown in Figure 4. This potential is twice deeper than that of $g = +0.59$ , and therefore we expect that the binding energy is larger.

As in the case of $g = +0.59$ , we show the result that heavy quark spin symmetry is preserved in Figure 5 and the result that it is broken in Figure 6 for $g = −0.59$ .

Figure 6 shows that all three states of spin 1/2 singlet and (1/2, 3/2) doublet are degenerate in the heavy quark

| TABLE I. Masses of relevant charmed and bottomed hadrons [59]. |
|---------|---------|---------|---------|---------|
| Mass[MeV] | $D$ | $\bar{D}^*$ | $B$ | $B^*$ |
|---------|---------|---------|---------|---------|
| 1867.21 | 2008.56 | 5279.48 | 5324.65 |
| 2453.54 | 2518.13 | 5813.4 | 5833.6 |
| $\Sigma_c$ | $\Sigma_c^*$ | $\Sigma_b$ | $\Sigma_b^*$ |

The higher order terms of the effective Lagrangians also break the heavy quark symmetry. However, we employ the leading term of Lagrangians in this study.
TABLE II. Values of parameters to include the effect of heavy quark spin symmetry breaking in Eqs. (39)-(42).

\[
\begin{array}{cccccc}
a [\text{GeV}^2] & b [\text{GeV}^2] & c [\text{GeV}^2] & d [\text{GeV}^2] & w [\text{GeV}^3] & x [\text{GeV}^3] \\
-2.0798 & -1.8685 & 1.9889 & 2.0814 & 3.1677 & -3.1729 \\
\end{array}
\]

TABLE III. Percentage of (singlet, doublet, triplet) components in wavefunctions of the spin \(1/2\) and \(3/2\) states in the case of \(g = +0.59\) and \(\Lambda = 1000\) MeV.

| \(\mu [\text{GeV}]\) | Spin 1/2 triplet | Spin 3/2 triplet | Spin 3/2 singlet |
|--------------------|-----------------|-----------------|-----------------|
| 1                  | (0.8%, 0%, 99.2%) | (1.6%, 0%, 98.4%) | No bound state |
| 2                  | (0%, 0%, 100%)   | (0.9%, 0%, 99.1%) | No bound state |
| 3                  | (0%, 0%, 100%)   | (0%, 0%, 100%)   | (100%, 0%, 0%) |

FIG. 3. Energies of the \(p^{(*)} \Sigma_c^{(*)}\) states with heavy quark spin symmetry breaking effect, obtained for \(\Lambda = 1000\) MeV (the upper figure) and \(1500\) MeV (the lower figure) with \(g = +0.59\). These energies are measured from \(P \Sigma_c\) threshold. The purple solid, green dotted and yellow dashed-dotted curves are the energies of spin \((1/2, 3/2, 5/2)\) triplet states respectively and the light blue dashed curve is that of spin \(3/2\) singlet state. For the sake of reference, we show the result for the case of keeping the heavy quark spin symmetry by the red dashed-dotted-dotted curve (Common mass).

FIG. 4. Attractive potential \(V(r) = \frac{29}{17} C_F\) for \(\Lambda = 1000\) (purple solid curve) and \(1500\) MeV (green dotted curve), where \(g = -0.59, g_1 = 0.942\) and \(f_\pi = 92.4\) MeV.

V. SUMMARY AND DISCUSSIONS

We showed that, in Sec. IV, the sign of a coupling constant of the heavy meson-pion interaction determines which multiplets have the attractive potential. In the case of \(g = +0.59\), spin \(3/2\) HQS singlet and spin \((1/2, 3/2, 5/2)\) HQS triplet have the attractive potential. On the other hand, in the case of \(g = -0.59\), spin \(1/2\) HQS singlet and spin \((1/2, 3/2)\) HQS doublet have the limit and the binding energy is \(-29.5\) MeV for \(\Lambda = 1000\) MeV and \(-48.1\) MeV for \(\Lambda = 1500\) MeV, which agree with the binding energies in the heavy quark limit shown in Figure 5. Unlike in the case of \(g = +0.59\), all states are bound even at \(\mu = 1.102\) GeV corresponding to \(\bar{P}\Sigma_c^{(*)}\) state and their binding energies are a few MeV.

The mixing ratio of wavefunction components for \(\Lambda = 1000\) MeV is shown in Table IV. The mixing ratio of the minor components induced by the heavy quark symmetry breaking effect is slightly larger than the case of \(g = +0.59\), however it is still small.

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TABLE IV. Percentage of (singlet, doublet, triplet) components in wavefunctions in the case of $g = -0.59$ and $\Lambda = 1000$ MeV.

| $\mu$ [GeV] | Spin 1/2 singlet | Spin 1/2 doublet | Spin 3/2 doublet |
|-------------|------------------|------------------|------------------|
| 1           | (3.9%, 96.1%, 0%)| (96.5%, 3.4%, 0.1%)| (0%, 99.9%, 0.1%)|
| 2           | (0.2%, 99.8%, 0%)| (99.9%, 0.1%, 0%)| (0%, 100%, 0%)|
| 3           | (0%, 100%, 0%)   | (100%, 0%, 0%)   | (0%, 100%, 0%)|

FIG. 5. Obtained binding energies for $\Lambda = 1000$ (purple solid curve) and 1500 MeV (green dotted curve) with $g = -0.59$. The energy is measured from the threshold of $\bar{P}_\text{ave} \Sigma_{\text{ave}}$. The mass parameter $\mu$ is changed from 1 GeV to 100 GeV.

This classification is explained by the light cloud spin structure in Eqs. (22)–(25). The light cloud spin of spin 3/2 singlet and spin (1/2, 3/2, 5/2) triplet is $[q[d]_1]_{3/2}$ and that of spin 1/2 singlet and spin (1/2, 3/2) doublet is $[q[d]_1]_{1/2}$. Because the pion exchange interaction is coupled to the light quark spin, the difference of the attractive multiplet comes from the difference of the light cloud spin structure. Moreover, we find the degeneracy of HQS singlet and triplet (singlet and doublet) in the case of $g = +0.59 (-0.59)$. It is a natural result because the OPEP does not depend on the heavy quark spin structure.

In the heavy quark limit, four (three) bound states exist for $g = +0.59 (-0.59)$. However, the heavy quark symmetry is broken for real charm / bottom hadrons, so that all four (three) bound states may not exist in reality as demonstrated in Sec. IV. But we expect that there exist some HQS partners of $P_c$ like pentaquarks. Especially, for bottom sector, the structure of HQS multiplet is more clearly than for charm sector, because the realization of the heavy quark symmetry is better. We expect the observation of the bottom pentaquarks to confirm the HQS multiplet structure of them.

3 We do not consider the tensor force in this study, but it is also determined by the light cloud spin structure. Not only the pion interaction, but also the other light meson interactions depend on the light cloud spin.

FIG. 6. Energies obtained with heavy quark spin symmetry breaking for $\Lambda = 1000$ MeV (the upper figure) and 1500 MeV (the lower figure) with $g = -0.59$. These energies are measured from $\bar{P}_\text{ave} \Sigma_{\text{ave}}$ threshold. The purple solid and light blue dashed curves are the energies of spin (1/2, 3/2) doublet states and the green dotted curve is that of spin 1/2 singlet state. For the sake of reference, we show the result for the case of keeping the heavy quark spin symmetry by the red dashed-dotted-dotted curve.

The discussion in the LCS basis can be compared to the quark model calculations, treating the constituent quarks as degrees of freedom of the system. In Refs. [34, 38], the short-range interaction in the $P_c$ pentaquarks are studied, which is derived based on the quark cluster model. The contributions from the color magnetic interaction of $c\bar{c}uud$ are evaluated, and they find that the $c\bar{c}uud$ configurations having the other spin $j = 3/2$ are important to produce an attraction. On the other hand, the con-
figurations with $j = 1/2$ give a repulsion. In this study, we also obtain that the states with $j = 1/2$ and $j = 3/2$ have a different role as shown in Eqs. (27), (29) and (31), namely one is attractive and the other one is repulsive. Thus, we find that a role of the interaction is characterized by the light cloud spin in both of the quark model and the hadronic molecular model. It indicates that the discussion of the HQS multiplet structure can be applied not only to the molecules, but also to the compact multiquark states. In Ref. [25], the $\epsilon\bar{c}u\bar{d}$ potential for $JP = 3/2^-$ with $(S,j) = (1,3/2)$ is stronger than that with $(S,j) = (0,3/2)$. This behavior also agrees with our results.

Focusing on the light cloud spin structure, there are constraints of the S-wave decay channel of the spin 3/2 HQS singlet and spin (1/2, 3/2, 5/2) HQS triplet. Since their light cloud spin is given by $[q|d]_{3/2}$, they cannot couple to the S-wave $[Q\bar{Q}] N$ and $\bar{P}^{(s)} \Lambda_Q$ states. Here $[Q\bar{Q}]$, $N$ and $\Lambda_Q$ denote the heavy quarkonium, spin 1/2 nucleon and HQS singlet heavy baryon like $\Lambda_c$, respectively.

Due to the heavy quark spin symmetry, heavy quark spin and light cloud spin are independently conserved. Therefore, $[q|d]_{3/2}$ having light cloud spin 3/2 does not couple to the nucleon of spin 1/2. Moreover, $[q|d]_{13/2}$ cannot construct the diquark spin 0 by the spin rearrangement. So, $[q|d]_{13/2}$ cannot couple to $\Lambda_Q$ with diquark spin 0 as well. As a result, the S-wave decay channels to $[Q\bar{Q}] N$ and $\bar{P}^{(s)} \Lambda_Q$ from spin 3/2 HQS singlet and spin (1/2, 3/2, 5/2) HQS triplet are prohibited in the heavy quark limit. There exist decay channels by the D-wave decay, however we expect that they are small.

On the other hand, there are no constraint of S-wave decay to $[Q\bar{Q}] N$ and $\bar{P}^{(s)} \Lambda_Q$ for spin 1/2 HQS singlet and spin (1/2, 3/2, 5/2) HQS doublet which have the light cloud spin of $[q|d]_{11/2}$ in the view of heavy quark spin symmetry. These restrictions are independent of the model and derived only from heavy quark symmetry. The difference in their S-wave decay channel restrictions should appear in the decay branching ratio of $Q\bar{Q}qqq$ pentaquark state. We expect the measurement of the branching ratio to $[Q\bar{Q}] N$ and $\bar{P}^{(s)} \Lambda_Q$ to confirm the heavy quark symmetry in $P_c$-like pentaquarks.

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