Interrelations of various soil types mechanical properties

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Abstract. The article aims at assessing relationships of deformation modulus, cohesion and angle of internal friction of various soil types with penetration resistance (cone index). Research methods include analysis of reference information, computational experiment and approximation of calculated data. The analysis is performed for calculated values of mechanical properties of coarse-grained sand, medium-grained sand, fine-grained sand, silty sand, sandy loam, loam and clayey soil. It is found out that angle of internal friction and cohesion of sandy soils, sandy loam, loam and clayey soil are related to deformation moduli by power dependences. Coefficients of the formulae depend on the soil type and, according to the results of calculated data processing, are invariant with respect to porosity coefficients and consistencies of the soils. It is shown that cone indices are related to deformation moduli by power dependences, the coefficients of which are determined by soil type and are invariant with respect to the porosity coefficients and consistencies. The obtained calculated values of the cone indices correspond well with reference data on the penetration resistance of mixed soils. The dependences intend to simplify procedure for estimating mechanical properties of the bearing surface, since, with a soil type known, using the value of deformation modulus or cone index, it becomes possible to calculate the soil parameters characterizing the shear resistance.

Introduction.

When calculating tractive performance of wheeled or tracked vehicles in off-road conditions, several characteristics of the mechanical properties of soils, the vehicles interact with, are used: deformation modulus, elastic modulus and shear modulus, cohesion and internal friction angle, specific weight of the soil [1], [2]. Their values are either taken as reference, or determined experimentally. Variability of the properties reduces accuracy of the reference estimates, and large number of the parameters used complicates the experimental soil conditions mapping. In order to avoid the difficulties observed, researchers seek to reveal interrelationships of physical and mechanical properties of soils or to introduce integral assessments of soil properties into mathematical models, where values are relatively easy to determine in field conditions. For example, in work [3] formulae were obtained for calculating deformation modulus, cohesion and angle of internal friction for various soil types according to the coefficient of porosity, consistency index (for cohesive soils), water saturation index (for sands). References [4], [5], [6] give formulae for calculating deformation modulus of soil depending on penetration resistance (cone index), which value is measured using a manual cone penetrometer. At the same time, dependences for determining the shear properties of soils by cone index need further consideration.
The aim of this study is to assess relationships of deformation modulus, specific adhesion and angle of internal friction on cone index for different soil types.

Research methods include analysis of reference information, computational experiment, and approximation of the calculated data.

Formulation of the problem. An analytical expression is known for calculating soil cone index CI [4], [5]:

\[ CI = -C \cot \varphi + \Theta \cdot \frac{24G^m (\tan \alpha + \tan \varphi)(1 + \sin \varphi)\tan \alpha}{d^2 \gamma^2 (m - 2)(m - 3)(3 - \sin \varphi)\tan^3 \varphi} \]

\[ \Theta = \left\{ C + (Z + L) \cdot \gamma \tan \varphi \right\}^{3-m} - \left\{ C + Z \cdot \gamma \tan \varphi \right\}^{12-m} \cdot \left\{ C + (Z + 3L - Lm) \cdot \gamma \tan \varphi \right\} \\

m = \frac{4 \sin \varphi}{3[l + \sin \varphi]} \]

where \( \Theta, m \) are auxiliary designations, \( C \) is cohesion, \( G \) is shear modulus, \( \varphi \) is angle of internal friction, \( \gamma \) is specific weight of the soil, \( \alpha \) is cone apex angle, \( d \) is diameter of the cone base, \( l \) is length of the conical part indenter, \( Z \) - depth of the cone penetration.

Geometrical parameters of standard cone penetrometer are: \( \alpha = 30^\circ \), \( d = 36.27 \text{ mm} \), \( l = 37.66 \text{ mm} \), \( Z \approx 2l \) [7].

Equation (1) allows calculating \( CI \) from values of \( C, G, \varphi, \gamma \), but, obviously, does not allow solving the inverse problem.

Reference [3] gives formulae for calculating deformation modulus \( E \), as well as \( C, \varphi \) according to coefficient of porosity \( e \) and consistency index \( I_L \) of the soil:

\[ E = (A_{E1} + B_{E1}I_L) \cdot e^{(A_{E2} + B_{E2}I_L)} \]  

\[ C = (A_{C1} + B_{C1}I_L) \cdot e^{(A_{C2} + B_{C2}I_L)} \]  

\[ \varphi = (A_{\varphi1} + B_{\varphi1}I_L) \cdot e^{(A_{\varphi2} + B_{\varphi2}I_L)} \]  

where \( A_{ad}, B_{ad}, A_{bd}, B_{bd}, A_{ac}, B_{ac}, A_{bc}, B_{bc}, A_{ap}, B_{ap}, A_{bp}, B_{bp} \) are numerical coefficients (presented in Table 1).

Note: when calculating the non-cohesive soils properties in formulae (2) - (4), \( I_L \) should be replaced by water saturation index \( I_W \).

**Table 1.** The coefficients for calculating mechanical properties of different soil types [3]

| coeff. | coarse sand | medium sand | fine sand | silty sand | sandy loam | loam | clayey |
|--------|-------------|-------------|-----------|------------|------------|------|-------|
| \( A_{E1} \) | 18.112 | 17.851 | 14.746 | 8.332 | 10.67 | 11.366 | 15.217 |
| \( B_{E1} \) | 0 | 0 | -6.974 | -2.282 | -7.273 | -7.575 | -10.68 |
| \( A_{E2} \) | -1.288 | -1.291 | -1.543 | -2.07 | -1.524 | -1.403 | -0.969 |
| \( B_{E2} \) | 0 | 0 | -0.671 | 0.24 | -0.974 | -1.089 | -1.975 |
| \( A_{C1} \) | 0.264 | 0.595 | 0.707 | 1.951 | 4.586 | 19.239 | 42.573 |
| \( B_{C1} \) | 0 | 0 | 0.0005 | -1.302 | -3.775 | -14.366 | -19.656 |
| \( A_{C2} \) | -2.784 | -2.126 | -2.46 | -1.788 | -1.475 | -1.204 | -1.136 |
| \( B_{C2} \) | 0 | 0 | -0.123 | -0.82 | -1.118 | -0.647 | -0.22 |
| \( A_{\varphi1} \) | 32.535 | 24.746 | 24.819 | 29.578 | 22.496 | 21.128 | 16.887 |
| \( B_{\varphi1} \) | 0 | 0 | -0.042 | -2.832 | -7.808 | -11.79 | -11.98 |
| \( A_{\varphi2} \) | -0.33 | -0.978 | -0.565 | -0.544 | -0.39 | -0.315 | -0.204 |
| \( B_{\varphi2} \) | 0 | 0 | 0.012 | 0.062 | -0.25 | -0.722 | -2.506 |
Shear modulus $G$ is associated with elasticity modulus $E_0$ by equation [6]:

$$ G = \frac{E_0}{2(1 + \nu)} $$

(5)

where $\nu$ is Poisson's ratio of the soil.

It was also found that the deformation modulus is approximately expressed in terms of $E_0$ [8]:

$$ E \approx 0.2E_0 $$

(6)

Thus, we get:

$$ G = \frac{5E}{2(1 + \nu)} $$

(7)

It is noted that $G$ affects results of calculating CI by equation (1) to a lesser extent than $C, \varphi$ [5]. In addition, when $\gamma$ varies from $1 \cdot 10^4$ to $2 \cdot 10^4$ N/m$^3$, its value practically does not affect the results of calculating CI by equation (1) [6]. Further, we take $\nu = 0.3, \gamma = 1.8 \cdot 10^4$ as constant values.

Calculation results and conclusions.

We have performed calculations for varying $e = 0.4 \ldots 1.2$, $I_L$ ($I_W$) = 0 \ldots 1 producing 1000 combinations of values for each soil type. Processing of the received data sets showed that mechanical parameters $\varphi \ [^\circ], C \ [kPa], CI \ [MPa]$ are connected with $E \ [MPa]$ by power dependencies ($R^2 > 0.9$):

$$ \varphi = A_{\varphi E}E^{B_{\varphi E}} $$

(8)

$$ C = A_{CE}E^{B_{CE}} $$

(9)

$$ CI = A_{CIE}E^{B_{CIE}} $$

(10)

where $A_{\varphi E}, B_{\varphi E}, A_{CE}, B_{CE}, A_{CIE}, B_{CIE}$ are numerical coefficients, the values of which are presented in Table 2.

| coeff. | coarse sand | medium sand | fine sand | silty sand | sandy loam | loam | clayey |
|--------|-------------|-------------|-----------|------------|------------|------|--------|
| $A_{\varphi E}$ | 15.49  | 2.79  | 10.73  | 16.44  | 10.2  | 5.51 | 1.57 |
| $B_{\varphi E}$ | 0.26 | 0.76 | 0.33 | 0.27 | 0.31 | 0.49 | 0.83 |
| $A_{CE}$ | 0.0005 | 0.0052 | 0.0164 | 0.102 | 0.249 | 1.64 | 8.15 |
| $B_{CE}$ | 2.16 | 1.65 | 1.48 | 1.26 | 1.18 | 0.95 | 0.62 |
| $A_{CIE}$ | 0.00851 | 0.00007 | 0.0056 | 0.033 | 0.021 | 0.038 | 0.073 |
| $B_{CIE}$ | 1.72 | 3.16 | 1.84 | 1.6 | 1.57 | 1.55 | 1.34 |

Equations (8) - (10) are clearly illustrated by the graphs in Figures 1 - 3.

In Figure 3, a graph of CI versus $E$ is added, which is constructed using equation [8], [9]:

$$ E = 3CI + 7 $$

(11)

The graph is given as a reference, the coefficients values of linear function (11) are taken in accordance with results of [8], [9] for mixed soils.
Figure 1. Dependence of angle of internal friction on deformation modulus for different soil types

Figure 2. Dependence of cohesion on deformation modulus for different soil types
1. Angle of internal friction and cohesion of sandy soil, sandy loam, loam and clayey soil are associated with the soils’ deformation moduli by power dependences. The coefficients of the formulae depend on the soil type and are invariant with respect to the coefficient of porosity and the soil consistency.

2. Soil penetration resistance (cone index) is related to the deformation modulus by power dependence, the coefficients of which are determined by the soil type and are invariant with respect to its coefficient of porosity and consistency. The obtained calculated values of the cone index are in satisfactory agreement with the reference data on the penetration resistance of mixed soils.

We suppose that the use of the obtained dependences in practice will simplify the procedure for estimating mechanical properties of the bearing surface, since with the soil type known, it is possible to calculate the soil parameters characterizing its shear resistance basing on value of the deformation modulus or cone index.

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