Dynamics of Fiberboids

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Fiberboids are active filaments trapped at the interface of two phases, able of harnessing energy (and matter) fluxes across the interface in order to produce a rolling-like self-propulsion. We discuss several table-top examples and develop the physical framework for understanding their complex dynamics. In spite of some specific features in the examples studied we conclude that the phenomenon of fiberboids is highly generic and robust across different materials, types of fluxes and timescales. Fiberboid motility should play a role from the macroscopic realm down to the micro scale and, as recently hypothesized, possibly as a means of biological self-propulsion that has escaped previous attention.

1 Introduction

When two bulk phases meet at an interface, more often than not, they are in a mutual thermodynamic disequilibrium. As a consequence, they exchange fluxes of energy and matter along the normal to the interface. If now a deformable filament is trapped in the interface plane, and if it is elastically responsive to the agent flowing across it, something truly astonishing happens: the filament bends into an arc, starts to axially “spin” and move along the interface, transforming itself from a passive object into a simple engine, which we here term “fiberboid”. Some extremely simple, table-top experimental realizations of fiberboids are shown in Fig. 1 and detailed in the next section.

The bidirectional active motion of fiberboids (albeit with a focus on the unidirectional rotational motion of such fibers closed into a torus) was described in a recent work. Yet our understanding of their general physics, in particular their dynamics, remains incomplete. Due to their self-organized nature that involves symmetry breaking, self-propulsion, bi-directionality of motion, interactions and collective effects, fiberboids are significantly more complex than their unidirectional variants closed into a torus or spiral. Yet, at the same time, fiberboids are extremely simple to generate as demonstrated by the examples shown in Fig. 1 and detailed in the next section.

In this work we will distill the generic features of fiberboid motility and contrast them to more particular features exhibited by specific experimental realizations. The outline of this work is as follows. After discussing two distinct examples in section 2 in section 3 we develop a simple dynamical model describing fiberboid motion. Comparing the predictions of this toy model to experimental data, we will see how specific observations for different fiberboid types naturally lead to model extensions. In section 4 we study the “social” behavior of fiberboids, i.e. how they interact with obstacles and other fiberboids, paving the way to studying their collective behavior in the near future.

2 Examples

To get a taste for the fiberboid phenomenon, let us start with two easy to reproduce table-top examples. Fig. 1b) shows snapshots of a nylon fiber – a piece of commercial fishing line – on an aluminum plate heated at \( \sim 140 \, ^\circ\text{C} \). Fig. 1b) displays a so-called kymograph (space-time plot generated from time-lapse images as shown in part a), clearly evidencing a rolling motion along the substrate with a typical velocity of \( \sim \text{cm/s} \). The direction of motion with respect to the fiber curvature depends on the coupling of the thermal gradient through the fiber (between the heated plate and the colder surroundings) to the fibers elastic deformation, cf. Fig. 2, i.e. it is determined by the sign of the coefficient of thermal expansion. For thermally contracting materials as nylon, the fiber curves away from the direction of motion, while for thermally expanding materials, such as polydimethylsiloxane (PDMS) fibers, it curves into the direction of motion. The same effect has been recently applied to propel monodomain liquid crystalline elastomer rods.

Figs. 1c)-f) show a second possibility to drive fibers: here, uncooked spaghetti (c), as well as Polyacrylamide (PAM) hydrogel rods (e), were put on a wet kitchen towel. The water then slowly enters the rods via the wet substrate, differentially swells and hence curves them. As in the case of heat flow in the first example, there is a constant solvent flow (water enters from the bottom and evaporates at the top) and the fibers start to roll, albeit much slower here, with typical velocities of few cm/hour, cf. the kymograph in Fig. 1f). The traveling direction with respect to curvature for hygroscopic swelling is analogous to the thermal expansion case. A related effect was reported earlier using PDMS rods. However, there, a solvent droplet was injected at what later becomes the back of the rolling rod, i.e. the spontaneous symmetry breaking and bi-directionality, characteristic for fiberboids (cf. Ref. [and as discussed below],

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was absent in these earlier examples.

### 3 Generic model

Looking at these examples, with different driving mechanisms and operating at (vastly) different timescales, a natural question arises: Is there a unified approach to understand the dynamics? Both systems have in common that there is a propelling agent “flowing” between two phases (baths) through the filament. The propellant, which stands for heat energy density or water concentration, respectively, in the two example cases, has a scalar density within the material denoted by \( \psi(\rho, \phi, t) \). The latter depends on the polar coordinate position \((\rho, \phi)\) in the fiber cross-section in the laboratory frame and on time \(t\). For the basic physics, however, only the differences of \( \psi \) in the normal (z-direction/top-to-bottom) and in-plane direction (x-direction/left-to-right) will be important, cf. Fig. 2.

The responsiveness of the filament is related to the coupling between the propellant density and elastic deformations. In the simplest case, the presence of the propellant gives rise to an isotropic material eigenstrain (also called prestrain) tensor

\[
\epsilon_{ij}^{\text{eig}} = \alpha \psi \delta_{ij},
\]

with \(i, j \in \{x, y, z\}\) and \(\delta_{ij}\) the Kronecker delta. Here \(\alpha\) is an isotropic linear expansion coefficient, describing thermal expansion or gel swelling, respectively. Note that this coefficient can also be negative, corresponding to contraction, as in case of heated nylon. The eigenstrain can be seen as the locally preferred strain in the absence of local and global constraints. It evolves in time with the propellant concentration and gives rise to all mechanical effects in fiberboids.

The propellant’s evolution is governed by three processes: (i) the propellant influx rate \(q\), (ii) its diffusive relaxation rate \(r\) and (iii) the rotation of the cross-section with angular velocity \(\omega\), in case the rolling motion has set in. The influx rate scales as

\[
q \sim D_R \Delta_t \psi^2,
\]

Fig. 1 Fiberboids in action. a) A nylon fiber rolling on a heated aluminium substrate. See supplementary movie 1. b) A kymograph (space vs. time plot) of the center of mass position of the nylon fiber shown in a). c) A stroboscopic picture of three initially dry spaghetti curving and then rolling on a wet towel (at \(\sim 60\%\) air humidity at \(25^\circ \text{C}\)). See supplementary movie 2. d) Kymographs for two of the rolling spaghetti shown in c). e) Polyacrylamide (PAM) hydrogel rods on a wet towel. f) Stroboscopic picture showing the rolling motion in e).

Fig. 2 a) Sketch of the mechanism underlying fiberboid rolling. The difference in propellant density \(\psi_{\text{top}} - \psi_{\text{bottom}}\) gives rise to a propellant flux establishing a gradient along the z-direction. Differential expansion then leads to a preferred out-of-plane eigen-curvature, which however cannot be realized due to the confinement to the interface. Instead, an in-plane curvature \(\kappa_{\text{eig}}^x\) is built-up and the constant phase difference between the propellant-induced strain and the realized strain results in a driving torque that induces a rotation of every cross-section with frequency \(\omega\). b) Depending on their sign of the expansion coefficient \(\alpha\), fiberboids fall into two classes: “vexers” having \(\alpha > 0\) and rolling towards their center of curvature, as exemplified by a PDMS rod. And “cavers” having \(\alpha < 0\) and moving away from it, as exemplified for a nylon fiber. Arrows show direction of motion.
i.e. it is given by the difference of the propellant concentration $\Delta_N \psi = \psi_{top} - \psi_{bottom}$ (between the top and the bottom of the fiber), the propellant diffusion coefficient $D_p$ and the fiber’s cross-sectional radius $R$. In the absence of rotation, the propellant diffusive relaxation rate

$$r \sim D_p R^{-2}$$

(3)
counterbalances the influx, giving rise to a typical z-gradient $\nabla \psi \sim \frac{\Delta_N \psi}{R}$. The latter induces an eigenstrain gradient in the axial component (i.e. $\varepsilon^{eig}_{yy}$, where $y$ lies along the fiber axis), which in turn leads to a preferred out-of-plane eigen-curve,

$$\kappa^{eig}_x \sim R^{-1} \Delta_N \varepsilon^{eig}_{yy}.$$  

(4)

This curvature $\kappa^{eig}_x$, however, is entirely “virtual”, i.e. it cannot be actually realized due to the presence of normal forces$^*$ confining the fiber to the interface plane – in the two macroscopic examples presented in Fig.1 the fiber is confined simply by its own weight. Given the confinement, a fiber that is constantly driven by a flux normal to the confinement plane is mechanically frustrated. Being unable to simply leave the plane, it follows an alternative path of least action and tends to axially rotate by $\sim 90^\circ$, so that it can realign its preferred curvature with the x-direction.

This results in the build-up of an in-plane curvature

$$\kappa^{eig}_x \sim R^{-1} \Delta_N \varepsilon^{eig}_{yy},$$

(5)

where $\Delta_N$ now stands for the left-right difference (in-plane, perpendicular to the fiber axis). Since the eigenstrain is proportional to the linear expansion coefficient $\alpha$, fiberboids can be divided into two classes: “vexers” having $\alpha > 0$ and “cavers” having $\alpha < 0$ (implying convex and concave shapes, respectively) as shown in the snapshots in Fig.2).

The frustration-induced in-plane curvature is the precursor for the occurrence of the torque driving fiberboid rolling: the constant phase difference between the propellant-induced strain (normal to the plane) and the realized strain (in-plane) results in a driving torque that induces a rotation of every cross-section with frequency $\omega$. The driving torque density (per unit length) is given as the product of the geometrically induced strain $\Delta_N \varepsilon^{eig}_{yy}$ and the thermally induced stress $Y \kappa^{eig}_x$, with $Y$ Young’s modulus, integrated over the cross-section (i.e. times $R^2$), yielding

$$m_{\text{drive}} \sim R^2 Y \left(\Delta_N \varepsilon^{eig}_{yy} \right) \left(\Delta_N \varepsilon^{eig}_{yy} \right).$$

(6)

Since the eigenstrain and the expansion coefficient $\alpha$ are linearly related and $m_{\text{drive}}$ is quadratic in strain, the torque is insensitive to the sign of $\alpha$. Together with the sign sensitivity of the in-plane curvature $\kappa^{eig}_x$, this explains the opposite relations of fiber curvature and direction of motion for vexers and cavers.

3.1 Minimal model

A minimal dynamic model, capturing the physical essence of most fiberdrive phenomena has three variables: the two non-dimensional eigen-strain gradients $x = R \kappa^{eig}_x$, $z = R \kappa^{eig}_z$ and the angular frequency $\omega$:

$$\dot{x} = -r x - \omega z$$

(7)

$$\dot{z} = -r z + \omega x + p$$

(8)

$$\omega = -\mu z$$

(9)

with constants $|r|, |p|, |\mu| = s^{-1}$ having dimension of a rate. The first two equations describe the kinetics of the propellant-induced eigenstrains in the cross-section, with $r$ and $p = aq$ the strain relaxation and strain pumping rates, respectively. The terms proportional to $\omega$ in Eqs. (7-8) originate from the interconversion of the two modes via rotation. Eq. (2) reflects the torque balance between $a$, here assumed Stokes-like, dissipative torque $-\frac{1}{2} \mu \omega$ with $\mu$ a friction constant, and the driving torque (density) from Eq. (6), which reads $R^2 \gamma xz$. The ratio

$$\mu \sim \frac{Y R^2}{\xi}$$

(10)

occurring in Eq. (9) can be interpreted as a “mechanical mobility” that captures the elastodynamic response rate of the fiber. We note that equations Eqs. (7-8), as well as the functional dependence of the driving torque can also be derived directly from the underlying equations of thermo-elasticity of a slender rod and thermal advection-diffusion$^$.

Let us study the steady states of the model. First, obviously, an immobile state $\omega = 0$ exists, with $x = 0$ and $z = p/r$. In the case of thermal driving this state corresponds to the thermal conduction state in absence of any advection. A second steady state emerges, provided the following relations hold

$$\omega^2 = \frac{|p| \sqrt{\mu r} - r^2}{r},$$

(11)

$$\lambda^2 = \frac{|p| \sqrt{\mu |r|} - r^2 \mu}{r^2 \mu}.$$  

(12)

For this motile state to exist, the r.h.s. of both relations has to be positive, implying the existence of a critical driving

$$|p| > \lambda_{\text{crit}}^2 = \mu^{-1/2} R^{3/2}.$$  

(13)

Physically, this relation states that the pumping rate has to be larger than a threshold value, namely large compared to the two forms of losses: $r$ (thermodinamic loss due to propellant

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$^*$Two generic cases of confinement may occur: In the case of short range, adhesive contact forces – e.g. Pickering-type pinning at the interface$^$ – the fiber stays fully confined as long as its bending energy density is smaller than the gain in adhesion energy. For macroscopic fibers confinement is due to gravity, which is a small and long-ranged force and additional effects come into play that can be fully confined as long as its bending energy density is smaller than the gain in~

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relaxation) and $\mu^{-1}$ (frictional loss). The other way round, motion will stop, if the mobility $\mu$ drops below a critical value $\mu_{\text{crit}} = r^2 P^{-2}$, e.g. in case when the friction increases due to a change of environment. Note that while the translational velocity $v = R\dot{\omega}$ always grows with $p$ and $\mu$, the in-plane curvature $\kappa = \frac{\bar{\psi}}{R}$ is non-monotonous in $\mu$ and vanishing both for small $\mu \leq \mu_{\text{crit}}$ and large mobilities $\mu \to \infty$. A maximum curvature $\kappa^* = \frac{\bar{\psi}}{2R} P^{-1}$ is attained at an intermediate mobility value $\mu = \mu_{\text{crit}}$.

The model hence predicts a pitchfork bifurcation in the (angular) velocity at $p = p_{\text{crit}}$, cf. Fig. 3. The rolling direction is spontaneously chosen, as observed for both the thermal and the hygroscopic fiberboids. Fiberboids are hence bidirectional and the onset of motion occurs via a spontaneous symmetry breaking. It should be noted that in Ref. 1 dry friction instead of Stokes friction was used in the model, which renders the onset of motion discontinuous.

To experimentally probe the model predictions, Eqs. (11) and (12) can be used to obtain the following steady state relation,

$$\kappa = -\frac{p \bar{\dot{\psi}}}{R \omega^2 + r^2},$$

connecting the two directly measurable variables, curvature $\kappa$ and angular velocity $\omega$. It should be noted that the validity of this relation is independent of $\mu$ and hence of the frictional mechanism – i.e. the same relation can be obtained if dry friction is assumed in Eq. 9.

Fig. 4 shows experimental results for both thermally driven nylon fibers in (a) and humidity-driven thin spaghetti (capellini) in (b). On the one hand, all the measured points, obtained both from transient dynamics and steady rolling, fall on the master curve given by Eq. 14. This hence allows to extract the pumping and relaxation rates $p$ and $r$, as given in the figure caption.

On the other hand, an interesting question arises: why do the points, even when obtained for a single sample, not condense into a single spot? In fact, a given mechanical mobility $\mu$ should select a single point on the master curve. This can be evidenced as in Fig. 5 where numerically obtained trajectories from Eqs. (7) and (9) are shown in the plane in-plane eigenstrain vs. angular velocity. All trajectories were initiated with zero strain (and hence curvature) and velocity. After a transient excursion they all land on the master curve, with low mobility values $\mu$ corresponding to the ascending branch and higher mobilities to the descending branch of the master curve.

The experimentally observed variance along the master curve could be partially rationalized by the inherent disorder of each sample (especially pronounced for the starch-based noodles used) and the lack of end-piece confinement of the filaments. The latter sometimes leads to a visible “hinging”-type of movement with a pronounced anti-phase oscillation of velocity and curvature (see also Fig. 2). This behavior can be interpreted in terms of a time-dependent mobility $\mu(t)$ that oscillates during one rotation cycle. Furthermore, inhomogeneities of the (here rigid) substrate surface can further contribute to variations in $\mu$. Both effects combined can lead to a “migration” along the $\kappa - \omega$ master curve for each sample.

However, in addition to such intrinsic disorder effects, there are more systematic reasons for this behavior necessitating extensions of the minimal model, discussed in the next section.

### 3.2 Refined models

For the sake of simplicity, in the minimal model we have neglected certain processes and materials behavior. First, besides $x$ and $z$, there is a third dynamic “mode” that can become important: namely, a storage or internal reservoir mode, related to the average amount of propellant density $\bar{\psi}$. The presence of this mode can be visualized experimentally by placing a hot fiber (formerly rolling on a heated plate) on a room temperature cold plate. The fiber will still roll transiently – until it has equilibrated with the plate – but now curves oppositely with respect to the direction of motion as before, see supplementary movie 3. The reason is that, now, heat is flowing out of the fiber.

A second effect concerns the material properties of the fiber. In case of thermal driving, the storage mode corresponds to the average temperature in the fiber cross-section and as such will affect the fiber’s mechanical properties: most importantly, the bending rigidity, determined by Young’s modulus $Y$, will vary with the average temperature (or the average density of propellant). In addition, other material parameters may also vary, for instance the (linear) thermal expansion coefficient depends on the reference temperature.

The storage mode, that was neglected in Ref. 3, can be included in the model on a generic level, as well as directly derived from the underlying thermal diffusion-convective equation, see Appendix A. Its dynamics is given by

$$\dot{\bar{\psi}} = -r_{\psi,\bar{\psi}} \bar{\psi} + p_{\psi},$$

hence analogously to the $z$-mode, the storage mode is pumped and relaxes, albeit with different (but related) rates $r_{\psi}, p_{\psi}$. Eqs. (7) and (9) remain unchanged, but now the mobility $\mu(\psi)$ is a function of $\bar{\psi}$, stemming from the $\psi$ – dependence of Young’s modulus $Y(\psi)$. The main effect of the storage mode is exemplified in Fig. 3 where we assumed that $\bar{\psi}$ relaxes slower than $x, z$ and leads to a substantial softening of the Young’s modulus. One can see that the system stays close to the master curve for a substantial amount of time while moving along it in a non-monotonic fashion (see colored dots).

Yet another complexity stems from the fact that the propel-
lant transfer is typically non-symmetric. This effect is especially pronounced when there is one solid-solid and one solid-air interface on the two sides of the fiber, giving rise to vastly different boundary conditions on the flux: considering thermal flux, typically, the conductive heat flux at the solid-solid interface is much faster than the diffusion-advection dominated flux at the solid-air interface. In that case it can be shown (see Appendix B) that Eq. (8) gets an additional term, coupling directly to the storage mode dynamics,

\[ \dot{\psi} = -\kappa \dot{x} + \omega x + \chi(-r_p \psi + p_p), \]  

with \( \chi \) a coupling coefficient, see appendix for details.

In case of a symmetric propellant transfer, i.e. perfect thermal contact at bottom and top with the boundary condition \( T_{\text{ext(bottom)}} = T_{\text{solid}} \) and \( T_{\text{ext(top)}} = T_{\text{air}} \), it can be shown by solving the rotationally advected heat equation in the fiber cross-section (see appendix A) that \( \chi = 0 \). In this simplest case, the mean propellant density, \( \psi \), influences the dynamics only via slow changes in the mechanical parameters, as described above. In contrast, if a more realistic boundary condition is employed, namely \( l_{ib} \frac{d}{d \psi} T + (T - T_{air}) = 0 \) accounting for convective heat transport in the surrounding liquid (in our case air), \( \chi \) has the following properties, see appendix B: \( \chi \) is (i) finite, (ii) proportional to the thermal expansion coefficient \( \alpha \) and (iii) increasing with the thermal length scale \( l_{ib} \), until saturating for large values of \( l_{ib}/R \).

**Fig. 4** Master curves, as described by Eq. (14), of curvature \( \kappa \) vs. angular velocity \( \omega \) for nylon fibers on a heated plate (a), and thin spaghetti (capellini) on a wet towel (b). Blue dots are experimental data obtained from both transient motion and of steady rolling fibers, the red curve is a fit to the theory, allowing to extract values for the propellant pumping rates and the propellant relaxation rates: one obtains \( p = 0.85 \text{s}^{-1} \) and \( r = 20.0 \text{s}^{-1} \) for nylon (diameter 0.4 mm, temperature 160°C) and \( p = 1.7 \text{h}^{-1} \) and \( r = 4.3 \text{h}^{-1} \) for capellini (diameter 1 mm, humidity 60%, temperature 25°C).

**Fig. 5** Shown are numerically obtained trajectories from Eqs. (7-9) in the plane in-plane eigenstrain \( x(t) \) vs. angular velocity \( \omega(t) \) and the corresponding master curve for the stationary state. All trajectories were initiated with zero strain (and hence curvature) and velocity, \( x(0) = 0, \omega(0) = 0 \). After a transient excursion they all land on the master curve, with low mobility values \( \mu \) corresponding to the ascending branch and higher mobilities to the descending branch of the master curve. The black dashed line is the master curve given by Eq. (14) with \( x = \kappa R \). Parameters: \( r = p = 1 \).

**Fig. 6** Shown are numerically obtained trajectories from Eqs. (7-9) as in Fig. 5 but accounting for the dynamics of the storage mode, Eq. (15). We assumed \( \mu = \mu_0(1 - 0.2\psi) \) and \( r_p = p_p = 0.3 \). Hence the time scale for the dynamics of the storage mode is slower than the \( x,z \)-modes by roughly a factor of three, and the Young’s modulus (and hence the mobility) is reduced due the increased temperature to 20% of its initial value (at room temperature). The black dashed line is the master curve given by Eq. (14) with \( x = \kappa R \). One can see that for both examples, after a rapid transient with high frequency and curvature excursions, the system stays close to the master curve for a substantial amount of time and moves in a non-monotonic fashion (see indicated times as colored dots).
Hence there are two important slow modes, not present in the minimal model: first, the dynamics of $\Psi$, which is slower than the one of the modes $x$ and $z$ (for geometrical reasons, $r_{\Psi} < r$ holds), influences the dynamics via slow changes in the mechanical parameters. And second, in the general, asymmetric flux case, $\Psi$ also induces a slow variation of the effective driving of the $z$-mode like $p_{\text{eff}}(t) = p + \chi(-r_{\Psi}\Psi(t) + p_{\Psi})$, cf. Eq. (16). Inspecting Eq. (15), as $\Psi$ is equilibrating, $p_{\text{eff}}(t)$ exponentially relaxes towards $p$.

When placing a nylon fiberboid having room temperature on a heated plate, one hence expects the following slow relaxations in time of the parameters: the thermal expansion of nylon is typically non-linearly increasing in the relevant region $T \in [100 – 180]^{\circ}$C, meaning it slowly increases with the mean temperature $\Psi$ until it reaches its stationary value. Hence the effective pumping rate of the $z$-mode, $p_{\text{eff}}(t)$, increases due to its proportionality in the expansion coefficient $\alpha$. However, as the driving flux is asymmetric, at the same time the pumping rate relaxes due to equilibration of $p_{\text{eff}}$ towards $p$. Altogether, the behavior of the pumping rate is hence non-monotonous in time in the general case. In contrast, the second material coefficient affected by the storage mode, the mobility $\mu$, always decreases in time: it is proportional to Young’s modulus $Y$, which softens with temperature.

Fig. 7 shows the time dependence of curvature (red) and rolling velocity (blue) for a thermally driven nylon fiber and a humidity-driven spaghetti fiberboid, respectively. In both cases, one can observe an anti-correlation between curvature and velocity, i.e. the shown transient events close to the onset of motion are located on the descending branches of the respective master curves shown in Fig. 4 (a), b). Interestingly, for nylon curvature decreases while velocity increases while noodles show the opposite trend, including oscillations related to the less strict confinement of the fiber ends.

4 “Social” behavior: Interaction with walls and other fiberboids

As we have seen, fiberboids are bidirectional and display spontaneous symmetry breaking at the onset of motion. This suggests that they should be able to respond to their environment, e.g. by inverting their direction of motion, and that they are poised to engage in interesting collective behavior, see supplementary movie 4. In the following we study the two-body behavior and answer the question: What happens when running fiberboids engage immovable obstacles or when they frontally collide with other members of their species?

The answer to this question depends on whether the fiberboids are “vexers” or “cavers”, cf. Fig. 2 (b), i.e. if they curve towards or away from their direction of motion. In general, by simple geometric reasoning, cavers (like thermally driven nylon) establish single-point contact at their center apex position, while vexers (like humidity-driven noodles) make contact at their distant end points. The contact in the former case is hence much more stable and leads to more predictable interactions. In the latter case, the two-point contact renders the interaction less predictable because the ends are often insufficiently confined to the plane.

4.1 Cavers

Fig. 8 shows the generic scenarios found for thermally driven fiberboids of “caver”-type hitting a wall. Experimentally, this can be simply studied by confining the rolling fiber between two glass slides (placed as obstacles) on the heating plate. Panel a) shows a weakly driven fiber ($T = 130^{\circ}$C) that is simply stopped by the wall. The reason is a combination of increased friction due to the presence of the wall and unfavorable in-plane heat transfer through the glass wall to the fiber. In fact, the back of a rolling nylon fiber is warmer than the front (cf. Fig. 2 (a)); for thermally driven fibers the color-code there means red=hot, blue=cold). But the front receives additional thermal energy from the glass wall, disfavoring the motion.

However, if the temperature is further increased, as shown in Fig. 8 (b) for $T = 160^{\circ}$C, the fiber gets effectively reflected from the wall: the middle part of the fiber is slowing down and “loosening” curvature just to an extent that the fiber ends are able to catch up and the thermal transfer through the glass can invert the direction of motion. As seen in the same figure, reflections are nicely reproducible at both walls. For even higher driving, cf. Fig. 8 (b), the fiber stubbornly and continuously rolls against the wall, while its ends, not in direct contact to the wall, perform a kind of flapping motion.

The model developed in section 3 can account for this generic
behavior of fiberboids colliding with walls (or with other fiberboids) as follows: in fact, the case shown in Fig. 8a) corresponds to weak driving (p small) and a rather rough obstacle (mobility µ low). The fiberboid stops since the first term in Eq. 11 decreases with the reduced µ resulting from the wall friction: if it becomes less than the second contribution, rolling is no longer possible. In the case of Fig. 8b), corresponding to intermediate p, a finite rotation speed is still possible. However, Eq. 12 then implies that the curvature is reduced due to friction, since the second, negative contribution has a stronger µ-dependence. This leads to the catch-up of the ends described above and then the fiber is ready to invert due to the additional heat influx from the wall into what will become the new back of the reversed fiber. If p is even larger, the reduction in µ is not sufficient to reverse the fiber and rolling against the wall persists, corresponding to the case shown in Fig. 8c).

For binary collisions between two fiberboids, quite similar effects to wall collisions are observed. Fig. 9 shows the main types of events for binary central collisions between two thermally driven fiberboids of equal lengths. Panels a), b) shows what we call “follower-leader” behavior: for a short transient, both fibers roll against each other; but then the “weaker” inverts direction and the pair now travels together. This pairing is quite robust against further collisions or perturbations. Panels c), d) shows what we call “dancing pair” behavior: here, the pair keeps on rolling against each other for long times. At the same time, the common center of mass goes back and forth, due to flapping-type curvature variations.

4.2 Vexers

Vexers, like the hygroscopically driven spaghetti, can exhibit similar “social behavior” as cavers, as exemplified in Fig. 10a) by an inversion event upon a collision. However, unlike cavers, vexer-type fiberboids exhibit more random, generically unstable, multipoint contact configurations. This leads more frequently to complex out-of-plane deformations, followed by mutual entanglement or amusing “tunneling” behavior where one filament lifts and bypasses the other, as exemplified in Fig. 10b). Such richness and variability of collision outcomes makes vexers overall less suitable for investigating their collective behavior.

5 Conclusions and Outlook

Fiberboids are a new member of the class of active, self-propelled particles that was surprisingly long hiding in the plain view, before being only recently described. They share some similarities with other actively rolling systems, like Quincke spinners and Leidenfrost droplets, but display their own and unique features. In particular they develop a continuously propagating internally rotating prestress mode akin to Bénard convection in fluids, yet surprisingly, running through a solid material.

Many everyday objects can be easily turned into fiberboids, as demonstrated here: pieces of fishing line spin quickly on a hot plate, while spaghetti noodles spin both rapidly on a hot plate, as well as much more slowly on wet cloth. The latter example also shows that different fluxes can be employed to drive fiberboids, and on vastly different timescales (differing by orders of magnitudes). Last but not least, fiberboids are
Fig. 9 Typical binary collision events of thermally driven nylon fiberboids of equal length. Two main outcomes for the head-on collision dominate: panel a) sketches an event, where after rolling against each other for a short transient time, one fiber switches directionality such that finally both move together as a pair with almost equal velocity. This can be seen in the kymograph shown in b) at the very left. Note that the kymograph also displays several collisions of the fiberboid pair with boundaries, where the pair is effectively reflected. Hence this “follower-leader” mode is quite robust against perturbations. Panel c) sketches a second common event, where the pair keeps on rolling against each other, with the center of mass of the “dancing pair” slightly going back and forth due to curvature variations (“flapping”). As can be seen from the time scale bar in d), this state can be stable for a substantial time (minutes).

Fig. 10 Typical binary collision events for vexers, realized by spaghetti on a wet towel. Panel a) shows a collision and inversion of short spaghetti. See supplementary movie 6, b) a collision of one long and one short spaghetti. In the latter case, the longer specimen transiently leaves the planar confinement and “jumps” over the shorter one. See supplementary movie 7.

anisotropic objects and hence should display interesting new collective effects compared to the existing spherical rollers.

Here we have laid out the basic phenomenology and proposed generic models of increasing complexity for the dynamics of fiberboids. We started from a simple two-mode model, describing the dynamics of the propellant gradients in two orthogonal directions of the fiber cross section. This simple model phenomenologically captures many of the features of fiberboid behavior, including the symmetry breaking at the bifurcation point and a curvature-velocity dynamics following a simple master curve. To treat all the experimentally observed phenomena, including dynamic transients, we extended the two-mode description and introduced a third degree of freedom – a “storage mode” corresponding to the mean propellant density in the cross section. Interestingly, this mode couples in several ways, namely both via the mechanics and the dynamics, to the other modes.

By studying fiberboid collision events, both with walls and other fiberboids of the same type, we have extracted the conceptual ingredients for future studies of the collective behavior of (many) fiberboids. Moreover, the generic fiberboid collision events also could be rationalized by the model.

We conclude with a simple message: fiberboids should be a general phenomenon in nature, as they require only a few physical ingredients, which are: a) An interface between two phases A and B, at least one of which is a fluid (to allow for mobility). b) The phases A and B should be out-of-equilibrium, giving rise to a flux of energy/matter across their interface. c) Straight filaments, with cylindrical cross-section, that are pinned (confined) at the interface and that mechanically respond to the flux.

These three conditions are indeed generic, also on the microscopic scale, as we are surrounded by interfaces that frequently are out-of-equilibrium. Naturally ordered, biological fibers can be trapped there by Van der Waals or electrostatic forces or Pickering pinning, and driven by a multitude of chemical and thermodynamic fluxes. Moreover, it has been shown that many biofilaments, like microtubules and intermediate filaments can spontaneously break their cylindrical sym-
we will later identify the final prestress and leads to symmetry broken zero energy (“wobbling”) modes. The latter zero modes are similar to the dissipatively emerging (rotating) curvature mode in fiberboids and could further facilitate the emergence of the fiberboid effect by eliminating the threshold for buckling. Related ideas of generating fluxes via various ratchet effects in circular DNA could further expand our toolbox to “motorize any filament”. Recently, we have speculated that filamentous viruses like ebola might utilize fiberboid physics to roll on surfaces. Considering the ease with which we generated fiberboids, in particular the motorization of simple starch fibers (noodles) in various ways, it seems in fact unlikely that nature could have overlooked the motif.

Appendix A: Symmetric thermal drive

In the case of thermal driving, one has to solve the rotationally advected thermal diffusion equation

$$\partial_t T = \frac{D}{R^2} \nabla^2 T + \omega \partial_\theta T.$$  \hfill (17)

Note that we rescaled space by the fiber radius $R$, such that in polar coordinates, $T(\rho, \phi)$, $\rho \in [0, 1]$. For the simplest case of symmetric thermal contact, the boundary condition (BC) at the cylinder surface reads

$$T(1, \phi) - T_{\text{ext}}(1, \phi) = 0.$$  \hfill (18)

To capture the essential features, this BC is approximated by

$$T_{\text{ext}}(1, \phi) = \frac{T_b - T_{\text{air}}}{2} \cos \phi + \frac{T_b + T_{\text{air}}}{2} = T^a \cos \phi + T^+, \hfill (19)$$

Like that $T_{\text{ext}}(1, 0) = T_b$ is the temperature at the bottom (‘surface’) and $T_{\text{ext}}(1, \pi) = T_{\text{air}}$ is the temperature at the top (‘air’).

We now are looking for a solution of the form

$$T = C(\rho) \cos \phi + S(\rho) \sin \phi + \tilde{T}(\rho).$$  \hfill (20)

We will later identify the $\phi$-mode of the main text (as $C$), the $\rho$-mode (as $S$) and the storage mode $\tilde{T}$ (as $\tilde{T}$), respectively. The symmetry-adapted functions to the problem are the so-called Zernike polynomials which have the general form $Z_n^m = R_n^m(\rho) \cos(m \phi)$, $Z_n^{-m} = R_n^m(\rho) \sin(m \phi)$. We will need only the first few which are given by

$$Z_0^0 = 1, \quad Z_1^0 = \sqrt{3} (3 \rho^2 - 2) \sin \phi, \quad Z_1^1 = \sqrt{3} (3 \rho^2 - 2) \cos \phi,$$

Putting the ansatz Eq. (20) with

$$C = c_1 R_1^1 + c_2 R_2^1, \quad S = s_1 R_1^{-1} + s_2 R_2^{-1}, \quad \tilde{T} = T_0 R_0^0 + T_2 R_2^0$$

into the BC (19) and neglecting higher modes ($m > 2$) yields the three conditions $C(1) = T^{-} = 0$, $S(1) = 0$ and $\tilde{T}(1) = T^{+} = 0$, that can be solved for $c_2, s_2$ and $T_2$ to yield the following ansatz,

$$T = s_1 \left[ Z_1^{-1} - \frac{1}{\sqrt{2}} Z_1^{-1} \right] + c_1 \left[ Z_1^1 - \frac{1}{\sqrt{2}} Z_1^1 \right] + \frac{T^+}{2\sqrt{3}} Z_2^0 + T_0 \left[ Z_0^0 - \frac{1}{\sqrt{3}} Z_0^0 \right] + \frac{1}{3} T^+ Z_2^0.$$  \hfill (21)

We now insert this ansatz into Eq. (17) and project on the different modes via $\phi' = \phi'_{Z_m} dp$ to project on $Z_1^{-1}$ (and analogously for $Z_1^1$ and $Z_0^0$). Using the relations for the Laplacian of the Zernike polynomials, $\Delta Z_1^0 = 0, \Delta Z_1^1 = 0, \Delta Z_2^0 = 2Z_2^0$, as well as orthogonality, yields

$$\dot{s}_1 = -\frac{24D}{\sqrt{2^2 R^2}} s_1 - \omega \phi,$$

$$\dot{c}_1 = -\frac{24D}{\sqrt{3^2 R^2}} \left( c_1 - \frac{T^+}{2} \right) + \omega s_1,$$

$$\dot{T}_0 = -\frac{8D}{\sqrt{3^2 R^2}} (T_0 - T^+).$$  \hfill (22)

Identifying $s = \alpha s_1$, $z = \alpha c_1$ and $\bar{T} = T_0$, this can be written as in the main text, i.e.

$$\dot{x} = -x - \omega z,$$

$$\dot{z} = -z + \alpha x + p,$$

$$\bar{T} = -\bar{r} \bar{T} + \bar{p},$$

with the rates

$$r = \frac{24D}{\sqrt{2^2 R^2}}, \quad p = \frac{24D}{\sqrt{2^2 R^2}} \alpha (T_b - T_{\text{air}})$$

for the $x, z$-modes (where $\alpha$ is the thermal expansion coefficient) and

$$\bar{r} = \frac{8D}{\sqrt{3^2 R^2}}, \quad \bar{p} = \frac{8D}{\sqrt{3^2 R^2}} (T_b + T_{\text{air}})$$

for the storage mode.

As it should be, the stationary state for the storage mode is $\bar{T} = \bar{T}^+_\text{th}$, i.e. the mean temperature. The stationary heat conducting state is given by $z = \frac{p}{\bar{r}} = \frac{\alpha (T_b - T_{\text{air}})}{4}$, i.e. proportional to the mean temperature difference.

Appendix B: Asymmetric thermal drive

As discussed in the main text, a more realistic boundary condition is

$$\frac{d}{dp} T(1, \phi) + [T(1, \phi) - T_{\text{ext}}(1, \phi)] = 0.$$  \hfill (26)

Here $l(\phi)$ is a thermal length scale describing convective transport in the fluid medium (‘air’) at the top, see Ref. We assume $l(\phi = \pi) = l_a$ at the top and at the bottom $l(\phi = 0) = 0$, i.e. still perfect thermal contact at the metal surface. The symmetry-adapted approximated version of this BC then reads

$$l(1, \phi) = (1 - \cos \phi) \frac{l_a}{2}.$$  \hfill (27)
and $T_{	ext{cen}}(1, \phi)$ as defined in Eq. (19). Using the same Zernike-mode expansion and projection yields the equations

$$
\dot{x} = -rx - \omega z, \quad (28)
$$

$$
\dot{z} = -rz + p + \omega x + \chi (r_y \psi + p_y - \varepsilon z), \quad (29)
$$

$$
\dot{\psi} = -r_y \psi + p_y - \varepsilon z. \quad (30)
$$

Now $r$, $p$, $r_y$, $p_y$ are all functions of $\lambda = \frac{l}{R}$ that can be given explicitly. More importantly, a coupling between $z$ and $\bar{\psi}$ arises, with

$$
\varepsilon = \frac{8}{\sqrt{3}} \frac{D}{\alpha R^2} \frac{\lambda}{1 + \frac{3}{2} \frac{\lambda}{R^2}}, \quad \chi = \frac{9\sqrt{3}}{2\sqrt{2}} \frac{\lambda}{\lambda^2 + 2}. \quad (31)
$$

It can be seen that for $l, \lambda \to 0$ we regain the symmetric case since $\varepsilon, \chi \to 0$. The term in brackets in Eq. (29) can be interpreted as a temporal modulation of the driving term $p$ of the $z$-mode, as discussed in the main text (where the contribution $\varepsilon z$ was omitted to simplify the discussion).

**Conflicts of interest**

There are no conflicts to declare.

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