Teleparallel Versions of Friedmann and Lewis-Papapetrou Spacetimes

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Abstract

This paper is devoted to investigate the teleparallel versions of the Friedmann models as well as the Lewis-Papapetrou solution. We obtain the tetrad and the torsion fields for both the spacetimes. It is shown that the axial-vector vanishes for the Friedmann models. We discuss the different possibilities of the axial-vector depending on the arbitrary functions \( \omega \) and \( \psi \) in the Lewis-Papapetrou metric. The vector related with spin has also been evaluated.

Keywords: Teleparallel Theory, Torsion.

1 Introduction

The dynamics of the gravitational field can be described with the help of Teleparallel theory (TPT) [1]. This theory is characterized by the vanishing of curvature identically but the torsion is taken to be non-zero. The basic entity of this theory is the non-trivial tetrad field \( h^a_\mu \) while in General Relativity (GR) the metric tensor plays the role of the basic entity. TPT corresponds to a gauge theory for the translation group [2,3] based on Weitzenböck geometry [4]. In spite of these fundamental differences, the two theories provide equivalent descriptions of the gravitational interaction [5].

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This implies that curvature and torsion might be simply alternative ways of describing the gravitational field. Consequently, these are related to the same degrees of freedom of gravity. This supports the fact that the symmetric energy-momentum tensor is a source in both the theories, i.e., the source of curvature in GR and the source of torsion in TPT. In some other theories [2,6], torsion is only relevant when spins are important [7]. This point of view indicates that torsion might represent additional degrees of freedom as compared to curvature. As a result, some new physics may be associated with it.

Even if GR is the unique true theory of gravity, consideration of close alternative models can shed light on the properties of GR itself. Theories of gravity based on the geometry of distance parallelism [3, 8-14] are commonly considered as the closest alternative to the GR. TP gravity models possess a number of attractive features both from the geometrical and physical viewpoints. Teleparallelism is naturally formulated by gauging external (spacetime) translation and underline the Weitzenböck spacetime characterized by the metricity condition and by the vanishing of the curvature tensor. Translations are closely related to the group of general coordinate transformations which underlies GR. Thus the energy-momentum tensor represents the matter source in the field equations of tetradic theories of gravity like in GR.

There is a literature available [15-21] about the study of TP versions of the exact solutions of GR. Recently, Pereira, at el. [22] obtained the TP versions of the Schwarzschild and the stationary axisymmetric Kerr solutions of GR. They proved that the axial-vector torsion plays the role of the gravitomagnetic component of the gravitational field in the case of slow rotation and weak field approximations. Also, Nashed [23-25] has found many TP versions of the exact solutions in GR and used them to calculate different quantities. In this paper, we extend the procedure to find the TP versions of the Friedmann models and the stationary axisymmetric Lewis-Papapetriou solution of GR. It turns out that the axial-vector has only two non-vanishing components for the Lewis-Papapetriou spacetime as expected. However, the axial-vector torsion vanishes for the Friedmann models due to spherical symmetry. This is similar to the case of Schwarzschild spacetime [22].

The structure of the paper is as follows. In section 2, we shall briefly review the main results of the teleparallel theory. Section 3 is devoted to determine the tetrad field, the Weitzenböck connection and the irreducible components of the torsion tensor for the Friedmann models. Section 4 pro-
vides the evaluation of the tetrad field, the Weitzenböck connection and the irreducible components of the torsion tensor for the Lewis-Papapetrou spacetime. These will give the vector and the axial-vector parts of the torsion tensor. We shall summarize and conclude the results in the last section.

2 An Overview of the Teleparallel Theory

We define the Weitzenböck connection as \[ \Gamma^\theta_{\mu\nu} = h^a_\theta \partial_\nu h_a^\mu, \] (1)
where the non-trivial tetrad \( h^a_\mu \) with its inverse field \( h^a_\nu \) satisfies the relations
\[ h^a_\mu h^a_\nu = \delta^\nu_\mu; \quad h^a_\mu h^b_\mu = \delta^a_b. \] (2)

In this paper the Latin alphabet \( (a, b, c, ... = 0, 1, 2, 3) \) will be used to denote the tangent space indices and the Greek alphabet \( (\mu, \nu, \rho, ... = 0, 1, 2, 3) \) to denote the spacetime indices. The Riemannian metric in TPT arises as a by product [3] of the tetrad field given by
\[ g_{\mu\nu} = \eta_{ab} h^a_\mu h^b_\nu, \] (3)
where \( \eta_{ab} \) is the Minkowski spacetime such that \( \eta_{ab} = \text{diag}(+1, -1, -1, -1) \).

In TPT, the gravitation is attributed to torsion [20], which plays the role of force. For the Weitzenböck spacetime, the torsion is defined as [27]
\[ T^\theta_{\mu\nu} = \Gamma^\theta_{\nu\mu} - \Gamma^\theta_{\mu\nu}, \] (4)
which is antisymmetric in nature. Due to the requirement of absolute parallelism the curvature of the Weitzenböck connection vanishes identically. The Weitzenböck connection also satisfies the relation given by
\[ \Gamma^{0\theta}_{\mu\nu} = \Gamma^\theta_{\mu\nu} - K^\theta_{\mu\nu}, \] (5)
where
\[ K^\theta_{\mu\nu} = \frac{1}{2}[T^\theta_{\mu\nu} + T^\theta_{\nu\mu} - T^\theta_{\nu\mu}] \] (6)
is the contortion tensor and \( \Gamma^{0\theta}_{\mu\nu} \) are the Christoffel symbols of GR given by
\[ \Gamma^{0\theta}_{\mu\nu} = \frac{1}{2}g^{0\rho}(g_{\mu\rho,\nu} + g_{\nu\rho,\mu} - g_{\nu,\mu,\rho}). \] (7)
The torsion tensor of the Weitzenböck connection can be decomposed into three irreducible parts under the group of global Lorentz transformations [3]:

- the tensor part
  \[ t_{\lambda \mu \nu} = \frac{1}{2}(T_{\lambda \mu \nu} + T_{\mu \lambda \nu}) + \frac{1}{6}(g_{\nu \lambda} V_{\mu} + g_{\nu \mu} V_{\lambda}) - \frac{1}{3} g_{\lambda \mu} V_{\nu}, \]  
  \( T_{\lambda \mu \nu} \)

- the vector part
  \[ V_{\mu} = T^{\nu}_{\nu \mu}, \]

- and the axial-vector part
  \[ A^{\mu} = \frac{1}{6} \varepsilon^{\mu \nu \rho \sigma} T_{\nu \rho \sigma}. \]

The torsion tensor can now be expressed in terms of these irreducible components as follows:

\[ T_{\lambda \mu \nu} = \frac{1}{2}(t_{\lambda \mu \nu} - t_{\lambda \nu \mu}) + \frac{1}{3}(g_{\lambda \mu} V_{\nu} - g_{\lambda \nu} V_{\mu}) \epsilon_{\lambda \mu \nu \rho} A^{\rho}, \]

where

\[ \epsilon_{\lambda \mu \nu \rho} = \frac{1}{\sqrt{-g}} \delta_{\lambda \mu \nu \rho}. \]

Here \( \delta = \{ \delta_{\lambda \mu \nu \rho} \} \) and \( \delta^* = \{ \delta_{\lambda \mu \nu \rho} \} \) are completely skew symmetric tensor densities of weight -1 and +1 respectively [3]. TPT provides an alternate description of Einstein’s equations which is given by the teleparallel equivalent of GR [10, 26-27]. It is worth mentioning here that the deviation of axial symmetry from the spherical symmetry is represented by the axial-vector torsion. It has been shown, in both GR and TPT, by many authors [3, 28] that the spin precession of a Dirac particle in torsion gravity is related to the torsion axial-vector by

\[ \frac{dS}{dt} = -\frac{3}{2} A \times S, \]

where \( S \) is the spin vector of a Dirac particle and \( A \) is the spacelike part of the torsion axial-vector. The Hamiltonian would be of the form [29]

\[ \delta H = -\frac{3}{2} A \cdot \sigma, \]

where \( \sigma \) is the particle spin.
3 Teleparallel Solution of the Friedmann Models

The Friedmann models of the universe are defined by the metric

\[ ds^2 = dt^2 - a^2(t)[d\chi^2 + f_\kappa^2(\chi)d\Omega^2], \]  

where

\[ f(\chi) = \sinh \chi, \quad k = -1, \]
\[ = \chi, \quad k = 0, \]
\[ = \sin \chi, \quad k = +1, \]  

(16)

\( \chi \) is the hyper-spherical angle and \( a(t) \) is the scale parameter, \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \) is the solid angle.

We use an isotropic form of the Friedmann metric to find the components of the tetrad field. The Friedmann metric in isotropic coordinates \((\tau, \rho, \theta, \phi)\) can be written as [30]

\[ ds^2 = d\tau^2 - \frac{a^2(\tau)}{(1 + \frac{1}{4}\kappa \rho^2)^2}(d\rho^2 + \rho^2 d\Omega^2), \]  

(17)

where \( \tau \) denotes the proper time. The proper radius is given by

\[ R(\tau) = \frac{\rho}{a(\tau)}\left\{1 + \frac{1}{4}\kappa \rho^2\right\}, \]  

(18)

For the sake of simplicity, we substitute \( A(\rho) = 1 + \frac{1}{4}\kappa \rho^2 \) in Eq.(17) so that

\[ ds^2 = d\tau^2 - \frac{a^2(\tau)}{A^2(\rho)}(d\rho^2 + \rho^2 d\Omega^2). \]  

(19)

The tetrad components of the Friedmann models can thus be evaluated by using a standard procedure [22, 26]. They are given as

\[ h^a_\mu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a/A & 0 & 0 \\ 0 & 0 & a/A & 0 \\ 0 & 0 & 0 & a/A \end{bmatrix}. \]  

(20)
Its inverse becomes

\[ h_a^\mu = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & A/a & 0 & 0 \\
0 & 0 & A/a & 0 \\
0 & 0 & 0 & A/a \\
\end{bmatrix}. \] (21)

If we replace \( dt^2 \) by \( \gamma^2 d\tau^2 \) in Eq.(15) and then compare it with Eq.(19), we get

\[ \gamma = 1, \quad d\rho = A(\rho)d\chi, \] (22)

\[ f_\kappa(\chi) = \frac{\rho}{A(\rho)}. \] (23)

Using general coordinate transformation law

\[ h_a^\mu = \frac{\partial X^\nu}{\partial X^\alpha} h_a^\alpha, \] (24)

it follows that

\[ h_a^\mu = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & asin\theta \cos \phi & a f \cos \theta \cos \phi & -af \sin \theta \sin \phi \\
0 & asin\theta \sin \phi & a f \cos \theta \sin \phi & af \sin \theta \cos \phi \\
0 & acos\theta & -af \sin \theta & 0 \\
\end{bmatrix}. \] (25)

Its inverse is

\[ h_a^\mu = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & a^{-1} \sin \theta \cos \phi & (af)^{-1} \cos \theta \cos \phi & -(af \sin \theta)^{-1} \sin \phi \\
0 & a^{-1} \sin \theta \sin \phi & (af)^{-1} \cos \theta \sin \phi & (af \sin \theta)^{-1} \cos \phi \\
0 & a^{-1} \cos \theta & -(af)^{-1} \sin \theta & 0 \\
\end{bmatrix}. \] (26)

Notice that Eqs.(2) and (3) can be verified by using Eqs.(25) and (26). When we use Eqs.(25) and Eq.(26) in Eq.(1), the following non-vanishing components of the Weitzenböck connection turn out:

\[ \Gamma^1_{10} = \Gamma^2_{20} = \Gamma^3_{30} = \frac{\dot{a}}{a}, \]

\[ \Gamma^1_{22} = -f_\kappa(\chi), \quad \Gamma^1_{33} = \Gamma^1_{22} \sin^2 \theta, \]

\[ \Gamma^2_{12} = \frac{1}{f_\kappa(\chi)}, \quad \Gamma^2_{21} = \Gamma^3_{31} = \Gamma^2_{12} f'_\kappa(\chi), \]

\[ \Gamma^2_{33} = -\sin \theta \cos \theta, \quad \Gamma^3_{23} = \cot \theta = \Gamma^3_{32}, \] (27)
where dot and prime denote the derivatives w.r.t. \( t \) and \( \chi \) respectively. The corresponding non-vanishing components of the torsion tensor are obtained by using Eq.(27) in Eq.(4). These are given by

\[
T^{1\,10} = T^{2\,20} = T^{3\,30} = -T^{1\,01} = -T^{2\,02} = -T^{3\,03} = -\frac{\dot{a}}{a},
\]

\[
T^{2\,21} = T^{3\,31} = -T^{2\,12} = -T^{3\,13} = \frac{1}{f_\kappa(\chi)}\{1 - f'_\kappa(\chi)\}. \tag{28}
\]

Since \( A^\mu \) gives the deviation from spherical symmetry [28], the axial-vector part of the torsion tensor vanishes identically for the Friedmann models, i.e,

\[
A^\mu = 0. \tag{29}
\]

One can verify this by using Eq.(28) in Eq.(10). This shows that the spin vector of the Dirac particle is constant and the corresponding Hamiltonian induced by the axial vector spin coupling vanishes. The non-zero components of the vector and tensor parts of the torsion tensor take the following forms:

\[
V_0 = -\frac{3\dot{a}}{a}, \tag{30}
\]

\[
V_1 = \frac{2}{f_\kappa(\chi)}\{1 - f'_\kappa(\chi)\}. \tag{31}
\]

and

\[
t_{202} = -\frac{1}{2}a\dot{a}f^2_\kappa(\chi), \tag{32}
\]

\[
t_{220} = -2t_{202}, \tag{33}
\]

\[
t_{303} = t_{202}sin^2\theta, \tag{34}
\]

\[
t_{330} = -2t_{303}, \tag{35}
\]

\[
t_{212} = \frac{1}{2}a^2f_\kappa(\chi)\{1 - f'_\kappa(\chi)\}, \tag{36}
\]

\[
t_{221} = -2t_{212}, \tag{37}
\]

\[
t_{313} = \frac{1}{2}a^2f_\kappa(\chi)\{1 - f'_\kappa(\chi)\}sin^2\theta, \tag{38}
\]

\[
t_{331} = -2t_{313}, \tag{39}
\]

respectively. Since the torsion plays the role of the gravitational force in TPT, a spinless particle will obey the force equation [22, 26] in the gravitational field

\[
\frac{du_\rho}{ds} - \Gamma_{\mu\rho\nu}u^\mu u^\nu = T_{\mu\rho\nu}u^\mu u^\nu. \tag{40}
\]
The left hand side of this equation is the Weitzenböck covariant derivative of $u_\rho$ along the world line of the particle. The presence of the torsion tensor on its right hand side means essentially that torsion plays the role of an external force in teleparallel gravity.

4 Teleparallel Solution of the Lewis-Papapetrou Spacetime

The class of stationary axisymmetric solutions of the Einstein field equations is the appropriate framework to include the gravitational effect of an external source in an exact analytic manner [31]. At the same time, such spacetimes are of obvious astrophysical importance, as they describe the exterior of the body in equilibrium. The line element of a stationary axisymmetric spacetime is given by the Lewis-Papapetrou metric as

$$ds^2 = e^{2\psi}(dt - \omega d\theta)^2 - e^{2(\gamma - \psi)}(d\rho^2 + dz^2) - \rho^2 e^{-2\psi}d\theta^2.$$  (41)

Here $\omega$ is the angular velocity and $\gamma, \psi, \omega$ are arbitrary functions of $\rho$ and $z$ only. The corresponding tetrad components are

$$h^a_{\mu} = \begin{bmatrix} e^{\psi} & 0 & -\omega e^{\psi} & 0 \\ 0 & e^{\gamma - \psi} \cos \theta & -\rho e^{\psi} \sin \theta & 0 \\ 0 & e^{\gamma - \psi} \sin \theta & \rho e^{\psi} \cos \theta & 0 \\ 0 & 0 & 0 & e^{\gamma - \psi} \end{bmatrix}.  \quad (42)$$

Its inverse is given by

$$h_a^{\mu} = \begin{bmatrix} e^{-\psi} & 0 & 0 & 0 \\ -\omega^{-1} e^\psi \sin \theta & e^{\gamma + \psi} \cos \theta & -\rho^{-1} e^\psi \sin \theta & 0 \\ 0 & e^{\gamma + \psi} \sin \theta & \rho^{-1} e^\psi \cos \theta & 0 \\ 0 & 0 & 0 & e^{-\gamma + \psi} \end{bmatrix}.  \quad (43)$$

We see that Eqs.(2) and (3) can be easily verified by using Eqs.(42) and (43). Using Eqs.(42) and (43) in Eq.(1), we obtain the following non-vanishing components of the Weitzenböck connection

$$\Gamma^0_{01} = \dot{\psi}, \quad \Gamma^0_{03} = \psi', \quad \Gamma^0_{12} = \omega \rho^{-1} e^\gamma,$$

$$\Gamma^0_{21} = \omega \rho^{-1} - (\dot{\omega} + 2\omega \dot{\psi}), \quad \Gamma^0_{23} = -\dot{\omega} + 2\omega \psi',$$

$$\Gamma^1_{11} = \dot{\gamma} - \dot{\psi} = \Gamma^3_{31}, \quad \Gamma^1_{22} = -\rho e^{-\gamma}, \quad \Gamma^1_{13} = \gamma' - \psi' = \Gamma^3_{33},$$

$$\Gamma^2_{12} = \rho^{-1} e^\gamma, \quad \Gamma^2_{21} = \rho^{-1}(1 - \rho \dot{\psi}), \quad \Gamma^2_{23} = -\psi'.  \quad (44)$$
where dot and prime denote the derivatives w.r.t. $\rho$ and $z$ respectively. The corresponding non-vanishing components of the torsion tensor are obtained by using Eq.(44) in Eq.(4). These are given by

$$
T_{01}^0 = -\dot{\psi} = -T_{10}^0, \quad T_{03}^0 = -\psi' = -T_{30}^0, \\
T_{12}^0 = \omega \rho^{-1}(1 - e^\gamma) - (\dot{\omega} + 2\omega \dot{\psi}) = -T_{01}^2, \\
T_{23}^0 = \omega' + 2\omega \psi' = -T_{32}^0, \quad T_{13}^1 = -\gamma' + \psi' = -T_{31}^1, \\
T_{12}^2 = \rho^{-1}(1 - e^\gamma) - \dot{\psi} = -T_{21}^2, \quad T_{23}^2 = \psi' = -T_{32}^2, \\
T_{31}^3 = -\dot{\gamma} + \dot{\psi} = -T_{13}^3.
$$

If we make use of Eq.(45) in Eq.(9), the following non-vanishing components of the vector torsion turn out

$$
V_1 = \dot{\psi} - \dot{\gamma} - \rho^{-1}(1 - e^\gamma), \quad (46) \\
V_3 = \psi' - \gamma'. \quad (47)
$$

In view of Eq.(45), it follows from Eq.(10) that the non-vanishing components of the axial-vector torsion are

$$
A^{(1)} = \frac{1}{3h}\left[g_{00}T_{32}^0 + g_{02}(T_{30}^0 + T_{32}^2)\right], \quad (48) \\
A^{(3)} = \frac{1}{3h}\left[g_{00}T_{12}^0 + g_{02}(T_{12}^2 + T_{01}^0)\right], \quad (49)
$$

where $h = \sqrt{-g} = \rho e^{(\gamma - \psi)}$. The component of axial vector along $\theta$ direction vanishes due to symmetry about $\rho z$-plan. Therefore, the spacelike axial-vector can be written as

$$
\mathbf{A} = \sqrt{-g_{11}}A^{(1)}\hat{e}_\rho + \sqrt{-g_{33}}A^{(3)}\hat{e}_z, \quad (50)
$$

where $\hat{e}_\rho$ and $\hat{e}_z$ are the unit vectors along radial and $z-$ directions respectively. Using Eqs.(45)-(49) together with the value of $h$ in Eq.(50), the axial-vector becomes

$$
\mathbf{A} = \frac{-1}{3\rho}e^{3\psi - \gamma}[(\omega' + 2\omega \psi')\hat{e}_\rho + \dot{\omega}\hat{e}_z]. \quad (51)
$$

From Eq.(51), we note the following special cases depending upon the values of $\omega$ and $\psi$. If $\omega$ is only a function of $z$, then the axial-vector will be symmetric about radial axis and takes the form

$$
\mathbf{A} = \frac{-1}{3\rho}e^{3\psi - \gamma}[(\omega' + 2\omega \psi')\hat{e}_\rho]. \quad (52)
$$
If $\omega$ is only a function of $\rho$, then it lies in $\rho z$-plane and is given by

$$A = -\frac{1}{3\rho} e^{3\psi-\gamma} [2\omega\dot{\psi}' \hat{e}_\rho + \dot{\omega}\hat{e}_z].$$

(53)

If $\omega$ is constant, $A$ becomes

$$A = -\frac{1}{3\rho} e^{3\psi-\gamma} [2\omega \dot{\psi}' \hat{e}_\rho],$$

(54)

that is, it is symmetric about radial direction. Finally, when $\omega$ is constant and also $\psi$ is a function of $\rho$ only, then the axial-vector vanishes, i.e,

$$A = 0,$$

(55)

that is, in this case the cylindrical symmetry will not be disturbed even if $\omega$ is non-zero. The spin precession of the Dirac particle in torsion gravity turns out to be

$$\frac{dS}{dt} = \frac{1}{2\rho} e^{3\psi-\gamma} [\left(\omega' + 2\omega \psi'\right) \hat{e}_\rho + \dot{\omega}\hat{e}_z] \times S.$$

(56)

The corresponding Hamiltonian will be

$$\delta H = \frac{1}{2\rho} e^{3\psi-\gamma} [\left(\omega' + 2\omega \psi'\right) \hat{e}_\rho + \dot{\omega}\hat{e}_z].\sigma.$$

(57)

5 Summary and Discussion

GR is very successful in describing long distance phenomena. However, this theory encounters serious difficulties on microscopic distances. The Lagrangian structure of GR differs, in principle, from the ordinary microscopic gauge theories. In particular, a covariant conserved energy-momentum tensor for the gravitational field can not be constructed in the framework of GR. Consequently, the study of alternative models of gravity is justified from the physical as well as from the mathematical point of view.

This paper is devoted to obtain TP versions for the Friedmann models and the stationary axisymmetric Lewis-Papapetrou solution. For this purpose, a tetrad having four unknown functions is applied to the field equation of the TP theory of gravity by using a coordinate transformation. The associated metrics of tetrad for the Friedmann and the Lewis-Papapetrou spacetimes are given by Eqs.(25) and (42) respectively. For the Friedmann models,
to spherical symmetry, the axial-vector torsion vanishes identically and there occurs no deviation in the spherical symmetry of the spacetime. Consequently there exists no inertia field with respect to Dirac particle and the spin vector of a Dirac particle becomes constant. The only non-vanishing components of the vector part are the time and the radial one. It is mentioned here that for the Schwarzschild metric, the axial vector is only in the radial direction [22]. The reason is that the Friedmann models are non-static while Schwarzschild metric is static.

For the teleparallel Lewis-Papapetrou solution, we obtain the vector and the axial-vector parts of the torsion tensor. The vector parts are in the radial and z-directions. This corresponds with the Kerr metric [22] where we get the vector part in the radial and \( \theta \)-directions. The axial-vector torsion turns out to be symmetric about \( \rho z \)-plane as its component along \( \theta \)-directions vanishes everywhere. The non-inertia force on the Dirac particle can be represented as a rotation induced torsion of spacetime. There arise three possibilities of the axial-vector depending upon the nature of the values of \( \omega \) and \( \psi \). When \( \omega \) is a function of \( z \) only or \( \omega \) is constant, the axial-vector will be along the radial direction, that is, it will be symmetric about radial direction. When \( \omega \) is a function of \( \rho \) only, the axial-vector has its components along the radial as well as z-direction, i.e., the axial-vector lies in \( \rho z \)-plane. For the third possibility when \( \omega \) is constant and \( \psi \) is a function of \( \rho \) only, the axial-vector vanishes identically. For the cases when \( \omega \) is constant, the non-inertia force on the Dirac particle remains same everywhere in the space [32]. From the spacetime geometry view, the torsion axial-vector represents the deviation from the symmetry of the underlying spacetime which corresponds to an inertia field with respect to Dirac particle as expressed by the Eq.(56).

Finally, we would like to mention here that tetrad formalism itself has some advantages. This comes mainly from its independence from the equivalence principle and consequent suitability to the discussion of quantum issues. Some classic solutions of the Einstein field equations have already been translated into the teleparallel language. This paper adds two more solutions. It is always enriching to look at known things from another point of view, so that the endeavor is in itself commendable.

The study of energy content of this tetrad is in progress [33].

**Acknowledgment**

We acknowledge the enabling role of the Higher Education Commission
Islamabad, Pakistan, and appreciate its financial support through the Indigenous PhD 5000 Fellowship Program Batch-I. We would like to thank anonymous referee for their useful suggestions.

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