Application of the theory of Large-Scale Quantization to the inner heliosheath

G Livadiotis
Southwest Research Institute, USA
E-mail: glivadiotis@swri.edu

Abstract. This paper addresses a remarkable feature of space plasmas that is related to their peculiar statistical behavior: The existence of a new quantization constant \( \hbar^* \), similar to the standard Planck’s quantization constant \( \hbar \), but \( \sim 12 \) orders of magnitude larger. The standard quantization through the Planck constant \( \hbar \) may occur to individual particles, while the large-scale quantization through the new constant \( \hbar^* \) applies only to clusters of correlated particles. In space plasmas, the value of \( \hbar^* \) can be approximated by the division of the magnetosonic energy over the plasma frequency. Having the actual value of \( \hbar^* \) been determined, it can be further applied in space plasma datasets to derive unknown plasma parameters when some key observable is missing, such as the magnetic field strength. This application is tested with solar wind data in the inner heliosphere, where the magnetic field is known, and then it is employed in the inner heliosheath plasma to estimate the unknown local magnetic field.

1. Introduction
Recent analyses of space plasmas revealed strong evidence for the existence of a new fundamental constant [1-7]. The new type of quantization constant \( \hbar^* \) is similar to the Planck constant \( \hbar \), but \( \sim 12 \) orders of magnitude larger. Planck’s constant constitutes the smallest possible phase-space parcel for individual and uncorrelated particles, while the new quantization constant describes the smallest possible phase-space parcel for collisionless particle systems characterized by local correlations. The majority of space plasmas throughout the heliosphere are such systems.

The system’s phase-space is represented by the \( 6N \)-dimensional space of the position \( \vec{x} \) and momentum \( \vec{p} \) of the \( N \) particles of the system. The phase-space geometry invokes that there always exists a non-zero phase-space minimum parcel [8]; in other words, Classical Mechanics has the uncertainty principle built-in. While the existence of such non-zero phase-space cell is mathematically unavoidable, the exact value of this phase-space cell cannot be determined by mathematical theorems, but strictly from observations of the physical universe. The Planck constant discovered and determined from such observations that involve individual and uncorrelated particles. There is no reason for the phase-space cell of more complex systems to be strictly determined, again, by the Planck’s constant, \( \hbar \).

Imagine an intellectual phase-space version of Maxwell’s demon that lives within the system and does not allow any type of physical structure of particles/quasi-particles to pass information through a phase-space smaller than a mega-cell given by \( \hbar^* \), where \( \hbar^* \) is larger than that of Planck’s \( \hbar \). This could be done, for example, by connecting various vicinal Planck’s phase-space cells, i.e., by correlating the
forces carrying information through these Planck’s cells (Figure 1). Indeed, if there were no physical ways to pass information through a phase-space cell $h$, then, would the relevant physical laws alter and depend on $h$ instead of $\hbar$? The first evidence that such a large-scale phase-space quantum may exist was shown in [1]. This mega-quantum characterizes systems with strong collective behavior and local correlations such as space plasmas.

The values of $\hbar$ were initially derived using four independent observational and theoretical analyses [1], that is, by examining: 1) a specific plasma with highly variant plasma parameters (Ulysses spacecraft measurements for solar wind); 2) a variety of 11 different space plasmas; 3) the limit of entropy $S\to 0$, where the classic Sackur-Tetrode equation of entropy that applies to particle systems with no correlations (e.g., Earthy gases) is generalized to apply also to systems with correlated particles (i.e., collisionless plasmas); and, 4) explosive events and other plasma phenomena which can be related to the new type of quantization constant. The physical meaning of $\hbar$, as the smallest phase-space parcel, did not imply that this is some universal constant similar to the Planck’s constant $\hbar$. Indeed, the values of $\hbar$ could be dependent on the plasma parameters and generally on the type of the plasma. Surprisingly, a similar value was found through all the measurement methods [1,6], supporting the constancy and universality of the value of $\hbar \sim 1.2 \times 10^{-22}$ Js.

![Figure 1. Sketch of phase-space, indicating the Planck’s non-zero phase-space parcel, $\hbar$ (upper sketch), but this may change with the presence of correlations (lower sketch).](image)

**2. Statistical behavior and Large-Scale Quantization in space plasmas**

Numerous independent developments in space plasma physics have revealed the peculiar statistical behavior of this category of plasmas: Particle populations in space plasmas reside in stationary states out of thermal equilibrium (e.g., [9] and refs therein). Thermal equilibrium is a special stationary state; systems at thermal equilibrium have their distribution functions of particle velocities stabilized into a Maxwell distribution, which is connected with the classical framework of Boltzmann-Gibbs statistical mechanics. However, Maxwell distributions are extremely rare in space plasmas. On the contrary, the vast majority of space plasmas resides out of thermal equilibrium, and is described by the kappa distributions (e.g., [10-14]). These non-Maxwellian distributions cannot embody the Boltzmann-Gibbs statistics, but instead, must be described within a generalized statistical framework (e.g., see [15,16]).

The origin of this vastly different statistical behavior between classical systems and space plasmas is the manifestation of correlations between the plasma particles. The stronger the correlation the further the plasma resides from thermal equilibrium [9]. On the other hand, the presence of correlations causes the phenomenon of the large-scale quantization.
The critical difference between the Planck constant $\hbar$ and the large-scale quantization constant $\hbar^*$, is that the latter applies – and it has been detected – in systems with local correlations between their particles. Local correlations are manifested by the presence of a correlation length between particles. For example, the Debye length in space plasmas gives a measure of the smallest correlation length, i.e., particles within such a length are always correlated, forming the Debye sphere [5]. The correlation length divides the system into an ensemble of clusters of correlated particles, e.g., Debye spheres (Figure 2). Particles within “correlation clusters” participate altogether to the large-scale quantization.

![Figure 2. Sketch of a particle system with no correlations (Left), and with local correlations that forms clusters of correlated particles - the correlation clusters (Right). Large-scale quantization characterizes such systems with local correlations between their particles (e.g., space plasmas).](image)

The scheme in Figure 3 demonstrates how the large-scale quantization gets related to the non-equilibrium statistical behavior of space plasmas and the competition between correlations and collisions. While correlations shift plasmas away from thermal equilibrium, collisions destroy correlations, recovering plasmas back at thermal equilibrium [1,9]. Thus, the large-scale quantization appears in collisionless plasmas, i.e., in plasmas where the correlation length is smaller than the mean free path – the average collision length.

![Figure 3. The non-equilibrium statistical behavior of space plasmas is related to the large-scale quantization. These physical phenomena are both caused by long-range interactions that induce local correlations. The particle correlations must be strong enough to defy particle collisions that lead the system back to thermal equilibrium.](image)

3. Large-scale quantization and Quantum Mechanics

3.1. The uncertainty principle

Quantum Mechanics requires a minimum phase-space volume, and this phase-space “quantum” is typically recognized as the Planck constant. This limitation of phase-space follows the Heisenberg uncertainty principle [17]. In particular, individual particles, or particle systems with no local
correlations, have a minimum energy level $\epsilon_p$ connected with the Planck constant, $\epsilon_p \sim \hbar\omega$, while the lifetime associated with this lowest energy level is $t_p \sim 1/(2\omega)$; the frequency $\omega$ denotes the particle’s basic oscillation mode. Therefore, for an arbitrary energy exchange that involves these particles, $\Delta \epsilon \geq \epsilon_p$ and for an arbitrary duration of this process, $\Delta t \geq t_p$, the uncertainty principle is given by

$$\Delta \epsilon \Delta t \geq \epsilon_p t_p = \frac{\hbar}{2} \cdot$$  \hspace{1cm} (1)

As explained in Introduction, no mathematical or analytical argument can point to a particular value of the phase-space cell, $\hbar$. Structured systems, more complex than simply a group of uncorrelated individual particles, may exist and be characterized by a quantization constant different than that of the Planck constant. Down to the scale of Debye length, plasmas cannot be described through the approximation of a continuous fluid that applies for larger scales; instead, a new type of interpretation must be given that treats the smallest parcels of plasma as phase-space “quanta”. This concept led to the development of the large-scale quantization that extends in scales of space and time larger than those of Planck’s quantization [1,6]. It has been shown that there is a minimum energy $\epsilon_c \sim \hbar\omega$ and a lifetime $t_c \sim 1/(2\omega)$ associated with the clusters of correlated particles, leading to a new type of quantum-cell and uncertainty principle,

$$\Delta \epsilon \Delta t \geq \epsilon_c t_c = \frac{\hbar}{2} \cdot \hbar \cdot, \ h \cdot \gg \hbar \cdot \ .$$  \hspace{1cm} (2)

The inequality $\hbar \cdot \gg \hbar$ acknowledges that the Planck constant is still unique as the smallest possible quantization in the case where more than one quantization values exist.

3.2. The energy-frequency relation

Uncorrelated particles can transfer their energy in Planck’s quanta of $\hbar\omega_{ph}$, that is the energy of a quasiparticle, e.g., a photon of frequency $\omega_{ph}$. In a similar way, correlated particles can transfer their energy in larger quanta, $\sim \hbar\omega_{pl}$, where each quantum corresponds again to a quasiparticle; in plasmas, the transmitted quasiparticle is typically a plasmon of frequency $\omega_{pl}$.

The smallest energy that can be transferred from an individual particle that is not subject to correlations into one quasiparticle, e.g., a photon, is $\epsilon_p \sim \hbar\omega_{ph}$, while the smallest energy that can be transferred from a particle that is subject to local correlations into one quasiparticle, e.g., a plasmon, is $\epsilon_c \sim \hbar\omega_{pl}$ (Figure 4) [1,6].

![Figure 4](image)

Figure 4. (a) The smallest energy that can be transferred from an uncorrelated particle is of the order of $\sim \hbar\omega$. The quasi-particle that carries this energy quantum may be a photon oscillating with frequency $\omega_{ph}$. (b) For a correlated particle in plasma, the smallest energy that can be transferred is of the order of $\sim \hbar\omega_{pl}$ (with $\hbar/\hbar \cdot \sim 10^{12}$). The quasi-particle is typically a plasmon oscillating with the plasma frequency $\omega_{pl}$. (Taken from [6])
The transformation of the plasma particle’s energy (e.g., kinetic, magnetic, etc) into the quasi-particle’s transmitted energy constitutes one of the basic methods of determining measurements of the large-scale quantization constant, i.e., \( h \sim \epsilon_i / \omega_{pl} \).

4. Application

4.1. The approximation of \( h \) in space plasmas

The amount of particle energy that can be transferred away from the correlation cluster (e.g., the Debye sphere - if the correlation length is given by the Debye length) via a quasiparticle can be estimated by the average particle energy. This sums the thermal and magnetic energy, leading to the magnetosonic energy \( E_{ms} = \frac{1}{2}(m_i + m_e)U_{ms}^2 \) [6], where \( U_{ms} \) is the fast magnetosonic speed (\( m_i, m_e \) are the ion/electron masses). Also, the plasmon’s frequency \( \omega \) is typically given by the primary plasma frequency, \( \omega_{pl} \). As a consequence, the value of \( h \) can be approximated by \( h \sim \frac{1}{2}(m_i + m_e)U_{ms}^2 / \omega_{pl} \), or more analytically [5,6], by

\[
h \approx 0.0705 n^{-2} B^2 + 2.447 \gamma n^{-2} T_e (1 + \alpha)/2 ,
\]

where the units are \( h [10^{-22} J \cdot s], B [nT = 10 \mu G], n [cm^{-3}], T_e [K] \); \( \gamma \) is the average polytropic index \( \gamma = (\gamma_i + \alpha \gamma_e) / (1 + \alpha) \), \( \alpha = T_e / T_i \), ratio between electron/ion temperatures.

The actual value of \( h \) can be approximated by \( h \sim \frac{1}{2}(m_i + m_e)U_{ms}^2 / \omega_{pl} \), only when the plasma wave spectrum is dominated by the plasma frequency, \( \omega_{pl} \). Indeed, typically, space plasmas exhibit a dominant oscillation at the plasma frequency. (As an example, Figure 5 plots the wave spectrum of solar wind taken by STEREO A [18]). In general, more frequencies may be important in the particle’s energy. Then, the energy \( \epsilon_i \) is distributed over all these frequencies, i.e.,

\[
\epsilon_i = h \overline{\omega} , \text{ where } \overline{\omega} = \int \omega D(\omega) d\omega , \int D(\omega) d\omega = 1 ,
\]

where the integration is over all the wave spectrum.

![Figure 5](image-url)  
*Figure 5.* Typical STEREO A wave spectrum. The dominant frequency is the plasma frequency \( \omega_{pl} \) [18].

The constancy of the approximation \( h \sim \frac{1}{2}(m_i + m_e)U_{ms}^2 / \omega_{pl} \) in space plasmas can be demonstrated with two examples of datasets, the ACE data for solar wind near Earth (~1AU), and Ulysses data for solar wind that is for heliocentric distances between Earth and Jupiter (1.1AU<\( r <5.5AU \)) (see Figure 6).
4.2. Derivation of the magnetic field

We can exploit the constancy of $h^*$ in space plasmas throughout the heliosphere, and use the approximation $h^* \sim \omega_p / \omega_m$ to derive unknown plasma parameters, e.g., magnetic field strength, density, ion and electron temperature \([4,6]\). Indeed, the dominant oscillation is given by the plasma frequency, $\omega_p \sim \omega_m$, hence the large-scale quantization can be approximated by the relation $h^* \sim \omega_p / \omega_m$, which is expanded in Eq.(3). This relation can be reversed, and expressed in terms of the unknown parameter, e.g., the magnetic field strength,

$$B \equiv \sqrt{16.74n^2 - 34.7\gamma nT/(1 + \alpha)/2},$$

with the same units as in Eq.(3).

First, we use Ulysses measurements of the solar wind over a broad range of heliocentric distances $1.1 \text{AU} < r < 5.5 \text{AU}$. In Figure 7(a), the observational values of $B$ are co-plotted with the modelled ones. Both sets of values follow similar distributions. The modelled $B$ values appear with some larger variance around the statistical mode (most frequent value) at $B \sim 0.5-0.6 \text{nT}$.

After this verification, the method can be also applied in space plasmas with unknown magnetic field. Such an example is the inner heliosheath. There, the temperature, density \([12]\), and polytropic index have been estimated, but the magnetic field strength is unknown. The plasma parameters have been estimated as radial averages for each sky direction. Figure 7(b) shows the results, that is a histogram of all the derived values of the magnetic strength. The mode of the distribution is $\sim 2 \mu \text{G}$, a value that characterizes the whole inner heliosheath. The derived value is consistent with the in situ measurements taken by Voyager 1 and 2.

**Figure 7.** Estimation of the magnetic field strength values, (a) for solar wind using Ulysses measurements, and comparison with the respective observational values \([6]\); (b) for the inner heliosheath using IBEX measurements.
5. Conclusion

Recent publications revealed strong evidence that space plasmas are characterized by a new type of quantization, called the Large-Scale Quantization, where the new quantization constant $\hbar^*$ is similar to the standard Planck’s quantization constant $\hbar$, but 12 orders of magnitude larger.

While the standard quantization through the Planck constant $\hbar$ characterizes interactions of individual particles, the large-scale quantization through the new constant $\hbar^*$ applies only to interactions involving clusters of correlated particles. Local correlations are realized when a correlation length is assigned between the system’s particles. The Debye length in space plasmas constitutes the smallest possible correlation length. The correlation length divides the system into an ensemble of clusters of correlated particles. The particles within each of these “correlation clusters” participate altogether to this new type of quantization.

In space plasmas, the value of $\hbar^*$ can be approximated by the division of the magnetosonic energy over the plasma frequency. Having the value of $\hbar^*$ already been determined by various theoretical and observational methods, it can be further applied in space plasma datasets to derive unknown plasma parameters when some key observable is missing, such as the magnetic field strength. This application is tested with solar wind data in the inner heliosphere, where the magnetic field is known, and then is used in the inner heliosheath plasma to estimate the unknown local magnetic field.

References

[1] Livadiotis G and McComas D J 2013 Evidence of Large-Scale Quantization in Space Plasmas Entropy 15 1116
[2] Witze A 2013 Space plasmas share a secret Nature News, doi:10.1038/nature.2013.13159
[3] Livadiotis G and McComas D J 2013 Large-Scale Quantization in plasmas APS Bul. Plasma Phys 58 G06.00005
[4] Livadiotis G and McComas D J 2014 Large-Scale Quantization in space: Summary and applications ASP Conf Ser 484 131
[5] Livadiotis G and McComas D J 2014 Electrostatic shielding in plasmas and the physical meaning of the Debye length J Plasma Phys 80 341
[6] Livadiotis G and McComas D J 2014 Large-Scale phase-space quantization from local correlations in space plasmas J Geophys Res 119 3247
[7] Livadiotis G 2014 Lagrangian temperature: Derivation and physical meaning for systems described by kappa distributions Entropy (In Press)
[8] Gromov M 1985 Pseudo holomorphic curves in symplectic manifolds Invent math 82 307
[9] Livadiotis G and McComas D J 2013 Understanding Kappa Distributions: A Toolbox for Space Science and Astrophysics Space Sci Rev 75 183
[10] Ogasawara K, et al. 2013 Characterizing the dayside magnetosheath using energetic neutral atoms: IBEX and THEMIS observations J Geophys Res 118 3126
[11] Nicolau G, Livadiotis G, and Moussas X 2014 Long-Term Variability of the Polytropic Index of Solar Wind Protons at 1 AU Sol Phys 289 1371
[12] Livadiotis G, et al. 2011 First sky map of the inner heliosheath temperature using IBEX spectra Astrophys J 734 1
[13] Livadiotis G, et al. 2012 Pick-up ion distributions and their influence on ENA spectral curvature Astrophys J 751 64
[14] Livadiotis G, et al. 2013 Pressure of the proton plasma in the inner heliosheath Astrophys J 762 134
[15] Livadiotis G 2009 Approach on Tsallis statistical interpretation of hydrogen-atom by adopting the generalized radial distribution function J Math Chem 45 930
[16] Tsallis C 2009 Introduction to Non-extensive Statistical Mechanics: Approaching a Complex World (New York: Springer)
[17] Heisenberg W 1927 Über den anschaulichen inhalt der quantentheoretischen Kinematik und Mechanik Z Phys 33 879
[18] Thejappa G, MacDowall R J, and Bergamo M 2012 In situ detection of strong Langmuir turbulence processes in solar type III radio bursts J Geophys Res 117 A08111