Entropy of Kerr-de Sitter black hole

Huai-Fan Li$^{a,b,*}$, Meng-Sen Ma$^{a,b}$, Li-Chun Zhang$^{a,b}$ and Ren Zhao$^b$

$^a$Department of Physics, Shanxi Datong University, Datong 037009, China
$^b$Institute of Theoretical Physics, Shanxi Datong University, Datong 037009, China

Based on the consideration that the black hole horizon and the cosmological horizon of Kerr-de Sitter black hole are not independent each other, we conjecture the total entropy of the system should have an extra term contributed from the correlations between the two horizons, except for the sum of the two horizon entropies. By employing globally effective first law and effective thermodynamic quantities, we obtain the corrected total entropy and find that the region of stable state for kerr-de Sitter is related to the angular velocity parameter $a$, i.e., the region of stable state becomes bigger as the rotating parameters $a$ is increases.

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I. INTRODUCTION

The astronomical observations show that our Universe is probably approaching de Sitter spacetime. The cosmological constant corresponds to vacuum energy and is usually considered as a candidate of dark energy. The accelerating universe will evolve into another de Sitter phase. In order to construct the entire history of evolution of the universe, we should have a clear perspective to the classical and quantum properties of de Sitter spacetime [1]. However, as is well known that in de Sitter space there is no spatial infinity and no asymptotic Killing vector which is globally timelike. Moreover, black holes in de Sitter spacetime cannot be in thermodynamic equilibrium in general. Because there are multiple horizons with different temperatures for de Sitter black holes.

The thermodynamic quantities on the black hole horizon and the cosmological horizon all satisfy the first law of thermodynamics, moreover the corresponding entropies both fulfill the area formulae [1-7]. There have been accumulated interest to the thermodynamics property

* Email: huaifan.li@stu.xjtu.edu.cn
of de Sitter spacetime in recent years [8–13]. In this way, the two horizons can be analyzed independently. Besides, one can also take a global view to construct the globally effective temperature and other effective thermodynamic quantities. No matter which method is used, the total entropy of de Sitter black hole is supposed to be the sum of both horizons, namely $S = S_+ + S_c$. [14–16]

We think that the correct method may to be so simple because the event horizon and the cosmological horizon are not independent. There may exist some correlations between them due to the following considerations. We can take the Kerr-de Sitter black hole as example. There are first laws of thermodynamics for both horizons. According to [17], the thermodynamic quantities on the two horizons satisfy the first law of thermodynamics system,

$$
dM = T_+ dS_+ + \Omega_+ dJ + V_+ d\Lambda, \\
\sum dM = -T_c dS_c + \Omega_c dJ + V_c d\Lambda, \tag{1.1}
$$

where $M$ is the conserved mass in de Sitter spacetime, $\Omega$ and $J$ stand for the angular velocities and angular momentum, and the cosmological constant $\Lambda$ as a pressure and define the thermodynamically conjugate variable to the thermodynamics volume, which can be different from the geometrical volume, the subscript "$+"$ denotes the black hole horizon and the subscript "$c$" denotes the cosmological horizon. Including the variable $\Lambda$, the above first laws can have corresponding Smarr formulae, which are also given in [18, 19]. The two laws in Eq.(1.1) are not truly independent. They depend on the same quantities $M$, $J$, $\Lambda$. All the geometric and thermodynamic quantities for the both horizons can be represented by $M$, $J$, $\Lambda$. Therefore, the size of black hole horizon is closely related to the size of the cosmological horizon, and the evolution of black hole horizon will lead to the evolution of the cosmological horizon. Considering the relation between the thermodynamic quantities on the two horizons is very important for studying the thermodynamic properties of de Sitter spacetime [20, 21].

Considering the correlation or entanglement between the black hole event horizon and the cosmological horizon, the total entropy of the Kerr-de Sitter black hole is no longer simply $S = S_+ + S_c$, but should include an extra term from the contribution of the correlations of the two horizons. In the Ref. [22], we calculate the extra term in the entropy for the spherically symmetric charged black hole in de Sitter spacetime. By the method we obtain
the reasonable interpretation for the R-N black hole in de Sitter spacetime. In this work, we will expand the method to the axisymmetric rotating black hole in de Sitter spacetime. We can obtain the extra term form and find the relates between the region of stable state and the rotating parameter $a$.

Our paper is organized as follows. In the next section we simple review the thermodynamics quantities of black hole horizon and cosmological horizon of the Kerr-de Sitter black hole, obtain the condition that the effective temperature of the horizon approaches to zero. In the base that the Kerr-de Sitter system satisfies the first laws of thermodynamics, the entropy and the effect temperature of Kerr-de Sitter black hole is obtained, we study the condition that Kerr-de Sitter system satisfies the stable equilibrium of the thermodynamics in Sec. III. Sec. IV is devoted to conclusions.

II. KERR-DE SITTER BLACK HOLE

Kerr-de Sitter black hole is a solution of Einstein equation in (3 + 1) dimensions with a positive cosmological constant $\Lambda = \frac{3}{l^2}$. It is characterised by parameters, mass $M$ and angular momentum $J$. The spacetime metric of the Kerr-de Sitter black hole in Boyer-Lindquist coordinates $(t, r, \theta, \phi)$ is given by [1]

$$ds^2 = -\frac{\Delta_r}{\rho^2} \left( dt - \frac{a \sin^2 \theta}{\Xi} d\phi \right)^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_{\theta}} d\theta^2 + \frac{\Delta_{\theta} \sin^2 \theta}{\rho^2} \left( a dt - \frac{r^2 + a^2}{\Xi} d\phi \right)^2,$$  \hspace{1cm} (2.1)

where the various functions entering the metric are given by

$$\rho^2 = r^2 + a^2 \cos^2 \theta, \quad \Xi = 1 + \frac{\Lambda}{3} a^2, \quad \Delta_{\theta} = 1 + \frac{\Lambda}{3} a^2 \cos^2 \theta,$$

$$\Delta_r(r) = (r^2 + a^2) \left( 1 - \frac{\Lambda}{3} r^2 \right) - 2mr,$$ \hspace{1cm} (2.2)

the black hole horizon and cosmological horizon satisfies the equation $\Delta_r(r_+) = 0$ and $\Delta_r(r_c) = 0$, here $r_+$ and $r_c$ are the location of black hole horizon and cosmological horizon, respectively. So we can obtain [21]

$$2m = \frac{(r_c + r_+)(r_c^2 + a^2)(r_+^2 + a^2)}{r_c r_+(r_c^2 + r_c r_+ + r_+^2 + a^2)},$$

$$\Xi = \frac{r_c r_+(r_c^2 + r_c r_+ + r_+^2 + 2a^2) - a^4}{r_c r_+(r_c^2 + r_c r_+ + r_+^2 + a^2)}, \quad \frac{\Lambda}{3} = \frac{r_c r_+ - a^2}{r_c r_+(r_c^2 + r_c r_+ + r_+^2 + a^2)}.$$ \hspace{1cm} (2.3)
In the de Sitter spacetime, by regarding respectively the black hole horizon and the cosmological horizon as the thermodynamics systems, we can have

\[
T_+ = \frac{r_+^3 r_+ + r_+^2 r_+^3 - 2 r_+ r_+^3 + a^4 + 3a^2 r_+^2 - a^2(r_+^2 + r_+^2 + r_+ r_+ + a^2) r_+ / r_+}{4 \pi r_+ (r_+^2 + a^2)(r_+^2 + r_+ r_+ + r_+^2 + a^2)},
\]

\[
T_c = - \frac{r_+^3 r_+ + r_+^2 r_+^3 - 2 r_+ r_+^3 + a^4 + 3a^2 r_+^2 - a^2(r_+^2 + r_+ r_+ + r_+^2 + a^2) r_+ / r_+}{4 \pi r_+ (r_+^2 + a^2)(r_+^2 + r_+ r_+ + r_+^2 + a^2)}.
\]

(2.4)

The entropy and energy of the black horizon and the cosmological horizon are respectively,

\[
S_+ = \frac{\pi (r_+^2 + a^2)}{2}, \quad S_c = \frac{\pi (r_+^2 + a^2)}{2}, \quad M = \frac{m}{2}. \quad \text{(2.5)}
\]

The angular velocity of the black hole horizon and the cosmological horizon are respectively,

\[
\tilde{\Omega}_+ = \frac{a}{r_+^2 + a^2}, \quad \tilde{\Omega}_c = \frac{a}{r_+^2 + a^2}, \quad \text{(2.6)}
\]

while the angular momentum and the angular velocity at infinity read

\[
J = \frac{a m}{2} = a \frac{(r_+ + r_+)(r_+^2 + a^2)(r_+^2 + a^2)}{2 r_+ r_+ (r_+^2 + r_+ r_+ + r_+^2 + a^2) \Xi}, \quad \Omega_\infty = \frac{\Lambda}{3} a. \quad \text{(2.7)}
\]

Taking the cosmological constant \( \Lambda \) as the pressure, the thermodynamics quantities of the both horizons satisfy the first laws of the thermodynamics \[2,8\],

\[
\delta M = T_+ \delta S_+ + \Omega_+ \delta J + V_+ \delta P, \quad \text{(2.8)}
\]

\[
\delta M = -T_c \delta S_c + \Omega_c \delta J + V_c \delta P, \quad \text{(2.9)}
\]

here

\[
\Omega_+ = \tilde{\Omega}_+ - \Omega_\infty = \frac{a(1 - \Lambda r_+^2 / 3)}{r_+^2 + a^2} = \frac{a(r_+ + r_+)(r_+^2 + a^2)}{r_+ (r_+^2 + a^2)(r_+^2 + r_+ r_+ + a^2)}, \quad \text{(2.10)}
\]

\[
\Omega_c = \tilde{\Omega}_c - \Omega_\infty = \frac{a(1 - \Lambda r_+^2 / 3)}{r_+^2 + a^2} = \frac{a(r_+ + r_+)(r_+^2 + a^2)}{r_+ (r_+^2 + a^2)(r_+^2 + r_+ r_+ + a^2)}. \quad \text{(2.11)}
\]

The black hole and cosmological thermodynamics volumes are respectively \[8\],

\[
V_+ = \frac{r_+ A_+}{3} \left[ 1 + \frac{a^2}{2 \Xi r_+^2} \left( 1 - \frac{\Lambda}{3} r_+^2 \right) \right], \quad \text{(2.12)}
\]

\[
V_c = \frac{r_c A_c}{3} \left[ 1 + \frac{a^2}{2 \Xi r_+^2} \left( 1 - \frac{\Lambda}{3} r_+^2 \right) \right]. \quad \text{(2.13)}
\]

From the Eq.\(2.4\), when \( a \ll r_+ \), we can obtain the approximate value for the temperature

\[
T_+ = -\frac{x^3 + x^2 - 2x + a^2(3x^2 - 1/x - x - 1)/r_+^2}{4 \pi r_+ [(x^2 + a^2/r_+^2)(1 + x + x^2) + x^2 a^2/r_+^2]},
\]

\[
T_c = -\frac{x^3 + x^2 - 2x + a^2(3x^2 - 1/x - x - 1)/r_+^2}{4 \pi r_+ [(1 + a^2/r_+^2)(1 + x + x^2) + a^2/r_+^2]}. \quad \text{(2.14)}
\]

where \( x = r_+ / r_c \).
III. THE EFFECTIVE THERMODYNAMICS OF KERR-DE SITTER BLACK HOLE

Generally, the temperatures at the black hole event horizon and cosmological horizon are not equal. Thus, the whole Kerr-de Sitter system cannot be in equilibrium thermodynamically. However, there are two special cases for the Kerr-de Sitter black hole, in which the temperatures at the both horizons are the same. One case is the so-called Nariai black hole, the other is the lukewarm black hole \[8–13\]. For the Nariai black hole, the event horizon and cosmological horizon coincide \textit{apparently} and have the same temperature, zero or nonzero \[23\].

Considering the connection between the black hole horizon and the cosmological horizon \[20, 21\], we can derive the effective thermodynamic quantities and corresponding first law of black hole thermodynamics\(^1\):

\[
dM = T_{\text{eff}} dS - P_{\text{eff}} dV + \Omega_{\text{eff}} dJ.
\]

(3.1)

Here the thermodynamic volume is that between the black hole event horizon and the cosmological horizon, namely

\[
V = V_c - V_+ \approx \frac{4\pi}{3} (r_c^3 - r_+^3) = \frac{4\pi}{3} r_c^3 \left(1 - x^3\right).
\]

(3.2)

The total entropy can be written as

\[
S = S_+ + S_c + S_{\text{ex}} = \pi r_c^2 \left[1 + x^2 + f(x)\right].
\]

(3.3)

When \( J \) is a constant, from \( J = \frac{am}{2\pi} = aM \), we can obtain

\[
\delta a = -\frac{a}{M} \delta M,
\]

(3.4)

\[
\delta M = \left(\frac{\partial M}{\partial r_c}\right)_{x,a} \delta r_c + \left(\frac{\partial M}{\partial x}\right)_{r_c,a} \delta x + \left(\frac{\partial M}{\partial a}\right)_{r_c,x} \delta a,
\]

(3.5)

When \( a \ll r_+ \), from the eqs.\((3.2)\), \((2.12)\) and \((2.13)\), we can obtain

\[
\delta V = \left(\frac{\partial V}{\partial r_c}\right)_{x} \delta r_c + \left(\frac{\partial V}{\partial x}\right)_{r_c} \delta x = 4\pi r_c^2 (1 - x^3) \delta r_c - 4\pi r_c^3 x^2 \delta x,
\]

(3.6)

\(^1\) One can also take the cosmological constant \( \Lambda \) as the pressure and then derive the effective volume. In this case the effective first law should be: \( dM = T_{\text{eff}} dS + V_{\text{eff}} dP + \Omega_{\text{eff}} dJ \). This has been done in another paper.
From the Eq.\(3.1\), we can have the effective temperature of the Kerr-de Sitter black hole, 

\[
T_{\text{eff}} = \left( \frac{\partial M}{\partial S} \right)_{J,V} = \left( 1 + \frac{a M}{\partial a} \frac{\partial V}{\partial r_c} \right) \frac{A}{\left[ (\partial S/\partial r_c)_{x,a} \left( \frac{\partial V}{\partial r_c} \right)_{r_c} - \frac{\partial M}{\partial a} \right]_{r_c,x} A} \tag{3.7}
\]

here, \(A = \left( \frac{\partial M}{\partial a} \right)_{r_c,a} \left( \frac{\partial V}{\partial r_c} \right)_{x} - \left( \frac{\partial M}{\partial r_c} \right)_{x,a} \left( \frac{\partial V}{\partial r_c} \right)_{r_c} \). When \(a \ll r_+\), \(\Xi \sim 1\). we can obtain via neglecting the higher order of \(a\),

\[
T_{\text{eff}} \approx \left( \frac{\partial M}{\partial S} \right)_{J,V} \approx \left[ (\partial S/\partial r_c)_{x,a} \left( \frac{\partial V}{\partial r_c} \right)_{r_c} - \frac{\partial M}{\partial a} \right]_{r_c,x} A \tag{3.8}
\]

From the eq. \(2.3\), we have

\[
M \approx \frac{(r_c + r_+)(r_c^2 + a^2)(r_+^2 + a^2)}{2r_c r_+ (r_c^2 + r_c r_+ + r_+^2 + a^2)} \approx r_c^2 \frac{(1 + x)(x^2 + (1 + x^2)a^2/r_c^2)}{2x(1 + x + x^2)}. \tag{3.9}
\]

Considering the expression for the entropy of both horizons \(3.3\), we can take the entropy of Kerr-de Sitter black hole as

\[
S = \frac{\pi \, r_c^2}{\Xi} \left( 1 + x^2 + f(x) + 2 \frac{a^2}{r_c^2} \right) \approx \pi r_c^2 \left( 1 + x^2 + f(x) + 2 \frac{a^2}{r_c^2} \right). \tag{3.10}
\]

Here the undefined function \(f(x)\) represents the extra contribution from the correlations of the two horizons. Then, how to determine the function \(f(x)\)? Inserting eqs. \(3.10\), \(3.9\) and \(3.2\) into eq.(3.8), we can obtain

\[
T_{\text{eff}} = \left. \frac{1}{2\pi r_c} \frac{T_1(x)}{T_2(x)} \right|_{J,V}, \tag{3.11}
\]

where

\[
T_1(x) = \frac{1}{x^2(1 + x + x^2)} \left[ x^2(1 + 2x)(1 - x) + x^5(1 + x) - \frac{a^2}{r_c^2} \left( (1 + x^4)(1 + x + x^2) - 2x^3 \right) \right],
\]

\[
T_2(x) = \left[ 2x(1 + x) + 2x^2 f(x) + (1 - x^3) f'(x) \right]. \tag{3.12}
\]

Neglecting the higher order terms of \(a\), we can rewrite the eq.\(2.4\),

\[
T_+ = \frac{x + x^2 - 2x^3 + a^2(3x^2 - 1/x - x - 1)/r_c^2}{4\pi r_c \left( x^2 + a^2/r_c^2 \right)(1 + x + x^2) + x^2 a^2/r_c^2},
\]

\[
T_c = -\frac{x^3 + x^2 - 2x + a^2(3 - x - x^2 - x^3)/r_c^2}{4\pi r_c x \left( 1 + a^2/r_c^2 \right)(1 + x + x^2) + a^2/r_c^2}. \tag{3.13}
\]
From the solution of (3.13), when

\[
\frac{a^2}{r_c^2} = \frac{x^4 + 5x^3 + 5x^2 + 5x + 1 - \sqrt{x^8 + 10x^7 + 31x^6 + 56x^5 + 69x^4 + 56x^3 + 31x^2 + 10x + 1}}{2(x^2 + 1)},
\]

the radiation temperature of both horizons is equal.

![Graph](image_url)

**FIG. 1:** \(T_+/c\) with respect to \(x\). When \(a^2/r_c^2\) satisfies the condition of (3.14), \(T_+\) and \(T_c\) is equal.

In general cases, the temperatures of the black hole horizon and the cosmological horizon are not the same, thus the globally effective temperature \(T_{eff}\) cannot be compared with them. However, in the special case, such as lukewarm case \([24, 25]\), the temperatures of the two horizons are the same. We conjecture that in this special case the effective temperature should also take the same value. On the basis of this consideration, we can obtain the information of \(f(x)\). For Kerr-de Sitter black hole,

\[
T_{eff} = \tilde{T}_c = \tilde{T}_+.
\]

From the eq. (3.11) and (3.15), we can obtain

\[
T_{eff} = \frac{T_1(x)}{T_1(x) \tilde{T}_+[\tilde{T}_c]},
\]

where \(\tilde{T}_1(x)\) is the value of \(T_1(x)\) with \(a^2/r_c^2\) in eq. (3.14). Substituting the eqs. (3.11) and (3.15) into the eq (3.16), we can obtain

\[
T_2(x) = \frac{1}{2\pi r_c \tilde{T}_c[\tilde{T}_+]} \tilde{T}_1(x),
\]

Inserting eqs. (3.12) and (3.13), we can obtain the equation of \(f(x)\)

\[
\frac{2x(1 + x) + 2x^2f(x) + (1 - x^3)f'(x) + 2[(1 + a^2/r_c^2)(1 + x + x^2) + a^2/r_c^2]}{x^2(1 - x)(1 + 2x) + x^5(1 + x) - [(1 + x^4)(1 + x + x^2) - 2x^3]a^2/r_c^2} = 0
\]

(3.18)
FIG. 2: $T_{\text{eff}}$ with respect to $x$. $T_{\text{eff}}$ has a maximum at $x = 0.21$ for $a = 0.2$, $x = 0.269$ for $a = 0.25$, $x = 0.331$ for $a = 0.3$.

FIG. 3: $f(x)$ with respect to $x$. $f(x)$ is zero at $x = 0.21$ for $a = 0.2$, $x = 0.269$ for $a = 0.25$, $x = 0.331$ for $a = 0.3$.

FIG. 4: $S$ with respect to $x$. the dashed(black) curve represents the sum of the two horizon entropy and the solid(red) curve depicts the result in Eq.(3.10). In the calculation we set $r_c = 1$.

In Fig[2, 3] and 4 we depict the effective temperature $T_{\text{eff}}$, $f(x)$ and $S$ as functions of $x$. It is shown that $T_{\text{eff}}$ tends to zero as $x \to 1$, namely the Nariai limit. Although this result
does not agree with that of Bousso and Hawking \[23\], it is consistent with the entropy. As is depicted in Fig[4] the entropy will diverge as \(x \to 1\).

The specific heat of kerr-de Sitter system can be defined as

\[
C_{r_c,a} = T_{eff} \left( \frac{\partial S}{\partial T_{eff}} \right)_{r_c,a}.
\]

(3.19)

\[
\begin{array}{c}
\text{FIG. 5: } C_{r_c,a} \text{ with respect to } x. \text{ In the calculation we set } r_c = 1.
\end{array}
\]

From Fig[5] when \(0 < x < x_c\), the specific heat of system is positive, while \(x_c < x < 1\), it is negative. This means that the Kerr-de Sitter black hole with \(x < x_c\) is thermodynamically stable. From the Fig[5] we can find that the region of stable state for kerr-de Sitter is related to the angular velocity parameter \(a\), i.e., the region of stable state becomes bigger as the rotating parameters \(a\) is increases.

**IV. CONCLUSION**

In this letter, we mainly studied the entropy of Kerr-de Sitter black hole. In addition, we analyzed the thermodynamic stability of this black hole. Firstly, we simply review the thermodynamic quantities of black hole horizon and cosmological horizon of the Kerr-de Sitter black hole. Considering the relation of the black hole horizon and the cosmological horizon, we conjecture the total entropy should take the form of Eq.(3.3). Secondly, we find that the entropy obtained by Eq.(3.3) monotonically increases as \(x\) increased. When \(x \to 1\), this entropy diverges. Finally, it is found that the Kerr-de sitter black hole is unstable due to negative heat capacity in some region for rotating parameter \(a\). In a word, we obtain the corrected entropy of Kerr-de Sitter black hole, which may make the method more acceptable.
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