Electroweak corrections to $e^+e^- \rightarrow f\bar{f}H$\footnote{This work was supported in part by the Swiss Bundesamt für Bildung und Wissenschaft and by the European Union under contract HPRN-CT-2000-00149.}

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Some of the most interesting Higgs-production processes at future $e^+e^-$ colliders are of the type $e^+e^- \rightarrow f\bar{f}H$. We present a calculation of the complete $O(\alpha)$ corrections to these processes in the Standard Model for final-state neutrinos and top quarks. Initial-state radiation beyond $O(\alpha)$ at the leading-logarithmic level as well as QCD corrections are also included. The electroweak corrections turn out to be sizable and reach the order of $\pm 10\%$ and will thus be an important part of precise theoretical predictions for future $e^+e^-$ colliders.

1. Introduction

One of the main future tasks in particle physics will be the investigation of the mechanism of electroweak symmetry breaking in general and the discovery of the Higgs boson and the determination of its properties in particular. Since the Higgs-boson mass is expected to be in the range from the lower experimental bound of $114.4 \text{ GeV}$ up to $1 \text{ TeV}$, with a light Higgs mass (below $\sim 200 \text{ GeV}$) favoured by electroweak precision data, the LHC will be able to discover it in the full mass range, provided it exists and has no exotic properties. However, for the complete determination of its profile, including its couplings to fermions and gauge bosons, experiments in the clean environment of an $e^+e^-$ linear collider are indispensable.

Here we concentrate on the associated production of a Higgs boson together with a pair of neutrinos or top quarks in $e^+e^-$ annihilation, which are among the most interesting Higgs-boson production processes at future $e^+e^-$ linear colliders.

2. The process $e^+e^- \rightarrow \nu\bar{\nu}H$

At $e^+e^-$ colliders the two main Higgs production processes are the Higgs-strahlung and W-boson-fusion processes. In the Higgs-strahlung process the Higgs boson is radiated off a $Z$ boson, with the corresponding cross section rising sharply at the threshold, located at a centre-of-mass (CM) energy of $\sqrt{s} = M_Z + M_H$, to a maximum a few tens of GeV above the threshold energy and then falling off as $1/s$. In the W-boson-fusion process the Higgs boson is produced via fusion of two $W$ bosons, each emitted from an incoming electron/positron. The corresponding cross section grows as $\ln s$ and thus is the dominant production mechanism at large energies. Both production mechanisms appear in the process $e^+e^- \rightarrow \nu\bar{\nu}H$, with $l = e, \mu, \text{ or } \tau$, though the W-boson-fusion process is only present for $l = e$.

For the process $e^+e^- \rightarrow ZH$ the $O(\alpha)$ electroweak radiative corrections have been calculated many years ago in Ref. [2]. Furthermore a Monte Carlo algorithm for the calculation of the real photonic corrections to this process was described in Ref. [3]. For the full process $e^+e^- \rightarrow \nu\bar{\nu}H$ there has been a lot of activity regarding the electroweak corrections recently. Within the Minimal Supersymmetric Standard Model (MSSM) the fermion and sfermion loop contributions have been evaluated in Refs. [4,5]. Analytical results for the one-loop corrections in the SM have been obtained in Ref. [6], though no numerical results have been given there. Finally, calculations
of the complete $\mathcal{O}(\alpha)$ electroweak corrections to $e^+e^- \to \nu\bar{\nu}H$ in the SM have been performed in Refs. [1,7]. A comparison of these calculations has revealed an agreement within 0.3%, which is of the same order as the integration error of Ref. [7]. Very recently also results on corrections to the Z-boson-fusion process $e^+e^- \to e^+e^-H$ have been presented in Ref. [8].

A sketch of the calculational setup and a summary of the main results of our calculation [1] of the complete electroweak corrections is given in the following. Apart from the $\mathcal{O}(\alpha)$ corrections we have also included the leading-logarithmic part of the higher-order initial-state radiation (ISR) using the structure-function approach. Furthermore by using the $G_\mu$ scheme we have absorbed corrections proportional to $m_t^2/M_W^2$ in the fermion–W-boson couplings and the running of $\alpha(Q^2)$ from $Q^2 = 0$ to the electroweak scale. The calculation has been performed mostly using standard techniques. However, the appearance of pentagon diagrams potentially leads to numerical instabilities related to leading inverse Gram determinants. We have therefore used the reduction scheme of Ref. [9].

For the extraction of the soft and collinear singularities in the real corrections we have used both the dipole subtraction method [10,11] and phase-space slicing following closely Ref. [12]. Two independent calculations have been made resulting in two independent computer codes for the numerical evaluation, one employing a multi-channel Monte-Carlo generator similar to Refs. [11,13,14] for the phase-space integration, the other one using VEGAS [15].

The results for the total cross section in lowest order and including the radiative corrections are shown in Figure 1 on the l.h.s. as a function of the CM energy for $M_H = 150$ GeV. The relative corrections shown on the r.h.s are large (≲−20%) and vary strongly in the ZH-threshold region while they are flat and about −10% for energies above 500 GeV. They are always negative because they are dominated by initial-state radiation and the cross section is monotonously rising. We have also constructed an improved Born approximation (IBA) which incorporates the leading-logarithmic part of the ISR using structure functions and furthermore contains the leading $m_t^2/M_W^2$ corrections from
the WWH-vertex. As shown in Figure 1 (r.h.s), the residual relative corrections normalized to the IBA are about 1–3%. Although they are systematically smaller than the corrections relative to the lowest order in the $G_\mu$ scheme, the inclusion of the full $O(\alpha)$ corrections is necessary for a precision analysis.

3. The process $e^+e^- \to t\bar{t}H$

We have also investigated the process $e^+e^- \to t\bar{t}H$, which is interesting since it permits a direct access to the top-quark Yukawa coupling $g_{t\bar{t}H}$, which is by far the largest Yukawa coupling ($g_{t\bar{t}H} \approx 0.5$) in the SM. This is possible because the process proceeds mainly through Higgs-boson emission off top quarks, while emission from intermediate $Z$ bosons plays only a minor role if the Higgs-boson mass is not too large, i.e. $M_H \sim 100$–200 GeV. For a light Higgs boson with a mass around $M_H \sim 120$ GeV, a precision of about 5% can be reached at an $e^+e^-$ linear collider operating at $\sqrt{s} = 800$ GeV with a luminosity of $\int L \, dt \sim 1000 \, \text{fb}^{-1}$ [16]. An even better accuracy can be obtained by combining the $t\bar{t}H$ channel with information from other Higgs-production and decay processes in a combined fit [17].

Within the SM the $O(\alpha_s)$ corrections have been calculated for the dominant photon-exchange channel in Ref. [18], while the full set of diagrams has been evaluated in Ref. [19]. The $O(\alpha_s)$ corrections to the photon-exchange channel in the MSSM have been considered in Ref. [20]. In Ref. [21] all QCD diagrams have been taken into account, while the SUSY-QCD corrections have been worked out in Ref. [22]. The evaluation of the electroweak $O(\alpha)$ corrections in the SM has made considerable progress recently. Results have been presented in Refs. [23–25], with agreement between Refs. [24,25] while Ref. [23] shows deviations close to threshold and at high energies.

We finally present some results of our calculation of the $O(\alpha)$ electroweak and the $O(\alpha_s)$ QCD corrections. Though the calculation of the virtual corrections for this process is much more involved than for the process $e^+e^- \to \nu\bar{\nu}H$, the same calculational techniques could be used.

Results for the total cross section in lowest order and the corrected cross section including both the electroweak and QCD corrections are shown in Figure 2 on the l.h.s. Away from the kinematic threshold at $\sqrt{s} = 2m_t + M_H$ the size of the cross

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Lowest-order and corrected cross sections (l.h.s.) as well as relative corrections (r.h.s.) in the $G_\mu$ scheme for a Higgs-boson mass $M_H = 150$ GeV.}
\end{figure}
section is typically a few fb, with a maximum at about 800 GeV. On the r.h.s. of Figure 2 the relative corrections are shown. The QCD corrections are large and positive close to threshold where soft-gluon exchange in the t\bar{t} system leads to a Coulomb-like singularity. For larger energies the QCD corrections decrease, eventually turn negative and reach about $-8\%$ at an energy of $\sqrt{s} = 1.5$ TeV. The electroweak corrections are about $-10\%$ and only vary weakly with energy away from the threshold region, and are thus of a comparable size as the QCD corrections. Close to threshold they reach about $-20\%$ due to the large ISR QED corrections in this region. The behaviour of the combined electroweak and QCD corrections is dominated by the Coulomb-like singularity close to threshold while turning negative and reaching about $-15\%$ at high energies.

4. Summary

Recently, the full $\mathcal{O}(\alpha)$ electroweak corrections have become available for the Higgs-production processes $e^+e^- \rightarrow \nu\bar{\nu}H$ and $e^+e^- \rightarrow t\bar{t}H$. In both cases at least two completely independent calculations have been performed by different groups, agreeing better than 0.3%. The corrections are sizeable and can reach $\pm 10\%$ and will thus be an important ingredient of precise theoretical predictions for future $e^+e^-$ colliders.

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