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Acceptance Sampling Plans from Life Tests Based on Percentiles of New Weibull–Pareto Distribution with Application to Breaking Stress of Carbon Fibers Data

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Abstract: In this paper, acceptance sampling plans (ASPs) are proposed for the new Weibull–Pareto distribution (NWPD) percentiles assuming truncated life tests at a pre-determined time. The minimum sample sizes essential to assert the specified percentile are calculated for a given consumer’s risk. The operating characteristic function values of the developed ASPs and producer’s risk are provided. A real data set related to the breaking stress of carbon fibers data are presented for illustration.

Keywords: truncated life test; operating characteristic function; acceptance sampling; producer’s risk; new Weibull–Pareto distribution; consumer’s risk

MSC: 62D05

1. Introduction

The quality of a product is important to long-serving customers, while at the same time, owners or producers of the product are interested in saving costs and time in the production process. These objectives have encouraged researchers in the field to find a tool in order to maintain the quality of products lots. Acceptance sampling plans are well known in industry to emphasize the acceptability of a lot based on a random sample selected from the product. Based on this sample, the consumer can accept or reject the lot. The process of the acceptance sampling plan (ASP) operates by first obtaining the minimum ample size that is important to emphasize a certain percentile or average life when the life test is terminated at a pre-specified time. These types of tests are called truncated lifetime tests.

Different types of ASP are known to practitioners as the single ASP, double ASP, group ASP, multiple ASP as well as other methods. Details regarding these types can be found in previous papers: single ASP to the exponential distribution by [1], the three-parameter Lindley distribution [2], ASP for the exponentiated Fréchet distribution [3], double ASP for the NWPD is suggested by [4], single ASP for the NWPD is proposed by [5], three parameters Kappa distribution [6], single ASP for the generalized Rayleigh distribution [7], single ASP for the weighted exponential distribution [8], ASP for length-biased weighted Lomax distribution [9,10] single ASP for generalized exponential distribution [9,11] for the Akash distribution. These works have considered the mean as a quality parameter. Further works include ASP for log-logistic distribution [12], for single ASP under exponentiated inverse Rayleigh distribution [10,13] for ASP based on generalized inverted exponential distribution see [14].

For ASP based on model percentiles, single ASP for percentiles under the linear failure rate distribution [15], ASP for percentiles under the inverse Rayleigh distribution [16], the
Birnbaum Saunders distribution for percentiles [17], for Log-Logistic distribution for percentiles [18,19] for the ASP percentile under Marshall–Olkin extended Lomax distribution.

The structure of the paper is as follows. In Section 2, the NWPD is introduced. Section 3 describes the suggested ASP under the NWPD. Section 4 provides Illustrative examples for the real data set. Finally, our conclusions are summarized in Section 5.

2. The NWPD

The NWPD is introduced by [20] as a new continuous lifetime distribution to be more flexible in fitting real data in various fields. Ref. [21] suggested the exponentiated NWPD as a modification of the NWPD. Ref. [22] used the ranked set sampling to estimate the parameters of the NWPD. The distribution function of the NWPD has the form

\[ F(x; \alpha, \theta, \eta) = 1 - e^{-\alpha (\frac{x}{\theta})^{\eta}}, \quad x > 0, \eta, \theta, \alpha > 0 \]  

(1)

with a probability density function provided by

\[ f(x; \alpha, \theta, \eta) = \frac{\alpha \eta}{\theta \eta} x^{\eta-1} e^{-\alpha (\frac{x}{\theta})^{\eta}}, \]  

(2)

The mean and the variance of the model, respectively, are

\[ E(X) = \theta \alpha^{-\frac{1}{\eta}} \Gamma \left( \frac{\eta + 1}{\eta} \right) \] 

\[ Var(X) = 2\theta \alpha^{-\frac{2}{\eta}} \Gamma \left( \frac{\eta + 2}{\eta} \right) - \left( \theta \alpha^{-\frac{1}{\eta}} \Gamma \left( \frac{\eta + 1}{\eta} \right) \right)^2 \]  

(3)

The NWPD has a hazard rate function and mode at \( x = x_0 \), respectively, provided by

\[ H(x) = \frac{\alpha \eta}{\theta \eta} x^{\eta-1} \]  

\[ x_0 = \theta \left( \frac{\eta - 1}{\alpha \eta} \right)^{\frac{1}{\eta}} \]

The 100q-th percentile of the NWPD is

\[ t_q = \theta \left( \frac{1}{\ln} \left( \frac{1}{1-q} \right) \right)^{\frac{1}{\eta}} \]

In Figure 1, we presented the plot of pdf and reliability functions of the NWPD for some selected parameters. Additionally, in Figure 2, the hazard function and the distribution function of the NWPD are offered. It is clear that the model is skewed to the right with decreasing reliability function for the selected parameter values. In Figure 2, it is noted that the hazard function increases for \( \eta = 2, \theta = 4 \) as \( \alpha = 1, 2, 3, 4, 5 \).

Figure 1. The pdf and reliability function of the NWPD with \( \eta = 2, \theta = 4 \).
3. The Suggested ASP

Assume that the life test is scheduled to be \( t \), and \( c \) is the maximum number of admissible bad lots to accept the lot, with at least \( p^* \) being the probability of rejecting a bad lot. The truncated life test ASPs for percentile is to maintain the minimum sample size \( n \) for a specified acceptance number \( c \) provided that the consumer’s risk (which is the probability of accepting a bad lot) doesn’t exceed \( 1 - p^* \). A bad lot that is the true is in the 100\( q \)th percentile, while \( t_q \) is less than the identified percentile \( t_0^q \). Thus, the probability of rejecting a bad lot with \( t_q < t_0^q \) is at least equal to \( p^* \). In this sense, the parameters of the offered sampling plan are \( (n, c, t_q/t_0^q) \) with a probability \( p^* \).

3.1. Minimum Sample Size

For a fixed \( p^* \) where \( p^* \in (0, 1) \), the suggested ASPs can be characterized by \( (n, c, t/t_0^0) \), assuming that the lot size is adequately large so that the binomial distribution can be employed. The smallest positive sample size \( n \) needed to assert that \( t_q > t_0^0 \) should satisfy the inequality

\[
\sum_{i=0}^{c} \binom{n}{i} p^i (1 - p)^{n-i} \leq 1 - p^* \tag{4}
\]

where \( p = F(t; \delta_0) \) is the probability of a failure observed through the time \( t \) given that a determined 100\( q \)th percentile for lifetime \( t_0^0 \) which depends only on \( \delta_0 = t/t_0^0 \).

\( F(t; \delta) \) is a non-decreasing function of \( \delta \), since \( \partial F(t; \delta)/\partial \delta > 0 \). Therefore, \( F(t; \delta) < F(t; \delta_0) \leftrightarrow \delta \leq \delta_0 \), which is equivalently

\[
F(t; \delta) \leq F(t; \delta_0) \leftrightarrow t_q \leq t_0^q \tag{5}
\]

The smallest sample size \( n \) that satisfies (3) can be obtained for any given \( q, \delta_0 = t/t_0^0 \), \( p^* \), \( \theta, \alpha, \eta \). For illustration, the required smallest sample sizes are obtained for \( q = 0.1 \) \( t/t_0^0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712, p^* = 0.75, 0.9, 0.95, 0.99 \) and \( c = 0, 1, 2, \ldots, 10 \). The results are shown in Table 1 for \( \eta = 2.793 \) and \( \alpha = 1.011 \) under the NWPD. Further, the minimum sample size values are presented in Table 2 for \( \eta = 2 \) and \( \alpha = 2 \).

3.2. OC of the Sampling Plan \( (n, c, t/t_0^q) \)

For the ASP \( (n, c, t/t_0^q) \), the OC function of the sampling plan is the probability of acceptance of a lot. The OC is defined as

\[
L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1 - p)^{n-i} \tag{6}
\]

where \( p = F(t; \delta) \). It is of interest that \( p = F(t; \delta) \) can be utilized as a function of \( \delta = t/t_q \).

Hence, \( p = F\left(\frac{t}{t_q^q d_q}\right) \), \( d_q = \frac{t_q}{t_0^q} \). With reference to Equation (6), the values of the OC

Figure 2. The hazard and distribution functions of the NWPD with \( \eta = 2, \theta = 4 \).
as a function of $d_q = \frac{t_q}{t_0}$ can be calculated for the sampling plan \( n, c = 2, \frac{t_q}{t_0} \) with the model parameter values. Table 3 is devoted to the OC values for the sampling plan \( (n, c = 2, \frac{t}{t_0}) \) when $\eta = 2.793$ and $\alpha = 1.011$ for the NWPD, and in Table 4 for $\eta = 2$ and $\alpha = 2$.

Table 1. Minimum sample sizes necessary to assure the percentile $q = 0.1$ life of a product to exceed a given $t_{0.1}$ with $\eta = 2.793$ and $\alpha = 1.011$ for the NWPD.

| $p^*$ | $c$ | \( \delta_{0.1} \) |
|-------|-----|---------------------|
|       |     | 0.628 | 0.942 | 1.257 | 1.571 | 2.356 | 3.141 | 3.927 | 4.712 |
| 0.75  | 0   | 49    | 16    | 7     | 4     | 2     | 1     | 1     | 1     |
|       | 1   | 95    | 31    | 15    | 8     | 3     | 2     | 2     | 2     |
|       | 2   | 138   | 45    | 21    | 12    | 5     | 3     | 3     | 3     |
|       | 3   | 180   | 59    | 28    | 16    | 7     | 5     | 4     | 4     |
|       | 4   | 221   | 73    | 34    | 19    | 8     | 6     | 5     | 5     |
|       | 5   | 261   | 86    | 40    | 23    | 10    | 7     | 6     | 6     |
|       | 6   | 301   | 100   | 46    | 27    | 11    | 8     | 7     | 7     |
|       | 7   | 341   | 113   | 53    | 30    | 13    | 9     | 8     | 8     |
|       | 8   | 380   | 126   | 59    | 34    | 15    | 10    | 9     | 9     |
|       | 9   | 420   | 139   | 65    | 37    | 16    | 11    | 10    | 10    |
|       | 10  | 459   | 152   | 71    | 41    | 18    | 12    | 11    | 11    |
| 0.9   | 0   | 81    | 26    | 12    | 7     | 2     | 1     | 1     | 1     |
|       | 1   | 136   | 45    | 21    | 11    | 4     | 3     | 2     | 2     |
|       | 2   | 187   | 61    | 28    | 16    | 6     | 4     | 3     | 3     |
|       | 3   | 235   | 77    | 36    | 20    | 8     | 5     | 4     | 4     |
|       | 4   | 281   | 92    | 43    | 24    | 10    | 6     | 5     | 5     |
|       | 5   | 326   | 107   | 50    | 28    | 11    | 7     | 6     | 6     |
|       | 6   | 370   | 122   | 56    | 32    | 13    | 9     | 7     | 7     |
|       | 7   | 414   | 136   | 63    | 36    | 15    | 10    | 8     | 8     |
|       | 8   | 457   | 150   | 70    | 40    | 16    | 11    | 9     | 9     |
|       | 9   | 499   | 164   | 76    | 43    | 18    | 12    | 10    | 10    |
|       | 10  | 542   | 178   | 83    | 47    | 20    | 13    | 11    | 11    |
| 0.95  | 0   | 105   | 34    | 16    | 9     | 3     | 2     | 1     | 1     |
|       | 1   | 166   | 54    | 25    | 14    | 5     | 3     | 2     | 2     |
|       | 2   | 221   | 72    | 33    | 18    | 7     | 4     | 3     | 3     |
|       | 3   | 272   | 89    | 41    | 23    | 9     | 5     | 4     | 4     |
|       | 4   | 321   | 105   | 48    | 27    | 11    | 7     | 5     | 5     |
|       | 5   | 369   | 121   | 56    | 31    | 13    | 8     | 6     | 6     |
|       | 6   | 416   | 136   | 63    | 35    | 14    | 9     | 8     | 8     |
|       | 7   | 462   | 151   | 70    | 40    | 16    | 10    | 9     | 9     |
|       | 8   | 507   | 166   | 77    | 44    | 18    | 11    | 10    | 10    |
|       | 9   | 552   | 181   | 84    | 47    | 19    | 13    | 11    | 11    |
|       | 10  | 596   | 196   | 91    | 51    | 21    | 14    | 12    | 11    |
| 0.99  | 0   | 161   | 52    | 24    | 13    | 4     | 2     | 1     | 1     |
|       | 1   | 232   | 75    | 34    | 19    | 7     | 4     | 3     | 2     |
|       | 2   | 294   | 96    | 44    | 24    | 9     | 5     | 4     | 3     |
|       | 3   | 352   | 115   | 52    | 29    | 11    | 6     | 5     | 4     |
|       | 4   | 406   | 133   | 61    | 34    | 13    | 8     | 6     | 5     |
|       | 5   | 459   | 150   | 69    | 38    | 15    | 9     | 7     | 6     |
|       | 6   | 511   | 167   | 77    | 43    | 17    | 10    | 8     | 7     |
|       | 7   | 561   | 183   | 84    | 47    | 18    | 11    | 9     | 8     |
|       | 8   | 610   | 200   | 92    | 51    | 20    | 13    | 10    | 9     |
|       | 9   | 659   | 216   | 99    | 56    | 22    | 14    | 11    | 10    |
|       | 10  | 707   | 231   | 107   | 60    | 24    | 15    | 12    | 11    |
Table 2. Minimum sample sizes necessary to assure the percentile \( q = 0.1 \) life of a product to exceed a given \( t_{0.1} \) with \( \eta = 2 \) and \( \alpha = 2 \) for the NWPD.

| \( p^* \) | \( c \) | 0.628 | 0.942 | 1.257 | 1.571 | 2.356 | 3.141 | 3.927 | 4.712 |
|---|---|---|---|---|---|---|---|---|---|
| 0.75 | 0 | 34 | 15 | 9 | 6 | 3 | 2 | 1 | 1 |
| | 1 | 66 | 30 | 17 | 11 | 6 | 4 | 3 | 2 |
| | 2 | 96 | 43 | 25 | 17 | 8 | 5 | 4 | 4 |
| | 3 | 125 | 57 | 33 | 22 | 11 | 7 | 6 | 5 |
| | 4 | 154 | 70 | 40 | 27 | 13 | 9 | 7 | 6 |
| | 5 | 182 | 82 | 48 | 32 | 16 | 11 | 8 | 7 |
| | 6 | 209 | 95 | 55 | 37 | 18 | 12 | 10 | 8 |
| | 7 | 237 | 108 | 62 | 41 | 21 | 14 | 11 | 9 |
| | 8 | 264 | 120 | 69 | 46 | 23 | 16 | 12 | 10 |
| | 9 | 292 | 132 | 77 | 51 | 26 | 17 | 13 | 12 |
| | 10 | 319 | 145 | 84 | 56 | 28 | 19 | 15 | 13 |
| 0.9 | 0 | 56 | 25 | 14 | 9 | 4 | 3 | 2 | 1 |
| | 1 | 95 | 43 | 24 | 16 | 8 | 5 | 4 | 3 |
| | 2 | 130 | 58 | 33 | 22 | 11 | 7 | 5 | 4 |
| | 3 | 163 | 73 | 42 | 28 | 14 | 9 | 6 | 5 |
| | 4 | 195 | 88 | 51 | 33 | 16 | 10 | 8 | 7 |
| | 5 | 226 | 102 | 59 | 39 | 19 | 12 | 9 | 8 |
| | 6 | 257 | 116 | 67 | 44 | 22 | 14 | 11 | 9 |
| | 7 | 287 | 130 | 75 | 49 | 24 | 16 | 12 | 10 |
| | 8 | 317 | 144 | 83 | 55 | 27 | 18 | 13 | 11 |
| | 9 | 347 | 157 | 90 | 60 | 30 | 19 | 15 | 13 |
| | 10 | 376 | 170 | 98 | 65 | 32 | 21 | 16 | 14 |
| 0.95 | 0 | 73 | 33 | 18 | 12 | 6 | 3 | 2 | 2 |
| | 1 | 115 | 52 | 30 | 19 | 9 | 6 | 4 | 3 |
| | 2 | 153 | 69 | 39 | 26 | 12 | 8 | 6 | 4 |
| | 3 | 189 | 85 | 49 | 32 | 15 | 10 | 7 | 6 |
| | 4 | 223 | 100 | 58 | 38 | 18 | 12 | 9 | 7 |
| | 5 | 256 | 115 | 66 | 44 | 21 | 13 | 10 | 8 |
| | 6 | 289 | 130 | 75 | 49 | 24 | 15 | 11 | 9 |
| | 7 | 320 | 145 | 83 | 55 | 27 | 17 | 13 | 11 |
| | 8 | 352 | 159 | 91 | 60 | 30 | 19 | 14 | 12 |
| | 9 | 383 | 173 | 99 | 66 | 32 | 21 | 16 | 13 |
| | 10 | 414 | 187 | 108 | 71 | 35 | 23 | 17 | 14 |
| 0.99 | 0 | 111 | 50 | 28 | 18 | 8 | 5 | 3 | 2 |
| | 1 | 161 | 72 | 41 | 27 | 12 | 7 | 5 | 4 |
| | 2 | 204 | 91 | 52 | 34 | 16 | 10 | 7 | 5 |
| | 3 | 244 | 109 | 62 | 41 | 19 | 12 | 8 | 7 |
| | 4 | 282 | 127 | 72 | 47 | 22 | 14 | 10 | 8 |
| | 5 | 318 | 143 | 82 | 53 | 26 | 16 | 12 | 9 |
| | 6 | 354 | 159 | 91 | 60 | 29 | 18 | 13 | 11 |
| | 7 | 389 | 175 | 100 | 66 | 32 | 20 | 15 | 12 |
| | 8 | 423 | 191 | 109 | 72 | 35 | 22 | 16 | 13 |
| | 9 | 457 | 206 | 118 | 77 | 37 | 24 | 17 | 14 |
| | 10 | 490 | 221 | 127 | 83 | 40 | 26 | 19 | 16 |

3.3. Producer’s Risk

The producer’s risk is the probability of rejecting the lot if \( t_q > t_{0.1}^q \). For a given value of the producer’s risk, say \( \phi \), the researchers were interested in determining the value of \( d_q \) to assert that the producer’s risk is less than or equal to \( \phi \) when the \( (n, c, t/t_q) \) is developed at a specified \( p^* \). Therefore, we aimed to achieve the smallest value of \( d_q \) satisfying \( L(p) \geq 1 - \phi \). In this case,
\[ P(\text{Rejecting a lot}) = \sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \] (7)

Table 5 shows the minimum ratios of \(d_{0.1}\) for the acceptability of a lot under \(\phi = 0.05\) when \(\eta = 2.793\) and \(\alpha = 1.011\) for the NWPD and in Table 6, the ratio values show when for \(\eta = 2\) and \(\alpha = 2\).

Table 3. OC values of sampling plans of \(c = 6\), for a given \(p^*\) with \(\eta = 2.793\) and \(\alpha = 1.011\) for the NWPD.

| \(p^*\) | \(d_{0.1}\) | \(n\) | 2 | 4 | 6 | 8 | 10 | 12 |
|---------|-------------|-------|---|---|---|---|----|----|
| 0.75    |             |       |   |   |   |   |     |     |
| 0.628   | 301         | 0.999701 | 1 | 1 | 1 | 1 | 1   | 1   |
| 0.942   | 100         | 0.999683 | 1 | 1 | 1 | 1 | 1   | 1   |
| 1.257   | 46          | 0.999700 | 1 | 1 | 1 | 1 | 1   | 1   |
| 1.571   | 27          | 0.999626 | 1 | 1 | 1 | 1 | 1   | 1   |
| 2.356   | 11          | 0.999629 | 1 | 1 | 1 | 1 | 1   | 1   |
| 3.141   | 8           | 0.998381 | 1 | 1 | 1 | 1 | 1   | 1   |
| 3.927   | 7           | 0.992162 | 1 | 1 | 1 | 1 | 1   | 1   |
| 4.712   | 7           | 0.929527 | 0.999998 | 1 | 1 | 1 | 1 | 1   |
| 0.9     |             |       |   |   |   |   |     |     |
| 0.628   | 370         | 0.998996 | 1 | 1 | 1 | 1 | 1   | 1   |
| 0.942   | 122         | 0.998960 | 1 | 1 | 1 | 1 | 1   | 1   |
| 1.257   | 56          | 0.998983 | 1 | 1 | 1 | 1 | 1   | 1   |
| 1.571   | 32          | 0.998874 | 1 | 1 | 1 | 1 | 1   | 1   |
| 2.356   | 13          | 0.998549 | 1 | 1 | 1 | 1 | 1   | 1   |
| 3.141   | 9           | 0.994680 | 1 | 1 | 1 | 1 | 1   | 1   |
| 3.927   | 7           | 0.992162 | 1 | 1 | 1 | 1 | 1   | 1   |
| 4.712   | 7           | 0.929527 | 0.999998 | 1 | 1 | 1 | 1 | 1   |
| 0.95    |             |       |   |   |   |   |     |     |
| 0.628   | 416         | 0.998049 | 1 | 1 | 1 | 1 | 1   | 1   |
| 0.942   | 136         | 0.998057 | 1 | 1 | 1 | 1 | 1   | 1   |
| 1.257   | 63          | 0.997956 | 1 | 1 | 1 | 1 | 1   | 1   |
| 1.571   | 35          | 0.998040 | 1 | 1 | 1 | 1 | 1   | 1   |
| 2.356   | 14          | 0.997481 | 1 | 1 | 1 | 1 | 1   | 1   |
| 3.141   | 9           | 0.994680 | 1 | 1 | 1 | 1 | 1   | 1   |
| 3.927   | 8           | 0.964743 | 1 | 1 | 1 | 1 | 1   | 1   |
| 4.712   | 7           | 0.929527 | 0.999998 | 1 | 1 | 1 | 1 | 1   |
| 0.99    |             |       |   |   |   |   |     |     |
| 0.628   | 511         | 0.994005 | 1 | 1 | 1 | 1 | 1   | 1   |
| 0.942   | 167         | 0.993995 | 1 | 1 | 1 | 1 | 1   | 1   |
| 1.257   | 77          | 0.993688 | 1 | 1 | 1 | 1 | 1   | 1   |
| 1.571   | 43          | 0.993472 | 1 | 1 | 1 | 1 | 1   | 1   |
| 2.356   | 17          | 0.990642 | 1 | 1 | 1 | 1 | 1   | 1   |
| 3.141   | 10          | 0.987026 | 1 | 1 | 1 | 1 | 1   | 1   |
| 3.927   | 8           | 0.964743 | 1 | 1 | 1 | 1 | 1   | 1   |
| 4.712   | 7           | 0.929527 | 0.999998 | 1 | 1 | 1 | 1 | 1   |

Table 4. OC values of sampling plans of \(c = 2\), for a given \(p^*\) with \(\eta = 2\) and \(\alpha = 2\) for the NWPD.

| \(p^*\) | \(d_{0.1}\) | \(n\) | 2 | 4 | 6 | 8 | 10 | 12 |
|---------|-------------|-------|---|---|---|---|----|----|
| 0.75    |             |       |   |   |   |   |     |     |
| 0.628   | 96          | 0.922106 | 0.997918 | 0.999798 | 0.999963 | 0.99999 | 0.999997 | 0.999997 |
| 0.942   | 43          | 0.923079 | 0.997949 | 0.999801 | 0.999963 | 0.99999 | 0.999997 | 0.999997 |
| 1.257   | 25          | 0.920043 | 0.997849 | 0.999791 | 0.999961 | 0.99999 | 0.999997 | 0.999997 |
| 1.571   | 17          | 0.912496 | 0.997595 | 0.999765 | 0.999957 | 0.999988 | 0.999996 | 0.999996 |
| 2.356   | 8           | 0.91682 | 0.997740 | 0.999780 | 0.999959 | 0.999989 | 0.999996 | 0.999996 |
| 3.141   | 5           | 0.917525 | 0.997740 | 0.999779 | 0.999959 | 0.999989 | 0.999996 | 0.999996 |
| 3.927   | 4           | 0.888453 | 0.996659 | 0.999668 | 0.999938 | 0.999984 | 0.999994 | 0.999994 |
| 4.712   | 4           | 0.768052 | 0.990960 | 0.999051 | 0.99982 | 0.999951 | 0.999984 | 0.999984 |
Table 4. Cont.

| $p^*$ | $\delta_{0.1}^0$ | $n$ | $\delta_{0.1}^0$ |
|-------|-----------------|-----|-----------------|
|       |                 |     | 2   | 4   | 6   | 8   | 10  | 12  |
| 0.9   | 0.628           | 130 | 0.847796 | 0.95115 | 0.999508 | 0.999908 | 0.999975 | 0.999992 |
|       | 0.942           | 58  | 0.849659 | 0.95194 | 0.999516 | 0.999910 | 0.999976 | 0.999992 |
|       | 1.257           | 33  | 0.84963  | 0.95202 | 0.999517 | 0.999910 | 0.999976 | 0.999992 |
|       | 1.571           | 22  | 0.842178 | 0.94871 | 0.999482 | 0.999903 | 0.999974 | 0.999991 |
|       | 2.356           | 11  | 0.819956 | 0.93858 | 0.999373 | 0.999882 | 0.999968 | 0.999989 |
|       | 3.141           | 7   | 0.798899 | 0.92813 | 0.999261 | 0.999861 | 0.999963 | 0.999987 |
|       | 3.927           | 5   | 0.789398 | 0.92250 | 0.999197 | 0.999848 | 0.999959 | 0.999986 |
|       | 4.712           | 4   | 0.768052 | 0.90960 | 0.999051 | 0.999820 | 0.999951 | 0.999984 |
| 0.95  | 0.628           | 153 | 0.788780 | 0.92353 | 0.999211 | 0.999851 | 0.999960 | 0.999986 |
|       | 0.942           | 69  | 0.786140 | 0.92217 | 0.999196 | 0.999848 | 0.999959 | 0.999986 |
|       | 1.257           | 39  | 0.788257 | 0.92232 | 0.999207 | 0.999851 | 0.999960 | 0.999986 |
|       | 1.571           | 26  | 0.777140 | 0.91745 | 0.999144 | 0.999838 | 0.999957 | 0.999985 |
|       | 2.356           | 12  | 0.782827 | 0.92028 | 0.999175 | 0.999844 | 0.999958 | 0.999984 |
|       | 3.141           | 8   | 0.730352 | 0.92346 | 0.999051 | 0.999820 | 0.999951 | 0.999984 |
|       | 3.927           | 5   | 0.768052 | 0.90960 | 0.999051 | 0.999820 | 0.999951 | 0.999984 |
|       | 4.712           | 4   | 0.768052 | 0.90960 | 0.999051 | 0.999820 | 0.999951 | 0.999984 |
| 0.99  | 0.628           | 204 | 0.647266 | 0.93466 | 0.998200 | 0.999654 | 0.999960 | 0.999986 |
|       | 0.942           | 91  | 0.648696 | 0.93857 | 0.998212 | 0.999657 | 0.999960 | 0.999986 |
|       | 1.257           | 52  | 0.643657 | 0.93819 | 0.998167 | 0.999648 | 0.999964 | 0.999986 |
|       | 1.571           | 34  | 0.637692 | 0.92827 | 0.998112 | 0.999637 | 0.999961 | 0.999986 |
|       | 2.356           | 16  | 0.625914 | 0.91766 | 0.997998 | 0.999614 | 0.999961 | 0.999986 |
|       | 3.141           | 10  | 0.589807 | 0.978632 | 0.997620 | 0.999539 | 0.999874 | 0.999957 |
|       | 3.927           | 6   | 0.569516 | 0.976613 | 0.997370 | 0.999489 | 0.999860 | 0.999952 |
|       | 4.712           | 5   | 0.606321 | 0.979689 | 0.997739 | 0.999562 | 0.999881 | 0.999959 |

Table 5. Minimum ratio of $d_{0.1}^0$ for the acceptability of a lot with producer’s risk 0.05 with $\eta = 2.793$ and $\alpha = 1.011$ for the NWPD.

| $p^*$ | $c$ | $\theta_{0.1}^0$ |
|-------|-----|-----------------|
| 0.75  | 0   | 3.2378          |
|       | 1   | 2.0711          |
|       | 2   | 1.7554          |
|       | 3   | 1.6058          |
|       | 4   | 1.5156          |
|       | 5   | 1.4536          |
|       | 6   | 1.4092          |
|       | 7   | 1.3755          |
|       | 8   | 1.3477          |
|       | 9   | 1.3263          |
|       | 10  | 1.3075          |
| 0.9   | 0   | 3.9193          |
|       | 1   | 2.3564          |
|       | 2   | 1.7532          |
|       | 3   | 1.6058          |
|       | 4   | 1.5156          |
|       | 5   | 1.4536          |
|       | 6   | 1.4092          |
|       | 7   | 1.3755          |
|       | 8   | 1.3477          |
|       | 9   | 1.3263          |
|       | 10  | 1.3075          |
### Table 5. Cont.

| $p^*$ | c   | $t_{0.01}^{ph}$ |
|-------|-----|-----------------|
| 0     | 4.3009 | 4.3084          |
| 2     | 2.0798 | 2.0817          |
| 4     | 1.7341 | 1.7352          |
| 0.95  | 1.6472 | 1.6491          |
| 1     | 1.5838 | 1.5834          |
| 2     | 1.5350 | 1.5340          |
| 3     | 1.4957 | 1.4953          |
| 4     | 1.4640 | 1.4641          |
| 5     | 1.4370 | 1.4384          |

| $p^*$ | c   | $t_{0.5}^{ph}$ |
|-------|-----|-----------------|
| 0     | 0.95 | 0.99            |
| 0     | 5.0121 | 5.0162          |
| 1     | 2.8545 | 2.8531          |
| 2     | 2.3045 | 2.3096          |
| 3     | 2.0446 | 2.0482          |
| 4     | 1.8871 | 1.8913          |
| 5     | 1.7819 | 1.7836          |
| 6     | 1.6463 | 1.6457          |
| 7     | 1.5988 | 1.6009          |
| 8     | 1.5606 | 1.5621          |
| 9     | 1.5283 | 1.5278          |

### Table 6. Minimum ratio of $d_{0.1}^p$ for the acceptability of a lot with producer's risk 0.05 with $\eta = 2$ and $\alpha = 2$ for the NWPD.

| $p^*$ | c   | $t_{0.01}^{ph}$ |
|-------|-----|-----------------|
| 0     | 5.2482 | 5.2289          |
| 1     | 2.7675 | 2.7857          |
| 2     | 2.1972 | 2.1912          |
| 3     | 1.9380 | 1.9486          |
| 4     | 1.7905 | 1.7962          |
| 0.75  | 1.6895 | 1.6863          |
| 5     | 1.6414 | 1.6178          |
| 6     | 1.5612 | 1.5664          |
| 7     | 1.5169 | 1.5196          |
| 8     | 1.4839 | 1.4820          |
| 9     | 1.4543 | 1.4564          |

| $p^*$ | c   | $t_{0.5}^{ph}$ |
|-------|-----|-----------------|
| 0     | 5.0121 | 5.0162          |
| 1     | 2.8545 | 2.8531          |
| 2     | 2.3045 | 2.3096          |
| 3     | 2.0446 | 2.0482          |
| 4     | 1.8871 | 1.8913          |
| 5     | 1.7819 | 1.7836          |
| 0.9   | 1.6463 | 1.6457          |
| 6     | 1.5988 | 1.6009          |
| 7     | 1.5606 | 1.5621          |
| 8     | 1.5283 | 1.5278          |

| $p^*$ | c   | $t_{0.01}^{ph}$ |
|-------|-----|-----------------|
| 0     | 6.7354 | 6.7505          |
| 1     | 3.3242 | 3.3438          |
| 2     | 2.5604 | 2.5528          |
| 3     | 2.2162 | 2.2118          |
| 4     | 2.0176 | 2.0200          |
| 5     | 1.8853 | 1.8867          |

| $p^*$ | c   | $t_{0.5}^{ph}$ |
|-------|-----|-----------------|
| 0     | 7.1924 | 7.1931          |
| 1     | 7.1720 | 7.1723          |
| 2     | 1.6643 | 1.6695          |
| 3     | 1.6196 | 1.6209          |
| 4     | 1.5808 | 1.5812          |
Table 6. Cont.

| $p^*$ | $c$ | $t(t_\eta^0)$ |
|-------|-----|---------------|
|       |     | 0.628 | 0.942 | 1.257 | 1.571 | 2.356 | 3.141 | 3.927 | 4.712 |
| 0     | 0   | 7.6901 | 7.7557 | 7.6433 | 7.7997 | 8.2711 | 7.7972 | 7.9595 | 9.5506 |
|       | 1   | 3.6591 | 3.6809 | 3.7172 | 3.6786 | 3.7366 | 4.0018 | 3.9774 | 4.0107 |
|       | 2   | 2.7793 | 2.7883 | 2.7811 | 2.8188 | 2.8003 | 2.9708 | 3.1263 | 2.8609 |
|       | 3   | 2.388  | 2.3902 | 2.4053 | 2.4083 | 2.3986 | 2.5288 | 2.5228 | 2.7185 |
|       | 4   | 2.159  | 2.1564 | 2.1749 | 2.1787 | 2.1737 | 2.2815 | 2.3691 | 2.3673 |
|       | 5   | 2.0078 | 2.0062 | 2.0109 | 2.0311 | 2.0289 | 2.0248 | 2.1194 | 2.1389 |
|       | 6   | 1.9019 | 1.901  | 1.9096 | 1.9069 | 1.9273 | 1.9290 | 1.9428 | 1.9771 |
|       | 7   | 1.8176 | 1.8229 | 1.8229 | 1.8329 | 1.8517 | 1.8576 | 1.9257 | 2.0223 |
|       | 8   | 1.7549 | 1.7567 | 1.7558 | 1.7598 | 1.7931 | 1.8020 | 1.8118 | 1.9120 |
|       | 9   | 1.7026 | 1.7039 | 1.7023 | 1.7155 | 1.7147 | 1.7574 | 1.8100 | 1.8231 |
|       | 10  | 1.6598 | 1.6607 | 1.6666 | 1.6665 | 1.6793 | 1.7207 | 1.7279 | 1.7495 |

Based on the results presented in Tables 1 and 2, we can see that the values of minimum sample sizes depend on the values of the distribution parameters.

4. Illustrative Examples

In this section, the performance of the suggested ASPs based on percentiles of the NWPD is investigated based on a real data set. The data set represents the breaking stress of carbon fibers (in Gba), which has already been studied by [23]. The observations are: 0.39, 0.81, 0.85, 0.98, 1.08, 1.12, 1.17, 1.18, 1.22, 1.25, 1.36, 1.41, 1.47, 1.57, 1.57, 1.59, 1.59, 1.61, 1.61, 1.69, 1.69, 1.71, 1.73, 1.80, 1.84, 1.84, 1.87, 1.89, 1.92, 2.00, 2.03, 2.03, 2.05, 2.12, 2.17, 2.17, 2.17, 2.35, 2.38, 2.41, 2.43, 2.48, 2.48, 2.50, 2.53, 2.55, 2.56, 2.59, 2.67, 2.73, 2.74, 2.76, 2.77, 2.79, 2.81, 2.81, 2.82, 2.83, 2.85, 2.87, 2.88, 2.93, 2.95, 2.96, 2.97, 2.97, 3.09, 3.11, 3.11, 3.15, 3.15, 3.19, 3.19, 3.22, 3.22, 3.27, 3.28, 3.31, 3.31, 3.33, 3.39, 3.39, 3.51, 3.56, 3.60, 3.65, 3.68, 3.68, 3.70, 3.75, 4.20, 4.38, 4.42, 4.70, 4.90, 4.91, 5.08, 5.56. Table 7 presents the summary statistics of the data.

Table 7. Descriptive statistics of the carbon fibers data.

| n  | Mean | SD  | Median | Kurtosis | Skewness | Min | Max |
|----|------|-----|--------|----------|----------|-----|-----|
| 100| 2.62 | 1.01| 2.7    | 0.04     | 0.36     | 0.39| 5.56|

The distribution parameters were estimated using the maximum likelihood estimation (MLE) method and maximized value of the log likelihood function based on the considered model were obtained. We used the criteria of Bayesian information (BIC), Hannan–Quinn information (HQIC), Akaike information (AIC), and consistent Akaike information (CAIC). The Kolmogorov–Smirnov (KS), and Anderson–Darling (AD) statistics were obtained. The fitting results are presented in Table 8.
Table 8. The BIC, AIC, HQIC, CAIC, AD, W, KS, and \(-2\text{LL}\) for the carbon fibers data.

| BIC     | AIC   | HQIC  | CAIC  | AD    | W     | \(-2\text{LL}\) | KS     | p-Value |
|---------|-------|-------|-------|-------|-------|----------------|--------|---------|
| 296.874 | 289.06 | 292.22 | 289.31 | 0.4158 | 0.0623 | 141.53 | 0.0605 | 0.8578  |

The MLE of the NWPD parameters are \(\hat{\alpha} = 1.0113\), \(\hat{\theta} = 2.9556\), and \(\hat{\eta} = 2.793\). The values of the criteria show that the NWPD fits well the carbon fibers data.

Assume that the researcher intends to emphasize that the true unknown 10th percentile lifetime for the time breaking stress of carbon fibers is at least 1000 h with probability \(p^* = 0.75\), and assume that the life test will be terminated at \(t = 942\) h, leading to the ratio \(\delta = t / \hat{t}_{0.1} = 0.942\). Hence, for the acceptance number \(c = 6\) and confidence level \(p^* = 0.75\), the corresponding sample size in Table 1 is \(n = 100\). Therefore, the ASP for the 10th percentile of NWPD should be \((n, c, t / \hat{t}_{0.1}) = (100, 6, 0.942)\). Based on the breaking stress of carbon fibers data, the researcher must make a decision about whether to reject or accept the lot. If a sample of 100 runoff amounts is selected, the lot is accepted when no more than six failures occur before breaking stress of carbon fibers 0.942. Based on this plan, the breaking stress of carbon fibers can be accepted because there are only three failures before the termination of the time.

The OC function values for the new ASP \((n, c, t / \hat{t}_{0.1}) = (100, 6, 0.942)\) when \(p^* = 0.75\) under the NWPD with \(\eta = 2.793\) and \(\alpha = 1.011\) from Table 2 are:

| \(t_{0.1}/\hat{t}_{0.1}\) | 2    | 4    | 6    | 8    | 10   | 12   |
|--------------------------|------|------|------|------|------|------|
| OC                       | 0.999683 | 1    | 1    | 1    | 1    | 1    |

This implies that if the true 10th percentile is two times the specified percentile life \((t_{0.1}/\hat{t}_{0.1} = 2)\), the producer’s risk is about 0.000317, and the producer’s risk is zero when \(t_{0.1} \geq \hat{t}_{0.1} = 4\).

It can be seen from Table 3, which provides the values of \(d_{0.1}\) for various choices of the acceptance \(c\) and \(t / \hat{t}_{0.1}\), that the producer’s risk should not more than 0.05. Thus, for the ASP \((n, c, t / \hat{t}_{0.1}) = (100, 6, 0.942)\) and \(p^* = 0.75\), the table entry is 1.4141. This means that the product should have a 10th percentile life of at least 1.4141 times the necessary 10th percentile lifetime based on the ASP \((n, c, t / \hat{t}_{0.1}) = (100, 6, 0.942)\) such that the product is accepted with a probability of 0.95 or more.

5. Conclusions

This paper suggests new ASPs for the percentiles of the NWPD based on truncated lifetime tests. Tables of minimum sample sizes, the operating characteristic function values as well as the associated producer’s risks are presented for selected values of the model parameters. An application example of real data is provided for illustration. It can be concluded that the developed ASP can be easily implemented for practitioners. The group acceptance sampling plans based on the NWPD can be considered for future research.

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