The Quark-Gluon Plasma in a Finite Volume

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The statistical mechanics of quarks and gluons are investigated within the context of the canonical ensemble. Recursive techniques are developed which enforce the exact conservation of baryon number, total isospin, electric charge, strangeness, and color. Bose and Fermi-Dirac statistics are also accounted for to all orders. The energy, entropy and particle number densities are shown to be significantly reduced for volumes less than 5 fm$^3$.

I. INTRODUCTION

The study of the quark gluon plasma in the laboratory through hadronic collisions represents one of the largest current initiatives in nuclear physics. Although these volumes are manifestly finite, modeling of the collisions seldom addresses the impact of the finiteness as most treatments are based on the grand canonical ensemble where the conservation constraints of baryon number, strangeness, isospin and color are ignored. Indeed, one expects that these constraints would become meaningless for larger volumes. The effective volume for a $pp$ collision might be only a few cubic fm, while the volume of a RHIC collision might be a few thousand cubic fm. But even in a RHIC collision, the duration of the quark gluon phase might be only a few fm/c, meaning that the effective volume for charge conservation may be a few dozen cubic fm.

Recently, the effects of local charge conservation have gained more attention due to the relationship of charge conservation to charge fluctuations and charge balance functions[1, 2, 3, 4, 5, 6, 7, 8, 9]. These observables are intimately related to the delay of hadronization expected should a quark-gluon plasma be created. Recent papers have focused on the importance of conserving electric charge, strangeness and baryon number, as well as the dynamics of charge equilibration[10]. The statistical mechanics of a finite-volume hadron gas have been studied in the context of pion flavor distributions[11].

The approaches for solving the canonical ensemble can be divided into two classes. In the first set of approaches, one can sum over all partition functions in the grand canonical ensemble assuming the chemical potentials are imaginary, $Z \sim \text{Tr} e^{-\beta H + i\beta \mu Q}$. By integrating over all $\mu$, the phase factor becomes a delta function which restricts the phase space to states with $Q = 0$. This approach can even be applied to non-Abelian symmetries such as SU(N)[12, 13, 14]. By associating $N-1$ charges, $Q_1 \cdot Q_{N-1}$, with the $N-1$-fold Cartan subgroup, one can restrict the ensemble to states with specific charges $Q$, which can then be associated with the SU(3) multiplet labels, $(p, q)$, by a transformation. These techniques work well for simple systems, e.g. massless partons, where the integral over complex phases can be performed analytically. Multiple charges can be readily incorporated, including conservation of overall momentum.

The second class of approaches for the canonical ensemble centers about recursion relations which were proposed by Chase and Mekjian[15, 16] for the study of nuclear fragmentation. These methods have been extended to include quantum statistics[17] and non-additive charges[11], such as angular momentum and isospin. In addition to partition functions, these methods can also generate multiplicity distributions[11, 20]. The advantage of these methods is that, given the one-particle partition functions, they provide exact answers. Since these methods involve sums, rather than integration over phases, they provide robust answers for arbitrary energy levels. Furthermore, symmetrization is included to all orders. If the number of particles to be considered remains below 100, numerical calculations tend to take only seconds or minutes at most.

We review recursive techniques for canonical ensembles and present extensions to incorporate conservation of color in the next section. In the subsequent section a simple example of a non-interacting quark-gluon gas is explored.

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Canonical partition functions are generated which enforce conservation of baryon number, strangeness and charge, while requiring the system to be in an isosinglet and in a color singlet. Bose and Fermi statistics are included to all order. Energy, entropy, and particle number densities are all shown to be significantly suppressed for small volumes, less than 5 fm$^3$. Arguments are made that these effects are unlikely to be important in a relativistic heavy ion collision, but might be quite important for pp collisions.

II. RECURSIVE TECHNIQUES FOR QUANTUM COLORED PARTICLES

For additive charges, canonical partition functions can be calculated in a straight-forward manner. Neglecting symmetrization, conservation of a set of charges $\vec{Q}$ can be enforced through the recursion relation,

$$Z_{A,\vec{Q}}(T) = \sum_k \frac{a_k \omega_k(T)}{A} Z_{A-\vec{Q},\vec{q}_k}(T).$$

(1)

Here, $k$ refers to all particles species of charge $\vec{q}_k$ whose single-particle partition functions are:

$$\omega_k = \sum_i d_i e^{-E_i/T},$$

(2)

where $i$ denotes the individual energy levels of energy $E_i$ and degeneracy $d_i$. The number $a_k$ must be positive for every species. For low energy nuclear physics applications, $a_k$ could be the baryon number and $A$ would represent one of the conserved charges. For a high-energy system where anti-particles must be included and there are no positive-definite conserved charges, $A$ can refer to the number of hadrons. This number is positive-definite, but as the number of hadrons is not conserved, one must consider all $A$ to obtain the partition function, $Z = \sum_A Z_{A,\vec{Q}}$.

Symmetrization can be incorporated by considering all permutations of identical particles \[19, 21\],

$$Z_{A,\vec{Q}} = \frac{1}{A!} \sum_{j_1 \cdots j_A, P(i)} \langle j_1 \cdots j_A | e^{-H/T} | P(j_1 \cdots j_A) \rangle (-1)^{N_P(P)},$$

(3)

where a sum over all $A!$ permutations of the single particle levels accounts for the symmetrization and $N_P(P)$ counts the number of pair-wise interchanges of fermions involved in the permutation $P$.

By considering the cyclic permutation,

$$C^{k,\ell}(T) = \sum_{j_1 \cdots j_{\ell} \cdots j_{\ell+1}} \langle j_1, j_2 \cdots j_\ell | e^{-H/T} | j_2, \cdots j_\ell, j_1 \rangle$$

$$= \sum_i e^{-\ell E_i/T} = \omega_k(T/\ell),$$

(4)

one can derive a modified recursion relation,

$$Z_{A,\vec{Q}}(T) = \sum_k \sum_\ell \frac{a_k C^{k,\ell}(T)}{A} (\pm 1)^{\ell+1} Z_{A-\ell \vec{q}_k,\vec{q}_k}(T).$$

(5)

By summing over all possible values of $\ell$, symmetrization is included to all orders.

Before addressing SU(3) symmetries, we review the techniques used for SU(2) as described in \[11\]. Extending the techniques described above to incorporate an SU(2) symmetry such as isospin or angular momentum can be accomplished in two ways. First, one can consider the projection $M$ as an additive charge and solve for $Z_M(T)$ with the method outlined above. Given the $(2I+1)$ degeneracy of all multiplets of size $I$, the partition function for states with fixed $I$ is easily obtained from the partition function for fixed $M$, $Z_I = Z_M - Z_{M+1}$. The second technique requires one to understand the decomposition of the cycle diagrams into SU(2) multiplets. In terms of the projection operator, $O_M^I$, which projects states with total isospin $I$ and projection $M$, one can define the isospin decomposition of the cycle diagram.

$$C^{i,\ell}_i(T) = \sum_{j_1 \cdots j_\ell} \langle j_1, j_2 \cdots j_\ell | e^{-H/T} O_m^i | j_2, \cdots j_\ell, j_1 \rangle$$

$$= \omega_k(T/\ell) \chi_i^{k,\ell},$$

(6)
where $\chi_{i}^{\kappa,\ell}$ is the operator $C_{\ell,i}^{\kappa}$ evaluated with the particles being combined to one quantum state of zero energy. The subscript $\kappa$ refers to a specific multiplet, not the individual states. For instance, $\kappa$ might refer to pions and $\omega_{\kappa}(T)$ would be the partition function for a given species of pion. The details about the energy levels are absorbed in $\omega_{\kappa}$ while $\chi$ is independent of the energy levels, but does depend on the isospin of the particular species, $i_{\kappa}$.

Once $\chi_{i}^{\kappa,\ell}$ is known, one can calculate the partition function for fixed $I$ recursively.

$$Z_{A,Q,i}(T) = \sum_{\kappa} \sum_{\ell} \frac{a_{\kappa} \omega_{\kappa}(T/\ell)}{A} (\pm 1)^{\ell+1} Z_{A-\ell \kappa,i,Q-\ell Q_{\ell i}}(T) \chi_{i}^{\kappa,\ell} \mathcal{N}(I', i; I).$$

(7)

One could have added indices $M$, $M'$ and $m$ onto $Z_{A,i}$, $Z_{A',i'}$ and $\chi_{i}^{\kappa,\ell}$ respectively. However, as each of these functions involve a trace over all states, and the Hamiltonian has no dependence on the isospin projection, none of these quantities has any dependence on the projection and the indices can be suppressed. The function $\mathcal{N}(I', i; I)$ counts the number of multiplets of size $I$ that one obtains by coupling $I'$ with $i$. For SU(2), these obey a simple form.

$$\mathcal{N}(I', i; I) = \sum_{M', m} (|I', M'| I', M', i, m)^{2}$$

$$= \begin{cases} 1, & |I' - i| \leq I \leq I' + i \\ 0, & \text{otherwise} \end{cases}$$

(8)

Before performing the recursive calculations described in Eq. (7), one needs a simple expression for $\chi_{i}^{\kappa,\ell}$. This can be accomplished by realizing that by summing over $i$ one obtains the projection of states with fixed $m$.

$$\sum_{i} \chi_{i,m}^{\kappa,\ell} = \sum_{j_{1} \cdots j_{\ell} = m} \langle j_{1}, j_{2}, \cdots j_{\ell} | \mathcal{O}_{m} | j_{2}, \cdots j_{\ell}, j_{1} \rangle.$$  

(9)

Since the states $|j_{1} \cdots j_{\ell}\rangle$ are eigenstates of $\mathcal{O}_{m}$, the bra and ket must be identical, i.e., all particles must be identical and have the same isospin projection.

$$\sum_{i} \chi_{i,m}^{\kappa,\ell} = \begin{cases} 1, & m = 0, \pm \ell, \pm 2\ell, \cdots \pm \ell_{\kappa} \\ 0, & \text{otherwise} \end{cases}$$

(10)

Since $\chi_{i,m}^{\kappa,\ell}$ is independent of $m$ when $m$ is a member of the isomultiplet $i$, the sum over $i$ in Eq. (10) can be limited to $i \geq m$. It is then straightforward to see that

$$\chi_{i}^{\kappa,\ell} = \begin{cases} 1, & i = 0, \ell, 2\ell, \cdots \ell_{\kappa} \\ -1, & i = -1, 2\ell - 1, \cdots \ell_{\kappa} - 1 \\ 0, & \text{otherwise} \end{cases}$$

(11)

Here, the index $m$ is suppressed since $\chi$ is independent of $m$ as long as $m$ is a member of the isomultiplet $i$. For the $i_{\kappa} = 1$ case, e.g., pions or $\rho$ mesons, $\chi_{i}^{\ell} = 1$ for $i = \ell$ and $i = 0$, and $\chi_{i}^{\ell} = -1$ for $i = \ell - 1$.

Restricting SU(3) color can be accomplished with the same procedure. Following the same steps as the SU(2) case, one can derive analogous expressions,

$$Z_{A,Q,(P,Q)}(T) = \sum_{\kappa,\ell,(P',Q'),(p,q)} \frac{a_{\kappa} \omega_{\kappa}(T/\ell)}{A} (\pm 1)^{\ell+1} Z_{A-\ell \kappa,Q-\ell Q_{\ell i}}((P', Q'), (p, q)) \chi_{(p, q),\mu}^{\kappa,\ell} \mathcal{N}'((P', Q'), (p, q); (P, Q)).$$

(12)

Here, $(P, Q)$ denotes an SU(3) multiplet, e.g., the gluon color octet is represented by $(P = 1, Q = 1)$. For an explanation for the notation see [22]. Again, the challenge in making Eq. (12) tractable is in finding expressions for $\chi_{(p, q),\mu}^{\kappa,\ell}$ and $\mathcal{N}'((P', Q'), (p, q); (P, Q))$. Coupling SU(3) multiplets, i.e., finding expressions for $\mathcal{N}$, follows rules based on manipulating Young tableaux [22]. For the calculations in this paper, these rules were programmed numerically.

Finding an expression for $\chi_{(p, q),\mu}$ requires performing a color decomposition of the cycle diagrams and can be done analogously as was shown above for SU(2). First, one must project states of a given $\mu$, where $\mu$ represents the eigenvalues of the projection operators in SU(3), e.g., hyper-charge and $I_{3}$ for SU(3) flavor. As with the SU(2) example above, this projection is realized by summing $\chi_{(p, q),\mu}^{\kappa,\ell}$ over all $(p, q)$ multiplets which include the projection $\mu$.

$$\sum_{(p, q),\mu} \chi_{(p, q),\mu}^{\kappa,\ell} = \sum_{j_{1} \cdots j_{\ell}} \langle j_{1}, j_{2}, \cdots j_{\ell} | \mathcal{O}_{\mu} | j_{2}, \cdots j_{\ell}, j_{1} \rangle.$$  

(13)
Again, the $\mu$ dependence in $\chi$ represents the known degeneracy of $\mu$ within a given $(p, q)$ multiplet.

At this point, we proceed by considering gluons as an example, and drop the index $\kappa$. For $\ell$ gluons in a given quantum state the trace of $\langle j_1, j_2 \cdots j_l| \mathcal{O}_\mu | j_2 \cdots j_l, j_1 \rangle$ will be zero unless $\mu$ corresponds to $\ell$ identical gluons. The upper-left panel of Fig. 1 displays the projections $\mu$ that result from this trace applied for five gluons. The upper-right and lower-left panels display the $(\ell, \ell)$ and $(\ell - 2, \ell + 1)$ multiplets. By careful inspection, one can see that the states along the diagonal, as shown in the lower-right panel of Fig. 1, are represented by the combination, adding two color singlets, one can see that

\[
\chi_{(p,q); (\mu, \nu)}^{\text{gluons}, \ell} = \begin{cases} 
1, & (p, q) = (\ell, \ell) \text{ or } (\ell - 3, \ell) \text{ or } (\ell, \ell - 3) \\
-1, & (p, q) = (\ell - 2, \ell + 1) \text{ or } (\ell + 1, \ell - 2) \text{ or } (\ell - 2, \ell - 2) \\
2, & (p = 0, q = 0) \\
0, & \text{otherwise}
\end{cases}
\]

Inserting this expression into Eq. (12) allows one to account for Bose effects in the gluonic partition function.

Quarks are represented by the multiplet $(p = 1, q = 0)$ while anti-quarks are represented by the multiplet $(0, 1)$. Following the same ideas as were illustrated for the gluons, one can show

\[
\chi_{(p,q); (\mu, \nu)}^{\text{quarks}, \ell} = \begin{cases} 
1, & (p, q) = (\ell, 0) \text{ or } (\ell - 3, 0) \\
-1, & (p, q) = (\ell - 2, 1) \\
0, & \text{otherwise}
\end{cases}
\]

By combining the recursion relations for additive charges, which are used to account for baryon and strangeness conservation, with the recursive techniques for SU(2), which are used to enforce conservation of isospin, and the methods for SU(3) which account for color, one can construct a recursive prescription which account for all the charges in a parton gas. One might also choose to write partition functions for subsystems, e.g., the strange quarks or the gluons, then convolute the partition functions together to find the partition function of the combined system. For instance, partitions for the two subsystems $a$ and $b$ can be combined to find the combined partition function,

\[
Z_{(P,Q)}(T) = \sum_{(p_a, q_a), (p_b, q_b)} Z_{(p_a, q_a)}^a(T) Z_{(p_b, q_b)}^b(T) N((p_a, q_a), (p_b, q_b); (P, Q)).
\]

### III. RESULTS AND CONCLUSIONS

The methods described in the last section were applied to the example of a parton gas in a volume $V$ at a temperature, $T = 250$ MeV. The gluons, up-quarks and down-quarks were assumed to be massless, while the strange quark was assumed to have mass of 150 MeV. The single-particle partition functions were calculated by integrating over the momenta,

\[
\omega(T) = \frac{2V}{(2\pi)^2} \int d^3p e^{-\beta \sqrt{p^2 + m^2}} \\
= \frac{m V}{2\beta^2 \pi^2} (\beta m K_0(\beta m) + 2K_1(\beta m)).
\]

The two helicities were accounted for by the factor of two preceding the expression. One could easily account for shell effects by replacing the integral over momentum with a discrete sum. Although discrete states are more consistent given the Bose and Fermi effects described above, the continuous form for $\omega$ is used here so that charge conservation effects can be viewed separately.

The calculations were performed according to the following prescription:

1. The gluon partition function, $Z_{(p,q); (\mu, \nu)}^{\text{gluon}}$ was calculated recursively by first calculating for all numbers of gluons $A$, then summing over $A$.

2. The partition function for strange quarks, $Z_{A; (p,q)}^s$, was generated. The partition function for anti-strange quarks, $Z_{A; (p,q)}^{\bar{s}}$, was then generated by switching $(p, q)$ with $(q, p)$. The partition function of the strange/anti-strange quark system was found by summing over all partition functions with equal numbers of $s$ and $\bar{s}$ quarks.

\[
Z_{(p,q); (\mu, \nu)}^{s}(T) = \sum_{A; (p_a, q_a), (p_b, q_b)} Z_{A; (p_a, q_a)}^s(T) Z_{A; (p_b, q_b)}^{\bar{s}}(T) N((p_a, q_a), (p_b, q_b); (p, q)).
\]
3. The partition function for up and down quarks were calculated separately. In terms of the isospin projection, $m$, and the net number of up/down quarks, $A$, the partition function for the up and down quarks is:

$$
Z^{ud}_{A,m,(p,q)}(T) = \sum_{(p_{a},q_{a})(p_{b},q_{b})} Z^{u}_{m+A/2,(p_{a},q_{a})}(T)Z^{d}_{m+A/2,(p_{b},q_{b})}(T)\mathcal{N}((p_{a},q_{a}),(p_{b},q_{b});(p,q)).
$$

(19)

4. The partition function for $\bar{u}\bar{d}$ quarks is calculated by interchanging $p$ and $q$. In the same manner as the $s\bar{s}$ quarks were convoluted, the $ud$ and $\bar{u}\bar{d}$ partition functions were convoluted to find the partition function for $u$, $d$, $\bar{u}$ and $\bar{d}$ quarks with the constraint of zero baryon number, $Z^{ud\bar{u}\bar{d}}_{M,(p,q)}$, where $M$ is the isospin projection.

5. The partition function for the $ud\bar{u}\bar{d}$ system constrained to an isosinglet is calculated by taking the difference of the $M = 0$ and $M = 1$ partition functions.

6. Using Eq. (10), the partition function for the $uds\bar{u}\bar{d}s$ system were generated. This was then convoluted with the partition function for gluons to find the partition function for the entire system.

Energy densities can be calculated with the well-known formula, $\langle E \rangle = (-\partial/\partial \beta) \ln(Z)$. This requires performing the calculation at two adjacent temperatures, doubling the CPU time of calculating the partition function. Thus, an alternative approach was developed where recursion relations were derived for $(\partial/\partial \beta)Z$. Recursion relations were also generated for the trace of $Ne^{-\beta H}$, where $N$ could be one of several number operators, $N_{\text{gluons}}$, $N_{s}$, $N_{u}$ or $N_{d}$. The recursion relations for these functions were calculated simultaneously with the recursion relations for the partition functions with little penalty in CPU time, because all calculations involve the same calculations of $\mathcal{N}((p,q),(P';Q');(P,Q))$.

Figure 2 shows the energy and entropy densities as a function of the volume. Both the entropy, $S = \ln(Z)+\beta\langle E \rangle$, and the energy are lowered by the reduction in the available states due to the conservation constraints. The grand canonical limit, represented by the dashed lines, is approached at large volumes. As shown in Fig. 6, restricting the matter to a color singlet reduces the partition function by a factor which scales as $V^{-4}$. The other charge conservation constraints result in additional reductions to the quark sectors. A reduction of $V^{-4}$ to the partition functions corresponds to a reduction of $-4\ln(V)$ to the entropy and energy densities. Since the bulk contributions scale as $V$ the logarithmic conservation penalties become irrelevant at large volumes. The chemical decomposition of the various species behave similarly as shown in Fig. 8. The color penalty for gluons is more severe than the color penalties for quarks since gluons effectively have larger color charges and are less likely to couple to a singlet.

From Fig.s 2 and 8, it is clear that conservation rules are important for volumes of 5 fm$^{3}$ or less. It is therefore important to take such considerations into account in pp or pA collisions. Heavy ion collisions at RHIC occupy thousands of cubic fm. However, the local nature of charge conservation results in an effective volume which is determined by the initial conditions, and the diffusion of the various charges. Given that the color of a projectile nucleon is spread over several units of rapidity by the initial stopping process, and given the spread of charge over a few fm in the transverse direction, it would be difficult to make a case that the effective volumes are less than a few dozen cubic fm. It should be emphasized that the determining factor for the importance of conservation constraints is the number of partons in the effective volume. For massless partons, the density of partons is expected to exceed 10 fm$^{-3}$ in a during the first one or two fm/c of a RHIC collision. If the degrees of freedom are restricted by another means, e.g., a large effective mass for partons, the relative penalty for color conservation would increase.

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\[ \sum_{(p,q)} \mathcal{H}^5 \]

\[ (p=5, q=5) \]

\[ \begin{array}{cccccc}
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 2 & 2 & 2 & 1 \\
1 & 2 & 3 & 3 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 4 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 5 & 5 & 4 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 5 & 8 & 5 & 4 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 5 & 4 & 3 & 2 & 1 \\
1 & 2 & 3 & 3 & 3 & 3 & 2 & 1 \\
1 & 2 & 3 & 3 & 3 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 \\
\end{array} \]

\[ (p=3, q=6) \]

\[ \begin{array}{cccccc}
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 2 & 2 & 2 & 1 \\
1 & 2 & 3 & 3 & 3 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 4 & 4 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 4 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 4 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 4 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 4 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 4 & 3 & 2 & 1 \\
\end{array} \]

\[ (5,5)+(4,4)-(3,6)-(6,3) \]

FIG. 1: Upper-left: A diagrammatic representation of the cyclic trace of five gluons in a single quantum level. Upper-right and lower-left: A diagrammatic representation of the (5, 5) and (6, 3) states. The integers refer to the degeneracy of states with a given projection \( \mu \). Lower-right: By combining the four multiplets as shown, one eliminates all states \( \mu \) except those along diagonals defined by the upper-right diagram. By taking the difference of these states with the analogous combination for \( \ell = 4 \), and adding two color singlets, one obtains the desired combination of states in the upper-left diagram.

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FIG. 2: The energy and entropy densities are shown for a parton system at a temperature of 250 MeV as a function of the volume. For small volumes, conservation of baryon number, isospin, strangeness and color restrict the phase space and significantly lower the entropy and energy density. For larger volumes, the results approach the grand canonical limit (dashed lines).

FIG. 3: Densities of gluons (circles), s and \(\bar{s}\) quarks (triangles), and u, d, \(\bar{u}\) and \(\bar{d}\) quarks (squares) are suppressed for small volumes due to charge and color conservation. The grand canonical limits are represented by dashed lines.