Potassium Abundances in Red Giants of Mildly to Very Metal-Poor Globular Clusters*†

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Abstract

A non-LTE analysis of K I resonance lines at 7664.91 and 7698.97 Å was carried out for 15 red giants belonging to three globular clusters of different metallicity (M 4, M 13, and M 15) along with two reference early-K giants (ρ Boo and α Boo), in order to check whether the K abundances are uniform within a cluster, and to investigate the behavior of the [K/Fe] ratio at the relevant metallicity range of −2.5 ≤ [Fe/H] ≤ −1. We confirmed that [K/H] (as well as [Fe/H]) is almost homogeneous within each cluster to a precision of ≤0.1 dex, though dubiously large deviations are exceptionally seen for two peculiar stars showing signs of considerably increased turbulence in the upper atmosphere. The resulting [K/Fe] ratios are mildly supersolar by a few tenths of dex for three clusters, tending to gradually increase from ~0.1–0.2 at [Fe/H] ~ −1 to ~+0.3 at [Fe/H] ~ −2.5. This result connects reasonably well with the [K/Fe] trend of disk stars (−1 ≤ [Fe/H]) and that of extremely metal-poor stars (−4 ≤ [Fe/H] ≤ −2.5).

That is, [K/Fe] appears to continue a gradual increase from [Fe/H] ~ 0 toward a lower metallicity regime down to [Fe/H] ~ −3, where a broad maximum of [K/Fe] ~ +0.3–0.4 is attained, possibly followed by a slight downturn at [Fe/H] ≤ −3.

Key words: stars: abundances (potassium) — stars: atmospheres — stars: globular clusters (M 4, M 13, M 15) — stars: late-type — stars: spectra

1. Introduction

The chemical evolution of potassium (K) in the Galaxy, which can be investigated by examining its photospheric abundances of old metal-poor stars, is still only insufficiently explored and not yet well understood, especially in the metal-poor regime of halo stars. The main reason for this unsatisfactory situation is presumably a difficulty involved with poor regime of halo stars. The main reason for this unsatisfactory situation is presumably a difficulty involved with determining the stellar potassium abundances, for which only unsatisfactory situation is presumably a difficulty involved with poor regime of halo stars. The main reason for this unsatisfactory situation is presumably a difficulty involved with poor regime of halo stars. The main reason for this unsatisfactory situation is presumably a difficulty involved with poor regime of halo stars. The main reason for this unsatisfactory situation is presumably a difficulty involved with poor regime of halo stars.

— Above all, they generally suffer an appreciably large non-LTE effect,1 which depends on the stellar atmospheric parameters as well as the line strength. Actually, since the extent of the (negative) non-LTE correction amounts from a few tenths dex even up to ~1 dex, it is requisite to take account of the non-LTE effect in deriving the stellar K abundances.

— Besides, they occasionally suffer serious blending with strong absorption lines originating from Earth’s atmosphere (especially for the 7665 line) because of being located in an unfavorable wavelength region crowded with such telluric lines. Hence, when encountered with such unfortunate cases, one has to recover the pure stellar spectrum by appropriately eliminating the blended telluric spectrum.

The first non-LTE study of K abundances in metal-poor stars was carried out by Takeda et al. (2002a, hereinafter referred to as Paper I). They showed that [K/Fe] values for the main-sequence stars in the galactic disk (−1 ≤ [Fe/H] ≤ 0) tend to show a rather tight relation of steadily increasing with a decrease of metallicity (such as like the trend of α–group elements). This was actually a reconfirmation of the results of calculations on a grid of models and published tables of non-LTE corrections applicable to F–G–K dwarfs through supergiants. Recently, Zhang et al. (2006a) performed a careful non-LTE investigation of solar potassium lines with a special attention to clarifying the important atomic parameter of neutral hydrogen collision cross section, which they further applied to determinations of K abundances for metal-poor stars (Zhang & Zhao 2005; Zhang et al. 2006b).

* Based on data collected at Subaru Telescope, which is operated by the National Astronomical Observatory of Japan.
† The electronic table E1 is available at the PASJ web site of (http://pasj.asj.or.jp/v61/n3/610321/).
1 While non-LTE line formation of K I 7665/7699 resonance lines specific to the solar atmosphere had begun already in 1980s (see Bruls et al. 1992 and the references therein), non-LTE studies of these K I doublet lines directed to stars other than the Sun gradually appeared in the last decade: Takeda et al. (1996) carried out a detailed study on the formation of this line in Procyon (along with the Sun). Ivanova and Shchamsik (2000) calculated non-LTE abundance corrections applicable to A–K stars of wide parameter ranges. Similarly, Takeda et al. (2002a) carried out extensive non-LTE calculations on a grid of models and published tables of non-LTE corrections applicable to F–G–K dwarfs through supergiants. Recently, Zhang et al. (2006a) performed a careful non-LTE investigation of solar potassium lines with a special attention to clarifying the important atomic parameter of neutral hydrogen collision cross section, which they further applied to determinations of K abundances for metal-poor stars (Zhang & Zhao 2005; Zhang et al. 2006b).
Chen et al.'s (2000) LTE analysis, despite significant non-LTE corrections of \( \sim -0.5 \) dex, because the corrections turned out to act similarly on the Sun and these disk dwarfs (i.e., \([\text{K}/\text{H}]\), the differential stellar potassium abundance relative to the Sun, was eventually not much affected). Such an \( \alpha \)-like trend of \([\text{K}/\text{Fe}]\) for disk stars was also confirmed by Zhang et al. (2006b).

Unfortunately, the situation becomes quite uncertain when we go into the much lower metallicity regime (\([\text{Fe}/\text{H}] \lesssim -1\]). It was already noticed in Paper I from a non-LTE reanalysis of the published data for several halo stars of \([\text{Fe}/\text{H}] \sim -2\) (Gratton & Sneden 1987a) that their \([\text{K}/\text{Fe}]\) values widely spread from \( \sim +0.1 \) to \( \sim +0.7 \) (cf. figure 4a of Paper I; see also figure 8 of Gratton & Sneden 1987b). Zhang and Zhao (2005) also suggested a considerably large diversity of \([\text{K}/\text{Fe}]\) (from \( \sim 0 \) to \( \sim 1 \)) for the halo stars of \( -2 \lesssim [\text{Fe}/\text{H}] \lesssim -1 \) that they studied (cf. figure 12 therein). On the other hand, however, five halo stars included in Zhang et al.'s (2006b) study have nearly the same \([\text{K}/\text{Fe}]\) ratios of around \( \sim +0.2 \) over the range of \( -2 \lesssim [\text{Fe}/\text{H}] \lesssim -1 \) (cf. figure 5 therein). Besides, Cayrel et al. (2004) obtained in their extensive study of extremely metal-poor stars (\( -4 \lesssim [\text{Fe}/\text{H}] \lesssim -2.5 \)) that \([\text{K}/\text{Fe}]\) shows a slightly supersolar trend (\( \sim +0.1-0.2 \)) of decreasing toward a very low metallicity with a fairly small scatter (cf. figure 9 therein). Thus, at present, we know little about how the \([\text{K}/\text{Fe}]\) ratio actually behaves itself at \([\text{Fe}/\text{H}] \leq 1\). Tight trend? Large diversity? Does it smoothly connect to the \( \alpha \)-like clear tendency seen in disk stars at \([\text{Fe}/\text{H}] \gtrsim -1\)?

For the purpose of checking the possibility of K abundance diversity in such metal-deficient halo stars, it would be interesting to study stars in globular clusters, for which any trial of potassium abundance determination has never been reported so far to our knowledge. Admittedly, the elemental abundances of old globular cluster stars may not necessarily be the same as those of field halo stars, given that specific abundance peculiarities are known for several elements (e.g., O, Na, Mg, Al) which are presumably due to dredge-up of nuclear-processed product caused by evolution-induced deep mixing. However, since K is considered to be synthesized mainly via oxygen burning in high-mass stars, whichever the process is explosive or hydrostatic (see, e.g., table 19 in Woosley & Weaver 1995), it is unlikely that the surface K abundance undergoes any appreciable changes during the course of stellar evolution in low-mass stars of globular clusters. Besides, the fact that no star-to-star variation in globular clusters is observed in the abundances of Ca (\( Z = 20 \), near to \( Z = 19 \) of K) may also suggest the inertness of K to nuclear processes in the stellar interior, presumably because of the higher Coulomb barrier compared to lighter elements. Therefore, as a reasonable working hypothesis, we may postulate (such as in the case for Fe in most clusters) that (i) the abundance of K was the same in any stars of a given globular cluster when they were born, that (ii) this uniformity has been retained up to now, and that (iii) the surface K abundances of cluster stars may be regarded to be equivalent to field halo stars of similar metallicity. Accordingly, if we could observationally confirm postulation (ii), this would assure the practical validity of (iii), which means that we may directly compare the results of cluster stars with those of other halo stars in general. Or, alternatively, if we found a markedly large diversity of K abundances within a cluster contrary to postulation (ii), we would have to consider a possibility of a real star-to-star variation (or cast doubt on the validity of our abundance determination method).

Motivated by this consideration, we decided to conduct a spectroscopic study on the intrinsically bright red-giant stars of three globular clusters (M 4, M 13, and M 15) covering a wide metallicity span (from \([\text{Fe}/\text{H}] \sim -2.5 \) to \(-1\)) based on high-dispersion spectra obtained with Subaru/HDS, in order to determine the abundance of K for each star while taking into account the non-LTE effect, after having established the atmospheric parameters including \([\text{Fe}/\text{H}]\). What we intend to clarify is, as described above, to check the homogeneity of \([\text{K}/\text{H}]\) within a cluster [postulation (ii)], and to study the behavior of \([\text{K}/\text{Fe}]\) in halo stars based on the results of these globular clusters [postulation (iii)] while comparing with those of other metallicity regime. This is the purpose of this study.

Besides, in connection with the main subject, we carried out a reanalysis of the equivalent-width data of K I \( \lambda 7665 \) and 7669 lines for \( \sim 30 \) extremely metal-poor stars published by Cayrel et al. (2004), in order to establish the \([\text{K}/\text{Fe}]\) vs. \([\text{Fe}/\text{H}]\) relation at the metallicity range of \(-4 \lesssim [\text{Fe}/\text{H}] \lesssim -2.5 \). The main motivation is to properly take into account the non-LTE effect, since their treatment in this respect does not appear to be sufficiently valid (i.e., they applied a tentatively assumed correction uniformly to all stars). This reanalysis is separately described in the Appendix.

### 2. Observational Data

Three globular clusters (M 4, M 13, and M 15) were chosen for this study, because (1) they have different metallicities from each other ([Fe/H]_{M15} \(< [\text{Fe}/\text{H}]_{M13} \leq [\text{Fe}/\text{H}]_{M14}\)), (2) they are located at near or reasonable distances from us (so that red giants of \( V \lesssim 12 \) mag exist), and (3) they are comparatively well studied, and thus a number of literature data may be found. Practically, we selected 5 red-giant stars for each cluster satisfying the criterion \( 4100 \text{ K} \lesssim T_\text{eff} \leq 4300 \text{ K} \) (\( T_\text{eff}^\text{pho} \)) is the effective temperature photometrically evaluated from (\( B - V \)) colors; cf. subsection 5.1): L 2406, L 2617, L 3209, L 3624, and L 4511 for M 4; I-13, II-76, III-52, III-59, and III-73 for M 13; K 144, K 341, K 431, K 634, and K 825 for M 15. These targets belong to the brightest-class group in each cluster (\( V \sim 11-12 \) mag for M 4, \( V \sim 12-13 \) mag for M 15). In addition, two early-K giant stars of near-solar or subsolar metallicity (\( \rho \) Boo and \( \alpha \) Boo) were also included as standard stars, though they are normal K giants of luminosity class III with log (\( L/L_\odot \)) \( \sim 2 \), while all 15 cluster stars are “tip giants” located on the tip of the giant branch at log (\( L/L_\odot \)) \( \sim 3 \).

The observations of these 17 stars (along with a rapid rotator Altair as a reference of telluric lines) were carried out on the night of 2008 August 20 (Hawaii Standard Time) by using the High Dispersion Spectrograph (HDS; Noguchi et al. 2002) placed at the Nasmusy platform of the 8.2-m Subaru Telescope, which can record high-dispersion spectra covering a wavelength portion of \( \sim 2600 \) Å (in the red cross disperser mode) with two CCDs of 2 K \( \times 4 \) K pixels at a time. With the slit width set at 0.6 (300 μm) and a binning of 2 \( \times 2 \) pixels, the resolving power of the obtained spectra is \( R \sim 60000 \). In the
standard “Ra” setting, our spectra cover the wavelength region of 5100–6400 Å (blue CCD) and 6500–7800 Å (red CCD), which was so chosen so as to make use of the yellow–red region of λ ≥ 5000 Å (where the sensitivity of CCD is large and many Fe lines usable for parameter determinations exist) while including the targeted K I 7665/7699 lines amply. The seeing size was 0.5–0.6′, and all of the targets could be successfully observed without any significant influence of neighboring stars. The integrated exposure time for each cluster star was from 10 min (≈ 5 min × 2; for M 4 stars) to 15–20 min (≈ 7.5–10 min × 2; for M 13 or M 15 stars).

The reduction of the spectra (bias subtraction, flat-fielding, scattered-light subtraction, spectrum extraction, wavelength calibration, continuum normalization) was performed by using the echelle package of the software IRAF\(^2\) in a standard manner. The resulting average S/N ratios were around ~100–150 for each of the 15 cluster stars, while much higher values were attained for the bright reference stars: ~300–400 (ρ Boo), ~200–300 (α Boo), and ~600–700 (Altair).

### 3. Atmospheric Parameters

The determination of the atmospheric parameters \(T_{\text{eff}}\) (effective temperature), \(\log g\) (surface gravity), \(v_t\) (microturbulence), and \([\text{Fe/H}] = A^{\odot}_{\text{Fe}} - A^{\odot}_{\odot}\); differential Fe abundance relative to the Sun, where \(A^{\odot}_{\text{Fe}}\) is 7.50 in the usual normalization of \(A_{\odot} = 12.00\) necessary for constructing model atmospheres was implemented by way of a spectroscopic approach using the equivalent widths (\(EW\)’s) of Fe I and Fe II lines, which has a merit of establishing these four parameters based only on the same spectrum to be further used for abundance determinations.

Practically, we used a computer program (named TGVIT) developed for this purpose (Takeda et al. 2005; cf. section 2 therein), which is based on the principle of searching for the most optimum solution in 3-dimensional \(T_{\text{eff}}, \log g, v_t\) space, such that simultaneously satisfying the three requirements: (1) \(x_{\text{low}}\)-dependence of Fe I abundances (excitation equilibrium, where \(x_{\text{low}}\) is the lower excitation potential), (2) equality of the mean abundance derived from Fe I lines and that from Fe II lines (ionization equilibrium), and (3) \(EW\)-dependence of the abundances (curve-of-growth matching), as described in Takeda, Ohkubo, and Sadakane (2002b). Since low-gravity K-type giants are specifically involved in this study, we newly computed a grid of data files [covering the parameter ranges of 3750–5000 K in \(T_{\text{eff}}\) (K), 0.0–2.5 in \(\log g\) (cm s\(^{-2}\)), 0.0–4.0 in \(v_t\) (km s\(^{-1}\)), and ~2.5 to +0.1 in \([\text{Fe/H}]\) (dex)] to be used in application of TGVIT by interpolation (or extrapolation).

First, we measured the \(EW\)’s of available lines by consulting a list of 330 Fe lines (cf. electronic table E1 in Takeda et al. 2005) by using the Gaussian fitting method, where only lines weaker than 100 mÅ were used in order to make sure that errors caused by damping wings/parameters are suppressed to a negligible level. Then, given these \(EW\) data as inputs, the

| Star | \(T_{\text{eff}}\) (K) | \(\log g\) (cm s\(^{-2}\)) | \(v_t\) (km s\(^{-1}\)) | [Fe/H] (dex) | \(EW_{7665}\) (mÅ) | \([K/H]_{7665}\) (dex) | \(\Delta_{\text{NLTE}}\) (dex) | \(EW_{7699}\) (mÅ) | \([K/H]_{7699}\) (dex) | \(\Delta_{\text{NLTE}}\) (dex) | [K/Fe] (dex) |
|------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| ρ Boo | 4363 | 2.09 | 1.29 | +0.05 | 287.7 | −0.19 | −0.08 | 238.6 | −0.25 | −0.12 | 0.27 |
| α Boo | 4281 | 1.72 | 1.49 | −0.55 | 284.5 | −0.45 | −0.12 | 235.9 | −0.55 | −0.19 | 0.19 |
| M 4 L2406 | 4048 | 0.61 | 2.16 | −1.22 | 364.3 | −0.39 | −0.09 | 319.5 | −0.42 | −0.12 | 0.08 |
| M 4 L2617 | 4256 | 1.48 | 1.38 | −1.12 | 223.1 | −0.99 | −0.21 | 197.9 | −0.98 | −0.26 | 0.13 |
| M 4 L3209 | 4025 | 1.13 | 1.49 | −1.12 | 255.4 | −0.98 | −0.13 | 222.5 | −1.01 | −0.18 | 0.12 |
| M 4 L3624 | 4269 | 1.47 | 1.42 | −1.12 | 216.8 | −1.07 | −0.24 | 189.5 | −1.10 | −0.30 | 0.04 |
| M 4 L4511 | 4173 | 1.20 | 1.44 | −1.13 | 230.8 | −0.99 | −0.18 | 202.9 | −1.02 | −0.23 | 0.12 |
| M 13 I-13 | 4155 | 0.73 | 1.75 | −1.58 | 215.4 | −1.36 | −0.26 | … | … | … | 0.22 |
| M 13 I-76 | 4159 | 0.27 | 1.87 | −1.77 | 186.8 | −1.70 | −0.25 | … | … | … | 0.07 |
| M 13 I-52 | 4271 | 0.80 | 1.55 | −1.58 | 168.1 | −1.67 | −0.36 | … | … | … | 0.09 |
| M 13 I-59 | 4206 | 0.54 | 1.42 | −1.58 | 190.7 | −1.31 | −0.26 | … | … | … | 0.27 |
| M 13 I-73 | 4145 | 0.38 | 1.93 | −1.73 | 252.8 | −1.03 | −0.23 | … | … | … | 0.70 |
| M 15 K144 | 4066 | −0.45 | 1.46 | −2.51 | 108.2 | −2.28 | −0.17 | 90.9 | −2.24 | −0.18 | 0.25 |
| M 15 K341 | 4085 | −0.15 | 2.10 | −2.51 | 131.2 | −2.26 | −0.20 | 102.0 | −2.28 | −0.19 | 0.24 |
| M 15 K431 | 4152 | −0.01 | 1.49 | −2.47 | 117.8 | −2.12 | −0.28 | 89.4 | −2.21 | −0.25 | 0.31 |
| M 15 K634 | 4161 | 0.23 | 1.78 | −2.36 | 126.4 | −2.19 | −0.27 | 111.8 | −2.09 | −0.27 | 0.22 |
| M 15 K825 | 4012 | −0.22 | 1.78 | −2.55 | 112.9 | −2.44 | −0.13 | 101.3 | −2.31 | −0.15 | 0.17 |

\(^*\) Columns 2–5 give the atmospheric parameters (the effective temperature, the surface gravity, the microturbulence, and the Fe abundance relative to the Sun) which were spectroscopically determined by using Fe I and Fe II lines and adopted in this study. The line equivalent width, the non-LTE potassium abundance relative to the Sun, and the relevant non-LTE correction are presented in columns 6–8 (K I 7665 line) and 9–11 (K I 7699 line). In the last column 12 is given the K-to-Fe logarithmic abundance ratio, \([K/Fe] = [K/H] - [Fe/H]\), where \([K/H]\) is the average of \([K/H]_{7665}\) and \([K/H]_{7699}\) in case both lines are available.

\(^{\dagger}\) Corrected values for the imperfect removal of telluric lines; \(\Delta EW_{\text{res}}\) corrections of 38.8 mÅ (ρ Boo), 30.9 mÅ (I-13), 37.9 mÅ (II-76), 47.9 mÅ (III-52), 72.9 mÅ (III-59), and 22.8 mÅ (III-73) have been subtracted from the directly measured \(EW\) (cf. footnote 3 in subsection 4.1).
converged solutions of $T_{\text{eff}}$, log $g$, and $v_1$ (along with $A_{\text{Fe}}$ as a product) were established by iteratively running TGVIT (cf. subsection 3.2 in Takeda et al. 2005). The resulting parameter solutions are summarized in table 1, while the detailed $EW$ data and the Fe abundances corresponding to the final parameters for each star are presented in electronic table E1. The trend concerning the Fe abundances corresponding to the final solutions is plotted against $EW$ as well as $\chi_{\text{low}}$ in figure 1, where we can see that the required conditions are reasonably accomplished.

The internal statistical errors involved with these solutions of $T_{\text{eff}}$, log $g$, $v_1$, and [Fe/H], which were derived by the procedure described in subsection 5.2 of Takeda, Okubko, and Sadakane (2002b), turned out to be $\sim 10-30$ K, $\sim 0.05-0.1$ dex, $\sim 0.1-0.4$ km s$^{-1}$, and $\sim 0.03-0.1$ dex, respectively. We will discuss in subsection 5.1 these spectroscopic parameters in comparison with those derived from the conventional method (i.e., use of photometric colors or evolutionary tracks).

4. Analysis of Potassium Lines

4.1. EW Measurement

Among our 17 target stars, 5 stars of M 13 were found to be associated with the most unfortunate case. The K I 7699 line could not be measured at all because it happened to fall just on the narrow inter-order gap of HDS (at 7687–7694 Å) due to M 13’s large (negative) radial velocity of $\sim -220$ to $-230$ km s$^{-1}$, while the K I 7665 line was seriously blended with the strong telluric O$_2$ line at 7659.3 Å. Such a contamination in the K I 7665 region is also seen for the case of two standard stars, $\alpha$ Boo and (especially) $\rho$ Boo, which turned out to be influenced by the telluric O$_2$ doublet at $\sim 7664$ 7665 Å. We, therefore, tried to eliminate the effect of telluric lines for these 7 stars by dividing the raw spectra by the spectrum of Altair (rapid rotator) by using the IRAF task telluretic. The resulting as well as the original spectra are shown in figure 2, where we can see that this elimination surely worked (though not perfectly). Figure 3 displays the spectra used for measurements of $EW_{7665}$ and $EW_{7669}$, which were measured by the Gaussian fitting method as for the case of Fe lines. The finally adopted values of $EW_{7665}$ and $EW_{7669}$ are summarized in table 1.

4.2. Abundance Determination

The determination of potassium abundances from the measured $EW_{7665}$ and $EW_{7669}$ by taking into account the non-LTE effect was carried out in almost the same manner as in Paper I. As the basic grid of model atmospheres, we used Kurucz’s (1993) ATLAS9 models (corresponding to $v_t = 2$ km s$^{-1}$), which were three-dimensionally interpolated (or extrapolated in special cases of negative log $g$) with respect to $T_{\text{eff}}$, log $g$, and [Fe/H] to generate an atmospheric model for each star.

The non-LTE statistical-equilibrium calculations were implemented for a grid of 90 models resulting from combinations of three $T_{\text{eff}}$ values (4000, 4250, 4500 K), five log $g$ values (0.0, 0.5, 1.0, 1.5, 2.0), and six [Fe/H] values ($-2.5$, $-2.0$, $-1.5$, $-1.0$, $-0.5$, 0.0), so that we could obtain depth-dependent non-LTE departure coefficients for any star by interpolating (or extrapolating) this grid. See Takeda et al. (1996) for the computational details. We suppressed the effect of neutral-hydrogen collisions to a negligible level by multiplying the classical value based on Drawin’s cross section (cf. Steenbock & Holweger 1984) by a factor of $10^{-4}$ according to the conclusion of that paper.

The derivation of the K abundance from a given $EW$ was done as in Paper I with Kurucz’s (1993) WIDTH9 program,
which had been considerably modified in various respects (especially to include the effect of departure from LTE). According to Takeda et al. (1996), the van der Waals damping parameter \((C_6)\) was increased by applying a correction of 
\[ \Delta \log C_6 = +1.00 \]
to the conventional Unsöld’s (1955) formula value (corresponding to using \( \Gamma_{\text{rad}} = 2.5 \sigma_{\text{Unsöld}} \)). We adopted log \( f / \sigma \) values of +0.13 (7665 line) and −0.17 (7699 line), and a radiation damping constant of \( \Gamma_{\text{rad}} = 0.38 \times 10^8 \, \text{s}^{-1} \), which were taken from Kurucz and Bell’s (1995) compilation.

Actually, we prepared two sets of non-LTE departure coefficients (grid of 90 models) corresponding to two different choices of the input K abundances \([\text{K}/\text{Fe}] = 0.0\) and \([\text{K}/\text{Fe}] = +0.5\), considering that the resulting \([\text{K}/\text{Fe}]\) is likely to be encompassed by these two values. Also, two kinds of non-LTE abundances \(A^K_0\) and \(A^K_5\) were obtained for a given \(E\) for each of the two sets. Then, these \(A^K_0\) and \(A^K_5\) were interpolated (or extrapolated) so that the final non-LTE solution \(A^K\) and the used departure coefficient would become consistent with each other (cf. subsection 4.2 in Takeda & Takada-Hidai 1994). In addition, we also derived the LTE abundance, \(A^K_{\text{LTE}}\), from which the non-LTE correction was derived as 
\[ \Delta A^K_{\text{NLTE}} = A^K - A^K_{\text{LTE}}. \]
Finally, the differential K abundance relative to the Sun was computed as \([\text{K}/\text{H}] = A^K - 5.12\), where Anders and Grevesse’s (1989) \(A^K\) value of 5.12 was adopted as the solar potassium abundance (which is also consistent with the result of Takeda et al. 1996). The resulting values of \([\text{K}/\text{H}]\) and \(\Delta A^K_{\text{NLTE}}\) for each line along with the \([\text{K}/\text{Fe}]\) ratio \((+[\text{K}/\text{H}] - [\text{Fe/H}] ; [\text{K}/\text{H}]\) is the average of two lines) are summarized in table 1. Regarding the typical errors in the atmospheric parameters or the damping constant, table 1 in Paper I (especially for the cases of metal-poor K giants such as HD 122563, HD 165195, and HD 221170) may be informative.

5. Discussion

5.1. Verifying Model Atmosphere Parameters

It may be worth comparing our spectroscopically determined atmospheric parameters with those by another method occasionally used for globular cluster stars; i.e., the photometric \(T_{\text{eff}}^\text{pho}\) and evolutionary \(g_\odot^\text{evol}\), where \(T_{\text{eff}}\) is derived from colors and \(g\) is evaluated from \(L\) (the stellar luminosity estimated from the apparent magnitude and the distance), \(T_{\text{eff}}\), and \(M\) (the stellar mass often assumed to be 0.8 \(M_\odot\) for globular cluster giants) by the relation of 
\[ g / g_\odot = (M / M_\odot) (L / L_\odot)^{-1} (T_{\text{eff}} / T_{\text{eff,\odot}})^4. \]

The basic data of \(V\) magnitude and \(B - V\) color were taken from Ivans et al. (1999: M 4), Smith and Briley (2006: M 13), and Sneden et al. (1997: M 15); the color excess \(E_{B-V}\) and the distance \(d\) (assumed to be the same for all targets in a cluster) were taken from an on-line database4 elaborated by the Padova group. For \(\rho\) Boo and \(\alpha\) Boo, \(E_{B-V}\) was assumed to be zero, and the other data were taken from the SIMBAD database. We first derived \(T_{\text{eff}}\) from the reddened \((B - V)_0\) color by using Alonso, Arribas, and Martínez-Roger’s (1999) equation (4) in their table 2, where \(0.0\) (\(\rho\) Boo), −0.6 (\(\alpha\) Boo), −1.2 (M 4 stars), −1.7 (M 13 stars), and −2.5 (M 15 stars) were assumed for \([\text{Fe}/\text{H}]\). Then, since the stellar luminosity, \(L\), can be obtained from the extinction-corrected \(V_0\) magnitude \((V - 3.1 \, E_{B-V})\), the distance \(d\), and the bolometric correction evaluated by Alonso, Arribas, and Martínez-Roger’s (1999) equations (17) and (18), we can derive \(g_\odot^\text{evol}\) from the above-mentioned relation by assuming \(M = 0.8 \, M_\odot\). The resulting \(T_{\text{eff}}^\text{pho}\) and \(g_\odot^\text{evol}\) (along with the corresponding \([\text{Fe}/\text{H}]\) values when these parameters were used; cf. table 3) are summarized in table 2, where the published values taken from various

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4 (http://dipastro.pd.astro.it/globulars/).
Given the existence of such an appreciable discrepancy, we tend to wonder whether our use of spectroscopic parameters is really justified. Which sets ($T_{\text{spe}}^\text{eff}$ / $\log g^\text{spe}$ vs. $T_{\text{eff}}^\text{spe}$ / $\log g^\text{spe}$) should be preferably used for abundance determinations? Before going into this, we point out that our targets of intrinsically bright red giants are on or near to the AGB tip where stars (in their late stage of evolution) are likely to have extended envelopes and show significant mass loss as well as time variability. Actually, as demonstrated in figure 4, not a few stars show prominent emissions and blue-shifted cores in H\alpha (especially, emissions are seen in all five M 15 stars), indicating the existence of such active phenomena.

In such cases, we would state that the spectroscopic method is more advantageous:

1) If stars are unstable and variable in time, parameters based on observational data (colors, magnitudes, etc.) simply taken from catalogues are not reliable any more. Meanwhile, spectroscopic parameters are established from the spectrum, itself, from which abundances are derived.

2) We must recall that most stellar model atmospheres widely used are based on the assumption of hydrostatic equilibrium and the plane-parallel approximation. In case where these assumptions failed, it would not be sensible any more to assign parameters derived from the true fundamental stellar quantities (e.g., $\log g^\text{evo}$) from realistic $L$, $M$, ..., $\ldots$). On the other hand, it is still possible to apply conventional model atmospheres, if we could choose their parameters so carefully as to reproduce the physical condition of the real atmosphere (while regarding that $T_{\text{eff}}$ and $\log g$ are not so much real physical quantities as rather adjustable parameters). Our $T_{\text{eff}}^\text{spe}$ and $\log g^\text{spe}$ should thus be interpreted in this sense.

Consequently, according to our opinion, the appreciably low $T_{\text{eff}}^\text{spe}$ and low $\log g^\text{spe}$ in M 13 and M 15 are nothing but a manifestation of the existence of an extended cooler region of lower density affecting the formation of spectral lines. As long as we invoke classical plane-parallel model atmospheres, such low parameter values must have been assigned to reproduce the relevant atmospheric condition. Since this kind of discordance is pronounced in M 13 ([Fe/H] $\sim$ –1.7) and especially in M 15 ([Fe/H] $\sim$ –2.5), while not in M 4 ([Fe/H] $\sim$ –1.1), this effect is considered to be metallicity-dependent, which may presumably be related to the observational fact pointed out by Meszaros, Duppee, and Szentgyorgyi (2008) that the outflow velocities of red giants are higher in metal-poor clusters (M 15) than in metal-rich clusters (M 4). To sum up, we consider that the use of $T_{\text{eff}}^\text{spe}$ and $\log g^\text{spe}$ is surely reasonable and preferable to other choices.5

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5 As an alternative interpretation, it may be possible to consider that the large discrepancy in $\log g$ (systematically low $\log g^\text{spe}$) seen in very metal-poor red giants is due to the non-LTE overionization effect (Fe I lines are weakened compared to the case of LTE, while Fe II lines are practically unaffected), which would naturally yield an underestimation of $\log g^\text{spe}$ if LTE is assumed. We can not exclude this possibility, as Rulant et al. (1980) once reported a sign of non-LTE ionization in low-excitation Fe I lines (while high-excitation lines are comparatively inert) in their analysis of the early-K giant Pollux ($T_{\text{eff}}$ $\sim$ 4800 K, $\log g$ $\sim$ 2.2, and the near-solar metallicity). On the other hand, theoretical investigations on the non-LTE effect in the formation of Fe lines in red giants are difficult, because the results are sensitively influenced by uncertainties in computational details (e.g., the treatment of UV photoionizing radiation field, collisional cross-...
5.2 Homogeneity of Fe and K Abundances

The results of [Fe/H], [K/H]7665, and [K/H]7699 (cf. table 1) for each star are graphically depicted in figure 5, from which we can read the following characteristics:

— The [Fe/H] values are considered to be nearly uniform in all three clusters (M 4, M 13, and M 15) to within a precision of ~0.05–0.1 dex. Actually, the mean ([Fe/H]) (and the standard deviation ±σ) averaged over 5 stars are −1.14 (±0.04), −1.65 (±0.09), and −2.48 (±0.07) for M 4, M 13, and M 15, respectively. Therefore, we can state that any of our M 4/M 13/M 15 targets have essentially the same metallicity within the cluster.

— Regarding [K/H], for which values derived from K I 7655 and 7699 lines are consistent with each other, we notice that two stars (M 4/L2406 and M 13/III-73) are evidently anomalous, because they show considerably discrepancy (i.e., larger [K/H] compared to other members. However, when these two stars are excluded, we can confirm a reasonable uniformity of [K/H] within a cluster; i.e., ([K/H]7665)/([K/H]7699) ±σ7665/±σ7699 is −1.01/−1.03 (±0.04/±0.04) for M 4 (4 stars), −1.51/−1.52 (±0.20/—) for M 13 (4 stars), and −2.26/−2.23 (±0.12/±0.09) for M 15 (5 stars). The larger dispersion (0.20) for the case of M 13 compared to two other clusters (~0.05–0.1) may be understood as being due to the availability of only one K I 7665 line, which is severely contaminated by telluric lines (cf. figure 2).

How should the discrepant nature of these two outliers be interpreted? Actually, we have a good reason to believe that this is nothing but a superficial effect caused by an inadequate treatment in the abundance determination (i.e., not the real abundance anomaly). We point out that the spectra of these M 4/L2406 and M 13/III-73 have markedly broad line widths (figure 3) and prominently strong Hα emission components (figure 4) compared to the remaining stars, which suggests a significant increase in the activity or turbulent velocity fields in the upper atmosphere (where the core of K I lines form) presumably related to dynamical phenomena often seen in AGB stars. We thus suspect the existence of considerably depth-dependent turbulent velocity dispersion that increases with height, which may have contributed an

7 More generally speaking, such an increasing tendency of the turbulent velocity field with height is a phenomenon occasionally seen in low-gravity stars (though its degree is different from case to case); see, e.g., Takeda (1992) for α Boo (K giant) or Takeda and Takada-Hidai (1994; appendix B) for A–F supergiants.

For reference, a simple averaging of the literature data given in table 2 (excluding our results) gives the mean cluster metallicities of ~−1.2 (M 4), ~−1.5 (M 13), and ~−2.3 (M 15).
Table 2. Comparison of the atmospheric parameters and metallicity with the literature values.*

| Star      | $T_{\text{eff}}$ (K) | $\log g$ (cm s$^{-2}$) | $v_t$ (km s$^{-1}$) | [Fe/H] | References† |
|-----------|----------------------|-------------------------|---------------------|--------|-------------|
| $\rho$ Boo | 4363                | 2.09                    | 1.29                | +0.05  | This study (adopted, spectroscopic parameters) |
|           | 4252                | 1.75                    | ...                 | 0.00   | This study ($T_{\text{eff}}^{\text{pho}}$ and $g_{\text{evol}}^{\text{pho}}$ for reference) |
|           | 4260                | 2.22                    | 2.10                | -0.17  | MCW90       |
| $\alpha$ Boo | 4281               | 1.72                    | 1.49                | -0.55  | This study (adopted, spectroscopic parameters) |
|           | 4303                | 1.56                    | ...                 | -0.65  | This study ($T_{\text{eff}}^{\text{pho}}$ and $g_{\text{evol}}^{\text{pho}}$ for reference) |
|           | 4490                | 2.01                    | 1.80                | -0.56  | LAM81       |
|           | 4425                | 1.06                    | 2.50                | -0.48  | GRA82       |
|           | 4350                | 1.60                    | 1.70                | -0.58  | KIR85       |
|           | 4330                | 1.50                    | 1.50                | -0.38  | GRA86       |
|           | 4400                | 1.70                    | 2.30                | -0.55  | KIR86       |
|           | 4250                | 1.70                    | 2.40                | -0.60  | LEE87       |
|           | 4375                | 1.97                    | 1.80                | -0.42  | EDV88       |
|           | 4300                | 2.00                    | 1.50                | -0.69  | FER90       |
|           | 4280                | 2.19                    | 2.30                | -0.60  | MCW90       |
|           | 4330                | 2.10                    | 1.60                | -0.58  | BRO92       |
|           | 4280                | 1.30                    | 1.40                | -0.54  | MCW94       |
|           | 4300                | 1.50                    | 1.70                | -0.47  | SNE94       |
|           | 4300                | 1.50                    | 1.70                | -0.51  | HIL97       |
|           | 4250                | 1.30                    | 1.70                | -0.68  | GON98       |
|           | 4345                | 2.05                    | 1.50                | -0.37  | THE99       |
|           | 4300                | 1.50                    | 1.70                | -0.63  | TOM99       |
|           | 4300                | 1.50                    | 1.72                | -0.49  | CAR00       |
| M 4 L2406 | 4048                | 0.61                    | 2.16                | -1.22  | This study (adopted, spectroscopic parameters) |
|           | 4125                | 0.76                    | ...                 | -1.23  | This study ($T_{\text{eff}}^{\text{pho}}$ and $g_{\text{evol}}^{\text{pho}}$ for reference) |
|           | 4100                | 0.45                    | 2.45                | -1.20  | IVA99       |
|           | 4150                | 0.15                    | 2.20                | -1.30  | YON08       |
| M 4 L2617 | 4256                | 1.48                    | 1.38                | -1.12  | This study (adopted, spectroscopic parameters) |
|           | 4238                | 1.24                    | ...                 | -1.21  | This study ($T_{\text{eff}}^{\text{pho}}$ and $g_{\text{evol}}^{\text{pho}}$ for reference) |
|           | 4200                | 0.95                    | 1.55                | -1.17  | IVA99       |
|           | 4275                | 1.25                    | 1.65                | -1.20  | YON08       |
| M 4 L3209 | 4025                | 1.13                    | 1.49                | -1.12  | This study (adopted, spectroscopic parameters) |
|           | 4162                | 0.83                    | ...                 | -1.33  | This study ($T_{\text{eff}}^{\text{pho}}$ and $g_{\text{evol}}^{\text{pho}}$ for reference) |
|           | 3975                | 0.60                    | 1.75                | -1.20  | IVA99       |
|           | 4075                | 0.75                    | 1.95                | -1.25  | YON08       |
| M 4 L3624 | 4269                | 1.47                    | 1.42                | -1.12  | This study (adopted, spectroscopic parameters) |
|           | 4290                | 1.26                    | ...                 | -1.22  | This study ($T_{\text{eff}}^{\text{pho}}$ and $g_{\text{evol}}^{\text{pho}}$ for reference) |
|           | 4225                | 1.10                    | 1.45                | -1.16  | IVA99       |
|           | 4225                | 1.05                    | 1.60                | -1.29  | YON08       |
| M 4 L4511 | 4173                | 1.20                    | 1.44                | -1.13  | This study (adopted, spectroscopic parameters) |
|           | 4251                | 1.16                    | ...                 | -1.21  | This study ($T_{\text{eff}}^{\text{pho}}$ and $g_{\text{evol}}^{\text{pho}}$ for reference) |
|           | 4150                | 1.10                    | 1.55                | -1.19  | IVA99       |
|           | 4150                | 1.05                    | 1.70                | -1.22  | YON08       |
| M 13 I-13 | 4155                | 0.73                    | 1.75                | -1.58  | This study (adopted, spectroscopic parameters) |
|           | 4200                | 0.91                    | ...                 | -1.57  | This study ($T_{\text{eff}}^{\text{pho}}$ and $g_{\text{evol}}^{\text{pho}}$ for reference) |
|           | 4290                | 1.00                    | 2.00                | -1.46  | KRA93, KRA92, SHE96 |
| M 13 II-76| 4159                | 0.27                    | 1.87                | -1.77  | This study (adopted, spectroscopic parameters) |
|           | 4212                | 0.91                    | ...                 | -1.65  | This study ($T_{\text{eff}}^{\text{pho}}$ and $g_{\text{evol}}^{\text{pho}}$ for reference) |
|           | 4350                | 1.00                    | 2.00                | -1.49  | KRA93, KRA92, SHE96, SM96 |
|           | 4300                | 0.85                    | 1.95                | -1.53  | COH05       |
additional increase to the strength of high-forming saturated K\textsc{i} lines. In such a case, an application of the \(v_t\) value determined from Fe lines with \(EW < 100\) mÅ would be no more adequate for the K\textsc{i} lines with \(EW\) of a few \(\times 100\) mÅ (especially intensified by the enhanced turbulence in the upper atmosphere), for which a somewhat larger \(v_t\) value should have been more relevant. Hence, we believe that the reason for the anomalously inappropriately small \(v_t\) values, which must have resulted in an overestimation of K abundances from such strongly saturated K\textsc{i} 7665/7699 lines.

Consequently, regarding that M 4/L2406 and M 13/III-73 correspond to exceptional cases of peculiar velocity fields in the upper atmospheric layer, we conclude that the abundance uniformity essentially holds for [K/H] (as well as for [Fe/H]) within M 4, M 13, and M 15.

5.3. Behavior of [K/Fe] over Wide Metallicities

Now that the homogeneity of the K (and Fe) abundance within each globular cluster has been confirmed, we can regard based on the argument in section 1 that the observed [K/Fe] ratios (which naturally also turn out to be uniform) of the

### Table 2. Continued

| Star     | \(T_{\text{eff}}\) (K) | \(\log g\) (cm s\(^{-2}\)) | \(v_t\) (km s\(^{-1}\)) | [Fe/H] (dex) | References\(^\dagger\) |
|----------|-------------------------|-----------------------------|--------------------------|--------------|------------------------|
| M 13 III-52 | 4271                    | 0.80                        | 1.55                     | -1.58        | This study (adopted, spectroscopic parameters) |
|           | 4328                    | 1.00                        | ...                      | -1.56        | This study (\(T_{\text{phot}}\) and \(\log g_{\text{evol}}\) for reference) |
|           | 4335                    | 1.00                        | 2.00                     | -1.50        | This study (adopted, spectroscopic parameters) |
| M 13 III-59 | 4206                    | 0.54                        | 1.42                     | -1.58        | This study (adopted, spectroscopic parameters) |
|           | 4260                    | 0.97                        | ...                      | -1.50        | This study (\(T_{\text{phot}}\) and \(\log g_{\text{evol}}\) for reference) |
|           | 4360                    | 1.10                        | 1.75                     | -1.45        | KRA93, KRA92, SHE96, SMI96 |
| M 13 III-73 | 4145                    | 0.38                        | 1.93                     | -1.73        | This study (adopted, spectroscopic parameters) |
|           | 4177                    | 0.81                        | ...                      | -1.66        | This study (\(T_{\text{phot}}\) and \(\log g_{\text{evol}}\) for reference) |
|           | 4300                    | 0.85                        | 2.25                     | -1.51        | KRA93, KRA92, SHE96, SMI96 |
| M 15 K144 | 4066                    | -0.45                       | 1.46                     | -2.51        | This study (adopted, spectroscopic parameters) |
|           | 4242                    | 0.74                        | ...                      | -2.25        | This study (\(T_{\text{phot}}\) and \(\log g_{\text{evol}}\) for reference) |
|           | 4390                    | 0.90                        | 2.00                     | -2.31        | SNE91 |
|           | 4460                    | 0.95                        | 2.00                     | -2.21        | SNE91 |
|           | 4425                    | 0.75                        | 2.10                     | -2.34        | SNE97 |
| M 15 K341 | 4085                    | -0.15                       | 2.10                     | -2.51        | This study (adopted, spectroscopic parameters) |
|           | 4210                    | 0.61                        | ...                      | -2.35        | This study (\(T_{\text{phot}}\) and \(\log g_{\text{evol}}\) for reference) |
|           | 4275                    | 0.45                        | 2.00                     | -2.34        | SNE97 |
| M 15 K431 | 4152                    | -0.01                       | 1.49                     | -2.47        | This study (adopted, spectroscopic parameters) |
|           | 4284                    | 0.75                        | ...                      | -2.30        | This study (\(T_{\text{phot}}\) and \(\log g_{\text{evol}}\) for reference) |
|           | 4430                    | 0.90                        | 2.00                     | -2.28        | SNE91 |
|           | 4500                    | 1.00                        | 2.00                     | -2.18        | SNE91 |
|           | 4375                    | 0.50                        | 2.30                     | -2.43        | SNE97 |
| M 15 K634 | 4161                    | 0.23                        | 1.78                     | -2.36        | This study (adopted, spectroscopic parameters) |
|           | 4242                    | 0.64                        | ...                      | -2.27        | This study (\(T_{\text{phot}}\) and \(\log g_{\text{evol}}\) for reference) |
|           | 4225                    | 0.30                        | 1.85                     | -2.34        | SNE97 |
|           | 4225                    | 0.60                        | 2.05                     | -2.30        | OTS06 |
| M 15 K825 | 4012                    | -0.22                       | 1.78                     | -2.55        | This study (adopted, spectroscopic parameters) |
|           | 4242                    | 0.63                        | ...                      | -2.30        | This study (\(T_{\text{phot}}\) and \(\log g_{\text{evol}}\) for reference) |
|           | 4275                    | 0.65                        | 1.75                     | -2.42        | SNE97 |

\(^\dagger\) Key to the references: BEL85 — Bell, Edvardsson, and Gustafsson (1985); BRO92 — Brown and Wallerstein (1992); CAR00 — Carr, Selgren, and Balachandran (2000); COH05 — Cohen and Melendez (2005); EDV88 — Edvardsson (1988); FER90 — Fernández-Villacáñiz, Rego, and Cornide (1990); GON98 — González and Wallerstein (1998); GRA82 — Gratton et al. (1982); GRA86 — Gratton and Ortolani (1986); HIL97 — Hill (1997); IDA99 — Ivanov et al. (1999); KRA92 — Kraft et al. (1992); KRA93 — Kraft et al. (1993); KVR86 — Kyröläinen et al. (1986); LAM81 — Lambert and Ries (1981); LEE87 — Leep, Wallerstein, and Oke (1987); MCW90 — McWilliam (1990); MCW94 — McWilliam and Rich (1994); OTS06 — Otsuki et al. (2006); SHE96 — Shetrone (1996); SMi96 — Smith et al. (1996); SNE91 — Sneden et al. (1991); SNE94 — Sneden et al. (1994); SNE97 — Sneden et al. (1997); THE99 — Thévenin and Idiart (1999); TOM99 — Tomkin and Lambert (1999); YON08 — Yong et al. (2008).

* In case that two kinds of log \(g\) values (\(g_{\text{phot}}\) and \(g_{\text{evol}}\)) are given for the same star (e.g., IVA99 for M 4 stars or SNE97 for M 15 stars), we preferably adopted the spectroscopic \(g_{\text{phot}}\).
Table 3. Abundance variations in case that the photometric $T_{\text{eff}}$ and the evolutionary $\log g$ are used.\(^a\)

| Star     | $\delta T_{\text{eff}}$ (K) | $\delta \log g$ (cm s\(^{-2}\)) | $\delta$[Fe t/H] (dex) | $\delta$[Fe t/H]\(_{\text{av}}\) (dex) | $\delta$[Fe/H]\(_{\text{av}}\) (dex) | $\delta$[K/H]\(_{\text{av}}\) (dex) | $\delta$[K/H]\(_{7665}\) (dex) |
|----------|------------------|------------------|------------------|----------------|----------------|----------------|----------------|
| $\rho$ Boo | -111             | -0.34            | -0.06            | -0.04          | -0.05          | -0.06          | -0.06          |
| $\alpha$ Boo | +22              | -0.16            | -0.04            | -0.15          | -0.10          | +0.07          | +0.05          |
| M 4 L2406 | +77              | +0.15            | +0.03            | -0.04          | 0.00           | +0.06          | +0.07          |
| M 4 L2617 | -18              | -0.24            | -0.05            | -0.13          | -0.09          | +0.03          | +0.02          |
| M 4 L3209 | +137             | -0.30            | -0.02            | -0.40          | -0.21          | +0.25          | +0.22          |
| M 4 L3624 | +21              | -0.21            | -0.02            | -0.17          | -0.10          | +0.07          | +0.05          |
| M 4 L4511 | +78              | -0.04            | +0.01            | -0.17          | -0.08          | +0.11          | +0.11          |
| M 13 I-13 | +45              | +0.18            | +0.02            | -0.02          | 0.00           | +0.05          | ...            |
| M 13 II-76 | +53              | +0.64            | +0.01            | +0.23          | +0.12          | -0.02          | ...            |
| M 13 II-52 | -23              | +0.20            | -0.03            | +0.07          | +0.02          | -0.03          | ...            |
| M 13 III-59 | +54             | +0.43            | +0.05            | +0.11          | +0.08          | +0.03          | ...            |
| M 13 III-73 | +32             | +0.43            | 0.00             | +0.15          | +0.07          | -0.04          | ...            |
| M 15 K144 | +176             | +1.19            | +0.16            | +0.37          | +0.26          | +0.08          | +0.10          |
| M 15 K341 | +125             | +0.76            | +0.11            | +0.23          | +0.17          | +0.06          | +0.07          |
| M 15 K343 | +132             | +0.76            | +0.13            | +0.22          | +0.17          | +0.08          | +0.09          |
| M 15 K634 | +81              | +0.41            | +0.10            | +0.09          | +0.10          | +0.11          | +0.11          |
| M 15 K825 | +230             | +0.85            | +0.28            | +0.24          | +0.26          | +0.20          | +0.21          |

\(^a\) Given are the abundance variations by using the photometric $T_{\text{eff}}$ and evolutionary $\log g$ instead of the spectroscopically determined “standard” parameters adopted in this study (cf. table 1). (Note that the same $E_{\lambda}$ data for Fe as well as K lines and the same microturbulence were used in this test calculation.) Columns 2 and 3 give the relevant changes in $T_{\text{eff}}$ ($T_{\text{eff}}^{\text{phot}} - T_{\text{eff}}^{\text{spec}}$) and $\log g$ ($\log g^{\text{evol}} - \log g^{\text{spec}}$), followed by the variation in the mean Fe abundance derived from Fe I lines (column 4), that from Fe II lines (column 5), and the average of both (column 6). Similarly, the resulting changes in the K abundance are presented in columns 7 (non-LTE K abundance from the 7665 line), 8 (non-LTE K abundance from the 7699 line), and 9 (the average of both). Finally, column 10 gives the variation in [K/Fe], which is simply the difference in the results of columns 9 and 6.

Cluster stars retain the original composition of the halo gas, from which they formed.

According to table 1 (where the mean of [K/Fe]\(_{7665}\) and [K/Fe]\(_{7699}\) is given), the averaged cluster values of ([K/Fe]) (and the standard deviation $\pm \sigma$) are $+0.10 \pm 0.04$ (M 4, [Fe/H] $\approx -1.1$, 4 stars, L2406 excluded), $+0.12 \pm 0.16$ (M 13, [Fe/H] $\approx -1.7$, 4 stars, III-73 excluded), and $+0.24 \pm 0.05$ (M 15, [Fe/H] $\approx -2.5$, 5 stars). We can state from these results that [K/Fe] is marginally supersolar at $\sim +0.1$--0.3 and [K/Fe] tends to gradually increase with a decrease in the metallicity, which means that a rather high and monotonic [K/Fe] vs. [Fe/H] relation (with almost no significant diversity) in the range of $-2.5 \lesssim [\text{Fe/H}] \lesssim -1$. These [K/Fe] vs. [Fe/H] relations for 15 stars of M 4, M 13, and M 15 (along with $\rho$ Boo and $\alpha$ Boo) are plotted in figure 6, where the results for nearby F-G disk dwarfs ($-1 \lesssim [\text{Fe/H}] \lesssim 0$; taken from Paper I) and those for extremely metal-poor stars ($-4 \lesssim [\text{Fe/H}] \lesssim -2.5$; obtained by reanalyzing the $E_{\lambda}$ data of Cayrel et al. 2004; cf. Appendix) are also shown for a comparison.

As can be clearly seen from this figure, our [K/Fe] results in this study for the cluster stars (excluding M 4/L2406 and M 13/III-73) and the reference objects ($\rho$ Boo and $\alpha$ Boo) over $-2.5 \lesssim [\text{Fe/H}] \lesssim 0$ connect with those of extremely metal-deficient stars as well as disk stars quite satisfactorily. We may thus conclude that [K/Fe] behaves itself in a rather clear manner over a wide [Fe/H] range of $\sim 4$ dex from near-solar to ultra-low metallicity. That is, the gradual increase of [K/Fe] with a decrease in [Fe/H] ($d[K/Fe]/d[Fe/H] \sim -0.3$) seen in disk stars of $-1 \lesssim [\text{Fe/H}] \lesssim 0$ (Paper I) continues further down at [Fe/H] $\lesssim -1$ with a slightly decreased slope until [Fe/H] $\sim -3$, where [K/Fe] appears to attain a broad maximum of $\sim +0.3$ (followed by a sign of weak downturn of [K/Fe] at [Fe/H] $\lesssim -3$).

Accordingly, we do not regard the considerably large diversity of [K/Fe] previously reported for halo stars of [Fe/H] $\sim -2$ (Zhang et al. 2005; Gratton & Sneden 1987b) to be real, which we suspect to be due to some improper treatment in abundance determinations (e.g., $E_{\lambda}$ errors caused by unsuccessful removal of telluric lines, or use of an inadequate microturbulence, such as we encountered in the present cases of M 4/L2406 and M 13/III-73).

This observational fact of mildly supersolar [K/Fe] over a wide metallicity range (with [K/Fe] $\sim 0$ at near-solar metallicity and a broad peak at [Fe/H] $\sim -3$) has to be explained by theoretical calculations. However, theoreticians did not find it easy to reproduce the trend of [K/Fe] $> 0$, especially at the very metal-poor regime, because the K yield is metallicity-dependent (as a characteristic of odd-Z element) and becomes too small there to bring [K/Fe] above zero. Therefore, comparatively plain calculations predicted only subsolar (or near-solar at most) [K/Fe] ratios for metal-poor stars. For example, Timmes, Woosley, and Weaver’s (1995) calculations resulted in a decrease of [K/Fe] $\lesssim 0$ with a lowering of [Fe/H], as shown figure 24 of their paper (also plotted in figure 6 as curve “T”), which markedly contradicts the observed tendency. Therefore, if the supersolar [K/Fe] is to be reenacted even
6. Conclusion

Given the confusing situation regarding the [K/Fe] ratio of metal-poor stars in the halo, where some work suggests a fairly tight tendency (such like disk stars) while others report a considerably large diversity amounting to ~1 dex, we carried out a spectroscopic study on 15 red giants of three mildly to very metal-poor globular clusters (M4, M13, and M15) along with two reference stars (∝ Boo and α Boo) based on the high-dispersion spectra obtained with Subaru/HDS, with a purpose of clarifying the behavior of [K/Fe] at the relevant metallicity range of −2.5 ≤ [Fe/H] ≤ −1.

The atmospheric parameters ($T_{\text{eff}}$, $\log g$, $v_t$, and [Fe/H]) were spectroscopically determined by using Fe I and Fe II lines, and the abundance of K was derived from the K I resonance lines at 7664.91 and 7698.97 Å while taking into account the non-LTE effect.

We confirmed that [K/H] (as well as [Fe/H]) is almost homogeneous within each of the three clusters to a precision of ≤0.1 dex, though superficially large deviations are exceptionally seen for two peculiar stars which show signs of considerably increased turbulence in the upper atmosphere.

The [K/Fe] ratios of these cluster stars turned out mildly supersolar by a few tenths of dex, tending to gradually increase from ~ +0.1–0.2 at [Fe/H] ~ −1 to ~ +0.3 at [Fe/H] ~ −2.5, which is a rather tight and clean trend. We thus consider that the previously reported large diversity of [K/Fe] in halo stars is not real, which we suspect to be due to some improper treatment in the analysis.

This result is quite consistent (i.e., smoothly connecting) with the [K/Fe] trend of disk stars (−1 ≤ [Fe/H]) and that of extremely metal-poor stars (−4 ≤ [Fe/H] ≤ −2.5).
is, [K/Fe] appears to continue a gradual increase from [Fe/H] \sim 0 toward a lower metallicity regime down to [Fe/H] \sim -3, where a broad maximum of [K/Fe] \sim +0.3–0.4 is attained, possibly followed by a slight downturn toward a further lower metallicity at [Fe/H] \lesssim -3.

This investigation is based on data obtained by observations with the Subaru Telescope, carried out during practical training of observational astronomy for graduate students as coursework of The Graduate University for Advanced Studies (SOKENDAI). We express our heartfelt thanks to S. S. Hayashi, R. Furuya, and A. Tajitsu for their continuous support and encouragement, as well as to C.-H. Peng and S. Honda for their collaboration in the observation.

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### Appendix. Reanalysis of Cayrel et al.'s (2004) Data

In an extensive spectroscopic study of a sample of 35 extremely metal-poor stars toward clarifying the abundance patterns of 17 elements from C to Zn recently carried out by Cayrel et al. (2004), the abundances of K were also determined by using K1 7665/7699 lines. However, they applied a constant non-LTE correction of \Delta \text{NLTE} = -0.35 \text{dex} to all stars, which they adopted as a rough estimate based on Ivanova and Shimasaki’s (2000) calculations. Because of this apparently imperfect treatment of the non-LTE effect, one can not be sure whether the behavior of [K/Fe] they obtained (i.e., near-solar or slightly supersolar with a decreasing tendency toward a lower metallicity from [K/Fe] \sim +0.2 at [Fe/H] \sim -2.5 to [K/Fe] \sim 0 at [Fe/H] \sim -4; cf. their figure 9) is reliable or not.

We therefore, decided, to reanalyze their EW data of K1 7665/7699 lines with the atmospheric parameters they used, while properly taking into account the non-LTE effect. Regarding the non-LTE departure coefficients to be included

![Table 4. Non-LTE reanalysis of Cayrel et al.’s (2004) data.](image-url)
in line-formation calculations for abundance determinations, we used the already calculated results for a grid of models \((T_{\text{eff}} \text{ from 4500 K to 6500 K}, \log g \text{ from 1.0 to 5.0}, v_{\text{t}} \text{ from 1 to 3 km s}^{-1}, \text{ and } [\text{Fe} / \text{H}] \text{ from 0.0 to } -3.0)\), which were originally computed for constructing extensive tables of non-LTE corrections,\(^8\) as explained in the Appendix of Paper I. Besides, we newly performed calculations corresponding to the case of \([\text{Fe} / \text{H}] = -4\) to be combined with the previous grids, so that we can handle any of the relevant 31 stars (for which Cayrel et al. (2004) measured \(E(W_{2665})\) and/or \(E(W_{665})\) in the parameter range of \(4500 \text{ K} \leq T_{\text{eff}} \leq 5300 \text{ K}, 0.7 \leq \log g \leq 2.7, 1.2 \text{ km s}^{-1} \leq v_{\text{t}} \leq 2.2 \text{ km s}^{-1}, \text{ and } -4.0 \leq [\text{Fe} / \text{H}] \leq -2.1\) by interpolation. Following the same manner as described in subsection 4.2, we determined \([\text{K} / \text{H}], \Delta_{\text{NLTE}}^{\text{K}}, \text{ and } [\text{K} / \text{Fe}]\) for each star, as given in table 4. The resulting values of \([\text{K} / \text{Fe}]\) (as well as \(\Delta_{\text{NLTE}}^{}\)) are plotted against \([\text{Fe} / \text{H}]\) in figure 6 (filled inverse triangles).

As we can see from figure 6b, the extents of the non-LTE corrections \((\Delta_{\text{NLTE}}^{})\) systematically decrease with a lowering of the metallicity; such as from \(\Delta_{\text{NLTE}}^{} 
early\sim 0.3 \text{ (at } [\text{Fe} / \text{H}] \nearly\sim -2.5)\) to \(\Delta_{\text{NLTE}}^{} \sim 0.2 \text{ (at } [\text{Fe} / \text{H}] \nearly\sim -4)\), which means that Cayrel et al.’s (2004) use of \(\Delta_{\text{NLTE}}^{} = -0.35\) must have somewhat overcorrected (i.e., underestimated) \([\text{K} / \text{Fe}]\) by \(0.1–0.2\) dex, such that the amount of overcorrection progressively increases with a decrease in \([\text{Fe} / \text{H}]\). Accordingly, regarding the \([\text{K} / \text{Fe}]\) vs. \([\text{Fe} / \text{H}]\) relation of extremely metal-poor stars, we consider that Cayrel et al.’s (2004) result \(([\text{K} / \text{Fe}]\text{ ratio is near-solar, or slightly above solar } \sim +0.1–0.2\) with an appreciable gradient of \(d[\text{K} / \text{Fe}] / d[\text{Fe} / \text{H}]\) should be revised as concluded in subsection 5.3 (cf. figure 6a): \([\text{K} / \text{Fe}]\) in this very metal-deficient regime still remains well supersolar at \(\sim +0.2–0.3\), with a marginal sign of decline (downturn) toward a further lower \([\text{Fe} / \text{H}]\).

The conclusion that \([\text{K} / \text{Fe}]\) remains supersolar even at \([\text{Fe} / \text{H}] \sim -4\) may provide theoreticians with an important constraint on the synthesis mechanism of K from the observational side, because considerable difficulties exist in theoretically reproducing the \([\text{K} / \text{Fe}]\) ratio (cf. section 5.3) also in this distinctly low metallicity regime, where the observed \([\text{K} / \text{Fe}]\) may be considered to simply reflect the composition of the first super/hyper-nova ejecta. For example, Tomina, Umeda, and Nomoto’s (2007) nucleosynthesis calculation based on population III supernova models predicted \([\text{K} / \text{Fe}] \sim -1 \pm 0.5\) at \(-4 \leq [\text{Fe} / \text{H}] \leq -3\) (cf. their figure 6), which apparently disagrees with the observational fact mentioned above. Hence, in order to resolve this situation, a new reassessment of assumptions or fundamental physics might as well be required. In this connection, Iwamoto et al.’s (2006) finding, that odd-Z elements such as K and Sc can be significantly overproduced if the proton-rich environment (large initial electron fraction \(Y_e\)) is realized, is quite interesting. Further investigation following this line may be worth a try.

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\(^8\) The anonymous ftp site to access these electronic tables described in the Appendix of Paper I is now not available any more. Instead, the same data materials are placed at the following web site: (http://optik2.mtk.nao.ac.jp/~takeda/potassium_nonlte/).
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