Direct Shear Test of Powder Beds

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Abstract

Interrelations among shearing stress, applied vertical stress (by external load), actual vertical stress at shearing plane, vertical displacement (powder bed expansion or contraction) and shearing displacement have been extensively investigated for a number of powders under various test conditions such as the method of preconsolidation, the shearing cell diameter, the initial void fraction of powder beds and so on, by utilizing a simple shear tester capable of testing under both constant-load and constant-volume conditions and of measuring the actual vertical stresses (6 points) in the vicinity of shearing plane, the shearing stress and the vertical displacement of a powder bed simultaneously.

It has been concluded that; if the internal friction factor having a universal validity and fairly independent of the test conditions is to be obtained, it is essential to measure the mean actual vertical stress in the vicinity of shearing plane, or it is indispensable otherwise to measure at least the vertical displacement of powder beds along with the shearing displacement and to make sure that the vertical displacement is being kept as small as possible during a course of shearing.

1. Introduction

Mechanical characteristics of powder beds have been evaluated by a direct shear tester and/or uni- and tri-axial compression testers. In the field of powder technology, the direct shear tester has gained a popularity because of: simplicity in its device structure, handling and operations, reproducibility for practical powder behaviors of non uniform strains and two dimensional deformation, and giving useful and fundamental information.

A direct shear test is conducted by; first applying a vertical load onto a sample powder bed confined in a cell and second measuring the shear stress generated upon horizontal shearing movement of the bed. Finally by plotting the shear stresses against various corresponding vertical stresses, we obtain a yield locus of the powder bed. The yield locus may represent the shearing-failure characteristics of the powder and the internal friction factor of the powder bed may be calculated from the slope of the locus. These characteristics have been widely utilized to evaluate the flowability of powders in various bulk handling processes such as transportation, storage, feeding and mixing and to establish rational criteria for design and analysis of such processes.

Although many studies on the direct shear test have been reported, the measured values of the internal friction factor, for example, are found to depend on experimental conditions, even if the same sample is tested by the same device. This is probably because the results of direct shear tests are likely to be affected by particle sizes, their distributions and packing structures of powder beds, since a powder bed consists of a great number of solid particles and

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Unfortunately, there are few reports which investigate what effect is dominant or whether a universal internal friction factor independent of testing methods and/or experimental conditions can be obtained.

The present paper summarizes the significant results of previous studies on direct shear tests for discussions and presents our recent experimental results obtained by the direct shear tester we have developed.

2. Background

A direct shear test simultaneously measures the vertical stress applied to a powder bed and the shear stress generated when the bed is forced to slip, as illustrated in Fig. 1. The shearing cell consists of two parts stacked, the upper and lower parts, the cross section of which may be circular or square with the same cross sectional shape and area. One of the cells is movable horizontally and the other is fixed.

Taneya et al. pointed out that a direct shear test would give larger internal friction factors with decreasing the cell diameter and/or the upper powder bed height. To clarify this reason, they observed the flow patterns of colored defatted milk powder in the shearing cell by taking cross sectional pictures, and recognized that the end effect near the side wall of the cell becomes appreciable as the cell diameter and/or the upper bed height decrease. They proposed that a shear test should be conducted under the conditions of a large bed height and a constant cell diameter to reduce such end effect. Yamafuji et al. summarized existing literatures to obtain the relationship between the internal friction factor and the initial void fraction of the powder bed and found that the internal friction factor increased as the initial void fraction decreased or the powder bed was packed denser. Recently, Tsunakawa et al. and Takagi et al. also reported similar results.

Ohtsubo suggested that a shear and vertical stresses relation, or a yield locus (YL), would not necessarily follow Coulomb's law of friction; a decrease or an increase in the void fraction of a powder bed, respectively, cause the YL to be concave or convex. Umeya et al. verified experimentally this phenomena. Shinhara et al. proposed a model of monodispersed powder bed made by ordered packing to investigate the effects of the void fraction on the internal friction factor. By taking surface friction at contact points between particles, mechanical gearing of particles and adhesion/cohesion of particles into considerations of the friction and cohesion mechanisms in the powder bed, they theoretically concluded that the dependency of the internal friction factor on the void fraction would be due to a nonuniformity of the powder bed structure.

Aoki et al. used a constant volume direct shear tester, which allowed the tests under constant void fraction conditions, to carry out a series of direct shear tests by changing the powder bed height in the shearing cell, and found that the slope of the YL (that could be obtained by a single trial with their tester), or the internal friction factor, increased as the bed height increased. Umeya et al. obtained a similar results by using a constant load direct shear tester. Aoki et al. claimed that a direct shear test cannot give a true internal friction factor but an apparent one, and proposed that the true internal friction factor can be obtained by extrapolating the relation between the bed height-to-cell diameter ratio and the corresponding internal friction factor to zero bed height according to Janssen's formula. This finding is completely contrary to the result of Taneya and implied that a direct shear test should be made by keeping the powder bed height as low as possible. Furthermore, Tsunakawa et al. have pointed out that the yield loci obtained by the constant volume shear tester have a tendency of being larger than those obtained by a constant load direct shear tester and this tendency becomes appreciable as the cohesiveness and compressibility of the
Ashton et al.\textsuperscript{12} conducted a series of shear tests for various cohesive powders by using Jenike cell\textsuperscript{13,14} (a kind of constant load direct shear testers). They changed the initial void fraction (or the bulk density) of the powder bed in the cell by adjusting the preconsolidation pressure to obtain a group of convex yield loci. They also derived Eq. (1) which is a modification of Coulomb's equation \(\tau = \mu \cdot \sigma + C\); \(\tau = \) shear stress, \(\sigma = \) vertical stress, \(\mu = \) internal friction factor, \(C = \) cohesive force on shearing:

\[
(\tau/C)^n = \sigma/\sigma_{\text{max}} + 1
\]

where \(\sigma_{\text{max}} = \) tensile strength of powder bed [Pa]

\(n = \) shear index [-]

This expression is sometimes called Farley-Valentin equation or Warren-Spring equation. Ashton et al. have found that the shear index is a representative factor of the flowability of a powder and takes the value of between 1 (for very flowable powders) and 2 (for highly cohesive powders). Eelkmanrooda\textsuperscript{17}, Nedderman\textsuperscript{18}, Kocova\textsuperscript{19} and Williams et al.\textsuperscript{20} have also paid their attention to the shear index. Farley et al.\textsuperscript{15} reported that the shear index decreased as the surface area of powder particles increased. Rumpf et al.\textsuperscript{21}, Farley et al.\textsuperscript{15}, Aoki et al.\textsuperscript{22} and Jimbo et al.\textsuperscript{23} measured the maximum tensile strength of powder beds to determine the shear index. Steinforth et al.\textsuperscript{34} tried to estimate the shear index only by fitting numerically an equation to theYL obtained. Recently, Tsunakawa et al.\textsuperscript{4} pointed out that the shear index cannot be an estimate of the flowability of powders because the different samples give almost the same value of the index within the experimental error of \(\pm 10\%\).

Umeya et al.\textsuperscript{7} conducted a series of direct shear tests by changing the applied vertical stress over a wide range and obtained a convex yield locus similar to the one obtained by Jenike\textsuperscript{14}. They expressed the locus by Eq. (2):

\[
\tau = K(\sigma + \sigma_0)^m
\]

where \(m = \) deformation index, [-]

\(K = \) deformation coefficient, [((Pa)\textsuperscript{-1} - (Kg/cm\textsuperscript{2})\textsuperscript{-1} - m)]

and found that the parameters, \(m\) and \(K\), depend on the properties and the packed state of the powders tested. Eq. (2) is equivalent to Eq. (1). They also have shown that the yield locus can be divided into three parts represented by \(m < 1, m > 1\) and \(m = 1\) and that a convex deviation of the locus from Coulomb's law may be due to mutual concessions of the powder particles under shearing.

Jenike et al.\textsuperscript{13,14} developed a flow factor tester (a kind of the constant load simple shear testers) which was designed to keep an initial packed state or an initial void fraction of the powder bed in the shearing cell, \(\varepsilon_0\), as constant as possible. They claimed that the tester can provide yield loci under constant void fraction conditions if the shear test is done at the vertical stress less than the preconsolidation pressure. A series of the tests under various preconsolidation pressures can also give a set of the yield loci, the end point of which is at the respective preconsolidation pressure. The envelop of Mohr circles which are in contact with the yield loci at each end point, becomes the straight line passing through the origin as shown in Fig. 2. This fact has also been confirmed by Ashton et al.\textsuperscript{13}. This straight line is called an effective yield locus (E.Y.L.) and its slope an effective friction angle. It is reasoned that the convex nature of the yield locus is due to the energy consumed at the slip plane for dilatation of the powder bed under shearing, which is added to the friction force between solid particles.

It can be noted from the survey of the literatures reported so far, that the vertical stress and its distribution in the vicinity of shearing plane have not yet been measured directly. All the yield loci reported so far are based on an apparent vertical stress calculated from the externally applied vertical load on the shearing cell, divided by the shearing area. It is also
noteworthy that little attempt has been made to measure the shear force and the dilatation and/or contraction of the powder bed in the shearing cell simultaneously, though the volume change in the bed has been long considered to take place as a result of shearing\(^1\)\(^{11}\)\(^{25}\)\(^{26}\)\(^{35}\).

Consider a "differential" area, \(dS\), on the shearing plane (area \(S\)) in a powder bed under shearing ("differential" in this case means very small but still as large as sufficient numbers of solid particles can be contained so that the identity as a powder is not lost) as shown in Fig. 3. The vertical stress applied on \(dS\), \(\sigma_z\), and the shearing stress, \(\tau_x\), can be related by Eq. (3), if Coulomb's law without cohesive force is assumed to hold:

\[
\tau_x \cdot dS = \mu \cdot \sigma_z \cdot dS
\]

where \(\mu\) represents an internal friction factor.

Integration of Eq. (3) over the whole shearing plane gives Eq. (4) if \(\mu\) is assumed to be a constant:

\[
T_x = \mu \cdot \Sigma_z
\]

where

\[
T_x = \int \tau_x \cdot dS = \int \sigma_z \cdot dS
\]

Eq. (4) may be rewritten by introducing the mean values as follows:

\[
\bar{\tau}_x = \mu \cdot \bar{\sigma}_z
\]

where

\[
\bar{\tau}_x = T_x / S, \quad \bar{\sigma}_z = \Sigma_z / S
\]

It should be noticed that the results of conventional shear tests have not been expressed in terms of \(\tau_x\) and \(\sigma_z\) but in terms of \(\bar{\tau}_x\) and \(\bar{\sigma}_z\). Although shearing does not take place on a single plane but in multiple layers\(^6\)\(^{27}\)\(^{28}\)\(^{29}\), it may be allowed that the mean value, \(\bar{\tau}_x\), of the measured shear force, \(T_x\), divided by the shearing area, \(S\), is regarded as \(\tau_x\). It is still controversial, however, whether the whole vertical load externally applied, \(\Sigma_w\) (the weight put on the powder bed in the shearing cell), can be assumed to be equal to \(\Sigma_z\) (the vertical load in the vicinity of the shearing plane). As is well known, the pressure force applied on the top of a powder bed never propagates isotropically throughout the bed and there is no universal and quantitative law on the propagation of pressure force through a powder bed. Whether \(\Sigma_z\) equals to \(\Sigma_w\) is dependent on materials, methods of measurements and experimental conditions. It is worthy to note that Aoki et al.\(^{10}\) have used Janssen's formula to correct the difference between \(\Sigma_z\) and \(\Sigma_w\). It has also been found, however, that the formula cannot be applied to the powder bed under shearing but to stationary powder beds\(^{30}\)\(^{31}\).

If the \(\tau_x - \sigma_z\) relation is linear, it may be possible to calculate a universal internal friction factor \(\mu\) from Eq. (5) by measuring the actual vertical stresses at several points in the vicinity of the shearing plane and regarding their arithmetic mean as \(\sigma_z\). With this in mind, we have developed an improved direct shear tester to test a variety of powders under various experimental conditions.

3. Experimental

The direct shear tester developed is shown in Fig. 4, which can test powders under both a constant load and a constant volume conditions. The tester has a pair of cylindrical shearing cells with the inner diameters of 40, 80, 100 and 120 mm; the upper part \(\circ\) is fixed and the lower one \(\triangle\) movable. The shear rate can be varied over the range of 0.25 to 15 mm/min.

In case of constant load shear tests, both the weight \(\natural\) and the length of lever on the beam \(\natural\) were adjusted to change the vertical load on the powder bed in the shearing cell from 0.78 to 147 kPa. In case of constant volume shear tests, on the other hand, the beam \(\natural\) was fixed tightly at the horizontal position by the clamp \(\natural\) installed on the
supporter ④ and the stopper ② to keep the powder bed volume in the shearing cell constant during a course of shearing. The compressive load transducer ⑦ connected to the lid of the shearing cell measures the vertical force $\Sigma w$ which is a part of the expanding force of the powder bed under shearing.

The proving ring ⑧ and the electric dial gauge ⑪ were used to determine the shear force $T_x$. The horizontal or shearing displacement $\delta_x$ was measured by using the electrical dial gauge ⑪ attached to the side wall of the movable cell ⑥. To observe dilatation and/or contraction of the powder bed in the shearing cell during a course of constant load shear tests, the dial gauge ⑪ was put on the part of the

| Powder          | Mean diameter [µm] | Density [kg/m³] | Bulk density [kg/m³] |
|-----------------|--------------------|----------------|---------------------|
| Zircon sand     | 130                | 4,660          | 2,910               |
| Toyoura sand    | 200                | 2,730          | 1,610               |
| Glass powder    | 220                | 2,620          | 1,240               |
| White alunnum   | 190                | 3,800          | 2,010               |
| Sodium carbonate| 180                | 2,550          | 1,030               |
| Potassium bicarbonate | 152   | 2,240          | 1,240               |
| P.V.C. powder   | 160                | 1,560          | 570                 |
| Glass spheres I | 140                | 2,560          | 1,580               |
| Glass spheres II| 960                | 2,690          | 1,570               |
| Carbondum       | 670                | 3,270          | 1,690               |
| Iron powders    | 80                 | 7,860          | 2,220               |
| Calcium carbonate (P-30)| 1.75 | 2,700          | 530                 |
| Talc            | 1.50               | 2,830          | 590                 |
| Sugar powder    | 1.15               | 1,510          | 410                 |
| Carbon black    | 1.44               | 1,800          | 170                 |
| Magnesium stearate | 1.75       | 1,040          | 200                 |
| Zinc oxide      | 0.73               | 2,070          | 530                 |
| Kanto loam (JIS #11)| 0.82    | 3,000          | 460                 |
| Corn starch     | 0.97               | 1,510          | 340                 |
| Flyash          | 2.42               | 2,140          | 530                 |
| Titanium oxide  | 0.2 ~ 0.4          | 3,930          | 350                 |
| Alumina         | 0.5 ~ 3            | 3,980          | 780                 |
| Soft flour      | 1 ~ 10             | 1,460          | 320                 |
| Kaolin          | 1 ~ 10             | 2,850          | 390                 |
beam (3) just above the center of the shearing cell.

In order to measure the actual vertical stresses in the vicinity of shearing plane, each three pressure transducers of a strain gauge type of 6 mm diameter were installed in both the upper and lower shearing cells, as shown in Fig. 5.

All the electric signals from these transducers were recorded on a multi-pen recorder and/or an X-Y recorder, or on a data-acquisition system with a micro computer, simultaneously from the beginning to the end of a shear test.

Since the shearing process in a direct shear test exhibits a variety of characteristics depending on the initial void fraction $\varepsilon_0$ of the powder bed in the shearing cell and hence it is indispensable to vary the void fraction positively over a wide range, it is essential to pack the sample powder into the cell as uniformly as possible. In case of the experiments under dense packing, the sample powder was charged into the cell little by little to a specified bed height and the surface of the powder bed was leveled by a flat plate at every moment of powder charge. Then, a preconsolidation pressure, $\sigma_{z,p}$, is applied to the bed until the bed height stops changing. After $\sigma_{z,p}$ was removed, the bed was sheared under various vertical stresses of less than $\sigma_{z,p}$.

In case of the experiments under loose packing, the sample powder was charged gently into the cell at a stretch, then the surface of the bed was leveled by a wiper and even no preconsolidation pressure was applied in some cases. The sample powders used in our experiments and their properties are summarized in Table 1.

4. Results and discussion

4.1 Constant load direct shear tests

The horizontal (shearing) displacement $\delta_x$ - shearing stress $\tau_x$ and $\delta_x$ - vertical displacement $\delta_z$ curves (will be called the shearing process characteristics in what follows) for all the sample powders under various initial void fractions could be classified into the three typical patterns as shown in Fig. 6: Pattern I (dense packing), Pattern II (optimal packing) and Pattern III (loose packing). The table below shows the effects of the shearing cell diameter $D$ and the height of the powder bed in the upper fixed cell $H_1$ on the interrelations between the external vertical stress applied, $\sigma_{z,w}$ and the actual mean vertical stress, $\sigma_{z,m}$ (the arithmetic mean of the vertical stresses measured at the points $R$, $O$ and $F$ in Fig. 5) in the vicinity of the shearing plane. The height of the bed in the lower movable cell has been kept constant at 5 mm, which has been deter-

| Shearing process | characteristic curve |
|------------------|----------------------|
| ![Graph](image) |

**Table 1**

| $D = 40\text{mm}$ | Coarse | Fine |
|-------------------|--------|------|
| $H_1 = 12$        | $\sigma_{z,w} > \sigma_{z,u}$ | $\sigma_{z,m} > \sigma_{z,u}$ | $\sigma_{z,m} > \sigma_{z,u}$ |
| $23$              | $\sigma_{z,u} > \sigma_{z,w}$ | $\sigma_{z,m} > \sigma_{z,u}$ | $\sigma_{z,m} > \sigma_{z,u}$ |
| $34$              | $\sigma_{z,u} > \sigma_{z,w}$ | $\sigma_{z,m} > \sigma_{z,u}$ | $\sigma_{z,m} > \sigma_{z,u}$ |

| $D = 100\text{mm}$ | Coarse | Fine |
|-------------------|--------|------|
| $12$              | $\sigma_{z,w} > \sigma_{z,u}$ | $\sigma_{z,m} < \sigma_{z,u}$ | $\sigma_{z,m} < \sigma_{z,u}$ |
| $23$              | $\sigma_{z,u} > \sigma_{z,w}$ | $\sigma_{z,m} < \sigma_{z,u}$ | $\sigma_{z,m} < \sigma_{z,u}$ |
| $34$              | $\sigma_{z,u} > \sigma_{z,w}$ | $\sigma_{z,m} < \sigma_{z,u}$ | $\sigma_{z,m} < \sigma_{z,u}$ |

![Fig. 6 Shearing process characteristics](image)
mined by another series of experiments. The interrelations between $\sigma_{z,w}$ and the mean actual vertical stress at the points $R'$, $O'$ and $F'$ in Fig. 5, $\sigma_{z,m}$ have been very similar to those between $\sigma_{z,w}$ and $\sigma_{z,m}$. As can be seen from Fig. 6, in the case of the Pattern I shearing process characteristics, the shear stress $\bar{\tau}_x$ increases steeply, and reaches a steady state value $\bar{\tau}_{x,s}$ after showing a clear maximum value $\bar{\tau}_{x,max}$. The vertical displacement $\delta_z$ is rather large in the positive direction and this means an expansion of the powder bed in the cell from a state of small initial void fraction (dense packing). In this case, the distribution of the vertical stress in the vicinity of shearing plane is not uniform and generally $\sigma_{z,m}$ is larger than $\sigma_{z,w}$ in the majority of cases. In the case of the Pattern III, on the other hand, $\bar{\tau}_x$ increases slowly and monotonically to reach a steady state value $\bar{\tau}_{x,s}$ at a large value of $\delta_z$ without showing a maximum. The vertical displacement $\delta_z$ is rather large in the negative direction, and this implies the contraction of the powder bed with a loose initial packing upon shearing. In this case, $\sigma_{z,m}$ is generally smaller than $\sigma_{z,w}$.

On the contrary, in the case of the Pattern II, where $\bar{\tau}_x$ reaches a steady state value $\bar{\tau}_{x,s}$ at a rather small value of $\delta_x$ without showing a maximum, and the vertical displacement $\delta_z$ is almost constant around nearly zero, $\sigma_{z,m}$ is very close to $\sigma_{z,w}$ in many cases.

These findings suggest that the internal friction factor can be correctly estimated by using $\sigma_{z,w}$ only in the limited cases when the shearing process characteristics show the Pattern II. It is highly desirable, however, to measure the actual vertical stresses in the vicinity of shearing plane, or otherwise the vertical displacement of the powder bed in the shearing cell, $\delta_z$, should be measured at least.

(1) Effect of powder bed height $H_1$ on $\bar{\tau}_x - \sigma_z$ relation

Figs. 7, 8 and 9 show the effects of the upper powder bed height $H_1$ on the yield loci in terms of $\bar{\tau}_x - \sigma_z$, $\bar{\tau}_{x,s} - \sigma_z$ relations. In Fig. 8, $\mu_{m,max}$ stands for the internal friction factor calculated from the $\bar{\tau}_{x,max} - \sigma_z,m$ relation and $\mu_{m,s}$ from $\bar{\tau}_{x,s} - \sigma_z,m$ relation, respectively. It is clear from Fig. 9 and a comparison of Fig. 8 with Fig. 7, that all the $\bar{\tau}_{x,max} - \sigma_z,m$ and $\bar{\tau}_{x,s} - \sigma_z,m$ relations with different values of $H_1$ fall into the respective single lines to give the internal friction factors independent fairly of $H_1$, though the internal friction factors evaluated from the $\bar{\tau}_x - \sigma_{z,w}$ relations have been influenced by $H_1$. This finding has also been observed in the shear tests of the other powders.
(2) Effect of shearing cell diameter $D$ on $\tau_x - \sigma_z$ relation

As shown in Fig. 10, the diameter of the shearing cell $D$ does not affect the $\tau_{x,s} - \sigma_{z,w}$ relations, and thus the internal friction factors independent of $D$ can be obtained.

(3) Effect of initial void fraction $\bar{\varepsilon}_0$ on $\tau_x - \sigma_z$ relation

The initial void fraction $\bar{\varepsilon}_0$ has been calculated by Eq. (6):

$$\bar{\varepsilon}_0 = 1 - \frac{M}{\rho V}$$  \hspace{1cm} (6)

where $M$ = mass of powder charged in shearing cell, [kg]

$V$ = initial volume of powder in shearing cell, [m$^3$]

$\rho$ = density of powder particle, [kg/m$^3$]

The effects of $\bar{\varepsilon}_0$ on the $\tau_x - \sigma_z$ relations are shown in Fig. 11 for a coarse powder and in Figs. 12 and 13 for fine powders. It can be seen from these figures, that the $\tau_{x,s} - \sigma_{z,m}$ relations are fairly independent of $\bar{\varepsilon}_0$, thus giving the unique or universal internal friction factor. The relations shown in Fig. 12 should be paid a special attention; although the $\tau_{x,s} - \sigma_{z,m}$ relations are convex for respective $\bar{\varepsilon}_0$’s as is often the case for fine powders, the $\tau_{x,s} - \sigma_{z,m}$ relation gives a single straight line and is independent of $\bar{\varepsilon}_0$. Since it is conventional to calculate the shear index $n$ according to Warren-Spring equation, Eq. (1), in the case of convex $\tau_{x,s} - \sigma_{z,w}$ relations, we have also done such calculations. The results, however, have been pessimistic and strongly influenced by the experimental conditions, and so scattered that the shear index $n$ cannot be any representative.
of the friction characteristics of powders. This has also been the case in calculating Jenike's flow factor.

(4) $\tau_x - \sigma_z$ relation under low vertical stresses

As a typical example of showing an appreciable difference between the $\tau_{x,s} - \sigma_{z,w}$ relation and the $\tau_{x,s} - \sigma_{z,m}$ relation, the results of the simple shear tests under low vertical stresses are shown in Fig. 14. In spite of convex nature in the $\tau_{x,s} - \sigma_{z,w}$ relation, the relation between $\tau_{x,s}$ and $\sigma_{z,m}$ has become a straight line as well. Besides, it has been verified that even some of fine powders give the shearing process characteristics of the Pattern I in Fig. 6 under low vertical stresses (one order of magnitude lower than the case of coarse powders). Unfortunate-

Table 2 Internal friction factors of powders tested

| Powder                  | $\mu_{m,s}$ | $\mu_{m,max}$ |
|------------------------|-------------|----------------|
| Coarse powders         |             |                |
| Zircon sand            | 0.68 ~ 0.70 | 0.81 ~ 0.84    |
| Toyoura sand           | 0.62        | 0.84           |
| Glass powder           | 0.87        | 0.92           |
| White alundum          | 0.75        | 0.80           |
| Sodium carbonate       | 0.75        | 0.83           |
| Potassium bicarbonate  | 0.83        | 0.86           |
| P.V.C. powder          | 0.70        | 0.80           |
| Glass spheres-I        | 0.30        | 0.35           |
| Glass spheres-II       | 0.31        | 0.39           |
| Carbon breadcrumbs     | 0.73        | 0.80           |
| Iron powder            | 0.85        | 0.91           |
| Fine powders           |             |                |
| Calcium carbonate(P-30)| 0.67        | --             |
| Talc                   | 0.42        | --             |
| Sugar powder           | 0.55        | --             |
| Carbon black           | 1.10        | --             |
| Magnesium stearate     | 0.64        | --             |
| Zinc oxide             | 1.02        | --             |
| Kanto loam (JIS #11)   | 0.78        | --             |
| Corn starch            | 0.81        | --             |
| Flyash                 | 0.46        | --             |
| Titanium oxide         | 1.23        | --             |
| Alumina                | 0.65        | --             |
| Soft flour             | 0.78        | --             |
| Kaolin                 | 0.80        | --             |
ly, the amount of data obtained so far is not sufficient and the details of the results under low vertical stresses will be reported in near future.

(5) Effect of shearing rate

Although no figure is provided, the $\tau - \sigma_z$ relations do not depend on the shearing rate except at extremely low rates.

The values of $\mu_{m,s}$ and $\mu_{m,\text{max}}$ obtained are summarized in Table 2. It is interesting to note that the sequential order of $\mu_{m,s}$ values reflects the empirical flowability of the powders tested, which is scored by our handling experiences.

4. 2 Constant volume direct shear tests

Constant load direct shear tests should be conducted under various vertical stresses by keeping the initial void fraction at a constant value to obtain an internal friction factor. It is not so easy, however, to adjust the initial void fraction at a specified value under various vertical stresses and it takes rather long to draw even a single yield locus. On the contrary, a yield locus can be obtained by a constant volume direct shear test only with a single trial. Since a stress is gradually developed in the shearing plane by shearing a powder bed under a constant volume condition, the stress is then resolved to the shearing and vertical stresses which are recorded simultaneously and continuously as shearing proceeds, and the powder bed experiences various vertical stress conditions during a course of shearing. However, almost no comparison of the results with those of the constant load shear tests has been made yet.
Table 3 Internal friction factors by constant-volume and constant-load shear testers

| Powder (Mean diameter, μm) | μ by const. volume shear test | μ<sub>m,s</sub> or μ<sub>m,max</sub> by const. load shear test |
|---------------------------|-------------------------------|-------------------------------------------------------------|
| Toyoura sand (200)        | 0.88                          | 0.84                                                        |
| Glass powder (180)        | 0.64                          | 0.57                                                        |
| White alundum (150)       | 0.50                          | 0.45                                                        |
| Glass sphere-Ⅱ (140)      | 0.40                          | 0.39                                                        |
| Magnesium stearate (1.75) | 0.63                          | 0.64                                                        |
| Sugar powder (1.15)       | 0.62                          | 0.55                                                        |

The results of the constant load and volume shear tests for a glass powder (210 ~ 350 μm) are compared with each other in Fig. 15. Since τ<sub>x,max</sub> is considered to be observed as the shear stress in case of the constant volume direct shear test, the τ<sub>x,max</sub> − σ<sub>z,w</sub> and σ<sub>z,m</sub> relations are shown in Fig. 15. The τ<sub>x,max</sub> − σ<sub>z,m</sub> relations in the both tests are almost identical with each other. The τ<sub>x,max</sub> − σ<sub>z,m</sub> relations in the both cases are also similar to each other and they have a less steep slope.

Figs. 16 and 17, respectively, compare the result of constant volume shear test with those of constant load test for sugar powder (1.15 μm) and for Magnesium Stearate (1.75 μm). In these figures, the yield loci in terms of τ<sub>x,s</sub> and σ<sub>z,m</sub> are shown by broken lines. Since τ<sub>x,s</sub> is considered to be observed as the shear stress in case of the constant volume shear test of fine powders, the τ<sub>x,s</sub> − σ<sub>z,w</sub> and σ<sub>z,m</sub> relations are plotted. It can be seen from Figs. 16 and 17, that the τ<sub>x,s</sub> − σ<sub>z,w</sub> relations obtained by the constant volume test are convex as if they deviate from Coulomb's law, the τ<sub>x,s</sub> − σ<sub>z,m</sub> relations, however, can be regarded to be almost linear with some data scattered. An agreement between the results of the constant load and volume shear tests is fairly well.

The internal friction factors obtained by the both shear tests are summarized in Table 3 for a couple of the powders. The internal friction factors of the coarse powders in this table are based on the τ<sub>x,max</sub> − σ<sub>z,m</sub> relations and those of the fine powders on the τ<sub>x,s</sub> − σ<sub>z,m</sub> relations. They may be considered to be in a good agreement with each other.

5. Conclusion

It has been made clear by illustrating some examples, that the measurements of the actual vertical stress and its distribution in the vicinity of shearing plane are indispensable for evaluating a universal internal friction factor of a powder bed, which is fairly independent of experimental methods and conditions, by simple direct shear tests. If the actual vertical stresses cannot be measured in case of the constant load shear test, the initial void fraction of the powder bed in the shearing cell should be varied over a certain range to find the experimental condition where almost no dilatation and/or contraction of the powder bed takes place. Therefore, it is at least necessary to measure the vertical displacement of the powder bed during a course of shearing.

If the vertical stresses in the vicinity of shearing plane cannot be measured in case of the constant volume shear test, on the other hand, there is no way to evaluate a universal internal friction factor but to know the effect of the upper powder bed height on the yield loci. Even after the effect is known, however, there still remains a problem unsolved in how the yield loci are to be extrapolated to zero bed height.

In either case, the flowability or similar characteristics of a powder can be assessed by the internal friction factor only if the factor is correctly evaluated and is fairly independent of the experimental methods and conditions.

Nomenclature

C : shearing cohesive force [Pa or Kg/cm²]
D : inside diameter of shearing cell [m]
H<sub>1</sub> : height of powder bed in upper shearing cell [m]
H<sub>2</sub> : height of powder bed in lower shearing cell [m]
K : deformation coefficient, defined in Eq. (2) [(Pa)<sup>1−m</sup> or (Kg/cm²)<sup>1−m</sup>]
m : deformation index, defined in Eq. (2) [--]
M : mass of powder in shearing cell [kg]
n : shear index, defined in Eq. (1) [--]
\[ S : \text{cross sectional area of shearing cell} \quad [m^2] \]
\[ T_x : \text{total shear force} \quad [N \text{ or } Kg] \]
\[ t_x : \text{shearing rate} \quad [m/s] \]
\[ V : \text{initial volume of powder in shearing cell} \quad [m^3] \]

\[ \delta_x : \text{shearing or horizontal displacement} \quad [m] \]
\[ \delta_z : \text{vertical displacement of powder bed in} \]
\[ \text{shearing cell} \quad [m] \]
\[ \varepsilon_0 : \text{initial void fraction of powder bed in} \]
\[ \text{shearing cell} \quad [-] \]
\[ \mu : \text{internal friction factor} \quad [-] \]
\[ \mu_{m, \text{max}} : \text{internal friction factor derived from} \]
\[ \bar{t}_{x, \text{max}} \text{ relation} \quad [-] \]
\[ \mu_{m, s} : \text{internal friction factor derived from} \]
\[ \bar{t}_{x, s} \text{ relation} \quad [-] \]
\[ \rho : \text{density of powder particle} \quad [Kg/m^3] \]
\[ \sigma_z : \text{vertical stress in powder bed} \quad [Pa \text{ or } Kg/cm^2] \]
\[ \sigma_{z,m} : \text{actual mean vertical stress in the vicinity} \]
\[ \text{of shearing plane} \quad [Pa \text{ or } Kg/cm^2] \]
\[ \bar{\sigma}_{z,p} : \text{preconsolidation pressure} \quad [Pa \text{ or } Kg/cm^2] \]
\[ \sigma_{z,w} : \text{apparent vertical stress calculated from} \]
\[ \text{external load applied, } \Sigma_z \text{ relation} \quad [Pa \text{ or } Kg/cm^2] \]
\[ \Sigma_z : \text{external vertical load} \quad [N \text{ or } Kg] \]
\[ \tau_x : \text{shearing force} \quad [N \text{ or } Kg] \]
\[ \bar{\tau}_x : \text{shearing stress calculated from} \]
\[ T_x \quad [Pa \text{ or } Kg/cm^2] \]
\[ \bar{t}_{x,\text{max}} : \text{maximum value of } \bar{t}_x \text{ during a course of} \]
\[ \text{shearing} \quad [Pa \text{ or } Kg/cm^2] \]
\[ \bar{t}_{x,s} : \text{steady state value of } \bar{t}_x \text{ during a course of} \]
\[ \text{shearing} \quad [Pa \text{ or } Kg/cm^2] \]

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