Applying Criteria Equations in Studying the Energy Efficiency of Pump Systems

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Abstract: This paper presents a method for evaluating the energy efficiency of pump systems used to transport fluids. It is mainly scientifically applied and engineering-applied in nature and aims to propose a new approach (method) to researchers in their study of the energy efficiency of such systems. By applying the well-known scientific method of Dimensional Analysis (Buckingham π-theorem), dimensionless complexes (π-criterions and their relevant equations, which are original innovative) and are offered for the first time in the scientific literature, used in achieving an energy assessment and analysis of such systems, are obtained. The criterion \( π_1 = \frac{e}{pgD} \) represents specific energy consumption in kWh/m³ for a given pipe system with an exemplary diameter D. The criterion \( π_2 = Q/(n(H_\text{st} - H_\text{st})D^2) \) represents a generalized parameter which is characterized by the selected method of flow rate (Q) regulation for a pump system with given static head \( H_\text{st} \)—by changing the speed of rotation (VFD, Variable Frequency Drive), by throttling, leading to an increase of the system hydraulic losses \( h_c = (H_p - H_\text{st}) \) or by diverting a part of the flows, known as “by-pass”, where the pump operates with the required system head \( H_p \), but ensures higher flow rates, i.e., \( Q_p > Q_s \). The flow rate criterion \( π_3 = Q/(\nu D) \) characterizes the flow rate for a pipe system with an exemplary diameter D, used to transport a liquid with known viscosity \( \nu \). An example for applying these dimensionless complexes in accomplishing a quantitative evaluation of the energy efficiency of a given pump system is presented. A method for determining the main parameters forming these criterions, used to describe the different methods of flow rate regulation, has been developed. To demonstrate the application of this method, newly proposed by the authors, including obtaining the relevant criteria equations of the type \( π_1 = f(π_2, π_3) \), a certain pump system was used. This original approach for studying pump systems used to transport fluids can be used both to accomplish an energy analysis of such systems as well as to solve for optimization or other engineering problems.

Keywords: pump systems; dimensional analysis; energy efficiency; flow rate regulation

1. Introduction

Increasing energy efficiency when operating machines and systems is an extremely significant task. This represents a great opportunity for reasonable consumption of the energy resources.

Fluid transport systems (pump and fan) are an integral part of many technical systems—water supply, heat supply, energy, etc. It is well-known that a significant amount of electricity is used to drive the electric pump and fan units. According to [1], in 2005, the annual energy consumption (in the European Union (EU)) of pumps and fans used to transport clean water was 109 TWh. This amount corresponds to 50 million tons of CO₂ emissions. If rules for limiting this consumption are not introduced, the forecast indicates that by 2021 it may be over 136 TWh.

It can be seen that these systems often operate in modes that are significantly different from their optimal ones. This usually occurs when a system flow rate changes, i.e., when a...
flow rate regulation is required. In literature sources, for example [2–5], these issues are well-described, although mainly in a qualitative aspect. As a result, methods and methodologies for accomplishing quantitative assessment of such systems are rarely proposed.

In [6], the authors present a hybrid optimization method to improve the energy efficiency of a water supply system towards more sustainable water management concerning the water-energy nexus. Meanwhile in [7], the main purpose of the research work is to indicate the world trends in trying to achieve more sustainable forms of pressurized water distribution and to present the obtained results. In [8], the authors pay attention to a significant problem of leakage levels in water distribution systems and focus on the necessity to apply an analysis of some indicators required for obtaining an improvement of the energy analysis of such systems. The authors in [9] have tried to identify and analyze some factors that may be responsible for the higher leakage exponents in systems used to transport water. Another paper [10] aims to review the methods and models developed in the past used in different phases of the process—from becoming aware of the leak’s existence to controlling the level of leakage in the system. According to [11], the variable demand requires highly efficient management of both water resources and energy resources—it has to be noticed that the study concerns large irrigation systems.

It is well-known that the energy consumption of a pump system depends on the selected method of flow rate regulation [12]. In practice, three ways of flow rate regulation are most often used—throttle, by increasing the hydraulic losses in the pipe system, frequency (VFD), by changing the speed n, and diverting a part of the flow from the system (“by-pass” control).

In [13], the most commonly used methods (concerning fixed and variable speed pumps) for controlling pump systems are studied based on their design. A methodology newly proposed by the authors is used for this aim. Another study [14] is focused on VFD flow rate regulation and, more specifically, aims to assess the accuracy of two well-known empirical equations used to estimate a pump system’s total efficiency and to present a new option for the estimation of efficiency when the pump system operates at new conditions. In [15], the results obtained, in terms of energy savings, by optimal regulation with VSPs versus CSPs (ON/OFF regulation) are analyzed, and optimizations are achieved by means of a specific Genetic Algorithm (GA) in both cases.

A general review and comparative analysis of the well-known methods for assessing the energy efficiency of water supply systems, which are described in the scientific literature and technical standards, is presented in [16].

The authors in [17] have demonstrated that in the era of IoT and big data, the data collected by Supervisory Control and Data Acquisition (SCADA) systems can be used to continuously monitor pumps’ performance and derive updated pump operating curves. In [18], the feasibility of using two parallel pumps in a multi-pump system instead of a single-pump system is examined, mainly taking into account the energy efficiency. For this aim, an optimization mathematical model is used.

To accomplish an analysis for proper energy assessment, pump operating curves (for commercial pumps available on the market) provided by the manufacturer can be used. In this regard, a study is presented in [19] where the conclusions are integrated in a gradient-based procedure using the Granados System as a base, giving rise to an optimized design method that accounts for the construction and operational costs.

The authors in [20] represent in details of four strategies for improving the energy efficiency of water pumping: control systems to vary pump speed drive according to water demand, pumped storage tanks, intermediary pumping stations integrated in the network, and elevated storage tanks floating on the system.

Another paper [21] represents a methodology for optimizing the operation of parallel pumping stations in an open-channel water transfer system, where a mathematical model is established for determining the minimum power with constraints on water level, flow rate and pump unit performance, and related factors. In [22], a novel management model is presented for the optimal design and operation of water pumping systems on
a real case study. The work is focused on the existence of significant opportunities to reduce pump system energy consumption through applying smart design, retrofitting, and operating practices.

According to [23], it can be seen that the presented results obtained after simulations indicate that the optimization scheme, newly proposed by the authors, leads to significant reduction in energy consumption as compared to other two alternative schemes, i.e., pumping with maximum and minimum flow rates.

This current work aims to obtain some criteria equations, including dimensionless complexes (criteria) that may be applied in analyzing the energy efficiency of pump systems used to transport liquids. To obtain such equations, the well-known and widely used scientific research method of Dimensional Analysis (DA) was applied.

2. Materials and Methods

LITERATURE SOURCES ANALYSIS

To assess the energy efficiency of pump units consisting of a pump and an electric motor or a pump, an electric motor, and a frequency inverter, the Energy Efficiency Index (EEI) is defined in [24]. It may be estimated as follows:

\[ \text{EEI} = \frac{P_{1,\text{avg}}}{P_{1,\text{ref}}} \]  

(1)

where \( P_{1,\text{avg}} \) is the weighted average value of the electrical power consumed by the pump unit, \( P_{1,\text{ref}} \) is the reference electrical power consumed by a pump unit when working at its nominal operating regime with constant speed of rotation.

A methodology for estimating the EEI of pump units working with constant or variable speed is proposed in [25]. The EEI indicator is used to evaluate and compare the energy efficiency of pump units produced by different manufacturers.

To accomplish a quantitative assessment of the energy consumption in regulating the flow rate of a pump system, it is appropriate to use the specific energy consumption, defined in [2]. It represents the consumed energy, \( E \), used for the transportation of a unit volume of liquid, \( V \), through the pipe system:

\[ e_v = \frac{E}{V}, \text{[kWh/m}^3\text{]} \].  

(2)

By presenting the system input energy as a multiplication between the pump power \( P_p \) and operating time \( t \) and by taking into account that the system volume flow rate \( Q_s \) is most often equal to the pump flow rate \( Q_p \), i.e., \( Q_s = V_t = Q_p \), it can be stated:

\[ e_v = \frac{P_p t}{1000 V} = \frac{\rho g H_p Q_p}{\eta V t^2} = \rho g \frac{H_p}{\eta}, \]  

(3)

where \( \rho \) is the density of the transported liquid, \( g \) is gravity acceleration, \( H_p \) and \( Q_p \) are pump head and flow rate, respectively, \( \eta \) is coefficient of efficiency of the motor-pump unit.

In the presence of flow rate losses (leaks) in Equation (3) the ratio \( Q_s/Q_p < 1 \) has to be included, which allows to determine the impact of these losses on the value of \( e_v \).

Replacing the power \( P_p \) and time \( t \), in Equation (3), in (kW) and (h) respectively, and taking into account that \( \rho = 1000 \text{ kg/m}^3\) (density of cold water) enables the estimation of the specific energy consumption by using the following equation:

\[ e_v = \frac{g H_p}{3600 \eta}, \text{kWh/m}^3. \]  

(4)

In [26], the authors propose a methodology for analyzing the energy efficiency of pump systems by using the proposed criteria of effectiveness, \( e_v \). The results obtained from an analytical study concerning the energy efficiency when applying different methods of
flow rate regulation, performed in accordance with the previously mentioned methodology, are presented in [12]. The results obtained after accomplishing similar types of studies are published in [18,27–29].

BASIC FEATURES CHARACTERIZING THE ENERGY CONSUMPTION OF PUMP SYSTEMS, USED TO TRANSPORT FLUIDS

An interesting approach for analyzing the energy efficiency of fan systems is proposed in [30]. The authors used Dimensional Analysis (DA), more specifically, the Vaschy–Buckingham method or Method of $\pi$, presented in [31], to obtain criterions for evaluating the energy efficiency of two methods for regulating the flow rate of fan systems, throttle and frequency (VFD).

In this current work, a similar approach for analyzing the energy efficiency of pump systems used to transport clean water was applied.

By using the $\pi$-criteria obtained in this work, three methods for regulating the flow rate of systems used to transport fluids—throttle, frequency (VFD), and diverting a part of the flow (by-pass)—were analyzed. Criteria equations for the three methods of flow rate regulation in pump systems were obtained.

The operating mode (regime) of a pump system is determined by studying the cooperation of a pump (or co-operating pump units) and the pipe system to which it is installed.

The classical approach is the graphical solution—determining the intersection of the pump head operating curve (or the equivalent characteristic of co-operating pumps) $H_p = f(Q_p)$ and the pipe system resistance curve, expressing the relationship between the required system head $H_s$ for ensuring a flow rate $Q_s$ passing through it: $H_s = f(Q_s)$.

It is clear that the parameters determining these two curves will have an impact on the energy consumption in each pipe system.

The operating curves of a pump represent the relationships between its head $H_p$ and coefficient of efficiency $\eta$, given as functions of the flow rate $Q$ in case of working at given constant speed.

The pump system head curve represents:

$$H_s = H_{st} + h_v,$$

where $H_{st}$ is the system static head, which is also known as system useful head, $h_v$ is energy loss (in form of head) as a result of the existing hydraulic resistances.

The hydraulic energy losses in a given pipe system, $h_v$, are represented by the equation:

$$h_v = k Q_s^2,$$

where $k$ is the coefficient of resistance describing the pipe system resistance curve, which is determined by the pipe sizes and existing hydraulic resistances.

Taking into account the above paragraphs, a list of the parameters that have an impact on the energy consumption of pump systems may be established:

**Pump operating parameters:**
- Head $H_p$, (m)
- Pump (system) flow rate $Q_p$, (m$^3$/s)
- Speed of rotation $n$, (s$^{-1}$)
- Coefficient of efficiency $\eta$

**Pipe system resistance curve parameters:**
- Pipe system conventional diameter $D$, (m)
- Hydraulic energy loss (in form of head) $h_v = H_p - H_{st}$, (m)
**Liquid type and properties:**

- density \( \rho \) (kg/m\(^3\))
- dynamic viscosity \( \mu \) (Pa·s)

**DIMENSIONLESS PARAMETERS (CRITERIONS) USED IN ANALYZING THE PUMP SYSTEMS OPERATING REGIMES**

As can be seen from the prepared list, the total number of operating parameters is \( N = 8 \), the dimensions of which are given by three basic units of the SI system—mass \( M \) in kilograms [kg], length \( L \) in meters [m] and time \( T \) in [s]. Therefore, the number of newly established dimensionless complexes (\( \pi \)-criterions) will be \( m = 5 \). The dimension equations concerning the mentioned parameters are given in Table 1.

**Table 1. Basic information concerning the used operating parameters.**

| No. | Parameter                  | Equation     |
|-----|----------------------------|--------------|
| 1   | Pump head \( H_p \)        | \([M^0 L^1 T^0]\) |
| 2   | Pump flow rate \( Q_p \)   | \([M^0 L^3 T^{-1}]\) |
| 3   | Speed \( n \)              | \([M^0 L^0 T^{-1}]\) |
| 4   | Coefficient of efficiency \( \eta \) | \([M^0 L^0 T^0]\) |
| 5   | Pipe system energy (head) loss \( h_v \) | \([M^0 L^1 T^0]\) |
| 6   | Pipe system conventional diameter \( D \) | \([M^0 L^1 T^0]\) |
| 7   | Density \( \rho \)         | \([M^1 L^3 T^0]\) |
| 8   | Dynamic viscosity \( \mu \) | \([M^1 L^{-1} T^{-1}]\) |

For new basic measuring units, the following ones were selected:

- density \( \rho = [M^1 L^{-3} T^0] \)
- pump head \( H_p = [M^0 L^1 T^0] \)
- dynamic viscosity \( \mu = [M^1 L^{-1} T^{-1}] \)

Using the well-known method of Dimensional Analysis (DA), the following five dimensionless complexes (\( \pi \)-criteria) are obtained:

\[
\pi_1 = \frac{\rho n H_p^2}{\mu} ; \quad \pi_2 = \frac{\rho Q_p}{\mu H_p} ; \quad \pi_3 = \frac{h_v}{H_p} ; \quad \pi_4 = \frac{D}{H_p} ; \quad \pi_5 = \eta.
\]

Combining the obtained criterions and multiplying and dividing with their appropriate constants enables to establish of the corresponding dimensionless parameters, denoted by \( \Pi \).

After combining \( \pi_4 = \frac{D}{H_p} \) and \( \pi_5 = \eta \) and then multiplying and dividing it with the volume weight \( (\rho g) \), the following equation is obtained:

\[
\Pi_1 = \frac{1}{\pi_4 \pi_5} = \frac{\rho g H_p}{\eta} \frac{1}{\rho g D} . \tag{7}
\]

Equation (7) clearly indicates that \( \rho g H_p / \eta \) represents the specific energy consumption, \( e_v \). Therefore, the first dimensionless parameter \( \Pi_1 \) is presented in the following way:

\[
\Pi_1 = e_v \frac{1}{\rho g D} . \tag{8}
\]
Combining the criterions \( \pi_1 \) with \( \pi_3 \) and \( \pi_4 \) enables to establish the second dimensionless parameter \( \Pi_2 \):

\[
\Pi_2 = \frac{\pi_2}{\pi_1 \pi_3 \pi_4} = \frac{\rho Q_p}{\mu D} \left( \frac{H_p}{H_p} \right) \frac{\rho H_p}{\mu H_p} \frac{h_v}{D^2},
\]

which can be presented as follows:

\[
\Pi_2 = \frac{Q_p}{n h_v D^2}.
\]  

Equation (3) clearly indicates that the hydraulic energy (head) losses \( h_v \) represent the difference between the pump head \( H_p \) and the static head \( H_{st} \) of the pump system:

\[
h_v = H_p - H_{st}.
\]  

Therefore, the following is obtained for \( \Pi_2 \):

\[
\Pi_2 = \frac{Q}{n (H_p - H_{st}) D^2}.
\]  

The criterion \( \pi_3 \), indicating the pipe system resistance curve, may be presented as \( \Pi_3 \):

\[
\Pi_3 = \pi_3 = \frac{h_v}{H_p} = \frac{H_p - H_{st}}{H_p},
\]

which takes the following final form:

\[
\Pi_3 = 1 - \frac{H_{st}}{H_p}.
\]  

The flow rate criterion \( \pi_2 \) is revised as follows:

\[
\Pi_5 = \frac{\pi_2}{\pi_4} = \frac{\rho Q_p}{\mu H_p} \frac{H_p}{D} = \frac{\rho Q_p}{\mu D}.
\]  

Taking into account that \( \mu/\rho = \nu \) is the liquid kinematic viscosity, the dimensionless complex \( \Pi_4 \) is determined as follows:

\[
\Pi_4 = \frac{Q_p}{\nu D}.
\]  

The efficiency criterion retains its form:

\[
\Pi_5 = \pi_5 = \eta.
\]  

The general criteria relationship has the form:

\[
f\left( \Pi_1; \Pi_2; \Pi_3; \Pi_4; \Pi_5 \right) = 0.
\]

Thus, the criterion \( \Pi_1 \), including the specific energy consumption \( e_v \), can be presented as follows:

\[
e_v \frac{1}{\rho g D} = f\left( \frac{Q_p}{n (H_p - H_{st}) D^2}; \left( 1 - \frac{H_{st}}{H_p} \right); \frac{Q_p}{\nu D}; \eta \right).
\]

Accomplishing an analysis of the dimensionless parameters (criterions) obtained in this work leads to the indication of the following features:

- The criterion \( \Pi_1 = e_v \frac{1}{\rho g D} \) is an indicator concerning the specific energy consumption for a pipe system having a conventional diameter \( D \);
- the criterion $\prod_2 = \frac{Q_p}{n(H_p - H_{st})D^2}$ represents a generalized parameter (feature) used in characterizing the selected method of pump system flow rate regulation—changing the pump speed $n$ (VFD), throttling that leads to an increase of hydraulic energy (head) losses in the pipe system, $h_v = H_p - H_{st}$, or diverting a part of the flow (the so-called “by-pass” regulation), where the pump operates with the required system head $H_s$, but ensures higher flow rates, i.e., $Q_p > Q_s$;

- the criterion $\prod_3 = 1 - \frac{H_{st}}{H_p}$ describes the ensured pump head $H_p$, representing the system useful head that is equal to its static head $H_{st}$;

- the flow rate criterion $\prod_4 = \frac{Q_p}{\nu D}$ indicates the ensured pump flow rate $Q_p$ in transporting a liquid with viscosity $\nu$ through a pipe having a conventional diameter $D$.

**SPECIFIC ENERGY CONSUMPTION IN REGULATING THE FLOW RATE OF PUMP SYSTEMS**

To calculate the values of the previously obtained dimensionless complexes (criterions), it is necessary to determine their constituent parameters: pump operating parameters for the given operating mode: head $H_p$, flow rate $Q_p$, and coefficient of efficiency $\eta_p$, used in determining the specific energy consumption $e_v$, as well as the pipe system resistance curve parameters: static head $H_{st}$ and hydraulic energy (head) losses $h_v$ for the ensured flow rate.

For the aims of this current theoretical research, it is necessary to know the equations describing the pump head $H_p = f(Q_p)$ and coefficient of efficiency $\eta_p = f(Q_p)$, which are obtained by using the manufacturers’ catalogues, where the pump operating curves are presented graphically. For centrifugal pumps, these equations are most often given in the following form:

\[
H = a + bQ_p + cQ_p^2, \quad (18)
\]

\[
\eta_p = d + eQ_p + fQ_p^2, \quad (19)
\]

where $a, b,$ and $c$ are the coefficients in the head equation, and $d, e,$ and $f$—the coefficients in the efficiency equation. These coefficients are obtained in accordance with the methodology given in [32]. For this purpose, the performance characteristics of the catalog data of the manufacturers were used. They were scanned using a specialized software product and further processed in Excel.

The specific energy consumption, $e_v$, at a certain pump flow rate is obtained using the above equations:

\[
e_v = \frac{g}{3600} \left( \frac{a + bQ_p + cQ_p^2}{d + eQ_p + fQ_p^2} \right). \quad (20)
\]

As it is known, the most commonly used methods for regulating the flow rate of a pump system consisting of centrifugal pumps are throttling and frequency (VFD). In some cases, the regulation is performed by diverting a part of the flow through a drain pipe (so-called “by-pass” method).

To determine the specific energy consumption when ensuring a given new lower flow rate, an individual approach concerning each of the selected methods is used.

**Specific energy consumption in regulating a pump system flow rate by throttling**

In throttling, the operating points that determine the pump operating modes are shifted to the left along the pump head curve compared to the operating point obtained in case that the throttle element is fully opened. In this case, the pump head increases, and the flow rate decreases. At the same time, the efficiency of the pump also changes.

At the lower flow rate $Q_{p,1}$ the specific energy consumption of the new pump operating mode, obtained by throttling, is determined as follows:

\[
e_{v,1, dr} = \frac{g}{3600} \frac{H_{p,1}}{\eta_{p,1}} = \frac{g}{3600} \left( \frac{a + bQ_{p,1} + cQ_{p,1}^2}{d + eQ_{p,1} + fQ_{p,1}^2} \right). \quad (21)
\]
Specific energy consumption in regulating a pump system flow rate by using VFD

When applying the frequency (VFD) method of flow rate regulation, the pump operating points are shifted by the pipe system resistance curve, and in case of speed reduction, the pump head and flow rate are decreased.

At the new lower flow rate \( Q_{p,1} \), the pump head \( H_{p,1} \) is determined by the pipe system resistance curve—\( H_{s,1} = H_{st} + kQ_{1}^{2} \). The coefficient \( k \) is determined by using the pump operating parameters when working with its nominal speed—nominal flow rate \( Q_{p,n} \) and nominal head \( H_{p,n} \):

\[
k = \frac{H_{p,n} - H_{st}}{Q_{p,n}^{2}}.
\]  

(22)

Therefore, the head of the new pump operating mode may be estimated as follows:

\[
H_{p,1} = H_{p,n} \left[ \frac{H_{st}}{H_{p,n}} + \left( 1 - \frac{H_{st}}{H_{p,n}} \right) \left( \frac{Q_{p,1}}{Q_{p,n}} \right)^{2} \right].
\]

(23)

The value of the pump coefficient of efficiency \( \eta_{p,1} \) for the new operating mode, obtained at lower speed \( n_{1} \), is determined by using the similarity equations. The similar pump operating mode is obtained at the nominal speed \( n_{n} \), as for this aim, the equation \( \eta_{p} = f(Q) \), given in the form of Equation (19), is preliminary known. This is accomplished by using the parabola of similarity, where its coefficient is determined by taking into account the flow rate \( Q_{p,1} \) and head \( H_{p,1} \). For the similar pump operating mode, obtained at the nominal speed \( n_{n} \), the flow rate \( Q_{N,1} \) is determined as follows:

\[
Q_{N,1} = \frac{1}{2} \left( \frac{H_{p,1}}{Q_{p,1}^{2} - c} \right) \left( \frac{1}{b_{p} + 4a} \left( \frac{H_{p,1}}{Q_{p,1}^{2} - c} \right) + b \right).
\]

(24)

For estimating the coefficient of efficiency when the pump operates at a certain mode, ensuring flow rate \( Q_{p,1} \) and head \( H_{p,1} \), the following equation is used:

\[
\eta_{p1} = d + e Q_{N,1} + f Q_{p,1}^{2}.
\]

(25)

The specific energy consumption in obtaining the new flow rate using VFD may be estimated by indicating the pump unit (aggregate) parameters, determined in accordance with Equations (23) and (25):

By analyzing the obtained equation it can be seen that the specific energy consumption depends on both the new flow rate \( Q_{p,1} \) and system static head \( H_{st} \).

Specific energy consumption in regulating a pump system flow rate by using “by-pass”

Using this method of regulation leads to the operating point being shifted to the right along the pump head curve compared to the operating point when working with a completely closed regulating element in the discharge pipe—flow rate \( Q \) and head \( H \). The ensured system flow rate (supplied to the user) is determined by using the pipe system resistance curve and the corresponding pump head.

The pump head \( H_{p,1} \) is determined by applying Equation (23), indicating the set system flow rate \( Q_{s,1} \) representing a part of the pump flow rate \( Q_{p,1} \) at this certain mode, which is estimated as follows:

\[
Q_{p,1} = \frac{1}{2c} \left( \sqrt{b_{p}^{2} - 4c(a - H_{p,1})} - b \right).
\]

(26)

Using the flow rate obtained previously allows estimating the pump coefficient of efficiency:

\[
\eta_{p1} = d + e Q_{p,1} + f Q_{p,1}^{2}.
\]

(27)
The specific energy consumption $e_{v,b}$ for this method of regulation is determined by indicating the fact that the flow rate $Q_s$ only represents a part of the pump flow rate $Q_p$.

Taking into account Equation (3), it can be concluded that:

$$e_{v,b} = \frac{P}{Q_1} = \frac{\rho g Q_p H_1}{3600 \ 1000 \eta_p Q_1} = \frac{g}{3600} \ \frac{H_1}{\eta_p} \ \frac{Q_p}{Q_1}. \quad (28)$$

By replacing Equations (19) and (23) in the equation above, the following equation is obtained:

$$e_{v1,b} = \frac{g}{3600} \ \frac{H_{p,1}}{(d + e Q_p + f Q_p^2)} \left[ \frac{H_{st}}{H_{p,1}} + \left(1 - \frac{H_{st}}{H_{p,1}}\right) \left(\frac{Q_1}{Q_p}\right)^2 \right] \frac{Q_p}{Q_1}, \text{ kWh/m}^3 \quad (29)$$

It can be seen that the system static head $H_{st}$ has impact on the specific energy consumption when applying both VFD and “by-pass” methods of flow rate regulation.

Equation (29) indicates that the ratio between the useful (supplied to the user) flow rate $Q_s$ and total pump flow rate $Q_p$ has a significant impact on the system’s energy consumption. As this ratio increases, the energy used for transporting a unit quantity of liquid increases in direct proportion.

### 3. Results and Discussion

**APPLICATION OF THE CRITERIONS OBTAINED IN ANALYZING THE ENERGY EFFICIENCY OF PUMP SYSTEMS**

To demonstrate the use of the dimensionless parameters (criterions) newly established in this work, a pump system consisting of a pump 50E50, produced by the manufacturer VIPOM-Bulgaria, was studied (analyzed). The coefficients related to Equations (18) and (19), describing the pump operating curves, obtained at a constant speed of $n = 2900 \text{ min}^{-1}$, are given in Table 2.

#### Table 2. Values of the equations describing the pump operating curves (determined in accordance with the methodology that is presented in [30]).

| a     | b     | c        | d     | e     | f        |
|-------|-------|----------|-------|-------|----------|
| 56.412 | 0.2432 | −0.0079  | 12.9  | 2.642 | −0.0259  |

The pump operating parameters when working at its nominal operating mode were set to be:

- nominal flow rate $Q_n = 50 \text{ L/s}$;
- nominal head $H_n = 49 \text{ mwc}$;
- coefficient of efficiency $\eta = 0.8$.

For this certain operating mode, the specific energy consumption determined by Equation (3) is obtained:

$$e_{v,n} = \frac{g}{3600} \ \frac{H_n}{\eta_n} = 0.166 \text{ kWh/m}^3. \quad (30)$$

When the pump operates at its nominal operating mode (regime), where the system static head is $H_{st} = 0$ ($\Pi_3 = 1$) and the pipe system conventional diameter is $D = 0.18 \text{ m}$, the values of $\Pi_1$, $\Pi_2$, and $\Pi_4$ are estimated by applying the following equations:

$$\Pi_{1,n} = \frac{e_{v,n}}{\rho g D} = 2.608 \times 10^{-5}; \quad (31)$$

$$\Pi_{2,n} = \frac{Q_n}{n \eta_n (H_n - H_{st}) D^2} = 1.09 \times 10^{-3}; \quad (32)$$
\[ \Pi_{4,n} = \frac{Q_n}{\sqrt{D}} = 0.278 \times 10^6. \tag{33} \]

To normalize the values of the criterions \( \Pi_1, \Pi_2, \) and \( \Pi_4 \), they are related to their corresponding parameters. However, when the pump operates at its nominal operating mode:

\[ \Pi_1^* = \frac{\Pi_1}{\Pi_{1,n}}, \quad \Pi_2^* = \frac{\Pi_2}{\Pi_{2,n}} \quad \text{and} \quad \Pi_4^* = \frac{\Pi_4}{\Pi_{4,n}}, \tag{34} \]

In Figure 1, the results obtained after applying the equation \( \Pi_1^* = f(\Pi_3^*) \) for the three selected methods of regulation are presented. The selected output operating mode was the one when the pump works with its nominal parameters—\( Q_n, H_n, \) and \( \eta_n \). This relationship indicates the change of the specific energy consumption, \( e_v \), in ensuring a certain flow \( Q \) by applying the three methods of regulation.

Figure 1. Presenting the relationship between the criterions \( \Pi_1^* \) and \( \Pi_4^* \).

The nature of the change in the criterion \( \Pi_1^* \) (concerning the specific energy consumption \( e_v \)) with decreasing \( \Pi_4^* \) (concerning the flow rate \( Q \)) fully corresponds to the well-known fact that the frequency method of flow rate regulation surpasses the other methods in terms of energy efficiency.

Figure 2 represents the relationship between the specific energy consumption (criterion \( \Pi_1^* \)) and the parameters of flow control—head loss (\( H-H_{st} \)) in applying throttle control (regulation), speed \( n \) in applying frequency (VFD) control, or the total pump flow rate, a part of which is supplied into the system when applying “by-pass” control.

It can be seen that when “by-pass” control is applied, the curve \( \Pi_1^*=f(\Pi_2^*) \) increases significantly upwards, which is due to the fact that the pump operates at a high flow rate, but only a part of it is sent to the user.

The lower the system flow rate \( Q_s \), the higher the pump flow rate \( Q_p \) and thus, a greater part of it is supplied back to the suction tank.

Since in this method of regulation the returned flow rate \( \Delta Q = Q_p - Q_s \) has the most significant impact on the determination of energy losses, this also leads to an increase in the criterion \( \Pi_1^* \).

To accomplish an energy analysis using the previously obtained dimensionless parameters (criterions), it is convenient to know the existing relationship between the main ones—\( \Pi_1, \Pi_2, \) and \( \Pi_3 \).
The dimensionless parameters (criterions) newly established in this work based on applying Dimensional Analysis (DA) represent an innovative approach to analyzing the energy efficiency of pump systems used to transport fluids. The criterion $\Pi_1$ (Equation (8)) was introduced as an indicator for the specific energy consumption. As a generalized parameter, characterizing the selected method for flow rate regulation, the criterion $\Pi_2$ (Equation (10)) was established. The pipe system resistance curve may be analyzed by using the dimensionless parameter $\Pi_3$ from Equation (13).

The criterion $\Pi_4$ enables the indication of the specifics of pump systems used to transport liquids other than water, i.e., those with different kinetic viscosity $\nu$.

The dimensionless complexes above can be modified, for example, by selecting another parameter as a basic geometric size. For instance, the pump impeller outside diameter $D_{p,2}$ can be used, which enables to study the impact of impeller blade trimming, known as a method of flow rate regulation (reduction), on energy consumption.

These dimensionless complexes, newly established by the authors, are used to solve various tasks, such as energy analysis of pump systems, optimization of individual elements (pump units, pipe network, etc.) of a pump system, as well as other engineering tasks concerning systems used to transport fluids.

Figure 2. Presenting the relationship between the criterions $\Pi_1^*$ and $\Pi_2^*$, including the parameters of flow control.

For the certain analyzed system, consisting of a pump 50E50, the following criteria equations concerning the three methods of flow control were obtained:

- in case of throttling:

$$\Pi_1^* = \frac{\Pi_3^{0.03}}{\Pi_4^{0.4}};$$  \hspace{1cm} (35)

- in case of using VFD (frequency):

$$\Pi_1^* = \frac{\Pi_3^{0.065}}{\Pi_2};$$  \hspace{1cm} (36)

- in case of using “by-pass”:

$$\Pi_1^* = 0.9707 \frac{\Pi_3^{0.5}}{\Pi_3^{0.12}}.$$  \hspace{1cm} (37)

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation, as well as the experimental conclusions that can be drawn.

4. Conclusions

The dimensionless parameters (criterions) newly established in this work based on applying Dimensional Analysis (DA) represent an innovative approach to analyzing the energy efficiency of pump systems used to transport fluids. The criterion $\Pi_1$ (Equation (8)) was introduced as an indicator for the specific energy consumption. As a generalized parameter, characterizing the selected method for flow rate regulation, the criterion $\Pi_2$ (Equation (10)) was established. The pipe system resistance curve may be analyzed by using the dimensionless parameter $\Pi_3$ from Equation (13).

The criterion $\Pi_4$ enables the indication of the specifics of pump systems used to transport liquids other than water, i.e., those with different kinetic viscosity $\nu$.
An expansion of the range of problem solutions can be obtained by using other specific linear dimensions. For example, using the outer impeller diameter $D_{p,2}$ of the analyzed centrifugal pump instead of the equivalent diameter $D$ of the pipe system as a specific linear size enables the analyzing the impact of impeller trimming (known as a method of flow rate regulation) on pump system operating modes.

An advantage of the novel method presented in this work, compared to other well-known previously established methods, is the possibility to theoretically obtain the criteria equations of the type (35), (36), and (37), which can be added to the catalog (passport) data of the pump units provided by manufacturers. This can be used by users to accomplish (quickly and easily) an assessment and analysis of the energy efficiency of a given pump system.

The results presented in this paper, demonstrating the application of the dimensionless criteria equations newly established by the authors, fully correspond to the well-known facts regarding the efficiency of the applied methods for regulating the flow rate of pump systems including centrifugal pumps. This is an indirect indicator of the adequacy (accuracy) of this innovative approach in developing a methodology for assessing the energy efficiency of systems used to transport fluids. This methodology can also be applied to systems with co-operating pump sets (complex systems consisting of parallel or serial connected pumps) by taking into account the different cases of location and types of pumps.

A limitation of the presented work methodology is that it is refers to systems with turbopumps (mainly centrifugal) containing one pump unit. In the case of two or more pump units working together, well-known hydraulic relationships (equations) have to be used for determining their operating parameters involved in the dimensionless criterion equations. For systems containing Hydraulic Positive Displacement Pumps, which are of limited use in practice, the specific differences (features) between these two types of machines have to be taken into account.

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