Event plane resolution correction for azimuthal anisotropy in wide centrality bins

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Abstract
We provide a method to correct the observed azimuthal anisotropy in heavy-ion collisions for the event plane resolution in a wide centrality bin. This new procedure is especially useful for rare particles, such as Ω baryons and J/ψ mesons, which are difficult to measure in small intervals of centrality. Based on a Monte Carlo calculation with simulated \(v_2\) and multiplicity, we show that some of the commonly used methods have a bias of up to 15%.

Keywords: Azimuthal anisotropy, flow, event plane

1. Introduction
Azimuthal anisotropy is one of the key observables to study the properties of matter created in high energy heavy-ion collisions (see e.g. [1]). It is characterized by the Fourier decomposition of the azimuthal particle distribution with respect to the reaction plane

\[
\frac{dN}{d(\phi - \Psi_{RP})} = \frac{N_0}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos \left[ n(\phi - \Psi_{RP}) \right] \right],
\]

where \(N_0\) is the number of particles, \(v_n\) is the \(n\)-th harmonic coefficient, \(\phi\) is the azimuthal angle of particles and \(\Psi_{RP}\) is the azimuthal angle of the true reaction plane which is determined by the beam axis and impact parameter.

One of the standard methods to extract \(v_n\) is the event plane method [2]. The most important task in this method is to estimate the reaction plane from the measured particles for each harmonic \(n\). We do not distinguish in this paper between reaction plane and participant plane since we discuss only resolution correction. The estimated reaction plane is defined as the event plane \(\Psi_n\) (\(-\pi/n \leq \Psi_n \leq \pi/n\)), but due to the finite multiplicity in nuclear collisions, the event plane can be different from the reaction plane. The observed \(v_n^{obs}(M)\) for a given, small, centrality range \(M\) must be corrected by the event plane resolution \(R_n(M)\) in order to take into account the difference between true reaction plane and event plane

\[
v_n(M) = \frac{\langle \langle \cos [n(\phi - \Psi_{m})] \rangle \rangle_M}{\langle \cos [n(\phi - \Psi_{RP})] \rangle_M} \equiv \frac{v_n^{obs}(M)}{R_n(M)},
\]

where \(m\) is the harmonic of the event plane and \(n = km\) is the harmonic of interest. Brackets denote the average over events, while double brackets denote the average over particles in all events. Subscript \(M\) on the bracket emphasizes that the average is taken for a given centrality \(M\). For simplicity, we will omit \(M\) from the observables, for example, we will write \(N_0\) instead of \(N_0(M)\).

In experiment the reaction plane angle \(\Psi_{RP}\) in Eq. (2) is not known. Therefore at least two subevent planes are necessary in order to calculate the event plane resolution [3]. An average \(v_n\) over a wider centrality range \(R\) can be calculated once \(v_n^{obs}\) and \(R_n\) are determined within \(R\)

\[
\frac{\int_R dM N_0 v_n^{obs}}{\int_R dM N_0} = \frac{\langle v_n^{obs} \rangle}{\langle R_n \rangle} = \langle v_n \rangle.
\]

We introduced brackets (…) for simplicity, which denote the average over a wide centrality range weighted by particle multiplicity \(N_0\) for a given phase space (e.g. for a given transverse momentum).

\(R_n\) depends on multiplicity and \(v_n\) itself, therefore Eq. (3) requires that \(v_n^{obs}\) and \(R_n\) are measured in sufficiently small centrality intervals. However, for rare particles (e.g. Ω, J/ψ) it is not always possible to do so. One of the conventional approaches (e.g. see Ref. [3]) is to average \(v_n^{obs}\) as well as \(R_n\) separately in a wide centrality range, weighted by the corresponding particle yields, and then take the ratio of \(\langle v_n^{obs} \rangle\) to \(\langle R_n \rangle\). However, this approach systematically overestimates \(v_n\) as we will discuss in the Section 3. The main point of this paper is the
following inequality
\[ \langle v_{\text{obs}}^n \rangle = \frac{1}{K_n} \neq \langle v_{\text{obs}}^n \rangle \neq \frac{1}{K_n} \langle v^n \rangle \]  (4)
where the left hand side is the correct average over a wide centrality bin, while the right hand side shows a commonly used approximation.

We will show the proper way to correct for the wide centrality bins. In Section 2, we derive the equations used to calculate \( v^n \) in our approach. We also show that the derived equations are equivalent to the average calculated from narrow centrality bins (see Eq. (4)). In Section 3 we show a fast Monte Carlo simulation to demonstrate the validity of the method, for the case of \( v_2 \).

2. Implementation

We now show several practical implementations to correct \( v^n \) for the event plane resolution in wide centrality bins. There are two or three steps to calculate \( v^n \) in wide centrality bins:

1. Determine the event plane resolution \( R_n \) as a function of narrow centrality ranges \( M \).
2. Analyse \( v^n \) with the weights \( 1/R_n \) for any, wide, centrality range of interest.
3. Perform an additional resolution correction (depends on implementation).

In the following two subsections, we discuss detailed implementations how to apply corrections separately for different types of particle identification. For the sake of simplicity, we assume that non-flow effects are negligible, and all correlations between particles are induced by flow. Namely, the azimuthal particle distribution with respect to the event plane can be written as
\[ \frac{dN}{d(\phi - \Psi_m)} = \frac{N_0}{2\pi} \left( 1 + 2 \sum_{n} v_{\text{obs}}^n \cos [n(\phi - \Psi_m)] \right) \]  (5)

2.1. Event-by-event particle identification

If the particle of interest can be identified on an event-by-event basis, one can directly calculate \( \cos [n(\phi - \Psi_m)] \) for every particle, corrected with the event plane resolution for the corresponding centrality \( M \)
\[ \cos [n(\phi - \Psi_m)] \frac{R_n}{K_n} \]  (6)
where \( K_n \) is supposed to be averaged over many events in advance. The event and centrality average of term (6) over \( \phi - \Psi_m \) reduces in this case to Eq. (3)
\[ \langle v_{\text{obs}}^n \rangle = \frac{\int dM \int_{-2\pi}^{2\pi} d\phi \frac{dN/\cos [n(\phi - \Psi_m)]}{d(\phi - \Psi_m)} \frac{dM}{R_n}}{\int dM \frac{dN}{R_n}} = \langle v^n \rangle \]  (7)
The main difference of our implementation to the conventional approach is the event-by-event \( R_n \) correction in Eq. (7). As we have mentioned earlier, event-by-event \( R_n \) correction does not mean that the \( R_n \) should be calculated event-by-event, but rather the correction is made event-by-event. Practically, the statistical error on the event plane resolution can be ignored if the event plane resolution is good. However, if the event plane resolution is very small then one needs to propagate also the statistical error on the event plane resolution into the final \( v^n \) values.

2.2. Statistical particle identification

There are two approaches in case the particle yield of interest can only be extracted statistically: the invariant mass fit method and the event plane method. The invariant mass fit method is almost equivalent to the one introduced in Section 2.1 while the event plane method needs one additional step to obtain the final \( v^n \), which will be discussed below.

2.2.1. Invariant mass fit method

The invariant mass method is quite useful to analyse particles that are detected through their decay products, such as \( K^0_s \rightarrow \pi^+\pi^- \), \( \Lambda \rightarrow p\pi^- \) and so on. The point of this method is to calculate the mean \( \cos [n(\phi - \Psi_m)] \) as a function of invariant mass \( M_{\text{inv}} \)
\[ v^n_{\text{inv}}(M_{\text{inv}}) = \langle \cos [n(\phi - \Psi_m)] \rangle_{\text{inv}}, \]  (8)
\[ v^n_{\text{inv}}(M_{\text{inv}}) = v^n_{\text{inv}} \frac{S}{S + B}(M_{\text{inv}}) + v^n_{\text{inv}} B \frac{B}{S + B}(M_{\text{inv}}). \]  (9)
where \( S \) is the signal yield, \( B \) is background yield, \( v_n^S \), \( v_n^B \) and \( v_{n+1}^B \) are the \( v_n \) for signal, background and total particles, respectively. Signal and background contributions are decomposed by taking into account the measured signal-to-background ratio and using a parametrization of the background \( v_n^B \) shape [4]. Since the average cosine is calculated in this approach, one can directly extract the \( v_n^S \) by subtracting the background contribution. The only modification is to add a weight \( 1/R_n \) on an event-by-event basis when one fills the histograms for \( \cos(n(\phi - \Psi_m)) \) versus invariant mass similar to the Eq. (7).

2.2.2. Event plane method

The first step in the event plane method is the signal extraction for a given \( \phi - \Psi_m \) bin

\[
N^R(\phi - \Psi_m) = \int_R dM \frac{1}{R_n} \frac{dN}{d(\phi - \Psi_m)}. \tag{10}
\]

where \( N^R(\phi - \Psi_m) \) is the number of particles for a given \( \phi - \Psi_m \) bin, weighted for each centrality bin with the inverse of the event plane resolution. The difference to the conventional method is the weight \( 1/R_n \) on the particle yields which will properly take into account the centrality dependence of the event plane resolution. Second, one integrates Eq. (10) over \( \phi - \Psi_m \) to calculate \( v_n \)

\[
\langle v_n^{\text{obs},R} \rangle \equiv \frac{\int_0^{2\pi} d(\phi - \Psi_m) N^R(\phi - \Psi_m) \cos[n(\phi - \Psi_m)]}{\int_0^{2\pi} d(\phi - \Psi_m) N^R(\phi - \Psi_m)}
= \frac{\int_R dM \int_0^{2\pi} d(\phi - \Psi_m) \frac{dN}{d(\phi - \Psi_m)} \cos[n(\phi - \Psi_m)]}{\int_R dM \int_0^{2\pi} d(\phi - \Psi_m) \frac{dN}{d(\phi - \Psi_m)} R_n}. \tag{11}
\]

Note that \( v_n^{\text{obs},R} \) in Eq. (11) is different from the conventional \( v_n^{\text{obs}} \) due to the \( 1/R_n \) weight on the particle yields. One could immediately notice that the numerator is identical to Eq. (7) with the additional normalization \( 1/(2\pi \int_R dMN_0) \). The denominator in Eq. (11) is

\[
\int_0^{2\pi} d(\phi - \Psi_m) N^R(\phi - \Psi_m)
= 2\pi \int_R dM N_0 \frac{1}{R_n} = 2\pi \left( \int_R dMN_0 \right) \left( \frac{1}{R_n} \right). \tag{12}
\]

Since the normalization factor \( 2\pi \left( \int_R dMN_0 \right) \) is cancelled out between numerator and denominator, the \( \langle v_n \rangle \) is obtained by multiplying \( (1/R_n) \) by the \( \langle v_n^{\text{obs},R} \rangle \)

\[
\langle v_n^{\phi - \Psi} \rangle \equiv \langle v_n^{\text{obs},R} \rangle \left( \frac{1}{R_n} \right) = \langle v_n \rangle. \tag{13}
\]

We would like to emphasize two important points for the event plane method. First, one must take an average of the inverse event plane resolution \( 1/R_n \) (not an average of \( R_n \)). Second, the integral \( \int_R dM \) in Eq. (11) for both numerator and denominator should be calculated in the same phase space. For instance, if one measures the \( v_n \) as a function of transverse momentum \( p_T \), one should calculate \( (1/R_n) \) for each \( p_T \) bin.

The new implementation can also be applied for the scalar product method [5, 6] because the scalar product method is a simple extension of event plane method where the magnitude of the Q-vector is considered for both \( v_n^{\text{obs}} \) and \( R_n \).

3. Simulation results

In this section we validate our implementation by a fast Monte Carlo simulation for \( v_2(p_T) \). As we have mentioned earlier, this approach can be applied for any harmonic of interest.

![Figure 1: Input multiplicity distribution in the fast Monte Carlo simulation. The shaded area show 0-5%, 5-10%, ..., 75-80% centrality classes.](image-url)
For every event a number of tracks according to the input multiplicity distribution was sampled. A Boltzmann-like distribution from 0.25–3 GeV/c was used. The input $v_2^{\text{in}}(p_T)$ for every centrality bin was fixed to a parametrization of the following form

$$v_2^{\text{in}}(p_T) = A(1 - e^{-p_T}) \left( \frac{a}{1 + e^{-(p_T - b)/c}} - d \right).$$  \hspace{1cm} (14)$$

The parameter $A$ was increased towards more peripheral centralities. Furthermore, we have added for every event a Gaussian smearing on $v_2^{\text{in}}$ with a width of 0.05.

Figure 2 depicts the used $v_2^{\text{in}}(p_T)$ for the 16 centrality bins. The event Q-vector was calculated according to Ref. [2].

Figure 3 shows the random subevent plane resolution as a function of centrality. For each event, particles are randomly divided into two different groups in order to evaluate the resolution $R_2$. The result shown here is the subevent plane resolution by using Eq. (14) in Ref. [2]. The resolution reaches a maximum of 0.45 around 30% centrality, it decreases towards more central and peripheral events because of a lower $v_2$ in central and less multiplicity in peripheral events.

Figure 4 shows an example of a particle azimuthal distribution with respect to the second harmonic event plane in a narrow $p_T$ bin. The particle yields are weighted by the inverse event plane resolution as indicated by $N^R$ in the y-axis title. The dashed line rep-
represents a fit with \(1 + 2v_{2}^{\text{obs}} \cos[2\phi - 2\Psi_{2}]\). The yield extraction and fit are repeated for all other \(p_{T}\) bins.

Figure 5 shows \(\langle v_{2} \rangle\) as a function of \(p_{T}\) in the 0-80% centrality bin. For comparison, we also plot the observed \(\langle v_{2}^{\text{obs}} \rangle\) without resolution correction as shown by solid grey circles. The difference between observed and corrected \(\langle v_{2} \rangle\) gives the size of the resolution correction.

The input \(\langle v_{2}^{\text{in}} \rangle\) of the simulation is shown as a magenta solid line. We tested both, the direct cosine calculation \(\langle v_{2}^{\cos} \rangle\) from Eq. (13) and the event plane method \(\langle v_{2}^{\phi} \rangle\) from Eq. (14), as shown by open blue stars and solid blue stars, respectively in panel (a) and (b). One can see that both direct cosine and event plane methods are consistent with the input \(\langle v_{2}^{\text{in}} \rangle\). The data points are in agreement with 0 within 0.3%. Panel (b) shows the observed \(\langle v_{2}^{\text{obs}} \rangle\), corrected by the inverse mean event plane resolution \((R)^{-1}\) (green line) and the mean inverse event plane resolution \((R^{-1})\) (red line). In panel (c) we show a correction similar to panel (b) but with the event plane resolution calculated for every single \(p_{T}\) bin. The relative difference \(\Delta \langle v_{2} \rangle\) of the corrected \(\langle v_{2} \rangle\) values to the input \(\langle v_{2}^{\text{in}} \rangle\) is shown in the lower panels. For the correction with \((R)^{-1}\) and \((R^{-1})\) we get a \(\Delta \langle v_{2} \rangle / \langle v_{2}^{\text{in}} \rangle\) of 5% and 15%, respectively. The difference becomes smaller if the average resolution is calculated for every \(p_{T}\) bin as shown in panel (c), but we still obtain relative deviations of 4.5% \((\langle R \rangle)^{-1}_{p_{T}}\) and 10% \((\langle R^{-1} \rangle)^{-1}_{p_{T}}\). In all cases the relative difference is almost independent of \(p_{T}\).

4. Summary

We have introduced an implementation to avoid event plane resolution effects of wide centrality bins on the \(v_{2}\) measurements. The new approach is essentially an extension of existing event plane methods. The basic modification is to add an inverse of event plane resolution weight to the cosine term or to the particle yields depending on the method. We confirmed that our approach reproduces the input \(\langle v_{2}^{\text{in}} \rangle\) in a wide centrality bin, whereas the conventional corrections lead to a bias as large as ~15% on the reconstructed \(\langle v_{2} \rangle\) with our fast Monte Carlo simulation.

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