HF Radar Sea-echo from Shallow Water

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Received: / Accepted: / Published:

Abstract: HF radar systems are widely and routinely used for the measurement of ocean surface currents and waves. Analysis methods presently in use are based on the assumption of infinite water depth, and may therefore be inadequate close to shore where the radar echo is strongest. In this paper, we treat the situation when the radar echo is returned from ocean waves that interact with the ocean floor. Simulations are described which demonstrate the effect of shallow water on radar sea-echo. These are used to investigate limits on the existing theory and to define water depths at which shallow-water effects become significant. The second-order spectral energy increases relative to the first-order as the water depth decreases, resulting in spectral saturation when the waveheight exceeds a limit defined by the radar transmit frequency. This effect is particularly marked for lower radar transmit frequencies. The saturation limit on waveheight is less for shallow water. Shallow water affects second-order spectra (which gives wave information) far more than first-order (which gives information on current velocities), the latter being significantly affected only for the lowest radar transmit frequencies for extremely shallow water. We describe analysis of radar echo from shallow water measured by a Rutgers University HF radar system to give ocean wave spectral estimates. Radar-derived wave height, period and direction are compared with simultaneous shallow-water in-situ measurements.

Keywords: HF radar oceanography, wave measurement, remote sensing.
1. Introduction

High Frequency (HF) radar systems are widely used internationally to provide continuous monitoring of ocean waves and currents for a large range of environmental conditions.

Within the U.S., coastal ocean current mapping with HF radar has matured to the point where it is now considered an essential component of regional ocean observing systems. A Mid-Atlantic HF radar network now provides high resolution coverage within five localized networks, which are linked together to cover the full range of the Mid-Atlantic coastal ecosystem. Similar regional networks throughout the country are in the process of coordinating to form a national HF radar network.

While much of the focus of these networks until now has been on offshore current mapping observations, a longer-term objective is to develop and evaluate near-shore measures of waves and currents. These investigations aim to understand the interaction of waves in the shallow coastal waters and how energy is transformed into the creation of dangerous rip currents along the New-Jersey/Long-Island shorelines. The radars owned by Rutgers University – whose data are used in this paper – cover these coastal regions at multiple frequencies from 4.5 to 25 MHz. Their echoes contain information on both currents and waves from deep water up into the shallow coastal zone, providing an excellent archive for such studies. This paper describes for the first time how radar observations in shallow water are related to the deep-water conditions that produced them.

Radar sea-echo spectra consist of dominant first-order peaks surrounded with lower-energy second-order structure. Analysis methods presently in use assume that the waves do not interact with the ocean floor. References dealing with this subject for deep-water wave extraction are found in [1, 2, 3] for phased-array-antenna beam-forming systems; and in [4] for systems with compact crossed-loop direction-finding antennas, such as the Seasonde.

The assumption of deep water is often invalid close to the coast and for broad continental shelves, and is particularly inadequate to describe the second-order sea-echo used to give information on ocean waves. Second-order echo is often visible above the noise only for close ranges. To interpret this echo correctly, the effects of shallow water must be taken into consideration.

In Section 2, we give the basic equations describing radar echo from shallow water, expanding on the previous description given in [5]. In Section 3, simulations are used to illustrate the effects of shallow water on waveheight, Doppler shifts and spectral amplitudes in radar sea-echo spectra, to investigate limits on the existing theory and to define depth limits at which shallow-water effects must be included in the analysis. In Section 4 methods are applied to the interpretation of measured radar echo from shallow water to produce wave directional spectral estimates which are compared with observations from a bottom-mounted acoustic Doppler profiler moored in the second radar range cell.

2. Radar Spectral Theory

In this section, a subscript or superscript s indicates a shallow-water variable.

Waves interact with the ocean floor when the water depth falls below a value given approximately by

$$2\pi d / L \leq 0.8$$  \hspace{1cm} (1)
where \( d \) is the water depth and \( L \) is the dominant ocean wavelength. The deep-water analysis must be modified to allow for shallow-water effects in the coupling coefficients, the dispersion equation refractive effects on wave direction, and the directional ocean wave spectrum itself. We only consider water of sufficient depth that effects of wave energy dissipation such as breaking and bottom friction may be ignored; thus we operate in the linear wave transformation regime. As a general rule, this assumption is valid when the water depth is greater than 5% of the deep-water wavelength.

Applying the lowest-order shallow-water dispersion equation to first-order scatter gives the following equations for \( \tilde{k}_s^1, \omega_s^1 \), the first-order spatial, temporal wavenumbers of the ocean waves in shallow water producing the backscatter.

\[
\tilde{k}_s^1 = -2\tilde{k}_0 \\
\omega_s^1 = m'\omega_b
\]  

(2)

where \( \omega_b \) is the Bragg frequency given by:

\[
\omega_b = \sqrt{2gk_0 \tanh(2k_0d)}
\]  

(3)

and where \( g \) is the gravitational constant. \( d \) is the water depth, \( k_0 \) is the radar wavenumber and \( m' \) is +1, -1 for waves moving toward, away from the radar respectively. The analogous relations for second-order backscatter are:

\[
\tilde{k}_s + \tilde{k}_s' = -2\tilde{k}_0 \\
\omega = m\sqrt{gk_s \tanh(k_sd)} - m'\sqrt{gk_s' \tanh(k_s'd)}
\]  

(4)

where here \( \tilde{k}_s, \tilde{k}_s' \) are the spatial wavevectors of the two shallow-water, first-order ocean waves interacting to produce the second-order backscatter.

The electromagnetic coupling coefficient has the same form as for deep water [5] but with shallow-water wavevectors:

\[
\Gamma_{EM}^s = 0.5 \left[ \frac{(\tilde{k}_s \cdot \tilde{k}_0)(\tilde{k}_s' \cdot \tilde{k}_0') / k_0^2 - 2\tilde{k}_s \cdot \tilde{k}_s'}{\sqrt{\tilde{k}_s \cdot \tilde{k}_s - k_0^2}} \right]
\]  

(5)

The hydrodynamic coupling coefficient, derived by Barrick and Lipa [6] through solution of the equations of motion and continuity, is a function of water depth:
\[
\Gamma'_{\text{it}} = -\frac{i}{2} \left[ k + k' - \frac{(kk' - \tilde{k}_s \cdot \tilde{k}_x)}{\sqrt{kk'}} \left( \frac{\omega^2 + \omega_b^2}{\omega^2 - \omega_b^2} \right) + \frac{\omega \left( \frac{m \sqrt{g k_s}}{k} \right)^3 \csc h^2 (k, d) + \left( \frac{m' \sqrt{g k'_s}}{k'} \right)^3 \csc h^2 (k'_d) \right) }{g(\omega^2 - \omega_b^2)} \right]
\]

(6)

The radar coupling coefficient \( \Gamma' \) is the coherent sum of the hydrodynamic and electromagnetic terms

\[
\Gamma' = \Gamma'_{\text{EM}} + \Gamma'_{\text{it}}
\]

(7)

The deep- and shallow-water spatial wavenumbers in (6) are related as follows (the former are written without subscripts):

\[
k = k_s \tanh(k_s d) \quad k' = k'_s \tanh(k'_s d)
\]

(8)

The coupling coefficient increases as the water depth decreases (at constant wavenumber). This results in an increasing ratio of second- to first-order energy as the depth decreases.

In the following analysis, it is assumed that the deep-water directional wave spectrum is spatially homogeneous and that any inhomogeneity arises from wave refraction in shallow water. It follows from linear wave theory that since the total energy of the wavefield, neglecting energy dissipation, must be conserved, the shallow-water wave spectrum expressed in the appropriate variables has exactly the same value as the deep-water spectrum [7]

\[
S_s(\tilde{k}_x) = S(\tilde{k})
\]

(9)

where the deep- and shallow-water wave vectors are related by Snell’s law and the dispersion equation:

\[
k \cos(\theta + \beta) = k_s \cos(\theta_s + \beta)
\]

(10)

\[
k = k_s \tanh(k_s d)
\]

(11)

Here \( \beta \) the angle between the radar beam and the depth contour and \( \theta_s, \theta \) the angles between the radar beam and the shallow-, deep-water ocean waves respectively. Figure 1 illustrates refraction at a discontinuity, although the equations apply over any region of depth change.
Figure 1. Schematic geometry of the radar beam and an ocean wave train at a depth discontinuity, denoted by the dashed line. Wave angle $\theta$ is measured counter-clockwise from the radar beam to the direction the wave is moving.

The shallow- and deep-water rms waveheights are given by:

$$H^2 = \int_0^{2\pi} S(k, \theta) dk d\theta$$  
$$H_s^2 = \int_0^{2\pi} S_s(k_s, \theta_s) dk_s d\theta_s$$

(12)

Substituting (10) and (11) into (12) gives the following relations which are useful for deriving the shallow- from the deep-water wave spectrum and vice versa:
\[ H^2 = \int_0^{2\pi} S(k_s, \theta_s) J(k_s, \theta_s) dk_s d\theta_s \quad H^2 = \int_0^{2\pi} S(k, \theta) \Gamma^{-1}(k_s, \theta_s) k_s dk d\theta \]

where the Jacobian \( J(k_s, \theta_s) \) is given by:

\[
J(k_s, \theta_s) = \begin{vmatrix}
\frac{\partial k_s}{\partial k} & \frac{\partial \theta_s}{\partial k} \\
\frac{\partial k_s}{\partial \theta} & \frac{\partial \theta_s}{\partial \theta}
\end{vmatrix}
= \left( \frac{\partial}{\partial \theta} \right)_\theta \left( \frac{\partial}{\partial k} \right)_k 
= \left[ 1 + \frac{k_s^2 \sec h^2(k_s^* d)}{\tanh(k_s^* d)} \right] \frac{\sin(\theta_s + \beta)}{\sin(\theta + \beta)}
\]

The equation for the first-order radar cross section in shallow water is given by:

\[
\sigma_1^s(\omega, \varphi) = k^4_0 \sum_{m=\pm 1} S(2k_0, \varphi + (m' + 1)\frac{\pi}{2}) \delta(\omega - m' \omega_B)
\]

where \( S(k, \alpha) \) is the directional ocean wave spectrum for wavenumber \( k \) and direction \( \alpha \).

The second-order radar cross section is given by:

\[
\sigma_2^s(\omega, \varphi) = k^4_0 \sum_{m,m' = \pm 1} \int_0^{2\pi} \int_{-\infty}^{\infty} \Gamma_s^2 |S(k_s, \theta_s + \varphi + m\pi)| \cdot S(k_s, \theta_s + \varphi + m\pi) \delta(\omega - m' \sqrt{gk_s \tanh(k_s^* d)} - m \sqrt{gk_s \tanh(k_s^* d)}) k_s dk_s d\theta_s
\]

where the coupling coefficient \( \Gamma_s \) is given by (7). The values of \( m \) and \( m' \) in (16) define the four possible combinations of direction of the two scattering waves. Common non-dimensional multiplicative constants in (15) and (16) have been omitted. The wavenumbers of the scattering waves are related through (4):

\[ k_s' = \sqrt{k_s^2 + 2k_s \cos(\theta_s) + 1} \]

To compute the second-order integral in (16), we use integration variables \( k_s' \) and the deep-water angle \( \theta \). In terms of these variables (16) becomes

\[
\sigma_2^s(\omega, \varphi) = k^4_0 \sum_{m,m' = \pm 1} \int_0^{2\pi} \int_{-\infty}^{\infty} I(k_s, \theta) \delta(\omega - h(k_s, \theta)) \left| \frac{\partial k_s'}{\partial \theta} \right| dhd\theta
\]

where
\[ h(k, \theta) = m\sqrt{gk} - m'\sqrt{gk_s}\tanh(k'd) \]  
(19)

and

\[ I(k_s, \theta) = \left| \Gamma_s^2 \right| S(k, \theta + \varphi + m\pi)S(k', \theta + \varphi + m'\pi)k_s \left( \frac{\partial \theta_s}{\partial \theta_s'} \right)_k \]  
(20)

and where we have substituted (9) for the shallow water directional spectra. The factors \( \left( \frac{\partial \theta_s}{\partial \theta_s'} \right)_k \) and \( \left| \frac{\partial k_s}{\partial h} \right| \) are obtained by differentiation using (10), (11) and (19).

To calculate the integral in (18), it is first reduced to a single-dimensioned integral using the delta function constraint. The remaining integral is computed numerically.

Frequency contours are defined by:

\[ \omega - h(k^*, \theta) = 0 \]  
(21)

which is solved for \( k^* \) as a function of \( \theta \). Due to wave refraction, the shallow water angle and wavenumber have discontinuities when the deep-water wave moves parallel to the depth contour, i.e. when

\[ \theta = -\beta, \quad \pi - \beta \]  
(22)

Frequency contours are hence also discontinuous due to this effect at deep-water wave angles defined by (22). Examples of frequency contours for deep- and shallow-water are shown in Figure 2, plotted in normalized deep-water spatial wavevector space \( \frac{k}{2k_0} \). Normalized components \( p, q \) are defined so that \( p \) is along the radar beam and \( q \) perpendicular:

\[ p = \left( k_0 + k\cos(\theta) \right) / (2k_0) \]
\[ q = k\sin(\theta) / (2k_0) \]  
(23)

The discontinuities in the frequency contours are more pronounced when the contour is drawn in shallow-water wavenumber space, as there are discontinuities in the shallow-water wave angle due to wave refraction, which follows from (10), (11).

**Figure 2.** Examples of frequency contours for water of depth 10m (continuous line) compared with the corresponding contours for deep water (dashed line). Angle between the radar beam and the depth contour is 60°. Normalized frequency: \( \omega / \omega_B = 1.2, \beta = 60^\circ \). Radar frequency: (a) 5Mhz, (b) 25Mhz
It can be seen from Figure 2 that the deep-water ocean wave numbers corresponding to a given radar spectral frequency change with depth: they become either greater or smaller than the deep-water values, depending on the wave direction. This results in the frequency of second-order peaks in the radar spectrum changing with water depth.

The effects of shallow-water on measured radar spectra are illustrated in Figure 3, which shows measured spectra from a 5 MHz radar at three ranges. As the water depth decreases, the second-order energy increases relative to the first-order and the frequency displacement between the first- and second-order peaks decreases.
Figure 3. (a) Spectra measured by the 5MHz SeaSonde at Sao Tome, Brazil, at 5:30am 8/27/2007. Range: (a) 18 km (b) 30 km (c) 42 km. Water depth around range cell: (a) 5m to 20m (b) 10m to 50m (c) 20m to 70m
3. Narrow-beam radar spectral simulations

To gain insight into the effects of shallow water, simulated radar echo spectra were calculated for a narrow-beam radar, using a model directional wave spectrum defined in [8] consisting of the sum of two terms: a continuous high-frequency wind wave spectrum and a swell component that is an impulse function in both wavenumber and direction. The swell component is defined by

$$S_s(k_s, \theta_s) = H_s^* \delta(\theta_s - \theta_s^*) \delta(k_s - k_s^*)$$

(24)

where $H_s^*$, $\theta_s^*$, $k_s^*$ are the specified rms waveheight, direction and wavenumber. For this model, four sharp spikes occur in the radar spectrum. In this section, we consider only the second-order sideband for which $m=1, m'=1$, assuming shallow-water depths in the range 5-100m. and radar transmit frequencies of 5Mhz and 25Mhz. It can be shown numerically that for these values of $m, m'$, Doppler frequencies are always greater than the positive Bragg frequency. The radar beam is taken to be pointing perpendicular to parallel depth contours (i.e $\beta = 90^\circ$ in Figure 1)

Significant waveheight is defined to be four times the rms waveheight.

3.1 Effect of water depth on waveheight

For our model it follows from (13) that the relationship between the shallow- and deep-water rms waveheight is given by:

$$H_s^* = H^* \sqrt{\frac{\sin(\theta_s^* + \beta) / \sin(\theta_s^* + \beta)}{[\tanh(k_s^* d) + k_s^* d \sec h^2(k_s^* d)]}}$$

(25)

This relationship is of course independent of radar frequency and has many angle symmetries. Figure 4 shows the ratio plotted as a function of depth for different wave directions.
Figure 4. The ratio of shallow- to deepwater waveheight plotted vs. depth for a 12 s wave.

Wave direction in deep water relative to the radar beam: Red 180°, Blue 135°

It can be seen from Figure 4 that the waveheight initially decreases with decreasing depth as the wave enters shallow water but increases at depths below about 20m, which agrees with [7].

3.2 Effect of water depth on Doppler shifts

It follows from (3) that the Bragg frequency decreases with depth, causing the Bragg peaks to move closer together. Figure 5 shows the Bragg frequency plotted as a function of depth.

Figure 5. Bragg frequency plotted as a function of depth.

Radar transmit frequency: Red 5Mhz, Blue 25Mhz
This shows that the change in the Bragg frequency with depth is small.

Figure 6 shows the displacement of the second-order peak from the Bragg frequency plotted as a function of depth for an 11-sec wave moving at different angles with respect to the radar beam.

**Figure 6.** The frequency shift of the second-order peak from the Bragg frequency for an 11s wave.

(a) 5Mhz (b) 25Mhz.

Angle between wave and radar beam: Yellow 0°, Blue 45°, Green 135°, Red 180°
It can be seen from Figure 6 that as the water depth decreases, the second-order peak shifts toward the Bragg frequency for waves moving toward the radar, and further away for waves moving away from the radar. This is consistent with the two branches of the contour plot as shown in Figure 2. This effect is more marked for lower radar frequencies and can be seen in the measured spectra from Sao Tome shown in Figure 3 in which the second-order peak moves closer to the first-order as the range from the radar and water depth decrease, with waves moving toward the radar.
3.3 Effect of water depth on radar spectral amplitudes

It is shown in [8] that for the impulse-function model defined by (24), the ratio $R$ of the second-order to first-order energy is given by:

$$ R = 2H_s^* |\Gamma_s^2| $$  \hfill (26)

where the coupling coefficient $\Gamma_s$ is evaluated at wavevectors defined by $\theta_s^*, k_s^*$. $\Gamma_s$ increases with decreasing depth and increasing wave period at a given radar frequency as illustrated in Figures 7 and 8, which also show that shallow water has a greater effect as the radar transmit frequency decreases.

**Figure 7.** The absolute value of the coupling coefficient vs. depth for different radar frequencies for a 9 sec wave. Radar frequency: Red: 5Mhz, Blue: 25Mhz.
Figure 8. The absolute value of the coupling coefficient vs. depth for waves of different period. Radar transmit frequency: 5Mhz. Wave period: Red 15s, Blue 12s, Green 9s.

As the coupling coefficient increases as the depth decreases, it follows from (26) that the second-order energy will increase with respect to the to first-order. This effect can be seen in the measured radar spectra shown in Figure 3. Figure 9 shows the theoretical ratio of the second- to the first-order energy obtained from (26) using our model for an 11-sec wave.
Figure 9. Ratio of second - to first-order energy for an 11 sec. Significant waveheight 2.4m. Radar transmit frequency: (a) 5 Mhz, (b) 25 Mhz.

Angle between wave and radar beam: Yellow 0°, Blue 45°, Green 135°, Red 180°

It can be seen from Figure 9 that for shallow depths and onshore waves, the ratio exceeds unity (i.e. the calculated second-order energy exceeds the first-order energy) for depths less than about 8 m for a 5MHz transmit frequency and for depths less than about 10 m for a 25 MHz transmit frequency.

This subsection demonstrates an important point. Since we have shown that the waveheight itself actually decreases slightly upon moving into shallow water, while the second-order echo increases...
significantly (due to the rapid growth of the coupling coefficient), wrongly using deep-water inversion theory to estimate waveheight will grossly overestimate this important quantity. Yet all treatments and demonstrations of wave extraction up to this paper have been based only on deep-water theory, when in fact many of the radar observations may actually have been made in shallow water.

3.4 Effect of water depth on breakdown of theoretical model

When the magnitude of the second-order energy approaches that of the first-order, it is apparent that the perturbation expansions on which Barrick’s equations are based are failing to converge and they therefore cannot provide an adequate description of the radar echo. This effect is similar to the well known radar spectral saturation occurring when the waveheight exceed a limit defined by the radar transmit frequency. Above this waveheight limit, the radar spectrum loses its definitive shape and the perturbation expansions fail to converge. The deep-water saturation limit on the significant waveheight \( W_{sat} \) is defined approximately by the relation:

\[
W_{sat} = \frac{2}{k_0}
\]  

(27)

For shallow-water, the saturation of the radar spectrum is exacerbated by the increase of the coupling coefficient. The radar spectrum saturates for waveheights less than that defined by (27). We here define the shallow-water saturation limit for the model to be that waveheight for which of second-order energy equals the first-order i.e. the ratio \( R \) is given by:

\[
R = 1
\]  

(28)

In practice the theory may fail before this limit is reached. The two saturation limits defined by (27) and (28) are plotted vs. depth in Figure 10 for two different radar frequencies. At depths of 30m the saturation limits are approximately equal. At depths less that 30m, the shallow-water limit drops off sharply, particularly for 5-MHz. Thus the radar spectrum can be expected to saturate at lower values of waveheight in shallow water.

For waveheights above the saturation limit, the theory is will over-predict the waveheight. However the theory cannot be applied at all when the second-order spectrum merges with the first.
Figure 10. Significant waveheight saturation limits for an 11-second wave coming straight down the radar beam. Radar transmit frequency: (a) 5 Mhz, (b) 25 Mhz
Significant waveheight for radar spectral saturation: Red: deep-water saturation limit defined by (27), Blue: shallow-water saturation limit defined by (28)
3.5 Depth limits when shallow-water effects become significant

We estimate these depths as follows: For first-order echo, the depth limit is defined by equality in (1). At this depth, the Bragg frequency defined by (3) for the Bragg frequency 96% of its deep-water value. For second-order echo, the depth limit is defined as the value for which the coupling coefficient defined by (7) exceeds 1.25 times the deep-water value, for a wave with an 11-second period. Figure 11 plots the depths at which shallow-water effects become significant vs, radar transmit frequency.

**Figure 11.** Depths at which shallow-water effects become significant vs. radar transmit frequency. Depth limit: Red, second-order echo. Blue, First-order echo.

Figures 10 and 11 help in assessing the validity of the existing (deep-water) methods. However the wave model used, as specified by (24), is quite restrictive: waves of a single wavelength are assumed to come down the radar beam. Also Figure 11 applies to an 11s wave. Performing similar studies for more general wave spectral models is beyond the scope of this paper. However, we observe that (1) Shallow water effects are stronger for longer ocean wavelengths (2) Second-order radar spectra for \( m=1, m'=1 \) are strongest for waves down the radar beam. (3) The stronger the second-order for a given waveheight, the sooner the radar spectrum will saturate as waveheight increases. Therefore shallow-water effects will be more marked at a given waveheight for a broad nondirectional spectrum which includes longer wavelengths e.g. a Pierson Moskowitz model. These differences would probably not be large however, due to the sharp cutoff of wave-spectral models for long wavelengths. The opposite effects would be expected for wave spectra which include directions that
are not directly down the radar beam e.g. a cardioid directional distribution. These effects are summarized in the Table 1.

### Table 1

| Condition                                      | Waveheight for spectral saturation | Depth for significant shallow-water effects |
|-----------------------------------------------|-----------------------------------|--------------------------------------------|
| Broad nondirectional spectrum                | < Fig. 10                         | >Fig. 11                                   |
| Broad directional distribution                 | > Fig. 10                         | < Fig. 11                                  |
| Waves nonparallel to radar beam               | > Fig. 10                         | < Fig. 11                                  |
| Wave period > 11s                             | > Fig. 11                         |                                            |
| Wave period < 11s                             | < Fig. 11                         |                                            |

### 4. Application to measured data

#### 4.1 Data set

The results presented here are based on analysis of 10-minute radar spectra measured by the 25MHz SeaSonde located at Breezy Point, NY. The time period from December 29 to 30, 2005, was chosen because of the simultaneous coverage provided by the SeaSonde and a bottom mounted acoustic Doppler profiler (ADCP), allowing a direct comparison to be made between results from the two sensors. The ADCP was located in the second radar range cell in water of depth 8m. This bathymetry in the area and the location of the mooring relative to the radar are shown in Figure 12.

**Figure 12.** The coastline and bathymetry (contours in meters) around Breezy Point, New Jersey. The SeaSonde is located by the red square, x marks the position of the bottom-mounted ADCP.
Depth contours near the radar are assumed in the following to be parallel to shore and the depth profile is obtained from Figure 12.

Figure 13 shows measured spectra from the Breezy Point SeaSonde at three ranges: the second-order energy increases relative to the first-order as the water depth decreases.

**Figure 13.** (a) Spectra measured by the 25MHz SeaSonde at Breezy Point. at 1:00pm 12/30/2005. Range: (a) 3 km (b) 6 km (c) 9 km.
4.2 Interpretation of the radar spectra

Lipa and Barrick [5] describe the extension of the narrow-beam theory described in Section 2 to apply to a broad antenna system such as the SeaSonde, assuming ideal antenna patterns. From the antenna voltage cross spectra, we form as intermediate data products the first five Fourier angular coefficients of the broad-beam return over a selected range ring surrounding the radar. These coefficients, $b^{1,2}_n(\omega)$ are defined in terms of the narrow-beam first and second-order return through the relation:
where the integration is performed over angle around the radar range cell between the coastline angles defined by $\gamma_1$, $\gamma_2$ and the superscripts refer to first- and second-order respectively. The narrow-beam radar cross sections are defined in terms of the ocean wave spectrum by (15), (16). Here the five Fourier coefficients are designated by the index $n = -2, -1, 0, 1, 2$ and following the notation in [5] the trigonometric functions $f_n(\varphi)$ are given by

$$f_n(\varphi) = \sin(n\varphi) \quad n < 0$$

$$= \cos(n\varphi) \quad n \leq 0$$

(30)

As described in [4], there are three steps in the interpretation of the radar spectrum to give deep-water wave information.

a) The first- and second-order regions are separated.

b) The first order region is analyzed to give the ocean wave spectrum at the Bragg wavenumber. It was assumed that deep-water theory is adequate for this step.

c) Second-order radar spectral data is collected from the four second-order sidebands of 10-minute averaged cross spectra and fit to a model of the deep-water ocean wave spectrum. Least-squares fitting to the radar Fourier coefficients is used to derive estimates of the significant wave height, centroid period and direction. During this step, the second-order spectrum is effectively normalized by the first-order, eliminating unknown multiplicative factors produced by antenna gains, path losses etc.

Shallow-water analysis requires a further step:

d) The shallow-water wave spectrum is calculated from the deep-water spectrum using (9)–(11).

4.3 Model ocean wave spectrum

We define a model for the deep-water ocean wave spectrum as the product of directional and nondirectional factors:

$$S(k, \varphi) = Z(k) \cos^4 \left( \frac{\varphi - \varphi^*}{2} \right)$$

(31)

The directional factor in (31) has a cardioid distribution around the dominant direction $\varphi^*$. For describing the second-order spectrum, $\varphi^*$ is taken to be the dominant long-wave direction. For describing the first-order spectrum, $\varphi^*$ is the short-wave direction, which is taken to be the same as the wind direction. For the nondirectional spectrum we use the Pierson-Moskowitz model for the nondirectional factor $Z(k)$:

$$Z(k) = \frac{A e^{-0.74(k/k_c)^4}}{k^4}$$

(32)

with parameters the cutoff wavenumber $k_c$ and a multiplicative constant $A$. The waveheight, centroid period and direction can be defined in terms of the model parameters. The significant waveheight follows from the directional spectrum through the relation:
\[ h = 4 \left( \int_{\gamma_i}^{\gamma} \int S(k, \varphi) dk d\varphi \right)^{1/2} \] (33)

The above model has proven satisfactory for use in deep-water wave extraction software that produces waveheight, period, and direction. It has been used for real-time SeaSonde systems for many years with good agreement [4].

4.4 Results

Figure 14 shows SeaSonde results for wind direction, wave height, period and wave direction in the second radar range cell together with the mooring results.

**Figure 14.** SeaSonde (red) and mooring (blue) results for
(a) Waveheight (b) Wave period (c) Wave direction (d) Wind direction
Southerly winds veer to the north-west after the passage of a front. After the passage of the front, wave height and period increase suddenly. Spectral saturation may be occurring at the peak of the storm, causing overestimates in the wave height. Wave direction remains about the same; due to wave refraction, wave directions in very shallow water approach perpendicular to the depth contours. Both radar and buoy are observing directions in shallow water, hence this perpendicular condition is being enforced on the longer waves, although the wind direction driving short waves is seen to change significantly over this storm period.

Table 2 gives the bias and standard deviation between the Seasonde and buoy measurements of wave height, wave period and direction, for the short-period waves before the storm and the longer-period waves afterwards.

### Table 2  Comparison statistics, radar vs. buoy

|                | Before storm | After storm |
|----------------|--------------|-------------|
| **Waveheight** |              |             |
| Standard deviation | 0.25m       | 0.35m       |
| Bias           | -0.23m       | 0.17m       |
| **Wave Period** |              |             |
| Standard deviation | 0.76s       | 0.60s       |
| Bias          | 0.70s        | -0.27s      |
| **Wave Direction** |            |             |
| Standard deviation | 13.8°   | 19.7°       |
| Bias          | -9.5°        | 17.0°       |
Figure 15 shows the Seasonde waveheight calculated assuming infinitely deep water compared with the buoy waveheight. Clearly waveheight is overestimated with this assumption. The simulations described in Section 3 indicate that the cause of this overestimate is the failure to account for the increase of the coupling coefficient in shallow water.

**Figure 15.** Significant waveheight:
Red: Seasonde calculated assuming infinite water depth. Blue: buoy.

Table 3 gives the bias and standard deviation between the Seasonde and buoy waveheight measurements, with the former calculated assuming infinitely deep water.

| Waveheight | Before storm | After storm |
|------------|--------------|-------------|
| Standard deviation | 0.25m | 0.95m |
| Bias | 0.19m | 0.90m |

**5. Conclusion**

We have presented the theory of narrow-beam HF radar sea-echo from shallow water and illustrated the effect of decreasing water depth using simulations for a simple swell model of the ocean wave
spectrum. The second-order spectral energy increases relative to the first-order as the water depth decreases, resulting in spectral saturation when the waveheight exceeds a limit defined by the radar transmit frequency. This effect is particularly marked for lower radar transmit frequencies. For waveheights above the saturation limit, the perturbation expansions on which Barrick’s equations are based fail to converge. The saturation limit on waveheight is less for shallow water. Shallow water affects second-order spectra (which gives wave information) far more than first-order (which gives information on current velocities). Figure 11 shows the depths at which shallow-water effects become significant plotted as a function of radar frequency for an 11s wave.

The shallow-water theory was then extended to apply to broad-beam systems such as the SeaSonde and applied to the interpretation of two days of radar data measured by a 25Mhz SeaSonde located on the New Jersey shore. A buoy was operated in the second radar range cell in water of depth less than 8m. Radar results were compared with simultaneous buoy measurements. During the measurement period, a storm passed over the area. Comparisons with simultaneous buoy measurements confirm aspects of the theory presented in Section 3. The standard deviation between SeaSonde and buoy and waveheight measurements decreased by a factor of three when the effects of shallow water were included in the analysis, and the bias decreased by a factor of five. Possible explanations for the remaining discrepancies between SeaSonde and buoy measurements are (1) the assumption of parallel depth contours (2) the assumption that the wave spectrum is homogeneous in the circular range cell (3) saturation in the radar spectrum around the peak of the storm, which, as discussed in Section 3, leads to the over-prediction of the waveheight.

Acknowledgements

Measured radar and buoy wave data from New Jersey as well as its analysis were funded through the New Jersey NOAA Sea Grant program.

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