QED correction to radiative tail from elastic peak in DIS

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We calculate DIS cross section under kinematical requirements when a final hadron state is a single proton. The process in the lowest order is known as radiative tail from elastic peak. We take into account correction, coming from emission of virtual and soft real photons, to the amplitude of this process as well as the one, induced by emission of additional hard photon and a hard pair. Resulting contributions of these channels are presented in the leading and, partly, next-to-leading logarithmic approximations. Some general expressions for the photon contributions are given. Numerical results are presented for kinematical conditions of the current experiments on DIS.

PACS number(s): 13.40.Ks

I. INTRODUCTION

Deep inelastic scattering (DIS) is one of the powerful tools in investigating a nucleon nature. Recent experiments at CERN, DESY and SLAC have provided very precise data in the wide kinematical region. The modern level of data analysis in experiments on DIS requires careful consideration of the QED radiative effects which can give substantial contribution to measured quantities. Usually the radiative photon cannot be registered in a detector. As is well understood the corrections due to soft photons and loop effects cannot be separated from observables in principle. Hence their contribution has to be calculated theoretically and subtracted from observed data. The lowest order radiative corrections were first calculated by Mo and Tsai. Covariant approach utilized in [2] was applied to unpolarized [3] and polarized [4,5] DIS. Second order correction to DIS cross section in the leading approximation is discussed in [6–8] and recently in [9]. For completeness we cite the papers [10–17] in which the correction was calculated within a framework of electroweak theory basically for HERA kinematics. The excellent review of the lowest order RC can be found, say, in [12].

One of the important contribution to RC comes from a so called radiative tail from elastic peak (or simply elastic radiative tail), when final hadronic state is pure nucleon but the invariant mass of unobserved particle system (a radiated photon and this nucleon) is the same as in the main Born-level process. Consequently events detected can correspond to the main process as well as to the elastic radiative tail. The calculation in the lowest order of QED is standard and the results are included in many FORTRAN codes intending to perform the RC procedure of experimental data (see review [13] and more recently developed codes [14–17]). Several papers were devoted to electroweak correction to elastic radiative tail. Numerical analysis of the elastic radiative tail shows that its contribution is very important and can exceed the main measured process at the Born level. Therefore the next step is to calculate QED correction to the elastic radiative tail with the maximal possible accuracy. So far only the leading correction to elastic radiative tail due to double bremsstrahlung, which is part of the total second order correction, was treated numerically [10,16] and the attempt to calculate the correction exactly was done in [17].

The structure of the cross section of elastic radiative tail is the following

\[ \sigma_{ERT} \sim \int \frac{dQ_h^2 K(Q_h^2, Q^2, W^2) F^2(Q_h^2)}{Q_h^2_{min}}, \]

where \( Q_h^2 \) is a momentum square transferred to hadronic system, and \( Q^2 \) and \( W^2 \) are leptonic kinematical variables measured. The quantity \( K \) is a kinematical factor known exactly and \( F \) is a nuclear form factor. Due to rapid fall of the form factor squared as a function of \( Q_h^2 \) the main integration region is close to the lower integration limit. In papers [15,16] this fact was used to construct an approximation where \( Q_h^2 \) is considered as a small parameter of order of the proton mass squared. In this paper we will also use this approximation to analyze the correction to elastic radiative tail. Application of Sudakov technique will allow us to obtain compact explicit formulae for processes considered. The first effect which has to be considered is the one-loop correction and the emission of additional real photon. We will analyze both
of them at leading and next-to-leading levels. Another
effect contributing to the cross section is a lepton pair
creation. We will calculate it in the leading log approxi-
mation.

Obtaining a second order correction to deep inelastic
scattering is the main motivation of this paper. How-
ever our results can be used in other cases. For instance,
they can be considered as a radiative correction in mea-
urements with hard photon detected in coincidence with
scattered lepton (see [2], for example), that allows one
to reach kinematical regions otherwise unreachable. That
is why we do not concretize our notation usually used in
DIS but instead try to keep it as general as possible. In
the next section we introduce our notation and obtain re-
ults for the cross section of single bremsstrahlung using
Sudakov technique. In section II we give results for one-
loop corrections. Double bremsstrahlung and contribu-
tions due to pair production are considered in sections IV
and V and final remarks are given in section VI. Some
technical details are discussed in Appendices.

II. SINGLE BREMSSTRAHLUNG

We study the process
\[ e(p_1) + p(p_2) \rightarrow e(p_1') + \gamma(k_1) + p(p_2'), \quad s = 2p_1p_2, \]
\[ Q^2_h = -(p_2 - p_2')^2, \quad Q^2 = 2p_1p_1', \quad k^2_1 = 0, \quad p_1' = p_1 = m^2, \quad p_2 = p_2' = M^2, \quad q^2 = -Q^2_h, \]
in the kinematical region
\[ s \gg Q^2 > Q^2_h \sim M^2, \quad 2p_2p_1' \sim s. \]
The expression for differential cross section in Born ap-
proximation looks (details are given in the Appendix A):
\[ 2\pi^2 \frac{d^3 \sigma^\gamma_{\gamma'}}{dp_1'^3} = \frac{4\alpha^3}{s} \int \frac{d^2 q}{(q^2 + Q^2_{\min})^2} \frac{1}{1 - b} \Phi^\gamma \Phi^{\text{prot}}, \]
with \( b = 2p_2p_1'/s \) the energy fraction of the scattered
electron. We imply the Sudakov parameterization of the
4-momenta in the problem (see Appendix A).

Note that due to gauge invariance condition
\[ q^\mu J^{(1)}_\mu(q^2) = (\alpha_q p_2 + q_1)q^\mu J^{(1)}_\mu = 0, \]
the quantity \( \Phi^\gamma \) is constructed out of \((1/s)p_2J^{(1)}\) which may be rearranged as follows:
\[ \frac{1}{s} J^{(1)}_\mu = -\frac{s_1}{s_1} |q| e^\mu q^{(1)}_\mu, \quad e^\mu q = q \frac{q^\nu q^\mu}{|q|^2}, \quad s_1 = s\alpha_q = (p_1' + k_1)^2 + q^2 - m^2. \]
Thus \( \Phi^\gamma \) vanishes for small \( q^2 \). The explicit expression
for \( \Phi^{\text{prot}} \) is found to be
\[ \Phi^{\text{prot}} = 2(F^2_1 + \frac{q^2}{M^2} F^2_2). \]

For \( \Phi^\gamma \) we have (we refer for further details to the Ap-
pendix A):
\[ \Phi^\gamma = \frac{(1 - b)^2 b(1 + b^2) q^2}{n_1 n}, \]
with
\[ n = (p_1' - bq)^2, \quad n_1 = (p_1 - q)^2. \]

Another fact is that both \( \Phi^\gamma/q^2 \) and \( \Phi^{\text{prot}} \) do not van-
ish in the limit of small momentum transfer \(|q|\), thus
providing the logarithmic enhancement upon performing
the \( Q^2_h \approx q^2 \) integration (Weizsäcker-Williams approxi-
mation). Indeed, the quantity \( Q^2_{\min} \) entering the cross
section is a small quantity,
\[ Q^2_{\min} = M^2 \left( \frac{Q^2}{(1 - b)s} \right)^2 \ll M^2. \]

For completeness we put the phase volume of the scattered
electron in terms of Sudakov variables:
\[ \frac{d^3 p_1'}{2\pi^3} = \frac{d\theta}{2\pi} d^3 p_1', \quad Q^2 = 2p_1p_1' = \frac{p_1'^2}{b}. \]

Note that the requirement \( Q^2 > Q^2_h \) provides the absence
of singularities while doing an integration over \( d^2 q \).

III. VIRTUAL AND SOFT PHOTONS EMISSION

CONTRIBUTION

The correction coming from the emission of virtual and
soft photons (in the cms reference frame) can be drawn
out of paper [7], in which the radiative corrections to the
Compton tensor were calculated
\[ 2\epsilon \frac{d^3 \sigma_{\text{B+V} + s}}{dp_1'} = -2\epsilon \frac{d^3 \sigma^\gamma_{\gamma'}}{dp_1'} \left[ 1 + \frac{\alpha}{2\pi} \tilde{\rho} \right. \]
\[ + \frac{\alpha}{4\pi} \left. \frac{1}{1 + b^2} \left( \tau_{11} + b(\tau_{12} + \tilde{\tau}_{12}) + b^2 \tilde{\tau}_{11} \right) \right], \]
with
\[ \tilde{\rho} = 2(L - 1)(2\ln \Delta - \ln b) + 3L_h - \ln^2 b \]
\[ - \frac{9}{2} \frac{\pi^2}{3} + 2L_{12} \left( \cos^2 \frac{\theta}{2} \right), \]
\[ L = \ln \frac{Q^2}{m^2}, \quad L_h = \ln \frac{Q^2_h}{m^2}, \]
\[ \Delta = \frac{\Delta E}{E}, \quad \text{Li}_2(x) = -\int_0^x \frac{\ln(1 - y)}{y} dy. \]

The Born cross section after substitution of Eq. (3) into
Eq. (4) and neglect of sub-leading terms becomes
The relevant contribution to the cross section looks \( q_i^2 \) and \( \tilde{q}_j \) are explicitly given in the Appendix D. It should be noted that they do not contain any large logarithms but include the quantity \( Q_i^2 \) which is small in our approximation. If one keeps only non-zero terms in the expansion over \( Q_i^2 \), then

\[
\frac{1}{2} \left( \tau_{11} + b(\tau_{12} + \tau_{12}) + b^2 \tau_{11} \right) = 3 \log \frac{Q_2^2}{Q_n^2(1-b)} - 1 \times \left( (1+b^2) + 4b \log(1-b) \right) + \left[ b^2 + (1-b)^2 \right] \times \left[ \log \frac{(1-b)}{b} + \pi^2 \right] + [1+(1-b)^2] \log^2(1-b) + (3-2b) \log b, \tag{14}
\]

The logarithms \( \log Q_i^2 \) cancel out exactly in the sum of \( \Phi \) and \( \tilde{\rho} \).

**IV. TWO HARD PHOTONS EMISSION CONTRIBUTION**

We will consider now the process of two hard photons emission:

\[ e(p_1) + p(p_2) \rightarrow e(p'_1) + \gamma(k_1) + \gamma(k_2) + p(p'_2). \tag{15} \]

The relevant contribution to the cross section looks

\[
\frac{2 \epsilon^2}{d^3 p'_1} = \frac{\alpha^4}{8\pi^4} \int \frac{d^2 q}{(q^2 + Q_{min}^2)^2} x_1 x_2 \Phi^{\gamma\gamma} \Phi^{\text{prot}},
\]

\[ Q_{min} = M^2 \left( \frac{s_{11}}{s} \right)^2, \quad s_1 = (p_1 + k_1 + k_2)^2, \tag{16} \]

with the expression for \( \Phi^{\text{prot}} \) given earlier. The explicit form of \( \Phi^{\gamma\gamma} \) can be found in the Appendix B. The integration over \( d^2 k_1 \) may be performed using the integrals given in the Appendix C.

Concerning the region \( Q_i^2 \ll Q^2 \), the result is found to be

\[
\Phi^{\gamma\gamma} = 16q^2 \left\{ \frac{Q^2}{s^2} \left[ \frac{s_1}{s_1 d_1^2 d_1' d_2^2} + \frac{d_1^2 + d_1'^2}{bs_1^2 d_2 d_2'} + \frac{d_2^2 + d_2'^2}{bs_1^2 d_1 d_1'} \right] \right. \]

\[ - \frac{2}{Q^2} (1 + P_{12}) \left[ \frac{m^2 x_2^2 (b^2 + (1-x_1)^2)}{d_1^2 b(1-x_1)} \right] \]

\[ + \frac{m^2 x_2^2 b(1 + (1-x_2)^2)}{(1-x_2)^3} \left\}. \right. \tag{17} \]

with the notations introduced

\[
s_1 = \frac{k_1^2}{x_1} + \frac{k_2^2}{x_2} + \frac{p_1^2}{x_1}, \quad d_1 = \frac{1}{x_1}(m^2 x_i + k_i^2), \tag{18} \]

\[ d'_i = \frac{1}{x_i b} (m^2 x_i^2 + (x_i p'_i - b k_i)^2), \]

where \( x_{1,2} \) are the energy fractions of hard photons, \( x_1 + x_2 + b = 1 \). Besides we use the relations

\[ k_1 + k_2 + p'_i = 0, \quad 2q p'_i = s_1 b, \quad s_1 = 2q p_i = s_{\alpha q}. \]

An integration over \( d^2 k_1 \) may be performed analytically and to a logarithmic accuracy it boils down to

\[
\int \frac{d^2 k_1}{2} \Phi^{\gamma\gamma} = \frac{16\pi q^2 l}{b(Q^2)^2} \left[ 1 + P_{12} \right] x_2^2 \left[ \left( 1 + \frac{1}{(1-x_1)^2} + \frac{b^2}{(1-x_2)^2} \right) (1 + b^2) + \frac{b^4}{(1-x_1)^2} + \frac{b^4}{(1-x_2)^2} \right], \tag{20} \]

\[
\Delta < x_i < 1-b-\Delta. \]

Carrying out the integration of Eq. (17) over \( \tilde{k}_1 \) and \( x_1 \) to a next-to-leading accuracy we obtain for the contribution to the differential cross section coming from emission of two hard photons,

\[
\frac{2 \epsilon^2}{d^3 p'_1} = \frac{2 \alpha^4}{8\pi^4} \int \frac{d^2 q}{(q^2 + Q_{min}^2)^2} \frac{q^2 (T_{LL} + T_{NLO})}{b(Q^2)^2} \Phi^{\text{prot}}, \tag{21} \]

where the leading and next-to-leading contributions read

\[
T_{LL} = (L - 1) \left[ 4(1-b)(1+b^2) \ln \frac{1-b}{\Delta} \right. \]

\[
+ (1-b)(1-b^2) \ln b - \frac{2}{3} (1-b)(7-2b+7b^2) \right], \tag{22} \]

\[
T_{NLO} = \frac{1}{2} \left( \frac{1}{3} (b^2 + 1) \log^2 b \right. \]

\[
+ \frac{1}{3} (3-b^2)(3b) \log b \]

\[
+ \frac{8}{3} (1-b)(b^2 + b + 1) \log(1-b) \]

\[
- (1-b) \left[ \frac{1}{3} (15b^2 - 2b + 15) \right. \]

\[
+ 2 \left( \log_2(b) - \frac{\pi^2}{6} \right) b^4 + \frac{6b^2 + 1}{1-b} \right]. \]

\[ \]
V. CONTRIBUTION OF LEPTON PAIR PRODUCTION

Consider now the hard pair production process that takes place at the same order of perturbation theory as the hard photon emission. In the same way we may conclude that the soft pair case as well as the case of double collinear kinematics does not contribute to the radiative tail. Therefore we may consider only semi-collinear kinematics of hard pair production of which there exist two different mechanisms [24]. One of these is the two photon mechanism of pair creation. An electron from that pair having momentum $p_1'$ is detected in experiment and the scattered electron moves close to the initial electron direction. This kinematics permits us to apply the Weizsäcker-Williams approximation,

$$2\varepsilon_1^2 \frac{d^3\sigma_{\text{pair}}}{d^3p_1'} = \frac{2\alpha^4}{\pi^3} \int \frac{d^2q}{(q^2 + Q_{\text{min}}^2)^2} \frac{q^2L}{b(Q^2)^2} \Phi_{\text{prot}}$$  

$$\times \frac{d\beta_-}{(1 - \beta_-)^4} ((1 - \beta_- - b)^2 + b^2)(1 + \beta_+^2),$$

$$s_1 = Q^2 \frac{1 - \beta_-}{\beta_+}, \quad b + \beta_- + \beta_+ = 1.$$  

The second mechanism is characterized by the bremsstrahlung mechanism of pair creation, with an electron from a pair to be detected. Leaving details to the Appendix E let us present here the result

$$2\varepsilon_1^2 \frac{d^3\sigma^{(2)}_{\text{pair}}}{d^3p_1'} = \frac{2\alpha^4}{\pi^3} \int \frac{d^2q}{(q^2 + Q_{\text{min}}^2)^2} \frac{q^2L}{(Q^2)^2} \Phi_{\text{prot}}$$

$$\times \frac{b(1 + \beta_-^2)d\beta_-}{(1 - \beta_-)^4} [(1 - b - \beta_-)^2 + b^2],$$

$$s_1 = Q^2 \frac{1 - \beta_-}{b\beta_-}.$$  

The integration over $\beta_-$ can be performed analytically with additional assumption that $Q_{\text{min}}^2$ has no $\beta_-$ dependence. The result for the sum of these contributions is found to be

$$2\varepsilon_1^2 \frac{d^3\sigma_{\text{pair}}}{d^3p_1'} = \frac{2\alpha^4}{\pi^3} \int \frac{d^2q}{(q^2 + Q_{\text{min}}^2)^2} \frac{q^2L(1 + b^2)}{b(Q^2)^2} \Phi_{\text{prot}}$$

$$\times (1 - b + 2(1 + b) \log b + \frac{4}{3b}(1 - b^4)).$$  

VI. DISCUSSION AND CONCLUSION

In the paper presented the correction to radiative tail from elastic peak is studied in the kinematics when a final lepton is measured. Using Sudakov technique the contributions of loops (12), double photon bremsstrahlung (21) and a pair production (23,24) are calculated.

In this section we analyze obtained contributions numerically. Both the relative contributions of the processes considered and the total correction to the lowest order process are investigated within kinematical conditions of experiments on electron DIS at TJNAF and DESY (both for HERA and for HERMES). It is convenient to define the following quantities:

$$\delta = \frac{\sigma_L + \sigma_N + \sigma_p}{\sigma_0}, \quad \delta_{L,N,p} = \frac{\sigma_{L,N,p}}{\sigma_0}. $$  

Here $\sigma_0$ stands for the cross section of radiative tail from elastic peak [4]. Other $\sigma$’s constitute the next order results. The quantity $\sigma_p$ is a direct sum of two mechanisms of pair creations (23,24), whereas $\sigma_L$ and $\sigma_N$ are the leading (including mass singularities terms log($Q^2/m^2$)) and next-to-leading (independent of leptonic mass) terms.

![FIG. 1. QED correction to radiative tail from elastic peak in electron DIS for the kinematics of TJNAF ($\sqrt{S} = 4$ GeV, $x = 0.05$), HERMES ($\sqrt{S} = 7$ GeV, $x = 0.5$) and HERA ($\sqrt{S} = 300$ GeV, $x = 0.0005$).]
the next-to-leading contribution completely fixes all uncertainties of leading log approximation thus leaving unknown only terms proportional to lepton mass squared and $Q^2_h$, which is effectively small due to behavior of form factors.

**ACKNOWLEDGEMENTS**

We would like to thank P.Kuzhir for useful discussions and comments. The work of IA was partially supported by the U.S. Department of Energy under contract DE-AC05-84ER40150. EAK and BGS acknowledge support of RFBR via grant No. 99-02-17730.

**APPENDIX A. DETAILS OF MATRIX ELEMENT CALCULUS: THE CASE OF SINGLE PHOTON BREMSSTRAHLUNG**

Using the Sudakov decomposition of the 4-vectors in the problem

$$p'_1 = \alpha'_1 \tilde{p}_2 + b \tilde{p}_1 + p'_1 \perp, \quad k_1 = \alpha_1 \tilde{p}_2 + x_1 \tilde{p}_1 + k_1 \perp,$$

$$q = p_2 - p'_2 = \alpha_2 \tilde{p}_2 + \beta_2 \tilde{p}_1 + q \perp,$$

$$p'_2 = \alpha'_2 \tilde{p}_2 + \beta'_2 \tilde{p}_1 + p'_2 \perp, \quad v \perp p_1 = v \perp p_2 = 0,$$

we have excluded parameters $\alpha_1, \alpha'_1, \beta_q$ using the on-shell conditions

$$p'_2 - M^2 = -s \beta_q (1 - \alpha_q) - q^2 - \alpha_q M^2 = 0, \quad (A.2)$$

$$p'_1 = s b \alpha_1 - p'_1 \perp = 0, \quad k_1^2 = s x_1 \alpha_1 - k_1^2 = 0,$$

besides

$$\Phi^\text{prot} = \frac{1}{s^2} \text{Sp}\{ \{ \tilde{p}_2 + M \} \Gamma_{\rho} \{ \tilde{p}_2 + M \} \tilde{\Gamma}_{\sigma} p_1^\rho p_1^\sigma \},$$

$$\Gamma_{\rho} = F_1(q^2) \gamma_{\rho} + \frac{\sigma_{\rho \mu}}{2M} F_2(q^2).$$

Here $F_1, F_2(q^2)$ are the Dirac and Pauli form factors of a proton. For $\Phi^\gamma$ we have:

$$\Phi^\gamma = -\frac{1}{s^2} \text{Sp}\{ \tilde{p}_2^\rho O_\mu \tilde{p}_1^\mu \},$$

$$O_\mu = \tilde{p}_2^\mu \tilde{p}_1^\rho - \tilde{k}_1^\rho \gamma_{\mu} + \gamma_{\mu} \tilde{p}_1^\rho + \tilde{k}_1^\rho \tilde{p}_2,$$

and then

$$q^2 = -Q^2_h = -\frac{1}{1 - \alpha_q} [q^2 + M^2 \alpha_q^2 \approx -[q^2 + Q^2_{\text{min}}]], \quad (A.5)$$

with $Q^2_{\text{min}}$ given in the text. The matrix element
\[ M = \frac{1}{q^2} f_\sigma^{(1)} \tilde{u}(p_2') \Gamma \rho u(p_2) g^{\rho\sigma}, \quad (A.6) \]

using the Gribov representation for the metric tensor

\[ g^{\rho\sigma} = g_\perp^{\rho\sigma} + \left(\frac{2}{s}\right) (\tilde{p}_2^\rho \tilde{p}_1^\sigma + \tilde{p}_2^\sigma \tilde{p}_1^\rho) \approx \left(\frac{2}{s}\right) \tilde{p}_2^\rho \tilde{p}_1^\rho, \quad (A.7) \]

may be put in a form

\[ M = \frac{2s}{q^2} \left( \frac{1}{s} p_2^\sigma j_1^{(1)} \right) \left( \frac{1}{s} \tilde{u}(p_2') \Gamma p u(p_2) p_1^\rho \right). \quad (A.8) \]

Note that each expressions in the parentheses on the RHS of Eq. (A.8) do not depend on \( s \) in the limit \( s \to \infty \). The expression for \( \Phi^\gamma \) may be transformed using the following reduced expression

\[ O_\mu = x_1 \left[ sb \gamma_\mu \left( \frac{1}{n} - \frac{1}{n_1} \right) + \frac{1}{n_1} b \gamma_\mu \hat{q} \hat{p}_2 - \frac{1}{n} \gamma_\mu \hat{p}_2 \hat{q} \right], \quad (A.9) \]

to take the form given in Eq. (8).

**APPENDIX B. DETAILS OF MATRIX ELEMENT CALCULUS: THE CASE OF DOUBLE PHOTON BREMSSTRAHLUNG**

Let’s first demonstrate that the matrix element of the process

\[ \gamma^* (q) + e(p_1) \to e(p'_1) + \gamma(k_1) + \gamma(k_2) \quad (B.1) \]

is explicitly proportional to \( q \) for small values of the latter, which is in fact the requirement of gauge invariance with respect to the virtual photon. The matrix element is described by six diagrams. With regard to the gauge invariance this set can be separated out to the two subsets in each of which the gauge condition is satisfied independently. Introducing the photon-permuting operator \( P_{12} \) we bring the matrix element to the form:

\[ M = (1 + P_{12}) Q, \quad Q = M_1 + M_2 + M_3, \quad (B.2) \]

where

\[ M_1 = \frac{1}{d_1 d_2} \tilde{u}(p'_1) \tilde{p}_2 (\tilde{p}_1 - \tilde{k}_1 + m) \tilde{e}_1^* \]

\[ \times \tilde{e}_2^*(\tilde{p}_1 - \tilde{k}_1 + \tilde{q} + m) \tilde{e}_1^* u(p_1), \quad (B.3) \]

\[ M_2 = \frac{1}{d_1 d_2} \tilde{u}(p'_1) \tilde{e}_2^*(\tilde{p}_1 - \tilde{k}_1 + \tilde{q} + m) \tilde{p}_2 \]

\[ \times (\tilde{p}_1 - \tilde{k}_1 + \tilde{q} + m) \tilde{e}_1^* u(p_1), \quad (B.4) \]

\[ M_3 = \frac{1}{d_1 d_2} \tilde{u}(p'_1) \tilde{e}_2^*(\tilde{p}_1 - \tilde{k}_1 + \tilde{q} + m) \]

\[ \times \tilde{e}_1^*(\tilde{p}_1 + \tilde{q} + m) \tilde{p}_2 u(p_1), \quad (B.5) \]

and

\[ d = d_1 + d_2 - \frac{1}{x_1 x_2} (x_1 \tilde{k}_2 - x_2 \tilde{k}_1)^2, \]

\[ d' = d'_1 + d'_2 + \frac{1}{x_1 x_2} (x_1 \tilde{k}_2 - x_2 \tilde{k}_1)^2. \]

The permutation operator \( P_{12} \) for the photons acts the following way

\[ P_{12} f(k_1, e_1; k_2, e_2) = f(k_2, e_2; k_1, e_1), \quad P_{12}^2 = 1. \]

The quantity \( Q \) is gauge invariant regarding the virtual photon \( k \) since all permutations of this photon have been taken into account. Therefore \( Q \) is proportional to \( q_\perp \) in the limit of \( q_\perp \to 0 \). Indeed, making use of the relations

\[ Q = p_\mu Q^\mu, \quad q_\mu Q^\mu = (\alpha q \tilde{p}_2 + q_\perp) u Q^\mu = 0, \quad (B.6) \]

we immediately obtain (neglecting the small contribution \( \beta p_\mu Q^\mu \sim 1/s \))

\[ Q = -\frac{q_\mu}{\alpha q} Q^\mu. \quad (B.7) \]

Then transform the quantities \( M_j \) to such a form that the noticed low \( q_\perp \) behavior is present in their sum \( Q \) explicitly. The reason is that in this case all individual large (compared to \( q_\perp \)) contributions are mutually cancelled. The first step is to use the Dirac equations \( \hat{p}_1 u(p_1) = mu_1, \tilde{u}(p'_1) \tilde{p}_1' = m\tilde{u}(p'_1) \) and to rearrange the amplitudes \( M_j \) of Eq. (B.3),

\[ M_1 = \tilde{u}(p'_1) \left\{ \frac{s}{d_1} \tilde{e}_2^*(\tilde{p}_1 - \tilde{k}_1 + m) \tilde{e}_1^* \right. \]

\[ - \frac{1}{d_1} \tilde{p}_2 \hat{q} \tilde{e}_2^*(\tilde{p}_1 - \tilde{k}_1 + m) \tilde{e}_1^* \}

\[ \left. \right\} u(p_1), \quad (B.8) \]

\[ M_2 = \tilde{u}(p'_1) \left\{ - \frac{s(1 - x_1)}{d_1 d_2} \tilde{e}_2^*(\tilde{p}_1 - \tilde{k}_1 + m) \tilde{e}_1^* - \frac{1}{d_2} \tilde{e}_2^* \tilde{p}_2 \tilde{e}_1^* \right. \]

\[ + \frac{1}{d_1 d_2} \tilde{e}_2^* \hat{q} \tilde{p}_2 (\tilde{p}_1 - \tilde{k}_1 + m) \tilde{e}_1^* \}

\[ \left. \right\} u(p_1), \quad (B.8) \]

\[ M_3 = \tilde{u}(p'_1) \left\{ \frac{s}{d_2} \tilde{e}_2^* (\tilde{p}_1 - \tilde{k}_1 + m) \tilde{e}_1^* + \frac{s}{d_2} \tilde{e}_2^* \tilde{q} \tilde{e}_1^* \right. \]

\[ + \frac{1}{d_1 d_2} \tilde{e}_2^* (\tilde{p}_1' + \tilde{k}_2 + m) \tilde{e}_1^* \tilde{q} \tilde{p}_2 \}

\[ \left. \right\} u(p_1). \quad (B.8) \]

From these formulae it can be noted that the last terms in \( M_1, M_2, M_3 \), up to terms of the order of

\[ \frac{m^2}{E^2}, \quad \theta^2, \quad \frac{m}{E} \theta, \]

are proportional to \( q_\perp \),

\[ \hat{p}_2 \hat{q} = \hat{p}_2 (\alpha q \hat{p}_2 + \beta \hat{q} + \tilde{q}_\perp) = \hat{p}_2 \hat{q}_\perp = -\tilde{q}_\perp \hat{p}_2. \quad (B.9) \]

Next, one can see that the sum of the first three terms in Eqs. (B.8) is also proportional to \( q_\perp \) since (for more details see [25])
\[ A \equiv \frac{b}{d_1} + \frac{1 - x_1}{d_1 d'_2} + \frac{1}{d_2 d'_2}, \quad A|_{q_\perp = 0} = 0. \] (B.10)

Finally we consider the sum of the second terms of the quantities \( \mathcal{M}_2, \mathcal{M}_3 \) given in Eqs. (B.8). Using the relations (23) and Eq.(21)) and

\[ (p'_1 + k_1 + k_2)^2 = (p_1 - k)^2 = m^2 - k^2 - s \alpha_k, \]

one immediately gets

\[ \frac{\hat{p}_2}{d'_2} + \frac{s(\alpha \beta \hat{p}_2 + \hat{q}_1 \hat{q}_2)}{d'_2 d'_2} = \frac{s q_\perp}{d'_2 d'_2} + \frac{\hat{p}_2 q_2}{d'_2}. \] (B.11)

Therefore, from Eqs. (B.9), (B.10), (B.11) it is clearly seen that the property illustrated by Eq. (B.7)

\[(\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3)|_{q_\perp = 0} = 0\]

is evidently satisfied and consequently the quantity

\[ Q = \overline{u}(p'_1) \left\{ A s \hat{e}^*_2 (\hat{p}_1 - \hat{k}_1 + m) \hat{e}_1^* + \frac{1}{d_1} \hat{p}_2 \hat{q}_1 \hat{e}^*_2 (\hat{p}_1 - \hat{k}_1 + m) \hat{e}_1^* - \frac{q^2}{d_2 d'_2} \hat{e}^*_2 \hat{e}^*_1 \hat{p}_2 + \frac{s}{d'_2} \hat{e}^*_2 \hat{q}_1 \hat{e}_1^* \right. \]

\[ + \frac{1}{d_1 d'_2} \hat{e}^*_2 \hat{q}_1 \hat{p}_2 (\hat{p}_1 - \hat{k}_1 + m) \hat{e}_1^* + \left. \frac{1}{d_1 d'_2} \hat{e}^*_2 (\hat{p}_1 + \hat{k}_2 + m) \hat{e}_1^* \right\} u(p_1). \] (B.12)

Calculating the contribution of the trace \( Sp\{p'_1 Q p_1 \hat{Q} \} \) we neglect masses whose contribution to the quantity \( \Phi^{\gamma} \) may be restored using the general prescription [23]. The corresponding correction has the form:

\[ \Delta_m \Phi^{\gamma} = (1 + P_{12}) \left\{ -\frac{4 m^2 x_2 y_1 (1 + y_1^2)}{d'^2} (1 - x_2)^2 \right. \]

\[ \times \frac{q^2}{(q - y_1 p'_1)^2 (q - p'_1/b)^2} - \frac{4 m^2}{d'^2} \beta_2^2 z_1 (1 + z_1^2) q^2 \left. \right\}, \] (B.13)

where

\[ y_1 = \frac{1 - x_2}{b}, \quad \beta_2 = \frac{x_2}{1 - x_1}, \quad z_1 = \frac{b}{1 - x_1}. \] (B.15)

**APPENDIX C. EVALUATION OF 2-DIMENSIONAL INTEGRALS**

The azimuthal integration may be performed making use of the following equality:

\[ J_{12...n} = \frac{1}{2\pi} \int_0^{2\pi} d\phi \prod_i [a_i + b_i \cos(\phi - \phi_i)]^{-1} \]

\[ = \sum_{k=1}^{n-1} \prod_{j \neq k} \frac{b_k}{r_k} \overline{r_k} \sin(\phi_k - \phi_j), \] (C.1)

with

\[ r_i = \sqrt{a_i^2 - b_i^2}, \quad |a_i| > |b_i|, \]

\[ b_{ij} = b_ia_j - b_ja_i \cos(\phi_i - \phi_j). \]

It is curious to note that the absence of the imaginary part provides an interesting algebraic identity. For \( n = 2, n = 3 \) it looks

\[ J_{12} = \frac{1}{d_{12}} \left( \frac{b_1}{r_1} b_{12} + \frac{b_2}{r_2} b_{12} \right), \quad d_{12} = a_{12}^2 - r_{12} r_{22} \] (C.2)

\[ a_{12} = a_1 a_2 - b_1 b_2 \cos(\phi_1 - \phi_2), \]

\[ J_{123} = \frac{b_{12}^2}{r_1} \frac{a_{12} a_{13} - r_{12} r_{13}}{d_{12} d_{13}} + \frac{b_{2}^2}{r_2} \frac{a_{21} a_{23} - r_{21} r_{23}}{d_{21} d_{23}} + \frac{b_{3}^2}{r_3} \frac{a_{31} a_{32} - r_{31} r_{32}}{d_{31} d_{32}}. \]

This form is convenient for a subsequent integration over \( dk^2_1 \).

**APPENDIX D. NLO CONTRIBUTIONS FROM VIRTUAL AND SOFT PHOTON EMISSION**

To avoid the misprints we use here the notations of the paper [8]

\[ s = d'_1, \quad t = -d_1, \quad u = -Q^2, \] (D.1)

\[ s + t + u = Q^2, \quad f(s, t) = f(t, s), \quad a = s + t, \]

\[ b = s + u, \quad c = u + t. \]

The quantities \( \tau_{ij} \) encountered in the text (see Eq. (12)) may be written as

\[ \tau_{11} = -G \left[ 1 + \frac{u^2}{s^2} \right] - \tilde{G} \left( 2 + \frac{b^2}{t^2} \right) + \frac{b^2}{u} \frac{2 u + a}{2 u - s + t} \]

\[ + \frac{2}{a^2} (u^2 - b t) l_{qu} + \frac{b^2}{a^2 t^2} (2 c + t) l_{qs} + \frac{2 u - s + t}{s} l_{qt} \] (D.2)

\[ + \frac{1}{q^2} \left( \frac{4}{a} (b t - u^2) - 4 u - 2 q^2 + t - \frac{b^2}{c} \right). \]

\[ \tau_{12} = \frac{c}{a s} (u - s) G + \frac{1}{t} \left( u q^2 - st \right) \tilde{G} - \frac{u q^2}{s} \frac{2 u - s + t}{a} \]

\[ + \frac{2}{a^2} (u^2 - c s) l_{qu} + \frac{2 c + t}{c^2} \left( s - \frac{u}{t} q^2 \right) l_{qs} \]

\[ - \frac{c}{b s} (2 u - s) l_{qt} + \frac{1}{q^2} \left[ \frac{4}{a} (u^2 - c s) + 8 u + 3 t \right] \]

\[ - s + \frac{2}{c} u s, \]
and the additional notations look
\[ l_{qu} = \ln \frac{q^2}{u}, \quad l_{qs} = \ln \frac{-q^2}{s}, \quad l_{qt} = \ln \frac{q^2}{t}, \quad l_{ut} = \ln \frac{u}{t}, \]
\[ G = l_{qu}(l_{qt} + l_{ut}) + 2Li_2 \left( 1 - \frac{t}{q^2} \right) \]
\[ -2Li_2 \left( 1 - \frac{q^2}{u} \right) - 2Li_2(1). \]

**APPENDIX E. SEMI-COLLINEAR KINEMATICS OF PAIR CREATION**

The matrix element in the kinematics \([2]\) may be put in a form (we extract the coupling constant):
\[ M^{(1)} = \frac{1}{q^2} J_\nu I_\mu g^{\mu\nu}, \quad J_\nu = \bar{u}(p_-)\gamma_\nu u(p_1), \] (E.1)
where the current \(I\) describes a pair production by the photon with momentum \(q_1\) off a proton. Using the Sudakov form of the 4-vectors \(p_-\) and \(q\) with basic 4-vectors \(p_1\) and \(p_2\),
\[ p_- = \alpha_\perp \vec{p}_2 + \beta_\perp \vec{p}_1 + p_\perp, \quad q = \alpha_q \vec{p}_2 + \beta_q \vec{p}_1 + q_\perp, \]
the representation of the metric tensor
\[ g_{\mu\nu} = g_{\mu\nu\perp} + \frac{2}{s} p_{2\nu} p_{1\mu}, \]
and the gauge condition
\[ Iq = I(\beta_q p_1 + q_\perp) = 0, \quad \beta_q + \beta_- = 1, \]
we obtain for the matrix element squared and summed over spin states of electron:
\[ \sum |M^{(1)}|^2 = \frac{1}{(q^2)^2} \left[ -2q^2 \mkappa^2 + \frac{8}{\beta_q^2} (p_\perp \mkappa)^2 \right]. \] (E.2)

To calculate the quantity \(\mkappa^2\), we again present it in a form
\[ I = e_{q_1} I = e_\mu^1 e_\nu^1 2s|q|^2 \rho_{2\mu} \bar{Y}_{\mu} \bar{u}(p_-') O_{\mu\nu} v(p_+), \]
\[ s_1 = \rho_2 (p_2 + q_1)^2, \quad Y_{\mu} = \bar{u}(p_2') \Gamma_{\nu} u(p_2'). \] (E.3)
The phase volume is transformed the way to take the following form
\[ d\Gamma_4 = (2\pi)^{-8} \frac{1}{8s\beta_-\beta_+ b} d^2q d^2p_- d\beta_- \] (E.4)

Using
\[ \sum |\bar{u}(p_-') O_{\mu\nu} v(p_+) e_\mu e_\nu|^2 = 8 \left[ \frac{b}{\beta_+} + \frac{\beta_+}{b} \right], \]
we obtain the result for the cross section given in the text.

For the kinematics of bremsstrahlung mechanism the matrix element has a form
\[ M^{(2)} = \frac{1}{k_1^2} I_\nu J_\mu g^{\mu\nu}, \quad k_1 = p_+ + p_1'. \] (E.5)
Here it is suitable to use alternative basis vectors of Sudakov parameterization
\[ p_+ = \alpha_+ q + b_+ \vec{p}_1' + p_+\perp, \quad k_1 = \alpha_1 q + b_1 \vec{p}_1' + k_1\perp, \] (E.6)
\[ g_{\mu\nu} = g_{\mu\nu\perp} + \frac{2}{s} q^\rho p_1'^\rho, \quad k_1^2 = \frac{p_1'^2 + m^2 b_1^2}{b_1 - 1} > 0. \]
Quite the same manipulations give
\[ \sum |M^{(2)}|^2 = 2k_1^2 \mkappa^2 - \frac{8}{b_1^2} (k_1 \mkappa)^2. \]
Performing the integration over \(d^2(p_+\perp)\) to a logarithmic accuracy and expressing the parameter \(b_1\) in terms of the standard Sudakov decomposition with basic 4-vectors \(p_1, p_2\)
\[ b_1 = \frac{1 - \beta_-}{b}, \]
we immediately obtain the result given in the text.

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