A close examination of cosmic microwave background mirror-parity after Planck

Assaf Ben-David\textsuperscript{1∗} and Ely D. Kovetz\textsuperscript{2†}

\textsuperscript{1}Niels Bohr International Academy and Discovery Center, The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen \textOmega, Denmark
\textsuperscript{2}Theory Group, Department of Physics and Texas Cosmology Center, The University of Texas at Austin, TX 78712, USA

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ABSTRACT

Previous claims of significant evidence for mirror-parity in the large-scale cosmic microwave background (CMB) data from the Wilkinson Microwave Anisotropy Probe (WMAP) experiment have been recently echoed in the first study of isotropy and statistics of CMB data from Planck. We revisit these claims with a careful analysis of the latest data available. We construct statistical estimators in both harmonic and pixel space, test them on simulated data with and without mirror-parity symmetry, apply different Galactic masks, and study the dependence of the results on arbitrary choices of free parameters. We confirm that the data exhibit evidence for odd mirror-parity at a significance which reaches as high as ∼ 99 per cent C.L., under some circumstances. However, given the inherent biases in the pixel-based statistic and the dependence of both pixel and harmonic space statistics on the particular form of Galactic masking and other a-posteriori choices, we conclude that these results are not in significant tension with the predictions of the concordance cosmological model.

Key words: cosmic background radiation – methods: data analysis – methods: statistical

1 INTRODUCTION

The importance of large scale symmetries in the cosmic microwave background (CMB), if those are shown to exist, cannot be overstated. From a theoretical standpoint, these scales provide us with a possible observational window to the very early Universe, and to the physics at very high energies. Deviations from the expectations of the concordance Λ cold dark matter (ΛCDM) cosmological model on these scales, such as the breaking of statistical isotropy, could indicate a special location, axis or direction, which in turn can point to the existence of exotic pre-inflationary relics or other effects in the pre-inflationary Universe.

Mirror-parity is an intriguing example of a symmetry which breaks statistical isotropy. As it involves a symmetry plane, it can be naturally attributed to various early universe models predicting large-scale modulation of the CMB (Gordon et al. 2005; Ackerman, Carroll & Wise 2007; Hou et al. 2010; Schmidt & Hui 2013). Another interesting class of models that could induce this symmetry involves a finite topology for the Universe (de Oliveira-Costa, Smoot & Starobinsky 1996). In such models, statistical isotropy on large scales may be broken by the finite fundamental domain (Levin, Riaziuelo et al. 2004).

When searching for a mirror-parity symmetry plane on real data, however, care must be taken to avoid spurious effects that may induce even mirror-parity due to experimental systematics (some of which involve the ecliptic plane) or foregrounds (which dominate the Galactic plane). Also, as always, inherent biases and a-posteriori choices in the definition of the statistical estimators may lead to unfounded conclusions regarding anisotropies in the data.

In this paper, we revisit recent claims of borderline significant detection of mirror-parity in the CMB (Land & Magueijo 2005; Ben-David, Kovetz & Itzhaki 2012; Finelli et al. 2012; Ade et al. 2013a) in data from both the Wilkinson Microwave Anisotropy Probe (WMAP; see e.g. Bennett et al. 2011) and Planck (Ade et al. 2013a) experiments. The different reports have been consistent so far in identifying two prominent directions in the sky, one which maximizes even mirror-parity and another corresponding to odd mirror-parity. The even mirror-parity direction found (Land & Magueijo 2005; Ben-David et al. 2012; Finelli et al. 2012) coincides well with the direction of the CMB dipole (Kogut et al. 1992; Bennett et al. 2003, Aghanim et al. 2013), and its reported significance was rather mild. Meanwhile, the direction in which odd mirror-parity is maximized (Ben-David et al. 2012; Ade et al. 2013a) has been assigned much higher levels of statistical significance. The various works, however, differ greatly in the methods used to analyse the data, and utilise different statistical estimators, Galactic masks and significance estimation methods.

This work aims to shed a clear light on these findings. We use the recent Planck data release to perform a robust search for mirror-parity using both a pixel-based statistic (de Oliveira-Costa et al. 1996, 2004) and a statistic in harmonic space (Ben-David et al. 2012; Finelli et al. 2012). We address several issues involved, from the statistical methods used in the analyses and their inherent
2 DATA AND SIMULATIONS

Our main focus in this work is the first data release of CMB sky maps from the Planck experiment (Ade et al. 2013a). We shall use two of the four component separation maps available – the SMICA and NILC maps. These two maps are the cleaner of the four, and are accompanied by smaller Galactic masks. The other two component separation maps, SEVEM and Commander-Ruler, both give similar results and were left out for brevity. The third map we use has recently been released by Bobin et al. (2014). It was created using the LGMCA component separation method, and it combines the data from the 9-year release of WMAP as well as the first Planck release (we use the map Bobin et al. (2014) refer to as WPR1). It is claimed to be a rather clean full sky map, that can be trusted, like the Planck SMICA and NILC maps, with small Galactic masks, and also completely unmasked. We refer to this map as LGMCA.

Since we are interested in analysing the large scales of the CMB sky, we smooth the data maps and degrade them to a low HEALPix (Górski et al. 2005) resolution of $N_{\text{side}} = 16$, corresponding roughly to a harmonic scale of $\ell_{\text{max}} = 48$. This also gives the added benefit of significantly reducing the computation time required for the analysis. Each map is first deconvolved in harmonic space with the beam and pixel window functions. It is then convolved with a Gaussian beam with FWHM of $\sim 11^\circ$, equivalent to 3 pixels of a $N_{\text{side}} = 16$ map. We retain only the harmonic coefficients up to $\ell_{\text{max}} = 64$ and convert the smoothed coefficients back to a $N_{\text{side}} = 16$ map.

We use three Galactic masks in our analysis, of various sizes, all released by the Planck team (Ade et al. 2013a). As the main mask we use the fairly large U73 mask, which is the union of the confidence masks for the individual component separation maps. This mask covers 27 per cent of the sky. In addition, we apply two smaller masks to the SMICA map – its confidence mask, covering 11 per cent of the sky, named CS-SMICA89 (Ade et al. 2013c) and the small inpainting mask defined by the Planck team for the SMICA map, which we denote as SMICA-INP. This mask covers only 3 per cent of the sky.

The degradation of these masks to a low resolution is done in a similar manner to the process used for the data maps. The masks are first smoothed with the same Gaussian kernel as the data maps, and converted to $N_{\text{side}} = 16$ maps. In addition, they are thresholded to produce a binary mask. We have chosen the threshold for each mask so that the masked area will not be changed by the process of smoothing and degradation. The threshold values we use are 0.68, 0.57 and 0.81 for the U73, CS-SMICA89 and SMICA-INP masks, respectively. The resulting three low-resolution masks are shown in Fig. 1.

In order to test the significance of our findings, we compare them to realisations of the statistically isotropic $\Lambda$CDM. We use the Planck maximum-likelihood power spectrum (Ade et al. 2013b) to generate $10^5$ random realisations with $\ell_{\text{max}} = 48$. The harmonic coefficients are smoothed with the same Gaussian kernel used to smooth the data maps and converted to $N_{\text{side}} = 16$ maps. No detector noise is added to these simulations (for the large scales we are studying, the dominant source of uncertainty is cosmic variance). We note that when comparing the data with the simulations, we always treat the simulations in exactly the same way as the data, i.e., if the data is masked, we compare it to randoms which have been masked with the same mask.

3 METHOD

In the various previous works testing the CMB sky for signs of mirror-parity, two different statistics were used. One, introduced by de Oliveira-Costa et al. (1996), uses the data in pixel-space, and the other, introduced by Ben-David et al. (2012), uses the harmonic decomposition of the CMB sky. In this work we use both approaches. This allows us to compare between the two statistics and present a broader analysis.

3.1 Pixel-Based Statistic

Under mirror-parity, a direction $\hat{r}$ in the sky transforms as

$$\hat{r} \rightarrow \hat{r}_n = \hat{r} - 2(\hat{r} \cdot \hat{n})\hat{n},$$

where $\hat{n}$ is the normal to the mirror plane. In order to check for mirror-parity in pixel-space, one can simply compare the sky hemispheres by using the statistic (de Oliveira-Costa et al. 2004; Pinelli et al. 2012; Ade et al. 2013c) 

$$S_p^2(\hat{n}) = \left[ \frac{T(\hat{r}) \pm T(\hat{r}_n)}{2} \right]^2,$$

where the over-bar denotes an average over all spatial directions $\hat{r}$. With this statistic, a large degree of even (odd) mirror-parity is given by a low $S_p^2(\hat{n})$.

In the pixel-based approach, it appears that applying a Galactic mask is straightforward, as the masked pixels can simply be...
We use two methods in order to estimate the significance of the decomposition of the masked sky. The method we use for harmonic orthogonal on the cut sky, one cannot simply perform a harmonic when transforming the sky maps to harmonic space after applying and count the number of random simulations with a higher value tic, i.e.,

\[
R_{\ell m} = \frac{\ell}{2r + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2
\]

is the actual observed power spectrum and the subtraction of \( \ell_{\text{max}} - 1 \) simply ensures that the ensemble mean is zero. The free parameter, \( \ell_{\text{max}} \), that determines the smallest scale taken into account, allows us to easily test the scale dependence of the results. In this work we consider \( \ell_{\text{max}} = 5, \ldots, 9 \). Using the harmonic statistic, a large degree of even (odd) mirror-parity is given by a high (low) value for \( S_h \).

Unlike in the pixel-based approach, here care must be taken when transforming the sky maps to harmonic space after applying a Galactic mask. Since the spherical harmonic functions are not orthogonal on the cut sky, one cannot simply perform a harmonic decomposition of the masked sky. The method we use for harmonic reconstruction is described in the appendix.

### 3.3 Significance Estimation

We use two methods in order to estimate the significance of the mirror-parity scores on the input data maps. In the first method, the ‘raw’ score is used. We define \( R \) to be the best score of each statistic, i.e., \( R \) is the minimum over all directions of \( S_n(\hat{n}) \) (\( S_o(\hat{n}) \)) for even (odd) parity using the pixel-based statistic, and the maximum (minimum) over all directions of \( S_n(\hat{n}) \) for even (odd) parity using the harmonic statistic.

\[
R = \begin{cases} 
\min S_n(\hat{n}) & \text{pixel-based even parity,} \\
\min S_o(\hat{n}) & \text{pixel-based odd parity,} \\
\max S_n(\hat{n}) & \text{harmonic even parity,} \\
\max S_o(\hat{n}) & \text{harmonic odd parity.}
\end{cases}
\]

We then compare \( R \) to its value on random simulations. Note that for the pixel-based statistic, this is the same estimator used by the Planck team in their search for anomalous mirror-parity (Ade et al. 2013). The second method was first introduced in Ben-David et al. (2012) (see also Rassat & Starck 2013). For each score map, we calculate its mean \( \mu \) and standard deviation \( \sigma \) over all directions. We then normalize the best score as

\[
R = \frac{R - \mu}{\sigma}
\]

and count the number of random simulations with a higher value of \( R \). This estimator tests how significant the best direction is relative to the rest of the score map, before comparing to simulations.

It is particularly well suited for a scenario in which the anomalous mirror-parity is of cosmological origin, e.g. induced by some large scale pre-inflationary effect\(^1\). In such a scenario, we expect to discover a single pronounced direction of mirror-parity in the sky. The \( R \) estimator therefore complements the \( R \) estimator, which only considers the best direction of each map.

### 4 TESTS ON RANDOM SIMULATIONS

Before using the pixel-based and harmonic statistics on the real CMB data, we test their performance on random simulations to compare their efficiency in detecting signs of mirror-parity and their sensitivity to the overall CMB power and to the use of large sky masks.

#### 4.1 Detectability of Mirror-Parity

In order to test the ability of our statistics to detect anomalous mirror-parity in the CMB data, we use random realisations that have been modulated to contain different traces of mirror-parity. Since we are searching for any signs of mirror-parity, without limiting ourselves to a specific model, we modulate the random realisations using very simple phenomenological models.

Before modulating each random map, we first choose a mirror-parity axis at random and use it as the \( z \)-axis to perform a harmonic decomposition of the map. We then rescale each of the harmonic coefficients \( a_{\ell m} \) by the non-negative factors \( f_{\ell m} \) given by

\[
f_{\ell m} = \begin{cases} 
e & \ell + m \text{ is even,} \\
o & \ell + m \text{ is odd.}
\end{cases}
\]

The total power is kept constant by the requirement that these even and odd coefficients satisfy

\[
(\ell + 1)c_x^2 + 6\sigma_x^2 = 2\ell + 1
\]

for each \( \ell \). We introduce a scale dependent amplitude \( x_\ell \in [-1,1] \), with \( x_\ell = 1 \) indicating maximal even parity and \( x_\ell = -1 \) maximal odd parity. To modulate with even (odd) mirror-parity, we set \( \sigma_\ell = 1 - x_\ell \) for \( x_\ell \geq 0 \) (\( \sigma_\ell = 1 + x_\ell \) for \( x_\ell < 0 \)), and use the power conservation equation \((9)\) to determine \( e_\ell (o_\ell) \).

We perform the modulation according to two different schemes:

(i) Constant mirror-parity, for which the amplitude is constant for all scales, \( x_\ell = x \).

(ii) Decaying mirror-parity, for which the amplitude is exponentially decaying. We set a pivot scale \( \ell_\star = 7 \) and, for a given global parity amplitude \( x \), set the parity level as \( x_\ell = \text{sgn}(x)|x|^\ell/\ell_\star \).

We vary the mirror-parity amplitude \( x \) and examine simulations with even mirror-parity at amplitudes \( x = 0.1, 0.2, \ldots, 0.8 \). Apart from this mirror-parity modulation, these simulations are created in exactly the same manner as the clean \( \Lambda \)CDM simulations discussed above. We draw \( 10^4 \) realisations and modulate each of them with the 8 amplitude values, in each of the two modulation schemes.

The results of testing the effectiveness of the parity estimators using the modulated random realisations are plotted in Fig.\( [2] \).

\(^1\) An extension of the model introduced in Fialkov, Itzhaki & Kovetz (2010) (see also Kovetz, Ben-David & Itzhaki 2010), in which a pre-inflationary particle in motion remains in the vicinity of the observable Universe after inflation, can be shown to induce traces of mirror-parity in the CMB sky.
Panels (a) and (b) show histograms of the $R$ estimator for different modulation amplitudes, in both the constant and decaying schemes, for the pixel-based and harmonic statistics, respectively. We see that as the parity amplitude is increased, the histograms shift further away from the unmodulated ($x = 0$) simulations.

This is summarised in panel (c) where we plot for each modulation amplitude $x$ the normalized distance between its histogram and the unmodulated histogram. We define this distance as

$$\eta(x) = \frac{|\mu(x) - \mu(0)|}{\sqrt{\sigma^2(x) + \sigma^2(0)}},$$

where $\mu(x)$ and $\sigma(x)$ are the mean and standard deviation of the $R$ estimator, respectively. This distance can be thought of as a rough estimate of the signal-to-noise ratio for the detection of such mirror-parity. We see that for both the constant and decaying modulation schemes, both types of statistics, the pixel-based and harmonic, perform very similarly, the former being slightly more efficient.

While so far we have tested the detectability of the mirror-parity modulation using the value of the $R$ estimator, we can also use the direction as our testing tool. For each modulated random realisation, we calculate the distance between the parity axis used to modulate the map and the direction which yields the best parity score $R$. If this distance is smaller than $8^\circ$ (i.e., within two $N_{\text{side}} = 16$ pixels), we consider this a ‘successful’ detection of the modulated parity. We count the number of realisations yielding a detection, and plot in Fig. 2(d) the probability for detecting the modulation by this direction-based test. We see that this test also shows that both estimators are effective in detecting mirror-parity with a high enough amplitude. Again, we see that the pixel-based statistic performs better than its harmonic counterpart.

Finally, comparing between panels (c) and (d) we see that both the intensity-based and direction-based tests yield similar results. The decaying modulation scheme is in general more easily detected than the constant scheme.

### 4.2 Sensitivity to the Total Power

When searching for some trait or property in the data, it is desirable to devise a statistic that is as sensitive as possible to this property, but that is also as insensitive as possible to any other property. Otherwise, the statistical significance of any finding is ‘contaminated’ by residual significance levels of other properties and oddities picked up by the statistic.

Following this reasoning, we wish to disentangle the question

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**Figure 2.** Effectiveness of the different parity statistics, measured using the $R$ estimator on random realisations modulated with various amplitudes of even parity. All harmonic scores are calculated here with $\ell_{\text{max}} = 7$. (a) Histograms of $R$ for the pixel-based statistic. (b) Histograms of $R$ for the harmonic statistic. (c) The normalized distance between modulated and unmodulated histograms, as defined in Eq. (10). (d) The probability for detecting the parity modulation based on the distance between the modulation axis and the direction of the best parity score, as described in the text.
of the existence of anomalous mirror-parity in the CMB from that of the normality or abnormality of the total measured power on large scales. Since mirror-parity is a trait related to the relative distribution of the phases of harmonic coefficients, their total power is irrelevant. As a lack of power on large scales compared to the \( \Lambda \)CDM expectation is evident (though not significant by itself, see Bennett et al. [2011] Ade et al. [2013b]) in the Planck maps, such entanglement could have an important effect on the search for mirror-parity.

Upon examination of our two statistics, Eqs. (2) and (4), we can naturally expect the harmonic statistic to be insensitive to the total power of each scale, as it is specifically normalized by this factor, \( \tilde{C}_l \) of Eq. (5). The pixel-based statistic, however, may be sensitive to this property.

To test that, we generate a third set of \( 10^4 \) simulations. Following Copi et al. (2013a), we generate constrained realisations of the measured \( \Lambda \)CDM power spectrum. For each realisation, we draw random values \( C_l \) from Gaussian distributions centred around the measured values, with standard deviations corresponding to the measurement errors. This produces a power spectrum consistent with the one measured by the experiment, and allows us to produce realisations of our Universe, as reported by observations, instead of realisations of the full \( \Lambda \)CDM model. As stressed in Copi et al. (2013a), for this aim cosmic variance is irrelevant, as is the fact that \( C_l \) are \( \chi^2 \)-distributed in \( \Lambda \)CDM. We use the Planck maximum-likelihood power spectrum (Ade et al. [2013b]), available unbinned for the large scales (\( l \leq 48 \)) we require. As the Planck measurement errors were not made available separate from the cosmic variance component, we use the (larger) measurement errors reported by WMAP (Larson et al. [2011]) instead, but this choice has a negligible effect on our results. We then constrain a given set of \( a_{lm} \) coefficients by calculating their power \( C_l \) using Eq. (5) and rescaling them by the factors \( \sqrt{\tilde{C}_l / \tilde{C}_l} \). Each random set of \( \tilde{C}_l \) is used to create a single constrained realisation. Apart from constraining their power, these simulations are created in exactly the same manner as our clean \( \Lambda \)CDM simulations.

This set of constrained realisations can now be used instead of the normal set to compute the significance level of either the \( \bar{R} \) or \( \tilde{R} \) parity estimators. We wish to measure how much the significance level for each estimator is influenced by the change of ensembles, assuming the data indeed show a tendency for mirror-parity. To do so, we use random realisations which have been mildly modulated with even mirror-parity (amplitude \( x = 0.2 \)), either in the constant or decaying modulation schemes. We take the median\(^3\) of the score over all modulated realisations using each of the mirror-parity estimators, and calculate its significance level \( \sigma \) compared to either the constrained or unconstrained random realisations. The differences between the two, \( \Delta \sigma \), are summarised in Table 1.

We see that while the harmonic statistic is hardly affected by the difference in ensembles and the significance level changes by only \( \sim 0.02 \sigma \), the pixel-based statistic is very sensitive to this change. When comparing to the ‘wrong’ ensemble, this statistic can give a significance level that is off by \( \sim 0.5 \sigma \). These results do not vary considerably between the two modulation schemes and the two significance estimators. They are also not strongly affected by the modulations amplitude \( x \), as long as it is not very high.

Since the large scales that we are testing here suffer from a large cosmic variance, the power spectrum from which they were drawn is not well known. We have been using the best-fitting \( \Lambda \)CDM power spectrum to draw random realisations out of the assumption that this model is valid for all cosmological scales. However, effects that could change the power spectrum on large scales cannot, at this time, be completely ruled out by the data. We therefore conclude that when searching for evidence of mirror-parity in the CMB, the harmonic statistic, which is insensitive to the total power (by construction) is preferred over the pixel-based statistic, that can cause a notable over- or under-estimation of the significance of the results.

### 4.3 Bias on Masked Sky

Our final test on the two mirror-parity statistics is for bias due to the shape of the Galactic masks. The introduction of a mask over the sky breaks the isotropy of the search for mirror-parity, and can bias the process of evaluating the significance of the results (Inoue et al. [2008] Copi et al. [2011]). Even in a completely isotropic setup, on a masked sky some directions can be more likely to get an extreme parity score than others. This concern is worsened by the fact that Galactic masks tend to be roughly band-shaped and highly parity-even in Galactic coordinates (Inoue et al. [2008] Copi et al. [2011]).

We therefore calculate, for both the pixel-based and harmonic parity statistics, the direction exhibiting maximal even parity for each of our \( 10^4 \) random simulations. We do this for unmasked simulations, as well as after masking the simulations with each of the three masks – SMICA-INP, CS-SMICA89 and U73. In Fig. 3 we plot histograms of the results.

Since the random simulations are realisations of a completely isotropic statistical model, any deviation of the results from isotropy must be associated with the Galactic mask, and shows the sensitivity of the parity statistic to this mask. We can calculate how consistent each histogram of Fig. 3 is with a uniform distribution on a sphere using, e.g., a simple \( \chi^2 \) test. We find that on the full sky, without masking, both the pixel-based and the harmonic statistics are isotropic, as expected. The pixel-based statistic, however, becomes less and less isotropic as the masked area is increased. With the small SMICA-INP mask its histogram is marginally isotropic, while for the larger CS-SMICA89 and U73 masks it is significantly anisotropic. The harmonic statistic, however, remains isotropic on the masked sky, even when a large mask such as U73 is used. This is due to our use of the maximum likelihood method to reconstruct the full sky harmonic coefficients before calculating the score. It is evident now that for the large scales we require here, the reconstruction method does not introduce a bias to the phases of the coefficients, leaving the harmonic statistic free of bias due to masking.

In summary, before moving on to the results on the real CMB data, we have shown that while both pixel-based and the harmonic

\[\begin{array}{l|cc}
\text{Table 1. The difference } \Delta \sigma \text{ between the significance level of the median score of random realisations modulated with even parity of amplitude } x = 0.2, \text{ when compared to the constrained ensemble and to the unconstrained ensemble. The harmonic scores are calculated with } \ell_{\text{max}} = 7. \\
\hline
\text{Constant} & \bar{R} & \tilde{R} \\
0.703 & 0.025 \\
0.537 & 0.008 \\
\hline
\text{Decaying} & \bar{R} & \tilde{R} \\
0.431 & 0.027 \\
0.616 & 0.016 \\
\hline
\end{array}\]
statistics are equally efficient in detecting CMB mirror-parity, the pixel-based statistic suffers from two major drawbacks compared to the harmonic one: it is sensitive to the total power on large scales, an effect that should be investigated and evaluated separately from mirror-parity, and it suffers from an anisotropy bias induced by a Galactic mask. Both effects alter the significance of the results. We therefore conclude that the harmonic statistic, which does not suffer from these two effects, should be preferred when analysing the CMB in search for signs of anomalous mirror-parity. The ability to investigate the scale dependence of the results via the harmonic statistic, with \( \ell_{\text{max}} \) estimated using the \( \tilde{S} \) estimator, the significance level already drops below 3\( \sigma \) or when the maps are tested without a Galactic mask, the significance levels never reach 3\( \sigma \) on any of the maps. As was demonstrated above (see Section 4.3), results of the pixel-based statistic on the masked sky are inherently biased, especially with a mask as large as U73.

5 RESULTS

For all map and mask combinations and for both statistics, we identify a single direction of maximal even mirror-parity, \((l, b) \sim (260^\circ, 48^\circ)\), and a corresponding one for odd mirror-parity, \((l, b) \sim (264^\circ, -17^\circ)\) in the CMB data. We now turn to the analysis of the significance levels of these findings using the two estimators discussed above, \(R\) and \(\tilde{R}\).

5.1 Using the Pixel-Based Statistic

In Fig. 4 we plot the results of the pixel-based statistic for our set of maps masked with the U73 mask using the \(R\) estimator, along with a histogram of the results on our set of random simulations, masked with the same mask. Considering panel (a), it is immediately clear that none of the maps show any significant evidence of even mirror-parity. Indeed, regardless of which map and mask are considered, using either the \(R\) or \(\tilde{R}\) estimators, no significance level reaches above 3\( \sigma \) (and most combinations do not even reach 2\( \sigma \)).

When inspecting panel (b) the results for odd mirror-parity appear to be more significant. We summarise the odd mirror-parity significance levels for all the maps and masks in Table 2. We see that as Fig. 4(b) suggests, the significance levels for the maps masked with the U73 mask are somewhat high when measured using the \(R\) estimator, crossing the 3\( \sigma \) level. This matches the results reported in Ade et al. (2013c). However, when the same results are evaluated using the \(\tilde{R}\) estimator, the significance level already drops to \( \sim 2.5\sigma \), for all maps. Furthermore, when a smaller mask is used, or when the maps are tested without a Galactic mask, the significance levels never reach 3\( \sigma \) on any of the maps. As was demonstrated above (see Section 4.3), results of the pixel-based statistic on the masked sky are inherently biased, especially with a mask as large as U73.

Finally, we refer the reader to the second histogram plotted in both panels of Fig. 4 showing the results on the set of constrained realisations of Section 4.2. This is done as a final demonstration of the effect of the total power of the pixel-based statistic. We can easily see how the same results of the CMB data maps are assigned a lower significance level using the set of constrained realisations, as the corresponding histograms are shifted towards smaller values. This sensitivity hampers our ability to evaluate the mirror-parity level separately from other effects by using the pixel-based statistic. As we have demonstrated above, the harmonic statistic is more robust and does not suffer from this disadvantage.

5.2 Using the Harmonic Statistic

Much like the pixel-based statistic, the harmonic statistic attributes very low significance levels to the even mirror-parity results on all combinations of maps and masks, using both the \(R\) and \(\tilde{R}\) estimators.

In Table 3 we present the significance levels of the odd mirror-parity search results, using the harmonic statistic. We include the
However, the analysis in harmonic space further allows us to check the scale dependence of the results by changing the $\ell_{\text{max}}$ parameter. We therefore plot in Fig. 5 the significance levels of the results as a function of $\ell_{\text{max}}$ for some examples of map and mask combinations. We can see that even the mildly significant map and mask combinations presented in Table 3 become completely insignificant when $\ell_{\text{max}}$ of the harmonic statistic is changed. The sensitivity of the results to the scale considered appears to be very strong. Furthermore, we see no apparent trend in any of the plotted lines. A cosmologically originated anomalous parity due to a pre-inflationary effect, for example, is expected to produce a decaying significance level when plotted against decreasing scales.

### 6 DISCUSSION AND CONCLUSION

Over the past decade, the pursuit of large-scale anomalies in CMB data has generated numerous claims of deviations from the expected behaviour according to the concordance $\Lambda$CDM model (see Bennett et al. 2011; Ade et al. 2013c and references within), spurring a tumultuous discussion of the statistical methods used in the analyses Bennett et al. 2011 and especially the approaches toward significance estimation. Some of the disagreements are grounded in principles, such as the age-long Bayesian vs. frequentist debate, and are likely to persist. Certain claims might be considered as questions of taste. For instance, some advocate that the statistical significance of each new reported result be normalized according to the total number of tests conducted on CMB data hitherto, in an attempt to compensate for the ‘look elsewhere’ effect (although it is not clear how this number is to be tracked and whether it should carry over to each new dataset). Regardless of personal convictions, it is important that results of great potential importance such as those cited above undergo careful scrutiny and robust examination if they are to advance our theoretical understanding of cosmology in a meaningful way.

In this work we have scrutinised previous claims of significant levels of mirror-parity in CMB maps from both the WMAP and Planck experiments. We have reproduced the results of Ade et al. (2013c) using a pixel-based statistic and the large U73 Galactic mask, indicating a $\sim 3\sigma$-anomalous odd-mirror-parity direction. However, we have shown that these results are biased due to the large mask, and are also sensitive to assumptions regarding the to-
tal power on large scales, which is weakly constrained by the data. Indeed, with a smaller mask, the significance level is much lower.

We have also tested the data for mirror-parity using a harmonic statistic. We have shown this statistic to be far more robust than its pixel-based counterpart – stable against the use of a Galactic mask and with regard to the power spectrum amplitude. The statistical significance of the harmonic results reaches a level of $\sim 2.5\sigma$ at its highest. This level, however, is sensitive to the choice of component separation method applied to the CMB data. In addition, we have shown that the significance level is highly sensitive to scale.

Furthermore, with the high quality of the Planck maps, and especially the LGMCA map, we have also allowed ourselves to test the sky maps completely unmasked. This way, as long as the maps are clean enough of Galactic foregrounds, both statistics are expected to perform the best, free of anisotropy bias in the case of the pixel-based statistic and of the CDM assumptions used for harmonic reconstruction in the case of the harmonic statistic. The results for the unmasked maps, however, are not statistically significant.

In light of these findings, we conclude that while there is some tendency for odd parity in the CMB data, which peaks at the scale of $\ell = 7$, when embracing a broader perspective and examining the complete set of data maps and Galactic masks and the properties of the statistical estimators, it appears that the evidence for anomalous mirror-parity is rather weak. Our conclusion is that it poses no real challenge to the concordance model, and should therefore not be considered a CDM anomaly.

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APPENDIX A: RECONSTRUCTION OF LARGE SCALES

Several approaches can be found in the literature to the question of estimating the full-sky harmonic coefficients from a masked sky. These include direct inversion of the harmonic coupling matrix (Efstathiou, Ma & Hanson 2010; Bielewicz, Wandelt & Banday 2013), Gaussian inpainting methods based on constrained realisations (Inoue et al. 2008; Kim, Naselsky & Mandolesi 2012; Copi et al. 2013b), filtering methods with or without preconditioning (Smith, Zahn & Doré 2007; Elsner & Wandelt 2013) and sparsity-based techniques (Starck, Fadili & Rassat 2013).

Since we are only interested in the largest scales in this work, we have chosen to use the maximum likelihood method (de Oliveira-Costa & Tegmark 2006; Efstathiou et al. 2010; Aurich & Lustig 2011; Ben-David et al. 2012) to reconstruct the harmonic coefficients. The behaviour of this method has been discussed extensively in the literature (de Oliveira-Costa & Tegmark 2006; Efstathiou et al. 2010; Feeney, Peiris & Pontzen 2011; Copi et al. 2011). Following Feeney et al. (2011) we use a Gaussian smoothing kernel instead of a top-hat, as this leads to superior performance on component separated maps.

In the maximum likelihood reconstruction method, the CMB data on the masked sky are represented as

$$x = Y a + n, \quad (A1)$$

where $x$ is a vector of the temperature in the valid pixels, $Y$ is a matrix of the spherical harmonics$^4$, evaluated on each valid direction ($Y_{ij} = Y_{i,j}(\hat{r}_i)$), $a$ is a vector of the harmonic coefficients we wish

$^6$ Without loss of generality, we use in this work the real form of both

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to reconstruct (i.e., containing only the scales $2 \leq \ell \leq \ell_{\text{rec}}$) and $n$ is a vector representing everything else that contributes to $x$ – including the CMB data of smaller scales ($\ell > \ell_{\text{rec}}$) and the detector noise – and acts as noise for the reconstruction process. Assuming Gaussianity, an estimator $\hat{a}$ of $a$, which is unbiased ($\langle \hat{a} \rangle = a$) and has minimal variance, can be calculated as $\hat{a} = Wx$, where the reconstruction matrix is

$$W = \left( Y^T C^{-1} Y \right)^{-1} Y^T C^{-1}$$

(A2)

and $C = \langle nn^T \rangle$ is the noise covariance matrix.

Under the $\Lambda$CDM assumption of statistical isotropy for the smaller scales of $\ell > \ell_{\text{rec}}$, we can estimate $C$ as

$$C_{ij} = \sum_{\ell=\ell_{\text{rec}}+1}^{L} \frac{2\ell + 1}{4\pi} P_{\ell}(\hat{r}_{i}, \hat{r}_{j}) w_{j}^{2} b_{\ell}^{2} C_{\ell} + N \delta_{ij},$$

(A3)

where $P_{\ell}$ are the Legendre polynomials and $C_{\ell}$ is the best-fitting $\Lambda$CDM power spectrum. $b_{\ell}$ is the Gaussian smoothing kernel, allowing us to cut the summation at $L = 48$, and $w_{j}$ is the pixel window function. Diagonal regularisation noise of $N = 2 \mu K^{2}$ is added to keep the covariance matrix from becoming singular and has little effect on modes that contribute to the inversion. We choose $\ell_{\text{rec}} = 9$ when reconstructing the harmonic coefficients, since this is the smallest scale we use to test for mirror-parity in this work.

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