Nonlinear Spinors as the Candidate of Dark Matter

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In order to explain the present accelerating expansion of the universe and the related dark energy with negative pressure, the scalar model and cosmological factor have been widely studied. Relatively, the spinor field is unreasonably overlooked. Considering the facts that all fermions are described by spinors and the uniform scalar field can hardly explain the galactic structure, so the dark spinors may be partially responsible for dark matter. In this paper, we give a detailed investigation on the state functions of spinors, such as the mass-energy density and equation of state, in the context of cosmology. Here the spinors are quantized and identified by nonlinear potentials, and their state functions are derived from standard statistical and variation principles. The results provide us some interesting consequences and new insights: (I) The negative pressure exists, and the equation of state can be designed by adjusting the nonlinear potential. (II) The initial singularity of the space-time can be removed by the nonlinear potential. (III) A short period of inflation exists and in this period the scale factor inflates much faster than the exponential function.

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I. INTRODUCTION

Since the discovery that the expansion of the universe is accelerating[1]-[6], the standard model of cosmology has shifted from a matter dominated and decelerating expansion picture to search for the dark energy and matter with repulsive gravity or modification of the general relativity. The present methods to analyze the observational data include direct measures of cosmic scales through Type Ia supernova luminosity distances, the angular distance scales of

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baryon acoustic oscillation and cosmic microwave background density perturbations, as well as indirect probes such as the effect of cosmic expansion on the growth of matter density fluctuations. The basic building blocks of the universe is now believed to consist of 4% baryons, 20% dark matter, and 76% dark energy. The cosmic accelerating expansion results in a profound mysteries in all of science, with deep connections to both astrophysics and particle physics.

The simplest choice for such dark energy with negative pressure is the cosmological constant $\Lambda$, a term consists with Einstein’s equations. This term acts like a fluid with equation of state (EOS) $p_\Lambda = -\rho_\Lambda$. Although the $\Lambda$CDM model is basically consistent with all observational data, the evaluated $\Lambda$ seems incoincident with the local Newtonian approximation of the galactic gravity, and a lot of theoretical efforts was paid to explain its origin and apparent value\[7\]–\[10\].

The widely studied dynamical models to describe dark energy are mainly the scalar fields, such as quintessence, K-essence, tachyon, phantom and dilaton, which are uniformly distributed in the universe\[10\]–\[19\]. These models have different theoretical origin and consequences, and occasionally can give some explanation for the observational data and the origin of $\Lambda$. However these models, viewed objectively, suffer from several shortcomings such as lack of predictive power and the potentials without natural theoretical justification, etc.\[11\],\[18\],\[20\].

Noting the facts that all fermions are described by spinors and the uniform scalar field can hardly explain the galactic structure\[21\], so the dark spinors may be partially responsible for dark matter. In this paper we give a detailed discussion for their state functions, such as the mass density and equation of state, in the context of cosmology. Here the spinors are quantized and identified by nonlinear potentials, and their state functions are derived from standard statistical and variational method. This work is a further research of the preceding papers\[22\]–\[25\]. Some similar works were once done in\[26\]–\[29\], where the authors also considered the nonlinear spinor field as candidate of dark energy, but only one spinor field was taken into account.

The nonlinear spinors provide us some interesting consequences and new insights: The negative pressure exists, and the EOS can be designed by adjusting the potential. The initial singularity of the universe is cancelled by the nonlinear potential. A short period of inflation without inflaton exists, in which the scale factor inflates much faster than the exponential function. The state functions are only the functions of scale factor, but independent of its derivatives. Combined with other models, the nonlinear spinors are hopeful to give more
natural explanation to the cosmic accelerating expansion and other observational data.

II. FUNDAMENTAL CONCEPTS AND EQUATIONS

At first, we introduce some standard concepts and equations in cosmology and field theory. In this paper, we adopt the conformal coordinate system for the universe, then the corresponding Friedmann-Robertson-Walker metric becomes

$$g_{\mu \nu} = a^2(t) \text{diag} \left[ 1, -1, -S^2(r), -S^2(r) \sin^2 \theta \right],$$

(2.1)

where

$$S = \begin{cases} 
\sin r & \text{if } K = 1, \\
S' & \text{if } K = 0, \\
\sinh r & \text{if } K = -1.
\end{cases}$$

(2.2)

The Lagrangian of the gravitational field is given by

$$\mathcal{L}_g = \frac{1}{16\pi G} (R - 2\Lambda), \quad R = 6 \frac{a'' + Ka}{a^3},$$

(2.3)

where $R$ is the scalar curvature, $\Lambda$ the cosmological factor. For the metric (2.1), we have the nonzero components of the Einstein’s tensor as follows

$$G^0_0 = -\frac{3}{a^2} \left( \frac{a'^2}{a^2} + K \right), \quad G^1_1 = G^2_2 = G^3_3 = -\frac{1}{a^2} \left( \frac{2a''}{a} - \frac{a'^2}{a^2} + K \right),$$

(2.4)

where $a' = \frac{d}{dt}a$, $S' = \frac{d}{dr}S'$. Corresponding to metric (2.1), the energy-stress tensor certainly takes the following form

$$T^\mu_\nu = \text{diag} [\rho, -P, -P, -P],$$

(2.5)

which is unnecessarily related to the perfect fluid model. In (2.5), $\rho$ stands for the total energy of all matter, but $P$ includes not only the usual pressure, but also all potentials of interaction[23, 24, 30].

By the Einstein equation, for the metric (2.1) we have the following two independent equations,

$$0 = G^0_0 + \Lambda + 8\pi G \rho = -\frac{3}{a^4} (a'^2 + Ka^2) + \Lambda + 8\pi G \rho,$$

(2.6)

$$0 = G^\mu_\mu + 4\Lambda + 8\pi G (\rho - 3P) = -\frac{6}{a^3} (a'' + Ka) + 4\Lambda + 8\pi G (\rho - 3P),$$

(2.7)
(2.6) and (2.7) lead to the energy-momentum conservation law

\[ P = -\frac{1}{3a^2} \frac{d(\rho a^3)}{da}. \]  

If reckoning in the effect of \( \Lambda \), we get the total energy-momentum tensor with vacuum energy

\[ T^\nu_{\mu}|_{\text{tot}} = \text{diag}[\rho_{\text{tot}}, -P_{\text{tot}}, -P_{\text{tot}}, -P_{\text{tot}}], \quad \rho_{\text{tot}} = \rho + \frac{\Lambda}{8\pi G}, \quad P_{\text{tot}} = P - \frac{\Lambda}{8\pi G}. \]

Correspondingly, the Einstein equation becomes

\[ G^\mu_\nu + 8\pi G T^\mu_\nu|_{\text{tot}} = 0, \]

and then the total EOS is defined by

\[ w = \frac{P_{\text{tot}}}{\rho_{\text{tot}}} = \frac{8\pi GP - \Lambda}{8\pi G \rho + \Lambda}. \]

Now we construct the Lagrangian of the nonlinear spinors. Denote the Pauli matrices by

\[ \vec{\sigma} = (\sigma^j) = \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}, \quad (j = 1, 2, 3). \]

Define \( 4 \times 4 \) Hermitian matrices as follows

\[ \alpha^\mu = \left\{ \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \right\}; \quad \gamma = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \quad \beta = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix}. \]

We consider the following Lagrangian for the same kind spinors in flat spacetime\[22, 23, 24]\]

\[ \mathcal{L} = \sum_k \phi^*_k (\alpha^\mu_k i\partial_\mu - \mu\gamma) \phi_k + F(\tilde{\gamma}_k, \tilde{\beta}_k, \tilde{\alpha}^\mu_k), \]

where we assign \( \phi_k \) to the \( k \)-th dark spinor \( S_k \), \( \mu > 0 \) is constant mass, which takes one value for the same kind particles. \( F \) is the nonlinear potential term.

Considering that all the particle-like solutions of the nonlinear spinor equation have a mean diameter about \( 10^2 \) Compton wave lengths, and all \( |\phi_k| \) decay exponentially with respect to the distance from the center\[23, 24\]. So we can neglect the nonlinear interaction terms among spinors, but keep the self coupling terms. Then the nonlinear potential \( F \) should take the following decoupling form

\[ F = \sum_k V_k, \quad V_k = V(\tilde{\gamma}_k), \]

where the self-coupling potential \( V \) is a differentiable and concave function.

The generalized form of (2.13) in curved space-time is given by\[31, 32\]

\[ \mathcal{L}_m = \sum_k \left( \Re \left( \phi^*_k e^{\mu i\partial_\mu \phi_k} - \mu \tilde{\gamma}_k + V_k \right) \right), \quad g^\mu = \left( \frac{\alpha^0}{a}, \frac{\alpha^1}{a f}, \frac{\alpha^2}{a f}, \frac{\alpha^3}{a f \sin \theta} \right). \]
The variation of (2.15) with respect to \( \phi_k^+ \) gives the dynamic equation for each \( S_k \)

\[
\varphi^\mu i(\partial_\mu + \Upsilon_\mu)\phi_k = (\mu - V'_k)\gamma\phi_k, \quad \Upsilon_\mu = \left(\frac{3a'}{2a}, \frac{S'}{S}, \frac{1}{2} \cot \theta, 0\right). \tag{2.16}
\]

It is easy to check the normalization condition holds for each spinor

\[
\int_\Omega |\phi_k|^2 a^3 d\Omega = 1, \quad (\forall k), \quad d\Omega = S^2 \sin \theta dr d\theta d\varphi, \tag{2.17}
\]

where \( d\Omega \) is the comoving volume element independent of \( t \).

Coupling (2.3) with (2.15), we get the total Lagrangian of the universe with dark spinors

\[
L = \frac{\sqrt{g}}{16\pi G}(R - 2\Lambda) + \sum_k \left( \Re (\phi_k^+ \varphi^\mu i\partial_\mu \phi_k) - \mu \gamma_k + V_k \right) \sqrt{g}, \tag{2.18}
\]

where \( \sqrt{g} = \sqrt{|\det(g_{\mu\nu})|} = a^4 S^2 \sin \theta \). Generally the variation of (2.18) with respect to \( g_{\mu\nu} \) gives the Einstein equation

\[
G^{\mu\nu} + \Lambda g^{\mu\nu} + 8\pi G T^{\mu\nu} = 0, \tag{2.19}
\]

where the energy momentum tensor \( T^{\mu\nu} \) is given by (only valid for diagonal metric[31])

\[
T^{\mu\nu} = \sum_k \left( \frac{1}{2} \Re (\phi_k^+ (\varphi^\mu i\nabla^\nu + \varphi^\nu i\nabla^\mu)\phi_k) + (V'_k \gamma_k - V_k) g^{\mu\nu} \right). \tag{2.20}
\]

By the dynamic equation (2.16), we get

\[
T^\mu_\mu = \sum_k \left( \Re (\phi_k^+ \varphi^\mu i(\partial_\mu + \Upsilon_\mu)\phi_k) + 4(V'_k \gamma_k - V_k) \right)
= \sum_k (\mu \gamma_k + 3V'_k \gamma_k - 4V_k). \tag{2.21}
\]

By (2.5), the following relation holds

\[
\rho - 3P = \bar{T}^\mu_\mu \equiv \frac{1}{\Omega} \int \sum_k (\mu \gamma_k + 3V'_k \gamma_k - 4V_k) d\Omega. \tag{2.22}
\]

### III. THE EQUATION OF STATE

In this section, we derive the state functions \( \rho(a) \) and \( P(a) \) corresponding to the following nonlinear potential \( V \), but the procedure is valid for more complex cases.

\[
V = \frac{w_1}{2} \gamma^2 + \frac{w_2}{3} \gamma^3 + \cdots + \frac{w_n}{n + 1} \gamma^{n+1}. \tag{3.1}
\]
Denote the present time by \( t_0 \) and \( a_0 = a(t_0) \). Making transformation

\[
\begin{align*}
t &= \frac{\bar{t}}{a_0}, \\
r &= \frac{\bar{r}}{a_0}, \\
\phi &= \left( \frac{a_0}{a(t)} \right)^{\frac{3}{2}} \psi,
\end{align*}
\]

and considering that in the region of spinor particle \( a_0 S(r) = a_0 S \left( \frac{r}{a_0} \right) \to \bar{r} \), we get

\[
\begin{align*}
V &= \frac{w_1}{2} \left( \frac{a_0}{a} \right)^6 \tilde{\gamma}^2 + \frac{w_2}{3} \left( \frac{a_0}{a} \right)^9 \tilde{\gamma}^3 + \cdots + \frac{w_n}{n+1} \left( \frac{a_0}{a} \right)^{3(n+1)} \tilde{\gamma}^{n+1},
\end{align*}
\](3.3)

\[
\begin{align*}
V' &= w_1 \left( \frac{a_0}{a} \right)^3 \tilde{\gamma} + w_2 \left( \frac{a_0}{a} \right)^6 \tilde{\gamma}^2 + \cdots + w_n \left( \frac{a_0}{a} \right)^{3n} \tilde{\gamma}^n,
\end{align*}
\](3.4)

\[
\begin{align*}
\alpha^\mu \bar{\partial}_\mu \psi &= \frac{a}{a_0} \left( \mu - w_1 \left( \frac{a_0}{a} \right)^3 \tilde{\gamma} + w_2 \left( \frac{a_0}{a} \right)^6 \tilde{\gamma}^2 - \cdots - w_n \left( \frac{a_0}{a} \right)^{3n} \tilde{\gamma}^n \right) \gamma \psi.
\end{align*}
\](3.5)

in which \( \tilde{\gamma} \equiv \psi^+ \gamma \psi \). The normalization condition (2.17) becomes

\[
1 = \int |\phi|^2 a^3 d\Omega = \int |\psi|^2 a_0^3 d\Omega = \int_{R^3} |\psi|^2 d^3 \bar{x},
\]

so \( \psi \) is normalized in the new coordinate system. By (3.2)-(3.4), equation (2.21) becomes

\[
T^\mu_{\mu} = \sum_k \left( \mu \tilde{\gamma}_k \left( \frac{a_0}{a} \right)^3 + w_1 \tilde{\gamma}_k^2 \left( \frac{a_0}{a} \right)^6 + \cdots + w_n \tilde{\gamma}_k^{n+1} \left( \frac{a_0}{a} \right)^{3n} \right) \mu_j \tilde{\gamma}_k^{j+1} d^3 \bar{x}, (j = 1, \cdots, n).
\]

(3.7) is a scalar equation, which is more convenient for local Lorentz transformation. We take (3.7) as an entry to derive state functions.

Denote \( \mu_j \) to be the static parameters defined in central coordinate system

\[
\mu_0 = \int_{R^3} \mu \tilde{\gamma}_k d^3 \bar{x}, \quad \mu_j = \int_{R^3} w_j \tilde{\gamma}_k^{j+1} d^3 \bar{x}, (j = 1, \cdots, n).
\]

(3.8)

Since \( a(t) \) is a slowly varying function, we can treat it as constant. Then all \( \mu_k \) in (3.8) only very weakly depend on \( a \), and they can be regarded as constants except for the case \( a \to 0 \).

Assuming the peculiar speed of the \( k \)-th spinor is \( v_k \), which depends on \( a(t) \). Solving the geodesic, we have\[30, 33\]

\[
v_k = \frac{b_k}{\sqrt{a^2 + b_k^2}}, \quad \text{or} \quad \sqrt{1 - v_k^2} = \frac{a}{\sqrt{a^2 + b_k^2}},
\]

(3.9)

where \( b_k \) is a constant determined by initial speed. By (3.9), we can calculate the mean value of (3.7) as

\[
\begin{align*}
\bar{T}^\mu_{\mu} &= \frac{1}{\Omega} \int_{\Omega} T^\mu_{\mu} d\Omega, \\
&= \frac{1}{a_0^3 \Omega} \int_{X_k \in \Omega} \left( \mu_0 + 2 \mu_1 \left( \frac{a_0}{a} \right)^3 + \cdots + (3n - 1) \mu_n \left( \frac{a_0}{a} \right)^{3n} \right) \sqrt{1 - v_k^2} dP \\
&= \frac{N}{a_0^3} \left( \mu_0 + 2 \mu_1 \left( \frac{a_0}{a} \right)^3 + \cdots + (3n - 1) \mu_n \left( \frac{a_0}{a} \right)^{3n} \right) \left( 1 - \frac{3 \sigma}{2(\sigma \bar{m} + kT)} \right),
\end{align*}
\]

(3.10)
where $\mathbf{X}_k$ stands for the central coordinate of $S_k$, and $d\mathcal{P}$ the kinetic energy distribution, $\mathcal{N} = \frac{N_0}{\Omega}$ the comoving number density of the spinors, $\bar{m}$ is mean mass of all particles, $\sigma$ is a parameter determined by distribution function. For Maxwell distribution, we have $\sigma = \frac{2}{5}$.

In [33], we derived the relation $T(a)$

$$kT(a) = \frac{\sigma \bar{m} b^2}{a(a + \sqrt{a^2 + b^2})} = \frac{\sigma \bar{m}}{a}(\sqrt{a^2 + b^2} - a),$$

where $b$ is a constant determined by the initial temperature.

It should be pointed out that, (3.9) and (3.11) only exactly hold for mass points, but approximately hold for nonlinear spinors with a tiny mass and a large potential[30, 34]. This may be the underlying reason why the ordinary baryons in a galaxy can be automatically separated with dark matter and shift into a disc to develop the spiral structure[21]. However, in the case $a \gg b$ and $(v_k \rightarrow 0, T \rightarrow 0)$, the following results hold to high precision. In the extreme case near $a \rightarrow 0$, only the trends of (3.8)-(3.11) are right.

In order to get more information and intuition, we calculate the following simple case in detail. Assume

$$\rho = \mathcal{N} \mu_0, \quad \varepsilon = \frac{\mu_1}{\mu_0} a^3 > 0, \quad \mu_j = 0, \ (j \geq 2),$$

(3.12)

where $\rho$ is the comoving density independent of $t$ or $a$. Substituting (3.11) and (3.12) into (3.10), we get

$$\bar{T}^\mu = \frac{\rho}{a^3} \left( 1 + \frac{2\varepsilon}{a^3} \right) \left( 1 - \frac{3\sigma(\sqrt{a^2 + b^2} - a)}{2\sqrt{a^2 + b^2}} \right),$$

(3.13)

and then

$$\int a^3 \bar{T}^\mu da = -3\rho \alpha + \rho a \left( 1 + \frac{3\sigma}{2a} (\sqrt{a^2 + b^2} - a) \right) - \frac{\rho \varepsilon}{a^2} \left( 1 - \frac{3\sigma}{2a} \frac{\sqrt{a^2 + b^2} - a}{\sqrt{a^2 + b^2 + a}} \right).$$

(3.14)

On the other hand, the trace of Einstein equation $(G^\mu_\mu + 4\Lambda + 8\pi G \bar{T}^\mu_\mu) = 0$ gives

$$8\pi G \int a^3 \bar{T}^\mu_\mu da = 3(a^2 + K a^2) - \Lambda a^4,$$

$$= -a^4 (G^0_0 + \Lambda) = 8\pi G \rho a^4.$$

(3.15)

Using (3.14), we solve the rigorous $\rho(a)$ as

$$\rho = \frac{1}{a^4} \int a^3 \bar{T}^\mu_\mu da$$

$$= \frac{\rho}{a^3} \left( 1 + \frac{3\sigma}{2a} (\sqrt{a^2 + b^2} - a) \right) - \frac{3\rho \alpha}{a^4} - \frac{\rho \varepsilon}{a^6} \left( 1 - \frac{3\sigma}{2a} \frac{\sqrt{a^2 + b^2} - a}{\sqrt{a^2 + b^2 + a}} \right),$$

(3.16)
where \( \alpha \) is a constant probably related to the gravitational potential, the first term corresponds to the mass energy and kinetic energy of the spinors, and the second and third terms correspond to the total potential energy including the gravitational one. Substituting (3.16) into the energy-momentum conservation law (2.8), we get the rigorous \( P(a) \) as
\[
P = -\frac{\rho \alpha}{a^4} + \frac{\rho \sigma b^2}{2a^4 \sqrt{a^2 + b^2}} - \frac{\rho \varepsilon}{a^6} \left( 1 - \frac{(2a + 3\sqrt{a^2 + b^2}) \sigma b^2}{2(\sqrt{a^2 + b^2} + a)^2 \sqrt{a^2 + b^2}} \right). \tag{3.17}
\]
We can check that, (3.16) and (3.17) also satisfy the equations (2.7) and (2.22), so they are the exact cosmic EOS for the spinors. The constants \( \rho, \alpha, \varepsilon, b, \sigma \sim \frac{2}{3} \) can be treated as macroscopic parameters to be determined by empirical data. However, it should be keep in mind that, the above results have only qualitative significance when \( a \to 0 \).

The constant \( \alpha \) is equal to \(-\frac{1}{3}C_0 \) in [22], where the explanation for its physical origin and meaning is superfluous and inexact. By (3.16) and (3.17), we find
\[
\frac{P}{\rho} \to -\frac{\alpha}{a} \to 0, \quad (a \to \infty), \tag{3.18}
\]
which proves the infinitesimal value of the dimensionless pressure for a spinor in flat space-time as worked out in [24].

**IV. GLOBAL PROPERTY AND ASYMPTOTIC BEHAVIOR**

Substituting the above EOS into (2.6), we get a closed dynamic equation for scale factor \( a(t) \),
\[
a^2 = -K a^2 + \frac{\Lambda}{3} a^4 + 2 \tilde{R} \left[ a + \frac{3\sigma b^2}{2(\sqrt{a^2 + b^2} + a)} - 3\alpha - \frac{\varepsilon}{a^2} \left( 1 - \frac{3\sigma}{2} \xi(a) \right) \right] \tag{4.1}
\]
\[
\equiv -\Phi(a),
\]
where \( \tilde{R} \) is the mean scale with length dimension.
\[
\tilde{R} = \frac{4\pi G \rho}{3}, \quad \xi = \frac{\sqrt{a^2 + b^2} - a}{\sqrt{a^2 + b^2} + a}. \tag{4.2}
\]
Of course, for practical resolution to the cosmological model, one should introduce the mass-energy density of other fields such as photons to the total potential \( \Phi(a) \). If taking \( a(t) \) as generalized coordinate of a mass point, then (4.1) have the form of Hamiltonian, and \( \Phi(a) \) is equivalent to the potential. The qualitative analysis for its solution is routine and convenient[19, 35].

Since \( \varepsilon > 0 \), by (4.1) we find that, if the nonlinear potential of the spinor exists, the initial singularity is absent due to \( \Phi \to +\infty \) as \( a \to +0 \), namely, the phase trajectory of \( (a', a) \)
can not reach $a \rightarrow 0$. In the cases of ordinary fluid model and linearized Dirac equation, we have actually assumed $\varepsilon = 0$, and then the fact is concealed.

For the behavior of solution of (4.1) and the destiny of the universe, the roots of $\Phi(a) = 0$ have decisive significance[35]. Since $\varepsilon > 0$, we have at least one positive root. Denote the least positive root by $\zeta_0$, the second positive root by $\zeta_1(\zeta_1 \leq +\infty)$, where $\zeta_1 = +\infty$ means the unbound trajectory and bouncing universe, but $\zeta_1 < +\infty$ corresponds to the cyclic universe. Then we only need to discuss the solution in the connected region $\zeta_0 \leq a < \zeta_1$.

Near the root $\zeta = \zeta_0$ or $\zeta = \zeta_1 < \infty$, we have the following Taylor expansion
\[
a^2 = A_0(a - \zeta) + A_1(a - \zeta)^2 + O((a - \zeta)^3). \tag{4.3}
\]

In contrast it with (4.1), we get the parameters expressed by $\zeta$ as
\[
\alpha = -\frac{\varepsilon}{3\zeta^2} \left(1 - \frac{3\sigma}{2} \sqrt{\zeta^2 + b^2 - \zeta} \right) + \frac{1}{2} \frac{\sigma b^2}{\zeta + \sqrt{\zeta^2 + b^2}} + \frac{\zeta + \sqrt{\zeta^2 + b^2}}{3} + \frac{-3K\zeta^2 + \zeta^4\Lambda}{18R}, \tag{4.4}
\]
\[
A_0 = \bar{R} \left(2 + \frac{4\varepsilon}{\zeta^3} \left(1 - \frac{3\sigma}{2} \sqrt{\zeta^2 + b^2 - \zeta} \right) - 3\sigma \sqrt{\zeta^2 + b^2 - \zeta} \right) - 2K\zeta + \frac{4}{3} \Lambda \zeta^3, \tag{4.5}
\]
\[
A_1 = \bar{R} \left(-\frac{3\varepsilon}{\zeta^4} \left(2 - \sigma \left(3 - \frac{\zeta(2b^2 + 3\zeta^2)}{\sqrt{(\zeta^2 + b^2)^3}} \right) \right) + \frac{3\sigma b^2}{2\sqrt{(\zeta^2 + b^2)^3}} \right) - K + 2\Lambda \zeta^2. \tag{4.6}
\]

The solution near the roots equals to
\[
a = \zeta + \frac{1}{2} A_0 t^2 + O(t^4). \tag{4.7}
\]

In the case $\zeta \rightarrow \zeta_1 < \infty$, we should have $A_0 < 0$. So for the cyclic universe, by (4.5) and $A_0 < 0$, we have the estimation $K > \frac{2}{3} \Lambda \zeta_1^2 + \frac{\bar{R}}{\zeta_1} > 0$, which implies $K = 1$, and then
\[
\Lambda < \frac{3(\zeta_1 - \bar{R})}{2\zeta_1^3} \leq \frac{2}{9\bar{R}^2}. \tag{4.8}
\]

In the case $\zeta \rightarrow \zeta_0 \sim +0$, we should have $A_0 > 0$, omitting the higher order terms, then we have
\[
\alpha > 0, \text{ if } \varepsilon < \left(1 - \frac{3\sigma}{2} \sqrt{\zeta_0^2 + b^2 - \zeta_0} \right)^{-1} \left(3 \frac{\sigma b^2}{2\zeta_0 + \sqrt{\zeta_0^2 + b^2}} + \zeta_0 \right) \zeta_0^2. \tag{4.9}
\]

If $\alpha > 0$, the trends of all functions are similar to those displayed in [25].

Denote $\tau = \int_0^t a(t)dt$ to be the proper time, by (4.3) we get
\[
\frac{d^2 a}{d\tau^2} \bigg|_{\tau = 0} = \frac{A_0}{2\zeta_0^2} \rightarrow \frac{\bar{R}}{\zeta_0^3} \rightarrow \infty, \text{ (} \zeta_0 \rightarrow 0 \text{).} \tag{4.10}
\]
So the inflation indeed exists for any nonsingular cosmological model, and the scale factor would initially expand much faster than the exponential function. The temperature parameter $b$ have little influence on the global behavior, because terms including it always have a large part, but it has important influences on the value of parameters $(\zeta_0, \alpha)$.

Although the above $w(a)$ for the nonlinear spinors is negative near $\zeta_0$, the trends is different from that of the fitted EOS previously derived from the observational data. The former is an increasing function of $a$ in the case of $\alpha > 0$, but the latter seems prefer to a decreasing function. However, according to the WMAP five-year data[6], if dark energy perturbation is included, the constraints on the time evolving equation of state of dark energy is consistent with an increasing $w(a)$[36].

On the other hand, similar to the quintessence model, by choosing suitable potential, we can almost get any wanted EOS[18]. Calculation shows that, if the potential of the spinors including terms like $w_{-1}(C^2 + \tilde{\zeta}^2)^{-\frac{1}{2}}$, the corresponding EOS is decreasing function similar to the fitted functions. But few people would believe the Nature works in such manner.

V. DISCUSSION AND CONCLUSION

According to the above calculation, we get some important consequences and insights.

1. By (3.17), for the nonlinear spinors, the negative pressure definitely exists, and the equation of state can be designed by adjusting the potential.

2. By (4.1), we find the initial singularity of the universe is cancelled by the nonlinear potential of the spinors due to $\Phi(a) < 0$ when $a \to 0$.

3. A short period of inflation of the universe exists and in which the scale factor inflates much faster than the exponential function[25]. Such inflation implies the universe only spends a very short time to cross the turning point.

4. Although the dynamical equation of the spinors is more complex than that of the scalar fields, the state functions of the spinors are even relatively simpler due to the normalizing condition (3.6). These functions are only the functions of scale factor, but independent of its derivatives.

5. By (3.16), we find that, different from the point particle and fluid model, mass-energy density of the spinors is not a monotone decreasing function of $a$. Spinors have strong interaction with the heavily curved space-time.
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