**On particle incipient motion from the sedimentation layer in shear flow**

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**Abstract.** This paper presents the results of numerical modelling for a laminar flow around a spherical particle located near a sedimentation layer. The particle being at the minimum distance from the rough layer is affected by a fluid shear flow with Reynolds numbers varying from 2 to 200. The considered fluids included Newtonian and power law ones. Depending on a shear Reynolds number, the critical Shields numbers have been determined to describe the particle’s incipient motion. The obtained parameters have been compared against those for a smooth surface to demonstrate that in some cases the differences in critical stress coefficients may reach up to 70%.

1. **Introduction**

Cuttings transport has been one of the topical issues for drilling of inclined and horizontal wells to penetrate hydrocarbon reservoirs. Ineffective clearing of the inner annulus from cuttings particles results in ring channel clogging, oil seal, string sticking and increased bottom-hole pressure [1]. To estimate the movability of the layer of dispersed debris in a well, one uses the dependencies obtained from the analysis of a simplified model in which a spherical cuttings particle starts moving over a motionless sedimentation layer. The main integral characteristics of this motion include the drag and lift forces, and torque that depends on the Reynolds number. These characteristics are used as input parameters in different drilling support software packages to determine the mud weight and rate of used drilling fluid, penetration rate, etc. [2].

The theoretical aspect of the issue to be the problem of particle-bearing flow that determines the parameters of particle incipient motion has been considered in a number of studies [3]–[5]. All of them describing the incipient motion of a spherical particle near a smooth wall.

However, since in borehole conditions cuttings transport occurs over a rough sedimentation layer, a case of a particle-bearing flow over such a rough boundary is of sufficient interest to study. Experiments to study the flow around granular pack located on the sedimentation layer has been described in [6],[7]. The increased computation capacities during the last decades make it possible to perform 3D numerical modeling of a particle hydrodynamics over a rough sedimentation layer. Thus in [8], Lee and Balachandar present their results for a shear Newtonian particle-bearing flow over a layer of hemispheres whose radius is similar to the one of the particle. In our study, we used a similar problem statement and applied it to calculate particle incipient motion for power law fluid.
2. Problem statement

We consider a particle of diameter \( d \) that is placed over a surface composed of hemispheres of the same diameter (figure 1, a). The hemispheres are honeycombed on a substrate, and the particle is placed over a gap formed by the three hemispheres colored in green (see figure 1, b). Due to the preliminary calculations demonstrating that the flow of viscous, incompressible fluid over a rough surface is completely formed behind the fourth raw of hemispheres; the particle has been placed just there.

![Figure 1](image1.png)

**Figure 1.** Problem statement for a shear flow bearing a particle over a rough surface (a). The bed level corresponds to the value \( \delta = 0 \), when the particle touches the two bottom hemispheres marked in green (b).

The spherical particle and sedimentation layer are flown by a shear flow moving along the x-axis. The sphere remains steady. The velocity profile \( \bar{u} \) is set as input data and is characterized by its shear parameter \( G = \frac{\partial u}{\partial z} \). The distance from the particle to the rough surface \( \delta \) is determined based on the condition that at \( \delta = 0 \), the particle touches the three bottom hemispheres.

The system of hydrodynamic equations for viscous incompressible fluid in dimensionless form can be written as:

\[
\begin{align*}
\nabla \cdot \bar{u} &= 0 \\
\frac{\partial \bar{u}}{\partial t} + (\bar{u} \cdot \nabla)\bar{u} &= -\nabla p + \frac{1}{Re_{sh}} \nabla (2\mu \mathbf{D}) + \bar{f} 
\end{align*}
\]

where \( \mathbf{D} = \frac{1}{2}[\nabla \bar{u} + (\nabla \bar{u})^T] \) denotes the deformation rate tensor; \( \mu = \mu(D) \) is the dimensionless viscosity being the function of deformation rate tensor components for non-Newtonian fluids; \( \bar{f} \) is the dimensionless density of mass forces. The shear Reynolds number \( Re_{sh} \) for power law fluid is determined as \( Re_{sh} = \rho_f G^{n-1}d^2/k \), where \( n \) denotes the power; \( \rho_f \) is the fluid density, \( d \) is the diameter of the particle, \( k \) is the fluid consistency index.

In our study, a laminar flow of Newtonian and power law (pseudo-plastic) fluids was considered, whose \( Re_{sh} \) varies from 2 to 200. The geometry and mesh were constructed in ANSYS Meshing, then the prepared setup was used for calculations in the ANSYS Fluent and OpenFOAM packages.

The main integral parameters characterizing the way the flow affected the spherical particle such as the drag force \( F_D \), lift force \( F_L \) and torque \( M \), were calculated using corresponding surface integrals of a stress tensor over the particle. The coefficients of the drag (\( C_D \)), lift (\( C_L \)) forces and of the torque (\( C_M \)) were determined through normalization by the characteristic force \( F_0 \) affecting the particle’s cross-section: \( F_0 = 0.5 \cdot u_p^2 \rho_f \pi (d/2)^2 \), where \( u_p \) denotes the “undisturbed” flow velocity in the particle’s center that is obtained from a shear flow above the rough wall without a particle. The coefficients \( C_D \), \( C_L \), \( C_M \) are calculated following the formula: \( C_D = F_D / F_0 \), \( C_L = F_L / F_0 \), \( C_M = 2M / (d \cdot F_0) \).
Since the main objective of our study was to determine the particle incipient motion parameters, the particle’s position was considered to be as close as possible to the rough wall. Using ANSYS Meshing it became possible to make a mesh allowing the distance $\delta=0.005d$ between the particle and hemispheres. To test the correctness of the selected modeling and meshing parameters, a case of shear Newtonian flow over rough sedimentary layer with particle was calculated so that the obtained integral parameters $C_D$ and $C_L$ could be compared against those calculated in [8]. A satisfactory comparison was obtained when using elements of 0.005$d$ in size on the particle’s surface, inflation layers of 0.005$d$ in thickness and cells of 0.05$d$ in height on the hemisphere’s surface. In this case, the characteristic size of the mesh was about $6 \times 10^6$ tetrahedral elements.

3. Newtonian fluid

The drilling support packages to calculate cuttings transport parameters, as a rule, use correlations for the forces and torques obtained for a flow around particle near smooth wall. For that reason, the effect produced by the wall’s roughness on particle incipient motion in compare to the smooth wall is a matter of great interest and relevance. Newtonian fluid being the simplest one in terms of rheology and was chosen on the first step. For a flow around a sphere over a smooth wall, the modeling results for $C_D$, $C_L$, and $C_M$ were published in [5].

The calculations for rough surface demonstrated that for the range of $Re_{sh}$ from 2 to 200 both the drag and lift forces increased. The increase of $F_D$ was due to a wider area of lower pressure behind the particle. The area widened because the particle’s lower part was placed in the gap between the sedimentation layer’s hemispheres. The gap became a place for a stagnation region to form, so its outflow restricted by the channel between the particle and the hemisphere made the pressure behind the particle lower (figure 2).

![Figure 2](image)

**Figure 2.** Distribution of the dimensionless hydrodynamic pressure $p$ in a laminar shear flow (Newtonian fluid) around a particle, $Re_{sh}=10$. The particle is placed over a smooth (a) and rough (b) surfaces.

The increased lift force $F_L$ can be explained by forming an area of excessive pressure before the gap and that this pressure is higher than that near a smooth wall. Moreover, reducing the pressure in the suction area, whose minimum is the particle’s upper boundary behind the flow also contributes to the $F_L$ increase. The torque, in this case, increases as well because the flow does not go around the whole particle: the reduced flow in the gap doesn’t counterwork the torque created at the particle’s upper boundary.

As for the force coefficients, things are not so clear, because the profile of undisturbed velocity is different from a linear one: the velocity $u_p$ over a rough wall is higher than that over a smooth one, which has also been confirmed in [8]. The dimensional specific force depends on $u_p^2$, so in Newtonian
fluid $C_D$ reduces, $C_L$ increases, and $C_M$ slightly changes if compared to the smooth wall case. In total, $C_D$ changes up to 5%, $C_L$ – up to 17%, and $C_M$ – up to 7%. Since coefficients demonstrate certain invariant behavior when modeled for the rough wall, in Newtonian case the smooth wall approximations can be used to determine the forces affecting a particle in a flow.

4. Power law fluid

Analysis of the calculations for power law fluid demonstrates that the tendency to increase the torque and drag and lift forces is preserved.

![Figure 3](image1.png)

**Figure 3.** Drag $C_D$ (a), lift $C_L$ (b) and torque $C_M$ (c) coefficients and their behaviour depending on Re_{SH} for a particle over smooth and rough surfaces in power law fluid ($n=0.6$).

At the same time, all three coefficients get sufficiently smaller than those calculated for the smooth wall, mainly because of the increased difference in undisturbed velocity between the flows over smooth and rough surfaces. Figure 3 demonstrates the coefficient curves calculated for power law fluid ($n=0.6$). As you can see, the most significant difference in the lift force (up to 40%) was registered for small Reynolds numbers (2-10). The maximum deviations of the other coefficients for the whole Re_{SH} range comprise 25% for $C_D$ and 35% for $C_M$.

5. Incipient motion conditions

The conditions that initiate the particle’s motion along the sedimentation layer can be determined if we write down the balance of moments relative to point A (figure 4) where the particle touches a hemisphere being a part of the sedimentation layer [9],[10].

![Figure 4](image2.png)

**Figure 4.** Incipient motion condition that makes a particle move along a rough sedimentary layer.

When the sum of these moments exceeds zero, the particle starts roll out from the gap between the hemispheres: $M + F_D \delta_\theta + F_L \delta_\phi - F_A \delta_\delta - F_A \delta_\phi \geq 0$, where $F_g = \rho_p V_p g$ denotes the gravity, $\rho_p$ is the particle’s density, $V_p = 4/3 \cdot \pi \cdot (d/2)^3$, $F_A = \rho_p V_p g$ is buoyancy force. In the case of dense layer
packing and if the particle and hemispheres are of the same diameter, the angle $\alpha$ will be close to 60° while $\delta_s = d/4, \delta_h = d\sqrt{3}/4$. Having taken that into account and divided by $F_0$, we obtain:

$$\frac{\sqrt{3}}{16} \rho \, d^2 \left(2C_M + C_D + \sqrt{3}C_L\right) \geq g d (\rho_p - \rho_f).$$

Now, let consider another kind of incipient motion when a particle is taken away from a rough surface by a flow. In this case, the inequality for the forces is written as $F_L - F_f + F_A \geq 0$.

After the transformations analogous to those in (2) we obtain:

$$C_L \geq \frac{16}{3} \frac{g d (\rho_p - \rho_f)}{\rho_f G^2 d^2}.$$  

When analyzing the incipient motion conditions, it is convenient to consider the so-called Shields number $\tau_B$ that represents the ratio of the characteristic shear stress on the wall $\tau_{wall}$ and the difference between the buoyancy force $F_A$ and the gravity $F_f$ [9],[10]. Having defined the characteristic shear stress as $G$ and the viscosity as $\mu$, the following expression for $\tau_B$ can be obtained:

$$\tau_B = \frac{G \mu}{g d (\rho_p - \rho_f)}.$$  

When transforming expressions (2), (3) for the Shields number, we obtain that the particle’s rolling motion on a rough surface starts when $\tau_B \geq \tau_R$, and the particle is detached from surface when $\tau_B \geq \tau_L$, where

$$\tau_R = \frac{16}{\sqrt{3} \text{Re}_\text{SH} \left(2C_M + C_D + \sqrt{3}C_L\right)}, \quad \tau_L = \frac{16}{3 \text{C}_L \text{Re}_\text{SH}}.$$  

Comparing the smooth and rough wall cases for Newtonian fluid it should be noted that the parameter $\tau_R$ changes insignificantly (up to 3-5%) in the range of $\text{Re}_\text{SH}$ from 2 to 150, and the changes become significant (up to 17%) only at $\text{Re}_\text{SH} = 200$. At the same time $\tau_L$ reduces by 9-13% for the whole range of Reynolds numbers.

![Figure 5](image-url)  

**Figure 5.** Comparison of the critical Shields numbers that produce the rolling motion $\tau_R$ (a) and detaching $\tau_L$ (b) of a particle from smooth and rough surfaces in a shear flow of power law fluid ($n=0.6$).

The critical Shields numbers for smooth and rough surfaces in a flow of power law fluid ($n=0.6$) can be seen in figure 5. Unlike the Newtonian case, the differences between the smooth and rough
walls here are much more pronounced. For example, the parameter $\tau_R$ is higher for the rough than for the smooth wall, the difference comprises up to 27%. Changes of the parameter $\tau_L$ reach their maximum at $\text{Re}_{sh} = 2$ and comprise 70%. As the Reynolds number increases, the difference in $\tau_L$ between the two kinds of walls becomes smaller.

The way the coefficients $\tau_R$, $\tau_L$ change reflects the fact that for non-Newtonian fluid it is far more difficult to make a particle move from a rough wall than from a smooth one. At the same time, the particles take-off or their mixing occurs when the parameter $\tau_B$ increases. For a power law fluid, this parameter can be written as $\tau_B = \rho_f G^2 \left[ \text{Re}_{sh} \cdot g(\rho_r - \rho_f) \right]$.

With that in mind, the particle’s motion can be induce first of all by increasing of G with a simultaneous changing of the fluid’s consistency k (to meet the same values of $\text{Re}_{sh}$). The second way to increase $\tau_B$ is the reduction of the particles density.

6. Conclusion

The obtained results can be summarized in the following highlights. Numerical simulation has been performed that describes a shear flow of Newtonian and non-Newtonian fluid around a particle on a rough sedimentary layer for Reynolds numbers ranging from 2 to 200. The key integral characteristics have determined from the simulation describing the way the particle behaves in a flow. In the engineering formulas for Newtonian fluids, devised to determine the force parameters and conditions of a particle incipient motion, approximations for a smooth wall can be used. It has been demonstrated that for non-Newtonian power law fluid near a rough wall, the drag and lift force coefficients, as well as the torque coefficient, significantly differ from those calculated for a smooth wall. For a pseudo-plastic fluid ($n=0.6$), critical shear stresses that characterize a particle incipient motion from a rough surface have been determined and their difference from the smooth wall case has been demonstrated.

The performed study allows one to specify the movability conditions for a cuttings layer in a pipe, which will facilitate solving the problem of annulus clearing while drilling.

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