AdS/CFT for Accelerator Physics
or
Building the Tower of Babel

Arthur Hebecker

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 19,
D-69120 Heidelberg, Germany
(a.hebecker@thphys.uni-heidelberg.de)

Abstract

The crucial property of particle colliders is their ability to convert (e.g. electrical) energy into the mass of heavy particles. We have become used to the extremely low efficiency of this conversion and the severe limitations on the mass scale of heavy particles which can be reached. In view of this situation, it appears reasonable to ask whether a perfect conversion machine of this type (a perfect ‘collider’) exists even in principle and whether there is a highest mass scale which can be reached by such a machine. It turns out that, with a number of assumptions, such a machine is conceivable in a world with a strongly-coupled, approximately scale invariant 4d field theory with 5d gravity dual. This machine can be realized as a 5d tower built on the IR brane (in Randall-Sundrum model language). Transporting mass to the tip of this tower is, under certain conditions, equivalent to producing heavy point-like 4d particles. Hence, this can be thought of as a perfect ‘collider’. However, this machine can only reach a certain maximal energy scale, which falls as the gravity-dual of the 4d QFT approaches the strong coupling domain. On these grounds, one might expect that a no-go theorem (in the spirit of that of Carnot for the conversion of heat into work) exists for weakly-coupled QFTs. We end with some speculations about collider efficiencies at weak coupling, involving possibly the concept of entanglement entropy.
1 Introduction

The production of very heavy particles is one of the main goals of modern experimental particle physics. The method of choice is the acceleration of beams of charged particles (i.e. the conversion of electrical into kinetic energy) and their subsequent collision (i.e. the conversion at least a small fraction of that kinetic energy into the mass of heavy particles). While in practice the longevity of these particles has always been very limited, the production of stable very heavy states (such as the famous WIMP possibly making up dark matter) is most certainly conceivable.

In this context, one naturally encounters the following apparently very basic and general question: Does there exist, at least in principle, a perfect machine for the conversion of work into mass of heavy particles? To give an extreme example, is it conceivable to take just $543 \text{kWh} = 1.22 \times 10^{19} \text{GeV}$ from the electrical grid and covert them into one Planck mass particle?

It is, of course, well known that conventional colliders with this energy reach are very hard to imagine. Furthermore, even the production of e.g. 100 Higgs bosons is energetically much more expensive than the equivalent amount of electrical energy would suggest. But the question remains whether this is just due to our insufficient ingenuity or the limited technological progress made so far by mankind, or whether there exists some fundamental limitation.

Unfortunately, the present paper fails by a large margin to answer this extremely interesting question. However, it will at least outline a somewhat unusual (AdS/CFT-based [1]) way to think about problems of this type. In this context, a suggestion for a perfect energy conversion machine can be made. It will turn out that the reach of this type of machine is limited and that this range becomes small as the underlying 4d QFT becomes weakly coupled. This appears to at least hint at the existence of a fundamental no-go theorem for a prefect machine in 4d weakly coupled QFT (which is what we are apparently stuck with in this part of the multiverse).

The paper is organized as follows: Sect. 2 shows that, if our world were described by a Randall-Sundrum (RS) model [2], a 5d tower built on the IR brane (and ideally reaching the UV brane) can be thought of as a perfect (Planck scale) collider. This is almost obvious since a simple 5d elevator, using electrical energy with an energy conversion efficiency near unity, could now be employed to ‘UV-shift’ massive particles. Thus, as a first step, a very simple ‘toy-tower’ (a horizontal mirror supported by radiation pressure) is considered. It turns out that only a certain limited height can be reached.

Sect. 3 discusses an actual tower (made from some imagined 5d analogue of solid matter consisting e.g. of 5d ‘atoms’) and its maximal height. The result turns out to be similar to that of the previous Section. Thus, both models suggest that only a certain limited maximal energy can be reached by our ideal collider. The maximal height of the tower (and hence the ‘collider’ energy) falls with growing 5d curvature. This suggests that in weakly coupled 4d QFT (where the AdS dual is strongly curved), no ‘perfect collider’ can be built even in principle. We attempt to support this by some simple
considerations concerning the maximal energy conversion efficiency of colliders directly in 4d. We will also comment on some of the previous ideas concerning ‘Planck scale colliders’, see e.g. [3–5].

The final section is devoted to a brief summary, a discussion of open questions, and further speculations.

2 Colliders vs. elevators in the Randall-Sundrum model

2.1 How towers in RS models can be used to produce heavy particles

Our use of the AdS/CFT proposal will be limited to its simple yet very concrete and intuitive implementation in RS type models. To be very specific, we take the AdS metric in the form

$$ds^2 = e^{2ky} dx^2 + dy^2,$$

(1)

where $k$ sets the AdS curvature scale. Our discussion is based on the action [2]

$$S = \int_0^{y_{UV}} d^4xdy\sqrt{-g_5} \left( \frac{1}{2} M_5^2 R - \mathcal{L}_{5d} \right) + \int d^4x\sqrt{-g_{IR}}\mathcal{L}_{IR} + \int d^4x\sqrt{-g_{UV}}\mathcal{L}_{UV}.$$

(2)

Here the compact space is the interval $y \in [0, y_{UV}]$, with gravity and some 5d field theory in the bulk and two 4d theories at the boundaries (coupled to the induced metrics $g_{IR}$ and $g_{UV}$). Appropriate 5d and 4d cosmological constants have been absorbed in the lagrangians for brevity. As in the celebrated proposal for the solution of the hierarchy problem [2], we take ‘our’ QFT to be IR-brane-localized. Furthermore, and this is the crucial and non-trivial step, we imagine that future technology will allow us to penetrate the bulk and construct ‘5d robots’ capable of manipulating structures in 5d, at least near the IR brane (cf. Fig. 1).
To be very clear, the point here is not that an RS model will actually be discovered at the LHC. Neither do we really hope that we will learn to manipulate structures at length scales of $\text{TeV}^{-1}$ (which is equivalent to manipulating structures in the bulk). We are here considering a ‘model universe’, not too dissimilar from our own, where the question of probing the Planck scale appears with an interesting twist (as we will presently explain).

Before doing so, we recall some familiar facts about the setting described above (so far without any robots) and its AdS/CFT interpretation (see e.g. [6,7]): First, we dimensionally reduce to 4d and Weyl-rescale the 4d metric $g_4$ to ensure that $g_4 = g_{\text{IR}}$. The resulting 4d effective theory of this compactification includes 4d gravity (with a Planck scale set by $M_4^2 \sim k M_5^3 \exp(2k y_{\text{UV}})$) and a strongly coupled sector (the KK modes of 5d gravity and $L_{5d}$). This sector is approximately conformal in the energy range $k \ll E \ll k \exp(k y_{\text{UV}})$. Furthermore, the 4d effective theory also includes the two (by assumption weakly coupled) 4d field theories governed by $L_{\text{IR}}$ and $L_{\text{UV}}$. If these two lagrangians, as they appear in [2], are governed by mass parameters $M_1$ and $M_2$, then the two corresponding sectors of the resulting 4d effective theory will be governed by mass parameters $M_1$ and $M_2 \exp(k y_{\text{UV}})$ respectively. This is due to the different induced metrics at the two boundaries of our slice of AdS space. For simplicity, we set $M_1 = M_2 = M$ from now on.

From the 4d perspective, this setting looks rather conventional: One may think of it as of the ‘Standard Model’ ($L_{\text{IR}}$ with mass scale $M \sim \text{TeV}$), some form of technicolor, 4d gravity, and a weakly coupled sector with very heavy particles ($L_{\text{UV}}$ with mass scale $M \exp(k y_{\text{UV}})$). The point is that, if we can build a 5d tower (in the AdS interpretation of this model, cf. Fig. 1), then this corresponds to a perfect collider (in the sense of a machine for producing very heavy point-like particles) on the 4d side. We will shortly estimate the maximal height our 5d tower can reach, but before doing so let us argue in some detail that such a tower would be able to do the job of a conventional particle collider: Indeed, let us assume that $L_{5d}$ contains some fundamental field of mass $m$ ($m \sim M$ for simplicity). Corresponding particles can hence be produced by a conventional (i.e. IR-brane-bound) collider. This 5d field may also couple to a set of UV-brane fields, allowing e.g. its decay to two UV-brane particles of mass $\epsilon m$ and $(1 - \epsilon)m$. Thus, if a 5d tower reaching the UV brane could be build, this would be equivalent to a perfect Planck scale collider: One would just have to create our 5d particles with a TeV-scale machine, transport them up the tower using conventional mechanical energy (e.g. in an elevator) and eventually let them decay to UV-brane particles of mass almost equal to $m$.

From a 4d perspective, this corresponds to producing heavy, point-like particles (since $L_{\text{UV}}$ is supposed to be a weakly-coupled local lagrangian) of mass $m \exp(k y_{\text{UV}})$ with energy conversion efficiency $\eta_{\text{coll.}} \sim 1$. (Here we ignore the (in)efficiency of our original 4d collider taking us up to the TeV domain.)

2.2 Toy model of a suspended mirror

Now it is unfortunately clear that a tower of some particular desired height (e.g. reaching the UV brane) can not be built in general. To understand the limitations, let us first
Figure 2: Mirror supported by a ‘5d photon’ beam above the IR brane.

focus on an (at least calculationally) simpler device which is sufficient for suspending an elevator: We add 5d photons to our list of assumptions and let a mirror float above the IR brane, supported by the pressure of photons bouncing back and forth between brane and mirror (cf. Fig. 2). Obviously, to construct the mirror and the elevator, we also have to assume that some form of structured, stable matter exists in 5d.\footnote{This is non-trivial since all structures we manipulate every day in 4d rely microscopically on renormalizable gauge theories, which are not available in 5d. Let us nevertheless make this assumption and press ahead.} Governed by our 4d experience, we take this matter to consist of some small units (‘atoms’). To simplify our analysis, we assume these ‘atoms’ to have mass and inverse size\footnote{Obviously, our familiar 4d atoms have a mass and size which are parametrically different since the former is governed by the mass of the nucleus while the latter depends on electron mass and gauge coupling. In this language, our 5d model of matter corresponds to taking $m_N \sim m_e$ and $\alpha_e \sim 1$ in 4d.} $M$. We are interested in the lightest possible mirror, which will nevertheless have a thickness of at least a few ‘atoms’. The (hyper)surface density of this object will hence be $\rho_{s} \sim M^4$. (Note that this mirror extends in 3 spatial dimensions and hence the corresponding surface density has units of mass/(length)$^3$.)

To determine the force required to support such a mirror, consider first a particle with mass $m$ that is stationary at some height $y$. Its action is

$$S_y = -m \int_{y=\text{const.}} d\tau = -m e^{ky} \int dt,$$

where $\tau$ and $t$ are the eigentime and the time at the IR brane respectively. It is apparent that the same particle, if stationary at height $y + \delta y$, has an action enhanced by a factor $\exp(k \delta y)$. Thus, ‘lifting’ a particle a distance $\delta y$ costs an energy

$$\delta E = m e^{k(y+\delta y)} - m e^{ky} \simeq m e^{ky} k \delta y$$

from the perspective of the IR brane. Here the factor $e^{ky}$ appears as a ‘blue-shift’, because we took the IR-brane point of view. For a local observer at height $y$, lifting the same particle by $\delta y$ costs an energy $\delta E \simeq m k \delta y$. The force required to support a particle $m$, and now we use the local perspective, is hence $km$.

Our mirror is supported by the vertically directed (both up and down) photon stream.
with energy momentum tensor

\[ T_{MN} \sim \text{diag}(\rho, p, p, p, p) = \text{diag}(\rho, 0, 0, 0, \rho), \quad \text{where} \quad M, N \in \{0, 1, 2, 3, 5\}, \quad (5) \]

which is here given in a coordinate system with Minkowski metric in the vicinity of the mirror. To keep the mirror stationary, we need

\[ p = \frac{F}{A} = \frac{\rho_s A k}{A} = \rho_s k \sim M^4 k, \quad (6) \]

in self-explanatory notation. This is the pressure (and hence energy density) at the position \( y \) of the mirror. Since each photon travels vertically (at constant \( \vec{x} \)), the number of photons per unit-brane-surface (in the vicinity of the IR brane) is enhanced by \( \exp(3ky) \). Furthermore, due to the gravitational redshift, each photon has an energy enhanced by \( \exp(ky) \) when it is reflected by the IR brane. Thus, the energy density of our beam near the brane is

\[ \rho_{IR} \sim M^4 k e^{4ky}. \quad (7) \]

Assuming that the reflection of photons both at the IR brane and at our mirror is perfect, we can imagine that this configuration is stationary, without the need of continuous energy input. Nevertheless, the mirror had to be raised to its position \( y \), which required the input of energy into the photon beam near the IR brane. Since we assume that such an energy input can be realized maximally at a scale \( M \), we have the constraint \( \rho_{IR} < M^5 \). Comparing this with (7), we see that the maximal height \( y_{\text{max}} \) which can be achieved is set by

\[ e^{ky_{\text{max}}} \sim \left( \frac{M}{k} \right)^{1/4}. \quad (8) \]

In fact, there is an additional constraint arising from the danger of black hole formation (or, more generally, strong deformation of the 5d metric) in the region of high beam density. To see this, note that we actually have a layer of thickness \( \sim 1/k \) of an approximate energy density \( \rho_{IR} \) directly above the IR brane. We now estimate how large \( \rho_{IR} \) can become before black holes are formed in this region. To do so, recall that the mass of a \( d \)-dimensional black hole of radius \( R \) is (see e.g. [8])

\[ M_{BH} \sim M_{P,d}^{d-2} R^{d-3}. \quad (9) \]

This has to be compared to the relation between mass and radius of the corresponding smooth energy distribution:

\[ M_{BH} \sim R^{d-1} \rho. \quad (10) \]

Eliminating \( M_{BH} \) from (9) and (10) and specifying \( d = 5 \), we determine the critical radius for black hole formation,

\[ R_c \sim \sqrt{\frac{M_5^3}{\rho}}, \quad (11) \]
where $M_5$ is the 5d Planck mass. Now we substitute $R_c \sim 1/k$ and $\rho \equiv \rho_{IR}$ (cf. (7)), in (11) and solve for $\exp(ky)$. This gives us another bound on the achievable height $y$,

$$e^{ky} \sim \frac{M_5^{3/4}k^{1/4}}{M},$$

(12)

supplementing (8).

One possible interpretation is that (8) remains our basic formula for the maximal height but, due to (12), we in addition need to demand

$$M_5^3 > \frac{M^5}{k^2},$$

(13)

i.e., 5d gravity has to be sufficiently weak. As outlined earlier, we assume that our mirror has been lifted together with an attached 5d elevator, such that we are now in possession of a collider with ‘energy reach’ $(M/k)^{1/4}$. In other words, we can use e.g. photons at energy $M$ to produce particles with mass $M(M/k)^{1/4}$, with 100% energy efficiency (at least in principle). Obviously, we here do not include the one-time energy investment required for the construction of this ‘collider’.

For example, the UV brane or ‘Planck brane’ of [2] could be located at the height $y_{max}$ given by (8), in which case we could ‘lift’ energy to the Planck brane. Note that, due to the constraint (13), the 4d Planck mass ($M_4^2 \sim M_5^2/k$ using the UV-brane induced metric) remains higher than $M$, such that we can never actually reach the 4d Planck scale using this type of ‘perfect collider’.

It is obvious that our construction with a horizontal mirror and a vertical photon beam is far from optimal. It can be improved by making the floating mirror as small (in brane-parallel direction) as possible, curving it appropriately, and supporting it by a tapering photon beam arrangement. This clearly requires an appropriate mirror array at the IR brane. We do not pursue this analysis here but turn right-way to the construction of an (also tapering) ‘real’ tower made from solid material.

3 Maximal-height 5d towers and possible implications for 4d colliders at weak coupling

3.1 Optimal towers

An optimal tower will use the strongest 5d material available, i.e. that with the largest ratio $p/\rho$. We will henceforth assume that this ratio is maximized for one particular substance, which we will use to build our tower. Most naively, one would try to adapt

---

3 Presumably $p/\rho \ll 1$ holds even for the strongest available material, at least if this material is made from point-like weakly-interacting particles, as in our 4d world. Note, however, that our world is not weakly-coupled throughout and that much stronger materials, such as the neutron star crust, appear to exist.

---

7
Weisskopf’s famous argument \cite{10} for the maximal height of mountains (expressed in terms of fundamental constants) to our situation. While his argument is energetic (sinking of the mountain vs. melting of the rock at the bottom of the mountain), we make an essentially equivalent force-based estimate:

First, as a warm up, let $ky \ll 1$ such that $\exp(ky) \simeq 1 + ky$. A rectangular 5d mountain with (constant!) cross section $A$ and height $y$ has mass $Ay\rho$ and exerts a force $Aypk$ on its base. The base can provide a force $Ap$. Hence, for a given constant $p/\rho$ the maximal height is

$$y_{max} = \frac{1}{k} \cdot \frac{p}{\rho}.$$  \hspace{1cm} (14)

While self-consistent with our linearization (since $p/\rho < 1$), this is clearly not interesting: The crucial energy reach of our ‘collider’ is $\exp(ky_{max})$, which can hence not become large in our toy-model with constant cross section.

An optimal tower will taper towards its tip, such that each cross section is just large enough (assuming maximal vertical pressure at each point of the cross section) to support the part of the tower above. This clearly can be cast in the form of a differential equation for the cross section $A(y)$, and we will do so shortly. The solution then determines the shape of the tower and, as we will see, its maximal height.

Naively, one might expect to find a complete solution of this simple and fundamental problem in engineering textbooks or papers. However, in real-world towers, wind pressure is the most important issue and (unlike our case) the gravitational field can be treated either as linear ($\exp(ky) \to gy$) or, if one considers extremely high towers, according to the $1/r^2$ force-law. The closest related ideas and calculations in the literature appear to be related to either the Tsiolkovsky tower or the space elevator \cite{11} suspended from a point in geostationary orbit (in the latter case, the tapering is towards the bottom for obvious reasons). In any case, we were not able to find a treatment of a situation exactly equivalent to ours.

Fortunately, the corresponding equations are simple even in our exotic case. Everything can be derived from an equation relating the vertical forces at heights $y$ and $y + \delta y$:

$$F(y) = F(y + \delta y) \cdot (1 + k\delta y) + k\rho A(y)\delta y.$$  \hspace{1cm} (15)

Except for the factor $(1 + k\delta y)$, this is self-evident: Going down the tower by a distance $\delta y$, the force grows by the weight of an additional layer of material. The factor $(1 + k\delta y)$ comes from the warping: As explained earlier, raising a mass from $y$ to $y + \delta y$ costs an energy $mk\delta y\exp(ky)$ from the perspective of $y = 0$. This means that this mass exerts a force $mk\exp(ky)$ at any support at $y = 0$, while it obviously only exerts a force $mk$ at any support at its own height. In other words, vertical forces are subject to warping in the very same way as energies. Thus, the weight of all the tower material above $y + \delta y$ exerts a force on the surface at height $y$ which is enhanced by a factor $\exp(k\delta y) \simeq (1 + k\delta y)$. This is the content of the first term on the r.h. side of (15).

With $F(y) = pA(y)$ and $p = \text{const.}$ (an optimal tower will have maximal pressure at
any layer), one then immediately derives a differential equation for $A$,

$$- A'(y) = A(y)k(1 + \rho/p) ,$$  

(16)

where $\rho$ is constant by assumption. The solution is

$$A(y) = A_0 e^{-(1+\rho/p)ky} .$$  

(17)

Just to prevent any possible confusion: As should be clear from the derivation, this function $A(y)$ characterizes the $y$-dependence of the cross-section of our tower as a locally well-defined 5d physical quantity. For example, it could be the cross section in 5d Planck units. It is very different from the cross section as measured in the coordinates $x^\mu$ of $\Pi$.

In our analysis of the shape of the tower we have neglected any horizontal force components. This is only justified as long as the tower is a ‘thin object’, i.e., $A(y)$ does not change too rapidly with $y$. Quantitatively, this will certainly hold if the angle between the tower surface and the vertical axis is small. Most naively, one would estimate this angle as (minus) the derivative of the tower radius with respect to the height: $- [A^{1/3}(y)]'$. However, due to warping this derivative is non-zero even for a vertical tower, i.e. for a tower the surface of which is made from lines at $\vec x = \text{const}$. In fact, the cross section of such a vertical tower is given by $A_v(y) = A_0 \exp(3ky)$. Thus, when estimating the angle at the base of the tower and requiring it to be parametrically small, we have to do so relative to vertical tower:

$$- \left\{ [A^{1/3}(y)]' - [A^{1/3}_v(y)]' \right\} = \left\{ \frac{(1 + \rho/p)k}{3} + k \right\} A_0 \ll 1 .$$  

(18)

This translates into an estimate of the maximal $A_0$ allowed:

$$A_0^{1/3} \sim \frac{3}{(4 + \rho/p)k} .$$  

(19)

At its tip, our tower can certainly not become thinner than $1/M$. Thus, substituting $A_0^{1/3}$ from (19) and $A(y)^{1/3} \sim 1/M$ in (17), we eventually find that the maximal height $y_{max}$ is determined by

$$e^{ky_{max}} \sim \left( \frac{3M}{(4 + \rho/p)k} \right)^{\frac{3}{1+\rho/p}} .$$  

(20)

Note that this is rather similar to our ‘floating mirror’ result of (8): Since we did not keep track of $\mathcal{O}(1)$ factors, the prefactor $3/(4 + \rho/p)$ accompanying the ratio $M/k$ is most probably irrelevant. The only difference is then in the exponent. For an isotropic 5d radiation gas, which is presumably close to the stiffest possible matter, we have $p = \rho/4$ and hence an exponent $3/5$. This is better than the $1/4$ of (8), although we have to remember that we did not try to optimize the shape of the beam in Sect. 2. Thus, the competition between the two ‘perfect collider technologies’ of Sects. 2 and 3 can not be decided at this level of precision.

It is interesting to note that the approximate agreement arises in spite of the two configurations being distinctly different: The tower we are presently constructing becomes
wider towards its base. By contrast, the region of the IR brane from which the photon beam of Sect. 2 is reflected is much smaller than the floating mirror.

Finally, we expect a bound on $M_5$ arising from the danger of black hole formation at the base of the tower. It is easy to obtain by requiring that the critical radius of (11) is smaller than the width of the tower at its base, given by (19). One finds

$$M_5^3 > \frac{9\rho}{(4 + \rho/p)^2 k^2},$$

(21)

which, for the natural value $\rho \sim M_5$, once again becomes extremely similar to the analogous bound of (13).

3.2 Strong coupling appears to be unavoidable

To sum up, we have seen in two independent ways that a perfect collider with energy reach $(M/k)^\alpha$ (with $\alpha \sim O(1)$) can be built in principle. Since we are using a weak-coupling analysis on the gravity side, the corresponding 4d theory has to be strongly coupled. Let us try to be more precise by recalling that $\lambda \sim g_N^2 N \sim \left(\frac{M_s}{k}\right)^4$ in ‘proper’ AdS/CFT, i.e. in the duality between 4d $\mathcal{N} = 4$ Super-Yang-Mills theory and type IIB string theory on $AdS_5 \times S^5$. Here $\lambda$ is the ‘t Hooft coupling, i.e. the actual control parameter of perturbation theory on the 4d side, and $M_s \sim 1/l_s$ is the string scale. Thus, if we were to identify our ‘scale of 5d structure’ $M$ with the string scale $M_s$, we would find that the energy reach of our collider grows as $\lambda \to \infty$. By contrast, it approaches unity for $\lambda \to 1$. In other words, our perfect collider exists precisely because the 4d theory is strongly coupled and it ceases to exist as we approach the boundary between strong and weak coupling. Clearly, this does not exclude the existence of perfect colliders of some totally different type in the weak coupling domain, but it is a hint to the contrary.

We continue the discussion of a possible stringy realization of our 5d collider by recalling that, dimensionally reducing from 10d to 5d on $S^5$, we have

$$M_5^3 \sim M_s^8/k^5.$$

(23)

This is consistent with the constraint (13) or, equivalently, (21) if one identifies $M$ with $M_s$ as suggested above. Unfortunately, there is a different problem with this identification: At the scale $M_s$, we are deeply in the 10d regime since the compactification scale from 10d to 5d is $k$. Thus, we would either have to redo our ‘tower building exercise’ in 10d or construct the tower on branes extending along the radial direction of the $AdS_5 \times S^5$ throat. Furthermore, it may be necessary to use a ‘structure scale’ below $M_s$ to avoid back-reaction. We leave the investigation of these interesting questions to future work.
We end this line of thought by recalling that, to introduce an IR scale, all of the above would actually have to be done in a Klebanov-Strassler throat \[12\] (or one of its variants) rather than in the \(AdS_5 \times S^5\) throat. The former has a well defined IR end (an ‘IR brane’, if one wishes) and can be consistently glued to a Calabi-Yau\[4\] at its UV end, thereby inducing 4d gravity. Its length is automatically stabilized, which can be viewed as a variant of Goldberger-Wise stabilization \[14, 15\]. Thus, including the embedding of branes, it might be possible to realize all of the above in almost realistic settings. The trouble is only that, up to now, the LHC gives us very little hope that we actually live in such a geometry with low string scale.

### 3.3 Towards the weak coupling regime

Given that our world is apparently weakly-coupled, it is most interesting to consider implications for the weak coupling regime. Of course, realistic accelerator technology is a highly-developed field of research (see e.g. \[17\]) and the present author is completely ignorant in this important field. Hence this subsection only serves to share some vague ideas inspired by the previous ‘fundamental’ perspective. Three such ideas or potential relations between the strong-coupling and weak-coupling situation are detailed below.

Before proceeding, we note that an analysis related in spirit to this subsection has appeared in \[3\]. In contrast to \[3\], our emphasis is on the efficiency of energy conversion rather than on fundamental achievability of Planck scale energies (which we take for granted by linear collider technology, at least in a flat universe and with \(M_4 \rightarrow \infty\)).

Our first point is related to the possibility that, in between the IR and UV brane, other branes \[16\] (stabilized by the Goldberger-Wise mechanism \[14\] at certain 5d positions \(y_i\)) may exist. Given appropriate 4d field theories on those branes and couplings between these 4d theories and the 5d bulk fields, each brane may serve as a ‘base’ for the construction of a tower, cf. Fig. \[3\]. In other words, if the first brane is low enough to be reachable by a tower, a second tower can be constructed starting from this brane. This tower could, in principle, reach yet another brane, and so on. To summarize, it is conceivable that in such a generalized RS model an arbitrarily high energy scale can be reached by a perfect collider.

This peculiar construction has a surprisingly obvious weakly-coupled 4d analog. Indeed, imagine the LHC finds stable TeV-scale charged particles and some future, purpose-built collider produces them copiously. It is then conceivable to fill a further collider (storage ring) with those particles and accelerate them (very efficiently because of their huge mass and hence small synchrotron radiation) to a much higher energy\[5\] If, at this (say PeV) energy, another yet heavier species of stable charged particles is found and can be copiously produced, one may go on and reach the Planck scale (or at least a very high energy scale). While this collider cascade would presumably not be perfect (i.e. \(\eta_{\text{coll.}} \ll 1\) parametrically), it would nevertheless be much better than anything we

---

\[4\] For a different view on fundamental limitations in learning about the internal geometry of string compactifications see e.g. \[13\].

\[5\] In analogy to the muon-collider, just with much heavier and, most importantly, stable muons.
can imagine without new stable particles. It appears that the collider cascade works for the same reason as the tower cascade described above: If the fundamental theory is not approximately conformal, reaching high energies appears to be more doable, both at weak and strong coupling.

The second connection with 4d weakly-coupled theories is more straightforward: Let us ignore all technical objections and envision an extremely long (built in open space, far away from stars or planets\textsuperscript{6}) linear collider reaching a very high energy scale $M_{UV}$. In principle, the energetic efficiency of the accelerator part can be close to unity. However, the production of heavy particles (i.e. the collider part) presumably works much less efficiently for the following reason: The quality of beam-focusing may be limited \textit{in principle} (we will return to this in a moment). Let us assume that there is some minimal area $1/m^2$ to which the beams can at best be focused near the interaction point. The cross section for the production of particles of mass $M_{UV}$ is $\sim 1/M_{UV}^2$. Since, due to electromagnetic interactions, the beams are lost after the interaction point, the efficiency with which energy is converted into mass of heavy particles can not be better than

$$\eta_{\text{coll.}} \sim m^2/M_{UV}^2 .$$

(24)

Now, the crucial issue is the value of the ‘optimal focusing scale’ $m$. Even if we assume that transverse oscillations of the beam particles can in principle be completely removed by cooling, it is hard to imagine how the focusing scale can ever exceed the mass of those particles. This expectation follows simply from the uncertainty principle and the limitations on the electromagnetic field focusing the beam. (Note that for our purposes we identify electron and proton mass and consider the electromagnetic coupling as an $O(1)$ parameter.) Thus, we identify $m$ in (24) with the ‘smallest scale at which we can manipulate structures’ (e.g. electron mass etc.). In other words, $\eta_{\text{coll.}}$ falls below unity to the extent that $M_{UV}$ exceeds this ‘structure scale’. But this is exactly what the holographic point of view discussed earlier had suggested: In that case, the structure scale of the full theory is the IR-brane structure scale multiplied by the warp factor of the maximal-height tower. This is the mass scale that could be reached by our ideal ‘collider’. Anything beyond can only be reached by sacrificing efficiency.

\textsuperscript{6} We also ignore the gravitational effects emphasized in \textsuperscript{3}. 

---

Figure 3: If additional branes in between the IR and UV brane exist, a cascade of towers can, in principle, reach arbitrarily high energy scales.
According to the above, setups with point-like heavy particles bear some resemblance to systems with negative temperature [18]. Indeed, the transformation of work into thermal energy of a negative-temperature system can be extremely inefficient. This fundamental inefficiency is apparently shared by the transformation of work into mass of heavy particles. To be more specific, let us assume that our heavy particles correspond to energy with negative temperature $T_2$. Consider a Carnot cycle with input heat $Q_1$ at positive temperature $T_1$ (e.g. $\sim 10^3 K$, as in power stations), output ‘heat’ $Q_2$ at $T_2$, and output work $W$. Its efficiency,

$$\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1},$$

(25)
can be greater than unity. This is intuitive since the negative-temperature system can deliver energy while increasing its own entropy (see e.g. [19]). It can thus contribute positively to the delivered work, $Q_2 < 0$, while respecting reversibility (no entropy change, $\delta S = 0$). Our previous collider efficiency was that for the transformation of (input) work $W$ to ‘heat’ at $T_2$, corresponding to the reversed process. Hence

$$\eta_{coll.} = \frac{-Q_2}{W} = \frac{W - Q_1}{W} = 1 - \frac{1}{\eta} = \frac{-T_2}{T_1 - T_2}.$$  

(26)

This quantity can thus be extremely small as a matter of principle (as conjectured in this paper) if the ‘equivalent’ temperature of heavy point-like particles is negative and tiny.

Finally, the third connection between the strongly- and weakly coupled situations goes as follows: Let us to try imagine, at least very roughly, what a direct 4d weak-coupling analogue of our 5d-tower would look like. For example, one could think of a spherical mirror inside which a standing wave of electromagnetic energy was trapped. The energy density in this spherical standing wave would have to grow to extreme values as one approaches the center. Let alone the question of how to create a sufficiently perfect mirror and to set up the relevant field configuration, one immediately sees that it could never be stable: The reason is simply that in the inner part (where the energy density would have to be extremely high) electron-positron pairs would be created and escape to the outside. This is just a result of the non-vanishing interaction between UV and IR modes, which is unavoidable in a weakly-coupled local field theory. By contrast, in our tower the highly-concentrated (in 4d language) energy at the tip of the tower is prevented from decaying to IR-brane degrees of freedom by 5d locality. Thus, we here see another hint that the construction of perfect colliders is presumably even harder at weak than at strong 4d coupling.

4 Conclusions

We have presented some, admittedly rather speculative, ideas concerning the (im)possibility of a perfect collider. Our main technical point was very simple: For theories having a 5d gravity dual, reaching for UV energy scales corresponds to building 5d towers based on the IR brane and pointing to the UV brane. The height of such towers
appears to be limited at a rather fundamental level (quite analogously to the limited height of mountains, given the limited strength of granite). We estimated this maximal height and conjectured (given the parametric behaviour of our result) that in 4d weakly-coupled theories it is completely impossible to build a perfect machine (i.e. a machine with energetic efficiency near unity) which transforms energy, starting from the 'structure scale' of our theory, towards the UV.

Clearly, in our holographic approach, accelerator physicists are ‘tower builders’, struggling with the 5d gravitational potential. The 4d weak-coupling analogue of their problem is apparent: The tendency of massive objects to fall translates into the tendency of energy to transfer from the UV to the IR in conventional QFT.

Many interesting questions are still open: For example, it remains to be proven that, in a slice of AdS with a certain 5d lagrangian, it is truly impossible to build an arbitrarily high tower. Similarly, it has to be clarified whether a tower is really the only or at least the best way to transfer energy reversibly (with efficiency near unity) from the IR brane to an arbitrarily high position above it. It is tempting to speculate that the presently very popular concept of holographic entanglement entropy (see e.g. [20–22]) has something to do with this: Obviously, by transferring energy to the UV we concentrate it in a small 4d area, with a low entanglement entropy. This is nicely visualized as concentrating the energy inside the minimal surface measuring the entanglement according to [20].

Also independently of the holographic interpretation, entanglement entropy may be relevant. For example, an extreme growth of entanglement entropy in particle decay (taking the perspective of one of the decay products) has been obtained in the analysis of [24]. But particle decay is just the inverse of particle production in a collider. Thus, one may hope that the difficulties in constructing (even in a Gedankenexperiment) a perfect collider can be understood, at the fundamental level, as follows: Such a collider creates very energetic states with near-zero entanglement entropy (the very heavy, point-like particles to be produced). This may be impossible with high energetic efficiency due to limitations analogous to those familiar from the Carnot cycle and its standard entropy-based analysis. Thus, an ‘entanglement entropy analysis’ of an ideal machine transforming (macroscopic, entropy-free) work into heavy point-like particles may be worthwhile. Alternatively, as we discussed in some detail, a more direct Carnot-type obstruction to perfect colliders arises if one can argue that settings with heavy point-like particles correspond to negative-temperature systems.

Finally, returning to the holographic perspective, the following consideration appears interesting: Envision a universe with structure, planets, live etc., but with a relatively large cosmological constant. If the de-Sitter curvature is high enough, it may prevent the construction of a very long linear accelerator as a matter of principle (see [3] for related arguments). Let the field theory in addition be strongly coupled, with a gravity dual of the type of a Randall-Sundrum model. In that situation also the construction of sufficiently

---

7 Of course, this is far from obvious since our source of energy is conventional work with entropy zero. If the gravitational IR attraction in the AdS slice could be given a clear thermodynamic meaning, e.g. along the lines of [23], a more direct application of the 2nd law of thermodynamics may become possible.
high 5d tower (as detailed in the paper) may be impossible. As a result, reaching the UV or Planck brane may be ruled out altogether. This could be a fundamental obstruction to unraveling microscopic details of 4d quantum gravity - a ‘UV protection’ mechanism reminiscent of \[25\], but different at the technical level. Thus, a world in which the UV completion of its field theory is absolutely protected from observation (and hence not part of physical reality) may be conceivable. We find it intriguing to think that this form of ‘UV protection’ is due to the overwhelming force of gravity in the dual AdS geometry.

**Acknowledgments**

I would like to thank Stefan Theisen and Timo Weigand for helpful discussions.

**References**

[1] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231 [hep-th/9711200];

O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” Phys. Rept. 323 (2000) 183 [hep-th/9905111].

[2] L. Randall and R. Sundrum, “A Large mass hierarchy from a small extra dimension,” Phys. Rev. Lett. 83 (1999) 3370 [hep-ph/9905221].

[3] A. Casher and S. Nussinov, “Some speculations on the ultimate Planck energy accelerators,” hep-ph/9510364 and “Is the Planck momentum attainable?,” hep-th/9709127.

[4] L. Labun and J. Rafelski, “Strong Field Physics: Probing Critical Acceleration and Inertia with Laser Pulses and Quark-Gluon Plasma,” Acta Phys. Polon. B 41 (2010) 2763 [arXiv:1010.1970 [hep-ph]].

[5] M. Banados, J. Silk and S. M. West, “Kerr Black Holes as Particle Accelerators to Arbitrarily High Energy,” Phys. Rev. Lett. 103 (2009) 111102 [arXiv:0909.0169 [hep-ph]]; S. T. McWilliams, “Black Holes are neither Particle Accelerators nor Dark Matter Probes,” Phys. Rev. Lett. 110 (2013) 011102 [arXiv:1212.1235 [gr-qc]].

[6] R. Rattazzi and A. Zaffaroni, “Comments on the holographic picture of the Randall-Sundrum model,” JHEP 0104 (2001) 021 [hep-th/0012248].

---

\[8\] One may envision producing a large black hole and waiting for it to evaporate, thus probing quantum gravity through the observation of the final moments of its evaporation. However, the mass inside the horizon may be too small to produce a black hole or the universe may not live long enough for complete evaporation.
I. Heemskerk, J. Penedones, J. Polchinski and J. Sully, “Holography from Conformal Field Theory,” JHEP **0910** (2009) 079 [arXiv:0907.0151 [hep-th]]; R. Sundrum, “From Fixed Points to the Fifth Dimension,” Phys. Rev. D **86** (2012) 085025 [arXiv:1106.4501 [hep-th]].

F. R. Tangherlini, “Schwarzschild field in n dimensions and the dimensionality of space problem,” Nuovo Cim. **27** (1963) 636; R. C. Myers and M. J. Perry, “Black Holes in Higher Dimensional Space-Times,” Annals Phys. **172** (1986) 304.

C. J. Horowitz and K. Kadau, “The Breaking Strain of Neutron Star Crust and Gravitational Waves,” Phys. Rev. Lett. **102** (2009) 191102 [arXiv:0904.1986 [astro-ph.SR]].

V. F. Weisskopf, “Modern physics from an elementary point of view,” CERN-70-08.

K. E. Tsiolkowski, “Dreams of Earth and Sky” (1895), reissue (Athena Books, Barcelona-Singapore, 2004); Y. Artsutanov, “V Kosmos na Elektrovoze” (To Space by Funicular Railway), Kom-somolskaya Pravda, July 31, 1960.; J. Pearson, “The Orbital Tower: A Spacecraft Launcher Using the Earth’s Rotational Energy”, Acta Astronautica **2** (1975) 785; G. A. Landis and C. Cafarelli, “The Tsiolkovski Tower Re-Examined”, JIBIS, 52, (1999) 175; B. C. Edwards, “Design and Deployment of a Space Elevator”, Acta Astronautica **47** (2000), No. 10, 735; P. K. Aravind, “The Physics of the Space Elevator”, Am. J. Phys. 75, 125 (2007).

I. R. Klebanov and M. J. Strassler, “Supergravity and a confining gauge theory: Duality cascades and chi SB resolution of naked singularities,” JHEP **0008** (2000) 052 [hep-th/0007191].

J. J. Heckman, “Statistical Inference and String Theory,” [arXiv:1305.3621 [hep-th]].

W. D. Goldberger and M. B. Wise, “Modulus stabilization with bulk fields,” Phys. Rev. Lett. **83** (1999) 4922 [hep-ph/9907447].

F. Brummer, A. Hebecker and E. Trincherini, “The Throat as a Randall-Sundrum model with Goldberger-Wise stabilization,” Nucl. Phys. B **738** (2006) 283 [hep-th/0510113].

I. Oda, “Mass hierarchy from many domain walls,” Phys. Lett. B **480** (2000) 305 [hep-th/9908104] and “Mass hierarchy and trapping of gravity,” Phys. Lett. B **472** (2000) 59 [hep-th/9909048]; H. Hatanaka, M. Sakamoto, M. Tachibana and K. Takenaga, “Many brane extension of the Randall-Sundrum solution,” Prog. Theor. Phys. **102** (1999) 1213 [hep-th/9909076].
[17] C. M. Ankenbrandt, M. Atac, B. Autin, V. I. Balbekov, V. D. Barger, O. Benary, J. S. Berg and M. S. Berger et al., “Status of muon collider research and development and future plans,” Phys. Rev. ST Accel. Beams 2 (1999) 081001 [physics/9901022]; J. -P. Delahaye et al. [CLIC Study Group Collaboration], “CLIC: A Two beam multi-TeV e+- linear collider,” eConf C 000821 (2000) MO201 [physics/0008064 [physics.acc-ph]]; J. Brau et al. [ILC Collaboration], “ILC Reference Design Report: ILC Global Design Effort and World Wide Study,” [arXiv:0712.1950 [physics.acc-ph]]; I. Blumenfeld, C. E. Clayton, F. -J. Decker, M. J. Hogan, C. Huang, R. Ischebeck, R. Iverson and C. Joshi et al., “Energy doubling of 42 GeV electrons in a metre-scale plasma wakefield accelerator,” Nature 445 (2007) 741.

[18] N. F. Ramsey, “Thermodynamics and Statistical Mechanics at Negative Absolute Temperatures,” Phys. Rev. 103 (1956) 20; M. J. Klein, “Negative Absolute Temperatures,” Phys. Rev. 104 (1956) 589.

[19] A. Rapp, S. Mandt, A. Rosch, “Equilibration rates and negative absolute temperatures for ultracold atoms in optical lattices,” Phys. Rev. Lett. 105 (2010) 220405 [arXiv:1008.0468 [cond-mat]].

[20] S. Ryu and T. Takayanagi, “Holographic derivation of entanglement entropy from AdS/CFT,” Phys. Rev. Lett. 96 (2006) 181602 [hep-th/0603001].

[21] T. Takayanagi, “Entanglement Entropy from a Holographic Viewpoint,” Class. Quant. Grav. 29 (2012) 153001 [arXiv:1204.2450 [gr-qc]].

[22] M. Nozaki, T. Numasawa and T. Takayanagi, “Holographic Local Quenches and Entanglement Density,” JHEP 1305 (2013) 080 [arXiv:1302.5703 [hep-th]].

[23] T. Jacobson, “Thermodynamics of space-time: The Einstein equation of state,” Phys. Rev. Lett. 75 (1995) 1260 [gr-qc/9504004]; E. P. Verlinde, “On the Origin of Gravity and the Laws of Newton,” JHEP 1104 (2011) 029 [arXiv:1001.0785 [hep-th]].

[24] L. Lello, D. Boyanovsky and R. Holman, “Entanglement entropy in particle decay,” [arXiv:1304.6110 [hep-th]].

[25] G. Dvali and C. Gomez, “Self-Completeness of Einstein Gravity,” [arXiv:1005.3497 [hep-th]]; G. Dvali, G. F. Giudice, C. Gomez and A. Kehagias, “UV-Completion by Classicalization,” JHEP 1108 (2011) 108 [arXiv:1010.1415 [hep-ph]]; G. Dvali, C. Gomez and D. Lust, “Black Hole Quantum Mechanics in the Presence of Species,” [arXiv:1206.2365 [hep-th]].