Pair Production of Majorana Neutrinos by Annihilation of Charged Particles in High Energy Collision

Y. M. Goh†, H. K. Lee‡, W.-G. Paeng§ and Y. Yoon†
Department of Physics, Hanyang University, Seoul 133-791
(Received)

Assuming that neutrinos have non-vanishing magnetic moments, we discuss the possibility of pair production through annihilation of charged fermions in high-energy collisions. Adopting the Pauli interaction for photon-neutrino coupling, we calculate the neutrino pair production cross section in the photon channel and compare the result with the standard model in $Z^0$ channel. We demonstrated that the enhancement of the production rate for Majorana neutrino pairs over the standard model rate can be possible at the center-of-mass energy of $10 - 100$ TeV for the Large Hadron Collider or the ultra-high-energy cosmic Ray when the transition magnetic moment is not smaller than $10^{-9} - 10^{-10} \mu_B$.

PACS numbers: 13.40.Em, 13.66.Hk, 14.60.St

Keywords: Majorana neutrinos, High energy collision

I. INTRODUCTION

Atmospheric and solar neutrino observations [1, 2], as well as reactor experiments [3], provide us strong evidence of the oscillation between different flavors of neutrinos, which can’t be possible if neutrinos are massless. Neutrinos are massive, electrically neutral fermions with spin 1/2. Although they are neutral fermions, it has been an interesting question how neutrinos can couple to photons. One of the possibilities is the non-vanishing magnetic moment of neutrinos, which can induce a spin-dependent coupling to photons.

Experimental bounds for the neutrino magnetic moments have been obtained from experiments for the solar neutrinos, accelerator neutrinos and reactor neutrinos [4]. The upper bound of the neutrino magnetic moment is found to be in the range of $10^{-10} - 10^{-7} \mu_B$. Here, $\mu_B$ is the Bohr magneton. A model-dependent bound can also be obtained, for example, in big-bang nucleosynthesis and SN87a, to be $10^{-12} - 10^{-10} \mu_B$ [5, 6]. The theoretical bounds [8] were also discussed recently to get a rather wider range of upper bounds, $10^{-15} - 10^{-7} \mu_B$. In the standard model, the neutrino magnetic moment induced by the one-loop effect [9] is $\mu_\nu = 3 \times 10^{-19} \left( \frac{m_\nu}{\text{eV}} \right) \mu_B$, which is much smaller than the above bounds.

In this work, we take the magnetic moment as a parameter, which might probe the physics beyond the standard model. It is interesting to note that the effect of the magnetic moments of neutrinos on the vacuum instability has recently been investigated in the presence of a strong external magnetic field [10] to find out that with non-vanishing magnetic moment the vacuum instability appears beyond the critical field strength, $B_c = \frac{2e}{m_e}$, against the pair production of neutrinos.

We consider a process of charged fermion-antifermion annihilation into neutrino pairs. The standard process is a pair production through the $Z^0$ channel. If neutrinos have non-vanishing magnetic moments, they can also be produced in the photon channel through the Pauli interaction [11] on top of the standard process. Previously, the production of a massive neutrino through a magnetic interaction has been calculated and discussed for obtaining the experimental bounds [12–15].

In this work, we calculate the differential and the total cross sections for pair production of Majorana neutrinos with transition magnetic moments in the photon channel through the Pauli interaction. The Majorana neutrino is known to have only a transition magnetic moment, which implies that the lepton flavor number is not conserved: a pair produced via the transition magnetic moment consists of two different lepton flavors. We consider an extreme process with ultra-high energy, $E_{CM} > 10$ TeV, which is possible for the hadronic collision in the Large Hadron Collider (LHC) and $E_{CM} \approx 100$ TeV in the ultra-high-energy cosmic ray (UHECR). Because of the momentum-dependent coupling in the Pauli interaction, the total cross section in the photon channel becomes constant in the high-energy region, $E \gg m_i$, while the cross section in the $Z^0$ channel decreases as the energy scale increases. Hence, there is a critical energy beyond which the Pauli interaction dominates the SM process. We demonstrate that the critical energy can be in the energy region of the LHC or the UHECR provided the transition magnetic moment is not much smaller than $10^{-9} - 10^{-10} \mu_B$. It is also found that the angular distribution of the differential cross section in the Pauli interaction has a maximum at $\theta = \pi/2$ in the center-of-mass system.

In Section II, The basic feature of the Pauli interaction and the cross section in the center-of-mass system are discussed. In section III, the cross section for Majorana neutrino pairs is discussed in detail, and discussions are given in Section IV.
II. PAULI INTERACTION

In relativistic quantum theory, the standard picture is that the motion of a charged fermion is governed by the Dirac equation with an interaction with the external electromagnetic field, $A^\mu$, in a gauge-covariant way:

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ c\alpha \cdot \left( p - \frac{e}{c} A \right) + \beta mc^2 + e\Phi \right] \psi. \quad (1)$$

Pauli introduced a new form of the interaction with a magnetic moment [11]:

$$\mathcal{L}^{\text{Pauli}} = \frac{1}{2} \bar{\psi} \gamma^\mu \gamma^\nu \sigma^{\mu\nu} F_{\mu\nu} \psi, \quad (2)$$

where the $F_{\mu\nu}$ are the external field strengths in natural units and $l$ has the dimension of a length. The corresponding Dirac-Pauli Lagrangian is given by

$$\mathcal{L} = \bar{\psi} \left( \gamma^\mu \partial_\mu - \frac{\mu}{2} \sigma^{\mu\nu} F_{\mu\nu} - m \right) \psi, \quad (3)$$

where $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$ and $g_{\mu\nu} = (+, -, -, -)$. $\mu$ in the Pauli term measures the magnitude of the magnetic moment of the neutral fermion. This Lagrangian describes the interaction of the neutral fermion, but with a non-vanishing magnetic moment coupled with the external electromagnetic field through the Pauli interaction. The quantum mechanics with the Pauli interaction has been investigated for various types of electromagnetic fields [12]. The vacuum instability against the pair production rate of neutral fermions in linear magnetic fields through the Pauli interaction has been calculated [10].

Neutrinos do not have electric charges, but they are found to have a non-vanishing mass. Also, it is natural to ask about the possibility of magnetic moments. However, it is not clear so far whether they have non-vanishing magnetic moments through which they can interact with photons directly. We assume, in this work, the case where the massive neutrinos have non-vanishing magnetic moments or transition magnetic moments. Then, we can consider a process in which the neutrino pairs can be produced through photon exchange, in addition to the weak process by $Z_0$ boson exchange.

In this work, we calculate the cross section of neutrino pair production through the Pauli interaction to investigate the observational effect of the magnetic moment of neutrinos. We first consider a process in which neutrino pairs are produced by the annihilation of a charged fermion, $q$, with charge $Qe$, which has minimal coupling to a photon, lepton, or parton in the hadron collision. In high-energy hadron collisions, the charged fermions can be considered to be those of partons in hadrons.

The process we are considering is the annihilation of charged fermions $q$ and $\bar{q}$ into a neutrino through the photon channel with the Pauli interaction. The interaction Lagrangian for neutrino pair production is

$$\mathcal{L}_{\text{int}} = \frac{\mu}{2} \bar{\psi}_\nu \sigma^{\mu\nu} F_{\mu\nu} \psi_\nu. \quad (4)$$

In the center-of-mass frame, we get the differential cross section [13]:

$$\left( \frac{d\sigma}{d\Omega} \right)_D = \frac{Q^2 \alpha \mu^2}{16\pi^2} \left[ \frac{1}{1 - m^2_q E^2} \right] \left( 1 + \frac{m^2_q}{m^2_{\nu_e}} \right) \left( 1 + \frac{m^2_{\nu_\mu}}{m^2_{\nu_e}} \right) \cos^2 \theta \quad (5)$$

and the total cross section

$$\sigma_D = \frac{1}{6} Q^2 \alpha \mu^2 \sqrt{\frac{1 - m^2_q}{1 - m^2_{\nu_e}}} \left( 1 + \frac{m^2_q}{m^2_{\nu_e}} \right) \left( 1 + \frac{m^2_{\nu_\mu}}{m^2_{\nu_e}} \right). \quad (6)$$

Here, $D$ denotes Dirac type neutrinos. A similar calculation has been done by Barut et al. [12], where the total cross section is calculated as

$$\sigma_B = \frac{Q^2 \alpha \kappa^2}{6} \sqrt{\frac{1 - m^2_q}{1 - m^2_{\nu_e}}} \left( 1 + \frac{m^2_q}{m^2_{\nu_e}} \right) \left( 1 + \frac{m^2_{\nu_\mu}}{m^2_{\nu_e}} \right). \quad (7)$$

where $\kappa$ is the neutrino magnetic moment in their convention. There is a small difference in the last term, which might be due to the additional $(1 + \gamma_5)$ term in their interaction Lagrangian whereas we consider the case without chirality for the Pauli coupling. At high energy, $m_q, m_\kappa, m_e \ll E$, which is the scale at the LHC or the UHECR of our interest, they give the same result modulo the coupling constants:

$$\left( \frac{d\sigma}{d\Omega} \right)_i = \eta_i \sin^2 \theta, \quad \sigma_i = \frac{8\pi}{3} \eta_i, \quad (8)$$

where $i$'s stand for the choice of coupling constants, $\eta_D = Q^2 \alpha \mu^2 / 16\pi$, and $\eta_B = Q^2 \alpha \kappa^2 / 8\pi$. The cross section becomes constant at high energy. If these are valid all the way to higher energy, then there is the problem of violation of the unitary bound. However, the Pauli coupling is an effective interaction term that is valid only up to some scale, and we assume it to be higher than the scale we are considering in this work. Now, it is interesting to note that energy dependence is quite different from that of pair production through the $Z$-boson channel in the standard model, where the cross section decreases with increasing colliding energy. The comparison and the possible implication will be discussed in detail in the final section.

III. PAIR PRODUCTION CROSS SECTION OF MAJORANA NEUTRINOS

The Majorana field is basically represented by a two-component spinor, $\chi$. For a free particle, the Lagrangian of the two-component Majorana field is given by

$$\mathcal{L}_M = \chi^\dagger i\sigma \cdot \partial \chi - \frac{m}{2} \left[ (\chi^C)^\dagger \chi + \chi^\dagger \chi^C \right]. \quad (9)$$
trino with the Pauli interaction can be written as
\[ \chi = \sum_s \int \frac{d^3 \tilde{p}}{(2\pi)^3/2} \frac{1}{(2E_p)^{1/2}} \left[ f (\tilde{p}, s) a (\tilde{p}, s) e^{-ip \cdot x} + g (\tilde{p}, s) a^\dagger (\tilde{p}, s) e^{ip \cdot x} \right], \]
where \( f \) and \( g \) are two-component spinors that satisfy the equation of motion for Majorana field,
\[ i\tilde{\sigma} \cdot \partial \chi - ima^2 \chi^2 = 0. \]

Then, it is possible to construct a four-component Majorana field:
\[ \Psi_M = \sum_s \int \frac{d^3 \tilde{p}}{(2\pi)^3/2} \frac{1}{(2E_p)^{1/2}} \left[ u (\tilde{p}, s) a (\tilde{p}, s) e^{-ip \cdot x} + v (\tilde{p}, s) a^\dagger (\tilde{p}, s) e^{ip \cdot x} \right], \]
where
\[ u \equiv \left( \begin{array}{c} f (\tilde{p}, s) \\ g \end{array} \right), \quad v \equiv \left( \begin{array}{c} g (\tilde{p}, s) \\ f \end{array} \right). \]

Now, the interaction Lagrangian for the Majorana neutrino with the Pauli interaction can be written as
\[ \mathcal{L}_{\text{int}} = i \frac{\mu ij}{2} \bar{\Psi}^i_M \sigma_{\mu \nu} \Psi_j M^\mu \gamma^\nu, \]
where \( \sigma_{\mu \nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] \) and \( g_{\mu \nu} = (+, -, -, -) \). \( \mu ij \) is a transition magnetic moment that is antisymmetric for a Majorana neutrino, \( \mu_{ij} = -\mu_{ji} \).

The differential cross section in the center of mass (CM) can be calculated in a straightforward way:
\[ \left( \frac{d\sigma}{d\Omega} \right)_M = \frac{\alpha Q^2 \mu_{ij}^2}{4\pi} \sqrt{\frac{E_1^2 - m_{e1}^2}{E_1^2 - m_{\nu_e}^2}} \frac{E_2^2 - m_{\nu_e}^2}{E_2^2 - m_{\nu_e}^2} \times \left[ \frac{E_1 E_2}{E_1^2} + \frac{E_1 E_2 m_{\nu_e}^2}{2E_1^2} + \frac{m_{\nu_e} m_{\nu_e} m_{\nu_e}}{2E_1^2} \frac{m_{\nu_e} m_{\nu_e} m_{\nu_e}}{2E_1^2} \frac{m_{\nu_e} m_{\nu_e} m_{\nu_e}}{2E_1^2} \right] \frac{1}{E_1^2} \left[ 1 - \frac{m_{\nu_e}^2}{E_1^2} \right], \]
where \( \alpha \) is a fine-structure constant and \( M \) denotes the Majorana neutrino. After integrating over \( d\Omega \), the total cross section is given by
\[ \sigma_M = \frac{\alpha Q^2 \mu_{ij}^2}{6} \frac{E_1}{E_1^2} \left( 2 + \frac{m_{\nu_e}^2}{E_1^2} \right) \sqrt{1 - \frac{m_{\nu_e}^2}{E_1^2}} \frac{m_{\nu_e}^2}{E_1^2} \frac{m_{\nu_e}^2}{E_1^2} \times \left[ \frac{3E_1 E_2}{E_1^2} + \frac{3m_{\nu_e} m_{\nu_e}}{E_1^2} \right] \frac{E_1 E_2}{E_1^2} \sqrt{1 - \frac{m_{\nu_e}^2}{E_1^2}} \frac{m_{\nu_e}^2}{E_1^2} \frac{m_{\nu_e}^2}{E_1^2}, \]

In the high-energy region, where the particle masses are very small compared to the energy scale, \( m_i \ll E \), the differential cross section and the total cross section, respectively, converge to simple expressions:
\[ \left( \frac{d\sigma}{d\Omega} \right)_M = \frac{\alpha Q^2 \mu_{ij}^2}{4\pi} \sin^2 \theta \]
and
\[ \sigma_M = \frac{2\alpha Q^2 \mu_{ij}^2}{3}, \]
which are similar to the results for a Dirac neutrino with magnetic moment.

For comparison with the neutrino pair production in the SM, in the high-energy limit \( E \gg m_\nu, M_Z \), the differential cross section and the total cross section in the standard model are known to behave as
\[ \left( \frac{d\sigma}{d\Omega} \right)_{SM} \propto (1 + \cos^2 \theta), \quad \sigma_{SM} \propto \left( \frac{1}{E} \right)^2. \]

We can see that the total cross section behaves as \( \frac{1}{E} \) in the high-energy limit. The angular distribution of the differential cross section is maximum for \( \theta = 0 \) and \( \pi \) and minimum for \( \theta \sim \pi/2 \). These features are quite different from those with the Pauli interaction, Eqs. (17) and (18).

IV. DISCUSSION

We calculate the high-energy behavior of the cross section for Majorana neutrino pair production, assuming that the neutrinos have non-vanishing transition magnetic moments and are interacting electromagnetically with the Pauli interaction. We found that the production cross section for Majorana-type neutrino production is similar to that of Dirac-type neutrino production. The angular distribution in the center-of-mass frame peaks at \( \theta = \pi/2 \) while the angular distribution in the standard model has a minimum at \( \theta \sim \pi/2 \). The total cross section turns out to be independent of energy:
\[ \sigma_M = 1.66 \times 10^{-29} Q^2 m^2 (\tilde{\mu})^2, \]
where \( \mu_{12} \equiv \tilde{\mu} B \). This is basically because the neutrino interaction vertex carries a momentum factor of the virtual photon, which cancels the energy dependence, which is otherwise inversely proportional to the square of the energy in the center-of-mass frame as is the case for the standard model. Hence, we can expect the neutrino production through the Pauli interaction to compete with that of the SM for high-energy collisions. The energy scale, \( E_{0.1} \), for which \( \sigma_M \) becomes 0.1 \( \sigma_{SM} \), can be estimated as
\[ E_{0.1} \sim 10^2 \left( \frac{10^{-10}}{\tilde{\mu}} \right) \text{TeV}. \]
For the Majorana-type neutrino, the upper bound is somewhat less stringent (although model dependent) than it is for the Dirac neutrino. If we take $\tilde{\mu} \lesssim 10^{-9} - 10^{-10}$, we get $E_0 \sim 10 - 100 \text{ TeV}$, which can be reached in LHC and in UHECR experiments. However, if $\tilde{\mu} < 10^{-11}$, the corresponding energy scale becomes higher, beyond the GZK cutoff. One of the characteristics of neutrino production by Pauli coupling is that the differential cross section has a maximum value $\theta = \pi/2$ in the center-of-mass frame, compared to the SM, which predicts a minimum at $\theta = \pi/2$.

Since the magnetic moment for a Majorana neutrino in this process is not diagonal and can have only a transition magnetic moment, if the neutrinos produced are Majorana type, then pairs should be produced with different flavors. This difference gives us an additional way to find out which type of neutrino is produced, Majorana or Dirac. However, it should be noted that most of the present experimental detector systems are such that the neutrinos produced in high-energy collisions escape detection. Hence, for this purpose, we need detecting systems dedicated to high-energy neutrinos, for example, in high-energy cosmic ray experiments.

In summary, we discuss an observational possibility of a neutrino magnetic moment at high-energy experiments and/or high-energy cosmic-ray experiments. Although the present energy scale is found not to be sufficiently high enough for magnetic moments smaller than $\sim 10^{-11}\mu_B$, the neutrino magnetic moment with Pauli coupling can open an interesting channel in future experiments, through which the type of neutrino can be distinguished.

Acknowledgments

The authors would like to thank Byung-Gu Cheon for useful discussions. This work is supported by the World Class University (WCU) project of the Korean Ministry of Education, Science, and Technology (R33-2008-000-10087-0).

[1] Y. Ashie et al. [The Super-Kamiokande Collaboration], Phys. Rev. Lett. 93, 101801 (2004).
[2] B. Aharmim et al. [SNO Collaboration ], Phys. Rev. C 72, 055502 (2005).
[3] T. Araki et al. [KamLAND Collaboration], Phys. Rev. Lett. 94, 081801 (2005).
[4] H. T. Wong and H-B Li, Mod. Phys. Lett. A 20, 1103(2005).
[5] G. G. Raffelt, Phys. Rep. 320, 319(1999).
[6] Per Elmfors, Kari Enqvist, Georg Raffelt, and Gunter Sigl, Nucl. Phys. B 503, 3(1997).
[7] Ashok Goyal, Sukanta Dutta, and S. R. Choudhury, Phys. Lett. B 346, 312(1995).
[8] N. F. Bell, Mikhail Gorchtein, Michael J. Ramsey-Musolf, Petr Vogel, and Peng Wang, Phys. Lett. B 642, 4 (2006).
[9] K. Fujikawa and R. E. Shrock, Phys. Rev. Lett. 45, 963 (1980); M. Dvornikov and A. Studenikin, Phys. Rev. D 69, 073001 (2004) and references therein.
[10] H. K. Lee and Y. S. Yoon, JHEP 0603:078 (2006); H. K. Lee and Y. S. Yoon, JHEP 0703:086 (2007).
[11] W. Pauli, Rev. Mod. Phys. 13, 203 (1941).
[12] A. O. Barut, Z. Z. Aydin, and I. H. Duru, Phys. Rev. D 26, 1794 (1982).
[13] L. M. Sehgal and A. Weber, Phys. Rev. D 46, 2252 (1992).
[14] N. G. Deshpande and K. V. L. Sarma, Phys. Rev. D 43, 943 (1991).
[15] H. K. Lee and Y. Yoon, Mod. Phys. Lett. A 23, 1447 (2009).
[16] $E^\text{GZK}_{\text{CM}}$ is the center of mass energy of UHECR ($E^\text{GZK}_{\text{CM}} \sim 10^{19} \text{ eV}$) and a proton at rest ($E_p = m_p$), $\sqrt{2E^\text{GZK}_{\text{CM}} m_p} \sim 100 \text{ TeV}$.
[17] See, for example, C. L. Ho and P. Roy, Ann. Phys. 312, 161 (2004) and references therein.
[18] N. F. Bell, Int. J. Mod. Phys. A 22, 4891 (2007).