Parametric optimization of pre-stressed steel arch-shaped trusses with ties

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Abstract. An algorithm of searching for the efficient values of the parameters of pre-stressed steel arch trusses with high-strength ties is currently being developed. The task in focus is to reduce the cost of the operating trusses while taking into consideration the strength, rigidness, and stability limitations as per regulatory requirements. It is assumed that a truss shall be fastened to be displaced from its plane by the nodes, and its bars shall generally be subjected to stretching-compression and in-plane bending. The cross-section areas of bars and ties along with the pretension force vary within discrete sets of allowable values. The optimization is performed via a genetic algorithm. The strain-stress state of the structure variants is calculated basing upon the finite element method. The feasibility of the suggested method was illustrated for optimal engineering of a steel truss with a 60 m span, pre-stressed with a double-lay rope.

1. Introduction

Arch-shaped trusses with pre-stressed ties are efficient structural systems essential for long-span buildings and structures. The most important condition for engineering low cost bearing structures of this type is their optimal design. A number of research works [1-6] were dedicated to the optimization of pre-stressed metal structures. For this purpose, research papers [1-4] use mathematical programming methods where optimal design is divided into several stages, where a search is completed at each stage after varying values of a specific group of parameters. Introduction of such stage-by-stage procedures may in many cases distort the conditions of optimization tasks. It should be noted that one of the most universal methods for searching for efficient solutions for optimizing load-bearing systems is the use of metaheuristic computation schemes [5-18] that normally do not require consideration of function derivatives, yet provide the possibility to search for variable parameters within discrete sets, and are efficient for finding global extremums. Particularly, such procedures are represented by genetic algorithms [9,11,18]. Use of metaheuristic schemes for optimal design of pre-stressed steel trusses is shown in the following articles [5,6]. At the same time, these researches also use several optimization stages. This paper presents an algorithm for optimizing steel trusses with pre-stressed ties that performs a simultaneous search both for bar profiles and tie cross-sections along with pre-stress forces. The results are obtained by a single run of the program and may be used directly in design solutions.
2. Formulation of the problem

Let’s consider a pre-stressed flat steel arch-shaped truss with a high-strength tie. We can assume that the truss bars may generally be subjected to tension-compression and plane bending, whereas the ropes are subjected to stretching deformation. We shall assume that the truss nodes are fastened to be displaced away from its plane. Pre-stress is applied to the structure. The sequence of actions on the truss in the form of pre-stress and design loads is considered to be pre-determined. At that, the gravity forces of the structure itself shall be corrected depending on parameter values. The task is to minimize cost $C$ of the operated truss:

$$C\left(\left(D_s, \alpha_s \right), A_1, A_2, \ldots, A_n \right) \Rightarrow \min$$

where $\left(D_s, \alpha_s \right)$ is a pair of numbers determining, respectively, the cross-section diameter of the tie and the portion of its pre-stress force $T_s$ from the breaking force $R_s$ ($T_s = \alpha_s R_s$), $A_j$ is the cross-section area of the $j$th group of bars, within each of which the same cross-section is considered; $n$ is the total number of such groups.

Value $C$ shall include the cost of materials as well as transportation and processing costs for manufacturing the main load-bearing structure of the truss and pre-stressing the object. According to norms [19], we shall consider the following limitations, the compliance with which shall be checked at each of the considered stages of force actions:

A). Bar stress limitations shall be as follows:

$$|\sigma| \leq R_y$$

where $\sigma$ is the normal stress in the bar cross-section, $R_y$ is the design resistance of steel calculated based on the yield limit.

B). Rigidness limitation:

$$|\delta| \leq f$$

where $\delta$ – is the projection of the truss nodal displacement vector on the vertical or horizontal axis, $f$ is the allowable modular value of the movement.

C). Stability of bars.

D). Tie force limitation:

$$0 \leq N_s \leq R_s / k$$

where $N_s$ is the force value, $k$ is the safety factor assumed to be 1.6, $R_s$ is the breaking force of the tie.

3. Optimal search methodology

Within the genetic algorithm, we suppose that one of the genes in a chromosome contains information about the pair realization $\left(A_j, T_j \right)$ number in the set of allowable variants, whereas each of the other genes contains information about the number of the profile for a group of bars in the system of numeration of allowable variants for cross-sections of this group. We shall use the main provisions of the genetic algorithm development strategy presented in researches [20-22]. At that, we shall consider the main population $\Omega_\alpha$ and the elite population $\Omega_\beta$. Population $\Omega_\beta$ serves to store efficient genetic material which is occasionally taken into account in population $\Omega_\alpha$. A mixed selection scheme is used which eliminates distortion in the statement of problem considering the limitations, due to penalty functions. The population $\Omega_\alpha$ is divided into $\Pi_1$ and $\Pi_2$ groups of individuals. If any individual of group $\Pi_1$ does not comply with any of the set limitations, it shall be replaced with an individual from
population $\Omega_b$ not used by the population or by a newly-formed variant of the load-bearing system. If the limitations are not satisfied for an individual from group $\Pi_2$, a penalty is applied by way of multiplying the value of the objective function using a coefficient which depends on the degree of dissatisfaction of the set limitations. A combined mutation scheme is also applied which provides random replacement of parameter values alternating it with the choice of the closest options based on the number in the chromosome and from elements located arbitrarily in the chromosome. Crossover is performed based on a single-point scheme [23].

Limitation check calculation is performed by the finite element method for each accepted structure variant at the pre-stress and loading stages. We shall assume that calculations within some specific actions may be performed via linear formulation. For each $i$th action, incremental vector $\{\Delta i\}$ of nodal displacements shall be applied which shall be considered in the following formula:

$$\{\delta\} = \{\delta\}^{(i-1)} + \{\Delta i\} \quad (i = 1, ..., i_o)$$

(5)

where $\{\delta\}^{(i-1)}$, $\{\delta\}^{(i)}$ are full vectors of nodal displacements resulting from actions $i$ and $i-1$ \cite{delta}, $i_o$ is the number of actions.

We shall assume that after pre-stress, the force in the tie shall take a set value $T_o$ considering losses after anchoring. The calculation step from the structure pre-stressing shall be implemented in two stages. At first, initial test deformation $\varepsilon_o^{(1)}$ in the tie shall be set in a way that results in the occurrence of auxiliary nodal forces $F_o^{(1)}$ at its ends (figure 1). Structure calculation shall be made considering only the mentioned forces followed by calculation of the force $T_o^{(1)}$ in the tie. Then, the actual force calculated from this initial deformation may be determined by using the following formula:

$$T^{(1)} = T_o^{(1)} + F_o^{(1)}$$

(6)

where the forces are considered as algebraic values.

![Figure 1. Auxiliary forces at stage $r$ of pre-stress consideration ($r=1, 2$): $S$ is the tie.](image)

The value of the auxiliary force shall be updated at the second stage:

$$F_o^{(2)} = \left| F_o^{(1)} \right| \frac{T_o}{T^{(1)}}$$

(7)

Then the structure shall be re-calculated. Since the calculations are made in linear formulation, then after stressing the structure with forces $F_o^{(2)}$, we shall have: $T^{(2)} = T_o$. As net load in action $i$ is applied, the force in the tie shall be calculated as follows:

$$T_{(i)} = T_{(i-1)} + T_{(i)}^*$$

(8)

where $T_{(1)}$, $T_{(i-1)}$ are forces in the tie before and after action $i$, $T_{(i)}^*$ is the tension in the tie resulting from action $i$. 
4. Optimization example

The capability of the presented computation scheme can be illustrated with an example describing the optimization of an arch truss with a 60 m span furnished with a tie (figure 2). Net load is represented by the concentrated force system were \( P = 54 \) kN. It was assumed that the bars are made of steel pipes per GOST 32931-2015 “Profile steel pipes for metal constructions. Specifications”. Material of the truss bars: steel VSt3sp per GOST 380-2005 “Common quality carbon steel. Grades”. Ropes per GOST 3081-80 “Two lay rope type LK-O construction 6х19(1+9+9)+7х7(1+6)” were considered. Material properties were accepted as per norms [19] and GOST 3081-80. The value \( R_y = 245 \) N/mm\(^2\) was accepted for the pipes. The modulus of elasticity for the bar materials was set at \( E = 2.06 \cdot 10^5 \) MPa, for rope material – at \( E_s = 1.47 \cdot 10^5 \) MPa. The cost of the structure was estimated using up-to-date prices.

![Figure 2. Pre-stressed arch truss: 1 – 41 are bar numbers.](image)

Allowable parameter values are provided in table 1, where the bars have the indicated outer diameter \( D \) and pipe thickness \( t \), and pair options \((D_s, \alpha_s)\) are given for the rope. The actions were considered as per the following sequence: gravity forces of the structure, pre-stressing, net load. The bars were combined into 12 groups. Grouping of the bars is shown in table 2.

After optimization, the following values were obtained: \( D_s = 34 \) mm, \( \alpha_s = 0.2 \) and pipe cross-sections are shown in table 2. The iteration process convergence graph for this variant of the load-bearing system is shown in figure 3. By the 5905th iteration, value \( C = 289.1 \) thousand rubles was reached, then this value was not corrected until the 10000th iteration.

**Table 1. Allowable values of variates.**

| \( D \times t \) (mm) | \( (D_s, \alpha_s) \) |
|----------------------|----------------------|
| 89×3.5, 89×4, 140×3, 140×3.5, 114×5, 114×5.5, 133×5.5, 152×5.5, 168×6, 168×7, 177.8×8, 219×10, 219×11, 219×13 | (27.5, 0.1), (29.5, 0.1), (31.5, 0.1), (34, 0.1), (35.5, 0.1), (27.5, 0.2), (29.5, 0.2), (31.5, 0.2), (34, 0.2), (35.5, 0.2), (27.5, 0.3), (29.5, 0.3), (31.5, 0.3), (34, 0.3), (35.5, 0.3), (27.5, 0.4), (29.5, 0.4), (31.5, 0.4), (34, 0.4), (35.5, 0.4), (27.5, 0.5), (29.5, 0.5), (31.5, 0.5), (34, 0.5), (35.5, 0.5), (27.5, 0.6), (29.5, 0.6), (31.5, 0.6), (34, 0.6), (35.5, 0.6) |
Table 2. Grouping of bars and obtained pipe cross-sections.

| Group | Numbers of rods | $D \times t$ (mm) |
|-------|-----------------|------------------|
| 1     | 1, 2, 9, 10     | 168×6            |
| 2     | 3, 4, 8, 7      | 168×6            |
| 3     | 5, 6            | 140×3            |
| 4     | 13, 14, 21, 22  | 177.8×8          |
| 5     | 15, 16, 19, 20  | 140×3.5          |
| 6     | 17, 18          | 152×5.5          |
| 7     | 11, 12          | 89×4             |
| 8     | 23, 24, 30, 31  | 89×3.5           |
| 9     | 25, 26, 28, 29  | 89×3.5           |
| 10    | 27              | 114×5            |
| 11    | 32, 33, 34, 39, 40, 41 | 89×4 |
| 12    | 35, 36, 37, 38  | 152×5.5          |

Figure 3. Iteration process convergence graph.

5. Conclusions
A computation procedure was suggested to optimize pre-stressed structures of arch-type steel trusses with ties within discrete sets of allowable variants of bar cross-sections, the cross-section area of ties and their pre-stress forces. The problem solving methodology is based on the use of a genetic algorithm which includes mixed selection and mutation schemes. The possibility was provided to obtain practically significant results within a single calculation process via selecting parameter values with regards to the requirements of standards and construction norms.

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