Method for calculating endurance based on analytical deformations of concrete and reinforcement

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Abstract. The strength of reinforced concrete structures under repeated cyclic loads is determined by the strength of concrete or reinforcement, and depends on the conditions of their joint work, as well as the parameters and loading regime. Depending on many factors fatigue failure of the reinforced concrete bending element can occur from: exhaustion of the normal section resource due to fatigue rupture of the longitudinal main reinforcement, or fatigue failure of concrete in the compressed zone; exhaustion of the resource of inclined sections due to fatigue rupture of transverse reinforcement or fatigue failure of concrete over the vertex of the inclined cracks. The results of the study of the fatigue strength of bent reinforced concrete structures indicate that the most common failure occurs from the exhaustion of the fatigue resource of normal sections. To evaluate the fatigue resource of a normal section of a bending reinforced concrete element, a calculation method is proposed based on analytical diagrams of the concrete and reinforcement deformation. The proposed method of calculation makes possible to evaluate the resource of normal sections of bending elements under cyclic loading.

Key words: cyclic loading, bending reinforced concrete element, normal cross-section, fatigue failure, fatigue resource, deformation diagrams of concrete and reinforcement, ultimate forces, cycle asymmetry coefficient.

1 Introduction

The stress-strain state of reinforced concrete bending elements under repeated cyclic stresses is extremely complex. Repeated cyclic loads cause changes in both the physical and mechanical properties of steel and concrete, as well as the nature of their interaction in reinforced concrete elements, which makes the assessment of the endurance and stress-strain state of the structure significantly complicated [1-5]. The exact solution of the problem of changing the stress-strain state and fatigue strength under cyclic loading requires taking into account a large number of factors, some of which cause opposite effects, and do not overlap with each other [6-10]. Under the action of repeated cyclic loads a continuous change of stress state of reinforced concrete bending elements is understood. It results in increased stress and asymmetry of stress cycle in the stretched reinforcement and the reduction of stress and asymmetry of stress cycle in concrete compressed zone [11-15]. To evaluate the strength of reinforced concrete structures, it is necessary to determine the stress in the reinforcement and concrete at all stages of loading [16-18]. Under the action of cyclic loads, to determine sufficiently accurate stress values in concrete and reinforced concrete element reinforcement in conditions of continuous redistribution of forces between them and the most preferred method for estimating fatigue resource is a deformed calculation model based on analytical diagrams of concrete and reinforcement deformation [19-21].
2 Materials and methods

2.1 General physical ratios for calculating endurance

The General calculation model for calculating the endurance of normal sections of reinforced concrete core elements was developed on the basis of analytical diagrams of concrete and reinforcement deformation. This approach allows calculating the strength, crack resistance and deformations of structures from a single position of view, taking into account the nonlinear properties of materials under short-term static, as well as under various regimes of external cyclic loading.

Figure 1. Scheme of forces, stress and strain diagrams for calculating the normal cross-section endurance based on analytical diagrams of material deformation.

Figure 2. Diagrams of deformation of materials; a – concrete; b, c – reinforcement.
Calculating of the strength of reinforced concrete elements, before the appearance of normal cracks is made on the basis of the following assumptions:
- sections normal to the longitudinal axis of the element are considered;
- the relationship between axial stresses and relative strains of concrete is represented in the form of diagrams $\sigma - \varepsilon$ (figure 2), transformed to account for the influence of cyclic loading and its regimes;
- the relationship between the axial stresses and the relative strains of the reinforcement is taken as diagrams shown in (figure 2);
- general relative strains consist of stress-induced strains and residual vibration-creep strains;
- the distribution of General relative deformations over the section height subject to the hypothesis of flat sections;
- the condition of compatibility of axial deformations of concrete and reinforcement is accepted;
- the calculated moment of normal crack formation is taken as the moment when the concrete strains in the stretched section zone reach the maximum tensile strains of concrete ($\varepsilon_{cr}$).

When calculating elements with normal cracks we proceed from the following assumptions:
- sections normal to the longitudinal axis of the element are considered;
- as calculated normal stresses are assumed in the section with the crack, the work of stretched concrete is not taken into account, and all forces in the stretched zone are transferred to the reinforcement;
- Stresses in the section with a crack are associated with the average relative strains of the element in the areas between the cracks in the diagrams $\sigma_{a-cr}, \sigma_{a-e}$ (figure 2);
- the relationship between axial stresses and relative strains of concrete is represented as diagrams $\sigma - \varepsilon$ (figure 2) transformed to account for the influence of cyclic loading and its regimes;
- the relationship between the axial stresses and the relative strains of the reinforcement is assumed in the form of diagrams shown in (figure 2);
- for average strains of concrete compressed zone and stretched reinforcement the hypothesis of flat sections is considered;
- the condition for compatibility of reinforcement and concrete strains is observed only in the areas between cracks, and in sections with cracks, this condition is replaced by the scheme of coupling of reinforcement according to V.I. Murashev.

The stress cycle asymmetry coefficients $\rho_{bt}$ and $\rho_{s}$ are determined from the current stresses in the concrete of the compressed zone and the stretched reinforcement.

Based on the hypothesis of plane-sections and transformed dependence diagrams $\sigma_{a-cr}$ and $\sigma_{a-e}$ the corresponding strains are used to determine the stresses in concrete and reinforcement. The stresses in concrete $\sigma_{ct}$ and the stresses in rebar $\sigma_{st}$ determine the internal forces in the section for any considered cycle:

$$
N_x = \int_0^X \sigma_b [\varepsilon_b(x_i)] \cdot b(x) dx - \sigma_s (\varepsilon_s) A_s + \int_0^{h_0} \sigma_{bt} [\varepsilon_{bt}(x)] \cdot b(x) dx ;
$$

$$
M_x = \int_0^X \sigma_b [\varepsilon_b(x_i)] b(x) \cdot x_i dx + \sigma_s (\varepsilon_s) A_s (h_0 - x_i) + \int_0^{h_0} \sigma_{bt} [\varepsilon_{bt}(x)] b(x) (h_0 - x_i) dx
$$

(1)

Where $\sigma_b (\varepsilon_b)$ and $\sigma_s (\varepsilon_s)$ – dependences of stress–strain of concrete and reinforcement; $\varepsilon_b(x_i)$ – Regularities of changes in strains along the section height; $b(x)$ – function for changing the width of the cross section by height; $x_i$ – height of the compressed zone for the considered cycle.

Calculation of the internal forces formula is produced by the method of successive approximations, until the condition is met: $|\Delta N_x| \leq \delta$; where $\delta$ is the specified calculation accuracy. The endurance of a reinforced concrete construction at all stages of loading is evaluated based on the condition:

$$
M_x^{max} + \Delta M_{st} + \Delta M_{bt} \leq M_x
$$

(2)
\[ \Delta M_s = \varepsilon_{pl}(N) \cdot \frac{(h_0 - x)^2}{x} \cdot E_s A_s \]

\[ \Delta M_b = \varepsilon_{pl}(N) \frac{h_0 - x}{x} E_s A_s \left[ \frac{1}{A_{ud}} - \frac{S_{ud}}{A_{ud}} \left( \frac{h}{A_{ud}} - \frac{S_{ud}}{A_{ud}} \right) \right] \cdot x \cdot x_p - \frac{2b(x)}{3} \]

Where \( M^\text{max}_i \) is the bending moment from the maximum value of the external load cycle in the in the considered block; \( \Delta M_s \) – additional bending moment due to the occurrence and development of residual strains in the reinforcement; \( \Delta M_b \) – additional bending moment due to additional stresses in the concrete of the compressed zone; \( x_p \) – height of the compressed zone during unloading; \( A_{ud}, S_{ud}, J_{ud} \) – accordingly, the area, static moment, and moment of inertia of the reduced section. The current values of stress cycle asymmetry coefficients in compressed zone concrete (\( \rho_{bt} \)) and in longitudinal stretched reinforcement (\( \rho_{st} \)) at the considered time \( t \) are represented as:

\[ \rho_{bt} = \frac{M^\text{max}_i \cdot p_M + M_b}{M^\text{max}_i + \Delta M_b} \]  
(3)

\[ \rho_{st} = \frac{M^\text{max}_i \cdot p_M + \Delta M_s}{M^\text{max}_i + \Delta M_s} \]  
(4)

where \( \rho = \frac{M^\text{min}_i}{M^\text{max}_i} \). Eqs (1), (2), (3), and (4) is fairied for all stages of the stress-strain situation of the element, including the stage of fatigue failure. The endurance of a normal section is considered to be provided if condition is satisfied Eq (2).

3 Results and discussions

3.1 Analytical dependence to describe the deformation diagrams of concrete

To describe the diagrams of concrete deformation under cyclic loading, the initial diagrams \( \sigma - \varepsilon \) are used for the case of a single short-term static loading. There are various proposals for the approximation of the specified diagram.

The most applicable dependencies are V.N. Baykov, S.V. Gorbatov, and Z.A. Dmitrov:

\[ \sigma_b = A \frac{\varepsilon_b}{\varepsilon_{bR}} + B \left( \frac{\varepsilon_b}{\varepsilon_{bR}} \right)^2 + C \left( \frac{\varepsilon_b}{\varepsilon_{bR}} \right)^3 + D \left( \frac{\varepsilon_b}{\varepsilon_{bR}} \right)^4 + F \left( \frac{\varepsilon_b}{\varepsilon_{bR}} \right)^5 \]  
(5)

Where \( A, B, C, D, F \) – are constants defined from boundary conditions. Dependencies of, V.Ya. Bachinsky and A.I. Bambura:

\[ \sigma_b = 2,65 \frac{\varepsilon_b}{\varepsilon_{bR}} - 2,2 \left( \frac{\varepsilon_b}{\varepsilon_{bR}} \right)^2 + 0,6 \left( \frac{\varepsilon_b}{\varepsilon_{bR}} \right)^3 - 0,05 \left( \frac{\varepsilon_b}{\varepsilon_{bR}} \right)^4 \]  
(6)

N.I. Karpenko, T.A. Mukhamedieva and A.I. Petrov:

\[ \sigma_b = \varepsilon_b E_b V_b \]  
(7)

P.O. Krasnovsky, I.S. Krol, S.A. Tikhomirov:

\[ \frac{\sigma_b}{\varepsilon_{bR}} = 1 - \left( 1 - \frac{\varepsilon_b}{\varepsilon_{bR}} \right)^k \]  
(8)

M. Sarjina recommended by the Euro-international Committee on concrete /EKB-FIP/ for the calculation of reinforced concrete construction:

\[ \frac{\sigma_b}{\varepsilon_{bR}} = \frac{k \left( \frac{\varepsilon_b}{\varepsilon_{bR}} \right) - \left( \frac{\varepsilon_b}{\varepsilon_{bR}} \right)^2}{1 + (k - 2) \left( \frac{\varepsilon_b}{\varepsilon_{bR}} \right)} \]  
(9)

S. Yu. Zeitlin:

\[ \frac{\sigma_b}{\varepsilon_{bR}} = k \frac{\varepsilon_b}{\varepsilon_{bR}} \cdot \left( 1 - \frac{k}{4} \cdot \frac{\varepsilon_b}{\varepsilon_{bR}} \right) \]  
(10)
Analysis of the results of numerical experiments shows that different methods of describing the concrete diagram \( \sigma - \varepsilon \) under compression lead to almost little different results when evaluating the strength of normal sections of reinforced concrete bending elements under static loading. Therefore, when choosing the initial basic function \( \sigma - \varepsilon \) proceeded from the following requirements:

- Ease of analytical connection;
- Ability to transform the dependence to account for various factors, such as vibration-creep, continuous changes in strength and stress-strain situation, loading regimes;
- The content of the minimum number of normalized characteristic points.

Based on the above, dependencies are assumed to be the original base function, EKB – FIP (figure 2):

\[
\frac{\sigma_b}{R_b} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta} \quad (11)
\]

Where \( \eta = \frac{\varepsilon_b}{\varepsilon_{br}}, k = \frac{E_K R_{br}}{R_b} \).

N.I. Karpenko, T.A. Mukhamedieva, A.N. Petrov:

\[
\sigma_{b0} = \varepsilon_{b0} \frac{E_{b0} V_b}{b,rep} \quad (12)
\]

Where \( V_b = \tilde{V}_{bm} \pm (V_0 - \tilde{V}_m) \sqrt{1 - \omega_1 m \eta - \omega_2 m \eta^2} \) on the ascending branches of the diagram \( V_0 = 1, \omega_1 m = 2, 0 - 2,5 \tilde{V}_m \) on the descending branch of the diagram \( V_0 = 2,05 \tilde{V}_m, \omega_1 m = 1,95 \tilde{V}_m - 0,138; m = b, bt; \tilde{V}_{bt} = 0,6 \cdot (1 + 0,1 \frac{R_{bt}}{1MPa}); \tilde{V} = \) the value of \( V_m \), with \( \sigma_m = \tilde{\sigma}_m (\tilde{\sigma}_m = \sigma_b = R_b) - \) at the vertex of the diagram); \( \omega_2 m = 1 - \omega_1 m \).

The numerical values of the relative concrete strain at the top of the diagram are calculated using the Formula:

\[
\dot{\varepsilon}_b = \frac{a + b}{b - cR} \cdot \frac{62R + 0,68R^2 + 22}{7R + R^2 + 22} \quad (13)
\]

Where \( R \) is assumed to be \( \bar{R}_b, \overline{R}_b \) or \( R_{b,rep} \) respectively, for the average experimental diagram, diagrams for calculations for the first and second groups of limit situation; \( \lambda, a, b, c - \) empirical coefficients; for heavy concrete \( \lambda = 1, a = 18, b = 53000, c = 62. \)

### 3.2 Diagram of concrete compression under cyclic loading of stationary regime

Analytical dependencies for describing concrete strain diagrams under stationary repeated cyclic loading are obtained by transformation of the source diagrams under short-term static loading. The transformed diagram is assumed to look similar to the source diagram, taking into account the following additional provisions (figure 3). The parameters of the main node point of the diagram are taken as stresses in concrete equal to the endurance limit \( \sigma_{b,rep} = \varepsilon_{b,rep} \); for an additional nodal point that defines the boundaries of the diagram, the strains are assumed to be equal to the limit strains under static loading \( \varepsilon_{b,rep} = \varepsilon_{b0} \) and the stresses when using the FIP-EKB diagram are calculated by the dependencies Eq. (14). The coordinates of the beginning of the diagrams are assumed to be variables, namely, shifted by an amount equal to the vibration creep strain at the time in Eq. (14):

\[
\varepsilon_{pl}(N) = c(t, \tau) \cdot \sigma_{pl}^{max}(t, t_b) \cdot f(N). \quad (14)
\]

The angle of inclination of the diagrams takes into account the change in the elastic modulus of concrete under cyclic loading. It follows from the above that each loading cycle is implemented with its own deformation diagram, taking into account the influence of previous cycles. Imagine a graphical change in the diagram \( \sigma - \varepsilon \) under cyclic loading. To do this, a third axis, which takes into account the number of the loading cycle, is added to the two normal coordinates, \( \sigma \) and \( \varepsilon \). Then a surface formed by a continuous series of parallel planes will appear. Each of them will have a graph of the dependence \( \sigma - \varepsilon \) only for a specific loading cycle (figure 4). Vertices of transformed diagrams constructed for different loading cycles are designed for a common line-the concrete endurance line \( \sigma - N \) and the starting points of these diagrams form the vibration creep curve \( \varepsilon_{pl} - N \). Dependence between the vertex coordinate of the diagram and the number of loading cycles and the stress cycle asymmetry coefficient is described by Eq. (14). The concrete vibration creep deformations at the time
under consideration are calculated using the Eq. (14). In this case, the strains at the characteristic points of the transformed diagrams are assumed to be equal (figure 3).

\begin{align*}
\varepsilon_{bR}(t, t_0) &= \varepsilon_{bR} - \varepsilon_{pl}(N); \\
\varepsilon_{bu}(t, t_0) &= \varepsilon_{bu} - \varepsilon_{pl}(N)
\end{align*}

Where \( \varepsilon_{bR} \) is the strains at the vertex of the original base diagram under static loading; \( \varepsilon_{bu} \) – limit strains under static loading.

4 Conclusion
The deformational method for calculating the endurance of normal cross-sections of reinforced concrete core elements under cyclic loading based on analytical diagrams of concrete and reinforcement deformation is developed. It takes into account the actual stress-strain state of the cross-sections. Stress and stress cycle asymmetry coefficients in concrete of the compressed zone and longitudinal stretched reinforcement are calculated taking into account their changes in the process of

Figure 3. Transformed concrete deformation diagrams under stationary cyclic loading.

Figure 4. Transformed concrete deformation diagrams at various stages of work.

\begin{align*}
\varepsilon_{bR}(t, t_0) &= \varepsilon_{bR} - \varepsilon_{pl}(N); \\
\varepsilon_{bu}(t, t_0) &= \varepsilon_{bu} - \varepsilon_{pl}(N)
\end{align*}
cyclic loading due to the manifestation of vibration creep of concrete of the compressed zone under related conditions. The endurance of normal cross sections is estimated based on the initial and transformed diagrams of concrete and reinforcement deformation. The presented calculation method allows us to estimate with high accuracy the stress-strain state and the endurance of normal cross sections in rod reinforced concrete bending elements at all stages of cyclic loading.

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