Theoretical study on fatigue cumulative damage model of nanometer concrete

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Abstract. In order to follow-up nano concrete freeze-thaw damage model can have a solid theoretical foundation, this paper elaborates the concrete damage theory and fatigue damage model, and analyzes the characteristic and law of development of concrete fatigue cumulative damage, in order to fully understand the deteriorating law of the fatigue damage of concrete freezing and thawing, guide the establishment of concrete freeze-thaw damage model.

1. Freeze-thaw cycle test of nanometer concrete
The 100×100×400mm prism specimen was used in the freeze-thaw cycle test. When the concrete age reached 24 days, the specimen was immersed in 20±2°C clean water for 96h. When the concrete age reached 28 days, the specimen was taken out to test the initial mass and initial dynamic elastic modulus. Through calculation and analysis, it can be seen that the influence of the dosage of nano-materials on the dynamic elastic modulus and mass loss rate of concrete after freeze-thaw, as shown in Figure. 1 and Figure. 2.

Figure.1 Effect of nano materials on the dynamic elastic modulus of concrete after freezing and thawing
2. Concrete fatigue cumulative damage model

An appropriate cumulative damage model is the basis for describing the development process of cumulative fatigue damage of structures or components under cyclic loading, and for anti-fatigue design of structures or life assessment of structures. At present, the cumulative damage model of materials is studied mainly through fatigue test and the method of building the cumulative damage model based on the continuum damage mechanics theory model. Most of the theories and models used to describe the fatigue cumulative damage of concrete materials are based on the theory and calculation model of fatigue cumulative damage of homogeneous materials.

Up to now, many theories and calculation models of fatigue cumulative damage have been proposed. Because of the complexity of the fatigue cumulative damage of concrete materials, the unified theory and calculation model have not been formed. After reading the relevant literature, this paper makes a brief evaluation of the cumulative damage model.

3. Cumulative damage model based on cycle cycle or cycle life ratio

In 1924, Palmgren first proposed the linear fatigue cumulative damage criterion based on cyclic cycle ratio, namely Palmgren-Miner linear cumulative damage criterion. In 1945, when Miner studied the fatigue cumulative damage of aluminum alloy, he also assumed that the cumulative damage of the material was linearly related to the number of cyclic stresses, and that the cumulative damage value of the material was 1, it was considered that the fatigue damage of the material occurred. The Miner's Rule defines cumulative fatigue damage of materials as:

$$ D = \frac{n}{N_f} $$

(1)

When the material is subjected to multiple levels of different stress amplitudes, its damage D under fatigue failure is:

$$ D = \sum \Delta D_i = \sum \frac{n_i}{N_{f_i}} $$

(2)
Palmgren-Miner’s linear cumulative damage criterion is described by formula, which simplifies the fatigue damage process of materials, and the model is very simple. However, the damage criterion does not consider the relationship between material damage and load state, the relationship between cumulative damage and loading sequence and times, and the interaction between various loads.

Given the problems with the Miner Rule, Manson\cite{1}, it is considered that there are two stages in the process of cumulative fatigue damage: crack initiation and crack propagation. It is assumed that the Miner linear cumulative damage model is not suitable for describing the whole process of material fatigue damage, but it still conforms to the linear cumulative damage law for two different damage stages. Therefore, based on the linear fatigue cumulative damage model and test data, Manson established a bilinear fatigue cumulative damage model by defining damage variables with cyclic cycle ratio. Using Manson damage curve model (DCA), the nonlinear fatigue cumulative damage model under secondary loading can be obtained. Its expression is:

\[
\frac{n_2}{N_{f1}} = 1 - \left( \frac{n_1}{N_{f1}} \right)^{\left( \frac{N_{f1}}{N_{f2}} \right)^{0.4}} = 1 - D
\]

That is:

\[
D = \left( \frac{n_1}{N_{f1}} \right)^{\left( \frac{N_{f1}}{N_{f2}} \right)^{0.4}}
\]

The method for calculating the two-stage turning point of Manson bilinear cumulative damage model (DLDR) can be expressed as follows:

\[
\left( \frac{n_1}{N_1} \right) = 0.35 \left( \frac{N_1}{N_2} \right)^{0.25}
\]

\[
\left( \frac{n_2}{N_2} \right) = 0.65 \left( \frac{N_1}{N_2} \right)^{0.25}
\]

Manson model considers the relationship between damage and loading sequence, and the form is simple. The data results of the two-stage loading test have a high degree of agreement, but whether the multi-stage or random loading is consistent with the actual material damage has not been confirmed. In addition, the two-stage damage process has not been accurately defined. Just like the Miner damage model, it ignores the interaction between loads, lacks clear physical meaning and is difficult to determine the demarcation point of the two stages, which is not convenient for direct application in practical engineering.

Marco-Starkey\cite{2} the damage model is a nonlinear fatigue cumulative damage model which changes with the cycle ratio in a power function relationship. Under constant amplitude loading, the fatigue damage D of the material is defined as:

\[
D = \left( \frac{n}{N} \right)^{C_i} \quad (C_i > 1)
\]

4. Fatigue accumulative damage model based on macroscopic property parameters of materials

In order to better reflect the fatigue damage behavior of concrete material, using the damage equivalent principle and irreversible damage mechanism assumption, such as selecting the parameters of macroscopic measurable easy access to (such as deformation, strength, stiffness and elastic modulus,) to construct the damage variable to characterization of damage mechanism of concrete internal irreversible change and quantitatively describe the damage evolution behavior, and build up in
the process of constant amplitude and variable amplitude fatigue damage degradation model of macroscopic mechanical parameters to estimate the residual fatigue life of concrete or intensity.

(1) Fatigue cumulative damage model based on residual strain method

Fatigue residual strain of concrete represents the degree of irreversibility of micro-damage (plastic deformation and micro-crack). Gao Haijing defined the relative residual strain as the damage variable based on the residual deformation of concrete. Combined with the uniaxial compression fatigue load test data of concrete for regression analysis, Gao Haijing established the low-cycle uniaxial compression fatigue cumulative damage model of high-strength concrete after different temperature processes[3]; Based on the influence of residual deformation and stiffness degradation on concrete fatigue damage, scholar Wei Jun et al. Introduced residual deformation influence factor and deformation modulus damage factor with clear physical significance, and established concrete fatigue damage constitutive model[4].

(2) Fatigue cumulative damage model based on elastic modulus method

The fatigue cumulative damage model established based on the elastic modulus method, the damage variable is defined as:

$$D = 1 - \frac{E_d}{E}$$

(8)

(3) Fatigue cumulative damage model based on energy dissipation method

Due to the application of basic theories such as energy dissipation theory, materials will be damaged to a certain extent in a single cycle under cyclic loading, and the damage will lead to the dissipation of energy inside the material[5]. Miroslaw[6] Based on the principle of conservation of energy, it is considered that the fatigue damage of concrete is accompanied by energy release. On this basis, energy dissipation is proposed to define the damage variable of concrete, namely:

$$D = \frac{E}{E_{tot}}$$

(9)

(4) The fatigue cumulative damage model was constructed based on the maximum fatigue strain damage variable

Compared with the damage variable defined by the residual strain of concrete, the maximum fatigue should be changed to represent the degree of internal microcrack propagation of concrete under the upper fatigue load. Based on the damage variable defined by the fatigue maximum strain method, its expression is as follows:

$$D = \frac{\varepsilon_{\max}^n - \varepsilon_{\max}^0}{\varepsilon_{\max}^f - \varepsilon_{\max}^b}$$

(10)

(5) Fatigue cumulative damage model based on residual strength method

Scholars He Tianqin according to concrete after fatigue load and coupled action of freezing and thawing damage deterioration law of the residual flexural tensile strength is pointed out that the degradation of flexural strength can be described well road surface and internal damage law of coagulation, and define it as damage variable to study the pavement concrete in the fatigue load and under the coupled action of freezing and thawing cumulative damage rule[7]. The damage variable established based on the residual strength method is expressed as:

$$D = 1 - \frac{\sigma}{f_c}$$

(11)

(6) Fatigue cumulative damage model based on ultrasonic velocity method

Because concrete is affected by the external action (load action, environmental temperature change.) will cause its internal organizational structure to have obvious changes, such as micro-crack initiation and expansion, cavity formation and lattice dislocation. In this case, the ultrasonic wave velocity can
be measured by nondestructive testing instruments, and the damage variable can be defined, and the corresponding cumulative damage model can be established to study the deterioration degree of concrete material. Based on the damage variable of ultrasonic wave velocity method, its expression is as follows:

\[ D = \frac{V_{fu}^2}{V_T^2} \]  

(12)

To sum up, the above damage variables can reflect the process and law of nonlinear fatigue damage of concrete to a certain extent. Definition of concrete fatigue damage variable has not form a unified, clear macroscopic measurable parameters, as long as the damage variable can be adopted in accordance with the concept of damage mechanics, better characterization of the fatigue damage mechanism of concrete material internal and degradation, and convenient for engineering application can establish corresponding cumulative damage model.

5. Fatigue accumulative damage model based on continuum damage mechanics

Chaboche[8] based on the theory of continuous damage mechanics and irreversible thermodynamics, the nonlinear fatigue cumulative damage model is established, which is one of the representatives of the theory of continuous fatigue cumulative damage. The theory assumes that the amount of fatigue damage \( D \) is related to the strain in the microplastic zone inside the material. In each cycle, the material damage increment is:

\[ \frac{dD}{dN} = \left[ 1 - (1 - D)^{1+\beta} \right]^{\alpha} \left[ \frac{\Delta \sigma}{M (1 - D)} \right]^\beta \]  

(13)

According to Equation (13), we can get:

\[ D = 1 - \left\{ 1 - \left( \frac{N}{N_F} \right)^{1-\alpha} \right\}^{1+\beta} \]  

(14)

In the formula: \( N_F \) is the limit cycle number of material fatigue failure. namely:

\[ N_F = N_F \left( \Delta \sigma, \sigma_m \right) - (1 - \alpha)^{-1} \left( 1 + \beta \right)^{-1} \left( \frac{\Delta \sigma}{M} \right)^{-\beta} \]  

(15)

The model is a damage evolution equation for the residual life of the material, which takes into account the stress history, the number of cycles and the material parameters related to temperature. It can be used to predict the residual life and strength of materials under simple loads. It is not suitable for predicting the residual life and residual strength of materials under complex loads. Because the model is too complicated, the theory is strong, the model parameter is difficult to determine, so it is not easy to apply in the engineering practice.

6. Probabilistic fatigue cumulative damage model

The initial defect randomness, defect derivation randomness, load action randomness, damage and fracture randomness, as well as the time-varying and randomness of the static load strength of concrete materials are caused by the construction factors of heterogeneous and non-uniform concrete materials[9], which endows the randomness and discretization of the fatigue damage of concrete. Therefore, it is not rigorous enough to consider the fatigue damage (such as micro-cracks, pores and other defects) inside the concrete material in isolation. Based on the theory of probability, the fatigue cumulative damage of concrete is described, and the fatigue damage mechanism and inherent discreteness of concrete are described macroscopic. The fatigue cumulative damage model of concrete
established by this method is more objective in theory than that established by the deterministic method.

7.Conclusion
By summarizing and analyzing the common concrete fatigue cumulative damage models, this paper gives the common understanding and the difference understanding. The present research status of concrete fatigue cumulative damage model and the macroscopic measurable parameters related to the construction of damage variables are summarized systematically, and the simple fatigue cumulative damage model is classified, which lays the foundation for the establishment of the freeze-thaw cumulative damage model in the following paper. Through comparative analysis, the fatigue damage theory describing the rule of cumulative damage of materials is generally established on the basis of tests and theoretical analysis. Therefore, this paper decided to adopt the fatigue damage theory based on the continuum damage mechanics to establish the freeze-thaw damage model of concrete.

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