SPIN POLARIZATION PHENOMENA IN DENSE NUCLEAR MATTER

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Abstract

Spin polarized states in nuclear matter with an effective nucleon-nucleon interaction are studied for a wide range of isospin asymmetries and densities. Based on a Fermi liquid theory, it is shown that there are a few possible scenarios of spin ordered phase transitions: (a) nuclear matter undergoes at some critical density a phase transition to a spin polarized state with the oppositely directed spins of neutrons and protons (Skyrme SLy4 and Gogny D1S interactions); (b) at some critical density, a spin polarized state with the like-directed neutron and proton spins appears (Skyrme SkI5 interaction); (c) nuclear matter under increasing density, at first, undergoes a phase transition to the state with the opposite directions of neutron and proton spins, which goes over at larger density to the state with the same direction of nucleon spins (Skyrme SkI3 interaction).

The issue of spontaneous appearance of spin polarized states in nuclear matter is a topic of a great current interest due to its relevance in astrophysics. In particular, the scenarios of supernova explosion and cooling of neutron stars are essentially different, depending on whether nuclear matter is spin polarized or not. On the one hand, the models with the effective nucleon-nucleon (NN) interaction predict the occurrence of spin instability in nuclear matter at densities in the range from \( \rho_0 \) to 6\( \rho_0 \) for different parametrizations of the NN potential [1]–[4] (\( \rho_0 = 0.16 \text{ fm}^{-3} \)). On the other hand, for the models with the realistic NN interaction, the ferromagnetic phase transition seems to be suppressed up to densities well above \( \rho_0 \) [5]–[7].

Here the issue of spin polarizability of nuclear matter is considered with the use of an effective NN interaction. The main objective is to study the possible scenarios of spin ordered phase transitions in dense nuclear matter with Skyrme and Gogny forces. In particular, we choose Skyrme SLy4 effective interaction, constructed originally to reproduce the results of microscopic neutron matter calculations [8]. We utilize Skyrme SkI3 and SkI5 parametrizations as well, giving a correct description of isotope shifts in neutron-rich medium and heavy nuclei [9]. Besides, we employ Gogny D1S interaction, widely used in nuclear structure calculations. The basic formalism is presented in detail in Ref. [4]. We are interested in studying spin polarized states with like-directed and oppositely directed spins of neutrons and protons. One should solve the self-consistent equations for the coefficients \( \xi_{00}, \xi_{30}, \xi_{03}, \xi_{33} \) in the expansion of the single particle energy in Pauli matrices in spin and isospin spaces

\[
\begin{align*}
\xi_{00}(p) &= \varepsilon_0(p) + \bar{\varepsilon}_{00}(p) - \mu_{00}, \quad \xi_{30}(p) = \bar{\varepsilon}_{30}(p), \\
\xi_{03}(p) &= \bar{\varepsilon}_{03}(p) - \mu_{03}, \quad \xi_{33}(p) = \bar{\varepsilon}_{33}(p).
\end{align*}
\]

Here \( \varepsilon_0(p) \) is the free single particle spectrum, \( \mu_{00} \) and \( \mu_{03} \) are half of a sum and half of a difference of neutron and proton chemical potentials, respectively, and \( \bar{\varepsilon}_{00}, \bar{\varepsilon}_{30}, \bar{\varepsilon}_{03}, \bar{\varepsilon}_{33} \)
are the Fermi liquid (FL) corrections to the free single particle spectrum, related to the normal FL amplitudes \( U_0(k), ..., U_3(k) \) by formulas

\[
\bar{\epsilon}_{00}(p) = \frac{1}{2\sqrt{3}} \sum_q U_0(k) f_{00}(q), \quad \bar{\epsilon}_{30}(p) = \frac{1}{2\sqrt{3}} \sum_q U_1(k) f_{30}(q), \quad \mathbf{k} = \frac{\mathbf{p} - \mathbf{q}}{2},
\]

\[
\bar{\epsilon}_{03}(p) = \frac{1}{2\sqrt{3}} \sum_q U_2(k) f_{03}(q), \quad \bar{\epsilon}_{33}(p) = \frac{1}{2\sqrt{3}} \sum_q U_3(k) f_{33}(q).\]

The distribution functions \( f_{00}, f_{03}, f_{30}, f_{33} \), in turn, can be expressed in terms of the components \( \xi \) of the single particle energy and satisfy the normalization conditions for the total density \( \varrho_n + \varrho_p = \varrho \), excess of neutrons over protons \( \varrho_n - \varrho_p \equiv \alpha \varrho \), ferromagnetic (FM) \( \varrho_\uparrow - \varrho_\downarrow \equiv \Delta \varrho_\uparrow \downarrow \) and antiferromagnetic (AFM) \( \varrho_{n\uparrow} + \varrho_{p\downarrow} \equiv \Delta \varrho_{n\uparrow} \downarrow \) spin order parameters, respectively (\( \alpha \) being the isospin asymmetry parameter, \( \varrho_\uparrow = \varrho_{n\uparrow} + \varrho_{p\uparrow} \) and \( \varrho_\downarrow = \varrho_{n\downarrow} + \varrho_{p\downarrow} \), \( \varrho_{n\uparrow}, \varrho_{n\downarrow} \) and \( \varrho_{p\uparrow}, \varrho_{p\downarrow} \) being the neutron and proton number densities with spin up and spin down). The quantities of interest are the neutron and proton spin polarization parameters \( \Pi_n = \frac{\varrho_{n\uparrow} - \varrho_{n\downarrow}}{\varrho_n}, \Pi_p = \frac{\varrho_{p\uparrow} - \varrho_{p\downarrow}}{\varrho_p} \), characterizing spin ordering in neutron and proton subsystems.

Fig. 1a shows the density dependence of the neutron and proton spin polarization parameters at zero temperature for SLy4 force. The main qualitative feature is that for SLy4 force there are only solutions corresponding to the oppositely directed spins of neutrons and protons in a spin polarized state. The reason is that for SLy4 force the FL amplitude \( U_1 \), determining spin–spin correlations, is repulsive for all relevant densities, while the FL amplitude \( U_3 \), describing spin–isospin correlations, becomes quite attractive at high densities. The critical density of spin instability in symmetric nuclear matter \( (\alpha = 0) \), corresponding to AFM spin ordering \( \Delta \varrho_{n\uparrow} \downarrow \neq 0, \Delta \varrho_{n\uparrow} \downarrow = 0 \), is \( \varrho_c \approx 0.33 \text{ fm}^{-3} \). It is less than the critical density of FM instability in neutron matter, \( \varrho_c \approx 0.59 \text{ fm}^{-3} \). Even small admixture of protons to neutron matter leads to the appearance of long tails in the density profiles of the neutron spin polarization parameter near the transition point to a spin ordered state. As a consequence, a spin polarized state is formed much earlier in density than in pure neutron matter.

As seen from Fig. 1b, for SkI5 force, oppositely to SLy4 force, there are only solutions corresponding to the same direction of neutron and proton spins in a polarized state. In the case under consideration the FL amplitude \( U_3 \) is repulsive for all relevant densities, while the FL amplitude \( U_1 \) becomes quite attractive at high densities. For SkI5 force, a phase transition to the FM spin state in neutron matter takes place at the critical density \( \varrho_c \approx 0.28 \text{ fm}^{-3} \). It is less than the critical density of spin instability in symmetric nuclear matter \( \varrho_c \approx 0.43 \text{ fm}^{-3} \), corresponding to FM spin ordering \( \Delta \varrho_{n\uparrow} \downarrow \neq 0, \Delta \varrho_{n\uparrow} \downarrow = 0 \). There are no long tails in the density profiles of the neutron spin polarization parameter at large isospin asymmetry. In the given case, a small admixture of protons to neutron matter even leads to the increase of the critical density of spin instability.

Fig. 2 shows the neutron and proton spin polarization parameters as functions of density at zero temperature for SkI3 force. There are two types of solutions of the self-consistent equations in symmetric nuclear matter, corresponding to FM and AFM ordering of neutron and proton spins. Due to proximity of FL amplitudes \( U_1 \) and \( U_3 \), the respective critical densities are very close to each other \( \varrho_c \approx 0.910 \text{ fm}^{-3} \) for FM ordering and \( \varrho_c \approx 0.917 \text{ fm}^{-3} \) for AFM ordering) and larger than the critical density of spin instability in neutron matter \( \varrho_c \approx 0.37 \text{ fm}^{-3} \). When some admixture of protons is added to neutron
Figure 1. Neutron and proton spin polarization parameters as functions of density at zero temperature for (a) SLy4 force and (b) SkI5 force.

matter, the last critical density is shifted to larger densities and a spin polarized state with the oppositely directed spins of neutrons and protons appears. Under increasing density of nuclear matter, the neutron spin polarization continuously increases till all neutron spins will be aligned in the same direction. Protons, at first, become more polarized with density and their spin polarization is opposite to the spin polarization of neutrons. But, after reaching the maximum, spin polarization of protons decreases and at some critical density spins of protons change direction, so that the spin ordered phase with the like-directed spins of neutrons and protons is formed. Then, beyond the critical density, the spin polarization of protons is continuing to increase until the totally polarized state with parallel ordering of neutron and proton spins will be formed. Thus, for SkI3 force nuclear matter undergoes at some critical density a phase transition from the state with antiparallel ordering of neutron and proton spins to the state with parallel ordering of spins. With increasing isospin asymmetry, this critical density increases as well. Note that there are no long tails in the density profiles of the neutron spin polarization parameter
Now we present the results of the numerical solution of the self–consistent equations with the D1S Gogny effective force for symmetric nuclear matter. The main qualitative feature is that for the D1S force there are only solutions corresponding to AFM spin ordering and there are no solutions corresponding to FM spin ordering. In Fig. 3, it is shown the dependence of the AFM spin polarization parameter $\Delta \rho_{\uparrow\downarrow}/\rho$ as a function of density at zero temperature. The AFM spin order parameter arises at density $\rho \approx 3.8\rho_0$ for the D1S potential. A totally antiferromagnetically polarized state ($\Delta \rho_{\uparrow\downarrow}/\rho = 1$) is formed at $\rho \approx 4.3\rho_0$. The neutron and proton spin polarization parameters for the AFM spin ordered state are opposite in sign and equal to

$$\Pi_n = -\Pi_p = \frac{\Delta \rho_{\uparrow\downarrow}}{\rho}.$$ 

For comparison, we plot in Fig. 3 the density dependence of the AFM spin polarization parameter for the Skyrme effective forces SkM$^*$ and SGII being relevant for calculations at small isospin asymmetry.

It is necessary to emphasize that different behavior at high densities of the interaction amplitudes, describing spin–spin and spin–isospin correlations, lays behind this divergence in calculations with different effective forces. These results clearly indicate the necessity to construct a new generation of the energy functionals with the properly constrained time-odd part at high densities. Probably, these constraints will be obtained from the data on the time decay of magnetic field of isolated neutron stars [10].

Figure 2. Same as in Fig. 1, but for SkI3 force. Also the curves, corresponding to FM and AFM ordering in symmetric nuclear matter, are shown.

at large asymmetries.
Figure 3. AFM spin polarization parameter as a function of density at zero temperature for the D1S Gogny force and the SkM*, SGII Skyrme forces.

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