Vacuum Polarization in the Spacetime of a Scalar-Tensor Cosmic String

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Abstract

We study the vacuum polarization effect in the spacetime generated by a magnetic flux cosmic string in the framework of a scalar-tensor gravity. The vacuum expectation values of the energy-momentum tensor of a conformally coupled scalar field are calculated. The dilaton’s contribution to the vacuum polarization effect is shown explicitly.
1 Introduction

One of the most interesting features of the spacetime generated by a static, straight axially symmetric cosmic string in General Relativity [1] is that particles and fields are sensitive to its global (conical) structure and, therefore, some physical effects may arise due solely to the conicity of this geometry. In particular, many authors [2] have already considered the vacuum polarization effect in connection with the Casimir effect [3] in which the conducting planes form an angle equal to the deficit angle $\Delta = 8\pi \mu G$ associated with metric (1). In the papers [4, 5] a more general situation has been carried out. Namely, a cosmic string carrying a magnetic flux interacting with a charged scalar field placed in background (1) was considered. In this case, the vacuum polarization effect arises not only in connection with the non-trivial gravitational interaction but also with the Aharonov-Bohm interaction.

It is interesting to notice that all the above mentioned implications of the interactions between a quantum field and a cosmic string have been done in the framework of Einstein’s gravity. However, it has been argued that gravity may not be described by a purely tensorial field $g_{\mu \nu}$. In particular, the existence of a scalar partner $\phi$ (for instance, the dilaton field) for the graviton arises naturally in all attempts to unifying gravity with the other fundamental interactions [6]. Although Einstein’s theory agrees with its experimental tests with an accuracy of one percent or better, this present agreement between
theory and experiments is compatible with the existence of a long-range scalar (gravitational) field: it has been shown that General Relativity acts as an attractor to the scalar-tensor gravities as a consequence of the cosmological expansion which drives the scalar couplings towards zero [7, 8].

In this letter, we are interested in studying the vacuum polarization effect of a charged, (massless) scalar field due to a magnetic flux cosmic string in the framework of the scalar-tensor theories of gravity. For this purpose, we will first present the metric generated by a cosmic string in scalar-tensor gravities [9] and we will show that this metric is conformally flat, to linear order of $G_0\mu$. Then, we compute the vacuum expectation values (v.e.v.) of the components of the energy-momentum tensor of a conformal scalar field. Whenever convenient, we reduce our results to the particular case of the Brans-Dicke gravity and we compare these results with the ones obtained in the framework of Einstein’s gravity. We anticipate that our main result is to derive explicitly the dilaton’s contribution to the vacuum polarization effect.

This work is outlined as follows. In section 2 we recall some results which will be used throughout this paper compute the v.e.v. of the stress-energy tensor for a conformally coupled scalar field. We reduce our results to the particular case of the Brans-Dicke theory of gravity and we present graphs of the energy density for a neutral ($\gamma = 0$) and a twisted ($\gamma = 1/2$) conformal scalar fields in the Brans-Dicke theory and we show that the Aharonov-Bohm interaction is the leading interaction between the scalar field and the magnetic flux cosmic string. Finally, in section 3 we end with some conclusions and discussions on the results of the present work.
2 The Vacuum Polarization Effect in Scalar-Tensor Gravities

The metric of a static, straight axially symmetric cosmic string in scalar-tensor gravity is

\[
\begin{align*}
\mathbf{ds}^2 &= \left[1 + 8G_0\mu\alpha^2(\phi_0)\ln \rho/\rho_c\right] \left[ -dt^2 + dz^2 + d\rho^2 + (1 - 8G_0\mu)\rho^2 d\varphi^2 \right], \\
\end{align*}
\]

(2)

where \( G_0 \) is defined as \( G_0 \equiv G_\ast A^2(\phi_0) \) and \( \alpha(\phi) = \partial \ln A/\partial \phi \) is the coupling between matter and the dilaton field. All quantities here are computed up to first order in \( G_0\mu \). The constant \( \rho_c \) appearing in metric (2) is a constant of integration and is, conveniently, of the same order of magnitude of the string’s radius. \( \phi_0 \) denotes the cosmologically-determined value of the dilaton field far away from the solar system.

Let us define the conformal factor

\[
\Omega \equiv 1 + 4G_0\mu\alpha^2(\phi_0)\ln \rho/\rho_c,
\]

and denoting \( B = 1 - 4G_0\mu \), metric (2) can be re-written as

\[
\mathbf{ds}^2 = \Omega_{\text{lin}}^2 \left[ -dt^2 + dz^2 + d\rho^2 + B_{\text{lin}}^2\rho^2 d\varphi^2 \right].
\]

(3)

\( \Omega_{\text{lin}}^2 \) is the linearised conformal factor; its expression being \( \Omega_{\text{lin}}^2 = 1 + 8G_0\mu\alpha^2(\phi_0)\ln \rho/\rho_c \) and \( B_{\text{lin}}^2 \) is given by \( B_{\text{lin}}^2 = 1 - 8G_0\mu \). Let \( \theta \) be the new azimuthal angle \( \theta = (1 - 4G_0\mu)\varphi \). Then, metric (3) becomes conformally flat with deficit angle equal to \( \Delta \theta = 8\pi \mu G_0 \).

Since metric (3) is conformally flat, we can apply an alternative expression to compute the v.e.v. of the components of the energy-momentum tensor \( < T_\mu^\nu > \), instead of making use of the Green’s functions [3]. Namely,
in the particular case of a conformally coupled scalar field ($\xi = 1/6$ in 4-dimensions), we have

$$< T_{\mu}^{\nu} >_g = (\frac{g}{\bar{g}})^{1/2} < T_{\mu}^{\nu} >_{\bar{g}} - \frac{1}{2880\pi^2} \frac{1}{6} (1) H_{\mu}^{\nu} - (3) H_{\mu}^{\nu};$$

(4)

where

$$(1) H_{\mu\nu} \equiv 2R_{\mu\nu} - 2\bar{g}_{\mu\rho} \Box_{\bar{g}} R - \frac{1}{2} \bar{g}_{\mu\nu} R^2 + 2RR_{\mu\nu}$$

$$(3) H_{\mu\nu} \equiv \frac{1}{12} R^2 \bar{g}_{\mu\nu} - R^{\rho\sigma} R_{\rho\mu\sigma\nu}.$$  

For the seek of clarity, we have denoted metric (3) as $\bar{g}_{\mu\nu}$ in order to distinguish from the metric $g_{\mu\nu}$ of the flat spacetime (1). The term $< T_{\mu}^{\nu} >_g$ appearing in the r.h.s. of expression (4) is the energy-momentum tensor computed with respect to metric (1) and has been already calculated in the ref. [5]

$$< T_{\mu}^{\nu} >_g = \left[ \omega_4(\gamma) - \frac{1}{3} \omega_2(\gamma) \right] \frac{1}{\rho^4} diag(1,1,1,-3),$$

The quantities $\omega_2(\gamma)$ and $\omega_4(\gamma)$ were evaluated by Dowker [4, 10]

$$\omega_2(\gamma) = -\frac{1}{8\pi^2} \left\{ \frac{1}{3} - \frac{1}{2B^2} \left[ 4 \left( \gamma - \frac{1}{2} \right)^2 - \frac{1}{3} \right] \right\},$$

$$\omega_4(\gamma) = -\frac{1}{720\pi^2} \left\{ 11 - \frac{15}{B^2} \left[ 4 \left( \gamma - \frac{1}{2} \right)^2 - \frac{1}{3} \right] \right. \right.$$ 

$$\left. + \frac{15}{8B^4} \left[ 16 \left( \gamma - \frac{1}{2} \right)^4 - 8 \left( \gamma - \frac{1}{2} \right)^2 + \frac{7}{15} \right] \right\}.$$

Both expressions are valid only if $B > 1/2$. $\gamma$ is the fractional part of $\Phi/\Phi_0$, $\Phi_0$ being the quantum flux $2\pi/e$, and lies in the interval $0 \leq \gamma < 1$. The particular values of $\gamma = 0$ and $\gamma = 1/2$ correspond to the cases of a vanishing flux and a twisted field around the axis $\rho = 0$, respectively.
Therefore, for a conformally coupled scalar field in the spacetime \( (3) \), we have, up to second order in \( G_0\mu \):

\[
< T^\mu_\nu >_{\bar{g}} = \left( 1 - 16G_0\mu \alpha^2(\phi_0) \ln \rho/\rho_c + 128G_0^2\mu^2 \alpha^4(\phi_0) \ln^2 \rho/\rho_c \right) < T^\mu_\nu >_g \\
- \frac{1}{15\pi^2 \rho^4} G_0^2 \mu^2 \alpha^4(\phi_0) \text{diag}(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, 1).
\]

The vacuum polarization effect expressed by (5) is a consequence of the conical geometry (non-trivial gravitational interaction), of the Aharonov-Bohm interaction between the quantum scalar field and the magnetic flux string, and of the presence of the dilaton field in this theory. Expression (5) is convenient because it expresses the contribution of the dilaton field to the vacuum polarization effect explicitly. The second term in the r.h.s. of (5) is a contribution due solely to the dilaton in comparison to the first term which is a combination of all the interactions. It is interesting to notice that such a contribution is fully described by one dimensional coupling strength \( G_0 \) and one post-Newtonian parameter \( (\alpha(\phi_0)) \). Finally, we point out that the trace anomaly appears up to second order in \( G_0\mu \).

The Particular Case of the Brans-Dicke Theory

It is very illustrative to consider a particular form for the coupling function \( \alpha(\phi) \), corresponding to the Brans-Dicke theory. Namely, \( \alpha^2 = \frac{1}{2\omega + 3}, (\omega = cte) \). In this case, the metric of a cosmic string is given, to first order in \( G_0\mu \), by \[11]\:

\[
 ds^2 = \left[ 1 + \frac{8\mu G_0}{2\omega + 3} \ln \frac{\rho}{\rho_c} \right] \left[ -dt^2 + dz^2 + d\rho^2 + (1 - 8\mu G_0)\rho^2 d\theta^2 \right].
\]

\(^1\)Expression (5) was obtained with the help of the computer algebra program Maple.
Besides, we have that $G_0 = \left(\frac{3\omega^3 + 3}{2\omega + 4}\right)G$ where $G$ is the Newtonian constant \[12\]. Therefore, expression (5) reduces to:

$$< T^\mu_\nu >_{\bar{g}} = \left[1 - \frac{16G\mu}{2\omega + 4} \ln \rho/\rho_c + \frac{128G^2\mu^2}{(2\omega + 4)^2} \ln^2 \rho/\rho_c\right] \left[\omega_4(\gamma) - \frac{1}{3} \omega_2(\gamma)\right] \frac{1}{\rho^4} \text{diag}(1, 1, 1, -3)$$

$$- \frac{G^2\mu^2}{15\pi^2 \rho^4 (2\omega + 4)^2} \text{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 1).$$

(6)

We can verify that in the limit where $\omega \to \infty$ our result agrees with the one obtained in the framework of General Relativity, as expected. For values of $\omega$ such that $\omega > 2500$ (consistent with solar system experiments made by Very Long Baseline Interferometry (VLBI) \[13\]), we can see that the corrections due to the presence of the dilaton are very small in comparison with the previous situation in General Relativity.

Figures 1 and 2 present the behaviour of the energy density of a massless, conformally coupled ($\xi = 1/6$) scalar field in the particular cases of a vanishing flux ($\gamma = 0$) and a twisted field ($\gamma = 1/2$) in the Brans-Dicke gravity, respectively. We can notice that the Aharonov-Bohm interaction is the leading interaction between the scalar field and the magnetic flux cosmic string.

3 Conclusions

In this work, we have computed the vacuum expectation values of the energy-momentum tensor of a conformally coupled scalar field, by noting that space-time (3) is conformally flat. The expression for this tensor (5) reveals explicitly the dilaton’s contribution to the vacuum polarization effect. As an
example, we considered the particular case of the Brans-Dicke theory and we presented the behaviour of the energy density for a conformally coupled scalar field for both $\gamma = 0$ and $\gamma = 1/2$ cases. The Aharonov-Bohm interaction is the predominant interaction between the (charged) scalar field and the magnetic flux cosmic string, a result which is also valid in the framework of the General Relativity theory [4].

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Figure 1: Graph of $x^2 < T_0$, with $\gamma = 0$ and $\omega = 3000$.
Figure 2: Graph of $\pi^2 < T_{00} >$, with $\gamma = \frac{1}{2}$ and $\omega = 3000$