Optimal shipping schedule for a near-sea liner service route with time windows

Xi Jiang*
School of Transportation Southeast University Nanjing, 21189, China
* Corresponding author: jiangxi178727383@163.com

Abstract—This paper proposes a shipping schedule design problem arising in near-sea liner shipping that aims to determine containership deployment plan, the arrival time at each port of call, and sailing speed for a given liner service route with hard time windows. The problem is transformed into a mixed integer nonlinear nonconvex optimization model, and further transformed into a mixed integer linear optimization model, which can be effectively solved by commercial solvers. The case studies based on a real-life service route show the effectiveness and efficiency of the proposed solution method. Results also demonstrate that the port time windows affect the total cost of a ship route, the optimal number of ships to deploy, and the optimal schedule.

1. INTRODUCTION
Liner container shipping is a modern transportation mode with high efficiency and the main means of transportation in international trade. Compared with dry-bulk shipping, liner container shipping usually carries small quantities of goods with higher value. Although the volume it carries only accounts for 20% to 30% of the total global trade, the value of the goods it carries accounts for 70% to 80%. Similar to public transport services, a container liner service transports containers following a fixed port call sequence and fixed arrival and departure times at each port of call. The railway service transporting customers usually runs with daily frequency, while container liner service needs to maintain weekly service frequency. Berths in a port are not always available. If the port's berths are all busy, a ship that has already arrived at the port has to wait until there is an available berth. Therefore, a ship's berthing time at a certain port is not arbitrary. The available time can be regarded as port hard time windows. Hence, when designing shipping schedule for a service route, port time windows should be taken into account.

The vessel scheduling problem is a tactical level decision problem to determine the containership sailing speed per shipping leg, ship arrival and departure time per port of call, and number of ships deployed on the designed service route. Liner shipping scheduling problem has been receiving a constant attention in academia [1-3]. Fagerholt [4] was the first to raise the issue of soft time windows in tramp shipping scheduling problem with the objective of minimizing the total transportation and inconvenience cost. A set partitioning based algorithm was proposed to solve the problem. Christiansen and Fagerholt [5] treated the cargo time windows as multiple time windows because of limitation in port operating time. To avoid idle times in port during weekends, penalty cost was imposed for arrivals at risky times. Agarwal and Ergun [6] examined the ship scheduling and cargo routing problem in liner shipping network design with consideration of weekly service frequency. Wang and Meng [7] presented the concept of market-level transit time in liner shipping scheduling problem. They designed
an optimal schedule with fixed sailing speed for a shipping network to minimize the total transshipment cost, penalty cost for longer transit time and the bonus for shorter transit time. Wang and Meng [8] examined the liner shipping scheduling problem by taking into account time uncertainties at sea and at port. Wang et al. [9] considered elastic demand in containership scheduling problem. They assumed that the shipping demands vary with transit time from their origin ports to destination ports. Wang et al. [10] discussed the liner ship route schedule design problem with time windows. Wang et al. [11] examined the liner ship schedule design problem to a liner shipping network. Aydin et al. [13] incorporated the waiting costs of early arrivals in sailing speed optimization problem with time windows on a single voyage. Charisis et al. [14] dealt with the liner shipping routing and scheduling problem incorporating port time windows. Tabu search algorithm was designed to solve the problem. Yu et al. [15] examined the speed optimization problem with the fuzzy time window in the field of tramp shipping. A bi-objective model was proposed to jointly minimize the operating cost and shipper’s satisfaction.

The above literature review shows that only a few studies have considered time windows of liner shipping. However, these articles have not considered the case that a ship arrives at a port and leaves the same port within one day, which is very common in the near-sea liner shipping. This is because the volume of loading and unloading containers at the port on a near-sea route is usually small, and it only takes half a day for a ship to finish the loading and unloading operations. Hence, this article considers such a case in the planning decision, which has practical significance for liner shipping companies operating near-sea shipping routes.

2. MATHEMATICAL MODEL
This section presents a mathematical model of the problem that aims to minimize the total cost composed of vessel operating cost, fuel cost, and container inventory cost. We consider a typical near-sea shipping route CPX6 service operated by SITC Container Lines, which includes \( P = \{1,2,\ldots,N\} \) ports of call. The port call order of this service is as follows: 1(Qingdao)-2(Shanghai)-3(Subic Bay)-4(Manila North)-5(Davao)-6(General Santos)-7(Cagayan De Oro)-8(Shanghai)-1(Qingdao). It is noticeable that the port Shanghai is visited twice in two different directions in a round-trip journey. Hence, we directly define two ports of call corresponding to the above situation. The voyage from the \( i \)th port of call to the \( (i+1) \)th is called leg \( i \), and the voyage from \( N \)th port of call back to the first one is called leg \( N \). Let \( D_i \) be the length of leg \( i \). We define \( r_i \in R \) as a kind of sailing speed corresponding to leg \( i \). Let \( F_{r_i} \) and \( T_s \) be the bunker consumption and sailing time that corresponds to the sailing speed \( r_i \) on leg \( i \), respectively. We set the binary variables \( X_{r_i} \in \{0,1\} \) to indicate whether the sailing speed \( r_i \) is used. Let \( \beta \) be the unit bunker price, and \( \alpha \) be the unit inventory cost. Let \( C_{operating} \) represent the fixed weekly operating cost of one container ship, and let the integer variable \( Y \) represent the number of ships deployed on the designed shipping route. We use “half day” as the unit for liner shipping schedule design, and we define the time at 00:00 of a certain Sunday as time 0. Let \( T_{wait} \) be the fixed waiting time at \( i \)th port of call. Let \( T_{arr} \) and \( T_{arr}' \) be the arrival time and the arrival time in the first week at \( i \)th port of call. Let \( \Omega_i \) represent the time windows at port of call \( i \) and \( V_i \) be the number of containers transported on leg \( i \). Thus, the problem can be expressed as follows:

\[
\min \{C_{operating}Y + \beta \sum_{i \in P} \sum_{r \in R} D_i F_{r_i} X_{r_i} + \alpha \sum_{i \in P} \sum_{r \in R} V_i T_{s_i} X_{r_i}\} \tag{1}
\]

subject to:

\[
T_{arr} = T_{arr} + \sum_{i \in P} \sum_{r \in R} T_{s_i} X_{r_i}, \quad \forall i \in P \tag{2}
\]

\[
\sum_{i \in P} \sum_{r \in R} T_{s_i} X_{r_i} \leq 14Y \tag{3}
\]

\[
\sum_{r \in R} X_{r_i} = 1, \quad \forall i \in P \tag{4}
\]
The objective function (1) minimizes the sum of the vessel operating cost, fuel cost, and container inventory cost. Constraint (2) defines the relationship of arrival times between the two adjacent ports of call. Constraint (3) ensures that the shipping service should maintain a weekly frequency. Constraint (4) indicates that only one kind of sailing speed can be selected on each shipping leg. Constraint (5) defines the arrival time at the first port of call. Constraint (6) defines \( Y \) as an integer variable. Constraint (7) defines \( X_i \) as a binary variable. Constraint (8) defines the arrival time at each port of call except the first in the first week. Constraint (9) defines the port time windows at each port of call.

3. SOLUTION METHOD

3.1. Linear constraints excluding infeasible solutions

Since the set \( \Omega \) may be a discrete domain that results in the model nonconvex, we try to add a series of linear constraints that can exclude the infeasible solutions while keeping all other solutions. First, we add the following constraints:

\[
T_i = \tilde{T}_i + 14w_i, \quad i \in P
\]

(10)

\[
1 \leq \tilde{T}_i \leq 14, \quad i \in P
\]

(11)

\[
w_i \in \mathbb{Z}^+, \quad i \in P
\]

(12)

It is easy to see that the above constraints are equivalent to constraint (8). \( T_i \) and \( w_i \) are defined to be integers and, consequently, \( \tilde{T}_i \) is an integer. Then we can rewrite \( \tilde{T}_i \) using binary variables:

\[
\tilde{T}_i = w_i^1 + 2w_i^2 + 4w_i^3 + 8w_i^4, \quad i \in P
\]

(13)

\[
w_i^1, w_i^2, w_i^3, w_i^4 \in \{0, 1\}, \quad i \in P
\]

(14)

Let the vector \((w_i^1, w_i^2, w_i^3, w_i^4, i \in P, j_i \in J_i)\) corresponds to one half day outside the time windows \( \Omega_i \), then, we have:

\[
w_i^1(1 - \tilde{\hat{w}}_i^1) + (1 - w_i^1)\tilde{\hat{w}}_i^1 + w_i^2(1 - \tilde{\hat{w}}_i^2) +
(1 - w_i^2)\tilde{\hat{w}}_i^2 + w_i^3(1 - \tilde{\hat{w}}_i^3) + (1 - w_i^3)\tilde{\hat{w}}_i^3 +
w_i^4(1 - \tilde{\hat{w}}_i^4) + (1 - w_i^4)\tilde{\hat{w}}_i^4 \geq 1
\]

(15)

3.2. Global optimal solution

Now we can obtain a new model with the objective function (1) and constraints (2)-(7), (10)-(15). The developed model is a mixed integer linear programming model that can be addressed directly and efficiently by the state-of-the-art solvers.

4. CASE STUDY

In order to evaluate the applicability of the proposed models and algorithms, we conduct case studies based on the service CPX6 mentioned in section 2. The operating cost \( \text{operating} = \$200,000/\text{week.} \) the bunker price \( \beta = \$300/\text{t,} \) and the unit inventory cost \( \alpha = \$12/\text{half day.} \) The waiting time at each port of call, the length of each shipping leg, and the number of containers on each shipping leg are shown in Table I. The time windows at each port of call are shown in case 1 of Table II.

| Table I. Parameters in the case study |
|---------------------------------------|
| Port ID | Port | Waiting time (half day) | Length (nm) | Containers (TEU) |

3
| Port ID | Qingdao  | 3 | 424.4 | 500 |
|--------|----------|---|--------|-----|
| 2      | Shanghai | 2 | 1,106.6| 1,500|
| 3      | Subic Bay| 1 | 74.5   | 1,300|
| 4      | Manila(N)| 4 | 887.3  | 1,200|
| 5      | Davao    | 2 | 199.3  | 1,100|
| 6      | General Santos | 1 | 509.9  | 1,200|
| 7      | Cagayan De Oro | 1 | 1,496.3| 1,100|
| 8      | Shanghai | 2 | 424.4  | 300 |

**TABLE II. TIME WINDOWS AT EACH PORT OF CALL**

| Port ID | Case 1 | Case 2 |
|---------|--------|--------|
| 1       | free   | free   |
| 2       | free   | free   |
| 3       | free   | free   |

**TABLE III. OPTIMAL ARRIVAL TIME AT EACH PORT OF CALL**

| Port ID | Case 1 | Case 2 |
|---------|--------|--------|
| 1       | 14     | 14     |
| 2       | 22     | 8      |
| 3       | 32     | 4      |
| 4       | 35     | 7      |
| 5       | 48     | 6      |
| 6       | 52     | 10     |
| 7       | 60     | 4      |
| 8       | 69     | 13     |
| 1       | 84     | 14     |
| Case 2  | 2      | 2      |
| 3       | 19     | 34     |
| 4       | 21     | 38     |
| 5       | 34     | 45     |
| 6       | 52     | 58     |
IBM ILOG CPLEX 12.8.0 programmed by yalmip toolbox in MATLAB on the personal computer with Intel Core i5 2.5 GHz CPU and 16 GB RAM is called to solve the developed mixed integer linear programming model. The test can be completed within 5 seconds.

From results of case 1 in Table III, we can see that the arrival time at each port of call in the first week does not violate any port time window constraints. The solution is feasible and optimal to the developed model, and thereby the original one. We subsequently investigate how the variation of port time windows at each port influences the optimal sailing speed, the number of ships deployed on the designed route, and the total cost. From case 1 to case 3, more and more available times are provided as shown in Table II. The results are shown in Tables III and IV. It is noticeable that different settings of port time windows affect the sailing speeds on shipping legs and the arrival times at ports. The results also demonstrate that more relaxed time windows at each port lead to a lower total cost and less ships deployed on the designed route.

5. CONCLUSIONS AND FUTURE WORK

This paper examines the liner shipping schedule design problem for a near-sea liner service route with time windows, which is of practical significance for container lines operating near-sea routes or feeder routes. The problem can be expressed as a mixed integer nonlinear nonconvex optimization model which is then transformed into a mixed integer linear programming model by integer programming tricks. The model is applied to a real-world case study involving SITC Container Lines. The result shows that the model can be solved efficiently by the current mainstream optimization solver to obtain the optimal solution, which proves the applicability of the model to the problem. Finally, we conducted sensitivity analysis of port time windows. The results demonstrate that different port time windows may affect the optimal sailing speeds on shipping legs, the optimal arrival times at ports, the optimal number of ships deployed on the service route, and thereby affect the total cost.

In this study, we assume that the vessel fleet was homogeneous. In future studies, we will extend the problem to the one with heterogeneous fleet, which is more practical to the liner shipping industry.

ACKNOWLEDGMENT

This research was supported by the Postgraduate Research & Practice Innovation Program of Jiangsu Province (KYLX16_0281).

REFERENCES

[1] M. Christiansen, K. Fagerholt, B. Nygreen, and D. Ronen, "Ship routing and scheduling in the new millennium," European Journal of Operational Research, vol. 228, pp. 467-483, 2013.

[2] Q. Meng, S. Wang, H. Andersson, and K. Thun, "Containership Routing and Scheduling in Liner Shipping: Overview and Future Research Directions," Transportation Science, vol. 48, pp. 265-280, 2014.
[3] N. K. Tran and H. Haasis, "Literature survey of network optimization in container liner shipping," Flexible Services and Manufacturing Journal, vol. 27, pp. 139-179, 2015.

[4] K. Fagerholt, "Ship scheduling with soft time windows: An optimisation based approach," European Journal of Operational Research, vol. 131, pp. 559-571, 2001.

[5] M. Christiansen and K. Fagerholt, "Robust ship scheduling with multiple time windows," Naval Research Logistics (NRL), vol. 49, pp. 611-625, 2002.

[6] R. Agarwal and O. Ergun, "Ship Scheduling and Network Design for Cargo Routing in Liner Shipping," Transportation Science, vol. 42, pp. 175-196, 2008.

[7] S. Wang and Q. Meng, "Schedule Design and Container Routing in Liner Shipping," Transportation Research Record: Journal of the Transportation Research Board, vol. 2222, pp. 25-33, 2011.

[8] S. Wang and Q. Meng, "Liner ship route schedule design with sea contingency time and port time uncertainty," Transportation Research Part B: Methodological, vol. 46, pp. 615-633, 2012.

[9] S. Wang, Q. Meng and Z. Liu, "Containership scheduling with transit-time-sensitive container shipment demand," Transportation Research Part B: Methodological, vol. 54, pp. 68-83, 2013.

[10] S. Wang, A. Alharbi and P. Davy, "Ship Route Schedule Based Interactions Between Container Shipping Lines and Port Operators," in Handbook of Ocean Container Transport Logistics: Making Global Supply Chains Effective, C. Lee and Q. Meng, Eds. Cham: Springer International Publishing, 2015, pp. 279-313.

[11] S. Wang, A. Alharbi and P. Davy, "Liner ship route schedule design with port time windows," Transportation Research Part C: Emerging Technologies, vol. 41, pp. 1-17, 2014.

[12] A. Alharbi, S. Wang and P. Davy, "Schedule design for sustainable container supply chain networks with port time windows," Advanced Engineering Informatics, vol. 29, pp. 322-331, 2015.

[13] N. Aydin, H. Lee and S. A. Mansouri, "Speed optimization and bunkering in liner shipping in the presence of uncertain service times and time windows at ports," European Journal of Operational Research, vol. 259, pp. 143-154, 2017.

[14] A. Charisis, N. Mitrovic and E. Kaisar, "Container shipping route and schedule design with port time windows and coordinated arrivals," in 21st International Conference on Intelligent Transportation Systems, Maui, Hawaii, USA, 2018, pp. 2538-2543.

[15] B. Yu, Z. Peng, Z. Tian, and B. Yao, "Sailing speed optimization for tramp ships with fuzzy time window," Flexible Services and Manufacturing Journal, vol. 31, pp. 308-330, 2019.