Non-Paradoxical Loss of Information in Black Hole Evaporation

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We consider the black hole information loss issue within the context of modified versions of quantum theory known as dynamical reduction theories. In this setting the Hawking evaporation is accompanied by a large amount of information loss in a clear manner involving no paradox. Quantum gravity’s role in this picture is restricted to the resolution of the singularity. The specific analysis is carried on using a well known two dimensional model known as CGHS and a specific dynamical reduction theory known as CSL.

The essence of the information loss paradox in black hole (BH) evaporation is that starting with a system in a pure initial quantum state that forms the BH, it then evolves into something that, at the quantum level, can only be characterized as a highly mixed quantum state, while, the standard quantum mechanical considerations lead one to expect a fully unitary evolution \[1\]. There is even a debate as to whether or not this should be considered as a paradox \[2\]. In \[3\] it is argued that the debate arises due to basic differences of outlook regarding the fate of the BH singularity in the context of a quantum theory and related issues.

Many researchers in the (QG) field have been trying to address the black hole information conundrum, within the context of their preferred approach. After all, the “paradox”, truly emerges only if one assumes that a QG will remove the singularities that appear in association with black holes in general relativity (GR), otherwise the singularity could be viewed as representing an additional boundary of space-time, where the missing information could be registered. The proposals to address the issue involve various schemes whereby the complete evolution respects quantum mechanical unitarity and thus the information is strictly conserved \[1\]-\[6\]. Recent studies along those lines \[7\] connect the resolution of the issue with the emergence of “firewalls”, creating a serious tension between the equivalence principle of GR and unitarity of quantum mechanics.

We present here a proposal based on a different point of view, where information is lost, and quantum mechanical evolution is not strictly unitary. The scheme is based on a basic feature of the dynamical reduction theories that have been introduced as a way to address conceptual difficulties of quantum theory, namely the so called “measurement problem” \[10\]-\[13\]. These ideas date back to \[14\] with the first specific toy model proposals in \[15\] and the first viable proposals in \[17\] with the theory of Spontaneous Localization, and modified quantum theory known as Continuous Spontaneous Localization or CSL \[19\], \[20\]. Not long after that Diosi \[21\] and Penrose \[22\] proposed the connection of these ideas with quantum aspects of gravity. It should be stressed that resolving the BH information paradox within such approach would require explaining how a pure state becomes a (quantum) thermal state corresponding to a proper mixture (rather than an improper one, see \[8\] for terminology) as a result of the eventual disappearance of the inside region. We will see this at work in the analysis below.

Given the complexity of the problem we will present our analysis using a simplified 2-dimensional model known as the CGHS black hole \[24\] and an adapted version of CSL \[25\]. The quantum dynamical evolution, dictated by CSL, is specified by two equations: i) A modified Schrödinger equation, whose solution is:

\[
|\psi, t\rangle_w = \mathcal{T} e^{-\int_0^t dt' \left[ i\mathcal{H} + \frac{1}{16\lambda}\sqrt{2\pi|w(t')|^2}\right]|\psi, 0\rangle,
\]

where (\(\mathcal{T}\) the time-ordering operator), \(w(t)\) is a random, white noise type classical function of time whose probability is given by the second equation, ii) the Probability Rule:

\[
PDw(t) = w(\psi, t|\psi, t)w(t) \prod_{t_i = 0}^t \frac{dw(t_i)}{\sqrt{2\pi\lambda/dt}}.
\]

Thus the standard Schrödinger evolution and the corresponding measurement of the observable \(\hat{A}\) are unified. For non-relativistic quantum mechanics of a single particle, the proposal assumes that, (without invoking any measurement) \(\hat{A} = \hat{\chi}\), where \(\hat{\chi}\) is a suitably smeared version of the position operator. When this is generalized to multi-particle systems and everything including apparatuses are treated quantum mechanically, the theory seems to successfully address the measurement problem \[10\]-\[13\]. The collapse parameter \(\lambda\) must be small enough not to conflict with tests of QM in the domain of subatomic physics, and big enough to result in rapid localization of “macroscopic objects”. The GRW suggested value is \(\lambda \sim 10^{-16}\) sec\(^{-1}\).

Unlike the previous studies, here we need to adapt the approach in situations involving both quantum fields and gravitation. Dynamical reduction in the quantum state requires the notion of “time” (the collapse takes place in time). As QM has a problem with time, we make our
analysis assuming we can rely on a semiclassical framework. Our point of view is that even if at the deepest levels gravitation must be quantum mechanical in nature, at the meso/macro scales, it corresponds to an emergent phenomena, with traces of the quantum regime surviving in the form of an effective dynamical state reduction for matter fields.

Our analysis will be based on: i) the CGHS black hole, ii) a toy version of CSL adapted to a field theory on a curved space-time, iii) some simplifying assumptions about what happens when QG cures a singularity, and iv) an assumption that the CSL collapse parameter is not fixed but depends (increases) with the local curvature.

Now we provide the basic features of the CallanGiddings-Harvey-Strominger (CGHS) model that will be needed. For more details we refer the reader to [20]. The CGHS action is given by

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ e^{-\phi} \left( R + 4(\nabla^2\phi) + 4\Lambda^2 \right) - \frac{1}{2}(\nabla f)^2 \right]$$

where $\phi$ is the dilaton field, $\Lambda^2$ is a cosmological constant, and $f$ is a scalar field, representing matter. The generic construction of the CGHS model is shown in Fig. 1. At $x^+ < x_0^+$, the metric is Minkowskian, usually known as the dilaton vacuum (region I and I'), given by $ds^2 = -\frac{dx^+dx^-}{\Lambda^2 x^+(x^-+\Delta)}$, whereas, at $x^+ > x_0^+$ it is represented by the black hole metric (region II, III) $ds^2 = -\frac{dx^+dx^-}{\Lambda^2 x^+(x^-+\Delta)}$. These (null) Kruskal coordinates $(x^+, x^-)$ are useful to know the global structure of the spacetime. On the other hand for physical studies involving Quantum Field Theory (QFT) in curved space-time it is necessary to use physical coordinates which define physical observers in various regions. In the dilaton vacuum region, natural coordinates which resembles with the Minkowski definition, are $y^+ = \frac{1}{\Lambda} \ln(\Lambda x^+), y^- = \frac{1}{\Lambda} \ln(-\frac{x^-}{\Lambda})$, and the metric is expressed as $ds^2 = -dy^+dy^- + \infty < y^- < \infty; -\infty < y^+ < \frac{1}{\Lambda} \ln(\Lambda x^+)$.

On the other hand, the BH exterior (region II), where physical observers exist, has coordinates $\sigma^+ = \frac{1}{\Lambda} \ln(\Lambda x^+(x^-+\Delta))$, so that the metric is $ds^2 = -\frac{dx^+dx^-}{x^+(x^-+\Delta)}$ with $-\infty < \sigma^- < \infty$ and $\sigma^+ > \sigma_0^+ = \frac{1}{\Lambda} \ln(\Lambda x_0^+)$. To check the asymptotic flatness, we express the BH metric in Schwarzschild like coordinates $(t, r)$ defined as $\sigma^+ = t \pm \frac{1}{\Lambda} \ln(e^{2\Lambda r} - M/\Lambda)$ so that, we have $ds^2= -(1 - \frac{\Lambda}{2\pi} e^{-2\sqrt{\frac{M}{2\pi}}} r^2 + \frac{\pi^2}{4\Lambda^2} r^2 e^{-2\sqrt{\frac{M}{2\pi}}} r^2 dx^+dx^-)$.

The Kruskal coordinates $2T = x^+ - x^- + \Delta$. $2X = x^+ - x^- - \Delta$ can be related with Schwarzschild like time $t$ and space $r$ coordinates using tanh($M_\Lambda r$) = $T/X$ and $\Lambda/2\pi = (e^{2\Lambda r} - M/\Lambda) = T^2 - X^2$.

Next we consider the quantum description of $f$ for which one uses the $\mathcal{F}_R^*$ and $\mathcal{F}_L^*$ as the asymptotic past (in) region and the black hole (exterior and interior) region as the asymptotic out region. In the in basis the field operator can be expanded as $\hat{f}(x) = \sum_\omega (\hat{f}_R^R(x) + \hat{f}_L^L(x))$, where $\hat{f}_R^R = \tilde{b}_R^R \psi_{R/L} + \tilde{b}_R^{R/L} \psi_{R/L}^*$. Here, the basis functions (modes) are $u_0^R = \frac{1}{\sqrt{2\pi}} e^{-i\omega y^+}$ and $u_0^L = \frac{1}{\sqrt{2\pi}} e^{-i\omega y^-}$, with $\omega > 0$. The superscript $R$ and $L$ means right and left moving modes. These modes will define an in vacuum right ($|0_{in}\rangle_R$) and in vacuum left ($|0_{in}\rangle_L$) whose tensor product ($|0_{in}\rangle_R \otimes |0_{in}\rangle_L$) defines our in vacuum. One can also expand the field in the out region in terms of the complete set of modes that have support both outside (exterior) and inside (interior) the event horizon. Therefore the field operator has the form $\hat{f}(x) = \hat{f}_R^R(x) + \hat{f}_L^L(x)$

$$\hat{f}_R^R(x) = \sum_\omega \tilde{b}_R^R \psi_{R/L} + \tilde{b}_R^{R/L} \psi_{R/L}^* + \sum_\omega \tilde{b}_R^R \psi_{R/L} + \tilde{b}_R^{R/L} \psi_{R/L}^* ,$$

where we use the convention in which modes and operators with and without tilde are defined inside and outside the horizon, respectively. We note that the arbitrariness in the choice of basis inside the horizon does not affect our physical results. The mode functions in the exterior to the horizon that we will use are: $v_\omega^R = \frac{1}{\sqrt{2\pi}} e^{-i\omega \sigma^+} \Theta(-x^-+\Delta)$ and $v_\omega^L = \frac{1}{\sqrt{2\pi}} e^{-i\omega \sigma^-} \Theta(x^-+\Delta)$. Similarly one can define a set of modes in the black hole interior so that the basis of modes in the out region is complete. The left moving mode remains unchanged, while for the right moving mode we use $\tilde{v}_\omega^R = \frac{1}{\sqrt{2\pi}} e^{i\omega \sigma^+} \Theta(x^-+\Delta)$. Following [20], we replace the above delocalized plane wave modes by a complete orthonormal set of discrete wave packets modes, $v_{nj}^{R/L} = \frac{1}{\sqrt{\pi}} \mathcal{F}^{(j+1)} f_{\omega} e^{2\pi i \omega / \epsilon} v_{nj}^{R/L}$, where the integers $j \geq 0$ and $-\infty < n < \infty$. These wave packets are peaked about $\sigma^+/- = 2\pi n/\epsilon$ with width $2\pi/\epsilon$ respectively.

The non-trivial Bogolubov transformations are only
relevant in the right moving sector, and the corresponding transformation from in to exterior modes. This results in the fact that the initial state, corresponding the vacuum for the right moving modes and the left moving pulse forming the black hole $|\Psi_{in}\rangle = |0_{in}\rangle_R \otimes |\text{Pulse}\rangle_L$ can be written as:

$$N \sum_{F_{nj}} C_{F_{nj}} |F_{nj}\rangle^{\text{ext}} \otimes |F_{nj}\rangle^{\text{int}} \otimes |\text{Pulse}\rangle_L,$$  \hspace{1cm} (4)

where a particle state $F_{nj}$ consists of arbitrary but finite number of particles, $N$ is a normalization constant, and the coefficients $C_{F_{nj}}$’s are determined using the Bogolubov transformations. Tracing over the interior DOF, we end up with an improper thermal state.

With that, we are in a position to proceed to show how a proper thermal state is obtained using CSL, and some reasonable assumptions about QG. As the CSL theory involves a modification of the time evolution of the quantum states, we need a foliation of our space-time associated with a “global time parameter”. As is customary in QFT, we will be using an interaction-type picture, where the free part of the evolution is encoded in the field operators, and the interaction, which in our case is just the new CSL part, drives the evolution of the states.

We now construct the foliation. We choose a $r = \text{const.}$ and a $t = \text{const.}$ surfaces in the inside and outside of the horizon, respectively, and then join them using a surface $T = \text{const.}$ We specified puncture points by two curves, $T_{1,2}(X)$. We take $T_1(X) = \left( X^2 + \frac{M}{\Lambda e^{-2\Lambda/\sqrt{X}}} \right)^{1/2}$ and $T_2(X)$ is found by reflection of $T_1(X)$ with respect to the horizon $T = X$.

Using this construction, we introduce a time coordinate $\tau$ specifying the hyper surfaces of the foliation by the value of its intersect with the $T$ axis. This foliation covers the complete BH (exterior and interior) region and one can extend it to vacuum region arbitrarily with no effect in our study, since, the CSL modification is significant only close to singularity.

Next we must determine the operators driving the collapse in our version of CSL. We note first that the CSL equations can be generalized to drive collapse into a state of a joint eigen-basis of a set of commuting operators $\{A^\alpha\}$ which we call as collapse operators. For each $A^\alpha$ there is one $w^\alpha(t)$ and the evolution equation will be

$$|\Psi_{t},\tau\rangle = \tilde{T} e^{- \int_0^\tau dt' \left[ i\hat{H} + \frac{1}{\hbar} \sum_\alpha [w^\alpha(t') - 2\lambda \hat{A}^\alpha]^2 \right]} |\Psi, 0\rangle.$$  \hspace{1cm} (5)

In this work we choose a special set of collapse operators following three principles: i) States will collapse to a state of definite number of particles in the inside region. ii) The rate of collapse is enhanced by the curvature of the space-time, so that, as the evolution approaches the singularity (in a finite time), the rate of collapse will diverge. Far from the singularity the rate of collapse will be much smaller, and the direct effects of CSL evolution will be negligible. iii) We are working in the interaction picture which requires the replacement $\hat{H} \rightarrow 0$ in the above equation.

The next aspect we must consider is a curvature dependent form of the coupling $\lambda$ in modified CSL evolution. As we have said, we assume that the CSL collapse mechanism will be amplified by the curvature of space-time. That is, we will assume that the rate of collapse $\lambda$, will depend, in this case, on the Ricci scalar:

$$\lambda(R) = \lambda_0 \left[ 1 + \left( \frac{R}{\mu} \right)^\gamma \right]$$

where $R$ is the Ricci scalar of the CGHS space-time and $\gamma \geq 1$ is a constant, $\mu$ provides an appropriate scale.

Note that the hypersurfaces given by the foliation have constant $R$ inside the black hole (in almost all the part of $\Sigma_\tau$ that lies inside). Then, for the region of interest we have $\lambda = \lambda(\tau)$.

The resulting evolution will achieve in the finite time to the singularity, what ordinary CSL achieves in infinite time. That is, to drive the state to one of the eigenstates of the collapse operators.

Recall that the particle content of state $|F\rangle$ is given by the particle distribution $F = \{\ldots F_{nj}, \ldots \}$ where $F_{nj}$ is the number of particles in mode $v_{nj}$ (both for the inside of the black hole and the outside). Note, we avoid singularities by using smeared versions of these operators. The action of the number operator $N_{nj}$ acting on $\mathcal{F}^{\text{bh}}$ is $N^{\text{bh}}_{nj} |F\rangle^{\text{bh}} = F_{nj} |F\rangle^{\text{bh}}$ The set of collapse operators we are proposing for the quantization is $\hat{A}^\alpha = N^{\text{bh}}_{nj} \otimes \text{I}_{\text{out}}$ for all $n, j$, where $\text{I}_{\text{out}}$ is the identity operator in $\mathcal{F}^{\text{out}}$.

The fact that CSL evolves states towards one of the eigenstates of the collapse operators ensures that, as the result of the evolution, the state at hypersurfaces $\tau = \text{const.}$ but very close to the singularity (on $\Sigma_\tau$, in Fig. 2) would be of the form:

$$|\Psi_{in,\tau}\rangle = NC_{F_{nj}} |F_{nj}\rangle^{\text{ext}} \otimes |F_{nj}\rangle^{\text{int}} \otimes |\text{Pulse}\rangle_L,$$  \hspace{1cm} (6)

Note that there is no summation so the state is pure, even though it is undetermined, because we don’t know the actual realization of the stochastic function $w$.

Note that, although individual states with definite occupation number in the ext and int modes such as the one above lead to singular ($T_{\text{bh}}$), those states are only approached asymptotically as $\tau \rightarrow \tau_s$. The dynamics of CSL generate only smooth states prior to the singularity \cite{27}. The situation is analogous to the measurement of a precise number of Rindler particles in the Minkowsky vacuum in a finite time, which is impossible unless the field-detector interaction is singular \cite{28}.

Next we consider the role of quantum gravity to pass to the final hypersurface $\Sigma_\tau$, in Fig. 2. We will assume that a reasonable theory of QG will resolve the singularity and lead, on the other side, to a reasonable space-time. Moreover, we will assume that such a theory will not lead to large violations of the basic space-time conservation laws.
The above result is a bit unsettling because we end up with a pure quantum state, but we do not know which one. That depends on the particular realization of the random functions \( w^\alpha \) that appear in the CSL evolution equation, which, as we know, is a stochastic equation. So let us consider now an ensemble of systems identically prepared in the same initial state \( |\Psi_{in}\rangle = |0_{in}\rangle_R \otimes |\text{Pulse}\rangle_L \). We describe this ensemble by the pure density matrix \( \rho(\tau_0) = |\Psi_{in}\rangle \langle \Psi_{in}|. \) Next we consider its CSL evolution up to the hypersurface just before the singularity. The evolution from initial hypersurface \( \Sigma_{\tau_0} \), and up to the hypersurface \( \Sigma_{\tau} \) yields:

\[
\rho(\tau) = T e^{-\int_{\tau_0}^\tau d\tau' \lambda(\tau') \sum_{n_j} |N_{\tau} - N_{\tau_0}|^2} \rho(\tau_0).
\]

We express \( \rho(\tau_0) = |0\rangle_{in} \langle 0|_{in} \) in terms of the out quantization (ignoring left moving modes):

\[
\rho(\tau_0) = N^2 \sum_F e^{-\frac{\pi}{2}(E_F + E_G)} \langle F\rangle^{bh} \otimes \langle F\rangle^{out} \langle G\rangle^{bh} \otimes \langle G\rangle^{out},
\]

where \( \Lambda \) is the parameter of the CGHS model and \( E_F \equiv \sum_{n_j} \omega_{n_j} F_{n_j} \) is the energy of either state \( |F\rangle^{bh} \) or \( |F\rangle^{out} \) with respect to late-time observers near \( \mathcal{I}^+ \).

The operators \( \tilde{N}_{n_j} \) and their eigenvalues are independent of \( \tau \). Thus,

\[
\rho(\tau) = N^2 \sum_{F,G} e^{-\frac{\pi}{2}(E_F + E_G)} B|F\rangle^{bh} \otimes |F\rangle^{out} \langle G\rangle^{bh} \otimes \langle G\rangle^{out},
\]

where \( B = e^{-\sum_{n_j} (F_{n_j} - G_{n_j})^2 \int_{\tau_0}^\tau d\tau' \lambda(\tau')/2} \). In general, this equation does not represent a thermal state. Nevertheless, as \( \tau \) approaches the singularity, say at \( \tau = \tau_s \), the integral \( \int_{\tau_0}^{\tau_s} d\tau' \lambda(\tau')/2 \) diverges since \( \lambda(\tau) \) is evaluated at hypersurfaces of higher curvature. Then, as \( \tau \rightarrow \tau_s \) the non diagonal elements of \( \rho(\tau) \) cancel out and we have:

\[
\lim_{\tau \rightarrow \tau_s} \rho(\tau) = N^2 \sum_F e^{-\frac{\pi}{2}E_F} |F\rangle^{bh} \otimes |F\rangle^{out} \langle F\rangle^{bh} \otimes \langle F\rangle^{out}.
\]

If we now include the left moving pulse and take into account what we have assumed about the role of QG, we obtain a density matrix characterizing the ensemble after the singularity (on \( \Sigma_{\tau_s} \) in Fig. 2), which is given by

\[
\rho^{\text{final}} = N^2 \sum_F e^{-\frac{\pi}{2}E_F} |F\rangle^{out} \otimes |0_{p.s.}\rangle \langle 0_{p.s.}| \langle F\rangle^{out} \otimes \langle 0_{p.s.}|.
\]

This is just \( |0_{p.s.}\rangle \langle 0_{p.s.}| \otimes \rho^{\text{thermal}} \). That is, we started with an initial pure state of the quantum field corresponding to an initially collapsing pulse, and the corresponding space-time initial data on past null infinity, and ended up, with a \textit{proper} thermal state on future null infinity followed by an empty region.

Of course, we assumed that a QG theory would resolve the singularity and otherwise be reasonable so that it does not lead to gross violations of conservation laws, with potentially observable implications in the regions where something close to a classical space-time description is expected. We reiterate that, at this point, this is
only a toy model, but we believe that reasonable models with the basic features we have discussed here do offer perhaps the best hope to resolving the long standing conundrum known as the “Black Hole Information Loss Paradox”.

Acknowledgment: We thank Robert Wald, Philip Pearle and Elias Okon for discussions. DS is supported by the CONACYT-México Grant No. 220738 and UNAM-PAPIIT Grant No. IN107412. SKM and LO are supported by DGAPA fellowships from UNAM.