Abstract
We study two reciprocal thermal effects in the ferromagnetic semiconductor (Ga,Mn)As by scattering theory: domain wall motion induced by a temperature gradient as well as heat currents pumped by a moving domain wall. The effective out-of-plane thermal spin transfer torque parameter $P Q \beta Q$, which governs the coupling between heat currents and a magnetic texture, is found to be of the order of unity. Unpinned domain walls are predicted to move at speed 10 m/s in temperature gradients of the order 10 K/\mu m. The cooling power of a moving domain wall only compensates the heating due to friction losses at ultra-low domain wall velocities of about 0.07 m/s. The Seebeck coefficient is found to be of the order 100-500 \mu V/K at T=10 K, in good agreement with recent experiment.

Keywords: A. Ferromagnets; D. thermoelectrics; D. domain wall; D. spin caloritronics

PACS: 72.15.Jf, 75.30.Sg, 75.78.-n, 75.50.Pp

A magnetic domain wall is a region with a gradual reorientation of the local magnetic moments between magnetic domains with different magnetization directions (Fig. 1). The position of a domain wall in a magnetic wire can be manipulated by an external magnetic field or an electric current [1, 2]. The latter mechanism makes electronic shift registers possible in which the magnetic domains separate different bits which are collectively moved by a current ("racetrack memory") [3].

Figure 1: (Color online) A ferromagnetic wire containing a domain wall in which the magnetic texture rotates in the transverse xz-plane (Bloch wall) [3].

When a current, which in a metallic ferromagnet is spin polarized, traverses a magnetic domain wall, it exerts a torque on the domain wall, which can be decomposed into an in-plane and an out-of-plane component [1]. In the adiabatic limit, the in-plane torque component can be understood easily in terms of conservation of angular momentum. The out-of-plane component is parameterized by a so-called $\beta$-factor [4]. Together with the Gilbert damping parameter $\alpha$, which describes magnetic friction processes, it determines the current-driven domain wall mobility: an increasing $\beta$-factor increases the mobility, while it decreases with increasing $\alpha$ [1].

The $\beta$-factor associated with a voltage gradient is denoted $\beta c$. In transition metal ferromagnets, $\beta c$ and $\alpha$ are found to be of the same order of magnitude ($\sim 10^{-3}$ – $10^{-2}$) [4, 1]. These parameters have recently also been studied in ferromagnetic semiconductors [5, 8, 9], in which the strong spin-orbit coupling in the valence band is found to be responsible for a drastically larger $\beta c / \alpha \sim 100$ [8, 9].

In the last few years a new research field called Spin Caloritronics, loosely speaking the study of thermal effects in spintronics, has emerged leading to renewed interest in domain wall motion induced by temperature gradients. The effect was first studied a couple of decades ago [10], and has recently enjoyed a renaissance [11, 12, 13, 14, 15]. Since currents can be induced by both voltage and temperature gradients, there are two distinct contributions to $\beta$: (1) $\beta c$ linked to a voltage gradient, and (2) the recently predicted thermal $\beta Q$ associated with a temperature gradient [13, 14, 16]. Analogous to electrically induced domain wall motion, the ability to move the domain wall by a temperature gradient is controlled by the $\beta Q / \alpha$-ratio [13, 14]. It can arise from ballistic non-adiabaticity in narrow domain walls [16]. Here we discuss the adiabatic and dissipative contribution to $\beta Q$ by microscopic scattering theory based on the Luttinger Hamiltonian of the GaAs valence band.

In (Ga, Mn)As, strong spin-orbit interaction complicates the definitions of spin-dependent conductivities and Seebeck coefficients. Instead, we calculate the well-defined
quantities $P_Q\beta_Q$ and $P_r\beta_r$ ($P_r$ and $P_Q$ defined below). The $P_Q\beta_Q$ ($P_r\beta_r$) parameter we calculate, describes the effective out-of-plane thermal (electric) spin transfer torque parameter experienced by a rigidly moving domain wall, i.e. a global parameter depending on the magnetization profile, in particular the domain wall length.

$\langle$Ga,Mn$\rangle$As has a large thermopower with Seebeck coefficients of the order $S \sim 100 - 300 \mu V/K$ [17]. Here we calculate the thermopowers of different alloys and find good agreement with experiments, implying a strong particle-hole asymmetry in the system. Combined with the large $P_r\beta_r$ values found before, we also expect a high $P_Q\beta_Q$ in these materials. Here we present the first study of domain wall motion induced by a temperature gradient in the ferromagnetic semiconductor (Ga,Mn)As. We find a large thermally induced domain wall response with $P_Q\beta_Q \sim 1$, which is the same order of magnitude as $P_r\beta_r$. This leads to the prediction that domain walls move at a speed of 10 m/s with temperature gradients as low as 100 K/µm, much smaller than what has been predicted for transition metals [14].

The paper is organized as follows: We first review the derivation of $P_Q\beta_Q$ and $P_r\beta_r$ in terms of the scattering matrix following Refs. [8, 14]. Then $P_Q\beta_Q$ is calculated numerically for disordered (Ga,Mn)As, which enables us to predict the temperature gradients needed to move domain walls. We also discuss the heat pumping by domain wall systems [13, 14].

Magnetization dynamics is well described phenomenologically by the generalized LLG equation [1]

$$\dot{\mathbf{m}} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \dot{\mathbf{m}} + \mathbf{\tau}$$

(1)

where $\mathbf{m} = \mathbf{M}(r, t)/M_s$ is the unit direction vector of the magnetization $\mathbf{M}$, $\mathbf{H}_{\text{eff}} = -\delta F[\mathbf{M}(r, t)]/\delta \mathbf{M}$ is the effective field given by the free energy functional $F[\mathbf{M}(r, t)]$, $\gamma$ is (minus) the gyromagnetic ratio, and $\alpha$ the Gilbert damping constant. In conducting ferromagnets, the itinerant quasiparticles are strongly coupled to the magnetization via the exchange interaction. Through this coupling a spin current can induce a torque $\mathbf{\tau}$ on a magnetic texture. The torque can be generated by a temperature gradient $\Delta T \equiv T_L - T_R$ ($\tau_Q$) or a voltage gradient $\Delta V \equiv V_L - V_R$ ($\tau_c$). To lowest order in magnetization gradients these two torques have the form [13, 14]

$$\tau_c = \frac{\hbar P_r G}{AeS_0} \Delta V (1 - \beta_r \mathbf{m} \times) \partial_y \mathbf{m}$$

(2)

$$\tau_Q = \frac{\hbar P_Q TS G}{AeS_0} \Delta T (1 - \beta_Q \mathbf{m} \times) \partial_y \mathbf{m}$$

(3)

where $S_0 = M_s/\gamma$ is the spin density along $-\mathbf{m}$, $G$ is the conductance, $S = -e\mathcal{L}T_0 \ln G$ is the Seebeck coefficient, $\mathcal{L} = (k_B T)^2/(3e^2)$ is the Lorenz constant and $\partial_E \equiv \partial / \partial E$ at the Fermi energy, $e$ the modulus of the electron charge, $2P_r \equiv \sigma^1(\sigma^1 + \sigma^2)/2(\sigma^1 + \sigma^2)$ where $\sigma^{i(j)}$ are the spin-dependent conductivities, $P_Q \equiv P_r + P_1 (1 - 4P_r^2)$ [13] with $2P_r \equiv (S^1 - S^2)/(S^1 + S^2)$ where $S^{i(j)}$ are the spin-dependent Seebeck coefficients, and $A$ is the cross-sectional area of the conductor. The first terms in Eqs. (2) and (3) are the in-plane torques, which describe the angular momentum transfer on $\mathbf{m}$ by a spin current following the magnetic texture adiabatically. The second terms, parameterized by $\beta_r$ and $\beta_Q$, describe dissipative out-of-plane torques. $\beta_r, \beta_Q$ and $\alpha$ govern the domain wall response to an applied temperature or voltage gradient.

We adopt a magnetic free energy functional for a wire along the $y$-axis

$$F[\mathbf{M}] = M_s \int dr \left( \frac{K_n}{2} (\nabla \theta)^2 + \sin^2(\theta) (\nabla \phi)^2 \right) +$$

$$\frac{K_{\perp}}{2} \sin^2 \theta \sin^2 \phi - \frac{K_z}{2} \cos^2 \theta - H_{\text{ext}} \cos \theta$$

(4)

where $K_s$ is the spin wave stiffness, $K_{\perp}$ and $K_z$ are anisotropy constants, $H_{\text{ext}}$ is an external magnetic field, and $\theta$ and $\phi$ are the polar and azimuth angles describing the local magnetization directions, respectively. A local minimum of this functional is a Bloch wall [4] rotating in the (transverse) $x$-$z$ plane with local magnetization direction given by $\cos \theta = \tan h[(y - r_w)/\lambda_w]$, $\sin \theta = 1/\cosh[(y - r_w)/\lambda_w]$, where $r_w$ is the position of the wall and $\lambda_w$ the domain wall length. The position can be manipulated either by a magnetic field or a spin current. For low current densities and magnetic fields the wall moves rigidly with constant tilt angle $\phi$ and domain wall length $\lambda_w = \sqrt{K_s/(K_z + K_\perp \sin^2(\phi))}$, while the domain wall position $r_w$ is given by:

$$\frac{\alpha r_w}{\lambda_w} = -\gamma H_{\text{ext}} - \frac{\hbar P_r \beta_r G}{AeS_0 \lambda_w} \Delta V - \frac{\hbar P_Q TS \beta_Q G \Delta T}{AeS_0 \lambda_w}$$

(5)

Eq. (5) is obtained by substituting the Bloch wall ansatz with a time-dependent tilt angle $\phi(t)$ into Eq. (1), and consider the regime where $d\phi/dt = 0$. The values of $\Delta T/T$, $\Delta V$ and $H_{\text{ext}}$ at which this regime breaks down are known as the Walker thresholds, above which also the parameters $\lambda_w$ and $\phi$ have to be treated as dynamic variables. In this paper we will always assume $\Delta T/T$, $\Delta V$ and $H_{\text{ext}}$ to be below the threshold fields, so that $r_w(t)$ is the only time-varying parameter. In (Ga,Mn)As the Walker ansatz breaks down for velocities typically of the order 10 m/s [18].

Eq. (5) shows that the magnetic system responds to temperature and voltage gradients. From Onsager's reciprocal relations [19] we know that the reciprocal processes heat and charge pumping by a moving domain wall also exist, and that the response coefficients have to obey Onsager symmetry relations. The response coefficients for heat and charge pumped by a moving domain wall thus gives us expressions for the parameters $P_Q\beta_Q$ and $P_r\beta_r$. This strategy is used in Refs. [8, 14] to derive scattering matrix expressions for $P_r\beta_r$ and $P_Q\beta_Q$. The response coefficients $L_{QW} (L_w)$ and $L_{eQ} (L_{wc})$, describing heat (charge) current pumped by a moving domain wall and domain wall
Heat pumping by a moving domain wall reported in Ref. [8]. We investigate the reciprocal effects: aggregating modes in the leads, and $\lambda_P$ and calculate the study is beyond the scope of this manuscript.

In the following, we focus on the (Ga,Mn)As system, and calculate $P_Q\beta_Q$ using Eq. (6) and $P_c\beta_c$ by the methods reported in Ref. [8]. We investigate the reciprocal effects: Heat pumping by a moving domain wall as well as torque induced by a temperature gradient.

To model the band structure of (Ga,Mn)As we use the Hamiltonian:

$$H = H_L + h(r) \cdot J + V(r),$$

where $H_L$ is the $4 \times 4$ Luttinger Hamiltonian for zincblende semiconductors in the spherical approximation. $J$ is a vector of 4x4 spin matrices for $J=3/2$ spins. The $h \cdot J$ term is a mean field approximation of the interaction between the itinerant holes and the local magnetic moment of the Mn dopants. The exchange field $h$ is antiparallel to the magnetization direction $m$. $V(r) = \sum_i V_i \delta(r - R_i)$ is the impurity scattering potential, where $R_i$ is the position of impurity $i$, and $V_i$ its scattering strength [23]. The $V_i$ are randomly and uniformly distributed in the interval $[-V_0/2, V_0/2]$.

The response coefficients $L_{cw}$ and $L_{Qw}$ are calculated from the scattering matrix expressions in Eq. (6) and Eq. (7). The scattering matrix is calculated numerically using a stable transfer matrix method [24]. Disorder effects are included fully and nonperturbatively by the ensemble average $\langle L_{ij} \rangle = \sum_{N_i=1}^N \langle L_{ij} \rangle_{N_i}$, where $N_i$ is number of different impurity configurations. All coefficients are averaged until an uncertainty $\delta \langle L_{ij} \rangle = \sqrt{\langle (L_{ij}^2) - \langle L_{ij} \rangle^2 \rangle} / N_i$ of less than 10% is achieved.

The mean free path $l$ for impurity strength $V_0$ is calculated by fitting the average transmission probability $T = \langle G \rangle / G_{sh}$ to $T(l) = l/(l + L_y)$ [25], where $G_{sh}$ is the Sharvin conductance and $G$ is the conductance for a system of length $L_y$.

We consider a discrete (Ga,Mn)As system with transverse dimensions $L_x = 19$ nm, $L_z = 15$ nm and $L_y \in [100, 350]$ nm connected to infinite ballistic leads. The lattice constant is 1 nm, much less than the Fermi wavelength $\lambda_F \sim 10$ nm. The Fermi energy is 0.077 eV when measured from the lowest subband edge. The Luttinger parameters in $H_L$ are $\gamma_1 = 7.0$ and $\gamma_2 = 2.5$, and $|h| = 0.032$ eV [22]. $\gamma_w \in [10, 20, 40]$ nm are the wall widths. To estimate a typical saturation value of the magnetization we use $M_s = 10^\gamma |h|/a_{GaAs}^3$ [25], with $x = 0.05$ as the doping level, and where $a_{GaAs}$ is the lattice constant for GaAs.

Since we used the Sommerfeld approximation, the scattering matrix expressions for the Onsager coefficients in Eq. (6) and Eq. (7) are valid only for thermal energies that are small compared to the Fermi energy. For instance $k_B T / E_F = 0.01$, implies a temperature of the order 10 K with the Fermi energy specified above.

Fig. 2 (a) and (b) show the out-of-plane torque parameters $P_c\beta_c$ and $P_Q\beta_Q$ as a function of disorder. We observe that in the diffuse regime ($l \ll \lambda_w$) $P_Q\beta_Q \sim 1.0$ is of the same order as $P_c\beta_c$. As explained in Ref. [3], the high $P_c\beta_c$ of (Ga,Mn)As arises from large hole reflection at the domain wall due to the intrinsic domain wall resistance caused by spin-orbit coupling [27]. This intrinsic resistance decreases with disorder, reducing $P_c\beta_c$ with increasing disorder. For even smaller mean-free paths, $P_c\beta_c$ saturates or increases slightly due to the increasing spin-flip rate caused by the impurity scattering. In the ballistic limit ($l \gg \lambda_w$) $P_Q\beta_Q$ is nearly one order of magnitude larger than $P_c\beta_c$, but decreases rapidly when disorder is
mean free path $l$ for mean free path than in the diffuse regime. Since $P_Q\beta_Q$ is a measure of the effect of particle-hole asymmetry on the out-of-plane torque efficiency, this means that the particle-hole asymmetry of the torque efficiency is stronger in the ballistic than in the diffuse regime.

In the diffuse regime we find Seebeck coefficients $S = 100 \pm 500 \mu V / K$ ($S/T = 10 \pm 50 \mu V / K^2$) at $T = 10$ K for mean free path $l \approx 7 - 27$ nm [28], in good agreement with the experimental values $S \sim 100 - 300 \mu V / K$ [17]. The Gilbert damping calculated from Eq. (10) is of the order $10^{-3} - 10^{-2}$ [8].

The high $P_Q\beta_Q$ combined with a large Seebeck coefficient implies a strong coupling between thermally induced currents and domain wall motion in (Ga,Mn)As. This increases the ability to manipulate the domain wall position with a temperature gradient, and oppositely, to transfer heat between the left and right contacts by moving the domain wall. These two effects are illustrated in Fig. 3 where we plot the reciprocal thermal effects: Domain wall motion induced by a temperature gradient and heat current pumped by a moving domain wall. Fig. 3 (a) shows the domain wall velocity divided by $T$ as a function of temperature gradient, while Fig. 3 (b) shows the “thermal motive force” $V_Q = Q/G$, a system-size independent quantity determining how efficient the domain wall system pumps a heat current in the absence of a temperature or voltage bias, divided by $T^2$ as a function of domain wall velocity.

To get a better feeling of the importance of these two reciprocal processes, let us consider a wire at temperature $T = 10$ K with $L_y = 60 \mu m$, and conductivity $\sigma \sim 270 \Omega^{-1} cm^{-1}$, containing a Bloch wall with $\lambda_w = 10$ nm. We limit attention to the regime in which the domain wall moves rigidly, i.e. below the Walker thresholds. We see from Fig. 3 (a) that a temperature gradient of around 10 K/µm is needed to drive the domain wall to speed 10 m/s. According to Eq. (5), an electric field-induced current density of the order $10^4 A/cm^2$ will give an equivalent torque. However, in a real (Ga,Mn)As system, we know that current densities of the order $10^3 - 10^6 A/cm^2$ are needed to achieve such an effect [29]. The disagreement is caused by reduction of the domain wall mobility by extrinsic pinning effects [30]. Thus, taking into account
this effect, one may expects that a temperature gradient of the order 100 – 1000 K / μm needed for driving the wall at speed 10 m / s is a more realistic estimate.

As mentioned above, there is also Joule heat generation associated with the magnetization damping. Note that this heating does not lead to a temperature difference between the left and right contact for a mirror symmetric system since an equal amount of energy is pumped into the right and left contact, but equally heats both two contacts. Assuming a Gilbert damping of the order \( \alpha = 10^{-2} \) implies that the domain wall dissipates a heat current density of 30 W/m² due to magnetic friction \( J^w \) into each reservoir, while the moving domain wall pumps 0.2 W/m² between the left and right contact. Thus, close to the Walker threshold this effect, one may expects that a temperature gradient of the order 1000 K / μm needed for driving the wall at speed 10 m / s.

In conclusion, we have studied thermally induced domain wall motion, and the reciprocal effect, a heat pump operated by moving the domain wall, in the ferromagnetic semiconductor (Ga,Mn)As. The heat current density pumped by the moving domain wall \( (J_Q) \) scales quadratically with \( \dot{r}_w \), while the energy current density associated with magnetization damping \( (J_{Qmagn}) \) scales quadratically. For instance, for the system considered above \( J_Q = 0.02 \dot{r}_w \) W/m² and \( J_{Qmagn} = 0.3 \dot{r}_w^2 \) W/m². The domain wall system is therefore not an efficient cooler except for very small domain wall velocities \( (\dot{r}_w < 0.07 \text{ m/s}) \), where the domain wall pumped energy will dominate over the magnetic friction process. The situation is the same in transition metal ferromagnets, in which the out-of-plane torque parameter is of the same order as the Gilbert damping \( \alpha \). Insulating ferromagnets such as the Yttrium-Iron-Garnets have very low Gilbert damping and are possibly better suited for cooling purposes.

We thank Anh Kiet Nguyen for developing the numerical transfer matrix code. This work was supported in part by computing time through the Notur project and EC Contract IST-033749 "DynaMax".

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