We discuss a few situations related to non separable correlations in multiterminal hybrid structures. We show that the existence of such correlations can modify the strength of the gap of the superconductor. We discuss linear combinations of non local Cooper pairs. We discuss a possible transport experiment intended to probe non separable correlations. The models are worked out at the heuristic level of effective single site Green’s functions.

1 Introduction

Non locality and non separable correlations are one of the deepest features of quantum mechanics. The presence of non separable correlations in quantum mechanics was identified by Einstein, Podolsky and Rosen\(^1\) (EPR) in 1935. One of the possible alternatives to quantum theory could have been hidden variable theories. Bell has shown in 1964 that there exists a basic difference between hidden variable theories and quantum mechanics\(^2\). A. Aspect has shown experimentally in 1981 that Bell inequalities were violated with photons\(^3\). This constitutes the ultimate test of quantum mechanics, which is a non local theory without hidden variables.

Our goal is to describe the physics of heterostructures in which a superconductor is connected to several electrodes, and discuss the consequences for electronic EPR-like experiments. In these systems, non separable correlations are due to superconducting pairs correlations\(^6\),\(^7\). The approach is heuristic in the sense that it contains the correct phenomenology but does not rely on full mathematical rigor. We use a single site Green’s function toy-model in which charge conservation is enforced “by hand”\(^5\). This approach is complementary to lowest order perturbation theory\(^8\). We already worked out part of the rigorous formulation of microscopic transport theory which will be presented elsewhere\(^9\).

2 Implications for thermodynamics

Let us start to discuss a model in which a superconductor \(x\) is connected to \(N\) ferromagnetic electrodes \(\alpha_k, k = 1, \ldots, N\)\(^4\). Each of these elements is represented by an effective single site Green’s function\(^10\). We calculate the Gorkov function \(\hat{G}^{+,-}(\omega)\)\(^{1,2}\) of the superconducting site connected to the ferromagnetic electrodes\(^4\). The hopping matrix elements can be arbitrarily large. The gap should satisfy the self-consistent equation \(\Delta = U \int_{-\infty}^{+\infty} d\omega/(2i\pi) \left[\hat{G}^{+,-}(\omega)\right]_{1,2}\)\(^1\), where \(U\) is the microscopic attractive interaction. This leads to the modified BCS relation

\[
\Delta = D \exp \left[-\frac{1}{\rho_N U} (1 + \pi \rho_N \Gamma_{\uparrow}) (1 + \pi \rho_N \Gamma_{\downarrow})\right],
\]

where \(D\) is the bandwidth, \(\rho_N\) is the normal state density of states and \(\Gamma_{\sigma}\) is the spectral linewidth of spin-\(\sigma\) electron: \(\Gamma_{\sigma} = \sum_{k=1}^{N} \Gamma_{k,\sigma}\), where \(\Gamma_{k,\sigma} = |t_{x,\alpha_k}|^2 \rho_{k,\sigma}\). The spectral linewidths measure the coupling between the superconductor and the ferromagnetic electrodes. With two electrodes only, one has \(\Gamma_{\uparrow} = 2\gamma, \Gamma_{\downarrow} = 0\) in the parallel alignment. In the antiparallel alignment, one has \(\Gamma_{\uparrow} = \Gamma_{\downarrow} = \gamma\). We deduce from Eq.\(^4\) the value of the ratio of the gaps:

\[
\frac{\Delta_{AP}}{\Delta_P} = \exp \left(-\frac{\pi^2 \rho_N \gamma^2}{U}\right).
\]
Figure 1: Representation of a device in which three ballistic ferromagnetic regions $\alpha$, $\beta$, $\gamma$ are connected to a superconductor. The additional electrodes a, b, c are used to perform a measurement of the linear superposition.

The gap is stronger if the electrodes are in a parallel alignment. This is because the proximity effect is stronger in the antiparallel alignment. This behavior coincides with a diffusive model solved recently\[11.\]

3 Linear superpositions of correlated pairs of electrons

Now we consider a system in which a superconductor is connected to three ferromagnetic ballistic regions by high transparency contacts. The three ferromagnetic ballistic regions are connected to three ferromagnetic reservoirs by low transparency contacts (see Fig. 1). We want to show that transport in this system can be interpreted in terms of projections of a BCS-like wave function, which is a linear superposition of correlated pairs\[5.\].

Transport formula: We assume that a voltage $V$ is applied on the superconductor. The Andreev current is found to be the sum of all Cooper pair transmissions\[4.\]:

$$I_{n,\sigma}^A = 4\pi^2 \int d\omega \left[ n_F(\omega - eV) - n_F(\omega) \right] \sum m \hat{Q} \left[ u_{n,\sigma,m,-\sigma,m} u_{m,-\sigma,m} \right] |G_{x,x,1,2}^A|^2,$$

where $u_{n,\sigma,m}$ = $\pi^2 |t_{x,\sigma,n}|^2 |t_{a,\sigma,n}|^2 \rho_{\alpha,n,\sigma} \rho_{\alpha,n,\sigma}/(1 + \pi^2 |t_{a,n,\sigma,n}|^2 \rho_{\alpha,n,\sigma} \rho_{\alpha,n,\sigma})^2$. The operator $\hat{Q}$ is used to enforce charge conservation\[5.\].

BCS-like wave function: It is tempting to look for a BCS-like wave function associated to the situation on Fig. [1]. For instance in the presence of only two electrodes $\alpha$ and $\beta$ having a spin orientation $\sigma_\alpha = \uparrow$ and $\sigma_\beta = \downarrow$, the superconducting wave function is the product of two contributions: (i) Local Cooper pairs (residing in S); (ii) Non local Cooper pairs (residing in the ferromagnets $\alpha$ and $\beta$). We drop out the contribution of local Cooper pairs and write the wave function under the form $|\psi\rangle = c_{\alpha,\uparrow}^+ c_{\beta,\downarrow}^+ |0\rangle$. In the presence of three ballistic regions $\alpha$, $\beta$, $\gamma$, the BCS-like wave function takes the form

$$|\psi\rangle = c_{\alpha,\uparrow}^+ \left[ \sqrt{t_{\beta,\gamma,\downarrow} \rho_{\beta,\gamma,\downarrow}} c_{\beta,\uparrow}^+ + \sqrt{t_{\gamma,\beta,\downarrow} \rho_{\gamma,\beta,\downarrow}} c_{\gamma,\uparrow}^+ \right] |0\rangle,$$

where we assume that $\alpha$, $\beta$ and $\gamma$ are three fully spin polarized ferromagnets with spin orientations $\sigma_\alpha = \uparrow$, $\sigma_\beta = \sigma_\gamma = \downarrow$ (see Fig. [1]).
Interpretation of the transport formula: In the presence of an arbitrary number of electrodes, the BCS-like wave function is a superposition of all possible Cooper pairs:

$$|\psi\rangle = N^{-1/2} \sum_{p,q} \sqrt{t_{\alpha_p} t_{\alpha_q} \rho_{\alpha_p,\uparrow} \rho_{\alpha_q,\downarrow}} |\gamma_{\alpha_p,\uparrow} \gamma_{\alpha_q,\downarrow} \rangle |0\rangle,$$

where the normalization coefficient is $N = \sum_p t_{\alpha_p} \rho_{\alpha_p,\uparrow} \sum_q t_{\alpha_q} \rho_{\alpha_q,\downarrow}$. One can define a projection operator associated to the Cooper pair $(p,q)$: $\hat{P}_{p,q} = c_{\alpha_p,\uparrow} c_{\alpha_q,\downarrow} + c_{\alpha_p,\downarrow} c_{\alpha_q,\uparrow}$. The spin-up current through electrode $a_p$ is found to be

$$I_p = 4\pi^2 N^2 \sum_q \int d\omega \left[ n_F(\omega - eV) - n_F(\omega) \right] |\gamma_{\alpha_p,\uparrow} \gamma_{\alpha_q,\downarrow} \rangle \langle \gamma_{\alpha_p,\uparrow} \gamma_{\alpha_q,\downarrow} | \hat{P}_{p,q} |\psi\rangle |^2 \frac{G_{x,x,1,2}}{|a_p|},$$

where $\gamma_{\alpha_p,\uparrow} = \pi |t_{\alpha_p} |^2 \rho_{\alpha_p,\uparrow}$. We interpret the transport formula (4) as follows: (i) Before the tunneling event, the Cooper pairs are described by the BCS-like wave function. (ii) Short after the tunneling event, there is one Cooper pair transferred in electrodes $(a_p, a_q)$. (iii) Long after the tunneling event, there is a spin-up electron in electrode $p$ and a spin-down electron in electrode $q$ without phase coherence between the two.

The situation in which $S$ is replaced by a normal metal is already non trivial. In this case, the wave function is a linear superposition of single electron states while we pointed out here the existence of a linear superposition of two-electron states.

4 Experiment proposal

Geometry: Now we discuss a more realistic experiment that can be made without ferromagnets. The idea is to connect two electrodes to a superconductor. One of these electrodes contains an Aharonov-Bohm loop (see Fig. 2).

Physical picture: Let us assume that the Aharonov-Bohm loop is weakly connected to the rest of the circuit. In the tunnel regime, there are discrete energy levels $\epsilon_n(\phi)$ on the Aharonov-Bohm loop, which are periodic functions of $\phi$. One-electron conduction is maximal when one level is resonant. The Aharonov-Bohm loop can be represented by an effective transmission amplitude $T(\phi)$. The Cooper pair current is proportional to $|t_{x,\beta}|^2 |T(\phi)|^2$, which oscillates with $\phi$. As a consequence, the current through the left electrode on Fig. 2 oscillates as a function of the flux enclosed in the right electrode.
Single site calculation: The non local contribution to the current through the left electrode on Fig. 2 takes the form:

\[ I_{A,n.l.}^{\beta,\sigma} = \int Q \left[ \frac{4\pi^2 |t_{x,\beta}|^2 |t_{x,a}|^2 \rho_{\beta,\sigma} \rho_{a,-\sigma} \rho_{b,-\sigma} \left[ (\gamma_{-\sigma} + \gamma_{-\sigma}')^2 + 2\gamma_{-\sigma} \gamma_{-\sigma}' (1 - \cos (2\pi \phi/\phi_0)) \right]}{1 + (\rho_{a,-\sigma} + \rho_{b,-\sigma}) (\gamma_{-\sigma} + \gamma_{-\sigma}') + 2\gamma_{-\sigma} \gamma_{-\sigma}' \rho_{a,-\sigma} \rho_{b,-\sigma} (1 - \cos (2\pi \phi/\phi_0))^2} \right] \times \left| n_F(\omega - eV) - n_F(\omega) \right| |G_{A,x,x,1,2}^A|^2 d\omega. \]  

The current (5) oscillates with a period \( \phi_0 = h/e \), which is in agreement with the tunnel limit behavior. The magnitude of the oscillatory component is of order \( |t_{x,\beta}|^2 |t_{x,a}|^2 \). Note that the local current

\[ I_{\beta,\text{local}}^{A} = 4\pi^2 |t_{x,\beta}|^4 \rho_{\beta,\uparrow} \rho_{\beta,\downarrow} \int d\omega \left| n_F(\omega - eV) - n_F(\omega) \right| |G_{x,x,1,2}^A|^2, \]  

also oscillates with \( \phi \) because of the renormalization of the Green’s function \( G_{x,x,1,2}^A \). These oscillations in the local contribution correspond to processes in which a Cooper pair is extracted from the superconductor and is transferred into the right reservoir. The Cooper pair couples to the Aharonov-Bohm flux, comes back in the superconductor and is transferred into the left electrode. This process appears at order \( |t_{x,\beta}|^4 |t_{x,a}|^8 \) while the current due to non separable correlations appears at order \( |t_{x,\beta}|^2 |t_{x,a}|^2 \). If at least one of the interfaces has a low transparency, it would be possible to retain mostly the current due to non separable correlations.

5 Conclusions

To summarize, we have pointed out a few situations intended to probe non separable correlations in electronic systems. The dependence of the superconducting gap upon the spin orientation of a ferromagnetic environment may be tested in experiments. It would be harder to test the linear superposition of correlated pairs of electrons. The Aharonov-Bohm effect can be probed in experiments because phase coherence can propagate over sufficiently large distances in normal metals. Finally, we note that an electronic EPR experiment based on ballistic electrons in semiconductors has been proposed recently [12].

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