GLOBAL GENERAL RELATIVISTIC MAGNETOHYDRODYNAMIC SIMULATION OF A TILTED BLACK HOLE ACCRETION DISK

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ABSTRACT

This paper presents a continuation of our efforts to numerically study accretion disks that are misaligned (tilted) with respect to the rotation axis of a Kerr black hole. Here we present results of a global numerical simulation which fully incorporates the effects of the black hole spacetime as well as magnetorotational turbulence that is the primary source of angular momentum transport in the flow. This simulation shows dramatic differences from comparable simulations of untilted disks. Accretion onto the hole occurs predominantly through two opposing plunging streams that start from high latitudes with respect to both the black hole and disk midplanes. This is due to the aspherical nature of the gravitational spacetime around the rotating black hole. These plunging streams start from a larger radius than would be expected for an untilted disk. In this regard, the tilted black hole effectively acts like an untilted black hole of lesser spin. Throughout the duration of the simulation, the main body of the disk remains tilted with respect to the symmetry plane of the black hole; thus, there is no indication of a Bardeen-Petterson effect in the disk at large. The torque of the black hole instead principally causes a global precession of the main disk body. In this simulation, the precession has a frequency of $3(M_{\odot}/M)\,\text{Hz}$, a value consistent with many observed low-frequency quasi-periodic oscillations. However, this value is strongly dependent on the size of the disk, so this frequency can be expected to vary over a large range.

Subject headings: accretion, accretion disks — black hole physics — galaxies: active — MHD — relativity — X-rays: stars

1. INTRODUCTION

Black hole accretion has long been postulated to power the energetic emissions seen from quasars, active galactic nuclei (AGNs), and many galactic X-ray sources; there is now ample observational evidence to support such claims (e.g., Krolik 1999; McClintock & Remillard 2006). Black hole accretion flows are also of interest as laboratories to test predictions of general relativity. However, the nature of such flows is complex, involving time-dependent, multidimensional dynamics with generically little symmetry. Hence, numerical simulations play an integral role in advancing our understanding.

Many simulations of black hole accretion flows have been carried out over the past three decades, both in the hydrodynamic (e.g., Wilson 1972; Hawley et al. 1984; Hawley 1991) and magnetohydrodynamic (MHD; e.g., Koide et al. 1999; Gammie et al. 2003; De Villiers & Hawley 2003b) regimes. A common assumption in nearly all of the work to date has been that the symmetry plane of the central black hole is aligned with the midplane of the accretion flow, at least in some averaged sense. However, there is compelling observational evidence in several black hole X-ray binaries, e.g., GRO J1655–40 (Orosz & Bailyn 1997) and XTE J1550–564 (Hannikainen et al. 2001; Orosz et al. 2002), and AGNs, e.g., NGC 3079 (Kondratko et al. 2005), NGC 1068 (Caproni et al. 2006), and NGC 4258 (Caproni et al. 2007), suggesting that misaligned (or tilted) black holes may be common (see also Maccarone 2002). This claim relies on the observation of relativistic bipolar jets (thought to be aligned with the spin axis of the black hole) that are not perpendicular to the plane of the accretion disk observed at large scales.

There are also compelling theoretical arguments that many black holes should be tilted. First, the formation avenues for many black hole–disk systems favor, or at least allow for, a tilted configuration (Fragile et al. 2001). In stellar-mass binaries, the orientation of the outer disk is fixed by the binary orbit, whereas the orientation of the black hole is determined by how it became part of the system, whether through a supernova explosion or multibody interaction. If the black hole formed from a member of a preexisting binary through a supernova, then the black hole could be tilted if the explosion were asymmetric. If the black hole joined the binary through multibody interactions, such as binary capture or replacement, then there would have been no preexisting symmetry, so the resulting system would nearly always harbor a tilted black hole. This same argument can be extended to AGNs in which merger events reorient the central black hole or its fuel supply and result in repeated tilted configurations.

If an accretion disk is misaligned or tilted, it will be subject to Lense-Thirring precession. For an ideal test particle in a slightly tilted orbit at a radius $r$ around a black hole of mass $M$ and specific angular momentum $a$, this precession occurs at an angular frequency $\Omega_{LT} \approx 2aM/r^3$. Close to the black hole, this is comparable to the orbital angular frequency $\Omega = (M/r^3)^{1/2}/[1 + a(M/r^3)^{1/2}] \approx \Omega_{Kep}$. However, because of its strong radial dependence, Lense-Thirring precession becomes much weaker far
from the hole. Therefore, a disk will experience a differential pre-
cession that will tend to twist and warp it.

A warping disturbance can be communicated through a disk in
either a diffusive or wavelike manner. In the diffusive case, the
warping is limited by secular (i.e., “viscous”) responses within the
disk. In such a case, Lense-Thirring precession is expected to do-
minate out to a unique, nearly constant transition radius (Bardeen &
Petterson 1975; Kumar & Pringle 1985), inside of which the disk
is expected to be flat and aligned with the black hole mid-
plane and outside of which the disk is also expected to be flat but
in a plane determined by the angular momentum vector of the gas
reservoir. This is what we term a “Bardeen-Petterson” configu-
ration. Interestingly, data for the two black hole X-ray binaries
previously mentioned are best fit by disk components with in-
clinations that differ from their binary measurements. The best-
fit inclinations are more consistent with inclination constraints
derived from the radio jets (Davis et al. 2006), possibly sug-
gestng Bardeen-Petterson configurations. Caproni et al. (2006)
also claim that the observations of NGC 1068 are consistent with
the warping only occurred inside a radius in the disk at which the
accretion onto rapidly rotating (Kerr) black holes. We found that,
although Lense-Thirring precession did cause the disk to warp,
neither a diffusive or wavelike manner. In the diffusive case, the
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and

\[ B^\mu = W (B^\mu - B^0 V^\mu) \]  

(8)

is the divergence-free \((\partial B^\mu / \partial x^i = 0)\), spatial \((B^0 = 0)\) representation of the field. The time component of the magnetic field \(B^0\) is recovered from the orthogonality condition \(B^\mu u_\mu = 0\),

\[ B^0 = -\frac{W}{g} \left( g_{00} B^0 + g_{ij} B^i V^j \right). \]  

(9)

The relativistic enthalpy is

\[ h = 1 + \frac{\Gamma P}{(\Gamma - 1) \rho} + \frac{Q}{\rho c^2}, \]  

(10)

where we have assumed an equation of state of the form \(P = (\Gamma - 1) \rho e\). Finally, \(P_B = |B|^2/8\pi = g_{\mu\nu} B^\mu B^\nu / 8\pi\) is the magnetic pressure. We use the scalar \(Q\) from Anninos et al. (2005) with \(k_2 = 2.0\) and \(k_1 = 0.3\). We fix the divergence cleanser coefficients to be \(c_0 = c_{\text{eff}} \Delta \min / \Delta t\) and \(c_p = c_h\), where \(c_{\text{eff}} = 0.7\) is the Courant coefficient, \(\Delta \min\) is the minimum covariant zone length, and \(\Delta t\) is the evolution time step. For simplicity, we hold the time step fixed at \(\Delta t = c_{\text{eff}} \Delta \min\) throughout the simulation.

These GRMHD equations are evolved in a “tilted” Kerr-Schild polar coordinate system \((t, r, \vartheta, \varphi)\). This coordinate system is related to the usual (untilded) Kerr-Schild coordinates \((t, r, \theta, \phi)\) through a simple rotation about the \(y\)-axis by an angle \(\beta_0\), such that

\[
\begin{pmatrix}
\sin \vartheta \sin \varphi \\
\cos \vartheta \\
\sin \beta_0 \\
0
\end{pmatrix} = \frac{1}{\sin \beta_0} \begin{pmatrix}
\cos \beta_0 & 0 & -\sin \beta_0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \cos \beta_0 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
\sin \vartheta \sin \varphi \\
\cos \vartheta \\
\sin \beta_0 \\
0
\end{pmatrix}.
\]  

(11)

The full tilted metric terms are provided in Fragile & Anninos (2005; see also Fragile & Anninos 2007). The computational advantages of the “horizon-adapted” Kerr-Schild form of the Kerr metric were first described in Papadopoulos & Font (1998) and Font et al. (1998). The primary advantage is that, unlike Boyer-Lindquist coordinates, there are no singularities in the metric terms at the event horizon, so the computational mesh can extend into the hole’s interior. In principle, this should keep the inner boundary causally disconnected from the flow, although numerically there is still some communication.

The simulation is carried out on a spherical polar mesh with nested resolution layers. The base grid contains 323 mesh zones and covers the full \(4\pi r_\text{g}^2\) sr. Varying levels of refinement are added on top of this base layer; each refinement level doubles the resolution relative to the previous layer. The main simulation, referenced as model 915h, has two levels of refinement, thus achieving a peak resolution equivalent to a \(128^3\) simulation. For comparison we also discuss results from an equivalent untilted simulation (model 90h) with the same resolution. As an argument that our results are reasonably well converged, we also include results from two other tilted simulations: one with a single refinement layer and an equivalent resolution of \(64^3\) (model 915m) and another that starts from a base grid of \(24 \times 24 \times 32\) and adds three layers of refinement for an equivalent resolution of \(192 \times 192 \times 256\) (model 915vh). The evolution times for these simulations differ as described below. In all cases, the full refinement covers the region \(r_{\min} \leq r \leq r_{\max}\), \(0.075\pi = \vartheta_1 \leq \vartheta \leq \vartheta_2 = 0.925\pi\), and \(0 \leq \varphi \leq 2\pi\), where \(r_{\min} = 0.98r_{\text{BH}} = 1.41r_\text{g}\) and \(r_{\max} = 120r_\text{g}\) are the inner and outer boundaries of the grid, respectively, and \(r_{\text{BH}} = 1.43r_\text{g}\) is the black hole horizon radius. The primary motivation for using a tilted grid is to allow us to maintain a reasonable Courant-limited time step without sacrificing any spatial resolution within the disk or completely excluding the region near the pole. The gain in computational efficiency is significant, since for a polar mesh, the time step scales as \(\Delta t \sim r_{\min} \sin \vartheta_{\min} \Delta \varphi\). By underresolving the polar region, we gain by increasing both \(\vartheta_{\min}\) and \(\Delta \varphi\). With two levels of refinement, we are able to use a time step that is a factor of 11.8 larger than what we could use if our most refined layer extended all the way to the pole. The main drawback of this approach is that we are unable to resolve the region in which jets are expected to form.

In the radial direction we use a logarithmic coordinate of the form \(\eta \equiv 1.0 + \ln(r/r_{\text{BH}})\). The spatial resolution near the black hole horizon is \(\Delta r \approx 0.05r_\text{g}\); near the initial pressure maximum of the torus, the resolution is \(\Delta r \approx 0.5r_\text{g}\). Both are considerably smaller than the initial characteristic MRI wavelength \(\lambda_{\text{MRI}} \equiv 2\pi r_\text{A}/\Omega \approx 2.5r_\text{g}\). This also gives us a large number of zones inside the plunging region. In the angular direction, in addition to the nested grids, we use a concentrated latitude coordinate \(x_2\) of the form \(\varphi = x_2 + \frac{1}{2}(1 - h) \sin(2x_2)\) with \(h = 0.5\), which concentrates resolution toward the midplane of the disk. As a result, \(r_{\text{cen}} \Delta \varphi = 0.3r_\text{g}\) near the midplane, while it is a factor of \(\sim 3\) larger for the fully refined zones near the pole. The grid used in models 915h and 90h is shown in Figure 1.

Since we cover the full \(4\pi r_\text{g}^2\) sr, the only “external” boundaries are the inner and outer radial boundaries, where we apply outflow conditions. Fluid variables are set the same in the external boundary zone as in the neighboring internal zone, except for velocity, which is chosen to satisfy

\[ V^r_{\text{ext}} = \begin{cases} 
V^r_{\text{int}}, & \text{if } V^r \text{ points off the grid,} \\
- V^r_{\text{int}}, & \text{if } V^r \text{ points onto the grid.} 
\end{cases} \]  

(12)

In the azimuthal direction we apply periodic boundaries at \(\varphi = 0\) and \(2\pi\). Since Cosmos++ is a zone-centered code, we do not
have to treat the pole ($\vartheta = 0$ or $\pi$) directly. Instead, unboosted scalar quantities, such as the gas pressure $P$, in the “ghost” zones across the pole are filled with real data from the corresponding zone located $180^\circ$ away in azimuth. Unboosted vector quantities, such as velocity $V^i$, are similarly filled with data from appropriate real zones, albeit with the signs reversed for the $\vartheta$ and $\varphi$ components to maintain a consistent sense of coordinate direction across the pole. Boosted quantities, since they contain the metric determinant $(-g)^{1/2}$, are reflected across the pole so they extrapolate to zero there. This treatment differs from the pure reflecting boundaries used in other works (e.g., De Villiers et al. 2003; McKinney 2006) in its treatment of the unboosted variables. For untilded black holes the difference is relatively minor. However, for tilted black holes our approach makes the pole more transparent to the fluid.

We initialize these simulations starting from the analytic solution for an axisymmetric torus around a rotating black hole (Chakrabarti 1985). To provide a link with an untilted model solution for an axisymmetric torus around a rotating black hole more transparent to the fluid.

However, for tilted black holes our approach makes the pole reflecting boundaries used in other works (e.g., De Villiers et al. 2003). As in model KDP, we take $l_{\text{in}} = u_l(r_{\text{in}})$, the surface binding energy of the torus, from $u_l^2 = g^\mu_\nu - 2l^\nu g^\mu_\phi$. The solution of the torus variables can now be specified. The internal energy of the torus is (De Villiers et al. 2003)

$$\epsilon(r, \theta) = \frac{1}{\Gamma} \left[ \frac{u_m f(l_{\text{in}})}{u_l(r, \theta) f(l(r, \theta))} \right].$$

Finally, having chosen $r_{\text{in}}$ we can obtain $u_m = u_l(r_{\text{in}})$, the surface binding energy of the torus, from $u_l^2 = g^\mu_\nu - 2l^\nu g^\mu_\phi$. It is given by the surface density of the torus $\rho_{\text{surf}}$, which corresponds to $10^5$ to $10^6$ of the initial background is replaced by evolved disk material. These floors are very seldom applied once the initial background is replaced by evolved disk material.

This introduces inflow through the horizon without creating large velocity jumps at the torus surface. This background is initially more dense than the static background used by De Villiers et al. (2003). However, since this background reservoir is not replenished at the outer boundary, it is rapidly depleted and has virtually no long-term dynamical impact on the problem. Numerical floors are placed on $\rho$ and $e$ at approximately $10^{-10}$ and $10^{-16}$ of their initial maxima, respectively. These floors are very seldom applied once the initial background is replaced by evolved disk material.

The final step of the initialization is to tilt the black hole by an angle $\beta_0 = 15^\circ$ relative to the disk (and the grid) by transforming the Kerr metric. The full transformation is provided in Fragile & Anninos (2005; see also Fragile & Anninos 2007). Thus, while the torus is responding to the action of the MRI, it will also experience a gravitomagnetic torque from the tilted black hole.

3. RESULTS

In the main simulation (915h), the torus is evolved for a total of 10 orbital periods ($10T_{\text{orb}}$) as measured at $r = r_{\text{cen}}$, which corresponds to $\sim 350$ orbits near $r_{\text{isco}} = 2.32 r_g$, the coordinate radius of the innermost stable circular orbit (ISCO). For prograde and retrograde orbits in the symmetry plane of the black hole. The very high
resolution simulation (915vh) is only run for half as long (5t_orb),
while the lower resolution simulation (915m) is run for twice as
long (20t_orb). Figure 2 shows snapshots of the disk from model
915h at times \( t = 0, \ t_{\text{orb}}, 2t_{\text{orb}}, 4t_{\text{orb}}, 7t_{\text{orb}}, \) and \( 10t_{\text{orb}} \). The first
orbit is dominated by winding of the magnetic field lines and non-
linear growth of the MRI. Both of these cause rapid redistribu-
tions of disk material and angular momentum. The initial torus is
stretched radially and material begins to accrete onto the hole and
is also carried out to large radii. A strong current sheet forms in the
initial symmetry plane of the disk through differential winding.

From orbits 1–2, MRI-driven turbulence begins to grow in the
inner parts of the disk. At the same time, some bending of the
disk due to the differential precession from the hole becomes ap-
parent. The MRI is fully developed through most of the disk
around orbit 2.

By about orbit 7–8, the disk has reached a quasi-steady state.
In the remainder of this section we detail the properties of the re-
sultant structure. We follow an “inside-out” track, starting from
key features of the flow near the hole and working toward larger
radii. Where practical, we draw attention to similarities and dif-
fences between the quasi-steady structure that results in this
simulation and the unilluminated simulations of De Villiers et al. (2003).
In particular, we draw attention to the fact that some features, such
as the inner torus and plunging region, are significantly altered,
while others, such as the main body and coronal envelope, show
very similar properties. Again, because of the varying levels of
refinement along the poles, we do not discuss the evacuated funnel
or funnel-wall jet in this paper.

### 3.1. Global Structure

#### 3.1.1. Plunging Streams

Perhaps the most striking feature in the tilted disk at late times
are the two opposing streams that start from high latitudes both
with respect to the black hole symmetry plane and the disk mid-
plane (Fragile et al. 2007). Figure 3 shows a zoomed-in view of
the region around the black hole including these streams. Note that
stream 1 remains entirely above the black hole symmetry plane,
while stream 2 remains below. Clearly, the material in each stream
is in a plunging orbit into the black hole. Hence, we refer to these
features as the “plunging streams.”

Figure 4 captures the plunging streams from a different per-
spective. This image is a view looking down the angular mo-
mentum axis of the black hole onto a single isodensity surface.
The two opposing streams are clearly visible in the interior region
of the disk as well as two relatively evacuated lobes.

As material passes through the plunging streams it undergoes
strong differential precession. As we show below, the precession
totals approximately 180°, accounting for how the material in the
plunging streams is able to enter the black hole from the opposite
azimuth from which it began its plunge without ever passing
through the symmetry plane of the hole.
Two very important points to make about these streams is that they appear to be stable and stationary. They begin forming as early as $t = T_{\text{orb}}$ and last until the end of the simulation. During this time their azimuthal location does not change appreciably. The interesting questions are why do these opposing plunging streams form and why do they start from such high latitude with respect to the black hole symmetry plane and disk midplane? The answers, of course, are related and the fundamental cause is the aspherical nature of the gravitational spacetime around the rotating black hole. This is best illustrated by considering the dependence of $r_{\text{ISCO}}$ on inclination for orbits that are circular in the sense that they have constant coordinate radius. Briefly, $r_{\text{ISCO}}$ is the radius at which the quantity

$$R \equiv A^2 \left( \frac{d\tau}{d\theta} \right)^2$$

and its first two derivatives equal zero, i.e., $R = R' = R'' = 0$, where $E$, $L_z$, and $Q$ are the energy, angular momentum, and Carter constant, respectively, describing orbits around Kerr black holes (Hughes 2001), $A = r^2 + a^2 \cos^2 \theta$, and $\Delta = r^2 - 2Mr + a^2$. Following Hughes (2001) we can eliminate $Q$ in favor of the inclination $i$ defined as

$$\cos i = \frac{L_z}{(L_z + Q)^{1/2}}.$$  

Figure 5 illustrates this dependence for a few selected cases of $a$. The key point of the formula and the plot is that orbital stability around a rotating black hole is strongly dependent on the inclination of the orbit. Note that the unstable region increases monotonically for increasing inclination.

We can make better use of the information in Figure 5 by converting it to a polar plot (using only the prograde orbits) and overlaying it onto a plot of data from the simulation, as is done in Figure 6. Such a polar plot creates a representation of the prograde “ISCO surface” (symmetric about the spin axis of the black hole), which gives a clear indication of where the most unstable regions of the spacetime are. Note that the plunging orbits highlighted previously start near where the disk first encounters the
For both simulations, the data has been time-averaged over the interval \( t = 10t_{\text{orb}} \) approximately 180° away in azimuth (black arrows). The plot is overlaid with a polar plot of the “ISCO surface” for prograde orbits about an a = 0.9 black hole (white line). This surface is symmetric about the spin axis of the hole. Note that the plunging streams from Figs. 3 and 4 start near the largest cylindrical radius \((r \cos \vartheta)\) of this surface (white arrows) and connect with the horizon approximately 180° away in azimuth (black arrows).

ISCO surface. More precisely, the streams start near the largest cylindrical radius \((r \cos \vartheta)\) of the ISCO surface, measured with respect to the angular momentum axis of the disk. This explains why the plunging streams start at such high inclinations relative to the black hole symmetry plane and the disk midplane and why there are only two streams. The plunging region is no longer azimuthally symmetric from the perspective of the disk.

Another point to take away from Figures 5 and 6 is that \( r_{\text{ISCO}} \) is larger for larger inclinations. Thus, for a given black hole spin plunging orbits will always start further away from the hole for more tilted disks. The tilted black hole effectively acts like an untilted black hole of lower spin, which would likewise have a larger \( r_{\text{ISCO}} \).

3.1.2. Inner Torus

In our tilted simulation, the plunging streams appear to connect directly to the main disk body without a clearly identifiable intermediate “inner torus.” This appears to be a particular result of the tilted simulation and not, for instance, due to the differences in the coordinates used in our simulation (Kerr-Schild) versus those used in De Villiers et al. (2003; Boyer-Lindquist) or numerical techniques. We base this statement on the fact that our own untilted simulation in Kerr-Schild coordinates shows an inner torus very similar to the one described in De Villiers et al. (2003). For instance, Figure 7 shows the shell-averaged density and pressure as a function of radius for our tilted and untilted simulations. Shell-averaged quantities are computed over the most refined grid as

\[
\langle Q \rangle_A(r, t) = \frac{1}{A} \int_0^{2\pi} \int_{\theta_1}^{\theta_2} Q \sqrt{-g} d\vartheta d\varphi,
\]

where \( A = \int_0^{2\pi} \int_{\theta_1}^{\theta_2} (-g)^{1/2} d\vartheta d\varphi \) is the surface area of the shell. The data in Figure 7 has also been time-averaged over the final orbit, \( 9t_{\text{orb}} \leq t \leq 10t_{\text{orb}} \), where time averages are defined as

\[
\langle Q \rangle_t = \frac{1}{t_{\text{max}} - t_{\text{min}}} \int_{t_{\text{min}}}^{t_{\text{max}}} Q dt.
\]

In the untilted simulation, both the density and the pressure show local maxima near 4.5\( R_G \), indicating an inner torus. The tilted simulation, on the other hand, shows only marginal evidence for local maxima near 10\( R_G \).

Another check of the presence of an inner torus is to look at the distribution of specific angular momentum in the disk. Because the inner torus is partially supported by pressure gradients, some portion of the flow must be locally supergeodesic. In Figure 8 we plot the density-weighted shell average of the specific angular momentum \( \langle \ell \rangle_A = \langle \rho \ell \rangle_A / \langle \rho \rangle_A \) as a function of radius, again time-averaged over the interval \( t = 9t_{\text{orb}} - 10t_{\text{orb}} \). We compare this against...
the specific angular momentum distribution of circular orbits with inclinations of 15° and 0°. These are calculated from the expression

\[ l = \frac{N_1 + \Delta(Mr)^{1/2}N_2^{1/2} \cos i}{D}, \tag{26} \]

where

\[ N_1 = -aMr(3r^2 + a^2 - 4Mr) \cos^2 i, \tag{27} \]
\[ N_2 = r^4 + a^2 \sin^2 i(a^2 + 2r^2 - 4Mr), \tag{28} \]
\[ D = a^2(2r^2 + a^2 - 3Mr) \sin^2 i + r^4 + 4M^2r^3 - 4r^2M - Mra^2, \tag{29} \]

which comes from noting that for circular orbits \( R = R' = 0 \) from equation (22) and from the definition \( l = L_z/E \). Both simulations show a nearly geodesic angular momentum distribution through most of the disk. The untilted simulation shows a small region of supergeodesic flow inside \( 4r_G \), corresponding to the inner torus. The tilted simulation, on the other hand, remains subgeodesic throughout, consistent with the absence of an inner torus.

Another indication that the inner torus is less prominent in the tilted simulation than the untilted one comes from comparing the total rest mass in the near-hole region \( (r < r_{cut} = 10r_G) \). This is done in Figure 9, where we plot the time histories of the total (volume-integrated) rest mass

\[ \langle \rho a^0 \rangle_t = \int_0^{2\pi} \int_0^\pi \int_{r_{cut}}^{r_{t_{orb}}} D \, dr \, d\theta \, d\phi. \tag{30} \]

At \( t = 10t_{orb} \), the inner torus is 59% less massive in model 915h.

When present, the inner torus usually performs two functions: regulating the accretion of matter into the black hole and serving as the launching point for the funnel-wall jet. Therefore, we may expect a weaker funnel-wall jet (to be discussed in future work) and a higher mass accretion rate in our tilted-disk simulation relative to the untilted simulation due to the less prominent inner torus in the former. We compute the mass accretion rate

\[ \dot{M}(r) = \int_0^{2\pi} \int_0^\pi DV_r \, d\theta \, d\phi \tag{31} \]

100 times per \( t_{orb} \) (about every \( 8M \)) at each of the external grid boundaries. Figure 10a shows a plot comparing \( \dot{M}(r_{min}) \) for our equivalent tilted and untilted simulations. When averaged over the quasi–steady state of each simulation (from \( t = 7t_{orb} - 10t_{orb} \)), \( \langle \dot{M} \rangle_t \) into the hole for the tilted simulation (915h) is \( 7.2 \times 10^{-6} \), while for the untilted one (90h), it is \( 4.9 \times 10^{-6} \). There is a clear tendency toward a higher \( \dot{M} \) in the tilted-disk simulation.

Figure 10b compares \( \dot{M} \) of the tilted-disk simulation at three different resolutions. Due to the chaotic nature of the mass accretion we do not expect the individual peaks to match; yet we are encouraged that the overall shape and magnitude of the two high-resolution models (915h and 915vh) are very consistent, suggesting

![Fig. 8.—Plot of the density-weighted time- and shell-averaged specific angular momentum \( \langle |l| \rangle_t \) (thick lines) as a function of radius for equivalent (a) tilted \( \beta_0 = 15° \) (915h) and (b) untilted \( \beta_0 = 0° \) (90h) simulations. For both simulations, the data has been time-averaged over the interval \( t = 9t_{orb} - 10t_{orb} \). In each plot, a comparison is provided with the specific angular momentum distribution of circular orbits with inclinations of 15° and 0° , respectively (dashed lines). For reference we also include the initial angular momentum distribution in the midplane of the torus (thin lines).](image)

![Fig. 9.—Total rest mass in the near-hole region \( (r < 10r_G) \) as a function of time for the tilted (915h and 915vh) and untilted (90h) simulations. The mass and time are normalized by the initial mass and orbital period of the torus, respectively.](image)
we are reasonably well converged. The medium-resolution simulation (model 915m), on the other hand, is clearly underresolved.

3.1.3. Main Disk Body and Coronal Envelope

The main disk body does not differ substantially between the tilted and untilted simulations, except in the notable fact that the tilted disk precesses (as discussed in § 3.2.2 below). Likewise, the coronal envelope, which extends above and below the disk, shows very similar properties in all our simulations. The material in the coronal envelope is characterized by low density and rough magnetic equipartition ($\beta_{\text{mag}} \approx 1$). By contrast, the main body of the disk is generally gas pressure-dominated ($\beta_{\text{mag}} < 1$). Therefore, a plot of $\beta_{\text{mag}}$ and $\beta$, such as Figure 11, provides a convenient means to identify these two regions. As found in De Villiers et al. (2003), the material in the coronal envelope moves mostly radially outward, yet has $-h_{\text{ut}} < 1$. This suggests that the material may be gravitationally bound, in which case it must circulate back to the disk at large radii. However, we point out that this definition of binding energy ignores the contribution of the magnetic fields, so some of this material may in fact escape the system. We plan to examine outflows from tilted disks more thoroughly in future work.

Because the disk is precessing, its angular momentum axis does not remain aligned with the grid. Therefore, an azimuthal slice through the disk at late times, such as Figure 11, may give the impression that the disk has aligned with the symmetry plane of the black hole when indeed this is not the case. We now turn to the question of disk alignment and precession.

3.2. Results Specific to a Tilted Disk

3.2.1. Tilt

One key diagnostic for describing the global response of a tilted disk subject to Lense-Thirring precession is the tilt between the angular momenta of the black hole and disk as a function of radius and time. For example, in the Bardeen-Petterson solution no time variability is observed, and the tilt transitions from nearly zero close to the black hole to a nonzero asymptote at large radii.

As in Fragile & Anninos (2005) we recover the tilt from the simulation data using the definition

$$\beta(r) = \arccos \left( \frac{\mathbf{J}_{\text{BH}} \cdot \mathbf{J}_{\text{disk}}(r)}{||\mathbf{J}_{\text{BH}}|| ||\mathbf{J}_{\text{disk}}(r)||} \right),$$

where

$$\mathbf{J}_{\text{BH}} = (-aM \sin \beta_0 \mathbf{x}, 0, aM \cos \beta_0 \mathbf{z})$$

is the angular momentum vector of the black hole and

$$\mathbf{J}_{\text{disk}}(r) = [(\mathbf{J}_{\text{disk}})_1 \mathbf{x}, (\mathbf{J}_{\text{disk}})_2 \mathbf{y}, (\mathbf{J}_{\text{disk}})_3 \mathbf{z}]$$

is the angular momentum vector of the disk in an asymptotically flat space. This is given by

$$\mathbf{J}_{\text{disk}}(r) = \epsilon_{\mu\nu\sigma} L_{\mu\nu} S_\sigma \frac{2 \sqrt{-S_\sigma S_\sigma}}{S_\mu}.$$

Fig. 10.—(a) Plot of the mass accretion history from model 915h with $\beta_0 = 15^\circ$ (thick line) and model 90h with $\beta_0 = 0^\circ$ (thin line). The accretion rate and time are normalized by the initial mass and orbital period of the torus, respectively. (b) Plot of mass accretion rate, comparing our medium (915m), high (915h), and very high (915vh) resolution tilted-disk simulations. The very high resolution simulation was only run to $t = 5 \sigma_{\text{orb}}$.

Fig. 11.—Azimuthal slice through the simulation along $\varphi = 0$ taken from the final dump ($t = 10 \sigma_{\text{orb}}$). The ratio of magnetic pressure to gas pressure ($\beta_{\text{mag}}$) is represented as a pseudocolor plot. The colors are scaled logarithmically and cover the range $10^{-2} \leq \beta_{\text{mag}} \leq 10^2$. The gas density is given by isocontours at $\rho = 10^{-2}, 10^{-1.5}, 10^{-1}$, and $10^{-0.5} \rho_{\text{max,0}}$. As with Fig. 6, this figure is oriented in the sense of the grid, so that the black hole is tilted $15^\circ$ to the left. The apparent tilt of the disk is actually due to its precession about the black hole spin axis, such that the angular momentum axis of the disk is no longer in the plane of this image; the disk has not actually realigned with the hole. We remind the reader that the region near the poles is not sufficiently resolved, so caution should be used when interpreting results there.
where
\[ L^\mu = \int (\nabla^\mu T^{\nu\theta} - x^\nu T^{\mu\theta}) \, d^3 x \] (36)
and \[ S^\sigma = \int T^{\sigma\theta} \, d^3 x. \] The equations for \( L^\mu \) and \( S^\sigma \) are integrated over concentric radial shells of the most refined grid layer, e.g.,
\[ S^\sigma(r) = \int_0^{2\pi} \int_{\theta_1}^{\theta_2} \int_0^{\infty} T^{\sigma\theta} r^\sigma \Delta r \, d\theta \, d\varphi. \] (37)

The unit vector \( \hat{y} \) points along the axis about which the black hole is initially tilted, and \( \hat{z} \) points along the initial angular momentum axis of the disk.

In Figure 12 we show the radial profile of \( \beta \) time-averaged over the interval \( 9t_{\text{orb}} \leq t \leq 10t_{\text{orb}} \). Recall \( \beta_0 = 15^\circ \) for this simulation. This profile remains fairly constant over many orbital times once the quasi-steady state is reached, so the time-averaged data gives a good representation for all \( t \geq 7t_{\text{orb}} \). The variability from this time-averaged profile is generally \( \lesssim 20\% \) and is generally carried by moderate-amplitude waves traveling through the disk. The increase in tilt at \( r \approx 10r_G \) is attributable to the high-latitude plunging streams described in \( \S \) 3.1.1.

One very obvious characteristic of the profile in Figure 12 is that \( \beta \) does not approach zero except perhaps very close to the hole. Thus, we do not see evidence for the Bardeen-Petterson effect in this simulation. This is not surprising since the Bardeen-Petterson solution is only expected for thin disks \( (H/r < \alpha) \). This is not the applicable regime for this simulation, as we illustrate in Figure 13, which shows \( H/r \) and \( \alpha \) plotted as functions of \( r \).

The scale height \( H(r) \) is defined in each radial shell as one-half the distance \( (0.5r\Delta\theta) \) between the two points where \( \rho = \rho_{\text{max}}/e \), where we use the time-averaged density along the \( \varphi = 0 \) azimuthal slice. The dimensionless stress parameter \( \alpha \) in the disk is taken to be
\[ \alpha = \left\langle \frac{\left| ||u'\nu'||B'|| - B'\nu| \right|^2}{4\pi P} \right\rangle_A. \] (38)

We restrict the calculation of \( \alpha \) to only bound material \( (-hu_r < 1). \) Using these definitions we find \( H/r \sim 0.2 \) and \( \alpha \lesssim 0.01 \) through most of the disk.

Since warps in slim disks are expected to propagate as bending waves, it may seem unusual at first that we see little evidence for such waves in Figure 12. For instance, Lubow et al. (2002) provides an analysis of the theory of bending waves in nearly Keplerian, weakly inclined disks and predicts that the tilt \( \beta \) should be a \textit{time-independent, oscillatory} function of radius (see also Marković & Lamb 1998). However, using equation (16) of Lubow et al. (2002) we estimate the wavelength of such oscillations for our simulation to be
\[ \lambda \approx \frac{\pi a^{9/4}}{(6a)^{1/2}} \left( \frac{H}{r} \right) \sim 50M \] (39)
at \( r = 10r_G \). This is strongly radially dependent \( (\lambda \propto r^{9/4} \) with \( H/r \sim \text{const} \)), so oscillations of \( \beta \) are essentially absent outside \( r = 10r_G \), consistent with what is shown in Figure 12.

The same conclusion, that \( \beta \) is not expected to oscillate outside \( r = 10r_G \) for this simulation, is also reached by considering equation (22) of Lubow et al. (2002). That equation defines a dimensionless variable
\[ x = \left( \frac{24a}{c^2} \right)^{1/2} e^{-\frac{(h+1)/4}{h+1/4}}, \] (40)
which is used to identify the transition radius between oscillatory behavior and asymptotic behavior, where \( h \) and \( c \) are used to parameterize the radial dependence of the disk scale height \( H/r = c^{p-h} \). Whenever \( x \gg 1 \) (small \( r \)), oscillations should be prominent, whereas whenever \( x \ll 1 \) (large \( r \)), \( \beta \) tends to the outer boundary value. For our simulation with \( c \approx 0.2 \) and \( h \approx 0.5 \) at \( r \approx 10r_G \), thus, from both approaches it is clear that our simulation does not satisfy the criteria to develop large oscillations in \( \beta \) within the main body of the disk.

Inside \( r = 10r_G \), the density of the disk drops off rapidly, and the dynamics is dominated by the plunging streams, which are not accounted for in the model of Lubow et al. (2002). Nevertheless, we appear to capture one-half of one wavelength of a bending wave oscillation inside \( r = 10r_G \), based on Figure 12. Thus, overall our results seem to be generally consistent with the predictions of Lubow et al. (2002).
3.2.2. Precession

A second useful diagnostic for tilted disks is the twist $\gamma$ of the disk as a function of radius and time. We define the precession angle (twist) as

$$
\gamma(r) = \arccos\left[ \frac{\mathbf{J}_{BH} \times \mathbf{J}_{disk}(r)}{\left| \mathbf{J}_{BH} \times \mathbf{J}_{disk}(r) \right|} \cdot \mathbf{\hat{y}} \right],
$$

From this definition, $\gamma(r) = 0$ throughout the disk at $t = 0$. In order to capture twists larger than 180°, we also track the projection of $\mathbf{J}_{BH} \times \mathbf{J}_{disk}(r)$ onto $\mathbf{\hat{x}}$, allowing us to break the degeneracy in arccos. A time-averaged plot of $\gamma$ is provided in Figure 14.

As described in our previous work (Fragile & Anninos 2005), we expect differential Lense-Thirring precession to dominate whenever the precession timescale $t_{LT} = \frac{r_{G}^{2}}{r} \approx \frac{g^{3/2}g^{5/2}}{g}$ is shorter than local dynamical timescales in the disk (Bardeen & Petterson 1975; Kumar & Pringle 1985). We consider three possible limiting timescales: the mass accretion timescale $t_{acc} = rV$, where $V = \langle \rho V_{i} \rangle_{a}(\rho_{a})_{j}$ is the density-weighted average inflow velocity; the sound crossing time $t_{cs} = r/c_{s}$, where $c_{s} = \langle \rho_{cs} \rangle_{a}(\rho_{a})_{j}$ is a density-weighted average of the local sound speed; and the Alfvén crossing time $t_{A} = rV_{A}$, where $V_{A}$ is a density-weighted average of the local Alfvén speed.

The local sound speed is recovered from the fluid state through the relation $c_{s}^{2} = \Gamma(\Gamma - 1)P/[(\Gamma - 1)\rho + \Gamma P]$. The Alfvén speed is

$$
v_{A} = \sqrt{\frac{|B|^{2}}{4\pi \rho h + |B|^{2}}}.
$$

Since $c_{s}$ and $v_{A}$ are defined in the frame of the fluid, it is not strictly accurate to compare $t_{cs}$ and $t_{A}$ to quantities defined using the coordinate time (such as $t_{LT}$ and $\Omega^{-1}$). However, we are mostly concerned with the timescales in the main body of the disk where such discrepancies are small. From Figure 15 we can see that the Lense-Thirring precession timescale is longer than the sound crossing time at virtually all radii.

Since the sound crossing time is short compared to the precession timescale throughout the bulk of the disk, pressure waves strongly couple the disk material. The disk, thus, responds as a single entity to the torque of the black hole and precesses as a global structure. Such global precession has been noted before in low-Mach number hydrodynamic disks (Nelson & Papaloizou 2000; Fragile & Anninos 2005). To estimate the precession period, we have plotted $\gamma$, averaged over the bulk of the disk ($20 \leq r/r_{G} \leq 50$), as a function of time in Figure 16. A linear fit to this plot yields a precession period of $T_{prec} \approx 0.3(M/M_{\odot})$ s, which corresponds to about 80$\tau_{orb}$. This is longer than the evolution time of all of our models, so we have had to extrapolate the full precession period. However, model 915m is run to 20$\tau_{orb}$ and shows a nearly linear growth of precession over the full simulation.

Classically, we expect the precession period for a solid-body rotator with angular momentum $J$ subject to a torque $\tau$ to be $T_{prec} = 2\pi \sin(\beta)(J/\tau)$ (Liu & Melia 2002). Assuming a radial dependence to the surface density of the form $\Sigma = \Sigma_{0}(r/r_{i})^{-k}$ and ignoring higher order general relativistic corrections, we

\[ T_{prec} \approx \frac{2\pi \sin(\beta)J_{i}}{\tau_{A}} \]

\[ \approx \frac{2\pi \sin(\beta)J_{i}}{\tau_{A}} \]

where $J_{i}$ is the angular momentum at the inner boundary of the disk and $\tau_{A}$ is the Alfvén torque. The classical precession period is given by

\[ T_{prec} = \frac{2\pi \sin(\beta)J_{i}}{\tau_{A}} \]

Fig. 14.—Plot of the twist ($\gamma$), as a function of radius through the disk. The data for this plot has been time-averaged from $t = 9\tau_{orb}$ to $10\tau_{orb}$. Initially the twist was zero throughout the disk. The disk matter has precessed ~180° by the time it reaches the hole. The shape of this twist profile remains fairly constant throughout the simulation.

Fig. 15.—Plot comparing various timescales within the disk, including the Lense-Thirring precession timescale $t_{LT}$, the accretion timescale $t_{acc}$, the sound crossing time $t_{cs}$, and the Alfvén crossing time $t_{A}$. All timescales are normalized by the local orbital period in the midplane of the black hole, $\Omega^{-1}$. The data for this plot has been time-averaged from $t = 9\tau_{orb}$ to $10\tau_{orb}$.

Fig. 16.—Plot of the twist $\gamma$, averaged over the bulk of the disk ($20 \leq r/r_{G} \leq 50$), as a function of time. The slope of this plot can be used to estimate the precession period of the disk as a whole, which is $0.3(M/M_{\odot})$ s.
have \( J = 2\pi M^{1/2}\Sigma ri_0^{5/2-\zeta}[1 - (r_0/r_a)^{5/2-\zeta}]/(5/2 - \zeta) \) and \( \tau = 4\pi(\sin \beta) a M^{5/2} \Sigma(1 - (r_0/r_a)^{-1/2+\zeta})[(r_0/r_a)^{1/2 + \zeta}] \), where \( r_i \) and \( r_o \) are the inner and outer radii of the evolved disk, respectively. Therefore,

\[
T_{\text{prec}} = \frac{\pi(1 + 2\zeta)}{(5 - 2\zeta)} \frac{r_o^{5/2-\zeta} r_i^{1/2+\zeta}}{a M [1 - (r_i/r_o)^{1/2-\zeta}]}.
\]

Equation (43) differs from the test particle Lense-Thirring precession period, because \( T_{\text{prec}} \) depends on the total torque integrated over the entire disk.

4. DISCUSSION

In this paper we studied the evolution of an MRI turbulent disk that was tilted with respect to the spin axis of a modestly fast rotating black hole. Although this prescription can lead to a Bardeen-Petterson configuration for some disk parameters, we did not see evidence for this in our simulation, as alignment of the disk with the equatorial plane of the black hole did not occur. This is not surprising since this simulation was carried out in the thick-disk regime where \( H/r > \alpha \) and warps produced in the disk propagate as waves (Papaloizou & Lin 1995), rather than diffusively as in the Bardeen-Petterson case. Since the expected bending wavelength (Lubow et al. 2002) turned out to be longer than the radial extent of the disk in the simulation, little warping of the disk was observed. Instead, the unwarped disk precessed uniformly. The extrapolated precession period \( T_{\text{prec}} \approx 0.3(M/M_\odot) \text{ s} \) equates to periods of \( \approx 3 \text{ s} \) and \( \approx 3 \text{ days} \) for black holes of mass \( M = 10 M_\odot \) and \( M = 10^8 M_\odot \), respectively. Such global disk precession could explain certain variability features observed from accreting black holes, such as low-frequency QPOs (LFQPOs; Stella et al. 1999; Liu & Melia 2002; Schnittman et al. 2006), since the observer’s viewing angle of the inner, X-ray-emitting region of the disk would vary periodically.

If the inner disk is optically thick enough to produce relativistically broadened reflection features, such as an iron K\( \alpha \) line, then such precession should also be observable through periodic changes in both the shape and strength of the lines (Fragile et al. 2005). These changes should be correlated with the phase of the corresponding LFQPO. Such a correlation has been observed in GRS 1915+105 (Miller & Homan 2005), although only between line strength and QPO phase; those data were not sufficiently resolved to determine the line shape.

Generally, we expect the precession period to be given by equation (43), which has a strong dependence on the radial distribution of the disk \( \propto r_a^{5/2-\zeta} r_i^{-1/2+\zeta} \). One idea to consider is that the outer radius may correspond to the truncation radius proposed to explain the hard state of black hole X-ray binaries (e.g., Esin et al. 1997; but see also Rykoff et al. 2007). In this case, our simulated disk would represent the hot, geometrically thick flow that fills the region inside the truncation radius. The LFQPO would then correspond to the precession frequency of this inner flow, in which case it should scale as \( r_a^{-5/2+\zeta} \). Sobczak et al. (2000) explored the dependence of the LFQPO frequency on spectral fitting parameters, including what would be the truncation radius in the context of the suggested hard-state model. They studied two sources, XTE J1550–564 and GRO J1655–40, and found opposite trends between frequency and radius. For XTE J1550–564, the observed frequency was \( \nu_{\text{LFQPO}} \approx 5 \text{ Hz} \), and the observed truncation radius was \( r_o/r_G = 2.7(10 M_\odot/M)(D/6 \text{ kpc})(\cos \theta)^{-1/2} \). From equation (43) we would expect

\[
\frac{r_o}{r_G} = \left[ \frac{5 - 2\zeta}{\pi(1 + 2\zeta)} \right]^{2/(5-\zeta)} \left[ \frac{a}{M} \right]^{2/(5-\zeta)} \times \left[ \frac{\nu}{(\nu M)^{-2/(5-\zeta)}} \right].
\]

In our simulation we found \( \zeta \approx 0 \), which gives \( r_o \approx 33r_G \) for \( M = 10 M_\odot \) and \( \nu = 5 \text{ Hz} \). This is considerably larger than the observed value. However, some of the discrepancy may be attributable to the large uncertainties in the parameters used to describe this source, including its distance, mass, and inclination. In addition, if the surface density in XTE J1550–564 depends strongly on radius, which was not the case for our simulated disk, then our prediction would change significantly. Further observational studies along this line are needed to test this prediction more thoroughly.

Although the main body of the disk was not significantly altered by the tilt, we did not see significant differences in the inner regions of the flow when compared with untitled simulations. First, a tilted disk encounters the generalized ISCO surface at a larger radius than an untitled disk. This causes the plunging region to start further out. The binding energy of the innermost material in the disk is therefore less than it would be for an aligned disk, and the overall radiative efficiency should then be reduced.

On the other hand, tilting the disk appears to produce a higher overall mass accretion rate (shown here in Figure 10a; also discussed in Lodato & Pringle 2006). A tilted accretion disk will therefore have a lower surface density than an untitled disk with the same accretion rate. This may affect the emergent spectrum, especially for hot, optically thin flows. On the other hand, for flows that are effectively optically thick Davis et al. (2005) found that the emergent spectra are remarkably independent of the overall stress and surface density.

We also found that the plunging region is not axially symmetric. Instead, accretion onto the hole in the tilted-disk case occurs through two discrete streams of material that leave the disk at high latitudes with respect to the black hole and disk symmetry planes. This may affect the magnitude of magnetic torques exerted by the plunging region on the disk. An interesting question for future work is how these streams vary on the timescale of the precession of the disk. We intend to explore the detailed properties of the plunging region and innermost disk in a future paper.

The tilted disk also seems not to have formed a clearly identifiable inner torus. This could be significant because the inner torus serves as a launching point for the matter-dominated, funnel-wall jet. The absence of a prominent inner torus may lead to a weaker matter jet. However, the present simulation is not suited to addressing this issue because of the poor and varying resolution used near the pole. Instead, we plan to explore jets and outflows from tilted disks in future work.

In many respects, the tilted-disk simulation exhibited properties consistent with an untitled disk around a black hole of lower spin. These included the larger plunging radius, higher mass accretion rate, and less prominent inner torus. Thus, black hole tilt could hamper efforts to estimate black hole spin based on such properties. Indeed, it is commonly stated that astrophysical black hole spacetimes depend on just two parameters: mass and spin. But it should be remembered that the observed properties of black hole accretion disks also depend on their inclinations with
respect to the spin axes of their central black holes. This inclination should be a target of future observational programs that use accretion disks as surrogates to study properties of black holes.

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