Asymmetries involving dihadron fragmentation functions: from DIS to $e^+e^-$ annihilation

Alessandro Bacchetta,1,∗ Federico Alberto Ceccopieri,2† Asmita Mukherjee,3‡ and Marco Radici4§

1Theory Center, Jefferson Lab, 12000 Jefferson Ave, Newport News, VA 23606, USA
2Dipartimento di Fisica, Università di Parma, and INFN, Gruppo Collegato di Parma, I-43100 Parma, Italy
3Physics Department, Indian Institute of Technology Bombay, Povai, Mumbai 400076, India
4Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy

Using a model calculation of dihadron fragmentation functions, we fit the spin asymmetry recently extracted by HERMES for the semi-inclusive pion pair production in deep-inelastic scattering on a transversely polarized proton target. By evolving the obtained dihadron fragmentation functions, we make predictions for the correlation of the angular distributions of two pion pairs produced in electron-positron annihilations at BELLE kinematics. Our study shows that the combination of two-hadron inclusive deep-inelastic scattering and electron-positron annihilation measurements can provide a valid alternative to Collins effect for the extraction of the quark transversity distribution in the nucleon.

PACS numbers: 13.87.Fh, 11.80.Et, 13.60.Hb

I. INTRODUCTION

Dihadron fragmentation functions (DiFF) describe the hadronization of a quark in two hadrons plus other unobserved fragments. In their simplest form, they represent the probability that at some hard scale a parton hadronizes in two hadrons with fractional energies $z_1$ and $z_2$. They were introduced for the first time when studying the $e^+e^- \rightarrow h_1h_2X$ process in the context of jet calculus [1]. They are in fact necessary to guarantee the factorization of all collinear singularities for such a process at next-to-leading order (NLO) in the strong coupling constant [2].

In experiments, often not only the fractional energies of the two hadrons are measured, but also their invariant mass $M_{h}$ (see, e.g., Refs. [3, 4, 5]). Hence, it is useful to introduce extended DiFF (in analogy with extended fracture functions [6]), which are explicitly depending on $M_{h}$. Their definition and properties have been analyzed up to subleading twist [7, 8]. Their evolution equations are known [9] and presently solved in the leading logarithm approximation (LL), and there are valid arguments to assume that they can be factorized and are universal, similarly to what happens for extended fracture functions [6]. In fact, (extended) DiFF can appear also in two-particle-inclusive deep-inelastic scattering (SIDIS) and in hadron-hadron collisions.

DiFF can be used as analyzers of the polarization state of the fragmenting parton [10, 11, 12]. Because of this, they have been proposed as tools to investigate the spin structure of the nucleon, in particular to measure the transversity distribution $h_t^q$ of a parton $q$ in the nucleon $N$ (see Ref. [13] for a review). The $h_t^q$, together with the momentum $f_t^q$ and helicity $g^q$ distributions, fully characterizes the (leading-order) momentum/spin status of $q$ inside $N$, if quark transverse momentum is integrated over. Transversity is a chiral-odd function and needs to appear in a cross section accompanied by another chiral-odd function. The simplest example is the fully transversely polarized Drell-Yan process, where $h_t^q$ appears multiplied by its antiquark partner $\overline{h}_t^\bar{q}$ [14]. Although this process is theoretically very clean, it appears to be experimentally very challenging [15], at least at present facilities (the same finding is confirmed for proton-proton collisions leading to prompt photon [16] and semi-inclusive pion production [17]).

An alternative approach, so far the only fruitful one, is to turn to SIDIS and measure the correlation between the transverse polarization of the target and the transverse momentum of the final hadron, which involves a convolution of $h_t^q$ with the chiral-odd Collins fragmentation function $H^q_\perp$ [18]. The resulting asymmetry has already been measured at HERMES [19, 20], and at COMPASS [21, 22, 23]. The knowledge of the Collins function is required to extract the transversity distribution. This can be obtained through the measurement of azimuthal asymmetries.

∗Electronic address: alessandro.bacchetta@jlab.org
†Electronic address: federicoalberto.ceccopieri@fis.unipr.it
‡Electronic address: asmita@phy.iitb.ac.in
§Electronic address: marco.radici@pv.infn.it
in $e^+e^- \rightarrow \pi^+\pi^-X$ with almost back-to-back pions \cite{24}. The BELLE collaboration at KEK has measured this asymmetry \cite{25}, making the first-ever extraction of $h_1^q$ possible from the global analysis of SIDIS and $e^+e^-$ data \cite{26}.

At present, large uncertainties still affect this analysis and the resulting parametrization of $h_1^q$. The most crucial issue is the treatment of evolution effects, since the BELLE and the HERMES/COMPASS measurements happened at two very different scales: $Q^2 \sim 100$ and $\langle Q^2 \rangle = 2.5$ GeV$^2$, respectively. Both $h_1 \otimes H_T^+ \otimes \mathcal{P}_1^+$ and $h_1^q \otimes \mathcal{P}_1^o$ convolutions involve transverse-momentum dependent functions (TMD) \cite{27, 28} whose behaviour upon scale change should be described in the context of Collins-Soper factorization \cite{29, 30} (see also Refs. \cite{31, 32}). However, the global analysis of Ref. \cite{26} neglects any change of the partonic transverse momentum with the scale $Q^2$ leading to a possible overestimation of $h_1$ \cite{28, 33, 34}. It would be desirable, therefore, to have an independent way to extract transversity, involving collinear fragmentation functions. Here, we consider the semi-inclusive production of two hadrons inside the same current jet.

As already explained, the fragmentation $q \rightarrow (\pi^+\pi^-)X$ is described by an (extended) DiFF. When the quark is transversely polarized, $q^\perp$, a correlation can exist between its transverse polarization vector and the normal to the plane containing the two pion momenta. This effect is encoded in the chiral-odd polarized DiFF $H_1^{q, \perp}$ via the dependence on the transverse component of the pion pair relative momentum $R_T$ \cite{7}. The function $H_1^{q, \perp}$ can be interpreted as arising from the interference of $(\pi^+\pi^-)$ being in two states with different angular momenta \cite{35, 36, 37, 38}. Since the transverse momentum of the hard parton is integrated out, the cross section can be studied in the context of collinear factorization and its polarized part contains the factorized product $h_1^q H_1^{q, \perp} \mathcal{P}_1^o \otimes \mathcal{P}_1^p$. The HERMES collaboration has recently measured such spin asymmetry using transversely polarized proton targets \cite{40}; the COMPASS collaboration performed the same measurement on a deuteron target \cite{41} and should soon release data using a proton target.

Similarly to the Collins effect, the unknown $H_1^{q, \perp}$ has to be extracted from electron-positron annihilation, specifically by measuring the angular correlation of planes containing two pion pairs in the $e^+e^- \rightarrow (\pi^+\pi^-)_{\text{jet1}}(\pi^+\pi^-)_{\text{jet2}}X$ process \cite{12, 32}. The BELLE collaboration is analyzing data for this angular correlation \cite{13, 14}. Therefore, it seems timely to use available models for extended DiFF to make predictions for the $e^+e^-$ azimuthal asymmetry at BELLE kinematics. Since evolution equations for extended DiFF are available at NLO \cite{9}, at variance with the Collins effect the asymmetries with inclusive hadron pairs in SIDIS and $e^+e^-$ can be correctly connected when the scale is ranging over two orders or magnitude. Therefore, the option of using the semi-inclusive production of hadron pairs inside the same jet seems a theoretically clean way to extract transversity \cite{31}. Finally, we point out that pair production in polarized hadron–hadron collisions allows in principle to "self-sufficiently" determine all the unknown DiFF and $h_1$ \cite{42}. The STAR collaboration has recently presented data on this kind of measurement \cite{43}.

The paper is organized as follows. In Sec. \ref{sec:fit} using the model calculation of DiFF from Ref. \cite{38}, we fit the spin asymmetry recently extracted by HERMES for the SIDIS production of $(\pi^+\pi^-)$ pairs on transversely polarized protons \cite{40}. In Sec. \ref{sec:fit}, we describe how we calculate the evolution of the involved extended DiFF starting from the HERMES scale up to the BELLE scale. In Sec. \ref{sec:pred} we illustrate the predictions for the correlation of angular distributions of two pion pairs produced in $e^+e^-$ annihilations at BELLE kinematics. Finally, in Sec. \ref{sec:conc} we draw some conclusions.

\section{Fit to Deep Inelastic Scattering Data}

We consider the SIDIS process $e(l) + N^\perp(P) \rightarrow e(l') + \pi^+(P_1) + \pi^-(P_2) + X$, where $P$ is the momentum of the nucleon target with mass $M$, $l, l'$ are the lepton momenta before and after the scattering and $q = l - l'$ is the space-like momentum transferred to the target. The final pions, with mass $m_\pi = 0.14$ GeV and momenta $P_1$ and $P_2$, have invariant mass $M_h$ (which we consider as much smaller than the hard scale $Q^2 = -q^2 \geq 0$ of the SIDIS process). We introduce the pair total momentum $P_h = P_1 + P_2$ and relative momentum $R = (P_1 - P_2)/2$. Using the traditional Sudakov representation of a 4-momentum $a$ in terms of its light-cone components $a^\pm = (a^0 \pm a^3)/\sqrt{2}$ and transverse spatial components $a_T$, we define the light-cone fractions $x = p^+/P^+$ and $z = P_h^-/k^-$, where $p$ and $k = p + q$ are the momenta of the parton before and after the hard vertex, respectively.

In this process, the following asymmetry can be measured (for the precise definition we refer to Refs. \cite{38, 40}),

$$A_{UT}^{\sin(\phi_H + \phi_e)} \sin \theta (x, y, z, M_h^2) = \frac{1 - y - z/2 + y^2/4}{x^2 (1 - y + z/2 + y^2/4)} \left(1 - \frac{\gamma^2}{2x} \right) \frac{1}{2} \frac{M_h}{\sqrt{2}} \left| R \right| \sum_q c_q^2 h_1^q(x) \left( \frac{H_1^{q, \perp}(z, M_h^2)}{4m_H^2} \frac{D_{1, q}(z, M_h^2)}{D_{1, q}(z, M_h^2)} \right),$$

where $y = P \cdot q / P \cdot l$ is related to the fraction of beam energy transferred to the hadronic system, $\gamma = 2Mx/Q$, $f_1^q$ and $h_1^q$ are the unpolarized and transversely polarized parton distributions, respectively, and

$$\left| R \right| = \frac{M_h}{2} \sqrt{1 - \frac{4m_H^2}{M_h^2}}.$$
The spin asymmetry $A_{LL}$ is related to an asymmetric modulation of pion pairs in the angles $\phi_S$ and $\phi_R$, which represent the azimuthal orientation with respect to the scattering plane of the target transverse polarization and of the plane containing the pion pair momenta, respectively (see Ref. 33 for a precise definition, which is consistent with the Trento conventions 43).

The polar angle $\theta$ describes the orientation of $P_h$ in the center-of-mass frame of the two pions, with respect to the direction of $P_h$ in the lab frame. DiFF depend upon $\cos \theta$ via the light-cone fraction $\zeta = 2R^+/P^+_h = 2 \cos \theta |\mathbf{R}|/M_h$, which describes how the total momentum of the pair is split between the two pions 33. DiFF can be expanded in terms of Legendre polynomials of $\cos \theta$ and the expansion can be reasonably truncated to include only the $s$ and $p$ relative partial waves of the pion pair, since their invariant mass is small (typically $M_h \lesssim 1$ GeV). At leading twist and leading order in $\alpha_s$, the spin asymmetry of Eq. (1) contains only $D_{1,q} = D_{1,u}^q + D_{1,d}^q$, which includes the diagonal pure $s$- and $p$-wave contributions, and $H_{1,q}^\perp$, which originates from the interference between them 33, 39. Subleading-twist terms have different azimuthal dependences.

The polarized DiFF $H_{1,q}^\perp$ represents the chiral-odd partner to isolate the transversity distribution $h_1^T$ in the spin asymmetry of Eq. (1). It describes the interference between the fragmentations of transversely polarized quarks into pairs of pions in relative $s$ and $p$ waves 39. Together with $D_{1,q}$, it was analytically calculated in a spectator-model framework for the first time in Ref. 37, and later in a refined version 38. The analytical expressions for $D_{1,q} \equiv D_{1,cc}$ and $H_{1,q}^\perp = H_{1,ct}^\perp$ can be found in Eqs. (23) and (26) of Ref. 38, respectively. The model parameters were fixed by adjusting $D_{1,q}$ to the output of the PYTHIA event generator tuned to the SIDIS kinematics at HERMES 10; their values are listed in Eqs. (32-35) of Ref. 38. Note that the spectator model by construction gives $D_{1,u} = D_{1,d} = D_{1,i} = D_{1,i}$, and $H_{1,u}^\perp = -H_{1,d}^\perp = -H_{1,i}^\perp = H_{1,i}^\perp$.

The calculated spin asymmetry follows the same trend of the data. In particular, the shape of the invariant mass dependence is dominated by a resonance peak at $M_h \approx m_{\rho}$, which is due to the interference between a background production of pion pairs in $s$ wave and the $p$-wave component dominated by the decay $\rho \rightarrow \pi \pi$ of the $\rho$ resonance. Similarly, the model displays also another broad peak at $M_h \approx 0.5$ GeV due to the $\omega \rightarrow \pi \pi \pi$ resonant channel. Both predictions and data show no sign change in $A_{LL}^{\sin(\phi_R + \phi_S) \sin \theta}$ as a function of $M_h$ around $M_h \approx m_{\rho}$, contrary to what was predicted in Ref. 56. However, the results of Ref. 38 systematically overpredict the experimental data at least by a factor of two 40.

To correctly reproduce the size of the asymmetry, we multiply the model prediction of $H_{1,q}^\perp$ by an extra parameter $\alpha$, while we use the model prediction for $D_{1,q}$ without further changes, since its parameters have been already fitted to reproduce the unpolarized cross section, as predicted by PHOTIA. We use also the GRV98 LO parametrization for $f_1^\perp$ at the HERMES scale $Q^2 = 2.5$ GeV$^2$. For $h_1^T$, we use the recently extracted parametrization from Ref. 48, whose central value is basically the same as the former one from Ref. 26 in the region $x < 0.2$ of interest here. The asymmetry is calculated by averaging the numerator and denominator of $A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta}$ in each experimental bin, while integrating in turns all remaining variables in the ranges

$$0.023 < x < 0.4,$$
$$0 < z < 0.99,$$
$$0.5 \text{ GeV} < M_h < 1 \text{ GeV}.$$  

The variable $y$ is always integrated in the range

$$y_{\min} = \text{Max}[0.1, Q^2_{\min}/(x(s - M^2)), (W^2_{\min} - M^2)/(1 - x)(s - M^2)],$$  

with $s = 56.2$ GeV$^2$ and $W^2_{\min} = 4$ GeV$^2$.

The best value for the parameter $\alpha$ is found by means of a $\chi^2$ fit; the results are shown in Fig. 1. We took into consideration the experimental errors by adding in quadrature the statistical and systematic errors (the error bands in Fig. 1 represent such a sum). We did not include the small theoretical errors coming from the uncertainty on the other model parameters of the fragmentation functions, from the uncertainties of the parton distribution functions, and from the choice of factorization scale. The best-fit value of our reduction parameter turns out to be $\alpha = 0.32 \pm 0.06$ corresponding to $\chi^2/d.o.f. = 1.24$. In summary, the model calculation of the $H_{1,q}^\perp$ function has to be reduced by a factor 3 to reproduce the HERMES data, if the transversity from Ref. 48 is used.

### III. Evolution of DiHadron Fragmentation Functions

In order to predict the azimuthal asymmetry in the distribution of two pion pairs produced in $e^+e^-$ annihilation, we need to evolve the DiFF $D_{1,q}$ and $H_{1,q}^\perp$ from the HERMES scale to the BELLE scale.

DiFF usually depend on $z$, $\zeta = 2 \cos \theta |\mathbf{R}|/M_h$ [or, alternatively, on $z_1 = z(1 + \zeta)/2$, $z_2 = z(1 - \zeta)/2$, and on $R^2_T$,...
which is connected to the pair invariant mass by \[ R_T^2 = \frac{(P_1 - P_2)^2}{4} = \frac{z_1 z_2}{z_1 + z_2} \left[ \frac{M_h^2}{z_1} - \frac{M_1^2}{z_1} - \frac{M_2^2}{z_2} \right]. \] 

The further dependence on the scale \( Q^2 \) of the process is described by usual DGLAP evolution equations; at LL, they read \[ \frac{d}{d \log Q^2} D_q(z_1, z_2, R_T^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_1 + z_2}^1 \frac{du}{u^2} D_q\left(\frac{z_1}{u}, \frac{z_2}{u}, R_T^2, Q^2\right) P_{q'q}(u), \] 

where \( P(u) \) are the usual leading-order splitting functions \([49]\). A similar equation holds for \( H^{\text{vtx}}_1 \) involving the splitting functions \( \delta P(u) \) for transversely polarized partons \([50, 51]\) (see also the Appendix of Ref. [9], for convenience).

The same strategy can be applied to study evolution of single components of extended DiFF in the expansion in relative partial waves of the pion pair. In fact, Eq. (6) can be rewritten as 

\[
\frac{d}{d \log Q^2} D_q(z, \zeta, M_h^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{du}{u} D_q'\left(\frac{z}{u}, \zeta, M_h^2, Q^2\right) P_{q'q}(u). \tag{7}
\]

Note that the evolution kernel affects only the dependence on \( z \), leaving untouched the dependence on \( \zeta \). That is, it affects the dependence on the fractional momentum of the pion pair with respect to the hard fragmenting parton, but not the dependence on the nonperturbative processes that make the fractional momentum split inside the pair itself.

The net effect is that extended DiFF display evolution equations very similar to the single-hadron fragmentation case. Using the above identity \( \zeta = 2 \cos \theta |\mathbf{R}|/M_h \), we can again expand both sides of Eq. (7) in terms of Legendre functions of \( \cos \theta \) and apply the evolution kernel to each member of the expansion. By integrating in \( d \cos \theta \) both sides we come to the final result

\[
\frac{d}{d \log Q^2} D_{1,q}(z, M_h^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{du}{u} D_{1,q}'\left(\frac{z}{u}, M_h^2, Q^2\right) P_{q'q}(u). \tag{8}
\]

that involves the DGLAP evolution of the single diagonal component \( D_{1,q} = D_{1,q}^s + D_{1,q}^p \) related to the pure \( s \) and \( p \) relative partial waves of the pion pair. Analogously, we can get an evolution equation similar to Eq. (8) for \( H^{\text{vtx}}_{1,q} \) provided that \( P(u) \) is replaced by \( \delta P(u) \).

Equation (8) shows that also the dependence on the pair invariant mass \( M_h \) is not affected by the evolution kernel, as is reasonable, since \( M_h \) is a scale much lower than \( Q^2 \). However, in order to get the \( M_h \) dependence at a different scale \( Q'^2 \neq Q^2 \) it is important to completely integrate away the \( z \) dependence. Usually, experimental phase spaces are limited by the geometry of the apparatus and, in this case, the integration in \( dz \) is performed in the interval \([z_{\min}, 1]\) with \( z_{\min} \neq 0 \). In Fig. 2 we show \( D_{1,u}(M_h) \) for the up quark at the HERMES scale \( Q^2 = 2.5 \text{ GeV}^2 \) (dot-dashed line) and at the BELLE scale of \( Q^2 = 100 \text{ GeV}^2 \) (solid line). In the left panel, results are obtained using \( z_{\min} = 0.02 \),
in the right panel with $z_{\text{min}} = 0.2$ as in the BELLE setup. Since the DGLAP evolution shifts the strength at lower $z$ for increasing $Q^2$, cutting the $z$ phase space from below makes the final result miss most of the strength and, consequently, display a reduced $D_{1,\mu}(M_h)$. The apparent (and contradictory) effect of perturbative evolution on the dependence upon the nonperturbative scale $M_h$ in the right panel is actually spurious, and it disappears as soon as the phase space for $z$ integration is properly enlarged to include the lowest $z$ for $z_{\text{min}} \to 0$, as shown in the left panel. When extracting azimuthal (spin) asymmetries, it is, therefore, crucial to keep in mind these features to estimate the effect of evolution.

As a last general comment, we stress that the analysis of evolution is facilitated by the fact that azimuthal asymmetries based on the mechanism of dihadron fragmentation can be studied using collinear factorization. This feature makes them a cleaner observable than the Collins effect, from the theoretical point of view.

In the next section, we compute azimuthal asymmetries for two pion pairs production in $e^+e^-$ annihilations using evolved extended DiFF. The goal is to make model predictions at BELLE kinematics, and, at the same time, to estimate the evolution effects, both the pure one on the $z$ dependence and the spurious one on the $M_h$ dependence, due to the limited experimental phase space.

![Diagram](image_url) 

**FIG. 2:** The unpolarized extended DiFF $D_{1,u}(M_h, Q^2)$ in arbitrary units, after integrating the $z$ dependence away in the interval $[0.02, 1]$ (left panel) and $[0.2, 1]$ (right panel). Dot-dashed line for $Q^2 = 2.5$ GeV$^2$ at HERMES, solid line for $Q^2 = 100$ GeV$^2$ at BELLE (see text).

![Diagram](image_url) 

**FIG. 3:** Momenta and angles involved in the process $e^+e^- \to (\pi^+\pi^-)_{\text{jet1}}(\pi^+\pi^-)_{\text{jet2}}X$. 

As a last general comment, we stress that the analysis of evolution is facilitated by the fact that azimuthal asymmetries based on the mechanism of dihadron fragmentation can be studied using collinear factorization. This feature makes them a cleaner observable than the Collins effect, from the theoretical point of view.
IV. PREDICTIONS FOR ELECTRON-POSITRON ANNIHILATION

For the process $e^+e^- \rightarrow (\pi^+\pi^-)_{\text{jet}1}(\pi^+\pi^-)_{\text{jet}2}X$, the momentum transfer $q = l + l'$ is time-like, i.e. $q^2 = Q^2 \geq 0$, with $l, l'$, the momenta of the two annihilating fermions. We have now two pairs of pions, one originating from a fragmenting parton and the other one from the related antiparton. Therefore, we will use in analogy to Sec. [11] the variables $\phi_R, \theta, P_1, P_2, P_h, R, M_h, z, \zeta$, for one pair, adding the variables $\phi_R, \theta, P_1', P_2', P_h', R, M_h', z, \zeta$, for the other pair. Since we assume that the two pairs belong to two back-to-back jets, we have $P_h \cdot P_h' \approx Q^2$. The momenta and angles involved in the description of the process are depicted in Fig. 3. The azimuthal angles $\phi_R$ and $\phi_R'$ are defined by

$$
\phi_R = \frac{(l_{e+} \cdot P_h)}{|(l_{e+} \times P_h)|} \arccos \left( \frac{(l_{e+} \times P_h)}{|l_{e+} \times P_h|} \right),
$$

$$
\phi_R' = \frac{(l_{e+} \times P_h)}{|(l_{e+} \times P_h)|} \arccos \left( \frac{(l_{e+} \times P_h)}{|l_{e+} \times P_h|} \right),
$$

(9)

where $l_{e+}$ is the momentum of the positron, and $R, \overline{R}$ indicate the transverse component of $R, \overline{R}$ with respect to $P_h, \overline{P}_h$, respectively. They are measured in the plane identified by $l_{e+} \times P_h$ and $(l_{e+} \times P_h) \times \hat{z}$, with $\hat{z} \parallel -P_h$ in analogy to the Trento conventions [47]; this plane is perpendicular to the lepton plane identified by $L_{e+}$ and $\overline{P}_h$ (see Fig. 3). Note that the difference between $\phi_R$, as defined in Eq. (9), and the azimuthal angle of $R_T$, as measured around $\overline{P}_h$, is a higher-twist effect. The invariant $y = P_h/l/P_h \cdot q$ is now related, in the lepton center-of-mass frame, to the angle $\theta_2 = \arccos(l_{e+} \cdot P_h/(l_{e+} \cdot |P_h|))$ by $y = (1 + \cos \theta_2)/2$.

Starting from Eq. (21) of Ref. [42], we define the so-called Artru-Collins azimuthal asymmetry $A(\cos \theta_2, z, \zeta, M_h^2, \overline{M}_h^2)$ as the ratio between weighted leading-twist cross sections for the $e^+e^- \rightarrow (\pi^+\pi^-)_{\text{jet}1}(\pi^+\pi^-)_{\text{jet}2}X$ process, once integrated upon all variables but $\cos \theta$ and $\cos \theta'$. By further integrating upon $d \cos \theta$ and $d \cos \theta'$, we deduce the analogue of Eq. (32) of Ref. [42] for the specific contribution of $s$ and $P$ partial waves to the Artru-Collins azimuthal asymmetry, namely

$$
A(\cos \theta_2, z, M_h^2, \zeta, \overline{M}_h^2) \equiv \frac{\langle \cos(\phi_R + \phi_R') \rangle}{\langle 1 \rangle} = \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \frac{z^2}{M_h M_{\overline{M}_h}} \sum_q c_q^2 H_{1,q}^{\text{esp}}(z, M_h^2) \overline{H}_{1,q}^{\text{esp}}(\zeta, \overline{M}_h^2). \tag{11}
$$

The extended DiFF $D_{1,q}$ and $H_{1,q}^{\text{esp}}$ are the same universal functions appearing in the SIDIS spin asymmetry of Eq. (1). Hence, tuning model predictions for them at BELLE kinematics would help in reducing the uncertainty in the extraction of the transversity $h_1$ at the HERMES scale. In this strategy, a crucial role is played by evolution. At variance with the Collins effect, the dihadron fragmentation mechanism is fully collinear, since only $R_T$ matters and $P_{h,k}$ can be integrated. Hence, evolution equations for extended DiFF are easily under control, presently at LL level [9], and are represented by Eq. (8) and its analogue for $H_{1,q}^{\text{esp}}$.

In Fig. 4 the azimuthal asymmetry of Eq. (11) is displayed as a function of $M_h$ for the $z$ bins $[0.01, 0.1], [0.1, 0.2], [0.2, 0.3], [0.3, 0.4], [0.4, 0.55]$, and $[0.55, 0.75]$, after integrating upon the other variables $0.4 \leq \overline{M}_h \leq 1.2$ GeV, 

\[\text{with respect to Ref. [42], we use the modified definition } \zeta = 2 \xi - 1.\]
FIG. 4: The azimuthal asymmetry for two pion pairs production in $e^+e^-$ annihilation as a function of the invariant mass $M_h$ of one pair for the indicated bins in its momentum fraction $z$. Notations as in Fig. 2. The uncertainty band around the solid line originates from the fit error of Fig. 1 through error propagation. For each panel, the lower plot shows the modification factor of the final result because of DGLAP evolution (see text).

$0.2 \leq \overline{z} < 0.3$, and $-0.6 \leq \cos \theta_2 \leq 0.9$, according to the BELLE experimental phase space. In particular, according to Ref. [25] for each bin we have assumed the following coefficient

$$
\frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} \approx 0.7.
$$

(12)
In each panel, the upper plot shows the $A(z_{\text{bin}}, M_{h}^2)$ at the HERMES scale $Q^2 = 2.5$ GeV$^2$ (dot-dashed line) and at the BELLE scale $Q^2 = 100$ GeV$^2$ (solid line), the latter being supplemented by the uncertainty band propagated from the SIDIS fit error of Fig. 1. The lower plot shows

$$\Delta = \frac{A(z_{\text{bin}}, M_{h}^2, Q^2 = 100) - A(z_{\text{bin}}, M_{h}^2, Q^2 = 2.5)}{A(z_{\text{bin}}, M_{h}^2, Q^2 = 100)},$$

(13)

i.e. the modification factor of the final result due to the evolution starting at the HERMES scale.

Some comments are in order about Fig. 4. First of all, the absolute size of the Artru–Collins asymmetry is small, reaching at most the percent level (see Fig. 6). However, it should be within the reach of the BELLE experimental capabilities, if compared with the corresponding Collins effect for two separated single-hadron fragmentations [22]. The error band originates from the uncertainty in the size of DiFF due to the fit of the SIDIS data for the spin asymmetry of Eq. (1). This error band is always much larger than the effects due to evolution.

After the comments about Fig. 2, one would be tempted to attribute this sensitivity of the $M_h$ dependence to the hard scale $Q^2$ as coming from a spurious effect; indeed, in each panel of Fig. 4 the asymmetry is integrated in the indicated $z$ bin, which is obviously just a small fraction of the available phase space. However, the asymmetry of Eq. (11) is the ratio of two objects that behave very differently under evolution because of their kernels $P(u)$ and $\delta P(u)$, respectively. Hence, there is no fundamental reason to expect the pure $M_h$ dependence of the asymmetry be preserved by DGLAP evolution, even after integrating upon the whole phase space of the other variables. Moreover, the moderate sensitivity of $A(z_{\text{bin}}, M_{h}^2)$ to the hard scale of the process is the result of a compensation of two very large sensitivities both in the numerator and in the denominator, as is clear from Fig. 5.

FIG. 5: Numerator (left panels) and denominator (right panels) of the azimuthal asymmetry in the same conditions and with the same notations as in Fig. 4, for the indicated boundary bins in $z$. The result at $Q^2 = 100$ GeV$^2$ (solid line in the upper plot of each panel) is emphasized by the factor 10.
In Fig. 5 the numerator (left panels) and denominator (right panels) of the asymmetry \((11)\) are shown with the same notations as in Fig. 4 for the 0.01 \(\leq z \leq 0.1\) (upper panels) and 0.55 \(\leq z \leq 0.75\) bins (lower panels). The solid line, corresponding to the result at \(Q^2 = 100\) GeV\(^2\), is amplified by a factor 10. Therefore, the effect of DGLAP evolution is enormous, both in the numerator and in the denominator, where, in particular, it can reach a reduction factor of more than two orders of magnitude. Also the shape of the \(M_h\) dependence is altered, making the more or less stable trend of \(A(z_{\text{bin}}, M_h^2)\) at different \(Q^2\) just a fortuitous case.

In summary, even if DGLAP evolution of extended DiFF seems to mildly affect the predictions for the azimuthal asymmetry at BELLE, this small sensitivity arises from a dramatic compensation between big modifications in the numerator and in the denominator of the asymmetry. Therefore, it is wise to carefully consider such effect, because it could provide more sizeable modifications in other portions of the phase space.

For sake of completeness, in Fig. 6 the azimuthal asymmetry \((11)\) is displayed as a function of \(z\) for the \(\tau\) bins \([0.01, 0.1], [0.1, 0.2], [0.2, 0.3], [0.3, 0.4], [0.4, 0.55]\), and \([0.55, 0.75]\), with the same notations as in Fig. 5 and again after integrating upon \(0.4 \leq M_h, \overline{M}_h \leq 1.2\) GeV, and \(-0.6 \leq \cos \theta_2 \leq 0.9\). It shows a rising trend for increasing both \(z\) and \(\tau\). The effect of DGLAP evolution is small and within 10\%, except for the lowest \(z\) values.

\[V. \quad \text{CONCLUSIONS}\]

In this paper, using the model calculation of extended dihadron fragmentation functions (DiFF) from Ref. 38 we fitted the spin asymmetry recently extracted by the HERMES collaboration for the semi-inclusive deep-inelastic scattering (SIDIS) production of \((\pi^+\pi^-)\) pairs on transversely polarized protons 40. Then, using the results of Ref. 39 we calculated the evolution of extended DiFF at leading logarithm level, starting from the HERMES scale up to the scale of the process \(e^+e^- \rightarrow (\pi^+\pi^-)_{\text{jet1}} (\pi^+\pi^-)_{\text{jet2}}X\) at BELLE kinematics. Finally, we made our predictions for the so-called Artru–Collins asymmetry, describing the correlation of angular distributions of the involved two pion pairs. The BELLE collaboration plans to measure this asymmetry in the near future.

The absolute size of the Artru–Collins asymmetry turns out to be small, but it should be within reach of the BELLE experimental capabilities, if compared with the corresponding Collins effect for two separated single-hadron fragmentations 25. The theoretical error band, originating from the uncertainty in the fit of the SIDIS data, is always larger than the effects produced by the evolution of DiFF. The latter seems to mildly affect the predictions for the azimuthal asymmetry at BELLE. Nevertheless, this small sensitivity arises from a dramatic compensation between big modifications in the numerator and in the denominator of the asymmetry. Therefore, it is wise to carefully consider such effect, because it could provide more sizeable modifications in other portions of the phase space.

We stress that azimuthal asymmetries based on the mechanism of dihadron fragmentation can be studied using collinear factorization, which facilitates the analysis of, e.g., evolution. From the theoretical point of view, this feature makes them a cleaner observable than the Collins effect in single-hadron fragmentation, where transverse-momentum dependent (TMD) functions are involved, whose evolution is yet not taken into account. All this procedure would not be plagued by theoretical uncertainties about factorization and evolution of TMD parton densities, which currently affect the analysis of single-hadron fragmentation. As a consequence, the option of using the semi-inclusive production of hadron pairs inside the same jet seems theoretically the cleanest way to extract the transversity distribution \(h_1\), at present 34.

When BELLE data will be available, it will be possible to constrain extended DiFF on \(e^+e^-\) data, to evolve them back to the HERMES scale, and to use them in the formula for the SIDIS spin asymmetry to directly extract \(h_1\).

\[\text{Acknowledgments}\]

This work is part of the European Integrated Infrastructure Initiative in Hadronic Physics project under Contract No. RII3-CT-2004-506078.

Authored by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177. The U.S. Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce this manuscript for U.S. Government purposes.

A.M. acknowledges support from BRNS, government of India, and hospitality of INFN - Sezione di Pavia (Italy) and Jefferson Laboratory (Virginia - USA), where part of this work was done.

[1] K. Konishi, A. Ukawa, and G. Veneziano, Phys. Lett. B78, 243 (1978).
FIG. 6: Same as in Fig. 4 but as a function of $z$ for the indicated $\tau$ bins.

[2] D. de Florian and L. Vanni, Phys. Lett. B578, 139 (2004), hep-ph/0310196.
[3] OPAL, P. D. Acton et al., Z. Phys. C56, 521 (1992).
[4] DELPHI, P. Abreu et al., Phys. Lett. B298, 236 (1993).
[5] ALEPH, D. Buskulic et al., Z. Phys. C69, 379 (1996).
[6] M. Grazzini, L. Trentadue, and G. Veneziano, Nucl. Phys. B519, 394 (1998), hep-ph/9709452.
[7] A. Bianconi, S. Boffi, R. Jakob, and M. Radici, Phys. Rev. D62, 034008 (2000), hep-ph/9907475.
[8] A. Bacchetta and M. Radici, Phys. Rev. D69, 074026 (2004), hep-ph/0311173.
[9] F. A. Ceccopieri, M. Radici, and A. Bacchetta, Phys. Lett. B650, 81 (2007), hep-ph/0703265.
[10] A. V. Efremov, L. Mankiewicz, and N. A. Tornqvist, Phys. Lett. B284, 394 (1992).
[11] J. C. Collins, S. F. Heppelmann, and G. A. Ladinsky, Nucl. Phys. **B420**, 565 (1994), hep-ph/9305309.
[12] X. Artru and J. C. Collins, Z. Phys. **C69**, 277 (1996), hep-ph/9504220.
[13] V. Barone and P. G. Ratcliffe, *Transverse Spin Physics* (World Scientific, River Edge, USA, 2003).
[14] J. P. Ralston and D. E. Soper, Nucl. Phys. **B152**, 109 (1979).
[15] O. Martin, A. Schafer, M. Stratmann, and W. Vogelsang, Phys. Rev. **D60**, 117502 (1999), hep-ph/9902250.
[16] A. Mukherjee, M. Stratmann, and W. Vogelsang, Phys. Rev. **D67**, 114006 (2003), hep-ph/0303226.
[17] A. Mukherjee, M. Stratmann, and W. Vogelsang, Phys. Rev. **D72**, 034011 (2005), hep-ph/0506315.
[18] J. C. Collins, Nucl. Phys. **B396**, 161 (1993), hep-ph/9208213.
[19] HERMES, A. Airapetian et al., Phys. Rev. Lett. **94**, 012002 (2005), hep-ex/0408013.
[20] HERMES, M. Diefenthaler, (2007), arXiv:0706.2242 [hep-ex], Proceedings of the 15th International Workshop on Deep Inelastic Scattering (DIS 2007), Munich, Germany, 16 - 20 Apr 2007.
[21] COMPASS, V. Y. Alexakhin et al., Phys. Rev. Lett. **94**, 202002 (2005), hep-ex/0503002.
[22] COMPASS, E. S. Ageev et al., Nucl. Phys. **B765**, 31 (2007), hep-ex/0610068.
[23] COMPASS, S. Levorato, (2008), 0808.0086.
[24] D. Boer, R. Jakob, and P. J. Mulders, Nucl. Phys. **B504**, 345 (1997), hep-ph/9702281.
[25] Belle, R. Seidl et al., Phys. Rev. **D78**, 032011 (2008), 0805.2975.
[26] M. Anselmino et al., Phys. Rev. **D75**, 054032 (2007), hep-ph/0701006.
[27] D. Boer and P. J. Mulders, Phys. Rev. **D57**, 5780 (1998), hep-ph/9711485.
[28] D. Boer, (2008), 0804.2408.
[29] J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981).
[30] X. Ji, J.-P. Ma, and F. Yuan, Phys. Rev. **D71**, 034005 (2005), hep-ph/0404183.
[31] F. A. Ceccopieri and L. Trentadue, Phys. Lett. **B636**, 310 (2006), hep-ph/0512372.
[32] A. Bacchetta, L. P. Gamberg, G. R. Goldstein, and A. Mukherjee, Phys. Lett. **B659**, 234 (2008), 0707.3372.
[33] D. Boer, Nucl. Phys. **B603**, 195 (2001), hep-ph/0102071.
[34] D. Boer, (2008), 0808.2886.
[35] J. C. Collins and G. A. Ladinsky, (1994), hep-ph/9411444.
[36] R. L. Jaffe, X. Jin, and J. Tang, Phys. Rev. Lett. **80**, 1166 (1998), hep-ph/9709322.
[37] M. Radici, R. Jakob, and A. Bianconi, Phys. Rev. **D65**, 074031 (2002), hep-ph/0110252.
[38] A. Bacchetta and M. Radici, Phys. Rev. **D74**, 114007 (2006), hep-ph/0608037.
[39] A. Bacchetta and M. Radici, Phys. Rev. **D67**, 094002 (2003), hep-ph/0212300.
[40] HERMES, A. Airapetian et al., JHEP **06**, 017 (2008), 0803.2307.
[41] COMPASS, A. Martin, (2007), hep-ex/0702002.
[42] D. Boer, R. Jakob, and M. Radici, Phys. Rev. **D67**, 094003 (2003), hep-ph/0302232.
[43] BELLE, K. Abe et al., Phys. Rev. Lett. **96**, 232002 (2006), hep-ex/0507063.
[44] K. Hasuko, M. Grosse Perdekamp, A. Ogawa, J. S. Lange, and V. Siegle, AIP Conf. Proc. **675**, 454 (2003).
[45] A. Bacchetta and M. Radici, Phys. Rev. **D70**, 094032 (2004), hep-ph/0409174.
[46] STAR, R. Yang, (2008), PKU-RBRC Workshop on Transverse Spin Physics, Beijing, June 30th-July 4th, 2008, http://rchep.pku.edu.cn/workshop/0806/20080627-if.pdf.
[47] A. Bacchetta, U. D’Alesio, M. Diehl, and C. A. Miller, Phys. Rev. **D70**, 117504 (2004), hep-ph/0410050.
[48] M. Anselmino et al., (2008), 0807.0173.
[49] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).
[50] M. Stratmann and W. Vogelsang, Phys. Rev. **D65**, 057502 (2002), hep-ph/0108241.
[51] X. Artru and M. Mekhfi, Z. Phys. **C45**, 669 (1990).