Rare regions and annealed disorder in quantum phase transitions

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Abstract. The Griffiths region that is due to rare regions and the resulting local moments in disordered itinerant quantum magnets, and its influence on the critical behavior, is considered within the framework of an effective field theory. It is shown that the local moments can be described in terms of static, annealed disorder, and the physical consequences of this description are discussed.

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1 Introduction

In systems with impurities, one distinguishes between two types of disorder: The impurities may be in equilibrium with the other degrees of freedom ("annealed" disorder), or ‘frozen in’ ("quenched" disorder). In the former case the free energy is obtained in terms of the disorder averaged partition function, while in the latter usually the free energy is a self-averaging quantity. In magnets, quenched disorder reduces the critical temperature from its clean value \( T_c^0 \) to \( T_c < T_c^0 \), and the magnetization \( M \) is a nonanalytic function of the external magnetic field \( B \) everywhere in the “Griffiths region” \( T_c < T < T_c^0 \). This is a result of the existence of rare regions that are devoid of any impurities and lead to the formation of locally ordered regions, or local moments, even though the system as a whole does not display global order. Since large rare regions are exponentially unlikely, this is a weak effect in generic classical systems; the singularity in the free energy is only an essential one.

A stronger effect of the same type is encountered in classical models with correlated disorder. McCoy and Wu studied a 2-d Ising model with identical bonds in \( y \)-direction, and bonds in \( x \)-direction that are random, but identical within each column. Here the rare regions lead to a hierarchy of temperatures \( T_n \) (\( n = 1, 2, \ldots \)) between \( T_c \) and \( T_c^0 \), with the disorder averaged \( n \)-th order susceptibility, \( \chi_n = \langle \partial^n M/\partial B^n \rangle \), diverging for all \( T < T_n \). Even the susceptibility proper, \( \chi = \chi_1 \), diverges for \( T_c < T < T_1 \), even though \( \langle M \rangle \) becomes nonzero only for \( T < T_c \), see Fig. The reason for this are atypical fluctuations in the distribution of \( \chi \) which dominate the mean.
A generalization to $d$ dimensions, with the disorder correlated in one of them, is very similar to models of quantum magnets in $d - 1$ spatial dimensions with uncorrelated disorder, since imaginary time plays the role of the correlated direction. One therefore expects strong effects of rare regions on generic quantum magnets. This is borne out by studies of quantum spin system in $d = 1$. However, little is known about the Griffiths phase and its influence on the critical properties in quantum systems in $d > 1$.

Motivated by similar considerations, we have recently considered rare regions in quantum magnets in $d > 1$. Previous theories for itinerant antiferromagnets and ferromagnets had found power-law critical behavior. It was found in Ref. that rare regions destroy the conventional fixed point in the former case, but not in the latter. In this paper we discuss the basic physics behind these findings, in particular the notion of local moments as static annealed disorder, and the relation between this physics and some important technical points that arise in the derivation of a model that describes local moments.

## 2 Itinerant quantum antiferromagnets

We consider the following action for an itinerant quantum antiferromagnet,

\[
S = \int dx \, dy \, \phi(x) \left[ \Gamma^{(0)}(x - y) + \delta(x - y) \delta t(x) \right] \phi(y) + u \int dx \, (\phi(x) \cdot \phi(x))^2. \tag{1}
\]

Here $x \equiv (x, \tau)$ comprises position $x$ and imaginary time $\tau$, $\int dx \equiv \int d\mathbf{x} \int_0^{1/T} d\tau$, and we put $\hbar = k_B = 1$. The Fourier transform of $\Gamma^{(0)}$ is (omitting constant prefactors)

\[
\Gamma^{(0)}(\mathbf{q}, \omega_n) = (t + q^2 + |\omega_n|)/2. \tag{2}
\]

Here $t$ denotes the distance from the critical point, and $\delta t$ in Eq. (1) is a random function of position.

In analogy to the treatment of classical random magnets by Dotenko et al., we now formally consider inhomogeneous saddle-point solutions of our field theory.
in the disordered phase \((t > 0)\). Such solutions exist since the disorder allows for order parameter configurations that are nonzero on ‘islands’ where \(t + \delta t(x) < 0\), and they describe a Griffiths phase. Let there be \(N\) such islands. Since they are far apart, we actually have \(2^N\) almost degenerate saddle-point solutions, \(\phi_{sp}^{(a)}\) \((a = 1, \ldots, 2^N)\), that are obtained by flipping the local magnetizations on the individual islands. The partition function can be calculated by expanding about a particular saddle point, say, \(\phi_{sp}^{(1)}\): \(Z = \int D[\varphi] \exp[-S(\phi_{sp}^{(1)} + \varphi)]\). This is exact as long as the functional integral is extended over all fluctuations \(\varphi\) of the order parameter field.

In practice, however, one can perform the integral only perturbatively, taking into account only small fluctuations. Simple energy and statistics considerations show that, in the thermodynamic limit, almost all of the nearly degenerate saddle point configurations are separated from one another by macroscopic energy barriers, so any perturbative evaluation of the integral defining \(Z\) will miss a macroscopic number \((2^N - 1)\) of contributions to \(Z\) that are equally important as the one obtained by expanding about \(\phi_{sp}^{(1)}\). However, it also means that, as long as we confine ourselves to a perturbative evaluation of the functional integral, we can simply add up these contributions:

\[
Z \approx \sum_{a=1}^{2^N} \int D[\varphi] e^{-S[\phi_{sp}^{(a)}+\varphi]} = \int D[\phi_{sp}] P[\phi_{sp}] \int D[\varphi] e^{-S[\phi_{sp}+\varphi]} . \tag{3}
\]

Here \(\int_{<}\) indicates an integration over small fluctuations only, and we have replaced the sum over saddle points by an integration over a suitable distribution \(P[\phi_{sp}]\).

The local moments \(\phi_{sp}(x)\) are described by a random function of position, since they are ultimately determined by the random \(\delta t(x)\). However, since they are generated by the electron system itself, in response to the random potential, they are in equilibrium with the rest of the electronic degrees of freedom. Therefore, the partition function is averaged over the saddle-point configurations, which hence represent static, annealed disorder.

We now perform the averages over the disorder. In a cumulant expansion, any reasonable distribution \(P[\phi_{sp}]\) will in particular produce a term that is produced by a Gaussian distribution, viz. \(w \int dx \int d\tau d\tau' (\varphi(x, \tau))^2 (\varphi(x, \tau'))^2\) with some coupling constant \(w\). The quenched disorder we handle by means of the replica trick \([1]\). In a Landau expansion, and with \(\Gamma(0)\) from Eq. (3), we obtain the following effective action:

\[
S_{eff} = \sum_{\alpha} \int dx\, dy \, \varphi^\alpha(x) \Gamma(0)(x,y) \varphi^\alpha(y) + u \sum_{\alpha} \int dx \, (\varphi^\alpha(x) \cdot \varphi^\alpha(x))^2
- \sum_{\alpha,\beta} (\Delta + w \delta_{\alpha,\beta}) \int dx dy \, \delta(x-y) \, (\varphi^\alpha(x))^2 (\varphi^\beta(y))^2 + O(\varphi^4) . \tag{4}
\]

Here \(\Delta\) is the variance of the \(\delta t\)-distribution, and \(\alpha\) and \(\beta\) are replica indices.

An RG analysis of \(S_{eff}\) reveals that the \(w\)-dependent terms in the flow equations contain factors of \(1/T\), and hence do not exist at \(T = 0\). This can be understood by means of a simple zero-dimensional toy model that couples Gaussian disorder \(v\), with variance \(w\), to an order parameter \(m\). Integrating out the disorder yields

\[
\int dv \, e^{-v^2 - 2vm/T} \propto e^{wm^2/T^2} \tag{5}
\]
This leads to a term in the free energy that is proportional to $1/T$. The reason for the occurrence of these unphysical terms is the unboundedness of the disorder distribution: Since annealed disorder allows for full equilibration, the system can always lower its free energy by choosing deeper and deeper potential wells. This unphysical feature can be easily cured by using a distribution that is bounded below. A mathematically and physically equivalent procedure is to make the variance of the unbounded distribution a linear function of temperature. We therefore write $w = \tilde{w}T$, and consider $\tilde{w}$ as the coupling constant that represents the presence of local moments. We note that this is a fundamental feature of annealed disorder. The fact that the variance of any annealed disorder distribution effectively vanishes as $T \to 0$ has important consequences for all problems involving local moments, since it implies that the mass that some modes acquire due to the local moments vanishes at zero temperature. The massive modes therefore do not simply drop out of the problem, as one might naively expect.

A standard one-loop RG analysis shows that the critical fixed point discussed in Ref. [8] is unstable with respect to the new coupling constant $\tilde{w}$ that represents the rare regions, and that one finds runaway flow in all physical regions of parameter space. The interpretation of this runaway flow is currently not clear. Possibilities include, the absence of a phase transition, or, more likely, the existence of a fixed point that is not accessible by means of a perturbative analysis of our effective action.

An analysis of itinerant quantum ferromagnets can be performed along the same lines[7]. In this case, an effective long-range interaction between the order parameter fields suppresses all fluctuations, including those due to the rare regions. As a result, the critical behavior of itinerant quantum ferromagnets as determined in Ref. [9] is not affected by rare regions, in sharp contrast to the antiferromagnetic case.

Finally, we point out that the treatment of rare regions sketched above, and the physical consequences of the properties of annealed disorder we have discussed, are very general and have important consequences for other problems where local moments are important, e.g., for metal-insulator transitions. Results for such problems will be reported elsewhere.

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