On the use of DMT approximations in adhesive contacts, with remarks on random rough contacts

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Abstract

The contact between rough surfaces with adhesion is an extremely difficult problem, and the approximation of the DMT theory (to neglect deformations due to attractive forces), originally developed for spherical contact of very small radius, are receiving some new interest. The DMT approximation leads to extremely large overestimate of the adhesive forces in the case of spherical contact, except at pull-off. For cylindrical contact, the opposite trend is found for larger contact areas. These findings suggest some caution in solving rough contacts with DMT models, unless the Tabor parameter is really low. Further approximate models like that of Pastewka & Robbins’ may be explained to work only for a coincidence of error cancellation in their range of parameters.

Keywords: Adhesion, Maugis’ theory, rough surfaces, DMT theory, JKR theory

1 Introduction

The Derjaguin-Muller-Toporov (DMT) theory (Derjaguin et al., 1975, Muller et al., 1980, 1983), for the contact of elastic spheres with adhesion, has a long history. After Bradley (1932) and Derjaguin (1934) obtained the adhesive force between two rigid spheres, equal to $2\pi R w$, where $w$ is the work of adhesion, and $R$ is the radius of the sphere, JKR (Johnson Kendall and Roberts 1971) developed a theory for elastic spheres, assuming adhesive forces occur entirely within the contact area, obtaining $3/4$ of the Bradley pull-off value, and hence the independence on the elastic modulus raised a long debate about the a comparison of the pull-off prefactor.

As the main attention in the diatribe between JKR and DMT was limited to
the pull-off value, it is often believed that DMT is the limit for Tabor parameter (Tabor, 1977)

\[
\mu = \left( \frac{Rw^2}{E^* \Delta r^3} \right)^{1/3} = \left( \frac{Rl_a^2}{\Delta r} \right)^{1/3} = \frac{\sigma_{th}}{E^*} \left( \frac{R}{l_a} \right)^{1/3} \rightarrow 0 \tag{1}
\]

where \(\Delta r\) is the range of attraction of adhesive forces, close to atomic distance, and \(E^*\) the plane strain elastic modulus. Also, we have introduced the length \(l_a = w/E^*\) as an alternative measure of adhesion, and \(\sigma_{th}\) is the theoretical strength of the material. Now, while it is true that DMT predicts the Bradley result for the force at pull-off also for elastic spheres, the DMT theories have been much less compared with exact results, when considering the entire load-displacement curves. In both DMT methods,

- the attraction forces act exclusively outside the contact, and
- the repulsive forces only are responsible for deformation.

Then, in the DMT "force method":

- the force of adhesion can be simply obtained by integrating, according to Derjaguin’s approximation, the forces of facing elements outside of the contact, separated by a gap which is given by Hertz theory.

We shall concentrate on the latter (force) method, which is what is commonly used when DMT approximation is considered in the generalized context of rough contact (see Persson & Scaraggi, 2014). In one looks at the force of adhesion not at pull-off, it decreases from \(2\pi Rw\) to \(\pi Rw\), in the "thermodynamic method"\(^1\) while it increases in the "force method", as shown by (Müller et al., 1983), and Pashley (1984). Pashley (1984) in particular notices that in the force method, the adhesive force should be always larger than \(2\pi Rw\), the value obtained for a truncated rigid sphere independently on the contact radius, as the Hertzian profile is closer to the flat surface than the rigid spherical profile.

Maugis (2000), in his Maugis-Dugdale analysis (which does not make the DMT approximations) called the low \(\mu\) end the "DMT theory", which in fact is now the version most commonly associated with DMT, and sometimes called DMT-M. In this version, the attractive forces are constant, and equal to the pull-off value, \(2\pi Rw\). This is indeed what comes out from DMT theory, but only in the limit of \(\mu = 0\); therefore, DMT is exact only in this limit case, and for any finite value of \(\mu\), DMT theories give an error which we shall estimate in fact in details in the present paper as a function of the Tabor parameter, since the previous estimates of (Müller et al., 1983), and Pashley (1984) do not clarify

\(^1\)In the "thermodynamic method", the force is computed by the rate of change of surface energy as the sphere is pressed with approach \(\alpha\), i.e. \(dW_s/\alpha\). It turns out that the "thermodynamic" method tends to give opposite error with respect to the "force method", and it is also more complicated to use, so it has not received much attention.
clearly the role of Tabor parameter. Greenwood (2007) also has discussed more
details of the DMT theory in the limit $\mu \to 0$.

But we shall not limit ourselves to the spherical contact case, since this case
has been given already much attention, and is only one special case. The DMT
approximation is gaining relevance more recently again, in the context of rough
contact, where there is a lot of interest in simplifying the problem since the JKR
assumption leads to very complicated and hysteretic behaviour, which so far, has
not been included in a framework of any theory, despite some attempts (Persson,
2002, Ciavarella, 2015). Moreover, as roughness at the small scales seems to
point to low values of Tabor parameter, the “almost rigid” behaviour has some
fundamental interest. Persson & Scaraggi (2014) have indeed attempted using
the DMT approximations using the Persson’s theory for adhesionless contact,
and seemed to find some reasonable accuracy at least for the range of parameters
they observed. Also, Pastewka & Robbins (2014, PR in the following) make
some scaling predictions which seem to fit well some limited range of their
extensive full numerical simulations involving atomistics rough solids. We made
a first attempt to discuss PR findings in Ciavarella (2016) where we noticed that,
if PR were concerned with spherical contact, using the DMT approximation
with the additional simplification of using only the asymptotic first term in the
expansion of the gap outside the Hertzian contacts, they would find easily
large errors. But one limit to this estimate is that we assumed circular contact,
whereas PR calculation shows more like 2D fractal contact area, perhaps closer
to very elongated contacts like in 2D cylindrical contact — indeed, as we will
discuss below, they find a characteristic diameter of the contact independent on
load, and load only affects the elongation of the contact area. Therefore, in the
present note we develop a simple 2D line contact DMT model, we give more
details about the DMT limit for the sphere, and make further comparisons with
the DMT rough contact results.

2 A 2D DMT-Maugis line contact model

For 2D contact with ”repulsive” diameter $d_{rep} = 2a$, the full form of the gap
outside the contact is

$$h(c) = \frac{a}{R} f \left( \frac{c}{a} \right)$$

where $c > a$ and (Johnson and Greenwood, 2008)

$$f \left( \frac{c}{a} \right) = \frac{1}{2} \left[ \frac{c}{a} \sqrt{\left( \frac{c}{a} \right)^2 - 1} \right]$$

whose first term in the series expansion near $c = a$ is $f_{as} \left( \frac{c}{a} \right) = \frac{\sqrt{\pi} a}{3 R} \left( \frac{c}{a} - 1 \right)^{3/2}$,
is used in the PR version of DMT method, as commented in (Ciavarella, 2016),
and in the later paragraph. We shall assume for the potential, a Maugis simple
law. This will permit a direct comparison with the "exact" Maugis solution including deformations induced by the adhesive stresses, given by Baney and Hui (1997) Morrow and Lovell (2005) and Johnson and Greenwood (2008), whereas Jin et al. (2014) give a double Hertz solution which show that results will not differ much with those with other choices of potential.

The pull-off force is not a simple multiple of $Rw$ as with circular contacts, but varies from $P_{\text{rigid}} = \sqrt{8R\sigma_{th}w}$ to $P_{\text{JKR}} = \frac{3}{4} \left(4\pi E^*Rw^2\right)^{1/3} = \frac{3}{4} \left(\pi E^*Rw^2\right)^{1/3}$, so it depends on elastic modulus.

We define the following non-dimensional contact radius and load

$$a^* = \frac{a}{2\pi^{1/3} R^{2/3}a}$$

$$P^* = \frac{P}{(\pi E^*Rw^2)^{1/3}} = \frac{P}{4^{2/3}P_{\text{JKR}}/3}$$

and accordingly the Hertz and JKR limits are found as (Johnson and Greenwood, 2008)

$$P^*_{\text{Hertz}} = a^{*2}$$

$$P^*_{\text{JKR}} = a^{*2} - 2\sqrt{a^*} = P^*_{\text{Hertz}} - 2\sqrt{a^*}$$

whereas the Maugis-Dugdale model shows a smooth transition between the Hertz and JKR limit — unlike the 3D case, where there is a transition from Bradley rigid to JKR model. Notice that the rigid limit is subtle: while there is a tendency to the Hertz regime, the actual pull-off in rigid limit is not zero.

Moving to a DMT force method estimate, setting the gap to $\Delta r$ gives

$$f\left(\frac{c}{a}\right) = \frac{R\Delta r}{a^2}$$

and it is clear that the approximation is good until $\frac{R\Delta r}{a^2} < 1$. In dimensionless notation, $R\Delta r/a^2 = \left(4\pi^{2/3}a^{*2}\mu\right)^{-1}$.

Using the asymptotic term, the lateral distance defining the size of attractive region (which is composed of two strips of size $d_{\text{att}}$) is

$$d_{\text{att,asym}} = \left[\frac{\left(\frac{3}{2} R\Delta r\right)^2}{d_{\text{rep}}}\right]^{1/3}$$

and when contact radius is small, we require a correction from the solution of (8), $\frac{d_{\text{rep}}}{a} = \frac{5}{6} - 1 = \beta^{d_{\text{att,asym}}}$. As we are using the Maugis potential, the attractive load is therefore simply the product of the theoretical strength and the area of the adhesive strips, $P_{DMT,\text{att}} = 2d_{\text{att}}\frac{w}{\Delta r}$. Using (9), the attractive load is obtained as

$$P_{DMT,\text{att}}^3 = -2^3 \beta^3 d_{\text{att}}^3 \left(\frac{w}{\Delta r}\right)^3 = \frac{-2^3 \beta^3}{2\pi} R^2 \Delta r^2 \frac{1}{2a} \left(\frac{w}{\Delta r}\right)^3$$

(10)
Using (4, 5) and Tabor parameter (1),

\[ P_{DMT, att}^* \simeq -\beta \left( \frac{\mu}{a^*} \right)^{1/3} \]  \hspace{1cm} (11)

The results of the DMT theory are presented in Fig.1a (dashed lines) for \( \mu = 0.05, 0.25, 1, 5 \), together with the Maugis solution of Johnson and Greenwood (2008) which we take as reference as "exact". In Fig.1b, we compare the JG Maugis solution with the further approximation of taking only the first term in the gap profile, which clearly leads to serious errors even at low Tabor parameter. We shall explain why in the PR use of this further approximation, the error was probably balanced by another approximation.
It is clear that the DMT theory gives a reasonable result only for $\mu < 0.05$ as at $\mu = 0.25$, the error is already significant, of the order of 20% at pull-off. Errors become large at $\mu > 1$, particularly at pull-off, as larger than 100%. This is clearer from fig. 2, where the pull-off values are plotted.
Fig.2 Line contact. Comparison of pull-off values for Maugis "exact" solution (solid blue line), with the DMT model with full shape of the gap (dashed line).

3 The spherical DMT model

As this case is classical, DMT has been compared with JKR and other models in a large number of papers. However, the comparison is mostly done for the value at pull-off, where of course the DMT model gives the correct Bradley result for $\mu = 0$: only perhaps (Muller et al., 1983), Pashley (1984), and Greenwood (2007) discuss more details of DMT at compressions larger than zero, and even then, do not fully clarify the error as a function of Tabor parameter.

We shall assume for the potential, a Maugis simple law, consistently to the line contact of the previous paragraph. We have already discussed a further simplified form of this model in (Ciavarella, 2016) inspired by PR paper (Pastewka & Robbins, 2014), namely using the first term asymptotic form of the gap function outside of the contact, and of computing the area of attraction by multiplying the perimeter of the contact by a $d_{\text{att}}$ size of attractive region. As we already discussed the very strong effect of both these approximations on the DMT solution, we shall try here to give a fair assessment of a full DMT model, and we do not make such further assumptions.

Therefore, we introduce the following dimensionless notation for the approach, and the load in spherical contact

$$\hat{\delta} = \delta / (\mu \Delta r) \quad ; \quad \hat{P} = P / (\pi Rw)$$ (12)

We know the expression of the gap outside the contact area (Muller et al.,
Imposing the gap is equal to the range of attractive forces, gives the size of attractive region. Writing the size of the region using the first order expansion of the gap is possible in closed form, giving

\[ d_{att, asym} = d_{rep} \frac{\left( \frac{3\pi R \Delta r}{2 \sqrt{2} d_{rep}} \right)^{2/3}}{2} \],

and the correct solution with the full expression of the gap requires a numerical correction for small contact areas, which is \( d_{att} = \beta_s d_{att, asym} \). We write \( \beta_s \) to avoid confusion with the equivalent coefficient in line contact. For the circular geometry Hertz theory gives \( \delta = \frac{d_{rep}}{4R} \) and hence, using the notation of \( PR \), \( d_{rep} \) for the repulsive contact diameter, and \( d_{att} \) for the size of attractive region (not a diameter), we have

\[ P_{att} = -\frac{\pi}{4} \left[ (d_{rep} + 2\beta_s d_{att})^2 - d_{rep}^2 \right] \frac{w}{\Delta r} \]

Normalizing with (12), we have

\[ \hat{P}_{att} = -\mu \hat{\delta} \left[ \left( 1 + \beta_s \left( \frac{3\pi}{8\sqrt{2}\mu \hat{\delta}} \right)^{2/3} \right)^2 - 1 \right] \] (14)

The result can be compared with the Maugis solution for \( \mu \to 0 \) (which indeed is the "DMT" limit for \( \mu \to 0 \)), and the JKR theory (Johnson, et al., 1971).\(^2\)

\[ \hat{P}_{\mu=0} = \frac{4}{3\pi} \hat{\delta}^{3/2} - 2 \] (15)

\[ \hat{P}_{DMT} = \frac{4}{3\pi} \hat{\delta}^{3/2} - \left( \frac{4}{3\pi} \left( \frac{3\pi}{8\sqrt{2}\mu \hat{\delta}} \right)^{4/3} - 1 \right) \beta_s \hat{\delta} \mu \] (16)

\[ \hat{P}_{JKR} = \hat{P}_0 - 1.1 \left( \hat{\delta} - \hat{\delta}_0 \right)^{1/2} + 0.43 \left( \hat{\delta} - \hat{\delta}_0 \right)^{3/2} \] (17)

where \( \hat{\delta}_0 = -\frac{3}{4} \pi^{2/3} \), \( \hat{P}_0 = -5/6 \) are the JKR values at pull-off in displacement control.

Fig.3 reports the DMT solution, with the correct Maugis solution for various values of \( \mu \), and immediately it appears that all of the DMT solutions are incorrectly below the limit for \( \mu \to 0 \), while the correct Maugis solutions are all above this limit — indeed, already at \( \mu = 0.05 \) the Maugis solution is quite distinct from the limit solution \( \mu \to 0 \) (which, adding to the confusion, Maugis calls the DMT limit!). Not shown is also that the correction for the full form of the gap is necessary at small indentations, as otherwise there is an additional spurious increase of adhesive forces estimate.

\(^2\)JKR is presented in a curve fitted form in order to be easily used.
Fig. 4 reports the values of the pull-off estimate with the DMT method, as compared with the Maugis solution.

Fig. 3 Spherical contact. Comparison of Maugis "exact" solution (solid blue line), with the DMT model with full shape of the gap (dashed line). The black and red solid line are $\mu = 0$ (what Maugis calls the DMT limit), and JKR limits. The arrow shows increasing values of Tabor parameter $\mu = 0.05, 0.25, 1, 5$. 

$\hat{P}$

$\delta$

Maugis

DMT $\mu = 0.05, 0.25, 1, 5$
4 DMT for rough contacts

These simple estimates suggest that the order of error in using the DMT approximation in actually solving rough adhesive contact problems, like done by Persson & Scaraggi (2014) for example, could be very large, unless Tabor parameter is quite low, and the body very close to "rigid". Incidentally, this means a condition stricter than just Tabor parameter lower than, say, 0.2 at the smallest scale in the model, since at the large scale, the contact may result hysteretic, and the DMT model would lead to very large errors. This may be in practice a very strong condition and would therefore limit significantly the use of such methodology. In the particular case, the Persson-Scaraggi method, which relies on use of additional approximations in solving the adhesiveless contact and integrating for the adhesive force, doesn’t lead to simple analytical results so it is also important to check, in the end, if there is a significant advantage with respect to a full numerical solution of the problem, like done by Pastewka and Robbins (2014).

Speaking of PR, they interpreted their "exact" results with some very simple estimates based on DMT approximation, and further simplifications. One may wonder why PR obtained a reasonable fit of their data, with assuming first order expansion of the gap function, which for example would lead to completely wrong results for very small contact area (see Fig.1b). In particular, this is surprising since their Tabor parameter are said to be of the order of 1, and
therefore not very small.

PR, in trying to find some scaling equations, suggested to use a DMT-like force method calculation of the attractive force during the loading stage of what they call "non-sticky" cases, based on an asymptotic first term for of the gap,

$$2 \frac{b(x)}{d_{rep}} = \sqrt{\frac{\pi}{3}} h'_{rms} \left( \frac{a}{d_{rep}} \right)^{3/2}$$  \hspace{1cm} (18)

where $h'_{rms}$ is not the slope at the contact edge obviously equal to $a/R$, as in a standard Hertzian contact, but a slope value they say estimated from the random roughness, and so anyway it is a fixed geometric quantity. This apparently innocuous assumption hides a very important effect: while taking the first order in the gap leads to very large error (see Fig.1b) at small areas, here PR take a representative value which can only permit to fit results of their cases — as they have $h'_{rms} = 0.1$ or $0.3$ this corresponds to a DMT model at $a/R = 0.1, 0.3$ respectively. If they had much smaller $h'_{rms}$ or much higher (as indeed it is very possible), then we would not know the order of the error. Therefore, we suspect that it is a pure coincidence that they were able to make reasonable fits.

However, since the rough contact is such that they find (numerically) an average representative diameter of the elongated areas of contact of

$$d_{rep} = 4 h'_{rms}/h''_{rms} = 2 h'_{rms} R$$  \hspace{1cm} (19)

where $h'_{rms}, h''_{rms}$ are rms slopes and curvatures, and we used $R = 2/h''_{rms}$. Hence, $a/R = h'_{rms}$, and it is not just an estimate that PR do when exchanging $h'_{rms}$ with $a/R$ in (18), but a quite correct substitution. Their contacts have elongated shapes whose representative size is of the order of the smallest wavelength. Indeed, we can estimate from random process theory that $\frac{h''_{rms}}{h'_{rms}} \simeq \sqrt{\frac{3+H}{2-H}} \left( \frac{2 \pi}{\lambda_s} \right) \simeq \theta \frac{\pi}{\lambda_s}$, where $\lambda_s$ is the smallest wavelength of roughness, $H$ is the Hurst exponent, and $\theta = 0.707, 1.0, 1.11$, respectively for $H = 0.3 - 0.5 - 0.8$. Therefore, geometry fixes $d_{rep}$ independently on load, and also the gap function is therefore only dependent on geometry.

If the contact areas were circular, applying Hertz theory would result in a constant compression $\delta = d_{rep}/R = 4 h'_{rms}/h''_{rms}$, or else

$$\delta \simeq \frac{4}{\pi \theta} h'_{rms} \lambda_s.$$  \hspace{1cm} (20)

In their simplified DMT calculation using first order term for the gap we estimated (Ciavarella, 2016)

$$P_{att} = \frac{\pi w R}{2} \left( \frac{\delta}{\Delta r} \right)^{1/3}$$  \hspace{1cm} (21)

Finally, as $\Delta r = 1.1 a_0$, for their case of the attractive potential $l_a/a_0 = 0.05$
(a₀ is atomic distance), and \( \frac{\lambda_s}{a_0} = 4, 8, 16, 32, 64, \)

\[
\frac{P_{\text{att}}}{2\pi \mu R} \approx \frac{3^{2/3}}{2} \left( \frac{4}{\pi \theta} \frac{\lambda_s}{a_0} \right)^{1/3}
\]

\[
\approx 0.83...2.08 \ (h'_{r_{rms}} = 0.1)
\]

\[
\approx 1.2...3.02 \ (h'_{r_{rms}} = 0.3)
\]

which is clearly up to 2 or 3 times higher than the value expected by the Maugis limit for \( \mu = 0 \) (Maugis, 2000) – which in turn means the error is probably much larger than 2 or 3. In other words, this suggests their contacts cannot be circular.

It remains to see therefore an estimate of a line contact geometry. Then, from our DMT estimate with \( \beta = 1 \) (only first term in the gap function)

\[
P_{\text{DMT}, \text{att}}^* \approx -\left( \frac{\mu}{a^*} \right)^{1/3}
\]

However, the dimensionless contact areas (4) is, using again (19), Tabor parameter (1), and \( R = 2/h''_{rms} \)

\[
\alpha^* = \frac{h'_{rms} \mu E}{2\pi^{1/3} \sigma_{th}}
\]

and hence, the PR-DMT 2D model becomes independent on Tabor, but only on slope

\[
P_{\text{PR}, \text{att}}^* = -\left( \frac{2\pi^{1/3} \sigma_{th}}{h'_{rms} E} \right)^{1/3}
\]

which clearly becomes arbitrarily large at small slopes.

The real total load curves for PR are therefore (from their potential, we find \( \frac{R}{\sigma_{th}} \approx \frac{1}{0.07} = 14.3 \) for \( \frac{h''}{a_0} = 0.05 \))

\[
P_{\text{PR}}^* = P_{\text{Hertz}}^* - 0.59 \frac{1}{h'_{rms}^{1/3}}
\]

We estimate the Tabor parameter at small scale for the PR data is about 0.4-0.5 for \( \lambda_s/a_0 = 4, \) and about 1 for \( \lambda_s/a_0 = 64, \) when \( h'_{rms} = 0.1. \) For \( h'_{rms} = 0.3 \) values are smaller by a factor close to 2 (0.27 and 0.67). This is for \( l_a/a_0 = 0.05 \) whereas for \( l_a/a_0 = 0.005, \) they are slightly larger. However, notice that the DMT-PR estimate does not depend on \( \lambda_s/a_0 \) wavelength, whereas the Maugis solution does, and therefore this trend is not captured with the simple DMT estimate.

Therefore, for \( l_a/a_0 = 0.05 \) we report in Fig.5a the case \( h'_{rms} = 0.1 \) together with Maugis with \( \mu = 0.5, 1, \) whereas in Fig.5b, the case \( h'_{rms} = 0.3 \) together with Maugis with \( \mu = 0.25, 0.5. \) It is clear that the agreement is reasonable and may explain the fit done by PR. In particular, the lower Tabor parameter seem to be better fitted by the simple DMT-PR law. In the PR results, these
are the low $\lambda_s/a_0$ which are the stickiest cases. For the high $\lambda_s/a_0$, we expect that DMT-PR underestimates the force of attraction, and therefore surfaces are stickier than the PR criterion would suggest. This seems in agreement with the PR findings for pull-off, which do not fit their stickiness criterion (see Ciavarella, 2016).

However, more investigation is needed to understand within which limit one can use this approximation for rough contacts. For very small slopes, PR-DMT would tend to extremely large attractive forces without limit, whereas when the Tabor parameter increases to values larger than 5, the correct adhesion solution becomes JKR and doesn’t show an unbounded attractive force.
Fig. 5. Line contact vs. rough contact in PR numerical simulations. Comparison of Maugis “exact” solution (solid blue line) for estimated range of Tabor parameter, with the DMT-PR model with first order term of gap (dashed line) for $h_{\text{rms}} = 0.1$ (a), and $h_{\text{rms}} = 0.3$ (b). In both figures, $l_a/a_0 = 0.05$. 

$\text{Maugis}$

$\text{JKR}$

$\text{Hertz}$

$\text{DMT-PR}$

$P^+$

$a^*$

$\text{a)}$

$\text{b)}$
5 Conclusion

We have examined in details the effect of the DMT approximations ("force method") in the case of a spherical and line contact, and we have shown that:

• for spherical contact, DMT leads to an error overestimating the attractive forces, which is very significant already at very low Tabor parameters, such as $\mu = 0.05$, but becomes extremely large at Tabor of the order of 1 — much larger than the error for the pull-off value, which occurs for a wrong value of separation;

• for line contact, DMT leads to a different error: the attractive force tends to be underestimated, except near pull-off, where now the error can be very significant at $\mu = 1$

• it could be that for elongated contacts, like those found in rough contact, the error could be intermediate between these two extreme conditions,

• in general, it is highly unlikely that DMT can be a good approximation for values of Tabor parameter greater than, say, 0.25.

• in rough contacts, due to special features of the contact area (far from set of spherical asperities like in the classical Fuller-Tabor (1975) model based on Nayak (1971) geometries), the contact appears a set of elongated contacts, whose representative diameter is relatively independent on load — but this is probably only true during loading.

• the use of only asymptotic term of the gap function leads to serious errors, however when adding the further crude approximation made by Pastewka & Robbins that the slope at the contact edge was given by geometrical considerations only, it leads to some more reasonable behaviour. However, quantitatively the fit they may have obtained is only a pure coincidence

• the scaling equations obtained from DMT type of model by Pastewka and Robbins should not be used outside their range of parameters, as extrapolation is not likely to work. In particular, for very small rms slopes of surface, the limit results do not seem realistic. Also, the dependence on Tabor parameter is ignored for the data with different short wavelengths than in the PR surfaces.

6 References

R.S. Bradley, (1932) The cohesive force between solid surfaces and the surface energy of solids. Phil. Mag. 13, 853.

M. Ciavarella, (2015). Adhesive rough contacts near complete contact. International Journal of Mechanical Sciences, 104, 104-111.
M. Ciavarella, (2016). On a recent stickiness criterion using a very simple generalization of DMT theory of adhesion. Journal of Adhesion Science and Technology, 30(24), 2725-2735.

B.V. Derjaguin, (1934) Theorie des Anhaftens kleiner Teilchen. Kolloid Zeitschrift 69, 155.

B.V. Derjaguin, V.M. Muller and Yu.P. Toporov, (1975) Effect of contact deformations on the adhesion of particles. J. Colloid Interface Sci. 53, 314.

K. N. G. Fuller, & D.Tabor, (1975). The effect of surface roughness on the adhesion of elastic solids. In Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences (Vol. 345, No. 1642, pp. 327-342).

J.A. Greenwood, (2007). On the DMT theory. Tribology Letters, 26(3), 203-211.

F. Jin, W. Zhang, S. Zhang, X. Guo, (2014). Adhesion between elastic cylinders based on the double-Hertz model. International Journal of Solids and Structures, 51(14), 2706-2712.

K.L. Johnson, J.A. Greenwood, (2008). A Maugis analysis of adhesive line contact. J. Phys. D: Appl. Phys. 41, 155315-1–155315-6.

V.M. Muller, V.S. Yuschenko and B.V. Derjaguin, (1980). On the influence of molecular forces on the deformation of an elastic sphere and its sticking to a rigid plane. J. Colloid Interface Sci. 77, 91.

V.M. Muller, B.V. Derjaguin and Yu.P. Toporov, (1983) On two methods of calculation of the force of sticking of an elastic sphere to a rigid plane. Colloids Surf 7. 251.

M.D. Pashley, (1984). Further consideration of the DMT model for elastic contact. Colloids Surf 12, 69.

K.L Johnson, K. Kendall, and A. D. Roberts. (1971). Surface energy and the contact of elastic solids. Proc Royal Soc London A: 324. 1558.

D Maugis, (2000). Contact, adhesion and rupture of elastic solids (Vol. 130). Springer, New York.

C.A. Morrow, M.R. Lovell, (2005). An extension to a cohesive zone solution for adhesive cylinders. J. Tribol. 127, 447–450.

V.M. Muller, V.S. Yuschenko and B.V. Derjaguin, (1980). On the influence of molecular forces on the deformation of an elastic sphere and its sticking to a rigid plane. J. Colloid Interface Sci. 77, 91.

V.M. Muller, B.V. Derjaguin and Yu.P. Toporov, (1983) On two methods of calculation of the force of sticking of an elastic sphere to a rigid plane. Colloids Surf 7. 251.

P.R. Nayak, Random process model of rough surfaces in plastic contact, Wear 26 (1973) 305–333.

M. D. Pashley, (1984). Further consideration of the DMT model for elastic contact. Colloids and surfaces, 12, 69-77.
L. Pastewka, & M.O. Robbins, (2014). Contact between rough surfaces and a criterion for macroscopic adhesion. Proceedings of the National Academy of Sciences, 111(9), 3298-3303.

B.N.J. Persson, (2002). Adhesion between an elastic body and a randomly rough hard surface, Eur. Phys. J. E 8, 385–401

B. N. Persson, & M. Scaraggi, (2014). Theory of adhesion: Role of surface roughness. The Journal of chemical physics, 141(12), 124701.

D. Tabor, (1977) Surface forces and surface interactions. J. Colloid Interface Sci. 58, 2.
\( h'_{\text{fms}} = 0.01, 0.1, 0.3, 1 \) \\
\( \mu = 0.05, 0.25, 1, 5 \) \\
Hertz (\( \mu = 0 \)) \\
JKR (\( \mu > 5 \))