Domain Discrepancy Aware Distillation for Model Aggregation in Federated Learning

Shangchao Su, Bin Li, Xiangyang Xue
Fudan University

Abstract
Knowledge distillation has recently become popular as a method of model aggregation on the server for federated learning. It is generally assumed that there are abundant public unlabeled data on the server. However, in reality, there exists a domain discrepancy between the datasets of the server domain and a client domain, which limits the performance of knowledge distillation. How to improve the aggregation under such a domain discrepancy setting is still an open problem. In this paper, we first analyze the generalization bound of the aggregation model produced from knowledge distillation for the client domains, and then describe two challenges, server-to-client discrepancy and client-to-client discrepancy, brought to the aggregation model by the domain discrepancies. Following our analysis, we propose an adaptive knowledge aggregation algorithm FedD3A based on domain discrepancy aware distillation to lower the bound. FedD3A performs adaptive weighting at the sample level in each round of FL. For each sample in the server domain, only the client models of its similar domains will be selected for playing the teacher role. To achieve this, we show that the discrepancy between the server-side sample and the client domain can be approximately measured using a subspace projection matrix calculated on each client without accessing its raw data. The server can thus leverage the projection matrices from multiple clients to assign weights to the student (aggregated) model will drop significantly. Some works (Zhang et al. 2021; Zhang and Yuan 2021) employ the data-free distillation by restoring pseudo samples on the server from the local models that fit the original domain data distributions. To use the data-free method in real natural images, the running mean and variance of the batch normalization layers must be obtained. However, some studies (Yin et al. 2020, 2021) show that the mean and variance may leak the user’s training data. In addition, the data-free method requires a large amount of additional calculation cost. Therefore, in this paper, we focus on the setting that the server has public unlabeled data. Now the problem is: how can we extract as much knowledge as possible from available unlabeled data on the server to maximize the performance of the aggregated model obtained through distillation?

To answer this question, we define a domain as a pair \((D, f)\) consisting of a distribution \(D\) on the input data \(\mathcal{X}\) and a labeling function \(f\), and then analyze the generalization error of the aggregated model obtained by KD in the client domains. We find that when the teacher models (local models) originate from different domains, there are two main factors that affect the performance of the aggregated model: 1) Server-to-Client (S2C) discrepancy. When the do-
main discrepancy between the server domain and each client domain increases, the performance of the aggregated model on the client side will decrease. 2) Client-to-Clients (C2C) discrepancy. There is a knowledge conflict among multiple client models. Since the teacher models are learned from different client domains, the sample in the server domain is likely to obtain different predictions from different teacher models, which may severely interfere with each other. Based on these two factors, we can see that if we can reasonably take into account the similarity between the server domain and different client domains, and assign the most appropriate teacher models to different samples in the server domain, the generalization error upper bound can be reduced.

Motivated by our analysis, we propose an adaptive knowledge aggregation method, Domain Discrepancy Aware Distillation, called FedD3A. During distillation, we assign independent teacher weights to a server-side sample based on how similar this sample is to the client domains. Then we can reduce the knowledge conflict between different teacher models. In each round, the client extracts the features of the local data using the backbone of the global model, and then calculates the subspace projection matrix of the local feature space. The server obtains the projection matrices from the clients, and calculates the angles between the server data and the local feature space to measure similarity without accessing client data features. Overall, our contributions are as follows:

- By analyzing the generalization error of the aggregated model in the client domains, we point out two possible reasons why distillation-based model aggregation performance drops when there is a discrepancy between the server unlabeled data and the client domain data, namely Server-to-Client (S2C) discrepancy and Client-to-Clients (C2C) discrepancy.

- Motivated by our analysis, we propose an aggregation method FedD3A based on domain discrepancy aware distillation, which further exploits the potential of abundant unlabeled data on the server.

- To validate our method, we conduct extensive experiments on several datasets. The results show that compared with baselines, our method has a significant improvement in both cross-silo and cross-device FL settings.

2 Related Work

**Federated Learning.** FedAvg (McMahan et al. 2017) proposes the federated averaging method. In each round of communication, a group of clients is randomly selected, the initial model is sent to all clients for training, and then the models trained by the clients are collected by the server. The aggregated model is obtained by averaging the model parameters of the clients. Some works (Li et al. 2019; Sahu et al. 2018) demonstrate the convergence of FedAvg and point out that the performance of FedAvg will degrade when different client datasets are non-iid distributed. A lot of research has tried to solve the non-iid problem encountered by FedAvg. One type of work (Li et al. 2020a; Karimireddy et al. 2020; Reddi et al. 2020; Wang et al. 2020c; Singh and Jaggi 2020; Su, Li, and Xue 2022) attempts to improve the fitting ability of the global aggregated model. The other type of work (Dinh, Tran, and Nguyen 2020; Fallah, Mokhtari, and Ozdaglar 2020; Hanzely et al. 2020; Li et al. 2021) seeks to establish the personalized federated learning (pFL), in which the clients can train different models with different parameters.

**Knowledge Distillation.** Knowledge distillation (Hinton, Vinyals, and Dean 2015) is a knowledge transfer approach and is initially proposed for model compression. Usually, a larger model is used as the teacher model, and the knowledge of the teacher model is transferred to the student model by letting the smaller student model learn the output of the teacher model. The techniques of KD are mainly divided into logits-based distillation (Hinton, Vinyals, and Dean 2015; Li et al. 2017), feature-based distillation (Romero et al. 2014; Huang and Wang 2017; Yim et al. 2017), and relation-based distillation (Park et al. 2019; Liu et al. 2019; Tung and Mori 2019). Some works (Du et al. 2020; Shen, He, and Xue 2019) have also conducted research on multi-teacher distillation. (Du et al. 2020) tries to make the gradient of the student model close to that of all teacher models by multi-objective optimization. (Shen, He, and Xue 2019) uses adversarial learning to force students to learn intermediate features similar to multiple teachers. There are some studies (Yin et al. 2020; Lopes, Fenu, and Starner 2017; Chawla et al. 2021) on data-free distillation, which use pseudo samples generated by the teacher model to replace real data.

**Federated Learning with Knowledge Distillation.** There are several ways of applying KD in FL: 1) The first is to perform distillation on the clients (Yao et al. 2021; Wu et al. 2022; Zhu, Hong, and Zhou 2021), treating the global aggregated model as the teacher. 2) In (Gong et al. 2021, 2022; Sun and Lyu 2020), the server and all the clients share a public unlabeled dataset. The predictions of different models on the dataset are transmitted among all parties to perform distillation. This is often used for personalized federated learning. 3) The third way is to directly use the client models as the teachers. (Lin et al. 2020; Guha, Talwalkar, and Smith 2019) suppose the server has unlabeled data and use an ensemble of multiple client models as the teacher. The average output of the teacher model is then used to calculate the distillation loss. (Sturluson et al. 2021) proposes using the median-based scores instead of the average logits of teacher outputs for distillation. In addition, the date-free method has also been applied on the server. (Zhang et al. 2021; Zhang and Yuan 2021) try to learn a generative model based on the ensemble of client models. However, high-quality pseudo samples rely on the running mean and variance carried by BN layers in the client models, which may reveal privacy (Yin et al. 2020; 2021). The data-free method requires a large amount of computational cost, and there is no evidence that it can be applied to tasks other than image classification. Nevertheless, data-free method is orthogonal to FedD3A, and all pseudo samples can be used as our training data.

3 Proposed Method

**Notations and Analysis**

Following the domain adaptation (Ben-David et al. 2010) field, during the analysis, we consider the binary classifica-
tion task, where a domain is defined as a pair consisting of a distribution $D$ on the input data $\mathcal{X}$ and a labeling function $f: \mathcal{X} \to \{0, 1\}$. In FL, we have $K$ client domains, $(T_k, f_k)$, $k = 1, \cdots, K$. The server domain is $(S, f_s)$, where $f_s$ is unknown because the server has only unlabeled data.

Knowledge distillation on the server takes $K$ client models $h_k, k = 1, \cdots, K$ as the teacher models, and uses the unlabeled data $S$ to train the aggregated model $h$ (student). KD aims to minimize the generalization error of the aggregated model on multiple clients in each round:

$$\sum_k \lambda_k R_{T_k}(h) = \sum_k \lambda_k \mathbb{E}_{x \sim T_k} \|f(x) - f_k(x)\|$$  \hspace{1cm} (1)

where $\lambda_k$ denotes the importance of the $k$-th client domain and defaults to $\frac{1}{K}$. $R$ is the risk. When performing multi-teacher distillation, it can be viewed as adding pseudo-labels to $S$. That is, we can view $f_s$ as $\tilde{f}_s(x) = \text{sign}(\frac{1}{K} \sum_k h_k(x))$. Next, we analyze the generalization error of the aggregated model in the client domains through the theory of domain adaptation [Ben-David et al. 2010].

**Definition 1** ($L^1$-distance [Ben-David et al. 2010]). $B$ is the set of measurable subsets under $D_1$ and $D_2$. The $L^1$-distance between two distributions $D_1$ and $D_2$ is defined as:

$$d_1(D_1, D_2) = 2 \sup_{B \in B} \left| \Pr_{D_1}(B) - \Pr_{D_2}(B) \right|$$  \hspace{1cm} (2)

Then we have:

**Theorem 1.** Given the server data $S$ , the client domains $(T_k, f_k)$, $k = 1, \cdots, K$, and a hypothesis space $\mathcal{H}$, $R_S(h) = \mathbb{E}_{x \sim S} \ell(h(x), \tilde{f}_s(x)) : \forall h \in \mathcal{H}$, the following holds:

$$\sum_k \lambda_k R_{T_k}(h) \leq R_S(h) + \sum_k \lambda_k d_1(S, T_k) \hspace{1cm} (\text{Server risk})$$

$$\sum_k \lambda_k \mathbb{E}_{x \sim S} \mathcal{F}(x), \sum_k \lambda_k \mathbb{E}_{x \sim T_k} \mathcal{F}(x) \hspace{1cm} (\text{S2C discrepancy})$$

$$\min \left\{ \sum_k \lambda_k \mathbb{E}_{x \sim S} \mathcal{F}(x), \sum_k \lambda_k \mathbb{E}_{x \sim T_k} \mathcal{F}(x) \right\} \hspace{1cm} (\text{C2C discrepancy})$$

where $\mathcal{F}(x) = |\tilde{f}_s(x) - f_k(x)|$.

See Supplementary Materials for proof. Theorem 1 implies that the error upper bound is related to three terms, from which we can summarize two major challenges for model aggregation through KD. The first challenge is related to the second term, that is server-to-client (S2C) discrepancy. When the difference between server domain data and client domain data is large, the risk upper bound will also increase. This domain discrepancy can naturally be reduced if we can access the client domain data and perform data augmentation through image mixing. Unfortunately, in the context of federated learning, we cannot obtain client data. The second challenge is related to the third term, we call it client-to-clients (C2C) discrepancy, that is, the difference between a single client and all clients. In the extreme case that we have only one client ($T_1, f_1$), the third term becomes:

$$\min \left\{ \mathbb{E}_{x \sim S} [\tilde{f}_s(x) - f_1(x)] , \mathbb{E}_{x \sim T_1} [f_1(x) - \tilde{f}_s(x)] \right\}$$

where $\mathbb{E}_{x \sim T_1} [f_1(x) - \tilde{f}_s(x)] = \mathbb{E}_{x \sim T_1} [f_1(x) - h_1(x)] = R_{T_1}(h_1)$, which is a fixed small value, because the client model $h_1$ has been trained on the client domain $T_1$. However, when we have multiple client models of different domains, due to the internal conflict between teacher models, the difference between the label function $f_1$ and each client-side label function $f_k$ will be accumulated.

Based on the above analysis, we propose to assign the most reasonable pseudo-label to each sample in $S$, thereby reducing the difference between the server’s labeling function and the labeling functions $\{f_k\}_k$ of other client domains. Suppose there are samples in $S$ that are similar to some client domains, and assume that we have a domain discriminant function $\text{dom}()$ by which all samples in $S$ can be labeled with their closest domain identity. Then $S$ can be viewed as $S = \sum_k \lambda_k S_k, k = 1, \cdots, K$, the samples in $S_k$ are similar in distribution to $T_k$, so the hypothesis $h_k(x)$ learned in the $k$-th domain can be used to approximate its labeling function. Now Theorem 1 can be modified as:

**Theorem 2.** For $\forall h \in \mathcal{H}$:

$$\sum_k \lambda_k R_{T_k}(h) \leq R_S(h) + \sum_k \lambda_k d_1(S_k, T_k) + \min \left\{ \sum_k \lambda_k \mathbb{E}_{x \sim S_k} \mathcal{F}_k(x), \sum_k \lambda_k \mathbb{E}_{x \sim T_k} \mathcal{F}_k(x) \right\} \hspace{1cm} (4)$$

where $\mathcal{F}_k(x) = |\tilde{f}_s(x) - f_k(x)|$.

In Theorem 2 $\tilde{f}_s(x)$ is approximated by $h_k(x)$. Since $h_k(x)$ is a hypothesis learned in $T_k$, $\mathbb{E}[|\tilde{f}_s(x) - f_k(x)|]$ is always a small value. Therefore, $\mathbb{E}[|\tilde{f}_s(x) - f_k(x)|]$ will also remain at a small value.

**Domain Discrepancy Aware Distillation**

From the above analysis, we establish an improved route, that is, use the domain discriminator $\text{dom}(\cdot)$ to perform teacher screening for individual samples on the server. Below we introduce the specific implementation method.

The ideal implementation is to obtain the data of all clients and directly calculate the data similarity, but it is obviously impossible in the FL scenario. One possible implementation of the domain discriminator is multi-party joint modeling. That is, one labels all data in $T_k$ with a label $k$, and then $K$ clients jointly train a $K$-class classifier by FL. This special setting has been studied in FedAws [Yu et al. 2020]. FedAws performs multiple rounds of federated training to jointly learn a classifier when each client has only one class of data. However, if FedAws is adopted, an additional auxiliary FL task for domain classification has to be introduced before the main FL task, which greatly increases the communication cost and computational overhead. Therefore, below we propose a lightweight approach based on subspace projection to implement $\text{dom}(\cdot)$. The overview is illustrated in Figure 1. The overall framework consists of five steps. Steps 1 and 4 are the communication between the server and the clients. The other steps are as follows:

**Subspace Projection** (Step2). Suppose that each client has dataset $D = \{(x_i, y_i)\}_{i=1}^n$, $n$ is the number of samples, for simplicity, we omit the client index. The feature
matrix is \( Z \in \mathbb{R}^{n \times d} \), each row vector \( z_i \in \mathbb{R}^d \) in \( Z \) is the feature of \( x_i \) extracted by backbone. Denote the subspace spanned by \( Z \) as \( S \), then the projection matrix of \( S \) is \( \hat{P} = Z^\top (ZZ^\top + \alpha I)^{-1} Z \). For any vector \( u \), \( \hat{P} \) can project \( u \) onto space \( S \), i.e., \( \hat{P} u \). The angle between the original vector \( u \) and the projection vector \( \hat{P} u \) reflects the proximity between \( u \) and \( Z \). We can use \( \cos(\hat{P}, \hat{P} u) \) to measure the proximity. To avoid the matrix-inverse operation, we calculate \( \hat{P} \) in an iterative manner (Zeng et al. 2019):

\[
\hat{P}_0 = \frac{I}{\alpha}, \quad \hat{P}_t = \hat{P}_{t-1} - \frac{\hat{P}_{t-1} z_i z_i^\top \hat{P}_{t-1}}{1 + z_i^\top \hat{P}_{t-1} z_i}
\]

where \( z_i \) is the mean feature of the \( i \)-th batch. After getting \( \hat{P} \), we have \( \hat{P} = \frac{I}{\alpha} \). As shown in Figure 1 in each round, before local training, the client first extracts the features matrix \( Z \) of local data based on the global model, and then calculates the projection matrix \( \hat{P} \), according to Eq.5 \( \hat{P} \) will be uploaded to the server along with the model parameters.

**Local Training** (Step3): After obtaining the projection matrix, the client uses the local dataset to train the local model. When the global model and the local model are homogeneous, the local model is initialized with the global model parameters. When the global model and the local model are heterogeneous, the global model and the local models have different structures. The client needs to use the global model as a teacher for training, that is, using the cross entropy loss and a distillation loss to guide the local model training at the same time. In the experimental part, we will show the effectiveness of FedD3A under both the homogeneous and heterogeneous setting.

**Server Training** (Step5): On the server, suppose the client indexes chosen in this round is \( M_1, \ldots, M_m \), the backbone of global model is \( B \). For each sample, the corresponding teacher weights \( \alpha \) is calculated as follows:

\[
r = [\cos \langle B(x), P_{M_1} B(x) \rangle, \ldots, \cos \langle B(x), P_{M_m} B(x) \rangle]
\]

\[
\alpha = \text{SoftMax} \left( \frac{r - \text{mean}(r)}{\sqrt{\text{var}(r)}} \right)
\]

The local models \( h_{k_i}, k = M_1, \ldots, M_m \) are used as the teachers in distillation. Then for each sample \( x \) on the server, we have the pseudo-label:

\[
y = \sum_{i=1}^m \alpha_i h_{M_i}(x)
\]

Now the loss function for distillation is:

\[
E_{KD} = D_{KL}(h(x)||y)
\]

where \( D_{KL} \) is the Kullback-Leibler divergence, \( h \) can be initialized using the average parameters of \( h_{M_1}, \ldots, h_{M_m} \). The overall method is described in Algorithm 1.
Discussions

Communication Cost. Compared to FedAvg (McMahan et al. 2017), our method only needs to send the additional projection matrix to the server. Taking ResNet34 as an example, the number of parameters of the model is 21.8M. Assuming that the data are $224 \times 224$ images and the feature extracted by the backbone is a 512 dimensional vector. In this case, the number of parameters of the projection matrix is $512 \times 512$, only accounting for $1.2\%$ ($512^2/21.8M$) of the total parameters of the model. If a technique such as SVD is used for matrix compression, this ratio can be further reduced. In addition, FedD3A can speed up the convergence and make the aggregated model have better performance, which makes the extra $1.2\%$ communication cost worthwhile.

Privacy Protection. Our method only needs to exchange model parameters and projection matrices, without sharing the raw client-side datasets or features. Compared with the operation of directly transmitting the running mean images or the features extracted from the backbone in some works (Peng et al. 2019b, Yoon et al. 2021), the projection matrix can avoid directly further disclosing the user data features. It is also worth noting that, in the proposed method, it is not required to transmit the mean and variance in the BN layers of local models. Therefore the risk of restoring raw data (Yin et al. 2020, 2021) can be avoided.

Application to Heterogeneous Models. Knowledge distillation based model aggregation methods only involve the outputs of the local models, thus allowing local models to have different structures. In this paper, we focus on the following heterogeneous model aggregation scenario: the clients have different model structures, the server aggregates these models into a public model structure. After the aggregated model is sent to the clients, each client uses KD for knowledge transfer. Ultimately, the server can obtain the global model while the clients retain their personalized models.

4 Experiments

In this section, to verify the ability of FedD3A, we first compare the overall performance of compared methods in cross-silo and cross-device settings. Then, we demonstrate the efficacy of FedD3A in heterogeneous model aggregation. Finally, we validate the effect of the proposed teacher model weighting strategy.

Datasets and Models. For federated learning, we conduct experiments based on Digit-5 (Zhao et al. 2020) which has five domains with 10 categories $32 \times 32$ images per domain, and DomainNet (Peng et al. 2019a) which is a large-scale multi-domain dataset containing 6 domains with 345 categories $224 \times 224$ images per domain. The image examples in different domains are shown in Figure 2. By default, ResNet50 (He et al. 2016) is selected for DomainNet, ResNet18 is selected for Digit-5. All models are initialized using ImageNet pre-trained parameters.

Baselines. We take the following three different types of methods as our competitors: 1) FedAvg (McMahan et al. 2017), a classical aggregation method based on parameter averaging. 2) FedProx (Li et al. 2020b), an improved method based on FedAvg for non-iid datasets. 3) FedDF (Lin et al. 2020), a SOTA distillation based model aggregation method, where each teacher is treated equally.

Implementation Details. We take the momentum SGD as the optimizer. The initial learning rate is set to 0.01, the hyperparameter $\mu$ in FedProx is set to 0.1. With the increase of communication rounds, the learning rate gradually decreases to $1e^{-4}$ according to the cosine schedule. We implement our method based on PyTorch (Paszke et al. 2019) with two NVIDIA GeForce RTX 3090 GPUs. The batch size is 256.

Cross-silo Setting. We select different domains as the server domain in turn, and the rest as the client domains. Take DomainNet for example, we can take the real domain as the server, the remaining five domains serve as five clients. The number of local epochs and global rounds are 1 and 30, respectively. The distillation on the server also perform 1 epoch. The main results are shown in Figure 3. Compared with the baseline methods, we can see: 1) In general, FedD3A has obvious improvement and can use fewer communication rounds to achieve higher accuracy, especially on the more challenging DomainNet dataset. 2) FedDF outperforms FedAvg. However, it is worse than FedProx. Also based on distillation, the accuracy of FedD3A can lead FedDF by nearly $10\%$ on DomainNet, much higher than FedProx, which indicates that addressing domain discrepancy problem is critical to improve the distillation.

In order to observe the performance of the aggregated model in extreme cases when the server domain is completely consistent with a client domain, we replace the data of the server in the above experiment Figure 3(a) with clipart, that is, the data of the server and the first client are completely consistent. The result is shown in Figure 5 ‘All_before’ denotes the mean accuracy on all clients before replacing the server data. It can be seen that: 1) In FedD3A, the mean accuracy of all clients is slightly improved compared with the original results and this improvement mainly comes from the clipart domain. The performance of the other four domains has hardly changed. This is because the server data coincides with the first client (clipart), so that the student can learn more from the teacher corresponding to that client. 2) In FedDF, all domains have a slight decrease.

Cross-device Setting. We split each local domain into five clients, then the DomainNet is divided into 25 clients. In order to simulate the non-IID case, following the common practices (Lin et al. 2020), we sample $p_c \sim \text{Dir}(\beta1_K)$,
$K = 5$, and allocate a $p_{c,k}$ proportion of the instances with label $c$ to the training set of the $k$-th local model in each domain. When $\beta$ is small, the dataset is more challenging. In each round, 5 clients are selected for aggregation.

Figure 3 shows the results of $\beta = 0.2$ and $\beta = 1$. We can see: 1) Compared with $\beta = 1$, the gap between FedAvg and other methods is greater when $\beta = 0.2$. It can be seen that non-IID has a great impact on the performance of model averaging. 2) Compared with FedAvg, FedProx has a significant improvement, but its performance is still inferior to that of the distillation based method FedDF. While in the cross-silo experiment, FedDF is worse than FedProx. This indicates that the distillation based method is more effective when the clients are extreme non-IID. 3) In general, our FedD3A clearly outperforms other competitors with higher convergence speed and higher accuracy.

**Heterogeneous Model Aggregation.** In order to demonstrate the ability of FedD3A in heterogeneous model aggregation, we do experiments using VGG16 (Simonyan and Zisserman 2014), ResNet34 (He et al. 2016), ResNet18 (He et al. 2016) and MobileNetV3 (Howard et al. 2019) on different clients, respectively. For cross-device setting, five clients within each domain use the same kind of model. We take ResNet50 as the global model. In each round, the clients take the global model as a teacher, and then trains its own local model by cross-entropy loss and distillation loss (KL divergence). The server uses the heterogeneous models uploaded by the clients for multi-teacher distillation to obtain the global model. The result is in Figure 6. Compared with FedDF, FedD3A can converge to better performance and has a stable learning process. Only four rounds are needed to...
Table 1: Comparison of different weighting strategies.

|                | Client1 (infograph) | Client2 (painting) | Client3 (quickdraw) | Client4 (real) | Client5 (sketch) | Avg  |
|----------------|---------------------|--------------------|--------------------|---------------|-----------------|------|
| Ceiling        | 0.272               | 0.344              | 0.188              | 0.707         | 0.363           | 0.445|
| Avg            | 0.192               | 0.436              | 0.188              | 0.593         | 0.486           | 0.369|
| Random         | 0.194               | 0.443              | 0.126              | 0.583         | 0.466           | 0.363|
| FedD3A-onehot  | 0.264               | 0.501              | 0.162              | 0.634         | 0.540           | 0.420|
| FedD3A-soft    | 0.256               | 0.535              | 0.176              | 0.673         | 0.554           | 0.439|

Table 2: Classification ability of subspace projection.

|                  | prototype-based | projection-based |
|------------------|-----------------|------------------|
| MNIST (10 clients) | 0.693 ± 0.005  | 0.907 ± 0.004    |
| DomainNet (345 clients) | 0.382 ± 0.007 | 0.433 ± 0.009    |
| CIFAR10 (10 clients)  | 0.494 ± 0.003  | 0.628 ± 0.005    |
| CIFAR100 (100 clients) | 0.355 ± 0.007 | 0.444 ± 0.003    |
| Tiny Imagenet (200 clients) | 0.411 ± 0.024 | 0.504 ± 0.036    |

obtain the convergence accuracy of FedDF in Figure 6(a).

**Weighting Strategies.** The main idea of FedD3A is adaptive filtering of teachers' knowledge. To see how much this contributes, we compare the following different weighting methods: 1) **Avg.** Take the average output of all teachers. 2) **Random.** Add random weights to the different teachers, i.e., \( \alpha = \text{SoftMax}(z) \), where \( z \) is a random noise. 3) **FedD3A-onehot.** For each input sample, only the single teacher model with the largest weight is retained. 4) **FedD3A-soft.** Weight the different teachers using our proposed weighting method, see Eq 6. This is used in the previous experiments. 5) **Ceiling.** Train a 5-class classifier by aggregating the datasets from 5 client domains. Note that this classifier is not actually available due to privacy issues. We do a one-round experiment (1 global round, 50 local epochs) using these five strategies.

Take *clipart* as server. The results are reported in Table 1. It can be seen that random weighting is close to the average strategy, both of which have lower accuracy. In contrast, our FedD3A method has an obvious performance improvement. This shows that it is very important to carry out adaptive knowledge filtering for teacher models. In addition, FedD3A-soft is better than FedD3A-onehot, which implies that onehot strategy may ignore the overlapping knowledge among local models.

**Classification Ability of Subspace Projection.** In the above experiments, we verify the effectiveness of FedD3A in the FL context. One remaining problem is that, based on subspace projection, we can really judge which domain the test image comes from? To intuitively verify this, we design the following experiments. We divide the dataset according to category, and each client has one class. For example, the training set of CIFAR100 is divided into 100 clients according to category, and the client index is consistent with the class label. By uploading the subspace projection matrices, we try to classify the test set in the server with only one round. In this experiment, each category can be regarded as a domain, there is no need to consider cross-domain datasets. Therefore, we add several more common datasets in addition to Digit-5 and DomainNet. The backbone is a 3-layer MLP for MNIST, and a ResNet18 for CIFAR10/100, a ResNet50 for the other datasets. In this experiment, we take the prototype-based method as a baseline, in which, each client upload the mean

Figure 7: Feature visualization of global model. Different numbers represent different classes. Different colors denote different domains.

feature as the class prototype. The server calculates which prototype the feature of the test image is most similar to. We take three random seeds and average the results. See Table 2, the projection-based method does not need to upload the raw features, and achieves better test accuracy.

**Visualization.** We randomly select four categories of data from all domains and visualize the features extracted by the global model in the cross-silo experiment. The results are in Figure 7. ‘Ceiling’ combines all client data to train a global model. It can be seen that: 1) Compared with the baseline method, FedD3A can learn better feature representations that are more conducive to classification. On the contrary, the features learned by FedAvg are greatly affected by domain discrepancy. 2) Domain information can be distinguished from the extracted features. In the same cluster, data from different domains are not mixed, which indicates that the subspace projection can be used to distinguish data domains.

**5 Conclusions**

In this paper, we explore the data domain discrepancy issue faced when leveraging server unlabeled data for model aggregation in federated learning. We show that the generalization error upper bound of the aggregation model on the clients can be reduced by assigning appropriate teacher weights to the unlabeled data on the server. Based on our analysis, we propose a domain discrepancy aware knowledge distillation algorithm FedD3A. Experimental results show that FedD3A outperforms the baseline methods in both cross-silo and cross-device FL settings, and heterogeneous model aggregation scenarios. Moreover, the experiments also verify that the adaptive filtering of teacher knowledge is very important to improve the performance of the aggregated model, which validate the claims in the contributions in Section 4.
References

Ben-David, S.; Blitzer, J.; Crammer, K.; Kulesza, A.; Pereira, F.; and Vaughan, J. W. 2010. A theory of learning from different domains. *Machine learning*, 79(1): 151–175.

Chawla, A.; Yin, H.; Molchanov, P.; and Alvarez, J. 2021. Data-free knowledge distillation for object detection. In *Proceedings of the IEEE/CVF Winter Conference on Applications of Computer Vision*, 3289–3298.

Dinh, C. T.; Tran, N. H.; and Nguyen, T. D. 2020. Personalized Federated Learning with Moreau Envelopes. In Larochelle, H.; Ranzato, M.; Hadsell, R.; Balcan, M.; and Lin, H., eds., *Advances in Neural Information Processing Systems 33: Annual Conference on Neural Information Processing Systems 2020, NeurIPS 2020*, December 6-12, 2020, virtual.

Du, S.; You, S.; Li, X.; Wu, J.; Wang, F.; Qian, C.; and Zhang, C. 2020. Agree to disagree: Adaptive ensemble knowledge distillation in gradient space. *Advances in neural information processing systems*, 33: 12345–12355.

Fallah, A.; Mokhtari, A.; and Ozdaglar, A. 2020. Personalized federated learning with theoretical guarantees: A model-agnostic meta-learning approach. *Advances in Neural Information Processing Systems*, 33: 3557–3568.

Gong, X.; Sharma, A.; Karanam, S.; Wu, Z.; Chen, T.; Doerrmann, D.; and Innanen, A. 2021. Ensemble Attention Distillation for Privacy-Preserving Federated Learning. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, 15076–15086.

Gong, X.; Sharma, A.; Karanam, S.; Wu, Z.; Chen, T.; Doerrmann, D.; and Innanen, A. 2022. Preserving Privacy in Federated Learning with Ensemble Cross-Domain Knowledge Distillation.

Guha, N.; Talwalkar, A.; and Smith, V. 2019. One-shot federated learning. *arXiv preprint arXiv:1902.11175*.

Hanzely, F.; Hanzely, S.; Horváth, S.; and Richtárik, P. 2020. Lower bounds and optimal algorithms for personalized federated learning. *Advances in Neural Information Processing Systems*, 33: 2304–2315.

He, K.; Zhang, X.; Ren, S.; and Sun, J. 2016. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, 770–778.

Hinton, G.; Vinyals, O.; and Dean, J. 2015. Distilling the knowledge in a neural network. *arXiv preprint arXiv:1503.02531*.

Howard, A.; Sandler, M.; Chu, G.; Chen, L.-C.; Chen, B.; Tan, M.; Wang, W.; Zhu, Y.; Pang, R.; Vasudevan, V.; et al. 2019. Searching for mobilenetv3. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, 1314–1324.

Huang, Z.; and Wang, N. 2017. Like what you like: Knowledge distill via neuron selectivity transfer. *arXiv preprint arXiv:1707.01219*.

Karimireddy, S. P.; Kale, S.; Mohri, M.; Reddi, S.; Stich, S.; and Suresh, A. T. 2020. SCAFFOLD: Stochastic controlled averaging for federated learning. In *International Conference on Machine Learning*, 5132–5143. PMLR.

Li, T.; Sahu, A. K.; Zaheer, M.; Sanjabi, M.; Talwalkar, A.; and Smith, V. 2020a. Federated Optimization in Heterogeneous Networks. In Dhillon, I. S.; Papailiopoulos, D. S.; and Sze, V., eds., *MLSys. mlsys.org*.

Li, T.; Sahu, A. K.; Zaheer, M.; Sanjabi, M.; Talwalkar, A.; and Smith, V. 2020b. Federated optimization in heterogeneous networks. *Proceedings of Machine Learning and Systems*, 2: 429–450.

Li, X.; Huang, K.; Yang, W.; Wang, S.; and Zhang, Z. 2019. On the convergence of fedavg on non-iid data. *arXiv preprint arXiv:1907.02189*.

Li, X.; Jiang, M.; Zhang, X.; Kamp, M.; and Dou, Q. 2021. FedBN: Federated Learning on Non-IID Features via Local Batch Normalization. In *9th International Conference on Learning Representations, ICLR 2021*, Virtual Event, Austria, May 3-7, 2021. OpenReview.net.

Li, Y.; Yang, J.; Song, Y.; Cao, L.; Luo, J.; and Li, L.-J. 2017. Learning from noisy labels with distillation. In *Proceedings of the IEEE International Conference on Computer Vision*, 1910–1918.

Lin, T.; Kong, L.; Stich, S. U.; and Jaggi, M. 2020. Ensemble distillation for robust model fusion in federated learning. *Advances in Neural Information Processing Systems*, 33.

Liu, Y.; Cao, J.; Li, B.; Yuan, C.; Hu, W.; Li, Y.; and Duan, Y. 2019. Knowledge distillation via instance relationship graph. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 7096–7104.

Lopes, R. G.; Fenu, S.; and Starner, T. 2017. Data-free knowledge distillation for deep neural networks. *arXiv preprint arXiv:1710.07535*.

McMahan, B.; Moore, E.; Ramage, D.; Hampson, S.; and y Arcas, B. A. 2017. Communication-efficient learning of deep networks from decentralized data. In *Artificial Intelligence and Statistics*, 1273–1282. PMLR.

Park, W.; Kim, D.; Lu, Y.; and Cho, M. 2019. Relational knowledge distillation. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 3967–3976.

Paszke, A.; Gross, S.; Massa, F.; Lerer, A.; Bradbury, J.; Chanan, G.; Killeen, T.; Lin, Z.; Gimelshein, N.; Antiga, L.; et al. 2019. Pytorch: An imperative style, high-performance deep learning library. *Advances in neural information processing systems*, 32.

Peng, X.; Bai, Q.; Xia, X.; Huang, Z.; Saenko, K.; and Wang, B. 2019a. Moment matching for multi-source domain adaptation. In *Proceedings of the IEEE/CVF International conference on computer vision*, 1406–1415.

Peng, X.; Huang, Z.; Zhu, Y.; and Saenko, K. 2019b. Federated adversarial domain adaptation. *arXiv preprint arXiv:1911.02054*.

Reddi, S.; Charles, Z.; Zaheer, M.; Garrett, Z.; Rush, K.; Konečný, J.; Kumar, S.; and McMahan, H. B. 2020. Adaptive federated optimization. *arXiv preprint arXiv:2003.00295*.

Romero, A.; Ballas, N.; Kahou, S. E.; Chassang, A.; Gatta, C.; and Bengio, Y. 2014. Fitnets: Hints for thin deep nets. *arXiv preprint arXiv:1412.6550*.
Sahu, A. K.; Li, T.; Sanjabi, M.; Zaheer, M.; Talwalkar, A.; and Smith, V. 2018. On the convergence of federated optimization in heterogeneous networks. *arXiv preprint arXiv:1812.06127*, 3: 3.

Shen, Z.; He, Z.; and Xue, X. 2019. Meal: Multi-model ensemble via adversarial learning. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 33, 4886–4893.

Simonyan, K.; and Zisserman, A. 2014. Very deep convolutional networks for large-scale image recognition. *arXiv preprint arXiv:1409.1556*.

Singh, S. P.; and Jaggi, M. 2020. Model fusion via optimal transport. *Advances in Neural Information Processing Systems*, 33.

Sturluson, S. P.; Trew, S.; Muñoz-González, L.; Grama, M.; Passerat-Palmbach, J.; Rueckert, D.; and Alansary, A. 2021. FedRAD: Federated Robust Adaptive Distillation. *arXiv preprint arXiv:2112.01405*.

Su, S.; Li, B.; and Xue, X. 2022. One-shot Federated Learning without Server-side Training. *arXiv preprint arXiv:2204.12493*.

Sun, L.; and Lyu, L. 2020. Federated model distillation with noise-free differential privacy. *arXiv preprint arXiv:2009.05537*.

Tung, F.; and Mori, G. 2019. Similarity-preserving knowledge distillation. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, 1365–1374.

Wang, H.; Yurochkin, M.; Sun, Y.; Papailiopoulos, D. S.; and Khazaeni, Y. 2020a. Federated Learning with Matched Averaging. In *International Conference on Learning Representations*.

Wang, J.; Liu, Q.; Liang, H.; Joshi, G.; and Poor, H. V. 2020b. Tackling the objective inconsistency problem in heterogeneous federated optimization. *Advances in Neural Information Processing Systems*, 33.

Wu, C.; Wu, F.; Lyu, L.; Huang, Y.; and Xie, X. 2022. Communication-efficient federated learning via knowledge distillation. *Nature Communications*, 13(1): 1–8.

Yang, Q.; Liu, Y.; Chen, T.; and Tong, Y. 2019. Federated machine learning: Concept and applications. *ACM Transactions on Intelligent Systems and Technology (TIST)*, 10(2): 1–19.

Yao, D.; Pan, W.; Dai, Y.; Wan, Y.; Ding, X.; Jin, H.; Xu, Z.; and Sun, L. 2021. Local-Global Knowledge Distillation in Heterogeneous Federated Learning with Non-IID Data. *arXiv preprint arXiv:2107.00051*.

Yim, J.; Joo, D.; Bae, J.; and Kim, J. 2017. A gift from knowledge distillation: Fast optimization, network minimization and transfer learning. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 4133–4141.

Yin, H.; Mallya, A.; Vahdat, A.; Alvarez, J. M.; Li, Z.; Molchanov, P.; and Kautz, J. 2020. Dreaming to distill: Data-free knowledge transfer via deepinversion. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 8715–8724.

Yoon, T.; Shin, S.; Hwang, S. J.; and Yang, E. 2021. FedMix: Approximation of Mixup under Mean Augmented Federated Learning. In *9th International Conference on Learning Representations, ICLR 2021, Virtual Event, Austria, May 3-7, 2021*. OpenReview.net.

Yu, F.; Rawat, A.; Menon, A.; and Kumar, S. 2020. Federated learning with only positive labels. In *International Conference on Machine Learning*, 10946–10956. PMLR.

Zeng, G.; Chen, Y.; Cui, B.; and Yu, S. 2019. Continual learning of context-dependent processing in neural networks. *Nature Machine Intelligence*, 1(8): 364–372.

Zhang, J.; Chen, C.; Li, B.; Lyu, L.; Wu, S.; Xu, J.; Ding, S.; and Wu, C. 2021. A Practical Data-Free Approach to One-shot Federated Learning with Heterogeneity. *arXiv preprint arXiv:2112.12371*.

Zhang, L.; and Yuan, X. 2021. FedZKT: Zero-Shot Knowledge Transfer towards Heterogeneous On-Device Models in Federated Learning. *arXiv preprint arXiv:2109.03775*.

Zhao, S.; Wang, G.; Zhang, S.; Gu, Y.; Li, Y.; Song, Z.; Xu, P.; Hu, R.; Chai, H.; and Keutzer, K. 2020. Multi-source distilling domain adaptation. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, 12975–12983.

Zhao, M.; Hong, J.; and Zhou, J. 2021. Data-free knowledge distillation for heterogeneous federated learning. In *International Conference on Machine Learning*, 12878–12889. PMLR.