Goulart, J. H. de M.; Comon, P.
On the minimal ranks of matrix pencils and the existence of a best approximate block-term tensor decomposition. (English) Zbl 1403.15008
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Summary: Under the action of the general linear group with tensor structure, the ranks of matrices $A$ and $B$ forming an $m \times n$ pencil $A + \lambda B$ can change, but in a restricted manner. Specifically, with every pencil one can associate a pair of minimal ranks, which is unique up to a permutation. This notion can be defined for matrix pencils and, more generally, also for matrix polynomials of arbitrary degree. In this paper, we provide a formal definition of the minimal ranks, discuss its properties and the natural hierarchy it induces in a pencil space. Then, we show how the minimal ranks of a pencil can be determined from its Kronecker canonical form. For illustration, we classify the orbits according to their minimal ranks (under the action of the general linear group) in the case of real pencils with $m, n \leq 4$. Subsequently, we show that real regular $2k \times 2k$ pencils having only complex-valued eigenvalues, which form an open positive-volume set, do not admit a best approximation (in the norm topology) on the set of real pencils whose minimal ranks are bounded by $2k - 1$. Our results can be interpreted from a tensor viewpoint, where the minimal ranks of a degree-$(d - 1)$ matrix polynomial characterize the minimal ranks of matrices constituting a block-term decomposition of an $m \times n \times d$ tensor into a sum of matrix-vector tensor products.

MSC:
15A22 Matrix pencils
15A69 Multilinear algebra, tensor calculus
41A50 Best approximation, Chebyshev systems

Keywords:
matrix pencil; Kronecker canonical form; matrix polynomial; tensor decomposition

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