The Concept of a Dew Collection Device Based on the Mathematical Model of Sliding Liquid Drops on an Inclined Solid Surface

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Abstract. Dew collection, and devices for such, may play an important role in regions of our planet that are arid and lack clean water. Usually, dew collection devices are represented as an inclined plane, which is a trivial topology. In this work, we first propose the concept a dew collection device with a helicoidal structure in order to increase its surface area, and we suggest taking into account Frenkel’s mathematical model of sliding drops on an inclined surface as the fundamental idea in designing dew collection devices. We also believe that in the future this mathematical model can be used to investigate the possibility of condensing liquid drops within other planets’ atmospheres that contain hydrogen and oxygen. Finally, we represent our concept as a three-dimensional Rhino model.

1. Introduction
Our planet contains many sources of fresh water that are distributed heterogeneously. While some countries, such as Canada, Russia, the USA, Latin American countries, etc., have a surplus of fresh water, most African countries suffer from a lack of fresh and clean water. The structure of some African land can cause rivers to turn into streams of mud, resulting in a high-level of infections, poor harvests and, consequently, malnutrition and high mortality.

Many deserts in Africa are located near the coast, where rainfall is very low but humidity is relatively high. These regions can be considered effective for the collection of dew, which is water in the form of droplets that appear on thin, exposed objects in the morning or evening due to condensation.

Nilsson et al. [1] investigated the possibility of implementing a dew collector to condense atmospheric water vapour using the radiative cooling effect – in particular, they studied the use of pigmented polymer foils with high solar reflectance and high thermal emittance. Nilsson [2] believes that moisture in the air can be condensed as dew and used not only for drinking but also for irrigation. Based on the experiments carried out in Sweden and Tanzania, his paper concerns observations of dew formation on radiatively cooled pigmented polyethylene foils. Similarly, Gandhidasan et al. [3] studied dew formation on radiatively cooled pigmented polyethylene foils.
Jacobs et al. [4] proposed a 1 m$^2$ insulated planar dew collector, set at a 30° angle between the collector and ground, and a dew collector in the shape of an inverted pyramid, which was constructed to reduce the view angle to the night-time sky. They found that the pyramid collector design was able to collect about 20% more dew than the inclined planar collector. A more sophisticated method of dew collection was proposed by Nikolayev et al. [5], who revised previous approaches in the light of recent investigations considering the basic physical phenomena involved in the formation of dew.

Rajvanshi et al. [6] proposed a scheme for large-scale dew collection as a supply source of fresh water. The schematic requires cold seawater to be pumped from the neritic zone to a heat exchanger field. In addition, since it requires electricity for a pump, a wind-powered generator is used. Muselli et al. [7] investigated the relative contributions of dew and rainwater in the Mediterranean Dalmatian coast and the islands of Croatia, especially in the dry summer season. In addition, the authors evaluated the possibility of transforming roof rain collectors to dew water collectors as well.

Sharan et al. [8] determined the amount of dew water that could be collected with little investment by adapting plain, uninsulated, corrugated galvanized iron roofs that are common in most rural regions of India. Berkowicz et al. [10] measured dew water condensation in the city of Jerusalem using a condenser unit, such as the one in the Kothara village in India [8, 9] and reported a dew collection of 33 mm over a 12-month period from more than 176 dew events with a nightly maximum of 0.5 mm.

Beysens [11] reviewed the aspects related to heterogeneous nucleation and subsequent growth of water droplets, while condensation-induced water-drop growth on a super-hydrophobic spike surface was studied by Narhe [12]. Beysens et al. [13] experimentally studied dew collection from several passive foil-based radiative condensers and noted that chemical and biological analyses established that dew is generally potable.

The rest of this paper is organised as follows. Section 2 briefly reviews the model of a standard dew collection device. Section 3 discusses Frenkel’s mathematical model of liquid drops sliding down an inclined solid surface. Section 4 explains the geometry of a helicoid and provides its parametric equations. Section 5 proposes a new model for a dew collection device based on a helicoidal structure. Section 6 concludes our work and proposes some ideas for future works.

2. A Standard Dew Collection Device
A standard dew collection device has a very simple structure and usually consists of one or more inclined planes. Figure 1 (left) shows its schematic view.

![Figure 1](image1.png)

**Figure 1.** A standard dew collection device scheme (left) and rendered three-dimensional model (right).

This device usually has three components: a panel (made of materials including polyethylene film, polyethylene mixed with titanium oxide and barium sulfate film, fibre-reinforced plastic sheet and poly-carbonate sheet), a mounting frame, and collection accessories. A planar panel is mounted on a metal frame constructed of welded angles. The collection accessories (channel and tube) are also...
supported on the frame. The panel is mounted at an angle, \( \alpha \), with the horizontal. Flow is channelled via a flexible rubber tube into a plastic bottle securely placed on the ground. It is supposed that; however, the optimal values of \( \alpha \) have yet to be determined.

### 3. Mathematical Model of Liquid Drops Sliding Down an Inclined Solid Surface

Dew collection includes a fundamental problem of liquid drops sliding down an inclined solid surface, which was first investigated by Frenkel [14]. Frenkel mentioned that one of the well-known facts in hydromechanics is that a running fluid adheres to a solid surface. Also, the mechanism of a running fluid is that it is not sliding down but rather flush to the rim (front to back). This phenomenon starts only when gravity is stronger than or equal to the adhesive strength of the rim at the back of the drop.

In two-dimensions, the limit of commencing the slide down a flat surface with a tilt angle \( \alpha = \alpha^* \) (\( \alpha^* \) is the angle value when the drop starts sliding down) is decided by \( \Delta \sigma = g m \sin \alpha^* \), where \( \Delta \sigma = \sigma_{20} + \sigma_{10} - \sigma_{12} \) is the energy of the adhesive strength per unit area of the fluid on solid matter.

Here, \( g m \sin \alpha^* \) can be replaced by \( \sigma (\cos \theta_1^* - \cos \theta_2^*) \). If \( m \leq \frac{2 \sigma}{g} \) (in other words, \( \sin \alpha^* \leq 1 \)), the drop will not slide down. Frenkel [14] formulated the following equation to predict the angle, \( \alpha \), at which a liquid drop begins sliding down an inclined surface:

\[
mg \sin \alpha = \omega (\gamma_{LV} + \gamma_{SV} - \gamma_{SL})
\]  

(1)

where \( m \) is the mass of a liquid drop; \( g \) is gravitational acceleration \( g \approx 9.8 m/s^2 \); \( \alpha \) is the tilt angle; \( \omega \) is the maximum width of the contact area between the drop and the surface; \( \gamma_{LV} \) is the liquid–vapour interfacial tension; \( \gamma_{SV} \) is the solid–vapour interfacial tension; and \( \gamma_{SL} \) is the solid–liquid interfacial tension. The expression \( (\gamma_{LV} + \gamma_{SV} - \gamma_{SL}) \) in Eq. 1 is known as the reversible work of adhesion, and the values for liquid drops on different surfaces are found in [15]. Subsequently, Olsen et al. [16] experimentally confirmed that Frenkel’s theory [14] has high correlation with the actual experimental values in his thesis.

**Figure 2 (a, b).** Tilt angle (\( \alpha \)) versus mass of a liquid drop (\( m \)). The vertical axes show angles in radians.
As can be seen in Fig. 2, liquid drops with greater masses need smaller angles to commence sliding. If the mass is too small, the angles tend bend towards 90°. Larger angles are required as the contact surface area increases to commence sliding.

4. The Geometry of a Helicoid

A helicoid is an inclined surface that was first defined by Euler in 1774. It can be described by the following parametric equations in Cartesian coordinates:

\[ r(u, v) = (v \cos \alpha u, v \sin \alpha u, cu), \]

where \( v \) and \( u \) range from negative infinity to positive infinity; while \( \alpha \) and \( c \) are constants. If \( \alpha \) is positive, then the helicoid is right-handed; if it is negative, then the helicoid is left-handed.

![Figure 3. Helicoids with different values of \( \alpha = \{2, 5, 10\} \) (from left to right) and \( c = 0.1 \).](image)

The constants, \( \alpha \) and \( c \), determine the number of turns and the height of a helicoid, respectively. Moreover, varying these parameters presents the opportunity to obtain a helicoid with a large surface area, and this fact is considered one of the advantages of a helicoidal surface.

The tilt angle at any point can be computed as the angle between the normal vector of the parameterised surface and the tangent plane [17].

\[ \vec{n} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|} = \frac{1}{\sqrt{(a^2v^2 + c^2)(-c \sin \alpha u, c \cos \alpha u, -av)}}. \]

The surface area of a helicoid can be calculated by integrating the length of the normal vector (3) to the surface over the appropriate region, \( D \), in the parametric \( uv \) plane:

\[ S(D) = \int_{D} |\vec{n}_u \times \vec{n}_v| dudv = \int_{D} \frac{1}{\sqrt{a^2 v^2 + c^2}} dudv. \]

Integrating over \( v \in [0, \theta] \) and \( u \in [0, r] \) then gives:

\[ S = \frac{1}{2} \int_{0}^{\theta} \left[ r \sqrt{c^2 + r^2} + c^2 \ln \left( \frac{r + \sqrt{c^2 + r^2}}{r} \right) \right]. \]
5. Helicoidal Dew Collection Device Model
The structure of this model is as follows. The helicoid can be considered the main part of the model and should be covered by a protective shield to avoid falling leaves, dust, and other debris. The protective shield has a spherical form and rests on a torus-shaped beam based on three legs. Condensed water slides down the helicoidal surfaces and enters the collection channel before being collected in a water tank. The water is filtered, and the filter can be easily replaced. The parts of this device can be made of plastic, but the material used for the helicoid should be appropriate for the geographical region in which the device is used. D'urso et al. [18] proposed hydrophobic material as an option. Superhydrophobic surfaces, such as the leaves of the lotus plant, are extremely water-resistant, so they are perfect for collecting condensed water.

Dew collection using helicoidal surfaces is more complex and might be more efficient than standard dew collection. Such a device can be very compact while possessing a large collection surface area.

![Figure 4. A helicoidal dew collection device model concept created in Rhino 3D.](image)

6. Discussion and Limitations
Until now, no serious research existed on designing dew collection devices that considers the theory of sliding liquid drops. As pioneers in this research, we only propose the concept for such devices and
show that the tilt angle values are not constant but depend on the environment, drop size, and material used.

In this work, we:

- do not propose a device that is ready for production,
- do not know the optimal value of a tilt angle (because this value can vary), or
- are aware of another surface that is more efficient than a helicoid, but we are willing to investigate this problem in our future works.

7. Conclusions and Future Work

In this work, we propose a novel concept for the mathematical design of a dew collection device with a helicoidal structure. The constants, $\alpha$ and $c$, from Eq. 2 allow the height and diameter of the helicoid to be easily adjusted, resulting in the angle to change appropriately for sliding drops. Moreover, we can increase the surface area by using more turns, making it possible to collect drops of water with different masses.

Additionally, we are interested in using various materials with different values of reversible work of adhesion, as this can help us to create an optimal model that will allow to maximize the collection of dew. We also plan to study Frenkel’s mathematical model, considering the change of gravitational acceleration – for example, on planets with an atmosphere (Mars, Venus, etc.). As we believe this research might be useful for understanding the possibility of collecting water or other liquids if humans go on to colonise other worlds.

Finally, we are willing to investigate the behaviour of sliding liquid drops on kinematic superspiral surfaces [19] and propose new surfaces for dew collectors in NURBS [20] or Bernstein-Bézier form [21].

8. References

[1] Nilsson T M J, Vargas W E, Niklasson G A and Granqvist C G 1994 Condensation of water by radiative cooling Renewable Energy 5(1) 310-317
[2] Nilsson T 1996 Initial experiments on dew collection in Sweden and Tanzania Solar Energy Materials and Solar Cells 40(1) 23-32
[3] Gandhidasan P, Abualhamayel H I 2005 Modeling and testing of a dew collection system Desalination 180(1) 47-51
[4] Jacobs A F G, Heusinkveld B G and Berkowicz S M 2008 Passive dew collection in a grassland area, The Netherlands Atmospheric Research 87(3) 377-385
[5] Nikolayev V S, Beysens D, Gioda A, Milimouk I, Katiushin E and Morel J P 1996 Water recovery from dew Journal of Hydrology 182(1) 19-35
[6] Rajvanshi A K 1981 Large scale dew collection as a source of fresh water supply Desalination 36(3) 299-306
[7] Muselli M, Beysens D, Miletta M and Milimouk I 2009 Dew and rain water collection in the Dalmatian Coast, Croatia Atmospheric Research 92(4) 455-463
[8] Sharan G, Beysens D and Milimouk-Melnytchouk I 2007 A study of dew water yields on Galvanized iron roofs in Kothara (North-West India) Journal of Arid Environments 69(2) 259-269
[9] Sharan G 2011 Harvesting Dew with Radiation Cooled Condensers to Supplement Drinking Water Supply in Semi-arid International Journal for Service Learning in Engineering, Humanitarian Engineering and Social Entrepreneurship 6(1) 130-150
[10] Berkowicz S M, Beysens D, Milimouk I, Heusinkveld B G, Muselli M, Wakshal E and Jacobs A F 2004 Urban dew collection under semi-arid conditions: Jerusalem Proc. of the Third International Conference on Fog, Fog Collection and Dew (Pretoria) p. E4
[11] Beysens D 1995 The formation of dew Atmospheric Research 39(1) 215-237
[12] Narhe R D, Beysens D A 2006 Water condensation on a super-hydrophobic spike surface Europhysics Letters 75(1) 98-104
[13] Beysens D, Muselli M, Milimouk I, Ohayon C, Berkowicz S M, Soyeux E,... and Ortega P 2006 Application of passive radiative cooling for dew condensation Energy 31(13) 2303-2315
[14] Frenkel Y I 1948 On the behavior of liquid drops on a solid surface. 1. The sliding of drops on an inclined surface Zhur. Ekspt. i Teoret. Fiz. (USSR) 18(7) 659
[15] Aron Y B, Frenkel Y I 1949 On the behavior of liquid drops and bubbles on a solid body surface Zhur. Ekspt. i Teoret. Fiz. (USSR) 19(9) 807
[16] Olsen D A, Joyner P A, and Olson M D 1962 The sliding of liquid drops on solid surfaces. The Journal of Physical Chemistry 66(5) 883-886
[17] Pogorelov A 1974 Differential Geometry Moscow USSR: Nauka
[18] D'urso B R, Simpson J T 2007 U.S. Patent No. 7,258,731 Washington DC: U.S. Patent and Trademark Office
[19] Ziatdinov R 2012 Family of superspirals with completely monotonic curvature given in terms of Gauss hypergeometric function Computer Aided Geometric Design 29(7) 510-518
[20] Piegl L, Tiller W 2012 The NURBS book Springer Science & Business Media
[21] Farin G E 2002 Curves and surfaces for CAGD: a practical guide Morgan Kaufmann

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