Guiding of vortices and ratchet effect in superconducting films with asymmetric pinning potential

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Two-dimensional vortex dynamics in a ratchet washboard planar pinning potential (PPP) in the presence of thermal fluctuations is considered on the basis of a Fokker-Planck equation. Explicit expressions for two new nonlinear anisotropic voltages (longitudinal and transverse with respect to the current direction) are derived and analyzed. The physical origin of these odd (with respect to magnetic field or transport current direction reversal) voltages is caused by the interplay between the even effect of vortex guiding and the ratchet asymmetry. Both new voltages are going to zero in the linear regimes of the vortex motion (i.e. in the thermally activated flux flow (TAFF) and ohmic flux flow (FF) regimes) and have a bump-like current or temperature dependence in the vicinity of the highly nonlinear resistive transition from the TAFF to the FF.

I. INTRODUCTION

Now vortex ratchets, which exploit asymmetric vortex dynamics have been attracting considerable attention [1-5]. The common feature of superconducting ratchet systems is their rectifying property: the application of the alternating current to a superconductor patterned with a periodic asymmetric pinning potential can produce vortex motion whose direction is determined only by the asymmetry of the pattern.

Although considerable theoretical work exists [1], only a few experiments have been realized. Recently a vortex lattice ratchet effect has been investigated in Nb films sputtered on arrays of nanometric Ni triangles, which produce the periodic asymmetric pinning potential [2]. Similar effects were also discussed for YBCO films with antidots [3]. Earlier it has been proposed in [4] how the ratchet effect can be used to remove vortices from low-temperature superconductors.

Unfortunately, a full temperature-dependent theoretical description of the superconducting devices proposed in [1], is not available due to the complexity of the two-dimensional periodic pinning potential in [1] used. In particular, a theoretical explanation of the experimentally available study of the vortex flow along the vortex channeling directions in above-mentioned structures is a difficult problem. Due to this reason we propose below to study experimental ratchet properties of superconductors on the basis of a more simple ratchet device for which exist a full theoretical description (at least, in the single-vortex approximation) of its two-dimensional vortex dynamics within the framework of a Fokker-Planck approach.

It is noticeable, that such a device has already been exploited many years ago by Morrison and Rose in their experiments on controlled asymmetric (as now we say “ratchet”) surface pinning in the superconducting-alloy films [6]. Recent progress in fabrication of submicrometric structures with a periodic ratchet modulation of their thickness by methods of electron-beam lithography [7] or molecular-beam epitaxy on faceted substrates [8] allows to prepare Nb films with a similar well-controlled asymmetric washboard pinning structure. Note also, that the main feature of similar structures is the existence of well-defined guiding of vortices along the channels of the washboard pinning potential at relatively low temperatures.

One of the first experimental observations of guided vortex motion in the flux-flow regime was made by Niessen and Weijensfelf still in 1969 [9]. They studied guided vortex motion in the cold-rolled sheets of a Nb-Ta alloy by measuring transverse voltages of the pattern for different magnetic fields $H$, transport current densities $J$, temperatures $T$, and different angles $\alpha$ between the rolling and current direction. The $(H,J,T,\alpha)$-dependences of the cotangent of the angle $\beta$ between the average vortex velocity $\langle \nu \rangle$ and the $j$ direction were presented. For the discussion, a simple theoretical model was suggested, based on the assumption that vortex pinning and guiding can be described in terms of an isotropic pinning force plus a pinning force with a fixed direction which was perpendicular to the rolling direction. The experimentally observed dependence of the transverse and longitudinal voltages on the magnetic field in the flux flow regime as a function of the angle $\alpha$ was in agreement with this model. However, the dynamics of the vortex that is moving transverse to the pinning channels has substantively nonlinear behavior and cannot be entirely explained within the flux-flow approach.

The nonlinear guiding problem was exactly solved at first only for washboard PPP within the framework of the two-dimensional stochastic model of anisotropic pinning which takes into account the vortex and the Hall viscosity coefficients and based on the Fokker-Planck equation with a concrete form of the symmetric pinning potential $\rho_{H_{i}}^{\pm}(j,t,\alpha,\epsilon)$.

Rather simple formulas were derived in [11] for the experimentally observable nonlinear even (+) and odd (-) (with respect to the magnetic field reversal) longitudinal and transverse magnetoresistivities $\rho_{H_{i}}^{\pm}(j,t,\alpha,\epsilon)$ as
functions of the dimensionless transport current density $j$, dimensionless temperature $t$, and relative volume fraction $0 < \varepsilon < 1$ occupied by the parallel twin planes directed at an angle $\alpha$ with respect to the current direction. The $\rho_{\parallel, \perp}^\pm$-formulas were presented in [11] as linear combinations of the even and odd parts of the function $\nu(j, t, \alpha, \varepsilon)$ which can be considered as the probability of overcoming the potential barrier of the pinning channel; this made it possible to give a simple physical treatment of the nonlinear regimes of vortex motion.

In addition to the appearance of well known a relatively large even transverse $\rho_\parallel^+$ resistivity [9], generated by the guiding of vortices along the channels of the washboard PPP, explicit expressions for two new nonlinear anisotropic Hall resistivities $\rho_\parallel^-$ and $\rho_\perp^-$ were derived and analyzed. The physical origin of these odd contributions caused by the subtle interplay between even effect of vortex guiding and the odd Hall effect. Both new resistivities were going to zero in the linear regimes of the vortex motion (i.e. in the thermally activated flux-flow and ohmic flux-flow regimes and had a bump-like current or temperature dependence in the vicinity of highly nonlinear resistive transition from the thermally activated flux-flow to flux-flow regimes. As the new odd resistivities arose due to the Hall effect, their characteristic scale was proportional to the small Hall constant as for ordinary odd Hall effect investigated earlier in [10].

In contrast to the model which uses the uniaxial symmetric PPP [11] with the Hall effect, we consider below the more simple modified model with asymmetric (ratchet) sawtooth washboard pinning potential where the Hall effect is absent. It will be shown the appearance of two step-like and two bump-like singularities in the $\rho_\parallel^+$ and $\rho_\perp^-$ (Hall-like) resistive responses in this model, even in the absence of the Hall effect.

The objective of this paper is to present results of a temperature-dependent theory for the calculation of the nonlinear magnetoresistivity tensor for asymmetric sawtooth washboard pinning potential at arbitrary value of asymmetry parameter $0 < \varepsilon < 1$ for the case of in-plane geometry of experiment. This approach will give us the experimentally important theoretical model which demonstrates the $\rho_{\parallel, \perp}^\pm$ magneto-resistivities for all corresponding values of the modeling parameters and predicts an appearance of the nonlinear magnetoresistivity $\rho_\perp^-$ at some set of parameters $\varepsilon$ (when the Hall coefficient is zero) due to the asymmetry of the washboard PPP.

The organization of the article is as follows. The section [11] presents those general results in the stochastic model of anisotropic pinning which don’t require specification of the form of the pinning potential: the Fokker-Planck method in the two-dimensional model of anisotropic pinning and the nonlinear resistivity and conductivity tensors. In subsection III A we substitute a specific sawtooth form of the pinning potential into the general formulas of the preceding section. It enables us to find the exact analytical solution of our model. It will be analyzed there the behavior of the model depending on the asymmetry parameter $0 < \varepsilon < 1$ for the case of in-plane geometry of experiment. This approach will give us the experimentally important theoretical model which demonstrates the $\rho_{\parallel, \perp}^\pm$ magneto-resistivities for all corresponding values of the modeling parameters and predicts an appearance of the nonlinear magnetoresistivity $\rho_\perp^-$ at some set of parameters $\varepsilon$ (when the Hall coefficient is zero) due to the asymmetry of the washboard PPP.

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on the model’s parameters: \( j, t, \alpha, \varepsilon \). Some formulas will be obtained for the \( \rho_{\eta j}^T j, t, \alpha, \varepsilon \). Subsection III.B is dedicated to an analysis of the nonlinear guiding effect in presence of the PPP asymmetry, and Subsection II.D discusses the behavior of resistive responses due to the asymmetry of pinning potential. Subsection II.E discusses a magnetoresitivity stability with respect to small deviations of the angle \( \alpha \) from its values adopted in the longitudinal (L) and transverse (T) geometries of experiment. Subsection II.F gives a short discussion of main new features for the case of weak asymmetry. Finally, the last section IV represents the obtained results and formulates the conclusions.

II. GENERAL RESULTS

A. The Fokker-Plank method in the anisotropic pinning model

The Langevin equation for a vortex moving with velocity \( \mathbf{v} \) in a magnetic field \( \mathbf{B} = n \mathbf{B} (B \equiv |\mathbf{B}|, n = nz, \mathbf{z} \) is the unit vector in the \( z \)-direction and \( n = \pm 1 \) has the form

\[
\eta \mathbf{v} = \mathbf{F}_L + \mathbf{F}_p + \mathbf{F}_{th},
\]

where \( \mathbf{F}_L = n(\Phi_0/c)j \times \mathbf{z} \) is the Lorentz force \( (\Phi_0 \) is the magnetic flux quantum, \( c \) is the speed of light, and \( j \) is the transport current density), \( \mathbf{F}_p = -\nabla U_p(x) \) is the anisotropic pinning force \( (U_p(x) \) is the uniaxial and asymmetric \( U_p(x) \neq U_p(-x) \) planar pinning potential), \( \eta \) is the electronic viscosity constant. The thermal fluctuation force \( \mathbf{F}_{th} \) is represented by a Gaussian white noise, whose stochastic properties are assigned by the relations

\[
\langle F_{th,i}(t) \rangle = 0, \quad \langle F_{th,i}(t)F_{th,j}(t') \rangle = 2T\eta \delta_{ij}\delta(t - t'),
\]

where \( T \) is the temperature in energy units. Employing relation \( 2 \), we can reduce Eq. \( 1 \) to a system of Fokker-Plank equations:

\[
\frac{\partial P}{\partial t} = -\nabla \cdot \mathbf{S}
\]

\[
\eta \mathbf{S} = (\mathbf{F}_L + \mathbf{F}_p)P - T \nabla P,
\]

where \( P = P(\mathbf{r}, t) \) is the probability density associated with finding the vortex at the point \( \mathbf{r} = \mathbf{r}(x, y) \) at the time \( t \), and

\[
\mathbf{S}(\mathbf{r}, t) \equiv P(\mathbf{r}, t)\mathbf{v}(\mathbf{r}, t),
\]

is the probability flux density of the vortex. Since the anisotropic pinning potential is assumed to depend only on the \( x \) coordinate and is assumed to be periodic \( U_p(x) = U_p(x + a) \), where \( a \) is the period), the pinning force is always directed along the \( x \) axis (with the unit anisotropy vector \( \mathbf{x} \), see Fig. \( 2 \) so that it has no component along the \( y \) axis \( [F_{py} = -\partial U_p(x)/\partial y = 0] \). Thus, Eq. \( 3 \) in the stationary case for the functions \( P = P(\mathbf{x}) \) and \( \mathbf{S} = (S_x, S_y) = S_x(x)\mathbf{x} + S_y(y)\mathbf{y} \) reduces to the equations

\[
\eta S_x = \langle \Phi F_{ξ} \rangle \frac{dU_p}{dx} - T \frac{dP}{dx},
\]

\[
\eta S_y = \langle \Phi F_{ξ} \rangle F_{1y},
\]

where \( F_{1x} = n(\Phi_0/c)j \cos \alpha \) and \( F_{1y} = -n(\Phi_0/c)j \sin \alpha \) are the \( x \) and \( y \) components of the Lorentz force, respectively, and \( \alpha \) is the angle between the direction of the transport current density \( j \) and the \( y \) axis (see Figs. \( 1,2 \) so that it has no component along the \( y \) axis \( [F_{py} = -\partial U_p(x)/\partial y = 0] \). Thus, Eq. \( 3 \) in the stationary case for the functions \( P = P(\mathbf{x}) \) and \( \mathbf{S} = (S_x, S_y) = S_x(x)\mathbf{x} + S_y(y)\mathbf{y} \) reduces to the equations

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Invoking the condition of stationarity for Eq. \( 3 \) and eliminating \( S_y \) from Eq. \( 4 \) and \( 5 \) we obtain

\[
T \frac{dP}{dx} + \langle \Phi F_{ξ} \rangle \frac{dU_p}{dx} - F_{1x} P = -S_x \eta
\]

From the mathematical point of view, Eq. \( 6 \) is the Fokker-Planck equation of the one-dimensional vortex dynamics. The solution of Eq. \( 6 \) for periodic boundary conditions \( P(0) = P(a) \) and one-dimensional periodic pinning potential of general form is

\[
P(x) = \frac{\eta S_x}{T} \frac{f(a)f(x)}{f(a) - f(0)} \int_x^{x+a} \frac{d\xi}{f(\xi)}
\]

where \( f(\xi) = \exp[(F_{ξ} - U_p(\xi))/T] \).

Using the definition of the mean vortex velocity

\[
\langle \mathbf{v} \rangle = \int_0^a \mathbf{S}(x)dx / \int_0^a P(x)dx,
\]

we obtain an expression for the \( x \) and \( y \) components of the vortex mean velocity

\[
\langle v_x \rangle = \int_0^a S_x(x)dx / A = S_x a / A = F_{1x} \nu(F_{1x}) / \eta,
\]

FIG. 3: Asymmetric sawtooth pinning potential \( U_p(x) \): \( b \) is the potential period (width of the potential channels), \( a \) is the \( x \)-coordinate of the minimum of the potential well, \( U_0 \) is the depth of the potential well.
\[ \langle v_y \rangle = \int_0^a S_y(x)dx/A = F_{Lx}/\eta, \]  

where \( A = \int_0^a P(x)dx \) and 
\[
\frac{1}{\nu(F_{Lx})} = \frac{F_{Lx}}{T \alpha(1-\exp(-F_{Lx}a/T))} \int_0^a dx \\
\quad \times \int_0^a dx' \exp \left( -\frac{F_{Lx}x}{T} \right) \\
\quad \times \exp \left( \frac{U_p(x + x') - U_p(x')}{T} \right).
\]  

The dimensionless function \( \nu(F_{Lx}) \) in the limit \( F_{Lx} \to 0 \) coincides with the analogous quantity introduced in [10]. It has the physical meaning of the probability of the vortex overcoming the potential barrier, the characteristic value of which we denote as \( U_0 \). This can be seen by considering the limiting cases of high \((T \gg U_0)\) and low \((T \ll U_0)\) temperatures. In the case of high temperatures we have \( \nu \approx 1 \), and expression (13) corresponds to the flux-flow regime (FF regime). Indeed, in this case the influence of pinning can be neglected. In the case of low temperatures \( \nu \) is a function of the transport current. For strong currents \((F a \gg U_0)\) the potential barrier disappears, \( \nu \approx 1 \), and the FF regime is realized. For weak currents \((F a \ll U_0)\) we have \( \nu \sim \exp(-U_0/T) \), which corresponds to the thermally activated flux-flow regime (TAFF regime). The transition from the TAFF regime to the FF regime is associated with a lowering of the potential barrier with growth of the current.

B. The nonlinear conductivity and resistivity tensors

The average electric field in the \( xy \) coordinate system induced by the moving vortices is given by
\[
\mathbf{E} = \frac{1}{c} \mathbf{B} \times \langle \mathbf{v} \rangle = n(B/c)(-\langle v_y \rangle \mathbf{x} + \langle v_x \rangle \mathbf{y}).
\]  

Taking Eqs. (11), (12) and (14) we obtain the dimensionless magnetoresistivity tensor \( \hat{\rho} \) (having components measured in units of the flux-flow resistivity \( \rho_f \)) for the nonlinear law \( \mathbf{E} = \hat{\rho}(j)\mathbf{j} \)
\[
\hat{\rho} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \nu(f) \end{pmatrix},
\]  

where the dimensionless components of the electric field are measured in units of \( E_0 = B U_0 / \eta a n \), and of the current, in units of \( j_0 = c U_0 / \eta b \), and
\[
f = -\frac{F_{Lx}}{U_0} = m \cdot n \cdot j_y = m \cdot n \cdot j \cdot \cos \alpha = p \cdot j \cdot \cos \alpha,
\]  

where \( m = \pm 1 \) determines the transport current reversal \((j = m|j|)\), \( n = \pm 1 \) determines the magnetic field direction reversal \((\mathbf{B} = n|\mathbf{B}|)\), \( p = m \cdot n \) is the combination for simplification of the current and magnetic field directions reversal and \( \alpha \) is the angle between the current direction and asymmetric PPP channels.

The conductivity tensor \( \hat{\sigma} \) (the components of which are measured in units of \( 1/\rho_f \)), which is inverse of the tensor \( \rho \), has the form
\[
\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \nu^{-1}(f) \end{pmatrix}.
\]  

From Eqs. (15) and (17) we see, that off-diagonal components of the \( \rho \) and \( \hat{\sigma} \) tensors are zero, and the nonlinear components of the \( \hat{\rho} \)-tensor and \( \hat{\sigma} \)-tensor are functions of the external force value \( f \) through the external current density \( j \), the temperature \( T \), and the angle \( \alpha \).

The experimentally measurable resistive responses refer to a coordinate system tied to the current (see Fig. 2). The longitudinal and transverse (with respect to the current direction) components of the electric field, \( E_l \) and \( E_\perp \), are related to \( E_x \) and \( E_y \) by the simple expressions
\[
\begin{cases}
E_l = E_x \sin \alpha + E_y \cos \alpha, \\
E_\perp = -E_x \cos \alpha + E_y \sin \alpha.
\end{cases}
\]  

Then according to Eqs. (15) and (18), the expressions for the experimentally observable longitudinal and transverse (with respect to the \( j \)-direction) magnetoresistivities \( \rho_l \equiv E_l / j \) and \( \rho_\perp \equiv E_\perp / j \) have the form:
\[
\begin{cases}
\rho_l = \sin^2 \alpha + \nu(f) \cos^2 \alpha, \\
\rho_\perp = (\nu(f) - 1) \cos \alpha \sin \alpha.
\end{cases}
\]  

We introduce the L and T geometries in which \( j \parallel \mathbf{x} \) and \( j \perp \mathbf{x} \), respectively. From Eq. (19) follows, that in the L geometry vortex motion takes place along the pinning channels (the guiding effect), and in the T geometry - transverse to the pinning channels direction (the slipping effect). In the L geometry the crossover (or critical at \( T > 0 \)) current is equal to zero since the FF regime is realized for guided vortex motion along pinning channels direction. In the T geometry, i.e., for the vortex motion transverse to the pinning channels, a pronounced nonlinear regime is realized for \( T < U_0 \), in the range \( j_{cr1} < j < j_{cr2} \) (we denote it by the \( cr \) (crossover) index). In our case, when the temperature \( T > 0 \), strictly speaking there is no exact value of the crossover currents \( j_{cr1} \) and \( j_{cr2} \), as at any relatively low temperature vortices can move transverse to the pinning channels. However, when \( T < U_0 \), the existence of the crossover current might make sense which separates the guiding region of motion of the vortices, where they move only along the pinning channels, and the slipping region when vortices also slip over pinning barriers.
It is evident that the presence of different crossover currents for mutually opposite directions along the vector $x$ is a direct consequence of an asymmetric pinning potential. Let us consider a diagram of the dynamical states of the vortex system in the $(j_x, j_y)$ plane (Fig. 4). For arbitrary angle $\alpha$ the tip of the vector $\mathbf{j}$ can lie in two different regions which are different in their physical meanings. If $j_y < j_{cr1}$ or $j_y < j_{cr2}$ the guided vortex motion takes place (the guiding region). For $j_y > j_{cr1}$ or $j_y > j_{cr2}$ the guided motion along the pinning channels is joined by motion transverse to the pinning channels (the slipping region). It is clear, that if we apply an alternating current $j_{y}^{AC}$ along the $y$ axis such that the amplitude of a current satisfies to the relation $j_{cr1} < j_{y}^{AC} < j_{cr2}$, it will lead to a motion of vortices along the $x$ axis as they can overcome a pinning potential only in one direction. It is the occurrence of the ratchet effect.

### III. VORTEX PINNING ON ASYMMETRIC PPP AND ANALYSIS OF NONLINEAR REGIMES.

#### A. Pinning potential and $\nu$-function behavior.

The nonlinear properties of the resistivity tensor $\hat{\rho}$, as can be seen from formula (14), are completely determined by the behavior of the function $\nu$, which has the physical sense of the probability of a vortex overcoming the potential barriers created by the channels of asymmetric pinning potential. In turn, the function $\nu$, according to formula (13), depends on the form of the pinning potential. Above we considered the simplest case of the asymmetric PPP that has sawtooth-like form (see Fig. 3)

\[
U_p(x) = \begin{cases} 
-F_{p1}x, & 0 \leq x < a \\
-F_{p2}(x - b), & a < x \leq b, 
\end{cases}
\]

where $F_{p1} = U_0/a$ and $F_{p2} = -U_0/(b - a)$ are the pinning forces in the different directions of the $x$-axis and $U_0$ is the depth of a potential well, $b$ is the period of the asymmetric PPP ($b \geq a$), $\varepsilon = a/b$ is the parameter characterizing the asymmetry of the pinning potential ($0 < \varepsilon < 1$ and $\varepsilon = 1/2$ corresponds to the symmetric well).

Substituting the potential (20) into formula (13) for the probability function $\nu$ gives the following expression:

\[
\nu(f, t, \varepsilon) = \frac{(\exp(f/t) - 1)(1 - 2f\varepsilon + 2f^2 + f^2\varepsilon^2)}{(f(-1)} \\
-2f^2\varepsilon + f^2)(f\varepsilon - 1)^2 / (f(-1) \\
+ 6\exp(f/t) f^2\varepsilon^2 + 2\varepsilon + 2f^2\varepsilon \\
-5f^2\varepsilon^2 + t\exp((f\varepsilon - f - 1)/t) - t \\
+ t\exp((f\varepsilon - 1)/t) - f - \varepsilon^2f^3 + 2\varepsilon^3f^3 \\
+ 5f\varepsilon + f\exp(f/t) + 4\varepsilon^3f^2 \\
-2f^2\varepsilon\exp(f/t) + f^3\varepsilon^2\exp(f/t) \\
+f^3\varepsilon^4\exp(f/t) - \varepsilon^4f^3 + \exp(f/t) \\
t\exp(f/t) - 4f^2\varepsilon^3\exp(f/t) \\
-2f^3\varepsilon^3\exp(f/t) - 6f^2\varepsilon^2 - 2\varepsilon\exp(f/t) \\
+ 5f\varepsilon^2\exp(f/t) - 5f\varepsilon\exp(f/t),
\]

where $f = -F_{tot}/U_0$ is the dimensionless external force which gives ratio of this force to the average pinning force $U_0/\nu$, $T = T_0/U_0$ is the dimensionless temperature which gives the ratio of the energy of the thermal fluctuations of the vortices to the depth of the potential wells $U_0$. In our case the dimensionless external force $f$ also coincides with the dimensionless transport current $j_y$, which is given by formula (15).

At function evaluation (21) we assumed, that the asymmetry parameter $\varepsilon$ changes from zero value, that corresponds to shift of the pinning potential minimum to the left, up to unity value that corresponds to shift of potential to the right and that naturally leads to as

![Diagram of dynamic states of the vortex system in the xy plane.](image.png)
much as possible asymmetry of the pinning potential in appropriate direction.

Let us consider in turn the dependence of the probability function $\nu(f, t, \varepsilon)$ on each of the quantities $f$, $t$ and $\varepsilon$ for the remaining quantities held fixed (denoted by the subscript "0").

The dependence $\nu(f, t) = \nu(f, t, \varepsilon_0)$ (see Fig. 5) characterizing $\nu$ as a function of the external force acting on a vortex at constant asymmetry parameter. The influence of the external force $f$ acting on the vortices is that it lowers the height of the potential barrier for vortices localized along the channels of the asymmetric PPF and, consequently, increases the probability of escape from them. Raising the temperature also increases the probability that a vortex will escape from a potential well through an increase in the energy of the thermal fluctuations of the vortices. Thus, the pinning potential, leading as $f, t \to 0$ to localization of vortices, can be suppressed both by an external force and by an increase in the temperature. From Eq. (21) follows that

$$
\lim_{t \to \infty} \nu(f, t, \varepsilon) = \lim_{f \to \infty} \nu(f, t, \varepsilon) = 1. \quad (22)
$$

The function

$$
\nu_0(f, \varepsilon) = \lim_{t \to 0} \nu(f, t, \varepsilon)
$$

is equal to

$$
\nu_0(f, \varepsilon) = \begin{cases}
0 & \frac{1}{4} < f < \frac{1}{2} \\
\frac{1}{f^2 - f - 2 + 1} & (f < \frac{1}{4}) \cup (f > \frac{1}{2})
\end{cases}
$$

(23)

This corresponds to the zero-temperature limit (see Fig. 5).

From Eqs. (23) and (16) follows, that the crossover transport current and, respectively, crossover external force (in the dimensionless units) in both directions is

$$
f_{cr1} = j_{cr1} = 1/(\varepsilon - 1) \quad \text{and} \quad f_{cr2} = j_{cr2} = 1/\varepsilon \quad (\text{see also the diagram of dynamic states in Fig. 1}).
$$

If $0 < \varepsilon < 1/2$, then $|j_{cr1}| < |j_{cr2}|$, and for $1/2 < \varepsilon < 1$ we have $|j_{cr1}| > |j_{cr2}|$.

In the zero-temperature limit, for $|j_y| < |j_{cr1}|, |j_{cr2}|$ the vortices are trapped in the potential wells of the pinning channels and they cannot move across pinning barriers, while for $|j_y| > |j_{cr1}|, |j_{cr2}|$ the potential barrier disappears and the vortices begin to move in the one or both directions.

Let’s note also, that value $\varepsilon = 1/3$ which we use in some figures of this article, corresponds to a case when $j_{cr1} = 2j_{cr2}$.

It is easy to understand the influence of the temperature on the qualitative form of $\nu(j_y, \varepsilon)$. Specifically at low temperatures ($T \ll U_0$) a nonlinear transition takes place from the TAFF regime of vortex motion perpendicular to the pinning channels to the FF regime with growth of the external force, wherein the function $\nu(j_y, \varepsilon)$ has a characteristic nonlinear shape (see Fig. 5). At high temperatures ($T \gg U_0$) the FF regime is realized over the entire range of variation of the external current. At nonzero temperature the $j_{cr1}$ and $j_{cr2}$ disappear because $\nu(j_y, \varepsilon)$ is not vanishing at any value of the parameters $j_y, \varepsilon$. Increase of the temperature leads to smoothing of the function $\nu(j_y, \varepsilon)$ and in the limit $t \to \infty$ it simply degenerates in plane $\nu(j_y, \varepsilon) = 1$ that corresponds to the free motion of the vortices.

In the limit $f \to 0$ we have:

$$
\nu_0(t, \varepsilon) = \frac{\exp(1/t)}{t^2(\exp(1/t) - 1)^2}. \quad (24)
$$

Note, that the value of $\nu_0(t, \varepsilon)$ does not depend on the parameter of asymmetry of a pinning potential. Physically it means that effects related to asymmetry of a pinning potential are relevant when the vortices are moving...
(they are moving always except the case when $T = 0$ and $|j| < |j_{cr1}|, |j_{cr2}|$).

As follows from Eqs. (16), (19) and (21) the dynamics of a vortex system depends substantially on the direction of the current flow and the magnetic field reversal. According to Eq. (16), an external transport current density $j$ or direction of the external magnetic field $\mathbf{B}$ equally cause the reversal of the Lorentz force $F_L$ which change the magnitude of the $\nu(f, t, \varepsilon)$ function due to the inversion of $f$. In order to consider only $p$-independent magnitudes of the $\rho_\parallel$ and $\rho_\perp$ resistivities in Eqs. (19) we should introduce the even(+) and odd(-) magnetoresistivities with respect to $p$-inversion ($(\rho(p)^\pm = (\rho(p) \pm \rho(-p))/2)$). From this point of view follows that we should present the function $\nu(f)$ as a sum of the even (+) and odd (-) parts with respect to inversion of the moving force:

$$\nu(p) = \nu^+(p) + \nu^-(p),$$

$$\nu^\pm(p) = \frac{\nu(pf, t, \varepsilon) \pm \nu(-pf, t, \varepsilon)}{2},$$

where $\nu^\pm$ are even and odd parts of the $\nu$ function respectively.

As we can make sure, the dependence $\nu^+(f, t)$ is closely similar to the $\nu(f, t)$. The qualitative behavior and the limits of the component $\nu^+(f, t)$ as $f, t \to 0, \infty$ coincide with the corresponding limits of $\nu(f, t)$ as it is follows from Eq. (22) and Eq. (26).

The dependences of the $\nu^\pm(f, \varepsilon) = \nu^\pm(f, t_0, \varepsilon)$ as a function of the external motive force and asymmetry parameter at a constant temperature are shown in Figs. 7 and 8. At comparatively low temperatures ($t \ll 1$) the asymmetry parameter $\varepsilon$ influences to shape of the $\nu^+$ function (see Fig. 7). It is appears as step-like dependence of the $\nu^+(f)$ at $\varepsilon \neq 1/2, 0, 1$. The origin of this steps corresponds to critical currents density $j_{cr1,2}$.

Such, that if $\varepsilon = 1/2$ than we have one step with origin at $f_{cr1} = f_{cr2} = 2$ and function will tend up to unity if $f \to 0$. In case of $\varepsilon = 0$ or $\varepsilon = 1$ corresponding values of the critical current density are equal to $f_{cr1} = -1, f_{cr2} = +\infty$ or $f_{cr1} = -\infty, f_{cr2} = 1$ respectively, and we should see one step again and the value of the $\nu^+$ function does not exceed $1/2$. In the other case we have two step in the $\nu^+(f)$ dependence as consequence of the asymmetry of the pinning potential. It is worth noticing, that function under consideration is symmetrical relatively to planes $f = 0$ and $\varepsilon = 1/2$.

The component $\nu^-(f, t)$ tends to zero in the linear regimes (as $f, t \to 0, \infty$) and is nonzero in the region of nonlinearity of $\nu$ (see Fig. 8). It means that $\nu^-(f, t)$ can be suppressed with increasing of the external motive force or temperature. When the value of temperature grows, function $\nu^-(f, t)$ tends to zero since the temperature fluctuations at $t \gg 1$ in a superconductor suppress the influence of the pinning potential and the vortex, thus, can move freely in any direction (flux-flow regime is realized). At small temperature, when the contribution of pinning potential is comparable or more than the contribution of temperature fluctuations ($t \ll 1$), the $\nu^-$ behavior will explain by interplay of the external motive force $f$ and asymmetry pinning potential (which causes to appearance the critical currents $j_{cr1,2}$). If $f < f_{cr1}, f_{cr2}$ and the temperature value is close to zero, then $\nu^-(f, t)$ also is close to zero because the vortex cannot move across pinning channels in any direction. When $f_{cr1} < f < f_{cr2}$ (or vice versa $f_{cr2} < f < f_{cr1}$ that depends on the value of the asymmetry parameter $\varepsilon$) than the thermally activated flux-flow regime is realized and the curve $\nu^-(f, t)$ accepts a bump-like form (see Fig. 9). In this case the ratchet effect occurs and vortices start to move in a di-

---

**FIG. 7:** The dependence $\nu^+(f, \varepsilon)$ for fixed value of the temperature $t_0 = 0.05$.

**FIG. 8:** The dependence $\nu^-(f, \varepsilon)$ for fixed value of the temperature $t_0 = 0.05$. 
rection that associated with the minimal pinning force. At strong external motive force, when \( f \gg f_{cr1}, f_{cr2} \), the function \( \nu^-(f) \) also is vanishing because external current suppresses influence of the pinning potential on the vortex (flux-flow regime arises) as it follows from the general properties of the \( \nu(f) \) function and the vortices can freely move in any direction across the pinning channels. The width of the base of a bump-like curve is associated with asymmetry parameter \( \epsilon \) and also can be simply presented as \( 2\Delta f_{cr} = 2||f_{cr1}|-|f_{cr2}|| = (|\epsilon| - |\epsilon - 1|)/(|\epsilon - 1|\epsilon) \). It is means when \( \epsilon \to 0, 1 \) the width of the peak will tend to infinity and in Fig. 8 we seeing a change the bump-like to step-like dependence of \( \nu^-(f) \). The maximum of the \( \nu^-(f) \) function with respect to the external motive force \( f \) (see Figs. 8, 9) corresponds to the maximum pinning force of both the \( f_{cr1} \) and \( f_{cr2} \), respectively.

Appearance of the \( \nu^-(f, t) \) is a direct consequence of asymmetry of the pinning potential. When the asymmetry parameter \( \epsilon \) is not equal to 1/2, then the same absolute value of a motive force enclosed in mutually opposite directions leads to different values of the function \( \nu \) that leads to occurrence of an odd component \( \nu^-- \).

When the pinning potential is symmetric (\( \epsilon = 1/2 \)), the function \( \nu^-- \) is equal to zero (see Figs. 8, 10). Also, for \( \epsilon = 1/2 \) we regain the results of Ref. 12:

\[
\nu(f, t, 1/2) = (u^2 - 1)/u^2(u^2 - 1) - 2ut(\cosh(u/t) - \cosh(1/t))/\sinh(u/t), \quad (27)
\]

where \( u = 2f \).

In Ref. 11 the influence of the Hall effect on occurrence of the \( \nu^-- \) in presence of symmetric pinning potential has been discussed. It has been shown that if the Hall constant is distinct from zero then \( \nu^- \) is distinct from zero too, and vice versa. Now we see that the asymmetric PPP leads to occurrence of an odd component \( \nu^-- \) if we neglect the Hall effect.

The dependence \( \nu^-(f, t) = \nu^-(f, t, \epsilon_0) \) and \( \nu^-(t, \epsilon) = \nu^-(f_0, t, \epsilon) \) explain behavior of \( \nu^- \) relative to the temperature, external motive force and asymmetry parameter \( \epsilon \) for the fixed values of the asymmetry parameter (Fig. 9) and the external motive force (Fig. 10). From Fig. 9 it is follows that \( \nu^- \) function is tend to maximum if temperature tend to zero, but in Fig. 10 we can observe extremum by temperature. It is happens because temperature’s raising firstly lead to activation of the vortices overcoming across channel’s walls of the pinning potential and suppres the \( \nu^- \) function when \( t \gg 1 \).

Now we will obtain expressions from formulas 19, 21, 16 and 26 for the experimentally observed longitudinal and transverse resistivities (relative to the current direction) with the asymmetric pinning potential taken into account. We separate out their even and odd components relative to the current direction:

\[
\rho_+^\parallel = \nu^+ \cos^2 \alpha + \sin^2 \alpha, \quad (28)
\]

FIG. 9: The dependence \( \nu^-(f, t) \) for fixed value of the asymmetry parameter \( \epsilon_0 = 1/3 \).

\[
\rho_-^\parallel = \nu^- \sin 2\alpha/2, \quad (30)
\]

FIG. 10: The dependence \( \nu^-(t, \epsilon) \) for fixed value of the external motive force \( f_0 = 0.7 \).

\[
\rho_+^\perp = (\nu^+ - 1) \sin(2\alpha)/2, \quad (29)
\]

\[
\rho_-^\perp = \nu^- \sin 2\alpha/2, \quad (30)
\]

\[
\rho_\perp = \nu^- \cos^2 \alpha, \quad (31)
\]

where \( \nu^\pm \) are the above-defined even and odd components relative to the current direction of the function \( \nu(f, t, \epsilon) \). In formulas (28), (29) the nonlinear and linear terms separate out in a natural way. The physical reason for the appearance of linear terms is that in the
model under consideration for $\alpha = \pi/2$ there is always a flux-flow regime of vortex motion along the pinning channels.

B. The peculiarities of nonlinear guiding effect in presence of the PPP asymmetry.

As is well known [9], the specifics of anisotropic pinning consist in the noncoincidence of the directions of the external motive force acting on the vortex, and its velocity. From Fig. 2 and from Eqs. (14) and (19) follows formula

$$\beta = \beta(j, t, \alpha) = \arccot(\rho_{\perp}/\rho_{\parallel}) = \arccot((1 - \nu(j \cos \alpha, t, \varepsilon))/(\tan \alpha + \nu(j \cos \alpha, t, \varepsilon) \cot \alpha)),$$

which is used to describe the guiding effect, where $\beta$ is the angle between the average vortex velocity vector $\mathbf{v}$ and the current density vector $j$ (see Fig. 2). The guiding effect is expressed that much more strongly, the larger is the difference in directions as it's follows from subsection III. Unfortunately, but the odd $\beta^{-}$ part hasn’t pictorial view, like $\beta^{+}$, and quantitative experimental measurement of the $\beta^{-}$ with difficulty.

Now, we had to underline that guiding of the vortices in the pinning channels is the necessary condition for appearance ratchet effects’s, but effect’s magnitude entirely depends from distinction between probability of the vortices overcoming over pinning potential walls in both directions as it’s follows from subsection [11].

C. The resistive responses due to asymmetry of the pinning potential.

In this subsection we consider peculiarities of the resistive characteristics in the investigated model due to the asymmetry of the pinning potential. Experimentally, two types of measurements of the observed resistive characteristics are possible in a prescribed geometry defined by a fixed value of the angle $\alpha$: CVC measurements and resistive measurements, which investigate the dependence of the observed resistivities on the current density at a fixed temperature $\rho_{\parallel,\perp}^{+}(j, t_0)$ and on the temperature for fixed current density $\rho_{\parallel,\perp}^{+}(j_0, t)$. The form of these dependences is governed by a geometrical factor — the angle $\alpha$ between the directions of the current density vector $j$ and the PPP channels. There are two different forms of the dependence of $\rho_{\parallel,\perp}^{+}$ on the angle $\alpha$ (see formulas (28) - (31)). The first of these is the "tensor" dependence, also present in the linear regimes (TAFF, FF and strong FF regimes), which is external to the function $\nu$. The second is through the dependence of the function $\nu$ on its argument $f = j_y = |j| \cos \alpha$, which in the region of the transition from the thermally activated flux-flow to the flux-flow regime is substantially nonlinear (see Eq. (21)).

First recall that in the absence of an asymmetry of the pinning potential ($\varepsilon = 1/2$) there exist only even resistivities $\rho_{\parallel,\perp}^{-}$ in the magnetic field, whereas the odd resistivities $\rho_{\parallel,\perp}^{+}$ are zero (see formulas (28) - (31)). The presence of $\varepsilon \neq 1/2$ leads to the appearance of the odd component $\nu^{-}$, which has a maximum in the region of the nonlinear transition from the TAFF to the FF regime and is essentially equal to zero outside of this transitional region (see Figs. 3, 9).

Let us analyze the resistive dependences $\rho_{\parallel,\perp}^{+}(j)$ and $\rho_{\parallel,\perp}^{+}(t)$ with allowance for the asymmetric pinning potential. The nature of the behavior of the current and temperature dependence of $\rho_{\parallel,\perp}^{+}$ is completely determined by the behavior of the dependences $\nu^{+}(j)$ and $\nu^{+}(t)$. As follows from formulas (28) - (31), the even resistivities $\rho_{\parallel,\perp}^{+}$ depend only on the even function $\nu^{+}$ and similarly, $\rho_{\parallel,\perp}^{-}$ depend only on the odd function $\nu^{-}$.

The limiting values of the qualitatively similar dependences $\rho_{\parallel}^{+}(j_0)$ and $\rho_{\parallel}^{+}(t)$ corresponding to the TAFF regime of vortex motion transverse to the pinning channels are determined by guided vortex motion along the
pinning channels and grow with increasing magnitude of the angle \( \alpha \) since in this case the component of the Lorentz force along the pinning channels increases. In the FF regime, as the pinning viscosity becomes isotropic the contribution to the dependences \( \rho^+_{\parallel}(j, \varepsilon) \) and \( \rho^+_{\perp}(t) \) due to vortex motion transverse to the PPP channels becomes substantial, and the limiting values of these dependences are equal to unity (see Figs. 11 - 14).

The main contribution to the even transverse resistivity \( \rho^+_{\perp} \) is proportional to the factor \( \sin(2\alpha)/2 \); therefore, the angle most favorable for its observation is \( \alpha = \pi/4 \). The current dependence \( \rho^+_{\perp}(j) \) and the temperature dependence \( \rho^+_{\perp}(t) \) have their maximum absolute values in the TAFF regime of vortex motion transverse to the PPP channels (the same value is approached if the angle is replaced by its complement in the limit \( j \to 0 \text{ and } t \to 0 \)) and go to zero with the onset of the FF regime as a consequence of isotropization of the pinning viscosity (see Fig. 13). The resistivity \( \rho^+_{\perp} \) can serve as a measure of the anisotropy of the pinning viscosity since it is determined by the difference of the pinning viscosities transverse to and along the pinning channels (see also Eqs. 28, 29).

As can be seen from Figs. 7, 12 and 14 that behavior of the \( \rho^+_{\parallel,\perp}(j, \varepsilon) \) resistivities is closely equal to the behavior of the \( \nu^+(f, \varepsilon) \). It is also follows from Eqs. (28), (29), that all that had told about behavior of the \( \nu^+(f, \varepsilon) \) function can be repeated by analogy here. Hence, the step-like appearance of the \( \rho^+_{\parallel,\perp}(j) \) is direct consequence the asymmetry of the pinning potential. A unique distinction is the influence of the angle \( \alpha \) on the \( \rho^+_{\parallel,\perp}(j, \varepsilon) \) resistivities by means of internal angle dependence. The internal angle dependence is reduces current’s influence to \( \nu^+(f, \varepsilon) \) function and cause to expansion of the even resistivities along \( j \) axis when \( \alpha \) increases.

As was noted above, the odd longitudinal \( \rho^-_{\parallel} \) and transverse \( \rho^-_{\perp} \) magnetoresistivities arise thanks to the asymmetry of the pinning potential, and therefore their characteristic scale is proportional to \( \nu^- \) (see Eqs. 30, 31). Therefore, their qualitative form is inherited completely by the behavior of \( \nu^- \) as a function of the current, asymmetry parameter and temperature (Figs. 15 - 18).

A characteristic peak appears in the dependencies \( \rho^-_{\parallel}(j) \) in the region of nonlinearity of \( \nu^- \) as a function of the current and parameter of asymmetry while in the TAFF and FF regimes of vortex motion transverse to the pinning channels they vanish (Figs. 15, 16). The temperature behavior of the resistivities \( \rho^-_{\perp} \) and \( \rho^-_{\parallel} \) is similar to \( \nu^-(t) \) (see Figs. 9, 10). As the main contribution to
the odd transverse resistivity \( \rho_\perp^- \) is proportional to the factor \( \sin(2\alpha)/2 \), then the angle most favorable for its observation is \( \alpha = \pi/4 \). It can be important for experiment, that the maximal value of the resistivity \( \rho_\perp^- \) does not exceed 1/2, as it follows from Eqs. (29), (30).

As was noted above, the resistivity internally depends from the angle \( \alpha \) such as \( f = j_y = j \cos \alpha \), and it follows from this, that value of the transport current density, when resistivity \( \rho_\perp^- \) is maximal, will be expressed as:

\[
j_{\text{max}} = \min(j_{cr1}, j_{cr2})/ \cos \alpha.
\]

If \( \alpha \) tends to \( \pi/2 \), when \( j_{\text{max}} \) tend to infinity. It physically means, that the Lorentz force, which is acting to the vortices, there is parallel to the pinning channels and can’t drags the vortices across the pinning channels.

It is worth noticing, that the \( j_{cr1} \) and \( j_{cr2} \) are functions of the asymmetry parameter \( \varepsilon \) as it was proved in Section IIIA. This explains the fact, that if \( \varepsilon \to 0 \), the \( \rho_{||,\perp}^- \) tends to step-like or to bump-like form. It happens because one of the pinning forces is becoming infinite.

D. The angular stability of the resistivities in LT geometries.

Let us consider the observed resistivities in the T and L geometries, where the current is directed exactly parallel (\( \alpha = 0 \)) or perpendicular (\( \alpha = \pi/2 \)) to the PPP channels. It follows from formulas \( \{28\} \) - \( \{31\} \) that in these limiting
cases \( \rho^+_{\perp} = 0 \), and we obtain for \( \rho^+_{\parallel} \) and \( \rho^-_{\parallel} \)

\[
\rho^+_{\parallel,T} = \nu^+_T, \quad \rho^-_{\parallel,T} = \nu^-_T \quad (\alpha = 0, T \text{ geometry}), \tag{34}
\]

\[
\rho^+_{\parallel,L} = 1, \quad \rho^-_{\parallel,L} = 0 \quad (\alpha = \pi/2, L \text{ geometry}), \tag{35}
\]

where longitudinal even \( \rho^+_{\parallel,T} \) and odd \( \rho^-_{\parallel,T} \) resistivities are due to vortex motion transverse to the PPP channels, and described by the functions \( \nu^+_T = \nu^+(j, t, \varepsilon) \) and \( \nu^-_T = \nu^-(j, t, \varepsilon) \) respectively. In the limit \( j, t \to \infty \) we have \( \rho^+_{\parallel,T} = 1, \rho^-_{\parallel,T} = 0 \). The resistivity \( \rho^+_{\parallel,L} \) in the L geometry is equal to unity due to guided vortex motion along the PPP channels, for which pinning is absent.

Formula \( \rho_\pm \) expresses simple relations between the observable resistivities \( \rho^+_{\parallel,T} \) and \( \rho^-_{\parallel,T} \) in the T geometry. The form of the functions \( \nu^\pm_T \) can be reconstructed, as can be seen from formulae \( \rho_\pm \), from the measurements of \( \rho^+_{\parallel,T} \) and \( \rho^-_{\parallel,T} \).

Therefore, it makes sense to consider the question of the stability of the measurements in these geometries since the preparation of the samples can lead to small deviations \( \delta \alpha \) from the values \( \alpha = 0, \pi/2 \). Here it should also be borne in mind that besides the resistivities \( \rho^\pm_{\parallel} \), and \( \rho_{\perp} \) assigned by formulas \( \rho_\pm \) and \( \rho_{\perp} \), in the presence of an angle deviation, \( \alpha \), the resistivities \( \rho^\pm_{\parallel} \) and \( \rho_{\perp} \), not present in the L and T geometries, also appear. The expansions of \( \rho^\pm_{\parallel} \) in \( \alpha \) about \( \alpha = 0 \) (in the T geometry) and in \( \Delta \alpha = \pi/2 - \alpha \) about \( \alpha = \pi/2 \) (in the L geometry) out to the first nonvanishing terms have the form:

\[
\rho^+_{\perp,T} = \nu^-(j)\alpha + o(\alpha^3), \tag{36}
\]

\[
\rho^-_{\perp,T} = \nu^+_T(j) - \left( \frac{1}{2} \frac{\partial \nu^+_T(j)}{\partial j} j + \nu^-_T(j) \right) \alpha^2 + o(\alpha^3), \tag{37}
\]

\[
\rho^+_{\parallel,T} = (\nu^+_T(j) - 1)\alpha + o(\alpha^3), \tag{38}
\]

\[
\rho^-_{\parallel,T} = \nu^+_T(j) + \left( 1 - \frac{1}{2} \frac{\partial \nu^+_T(j)}{\partial j} j - \nu^+_T(j) \right) \alpha^2 + o(\alpha^3), \tag{39}
\]

\[
\rho^+_{\perp,L} = -\frac{\partial \nu^-_L(j)}{\partial j} |_{j=0} (\Delta \alpha)^3 + o((\Delta \alpha)^4), \tag{40}
\]

\[
\rho^-_{\perp,L} = \frac{\partial \nu^-_L(j)}{\partial j} |_{j=0} (\Delta \alpha)^3 + o((\Delta \alpha)^4), \tag{41}
\]

\[
\rho^+_{\parallel,L} = (\nu^+(0) - 1)(\Delta \alpha) + o((\Delta \alpha)^2), \tag{42}
\]

\[
\rho^+_{\parallel,L} = 1 + (\nu^+(0) - 1)(\Delta \alpha)^2 + o((\Delta \alpha)^3). \tag{43}
\]

Below we will use simple physical arguments in order to estimate a value and to explain all main features of the resistivities \( \rho_{\pm} \). The main cause of the presented behavior of the resistivities in the L geometry is extremely small inner dependence \( (f \approx j\alpha) \) of the \( \nu_{\pm} \) functions from the transport current density. Besides, it is easy to see that resistivities \( \rho^+_{\perp,L} \) and \( \rho^-_{\parallel,L} \) are close to zero for \( t \ll 1 \) and \( j \ll 1 \). It happens because the derivative of the \( \nu^- \) is nonzero only in the vicinity of transition from the full guiding regime to the TAFF regime and from the TAFF to the FF regime. If temperature and current will be rise, it causes the derivative and appropriate resistivities grows until the FF regime does not happen. On the other hand, the resistivity \( \rho^+_{\parallel,L} \) in the L geometry varies linearly for small deviations of the \( \alpha \) and does not depend on the current density. In the same way the \( \rho^-_{\parallel,L} \) does not depend on the small deviation of \( \alpha \) and on the current density.

In the T geometry the inner dependence of the \( \nu_{\pm} \) from the current density is strong \( (f \approx j\alpha) \). The resistivities \( \rho^+_{\parallel,T} \) and \( \rho^-_{\parallel,T} \) depend only on \( \nu^+ \) and \( \nu^- \) functions and have a weak angle dependence accordingly. The resistivities \( \rho^+_{\perp,T} \) and \( \rho^-_{\parallel,T} \) are proportional to \( \alpha \) deviation. Similarly to foregoing we can conclude, that the resistivity \( \rho^+_{\perp,T} \) will be more unstable in comparison with \( \rho^+_{\parallel,L} \) for a small deviation of the angle \( \alpha \) from the T geometry.

The relative deviation of the resistivity for a small deviation from the T and L geometries for \( \rho^+_{\parallel} \) it is of the order \( \Delta \rho^+_{\parallel,T}/\rho^+_{\parallel,T} \sim \alpha^2/\nu(j,t) \) in the T geometry and \( \Delta \rho^+_{\parallel,L}/\rho^+_{\parallel,L} \sim \Delta \alpha \) in the L geometry. Thus, \( \rho^+_{\perp,L} \) is the most unstable in the TAFF regime of vortex motion transverse to the pinning channels, where \( \nu(j,t) \ll 1 \).

The physical reason for this behavior is the rapid variation of the angle \( \beta \) from \( \alpha = 0 \) in the T geometry, where \( v_y = 0 \), to the angle corresponding to the guiding regime with \( v_x \gg v_y \).

The behavior of the resistivities in the L geometry is physically clear from the fact that for \( \alpha \approx \pi/2 \) the angle \( \beta \) varies hardly at all, i.e., the direction of the velocity vector \( \mathbf{v} \) varies only slightly (in contrast to the case of the T geometry) and thermally activated transitions of the vortices through pinning potential barriers play main role here.

As was stated above, in an actual experiment small deviations of the angle \( \alpha \) from the values \( \alpha = 0, \pi/2 \) corresponding to the L and T geometries are always present. Utilizing experimental measurements of \( \rho^+_{\perp,L} \), these deviations can be found using the following scheme. First, neglecting small quadratic contributions in \( \alpha \) to the resistivities \( \rho^+_{\parallel,T} \) and \( \rho^-_{\parallel,T} \) (in the region where they are stable), it is possible to solve the inverse problem using formulas \( \rho^+_{\perp,L} \) and \( \rho^-_{\parallel,L} \), i.e., to reconstruct the function \( \nu \). Knowing this, from the formulas for the resistivity
\( \rho_{+}^{+} \), which vanish in the L geometry and are linear for small deviations \( \alpha \), it is possible to find the corresponding value of \( \alpha \) deviations. The self-consistency of this scheme is checked by calculating the quadratic corrections in \( \alpha \) and \( \Delta \alpha \), which should be small relative to the main contribution in the T and L geometries.

E. Weak and strong asymmetry.

Let us discuss firstly the case when asymmetry of the pinning potential is very small, i.e.

\[
\varepsilon = 1/2 + z, \tag{44}
\]

where \( z \to 0 \) is the small deviation of the asymmetry parameter from the symmetric case. Substituting Eq. (44) into Eq. (21) we can expand the \( \nu(f, t, \varepsilon) \) in a Taylor series about small deviation \( z \) up to the second-order terms. A convenient result can be presented in the following form:

\[
\nu \approx \tilde{\nu} = \tilde{\nu}^+ + \tilde{\nu}^-, \tag{45}
\]

corresponds to the even component of the \( \tilde{\nu} \) function expansion into a Taylor series, whereas

\[
\tilde{\nu}^- = Wz, \tag{47}
\]

corresponds to the odd component of the \( \tilde{\nu} \),

\[
G = 16ft(\cosh(f/(2t)) - \cosh(1/t))/\sinh(f/(2t)), \quad W = 16(4 - f^2)(G + (4 - f^2)(f \sinh(1/t)/\sinh(f/(2t)) + 2)/(f(4 - f^2) + G)^2). \tag{46}
\]

Notice now, that \( \tilde{\nu}^+ \) function in Eq. (45) is the even function of the external motive force \( f \) and coincides with the similar expression given by Eq. (27) which earlier was pointed out in [12]. It is easy also to prove, that \( \tilde{\nu}^- \) in Eq. (47) is odd with respect to the \( f \) and \( z \) respectively.

From Eqs. (47), (40) and (41) we can calculate an expression for the asymmetry parameter \( \varepsilon \):

\[
\varepsilon = 1/2 + \rho_{-}^-T/W. \tag{48}
\]

Note that Eq. (48) can be used for calculating the value of the \( \varepsilon \) from the experimental data in the limit \( \alpha \to 0 \) and \( \varepsilon \approx 0 \) (the case \( \varepsilon \approx 1 \) it is possible to consider by substitution \( \varepsilon \) to 1 and \( f \) to \( -f \) in formulas (49)–(52)).

IV. CONCLUSION.

In this work we proposed exactly solvable [11] two-dimensional model structure for study of the ratchet effect in superconducting film in presence of the asymmetric planar pinning potential as was studied by experiment firstly in [3].

We have theoretically examined the strongly nonlinear resistive behavior of the two-dimensional vortex system of a superconductor as a function of the transport current density \( j \), the temperature \( t \), and the angle \( \alpha \) between the directions of the current and the PPP channels. The nonlinear (in \( j \)) resistive behavior of the anisotropic vortex ensemble can be caused by factor of a “pinning” origin which takes into account the presence of anisotropic pinning with asymmetry of the PPP. It is physically obvious that such pinning at low enough temperatures leads to anisotropy of the vortex dynamics since it is much easier for vortices to move along the pinning channels (the guiding effect in the flux-flow regime, which is linear in the current) than in the perpendicular direction, where it is necessary for them to overcome the pinning potential barriers from the pinning channels, which also is a source of resistive nonlinearity. If under variation of one of the ”external” parameters \( j \), \( t \), \( \alpha \) the intensity of manifestation of the indicated nonlinearity is weakened, then this weakening will lead to an ”effective isotropization” of the vortex dynamics, i.e., to a convergence (and in the
limit of the absence of nonlinearity, to coincidence) of the directions of the mean velocity vector of the vortices and the Lorentz force.

It is physically clear that the current, temperature, and angle \( \alpha \) have a qualitatively different effect on the weakening of pinning and the corresponding transition from anisotropic vortex dynamics to isotropic. With growth of \( j \) the Lorentz force \( F_L \) grows and the height of the potential barrier decreases, so that for \( j \gtrsim j_{cr1}, j_{cr2} \) (where \( j_{cr1}, j_{cr2} \) are the crossover currents of the indicated transition, whose width grows with growth of \( t \)) this barrier essentially disappears. The quantities \( j_{cr1}, j_{cr2} \) depend on \( \alpha \) by virtue of the fact that the probability of overcoming the barrier is governed not by the magnitude of the force \( F_L \), but only by its transverse component \( F_L \cos \alpha \), so that \( j_{cr1,2}(\alpha) = j_{cr1,2}(0)/\cos \alpha \) grows with growth of \( \alpha \). Since an increase in the temperature \( t \) always increases the probability of overcoming the pinning barrier, the transition to isotropization of the vortex dynamics is that much steeper in \( t \), the smaller is the pinning barrier.

In order to theoretically analyze the above-described physical picture of a nonlinear anisotropic resistive response, Sections II A and III employed a comparatively simple, but at the same time quite realistic, planar model of stochastic pinning. It allows one to reduce the calculations to the evaluation of analytical formulas (28)-(31), which have a simple physical interpretation. A distinguishing feature of this model is the possibility, within the framework of a unified approach, to describe consistently the nonlinear transition from the anisotropic dynamics of a vortex system (for currents \( j < j_{cr1,2}(\alpha) \) at relatively low temperatures) to isotropic behavior (for currents \( j > j_{cr1,2}(\alpha) \) at relatively high temperatures). In the model under consideration this approach corresponds (for \( t > 0 \)) to a substantially nonlinear crossover from the linear low-temperature thermally activated flux-flow regime to the ohmic flux-flow regime of vortex motion.

Proceeding now to a brief description of the main theoretical results, we note here that an analytical representation of the nonlinear resistive response of the investigated system in terms only of elementary functions was possible thanks to the use of a simple but physically realistic model of anisotropic pinning with asymmetric sawtooth PPP (see Sec. III) and Fig. 9. The exact solution obtained made it possible for the first time to consistently analyze not only the qualitatively clear dynamics of the nonlinear guiding effect, but also the nontrivial question of the interaction of guided vortex motion along PPP channels and the ratchet effect. The most important result in our opinion is the conclusion that the appearance of novel \( \rho_{\parallel,\perp} \) magnetoconductivities does not require (as it was in (14)) the Hall effect (see Sec. III). The nonlinear formulas (30) and (31) in agreement with physical intuition (now already nonlinear) clearly demonstrate that the most natural and "sufficient" reason for the relatively large novel \( \rho_{\parallel,\perp} \)-effects is the asymmetry of the pinning wells. At comparatively low temperatures and weak currents it leads to the realization of a quite intense (over a wide interval of angles around \( \alpha = \pi/4 \)) guided vortex motion along the pinning channels in the thermally activated flux-flow regime, i.e., to the appearance of \( \rho_{\parallel} \)-effects, and at currents \( j \approx j_{cr,1,2}(\alpha) \), to the appearance of characteristic maxima in the curves of the odd components of the resistivities \( \rho_{\parallel,\perp} \) (see Subsection III C and Figs. 13-15).

A completely novel result of the present work is also contained in formulas (30) and (31). It is a quantitative description of the interaction of the guiding effect and the ratchet effect, which is valid for all possible values of the asymmetry parameter \( \theta < \epsilon < 1 \). Formally, this interaction arises as a result of the fact that in the case of anisotropic pinning on asymmetric PPP the force of the overcoming the pinning well (see Eq. (20)), which determines the probability of overcoming the potential barrier (and therewith also determines the magnitude of the component of the vortex velocity perpendicular to the pinning channels), is different in the opposite directions of the \( x \)-axis. Then arising of the odd resistivities defined by Eqs. (30), (31), appears only due to the ratchet form of the PPP and to the change of their sign with the current or magnetic field reversals (see Eq. (29)). Their origin follows from the emergence of a certain equivalence of the \( xy \)-direction for the case, that a guiding of vortices along the channels of the washboard PPP is realized at \( \alpha \neq 0, \pi/2 \). Note also that for \( \alpha = 0 \) Eq. (31) gives in fact the ratchet signal measured in \( \xi \). The key point in the physical interpretation of these formulas is our treatment of the function \( \nu(f, t, \varepsilon) \) as the probability of overcoming the potential barrier of the pinning channel, from which follows an understanding of the evolution of the functions associated with it, \( \nu^{\pm} \) (see Subsection III), as functions of the magnitude of the current density \( j \), temperature \( t \), and angle \( \alpha \). Note that this treatment is not a unique property of the stochastic model of anisotropic pinning considered in this work, but can also be consistently realized within the framework of the nonlinear phenomenological approach under much broader assumptions, including, in particular, an account of the inter-vortex interaction.

If, as is usually the case in experiment \( \bar{\xi} \), that the asymmetry of the pinning potential are sufficiently small (\( \varepsilon \approx 1/2 \)), then formulas (28), (30) simplify substantially since under these conditions \( \nu^{\pm} \sim (1/2 + z) \), \( z \to 0 \) (see Subsection III E).

In conclusion, it should also be noted that ratchet effect opens up the possibility for a variety of experimental studies of directed motion of vortices simply by measuring longitudinal and transverse voltages. Experimental control of amplitude and frequency of the external force, damping, anisotropy parameters, and temperature can be easily provided. In contradiction with other vortex-based ratchet models, the one presented here allows to separate the Hall and ratchet voltages which are similar in their \((j, t)\) behavior, but have different origin and magnitude. Note also that the new ratchet voltages dis-
appear during the procedure of the "current averaging" frequently used in experiments [13] for the cancelation of parasitic thermoelectric voltages.

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