Probability distributions of three-antenna efficiency measurement in a reverberation chamber

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Abstract
Analytical expressions are derived for probability distributions of the three-antenna efficiency measurement. The probability density function (PDF), the expected value, and the standard deviation are derived analytically. The Mellin transform is used in the PDF derivation to obtain the product of random variables, and the analytical distributions for the measured antenna efficiency are obtained. Measurements are performed to verify the results, which agree well with the analytical expressions.

1 | INTRODUCTION

Antenna efficiency is an important parameter in many applications. In mobile communications, antenna efficiency is a key parameter because of the random multipath environment [1]; in the calibration of electromagnetic compatibility tests [2] and over-the-air measurements [3], antenna loss plays an important role in the entire measurement process.

With the development of statistical analysis of antenna measurements [4–8], two primary antenna-efficiency methods have been proposed for reverberation chambers (RCs), reference [1–3, 9] and non-reference [10, 11]. The key to the problem is the efficiency of the reference antenna. When a reference antenna with known efficiency is available, the efficiency of an antenna under test (AUT) will be easy to measure. In an RC, three approaches have been proposed for non-reference antenna measurement—the one-antenna, two-antenna, and three-antenna methods [10]. The one-antenna and two-antenna methods have preconditions on the enhanced backscatter coefficient ($e_b$) [10], whereas for the three-antenna method, no precondition is required for $e_b$. Ideally, the three-antenna method only requires that the chamber transfer function of the RC (after correcting the antenna efficiency) is uniform, and the RC is well stirred [12].

Since an RC is a statistical environment, physical quantities should be analysed using statistical methods. Existing literature has included statistical analyses of Q factors [13, 14], impedance mismatch [15], phase [16], fields in nested RCs [17], and level crossing excursions [18]. The uncertainty of the three-antenna method in an anechoic chamber (gain measurement) [19–21] and in an RC (efficiency measurement) [10] has been estimated numerically using the error propagation method. The analytical distributions have also been given for the reference antenna method in an RC [22–25].

We present the analytical distributions for the antenna efficiency of the three-antenna method in an RC and relate the results to the independent sample number $N$. Although the error propagation can be estimated numerically in an empirical way, analytical results are still meaningful and useful, as they can provide probability density functions (PDFs) and cumulative distribution functions (CDFs). The unbiased estimator, the theoretical lower bound for a given $N$ (or the minimum $N$ required for a given uncertainty), can be obtained from analytical solutions. Unlike [24], the approach used here is different (the Mellin transform), and the PDFs and CDFs are verified experimentally.

This paper is organized as follows. In Section 2, we review the three-antenna method in an RC and present the analytical analysis. Measurements are performed to validate the analytical
model, and the non-uniformity effect is discussed, in Section 3. Conclusions are given in Section 4.

2 | METHODS AND ANALYTICAL ANALYSIS

In an overmoded and well-stirred RC (Figure 1), suppose that \( \eta_{1\text{tot}}, \eta_{2\text{tot}}, \text{ and } \eta_{3\text{tot}} \) are the total efficiencies (defined as the ratio of the power radiated to the power available at the antenna port) of three antennas; from the three-antenna method, we have [10, 11]:

\[
\begin{align*}
\eta_{1\text{tot}} & \eta_{2\text{tot}} T_{RC} = (|S_{12,s}|^2)_N \\
\eta_{1\text{tot}} \eta_{3\text{tot}} T_{RC} = (|S_{13,s}|^2)_N \\
\eta_{2\text{tot}} \eta_{3\text{tot}} T_{RC} = (|S_{23,s}|^2)_N
\end{align*}
\]

and

\[
T_{RC} = \frac{\lambda^3 Q}{16\pi^2 V} = \frac{\lambda^3 \omega r_{RC}}{16\pi^2 V}
\]

where \( T_{RC} \) is the chamber transfer function, which can be calculated using (2) [10, 26], \( \lambda \) is the wavelength, \( Q \) is the average factor of the RC measured in the time domain, \( \omega \) is the angular frequency, \( r_{RC} \) is the time constant of the RC, and \( V \) is the volume of the RC. Note that when the RC is ideally well stirred, \( T_{RC} \) is uniform and does not depend on the position of antennas. We will discuss the effect of the non-uniformity of \( T_{RC} \) in Section 3. The right-hand sides of (1) are the measured stirr parts of the normalized transferred power, in which \( N \) represents the number of independent samples, and \( \langle \cdot \rangle_N \) means the average value from \( N \) independent samples. From (1), the total efficiency of the three antennas can be solved as [10, 11]:

\[
\begin{align*}
\eta_{1\text{tot}} & = \sqrt{(|S_{12,s}|^2)_N (|S_{13,s}|^2)_N (|S_{23,s}|^2)_N} / \sqrt{T_{RC}} \\
\eta_{2\text{tot}} & = \sqrt{(|S_{23,s}|^2)_N (|S_{12,s}|^2)_N (|S_{13,s}|^2)_N} / \sqrt{T_{RC}} \\
\eta_{3\text{tot}} & = \sqrt{(|S_{13,s}|^2)_N (|S_{23,s}|^2)_N (|S_{12,s}|^2)_N} / \sqrt{T_{RC}}
\end{align*}
\]

In this section, we start from (3), and the final equations are given in (14)–(16). To derive the PDF of \( \eta_{1\text{tot}} \) in (3) analytically, we start from the PDF of \( (|S_{12,s}|^2)_N / T_{RC} \) and focus on \( \eta_{1\text{tot}} \). Since the three equations in (3) are similar, the results can be easily applied to \( \eta_{2\text{tot}} \) and \( \eta_{3\text{tot}} \). It is well known that \( |S_{12,s}|^2 / T_{RC} \) has an exponential distribution, and the PDF can be expressed as [26]

\[
p_{X_1}(x_1) = e^{-x_1 / T_X} / T_X, \quad X_1 = |S_{12,s}|^2 / T_{RC}
\]

where both the expected value and the standard deviation are \( T_X \). The PDF of the average value from \( N \) independent samples of \( |S_{12,s}|^2 / T_{RC} \) can be obtained as [23, 27]

\[
p_{X_1}(x_2) = \left( \frac{N}{T_X} \right)_N \Gamma \left( \frac{N}{2} \right) e^{-x_2 N / T_X}, \quad X_2 = |S_{12,s}|^2 / T_{RC}
\]

which is a gamma distribution (or Erlang distribution). The expected value of the PDF in (5) is still \( T_X \), and the standard deviation is \( T_X / \sqrt{N} \). Similarly, we have the same PDFs for the random variables \( (|S_{13,s}|^2)_N / T_{RC} \) and \( (|S_{23,s}|^2)_N / T_{RC} \), but the expected values are denoted as \( T_Y \) and \( T_Z \), respectively.

We can further derive the PDF of \( 1 / X_2 \) as \( p_{X_2}(1/x_2) / x_2^2 \) in (5) (using the variable transformation in statistics [28]) and replace \( T_X \) with \( T_Z \)

\[
p_{X_2}(x_3) = \left( \frac{N}{T_Z} \right)_N \Gamma \left( \frac{N}{2} \right) e^{-x_3 N / T_Z}, \quad X_3 = T_{RC} / (|S_{23,s}|^2)_N
\]

the expected value is \( N / [T_Z (N-1)] \), and the standard deviation is \( N / [T_Z (N-1)] \sqrt{N-2} \). Until now, we have obtained the PDFs for the random variables \( (|S_{12,s}|^2)_N / T_{RC}, \ (|S_{13,s}|^2)_N / T_{RC} \) and \( T_{RC} / (|S_{23,s}|^2)_N \). Note that the Mellin transform can be used to obtain the PDF of the product of random variables—the Mellin transform of the PDF of a product of random variables is the product of the Mellin transforms of the PDF of individual random variables [29, 30]. The Mellin transform of function \( f(t) \) is defined as \( M(s) = M[f(t)] = \int_0^\infty f(t)t^{s-1}dt \), where \( s \) is defined in the complex domain. The inverse Mellin transform is given as \( f(t) = M^{-1}[M(s)] = \frac{1}{2\pi j} \int_{C-\infty}^{C+\infty} M(s)t^sds \) and \( C \) is a real number [31]. Note that if a random variable \( X \) has a PDF of \( f(t) \), the expected value and variance of \( X \) are just \( M(2) \) and \( M(3) - M(2)^2 \), respectively. Another useful property is that the Mellin transform of the PDF of variable
where the integral path value of (10) can be obtained by substituting which is transforms some useful properties used in this paper. The Mellin transform \( \mathcal{M} \) of (5) can be obtained as

\[
\mathcal{M}\left[p_X(x_2)\right] = \int_0^\infty p_X(x_2) x_2^{s-1} dx_2 = \frac{\Gamma(N + s - 1)}{\Gamma(N)} \left( \frac{T_X}{N} \right)^{s-1} \quad (7)
\]

and the Mellin transform of (6) can be derived as

\[
\mathcal{M}\left[p_X(x_3)\right] = \int_0^\infty p_X(x_3) x_3^{s-1} dx_3 = \frac{\Gamma(N - s + 1)}{\Gamma(N)} \left( \frac{N}{T_Z} \right)^{s-1} \quad (8)
\]

and thus, the Mellin transform of the PDF of the product of the random variables \( \langle |S_{12,x}|^2 \rangle_N/T_{RC}, \langle |S_{13,x}|^2 \rangle_N/T_{RC}, \) and \( T_{RC}/\langle |S_{23,x}|^2 \rangle_N \) can be obtained from (7) and (8):

\[
\mathcal{M}\left[p_X(x_4)\right] = \frac{\Gamma(N + s - 1)^2 \Gamma(N - s + 1)}{\Gamma(N)^3} \left( \frac{T_X T_Y}{N T_Z} \right)^{s-1} \quad (9)
\]

the PDF of \( X_4 \) in (9) can be obtained by applying the inverse Mellin transform to (9), which is

\[
p_{X_4}(x_4) = \frac{x_4^{N-2}}{\Gamma(N)} \left( \frac{N T_Z}{T_X T_Y} \right)^{N-1} G(x_4) \quad (10)
\]

where \( G(x_4) \) belongs to the Meijer G-functions [32], and

\[
G(x_4) = \frac{1}{2\pi i} \int_L (1-s)^2 \Gamma(2N - 1 + s) \left( \frac{N T_Z}{T_X T_Y} \right)^s ds \quad (11)
\]

where the integral path \( L \) is defined in [32]. The expected value of (10) can be obtained by substituting \( s = 2 \) into (9), which is

\[
\frac{\Gamma(N + 1)^2 \Gamma(N - 1)}{\Gamma(N)^3} \left( \frac{T_X T_Y}{N T_Z} \right) = \frac{N}{N - 1} \left( \frac{T_X T_Y}{T_Z} \right) \quad (12)
\]

the standard deviation of (10) can be obtained by substituting \( s = 2 \) and \( s = 3 \) into (9) and calculating \( \sqrt{M(3) - M^2(2)} \), which is

\[
\sqrt{\frac{3N^2 - N - 1}{(N - 1)\sqrt{N - 2}} T_X T_Y} \quad (13)
\]

finally, the PDF of \( \eta_{10e} \) in (3) can be obtained from the square root of \( X_4 \) in (10) as

\[
p_\eta(\eta) = \frac{2\eta^{2N-3}}{\Gamma(N)^3 \left( \frac{T_X T_Y}{T_Z} \right)^{N-1}} \left( \frac{N T_Z}{T_X T_Y} \right)^{N-1} G(\eta^2), \quad \eta = \eta_{10e} \quad (14)
\]

The expected value can be obtained by substituting \( s = 3/2 \) into (9), which is

\[
E(\eta_{10e}) = \frac{2\Gamma(N + 1/2)^3}{(2N - 1)\Gamma(N)^3} \sqrt{\frac{T_X T_Y}{T_Z}} \quad (15)
\]

and the standard deviation is

\[
\text{std}(\eta_{10e}) = 2 \sqrt{\frac{N^2 \Gamma(N)^6(N - 1/2)^2 - (N - 1)\Gamma(N + 1/2)^6}{(2N - 1)\sqrt{N(N - 1)} \Gamma(N)^3}} \times \sqrt{\frac{T_X T_Y}{T_Z}} \quad (16)
\]

The relative standard deviation (coefficient of variation) can be calculated from the ratio of (16) and (15), which is

\[
\text{std}_{rel}(\eta_{10e}) = \sqrt{\frac{N^2 \Gamma(N)^6(N - 1/2)^2 - (N - 1)\Gamma(N + 1/2)^6}{(2N - 1)\sqrt{N(N - 1)} \Gamma(N)^3}} - 1 \quad (17)
\]

note that \( \Gamma(N + 1/2)/\Gamma(N) \approx \sqrt{N[1 - 1/(8N) + 1/(128N^2) + \ldots]} \), and thus

\[
\lim_{N \to \infty} E(\eta_{10e}) = \sqrt{\frac{T_X T_Y}{T_Z}} \quad (18)
\]

\[
\lim_{N \to \infty} \frac{\text{std}_{rel}(\eta_{10e})}{1/\sqrt{N}} = \sqrt{\frac{1}{[1 - 1/(8N) + 1/(128N^2) + \ldots]^6} - 1} \quad (19)
\]

by using \( \sqrt{1/(1-x)^6} - 1 \approx 6x + 7x\sqrt{6x/4} + \ldots \), we have

\[
\lim_{N \to \infty} \frac{\text{std}_{rel}(\eta_{10e})}{1/\sqrt{N}} = \sqrt{N\sqrt{6/(8N)}} = \frac{\sqrt{3}}{2} \quad (20)
\]
and thus, \( \text{std}_\text{rel}(\eta_{\text{tot}}) \approx \sqrt{3}/(2\sqrt{N}) \). Although the analysis is for \( \eta_{\text{tot}} \) in (3), the results are similar for \( \eta_{\text{200}} \) and \( \eta_{\text{300}} \) with permutations of \( T_X, T_Y, \) and \( T_Z \). The coefficients in (15) and (17) are plotted in Figure 2(a) and Figure 2(b), respectively. Note that if there is no statistical fluctuation \( (N \rightarrow \infty) \), the total efficiency of Antenna 1 is just \( \sqrt{T_X T_Y/T_Z} \). For \( N \) finite samples, the three-antenna method is not unbiased, but the coefficient in (15) is very close to 1 (almost unbiased, especially for large \( N \)). The coefficient of variation from the Lindeberg–Levy central limit theorem \( (1/\sqrt{N}) \) is also given [28]. Intuitively, the relative standard deviation should be \( 1/\sqrt{N} \) for a large sample number. However, for the three-antenna method, when three random variables are combined and when \( N \) is large, a factor of \( \sqrt{3}/2 \) is introduced.

3 | MEASUREMENTS

To verify the results, we performed the RC measurements shown in Figure 1. The AUTs are wideband dipole antennas. Two antennas were measured each time while the third antenna was loaded with 50 \( \Omega \) in the RC, and the antennas were located a sufficient distance (at least \( \lambda/4 \)) from the RC boundaries and stirrers. A computer synchronized the rotation of the stirrers and the trigger of the vector network analyser, two stirrers were rotated with 360 stirrer angles \( (1^\circ/\text{step}) \), and 100,001 S-parameters for each stirrer position were recorded in the frequency range of 10 MHz–6.5 GHz. The measurement time for one pair of antennas is about 4 h and the total measurement time is about 12 h.

### Table 1

| Random variable | Probability distribution function (PDF) | Mellin transform of PDF | Mean | Variance |
|-----------------|----------------------------------------|-------------------------|------|----------|
| \( X \)         | \( p_X(x) \)                            | \( M_X(s) \)            | \( M_X(2) \) | \( M_X(3) - M_X^2(2) \) |
| \( X^{1/m} \)    | \( nx^{n-1} p_X(x^n) \)                | \( M_X \left( \frac{s}{n} + 1 \right) \) | \( M_X \left( \frac{s}{n} + 1 \right) \) | \( M_X \left( \frac{s}{n} + 1 \right) - M_X^2 \left( \frac{s}{n} + 1 \right) \) |
| \( X \times Y \) | \( M_{XY}(s) = M_X(s)M_Y(s) \)        | \( M_{XY}(2) \)         | \( M_{XY} \left( \frac{s}{m} + 1 \right) \) | \( M_{XY} \left( \frac{s}{m} + 1 \right) - M_X^2 \left( \frac{s}{m} + 1 \right) \) |
| \( X^{1/m} \times Y^{1/n} \) | \( M_X \left( \frac{s}{n} + 1 \right) \times M_Y \left( \frac{s}{m} + 1 \right) \) | \( M_X \left( \frac{s}{n} + 1 \right) \times M_Y \left( \frac{s}{m} + 1 \right) \) | \( \cdots \) |

Note: \( m, n \) can be positive or negative, \( X \) and \( Y \) are independent variables. The variance of \( X^{1/m} \times Y^{1/n} \) can be derived in a similar way not shown here.

![Figure 2](image-url)  

**Figure 2** (a) Coefficient in (15), (b) Coefficient of variation in (17); the curves for \( 1/\sqrt{N} \) and \( \sqrt{3}/(2\sqrt{N}) \) are also presented.
The measurement results are shown in Figure 3. The extracted time constant is illustrated in Figure 3(a), the inverse Fourier transform [10] is applied to the measured $S$-parameters to extract the power delay profile, and a moving window of $B = 100$ MHz bandwidth is used. There are 1541 frequency samples in the bandwidth. Note that a different window bandwidth means a different resolution in the frequency domain, and the results may be slightly different. Since $\tau_{RC}$ varies slowly with frequency and the frequency stirring technique is used, the choice of bandwidth is not sensitive to the final results when the resolution is sufficiently high ($B < 200$ MHz in this RC). The averaged $S$-parameters over

![Figure 3](image.png)

**Figure 3** (a) Measured $\tau_{RC}$, (b) Measured power transfer functions (stirrer part), (c) Measured total efficiency of Antenna 1; ‘smoothed’ means that frequency stirring is applied, (d) Zoomed plot of (c)
360 stirrer positions are given in Figure 3(b), and the measured \( \eta_{\text{tot}} \) is presented in Figure 3(d), in which the average value is about 76%. Because the mean value of the measured \( \eta_{\text{tot}} \) at 4.8–5 GHz is relatively stable, we choose this frequency range to verify the PDF and CDF. In this frequency range, the measured \( T_{\text{RC}} \approx 610 \text{ ns} \). The average mode bandwidth is about \( \Delta f = f/Q = 1/(2\pi T_{\text{RC}}) \approx 261 \text{ kHz} \). Thus the independent sample number in the frequency domain is about \( N_f = 200 \text{ MHz}/261 \text{ kHz} \approx 766 \). As the antenna efficiency is relatively flat in this frequency range, these samples are used to generate the measured PDFs and CDFs (in Figure 4a and Figure 4b). The PDFs and CDFs are compared with theoretical curves calculated numerically from (14). Results from different stirrer position numbers \( (N = 60, N = 120, \text{and } N = 360) \) are also presented, and the independent sample numbers have been confirmed using the autocorrelations defined in [2]. As expected, the measurement results and theoretical equations are in perfect agreement. We also performed the Kolmogorov–Smirnov (KS) test, and the results show that the KS test fails to reject the hypothesis (theoretical CDF) at a 5% significance level. If the frequency stirring technique is further applied, the hybrid independent sample number \( N \) can be replaced by \( N \times N_f \) based on the assumption that antenna efficiency is nearly constant in the frequency stirring bandwidth used.

The standard deviation in (20) can also be verified using the classical error propagation method [33]. We can generalize the model to include the non-uniformity of \( T_{\text{RC}} \). Suppose \( \eta_{\text{tot}} = \sqrt{x_1 x_2 / x_3} \times x_4 \), where \( x_1 = T_x, x_2 = T_y, x_3 = T_z \), and \( x_4 = T_{\text{RC}} \), respectively, when the covariance terms are zero. Therefore, the standard deviation (uncertainty) of \( \eta_{\text{tot}} \) can be estimated as

\[
\text{std}(\eta_{\text{tot}}) \approx \sqrt{\sum_{i=1}^{4} \left[ \frac{\partial \eta_{\text{tot}}}{\partial x_i} \right]^2 \text{std}(x_i)^2}
\]

by substituting the standard deviation of each term \( \text{std}(x_i) = x_i / \sqrt{N} \), \( i = 1, 2, 3 \) into (21), we have

\[
\text{std}_r(\eta_{\text{tot}}) = \frac{\text{std}(\eta_{\text{tot}})}{\eta_{\text{tot}}} \approx \sqrt{\frac{3}{4N} + \frac{\text{std}_r^2(T_{\text{RC}})}{4}}
\]

where \( \text{std}_r(T_{\text{RC}}) \) is the relative standard deviation of \( T_{\text{RC}} \). Not surprisingly, (22) confirms (17) and (20) when the RC is statistically homogeneous and isotropic (the uncertainty of \( T_{\text{RC}} \) is zero). Note that even when \( T_{\text{RC}} \) is not spatially uniform, as long as the PDF of the exponential distribution holds, it does not affect the PDF of \( \eta_{\text{tot}} \) (but with a different mean value).

This systematic error can be reduced by repeating the three-

![Figure 4](image-url)

**Figure 4** (a) Measured and analytical probability density functions (PDFs) and cumulative distribution functions (CDFs). (b) Measured and analytical PDFs of the normalized \( \eta_{\text{tot}} \) (normalized to the mean value). The empirical PDFs have been normalized to satisfy \( \int_{-\infty}^{\infty} p(x) dx = 1 \), and the bin width is 0.01.
antenna method multiple times at different antenna positions. Since no physical PDF of $T_{RC}$ is available currently (normal distribution is an empirical estimation), deriving the analytical PDF considering the statistical distribution of $T_{RC}$ remains unsolved and may not have closed-form results. However, this does not affect the use of (22) to estimate the error propagation empirically.

The relative standard deviation of $T_{RC}$ can be estimated empirically using an approach similar to that used in field uniformity measurement [2], but we must note that $T_{RC}$ is already the expected value (the FU of the mean value). To identify $std_{rel}(T_{RC})<\varepsilon$, the independent sample number should be larger than $1/\varepsilon^2$; otherwise, the fluctuation of $T_{RC}$ will be overwhelmed by the inherent statistical fluctuation of the measured results. Figure 5 shows the measured $std_{rel}(T_{RC})$ with 180 stirrer positions and 766 points of frequency stirring (the same as in the antenna efficiency measurement at 4.8–5 GHz). The results show that $std_{rel}(T_{RC}) \approx 1.5\% \ (0.06 \text{ dB})$ at 4.9 GHz, which is higher than $1/\sqrt{180} \times 766 \approx 0.26\% \ (0.01 \text{ dB})$, and hence the results are well above the statistical noise. Note that the measured $std_{rel}(T_{RC})$ slightly increases at higher frequencies; this could be due to the movement of cables in the measurement of different positions.

4 | CONCLUSIONS

The PDF of measured efficiency from the three-antenna method has been derived analytically. The Mellin transform has been applied to derive the PDF of the product of random variables. The approach used here could easily be generalized to the uncertainty analysis of the product (or division) of random variables in RC measurements. The analytical results show that the average value of the measured efficiency is very close to the expected value (almost unbiased), the standard deviation of which is approximately $\sqrt{3}/(2\sqrt{N})$. Systematic error from the inhomogeneity of the RC has also been discussed, while the analytical results remain unsolved (and may not have a closed form).

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CONFLICT OF INTEREST

No.

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