Correlation Study Of Some Solar Activity Indices
In The Cycles 21 - 23

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\textbf{Abstract.} The correlation coefficients of the linear regression of six solar indices versus 10.7 cm radio flux $F_{10.7}$ were analyzed in solar cycles 21, 22 and 23. We also analyzed the interconnection between these indices and $F_{10.7}$ with help of the approximation by the polynomials of second order. The indices we’ve studied in this paper are: relative sunspot numbers - SSN, 530.3 nm coronal line flux - $F_{530}$, the total solar irradiance - TSI, Mg II 280 nm core-to-wing ratio UV-index, Flare Index - FI and Counts of flares. In the most cases the regressions of these solar indices versus $F_{10.7}$ are close to the linear regression except the moments of time near to the minimums and maximums of 11-year activity. For the linear regressions we found that the minimum values of correlation coefficients $K_{\text{corr}}(t)$ for the solar indices versus $F_{10.7}$ and SSN occurred twice during the 11-year cycle.

\textbf{Key words.} Solar cycle: observations, solar activity indices.

\textbf{1. Introduction}

Magnetic activity of the Sun is called the complex of electromagnetic and hydrodynamic processes in the solar atmosphere. The analysis of active regions (plages and spots in the photosphere, flocculae in the chromosphere and prominences in the corona of the Sun) is required to study the magnetic field of the Sun and the physics of magnetic activity. This task is of fundamental importance for astrophysics of the Sun and the stars. Its applied meaning is connected with the influence of solar active processes on the Earth’s magnetic field.

We have studied monthly averaged values of six global solar activity indices in magnetic activity cycles 21, 22 and 23. Most of these observed data we used in our paper were published in (Solar-Geophysical Data Reports 2009) and (National Geophysical Data Center. Solar Data Service 2013).

All the indices studied in this paper are very important not only for analysis of solar radiation formed on the different altitudes of solar atmosphere, but for solar-terrestrial relationships as the key factors of the solar radiation influence (Extreme Ultra Violet and Ultra Violet EUV/UV) solar radiation is the most important) on the different layers of terrestrial atmosphere also.
It’s known that the various parameters of solar activity correlate quite well with the popular sunspot number index - SSN (or relative sunspot numbers) and with each others over long time scales. (Floyd et al. 2005) showed that the mutual relation between sunspot numbers and three solar UV/EUV indices, the $F_{10.7}$ flux and the Mg II core-to-wing ratio, which is the important chromospheric UV index (Viereck et al. 2001), Viereck et al. 2004), remained stable for 25 years until 2000. At the end of 2001 these mutual relations dramatically changed due to a large enhancement which took place after actual sunspot maximum of the cycle 23 and the subsequent relative quietness intermediate called the Gnevyshev gap.

In our issue for all the indices we used the monthly averages values. Such averages allowed us to take into consideration the fact that the major modulation of solar indexes contains a periodicity of about 27-28 days (corresponding to the mean solar rotation period). So we reduced the influence of the rotational modulation of the data sets.

(Vitinsky et al. 1986) have analyzed solar cycles 18 - 20 and pointed out that correlation for relative sunspot numbers versus radio flux $F_{10.7}$ does not show the close linear connection during all the activity cycle. Also it was emphasized the importance of statistical study in our solar activity processes understanding. To achieve the best agreement in approximation of spot numbers values by $F_{10.7}$ observations (Vitinsky et al. 1986) proposed to approximate the dependence SSN - $F_{10.7}$ by two linear regressions: the first one - for the low solar activity (where $F_{10.7}$ less than 150 sfu) and the second one - for the high activity ($F_{10.7}$ more than 150 sfu). A solar flux unit ($sfu$) $= 10^{-22} \cdot W \cdot m^{-2} \cdot Hz^{-1}$

In this paper we found out that the linear correlation was violated not only for maximums of solar activity cycles but for minimums of the cycles too. Our analysis of the interconnection between these indices and $F_{10.7}$ with help of the approximation by the polynomials of second order confirmed this fact.

We also analyzed the dependence of time of a three-year determined correlation coefficients $K_{corr}(t)$ (in linear regression assumption) for solar activity indices versus $F_{10.7}$ and versus SSN.

Since the nature of solar activity is very complex so we have to keep in mind that there are different sources which give the contribution to the value of the indices studied in this paper.

The magnetic activity of the Sun is called the complex of electromagnetic and hydrodynamic processes in the solar atmosphere and in the underphotospheric convective zone, see Rozgacheva & Bruevich 2002. The analysis of active regions (plages and spots in the photosphere, flocculae in the chromosphere and prominences in the corona) requires to study the magnetic field of the Sun and the physics of magnetic activity. This task is of fundamental importance for astrophysics of the Sun and the stars. Its applied meaning is connected with the influence of solar active processes on the Earth’s magnetic field.
For solar energy coming to the atmosphere of the Earth, it is desirable to have solar indices and proxies that vary differently through time. This strategy of using multiple solar indices has significantly improved the accuracy of density modeling of the atmosphere of the Earth has reported by (Bowman, et al. 2008). Use of these solar indices in their thermospheric density model produces significant improvements in previous empirical thermospheric density modeling.

2. Global activity indices

![Time series of monthly average values of sunspot numbers SSN, F_{10.7}, Mg II core-to-wing ratio, F_{530}, Flare Index and Counts of flares.](image)

Figure 1: The time series of monthly average values of sunspot numbers SSN, $F_{10.7}$, Mg II core-to-wing ratio, $F_{530}$, Flare Index and Counts of flares. The upper index N indicates that solar activity indices are normalized to their values averaged over the analyzed time interval.

Then we have to say a few words about solar indices studied in this paper. At Figure 1 we show the activity indices which are normalized to their values averaged over the analyzed time interval. We can see in most cases that the relative variation of an index in a solar cycle is about 2-3 times. However, the magnitude of Mg II - index (as well as TSI, not shown in this figure) changes very little - about shares of percent.
The sunspot number SSN (also known as the International sunspot number, relative sunspot number, or Wolf number) is a quantity that measures the number of sunspots and groups of sunspots present on the surface of the sun.

The historical sunspot record was first put by Wolf in 1850s and has been continued later in the 20th century until today. Wolf’s original definition of the relative sunspot number for a given day as $R = 10 \cdot \text{Number of Groups} + \text{Number of Spots}$ visible on the solar disk has stood the test of time. The factor of 10 has also turned out to be a good choice as historically a group contained on average ten spots. Almost all solar indices and solar wind quantities show a close relationship with the SSN. (Svalgaard et al. 2011; Svalgaard & Cliver 2010). In our paper we use the proper homogeneity calibrations of SSN from (National Geophysical Data Center. Solar Data Service 2013).

At the present time the 10.7 - cm solar radio flux $F_{10.7}$ is measured at the Dominion Radio Astrophysical observatory in Penticton, British Columbia by the Solar Radio Monitoring Programme. $F_{10.7}$ is a useful proxy for the combination of chromospheric, transition region, and coronal solar EUV emissions modulated by bright solar active regions whose energies at the Earth are deposited in the thermosphere. (Tobiska et al. 2008) pointed the high EUV - $F_{10.7}$ correlation and used this in the Earth’s atmospheric density models.

According to Tapping & DeTracey 1990 the 10.7 - cm emission from the whole solar disc can be separated on the basis of characteristic time-scales into 3 components: (i) transient events associated with flare and similar activity having duration less than an hour; (ii) slow variation in intensity over hours to years, following the evolution of active regions in cyclic solar activity designated as S-component; (iii) a minimum level below which the intensity never falls - the “Quiet Sun Level”. The excellent correlation of S-component at 10.7 cm wavelength with full-disc flux in Ca II and MgII was discussed by (Donnelly et al. 1983 ). The 10.7 cm flux resembles the integrated fluxes in UV and EUV well enough to be used as their proxy (Chapman & Neupert 1974; Donnelly et al. 1983; Bruevich & Nusinov 1984; Nicolet & Bossy 1985; Lean 1987).

This radio emission comes from high part of the chromosphere and low part of the corona. $F_{10.7}$ radio flux has two different sources: thermal bremsstrahlung (due to electrons radiating when changing direction by being deflected by other charged particiles - free-free radiation) and gyro-radiation (due to electrons radiating when changing direction by gyrating around magnetic fields lines). The (iii) a minimum level component (when SSN is equal to zero as it was at the minimum of the cycle 24 and local magnetic fields are negligible) is defined by free-free source. When the local magnetic fields become strong enough at the beginning of the rise phase of solar cycle and solar spots appear the gyro-radiation source of $F_{10.7}$ radio flux begins to prevail over free-free so (i) and (ii) components begin to grow strongly.
The S-component comprises the integrated emission from all sources on solar disc. It contains contribution from free-free and gyroresonance processes, and perhaps some non-thermal emission (Gaizauscas & Tapping 1998). The relative magnitude of these processes is also a function of observing wavelength. Observations of emission from active regions over the wavelength range 21-2 cm suggest that at 21 cm, free-free emission is dominant, whereas at 6 cm, the contribution from gyroresonance is larger. At a wavelength of 10 cm, the two processes are roughly equal in importance. At a wavelength of 2-3 cm, the emission is again mainly free-free, possible with a non-thermal component (Gaizauscas & Tapping 1998). The spatial distributions of two thermal processes are different; the gyroresonant emission originates chiefly in the vicinity of sunspots, where the magnetic fields are strong enough, while the free-free emission is more widely-distributed over the host region complex (Tapping & DeTracey 1990).

The intensities of the Ca II and Mg II spectral lines are primary functions of chromospheric density and temperature, while the soft X-rays are produced in the corona. The high degree of correlation of the 10.7 cm flux with all these quantities suggests some dependence upon common plasma parameters and that their sources are spatially close. Another strong correspondence is between 10.7 cm flux and full-disc X-ray flux. When activity is high, they are well-correlated; however, when activity is low, the X-rays are too weak to be detected, while some 10.7 cm emission in excess of the "Quiet Sun Level" is always present (Kruger 1979). Our study of the connection between 10.7 cm flux and full-disc X-ray flux (Bruevich, & Yakunina 2011) also confirm the conclusions of (Kruger 1979).

Thus we have enhanced 10.7 cm radiation when the temperature, density and magnetic fields are enhanced. So \( F_{10.7} \) is a good measure of general solar activity.

The green and red coronal lines observations was regularly started from 1960 and the new solar index \( F_{530} \) - the averaged intensity of coronal flux at 530.3 nm was introduced. We used NASA data from several observatories and from satellite’s instruments. These data were modified to the common uniform system and are available as archive data of (National Geophysical Data Center. Solar Data Service 2013).

The 280 nm Mg II solar spectrum band contains photospheric continuum and chromospheric line emissions. The Mg II \( h \) and \( k \) lines at 279.56 and 280.27 nm, respectively, are chromospheric in origin while the weakly varying wings or nearby continuum are photospheric in origin. The instruments of the satellites observe both features. The ratio of the Mg II variable core lines to the nearly non-varying wings is calculated. The result is mostly a measure of chromospheric solar active region emission that is theoretically independent of instrument sensitivity change through the time.

Mg II core-to wing ratio (crw) observations were made at NOAA series operational satellites (NOAA-16-18), which host the Solar Backscatter Ultra Violet
(SBUV) spectrometer (Viereck, et al. 2001). This instrument can scatter solar Middle Ultra Violet (MUV) radiation near 280 nm. The Mg II observation data were also obtained from ENVISAT instruments. NOAA started in 1978 (during the 21st, 22nd and the first part of the 23rd solar activity cycles), ENVISAT was launched on 2002 (last part of the 23rd solar activity cycle). Comparison of the NOAA and ENVISAT MgII index observation data shows that both the Mg II indexes agree to within about 0.5% (Puga & Viereck 2004), (Scupin et al. 2005). We used both the NOAA and ENVISAT Mg II index observed data.

The Mg II index is especially good proxy for some Far Ultra Violet (FUV) and Extreme Ultra Violet (EUV) emissions (Scupin et al. 2005). It well represents photospheric and lower chromospheric solar FUV Schumann-Runge Continuum emission near 160 nm that maps into lower thermosphere heating due to O2 photodissociation (Bowman, et al., 2008). Since a 160 nm solar index is not produced operationally, The Mg II index proxy is used for comparison with the other solar indices (Tobiska et al. 2008).

Solar irradiance is the total amount of solar energy at a given wavelength received at the top of the earth’s atmosphere per unit time. When integrated over all wavelengths, this quantity is called the total solar irradiance (TSI) previously known as the solar constant. Regular monitoring of TSI has been carried out since 1978. From 1985 the total solar irradiance was observed by Earth Radiation Budget Satellite (EBRS). We use the NGDC TSI data set from combined observational data of several satellites which were collected in NASA archive data (National Geophysical Data Center. Solar Data Service 2013). The importance of UV/EUV influence to TSI variability (Active Sun/ Quiet Sun) was pointed by (Krivova & Solanki 2008). There were indicated that up to 63.3% of TSI variability is produced at wavelengths below 400 nm. Towards activity maxima SSN grows dramatically. But on average the Sun brightens about 0.1% only. This is due to the increase of amounts of bright and dark features: faculae and network elements on the solar surface on the one hand and spots on the other hand. The total area of the solar surface covered by such features rises more strongly as the cycle progresses than area of dark sunspots. The TSI (from Earth Radiation Budget Satellite) maxima are fainter than those of the other indices because the solar irradiance variation in solar cycle is approximately equal to 0.14%. This value seems very small but is normal. Some TSI physics-based models have been developed with using the combined proxies describing sunspot darkening (sunspot number or areas) and facular brightening (facular areas, Ca II or Mg II indices), see (Frontenla et al. 2004), (Krivova et al. 2003).

We also analyzed two activity indices which describe rapid processes on the Sun - Flare Index (FI) and monthly Counts of grouped solar flares (Count of flares). According to (Solar-Geophysical Data Reports 2009) the term 'grouped' means observations of the same event by different sites were lumped together and
counted as one.

(Kleczez 1952) defined the value $FI = \text{it}$ to quantify the daily flare activity over 24 hours per day. He assumed that relationship roughly gave the total energy emitted by the flare and named it flare index (FI). In this relation $i$ represents the intensity scale of importance of the flare and and $t$ the duration of the flare in minutes. In this issue we also used the monthly averaged FI values.

So it should be noted that the data used in our article are not uniform enough but we neglect this. We study the behavior of solar indexes during cycles of activity as a whole. Thus we analyze the general trend in their relationship.

3. Recent changes in the Sun

The recent solar cycle 23 was the outstanding cycle for authentic observed data from 1849 year. It lasted 12.7 years and was the longest one for two hundred years of direct solar observations. This cycle is the second component in the 22-year Hale magnetic activity cycle but the $23^{rd}$ cycle was the first case of modern direct observations (from 1849 to 2008) when Gnevyshev-Ol’s rule was violated: activity indices in cycle 23 had their maximum values less then the values in the cycle 22 (but according to Gnevyshev-Ol’s rule the cycle 23 must dominate).

(Ishkov 2009) pointed that in this unusual cycle 23 the monthly averages values for SSN during 8 months exceeded 113 and most of sunspot groups were less in size, their magnetic fields were less composite and characterized by the greater lifetime near $2^{nd}$ maximum in comparison with values near the $1^{st}$ maximum. SSN reaches its first maximum 3.9 years after the beginning. After the first maximum, the index decrease by 14% (of that maximum). The two maxima of this index have the same amplitude.

(Nagovitsyn et al. 2012) showed that for sunspot numbers SSN we see the opposite long-scale trends (which influence to increase or decrease of SSN) during over the last several solar cycles. In (Nagovitsyn et al. 2012) it was analyzed the data set of SSN from (Penn & Livingston 2006, Pevtsov et al. 2011) and it was shown that during the cycle 23 and the beginning of the cycle 24 the number of large sunspots gradually decreased, while the number of small sunspots steadily increased. It was suggested that this change in the fraction of small and large sunspots (perhaps, due to changes in the solar dynamo) can explain the gradual decline in average sunspot field strength as observed by (Penn & Livingston 2006).

In the cycle 23 the $F_{10.7}$ radio flux and the TSI have the lowest values from 2007 to 2009 (the beginning of the cycle 24) all over of these indices observation period. The $F_{10.7}$ radio flux index shows the second maximum is 8.4% stronger than the first one.

It’s known that all solar indices have been closely correlated as they all derive
from the same source: the variable magnetic field. But while there has long been a stable relationship between the 10.7 cm flux and SSN this relationship has steadily deteriorated in the past decade to the point where the sunspot number for a given flux has decreased by about a third.

Observations by Livingston since 1998 until the present show that the average magnetic field in sunspots has steadily decreased by 25% (Livingston et al., 2012). Since their magnetic fields cool sunspots, a decreasing field means that sunspots are getting warmer and that their contrast with the surrounding photosphere is getting smaller, making the spots harder to see. Without the dark spots, TSI might even be a bit higher, see (Svalgaard et al. 2011). We can see that the relation TSI/$F_{10.7}$ is larger a little for the cycle 23 (Fig. 3c) compared to the previous cycle 22. It is not clear what this will mean for the impact of solar activity on the Earth’s environment.

(Janardhan et al. 2010) have examined polar magnetic fields for the last three solar cycles 21, 22 and 23 using NSO Kitt Peak synoptic magnetograms and showed a large and unusual drop in the absolute value of the polar fields during cycle 23 compared to previous cycles and also it’s association with similar behavior in meridional flow speed.

In the cycle 23 the Flare index has a higher first maximum. This shows that the flares can be more efficiently generated during the first maximum, and it seems that the generation is decreasing towards the end of the cycle.

Figure 1 demonstrates that for all activity indices in the 23rd solar activity cycle we can see two maximums separated one from another on 1.5 year approximately. We see the similar double-peak structure in cycle 22 but for the cycle 21 the double-peak structure is not evident. We see that there are displacements in both maximum occurrence time of all these indices in the 23rd solar cycle.

Figure 1 also shows that for all solar indices in the cycle 23 the relative depth of the cavity between two maximums is about 10 – 15%.

Ishkov (2009) pointed that there was very high level of flared activity in the cycle 21 and very low level of flared activity in the cycle 23. The Sun’s flare activity is an important indicator of the general level of activity of the atmosphere is also described in other activity indices, in particular around the solar disk index and a locally varying flux in the H-alpha, see Bruevich (1995).

4. Changed relation between $F_{10.7}$ and solar activity indices in the cycles 21-23

Figure 2 illustrates the high level of interconnection between solar activity indices versus $F_{10.7}$. We see that coefficients of linear regression (slope - $A$ and intercept - $B$) differ among themselves for the activity cycles 21 - 23. The most differences shows the Counts of flares index.

When studied solar activity indices in 21, 22 and 23 solar activity cycles we
Figure 2: Correlation between monthly averages for solar indices versus $F_{10.7}$ individually determined for the cycles 21, 22 and 23. (a) $F_{530}$, (b) Mg II core-to-wing ratio, (c) TSI and (d) Counts of flares/10.

separated out rise phases, cycle’s maximum phase, cycle’s minimum phase and decline phase. In case of linear regression we’ve found that the maximum values of correlation coefficients $K_{corr}$ reached for the rise and decline phases of the cycles. According to our calculations the highest values of correlation coefficients $K_{corr}$ we see in connection between SSN and $F_{10.7}$. Correlation coefficients $K_{corr}$ for linear regression for TSI versus $F_{10.7}$ are the minimal of all correlation coefficients determined here.

The cyclic variation of fluxes in different spectral ranges and lines at the 11-year time scale are widely spread phenomenon for F, G and K stars (not only for the Sun), see (Bruevich & Kononovich 2011, Bruevich & Ivanov-Kholodnyj 2011). The chromospheric activity indices (radiative fluxes at the centers of the $H$ and $K$ emission lines of Ca II - 396.8 and 393.4 nm respectively) for solar-type stars were studied during HK-project by (Baliunas et al. 1995) at Mount Wilson observational program during 45 years, from 1965 to the present time. Authors of the HK-project supposed that all the solar-type stars with well determined cyclic activity about 25% of the time remain in the Maunder minimum conditions. Some
scientists proposed that the solar activity in the cycle 24 will be very low and similar to activity during the Maunder minimum period. We can see now that although the new cycle of activity is not characterized by very high SSN values, however, the activity in the cycle 24 is significantly higher than in the Maunder minimum period.

Figure 3: Correlation between monthly averages of solar indices versus $F_{10.7}$ in the cycles 21 - 23 (1975 - 2008). (a) SSN, (b) $F_{530}$, (c) Mg II core-to-wing ratio, (d) Flare Index, (e) Counts of flares and (f) TSI.

At Figure 3 we presented the interconnection between solar indices and radio
flux $F_{10.7}$ in the cycles 21 - 23. We analyzed the interconnection between activity indices SSN, $F_{530}$, Mg II core to wing ratio, Flare Index, Counts of flares and TSI versus $F_{10.7}$ for the 21st, 22nd and 23rd solar cycles. We have studied both the linear and polynomial dependencies.

The linear model corresponds to the linear regression equation:

$$F_{\text{ind}} = A_{\text{ind}} + B_{\text{ind}} \cdot F_{10.7}.$$  \hspace{1cm} (1)

were $F_{\text{ind}}$ is the activity index flux,

$A_{\text{ind}}$ is the intercept of a linear regression,

$B_{\text{ind}}$ is the slope of a linear regression.

Table 1: Solar activity indices versus $F_{10.7}$. Coefficients of linear regressions: $A$, $B$ and their standard errors. Observational data 1975 - 2010.

| Act. indices versus $F_{10.7}$ | $A_{\text{ind}}$ | $B_{\text{ind}}$ | Err. $\sigma_A$ | Err. $\sigma_B$ |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|
| SSN                           | -62.28          | 1.03            | 1.57            | 0.011           |
| $F_{530}$                     | -2.93           | 0.084           | 0.27            | 0.002           |
| Mg II                         | 0.258           | $1 \cdot 10^{-4}$ | $2.6 \cdot 10^{-4}$ | $1.8 \cdot 10^{-7}$ |
| Flare Index                   | -7.37           | 0.106           | 0.51            | 0.0036          |
| Counts fl/10                  | -269.03         | 4.31            | 20.14           | 0.146           |
| TSI                           | 1365.07         | 0.0066          | 0.044           | $3 \cdot 10^{-4}$ |

In the Table 1 we present the coefficients of the linear regressions and their standard errors $\sigma$ of intercept and slope values.

The polynomial model corresponds to the following equation of a second order polynomial:

$$F_{\text{ind}} = A_{\text{ind}} + B_{1\text{ind}} \cdot F_{10.7} + B_{2\text{ind}} \cdot F_{10.7}^2.$$  \hspace{1cm} (2)

were $F_{\text{ind}}$ is the activity index flux,

$A_{\text{ind}}$ is the intercept of a polynomial regression,

$B_{1\text{ind}}$ and $B_{2\text{ind}}$ are the coefficients of a polynomial regression.

In the Table 2 we present the coefficients of polynomial regressions: $A$, $B_1$, $B_2$ and their standard errors $\sigma$ of intercept and slope values.

We have to point out that close interconnection between radiation fluxes characterized the energy release from different atmosphere’s layers is the widespread phenomenon among the stars of late-type spectral classes. (Bruevich & Alekseev 2007) confirmed that there exists the close interconnection between photospheric and coronal fluxes variations for solar-type stars of F, G, K and M spectral classes with widely varying activity of their atmospheres. It was shown that the summary areas of spots and values of X-ray fluxes increase gradually from the sun...
Table 2: Solar activity indices versus \( F_{10.7} \). Coefficients of polynomial regressions: \( A, B_1, B_2 \) and their standard errors. Observational data 1975 - 2010.

| Act. indices versus \( F_{10.7} \) | A      | B1  | B2         | Err. \( \sigma_A \) | Err. \( \sigma_{B1} \) | Err. \( \sigma_{B2} \) |
|-----------------------------------|--------|-----|------------|----------------------|----------------------|----------------------|
| SSN                               | -87.26 | 1.45| -0.0015    | 4.86                 | 0.078                | 3 \cdot 10^{-4}     |
| \( F_{530} \)                     | -7.38  | 0.158| -2.6 \cdot 10^{-4} | 0.84                 | 0.013                | 4.8 \cdot 10^{-5}   |
| Mg II                             | 0.25   | 2 \cdot 10^{-4} | -3 \cdot 10^{-7} | 7 \cdot 10^{-4}   | 1 \cdot 10^{-5}    | 4 \cdot 10^{-8}     |
| Flare Index                       | -4.36  | 0.057| 1.7 \cdot 10^{-4} | 1.58                 | 0.024                | 7 \cdot 10^{-5}     |
| Counts fl/10                      | -361.4 | 5.83 | 0.005     | 64.01               | 1.01                 | 0.0035               |
| TSI                               | 1364.4 | 0.017| -3.7 \cdot 10^{-5} | 0.13                 | 0.002                | 7 \cdot 10^{-7}     |

and HK-project stars with the low spotted discs to the highly spotted K and M-stars for which (Alekseev & Gershberg 1996) constructed the zonal model of the spots distributed at the star’s disks. The variations of activity indices in the whole 11-yr cycle of the Sun are very similar to the cyclical variations of the chromospheric fluxes on the stars. So we can simulate the dependencies which describe the variations of the indices during the activity cycle for the stars as for the Sun, see Bruevich and Bruevich (2004).

5. The time variations of correlation coefficient \( K_{corr}(t) \) for the linear regression of solar activity indices versus \( F_{10.7} \) and versus SSN.

We’ve calculated values \( K_{corr}(t) \) of linear regression for solar activity indices versus \( F_{10.7} \) for the cycles 21, 22 and 23. The values \( K_{corr}(t) \) were determined for each moment of time \( t \) from \( K_{corr}(t) \) calculation during the 3-year time interval \( t - 1.5yr \leq \Delta T \leq t + 1.5yr \).

Figure 4 demonstrates the results of our correlation calculations of these solar activity indices versus \( F_{10.7} \) and versus SSN - \( K_{corr}(t) \) variations during the cycles 21 - 23. We can see that all the \( K_{corr}(t) \) values have the maximum amounts at the rise and at the decline phases. The minimum values of the \( K_{corr}(t) \) we see at the minimum and the maximum phases of solar cycles.

We can see that the minimum values of correlation coefficients \( K_{corr}(t) \) for the solar indices versus \( F_{10.7} \) and SSN occurred twice during the 11-year cycle. We assumed that this fact must be considered for the understanding of the solar indices interconnections and for successful forecasts of different activity indices using \( F_{10.7} \) or SSN observations.

Note that the linear correlation (see Figure 4) of activity indices \( F_{530} \), MgII index, Flare Index and TSI versus \( F_{10.7} \) is a little stronger than the linear correlation of these indices versus SSN. We assumed that it’s a logical result: these indices (as well \( F_{10.7} \)) characterize the solar irradiance proceeded from different altitudes.
Figure 4: Correlation coefficients of linear regression $K_{corr}(t)$ for (a) TSI, (b) Flare Index, (c) Counts of flares and (d) Mg II UV-index versus SSN and versus $F_{10.7}$. $K_{corr}(t)$ was calculated during three-year interval. Observational data 1975 - 2010).

of solar atmosphere. But SSN and Counts of flares are not connected directly with solar irradiance at different wavelengths or spectral intervals. So the linear correlation of Counts of flares (see Figure 4) has no difference versus $F_{10.7}$ or versus SSN because the Counts of flares index describes the fast flared processes (not irradiance) on the Sun while SSN is the relatively subjective measure of the total level of solar activity.

The cyclic behavior of $K_{corr}$ can be explained by following assumption: we imagine that some activity index flux depends on time $t$ by the expression:

$$F_{ind}(t) = F^{background}_{ind}(t) + \Delta F^{AR}_{ind}(t).$$  \hspace{1cm} (3)

were $F^{background}_{ind}(t)$ is the background flux which continuously rising with increasing of solar activity. Background flux consists of two components - (1) slow variation in intensity over hours to years, following the evolution of active regions in cyclic solar activity and (2) a minimum level below which the intensity never
falls - the "Quiet Sun Level". In case of the radio flux $F_{10.7}$ the component (1) is (ii), the component (2) is (iii) respectively. It can be note that all indices have the "Quiet Sun Level" different from zero except SSN, which at the minimum of the cycle has the "Quiet Sun Level" value equal to zero.

$\Delta F_{\text{AR}}^{\text{ind}}(t)$ is the additional flux to the overall flux from the active regions.

The previous correlation study allows us to consider that $F_{\text{ind}}^{\text{background}}(t)$ and $\Delta F_{\text{ind}}^{\text{AR}}(t)$ are the linear functions of the background and activity regions levels of solar activity. In our case we choose the radio flux $F_{10.7}$ as the best basal indicator of solar activity levels:

$$F_{\text{ind}}^{\text{background}}(t) = a_1 + b_1 \cdot F_{\text{background}}^{10.7}(t).$$  \hspace{1em} (4)

$$\Delta F_{\text{ind}}^{\text{AR}}(t) = a_2 + b_2 \cdot \Delta F_{\text{AR}}^{10.7}(t).$$  \hspace{1em} (5)

The coefficients $a_1$ and $b_1$ vary from $a_2$ and $b_2$ in different power for our different activity indices. For SSN this difference is small, but for Counts of flares index the difference between $a_1$, $b_1$ and $a_2$, $b_2$ is more significant than for SSN.

During the rise and decline cycle’s phases the dependence $F_{\text{ind}}(t)$ versus $F_{10.7}(t)$ is approximately linear and relative addition flux from active regions $\Delta F_{\text{ind}}^{\text{AR}}(t)$ is neglect with respect to $F_{\text{ind}}^{\text{background}}(t)$. So additional flux from active regions cannot destroy a balance in the correlation close to linear between $F_{\text{ind}}(t)$ and $F_{10.7}(t)$ and respective values of $K_{\text{corr}}(t)$ reach their maximum values during all over the cycle.

During the minimum of activity cycle both values $F_{\text{ind}}^{\text{background}}(t)$ and $\Delta F_{\text{ind}}^{\text{AR}}(t)$ are small, but additional flux from active regions is not neglect in relation to background flux that has the minimum values during all over the cycle.

During the maximum of activity cycle $\Delta F_{\text{ind}}^{\text{AR}}(t)$ often exceeds $F_{\text{ind}}^{\text{background}}(t)$ so disbalance in linear regression between activity indices increases and values of $K_{\text{corr}}(t)$ also reach their minimum values during all over the cycle too.

6. Conclusions

For a long time the scientists were interested in the simulation of processes in the earth’s ionosphere and upper atmosphere. It’s known that the solar radiance at 30.4 nm is very significant for determination of the Earth high thermosphere levels heating. (Lukyanova & Mursula 2011) showed that the for solar 30.4 nm radiance fluxes forecasts (very important for Earth thermosphere’s heating predictions) there were more prefer to use Mg II 280 nm observed data.

Although $F_{10.7}$ does not actually interact with the Earth atmosphere $F_{10.7}$ is a useful proxy for the combination of chromospheric, transition region and coronal solar EUV emissions modulated by bright solar active regions whose energies at
the Earth are deposited in the thermosphere (Tobiska et al. 2008). $F_{10.7}$ dependence on few processes, combined with it localized formation in the cool corona, i.e. region that is closely coupled with magnetic structures responsible for creating the XUV-EUV irradiances, make this a good generalized solar proxy for thermospheric heating.

(Tobiska et al. 2008) presented the improved thermospheric density model, where four solar and two geomagnetic indices were used. Solar indices are $F_{10.7}$, 26-34 nm EUV emission, Mg II core-to-wing ratio, X-rays in the 0.1-0.8 nm. The geomagnetic indices are ap index (amplitude of planetary geomagnetic activity - which is derived from geomagnetic field measurements made at several locations around the world) and Dst index (Disturbance Storm Time - as indicator of the storm-time ring current in the inner magnetosphere). It was proved the efficiency of of the simultaneous use of multiple indexes of solar and geomagnetic activity.

In this paper we found out the cyclic behavior of a calculated during three-year interval values of correlation coefficients $K_{corr}(t)$ of linear regression for TSI, Flare Index, Mg II 280 nm and Counts of flares versus $F_{10.7}$ and SSN during solar activity cycles 21,22 and 23 (see Figure 4). We showed that $K_{corr}(t)$ have the maximum values at the rise and decline phases - the linear connection between indices is more strong in these cases. It means that the forecasts of solar indices, based on $F_{10.7}$ observations will be more successful during the rise and decline cycle’s phases. We showed that the linear correlation of activity indices $F_{530}$, Mg II index, Flare Index and TSI versus $F_{10.7}$ is stronger than the linear correlation of these indices versus SSN but the linear correlation of Counts of flares has no difference versus $F_{10.7}$ or versus SSN. This may be due, in particular, that all indices have the ”Quiet Sun Level” different from zero except SSN, which has the minimum value equal to zero.

We also determined that a calculated during three-year interval values of correlation coefficients $K_{corr}(t)$ show (for the linear regressions assumption) that for the solar indices versus $F_{10.7}$ and SSN the minimum values were achieved two times during the 11-year cycle.

Our study of linear regression between solar indices and $F_{10.7}$ confirms the fact that at minimum and at maximum cycle’s phases the nonlinear state of interconnection between solar activity indices (characterized the energy release from different layers of solar atmosphere) increases.

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