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Correlation between Conventional Spacing Factors of Air Voids in Concrete and Characteristic Distances Defined by Point Process Statistics

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Abstract

The most important parameter used to determine the frost resistance of concrete is the distance between air voids. The linear traverse method has been used to obtain this spacing factor as a representative distance parameter. One of the present authors has proposed a new method of evaluating the distance using point process statistics. In this study, the spacing factors independently obtained for 52 extant mixtures of concrete are compared with the characteristic distance defined by the nearest neighbor distance distribution function. For that, a procedure for obtaining the necessary parameters for the point process method from records of linear traverse measurements is proposed. There exists a strong correlation between the spacing factor and the characteristic distance. The difference between two distance parameters is at most a few tens of micrometers. Furthermore, the median distance simply defined by the nearest neighbor distance distribution function is also found to have a good correlation with the spacing factor. The characteristic distance or the median distance is representative of an air void system and is an alternate to the conventional spacing factor.

This paper is the English translation from the authors’ previous work [Igarashi, S., Taniguchi, M. and Yamashita, S., (2021). “Derivation of characteristic distances of the point process method from records of linear traverse measurement and their correlation with spacing factors.” Cement Science and Concrete Technology, 74, 131-138. (in Japanese)].

1. Introduction

Because of its porous nature, concrete is susceptible to frost damage. The fundamental means of providing resistance to freezing and thawing cycles is the addition of air-entraining agents. The resistance mechanism provided by the air entrainment is rather complicated, because many factors are involved in the deterioration, owing to frost attack (Valenza II and Scherer 2007). However, it is empirically known that the resistance is related to not only the entire volume of air voids but also the distances between them. This is sometimes explained as the protected paste volume concept (Wawrzenczyk and Kozak 2016). It is assumed that each air void protects the surrounding cement paste matrix within a certain distance from its surface. To give sufficient resistance against frost action, any location in the cement paste matrix should be included within the protected region. In other words, the whole cement paste matrix must be covered with the protected regions formed around air bubbles. Thus, the evaluation of the farthest distance from air voids within concrete is of vital significance in ensuring the frost resistance of concrete.

For the past 60 years, to evaluate the distance between air voids in concrete, a linear traverse measurement has often been made. This test method is based on the proposal by Powers (1949). The fundamental background of the evaluation procedure was its so-called model-based stereology or a simplified technique of Delesse’s rule (Delesse 1847). The average maximum distance from the surfaces of air voids is calculated as a spacing factor, using an area fraction and the specific surface of air voids obtained via lineal analysis. The linear traverse measurement, standardized as ASTM C457 (ASTM 1998), is a simple procedure that detects and tallies encounters with air bubbles along traverse lines. However, the manual standardized procedure is difficult to perform routinely, because it is very time-consuming and tedious. Currently, computerized equipment capable of identifying air voids and analyzing their distribution automatically has been made available (Jakobsen et al. 2006). If modern techniques were also available, the time required for measurement would be greatly shortened, and the dependence on the skill of concrete petrographers would be minimized. For example, nowadays, the 3D imaging technique for air voids in concrete is becoming more common than it was (Promentilla and Sugiyama 2010; Schock et al. 2016). However, despite using the most advanced techniques and tools, some workers still evaluate the spacing factor as a virtual distance between air voids even though a real structure of air voids is re-
Therefore, in this study, the proposed method is verified usefulness of the point process method statistically. Conventional spacing factor and the characteristic distance measurements comparing the two parameters (i.e., the conventional spacing factor) were taken as a representative of interparticle distances. Therefore, it is impossible to specify a representative distance (e.g., simple mean of all distances) among them. To evaluate its random distribution in terms of spatial statistics and obtaining a representative distance that can be used as a judgment of frost resistance, several equations for calculating air void spacing have been proposed (Lu and Torquato 1992; Mayercsik et al. 2014; Snyder 1998). One of the present authors has proposed a new method based on point process statistics (Murotani et al. 2019). In the proposed method, all air voids in an image were converted to points dispersed on a 2D plane. Their spatial distribution as a planar point process was evaluated using the nearest neighbor distance distribution function, measuring the shortest distance from each air void to the other. Notably, the characteristic distance defined by the spatial distance function has a strong correlation to the conventional spacing factor. Further, the proposed method can be applied to any point patterns for air voids regardless of the conditions of image acquisition as far as the air voids are adequately segmented by general procedures of image analysis. It takes a short time to determine the characteristic distance if a binary image of air voids is prepared for subsequent analysis. All the reliable tools needed to calculate spatial statistics for those random points are freely available. More importantly, the characteristic distance proposed is a parameter that reflects a real distribution, not that of a virtual regular arrangement. Specific assumptions about the original arrangement of air voids are unnecessary. A distance that is practically measured easily in concrete is taken as a representative of interparticle distances. Moreover, it would be beneficial if the decision criterion for ensuring frost resistance was empirically associated with proven results. This would be further advantageous to practical examinations.

These advantages were described in a previous study (Murotani et al. 2019). However, the number of measurements comparing the two parameters (i.e., the conventional spacing factor and the characteristic distance newly proposed) was not large enough to support the full usefulness of the point process method statistically. Therefore, in this study, the proposed method is verified using results of linear traverse measurements carried out over the past few decades independently. First, the proposed point process method is briefly reviewed for a better understanding of the subsequent description. Then the procedure for obtaining fundamental parameters of the point process method from the records of the conventional linear traverse measurement is explained since images of air voids associated with their records were not taken at the measurement in general. After reviewing the records of linear traverse measurements for different concrete mixtures, a correlation between the conventional spacing factor and the characteristic distance is discussed. Then to evaluate air void systems easily using the point process method, a procedure to estimate the characteristic distance graphically is proposed. Finally, a correlation between the distance simply determined by the nearest neighbor distance distribution function and the spacing factor is also pointed out based on an interesting fact on the characteristic distance.

2. Derivation of point process parameters from the result of linear traverse measurement

2.1 Records of linear traverse measurement

The authors collected 52 records of linear traverse measurement that followed the ASTM C457, Procedure A, linear traverse method. The main properties of those concrete mixtures are as follows. Most of them are ordinary concretes used for general purposes although a few high strength concretes are included.

a) Water to cement ratio: 38 to 60%
b) Air content in fresh concrete: 2.5 to 7.7%
c) Maximum aggregate size: 20 or 25 mm
d) Paste content: 26.0 to 42.1%

Of those, 49 measurements were made by experienced concrete petrographers who manually measured the air void systems following the given procedure. In the measurement, the petrographer, i.e., operator counts the number and lengths of chords that traverse air voids in a cross section of concrete viewed under a microscope. The air content is calculated by the ratio of the traverse length through air voids to the total length of the traverse. Further, the specific surface of air voids is easily determined by the average chord length. Using the air content and the specific surface, a cubic lattice arrangement of mono-sized air bubbles is assumed, and the average maximum distance from any point in the paste to the surface of a void is defined as the spacing factor. The other three used the latest equipment, comprising a high-resolution stereomicroscope and new software used to automatically calculate the spacing factor. The frame of each image is 6.14×6.14 mm. Its resolution is 3 µm/pixel. The whole observation field is approximately 60×60 mm, which consists of 121 frames. In this automatic measurement equipment, air voids are identified and segmented using the differences in shadows between vertical and diagonal lightening to the polished surfaces of concrete. The equipment was operated by trained
2.2 Point process analysis to determine the characteristic distance

(1) Point intensity $\lambda_i$

When a binary image of air voids is processed by the
given procedure, the area fraction of air is obtained via image analysis. Centroids \( x_i \) of individual air voids are also obtained. This set of the centroid points is regarded as a spatial point process, \( X = \{ x_i : i = 1, \ldots, n \} \) (Fig. 1). The point intensity \( \lambda \) is a basic descriptive characteristic of a point process and is defined as follows:

\[
\lambda = \frac{n}{|W|} \quad (1)
\]

where \( n \) is the number of points (i.e., air void profiles) in the observation window \( W \). \( |W| \) is the area of \( W \) that includes both fine and coarse aggregate particles. When a measurement of air voids is made following the procedure of ASTM C457, the intensity \( \lambda \) is unknown, because neither \( n \) nor \( |W| \) are recorded. It should be noted that the number of air voids within area \( W \) is different from the number of chords on the air voids. Therefore, the most significant parameter \( \lambda \) of the point process statistics must be estimated using a different procedure, which is described later.

(2) Characteristic distance defined by the nearest neighbor distance distribution function \( G(r) \)
For a point \( x_i \in X \), its shortest distance to the other point \( x_j \in X \) is written as \( d = d(x_i, X \setminus x_j) \). Then, for any \( r \geq 0 \), the nearest neighbor distance distribution function \( G(r) \) is defined by the following probability equation (Baddeley et al. 2016):

\[
G(r) = \Pr\{ d = d(x_i, X \setminus x_j) \leq r | x_i \in X \} \quad (2)
\]

This function is a cumulative probability function of the distance, \( r \) (Fig. 2). It converges to 1 at long distances. The distance that corresponds to the probability of 0.5 is defined as a median distance \( R_{50} \) [Fig. 2(a)]. The distance \( d = d(x_i, X \setminus x_j) \) is the distance between two points included in the point process, \( X \). Meanwhile, the spacing factor is defined as the distance from the farthest location in the paste to the nearest surface of a void, arranged regularly. It is not a distance between two air voids. For the median distance \( R_{50} \) to have the same meaning as the spacing factor, \( R_{50} \) is modified to define a characteristic distance \( L' \) [Fig. 2(b)] (Murotani et al. 2019):

Fig. 1 Conversion from a microscope image of air voids to a point pattern.
(a) original image of air voids; (b) binary segmentation of air voids; (c) point pattern of air voids; (d) schematic of nearest neighbor distance from a point \( x_i \in X \) to \( x_j \in X \) and spherical contact distance from a point \( u \notin X \) to \( x_i \in X \).
where $D_\lambda$ is the mean diameter of air voids. This is easily obtained via simple image analysis. The background of this definition is the equivalence of the nearest neighbor distance distribution function to the spherical contact distribution function in a stationary Poisson point process (Baddeley et al. 2016). Therefore, the value $L'$ may also be interpreted as the distance from an arbitrary point $u \in W \setminus X^\prime$, to the edge of the nearest air void. Again, neither $R_{50}$ nor $D_\lambda$ is measured in the conventional linear traverse measurement if the standardized original procedure is followed. Thus, these two values in addition to the point intensity must be determined differently.

(3) Formation of a 3D cubic lattice from 2D characteristics of the point process

Using the DeHoff and Rhines relation (DeHoff and Rhines 1961) [Eq. (4)] of a fundamental stereological rule, the number density of air voids observed on a plane section is equal to the product of the mean height of air voids $E[H]$ and their number density per unit volume of concrete $N_v$ in a 3D space:

$$Q_\lambda = E[H] N_v$$

where $Q_\lambda$ is the number of air void profiles per unit area on a plane section. In this study, $Q_\lambda$ is replaced with the $\lambda_v = N_v$. The mean height $E[H]$ is the average height of air voids projected onto a line normal to the section plane of observation. Strictly speaking, it must be determined from the 3D particle size distribution of air voids. However, air void systems are usually assumed to be statistically homogeneous and isotropic, as understood from the fact that the Delesse rule has been used in the linear traverse method. Furthermore, the particle size distribution in a cross-section gives useful information on the real particle sizes in 3D space, because air voids can be assumed to be spherical particles (Baddeley and Jensen 2005). If all the particles are spheres, then the mean projected height $E[H]$ is the same as the mean sphere diameter of all the particles in the 3D space. Furthermore, any planes containing many cross-sections of air voids from tiny to large are representative of all the sections throughout the 3D concrete structure. These fundamental assumptions, which are common practice, allow us to regard the mean diameter $D_\lambda$ of air voids within the observation window as the mean sphere diameter $E[H]$. Then the 3D point intensity per unit volume of cement paste $\lambda_v = N_v$ is simply given by

$$\lambda_v = \frac{\lambda_\lambda}{p D_\lambda}$$

where $p$ is the volume fraction of cement paste. As with in the definition of the spacing factor, if a regular arrangement of cubic lattices is formed, the reciprocal of $\lambda_v$ is a volume of cubic allocated to each point in 3D space (Fig. 3). Its edge length is determined such that half the diagonal of the cubic unit $R_{50}$ is given by

$$\frac{R_{50}}{2} = \frac{\sqrt{3}}{2} \sqrt{\frac{1}{\lambda_v}} = \frac{\sqrt{3}}{2} \sqrt{\frac{p D_\lambda}{\lambda_\lambda}}$$

Thus, the spacing factor $L$ for the cubic structure formed by the use of information of a 2D random point process is given by

$$L = \frac{R_{50}}{2} - \frac{D_\lambda}{2} = \frac{\sqrt{3}}{2} \sqrt{\frac{p D_\lambda}{\lambda_\lambda}} - \frac{D_\lambda}{2}$$

This length is a spacing factor that does not assume mono-sized spherical systems. As shown in the previous study (Murotani et al. 2019), the characteristic distance $L'$ of Eq. (3) determined by a point pattern is comparable to the spacing factor $L$ given by Eq. (7). Therefore, the characteristic distance $L'$ gives the spacing factor $L$ if a 3D lattice arrangement is constructed from the random 2D point pattern. Thus, equating Eqs. (3) to (7), the median distance $R_{50}$ of a point process is equal to half the diagonal of the cubic unit:

$$R_{50} = \frac{\sqrt{3}}{2} \sqrt{\frac{p D_\lambda}{\lambda_\lambda}}$$

If the median distance $R_{50}$ is determined from a point pattern with intensity $\lambda_v$, the mean diameter of air voids is rewritten using $R_{50}$:
Thus, the mean diameter is estimated using three parameters: point intensity, median distance, and volume fraction of cement paste. By substituting $D_A$ into the original definition of $L'$ [Eq. (3)], the characteristic distance is given by,

$$L' = R_{50} - \frac{1.54}{p} A_R p R_{50}^3 \left( 1 - 0.77 \frac{A_R}{p} R_{50}^3 \right) R_{50}$$

The characteristic distance $L'$ in Eq. (10) is a different expression of $L'$ based on its correspondence to the diagonal of the cubic lattice. If the volume fraction $p$ of cement paste is known in advance by a mix proportion or image analysis, the characteristic distance $L'$ can be determined by the point intensity and the median distance. In the previous study (Murotani et al. 2019), it was also shown that the characteristic distance obtained by Eq. (10) was nearly the same as the conventional spacing factor.

If air voids are assumed to disperse completely spatially randomly throughout the cement paste matrix, the distance $R_{50}$ can be uniquely estimated using the nearest neighbor distance distribution function $G(r)$ without observing a real point pattern. The $G(r)$ function for a 2D stationary Poisson process having an intensity $\lambda_i$ is given by

$$G(r) = 1 - \exp(-\lambda_i \pi r^2)$$

By equating $G(r)$ to 0.5, the median distance $R_{50}$ of the complete random point pattern is given as

$$R_{50} = \frac{\ln(0.5)}{-\lambda_i \pi} \approx \frac{0.47}{\sqrt{\lambda_i}}$$

where $R_{50}$ is the median distance of the Poisson point process with intensity $\lambda_i$. Substituting it into Eq. (10), the characteristic distance $L'$ is given by

$$L' = \left[ 1 - 0.77 \left( \frac{A_R}{p} \right) \left( \frac{0.47}{\sqrt{\lambda_i}} \right) \right] \left( \frac{0.47}{p} \right) \left( \frac{1}{p} \right) \left( \frac{1}{p} \right)$$

The characteristic distance $L'$ is finally determined by the volume fraction of cement paste $p$ and the point intensity $\lambda_i$ (i.e., the number of air voids per unit area). In other words, the characteristic distance, which is analogous to the spacing factor, is given by Eq. (13), even when a point pattern is not observed. The agreement of the spacing factor and the characteristic distance given by Eq. (13) was also suggested in the previous study (Murotani et al. 2019).

(4) Determination of parameter $\lambda_i$ from the results of linear traverse measurement

As is understood in ASTM C457, using the model-based stereology rule, air voids can be assumed to disperse randomly in the cement paste matrix. Thus, its spatial distribution is naturally assumed to be random without examining a real point pattern. Then the characteristic distance $L'$ is estimated using Eq. (13). However, it should be noted that the point intensity $\lambda_i$ is unknown in the linear traverse measurement. A comparison of the characteristic distance with the spacing factor assumes that the intensity $\lambda_i$ is known. Thus, this intensity must be estimated in advance from the records of linear traverse measurements. This can be done as follows (Igarashi et al. 2021).

If the number of air voids $n$ in the observation window and the mean area $\bar{a}$ of them are measured together with other air void characteristics during the linear traverse measurement, the measured air content $A_d$ would be expressed as the following equation:

$$A_d = A_w \frac{n \bar{a}}{A_w \bar{a} \times T_w / T_i}$$

where $R_{50}$ is the median distance of the Poisson point process with intensity $\lambda_i$. Substituting it into Eq. (10), the characteristic distance $L'$ is given by

$$L' = \left[ 1 - 0.77 \left( \frac{A_R}{p} \right) \left( \frac{0.47}{\sqrt{\lambda_i}} \right) \right] \left( \frac{0.47}{p} \right) \left( \frac{1}{p} \right) \left( \frac{1}{p} \right)$$

The characteristic distance $L'$ is finally determined by the volume fraction of cement paste $p$ and the point intensity $\lambda_i$ (i.e., the number of air voids per unit area). In other words, the characteristic distance, which is analogous to the spacing factor, is given by Eq. (13), even when a point pattern is not observed. The agreement of the spacing factor and the characteristic distance given by Eq. (13) was also suggested in the previous study (Murotani et al. 2019).
where \( A_A \) is the total area of all air voids, and \( A_c \) is the area of the reference window that is equal to the observation field, \( W \). Both are unknown. \( T \) is the traverse length through air voids, and \( T_t \) is the total length of traverse lines.

From Eq. (14), the unknown parameter \( \lambda_a \) is alternatively expressed:

\[
\lambda_a = \frac{n}{A_c} = \frac{A_A}{\bar{A}} \tag{15}
\]

Thus, to ascertain the point intensity, the average area of air voids \( \bar{A} \) is necessary. However, any areas of air voids are not measured in the linear traverse measurement as long as the standardized procedure of the linear traverse is followed. However, all chord lengths and averages are successively recorded in the linear traverse measurement method. The average chord length \( \bar{T} \) determines the specific surface \( \alpha \). The average area \( \bar{A} \) of air voids is also related to the same specific surface \( \alpha \). As given in Eq. (16) (Murotani et al. 2019), the average record of \( \bar{T} \) determined in the linear traverse measurement is related to the average area \( \bar{A} \), which is not measured.

\[
\alpha = \frac{4}{\bar{T}} = \sqrt{\frac{6\pi}{\bar{A}}} \tag{16}
\]

Using Eq. (16), the average area is determined from the average chord length. As a result, the important parameter (i.e., point intensity \( \lambda_a \)) is determined by Eq. (15). When the intensity \( \lambda_a \) is estimated, the characteristic distance \( L' \) is estimated using Eq. (13), even if any area measurements are not made for each air void.

3. Results and Discussion

3.1 General tendencies of results of the linear traverse measurements

All results in Table 1 are separately shown as histograms in Fig. 4. The air contents range from 1.0 to 8.0% [Fig. 4(a)]. The greatest frequency is found in the range between 3.0 and 4.0%. About 85% of the mixtures fall in the range between 2.0 and 5.0%. The average chord length ranges relatively narrow, as shown in Fig. 4(b). Most of them are within the range between 0.1 and 0.2 mm. The average chord length reflects the size of the air bubbles. The size of mono-sized spheres assumed for calculating the spacing factor is determined by this length. As long as the commercial air-entraining agents are produced under proper quality control, the results in Fig. 4(b) suggest that the size of entrained air bubbles may not greatly change among different manufacturers. The specific surface of air bubbles is distributed almost symmetrically, whereas the frequency of smaller bubbles (i.e., those greater the specific surface) is slightly greater [Fig. 4(c)]. The distribution of paste contents is shown in Fig. 4(d). Its frequency tends to concentrate on a range between 28 and 30%, because the paste contents (i.e., mix proportions of concrete) are empirically determined by their required workability and strength. The distribution of the spacing factors is shown in Fig. 4(e). About 2/3 of the mixtures satisfy a general requirement for values less than 0.25 mm.

3.2 Comparison of the distance parameters

The linear traverse procedure is analogous to examining air voids as an occurrence of a specific event in one dimension (i.e., one direction). The sequence of the random presence of each air void is recorded as a particular event along the traverse lines. The probability of encountering an event is expressed by its rate or intensity, which corresponds to the number of air voids in a unit length. Thus, the density of chords through air voids is a parameter used to determine the probability of random events (Ang and Tang 2007). This density is obtainable from the record of the conventional procedure of linear traverse measurement. This idea can be generalized to a 2D space. The presence of a point in an arbitrary 2D region is regarded as a particular event. Thus, the point intensity \( \lambda_p \) is essentially a parameter corresponding to the chord density. The chord density is a parameter using 1D probes of lines, whereas the point intensity is the one using 2D probes of areas. Both can be used for scanning the dispersion of air voids. Thus, their results must be consistent with each other as long as the same dispersion of air voids is quantified. Figure 5 shows the relationship between the point intensity \( \lambda_p \) and the chord density measured with the linear traverse procedure. As expected, there exists a strong correlation between them, i.e., the more chords, the more points.

Figure 6 shows the relationship between the characteristic distance \( L' \) and the spacing factor \( L \). There also exists a good correlation between them. The characteristic distance is a bit smaller than the spacing factor. However, most of the difference is, at most, a few tens of micrometers. This falls within the range of variation allowable for the spacing factor when a durability factor is specified (Mindess et al. 2003). Therefore, the characteristic distance can be regarded as a parameter consistent with the spacing factor. Furthermore, if the tolerance of errors in the measurement is taken into account, this difference may not matter much when judging the frost resistance (Snyder et al. 1991). Thus, the same criteria as the spacing factor (e.g., 250 \( \mu \)m) can be applied to the characteristic distance in judging whether adequate frost resistance can be ensured.

This similarity between the characteristic distance \( L' \) and the spacing factor \( L \) is deduced from the fact that the size of cubic lattice estimated based on the DeHoff equation and the cumulative probability of 0.5 in the nearest neighbor distance distribution function \( G(r) \) is nearly the same as the size as the lattice assumed for the spacing factor. The DeHoff equation is a fundamental and well-known relationship in stereology. The average area plays the same role as the average chord length in evaluating a specific surface, as mentioned above.
However, the characteristic distance does not assume mono-sized systems of spheres, and it gives a representative distance observed in a real random arrangement. Only counting air voids in an image is enough for evaluating the distribution if the paste content is known [Eq. (13)]. Most image analysis software, even freeware, has a function that can be used to perform the necessary evaluation for the proposed method as particle analysis (ImageJ 2020). If necessary, tools needed to calculate the spatial statistics functions (e.g., the nearest neighbor distance distribution function) are also available (Baddeley et al. 2016). Considering these circumstances, with its easy measurement and the availability of the same criteria as the spacing factor, the characteristic distance is regarded as a promising parameter of air void systems from a practical perspective. The evaluation of air void systems in concrete is thus a proper application of the point process statistics.

3.3 Graphical determination of the characteristic distance

In the procedure to determine the characteristic distance,
the point intensity and the paste content are necessary [Eq. (13)]. In practical petrographical examinations of concrete, the paste content of a sample is sometimes known in advance. In that case, it is convenient if frost resistance is judged only by tallying the air voids from the observation fields. A counting function is commonly included, even in a public domain image analysis software. Figure 7 shows the curves in which the characteristic distance is plotted against the point intensity in a specific paste content. Using these curves of each paste volume fraction, the point intensity necessary for finding the adequate characteristic distance or spacing factor can be graphically estimated. If a point intensity is determined by an image examination in concrete having a specific paste content, then one can select the point intensity on the horizontal axis and move upward to the curve. Then, one moves horizontally toward the vertical axis. The intercept gives the characteristic distance, which is comparable to the spacing factor, as mentioned above. If the opposite direction is followed, a required point intensity can be estimated for ensuring a characteristic distance specified from the frost resistance required. The spacing factors obtained by the linear traverse measurements are also superimposed onto Fig. 7. As expected, they are plotted a bit high, because they are somewhat greater than the characteristic distance (Fig. 6).

3.4 Further simplification of the procedure to determine the distance parameters
As shown in Fig. 6, the characteristic distance inferred from the records of linear traverse measurements has a strong correlation with the spacing factor directly measured by the linear traverse procedure. However, the spacing factor is slightly greater than the characteristic distance (Fig. 7). The difference is at most a few tens of micrometers, which is almost comparable with the size of an air bubble. Considering the original definition of the characteristic distance $L'$ in Eq. (3), the difference seems to approximately correspond to half the diameter $D_1/2$. In this study, there are no actual point patterns for each linear traverse measurement. Thus, the median distance $R_{50}$ is not measured for real point patterns. Nonetheless, the point intensity in the cement paste matrix can be estimated from the records. Thus, as mentioned earlier, if a completely random distribution, such as Poisson distribution, is assumed for the air bubbles in the 52 mixtures, the nearest neighbor distance distribution function $G(r)$ can be calculated to specify $R_{50}$. Figure 8 shows the relationship between the spacing factor obtained by the linear traverse procedure and the distance parameters.
Further study is needed to elaborate on the reason why those two parameters are almost the same. At least, this heuristic fact confirmed by the database of linear traverse measurement means that the edge length of the cubic lattice arrangement to define the spacing factor in 3D space is comparable to a distance parameter of the cubic structure that is derived from a point intensity of a random point process in the 2D plane. If this finding is approved or accepted empirically, the actual measurement for the spacing factor may be further simplified. The nearest neighbor distance distribution function of air voids can be directly evaluated from the images without the knowledge of the paste volume. Determining the paste content is not necessarily easy in actual petrographic examinations. However, Fig. 8 suggests that the evaluation of the volume fraction of the cement paste matrix and diameters of air voids can be omitted. In particular, once the median distance $R_{0.5}$, defined on a 2D plane, is specified from the measured function $G(r)$, it can already become a distance parameter comparable to the conventional spacing factor, which is obtained by 3D reconstruction of the regular arrangement of air voids. This process is quite convenient for old concrete specimens, whose documentation (e.g., mixture proportion) is not available. These simplifications may make the practical measurement of air void structures considerably easier.

The results in Figs. 6 and 8 confirm the significance and usefulness of the proposed method from a statistical point of view. Besides, if the procedure described in this study is followed, any records of the linear traverse measurement can be easily converted to the characteristic distance are different from each other.

4. Conclusions

The authors proposed a new method of evaluating air void systems based on spatial point process statistics. In this study, the usefulness of the proposed method and its consistency with the results of the conventional linear traverse measurement was discussed based on the comparison of the distance parameters. The major results obtained in this study are summarized below.

(1) The derivation of the fundamental parameters of point process statistics from the linear traverse measurement was described. It is based on the fact that the specific surfaces of air voids can be uniquely determined, regardless of the probes used for examining a distribution pattern. This procedure enables the practitioners to obtain the characteristic distance from their records of the conventional linear traverse measurement. It is also useful for verifying the proposed method by themselves.

(2) Validation of the proposed method’s capabilities was presented with a larger database. There is a strong correlation between the characteristic distance proposed and the conventional spacing factor. They are comparable to one another. This means that the size of the cubic lattice assumed hitherto for the spacing factor corresponds to the distance at the cumulative probability of 0.5 in the nearest neighbor distance distribution function of a completely random point pattern having a specific point intensity.

(3) A simple way of determining the characteristic distance was graphically shown. If necessary, the conventional spacing factor can be estimated by shifting the characteristic distance by a few tens of micrometers. Thus, the spacing factor can be estimated by counting the air voids.

(4) The comparison of the characteristic distance with the spacing factor suggests that the inferred median distance is also comparable with the spacing factor. This fact implies the possible estimation of the distance equivalent to the conventional spacing factor without specifying the volume of the cement paste and the representative diameter of air voids. This could lead to further simplification of the proposed method using the point process statistics.

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