Optimizing Vehicle Routing for Simultaneous Pick-up and Delivery considering the Reusable Transporting Containers: Case of Convenient Stores

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Abstract—A lot of previous research have proposed various frameworks and algorithms to optimize routes to reduce the total transportation cost, which accounts for over 70\% of overall logistics cost. However, it is very hard to find the cases applied the mathematical models or algorithms to the practical business environment cases, especially daily operating logistics services like convenient stores. Most of previous research have considered the developing an optimal algorithm which can solve the mathematical problem within the practical time while satisfying all constraints such as the capacity of delivery and pick-up, and time windows. For the daily pick-up and delivery service like supporting several convenient stores, it is required to consider the unit transporting container as well as the demand, capacity of trucks, traveling distance and traffic congestion. Especially, the reusable transporting container, trays, should be regarded as the important asset of logistics center. However, if the mathematical model focuses on only satisfying constraints related delivery and not considering the cost of trays, it is often to leave the empty trays on the pick-up points when there is not enough space in the track. In this research, it has been proposed to build the mathematical model for optimizing pick-up and delivery plans by extending the general vehicle routing problem of simultaneous delivery and pickup with time windows while considering left-over cost. With the numerical experiments, it has been proved that the proposed model may reduce the total delivery cost. It may be possible to apply the proposed approach to the various logistics business which uses the reusable transporting container like shipping containers, refrigerating containers, trays, and pallets.

Keywords: Vehicle Routing Problem, Delivery and Pickup, Time Windows, Left-over Cost, Reusable Container

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I. INTRODUCTION

Currently, in response to the rising cost of goods, many companies committed to minimize logistics cost through improved logistics process. Especially, because the transportation cost accounts for 70% of total cost of logistics service, many researchers and companies have developed the methodologies to find the optimal solutions and algorithms to reduce the transportation cost. In addition, due to the change of awareness in the environment and efficiency of resource and energy, it has been integrated the reverse logistics with current one focused on forwarding flow of material. The integrated logistics process can be devised at the various points from sourcing and manufacturing to transporting products along the supply chain (S. Dowlatshahi, 2000). Also, most of logistics service providers try to reduce the transportation cost by re-engineering the current silo processes to simultaneous pick-up and delivery.

However, although many researchers and companies have tried to devise the various optimal vehicle routing algorithms and solutions to reduce the transportation cost, in real-world business case which are applied these optimal algorithms or solutions are hard to find. Although it has been developed lots of optimization algorithms and solutions, there is a limitation which are difficult to apply to actual business due to the problems that occur in operational. Therefore, it can be said that proposing an optimal vehicle routing planning solution applied real problem is important. However, it is very hard to find the cases applied the mathematical models or algorithms to the practical business environment cases, especially daily operating logistics services like convenient stores. Most of previous research have considered the developing an optimal algorithm which can solve the mathematical problem within the practical time while satisfying all constraints such as the capacity of delivery and pick-up, and time windows. The mathematical model to find the optimal route for picking up and delivery at the same time (VRPSDP) (J. Potvin, J. Roussee et al, 1993) has been extended according to the constraints such as time window (VRPSDP-TW) and conditions of vehicles (R. Bent, P. Van Hentenryck 2006). For the daily pick-up and delivery service like supporting several convenient stores, it is required to consider the reusable unit transporting container as well as the capacity of trucks, traveling distance and traffic congestion. Especially, the reusable transporting containers such as trays, shipping containers, packages, should be regarded as the important assets of logistics centers. However, if the mathematical model focuses on only satisfying constraints related delivery and does not consider the cost of managing reusable assets, it is often to leave the empty assets at the pick-up points. For example, in the daily distribution service for supporting convenient stores, the empty trays are often left at the outside of the stores. In addition, the trays are sometimes broken or missing. All these events cause additional cost to re-arrange trucks to return trays, fix the broken trays, and keep more trays than expected.

In this research, it has been proposed the vehicle routing problem with simultaneous delivery and pick up under time window considering left over cost (VRPSDP-TW-LO). The mathematical model has been devised to extend the previous model VRPSDP-TW by adding more cost factors and constrains related to the left trays. And the model was applied to the small business case with the practical locations of stores and demand in terms of trays. With the numerical example of independent daily operation and consecutive operation for one month
by generating random demand, it has been compared the performance of the proposed model with various quantitative metrics. Based on the result of statistical tests, it may be possible to develop the optimal strategy for finding the vehicle for picking up and delivering the trays. Also, it is possible to reduce the total cost including transportation cost, left over cost and penalty from violating the time window constraints. The rest of this paper is organized as follows: in the following section, it has been summarized and criticized the previous research related to VRP and VRPSDP. In section III, it has been described how to build the mathematical model with the assumptions, the objective function and constraints by considering the practical business environment. In section IV, with the numerical experiment, the performance and practical applicability has been proved. Finally, it concluded with the contributions and limitations in section V.

II. LITERATURE REVIEW

It has been proved that VRP is one of the NP-hard problems by Danzing and Ramser (1959). Therefore, the early VRP models assumed that only one vehicle is assigned for each route, starting from a single depot and returning to the same depot. Burak Eksioglu et al. (2009), Kris Brackers et al. (2016) have classified various VRP models according to the nature of the problems. VRP models can be classified into the Capacitated VRP(CVRP), VRP with Time Windows (VRPTW), the multi-depot VRP(MDVRP), and the VRP with Pickup and Delivery (VRPDP).

In this study, it has been focused on VRPTW and VRP with simultaneous delivery and picking up as described in Section I. Different from other groups of VRP, in VRPTW, it assumed that the vehicles have to visit customers within the predefined time schedule (Time Windows). VRPTW models can be characterized based on how relax the time windows constraints (Vahid Baradaran, Amir Shafaei, Amir Hossein Hoseinian, 2019). If it is allowed to violate the time windows constraints by paying the penalty, this model has the soft time windows. Otherwise, the model has the hard time windows constraints. Several previous research have presented the derived models considering different conditions. Michel Gendreau et al., (1997) has developed a cross-exchange method for Tabu Search among metaheuristics to improve the performance of VRP with soft time windows (VRPSTW). A.C. Wade et al., (2002) studied the problem of vehicle routing problem with backhaul (VRPB) service after servicing the linehaul customer. They applied R-INS heuristic algorithm to collect the vehicle’s capacity-based linehaul customer service in the middle of the service. João Teixeira et al., (2004) presented a step-by-step approach heuristic method to address the periodic vehicle routing problem about the reusable waste in Colombia. This method is that the first step is to separate each area into zones, and the second step is to set the type of waste to collect and the area to visit, and the last step is to determine the route. Byung-In Kim et al., (2006) studied VRPTW under the time constraints for actual garbage collection taking into account the compactness and workload balancing. He presented solution using first clustering and an extended insertion algorithm. Katja Buhrkal et al., (2012) studied the problem of optimizing the route of the waste collection vehicle considering the rest and lunch time of the vehicle drivers, and the proposed an ALNS algorithm using clustering and actual data from the waste collection company.

Aldair Albarez et al., (2017) presented hybrid method of metaheuristic using Large Neighborhood Search (LNS) and Iterated Local Search (ILS) for VRP with time windows and multi delivery service providers (VRPTWMD). It showed better performance than branch-price-and-cut method. Galina Iassinovskaia et al., (2017) studied a
pickup and delivery inventory-routing problem within time windows (PDIRTW) in situations where each partners share returnable transport items (RTIs) in an integrated supply chain network due to environmental impact and corporate regulation. They developed a cluster first-route second metaheuristic and then it produced a good solution for realistic instance. Yangkun Xia et al., (2019) conducted a study on the open VRP with soft time window and pattern rate (OVRPSTWSR) that increase customer satisfaction in OVRP situations. They designed the embedded Tabu Search algorithm (ITSA), which showed the better result than traditional heuristics.

Different from VRPTW, it has been considered the simultaneous pick-up and delivery service when visiting customers, not the visiting time for VRPSDP which was first introduced by Min(1989). Mustafa Avci et al. (2016) considered problems with heterogenous VRP with Multiple Pickup and Delivery (HVRPSDP). They developed a HLS algorithm by applying a simulated annealing (SA) and Tabu Search (TS) to solve the problem and proposed that a strategy to relocate goods in a vehicle can reduce transportation costs for VRPSDP.

Henrietee Koch et al. (2018) studied a vehicle routing problem with multiple delivery and pickup, time windows and three-dimensioning loading(3L-VRPDPPTW) that take into account the volume of backhaul products collected when vehicle visits a line haul node. To solve the problem of 3L-VRPDPPTW, they applied the heuristic algorithm (LS_OS, LS_DBLF) and ALNS. Min J et al.(2017) studied the VRPSDP model taking into account the set-up cost, distance cost and penalty cost for violating time windows of the vehicle, they designed two step approach to solve the problem. The first step was to apply the I-GA heuristic to minimize the set-up and distance costs of the vehicle, and I-GA2 to find the global optimum through I-GA.

Most research related VRP have presented the derived models like VRPTW, VRPSDP and VRPSDPTW by addressing the several different conditions such as time constraints, violation penalties, the predefined visiting sequences, and capacity of vehicles. However, it is very hard to find the previous research which considered the reusable assets for pick-up and delivery service as the cost factors. In the practical business environment like daily delivery service for supporting convenient stores, it is very important factors to develop the optimal vehicle routing plan the capacity of trucks in terms of trays as well as the traveling distance and time. The available capacity of trucks is variable according to the demand for delivery and pick-up which determined by the sequence of visiting customers. If the model is developed by not considering the available capacity for pick-up, it cannot be avoided to leave the trays. To consider this condition, it has been presented the vehicle routing problem with simultaneously pick-up and delivery, time windows, and left-over cost (VRPSDPW-LO).
III. MODEL FOR VEHICLE ROUTING PROBLEM

A. Basic assumptions and constraints for building model

In order to build the mathematical model, it has been assumed that there are K vehicles with which N stores, noted as customers. The demand of products delivered to each customer (i) will be counted as the number of trays, \( g_i \). These trays should be delivered to the customer \( i \) within the time window \([e_i, l_i]\) for \( i = 1, ..., N \).

A vehicle delivers any stuffs requested by some customers in trays, but the number of trays are limited by \( C \), which denotes the capacity of vehicles. It has been assumed that each vehicle starts its service at the depot and returns to the depot after completing picking and delivery service. Also, each customer can be served by only one vehicle. It means that the number of trays loaded in a vehicle from the depot is equal to the sum of number of trays which should be delivered to the customers included in the route of the vehicle.

Once again, it should be noted that all stuffs will be delivered packed in the trays. Thus, it is required the enough number of empty trays at the depot. The empty trays can be collected from the customers the day after delivery. Therefore, it should be devised the optimal picking and delivery plan considering the number of trays delivered and will be delivered at the same time. As the maximum capacity of vehicles are fixed to \( C \), some of delivered trays may be left at the customer if there is not enough space in the vehicle. The number of trays in a vehicle at any time instant consists of empty and non-empty trays, which is less than or equal to \( C \). The picked-up trays are returned to the depot and the total number of picked-up trays by vehicles should be more than a lower bound, \( L \).

In this paper, there are two objectives; (1) a sequence of customers that a vehicle visits and (2) the number of empty trays picked up from each customer when the vehicle visits. These objectives can be merged into a single problem to minimize the total cost which consists with travelling cost for visiting customers, left-over cost which comes from the empty trays not returned from the customers, and penalty cost from the early or late arrivals of vehicles. This problem of vehicle routing can be denoted as VRPSDP-TW-LO which means the vehicle routing problem in which pickup and delivery are simultaneously considered with the time window and left-over cost.

B. Mathematical Model for VRPSDP-TW-LO

In this section, it has been devised the mixed integer programming (MIP) model for VRPSDP-TW-LO. Table 1 summarizes the notations used in the MIP formulation.

| Notation | Description |
|----------|-------------|
| \( N \)  | Set of customers (0th customer represents the depot) |
| \( K \)  | Set of vehicles |
| \( C \)  | Set of capacities of vehicles |
| \( T \)  | Set of maximum travel time of vehicles |
| \( c_{ij} \) | Travelling cost from customer \( i \) to \( j \) |
| \( p_i \) | Number of empty trays at customer \( i \) from the previous planning horizon |
| \( g_i \) | Number of non-empty trays requested for customer \( i \) |
| \([e_i, l_i]\) | Time window for customer \( i \) |
| $s_i$  | Service time for customer $i$ |
|-------|-----------------------------|
| $t_{ij}$ | Travel time from customer $i$ to $j$ |
| $C_{lo}$ | Penalty cost per left-over tray |
| $C_e$ | Penalty cost for violating time window to arrive earlier |
| $C_l$ | Penalty cost for violating time window to arrive late |
| $M$ | Large number |

Using the flow-arc formulation [5], vehicle routing problem can be described as follows. Let $G = (V,A)$ be a graph where $V = (v_0,\cdots,v_{n+1})$ is the set of vertices and $A = \{(v_i,v_j)\mid i \neq j \land i,j \in V\}$ is the set of arcs. Note that both $v_0$ and $v_{n+1}$ denote the depot. Thus, any tour which starts and ends at the depot can be represented as a path from $v_0$ to $v_{n+1}$. Each vertex $v_i$ in $V$ has an associated demand $d_i \geq 0$, a pickup quantity $p_i \geq 0$ and a time window $[e_i,l_i]$. Especially, the depot has $g_0 = g_{n+1} = 0$, $p_0 = p_{n+1} = 0$, and $[\infty,\infty]$. For notational convenience, we introduce the set of customers, $N = \{v_1,\cdots,v_n\}$. Each arc $(v_i,v_j)$ has an associated cost $c_{ij} \geq 0$ and travel time $t_{ij} \geq 0$.

The decision variables for the proposed model are defined as follows:

- $x_{ij}^k$: the binary variable whose value is 1 if a vehicle $k$ travel from customer $i$ to $j$ and 0 otherwise; 
- $f_k^i$: the number of empty trays loaded on the vehicle $k$ at customer $i$;
- $y_k^i$: the number of trays (both empty and non-empty trays) a vehicle $k$ loads when it departs from customer $i$;
- $t_k^i$: the time at which a vehicle $k$ departs from customer $i$;
- $ze_k^i$: the binary variable whose value is 1 if a vehicle $k$ arrives at customer $i$ earlier than $e_i$ and 0 otherwise;
- $zt_k^i$: the binary variable whose value is 1 if a vehicle $k$ arrives at customer $i$ later than $l_i$ and 0 otherwise;
- $zw_k^i$: the binary variable whose value is 1 if a vehicle $k$ arrives at customer $i$ within the time window $[e_i,l_i]$ and 0 otherwise.

The objective function is defined as follows:

$$
\text{minimize} \quad \sum_{k=1}^{K} \sum_{i=0}^{N} \sum_{j=0}^{N+1} c_{ij} x_{ij}^k + C_{lo} \sum_{k=1}^{K} \sum_{i=1}^{N} (p_i - f_k^i) + M \sum_{k=1}^{K} \sum_{i=1}^{N} (C_e z_e^k + C_l z_l^k)
$$

A feasible solution of the proposed model satisfies the following constraints.

1. **Path constraints:** For each vehicle, a tour which starts and ends at the depot can be represented as a path from $v_0$ to $v_{n+1}$. The feasible movements of vehicles satisfy the following three set of constraints:

   $$
   \sum_{j=1}^{N+1} x_{0j}^k = \sum_{i=0}^{N} x_{iN+1}^k = 1, \forall k = 1, \ldots, K
   $$

   $$
   \sum_{i=0}^{N} x_{ih}^k - \sum_{j=0}^{N+1} x_{kj}^h = 0, \forall h \in \{1, \cdots, N\}, \forall k = 1, \ldots, K
   $$

2. **Disjoint path constraints:** By the assumption that each customer is served by exactly one customer, the feasible movements of vehicles must be disjoint paths in the underlying graph.
\[ \sum_{k=1}^{K} \sum_{i=1}^{N} x_{ij}^k = 1, \forall j = 1, ..., N \]

3) Tray capacity constraints: At any instant of times, the total number of trays in a vehicle cannot exceed the capacity.

\[
y_j^k - y_i^k = f_j^k - g_jx_{ij}^k, \forall (v_i, v_j) \in A, k = 1, ..., K
\]

\[
y_j^k \leq C, i = 0, 1, ..., N, k = 1, ..., K
\]

\[
y_0^k = \sum_{i=0}^{N+1} \sum_{j=0}^{N+1} g_ix_{ij}^k, k = 1, ..., K
\]

4) Loading empty-tray constraints: The number of empty trays a vehicle which visit a customer can load is limited.

\[
0 \leq f_i^k \leq p_i \sum_{j=0}^{N+1} x_{ij}^k, \forall i = 1, ..., N, k \in K
\]

5) Departure time constraints: The time at which a vehicle departs from a customer satisfies the following set of constraints.

\[
s_j + t_{ij}^k + M(x_{ij}^k - 1) \leq t_j^k - t_i^k, \forall i, j = 1, ..., N, k = 1, ..., K, i \neq j
\]

\[
t_0^k = 0, k = 1, ..., K
\]

\[
t_{k+1}^k \leq T \sum_{i=0}^{N} x_{iN+1}^k, k = 1, ..., K
\]

6) Early and late arrival constraints: According to the set of customers and sequence of them a vehicle choose, there are three possibilities: early arrival, late arrival, or on-time arrival.

\[
t_i^k - s_j \leq e_j^kze_j^k + T(ze_j^k + zl_j^k), \forall j = 1, ..., N, k = 1, ..., K
\]

\[
-T(ze_j^k + zl_j^k) + e_j^kzw_j^k \leq t_j^k - s_j \leq l_j^kzw_j^k + T(ze_j^k + zl_j^k), \forall j = 1, ..., N, k = 1, ..., K
\]

\[
l_j^kzl_j^k \leq t_j^k - s_j, \forall j = 1, ..., N, k = 1, ..., K
\]

\[
ze_j^k + zw_j^k + zl_j^k = 1, j = 1, ..., N, k = 1, ..., K
\]

7) Binary variable constraints:

\[
x_{ij}^k \in \{0, 1\}, \forall i, j = 1, ..., N, i \neq j, k = 1, ..., K
\]

\[
ze_j^k, zw_j^k, zl_j^k \in \{0, 1\}, \forall j = 1, ..., N, k = 1, ..., K
\]

\[
f_j^k \in \mathbb{Z}^+, \forall j = 1, ..., N, k = 1, ..., K
\]
IV. COMPUTATIONAL EXPERIMENTS

At first, the proposed model, VRPSPD-TW-LO, has been verified with the small instances by checking feasibility and practical profitability based on the statistical test. All of the computational experiments are executed on a personal desktop computer (Intel i7-7000 3.60GHz, 16GB) and the solutions of VRPSDP-TW-LO and VRPSDP-TW are obtained by Gurobi 8.1.

A. Instance generation

For the statistical test, all parameters are assumed based on the case study of delivery service for convenient stores as follows:

- A set of customers, \( n = 11 \): \( i, j \in \{0, ..., 10\} \);
- A set of vehicles, \( v = 3 \): \( v \in \{1, 2, 3\} \);
- Capacity of all vehicles has a capacity \( Q = 25 \);
- Average speed of a vehicle \( s = 60 \text{km/h} \);
- Service time at each customer: 10min.;
- Penalty cost for violating time windows both early arrival and late arrival: \( \text{₩}1,000 \);
- Penalty cost for left over tray: \( \text{₩}500 \);

For the simple numerical examples, a new set of spatial coordinates for each customer is randomly generated as integers in a square of 100 units, and the location of depot is set at (0, 0) in the center of the square. The distances between all visiting points including the depot and customers is calculated with Haversine method because the well-known method such as Euclidean and Manhattan distance have limitations in calculating the actual distance based on the actual latitude and longitude coordinates. The driving speed of vehicles speed is set as 60km/hr. Also, it has been assumed the fuel mileage as 6km/l, and unit oil price as 1,000KRW.

The demands of customer including picking and delivery are randomly generated between 1 and 8 trays. It has been assumed that the number of trays follows the uniform distribution. The time window constraint for visiting each customer is determined by considering the number of visiting nodes on the path combined with the arrival time of the previous node, 30 minutes for operation like delivery and picking up. If the vehicle could not meet the predefined time windows, the penalty, 1,000 KRW will be charged. Also, if the vehicle does not pick up some tray previously delivered at a certain node, the penalty for left-over tray, 500KRW per tray in a day, will be charged. To get rid of the effect from the randomness, the experiment was repeated 30 times.

B. Result of small instances

Based on the independent 30 instances, the performance of proposed models, VRPSDP-TW-LO, was evaluated by comparing with VRPSDP-TW which does not consider the left-over cost. Following Table 2 shows the result of each instance of the VRPSDP-TW and the VRPSDP-TW-LO models. The performance was evaluated with the metrics such total cost, total travelling distance, number of time window penalty, and number of left-over trays. At first, the result shows that the average total cost of VRPSDP-TW is higher than VRPSDP-TW-
LO as much as 2.9%. It can be found that there exists the trade-off among total travelling distance, the number of time window penalty and number of left-over trays.

### TABLE 1. Comparisons of VRPSDP-LO vs VRPSDP-TW

| Instance | Total Cost | Total Travelling distance | Time window violations | Num. of left-over trays | Total Cost | Travelling distance | Time window violations | Num. of left-over trays | Gap of Total Cost (%) |
|----------|------------|---------------------------|------------------------|-------------------------|------------|---------------------|------------------------|-------------------------|------------------------|
| 1        | 69,934.7   | 368.6                     | 3                      | 5                       | 71,434.7   | 368.6               | 3                      | 8                       | 2.1                    |
| 2        | 69,160.6   | 367.0                     | 3                      | 4                       | 69,160.6   | 367.0               | 3                      | 4                       | -                      |
| 3        | 70,414.4   | 365.5                     | 4                      | 5                       | 70,414.4   | 365.5               | 4                      | 5                       | -                      |
| 4        | 70,033.3   | 369.2                     | 4                      | 5                       | 70,033.3   | 369.2               | 4                      | 5                       | -                      |
| 5        | 70,135.0   | 369.8                     | 4                      | 3                       | 70,672.7   | 367.0               | 3                      | 7                       | 0.8                    |
| 6        | 76,032.8   | 387.2                     | 5                      | 7                       | 77,532.8   | 387.2               | 4                      | 12                      | 1.9                    |
| 7        | 72,108.6   | 357.7                     | 4                      | 11                      | 72,259.1   | 346.6               | 5                      | 13                      | 0.2                    |
| 8        | 74,144.3   | 384.9                     | 5                      | 6                       | 74,144.3   | 384.9               | 5                      | 6                       | -                      |
| 9        | 83,500.2   | 426.0                     | 7                      | 5                       | 87,000.2   | 426.0               | 5                      | 16                      | 4.0                    |
| 10       | 64,927.3   | 341.6                     | 5                      | 2                       | 64,927.3   | 341.6               | 5                      | 2                       | -                      |
| 11       | 73,347.7   | 380.1                     | 5                      | 4                       | 104,998.1  | 561.0               | 5                      | 7                       | 30.1                   |
| 12       | 73,179.5   | 361.1                     | 3                      | 14                      | 73,179.5   | 361.1               | 3                      | 14                      | -                      |
| 13       | 73,960.5   | 383.8                     | 3                      | 8                       | 75,694.8   | 376.2               | 2                      | 16                      | 2.3                    |
| 14       | 76,232.6   | 391.4                     | 7                      | 2                       | 76,732.6   | 391.4               | 6                      | 5                       | 0.7                    |
| 15       | 62,101.1   | 318.6                     | 4                      | 4                       | 63,149.7   | 300.9               | 4                      | 12                      | 1.7                    |
| 16       | 64,719.9   | 340.3                     | 2                      | 6                       | 64,915.3   | 341.5               | 1                      | 8                       | 0.3                    |
| 17       | 82,085.0   | 420.5                     | 6                      | 6                       | 85,274.0   | 412.6               | 5                      | 17                      | 3.7                    |
| 18       | 73,295.3   | 376.8                     | 5                      | 7                       | 74,374.4   | 371.2               | 4                      | 13                      | 1.5                    |
| 19       | 56,807.1   | 295.8                     | 1                      | 7                       | 56,807.1   | 295.8               | 1                      | 7                       | -                      |
| 20       | 67,720.1   | 343.3                     | 5                      | 5                       | 69,808.1   | 340.8               | 4                      | 12                      | 3.0                    |
| 21       | 71,968.8   | 374.8                     | 4                      | 5                       | 73,555.1   | 369.3               | 3                      | 12                      | 2.2                    |
| 22       | 76,089.9   | 369.5                     | 4                      | 15                      | 76,089.9   | 369.5               | 4                      | 15                      | -                      |
| 23       | 73,935.9   | 368.6                     | 5                      | 9                       | 75,935.9   | 368.6               | 4                      | 15                      | 2.6                    |
| 24       | 68,407.1   | 344.4                     | 3                      | 10                      | 68,426.8   | 335.6               | 4                      | 11                      | 0.03                   |
| 25       | 62,184.4   | 319.1                     | 5                      | 2                       | 64,684.4   | 319.1               | 4                      | 9                       | 3.9                    |
| 26       | 74,342.3   | 365.1                     | 6                      | 11                      | 74,342.3   | 365.1               | 6                      | 11                      | -                      |
| 27       | 73,391.2   | 365.3                     | 6                      | 7                       | 75,342.0   | 365.1               | 5                      | 13                      | 2.6                    |
| 28       | 72,930.5   | 368.6                     | 6                      | 5                       | 79,930.5   | 368.6               | 4                      | 23                      | 8.8                    |
| 29       | 84,030.9   | 441.2                     | 5                      | 5                       | 88,894.0   | 428.4               | 6                      | 17                      | 5.5                    |
| 30       | 77,018.2   | 405.1                     | 3                      | 7                       | 81,795.5   | 391.8               | 4                      | 19                      | 5.8                    |
| Avg.     | 71,938.0   | 369.0                     | 4.4                    | 6.4                      | 74,383.6   | 371.9               | 4.0                    | 11.1                     | 2.8                   |

Following Figure 1 and 2 show the details of the difference in the number of time window violations and number of left-over trays.
Generally, the total travelling distance of VRPSDP-TW-LO model longer than the VRPSDP-TW model because the blue dotted line located under the zero. Also, it is possible to find the number of time-window violations (KS1) is bigger than the VRPSDP-TW. However, the total number of left-over trays is lower than the VRPSDP-TW because it is included in the objective function. In order to secure the statistical validity, it has been conducted the paired t-test, which shows the statistically significant difference of the total cost between VRPSDP-TW and VRPSDP-TW-LO. Table 2 show pairs mean difference for instances and p value. The mean difference of VRPSDP-TW and VRPSDP-TW-LO is -2,445.7 with p value 0.014. This indicates that VRPSDP-TW-LO is statistically significantly different and can show better solutions.

Table 2. Result of paired t-test

| Model                  | Mean Difference | p-value |
|------------------------|-----------------|---------|
| VRPSDP-TW-LO - VRPSDP-TW | -2,445.7        | 0.014   |
Therefore, it can be said that VRPSDP-TW-LO may guarantee the better solution than VRPSDP-TW, which can accept the alternative hypothesis, $H_1$, given below:

$$H_1: \mu_{\text{VRPSDP-TW-LO}} - \mu_{\text{VRPSDP-TW}} < 0$$

V. CASE STUDY FROM PRACTICAL BUSINESS

In this section, it has been described the result of validity and efficiency of the proposed VRPSDP-TW-LO model based on the practical distribution business of Company A. Company A is currently supporting the convenience stores in a region of Pusan, Korea. A vehicle visits average 30 stores a day. This company has the similar problem due to the left-over trays which could not picked up by the vehicle. They get a lot of complaints from the store, cost from the missed or damaged trays, and usually dispatches the additional vehicles to pick up the left-over trays. To make the experiments simple without loss of generality, it has been assumed that the capacity of vehicles is 30 trays, total number of visiting nodes is 12, the maximum operating time is 240 minutes (6hrs). The daily demand of delivery to each node is randomly generated from the uniform distribution, 1 to 8 trays. From the actual location of convenient stores, the distances between visiting nodes are calculated considering the actual travelling route using ArcGIS. The demand of picking up is determined as the number of trays which are previously delivered and left over, not picked up yet. To analyze the cumulative effect of daily operations, it has been analyzed the performance of consecutive 30 days. For the first day, it was set that there exists no left-over tray.

Following Figure 3 shows the one example of 30 daily operations. For both VRPSDP-TW and VRPSDP-TW-LO, all 12 stores in a same region are supported by three vehicles. The routes for vehicles are different. The results of daily operations for 30 days are acquired based on these different vehicle routes.

![Figure 3](image-url)
Following Figure 4 shows the daily total operating cost including travelling, penalty for violating time windows, and penalty for left-over trays for 30 days.

![Figure 4. Daily total operating cost of VRPSDP-TW-LO and VRPSDP-TW](image)

The difference between two models in terms of total cost on each day are not significant during the initial 10 days, not greater than 14.9%. However, it gets gradually increasing after 10 days. Eventually, the difference of total cost became 61.8% on the final 30th day. It can be inferred that the cost difference arises from the number of left-over trays. Figure 5 shows the number of time window violations (a), and number of left-over trays (b). Also, Figure 6 shows the result of integrated comparison of difference of travelling distance, number of time window violations, and number of left-over trays. Whereas the number of time window violations is not significantly different, the difference of number of left-over trays gets greatly bigger as time goes on. In addition, the total travelling distance does not show the significant difference between two models. Because there exists the limitation of trays which each distribution center can manage, the additional vehicles should be arranged to only pick up the left-over trays. It causes additional cost and several issues for keeping and managing trays.

![Figure 5 the comparison of a case study of time penalty and left-over tray](image)
From the result, it can be clearly concluded that the proposed VRPSDP-TW-LO model may improve efficiency of the service operation and reduce the total cost by considering the left-over cost which was not included in previous research. For the example of distributing service supporting convenience stores in this case study, it is possible to secure the significant benefit for daily operations by reducing the cost from left-over trays, improve the service for stores. The proposed model can be applied to not only the distribution service for supporting convenience stores, but also to other services using unit containers such as grocery stores, pharmacies, and marine container shipping business.

VI. CONCLUSION

It becomes the most important function the efficient logistics service for securing the competitive advantages. In particular, the transportation, which accounts for the largest cost in logistics activities, is critical for enhancing the service level and saving the cost. However, current Transport Management System (TMS) focuses on designing the optimal route which minimize the objective function consisted with different factors such as travelling distance, required vehicles and drivers while meeting several different constraints like the maximum capacities, time windows to visit, and demand for delivery and picking up. A lot of previous research presented very advanced models, methods, and algorithms to solve the mathematical problem related the VRP. Still, it is not easy to find the previous research which considers the valuable assets for transportation service like containers and trays except vehicles and drivers, even though these assets are very critical to enhance the service level and causes a great deal of cost.

In this research, it has been devised a novel mathematical problem, VRPSDP-TW-LO, which considers the cost from the left-over trays, travelling distance, and time-window penalty, at the same time. This model may reflect the practical circumstances of daily distributing services by relaxing some constraints of traditional VRPSDP or VRPSDP-TW and adding more factors.
With the small and simple numerical experiments and statistical test, it has been verified the practical applicability of the proposed model by comparing with the various performance metrics such as travelling distance, number of time-window penalty, and number of left-over trays, not only the total cost. From the result, it has been shown that the proposed model may save more total cost as much as 2.9% while comparing with VRPSDP-TW model. In addition, it has been evaluated the performance based on the practical business environment for 30 days to check the cumulative effect of daily operations. Although considering the trade-off of cost from the time window penalty and left-over trays, it is very clear that the proposed model, VRPSDP-TW-LO, may guarantee the better service level as well as the reduce the cost. The proposed model may be applied to the various industries and services in which it is regarded as very important reusable assets such as unit containers and trays.

Still, this research has some limitations to be applied to real practical business environment. It has not considered the external environmental factors like the traffic conditions which have great impacts on the arrival time to the customer and transportation cost. Also, the number of left-over trays could not be deterministic because the trays may be missed or damaged. Therefore, in the further research, it should be considered the constraints of external environmental factors and stochastic variables and parameters which are out of scope of this research. In addition, in this research, the proposed model was verified with only small numerical experiments. Thus, to apply the proposed model to the real business environment and find the optimal, it is required to design heuristic algorithms like Tabu search, genetic algorithm or swarm optimization approach to solve the bigger problem with larger number of delivery points.

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