General Relativity as Geometro-Hydrodynamics *

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Abstract

In the spirit of Sakharov's 'metric elasticity' proposal [1], we draw a loose analogy between general relativity and the hydrodynamic state of a quantum gas, and examine the various conditions which underlie the transition from some candidate theory of quantum gravity to general relativity, specifically, the long wavelength, low energy (infrared) limits, the quantum to classical transition, the discrete to continuum limit, and the emergence of a macroscopic collective state from the microscopic constituents and interactions of spacetime and fields. In the 'top-down' approach, we mention how general relativity arises as various limits are taken in some popular candidate theories of quantum gravity, such as string theory, quantum geometry via the Ashtekar variables, and simplicial quantum gravity. Our emphasis here is more on the 'bottom-up' approach, where one starts with the semiclassical theory of gravity and examines how it is modified by graviton and quantum field excitations near and above the Planck scale.

We mention three aspects based on our recent findings: 1) Emergence of stochastic behavior of spacetime and matter fields depicted by an Einstein-Langevin equation. The backreaction of quantum fields on the classical background spacetime manifests as a fluctuation-dissipation relation. 2) Manifestation of stochastic behavior in effective theories below the threshold arising from excitations above. The implication for general relativity is that such Planckian effects, though exponentially suppressed, is in principle detectable at sub-Planckian energies. 3) Decoherence of correlation histories and quantum to classical transition. From Gell-Mann and Hartle's observation that the hydrodynamic variables which obey conservation laws are most readily decohered, one can, in the spirit of Wheeler, view the conserved Bianchi identity obeyed by the Einstein tensor as an indication that general relativity is a hydrodynamic theory of geometry. Many outstanding issues surrounding the transition to general relativity are of a nature similar to hydrodynamics and mesoscopic physics.

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The abstract above outlines the philosophy and emphasis of our current investigation. Details of the three main directions are contained in the following references:

1) Einstein-Langevin equations and backreation in semiclassical gravity as fluctuation-dissipation relation: [2, 3, 4, 5] (see also [6])

2) Stochastic behavior of effective field theory across the threshold. [7, 8, 9]

3) Decoherent correlation history and general relativity as geometro-hydrodynamics. It is based on the ideas of Gell-Mann and Hartle [10, 11] (expanded in [12, 13]) and Calzetta and Hu [14, 15] (detailed in [16]).

Because of limited space, I will focus on only the last of these three directions mentioned above.

Let me begin with the bigger picture of where we stand in understanding the relation between the known classical theory of general relativity, semiclassical gravity (where quantum fields are considered with classical background spacetime) [17], and the unknown but increasingly revealing domain of quantum gravity. Generally speaking, one can group all approaches into two categories:

0.1 ‘Top-down’: How to reach the correct limits

The transitional aspects mentioned in the abstract, i.e., quantum to classical transition, low energy, long wavelength (infrared) limits, discrete to continuum limit, extended structure to point structure, and micro/constituents versus macro/collective states, manifest in varying degrees of transparency in three leading types of candidate theories of quantum gravity: the superstring theory [18], the loop representation of quantum geometry via spin connections [19], and simplicial quantum gravity [20]. In string theory, a spin-two particle is contained in the string excitations, and it is easy to see the limit taken from an extended structure such as a p-brane to a point. The Bekenstein-Hawking expression for the black hole entropy [21] is obtained as the tree level result of many quantum theories of gravity [22]. But in the construction of a statistical mechanical entropy [24], it is not so clear which of the many internal degrees of freedom of string excitations contribute to the leading quantum correction term. It is likely to be some collective excitation state which dominate at low energy, like the collective degrees of freedom depicting the vibrational and rotational motion of a nucleus, which, though intuitively clear from low energy, are not so easily constructable from nucleon wavefunctions. The larger problem of how the target space (e.g., spacetime of 26 dimensions for bosonic string) can be deduced from, or at least treated on the same footing as, the world-volume of fundamental p-branes, still remains elusive. In the quantum relativity approach using Ashtekar’s spin connection and Rovelli-Smolin’s loop representation, the picture of a one-dimensional quantum weave behaving like a polymer is evoked [25]. When viewed at a larger scale the weaves appear to ‘knit’ a higher dimensional spacetime structure. This is an interesting picture, but how this collective process comes about – i.e., how the physical spacetime becomes a dynamically preferred entity and an infrared stable structure – remains to be explicated (cf. protein-folding?). In simplicial quantum gravity [20], the classical limit might be obtained more easily in some versions (e.g., in the Ponsano-Regge 6j calculus [26],

\[1\] Jacobson [23] has used the thermodynamic expression for black hole entropy to show how Einstein’s equation can be derived as a thermodynamic equation of state. The underlying philosophy of this view is similar to ours.
it is quite similar to the treatment of ordinary spin systems via group-theoretical means, in place of the more involved considerations of environment-induced decoherence\(^2\), but essential properties like diffeomorphism invariance in the continuum limit are not guaranteed, such as in Regge calculus. Dynamical triangulation procedure\(^2\) was believed to work nicely in these respects. But recently it was reported\(^2\) that a first order transition may arise which destroys the long wavelength niceties.\(^3\)

Many structural aspects of these theories in their asymptotic regimes (defined by the above-mentioned limits) near the Planck scale bear sufficient resemblance to the physics in the atomic and nuclear scales that I think it is useful to examine the underlying issues in the light of these better-understood and well-tested theories. These include on the one hand theories of ‘fundamental’ interactions and constituents, such as quantum electrodynamics (QED), quantum chromodynamics (QCD), supersymmetry (SUSY), and grand unified theories (GUT) which are indeed what piloted many of today’s candidate theories of quantum gravity, and on the other hand theories about how these interactions and constituents manifest in a collective setting – theories traditionally discussed in condensed matter physics using methods of statistical mechanics and many-body theories. These two aspects are not disjoint, but are interlinked in any realistic description of nature (see\(^3\)). They should be addressed together in the search for a new theory describing matter and spacetime at a deeper level. The collective state description has not been emphasized as much as the fundamental interaction description. We call attention to its relevance because especially in this stage of development of candidate theories of quantum gravity, deducing their behavior and testing their consequences at low energy constitute an important discriminant of their viability. Low energy particle spectrum and black hole entropy are among the currently pursued topics.

Take, for example, the interesting observations related above, that four-dimensional spacetime is an apparent (as observed at low energy) rather than a ‘real’ (at Planck energy scale) entity (this is also highlighted in Susskind’s\(^3\) world as hologram and ‘t Hooft’s\(^3\) view of the string theoretical basis of black hole dynamics and thermodynamics, or the relation between the weave and the cloth, in Ashtekar’s program). General relativity could be an emergent theory in some ‘macroscopic’, averaged sense at the low energy, long wavelength limit. The fact that fundamental constituents manifest very different features at lower energies is not such a surprising fact, they are encountered in almost all levels of structure – molecules from atoms, nuclei from quarks. These are what I have referred to categorically as ‘collective states’. How relevant and useful these variables or states are depend critically on the scale and nature of the physics one wants to probe. One cannot say that one is better than the other without stipulating the range of energy in question, the nature of the probe or the precision of the measurement. Just as thermodynamic variables are powerful and economical in the description of long wavelength processes, they are completely useless at molecular scales. Even in molecular kinetic theory, different variables (distribution and correlation functions) are needed for different ranges of interactions. In treating the relation of quantum gravity to general relativity it is useful to bear in mind these general features

\(^2\)I will not attempt to quote standard references in each of these areas, as there are too many and fast-accumulating. The reader can browse through this proceeding and get a fairly good picture of the current status.
we learned from more familiar processes.

Even when one is given the correct theory of the constituents, it is not always an easy task to construct the appropriate collective variables for the description of the relevant physics at a stipulated scale. Note, e.g., how different the dynamical variables of the nuclear collective models are from the independent particle model. Not only are the derived structures different from their constituents, their effective interactions can also be of a different nature. There used to be a belief (myth) that once one has the fundamental theory it is only a matter of details to work out an effective theory for its lower-energy counterparts. Notice how nontrivial it is to deduce the nuclear force from quark-gluon interactions, despite our firm knowledge that QCD is the progenitor theory of nucleons and nuclear forces. Also, no one has been clever enough to have derived, say, elasticity from QED yet. Even if it is possible to introduce the approximations to derive it, we know it is plain foolish to carry out such a calculation, because at sufficiently low energy, one can comfortably use the stress and strain variables for the description of elasticity. (Little wonder quantum mechanics, let alone QED, is not a required course in mechanical engineering!)

0.2 ‘Bottom-Up’: Tell-tale signs from low energy

How the low energy behavior of a theory is related to its high energy behavior (issues of effective decoupling and renormalizability naturally would arise [35]), whether one can decipher traces of its high energy interactions or remnants of its high energy components, have been the central task of physics since the discovery of atoms in the last century and subatomic particles in this century to today’s attack on unified theories at ultrahigh energy. The symmetry of the particles and interactions existing at low energies are the raw data we need to construct (and rate the degree of success of) a new unified theory. (Such is the central mission of e.g., string phenomenology.) Some salient features of general relativity such as diffeomorphism invariance, Minkowsky spacetime as a stable low energy limit, etc., are necessary conditions for any quantum theory of gravity to meet. Approaching Planck energy from below, the beautifully simple yet deep theory of black hole thermodynamics [21] based on semiclassical gravity, has, and still is, providing a checkpoint for viable quantum gravity theories – the task now is to obtain a microscopic (statistical mechanical) description of black hole entropy beyond the tree-level. I would like to supplement these two ongoing efforts (i.e., low energy particle spectrum and black hole entropy) by proposing some new directions, related to the existence of stochastic and fluctuation effects at the cross-over regime.

0.2.1 fluctuations and noise at the threshold

An important feature of physics at the Planck scale depicted by semiclassical gravity is the backreaction of quantum effects of particles and fields, such as vacuum polarization and particle creation, on the classical gravitational spacetime. This is an essential step beyond classical relativity for the linkage with quantum gravity. For example, generalization to the $R + R^2$ theory of gravity is a necessary product from the renormalization considerations of quantum field theory in curved spacetimes. It should also be the low energy form of string theory (plus dilaton and antisymmetric fields). Backreaction demands more, in that the
quantum matter field is solved consistently with the classical gravitational field \[36\]. The consistency requirement in a backreaction calculation brings in two new aspects:

1) The classical gravitational field obeys a dynamics which contains a dissipation component arising from the backreaction of particle creation in the quantum field. The dissipation effect is in general nonlocal, as it is influenced by particle creation not only occurring at one moment, but also integrated over the entire history of this process \[37\].

2) Creation of particles in the quantum matter field at the Planck energy (which is responsible for the dissipative dynamics of the gravitational field) can be depicted as a source which has both a deterministic and a stochastic component. The first part is the averaged energy density of created particles, which is known in previous treatments. The second part is new – it measures the difference of the amount of particles created in two neighboring histories and is depicted by a nonlocal kernel, the correlator of a colored noise \[3, 38\]. The dissipation and noise kernels are related by a fluctuation-dissipation relation which is a consequence of the unitarity condition of the original closed (gravitation + matter field) system.

These processes are captured nicely by way of the influence functional \[39\], which is structurally equivalent to the Schwinger-Keldysh (closed-time-path or in-in) effective action \[10\]. It also elicits clearly the statistical mechanical meaning of these quantum processes. (For a review of these recent findings in the backreaction problems of semiclassical gravity, see, e.g., \[11]\.) The backreaction equation is in the form of a Langevin equation, which we call the Einstein-Langevin equation \[5, 4\]. The stochastic source term signifying Planckian quantum field processes integrates to zero when one takes the ensemble average with respect to the noise, reproducing the traditional semiclassical Einstein equation. It is in the sense that we call general relativity a mean field theory. The Einstein-Langevin equation constitutes a new frontier for us to explore possible phase transition and vacuum instability issues, which we believe many of the ‘top-down’ approaches would also encounter in this cross-over regime.

0.2.2 stochastic behavior below the threshold

What are the tell-tale signs for a low energy observer of the existence of a high energy sector in the context of an effective field theory? We wish to adopt an open system viewpoint to effective theory and explore its statistical mechanical properties. The question is to compare the difference between a theory operative, i.e., giving an adequate description at low energies (an open system, with the high energy sector acting as the environment) to that of an exact low energy theory by itself taken as a closed system. We know that there are subtle differences between the two, arising from the backreaction of the heavy on the light sector. Though not obvious, the stochastic behavior associated with particle creation above the threshold (which for gravitational processes is the Planck energy) is related to the dissipative behavior of the background spacetime dynamics. This was known for some time \[2\]. Schwinger’s result \[12\] for pair production in a strong electromagnetic field is well-known. This effect at very low energy has however been ignored, as it is usually regarded as background noise covered by very soft photons. That such a noise carries information about the field at high energy was pointed out only recently. Using a simple interacting field model, Calzetta and I found \[7\] that even at energy way below the threshold, stochastic effects, albeit at extremely small amplitudes, can reveal some general (certainly not the specifics) properties of the high energy sector. We won’t have space to discuss the details, but refer the reader to \[7\] and
earlier references therein. We now turn to the decoherence aspects of quantum theories and their classical limits to show why general relativity can be viewed as the hydrodynamic limit of quantum gravity.

1 Decoherence of Correlation History

(This section contains a summary of work in [14, 15].) The basic tenet of the consistent histories approach to quantum mechanics [43] is that quantum evolution may be considered as the result of the coherent superposition of virtual fine-grained histories, each carrying full information on the state of the system at any given time. If we adopt the ‘natural’ procedure of specifying a fine-grained history by defining the value of the field \( \Phi (x) \) at every space time point, these field values being c numbers, then the quantum mechanical amplitude for a given history is \( \Psi[\Phi] \sim e^{iS[\Phi]} \), where \( S \) is the classical action evaluated at that particular history. The virtual nature of these histories is manifested through the occurrence of interference phenomena between pairs of histories. The strength of these effects is measured by the ‘decoherence functional’

\[
D_F[\Phi, \Phi'] \sim \Psi[\Phi]\Psi[\Phi']^* \sim e^{i(S[\Phi] - S[\Phi'])}
\]

In reality, actual observations in a classical world correspond to ‘coarse-grained’ histories. A coarse-grained history is defined in terms of a ‘filter function’ \( \alpha \), which determines which fine-grained histories belong to the superposition, and their relative phases. \( \Box \) The quantum mechanical amplitude for the coarse-grained history is defined as

\[
\Psi[\alpha] = \int D\Phi e^{iS[\Phi]} \alpha[\Phi]
\]

where the information on the quantum state of the field is assumed to have been included in the measure and/or the boundary conditions for the functional integral. The decoherence functional for two coarse-grained histories is \( \Box \)

\[
D_F[\alpha, \alpha'] = \int D\Phi D\Phi' e^{i(S[\Phi] - S[\Phi'])} \alpha[\Phi]\alpha'[\Phi']^*
\]

In this path integral expression, the two histories \( \Phi \) and \( \Phi' \) are not independent; they assume identical values on a \( t = T = \) constant surface in the far future. These are the same boundary conditions satisfied by the histories on each branch of the time path in the closed-time-path (CTP) formalism \( \Box \). There, one considers the specified kernels as products of fields defined on a closed time-path. As such, one may define up to four different kernels \( G^{ab} (a, b = 1, 2 \text{ are the CTP indices}) \) independently, to be identified with the four different possible orderings of the fields. If the kernels \( G^{ab} \) can actually be decomposed as products of c-number fields on the CTP, then we associate to them the quantum amplitude for correlation histories (of second order here) as

\[
\Psi[G^{ab}] = \int D\Phi^a e^{iS} \prod_{x \geq x', ab} \delta(\Phi^a(x)\Phi^b(x') - G^{ab}(x, x'))
\]

\( ^{3} \)For example, we may define a coarse-grained history of a system with two degrees of freedom \( x \) and \( y \) by specifying the values \( x_0(t) \) of \( x \) at all times. Then the filter function is \( \alpha[x, y] = \prod_{t \in R} \delta(x(t) - x_0(t)) \).
where $S$ stands for the CTP classical action. The path integral can be manipulated to yield

$$\Psi[G^{ab}] \sim \det\left\{ \frac{\partial^2 \Gamma}{\partial G^{ab} \partial G^{cd}} \right\}^{1/2} e^{i \Gamma[G^{ab}]}$$

(1.5)

where $\Gamma$ stands for the closed-time path two-particle irreducible (CTP 2PI) effective action. This last expression can be analytically extended to more general propagator quartets (and, indeed, even to kernels which do not satisfy the relationships $G^{11}(x, x') = G^{21}(x, x') = G^{12*}(x, x') = G^{22*}(x, x')$ for $t \geq t'$, which follow from their interpretation as field products).

Considering two histories associated with kernels $G(x, x')$ and $G'(x, x')$, which can in turn be written as products of fields, we can construct the decoherence functional for the second correlation order as

$$D[G, G'] = \int \mathcal{D}\Phi D\Phi' e^{i(S[\Phi] - S[\Phi'])} \prod_{x \gg x'} \delta((\Phi(x)\Phi(x') - G(x, x'))\delta((\Phi'(x)\Phi'(x') - G'^{*}(x, x')))$$

(1.6)

Using the expression Eq. (1.4) for the quantum amplitude associated with the most general binary correlation history, we can rewrite Eq. (1.6) as

$$D[G, G'] = \int DG^{12} DG^{21} \Psi[G^{11} = G, G^{22} = G'^{*}, G^{12}, G^{21}]$$

(1.7)

In the spirit of our earlier remarks, we can use the ansatz Eq. (1.5) for the CTP quantum amplitude and perform the integration by saddle point methods to obtain

$$D[G, G'] \sim e^{i \Gamma[G^{11} = G, G^{22} = G'^{*}, G^{12}, G^{21}]}$$

(1.8)

where the Wightman functions are chosen such that

$$\frac{\partial \Gamma}{\partial G^{12}_{0}} = \frac{\partial \Gamma}{\partial G^{21}_{0}} = 0$$

(1.9)

for the given values of the Feynman and Dyson functions.

One can generalize this construction to higher correlation orders. Indeed, in appealing to Haag’s ‘reconstruction theorem’, (which states that the set of all expectation values of time-ordered products of fields carries full information about the state of the system) one can consider fine-grained histories as specified by the given values of the irreducible time-ordered correlation functions. These histories include those defined by the local value of the field, as those where all irreducible Feynman functions vanish, but allow also more general possibilities. Coarse grained histories will be specified by finite sets of Feynman functions, and correspond to the truncated theories described in detail in [15].

In particular, consider two histories defined by two sets of mean fields, Feynman propagators and correlation functions up to $l$ particles, $\{\Phi, G, C_{3}, \ldots C_{l}\}$ and $\{\Phi', G', C'_{3}, \ldots C'_{l}\}$, respectively. Then the decoherence functional between them is given by

$$D_{F}[\{\Phi, G, C_{3}, \ldots C_{l}\}, \{\Phi', G', C'_{3}, \ldots C'_{l}\}] \sim [\text{prefactor}] e^{i \Gamma_{l}}$$

(1.10)
where $\Gamma_l$ is the $l$-loop CTP effective action evaluated at the following history: (the prefactor is not important for our discussion) a) Correlation functions on the ‘direct’ branch are defined according to the first history: $\Phi = \phi$, $G^{11} = G$, etc. b) Those on the ‘return’ branch are identified with the time-reverse of those in the second history: $\Phi' = \phi'$, $G^{22} = (G')^*$, etc. c) All others are slaved to these. (See [15] for the meaning of ‘slaving’)

The decoherence functional for correlation histories is a generalization of the Griffith-Omnes and Gell-Mann-Hartle decoherence functional [14] between histories: it reduces to the latter when all irreducible Green functions are chosen to vanish. It is consistent, in the sense that further integration over the higher correlation functions $\{C_{k+1}, \ldots C_l\}$, say, gives back the decoherence functional appropriate to the $k$ loop theory in the saddle point approximation to the trace.

Decoherence means physically that the different coarse-grained histories making up the full quantum evolution are individually realizable and may thus be assigned definite probabilities in the classical sense. Therefore, the quantum nature of the system will be shed to the degree of accuracy afforded by the coarse-graining procedure, and the dynamics is described by a self-consistent, coupled set of equations of a finite number of (nonlocal) $c$-number variables.

In this finite, truncated theory, decoherence is associated with information degradation and loss of full predictability [11]. The effective dynamics of the open system becomes dissipative and acquires a stochastic component. The noise incurred from the truncation of the Dyson-Schwinger hierarchy and the slaving of higher correlation functions is called ‘correlation noise’. We have shown [16] that the relationship between dissipation and noise is embodied in the fluctuation-dissipation relation. In the following we shall show how this formalism can be conveniently applied to the consideration of hydrodynamic variables.

## 2 Decoherence and Classicality in Hydrodynamic Variables

### 2.1 Energy-Momentum Tensor of Scalar Fields

(Discussion in this subsection is based on work currently in progress with Calzetta and Paz [16].) As an example of the correlation history formalism, let us consider operators defined in terms of quadratic products of the fields

$$T_{\mu\nu}(x, x') = \mathcal{T}_{\mu\nu}(x)\Phi(x)\Phi(x')$$

where $\mathcal{T}$ denotes some differential operator on fields defined at two points. For the energy momentum operator, the usual expression $T_{\mu\nu}(x)$ is the coincidence limit of $\mathcal{T}_{\mu\nu}(x, x')$, when the two points $x, x'$ are identified. Let the diagonal components of the energy-momentum tensor be the energy density $\rho(x)$ and the momentum density $p$, then

$$\rho(x) = \lim_{x \to x'} T_{00}(x, x')G(x, x')$$

where the Green function $G(x, x')$ is one of the two point functions constructed from the product of $\Phi(x)$ and $\Phi(x')$. We want to show that the decoherence functional $\mathcal{D}$ of correlation
histories peaks for operators of fields $\mathcal{T}$ which obey a conservation law

$$T_{\mu\nu}^{\mu\nu} = 0$$  \hfill (2.3)

Physically this means that quantities which are conserved would be most likely to be decohered and acquire a classical attribute. Using the expression we derived in the previous section for correlation histories of the second order, we can write down the decoherence functional for $\mathcal{T}$ to be

$$\mathcal{D}[T_{\mu\nu}, T'_{\mu\nu}] = \int D\Phi D\Phi' e^{i(S[\Phi] - S[\Phi'])} \prod_{x \gg x'} \delta(T_{\mu\nu}(x, x') - T_{\mu\nu}(x, x')\Phi(x)\Phi(x'))$$

$$\delta(T_{\mu\nu}(x, x') - T_{\mu\nu}(x, x')\Phi(x)\Phi(x'))$$  \hfill (2.4)

Introducing the integral representation of the delta functional and using the CTP indices $a, b, c = 1, 2$ to include both the forward and backward integrations, we can write this expression as,

$$\mathcal{D}[T_{\mu\nu}, T'_{\mu\nu}] = \int D\Phi^a \int DK^\mu_{\nu} e^{iS[\Phi^a]} e^{iK^\mu_{\nu}(x,x')} [T_{\mu\nu} - T_{\mu\nu}(x, x')\Phi^a(x)\Phi^b(x')$$

$$\sim e^{i\Gamma[T_{\mu\nu}(x, x'), T'_{\mu\nu}(x, x')]}$$  \hfill (2.5)

For an action of the form $S[\Phi] = \Phi^a \Delta_{ab} \Phi^b$, we get

$$\mathcal{D}[T_{\mu\nu}, T'_{\mu\nu}] = \int DK^\mu_{\nu} e^{i\Phi^a [\Delta + K^\mu_{\nu}(x,x')T^c_{\mu\nu}(x,x')]_{ab} \Phi^b} e^{iK^\mu_{\nu}(x,x')}$$

$$\sim e^{i\Gamma[T_{\mu\nu}(x, x'), T'_{\mu\nu}(x, x')]}$$  \hfill (2.6)

This is the framework which one could use to validate the statement above, i.e., that by virtue of the conservation law obeyed by $T_{\mu\nu}(x, x')$, the decoherence functional peaks with respect to the hydrodynamic variables $(\rho, p)$. These variables are most readily decohered and have the greatest chance of becoming classical. (In the words of Gell-Mann and Hartle, they have the greatest ‘inertia’.) Using the relation between the decoherence and influence functional as shown in [3], one can further evaluate the noise and fluctuations around the classical trajectory defined by the equations of motion derived from the influence functional in a stochastic form.

### 2.2 Hydrodynamics of Geometry: Bianchi Identities and Classicality

Using the argument above, one can ask the same question about the properties of geometric quantities such as the Einstein tensor $G_{\mu\nu}$, which shares similar properties as $T_{\mu\nu}$. Its conservation by virtue of the Bianchi identity $G_{\mu\nu}^{\mu\nu} = 0$ would likewise make the decoherence functional peaks with respect to the hydrodynamic variables $(\rho, p)$. An alternative approach is to rely on a kinetic theory proof of how hydrodynamics is derived from the BBGKY hierarchy, i.e., by taking the long wavelength and collision-dominated limit of equations of motion from the nPI effective action in the Schwinger-Dyson hierarchy with conservation laws imposed on the hydrodynamical variables.

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4This is not a straightforward proof and we have not succeeded in this way. An alternative approach is to rely on a kinetic theory proof of how hydrodynamics is derived from the BBGKY hierarchy, i.e., by taking the long wavelength and collision-dominated limit of equations of motion from the nPI effective action in the Schwinger-Dyson hierarchy with conservation laws imposed on the hydrodynamical variables.
functional between various quantum geometries peak, and be placed as the hydrodynamic variables of geometries. It is in this sense (amongst other considerations) that we can regard general relativity (Einstein’s equations governing the dynamics of geometry – or geometrodynamics [45]) as gemetro-hydrodynamics. Wheeler has long since emphasized the significance of the Bianchi identity in relation to the basic laws of physics QED, QCD. The geometric meaning of it amounts to the almost naive statement of “the boundary of a boundary is zero” [46]. This is what constitutes his “Austerity Principle” which governs all laws of physics [47]. Here, the significance of the Bianchi identity is in providing a criterion for picking out the variables with the largest ‘inertia’ which can decohere most readily [11] – the ‘collective’ or ‘hydrodynamic’ variables of geometry. These are the variables which enter into the classical laws governing the dynamics of geometry, i.e., general relativity.

In considering classical geometry as hydrodynamic variables, a natural question evoked from this view is: What would correspond to quantum geometry? We are allowing for the possibility that geometry can still be a valid concept in a progenitor theory for a quantum description of spacetime, although at small distances, most likely it would have lost its smoothness property. It could involve non-commutative characters [48] and non-trivial topologies, of which string theory and spin network [49] are capable of addressing. New ideas like quantum topology and logic have also been proposed, and related to the Gell-Mann Hartle decoherent history conceptual scheme [50].

In light of the connection between the quantum and the classical provided by the decoherence conceptual scheme, one should view the classical world as an emergent structure, the conservation laws are what give them the relative stability and persistence characters. In addition to recovering the classical equations of motion from the extremum of the decoherence functional, one can also get a measure of how stable it is with respect to the influence of noise and fluctuations from the ‘environment’ (in the correlation approach, it is provided by the higher-order correlation functions). Fluctuations and dissipation in the dynamics of the effective system provide us with a window to look beyond the macro-classical world into its micro-quantum basis. Even starting from pure thermodynamics, without the knowledge of statistical mechanics based on quantum levels, one can obtain some important properties of the system from its thermodynamic functions. Many important thermodynamic properties of a system can be obtained in the linear response regime, by considering how the system reacts to small stimulus from an external force. This approach is useful for studying possible phase transitions at Planck energy in the early universe [3, 4, 5, 6] or to obtain a statistical mechanics generalization of the Bekenstein-Hawking relation for black hole entropy [22], problems we are currently investigating. Let us return to some more general questions generated from this new way of viewing general relativity as low energy physics.

One could gain quite a bit of information from studying the fluctuations of a system. This was the way critical phenomena was studied in the Fifties and Sixties – e.g., the heat capacity at constant volume measures the thermodynamic stability of the canonical ensemble, the compressibility at constant pressure measures the stability of the grand canonical ensemble.
3 Implications of this viewpoint: low energy collective state physics and beyond

Suppose one takes this viewpoint seriously, what are the possible implications? We can make a few observations here.

3.1 Quantizing metric may yield only phonon physics

First, the laws governing these collective variables are classical, macroscopic laws. It may not make full sense to assume that by quantizing these variables directly one would get the micro-quantum basis of the macro-classical theory, as has been the dominant view in quantum gravity. Just like the energy density $\rho$ and momentum densities $p$ in $T_{\mu\nu}(x, x')$, which are the hydrodynamic variables of a matter field, quantization should only be performed on the microscopic fields $\Phi(x)$ from which they are constructed. If one did so for the metric or the connection variables, one would get the quantum excitations of geometry in the nature of phonons in relation to atoms (or other quantum collective excitations in condensed matter physics). That may be the next order of probe for us, and may yield some new physics, but it is still very remote from seeing the nucleon structure in the solid lattice or the attributes of quantum electrodynamics. In the analogy we mentioned above, quantum elasticity tells us nothing about QED!

Second – and this is perhaps the more interesting aspect – assuming that the metric and connections are the collective variables, from the way they are constructed, what can one say about their microscopic, quantum basis? Historically this question was asked repeatedly when one probes from low to high energy scales, trying to decipher the microscopic constituents and laws of interactions from macroscopic phenomena. This is like going from phonons to the structure of atoms, from nuclear rotational spectrum to nucleon structure – not an easy question to answer. But there are nevertheless ways to guide us, e.g., in terms of the tell-tale signs mentioned earlier. In the above analogies, recall that atomic spectroscopy reveals many properties about the electron-electron and electron-nucleus interactions, low temperature anomalous behavior of specific heat reveals the quantum properties of electrons, the intermediate boson model bridges the symmetry of the collective modes with that of the independent nucleons. To address questions like this, one needs to proceed from both ends: One needs to postulate a theory of the microscopic structure, and work out its collective states at large scale and low energies. One also needs to comb through the consequences of the known low energy theory, paying attention to subtle inconsistencies or mistakenly ignored trace effects from higher energy processes. Indeed, this is what is going on today, with string theory as the micro theory, and semiclassical gravity as its low energy limit. The viewpoint we are proposing would suggest focusing on collective states (solitons?) of excitations of the fundamental string on the one hand and a detailed study of the possible new phenomena in quantum field theory in curved spacetime, such as fluctuations and phase transitions around the Planck energy, and quantum corrections to the black hole entropy.
3.2 Common features of collective states built from different constituents

As mentioned above, there are two almost orthogonal perspectives in depicting the structure and properties of matter. One is by way of its constituents and interactions, the other according to its collective behaviors. The former is the well-known and well-trodden path of discovery of QED, QCD, etc. If we regard this chain of QED - QCD - GUT - QG as a vertical progression depicting the hierarchy of basic constituents, there is also a horizontal progression in terms of the stochastic - statistical - kinetic - thermodynamic/hydrodynamic depiction of the collective states. It should not surprise us that there exist similarities between matters in the same collective state (e.g., hydrodynamics) but made from different constituents. Macroscopic behavior of electron plasmas are similar in many respects to the quark-gluon plasma. Indeed, one talks about magneto-hydrodynamics from Maxwell’s theory as well as magneto-chromo hydrodynamics from QCD. In this long wavelength, collision-dominated regime, they can both be depicted by the hydrodynamics of fluid elements, which are governed simply by Newtonian mechanics. The underlying micro-theories are different, but the hydrodynamic states of these constituents are similar. Here we are proposing that general relativity could be viewed in a similar way, i.e., it is the geometro-hydrodynamics from some candidate theory of quantum gravity. It is an effective theory in the way that nuclear physics is with regard to QCD, and atomic physics is with regard to QED. They are all low energy collective states of a more fundamental set of laws. The macroscopic, hydrodynamic equations and their conservation laws like the Naviers-Stoke and the continuity equations of hydrodynamics are all based on dynamical and conservation laws of microphysics (e.g., Newtonian mechanics), but when expressed in terms of the appropriate collective variables, they can take on particularly simple and telling forms. Thermodynamic variables like temperature, entropy, etc. (think black hole analogy – mass, surface area) are derived quantities with their specific laws (three laws) traceable via the rules of statistical mechanics (of Gibbs and Boltzmann) to the laws of quantum mechanics. Rules of statistical mechanics are important when we probe into a deeper layer of structure from known low energy theories such as semiclassical gravity: we need to know how to disentangle the collective states to sort out how the microphysics works. It is hard to imagine how a complete theory of microphysics can be attained without going through this step.

One comment about symmetry laws: If we view general relativity as a hydrodynamic theory in the same sense as the nuclear rotational and vibration states in the collective or liquid drop model, we can see that the symmetries of rotational and vibrational motion are useful description of the large scale motion of a nucleus, but has no place in the fundamental symmetries of nucleons, much less their constituents, the quarks and gluons. In this sense one could also question the necessity and legitimacy of familiar concepts like Lorentz invariance and diffeomorphism invariance in the more fundamental theory. It should not surprise us if they no longer hold for trans-Planckian physics.

\[^6\] Savour the importance of, say, coming up with a statistical mechanical definition of temperature in a canonical ensemble as the rate of change of the accessible states of a system in contact with a heat reservoir with respect to changes in energy, and we can appreciate the importance of Gibbs’ work in relation to quantum physics.
3.3 Hydrodynamic fluctuations applied to black holes and cosmology

A problem where this analogy with collective models may prove useful is that of black hole entropy. If we view the classical expression for black hole entropy to be a hydrodynamic limit, and the corrections to it as arising from hydrodynamic fluctuations, one could use linear response theory to approach conditions near thermodynamic equilibrium and construct a non-equilibrium theory of black hole thermodynamics. It also seems to us that many current attempts to deduce the quantum corrections of black hole entropy from the micro-quantum theory of strings could be missing one step. This is like the correspondence between results predicted from the independent particle (nucleon) model (where one can construct the shell structures), and that from the liquid drop model (where one can construct the collective motions) – a gap exists which cannot easily be filled by simple extensions of either models operative in their respective domains of validity. This involves going from the individual nucleon wavefunctions to the collective states of a nucleus. It is likely that only some appropriate combinations of fundamental string excitation modes which survive in the long wavelength limit can contribute to the excitations of the collective variables (area and surface gravity of black hole) which enter in the (semiclassical gravity) black hole entropy.

Another example where viewing the classical GR as a hydrodynamic limit would help us orient our conceptual inquiries is in semiclassical cosmology at the Planckian scale. As mentioned above, the abundance of particle creation at that energy makes the consideration of their backreactions important, and to accommodate the fluctuations of the field and the geometry the conventional semiclassical Einstein equation needs be generalized to the form of an Einstein-Langevin equation. In the view proposed here, this would correspond to the hydrodynamic fluctuations of spacetime dynamics as induced by these quantum field processes. One could study the behavior of metric and field fluctuations with this Langevin equation in a way similar to that of critical dynamics for fluids and condensed matter.

3.4 Planck scale resonance states

Finally, following the progression from hydrodynamics to kinetic theory and quantum micro-dynamics, one may ask if there could exist quasistable structures at energy scales slightly higher than (or observation scales finer than) the semiclassical scale. Assuming that string theory is the next level micro-theory, does there exist quasi-stable structures between that and general relativity? This is like the existence of resonance states (as quasi-stable particles) beyond the stable compounds of quarks (baryons) or quark-antiquarks (mesons). Viewed in the conceptual framework of kinetic theory, there could exist such states, if the inter-particle reaction times (collision and exchange) and their characteristic dynamics (diffusion

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7 Black hole backreaction problem has been studied by many authors before, notably by York [51], Anderson and Hiscock [52] and their collaborators. We are taking a non-equilibrium statistical field theory approach. We aim to get the fluctuations of the energy momentum tensor of a quantum field in a perturbed Schwarzschild spacetime [53], examine how they might induce dissipations of the event horizon and deduce a susceptibility function of the black hole [54]. This would realize the proposal of Sciama that a black hole in equilibrium with its Hawking radiation can be depicted in terms of a fluctuation-dissipation relation [55]. (See also [56, 57])
and dissipation) become commensurate at some energy scale. (Turbulence in the nonlinear regime are abundant in these intermediate states). In the framework of decoherent history discussed above, it could also provide metastable quasiclassical structures. It would be interesting to find out if such structures can in principle exist around the Planck scale. This question is stimulated by the hydrodynamic viewpoint, but the resolution would probably have to come from a combination of efforts from both the top-down and the bottom-up approaches. Deductions from high energy string theories would also need the guidance from the different collective states which can exist in the low energy physics of general relativity and semiclassical gravity.

In closing, we note that progress of physics – the probing of the structure and dynamics of matter and spacetime – has always moved in the direction from low to high energies. One needs to pay attention to the seemingly obvious facts at low energies and probe into any discrepancy or subtleties not usually observed to find hints to the deeper structures. By examining how certain common characteristics of all successful low energy theories (here, we only discuss the hydrodynamic and thermodynamic aspects) may recur in a new theory at a higher energy, and how they differ, we can perhaps learn to ask the right questions and focus on some hitherto neglected aspects.

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