FINSLERIAN EXTENSION OF LORENTZ TRANSFORMATIONS
AND FIRST-ORDER CENSORSHIP THEOREM

G.S. Asanov

Division of Theoretical Physics, Moscow State University
117234 Moscow, Russia
Abstract

Granted the post-Lorentzian relativistic kinematic transformations are described in the Finslerian framework, the uniformity between the actual light velocity anisotropy change and the anisotropic deformation of measuring rods can be the reason proper for the null results of the Michelson-Morley-type experiments at the first-order level.

Key words: Finsler metric, non-Lorentzian transformations, relativistic effects.
1. Introduction

One of the important goals of modern physics is to accurately investigate and measure possible post-Lorentzian relativistic parameters [1,2]. In this respect, however, the outcomes of the Michelson-Morley-type experiments (including various optical interferometer experiments [3-9] as well as the modern laser high-precision experiments with improved results [10-13]) seems to be pessimistic rather than optimistic: experiments steadily reproduce “no fringe shift” and, therefore, the “null result” for deviations from the traditional theory of Special Relativity (SR).

If it is just the Finslerian extension of SR that turns out to be the case, then it will perhaps be possible to infer rigorously, and geometrically, that the light velocity value should be anisotropic in moving reference frames. Would it be testable?

The important point is that the experiments don’t measure directly the differences between the velocities of light signals sending in different directions. Primarily, the experiments measure the fringe shifts corresponding to differences between the times of signal flights. The values of the times are, however, results of comparing between the light velocity values, on the one hand, and the lengths of interferometer legs (optic paths), on the other hand. There is an open possibility that these two major factors can compensate one another (here the analogy with the Lorentz-Fitzgerald contraction is pertinent; [14,15]) and lead, therefore, to the conclusion that no apparent fringe shifts should come about.

Mixing the two aspects of anisotropy would, generally, lead to erroneous identifications. Alas, having assumed that, though the isotropical symmetry is implied for a preferred rest frame $S_0$, the observation space in a moving reference frame $S(v)$ may devoid of spatial isotropy and that, therefore, the anisotropy of light velocity value in $S(v)$ may well be expected, the assumption that the length of a measuring rod (as well as of a interferometer leg or a optic path) should not be effected by rotation performed in $S(v)$ becomes neither obvious nor cogent.

On explaining relevant arguments and evaluation methods in Section 2, we are able in Section 3 to assign the theoretical realism to

**THE RELATIVISTIC FIRST-ORDER CENSORSHIP THEOREM.** The Finslerian extension of Lorentz transformations retains conspiracy of the first-order effects in the Michelson-Morley-type experiments.

We use the particular special-relativistic Finslerian metric function $F_{SR}(g; T, X, Y, Z)$, where $g$ is a parameter, subject to the following attractive conditions:

**(P1)** The indicatrix-surface $\mathcal{I}$ defined by the equation $F_{SR}(g; T, X, Y, Z) = 1$ is a regular space of a constant negative curvature, $R_I$,

which is a convenient stipulation to seek for the nearest Riemannian-to-Finslerian relativistic generalization;

**(P2)** The Finslerian metric function is compatible with the principle of spatial isotropy;

**(P3)** The associated Finslerian metric tensor is of the time-space signature $(+ -- -)$;

and

**(P4)** The principle of correspondence holds true,

that is, the associated Finslerian metric tensor [16,17] reduces exactly to its ordinary known special- or general-relativistic prototype when $R_I \to -1$, which physical significance is quite transparent.
All the items (P1)–(P4) are obeyed whenever one makes the choice

\[ F_{\text{SR}}(g; T, X^1, X^2, X^3) = TV(g; |X|/T) \quad (0.1) \]

with

\[ V(g; w) = [Q(g; w)]^{1/2} \left( \frac{1 + g_-w}{1 + g_+w} \right)^{-g/4h} \equiv (1 + g_-w)^{g_+/2h}(1 + g_+w)^{-g_-/2h}, \quad (0.2) \]

where

\[ Q = 1 - gw - w^2 \equiv (1 + g_-w)(1 + g_+w) \quad (0.3) \]

and

\[ g_\pm = -\frac{1}{2}g \pm h, \quad h = \left( 1 + \frac{1}{4}g^2 \right)^{1/2}. \quad (0.4) \]

Vice versa, we can claim the following

**THE UNIQUENESS THEOREM.** The properties (P1)–(P4), when treated as conditions imposed on the Finslerian metric function, specify it unambiguously in the form given by Eqs. (1.1)–(1.4), which is valid due to Theorem 5 of the work [18] (see also [19–21]) in which the Finsler spaces compatible with the properties (P1) and (P2) have been studied. The indicatrix curvature is

\[ R_I = -\left( 1 + \frac{1}{4}g^2 \right) \leq -1. \quad (0.5) \]

**2. RELATIVISTIC KINEMATIC PATTERNS**

**2.1. The Lorentz transformations** have been serving over a century to predict and describe new relativistic effects. Despite the general feeling of a high degree of accuracy between predictions and measurements, various modifications, including the well-known cases

(I) The Robertson Transformations [22];

(II) The Edward Transformations [23, 24];

(III) The Mansouri–Sexl Transformations [25];

(IV) The Tangherlini Transformations [26];

and

(V) The Selleri Transformations [27]

(listed here in the chronological order) have been used to overcome traditional Lorentzian patterns. The transformations (I)–(III) have clearly been compared with each other in [28]; a systematic review of various kinematics relations stem from the choice (IV) can be found in [29]. In fact, the transformations (I)–(V) have been introduced primarily to reanalyze the role of synchronization procedure (see [30–33]), and examine possible observable difference which would result if the light speed were anisotropic. The case (V) was examined systematically in [27] under attractive conditions: namely, assuming that the two-way light velocity is invariant under transformations among inertial reference frames and that the clock relativistic retardation takes place with the ordinary square-root
factor, $\sqrt{1 - \beta^2}$, the transformations (V) ensue. No Finslerian metric function matching a member of the set of non-Lorentzian transformations (I)–(V) has been proposed.

However, the geometrically-motivated status of SR can be retained under post-Lorentzian extension if the Finsler geometry is invoked as a necessary basis to proceed, which leads to choose the case $F = F_{SR}$ as given by (1.1)–(1.4).

### 2.2. The Finslerian kinematic transformations

\[
T = \frac{1}{V(g; v)} t + \frac{1}{V(g; v)} vx, \quad X = \frac{1}{V(g; v)} vt + \frac{1}{V(g; v)} (1 - gv)x, \quad (2.1)
\]

\[
Y = \sqrt{\frac{Q(g; v)}{V(g; v)}} y, \quad Z = \sqrt{\frac{Q(g; v)}{V(g; v)}} z, \quad (2.2)
\]

can well be used to accomplish the special-relativistic transition from a preferred isotropical rest frame $S_0$ to a reference frame $S(v)$ moving inertially at a relative velocity $v > 0$ with respect to $S_0$ along the common direction of the $x$- and $X$-axes. The Capital letters, $\{T, X, Y, Z\} \in S_0$, represent measurements in $S_0$, while the small letters, $\{t, x, y, z\} \in S(v)$, represent measurements in $S(v)$.

The transformations (2.1)-(2.2) can unambiguously be explicated from the Finslerian metric function $F_{SR}$ given by (1.1)-(1.4): to this end one should merely calculate the tetrads of the associated Finslerian metric tensor to use them for connecting the proper reference systems of the RFs $S_0$ and $S(v)$ (cf. the general relativistic role of tetrads, [34-35]).

Inversion yields

\[
t = a(g; v)T + e(v)x, \quad x = b(g; v)(X - vT), \quad y = d(g; v)Y, \quad z = d(g; v)Z \quad (2.3)
\]

with

\[
a = V(g; v), \quad b(g; v) = \frac{V(g; v)}{Q(g; v)}, \quad d(g; v) = \frac{V(g; v)}{\sqrt{Q(g; v)}}, \quad (2.4)
\]

and

\[
e(v) = -v. \quad (2.5)
\]

The kinematic meaning of the caracteristic Finslerian parameter $g$ is the following:

\[
g = (db/dv)|_{v=0}; \quad (2.6)
\]

the normalization conditions $a(0) = b(0) = V(0) = 1$ have been implied; we have also put $(da/dv)|_{v=0} = 0$ to agree perfectly with the low-velocity experimental evidence.

Whenever $v \ll 1$, we obtain

\[
a(g; v) = 1 - \frac{1}{2}v^2 - \frac{1}{3}gv^3 - \frac{1}{8}(1 + 2g^2)v^4 + O(5), \quad (2.7)
\]

\[
b(g; v) = 1 + gv + \frac{1}{2}(1 + 2g^2)v^2 + O(3), \quad (2.8)
\]

\[
d(g; v) = 1 + \frac{1}{2}gv + \frac{3}{8}g^2v^2 + O(3), \quad (2.9)
\]
where the symbol \( O(N) \) in (2.7)–(2.9) denoted the terms proportional to \( v^K \) with \( K \geq N \).

The neglect of the third-order term in the low-velocity expansion of the time dilatation factor \( a \) would entail the traditional pseudo-Euclidean case: \( \{ g = 0, \ a(v) = \sqrt{1 - v^2}, \ b(v) = 1/a(v) \} \).

3. METROSURFACE AND LIGHTSURFACE

In each RF \( S(v) \) an observer is assumed to be equipped with a Metrosurface, \( M_v \), to assign the length values to roads (or legs, or optic paths,...) pointed in various directions. The Finslerian approach proposes a geometrically-motivated method to introduce the device according to the definition

\[
\frac{1}{V(g; v)} F_{SR}(g; vx, (1 - gv)x, \sqrt{Q(g; v)} y, \sqrt{Q(g; v)} z) = 1. \tag{3.1}
\]

Indeed, it is natural to consider \( M_v \) to be a geometric place of the ends of segments directed from the origin, \( O \), of \( S(v) \) subject to the conditions that the transforms of the segments from \( S(v) \) into \( S_0 \) have the unit Finslerian length: \( F_{SR}(g; T, X, Y, Z) = 1 \). When we use here the substitutions (2.1)-(2.2) and put \( t = 0 \), we just receive (3.1).

On using (3.1), simple calculations show that in the approximation keeping only first powers of the parameter \( g \), the polar-angle representation for \( M_v \) reads

\[
r(g; v; \theta) = 1 + \frac{1}{2} gv(1 - \cos \theta)^2, \tag{3.2}
\]

where \( \theta \) is the angle made in \( S(v) \) between the \( x \)-axis and the direction of the measuring segment (rod, leg, optical way,...). In the pseudo-Euclidean limit, that is when \( g = 0 \), the Metrosurface is simply the unit sphere, \( r = 1 \), in any \( S(v) \).

Let us now pose the question of what is the form of a similar representation for the Lightsurface, \( L_v \), in \( S(v) \)? The due understanding of the question can be gained on borrowing the ordinary special- and general-relativistic treatment of the Lightsurface to be the isotropic surface of the fundamental metric function, that is, in our present study, the surface defined by \( F_{SR} = 0 \). Then from (1.2) it follows that the light velocity in \( S_0 \) is the constant

\[
c = g_+ \tag{3.3}
\]

(because of \( V(g; g_+) \equiv 0 \)). Transforming the resultant equation \( X^2 + Y^2 + Z^2 = (g_+)^2 T^2 \) with the help of (2.1)-(2.2)) yields the light velocity value \( c(g; v; \theta) \) in \( S(v) \):

\[
\frac{c(g; v; \theta)}{g_+} = 1 + \frac{1}{2} gv(1 - \cos \theta)^2. \tag{3.4}
\]

Comparing between (3.2) and (3.4) reveals the uniformity of anisotropy of the two types mentioned, such that

\[
\frac{r(g; v; \theta)}{c(g; v; \theta)} = \text{const} \tag{3.5}
\]

in any admissible reference frame \( S(v) \), which thereby proves the Censorship Theorem formulated above in Section 1.

4. CONCLUSIONS
The Finslerian approach explicitly provides rigorous definitions of the means whereby the special-relativistic quantities can be measured and, therefore, invites the re-consideration of the SR to substitute the Lorentz transformations with the Finslerian kinematic transformations.

In the foregoing, we have defined due devices, namely the Metrosurface and the Lightsurface, built on the post-Lorentzian Finslerian transformation properties of standards rods and light rays, to set forth the operational basis for the standard relativistic measurements. The light velocity anisotropy and the spatial length anisotropy may occur to be different concepts in the RF $S$, so that, when analyzing some relativistic observations, one should bring to evidence what kind of the anisotropy is considered in the corresponding experimental set-up.

**Nature may reveal conspiracy of anisotropy of space-time!** The breakdown of spatial isotropy in a moving RF should, in principle, contribute to both the light-velocity anisotropy and to the length standard anisotropy. In the context of the analysis and calculations described above, these two effects just compensate one another in the first-order treatment, thereby making one obtains in any inertial reference frame the null result expected for spatial isotropy. To settle the matter outright, it is required to conduct experiments measuring the second-order relativistic effects.

### REFERENCES

1. M.P. Haugan and C.M. Will: *Physics Today* **40**, 69 (1987).
2. C.M. Will: *Phys. Rev. D* **45**, 403 (1992).
3. A.A. Michelson and E.W. Morley, *Amer. J. Sci.* **34**, 333 (1887).
4. B. Jaffé: *Michelson and the Speed of Light* (Anchor Books, Doubleday, N.Y., 1960).
5. R. J. Kennedy and E. M. Thorndike, *Phys. Rev. B* **42**, 400 (1932).
6. H. E. Ives and G. R. Stillwelt, *J. Opt. Soc. Amer.* **28**, 215 (1938); **31**, 369 (1941).
7. E.W. Silvottooth, *J. opt. Soc. Am.* **62**, 1330 (1972).
8. S. Marinov, *Czech. J. Phys. B* **24**, 965 (1974).
9. D.J. Mora, *Not. T. Astr. Soc.* **173**, 33 (1975).
10. T.S. Jaseja, A. Javan, J. Murray, and C.H. Townes: *Phys. Rev. A* **133** (5), 1221 (1964).
11. A. Brillet and J.L. Hall: *Phys. Rev. Lett.* **42**, 549 (1979).
12. T.P. Krisher et al.: *Phys. Rev. D* **42**, 731 (1990).
13. D. Hils and J.L. Hall: *Phys. Rev. Lett.* **64**, 1697 (1990).
14. H.A. Lorentz: *The Theory of Electrons and its Application to the Phenomena of Light and Radiant Heat*, 2nd edition (Dover, N.Y., 1952), p. 178.
15. C. Møller: *The Theory of Relativity*, 2nd edition (Claredon, Oxford, 1972), p. 21.
16. H. Rund: *The Differential Geometry of Finsler Spaces* (Springer, Berlin, 1959).
17. G.S. Asanov: *Finsler Geometry, Relativity and Gauge Theories* (D. Reidel Publ. Comp., Dordrecht, 1985).
18. G.S. Asanov: *Aeq Math.* **49**, 234 (1995).
19. G.S. Asanov: *Rep. Math. Phys.* **39**, 69 (1997); **41**, 117 (1998); **46**, 383 (2000); **47**, 323 (2001).
20. G.S. Asanov: *Moscow University Physics Bulletin* **49** (1), 18 (1994); **51** (1), 15 (1996); **51** (2), 6 (1996); **51** (3), 1 (1996); **53** (1), 15 (1998).
21. G.S. Asanov: “The Finsler-type recasting of Lorentz transformations.” In: Proceedings of Conference *Physical Interpretation of Relativity Theory*, September 15-20 (London,
Sunderland, 2000), pp. 1-24.
22. H.P. Robertson: Rev. Mod. Phys. 21, 378 (1949).
23. W.F. Edwards: Am. J. Phys. 31, 482 (1963).
24. J.A. Winnie: Philos. Sci. 37, 81 and 223 (1970).
25. R. Mansouri and R. Sexl: Gen. Rel. Grav. 8, 496, 515, and 809 (1977).
26. F.R. Tangherlini: Suppl. Nuovo Cimento 20, 1 (1961).
27. F. Selleri: “Space and time should be preferred to spacetime, 1 and 2”. In: Redshift and gravitation in a relativistic universe (K. Rudnicki, ed.) (Apeiron, Montreal, 2001), pp. 63-71 and 81-94.
28. Y.Z. Zhang: Gen. Rel. Grav. 27, 475 (1995).
29. G. Spavieri: Phys. Rev. A 34, 1708 (1986).
30. H. Reichenbach: The Philosophy of space and time (Dover Pupl., Inc., N. Y., 1958).
31. A. Grünbaum: The Philosophy of Science, A. Danto and S. Morgenbesser, eds. (Meridian Books, N. Y., 1960).
32. A. Grünbaum: The Philosophy of Space and Time (Redei, Dordrecht, 1973).
33. R. Anderson, I. Vetharaniam and G.E. Stedman: Phys. Reports 295, 93-180 (1998).
34. H.-J. Treder, H.-H. von Borzeszkowski, A. van der Merwe, and W Youngrau: Fundamental Principles of General Relativity Theories. Local and Global Aspects of Gravitation and Cosmology (Plenum, N. Y., 1980).
35. J.L. Synge: Relativity: the General Theory (North-Holland, Amsterdam, 1960).