The Ontology and Semantics of Quantum Theory for Quantum Gravity

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Abstract

Based on a clear ontology of material individuals, we analyze in detail the factual semantics of quantum theory, and argue that the basic mathematical formalism of quantum theory is just okay with (a certain form of) realism and that it is perfectly applicable to quantum gravity. This is basically a process about “cleansing” the formalism from semantic assumptions and physical referents that it doesn’t really need (we use the term “semantics” in the sense of the factual semantics of a physical theory, and not in the sense of model theory of abstract mathematics or logic). We base our study on the usual non-Boolean lattice of projectors in a Hilbert space and probability measures on it, to which we give a careful physical interpretation using the mentioned tools in order to avoid the usual problems posed by this task. At the end, we study a possible connection with the theory of quantum duration and time proposed in [4], for which this paper serves as a philosophical basis, and argue for our view that quantum gravity may show that what we perceive as change in the classical world was just (an ontologically fundamental) quantum collapse all along.

Introduction

In the following, we will present some of the tools developed in [1] to deal with the philosophical issues often found in science. The goal is to formulate a purely physical, independent and objective interpretation of quantum theory. Purely physical in the sense that it will only refer to independent physical objects and not to subjective or human elements (such as minds, knowledge, uncertainty in knowledge, etc.). Independent and objective in the sense that only genuine properties of the system are taken into account and not elements that, although physical, are superfluous to it (such as measuring devices, etc.).

These tools are based on two basic pillars of philosophy: ontology and semantics. In scientific theories, they are embedded in them, although they are not always made explicit. The fact that scientific theories use these tools should not be surprising. After all, one of the central problems of ontology is to establish what the world is really made of, which is also a problem of interest to science (not to say equally central.) However, in science, ontology cannot be arbitrary: it must always be compatible with materialistic realism, which states that reality exists objectively and its fundamental substantial components are only material entities. Our focus, however, will be primarily on semantics; some additional ontological hypotheses will be mentioned to the extent that they are necessary.

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1. Scientific Ontologies

Science assumes that an external and independent Reality exists (its famous method is just a consequence of this hypotheses; by the same reason, one cannot use science’s method to prove that ontological hypotheses, one can just judge its correctness in terms of its fertility and success.) Furthermore, Physical Theories assume that Reality is made of something. In ontology, this something is called a Substance. In science, it’s assumed that Reality is made of a single type of Substance, which is called Material Substance. Therefore, science’s ontology is monistic. In ordinary language, we refer to this Substance simply as Matter. Nevertheless, be aware that the “matter” of which we are talking about is not necessarily some chunk of something with some mass, as, say, a piece of steel. The notion of Matter used in science can become quite subtle and unintuitive.

A key property of Material Substance is that two material individuals can Associate to form a third material individual. If a and b are these two material individuals, we will denote the association operation as $\oplus$, and, in this sense, if $c$ is the third material individual mentioned before, we write (we will denote as $S$ the set which contains our material individuals):

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c = a \oplus b.
\]

That is, in science we have different types of “matter”, which are each characterized by the binary association property, $\oplus : S \times S \to S$, among its elements. Mathematically, we can postulate a Boolean $\sigma-$algebra structure for the triple $(S, \oplus, \emptyset)$ (where $\emptyset$ represents the null element, or minimal element, $R$ the maximal element, see below, and $\lor$ the “disjunction” or $\text{sup}$, that is, the minimal upper bound between elements; one also has the $\text{inf}$ or “conjunction”, which is interpreted as the Interposition $\times$ of material individuals.) In physics, two typical examples are the following:

- if the elements of the set $S$ are pieces of charged material and the association of two pieces to form a third is modeled by the operation $\oplus$, then the electric charge, $Q : S \to \mathbb{R}^+$, is additive on $\oplus$, that is, $Q(x \oplus y) = Q(x) + Q(y), \forall x, y \in S$;

- if the elements of the set $S$ are static electric fields and the, pointwise, superposition of two fields to form a third is modeled by the operation $\oplus$, then the intensity of the electric field at point $p$ of space, $E_p : S \to \mathbb{R}^+$, is additive on $\oplus$, that is, $E_p(x \oplus y) = E_p(x) + E_p(y), \forall x, y \in S$.

In this way, we see how the proposed ontological theory is consistent with scientific knowledge. In addition, we see how, in general, scientific theories necessarily require some kind of ontological theory (of course, the scientific theories themselves are the ones which suggest the structure of the latter, although they do not usually determine them completely.)

The above is a non-trivial characteristic which we must postulate. But, it also must be so that, when individuals associate, matter doesn’t disappear: Material Substance is always conserved; that is, if it exists, then it cannot suddenly disappear.

Other metaphysical theories may introduce other types of Substances, like “Immaterial Souls”, “Mental Substance” (this one is famously associated with Descartes and his dualist, in terms of Substance, solution to the, also famous, Mind-Body problem, i.e., what’s the mind and how does it interact with the physical body?), etc. But, in science, only matter interacts and associates with matter.

\[\text{\footnotesize{1}}\text{The reader may be thinking in the supposed “annihilation” of matter with antimatter, but that’s not an annihilation, but rather a transformation of one type of material individuals (say, electrons and antielectrons) into another type of material individuals (in this example, photons; we can see how the naive picture of “matter is mass” can be quite misleading.) Note that, in this example, conservation of matter is related to conservation of energy (since the mass of the initial particles is transformed into the light energy of the massless photons via } E = mc^2); \\text{we will come back to this latter.}}\]
All of this may seem a little too vague. Is there some way to characterize matter in a more precise way? How do we know what is actually matter and what cannot be considered as genuine matter? Fortunately, there’s a way. We postulate that all Material Substance carries Energy, where this concept is defined according to the best physical theory at disposition. In general, it’s a Property (see below) of matter which is represented, in one way or another, by an additive, real function on matter (which depends on the reference frame adopted, though), i.e., an $h(a)$ such that

$$h(a + b) = h(a) + h(b),$$

for any two elements $a, b$ from $S$, and which “generates time translations” in the context of that theory (this is usually done in what is called the “Hamiltonian formulation” of a dynamical theory.) What this means is that Energy is the Property of a Thing that allows us to determine, in a physical theory, how the Properties of this Thing change in time (see below for these terms.)

Thus, to us, if something doesn’t possess energy, then it cannot be considered matter.

Of course, the things that inhabit Reality are much more than mere undifferentiated bunches of material substance. The “fauna” of Reality is very rich, with individuals that can have very peculiar and different Substantial Properties. A Substantial Property is a property/quality that a material element possess or bears and it’s as objective as the existence of the material element itself (properties are not substantial elements themselves, though!) In order to characterize a property and have a clear understanding of its peculiarities, we need a physical theory about the material element at issue. In this way, the property is characterized by all of the logical consequences in the theory that involve this property. For example, for a charged body, its electric charge is one of its substantial properties and, in order to understand what electric charge exactly is, we need to consider logical consequences in a theory that involves the electric charge; for example, consider the following logical consequence from classical electricity: “two bodies with charges of equal sign repel each other while two bodies with charges of opposite sign attract each other”; evidently, this proposition gives us valuable information that helps to elucidate what the property “charge” is. We will call Thing to a matter element which possess substantial properties. Things also associate to produce other Things. We denote the set of all Things that exist as $\Theta$. We mention that a very particular type of properties are the so-called Emergent Properties, that is, properties which are present in the aggregate Thing $c = a + b$, but not in its component Things, $a$ and $b$, before the aggregation.

Reality is the Thing $R \in \Theta$ which consists in the association or aggregation of all Things in $\Theta$.

2. Scientific Semantics

2.1 Reference

A mathematical theory without a factual interpretation is only mathematical, not physical. A scalar field satisfying the Laplace equation may be the potential for a gravitational field or for a static electric field. It is the factual interpretation (which interprets this field as the potential for a gravitational field or

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2 This in contrast to Idealism, which states that properties are mental constructs. As we will see below, in physical theories, we will represent properties by mental constructs, but we assume that the property exists without us, and that, in fact, this representation may be only approximative. Idealism identifies as a single thing the property and the mental construct (although it accepts the existence of matter.)

3 One could say, what about time and space, are not they part of Reality too? This is a complex issue. They are intrinsically tied to Things in a certain specific sense, as explained in 4. Also, what about logic? We use logic to understand Reality, but we don’t take it as part of Reality (it’s not even a thing.)
for a static electric field) the thing that transforms it into a theory of physics. And, of course, it is this map of interpretation (which maps mathematical objects to objects of the real world) the source of most of the problems, due to the number of different, sometimes contradictory, positions that are adopted in everyday practice and the literature.

But we need the map to define and establish the theory, what do we do then? How do we formulate a physical theory?

There are two types of semiotic interpretations or semiotic maps:

1. the purely semantic ($\varphi$);
2. the pragmatic one ($\pi$).

The purely semantic map, (1), assumes a certain kind of realism (which is an ontological hypothesis), simply the same that is needed to give meaning to the fundamental maxim of the scientific method (not to be confused with classical naive realism, in which all the properties of a given physical system always have to possess defined values, etc.;

the realism we are considering here only assumes that there are entities/things that exist and that have well defined and independent physical properties; we stress: just properties, not necessarily a definite value for ALL of these properties; properties that have a definite value are called manifest, while, if they do not have it, are called latent; furthermore, to be considered as existing, a thing must have at least one manifest property; if a thing ceases to have any manifest properties, then it ceases to exist according to our view; we will come back to this in later.)

In this way, the interpretation here is very simple: factual item $y$ of this physical system will be represented mathematically by, say, the mathematical function $f$ (which is a construct.) And that’s all! We don’t need anything else, we simply refer to the factual item because it exists, it is simply there, we simply point to it (semantically speaking, not pointing in a literal sense.)

For example, if the physical entity is a particle and the property is its classical electric charge, then at the abstract and mathematical level we have a set $P$ and an additive function $Q : P \rightarrow \mathbb{R}$; the semantic map $\varphi$ relates these abstract mathematical elements to objects of the real world:

i) $\varphi(P) = \text{particles};$

ii) $\varphi(Q(x)) = \text{electric charge of } x \in P.$

4 Usually, the invention of some mathematical concepts is inspired by the physical reality and are used to represent the elements behind that inspiration; however, in many cases, the concepts prove to be useful for representing other elements of physical reality, which can be unrelated to the ones that inspired the invention of the concept; a typical example is the concept of Hilbert space, which was originally used to describe ordinary physical euclidean space, but later appeared as the space of states of a quantum system and having infinite dimensions instead of the usual three.

5 These mathematical objects are mere mental constructs made by human brains. In this way, we do not adhere to mathematical Platonism (which says they exist in their “own objective reality”). However, this does not imply a free subjectivism, since different humans can manage to understand each other through the same concepts. Then, in practice, we can pretend as if Platonism were true. In addition, it is assumed that classical Predicate Logic is applicable to both purely mathematical predicates and to predicates with factual content. Hence the utility of mathematics in the factual sciences, there’s no mystery: given a factual interpretation of the kind discussed here, we can take advantage of the abstraction, precision, and systematic nature of mathematics to study more accurately and systematically the physical world (indeed, since we assume that classical logic applies to predicates at both ends, then the conclusions derived in the purely mathematical plane should in fact predict or indicate a novelty in the factual plane once the initial factual interpretation is applied to these conclusions.)

6 There is a third type of semiotic interpretation called “mathematical interpretation”, in which abstract mathematical constructs are interpreted in terms of other abstract mathematical constructs.
Note that, in this kind of interpretation, if one introduces factual elements which, however, lack a mathematical correlate in the theory, then they are not true factual elements pertaining to this physical theory and one can dispense with them. One should be careful to only include in the factual base of the theory those elements for which the semantic map is well defined and assigns a definite mathematical correlate to them. Otherwise, we would be dealing with “ghosts”, i.e., entities that are not modeled by the physical theory in consideration. Of course, these ghosts are just superfluous elements and should be, as we said, cleared up from the factual base of the theory. This is a key philosophical principle that one must always have in mind when trying to provide a physical interpretation for the mathematical formalism of quantum mechanics (or any other physical theory for that matter), one really cannot overstate this point. But this can be very tricky if we don’t have our physical theory precisely and clearly formulated (which is, unfortunately, the exception rather than the norm in today’s physics.) This trickiness (together with the difficulty in trying to measure directly the referents of a theory) is the reason why most physicists tend to distrust ontology, particularly in matters related to quantum theory; they prefer, instead, to resort to operationalist approaches (which, ironically, actually makes matters even worse, since this just opens the doors for ghosts to held a party inside the theory!) Avoiding a problem doesn’t solve it nor makes it disappear!

The pragmatic interpretation, (2), does not assume an objective property of the system; instead, it begins with a series of laboratory operations. The interpretation would be as follows: the measurements made by these particular laboratory operations will be represented mathematically by, say, the function \( f : \mathbb{R} \rightarrow \mathbb{R} \). Notice how drastically different these two interpretations are.

For example, consider the color change on a litmus paper. We know that this color change indicates acidity; but if we abstain only to the spirit of (1), we cannot say that this change is acidity (as (2) would say.) Acidity is an objective property of the chemical medium, while the color change of the litmus paper measures it.

It should be noted that in the semiotic interpretations normally found in science, not all the objects of the mathematical theory or structure in question necessarily have a factual interpretation. This is called a “partially interpreted theory” and is what is usually used in scientific theories, which only make use of “pieces” of some mathematical theory.

2.2 Meaning

When formulating a theory, it is only acceptable to use interpretations of type (1). This is because we are only looking for one very basic thing: basic physical meaning, anything else would be superfluous here. The formulation and interpretation of a theory is something that belongs to the purely theoretical and conceptual plane (which does not mean that reference is not made to factual items); a priori, it has nothing to do with laboratories and laboratory operations. Laboratories and laboratory operations become relevant in the stage of empirical testing of theory, which comes later, once the theory has been formulated. When we make empirical tests of theories, then it’s there when we turn to interpretations of type (2). But, in fact, the operations that one considers in (2) will depend on the basic meaning of the concept according to (1). It is precisely the meaning of the concept the thing that justifies, and allows us to elucidate, the kind of operations that we need to consider.

The word meaning has been used; it is, therefore, convenient to analyze it in more detail. The purely semantic interpretation takes a mathematical construct and relates it to the real world. But this is not enough to give physical meaning to this construct. The purely semantic interpretation gives us, at best, the

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7Typical of the so-called “instrumentalist” and “subjectivistic” interpretations of quantum theory, which introduce things like “observers”, “apparatuses”, “ignorance”, “uncertainty”, and “information”, all things that have no actual mathematical correlate in the theory.
physical referents of the construct, that is, those physical entities (i.e., the Things that comprise the Reality hypothesized by the physical theory in consideration) to which the physical interpretation refers. What is important to note is that, in a full theory of physics, this construct is not isolated. Indeed, there are also other mathematical constructs in the theory, and, in general, they are all inter-related with one another (in the logical sense.) These interrelations constitute what is called the mathematical Sense of the construct. Some of these other constructs are also physically interpreted in terms of purely semantic interpretations, and, thus, the mathematical sense becomes also a physical sense. In this way, given a physical theory, we will adopt the point of view in which the meaning of a construct is fully established by its total Sense and its Reference class ($\Sigma \ni \sigma$ and $\Sigma \subseteq \Theta_{Universe}$) of Things ($\sigma$), both which can only be read once the theory has been fully established in its mathematical axioms and semantic interpretations. This is the reason why we need to know all the logical consequences that involve a Property in order to understand the Meaning of that Property according to the physical theory in consideration. Also note that, in a completely axiomatized theory, certain basic constructs will determine the meaning of the other constructs of the theory (in general, these basic constructs will be those whose physical interpretations are made in terms of factual elements that are usually taken as factual primitives, such as the notions of length, lapses of time, facts, propensity, etc.) It’s only after one has a clearly formulated theory that one uses the meaning of its concepts according to it to design experiments to measure them, and not the other way around, as in operationalism, where semantics, meaning, and experimental testing of a theory are all conflated and confounded.

For example, the concept of mass in classical mechanics is mathematically an additive function $M : P \rightarrow \mathbb{R}^+$. The semantic map $\varphi$ gives to it a physical interpretation as follows:

i) $\varphi(P) =$ particles; where, also, $\Sigma =$ particles;

ii) $\varphi(M(x)) =$ inertia of $x \in P$;

iii) $M$ appears in the equations of motion multiplying the acceleration (which is another construct, with its own factual interpretation and so on.)

Clearly, i) and ii) concern the semantic interpretation part, while iii) is related to the sense of the construct. Thus, it is the complete theory of classical mechanics (its mathematical formalism, its semantic interpretations, its laws, etc.) the thing which determines the meaning of the construct in this example.

It is well known that the three previous items in the mass meaning are enough to propose, in a justified way, operational methods that can allow the determination of the concrete numerical value of this mass in the context of the empirical test stage of the theory. For example, if $M$ is the value of the mass of a test particle and $M_0$ the value for a fixed particle taken as a reference for unity, we know from the third law of motion that if the two particles interact, then (in magnitude) $Ma_M = M_0a_0$. In this way, taking $M_0 = 1$ by convention, the value of $M$ can be calculated in terms of the measured values of these accelerations through $M = a_0/a_M$ (this is the supposed “interpretation” of the concept of mass given by Mach; from our point of view, it’s only one way of measuring the value of mass, not a “definition” of mass, much less its physical interpretation.) Thus, the meaning of mass is inertia, i.e., the same force applied to a body of some mass gives it an acceleration which is greater than the one it would give to a body with a mass greater than the one of the initial one, i.e., inertia or mass is a “resistance” to being accelerated by a force. As mentioned, this is not in any way a “definition” of mass, which is actually a specific primitive (i.e., beyond stating that it’s a property and that it appears in the mentioned law, it’s left undefined) of this theory. What we are doing is studying the logical consequences of the axioms in order to understand what role this concept plays in the theory. In Special Relativity, the meaning of mass, besides also being inertia here, is expanded because of new features introduced by this theory, like the one established in the famous equation $E = mc^2$, which

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8Space and time, in particular, are quite generic to most theories. In Scientific Ontology, one can actually make theories that give them a precise physical meaning. These theories are appropriately called “proto-physics” (see [3] for a proto-physical theory of spacetime.) Although, one often needs to insert them into a physical theory in order to have a more exact (in a quantitative sense) meaning.
expresses that the (rest) mass, the inertia of a particle, can be transformed into the energy (e.g., kinetic energy, electromagnetic energy) of another particle, possibly massless (of course, these processes are not allowed in classical mechanics, where the sum, \( M_o = \sum m_{o(i)} \), of the total rest masses of the particles in a system is always conserved.)

Of course, there may be many other operational methods, since this is not the way to give meaning to the concept of mass, it is only a way of calculating its concrete numerical value in a given experimental context. It is in fact the well-defined meaning the thing that alone can and should justify all these possible and different operational methods. We have used the theory to justify the method proposed in the example, which might suggest that there is some risk of circularity. But this is not the case because the positive truth of physical theories is not something that can be definitively proved and in terms of considerations completely alien to it. Instead, or it is established that a theory of physics is self-consistent and in accordance with observed facts (of course, this will never be enough to “prove” the theory “once and for all”, this would fall into the severely criticized fallacy of Empirical Induction), or it is established that it is inconsistent and in disagreement with the observed facts. In this second case, of course, the theory has been empirically falsified (in the Popperian sense of the term; of course, this is also just a simplification of a much more complex process in which theories are first amended many times.) In this way, the method proposed in the example may actually help to establish whether the theory is self-consistent in the empirical context. In fact, if, once the values of the masses have been obtained, the dynamics observed in a new experiment does not agree with what is stipulated by the second law, where the previous values measured for the masses are used in this law, then something goes wrong in theory, it is not a correct description of nature. If there is agreement, then we simply obtain a transient verification of it (which, as we have already said, is by no means a definitive proof of the theory), and this is indeed the standard scientific practice.

In a mathematical interpretation, the role of the referent is given by the more specific mathematical construct. In general, this can be precisely defined using mathematical axioms. In the case of the factual interpretation, however, the referents are factual items, which cannot be characterized by such definitions. The factual items can only be pointed out or mentioned, not defined. In general, factual items (entities, properties of entities, facts involving entities) are characterized in a partial way by predicates, statements and propositions. If these referents indeed exist in the physical reality, they can, at best, be discovered in some experiment (although this is not necessary for the formulation of the semantics of the physical theory in question; not even its actual existence in the physical reality is needed, since the physical theory and its referent are, in principle, just hypotheses about the physical reality; concern about how to measure the existence of referents at the formulation stage of the theory is unnecessary; once the theory is empirically tested, it may in fact be proven that the referents do not exist; however, this does not show that the theory affirms its non-existence; in general, the theory needs them for its formulation; what would happen if it were shown that the referents do not exist is the falsification of the theory itself. All this should not be surprising since, for example, when formulating theories such as classical mechanics, the existence of its referents, classical particles, is taken for granted; however, if we conform to the formalism and standard interpretations of quantum mechanics, these referents are pure fiction, there is no such thing as a classical particle in the real world.)

2.3 Operationalism in Modern Physics

In the operationalist philosophy of physics (like the usual “Copenhagen interpretation” of quantum mechanics), the very formulation of the theory is performed in terms of pragmatic interpretations of type (2).

This is very problematic due to the following reasons: in a pragmatic interpretation, a large number of elements (such as measuring devices, human observers, etc.) are usually brought to the scene and, although they are necessary to obtain the concrete experimental value of a property, they are completely alien and superfluous to what is really necessary to simply give basic physical meaning to the property being measured.
As mentioned, this confuses two very different levels: the formulation of theory (which is a purely conceptual and theoretical process) and the stage of empirical testing of it (which should in principle be a posteriori of the first.) This confusion may even introduce incoherence into the theory, as exemplified by the Copenhagen interpretation of quantum mechanics (according to which classical mechanics would be among the hypotheses of quantum mechanics; this is absurd since quantum mechanics is a theory which supercedes classical mechanics.) The value obtained in a pragmatic interpretation will, in general, depend on the system being measured, of course, but also on the measuring devices used and possibly on other details of the experimental context. This, per se, is not problematic, provided it is clear that we are in the stage of empirical testing of theory and not in its formulation.

Consider a simple example. Let \( \hat{A} \) be a quantum operator that represents some property \( A \) of a quantum system. The standard operational interpretation states the following: “the eigenvalue \( a \) of operator \( \hat{A} \) is a possible result/value of measuring \( A \) in some given experimental context”.

Now, let’s analyze what quantum theory actually says (that is, let’s analyze how the theory is actually used by professionals in their routine calculations.) Given that the theory provides an interpreted mathematical formalism (in the semantic sense), then it is possible to use an elementary tool of the semantics outlined here: identification of the factual referents. As mentioned above, when a mathematical concept is interpreted in terms of physical elements, reference is made to certain physical entities; these entities are the factual referents. A quantum particle can be a referent, as well as a measuring device and even the “mind” of a human observer.

In this way, let’s take the operator \( \hat{A} \), as it is usually given by quantum theory, and let’s investigate its factual referents (to make the analysis more concrete, the operator in question could be taken, for example, as the quantum z-spin; the Hamiltonian can contain the environment in its variables, so it is an example to be taken as a separate case.) And this is where the inadequacy of the usual operational interpretations is evident: in the standard formalism of quantum mechanics, both \( \hat{A} \) and its eigenvalues are only functions (in the semantic sense) of the quantum particle; no reference is made to measuring devices (much less minds), nor is the mathematical form of the operator dependent on such elements. In more precise terms, let \( P \) be the “observable” object of study, \( \Sigma \ni \sigma \) a factual reference class and \( \Sigma' \ni \sigma' \) the class of physical environments or ambients of the elements of \( \Sigma \). If \( P \) is a real and objective property of its referent \( \sigma \in \Sigma \), then we denote \( P = P(\sigma) \). Operational interpretations often state that, for any quantum property \( P \), it is generally given that \( P = P(\sigma, \sigma') \), where \( \sigma' \in \Sigma' \) is the environment of referent \( \sigma \). However, this is usually the exception rather than the rule. If we take the example of z-spin, standard quantum mechanics states that the z-Pauli matrix continues to represent the z-spin, regardless of which environment the quantum particle is immersed in: the theory itself states that the same matrix represents this property for both the free (Hamiltonian only kinematic) particle and the particle in interaction with, for example, a field. In view of examples such as these, it is clear that it is capricious to insist that the referent of quantum mechanics is an irreducible “object-apparatus” (or even “object-mind”) entity rather than physical systems (and their corresponding objective properties) that can, in principle, be considered independently (at least ideally) from their surroundings.

Thus, we know that the referent has to be just the system (and not a system-apparatus, system-instrument, system-observer, system-mind, etc., complex) because one can see explicitly in the mathematical description of the properties in this theory that there’s no room for the environment in them. Let’s take another example. For a particle, interactions with external entities are introduced in the theory by adding a “potential energy” term \( \hat{V} \) to the kinetic one. The total energy one gets is, then: \( \hat{H} = \frac{p^2}{2m} + \hat{V} \). By studying the formalism, one can see that the mathematical form of \( \hat{p} = -i\frac{\partial}{\partial x} \) (a property of the particle) is left unchanged if we change the type of interaction characterized by \( \hat{V} \) (where one has to put apparatuses, instruments, observers, etc.; “ideal measurements/devices”, which wouldn’t appear here, are non-sense, physically.) This can only be the case if \( \hat{p} \) is a property of the quantum system alone, i.e., a property that exists objectively and not something that arises via the interaction with an instrument (as operationalists,
and perhaps also radical relationalists, believe.) The same analysis is valid for other properties. Now, when there’s an interaction, evidently, the probabilities will change. But, for example, for a particle, the theory says that there can indeed be free particles (just take $\hat{V} = 0$), and that probabilities in the theory still make sense in that case\(^9\) (of course, they do not depend on any environment, since there’s none; the typical ones are the “wave-packets”, which are probability distributions that center around, e.g., a position value and only depend on that number, the dispersion $\Delta x$ around it, and time.) This can only be the case if probabilities in quantum theory refer to an actual Property of the system alone in the first place, and not something else (for example, if they were mere “information” that some observer has about the system, then some variable there should say which observer); in particular, if, for a free particle, the probability is 1 for some value of a property, then the only possible interpretation (i.e., one that doesn’t introduce superfluous elements lacking correlates in the variables) is that the value is simply being possessed by the particle, in the same way in which a classical particle possess a value, even when there’s nothing measuring it (this point of view assuming that the description given by quantum physics is complete.) In both cases, for probabilities values and for the values of other properties, it’s the mathematical formalism the one that suggests which interpretation suits it and which doesn’t; in these cases, we saw that it suggests realist, objective and non-operational interpretations. All of these semantic considerations are necessary since we want to know what’s exactly the referent of this theory. Furthermore, the only way to shed light to this matter is by a semantic analysis, as the one we just did.

We can see, then, that the operational interpretations only manage to attach to $A$ useless and unnecessary referents, which make no difference in the predictions made by the concrete theory. When this happens, the interpretation is said to be adventitious. Most pragmatic interpretations make this conceptual mistake massively when they are used to give a supposed physical meaning to some construct; they introduce physical entities (like the physical environment of the particle, in the previous example) that remain, however, orphaned from a mathematical correlate in the theory; that is, entities that are thus transformed into phantoms, which are smuggled into theory (and removed with equal ease when it is convenient, since they make no real difference.) Then, in the spirit of (1), we rephrase as: “the eigenvalue $a$ of the operator $\hat{A}$ is a possible value of the objective property $A$ (of the referred system, of course) represented by this operator”. That is, reference is made only to the quantum system and its objective properties.

Note that, in principle, it would be possible to develope a theory in which the operator $\hat{A}$ in fact depends on the measuring apparatus; moreover, further research could prove experimentally that this new theory is superior to standard quantum mechanics from the empirical point of view. However, this new theory would be just that: a new theory (superior and distinct from standard quantum mechanics.) What the previous semantic analysis shows is that, as far as the formalism of standard quantum mechanics concerns, there are no references to measuring apparatuses in the operator $\hat{A}$ or its eigenvalues.

In this way, we see that semantic interpretations (and, in fact, the whole theory) are actually also hypotheses about reality, which could eventually prove to be merely approximations to the latter. However, theories must be formulated at some point, which implies making these hypotheses explicit. In the case of quantum mechanics, this conveys to make explicit the hypothesis that establishes that the properties of the system will not be considered as dependent on its surroundings. Perhaps this is not correct, perhaps it is only an idealization of reality, but this is work for a new theory; these are two distinct levels that should not be confused. The concern about how to measure the existence of referents in the formulation stage of the theory is unnecessary; once the theory is empirically tested, this can in fact prove that the referents are non-existent; however, this does not show that the theory affirms its nonexistence in general, the theory needs them for its formulation; what would happen if it is shown that the referents do not exist is the falsation of the theory itself. All this should not be surprising since, for example, in formulating theories such as

\[^9\]As an analogy, consider the position $x_t(\sigma; K)$ of a particle $\sigma$ moving under the influence of a spring of spring constant $K$. Thus, of course, the explicit form of $x_t(\sigma; K)$ will depend on the environment. Indeed: $x_t(\sigma; K) = x_0 \sin \left(\sqrt{\frac{K}{m(\sigma)}} t\right)$. But this is only an effect of the dynamics; if $K = 0$, there’s no force, no spring, the particle is free, but $x_t(\sigma)$ still remains there, this means that the position was actually a property of the particle in the first place. The same is valid for the probability $P_t(\sigma)$ of a free quantum particle $\sigma$ (see next section.)
classical mechanics, the existence of their referents (classical particles) is taken for granted; however, if we adapt to the formalism and standard interpretations of quantum mechanics, these referents are pure fiction, there is no such thing as a classical particle in the real world.

The above considerations do not, however, mean that the environment (considered as a physical entity that exists objectively and in which we include, for example, measuring devices, human observers, a photon gas, etc.) cannot interact with a quantum system: it can, but the form of this interaction is something that must be modeled by the quantum theory itself, that is, it is not a part of the formulation of the latter; instead, the corresponding, and possibly not trivial, temporal evolution of the probabilities (for example, they may become more acute distributions on a given value after interaction with the apparatus, the surrounding environment, etc.) is something to be explained by the theory once it has been formulated and the environment included in its Hamiltonian. In fact, the way in which the physical environment of a quantum system can cause the sharpening of probabilistic distributions of the latter is a topic of debate and research in contemporary physics (i.e., the need for an adequate measurement theory.)

3. (Objective) Probability and Quantum Theory

Let us consider the facts, \(
\begin{align*}
\text{\textit{h}}_{1} & \quad \text{that can happen to a thing} \\
\text{\textit{h}}_{2} &
\end{align*}
\)
(we take “happening” as something dichotomic: a fact either happens or not, we don’t accept “partial happening”, whatever that could be.) We include under this notion the facts that, although they can be in principle possible and accessible for this concrete thing, never happen to the system in the real world; facts that actually happen are called “real”, while the rest are called “potentials” (in this way, real facts are a proper subset of potentials); given the potential facts, \(\text{\textit{h}}_{1}\) and \(\text{\textit{h}}_{2}\), the disjunctive fact “\(\text{\textit{h}}_{1}\) or \(\text{\textit{h}}_{2}\)” only makes sense as a potential fact and not as real, since only conjunctive facts of the type “\(\text{\textit{h}}_{1}\) and \(\text{\textit{h}}_{2}\)” (that is, the fact in which both facts occur simultaneously) can actually occur in a given system; thus, disjunction only makes sense as potential, not as reality; when we speak of the “set of facts”, in general we will be referring to the total set of potential facts (note that these physical conjunctions and disjunctions of facts must, and should, always be Boolean operations, in order to make any sense and match intuition.) This is so because physical probability theories always operate at the level of potential facts; in particular, to include such a disjunction of potential facts. The distinction between potential and actual facts becomes relevant when one wants to empirically contrast the theory, since it only makes sense to empirically test situations related to real facts.

In this way, the facts that can occur to the system are elements that are also capable of appearing as the corresponding factual objects in a possible semantic interpretation. This is the case in physical probability theories, where, physically, probability provides the instantaneous intensity of the propensity of a single system towards the occurrence of a given fact; this is a key insight by, among others, Popper into the whole debate about probabilities and their interpretations. That is, for us, the propensity, the mere potentiality, of a system towards the possible occurrence of a given fact that involves it, is a real, manifest, actual, instantaneous and objective property of it. This categorization is, of course, a hypothesis of the ontological type.

Now, given a theory and its respective laws, the temporal evolution of probability will dictate which facts are more likely than others as the system evolves; in analogy to mechanics, we could say that there is a “dynamic” part, an evolution “constrained by the laws of motion” (that is, the laws of physical theory), while the set of all potential facts and all probability measures are the “kinematic” part of a theory of probabilities. In general, one first must consider the system and all of the physical properties that the physical theory attributes to it; if it’s a quantum field, these could be the quantum energy or momentum

10These facts usually involve some property of the system, e.g., a possible fact can be that “the position of the system is \(x = 5\text{ m}\)”. In the case of completely classical theories, one usually adopts the point of view according to which the values of properties are always defined. However, results like the Kocher-Specker theorem, establish that this is impossible in quantum physics (at least for all properties at the same time.)
of the field, etc. Thus, the potential facts for this system will all be occurrences which involve the eventual definition/acquisition of values for some properties (or that this value is in some subset of the real numbers.)

For example, consider a spin particle \( \frac{1}{2} \). The latter implies that given a direction \( m \in \mathbb{R}^3 \), the projection of the spin of the particle in that direction can only take the values \( S_m = \pm \frac{1}{2} \). Then, the facts \( h_{\pm \frac{1}{2}} \) in which the value of this projection is indeed \( \pm \frac{1}{2} \) are possible facts for this particle and therefore fall into our definition of potential facts (note that it will be the dynamics of theory the one that will say if the particle indeed acquires some of these values or not.) On the other hand, the alleged fact \( h_{\frac{1}{2}} \), in which the value of the projection is \( S_m = \frac{1}{3} \), will never happen; but, this, not because the fact is a potential one and the dynamics dictamines its non-occurrence, but, rather, because, being the particle of spin \( \frac{1}{2} \), it is not even in its intrinsic potentiality to acquire that value for the projection; that is, the fact \( h_{\frac{1}{2}} \) is not a potential fact for this particular system under consideration.

In the experimental testing stage of the theory, it is evident that, given this interpretation of probability, the experimental frequency obtained by measuring many identical systems will approximate the theoretical value of probability (this is due to the law of large number\(^{11}\). However, we emphasize that this frequency cannot be taken as the meaning of probability; instead, it is simply a particular way of measuring its value in the laboratory; for example, another method, other than the above, often used in standard experimental practice is to measure the intensities of spectral lines in a spectroscopic apparatus. Clearly, these two distinct methods are simultaneously justified only by adopting the objective probability interpretation, and for a single system, mentioned in this section.

More precisely, given a physical system and a notion or pre-theoretical conception of some of its properties\(^{12}\), we can exactify and elucidate it (and thus construct a physical theory) if we can find a given mathematical structure \( M \) (exactification) such that some of its elements can be interpreted factually (elucidation) in terms of the physical notion under consideration. For example, for the case of probability, a physical theory of probability is obtained by specifying elements \( m \) and \( m' \) en \( M \) such that the semantic map \( \varphi : M \rightarrow \text{factual items} \) acts on them as follows:

\[
\varphi(m) = \text{fact} \quad \text{and} \quad \varphi(m') = \text{intensity of the propensity of this fact}.
\]

We’ll call the pair \( (M, \varphi) \) (and in which factual referents and their properties are implicit) an exactification-elucidation and possible physical theory of probabilities. Thus, the “exactification” of the concept of intensity of propensity is, once given this notion, the process of finding this semantic map and mathematical theory \( M \) for a given system (or class of systems.)

Of course, there may be many different mathematical structures that allow us to do this, which will predict different behaviors for probability. In the definitions below, we will take the pre-theoretical concept of probability as intensity of propensity and look for different exactifications for it. In particular, we will see that what fundamentally differentiates quantum theories from classical ones is precisely that they exactificate this notion of probability with different mathematical structures. This simple distinction will be, for us, the starting point of quantum theories.

**Definition 3.1 (Physical Theory of Classical Probabilities)** Let \( (X, \Sigma(X), \mu) \) be an abstract theory of

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\(^{11}\)Which is a mathematical theorem derived from the mathematical machinery of probability theories (we stress, it’s neither a definition nor an interpretation of probabilities, it’s just a theorem) that, in simple situations (such as the throw of a dice), can be physically interpreted in the sense that, if one makes \( N \) identical trials, and where the dice has the same intensity of the propensity for a given fact in each of them, then the number \( n \) of identical trials in which the fact actually happens increases when the mentioned intensity increases, which is what we would expect. Even more, as the number of identical trials gets very large, then the frequency \( \frac{n}{N} \) approximates better and better the theoretical value for the intensity of the propensity.

\(^{12}\)As is the previous notion of propensity; it is said pre-theoretical in the sense that we will define a physical theory as a mathematical formalism interpreted through factual semantics and then the previous physical conception is only halfway, still orphaned of a mathematical formalism.
functions, that is, \( f \) such that their probability distributions are identical and independent (and for which of the propensities to occur of these facts. Also, given two facts \( h_1 \) and \( h_2 \), we will denote as \( h_1 \vee f h_2 \) to the fact that consists in their physical conjunction, and as \( h_1 \land f h_2 \) to the (exclusively potential) fact that consists on their physical disjunction. Then, if one has a factual semantic interpretation of the previous abstract theory (or an exactification-elicitation of the physical concepts in terms of this abstract theory) such that the semantic map \( \varphi \) acts as:

\[
\varphi(E_h) = h \quad \text{(with } E_h \text{ arbitrary element of } \Sigma(X) \text{ and } h \text{ its corresponding fact in } H),
\]

\[
\varphi(\mu(E_h)) = I_h,
\]

\[
\varphi(E_{h_1} \cup E_{h_2}) = h_1 \lor f h_2 \quad \text{and} \quad \varphi(E_{h_1} \cap E_{h_2}) = h_1 \land f h_2 \text{ for all pair of elements } E_{h_1}, E_{h_2} \in \Sigma(X),
\]

we call the pair \([X, \Sigma(X), \mu, \varphi]\) a Physical Theory of Classical Probabilities. □

Given the notion of this last definition, two comments come to place:

- First, it is only in the context of physical probability theories that the usual names of “event space” for the measure space and “random variables” for measurable functions are consistent; in a purely abstract mathematical theory, they are completely out of place.

- Second, it is now possible to see the powerful physical implications of the law of large numbers. Suppose we have a physical system described mathematically by a classical theory of probabilities and we want to measure experimentally the numerical value of some average value \(< f >_\mu\) (of a property of the system represented by the function \( f \)) to compare it with the value predicted by the theory, in order to make an empirical contrastation of the latter. A priori, the physical theory does not suggest any experimental method that we can apply to measure this value. However, the law of large numbers offers us another mathematical way to calculate this value (namely, in terms of “frequencies” of certain infinite sequences.) Given the physical interpretation of probability as intensity of the propensity, what is important is that this does suggest an operational method (one of many possible) to obtain an empirical estimation of the value at issue and which consists simply in taking a very large number of identical physical systems, and prepared in the same initial state (since, when interpreted physically, the condition \(< f >_\mu = M\) means that all the systems must have the same propensity and probability distribution for the property being measured, which translates into them all having the same initial state; note that the use of different functions \( f_n \) for the same property rather than a single one is just an artifact to model the different values of the outcome of the measurement, what is relevant here is just the probability distribution for these values), in measuring the value\(^{15}\) of the property at issue, and finally in calculating the value of the frequency and empirical averages obtained from the

\(^{13}\)A well known purely mathematical result that will be mentioned later is the following. (Weak Law of Large Numbers) Let \((X, \Sigma(X), \mu)\) be an abstract theory of probabilities. \(\{f_n\}_{n \in \mathbb{N}}\) an infinite sequence of measurable functions \(f_n : X \rightarrow \mathbb{R}\), and such that their probability distributions are identical and independent (and for which \(< f_n >_\mu = M\) and \(\text{Var}(f_n) \leq \sigma^2 \) exist for all functions, that is, \(< f_n >_\mu < \infty \text{ and } \sigma^2 < \infty \); thus, it is obvious that \(< f_n >_\mu = M, \forall n \in \mathbb{N}, \text{ where } M \text{ is some fixed positive real number.} \) Then:

\[
Pr \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} f_k = M,
\]

\(^{14}\)Note how we obtain the intuitive notion of the addition of probabilities for mutually exclusive (i.e., \(E_{h_1} \cap E_{h_2} = \emptyset\)) facts:

\[
I_{h_1 \lor f h_2} = \varphi(\mu(E_{h_1} \cup E_{h_2})) = \varphi(\mu(E_{h_1}) + \mu(E_{h_2})).
\]

\(^{15}\)While this is itself a definition, it contains elements that are actually physical hypotheses, such as the existence of a semantic map that acts in a given way, etc.

\(^{16}\)That is, we assume here that the system is classical and that always has defined values. In this way, by measuring it, one simply reveals the value that the property already possessed. The situation is not so clear and simple for quantum systems; this is discussed in detail later on.
measurements in question. According to the law of large numbers\(^\text{17}\), the larger the number of identical experiments, the empirical values should be closer and closer to the theoretical value for the average value of the property predicted by the theory (if the latter is in fact correct, of course.) That is, it is the physical theory of probabilities (complete mathematical formalism plus its semantic interpretation) the thing that justifies the operational methods to put it to empirical test. As explained before, this is typical of theories in which their concepts have well-defined physical meanings.

**Definition 3.2 (Quantum Probability Measure)** Let \((\mathcal{H}, \langle \cdot, \cdot \rangle)\) be a complex Hilbert space and \(\mathfrak{P}(\mathcal{H})\) its lattice of orthogonal projectors. A quantum probability measure over \(\mathfrak{P}(\mathcal{H})\) is a function \(\rho : \mathfrak{P}(\mathcal{H}) \rightarrow [0, 1]\) such that:

a) \(\rho(0) = 0\) and \(\rho(1) = 1\);

b) \(\rho (\lor_{k \in \mathbb{N}} P_k) = \sum_{k=1}^{\infty} \rho(P_k)\) if \(P_k \in \mathfrak{P}(\mathcal{H})\), \(\forall k \in \mathbb{N}\), and \(P_k \cdot P_j = 0\) when \(k \neq j\). \(\square\)

It is simple to prove that, under the assumptions of (b), \(\lor_{k \in \mathbb{N}} P_k = s - \sum_{k=1}^{\infty} P_k\).

Note that, on each maximal set of commuting projectors, \(\mathfrak{P}_0(\mathcal{H}) \subset \mathfrak{P}(\mathcal{H})\), the restriction of \(\rho\) to any of them is simply the equivalent of a classical probability measure in \(\mathfrak{P}_0(\mathcal{H})\). However, by switching to a different \(\mathfrak{P}_0(\mathcal{H})\), in general, this restriction results in a classical measure which is different from the one arising from the restriction on the first subset considered. In this way, we can see the theory of the classical measure as contained and also generalized in our previous definition.

**Definition 3.3 (Abstract Theory of Quantum Probabilities)** We will call abstract theory of quantum probabilities to the triple \((\mathcal{H}, \mathfrak{P}(\mathcal{H}), \rho)\), where \(\mathcal{H}\) is a complex Hilbert space and \(\rho\) a quantum probability measure\(^\text{18}\) on \(\mathfrak{P}(\mathcal{H})\). \(\square\)

In line with the previous comment, we can see that one of the fundamental characteristics of quantum probability theories is that they introduce a "new mathematical degree of freedom" (absent in the classical case), namely the notion that two elements \(P, Q \in \mathfrak{P}(\mathcal{H})\) can commute among them or not. As we began to see, this brings important consequences, like what was said in the previous comment. This motivates the following definition:

**Definition 3.4 (Compatible and Incompatible Elements)** Let \((\mathcal{H}, \langle \cdot, \cdot \rangle)\) be a complex Hilbert space and \(\mathfrak{P}(\mathcal{H})\) its lattice of orthogonal projectors. Two elements \(P, Q \in \mathfrak{P}(\mathcal{H})\) are called compatible whenever they commute with each other, that is, \(PQ = QP\). Two elements \(P, Q \in \mathfrak{P}(\mathcal{H})\) are called incompatible if they do not commute with each other, that is, \(PQ \neq QP\). \(\square\)

**Definition 3.5 (Physical Theory of Quantum Probabilities)** Let \((\mathcal{H}, \mathfrak{P}(\mathcal{H}), \rho)\) be an abstract theory of quantum probabilities. Consider now a physical system, the set \(\mathcal{H}\) of all the potential facts that involve it and the intensities, \(h_h\), where \(h \in \mathcal{H}\), of the propensities to occur of these facts. Given two certain facts (i.e., not just any two given facts; this will become clear below) \(h_1\) and \(h_2\), we denote as \(h_1 \wedge h_2\) to the fact that consists in their physical conjunction, and as \(h_1 \vee h_2\) to the (exclusively potential) fact that consists on their physical disjunction. Then, if one has a factual semantic interpretation of the previous abstract theory (or an exactification-elucidation of the physical concepts in terms of this abstract theory) such that the semantic map \(\varphi\) acts as:

\(^{17}\)Actually, the theorem doesn’t say anything about finite numbers of experiments, it only mentions the infinite limit. Thus, we are extracting here from it more than what it says. More refined theorems, that tackle this subtle issue, exist. Here, we just mention the basic reasoning here with this version of the theorem, even if it’s not strictly correct, in order to illustrate what’s the actual relation between probabilities and experimental frequencies, which are often confounded (more on this later).

\(^{18}\)By Gleason’s theorem, there is a positive, trace-class operator, of trace 1, \(T\), (determined completely by \(\rho\)) such that \(\rho(P) = \text{tr}(TP), \forall P \in \mathfrak{P}(\mathcal{H})\). As a corollary (Kochen-Specker), then \(\rho\) cannot take values only in \(\{0, 1\} \subset [0, 1]\).
\[ \varphi(P_h) = h \] (with \( P_h \) arbitrary element of \( \Psi(\mathcal{H}) \) and \( h \) its corresponding fact in \( H \); furthermore, the correspondence in question must be a bijection),

\[ \varphi(\rho(P_h)) = I_h, \]

\[ \varphi(P_{h_1} \wedge P_{h_2}) = h_1 \wedge f h_2 \text{ and } \varphi(P_{h_1} \vee P_{h_2}) = h_1 \vee f h_2 \] for all pair of compatible elements \( P_{h_1}, P_{h_2} \in \Psi(\mathcal{H}), \)

we call the pair \( [(\mathcal{H}, \Psi(\mathcal{H}), \rho), \varphi] \) a Physical Theory of Quantum Probabilities.  

Given this last definition, two comments come to place:

- First, when describing different physical systems by means of physical theories of quantum probability, we are talking about different theories, because their concrete referent is different, even if the background is always a physical theory of quantum probability. This is why it would be more appropriate to talk about quantum theories, in the plural, instead of a quantum theory. In fact, what one does is to take this notion of physical theory of quantum probability, apply it to different types of concrete systems (namely, particles, fields, etc.), obtain the corresponding physical theories and study their particular properties. Note that we were relatively vague when talking about what types of facts are contained in \( H \). In principle, we will consider that this set contains all the possible facts that can happen to the physical system in consideration (in particular, if it has a physical property valued in the real numbers, the facts in which the value of this property is defined and contained in some \( B \in \mathcal{B}(\mathbb{R}) \)). These facts will depend on which are the particular properties of the concrete system under consideration; in this way, the facts alluded to refer to the physical entity as it is modeled and described by the theory (for example, if the referent is a quantum field, possible facts can be those referred to properties such as the energy or momentum of the field, both properties introduced by the theory itself); we can see, then, that theories introduce their referent and its properties; in this way, its existence in reality is hypothetical (note that this hypothetical existence is something that has no relation to the use of the concept of potential fact, which was used in the previous definition, as something that can potentially occur to the referent once it is hypothesized that it exists.) In other cases, we will not stop to try to build or make explicit \( H \) for each particular system, but we will assume that \( [(\mathcal{H}, \Psi(\mathcal{H}), \rho), \varphi] \) exists; we will do this since we are interested in studying quantum theories in a general way; that is, the object of study will almost always be a generic quantum theory. We will refer to the factual referent \( \sigma \) of a generic Physical Theory of Quantum Probabilities as a Quantum System.

- We will not bother in trying to define (beyond the pre-theoretical realm) \( h_1 \vee f h_2 \) or \( h_1 \wedge f h_2 \) when \( P_{h_1}, P_{h_2} \in \Psi(\mathcal{H}) \) are incompatible elements (and, thus, much less the action of the semantic map), since, as we will argue later in Proposition 3.2 below, these are not valid potential facts for a quantum system; also, we will not give a factual interpretation to the lattice operations \( P_{h_1} \wedge P_{h_2} \) or \( P_{h_1} \vee P_{h_2} \) when \( P_{h_1}, P_{h_2} \) are mutually incompatible (although, the projectors \( P_{h_1} \wedge P_{h_2} \) or \( P_{h_1} \vee P_{h_2} \) themselves can indeed represent some fact, what we are saying is that we don’t give a physical interpretation, in terms of conjunction or disjunctions of the facts \( h_1 \) and \( h_2 \), to the lattice operations, in this case; in this way, the non-Boolean lattice we use is partially interpreted.)

We will discuss later the so-called measurement problem, the role of the law of large numbers in quantum probabilities, and other related topics. We don’t discuss them here since they are not part of the basic semantic axioms of quantum probabilities.

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\[19\]In this way, on each \( \Psi_0(\mathcal{H}) \subset \Psi(\mathcal{H}) \), the physical theories of quantum probabilities are reduced to a classical physical theory of probabilities.
Proposition 3.1 (Characterization of Incompatible Events) Let \( \rho \) be a quantum probability measure and \( P \), an event such that \( \rho(P) = 1 \). Then, if \( Q \) is (pure)\(^{20}\) incompatible with \( P \), it always results that \( 0 < \rho(Q) < 1 \), where the inequality is strict. \( \square \)

In this way, we can see why the equivalent of global Dirac measures in quantum probability theories do not exist: as the previous proposition shows, it is because of the existence of incompatible events. This result, valid for any quantum probability measure, is not possible in classical probability theories (both because the existence of global Dirac measures and by the fact that quantum measures always distinguish when events are incompatible, which is remarkable since in the definition of the latter only properties of the projectors intervene, they have nothing to do with the measures) and shows us the radical mathematical and physical consequences of the existence of incompatible elements/events in theories of quantum probability.

Now, a key result that one can rigorously prove only in the framework we have adopted (or a similar one), since it’s a result that involves not only the mathematical apparatus of the theory, but also its physical semantical interpretation and the elements related to it. To most physicists, it’s obvious; nevertheless, it’s rarely stated clearly so that one can identify all of the very different elements that intervene.

Proposition 3.2 There is no projector \( P_{h_1 \land h_2} \in \mathfrak{P}(\mathcal{H}) \) such that \( \varphi(P_{h_1 \land h_2}) = h_1 \land h_2 \) for (supposed) potential facts \( h_1 \land h_2 \) given by the conjunction of two incompatible facts, \( h_1 \) and \( h_2 \). That is, a physical theory of quantum probabilities does not admit, or cannot model, the mentioned facts, and, therefore, they are not in the first’s factual base.

Proof: Consider \( h_{1,2} = h_1 \land h_2 \), defined at the pre-theoretical level as a fact which happens iff both \( h_1 \) and \( h_2 \) happen simultaneously. Now, in classical probability, given an event \( E_h \neq 0 \) that models the potential fact \( h \), there’s a Dirac measure \( \delta_s \), \( s \in E_h \), which is such that \( \delta_s(E_h) = 1 \); in quantum probability, given an event \( P_h \in \mathfrak{P}(\mathcal{H}) \), \( P_h \neq 0 \), that models the potential fact \( h \), the projector can always be decomposed as \( P_h = \sum_{i=1}^{\infty} u_i(u_i, \cdot) \), where \( \{u_i\}_{i=1}^{\infty} \) is an orthonormal basis of the projection space, and then there is a \( T_h = \sum_{i=1}^{\infty} C_i u_i(u_i, \cdot) \), where \( \{C_i\}_{i=1}^{\infty} \) is any series such that \( \sum_{i=1}^{\infty} C_i = 1 \) (for example, the geometric series \( \{\left(\frac{1}{2}\right)^i\}_{i=1}^{\infty} \)), such that \( pr_h(P_h) = tr(T_h P_h) = 1^{21} \). Suppose \( h_{1,2} \) is modeled by \( P_{h_{1,2}} \in \mathfrak{P}(\mathcal{H}) \), \( P_{h_{1,2}} \neq 0 \) (thus, it’s indeed in the potentiality of the system); then, there are states of the system, say \( \rho_{h_{1,2}} \), which give a probability equal to 1 to this fact. Suppose the system is in one of these states. Then \( h_{1,2} \) has maximum intensity in its propensity to happen. By our definition of \( h_{1,2} \), this, in turn, requires that both facts, \( h_1 \) and \( h_2 \), must also have maximum intensity of their propensity to happen, i.e., probability equal to 1, each of them, on that same state \( \rho_{h_{1,2}} \). However, as we saw earlier in Proposition 3.1, there is no such state for pairs of incompatible facts. Thus, any state \( \rho_{h_{1,2}} \), whose supposed existence implied this contradiction in the first place, cannot actually exist, and this implies that the fact \( h_{1,2} \) cannot be modeled by some \( P_h \in \mathfrak{P}(\mathcal{H}) \), \( P_h \neq 0 \), since, if this were the case, then these states have to exist, as we mentioned above. Furthermore, it cannot be modeled by \( P_{h_{1,2}} = 0 \) either. Indeed, we have \( P_{h_1} \neq 0 \) and \( P_{h_2} \neq 0 \) (otherwise, they would commute), and \( \rho(P_{h_{1,2}}) = 0 \), for any measure. Since \( P_{h_1} \neq 0 \), there must be a measure \( \rho_{h_1} \), such that \( \rho_{h_1}(P_{h_1}) = 1 \). Suppose the system is in that state, then \( \rho_{h_1}(P_{h_2}) = 0 \), since \( \rho_{h_1}(P_{h_{1,2}}) = 0 \) and the definition of \( h_{1,2} \), but this contradicts Proposition 6. Thus, the fact \( h_{1,2} \) cannot be modeled by the theory. \( \square \)

An analogous result holds for the disjunction \( h_1 \lor h_2 \) of incompatible facts. In this way, since a possible factual element but lacking a mathematical correlate in the formalism of the theory cannot be considered as a genuine factual element belonging to the whole of which the theory makes semantic allusion, we conclude that it is not in the intrinsic potentiality of a quantum system the possibility of the occurrence of the alleged facts \( h_1 \land h_2 \) or \( h_1 \lor h_2 \) when \( h_1 \) and \( h_2 \) are incompatible with each other.

\(^{20}\)That is, it cannot be further decomposed as the sum of a noncommuting and a commuting part with \( P \).

\(^{21}\)Indeed, intuitively, if the fact \( h \) is genuinely in the potentiality of the system (and, thus, actually modeled by the theory), that is, it can indeed happen, then its eventual occurrence (although the dynamics may establish that it does not happen) has to be able to be described by the theory; just to be in agreement with this. For this particular theory, if an event occurs, then the state of the system must be such that, at the instant that this happens, it assigns probability equal to 1 to this fact, since it is actually occurring (note that we are not saying that this probability has to be equal to 1 before the fact occurs.) Thus, if a fact can happen, there must be at least one state in the theory in which, if the system is in that state, then this fact happens.
We can see that this result is not a consequence of the non-distributivity of the lattice alone, but more a consequence of the whole formalism of quantum theory itself, no matter how we frame it: in a quantum system, these facts simply do not have a place and we must just live with that.

In relation to the material above, we make some pertinent comments here. The fact that the lattice \( \mathcal{P}(\mathcal{H}) \) is non-distributive has brought, throughout the history of the concept (that goes back to the years 1930s with von Neumann), diverse and quite extravagant interpretations, all of them incorrect, to us, and based on (to our point of view) elementary semantic misunderstandings. The most widespread, and old, of these is to denominate the physical theories of quantum probability as “Quantum Logic” and to make the interpretation that the non-distributivity of the lattice implies in some way a “new non-distributive logic for reality”.

First, note that we have completely ruled out the usual practice of naming the facts \( h \) as “possible experimental propositions about the system” (this is a key distinction we make here, i.e., propositions vs. facts) and the probability as “probability that the proposition is true in an experiment”. This is so because, in physics, one speaks of the probability of a fact, not of a “proposition”; in addition, bringing up the concept of “truth” entangles our vocabulary even further: the notion that a theory of physics is true or not is something that is established, in general, when comparing its predictions with the experiment, it has absolutely nothing to do with the (purely conceptual) stage of its formulation; this latter comment also applies if one takes the notion of “experiment” used in these propositions as belonging to some kind of operationalist physical interpretation. Facts and propositions are two different concepts. For example, the fact that the position of the system is defined and is \( q = 5m \) is something other than the proposition \( P = \text{“The position of the system is } q = 5m\text{”} \); in the first case, we are talking about a factual item that inhabits and occurs in the physical reality (i.e., those things that physical theories aim to describe), while the second is a mere statement, which can admit different types of truth (Theoretical truth, if compared with theoretical statements derived from a theory; Factual truth, if compared with statements obtained from empirical results). Physical theories of probability apply to the first, since they seek to mathematically model factual items; no physicist speaks of “the probability of a statement”: s/he speaks of the probability that a fact will happen.

The set of propositions (which includes statements derived from/about facts \( h \), not the fact itself; note that a fact has always associated at least one statement, the one it says that the fact occurs, while the converse is not necessarily valid) is in fact a Boolean algebra under the operations of logical conjunction and disjunction of statements, but this structure inhabits the abstract plane and in fact logic itself does, it’s not even possible of being refuted empirically since it does not refer to any physical system in the first place! (note that logic is about connectives and propositions and it’s completely independent from the propositions’ truth values.) In fact, the interpretation of a Boolean lattice as the logic of propositions is

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22 Even more, the concept of factual truth applies to propositions and may be different from the value of probability, according to theory, of the associated fact; for example, a theory can predict the value 1,0 for the probability of \( h \) and an experiment may determine, however, that the (factual) truth value of the corresponding statement, “the fact \( h \) occurs for sure”, is, for example, false.

23 A given proposition can be true according to one body of knowledge (either empirical or theoretical), but false according to another. This point of view is particularly adequate in the sciences, where one first makes deductions from hypotheses (which we take provisionally as true) to later put these deductions to some test; if the test says the deductions affirm something which is not true (in relation to some body of knowledge), then we correct the hypotheses, not the deductions (assuming, of course, that they are logically correct.)

In particular, we don’t take the point of view in which all propositions are “born” with an “intrinsic, definite, objective and eternal” truth value; instead, the given body of knowledge in consideration will establish if some proposition has a truth value and which one it is, i.e., it will assign, if possible, a truth value to the proposition. Compare this with the platonist view on logic, in which all propositions are born with an intrinsic and unalterable truth value (to the point in which a proposition and the statement that says that this proposition is true are basically taken to be the same thing) and where one can always state things like “\( p \) and \( q \) is true iff \( (p \text{ is true}) \) and \( (q \text{ is true}) \)” (for this, the truth value of \( p \) and also \( q \) must, of course, be defined from the beginning, as in the platonist view.)

24 The interpretation of these connectives is just the standard, logical one, which is a purely conceptual notion and, in particular, they don’t necessarily have any relation to the conjunction and disjunction of physical facts about a physical system (this possible relation, if any, should be established by the semantic interpretations of the physical theory.)
a typical example of what we called an abstract or mathematical interpretation. The fact that a Boolean lattice can be used to model propositional logic and also the set of facts in a classical classical theory of probability only shows the flexibility and vast applicability of the abstract mathematical concept of Boolean lattice. As should be obvious, in all the demonstrations and arguments one does in quantum physics, one always uses the ordinary (Boolean) usual propositional logic.

In other words, if \( e(h) \) represents the propositional statement of fact \( h \) (i.e., \( e(h) = "h \) happens") , the logical conjunction and disjunction of \( e(h_1) \) and \( e(h_2) \) are always defined and are Boolean operations for every pair of facts in both classical and quantum. If in physics we then decide to use another lattice in our physical theory of probabilities, this implies exactly nothing in relation to ordinary logic. If \( P_{h_1}, P_{h_2} \in \Psi(\mathcal{H}) \) are compatible, then we will postulate the point of view in which the propositional statement of \( h_1 \land_f h_2 \) (respectively, \( h_1 \lor_f h_2 \)) is given by the logical conjunction of \( e(h_1) \) and \( e(h_2) \) (respectively, disjunction).

Thus, with this and the results of the main text, we can see then that the value of theoretical truth (with respect to the quantum formalism developed so far) of the proposition or statement that affirms \( e(h_1) \land_L e(h_2) \), with \( h_1 \) and \( h_2 \) incompatible with each other, is always defined and is simply always false.

Note that we only claim that the proposition \( e(h_1) \land_L e(h_2) \) is false (because the fact \( h_1 \land_f h_2 \) is not in the potentiality of the system, since it’s assumed that \( \Psi(\mathcal{H}) \) exhausts all the potential events, and thus it can never happen) but not that this truth value has been obtained by analysing the (probability, i.e., with respect to a given state or probability measure) truth values of \( e(h_1) \) and \( e(h_2) \) separately for this case. This because, since \( h_1 \land_f h_2 \) is not modeled by the theory, the probability truth of \( e(h_1) \land_f h_2 \equiv e(h_1) \land_L e(h_2) \) is not defined and thus trying to analyze it and calculate it via the standard rules would be as inconclusive as trying to find a state \( \rho \) in classical mechanics such that for the fact \( h_3 \), in which the position of the particle is \( x = 3 \), gives a probability \( \rho(h_3) = 0 \) for a particle that can only move in the interval \([0, 2]\): this cannot happen since \( h_3 \) is not in the domain of any probability measure (which act on potential facts) because \( h_3 \) is not a valid potential fact for this particle (\( x = 3 \) is not in the kinematics of the particle.) Also note that the conclusion we reached here was done by analysing both the mathematical formalism and its physical interpretation; in particular, the meaning of the notion that the fact \( h_1 \land_f h_2 \) is not in the potentiality of the system, since \( \Psi(\mathcal{H}) \) exhausts all the potential events, and thus it can never happen.

4. **Is the Reality of Quantum Theory Fundamental?**

Note that physical probability theories, such as those in the previous section, only attribute to its referent new (i.e., besides the ones already considered and which depend on the nature of the system) properties such as the propensity, nothing more and nothing less. That is, the propensity, the mere potentiality, of a system for the occurrence of a given fact that involves it, is a real, actual, and objective property of it. What is real, is perfectly defined and exists at all times is the quantum system itself and its different propensities; this quantum system is the bearer of these propensities.

The question that names this section is actually rather tricky and rivers of ink have flowed in trying to address it. From one side, it’s indeed a fundamental theory, in the sense that its laws and predictions have not been superseded by a new and more precise theory. The debate revolves, instead, around the question...
about what is exactly the underlying reality “behind the surface” of a quantum probability theory, in a sense which we will explain in the next paragraph.

In a classical probability theory, there are probability measures $\delta$ (called Dirac measures) such that, for any fact they assign to it a probability of, strictly, either 0 or 1, that is, absolute certainty that the fact occurs or that it doesn’t. The fact could be, e.g., about a given value of energy, and these measures will tell us with precision if that’s actually the value of energy or not. In this way, all properties of the system have a well defined value. Thus, we can forget about probability, just take the trajectory $x(t)$, which we know is well defined, and build our mechanics around it. In this sense, if we have a probability here, we could say that, while which values occur and which do not occur was already intrinsically clear, for some reason or another we cannot describe the system with precision and thus we have to conform ourselves with a crude approximative theory with a referent only having a propensity for these values\(^{26}\) (determinism and defined values, though, are still still there, they are just behind the “surface”; thanks to this, in experimental situations, one can interpret the law of large numbers in a way similar to the example of the throw of a dice.) When the theory is compatible with an underlying reality of this type (where all the properties have defined values), one says that the theory admits “hidden variables” (them being, of course, the ones that make all the properties to have defined values.) Note that the “ignorance of information” take on probabilities, where they are not considered as real properties of the system, and, instead, after an experiment, one simply checks the system to “update” our information about its definite values, seems to suggest an underlying hidden variable reality (this is its true content.)

Now, the issue with quantum probability theories is that probability measures of the type of $\delta$ cannot exist now! (this key result is the Kochen-Specker theorem.) This means we can’t do here the trick that we did in the classical case, that is, a full, completely objective underlying hidden variable reality is simply not possible in quantum physics (there’s still a possibility for a restricted type, see discussion about the measurement problem below). Thus, at any given situation, there will be facts for which the intensity of their propensity to happen is greater than 0 but less than 1\(^{27}\). In this way, we seem forced to conclude that, if the value of a property is associated to one of these facts, it will be undefined. That is, not all properties of the system will have defined values, i.e., the very core of classical physics turns to be invalid. Now probabilities become the primary variable in the theory rather than, e.g., trajectories. In particular, it also means that the conjunction, $h_1 \land h_2$, of two incompatible facts, $h_1$ and $h_2$, is not an admissible fact for a quantum system, i.e., its eventual occurrence is not allowed even in principle (since, if the occurrence of their conjunction were permissible, this eventual occurrence would mean the simultaneous happening of the two component incompatible facts, something which the quantum formalism doesn’t allow even by a trick like the one of the classical case, because Dirac measures cannot exist on sets of facts which include facts incompatible with each other; thus, if the value of a property associated with one of these facts is defined, then, the value of a property associated with the other is, not, and viceversa.)

Properties that have a definite value are called manifest, while those which don’t, are called latent\(^{28}\).

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\(^{26}\)That is, we have two theories here: one whose referent only has propensities (of intensity less than one) and another whose referent has a definite trajectory. The first one is an approximation of the second one. In particular, from the point of view of the first theory, the propensities are true real properties of its referent, and not “information”. From the point of view of the second theory, the first theory is, of course, incomplete, and this is often loosely stated by saying that the propensities of the first theory are just “(incomplete) information that the experimenter has about the system” or that the propensities are just “epistemic”; the correct interpretation is the one we mentioned and not these last ones, there’s only one type of probabilities (interpreted physically as propensities, which are considered real properties of the system; thus, it’s not a question of semantics), the epistemic part is in the approximation process of one theory by another.

\(^{27}\)This is caused by the existence of incompatible facts in quantum physics: indeed, it’s impossible for two incompatible facts to have, each one, an intensity of their propensity equal to 1, and, even more, for some type of incompatible facts, the one which has probability less than 1 cannot have probability equal to zero, either.

\(^{28}\)Based on classical intuition, one may think that this distinction doesn’t seem to make much sense, since, if a property is not manifest, then one would think it’s not there, that it doesn’t exist. But, in classical physics, we can consider that the time evolution of all the defined values is given by a time dependent Dirac measure $\delta_t$; thus, the difference in quantum physics is that the dynamics is given by a time dependent general measure $\mu_t$. With this we can see that there are actually two layers, one is the fixed set of all the possible values that a property can take (which can be discrete), and other is the time dependent probability measure which indicates what’s the propensity for these values. It’s in the measure where the interactions with other things is included and this is the way in which they interfere with the dynamics. Nevertheless, all the machinery of the
furthermore, to be considered as existing, a thing must have at least one manifest property; if a thing ceases to have any manifest properties, then it ceases to exist according to our view. Some say that the implication, in quantum physics, that not all properties can be manifest at once implies the “death of realism in physics”. But this is a naive classical view of realism. The fact that a system can have some property whose value is not defined does not imply that the system “does not exist”, the system always exists and always has certain real and actual manifest properties defined: the very propensities of the system for the occurrence of facts. The position value it can adopt is not a reliable indicator of its continuous existence as in classical mechanics.

Nevertheless, the view in which values are not defined implies an evident problem: when and how these systems acquire a definite value, then? There are some cases in which this is very clear and non-controversial (for example, an electron in a homogeneous magnetic field, along the $z-$axis, is such that the value of the $x-$component of its spin periodically oscillates back and forth between being defined and undefined; this is a very simple exact solution to the equations for the time evolution of a quantum system.) But, there are some other cases which seem less straightforward to analyze. Indeed, the physical evidence seems to suggest the following. Given two Things $x$ and $y_{Q|R}$ ($R$ being a property of $y_{Q|R}$), if Thing $x$ has Propensities for the different values of a Property $Q$ of it, then the aggregation with $y_{Q|R}$, to form $x + y_{Q|R}$, will make the fact $h_q$, for some possible value $q$ of $Q$, to happen at some point, but one cannot predict with previous information which particular value will take place: that will depend on the Intensity of the Propensity towards each value that the system had, but it only depends on the value for this intensity that it had at the very moment before the aggregation, that is, the aggregation doesn’t modify the system’s intrinsic tendency, it only makes the fact $h_q$ to happen in an apparently unpredictable way. Thus, at the end of the aggregation, the situation becomes similar to the application of probabilities in the classical case: one has a definite value, but only a probability to predict which. In physical experiments, where the previous behaviour is observed, the law of large numbers (which is also valid in quantum mechanics, since every fact is contained in a Boolean sublattice of the lattice of projectors; in this case, the sublattice is the one for which $P_{h_q} \in \mathcal{F}^h_0(\mathcal{H})$) is then used to get an empirical estimate of the probability, which is compared with the theoretical value, and this is the standard procedure to test the predictions of quantum physics.

The problem of definite outcomes in this type of interactions is often called or known as “the measurement problem”, because of its role in experimental measurements of quantum systems (it’s also called the “collapse of the probability function” problem, since, in these processes, the function seems to pass abruptly from a value which is less than 1 to the value 1; although, this may be an incorrect view, since, at the microscopic level, the value may be still undefined and what makes it to seem defined is the interaction together with an idealized classical interpretation of the result.) Others think that the problem is much more far reaching and that it consists in the very problem about how quantum indefiniteness reconciles with the obvious definiteness of the classical, macroscopic world. In any case, the problem is rather complex and admits, at the very least, two layers: i) it’s rather unclear in what exactly consists the full and complex interaction process that makes a quantum system acquire a definite value (or appear to do it) when interacting with another physical system (this other system can be, but not necessarily, a measurement apparatus; the key aspect, however, seems to be that this second system is usually a macroscopic one) and if it can be explained by quantum physics itself (including modeling the apparatus itself as a quantum system, too) or

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29 Even when assuming this is needed to measure quantum systems, it’s not a semantical assumption here (it may be in operationalist approaches, where probability is semantically interpreted in terms of experimental frequences, but this is untenable since, as we mentioned, probabilities in quantum physics are only a function of the system alone, and, also, frequences only appear mentioned in a theorem derived from the basic mathematical machinery of probability theory, it plays no role in its axioms), instead, it’s just a physical assumption or hypothesis that it’s done when measuring quantum systems, and one which is supported by the evidence. It’s needed to give a usable physical interpretation to the law of large numbers, but it’s not mandatory for the basic formalism (with its semantic assumptions) to necessarily give this theorem a physical interpretation, if not by the mere fact, as we have seen here, that, unlike the dice example, it’s not a trivial thing to do. These situations are common in physical theories (for example, one cannot measure temperature with a gas thermometer without some additional physical information, not contained in the theory of thermodynamics, about the behaviour of this gas.)
another supplementary theory is needed (those who believe that quantum physics, as it’s known today, is the fundamental theory and that no hidden variables or modifications to the current formalism are needed, are naturally inclined to the first approach, which is rather problematic because the way in which quantum systems evolve in time seems precisely to forbid processes of this kind, but, on the other hand, this also is relative to if one is considering that the system actually acquires the value or just appears to do so from a classical, macroscopic perspective; the problem is often stated as a trilemma in which at least one of the following three options always has to be rejected in order to avoid a logical contradiction in the known mathematical formalism of quantum physics: a) there are no hidden variables, b) measurements always yield single, defined outcomes, c) the standard time evolution of quantum physics holds... the problem is that all the three options seem reasonable and difficult to give up! this is the root of all the controversies around the problem, since any way out from it implies the sacrifice of a cherished belief; thus, in particular, if one doesn’t want hidden variables, see paragraph below, and believes that the interaction implies a quantum dynamical process in which the probability distribution becomes sharp, then it seems a modification of the standard quantum dynamics will be needed); ii) its central role in the very procedure with which quantum physics’ predictions are experimentally tested. The “measurement problem” is one of the most infamous, controversial and discussed aspects in the foundations of quantum physics (it has been around, without an universally accepted solution, since the very origin of quantum physics itself, in the late 1920s; both Einstein and Schrödinger famously condemned the standard interpretations of quantum physics because of it; the latter with his famous experiment in which the “value of life”, which can be either dead or alive, of a cat, a macroscopic thing, is undefined if described by quantum physics alone.) It should also be noted that, while there seems to be indeed some sort of indeterminism (since one can’t predict which value a property is going to take because we only have an intensity for the propensity towards the occurrence of each of these values), it’s still unclear if quantum systems acquire a manifest property only after an interaction or if they can spontaneously do so (which is an even more radical form of indeterminism; although, one may consider that an interaction with the ever present, but diffuse, environment causes it.)

We mention, though, that there’s actually a final attempt to give some classical flavor, i.e., hidden variables, to quantum physics. As we mentioned, classical probability is locally valid over sets of facts related to properties that can take definite values simultaneously, that is, compatible facts. Thus, at each of these local sets, we have local Dirac measures and, then, we can do the trick we did in the classical case: for the properties involved in the local set, all values are well defined and the probability given by quantum physics is just an approximation. But, we can only do this at only one set at a time, since a same property may belong to two different local sets but the values assigned to it in each of them cannot be, at least for one property, the same (since, otherwise, we risk at getting a global Dirac measure.) Thus, this, by itself, is not very helpful. The move is to bring back operationalism, to some extent, and say that a set of measuring devices/apparatuses (or, maybe, a very particular type of environment, just to avoid talking about instruments in an abstract way and the usual conceptual fog that this brings) prepared to measure the properties for a given local set “select/create” one of these local valuations for it. Now, as we already saw, the mathematical form of an observable (and, thus, its possible values) doesn’t depend, in the quantum formalism, on the “context”, and, then, the role of the measuring instruments, the context, is just to create (each time we measure) the valuations which give different values each time and that we cannot predict with the quantum formalism alone (presumably, which, exactly, are the actual assigned values will depend on the details of the instruments’ composition; note that this environment still can appear in the total energy function $H$ of the quantum theory and, in this way, may still produce at least some effects in the dynamics that can be described by it.) As a first point, the reader may already have noticed the similarity betwen this and what we called the measurement problem, it’s still here, just disguised in another form: while values are technically defined, when we pass to another context, the value assigned to a same property changes abruptly, or, properties incompatible with the ones of the previous context, suddenly acquire values; the difference is that, here, the explanation of how this happens is outside the quantum formalism, while in the standard collapse approach it’s assumed that the collapse process should be explained by the formalism, since its reality is assumed fundamental. Having said all this, if we restrict to situations in which

30For example, in the radioactive decay of radioactive materials, the nucleus of an atom seems to spontaneously split into two lighter nuclei, and, of course, while emitting radiation in the process.
there’s always some measuring setup around the quantum system, then the difference between the two different approaches in discussion reduces to: either, 1) the measuring context creates a valuation (via some “contextual hidden variables”, i.e., hidden variables that change from context to context), which we don’t know with precision, and the quantum formalism just serves to give a crude probabilistic approximation, or 2) the measuring context makes the propensity to go sharp, or to “collapse”, at some values, which we don’t know with precision, for the different compatible properties of that context and where the occurrence of a given value depends on the intensity of the propensity at the instant before the measurement (intensity whose value is exactly equal to the probability value of the previous point.) A thing to note is that, in the second approach, if the probability is 1 for some fact, then one interprets that it’s happening, since the probability measure provides a complete description of the system; but, in the first approach, it can only be considered to be happening if measuring devices are giving a valuation to the context to which it belongs (if they are measuring another context, then the values associated to the first one are undefined, since the valuation is now in the second context; thus, the probability provided by quantum physics should be interpreted with care here, since the interpretation in which they are complete descriptions will give a misleading picture of the underlying reality.) Another thing to notice with the first approach (besides the fact that it, still, cannot assign, simultaneously, definite values to incompatible properties, since their contexts are different) is that it’s not valid for a completely isolated particle, which is a widely used notion in, e.g., particle physics and perturbative QFT. A further questionable aspect of these interpretations is the completely ad hoc nature of the hidden variables (in fact, most of what happens would actually be described by something outside quantum physics, the latter being reduced to a rather modest role) and their rather peculiar “contextual” dependence, which is anathema to classical physics, that is, precisely the thing that one supposedly wanted to recover in the first place with all this approach; thus, the victory is at best a pyrrhic one.

Thus, quantum theory can be interpreted either with contextual hidden variables or with propensity as fundamental (that is, without an underlying hidden variable reality.) That is, in either case, if we want interpretations with a coherent semantics, the result is a realist ontology, since at every moment at least one property has a defined value (in the first, given by the “context” properties used to define the contextual hidden variables, and, in the second, simply the propensity itself.) Furthermore, in both cases, the so-called wavefunction collapse can be added as a non-semantic axiom (unlike the operational interpretations, where the collapse is indeed a semantic axiom, and, therefore, the theory cannot even be formulated without postulating it, nor can the theory be used to explain it; in the realist approaches, is not a semantic axiom but it can be postulated as a law if necessary, and, in this way, it still can intervene in the meaning of, e.g., probability, the law of large numbers, and experimental frequencies.) It should be stressed, though, that in whatever of the two cases, the interpretation of probability in quantum theory must be done in terms of propensity if one pretends semantic coherence (of course, in the hidden variables case, it’s not going to be the fundamental reality, but only an effective one.)

5. Quantum Gravity: a Possible Way Out

Thus, regarding collapse, we have, as discussed in the previous section, two options with two sub-options each. That is, when a certain context-defining interaction occurs, then, either the quantum probability distribution (taken as fundamental and objective) becomes sharp, or the values of some contextual hidden-variables define (for this particular context, and get undefined for incompatible contexts). Furthermore, this sharpening/value-definition is either modeled as some explicit time evolution or dynamics (in the first case, necessarily non-linear; in the second, some dynamical equations for the hidden values must be provided), or, in both cases, it’s added as an ad-hoc postulate without further clarification (of course, this breaks the usual Schrödinger evolution by definition).

31Note that, unlike the above, in the classical case, where the (objective) valuation is just a single and global one, if the probability is one, the valuation also agrees (for this value) with the picture given by the approximative theory.
Now, a possible phenomenon in quantum gravity may be the discretization of time duration into a succession of finite fundamental processes. Furthermore, each of these processes is subjected to the usual quantum randomness in their possible occurrence. That is, a context-defining interaction will randomly produce these occurrences. Physically, for this picture to be coherent, it must be such that time advances as a succession of instants which are very close to each other in proper time distance and in which the duration of the instants themselves is very small. Thus, at the macroscopic scale, this is perceived as a succession of instants, each of duration zero, which forms a continuum whose subsets have finite duration, and which monotonically increases (since, upon change, almost all intermediate steps are visited, and, thus, the system can interact with whatever thing that resides at those steps), that is, the classical picture of time. The quantum system at an event is surrounded by a dispersion cloud of events which can visit next, and the classical proper time is just some average $<\tau>$ of that dispersion, and the actual quantum transitions measure how much the actual quantum time deviates from this average $<\tau>$.

But this means that quantum collapse modeled as some explicit time evolution or dynamics is untenable here, and that only the remaining one is consistent. Indeed, in our view, the standard classical time, which, among other things, is used as the external time parameter to define, e.g., the Schrödinger time evolution or any modification, is only an emergent feature at the macro level, and fueled at the micro, fundamental level by an irreducible collapse. Thus, the time evolution under some classical time parameter $\tau \approx <\tau>$ will break when something from the fundamental level leaks to the macroscopic level. And, of course, this is precisely the situation in a quantum measurement, when the state collapses when certain two systems interact: the classical time will never be able to explain this process since the latter intervenes precisely in making possible the quantum, and therefore also the classical, time.

In classical GR, time is a layered notion consisting in \cite{3}: i) mutability; ii) duration; iii) change. Only the first two are explicitly modeled by GR’s formalism, and its results are read as those for a changing world, but change remains a baffling thing that has to be taken as ontologically fundamental and without further explanation. But now, in the previous QG possible scenario and unlike the classical case, we don’t need to introduce change in an ad-hoc manner here, since change can be seen as arising from quantum collapse \cite{4} (now taken as ontologically fundamental and irreducible) after an interaction. We take the collapse as the only source of change and actually identify it with it (in other views, collapse, of course, implies change, but the converse is not necessary; here we say they are indeed the same thing.) Thus, the change in the classical theory actually comes from the fundamental quantum theory, of which the former is a limit. Furthermore, in light of this, then the argument used to show the necessity of the collapse\cite{4} in QM can be now used to show the necessity of change, which then becomes a quantum phenomena and very tied to the characteristic non-commutativity of quantum properties.

Thus, QG may achieve an ontological synthesis in the ontology of today’s physics. At one hand, QM

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The argument consists in noting that, due to the Kochen-Specker theorem, not all values can be defined simultaneously for a same state, it’s only possible locally for any maximal set of commuting properties; thus, if the value of a property $A$ is defined in an initial state as well as that of another property $C$, which commutes with the first one but also with another property $B$ (whose value is undefined in the initial state) which is non-commuting with $A$, a state that makes $B$ to have a value cannot be the same as the initial one, since $A$ cannot have a defined value in this second state due to the non-commutation, and, in addition, the value of $C$ in the second state may be different from the first value. In this way, it’s clear that the passing from the initial state to the second one will involve a jump in the definition/indetermination of the values of $A$ and $B$, and a possible jump in the value of $C$. But this doesn’t say if this jump actually ever happens in reality. Now, experimentally, it’s known that certain type of interactions select a state for the system. Thus, if we subject the system at the same time to two interactions, the first which selects the previous initial state and the second which selects the other state, then we reach a paradox since those states cannot be the same. What this means, of course, is that we cannot have at the same time the two previous interactions happening to the system. One solution is that only one of the interactions happens in the present universe and then the other is forever forbidden to happen to the system. A second one is that a particular type of change given by the mentioned jump does exist and that one interaction can only happen after the other in this case (in our approach, this type of change is, furthermore, the only one that actually exists.) Of course, given the experimental results, it’s clear that the correct option is the second one. What’s interesting of this is that one is forced to postulate it by the quantum formalism itself (although, supplemented with some experimental inputs, as we saw.) In classical physics, where all values are always defined, no paradox is present, and then one can add or subtract the notion of change without producing any logical contradiction inside the formalism. We note that quantum collapse cannot be “illusory”, since its necessity comes from very basic features of quantum theory. This means that, in our approach, change cannot be “illusory” either, then.
informs GR about the quantization of proper time, on the other, GR, by putting time itself as something to quantize, brings its layers, change in particular, to inform QM.

References

[1] Bunge, M. (1977). Treatise on Basic Philosophy (in 8 Tomes; D. Reidel Publishing Company, Dordrecht, Holland); Bunge, M. (1967). Foundations of Physics (Springer).

[2] Moretti, V. (2013). Spectral Theory and Quantum Mechanics: With an Introduction to the Algebraic Formulation, UNITEXT, vol. 64 (Springer-Verlag, Berlin); Landsman, N.P. (2017). Foundations of Quantum Theory: From Classical Concepts to Operator Algebras (Springer).

[3] Ascárate, A. (2021). Spacetime Relationalism in GR and QG (URL https://arxiv.org/abs/2109.03297).

[4] Ascárate, A. (2021). The Spacetime Picture in Quantum Gravity II (URL https://arxiv.org/abs/2107.06693); Ascárate, A. (2021). The Spacetime Picture in Quantum Gravity (Class. Quantum Grav. 38 165010; URL https://arxiv.org/abs/2012.03994).