Weight Optimization of Thick Plate Structures Using Radial Basis Functions Parameterization

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Abstract. This study sought to optimize the weight of thick plate structures with topology optimization. Topology optimization is a numerical method for identification of voids in the design domain, to aid weight and material reduction. The level set approach is a new method in topology optimization. In this work, the radial basis functions level set approach is used to parameterize the continuum domain. The radial basis functions are used to parameterize the level set functions, which comprise axial symmetric, real value functions. These functions are suitable tools for function approximation. Thick plates are one of the most useful and familiar geometries in structural and mechanical engineering applications. Topology optimization of such structures would be very useful to identify efficient and lightweight plate structures. In this paper, the effectiveness of the parametric level set approach in topology optimization of clamped plate is demonstrated. The results show smooth boundaries and there are no intermediate densities, which constitutes a major advantage.

Keywords. Topology Optimization, Level Set Approach, Radial Basis Functions Approximation

1. Introduction

Topology optimization a computational method which helps to decrease the material and weight in solid structures. It was introduced by Bendsøe and Kikuchi [1]. There are different approaches by which to parameterize the continuum design domain. The most famous approaches are density based, homogenization approaches [2], ground structures [3] and the level set method [4]. Homogenization-based approaches are highly suitable for determining the optimal topologies for cellular structures [5]. However, due to problems in controlling the length scale, using these methods may lead to checkerboards and single node hinges in compliant mechanisms. Density-based approaches same as solid isotropic material with penalization (SIMP) are well-known approaches in topology optimization [6]. The problem with these methods is the dependency of the results on the number of meshes and the penalty factor.

A novel method for analysis of front propagation in continuous media is proposed by Osher and Sethian [7] and called the level set approach. This method was first used in topology optimization by
Sethian and Wiegmann [8]. Allaire used intermediate densities for voids in the level set and sensitivity analysis for updating the level set function [9]. A discrete form of level set approach was presented by Challis [10]. Topological derivatives are used to modify the Hamilton Jacobi equation in this paper.

Recently, reaction diffusion equation has been used for updating the level set function in topology optimization [11]. An explicit representation of level set by means of moving morphable components (MMC) is the approach proposed by Guo et al [12]. They used a method of moving asymptotes (MMA) [13] to update the positions, degrees, lengths and widths of level set components for two- and three-dimensional structures [14]. This method has also been used for topology optimization of rib stiffeners for plate structures [15], and was also effective for optimization of curved skeletons [16].

Another explicit idea for level set topology optimization is to consider voids in the level set domain and move them in order to find the optimized topologies. This is called moving morphable voids (MMV) [17]. Bujny et al adopted the evolutionary algorithms with MMC for topology optimization of structures under crash loaded cases [18,19]. The algorithm shows good performance in the design of thin walled structures in automotive applications [20].

Recently, Wang [21] used radial basis functions to parameterize the level set function. The method was shown to be powerful and efficient.

In this paper, radial basis function parameterization is used for topology optimization of thick plates. Mindlin theory is used for analysis of thick plates under bending loads. Finite element analysis, based on Mindlin theory, is utilized for the analyses of bending. The aim of this work is to investigate the effectiveness of radial basis function parameterization in the topology optimization of Mindlin plate structures.

2. Thick plate theory

First, the Mindlin plate theory for analysis of bending plates is described briefly. In this theory, the transverse shear deformations will be taken into account and transverse displacements and slopes are independent. There are some assumptions in this plate theory, which are as follows [22]:

1. Vertical elements will remain perpendicular to middle surface after bending.
2. The transverse normal lines will rotate but they will not remain normal to middle surface.
3. A rotated line which was normal to mid-surface will have two rotational components as \( \theta_x, \theta_y \).

The displacement field of bending plate \( u, v, w \) are as follows [22]:

\[
\begin{align*}
    u &= z \theta_x, \\
    v &= z \theta_y, \\
    w &= w(x, y),
\end{align*}
\]

(1)

In which, \( x, y \) are the directions and \( \theta_x, \theta_y \) are the rotation terms. The strain field can be explained as follows:
By assuming the homogeneous isotropic material, the shear strains \( (r_{yz}, r_{xz}) \) are related to each other as follows:

\[
\begin{bmatrix}
{r}_{yz} \\
{r}_{xz}
\end{bmatrix} = \begin{bmatrix}
\frac{E}{2(1-\nu)} & 0 \\
0 & \frac{E}{2(1-\nu)}
\end{bmatrix} \begin{bmatrix}
{\gamma}_{yz} \\
{\gamma}_{xz}
\end{bmatrix},
\]

(3)

The moments in the Mindlin plate theory are as follows:

\[
\begin{bmatrix}
{M}_{x} \\
{M}_{y} \\
{M}_{xy} \\
{Q}_{y} \\
{Q}_{x}
\end{bmatrix} = \begin{bmatrix}
\frac{Eh^3}{12(1-\nu)} & \frac{\nu Eh^3}{12(1-\nu)} & 0 & 0 & 0 \\
\frac{\nu Eh^3}{12(1-\nu)} & \frac{Eh^3}{12(1-\nu)} & 0 & 0 & 0 \\
0 & 0 & \frac{Eh^3}{24(1+\nu)} & 0 & 0 \\
0 & 0 & 0 & \frac{Eh}{2(1-\nu)} & 0 \\
0 & 0 & 0 & 0 & \frac{Eh}{2(1-\nu)}
\end{bmatrix} \begin{bmatrix}
{\frac{\partial \theta_x}{\partial x}} \\
{\frac{\partial \theta_y}{\partial y}} \\
{\frac{\partial w}{\partial y}} + \frac{\partial \theta_y}{\partial y} \\
\frac{\partial w}{\partial y} + \frac{\partial \theta_x}{\partial x} \\
\frac{\partial \theta_x}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial \theta_y}{\partial y}
\end{bmatrix},
\]

(4)

The strain energy which consists of bending and shear energies is described in equation (5):

\[
U = U_B + U_S = \frac{1}{2} \int_{A} X_B^T D_B X_B \, dA + \frac{\kappa}{2} \int_{A} X_s^T D_s X_s \, dA,
\]

(5)

In which the energy coefficient \( \kappa \) is a constant parameter. And also:

\[
X_B = \begin{bmatrix}
{\frac{\partial \theta_x}{\partial x}} \\
{\frac{\partial \theta_y}{\partial y}} \\
\frac{\partial w}{\partial y} + \frac{\partial \theta_y}{\partial y}
\end{bmatrix},
\]

\[
X_s = \begin{bmatrix}
\theta_y - \frac{\partial w}{\partial y} \\
\theta_x - \frac{\partial w}{\partial y} \\
\frac{\partial w}{\partial y} + \frac{\partial \theta_x}{\partial x} + \frac{\partial \theta_y}{\partial y}
\end{bmatrix},
\]

\[
D_B = \frac{Eh^3}{12(1-\nu^2)} \begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{bmatrix},
\]

\[
D_s = G \begin{bmatrix}
h & 0 \\
0 & h
\end{bmatrix},
\]
3. Finite element analysis

For finite element analysis, displacement and rotation terms should be interpolated. Mindlin plate under bending loads has three degrees of freedom, which can be interpolated for \( n \) nodes as follows [22]:

\[
\begin{align*}
\mathbf{w} &= \sum_{i=1}^{n} N_i(\xi, \eta) \mathbf{w}_i, \\
\mathbf{\theta}_x &= \sum_{i=1}^{n} N_i(\xi, \eta) \mathbf{\theta}_{xi}, \\
\mathbf{\theta}_y &= \sum_{i=1}^{n} N_i(\xi, \eta) \mathbf{\theta}_{yi},
\end{align*}
\]

(6)

In the above equation, \( N_i \) is the shape function for Mindlin elements.

4. Level set topology optimization

The objective function of this optimization is the compliance and the volume is the constraint. Therefore, the problem can be formulated as:

\[
\begin{cases}
\text{Minimize} & C = \sum_{i=1}^{n} \int f^i \cdot u dV + \int t \cdot udS \\
\text{subject to} & V \leq V^T
\end{cases}
\]

(7)

Where \( C \) is the compliance, \( f^i \) is the body force and \( t \) is the surface traction, \( u \) is displacement field and \( V^T \) is the target volume fraction. The level set function is interpolated based on multi quadratic (MQ) splines as follows [23]:

\[
\phi(x, t) = \sum_{i=1}^{n} \alpha_i(t) g_i(x) + p(x, t),
\]

(8)

In which, \( p \) is a polynomial and \( g_i \) is the MQ spline (which is a radial basis function [23]) with coordinate of \( x_i = (x_i, y_i) \), formulated as follows:

\[
g_i(x) = \sqrt{(x - x_i)^2 + c^2}, x_i \in D
\]

(9)

In equation (8), \( \alpha_i \) is the expansion coefficient, formulated as:

\[
\alpha(t) = \left[ A - P \right]^{-1} \phi(t),
\]

(10)
in which:
\[
A = \begin{bmatrix}
g_1(x_1) & \ldots & g_n(x_1) \\
\vdots & \ddots & \vdots \\
g_1(x_n) & \ldots & g_n(x_n)
\end{bmatrix},
P = \begin{bmatrix}
1 & x_1 & y_1 \\
\vdots & \ddots & \vdots \\
1 & x_n & y_n
\end{bmatrix}
\]

Now, the parameterized level set function can be formulated as:
\[
\alpha(t) = G^{-1}\alpha(t),
\]

(11)

The Hamilton-Jacobi equation for updating the parameterized level set function is based on Equation (12).
\[
g(x) \frac{d\alpha(t)}{dt} - V_n \| \nabla g(x) \alpha(t) \| = 0,
\]

(12)

In equation (12), \( V_n \) is the normal velocity for changing the level set function, described in Equation (13).
\[
V_n = \varepsilon(u) : C : \varepsilon(u) - \lambda,
\]

(13)

In which \( \lambda \) is the Lagrange multiplier which updates as follows [23]:
\[
\lambda^{k+1} = \begin{cases}
\mu G^k & k \leq n_R \\
\lambda^k + \gamma G & k > n_R
\end{cases}
\]

(14)

In which:
\[
G^k = \int_D H(\phi) d\Omega - \left[ V_0 - (V_0 - V_{max}) \frac{k}{n_R} \right],
\]

(15)

5. Results

The problem considered in this paper is a plate, fixed from all edges, and there is a force in the middle of it as shown in Figure 1. The prescribed volume is equal to 0.7 (70 percent of the whole model). The length of the square plate is equal to 1 and the width is considered as 0.1. The material is considered to be homogeneous and the load is one kilonewton.

**Figure 1.** A plate with all edges fixed under a bending load.

The results of optimization process are illustrated in Figure 2. The level set method starts with initial holes and volume of 0.93 and finally converges to the value of 0.66 for volume. It should be noted that in this work, the topological derivatives are not going to be used in the Hamilton-Jacobi equation. So, some initial holes, which are illustrated in Figure 3a, should be defined to improve the convergence properties.
The compliance is also converged to value of 148. It should be noted that the iteration 71 in the related figure is the step in which the algorithm is converged to the solution. The material properties are similar to the paper presented by Sigmund [6]. MATLAB programming is used for this optimization process. The material domain and corresponding level set functions are also available in Figure 3 and the optimum value is equivalent to Figure 3e.

Figure 2. The convergence plot for Mindlin plate bending.
Figure 3. The optimized material distribution and corresponding level set function. (a) Material distribution in iteration 1 (b) Level set function in iteration 1 (c) Material distribution in iteration 30 (d) Level set function in iteration 30 (e) Material distribution in iteration 71 (f) Level set function in iteration 71.

6. Conclusion

In this paper, the parametric level set method is used for topology optimization of thick plates. Mindlin plate theory is adopted with finite element analysis for analysis of the structure under bending loads. The results demonstrate that the level set approach is a suitable tool for topology optimization of plate structures. The results are presented based on convergence plot, material domain and evolution of level set function. In comparison with state-of-the-art methods, using the parametric level set leads to smooth boundaries without intermediate densities. The results have also good manufacturing properties.

7. References

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