Optical bistability in a nonlinear-shell-coated metallic nanoparticle

Hongli Chen1,2, Youming Zhang2, Baile Zhang3,4 & Lei Gao1,4

We provide a self-consistent mean field approximation in the framework of Mie scattering theory to study the optical bistability of a metallic nanoparticle coated with a nonlinear shell. We demonstrate that the nanoparticle coated with a weakly nonlinear shell exhibits optical bistability in a broad range of incident optical intensity. This optical bistability critically relies on the geometry of the shell-coated nanoparticle, especially the fractional volume of the metallic core. The incident wavelength can also affect the optical bistability. Through an optimization-like process, we find a design with a broader bistable region and lower threshold field by adjusting the size of the nonlinear shell, the fractional volume of the metallic core, and the incident wavelength. These results may find potential applications in optical bistable devices such as all-optical switches, optical transistors and optical memories.

Optical bistability has attracted remarkable attention in recent years because of its promising applications in logic functions1, metamaterials2–5, all-optical switching6 and low power lasing7. An optical bistable system has two stable transmission states to switch on and off, depending upon the history of input signal8–10. As a conventional type of nonlinear materials, Kerr materials are widely studied in optical bistability. In view of its weak nonlinearity, sophisticated structural design is needed in Kerr nonlinear devices, aiming for faster switching speed and broader range of operation for incident intensity11.

In this report, we study the optical bistability of a metallic nanoparticle coated with a nonlinear shell based on a self-consistent mean field approximation in the framework of Mie scattering theory. We decompose the scattered fields of the coated nanosphere into spherical waves with Debye potentials in order to establish the relationship between the local field of the shell and the incident field. The self-consistent mean field approximation is then adopted to study the optical bistability. Our results match well with the previous quasi-static solution of the Laplace equations at a deep-subwavelength scale12. On the other hand, our full-wave solutions are valid in a more strict sense, and thus are more suitable for structural design of optical bistable devices, in which various parameters need to be varied in a wide parameters space.

Results

Theoretical development. We first consider electromagnetic wave scattering from the coated metallic nanoparticle. The coated particle, as shown in Fig. 1, has a metallic core with radius a and a shell with outer radius b. The relative permittivity and permeability of the core (the shell) are \( \varepsilon_c \) and \( \mu_c \) (\( \varepsilon_s \) and \( \mu_s \)). The surrounding medium has a relative permittivity \( \varepsilon_m \) and a relative permeability \( \mu_m \). We assume that the incident plane wave propagates in the \( +z \) direction, with electric field polarized in the \( x \) direction:

\[
E^m = \mathbf{E}_0 e^{j k_m r \cos(\theta)},
\]

where

\[
k_m = k_0 \sqrt{\varepsilon_m \mu_m} = \frac{\omega}{c} \sqrt{\varepsilon_m \mu_m}.
\]

In time-harmonic cases, Maxwell’s equations in the core, shell and surrounding media can be written as

\[
\nabla \times \mathbf{H}_i = -j \omega \varepsilon_m \mathbf{E}_i, \quad \nabla \times \mathbf{E}_i = j \omega \mu_m \mathbf{H}_i,
\]

where \( i = c, s, m \), denote ‘core’, ‘shell’ and ‘surrounding medium’, respectively.
respectively. When all media are linear, fields can be expressed with Debye potentials. By matching the boundary conditions at $r=a$ and $r=b$, the scattered field in the surrounding medium and the fields in the core and shell can all be solved for both transverse-electric (TE) and transverse-magnetic (TM) waves (see details in Methods). Since the nanoparticle under consideration is much smaller than the wavelength, the 1st order TM wave is generally sufficient for numerical calculation. More orders can be included if more accurate results are required.

Now, we consider the case in which the coated shell is a Kerr material with weak nonlinearity. The electric displacement $D_s$ and the electric field $E_s$ in the shell can be written as:

$$D_s = \varepsilon_0 \varepsilon_s E_s \approx \varepsilon_0 (\varepsilon_s + \chi_s |E_s|^2) E_s,$$

where $\varepsilon_s$ is the nonlinear permittivity of the shell, which is related to the linear permittivity $\varepsilon_s$, the nonlinear susceptibility $\chi_s$ and the local electric field intensity $|E_s|^2$ of the coated shell. To solve the field in the nonlinear shell, we adopt self-consistent mean field approximation. The nonlinearity of the shell is very weak, meaning that the linear part $\varepsilon_s$ is much larger than the nonlinear part $\chi_s |E_s|^2$. Thus, the nonlinear permittivity of the shell can be expressed as:

$$\varepsilon_s = \varepsilon_s + \chi_s |E_s|^2 \approx \varepsilon_s + \chi_s \langle |E_s|^2 \rangle_s,$$

where $\langle |E_s|^2 \rangle_s$ corresponds to the average of the field intensity in a linear shell. It can be calculated as:

$$\langle |E_s|^2 \rangle_s = \frac{1}{V_s} \int \int \int (E_{0s} E_{0s}^* + E_{as} E_{as}^* + E_{bs} E_{bs}^*) dV_s.$$

Following Refs 15 and 16, we obtain,

$$\langle |E_s|^2 \rangle_s \approx 9 \eta_0^2 D_1^TM \left[ \frac{\psi_1(k,b)}{(k,b)^2} - \frac{\psi_1(k,a)}{(k,a)^2} \right]^2 + E_1^TM \left[ \frac{\chi_1(k,b)}{(k,b)^2} - \frac{\chi_1(k,a)}{(k,a)^2} \right]^2,$$

where $\eta = a^3/b^3$ is the fractional volume of the metallic core, $k = \kappa_0 \sqrt{\mu_0 / \varepsilon_0}$, $D_1^TM$ and $E_1^TM$ are the expansion coefficients of Debye potentials for TM waves. Detailed expressions can be found in Method. After replacing the linear permittivity $\varepsilon_s$ in Eq. (6) with the field-dependent nonlinear permittivity $\varepsilon_s$ in Eq. (4), we can obtain a bistable relation between electric field intensity of the incident wave $E_0^2$ and the average electric field intensity in the nonlinear shell $\langle |E_s|^2 \rangle_s$.

The optical bistable response of a coated sphere has been studied previously using quasi-static approximation which shows the relation between the local field average in the shell and the external applied field as

$$\langle |E_s|^2 \rangle_s = \langle |\tilde{R}_s|^2 + 2\eta |\tilde{C}|^2 \rangle E_0^2,$$

where

---

**Figure 1. Geometry of scattering of a plane wave by a coated sphere.** The radius of the core is $a$ and the outer radius is $b$. The incident plane wave is polarization along the $x$-direction and propagates along $z$-direction. The relative permittivity and permeability of the core (the shell) are $\varepsilon_c$ and $\mu_c$ ($\varepsilon_s$ and $\mu_s$). The surrounding medium has a relative permittivity $\varepsilon_m$ and a relative permeability $\mu_m$. 

Comparing the relation curves of the electric field amplitude of $\varepsilon$ with respect to frequency, bistable responses can be clearly seen over the whole range of the fractional volume of metallic core $\eta$. The near field properties of the linear coated nanoparticle at different wavelengths are shown in Fig. 3. The excited surface plasmons bring out enhanced field at the resonant wavelength in the shell, justifying the consideration of nonlinearity in the shell.

Next, we introduce weak nonlinearity into the shell. The nonlinear relative permittivity of the shell is set as $\varepsilon_s = \varepsilon_0 + \chi_s |\mathbf{E}|^2$, where $\varepsilon_0 = 2.2$, $\chi_s = 4.4 \times 10^{-20}$ m$^2$/V$^2$. The relations between the electric field amplitude of the incident wave $E$ and the average electric field amplitude in the nonlinear shell $E_\mu \equiv \langle ||E|^2|\rangle^{1/2}$ for different shell sizes ($b = 5$ nm, 10 nm, 15 nm) and the relation calculated with quasi-static approximation in different surrounding media ($\varepsilon_m = 1$ and 4) are shown in Fig. 4. The fractional volume of the core $\eta$ increases as the incident field increases from zero. When the incident field amplitude reaches the switching-up threshold field $E_{\text{up}} \approx 3.39 \times 10^{7}$ V/m ($E_{\text{up}} \approx 9.01 \times 10^{5}$ V/m) for $\varepsilon_m = 1$ ($\varepsilon_m = 4$), the electric field amplitude in the shell will discontinuously jump to the upper stable branch. If the incident field is decreased back from a large value to zero, the electric field in the shell will first decrease continuously, and then jump down to the lower stable branch.}

**Figure 2.** The scattering efficiency vs wavelength diagrams when (a) $\eta$ is fixed at 0.5 and (b) $b$ is fixed at 10 nm. The permittivity of the media $\varepsilon_m = 1$.
\( \varepsilon_m = 4 \), as shown in Fig. 5(b,c), the bistability disappears when the fractional volume of metallic core \( \eta \) goes beyond a critical value \( \eta_c = 0.53 \). If we keep \( \varepsilon_m = 1 \) and lower the wavelength to \( \lambda = 365 \text{ nm} \), as shown in Fig. 5(c,f), we find that the bistability disappears when \( \eta \) decreases below a critical value \( \eta_c = 0.48 \).

The shell size can also affect the bistable behavior. As shown in Fig. 6(a), the switching-up and switching-down threshold fields are almost unchanged as the shell size increases for \( \lambda = 445 \text{ nm} \), \( \varepsilon_m = 1 \). However, as shown in Fig. 6(b), the maximum critical fractional volume of metallic core \( \eta_c \) for \( \lambda = 445 \text{ nm} \), \( \varepsilon_m = 4 \) decreases as the shell size increases. As shown in Fig. 6(c), the minimum critical fractional volume of metallic core \( \eta_c \) for \( \lambda = 365 \text{ nm} \), \( \varepsilon_m = 1 \) increases as the shell size increases. As a consequence, the bistable region becomes broader as the shell size decreases.

Figure 3. Distributions of the electric field for (a) \( \lambda = 362 \text{ nm} \), (b) \( \lambda = 354 \text{ nm} \) and (c) \( \lambda = 370 \text{ nm} \). The other parameters are \( \eta = 0.5 \) and \( b = 10 \text{ nm} \).

Figure 4. The average local field \( E_s \) as a function of the incident field \( E_0 \) for various sizes of the sphere. The relevant parameters are (a) \( \varepsilon_m = 1 \) (b) \( \varepsilon_m = 4 \). And the core-shell-volumes ratio \( \eta = 0.4 \) and the incident wavelength \( \lambda = 445 \text{ nm} \) (the resonance wavelength for \( \varepsilon_m = 4, b = 15 \text{ nm} \) and \( \eta = 0.5 \)).
In Fig. 5(f), we find that the switching-up threshold field increases and the bistable region widens as the fractional volume $\eta$ increases when the permittivity of the surrounding medium is $\varepsilon_m = 1$. On the contrary, in Fig. 5(e), the switching-up threshold field decreases and the bistable region becomes narrow as the fractional volume $\eta$ increases when permittivity of the surrounding medium is $\varepsilon_m = 4$. We thus speculate that there is critical surrounding permittivity $\varepsilon_{mc}$ for such a transition. We plot the electric field amplitude in the shell versus the incident field amplitude for different linear part of the shell permittivity ($\varepsilon_s = \ldots, 2, 2.2, 2.3$). As can be seen in Fig. 7(a–c), the contour plots of $E_{0\text{-up}}$ are nearly trapezoids. Thus, at the critical surrounding permittivity, the contour of the switching-up threshold fields should be vertical. In Fig. 7(a), the critical surrounding permittivity nearly equals 2.4 when $\varepsilon_s = 1.1$. When the surrounding permittivity $\varepsilon_m = 2.4$ [Fig. 7(g)], the switching-up threshold field is nearly unchanged with increasing the fractional volume $\eta$. When the surrounding permittivity is less than the critical one, as shown in Fig. 7(d), the switching-up threshold field increases with increasing the fractional volume $\eta$. In Fig. 7(j), the switching-up threshold field decreases with increasing the fractional volume $\eta$ because the surrounding permittivity is more than the critical one. These properties also apply to the cases when $\varepsilon_s = 2.2, 2.3, 3.3$, as shown in the lower two rows of panels in Fig. 7.

At last, we fix the incident field amplitude $E_0 = 10^8 \text{ V/m}$, and study how the shell size affects the relationship between the average local field in the shell and the incident wavelength. It can be seen from Fig. 8 that, for $\varepsilon_m = 1$ and 4, the switching-up wavelength at the fixed input power blue-shifts when the shell sizes increase, while the
switching-down wavelength at the fixed input power red-shifts when the shell sizes increase. In Fig. 8(b), when the wavelength reaches about 775 nm (the switching-up wavelength for $b = 5$ nm and the switching-up wavelength is about 750 nm for $b = 15$ nm), the electric field amplitude in the shell will discontinuously jump to the lower stable branch. However, if one decreases the wavelength to about 520 nm (the switching-down wavelength for $b = 5$ nm and the switching-down wavelength is about 540 nm for $b = 15$ nm), the electric field amplitude in the shell will discontinuously jump up to the upper stable branch.

**Conclusions**

In this report, we adopt self-consistent mean field approximation within the framework of Mie scattering theory to study the optical bistability of a nonlinear coated metallic nanoparticle. Introducing weak nonlinearity to the shell, we demonstrate numerically that the metallic nanoparticle coated with a nonlinear shell has broad bistable region. We study the effect of the size of the coated spheres, the fractional volume of the metallic core, the permittivity of the surrounding medium, as well as the incident wavelength on the hysteresis loops and the switching-up and switching-down threshold fields. While our results match well with the previous quasi-static results, our full-wave solutions based on Mie scattering are valid in a much wider range of parameters, and thus are more suitable for design of optical bistable devices in optimization.

**Methods**

**Debye potentials.** We express the Debye potentials of the incident fields\(^\text{19}\),

\[
\begin{align*}
\text{Figure 6.} & \quad \text{The switching-up and switching-down threshold fields as a function of } \eta \text{ for (a) } \lambda = 445 \text{ nm, } \varepsilon_m = 1 \\
& \quad \text{(b) } \lambda = 445 \text{ nm, } \varepsilon_m = 4 \text{ and (c) } \lambda = 365 \text{ nm, } \varepsilon_m = 1.
\end{align*}
\]
Figure 7. (a–c) Contour plots of switching-up threshold $E_{0,\text{up}}$ as function of $\varepsilon_m$ and $\eta$. The dark gray region represents that the values are less than the color scale, while those in the light gray region surpass the color scale. (d–l) The average local field $E_s$ as a function of the incident field $E_0$. The size of the sphere $b = 10$ nm and the incident wavelength $\lambda = 480$ nm. The linear relative permittivity $\varepsilon_s = 1.1$ for the first row of panels, $\varepsilon_s = 2.2$ for the second row of panels, and $\varepsilon_s = 3.3$ for the third row of panels.

Figure 8. The average local field $E_s$ versus wavelength for different sizes and the permittivity of the surrounding medium (a) $\varepsilon_m = 1$ (b) $\varepsilon_m = 4$.

\[
\begin{align*}
\phi_{\text{TM}}^{\text{om}} &= \frac{1}{k_m} \sum_{n=1}^{\infty} \frac{2n + 1}{n(n + 1)} \psi_n(k_m r) P_n^{(1)}(\cos \theta) \cos(\phi), \\
\phi_{\text{TE}}^{\text{om}} &= \frac{\sqrt{\varepsilon_0 \varepsilon_m / \mu_0 \mu_m}}{k_m} \sum_{n=0}^{\infty} \frac{2n + 1}{n(n + 1)} \psi_n(k_m r) P_n^{(1)}(\cos \theta) \sin(\phi).
\end{align*}
(10)
The Debye potentials of the scattering wave are

\[
\begin{align*}
\phi_{TM}^{sc} & = - \frac{1}{k_m} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} A_n \zeta_n(k_m r) P_n^{(1)}(\cos \theta) \cos(\phi), \\
\phi_{TE}^{sc} & = - \frac{\sqrt{\varepsilon \varepsilon_n H_0^1}}{k} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} A_n \zeta_n(k_0 r) P_n^{(1)}(\cos \theta) \sin(\phi).
\end{align*}
\]

(11)

Then, in the shell, the Debye potentials are

\[
\begin{align*}
\phi_{TM}^{s} & = - \frac{1}{k_s} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[ D_n^{TM} \psi_n(k_s r) + E_n^{TM} \chi_n(k_s r) \right] P_n^{(1)}(\cos \theta) \cos(\phi), \\
\phi_{TE}^{s} & = - \frac{\sqrt{\varepsilon \varepsilon_n / H_0^1}}{k_s} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[ D_n^{TE} \psi_n(k_s r) + E_n^{TE} \chi_n(k_s r) \right] P_n^{(1)}(\cos \theta) \sin(\phi).
\end{align*}
\]

(12)

In the core, they should be written as

\[
\begin{align*}
\phi_{TM}^{c} & = - \frac{1}{k_c} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} F_n^{TM} \psi_n(k_c r) P_n^{(1)}(\cos \theta) \cos(\phi), \\
\phi_{TE}^{c} & = - \frac{\sqrt{\varepsilon \varepsilon_n / H_0^1}}{k_c} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} F_n^{TE} \psi_n(k_c r) P_n^{(1)}(\cos \theta) \sin(\phi).
\end{align*}
\]

(13)

where \( \psi_n(x), \chi_n(x) \) and \( \zeta_n(x) \) are the Ricatti-Bessel functions and they can be defined by \( \psi_n(x) = \sqrt{\pi} J_{n+1/2}(x) \), \( \chi_n(x) = -\sqrt{\pi} N_{n+1/2}(x) \) and \( \zeta_n(x) = \sqrt{\pi} H_1^{n+1/2}(x) \). Here, \( J_{n+1/2}(x), N_{n+1/2}(x) \) and \( H_1^{n+1/2}(x) \) are the Bessel functions, Neumann functions and the first-kind Hankel functions. \( P_n^{(1)}(\cos \theta) \) are the associated Legendre polynomials. In addition, we denote \( k_m = k_0 \sqrt{\varepsilon \varepsilon_m \mu_m}, k_c = k_0 \sqrt{\varepsilon \varepsilon_m \mu_c} \) and \( k_s = k_0 \sqrt{\varepsilon \varepsilon_n \mu_s} \).

**Boundary conditions and expansion coefficients of Debye potentials.** To solve the coefficients, we apply the boundary conditions on \( r = a \) and \( r = b \). They are

\[
\begin{align*}
\varepsilon_r \phi_{TM}^{s} + \varepsilon_m \phi_{TM}^{c}, & \quad \mu_r \phi_{TM}^{s} = \mu_m \phi_{TM}^{c}, \\
\frac{\partial (r \phi_{TM}^{s})}{\partial r} & = \frac{\partial (r \phi_{TM}^{c})}{\partial r} = \frac{\partial (r \phi_{TE}^{s})}{\partial r} = \frac{\partial (r \phi_{TE}^{c})}{\partial r} \text{ at } r = a,
\end{align*}
\]

(14)

and

\[
\begin{align*}
\varepsilon_r \phi_{TE}^{s} + \varepsilon_m \phi_{TE}^{c}, & \quad \mu_r \phi_{TE}^{s} = \mu_m \phi_{TE}^{c}, \\
\frac{\partial (r \phi_{TM}^{s})}{\partial r} & = \frac{\partial (r \phi_{TM}^{c})}{\partial r} = \frac{\partial (r \phi_{TE}^{s})}{\partial r} = \frac{\partial (r \phi_{TE}^{c})}{\partial r} \text{ at } r = b.
\end{align*}
\]

(15)

Substituting the boundary conditions into the Debye potentials, we can obtain the coefficients as follows,

\[
A_n^{TM} = \begin{bmatrix}
\mu_s \psi_n(k_m b) & \mu_m \psi_n(k_m b) & \mu_m \chi_n(k_m b) & 0 \\
\kappa \psi_n'(k_m b) & \kappa \psi_n'(k_m b) & \kappa \chi_n'(k_m b) & 0 \\
0 & \mu_s \psi_n(k_m a) & \mu_m \chi_n(k_m a) & -\mu_s \psi_n(k_m a) \\
0 & \kappa \psi_n'(k_m a) & \kappa \chi_n'(k_m a) & -\kappa \psi_n'(k_m a)
\end{bmatrix}
\]

and

\[
A_n^{TE} = \begin{bmatrix}
\mu_s \zeta_n(k_m b) & \mu_m \zeta_n(k_m b) & \mu_m \chi_n(k_m b) & 0 \\
\kappa \zeta_n'(k_m b) & \kappa \zeta_n'(k_m b) & \kappa \chi_n'(k_m b) & 0 \\
0 & \mu_s \zeta_n(k_m a) & \mu_m \chi_n(k_m a) & -\mu_s \zeta_n(k_m a) \\
0 & \kappa \zeta_n'(k_m a) & \kappa \chi_n'(k_m a) & -\kappa \zeta_n'(k_m a)
\end{bmatrix}
\]
\[
A_{n}^{\text{TE}} = \begin{pmatrix}
\varepsilon_{\text{z}} \psi_{n}^{\prime}(k_{m}b) & \varepsilon_{m} \psi_{n}^{\prime}(k_{b}) & \varepsilon_{m} \chi_{n}^{\prime}(k_{b}) & 0 \\
k_{z} \psi_{n}^{\prime}(k_{m}b) & k_{z} \psi_{n}^{\prime}(k_{b}) & k_{z} \chi_{n}^{\prime}(k_{b}) & 0 \\
0 & \varepsilon_{z} \psi_{n}(k_{b}) & \varepsilon_{m} \chi_{n}(k_{b}) & -\varepsilon_{z} \psi_{n}(k_{b}) \\
0 & k_{z} \psi_{n}(k_{b}) & k_{z} \chi_{n}(k_{b}) & -k_{z} \psi_{n}(k_{b}) \\
\end{pmatrix},
\]

\[
D_{n}^{\text{TM}} = \begin{pmatrix}
k_{m} \varepsilon_{\text{z}} \left[ A_{n}^{\text{TM}} \zeta_{n}^{\prime}(k_{m}b) - \psi_{n}(k_{m}b) \right] & k_{m} \varepsilon_{m} A_{n}^{\text{TM}} \chi_{n}(k_{b}) \\
k_{z} \varepsilon_{\text{z}} \left[ A_{n}^{\text{TM}} \zeta_{n}^{\prime}(k_{m}b) - \psi_{n}(k_{m}b) \right] & k_{m} \varepsilon_{m} A_{n}^{\text{TM}} \chi_{n}(k_{b}) \\
k_{m} \psi_{n}(k_{b}) & k_{m} A_{n}^{\text{TM}} \chi_{n}(k_{b}) \\
k_{m} \psi_{n}(k_{b}) & k_{m} A_{n}^{\text{TM}} \chi_{n}(k_{b}) \\
\end{pmatrix},
\]

\[
D_{n}^{\text{TE}} = \begin{pmatrix}
k_{m} \varepsilon_{\text{z}} \left[ A_{n}^{\text{TE}} \zeta_{n}^{\prime}(k_{m}b) - \psi_{n}(k_{m}b) \right] & k_{m} \varepsilon_{m} A_{n}^{\text{TE}} \chi_{n}(k_{b}) \\
k_{z} \varepsilon_{\text{z}} \left[ A_{n}^{\text{TE}} \zeta_{n}^{\prime}(k_{m}b) - \psi_{n}(k_{m}b) \right] & k_{m} \varepsilon_{m} A_{n}^{\text{TE}} \chi_{n}(k_{b}) \\
k_{m} \psi_{n}(k_{b}) & k_{m} A_{n}^{\text{TE}} \chi_{n}(k_{b}) \\
k_{m} \psi_{n}(k_{b}) & k_{m} A_{n}^{\text{TE}} \chi_{n}(k_{b}) \\
\end{pmatrix},
\]

\[
E_{n}^{\text{TM}} = \begin{pmatrix}
k_{m} \varepsilon_{\text{z}} \left[ A_{n}^{\text{TM}} \zeta_{n}^{\prime}(k_{m}b) - \psi_{n}(k_{m}b) \right] & k_{m} \varepsilon_{m} A_{n}^{\text{TM}} \chi_{n}(k_{b}) \\
k_{m} \psi_{n}(k_{b}) & k_{m} A_{n}^{\text{TM}} \chi_{n}(k_{b}) \\
k_{m} \psi_{n}(k_{b}) & k_{m} A_{n}^{\text{TM}} \chi_{n}(k_{b}) \\
k_{m} \psi_{n}(k_{b}) & k_{m} A_{n}^{\text{TM}} \chi_{n}(k_{b}) \\
\end{pmatrix},
\]

\[
E_{n}^{\text{TE}} = \begin{pmatrix}
k_{m} \varepsilon_{\text{z}} \left[ A_{n}^{\text{TE}} \zeta_{n}^{\prime}(k_{m}b) - \psi_{n}(k_{m}b) \right] & k_{m} \varepsilon_{m} A_{n}^{\text{TE}} \chi_{n}(k_{b}) \\
k_{m} \psi_{n}(k_{b}) & k_{m} A_{n}^{\text{TE}} \chi_{n}(k_{b}) \\
k_{m} \psi_{n}(k_{b}) & k_{m} A_{n}^{\text{TE}} \chi_{n}(k_{b}) \\
k_{m} \psi_{n}(k_{b}) & k_{m} A_{n}^{\text{TE}} \chi_{n}(k_{b}) \\
\end{pmatrix},
\]

\[
E_{n}^{\text{TM}} = \frac{k_{z} \varepsilon_{\text{z}} \left[ D_{n}^{\text{TM}} \psi_{n}(k_{m}a) + E_{n}^{\text{TM}} \chi_{n}(k_{m}a) \right]}{k_{m} \varepsilon_{\text{z}} \psi_{n}(k_{m}a)},
\]

and

\[
E_{n}^{\text{TE}} = \frac{k_{z} \varepsilon_{\text{z}} \left[ D_{n}^{\text{TE}} \psi_{n}(k_{m}a) + E_{n}^{\text{TE}} \chi_{n}(k_{m}a) \right]}{k_{m} \varepsilon_{\text{z}} \psi_{n}(k_{m}a)}.
\]

References
1. Hurtado, A., Nami, M., Henning, I. D., Adams, M. J. & Lester, L. F. Bistability patterns and nonlinear switching with very high contrast ratio in a 1550 nm quantum dash semiconductor laser. Appl. Phys. Lett. 101, 161117 (2012).
2. Litchinitser, N. M., Gabitov, I. R., Maimistov, A. I. & Shalaev, V. M. Effect of an optical negative index thin film on optical bistability. Opt. Lett. 32, 151–153 (2007).
3. Litchinitser, N. M., Gabitov, I. R. & Maimistov, A. I. Optical bistability in a nonlinear optical coupler with a negative index channel. Phys. Rev. Lett. 99, 113902 (2007).
4. Chen, P. Y., Farhat, M. & Alù, A. Bistable and self-tunable negative-index metamaterial at optical frequencies. *Phys. Rev. Lett.* **106**, 105503 (2011).
5. Tuz, V. R., Prosvirnin, S. L. & Kochetova, L. A. Optical bistability involving planar metamaterials with broken structural symmetry. *Phys. Rev. B* **82**, 233402 (2010).
6. Mazurenko, D. A., Kerst, R., Dijkhuis, J. I., Akimov, A. V., Golubev, V. G., Kurdyukov, D. A., Pevtsov, A. B. & Sel’kin, A. V. Ultrafast Optical Switching in Three-Dimensional Photonic Crystals. *Phys. Rev. Lett.* **91**, 213903 (2003).
7. Almeida, V. R. & Lipson, M. Optical bistability on a silicon chip. *Opt. Lett.* **29**, 2387–2389 (2004).
8. Dai, X. Y., Jiang, L. Y. & Xiang, Y. J. Low threshold optical bistability at terahertz frequencies with graphene surface plasmons. *Scientific reports* **10**, 1038 (2015).

Acknowledgements

This work was supported by the NNSF of China No. 11374223, the National Basic Research Program (No. 2012CB921501), the Ph.D. Program Foundation of the Ministry of Education of China (Grant No. 20123201110010), and PAPD of Jiangsu Higher Education Institutions. Y.Z. and B.Z. acknowledge the support from NTU NAP Start-Up Grant, Singapore Ministry of Education under Grant No. MOE2015-T2-1-070 and MOE2011-T3-1-005.

Author Contributions

L.G. conceived the idea. H.C. performed most theoretical and numerical calculations. H.C., Y.Z. and B.Z. analyzed the data. All authors joined discussion extensively and revised the manuscript before the submission.

Additional Information

Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Chen, H. et al. Optical bistability in a nonlinear-shell-coated metallic nanoparticle. *Sci. Rep.* **6**, 21741; doi: 10.1038/srep21741 (2016).

This work is licensed under a Creative Commons Attribution 4.0 International License. The images or other third party material in this article are included in the article’s Creative Commons license, unless indicated otherwise in the credit line; if the material is not included under the Creative Commons license, users will need to obtain permission from the license holder to reproduce the material. To view a copy of this license, visit http://creativecommons.org/licenses/by/4.0/