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\title{\textit{D_s^0(2317) as a Tetraquark State with QCD Sum Rules in Heavy Quark Limit}}

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\maketitle

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\begin{abstract}
In this article, we take the point of view that the charmed scalar meson $D_{s0}(2317)$ be a tetraquark state and devote to calculate its mass within the framework of the QCD sum rules approach in the heavy quark limit. The numerical values for the mass of the $D_{s0}(2317)$ are consistent with the experimental data, there must be some tetraquark component in the scalar meson $D_{s0}(2317)$. Detailed discussions about the threshold parameter and Borel parameter for the multiquark states are also presented.
\end{abstract}

PACS number: 12.38.Aw, 12.38.Qk

Key words: $D_s(2317)$, QCD sum rules

\section{Introduction}

The discovery of the two strange-charmed mesons $D_{s0}(2317)$ and $D_{s1}(2460)$ with spin-parity $0^+$ and $1^+$ respectively has triggered hot debate on their nature and under-structures \cite{1}. There have been a lot of explanations for their nature, for example, conventional $c\bar{s}$ states \cite{2,3}, two-meson molecular states \cite{4}, $D - K$ mixing states \cite{5} and four-quark states \cite{6,7}, etc. The mass of the $D_{s0}(2317)$ is significantly lower than the values of the $0^+$ state mass from the quark models and lattice simulations \cite{3}. Those two states $D_{s0}(2317)$ and $D_{s1}(2460)$ lie just below the $DK$ and $D^*K$ threshold respectively which are analogous to the situation that the scalar mesons $a_0(980)$ and $f_0(980)$ lie just below the $K\bar{K}$ threshold and couple strongly to the nearby channels. If we take the scalar mesons $a_0(980)$ and $f_0(980)$ as four-quark states with the constituents of scalar diquark-antidiquark sub-structures, the masses of the scalar nonet mesons below 1GeV can be naturally explained. The mechanism responsible for the low-mass charmed scalar meson may be the same as the light scalar nonet mesons, the $f_0(600)$, $f_0(980)$, $a_0(980)$ and $K^*_0(800)$ \cite{9,10}. The one-gluon exchange force and the instanton induced force lead to significant attractions between the quarks in the $0^+$ diquark channels, we can take the scalar diquark and antidiquark as the basic constituents in constructing the interpolating current \cite{11}, furthermore, in the color superconductivity theory, the attractive interactions in this channel lead to the formulation of nonzero condensates and the breaking of both the color and flavor $SU(3)$ symmetries for the light flavors \cite{12}.

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In this article, we take the point of view that the charmed scalar meson $D_{s0}(2317)$ be a tetraquark state consist of scalar diquark and antidiquark, and devote to calculate its mass in the heavy quark limit with the QCD sum rules [13]. The masses of the ground state and lowest excited state heavy mesons have been studied with QCD sum rules in heavy quark effective theory via the $1/m_Q$ expansion [13] [15].

The article is arranged as follows: we derive the QCD sum rules for the bound energy $\bar{\Lambda}$ of the $D_{s0}(2317)$ in the heavy quark limit in section II; in section III, numerical results and discussions; section VI is reserved for conclusion.

2 QCD sum rules for the $D_{s0}(2317)$ in the heavy quark limit

In the following, we write down the two-point correlation function $\Pi$ in the framework of the QCD sum rules approach [7],

$$\Pi = i \int d^4x e^{ikx} \langle 0 | T \{ J(x) J^+(0) \} | 0 \rangle,$$

$$J(x) = \frac{\epsilon^{kij} \epsilon^{kmn}}{\sqrt{2}} \left\{ u_i^T(x) C \gamma_5 c_j(x) \bar{u}_m(x) \gamma_5 C \bar{s}_n(x) + \bar{d}_m(x) \gamma_5 C \bar{s}_n(x) \right\},$$

here the $i, j, k, m, n$ are color indices and the $C$ is the charge conjugation matrix. In the heavy quark limit, the $c$ quark field can be approximated by the static heavy quark field $h_v(x)$ with the propagator,

$$\langle 0 | T \{ h_v(x) \bar{h}_v(0) \} | 0 \rangle = \frac{1 + \gamma_\mu}{2} \int_0^\infty \delta(x - vt) dt,$$

here the $v_\mu$ is a four-vector with $v^2 = 1$. The calculation of the operator product expansion can be performed in the coordinate space, and does not need the mixed picture both in coordinate and momentum spaces [7], in the heavy quark limit, the calculation can be greatly facilitated.

According to the basic assumption of current-hadron duality in the QCD sum rules approach [13], we insert a complete series of intermediate states satisfying the unitarity principle with the same quantum numbers as the current operator $J(0)$ into the correlation function in Eq.(1) to obtain the hadronic representation. After isolating the pole term of the lowest $D_{s0}(2317)$ state, we obtain the following result in the heavy quark limit,

$$\Pi = \frac{F^2}{\bar{\Lambda} - \omega} + \cdots,$$

$$\langle 0 | J(0) | D_{s0} \rangle = \sqrt{2} F,$$

$$\bar{\Lambda} = \text{limit}_{m_c \to \infty} m_{D_{s0}} - m_c,$$ (4)
here \( \omega = v \cdot k \) and \( \bar{\Lambda} \) is the bound energy.

We perform operator product expansion up to the vacuum condensates of dimension-9 to obtain the correlation function \( \Pi \) at the level of quark-gluon degrees of freedom. Once the analytical results are obtained, then we can take the current-hadron duality below the threshold \( \omega_c \) and perform the Borel transformation with respect to the variable \( \omega \), finally we obtain the following sum rule,

\[
F^2 e^{-\frac{\bar{\Lambda}}{T}} = \int_{\omega_c}^{\infty} d\omega e^{-\frac{\bar{\Lambda}}{T}} \left\{ \frac{\omega^8}{3360\pi^6} - \frac{\langle \bar{q}q \rangle \omega^5}{60\pi^4} - \frac{m_s (2\langle \bar{q}q \rangle - \langle \bar{s}s \rangle) \omega^4}{48\pi^4} \right. \\
\left. + \frac{\omega^4}{384\pi^4} \left( \frac{\alpha_s GG}{\pi} \right) + \frac{\langle \bar{q}g_s \sigma Gq \rangle \omega^3}{48\pi^4} + \frac{m_s (3\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle) \omega^2}{96\pi^4} \right. \\
\left. + \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle \omega^2}{6\pi^2} + \frac{m_s (2\langle \bar{q}q \rangle^2 - \langle \bar{q}q \rangle \langle \bar{s}s \rangle) \omega}{12\pi^2} - \frac{\langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \delta(\omega)}{18} \right\},
\]

(5)

here the \( T \) is the Borel parameter. Differentiate the above sum rule with respect to the variable \( \frac{1}{T} \), then eliminate the quantity \( F \), we obtain

\[
\bar{\Lambda} = \int_{m_s}^{\omega_c} d\omega e^{-\frac{\bar{\Lambda}}{T}} \left\{ \frac{\omega^9}{3360\pi^6} - \frac{\langle \bar{q}q \rangle \omega^6}{60\pi^4} - \frac{m_s (2\langle \bar{q}q \rangle - \langle \bar{s}s \rangle) \omega^5}{48\pi^4} \right. \\
\left. + \frac{\omega^5}{384\pi^4} \left( \frac{\alpha_s GG}{\pi} \right) + \frac{\langle \bar{q}g_s \sigma Gq \rangle \omega^4}{48\pi^4} + \frac{m_s (3\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle) \omega^3}{96\pi^4} \right. \\
\left. + \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle \omega^3}{6\pi^2} + \frac{m_s (2\langle \bar{q}q \rangle^2 - \langle \bar{q}q \rangle \langle \bar{s}s \rangle) \omega^2}{12\pi^2} \right. \\
\left. - \frac{\omega^2}{\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) + \frac{\langle \bar{q}g_s \sigma Gq \rangle \omega}{384\pi^4} + \frac{m_s (3\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle) \omega^2}{96\pi^4} \\
\right. \\
\left. + \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle \omega^2}{6\pi^2} + \frac{m_s (2\langle \bar{q}q \rangle^2 - \langle \bar{q}q \rangle \langle \bar{s}s \rangle) \omega}{12\pi^2} - \frac{\langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \delta(\omega)}{18} \right\}.
\]

(6)

It is easy to integrate over the variable \( \omega \), we prefer this formulation for simplicity.

### 3 Numerical results and discussions

The parameters for the condensates are chosen to be the standard values at the energy scale \( \mu = 1 \text{GeV} \), although there are some suggestions for updating those values, for reviews, one can consult Ref.\cite{16}. \( \langle \bar{s}s \rangle = 0.8\langle \bar{q}q \rangle, \langle \bar{q}q \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -(240 \text{MeV})^3, \langle \bar{q}g_s \sigma Gq \rangle = m_0^2\langle \bar{q}q \rangle, \langle \bar{s}g_s \sigma Gs \rangle = m_0^2\langle \bar{s}s \rangle, m_0^2 = 0.8 \text{GeV}^2, \langle \alpha_s GG \rangle = (0.33 \text{GeV})^4 \) and \( m_u = m_d = 0 \). Small variations of those condensates will not lead to large changes for the numerical values and impair the predictive ability, we can neglect the uncertainties for the vacuum condensates for simplicity. The mass of the \( s \) quark from the Particle Data Group is about \( m_s (\mu = 2 \text{GeV}) = (80 - 155) \text{MeV} \).
the values (listed in the first article of Ref. [16]) from the QCD sum rules, lattice QCD and \( \tau \) decays vary in a large range, \( m_s(\mu = 1 GeV) \approx (117 - 203) MeV \), here we take the average values from lattice QCD, \( m_s(\mu = 1 GeV) = (140 \pm 10) MeV \). The variations of the \( m_s \) about 20 MeV can only lead to tiny changes for the final result, the uncertainties can be safely neglected, we take the value \( m_s(\mu = 1 GeV) = 140 MeV \) in numerical calculation for simplicity. The values of the mass of the \( c \) quark are \( m_c = (1.0 - 1.4) GeV \) at the energy scale \( \mu = 2 GeV \) from the Particle Data Group [17]. The average values (listed in the first article of Ref. [16]) at the energy scale \( \mu = m_c \) are \( m_c(m_c) = (1.3 \pm 0.1) GeV \) from the QCD sum rules and lattice QCD. We take the values \( m_c(m_c) = (1.3 \pm 0.1) GeV \) and evolve them to the energy scale \( \mu = 1 GeV \) with the renormalization group equation, \( m_c(\mu = 1 GeV) = (1.4 \pm 0.1) GeV \).

In the following, we discuss the criterion for selecting the threshold parameter \( s_0 \) (or \( \omega_c \)) and Borel parameter \( M_B \) (or \( T \)) in the QCD sum rules dealing with the multiquark states. The QCD sum rules have been extensively applied to the hadronic physics and given a lot of successful descriptions [13, 16]. For the conventional (two-quark) mesons and (three-quark) baryons, the hadronic spectral densities are experimentally well known, the separations between the ground state and excited states are large enough, the "single-pole + continuum states" model works well in representing the phenomenological spectral densities. In the phenomenological analysis, the continuum states can be approximated by the contributions from the asymptotic quarks and gluons, and the single-pole dominance condition can be well satisfied,

\[
\int_{s_0}^{\infty} \rho_{pert} e^{-\frac{s}{M_B}} ds < \int_{0}^{s_0} (\rho_{pert} + \rho_{cond}) e^{-\frac{s}{M_B}} ds
\]  

for the conventional QCD sum rules, and

\[
\int_{\omega_c}^{\infty} \rho_{pert} e^{-\frac{\omega}{T}} d\omega < \int_{0}^{\omega_c} (\rho_{pert} + \rho_{cond}) e^{-\frac{\omega}{T}} d\omega
\]  

for the QCD sum rules in the heavy quark limit, here the \( \rho_{pert} \) and \( \rho_{cond} \) stand for the contributions from the perturbative and non-perturbative part of the spectral density respectively. From the conditions in Eqs. (7-8), we can obtain the maximal value for the Borel parameter \( M_B^{max} \) (or \( T^{max} \)), exceed this value, the single-pole dominance will be spoiled. On the other hand, the Borel parameter must be chosen large enough to warrant the convergence of the operator product expansion and contributions from the high dimension vacuum condensates which are poorly known are of minor importance, the minimal value for the Borel parameter \( M_B^{min} \) (or \( T^{min} \)) can be determined.

For the conventional (two-quark) mesons and (three-quark) baryons, the Borel window \( M_B^{max} - M_B^{min} \) (or \( T^{max} - T^{min} \)) is rather large and the reliable QCD sum rules can be obtained. However, for the multiquark states i.e. tetraquark states, pentaquark states, etc, the spectral densities \( \rho \sim s^m \) with \( m \) is larger than the
corresponding ones for the conventional hadrons, the integral \( \int_0^\infty s^m \exp \left( -\frac{s}{M_B^2} \right) ds \) (or \( \int_0^\infty \omega^m \exp \left( -\frac{s}{M_B^2} \right) d\omega \)) converges more slowly \[18\]. If one do not want to release the conditions in Eqs.(7-8), we have to either postpone the threshold parameter \( s_0 \) (or \( \omega_c \)) to very large values or choose very small values for the Borel Parameter \( M_B^{max} \) (or \( T_{max} \)). With large values for the threshold parameter \( s_0 \) (or \( \omega_c \)), for example, \( s_0 \gg M^2 \), here the \( gr \) stands for the ground state, the contributions from the excited states are already included in if there are really some ones, the single-pole approximation for the spectral densities is spoiled; on the other hand, with very small values for the Borel parameter \( M_B^{max} \) (or \( T_{max} \)), the operator product expansion is broken down, and the Borel window shrinks to zero and beyond. This may lead to the pessimistic opinion that the QCD sum rules can not be successfully applied to the multiquark states, the sum rules concerning the tetraquark states and pentaquark states should be rejected, however, we are optimistical participators for the QCD sum rules and take the point of view that the QCD sum rules can be successfully applied to the multiquark states, and one should resort to the "multi-pole + continuum states" to approximate the phenomenological spectral densities. The onset of the continuum states is not abrupt, the ground state, the first excited state, the second excited state, etc, the continuum states appear sequentially; the excited states may be loose bound states and have large widths. The threshold parameter \( s_0 \) (or \( \omega_c \)) is postponed to large value, at that energy scale, the spectral densities can be well approximated by the contributions from the asymptotic quarks and gluons, and of minor importance for the sum rules.

The present experimental knowledge about the phenomenological hadronic spectral densities for the tetraquark states is rather vague, even the existence of the tetraquark states is not confirmed with confidence, and no knowledge about either there are high excited states or not. In this article, the following criteria are taken. We choose the suitable values for the Borel parameter \( M_B \) (or \( T \)), on the one hand the minimal values \( M_B^{min} \) (or \( T_{min} \)) are large enough to warrant the convergence of the operator product expansion, on the other hand the maximal values \( M_B^{max} \) (or \( T_{max} \)) are small enough to suppress the contributions from the high excited states and continuum states i.e. we choose the naive analysis \( M_B^{max} < M_{gr} \) (or \( T_{max} < \bar{\Lambda} \)), furthermore, there exist a Borel platform which is insensitive to the variations of the Borel parameter. For the hadronic spectral density, the more phenomenological analysis is preferred, we approximate the spectral densities with the contribution from the single-pole term, the threshold parameter \( s_0 \) (or \( \omega_c \)) is taken slightly above the ground state mass, \( s_0 > M^2 \) (or \( \omega_c > \bar{\Lambda} \)), to subtract the contributions from the excited states and continuum states. One may reject taking the values from the more phenomenological analysis as quantitatively reliable, the results are qualitative at least.

In the heavy quark limit \( \sqrt{s_0} \sim m_c + \omega_c \), the threshold parameter \( \omega_c \) is chosen to vary between \((1.3 - 1.5)GeV\). The values are reasonable for the scalar meson \( D_{s0}(2317) \) with narrow width, \([m_c + (1.3 - 1.5)GeV] \geq (2.6 - 2.8)GeV > M_{D_{s0}} + \Gamma_{D_{s0}}, \)
the contributions from the $D_{s0}(2317)$ can be correctly taken into account. For the values $\omega_c \geq 1.3\, GeV$, the contributions from the perturbative term and the linear quark condensate terms proportional to the $\langle \bar{q}q \rangle$, $\langle \bar{s}s \rangle$ are dominating i.e. $> 60\%$ with the minimal Borel parameter $T_{min} \geq 0.5\, GeV$, in this region, we can warrant the convergence of the operator product expansion. For the intervals, $T = (0.5 - 1.1)\, GeV$ and $\omega_c = (1.3 - 1.5)\, GeV$, the main contributions to the bound energy $\bar{\Lambda}$ come from the linear quark condensates terms proportional to the $\langle \bar{q}q \rangle$ and $\langle \bar{s}s \rangle$, about $(60 - 70)\%$, the perturbative contributions are suppressed by the large numerical denominator and of minor importance, about $(10 - 15)\%$, which is significantly in contrary to the ordinary QCD sum rules for the conventional mesons and baryons where the main contributions come from the perturbative terms. Furthermore, the contributions from the gluon condensates are of minor importance due to the large numerical denominator [19]. From the Fig.1, we can see that the predicted bound energy $\bar{\Lambda}$ is almost independent on the Borel parameter $T$ in the region $0.5\, GeV \leq T \leq 1.1\, GeV^2$. If we restrict the values of the Borel parameter $T$ to the naive analysis $T_{max} < \bar{\Lambda}$, the predicted values are $\bar{\Lambda} = (0.8 - 1.0)\, GeV$ with $\omega_c = (1.3 - 1.5)\, GeV$. For $T_{max} = 0.8$, the contributions from the perturbative term and the linear quark condensate terms proportional to the $\langle \bar{q}q \rangle$, $\langle \bar{s}s \rangle$ are larger than $85\%$ with the threshold parameter $\omega_c > 1.5\, GeV$, the high dimensional condensates (non-perturbative terms) are greatly suppressed, this may be an indication the onset of the continuum states. For $\omega_c = (1.3 - 1.5)\, GeV$ and $T = (0.5 - 0.8)\, GeV$,

$$\bar{\Lambda} = (0.8 - 1.0)\, GeV,$$

$$m_{D_{s0}} = [m_c + (0.8 - 1.0)]\, GeV,$$

$$= [(1.3 - 1.5) + (0.8 - 1.0)]\, GeV,$$

$$= (2.1 - 2.5)\, GeV,$$

$$= (2.3 \pm 0.2)\, GeV. \hspace{1cm} (9)$$

Comparing with the experimental data, the tetraquark configuration gives reasonable values for the mass of the scalar meson $D_{s0}(2317)$; there must be some tetraquark component in the charmed scalar meson $D_{s0}(2317)$.

4 Conclusion

In this article, we take the point of view that the charmed scalar meson $D_{s0}(2317)$ be a tetraquark state and devote to calculate its mass within the framework of the QCD sum rule approach in the heavy quark limit. The numerical values for the mass of the $D_{s0}(2317)$ are consistent with the experimental data. There must be some tetraquark component in the scalar meson $D_{s0}(2317)$. 

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Acknowledgments

This work is supported by National Natural Science Foundation, Grant Number 10405009, and Key Program Foundation of NCEPU. The authors are indebted to Dr. J.He (IHEP), Dr. X.B.Huang (PKU) and Dr. L.Li (GSCAS) for numerous help, without them, the work would not be finished.

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