Deep Inelastic Structure Functions in Bag–Like Models

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Abstract
A new method of calculating deep inelastic structure functions for the nucleon in independent particle models is presented. The method is applied to the bag–like model and its predictions are compared with the parametrisation of Gluck, Reya and Vogt.

1 Introduction
Deep inelastic scattering provides worthy information about the structure of nucleons and nuclei. The papers of Gluck, Reya and Vogt [1, 2] have shown that quite good agreement with the data (and moreover some predictions) may be obtained assuming valence–like parton distributions at some small scale $Q^2 = \mu^2$ and evolving them using perturbative QCD. Nowadays it is not possible to calculate the initial distributions using directly QCD and we are left with various, less or more phenomenological, models. The idea of calculating leading twist parton distributions using quark models has over twenty years [3] and there were many implementations of it using the bag model, soliton models, the colour–dielectric model and others [5].

It is reasonable to assume that interactions between quarks may be approximately described by a mean field confining them. In fact, even one of the most prominent quark models, the bag model [11] (in the static cavity approximation) may be treated in such a way.

In the mean field approach quarks move independently and there arises the so called centre of mass problem, resulting from the simple observation, that independently moving particles cannot stand for a composite state of definite total momentum. The problem manifests itself in the fact, that quark distributions, calculated in such a model, spread over all Björken $x$ from $-\infty$ to $+\infty$ instead of having support in the region from 0 to 1. There are many methods of curing this problem, but all of them have some drawbacks. For example
the method using Peierls–Yoccoz or Peierls–Thouless projection gives badly normalised quark distributions. In this paper we follow the idea of Szymacha [3] that the independent particle model of a composite particle corresponds to the particle moving in an auxiliary external potential.

2 Deeply inelastic nucleon structure functions

Tensor $W^{\mu\nu}$ describing inclusive photon-nucleon scattering may be written as

$$\frac{2p^0}{4\pi} \int d^3x \int d^3y \sum_n \langle h|J^\mu(x)|n\rangle \langle n|J^\nu(y)|h\rangle e^{-i(q(x-y))} 2\pi\delta(p^0 + q^0 - P_n),$$

where $q^0$ and $\vec{q}$ are the photon energy and momentum, $|n\rangle$ are normalised to 1 final states with energy $P_n^0$, and $|h\rangle$ is the initial (normalised to 1) nucleon state with energy $p^0$.

For $|h\rangle$ being an eigenstate of definite momentum $\vec{p}$ and spin $s$,

$$|h\rangle = \frac{1}{\sqrt{2p^0V}}|h(p, s)\rangle,$$

we obtain usual form for $W^{\mu\nu}$

$$W^{\mu\nu} = \frac{1}{4\pi} \sum_n \langle h|J^\mu(0)|n\rangle \langle n|J^\nu(0)|h\rangle (2\pi)^4\delta^4(p + q - P_n).$$

If we approximate the real nucleon by independent particles (as in the bag model), then $|h\rangle$ cannot be a state of definite momentum and (1) is much more useful then (3).

$W^{\mu\nu}$ may be expressed in terms of scalar functions $F_1$, $F_2$, $g_1$ and $g_2$:

$$W^{\mu\nu} = -(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2})F_1 + (p^\mu - \frac{pq}{q^2} q^\mu)(p^\nu - \frac{pq}{q^2} q^\nu)\frac{F_2}{pq} + \frac{1}{m}e^{\mu\nu} \rho_\sigma q^\rho \left[ s^\sigma \frac{m^2}{pq}(g_1 + g_2) - (sq)p^\sigma \frac{m^2}{pq} g_2 \right],$$

where $m$ is the nucleon mass. For specific choices of $\mu$ and $\nu$ we obtain

$$W^{tt} = W_{\mu\nu} n^\mu_t n^\nu_t = \frac{F_2}{2x} - F_1,$$

$$W^{\perp\perp} = F_1,$$

where $Q = \sqrt{-q^2}$, $x = Q^2/2pq$, $n^\mu_t = \left( p^\mu - \frac{pq}{q^2} q^\mu \right) / \sqrt{m^2 - (pq)^2/q^2}$, and $\perp, \perp'$ denote components perpendicular to $n^\mu_t$ and $q^\nu$. 

2
3 “Naive” structure functions in the bag model

As was already stated, independent particles cannot correspond to a state which is a momentum eigenstate. Following Szymacha et al. we will assume that they describe the nucleon with (yet unknown) wavefunction \( \Psi_{CM} \), bounded by an auxiliary external potential. Nevertheless our first step is to calculate structure functions of the bag model (in the static cavity approximation) without any improvements for the centre of mass motion.

We assume that in DIS photon interacts only with a given quark from a nucleon, without any influence on other quarks, exiting it into the final state \( |n_q\rangle \). Then Eq. (1) may be rewritten as (we add the subscript \( q \) to indicate that this \( W^{\mu\nu} \) is calculated directly from the quark wave function)

\[
W^{\mu\nu}_q = \frac{2m}{4\pi} \int \frac{d^3x}{\pi} \int \frac{d^3y}{\pi} \sum_{n_q} \langle q|J^{\mu}(\vec{x})|n_q\rangle \langle n_q|J^{\nu}(\vec{y})|q\rangle e^{-i\vec{q}(\vec{x}-\vec{y})} 2\pi \delta(p^0 + q^0 - P^0_n) = \\
m \sum_{n_q} \delta(p^0 + q^0 - P^0_n) \langle q|J^{\mu}(\vec{x})|n_q\rangle \langle n_q|J^{\nu}(\vec{y})|q\rangle e^{-i\vec{q}(\vec{x}-\vec{y})},
\]

where it was assumed that for higher excitation, final states \( |n_q\rangle \) may be approximated by plane waves \( u_{n_q} e^{-i\vec{P}_{n_q} \cdot \vec{x}} \bar{\psi}_q(\vec{x}) \gamma^\mu u_{n_q} \) and \( \vec{P}_{n_q} \) is the initial wave function of the quark \( q \).

Since bispinor \( u_{n_q} \) is uniquely described by corresponding momentum \( \vec{P}_{n_q} \) and mass \( M = \sqrt{\left( P_{n_q}^0 \right)^2 - \left( \vec{P}_{n_q} \right)^2} \), we may replace \( \sum_{n_q} u_{n_q} \bar{u}_{n_q} \) by \( \sum_M \int d^3\vec{P} \frac{\gamma_{\vec{P}+M}}{(2\pi)^3 2\sqrt{M^2 + \vec{P}^2}} \). For large \( Q^2 \) we obtain

\[
W^{\mu\nu}_q = \frac{m}{4\pi} \sum_M \int d\varepsilon e^{-ip^\varepsilon a} \int d^3\vec{P} \bar{\psi}_q(\vec{P}) \Gamma^{\mu\nu}_q \psi_q(\vec{P} + \vec{e}_z a),
\]

where \( p^\varepsilon = \varepsilon_q - m\varepsilon (1 + \frac{M^2}{Q^2}) \) (\( \varepsilon_q \) is the initial quark energy), \( \vec{e}_z = \frac{\vec{p}}{|\vec{p}|} \), and

\[
\Gamma^{tt}_q = \frac{M^2}{Q^2} (\gamma^0 - \gamma^z), \\
\Gamma^{\perp\perp}_q = \gamma^0 - \gamma^z.
\]

We have to take into account that mass of the exited quark \( M \) is the same as the mass of quark in the ground state, hence there is only one term, with \( M = m_q \approx 0 \), in the sum \( \sum_M \).
4 Taking into account the centre of mass motion

The structure functions calculated in the previous section should be considered as structure functions for the nucleon in an artificial external potential. Photon interacting with such nucleon may excite it into higher eigenstate in the artificial potential, or excite its internal degrees of freedom, or do the both. Since the nucleon is a composite object, its coupling to the photon cannot be described by simple matrix $\gamma^\mu$. We assume this coupling to be of the form

$$f(q^2, P^2, Pq)\gamma^\mu,$$

where $P^\mu$ is momentum of the final state.

In general this coupling may have more complicated spinor structure, but for large $Q^2$ other choices are effectively equivalent with ours provided we allow $f(q^2, P^2, Pq)$ to be dependent on the photon polarisation, i.e. we take $f(q^2, P^2, Pq) = f^A(q^2, P^2, Pq)$, where $A = \mu\nu = tt, \perp\perp$. For large $Q^2$, due to the energy conservation, we obtain $Pq = M^2 - Q^2$, so in fact our formfactor depends only on the polarisation, $Q^2$ and $M^2 = P^2$. For a given process (fixed $q^\mu$) $Q^2$ remains unchanged and we may write $W^\mu\nu$ for $\mu\nu = tt, \perp\perp$ in analogous to (8) form (hereafter $A = \mu\nu$)

$$W^A = \sum_M |f^A(M^2)|^2 \frac{m}{4\pi} \int d^3\vec{r} \bar{\Psi}_{CM}(\vec{r})\Gamma^A\Psi_{CM}(\vec{r} + \vec{e}_z a),$$

with $p^z = E - mx(1 + M^2/Q^2)$, and $\Gamma^{tt}, \Gamma^{\perp\perp}$ given by Eqs. (9), (10). ($\Psi_{CM}$ is the nucleon wave function, and $E$ stands for its energy.)

Introducing variable $y = 1/(1 + \frac{Q^2}{M^2})$, which by definition satisfies the condition $0 \leq y \leq 1$, the above equations may be rewritten in the form

$$W^A(x, Q^2) = \int_0^1 \frac{dy}{y} W^A_{eff}(y, Q^2) W^A_{CM}(\frac{x}{y}, Q^2),$$

where

$$W^{tt}_{eff}(y) = |f^{tt}(y)|^2 \frac{M^2(y)}{Q^2} M(y),$$

$$W^{\perp\perp}_{eff}(y) = |f^{\perp\perp}(y)|^2 M(y).$$

($M(y)$ is a measure coming from replacing $\sum_M$ by $\int \frac{dy}{y}$, $M(y) = Q\sqrt{\frac{1}{y} - 1}$) and

$$W^A_{CM}(\frac{x}{y}, Q^2) = \frac{m}{4\pi} \int da e^{-i(E - mx\frac{y}{Q})a} \int d^3\vec{r} \bar{\Psi}^A_{CM}(\vec{r})\Gamma^A_{CM}\Psi_{CM}(\vec{r} + \vec{e}_z a),$$

with (note difference with (9))

$$\Gamma^{tt}_{CM} = \Gamma^{\perp\perp}_{CM} = \gamma^0 - \gamma^z.$$
For a constant $\Psi_{CM}$ (corresponding to not moving, unbounded nucleon), we simply obtain

$$W^A_{CM}(\frac{x}{y}, Q^2) = \frac{m}{4\pi} 2\pi\delta(E - m\frac{x}{y}). \quad (18)$$

In that case $E = m$ and hence from (13)

$$W^A = \frac{1}{2} W^A_{eff}. \quad (19)$$

Comparing the above expression with (5), (6) we obtain

$$W^{tt}_{eff} = F_2 x^2 - 2F_1, \quad (20)$$

$$W^{\perp\perp}_{eff} = 2F_1. \quad (21)$$

This means that to obtain predictions for structure functions in our model, we have to find $W^A_{eff}$. Since for massless quarks $W^{tt}_q = 0$ (the Callan—Gross relation), we are interested only in $W^{\perp\perp}_{eff}$, which is directly related to the quark distribution function. To obtain $W^{\perp\perp}_{eff}$ we have to perform three steps:

a) calculate the naive structure function $W^{\perp\perp}_q$, using given quark wavefunction;

b) calculate $W^{\perp\perp}_{CM}$, using the centre of mass wavefunction $\Psi_{CM}$ (function of the nucleon in the auxiliary potential);

c) solve the equation Eq. (13) for $W^{\perp\perp}_{eff}$ using found $W^{\perp\perp}_{CM}$ and $W^{\perp\perp}_q$ and taking $W^{\perp\perp} = W^{\perp\perp}_q$.

Of course as far as we don’t know the centre of mass wavefunction $\Psi_{CM}$, this procedure cannot be performed. We will learn from the next section, how to find $W^{\perp\perp}_{CM}$ without calculating explicit form of $\Psi_{CM}$.

5 Calculating $W^{\perp\perp}_{CM}$

To obtain $W^{\perp\perp}_{CM}$ from Eq. (16), we have to calculate $\int d^3\vec{r} \bar{\Psi}_{CM}(\vec{r}) \Gamma^{\perp\perp}_{CM} \Psi_{CM}(\vec{r} + \vec{e}_z a)$. Since $\Gamma^{\perp\perp}_{CM} = \gamma^0 - \gamma^z$, we may write

$$\int d^3\vec{r} \bar{\Psi}_{CM}(\vec{r}) \Gamma^{\perp\perp}_{CM} \Psi_{CM}(\vec{r} + \vec{e}_z a) = \int d^3\vec{r} \bar{\Psi}_{CM}(\vec{r}) \Psi_{CM}(\vec{r} + \vec{e}_z a) - \int d^3\vec{r} \bar{\Psi}_{CM}(\vec{r}) \alpha^z \Psi_{CM}(\vec{r} + \vec{e}_z a). \quad (22)$$

The term $\int d^3\vec{r} \bar{\Psi}_{CM}(\vec{r}) \Psi_{CM}(\vec{r} + \vec{e}_z a)$ should correspond to overlapping function between translated states of the nucleon treated as a composite object. In our model the nucleon consist of three quarks with wavefunctions $\psi_q$, so

$$\int d^3\vec{r} \bar{\Psi}_{CM}(\vec{r}) \Psi_{CM}(\vec{r} + \vec{e}_z a) = (\int d^3\vec{r} \bar{\psi}_q_1(\vec{r}) \psi_q_1(\vec{r} + \vec{e}_z a)) \cdot (\int d^3\vec{r} \bar{\psi}_q_2(\vec{r}) \psi_q_2(\vec{r} + \vec{e}_z a)) \cdot (\int d^3\vec{r} \bar{\psi}_q_3(\vec{r}) \psi_q_3(\vec{r} + \vec{e}_z a)). \quad (23)$$
To obtain analogous expression for \( \int d^3\vec{r}\Psi_{CM}(\vec{r})^\dagger \alpha^z \Psi_{CM}(\vec{r}+\vec{e}_z a) \) let us remind that for a Dirac particle, mean value of the matrix \( \vec{\alpha} \) may be interpreted as a mean velocity. It is thus natural to take

\[
\vec{\alpha}_{CM} = \frac{1}{E} (\epsilon_{q_1} \vec{\alpha}_{q_1} + \epsilon_{q_2} \vec{\alpha}_{q_2} + \epsilon_{q_3} \vec{\alpha}_{q_3}),
\]

where \( \epsilon_{q_i} \) is the energy of the quark \( q_i \), and \( E \) is the total energy of the system.

For identical quark wavefunctions we obtain

\[
\int d^3\vec{r}\Psi_{CM}(\vec{r})^\dagger \alpha^z \Psi_{CM}(\vec{r}+\vec{e}_z a) = \frac{3\epsilon_{q}}{E} \left( \int d^3\vec{r}\psi_{q_1}(\vec{r})^\dagger \alpha^z \psi_{q_1}(\vec{r}+\vec{e}_z a) \right)^2.
\]

Equations (23) and (25) mean that we can calculate \( W^\perp_{\perp CM} \) just using \( \psi_{q} \), without introducing \( \Psi_{CM} \).

![Graph](image_url)

Figure 1: Functions \( W^\perp_{q} \equiv W^\perp_{\perp q} \) and \( W^\perp_{CM} \equiv W^\perp_{\perp CM} \) in our bag model; \( \epsilon_{q} = 2.04\frac{1}{R}, E = 8.17\frac{1}{R}, m = 7.62\frac{1}{R} \), where \( R \) is the bag radius.

Functions \( W^\perp_{q}(x) \) and \( W^\perp_{\perp CM}(x) \) calculated using the bag model wavefunctions are shown in Fig. 1.

### 6 Consistency of the model—sum rules

Taking Fourier transform of Eq. (13) we obtain

\[
T^\perp_{q}(a) = \int T^\perp_{\perp CM}(ax)W^\perp_{\perp f}(x) \, dx,
\]

(26)
where
\begin{align}
T_q^{\perp\perp}(a) &= e^{ie_a} \int d^3 \vec{r} \psi_q(\vec{r}) \gamma^0 \Gamma_{q}^{\perp\perp} \psi_q(\vec{r} + e_z a), \\
T_{CM}^{\perp\perp}(a) &= e^{iE_a} \int d^3 \vec{r} \Psi_{CM}(\vec{r}) \gamma^0 \Gamma_{CM}^{\perp\perp} \Psi_{CM}(\vec{r} + e_z a).
\end{align}

For wavefunctions normalised to 1 we obtain
\[ T_q^{\perp\perp}(0) = 1 = T_{CM}^{\perp\perp}(0), \]
and hence from (26)
\[ \int W_{\text{eff}}^{\perp\perp}(x) dx = T_q^{\perp\perp}(0) T_{CM}^{\perp\perp}(0) = 1. \]

Since \( W_{\text{eff}}^{\perp\perp} \) has an interpretation as the quark distribution function, the above equation stands for obvious normalisation condition (or from the experimental point of view Björken [12] or Adler [13] sum rule). It should be stressed that other methods of obtaining the structure functions from the bag model have some problems with fulfilling this condition [4].

Differentiating (26) \( n \) times and putting \( a = 0 \) we obtain
\[ \int W_{\text{eff}}^{\perp\perp}(x)x^{n-1} dx = \frac{T_q^{\perp\perp}(n-1)(0)}{T_{CM}^{\perp\perp}(n-1)(0)}, \]
where \( T^{\perp\perp}(n-1) = \left( \frac{d}{da} \right)^{n-1} T^{\perp\perp} \). The above expression may be rewritten in the form
\[ M_{\text{eff}}^{\perp\perp}(n) = \frac{M_q^{\perp\perp}(n)}{M_{CM}^{\perp\perp}(n)}, \]
where \( M^{\perp\perp}(n) = \int W^{\perp\perp}(x)x^{n-1} dx \) (Mellin moments). It is well known that in the bag model without any improvements for the centre of mass motion one gets \( 2M_q^{\perp\perp}(2) = \frac{1}{3} \). This remains true in any independent particle model with the “volume” energy with energy-momentum tensor of the form:
\[ \mathcal{T}_V^{\mu\nu} = b(r)g^{\mu\nu}, \]
where \( b(r) \) is chosen so to guarantee conservation of the total (quarks + “volume”) energy-momentum tensor
\[ \partial_\nu (3 T_q^{\mu\nu} + T_V^{\mu\nu}) = 0. \]

On the other hand energy-momentum tensor \( \mathcal{T}_{CM}^{\mu\nu} \) of our nucleon moving in the auxiliary potential cannot be conserved, and so corresponding \( 2M_{CM}^{\perp\perp}(2) \neq 1 \). But if we modify the condition [24] by taking into account CM motion
\[ \partial_\nu (3 T_q^{\mu\nu} + T_V^{\mu\nu}) = \partial_\nu T_{CM}^{\mu\nu}, \]
then we obtain

\[ M_{\perp\perp}^{\perp\perp}(2) = \frac{M_{q}^{\perp\perp}(2) + M_{CM}^{\perp\perp}(2)}{2} = \frac{1}{3}, \] (34)

although in such case \(2M_{q}^{\perp\perp}(2) \neq \frac{1}{2}\).

On the other hand, equation (33) together with (25) allows us to find improved energy \(E\) of our bound nucleon. For \(N\) identical quarks it satisfies the following condition

\[ E = E_0 - \frac{N\epsilon_q - N\epsilon_q}{E}, \] (35)

where \(E_0\) is the nucleon energy calculated using (32).

For the bag model with massless quarks we obtain

\[ \epsilon_q = 2.04 \frac{1}{R}, \quad E_0 = 8.17 \frac{1}{R}, \quad E = 7.62 \frac{1}{R}, \] (36)

where \(R\) is the bag radius.

### 7 Extracting effective quark distributions

To calculate effective quark distributions, we may use equation (13) or (26), but since \(W_{CM}^{\perp\perp}\) and \(W_{q}^{\perp\perp}\) are slowly decreasing functions (see Fig. 1), Eq. (26) is much more useful for numerical calculations. This equation is in fact Fredholm equation of the first kind, generally ill-posed — its solutions may not exist or may be not unique. The standard method of solving such problems is to convert them into variational ones with additional regularization term [7]. Following [8] we introduce a smoothing functional

\[ \mathcal{F}_\lambda[W_{\text{eff}}^{\perp\perp}] = \int \left| T_q^{\perp\perp}(a) - \int T_{CM}^{\perp\perp}(ax)W_{\text{eff}}^{\perp\perp}(x)\,dx \right|^2 \, da + \lambda \int \left| W_{\text{eff}}^{\perp\perp}(x) \right|^2 \, dx, \] (37)

where \(\lambda > 0\) is a regularization parameter.

Now our goal is to find minimum of the functional \(\mathcal{F}_\lambda\). Since for physical solution \(W_{\text{eff}}^{\perp\perp} \geq 0\), we impose this condition as a constraint for our variational problem.

We have solved our problem using procedure PTIPR from [8]. Results are plotted in Fig. 3.

In Fig. 3 we have plotted l.h.s. of the equation (26), which can be written in the shorthand form as

\[ T_q = T_{CM} * W_{\text{eff}} \] (38)

and its r.h.s. corresponding to solutions for \(\lambda = 10^{-5}\) and \(\lambda = 10^{-6}\) plotted in Fig. 3.
Figure 2: Effective quark distributions $q(x)$ in the bag model found using variational method with different regularization parameter $\lambda = 10^{-5}, 10^{-6}, 10^{-7}$; $\epsilon_q = 2.04 \frac{1}{\pi}$, $E = 7.62 \frac{1}{\pi}$, $m = E$: For comparison there is also shown “naive” quark distribution $q(x) = 2W_{q\perp\perp}(x)$ for $\epsilon_q = 2.04 \frac{1}{\pi}$, $E = 8.17 \frac{1}{\pi}$, $m = 7.62 \frac{1}{\pi}$.

8 QCD evolution

The obtained quark distributions in the proton cannot be compared directly with data. According to the usual procedure we assume them to be twist two contribution to the structure functions at small scale $Q^2 = Q_0^2$, and then we evolve them to a higher scale. Results of the leading order (LO) evolution for $F_2$ to $Q^2 = 4\text{GeV}^2$ are plotted in the Fig. 4. There also is shown for comparison parametrisation of GRV $\frac{1}{4}$. The chosen initial scales are $Q_0^2 = 0.23\text{GeV}^2$ (the initial scale of GRV in the LO), and $Q_0^2 = 0.35\text{GeV}^2$, which we found as giving better predictions in the small $x$ region.

It may be easily seen that evolution of our quark distributions from $Q_0^2 = 0.23\text{GeV}^2$ gives too steep $F_2$ in the small $x$ region, but it is possible to find $Q_0^2$ giving much better agreement with the data. On the other hand, our $F_2$ differs drastically from the data for $x > 10^{-3}$. We attempted to cure this problem taking into account target mass effects.

9 Reinterpretation of the results in terms of the Politzer’s variable $\xi$

Let us start with the remark, that since GRV parametrisation was obtained by evolving (according to QCD) initial distributions given at small $Q^2 = \mu^2$, we don’t have to perform the evolution ourselves—it is enough to compare our
predictions with that of GRV at some small scale larger then $\mu^2$.

Described in the previous section procedure of finding quark distributions assumes that their entire dependence (with fixed Björken $x$) on $Q^2$ comes from QCD. But due to simple kinetic target mass effects it cannot be true for smaller $Q^2$. It was found many years ago [9], that much better scaling may be obtained in terms of the variable $\xi$ related to the Björken $x$ by

$$\xi = \frac{2x}{1 + \sqrt{1 + \frac{4m_p^2x^2}{Q^2}}}, \quad (39)$$

where $m_p$ stands for target (proton) mass.

The connection between usual $F_2(x, Q^2)$ and the scaling function $F_2^S(\xi, Q^2)$, which for large $Q^2$ coincides with $\frac{F_2(x, Q^2)}{x}$, in that limit $\xi = x$ is given by [10]:

$$F_2(x, Q^2) = \frac{x^2}{(1 + 4x^2m_p^2/Q^2)^{3/2}} F_2^S(\xi, Q^2)$$

$$+ 6 \frac{m_p^2}{Q^2} \frac{x^3}{(1 + 4x^2m_p^2/Q^2)^2} \int_\xi^1 d\xi' F_2^S(\xi', Q^2)$$

$$+ 12 \frac{m_p^4}{Q^4} \frac{x^4}{(1 + 4x^2m_p^2/Q^2)^{5/2}} \int_\xi^1 d\xi' \int_\xi^{\xi'} d\xi'' F_2^S(\xi'', Q^2), \quad (40)$$

where $\xi$ is the function of $x$, see Eq.(39).

$F_2(x, Q^2)$ obtained from the above expression with $F_2^S(\xi) = \frac{F_2^{eff}(\xi)}{\xi^2}$, where $F_2^{eff}(x) = xW_{eff}(x)$ is the effective bag structure function obtained in the
section 7, compared with GRV parametrisation for the same $Q^2$, is shown in the Fig. 5.

10 Conclusions

We have proposed here a method of finding hadron structures functions in independent particle models. The method leads to the structure functions with the correct support and exactly fulfils the normalisation condition for valence quark distributions.

The structure functions found for the bag model in the static cavity approximation differs drastically from the "naive" ones, calculated without any improvement for the centre of mass motion. They disagree with the data, but it is possible to obtain rough agreement in the small $x$ region, evolving them using perturbative QCD.

Much better agreement with parametrisation of GRV may be obtained, taking into account that structure functions scale for small $Q^2$ in terms of the Politzer’s variable $\xi$.

It should be stressed that our disagreement with the large $Q^2$ data arises mainly from quite different behaviour of our valence quark distributions at low $Q^2$, which are finite for $x \to 0$, and the ones used by GRV, which are singular in that limit (although their $xq(x) \to 0$). In fact, any independent particle model gives finite $q_v(x \to 0)$. To obtain singular $q(x)$ one has probably to take into account Regge trajectories for final states in the deeply inelastic scattering.
Figure 5: Valence quark part of $F_2(x)$ calculated using Eq.(43) for $Q^2 = 0.35\text{GeV}^2$ and $Q^2 = 4\text{GeV}^2$ compared with the parametrisation of GRV.

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References
[1] M. Gluck, E. Reya, A. Vogt, Z. Phys. C 48C (1990), 471; C 53 (1992), 127;
[2] M. Gluck, E. Reya, A. Vogt, Z. Phys. C 67 (1995), 433;
[3] R. L. Jaffe, Phys. Rev. D 11 (1975), 1953;
[4] F. M. Steffens and A. W. Thomas, Nucl. Phys. A568 (1994), 798.
[5] A. W. Streiber and A. W. Thomas, Phys Rev. D 49 (1994), 127;
[6] A. Szymacha, Phys. Lett. 146 B (1984), 350;
[7] W. H. Press et al., Numerical Recipes in C, Cambridge University Press, 1992;
[8] A. N. Tikhonov et al., Numerical Methods for the Solution of Ill-Posed Problems, Kluwer Academic Publishers, 1995;
[9] H. Georgi, H. D. Politzer, Phys. Rev. Lett. 36 (1976), 1281;
[10] A. De Rujula, H. Georgi, H. D. Politzer, Ann. Phys. 103 (1977), 315;

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[11] A. Chodos, R. L. Jaffe, K. Johnson and C. B. Thorn, Phys. Rev. D 10 (1974), 2599;

[12] J. D. Björken, Phys. Rev. 163 (1967), 1767;

[13] S. L. Adler, Phys. Rev. 143 (1966), 1144.