How the curvaton scenario, modulated reheating and an inhomogeneous end of inflation are related

Laila Alabidi *
Astronomy Unit, School of Mathematical Sciences, Queen Mary University of London,
Mile End Road, E1 4NS, UK
E-mail: l.alabidi@qmul.ac.uk

Karim Malik†
Astronomy Unit, School of Mathematical Sciences, Queen Mary University of London,
Mile End Road, E1 4NS, UK
E-mail: k.malik@qmul.ac.uk

Christian T. Byrnes
Fakultät für Physik, Bielefeld University, Universitätstrasse, D-33615 Bielefeld,
Germany

Ki-Young Choi‡
Department of Physics, Pusan National University, Busan, 609-735, Korea

Abstract: In this paper we analyse three models of the early universe, for which the respective mechanisms for generating the curvature perturbation are considered disparate. We find that in fact the mechanisms are very similar, and hence explain why they give rise to a large non-gaussianity. We show that the mechanism for generating the primordial curvature perturbation, and hence the observable non-gaussianity, is similar in both the Curvaton and Modulated Reheating models. In both cases the model can be written in terms of an energy transfer between the constituting fluids. We then show that this is also true for the mechanism of generating the curvature perturbation by symmetry breaking the end of inflation. We then relate this to the non-gaussian contribution to the curvature perturbation and find that it is inversely proportional to the efficiency with which the curvature perturbation is transferred between the fluids. For the first time, we generalise models of modulated reheating to allow for a non-linear energy transfer rate.

Keywords: inflation, perturbation theory, non-gaussianity.

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1. Introduction

One of the greatest successes of inflationary cosmology is to provide a mechanism to generate the spectrum for the primordial density perturbations, in excellent agreement with recent observations of the Cosmic Microwave Background (CMB) and galaxy surveys. In the most popular single field inflation models, the same field that drives inflation is also responsible for the generation of the spectrum during inflation. However, in recent years models in which these tasks are separated have become prominent, in particular the curvaton scenario, modulated reheating, and an inhomogeneous end of inflation (here the separation is incomplete). Although these models were usually considered to be very different, taking the mechanism that generates the primordial spectrum of perturbations to categorise the models we show that these models are in fact very similar, as the generation mechanism of the perturbations is the same. This also explains why all three models can give rise to large non-gaussianity.

The same observations cited above also show that the spectrum of primordial density perturbations is predominantly gaussian in nature with a possible minor deviation. Single field canonical models of inflation are known to give a non-gaussian contribution which is slow roll suppressed; this level of non-gaussianity is undetectable by current technology. It has been shown that if the field that generated the primordial curvature perturbation interacts with another fluid, such as in the curvaton or modulated...
reheating \cite{2, 7, 8, 9}, or if inflation is driven by more than one field, see e.g. \cite{20, 21, 22}, then the non-gaussianity can be enhanced to a detectable level.

In this paper we evaluate the curvature perturbation $\zeta$ and the non-linearity parameter $f_{NL}$ for models of modulated reheating, the curvaton model, and the inhomogeneous end of inflation. We investigate the relationship between these models and highlight the underlying mechanism of generation and the conditions for enhancing $f_{NL}$. Although at first glance these models appear to be very disparate, we show that the generation mechanism in all three cases is similar: all three models can be written in terms of an energy transfer between the constituting fluids; the energy transfer itself is controlled by a light scalar field; the fluctuations of the scalar field are then “inherited” after its decay by the curvature perturbation. In the curvaton and modulated reheating models, the curvature perturbation can be parametrised by the efficiency with which the curvature perturbation is translated from the field that generates the primordial curvature perturbation to the radiation field and hence is the determining factor in $f_{NL}$. The same is true for the inhomogeneous end of inflation case, however the efficiency parameter here refers to the efficiency by which the curvature perturbation is translated from one scalar field to the one that generates the primordial density perturbation.

For the first time we consider an energy transfer which depends non-linearly on the energy density in the modulated reheating scenario. We show how significantly more general scenarios lead to a curvature perturbation with the same functional form but modified coefficients, which affects the value of $f_{NL}$ but it still becomes large in the same limit as in the standard case of a linear transfer. Unlike previous calculations, we do not resort to solving the equations of modulated reheating numerically, which makes the derivation more transparent.

This paper is organised as follows: in Section (2) we present the equations governing the energy transfer between fluids, in Section (3) we evaluate the curvature perturbation for models in which a scalar field decays into radiation, in Section (4) we calculate the curvature perturbation using a different approach to that derived in previous work for the inhomogeneous end of inflation. We present the results for the non-gaussianity parameter $f_{NL}$ in Section (5) and discuss them in Section (6). We use the convention of decomposing the curvature perturbation in terms of the first order $\zeta_1$ and second order $\zeta_2$ contributions as follows

$$\zeta = \zeta_1 + \frac{1}{2}\zeta_2,$$

where $\zeta_1$ is the linear (gaussian) perturbation and $\zeta_2$ is a gaussian squared, see e.g. Ref. \cite{23}.  

2. Energy Transfer

The line element for a Friedmann-Robertson-Walker spacetime including scalar perturbations, without yet specifying a particular gauge, is

$$ds^2 = -(1 + 2\phi)dt^2 + 2aB_{,i}dt dx^i + a^2 [(1 - 2\psi)\delta_{ij} + 2E_{,ij}] dx^i dx^j,$$
where we use the notation and conventions of Refs. \[23\] with the curvature perturbation, $\psi$, the lapse function, $\phi$, and scalar shear, $a^2 \dot{E} - aB$, and where $\delta_{ij}$ denotes the flat background metric and $X_i \equiv \partial X / \partial x^i$.

Allowing for the exchange of energy between fluids, the equations governing the evolution of the density perturbation are given by Refs. \[24, 25, 23\]. From the local energy-momentum equation,

$$\nabla_\mu T^{\mu\nu}_{(\alpha)} = Q^\nu_{(\alpha)}, \quad (2.2)$$

we have the evolution equation for the energy density of a particular fluid in the background

$$\dot{\rho}_\alpha + 3H(\rho_\alpha + P_\alpha) = Q_\alpha, \quad (2.3)$$

and at first order in the perturbations on superhorizon scales

$$\dot{\delta\rho}_\alpha + 3H(\delta\rho_\alpha + \delta P_\alpha) - 3(\rho_\alpha + P_\alpha)\dot{\psi} + Q_\alpha\psi - \delta Q_\alpha = 0, \quad (2.4)$$

where $\rho$ is energy density, $P$ is pressure, subscript $\alpha$ denotes the fluid species, $Q_\alpha$ is the energy transfer to the $\alpha$ fluid and $T^{\mu\nu}$ is the stress-energy tensor.

Note that in Sections (3.1) to (3.3) we use the longitudinal gauge, $a^2 \dot{E} - aB = 0$, and we ignore anisotropic stress so $\phi = \psi$, whereas in Sections (3.4) and (4) we use the flat gauge, $\psi = 0$, for convenience.

3. Models in which a scalar field decays into radiation

We now consider three different forms for the energy transfer: in model (3.1) $Q_m$ is a linear function of $\rho_m$, in model (3.2) it is a higher order function of $\rho_m$ and in model (3.3) it is a function of a combination of powers. These three models are all examples of modulated reheating, in which the decay rate from the inflaton to radiation is modulated by a subdominant scalar. Since the Universe is matter dominated before decay and radiation dominated after decay, changes in the decay time lead to changes in the expansion and hence effect the curvature perturbation \[6\]. In section (3.4) we present the generic results for the curvaton scenario, in which the rate of decay is homogeneous.

3.1 $Q_m$ as a linear function of $\rho_m$

We model the energy transfer $Q_\alpha$ according to the standard choice

$$Q_\alpha = -\Gamma \rho_\alpha, \quad (3.1)$$

where $\Gamma$ is the decay rate of the $\alpha$ field into radiation. In this section we consider modulated reheating in which the decay rate of a light scalar field $\varphi$ varies, that is $\Gamma = \Gamma(\varphi)$, and in Section (3.4) we analyse the curvaton ($\sigma$) model in which $\Gamma$ is taken to be constant. For both these models the background evolution equations for an oscillating scalar field (i.e. matter type) fluid and radiation are \[23, 4\]

$$\dot{\rho}_m = -3H\rho_m - \Gamma \rho_m, \quad (3.2)$$

$$\dot{\rho}_r = -4H\rho_r + \Gamma \rho_m, \quad (3.2)$$
and we assume that the decay is into Standard Model radiation. Evaluating Eq. (2.3) for the oscillating scalar field, noting that \( P_m = \delta P_m = 0 \) and \( \delta Q_m = -\Gamma \rho_m - \Gamma \delta \rho_m \), which follows from (3.2), we get

\[
\frac{\dot{\delta \rho_m}}{\rho_m} + 3H \frac{\delta \rho_m}{\rho_m} - 3\dot{\phi} + \Gamma \phi + \Gamma \frac{\delta \rho_m}{\rho_m} + \delta \Gamma = 0.
\] (3.3)

Introducing the density contrast \( \delta_{\alpha} = \delta \rho_{\alpha}/\rho_{\alpha} \) to rewrite the above equation in a more compact form, and using Eq. (3.2) we get \( \dot{\delta \rho_m}/\rho_m = \dot{\delta}_m - \delta_{\Gamma} (3H + \Gamma) \) and

\[
\dot{\delta}_m = 3\dot{\phi} - \Gamma (\phi + \delta_{\Gamma}),
\] (3.4)

where \( \delta_{\Gamma} = \delta \Gamma/\Gamma \). Similarly, for the radiation fluid, where \( P_r = \rho_r/3 \) and \( Q_r = \Gamma \rho_m \), we find

\[
\dot{\delta}_r = 4\dot{\phi} + \Gamma \frac{\rho_m}{\rho_r} (\phi + \delta_{\Gamma} + \delta_m - \delta_r).
\] (3.5)

Finally we have from the time component of the Einstein equations \( G_{\mu\nu} = 8\pi GT_{\mu\nu} \) on large scales [25]

\[
H \dot{\phi} + H^2 \phi = -\frac{4\pi G}{3} (\rho_m \delta_m + \rho_r \delta_r).
\] (3.6)

In Ref. [6] Modulated Reheating is studied in the so called ‘purely forced’ case, where the metric perturbations are sourced only by the perturbations in the rate of decay. To derive the curvature perturbation for this case we assume that the perturbations in the matter and radiation fields are subdominant, hence leaving us with two equations

\[
\begin{align*}
H \dot{\phi}_{\text{mod}} + H^2 \phi_{\text{mod}} &= 0, \\
3\dot{\phi}_{\text{mod}} &= \Gamma (\phi_{\text{mod}} + \delta_{\Gamma}),
\end{align*}
\] (3.7)

where the subscript “mod” refers to ‘modulated reheating’. The above equations then give

\[
\phi_{\text{mod}} = -\frac{\Gamma}{\Gamma + 3H} \delta_{\Gamma} = -\beta \frac{\delta \Gamma}{\Gamma}.
\] (3.8)

Here \( \beta \) is proportional to the decay rate with \( \beta \simeq 1 \) corresponding to instant reheating.

The curvature perturbation on uniform density hypersurfaces is defined as

\[
\zeta_{\text{mod}} = -\phi_{\text{mod}} - H \frac{\delta \rho}{\dot{\rho}}.
\] (3.9)

where \( \delta \rho/\dot{\rho} = \sum_i \delta \rho_i / \sum_j \dot{\rho}_j \). Evaluating the RHS of Eq. (3.9) in the “forced case” (i.e. when \( \delta \rho/\rho \ll \phi \)) gives

\[
\zeta_{\text{mod}} = -\phi_{\text{mod}},
\] (3.10)

and finally, substituting Eq. (3.8) into Eq. (3.10) we get

\[
\zeta_{\text{mod}} = \beta \frac{\delta \Gamma}{\Gamma},
\] (3.11)
Assuming now that \( \Gamma \) depends only on one light scalar field, \( \varphi \), (Ref. \[9\] considers a more general case) a simple Taylor expansion results in

\[
\Gamma = \Gamma_0 + \frac{\partial \Gamma}{\partial \varphi_*} \delta \varphi_* + \frac{1}{2} \frac{\partial^2 \Gamma}{\partial \varphi_*^2} \delta \varphi_*^2 ,
\]

(3.12)

where the subscript * refers to horizon exit, this gives the curvature perturbation as

\[
\zeta_{\text{mod}} = \frac{\beta}{\Gamma} \left( \Gamma_{\varphi_*} \delta \varphi_* + \frac{\Gamma_{\varphi_* \varphi_*}}{2} \delta \varphi_*^2 \right) ,
\]

(3.13)

where \( \Gamma_{\varphi_*} \) and \( \Gamma_{\varphi_* \varphi_*} \) are the first and second derivatives with respect to \( \varphi_* \).

In sections (3.4) and (4) we define the efficiency as the ratio of the first order curvature perturbation of a field prior to its decay with respect to its value post decay, and hence we construct similar parameters for this model for completeness. The field \( \varphi_* \) can be related to a curvature perturbation of \( \zeta_{\varphi} = (\ddot{\varphi}_* + V_{,\varphi_*}) \delta \varphi_* / (3 \dot{\varphi}_*^2) \equiv b \delta \varphi_* \), see e.g. Ref. \[26\], then the efficiency parameter is

\[
c_1 = \frac{\zeta_{\text{mod} 1}}{\zeta_{\varphi}} = \frac{\beta \Gamma_{\varphi_*}}{b \Gamma} ,
\]

(3.14)

\[
c_2 = \frac{\zeta_{\text{mod} 2}}{\zeta_{\varphi}^2} = \frac{\beta \Gamma_{\varphi_* \varphi_*}}{b^2 \Gamma} .
\]

(3.15)

The overall curvature perturbation for this model can now be written as

\[
\zeta_{\text{mod}} = b c_1 \delta \varphi_* + b^2 c_2 \delta \varphi_*^2 .
\]

(3.16)

### 3.2 \( Q_m \) as a higher order function of \( \rho_m \)

In this case we define the energy transfer as a function of a higher power of the energy density, similar to the model recently considered in the context of dark energy decay in \[27\],

\[
Q_m = \Gamma \rho_m^n .
\]

(3.17)

In order to avoid singular behaviour in the limit of small \( \rho_m \) we require \( n > 0 \).

Following the same steps as in section (3.1), i.e. only considering \( \delta \rho = 0 \), we find that the curvature perturbation on superhorizon scales has the same form as Eq. (3.13), with \( \beta \) now also a function of the energy density:

\[
\beta = \frac{\Gamma \rho_m^{n-1}}{3H + \Gamma \rho_m^{n-1}} .
\]

(3.18)

In order for \( \rho_m \) to decay then, the decay rate must exceed the expansion rate of the Universe, \( \rho_m \ll 10^{-12} \) in Planck Units, and since \( H \) decays as \( \rho_m^{1/2} \), then in order for \( H / \Gamma \rho_m^{n-1} < 1 \) we require \( n < 3/2 \).
3.3 $Q_m$ as a function of a combination of powers of $\rho_m$

In this case we define the energy transfer function as

$$Q_m = \Gamma (A\rho_m^p + B\rho_m^q),$$

(3.19)

where $0 < p < q$. Then $\zeta$ has the same functional form as Eq. (3.13) but with $\beta$ redefined as

$$\beta = \frac{\Gamma \left( A\rho_m^{p-1} + B\rho_m^{q-1} \right)}{3H + \Gamma \left( A\rho_m^{p-1} + B\rho_m^{q-1} \right)}.$$

(3.20)

3.4 The curvaton scenario: $\delta \Gamma = 0$

We now consider the second scenario, where $\delta \Gamma = 0$. This is the case in the curvaton model. The energy density of the curvaton is highly subdominant compared to the inflaton field during inflation, but afterwards, while it oscillates about the (quadratic) minimum of its potential, its energy density decays like matter. This energy component grows relative to the radiation density generated by the inflaton decay products until the curvaton also decays into radiation \[ \text{[3, 4, 5]} \]. In this scenario the curvature perturbation is solely generated from perturbations in the curvaton field. Using a quadratic potential for the curvaton, the curvature perturbation in this model was found to be

$$\zeta_{\text{curv}} = \frac{2r_1}{3} \left( \frac{\delta \sigma_1}{\sigma_0} + \frac{1}{2} \left( \frac{\delta \sigma_1}{\sigma_0} \right)^2 \right),$$

(3.21)

where $\sigma$ has been split as $\sigma = \sigma_0 + \delta \sigma_1$, and $r_1$ parametrises the contribution of the $\sigma$ field to the overall curvature perturbation post $\sigma$ decay

$$r_1 = \frac{\zeta_{\text{curv, post } \sigma \text{ decay}}}{\zeta_{\text{curv, pre } \sigma \text{ decay}}}. \quad (3.22)$$

Using second order perturbation theory $\zeta_{\text{curv}}$ is given by \[ \text{[19]} \]

$$\zeta_{\text{curv}} = \frac{2}{3} r_1 \frac{\delta \sigma_1}{\sigma_0} + \frac{1}{3} \left( r_1 + \frac{2}{3} r_2 \right) \left( \frac{\delta \sigma_1}{\sigma_0} \right)^2, \quad (3.23)$$

where

$$r_2 = \frac{\zeta_{\text{curv, post } \sigma \text{ decay}}}{\zeta_{\text{curv, pre } \sigma \text{ decay}}} \quad (3.24)$$

4. Inhomogeneous End of Inflation

In previous work \[ \text{[10, 11, 28]} \], it has been shown that the curvature perturbation can be generated at the end of inflation if there is an ultra light scalar field sub-dominant to the inflaton during inflation. This second field does not play a role in the inflationary dynamics, but serves to perturb the inflaton trajectory from a straight line, potentially resulting in the generation of non-gaussianity. This concept was generalised to a two-field hybrid model
of inflation in \cite{12} and was found to give a measurable $f_{NL}$. The potential describing this model is given by:

$$V = V_0 - \frac{m^2}{2}(\phi^2 + \sigma^2) - \left(f \phi^2 + g \phi \sigma + h \sigma^2\right) - \left(f \phi^2 + g \phi \sigma + h \sigma^2\right) + \frac{m^2}{2} \chi^2,$$

where $\chi$ is the waterfall field which is held at zero during inflation, and $f$, $g$ and $h$ define the coupling of the inflaton fields to the waterfall field. In this case $f \neq g \neq h$ and hence the surface at the end of inflation is best defined by an ellipse. For simplicity we assume an instantaneous decay of the scalar fields into radiation at the end of inflation, for a more detailed discussion of this point see Refs. \cite{28, 13}.

In contrast to previous work carried out on this and similar models \cite{12, 10, 11, 13, 14} which used the $\delta N$ formalism, we use the perturbative approach in calculating $f_{NL}$. Starting with the definition of the curvature perturbation at horizon exit,

$$\zeta_e = -H \frac{\delta \rho_e}{\rho_e},$$

where the subscript “e” refers to the end of inflation. We take the $\sigma = 0$ trajectory and split $\varphi_e$ as $\varphi_e = \varphi_{0e} + \delta \varphi_e$, and thus,

$$\zeta_e = \frac{1}{2 \eta_s} \left(2 \frac{\delta \varphi_e}{\varphi_{0e}} + \left(\frac{\delta \varphi_e}{\varphi_{0e}}\right)^2\right).$$

In order to evaluate the spectrum and bi-spectrum for this model we will need to calculate terms of the form $< \delta \varphi_e(k_1) \delta \varphi_e(k_2) >$, but since we only know their values at horizon exit we re-write $\delta \varphi_e$ in terms of $\delta \varphi_e$:

$$\delta \varphi_e = \delta \varphi_e(\varphi_e + \delta \varphi_e, \sigma_e + \delta \sigma_e) = \frac{\partial \delta \varphi_e}{\partial \varphi_e} \delta \varphi_e + \frac{\partial \delta \varphi_e}{\partial \sigma_e} \delta \sigma_e + \frac{\varphi ''_e}{2} \delta \sigma_e^2,$$

where $\varphi_e' = \partial \varphi_e/\partial \sigma = -g/2f$, $\varphi_e'' = -(2h - g^2/(2f))/2f \varphi_e$, $\partial \varphi_e/\partial \varphi_e = e^{\eta_s N}$ \cite{12}, $\eta_s$ is the second derivative slow roll parameter and $N$ is the logarithmic ratio of the scale factor at the end of inflation with respect to its value when scales of cosmological interest exited the horizon \cite{29}.

Assuming that the curvature perturbation is generated predominantly from the $\sigma$ field, then Eq. (4.3) becomes

$$\zeta_e = \frac{\varphi_e'}{\eta_s \varphi_e} \delta \sigma_e + \frac{1}{2 \eta_s \varphi_e^2} \left(\varphi_e'' + \varphi_e \varphi_e''\right) \delta \sigma_e^2,$$

and since we are mainly interested in highlighting the underlying physical mechanism, we have used a simplifying \cite{18} calculation to get the second order contribution, which explains why our results do not match those of Ref. \cite{12} at second order exactly.

We now define the parameters

$$b_1 = \frac{\zeta_e}{\varphi_e} = \varphi_e' e^{-\eta_s N},$$
and
\[ b_2 = \frac{\zeta_2}{\zeta_1} = \frac{\eta_s}{2} e^{-2\eta_s N} \left( \varphi_e' + \varphi_e'' \right), \quad (4.7) \]
in analogy with the parameters \( c_1 \) and \( c_2 \) above. Therefore the curvature perturbations generated by the \( \sigma \) field can be written as
\[ \zeta_{\sigma} = \frac{1}{\eta_s} \left( b_1 \left( \frac{\delta \sigma}{\varphi} \right) + b_2 \left( \frac{\delta \sigma}{\varphi} \right)^2 \right). \quad (4.8) \]

This model can be recast in the form of two interacting fluids by writing Eqs. (2.3) and (2.4) in the slow roll approximation for this case
\[ \dot{\rho}_\phi = -Q_{\sigma} = -V_{\sigma} \dot{\sigma} = 0, \]
\[ \delta \rho_{\phi} = -\delta Q_{\sigma} = \frac{V_{\sigma}^2}{3H} \delta \sigma^2, \quad (4.9) \]
showing more clearly the relation to the previous models and hence explaining the similar results in terms of the non-linearity parameter, as detailed in the next section. Note that we are only modelling the energy transfer between fields and not the decay of the field into radiation. We assume an instantaneous decay into radiation at the end of inflation, for a discussion of this point see Refs. [28, 13]. The energy of the \( \phi \) field is transferred to the radiation at the end of inflation and the perturbation of this energy transfer is dominantly controlled by the perturbation of the \( \sigma \) field. After the decay of the \( \phi \) field, the energy of the radiation was endowed from the \( \phi \) field but the perturbation is inherited from the \( \sigma \) field.

5. The Non-Gaussianity Parameter \( f_{NL} \)

From Eq. (3.16) we find that the non-linearity parameter \( f_{NL} \) for the modulated reheating case is given by
\[ f_{NL} = \frac{5 c_2}{6 c_1}, \quad (5.1) \]
which in terms of \( \beta \) is
\[ f_{NL} = \frac{5 \Gamma_{\phi,\varphi}}{12 \beta \Gamma_{\varphi}^2}. \quad (5.2) \]
Using the functional form \( \Gamma = \Gamma_1 (1 + \delta \varphi/\varphi) \), following [3], Eq. (5.2) gives
\[ f_{NL} = \frac{5}{24 \beta}, \quad (5.3) \]
and \( \beta \) depends on the decay rate. This inverse dependence on \( \beta \) is also valid for the new models discussed in Sections (3.2) and (3.3).

The non-linearity parameter \( f_{NL} \) for the curvaton model is derived in the simplest case from Eq. (3.21) to be [18]
\[ f_{NL} = \frac{5}{4r_1}, \quad (5.4) \]
and using second order perturbation theory \[19\]

\[ f_{NL} = \frac{5}{4r_1} + \frac{5r_2}{6r_1^2} - \frac{5}{3}. \]  

(5.5)

Eqs. (5.4) and (5.5) were evaluated for the potential \( V(\sigma) = m^2\sigma^2/2. \)

Finally we have for the inhomogeneous end of inflation:

\[ f_{NL} = \frac{5}{6} \frac{b_2}{b_1^2}. \]  

(5.6)

6. Discussion

In this paper we have studied models of the early universe dominated by a scalar field that then decays into radiation, and a model in which two scalar fields drive inflation where the curvature perturbation is generated primarily at the end of inflation. Using the energy transfer \( Q = \Gamma \rho \) to characterise the models, the former case can be further split into models with a homogeneous \( \Gamma \) and varying \( \rho \), Curvaton models, and models with a varying decay rate \( \Gamma \) and homogeneous energy density \( \rho \), Modulated reheating.

We then evaluated \( f_{NL} \) for the simplest case of a homogeneous \( \Gamma \), and for various functional forms of the energy transfer parameter \( Q_\phi \). We found that in the varying \( \Gamma \) case \( f_{NL} \) is inversely proportional to the decay efficiency \( \beta \), even in the new case we consider of a non-linear energy transfer, \( Q = \Gamma \rho^a \) or \( Q = \Gamma (A\rho_m^p + B\rho_m^q) \), which parametrises the rate at which the scalar field decays into radiation, as well as the energy density of the scalar field prior to its’ decay. Reparametrising our equations shows that \( f_{NL} \) is inversely proportional to \( c_1^2 \), where \( c_1 \) is the ratio of \( \zeta \) prior to the decay of the light scalar \( \varphi \) to it’s value post \( \varphi \) decay. Similarly in the case where the curvature perturbation is generated by the inhomogeneous end of inflation, we found that \( f_{NL} \) is inversely proportional to the square of the parameter \( b_1 \), the ratio of the first order curvature perturbation sourced by \( \sigma \) to the curvature perturbation sourced by \( \varphi \) at horizon exit. Previous work \[13\] (which we reproduced here) already showed a similar result for the curvaton model; \( f_{NL} \) is inversely proportional to \( r_1 \), the ratio of the first order curvature perturbation after the decay of \( \sigma \) with respect to the value of the curvature perturbation prior to its decay, Eq. (3.22).

Our results show that the mechanism of generating the curvature perturbation in these models is similar, and can be modelled as energy transfer between interacting fluids: the fluctuations are generated in the first fluid, a massless scalar field, and are then inherited by the second one into which it decays. This therefore explains why the condition for enhancing \( f_{NL} \) is similar; it is the efficiency with which the energy density or curvature perturbation is translated between the fluids that determines the level of non-gaussianity.

Although the three models we have discussed are physically very different and neither share the same fields nor generate the curvature perturbation during the same epoch, in all cases the curvature perturbation is related to an energy transfer. This energy transfer is from a light scalar field into some other fluid into which it (and any other fields present) decay, see Table(1). We have explicitly shown that in all three cases it is the efficiency with which the energy density or curvature perturbation is translated between the fluids that determines the level of non-gaussianity.
Table 1: We summarise the generation of large non-Gaussianity in the three models analysed in this paper. The second and third column show the transfer of energy and the fourth column shows the source fluid which modulates the energy transfer and generates the curvature perturbation. The fifth column shows the source of the dominant background energy density. Large non-Gaussianity is only possible when the source of the background energy and that of the perturbations are different.

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References

[1] E. Komatsu et. al., Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation, arXiv:1001.4538.

[2] SDSS Collaboration, M. Tegmark et. al., Cosmological Constraints from the SDSS Luminous Red Galaxies, Phys. Rev. D74 (2006) 123507, astro-ph/0608632.

[3] D. H. Lyth and D. Wands, Generating the curvature perturbation without an inflaton, Phys. Lett. B524 (2002) 5–14, hep-ph/0110002.

[4] T. Moroi and T. Takahashi, Cosmic density perturbations from late-decaying scalar condensations, Phys. Rev. D66 (2002) 063501, hep-ph/0206026.

[5] K. Enqvist and M. S. Sloth, Adiabatic CMB perturbations in pre big bang string cosmology, Nucl. Phys. B626 (2002) 395–409, hep-ph/0109214.

[6] G. Dvali, A. Gruzinov, and M. Zaldarriaga, A new mechanism for generating density perturbations from inflation, Phys. Rev. D69 (2004) 023505, astro-ph/0303591.

[7] L. Kofman, Probing string theory with modulated cosmological fluctuations, astro-ph/0303614.

[8] F. Bernardeau, L. Kofman, and J.-P. Uzan, Modulated fluctuations from hybrid inflation, Phys. Rev. D70 (2004) 083004, astro-ph/0403315.

[9] M. Zaldarriaga, Non-Gaussianities in models with a varying inflaton decay rate, Phys. Rev. D69 (2004) 043508, astro-ph/0306008.

[10] F. Bernardeau and J.-P. Uzan, Inflationary models inducing non-gaussian metric fluctuations, Phys. Rev. D67 (2003) 121301(R), astro-ph/0209330.

[11] F. Bernardeau, L. Kofman, and J.-P. Uzan, Modulated fluctuations from hybrid inflation, Phys. Rev. D70 (2004) 083004, astro-ph/0403315.
[12] L. Alabidi and D. Lyth, *Curvature perturbation from symmetry breaking the end of inflation*, JCAP 0608 (2006) 006, astro-ph/0604569.

[13] M. Sasaki, *Multi-brid inflation and non-Gaussianity*, Prog. Theor. Phys. 120 (2008) 159–174, arXiv:0805.0974.

[14] A. Naruko and M. Sasaki, *Large non-Gaussianity from multi-brid inflation*, Prog. Theor. Phys. 121 (2009) 193–210, arXiv:0807.0180.

[15] K. M. Smith, L. Senatore, and M. Zaldarriaga, *Optimal limits on f_{NL}^\text{local} from WMAP 5-year data*, JCAP 0909 (2009) 006, arXiv:0901.2572.

[16] O. Rudjord *et. al.*, *An Estimate of the Primordial Non-Gaussianity Parameter f_{NL} Using the Needlet Bispectrum from WMAP*, Astrophys. J. 701 (2009) 369–376, arXiv:0901.3154.

[17] J. M. Maldacena, *Non-Gaussian features of primordial fluctuations in single field inflationary models*, JHEP 05 (2003) 013, astro-ph/0210603.

[18] D. H. Lyth, C. Ungarelli, and D. Wands, *The primordial density perturbation in the curvaton scenario*, Phys. Rev. D67 (2003) 023503, astro-ph/0208055.

[19] K. A. Malik and D. H. Lyth, *A numerical study of non-gaussianity in the curvaton scenario*, JCAP 0609 (2006) 008, astro-ph/0604387.

[20] L. Alabidi, *Non-gaussianity for a two component hybrid model of inflation*, JCAP 0610 (2006) 015, astro-ph/0604611.

[21] C. T. Byrnes, K.-Y. Choi, and L. M. H. Hall, *Conditions for large non-Gaussianity in two-field slow- roll inflation*, JCAP 0810 (2008) 008, arXiv:0807.1101.

[22] C. T. Byrnes and G. Tasinato, *Non-Gaussianity beyond slow roll in multi-field inflation*, JCAP 0908 (2009) 016, arXiv:0906.0767.

[23] K. A. Malik and D. Wands, *Cosmological perturbations*, Phys. Rept. 475 (2009) 1–51, arXiv:0809.4944.

[24] H. Kodama and M. Sasaki, *Cosmological Perturbation Theory*, Prog. Theor. Phys. Suppl. 78 (1984) 1–166.

[25] K. A. Malik and D. Wands, *Adiabatic and entropy perturbations with interacting fluids and fields*, JCAP 0502 (2005) 007, astro-ph/0411703.

[26] A. J. Christopherson and K. A. Malik, *The non-adiabatic pressure in general scalar field systems*, Phys. Lett. B675 (2009) 159–163, arXiv:0809.3513.

[27] C. G. Boehmer, G. Caldera-Cabral, N. Chan, R. Lazkoz, and R. Maartens, *Quintessence with quadratic coupling to dark matter*, arXiv:0911.3085.

[28] D. H. Lyth, *Generating the curvature perturbation at the end of inflation*, JCAP 0511 (2005) 006, astro-ph/0510443.

[29] D. H. Lyth and A. R. Liddle, *The primordial density perturbation*, .