1. Introduction

It is well known that the Poincaré symmetry, which is the spacetime symmetry of the orthodox relativistic quantum field theory (QFT), originates from the isotropy and homogeneity of Minkowski spacetime and is based on observations of macro- and micro-phenomena concerning observable physical bodies and particles. Today the Poincaré symmetry is a basis of any theory of fundamental interactions for quantum particles, for instance, the Standard Model of weak, electromagnetic and strong interactions (SM). In particular, the quantum chromodynamics (QCD), which is the QFT for quarks and is the part of the SM, incorporates the Poincaré group as the group of spacetime symmetries. The QCD describes well numerous experimental data at high energies where the perturbation theory works [1]. But the confinement of quarks is outside of the QCD perturbation theory and may be the QCD itself. So it is desirable to generalize the Poincaré group of spacetime symmetries for description of quark properties taking into account that quarks have not been observed as free particles in the Minkowski spacetime.

In general case it is good reason to consider various generalizations of the Poincaré symmetry bearing in mind complicated features of known and possibly unknown interactions of elementary particles.

Studies along these lines have been carried out in the context of theories with new fundamental physical constants additional to the well known $c$ and $\hbar$. Beginning with Snyder’s work [2], the theory with a fundamental length has been elaborated [3, 4]. However, in a theory of this sort, the reciprocity between coordinate and momenta, proposed by Born [5], was broken. The reciprocity was restored in by Yang [6], who added a fundamental mass to the modified theory (see also [7]). More general algebras depended on the constants with the dimensions of length (L), mass (M), and action (H) were considered in the works [8].

In the present paper some characteristics of spacetime symmetries for quantum particles that generalize Poincaré group characteristics are found. As an example of such particles we use quarks, which have some inexplicable so far properties. We investigate the general Lie algebra for quantum operators of coordinate, momentum and angular momentum of a
quantum particle that in a limiting case is reduced to the Lie algebra of observables of the canonical QFT. Quantum fields, which can be constructed with the help of representations of the general algebra, are referred to as HLM generalized quantum fields and the associated particles as HLM quantum particles. We present relations between spacetime observables of a HLM quantum particle, which depend on the new constants L, M and H. Some of these relations lead to the equations, which generalize the Poincaré invariance for canonical scalar and spinor fields. Operators of a momentum and coordinate of a HLM quantum particle, which depend on the new constants with the dimensions of length, mass and action, and the commutation relations can be written as \((i, j, k, l = 0, 1, 2, 3)\)

\[
[F_{ij}, F_{kl}] = if(g_{jk}F_{il} - g_{ik}F_{jl} + g_{il}F_{jk} - g_{jl}F_{ik}),
\]

\[
[F_{ij}, p_k] = if(g_{jk}p_i - g_{ik}p_j),
\]

\[
[F_{ij}, x_k] = if(g_{jk}x_i - g_{ik}x_j),
\]

\[
[F_{ij}, I] = 0, \quad [p_i, p_j] = (if/L^2)F_{ij},
\]

\[
[x_i, x_j] = (if/M^2)F_{ij},
\]

\[
[p_i, x_j] = if(g_{ij}I + F_{ij}/H),
\]

\[
[p_i, I] = if(x_i/L^2 - p_i/H),
\]

\[
[x_i, I] = if(x_i/H - p_i/M^2).
\]

The first relation specifies the algebra \(l\), while the second, third and fourth relations specify the tensor character for the well-known physical quantities. The fifth and sixth relations lead to noncommutativity of components as \(p\) and \(x\). The seventh, eighth and ninth relations generalize the Heisenberg relations. The system of relations (1) is written in units with \(c = 1\) (\(c\) is the velocity of light), it contains four dimensional parameters: \(f\) (action), \(M\) (mass), \(L\) (length) and \(H\) (action). But in the limiting case \(M \to \infty, L \to \infty, H \to \infty\), the system (1) should transform to the system of relations for the canonical quantum theory, so \(f = h\). More generally, \(f = f(M, L, H)\) and in the limiting case \(f(M, L, H) \to h\).

The algebra \(\Pi\) contains as special cases diverse Lie algebras of generalized space-time symmetries. The condition for the algebra \(\Pi\) to be semisimple can be presented in the form: \((M^2L^2 - H^2)/M^2L^2H^2 \neq 0\). If this condition is fulfilled, \(g\) is isomorphic to a pseudo-orthogonal algebra \(o(p, q)\), \(p + q = 6\) depending on selected \(M, L\) and \(H\) values (\(M\) and \(L\) can be real and pure imaginary). In other cases it is isomorphic to a direct or a semidirect product of a pseudoorthogonal algebra and an Abelian or integrable algebra \([8]\).

Any one of the algebras \(\Pi\) can be a basis for a new field theory. Some possible applications of the algebras \(\Pi\) at different scales are indicated in the works \([9] [10] [11]\). The central problem is finding a really suitable model for description of a new phenomenon in our world using appropriate one of these algebras. For instance, in Ref. \([11]\) the properties caused of the commutation relations \(\Pi\) attribute to the spacetime...
manifold itself. The following sections of the present paper are related to a possible application of the algebras \([\Xi]\) for description of properties of hypothetical quantum particles. Equations for scalar and spinor particles are given, in the last case it is thought of especially quarks.

3. Equation for HLM generalized quantum scalar field

Representations of the HLM algebra are characterized with values of three Casimir operators: 
\[ C_1 = \varepsilon_{ijklmn} F^{ij} F^{kl} F^{mn}, \quad C_2 = F_{ij} F^{ij}, \quad C_3 = (\varepsilon_{ijklmn} F^{ij} F^{kl} F^{mn})^2 \]

where \( i,j,k,l,m,n \) are values of three Casimir operators: 
\[ F_{ij} = (1/M^2 L^2) + I^2 + \frac{(x^i p^j + p_i x^j) / H - x_i x^j / L^2 - p_i p_j / M^2) \Phi(\xi) = 0} \]

An interesting case is when the \( p_i \) and \( x_j \) operators are commutative in a similar way it is in the standard theory framework and the only one new constant \( H \) remains. Then the algebra constitutes from the following operators in the \( \xi \)-representation \([10]\):

\[ F_{ij} = i h (\xi_j \frac{\partial}{\partial \xi_i} - \xi_i \frac{\partial}{\partial \xi_j}) \]

with \( a \) is an arbitrary real number.

4. The generalization of spacetime symmetries for quarks

Below we apply the algebra \([\Xi]\) for description of quarks and find some new features which arise in quark spacetime properties in this case. First of all quarks ought be characterized with new invariants instead of the Poincare algebra invariants connected with mass and spin. So a generalized quark field will be given with values of three Casimir operators: \( C_1, C_2 \) and \( C_3 \).

In the presence of the constant \( H \) the CP-invariance does not hold for the algebras \([\Xi]\) [8]. So if we take into account the \( CP \)-invariance of strong interactions we must put \( H = \infty \) for quarks and obtain the following nonzero commutation relations for \( p_i, x_j \) and \( I \) (below we use the natural units \( c = \hbar = 1 \)):

\[ [p_i, p_j] = (i / L^2) F_{ij}, \]

\[ [x_i, x_j] = (i / M^2) F_{ij}, \]

\[ [p_i, x_j] = ig_{ij}, \]

\[ [p_i, I] = i(x_i / L^2), \]

\[ [x_i, I] = i(-p_i / M^2). \]

Let us name the quantum fields which can be formed with the help of an irreducible representation of the LM algebra \([\eta]\) as the LM generalized quantum fields. Using of the Casimir operator of the second order \( C_2 \) we suppose that LM field obeys the following equation \([14]\):

\[ (\sum_{i<j} F_{ij} (1/M^2 L^2) + I^2 - x_i x^j / L^2 - p_i p^j / M^2) \Phi(\xi) = 0 \]

5. Equations for LM generalized quantum spinor fields

When a LM generalized quantum field have a nonzero spin value there are other equations in addition to the equation \([5]\). Two equations can be given for spinor fields. The equation \([8]\) is not invariant with respect to the space parity \( P \), while the equation \([7]\) is invariant with respect to this discrete transformation.

\[ (\gamma_i p^j - \gamma_i \gamma_5 x^j \zeta_1 \zeta_2 \sqrt{-M^2 / L^2} - \gamma_5 I \zeta_2 \sqrt{-M^2} \]

\[ - \sum_{i<j} \gamma_i \gamma_j F^{ij} \zeta_1 \sqrt{L^2} - n) \psi(\xi) = 0. \]

\[ (\sigma_0 \otimes \gamma_i p^j - \sigma_3 \otimes \gamma_i \gamma_5 x^j \zeta_1 \zeta_2 \sqrt{-M^2 / L^2} \]

\[ - \sigma_3 \otimes \gamma_5 I \zeta_2 \sqrt{-M^2} \]

\[ - \sigma_0 \otimes \sum_{i<j} \gamma_i \gamma_j F^{ij} \zeta_1 \sqrt{L^2} - \sigma_0 \otimes n) \Psi(\xi) = 0. \]

where \( F^{ij} \) are Lorentz generators, \( n \) is an arbitrary number, \( \sigma_0 \) and \( \sigma_3 \) are Pauli matrices, \( \gamma_i, i = 0, 1, 2, 3 \) are Dirac matrices, \( \zeta_1 = \pm 1, \zeta_2 = \pm 1 \).
The coefficients $\Xi$, $\Delta$ and $\Sigma$ of the equation (5) from Ref. [10] are made more precise with the coefficients of the equations (6) and (7). The properties of solutions of Eqs. (6) and (7) are largely dependent on values of the coefficients of these equations and should be studied closely in each case.

6. Dispersions for noncommutative observables of a LM quantum particle

In this section a brief description of modification of a quantum measurement procedure for quark spacetime observables is outlined. The especial feature consists in appearance of corrections dependent on time observables is outlined. The especial feature for achievement of mathematical completeness of the HLM generalized quantum fields approach. A further investigation in this direction is in progress now.

7. Conclusions

We have considered in some detail the general HLM algebra of the physical observables which depends on additional constants with the dimensions of length $L$, mass $M$ and action $H$. It has been supposed that the representations of the HLM algebra (11) can be used for construction of quantum fields which describe hypothetical HLM particles. In particular the possibility has been considered that the strongly interacting fundamental particles (quarks) can be particles of such type.

Some relations between spacetime observables of a HLM quantum particle depended on the new constants has been presented including the equations for scalar and spinor fields (2), (5), (6) and (7). The modification of a quantum measurement procedure relates to the nonzero lower limits to products of dispersions for nonzero lower limits to products of observables of quark space-time observables (9). These characteristics take into account inherent features of HLM quantum particles and can be used for their search.

In summary, it can be said that a study of the generalized algebra (11) and the properties of solutions of Eqs. (2), (5), (6) and (7) are important objectives for achievement of mathematical completeness of the HLM generalized quantum fields approach. A further investigation in this direction is in progress now.

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