Load Value Verification of Crankshaft 3D Model according to Stiffness Matrix Boundary Conditions

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Abstract. A crankshaft 3D model of bulk carrier 21100 DWT is studied. Moving masses, a chain wheel mass and a turning wheel mass are applied. A matrix displacement method is used to validate simulation loads of the crankshaft model. Boundary conditions and mesh control are entered for the model study in order to minimize the difference between values of analytic calculations and the 3D simulation loads. A percentage difference is presented between an analytical model and a simulation model. Pictures of stress and displacements of the shaft and the flange are pointed in the paper.

1. Introduction

Defining and observation of crankshaft performance is a complex task required an extensive research. The engine power that is transformed into a rotary movement of the crankshaft generate a torque with considerable loads. The crankshaft has to withstand of bending, shear and torsional stresses. Further the stresses and deformations [1] arisen from the torsional vibrations, regard to the constantly abruptly acceleration of the rotary movement.

A stiffness matrix method is used to calculate reaction forces and bending moments of the moving masses section (figure 1). The method is applied for the simply and symmetrically loaded section of crankshaft. The calculation results are enough condition to satisfy the load values of crankshaft 3D model.

3D models are made [2], regarded to a section with diameter $\varnothing = 302 \, mm$ and the entire crankshaft model. The first one is studied to validate the stiffness matrix load values of crankshaft section. The second one is studied to get values of the reaction forces and the bending moment, according to the boundary conditions and the mesh control used in the validation study.

In this paper the crankshaft of main engine of bulk carrier 21100 DWT is represented as a straight shaft with variable cross section. The forces of moving masses including crank throw are placed in the middle of rod bearing journals. The chain force minus chain wheel mass and the turning wheel mass are pointed on the diameters $\varnothing = 1080 \, mm$ and $\varnothing = 903 \, mm$ of crankshaft (figure 1).
Figure 1. Crankshaft scheme (left side) and displacement of moving masses section (right side)
2. Load values verification with stiffness matrix method

Resources are used basis on [3] and the moving masses section accorded to the possible joints displacement is presented in figure 1.

The total shaft stiffness matrix is illustrated on figure 2 [4, 5].

![Stiffness Matrix](image)

**Figure 2.** Total stiffness matrix of moving masses section

The internal reactions are

\[ IR_{AB} = IR_{BC} = IR_{CD} = IR_{DE} = \begin{bmatrix} \frac{F}{2} \\ \frac{FL}{2} \end{bmatrix} = \begin{bmatrix} 51250 \\ 51250 \times 0.748 \end{bmatrix} = \begin{bmatrix} 25625 \\ 4791.88 \end{bmatrix} \]

\[ Nm, \theta = \begin{bmatrix} -1.7705 \times 10^{-5} \\ 1.87 \times 10^{-6} \end{bmatrix} \text{rad.} \]

Slope and deflections are pointed in figure 3.
After calculation processes based on [3], the deformations and the reactions forces of each joint are:

\[
\begin{align*}
    d_{AB} &= \begin{bmatrix} 0 \\ -1.7705 \times 10^{-5} \end{bmatrix} \text{ rad}, \quad d_{BC} = \begin{bmatrix} 0 \\ 1.87 \times 10^{-6} \end{bmatrix} \text{ rad}, \\
    d_{CD} &= \begin{bmatrix} 0 \\ -1.87 \times 10^{-6} \end{bmatrix} \text{ rad}, \quad d_{DE} = \begin{bmatrix} 0 \\ 1.7705 \times 10^{-5} \end{bmatrix} \text{ rad}, \\
    F_{AB} &= \begin{bmatrix} 18526.68 \\ 32723.32 \\ -5309.54 \end{bmatrix} \text{ N}, \quad F_{BC} = \begin{bmatrix} 26463.26 \\ 5309.54 \\ -4682.52 \end{bmatrix} \text{ Nm}, \\
    F_{CD} &= \begin{bmatrix} 24786.74 \\ 4682.52 \\ 26463.26 \\ -5309.54 \end{bmatrix} \text{ N}, \quad F_{DE} = \begin{bmatrix} 32723.32 \\ 5309.54 \\ 18526.68 \end{bmatrix} \text{ Nm}
\end{align*}
\]

Finally, the reaction forces and the bending moments are:

\[
\begin{align*}
    R_A &= 18526.68 \text{ N}, \quad R_B = 59186.58 \text{ N}, \quad R_C = 49573.48 \text{ N}, \quad R_D = 59186.58 \text{ N}, \quad R_E = 18526.68 \text{ N}, \\
    M_A &= M_B = M_C = M_D = M_E = 0 \text{ Nm}.
\end{align*}
\]

Achieving the results of stiffness matrix calculations is related to correct boundary conditions and a finite element mesh [6] when simulating the loaded 3D crankshaft model.

An important condition is the places of fix (supports places). This should be carried out on a sufficiently small surface. The small surfaces restraints can avoid the concentration of stress and the significant deformations in the supports (if the supports are theoretically fixed in a point or line).

The restraints of entire model are placed over the auxiliary surfaces (figure 4).
A symmetric 3D model of moving masses section is made (figure 5) to reduce the time of simulation study.

![Symmetric 3D model of moving masses section](image)

**Figure 5.** Symmetric 3D model of moving masses section

Restraints has been made to (figure 6) [7]:

- fixing displacement by axis $Z$ (illustrated in red color);
- fixing displacement by axis $Y$ (illustrated in orange color);
- fixing displacement of the semi-cylindrical surfaces by axis $X$ (illustrated in green color);
- fixing displacement of the diametrical surfaces by coordinate system $YZ$ (illustrated in yellow color).

![Restraints of symmetric 3D model of moving masses section](image)

**Figure 6.** Restraints of symmetric 3D model of moving masses section

The fixed model thus approximates sufficiently to the boundary conditions [8] characterizing the mathematical calculation explained above and determines the proper arrangement of finite elements [9]. The mesh of finite elements is represented on figure 7.

![FEM of symmetric 3D model of moving masses section](image)

**Figure 7.** FEM of symmetric 3D model of moving masses section
The displacements are:

![Displacements of symmetric 3D model of moving masses section](image)

**Figure 8.** Displacements of symmetric 3D model of moving masses section

The results of support reactions and bending moments by the stiffness matrix method are presented, as well as the simulation data of loaded symmetric 3D model of moving masses section (table 1).

| IR | Stiffness matrix method | Symmetric 3D model of moving masses section | Percentage difference |
|----|-------------------------|---------------------------------------------|-----------------------|
| $R_A$ (N) | 18526.68 | 18577 | 0.27 |
| $R_B$ (N) | 59186.58 | 59118 | 0.12 |
| $R_C$ (N) | 49573.48 | 49596 | 0.05 |
| $R_D$ (N) | 59186.58 | 59145 | 0.07 |
| $R_E$ (N) | 18526.68 | 18548 | 0.12 |
| $M_A$ (Nm) | 0 | 0 | 0 |
| $M_B$ (Nm) | -5309.54 | -5301.1 | 0.16 |
| $M_C$ (Nm) | 5309.54 | 5301.1 | 0.16 |
| $M_D$ (Nm) | -4682.52 | -4694.6 | 0.26 |
| $M_E$ (Nm) | 4682.52 | 4694.6 | 0.26 |
| $M_F$ (Nm) | -5309.54 | -5316.8 | 0.14 |
| $M_G$ (Nm) | 5309.54 | 5316.8 | 0.14 |
| $M_H$ (Nm) | 0 | 0 | 0 |

The table values of percentage difference between the stiffness matrix and the 3D model is small enough and it can be assumed that the model boundary conditions with FE mesh is sufficient to verify the loaded 3D model of moving masses section. About those considerations, the crankshaft model is created (figure 9) and its load simulation is performed with relatively uniform boundary conditions and FE mesh values [10].

![Crankshaft 3D model](image)

**Figure 9.** Crankshaft 3D model
The boundary and load conditions of crankshaft are shown in figure 10 and they are:

- fixing displacement by axis \( Z \) (marked with red arrows);
- fixing displacement by axis \( Y \) (marked with orange arrows);
- fixing displacement of the cylindric faces by axis \( X \) (marked with green arrows);
- load of the chain wheel mass and the turning wheel mass (marked with pink arrows);
- load of the moving masses (marked with pink arrows);
- gravity (marked with blue arrow).

![Figure 10. Boundary conditions and FE mesh of crankshaft 3D model](image)

Displacements of crankshaft 3D model is pictured in figure 11.

![Figure 11. Displacement of crankshaft 3D model](image)

The values of crankshaft load simulation are given in table 2 and table 3.

**Table 2. Reaction forces of crankshaft 3D model**

| Reaction forces | \( R_A \) | \( R_B \) | \( R_C \) | \( R_D \) | \( R_E \) | \( R_F \) |
|-----------------|----------|----------|----------|----------|----------|----------|
| Values \((N)\)  | 29656    | 14582    | 56721    | 55050    | 63391    | 20300    |
Table 3. Bending moments of crankshaft 3D model

| Bending moment | $M_A$  | $M_B$  | $M_C$  | $M_D$  | $M_E$  | $M_F$  |
|----------------|--------|--------|--------|--------|--------|--------|
| Values (Nm)    | -9461  | -3687.6| -4858.3| -5089.4| -5554.4| 0      |

3. Conclusion

Stiffness matrix method is used according to shear forces influence to calculate reaction forces and bending moments of crankshaft moving masses section. The section is represented as simple sections with constant moment of inertia.

The created boundary conditions, finite element mesh and loading modes of symmetric 3D model of moving masses section are optimal for obtaining approximately the same values of reaction forces and bending moments accorded to stiffness matrix results.

The percentage difference is below 0.30 and satisfy the crankshaft 3D model verification.

The reaction forces and bending moments are shown in table 2 and table 3 regarded to the procedures applied in the stiffness matrix method.

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