Data Article

Universal (meta-)logical reasoning: The Wise Men Puzzle (Isabelle/HOL dataset)

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A R T I C L E I N F O

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The authors universal (meta-)logical reasoning approach is demonstrated and assessed with a prominent riddle in epistemic reasoning: the Wise Men Puzzle. The presented solution puts a particular emphasis on the adequate modeling of common knowledge and it illustrates the elegance and the practical relevance of the shallow semantical embedding approach when utilized within modern proof assistant systems such as Isabelle/HOL. The contributed dataset provides supporting evidence for claims made in the article "Universal (meta-)logical reasoning: Recent successes" (Benzmüller, 2019).

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The data is provided in form of three Isabelle/HOL source files (HOMML.thy, Relations.thy and WiseMenPuzzle.thy), which are bundled together in a single zip-file; the theory import dependencies of these files is depicted in Fig. 2.

HOMML.thy contains an encoding of higher-order multimodal logic K in classical higher-order logic following the authors shallow semantical embedding approach. Constant domain quantifiers are provided for all types in the HOL type hierarchy. The notion of logic combination adopted is that of a logic fusion [3]. The encoding supports both a local and global notion of modal validity (and logical consequence).
Fig. 1. Higher-order definition of the transitive closure of a relation; encoding in Isabelle/HOL.
Relations.thy contains some basic and useful properties and operations on relations. Example properties are reflexivity, symmetry, transitivity and euclideaness; example operations are the union and intersection of relations. Most importantly, however, Relations.thy provides a higher-order definition for the transitive closure of a relation (cf. Fig. 1). This definition is utilized in file WiseMenPuzzle.thy to provide an adequate notion of common knowledge (of a group of agents).

WiseMenPuzzle.thy, which imports the two other files, contains the encoding of the Wise Men Puzzle. Utilizing the basic concepts from HOMML.thy, three indexed KT45 modal box operators are introduced in order to model the individual knowledge of the three agents in the given puzzle scenario (the introduction of further box operators is straightforward). Another indexed KT45 modal box operator is then defined to capture the common knowledge of these three agents (the modeling thus closely follows the suggestions of Sergot [5]).

File WiseMenPuzzle.thy also demonstrates how the logic riddle is solved automatically by the automated theorem provers integrated with Isabelle/HOL.

The Isabelle/HOL system is needed to properly interpret and verify the provided dataset; it can be obtained from https://isabelle.in.tum.de.

2. Experimental design, materials, and methods

The data was acquired through manual encoding of the problem in the Isabelle/HOL [4] proof assistant system. The motivation has been to empirically assess the expressivity and automated reasoning capabilities of Isabelle/HOL (with its various integrated automated reasoning tools) in epistemic reasoning when utilizing the shallow semantical embedding approach [2]. The encoding of the data was conducted in form of dialog between the author and the proof assistant. In this process the author provided type declarations, definitions and abbreviations, axiom postulates, lemmas and theorems. The proof assistant constantly monitored these activities, automatically analyzed the input, and offered permanent feedback. In addition to syntax checks and type checking, this also included the automated formal assessment of the conjectured lemmas and theorems with automated theorem provers. Moreover, the consistency of the provided premises was permanently monitored by model finding tools. The final result of the encoding process are the formally verified data documents as provided with this data article.

The automated assessment of the data in the described process was supported by a range of automated reasoning tools that are integrated with the Isabelle/HOL proof assistant via its Sledgehammer tool (cf. [4] for further details and references). This includes state-of-the-art first-order automated theorem provers (such as prover E, Vampire and SPASS), higher-order automated theorem provers (such as LEO-II and Satallax), satisfiability modulo solvers (such as CVC4 and Z3), and the model and countermodel finders Nitpick and Nunchaku. The feedback offered by these systems during the data encoding process was guiding the continuous emendations of the data until a fully verified
data document was finally obtained. Fully verified in this context means that all proofs that were automatically constructed during the data encoding process have been reproduced and rechecked without failure in Isabelle/HOL’s trusted kernel (a small set of trusted proof rules). Whenever the provided data files are read again by the Isabelle/HOL system the described data verification process is repeated.

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Transparency document

Transparency document associated with this article can be found in the online version at https://doi.org/10.1016/j.dib.2019.103823.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.dib.2019.103823.

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