Common envelope events with low-mass giants: understanding the energy budget

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ABSTRACT

Common envelope events are important interactions between two binary stars that lead to the formation of close binary systems. We present here a systematic three-dimensional study in which we model common envelope events with low-mass giant donors. The results allow us to revise the energy formalism that is usually used to determine common envelope event outcomes. We show that the energy budget for this type of system should include the recombination energy, and that it also must take into account that a significant fraction of the released orbital energy is taken away by the ejecta. We provide three ways in which our results can be used by binary population synthesis studies: a relation that links the observed post-common envelope binary with the initial binary parameters, a fitting formula for the $\alpha_{\text{CE}}\lambda$ parameter of the standard energy formalism, and a revised energy formalism that takes into account both the recombination energy and the energy that is taken away by the ejecta.

Key words: hydrodynamic – binaries: close – white dwarfs – subdwarfs

1 INTRODUCTION

It is believed that the common envelope event (CEE) is the most important phase in the evolution of a wide range of different types of close binary systems. It most likely plays a crucial role in the formation of X-ray binaries, Type Ia supernovae progenitors, double degenerate stars, and more (for a review, see Ivanova et al. 2013). CEE is a short-lived physical process when two stars orbit inside a single, shared envelope. The outcome of a CEE is either a new binary with a reduced orbit, or a merger of the two stars.

One of the standard ways to predict an outcome of a CEE is by using the energy formalism. This method equates the binding energy of the envelope of the donor with the orbital energy before and after the event (see for example van den Heuvel 1976; Webbink 1984):

\[ E_{\text{bind}} = \Delta E_{\text{orb}}, \]

where $E_{\text{bind}}$ is the binding energy of the envelope of the donor, and $\Delta E_{\text{orb}}$ is the change in the orbital energy. Recognizing that not all the available orbital energy can be used to eject the envelope of the donor, Livio & Soker (1988) proposed a common-envelope efficiency parameter, $\alpha_{\text{CE}}$, defined as the fraction of released orbital energy that has been effectively used to eject the envelope of the donor. This $\alpha_{\text{CE}}$ parameter is now widely used in binary population synthesis studies (see, e.g. Han et al. 2002). A better understanding of the energy budget of a CEE, better than a simple parameterization, could help us to better predict the population of close binaries.

The systems where the parameters of the CEE can be best constrained are double-white-dwarf (DWD) binaries. It is widely believed that the last episode of mass transfer leading to DWD formation was an unstable MT, a CEE, where the donor was a red giant (RG) star (e.g., Tutukov & Yungelson 1981; Iben & Tutukov 1984; Webbink 1984; Tutukov & Yungelson 1988). For RGs, a well-defined relation between their core masses and radii exists. From the observations of DWD systems, we know that one of the white dwarfs (WDs) is usually younger, and therefore is the remnant of the pre-CE RG donor. However, the mass of the donor can not be uniquely determined, as long as $\alpha_{\text{CE}}$ is uncertain.

In this paper, we perform three-dimensional numerical simulations of CEEs between low mass RG stars and WD companions. This work is the extension of our preliminary study devoted to the formation of the DWD WD 1101+364 via a CEE Nandez et al. (2015). Here, we consider a wide parameter space based on the mass of the RG donor, the RG core mass and the companion mass. Our DWD binaries have a mass ratio $q = M_1/M_2$ between 0.8 and 1.125, where $M_1$ is the mass of the younger WD (formed during a CEE), and $M_2$ is mass of the older WD. The main goal of this paper is to understand the energy budget at the end of the CEE. We pay particular attention to the usage of the recombination energy, and to the energy that is taken away by the ejecta. Those energies are not usually taken into account by the standard energy formalism, and should explain the deviation of $\alpha_{\text{CE}}$ from 1.

We describe the initial conditions, the parameter space and the numerical methods in § 2. § 3 contains the definitions for the...
energies. In § 4, we give an overview of the final states of the simulations in terms of the mass of the ejecta, energy evolution during a spiral-in, orbital parameters at the end of a CEE, and discuss how the outcomes of the CEE can be parameterized. Finally, § 5 gives a brief discussion on how our results can be used in population synthesis studies, as well as comparison with the observed binaries that are known to have a post-CE WD.

2 PARAMETER SPACE

We study the progenitor systems that have likely formed the observed DWDs, in terms of the observed WD masses and the orbital separations. We adopt that immediately before a CEE, the DWD progenitor binaries consisted of a low-mass RG with a core mass close to the observed new (second-formed) WD, and of an older (first-formed) WD. We consider 24 binaries in the parameter space defined by the RG donor mass, the mass of the newly formed WD (RG core), and the mass of the old WD. For the RG donor mass, $M_{\text{d,1}}$, we take 1.2, 1.4, 1.6, and 1.8 $M_\odot$. Each low-mass RG is considered at two evolutionary points on its RG branch, namely when their degenerate He core masses $M_{\text{c,1}}$ are 0.32,$M_\odot$ and 0.36$M_\odot$ cores (we expect that the mass of the new WD will be very similar to the He core mass of the RG donors). For the mass of the old WD, $M_{\text{d,2}}$, we take 0.32, 0.36, and 0.40 $M_\odot$, for each case of the RG donor. Table 1 shows the summary of the considered parameter space, and the initial conditions for each binary.

To create the initial RG donor stars, we use the TWIN/EV stellar code (Eggleton 1971, 1972, recent updates are described in Giebelbeek et al. 2008). This allows us to obtain a realistic initial one-dimensional (1D) stellar profile for each RG donor. Stars are evolved until their degenerate He cores have grown close to 0.32$M_\odot$, and 0.36$M_\odot$.

To model a CEE between a RG and a WD, we use STARSmasHER (Gaburov et al. 2010; Lombardi et al. 2011), a Smoothed Particle Hydrodynamics (SPH) code. Technical details on using this code to model CEEs can be found in Nandez et al. (2014). We reiterate the point made in Nandez et al. (2015), that when a 1D star is transferred to a 3D code via the relaxation process in STARSmasHER, the core of the RG, $M_{\text{c,1}}$, must be increased slightly by about 0.01$M_\odot$ so that the resulting profile of the 3D star for pressure, density, internal energy and other quantities would match that of a 1D star. The RG envelope is modeled using 10^3 particles, and the RG core is modeled as a point mass, as is the WD (note that a point mass only interacts gravitationally with normal SPH particles). The envelope mass in our three-dimensional star is $M_{\text{env,1}} = M_{\text{d,1}} - M_{\text{c,1}}$. We found that for most RGs with cores close to 0.4$M_\odot$, the profiles could not be matched well with 1D stars after the relaxation, and hence RGs with core masses > 0.4$M_\odot$ were excluded from the considered parameter space. Improving the match of the profiles in those more evolved donors is computationally unfeasible now, as this requires such a change in the number of SPH particles and in their smoothing length, that the GPU time would be increased by at least 64 times.

The photospheric radius of the star in SPH, $R_{\text{SPH}}$, is defined as $R_{\text{SPH}} = R_{\text{out}} + 2h_{\text{out}}$, where $R_{\text{out}}$ is the position of the outermost particle and $h_{\text{out}}$ is the smoothing length of that particle (for a detailed discussion on how to define the photospheric radius of a three-dimensional star, see Nandez et al. 2014). Defining the photospheric radius this way ensures that all envelope particles are enclosed within $R_{\text{SPH}}$ for the non-synchronized cases.

The initial orbital separation, $a_{\text{orb.ini}}$, for the non-synchronized cases is found from the assumption that $R_{\text{SPH}}$ is equal to the Roche Lobe (RL) radius, $R_\text{RL}$, and using the approximation for the RL radius by Eggleton (1983). The initial orbital period, $P_{\text{orb.ini}}$, is found assuming a Keplerian orbit.

For the synchronized case, the initial orbital period and separation are found at the moment when the outermost particles overflow the donor’s RL during the scan process (see §2.3 of Lombardi et al. 2011, also Nandez et al. 2015). During the scan process, the envelope’s angular momentum is steadily boosted. This leads to the expansion of the radius of the donor as compared to the non-rotating case. As a result, the orbital separation in a synchronized and a non-synchronized case may not match, however, the difference in these quantities is not large. The photospheric radius is $R_{\text{SPH}} = R_{\text{out}}$ for the synchronized case.

We use the tabulated equation of state (TEOS) incorporated from MESA (see §4.2 of Paxton et al. 2011) and implemented as described in (Nandez et al. 2015). This TEOS includes recombination energy for H, He, C, N, O, Ne, and Mg. The dominant contribution to the recombination energy comes from H, which account for about 59% of the total energy, followed by He with about 38%, and 3% for the rest of the elements, in all our simulations.

3 DEFINITIONS

In this Section we declare definitions for the most important quantities. Definitions are adopted from (Nandez et al. 2015), unless stated otherwise.

Energy formalism. The energy formalism compares the donor’s envelope binding energy $E_{\text{bind}}$ with the orbital energy before the CEE, $E_{\text{orb.ini}}$, and after the CEE, $E_{\text{orb.fin}}$ (Webbink 1984; Livio & Soker 1988):

$$E_{\text{bind}} = a_{\text{bind}} (E_{\text{orb.fin}} - E_{\text{orb.ini}}) \equiv a_{\text{bind}} \Delta E_{\text{orb}}.$$

Here $a_{\text{bind}}$ is the fraction of the orbital energy effectively used to expel the CE. This parameter is equivalent to the commonly used $a_{\text{CE}}$, and is usually assumed to be $0 \leq a_{\text{bind}} \leq 1$.

The potential energy of the donor’s envelope in SPH is

$$E_{\text{pot.ini}} = \frac{1}{2} \sum_i m_i \phi_i,$$

where $m_i$, and $\phi_i$ are the mass, and specific gravitational energy, respectively, for each SPH particle $i$ in the initial RG profile, including the core. Note that this quantity is computed before the star is placed in the binary configuration. In our SPH method, $\phi_i$ is calculated as in Hernquist & Katz (1989).

The internal energy of the donor’s envelope in SPH is

$$E_{\text{int.ini}} = \sum_i m_i \left( \frac{3}{2} \frac{k T_i}{\mu_i m_i} + \frac{a T_i^4}{\rho_i} \right),$$

where $T_i$, $\rho_i$ and $\mu_i$ are the temperature, density, and mean molecular mass, respectively, for each particle $i$ in the initial RG profile. The constants $k$, $a$, and $m_i$ are the Boltzmann constant, radiation constant, and hydrogen atom mass.
The models names are composed as following: two digits representing the RG mass are followed by “G”, three digits representing the RG core mass followed by “C”, three digits representing the old WD mass followed by “D”; “S” stands for synchronized case, otherwise the simulation is non-synchronized case. \( M_{d1} \) and \( M_{d2} \) are the total, and core mass of the RG, whilst \( M_{i2} \) is the mass of the old WD, in \( M_{d1} : R_{\text{tot}} \) is the radius of the donor Roche lobe, in \( R_{\text{tot}} : \) \( \alpha_{\text{orb,ini}} \) is the initial orbital separation in \( R_{\odot} \). \( \rho_{\text{ini}} \) is the initial orbital period in days. \( \widetilde{E}_{\text{orb,ini}}, \widetilde{E}_{\text{int,ini}}, \widetilde{E}_{\text{bind,ini}}, \widetilde{E}_{\text{rec,ini}}, \widetilde{E}_{\text{orb,ini}} \) and \( \widetilde{E}_{\text{int,ini}} \) are the potential energy of the RG, the internal energy of the RG without recombination, the binding energy of the RG envelope without recombination energy, the total recombination energy of the RG envelope, initial orbital energy, and initial total energy, defined as the sum of the binding, recombination, and initial orbital energies, respectively, in the units of 10^68 erg. \( \lambda \equiv -G M_{d1}(M_{d1} - M_{d2})/(E_{\text{bind,ini}} R_{\text{tot}}) \) is a dimensionless star structure parameter (de Kool 1990).

The binding energy of the RG, without the recombination energy, is

\[ E_{\text{bind}} = E_{\text{pot,ini}} + E_{\text{int,ini}}. \]  

This binding energy was historically parameterized using the parameter \( \lambda \) (de Kool 1990; Ivanova et al. 2013),

\[ E_{\text{bind}} = - \frac{G M_{d1}(M_{d1} - M_{d2})}{\lambda R}. \]  

This equation, combined with the energy formalism equation 2, provides the most used equation to find CEE outcomes in binary population synthesis studies, where \( \alpha_{\text{bind,ini}} \lambda \) are used together as one single parameter:

\[ \Delta E_{\text{orb}} = - \frac{G M_{d1}(M_{d1} - M_{d2})}{\alpha_{\text{bind,ini}} R}. \]  

The orbital energy of the binary system in SPH takes the following form:

\[ E_{\text{orb}} = \frac{1}{2} \mathbf{v}^{2}_{12} + \frac{1}{2} \sum_{j} m_{j} v_{j}^{2} - \frac{1}{2} \sum_{k} m_{k} \phi_{k}^{\text{R1},2} - \frac{1}{2} \sum_{l} m_{l} \phi_{l}^{\text{R1},2}, \]  

where \( \mu = M_{1} M_{2} / (M_{1} + M_{2}) \) is the reduced mass, and \( \mathbf{v}_{12} = \mathbf{v}_{1} - \mathbf{v}_{2} \) is the relative velocity of the two stars. The first, second, third and fourth terms give the orbital kinetic energy, the total gravitational energy of the binary (with the sum being over all particles \( i \) in the binary), the self-gravitational energy of the donor (the sum being over all particles \( j \) in star 1), and of the WD (the sum being over all particles \( k \) in star 2, initially just the one particle representing the WD), respectively.

Recombination energy. The recombination energy is included in the total value of the specific internal energy provided by TEOs, and can be found as

\[ E_{\text{rec,ini}} = \sum_{i} \left( u_{i}^{i} - \frac{3}{2} k_{T,i} \right) \left( \frac{1}{\mu_{i0}} - \frac{1}{\mu_{i1}} \right) \equiv \alpha_{\text{rec}} \Delta E_{\text{orb}}, \]  

where \( \alpha_{i} \) is the SPH specific internal energy for each particle. \( \alpha_{\text{rec}} \) is the ratio between the recombination energy and the released orbital energy. Since the recombination energy acts as an additional (to the orbital energy) source of energy, \( \alpha_{\text{rec}} < 0 \). This energy is not part of the usually considered binding energy, as it is not available immediately, and its release must be triggered (Ivanova et al. 2015). The amount of stored recombination energy is proportional to the mass of the envelope \( E_{\text{rec,ini}} = \eta (M_{d1} - M_{d2}). \) In a fully ionized gas that consists of only helium (0.3 mass fraction) and hydrogen (0.7 mass fraction), \( \eta \approx 1.5 \times 10^{13} \text{ erg/g}. \) Our gas chemical composition is a bit different, and also our TEO takes ionization of heavier elements into account as well. For our donors, we find \( \eta \approx 1.6 \times 10^{13} \text{ erg/g}. \) The version of STARMasher we use evolves, for each SPH particle, the specific internal energy \( u_{i} \) and
density $\rho_i$ (see Eqs. A18 and A7 of Gaburov et al. 2010). The pressure is then found from the internal energy, density, and the adopted equation of state.

**Total energy.** The total initial energy, $E_{\text{tot,ini}}$, is

$$E_{\text{tot,ini}} = E_{\text{orb,ini}} + E_{\text{bind}} + E_{\text{rec}}.$$  \hspace{1cm} (10)

This quantity is conserved during the evolution of all our models.

**Bound and unbound material.** For each particle, its total energy is defined as

$$E_{\text{tot,uni}} \equiv 0.5m_i v_i^2 + m_i U_i + m_i Y_i,$$

where the first, second and third terms are the kinetic, potential, and internal energies, respectively. We classify our particle as

- (i) the *ejec*ta, $m_{\text{unb}}$ – the particles that have positive energy,
- (ii) the circumbinary material, $m_{\text{cir}}$ – the matter that is bound to the binary, but is located outside of both RLs, and
- (iii) the binary material, $m_{\text{bin}}$ – the particles that are inside either of the two RLs.

The total energy of the unbound material at infinity is found when the unbound mass is in a steady state after the CEE. It is computed as

$$E_{\text{tot,unb}} = \sum_i E_{\text{tot,uni}}\bigg|_{\infty} \equiv -\alpha_{\text{unb}} \Delta E_{\text{orb}}.$$  \hspace{1cm} (11)

Note that $E_{\text{tot,unb}}$ includes the recombination energy of the unbound material. $\alpha_{\text{unb}}$ is the ratio of the energy taken away by the unbound material to the released orbital energy.

**Final energies.** The total energy at the end of the simulation is distributed in the “binding” energy of the gas bound to the binary, $E_{\text{bind,fin}}$, the final orbital energy of the binary, $E_{\text{orb,fin}}$, and the total energy of the unbound material at infinity, $E_{\text{tot,unb}}$:

$$E_{\text{tot,fin}} = E_{\text{bind,fin}} + E_{\text{orb,fin}} + E_{\text{tot,unb}}.$$  \hspace{1cm} (12)

where $E_{\text{tot,unb}}$ is composed of $E_{\text{kin,unb}}$, $E_{\text{int,unb}}$, and $E_{\text{pot,unb}}$ – the kinetic, internal and potential energies of the unbound material, respectively.

Generally, $E_{\text{bind,fin}}$ has a fairly small absolute value at the end of the simulation, and so can be safely disregarded. In addition, the particles around the WD may be accreted during a CEE, and hence their presence there may not have any physical meaning. The extended energy formalism, following to Nandez et al. (2015), can then be written as follows:

$$\alpha_{\text{bind}} + \alpha_{\text{rec}} + \alpha_{\text{unb}} \approx 1.$$  \hspace{1cm} (13)

If $\alpha_{\text{rec}} = \alpha_{\text{unb}} = 0$, then Equation 13 reduces to the standard energy formalism.

For additional analysis of the energies at the end of a CEE, we introduce 3 more quantities:

- $\alpha_{\text{pot}} \equiv E_{\text{pot,unb}} / E_{\text{pot,ini}}$ – the ratio of the potential energy taken away by the ejecta, to the initial potential energy of the RG envelope,
- $\alpha_{\text{th}} \equiv E_{\text{int,unb}} / (E_{\text{int,ini}} + E_{\text{rec}})$ – the ratio of the internal energy (including recombination) taken away by the ejecta, to the sum of the initial internal energy and the recombination energy of the RG envelope,
- $\alpha_{\text{kin}} \equiv -E_{\text{kin,unb}} / \Delta E_{\text{orb}}$ – the ratio of the kinetic energy taken away by the ejecta, to the released orbital energy.

We point out that $\alpha_{\text{th}}$ is a part of $\alpha_{\text{unb}}$, however, $\alpha_{\text{pot}}$ and $\alpha_{\text{th}}$ are not a part of $\alpha_{\text{unb}}$, as they describe fractions of their corresponding initial energies.

All our simulations conserved quite well the total angular momentum and the total energy. We have checked and found that the error in the energy conservation in all our simulations is less than 0.1% of the initial total energy, while the error in the angular momentum conservations in all our non-synchronized cases is less than 0.001% of the initial total angular momentum value, and the error in the angular momentum conservation in the only synchronized case is 0.4%.

4 RESULTS

4.1 Overview

**Masses.** At the end of each simulation, we form a binary consisting of $M_1$ and $M_2$ (see Table 2). We note that $M_1$ and $M_2$ in Table 2 differ from the values given for $M_{\text{c,1}}$ and $M_{\text{c,2}}$ in Table 1, respectively, as a few SPH gas particles remain within the RLs of the DWD binary. Ultimately, the ejected material $M_{\text{unb}}$ is at least 99.4% of the initial RG envelope, and there is no circumbinary matter around the newly formed DWD binary.

**Final Time.** We stop our simulations no less than 800 orbits after the end of the plunge-in, and typically we stop the simulations after more than 2000 orbits. The plunge-in is the fastest phase of the spiral-in, during which the instantaneous separation (distance) between the RG core and the WD changes substantially on the timescale comparable to its inferred orbital period. At the moment we stop, the orbital separation is changing by less than $|\delta a_{\text{orb}}/a_{\text{orb}}| < 0.002$, where $\delta a_{\text{orb}}$ is found over one binary orbital period. Some simulations were calculated for much longer, e.g. the case 1.8G0.32C0.32D is calculated for more than 10 000 orbits after the end of the plunge-in, and the change of the orbital separation over the binary orbital period, at the end of the simulation, is $|\delta a_{\text{orb}}/a_{\text{orb}}| \approx 0.0002$. The final parameters provided in Table 2 are expected to be time-converged values.

**Energies.** Figure 1 shows how the energies change during the spiral-in phase for the case 1.2G0.32C0.32D. After the spiral-in phase is complete and there is no circumbinary matter left, the circumbinary matter material is at least 99.4% of the initial RG envelope, and there is no circumbinary matter around the newly formed DWD binary.
Let us consider in detail why the dominant energy source that drives the final ejection of the puffed-up envelope is the recombination energy. For example, the model 1.8G0.32C0.32D has more energetic ejecta than other models. We note that the overall energy budget, and the energy that was extracted from the formed binary, are much higher in this model than in any other model.

### 4.2 Role of the recombination energy.

In brief, we found that the circumbinary recombination energy has a maximum during the plunge-in phase. At this moment, almost the entire envelope is no longer within the Roche lobe of the binary and becomes circumbinary material (see an example in Figure 2).

The ejection of the circumbinary envelope then takes place on its dynamical time-scale, which, e.g., for the case 1.2G0.32C0.32D, is about 100 days.

Let us consider in detail why the dominant energy source that drives the final ejection of the puffed-up envelope is the recombination energy but not the binary orbital energy. For that, we will trace the evolution of specific energies during the crucial time at about the plunge-in, when the puffed-up circumbinary envelope formed initially.

The binary orbital energy can be expected to boost the envelope’s ejection by being transferred into kinetic energy of the envelope. However, the acceleration of material by the binary’s motion, compared to the overall energy, is much smaller.

The evolution of the total recombination energy (grey solid line), shown for the case 1.2G0.32C0.32D.

![Figure 2](image_url)
The closest and farthest orbital separations are $r_p$ and $r_a$, respectively, while $\alpha_{\text{orb,fin}}$ is the semimajor axis (all in $R_\odot$). The orbital period $P_{\text{orb,fin}}$ is given in days, and $e$ is the eccentricity of the orbit. The energy fractions $\alpha_{\text{bind}}$, $\alpha_{\text{rec}}$, and $\alpha_{\text{kin}}$ are defined in Eq. 2, Eq. 9, and Eq. 11, respectively. $\alpha_{\text{tot}}$ is the fraction of potential energy taken by the ejecta with respect to the initial potential energy, $\alpha_{\text{th}}$ is the ratio of the thermal energy taken by the ejecta to the initial thermal energy (including recombination energy) and $\epsilon_{\text{kin}}$ is the kinetic energy scaled with the released orbital energy.

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boundary between the outer and inner envelopes, drawn at 3\(a_{\text{orb}}\), oscillates as well as the orbital separation itself, the defined masses of the envelopes and their energies oscillate during the plunge-in.

In Figure 3, we can see that the specific kinetic energy of the outer envelope is settled by the end of the plunge-in. This outer envelope is bound by the conventional definition, in which the total energy excluding recombination is negative. At the same time, the outer envelope is effectively decoupled from the binary and is not receiving further boosts to its kinetic energy.

At the moment of the end of the plunge-in, the outer envelope possesses most of the mass that remains bound to the binary. The outer envelope has obtained some kinetic energy from its previous interaction with the shrinking binary during the plunge-in. That non-zero kinetic energy leads to the envelope expansion, on the dynamical timescale of the expanded envelope, where every SPH particle in the outer envelope can have only a parabolic (bound) trajectory with respect to the binary, if the recombination energy is not released.

However, once the material expands and cools down enough to start recombination, an SPH particle gains enough energy to become unbound -- it can be seen from Figure 3 that at the end of the plunge-in the stored potential recombination energy is sufficient to unbind the material of the outer envelope. The outer envelope is now flowing away; as more of its material feels a pressure differential between open space above and the remaining envelope below, it expands further, cools down and becomes unbound after recombination. This recombination-driven ejection is gradual and non-explosive, vs. the rather explosive, or dynamical, ejection that takes place during the plunge-in, as described in detail by Ivanova & Nandez (2016). The radius at which the released recombination energy can remove a particle out of the potential well is the “recombination” radius, and was derived in (Ivanova & Nandez 2016).

We can clarify that there is no recombination energy stored in the ejected material at the end of the simulations.

It is important to mention that the recombination takes place at large optical depths. Using our 3D models, we estimate that typical optical depths have values of at least 10, and 1D studies showed that hydrogen recombination can take place at optical depths above 100 (Ivanova et al. 2015).

### 4.3 Post-CE orbital parameters

We find the final orbital separation in a geometrical way as

\[ a_{\text{orb, fin}} = \left( r_a + r_p \right) / 2, \]

where \( r_p \) is the periastron, and \( r_a \) is the apastron. We ensure that these two quantities, \( r_p \) and \( r_a \), are no longer changing with time at the moment when we extract them from the simulations. We find the final orbital period of the binary assuming a Keplerian orbit, \( P_{\text{orb, fin}} \). Another important orbital parameter is the eccentricity, \( e \), which is found as \( e = (r_a - r_p) / (2a_{\text{orb, fin}}) \). The final orbital parameters are provided in Table 3.

Note that the final separation found using the orbital energy, \( a_{\text{orb, fin}}^{\text{En}} = -GM_1 M_2 / (2E_{\text{orb, fin}}) \), differs from the final orbital separation found in the geometric way, \( a_{\text{orb, fin}} \). This is for two reasons:

(i) There is still mass within the RLs of both point masses, as can be seen by the non-zero value of \( E_{\text{bind, fin}} \). The presence of these particles, and their not fully stable orbits around the point masses, makes the energy-based way to calculate \( a_{\text{orb, fin}}^{\text{En}} \) uncertain. Note that these few particles make the stars aspherical and the equation for \( a_{\text{orb, fin}}^{\text{En}} \) is formally not valid.

(ii) The distance between the two point masses (WD and RG core) is less than two times their smoothing lengths, which means that there is some extra smoothing in the gravitational potential equation (see the Appendix of Hernquist & Katz 1989). The smoothing length of the point masses acts as the softening term defined by Hernquist & Katz (1989). For details on the definition and how to determine the smoothing length in the case modeled here, see Lombardi et al. (2011). As an example, in the model \( 1.8G0.32C0.32D \), the smoothing length for the RG core is \( h_{a_{\text{core}}} = 0.35r_c \), and the smoothing length for the WD is \( h_{a_{\text{WD}}} = 0.73R_c \).

The difference in orbital separations between the geometrical way \( a_{\text{orb, fin}} \) and the energy way \( a_{\text{orb, fin}}^{\text{En}} \) varies from 7.1% (1.2G0.32C0.36D) to 18.1% (1.8G0.32C0.32W), where the separation derived via the geometrical way is always smaller than the separation derived via the energy way. There is a very small discrepancy for the initial orbital separations using the two methods, < 0.2%.

The two values for the orbital separation would be closer to each other if the potential in the SPH code were calculated without a softening term. Note however that due to the first reason above, which is the presence of SPH particles inside the RLs, the two terms will never be completely the same. The discrepancy between the two values due to the softening term is expected to decrease if the smoothing length is decreased, and that can be done if the number of the particles is increased, although it is not intuitive to state whether the separation found by the geometrical way would increase or decrease. Only one test was made for the CEE study of the formation of the specific binary, a simulation with 200k particles resulted in a 7 per cent smaller final orbital separation than the same case modelled with 100k particles (Nandez et al. 2015). To clarify, the smoothing length was smaller in the case of 200k by 20 per cent compared to the case of 100k, but the relative difference between the final separations derived in the two ways was smaller. The models presented in this study might be affected similarly, but it is likely that the relative change in the final results will be small even if the resolution will be doubled.

Even though the smoothing lengths of the point masses are partially responsible for the discrepancy between the orbital separations found by the two methods, the smoothing length values cannot explain the unbinding of the puffed-up envelope. For instance, let us consider the model mentioned above with the maximum discrepancy of 18%, 1.8G0.32C0.32W. The smoothing lengths for the RG core and WD are \( h_{a_{\text{core}}} = 0.35r_c \), and \( h_{a_{\text{WD}}} = 0.73R_c \). The softening in this case starts to work when the distance between the RG core and the WD is 2.16R_c. At that moment, most of the mass is located at an average distance of 14 R_c, except for a few strongly bound particles which remain bound within about 3 R_c from the center of mass. The final separation therefore can be dependent on the mass resolution of the particles that were initially strongly bound and were in the close neighborhood of the RG core (where the smoothing length become important). However, as was discussed previously in § 4.2, the binary is decoupled with the puffed-up outer envelope, which is too far from the binary to be able to effectively transfer away its orbital energy, and the envelope ejection only depends on the stored kinetic and recombination energy, and does not affect the final parameters of the binary.

Table 3 shows that the bigger the initial mass of the RG, the tighter is the final orbit, for each fixed companion mass. For each initial RG mass and different companion mass, usually, the smaller the mass of the companion, the tighter the final orbital separation. However, there are two exceptions:

- In the case of the 1.2M_⊙ RG with a 0.32M_⊙ core, the largest final orbital period is for the 0.36M_⊙ WD companion, instead of...
the 0.40\,M_⊙ WD. This could be because the 1.2G0.32C0.36D case deposited the least of the kinetic energy in the ejecta, as compared to the other two cases during the spiral-in phase (see Table 2).

- In the case of the 1.4\,M_⊙ RG with a 0.32\,M_⊙ core, the tighter final orbit is for the 0.36\,M_⊙ WD companion instead of the 0.32\,M_⊙ WD. This could be because the 1.4G0.32C0.36D case deposited more kinetic energy in the ejecta than the other two cases. The final binary in this case also has less remaining bound mass than in the other two cases (see Table 2).

We could not identify any other initial condition that could discriminate why the final orbital separation in the two discussed cases did not follow the trend. During a spiral-in, we find that in those two cases the velocity at which the companion plunges into the envelope was higher than in other cases, which is consistent with the ejecta taking away more angular momentum. However, what causes this deviation in the ejecta’s angular momentum is not fully clear.

Figure 4 shows the final orbital periods for all the simulations, as a function of the initial RG mass. It can be seen that qualitatively there are two populations, mainly defined by the mass of the RG core, and with a smaller dependence on the mass of the companion. In each of these two populations, the final orbital period appears to depend almost linearly on the initial RG mass.

Figure 5 shows the final orbital periods as a function of the mass of the core, the initial RG mass, and the mass of the companion. This appeared to produce the relationship that can be expressed as follows:

\[ P_{\text{orb,fin}} = 10^{-2.46 \pm 0.05} \left( P_{\text{orb ini}} \times \frac{M_2}{M_{\text{ini}} M_1} \right)^{1.18 \pm 0.04} , \]  

(14)

Here \( \pm \) indicates the standard error for each coefficient. The units for the quantities are \( \text{M}_\odot \) for all the masses, and days for periods.

4.4 \( \alpha_{\text{bind}} \lambda \) formalism

In population synthesis models, a crucial parameter is \( \alpha_{\text{bind}} \lambda \), which can be found from the results of our simulations as follows (see also Equation 7):

\[ \alpha_{\text{bind}} \lambda = \frac{G M_{\text{ini}} (M_{\text{ini}} - M_{\text{c1}})}{R_{\text{ini}} \Delta E_{\text{orb}}} . \]

Note that this quantity does not imply a separate consideration of the recombination energy as it is simply a fit to the standard energy formalism. Figure 6 shows the behavior of \( \alpha_{\text{bind}} \lambda \) in our models. Our best fit for \( \alpha_{\text{bind}} \lambda \) with the assumed multi-linear regression model is:

\[ \alpha_{\text{bind}} \lambda = 0.92 + 0.55 \frac{M_{\text{d1}}}{\text{M}_\odot} - 0.79 \frac{M_2}{\text{M}_\odot} + 1.19 \frac{M_{\text{c1}}}{\text{M}_\odot} \]  

(15)

This equation accurately represents all our models, and the maximum deviation between this equation and any data point of 0.13 (1.2G0.32C0.4D), and a minimum deviation of 0.002 (1.8G0.36C0.40D).

4.5 Energy carried away by the ejecta

The total energy carried by the ejecta is not negligible, and is comparable, within an order of magnitude, to the initial binding energy of the RG star. Figure 7 shows the ratio of the energy taken away by the unbound material to the released orbital energy, \( \alpha_{\text{unb}}^{\text{tot}} \). It can be seen that \( \alpha_{\text{unb}} \) decreases with the mass of the RG. A multilinear regression that uses all the points from the simulations (assuming that all the variables presented have linear trends with respect to each other) gives the following dependence:

\[ \alpha_{\text{unb}}^{\text{tot}} = -0.16 - 0.36 \frac{M_{\text{d1}}}{\text{M}_\odot} + 0.49 \frac{M_2}{\text{M}_\odot} + 2.27 \frac{M_{\text{c1}}}{\text{M}_\odot} . \]
of the RG, very similarly to $\alpha_{\text{unb}}$. The multilinear fitting equation takes the following form

$$
\alpha_{\infty} = -\frac{E_{\text{kin,unb}}}{\Delta E_{\text{orb}}} = 0.2 - 0.26 \frac{M_{\text{e1}}}{M_\odot} + 0.44 \frac{M_{\text{e2}}}{M_\odot} + 0.92 \frac{M_{\text{e1}}}{M_\odot} \quad (17)
$$

where this equation fits all the points presented in Table 3. The maximum deviation between this equation and any listed value in Table 3 is 0.07 (1.4G0.32C0.32D), while the minimum deviation is 0.0002 (1.2G0.32C0.32D).

The potential energy of the ejecta, compared to the initial potential energy, is not really significant, as in all cases $\alpha_{\text{pot}} \lesssim 0.04$. The thermal energy the ejecta still has at infinity, as compared to the initial thermal energy plus the recombination energy, is several times larger, albeit also limited to $\alpha_{\text{pot}} \lesssim 0.16$. The thermal energy of the ejecta is comparable to the kinetic energy of the ejecta, therefore, the internal energy still plays a role in supporting the ongoing expansion of the material even after all the material is unbound.

Figure 9 shows how the specific kinetic energy of the ejecta changes with the initial mass of the RG. We can see that overall this energy decreases as the RG mass increases, but no clear trend is observed.

Figure 10 shows how the sum of the potential and thermal specific energies of the ejecta changes with the mass of the RG. We cannot really see a trend, except that for RGs with the initial mass of $1.8 M_\odot$, this quantity is higher than for the rest. Note that this quantity is always smaller than the specific kinetic energy of the ejecta.

5 DISCUSSION

To study the formation of a DWD binary via a CEE, we have simulated 25 three-dimensional hydrodynamical interactions between a low-mass RG and a WD companion. We considered for the initial masses of the low-mass RG star 1.2, 1.4, 1.6, or $1.8 M_\odot$, with a He
Our results show that the standard energy formalism should be modified to take into account: (i) the energy that is taken away by the ejecta, as it is a substantial fraction of the released orbital energy, and (ii) the recombination energy, which plays a crucial role in ejection of the formed circumbinary envelope. The role of the recombination energy for the CEE with a low-mass RG donor is not that it is necessary for the overall energy budget, as none of the considered systems were expected to merge by the standard energy formalism, but because the recombination occurs exactly at the time when the shrunk binary is no longer capable of transferring its orbital energy to the expanded envelope.

For future population synthesis studies, we provide three ways in which our results can be used.

First of all, we provide a fitting formula (Equation 14) that relates the final orbital period, the initial orbital period, the companion mass, the initial RG mass and the RG core. The RG radius and its core mass are coupled for each donor mass (these can be found using single stellar evolution tracks). The initial RG mass, its radius and the initial orbital period are also related (e.g., by using the Roche lobe radius approximation from Eggleton 1983). Therefore our fitting formula provides the relation between the observed parameters—the post-CE orbital period and the observed masses of both WDs—and the RG mass and radius before the CE. And vice versa, a population study could use this fitting formula to obtain the post-CE orbital period from known binary parameters at the start of a CE.

Let us consider how this fitting formula can be used to interpret observed post-CE binaries. For example, take the WD 1101+364, which has observed parameters best matching the set of models we have calculated: $P_{\text{orb,obs}} = 0.145$ day, $M_1 = 0.31 M_\odot$ and $M_2 = 0.36 M_\odot$ (Marsh 1995). To find the radius of the donor core of $0.32$ or $0.36 M_\odot$, and WD companions with masses $0.32$, $0.36$, or $0.40 M_\odot$. We find that in all the cases, a DWD binary is formed, most of the envelope is ejected, and only a few SPH particles remain bound to the binary in some cases (the bound mass is less than 0.06% of the initial envelope mass). The envelope is ejected on the dynamical time-scale of the expanded envelope.

Figure 9. Values for $E_{\text{kin,unb}}/M_{\text{unb}}$ with respect to the initial RG mass. The open circles correspond to simulations with $M_{\text{ini}} = 0.32 M_\odot$, while the open triangles correspond to $M_{\text{ini}} = 0.36 M_\odot$. The small, medium and big symbols are for companions with $0.32 M_\odot$, $0.36 M_\odot$, and $0.40 M_\odot$, respectively.

Figure 10. Values for $(E_{\text{kin,unb}} + E_{\text{pot,unb}})/M_{\text{unb}}$ with respect to the initial RG mass. The open circles correspond to simulations with $M_{\text{ini}} = 0.32 M_\odot$, while the open triangles correspond to $M_{\text{ini}} = 0.36 M_\odot$. The small, medium and big symbols are for companions with $0.32 M_\odot$, $0.36 M_\odot$, and $0.40 M_\odot$, respectively.

Figure 11. Final orbital periods of sdB stars (from Kupfer et al. 2015) and their pre-CE conditions, as inferred from Equation 14. The solid black line is the relation provided by Equation 14. The crossing horizontal lines show the positions for different initial donor masses, using observed older WD masses. Solid symbols are for RG donors (ZAMS masses from 1.0 to 1.8 $M_\odot$, with an increment of 0.1 $M_\odot$, with the lowest mass on the right.) The RG donors are selected when their cores are $0.47 M_\odot$. Open symbols are for AGB donors (ZAMS masses 1.8, 1.9 and 2.0 $M_\odot$, the smallest on the right). The AGB donors are selected when their cores are $0.53 M_\odot$. We find that in all the cases, a DWD binary is formed, most of the envelope is ejected, and only a few SPH particles remain bound to the binary in some cases (the bound mass is less than 0.06% of the initial envelope mass). The envelope is ejected on the dynamical time-scale of the expanded envelope.
at the time when it had a core of the same mass as a younger WD
in the observed sample, we used parameterized evolutionary tracks
from Hurley et al. (2000). For WD 1101+364, the fitting formula
predicts a pre-CE donor mass of 1.5 $M_\odot$, and pre-CE orbital period
of 33 days. We note that more detailed studies devoted to the simu-
lations specifically of WD 1101+364 gave a similar pre-CE donor
mass, 1.5 $M_\odot$, as mentioned by Nandez et al. (2015). We note that since the pre-CE
radius is a strong function of the core mass, uncertainty in the mass
in the donor's envelope, and the fraction of the recombina-
tion energy in the total binding energy of the initial envelope, are
both
\[ \alpha \] fully trusted, as the fitting formula may not work well for them;
\[ \alpha \] implies that an energy additional to the orbital energy was used,
in our case it is the recombination energy. Some past population
synthesis studies have considered $\alpha_{\text{bind}} = 2$ (see e.g. Toonen et al.
2012) for all CEEs leading to DWD binary formation, but the re-
results of our simulations do not confirm that such a very high value
is plausible, at least in the case of CEEs with low-mass RG donors.

And, finally, we give the preferred way to use our results,
which is the most trusted method when one wants to extrap-
olate our result outside of the parameter space we considered. We advise
population synthesis studies to use the energy conservation equa-
tion that accounts for all energy sinks and sources. In the energy
conservation equation, all initially available recombination energy
can be used as an energy source. (Note that this statement is not yet
fully justified to extend our results for low-mass giants to the case
of more massive or more evolved donors, and shall require further
studies.) The ejected material can take away 20% to 40% of the
released orbital energy, both as thermal energy and as kinetic en-
ergy, and this is an energy loss. It is this energy that powers those
Luminous Red Novae which are produced by a CEE (Ivanova et al.
2015) for all CEEs leading to DWD binary formation, but the re-
sults of our simulations do not confirm that such a very high value
is plausible, at least in the case of CEEs with low-mass RG donors.

Table 4 summarizes the possible progenitors for each sdB bi-
ary. We note that we list the values for the closest point on the
Figure 11, not the exact intersection between the model and the
line with constant final orbital period; the donor mass for each case
will not change much from the listed value in the Table, only the
radius of the donor and its initial orbital period will change. We
can see from Table 4 or Figure 11 that the evolutionary stage of
the donor star can be associated with the final orbital period. For
$P_{\text{orb, fin}} \leq 1$ day, the donor star is likely a (relatively more massive)
AGB star, while for $P_{\text{orb, fin}} \geq 1$ day, the donor star is more likely to be a (relatively less massive) RG.

The second observable type of post-CE binaries for which we
can test the fitting equation are hot subdwarf B stars (sdBs). These
stars are hot core helium-burning stars with masses around 0.5 $M_\odot$.
We note that the post-CE remnants of this mass are beyond the set
of our current simulations (modeling a CEE with a more evolved
donor requires substantially more GPU time than is available at ex-
isting Compute/Calcul Canada facilities, and therefore is not fea-
sible yet), but we will try to look at the post-CE binaries to see if we
can place any constraint on their path. We use 12 sdB binaries for
which Kupfer et al. (2015) have found orbital periods, and in-
ferred the minimum companion masses in these systems from the
assumption of a canonical mass of 0.47 $M_\odot$ for the sdB stars. In
Figure 11 we show the predictions of our fitting formula. In addi-
tion to checking RG donors, we also took into account AGB donors.
A prediction for a post-CE outcome for an AGB donor can not be
fully trusted, as the fitting formula may not work well for them;
both \( \lambda \) in the donor’s envelope, and the fraction of the recombina-
tion energy in the total binding energy of the initial envelope, are
not the same as in the case of a low-mass RG. However, it is im-
portant that at least half of the considered sdB binary systems can
be better explained by a RG donor.

Table 4 summarizes the possible progenitors for each sdB bi-
ary. We note that we list the values for the closest point on the
Figure 11, not the exact intersection between the model and the
line with constant final orbital period; the donor mass for each case
will not change much from the listed value in the Table, only the
radius of the donor and its initial orbital period will change. We

### Table 4. sdB binary predictions

| Object | $M_{\text{ZAMS}}$ | $M_{d,1}$ | $M_{c,1}$ | $R_{d,1}$ | $M_2$ | $P_{\text{orb, ini}}$ | $P_{\text{orb, fin}}$ | Stage |
|--------|-----------------|---------|---------|---------|------|-----------------|-----------------|------|
| J0800+47515 | 1.80 | 1.75 | 0.53 | 110 | 0.14 | 212.03 | 0.15 | AGB |
| J0823+11364 | 1.90 | 1.87 | 0.53 | 64 | 0.44 | 106.94 | 0.21 | AGB |
| J1021+30101 | 1.90 | 1.87 | 0.53 | 64 | 0.30 | 100.42 | 0.30 | AGB |
| J09510+03475 | 1.80 | 1.75 | 0.53 | 110 | 0.23 | 229.56 | 0.43 | AGB |
| J1522+01301 | 1.80 | 1.75 | 0.53 | 110 | 0.27 | 235.69 | 0.67 | AGB |
| J15082+04045 | 1.80 | 1.75 | 0.53 | 110 | 0.39 | 250.38 | 0.97 | AGB |
| J11324+06365 | 1.30 | 1.20 | 0.47 | 157 | 0.14 | 461.31 | 1.06 | RG |
| J01185+00254 | 1.40 | 1.32 | 0.47 | 153 | 0.22 | 449.81 | 1.30 | RG |
| J13463+28172 | 1.70 | 1.67 | 0.47 | 139 | 0.50 | 565.64 | 5.40 | RG |
| J1522+01301 | 1.30 | 1.20 | 0.47 | 157 | 0.58 | 577.70 | 6.98 | RG |
| J09523+62581 | 1.00 | 0.78 | 0.47 | 176 | 0.31 | 824.86 | 7.43 | RG |

$M_{\text{ZAMS}}, M_{d,1}, M_{c,1},$ and $M_2$ are the ZAMS mass of the donor from our parameter space, inferred initial donor mass right before the CEE, mass of the donor’s core (which is assumed to become a sdB star after the CEE), and the companion mass (assumed to be the minimum companion mass from Kupfer et al. 2015), respectively. All the masses are in $M_\odot$. $R_{d,1}$ is the inferred radius of the donor star right before the CEE in $R_\odot$, $P_{\text{orb, ini}}$ and $P_{\text{orb, fin}}$ are the inferred initial orbital period, and the observed orbital period, respectively, in day. Stage is the evolutionary stage of the donor right before the CEE, AGB star or RG star.
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