An autocognitive conflict and its mapping in solving mathematics problem

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Abstract. An autocognitive conflict (ACC) is an internally anomaly situation based cognitive conflict in doing mathematical problem. It can be described by using a cognitive map. Therefore, this research aims to investigated the ACC that occurs in the thinking structure of undergraduate students while completing mathematics problem and made the ACC map. Based on the think aloud method while doing the task, 68 participants in this research were grouped into two groups, a group of students who had the ACC and who did not experienced the ACC. Subject in this research consist of two subjects form the first group. Based on our in-depth interview, we identified that there were two types of internally anomaly situation (IAS), mis-analogical construction based IAS and previous work based IAS. Another finding is that there were three types of anomaly situation management’s result: anomaly situation that could not be solved by the student, anomaly situation that could be a trigger of another anomaly situation, and anomaly situation that could be solved by the student. We also indicated that there were three types of the implementations of autocognitive conflict process. They are knowledge updates, the emergence of new anomaly situation, and the divorced of knowledge structure.

1. Introduction

An autocognitive conflict (ACC) is a cognitive conflict that occurs internally and also internally solved by student in their problem solving process. The term autocognitive conflict is more emphasized in anomalous situations presented by student themselves. Previous research on internal conflict has been introduced by [1,2]. According to Siegel, an internal cognitive conflict happened when there were two conflicting ideas [1], meanwhile [2] introduced the metacognitive conflict to illustrate the contradiction between the schemes in students’ thinking structures. There were many research about cognitive conflict in learning process [3-11]. Specifically, cognitive conflict in mathematical learning process has been discussed in [12,13]. However, those studies did not specifically highlight the existence of anomalous situations that emerged from the learners, which were then managed by themselves, and efforts to resolve the conflict which were also conducted by the learners themselves.

Anything that happened in a person’s thinking structure can be represented into cognitive maps. Pena A, at al. indicates that students’ understanding (perspective and belief in thinking structure and also their ability to maintain their own knowledge to the new knowledge) can be exposed in the cognitive map [14]. Cognitive maps are a tool to understanding human cognitive structure. It means that by creating students cognitive map, we can explore their thinking process [15]. We can trace the error that may
occur in their constructing process in the solving problem process by using this cognitive map. Therefore, this research aims to investigated the autocognitive conflict that occurs in their thinking structure while completing mathematics problem and make the autocognitive conflict map.

2. Method
This is a descriptive qualitative study that conducted to 68 seventh semester mathematics students at Universitas Negeri Malang. In this study, we asked them to determine the solution of $\frac{1}{x} + \frac{1}{y} = \frac{1}{p}$, $x, y \in \mathbb{Z} - \{0\}$ and the prime number $p$. We asked them to verbalize what they think during the completion of the task. Based on the results of their verbalization and work, we grouped these 68 students into two groups. There are a group of students who had an autocognitive conflict (5 students) and group of students who did not experience the autocognitive conflict (63 students). The group of students who experiencing the autocognitive conflict were devide again into two subgroups. They are a group that can resolve their autocognitive conflict (2 students) and a group that can not resolve their autocognitive conflict (three students). Based on this grouping result, we selected one student from each group as our subject in this paper. They were choosen by using the purposive sampling systems. The standard criteria that used in subject selection were their activity that shows the occurence of cognitive conflict [10] during think aloud process and the riches of unique information required. They were selected to asses their autocognitive conflict by using indepth interview techniques. The interview process is based on the results of their think aloud recorded and worksheet. During the interview process, we noticed and noted all activities displayed and expressed by the subject that showing the characteristic of cognitive conflict, as revealed by [10]. Next, we described the data that obtained from the think aloud recording, worksheets, and interview, according to Kwon’s cognitive conflict process [11], and then we created their cognitive maps.

3. Results and discussion

3.1. The autocognitive conflict of Elsa (Subject A1)
Subject A1 completed the task by dividing the problem into four cases. The first case is when $x = y = 1$, the second case is when $x = y = 2$, the third case is when $x = y \neq 2$, while the last case is when $x \neq y$. She perceived the cognitive conflicts while solving the second and the third case. The illustration of the cognitive conflict is presented in relation to Kwon’s cognitive conflict stages [11]. Based on Kwon’s theory about the cognitive conflict process, there are three stage in cognitive conflict process [11]. They are preliminary stage, conflict stage, and the resolution stage. The illustration of her cognitive conflict for the second case can be seen in Figure 1.

![Figure 1. A1’s autocognitive conflict process for the second case.](image-url)
Figure 1 shows that subject A1 meet her cognitive conflict at the end of the preliminary stage. She found that there was $4 \in \mathbb{Z}$ but $4 \notin P$, after she believed that $k \in \mathbb{Z} \rightarrow k \in P$ previously. The cognitive conflict experienced by the subject after she realizes that there is a discrepancy between what was previously considered true, that $k \in \mathbb{Z} \rightarrow k \in P$, with what she found that $4 \in \mathbb{Z}$ but $4 \notin P$. $4 \in \mathbb{Z}$ but $4 \notin P$ as an anomalous situation, its presence comes from subject herself. Lee G et al. and Stacey K et al. notes that it was the sign of the cognitive conflict occurrence when the students knows the discrepancy in their thinking structure [10,5]. If students did not know this or if students ignored this, it can be said that there was no cognitive conflict in their thinking structure [7].

Awareness of the presence of this discrepancy prompted her to manage it. So, she tried to manage this anomaly situation at the second stage. She seems very interested to the oddities that arise, so she try to find another appropriate number and support that $c \in \mathbb{Z}$ but $c \notin P$. Students’ interest to the emerging of anomaly situation is one of cognitive conflict characteristics [10]. She also sought to figure out why there was a discrepancy between her belief and what she had just discovered, by creating a truth table for implications along with its negations. Once a truth table has been created and adapted to the found numbers, she recognizes her misconception and confirms it. Then he made a new rationale that not all integers are $P$. In this stage, she changed her convictions. There were many responses when student meet their anomalous situation [3,5,6,8,9]. Subject shown a response changed her conviction, which is included in one type of response that proposed by Kang S et al. [9].

Finally, after changing her belief, A1 has managed to update her understanding based on this cognitive conflict that not all integers are prime numbers. In addition, she recognizes that she made a mistake when he tried to make a negation of the implication. At the end of the cognitive conflict process for the second case, she can correct the mistake of her answer and her reasoning for the given task.

A1 also had her autocognitive conflict while trying to solve the third case Figure 2. Generally, Figure 2 shows that subject has her cognitive conflict twice. The first cognitive conflict occured after she had come to the conclusion that the solution for the third case was an empty set, since $p = \frac{x}{2} \notin \mathbb{Z}$. While the second cognitive conflict occured after she found and think that $X$ was a sequence but did not match to her previous knowledge about sequence. The anomaly situation in these two cognitive conflict for the third case arises after she is reminded about what she has done before.

![Figure 2. A1’s autocognitive conflict process for the third case.](image-url)
The anomaly situation for the first cognitive conflict present after she think that \( p = \frac{x}{2} \notin \mathbb{Z} \). It was because she think that \( \exists x \in \mathbb{N} \exists p = \frac{x}{2} \in \mathbb{Z} \). Then she found that \( \exists x \in X \in \{4, 6, 8\} \subset \mathbb{N}, \exists p = \frac{x}{2} \in \mathbb{Z} \) obtained from the second case. She aware of this and then managed it to obtain the conflict resolution. She then attempts to perform the calculation on some of \( x \in \mathbb{N} \). After that, she selected the number and sorted it.

An anomaly situation returns when she attempts to find the pattern of \( X \). She tried to recollect and recall her memory of her prior knowledge about the sequence. In her thinking structure, the sequence just consist of two types, geometry and arithmetic sequence. Based on her prior knowledge, she believed that there are just two kind of sequence. But it becoming contradicted when she found \( X \). She think that it was a sequence too, but unsure about it. Subject trapped on her prior knowledge about the kind of sequence, and forget to recall her memory about sequence definition. Actually she has a concept schema about the sequence definition. This schema should help her to avoid her cognitive conflict. But the reality, she did not succed in connecting the concept she had to her second cognitive conflict. Consequently, she failed to apply the concept that is already owned.

After all, she could not solve her cognitive conflict and led to her failure to solve the third case. The bigger impact of this protraced conflict was that she feel saturated, frustrated and unwilling to try to solve the fourth case so that she can not complete the given task.

![Figure 3. A1’s autocognitive conflict map.

A1’s autocognitive conflict map can be seen at Figure 3. It shows us that the anomaly situation of both autocognitive conflict were present from her previous work. The anomaly situation of the cognitive conflict at the second case comes from her previous work in the first case. Meanwhile for the first cognitive conflict in the third case, the anomaly situation comes from her work on the second case. However, for the second cognitive conflict in the third case, the anomalous situation comes from her failure to connect the concept of sequence to the sequence encountered. Because of her failure to connect this, consequently she failed to apply the concept that is already owned.

There are two things that can be learned from the protracted of subject’s autocognitive conflict. Firstly, she could not complete the task. Because of her failure to resolve the second cognitive conflict for the third case, she was also unwilling to solve the fourth case. This is due to her frustration and saturation during the process of completion of her autocognitive conflict. “I give up, maam. I could not afford to finish the third case. It was frustrating since I found the strange sequence”

Secondly, she failed to correct her concept construction about sequence. Based on the interview result, actually she has the correct concept about sequence (definition of the sequence). But she was caught up in another concept about the type of sequence that she has since she was in the high school. She could not mix and match these two concepts so that she failed to find the pattern of the sequence.
3.2. The autocognitive conflict of Abdul (Subject A2)

Subject A2 experienced the cognitive conflict after completing the assigned task and concluded that the solution of the given task was an empty set. All anomaly situations was present when he tried to check the truth of the answer, and presented by himself. He did calculations by replacing $x$ and $y$ with some small numbers on his checking process. In this checking process, he found that there were $x$ and $y$ so that the given task solution is not an empty set. Then he tried to find the answer of what he has found. These events occur repeatedly up to three times. Generally, his autocognitive conflict process can be seen in Figure 4.

Figure 4 shows that the anomalous situation was presented three times in the process of solving problem. Generally, both of the anomaly situation was present while he checked out his answer. First anomaly situation was “there exist $x = y = 4$ therefore the solution of the problem was not an empty set”. The second anomaly situation was “there exist $x = 3$ and $y = 6$ therefore the solution of the problem was not only $(2p, 2p)$, $(p + 1, p(p + 1))$, $(p(p + 1), p + 1)$”. The first anomaly situation comes after he made the conclusion that the solution of the task was an empty set. He was very sure of this answer. Then he checked it out to be more convinced himself about his answer. In this checking process, he found that there is $x = y = 4$ thus the solution of the problem was not an empty set. This was contrary to what he believed earlier. It was attracted his attention, thus encouraging him to look for another number that satisfies the equation. He also found $x = y = 6$ and $x = y = 10$. He made them into a sequence. Subsequently, used his previous knowledge about sequence to find out the pattern of the sequence then found that the pattern was $x = y = 2p$. After that, he tried to substitute another number $2p$ to the equation. This trial process makes him sure that the solution of the equation was $\{(x, y) | (2p, 2p)\}$. He was very sure of this answer. Then he checked it out to be more convinced himself about his answer. In this checking process, he found another number, and they were $x = 3$ and $y = 6$. This turned out to be contrary to what he had believed. In this time, he has his second cognitive conflict.

He tried to managed his anomaly situation at the second conflict stages. He repeated all processes that he has done at the first conflict stage, which consist of trying to find another number by using his counting skills, and then creating a sequence and finding its pattern by using his previous knowledge about sequence. Additional point at this stage that he used his prior knowledge about commutative in addition to find another solution. At the end, he found that the solution of the equation was $\{(x, y) | (2p, 2p), (p + 1, p(p + 1)), (p(p + 1), p + 1)\}$. He was absolutely sure of the answer, and checked it out to be more convinced him.
The third anomaly situation comes in the checking process. He found that there were $(2, -6)$ and $(1, -2)$ as another solution of the equation. It was against to what he believed. Furthermore, he did the same step in the second stage to resolve the conflict. At the end of this stage, he found the solution of the equation was \{$(x, y) | (2p, 2p), (p + 1, p(p + 1)), (p(p + 1), p + 1), (p − 1, −p(p − 1), −p(p − 1), p − 1)$\}. Finally, he success found the solution of the equation. All the process of his autocognitive conflict can be described in the cognitive mapping as seen in Figure 5.

Figure 5. A2’s autocognitive conflict map.

Figure 5 shows us that the whole anomaly situation present when he try to looking back to his answer. It means that he was in the fourth stage of problem solving stage according to [16]. He used his procedural knowledge and also some prior knowledge like commutative in addition, sequence, and the pattern of the sequence to resolve his autocognitive conflict. Finally, he succeeded in resolving his conflict and also in completing the task.

Anomaly situation is one of the important points of cognitive conflict. It also happens to autocognitive conflict. The presence of anomalous situations that arised from students themselves and realized by themselves too, is a key point that determines the occurrence of autocognitive conflict. Based on the result of this study, the anomaly situations in solving mathematics problem can be presented by the students themselves. Its emersion can be derived from their previous work or from their previous concepts. We call this anomaly situation as autoanomaly situation. The presence of this autoanomaly situation is very important when students solve math problem. It can be seen at Subject A2, who made mistake at the begining of his solving process, but success to find the correct solution of the task at the end of the solving process. This also can be seen from Subject A1 who succeed in repairing her concept construction although she failed to solve the problem at the end of the process.

There are several open topics that can be studied related to this research. For example, on what condition are required so that students can generate their anomaly situation and how to familiarize them to lead their autoanomaly situation when solving math problem. Therefore, there is still an opportunity for further research about it.
4. Conclusions
The autocognitive conflict process begins with an anomalous situation whose presence comes from the students themselves (internally anomaly situation). There were two types of internally anomaly situation that we found, i.e mis-analogical construction based internally anomaly situation and previous work based internally anomaly situation. Subject realized it and then managed it themselves. We identified that there are three types of result of the anomaly situation management. First, the anomaly situation could not be solved by the student. Second, the anomaly situation could be a trigger of another anomaly situation. Finally, the anomaly situation could be solved by the student. This situation gives some impact to the implementation of their autocognitive conflict. We found that there were three types of the implementations of autocognitive conflict process. They are knowledge updates, the emergence of new anomaly situation, and the divorced of knowledge structure. The whole process was done independently and automatically by the student. Beside that, all the process can be illustrated in cognitive map as seen in Figure 3 dan Figure 5.

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