Quantitative connection between the nanoscale electronic inhomogeneity and the pseudogap of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ superconductors

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We have found a quantitative connection between the evolution of the inhomogeneous nanoscale electronic gaps (INSEG) state detected in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ by scanning tunneling microscopy/spectroscopy (STM/S) and the two, the upper and the lower, pseudogaps in high-temperature cuprate superconductors (HTCS). When the doping and the temperature dependent INSEG map are analyzed by using our proposed hole-scale, we find that two pseudogaps are quantitatively connected to specific coverage of the CuO$_2$ plane by INSEG: the 50% and the 100% coverage of the CuO$_2$ planes by INSEG correspond to the upper and the lower pseudogaps, respectively. This quantitative connection to the two pseudogaps indicates that the origin of the measured pseudogap energies and temperatures are intimately related to the geometrical coverage of the CuO$_2$ planes by the INSEG state. We find that INSEG and superconductivity coexist in the underdoped to the overdoped regimes. In contrast to common belief we showed that YBa$_2$Cu$_3$O$_{6.5}$ is electronic inhomogeneous. We suggest that pseudogap states are microscopically inhomogeneous and 100% coverage of the CuO$_2$ planes by the INSEG is a necessary condition for the high-$T_c$ superconductivity.

I. INTRODUCTION

One of the long-standing puzzles of the hole-doped high-temperature cuprate superconductors (HTCS) is the existence of the ubiquitous pseudogap state that precedes the superconducting state. The pseudogap state, a partial suppression of the spectral density, generally are detected as either a pseudogap temperature ($T^*$) or a pseudogap energy ($E^*$). The initial reported $T^*$ or $E^*$ differed in details from material to material at, presumably, the same doping level and, sometimes, it is not even consistent with each other in the same material at the same doping level if determined by different experimental probes. We also showed that, using our proposed universal $P_{pl}$-scale of the two-dimensional (2D) doped-hole concentration $P_{pl}$, all measured $T^*$’s and $E^*$’s of hole-doped HTCS fell on either of the two, the upper or the lower, pseudogap lines. Furthermore, a unified electronic phase diagram (UEPD) was constructed in which there are four characteristic temperatures (energies) for hole-doped HTCS with an optimal superconducting transition temperature $T_{c}^{max}$ of $\sim$90 K, such as Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, YBa$_2$Cu$_3$O$_{6.5}$ and HgBa$_2$CuO$_{4+\delta}$. In Fig. 1 we plot various pseudogap measurements of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ together with schematic sketch of the UEPD. Three very distinct features can be clearly summarized in Fig. 1 (i) there are three characteristic temperatures (energies): the lower pseudogap $T_{up}$ ($E_{up}$), the upper pseudogap $T_{lp}$ ($E_{lp}$) and the hump $T_{hump}$ ($E_{hump}$) in the underdoped to the slightly overdoped regimes; (ii) all three characteristic temperatures (energies) merge together with superconducting transition temperature $T_c$ (superconducting gap energy $\Delta_c$), at the slightly overdoped level; (iii) $T^*$ and $E^*$ are connected by $2E^*/k_BT^* = 7 \pm 1\Delta_c$ where $k_B$ is the Boltzmann’s constant. Therefore all three energy-scales, which are correlated with corresponding characteristic temperatures measured by vast different experimental probes on various HTCS with $T_{c}^{max}$ of $\sim$90 K, are unified in the UEPD.

Scanning tunneling microscopy/spectroscopy (STM/S) had made a unique contribution to the study of HTCS through the direct observation of the inhomogeneous nanoscale electronic gaps (INSEG) in the superconducting states. A recent STM/S study pushed this electronically heterogeneous picture well into the pseudogap state by showing that nanoscale local gaps persist at a temperature well above $T_c^{max}$. It was shown that in the optimal to overdoped regimes the local INSEG appear (vanish) at a temperature $T_f$ upon cooling (warming) where $T_f$ and the INSEG energy ($\Delta_f$) are universally connected by $2\Delta_f/k_BT_f = 7.9 \pm 0.5\Delta_c$. In the underdoped regime, the situation is more complicated: two gap-like structures, the pseudogap and the pairing gap, are observed and, therefore, the simple relation, between local-gap vanishing temperature and gap size, namely, $2\Delta_f/k_BT_f = 7.9 \pm 0.5\Delta_c$ could not be clearly pinned down. Temperature dependent STM measurements showed that the CuO$_2$ plane is gradually covered by the INSEG with decreasing temperature. Although the INSEG have been subjected to intense studies in the past decade, the origin and the physical significance of INSEG to the ubiquitous pseudogap state and the high-$T_c$ superconductivity is largely unexplored.

In this report we analyzed the STM data of the purely oxygen-doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ by using the $P_{pl}$-scale and compared the results with our proposed UEPD with $T_{c}^{max}$ of $\sim$90 K. We show that in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ the specific coverages, 50% and 100%, of the CuO$_2$ plane by INSEG are quantitatively connected to the upper and
lower pseudogaps of the UEPD. We further show that the previous conclusion based on nuclear magnetic resonance (NMR) studies that YBa$_2$Cu$_3$O$_{6+\delta}$ is electronically homogeneous is not correct under $P_{pl}$-scale. The electronic inhomogeneity is generic in the hole-doped HTCS. We conjecture that the origin of pseudogaps is the geometrical coverage of the CuO$_2$ planes by the INSEG state.

II. ANALYSIS

In analyzing STM data of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, we selected the single crystal data with either thermolectric power at 290 K ($S^{290}$) or $T_c$ reported in the publications. We use two criteria to extract $P_{pl}$: as the first method, $P_{pl}$ is determined from the value of $S^{290}$ by using $P_{pl}$-scale. This is more reliable.\cite{7} As the second method, $P_{pl}$ is determined from the value of $T_c$ by comparing it with the asymmetrical half-dome-shaped $T_c$-curve as shown in FIG. 5 in Ref. 7.\cite{7} We always selected the paper that reports the value of $S^{290}$ and used the data with the value of $T_c$ when $S^{290}$ is not available. However, it limits the available data sets. In this report we analyzed and compared the electronic inhomogeneity of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ and YBa$_2$Cu$_3$O$_{6+\delta}$. For YBa$_2$Cu$_3$O$_{6+\delta}$, the value of $P_{pl}$ was estimated from the double plateau $T_c$-curve.\cite{11} For HgBa$_2$CuO$_{4+\delta}$, the value of $P_{pl}$ was estimated from a relation of $T_c$ versus $P_{pl}$ obtained from $T_c$ versus $S^{290}$ in Ref. 12.

III. RESULTS AND DISCUSSION

A. The quantitative connection between the INSEG coverage and the pseudogaps in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$

To compare the temperature and doping evolution of INSEG, upon cooling or warming, with the pseudogaps in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ we define the temperatures corresponding to 0%, 50% and 100% coverage of the CuO$_2$ plane by INSEG as $T_{0\%}$, $T_{50\%}$ and $T_{100\%}$, respectively. Similarly for CuO$_2$ plane that is completely covered by the INSEG we define the energies corresponding to 0%, 50% and 100% coverage of the CuO$_2$ plane by INSEG as

![FIG. 1. (Color online) Electronic phase diagram of (a) $T - P_{pl}$ and (b) $E - P_{pl}$ in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. The colored solid lines are characteristic (a) temperatures and (b) energies in the unified electronic phase diagram (UEPD). The discrete symbols are some representative data points of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ measured by various experimental probes used to construct the UEPD. For details see text and Ref. 8 and references therein.](image1)

![FIG. 2. (Color online) (a) Operational definition of $E_{0\%}$, $E_{50\%}$ and $E_{100\%}$. For details see text. (b) $T_{0\%}$, $T_{50\%}$ and $T_{100\%}$ versus $P_{pl}$ of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. The plotted data are coming from Ref. 8 (c) $E_{0\%}$, $E_{50\%}$ and $E_{100\%}$ versus $P_{pl}$ of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. The data are coming from Refs. 12 and 13 $E_{0\%}$ at 100 K by Gomes is a lower bound of $E_{0\%}$.](image2)
$E_{0\%}, E_{50\%}$ and $E_{100\%}$, respectively. For all STM data we analyzed, we confirmed that the INSEG distributions are Gaussian suggesting that the gap distribution is driven by randomness. The value of squared multiple correlation coefficient adjusted for the degrees of freedom was always over 0.97, except of some Gaussian fittings of $0.9 \sim 0.95$.

In Fig. 2(a), we plot the red curve as the probability $P(E)$ of finding a nanoscale gap at energy $E$ and the green curve as the probability $P(> E)$ to find a nanoscale gap that is larger than $E$ of a typical INSEG map. The percentage of the area covered by the gaps larger than $E$ out of the total gapped area is calculated by integrating $E_{gap}$ that is larger than the green curve as the probability $P(> E)$ to find a nanoscale gap that is larger than $E$ of a typical INSEG map. The intersection of green curve with $P(> E) = 0\%$, $50\%$ and $100\%$ are $E_{0\%}$, $E_{50\%}$ and $E_{100\%}$, respectively.

In Fig. 2(b), we plot the temperatures $T_{0\%}$, $T_{50\%}$ and $T_{100\%}$, as a function of $P_{pl}$, determined from the reported temperature dependence of gap distribution reported by Gomes et al. It is clearly seen that $T_{100\%}$ and $T_{50\%}$ correspond to $T_{ip}$ and $T_{up}$, respectively. At the slightly overdoped regime, $T_{hump}$ is associated with the onset temperature, the $T_{0\%}$, of the INSEG. In Fig. 2(c) with an error band defined by $2E^*/k_B T^* = 7 \pm 1$, we plot the energies $E_{0\%}$, $E_{50\%}$ and $E_{100\%}$, extracted from the gap distribution measured at 100 K, 90 K, 80 K and 60 K by various groups. Note that in order to extract the corresponding energies for $0\%$, $50\%$ and $100\%$ coverage of the CuO$_2$ planes, we have to use the gap map that has completely covered the CuO$_2$ plane since in order to determine $50\%$ coverage, we first need to know the $100\%$ coverage. Therefore, all the subsequent gap distribution data we analyzed are collected below $T_{100\%} = T_{ip}$. It is clearly seen that, from the underdoped regime to the slightly overdoped regime, $E_{100\%}$ and $E_{50\%}$ correspond to $E_{ip}$ and $E_{up}$, respectively. At the slightly overdoped regime, $E_{0\%}$ lies on $E_{hump}$. Here is one of the most important conclusions of this paper, namely, in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, we found that the temperature and doping dependent coverage of CuO$_2$ plane by INSEG is intrinsically connected to the electronic phase diagram shown in Fig. 3 where $50\%$ and $100\%$ coverages correspond to upper and lower pseudogap, respectively.

In Fig. 3(a), we plot the expectation value, i.e. the peak value ($E_{GP}$), of the fitted Gaussian distribution versus the peak value ($E_{peak}$) read directly from the gap distribution observed in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ by various groups. It can be clearly seen that when we treat the gap distribution of whole INSEG map as a single Gaussian distribution, the $E_{GP}$ closely traces the $E_{peak}$. This validates, to the zeroth order, our choice of using a single Gaussian distribution to analyze INSEG maps.

In Fig. 3(b), we plot the $E_{50\%}$ versus $E_{peak}$ observed in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ for $T < T_{100\%} = T_{ip}$, respectively. $E_{50\%}$ is almost the same as the $E_{peak}$ and, therefore, $E_{GP}$. Accordingly, the $E_{peak}$ and $E_{GP}$ are also corresponding to the upper pseudogap energy observed in the UEPD. Since the upper pseudogap temperature is observed by the dc resistivity, as shown in Fig. 3(a), which is a typical bulk probe, the present result of $E_{peak} = E_{50\%} = E_{GP}$ implies that the three energies $E_{peak}$, $E_{GP}$ and $E_{50\%}$ share the identical physical meaning of the expectation value measured by the experimental probes.
were detected by both surface-sensitive probe such as STM and NMR in HgBa$_2$Cu$_2$O$_{6+\delta}$ reported by Broff et al. In the following we show that the absence of electronic inhomogeneity in YBa$_2$Cu$_3$O$_6+\delta$ is due to the use of a popular scale, we call $P_{dome}$-scale based on a well-known dome-$T_c$ curve with empirical formula of $T_c/T_c^{max} = 1 - 82.6(P_{dome} - 0.18)^2$, where the $P_{dome}$ is the hole density. In order to justify the use of our scale for the YBa$_2$Cu$_3$O$_6+\delta$ system we show (I) the well-known two-plateau behavior of $T_c$ is preserved in $P_{dome}$-scale but not in $P_{dome}$-scale and (II) when using $89K$ Knight shift as a measure of the hole concentration $89K$ increases linearly with $P_{pl}$ within the error bar of the $P_{pl}$-scale. However $89K$ is barely linearly proportional to $P_{dome}$ with a large scattering.

(I) In YBa$_2$Cu$_3$O$_6+\delta$, a two-plateau behavior of $T_c$ was reported in the plot of $T_c$ versus oxygen-content $\delta$. However one can never observe two-plateau behavior in the plot of $T_c$ versus $P_{dome}$ as shown in Fig. 4(a). If using $P_{pl}$-scale, the characteristic two-plateau behavior (FIG. 2(a) in Ref. 11) is naturally preserved as shown in the $T_c$-$P_{pl}$ plot of Fig. 4(b). In the plateau regions of $T_c = 60$ K and $90$ K, both $dT_c/d\delta$ and $dT_c/dP_{pl}$ are very small value. Incidentally, in Bobroff’s paper five samples with $0.85 < \delta < 1$ sit on the $90$ K plateau, while the other samples also are close to $60$ K plateau.

(II) A quantitative measure of the degree of doping inhomogeneity in HTCS was inferred from NMR line width a decade ago. In the Y-based HTCS, such as YBa$_2$Cu$_3$O$_6+\delta$ and YBa$_2$Cu$_4$O$_8$, the doping dependence of the $89K$ at $300$ K was used to determine the hole concentration. In Fig. 5 we plot $89K$ at room temperature (RT) as a function of $P_{dome}$ in the top panel and as a function of $P_{pl}$ in the lower panel. In Fig. 5(a), we also draw the two lines with the linear slopes of $580$ ppm/hole and $823$ ppm/hole reported in the literatures. While each linear slope may represent the limited data selected in each paper, it clearly showed that the doping dependence of $89K$ cannot be represented by one straight line using $P_{dome}$-scale when we use all available $89K$ at RT in Fig. 5(a). This suggests the ambiguity of using $89K$ data to determine hole concentration based on the $P_{dome}$-scale. However, in the $P_{pl}$-scale as shown in Fig. 5(b), all $89K$ at RT lie on one straight line represented by $555P_{pl} - 68$. The deviation at $P_{pl} < 0.07$ is coming from the AF phase.

In Ref. 26 from $89Y$ NMR Fourier transform spectra at $300$ K, the doping distribution $\Delta P_{dome}$ of the slightly overdoped sample YBa$_2$Cu$_3$O$_7$ and the underdoped YBa$_2$Cu$_3$O$_6+\delta$ were deduced by using the relation of the slope of $580$ ppm/hole in the $P_{dome}$-scale. The full width at half maximum (FWHM) of $89Y$ NMR spectra for YBa$_2$Cu$_3$O$_6+\delta$ gave us the doping distribution $\Delta P_{dome} < 0.025$ for YBa$_2$Cu$_3$O$_7$ and $\Delta P_{dome} < 0.01$ for YBa$_2$Cu$_3$O$_6+\delta$. It was claimed that these values are much smaller than $\Delta P_{dome} = 0.1$ in LSCO and Bi$_2$Sr$_2$Ca$_2$Cu$_2$O$_8+\delta$. If we calculate our doping dis-
distribution $\Delta P_{pl}$ from their FWHM by using of the relation of $555P_{pl} - 68$ based on the $P_{pl}$-scale, the $\Delta P_{pl}$ of YBa$_2$Cu$_3$O$_{6.6}$ is 0.07 (see Fig. 4(b)) and that of YBa$_2$Cu$_3$O$_7$ is $\sim 0.03$. We reproduce the doping distribution and $T_c$ of Fig. 2 of Bobroff et al. in Fig. 4(a) and re-plot the same data set by directly converting their hole density to $P_{pl}$ in Fig. 4(b). Therefore, as seen in Figs. 4(a) and 4(b), using the SAME NMR data, the electronic spread determined by $P_{pl}$-scale is seven times and one and a half as large as those determined by the $P_{dome}$-scale for underdoped and slightly overdoped YBa$_2$Cu$_3$O$_{6+\delta}$, respectively.

Similarly, if we calibrate the doping dependence of $^{63}$Cu nuclear quadrupole resonance (NQR) frequency in YBa$_2$Cu$_3$O$_{6+\delta}$ against $P_{pl}$-scale, the maximum hole-doping distribution $\Delta P_{pl}$ at 100 K estimated from the FWHM of $^{63}$Cu NQR spectrum is 0.03 $\sim$ 0.05 in YBa$_2$Cu$_3$O$_{6.7}$ and $\sim 0.04$ in YBa$_2$Cu$_4$O$_8$. These values are not significantly smaller than the other HTCS in the same $P_{pl}$-scale. In Fig. 6(a), we plot the doping distribution $\Delta P_{pl}$ obtained from the $^{63}$Cu NQR spectrum data. We used the reported value in Ref. 22 to calculate the $\Delta P_{pl}$ of La$_{2-x}$Sr$_x$CuO$_4$. The other $\Delta P_{pl}$ are extracted from the reported spectrum data in Refs. 42 and 43. The observed temperatures are shown in the parenthesis.

FIG. 5. (Color online) The doping dependence of the $^{89}$Y Knight shift ($^{89}K_y$) at room temperature (RT) for Y-based HTCS. (a) $^{89}K_y$ at RT versus $P_{dome}$. (b) $^{89}K_y$ at RT versus $P_{pl}$. Both plotted data are same. The plotted data are from Refs. 22, 24, 51, 49.
out oxygen disorder have almost same value of $\Delta P_{pl}$. For YBa$_2$Cu$_3$O$_{6+\delta}$ with the oxygen disorder, $\Delta P_{pl}$ is larger than that of HTCS without oxygen disorder. In Fig. 6(b), we plot same $\Delta P_{pl}$ of YBa$_2$Cu$_3$O$_{6+\delta}$ as a function of the oxygen content $6 + \delta$. At around $6 + \delta = 6.5$ and 7, $\Delta P_{pl}$ has the minimum value corresponding to ortho-I phase at $\delta \sim 1$ and the ortho-II phase at $\delta \sim 0.5$, respectively. Except at $\delta$ or $P_{pl}$ where the sample has special oxygen ordering, $\Delta P_{pl}$ of YBa$_2$Cu$_3$O$_{6+\delta}$ with oxygen disorder is about five times larger than that of the other HTCS without oxygen disorder. Accordingly, although the materials with the oxygen disorder have the $\Delta P_{pl}$ over 0.2, the materials without oxygen disorder will still have a $\Delta P_{pl} \sim 0.04$. Singer et al. pointed out $\Delta P_{pl} \sim 0.05$ of La$_{2-x}$Sr$_x$CuO$_4$ is related to the nanoscale spatial inhomogeneity.

Now we can distinguish two distinct origins of electronic inhomogeneities: an extrinsic dopant-induced electronic inhomogeneity and an intrinsic electronic inhomogeneity. For example, in the case of La$_{2-x}$Sr$_x$CuO$_4$, the extrinsic dopant-induced electronic inhomogeneity is generated by the degree of the inhomogeneity of the dopant distribution within the La$_2$O$_2$ layer. The intrinsic electronic inhomogeneity is generated by the intrinsic tendency toward an electronic phase separation within the CuO$_2$ layer due to doped holes alone as seen in Refs. 11 and 12. We believe that both YBa$_2$Cu$_3$O$_7$ and YBa$_2$Cu$_4$O$_8$ are dominated by the intrinsic electronic inhomogeneity, while YBa$_2$Cu$_3$O$_{6+\delta}$ has both intrinsic and extrinsic electronic inhomogeneities. In Fig. 6 $\Delta P_{pl}$ below 0.05 is originated from the intrinsic electronic inhomogeneity, while the $\Delta P_{pl}$ over 0.2 is due to the extrinsic electronic inhomogeneity or both.

Since all the previous arguments hinged heavily on the correctness of the $P_{pl}$-scale we want to emphasize that although there is no theoretical justification for any of the hole-scales used for the cuprates, we have shown that $P_{pl}$-scale is quantitatively consistent with the hole density determined in various cuprate systems by different spectroscopic probes (FIG. 3 in Ref. 7). We also showed that $P_{domse}$-scale is quantitatively different from $P_{pl}$-scale and, therefore, is not consistent with the hole density independently determined by many other probes. Since any experiment that addresses the doping distribution has to involve a hole-scale, using the quantitatively correct hole-scale is of fundamental importance to arrive at physically correct conclusions. Using our scale together with Bobroff’s data 25 and other published results 11,13 indicates that the YBa$_2$Cu$_3$O$_7$ is electronically inhomogeneous.

### C. The INSEG state is generic to HTCS with $T_{c,max} \sim 90$ K

In the $P_{pl}$-scale the optimal doped-hole concentration $P_{pl}^{opt}$ depends on the individual HTCS material 7. However when using the reduced temperature $T/T_{c,max}$ (the reduced energy $E/3.5k_BT_{c,max}$) and the reduced hole-concentration $p_n \equiv P_{pl}/P_{pl}^{opt}$ various HTCS can be easily compared with each other in spite of the variations in $P_{pl}^{opt}$ and $T_{c,max}$. Using $p_n$ and $T/T_{c,max}$ ($E/3.5k_BT_{c,max}$), for HTCS with $T_{c,max} \sim 90$ K, such as YBa$_2$Cu$_3$O$_{6+\delta}$, HgBa$_2$CuO$_{4+\delta}$ and Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, various characteristic temperatures (energies) can be unified into a UEPD shown as the three lines with shaded area in Fig. 2. This UEPD was constructed based on the analysis of the experimental data measured by fifteen different macroscopic and microscopic experimental probes.

Within these fifteen probes, five were surface-sensitive probes and ten were bulk probes. Therefore, UEPD represents a true intrinsic electronic phase diagram for HTCS with $T_{c,max} \sim 90$ K. We plot $T_{c,50}$, $T_{c,50}$ and $T_{c,100}$ in Fig. 7(a) and $E_{0\%}$, $E_{50\%}$ and $E_{100\%}$ in Fig. 7(b) together with data plotted in Figs. 2(b) and 2(c), respectively. We confirm again that both the 50% and 100% coverage of CuO$_2$ planes are consistent with the intrinsic, universal upper and lower pseudogaps, respectively and the 0% coverage corresponds to the hump. In Fig. 7(a), we also include the most recent experimental results performed on YBa$_2$Cu$_3$O$_{6+\delta}$ 31-33, HgBa$_2$CuO$_{4+\delta}$ 31-33 and Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ 34. We can see that: (1) the polarized elastic neutron scattering experiments suggesting a novel translational-symmetry-preserving magnetic transition falls on $T_{c,pl}$, (2) the Nernst effect measurements indicating a breaking of the 90°-rotational ($C_{4v}$) symme-

![FIG. 7](image_url) (Color online) (a) $T_{c,50}$, $T_{c,50}$ and $T_{c,100}$ of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ on the UEPD. We use the same symbols as that in Fig. 2(b). The new data are from Refs. 31-33. (b) $E_{0\%}$, $E_{50\%}$ and $E_{100\%}$ of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ on the UEPD. We use the same symbols as that in Fig. 2(c). The $T_{c,pl}$-curve for YBa$_2$Cu$_3$O$_{4+\delta}$ is from Ref. 11. The $T_{c,pl}$-curve for HgBa$_2$CuO$_{4+\delta}$ is obtained by analyzing the data from Ref. 12 using $P_{pl}$-scale.
try occurs at $T^*$ and (3) the Kerr-effect measurements signaling a time-reversal symmetry breaking corresponds to $T^c_T$. These experimental observations of very subtle changes of physical properties by different probes in different materials that universally fall on either the upper or the lower pseudogap reconfirm the two-pseudogap scenario reported in Ref. [6]. This further validates the UEPD reported in Ref. [6]. The novel connection between pseudogaps and nanogap distribution is a natural consequence of plotting the published experimental results by using the quantitatively correct and accurate $P_{pl}$-scale. Based on this inhomogeneous nanogap distribution picture and its quantitative connection to the bulk pseudogap, it is interesting to point out that these “phase transitions” are highly unusual that (1) and (2) appeared right at a 50% coverage and (3) occurred when 100% of the CuO$_2$ planes are covered by the nanogaps. How does a phase transition emerge from such an inhomogeneous background at the specific nanogap coverage require further studies.

To understand why various probes can detect the pseudogap of either the 50% or 100% coverage of CuO$_2$ planes by INSEG, we point out that the relationship between $E^*$ and $T^*$, $2E^*/k_B T^* = 7 \pm 1$, revealed in the UEPD plot, and that between $\Delta_g$ and $T_p$, $2\Delta_g/k_B T_p = 7.9 \pm 0.5$ reported in Ref. [3] are surprisingly similar, within the error band of the original construction of the UEPD. The UEPD was constructed by the data collected from many experimental techniques which probe an area with a length-scale that is much larger than the characteristic length-scale, $\sim 10^{-9} m$, of INSEG. This strongly suggests that the pseudogaps revealed in UEPD are the expectation value of the gap map sampled in the characteristic length-scale of the experimental probe. Indeed, it is intriguing to see that the gap map identified in the STM/S can be naturally related to the “bulk” pseudogap measured by various bulk probes: $E^*_{up}$ ($T^*_{up}$) and $E^*_{lp}$ ($T^*_{lp}$) are the gap energies (temperatures) when one half or the entire CuO$_2$ plane is covered by the INSEG, respectively. The former is detected by the deviation from the high-temperature trend that was measured by in-plane dc resistivity, and Nernst effect, and the latter are properties that are detected by various bulk and surface-sensitive probes. It is interesting to note that the 50% coverage corresponds to 2D percolation limit of a square lattice. Therefore, we conclude that the upper and lower pseudogaps detected by different experimental probes must also be related to, besides the characteristic length-scale, the energy-scale of the experimental probes. In this context, the pseudogaps in the UEPD are the spatially “averaged” response of the gap map measured by the individual experimental probe. Depending on the characteristic energy-scale and length-scale of the experimental probes: some of the probes are sensitive enough to pick up the incipient inhomogeneous nanoscale electronic state, some probes pick up the percolation path and the others measure the true bulk property when the CuO$_2$ plane is completely covered by INSEG.

There are two mutually exclusive scenarios regarding the connection between INSEG coverage and the pseudogaps: one is that the connection between 50% (100%) coverage to the upper (lower) pseudogap is only a surface manifestation of a intrinsically bulk pseudogap, therefore, the nanoscale inhomogeneity is just a surface state that is distinct from bulk. The other is that it is an intrinsic property of the CuO$_2$ plane that the INSEG is not confined to the surface but also exists throughout the bulk. We are not aware of any bulk transition that induces the surface coverage properties as we have observed here. It is very difficult to envision that the connection between the specific coverage of CuO$_2$ planes and the two pseudogaps over such wide doping range is only a surface property. In fact, similar characteristics and similar nanoscale inhomogeneity were observed in very different HTCS that we expect, if the gap map data sets are available in the literature, then the gap map would lead to the same coverage as we observed. Furthermore, UEPD is constructed from the reported $T^*$ and $E^*$, of many HTCS with $T^c_{max} \sim 90 \text{ K}$, measured by various surface-sensitive probes and bulk probes that cannot be just coming from a surface state. In light of the aforementioned observations and the further connection between the 100% and 50% coverage of CuO$_2$ planes by INSEG to the lower and the upper pseudogaps, respectively, we argue that the specific coverage of the CuO$_2$ planes by INSEG should be intrinsic, at least, to Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ and, more likely, to be generic to HTCS with $T^c_{max} \sim 90 \text{ K}$. This conclusion is also consistent to the fact that pseudogap is a pure 2D property as we have shown in Ref. [6].

D. The entropy associated with the lower pseudogap state ends with the disappearance of high-$T_c$ superconductivity

High-$T_c$ superconductivity appears at 100% gap coverage for optimally and overdoped HTCS as can be clearly seen in FIG. 6 in Ref. [3]. However superconductivity in the underdoped regime, as seen in Fig. 2, appears at an even lower temperature after the CuO$_2$ plane is completely covered by the INSEG. Therefore, combining all the above observations and for the entire doping range, we conclude that 100% coverage of the CuO$_2$ planes by the INSEG is a necessary condition for generating the high-$T_c$ superconductivity in cuprate superconductors. We emphasize that our conclusion is fundamentally different from other two-energy-scale scenarios, where the charge order is competing against superconductivity. In contrast, we proposed that the 100% coverage of the CuO$_2$ planes by the INSEG is a necessary condition for the high-$T_c$ superconductivity, and the high-$T_c$ superconductivity is “realized” on a texture of a globally coupled INSEG of the lower pseudogap state.

To further elaborate our point we examine the doping dependence of the entropy associated with the lower pseudogap state. For a Gaussian distribution the stan-
standard deviation ($\sigma$) is a measure of the entropy of the disordered state. In Fig. 8 we plot the $\sigma$ of the INSEG state in the superconducting state. Figs. 8(a) and 8(b) show the standard deviation in the superconducting state $\sigma(T < T_c)$ and in the normal state $\sigma(T > T_c)$, respectively. In the superconducting state, we see that the $\sigma(T < T_c)$ decreases monotonically with increasing $P_{pl}$ from underdoped to overdoped regime and tends to fall down to zero at the end ($p_u \sim 1.3$) of the superconducting half-dome of the UEPD. Since the $E_{50\%}(T_{50\%})$ and $E_{100\%}(T_{100\%})$ follow the upper and the lower pseudogap energies (temperatures), respectively, we can clearly see from Fig. 2 that $E_{50\%}(T_{50\%})$ and $E_{100\%}(T_{100\%})$ also disappear with $T_c (\Delta_c)$ at $p_u \sim 1.3$. The disappearances of $E_{50\%}(T_{50\%})$, $E_{100\%}(T_{100\%})$ and $\sigma(T < T_c)$ at the doping level when superconductivity ceases to exist, coupled to the monumental evidences that superconductivity coexists with the pseudogap state again suggests that the lower pseudogap state, the state that the CuO$_2$ plane is completely covered by INSEG, is a necessary condition for high-$T_c$ in cuprates.

In Fig. 8(b), we plot the $\sigma(T > T_c)$ as a function of $P_{pl}$. Although there are some scattering, $\sigma(T > T_c)$ tends to increase with doping. The trend in the doping dependence of $\sigma(T > T_c)$ is opposite to that of $\sigma(T < T_c)$. Although we do not know the scaling rule between $\sigma$ and $\Delta P_{pl}$, the trend of $\sigma(T > T_c)$ is similar to that of the intrinsic $\Delta P_{pl}$ in the normal state observed by NQR. Accordingly, $\sigma(T > T_c)$ may be corresponding to the intrinsic electronic inhomogeneity. $\sigma(T < T_c)$ was observed at below 60 K, while $\sigma(T > T_c)$ was observed also at above 80 K. However, the $\sigma(T < T_c)$ is comparable to $\sigma(T > T_c)$, although the extrinsic $\Delta P_{pl}$ is about five times larger than the intrinsic $\Delta P_{pl}$. Therefore, the observed $\sigma(T < T_c)$ is not due to the extrinsic electronic inhomogeneity. When the intrinsic electronic inhomogeneities in the superconducting state disappear, the high-$T_c$ also disappears.

**FIG. 8.** (Color online) Doping dependence of the standard deviation ($\sigma$) for the Gaussian fitting with the $T_c$-curve for the purely oxygen-doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. On the left, (a) $\sigma$ in the superconducting state versus $P_{pl}$. On both right, $T_c/T_c^{max}$ versus $P_{pl}$. The symbols used in Fig. 8 are the same as that used in the Fig. 6. The plotted data in (a) are the data point observed below $T_c$ in each group and the corresponding temperatures are shown in the parenthesis. The plotted data in (a) are the data point observed above $T_c$. For the energy-scale on the left, the green line is a guide for the eyes. For the temperature-scale on the right, the red $T_c$-curve is the universal half-dome shaped $T_c$-curve from the UEPD in Ref. 4.

**IV. SUMMARY**

The topographic coverage interpretation of the pseudogaps provides a microscopic inhomogeneous electronic picture for the origin of the pseudogap and superconductivity. Based on this picture, the pseudogaps, an observable due to the averaged response of the topographic coverage at 50% or 100% in the gap map detected by a specific experimental probe, loses its conventional meaning of a “gap”. It is in this context that the gap is “pseudo”, and accordingly, all properties measured on HTCS should be addressed with the characteristic length-scale and energy-scale of the experimental probes, and the underlying INSEG state in mind. Indeed, the photon-energy-dependence of the angle-resolved photoemission spectroscopy (ARPES) spectra were only recently observed when laser-based ARPES had achieved a unprecedented high-resolution, indicating that the probe energy should be as low as possible in addressing the low energy quasiparticle states. Furthermore, the NMR and NQR analysis show that the INSEG state observed in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ is extended...
to the other HTCS. While our results link the specific INSEG coverage of CuO$_2$ planes to the two pseudogaps, the origin of and how the high-$T_c$ superconductivity emerges from such a robust INSEG state remain to be a challenging problem of the mechanism for HTCS.

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