Reflection of sound pulses from the boundary of the bubble medium

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Abstract. The reflection of sound and radiation of a bubble medium under excitation by a short sound wave are studied for different values of the volume concentration of bubbles. It is shown that at the beginning of the interaction of the wave with the medium, reflection occurs as from a structureless soft boundary, which at the end of the interaction is replaced by radiation from an excited medium. The dependences of the reflection coefficient are obtained and the spectral characteristics of the radiation of the bubble medium are determined at the initial stage of the process and at large times when the radiation is steady-state. It is shown that the emission spectrum of the layer is continuous with a maximum in the frequency range of volume pulsations of free bubbles.

1. Introduction

Nonlinear processes in the interaction of intense sound with bubbles in water give additional information about the characteristics of the medium in acoustic sonication in the ocean [1]. In various processes where bubbles are an important component of the flow, intense sound is used because of strong absorption to determine the volume concentration of bubbles and their spatial distribution [2]. In bubble media, nonlinear effects are much stronger than in media where the oscillations are dipole in nature and this allows obtaining additional information about the medium. An interesting effect was discovered in [3] when studying the absorption of sound by a bubble layer with a thickness of one bubble. When irradiated with ultrasound in the low-frequency range, the recorded shift (increase) in the resonance frequency to the high-frequency region was 2.7 times relative to the Minnaert frequency. It was shown in [4] that a bubble layer in water near a free surface is the resonator due to the limited spatial size that emits a sound with a line spectrum after excitation by a pulsed sound wave.

The aim of the work is to study the features of reflection and radiation of a bubble medium when it is excited by powerful sound pulses of different intensities for various values of the volume concentration of bubbles.

2. Physical statement of the problem

The space is filled with water and divided into two areas. In one region, water without bubbles and, in the second region, gas bubbles of the same size are uniformly distributed over the medium. The boundary between the areas is flat. A sound wave is incident on this boundary in the form of a short signal from the side of pure water. The wave is partially reflected from the interface and partially penetrates into the bubble medium, and the medium begins to emit sound.
The subject of the study is the reflected sound wave and the subsequent radiation of sound by the bubble medium for different intensities of the exciting pulse and different values of the volume concentration of bubbles in the liquid.

3. Nonlinear wave system of equations

The bubble layer radiation characteristics were studied in a one-dimensional statement using a wave model [5]. The wave system of equations is as follows:

\begin{equation}
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = - \frac{1}{c^2} \frac{\partial}{\partial t} \left( p \frac{\partial}{\partial t} \ln(1-\alpha) \right)
\end{equation}

\begin{equation}
R_k \frac{d^2 R_k}{dt^2} + \frac{3}{2} \left( \frac{dR_k}{dt} \right)^2 + \frac{4\mu}{\rho_0 R_k} \frac{dR_k}{dt} + \frac{2\sigma}{\rho_0 R_k} = \frac{1}{\rho_0} \left[ P_0 + \frac{2\sigma}{R_k} \right] \left( \frac{R_k}{R_0} \right)^{3\gamma} - \frac{P_0}{\rho_0} - p(x_k,t)
\end{equation}

\begin{equation}
\alpha(x,t) = \sum_k v_k(t) \cdot \delta(x-x_k(t))
\end{equation}

\begin{equation}
v_k(t) = \frac{4}{3} \pi R_k^3(t)
\end{equation}

\( k = 1, \ldots, N \) defines the ordinal number of the bubble in the study area; \((x-x_k)\) characterizes the position of the \(k\)th bubble in space; \(P(x,t)\) is pressure in wave; \(P_0\) is initial pressure in the medium; \(c\) is speed of sound in water; \(\alpha\) is volume concentration of bubbles; \(R_k\) is radius of the \(k\)th bubble; \(v_k\) is volume of \(k\)th bubble; \(\rho\) is density of liquid; \(\sigma\) is surface tension of liquid; \(\mu\) is viscosity of liquid; \(t\) is time; and \(x\) is spatial coordinate. Gas in bubbles follows the adiabatic law, and pressure and density of water are related by expression \(P(x,t)= (x, t) c^2\).

For numerical solutions, system (1) - (4) was reduced to a dimensionless form using the expressions

\[ \delta R_k = R_k / R_0, \quad \delta P = P / P_0, \quad \delta t = t / (R_0 \sqrt{(\rho_0 / P_0)}), \quad \delta x = x / (cR_0 \sqrt{(\rho_0 / P_0)}) \]

The amplitude of the sound pulse \(P_w\) varied in the range from 100 Pa to 0.5 MPa. The duration of the sound pulse \(\tau\) in all calculations was taken equal to 60·10^-6 s. The initial pressure in water \(P_0=0.1\) MPa. The radius of the bubbles \(R_0=0.25\)·10^-3 m. The volume concentration of bubbles \(\alpha\) varied in the range from 10^-2 to 10^-1. The air in the bubble followed the adiabatic law with an exponent \(\gamma=1.4\). The density of water was \(\rho=1000\) kg/m^3. The speed of sound in water \(c=1500\) m/s. The width of the calculation area was 2 m. The width of the bubble region \(h=1\) m. To exclude interference from reflections at the boundaries of the computational domain, boundary conditions were applied that excluded reflection.

Boundary conditions excluding reflection were applied to model the boundless environment at the boundaries of the calculated area.

An algorithm for numerically solving the system of equations (1) - (4) and checking the solutions for compliance with experimental data of various authors is given in [5].

4. Discussion of the results

It is convenient to demonstrate the mechanism of nonlinear interaction of sound pulses with a bubble medium using large-amplitude broadband sound pulses. It is shown in [4] that the bubble layer near the soft boundary is a complex resonator. The soft boundary is an absolutely reflective surface, and the boundary between the clear liquid and the bubble layer is partially transparent. Under the influence of a sound pulse, a dynamic structure of pulsating bubbles is formed inside the layer, emitting a complex periodic sound wave through the interface into a clean liquid. The spectral characteristics of the
radiation of the bubble layer depend on the width of the layer and the collective pulsations of the bubbles inside the layer. The emission spectrum consists of lines.

In this paper, we consider a semi-infinite bubble medium. The studied pressure range of the incident wave lies in the range of 100 Pa - 0.5 MPa. Figure 1 shows the solutions of the system of equations (1) - (4) in a pure liquid at a distance of 0.1 m from the boundary of the bubble medium. At this point, the reflected wave is represented by a function of time. The exciting sound wave has the spatial size of 1.8·10^-3 m. Curve (a) represents the excitation wave δPw(t), and curve (b) represents the wave arriving at the same point after the interaction of the excitation wave with the bubble medium. The development of the process is considered in the time interval equal to the duration of the excitation wave. A bubble medium is a medium with a dynamic microstructure. Under the influence of a sound wave, the bubbles begin to compress with acceleration and begin to radiate sound.

![Figure 1.](attachment:image1.png)

**Figure 1.** Exciting and reflected waves in a pure liquid.

Figure 2 shows the dynamic characteristics of the bubble in the layer at the boundary of the bubble medium.

![Figure 2.](attachment:image2.png)

**Figure 2.** The dynamic characteristics of the bubble in the layer at the boundary of the bubble medium.

Due to the inertia of the attached mass of bubbles, their reaction lags behind the force of action of the sound wave, and reflection for some time occurs as from a uniform soft border that does not have a dynamic microstructure. This reflection mode lasts until the moment indicated on the graph as δt1. In the interval between δt1 and δt2, the wave profile is a superposition of the exciting wave and the radiation of the bubble medium. After the time moment δt2, the radiation amplitude of the bubble medium exceeds the pressure in the incident wave. Therefore, the moment δt2 is considered as the end of the process of reflection of the exciting pulse from the bubble medium. In figure 2, curve (b) represents the change in radius δR(t) in the layer at the interface between pure liquid and the bubble medium in the field of the exciting pulse δP(t) (a). Curves (c) and (d) describe the velocity of the bubble boundary δv(t) and the acceleration of the bubble boundary δa(t). The line denoting the moment δt1 is taken as the end of the wave reflection from the bubble medium being a medium with no structure and the beginning of the phase when the radiation begins to dominate.

![Figure 3.](attachment:image3.png)

**Figure 3.** Energy spectra of the exciting and reflected waves.
Figure 3 presents the spectra of the exciting wave and the reflected wave. The expansion of the spectrum of the reflected wave is associated with a reduction in the duration of the reflected wave due to the emission of the layer in the accepted formulation. The total energy of the exciting wave is $E = 2.89 \, J$, and the energy of the reflected wave is $E_{\text{ref}} = 0.667 \, J$. The reflected wave contains 23.1% of the energy of the exciting wave.

The analysis of the reflection of a sound pulse from a bubble medium allows one to calculate the reflection coefficient of sound pulses $r$ as the ratio $E_{\text{ref}} / E$, where $E_{\text{ref}}$ is the energy of the reflected wave and $E$ is the energy of the excitation wave. Energy of the reflected pulse is calculated in the time interval from the beginning of the wave to the moment $\delta t_2$.

Figure 4 shows the dependence of the reflection coefficient $r$ on the volumetric concentration of bubbles for three values of the amplitude of the incident wave 100 Pa, 0.1 MPa, 0.5 MPa. The greatest change of reflection coefficient $r$ due to volume concentration of bubbles lies in the range of $\alpha$ from $10^{-7}$ to $5 \cdot 10^{-3}$. In the range of $\alpha$ values from $10^{-3}$ to $10^{-1}$, the dependence is much weaker and the curve has the character of saturation.

Figure 5 shows the dependence of the reflection coefficient $r$ on the amplitude of the excitation wave at a fixed value of the volume concentration of bubbles.

The dependence of the reflection coefficient $r$ on the excitation wave amplitude is consistent with the experiments published in [4,5].

Figures 6 and 7 show the dimensionless emission spectra of the bubble medium excited by sound pressure pulses with amplitudes of 100 Pa and 0.5 MPa, respectively. The base frequency for the bubble medium is the frequency of free bubble pulsations. This frequency is called Minnaert frequency [7], which is obtained in solution (1) – (4) at $\alpha = 0$ and is $13.53 \, kHz$.

For excitation by a wave with an amplitude of 100 Pa, the main radiation frequency is lower than the Minnaert frequency and is equal to $12.96 \, kHz$. 

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**Figure 4.** Relationship of reflection coefficient to volume concentration of bubbles.  
**Figure 5.** Dependence of reflection coefficient on amplitude.
Figure 6. Radiation spectrum. Weak wave.

Figure 7. Radiation spectrum. Strong wave.

At a small amplitude of the exciting wave, the pulsations of bubbles are weakly nonlinear and a constant component typical of nonlinear processes appears in the radiation spectrum, but harmonics are not distinguished. Frequencies $\delta f_1$ and $\delta f_2$ denote the boundaries of the absorption band of the bubble medium. The frequency absorption band in dimensional form lies between a frequency of 12.5 kHz and a frequency of 53.8 kHz. For the amplitude of the exciting wave of 0.5 MPa, the general character of the structure of the spectrum is preserved, but higher harmonics are clearly manifested in it and subharmonics appear. The constant component sharply increases.

The frequency of the lower absorption limit has shifted to the high frequency region relative to the Minnaert frequency and is equal to 13.85 kHz. The upper frequency of the absorption band boundary has also shifted slightly to the high frequency region and is equal to 54 kHz. The decrease in the radiation frequency is due to the interaction of the bubbles with each other through a self-consistent sound field, which is formed by the collective dynamics of the bubbles and is described by the system of equations (1) - (4). The frequency shift of the main radiation line to the high-frequency region at high energies is associated with an increase in the stiffness of the bubble medium and is also due to the interaction of the bubbles. These results qualitatively agree with the experimental results of [3], in which the shift of the resonant frequency was significantly higher than the Minnaert frequency.

Conclusions
Studies have shown that a bubble medium with uniformly distributed identical bubbles transforms the reflected waves by superimposing on the reflected wave the intrinsic radiation of the bubbles excited by the wave within the time that the pulses travel through the boundary. The radiation of a bubble medium, after excitation by a broadband wave in the form of a single pulse, is in the nature of a complex periodic wave. The sound reflection coefficient strongly depends on the volume concentration of bubbles and wave energy. The curves describing the dependence of the reflection coefficient have the character of saturation. The emission spectrum has a maximum value at frequencies close to the natural frequency of free bubble pulsations. When excited by a broadband pulse for low energies, the spectrum is shifted to the low-frequency domain, and at high wave energies, the base frequency is shifted to the high-frequency domain, and subharmonics appear in the emission spectrum.

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