Indicating interdisciplinarity:
A multidimensional framework to characterize Interdisciplinary Knowledge Flow (IKF)

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UNDER REVIEW
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ABSTRACT

This study contributes to the recent discussions on indicating interdisciplinarity, i.e., going beyond catch-all metrics of interdisciplinarity. We propose a multi-dimensional and contextual framework to improve the granularity and usability of the existing methodology for quantifying the interdisciplinary knowledge flow (IKF) in which scientific disciplines import and export knowledge from/to other disciplines. To characterize the knowledge exchange between disciplines, we recognize three dimensions under this framework, namely, broadness, intensity, and heterogeneity. We show that each dimension covers a different aspect of IKF, especially between disciplines with the largest volume of IKF, and can assist in uncovering different types of interdisciplinarity. We apply this framework in two use cases, one at the level of disciplines and one at the level of journals, to show how it can offer a more holistic and detailed viewpoint on the interdisciplinarity of scientific entities than unidimensional and context-unaware indicators. We further compare our proposed framework, an indicating process, with established indicators and discuss how such information tools on interdisciplinarity can assist science policy practices such as performance-based research funding systems and panel-based peer review processes.

1. INTRODUCTION

How scientific disciplines import and export knowledge from/to others defines their role in the scientific community and informs their values and actions on interdisciplinarity. Understanding interdisciplinarity from the perspective of knowledge flow (linkage) between disciplines is steeped in the history of quantitative science studies. In 1978, Garfield, Malin, and Small (1978) pointed out that interdisciplinary activity can be studied using “linkages between specialties of diverse subject matter.” Their main focus back then was to discover specialties of science and linkages among them to arrive at a “map of science”. The study of interdisciplinarity of a certain discipline (or other entities, e.g. researchers or institutions) has bloomed since the 1980s, after which many empirical indicators of interdisciplinarity have been proposed and employed.

The trajectory of quantifying interdisciplinarity can be divided into two phases. Indicators proposed in the first phase (roughly before 2000) were based on cross-disciplinary citations (knowledge flow) to infer the exchange of knowledge/resources. According to Rafols & Meyer (2010), the percentage of citations outside categories (PCOC), proposed by Porter & Chubin (1985), was the “most common indicator of interdisciplinarity”. The “category” here
is normally operationalized as a set of journals to represent a discipline. For instance, van Leeuwen & Tijssen (2000) investigate cross-disciplinary citation links between 119 disciplines using PCOC and identified disciplines such as meteorology and atmospheric sciences as being more interdisciplinary. Another important indicator in this phase is the percentage of citations towards category (PCTC), which has a longer applied history and has attracted a broader audience in various fields. As early as 1952, Broadus (1952) analyzed the percentage of articles from different subjects (disciplines) cited in the American Sociological Review in 1950. Since then, this indicator is employed by researchers from various disciplines and topics, such as linguistics (Rappaport, 1971), marketing (Goldman, 1979), consumer research (Leong, 1989), agricultural education (Radhakrishna, 1992), and social science in general (Rigney & Barnes, 1980). It is typically referred to as citation analysis or citation study by general researchers, whose primary goal is to unveil the knowledge constitution of a certain discipline or journal, instead of quantifying the level of interdisciplinarity directly. Although simple and intuitive, the two indicators in the first phase are versatile in delivering contextual understandings of interdisciplinarity and are still utilized (directly or in modified form) in recent studies (Angrist et al., 2020; Huang et al., 2022; Truc et al., 2020).

The second phase of quantifying interdisciplinarity is marked by the introduction of the diversity framework (Stirling, 2007). Since 2000, some researchers have started to operationalize interdisciplinarity as cognitive diversity, or components of diversity, for instance, diversity of relationships with other disciplines using the Pratt index (Morillo et al., 2003). A milestone article in this phase is published by Rafols & Meyer (2010), in which a framework to quantify interdisciplinarity using diversity and its three components are explicitly proposed. From then on, a significant proportion of research on interdisciplinarity adopt this framework and different variants of indicators are proposed (Leydesdorff et al., 2019; Rousseau, 2018; Zhang et al., 2016). The main objective of research in this phase is to find an ideal aggregated indicator to gauge interdisciplinarity for various purposes.

However, recently some researchers voiced concerns about the current interdisciplinarity measures through both empirical analysis and theoretical discussions. Wang & Schneider (2020) survey 16 existing interdisciplinarity measures in an empirical study and concluded that the current measurements are “both confusing and unsatisfying” due to a lack of consistency in results. They encourage “something different” instead of “more of the same” in methodology to measure “the multidimensional and complex construct of interdisciplinarity” (Wang & Schneider, 2020, p. 239). In a similar spirit, Rafols (2020b) discusses the failed efforts of universal indicators and suggests the notion of “indicating interdisciplinarity”, which is “a contextualized process … indicating where and how interdisciplinarity develops as a process”. Marres and de Rijke (2020) describe “indicating” as “participatory, abductive, interactive, and informed by design” and propose digital mapping as a promising tool to indicate interdisciplinarity (p. 1042).

At the center of these debates is the complexity of understanding interdisciplinarity and the difficulty in operationalizing the concept. We agree with the recent concerns that the multidimensionality of interdisciplinarity cannot be entirely represented by a binary label
(interdisciplinary or disciplinary) or a single value in a continuous range (more or less interdisciplinary). In this article, we would like to contribute to the literature a possible methodology of “indicating interdisciplinarity” by characterizing interdisciplinary knowledge flow, following the theoretical objective of the first phase of quantifying interdisciplinarity.

The volume of knowledge flow (citations) from one discipline to another is determined by multiple factors. For instance, the size of a discipline or the span of research topics it hosts will significantly affect its propensity of citing or being cited by others. Comparing Psychology and Law, the former is more likely to be cited by a random article since articles in Psychology are in larger volumes and address a wider span of topics, from biological topics (e.g. neuropsychology) to social topics (e.g. social psychology). Also, the citation volume from one entity to another is heavily influenced by the research purpose of the citing entity and the characteristics of the cited entity. For instance, on the side of citing behaviors, a type of research called systematic review or meta-analysis in medical science often cites a cluster of clinical studies on the same topic and integrates their empirical results to draw comprehensive evidence for drug development. Research devised to improve certain machine learning algorithms would also more likely cite a cluster of prior studies on the same topic to prove their superiority in prediction power, yielding many citations to a single discipline; on the other hand, research introducing certain machine learning algorithms to a new field may cite a diverse collection of studies applying this algorithm to different fields to show its universality and support their decision on using it, yielding a lower citation volume to a broader set of disciplines. On the side of cited entities, citation volume can also be inflated by the intrinsic characteristics of the cited entity. For instance, some theory-oriented subfields within social science are believed to suffer from an “incoherence problem” caused by numerous incompatible theories and a lack of unified theoretical frameworks (Clancy, 2021; Watts, 2017). Other research may need to cite a wide spectrum of different theories under the same topic, which leads to a larger citation volume to a single discipline than citing fields with more unified theories and applications. Last but not least, the cognitive distance or similarity between the knowledge base of two disciplines is essential to the knowledge flow between them. Affinity in purpose, knowledge, methods, and epistemology may induce greater relevance, trust, and familiarity, which in turn increases the likelihood of citation.

The objective of this study is to propose a multidimensional framework to distill different dimensions of interdisciplinary knowledge flow, which can further indicate the scale and directionality of interdisciplinarity of various entities.

2. THE FRAMEWORK

2.1 Introducing the framework

The calculation of this framework is conducted within a citation matrix $M (n \times n)$, with $n$ denoting the total number of all publications in a given period. For any two publications $i$ and $j$, $M_{ij} = 1$ if $i$ cites $j$, and 0 otherwise. An entity $X$ (e.g. field, journal, or set of publication output of institutions or researchers) is represented by the set of publications classified under it: $X = \{x_1, ..., x_{|X|}\}$, with each element of $X$ an integer between 1 and $n$. We use $|X|$ to
denote the number of publications in $X$. The proposed framework aims to describe the citation relationship between two entities of interest. The citing and cited entities can be homogenous, i.e., both are the same type of entities such as fields, or they can be heterogeneous, e.g., the citing entities are journals while the cited entities are disciplines. Although the framework can be used for various purposes, in this study we focus on studying the interdisciplinary knowledge flow (IKF), in which case the cited entities are disciplines.

For every two entities $X$ and $Y$, we recognize three dimensions of knowledge flow from $X$ to $Y$, namely, broadness, intensity, and heterogeneity. We define each element of the proposed IKF framework in the following section.

The broadness ($B$) of IKF of $X$ citing $Y$ captures the percentage of papers in entity $X$ that cite entity $Y$. It is defined as:

$$B(X,Y) = \frac{\sum_{i\in X} \delta_i}{|X|}$$

where $\delta_i = \begin{cases} 1, & \text{if } \sum_{j\in Y} M_{ij} > 0 \\ 0, & \text{if } \sum_{j\in Y} M_{ij} = 0 \end{cases}$.

The intensity dimension ($I$) is a modified version of PCTC, with both quantifying the volume of citation links between entities. The calculations of the two follow the same logic with one additional restraint added for intensity, as shown in Equations 2 and 3. PCTC quantifies the percentage of citations from the citing entity $X$ to the cited entity $Y$ in the entire outward citations made by $X$, regardless of who the recipient entities are in the denominator. The intensity, on the other hand, controls the scope of the denominator by introducing $\delta_i$, which limits the inclusion of outward citations from $X$ only for those who cite $Y$.

$$I(X,Y) = \frac{\sum_{i\in X, j\in Y} M_{ij}}{\sum_{i\in X, j = 1, \ldots, n} (M_{ij} \delta_i)}$$

$$PCTC(X,Y) = \frac{\sum_{i\in X, j\in Y} M_{ij}}{\sum_{i\in X, j = 1, \ldots, n} M_{ij}}$$

The heterogeneity ($H$) dimension of IKF categorizes the difference in knowledge base between $X$ and $Y$, which compares the sets of publications cited by $X$ with those cited by $Y$. The heterogeneity of $X$ citing $Y$ is the percentage of unique co-cited publications by the two entities in all unique publications cited by $X$. It is de facto a normalized variant of bibliographic coupling (Kessler, 1963) between two disciplines, having the numerator as the coupling strength normalized by the size of knowledge base of the citing discipline. It is defined as:

$$H(X,Y) = 1 - \frac{|\{j \mid M_{ij} > 0, \ i \in X\} \cap \{j \mid M_{ij} > 0, \ i \in Y\}|}{|\{j \mid M_{ij} > 0, \ i \in X\}|}$$

First, we compare PCTC with the broadness and intensity dimensions. We believe PCTC is a function of both dimensions without considering them separately, which de facto carries different connotations and implications. For instance, in Figure 1, the citation relationship
between disciplines A and C is shown using a citation matrix. The vertical rows denote citing publications from discipline A (A_1, A_2, A_3, A_4, A_5) and C (C_1, C_2, C_3, C_4, C_5), whereas the horizontal columns represent the cited publication from discipline B (B_1, B_2, ..., B_{10}). The last column named “Total” records the total number of outward citations by the corresponding citing publication. If we quantify the two selected relationships, we find that PCTC(A citing B) is equal to PCTC(C citing B), therefore arriving at the same measured volume of knowledge flow for the two pairs. However, under the inspection of our proposed framework, both the broadness and intensity of IKF are assigned different values. The broadness of A citing B is larger than that of C citing B since four out of five papers in total from A cite at least one paper from B, compared to one out of five in the case of C citing B. The intensity of A citing B, on the other hand, is less than that of C citing B since only C_1 cites discipline B, but more intensely (with five citations in one paper).

|                | A citing B | C citing B |
|----------------|------------|------------|
| PCTC           | 5          | 5          |
| Broadness      | 4          | 1          |
| Intensity      | 5          | 5          |

Figure 1. Demonstration of the framework (broadness and intensity)

To sum up, we first employ broadness to determine the height of a sub-matrix in which only papers having citations from the focal discipline to the target discipline are included. For the recognized sub-matrix, we then quantify the intensity of citations from the focal discipline to the target discipline in the focal discipline’s complete citation portfolio. Besides empirically representing two distinct axes of the citation matrix between two disciplines, the two dimensions also capture different connotations in discipline-wise relationships. For instance, consider the following two scenarios:

- Scenario 1: \( B(X, Y) = 0.9, \ I(X, Y) = 0.1 \)
- Scenario 2: \( B(X, Y) = 0.1, \ I(X, Y) = 0.9 \)

From the perspective of X, Y plays completely different roles in the two scenarios. In scenario 1, Y contributes to X as a universal ingredient that 90% of publications in X include Y in the knowledge base. But when X cites Y, X only includes 10% of knowledge from Y, which shows that the knowledge borrowing is selective, concentrating on a small number of influential publications from Y. Scenario 1 may correspond to, e.g., the relation between the quantitative branch of social sciences and statistics. The latter may be needed in the majority of quantitative papers, but only a few references are typically sufficient enough to introduce the methodology. In scenario 2, Y only influences a small cluster of X yet rather intensively. This may correspond to how Philosophy influences AI research: only a small local cluster in
AI, namely AI ethics, cites Philosophy but rather intensively. To use cooking metaphors to classify the role of Y to X, in scenario 1 Y could be *salt*, which is necessary for most dishes yet in small volumes; in scenario 2, Y could be *turmeric*, which is not pervasive in cooking but used intensively when making curry-based cuisine.

The inclusion of the third dimension, namely *heterogeneity*, is to provide a foundation for our framework by quantifying the cognitive distance between discipline pairs. Therefore, the objective of heterogeneity is not to directly quantify the IKF between disciplines but to transform prior knowledge regarding the cognitive distance between disciplines into concrete measurements. It can be utilized as a “base map” when validating findings or deciphering universal patterns of IKF.

Through mathematical deduction and empirical tests, we found that the three dimensions enjoy the property of monotonicity and two of them enjoy size independence according to our definition, except for heterogeneity. The details are shown in Appendix B.

The framework has several advantages. First, it is *contextual*. It roots deeply in the context of pairwise relationships of disciplines and therefore can deliver contextual understandings of IKF. Second, it is *multidimensional*. It investigates three dimensions of IKF, instead of only one aggregated indicator (PCTC) in previous studies, thereby enabling a more holistic and refined characterization of interdisciplinarity. Last but not least, the framework is *asymmetrical* (e.g. A citing B vs. B citing A). Our analysis relies heavily on the directionality of citation flow. One can analyze both the citation side of IKF and the citing side to unveil patterns in knowledge diffusion as well as knowledge integration/ adoption (Liu et al., 2012).

2.2 Notes on the theoretical foundation of the framework

Having introduced the practical design of the framework, we would like to further connect it with several theoretical elements in the current understanding of interdisciplinarity: How does the framework relate to interdisciplinarity? How does the framework connect to the current scientometric indicators of interdisciplinarity? Why do we categorize our framework as “indicating interdisciplinarity”?

2.2.1 Relations with interdisciplinarity

Interdisciplinarity is often framed as a process of integrating different bodies of knowledge (Marres & de Rijcke, 2020; Porter & Rafols, 2009; Wagner et al., 2011), which foregrounds the notion of “knowledge integration” as the key interpretation and conceptualization. Most indicators are therefore invented to capture the degree of integration, in order to understand how frequently a certain entity engages with extramural knowledge clusters. This quantitative methodology studying the engagements between the focal entity with other fields is found acceptable and endorsed by researchers from Science and Technology Studies (STS) as they believed such a “relational approach” is adaptive to interpretative traditions (Marres & de Rijcke, 2020, p. 1045). Our goal in this study is to provide the contexts of knowledge integration in the knowledge base of entities and, by proposing three dimensions, attempt to indicate the scale and directionality of interdisciplinarity.
For a certain entity of interest, the framework yields three arrays of values \((B_n, I_n, H_n)\) representing each of the three dimensions, where the array size \(n\) is the number of potential extramural fields from which the entity under scrutiny can import knowledge. Based on the conceptualization of interdisciplinarity discussed above, entities with greater interdisciplinarity are therefore expected to associate with greater broadness (widely influenced by other fields), greater intensity (intensively influenced by other fields), and lower heterogeneity (greater overlap in the knowledge base with other fields) since they engage more actively in knowledge integration.

We would also like to emphasize that our framework cannot and should not provide one-off judgments on interdisciplinarity without clearly defining the context. We suggest a comparative understanding of interdisciplinarity in the sense that one could deduce the relative scale, type, and orientation of interdisciplinarity of an entity with comparisons with its comparable pairs, yet cannot be inferred on its own. Interdisciplinarity is a dynamic process that varies among disciplines and evolves over time. One cannot be classified as interdisciplinary without benchmarks to its disciplinary and temporal norm. Therefore, a second operationalization of this framework is proposed as the difference in knowledge integration between a certain entity and a comparative benchmark \((B_n - B'_n, I_n - I'_n, H_n - H'_n)\). If the analyzed entity possesses greater knowledge integration in comparison to the chosen benchmark for a certain dimension, we can say it associates with greater interdisciplinarity in relative terms. An example of such a benchmark comparison is provided in section 5.2.2.

2.2.2 Relations with current scientometric indicators of interdisciplinarity

The framework shares some ingredients and conceptualization with the established scientometrics indicators, including both knowledge-flow-based (Porter & Chubin, 1985) and diversity-based indicators (Rafols & Meyer, 2010). The calculation for both types of indicators is centered around the proportion/volume of citations from the entity of interest to other fields, which is equivalent to PCTC and \(p_i\) in diversity-based IDR indicators (e.g. Rao-Stirling diversity in equation 5). In our framework, we further decompose \(p_i\) to two dimensions, namely broadness and intensity, to provide a detailed lens for capturing interdisciplinary knowledge flow. The second ingredient that some diversity-based IDR indicators employed is the dissimilarity between disciplines, i.e. disparity \((d_{ij}\) in equation 5). We also include this dimension since it offers insights on the cognitive proximity between disciplines. Previous studies have employed several measurements for disparity such as bibliographic coupling, co-citation, and cross-citation. We try to indicate disparity using the proposed heterogeneity dimension, which is also a form of bibliographic coupling. The differences between ours and existing ones are twofold: we explicitly designed our framework as asymmetrical while previous disparity indicators yield the same results for A citing B and B citing A; the second difference is that we purposely design this dimension in a simple, intuitive and empirically meaningful form, which is the degree of overlap in the knowledge base.
\[ RS_{iDR} = \sum_{i,j(i\neq j)} d_{ij}(p_i p_j) \]  

Our framework contains all the elements in previous indicators on IDR. The framework directly modifies upon the knowledge-flow-based indicators on interdisciplinarity. For diversity-based interdisciplinarity, three elements of interdisciplinarity, namely variety, balance, and disparity, are also explicitly or implicitly indicated in our framework with improvements. 1) Variety is equivalent to the number of non-zero elements in the array of broadness and intensity, which is the number of disciplines cited by the evaluated entity. Our approach offers more information on variety in the sense that the volume of citations are also depicted. In addition, contrary to variety, which increases with even only one citation to a new extramural field, our approach is less sensitive to minor changes. 2) Balance is implicitly indicated by the distribution of broadness and intensity: a skewed distribution over fields corresponds to a lower balance and a uniform distribution shows a higher balance. 3) Disparity, or dissimilarity between disciplines, is explicitly included in our framework as heterogeneity which quantifies the difference in the knowledge base of the entity of interest and another field.

2.2.3 Relations with indicating interdisciplinarity

“Indicating interdisciplinarity” has been first proposed in the STS community, which primarily follows an interpretative tradition. Discussions on indicating interdisciplinarity are in a rather nascent status but touch on two questions: What is indicating interdisciplinarity? What is indicating in general?

In answer to the first question, Rafols (2020b) framed indicating as “indicating where and how interdisciplinarity develops as a process, given the particular understanding relevant to the specific policy goals”. We understand the core of this definition is to examine knowledge portfolios and delineate the process of knowledge integration, instead of one-off indicators without context. In addition, Rafols also stressed the importance of directionality in research and innovation and the evaluation of interdisciplinarity. He suggests going beyond values of interdisciplinarity (unidimensional indicators) to indicate “orientations of the research contents” (distributions). We categorize our framework as indicating interdisciplinarity since it also provides the contexts of diversity in the knowledge base and indicates the exact (disciplinary) directions from which the evaluated entity imports knowledge, by showing their distributions.

As for the second question, Marres and de Rijcke (2020) devote more focus to “indicating” or, in their words, the deployment of “engaging indicators”. Four elements of indicating are discussed, namely participatory, abductive, interactive, and designed. Our framework conforms with the abductive element of indicating in the sense that we sketch the contexts of interdisciplinary knowledge integration and do not only provide one-off and closed-end answers. Various stakeholders can generate interpretations with their own agenda. Participants can then infer from their perspectives on interdisciplinarity.
3 OUTLINE OF EMPIRICAL STUDY

To test the validity of this framework, we first study the relationships between the three dimensions of IKF using empirical citation data (section 4.1.1). We examine the correlations between every pair of dimensions by generating linear fits to their relationships. Discipline pairs that deviate significantly from linear fits are highlighted and discussed through case studies. Besides analysis for all discipline pairs, we look specifically into the IKF between disciplines that possess the 10% highest volume of knowledge flow, i.e. for every discipline, we study the 10% most cited disciplines separately. In a second validation analysis (section 4.1.2), we examine the relationships between the three dimensions and the volume of citations in IKF. Through these two analyses, we aim to demonstrate how the proposed framework can uncover different types of IKF and what kind of discriminative power it can contribute.

We further put the framework into practice with two use cases and illustrate how to indicate and compare the interdisciplinarity of disciplines (direct links with all disciplines) and journals (relative scales in a local discipline setting) using our proposed framework. In the use case of indicating discipline level interdisciplinarity (section 4.2.1), we provide examples for eight disciplines to illustrate how our model can detect their unique patterns of IKF. We also compare results from our framework with existing interdisciplinarity indicators such as True Diversity (Zhang et al., 2016) to show how the proposed multidimensional framework is empirically related to established indicators of interdisciplinarity. In the use case of indicating journal-wise interdisciplinarity in a local discipline setting (section 4.2.2), we select 87 journals from Library & Information Science (LIS) to constitute a sample representing this discipline at large. We then choose seven flagship journals and compare their relative interdisciplinary knowledge preferences with that of LIS in general. We also examine the correlation between a journal’s deviation in interdisciplinary knowledge from its affiliated discipline at large and its level of interdisciplinarity globally. The results illustrate how the framework enables a more granular and in-depth understanding of journal-level interdisciplinarity.

We place our focus in this study on the citing side of IKF, which can also be interpreted as knowledge borrowing.

4 DATA

The data in this study is harvested from the in-house version of Web of Science hosted at ECOOM KU Leuven. We select all citable items (e.g., article, review, and letter) published in 2015 and 2016 from all 74 disciplines recognized in the Leuven-Budapest classification scheme (Glänzel et al., 2016; Glänzel & Schubert, 2003). We investigate 2,995,186 publications and 89,956,048 references made by them. For each discipline pair, we calculate the aforementioned three dimensions, as well as the number of citations from the citing discipline to the cited discipline and the interdisciplinarity score of the citing discipline. For the use case of journals, we selected 87 journals in LIS from the Journal Citation Reports 2019 (JCR 2019) and all 8,359 articles published in these journals in 2015 and 2016.
5 RESULTS

5.1 Examining the framework

5.1.1 Relationship among the three dimensions

The relationships between the three dimensions are shown in Figure 2. The $R^2$ values of linear fits are denoted in the plots. It is clear that the intensity dimension in particular offers a disparate perspective on IKF from the other two dimensions. Its $R^2$ values with both broadness and heterogeneity are relatively small (0.214 and 0.272, respectively), which demonstrates once again the uniqueness of this dimension. As expected, a clear and relatively strong negative relationship can be found between broadness and heterogeneity, i.e. greater dissimilarity in knowledge base is associated with less broadness in impact. Nevertheless, they are not isomorphic, either theoretically or empirically. The broadness dimension describes the width of citing behavior, whereas the heterogeneity dimension quantifies the degree of difference in the knowledge base. We can also differentiate them in empirical analyses. In Figure 2b, some special data points are highlighted as case studies since they deviate significantly from the linear fit and the main cluster. The details of these cases are shown in Table 1. For the red dots in Figure 2b, the relationships between the citing and cited discipline possess higher heterogeneity and higher broadness than expected. This indicates that although the two disciplines have disparate knowledge bases, knowledge from the cited discipline is still broadly adopted by the citing discipline resulting in a broader diffusion than what one might expect. For instance, in cases 1, 2, and 3 (Figure 2b), Applied Mathematics and Computer Science/Information Technology, two methodology-oriented disciplines, constitute significant knowledge exporters with a broader impact on Genetics & Developmental Biology and Pure & Applied Ecology than average scenarios, despite the substantial cognitive distance between them. Two cited disciplines with multidisciplinary nature in cases 4 and 5 also have broader yet cognitively different impacts on the citing disciplines. Even though their knowledge base is different from that of citing disciplines due to the diversity and span of topics, they also contribute a valuable and broader knowledge source from which citing disciplines can learn. On the other hand, cases 6 and 7 exhibits lower heterogeneity and lower broadness than expected. They represent discipline pairs that are built on similar knowledge constructs yet cite each other in a disproportional broadness. In case 6, Architecture shares 59.6% of references with General & Traditional Engineering, however, only 40.4% of its publications cite this discipline. In case 7, the knowledge base of Pure Mathematics is only 32.8% different from that of Applied Mathematics, but the broadness of its citing behavior vis-à-vis Applied Mathematics is lower than expected. Among other factors, such deviations can partly relate to imbalanced discipline sizes (see Appendix Figure. A1) and the epistemic characteristics of the disciplines involved (e.g., fundamental versus applied research).

We also observe some general typology of interdisciplinary citations that illustrates the characteristic functions of disciplines. For instance, in Figure 2a, Psychology & Behavioral Sciences (red dots) is cited by two disciplines with higher intensity than expected, which
means it contributes intense knowledge to a subgroup of the citing discipline, such as studies relating to Forensic Psychology in Law or Neuroaesthetics in Arts & Design. On the other hand, Applied Physics contributes a broader yet less intense impact on knowledge flow to Physical Chemistry and Polymer Science, which shows that its knowledge serves as a common tool that does not require high maintenance, i.e. complex theoretical underpinning or heavy argument.

We further look into disciplines’ pairwise relationships with the highest volume of knowledge flow. For each discipline, based on the number of citations from them to the other disciplines, we limit the analysis to only the relationships between the focal disciplines and their 10% most cited counterparts. By doing this, we aim to check if our framework will be able to unveil different patterns of IKF in distinct citation classes. The R² value is reduced in all three plots (Figure 2. d-f), especially for intensity and broadness with R² close to zero. The broadness and intensity dimensions become orthogonal to each other in the highest citation group. This shows that, for the most prominent interdisciplinary relationships, discipline pairs with high broadness are not necessarily characterized by high intensity, which validates the discriminative power of our model.

![Figure 2. Relationship among the three dimensions. (a-c) all discipline pairs; (d-f) discipline pairs with 10% highest citation percentile. The numbered data points in panel (b) are discussed in Table 1. Linear fits for each subplot and their R² are visualized and reported.](image)

**Table 1. Case studies for selected discipline pairs from Figure 2**

**Figure (a)**
• Lower broadness & Higher intensity

| No. | Citing discipline          | Cited discipline                  |
|-----|----------------------------|----------------------------------|
| 1   | Arts & Design              | Psychology & Behavioral Sciences |
| 2   | Law                        | Psychology & Behavioral Sciences |

• Higher broadness & Lower intensity

| No. | Citing discipline          | Cited discipline                  |
|-----|----------------------------|----------------------------------|
| 3   | Physical Chemistry         | Applied Physics                   |
| 4   | Polymer Science            | Applied Physics                   |

**Figure (b)**

• Higher heterogeneity & Higher broadness

| No. | Citing discipline          | Cited discipline                  |
|-----|----------------------------|----------------------------------|
| 1   | Genetics & Developmental Biology | Applied Mathematics               |
| 2   | Pure & Applied Ecology     | Applied Mathematics               |
| 3   | Genetics & Developmental Biology | Computer Science/Information Technology |
| 4   | Pure & Applied Ecology     | Multidisciplinary Biology         |
| 5   | Particle & Nuclear Physics | Multidisciplinary Physics         |

**Lower heterogeneity & Lower broadness**

| No. | Citing discipline          | Cited discipline                  |
|-----|----------------------------|----------------------------------|
| 6   | Architecture              | General & Traditional Engineering |
| 7   | Pure Mathematics           | Applied Mathematics               |

**Figure (c)**

• Higher heterogeneity & Higher intensity

| No. | Citing discipline          | Cited discipline                  |
|-----|----------------------------|----------------------------------|
| 1   | Biomaterials & Bioengineering | Dentistry                       |
| 2   | Literature                 | History & Archaeology            |

• Lower heterogeneity & Lower intensity

| No. | Citing discipline          | Cited discipline                  |
|-----|----------------------------|----------------------------------|
| 3   | Polymer Science            | Physical Chemistry               |
| 4   | Polymer Science            | Applied Physics                   |

5.1.2 Relationship between the three dimensions and the volume of citation flow

We further examine the relationships between the proposed three dimensions and the volume of citation flow (counts) for discipline pairs. The results are shown in Figure 3a, where the corresponding value for each discipline pair is displayed in scatter plots. An OLS linear fit is
conducted using the ‘regplot’ function from the seaborn Python package (Waskom, 2021), and its $R^2$ value is reported in the plots.

The broadness dimension exhibits a monotonous positive relationship with citation volume between discipline pairs, with 55.4% variance explained. The intensity dimension, however, contributes a significant amount of variations (responsible for only 16% variance) which shows that it has a less correlated relationship with citation volume. The upper outliers with high intensity possibly denote scenarios where the total volume of citations between discipline pairs is small, yet it constitutes an intense citing environment. In other words, it signals a narrow yet intense relationship between disciplines in which a small branch of publications from the citing discipline engages intensively with the knowledge from the cited discipline. When publications from this branch cite them, publications from the cited discipline account for a significant proportion in the reference which is not proportional to expectation (average percentage). The heterogeneity dimension shows a monotonous negative relationship with citation volume, which makes intuitive sense since disciplines are more likely to import knowledge from counterparts with similar knowledge bases.

**a. All citation**

![graphs](image)

**b. Top 10% citation**

![graphs](image)

Figure 3. Relationship between the three dimensions and the volume of citation flow. Linear fits for each subplot and their $R^2$ are visualized and reported. (a) relationship for all discipline pairs; (b) relationships for discipline pairs with top 10% citation volumes.

We once again examine discipline pairwise relationships in the top 10% citation group separately and arrive at similar findings as to the previous one. In the 10% citation group
(Figure 3b), all three dimensions are found to be less correlated with citation volume, as indicated by lower values in $R^2$. This shows that our framework possesses greater discriminative power among discipline relationships with the largest volume of citations, which are normally the foci in the study of interdisciplinarity.

5.2 Use cases

To demonstrate how to use our framework for quantitative science studies or research evaluation, we test the capability of our proposed framework through two use cases. The first use case utilizes this framework to investigate and demonstrate unique patterns of IKF for disciplines. The second use case aims to indicate the relative interdisciplinarity of scientific journals from Library and Information Science (LIS) in comparison to the LIS field at large and visualize the process of knowledge integration.

5.2.1 Indicating the interdisciplinarity of disciplines

Here we choose eight disciplines as examples to illustrate how the framework can be employed to unveil different patterns of IKF and interdisciplinarity. They are (1) Applied Mathematics, (2) Business, Economics, Planning, (3) Cell Biology, (4) Computer Science/Information Technology, (5) Environmental Science & Technology, (6) General & Traditional Engineering, (7) Immunology, and (8) Linguistics. The scatterplots (Figure 4) illustrate how the focal discipline (indicated in the subtitle) cites the other disciplines under our framework.

In general, all disciplines rely heavily on a few disciplines in citing behavior, with more extreme values for all three dimensions. Nevertheless, disciplinary discrepancies can be spotted. Some disciplines, for instance, Computer Science/Information Technology, Linguistics, and Cell Biology have a significantly stronger relationship with only one discipline for all three dimensions. On the other hand, disciplines like Immunology and General & Traditional Engineering have a more balanced relationship with several disciplines, shown by a more scattered distribution. Potentially, if one desires an aggregated indicator for interdisciplinarity under our framework, the degree of variation (or scattering) of data points in these plots could be a candidate solution. For instance, we calculate the sum of standard deviations of three dimensions for each citing discipline and correlate it with the median interdisciplinarity values for all publications in it, namely true diversity or TD (Zhang et al., 2016) and DIV (Leydesdorff et al., 2019). The Pearson coefficients between the sum of standard deviations and some interdisciplinarity indicators are relatively high: 0.765 for DIV and 0.691 for TD, which indicates a relatively strong positive relationship between them (see Appendix A for scatter plot). Our goal in this analysis is not to propose a new indicator of interdisciplinarity, but to see the connection between our proposed multidimensional framework and established indicators.

Another valuable insight that can be drawn from here is the distinct patterns of IKF in pairwise discipline relationships. For Business, Economics, Planning, its relationships with Psychology & Behavioral Sciences and Applied Mathematics are apparently different in broadness and intensity. Psychology & Behavioral Sciences appear to possess a more intense
impact on the focal discipline, while Applied Mathematics more broadly influences its research. Applied Mathematics has become an indispensable and pervasive component in economics research, but it does not occupy a large proportion of references. In contrast, Psychology & Behavioral Sciences only relates closely to some dedicated branches of Business, Economics, Planning (e.g. marketing) yet contributes a significant amount of knowledge. Such patterns can also be found in Applied Mathematics, Environmental Science & Technology, and Linguistics, whose relationships with two disciplines (highlighted dots) have opposite trade-offs on broadness and intensity. This corroborates with the orthogonality between broadness and intensity in the higher 10% citation group. With our multidimensional framework, one can uncover more contextual and unique features of IKF and interdisciplinarity than a single dimension like citation counts or PCTC from previous studies. We would also like to point out what our framework can contribute to established interdisciplinarity indicators. For instance, Cell Biology and Immunology are associated with similar high interdisciplinarity values using existing indicators. However, the former’s high interdisciplinarity is associated with strong extramural citations with a limited number of disciplines, while the latter cites more widely and impartially from a cluster of disciplines. By using both established indicators and our indicating process, one can both quickly assess the interdisciplinarity of entities and infer the reason in detail.

![Figure 4. IKF of disciplines under our framework. These scatterplots illustrate how the focal discipline (indicated in the subtitle) cites the other disciplines (denoted as dots) under our framework. Citations toward the discipline itself are not included. Some special data points are highlighted and identified. The interdisciplinarity value (TD, True diversity) is quantified and reported for each discipline as the median TD value for all its publication.](image)

5.2.2 Indicating the interdisciplinarity of journals

To further demonstrate the utility of our framework, we examined the relative IKF of selected scientific journals in LIS under our framework in comparison to LIS publications in general, in other words, how certain LIS journals cite other disciplines differently from LIS at large. We choose 87 journals from JCR 2019 categorized under the field “Information Science &
Library Science” as representative of the LIS field and select seven journals as examples for study, namely Information and Organization (IO), Information Processing & Management (IPM), Journal of the Association for Information Science and Technology (JASIST), Journal of Informetrics (JOI), MIS Quarterly (MISQ), Research Evaluation (RE), and Scientometrics (SCIM).

For the calculation of three dimensions $D(J, Y)$, we set the citing side as one of the journals ($J_i$) or all journals from LIS in our dataset ($J_{all}$) and cited side as cited disciplines ($Y$). We then calculate the difference for each dimension between the focal journal and the LIS field as $D_{diff}(J_i, Y)$, where:

$$D_{diff}(J_i, Y) = D(J_i, Y) - D(J_{all}, Y)$$

In doing so, we aim to elucidate how the knowledge base of these journals deviates from the norm of the field of origin and try to describe the distinctive nuance in their knowledge portfolios.

Figure 5. A case study of LIS journals. (a). The IKF of the LIS field at large under our framework; (b-h). The relative IKF of seven LIS journals under our framework, compared to the LIS field. The sum of standard deviations of three dimensions for each journal is reported in the bottom-right box as an indication of deviation. The interdisciplinarity value (TD) is quantified and reported for each journal as the median TD value for all its publication.

Figure 5a illustrates the IKF of the LIS field under our framework; Figure 5 b-h showcases the deviations of the knowledge base (relative IKF) for seven LIS journals from their field of origin. A positive value in broadness or intensity dimension in b-h means the focal journal has a stronger relationship with the cited discipline than the LIS field in general. A negative value in heterogeneity denotes a greater cognitive affinity between the focal journal and the cited discipline than the LIS field in general. The more extreme the value, the greater the focal journal deviates from LIS as a whole. Seven cited disciplines are annotated as examples, namely:

- Applied Mathematics (ApplMath)
We now discuss the differences between the seven journals regarding their relationships (deviations) with these seven cited disciplines.

IPM, with a technological orientation, is not surprisingly found to possess stronger relationships with Computer Science/Information Technology in all three dimensions, and so do JASIST, JOI, and SCIM. The broadness of the aforementioned four journals is close to 100%, which means almost all papers from these journals cite Computer Science/Information Technology. The intensity, on the other hand, varies among them with IPM possessing the highest intensity value while JASIST, JOI, and SCIM exhibit intensity values in descending order. IPM also cites Electrical & Electronic Engineering with a greater broadness and intensity and less heterogeneity, which is lacking in the rest of the journals. Its relationship (especially broadness and heterogeneity) with the four social science disciplines is weaker compared to other journals.

In contrast, MISQ clearly has a stronger connection with social science disciplines in broadness. Particularly, MISQ cites Business, Economics, Planning more heavily in all three dimensions (same for IO). But it is the only journal in our selected examples that cite Business, Economics, Planning heavily in conjunction with Applied Mathematics in great broadness, characterizing one of the most unique features of MISQ. A greater percentage of MISQ publications cite Applied Mathematics than the general LIS fields but in a similar intensity.

IO, as a journal with a focus on the relationship between information technology and social organizations, also shares a greater affinity to several social science disciplines and less affinity to STEM disciplines such as applied mathematics and Electrical & Electronic Engineering. IO is the only journal in the selected seven that cites Applied Mathematics in lower broadness than average.

Another journal that possesses higher IKF values with Business, Economics, Planning for all three dimensions is RE, although smaller than that of MISQ and IO. It has a stronger connection with other social science disciplines, for instance, Community & Social Issues, Philosophy & Religion, and Political Science & Administration, instead of the other two we have labeled in Figure 5b. Its relationships with the three technical disciplines are not significantly different from general LIS fields.

JASIST and SCIM are the closest to the LIS field in terms of knowledge base and IKF, which is also shown by the lowest value for the sum of standard deviations of the three dimensions. Their relationships with most of the disciplines are not quite different from that of the LIS field. They both cite Computer Science/Information Technology in a greater broadness, but with a greater intensity only for JASIST. SCIM cites Psychology & Behavioral Sciences with
a smaller intensity, whereas JASIST cites Business, Economics, Planning with a smaller intensity. JASIST’s knowledge base is most similar to the knowledge base of the general LIS field, which makes intuitive sense since it is designed to cover a wide range of topics in LIS.

Finally, JOI shares a similar knowledge portfolio with JASIST and SCIM, but with a few discrepancies. It cites Multidisciplinary Sciences (the right-most point) at a significant level of broadness, which is absent in other journals. Additionally, Education, Media & Information Science appears to be vital knowledge source with greater intensity for JOI, yet that of both Psychology & Behavioral Sciences and Business, Economics, Planning are lower than the general LIS field.

Through this case study on the relative IKF of journals, we demonstrate how our multidimensional framework on pairwise relationships can vividly indicate the interdisciplinarity of the knowledge base for a certain entity, which is lacking in unidimensional and integrated indicators. For instance, existing indicators assign similar values to journals such as IO and JASIST, although they exhibit quite different characteristics in relative interdisciplinary contexts. We once again calculate the Pearson correlation between interdisciplinarity values and the sum of standard deviations for the difference of three dimensions compared to the LIS at large (in Figure 5 b-h) and arrived at a coefficient of 0.041 (see Appendix A for details). This shows that the relative IKF of LIS journals, the deviations in interdisciplinarity from LIS (relative interdisciplinarity), is different from its interdisciplinarity in a global context; that is to say, a journal that is found to be more interdisciplinary does not necessarily deviate significantly from its disciplinary norm. This illustrates the necessity of our proposed approach in relative terms that can unveil the relative interdisciplinarity of entities in comparison with their peers. Such operationalization of relative IKF can highlight some of the most significant interdisciplinary features of entities and facilitate valid peer comparisons.

6 DISCUSSION AND CONCLUSION

This paper proposes a multidimensional framework quantifying the interdisciplinary knowledge flow to infer the pattern and dynamics of interdisciplinarity. Three dimensions are recognized under this framework, namely, broadness, intensity, and heterogeneity. Using this conceptualization, we manage to offer a contextual and holistic understanding of the knowledge exchanges between disciplines and provide more details and scrutiny than simple citation counts. In order to validate this framework, we investigate the relationships among the three dimensions for discipline pairs as well as the relationships between the three dimensions and citation counts. We showcase that the three dimensions can capture distinct aspects of IKF, especially in the highest citation groups. As two use cases, we apply this framework to examine the disparate patterns of IKF in academic disciplines and scientific journals. We argue that our framework carries great power and potential in indicating interdisciplinarity and avoids some of the problems of composite indicators of interdisciplinarity.
The inspiration for this article is triggered by our previous research (Zhou et al., 2021, 2022), in which we studied the evolution of interdisciplinarity in the social sciences using several aggregated interdisciplinarity indicators. The study offers a succinct and general perspective on interdisciplinarity by approaching it as diversity in the knowledge base, which can be subsumed into a single measurement. In the task of finding a proxy to approximate interdisciplinarity, it is essential that one pursues simplicity and accuracy, by which we mean to improve the indicators of interdisciplinarity until it is closest to the ideal definition of interdisciplinarity. On the other hand, we should also realize that it is perhaps a “mission impossible” in the first place; complexity and comprehensiveness also matter in certain scenarios. Rafols (2020a) stresses the importance of “directionality”, i.e. the orientation of research contents in science and innovation. In the context of interdisciplinarity, we argue that an equally important question, if not more, could be “what is interdisciplined” compared to our current objective to quantify “how interdisciplinary”. Most attention and efforts have been directed to the latter question. This article attempts to offer a tentative solution to the less explored question, seeking to propose and utilize several indicators to arrive at disaggregated and more contextual understandings of interdisciplinarity. We would also argue that it’s unnecessary to completely abandon interdisciplinarity indicators, although they are currently under criticism. A combination of both indicators and the indicating processes of interdisciplinarity can deliver better and more accurate implications, just like what we did in this study to report both interdisciplinarity values and the three dimensions of IKF at the same time.

Furthermore, besides its kaleidoscopic connotations on the conceptual level, we argue that interdisciplinarity is a multifaceted construct also in the context of research evaluation and management that exists in different forms, delivers different interests to different parties, and should be approached by different methodologies. For instance, funding institutions adopt various policy tools to facilitate interdisciplinary research in different contexts and should be provided with suitable empirical evidence to assist decision-making. To begin with, evaluations of interdisciplinarity can be embedded in performance-based research funding allocation schemes as one of the factors that will affect the amount of funding each university obtain. In Flanders, the Dutch-speaking region of Belgium, a parameter for interdisciplinary research is scheduled to be added to the funding partition formula from 2024 onwards (Luwel, 2021). Under such a scenario, quantitative indicators could be more suitable to ensure a tangible and relatively unbiased evaluation process and demonstrate the funding body’s encouragement of interdisciplinary research. On the other hand, the review process for interdisciplinary research projects commonly involves a panel of referee experts (evaluation committee) and requires an abductive and participatory approach to evaluate interdisciplinarity. At Ghent University in Flanders, for example, this implies that applications for interdisciplinary research projects are firstly pre-selected and evaluated on the “level of interdisciplinarity” of the proposals¹. Three criteria for interdisciplinarity are stipulated: dissimilarity in involved disciplines or expertise, equally essential and integrated inputs from involved disciplines, and potential for the development of a new field of study or

¹ https://www.ugent.be/en/research/funding/bof/iop
new insights in both disciplines. Under such a scenario, we suggest that interdisciplinarity is better evaluated and presented in a contextual process where the decision-making process is supported by quantitative and qualitative background material: e.g. interdisciplinary profiles at the international level; disciplinary and interdisciplinary strengths, weaknesses, and planning at the national level; existing disciplinary and interdisciplinary strengths and outputs at the university level. The reviewing process can therefore be more informative and be situated in grander initiatives such as international competition, national R&D strategic planning, and university management. On the other hand, as more funding mechanisms are requiring interdisciplinary elements in the research design of applications, funding bodies should also be vigilant to prevent inflationary claims of interdisciplinarity wherever the proposed research is already subsumed under disciplinary practices. Initial filtering on applications aided by information tools can perhaps partly offset such inflationary tendencies and reduce the cost of evaluation. Besides research evaluation, funding bodies can also play important roles in “identifying questions that need an interdisciplinary approach”, “launching and shaping initiatives” (funding), and “establishing the architecture of an interdisciplinary programme” or research centers (Marsden et al., 2011). By proposing our multidimensional framework on interdisciplinary knowledge flow, we aim to contribute another scientometric tool to research evaluation practices that require a contextual understanding of the knowledge portfolio of scientific entities, not only disciplines and journals, but also research teams, research institutes, countries, or other groups of entities. We would also like to call for more diverse quantitative tools for indicating interdisciplinarity to cope with the diverse policy needs in facilitating interdisciplinary research in research evaluation and management.

Our study has several limitations and room for future studies. First, the proposed heterogeneity dimension still suffers from size dependence, although with only a minor effect, using our current operationalization of the knowledge base. Second, we focus on the citing side of disciplines or journals, i.e. references, which only entails one side of the interdisciplinary knowledge flow. Since the framework is designed to capture asymmetric relationships between entities, in future studies, we will also investigate the cited side of knowledge flow and unveil the dynamics of knowledge diffusion under our framework. In addition, in order to propose and validate this model first, we conduct analyses from a static perspective that studies the relationships between disciplines within a set time frame. Temporal evolutions of interdisciplinarity under our model will be explored in a future study.

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REFERENCES

Angrist, J., Azoulay, P., Ellison, G., Hill, R., & Lu, S. F. (2020). Inside Job or Deep Impact? Extramural Citations and the Influence of Economic Scholarship. *Journal of Economic Literature, 58*(1), 3–52. https://doi.org/10.1257/jel.20181508

Broadus, R. N. (1952). An Analysis of Literature Cited in the American Sociological Review. *American Sociological Review, 17*(3), 355–357. https://doi.org/10.2307/2088083

Clancy, M. (2021, August 2). What ails the social sciences. *Works in Progress*. https://www.worksinprogress.co/issue/what-ails-the-social-sciences/

Glänzel, W., & Schubert, A. (2003). *A new classification scheme of science fields and subfields designed for scientometric evaluation purposes*. 11.

Glänzel, W., Thijs, B., & Chi, P.-S. (2016). The challenges to expand bibliometric studies from periodical literature to monographic literature with a new data source: The book citation index. *Scientometrics, 109*(3), 2165–2179. https://doi.org/10.1007/s11192-016-2046-7

Goldman, A. (1979). Publishing Activity in Marketing as an Indicator of Its Structure and Disciplinary Boundaries. *Journal of Marketing Research, 16*(4), 485–494. https://doi.org/10.1177/002224377901600405

Huang, Y., Lu, W., Liu, J., Cheng, Q., & Bu, Y. (2022). Towards transdisciplinary impact of scientific publications: A longitudinal, comprehensive, and large-scale analysis on Microsoft Academic Graph. *Information Processing & Management, 59*(2), 102859. https://doi.org/10.1016/j.ipm.2021.102859

Kessler, M. M. (1963). Bibliographic coupling between scientific papers. *American Documentation, 14*(1), 10–25. https://doi.org/10.1002/asi.5090140103
Leong, S. M. (1989). A citation analysis of the Journal of Consumer Research. *Journal of Consumer Research, 15*(4), 492–497.

Leydesdorff, L., Wagner, C. S., & Bornmann, L. (2019). Interdisciplinarity as diversity in citation patterns among journals: Rao-Stirling diversity, relative variety, and the Gini coefficient. *Journal of Informetrics*. https://doi.org/10.1016/j.joi.2018.12.006

Liu, Y., Rafols, I., & Rousseau, R. (2012). A framework for knowledge integration and diffusion. *Journal of Documentation, 68*(1), 31–44.
https://doi.org/10.1108/00220411211200310

Luwel, M. (2021). Performance-based Institutional Research Funding in Flanders, Belgium. *Scholarly Assessment Reports, 3*(1), Article 1. https://doi.org/10.29024/sar.29

Marres, N., & de Rijcke, S. (2020). From indicators to indicating interdisciplinarity: A participatory mapping methodology for research communities in-the-making. *Quantitative Science Studies, 1*(3), 1041–1055. https://doi.org/10.1162/qss_a_00062

Marsden, W., Lyall, C., Bruce, A., & Meagher, L. (2011). A short guide for funders of Interdisciplinary research. *Londres, Inglaterra.*

Morillo, F., Bordons, M., & Gómez, I. (2003). Interdisciplinarity in science: A tentative typology of disciplines and research areas. *Journal of the American Society for Information Science and Technology, 54*(13), 1237–1249.
https://doi.org/10.1002/asi.10326

Porter, A. L., & Chubin, D. E. (1985). An indicator of cross-disciplinary research. *Scientometrics, 8*(3), 161–176. https://doi.org/10.1007/BF02016934

Porter, A. L., & Rafols, I. (2009). Is science becoming more interdisciplinary? Measuring and mapping six research fields over time. *Scientometrics, 81*(3), 719–745.
https://doi.org/10.1007/s11192-008-2197-2
Radhakrishna, R. B. (1992). *Characteristics of Literature Cited in the "Journal of Agricultural Education": An Empirical Study.*

Rafols, I. (2020a). Knowledge integration for societal challenges: From interdisciplinarity to research portfolio analysis. https://leidenmadtrics.nl/articles/knowledge-integration-for-societal-challenges-from-interdisciplinarity-to-research-portfolio-analysis

Rafols, I. (2020b, November 3). On “measuring” interdisciplinarity: From indicators to indicating. https://leidenmadtrics.nl/articles/on-measuring-interdisciplinarity-from-indicators-to-indicating

Rafols, I., & Meyer, M. (2010). Diversity and network coherence as indicators of interdisciplinarity: Case studies in bionanoscience. *Scientometrics, 82*(2), 263–287. https://doi.org/10.1007/s11192-009-0041-y

Rappaport, M. W. (1971). *Citation Patterns in Selected Core Journals for Linguistics. LINCS Project Document Series.* https://eric.ed.gov/?id=ED047322

Rigney, D., & Barnes, D. (1980). Patterns of interdisciplinary citation in the social sciences. *Social Science Quarterly, 61*(1), 114–127.

Rousseau, R. (2018). A new method for diversity measurement: Taking similarity between cells seriously. In *STI 2018 Conference Proceedings* (pp. 793–798). Centre for Science and Technology Studies (CWTS). https://hdl.handle.net/1887/65284

Stirling, A. (2007). A general framework for analysing diversity in science, technology and society. *Journal of The Royal Society Interface, 4*(15), 707–719. https://doi.org/10.1098/rsif.2007.0213

Truc, A., Santerre, O., Gingras, Y., & Claveau, F. (2020). The Interdisciplinarity of Economics. *SSRN Electronic Journal.* https://doi.org/10.2139/ssrn.3669335
van Leeuwen, T., & Tijssen, R. (2000). Interdisciplinary dynamics of modern science: Analysis of cross-disciplinary citation flows. *Research Evaluation, 9*(3), 183–187. https://doi.org/10.3152/14715440781777241

Wagner, C. S., Roessner, J. D., Bobb, K., Klein, J. T., Boyack, K. W., Keyton, J., Rafols, I., & Börner, K. (2011). Approaches to understanding and measuring interdisciplinary scientific research (IDR): A review of the literature. *Journal of Informetrics*. https://doi.org/10.1016/j.joi.2010.06.004

Wang, Q., & Schneider, J. W. (2020). Consistency and validity of interdisciplinarity measures. *Quantitative Science Studies, 1*(1), 239–263. https://doi.org/10.1162/qss_a_00011

Waskom, M. L. (2021). seaborn: Statistical data visualization. *Journal of Open Source Software, 6*(60), 3021. https://doi.org/10.21105/joss.03021

Watts, D. J. (2017). Should social science be more solution-oriented? *Nature Human Behaviour, 1*(1), Article 1. https://doi.org/10.1038/s41562-016-0015

Zhang, L., Rousseau, R., & Glänzel, W. (2016). Diversity of references as an indicator of the interdisciplinarity of journals: Taking similarity between subject fields into account. *Journal of the Association for Information Science and Technology, 67*(5), 1257–1265. https://doi.org/10.1002/asi.23487

Zhou, H., Guns, R., & Engels, T. C. E. (2021). The evolution of interdisciplinarity in five social sciences and humanities disciplines: Relations to impact and disruptiveness. *Proceedings of the 18th International Conference on Scientometrics and Informetrics*, 1381–1392.

Zhou, H., Guns, R., & Engels, T. C. E. (2022). Are social sciences becoming more interdisciplinary? Evidence from publications 1960–2014. *Journal of the Association for Information Science and Technology*. https://doi.org/10.1002/asi.24627
APPENDIX A. Additional plots mentioned in the article.

a. Three dimensions and discipline size ratio

![Graph A](image1.png)

b. Larger discipline citing smaller: log(size_ratio) > 1

![Graph B](image2.png)

c. Smaller discipline citing larger: log(size_ratio) < -1

![Graph C](image3.png)

Figure A1. Considering discipline size in the relationship between dimensions. The size ratio is quantified as the logarithmic ratio of the number of publications for the citing discipline and the cited discipline, denoted by colors – blue if the citing discipline is larger (log(size_ratio) > 0), red if cited discipline is larger (log(size_ratio) < 0). (a) relationships between three dimensions and discipline size ratio for all discipline pairs. (b-c) Disciplines pairs with significant size imbalances are selected (|log(size_ratio)| > 1). Linear fits for all discipline pairs (grey solid line) and selected ones (colored dashed line) are reported.
Figure A2. Relationship between the “scattering” of three dimensions and the interdisciplinarity value (TD median)

Figure A3. Relationship between the “scattering” of three dimensions in relative format and interdisciplinarity value (TD median)
APPENDIX B. Properties of the proposed three dimensions

1. Monotonicity

1.1 Broadness

The broadness of IKF of $X$ citing $Y$ is defined as a fraction as follows,

$$B(X, Y) = \frac{\sum_{i \in X} \delta_i}{|X|} = \frac{|X'|}{|X|}$$

where $X'$ denotes a subset of $X$ that cites $Y$. Having the size of $X$ fixed, $B(X, Y)$ increases monotonically with $|X'|$. On the other hand, having the size of $X'$ fixed, $B(X, Y)$ decreases monotonically with $|X|$.

1.2 Intensity

The intensity of IKF of $X$ citing $Y$ is defined as a fraction as follows,

$$I(X, Y) = \frac{\sum_{i \in X, j \in Y} M_{ij}}{\sum_{i \in X, j = 1, \ldots, n} (M_{ij} \delta_i)} = \frac{\text{Cit}(X, Y)}{\sum \text{Cit}(X', d)} = \frac{\text{Cit}(X', Y)}{\sum \text{Cit}(X', d)}$$

where a function $\text{Cit}$ quantifies the number of citations from one publication set to another one. The numerator $\text{Citation}(X, Y)$ is the number of citations from $X$ to $Y$, equivalent to $\text{Citation}(X', Y)$, whereas the denominator is the total number of outward citations (to all other entities $d$) by $X$. Having $\sum \text{Citation}(X', d)$ fixed, $I(X, Y)$ increases monotonically with $\text{Citation}(X', Y)$. On the other hand, having the numerator fixed, $I(X, Y)$ decreases monotonically with the denominator.

1.3 Heterogeneity

The heterogeneity of $X$ with $Y$ is defined as a fraction as follows:

$$H(X, Y) = 1 - \frac{|\{j \mid M_{ij} > 0, i \in X \} \cap \{j \mid M_{ij} > 0, i \in Y \}|}{|\{j \mid M_{ij} > 0, i \in X \}|}$$

$$= 1 - \frac{|R(X) \cap R(Y)|}{|R(X)|}$$

where a function $R$ quantifies the number of unique references cited by publication sets, i.e. each reference will appear only once even if cited by multiple publications. The denominator is the number of unique references by $X$ publications, whereas the numerator is the size of overlap of two unique reference sets (herein after named knowledge base) by $X$ and $Y$, respectively. Having the size of the knowledge base of $X$ fixed, an increasing overlap in the knowledge base with $Y$ is monotonously associated with decreasing $H(X, Y)$. On the other hand, having the overlap fixed (numerator), the size of the knowledge base of $Y$ (denominator) increases monotonically with $H(X, Y)$.

2. Size independence

2.1 Definition
For entity A, an identical subset of A with only a difference in size should obtain the same values for the three dimensions.

2.2 Mathematical proof

2.2.1 Broadness

Following the definition in 2.1, for entity X, we define an identical subset of X named \( X_s \). According to 2.1 and the definition of broadness, we should have:

\[
|X_s| = \alpha|X| \quad \text{and} \quad |X'_s| = \alpha|X'|, \quad (0 < \alpha \leq 1)
\]

Then we have:

\[
B(X, Y) = \frac{|X'|}{|X|}
\]

and,

\[
B(X_s, Y) = \frac{\alpha|X'|}{\alpha|X|} = \frac{|X'|}{|X|}
\]

hence,

\[
B(X, Y) = (X_s, Y)
\]

We conclude that \( B(X, Y) \) is independent of \( |X| \).

2.2.2 Intensity

Similar to 2.2.1, we conclude that \( I(X, Y) \) is independent of \( |X| \).

2.2.3 Heterogeneity

\( R(X) \) can be divided into two sub-sets, namely \( R(X) \cap R(Y) \) and \( R(X) \setminus R(Y) \).

Define:

\[
|R(X) \cap R(Y)| = \alpha|R(X)|, \quad (0 < \alpha \leq 1)
\]

Then we have:

\[
|R(X) \setminus R(Y)| = (1 - \alpha)|R(X)|
\]

For a given element in \( x \in R(X) \), it has \( \alpha \) chance to be in the intersection \( R(X) \cap R(Y) \).

Following the definition of size independence (section 2.1), let’s define \( R(X)' \) as a randomly selected subset of \( R(X) \), with only difference in size with \( R(X) \). \( R(X)' \) should have the following properties:

1. For each element in \( R(X)' \), it must belong to \( R(X) \) as well;
2. Each element \( x \in R(X) \) has the same probability to be in \( R(X)' \); and
3. The distribution of \( R(X)' \), is the same as \( R(X) \). Meaning: For a specific property (say \( D \)) of elements in \( R(X) \), suppose that the distribution of \( D \) follows this function: \( P(D = k) \). Then the distribution of \( D \) of elements in \( R(X)' \) should equal: \( P'(D = k) = P(D = k) \).
$R(X)'$ can also be divided into two sub-sets, namely $R(X)' \cap B$ and $R(X)' \setminus B$.

According to property (3) of size independence:

$$|R(X)' \cap R(Y)| = \alpha |R(X)'| \text{ and } |R(X)' \setminus R(Y)| = (1 - \alpha) |R(X)'|$$

Hence we have:

$$\frac{|R(X) \cap R(Y)|}{|R(X)'|} = \alpha$$

And,

$$\frac{|R(X)' \cap R(Y)|}{|R(X)'|} = \alpha$$

We conclude that $H(X,Y)$ is independent of $|R(Y)|$ if all three properties apply.

2.3 Empirical tests

2.3.1 Simulations

We further test the property of size independence in empirical settings. For each of the 74 disciplines in our dataset, we randomly select 10%, 20%, ..., 80%, and 90% of their publications to represent a subset of that discipline. We then calculate the three dimensions for each discipline pair in each random subset and quantify the difference between simulated results and with original results using the full set. We plot the average differences for each dimension and each random trial. The random process is repeated 10 times and the distribution of average differences is shown in Figure B1.

![Figure B1](image)

Figure B1. The average difference in the value of three dimensions between the full set and randomly simulated sub-sets. The x-axis denotes different sample sizes (percentage of the full set), while the y-axis denotes the difference between results from the full set and random samples. The mean difference of ten random trials is shown as the black dots and the grey bar represents the standard deviation. A horizontal red dashed line is shown to represent $y=0$, i.e. no difference between the full set and random subsets, hence size independence.

We find that broadness and intensity are indeed independent of the size of citing entities since values for various subsets of the entity remain around zero with minor fluctuations. So if a
discipline has \( n \) publications with a broadness (or intensity) distribution of \( D \), a random sample of that discipline having 0.5\( n \) publications is also expected to follow \( D \).

However, for heterogeneity, it seems that a smaller random sample is associated with a lower heterogeneity value which increases with sample size. Although the differences are rather minor (−.01 level), it does signal size dependence. Yet upon examination, we find that the property of size dependence still holds theoretically and the found association between sample size and difference in value is produced by the random sampling strategy.

2.3.2 Examples of the size independence property of heterogeneity

For instance, we have two journals in the same discipline category, A publishes five times more articles than B. Let’s consider two scenarios:

**Scenario 1:** A publishes more articles than B. But A and B publish similar articles using similar knowledge bases. B is equivalent to a representative subset of A, hence the general profile of heterogeneity is also similar according to section 2.2.3 in this Appendix.

**Scenario 2:** A publishes more articles than B and also covers more topics in the discipline yielding a more diverse and larger knowledge base for A.

In the first scenario, according to section 2.2.3 in Appendix, the size of journals does not cause a difference in heterogeneity between A and B, hence size independence. In the second scenario, the bigger journal A is expected to associate with lower heterogeneity on average since it situates on a large and diverse knowledge base and therefore is more likely to share common knowledge with other disciplines. So the difference in heterogeneity between journals with different sizes in this case is the direct result of a diverse knowledge base and the indirect result of journal size.

Then how to explain the difference between mathematical proof and empirical tests? We argue that Scenario 1 corresponds to the case made in mathematical proof because of property (3), which ensures the full set and the sub-set are identical in terms of the knowledge base with only a difference in size. On the other hand, Scenario 2 corresponds to the case made in empirical random tests in which property (3) is not guaranteed. Random samples are generated in a way that each paper gets an equal probability to be selected. Yet heterogeneity at the article level follows a skewed distribution, where high heterogeneity is quite rare. They could be underrepresented in such a random sample, which explains the lower heterogeneity value than the full set. As the sample size grows, the heterogeneity value increases as shown in the empirical tests.

However, we agree that, in practice, property (3) is hard to achieve and most analyses follow scenario 2, rather than scenario 1, hence size dependence. For two journals with similar knowledge bases and different sizes, the larger journal has a higher probability of citing a new and never-used-before article, which leads to a larger knowledge base in our operationalization. So the property of size independence is achieved only theoretically, but rarely empirically under our current operationalization of the knowledge base.
2.3.3 A possible solution to address the size dependence of heterogeneity

We would like to conclude the discussion with a potential solution to offset the dependence on size for the heterogeneity dimension under our current operationalization.

In the current study, we operationalize the knowledge base as a set of references. However, more references do not always signal more knowledge. To infer the “true” size of one’s knowledge base, it is necessary to quantify the amount of knowledge/concepts the references carry, not the number of references. To address this issue, one may need to extract the cited concepts in references using, for example, NLP techniques and perform harmonization to remove duplicates. In this way, we will get a more reasonable proxy for the knowledge base which suffers less from size dependence.