Acceleration-based PSO for Multi-UAV Source-Seeking

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Abstract—This paper presents a novel algorithm for a swarm of unmanned aerial vehicles to search for an unknown source. The proposed method is inspired by the well-known particle swarm optimization (PSO) algorithm and is called acceleration-based particle swarm optimization (APSO) to address the source-seeking problem with no a priori information. Unlike the conventional particle swarm optimization algorithm, where the particle velocity is updated based on the self-cognition and social-cognition information, here the update is performed on the particle acceleration. A theoretical analysis is provided, showing the stability and convergence of the proposed acceleration-based particle swarm optimization algorithm. Conditions on the parameters of the resulting third-order update equations are obtained using Jury’s stability test. High-fidelity simulations performed in CoppeliaSim, show the improved performance of the proposed acceleration-based particle swarm optimization algorithm for searching an unknown source when compared with the state-of-the-art particle swarm-based source-seeking algorithms. From the obtained results, it is observed that the proposed method performs better than the existing methods under scenarios like different inter unmanned aerial vehicle communication network topologies, varying numbers of unmanned aerial vehicles in the swarm, different sizes of search regions, restricted source movement, and in the presence of measurements noise.

I. INTRODUCTION

The objective of source-seeking problem in robotics is to locate a stationary or moving source by measuring the signal strength that is emanated from the source. In large and unknown outdoor environments, unmanned aerial vehicles (UAVs) becomes the preferred robotic platform for such applications. Deploying multiple UAVs for such a mission proves to be more efficient than using a single UAV [5]. However, source seeking based on multiple UAVs has its own challenges, like developing an efficient cooperative framework among UAVs to complete the task. Stochastic search method is a natural choice when there is no a priori information available on the probability distribution of the source location. Swarm intelligence based algorithms, such as Particle Swarm Optimization (PSO) [11] and its variants are widely used for solving the source-seeking problem. This study focuses on the PSO algorithm and its variants for solving the source-seeking problem.

Particle swarm optimization (PSO) is a swarm-based algorithm developed by Kennedy et al. [7], which is based on the collective social behavior of the particles. Over the last decade, a lot of the research has focused on using PSO-based algorithms for the source-seeking problem. This research has led to many variants of PSO such as the Darwinian PSO (DPSO) [9], Robotic PSO (RPSO), Robotic Darwinian PSO (RDPSO) [1], standard-PSO (SPSO) [12] and the Adaptive-Robotic PSO (ARPSO) algorithm [2]. The DPSO algorithm overcomes the local optima problem of conventional PSO by initializing multiple swarms that compete with each other in order to simulate a phenomenon that is similar to natural selection. RPSO consists of a population of robots that collectively searches source in the search space. RDPSO is an extension of RPSO that introduces a “punish-reward” mechanism to reduce the shared information between the UAVs since multiple swarms are formed. The main goal behind this technique is to prevent robots from erroneously converging towards local optima. The SPSO implements the optimized hyperparameters for the multi-robot source-seeking problem and proves to be better than the conventional PSO-based algorithm [12]. It also evaluates the performance of the algorithm under three different inter-robot communication network topologies. In the above-mentioned PSO variants, the hyperparameters are kept constant during the entire robotic search mission. Whereas, the hyperparameters are adaptively updated in the Adaptive-Robotic PSO algorithm. In [2] the authors showed that ARPSO performed better than RPSO and RDPSO. Hence, in this paper, we compare the proposed APSO algorithm with ARPSO-based and SPSO-based source-seeking algorithms.

This paper presents a novel update rule for the particle’s position. The original PSO can be portrayed as a second-order system with the particle’s velocity. The optimal solution is a third-order equation which has the particle’s position being dependent on the particle’s acceleration. A theoretical analysis is performed to show the stability and consensus of the proposed update rule. DPSO algorithm [9] is a higher-order system with the particle’s position being dependent on the particle’s velocity and acceleration. A theoretical analysis is performed to show the stability and consensus of the proposed update rule.

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The major contributions of this paper are the following.

- A new PSO-based source-seeking algorithm is presented that utilizes a third-order update rule for the particle’s position. The acceleration of the particle is updated using the self-cognition and social-cognition component, unlike the existing PSO-based methods where the particle’s velocity is updated through the self-cognition and social-cognition component.
- Detailed stability and convergence analysis of the proposed algorithm is provided.
- The condition on the values to be taken by the hyperparameters of the algorithm to ensure stability is established.
through the Jury’s stability test.

- A detailed performance comparison is provided with the state-of-the-art PSO-based source-seeking algorithms, showing the improved performance of the proposed method quantified in terms of average source-seeking time, the average number of iterations taken and the average swarm distance. A high-fidelity simulation is executed in CoppeliaSim for the performance comparison. The improvement in the performance of the proposed algorithm is shown to be consistent across scenarios like different inter-UAV communication network topologies, varying numbers of UAVs in the swarm, different sizes of search regions, restricted source movement and in the presence of measurements noise.

The paper is structured in the following manner. Section 2 briefly describes PSO, SPSO, and ARPSO. Section 3 details the APSO algorithm along with its convergence and stability analysis. The supporting results and discussions are presented in section 4. In section 5, the paper is concluded.

II. BACKGROUND

A. UAV and sensor description

The swarm of $n$ UAVs is randomly initialized at different locations in the search space to locate the source position solely based on the sensor information. The position of the UAVs is denoted with respect to an inertial frame of reference as $X_t - Y_t - Z_t$. All the UAVs are assumed to be flying at a fixed altitude different from each other to avoid the inter-UAV collision, and hence the $Z_t$ coordinate is not discussed here. The position coordinate of the $i^{th}$ UAV at any given time $t$ is denoted by $P_i(t) = [x_i(t), y_i(t)]^T$. The UAVs fly at a constant speed between successive waypoints.

The source is assumed to be located within the search space and emits a signal having a power that exponentially decays with the distance from it. Each UAV carries a sensor that measures the signal strength emitted by the source using the below model.

$$S_{mi}(t) = S_{m} e^{-\alpha d_i(t)^2} + \delta S$$

where $S_{mi}(t)$ is the power measured by the $i^{th}$ UAV located at a distance $d_i(t)$ from the source, $S_{m}$ is the source power and $\alpha > 0$ is a constant. The measurement noise is denoted by $\delta S$ and is random with a magnitude proportional to the measured power $S_{mi}$. The discrete-time version of the position coordinates are used in the subsequent sections and is denoted by $P_i(k) = [x_i(k), y_i(k)]^T$, where $t = kT$, $k$ is the discrete-time instant and $T$ is the sampling time in seconds.

B. Algorithms for PSO-based source-seeking

A brief overview of the popular PSO-based source-seeking algorithms is presented below. Equations are provided for a single-axis $X_t$ and analogous equations follow for the $Y_t$ axis also.

a) Regular PSO: The conventional PSO used for optimization is implemented for the source-seeking problem in [3]. The velocity ($v_i(k)$) and the position ($x_i(k)$) updates for the $i^{th}$ robot for $k^{th}$ sampling time instant is given by,

$$v_i(k + 1) = v_i(k) + R(0, c_1)(x_{ib}(k) - x_i(k)) + R(0, c_2)(x_{gb}(k) - x_i(k))$$ \hspace{1cm} (2)

$$x_i(k + 1) = x_i(k) + v_i(k + 1)T$$ \hspace{1cm} (3)

Here $T$ is a scaling factor (typically $T=1$ is used), $c_1 > 0$, $c_2 > 0$, $R(a, b)$ denotes a random number from uniform probability distribution within the interval $[a, b], x_{ib}(k)$ is the best individual position (given in (4)) and $x_{gb}(k)$ (given in (5)) is the best global position.

$$x_{ib}(k) = \max_{1 \leq j \leq k} S_{mi}(j)$$ \hspace{1cm} (4)

$$x_{gb}(k) = \max_{1 \leq j \leq n} x_{ib}(j)$$ \hspace{1cm} (5)

b) Standard Particle Swarm Optimization (SPSO): SPSO is an extension to the regular PSO algorithm [12] with an inertia factor added in the velocity update of the $i^{th}$ particle is given by,

$$v_i(k + 1) = \omega v_i(k) + R(0, c_1)(x_{ib}(k) - x_i(k)) + R(0, c_2)(x_{ib}(k) - x_i(k))$$ \hspace{1cm} (6)

The parameter values used are $\omega=0.721$ and $c_1 = c_2 = 1.93$ to achieve the best performance.

c) Adaptive-Robotic PSO (ARPSO): The ARPSO algorithm prevents the swarm from converging at local minima. The algorithm is also capable of handling obstacles that are present in the environment [2].

$$v_i(k + 1) = \omega_i v_i(k) + R(0, c_1)(x_{ib}(k) - x_i(k)) + R(0, c_2)(x_{gb}(k) - x_i(k)) + R(0, c_3)(x_a - x_i(k))$$ \hspace{1cm} (7)

where $x_a$ is an attractive position that is located away from the obstacle. The parameter $c_3$ is set to 0 for obstacle free environment. Unlike in SPSO, the parameter $\omega_i$ in ARPSO is not constant. The value of $\omega_i$ is different for different UAVs and also changes for each iteration.

III. ACCELERATION-BASED PSO (APSO) FOR SOURCE-SEEKING

In the proposed APSO, the particle acceleration is also updated apart from velocity as in the above-mentioned PSO and its variants to generate a new waypoint. This additional equation transforms the input-to-output relation to a third-order system. The following equations from (8) to (10) shows the waypoint update equations for the $i^{th}$ UAV.

$$a_i(k + 1) = w_1 a_i(k) + R(0, c_1)(x_{ib}(k) - x_i(k)) + R(0, c_2)(x_{gb}(k) - x_i(k))$$ \hspace{1cm} (8)

$$v_i(k + 1) = w_2 v_i(k) + a_i(k + 1)T$$ \hspace{1cm} (9)
\[ x_i(k+1) = x_i(k) + v_i(k+1)T \] (10)

where \( a_i(k) \) denotes the particle acceleration for the \( i \)th UAV. By simplifying (8) to (10), the following relation is obtained with \( r_1 = R(0, c_1) \) and \( r_2 = R(0, c_2) \).

\[
x_i(k+1) = (1 + w_1 + w_2 - r_1 T - r_2 T)x_i(k) + (-w_1 - w_2 - w_1 w_2)x_i(k-1) + w_1 w_2 x_i(k-2) + T(r_1 x_{ib}(k) + r_2 x_{gb}(k))
\] (11)

A. Stability Analysis

The equation describing the evolution of \( x_i(k) \) given in (11) represents a third-order dynamical system. Applying z-transform on (11) yields the following transfer function.

\[
G(z) = \frac{X(z)}{U(z)} = \frac{z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}
\] (12)

where the coefficients \( a_1 = -1 - w_1 - w_2 + r_1 T + r_2 T \), \( a_2 = w_1 + w_2 + w_1 w_2 \), \( a_3 = -w_1 w_2 \) and \( U(z) \) is the z-transform of the bounded input \( u(k) = T(r_1 x_{ib}(k) + r_2 x_{gb}(k)) \). The stability of \( G(z) \) depends upon the roots of the polynomial given below.

\[
H(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}
\] (13)

Since \( r_1 \) and \( r_2 \) are random numbers varying between \([0, c_1]\) and \([0, c_2]\) respectively, the coefficient \( a_1 \in [-1 - w_1 - w_2, -1 - w_1 - w_2 + T(c_1 + c_2)] \). Noting that \( H(z) \) is a third-order uncertain polynomial, it is enough to confirm that the extreme polynomials \( H(z) \) at \( a_1 = -1 - w_1 - w_2 \) (the coefficient and corresponding polynomial denoted by \( a_{11} \) and \( H(z, a_{11}) \) respectively) and \( H(z) \) at \( a_1 = -1 - w_1 - w_2 + T(c_1 + c_2) \) (the coefficient and corresponding polynomial denoted by \( a_{1u} \) and \( H(z, a_{1u}) \) respectively) does not have roots outside the unit circle [6]. Applying the Jury’s stability criterion for \( H(z, a_{11}) \) and \( H(z, a_{1u}) \), the following conditions are obtained on the parameters \( w_1, w_2, T, c_1 \) and \( c_2 \).

\[
2T(1 + w_1 + w_2 + w_1 w_2) > c_1 + c_2
\] (14)

\[
|w_1 w_2| < 1
\] (15)

\[
|(1 - w_1 w_2)(w_1 + w_2) + (w_1 w_2)(c_1 T + c_2 T)| < |1 - (w_1 w_2)^2|
\] (16)

\[
|w_1 + w_2| < |1 + w_1 w_2|
\] (17)

B. Convergence Analysis

Ensuring the stability conditions given in (14)-(17) enables the states of the dynamical system given in (11) to converge. The convergence condition for constant \( r_1 \) (denoted by \( r_{1c} \)) and \( r_2 \) (denoted by \( r_{2c} \)) is analyzed first as given below. A steady state condition for \( x_i(k) \) (denoted by \( x_i(ss) \)) in (11) can be obtained as given below.

\[
x_i(ss) = \lim_{k \to \infty} x_i(k) = \frac{r_{1c} x_{ib}(ss) + r_{2c} x_{gb}(ss)}{r_{1c} + r_{2c}}
\] (18)

where \( x_{ib}(ss) \) and \( x_{gb}(ss) \) are the steady-state values of \( x_{ib}(k) \) and \( x_{gb}(k) \) respectively.

Since for at least one of the UAVs, the condition \( x_{ib}(ss) = x_{gb}(ss) \) is satisfied, the right-hand-side (RHS) of (18) becomes independent of \( r_{1c} \) and \( r_{2c} \) and the following steady-state condition is achieved.

\[
x_i(ss) = x_{ib}(ss) = x_{gb}(ss)
\] (19)

When \( r_1 \) and \( r_2 \) are kept random, then the convergence of the mean value of \( x_i(k) \) denoted by \( \bar{x}_i(ss) = E(x_i(ss)) \) (where \( E(.) \) is the mathematical expectation operator) is analyzed as given below [4]. Applying \( E(.) \) on both sides of (11) and as \( k \to \infty \), the following relation is obtained.

\[
(E(r_1) + E(r_2)) \bar{x}_i(ss) = E(r_1) \bar{x}_{ib}(ss) + E(r_2) \bar{x}_{gb}(ss)
\] (20)

Noting that \( E(r_1) = \frac{c_1}{2} \) and \( E(r_2) = \frac{c_2}{2} \) for uniform probability distribution, (20) can be simplified as given in (21).

\[
\bar{x}_i(ss) = \frac{c_1 \bar{x}_{ib}(ss) + c_2 \bar{x}_{gb}(ss)}{c_1 + c_2}
\] (21)

IV. RESULTS AND DISCUSSION

In this section, the performance of APSO is compared against that of ARPSO and the standard PSO (SPSO). The obtained results are verified by simulation performed in the CoppeliaSim simulator. CoppeliaSim (formerly known as “V-REP”) is high fidelity and scalable robotic simulator used for testing and verification of different algorithms before deployment in the actual hardware [8]. All the three search algorithms (APSO, ARPSO, and SPSO) were implemented using python 3.7 in a conda-based environment. The search space considered here is in the shape of a square. For each simulation, the UAVs start from randomly initialized locations along the boundary of the search space. All the UAVs move from a waypoint to the next with a constant speed of 10 m/s. For all the simulations, the source is considered to be at the center of the search space. The algorithm is terminated when any of the UAV reaches a distance of 0.1 m from the source, in the case of measurement noise this value is increased to 0.5 m. In the implementation of APSO, the values of \( w_1 \) and \( w_2 \) are taken as 0.675 and -0.285 respectively. Both of the parameters \( c_1 \) and \( c_2 \) are selected as 1.193, and the value of \( T \) is taken as 1. It can be verified that the above-mentioned parameter setting satisfies the stability conditions mentioned in (14) to (17). The values of the parameters of SPSO and ARPSO algorithm are selected from [12] and [2] respectively.

A. Metrics for the performance comparison of source-seeking algorithms

- **Average source seeking time** (\( \mu(T_s) \)): This metric quantifies the time efficiency of the underlying source seeking algorithm and is defined in (22). Source seeking time is taken as the time duration from the start of the search till the source has been located by one or more members of the UAV swarm.
where \( N \) is the total number of Monte Carlo runs and \( T_{s,i} \) is the source seeking time for the \( i^{th} \) run.

- **Average number of iterations (\( \mu(I) \))**: This metric quantifies the sampling efficiency of the algorithm and is defined in (23) as the average number of waypoints generated to locate the source.

\[
\mu(I) = \frac{\sum_{i=1}^{N} W_{s,i}}{N} \tag{23}
\]

where \( W_{s,i} \) is the number of waypoints generated for the \( i^{th} \) run to locate the source.

- **Average Swarm Distance (\( \mu(SD) \))**: This parameter is related to the fuel efficiency when UAVs are deployed for the search and is defined in (24). Swarm distance is defined as the total distance travelled by the UAVs in the swarm, till the source has been located.

\[
\mu(SD) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{n} D_{s,i}}{N} \tag{24}
\]

where \( n \) is the size of UAV swarm and \( D_{s,i,j} \) is the distance travelled by the \( j^{th} \) UAV in the \( i^{th} \) simulation run.

The performance comparison is done under different operating conditions as given below.

a) **Inter-UAV communication network topology**: The exchange of information between the UAVs depends on the inter-UAV communication network topology. We consider three scenarios viz. i) fully connected (FC), ii) ring (RG) and iii) adaptive topologies (AD). More details of these three topologies are available in [12]. The other operating conditions considered are b) **different size of the search region**, c) **varying number of UAVs in the swarm**, d) **restricted source movement**, and e) **measurement noise**.

\[\textbf{B. Performance comparison of source-seeking algorithms}\]

The average is computed for \( N = 1000 \), for a swarm size \( n = 5 \) for a search space of dimensions \( 100 \times 100 \) m unless otherwise stated explicitly.

1) **Effect of inter-UAV communication network topology**: The results shown in Table I indicate that APSO is performing better than SPSO and ARPSO in all the three metrics for all the three topologies. The fully connected topology gives the best performance as expected. The swarm under adaptive topology performs better than the ring topology. Due to its superior performance, for all the remaining experiments the Fully Connected (FC) topology is used for all three algorithms.

2) **Different size of search region**: The dimensions of the search space is varied from \( 10 \text{ m} \times 10 \text{ m} \) to \( 100 \text{ m} \times 100 \text{ m} \). Table II shows the performance of all three algorithms with UAVs communicating under fully connected topology. The APSO performs the best among the three algorithms. The improvement in the performance of APSO is higher with an increase in the search area when compared to SPSO and ARPSO algorithms.

3) **Number of UAVs in a swarm**: Here the UAV swarm size is increased from 5 to 30 under fully connected topology. The average source seeking time for the three algorithms is plotted in Fig. 1. The \( \mu(T_{s}) \) for APSO is almost twice better than SPSO and ARPSO for \( n = 5 \) and later the improvement decreases. The \( \mu(T_{s}) \) saturates after a certain swarm size for all the three algorithms. The comparison for average number of iterations is shown in Fig. 2. The APSO outperforms SPSO irrespective of the number of UAVs in a swarm. The performance of ARPSO converges to that of APSO for \( n \geq 20 \). The APSO outperforms the other two algorithms for the case of \( \mu(SD) \) for all the UAV swarm size as shown in Fig. 3.

4) **Restricted source movement**: In this experiment, the source is moving randomly with in a circle of radius \( 4m \) and the search space considered here is of dimensions \( 200 \text{ m} \times 200 \text{ m} \). The speed of the source is varied from 0 to 0.3 \text{ m/s} \) and the performance of all the three algorithms is compared. Figure 4 shows the effect of source speed on the average source seeking time indicating the better performance of APSO when compared to ARPSO and SPSO. The value of \( \mu(T_{s}) \) increases slowly with source speed for APSO, whereas it increases rapidly for ARPSO and SPSO. Similar result can be observed for the case of \( \mu(SD) \) as shown in Fig. 6. The increase in \( \mu(I) \) with the source speed is noted for all the algorithms from Fig. 5 with APSO performing the best among

| Algorithm | Top. | \( \mu(I) \) | \( \mu(T_{s}) \) (s) | \( \mu(SD) \) (m) |
|-----------|------|--------------|---------------------|------------------|
| SPSO      | RG   | 43.26 ± 13.9 | 35.14 ± 17.0        | 387.59 ± 82.3    |
| ARPSO     | RG   | 44.13 ± 19.3 | 35.21 ± 14.5        | 350.49 ± 78.0    |
| APSO      | RG   | 26.77 ± 10.1 | 16.85 ± 6.0         | 174.60 ± 24.4    |
| SPSO      | FC   | 19.82 ± 7.5  | 23.73 ± 10.2        | 295.72 ± 86.6    |
| ARPSO     | FC   | 15.65 ± 6.0  | 19.75 ± 7.99        | 217.03 ± 51.0    |
| APSO      | FC   | 10.83 ± 3.1  | 10.87 ± 2.8         | 117.45 ± 18.3    |
| SPSO      | AD   | 38.65 ± 17.1 | 32.51 ± 14.7        | 374.79 ± 80.4    |
| ARPSO     | AD   | 37.56 ± 21.2 | 31.04 ± 12.5        | 317.54 ± 72.7    |
| APSO      | AD   | 22.39 ± 5.8  | 16.47 ± 5.8         | 167.66 ± 25.4    |

| Algorithm | Area (m²) | \( \mu(I) \) | \( \mu(T_{s}) \) (s) | \( \mu(SD) \) (m) |
|-----------|-----------|--------------|---------------------|------------------|
| SPSO      | 100       | 7.02 ± 1.1   | 1.27 ± 0.7          | 14.9 ± 7.8       |
| ARPSO     | 100       | 6.18 ± 2.8   | 1.27 ± 0.6          | 14.2 ± 5.0       |
| APSO      | 100       | 5.21 ± 2.2   | 0.79 ± 0.2          | 8.3 ± 2.5        |
| SPSO      | 625       | 11.58 ± 5.6  | 4.63 ± 2.2          | 55.5 ± 21.7      |
| ARPSO     | 625       | 8.93 ± 3.9   | 3.94 ± 1.8          | 43.6 ± 13.1      |
| APSO      | 625       | 7.31 ± 2.6   | 2.42 ± 0.7          | 25.4 ± 5.7       |
| SPSO      | 2500      | 15.49 ± 6.8  | 10.02 ± 5.0         | 129.7 ± 45.7     |
| ARPSO     | 2500      | 11.69 ± 4.9  | 8.89 ± 3.8          | 99.1 ± 27.6      |
| APSO      | 2500      | 9.11 ± 2.9   | 5.32 ± 1.4          | 56.0 ± 9.9       |
| SPSO      | 5625      | 17.72 ± 7.3  | 16.60 ± 7.3         | 207.2 ± 66.4     |
| ARPSO     | 5625      | 13.94 ± 5.8  | 14.31 ± 6.2         | 156.9 ± 40.5     |
| APSO      | 5625      | 10.23 ± 2.9  | 8.19 ± 2.0          | 87.0 ± 13.3      |
| SPSO      | 10000     | 19.67 ± 7.7  | 23.41 ± 10.3        | 288.3 ± 86.4     |
| ARPSO     | 10000     | 15.19 ± 6.0  | 19.51 ± 4.5         | 215.8 ± 54.6     |
| APSO      | 10000     | 10.97 ± 3.2  | 10.98 ± 2.7         | 118.2 ± 18.9     |
the three algorithms. For higher source speed, the difference between the performance of APSO and SPSO reduces.

5) Sensor Noise: In real scenarios, the sensor measurement is corrupted by noise as modeled in (1). Gaussian noise with zero mean and standard deviation proportional to the power measured by the UAV is considered as the sensor noise. The percentage of noise added is increased from 0 to 10% to see its effect on the three source seeking algorithms. Additionally, the minimum distance of any UAV from the source for which the algorithm terminates is increased from 0.1 m to 0.5 m to account for the measurement noise. The results are tabulated in Table III.

The performance of the three algorithms degrades when the Gaussian noise is introduced. However, APSO is capable of handling the noise much better than the other two algorithms.

C. DISCUSSION

The selection of the parameters $w_1, w_2, c_1$ and $c_2$ plays a crucial role in the performance of APSO. The parameters $c_1$ and $c_2$ are selected to be the same as the case with the SPSO algorithm. The stability conditions limits the choice of $w_1$ and $w_2$ to be with in $(-1, 1)$. The values of $w_1$ and $w_2$ used here are the ones yielding the best performance. A detailed analysis on the performance of APSO with varying $w_1$, $w_2$, $c_1$ and $c_2$ is not presented here due to the page limit.

The performance of ARPSO algorithm remained closer to the proposed APSO algorithm when compared to the SPSO algorithm. This is mainly due to the adaptive hyperparameters of the ARPSO algorithm when compared to the SPSO algorithm. The improved measurement noise handling capability of APSO algorithm can be attributed to the underlying third order dynamical system in contrast to the second order dynamical system for the ARPSO and SPSO algorithms.

V. CONCLUSION AND FUTURE WORK

This paper presented a new algorithm based on PSO for the multi-UAV source seeking problem. The proposed algorithm implemented a third-order update rule when compared to the existing second-order update rule. The stability analysis provided a guideline for selecting the hyperparameter values of the algorithm. The convergence of the proposed algorithm is similar to the existing PSO-based source-seeking algorithms [10]. The proposed APSO algorithm showed improved performance when compared to SPSO and ARPSO algorithms.
under different operating conditions. Obstacle avoidance, dynamic target tracking, and hardware implementation will be interesting extensions of this research work.

VI. ACKNOWLEDGMENT

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| Algorithm  | N (%) | $\mu(I)$ | $\mu(T_s)$ (s) | $\mu(SD)$ (m) |
|------------|-------|----------|----------------|----------------|
| SPSO       | 0     | 10.55 ± 5.3 | 18.03 ± 9.4 | 214.01 ± 91.8 |
| ARPSO      | 5%    | 6.81 ± 2.5  | 9.66 ± 2.8   | 102.30 ± 23.4 |
| APSO       | 10%   | 22.60 ± 15.8 | 28.43 ± 15.1 | 288.75 ± 90.3 |
| SPSO       | 0     | 13.79 ± 8.1  | 20.73 ± 9.7   | 206.18 ± 52.8 |
| ARPSO      | 5%    | 10.20 ± 4.6  | 11.72 ± 3.2   | 114.56 ± 21.3 |
| APSO       | 10%   | 21.39 ± 12.4  | 27.68 ± 15.2  | 277.98 ± 92.4 |
| SPSO       | 0     | 21.39 ± 24.1  | 27.68 ± 15.2  | 277.98 ± 92.4 |
| ARPSO      | 5%    | 14.37 ± 12.4  | 20.55 ± 9.1   | 201.42 ± 51.6 |
| APSO       | 10%   | 10.71 ± 5.2   | 11.94 ± 3.4   | 115.43 ± 20.7 |

Fig. 5. Effect of source speed on $\mu(I)$

Fig. 6. Effect of source speed on $\mu(SD)$