Monitoring lensed starlight emitted close to the Galactic center

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\textbf{ABSTRACT}

We describe the feasibility of detecting the gravitational deflection of light emitted by stars moving under the influence of the massive object at the Galactic center. Light emitted by a star orbiting behind the central mass has a smaller impact parameter than the star itself, and suffers the effect of gravitational lensing, providing a closer probe of the central mass distribution and hence a stricter test of the black hole hypothesis. A mass of \(4.3 \times 10^6 M_\odot\) causes a 0.1–2 mas deviation in the apparent position of orbiting stars projected within 10\(^\circ\) of the line of sight to the galactic center. In addition, we may uniquely constrain the distance to the center of the galaxy because lensing deflections constrain the ratio \(r_g/R_0\) of the Schwarzschild radius to the distance to the black hole, \(R_o\), whereas the ratio \(r_g/R_3\) is obtained by fitting the orbit.

\textbf{Key words:}

\section{INTRODUCTION}

Observations of the stellar motions near the Galactic center clearly reveal the existence of a very massive compact dark object (Genzel et al. 2000, Gezari et al. 2002, Figer et al. 2003, Genzel et al. 2003, Hornstein 2003, Schödel et al. 2003, Ghez et al. 2003). The analysis of Ghez et al. (2003) of the orbital motions of 22 stars yields a central dark mass of \(4 \pm 0.3 \times 10^6 (R_{\text{sun}}/M_\odot)^3\) where \(R_o\) is our distance from the galactic center. This is nearly twice the mass obtained from earlier velocity dispersion measurements (Genzel et al. 2002, Ghez et al. 2003). The motion of the star with the smallest impact parameter indicates that the mass of the central object is confined to lie within a maximum distance of 90AU from the center, limiting or excluding some of the proposed alternatives to the black-hole model (Ghez et al 2002).

The deviations from strict Keplerian orbits discussed (Ghez, private communication) are tantalizing but small and will be very interesting if they prove significant. Further monitoring and future higher resolution measurements, may uncover relativistic effects (Jaroszyński 1998, Pfahl & Loeb 2003, Weinberg et al. 2004) but are best explored with higher resolution observations using space interferometry or a planned giant telescopes (e.g. the Thirty Meter Telescope, Weinberg et al. 2004). Here we consider the signature of light deflection by gravitational lensing on the trajectories of the stars orbiting the central massive object. Jaroszyński (1998) discussed the statistics of proper motions in the presence of weak lensing and examined the apparent distortions of the trajectories only in the strong lensing limit. Lensing in the weak limit is much more typical, and here we examine the deflection of light from stars moving behind the central mass, particularly in the case of inclined trajectories which are of large radius but pass close in projection to the central mass, so that the lens-source separation is larger than the impact parameter of the light, generating significant light deflection. We will show that these effects are comparable or only a little smaller than the relativistic effects reported by Weinberg et al. (2004) and hence at the very least represent an important correction for stars orbiting behind the central mass.

Light reaching us from stars with low inclination angles passes closer to the central mass than do the stars themselves. Therefore light deflection effects can probe the inner mass distribution allowing a stricter test of the black hole hypothesis. Here we study the way light is deflected with simple constrasting models for the distribution of matter within the currently unconstrained region interior to 100AU from the center of mass.
2 THE EQUATIONS OF LIGHT DEFLECTION

Let $\beta$ and $\theta$ be, respectively, the source and image angular distances from the center of the lens. Also let $D_L$ and $D_S$ be the distances from the observer to the lens and to the source, respectively. The distance between the lens and the source is $D_{LS}$. The light bending equation by a spherical mass distribution in the thin lens limit is

$$\theta = \beta + \frac{D_{LS}}{D_S} \frac{4GM(< \xi)}{c^2} ,$$

(1)

where $\xi = D_L \theta$ and

$$M(< \xi) = \int_0^\xi 2\pi \xi' d\xi' \int_{-\infty}^\infty \rho(\xi', z) dz .$$

(2)

We will base our results on (1) which is valid in the thin lens limit. This approximation is valid for our the distances and mass of relevance here, as can be seen by comparing the thin lens approximation with the full solution to the geodesic equation. Using Fermat’s principle in a Schwarzschild metric (Landau & Lifshitz 1975) the path of light is given by

$$\frac{d^2u}{d\phi^2} + u = \frac{3}{2} r_s u^2$$

(3)

where $r_s = 2GM/r$ is the Schwarzschild radius, $u \equiv 1/r$, and $\phi$ is the angular position in the plane defined by the path. We assume the observer to be at $\phi = \pi$. Given a direction for the propagation of light as it leaves the star this equation could be solve to obtain $r(\phi)$. However, we do not know a priori which initial direction corresponds to a light ray seen by the observer. The initial direction, however, could be varied until the desired solution is found. Instead of this iterative approach, we adopt this approximate scheme described in Landau & Lifshitz (1976, p84). In this scheme the desired solution is found as a boundary value problem without resorting to iterations. We have tested this scheme in several cases and found very close agreement with the solution obtain by numerical integration of (3).

In figure (1) we show plot the deflection angle $(\theta - \beta)$ obtained from the thin lens approximation (the solid lines) and the full path equation (the points). The deflections are computed for sources lying on three edge-on circles centered a point mass lens. The lens has a mass is $M = 4.3 \times 10^6 M_{\odot}$ and $R_0 = 8$kpc is used to the distance to the lens. The deflections are computed only for the far side of the circles as the deflection of light emitted by a source between the lens plane and the observer is negligible. This figure demonstrates that the thin lens approximation is sufficiently accurate when the deflections are larger than 0.1mas even for stars as close as 100AU to the lens.

3 RESULTS

In addition to light deflection by a point mass (black hole) we present results for the following mass distributions: a constant density core, and an isothermal density profile $1/r^2$, both truncated at $r = r_1$. The orbits of stars analyzed by Ghez et al. (2003) probe the mass distribution outside the current minimum observed impact parameter of $r = 90$Au. These orbits are claimed to be consistent with Keplerian orbits, so we can assume that all the mass governing the orbits of stars and so we set $r_1 = 90$AU. For the total mass inside $r_1$ we take $M = 4.3 \times 10^6 M_{\odot}$, corresponding to a Schwarzschild radius of $1.2 \times 10^7$km, in the black hole case. For all the mass distributions we consider here the integral in (2) can be expressed in terms of elementary functions.

Figure (2) shows the the deflection angle, $\theta - \beta$ as a function of the source position $\beta$, for three values of the source distance from the center: 100, 1000, and 3000AU, as indicated in the figure. We only consider the weak lensing limit and therefore show results only for the bright image. The figure shows deflection for the bright image The solid, dotted, and dashed curves, respectively, correspond to deflection by a point mass, a $1/r^2$ distribution, and by a constant density. The right and left arrows indicate the locations of the Einstein rings for the 3000AU and 1000AU solid curves, respectively.

In figure (3) we plot the projected trajectories of a circular orbit as they would be see in the sky. The radius of the orbit is 3000AU and the orbital plane has an angle of $1^\circ$ with the l.o.s. The thin solid lines correspond to un lensed trajectories. The thick solid, dotted, dashed correspond to the a point mass, 1/r^2 density and constant density, respectively.

These figures demonstrate that the deflections are measurable for a wide range of $\beta$ for astrometric resolution of 0.1mas. Deflections larger than 1mas are restricted to very small values of $\beta$ corresponding to sources lying within a projected separation of 300AU from the center and are therefore rather improbable (cf. Ghez et al. 2003). The deflections by the three mass distributions differ only for rays passing within $r = r_1 = 90$AU of the center, following from our definition.
4 DISCUSSION

We have shown, rather surprisingly, that the gravitational lensing of stars on orbits about the large mass at the galactic center is a potentially detectable effect by its perturbation to the observed path of stars passing behind the central mass. We may expect $\sim 0.1\,\text{mas}$ deflections and these should be measurable with a sufficient number of accurate observations made with the next generation of space based interferometers or giant ground based telescopes. In general the light from stars orbiting behind the central mass must have a smaller impact parameter than the actual stellar orbit, by virtue of the projection, hence gravitational lensing allows the distribution of the central mass to be explored to within a smaller radius than might seem detectable, however in these cases it becomes a significant correction to the higher order relativistic effects of frame dragging and orbital precession, for which the anticipated deflections may be comparable or somewhat larger.

In a coordinate system with an origin at the galactic center, the apparent deflection of a star lying at an angle $\phi$ with the line of sight, is $\theta - \beta = \tan(\phi)D_L/(2r_g)$, independent of the star’s distance from the center (see equation 1). Substituting $r_g = 1.2 \times 10^7\,\text{km}$, and $D_L = R_o = 8\,\text{kpc}$ this means that for an astrometric resolution of $\Delta_0$ we should be able to measure deflection of light emitted by all stars lying in a cone of $11^\circ(\Delta_0/0.1\,\text{mas})$ behind the galactic center. This corresponding range in the orbital inclination angle (i.e., the angle between the line of sight and the normal to the orbital plane) is $85^\circ \lesssim i \lesssim 95^\circ$. The distortions in the orbits should be detectable by monitoring the stars around the central dark mass with future telescopes. According to Weinberg et al. (2004) the proposed Thirty Meter telescope (TMT) will be able to measure the orbits of $\sim 100$ stars at distances of $300 - 3000\,\text{AU}$ from the galactic center. The astrometric accuracy of the TMT will be $0.5\,\text{mas}$ and could as good as $0.1\,\text{mas}$. According to Table 3 of Ghez et al. (2003) the orbits of 2 out of 7 stars have inclinations in the range $85^\circ \lesssim i \lesssim 95^\circ$. Therefore in a sample of a 100 stars we expect more than ten stars to lie within the range of interest here.

In practice the lensing distortions can be detected by determining the orbital parameters for each star from the un-lensed majority of the the orbit. The deflection can then be detected as the residual between the parameterized orbit and the lensed part of the orbit. Given the source’s angular position, the deflection are proportional to $M/R_o^2$. This differs from the constraints on $M/R_o^2$ $M/R_o$ derived, respectively, from the projected trajectory and from radial velocity measurements (Eisenhauer 2003). Note that the magnification of the emitting stars in the weak lensing limit may rise to a level of $10^{-3}$ which might seem detectable, however in practice crowding by fainter stars will limit the accuracy of relative flux measurements.

General relativistic effects perturbing the physical orbits of stars have been studied thoroughly by Weinberg et al. (2004). They are important for stars in the inner central regions and therefore contain complementary information to the distortions by light deflection. In terms of the deflection of light, the rotation of a black hole with angular momentum parameter $a$ (normalized such that $a = r_g/2$ for maximal rotation) produces a first order correction to (1) for light rays propagating in the equatorial plane of $\Omega^2 r_g a/(r_g u - 1)^2$ and the central dark mass with future telescopes. According to Weinberg et al. (2004) the proposed Thirty Meter telescope (TMT) will be able to measure the orbits of $\sim 100$ stars at distances of $300 - 3000\,\text{AU}$ from the galactic center. The astrometric accuracy of the TMT will be $0.5\,\text{mas}$ and could as good as $0.1\,\text{mas}$. According to Table 3 of Ghez et al. (2003) the orbits of 2 out of 7 stars have inclinations in the range $85^\circ \lesssim i \lesssim 95^\circ$. Therefore in a sample of a 100 stars we expect more than ten stars to lie within the range of interest here.

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added to the left hand side. In this $\Omega$ is the inverse of the nearest approach distance and since $a < r_g/2$, this correction is significant only for rays grazing spheres of radii of a few $r_g$ and so the chances of detecting black hole spin via lensing of orbiting stars is therefore very small.

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