Resource Letter BEC-1: Bose-Einstein Condensates in Trapped Dilute Gases

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This Resource Letter provides a guide to the literature on Bose-Einstein condensation in trapped dilute gases. Journal articles and books are cited for the following topics: history, technological advances, condensates as quantum fluids, effects of interatomic interactions, condensates as matter waves, condensate optics, multiple condensates, lower dimensions, spectroscopy and precision measurement, entanglement, and cosmology.

I. INTRODUCTION

It is amusing to note that the field of Bose-Einstein condensation in dilute gases, extremely rich and vibrant today, traces its origins to a rejected paper in 1924. The author of the paper, S. N. Bose, resubmitted his paper on what would become known as “Bose statistics” to no less an authority than Albert Einstein, who arranged for its publication. In a subsequent trilogy of papers, published in 1925, Einstein considered the implications of Bose statistics for an ideal gas and determined that at sufficiently low temperatures and high densities the gas atoms would collect predominantly in the ground state of the system. This macroscopic occupation of a single-particle state, now known as Bose-Einstein condensation (BEC), is peculiar to systems of particles of integer spin (bosons), being forbidden in systems of half-integer spin (fermions) by the Pauli exclusion principle.

The condition for the appearance of a Bose-Einstein condensate may be stated in terms of phase-space density $D$:

$$D = n \Lambda_{dB}^3,$$  \hspace{1cm} (1)

where $n$ is the particle density. Here

$$\Lambda_{dB} = \frac{\hbar}{\sqrt{2 \pi m k_B T}},$$  \hspace{1cm} (2)

is the thermal de Broglie wavelength, a measure of the average size of the atomic wavepacket for an atom of mass $m$ in a sample at temperature $T$. The quantity $\Lambda_{dB}$ is related to the “volume” such an atom occupies, and its product with the particle density is a dimensionless number representing the degree of overlap of the individual atomic wavepackets. When $D$ becomes greater than a number of order 1, the wavepackets begin to overlap and the effects of quantum degeneracy begin to emerge.

Seventy years would pass before Bose-Einstein condensates were experimentally realized in weakly-interacting, near-ideal gases. In the meantime, macroscopic quantum behavior such as superfluidity in liquid helium and superconductivity in metals were convincingly related to the existence of Bose-Einstein condensates (see Ref. 1 and references therein for a review). Theoretical progress had been steady but difficult owing to the complicated nature of the interactions between the particles of which these condensates are composed.

The experimental search for BEC in a weakly-interacting dilute gas began with spin-polarized hydrogen, which was predicted to remain a gas all the way to zero temperature. Two techniques that played central roles in this and subsequent work with the dilute alkalis (see Ref. 18 for a review) are the magnetic trap, which confines atoms by their magnetic moments in strongly inhomogenous magnetic fields, and evaporative cooling, whereby the gas sample is cooled by preferentially permitting the most energetic atoms to escape.

The invention of the magneto-optical trap (MOT) in 1987 heralded the beginning of the modern era of BEC research in the alkalis. A MOT permitted production of a cold atomic sample (10-100 $\mu$K) at reasonable densities (up to $10^{12}$ cm$^{-3}$) consisting of atoms with convenient atomic transitions, such as the alkalis and metastable excited noble gases. Progress in further cooling and increasing the density of the sample was hampered, however, by the rescattering of the very light used to cool the sample.

The next step was to turn off the light and proceed in the dark: MOT-cooled atoms were loaded into a trap and confined by the interaction between their magnetic moments and an inhomogenous applied magnetic field. Once inside this potential, the atoms could be further cooled using evaporative cooling. This process relied on driving transitions between trapped and untrapped internal states of the atom in a spatially-selective fashion — that is, selecting for removal those with the largest amplitude orbits, and hence the largest energies. The remaining atoms rethermalize through collisions at decreasing temperatures until they reach the phase transition and begin to pile up in the ground state of the trap.

The emergence of the condensate was first observed at JILA in 1995, followed closely by reports at Rice University and MIT. Experimental and theoretical work proceeded swiftly, aided by two considerations. On the experimental side, the data collection and analysis generally involved imaging the condensates with a camera, permitting nearly direct visualization of the macroscopically occupied quantum state. This led to some stunning images, such as those of the interference fringes that appear between two overlapping condensates. At the same time, the theoretical analysis remained fairly simple, with qualitatively new features arising from the...
inhomogeneity of the trapping potential.

The Nobel Prize in 2001 was awarded to Eric Cornell, Wolfgang Ketterle, and Carl Wieman for their work on BEC [20,21]. More than fifty groups worldwide now are producing dilute-gas condensates in a variety of different atoms, using an ever-expanding collection of tricks and techniques [22]. An enormous amount of theoretical work has accompanied and driven the experimental progress, with tendrils expanding into many branches of physics.

One consequence of these rapid developments is that the current literature on BEC in dilute gases can be overwhelming to novices and experts alike. This Resource Letter aims to complement the several theoretical review articles that already have appeared by maintaining a slight bias towards experimental work. Even so, the references are necessarily incomplete. The fantastic pace of progress in this field, moreover, insures that this certainly will not be the last Resource Letter on BEC in the weakly interacting dilute Bose gases.

II. JOURNALS

The majority of the technical articles on BEC in dilute gases are found in the first four journals below. Technical articles in both Nature and Science often are accompanied by more general descriptions of the work and its context for a wider audience.

Physical Review Letters
Physical Review A
Science
Europhysics Letters
Journal of Physics B
Journal of Low Temperature Physics
Physics Today
Physics World
Scientific American
Optics Express (electronically published at www.opticsexpress.org)

III. BOOKS

1. Bose-Einstein Condensation, edited by A. Griffin, D. W. Snoke, and S. Stringari, (Cambridge University Press, Cambridge, 1995). A general guide to the various manifestations of Bose-Einstein condensation, with some sections on BEC in dilute gases. (I)

2. Proceedings, Enrico Fermi International Summer School on Bose-Einstein Condensation in Atomic Gases, Varenna, Italy, edited by M. Inguscio, S. Stringari, and C. E. Wieman (IOS Press, Washington, 1999). These proceedings from the Varenna Summer School on Bose-Einstein condensation contain the most important collection of review articles to date. (I)

3. Bose-Einstein Condensates and Atom Lasers, S. Martel-lucci, edited by A. N. Chester, A. Aspect, and M. Inguscio (Kluwer Academic/Plenum Publishers, New York, 2000). These are the proceedings from the Erice lectures on Bose-Einstein condensation. (I)

4. Introduction to Statistical Physics, K. Huang (Taylor and Francis, New York, 2001). Chapter 11 of this textbook contains a concise introduction to Bose-Einstein condensation. (I)

5. Bose-Einstein Condensation in Dilute Gases, C. J. Pethick and H. Smith (Cambridge University Press, Cambridge, 2002). This textbook provides a comprehensive overview to the field. (I)

IV. REVIEW ARTICLES

There already are many review articles on Bose-Einstein condensation for both the novice and expert. Each article listed below covers several different aspects of BEC. Additional articles that provide comprehensive reviews on single topics are listed at the beginning of the relevant sections of this Resource Letter.

6. "Bose-Einstein Condensation with Evaporatively Cooled Atoms," K. Burnett, Contemp. Phys. 37, 1–14 (1996). (I)

7. "The Richtmyer Memorial Lecture: Bose-Einstein Condensation in an Ultracold Gas," C. E. Wieman, Am. J. Phys. 64, 847–855 (1996). (E)

8. Special Issue on Bose-Einstein Condensation, edited by K. Burnett, M. Edwards, and C. W. Clark, J. Res. NIST 101, 419–600 (1996). (I)

9. "Bose-Einstein Condensation," C. G. Townsend, W. Ketterle, and S. Stringari, Phys. World 10, 29–34 (1997). (E)

10. "Bose-Einstein Condensation," I. F. Silvera, Am. J. Phys. 65, 570–574 (1997). This review contains some simple text-book style problems. (E)

11. "The Bose-Einstein Condensate," E. A. Cornell and C. E. Wieman, Sci. Am. 278(3), 40–45 (March 1998). (E)

12. "The Physics of Trapped Dilute-Gas Bose-Einstein Condensates," A. S. Parkins and D. F. Walls, Phys. Rep. 303, 1–50 (1998). (I)

13. "Experimental Studies of Bose-Einstein Condensation," W. Ketterle, Phys. Today 52(12), 30–35 (December 1999). (E)

14. "The Theory of Bose-Einstein Condensation of Dilute Gases," K. Burnett, M. Edwards, and C. W. Clark, Phys. Today 52(12), 37–42 (December 1999). (E)

15. "Bose Condensates Make Quantum Leaps and Bounds," Y. Castin, R. Dum, and A. Sinatra, Phys. World 12(8), 37–42 (August 1999). (E)

16. "Theory of Bose-Einstein Condensation in Trapped Gases," F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 71, 463–512 (1999). (I)

17. "Bose-Einstein Condensation in the Alkali Gases: Some Fundamental Concepts," A. J. Leggett, Rev. Mod. Phys. 73, 307–356 (2001). (I)

18. "Bose-Einstein Condensation of Trapped Atomic Gases," Ph. W. Courteille, V. S. Bagnato, and V. I. Yukanov, Laser Phys. 11, 659–800 (2001). (A)

19. "Bose-Einstein Condensation of Atomic Gases," J. R. Anglin and W. Ketterle, Nature 416, 211–218 (2002). This is one of six articles on the physics of cold atoms in this issue of Nature. (E)

20. "Nobel Lecture: Bose-Einstein Condensation in a Dilute Gas, the First 70 Years and Some Recent Experiments," E. A. Cornell and C. E. Wieman, Rev. Mod. Phys. 74, 875–893 (2002). (E)

21. "Nobel Lecture: When Atoms Behave as Waves: Bose-Einstein Condensation and the Atom Laser," W. Ketterle, Rev. Mod. Phys. 74, 1131–1151 (2002). (E)

V. WEB SITES

22. The BEC Online Bibliography at Georgia Southern University: http://amo.phy.gsu.edu:80/bec.html/bibliography.html contains links to a large number of preprints and up-to-date information on the current status of the field.

23. The Electronic Preprint Archive (http://cul.arxiv.org/) also contains preprints. These papers typically are found in the cond-mat section.

24. The Physics 2000 web site, http://www.colorado.edu/physics/2000/bec/index.html, has
descriptions and information for the general audience, and some interesting computer simulations.

25. Eric A. Cornell, Wolfgang Ketterle, and Carl E. Wieman won the 2001 Nobel Prize in physics for “for the development of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates.” More details are available at the official Nobel web site http://www.nobel.se/physics/laureates/2001/index.html.

26. Recent experiments from the groups of E. A. Cornell and C. E. Wieman are detailed on the JILA web site http://jila.colorado.edu/bec/. The JILA group also maintains a comprehensive bibliography of published works concerning BEC, complementary to the preprint listing in Ref. 29.

27. Recent experiments from the group of W. Ketterle are discussed on the MIT web site http://cua.mit.edu/ketterle_group/.

28. There has been considerable experimental and theoretical work on BEC at NIST, as documented on their web site http://bec.nist.gov/.

VI. PARTICULAR TOPICS

A. History

Additional historical perspectives can be found in articles in Refs. 12 as well as Ref. 29.

29. Subtle is the Lord ... The Science and the Life of Albert Einstein, A. Pais (Oxford University Press, New York, 1982). Chapters 19 and 23 are of particular relevance. (E)

30. “Quantentheorie des Einatomigen Idealen Gases. Zweite Abhandlung,” A. Einstein, Sitzungsberichte der Preussischen Akademie der Wissenschaften 1, 3–14 (1925). This is the paper in which Einstein establishes the existence of a condensate in an ideal gas. Although it is written in German, it should eventually appear in translation as a part of Einstein’s collected works. (E)

31. “The Phenomenon of Liquid Helium and the Bose-Einstein Degeneracy,” F. London, Nature 141, 643–644 (1938). This foundation paper is available online through the Nature Publishing Group Physics Portal (http://www.nature.com/physics/). (E)

32. “The Possible Superfluid Behaviour of Hydrogen Atom Gases and Liquids,” C. E. Hecht, Physica 25, 1159–1161 (1959). (I)

33. “Possible ‘New’ Quantum Systems,” W. C. Stwalley and L. H. Noanow, Phys. Rev. Lett. 36, 910–913 (1976). (I)

34. “The Stabilization of Atomic Hydrogen,” T. F. Silvera and J. Walraven, Sci. Am. 240(1), 66–74 (January 1982). (E)

35. “Lectures on Spin-Polarized Hydrogen,” T. J. Greytak and D. Kleppner, in New Trends in Atomic Physics, Vol. 2, edited by G. Gryenberg and R. Stora (North-Holland, New York, 1984), pp. 1125–1230. This set of lecture notes is from the 38th Les Houches summer school in 1982. (I)

2. First Realizations

Dilute-gas Bose-Einstein condensates have been realized in $^8\text{Rb}$, $^7\text{Li}$, $^{23}\text{Na}$, $^{85}\text{Rb}$, $^{41}\text{K}$, metastable $^3\text{He}$, and $^{133}\text{Cs}$. The following papers give accounts of the production of the first condensates in these systems — an introduction to the BEC “family tree.”

36. “Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor,” M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science 269, 198–201 (1995). (I)

37. “Evidence of Bose-Einstein Condensation in an Atomic Gas with Attractive Interactions,” C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet, Phys. Rev. Lett. 75, 1687–1690 (1995); ibid. 79, 1170 (1997). (I)

38. “Bose-Einstein Condensation in a Gas of Sodium Atoms,” K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. 75, 2967–2973 (1995). (I)

39. “Bose-Einstein Condensation of Atomic Hydrogen,” D. G. Fried, T. C. Killian, L. Willmann, D. Landhuis, S. C. Moss, D. Kleppner, and T. J. Greytak, Phys. Rev. Lett. 81, 3811–3814 (1998). (I)

40. “Stable $^{85}\text{Rb}$ Bose-Einstein Condensates with Widely Tunable Interactions,” S. L. Cornish, N. R. Claussen, J. L. Roberts, E. A. Biech, A. Simonis, and M. Inguscio, Science 294, 1320–1322 (2001). (I)

41. “Bose-Einstein Condensation of Potassium Atoms by sympathetic Cooling,” G. Modugno, G. Ferrari, G. Roati, R. J. Brecha, A. Simonis, and M. Inguscio, Phys. Rev. Lett. 85, 1795–1798 (2000). (I)

42. “Bose-Einstein Condensates of Metallic Atoms,” A. Robert, O. Sirjean, A. Browaeys, J. Poupard, S. Nowak, D. Boiron, C. I. Westbrook, and A. Aspect, Science 292, 461–464 (2001). (I)

43. “Bose-Einstein Condensation of Metastable Helium,” F. P. Santos, J. Léonard, J. Wang, C. J. Barrelet, F. Perales, E. Rasel, C. S. Unnikrishnan, M. Leduc, and C. Cohen-Tannoudji, Phys. Rev. Lett. 86, 3459–3463 (2001). (I)

44. “Bose-Einstein Condensation of Cesium,” T. Weber, J. Herbig, M. Mark, H.-C. Nägerl, and R. Grimm, Science 299, 232–235 (2003). (I)

3. Prospects in Other Atoms

Experimenters are working hard on extending the dilute-gas techniques to other atoms, as this brief list suggests.

45. “Prospects for Bose-Einstein Condensation of Metastable Neon Atoms,” H. C. W. Beijerinck, E. J. D. Vredenburg, R. J. Stas, M. R. Doery, and J. G. C. Tempelmeaars, Phys. Rev. A 61, 026907/1–15 (2000). (I)

46. “Optical-Dipole Trapping of Sr Atoms at a High Phase-Space Density,” T. Ido, Y. Isoya, and H. Katori, Phys. Rev. A 61, 061403/1–4 (2000). (I)

47. “Evaporative Cooling of Atomic Chromium,” J. D. Weinstein, R. deCarvalho, C. I. Hancox, and J. M. Doyle, Phys. Rev. A 65, 021604/1–4 (2002). (I)

B. Technological Advances

Advances in technology have led to new methods of confining atoms, creating and imaging condensates, and transporting and guiding them into new experimental configurations. Using the newly available techniques, experimenters bring an exquisite level of control and manipulation to these systems.

48. “Resource Letter TNA-1: Trapping of Neutral Atoms,” N. R. Newbury and C. Wieman, Am. J. Phys. 64, 18–20 (1996). This Resource Letter contains a large number of references on the advances that led to Bose-Einstein condensation in dilute gases. (E)

49. “Evaporative Cooling of Trapped Atoms,” W. Ketterle and N. J. van Druten, Adv. At. Mol. Opt. Phys. 37, 181–236 (1996). Review article. (I)

50. “Bose-Einstein Condensation in a Tightly Confining DC Magnetic Trap,” M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, D. S. Durfee, and W. Ketterle, Phys. Rev. Lett. 77, 416–419 (1996). (I)

51. “Direct, Nondestructive Observation of a Bose Condensate,” M. R. Andrews, M.-O. Mewes, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. 80, 2027–2030 (1998). (I)

52. “Optical Confinement of a Bose-Einstein Condensate,” D. M. Stamper-Kurn, M. R. Andrews, A. P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, and W. Ketterle, Phys. Rev. Lett. 80, 5292–5295 (1998). (I)

53. “Bose-Einstein Condensation in a Quadrupole-Ioffe-Configuration Trap,” T. Esslinger, I. Bloch, and T. W. Hansch, Phys. Rev. A 58, R2664–R2667 (1998). (I)
The scattering length vanishes for Einstein’s noninteracting ideal gas ($a = 0$), turning the GP equation into a linear Schrödinger-like equation for the macroscopic wavefunction. In the opposite limit, that is, \[
\frac{8\pi a N}{a_{HO}} \gg 1
\]
(where $a_{HO} = \sqrt{\hbar/(m\omega)}$ for atoms in a trap of frequency $\omega$), the mean-field term of the GP equation dominates. One then can neglect the kinetic-energy term (first term on the left-hand side of Eq. (3)) to arrive at an algebraic equation for $\Psi$. This is the Thomas-Fermi approximation.

The GP equation is an approximation that is valid provided the gas is dilute, that is, $n|a|^3 \ll 1$; otherwise, corrections beyond the mean-field approximation are required.

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the density, \( \rho \approx |\Psi|^2 \), and the velocity field \( \mathbf{v} = \hbar \nabla \phi / m \), where \( \phi \) is the phase of the condensate order parameter: 
\[ \Psi = \sqrt{\rho} e^{i \phi} \]

For a condensate in a noninteracting (ideal) gas, the excitation spectrum is entirely particle-like:
\[ E = \frac{\hbar^2 q^2}{2m} \]  
(7)

where \( q \) is the wavenumber of the excitation. For condensates with repulsive interactions, the spectrum of long-wavelength excitations becomes phonon-like, with energies proportional to the speed of sound waves in the condensate:
\[ E \approx \hbar cq \]  
(8)

where \( c \) is the speed of sound in the condensate. This modification of the long-wavelength excitation spectrum to collective behavior is responsible for the phenomenon of superfluidity (see below).

Condensates with attractive interactions have an imaginary excitation spectrum and therefore possess an instability; references are listed in Sec. VI.D.2, below.

3. Finite Temperature

At finite temperatures, there also are thermal excitations as the condensate interacts with a surrounding cloud of noncondensed atoms. Two regimes of this interaction have been studied. At low temperatures and densities, the system is in the collisionless regime; the mean-field is responsible for the interactions. At higher temperatures and densities, the system is described by a pair of coupled hydrodynamic equations within a two-fluid model of the interactions. The coupling of the condensate to the reservoir of thermal excitations can both shift the frequencies of the collective modes and lead to their damping.
101. “Dark Solitons in Bose-Einstein Condensates,” S. Burger, K. Bongs, S. Detttmer, W. Ertmer, K. Sengstock, A. Sanpera, G. Shlyapnikov, and M. Lewenstein, Phys. Rev. Lett. 83, 5198–5201 (1999).

102. “Generating Solitons by Phase Engineering of a Bose-Einstein Condensate,” J. Denschlag, J. E. Simsarian, D. L. Feder, C. W. Clark, L. A. Collins, J. Cubizolles, L. Deng, E. H. Hagley, K. Helmer, W. P. Reinhardt, S. L. Rolston, B. I. Schneider, and W. Phillips, Science 287, 97–101 (2000). (I)

103. “Formation of a Matter-Wave Bright Soliton,” L. Khaykovich, F. Schreck, G. Ferrari, T. Bourdel, J. Cubizolles, L. D. Carr, Y. Castin, and C. Salomon, Science 296, 1290–1293 (2002). (I)

104. “Formation and Propagation of Matter-Wave Soliton Trains,” K. E. Strecker, G. B. Partridge, A. G. Truscott, and R. G. Hulet, Nature 417, 150–153 (2002). (I)

105. “Dynamics of Dark Solitons in Elongated Bose-Einstein Condensates,” A. Muryshev, G. V. Shlyapnikov, W. Ertmer, K. Sengstock, and M. Lewenstein, Phys. Rev. Lett. 89, 110401/1–4 (2002). (I)

5. Superfluidity

The crossover between the low-energy excitations and high-energy excitations mentioned above is the basis for the existence of superfluidity, or dissipationless mass flow, in a Bose-Einstein condensate. One criterion of superfluidity is that obstacles moving within the condensate must exceed a critical velocity before they can transfer momentum to the condensate. The Landau criterion for superfluidity states:

\[ v_{\text{critical}} = \min \left( \frac{E_p}{p} \right) \]  

(9)

where \( E_p \) and \( p \) are the energy and momentum of an excitation, and \( v_{\text{critical}} \) is the velocity below which no excitations can be generated and superfluid flow is possible.

For an excitation spectrum that is particle-like, Eq. [7] it is clear that the critical velocity is zero, that is, no superfluid flow is possible. For a phonon-like excitation spectrum, Eq. [8] however, the critical velocity is the speed of sound, and superfluidity becomes possible.

106. “Observation of the Scissors Mode and Evidence for Superfluidity of a Trapped Bose-Einstein Condensed Gas,” O. M. Maragó, S. A. Hopkins, J. Arlt, E. Hodby, G. Hechenblaikner, and C. J. Foot, Phys. Rev. Lett. 84, 2056–2059 (2000). (I)

107. “Suppression and Enhancement of Impurity Scattering in a Bose-Einstein Condensate,” A. P. Chikkatur, A. Görlitz, D. M. Stamper-Kurn, S. Inouye, S. Gupta, and W. Ketterle, Phys. Rev. Lett. 85, 483–486 (2000). (I)

108. “Observation of Superfluid Flow in a Bose-Einstein Condensed Gas,” R. Onofrio, C. Raman, J. M. Vogels, J. R. Abo-Shaeer, A. P. Chikkatur, and W. Ketterle, Phys. Rev. Lett. 85, 2228–2231 (2000). (I)

109. “Superfluid and Dissipative Dynamics of a Bose Condensate in a Periodic Optical Potential,” S. Burger, F. S. Cataliotti, C. Fort, F. Minardi, M. Inguscio, M. L. Chiofalo, and M. P. Tosi, Phys. Rev. Lett. 86, 4447–4450 (2001). (I)

110. “Direct Observation of Irrotational Flow and Evidence of Superfluidity in a Rotating Bose-Einstein Condensate,” G. Hechenblaikner, E. Hodby, S. A. Hopkins, O. M. Maragó, and C. J. Foot, Phys. Rev. Lett. 88, 070406/1–4 (2002). (I)

6. Vortices

According to the hydrodynamic formulation of the GP equation, the gradient of the phase of the condensate order parameter is related to the condensate velocity by

\[ \mathbf{v} = \frac{\hbar}{m} \nabla \phi. \]  

(10)

The phase is therefore a “potential” for the velocity field of the condensate, and thus does not permit rotational flow in a simply-connected geometry (since \( \nabla \times \mathbf{v} = 0 \) always). In a multiply-connected geometry, however, in which the condensate density goes to zero in some region (such as a toroidal trap), the only restriction on the order parameter is that it be single-valued. The phase variation in any loop traced around the excluded region is restricted to \( 2n\pi \), where \( n \) is any integer. The corresponding mass flow is a quantized vortex of winding number \( n \).

Vortices were one of the original signs of superfluidity in liquid helium, but their generation and detection in dilute-gas condensates eluded experimenters for several years. This initial frustration has led since to a recent blossoming of experimental techniques and realizations of vortices — from single vortices in multiple-spin state condensates to lattices of vortices within a single condensate. A sampling of the relevant literature is given below.

111. “Vortices in a Trapped Dilute Bose-Einstein Condensate,” A. L. Fetter and A. A. Svidzinsky, J. Phys.: Condens. Matter 13, R135–R194 (2001). Review article. (I)

112. “Vortex Stability and Persistent Currents in Trapped Bose Gases,” D. S. Rokhsar, Phys. Rev. Lett. 79, 2164–2167 (1997). (I)

113. “Excitation Spectroscopy of Vortex States in Dilute Bose-Einstein Condensed Gases,” B. J. Dodd, K. Burnett, M. Edwards, and C. W. Clark, Phys. Rev. A 56, 587–590 (1997). (I)

114. “Predicted Signatures of Rotating Bose-Einstein Condensates,” R. J. Butts and D. S. Rokhsar, Nature 397, 327–329 (1999). (I)

115. “Phase Diagram of Quantized Vortices in a Trapped Bose-Einstein Condensate,” S. Stringari, Phys. Rev. Lett. 82, 4371–4375 (1999). (I)

116. “Vortex Stability of Interacting Bose-Einstein Condensates Confined in Anisotropic Harmonic Traps,” D. L. Feder, C. W. Clark, and B. I. Schneider, Phys. Rev. Lett. 85, 4956–4959 (1999). (I)

117. “Vortices in a Bose-Einstein Condensate,” M. R. Matthews, B. P. Anderson, P. C. Haljan, D. S. Hall, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. 83, 2498–2501 (1999). (I)

118. “Vortex Formation in a Stirred Bose-Einstein Condensate,” K. W. Madison, F. Chevy, W. Kohnleben, and J. Dalibard, Phys. Rev. Lett. 84, 806–809 (2000). (I)

119. “Measurement of the Angular Momentum of a Rotating Bose-Einstein Condensate,” F. Chevy, K. W. Madison, and J. Dalibard, Phys. Rev. Lett. 85, 2223–2227 (2000). (I)

120. “Vortex Precession in Bose-Einstein Condensates: Observations with Filled and Empty Cores,” B. P. Anderson, P. C. Haljan, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. 85, 2857–2860 (2000). (I)

121. “Use of Surface-Wave Spectroscopy to Characterize Tilt Modes of a Vortex in a Bose-Einstein Condensate,” P. C. Haljan, B. P. Anderson, I. Coddington, and E. A. Cornell, Phys. Rev. Lett. 86, 2922–2925 (2001). (A)

122. “Watching Dark Solitons Decay into Vortex Rings in a Bose-Einstein Condensate,” B. P. Anderson, P. C. Haljan, C. A. Regal, D. L. Feder, L. A. Collins, C. W. Clark, and E. A. Cornell, Phys. Rev. Lett. 86, 2926–2929 (2001). (I)

123. “Stationary States of a Rotating Bose-Einstein Condensate: Routes to Vortex Nucleation,” K. W. Madison, F. Chevy, V. Bretin, and J. Dalibard, Phys. Rev. Lett. 86, 4443–4446 (2001). (I)

124. “Observation of Vortex Phase Singularities in Bose-Einstein Condensates,” S. Inouye, S. Gupta, T. Rosenband, A. P. Chikkatur, A. Görlitz, T. L. Gustavson, A. E. Leanhardt, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. 87, 080402/1–4 (2001). (I)
7. Formation

The process by which a Bose-Einstein condensate forms out of a thermal gas as it is cooled below the transition temperature is becoming increasingly well understood. It has been known for some time that there is an enhancement of scattering into the condensate proportional to the number of atoms already in it (Bosonic stimulated enhancement of scattering into the condensate proportionally). The tunnelling (and hence the binding energy is stored internally by flipping the magnetic field. This is the Feshbach resonance. Experiments have found that inelastic processes also depend upon the value of a parameter external to the condensate, such as the magnetic field. Loosely speaking, two atoms can tunnel into a bound state in which the tunnelling (and hence a, by the scattering cross-section) can be enhanced or reduced by simply varying the magnetic field. This is the Feshbach resonance.

Experiments have found that inelastic processes also tend to increase in the vicinity of a Feshbach resonance, in some cases to the point at which experiments to tune a are rendered impossible. Stable condensates with tunnelling interactions in $^{85}$Rb and $^{133}$Cs nevertheless have been created to date. Another use of Feshbach resonances has been to create bright solitons (see Refs. [103] and [104]).

1. Tunable Interactions: Feshbach Resonances

In certain instances the s-wave scattering length $a$ can depend upon the value of a parameter external to the condensate, such as the magnetic field. Loosely speaking, two atoms can tunnel into a bound state in which the binding energy is stored internally by flipping the spin of one of the atoms. The bound-state energies can depend on the value of the magnetic field, which means that the tunnelling (and hence $a$, by the scattering cross-section) can be enhanced or reduced by simply varying the magnetic field. This is the Feshbach resonance.

Experiments have found that inelastic processes also tend to increase in the vicinity of a Feshbach resonance, in some cases to the point at which experiments to tune $a$ are rendered impossible. Stable condensates with tunnelling interactions in $^{85}$Rb and $^{133}$Cs nevertheless have been created to date. Another use of Feshbach resonances has been to create bright solitons (see Refs. [103] and [104]).

D. Effects of Interatomic Interactions

As mentioned above, nearly every aspect of the condensate depends critically on the s-wave scattering length $a$. From measurements of macroscopic properties of the condensate, such as size and shape, one can gain insight into binary elastic collisions. Two and three-body inelastic collisions also play an important role in the fate of the condensate. The following review articles present some of the details of ultracold collisions that are relevant to Bose-Einstein condensates.

Atoms involved in elastic collisions ("good" collisions) retain their internal states but can redistribute momentum and energy. These collisions are responsible for the thermalization of the samples that permit evaporative cooling to proceed efficiently. Inelastic collisions ("bad" collisions) often result in heating and trap loss, although they are sometimes subjects of interest in their own right. See also Ref. [138] above, and Ref. [139] below.

136. "Vortex Nucleation in a Stirred Bose-Einstein Condensate," C. Raman, J. R. Abo-Shaeer, J. M. Vogels, K. Xu, and W. Ketterle, Phys. Rev. Lett. 87, 210402/1–4 (2001). (I)

137. "Driving Bose-Einstein Condensate Vorticity with a Rotating Normal Cloud," P. C. Haljan, I. Coddington, P. Engels, and E. A. Cornell, Phys. Rev. Lett. 87, 210403/1–4 (2001). (I)

138. "Observation of Vortex Lattices in Bose-Einstein Condensates," J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle, Science 292, 476–479 (2001). (I)

139. "Vortex Nucleation in Bose-Einstein Condensates in an Oblate, Purely Magnetic Potential," E. Hodby, G. Heckenblaikner, S. A. Hopkins, O. M. Maragó, and C. J. Foot, Phys. Rev. Lett. 88, 010405/1–4 (2002). (I)

140. "Formation and Decay of Vortex Lattices in Bose-Einstein Condensates at Finite Temperatures," J. R. Abo-Shaeer, C. Raman, and W. Ketterle, Phys. Rev. Lett. 88, 070409/1–4 (2002). (I)

141. "Formation of the Condensate in a Dilute Bose Gas," H. T. C. Stoof, Phys. Rev. Lett. 66, 3148–3151 (1991). (A)

142. "Initial Stages of Bose-Einstein Condensation," H. T. C. Stoof, Phys. Rev. Lett. 78, 768–771 (1997). (I)

143. "Kinetics of Bose-Einstein Condensation in a Trap," C. W. Gardiner, P. Zoller, R. J. Ballagh, and M. Davis, Phys. Rev. Lett. 79, 1783–1786 (1997). (I)

144. "Bosonic Stimulation in the Formation of a Bose-Einstein Condensate," H.-J. Miesner, D. M. Stamper-Kurn, M. R. Andrews, D. S. Durfee, S. Inouye, and W. Ketterle, Science 279, 1005–1007 (1998). (I)

145. "Reversible Formation of a Bose-Einstein Condensate," D. M. Stamper-Kurn, H.-J. Miesner, A. P. Chikkatur, S. Inouye, J. Stenger, and W. Ketterle, Phys. Rev. Lett. 81, 2194–2197 (1998). (I)

146. "Quantum Kinetic Theory. VI. The Growth of a Bose-Einstein Condensate," M. D. Lee and C. W. Gardiner, Phys. Rev. A 62, 033606/1–26 (2000); and references therein. (A)

147. "Direct Observation of Growth and Collapse of a Bose-Einstein Condensate with Attractive Interactions," J. M. Gerton, D. Strekalov, I. Prodan, and R. G. Hulet, Nature 408, 692–695 (2000). (I)

148. "Growth of Bose-Einstein Condensates from Thermal Vapor," M. Köhl, M. J. Davis, C. W. Gardiner, T. W. Hänsch, and T. Esslinger, Phys. Rev. Lett. 88, 080402/1–4 (2002). (I)
2. Negative Scattering Length

If the scattering length is negative, the interatomic interactions are attractive. In a homogeneous gas the excitation frequencies are imaginary, which corresponds to an instability that prevents condensates from forming. In an inhomogeneous gas, the kinetic energy can help stabilize against collapse, although for sufficiently large numbers of atoms the attractive interactions cause the condensate to collapse.

Recent experiments also have induced a collapse as the scattering length is switched rapidly from positive to negative using a Feshbach resonance (see above). The condensate rapidly implemes, generating subsequent dynamics that often exhibit explosions of atoms (the "Bosenova") and other behavior that is not yet fully understood.

E. Condensates as Matter Waves

1. Phase Coherence

The description of a Bose-Einstein condensate in terms of an order parameter carries with it the notion of a phase; indeed, we have already seen that the phase plays the role of a velocity-field potential in the hydrodynamic description of the BEC. The phase, by itself, is not observable. If two condensates are brought together, however, interference patterns are expected to emerge based on the relative phase between the two condensates.

Complicating matters slightly, the number-phase uncertainty relation,

$$\Delta N \Delta \phi \approx 1,$$

prevents complete knowledge of both the number of atoms in the condensate and its phase. A condensate in a number state has a vanishing order parameter and thus cannot be said to have a well-defined phase at all. Nevertheless, interference patterns still emerge as the detection process entangles the two condensates.

These papers examine interference between two (or more) condensates, both in space (where the interference manifests itself in the atomic density) and in time (using different internal states).

158. “Quantum Phase of a Bose-Einstein Condensate with an Arbitrary Number of Atoms,” J. Javanainen and S. M. Yoo, Phys. Rev. Lett. 76, 161–164 (1996). (I)

159. “Observation of Interference between Two Bose-Einstein Condensates,” M. R. Andrews, C. G. Townsend, H.-J. Miesner, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science 275, 637–641 (1997). (I)

160. “Transition from Phase Locking to the Interference of Independent Bose Condensates: Theory versus Experiment,” A. Röhrli, M. Naraschewski, A. Schenzle, and H. Wallis, Phys. Rev. Lett. 82, 4143–4146 (1997). (A)

161. “Measurements of Relative Phase in Two-Component Bose-Einstein Condensates,” D. S. Hall, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. 81, 1543–1546 (1998). (I)

162. “Phase Standard for Bose-Einstein Condensates,” J. A. Dunningham and K. Burnett, Phys. Rev. Lett. 82, 3729–3733 (1999). (I)

163. “Measurement of the Coherence of a Bose-Einstein Condensate,” E. Hagley, L. Deng, M. Kozuma, M. Trippenbach, Y. Band, M. Edwards, M. Doery, P. Julienne, K. Helmerson, S. Rolston, and W. Phillips, Phys. Rev. Lett. 83, 3112–3115 (1999). (I)

164. “Imaging the Phase of an Evolving Bose-Einstein Condensate Wave Function,” J. E. Simsarian, J. Denschlag, M. Edwards, C. W. Clark, L. Deng, E. W. Hagley, K. Helmerson, S. L. Rolston, and W. D. Phillips, Phys. Rev. Lett. 85, 2040–2043 (2000). (I)

165. “Measurement of the Spatial Coherence of a Trapped Bose Gas at the Phase Transition,” I. Bloch, T. W. Hängsch, and T. Esslinger, Nature 403, 166–170 (2000). (I)

166. “Exploring Phase Coherence in a 2D Lattice of Bose-Einstein Condensates,” M. Greiner, I. Bloch, O. Mandel, T. W. Hängsch, and T. Esslinger, Phys. Rev. Lett. 87, 160405/1–4 (2001). (I)

167. “Observation of Phase Fluctuations in Elongated Bose-Einstein Condensates,” S. Dettmer, D. Hellweg, P. Ryttty, J. J. Arlt, W. Ertmer, K. Sengstock, D. S. Petrov, G. V. Shlyapnikov, H. Kreutzmann, L. Santos, and M. Lewenstein, Phys. Rev. Lett. 87, 160406/1–4 (2001). (A)

168. “Time-Domain Atom Interferometry across the Threshold for Bose-Einstein Condensation,” F. Minardi, C. Fort, P. Maldascoli, M. Modugno, and M. Inguscio, Phys. Rev. Lett. 87, 170401/1–4 (2001). (I)

169. “Expansion of a Coherent Array of Bose-Einstein Condensates,” P. Pedri, L. Pitaevskii, S. Stringari, C. Fort, S. Burger, F. S. Cataliotti, P. Maldascoli, F. Minardi, and M. Inguscio, Phys. Rev. Lett. 87, 170402/1–4 (2001). (I)

170. “Dynamics of a Bose-Einstein Condensate at Finite Temperature in an Atom-Optical Coherence Filter,” F. Ferlaino, P. Maldascoli, S. Burger, F. S. Cataliotti, C. Fort, M. Modugno, and M. Inguscio, Phys. Rev. A 66, 011604/1–4 (2002). (I)
Bose-Einstein condensates also are expected to have perfect higher-order coherence as well as first-order phase coherence. These coherences manifest themselves in the two and three-body correlations, which are discovered experimentally by examining the mean-field energy and the three-body recombination rate, respectively.

171. “Coherence, Correlations, and Collisions: What One Learns about Bose-Einstein Condensates from Their Decay,” E. A. Burt, R. W. Ghrist, C. J. Myatt, M. J. Holland, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. 79, 337–340 (1997). (I)

172. “Coherence Properties of Bose-Einstein Condensates and Atom Lasers,” W. Ketterle and H.-J. Miesner, Phys. Rev. A 56, 3291–3293 (1997). (I)

173. “Characterizing the Coherence of Bose-Einstein Condensates and Atom Lasers,” R. Dodd, C. W. Clark, M. Edwards, and K. Burnett, Opt. Expr. 1, 284–292 (1997). (I)

174. “Spatial Coherence and Density Correlations of Trapped Bose Gases,” M. Naraschewski and R. J. Glauber, Phys. Rev. A 59, 4535–4607 (1999). (A)

3. Tunnelling, Bloch Oscillations, and Josephson Effect

A lattice of Bose-Einstein condensates can be produced by placing a single condensate into a series of microtraps formed by an optical standing wave. Typically, the condensate atoms can tunnel from site to site within the lattice, exhibiting collective behavior reminiscent of the Josephson effect in superconductors. They maintain a phase coherence between the lattice sites by occupying delocalized states. When the trap depths are increased, however, the tunnelling becomes suppressed, and the lattice sites contain number-squeezed states — that is, the phase coherence between sites is lost, and well-defined numbers of atoms occupy each lattice site.

175. “Boson Localization and the Superfluid-Insulator Transition,” M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B 40, 546–570 (1989). (I)

176. “Cold Bosonic Atoms in Optical Lattices,” D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. 81, 3108–3111 (1998). (I)

177. “Macroscopic Quantum Interference from Atomic Tunnel Arrays,” B. P. Anderson and M. A. Kasevich, Science 282, 1686–1689 (1998). (I)

178. “Squeezed States in a Bose-Einstein Condensate,” C. Orzel, A. K. Tuchman, M. L. Fengshan, M. Yasuda, and M. A. Kasevich, Science 291, 2386–2389 (2001). (I)

179. “Dynamical Tunnelling of Ultracold Atoms,” W. K. Hensinger, H. Häffner, A. Browaeys, N. R. Heckenberg, K. Helmerson, C. McKenzie, G. J. Milburn, W. D. Phillips, S. L. Rolston, H. Rubinsztein-Dunlop, and B. Yurkev, Nature 412, 52–55 (2001). (I)

180. “Josephson Junction Arrays with Bose-Einstein Condensates,” F. S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Tombesi, A. Smerzi, and M. Inguscio, Science 293, 843–846 (2001). (I)

181. “Bloch Oscillations and Mean-Field Effects of Bose-Einstein Condensates in 1D Optical Lattices,” O. Morsch, J. H. Müller, M. Cristiani, D. Ciampini, and E. Arimondo, Phys. Rev. Lett. 87, 140402/1–4 (2001). (I)

182. “Quantum Phase Transition from a Superfluid to a Mott Insulator in a Gas of Ultracold Atoms,” M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415, 39–44 (2002). (I)

4. Collapses and Revivals

A condensate wave packet can be expanded in states of definite number, \( N \), each with a related chemical potential \( \mu_N \). Each of these states has a phase that evolves in time as \( N \mu_N t/\hbar \). Calculation of the macroscopic wavefunction in terms of these states for relatively small numbers of atoms in the condensate reveals a periodic collapse and revival of the condensate phase. These collapses and revivals affect the interference properties of the condensate. In the thermodynamic limit (\( N \to \infty \)) the collapse time becomes infinitely long and the condensate phase is well-defined. It is in this limit that the macroscopic wavefunction can be considered an order parameter.

183. “Quantum Phase Diffusion of a Bose-Einstein Condensate,” M. Lewenstein and L. You, Phys. Rev. Lett. 77, 3489–3493 (1996). (A)

184. “Collapses and Revivals in the Interference between Two Bose-Einstein Condensates Formed in Small Atomic Samples,” E. M. Wright, T. Wong, M. J. Collett, S. M. Tan, and D. F. Walls, Phys. Rev. A 56, 591–602 (1997). (I)

185. “Collapse and Revival of the Matter Wave Field of a Bose-Einstein Condensate,” M. Greiner, O. Mandel, T. W. Hänsch, and I. Bloch, Nature 419, 51–54 (2002). (I)

F. Condensate Optics

1. Light and Matter Waves

The interaction between coherent light and coherent matter waves produces some of the most fascinating behavior involving Bose-Einstein condensates. Bragg diffraction involves the coherent scattering of light from one laser beam into another, with a corresponding atomic recoil that conserves energy and momentum. In superradiant Rayleigh scattering, the initial incoherent (spontaneous) scattering of light by a condensate establishes a moving matter-wave grating as a fraction of the condensate recoils to conserve momentum. This induced grating then reinforces itself by further stimulated scattering of the incident light beam such that additional atoms join the single moving mode. The matter-wave amplification occurring in superradiant scattering has also been initialized with a controlled “seed” condensate that forms a moving matter-wave grating in conjunction with a larger condensate serving as a “gain medium.” The stimulated scattering in this case reinforces the initially seeded mode.

Coherent matter waves can also interact with each other in a process called four-wave mixing. In this nonlinear effect, three condensates with different wave vectors \( k_1, k_2 \), and \( k_3 \) interact to generate a fourth condensate \( k_4 = k_1 - k_2 + k_3 \).

186. “Nonlinear and Quantum Atom Optics,” S. L. Rolston and W. D. Phillips, Nature 416, 219–224 (2002). Review article. (E)

187. “Four Wave Mixing in the Scattering of Bose-Einstein Condensates,” M. Trippenbach, Y. B. Band, and P. S. Julienne, Opt. Expr. 3, 530–537 (1998). (I)

188. “Amplifying an Atomic Wave Signal using a Bose-Einstein Condensate,” C. K. Law and N. P. Bigelow, Phys. Rev. A 58, 4791–4795 (1998). (A)
An atom laser is generated by coherently removing atoms from a Bose-Einstein condensate, typically by interaction with applied laser light or a radiofrequency electromagnetic field. Both pulsed and quasicontinuous atom-laser beams have been demonstrated. (See the preceding section for the demonstration of a coherent gain mechanism.) A mode-locked atom laser has also been demonstrated (see Ref. [52]).

2. **Atom Lasers**

The alkali atoms (and hydrogen) have two hyperfine levels in their ground states and numerous Zeeman sublevels (“spin states”). Atoms in different internal states are distinguishable from one another, and therefore can form independent (but interacting) condensates. Experimentally, there are several difficulties that arise. Atoms in different hyperfine states tend to undergo exoergic spin-exchange collisions that cause a great deal of heating and atom loss. A fortuitous coincidence in single and triplet scattering lengths suppresses these losses in $^87$Rb, making long-lived double condensates possible. Within the same hyperfine level, different Zeeman sublevels have different magnetic moments, and magnetic traps tend to confine them differently (or not at all). Optical traps, which confine atoms by the interaction of an electric-field gradient and induced electric-dipole moment, are one solution to this difficulty that is finding increasing use (see Ref. [52]).

The physics of condensates in multiple internal states is quite rich. Creation of a double condensate system from a single condensate typically is done by introducing a resonant electromagnetic field. Where interconversion of the spins is possible, metastable spin domains can form. Topological phases can be introduced by driving spatially-selective transitions from one state to the other — as was done in creating the first vortices identified in a dilute-gas condensate (see Ref. [117]).

**G. Multiple Condensates**

### 1. Different Internal States

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208. “Local Spin-Gauge Symmetry of the Bose-Einstein Condensates in Atomic Gases,” T.-L. Ho and V. B. Shenoy, Phys. Rev. Lett. 77, 2555–2559 (1996). (I)
209. “Binary Mixtures of Bose Condensates of Alkali Atoms,” T.-L. Ho and V. B. Shenoy, Phys. Rev. Lett. 77, 3276–3276 (1996). (I)
210. “Production of Two Overlapping Bose-Einstein Condensates by Sympathetic Cooling,” C. J. Myatt, E. A. Burt, R. W. Ghrist, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. 81, 586–589 (1997). (I)
211. “Collisional Stability of Double Bose Condensates,” P. S. Julienne, F. H. Mies, E. Tiesinga, and C. J. Williams, Phys. Rev. Lett. 78, 1880–1883 (1997). (I)
212. “Stability Signature’ in Two-Species Dilute Bose-Einstein Condensates,” C. K. Law, H. Pu, N. P. Bigelow, and J. H. Eberly, Phys. Rev. Lett. 79, 3105–3108 (1998). (I)
213. “Dynamical Response of a Bose-Einstein Condensate to a Discontinuous Change in Internal State,” M. R. Matthews, D. S. Hall, D. S. Jin, J. R. Ensher, C. E. Wieman, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. 78, 251–254 (1997). (I)
214. “Dynamics of Component Separation in a Binary Mixture of Bose-Einstein Condensates,” D. S. Hall, M. R. Matthews, J. R. Ensher, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. 81, 1539–1542 (1998). (I)
215. “Spin Domains in Ground-State Bose-Einstein Condensates,” J. Stenger, S. Inouye, D. M. Stamper-Kurn, H.-J. Miesmer, A. P. Chikkatur, and W. Ketterle, Nature 396, 345–348 (1998). (I)
216. “Dynamics of Two Interacting Bose-Einstein Condensates,” A. Sinatra, P. O. Fedichev, Y. Castin, J. Dalibard, and G. V. Shlyapnikov, Phys. Rev. Lett. 82, 251–254 (1999). (I)
217. “Observation of Metastable States in Spinor Bose-Einstein Condensates,” M. G. Moore and P. Meystre, Phys. Rev. Lett. 82, 5407–5411 (1999). (I)
Chikkarat, S. Inouye, J. Stenger, and W. Ketterle, Phys. Rev. Lett. 83, 661–665 (1999). (I)

219. “Watching a Superfluid Untwist Itself: Recurrence of Rabi Oscillations in a Bose-Einstein Condensate,” M. R. Matthews, B. P. Anderson, P. C. Haljan, D. S. Hall, M. J. Holland, J. E. Williams, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. 83, 3358–3361 (1999). (I)

220. “Preparing Topological States of a Bose-Einstein Condensate,” J. E. Williams and M. J. Holland, Nature 401, 568–572 (1999). (I)

221. “Collective Oscillations of Two Colliding Bose-Einstein Condensates,” P. Maddaloni, M. Modugno, C. Fort, F. Minardi, and M. Inguscio, Phys. Rev. Lett. 85, 2413–2417 (2000). (I)

222. “Skyrmions in a Ferromagnetic Bose-Einstein Condensate,” U. Al Khawaja and H. Stoof, Nature 411, 918–920 (2001). (I)

2. Different Atoms

Double condensates have been produced in a system of $^{41}$K and $^{87}$Rb. Sympathetic cooling of the Rb atoms was used to condense the K atoms; see also Ref. [41].

223. “Prospects for Mixed-Isotope Bose-Einstein Condensates in Rubidium,” J. P. Burke, Jr., J. L. Bohn, B. D. Esry, and C. H. Greene, Phys. Rev. Lett. 80, 2097–2100 (1998). (I)

224. “Two Atomic Species Superfluid,” G. Modugno, M. Modugno, F. Riboli, G. Roati, and M. Inguscio, Phys. Rev. Lett. 89, 190404/1–4 (2002). (I)

3. Atoms and Molecules

A molecule is chemically distinct from the two atoms that it comprises, and a process that produces molecules coherently in fact should create a Bose-Einstein condensate of molecules. Two approaches have been developed. The first is to create molecules by photoassociation, essentially using a stimulated Raman transition to place a pair of atoms into a particular molecular state. The other method uses Feshbach resonances to produce quasibound molecular states.

225. “Superchemistry: Dynamics of Coupled Atomic and Molecular Bose-Einstein Condensates,” D. J. Heinzen, R. Wynar, P. D. Drummond, and K. V. Kheruntsyan, Phys. Rev. Lett. 84, 5029–5033 (2000). (I)

226. “Molecules in a Bose-Einstein Condensate,” R. Wynar, R. S. Freeland, D. J. Han, C. Ryu, and D. J. Heinzen, Science 287, 1016–1019 (2000). (I)

227. “Formation of Pairing Fields in Resonantly Coupled Atomic and Molecular Bose-Einstein Condensates,” M. Holland, J. Park, and R. Walser, Phys. Rev. Lett. 86, 1915–1918 (2001). (A)

228. “Photoassociation of Sodium in a Bose-Einstein Condensate,” C. McKenzie, J. H. Denschlag, H. Höffner, A. Browaeys, L. E. de Araújo, F. K. Fatemi, K. M. Jones, J. E. Simsarian, D. Cho, A. Simon, E. Tiesinga, P. S. Julienne, K. Helmerson, P. D. Lett, S. L. Rolston, and W. D. Phillips, Phys. Rev. Lett. 88, 120403/1–4 (2002). (I)

229. “Atom-Molecule Coherence in a Bose-Einstein Condensate,” E. A. Donley, N. R. Claussen, S. T. Thompson, and C. E. Wieman, Nature 417, 529–533 (2002). (I)

230. “Ramsey Fringes in a Bose-Einstein Condensate between Atoms and Molecules,” S. J. J. M. F. Kokkelmans and M. J. Holland, Phys. Rev. Lett. 89, 180401/1–4 (2002). (A)

4. Bose-Fermi Mixtures

Fermions obey the Pauli exclusion principle and are therefore unable to form Bose-Einstein condensates with-
in motional line narrowing. The relatively high densities in condensates introduce mean-field shifts to atomic resonances, however; these can depend dramatically on the atomic density. The shifts have been used effectively to help determine the onset of degeneracy in hydrogen but present a problem for other precision measurements in which the transition frequency is of primary interest.

244. “Heisenberg-Limited Spectroscopy with Degenerate Bose-Einstein Gases,” P. Bouyer and M. A. Kasevich, Phys. Rev. A 56, R1083–R1086 (1997). (I)

245. “Cold Collision Frequency Shift of the 1S–2S Transition in Hydrogen,” T. C. Killian, D. G. Fried, L. Willmann, D. Landhuis, S. C. Moss, T. J. Greytak, and D. Kleppner, Phys. Rev. Lett. 81, 3807–3811 (1998). (I)

246. “Bragg Spectroscopy of a Bose-Einstein Condensate,” J. Steenberg, S. Inouye, A. P. Chikkatur, D. M. Stamper-Kurn, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. 82, 4569–4573 (1999); ibid. 84, 2283 (2000). (I)

247. “Contrast Interferometry using Bose-Einstein Condensates to Measure \(h/m\) and \(\alpha\),” S. Gupta, K. Dieckmann, Z. Hadzibabic, and D. E. Pritchard, Phys. Rev. Lett. 89, 140401/1–4 (2002). (I)

J. Entanglement

A quantum state of two or more particles that cannot be decomposed into a direct product of single-particle states is said to be entangled. Entanglement is therefore manifested in quantum-mechanical correlations between the particles. When these correlations exist in the momentum or position states of the particles they can be responsible for the “spooky action at a distance” that so bothered Einstein. Applications of entangled states to quantum information processing are common today.

Entanglement begins with particles in a pure state. The Bose-Einstein condensate, with its particles all in the single-particle ground state, is therefore an auspicious starting point. In these examples, binary collisions are used to create the entanglement between the internal (spin) states and the external (typically momentum) states.

248. “Entanglement of Atoms via Cold Controlled Collisions,” D. Jaksch, H. J. Briegel, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. 82, 1975–1978 (1999). (I)

249. “Creating Macroscopic Atomic Einstein-Podolsky-Rosen States from Bose-Einstein Condensates,” H. Pu and P. Meystre, Phys. Rev. Lett. 85, 3987–3990 (2000). (A)

250. “Squeezing and Entanglement of Atomic Beams,” L.-M. Duan, A. Sorensen, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 85, 3991–3994 (2000). (A)

251. “Many-Particle Entanglement with Bose-Einstein Condensates,” A. Sorensen, L.-M. Duan, J. I. Cirac, and P. Zoller, Nature 409, 63–65 (2001). (A)

252. “Creating Massive Entanglement of Bose-Einstein Condensed Atoms,” K. Helmerson and L. You, Phys. Rev. Lett. 87, 170402/1–4 (2001). (I)

253. “Generation of Macroscopic Pair-Correlated Atomic Beams by Four-Wave Mixing in Bose-Einstein Condensates,” J. M. Vogels, K. Xu, and W. Ketterle, Phys. Rev. Lett. 89, 020401/1–4 (2002). (I)

K. Cosmology

It is curious that some of the most energetic processes in the universe can be simulated with some of its least energetic entities. Theoretical proposals suggest that acoustic or optical analogues of black holes can be created using a condensate with a vortex rotating faster than the local speed of sound or light, respectively. Analogues of Hawking radiation also have been proposed. Still another example is the collapse of condensates with attractive interactions, which closely resembles the collapse of stars in supernovae; see Ref. [156].

254. “Tabletop Astrophysics,” P. Ball, Nature 411, 628–630 (2001). Review article. (E)

255. “Relativistic Effects of Light in Moving Media with Extremely Low Group Velocity,” U. Leonhardt and P. Piwnicki, Phys. Rev. Lett. 84, 822–825 (2000). (A)

256. “Bose-Einstein Condensates with 1/r Interatomic Attraction: Electromagnetically Induced ‘Gravity’ ”, D. O’Dell, S. Giovanazzi, G. Kurizki, and V. M. Akulin, Phys. Rev. Lett. 84, 5687–5790 (2000). (A)

257. “Sonic Analog of Gravitational Black Holes in Bose-Einstein Condensates,” L. J. Garay, J. R. Anglin, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 85, 4643–4647 (2000). (A)

VII. ACKNOWLEDGMENTS

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