Generalized Adversarial Examples: Attacks and Defenses

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Abstract—Most of the works follow such definition of adversarial example that is imperceptible to humans but can fool the deep neural networks (DNNs). Some works find another interesting form of adversarial examples such as one which is unrecognizable to humans, but DNNs classify it as one class with high confidence and adversarial patch. Based on this phenomenon, in this paper, from the perspective of cognition of humans and machines, we propose a new definition of adversarial examples. We show that imperceptible adversarial examples, unrecognizable adversarial examples, and adversarial patches are derivates of generalized adversarial examples. Then, we propose three types of adversarial attacks based on the generalized definition. Finally, we propose a defence mechanism that achieves state-of-the-art performance. We construct a lossy compression function to filter out the redundant features generated by the network. In this process, the perturbation produced by the attacker will be filtered out. Therefore, the defence mechanism can effectively improve the robustness of the model. The experiments show that our attack methods can effectively generate adversarial examples, and our defence method can significantly improve the adversarial robustness of DNNs compared with adversarial training. As far as we know, our defending method achieves the best performance even though we do not adopt adversarial training.

Index Terms—Adversarial examples, adversarial attacks, adversarial defenses.

I. INTRODUCTION

Recent deep neural networks (DNNs) make breakthroughs in many areas such as computer vision, speech recognition and so on. The power of DNNs brings people infinite reverie. Many significant and interesting works on DNNs bursting out. There are many kinds of DNNs as such convolution neural network (e.g. LeNet [1], AlexNet [2], ResNet [3]), recurrent neural network (e.g. LSTM [4]). As the rapid development of DNNs, more and more people focus on the security of DNNs. In particular, creating adversarial examples and defending adversarial attack are crucial techniques in the security of DNNs.

The vulnerability [5] of DNNs receives great attention since it has been found. Great researches are focusing on this topic in literature. These researches can be roughly classified into two categories, such as adversarial attacks [5-15] and adversarial defenses [5,16,22]. The methods of adversarial attack always want to create the adversarial examples which can fool the DNNs. Furthermore, the adversarial examples are usually imperceptible to a human. Inversely, adversarial defenses make DNNs more robustness to adversarial examples. The adversarial attacks and adversarial defences just like a zero-sum game.

Many works follow such definition of adversarial example that adversarial example is very close to original natural images but fool the DNNs [5]. In the context of this definition, an adversarial example is imperceptible to human (We call them imperceptible adversarial examples). However, there are some other forms of adversarial examples. For example, some researches add some patches to the original image for fooling the DNNs [23,26]. These examples are called adversarial patches which can fool the DNNs though they are perceptive to human. Moreover, the adversarial noise images manipulated willfully by humans can fool the DNNs [27]. Though these noise images are no meaning for human, DNNs still classify them as a certain class with high confidence (we call them unrecognizable adversarial examples). Therefore, the definition of adversarial examples, that is imperceptible to human and fool DNNs is not very suitable.

To solve this problem, this paper represents a novel definition of adversarial examples which is more generalized than that [5]. We want to look for more generalized definition which can adaptive current most of the adversarial examples. Then, based on our generalized definition, we proposed three types of attacks and a defensive mechanism. In this paper, we mainly focus on defensive mechanism. Firstly, we present that the adversarial example cannot change the objective reality since all machine tasks are in the service of humanity, and humans understand the world according to the objective reality. For example, as shown in "B" part of Fig. 1 the left subfigure is an English word, called "E". We assume it is an objective reality (e.g. human and machine all called it as "F"). Then we get medial subfigure by adding little pixel in the left subfigure. These perturbation does not bother humans recognition in pixel space, but the machine may be confused with other words in some feature space. We consider it an adversarial example if the model classifies it wrongly. Furthermore, the right subfigure also made by adding little pixel in the left subfigure. We find that in pixel space it becomes another word which is called "E". Secondly, it is obvious that not all adversarial examples are imperceptible to humans such as Adversarial patches and Unrecognizable adversarial examples. We point out that it is significantly different between human vision and DNNs to recognize the images. As shown in "A" part of Fig. 1.
we find that when humans identify clean or disturbing images, the answer is always unique, whereas machines cannot give a definite answer. If the machine misidentified the disturbing image, such a sample is known as an adversarial example. Besides, if an adversarial example is imperceptible to human, the “imperceptibility” is defined in pixel space. Therefore, we emphasize that the human recognizes an image in pixel space while the machine does that in a special space which is a machine “vision” space (“C” part of Fig. 1). We call as Feature space. Therefore, we draw two rules about generalized adversarial examples as followed:

1) The adversarial example cannot change the objective reality.

2) The adversarial example is imperceptible to the machine in Feature space rather than to human vision.

According to these two rules, we assume that existing a function which can map the original pixel space to the feature space. We describe adversarial examples as those that are imperceptible to human in the feature space and can fool the DNN. Then, we provided three types of attacks included most of the current attack methods based on our generalized form of adversarial examples. Finally, based on the generalized adversarial examples, we propose a defence mechanism to improve the adversarial robustness of the model. Our defence strategy will compress the information of input and filter many of that, including the adversarial perturbation. And we theoretically certify the effectiveness of the defence under a simple case. Besides, we provide a certain defensive function and empirically verify the effectiveness of our defence mechanism. It is noteworthy that our defensive model achieves state-of-the-art performance.

The contributions are listed as below:

1) We draw a novel and more generalized definition of adversarial examples by keeping the properties of imperception and fooling in a feature space rather than original space or pixel space since the cognition mechanism of human is remarkably different with the machine. Due to this manipulation, we give the attacker more freedom to modify the original image (e.g. adversary can add small imperceptible perturbation or adversarial patches or even using an adversarial noise image to instead original image).

2) Based on our generalized form of adversarial examples, we provide three types of adversarial attack to generate imperceptible adversarial examples, unrecognizable adversarial examples and adversarial patch, which are shown in Fig. 2. Most of the existed attack methods can be derived from our definition of adversarial examples.

3) According to the generalized form of adversarial examples, we propose a defensive mechanism against adversarial attacks. We point out that a good defensive strategy needs to have two properties to prevent the perturbation from pixel space. Our defence method is simple and significantly improve the adversarial robustness of the DNNs.

4) The attack effectivity of our attack method of generating imperceptible adversarial examples is better than most of attacks method such as FGSM [6], L2BIA [7], LBFGS [5] and so on. It drops the accuracy from 98.49% to 59.59%, which is better than CW [10] on MNIST dataset. Moreover, it drops the accuracy respectively from 88.92% to 46.69% on CIFAR10 and from 69.60% to 49.07% on ImageNet. Our performance is very close to or even better than that of PGD [16] and CW [10], which are state-of-the-art attack method. And even though we do not use adversarial training, our defending method achieves state-of-the-art performance which defeat all existed methods.

In the rest of this paper, Section II introduce the related work in literature. In Section III we define generalized adversarial examples. In Section IV we derive three types of adversarial attacks from the definition of generalized adversarial examples. In Section V we introduce our adversarial defence which is state-of-the-art without adversarial training. In Section VI we conduct large experiments and confirm that our methods are effective. Finally, Section VII concludes the paper.

II. RELATED WORKS

A. Threat model

Recently DNNs are powerful function and have made the advanced achievements in many domains such as Image Classify [28], NLP [29, 30], Object Detection [23, 31, 32], Semantic classification [31, 33, 34] and many more. With the burst spread of DNNs, more and more people pay attention to the security of DNNs. We may want to (or not want to) apply the DNNs depends on the extent to which the adversary generates the adversarial examples.

Natural language processing (NLP) make great progress in recent years. The model based on DNNs achieves state-of-the-art performance in various fields such as language modelling [35, 36], syntactic parsing [37], machine translation [29, 38] and many more. Then, the attention has transformed the risk of the DNNs in NLP. Recent work has shown [39] it is possible to generate adversarial examples by adding noises into texts, which fool the DNNs. This work firstly finds adversarial examples in texts, leading an arm begins between attack and defence in texts.

In the context of object detection, recent works find various interesting adversarial examples [31]. For example, C. Xie et al. propose an algorithm called Dense Adversary Generation (DAG) to generate imperceptible adversarial examples. To decrease computing time, they utilize regional proposal network to produce the possible targets and then sum up the loss from these targets. Another form of adversarial examples in object detection is an adversarial patch [23] which generate a small image to stick it on the original image. Even these adversarial examples can attack the physical world (e.g. attacking Yolo [82]).

In the semantic classification domain, the adversary is allowed manipulating fewer pixels than other domain since each perturbation is responsible for at least one-pixel segmentation [31, 33, 34]. These attacks include non-targeted attacks and targeted attacks. However, they generate adversarial examples which are imperceptible to humans.
This paper focuses on image classification. The model based on DNNs makes excellent progress in image classification since Olga Russakovsky et al. created AlexNet [40] which achieved champion in ILSVRC 2010. After that, many excellent DNNs (such as GoogleNet [41], VGG [42], ResNet [3], etc.) are proposed in this domain. However, when people indulge in the feast of DNNs, it is found that the DNNs are incredibly vulnerable to adversarial examples [5]. The study of adversarial examples becomes significant in this domain.

Usually, the successful rate of adversarial attack is related to how much distortion adversary can manipulate in original images. Szegedy et al. [5] use L2-norm to quantify the difference between adversarial images and original images. However, this metric is not necessarily applicable in other domain such as NLP.

According to the knowledge of adversary, adversarial attacks can be classified as white-box attacks and black-box attacks. In this paper, We suppose the adversary can access the detail of DNNs, including parameters and framework. With this strong assumption, we can construct aggressive adversarial examples and then also utilize them to black-box attacks since previous work [17] find the adversarial examples have a property called transferability that perturbations crafted on an undefended model often transfer to an adversarially trained one.

B. Adversarial example

Szegedy et al. [5] firstly find the vulnerability of the DNNs. They show that original image added small perturbation, called adversarial example, can fool DNNs. They firstly describe searching adversarial examples as a box-constrained optimization problem:

$$
\min \|r\|_2
$$

s.t. \( f(x + r) = l \)

\( x + r \in [0, 1]^m \)

(1)

\( f \) means a classifier (the DNN) mapping pixel space to a discrete category set. \( x \) is the raw image and \( r \) is the perturbation which is limited in \([0, 1]\). Moreover, \( l \) is a targeted label. Solving this formula, we can construct the adversarial example which the model classifies \( x + r \) as \( l \).

This intriguing property rises significant attention on the security of the DNNs. Then, a significant number of works [5–15] try to generate all kinds of adversarial examples which can fool the DNNs such as Fast Gradient Sign Method [6], Basic Iterative Method [7], Jacobian-based Saliency Map Attack [8], DeepFool [9], CW [10], PGD [16], One Pixel Attack [13] and so on. Goodfellow et al. [6] firstly propose based-gradient attack, which updates along the direction of the signal function of the pixel gradient to obtain the adversarial examples. It is a fast method since it updates one step. Based on FGSM, Kurakin et al. [7] propose a more powerful attack method which is multi-update to generate adversarial examples. Also using multiple iterations, Deepfool [9] utilize a linear approximation method to produce adversarial examples by searching the minimum distance from a clean example to an adversarial example. Looking for more powerful and non-gradient based attack method, Carlini et al. [10] propose various objective functions and distance metrics to generate adversarial examples which can disable the Distillation Defense [18]. PGD [16] is the first-order adversary which is the most potent attack method among the first-order attack methods. Madry et al. [16] study the adversarial robustness of DNNs and utilize projected gradient descent to search for more aggressive
adversarial examples. Their method can significantly improve resistance to most of the adversarial attacks. However, most of the researches search the adversarial examples follow the definition of adversarial examples which are imperceptible to humans but can fool DNNs. Those researches focus on studying the small perturbation.

Some particular adversarial examples have been found as the development of adversarial examples. Brown et al. firstly introduce adversarial patch attacks for image classifiers. They design a small patch than the size of the original image and put the small patch into the original image. Those adversarial examples are called adversarial patches those that are applied in object detection. Moreover, many works find that adversarial patches exist in the physical world. The adversarial patch is not imperceptible to humans. And the perturbation usually does not confuse humans judge. However, it can fool the DNNs.

Anh et al. find that some unrecognizable for humans can be classified as a class with high confidence by DNNs. Based evolutionary algorithms algorithm, they proposed a new algorithm called the multi-dimensional archive of phenotypic elites MAP-Elites, enable them to evolve the population better. Sara et al. show that the representation of the raw image in a DNN can be operated to approximate those of other natural images by adding a minor, imperceptible perturbation to the original image. They focus on the internal layers of DNN representations. They create an adversarial example which approximated the original image, but its internal representation appears remarkably different from the original image. These researches are of great significance and raise some questions about DNN representations, as well as the vulnerability of DNN.

C. Adversarial Defense

Adversarial defence makes DNNs more robust to adversarial examples. Papernot et al. propose Defensive Distillation to defend DNN against adversarial examples. By controlling the temperature in the distillation network, they train a DNN with hard-label and train other DNN with soft-label. And the two networks have the same framework. However, this defence will fail under CW attack method. Adversarial training is one of the most effective defence methods. Szegedy et al. and Madry et al. use their attack method to generate adversarial examples. Then combining clean examples and adversarial examples, they utilize them to train a DNN, which can significant improve adversarial robustness of the DNN. However, such defence method will fail under more powerful attack methods and is time-consuming. Tramer et al. generate adversarial examples by various adversarial attacks and propose adversarial ensemble training which augments training data with perturbations transferred from other models. However, recent work points out that adversarial training could cause obfuscated gradients that lead to a false sense of security in defences against adversarial examples. They propose three types of obfuscated gradients and design adversarial attacks that successfully attack all defensive papers in ICLR 2018 except that for CW.

D. Notation

In this subsection we will show some mathematical symbols used in this paper. The details are shown in Table.
III. GENERALIZED ADVERSARIAL EXAMPLES

In this section, to begin, we introduce three existed types of adversarial attack. These three types of adversarial attack include most of the current adversarial attack methods. By analyzing the characters of these attacks, we present a more generalized definition of adversarial examples which can explain the existent of adversarial examples in DNNs. Then, we introduce that the generalized adversarial examples can be degenerate currently existed adversarial examples by adding some restricted condition.

Many works follow the definition of adversarial examples drawn by Szegedy et al. [5]. We refer to this type of adversarial examples as Imperceptible Adversarial Examples which is very close to the original image but can fool the DNN. Moreover, Nguyen et al. [27] find a new type of adversarial examples which are unrecognizable to human. We refer to it as Unrecognizable Adversarial Examples that are hard to understand for human but is classified by a DNN with high confidence. Another type of adversarial examples is Adversarial Patches which are created by adding some small patches to original images [23]. These adversarial examples have a familiar character that can fool the DNN, but their original images are classified correctly by DNNs. The difference between these adversarial examples is the magnitude of change in input. The details are shown below:

- Imperceptible Adversarial Examples: allows the adversary to change the whole image limiting in bounded-norm.
- Unrecognizable Adversarial Examples: allows the adversary to change the whole image without limiting, but the adversarial examples after adding perturbation should be unrecognizable to human.
- Adversarial Patches: allows the adversary to change the image in a confined region.

Therefore, in this paper, we want to explore the unknown type of adversarial examples and give a definition of adversarial examples that will help us understand adversarial examples.

Let us begin with two questions: A) If an example of A class become an example of B class after adding perturbation and the DNNs misclassify it, is it an adversarial example? A) Are adversarial examples imperceptible to humans? For the former one, we argue that the adversary cannot change the objective reality since all machine tasks are in the service of humanity, and the humans understand the world according to the objective reality. In practice, an adversarial example usually consists of a clean sample with a slight perturbation, e.g., imperceptible adversarial sample. It is this imperceptibility that makes it difficult for the perturbation to change the objective reality of the original example. However, the Adversarial patch does not take into account this imperceptibility, which may cause the sample to become another type of sample after adding disturbance. For example, as shown in B part if Fig. 1 the left subfigure is an English word, called "F". We assume it is an objective reality (e.g. human and machine all called it as "F"). Then we get medial subfigure by adding little pixel in the left subfigure. These perturbation does not bother humans recognition in pixel space, but the machine may be confused with other words in some feature space. We consider it adversarial example if the model classifies it wrongly. Moreover, the right subfigure also made by adding little pixel in the left subfigure. We find that in pixel space it becomes another word which is called "E". It is meaningless that we call a sample as an adversarial example in this case if the model classifies it wrongly.

For the after one, it is obvious that we can perceive the perturbation of adversarial patches and unrecognizable adversarial examples. Furthermore, for Imperceptible Adversarial Examples, if the limitation of perturbation is relaxed enough, we also can perceive the small perturbation. We point out that it is significantly different between human vision and DNNs to recognize the images. It is a common phenomenon that the perturbation that hard to recognize by human but may be easy to recognize for a DNN. Inversely, the perturbation such as adversarial patches that are easy to recognize by human but maybe are hard to recognize for a DNN. As shown in "A" part of Fig. 1 we find that when humans identify clean or disturbing images, the answer is always unique, whereas DNN cannot give a definite answer. If the machine misidentified the disturbing image, such a sample is known as an adversarial example. Besides, if an adversarial example is imperceptible to human, the “imperceptibility” is defined in pixel space. Therefore, we emphasize that the human recognizes an image in pixel space while the machine does that in a particular space which is a machine ”vision” space ("C” part of Fig. 1). We call it as Feature space.

We now draw the definition of adversarial examples which refer to it as Generalized Adversarial Examples which include previous three types of adversarial examples. To begin, we draw two rules about adversarial examples as below:

1) The adversarial example can’t change the objective reality.
2) The adversarial example is imperceptible to machine in feature space rather than to human vision.

According to these two rules, we assume that existing a function which can map the original pixel space to the feature space. Then, we define generalized adversarial examples.

Define 1: The generalized adversarial examples are those that do not change original objective reality and are imperceptible to the machine in feature space and can fool the machine.
We try to describe it formally. Let $I$ represents the original natural image and $y$ represents a class of $I$. $T$ is a function which maps the pixel space to the feature space. Moreover, $f$ maps the feature space to the category space. Therefore, image recognition can be described as below:

$$f(T(I)) = f(F) = y$$

Where $T(I) = F$. Then, the generalized adversarial examples are described as:

- If $T$ is an identity function, Adversarial Tiny Perturbation is the degenerate case of Generalized Adversarial Examples.
- If $\Delta$ is limited in the shape of patches, then Adversarial Patches is the degenerate case of Generalized Adversarial Examples.
- If $I + \Delta$ is unrecognizable to human, in that case, Generalized Adversarial Examples degenerate to Adversarial Noise Perturbation.

IV. GENERALIZED ATTACK MECHANISM

According to the last section (Generalized adversarial examples), we get the formal definition of Generalized Adversarial Examples. In this paper, we mainly focus on adversarial defence. Therefore, adversarial attacks are briefly introduced. Now we formulate the problem of searching a generalized adversarial example for an image $I$ as follows:

$$\min \|T(I) - T(I + \Delta)\|_p$$

$$s.t. \quad f(T(I + \Delta)) \neq y$$

From the perspective of generalized adversarial example, we provided three versions about searching for generalized adversarial examples. The key idea of these attacks is searching adversarial examples by maximizing the gap between original images and adversarial examples. The first one is tiny perturbation version described as below:

$$\min \|\Delta\|_p - \alpha \cdot \|T(I) - T(I + \Delta)\|_p$$

$$s.t. \quad f(T(I + \Delta)) \neq y$$

$$\|\Delta\|_p \leq \delta$$

where $\alpha$ is the hyperparameter and $\delta$ is the small value for humans perception. Eq. 2 try to find an adversarial example which is imperceptible to human by minimizing $\delta$ and maximize $\|T(I) - T(I + \Delta)\|$. The pixel space for human vision is similar to the feature space for DNNs. It is sensitive for DNNs to the perturbation in feature space. Applying this type of attack, the adversarial examples closed to the original image are created which are imperceptible to humans. However, the features are extracted from the adversarial examples by DNNs are different from that of original images. That is why the adversarial examples can fool the DNNs.

The second one is called confidence attack, which makes DNN classifies a natural unrecognized image for human or a natural image from other domain with high confidence. The detailed formula is described as below:

$$\min \|T(I_{in}) - T(I_{out} + \Delta)\|_p$$

$$s.t. \quad f(T(I + \Delta)) \neq y$$

$I_{in}$ represents a natural image which comes from the original domain. And $I_{out}$ represents an image which comes from some domain or original domain. If $I_{out}$ is unrecognizable to human, then we can create an adversarial example which is unrecognizable for human but is classified by a DNN with high confidence. In that case, the attack is similar to the work [4] that produces images that are unrecognizable for humans, but a DNN believes to be recognizable objects with high confidence. If $I_{out}$ came from original domain, then this type attack is similar to the work [1] which shows that the representation of a raw image in a DNN can be operated to approximate those of other natural images by adding a minor, imperceptible perturbation to the original image. Referring to [1], $I_{in}$ is guided image, and $I_{out}$ is the source image.

The third is patch version described as below:

$$\min - \|T(I) - T(I + P_\Delta)\|_p$$

$$s.t. \quad f(T(I + P_\Delta)) \neq y$$

$p_\Delta$ is a small patch

In Eq. 5 $P_\Delta$ is a smaller patch than original image. This attack allows adversary to manipulate pixels in confined region (e.g. $P_\Delta$) but is no limit to choose the values yielding the max or min value. And $P_\Delta$ have a specific shape such as square, circle and so on. In practical, when optimizing the Eq. 5 we need to know where $P_\Delta$ locate in and what size that is. Usually, the position and size of $P_\Delta$ can be randomly initialized as long as the patch does not change the objective reality of original image. Therefore, the size of $P_\Delta$ always is much smaller than that of original image.

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V. GENERALIZED DEFENSE MECHANISM

In this section, we introduce a defence mechanism against adversarial examples based on generalized adversarial examples. In feature space, we define a function $D$ which can smooth the perturbation from pixel space. Recalled that $T$ is a function which maps pixels value to feature value and $f$ classifies the feature to a class, we construct the defence strategy as below:

$$\min_{(I,y) \sim D} E_{(I,y) \sim D} [L(f \cdot D \cdot T(I), y)]$$

where $D = \{(I^{(i)}, y^{(i)})\}_{i=1}^N$ is a dataset with $N$ samples. And $f \cdot D \cdot T(I) = f(D(T(I)))$. $L$ is a loss function.

According to the definition of generalized adversarial examples, the perturbations made by the adversary in pixel space are not strictly limited. The goal of the adversary is to manipulate perturbations which can cause significant discrepancy than
original that when they transferred from pixel space to feature space. If we can prevent the disturbance from propagating backwards, then the model will naturally be adversarial robust. The simple idea is to design a filter like function and then deploy it between the pixel space and the feature space. The purpose of this function is to filter and compress input information. Because DNNs usually have multiple convolution kernels, the extracted features are redundant, which provides feasible conditions for this design scheme. When the filter compresses the input information, it loses some of the original information, including adversarial perturbation. However, because the input information is redundant, this filter does not have much impact on the performance of the model. Secondly, the filter should be differentiable or segmented differentiable, which facilitates its combination with DNN. Therefore, we propose that function $D$ have some property from the inverse perspective of an adversary.

1) $D$ has sufficient smoothness. It is uniform continuity or uniform continuity in segments. This property ensures that the small perturbations in $T$ are limited in a fixed range after they are transferring.

2) The range of $D$ is a denumerable finite set. The critical problem of adversarial defence is how to prevent the small perturbations slowly amplifying in a DNN. If the interval of the domain of $D$ contains the interval of the range, the perturbations will be compressed. This compression naturally leads to the loss of information. However, it is this compression effect that makes the impact of small perturbations on the results even smaller.

Considering a single network $f(x) = Wx$ where $W$ is a $m \times n$ vector in feature space, $m$ is the number of categories. And $n$ is the number of features. Let us denote $I$ as pixel space and denote $F$ as feature space. For $I \in I$, we have $T(I) = F \in F$. $D$ is a defence strategy. Then, we have the following theorem:

**Theorem 1:** Suppose $D$ satisfies the Lipschitz continuity condition, then $f \cdot D$ also satisfies the Lipschitz continuity condition. Existing a constant $K$, we have

$$\|f(D(F + \Delta)) - f(D(F))\|_2 \leq K \cdot \|\Delta\|_2$$

**Proof 1:** Since $D$ satisfies the Lipschitz continuity condition, then existing a constant $c$, we have

$$\|D(F + \Delta) - D(F)\|_2 \leq c \|\Delta\|_2$$

Then,

$$\|f(D(F + \Delta)) - f(D(F))\|_2 \leq \|W \cdot D(F + \Delta) + b - (W \cdot F + b)\|_2$$

$$= \|W \cdot (D(F + \Delta) - D(F))\|_2$$

$$\leq \|W\|_2 \cdot \|D(F + \Delta) - D(F)\|_2$$

$$\leq c \|W\|_2 \cdot \|\Delta\|_2$$

Let us denote $K = c \|W\|_2$, then

$$[7] \leq K \cdot \|\Delta\|_2$$

Proof done.

In Theorem 1 we give a strong supposition that $D$ satisfies the Lipschitz continuity condition, leading $f \cdot D \cdot T$ also satisfies the Lipschitz continuity condition. With this conclusion, the effect of perturbation can be under-control.

Next, we will show how to design a reasonable function $D$. For satisfying the above two property, and we propose a specific function:

$$D(F) = \text{sign}(F) = \begin{cases} 1, & F_i \geq 0 \\ -1, & F_i < 0 \end{cases}$$

Since Eq. [8] is bounded, $D$ satisfies Lipschitz continuity condition. And the range of $D$ is $\{1, -1\}$. In section VI we will show that applying Eq. [8] can significantly improve the adversarial robustness of the model at the same time, keeping accuracy on clean examples.

By observing Eq. [8] it is found that the equation is a piecewise differentiable function whose non-differentiable point is the origin. The defence function $D$ is too simplistic and extreme. Because the input is all integers or all negative Numbers, the model loses all available information through this function, resulting in network convergence failure. In the experiment section VI, we will introduce that the deeper the network layer where function $D$ (Eq. [8]) is located, the worse the performance of the network under clean samples. Therefore, based on equation Eq. [8] we propose a more general defence function as following:

$$D(F) = \text{sign}(F) = \begin{cases} b_u, & F_i \in [a_u, +\infty] \\ \cdots, & F_i \in [\ldots, \ldots] \\ b_l, & F_i \in [a_l, -\infty] \end{cases}$$

where $b_u$ is the upper bound, $b_l$ is the lower bound, and $\{a_u, \ldots, a_l\}$ are the non-differentiable points.

**Algorithm 1 Defense Algorithm**

**Require:** An dataset $D = \{(I^{(i)}, y^{(i)})\}_{i=1}^N$; A trained network $G = f \cdot D \cdot T = (I; \theta)$

1: Initialize hyperparameter $k$
2: Initialize epoches
3: for $i=1$ to epoches do
4:   for $k$ steps do
5:     Sample $m$ samples $d = \{I^{(1)}, I^{(2)}, \cdots, I^{(m)}\}$
6:     Update the parameters by descending its stochastic gradient:
7:     $$\nabla \frac{1}{m} \sum_{i=1}^{m} y^{(i)} \cdot \log(G(I^{(i)}))$$
8:   end for
9: end for

VI. EXPERIMENTS

In this section, we use Python 3.6 and Jupyter Notebook to conduct three types of adversarial attack method. Using tool box Advertorch [50], we implement several adversarial attack methods as a baseline. For imperceptible adversarial examples,
we compared our method with existed attack methods on three datasets such as MNIST [51], CIFAR10 [52] and IMAGENET [40]. Secondly, we use our method to generate various types of unrecognizable adversarial examples. Finally, we test our method, which generates the adversarial patches. In the setting in which the patch size is same, we compared our method with existed methods.

MNIST dataset consists of 60,000 training samples and 10,000 samples, each of which is a 28x28 pixel handwriting digital image. CIFAR10 dataset is composed of 60,000 32x32 colour image, 50,000 for training and 10,000 for testing. ImageNet is large colour image dataset which consists of more than 1,400,000 natural images and 1000 classes.

For any sample from MNIST, CIFAR10 and ImageNet, we make a preprocess on it, e.g., normalization described as below:

\[ I = \frac{I - \mu}{\sigma} \]

where \( \mu \) is the mean pixel value of the whole dataset and \( \sigma \) is the standard deviation of that. The detail is showed in Table II.

### TABLE II
THE MEAN VALUE AND STANDARD DEVIATION IN MNIST, CIFAR10 AND IMAGENET

|     | \( \mu \)       | \( \sigma \)       |
|-----|----------------|-----------------|
| MNIST | 0.1307          | 0.3081          |
| CIFAR10 | 0.4914, 0.4822, 0.4465 | 0.2023, 0.1994, 0.2010 |
| ImageNet | 0.485, 0.456, 0.406 | 0.229, 0.224, 0.225 |

### A. Imperceptible Adversarial Examples

### TABLE III
A COMPARISON OF IMPERCEPTIBLE ADVERSARIAL EXAMPLES WITH VARIOUS ADVERSARIAL ATTACKS ON MNIST.

| Method  | Steps | Epsilon | Accuracy |
|---------|-------|---------|----------|
| Natural | -     | -       | 0.9849   |
| FGSM    | -     | 0.3     | 0.8239   |
| PGD     | 100   | 0.3     | 0.5838   |
| SinglePixel | -     | -       | 0.9556   |
| CW      | 100   | 0.3     | 0.6052   |
| LBFGS   | 100   | 0.3     | 0.9972   |
| L2BIA   | 100   | 0.3     | 0.9758   |
| Our     | 100   | 0.3     | **0.5995** |

In the best case, our attack method can drop the accuracy down to 59.95% (Table III) with \( \epsilon = 0.3 \) on MNIST. Our attack method is second only PGD. For more challenging cases, such as imperceptible adversarial examples on CIFAR10, our method drop the accuracy down to 3.03%, with \( \epsilon = 0.3 \) (Table IV) which is second only PGD, and 46.69% with \( \epsilon = 0.03 \) (Table IV) which is second only PGD and CW. Moreover, we test our tiny attack method on ImageNet with \( \epsilon = 0.03 \). The adversary drops the accuracy down to 49.07% (Table V), which is second only PGD and CW.

To validate the effectivity of our attack method, we repeat experiments by adjusting the \( \alpha \) value of Eq. 3 Fig. 3 shows the effectivity of attack with different \( \alpha \). It can be seen that the closer \( \alpha \) is to zeros, the worse the performance of attack. Moreover, we find that when locating in [0,1], the attack achieves the best performance. As \( \alpha \) gradually increases, the success of the attack first rises rapidly and then becomes stable.

![Fig. 3. A trend of attack’s power with various \( \alpha \) values. We find that closer \( \alpha \) to zero, worse performance of attack.](image-url)
B. Unrecognizable Adversarial Examples

In this subsection, we utilize Eq. 4 to create unrecognizable adversarial examples to fool VGG19 neural network, which is trained with ImageNet. We regard the images which came from ImageNet as an image from the domain (labelled as $I_{in}$). And any images which do not belong to $I_{in}$ are out of the domain (labelled as $I_{out}$). Our goal is creating some noise images and regular images which are classified by DNNs as a class with high confidence.

a) Irregular image: Firstly, we randomly generate a noise image. Then randomly choosing an image in the domain to guide the noise to close each other in feature space. We find that it is easy to create such a noise image. And the DNN classifies it as a class with high confidence (Fig. 4).

b) regular image: Once again, we utilize Eq. 4 to generate unrecognizable adversarial examples. However, those are regular images rather than noise images. To find suitable images from out of domain, we utilize the Colored Brodatz Texture (CBT) database and the Multiband Texture (MBT) database [53]. CBT is a coloured version of the original 112 Brodatz grayscale texture images. And MBT is a collection of 154 colour images. The colour of the MBT images is mainly the result of inter-band and intra-band texture variation. Based on those images, we search unrecognizable adversarial examples in the out domain by minimizing the gap between the image from domain and the image from out domain in feature space. Though those images are very different from that from ImageNet, the DNN still mostly believes that they are from the domain (Fig. 4).

C. Adversarial Patch

In this subsection, we try to generate adversarial patches. Adopting the setting in [23], we set the patch size as $q \times q$ where $q = \lfloor (\text{Length of image})^2 + c \rfloor^{0.5}$. And $c$ is a constant. Because the image size in MNIST and CIFR10 datasets are relatively small, the generated adversarial patches are easy to modify the objective reality, so we only generate the adversarial patches to attack VGG19 network on ImageNet. The pixel of the patch can be manipulated freely. And the position of the patch is randomly located. Some examples are shown in Fig. 4.

We show that our method can successfully generate adversarial patches. Since limiting facility, we randomly choose 1,000 images from validation dataset of ImageNet to generate their adversarial patches. Moreover, we test the accuracy in a different setting. The details are described in Fig. 5. We find that our attack can effectively generate adversarial patches and is better than the Baseline [23] in most of the setting.

D. Adversarial Defense

a) MNIST: We use the part of convolution layers as function $T$ and use the part of linear layers as classifier $f$. We set epochs as 30. The learning rate is 0.01 at the beginning, and then half every 10 epochs. The momentum is 0.95. The batch size is 64. The results of the experiment are shown in Table VI. We find that our defence strategy almost invalidates
all adversaries. At the same setting, our defending method can effectively defend gradient-based attacks (such as PGD) or non-gradient-based attacks (such as CW).

**TABLE VI**

| Method  | Steps | $\epsilon$ | Natural | Adv. Training | Our |
|---------|-------|------------|---------|---------------|-----|
| Natural | -     | 0.9849     | 0.9923  | 0.9711        |     |
| FGSM    | -     | 0.8239     | 0.9795  | 0.9712        |     |
| L2BIA   | 100   | 0.9758     | 0.9503  | 0.9712        |     |
| PGD     | 100   | 0.5838     | 0.9507  | 0.9547        |     |
| CW      | 100   | 0.6052     | 0.7813  | 0.9709        |     |

**TABLE VII**

| Method  | Steps | $\epsilon$ | Natural | Adv. Training | Our |
|---------|-------|------------|---------|---------------|-----|
| Natural | -     | 0.8892     | 0.8613  | 0.892         |     |
| FGSM    | -     | 0.743      | 0.7151  | 0.8930        |     |
| L2BIA   | 100   | 0.8775     | 0.8595  | 0.8766        |     |
| PGD     | 100   | 0.4586     | 0.6943  | 0.8927        |     |
| CW      | 100   | 0.3595     | 0.0185  | 0.8903        |     |

**TABLE VIII**

| Method  | Steps | $\epsilon$ | Natural | Adv. Training | Our |
|---------|-------|------------|---------|---------------|-----|
| Natural | -     | 0.7198     | 0.4922  | 0.6453        |     |
| FGSM    | 100   | 0.03      | 0.05    | 0.0010        |     |
| PGD     | 100   | 0.03      | 0.0003  | 0.0009        |     |
| CW      | 100   | 0.03      | 0.0003  | 0.0009        |     |

**VII. Conclusion**

We show that, according to our definition of adversarial examples, we can generate imperceptible adversarial examples, unrecognizable adversarial examples and adversarial patches and construct defending strategy. Our attack methods can effectively generate adversarial examples whose performances are better than the most attack methods (except for PGD and CW in some setting). Furthermore, our defending strategy achieve state-of-the-art performance. As far as we know, our defence is the best performance than existed methods such as adversarial training. Since we do not use adversarial training, our defence is less time-consuming than adversarial training.
In this paper, we define generalized adversarial examples to include three types of adversarial examples. Furthermore, our attack methods are focus on a single hidden layer. Starting from our definition, we propose three attack methods and one defence strategy. We present that we can construct more powerful attack methods by manipulating multi-hidden layers or adaptively manipulating multi-hidden layers. That is what we want to improve in the future. Our defence strategy achieves excellent performance. However, even if our defence method can defend against the attack, the improper setting will lead to the model’s accuracy decreased in the clean sample. Therefore, there is room for improvement in our defence in the future.

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