Higgs mass and vacuum stability with high-scale supersymmetry

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Abstract

In the high-scale (split) MSSM, the measured Higgs mass sets an upper bound on the supersymmetric scalar mass scale $M_{\text{SUSY}}$ around $10^{11}$ ($10^8$) GeV, for $\tan\beta$ in the standard range and the central value of the top quark mass $m_t$. This article discusses how maximal $M_{\text{SUSY}}$ is affected by negative threshold corrections to the quartic Higgs coupling arising from the sbottom and stop trilinear couplings. In the high-scale MSSM with very high $\tan\beta$, the electroweak vacuum decay due to the large bottom Yukawa coupling rules out the possibility of raising $M_{\text{SUSY}}$ beyond the above limit. In cases with large $A_b$ or $A_t$, $M_{\text{SUSY}}$ as a common mass of the extra fermions and scalars can be as high as $10^{17}$ GeV remaining consistent with $m_h$ and the vacuum longevity if $m_t$ is smaller than the central value by $2\sigma$. For the central value of $m_t$, the upper limit on $M_{\text{SUSY}}$ does not change very much owing to the metastability, which is the case also in the split MSSM even with $\pm 2\sigma$ variations in $m_t$. 

arXiv:1809.07774v1 [hep-ph] 20 Sep 2018
The Large Hadron Collider is giving continuous blows to the idea of natural supersymmetry, its discovery as a major objective of the machine notwithstanding. The pressures are both direct and indirect. Direct searches for the supersymmetric particles are pushing the mass bounds up \cite{1}. The measured Higgs mass is hinting indirectly at the stop mass scale (possibly orders of magnitude) higher than $\mathcal{O}(10)$ TeV \cite{2}. Nevertheless, supersymmetry remains one of the most elegant frameworks in which to structure a fundamental theory of the nature. The existence of supersymmetry, if not at the TeV scale, would be more plausible if the superstring theory is assumed to be the quantum mechanical description of gravity. Supersymmetry at high scales might also be motivated from model-building perspectives, for instance as a setting for EeV-scale gravitino dark matter \cite{3}, or as a selection out of the string landscape \cite{4}. This thought prompts the question of how high the supersymmetry scale might be.

Shortly after the Higgs mass $m_h$ was measured, it was pointed out within the Standard Model (SM) that renormalization group running drives the quartic Higgs coupling $\lambda$ negative above a scale around $10^{10}$ GeV, if the top quark mass $m_t$, the strong coupling $\alpha_s$, and $m_h$ are taken to be their central values \cite{5}. This implies that high-scale supersymmetry is disfavoured if $M_{\text{SUSY}}$, the mass scale of the supersymmetric particles including the extra Higgses, is significantly higher than the zero of $\lambda(Q)$ \cite{6,7,8}. In the effective field theory (EFT) formalism with the minimal supersymmetric standard model (MSSM) as the full theory, this is due to the matching condition,

$$\lambda = \frac{1}{4}(g_s^2 + g_Y^2)\cos^2 2\beta + \Delta\lambda, \quad (1)$$

where the leading term, given by squares of gauge couplings and cosine of the Higgs mixing angle $\beta$, is non-negative, and $\Delta\lambda$ is the threshold correction. As the matching scale, chosen to be $M_{\text{SUSY}}$, grows so high that $\lambda$ becomes negative, the above condition becomes difficult to fulfil unless $\Delta\lambda$ is large negative enough.

Proposed ways to circumvent the upper limit on $M_{\text{SUSY}}$ can be classified into the following three categories: (a) allowing for large enough uncertainties in low energy parameters such as $m_t$ and $m_h$ for $\lambda$ to stay non-negative up to the Planck scale \cite{10}, (b) extending the low energy EFT to make the matching condition easier to satisfy \cite{11,12}, or (c) realizing a large negative threshold correction \cite{13,14}. For the last option, Ref. \cite{13} considered significantly non-degenerate spectra of the supersymmetric particles. In this way, even a unification-scale $M_{\text{SUSY}}$ has been shown to be viable provided that the Higgsino mass $\mu$ is much lower than the common gaugino mass $m_{1/2}$ to the extent that $\mu/m_{1/2} \sim 10^{-4}$. This mass hierarchy would imply that $|\mu| \lesssim 10^{14}$ GeV as long as the heaviest particle has a sub-Planckian mass. In \cite{14} on the other hand, $\tan\beta$ was pushed up to very high values such that a bottom Yukawa coupling larger than unity can be a source of $\Delta\lambda$ that can overcome the tree-level part of $\lambda$. The sbottom threshold correction is enhanced due to the large coupling that multiplies the scalar trilinear interaction term,

$$\Delta\mathcal{L}_F = h_b\mu H_u \tilde{Q}^c \tilde{b}_R \quad (2)$$

A Landau pole around $10M_{\text{SUSY}}$ stemming from a large but still perturbative value of the bottom Yukawa coupling $h_b$ \cite{14}, may not be regarded as a critical flaw of the scenario. As mentioned in that reference, however, what is not clear is whether the electroweak vacuum would be stable enough or not.
In this article, the possibility of fitting $m_h$ shall be contemplated in the context of high-scale (split) supersymmetry using the above supersymmetric as well as the following soft supersymmetry breaking trilinear terms,

$$\Delta L_{\text{soft}} = -T_b H_d \tilde{Q} b_R^* + T_t H_u \tilde{t}_R^*, \quad (3)$$

wherein it is common to factor the MSSM Yukawa couplings $h_b$ and $h_t$ out of the $T$-parameters, yielding the familiar definitions of the $A$-parameters:

$$T_b \equiv h_b A_b, \quad T_t \equiv h_t A_t. \quad (4)$$

To express contributions to $\Delta \lambda$, it is common to define the left-right squark mixing parameters,

$$X_b \equiv A_b - \mu \tan \beta, \quad X_t \equiv A_t - \mu \cot \beta. \quad (5)$$

One could also use the trilinear couplings of staus instead of sbottoms, as the origins of negative threshold corrections. The results would then be similar to those using sbottoms. Throughout this article, it shall be assumed that $X_\tau \equiv A_\tau - \mu \tan \beta = 0$.

As large trilinear couplings can cause charge/color breaking (CCB) global minima \cite{15}, the vacuum metastability shall be required as follows:

$$(\Gamma_{\text{vac}}/V) T^4 < 1, \quad (6)$$

where $\Gamma_{\text{vac}}/V$ is the decay rate of the electroweak vacuum per unit volume and $T$ is the age of the Universe. In the semiclassical formulation \cite{16}, the vacuum decay rate reads

$$\Gamma_{\text{vac}}/V = A \exp(-S[\phi]), \quad (7)$$

where $A$ is a prefactor of mass dimension 4, $S$ is the Euclidean action, and the “bounce” $\bar{\phi}$ is an O(4)-symmetric stationary point of $S$ \cite{17}. Thanks to this O(4)-symmetry, $S$ can be put into the form,

$$S[\phi(\rho)] = 2\pi^2 \int_0^\infty d\rho \rho^3 \left[ \frac{1}{2} \left( \frac{d\phi}{d\rho} \right)^2 + V(\phi) \right]. \quad (8)$$

The boundary conditions on $\bar{\phi}(\rho)$ then become

$$\bar{\phi}(\rho \to \infty) = \phi_+ , \quad \frac{d\bar{\phi}}{d\rho} (\rho = 0) = 0, \quad (9)$$

where $\phi_+$ denotes the false vacuum. The bounce $\bar{\phi}(\rho)$ is found numerically using the CosmoTransitions package \cite{18}. The Euclidean action $S[\bar{\phi}]$ thus obtained has been compared with that from another numerical method described in \cite{19} at selected points in the parameter space, resulting in good agreement.

The tree-level MSSM scalar potential is substituted for $V(\phi)$ in (8) with the restricted set of fields,

$$\phi = \{h, H, \tilde{b}_L, \tilde{b}_R\} \text{ or } \{h, H, \tilde{t}_L, \tilde{t}_R\}, \quad (10)$$

where the enumerated elements are the SM-like lighter and the heavier $CP$-even Higgses, as well as the real parts of the left- and the right-handed sbottoms or stops, respectively. Either set is chosen depending on whether a sbottom or stop trilinear coupling is responsible
for the enhancement of $\Delta \lambda$. The real parts of the squarks in the above sets are normalized as real scalar fields.

The prefactor $A$ in (7) shall be estimated to be $(M_{\text{SUSY}})^4$ on dimensional grounds. Methods have been developed to calculate $A$ at one-loop level which reduce the renormalization scale dependence of $\Gamma_{\text{vac}}/V$ [20]. As the running parameters determine $S$ which in turn is exponentiated in (7), the uncertainty in $\Gamma_{\text{vac}}/V$ from the scale dependence is indeed exponentially amplified. Conversely, this implies that the limits on the trilinear couplings from [6] depend on the scale only logarithmically. Therefore, the above simple-minded estimate of $A$ should be enough at least to understand qualitatively the impact of metastability on $m_h$ from high-scale supersymmetry. Another issue with the calculation of $\Gamma_{\text{vac}}/V$ is its gauge dependence which has also been addressed [21]. It should be a meaningful future project to improve the present analysis resolving the scale and gauge dependence.

The bounce action $S[\phi]$ is classically invariant under scaling of the parameters in the scalar potential $V(\phi)$ [22]. Therefore, a large dimensionless ratio $X_{\alpha_I}/M_{\text{SUSY}}$ is well capable of disturbing the vacuum stability no matter how high $M_{\text{SUSY}}$ is, which happens to be the same ratio that controls dominant negative contributions to $\Delta \lambda$. This “non-decoupling” property of metastability has been demonstrated in the context of flavour physics as a probe of flavour-violating trilinear couplings [23]. An EFT formulation has also been employed to argue that disturbance to the vacuum lifetime is not simply suppressed by pushing up the new physics scale [24].

The SM-like Higgs mass $m_h$ is computed in the EFT approach using FlexibleSUSY [25] version 2.2.0 [26] grown out of SOFTSUSY [27], in combination with SARAH [28]. The bundled model definitions, HSSUSY [5, 11, 29, 30] and SplitMSSM [3, 11, 30], are used for the high-scale and the split MSSM, respectively, with the following modifications: (a) SplitMSSM is modified to compute $m_h$ using the FlexibleEFTHiggs method [31], (b) SplitMSSM is modified to include the one-loop sbottom and stau threshold corrections to $\lambda$, taken from HSSUSY, (c) the tan$\beta$-enhanced corrections to $h_b$ and $h_\tau$ proportional to $g_1^2$ or $g_2^2$ are added. These latter corrections are not included in the above model files as some of the implemented results are in the “gaugless” limit where loops controlled by $g_{Y,2}$ are neglected [32]. These gauge couplings become comparable to $g_3$ and may thus be non-negligible as the renormalization scale approaches the unification scale $M_{\text{GUT}}$. Details of the modifications are documented in the appendix.

The numerical analysis involves the following MSSM parameters as input: tan$\beta$, the scalar trilinears $A_{b,t}$, the Higgsino mass $\mu$, the CP-odd Higgs mass $m_A = \tilde{m}$, the soft sfermion masses $m_Q^2 = m_{b_R}^2 = m_{t_R}^2 = m_{L}^2 = m_{\tau_R}^2 = \tilde{m}^2$, the gaugino masses $M_{1,2,3} = m_{1/2}$. For a given pair of $(\tan \beta, A_q)$ with $q = b$ or $t$, $M_{\text{SUSY}} = \tilde{m}$ is found such that $m_h = 125.09$ GeV [33] as is calculated by FlexibleSUSY taking $M_{\text{SUSY}}$ as the matching scale. Further assumptions about the remaining mass parameters are: $|\mu| = m_{1/2} = M_{\text{SUSY}}$ in the high-scale MSSM, and $\mu = m_{1/2} = 1$ TeV in the split MSSM. To isolate the effect of each trilinear coupling, either of $X_{b,t}$ is fixed at zero when it is not being scanned. At each such $M_{\text{SUSY}}$, tan$\beta$ or $T_{\alpha}$ is subsequently varied, with $M_{\text{SUSY}}$ fixed, until the left-hand side of (7) becomes unity, yielding the metastability bound. For this, the additional MSSM parameters $g_Y, g_2, g_3, h_{b,t}$, at the scale $M_{\text{SUSY}}$ in the $\overline{\text{DR}}$ scheme, are put into the tree-level scalar potential $V(\phi)$ in [6] together with $\mu, T_{b,t}$ already specified above. The $\overline{\text{DR}}$ gauge and Yukawa couplings are obtained from the MS couplings output by FlexibleSUSY using the conversion formulae available from the SusyHD package [14]. The soft mass parameters $B_{\mu}, m_{H_d}^2, m_{H_u}^2$ are determined at tree level by $m_A$ and the electroweak symmetry breaking conditions.
FIG. 1. Higgs mass curves (blue) and (meta)stability limit (red) on the \((M_{\text{SUSY}}, \tan \beta)\) plane. Both plots are the same except that the right panel is restricted to the high-\(\tan \beta\) range. The thickness and pattern of each blue curve reproducing \(m_h = 125.09\) GeV, indicate the size and sign of the deviation of used \(m_t\) from the central value, respectively. The starred point, ruled out by metastability, results in the specimen bounce shown in Fig. 2.

First, the high-\(\tan \beta\) scenario from [14] is revisited. The one-loop sbottom threshold correction to \(\lambda\) looks like
\[
\Delta \lambda_b^{(1)} = -\frac{(h_b \mu)^4}{32 \pi^2 \tilde{m}^4}.
\]
Making use of this contribution, one can increase \(\tan \beta\) to the extent that a large enough MSSM Yukawa coupling \(h_b\) allows (1) to hold for an arbitrarily high \(|\mu| = \tilde{m} = M_{\text{SUSY}}\). Note however that the same product \(h_b \mu\) affects not only the above threshold correction but also the existence of CCB global minima as suggested by (2). One should therefore pay attention to the stability of the electroweak vacuum at the same time.

The \(m_h\) constraint on \(M_{\text{SUSY}}\) and \(\tan \beta\) is reproduced in Fig. 1 using FlexibleSUSY. Both panels are Fig. 6 of [14] with \(X_t = X_\tau = 0\) instead of \(A_t = M_{\text{SUSY}}/2\) and \(A_\tau = 0\) as well as the horizontal and vertical axes interchanged. The sbottom trilinear \(A_b\) is set to zero and \(\mu\) is negative, as in the original plot. The right panel magnifies the high-\(\tan \beta\) region. For an estimation of the theory uncertainty due to missing higher order threshold corrections, \(\lambda\) is matched at both one- and two-loop levels by switching the \texttt{LambdaLoopOrder} parameter [26], leading to the light blue and the blue curves, respectively. The close proximity of the one-loop matched curves to the corresponding two-loop matched curves renders the former hard to see thereby indicating that the truncation error is reasonably small. In the region above the red dashed curve, the scalar potential develops a global CCB minimum with non-vanishing sbottom vacuum expectation values due to large \(h_b \mu\). Within that region, the vacuum longevity condition (6) gives rise to the upper limit on \(\tan \beta\) delineated by the red solid curve. It is looser than the CCB limit, but still excludes the upper part of the blue curves with \(M_{\text{SUSY}} \gtrsim 10^8\) GeV.

An instance of the bounce is shown in Fig. 2 which corresponds to the point marked with the star in Fig. 1. With this bounce as its argument, the Euclidean action \(S[\phi]\) evaluates to 275, much smaller than its lower bound 527 for the indicated \(M_{\text{SUSY}}\).
An alternative way to enhance negative $\Delta \lambda$ is to increase $|X_b|$ far beyond $\sqrt{12} M_{\text{SUSY}}$ via $A_b$ \cite{34}. In this case, one can choose $\tan \beta$ to be 1 to minimize the tree-level contribution to $\lambda$ in \cite{1} and then fit $m_h$ by varying $A_b$ in the high-scale and the split MSSM. The results are shown in Fig. 3, for $\mu > 0$ and $A_b < 0$. The other sign combinations lead to similar outcomes. The vertical axis is chosen to involve $T_b$ instead of $X_b$ or $A_b$ to avoid displaying huge numbers. There exists indeed a value of $T_b$ yielding correct $m_h$ at any $M_{\text{SUSY}}$ within the selected range. However, the metastability puts a stringent constraint on $T_b$; thereby resulting in the limits (a) $M_{\text{SUSY}} \lesssim 10^{11}$ GeV in the high-scale MSSM and (b) $M_{\text{SUSY}} \lesssim 10^8$ GeV in the split MSSM, for the central value of $m_t = 173.34$ GeV \cite{35}. If $m_t$ is shifted from this by $-2 \sigma$ with $\sigma = 0.76$ GeV \cite{35}, one can find solutions for $M_{\text{SUSY}} \gtrsim M_{\text{GUT}}$ allowed by both $m_h$ and the vacuum lifetime in the high-scale MSSM. In the split MSSM by contrast, varying $m_t$ by $\pm 2\sigma$ does not change the upper limit on $M_{\text{SUSY}}$ very much. As in Fig. 1, CCB bounds could also be plotted, which leave $|T_b/M_{\text{SUSY}}| \lesssim 0.04$ or narrower ranges. Their boundaries would therefore lie much lower than the displayed region.

A negative threshold correction can also arise from $|X_t| \gtrsim \sqrt{12} M_{\text{SUSY}}$ \cite{34}. Values of $X_t$, leading to correct $m_h$ and the vacuum decay limits thereon are shown in Fig. 4, for $\mu > 0$ and $X_t < 0$. The other sign combinations lead to similar outcomes. To suppress the tree-level term in \cite{1}, $\tan \beta$ is fixed at 1. In the range $0 < |X_t| < \sqrt{12} M_{\text{SUSY}}$, the stop threshold correction to $\Delta \lambda$ tends to be positive with the maximum around $|X_t| \approx \sqrt{6} M_{\text{SUSY}}$, thereby pushing $M_{\text{SUSY}}$ down even lower than the value for $X_t = 0$. For the purpose of raising $M_{\text{SUSY}}$, $X_t$ is therefore better chosen to be zero rather than a nonzero value in this range. A large enough $|X_t|$ on the other hand can yield correct $m_h$ for an arbitrarily high $M_{\text{SUSY}}$. In the high-scale MSSM, the metastability bound however happens to range in the vicinity of $|X_t/M_{\text{SUSY}}| = \sqrt{12}$ with a crossing point around $M_{\text{SUSY}} \approx 2 \times 10^{11}$ GeV. This means that the limit $M_{\text{SUSY}} \lesssim 10^{11}$ GeV \cite{8, 9} resulting from vanishing $X_t$ and the central value of $m_t$, does not change very much even if $X_t$ is allowed to be nonzero. A
FIG. 3. Sbottom trilinear coupling fitting the Higgs mass (blue) and metastability limit (red) for $\tan \beta = 1$, as a function of $M_{\text{SUSY}}$ in (a) the high-scale MSSM and (b) the split MSSM with $\mu = m_{1/2} = 1$ TeV. The thickness and pattern of each blue curve reproducing $m_h = 125.09$ GeV, indicate the size and sign of the deviation of used $m_t$ from the central value, respectively.

FIG. 4. Stop trilinear coupling fitting the Higgs mass (blue) and (meta)stability limit (red) for $\tan \beta = 1$, as a function of $M_{\text{SUSY}}$ in (a) the high-scale MSSM and (b) the split MSSM with $\mu = m_{1/2} = 1$ TeV. The thickness and pattern of each blue curve reproducing $m_h = 125.09$ GeV, indicate the size and sign of the deviation of used $m_t$ from the central value, respectively. The horizontal dotted line marks the height $|X_t/M_{\text{SUSY}}| = \sqrt{12}$. 
viable parameter region with \( M_{\text{SUSY}} \gtrsim M_{\text{GUT}} \) can still be opened up if \( m_t \) is lowered by 2\( \sigma \) from the central value. Note that this "low-\( m_t \) region" would be excluded leading to the restriction, \( M_{\text{SUSY}} \lesssim 6 \times 10^{14} \) GeV, even for \( \Delta m_t = -2\sigma \), if the more stringent CCB bound were adopted instead. In the split MSSM by contrast, the vacuum decay excludes all parts of the blue curves with \( |X_t| > \sqrt{2} M_{\text{SUSY}} \). Therefore, the resulting maximal \( M_{\text{SUSY}} \) for any shown \( m_t \) is the same as the corresponding upper limit for \( X_t = 0 \).

To sum up, this article has attempted to address the question of how high the supersymmetry scale can be, given the measured SM-like Higgs mass as a constraint on the parameters of the MSSM. Two types have been considered as to the mass spectrum of the supersymmetric particles including the extra Higgses: (a) nearly degenerate fermions and scalars (high-scale MSSM), (b) split spectrum where the fermions are at the TeV scale (split MSSM). To satisfy the matching condition on the quartic Higgs coupling, the sbottom and the stop trilinear couplings have been employed as sources of the potentially large negative threshold corrections. In all cases, it is possible to reproduce \( m_h = 125.09 \) GeV, by choosing appropriate values of \( \tan \beta \) in combination with \( A_b \) or \( A_t \), for \( M_{\text{SUSY}} \) up to \( 10^{17} \) GeV. However, the lifetime of the electroweak vacuum places severe constraints on the trilinear couplings and mostly brings back the upper limits on \( M_{\text{SUSY}} \) for \( X_b = X_t = 0 \): \( M_{\text{SUSY}} \lesssim 10^{11} \) GeV and \( 10^{8} \) GeV, in the high-scale and the split MSSM, respectively, for the central value of \( m_t \). Nevertheless, a small extension of the viable parameter volume could be achieved via non-vanishing \( X_b \) and/or \( X_t \) in the high-scale MSSM: \( M_{\text{SUSY}} \gtrsim M_{\text{GUT}} \) becomes viable if \( m_t \) is allowed to be smaller than the central value by 2\( \sigma \). Note that this smaller \( m_t \) still causes \( \lambda \) to turn negative at a scale around \( 6 \times 10^{14} \) GeV and therefore that \( M_{\text{SUSY}} \) higher than this zero of \( \lambda \) requires negative threshold corrections. On the other hand, the vacuum decay rate rules out the scenario where very high \( \tan \beta \) reconciles arbitrarily high \( M_{\text{SUSY}} \) with \( m_h \). As already mentioned in the introductory part, these findings still leave the possibilities of going beyond the assumptions made in this work in order to lift the restrictions on the scale of supersymmetry.

The author thanks Oscar Vives, Deog Ki Hong, Pyungwon Ko, and Dominik Stöckinger for the helpful comments. He also thanks the KIAS Center for Advanced Computation for providing computing resources through the Abacus system.

**Appendix: Corrections to Yukawa couplings added to FlexibleSUSY model files**

The SM and the MSSM bottom Yukawa couplings, \( y_b \) and \( h_b \), are related at one-loop order by \cite{36}

\[
y_b = h_b \cos \beta (1 + \frac{\Delta \tilde{\chi}^0_b}{\tan \beta} + \frac{\Delta \tilde{\chi}^b}{\tan \beta} + \frac{\Delta \tilde{\lambda}^0}{\tan \beta}),
\]

where the \( \tan \beta \)-enhanced corrections read

\[
\frac{16\pi^2}{\tan \beta} \frac{\Delta \tilde{\chi}^0}{\tan \beta} = + \frac{1}{9} g_2^2 \frac{g_1^2}{\tan \beta} X_b M_1 I (m_{b_L}^2, m_{b_R}^2, M_1^2) - \frac{1}{6} g_2^2 M_1 \mu I (M_1^2, \mu^2, m_{b_L}^2)
\]

\[
- \frac{1}{3} g_2^2 M_2 \mu I (M_2^2, \mu^2, m_{b_R}^2) - \frac{1}{2} g_2^2 M_2 \mu I (M_2^2, \mu^2, m_{b_L}^2),
\]

\[
\frac{16\pi^2}{\tan \beta} \frac{\Delta \tilde{\chi}^b}{\tan \beta} = - g_2^2 M_2 \mu I (M_2^2, \mu^2, m_{b_L}^2) + \frac{1}{3} g_2^2 X_t \mu I (m_{t_L}^2, m_{t_R}^2, \mu^2),
\]

\[
\frac{16\pi^2}{\tan \beta} \frac{\Delta \tilde{\lambda}^0}{\tan \beta} = - \frac{8}{3} g_3^2 \frac{X_b}{\tan \beta} M_3 I (m_{b_L}^2, m_{b_R}^2, M_3^2),
\]
in terms of the loop function

\[ I(a, b, c) \equiv \frac{ab \ln(a/b) + bc \ln(b/c) + ac \ln(c/a)}{(a - b)(b - c)(a - c)}. \]  

Likewise, the SM and the MSSM tau Yukawa couplings, \( y_\tau \) and \( h_\tau \), are related by

\[ y_\tau = h_\tau \cos\beta (1 + \Delta \tilde{\chi}_0^{\pm} + \Delta \tilde{\chi}_0^0), \]

\[ \frac{16\pi^2}{\tan\beta} \Delta \tilde{\chi}_0^{\pm} = -g_2^2 \frac{X_\tau}{\tan\beta} M_1 I(m_{\tilde{\tau}_L}^2, m_{\tilde{\tau}_R}^2, M_1^2) + \frac{1}{2} g_1^2 M_1 \mu I(M_2^2, \mu^2, m_{\tilde{\tau}_L}^2) \]

\[ - g_2^2 M_1 \mu I(M_1^2, \mu^2, m_{\tilde{\tau}_L}^2) - \frac{1}{2} g_2^2 M_2 \mu I(M_2^2, \mu^2, m_{\tilde{\tau}_L}^2), \]

\[ \frac{16\pi^2}{\tan\beta} \Delta \tilde{\chi}_0^0 = -g_2^2 M_2 \mu I(M_2^2, \mu^2, m_{\tilde{\nu}_\tau}^2). \]

Among the above terms, those proportional to \( h_t^2 \) and \( g_3^2 \), i.e. (A.4) and the last term of (A.3) are already implemented in HSSUSY. For this work, HSSUSY and SplitMSSM have been modified to include all the above corrections.

[1] See e.g. talks at SUSY2018 such as Hannsjörg Weber, “Searches for strong SUSY production”, http://indico.cern.ch/event/689399/contributions/2945157; Reina Camacho Toro, “SUSY searches—electroweak production”, http://indico.cern.ch/event/689399/contributions/2945161.

[2] See e.g. P. Athron, J.-h. Park, T. Steudtner, D. Stöckinger and A. Voigt, JHEP 1701 (2017) 079 [arXiv:1609.00371 [hep-ph]].

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