Baryon masses in a loop expansion with form factor

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Abstract

We show that the average multiplet masses in the baryon octet and decuplet can be fitted with an average error of only $0.5 \pm 0.3$ MeV in a meson loop expansion with chiral SU(6) couplings, with the hadrons treated as composite particles using a baryon-meson form factor. The form factor suppresses unphysical short distance effects and leads to a controllable expansion. We find, in contrast to the results of standard chiral perturbation theory, that pion loops are as important as kaon or eta loops as would be expected when only intermediate- and long-distance contributions are retained. We also find that the contributions of decuplet intermediate states are important in the calculation of the masses, and those states must be included explicitly in a consistent theory. These results agree with those of our recent loop-expansion analysis of the baryon magnetic moments. We show, finally, that the parts of the loop contributions that change the tree-level structure of the baryon masses are small, but largely account for the violations of the baryon mass sum rules which hold at tree level.
I. INTRODUCTION

Baryon masses have been studied intensively in chiral perturbation theory (ChPT). The results obtained in the standard approach using dimensional regularization to control the divergences in the theory are not completely satisfactory \[1–3\]. The chiral loop corrections evaluated in that regularization scheme are very large, even of the order of the leading terms, and the convergence of the chiral expansion is at best very slow \[4\]. In addition, the results are dominated by the contributions from heavy mesons, while pion loops would be expected to give the similar contributions in the range of distances or momenta in which the calculations are supposed to be reliable. The same difficulties appear in other situations, for example, in the calculation of the baryon magnetic moments in chiral perturbation theory \[5–8\], where the convergence and usefulness of the chiral expansion is again questionable \[9\]. These shortcomings are due physically to the treatment of the hadrons as point particles in the standard approach. Loop integrals then involve high momenta and unphysical short-distance contributions, and tend to be large and dominated by heavy mesons when calculated using dimensional regularization. The extension of ChPT to higher orders also requires the introduction of new couplings in the effective field theory, and the theory loses its predictive power \[1\].

In this work, we reexamine the calculation of the baryon masses using a loop expansion rather than an expansion in powers of the chiral parameters. Our approach is motivated by the remarkable success of a loop expansion for the baryon magnetic moments which starts from a QCD-based quark model \[10,11\]. That model was derived in a quenched approximation to QCD following the Wilson-loop method of Brambilla et al. \[12\]. The inclusion of loop corrections was necessary to remove the quenching. The baryon Hamiltonian obtained by Brambilla et al. has been used to fit the baryon spectra with reasonable accuracy \[13,14\], but again in the quenched approximation. We concentrate here on the calculation of the one-loop corrections to the masses in the ground-state baryon octet and decuplet. As in our earlier work on the moment problem \[11\], we use chiral couplings for the low momentum interactions of the mesons and baryons, but treat those hadrons as composite. In particular, because of their extended structure, the baryons and mesons cannot absorb high recoil momenta and remain in the same state, so that high-momentum or short-distance effects are suppressed naturally by wave function effects. We model these by introducing a form factor at the baryon-meson vertex as in \[10,11,15\]. The divergences associated with point baryons are eliminated in this approach, and the residual loop corrections are reduced in magnitude and have similar magnitudes for pion, kaon, and eta loops.

We include the decuplet baryon states explicitly in our calculations as was done in \[1\] and in our earlier work \[11,15\]. We find that the contributions of octet and decuplet intermediate states are of comparable importance for either the octet or decuplet masses. This would be expected in a QM picture, since the octet and decuplet baryons differ only in their spin.

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1 The chiral expansion gives a complete parametrization of the static properties of the baryons when carried to high enough order, but no dynamical information unless the new couplings can be calculated in the underlying theory. See, for example, the analysis of the magnetic moment problem in \[9\].
structure. We note also that the octet-decuplet mass splitting is small on the scale of the
relevant loop momenta, so an expansion of the decuplet contributions in powers of the loop
momentum over the mass splitting as in some approaches to ChPT is poorly convergent
even when the calculation is restricted to the octet masses.

The results of our calculation are in excellent agreement with experiment. The loop
corrections are typically about 35 percent of the leading mass term, suggesting that the
loop expansion converges reasonably well. Moreover, the structure of the corrections is
such that they can largely be absorbed by adjustments of the input parameters, leaving
residual corrections to the baryon masses of less than 10 MeV. If we fit the octet and
decuplet masses simultaneously using the five parameters that appear in the theory at tree
level and the SU(6) couplings used in our earlier work, we reproduce the average masses of
the eight octet and decuplet baryon multiplets with an average deviation of only 0.5 ± 0.3
MeV. This is to be compared with the average deviation of 3.1 ± 0.3 MeV in a fit without
loop corrections. If we fix one of the parameters using a lowest order QM relation, the
average deviation increases to 4.0 MeV, to be compared with a deviation of 4.4 MeV in a
corresponding fit without loop corrections. The real improvement in the results becomes
clear when we analyze the structure of the loop corrections and the violations of the mass
relations that hold at tree level. The parts of the loop corrections that cannot be described
within the tree-level structure are small, but largely account for the violations of the mass
sum rules.

An approach to loop corrections similar to that used here and in our earlier work
[10,11,15] was recently introduced by Donoghue and Holstein [16] and Donoghue, Holstein,
and Borasoy [4] in the context of ChPT. Those authors show that the difficulties with di-

II. MODEL AND THEORETICAL RESULTS

A. Heavy baryon couplings

Heavy baryon perturbation theory (HBPT) was developed in [17] and extended to the
chiral context in [18]. It has been used to study a number of hadronic processes at mo-
mentum transfers much less than a typical baryon mass. The key ideas in HBPT involve
the replacement of the momentum $p^\mu$ of a nearly on-shell baryon by its on-shell momentum
$m_B v^\mu$ plus a small additional momentum $k^\mu$, $p = m_B v + k$, and the replacement of the
baryon field operator $B(x)$ by an velocity-dependent operator $B_v(x)$ constructed to remove
the free momentum dependence in the Dirac equation, $B_v(x) = e^{i m_B \gamma^\nu v^\mu x^\nu} B(x)$ [17]. In
these expressions $m_B$ is the SU(3)-symmetric mass of the baryon octet, $v^\mu$ is the on-shell
four velocity of the baryon, and it is assumed that $k \cdot v \ll m_B$. Velocity-dependent Rarita-
Schwinger decuplet fields $T_v^\mu$ are defined in the same manner, with $T_v^\mu(x) = e^{i m_B \gamma^\nu v^\mu x^\nu} T^\mu(x)$.
Note that we only extract the large octet-baryon mass \( m_B \) in this construction to avoid the appearance of phase factors in the octet-decplet interactions defined below, and will treat the small decplet-octet mass difference \( \delta m_B = m_T - m_B \) explicitly. The velocity-dependent perturbation expansion involves modified Feynman rules and an expansion in powers of \( k/m_B \) \[17\], \[18\].

Because the low-momentum couplings of mesons to baryons appear to be well described as derivative couplings with the standard chiral structure, we will use those couplings in our analysis. However, we emphasize that we will be making a loop expansion for the masses rather than the expansion in powers of the momentum and the symmetry breaking parameter \( m_s \) characteristic of ChPT. Higher-order effective couplings of ChPT will be implicit output of our dynamical calculation.

The Lagrangian for the modified baryon fields depends on the usual matrix of baryon fields, with \( B \) replaced by \( B_v \), and on the pseudoscalar pion octet normalized as

\[
\phi = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \\
\pi^- \\
-K^+ \\
-\frac{\eta}{\sqrt{6}} + \frac{\eta^0}{\sqrt{6}} \\
\eta^0 \\
-K^0 \\
\end{pmatrix}.
\] (2.1)

This couples to the baryon fields at low momenta through the vector and axial vector currents defined by

\[
V_\mu = f^{-2}(\phi \partial_\mu \phi - \partial_\mu \phi \phi) + \cdots, \quad A_\mu = f^{-1}\partial_\mu \phi + \cdots,
\] (2.2)

where \( f \approx 93 \text{ MeV} \) is the meson decay constant. We will retain, as indicated above, only leading term in the derivative expansion. The lowest order Lagrangian for octet and decplet baryons is then

\[
\mathcal{L}_v = i \mathrm{Tr} \, \bar{B}_v \, (v \cdot \mathcal{D}) \, B_v + 2 \, D \, \mathrm{Tr} \, \bar{B}_v \, S^\mu_v \{A_\mu, B_v\} + 2 \, F \, \mathrm{Tr} \, \bar{B}_v \, S^\mu_v \{A_\mu, B_v\}
\]

\[
- i \, \bar{T}_v \, (v \cdot \mathcal{D}) \, T_{v\mu} + \delta m_B \, \bar{T}_v \, T_{v\mu} + C \left( T_{\mu} \, A_\mu + \bar{B}_v \, A_\mu \right) + 2 \, H \, T_{v\mu} \, S_v \, T_{v\mu} + \mathrm{Tr} \, \partial_\mu \phi \partial^\mu \phi + \cdots
\] (2.3)

where \( \delta m_B \) is the decplet-octet mass difference and \( \mathcal{D}_\mu = \partial_\mu + [V_\mu, \cdot] \) is the covariant derivative. \( B_v \) is now the matrix of octet baryon fields, and the Rarita-Schwinger fields \( T_{\mu} \) \[18\] represent the decplet baryons. \( D, F, C, \) and \( H \) are the strong interaction coupling constants. The spin operator \( S_v^\mu \) is defined in Ref. \[18\]. The “mass term” \( \delta m_B \, T_{v\mu}T_{v\mu} \) for the decplet fields will be absorbed in the following calculations in the definition of the decplet propagator. The propagator for the fields \( B_v \) involves no mass.

We will introduce SU(3) symmetry breaking into the Lagrangian by including a quark mass matrix \( M = \text{diag}(m_u, m_d, m_s) \) in chiral combinations with the velocity-dependent fields. We will take \( m_u = m_d = 0 \) in the present calculations as far as explicit symmetry breaking is concerned, but will use the physical values of the meson masses in evaluating the contributions of meson loop corrections to the baryon masses. The tree-level masses splittings of the baryons from \( m_B \) are then defined to first order in \( M \) by the Lagrangian \[1\]

\[
\mathcal{L}_v^M = b_D \, \mathrm{Tr} \, \bar{B}_v \{M, B_v\} + b_F \, \mathrm{Tr} \, \bar{B}_v \{M, B_v\} + c \, \bar{T}_v \, M T_{v\mu} + \sigma \, \mathrm{Tr} \, M (\Sigma + \Sigma^\dagger) \, \mathrm{Tr} \, \bar{B}_v B_v - \bar{\sigma} \, \mathrm{Tr} \, M (\Sigma + \Sigma^\dagger) \, \bar{T}_v T_{v\mu},
\] (2.4)
where
\[ M = \xi^\dagger M \xi^\dagger + \xi M \xi, \quad \xi = \exp(i\phi/f). \] (2.5)

If, as here, we consider only baryon and not meson masses, the terms with coefficients \( \sigma \) and \( \tilde{\sigma} \) contribute mass terms of standard form for the octet and decuplet baryons, and can be absorbed by redefinitions of \( m_B \) and \( m_T = m_B + \delta m_B \). We will follow this procedure. The theory then involves five mass parameters at the tree level, \( m_B, \delta m_B, m_s b_D, m_s b_F, \) and \( m_s c \).

It is straightforward to show that the structure at this level encompasses that in the \( L = 0 \) quark model taken to first order in \( m_s \), but the QM provides an extra first-order constraint
\[ c = -3(b_D + b_F) \] (2.6)
which connects the octet and decuplet masses. The existence of the constraint is a consequence of the general spin structure of the mass terms at that order, and the assumption that the octet and decuplet baryons would have the same internal structure up to spin for \( m_s = 0 \). We will consider both the constrained and unconstrained cases.

**B. Compositeness and the form factor**

In our previous work [10,11] on a QCD-based quark model with chiral couplings, we introduced a form factor characterizing the structure of baryons considered as composite particles and showed how to evaluate the loop graphs with the form factor inserted at the baryon-meson vertices. We will follow the same approach here. In particular, we introduce a simple form factor \( F(k, v) \) at each meson-baryon vertex, with
\[ F(k, v) = \frac{\lambda^2}{\lambda^2 + (k \cdot v)^2 - k^2}. \] (2.7)

Here \( k = (k_0, \mathbf{k}) \) is the 4-momentum of meson and \( \lambda \) is a parameter characterizing a natural momentum scale for the meson-baryon wave function. The form factor reduces in the rest frame of the heavy baryon to a function of \( k^2 \) only,
\[ F(k, v) = \frac{\lambda^2}{\lambda^2 + k^2}, \quad v = (1, 0), \] (2.8)
and can be interpreted in terms of a meson-baryon wave function. It also respects crossing symmetry for the meson line under the substitution \( \mathbf{k} \to -\mathbf{k} \).

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2See, for example, the semirelativistic Hamiltonian for the baryons given in [12–14]. Differentiation of the Hamiltonian with respect \( m_s \) and use of the Feynman-Hellman theorem [19] gives first-order perturbations of the baryon masses with precisely the structure given in Eq. (2.4), but with \( \delta m_B, m_s b_D, m_s b_F, \) and \( m_s c \) expressed in terms of specific matrix elements. The relation for \( c \) in terms of \( b_D \) and \( b_F \) follows.
The loop integrals involving this form factor can be evaluated by making an appropriate \( \nu \)-dependent shift in the loop momentum \([11]\). Because of the structure of the vertices and the relation \( \nu \cdot S = 0 \), the result is equivalent for some of the one-loop graphs to that obtained with a covariant form factor or cutoff \( F(k) = \lambda^2 / (\lambda^2 - k^2) \) \([4]\), but the physical content or motivation is different.

In \([11]\), we found an excellent fit to the baryon magnetic moments using a value \( \lambda \approx 407 \) MeV in the form factor, but good fits could also be obtained for somewhat different values in this general range.

C. Expressions for the baryon masses

We will write the mass of baryon \( i \) in the form

\[
M_i = M_i^{(0)} + M_i^{(\delta m_B = 0)} + M_i^{(\delta m_B \neq 0)},
\]

(2.9)

where the leading term \( M_i^{(0)} \) includes the SU(3) symmetric mass plus the tree-level mass splittings calculated from \( \mathcal{L}^M_v \). The terms in \( M_i^{(\delta m_B = 0)} \) are contributions from the loop graphs in Figs. 1 and 2 that involve only octet or only decuplet baryons. These contributions are independent of the decuplet-octet mass difference \( \delta m_B \). The terms in \( M_i^{(\delta m_B \neq 0)} \) come from the loop graphs in Figs. 1b, 1d, 2c, and 2f that involve both octet and decuplet baryons, and depend explicitly on \( \delta m_B \).

The calculations with the form factor \( F(k, \nu) \) included are straightforward using the methods in \([11]\), and we find that\(^3\)

\[
M_i^{(0)} = m^{(\pm)} + \alpha_i m_s,
\]

(2.10)

where \( m^{(+)} = m_B, m^{(-)} = m_B + \delta m_B \),

\[
M_i^{(\delta m_B = 0)} = \sum_{X = \pi, K, \eta} \frac{1}{16 \pi^2 f^2} \left\{ -\pi \tilde{M}^3_i \beta_i^{(X)} \right. \\
+ 2m_s \left[ \gamma_i^{(X)} \tilde{L}_0(m_X, \lambda) + (\gamma_i^{(X)} - \lambda_i^{(X)} \alpha_i \tilde{L}_0(m_X, \lambda) \right] \right\},
\]

(2.11)

and

\[
M_i^{(\delta m_B \neq 0)} = \sum_{X = \pi, K, \eta} \frac{1}{16 \pi^2 f^2} \left\{ -\pi \tilde{L}_2(m_X, \pm \delta m_B, \lambda) \right. \\
+ m_s \left( \gamma_i^{(X)} - \lambda_i^{(X)} \alpha_i \tilde{L}_1(m_X, \pm \delta m_B, \lambda) \right) \right\}.
\]

(2.12)

The upper and lower signs are to be used for external octet and decuplet baryons, respectively. The functions \( \tilde{M}^3, \tilde{L}_2(m, \pm \delta m_B, \lambda) \), and \( \tilde{L}_0(m, \lambda) \) are defined by

\(^3\) For simplicity, we suppress all the subscripts and superscripts used to label the loops.
\[ \tilde{M}^3 = \frac{\lambda^4 \lambda + 2m}{2(\lambda + m)^2}, \]  
\[ \tilde{L}_2(m, \pm \delta m_B, \lambda) = \tilde{M}^3 + \frac{\delta m_B}{\pi} L_2(m, \pm \delta m_B, \lambda), \]  
\[ \tilde{L}_0(m, \lambda) = \frac{\lambda^4}{(\lambda^2 - m^2)^2} \left[ \lambda^2 - m^2 + \frac{m^2}{\lambda} F_0(m, \lambda) \right]. \]  

The remaining functions \( F_0, L_0, L_1, \) and \( L_2 \) are given in [1] and [3].

The coupling coefficients \( \alpha_i \) are identical to those in [1]. We list the remaining, mostly new, coupling coefficients \( \beta_i, \beta'_i, \tilde{\gamma}_i, \hat{\gamma}_i, \gamma_i, \) and \( \lambda'_i \) in Appendix A. These reduce in combination to the coefficients given in [1] in the limit considered there.

III. COMPARISON WITH EXPERIMENT

A. Fits to the data

We will choose the strong interaction couplings \( F, D, C, \) and \( H \) to satisfy the SU(6) relations \( F = 2D/3, \) \( C = -2D, \) and \( H = -3D \) as in our previous work [1, 13, 14]. For \( F = 0.5, \) we then have \( D = 0.75, C = -1.5, \) and \( H = -2.15, \) with \( F + D \approx |g_A/g_V| = 1.26. \) We use the values \( f_\pi = 93 \) MeV and \( f_K = f_\eta = 1.2f_\pi \) [4]. While we fit the cutoff parameter \( \lambda, \) the fitted value of is found to lie in a range consistent with that in our earlier fits the baryon magnetic moments [11] as it must. \( \lambda \) is therefore not a new or independent parameter.

1. Unconstrained case

In this case we have a total of five parameters \( m_B, \delta m_B, m_s b_D, m_s b_F, \) and \( m_s c \) to fit the average masses of the eight SU(2) baryon multiplets. An equal-weight least-squares fit to the masses using only these parameters and no loop corrections gives an average deviation of \( 3.1 \pm 0.3 \) MeV between the theoretical and experimental values of the masses for \( m_B = 1191.6 \) MeV, \( \delta m_B = 43.4 \) MeV, \( m_s b_D = 30.1 \) MeV, \( m_s b_F = -94.8 \) MeV, and \( m_s c = 220.5 \) MeV. When the loop corrections are included in the fit and the parameters readjusted [4] the average

4For example, the sum \( \tilde{\gamma}_i + \hat{\gamma}_i + \gamma'_i \) is denoted by \( \gamma_i \) in [1].

5We use the values of the parameters \( \delta m_B, m_s b_D, m_s b_F, \) and \( m_s c \) obtained in the fit without loop corrections in evaluating the latter. The mass insertions in the loops in Fig. 2 and corresponding insertions on external lines correct for the deviations of the baryon masses on the internal and external masses from the symmetric values \( m_B \) and \( m_T = m_B + \delta m_B. \) These are described quite
deviation from experiment drops to $0.5 \pm 0.3$ MeV, a value significantly smaller than the mass splittings within the multiplets. The results are summarized in Table I. The best-fit parameters are $m_B = 1590.9$ MeV, $\delta m_B = 86.6$ MeV, $m_s b_D = 28.7$ MeV, $m_s b_F = -76.7$ MeV, and $m_s c = 185.6$ MeV, with $\lambda = 441$ MeV. The results are not especially sensitive to the value of the cutoff parameter. Thus, a change to the value $\lambda = 407$ MeV used in our analysis of the baryon magnetic moments [11] changes the average deviation of the theoretical masses from experiment of 0.7 MeV. The fitted value of the parameter $m_s c = 185.6$ MeV is reasonably close to, but somewhat above the value $-3(b_D + b_F)m_s = 144$ MeV predicted by the QM relation in Eq. (2.4). The difference is consistent with the expected enhancement of the attractive hyperfine interaction in the octet, and the corresponding suppression of the repulsive interactions in the decuplet [14], which gives $m_s c > -3m_s (b_D + b_F)$.

A detailed breakdown of the contributions of the loop integrals to the fitted baryon masses is given in Table II. All of the loop corrections lower the baryon masses as expected in second order perturbation theory. The corrections to the masses are substantial, but are still small in comparison to the leading contributions, ranging from 31 to 37 percent of the tree-level value of $m_B$ and suggesting reasonable convergence of the loop expansion. These results are in marked contrast to the results obtained in HBChPT for the point baryons, where the loop contributions calculated using dimensional regularization are comparable in size to the leading terms [4].

The results in Table II also show that the contributions of decuplet intermediate states are very important, of the same order as the contributions from the octet. Because the mean tree-level decuplet-octet mass splitting $\Delta m_B = \delta m_B + m_s c - \frac{2}{3} m_s b_f = 106$ MeV is small on the scale of the typical momenta $k \approx \lambda = 441$ MeV determined by the form factor, it is not justified to consider the decuplet as massive relative to the octet, integrate out its explicit contributions, and attempt an expansion of the decuplet contributions in powers of $k/\Delta m_B$ as in [8].

Table III shows the contributions of loops involving specific mesons to the total loop corrections. We note that the pion loops are very important, contrary to the result in HBChPT with dimensional regularization. In that approach, the finite parts of the loop integrals are proportional to the square or cube of the mass of the meson in the loop. Pion loops are consequently strongly suppressed, and were ignored, for example, in [3]. The effect of the form factor is crucial here. The derivative couplings of mesons to baryons emphasize high-momentum or short-distance contributions in the loop integrals which are not given reliably by the theory. These contributions are cut off by the form factors, reflecting the compositeness and extended structures of the baryons and mesons. With the form factors present, the momentum scale in the loop integrations is set primarily by the parameter $\lambda$, and the integrals are dominated by intermediate-range contributions. The effects of the meson masses on the loop integrals are suppressed accordingly. For example, the ratios of the loop integrals $\tilde{M}^3$, Eq. (2.13), corresponding to the diagram in Fig. Ia are $\pi : K : \eta = 1 : 0.76 : 0.73$ for $\lambda = 441$ MeV, a result not very different from the unit ratios that would hold for equal masses, but very different from the ratios $\pi : K : \eta = 1 : 44 : 60$ in ChPT.

well by the tree-level fit. The greater part of the loop corrections preserves the tree-level structure and can be absorbed in the corresponding parameters as discussed in the following.
Similar results hold for the other integrals defined above.

The loop corrections shown in Tables II and III are much larger than the final changes in the baryon masses obtained after adjusting the parameters $m_B$, $\delta m_B$, $m_s b_D$, $m_s b_F$, and $m_s c$. This suggests that the loop corrections have approximately the tree-level structure, and can be parametrized up to small residual contributions $\epsilon_i$ using the tree-level mass relations with five parameters analogous to those above. We note that the total loop corrections introduce no new symmetry breaking when the meson masses in the loops are taken as equal. The parts of the individual loop contributions that do not have the tree-level structure therefore cancel, and the total corrections can be absorbed completely by a readjustment of the tree-level parameters. While this is no longer possible for $M_\pi \neq M_K \neq M_\eta$, the effect of unequal masses on the loop integrals is suppressed because of the form factors as discussed above. The corrections therefore retain the tree-level structure approximately, and the largest parts of the corrections can be absorbed by adjusting the input parameters.

In Table III we show the residual contributions $\epsilon_i$ obtained after subtracting out tree-level parametrizations of the individual and combined meson loops. The $\epsilon$’s for pion, kaon, and eta loops are much smaller than the loop corrections themselves, with magnitudes less than 20 MeV. The $\epsilon$’s for different mesons all have similar magnitudes, and a varying pattern of signs. There is no suppression of pion loops. The total residual $\epsilon_{\text{tot}}$ would vanish identically for equal meson masses, and the values given are significantly smaller than the individual loop contributions because of the expected cancellations.

We also find that the sum of the fitted values of $m_B$, $\delta m_B$, $m_s b_D$, $m_s b_F$, and $m_s c$ and the corresponding parameters for the loops reproduces the tree-level parameters. The only new contributions of the loop corrections are therefore in the $\epsilon$’s. We will return to this in Sec. III B.

2. Constrained case

If the coupling $c$ is constrained by the QM relation in Eq. (2.6), we have four instead of five parameters to fit average masses of the eight SU(2) baryon multiplets. An equal-weight least-squares fit to the octet and decuplet masses using only those four parameters gives a average deviation from the experiment of 4.4 ± 0.3 MeV. With the loop corrections included, we find a best fit for $m_B = 1555.7$ MeV, $\delta m_B = 100.8$ MeV, $m_s b_D = 20.5$ MeV, $m_s b_F = -81.4$ MeV, and $\lambda = 432.2$ MeV, with an average deviation from the experimental values of the multiplet masses of about 4.0 ± 0.3 MeV arising mainly from deviations in the $\Sigma$ and $\Xi$ masses in the octet. The value $m_s c = -3m_s(b_D + b_F) = 182.7$ MeV, which affects only the decuplet masses in first order, is in excellent agreement with the value of 185.6 MeV obtained in the unconstrained fit. The constraint affects mostly the value the of the difference $m_s(b_D - b_F)$, and worsens the fit to the octet masses. The final results are summarized in Table IV.

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6This was shown analytically by Holstein et al. [4] for the purely octet case in their discussion of the difference between a momentum cutoff and dimensional regularization. It remains true in the present calculation with decuplet contributions.
Our overall conclusions with respect to the loop corrections are the same as in the unconstrained case. The corrections to the masses are substantial, but are still small in comparison to the leading contributions, ranging from 30 to 35 percent of the tree-level value of $m_B$. This again suggests reasonable convergence of the loop expansion. The pion loops are as important as kaon or eta loops, and the contributions of decuplet intermediate states are very important, of the same order as the contributions from octet states.

**B. BARYON MASS RELATIONS**

The unconstrained fit to the baryon masses given above is excellent. However, the improvement over the tree level fit is small if measured solely by the average deviation between theory and experiment, and the overall fit does not give as sensitive a test of the loop corrections as might be expected. As emphasized above, the only parts of the loop contributions that go beyond the tree-level structure are those in the $\epsilon$’s in Table III. To get a stronger test of the theory, we need to emphasize mass relations that are independent of the tree-level parameters and of the large parts of the loop corrections that have the same structure. If the coupling $c$ is unconstrained, there are three such mass relations, specifically the Gell-Mann–Okubo formula

$$\frac{1}{4}(3M_\Lambda + M_\Sigma) - \frac{1}{2}(M_N + M_\Xi) = 0 \quad (3.1)$$

for the baryon octet, and two independent relations from Gell-Mann’s equal spacing rule for the baryon decuplet,

$$M_{\Sigma^*} - M_\Delta = M_{\Xi^*} - M_{\Sigma^*} = M_\Omega - M_{\Xi^*}. \quad (3.2)$$

We will rearrange the latter to eliminate all tree-level parameters, and will deal with the relations

$$M_{\Sigma^*} + M_{\Xi^*} - M_\Delta - M_\Omega = 0,$$
$$2M_{\Sigma^*} - M_\Delta - M_{\Xi^*} = 0,$$
$$2M_{\Xi^*} - M_{\Sigma^*} - M_\Omega = 0, \quad (3.3)$$

where the first is the sum of the remaining two.

In Table V, we show the violations of these mass relations obtained from the loop graphs evaluated using the couplings and the fitting parameters given in the unconstrained case of the previous section. The violations of the Gell-Mann–Okubo formula, Eq.(3.1), are quite small graph-by-graph, both absolutely and on the scale of the loop contributions. In particular, it is not necessary to have small loop contributions for the Gell-Mann–Okubo relation to be well satisfied by the final octet-baryon masses.

The violations of the decuplet mass relations Eq.(3.3) by the pion, kaon, and eta loop graphs are significant individually. However, the total violations are small because of cancellations. The theoretical results agree with the observed violations in sign, and also agree reasonably well in magnitude given the theoretical and experimental uncertainties. We conclude that the general structure of the loop corrections is correct, but further contributions to the final masses are clearly needed.
IV. CONCLUSIONS

In this paper, we have considered the one-loop corrections to the baryon masses in a loop expansion in which baryons are treated as composite particles by introducing the form factor. Using the parameters that appear in the theory at tree level and the SU(6) couplings used in our earlier work, we fit the average masses of the eight octet and decuplet baryon multiplets can be fitted with an average deviation of $0.5 \pm 0.3$ MeV. The fit is remarkably good given the absence of higher order contributions and the experimental uncertainties.

As discussed above, the smallness of loop contributions relative to the leading terms suggests that the perturbation series is under control. We find also that pion loops are quite important as would be expected in a calculation dominated by intermediate range contributions. Both results are in sharp contrast to the situation in conventional HBChPT with dimensional regularization, where there is little indication that the perturbation series converges, and pion contributions are strongly suppressed. In addition, we find that the decuplet states must be treated as light on the scale of the octet, and included explicitly. In particular, the contributions of decuplet intermediate states in loops to the octet masses are as important as the contributions of octet intermediate states.

Finally, we emphasize that the sensitivity of the loop contributions to the masses of the mesons in the loops is greatly reduced by the form factors. As a result, the loop contributions retain the basic structure of the tree-level masses, and can largely be absorbed by readjustment of the tree-level parameters. The overall precision of the fit on the scale of the baryon masses is therefore not a good test of the theory. However, the observed violations of the Gell-Mann–Okubo and decuplet mass relations do provide tests. We find that the corrected Gell-Mann–Okubo relation is satisfied quite well. The pattern of violations of the decuplet mass relations is also described correctly, and the relations are satisfied reasonably well numerically.

We believe that these results demonstrate the usefulness of treating baryons as composite particles. By introducing the form factor, the unphysical high-momentum effects that dominate conventional calculations are suppressed, and a theory is obtained that appears to describe low energy processes involving baryons quite well. The dynamical problem that remains is one of understanding the form factor at a deeper level.

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APPENDIX A: THE COUPLING COEFFICIENTS

In this appendix, we present the coupling coefficients explicitly. For simplicity, the superscript \((X)\) is suppressed. Our coupling coefficients \(\alpha_i\) are identical to those in \([1]\). There are the relations between \(\lambda_i\) and \(\beta_i\) and also between \(\lambda_i'\) and \(\beta_i'\)

\[
\lambda_i = \frac{3}{2} \beta_i \quad \lambda_i' = \frac{3}{2} \beta_i'.
\]  

(A1)

The coupling coefficients \(\beta_i\) are

\[
\begin{align*}
\beta^{(\pi)}_N &= \frac{3}{2} (D + F)^2, \quad \beta^{(\pi)}_\Sigma = \frac{2}{3} (D^2 + 6F^2), \\
\beta^{(\pi)}_\Xi &= \frac{3}{2} (D - F)^2, \quad \beta^{(\pi)}_\Lambda = 2D^2, \\
\beta^{(\pi)}_\Delta &= \frac{25}{54} \mathcal{H}^2, \quad \beta^{(\pi)}_\Sigma^* = \frac{20}{81} \mathcal{H}^2, \\
\beta^{(\pi)}_{\Xi^*} &= \frac{5}{54} \mathcal{H}^2, \quad \beta^{(\pi)}_{\Omega} = 0,
\end{align*}
\]  

(A2)

for the pion loops,

\[
\begin{align*}
\beta^{(K)}_N &= \frac{5}{3} D^2 - 2DF + 3F^2, \quad \beta^{(K)}_\Sigma = 2(D^2 + F^2), \\
\beta^{(K)}_\Xi &= \frac{5}{3} D^2 + 2DF + 3F^2, \quad \beta^{(K)}_\Lambda = \frac{2}{3} D^2 + 6F^2, \\
\beta^{(K)}_\Delta &= \frac{5}{27} \mathcal{H}^2, \quad \beta^{(K)}_{\Sigma^*} = \frac{40}{81} \mathcal{H}^2, \\
\beta^{(K)}_{\Xi^*} &= \frac{5}{9} \mathcal{H}^2, \quad \beta^{(K)}_{\Omega} = \frac{10}{27} \mathcal{H}^2,
\end{align*}
\]  

(A3)

for the kaon loops, and

\[
\begin{align*}
\beta^{(\eta)}_N &= \frac{1}{6} (D - 3F)^2, \quad \beta^{(\eta)}_\Sigma = \frac{2}{3} D^2, \\
\beta^{(\eta)}_\Xi &= \frac{1}{6} (D + 3F)^2, \quad \beta^{(\eta)}_\Lambda = \frac{2}{3} D^2, \\
\beta^{(\eta)}_\Delta &= \frac{5}{54} \mathcal{H}^2, \quad \beta^{(\eta)}_{\Sigma^*} = 0, \\
\beta^{(\eta)}_{\Xi^*} &= \frac{5}{54} \mathcal{H}^2, \quad \beta^{(\eta)}_{\Omega} = \frac{10}{27} \mathcal{H}^2,
\end{align*}
\]  

(A4)

for the eta loops.

The coefficients \(\beta_i'\) are

\[
\begin{align*}
\beta^{(\pi)}_N' &= \frac{4}{3} C^2, \quad \beta^{(\pi)}_\Sigma' = \frac{2}{9} C^2, \quad \beta^{(\pi)}_\Xi' = \frac{C^2}{3}, \quad \beta^{(\pi)}_\Lambda' = C^2, \\
\beta^{(\pi)}_\Delta' &= \frac{C^2}{3}, \quad \beta^{(\pi)}_{\Sigma^*}' = \frac{5}{18} C^2, \quad \beta^{(\pi)}_{\Xi^*}' = \frac{C^2}{6}, \quad \beta^{(\pi)}_{\Omega}' = 0,
\end{align*}
\]  

(A5)

for the pion loops,
\[ \beta'_{N}^{(K)} = \frac{C^2}{3}, \quad \beta'_{\Sigma}^{(K)} = \frac{10}{9}C^2, \quad \beta'_{\Xi}^{(K)} = C^2, \quad \beta'_{\Lambda}^{(K)} = \frac{2}{3}C^2, \]
\[ \beta'_{\Delta} = \frac{C^2}{3}, \quad \beta'_{\Sigma^*}^{(K)} = \frac{2}{9}C^2, \quad \beta'_{\Xi^*}^{(K)} = \frac{C^2}{3}, \quad \beta'_{\Omega}^{(K)} = \frac{2}{3}C^2, \]  
(A6)
for the kaon loops, and
\[ \beta'_{N}^{(\eta)} = 0, \quad \beta'_{\Sigma}^{(\eta)} = \frac{C^2}{3}, \quad \beta'_{\Xi}^{(\eta)} = \frac{C^2}{3}, \quad \beta'_{\Lambda}^{(\eta)} = 0, \]
\[ \beta'_{\Delta}^{(\eta)} = 0, \quad \beta'_{\Sigma^*}^{(\eta)} = \frac{C^2}{6}, \quad \beta'_{\Xi^*}^{(\eta)} = \frac{C^2}{6}, \quad \beta'_{\Omega}^{(\eta)} = 0, \]  
(A7)
for the eta loops.

The coefficients \( \tilde{\gamma}_i \) are
\[ \tilde{\gamma}_N^{(K)} = 3b_D - b_F + 4\sigma, \quad \tilde{\gamma}_{\Sigma}^{(K)} = 2b_D + 4\sigma, \quad \tilde{\gamma}_{\Xi}^{(K)} = 3b_D + b_F + 4\sigma, \quad \tilde{\gamma}_{\Lambda}^{(K)} = \frac{10}{3}b_D + 4\sigma, \]
\[ \tilde{\gamma}_{\Delta}^{(K)} = -c + 4\tilde{\sigma}, \quad \tilde{\gamma}_{\Sigma^*}^{(K)} = -\frac{4}{3}c + 4\tilde{\sigma}, \quad \tilde{\gamma}_{\Xi^*}^{(K)} = -\frac{5}{3}c + 4\tilde{\sigma}, \quad \tilde{\gamma}_{\Omega}^{(K)} = -2c + 4\tilde{\sigma}, \]  
(A8)
for the kaon loops, and
\[ \tilde{\gamma}_N^{(\eta)} = \frac{4}{3}(b_D - b_F + \sigma), \quad \tilde{\gamma}_{\Sigma}^{(\eta)} = \frac{4}{3}\sigma, \quad \tilde{\gamma}_{\Xi}^{(\eta)} = \frac{4}{3}(b_D + b_F + \sigma), \quad \tilde{\gamma}_{\Lambda}^{(\eta)} = \frac{4}{9}(4b_D + 3\sigma), \]
\[ \tilde{\gamma}_{\Delta} = \frac{4}{3}\tilde{\sigma}, \quad \tilde{\gamma}_{\Sigma^*}^{(\eta)} = \frac{4}{9}(-c + 3\tilde{\sigma}), \quad \tilde{\gamma}_{\Xi^*}^{(\eta)} = \frac{4}{9}(-2c + 3\tilde{\sigma}), \quad \tilde{\gamma}_{\Omega}^{(\eta)} = \frac{4}{3}(-c + \tilde{\sigma}), \]  
(A9)
for the eta loops. The coefficients \( \tilde{\gamma}_i \) vanish for the pion loops.

The coefficients \( \hat{\gamma}_i \) are
\[ \hat{\gamma}_N^{(\pi)} = -\frac{9}{2}(b_D - b_f)(D + F)^2 - 2\sigma \lambda_N^{(\pi)}, \quad \hat{\gamma}_{\Sigma}^{(\pi)} = -\frac{8}{3}b_D D^2 - 2\sigma \lambda_{\Sigma}^{(\pi)}, \]
\[ \hat{\gamma}_{\Xi}^{(\pi)} = -\frac{9}{2}(b_D + b_f)(D - F)^2 - 2\sigma \lambda_{\Xi}^{(\pi)}, \quad \hat{\gamma}_{\Lambda}^{(\pi)} = -2\sigma \lambda_{\Lambda}^{(\pi)}, \]
\[ \hat{\gamma}_{\Delta}^{(\pi)} = -2\tilde{\sigma} \lambda_{\Delta}^{(\pi)}, \quad \hat{\gamma}_{\Sigma^*}^{(\pi)} = \frac{20}{81}c\mathcal{H}^2 - 2\tilde{\sigma} \lambda_{\Sigma^*}^{(\pi)}, \quad \hat{\gamma}_{\Omega}^{(\pi)} = -2\tilde{\sigma} \lambda_{\Omega}^{(\pi)}, \]  
(A10)
for the pion loops,
\[ \hat{\gamma}_N^{(K)} = -\frac{2}{3}b_D(D + 3F)^2 - 2\sigma \lambda_N^{(K)}, \quad \hat{\gamma}_{\Sigma}^{(K)} = -6b_D(D^2 + F^2) - 12b_F DF - 2\sigma \lambda_{\Sigma}^{(K)}, \]
\[ \hat{\gamma}_{\Xi}^{(K)} = -\frac{2}{3}b_D(D - 3F)^2 - 2\sigma \lambda_{\Xi}^{(K)}, \quad \hat{\gamma}_{\Lambda}^{(K)} = -2b_D(D^2 + 9F^2) + 12b_F DF - 2\sigma \lambda_{\Lambda}^{(K)}, \]
\[ \hat{\gamma}_{\Delta}^{(K)} = \frac{5}{27}c\mathcal{H}^2 - 2\tilde{\sigma} \lambda_{\Delta}^{(K)}, \quad \hat{\gamma}_{\Sigma^*}^{(K)} = \frac{40}{81}c\mathcal{H}^2 - 2\tilde{\sigma} \lambda_{\Sigma^*}^{(K)}, \quad \hat{\gamma}_{\Omega}^{(K)} = \frac{20}{27}c\mathcal{H}^2 - 2\tilde{\sigma} \lambda_{\Omega}^{(K)}, \]  
(A11)
for the kaon loops, and
\[
\begin{align*}
\hat{\gamma}_{\Delta}^{(n)} &= -\frac{1}{2}(b_D - b_F)(D - 3F)^2 - 2\sigma\lambda_N^{(n)},
\hat{\gamma}_{\Sigma}^{(n)} = -2\sigma\lambda_{\Sigma}^{(n)}, \\
\hat{\gamma}_{\Xi}^{(n)} &= -\frac{1}{2}(b_D + b_F)(D + 3F)^2 - 2\sigma\lambda_{\Xi}^{(n)},
\hat{\gamma}_{\Lambda}^{(n)} = \frac{8}{3}b_DD^2 - 2\sigma\lambda_{\Lambda}^{(n)}, \\
\hat{\gamma}_{\Omega}^{(n)} &= \frac{5}{27}cH^2 - 2\sigma\lambda_{\Xi*}^{(n)},
\hat{\gamma}_{\Omega}^{(n)} = \frac{10}{9}cH^2 - 2\sigma\lambda_{\Omega}^{(n)}, \\
\end{align*}
\] (A12)

for the eta loops.

Finally, the coefficients \(\gamma_i\) are

\[
\begin{align*}
\gamma_{N}^{(\pi)} &= -2\tilde{\sigma}\lambda_N^{(\pi)},
\gamma_{\Sigma}^{(\pi)} = \frac{2}{9}cC^2 - 2\tilde{\sigma}\lambda_{\Sigma}^{(\pi)}, \\
\gamma_{\Xi}^{(\pi)} &= \frac{2}{3}cC^2 - 2\tilde{\sigma}\lambda_{\Xi}^{(\pi)},
\gamma_{\Lambda}^{(\pi)} = cC^2 - 2\tilde{\sigma}\lambda_{\Lambda}^{(\pi)}, \\
\gamma_{\Delta}^{(\pi)} &= (b_F - b_D)C^2 - 2\sigma\lambda_{\Delta}^{(\pi)},
\gamma_{\Sigma}^{(\Pi)} = \frac{2}{3}b_DC^2 - 2\sigma\lambda_{\Sigma}^{(\Pi)}, \\
\gamma_{\Omega}^{(\Pi)} &= \frac{1}{2}(b_D + b_F)C^2 - 2\sigma\lambda_{\Xi*}^{(\Pi)},
\gamma_{\Omega}^{(\Pi)} = -2\sigma\lambda_{\Omega}^{(\Pi)}, \\
\end{align*}
\] (A13)

for the pion loops,

\[
\begin{align*}
\gamma_{N}^{(K)} &= \frac{1}{3}cC^2 - 2\tilde{\sigma}\lambda_N^{(K)},
\gamma_{\Sigma}^{(K)} = \frac{4}{9}cC^2 - 2\tilde{\sigma}\lambda_{\Sigma}^{(K)}, \\
\gamma_{\Xi}^{(K)} &= \frac{7}{3}cC^2 - 2\tilde{\sigma}\lambda_{\Xi}^{(K)},
\gamma_{\Lambda}^{(K)} = \frac{4}{3}cC^2 - 2\tilde{\sigma}\lambda_{\Lambda}^{(K)}, \\
\gamma_{\Delta}^{(K)} &= -2\sigma\lambda_{\Delta}^{(K)},
\gamma_{\Sigma}^{(K)} = \frac{2}{3}b_DC^2 - 2\sigma\lambda_{\Xi}^{(K)}, \\
\gamma_{\Omega}^{(K)} &= -\frac{2}{3}b_DC^2 - 2\sigma\lambda_{\Xi*}^{(K)},
\gamma_{\Omega}^{(K)} = -2(b_D + b_F)C^2 - 2\sigma\lambda_{\Omega}^{(K)}, \\
\end{align*}
\] (A14)

for the kaon loops, and

\[
\begin{align*}
\gamma_{N}^{(\eta)} &= -2\tilde{\sigma}\lambda_N^{(\eta)},
\gamma_{\Sigma}^{(\eta)} = \frac{1}{3}cC^2 - 2\tilde{\sigma}\lambda_{\Sigma}^{(\eta)}, \\
\gamma_{\Xi}^{(\eta)} &= \frac{2}{3}cC^2 - 2\tilde{\sigma}\lambda_{\Xi}^{(\eta)},
\gamma_{\Lambda}^{(\eta)} = -2\tilde{\sigma}\lambda_{\Lambda}^{(\eta)}, \\
\gamma_{\Delta}^{(\eta)} &= -2\sigma\lambda_{\Delta}^{(\eta)},
\gamma_{\Sigma}^{(\eta)} = -2\sigma\lambda_{\Sigma}^{(\eta)}, \\
\gamma_{\Omega}^{(\eta)} &= -\frac{1}{2}(b_D + b_F)C^2 - 2\sigma\lambda_{\Xi}^{(\eta)},
\gamma_{\Omega}^{(\eta)} = -2\sigma\lambda_{\Omega}^{(\eta)}, \\
\end{align*}
\] (A15)

for the eta loops.
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FIGURES

FIG. 1. Diagrams that give rise to non-analytic $m_s^{3/2}$ corrections to the baryon masses in conventional HBChPT. The dashed lines denote the mesons, the single and double solid lines denote octet and decuplet baryons, respectively. A heavy dot with a meson line represents an insertion of the form factor $F(k, \nu)$ in Eq. (2.8), with meson momentum $k$.

FIG. 2. Diagrams that give rise to non-analytic $m_s^2 \ln m_s$ corrections to the baryon masses in conventional HBChPT. Two short straight lines denote an insertion of the tree level mass terms $b_D$, $b_F$, and $\sigma$ for the baryon octet, and the terms $c$ and $\tilde{\sigma}$ for the baryon decuplet.
TABLE I. Baryon masses in MeV for the unconstrained case. Here $\Delta M_i = M_i^{(\text{theory})} - M_i^{(\text{exp})}$. The average deviation $|\Delta M_i| = 0.5 \pm 0.3$ MeV.

| Baryon | $N$ | $\Lambda$ | $\Sigma$ | $\Xi$ | $\Delta$ | $\Sigma^*$ | $\Xi^*$ | $\Omega$ |
|--------|-----|-----------|----------|------|---------|---------|------|------|
| Theory | 939.2 | 1115.3 | 1193.0 | 1318.4 | 1231.5 | 1385.8 | 1532.6 | 1672.6 |
| Exp.   | 938.9 | 1115.7 | 1193.1 | 1318.1 $\pm$ 0.3 | 1232.0 $\pm$ 2.0 | 1384.6 $\pm$ 0.4 | 1533.4 $\pm$ 0.3 | 1672.5 $\pm$ 0.3 |
| $\Delta M_i$ | 0.3 | $-0.4$ | $-0.1$ | 0.3 $\pm$ 0.3 | $-0.5$ $\pm$ 2.0 | 1.2 $\pm$ 0.4 | $-0.8$ $\pm$ 0.3 | 0.1 $\pm$ 0.3 |

TABLE II. Detailed breakdown of the contributions of the loop integrals to the masses of the octet and decuplet baryons in MeV for the unconstrained case. The contributions are evaluated using the value $\lambda = 441$ MeV for the cutoff parameter, and the couplings $F = 0.5$, $D = -0.75$, $C = -1.5$, and $H = -2.15$. A best fit is obtained for $m_B = 1590.9$ MeV, $m_s b_D = 28.7$ MeV, $m_s b_F = -76.7$ MeV, $m_s c = 185.6$ MeV, and $\delta m_B = 86.6$ MeV. Contributions from the various loop graphs are labeled using the corresponding coupling coefficients defined in Sec. [11C].

| Baryon | $\alpha$ | $\beta$ | $(\hat{\gamma} + \tilde{\gamma} - \lambda \alpha)$ | $\beta'$ | $(\gamma - \lambda \alpha)$ | Loops | $M_i$ |
|--------|--------|--------|---------------------------------|---------|----------------|-------|------|
| $N$    | 1380.1 | $-268.5$ | 49.1 | $-301.5$ | 80.0 | $-440.9$ | 939.2 |
| $\Lambda$ | 1514.3 | $-214.7$ | 20.2 | $-270.9$ | 66.3 | $-399.1$ | 1115.3 |
| $\Sigma$ | 1590.9 | $-225.6$ | 10.9 | $-198.2$ | 15.0 | $-397.9$ | 1193.0 |
| $\Xi$ | 1686.9 | $-168.7$ | $-22.4$ | $-208.4$ | 31.0 | $-368.5$ | 1318.4 |
| $\Delta$ | 1677.5 | $-286.3$ | $-22.8$ | $-111.8$ | $-25.2$ | $-446.0$ | 1231.5 |
| $\Sigma^*$ | 1801.2 | $-239.3$ | $-48.6$ | $-105.1$ | $-22.4$ | $-415.5$ | 1385.8 |
| $\Xi^*$ | 1924.9 | $-204.2$ | $-74.5$ | $-93.3$ | $-20.3$ | $-392.3$ | 1532.6 |
| $\Omega$ | 2048.7 | $-181.1$ | $-100.4$ | $-76.3$ | $-18.3$ | $-376.0$ | 1672.6 |

TABLE III. The contributions in MeV of the loop integrals involving pions, kaons, and etas to the baryon masses for the parameters given in Table II. The quantities $\epsilon_i$ are the residual contributions of the loops after the parts that can be described by the tree-level structure are subtracted.

| Baryon | $\pi$ loops | $\epsilon_\pi$ | $K$ loops | $\epsilon_K$ | $\eta$ loops | $\epsilon_\eta$ | $\epsilon_{\text{tot}}$ |
|--------|---------|------|---------|--------|--------|--------|------|
| $N$    | $-413.3$ | 5.2  | $-43.1$ | $-8.0$  | 15.5  | 5.4   | 2.62 |
| $\Lambda$ | $-245.9$ | 7.9  | $-141.8$ | 12.0  | $-11.3$ | 8.0  | 3.93 |
| $\Sigma$ | $-168.5$ | 2.6  | $-180.4$ | 4.0  | $-49.0$ | 2.7  | 1.31 |
| $\Xi$ | $-63.5$ | 5.3  | $-223.8$ | $-8.0$ | $-81.1$ | 5.4  | 2.62 |
| $\Delta$ | $-316.8$ | 8.6  | $-107.0$ | $-17.0$ | $-22.2$ | 12.0  | 3.57 |
| $\Sigma^*$ | $-194.0$ | 8.7  | $-189.4$ | 17.1  | $-32.0$ | $-12.0$ | 3.61 |
| $\Xi^*$ | $-88.7$ | 8.4  | $-237.4$ | 16.8  | $-66.2$ | $-11.9$ | 3.49 |
| $\Omega$ | 0.0 | 8.5  | $-252.0$ | $-16.9$ | $-124.0$ | 12.0  | 3.53 |
TABLE IV. Baryon masses in MeV for the constrained case. Here \( \Delta M_i = M_i^{\text{theory}} - M_i^{\text{exp}} \). The average deviation \( |\Delta M_i| = 4.0 \pm 0.3 \) MeV.

| \( N \) | \( \Lambda \) | \( \Sigma \) | \( \Xi \) | \( \Delta \) | \( \Sigma^* \) | \( \Xi^* \) | \( \Omega \) |
|---|---|---|---|---|---|---|---|
| Theory | 936.6 | 1119.2 | 1184.2 | 1325.9 | 1234.5 | 1386.7 | 1531.7 | 1669.6 |
| Exp. | 938.9 | 1115.7 | 1193.1 | 1318.1 \( \pm 0.3 \) | 1232.0 \( \pm 2.0 \) | 1384.6 \( \pm 0.4 \) | 1533.4 \( \pm 0.3 \) | 1672.5 \( \pm 0.3 \) |
| \( \Delta M_i \) | \(-2.3\) | \(3.5\) | \(-8.9\) | \(7.8 \pm 0.3\) | \(2.5 \pm 2.0\) | \(2.1 \pm 0.4\) | \(-1.7 \pm 0.3\) | \(-2.9 \pm 0.3\) |

TABLE V. Violations of the baryon mass relations in MeV for the unconstrained case. Violations from the various loop graphs are labeled using the corresponding coupling coefficients. The tadpole graphs satisfy the mass relations.

| Relation | \( \beta \) | \( \beta' \) | \( \hat{\gamma} - \lambda \alpha \) | \( \hat{\gamma}' - \lambda' \alpha \) | Total | Experiment |
|---|---|---|---|---|---|---|
| Gell-Mann–Okubo | 1.2 | 2.2 | 4.5 | \(-2.0\) | 5.9 | 6.6 \( \pm 0.2 \) |
| \( M_{\Sigma^*} - M_{\Xi^*} - M_\Delta - M_\Omega \) | 23.8 | \(-10.4\) | 0.0 | 0.8 | 14.2 | 13.5 \( \pm 2.1 \) |
| \( 2M_{\Sigma^*} - M_\Delta - M_{\Xi^*} \) | 11.9 | \(-5.2\) | 0.0 | 0.7 | 7.4 | 3.8 \( \pm 2.1 \) |
| \( 2M_{\Xi^*} - M_{\Sigma^*} - M_\Omega \) | 11.9 | \(-5.2\) | 0.0 | 0.1 | 6.8 | 9.7 \( \pm 0.7 \) |
