Dependence structure of CAT bonds and portfolio diversification: a copula-GARCH approach

Adlane Haffar· Éric Le Fur

Revised: 25 April 2022 / Accepted: 13 May 2022 / Published online: 3 June 2022
© The Author(s), under exclusive licence to Springer Nature Limited 2022

Abstract
This paper analyzes advantages of investing in catastrophe bonds (CATs) in terms of portfolio diversification. Indeed, the increase in environmental disasters and their economic and financial consequences are still poorly covered by insurance and reinsurance companies. As a result, there is a rapid growth in the use of catastrophe bonds on the financial markets, which can allow the transfer of risks to the capital market. We use copula-GARCH models to test the time-varying dependence of CATs, in a portfolio composed of six stock markets (CAC 40, DJIA, EUROSTOXX 50, FTSE 100, HANGSENG, and NIKKEI 225). Our results reveal that the CATs display the highest risk-adjusted performer. This security may be a good complement to a portfolio for investors seeking to optimize their risk-adjusted returns. In addition, the CATs are one of the best diversifiers. Finally, the CATs are the asset that increases the lowest the probability of extreme co-variations with its benchmark portfolio.

Keywords CAT bonds · Copula-GARCH model · Portfolio diversification · Portfolio risk · Robust MCD portfolio

JEL Classification C58 · G11 · G12 · G22

Introduction
The academic literature has widely studied the phenomenon of dependence structure of assets (see, e.g., Hong et al., 2007; Ning, 2010; Garcia and Tsafack, 2011; Mensi et al., 2017; Ji et al., 2018; Shahzad et al., 2018). For instance, equity returns seem to be more dependent during downturn periods than during upturns. In addition, in times of crisis, the phenomenon of financial contagion leads to an increase in dependence between markets. Extreme events in one market can also induce by extreme events in other markets.

To distinguish dependence between assets, some approaches are not sufficient such as linear correlation (Patton, 2004), linear regression models and constant correlations. Other approaches are more relevant such as measure in the tails of the distribution (Frahm et al., 2005; Schmidt and Stadtmüller, 2006) and quantile regression (e.g, Baur, 2013; Zhu et al., 2016; Amédee-Manesme et al., 2020). Copulas can also provide a useful tool for obtaining accurate estimates of value at risk. Indeed, copula can capture linear correlation, asymmetric dependence, and upper and lower tail dependence. Copulas were introduced by Sklar (1959) as a tool to link diverse marginal distributions together in order to form a joint multivariate distribution. Then, copulas have been broadly studied in finance, insurance and actuarial science to examine dependencies among risks (see, e.g., Frees and Valdez, 1998; Li, 2000; Embrechts et al., 2003; Cherubini et al., 2004; Nelsen, 2007; Patton, 2009, 2012, 2013; Ausin and Lopes, 2010; Dissmann et al., 2013).

Relatively few studies focus on the dependence structure of catastrophe bonds (CATs). For example, some studies consider issue size, maturity and time to maturity to measure the liquidity of CAT in the primary and secondary CAT markets (Braun, 2016; Gürtler et al., 2016). Others measure the liquidity of the CATs market and the liquidity premium embedded in CAT spreads and indicate that the time to...
maturity, yield volatility, and yield dispersion in the primary market are the three most effective liquidity indicators (Zhao and Yu, 2019). Nevertheless, CATs that are a type of insurance-linked securities could help solve the inefficiency of the reinsurance market. Indeed, correlated CAT risks within insurance markets may be uncorrelated with other risks in the economy (Cummins and Trainar, 2009). Studies on this issue agree on the interest of integrating CAT into a portfolio. In terms of portfolio diversification, the inclusion of CATs significantly increases the time-varying Sharpe ratio and the maximum diversification ratio of Choueifaty and Coignard (2008). CATs seems to provide the necessary diversification during critical periods, particularly during episodes of crisis and high volatility (Demert-Bélanget and Lai, 2020). The relatively small effect of the financial crisis on CATs compared to other asset classes makes them a valuable source of diversification for investors (Carayannopoulos and Perez, 2015). On average, CATs have a diversification advantage over the debt of similarly rated companies (Kish, 2016). In terms of stability, CATs are characterized by lower volatility and relatively high return stability (Mariani and Amoruso, 2016). Finally, portfolios with CATs are outside the mean variance frontier that can be reached by portfolios containing only traditional asset classes (Clark et al., 2016). However, to our knowledge, there are no studies about the dependence structure and portfolio diversification of CATs using copula.

The purpose of our article is to examine the dependence structure of CATs as well as their potential diversification gains. In other words, we measure the impact of this alternative asset on the performance indicators of the portfolio. We consider CATs as a protection tool that can be used during the occurrence of extreme events. To answer this question, we use copula-GARCH models to test the time-varying dependence of CATs in a portfolio that includes six global stock market indices (CAC40, DJIA, EUROSTOXX 50, FTSE 100, the HANGSENG, and the NIKKEI 225). We choose the GAMSTAR Institutional USD Acc (GAMSTAR hereafter), the STAR Institutional USD Acc (GAMSTAR) as the representative CAT index. Our results indicate that first the CAT displays the highest risk-adjusted performer of the indices. Second, the CAT may be a good complement to a portfolio for investors seeking to optimize their risk-adjusted returns. Third, the CAT is one of the best diversifiers. Finally, the CAT is the asset that increases the lowest the probability of extreme co-variations with its benchmark portfolio. The contribution of our article is twofold. First, it is the first to study the dependence structure of CATs using the copula approach. Second, it confirms the interest of introducing this asset into a portfolio composed of traditional assets.

The remainder of the paper is structured as follows: "Data" section outlines the data. “Methodology” section describes the "methodology". "Results and discussion" section reports and discusses the results. "Student Copula-AGDCC results" section concludes.

**Data**

Our analysis draws on monthly price data for the GAMSTAR Institutional USD Acc (GAMSTAR hereafter), the CAC 40, the DJIA, the EUROSTOXX 50, the FTSE 100, the HANGSENG, and the NIKKEI 225. Data are extracted from DataStream and cover the period from November 30, 2011, to February 28, 2020.

The GAMSTAR was created by the management company GAM Fund Management Limited on March 26, 2012, and is domiciled in Ireland. The benchmark used is the FTSE WGBI EUR 100%. At the end of February 2021, the outstanding amount is 402 million US dollars. The Fund’s primary investment objective is to seek to generate returns through selective investment in a global portfolio of catastrophe bonds. Asset allocation of the GAMSTAR is composed by 92% of fixed income and 8% of cash essentially distributed in corporate companies (85%).

The return series \( r_t \) are computed as follows:

\[
r_t = 100 \times \ln \frac{P_t}{P_{t-1}}
\]

where \( P_t \) is the observed price at time \( t \) and \( P_{t-1} \) is the observed price at time \( t-1 \).

Table 1 reports the main descriptive statistics of the data-set. The returns of each index are significantly negatively skewed. Excess kurtosis is evident in GAMSTAR. The multivariate (Mardia’s test) normality assumption is rejected. The univariate (Jarque-Bera’s test) normality assumption is only rejected for the GAMSTAR. Figure 1 describes the return data and shows that GAMSTAR displays the lowest volatility.

**Methodology**

A \( n \)-dimensional copula \( C(u_1, \ldots, u_n) \) is a \( n \)-dimensional distribution in the unit hypercube \([0, 1]^n \) with uniform \( U(0, 1) \) marginal distributions. According to Sklar (1959), for a function \( C \) that is called a copula of \( F \), every joint distribution \( F(x_1, \ldots, x_n) \) can be written as in Eq. (1)

---

1 The detailed characteristics of this asset are available on the link (pages 281-290): [https://www.morningstarfunds.ie/ie/funds/snapshot/snapshot.aspx?id=F00000Q1G2&tab=14&DocumentId=c285e5e767fade1d8e17d95f14f3ee4&Format=PDF](https://www.morningstarfunds.ie/ie/funds/snapshot/snapshot.aspx?id=F00000Q1G2&tab=14&DocumentId=c285e5e767fade1d8e17d95f14f3ee4&Format=PDF)
If the marginal distributions are continuous, then there is a unique copula associated with the joint distribution $F$, which can be obtained from Eq. (2):

$$C(u_1, \ldots, u_n) = F(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n))$$  \hspace{1cm} (2)$$

The corresponding density function is defined in Eq. (3):

$$f(x_1, \ldots, x_n) = c(F_1(x_1), \ldots, F_n(x_n)) \prod_{i=1}^{n} f_i$$

where $f_i$ are the marginal densities and $c$ is the density function of the copula which is derived from (2) and is given by Eq. (4):

$$c(u_1, \ldots, u_n) = \frac{f(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n))}{\prod_{i=1}^{n} f_i(F_i^{-1}(u_i))}$$

---

### Table 1 Descriptive statistics

| N= 100 | Gamstar | Cac 40 | Djia | Eurostoxx 50 | Ftse 100 | Hangseng | Nikkei 225 |
|--------|---------|--------|------|--------------|----------|-----------|------------|
| Min    | -0.0378 | -0.0893| -0.1061| -0.0968      | 0.008    | -0.1204   | -0.1045    |
| Max    | 0.0224  | 0.0993 | 0.0847| 0.0653       | 0.1024   | 0.1298    | 0.1236     |
| Mean   | 0.0037  | 0.0056 | 0.0080| 0.0022       | 0.0041   | 0.0040    | 0.0100     |
| Median | 0.0035  | 0.0072 | 0.0089| 0.008        | 0.00745  | 0.0092    | 0.0142     |
| Sum    | 0.3746  | 0.5665 | 0.8073| 0.2217       | 0.4129   | 0.401     | 1.0022     |
| Variance | 0.0000  | 0.0015 | 0.0011| 0.0010       | 0.0016   | 0.0024    | 0.0024     |
| Standard deviation | 0.0065 | 0.0394 | 0.0335| 0.0316       | 0.0410   | 0.0497    | 0.0497     |
| Skewness | -2.1069| -0.1941| -0.7029| -0.4589     | -0.2354  | -0.2475   | -0.4048    |
| Kurtosis| 14.8217| -0.4627| 1.1421| 0.1552       | -0.4744  | -0.0302   | -0.1148    |
| Jarque–Bera | 1036.8*** | 1.3521 | 14.754| 3.819        | 1.6983   | 1.056     | 2.8282     |

* $p<.10$, ** $p<.05$, *** $p<0.01$

---

**Fig 1** Log-returns of indices
where $F^{-1}_i$ is the quantile functions of the margins

A $n$-dimensional vector of financial time series, $y_t = (y_{1t}, \ldots, y_{nt})$, follows a copula-GARCH model if the joint cumulative distribution function is given by Eq. (5):

$$F(y_t|\mu, h_t) = C(F_1(y_{1t}|\mu_1, h_{1t}), \ldots, F_n(y_{nt}|\mu_n, h_{nt})) \tag{5}$$

where $C$ is a $n$ -dimensional copula and $f_i$ is the conditional distribution function of the marginal series $y_{it}$.

$y_{it}$ follows a standard univariate GARCH model as described in Eqs. (6) and (7):

$$y_{it} = \mu + \sqrt{h_{it}} \epsilon_{it} \tag{6}$$

$$h_{it} = \omega + a_{i}(y_{i,t-1} - \mu_i)^2 + \beta_i h_{i,t-1} \tag{7}$$

where $\epsilon_{it}$ are independent and identically distributed random variables with zero mean; $h_{i,t}$ is the conditional variance of $y_{it}; \omega, \alpha_i, \beta_i > 0$ guarantees positivity of $h_{i,t}; \alpha_i + \beta_i < 1$ ensures covariance stationarity.

A standardized skew Student distribution is then selected, $\epsilon_{it} \sim f_i(0, 1, \xi, \nu)$, where $\xi$ is the skew parameter and $\nu$ the shape parameter (Fernandez and Steel, 1998). The dependence structure between marginal series is characterized by a time-varying Student copula with constant shape parameter $\eta$ and conditional correlation $R_{it}$. The conditional density $c_{it}$ is given by Eq. (8):

$$c_{i}(u_{i1}, \ldots, u_{in}|R_{it}, \eta) = \frac{f_i\left(F^{-1}_i(u_{i1}|\eta), \ldots, F^{-1}_i(u_{in}|\eta)|R_{it}, \eta\right)}{\prod_{i=1}^{n} f_i\left(F^{-1}_i(u_{it}|\eta)|\eta\right)} \tag{8}$$

where $u_{it} = F_i(y_{it}|\mu_i, h_{it}, \xi, \nu)$ is the probability integral transformation of each univariate series by its conditional distribution $F_{it}$ resulting from the first-stage GARCH process; $F^{-1}_i(u_{it}|\eta)$ is the quantile transformation of the uniform margins; $f_i(*)|R_{it}, \eta)$ is the multivariate density of the Student distribution; $f_i(*)|\eta)$ is the univariate margins of the multivariate Student distribution (Demarta and McNeil, 2005).

We then select the Student copula (Cappiello et al., 2006). The dynamics of $R_{it}$ follows an asymmetric generalized dynamic conditional correlations (AGDCC) model, which generalizes the dynamic conditional correlation (DCC) GARCH model of Engle (2002). The joint density of the 2-stage estimation is given by Eq. (9):

$$f(r_t|\mu, h_t, R_t, \eta) = c_i(u_{i1}, \ldots, u_{in}|R_{it}, \eta) \prod_{i=1}^{p} \frac{1}{\sqrt{h_{it}}} f_i(\epsilon_{it}, \nu, \xi) \tag{9}$$

where the GARCH-DCC component is defined as a GARCH (1,1)-DCC (1,1) model (Hansen and Lunde, 2005). To estimate each ARMA process independently, we model the conditional mean process for each index. The order of the ARMA models is based on the computation of the BIC and AIC information criteria for different ($p, q$) pairs. The log-likelihood function can be separated into the univariate ARMA-GARCH component and the joint copula-DCC component. We use the two-stage maximum likelihood estimation (MLE) approach. In a first stage, the individual ARMA-GARCH parameters and their distributional parameters are estimated by maximization of the univariate ARMA-GARCH component of the function. Second, the copula-DCC parameters are estimated by maximization of the other component of the function.

Then, for measuring the risk-adjusted performance of indices, we consider the modified Sharpe ratio, i.e., the ratio of expected excess return over the value-at-risk (VaR) and the expected shortfall (ES$^2$) (Artzner et al., 1999; Rockafellar and Uryasev, 2002; Ausin and Lopez, 2010). These two measures allow to estimate changes in portfolio risk and control for risk magnitudes. VaR$\alpha$($0 < \alpha < 1$) measures downside risk and is defined in Eq. (11):

$$\mathbb{P}(L \leq -VaR_\alpha) = \alpha \tag{11}$$

where $L$ is the loss and profit random variable defined in Eq. (12)

$$L = \sum_{j=1}^{d} r_j / d \tag{12}$$

where $d$ is equi-weighted returns; $r = (r_1, \ldots, r_d)$ is the random vector of returns.

$ES$ represents the mean expected loss when the loss exceeds the VaR and is given by Eq. (13)

$$ES_\alpha = -E(L|L \leq -VaR_\alpha) \tag{13}$$

$^2$ Also known as Conditional Value-at-Risk (CVaR) or Expected Tail Loss (ETL).
Dependence structure of CAT bonds and portfolio diversification: a copula-GARCH approach

Then, VaR and ES approaches are complemented with three diversification measures (Choueifaty and Coignard, 2008; Choueifaty et al., 2013). First, the diversification ratio (DR) is defined in Eq. (14)

\[
DR(\omega) = \frac{\sum_{i=1}^{N} \omega_i \sigma_i}{\sqrt{\sum_{i=1}^{N} \omega_i}}
\]

where \( N \) is the number of assets in portfolio; \( \Sigma \) is the variance-covariance matrix of asset returns.

The concentration ratio (CR) is described in Eq. (15)

\[
CR = \frac{\sum_{i=1}^{N} (\omega_i)^2}{\left(\sum_{i=1}^{N} \omega_i \sigma_i\right)^2}
\]

The volatility-weighted average correlation (Vwac) of the assets is defined in Eq. (16)

\[
Vwac(\omega) = \frac{\sum_{i=1}^{N} (\omega_i \sigma_i) \rho_{ij}}{\sum_{i=1}^{N} (\omega_i \sigma_i)^2}
\]

Then, we examine the co-moments of time-series, i.e., evidence of the marginal contribution of each asset to the portfolio’s resulting risk. The diversification effect is measured in terms of volatility risk (co-variance), risk of asymmetry (co-skewness) and extreme events (co-kurtosis). In other words, we investigate the potential diversification benefits of the CATs compared to the traditional stock market indices. Finally, we estimate the efficient frontier of a constrained mean-variance portfolio with robust MCD covariance. In this framework, the composition of the optimal portfolio depends on the portfolio weight, the covariance risk budget and the target return and risk.
Results and discussion

Student Copula-AGDCC results

These results are reported in Table 2. Panel A displays the estimation results of the univariate ARMA-GARCH model. Robust standard errors in brackets are based on the White method (1982). AR parameters are noted \( ar_1 \) and \( ar_2 \). MA parameters are noted \( ma_1 \). \( \omega \) is the estimated value of the variance intercept parameter from the standard GARCH model of Bollerslev (1986). ARCH(\( q \)) and GARCH(\( p \)) correlation persistence parameters are noted \( \alpha \) and \( \beta \), respectively. \( \xi \) is the skew parameter, and \( \nu \) is the shape parameter (Fernandez and Steel, 1998). Results indicate that the \( \alpha \), \( \beta \) correlation persistence parameters of the GARCH (1, 1) are practically always jointly non-significant. In other words, a GARCH (1, 1) is equivalent to a constant conditional variance. The skew parameter shows a high significance. In addition, the estimates of the shape parameter are low and often insignificant.

Panel B reports the copula-DCC parameter estimation results. For the parameter estimation, we use the R package “rmgarch” of Ghalanos (2019). \( a \) and \( b \) are the DCC parameters. \( g \) denotes the asymmetry parameter of the AGDCC model (Cappiello et al., 2006). \( \eta \) is the shape parameter of the Student-\( t \) copula. Results indicates that \( a \) and \( b \) are not both nonzero parameters. This is inconsistent with the results from the test of dynamic correlation in Table 1. The \( g \) asymmetry parameter is insignificant suggesting lower answer to bad news than to good news. Finally, Figure 2 presents the time-varying correlations between each pair of assets.

Risk-adjusted performance of portfolios

Table 3 reports the modified Sharpe ratios are based on the 1-step-ahead simulated returns. The modified VaR used as an input to the modified Sharpe ratio is estimated at the 90% confidence level. In Panel A, each asset is considered as a mono-asset portfolio. All indices display positive modified Sharpe ratio. GAMSTAR displays the highest risk-adjusted performer. In addition, Panel B highlights significant differences between the mono-asset portfolios. Panel B reveals

---

3 See Bosd et al. (2019) for a review of multivariate GARCH models.
that the GAMSTAR modified Sharpe ratio is always positive. The GAMSTAR risk-adjusted performance is higher than that of the traditional stock indices.

In order to incorporate all assets in portfolio taken collectively, the modified Sharpe ratios are complemented with the peer-performance ratios of Ardia and Boudt (2018). The peer-performance ratios are expressed as a probability of equal performance, outperformance and underperformance. Panel C reports these results. The underperformance ratio of the FTSE 100 is equal to 100%. In other words, the probability of this index underperforms all other indices are equal to one. Conversely, the probability is null of outperforming of all other indices. The NIKKEI 225 provides an exact counter-example. In addition, the probability of equal performance of HANGSENG is equal to one. Finally, the probability of equal performance of the GAMSTAR is equal to 98%. In other words, the underperformance and outperformance ratios of the GAMSTAR are close to zero.

### Risk-adjusted performance from the minimum acceptable return

The Sortino M-squared measures the downside risk instead of total risk. In other words, this ratio measures excess return per unit of downside risk. Table 4 reports the Sortino M-squared of all return distributions. We adopt a sequential approach.
For example, MAR1 represents the minimum acceptable return defined as the cumulative return of the GAMSTAR until MAR7, which denotes the cumulative return of the NIKKEI 225. The MSCI World index is the return vector of the benchmark asset used to derive the annualized downside risk. Results in Table 4 are consistent with those in Table 3. Indeed, all Sortino M-squared are positive. In addition, the ranking of the seven market indices changes very slightly when either or both of the measures in Tables 3 and 4 are used.

In Panel B, the MSCI World index is used both as the MAR and the benchmark asset. The same sequential approach as in Panel A is adopted for the annualized portfolio return. Each of the seven indices serves as a mono-asset portfolio. For example, if the cumulative return of DJIA is used as the minimum acceptance return, then MAR3 equals 0.8073 and the Sortino M-squared of the annualized DJIA portfolio return is equal to 0.0939. Unsurprisingly, no annualized mono-asset portfolio return outperforms the MSCI World index.

### Marginal contribution of each asset to the portfolio’s resulting risk

In order to identify the diversification potential, we analyze the co-moments of return distribution. In other words, we measure the contribution of each of the assets to different benchmark portfolios’ skewness and kurtosis. Co-skewness and co-kurtosis are defined as the skewness and kurtosis of a given asset analyzed with reference to the skewness and kurtosis of a benchmark (Ranaldo and Favre, 2005). Theory points out that rational investors prefer positive co-skewness and co-negative co-kurtosis. (Table 5)

Table 6 presents co-skewness and co-kurtosis in Panel A, while their corresponding betas are reported in Panel B. Seven equally weighted benchmark portfolios are considered and denoted P1, […] P7. Each portfolio is composed of six assets selected among the seven indices. The co-moments and beta co-moments are computed between a single asset and a benchmark portfolio. The single asset i used in the calculations is not included in the benchmark portfolio j. The goal is to isolate the individual effect of a single asset when it is adding to a benchmark portfolio. For instance, to compute the co-moments and beta co-moments between the GAMSTAR and the first benchmark portfolio P1, the GAMSTAR is excluded from this portfolio.

Results in Panel B allow analyzing the diversification effect in terms of volatility (beta co-variance), risk of asymmetry (beta co-skewness) and extreme events (beta co-kurtosis). There is a slight positive beta co-variance between the GAMSTAR and the benchmark portfolio P1. In other

| Table 5 Value-at-Risk, expected shortfall, and diversification measures |
|---------------------------------------------------------------|
| Panel A (value-at-Risk)                                      |
| mVaR<sub>99%</sub><sup>a</sup>                             | GamsTAR | Cac40 | Djia | Ftse100 | Eurostoxx50 | Hangseng | Nikkei |
| Pf<sub>mVaR</sub>                                          | −0.0335 | −0.0869 | −0.0902 | −0.0808 | −0.0930 | −0.1195 | −0.1162 |
| Comp<sub>mVaR</sub><sup>b</sup>                           | 0.0737  |        |        |         |         |         |         |
| Comp<sub>mVaR(%)</sub><sup>c</sup>                        | −0.0004 | 0.0116  | 0.0114  | 0.0098  | 0.0126  | 0.0137  | 0.0147  |

| Panel B (expected shortfall)                               |
| mES<sub>99%</sub><sup>e</sup>                             | −0.0335 | −0.0985 | −0.1196 | −0.0995 | −0.1053 | −0.1402 | −0.1397 |
| Pf<sub>mES</sub>                                          | 0.0986  |        |        |         |         |         |         |
| Comp<sub>mES</sub><sup>f</sup>                           | −0.0010 | 0.0123  | 0.0184  | 0.0140  | 0.0139  | 0.0183  | 0.0224  |
| Comp<sub>mES(%)</sub><sup>g</sup>                        | −0.1079 | 12.5462 | 18.703  | 14.247  | 14.174  | 18.637  | 22.769  |

| Panel C (diversification measures)                         |
| Equally weighted Portfolio                                 |
| Diversification Ratio                                      | 1.2306  |        |        |         |         |         |         |
| Concentration Ratio                                        | 0.1634  |        |        |         |         |         |         |
| Vwac                                                       | 0.5938  |        |        |         |         |         |         |
| Optimal Weights (%)                                        | 0.0000  | 0.2647  | 0.1481  | 0.0764  | 0.0000  | 0.1258  | 0.3850  |

Most-diversified Portfolio

| Diversification Ratio                                      | 1.2096  |        |        |         |         |         |         |
| Concentration Ratio                                        | 0.1876  |        |        |         |         |         |         |
| Vwac                                                       | 0.6102  |        |        |         |         |         |         |
| Optimal Weights (%)                                        |         |        |        |         |         |         |         |

<sup>a</sup>mVaR<sub>99%</sub> denotes the modified measures of VaR

<sup>b</sup>Pf<sub>mVaR</sub> symbolizes the VaR portfolio

<sup>c</sup>Comp<sub>mVaR</sub> measures the contribution to the VaR portfolio

<sup>d</sup>Comp<sub>mVaR(%)</sub> is the percent contribution to the VaR portfolio

<sup>e</sup>mES<sub>99%</sub> represents the modified measures of ES

<sup>f</sup>Pf<sub>mES</sub> denotes the ES portfolio

<sup>g</sup>Comp<sub>mES</sub> measures the contribution to the ES portfolio

<sup>h</sup>Comp<sub>mES(%)</sub> is the percent contribution to the ES portfolio
words, adding the GAMSTAR to this portfolio will not reduce its volatility. However, the GAMSTAR reports the lowest beta co-variance. The GAMSTAR also displays the lowest positive beta co-skewness, close to zero. A rational investor prefers positive co-skewness due to the lower downside risk. In addition, all beta co-kurtosis are positive. In other words, adding one of these assets increases the kurtosis of the benchmark portfolio. However, the GAMSTAR displays the lowest beta co-kurtosis, indicating that this asset that increases the lowest probability of extreme co-variations with its benchmark portfolio. Finally, given theirs low beta co-variance, beta co-skewness and beta co-kurtosis, the GAMSTAR presents diversification benefits in terms of volatility, asymmetry and extreme events.

**Efficient frontier and optimal portfolio**

Finally, we define the composition of the optimal portfolio according to portfolio weight (Panels A), covariance risk budget (Panel B) and target risk and return (Panel C). Table 7 presents this composition with a number of border points of 5. Figure 3 displays the efficient frontier of a long-only constrained mean–variance portfolio with robust MCD (left) and sample (right) covariance estimates. From the top to bottom, there are the weights and the weighted returns. In other words, the risk attribution is measured by the performance attribution and the covariance risk budget. The upper axis defines the target risk, while the lower indicates the target return. The thick vertical line separates the efficient frontier from the minimum variance place. Therefore, the risk axis increases in value to both sides of the separator line. The legend to the right links the assets names to color of the bars. In both methods (DCM or VOC), the distribution of the assets in the portfolio is identical, indicating the similarity of two methods. In other words, the DCM method based on a dynamic variance–covariance matrix does not significantly improve the initial VOC method based on a static variance–covariance matrix. As a result, there is a portfolio allocation including exclusively the GAMSTAR, the DJIA, and the NIKKEI 225. The proportion invested in the GAMSTAR is maximum at the level of the vertical line of the graphs and decreases each time the distance from this line is important.

**Estimation of changes in portfolio risk and control for risk magnitudes**

Table 5 reports results of value-at-risk, expected shortfall, and diversification measures. In Panel A, the component VaR denotes the risk contribution of each asset to the risk of the entire portfolio VaR. Component VaR adds up to the value of the whole portfolio VaR. All market indices are negative. The GAMSTAR displays the highest modified

### Table 6 Co-Moments and beta co-moments

| Benchmark Portfolio | P1 | P2 | P3 | P4 | P5 | P6 | P7 |
|--------------------|----|----|----|----|----|----|----|
| Skewness           | −0.6958 | −0.7491 | −0.6142 | −0.6565 | −0.7391 | −0.6547 | −0.5724 |
| Excess Kurtosis    | 0.3537 | 0.4191 | 0.0674 | 0.2700 | 0.4004 | 0.3365 | 0.2429 |
| **Panel A (co-moments)** |     |     |     |     |     |     |     |
| Co-variance (10^3) | 3.5231e-05 | 0.0009 | 0.0007 | 0.0006 | 0.0009 | 0.0008 | 0.0009 |
| Co-Skewness (10^3) | −1.2513e-07 | −1.7054e-05 | −1.8330e-05 | −1.5602e-05 | −1.7939e-05 | −1.8452e-05 | −2.2824e-05 |
| Co-Kurtosis (10^6) | 6.4942e-08 | 2.4580e-06 | 2.3051e-06 | 2.0771e-06 | 2.5165e-06 | 2.2414e-06 | 2.5272e-06 |
| **Panel B (beta co-moments)** |     |     |     |     |     |     |     |
| Beta Co-variance   | 0.0309 | 1.1724 | 0.8618 | 0.7731 | 1.2379 | 1.0655 | 1.2704 |
| Beta Co-Skewness   | 0.0046 | 1.0131 | 1.1640 | 0.8911 | 1.1109 | 1.2490 | 1.9021 |
| Beta Co-Kurtosis   | 0.0149 | 1.1339 | 0.9941 | 0.7973 | 1.2119 | 1.0537 | 1.3484 |

^aCAC40, DJIA, FTSE 100, EUROSTOXX 50, HANGSENG, NIKKEI 225
^bGAMSTAR, DJIA, FTSE 100, EUROSTOXX 50, HANGSENG, NIKKEI 225
^cGAMSTAR, CAC40, FTSE 100, EUROSTOXX 50, HANGSENG, NIKKEI 225
^dGAMSTAR, CAC40, DJIA, EUROSTOXX 50, HANGSENG, NIKKEI 225
^eGAMSTAR, CAC40, DJIA, FTSE 100, HANGSENG, NIKKEI 225
^fGAMSTAR, CAC40, DJIA, FTSE 100, EUROSTOXX 50, NIKKEI 225
^gGAMSTAR, CAC40, DJIA, FTSE 100, EUROSTOXX 50, HANGSENG
VaR at the 99% confidence level. In addition, all individual VaR are lower than the entire portfolio VaR. The component VaR indicates that the GAMSTAR displays the lowest level within the portfolio. In other words, it is expected that removing the GAMSTAR would increase the portfolio VaR. With the lowest component VaR (and alone negative), the GAMSTAR contributes -0.54% of the portfolio total risk.

Panel B complements the VaR calculation with expected shortfall. All ES are negative confirming the conclusions from the analysis of VaR. The GAMSTAR reports the lowest ES. In addition, contributing only −0.1% to the portfolio ES, the GAMSTAR is the smallest contributor to total portfolio risk.

VaR and ES analyses are complemented in Panel C with different diversification measures in order to verify whether
our previous conclusions about diversification benefits hold true. The diversification ratio, the concentration ratio, and the volatility weighted average correlation are first calculated for an equally weighted portfolio and then for the most-diversified portfolio. This latter is derived from the optimal weights in percentages. Surprisingly, the diversification ratio and the volatility-weighted average correlation of the equally weighted portfolio are both higher than those the most-diversified portfolio. However, the result is inverse for the concentration ratio. The weight of GAMSTAR in the portfolio equals zero at extreme risk and return levels. Indeed, the investment in cat-bond is only justified if the "Target return" is outside the interval [0.0022; 0.00838] and the "Target risk" outside the interval [0.00925; 0.0345]. Therefore, the GAMSTAR could be a risk-reduction alternative asset and by consequence might be crucial in portfolio diversification strategy.

Conclusion

Should we be alarmed at the increase in the number of environmental disasters? Academics are broadly debating this issue. Notably, they focus on the coverage and compensation of possible economic and financial consequences. This issue is also very important for policy makers as well as for insurers and reinsurers who will have to bear a broad part of the cost. The assessment of the consequences depends in particular on the type and severity of the event, the underlying composition of the economy and the affected area. In this framework, CAT bonds appear to be one element of the response, as evidenced by the growing volume of their use.

Quite recently, the debate for investors also revolves around the potential of this asset as an alternative asset in building a portfolio. Recent studies are unanimous in considering the CAT bond as an asset with diversification gains. However, not all econometric approaches have yet been used. Thus, we innovative by using the Copula-GARCH model over the period from November 30, 2011, to February 28, 2020. This methodology allows an analysis of dependence from CAT bonds to financial markets. In addition, this approach also allows the measure of the impact of this asset on the performance of the portfolio. Our results confirm those of previous studies by indicating that CAT bond is a very effective risk diversification asset and significantly reduces risk in equity portfolios. The CAT bond is particularly well suited to risk-averse investors. These results have very important implications for many actors. For example, insurers and reinsurers could cover themselves by limiting the risks associated with compensation arising from the occurrence of a disaster. Investors (professionals and individuals) could diversify their financial portfolio while limiting its risk. Policy-makers could also use these CAT bonds to hedge against possible environmental disasters such as floods, tsunamis, volcanic eruptions, storms, hurricanes as well as health disasters as the recent COVID-19 pandemic. Our study is limited by the number of observations collected and could be extended using high-frequency data.

Funding  The data that support the findings of this study are available from the corresponding author, upon reasonable request.
Declarations

Conflict of interest This contribution is free from any conflicts of interest, including all financial and non-financial interests and relationships. In addition, the authors specify that there are no commercial links with the provider of the GAMSTAR.

References

Amédée-Manesme, C.O., B. Faye, and E. Le Fur. 2020. Heterogeneity and fine wine prices: application of the quantile regression approach. Applied Economics 52 (26): 2821–2840.

Ardia, D., and K. Boudt. 2015. Testing equality of modified Sharpe ratios. Finance Research Letters 13: 97–104.

Ardia, D., and K. Boudt. 2018. The peer performance ratios of hedge funds. Journal of Banking & Finance 87: 351–368.

Artzner, P., F. Delbaen, J.M. Eber, and D. Heath. 1999. Coherent Measures of Risk. Mathematical Finance 9 (3): 203–228.

Barras, L., O. Scaillet, and R. Wermers. 2010. False discoveries in mutual fund performance: Measuring luck in estimated alphas. The Journal of Finance 65 (1): 179–216.

Baur, D.G. 2013. The structure and degree of dependence: A quantile regression approach. Journal of Banking & Finance 37 (3): 786–798.

Bollerslev, T. 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31 (3): 307–327.

Boudt, Kris, Alexios Galanos, Scott Payseur, and Eric Zivot. 2019. Multivariate GARCH models for large-scale applications: A survey. In Conceptual Econometrics Using R, 193. Elsevier.

Braun, A. 2016. Pricing in the primary market for CATs: new empirical evidence. Journal of Risk and Insurance 83 (4): 811–847.

Cappiello, L., R.F. Engle, and K. Sheppard. 2006. Asymmetric dynamics in the correlations of global equity and bond returns. Journal of Financial Econometrics 4 (4): 537–572.

Carayannopoulos, P., and M.F. Perez. 2015. Diversification through catastrophe bonds: lessons from the subprime financial crisis. The Geneva Papers on Risk and Insurance-Issues and Practice 40 (1): 1–28.

Cherubini, U., Luciano, E., and Vecchiato, W. 2004. Copula methods in finance. John Wiley & Sons.

Chougnet, Y., and Y. Coignard. 2008. Toward maximum diversification. The Journal of Portfolio Management 35 (1): 40–51.

Chougnet, Y., T. Froide, and J. Reynier. 2013. Properties of the most diversified portfolio. Journal of Investment Strategies 2 (2): 49–70.

Clark, Steven P., and Faith Roberts Neale. 2016. Portfolio diversification effects of catastrophe bonds. SSRN Electronic Journal. https://doi.org/10.2139/ssrn.2806432.

Cummins, J.D., and P. Trainar. 2009. Securitization, insurance, and reinsurancen. Journal of Risk and Insurance 76 (3): 463–492.

Demarta, S., and A.J. McNeil. 2005. The t copula and related copulas. International Statistical Review 73 (1): 111–129.

Demers-Bélanger, K., and V.S. Lai. 2020. Diversification benefits of CATs: An in-depth examination. Financial Markets, Institutions & Instruments 29 (5): 165–228.

Dissmann, J., E.C. Brechmann, C. Czado, and D. Kurowicka. 2013. Selecting and estimating regular vine copulae and application to financial returns. Computational Statistics & Data Analysis 59: 52–69.

Embrechts, P., A. Höing, and A. Juri. 2003. Using copulae to bound the value-at-risk for functions of dependent risks. Finance and Stochastics 7 (2): 145–167.

Engle, R. 2002. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. Journal of Business & Economic Statistics 20 (3): 339–350.

Favre, L., and J.A. Galeano. 2002. Mean-modified value-at-risk optimization with hedge funds. The Journal of Alternative Investments 5 (2): 21–25.

Fernández, C., and M.F. Steel. 1998. On Bayesian modeling of fat tails and skewness. Journal of the American Statistical Association 93 (441): 359–371.

Frahm, Gabriel, Markus Junker, and Rafael Schmidt. 2005. Estimating the tail-dependence coefficient: properties and pitfalls. Insurance: Mathematics and Economics 37 (1): 80–100. https://doi.org/10.1016/j.insmatheco.2005.05.008.

Frees, E.W., and E.A. Valdez. 1998. Understanding relationships using copulas. North American Actuarial Journal 2 (1): 1–25.

Garcia, R., and G. Tsakakis. 2011. Dependence structure and extreme comovements in international equity and bond markets. Journal of Banking & Finance 35 (8): 1954–1970.

Ghalanos, A. (2019). Package rmgarch: Multivariate GARCH models. Version 1.3-7. September.

Gregoriou, G.N., and J.P. Gueye. 2003. Risk-adjusted performance of funds of hedge funds using a modified Sharpe ratio. The Journal of Wealth Management 6 (3): 77–83.

Gürtler, M., M. Hilibel, and C. Winkelvos. 2016. The impact of the financial crisis and natural catastrophes on CATs. Journal of Risk and Insurance 83 (3): 579–612.

Hansen, P.R., and A. Lunde. 2005. A forecast comparison of volatility models: does anything beat a GARCH (1, 1) ?. Journal of Applied Econometrics 20 (7): 873–889.

Hong, Y., J. Tu, and G. Zhou. 2007. Asymmetries in stock returns: Statistical tests and economic evaluation. The Review of Financial Studies 20 (5): 1547–1581.

Huffman, S.P., and C.R. Moll. 2011. The impact of asymmetry on expected stock returns: An investigation of alternative risk measures. Algorithmic Finance 1 (2): 79–93.

Ji, Q., E. Bouri, D. Roubaud, and S.J.H. Shahzad. 2018. Risk spillover between energy and agricultural commodity markets: A dependence-switching CoVaR-copula model. Energy Economics 75: 14–27.

Kish, R. J. (2016). Catastrophe (CAT) bonds: risk offsets with diversification and high returns. Financial Services Review, 25(3).

Ledoit, O., and M. Wolf. 2008. Robust performance hypothesis testing with the Sharpe ratio. Journal of Empirical Finance 15 (5): 850–859.

Li, D.X. 2000. On default correlation: A copula function approach. The Journal of Fixed Income 9 (4): 43–54.

Mariani, M., Amoroso, P. (2016). The effectiveness of catastrophe bonds in portfolio diversification. International Journal of Economics and Financial Issues, 6(4).

Mensi, W., S. Hammoudeh, S.J.H. Shahzad, and M. Shahbazi. 2017. Modeling systemic risk and dependence structure between oil and stock markets using a variational mode decomposition-based copula method. Journal of Banking & Finance 75: 258–279.

Nelsen, R. B. (2007). An introduction to copulas. Springer Science & Business Media.

Ning, C. 2010. Dependence structure between the equity market and the foreign exchange market—a copula approach. Journal of International Money and Finance 29 (5): 743–759.
Patton, A.J. 2004. On the out-of-sample importance of skewness and asymmetric dependence for asset allocation. Journal of Financial Econometrics 2 (1): 130–168.

Patton, A.J. 2009. Copula–based models for financial time series. In Handbook of financial time series, 767–785. Berlin, Heidelberg: Springer.

Patton, A.J. 2012. A review of copula models for economic time series. Journal of Multivariate Analysis 110: 4–18.

Patton, A. 2013. Copula methods for forecasting multivariate time series. Handbook of Economic Forecasting 2: 899–960.

Ranaldo, A., & Favre, L. (2005). How to price hedge funds: From two- to four-moment CAPM. UBS Research Paper.

Rockafellar, R.T., and S. Uryasev. 2002. Conditional value-at-risk for general loss distributions. Journal of Banking & Finance 26 (7): 1443–1471.

Schmidt, R., and U. Stadtmüller. 2006. Non-parametric estimation of tail dependence. Scandinavian Journal of Statistics 33 (2): 307–335.

Shahzad, S.J.H., W. Mensi, S. Hammoudeh, M.U. Rehman, and K.H. Al-Yahyaee. 2018. Extreme dependence and risk spillovers between oil and Islamic stock markets. Emerging Markets Review 34: 42–63.

Sklar, M. 1959. Fonctions de répartition an dimensions et leurs marges. Publ. Inst. Statist. Univ. Paris 8: 229–231.

Sortino, Frank A., and Lee N. Price. 1994. Performance measurement in a downside risk framework. The Journal of Investing 3 (3): 59–64. https://doi.org/10.3905/joi.3.3.59.

White, H. 1982. Maximum likelihood estimation of misspecified models. Econometrica: Journal of the Econometric Society, 1–25.

Yang Zhao Yu Min-Teh. 2019. Measuring the liquidity impact on catastrophe bond spreads. Pacific–Basin Finance Journal 56 (197): 210.

Zhu, H., Y. Guo, W. You, and Y. Xu. 2016. The heterogeneity dependence between crude oil price changes and industry stock market returns in China: Evidence from a quantile regression approach. Energy Economics 55: 30–41.

**Publisher’s Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

**Adlane Haffar** is an Assistant Professor in Economics and Applied Statistics at the Faculty of Mathematics of USTHB Algiers, Algeria. Adlane is affiliated with the MSTD laboratory, and his research focuses on financial markets, actuarial science, and economics. He has published several articles, notably in the Quarterly Review of Economics and Finance, Gestion 2000, and two books in Editions Universitaires Européennes.

**Éric Le Fur** is Full Professor of Finance at INSEEC Grande Ecole, OMNES Education, Bordeaux, France. His research focuses on wine economics, wealth management, and alternative assets. Eric has published in prestigious academic journals in finance and economics, including Annals of Operations Research, Applied Economics, Economic Modelling, Management International, The Journal of Wine Economics, and Urban Studies.