Production of semi-inclusive doubly heavy baryons via $Z$ boson decays

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(Dated: August 17, 2022)

The rare decay channels of $Z$ boson to doubly heavy baryons offer vital opportunities to explore the production mechanisms of doubly heavy baryons. We study the semi-exclusive decay channels of $Z$ boson to doubly heavy baryons, i.e., $Z \rightarrow \Xi_{bQ} + \bar{b} + Q’$ ($Q’ = c$ or $b$ quark) within the nonrelativistic QCD framework. The contributions from the intermediate diquark states, $\langle bc \rangle [^3S_1]_{3/6}$, $\langle bc \rangle [^1S_0]_{5/6}$, $\langle bb \rangle [^1S_0]_{6}$ and $\langle bb \rangle [^3S_1]_{3}$, have been taken into consideration. The differential distributions and two main sources of the theoretical uncertainties have been discussed. At the Circular Electron Positron Collider, there will be about $3.6 \times 10^7$ events of $\Xi_{bc}$ and $1.2 \times 10^6$ events of $\Xi_{bb}$ produced per year. There are fewer events produced at the Large Hadron Collider, about $3.6 \times 10^4$ events of $\Xi_{bc}$ and $1.2 \times 10^3$ events of $\Xi_{bb}$ in operation.

PACS numbers: 13.25.Hw, 11.55.Hx, 12.38.Aw, 14.40.Be

I. INTRODUCTION

Doubly heavy baryons consisting of two heavy quarks ($b$ or $c$) and one light quark ($u$, $d$, or $s$) are expected within the quark model [1–4]. The study of the doubly heavy baryons is enthralling as it provides unique test for the perturbative Quantum Chromodynamics (pQCD) and the nonrelativistic QCD (NRQCD). In the past decades, research on the doubly heavy baryons related studies has developed rapidly, including both experimental and theoretical aspects.

From the experimental side, the $\Xi_{cc}^{++}$ baryon was firstly observed by the LHCb collaboration via the decay channel $\Xi_{cc}^{++} \rightarrow \Lambda_{c}^{+} K^{-} \pi^{+} \pi^{-}$ and $\Lambda_{c}^{+} \rightarrow pK^{-}\pi^{+}$ in 2017 [5], which was confirmed by Ref. [6] and also by Ref. [7] via measuring another decay channel $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+} \pi^{+}$ with $\Xi_{c}^{+} \rightarrow pK^{-}\pi^{+}$. Moreover, the observations of doubly charmed baryon $\Xi_{cc}^{+}$ was firstly reported in $\Xi_{cc}^{+} \rightarrow pD^{+}K^{-}$ decay channel by the SELEX collaboration. However, the large production rates released of $\Xi_{cc}^{+}$ by the SELEX Collaboration have not been confirmed by the other collaboration, such as the BABAR, Belle, and even the FOCUS that is at the same collider of SELEX [8–10]. In the past several years,
although the LHCb Collaboration has searched for $\Xi_{cc}^{+}$ for many times, and released their newest measurement on the $R_{cc}^{+}$ which is defined as $R_{cc}^{+} = \sigma(\Xi_{cc} \rightarrow \Lambda_{c}^{+}K^{-}\pi^{+})/\sigma(\Lambda_{c}^{+})$ [11], varying in the region $[0.9, 6.5] \times 10^{-3}$ for $\sqrt{s} = 8$ TeV, and $[0.12, 0.45] \times 10^{-3}$ for $\sqrt{s} = 13$ TeV, which are still significantly below the value measured $R_{cc}^{+} = 9\%$ by the SELEX Collaboration. As regards $\Xi_{bc}$, which is containing one bottom quark ($b$) and one charm quark ($c$). Due to its unique nature in the family of Baryons, $\Xi_{bc}$ baryon also attracts widely attention of experiment and theory. In 2020, the LHCb Collaboration seek for the doubly heavy $\Xi_{bc}^{0}$ baryon via its decay to the $D^{0}pK^{-}$, but no evidence was found [12]. Recently, $\Xi_{cb}$ and $\Omega_{cb}$ are detected via $\Lambda_{c}^{+}\pi^{-}$ and $\Xi_{c}^{+}\pi^{-}$ decay modes, but evidence of signal is not found [13]. $\Xi_{bb}$ is still not experimentally detected. In a nutshell, there is still no explicit evidence on the doubly heavy baryons $\Xi_{bc}$ and $\Xi_{bb}$ (throughout the paper, we label them as $\Xi_{bQ}'$, with $Q'$ representing the heavy $b$ or $c$ quark, respectively). A careful study on the $\Xi_{bQ}'$ production shall be helpful for confirming whether enough $\Xi_{bQ}'$ baryon events can be produced and for further testing of the quark model and the nonrelativistic QCD (NRQCD). Currently, $\Xi_{bQ'}$ baryon productions at various high-energy colliders have been studied extensively in the literatures [14–46], such as the $e^{+}e^{-}$ annihilation, the photoproduction in $ep$ collision, the hadroproductions in $pp$ (or $p\bar{p}$) collision.

Apart from the direct production, $\Xi_{bQ'}$ baryon indirect production is also interesting, which may inform us not only the nature of $\Xi_{bQ'}$, but also the characters of its parent particles. Production of $\Xi_{bQ'}$ baryons via the top quark decays was discussed in Refs. [47], and via $W^{+}$ boson decay was calculated in Ref. [48]. In the work of Ref. [49], the $H \rightarrow \Xi_{QQ'} + X$ process was calculated. Recently, the process $Z \rightarrow \Xi_{cc} + X$ have been finished [50]. The authors there found about $10^{4}(10^{7})$ $\Xi_{cc}$ events production events arisen from $Z$ decay would be detected in one running year at the LHC(CEPC). Besides, the $Z$ boson decay can also provide a good opportunity for the studies on $\Xi_{bQ'}$ baryon because of the large number of $Z$ events at the high energy colliders, e.g., the LHC can produce $\sim 10^{9}$ $Z$ events each year [51]. The proposed future $e^{+}e^{-}$ collider, CEPC [52], equipped with “clean” background and enormous $Z$ production events ($\sim 10^{12}$/year), the “clean” background of CEPC comparing to LHC may help us to more easily hunt the heavy baryon related processes. Thus, in the following, we shall first concentrate our attention on the indirect production of $\Xi_{bQ'}$ via $Z$ boson decay at the LHC and CEPC.

The rest of the paper is organized as follows: In Sec. II, we present the detailed calculation technology. Numerical results and discussions are given in Sec. III. Section IV is reserved for a summary.
FIG. 1: Typical Feynman diagrams for the process \( Z \rightarrow \langle bQ' \rangle[n] + \bar{b} + \bar{Q} \), where \( Q' \) denote as the heavy \( c \) or \( b \) quark.

II. CALCULATION TECHNOLOGY

It is a widely accepted view that the production of the doubly heavy baryons can be factorized into two steps, cf. Refs. [17, 40, 46, 53]:

1) The first step is that a binding diquark \( \langle QQ' \rangle \) with with color- and spin- configuration\([n]\) is produced, where \( Q(Q') \) representing the heavy \( b \) or \( c \) quark. According to the decomposition \( 3 \otimes 3 = \bar{3} \oplus 6 \) in \( SU_c(3) \) group and NRQCD [19, 54], the color quantum number is the color-antitriplet \( \bar{3} \) and the color-sextuplet \( 6 \), and the spin quantum number of the diquark \( \langle QQ' \rangle \) can be \([\frac{3}{2}S_1]\) or \([\frac{1}{2}S_0]\).

2) The second step is that the diquark fragments to a physical colorless baryon \( \Xi_{QQ'}q \) by grabbing a light quark from ‘environment’ with a fragmentation probability of almost one hundred percent. For convenience, throughout the paper, we label the doubly charmed baryon as \( \Xi_{QQ'} \) instead of \( \Xi_{QQ'q} \). Among this total “100%” probability, the diquark fragmenting into \( \Xi_{QQ'}d \) and \( \Xi_{QQ'}u \) both accounts for 43%, respectively, and the ratio for \( \Omega_{QQ's} \) is 14% [43, 55].

Based on the above steps, we draw the typical feynman diagrams for the process \( Z(p_0) \rightarrow \langle bQ' \rangle[n](p_1) + \bar{b}(p_2) + \bar{Q}'(p_3) \) in Fig. 1, where \( Q' \) denote as the heavy \( c \) or \( b \) quark. The differential decay width for the production of \( \Xi_{bQ'} \) can be factorized as the following form:

\[
d\Gamma = \sum_n d\hat{\Gamma}(Z \rightarrow \langle bQ' \rangle[n] + \bar{b} + \bar{Q}')\langle O^H(n) \rangle
\]

where \( H \) is short for any doubly heavy baryon \( \Xi_{bQ'} \), the non-perturbative long-distance matrix element \( \langle O^H(n) \rangle \) is proportional to the transition probability from the perturbative quark pair \( \langle bQ' \rangle[n] \) to the \( \Xi_{bQ'} \) baryon, and can be obtained by approximatively related to the origin value of the Schrödinger wave function or its derivative, \( \langle O^H(n) \rangle = |\Psi_{bQ'}(0)| \) for \( S \)-wave, or \( \langle O^H(n) \rangle = |\Psi'_{bQ'}(0)| \) for \( P \)-wave. \( |\Psi_{bQ'}(0)| \) and \( |\Psi'_{bQ'}(0)| \) for heavy hadrons are derived from the experiment data or some nonperturbative methods, e.g., the potential model [56], lattice QCD (LQCD) [57], or QCD sum rules [58].
The decay width \( \Gamma(Z \to \langle bQ' \rangle[n] + \bar{b} + \bar{Q}') \) represents the perturbative short-distance coefficients can be written as

\[
d\Gamma(Z \to \langle bQ' \rangle[n] + \bar{b} + \bar{Q}') = \frac{1}{32m_z} \sum |M[n]|^2 d\Phi_3
\]

(2)

where \( m_z \) is the mass of the \( Z \) boson, \( |M[n]| \) is the hard amplitude, the factor 1/3 comes from the spin average of the initial \( Z \) boson, and \( \sum \) means that we need to sum over the color and spin of all the final particles. The three-particle phase space \( d\Phi_3 \) can be expressed as

\[
d\Phi_3 = (2\pi)^4 \delta^4(p_0 - \frac{1}{2} \sum_{f} p_f) \prod_{f} \frac{d^3p_f}{(2\pi)^3 2p_f^0}
\]

(3)

The 1 \( \to \) 3 phase space with massive quark/antiquark in the final state can be found in Refs. [59, 60]. With the help of the formulas listed in Refs. [59, 60], the Eq. (2) can be rewritten as

\[
d\Gamma(Z \to \langle bQ' \rangle[n] + \bar{b} + \bar{Q}') = \frac{1}{2\pi^3} \frac{1}{m_z^3} \sum |M[n]|^2 ds_{12} ds_{23}
\]

(4)

here the definitions of the invariant mass are \( s_{ij} = (p_i + p_j)^2 \).

A. Amplitudes for the diquark production

For the production of doubly heavy baryons \( \Xi_{bQ'} \), subgraphs (a) \( \to \) (d) in Fig. 1 are specifically represented the channels \( Z \to b\bar{b}/Q'\bar{Q}' \to \Xi_{bQ'} + X \). After the action of the charge parity \( C = -i\gamma^2\gamma^0 \), the hard amplitude \( M[n] \) for the production of the intermediate diquark state can be related to the familiar meson production, which has been sufficiently demonstrated in Refs. [29, 42], and here we are going to give a brief descriptions.

When dealing with the hard amplitudes for doubly heavy baryons, we use the charge conjugation to reverse one fermion line. The fermion line which need to be reversed can be writing as \( L_1 = \pi_{s_1}(k_{12})\Gamma_{i+1}S_F(q_i, m_i) \cdots S_F(q_1, m_1)\Gamma_1v_{s_2}(k_2) \). Here \( \Gamma_i \) stands for the the interaction vertex, \( S_F(q_i, m_i) \) denote the fermion propagator, \( s_1 \) or \( s_2 \) is for spin index, and \( (i = 0, 1, ...) \) is the number of the interaction vertices in this fermion line. According to he charge parity \( C = -i\gamma^2\gamma^0 \), we have

\[
\begin{align*}
\nu_{s_2}(p)C &= -\bar{\mu}_{s_2}(p), \\
C^{-1}S_F^T(-q_i, m_i)C &= S_F(q_i, m_i), \\
C^{-1}(\gamma^u)^T C &= -\gamma^u,
\end{align*}
\]

(5)

\[
\begin{align*}
C^{-1}\Gamma_i^T C &= -\Gamma_i, \\
CC^{-1} &= I, \\
C^{-1}\bar{\mu}_{s_2}(p_{12}) &= \nu_{s_1}(p_{12}).
\end{align*}
\]
If the fermion line which need to be reversed contains no axial vector vertex, we can obtain the following

\[ L_1 = L_1^T = v^T_{s_2}(p_2)\Gamma^T_F S^T_F(q_1, m_1) \cdots S^T_F(q_1, m_1)\Gamma^T_{i+1}u^T_{s_1}(p_{12}) \]
\[ = v^T_{s_2}(p_2)CC^{-1}\Gamma^T_I CC^{-1}\Gamma^T_F S^T_F(q_1, m_1)CC^{-1} \cdots CC^{-1}\Gamma^T_I CC^{-1}\Gamma^T_{i+1}CC^{-1}u^T_{s_1}(p_{12}) \]
\[ = (-1)^{i+1}u_{s_2}(p_2)\Gamma_1 S_F(-q_1, m_1) \cdots S_F(-q_i, m_i)\Gamma_{i+1}u_{s_1}(p_{12}). \] (6)

Otherwise, after reversing the fermion line, we can transform the amplitude of the diquark production to familiar meson production except an additional \((-1)^{n+1}\) factor for pure vector case and \((-1)^{n+2}\) factor for containing an axial vector case. i.e. the amplitude of \(Z \rightarrow \langle bQ' \rangle [n] + \bar{b} + \bar{Q}'\) can be written as

\[ M_{diquark} = (M_1^u - M_1^v) + (M_2^u - M_2^v) + M_3 + M_4 \] (7)

where \(M_1, M_2, M_3, M_4\) are the amplitude of the familiar meson production, and \(M_1^u, M_1^v\) are those for the cases containing an axial vector or pure vector cases of \(M_i\) respectively.

According to Fig. 1, the hard amplitude \(M_l[n]\) with \(l = (a, ..., d)\) can be written as

\[ M_a[n] = -\kappa u(p_{12})(-i\gamma^\nu)v(p_2)\bar{u}(p_{11})(-i\gamma^\nu)(m_{Q'} + \not{p}_1 + \not{p}_2)\not{q}(p_0)(c_v + c_a\gamma^5)v(p_3) \]
\[ \frac{(p_{12} + p_2)^2}{(p_{12} - p_2)^2 - m_{Q'}^2} \]
\[ M_b[n] = -\kappa u(p_{12})(-i\gamma^\nu)(m_b + \not{p}_1 + \not{p}_2)\not{q}(p_0)(c_v + c_a\gamma^5)v(p_2)\bar{u}(p_{11})(-i\gamma^\nu)v(p_3) \]
\[ \frac{(p_{11} + p_3)^2}{(p_{11} - p_3)^2 - m_b^2} \]
\[ M_c[n] = -\kappa u(p_{12})\not{q}(p_0)(c_v + c_a\gamma^5)(m_b - \not{p}_1 - \not{p}_2 - \not{p}_3)(-i\gamma^\nu)v(p_2)\bar{u}(p_{11})(-i\gamma^\nu)v(p_3) \]
\[ \frac{(p_{11} + p_3)^2}{(p_{11} + p_3)^2 - m_b^2} \]
\[ M_d[n] = -\kappa u(p_{12})(-i\gamma^\nu)v(p_2)\bar{u}(p_{11})\not{q}(p_0)(c_v + c_a\gamma^5)(m_{Q'} - \not{p}_1 - \not{p}_2 - \not{p}_3)(-i\gamma^\nu)v(p_3) \]
\[ \frac{(p_{12} + p_2)^2}{(p_{12} + p_2)^2 - m_{Q'}^2} \] (8)

where \(p_{11}\) and \(p_{12}\) denote the momenta of \(b\) and \(Q'\) quark, and \(\kappa = -Cg_s^2\), \(C\) is the color factor \(C_{ij,k}\), which will be described in detail in Section II B, and \(c_v, c_a\) are vector and axial coupling constants of \(Z_{Q'\bar{Q}'}\) vertex. If \(Q'\) representing the heavy \(c\) quark, we have

\[ c_v = -\frac{e(8\sin^2\theta_w - 3)}{12\cos\theta_w \sin\theta_w}, \quad c_a = -\frac{e}{4\cos\theta_w \sin\theta_w}. \] (9)

and \(Q'\) denote the heavy \(b\) quark, we can obtain

\[ c_v = \frac{e(4\sin^2\theta_w - 3)}{12\cos\theta_w \sin\theta_w}, \quad c_a = \frac{e}{4\cos\theta_w \sin\theta_w}. \] (10)
Here $\theta_w$ is the Weinberg angle. With the help of the formulas listed in Eq. (6) and insert the spin projector $\Pi_{p_1}^{[n]}$, the Eq. (8) can be rewritten as

$$M_a[n] = -\kappa \bar{u}(p_2)(-i\gamma^\nu)\Pi_{p_1}^{[n]}(-i\gamma^\nu)(m_{Q'} + \not{p}_1 + \not{p}_2)\not{c}\not{v}(p_3)\left(\frac{(p_1 + p_2)^2}{(p_1 + p_2)^2 - m_{Q'}^2}\right)$$

$$M_b[n] = -\kappa \bar{u}(p_2)\not{c}\not{v}(p_0)(c_v \gamma^5 - c_\nu)(m_{b} - \not{p}_1 - \not{p}_3)(-i\gamma^\nu)\Pi_{p_1}^{[n]}(-i\gamma^\nu)v(p_3)\left(\frac{(p_1 + p_3)^2}{(p_1 + p_3)^2 - m_{b}^2}\right)$$

$$M_c[n] = -\kappa \bar{u}(p_2)(-i\gamma^\nu)(m_{b} + \not{p}_1 + \not{p}_2 + \not{p}_3)\not{c}\not{v}(p_0)(c_v \gamma^5 - c_\nu)\Pi_{p_1}^{[n]}(-i\gamma^\nu)v(p_3)\left(\frac{(p_1 + p_3)^2}{(p_1 + p_2 + p_3)^2 - m_{b}^2}\right)$$

$$M_d[n] = -\kappa \bar{u}(p_2)(-i\gamma^\nu)\Pi_{p_1}^{[n]}\not{c}\not{v}(p_0)(c_v + c_\nu \gamma^5)(m_{Q'} - \not{p}_1 - \not{p}_2 - \not{p}_3)(-i\gamma^\nu)v(p_3)\left(\frac{(p_1 + p_3)^2}{(p_1 + p_2 + p_3)^2 - m_{Q'}^2}\right)$$

(11)

the projector $\Pi_{p_1}^{[n]}$ has the form of [61]

$$\Pi_{p_1}^{[n]} = \frac{1}{\sqrt{2}M_{bQ'}}\varepsilon^{[n]}(\not{p}_1 + M_{bQ'})$$

(12)

where $\varepsilon^{[1S_0]} = \gamma^5$ and $\varepsilon^{[3S_1]} = \not{c}$ with $\varepsilon^a$ is the polarization vector of the $[3S_1]$ diquark state. $M_{bQ'} \simeq m_b + m_{Q'}$ is adopted to ensure gauge invariance. $p_{11}$ and $p_{12}$ are the specific momenta of these two constituent quarks of the diquark state:

$$p_{11} = \frac{M_b}{M_{bQ'}}p_1 + p, \quad p_{12} = \frac{M_{Q'}}{M_{bQ'}}p_1 + p$$

(13)

where $p$ is the relative momentum between these two constituent quarks and it is small enough to neglect in the amplitude of S-wave state for the non-relativistic approximation.

### B. Color factor

According to Fig. 1, the color factor $C_{ij,k}$ can be calculated as

$$C_{ij,k} = N \times \sum_{a,m,n} (T^a)_{im} (T^a)_{jn} \times G_{mnk}$$

(14)

where $a = (1, \cdots, 8)$ is the color indices of the gluon. $i, j, m, n = 1, 2, 3$ are color indices of the two outgoing anti-quarks $\bar{b}, \bar{Q}'$ and the two constituent heavy quarks in the diquark, respectively. The $k$ is the color indices of the diquark $\langle bQ' \rangle [n]$. The normalization constant $N = \sqrt{1/2}$. For the color-antitriplet $\bar{3}$ state, the function $G_{mnk}$ is equal to the antisymmetric function $\varepsilon_{mnk}$, while will be the symmetric function $f_{mnk}$ for the color-sextuplutt $\bar{6}$ state. The function $\varepsilon_{mnk}$ and $f_{mnk}$ satisfies

$$\varepsilon_{mnk}\varepsilon_{m'n'k'} = \delta_{mm'}\delta_{jj'} - \delta_{mj'}\delta_{jm'}$$
\[ f_{mjk}f_{m'j'k} = \delta_{mm'} \delta_{jj'} + \delta_{mj'} \delta_{jm'} \] (15)

Then, square of \( C_{ij,k}^2 \) equals to \( 4/3 \) for the color antitriplet \( \bar{3} \) diquark production, and \( 2/3 \) for the color sextuplet \( 6 \) diquark production, respectively.

### III. NUMERICAL RESULTS

In numerical calculations, the input parameters are taken as follows values[18, 62]

\[
\begin{align*}
m_c &= 1.8 \text{ GeV}, \\ m_b &= 5.1 \text{ GeV}, \\ m_w &= 80.385 \text{ GeV}, \\ m_z &= 91.1876 \text{ GeV}, \\ \Gamma_z &= 2.4952 \text{ GeV} \\
M_{\Xi_{cc}} &= 3.6 \text{ GeV}, \\ M_{\Xi_{bb}} &= 10.2 \text{ GeV}, \\ M_{\Xi_{bc}} &= 6.9 \text{ GeV}, \\ G_F &= 1.1663787 \times 10^{-5} \text{ GeV}^3 \\
|\Psi_{cc}(0)|^2 &= 0.039 \text{ GeV}^3, \\ |\Psi_{bc}(0)|^2 &= 0.065 \text{ GeV}^3, \\ |\Psi_{bb}(0)|^2 &= 0.152 \text{ GeV}^3
\end{align*}
\] (16)

where the quark masses and wave functions are consistent with Ref. [18] and the others can be obtained from the PDG [62]. We use FeynArts 3.9 [63] to generate the amplitudes. The renormalization scale \( \mu_r \) is set to be \( 2m_c \) and \( 2m_b \) for the production of \( \Xi_{bc} \) and \( \Xi_{bb} \) correspondingly.

According to the parameters mentioned before, two main \( Z \) decay channels for the production of \( \Xi_{bQ'} \) have been analyzed in detail, and the decay width of each channel is presented in Table I. From Table I, one can see that, for the production of the \( \Xi_{bb} \), the state of \([^3S_1]_\bar{3}\) plays the leading role, more than three times bigger in magnitude than that of \([^1S_0]_6\). As for the \( \Xi_{bc} \) productions, the situations become just the identical. Moreover, in the case of \( \Xi_{bc} \), the decay widths through \( Z \rightarrow c\bar{c} \) channels are very small and only a few percent compared to that through \( Z \rightarrow b\bar{b} \).

| \( \Gamma(\text{KeV}) \) | \( \Xi_{bc} \) | \( \Xi_{bb} \) |
|-----------------|-----------------|-----------------|
| \( \Xi_{bc} \) | \[|^3S_1\]_\bar{3} \] & \[|^3S_1\]_6 & \[|^1S_0\]_\bar{3} & \[|^1S_0\]_6 |
| \( Z \rightarrow c\bar{c} \) | 0.644 & 0.322 & 0.741 & 0.371 & - & - |
| \( Z \rightarrow b\bar{b} \) | 33.014 & 16.507 & 24.137 & 12.068 & 1.999 & 1.028 |

In order to estimate the events of doubly heavy baryon \( \Xi_{bQ'} \) produced at the LHC and CEPC, the decay width of the \( Z \) boson is needed to obtain the branching ratio correspondingly. Here the total decay width of the \( Z \) boson is considered to be 2.4952 GeV as suggested by Ref. [62]. At the LHC(CEPC), there are about \( 10^9(10^{12}) \) \( Z \) bosons can be produced per year [52, 64]. Under these conditions, we can estimate the produced events of \( \Xi_{bc}(\Xi_{bb}) \) at the LHC and the CEPC, respectively. By summing up the contribution from each intermediate diquark state, the total
TABLE II: The total decay width, branching ratio and the estimated events of the doubly heavy baryons $\Xi_{bQ'}$ by summing up the contribution from each intermediate diquark state.

| Process          | $\Gamma \times 10^{-6}$ (GeV) | $B \times 10^{-6}$ | LHC events        | CEPC events        |
|------------------|--------------------------------|--------------------|--------------------|--------------------|
| $Z \to \Xi_{bc}$ | 89.712                         | 35.954             | $35.954 \times 10^3$ | $35.954 \times 10^6$ |
| $Z \to \Xi_{bb}$ | 3.027                          | 1.213              | $1.213 \times 10^3$  | $1.213 \times 10^6$  |

FIG. 2: The invariant mass differential decay widths $d\Gamma/ds_{23}$ for the process $Z \to \Xi_{bc}(\Xi_{bb}) + X$, where $\bar{3}(6)$ stands for the contribution from the intermediate diquark state with the color quantum number is the $\bar{3}$ and the $6$ has been considered, "Total" denote the contribution from each intermediate diquark state have been summed.

...
FIG. 3: The differential decay widths $d\Gamma/dz$ for $Z \rightarrow \Xi_{bc}(\Xi_{bb}) + X$ within two typical renormalization scales $\mu_r = (2m_c, m_z/2)$, where $z$ is the energy fraction carried by the concerned baryon, subscript $3(6)$ stands for the contribution from the intermediate diquark state with the color quantum number is the $3$ and the $6$ has been considered. “Total” denote the contribution from each intermediate diquark state have been summed.

To make a complete analysis of the distributions for the production of $\Xi_{bc}(\Xi_{bb})$ through these considered channels and to be helpful as regards experimental detection, the invariant mass differential decay widths $d\Gamma/ds_{ij}$ and the differential decay widths of $\Xi_{bc}(\Xi_{bb})$ with respect to $z$ distributions are plotted in Figs. 2 and 3, we define the invariant mass $s_{ij} = (p_i + p_j)^2$ and the energy fraction $z = 2E_1/E_Z$, where $E_1$ is the energy of the $\Xi_{bc}(\Xi_{bb})$ and $E_Z$ is the energy of the $Z$ boson in the rest frame of the initial $Z$ boson.

Meanwhile, in Figs. 2 and 3, one can find in cases of $\Xi_{bb}$ productions, the state of $[^3S_1]$ plays the leading role. Especially, the contribution from the state of $[^3S_1]$ is more than two times bigger in magnitude than that of $[^1S_0]$ for the $\Xi_{bb}$ productions. In Fig. 3, it can be seen that, in the cases of $\Xi_{bb}$ productions, the peak of $\frac{d\Gamma}{dz}[^{3S_1}]$ is around $z = 0.75$ and $\frac{d\Gamma}{dz}[^{1S_0}]$ peaks near $z = 0.7$. As for the $\Xi_{bc}$, the peak of $\frac{d\Gamma}{dz}[^{3S_1}]$ is around $z = 0.8$ and $\frac{d\Gamma}{dz}[^{1S_0}]$ peaks near $z = 0.85$. That the peak of $\Xi_{bc}(\Xi_{bb})$ energy distribution in $Z \rightarrow \Xi_{bc(bb)} + X$ lies in the large $z$ region can
TABLE III: The total decay width $\Gamma$ (in unit: $10^{-6}$) within theoretical uncertainties for the production of baryons $\Xi_{bc}(\Xi_{bb})$ via $Z$ boson decays by varying $m_c = 1.80 \pm 0.05$ GeV.

| $\mu_r$  | $m_c$  | $\Xi_{bc}$ | $\Xi_{bb}$ |
|----------|--------|------------|------------|
| 1.75 GeV | 1.80 GeV | 38.337 | 19.169 | 28.078 | 14.039 | 1.999 | 1.028 |
| 2m_c     | 1.80 GeV | 34.396 | 17.198 | 25.412 | 12.706 | 1.999 | 1.028 |
| 1.85 GeV | 1.80 GeV | 30.681 | 15.341 | 23.054 | 11.527 | 1.999 | 1.028 |
| $m_z/2$  | 1.80 GeV | 10.472 | 5.236 | 7.669 | 3.835 | 1.053 | 0.542 |
| 1.85 GeV | 1.80 GeV | 9.552 | 4.776 | 7.057 | 3.528 | 1.053 | 0.542 |
|          | 1.85 GeV | 8.736 | 4.368 | 6.510 | 3.255 | 1.053 | 0.542 |

primarily be attributed to the dominance of the quark fragmentation mechanism.

A. Uncertainty analysis

In this subsection, the theoretical uncertainties for the production of $\Xi_{bc}(\Xi_{bb})$ via the $Z$ boson decays would be discussed. There are three main sources of the theoretical uncertainties: the quark mass, the renormalization scale $\mu_r$ and the $|\Psi_{bQ}(0)|$. The $|\Psi_{bQ}(0)|^2$ is an overall factor in the calculation, the uncertainty due to it can be calculated out easily. So we shall not discuss the uncertainty of the $|\Psi_{bQ}(0)|^2$ and concentrate our attention on the uncertainties from the heavy quark masses and the renormalization scale. The decay widths through these decay channels have been summed up for the total decay width. We shall analyze the caused quark mass uncertainties by varying $m_c = 1.80 \pm 0.05$ GeV and $m_b = 5.1 \pm 0.5$ GeV, which are presented in Table III and Table IV. Table III and Table IV show that:

- The decay width for the production of the $\Xi_{bc}$ baryon decreases with the increment of $m_c$, which is mainly due to the suppression of phase space. The decay width of $\Xi_{bc}[^3S_1]$ increases with the increment of $m_b$, and the decay width of $\Xi_{bc}[^1S_0]$ decreases with the increment of $m_b$, which is mainly due to the influence of the projector in Eq. (12). Moreover, the uncertainty from $m_c$ is relatively larger than those of $m_b$.

- In the predictions of the total decay width of $\Xi_{bb}$, the decay width of $\Xi_{bb}$ decreases with the increment of $m_b$. 
TABLE IV: The total decay width $\Gamma$ (in unit: $10^{-6}$) within theoretical uncertainties for the production of baryons $\Xi_{bc}(\Xi_{bb})$ via $Z$ boson decays by varying $m_b = 5.1 \pm 0.5$ GeV.

| $\mu_r$  | $m_b$  | $\Xi_{bc}$          | $\Xi_{bb}$          |
|---------|--------|----------------------|----------------------|
|         |        | $[^3S_1]_3$ | $[^3S_1]_6$ | $[^1S_0]_3$ | $[^1S_0]_6$ | $[^3S_1]_3$ | $[^1S_0]_6$ |
| 5.60 GeV| 3.510  | 34.915               | 17.458               | 25.001               | 12.501               | 1.345     | 0.697       |
| 5.10 GeV| 3.510  | 34.296               | 17.198               | 25.412               | 12.706               | 1.799     | 1.028       |
| 4.60 GeV| 3.510  | 33.892               | 16.946               | 25.914               | 12.957               | 3.044     | 1.385       |
| $m_c$   | 5.60 GeV| 9.696               | 4.848                | 6.943                | 3.471                | 0.750     | 0.389       |
|         | 5.10 GeV| 9.552               | 4.776                | 7.057                | 3.528                | 1.053     | 0.542       |
|         | 4.60 GeV| 9.412               | 4.706                | 7.196                | 3.598                | 1.515     | 0.773       |

IV. SUMMARY

Within the framework of NRQCD, the decay widths for the indirect production of $\Xi_{bc}, \Xi_{bb}$ via $Z$ boson decay have been analyzed. By summing up all the contributions from the intermediate diquark states, the total decay width for the process $Z \rightarrow \Xi_{bc}(\Xi_{bb}) + X$ can be obtained, i.e.

$$\Gamma_{Z \rightarrow \Xi_{bc} + X} = 89.712^{+9.911}_{-9.109} \times 10^{-6} \text{ GeV}$$

$$\Gamma_{Z \rightarrow \Xi_{bb} + X} = 3.027^{+1.402}_{-0.985} \times 10^{-6} \text{ GeV} \quad (17)$$

where the uncertainty is caused by varying the quark mass $m_c = 1.80 \pm 0.05$ GeV for the $\Xi_{bc}$ and $m_b = 5.1 \pm 0.5$ GeV for the $\Xi_{bb}$. To be helpful as regards experimental detection, the invariant mass differential decay widths $d\Gamma/ds_{ij}$ and the differential decay widths of $\Xi_{bc}(\Xi_{bb})$ with respect to $z$ distributions have also been presented. The corresponding produced events of the doubly heavy baryons $\Xi_{bQ'}$ are both estimated at the LHC and the CEPC. There are about about $10^4(10^7)$ $\Xi_{bc}$ events produced at the LHC(CEPC) events, about $10^3(10^6)$ $\Xi_{bb}$ events produced at the LHC(CEPC) events. Consider the fact the large number of doubly heavy baryon $\Xi_{bQ'}$ events can produce via $Z$ boson decay at the CEPC comparing to LHC, and the “clean” background of CEPC may help us to more easily hunt the heavy heavy baryon related processes. Thus we think, in addition to LHC, the CEPC can also be a good laboratory for investigating the properties of the doubly heavy baryons.
Acknowledgments

This work is supported in part by the Natural Science Foundation of China under Grant No. 11765007, and by the Project of Guizhou Provincial Department of Education under Grant No.KY[2021]030.

[1] M. Gell-Mann, *A schematic model of baryons and mesons*, Phys. Lett. **8**, 214-215, (1964).
[2] D. Ebert, R. N. Faustov, V. O. Galkin, A. P. Martynenko and V. A. Saleev, *Heavy baryons in the relativistic quark model*, Z. Phys. C **76**, 111-115 (1997). [hep-ph/9607314]
[3] S. M. Gerasyuta and D. V. Ivanov, *Charmed baryons in bootstrap quark model*, Nuovo Cim. A **112**, 261-276 (1999). [hep-ph/0101310]
[4] C. Itoh, T. Minamikawa, K. Miura and T. Watanabe, *Doubly charmed baryon masses and quark wave functions in baryons*, Phys. Rev. D **61**, 057502 (2000).
[5] R. Aaij et al. [LHCb], *Observation of the doubly charmed baryon \( \Xi_{cc}^{++} \)*, Phys. Rev. Lett. **119**, 112001 (2017). [arXiv:1707.01621]
[6] R. Aaij et al. [LHCb], *Measurement of \( \Xi_{cc}^{++} \) production in pp collisions at \( \sqrt{s} = 13 \) TeV*, Chin. Phys. C **44**, 022001 (2020). [arXiv:1910.11316]
[7] R. Aaij et al. [LHCb], *First Observation of the Doubly Charmed Baryon Decay \( \Xi_{cc}^{++} \to \Xi_{c}^{+}\pi^{+} \)*, Phys. Rev. Lett. **121**, 162002 (2018). [arXiv:1807.01919]
[8] R. Chistov et al. [Belle], *Observation of new states decaying into \( \Lambda_{c}^{+}K^{-}\pi^{+} \) and \( \Lambda_{c}^{+}K_{S}^{0}\pi^{-} \)*, Phys. Rev. Lett. **97**, 162001 (2006). [hep-ex/0606051]
[9] B. Aubert et al. [BaBar], *Search for doubly charmed baryons \( \Xi_{cc}^{+} \) and \( \Xi_{cc}^{++} \) in BABAR*, Phys. Rev. D **74**, 011103 (2006). [hep-ex/0605075]
[10] S. P. Ratti, *New results on c-baryons and a search for cc-baryons in FOCUS*, Nucl. Phys. B Proc. Suppl. **115**, 33-36 (2003).
[11] R. Aaij et al. [LHCb], *Search for the doubly charmed baryon \( \Xi_{cc}^{++} \)*, Sci. China Phys. Mech. Astron. **63**, 221062 (2020). [arXiv:1909.12273]
[12] R. Aaij et al. [LHCb], *Search for the doubly heavy \( \Xi_{bc}^{0} \) baryon via decays to \( D^{0}pK^{-} \)*, JHEP **11**, 095 (2020). [arXiv:2009.02481]
[13] R. Aaij et al. [LHCb], *Search for the doubly heavy baryons \( \Omega_{bc}^{0} \) and \( \Xi_{bc}^{0} \) decaying to \( \Lambda_{c}^{+}\pi^{-} \) and \( \Xi_{c}^{+}\pi^{-} \)*, Chin. Phys. C **45**, 093002 (2021). [arXiv:2104.04759]
[14] S. J. Brodsky, S. Groote and S. Koshkarev, *Resolving the SELEX-LHCb double-charm baryon conflict: the impact of intrinsic heavy-quark hadroproduction and supersymmetric light-front holographic QCD*, Eur. Phys. J. C **78**, 483 (2018). [arXiv:1709.09903]
[15] V. V. Kiselev, A. K. Likhoded and M. V. Shevlyagin, *Double charmed baryon production at B factory*,
[16] A. F. Falk, M. E. Luke, M. J. Savage and M. B. Wise, *Heavy quark fragmentation to baryons containing two heavy quarks*, Phys. Rev. D **49**, 555-558 (1994). [hep-ph/9305315]

[17] C. H. Chang, J. P. Ma, C. F. Qiao and X. G. Wu, *Hadronic production of the doubly charmed baryon X_{ic} with intrinsic charm*, J. Phys. G **34**, 845 (2007). [hep-ph/0610205]

[18] S. P. Baranov, *On the production of doubly flavored baryons in pp, ep and γγ collisions*, Phys. Rev. D **54**, 3228-3236 (1996).

[19] G. T. Bodwin, E. Braaten and G. P. Lepage, *Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium*, Phys. Rev. D **51**, 1125-1171 (1995). [hep-ph/9407339]

[20] D. A. Gunter and V. A. Saleev, *Hadronic production of doubly charmed baryons via charm excitation in proton*, Phys. Rev. D **64**, 034006 (2001). [hep-ph/0104173]

[21] V. V. Kiselev, A. K. Likhoded and M. V. Shevlyagin, *Production of doubly charmed baryons at energy √s = 10.58 GeV*, Phys. Atom. Nucl. **58**, 1018-1021 (1995).

[22] V. V. Braguta, V. V. Kiselev and A. E. Chalov, *Pair production of doubly heavy diquarks*, Phys. Atom. Nucl. **65**, 1537-1544 (2002).

[23] E. Braaten, M. Kusunoki, Y. Jia and T. Mehen, *Λ^+ c/Λ^− c asymmetry in hadroproduction from heavy quark recombination*, Phys. Rev. D **70**, 054021 (2004). [hep-ph/0304280]

[24] S. Y. Li, Z. G. Si and Z. J. Yang, *Doubly heavy baryon production at gamma gamma collider*, Phys. Lett. B **648**, 284-288 (2007). [hep-ph/0701212]

[25] J. W. Zhang, X. G. Wu, T. Zhong, Y. Yu and Z. Y. Fang, *Hadronic Production of the Doubly Heavy Baryon Ξ_{bc} at LHC*, Phys. Rev. D **83**, 034026 (2011). [arXiv:1101.1130]

[26] J. Jiang, X. G. Wu, S. M. Wang, J. W. Zhang and Z. Y. Fang, *A Further Study on the Doubly Heavy Baryon Production around the Z^0 Peak at A High Luminosity e^+e^- Collider*, Phys. Rev. D **87**, 054027 (2013). [arXiv:1302.0601]

[27] H. Y. Bi, R. Y. Zhang, X. G. Wu, W. G. Ma, X. Z. Li and S. Owusu, *Photoproduction of doubly heavy baryon at the LHeC*, Phys. Rev. D **95**, 074020 (2017). [arXiv:1702.07181]

[28] J. W. Zhang, X. G. Wu, T. Zhong, Y. Yu and Z. Y. Fang, *Hadronic Production of the Doubly Heavy Baryon Ξ_{bc} at LHC*, Phys. Rev. D **83**, 034026 (2011). [arXiv:1101.1130]

[29] J. Jiang, X. G. Wu, Q. L. Liao, X. C. Zheng and Z. Y. Fang, *Doubly Heavy Baryon Production at A High Luminosity e^+e^- Collider*, Phys. Rev. D **86**, 054021 (2012). [arXiv:1208.3051]

[30] J. Jiang, X. G. Wu, S. M. Wang, J. W. Zhang and Z. Y. Fang, *A Further Study on the Doubly Heavy Baryon Production around the Z^0 Peak at A High Luminosity e^+e^- Collider*, Phys. Rev. D **87**, 054027 (2013). [arXiv:1302.0601]

[31] A. P. Martynenko and A. M. Trunin, *Relativistic corrections to the pair double heavy diquark production in e^+e^- annihilation*, Phys. Rev. D **89**, 014004 (2014). [arXiv:1308.3998]

[32] Z. J. Yang and X. X. Zhao, *The Production of Ξ_{bb} at Photon Collider*, Chin. Phys. Lett. **31**, 091301 (2014). [arXiv:1408.5584]

[33] Z. J. Yang, P. F. Zhang and Y. J. Zheng, *Doubly Heavy Baryon Production in e^+e^- Annihilation*,
[34] A. P. Martynenko and A. M. Truin, *Pair double heavy diquark production in high energy proton–proton collisions*, Eur. Phys. J. C **75**, 138 (2015). [arXiv:1405.0969]

[35] W. K. Lai and A. K. Leibovich, $\Lambda^+_c/\Lambda^-_c$ and $\Lambda^0_c/\bar{\Lambda}^0_c$ production asymmetry at the LHC from heavy quark recombination, *Pair double heavy diquark production in high energy proton–proton collisions*, Phys. Rev. D **91**, 054022 (2015). [arXiv:1410.2091]

[36] S. Koshkarev and V. Anikeev, *Production of the doubly charmed baryons at the SELEX experiment – The double intrinsic charm approach*, Phys. Lett. B **765**, 171-174 (2017). [arXiv:1605.03070]

[37] S. Koshkarev, *Production of the Doubly Heavy Baryons, $B_c$ Meson and the All-charm Tetraquark at AFTER@LHC with Double Intrinsic Heavy Mechanism*, Acta Phys. Polon. B **48**, 163 (2017). [arXiv:1610.06125]

[38] S. Groote and S. Koshkarev, *Production of doubly charmed baryons nearly at rest*, Eur. Phys. J. C **77**, 509 (2017). [arXiv:1704.02850]

[39] X. Yao and B. Müller, *Doubly charmed baryon production in heavy ion collisions*, Phys. Rev. D **97**, 074003 (2018). [arXiv:1801.02652]

[40] C. H. Chang, C. F. Qiao, J. X. Wang and X. G. Wu, *Estimate of the hadronic production of the doubly charmed baryon $\Xi_{cc}$ under GM-VFN scheme*, Phys. Rev. D **73**, 094022 (2006). [hep-ph/0601032]

[41] G. Chen, X. G. Wu, J. W. Zhang, H. Y. Han and H. B. Fu, *Hadronic production of $\Xi_{cc}$ at a fixed-target experiment at the LHC*, Phys. Rev. D **89**, 074020 (2014). [arXiv:1401.6269]

[42] X. C. Zheng, C. H. Chang and Z. Pan, *Production of doubly heavy-flavored hadrons at $e^+e^-$ colliders*, Phys. Rev. D **93**, 034019 (2016). [arXiv:1510.06808]

[43] G. Chen, C. H. Chang and X. G. Wu, *Hadronic production of the doubly charmed baryon via the proton–nucleus and the nucleus–nucleus collisions at the RHIC and LHC*, Eur. Phys. J. C **78**, 801 (2018). [arXiv:1808.03174]

[44] A. V. Berezhnoy, I. N. Belov and A. K. Likhoded, *Production of doubly charmed baryons with the excited heavy diquark at LHC*, Int. J. Mod. Phys. A **34**, 1950038 (2019). [arXiv:1811.07382]

[45] G. Chen, X. G. Wu and S. Xu, *Impacts of the intrinsic charm content of the proton on the $\Xi_{cc}$ hadroproduction at a fixed target experiment at the LHC*, Phys. Rev. D **100**, 054022 (2019). [arXiv:1903.00722]

[46] X. G. Wu, *A new search for the doubly charmed baryon $\Xi_{cc}^+$ at the LHC*, Sci. China Phys. Mech. Astron. **63**, 221063 (2020). [arXiv:1912.01953]

[47] J. J. Niu, L. Guo, H. H. Ma, X. G. Wu and X. C. Zheng, *Production of semi-inclusive doubly heavy baryons via top-quark decays*, Phys. Rev. D **98**, 094021 (2018). [arXiv:1810.03834]

[48] P. H. Zhang, L. Guo, X. C. Zheng and Q. W. Ke, *Excited doubly heavy baryon production via $W^+$ boson decays*, Phys. Rev. D **105**, 034016 (2022). [arXiv:2202.01579]

[49] J. J. Niu, L. Guo, H. H. Ma and X. G. Wu, *Production of doubly heavy baryons via Higgs boson decays*, Eur. Phys. J. C **79**, 339 (2019). [arXiv:1904.02339]

[50] X. Luo, Y. Z. Jiang, G. Y. Zhang and Z. Sun, *Doubly-charmed baryon production in $Z$ boson decay*,}
[arXiv:2206.05965]

[51] Q. L. Liao, Y. Yu, Y. Deng, G. Y. Xie and G. C. Wang, *Excited heavy quarkonium production via Z⁰ decays at a high luminosity collider*, Phys. Rev. D 91, 114030 (2015). [arXiv:1505.03275]

[52] J. B. Guimarães da Costa et al. [CEPC Study Group], *CEPC Conceptual Design Report: Volume 2 - Physics Detector*, [arXiv:1811.10545]

[53] J. P. Ma and Z. G. Si, *Factorization approach for inclusive production of doubly heavy baryon*, Phys. Lett. B 568, 135-145 (2003). [hep-ph/0305079]

[54] A. Petrelli, M. Cacciari, M. Greco, F. Maltoni and M. L. Mangano, *NLO production and decay of quarkonium*, Nucl. Phys. B 514, 245-309 (1998). [hep-ph/9707223]

[55] Z. Sun and X. G. Wu, *The production of the doubly charmed baryon in deeply inelastic ep scattering at the Large Hadron Electron Collider*, JHEP 07, 034 (2020). [arXiv:2004.01012]

[56] E. Bagan, H. G. Dosch, P. Gosdzinsky, S. Narison and J. M. Richard, *Hadrons with charm and beauty*, Z. Phys. C 64, 57-72 (1994). [hep-ph/9403208]

[57] G. T. Bodwin, D. K. Sinclair and S. Kim, *Quarkonium decay matrix elements from quenched lattice QCD*, Phys. Rev. Lett. 77, 2376-2379 (1996). [hep-ph/9605023]

[58] V. V. Kiselev, A. K. Likhoded and A. I. Onishchenko, *Semileptonic B_c meson decays in sum rules of QCD and NRQCD*, Nucl. Phys. B 569, 473-504 (2000). [hep-ph/9905359]

[59] C. H. Chang, J. X. Wang and X. G. Wu, *Production of B_c or \bar{B}_c meson and its excited states via t quark or \bar{t} quark decays*, Phys. Rev. D 77, 014022 (2008). [arXiv:0711.1898]

[60] X. G. Wu, *Uncertainties in Estimating the Indirect Production of B_c and Its Excited States Via Top Quark Decays at CERN LHC*, Phys. Lett. B 671, 318-322 (2009). [arXiv:0805.4511]

[61] G. T. Bodwin and A. Petrelli, *Order-v⁴ corrections to S-wave quarkonium decay*, Phys. Rev. D 66, 094011 (2002). [hep-ph/0205210]

[62] M. Tanabashi et al. [Particle Data Group], *Review of Particle Physics*, Phys. Rev. D 98, 030001 (2018).

[63] T. Hahn, *Generating Feynman diagrams and amplitudes with FeynArts 3*, Comput. Phys. Commun. 140, 418-431 (2001). [hep-ph/0012260]

[64] G. Weiglein et al. [LHC/ILC Study Group], *Physics interplay of the LHC and the ILC*, Phys. Rept. 426, 47-358 (2006). [hep-ph/0410364]

[65] L. C. Deng, X. G. Wu, Z. Yang, Z. Y. Fang and Q. L. Liao, *Z_0 Boson Decays to B_c(∗) Meson and Its Uncertainties*, Eur. Phys. J. C 70, 113-124 (2010). [arXiv:1009.1453]