Diquark correlations in baryons: the Interacting Quark Diquark Model *

E. Santopinto
INFN, Sezione di Genova, via Dodecaneso 33, 16146 Genova (Italy)

A review of the underlying ideas of the Interacting Quark Diquark Model (IQDM) that assess the baryon spectroscopy in terms of quark diquark degrees of freedom is given, together with a discussion of the missing resonances problem. Some ideas about its generalization the heavy baryon spectroscopy is given. Some ideas about its generalization the heavy baryon spectroscopy is given. The results are compared to the existing experimental data.

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I. INTRODUCTION

The notion of diquark is as old as the quark model itself. Gell-Mann [1] mentioned the possibility of diquarks in his original paper on quarks, just as the possibility of tetra and pentaquark. Soon afterwards, Ida and Kobayashi [2] and Lichtenberg and Tassie [3] introduced effective degrees of freedom of diquarks in order to describe baryons as composed of a constituent diquark and quark. Since its introduction, many articles have been written on this subject [4–15] up to the most recent ones [16–18]. But most important, different phenomenological indications for diquark correlations have been collected during the years, such as some regularities in hadron spectroscopy, the ∆I = 1/2 rule in weak nonleptonic decays [19], some regularities in parton distribution functions and in spin-dependent structure functions [20] and in the Λ(1116) and Λ(1520) fragmentation functions. Finally, although the phenomenon of color superconductivity [21] in quark dense matter cannot be considered an argument in support of diquarks in the vacuum, it is nevertheless of interest since it stresses the important role of Cooper pairs of color superconductivity, which are color antitriplet, flavor antisymmetric, scalar diquarks. The introduction of diquarks in hadronic physics has some similarities to that of correlated pairs in condensed matter physics (superconductivity [22]) and in nuclear physics (interacting boson model [23]), where effective bosons emerge from pairs of electrons [24] and nucleons [25], respectively. Most important, any interaction that binds π and ρ mesons in the rainbow-ladder approximation of the DSE will produce diquarks as can be seen in Ref. [26], moreover there are some indication of diquark confinement [27].

Recently quark-diquark effective degrees of freedom have shown their usefulness also in the study of transversity problems and fragmentation functions (see Ref. [28]), even in an oversimplified form, i.e. with the spatial part of the quark-diquark ground state wave function parametrized by means of a gaussian. Even if the microscopic origin of that effective degree of freedom of diquark, it is not completely clear, nevertheless, as in nuclear physics, one may attempt to correlate the data in terms of a phenomenological model, and in many cases it has already shown it usefulness. In this article, we will review the Interacting Quark Diquark model in its original formulation [11], since the Point Form relativistic reformulation it has been already discussed in another contribution of the same conference (see Ferretti and Santopinto). We shall focus on its differences and extension to the strange spectra. We will point out some important consequences on the ratio of the electric and magnetic form factor of the proton, that is a zero at $Q^2 = 8 \text{ GeV}^2$, while impossible with three quark model such as that of Ref. Ref. [28].

The new 12 GeV$^2$ experiment planned at Jlab will eventually shed light on the diquark structure of the nucleon. Finally by comparing the number of Λ’s states predicted by the relativistic Interacting Quark Diquark model, that are only a subset of those predicted by three-quark models, we will try to suggest a next generation Pentaquark analysis that evaluates the systematic error on the background due to the missing Λ’s states. The missing Λ resonances can not change the structures seen in the Dalitz Plot but eventually only modify some parameters.

The various aspects of the hadron structures have been investigated by many experimental and theoretical approaches in the last years. The observations of the hadron states with an exotic structure have attracted a lot interest. In particular, regarding the light flavor region, we can remind the exotic states found in the accelerator facilities such as the scalar mesons $a_0(980)$ and $f_0(980)$, or the Λ(1405) which are expected to have an exotic structure as multiquarks, hadronic molecules, but also hybrid states and so forth [31–33]. While regarding the exotic heavy hadrons, we can cite states such as the Zc [36, 37] and $Z_b^{(0)}$ [38] which can not be explained by the simple quark model picture. In addition, there are the pentaquark states, seen by LHCb [39], with quark content $c\bar{c}uud$.

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In parallel, recently, theoretical approaches based on QCD have been strongly developed. The chiral effective field theory respecting the chiral symmetry provides the hadron-hadron scatterings at low energy with the Nambu-Goldstone bosons exchange. This is a powerful tool to investigate hadronic molecules as the meson-meson [10, 42], meson-baryon [43, 44], and baryon-baryon [45, 46] states appearing near thresholds. On the other side, Lattice QCD performs ab initio calculation for hadron spectroscopy, even if it is not easy to approach hadron states at the physical pion mass or with heavy flavor. Nevertheless, the recent progress of the Lattice simulations are really impressive and hadron structures and interactions have been discussed extensively in Refs. [47, 48]. Finally, in the last part of this article, we will discuss briefly some new results obtained within the formalism of the Unquenched Quark Model (UQM); when LQCD or Chiral effective models can not be applied, it can provide anyway predictions, making up with the three quark model defects.

II. THE INTERACTING QUARK DIQUARK MODEL

The model will be discussed as an attempt to arrive to a systematic description and correlation of data in term of q-diquark effective degrees of freedom by formulating a quark-diquark model with explicit interactions, in particular with a direct and an exchange interaction. We will show the spectrum which emerges from this model. In respect to the prediction shown in Ref. [10] we have extended our calculation up to 2.4 GeV, and so we have predicted more states.

Up to an energy of about 2GeV, the diquark can be described as two correlated quarks with no internal spatial excitations [10, 10]. Then, its color-spin-flavor wave function must be antisymmetric. Moreover, as we consider only light baryons, made up of up, down, strange quarks, the internal group is restricted to SU_{sf}(6). If we denote spin by its value, flavor and color by the dimension of the representation, the quark has spin $s = \frac{1}{2}$, flavor $F_2 = 3$, and color $C_2 = 3$. The diquark must transform as $[3]$ under $SU_{C}(3)$, hadrons being color singlets. Then, one only has the symmetric $SU_{sf}(6)$ representation $21_{sf}(S)$, containing $s_1 = 0$, $F_1 = 3$, and $s_1 = 1$, $F_1 = 6$, i.e. the scalar and axial-vector diquarks, respectively. This is because we think of the diquark as two correlated quarks in an antisymmetric nonexcited state. We assume that the baryons are composed of a valence quark and a valence diquark.

The relative configurations of two body can be described by the relative coordinate $\vec{r}$ and its conjugate momenta $\vec{p}$. The Hamiltonian contains a direct and an exchange interaction. The direct interaction is coulomb plus linear interactions as perturbations, so the model is completely analytical. The matrix elements of $\vec{r}$ can be evaluated in closed form as

\begin{equation}
H = E_0 + \frac{\vec{p}^2}{2m} - \frac{r}{\tau} + \beta r + (B + C\delta_0)\delta_{S_{12},1} \nonumber
\end{equation}

\begin{equation}
+ (-1)^{l+1}2Ae^{-\alpha r}[s_{12} \cdot s_3 + t_{12} \cdot t_3 + 2s_{12} \cdot s_3 t_{12} \cdot t_3],
\end{equation}

For a purely Coulomb-like interaction the problem is analytically solvable. The solution is trivial, with eigenvalues

\begin{equation}
E_{n,l} = -\frac{\tau^2 m}{2 n^2} , \quad n = 1, 2, ... .
\end{equation}

Here $m$ is the reduced mass of the diquark-quark configuration and $n$ the principal quantum number. The eigenfunctions are the usual Coulomb functions

\begin{equation}
R_{n,l}(r) = \sqrt{\frac{(n-l-1)!(2g)^3}{[2n(n+l)!]^3}} e^{-gr} L^{2l+1}_{n-l-1}(2gr),
\end{equation}

where for the associated Laguerre polynomials we use the notation of Ref. morse and $g = \frac{2m}{\tau}$. We treat all the other interactions as perturbations, so the model is completely analytical. The matrix elements of $\vec{r}$ can be evaluated in closed form as

\begin{equation}
\Delta E_{n,l} = \int_0^\infty \beta r |R_{n,l}(r)|^2 r^2 dr = \frac{\beta}{2m\tau} [3n^2 - l(l + 1)].
\end{equation}

Next comes the exchange interaction of Eq. (5). The spin-isospin part is obviously diagonal in the basis of Eq. (7)

\begin{equation}
\langle \vec{s}_{12} \cdot \vec{s}_3 \rangle = \frac{1}{2} [S(S + 1) - s_{12}(s_{12} + 1) - s_3(s_3 + 1)]
\end{equation}

\begin{equation}
\langle \vec{t}_{12} \cdot \vec{t}_3 \rangle = \frac{1}{2} [T(T + 1) - t_{12}(t_{12} + 1) - t_3(t_3 + 1)].
\end{equation}
To complete the evaluation, we need the matrix elements of the exponential. These can be obtained in analytic form

\[ I_{n,l}(\alpha) = \int_0^\infty e^{-\alpha r} [R_{n,l}(r)]^2 r^2 dr \]  

(6)

The results are straightforward. Here, by way of example, we quote the result for \( l = n - 1 \)

\[ I_{n,l=n-1}(\alpha) = \left( \frac{1}{1 + \frac{n}{2\pi n_m^2}} \right)^{2n+1} \]  

(7)

Our results are in present in Tables I and II.

### III. THE RELATIVISTIC INTERACTING QUARK DIQUARK MODEL

The extention of the Interacting quark diquark model in Point Form can be easily done. This is a potential model, constructed within the point form formalism, where baryon resonances are described as two-body quark-diquark bound states; thus, the relative motion between the two constituents and the Hamiltonian of the model are functions of the relative coordinate \( \vec{r} \) and its conjugate momentum \( \vec{q} \). The Hamiltonian contains just as in the 2005 paper, the two basic ingredients: a Coulomb-like plus linear confining interaction and an exchange one, depending on the spin and isospin of the quark and the diquark. The mass operator is given by

\[ M = E_0 + \sqrt{\vec{q}^2 + m_1^2} + \sqrt{\vec{q}^2 + m_2^2} + M_{\text{dir}}(r) + M_{\text{ex}}(r) \]  

(8)

where \( E_0 \) is a constant, \( M_{\text{dir}}(r) \) and \( M_{\text{ex}}(r) \) the direct and the exchange diquark-quark interaction, respectively, \( m_1 \) and \( m_2 \) stand for diquark and quark masses. The direct term we consider,

\[ M_{\text{dir}}(r) = \frac{r}{2} \left( 1 - e^{-\mu r} \right) + \beta r \]  

(9)
is the sum of a Coulomb-like interaction with a cut off plus a linear confinement term. We also have an exchange interaction, since this is the crucial ingredient of a quark-diquark description of baryons that has to be extended to contain flavor Λ matrices in such a way to be able to describe in a simultaneous way both the non strange and the strange sector [10, 18]. We have also generalized the exchange interaction in such a way to be able to describe strange baryons, simply considering

\[ M_{\text{ex}}(r) = (-1)^{l+1} e^{-\sigma r} \left[ A_S \bar{s}_1 \cdot \bar{s}_2 + A_F \bar{\lambda} \cdot \bar{\lambda}_2 + A_I \bar{t}_1 \cdot \bar{t}_2 \right], \]  

where \( \bar{\lambda} \) are the SU(3) Gell-Mann matrices. In a certain sense, we can consider it as a Gürtler-Radicati inspired interaction [17, 51]. In the nonstrange sector, we also have to keep a contact interaction [16] in the mass operator

\[ M_{\text{cont}} = \left( \frac{m_n m_s}{E_L E_S} \right)^{1/2+\epsilon} \frac{v^2 D}{\pi^{1/2}} e^{-\eta^2 r^2} \delta_{L,0} \delta_{s,1} \left( \frac{m_n m_s}{E_L E_S} \right)^{1/2+\epsilon}, \]

as necessary to reproduce the \( \Delta - N \) mass splitting.

The results for the strange and non-strange baryon spectra from Ref. [14, 15] (See Tables II and III) were obtained by diagonalizing the mass operator of Eq. (8) by means of a numerical variational procedure, based on harmonic oscillator trial wave functions. With a basis of 150 harmonic oscillator shells, the results converge very well.

It is clear that a larger number of experiments and analysis, looking for missing resonances, are necessary because many aspects of hadron spectroscopy are still unclear. In particular the number of \( \Lambda \) states reported by the PDG are still very few in respect to the predictions both of the Lattice QCD and by the models. In particular the relativistic version of the interacting quark diquark model predict seven \( \Lambda \) missing states belonging to the octet and other six missing states belonging to the singlet (considering only states under 2.0 GeV), otherwise much more states should be considered. Typical three quark models will predict much more \( \Lambda \) missing states. New experiments should be dedicated to the hunting of those elusive missing \( \Lambda \) states. Incidentally, we observe that in the \( \text{LHC}_b \) pentaquark finding [29], the authors have included in the fit, correctly, only the experimentally well established \( \Lambda \) states, that means that the one * star states have been excluded. We observe that the eventual future discover of extra \( \Lambda \) states or even the confirmation of the one * star states , even not able, for sure, to modify the structures present in the Dalitz plot, can eventually change the parameters of the two Pentaquark states.

It is also worthwhile noting that in our model [15] \( \Lambda(1116) \) and \( \Lambda^*(1520) \) are described as bound states of a scalar diquark \([n, n]\) and a quark \( s \), where the quark-diquark system is in \( S \) or \( P \)-wave, respectively [18]. This is in accordance with the observations of Refs. [11, 12] on \( \Lambda \)'s fragmentation functions, that the two resonances can be described as \([n, n] - s\) systems.

We should also underline that the interacting quark-diquark model gives origin to wave functions that can describe in a reasonable way the elastic electromagnetic form factors of the nucleon . In particular they give origin to a reproduction of the existing data for the ratio of the electric and magnetic form factor of the proton that predict a zero at \( Q^2 = 8 \text{ GeV}^2 \)(see Fig. 1) like in vector meson parametrizations. On the contrary, we have found impossible to get this zero with a three quark model (see Fig. 2). The new experiment planned at JLab will be able to distinguish between the two scenarios ruling out one of the two models.

### Table II: As Table I but for \( \Delta \)-type resonances.

| Baryon \( L_{2g_z} J \) Status | Mass (MeV) | State \( M_{\text{cal}} \) (MeV) |
|-------------------------------|------------|-----------------|
| \( \Delta(1232) P_{33} \)    | 1230-1234  | 3/2+            | 1235 |
| \( \Delta(1600) P_{33} \)    | 1500-1700  | 3/2+            | 1709 |
| \( \Delta(1620) S_{31} \)    | 1600-1660  | 1/2−            | 1673 |
| \( \Delta(1700) D_{33} \)    | 1670-1750  | 3/2−            | 1673 |
| \( \Delta(1900) S_{31} \)    | 1840-1920  | 1/2−            | 2003 |
| \( \Delta(1905) F_{35} \)    | 1855-1910  | 5/2+            | 1930 |
| \( \Delta(1910) P_{33} \)    | 1860-1910  | 1/2+            | 1967 |
| \( \Delta(1920) P_{33} \)    | 1900-1970  | 3/2+            | 1930 |
| \( \Delta(1930) D_{35} \)    | 1900-2000  | 5/2−            | 2003 |
| \( \Delta(1940) D_{33} \)    | 1940-2060  | 3/2−            | 2003 |
| \( \Delta(1950) F_{37} \)    | 1915-1950  | 7/2+            | 1930 |
| \( \Delta(2000) F_{35} \)    | \( \approx \) 2000 | 5/2+          | 2015 |

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distinguish the flavor asymmetry and strangeness of the proton. Finally, the UQM is applied to describe the wave functions given explicitly. In another contribution of the same workshop (see Hugo Garcia et al.) are on the spin of the proton in which the effects of the sea quarks were introduced into the CQM in a systematic way and of scalar meson nonet (\[63, 64\], while Van Beveren and Rupp used an t-matrix approach \[65, 66\]. These techniques were applied to study effects. The unquenching of the quark model for hadrons is a way to take these components into account.

The constituent quark model (CQM) \[16, 54–62\], but it neglects quark-antiquark pair-creation (or continuum-coupling) potential-model expectations.

In this work we present the latest applications of the UQM to study the orbital angular momenta contribution to the charmonium properties using the Cornell coupled-channel model \[55\] to assess departures from the single-channel intermediate states that each meson can access. It is via these hadronic loops that the bare states become “dressed” and the hadronic loop contributions totally dominate the dynamics of the process. A similar approach was developed by Pennington in Ref. \[68\], where they have investigated the dynamical generation of the scalar mesons by initially missing – –

\begin{tabular}{llcccccccc}
\hline
Resonance & Status & $M^{\text{exp.}}$ (MeV) & $J^P$ & $L^P$ & $s_1$ & $Q^2q$ & $F \ F_1$ & $I \ t_1$ & $n_r$ & $M^{\text{calc.}}$ (MeV) \\
\hline
$\Lambda(1116) \ P_{01}$ & **** & 1116 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 3 & 0 & 0 & 1116 \\
$\Lambda(1600) \ P_{01}$ & *** & 1560 - 1700 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 3 & 0 & 0 & 1518 \\
$\Lambda(1670) \ S_{01}$ & **** & 1660 - 1680 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 3 & 0 & 0 & 1650 \\
$\Lambda(1690) \ D_{03}$ & **** & 1685 - 1695 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 3 & 0 & 0 & 1650 \\
$\Lambda(1800) \ S_{01}$ & *** & 1720 - 1850 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 3 & 0 & 0 & 1732 \\
$\Lambda(1810) \ P_{01}$ & *** & 1750 - 1850 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 3 & 0 & 0 & 1666 \\
$\Lambda(1820) \ S_{05}$ & **** & 1815 - 1825 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 3 & 0 & 0 & 1896 \\
$\Lambda(1830) \ D_{05}$ & **** & 1810 - 1830 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 6 & 0 & 0 & 1785 \\
$\Lambda(1890) \ P_{03}$ & **** & 1850 - 1910 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 6 & 0 & 0 & 1896 \\

missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 3 & 0 & 0 & 1732 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 6 & 0 & 0 & 1785 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 3 & 0 & 0 & 1785 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 6 & 0 & 0 & 1955 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 3 & 0 & 0 & 1960 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 6 & 0 & 0 & 1969 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 8 & 6 & 0 & 0 & 1969 \\

$\Lambda^*(1405) \ S_{01}$ & **** & 1402 - 1410 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 1 & 3 & 0 & 0 & 1431 \\
$\Lambda^*(1520) \ D_{03}$ & **** & 1519 - 1521 & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 1 & 3 & 0 & 0 & 1431 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 1 & 3 & 0 & 0 & 1443 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 1 & 3 & 0 & 0 & 1443 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 1 & 3 & 0 & 0 & 1854 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 1 & 3 & 0 & 0 & 1854 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 1 & 3 & 0 & 0 & 1928 \\
missing & – & – & $\frac{1}{2}^+$ & 0+ & 0 & $[n, n]$ & 1 & 3 & 0 & 0 & 1928 \\

\hline
\end{tabular}

TABLE III: Mass predictions \[18\] for A-type resonances compared with PDG data; APS copyright.

IV. THE UNQUENCHED QUARK MODEL

The behavior of observables such as the spectrum and the magnetic moments of hadrons are well reproduced by the constituent quark model (CQM) \[16, 54\] \[62\], but it neglects quark-antiquark pair-creation (or continuum-coupling) effects. The unquenching of the quark model for hadrons is a way to take these components into account.

The unquenching of CQM were initially done by Törnqvist and collaborators, who used an unitarized quark model \[63, 64\], while Van Beveren and Rupp used an t-matrix approach \[65, 66\]. These techniques were applied to study of scalar meson nonet (\[60,61\]) in which the loop contributions are given by the hadronic intermediate states that each meson can access. It is via these hadronic loops that the bare states become “dressed” and the hadronic loop contributions totally dominate the dynamics of the process. A similar approach was developed by Pennington in Ref. \[68\], where they have investigated the dynamical generation of the scalar mesons by initially inserting only one “bare seed”. Also, the strangeness content of the nucleon and electromagnetic form factors were investigated in \[70\], whereas Capstick and Morel in Ref. \[71\] analyzed baryon meson loop effects on the spectrum of nonstrange baryons. In the meson sector, Eichten et al. explored the influence of the open-charm channels on the charmonium properties using the Cornell coupled-channel model \[55\] to assess departures from the single-channel potential-model expectations.

In this work we present the latest applications of the UQM to study the orbital angular momenta contribution to the spin of the proton in which the effects of the sea quarks were introduced into the CQM in a systematic way and the wave functions given explicitly. In another contribution of the same workshop (see Hugo Garcia et al.) are on the contrary discussed the flavor asymmetry and strangeness of the proton. Finally, the UQM is applied to describe
meson observables and the spectroscopy of the charmonium and bottomonium, developing the formalism to take into account in a systematic way, the continuum components.

V. THE UQM FORMALISM

In the UQM for baryons [70, 72–74] and mesons [75–78], the hadron wave function is made up of a zeroth order $qqq$ ($q\bar{q}$) configuration plus a sum over the possible higher Fock components, due to the creation of $3P_0$ $q\bar{q}$ pairs. Thus, we have

$$|\psi_A\rangle = N \left[ |A\rangle + \sum_{BC\ell J} \int d\vec{K} k^2 dk \ |BC\ell J; \vec{K}k\rangle \left\langle BC\ell J; \vec{K}k\mid T\right\rangle A \right],$$  

where $T$ stands for the $3P_0$ quark-antiquark pair-creation operator [75–78]. $A$ is the baryon/meson, $B$ and $C$ represent the intermediate state hadrons. $E_a$, $E_b$, and $E_c$ are the corresponding energies, $k$ and $\ell$ the relative radial momentum and orbital angular momentum between $B$ and $C$ and $\vec{J} = \vec{J}_b + \vec{J}_c + \ell$ is the total angular momentum. It is worthwhile noting that in Refs. [75–78], the constant pair-creation strength in the operator (12) was substituted with an effective one, to suppress unphysical heavy quark pair-creation.

The introduction of continuum effects in the CQM can thus be essential to study observables that only depend on $q\bar{q}$ sea pairs, like the strangeness content of the nucleon electromagnetic form factors [70] or the flavor asymmetry of the nucleon sea [72] it has been discussed in another contribution of the same conference (see García-Tecocoatzi et al.) . The continuum effects can give important corrections to baryon/meson observables, like the self-energy corrections to meson masses [75–78] or the importance of the orbital angular momentum in the spin of the proton [73].
VI. ORBITAL ANGULAR MOMENTA CONTRIBUTION TO PROTON SPIN IN THE UQM FORMALISM

The inclusion of the continuum higher Fock components has a dramatic effect on the spin content of the proton \[74\]. Whereas in the CQM the proton spin is carried entirely by the (valence) quarks, while in the unquenched calculation 67.6% is carried by the quark and antiquark spins and the remaining 32.4% by orbital angular momentum. The orbital angular momentum due to the relative motion of the baryon with respect to the meson accounts for 31.7% of the proton spin, whereas the orbitally excited baryons and mesons in the intermediate state only contribute 0.7%. Finally we note, that the orbital angular momentum arises almost entirely from the relative motion of the nucleon and \(\Delta\) resonance with respect to the \(\pi\)-meson in the intermediate states.

VII. SELF-ENERGY CORRECTIONS IN THE UQM

The formalism was used to compute the charmonium \((c\bar{c})\) and bottomonium \((b\bar{b})\) spectra with self-energy corrections, due to continuum coupling effects \[75–78\]. In the UQM, the physical mass of a meson,

\[
M_a = E_a + \Sigma(E_a),
\]

is given by the sum of two terms: a bare energy, \(E_a\), calculated within a potential model \[57\], and a self energy correction,

\[
\Sigma(E_a) = \sum_{BC\ell J} \int_0^\infty k^2 dk \left| \frac{\mathcal{M}_{A\to BC}(k)}{E_a - E_B - E_C} \right|^2,
\]

computed within the UQM formalism.

\[\text{FIG. 3: Charmonium spectrum with self energies corrections. Black lines are theoretical predictions and blue lines are experimental data available. Figure taken from Ref. [76]; APS copyright.}\]

\[\text{FIG. 4: Bottomonium spectrum with self energies corrections. Black lines are theoretical predictions and blue lines are experimental data available. Figure taken from Ref. [77]; APS copyright.}\]
Our results for the self energies corrections of charmonia and bottomonia spectra, are shown in figures and

In our framework the $X(3872)$ can be interpreted as a $c\bar{c}$ core [the $\chi_{c1}(2^{3}P_{1})$], plus higher Fock components due to the coupling to the meson-meson continuum. In Ref. we were the first to predict analogous states (as $X(3872)$) with strong continuum components in the bottomonium sector but in the $\chi_{b0}(3^{3}P_{1})$ sector, due to opening of threshold of $BB, B^{*}B^{*}$ and $B^{*}B^{*}$. We expect similar interesting effects near threshold also in the $N^{*}$ sector.

It is interesting to compare the present results to those of the main three-quark quark models. It is clear that a larger number of experiments and analyses, looking for missing resonances, are necessary because many aspects of hadron spectroscopy are still unclear.

The present work can be expanded to include charmed and/or bottomed baryons, which can be quite interesting in light of the recent experimental effort to study the properties of heavy hadrons.

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