Strong decays of molecular states $Z^+_c$ and $Z'_{c}^+$

Yubing Dong$^{1,2}$, Amand Faessler$^3$, Thomas Gutsche$^3$, Valery E. Lyubovitskiij$^{3*}$

$^1$ Institute of High Energy Physics, Beijing 100049, P. R. China

$^2$ Theoretical Physics Center for Science Facilities (TPCSF), CAS, Beijing 100049, P. R. China

$^3$ Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D–72076 Tübingen, Germany

(Dated: May 7, 2014)

The newly observed hidden-charm meson $Z^+_c(3900)$ and a possible partner state $Z'_{c}^+$ with quantum numbers $J^P = 1^+$ are considered as hadronic molecules composed of $\bar{D}D^*$ and $\bar{D}^*D^*$, respectively. We give predictions for the decay widths of the strong two-body transitions $Z^+_c \to H + \pi^+$ and $Z'_{c}^+ \to H + \pi^+$ with $H = \Psi(nS), h_c(mP)$ in a phenomenological Lagrangian approach.

PACS numbers: 13.25.Gv, 13.30.Eg, 14.40.Rt, 36.10.Gv
Keywords: charm mesons, hadronic molecules, strong decays

I. INTRODUCTION

Recently the three collaborations BESIII [1], Belle [2] and CLEO-c [3] reported about the observation of a new resonance $Z_c(3900)$ with a mass $(3899 \pm 3.6 \pm 4)$ MeV and a width of $(46 \pm 10 \pm 20)$ MeV [1]. The observation of this state already motivated a series of theoretical studies based on different assumptions (mainly hadronic molecular and tetraquark interpretations were discussed). Here we analyze the strong two-body decays of $Z^+_c$ and its possible partner state $Z'_{c}^+$ using a phenomenological Lagrangian approach [4]-[9] based on the compositeness condition [10]-[13], which was successfully applied for the study of hadrons and exotic states as bound states of their constituents using methods of quantum field theory.

The main idea of the compositeness condition [10]-[13] is to define the coupling strength of the field representing the bound state and their constituents from the equation $Z = 0$ [10, 11]. Here $Z$ is the wave function renormalization constant of the field describing the bound state. The quantity $Z^{1/2}$ is the matrix element between a physical particle state and the corresponding bare state. The compositeness condition $Z = 0$ enables one to represent a bound state by introducing a hadronic field interacting with its constituents so that the renormalization factor is equal to zero. This does not mean that we can solve the QCD bound state equations but we are able to show that the condition $Z = 0$ provides an effective and self-consistent way to describe the coupling of a hadron to its constituents. One starts with an phenomenological interaction Lagrangian written down in terms of the field describing bound states and their constituents. Then, by using Feynman rules, the $S$–matrix elements describing hadron-hadron interactions are given in terms of Feynman loop diagrams with constituents running in the loops. The compositeness condition enables one to avoid the problem of double counting. The approach is self-consistent and all calculations of physical observables are straightforward. There is a small set of model parameters: the values of the constituent masses and the scale parameters that define the size of the distribution of the constituents inside a given bound state.

We consider the $Z^+_c$ state as a hadronic molecule as also discussed previously and extensively among the theoretical interpretations collected in Refs. [14]. In addition we extend the considerations to a possible partner state $Z'_{c}^+$. In particular, we treat the charged hidden-charm meson resonances $Z^+_c$ and $Z'_{c}^+$ as a superposition of the molecular configurations $\bar{D}D^*$ and $\bar{D}^*D^*$ as

$$|Z^+_c\rangle = \frac{1}{\sqrt{2}}(|D^{*+}\bar{D}^0 + \bar{D}^{*0}D^+\rangle),$$
$$|Z'_{c}^+\rangle = |D^{*+}\bar{D}^{*0}\rangle. \quad (1)$$

---

* On leave of absence from Department of Physics, Tomsk State University, 634050 Tomsk, Russia
We adopt the spin and parity quantum numbers $J^P = 1^+$ for the two resonances $Z_c^+$ and $Z_c'^+$. Note the bottomia states ($Z_c^+$ and $Z_c'^+$) have been considered in our approach in Ref. [2].

In the present paper we proceed as follows. In Sec. II we briefly review the basic ideas of our approach where we set up the two new resonances $Z_c^+$ and $Z_c'^+$ as $\bar{D}D^*$ and $D^*\bar{D}^*$ molecular states. Then we proceed to consider the strong two-body decays $Z_c^+(Z_c'^+) \to H + \pi^+$ and $Z_c'(Z_c'^+) \to h_c(mP) + \pi^+$ based on a phenomenological interaction Lagrangian. In Sec. III we present the numerical results and discussion.

II. FRAMEWORK

Our approach to the $Z_c^+$ and $Z_c'^+$ states is based on interaction Lagrangians describing the coupling of the $Z_c^+$ and $Z_c'^+$ states to its constituents as

$$
\mathcal{L}_{Z_c}(x) = \frac{g_{Z_c}}{\sqrt{2}} M_{Z_c} Z_c^\mu(x) \int d^4 y \Phi_{Z_c}(y^2) \left( D(x + y/2) \bar{D}_\mu(x - y/2) + D^*_\mu(x + y/2) \bar{D}(x - y/2) \right),
$$

$$
\mathcal{L}_{Z_c'}(x) = \frac{g_{Z_c'}}{\sqrt{2}} i \epsilon_{\mu\nu\alpha\beta} \partial^\mu Z_c'^\alpha(x) \int d^4 y \Phi_{Z_c'}(y^2) D^*\alpha(x + y/2) \bar{D}_\beta(x - y/2),
$$

(2)

where $y$ is the relative Jacobi coordinate (difference of coordinates of the constituents), $g_{Z_c}$ and $g_{Z_c'}$ are the dimensionless coupling constants of $Z_c^+$ and $Z_c'^+$ to the molecular $\bar{D}D^*$ and $D^*\bar{D}^*$ components, respectively. Here $\Phi_{Z_c}(y^2)$ and $\Phi_{Z_c'}(y^2)$ are correlation functions, which describe the distributions of the constituent mesons in the bound states.

A basic requirement for the choice of an explicit form of the correlation function $\Phi_H(y^2)$ ($H = Z_c, Z_c'$) is that its Fourier transform vanishes sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. We adopt a Gaussian form for the correlation function. The Fourier transform of this vertex function is given by

$$
\tilde{\Phi}_H(p^2_E/\Lambda^2) = \exp(-p^2_E/\Lambda^2),
$$

(3)

where $p_E$ is the Euclidean Jacobi momentum. $\Lambda$ is a size parameter characterizing the distribution of the two constituent mesons in the $Z_c^+$ and $Z_c'^+$ systems, which also leads to a regularization of the ultraviolet divergences in the Feynman diagrams. For a molecular system where the binding energy is negligible in comparison with the masses of the constituents this size parameter is expected to be smaller than 1 GeV. From our previous analyses of the strong two-body decays of the $X, Y, Z$ meson resonances and of the $\Lambda_c(2940)$ and $\Sigma_c(2880)$ baryon states we deduced a value of maximally $\Lambda \sim 1$ GeV [3]. For a very loosely bound system like the $X(3872)$ a size parameter of $\Lambda \sim 0.5$ GeV [4] is more suitable. For heavy compact states such as tetraquark states, charmonia or a possible charmonium component in the $X(3872)$ the size parameter $\Lambda$ is typically much larger (for example in a range from 2.5 to 3.5 GeV as discussed in Ref. [5]). Here we choose values for $\Lambda$ in the range 0.5-0.75 GeV which reflect a weakly bound heavy meson system.

Once $\Lambda$ is fixed the coupling constants $g_{Z_c}$ and $g_{Z_c'}$ are then determined by the compositeness condition [4]-[13]. It implies that the renormalization constant of the hadron wave function is set equal to zero with:

$$
Z_H = 1 - \Sigma_H'(M_H^2) = 0.
$$

(4)

Here, $\Sigma_H'$ is the derivative of the transverse part of the mass operator $\Sigma_H^{\mu\nu}$ of the molecular states (see Fig.1), which is defined as

$$
\Sigma_H^{\mu\nu}(p) = g_L^{\mu\nu} \Sigma_H(p) + \frac{p^\mu p^\nu}{p^2} \Sigma_H^L(p), \quad g_L^{\mu\nu} = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}.
$$

(5)

Analytical expressions for the couplings $g_{Z_c}$ and $g_{Z_c'}$ are given in Appendix A. In the calculation the masses of $Z_c$ and $Z_c'$ are expressed in terms of the constituent masses and the binding energy $\epsilon$ (a variable quantity in our calculations):

$$
M_{Z_c} = M_D + M_{D^*} - \epsilon, \quad M_{Z_c'} = 2M_{D^*} - \epsilon,
$$

(6)

where $\epsilon$ is the binding energy.

In the calculation of the two-body decays of $Z_c^+(Z_c'^+) \to H + \pi^+$ where $H = \Psi(nS), h_c(mP)$ we generate the four-particle $DD^*H\pi^+$ and $D^*\bar{D}^*H\pi^+$ vertices by a phenomenological Lagrangian

$$
\mathcal{L}_{DDH\pi}(x) = ig_F \text{tr} \left( \bar{D}(x) [\mathcal{H}(x), P(x)] D(x) \right) + g_D \text{tr} \left( \bar{D}(x) \{ \mathcal{H}(x), P(x) \} D(x) \right),
$$

(7)
where \( g_F \) and \( g_D \) are effective coupling constants, \([\ldots]\) and \( \{\ldots\} \) denote the commutator and anticommutator, respectively.

The \( H \) is the heavy charmonia field; \( D \) is the superposition of isodoublets of open-charm mesons with \( J^P = 0^-, 1^- \) and \( 1^+ \); \( \mathcal{P} \) is the chiral field:

\[
\mathcal{H} = J^\mu \gamma_\mu + h^\mu \gamma_\mu \gamma_5 + \frac{g_H}{M_H} \left( J^\mu \sigma_{\mu\nu} + h^{\mu
u} \sigma_{\mu\nu} \gamma_5 \right), \\
\mathcal{D} = \partial_\mu \gamma_5 + D^\mu \gamma_\mu + D_\mu^1 \gamma_\mu \gamma_5, \\
\mathcal{P} = \frac{1}{2} \tilde{u} \gamma^5 + \frac{1}{2} [u^\dagger, \gamma \mu] u^\mu,
\]

where \( J \) and \( h \) denote the \( \Psi \) and \( h_c \) states; \( V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu \) is the stress tensor of the \( \Psi \) and \( h_c \) states; \( g_H \) is a phenomenological coupling defining the mixing of derivative and nonderivative terms in \( \mathcal{H} \); \( M_H \approx M_J \) is associated with the \( \J/\Psi \) mass; \( D = (D^+, D^0), \ D^* = (D^{*+}, D^{*0}) \) are the doublets of pseudoscalar and vector charmed \( D \) mesons; \( u_\mu \) is the chiral vielbein:

\[
u = i \{u^\dagger, \gamma \mu \}, \quad u^2 = U = \exp \left[ i \frac{\hat{\pi}}{F_\pi} \right], \quad \hat{\pi} = \tilde{\pi} \pi
\]

where \( F_\pi = 92.4 \text{ MeV} \) is the pion decay constant, \( \tilde{\pi} = (\pi_1, \pi_2, \pi_3) \) is the triplet of pions. Note the couplings \( g_D, g_F \) and \( g_H \) are phenomenological parameters. Below we show that we have two constraints on these couplings.

From Eq. 8 we deduce specific Lagrangians describing the couplings between heavy charmonia, charmed mesons and the pion which are relevant for the decays of the \( Z_c \) and \( Z'_c \) states:

\[
\mathcal{L}_{D^* D \pi}(x) = \frac{8g_F g_H}{F_\pi M_J} J^\mu(x) \tilde{D}^\nu_\mu(x) \partial_\nu \hat{\pi}(x) D(x) + \text{H.c.}, \\
\mathcal{L}_{D^* D^* D \pi}(x) = \frac{4g_D}{F_\pi} \varepsilon^{\mu
u\alpha\beta} J_\mu(x) i \partial_\nu \hat{\pi}(x) D_\alpha^* D_\beta^*(x), \\
\mathcal{L}_{D^* D^* h_c \pi}(x) = \frac{4g_F g_H}{F_\pi M_J} J^\mu(x) \partial_\mu \hat{\pi} D_\pi(x) + \text{H.c.}, \\
\mathcal{L}_{D^* D^* D^* h_c \pi}(x) = \frac{4g_F}{F_\pi} \tilde{D}^\nu_\mu(x) (h_{\mu\nu}(x) i \partial_\nu \hat{\pi}(x) - h_\mu(x) i \partial_\nu \hat{\pi}(x)) D^*_{\nu\mu}(x).
\]

The three-particle coupling \( g_{D^* D \pi} \) of the pion to charmed \( D \) mesons is defined by the phenomenological Lagrangian:

\[
\mathcal{L}_{D^* D^* D \pi}(x) = \frac{g_{D^* D^* D \pi}}{\sqrt{2}} \tilde{D}^* \nu\mu(x) \partial_\mu \hat{\pi}(x) D(x) + \text{H.c.},
\]

where the value \( g_{D^* D^* D \pi} = 17.9 \) has been determined from data on \( D^* \rightarrow D \pi \) decay [13]. The coupling \( D^* D^* D \pi \) has been calculated e.g. using QCD sum rules, first in Ref. [16] and it was updated in several papers. One of the latest estimates is given in Ref. [17]. The first estimate of this coupling from lattice QCD was done in Ref. [18] and updated in Ref. [19].

Next we discuss how we fix the couplings \( g_D, g_F \) and \( g_H \). As mentioned before these parameters can be further constrained. In particular, we can relate the coupling \( g_D \) and the product \( g_F g_H \) to the four-particle couplings \( g_{D^* D J^\pi} \) and \( g_{D^* D J^\pi} \) which appear in the phenomenological Lagrangian proposed in Ref. [20] for the analysis of \( J/\Psi \) absorption in hadronic matter (see details in Appendix B). Note, that coupling of \( D \) mesons with pion and \( J/\Psi \) were calculated also in Ref. [21].

Matching of the coupling constants leads to

\[
g_{D^* D J^\pi} = \frac{9g_D D^* D \pi}{2 \sqrt{2}} \simeq \frac{8g_F g_H}{F_\pi M_J \sqrt{6}} (M_{Z_c}^2 - M_J^2) \left( 1 + \frac{M_J^2}{2M_{Z_c}^2} \right), \\
g_{D^* D^* D \pi} = \frac{2g_D}{F_\pi} \frac{M_{Z_c}^2 - M_J^2}{M_{Z_c}^2}.
\]

The four-particle couplings \( g_{D^* D^* J^\pi} \) and \( g_{D^* D J^\pi} \) were expressed in Ref. [20] in a factorization of two three-particle couplings

\[
g_{D^* D J^\pi} = 2 \sqrt{M_D M_D^*} \ g_{D^* D \pi} = \frac{9g_D D^* D \pi}{2 \sqrt{2}}.
\]
Therefore, in the numerical evaluation we use an approximate condition \( g_D \simeq g_F \) and \( g_H \) is fixed by the condition (following the discussion above)

\[
g_H \simeq g_{JDD} g_D d \frac{F_F M_{f} \sqrt{3}}{16 g_F (M_{Z_c}^2 - M_{f}^2)} \left(1 + \frac{M_{f}^2}{2M_{Z_c}^2}\right)^{-1}.
\] (19)

### III. NUMERICAL RESULTS

With the phenomenological Lagrangians introduced and discussed we can proceed to determine the widths of the two-body decays \( Z_c^+ (Z_c^+) \to \Psi(nS) + \pi^+ \) and \( Z_c^+ (Z_c^+) \to h_c(mP) + \pi^+ \). The relevant diagrams are indicated in Fig.2. The standard evaluation leads to the corresponding decay widths:

\[
\begin{align*}
\Gamma_{Z_c^+ \to \Psi(nS)\pi^+} & \simeq \frac{g_{Z_c^+\Psi(nS)\pi}}{96\pi M_{Z_c}^2} \lambda^{3/2}(M_{Z_c}^2, M_{\Psi(nS)}^2, M_{\pi}^2) \left(1 + \frac{M_{\Psi(nS)}^2}{2M_{Z_c}^2}\right), \\
\Gamma_{Z_c'^+ \to \Psi(nS)\pi^+} & \simeq \frac{g_{Z_c'^+\Psi(nS)\pi}}{96\pi M_{Z_c}^2} \lambda^{3/2}(M_{Z_c}^2, M_{\Psi(nS)}^2, M_{\pi}^2) \left(1 + \frac{M_{\Psi(nS)}^2 - M_{\Psi(nS)}^2}{3M_{Z_c}^2}\right), \\
\Gamma_{Z_c^+ \to h_c(mP)\pi^+} & \simeq \frac{g_{Z_c^+h_c(mP)\pi}}{96\pi M_{Z_c}^2} \lambda^{3/2}(M_{Z_c}^2, M_{h_c}^2, M_{\pi}^2) \left(1 + \frac{M_{h_c}^2}{2M_{Z_c}^2}\right), \\
\Gamma_{Z_c'^+ \to h_c(mP)\pi^+} & \simeq \frac{g_{Z_c'^+h_c(mP)\pi}}{96\pi M_{Z_c}^2} \lambda^{3/2}(M_{Z_c}^2, M_{h_c}^2, M_{\pi}^2),
\end{align*}
\] (20)

where \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \) is the Källen function. The decay coupling constants \( g_{Z_c\Psi(nS)\pi} \), \( g_{Z_c'\Psi(nS)\pi} \), \( g_{Z_c^+h_c(mP)\pi} \) and \( g_{Z_c'^+h_c(mP)\pi} \) are expressed by

\[
\begin{align*}
g_{Z_c\Psi(nS)\pi} &= g_{Z_c^+h_c(mP)\pi} = 8 g_{Z_c} \frac{g_F g_H}{F_F M_{f}^2} J_1 M_{Z_c}, \\
g_{Z_c'^+\Psi(nS)\pi} &= g_{Z_c'^+h_c(mP)\pi} \frac{g_D}{g_F} \sqrt{\frac{3}{2}} = 4 \sqrt{\frac{3}{2}} g_{Z_c'} \frac{g_D}{F_F} J_2,
\end{align*}
\] (21)

where \( g_{Z_c} \), \( g_{Z_c'} \) and the loop integrals \( J_1 \) and \( J_2 \) are given in Appendix A. We present our results in Tables I-III. In Table I we display the predictions for the phenomenological couplings \( g_{Z_c\Psi(nS)} \) and \( g_{Z_c'h_c(mP)} \) as defined in Eq. (21) for different values of the binding energy \( \epsilon \) and \( \Lambda \) varied from 0.5 to 0.75 GeV. For convenience, in Table II we present the values for the Källen functions in order to explain the results for the widths given in Table III. The decay rates for the \( Z_c^+ \) states are larger than the corresponding ones of the \( Z_c^+ \) resonances, the decay hierarchies with \( \Gamma(1S) > \Gamma(2S) > \Gamma(1P) \) are identical for both states. An increase of the size parameter \( \Lambda \), more suitable for a compact bound state, would also lead to a sizable increase in the decay rates. Therefore, if experiment will deliver larger values for the rates than predicted in our approach, it would signal that \( Z_c^+ \) and \( Z_c'^+ \) are probably not molecular states.

In summary, using a phenomenological Lagrangian approach we give predictions for the two-body decay rates of the \( Z_c^+ \) and \( Z_c'^+ \) states interpreted as hadronic molecules. The results could be useful for forthcoming measurements of these decay modes. In future we plan to consider radiative and other strong decays of the \( Z_c^+ \) and \( Z_c'^+ \) mesons.

**Acknowledgments**

This work is supported by the DFG under Contract No. LY 114/2-1, National Sciences Foundations of China No.10975146 and 11035006, and by the DFG and the NSFC through funds provided to the sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD”. The work is done partially under the project 2.3684.2011 of Tomsk State University. One of us (YBD) thanks the Institute of Theoretical Physics, University of Tübingen for the warm hospitality and thanks the support from the Alexander von Humboldt Foundation. Discussions with Dr. D. Y. Chen are appreciated.
Appendix A: Matching of the coupling constants $g_D$, $g_F$ and $g_H$

The idea for matching the coupling constants $g_D$, $g_F$ and $g_H$ is based on the equivalence of matrix elements squared (or decay rates) calculated in different approaches — from one side using phenomenological Lagrangians \cite{12} and from other side the Lagrangians proposed in Ref. \cite{20}.

\begin{align}
    \mathcal{L}_{\pi J}(x) &= g_{D^*D^*J}(x) \bar{D}^{*\mu}(x) \pi^\mu(x) \cdot \bar{\tau} D(x) + \text{H.c.}, \\
    \mathcal{L}_{D^*D^*J}(x) &= i \epsilon_{\mu\nu\alpha\beta} g_{D^*D^*J}(x) \left( J^{\mu}(x) \bar{D}^{*\nu}(x) \partial_\nu \bar{\pi}(x) \cdot D^{*\alpha}(x) \\
    &\quad + \partial_\nu J^{\nu}(x) \bar{D}^{*\beta}(x) \bar{\pi}(x) \cdot D^{*\alpha}(x) \right). 
\end{align}

Evaluating the matrix elements squared and averaging over the polarizations of the particle spins we get in case of the $Z_c \to J/\psi + \pi$ transition using the Lagrangian of Ref. \cite{20}

\begin{align}
    \sum_{\text{pol}} |M_{\text{inv}}|^2 &= g_{D^*D^*J}(x) \left( 3 - \frac{M_\pi^2}{M_Z^2} + \frac{(p_1 p_2)^2}{M_Z^2 M_J^2} \right) = g_{D^*D^*J}(x) \left( 3 + \frac{\lambda(M_Z^2, M_\pi^2, M_J^2)^2}{4M_Z^2 M_J^2} \right) \approx 3 g_{D^*D^*J}. \tag{A3}
\end{align}

Based on our Lagrangians we have for the same averaged matrix element squared

\begin{align}
    \sum_{\text{pol}} |M_{\text{inv}}|^2 &= \left( \frac{8 g_{FgH}}{F_\pi M_J} \right)^2 \left( M_Z^2 M_J^2 \left( 1 - \frac{M_\pi^2}{M_Z^2} \right) + 2(p_1 p_2)^2 \left( 1 + \frac{M_J^2}{2M_Z^2} \right) \right) \\
    &= \left( \frac{8 g_{FgH}}{F_\pi M_J} \right)^2 \left( \frac{\lambda(M_Z^2, M_\pi^2, M_J^2)}{2} \left( 1 + \frac{M_J^2}{2M_Z^2} \right) + 3M_\pi^2 M_J^2 \right) \\
    &\approx \left( \frac{8 g_{FgH}}{F_\pi M_J} \right)^2 \left( \frac{M_Z^2 - M_\pi^2}{2} \left( 1 + \frac{M_J^2}{2M_Z^2} \right) \right). \tag{A4}
\end{align}

In above expressions we neglect the pion mass and drop the loop integral, which is the same in both approaches.

Matching the expressions (A3) and (A4) we derive the constraint on the product of the couplings $g_{FgH}$

\begin{align}
    g_{D^*D^*J} = \frac{g_{JDD} g_{D^*D^*J}}{2 \sqrt{2}} \approx \frac{8 g_{FgH}}{F_\pi M_J \sqrt{6}} (M_Z^2 - M_\pi^2) \sqrt{1 + \frac{M_J^2}{2M_Z^2}}. \tag{A5}
\end{align}

Here we use the framework of Ref. \cite{20} in that the $g_{D^*D^*J}$ coupling is expressed through the product of the $g_{JDD}$ and $g_{D^*D^*}$ couplings.

In complete analogy we derive the relation between the coupling constants $g_{D^*D^*J}$ and $g_D$ considering the mode $Z_c' \to J/\psi + \pi$:

\begin{align}
    g_{D^*D^*J} = \frac{g_{D^*D^*J}}{2 \sqrt{M_D M_J}} = \frac{g_{JDD} g_{D^*D^*J}}{2 \sqrt{2}} = \frac{2g_D M_{Z_c'} - M_J^2}{F_\pi M_{Z_c'}^2} \tag{A6}
\end{align}

with $g_{JDD} = 6.5$ fixed in \cite{5}, which is a universal constant for all radially-excited $J(nS)$ states. One can get a more accurate estimate for these couplings. We consider the Lagrangian

\begin{align}
    \mathcal{L}_{JDD}(x) = g_{JDD} J_\mu(x) \bar{D}(x) i \partial^\mu D(x) + \text{H.c.} \tag{A7}
\end{align}

The coupling constant $g_{J(nS)DD}$ is given by

\begin{align}
    g_{J(nS)DD} = M_{J(nS)} \tag{A8}
\end{align}

where $f_{J(nS)}$ is determined from the leptonic decays of the $J(nS)$ states as

\begin{align}
    \Gamma(\Psi(nS) \to e^+ e^-) = \frac{16 \pi \alpha_{\text{em}}^2}{27} \frac{f_{J(nS)}^2}{M_{J(nS)}}. \tag{A9}
\end{align}
and \( \alpha_{\text{em}} = 1/137.036 \) is the fine-structure constant. The relation (A8) is the analogue to the \( \rho \)-meson universality

\[
g_{\rho\pi\pi} = \frac{M_\rho}{f_\rho} = \frac{1}{g_{\rho\gamma}} \tag{A10}
\]

extended to the heavy quark sector in Ref. [22], where \( g_{\rho\gamma} \) is the \( \rho \to \gamma \) transition coupling.

For the last couplings we get \( f_{J(1S)} = 416.4 \) MeV, \( f_{J(2S)} = 295.6 \) MeV, \( f_{J(3S)} = 187.2 \) MeV, where we used the mass values \( M_{J(1s,2s,3s)} = 3096.92 \pm 0.011 \) MeV, 3686.11 \( \pm 0.012 \) MeV and 4039.6 \( \pm 4.3 \) MeV as well as the results for the leptonic decay widths of the \( J(nS) \) states

\[
\begin{align*}
\Gamma(\Psi(1S) \to e^+e^-) &= 5.55 \pm 0.14 \pm 0.02 \text{ keV}, \\
\Gamma(\Psi(2S) \to e^+e^-) &= 2.35 \pm 0.04 \text{ keV}, \\
\Gamma(\Psi(3S) \to e^+e^-) &= 0.86 \pm 0.07 \text{ keV}. 
\end{align*}
\tag{A11}
\]

Note that we explicitly take into account the \( M_{J(nS)} \) dependence of the \( f_{J(nS)} \) and \( g_{JDD} \) couplings. Finally, for the set of \( g_{J(nS)DD} \) couplings we get: \( g_{J(1S)DD} = 7.44, g_{J(1S)DD} = 12.47, g_{J(3S)DD} = 21.58. \)

### Appendix B: Coupling constants and structure integrals

The expressions for the coupling constants \( g_{zz}, g_{zz}' \) and structure integrals \( J_1, J_2 \) are

\[
g_{zz}^2 = \frac{M_{Z_2}}{32\pi^2\Lambda^2} \int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_1} \left( \alpha_1 + 2\alpha_1 \alpha_2 \right) \frac{1 + \frac{\Lambda^2}{M_{Z_2}^2, \Delta_1}}{\Delta_1},
\]

\[
g_{zz}'^2 = \frac{M_{Z_2}}{16\pi^2\Lambda^2} \int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_1} \left( \frac{\Lambda^2}{M_{Z_2}^2} + \frac{\alpha_1 + 2\alpha_1 \alpha_2}{2\Delta_1} \right) \frac{1 + \frac{\Lambda^2}{M_{Z_2}^2, \Delta_1}}{M_{Z_2}},
\]

\[
J_1 = \frac{1}{8\pi^2} \int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_2} \left( 1 + \frac{\Lambda^2}{M_{Z_2}^2, \Delta_2} \right) \frac{M_{Z_2}^2, \alpha_1 + M_{Z_2}^2 \alpha_2 + \frac{\alpha_1 + 2\alpha_1 \alpha_2}{M_{Z_2}^2, \Delta_2}}{2\Delta_2},
\]

\[
J_2 = \frac{1}{8\pi^2} \int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_2} \left( 1 + \frac{\Lambda^2}{M_{Z_2}^2, \Delta_2} \right) \frac{M_{Z_2}^2, \alpha_1 + M_{Z_2}^2 \alpha_2 + \frac{\alpha_1 + 2\alpha_1 \alpha_2}{M_{Z_2}^2, \Delta_2}}{4\Delta_2},
\]

where

\[
\Delta_1 = 2 + \alpha_{12}, \quad \Delta_2 = 1 + \alpha_{12}, \quad \alpha_{12} = \alpha_1 + \alpha_2. \tag{B5}
\]

[1] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 110, 252001 (2013) [arXiv:1303.5949 [hep-ex]].

[2] Z. Q. Liu et al. [Belle Collaboration], Phys. Rev. Lett. 110, 252002 (2013) [arXiv:1304.0121 [hep-ex]].
FIG. 1: Mass operators of $Z_c^+$ and $Z_c'^+$.  

FIG. 2: Two-body decays $Z_c^+ \rightarrow \Psi(nS), h_c(mP) + \pi$ and $Z_c'^+ \rightarrow \Psi(nS), h_c(mP) + \pi$.  

Table I. Phenomenological couplings $g_{Z_c H\pi}$ and $g_{Z_c' H\pi}$ in GeV$^{-1}$.  

| $\epsilon$ (MeV) | $g_{Z_c, \Psi(1S)\pi}$ | $g_{Z_c', \Psi(1S)\pi}$ | $g_{Z_c, \Psi(2S)\pi}$ | $g_{Z_c', \Psi(2S)\pi}$ | $g_{Z_c, h_c(1P)\pi}$ | $g_{Z_c', h_c(1P)\pi}$ |
|------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 5                | 0.81-1.10      | 0.83-1.26      | 4.78-6.47      | 3.60-6.47      | 0.81-1.10      | 0.68-1.03      |
| 10               | 0.88-1.31      | 0.96-1.44      | 5.27-7.89      | 4.23-7.89      | 0.88-1.31      | 0.79-1.18      |
| 15               | 0.94-1.41      | 1.05-1.58      | 5.75-8.66      | 4.65-8.66      | 0.94-1.41      | 0.86-1.29      |
| 20               | 0.99-1.49      | 1.10-1.68      | 6.20-9.38      | 4.95-9.39      | 0.99-1.49      | 0.90-1.37      |

Table II. Values of Källen functions for different binding energies in GeV$^4$ for $M_{\pi} \equiv M_{\pi^+}$.  

| $\epsilon$ (MeV) | $\lambda_{Z_c, \Psi(1S)\pi}$ | $\lambda_{Z_c', \Psi(1S)\pi}$ | $\lambda_{Z_c, \Psi(2S)\pi}$ | $\lambda_{Z_c', \Psi(2S)\pi}$ | $\lambda_{Z_c, h_c(1P)\pi}$ | $\lambda_{Z_c', h_c(1P)\pi}$ |
|------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 5                | 28.183         | 40.895         | 0.741          | 4.959          | 5.275          | 12.074         |
| 10               | 27.767         | 40.895         | 0.638          | 4.765          | 5.084          | 11.786         |
| 15               | 27.358         | 40.895         | 0.540          | 4.573          | 4.895          | 11.502         |

Table III. Predictions for the strong decay widths of $Z_c^+$ and $Z_c'^+$ states in MeV.  

| $\epsilon$ (MeV) | $\Gamma_{Z_c}(1S)$ | $\Gamma_{Z_c}(1S)$ | $\Gamma_{Z_c}(2S)$ | $\Gamma_{Z_c}(2S)$ | $\Gamma_{Z_c}(1P)$ | $\Gamma_{Z_c}(1P)$ |
|------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 5                | 7.45-13.63     | 11.50-26.60    | 1.47-2.70       | 8.26-19.1      | 0.68-1.25      | 1.02-2.36      |
| 10               | 8.53-19.15     | 15.33-34.39    | 1.48-3.32       | 10.81-24.23    | 0.76-1.70      | 1.34-3.00      |
| 15               | 9.55-21.66     | 17.85-40.64    | 1.41-3.21       | 12.32-28.06    | 0.82-1.86      | 1.53-3.49      |
| 20               | 10.43-23.89    | 19.47-45.11    | 1.28-2.94       | 13.16-30.48    | 0.87-1.98      | 1.65-3.81      |