Radio haloes from simulations and hadronic models – II. The scaling relations of radio haloes

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ABSTRACT
We use results from a constrained, cosmological magnetohydrodynamic (MHD) simulation of the Local Universe to predict radio haloes and their evolution for a volume-limited set of galaxy clusters and compare to current observations. The simulated magnetic field inside the clusters is a result of turbulent amplification within them, with the magnetic seed originating from starburst-driven, galactic outflows. We evaluate three models, where we choose different normalizations for the cosmic ray proton population within clusters. Similar to our previous analysis of the Coma cluster, the radial profile and the morphological properties of observed radio haloes cannot be reproduced, even with a radially increasing energy fraction within the cosmic ray proton population. Scaling relations between X-ray luminosity and radio power can be reproduced by all models; however all models fail in the prediction of clusters with no radio emission. Also the evolutionary tracks of our largest clusters in all models fail to reproduce the observed bi-modality in radio luminosity. This provides additional evidence that the framework of hadronic, secondary models is disfavoured to reproduce the large-scale diffuse radio emission of galaxy clusters. We also provide predictions for the unavoidable emission of γ-rays from the hadronic models for the full cluster set. None of such secondary models is yet excluded by the observed limits in γ-ray emission, emphasizing that large-scale diffuse radio emission is a powerful tool to constrain the amount of cosmic ray protons in galaxy clusters.

Key words: MHD – methods: numerical – galaxies: clusters: intracluster medium.

1 INTRODUCTION

The thermal gas, that is the dominant component in the Inter-Galactic Medium (IGM), is mixed with magnetic fields and relativistic particles, as proven by radio observations which detected Mpc-sized diffuse radio emission from the IGM, in the form of radio haloes and relics (e.g. Feretti 2003; Ferrari et al. 2008). These Mpc-scale radio sources are found in a fraction of massive clusters with complex dynamics, which suggests a connection between non-thermal emission and cluster mergers (e.g. Buote 2001; Venturi et al. 2008; Brunetti et al. 2009). Cluster mergers are the most energetic events in the Universe and a fraction of the energy dissipated during these mergers may be channelled into the amplification of the magnetic fields (e.g. Dolag, Bartelmann & Lesch 2002; Subramanian, Shukurov & Haagen 2006; Ryu et al. 2008) and into the acceleration of relativistic, primary, electrons and protons via shocks and turbulence (e.g. Ensslin et al. 1998; Sarazin 1999; Brunetti et al. 2001, 2004; Brunetti 2004; Petroisian 2001; Gabici & Blasi 2003; Ryu et al. 2003; Cassano & Brunetti 2005; Pfrommer et al. 2006; Brunetti & Lazarian 2007; Vazza et al. 2009).

Relativistic protons in the IGM have long lifetimes and remain confined within galaxy clusters for a Hubble time (e.g. Völk, Aharonian & Breitschwerdt 1996; Berezinsky, Blasi & Ptuskin 1997). As a consequence they are expected to be the dominant non-thermal particle component. Collisions between these relativistic protons and the thermal protons in the IGM generate secondary particles that combined with the primary relativistic particles are expected to produce a complex emission spectrum from radio to γ-rays (e.g. Blasi 2001; Brunetti 2009). Only upper limits to the γ-ray emission from galaxy clusters have been obtained so far (Reimer et al. 2003; Perkins & the VERITAS Collaboration 2006; Aharonian 2009a,b; The MAGIC Collaboration, Aleksic et al. 2009); however the Fermi Gamma-ray telescope will shortly allow a step forward, having a chance to obtain first detections of galaxy clusters or to put stringent constraints on the energy density of the relativistic protons. Most importantly, in a few years the Low Frequency Array (LOFAR) and the Long Wavelength Array (LWA) will observe galaxy clusters at low radio frequencies with the potential to discover the bulk of the cluster-scale synchrotron...
emission in the Universe (e.g. Enßlin & Röttgering 2002; Cassano, Brunetti & Setti 2006; Cassano et al. 2010).

The emerging theoretical picture is very complex and modern numerical simulations provide an efficient way to obtain detailed models of non-thermal emission from galaxy clusters to compare with present and future observations. Advances in this respect have been recently made by including aspects of cosmic ray physics into cosmological Lagrangian hydrodynamical simulations mostly focusing on the acceleration of relativistic particles at shocks and on the relative production of secondary electrons (e.g. Frommer, Enßlin & Springel 2008). In this work, we investigate the non-thermal emission from secondary particles in galaxy clusters extracted from cosmological Lagrangian magnetohydrodynamic (MHD) simulations and, most important for the first time, we report on an adequate comparison between our expectations and observations.

2 SIMULATIONS

The simulation was done using the cosmological simulation code GADGET-2 (Springel 2005) with a treatment for magnetic fields. It features an entropy conserving formulation of smooth particle hydrodynamics (SPH) (Springel & Hernquist 2002), which is supplemented with the formulation of ideal MHD presented in Dolag & Stasyszyn (2009). The implementation follows the induction equation and computes the back reaction of the magnetic field using a symmetric formulation of the Lorentz force. We used a divergence cleaning scheme presented in Børve, Omang & Trulsen (2001), which reduces numerical noise in shocks by subtracting the magnetic force which is proportional to the divergence of the field. It also helps to suppress the clumping instability particle-based MHD codes encounter in regions with small plasma β (i.e. where magnetic pressure considerably exceeds thermal pressure).

In non-radiative simulations like ours, regions with small plasma β are rare. Only the strong shocks in cores of galaxy clusters during major mergers produce enough compression to amplify the field to become dynamically dominant. These mergers are relatively brief events and are handled more accurately with our new numerical treatment (see Dolag & Stasyszyn 2009, for details).

Borgani et al. (2006) have shown that non-radiative simulations overpredict the gas density in cores of galaxy clusters. This affects our simulation as well and can be seen in density and magnetic field profiles as well as in X-ray luminosities. As cosmological MHD SPH simulations lack physical dissipation, radiative SPH MHD simulations are not feasible at the moment. On the other hand, secondary models have difficulties reproducing the outer parts of radio haloes correctly. Therefore, our main focus lies on these regions, where the simulations are not affected by the overpredicted gas density.

2.1 Initial conditions

We used a constrained realization of the local universe [see Dolag et al. (2005) and references therein]. The initial conditions are similar to those used in Mathis et al. (2002) to study the formation of the local galaxy population. They were obtained on the basis of the IRAS 1.2 Jy galaxy survey. Its density field was smoothed on a scale of 7 Mpc, evolved back in time to z = 50 using the Zeldovich approximation and assumed to be Gaussian (Hoffman & Ribak 1991).

The IRAS observations constrain a volume of ≈115 Mpc centered on the Milky Way. It was sampled with dark matter particles and embedded in a periodic box of ≈343 Mpc comoving. Outside of the inner region, the box is filled with dark matter particles with six times the mass (i.e. 1/6th of the resolution), to cover for long range gravitational tidal forces arising from the low-frequency constrains.

In the evolved density field, many locally observed galaxy clusters can be identified by position and mass. Especially, the Coma cluster (see Donnert et al. 2010) shows remarkable similarities in morphology. A fly-through of the simulation can be downloaded from the MPA web site.1

The initial conditions were extended to include gas by splitting dark matter particles in the high-resolution region into gas and dark matter particles of masses 0.69 × 10^5 M⊙ and 4.4 × 10^5 M⊙, respectively. Therefore, the biggest clusters are resolved by about a million particles. The gravitational softening length was set to 10 kpc. This is comparable to the inter-particle separation found in the centre of the largest clusters.

2.2 Magnetic fields from galactic outflows

The origins of magnetic fields in galaxy clusters are still under debate. It is assumed that some kind of early seed magnetic field is amplified by structure formation through adiabatic compression, turbulence and shear flows to values observed today (∼1–10 µG in clusters). Three main classes of models for the seed field exist: at first, the seed fields can be created in shocks through the ‘Biermann battery’ (Kulsrud et al. 1997; Ryu, Kang & Biermann 1998; Miniati et al. 2001). A second class of models invokes primordial processes to predict seed fields that fill the entire volume of the universe. The coherence length of these fields strongly depends on the details of the model (see Grasso & Rubinstein 2001, for a review). Finally, the seed can be produced by AGN (Enßlin et al. 1997; Furlanetto & Loeb 2001) or starbursting galaxies (Völk & Atayan 2000) at high redshift (z ∼ 4–6), whose outflows contaminate the proto-cluster region.

Cosmological simulations using SPH (Dolag, Bartelmann & Lesch 1999; Dolag et al. 2002, 2005) and grid-based ADAPTIVE MESH REFINEMENT (AMR) codes (Brüggen et al. 2005; Dubois & Teyssier 2008; Li, Li & Cen 2008) were able to show that observed Faraday rotations are compatible with a cosmological seed field of ≃10⁻¹¹ G. They also suggest that spatial distribution and structure of cluster magnetic fields are determined by the dynamics in the velocity field caused by structure formation (Dolag et al. 1999, 2002).

For this work, we follow Donnert et al. (2009) in terms of magnetic field origin. They use a semi-analytic model for galactic winds (Bertone, Vogt & Enßlin 2006) to seed magnetic fields in a constrained cosmological MHD SPH simulation. The continuous seeding process is approximated with an instantaneous seed at z ≈ 4. As they were able to show, the main properties of magnetic fields obtained in clusters were not influenced by that approximation.

The wind model used assumes adiabatic expansion of a spherical gas bubble with homogeneous magnetic energy density around every galaxy below a certain mass threshold. The magnetic bubble can be characterized by radius and field strength. The galaxy injects gas into the bubble carrying frozen-in magnetic field from the disc into the bubble over the starburst time-scale. Its final size is determined by the wind velocity, which is a function of the star formation rate and the properties of the ISM. Bertone et al. (2006) give an evolution equation for the magnetic energy in the bubble depending on the starburst time-scale. The energy is converted into a dipole

1 http://www.mpa-garching.mpg.de/galform/data_vis/index.shtml#movie12
moment and seeded once at a chosen redshift. The magnetic field is then amplified by structure formation to μG level. For details on the wind model refer to Bertone et al. (2006), Donnert et al. (2009). Fig. 1 shows full sky maps produced from the simulation, projecting the electron density, temperature and the magnetic field (see Fig. 2). The magnetic field closely follows the density distribution. Fig. 3 shows X-ray, radio and γ-ray surface brightness. The magnetic field is more patchy in the filaments compared to a cosmological seed because the seeding by individual galaxies does not overlap.

The simulation used in this work is based on the 0.1 Dipole parameter set from Donnert et al. (2009), shown in Table 1. It represents the best fit to observations of the Faraday rotation presented in Donnert et al. (2009). There the parameter space was explored based on observations from the wind in M82. It was demonstrated that the resulting magnetic field does not critically depend on the exact choice of the parameters.

3 MODELLING HADRONIC SECONDARY ELECTRONS IN CLUSTERS

In secondary models proposed for the origin of radio haloes (Dennison 1980; Blasi & Colafrancesco 1999), the emitting cosmic ray electrons (CRE) are a product of cosmic ray proton (CRP) collisions with thermal protons in the cluster. Possible mechanisms for the injection of CRPs in the IGM include shock waves caused by cluster accretion and mergers (e.g. Gabici & Blasi 2003; Ryu et al. 2003; Pfrommer et al. 2006; Vazza et al. 2009), outflows from radio galaxies (AGN) (e.g. Ensslin et al. 1998; Rachen 2008) and supernova-driven winds (Völk et al. 1996). The resulting CRp spectrum from these theoretical scenarios is a power law in particle momentum (e.g. Schlickeiser 2002), that is also consistent with observations of galactic cosmic rays.

We assume a power-law spectrum of CRp in the IGM and calculate the spectrum of high-energy secondary electrons under stationary conditions by considering only synchrotron and inverse-Compton losses and by neglecting re-acceleration processes in the IGM (Section 3.1).

Throughout the first two of our models, we use the magnetic fields obtained from the constrained cosmological simulation described in Section 2.2. In a third model we increased the magnetic field, especially at larger distances to the centre, by up-scaling the magnetic field in our simulation to $B \propto \sqrt{\rho}$, which allows for better comparison with results obtained in previous literature, as such a relation is often used there (as e.g. in Pfrommer et al. 2008).

3.1 Synchrotron emission from secondary models

We consider a spectral distribution of the population of relativistic CRp described by a power law in energy $E_p$:

$$\dot{N}(E_p) = K_p E_p^{-\alpha_p};$$

(1)

if not stated otherwise, we use a spectral index $\alpha_p = 2.6$. The normalization $K_p$ is model dependent and is defined in the following sections.

The main channel of hadronic interaction between the CR protons and the ambient medium is multi-pion production (Blasi & Colafrancesco 1999):

$$\rho_{\text{CR}} + p_{\text{th}} \rightarrow \pi^\pm + \pi^0 + \pi^- + \text{anything}$$

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu$$

$$\mu^\pm \rightarrow e^\pm + \nu_\mu + \nu_e$$

$$\pi^0 \rightarrow 2\gamma.$$

In this case, the injection spectral rate of secondary $e^\pm$ is (e.g. Brunetti & Blasi 2005)

$$Q_{e}(E) = \int_{E_a}^{E} Q_{e}(E\prime) dE_a \times \int dE_\mu F_{\mu}^E(E_\mu, \theta, \phi) F_{\mu}(E_\mu, E_a).$$

(2)

where $F_{\mu}^E(E_\mu, \theta, \phi)$ is the spectrum of electrons and positrons from the decay of a muon with energy $E_\mu$ produced in the decay of a pion with energy $E_\mu$ (taken from Blasi & Colafrancesco 1999), $F_{\mu}(E_\mu, E_a)$ is the muon spectrum generated by the decay of a pion of energy $E_\mu$ (e.g. Moskalenko & Strong 1998) and the pion injection rate due to p–p collisions is (e.g. Dermer 1986; Blasi & Colafrancesco 1999; Brunetti & Blasi 2005)

$$Q_{\pi}(E_a) = n_\text{th} c \int dE_p N(p) \beta_p F_{\pi}(E_a, E_p) \times \sigma_{\text{pp}}(E_p) \sqrt{1 - \frac{m_p c^2}{E_p}}.$$  

(3)

where $n_\text{th}$ is the number density of thermal protons, $\sigma_{\text{pp}}$ is the p–p cross-section and $F_{\pi}$ is the spectrum of pions from the collision between a CRp of energy $E_p$ and thermal protons (e.g. Moskalenko & Strong 1998; Blasi & Colafrancesco 1999; Brunetti & Blasi 2005).

The observed synchrotron emission in radio haloes at frequencies of 6–7 GHz requires energies of the secondary particles of several GeV, considering field strengths of 1–2 μG typical of the IGM (e.g. Carilli & Taylor 2002). In this case a scaling model is appropriate to describe the pion spectrum from p–p collisions, and we calculate $Q_{e}(E)$ from equations (2–3) following Brunetti & Blasi (2005) and using a pion spectrum:

$$F_{\pi}(E_a) = \frac{1}{2E_a} \left[ c_1 \left( 1 - \frac{E_\pi}{E_a} \right)^{3.5} + c_2 \exp\left( -18 \frac{E_\pi}{E_a} \right) \right]$$

(4)

where $c_1 = 1.22$ and $c_2 = 0.92$ (Berezinskii & Kudriavtsev 1990).

The resulting electron source function can be approximated by

$$Q_{e}(E) = c n_\text{th} K_p E_{\text{th}}^{-\alpha_p} f(E, \alpha_p)$$

(5)

where $f(E, \alpha_p)$ accounts for the log-scaling of the p–p cross-section at high energies and causes the spectral shape to be slightly flatter than $E^{-\alpha_p}$; an analytical expression for the asymptotic form of $f$ (for $E_\mu \gg m_\pi c^2, E_a \gg m_\pi c^2, E_p \gg m_\pi c^2$) is derived in Brunetti & Blasi (2005).

The steady-state spectrum of high-energy electrons in the IGM is given by (e.g. Dolag & Enßlin 2000)

$$N_{e}(E) = |E(E)|^{-1} \int_{E}^{\infty} dE' Q_{e}(E')$$

(6)

where the relevant cooling processes involve synchrotron and inverse-Compton losses:

$$\dot{E}(E) = \frac{4\pi c}{3m_e c^4} \left( \frac{E^2}{8\pi} + \frac{B_{\text{CMB}}^2}{8\pi} \right) E^2,$$

(7)

$B$ is the local magnetic field strength and $B_{\text{CMB}}^2/8\pi$ gives the energy density of the CMB expressed as an equivalent magnetic field.

We calculate the electron spectrum from equations (2–4) and (6–7); this can be approximated by

$$N_{e}(E) = \frac{6\pi m_e c^4}{\sigma_T} n_\text{th} K_p E_{\text{th}}^{-(1+\alpha_p)} g(E, \alpha_p) \frac{g(E, \alpha_p)}{B^2 + B_{\text{CMB}}^2}.$$
Figure 1. Full sky maps of the simulation in galactic coordinates. From top to bottom, electron density, temperature and magnetic field, projected through the whole box. The inlay shows a zoom on to a $3^\circ \times 3^\circ$ region around the Coma cluster, respectively. In the upper most map, the most prominent clusters of the local universe are labelled and the arrow in the inlay points towards north.
where \( g \) is related to \( f \) in equation (5) via

\[
g(E, \alpha_p) = \frac{1}{E^{1-\alpha_p}} \int E \, dX \, X^{-\alpha_p} f(X, \alpha_p)
\]

and \( \Delta \approx 0.2 \) for \( E \approx \) a few GeV. The radio emissivity is

\[
j_v = \sqrt{3} e^2 B \frac{\sin^2 \theta}{v_c} \int_{E_{min}}^{E_{max}} \int_0^\pi \, dE_c \, d\phi \, F \left( \frac{v_c}{v} \right) N_c(E)
\]

where \( v_c = \frac{3}{4\pi} p^2 e^2 B \sin \theta / (mc^3) \) is the critical frequency and \( F \) the integral over the synchrotron kernel:

\[
F(x) = x \int \frac{K_{1/2}(x)}{x} \, dx.
\]

Here \( K_{1/2} \) denotes the modified Bessel function of the order of 1/2.

For a power-law spectrum of CR protons, \( N_p(E_p) \propto E_p^{-\delta_b} \), the resulting synchrotron emission from secondary electrons is \( j_v \propto v^{-(\delta_b-\Delta)/2} \).

### 3.2 \( \gamma \)-rays from hadronic interactions

As already mentioned before, CR protons produce neutral pions, which in turn decay to two photons. \( \gamma \)-rays are a direct measure of the CR protons and provide a complementary constraint to the injection process of secondary electrons.

In order to allow for a prompt comparison with recent results we follow the formalism described in Pfrommer & Enßlin (2004) to estimate the \( \gamma \)-ray flux from CR in our simulations.

The \( \gamma \)-ray source function is

\[
q_{\gamma}(E_{\gamma}) \approx \frac{2^{1-\alpha_p}}{3\alpha_p} \frac{\sigma_{pp} c n_{th} K_p}{\alpha_p - 1} \left( E_{p,\text{min}} \right)^{-\alpha_p} \frac{E_{\gamma}}{\text{GeV}} \sqrt{\frac{m_p c^2}{\text{GeV}}} \left( \frac{2 E_{\gamma}}{m_p c^2} \right)^{\alpha_p - 1} \left( \frac{E_{p,\text{min}}}{m_p c^2} \right)^{-\alpha_p + b_{\gamma}} \left( 2 E_{\gamma} / m_p c^2 \right)^{-\alpha_p / 4 \delta_p}
\]

where \( \alpha_p \lesssim \alpha_p \) is the asymptotic slope of the \( \gamma \)-ray spectrum, which resembles the slope of the proton spectrum (Dermer 1986). The shape parameter, which describes the semi-analytic model near the pion threshold, is \( \delta_p = 0.14 \alpha_p^{1.6} + 0.44 \) by using an effective cross-section \( \sigma_{pp} = 32 \times [0.96 + \exp(4.4 - 2.4 \alpha_p)] \text{mbarn} \). The integrated \( \gamma \)-ray source density \( \lambda_{\gamma} \) is then obtained by integrating the source function over energy (Pfrommer & Enßlin 2004):

\[
\lambda_{\gamma} = \int_{E_1}^{E_2} dE_p q_{\gamma}(E_{\gamma})
\]

\[
= \frac{\sigma_{pp} c n_{th} K_p}{3\alpha_p \delta_p} \left( \frac{E_{p,\text{min}}}{\text{GeV}} \right)^{-\alpha_p - 1} \left( 2^{\alpha_p - 1} \right)^{b_{\gamma}} \left( \frac{m_p c^2}{\text{GeV}} \right)^{-\alpha_p} \times \left[ B_1 \left( \alpha_p + 1, \alpha_p - 1, -\frac{2 E_{\gamma}}{m_p c^2} \right) \right]_{x_1}^{x_2}
\]

where \( B_1(a, b) \) denotes the incomplete beta-function and \( [f(x)]^x_b = f(a) - f(b) \).

### 3.3 The three models

To investigate the dependence of the predicted properties of non-thermal emission of clusters on the underlying assumptions, we investigate three models for the distribution of magnetic fields and cosmic rays in clusters. They are chosen to encompass the reasonable range suggested by theoretical and observational findings.

We keep the spectral index fixed to \( \alpha_p = 2.6 \) in order to be able to match the typical spectrum of giant radio haloes, \( \alpha \sim 1.2 - 1.3 \) (e.g. Ferrari et al. 2008), although a fraction of presently known haloes has a steeper spectrum (e.g. Brunetti et al. 2008; Giovannini et al. 2009; Macario et al. 2010). Also, a spectral index \( \alpha_p = 2.6 \) allows to fit the spectral shape of the Coma halo at \( v < 1.4 \) GHz, although also in this case the spectrum steepens at higher frequencies (e.g. Thierbach, Klein & Wielebinski 2003; Donnert et al. 2010).

#### 3.3.1 Model 1: constant \( X_{CR} \)

In our first model, the energy density of the CR protons is taken as a constant fraction of the thermal energy density. This is reasonable if a constant fraction of the energy that is channelled into the IGM to heat the gas goes into the acceleration of CR protons.

Therefore, in this model, the normalization \( K_p \) is chosen to have a constant fraction, \( X_p = \text{const} \), of kinetic CR energy density \( \epsilon_p \) to thermal energy density \( \epsilon_{th} \) of the IGM:

\[
\epsilon_p = X_p \epsilon_{th}
\]

\[
= \frac{K_p}{\alpha_p - 2} \left( \frac{E_{p,\text{min}}}{\text{GeV}} \right)^{2-\alpha_p}.
\]

For the magnetic field distribution within the IGM we take directly the magnetic field extracted from the simulations.

#### 3.3.2 Model 2: varying \( X_{CR}(r) \)

In a second model, we adopt a radius-dependent cosmic ray energy density fraction, \( X_{CR}(r) \), as obtained from simulations of CR acceleration in structure formation shocks by Pfrommer et al. (2007). We also assume a constant CRp spectral index over the whole cluster volume, \( \alpha_p = 2.6 \).

Pfrommer et al. (2008) simulated the injection of CR protons by merger shocks during structure formation. They find that cosmic ray pressure increases relative to thermal pressure with increasing distance to the cluster centre. Assuming an ideal gas this directly translates into a radially increasing cosmic ray energy density fraction \( X_{CR}(r) = \epsilon_{th}/\epsilon_p \). We use these results to infer a radially varying normalization, so that equation (14) becomes

\[
\epsilon_p(r) = X_p(r) \epsilon_{th}(r)
\]

\[
= \frac{K_p(r)}{\alpha_p - 2} \left( \frac{E_{p,\text{min}}}{\text{GeV}} \right)^{2-\alpha_p}.
\]
Figure 3. Full sky maps of the simulation in galactic coordinates. From top to bottom, X-ray, radio and $\gamma$-ray surface brightness are shown. For the radio and $\gamma$-ray emission, the model with the constant cosmic ray energy fraction was chosen. The $\gamma$-ray emission is evaluated for VERITAS (e.g. $E > 100$ GeV). The inlay shows a zoom on to a $3^\circ \times 3^\circ$ region around the Coma cluster. In the upper most map, the most prominent clusters of the local universe are labelled and the arrow in the inlay points towards north.
to match the observed radio luminosity of the Coma halo where needed.

4.1 The magnetic field in our cluster set

The magnetic field obtained for our simulated Coma cluster is consistent with the one inferred from modelling Faraday Rotation Measures of the Coma cluster, as shown in Donnert et al. (2010).

To compare the ‘statistical’ properties of the magnetic field in the complete set of simulated clusters with observations and to show the effect of re-scaling the field (model 3), we plot in Fig. 6 radial profiles of the Faraday rotation. Here, we bin all clusters with gas mass $M > 3 \times 10^{14} M_{\odot}$ in radius and take the median in each bin. In green, we plot the field profile from simulations, and in red the scaled field profile (model 3). To compare we overplot observations of a sample of Abell clusters from Kim, Kronberg & Tribble (1991), Clarke, Kronberg & Böhringer (2001), observations of A119 from Feretti et al. (1999) and observations of the Coma cluster (Feretti et al. 1995). Because of the small number of points, the error from the Abell clusters was estimated using a bootstrapping technique.

For every bin we generate several samples, by selecting every data point a random number of times and computing the median of this subsample. We then compute the standard deviation of all samples to get an estimate of the error in each bin.

4.2 Radial profile of the radio emission

Since both the target thermal protons and the expected magnetic field strength in the IGM decrease with distance from the cluster centre, most of the synchrotron luminosity emitted by the cluster electrons should be produced in the cluster-core region. This causes the radial profile of secondary-generated radio haloes to be substantially steeper than those of the observed haloes (e.g. Brunetti 2004), although formally this discrepancy may be alleviated by assuming that both the magnetic field and CRp have flat spatial distributions (Pfrommer & Enßlin 2004).

Cassano et al. (2007) report radial profiles of the radio emission of five well-studied radio haloes. We convolve our synthetic radio maps with a Gaussian using the typical beam size (43 kpc) of the observations, and in Fig. 7 compare these profiles with the observed ones. Both simulated and observed profiles were normalized to one. We also include the radial profile of the Coma halo from Deiss et al. (1997).

In all cases (models 1–3), Fig. 7 shows that the radial profiles of the secondary-generated radio haloes in our simulated clusters are considerably steeper than the observed ones. In agreement with Brunetti et al. (2004), by assuming a constant $X_p$ (first panel), the simulated radio emission (black) for $r \geq 0.15 R_{\text{vir}}$ is about 100 times below that measured in real radio haloes (red, green).

By allowing $X_p$ to increase with radius as in 3.3.2 results in an increase of the simulated emission for larger radii (Fig. 7, middle panel), but still expectations account for $<10$ per cent of the observed emission in real radio haloes at $r \geq 0.15 R_{\text{vir}}$. Additional up-scaling of the magnetic field (model 3) further increases the size of the simulated haloes, especially at very large distances from the cluster centre, where our MHD simulations would predict a steeper scaling of the magnetic field with gas density. However, this up-scaling only mildly increases the level of the expected emission at intermediate distances, $r = 0.1–0.3 R_{\text{vir}}$, where the radio brightness of radio haloes is constrained by present observations.

In a recent paper, Pfrommer et al. (2008) claim that, according to cosmological simulations, the combined synchrotron emission from...
Scaling functions

Figure 4. Energy in CRp relative to thermal energy over cluster radius, inferred from Pfrommer et al. (2007). The dependence was derived assuming a linear relation between pressure and energy density.

Figure 5. Scaling functions \( f_{\text{scal}}(r) \) of all clusters. It modifies the magnetic field globally, so that its magnetic energy scales linear with thermal energy.

secondary electrons and primary electrons accelerated at large-scale shocks may produce diffuse emission with a fairly broad spatial distribution. Based on our results, the contribution from primary electrons [even assuming Pfrommer et al. (2008) results\(^2\)] is not expected to solve the discrepancy between models and observations in Fig. 7. Indeed, our model 3 is thought to mimic the secondary-generated emission in Pfrommer et al. (2008) and, based on their fig. 9, the contributions from primary electrons at \( r \approx 0.2R_{\text{vir}} \) is (at best) comparable with that of the secondary electrons leaving expectations well below observations.\(^2\)

In principle, for each cluster, it would be possible to allow the energy density of CRp to further increase at large distances from the cluster centre and find an ad hoc spatial distribution of CRp that allows for matching the brightness profiles of radio haloes. However, this would imply the untenable scenario in which CRp store very large energy budget outside the cluster core, for example in the case of the Coma halo the energy density of CRp at \( r \approx 0.2 \)– \( 0.3R_{\text{vir}} \) should be comparable to that of the thermal ICM (Donnert et al. 2010).

\(^2\) Which are based on the optimistic assumption that 10 per cent of the energy of shock accelerated particles are channelled into CR electrons.

\(^3\) It is more difficult to evaluate the ratio of secondary to primary electrons at \( r \approx 0.3R_{\text{vir}} \) from fig. 9 in Pfrommer et al. due to the presence of a very bright shock-like spot southwest of the centre of their simulated cluster (see their figs 7 and 8) that affects the azimuthal brightness profile, but that is not indicative of an upturn in the distribution of the truly diffuse halo emission.

4.3 Morphology of the radio emission

A complementary approach to compare expectations from simulations with observations is to derive a point-to-point plot between radio and X-ray brightness. A number of clusters indeed show morphological correlations between X-ray and radio surface brightness (Govoni et al. 2001). This provides a complementary approach to the comparison of the radial profiles and is not affected by the morphological asymmetries and possible imperfect alignment of the emission centres (Govoni et al. 2001). In Fig. 8, we show the luminosity in (166 kpc)\(^2\) sized square patches of our synthetic radio maps of the four biggest simulated clusters in our sample as a function of the same patches in the X-rays. The three panels again show constant (model 1, left), increased cosmic ray scaling (model 2, middle) and cosmic rays scaled combined with magnetic fields scaled (model 3, right).

We include the observed scaling for three clusters (Coma and the steepest halo A2255) from Govoni et al. (2001) (dashed lines) and add the correlations found from best fits to the simulated data, calculated inside the region corresponding to the range of observed radio-X-ray brightness (dashed dotted lines). In Table 2, we include the slopes from a best-fitting model to the simulations inside the observed region for the four largest clusters. We find that the models produce too steep scaling with only the third model approaching the sub-linear regime, where haloes are found in observations. Still no model produces substantially sub-linear scalings, which are typical for very extended haloes, like e.g. Coma, A2163 and A2744, which have a scaling slope \( b = 0.6 \) (Govoni et al. 2001; Feretti et al. 2001; Murgia et al. 2009). This reflects the point raised in the previous section, i.e. that the slope of the radial distribution of the synchrotron emission in our secondary-generated radio haloes is too steep.

4.4 Scaling relations

There are several observed correlations for radio haloes that relate thermal and non-thermal properties of the IGM: those between the radio power at 1.4 GHz, \( P_{\text{1.4 GHz}} \), and the X-ray luminosity, \( L_X \), temperature and cluster mass (Liang et al. 2000; Govoni et al. 2001; Cassano et al. 2006). In addition, by making use of a sample of 14 giant radio haloes, Cassano et al. (2007) found new scaling relations

\[ X_C(r) = e^X_{\text{th}} \quad \text{(arb. units)} \]

\[ B_{\text{vir}} = B_{\text{KM}} \]
provides a simple, but viable estimate 0.64 for COMA and A2255 in purple, 
and that the radial profiles of the radio emission are self-
radio and the size containing the 85 per cent 
We stress that the error in the slope of this scaling, as well as those in 
\( R \propto R_{\text{vir}} \) and 
\( X_{\text{CR}} = \text{const}, B_{\text{in}} \)  
\( X_{\text{CR}} = X_{\text{CR}}(r), B_{\text{in}} \)  
\( X_{\text{CR}} = X_{\text{CR}}(r), B_{\text{cluster}} \) 

Figure 7. Normalized radial profiles of radio emission from 17 simulated clusters (black). In the left-hand panel, the CR population was modelled using a constant \( X_{\text{CR}} \). The middle one uses the radius-dependent CR to thermal energy fraction (Fig. 4) adopted from Pfrommer et al. (2008). Additionally, we modified (Fig. 5) the magnetic field in the right-hand panel to be \( \propto \sqrt{B} \) (right-hand panel). We add observations of A2744, A2319, A545, A2163 and A2255 from Cassano et al. (2007) as red curves with error bars, Coma (green) is taken from Deiss et al. (1997).

Table 2. Slopes of the radio versus X-ray surface brightness correlation from patches for our three models and the four largest clusters. Govoni et al. (2001) and Feretti et al. (2001) find slopes of Coma and A2163: \( b = 0.64 \), A2391: \( b = 0.98 \) and A2255: \( b = 0.82 \).

| Cluster | Model M1 | Model M2 | Model M3 |
|---------|----------|----------|----------|
| 0       | 1.09     | 1.04     | 0.87     |
| Perseus | 1.33     | 1.10     | 0.90     |
| Coma    | 1.54     | 1.18     | 0.98     |
| 4       | 1.21     | 1.01     | 0.94     |

That connect the radio power, \( P_R \), of haloes to the size of the emitting region, \( R_H \) (see also Murgia et al. 2009), and to the total cluster mass within \( R_H, M_H \); a geometrical scaling was also found between \( M_H \) and \( R_H \). The observed scalings from Cassano et al. (2007) are

\[ P_R \propto R_H^{4.18 \pm 0.68} \]  
\[ P_R \propto M_H^{1.99 \pm 0.22} \]  
\[ M_H \propto R_H^{2.17 \pm 0.19}. \]

Specifically, \( M_H \) was computed from X-ray observations under the assumption of hydrostatic equilibrium and spherical symmetry. Radio haloes are found exclusively in disturbed, merging clusters, where these assumptions break down to some degree. Therefore, this procedure may lead to errors as large as 40 per cent in mass (Rosati et al. 2006) which are expected to be not dependent on cluster mass, so that these errors might introduce considerable scatter without affecting the real trend of the correlation. \( R_H \) was measured on the radio images, \( R_H = \sqrt{R_{\text{max}} \times R_{\text{min}}} \), where \( R_{\text{max}} \) and \( R_{\text{min}} \) are the minimum and maximum radii measured on the 3\( \sigma \) radio isophotes. We stress that \( R_H \) provides a simple, but viable estimate of the physical size of radio haloes; indeed a one-to-one correlation has been found between \( R_H \) and the size containing the 85 per cent of the radio halo flux, \( R_{\text{halo}} \), derived from the observed brightness profiles of haloes (Cassano et al. 2007).

A scaling was also found between the size of radio haloes and the virial radius of clusters (Cassano et al. 2007):

\[ R_H \propto R_{\text{vir}}^{2.63 \pm 0.50}. \]  

(22)

Given that massive clusters are almost self-similar (e.g. Rosati, Borgani & Norman 2002) one might have expected that \( R_H \) scales with \( R_{\text{vir}} \) and that the radial profiles of the radio emission are self-similar. On the contrary, this result proves that self-similarity is broken in the case of the non-thermal cluster components, as first noted by Kempner & Sarazin (2001).

As the synchrotron power depends on both magnetic field scaling and CR scaling with density; it is unclear what is responsible for the break in the observed properties. On the other hand, we know from previous works (Donnert et al. 2009) that the magnetic field scaling (with temperature or mass) flattens out for the largest clusters in our simulation, that would imply an expected break of self-similarity in the thermal versus non-thermal properties of our simulated clusters.

Cassano et al. (2007) showed that all the correlations explored so far for radio haloes can be derived by combining the \( R_H - R_{\text{halo}} \) and

4 We stress that the error in the slope of this scaling, as well as those in equation (19) and following accounts for both intrinsic scatter and measured errors in the data points in both variables.
$P_{1.4} - R_H$ scalings. This suggests that there are two main scaling relations that carry out the leading information on the physics of the non-thermal components in galaxy clusters. In what follows we shall investigate whether the properties of our simulated secondary-radio haloes are consistent with the observed scalings.

### 4.4.1 Mass versus size

As a first step, before comparing the observed scalings with those derived for our simulated clusters, we check whether our clusters inherit the same mass distribution of real clusters with radio haloes. We compare the observed and simulated scaling between the total mass inside $R_{1.4}$ ($M_{1.4}$) and $R_{85}$, that provides a geometrical scaling on the halo region. Therefore, we plot in Fig. 9 $M_{1.4}$ versus $R_{85}$ for simulated (open triangles) and observed (diamonds) clusters, together with the best-fitting power law to the observed scaling from Cassano et al. (2007).

The three models define different values of $R_{85}$ and consequently different volumes where the scaling can be tested. We find that in all cases simulated clusters lie on the thermal scaling described by observed clusters, although, as expected (Sections 4.2–4.3), simulations populate a region in the $M_{1.4} - R_{85}$ diagram with smaller values of $R_{85}$.

### 4.4.2 The size versus size relation

As stated in Section 4.4, observations of clusters with radio haloes show that the size of haloes scales not linearly with the cluster virial radius, suggesting that the non-thermal component in clusters is not self-similar.

In Fig. 10, we plot the radius containing 85 per cent of the clusters emission over virial radius for all three models. We include data from Cassano et al. (2007) and the fits to the simulated clusters distribution obtained for models 1–3 at $z = 0$. We find a correlation between $R_{85}$ and $R_{85}$ for our simulated hadronic haloes. Results suggest that self-similarity is preserved in the non-thermal components of simulated clusters, as the increase of the haloes’ radius is roughly proportional to the virial radius of the hosting clusters; the slopes of the correlations are $b_1 = 0.8, 1.2$ and $1.3$ for models 1, 2 and 3, respectively.

This is not in line with observations: the expected correlations are flatter than the observed one and we predict haloes systematically smaller than the observed ones. In contrast this is expected considering results reported in Section 4.2 and confirms that it is challenging to reproduce the spatial extension of the observed radio haloes with hadronic models, even by adopting a profile of the magnetic field that is flatter than that from our MHD simulations and by assuming a flat profile of the spatial distribution of CRp.

Donnert et al. (2010) have shown that matching the radio emission of the Coma halo at distance $\approx 0.2–0.3$ $R_{vir}$ with hadronic models (by further increasing the CRp energy content at larger radii; see also Section 4.2) would require the energy content of CRp to be roughly similar to the content of the thermal ICM at these distances (at least when constraints on the magnetic field from RM observations of the Coma cluster are used). The scalings found in Fig. 10 make the situation possibly more challenging, because they show that even more energy in the form of CRp would be required in the case of

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**Figure 9.** Total gravitational mass inside the radio emitting region over radius of the same region. We plot the correlation for all three models, constant and varying CR fraction and varying CR fraction and upscaled magnetic field (from left to right). We also plot observations of 14 galaxy clusters from Cassano et al. (2007) and the best-fitting correlations. Due to uncertainties in the mass estimation from X-ray data, the observations may show systematic errors. For both, simulations and observations, the coma cluster is marked in blue.

**Figure 10.** Radius of the radio emitting region as a function of the virial radius of the cluster. We plot (left to right) the correlation for all three models and include the best fit ($\chi^2 = 0.54, 0.29, 0.86$, respectively). We also plot observations of 14 galaxy clusters from Cassano et al. (2007) and the best-fitting correlation. For both, simulations and observations, the Coma cluster is marked in blue. For model 1 we include the correlation at redshift 0.4.
Hadronic radio haloes from simulations

4.4.3 The X-ray luminosity versus radio power relation and the evolution of radio haloes

Radio haloes follow a correlation between the monochromatic radio luminosity at 1.4 GHz, $P_{1.4}$, and the X-ray luminosity of the hosting clusters, $L_X$ (e.g. Liang et al. 2000; Enßlin & Röttgering 2002; Bacchi et al. 2003; Cassano et al. 2006). Recent radio observations of a statistical sample of X-ray selected galaxy clusters, the ‘GMRT radio halo survey’ (Venturi et al. 2007, 2008), allow us to study the distribution of clusters in the $P_{1.4} - L_X$ diagram. These observations suggest that the distribution of clusters in the $P_{1.4} - L_X$ diagram is bi-modal: radio-halo clusters trace the $P_{1.4} - L_X$ correlation, while the majority of clusters are found ‘radio quiet’ with the limits to their radio luminosities about 10 times smaller than the radio luminosities of haloes.

In order to investigate the behaviour of our simulated clusters in the $P_{1.4} - L_X$ diagram, in Fig. 11 we plot our simulated clusters together with observed clusters (from Brunetti et al. 2007, 2009; Venturi et al. 2008). The synthetic radio luminosities of our simulated clusters are scaled in order to have the simulated Coma cluster matching the observed one.

According to the secondary models, a correlation between radio luminosity and cluster X-ray luminosity (or temperature) is expected (e.g. Dolag & Enßlin 2000; Miniati 2001; Dolag et al. 2005; Pfrommer et al. 2008). We qualitatively confirm these expectations and in all three models find that the largest simulated clusters would naturally approach the observed correlation. In all three models, the smallest systems are significantly more scattered in radio power than the largest clusters. This is expected as the magnetic field in the central regions of our simulated clusters (where most of the synchrotron emission is generated) is found to be tightly correlated with cluster thermal properties only in the case of massive clusters, while only a steep trend is found for smaller systems (fig. 9 in Donnert et al. 2009).

The most relevant difference with respect to observations is that according to all three models (as for every secondary model) the synchrotron luminosity of the massive simulated clusters is equivalent to that of typical radio haloes, at least if the radio luminosity of the simulated Coma cluster is normalized to that of the real Coma halo. This is inconsistent with observations which, on the other hand, found radio haloes in only about 1/3 of massive clusters.

Most important, no radio bi-modality is expected in our simulated secondary haloes. Hadronic haloes in simulated massive systems would follow a tight correlation, while those in less massive systems would be more broadly distributed.

One possibility to reconcile the hadronic scenario with the observed halo-merger connection and with the bi-modal distribution of clusters in the $P_{1.4} - L_X$ diagram is to admit that the observed bi-modality is driven by the amplification and dissipation of the magnetic field in the merging and post-merging phase, respectively (Brunetti et al. 2007; Pfrommer, Enßlin & Springel 2008; Kushnir, Katz & Waxman 2009). However, Brunetti et al. (2009) have shown that the degree of amplification/dissipation of the magnetic field and the time-scale of this process that would be necessary to explain observations are difficult to reconcile with the observed properties of magnetic fields in the ICM (namely the field intensity and coherence scales from rotation measurements), and appear also disfavourable by energetic arguments.

The amplification of the magnetic field during cluster mergers is followed by our MHD cosmological simulations that offer a complementary approach to highlight this issue. To show the effect of magnetic field evolution and investigate the bi-modality we plot the time evolution of the simulated radio haloes in Fig. 12. Shown is the radio luminosity over X-ray luminosity of the simulated sample for redshift $z < 0.48$ (big triangles; $z = 0$, small triangles and black line: earlier redshifts); We highlight the path of the largest simulated cluster in cyan. Further we show the radio power versus temperature correlation and evolution in the Appendix. No hint of a bi-modality is found, simply because magnetic field amplification in massive systems is a gradual process that happens in a time-scale comparable with the lifetime of clusters themselves. For example, cluster 0 (cyan in image 12) increases its mass by a factor of 1.5 in the redshift range, staying close to the correlation. The evolution of magnetic field is reflected in the broad/scattered distribution of clusters in the $P_{1.4} - L_X$ diagram, especially in the case of smaller systems. These smallest clusters approach the lower end of the correlation with rather large scatter because the magnetic field is not saturated yet in their central regions and even smaller mergers yield a significant field amplification resulting in an increased radio luminosity. Although one would expect this behaviour to be slightly dependent on numerical resolution, increasing the numerical resolution would make the situation even more stringent. Resolving smaller gas motions leads to increase in the amplification of the magnetic

Figure 11. Radio power per frequency at 1.4 GHz over X-ray luminosity in erg s$^{-1}$ from our simulated clusters using all three models (triangles, lr.: constant fraction, scaled fraction, scaled fraction and scaled field). Observed scalings by Cassano et al. (2007) (diamonds, $z > 0.2$: red diamonds) and non-detections in red (Venturi et al. 2007, 2008). For both, simulations and observations, the Coma cluster is marked blue.
Figure 12. Radio power per frequency at 1.4 GHz over X-ray luminosity in erg s$^{-1}$ from our simulated clusters using the constant model. Observed scalings by Cassano et al. (2007) (diamonds, $z > 0.2$; red diamonds) and non-detections in red (Venturi et al. 2007, 2008). We also include the time evolution of clusters for $z < 0.48$ (black lines), and highlight the largest cluster of the simulated sample (0) cyan.

5 $\gamma$-RAY EMISSION FROM SIMULATED CLUSTERS

We use the formalism in Section 3.2 to compute predictions for the $\gamma$-ray luminosity of the simulated clusters according to all three models. Of interest are the two energy bands of Cherenkov telescopes, $E > 0.1$ TeV, and Fermi/EGRET telescopes, $E > 0.1$ GeV.

In Table 3 (Table 4) we present fluxes for the simulated sample in the VERITAS (Fermi) energy band. None of the clusters is in the observable range of the VERITAS experiment; however the largest ones (0, Coma, Virgo, Perseus, Centaurus) have a chance to be detected by Fermi in the next few years, at least for models with radially increasing CRp normalization (models 2 and 3).

In the left-hand panel of Figs 13 and 14, we plot the simulated clusters in $\gamma$-ray versus radio luminosity at 1.4 GHz, left) and bolometric X-ray luminosity of all cluster at redshift $z = 0$. We plotted the three models as different symbols (diamonds – model 1, triangles – model 2, boxes – model 3) and include upper limits from Perkins (2008), Perkins & the VERITAS Collaboration (2006) in blue. The Coma cluster is marked green.

In the right-hand panel of Figs 13 and 14, we plot the emission of our clusters in $\gamma$-ray versus X-ray luminosity for the three models. Also in this case we show two $\gamma$-ray upper limits from

| Cluster | M1       | M2       | M3       |
|---------|----------|----------|----------|
| 0       | $3.2 \times 10^{13}$ | $5.8 \times 10^{13}$ | $6.0 \times 10^{13}$ |
| Hydra   | $6.9 \times 10^{13}$ | $3.3 \times 10^{14}$ | $3.4 \times 10^{14}$ |
| 2       | $2.4 \times 10^{14}$ | $1.0 \times 10^{14}$ | $1.0 \times 10^{14}$ |
| 3       | $1.7 \times 10^{14}$ | $2.5 \times 10^{14}$ | $2.5 \times 10^{14}$ |
| 4       | $9.9 \times 10^{14}$ | $2.0 \times 10^{14}$ | $2.1 \times 10^{14}$ |
| 5       | $1.7 \times 10^{14}$ | $2.6 \times 10^{14}$ | $2.6 \times 10^{14}$ |
| Coma    | $6.7 \times 10^{14}$ | $9.4 \times 10^{14}$ | $9.6 \times 10^{14}$ |
| 7       | $8.6 \times 10^{15}$ | $9.8 \times 10^{15}$ | $1.0 \times 10^{14}$ |
| 8       | $1.8 \times 10^{14}$ | $2.0 \times 10^{14}$ | $2.1 \times 10^{14}$ |
| Virgo   | $2.9 \times 10^{13}$ | $6.7 \times 10^{13}$ | $6.9 \times 10^{13}$ |
| A3627   | $3.2 \times 10^{14}$ | $3.3 \times 10^{14}$ | $3.4 \times 10^{14}$ |
| 11      | $1.4 \times 10^{14}$ | $2.1 \times 10^{14}$ | $2.1 \times 10^{14}$ |
| 12      | $4.5 \times 10^{14}$ | $4.2 \times 10^{14}$ | $4.3 \times 10^{14}$ |
| Perseus | $9.7 \times 10^{14}$ | $2.3 \times 10^{14}$ | $2.3 \times 10^{14}$ |
| Centaurus | $5.3 \times 10^{14}$ | $5.8 \times 10^{14}$ | $5.9 \times 10^{14}$ |
| 15      | $5.0 \times 10^{15}$ | $5.5 \times 10^{15}$ | $5.6 \times 10^{15}$ |
VERITAS (Poulin, Nieto & Davoust 1992) and preliminary Fermi (Mori 2008) observations, respectively. A quasi-linear correlation is predicted, and is found less scattered than that between γ-ray and radio luminosities since it compares two purely thermal quantities (also in the case of models 2 and 3 CRp are scaled with thermal energy in accordance with the profile in Fig. 4).

The Fermi results are still consistent with the models for CRp presented in this paper. However, as shown by Donnert et al. (2010), the expected γ-ray flux increases substantially if we consider a spatial distribution of CRp that would the simulated radio emission to fit the observed profiles of radio halos. We therefore expect a rejection or confirmation of these models in the very near future.

6 CONCLUSIONS

We use a constrained, cosmological MHD SPH simulation with a semi-analytic model for galactic magnetic outflows to obtain a sample of 16 galaxy clusters with thermal properties similar to clusters in the Local Universe. Further, we assume three different models for secondary cosmic rays motivated by simulations (Pfrommer et al. 2007) using the proper high-energy approximation for the pion cross-section. In the first model, we assume a constant CRp normalization relative to the thermal density and the simulated magnetic field. In the second model, we keep the magnetic field and introduce a radius-dependent CRp normalization inferred from non-radiative simulations by Pfrommer et al. (2007). The third model uses the same CRs and flattens the simulated magnetic field to be $B \propto \sqrt{r}$.

Although our simulations do not include a (internally) self-consistent treatment of CRp as done in other simulations (e.g. Pfrommer et al. 2007), contrary to previous work they allow us to properly simulate the properties of the magnetic field in the ICM which is important for modelling the cluster-scale synchrotron emission.

For the first time, we carry out a detailed comparison between the observed properties of giant radio haloes and those of simulated haloes according to secondary models and under different assumptions for the spatial distribution of CRp. In an earlier paper we presented a detailed comparison of the simulated Coma cluster (Donnert et al. 2010). In this work we focus on global sample properties and compare with recent observations.

In particular, as a first step, we show the following.

(i) The radial profiles of the Faraday rotation of the median of our cluster sample is in line with that obtained from a number of observations of different clusters. This confirms that the properties of the cluster’s magnetic fields in our simulations are similar to the observed ones.

(ii) The normalized radial profiles of the radio emission at 1.4 GHz of our simulated hadronic haloes show a deficit at radii $\geq 0.1 \, r_{\text{vir}}$ with respect to the synchrotron profiles observed for a sample of well-studied radio haloes. This is in line with previous claims based on semi-analytic calculations in the context of the hadronic model (Brunetti 2004). In addition, our results show that, even by assuming a flat profile for both the magnetic field and CRp spatial distributions (model 3), secondary electrons may account for less than about 10 per cent of the observed emission at radii 0.15–0.3 $r_{\text{vir}}$.

(iii) A point-to-point comparison of radio versus X-ray emission, obtained for the four largest clusters of our sample, confirms the results obtained for the profiles showing that the radial distribution of radio emission is too steep. All three models do not fit the observations over the whole range. Furthermore, an excess in radio emission of the innermost patches suggests that haloes from the simulation are too centrally peaked.

As a second step we compare scaling relations obtained for the hadronic haloes in our simulated cluster sample with those given in Cassano et al. (2007) that are obtained from a sample of observed radio haloes. We find the following.

(i) The geometrical correlation between the radius of radio haloes and the cluster mass contained within this radius is well reproduced by all three models. Due to the expected self-similarity of cluster in thermal properties, this result implies that – at least – the simulated and observed clusters share similar thermal properties.

(ii) A quasi-self-similar behaviour is found for the non-thermal properties of our simulated clusters. In particular, the radius of our hadronic haloes is found to scale (approximately) with the virial radius of the simulated clusters. This is contrary to observations that found a steeper correlation between halo radius and virial radius of the hosting clusters and implies that our simulated haloes are systematically smaller than the observed ones.

(iii) A correlation between the monochromatic luminosity of our hadronic haloes and the X-ray luminosity of the simulated hosting clusters is found. As soon as the population of our hadronic haloes is normalized, by scaling the radio luminosity of the simulated Coma halo with that of the observed one, the correlation is similar to that observed for radio haloes. However, since at this point all the simulated clusters show radio emission at the level of the observed haloes, we find that the cluster radio bi-modality, observed for X-ray selected clusters, cannot be reproduced. A radio bi-modality in the radio–X-ray diagram would require a fast ($< \text{Gyr}$) evolution of the radio luminosity in connection with cluster mergers; on the other hand, we find that the time evolution of our simulated massive clusters in this diagram happens on cosmological, long, time-scales. Finally, we show that once the radio–X-ray correlation is approached at low X-ray luminosities, during their evolution clusters follow the correlation closely due to the saturation of the magnetic field.

As a final point, we calculate the γ-ray emission from our simulated clusters once the radio luminosity of the simulated Coma halo is anchored to that of the observed one (essentially by scaling...
the number density of CRp in simulated clusters). We find that the following.

(i) The γ-ray emission expected from our simulated clusters is well below the sensitivity of present Cherenkov Arrays, e.g. the VERITAS experiment, for all the adopted models for hadronic haloes. The γ-ray fluxes at $>100$ MeV expected from our simulated clusters would allow for a marginal detection by the Fermi telescope in next years, at least by assuming models 2 and 3.

(ii) The integrated γ-ray flux from our simulated clusters is expected to scale with their radio emission, although with rather large scatter. On the other hand, a tight correlation is found between γ-ray and X-ray fluxes, in which case (due to the scalings between thermal and non-thermal CRp in models 1–3) the correlation is essentially driven by the density of the thermal gas in our simulated clusters.

Considering all these results, as well as the outcome from our earlier work on the Coma cluster (Donnert et al. 2010), we conclude that hadronic models alone are not able to explain the observed properties of giant radio haloes, in terms of their radial extension, observed cluster’s radio bi-modality, scaling relations and spectral properties. Therefore, we conclude that radio emission observed in galaxy clusters in the form of giant radio haloes is a powerful tool to infer the amount of CRp within galaxy cluster, confirming previous results that the energy content of CRp in clusters cannot exceed per cent level (e.g. Brunetti et al. 2007; Churazov et al. 2008).

The problem of halo’s extension could be alleviated in extended hadronic models, where the contribution from primary (shock accelerated) electrons at the cluster outskirts is combined in the simulations with that from secondary electrons (Pfrommer et al. 2008). However, as shown in Section 4.2, the problem of the halo profiles arises at distances $0.1–0.3 \, R_{\text{vir}}$ from cluster centres where, even based on the same simulations, the contribution to the diffuse synchrotron emission from these shock accelerated electrons is not yet dominant. Further, we would like to note that the detailed morphology of the radio emission caused by electrons injected at shocks (e.g. Vazza et al. 2010; Roettiger, Burns & Stone 1999) will be significantly different from the morphology of giant radio haloes and therefore is in general not able to mimic large scale, truly diffuse cluster-centric radio emission. Thus, additional physical processes are required to explain observations.

The problem of the radio bi-modality in massive clusters can be alleviated by assuming that MHD turbulence (and the rms magnetic field) decays as soon as clusters approach a relaxed state after a major merger (Brunetti et al. 2009; Kushnir et al. 2009). In this case, it might also be thought that CRp in these relaxed clusters would undergo less scattering on magnetic field irregularities escaping from the cluster volume and reducing the source of secondary electrons. All these processes are not properly included in our MHD cosmological simulations. However, Brunetti et al. (2009) have shown that the fast decay of the rms field in galaxy clusters, that is necessary to explain the observed bi-modality, would imply a rather unphysical situation where the magnetic field power spectrum peaks at the smaller scales, and leads to the consequence that an extremely large flux of energy in clusters goes into magnetic field amplification/dissipation. In addition, because turbulent cascade should start to decay on largest scales, which are resolved by Faraday rotation measurements, this scenario would predict a bi-modality in rotation measures and depolarization in largest clusters, which is not observed (Govoni 2009).

The discovery of giant radio haloes with very steep spectrum ($\alpha \sim 1.8–2$) in merging clusters (Brunetti et al. 2008; Giovannini et al. 2009; Macario et al. 2010) proves that inefficient particle acceleration mechanisms are responsible for the origin of these sources and, based on simple energy arguments, disfavours hadronic models (Brunetti et al. 2008). Observations of the best studied halo, in the Coma cluster, have shown that its spectrum steepens at higher frequencies (Thierbach et al. 2003), due to the competition between energy losses and acceleration of the emitting particles, in which case the particle acceleration time-scale would be of the order of 0.1 Gyr. More recently, we have shown that the steepening cannot be a result of the inverse-Compton (SZ) decrement (Donnert et al. 2010), confirming these previous finding and supporting a scenario of inefficient particle acceleration mechanisms at the origin of the halo.

Particle acceleration due to micro-turbulence in merging galaxy clusters has been proposed for the origin of radio haloes (e.g. Brunetti et al. 2001; Petrosian 2001). In this case gentle particle acceleration mechanisms would generate radio haloes in connection with (massive) cluster mergers, while the radio emission would decay as soon as clusters approach a relaxed state, due to dissipation of (at least a fraction of) this turbulence and the fast electron radiative cooling. It will therefore be interesting to consider these processes in future simulations used to study radio haloes. That includes an estimation of the locally merger-injected turbulence as well as a more detailed description of CR electron spectra.

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Here, we provide additional scaling relations to substantiate the scalings shown before.

A1 X-ray luminosity–temperature relation

In Fig. A1, we show bolometric X-ray luminosity over mass-weighted temperature for the simulated cluster sample at redshift $z = 0$ (diamonds) and $z = 0.48$ (triangles). We also plot the observed correlation from Pratt et al. (2009).
Figure A2. Radio power at 1.4 GHz over mass-weighted temperature inside 0.1$r_{\text{vir}}$ for all clusters. We include various observations from the literature. The evolution of the simulated clusters is plotted as lines in black (grey) for $z < 0.48$ ($z > 0.48$), while the points correspond to $z = 0.0$ (diamonds) and 0.48 (triangles). We include band-corrected observed relation from Pratt et al. (2009) (broken line) and the same correlation normalized to the simulated Coma cluster (black line).

It is well known that non-radiative simulations tend to overpredict the X-ray luminosity by a fair amount (see e.g. Borgani et al. 2006). In addition, the effect of $K$-correction can be seen in comparison with the cluster at high redshift.

Still, after re-normalizing the X-ray luminosity there is a reasonably good agreement with observations. We therefore conclude that our simulated cluster sample shows similar thermal properties than the observations.

A2 Radio luminosity–temperature relation

In Fig. A2, we present the scaling of radio power at 1.4 GHz over cluster temperature as obtained from e.g. X-ray observations for constant CRp scaling (model 1). We include the evolution of the emission with time as lines in black (grey) for $z < 0.48$ ($z > 0.48$). Further, we plot a number of recent observations. The sample follows the correlation closely at redshift zero, and only two clusters show significant deviation from the correlation at temperature larger than 5 keV, and only at high redshift.

In simulations, temperature is a less merger-sensitive mass estimator compared to the X-ray luminosity, in terms of cluster mergers. We therefore conclude that the bimodality observed in large clusters is not a result of biased mass estimation.

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