SHAPES OF WANDERING DOMAINS

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Abstract. We study shapes of wandering domains for entire functions and we prove that very bounded connected regular open set, whose closure is polynomially convex, is a wandering domain of some entire function. In particular such domain can be realized as an escaping or an oscillating wandering domain. As a consequence we obtain that every Jordan curve is the boundary of a wandering Fatou component of some entire function.

1. Introduction

A general goal in discrete dynamical systems is to describe qualitatively the possible dynamical behaviour under iteration of maps satisfying certain conditions, may they be algebraic (e.g. polynomial or rational maps) or analytic (e.g. smooth, symplectic or holomorphic selfmaps). In this paper we consider the dynamical system given by the iterates of an entire function $f : \mathbb{C} \to \mathbb{C}$. We use $f^n$ to denote the $n$’th iterate of $f$. There is a natural dichotomy of the complex plane associated to such dynamical system. We say that a point $p \in \mathbb{C}$ belongs to the Fatou set $\mathcal{F}_f$ if and only if there exists an open neighbourhood of $p$ on which the sequence of iterates $(f^n)$ form a normal family. The Julia set $\mathcal{J}_f$ is then defined as a complement of the Fatou set. The connected components of the Fatou set are called the Fatou components. We say that a Fatou component $\Omega$ is pre-periodic if there are non-negative integers $n \neq m$ such that $f^n(\Omega) = f^m(\Omega)$. A Fatou component which is not pre-periodic is called a wandering Fatou component or a wandering domain.

One of the main goals in complex dynamics is to obtain a complete classification of all possible Fatou components for a given class of maps in terms of their intrinsic dynamics, extrinsic dynamics and their geometry.

In the class of entire functions we have a complete classification of the pre-periodic Fatou components, see [11, 12], and the recent developments in studies of wandering domains show that we are also getting closer to a complete classification of wandering domains.

By Sullivan’s non-wandering theorem, [13], we know that polynomials can not have wandering domains, hence they can only appear in the class of the entire transcendental functions. First example of such domain was given by Barker [1] who proved that a certain entire function has a multiply connected wandering domain. Since then several examples of simply connected wandering domains were constructed, see [9, 2, 6]. Nowadays we know that a wandering domain can topologically be either simply connected, doubly connected or infinitely connected.

In terms of extrinsic dynamics of wandering domains we can classify them in to three classes: escaping (the orbit converges to infinity), oscillating (one subsequence of the orbit tends to infinity and another tends to a finite point $p \in \mathbb{C}$) or bounded (the orbit of the domain is bounded). Note that Barker’s first wandering domain was of the escaping type. The first example of an oscillating wandering domain was given by Éremenko and Lyubich in [6]. In the same paper they have proved the existence of an entire function $f$ having a wandering domain $\Omega$, so that $f^n|_\Omega$ is univalent for all $n \geq 1$. They were also the first who used an approximation theory for the construction of entire functions in complex dynamics. It is still unknown whether wandering domains with bounded orbit exist.

A complete description of the intrinsic dynamical behaviour in a multiply connected wandering domain was given in [3]. In [4] the authors have classified simply connected wandering domains in

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terms of the hyperbolic distance between orbits of points and in terms of convergence to the boundary. They show that there are in total nine types of simply connected wandering domains all of which can be realized as an escaping wandering domain. Recently, in [7] the authors have shown that only six out of these nine types can be realized by an oscillating wandering domain. Let us just mention that there are several important works proving the existence of wandering domains for entire functions in the Eremenko-Lyubich class, t.i. entire functions with bounded singular set, see [5, 8, 10]. Whether or not all of the previously discussed types of wandering domains can be realized this class is presently unknown.

By the Riemann mapping theorem all simply connected domains are conformally equivalent to the unit disk, hence from this point of view all simply connected wandering domains look the same. In contrast we can compare the shapes of domains, t.i. how they lie in the ambient space. We say that two domains have the same shape if there is an automorphism of the ambient space that maps one domain to the other. In this paper we study what kind of shapes can be realized by a simply connected wandering domain of an entire function. The following are our main results.

**Theorem 1.** Let \( \Omega \subset \mathbb{C} \) be a bounded connected regular open set whose closure is polynomially convex. There exists an entire function \( f \) for which \( \Omega \) is an escaping wandering domain and the iterates \( f^n|\Omega \) are univalent.

**Theorem 2.** Let \( \Omega \subset \mathbb{C} \) be a bounded connected regular open set whose closure is polynomially convex. There exists an entire function \( f \) for which \( \Omega \) is an oscillating wandering domain and the iterates \( f^n|\Omega \) are univalent.

The condition of boundedness and polynomially convexity of \( \Omega \) in our theorems is need for the application of the following well known approximation theorem.

**Theorem 3.** (Runge approximation theorem) Let \( K_1, \ldots, K_n \subset \mathbb{C} \) be pairwise disjoint compact polynomially convex sets. Let \( A_k \subset K_k \) be a finite set of points and \( f_k : K_k \to \mathbb{C} \) a holomorphic map for every \( 1 \leq k \leq n \). For every \( \varepsilon > 0 \) there exists an entire function \( f \) satisfying:

1. \( \| f - f_k \|_{K_k} < \varepsilon \)
2. \( f(x) = f_k(x) \) for all \( x \in A_k \)
3. \( f'(x) = f'_k(x) \) for all \( x \in A_k \)

for every \( 1 \leq k \leq n \).

The remaining condition, that \( \Omega \) is a regular open set, implies the existence of a sequence of points \( (x_n) \in \mathbb{C} \setminus \overline{\Omega} \) which accumulates everywhere on the boundary of \( \Omega \), t.i. \( \omega((x_n)) = b\Omega \).

Using the Runge approximation theorem we inductively construct a sequence of entire functions that converges uniformly to the entire function \( f \) with the following properties. The \( f \)-iterates of \( \Omega \) have bounded diameter and \( f^j|\Omega \) is univalent for all \( j \geq 1 \). This implies that \( \Omega \) is contained in the Fatou set. We make sure that the points \( x_n \) are pre-images of an attracting fixed point of \( f \), and since they accumulate everywhere on \( b\Omega \) it follows that \( \Omega \) is a Fatou component. The fact that the orbit of \( \Omega \) is escaping/oscillating will follow directly from the construction.

As an immediate consequence of our results we get the following corollary.

**Corollary 4.** Every Jordan curve is the boundary of a wandering Fatou component of some entire function.

1.1. **Notations:** An open set \( U \) is called regular if it coincides with the interior of its closure. A compact \( K \) set is called polynomially convex if it is equal to its polynomial hull, t.i. to the compact set \( \hat{K} = \{ z \in \mathbb{C} \mid |p(z)| \leq \| p \|_K \text{ for all polynomials } p \} \). This is equivalent to saying that a compact set \( K \subset \mathbb{C} \) is polynomially convex if and only if \( \mathbb{C} \setminus K \) is connected. From here it follows that all the components of the interior of \( K \) are simply connected. Throughout this paper we will use \( \Delta(p,r) \) to denote an open disk of radius \( r \) centered at \( p \). We will also use the over-line \( \overline{U} \), to denote the closure of the set \( U \), and \( bU \) to denote the boundary of \( U \).
2. Proof of Theorem 1

We start the proof by choosing a sequence of points \((x_n) \in \mathbb{C} \setminus \Omega\) satisfying the following properties:

1. \(d(x_n, b\Omega) > d(x_{n+1}, b\Omega)\) for all \(n \geq 1\),
2. \(\omega(x_n) = b\Omega\), i.e. \((x_n)\) accumulates on the boundary of \(\Omega\).

As we have mentioned in the introduction the idea is to construct an entire function \(f\) so that the iterates of \(\Omega\) escape every compact set and have bounded diameter. Moreover the iterates of \(f\) are univalent on \(\Omega\) and the points \(x_n\) are pre-images of an attracting fixed point of \(f\). Such function will be obtained as a limit of an inductively constructed sequence of entire functions given by the following lemma.

**Lemma 5.** There exists a sequence \((f_k)_{k \geq 0}\) of entire function, disjoint sequences of points \((p_n)_{n \geq 0}\), \((x^j_n)_{j \geq 0}\), a point \(s_0\) and a sequence positive real numbers \((R_k)_{k \geq 0} \nearrow \infty\) and \(\rho > 0\), such that the following properties are satisfied:

- \((a)\) \(\|f_{k+1} - f_k\|_{\Delta(0, R_k)} \leq 2^{-k}\) for all \(k \geq 0\),
- \((b)\) \(\|p_k\| > R_k\) for all \(k \geq 1\),
- \((c)\) \(f^j_k(\Omega) \subset \Delta(p_j, \rho)\), for all \(1 \leq j \leq k\) and all \(k \geq 1\),
- \((d)\) \(f^j_k(\Omega)\) is univalent for all \(1 \leq j \leq k\) and all \(k \geq 1\),
- \((e)\) \(f^j_k(x^j_0) = x^j_l\) for all \(0 \leq l < j \leq k\) and all \(k \geq 1\),
- \((f)\) \(f^j_k(x^j_0) = s_0\) for all \(1 \leq j \leq k\) and all \(k \geq 1\),
- \((g)\) \(f_k(s_0) = s_0\) and \(f'_k(s_0) = \frac{1}{2}\) for all \(k \geq 1\).

**Remark 2.1.** Conditions \((e) - (g)\) of the Lemma imply that \(x^j_k = s_0\) for all \(j \geq k \geq 1\).

**Proof of the Lemma:** Let \((x_n)_{n \geq 1}\) be a sequence as above the lemma and define \(x^0_n := x_n\) for all \(n \geq 1\). Let \((\lambda_n)_{n \geq 0}\) be a sequence of compact polynomially convex neighbourhoods of \(\Omega\) satisfying:

- \((i)\) \(\lambda_{n+1} \subset \text{int}(\lambda_n)\) for all \(n \geq 0\),
- \((ii)\) \(x^0_n \in \lambda_n\) if and only if \(j > n \geq 0\).

Let \(R_0 = 0\), \(p_0 \in \Omega\) and \(f_0(z) = z\). Let \(\rho > 0\) such that \(\lambda_0\) is contained in the disk \(\Delta(p_0, \rho)\) and let us fix some point \(s_0 \in \mathbb{C} \setminus \overline{\Delta(p_0, \rho)}\). In this setting all conditions \((a) - (g)\) are trivially satisfied for \(k = 0\). We proceed with the constructions satisfying the conditions for \(k + 1\).

Let \(R_{k+1} > R_k + 1\), such that \(\Delta(0, R_k) \cup \Delta(p_k, \rho) \subset \Delta(0, R_{k+1})\). Choose any point \(p_{k+1} \in \mathbb{C}\), such that \(\overline{\Delta}(p_{k+1}, \rho)\) and \(\Delta(0, R_{k+1})\) are disjoint. Let us define \(x^{k+1}_\ell := f^\ell_k(x^k_0)\) for \(0 \leq \ell \leq k\).

We are now ready to construct the map \(f_{k+1}\). Let us define compact sets

\[
K_1 := \overline{\Delta}(0, R_k), \quad K_2 := s_0, \quad K_3 := f^k_k(U_{k+1}), \quad K_4 := x^{k+1}_k
\]

and functions

\[
h_1(z) := f_k(z), \quad h_2(z) := \frac{1}{2}(z - s_0) + s_0, \quad h_3(z) = z - p_k + p_{k+1}, \quad h_4(z) := s_0.
\]

By the Runge approximation theorem, for every \(\varepsilon_{k+1} > 0\) there exists an entire function \(f_{k+1}\), such that:

- \((I)\) \(\|f_{k+1} - h_j\|_{K_j} \leq \varepsilon_{k+1}\) for every \(1 \leq j \leq 4\),
- \((II)\) \(f_{k+1}(x^j_k) = f_k(x^j_k)\) for all \(0 \leq \ell < j \leq k + 1\),
- \((III)\) \(f_{k+1}(x^{k+1}_k) = s_0\),
- \((IV)\) \(f_{k+1}(s_0) = s_0\) and \(f'_{k+1}(s_0) = \frac{1}{2}\),

where we have chosen \(\varepsilon_{k+1} \leq \frac{1}{2\pi}\) small enough such that:

- \((i)\) \(f_{k+1}(U_{k+1}) \subset \Delta(p_j, \rho)\), for all \(1 \leq j \leq k + 1\)
- \((ii)\) \(f_{k+1}^{j+1}(U_{k+1})\) is univalent for all \(1 \leq j \leq k + 1\)
Figure 1. The figure shows the first three iterates of \( f \) acting on the wandering domain \( \Omega \), which lies inside the black disk. There are three points around this disk that represent points \( x_0^1, x_0^2, x_0^3, \) and \( x_0^4 \), which are part of the sequence of points that accumulates every on the boundary of \( \Omega \). Points \( x_0^1, x_0^2, x_0^3, \) and \( x_0^4 \), that belong to the orbit of a point \( x_0^1, x_0^2, x_0^3, \) and \( x_0^4 \) respectively, and are mapped to the attracting fixed point \( s_0 \). The action of \( f \) on the orbit of \( \Omega \) is approximatively a translation.

The entire function \( f_{k+1} \) satisfies all the conditions \((a) – (g)\) of the Lemma, hence this completes the inductive step.

Let us continue with the proof of Theorem 1. Let \((R_k)_{k \geq 0} \nearrow \infty\) be an increasing sequence of positive real numbers, let \( s_0 \) a point, let \((p_k)\) be sequences of points and let \((f_n)\) be a sequence of entire functions as given by the lemma above. The sequence of entire functions \((f_n)\) converges uniformly on compacts to an entire function \( f \) satisfying:

1. \( \|p_k\| > R_k \) for all \( k \geq 0 \),
2. \( f^k(\Omega) \subset \Delta(p_k, \rho) \) for all \( k \geq 0 \),
3. \( f^k|_\Omega \) is univalent for all \( k \geq 0 \)
4. \( f^n(x_n) = s_0 \) as for all \( n \geq 1 \),
5. \( f(s_0) = s_0 \) and \( f'(s_0) = \frac{1}{2} \).

It follows from (1) and (2) that the orbit of \( \Omega \) leaves every compact set and that the Euclidean diameter of \( f^k(\Omega) \) remains bounded for all \( k \geq 0 \), hence \( \Omega \) is contained in some Fatou component \( F_0 \) of \( f \). By (5) the point \( s_0 \) is an attracting fixed point of \( f \) and therefore \( s_0 \notin F_0 \). Finally (4) implies that the pre-images of \( s_0 \) accumulate everywhere on the boundary of \( \Omega \), hence \( F_0 = \Omega \).

Remark 2.2. According to the classification in [4] we can conclude from the proof of Theorem 1 that our escaping wandering domain is of the \textit{eventually isometric} type and the orbits of all points \textit{stay away} from the boundary. With a small adaptation of the proof we could easily obtain the other two possible types of behaviour of the orbits of points in terms of convergence to the boundary, i.e. the \textit{bungee} and the \textit{convergent} type.

3. Proof of Theorem 2

Without the loss of generality we may assume that \( \frac{2}{3} \notin \Omega \subset \Delta(\frac{2}{3}, \frac{1}{9}) \), otherwise we apply the linear change of coordinates. Let \( (x_n) \in \Delta(\frac{2}{3}, \frac{1}{9}) \setminus \Omega \) be a sequence of points satisfying the following properties:

1. \( d(x_n, h\Omega) > d(x_{n+1}, h\Omega) \) for all \( n \geq 1 \),
2. \( \omega((x_n)) = h\Omega \), i.e. \( (x_n) \) accumulates on the boundary of \( \Omega \).

Furthermore let \((U_n)_{n \geq 0}\) be a sequence of compact polynomially convex neighbourhoods of \( \Omega \) satisfying:

1. \( U_0 \subset \Delta(\frac{2}{3}, \frac{1}{9}) \)
2. \( U_{n+1} \subset \text{int}(U_n) \) for all \( n \geq 0 \),
3. \( x_j \in U_n \) if and only if \( j > n \geq 0 \).
We will obtain or function \( f \) by taking a limit of an inductively constructed sequence of entire functions given by the following lemma.

**Lemma 6.** There exists a sequence \((f_k)_{k \geq 0}\) of entire function, sequence \((x^j_k)_{j \geq 1, n \geq 0}\), point \( s_0 \), a sequence positive real numbers \((R_k)_{k \geq 0}\) \( \not\to \infty \) and strictly increasing sequence of integers \((N_k)_{k \geq 0}\) satisfying \( N_0 = 0 \) such that the following properties are satisfied:

(a) \( \|f_{k+1} - f_k\|_{\Delta(0,R_k)} \leq 2^{-k} \) for all \( k \geq 0 \),
(b) \( f_k^N_k(\Omega) \subset \Delta(2k,1) \) for all \( k \geq 1 \),
(c) \( f_k^{N_k+1}(\Omega) \subset \Delta(0,\frac{1}{2k+1}) \) for all \( k \geq 1 \),
(d) \( f_k((2k+1) \Delta(0,0)) = 0 \),
(e) \( f_k|_{\Omega} \) is univalent for all \( 1 \leq j \leq N_k \),
(f) \( f_k(x^j_k) = x^{j+1}_k \) for all \( 0 \leq \ell < N_k \), all \( 1 \leq j \leq k \),
(g) \( f_k(x^j_{N_k+1}) = s_0 \) for all \( 1 \leq j \leq k \),
(h) \( f_k(s_0) = s_0 \) and \( f'_k(s_0) = \frac{1}{2} \) for all \( k \geq 1 \).

**Remark 3.1.** Conditions (f) – (h) of the Lemma imply that \( x^j_k = s_0 \) for all \( j > N_{k-1} \) and \( k \geq 1 \).

**Proof of the Lemma:** Let us define an annuli \( A_k := \Delta(0,\frac{2k}{2k+3}) \) for all \( k \geq 0 \) and choose a point \( s_0 \in A_0 \setminus U_0 \). Observe that \( \Omega \subset U_0 \subset A_0 \). As before we define \( x^0_k := x_n \) for all \( n \geq 1 \).

Let \( R_0 = 0 \), \( N_0 = 0 \) and \( f_0(z) = z \). In this setting all conditions (a) – (h) are trivially satisfied for \( k = 0 \). We proceed with the constructions satisfying the conditions for \( k + 1 \).

Let us define \( R_{k+1} := 2k + 1 \), \( N_{k+1} := N_k + k + 2 \) and \( x^{k+1}_\ell := f'_k(x^{k+1}_\ell) \) for all \( 0 \leq \ell \leq N_k \).

We are now ready to construct the map \( f_{k+1} \). Let us define compact sets

\[ K_1 := \overline{A}(0,R_k), \quad K_2 := s_0, \quad K_3 := f_k^{N_k}(U_{k+1}), \quad K_4 := x^{k+1}_N, \quad K_5 := f_k^{N_{k+1}}\left(A\left(0,\frac{1}{2k+3}\right)\right) \]

and functions

\[ h_1(z) := f_k(z), \quad h_2(z) := \frac{1}{2}(z - s_0) + s_0, \quad h_4(z) := s_0, \quad h_5(z) := z + 2. \]

Finally let \( h_3 \) be a linear map that satisfies \( h_3(p_{N_k}) = \frac{1}{2k+4} \) and \( h_3(K_3) \subset A_{k+1} \).

By the Runge approximation theorem, for every \( \varepsilon_{k+1} > 0 \) there exists an entire function \( f_{k+1} \), such that:

(I) \( \|f_{k+1} - h_j\|_{K_j} \leq \varepsilon_{k+1} \) for every \( 1 \leq j \leq 5 \),
(II) \( f_{k+1}(x^j_k) = f_k(x^j_k) \) for all \( 0 \leq \ell < N_k \) and \( 0 \leq j \leq k + 1 \),
(III) \( f_{k+1}(x^{k+1}_N) = s_0 \),
(IV) \( f_{k+1}(s_0) = s_0 \) and \( f'_{k+1}(s_0) = \frac{1}{2} \),
(V) \( f'_{k+1}(0) = 2j \) for all \( 0 \leq j \leq k + 1 \)
(VI) \( f'_{k+1}(U_{k+1}) \subset \Delta(2k + 2,\frac{1}{2}) \)

where we have chosen \( \varepsilon_{k+1} \leq \frac{1}{2\pi} \) small enough such that:

(i) \( f'_{k+1}(U_{k+1}) \subset \bigcup_{j \geq 0} \Delta(2j,\frac{1}{2}) \), for all \( 1 \leq j \leq N_{k+1} \),
(ii) \( f'_{k+1} \) is univalent on \( \overline{A}\left(0,\frac{1}{2k+4}\right) \) for all \( 1 \leq j \leq N_{k+1} \).

Observe that (ii) implies that \( f'_{k+1}|_{U_{k+1}} \) is univalent for all \( 1 \leq j \leq N_{k+1} \). The entire function \( f_{k+1} \) satisfies all the conditions (a) – (h) of the Lemma, hence this completes the inductive step.

Let us continue with the proof of Theorem 2. Let \( s_0 \) be a point, let \( (N_k)_{k \geq 0} \) be strictly increasing sequence of integers and let \( (f_n) \) be a sequence of entire functions as given by the lemma above. The sequence of entire functions \((f_n)\) converges uniformly on compacts to an entire function \( f \) satisfying:

(1) \( f^{N_k}(\Omega) \subset \Delta(2k,\frac{1}{2}) \) for all \( k \geq 1 \),
Figure 2. The figure shows the first six iterates of $f$ acting on the wandering domain, which lies inside the black disk. There are three points around this disk which represent points $x_0^1, x_0^2$ and $x_0^3$ and which are part of the sequence of points that accumulates every on the boundary of $\Omega$. Points $x_{N_0}^1, x_{N_1}^2$, and $x_{N_2}^3$, that belong to the orbit of a point $x_0^1, x_0^2$ and $x_0^3$ respectively, and are mapped to the attracting fixed point $s_0$.

(2) $f^{N_k+1}(\Omega) \subset \Delta(0, \frac{1}{2k+1})$ for all $k \geq 0$,
(3) $f^k(\Omega) \subset \bigcup_{j \geq 0} \Delta(2i, \frac{1}{2})$, for all $k \geq 1$,
(4) $f^k|\Omega$ is univalent for all $k \geq 1$,
(5) $f^{N_k+1}(x_{k+1}) = s_0$ as for all $k \geq 0$,
(6) $f(s_0) = s_0$ and $f'(s_0) = \frac{1}{2}$
(7) $f^k(0) = 2k$ for all $k \geq 1$

These properties imply that $\Omega$ is contained in the Fatou set. Since the pre-images of an attracting fixed point $s_0$ accumulate everywhere on the boundary of $\Omega$ it follows that $\Omega$ is a Fatou component. Finally properties (1) and (2) imply that $\Omega$ is an oscillating wandering domain.

**Remark 3.2.** At the beginning of the proof we have made an assumption that $\Omega$ is contained in a certain disk, which can be achieved by a linear change of coordinates. Given such $\Omega$, we can deduce form the construction the entire function $f$, that the orbit of the oscillating wandering domain $\Omega$ accumulates on non-negative even integers, t.i. for every $p \in \Omega$ we have $\omega((f^n(p))) = 2N$. This implies that the general case, the accumulation of the oscillating orbit will be linearly equivalent to $\mathbb{N}$.

The oscillating wandering domain from Theorem 2 is of the *eventually isometric* type but the orbits of all points *converge* to the boundary. With a small adaptation of the proof we could also obtain a *bungee* type behaviour in terms of the convergence of the orbits of points towards the boundary.

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