Fundamental Limits of Biometric Identification Systems with Strong Secrecy

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Abstract—The fundamental limits of biometric identification systems under a strong secrecy criterion are investigated. In the previous studies of this scenario, the fundamental trade-off among secrecy, template, privacy- and secrecy-leakages has been revealed in the case where there is only one user, while the case of multiple users has not been discussed yet. In this study, we consider the system with exponentially many users, and we characterize the capacity region of the rate tuples including the user (identification) rate under the strong secrecy criterion. In the achievability proof, we derive a new method to incorporate multiple users by extending the random binning for one user’s case. The obtained result shows that the characterization of the capacity region does not vary regardless of the weak or strong secrecy criterion in terms of secrecy-leakage.

I. INTRODUCTION

Biometric identification system (BIS) indicates a system using physical characteristics of living individual to identify its identity from a large group of people. For example, border control systems, medical record systems, etc., are some familiar practical application of the BIS. As large number of individuals are involved, the issues of privacy protection and storage saving have been broadly discussed.

In general, from information theoretic perspectives, the studies on the BIS investigate the maximum number of individuals and secret keys (usability) while minimizing error probability (reliability), memory usages (storage), and information leakage of individuals (security). O’Sullivan and Schmid [1] and Willems et al. [2] firstly introduced the BIS where the bio-data sequence is stored in the plain form. Willems et al. [2] characterized the identification capacity of the BIS. Tuncel [3] added constraint (encoding) on the helper data, which we call template in this paper, stored in the database and clarified the fundamental trade-off between identification and template rates. Some extended works considering the rate distortion can be found in [4] and [5].

Ignatenko and Willems [6] studied the BIS with estimating both individual’s index and secret key. In their paper, two common BIS models: Generated Secret BIS (GS-BIS) model and Chosen Secret BIS (CS-BIS) model were analyzed. In the GS-BIS model, secret key is extracted from bio-data sequence, while in the CS-BIS model the secret key is chosen independently of it. They characterized the capacity regions of identification, secrecy, and privacy-leakage rates in both models.

Here, privacy-leakage is the amount of information leakage between bio-data sequence and its corresponding template. Another security measurement in the BIS is secrecy-leakage, which represents the leaked information through secret key and template generated by the same bio-data sequence. A GS-BIS model without privacy-leakage by associating a private key was studied in [7]. Instead of evaluating the privacy-leakage, the GS-BIS model considering the template rate can be found in [8]. Furthermore, the GS-BIS model with the presence of an adversary at the decoder was discussed in [9]. Recently, Yachongka and Yagi characterized the capacity regions of more general models in [10] for the GS-BIS model and in [11] for the CS-BIS model via two auxiliary RVs. Compared to the models proposed in [6]–[9], they treated a noisy enrollment channel (hidden source model). However, all the studies mentioned above [6]–[11] measured the secrecy-leakage of individual under the weak secrecy criterion.

Another stream of studies like [12]–[18] copes with only the estimation of one individual’s secret key. More specifically, the studies in [12] and [13] investigated the fundamental trade-off of secrecy and privacy-leakage. The fundamental limits of secrecy, template, and privacy-leakage rates are argued in [15] for non-perfect secrecy-leakage and in [14] for perfect secrecy-leakage under the weak secrecy criterion. Studies on the BIS considering secrecy-leakage under the strong secrecy criterion can be found in [16] for a hidden source model, in [17] for broadcast channel measurements, and in [18] for a multi-access system. However, the BIS with multiple users under the strong secrecy criterion have not yet been considered.

In this study, we aim to characterize the capacity region of the models proposed in [10] and [11] under the strong secrecy criterion. In the achievability proof, we deployed the techniques developed by Yassaee et. al. [19] to verify that the total variation between the joint distribution of secret key and template assigned by random binning is exponentially close to their uniform distributions. This technique is also used in the proof of [16] and [17] for the case of one individual, and we extend it to the case of exponentially many users. Compared to the analyses in [16] and [17], the challenges are the analyses of error probability and privacy-leakage rate. We find out that it is required the sum of template and common randomness rates to be larger than the condition shown in
these studies in order to ensure that both index and secret key of the identified individual are reliably estimated. For dealing with the difficulties of the privacy-leakage rate, we introduce a virtual decoder for deriving the bound in the achievability proof, whose functions will be made clear later on. As a result, It is shown that the characterizations of the capacity regions do not change regardless of the weak or strong secrecy criterion in respect of secrecy-leakage.

II. Preliminaries

In this section, we define notation used in this paper and introduce the GS-BIS and CS-BIS models.

A. Notation

Upper-case such that $A$ denotes a random variable (RV) taking values in $A$ and lower-case $a \in A$ denotes its realization. Calligraphic letter $\mathcal{A}$ stands for a finite set and its size is written as $|\mathcal{A}|$. Superscripts represent a string of RVs, e.g., $A^n = (A_1, \cdots, A_n)$, taking values in $A^n$, and subscripts represent the position of a RV in the string. $P_A(a) = \Pr[A = a]$, $a \in A$, represents the probability distribution on $A$ and $P_A^n$ denotes the uniform distribution on the set $A$. $P_{A^n}$ represents the probability distribution of RV $A^n \in A^n$. $P_{A^n|B^n}$ represents the joint probability distribution of a pair of RVs $(A^n, B^n)$ and its conditional probability distribution $P_{A^n|B^n}$ is defined as

$$P_{A^n|B^n}(a^n|b^n) = \frac{P_{A^n=\mathcal{B}^n}(a^n,b^n)}{P_{B^n}(b^n)}$$

for any $a^n \in A^n$, $b^n \in B^n$ such that $P_{B^n}(b^n) > 0$. The total variation between two probability mass functions $P_A$ and $Q_A$ on the same alphabet $\mathcal{A}$ is defined as

$$\forall(P_A, Q_A) = \frac{1}{2} \sum_{a \in \mathcal{A}} |P_A(a) - Q_A(a)|.$$  

Throughout this paper, logarithms are of base two. Integers $x$ and $y$ such that $x < y$, $[x, y]$ denotes the set $\{x, x+1, \cdots, y\}$. A partial sequence of a sequence $c^n$ from the first symbol to the $t$th symbol $(c_1, \cdots, c_t)$ is represented by $c^t$. $A_{\text{st}}^n(X)$ denotes the strongly typical set (cf. [20], [21]).

B. System Models

The GS-BIS and CS-BIS models considered in this paper are depicted in Fig. 1 and Fig. 2, respectively. Let $I = [1, M_I]$ be the set of indices of individuals. For any $i \in I$, we assume that $X_i^n$ (bio-data sequence of individual $i$) with realizations on $X^n$ is generated independently and identically distributed (i.i.d.) from a discrete memoryless source $P_X$, $P_{Y|X}$, and $P_{Z|X}$ are memoryless channels at the encoder and decoder, observing $Y_i^n$ with realizations on $Y^n$ and $Z^n$ with realizations on $Z^n$, respectively, when $X_i^n$ is input. Thus, the joint distribution is given by

$$P_{Y_i^n, X_i^n, Z^n}(y_i^n, x_i^n, z^n) = P_{Y_i^n|X_i^n}(y_i^n|x_i^n)P_{X_i^n}(x_i^n)P_{Z^n|X^n}(z^n|x_i^n) = \prod_{i=k}^n P_{Y|X}(y_{ik}|x_{ik})P_{X}(x_{ik})P_{Z|X}(z_{ik}|x_{ik})$$

In the GS-BIS model, upon observing $Y_i^n$, the encoder $f$ generates a secret key $S(i) \in S$ and template $J(i) \in J$ as $(S(i), J(i)) = f(Y_i^n)$, where $S = [1, M_S]$ and $J = [1, M_J]$ are sets of secret key and templates. $J(i)$ is stored at position $i$ in the system database and $S(i)$ is handed back to the individual $i$. The decoder $g$ sees $Z^n$ and estimates the pair of index and secret key by

$$(\hat{W}, \hat{S}(w)) = g(Z^n, J_{\mathcal{M}_I}),$$

where $J_{\mathcal{M}_I}$ denotes the set of templates (the database) $\{J(1), \cdots, J(M_I)\}$.

Next, we define $J'(i)$ and $S'(i)$ as the template and secret key of individual $i$ for the CS-BIS model. In the CS-BIS model, $S'(i)$ is chosen independently of other RVs, e.g., $Y_i^n, X_i^n, Z^n$, and the encoder generates the template as $J'(i) = f(Y_i^n, S'(i))$ for every individual. After receiving $Z^n$, the decoder uses $Z^n$ and all templates stored in the database to reveal the identified individual’s index and secret key as

$$(\hat{W}, \hat{S'}(w)) = g(Z^n, J'_{\mathcal{M}_I}),$$

where $J'_{\mathcal{M}_I} = \{J'(1), \cdots, J'(M_I)\}$. 

Fig. 1. The GS-BIS model

Fig. 2. The CS-BIS model
III. PROBLEM FORMULATIONS AND MAIN RESULTS

In this section, we state the definitions and main results about the GS-BIS and CS-BIS models.

**Definition 1. (Achievability for GS-BIS model)**

A tuple of an identification, secrecy, template, and privacy-leakage rates \((R_I, R_S, R_J, R_L)\) is said to be achievable for the GS-BIS model if for any \(\delta > 0\) and large enough \(n\) there exist pairs of encoders and decoders that satisfy

\[
\max_{i \in I} \Pr\{ (\hat{W}, S(W)) \neq (W, S(W)) | W = i \} \leq \delta, \\
\frac{1}{n} \log M_I \geq R_I - \delta, \\
\frac{1}{n} \log M_J \leq R_J + \delta, \\
\min_{i \in I} \frac{1}{n} H(S(i)) \geq R_S - \delta, \\
\max_{i \in I} \frac{1}{n} I(X_i^n; J(i)) \leq R_L + \delta, \\
\max_{i \in I} I(S(i); J(i)) \leq \delta. 
(11)
\]

In the previous studies on the GS-BIS model, the secrecy-leakage were measured by \(\max_{i \in I} \frac{1}{n} I(S(i); J(i)) \leq \delta\), e.g. [6], [7], [9], and [11]. This constraint is generally called weak secrecy. Compared to the weak secrecy criterion, the condition \((11)\) is more rigorous since it is required that the quantity of secrecy-leakage for every individual \(i\) must not increase in accordance with the block length \(n\) (arbitrarily small constant). The condition \((11)\) is known as strong secrecy.

**Remark 1.** In [6] and [7], for bounding the sum rate of identification and secret key rates in the proof of converse part, a stronger requirement that the distribution of secret key of every individual must be almost uniform, i.e. \(\min_{i \in I} \frac{1}{n} \log M_S(i) \geq \delta \geq \frac{1}{n} \log M_S\), is included in [7]. However, this requirement was not necessary in the general problem formulation of the GS-BIS model.

**Definition 2. (Achievability for CS-BIS model)**

A tuple \((R_I, R_S, R_J, R_L)\) is said to be achievable for the CS-BIS model if there exist pairs of encoders and decoders that satisfy all the requirements in Definition [7] for any \(\delta > 0\) and large enough \(n\) by replacing \(S(i)\) and \(J(i)\) with \(S'(i)\) and \(J'(i)\).

**Definition 3.** \(\mathcal{R}_G\) and \(\mathcal{R}_C\) are defined as the closure of the set of all achievable rate tuples for the GS-BIS and CS-BIS models, respectively, called the capacity regions.

**Remark 2.** Note that all individuals are encoded by the same encoding function \(f(:)\). Another possible system is that each individual is encoded by a different encoding function, i.e., the encoder has a set of encoding functions \( \{f_i(:)\}_{i=1}^{M_1} \) for enrollment and those functions are shared between the encoder and decoder. However, either system model corresponds to the equivalent characterization of the capacity region.

**Theorem 1.** The capacity region for the GS-BIS is given by

\[
\mathcal{R}_G = \{(R_I, R_S, R_J, R_L) : \\
R_I + R_S \leq I(Z; U), \\
R_J \geq I(Y; U) - I(Z; U) + R_I, \\
R_L \geq I(X; U) - I(Z; U) + R_I, \\
R_I \geq 0, R_S \geq 0 \\
\text{for some } U \text{ s.t. } Z - X - Y - U \}
\]

where auxiliary RV \(U\) takes values in a finite alphabet \(U\) with \(|U| \leq |Y| + 2\).

**Theorem 2.** The capacity region for the CS-BIS is given by

\[
\mathcal{R}_C = \{(R_I, R_S, R_J, R_L) : \\
R_I + R_S \leq I(Z; U), \\
R_J \geq I(Y; U), \\
R_L \geq I(X; U) - I(Z; U) + R_I, \\
R_I \geq 0, R_S \geq 0 \\
\text{for some } U \text{ s.t. } Z - X - Y - U \}
\]

where \(U\) satisfies the same conditions as in Theorem 1.

The characterizations of Theorem 1 and Theorem 2 are identical to the results given in [10] Theorem 1] and in [11] Theorem 1] under the weak secrecy criterion, respectively. This indicates the capacity regions under weak or strong secrecy is characterized in the same form. Furthermore, the correspondence to the one given in [16] Theorem 5] can be confirmed by setting \(R_I = 0\).

**Remark 3.** The observations given in [14] Theorem 2] and [22] Theorem 2] for the case of single and multiple users, respectively, indicate that \(\mathcal{R}_G\) is clearly wider than \(\mathcal{R}_C\), which is due to the range of \(R_J\). A remark given in [8] implied that in case where the enrollment channel is noiseless \((X = Y)\), the fundamental limit of \(R_I\) and \(R_J\) is identical for the GS-BIS model under the weak secrecy criterion. The claim also holds for the GS-BIS model under the strong secrecy criterion. However, this claim can not be applied to the CS-BIS model.

In the following section, we highlight only some key points regarding the proof Theorem 1. Indeed, the proof of Theorem 2 (CS-BIS model) is based on the arguments of Theorem 1. The additional task is to use the secret key generated from the GS-BIS encoder to mask the chosen secret key of the CS-BIS model by implementing a one-time pad operation as shown in Fig. 3. For details, readers should refer to [6], [10], and [14] for example to see how the arguments for the GS-BIS model is applied to the CS-BIS model.

IV. PROOF OF THEOREM 1

**A. Achievability (Direct Part)**

First, fix \(\delta > 0\) and the length of codeword \(n\). We choose a test channel \(P_{U|Y}\) and set \(0 < R_I < I(U; Z), R_S = I(Z; U) - R_I - \delta, R_J = I(Y; U) - I(Z; U) + R_I + 2\delta, R_L = I(X; U) - I(Z; U) + R_I + 2\delta, \) and \(R_C = H(U|Y) - 2\delta.\)
also, we set \( M_I = 2^{n R_I}, M_S = 2^{n R_S}, M_J = 2^{n R_J}, \) and \( M_C = 2^{n R_C}. \) Here, \( R_C \) and \( M_C \) denote the rate and size of common randomness.

**Binning:** For each sequence \( u^n \in A^n(U) \), draw a index tuple \((s, j, c)\) uniformly at random from \([1, M_S] \times [1, M_J] \times [1, M_C]\), where \( s, j, \) and \( c \) represent the secret key, template, and common randomness, respectively. The sequences with the same indexes of template \( j \) and common randomness \( c \) form a bin \( B(j, c). \) The index assignments define a mapping function \( e : \mathbb{U}^k \rightarrow [1, M_S] \times [1, M_J] \times [1, M_C] \), which is revealed to the encoder and the decoder.

This is random binning with common randomness, where the index of common randomness \( c \) is fixed and shared by the encoder and the decoder.

**Encoding (Enrollment):** Given an observation \( y^n_i \) and \( c \), the encoder looks for \( u^n \) in the bin \( B(j, c) \) such that \((y^n_i, u^n) \in A^n(Y U). \) It determines \((s(i), j(i))\) obtains based on \( u^n \), i.e. \((s(i), j(i), c) = e(u^n). \) \( j(i) \) is stored at the position \( i \) in the database and \( s(i) \) is handed back to the individual. If there are several such \( u^n \), it picks one of them at random. In case, there is no such \( u^n \), it randomly selects a sequence from the bin \( B(1, 1, c). \)

**Decoding (Identification):** Observing \( z^n \) (the noisy sequence of \( x^n_i \)) and the database \( J_{M_J} \), it looks for \( u^n \in B(j, i), c \) such that \((z^n, u^n) \in A^n(ZU) \) for some \( i. \) If there is a unique pair \( i \) and \( u^n \), the decoder outputs \( \hat{w} = i \) and \( s(\hat{w}) = s \) as the corresponding secret key of the unique \( u^n \), i.e., \( e(u^n) = (s, j(i), c). \) Otherwise, it declares error.

**Analysis of Error Probability:** We bound the probability of error averaged over bin assignments. Let \( C \) be the RV of common randomness. Let \( S(i) \) and \( J(i) \) denote the random indices for \( U^n_i \) chosen by the encoder based on \( Y^n_i \) for given \( C. \) Notice that \( U^n_i \in B(J(i), C) \). For \( W = i, \) the error event at encoder is

1) \( E_1: (Y^n_i, U^n_i) \notin A^n(U Y) \)
2) \( E_2: (Z^n, U^n_i) \notin A^n(Z U) \)
3) \( E_3: \exists i' \neq i \) such that \( \hat{u}^n \in B(J(i'), C) \) and \((Z^n, \hat{u}^n) \in A^n(Z U) \) for some \( \hat{u}^n \).

4) \( E_4: \hat{u}^n \in B(J(i), C) \) and \((Z^n, \hat{u}^n) \in A^n(Z U) \) for some \( \hat{u}^n \neq \hat{u}^n. \)

Thus, we have

\[
\text{Pr}\{(\hat{W}, \hat{s}(\hat{W})) \neq (W, S(W))|W = i\} = \Pr\Big\{\bigcup_{i=1}^{4} E_i|W = i\Big\} \leq \sum_{i=1}^{4} \Pr\{E_i|W = i\} \tag{14}
\]

Thus, \( \Pr\{E_3|W = i\} \) goes to zero as \( n \) tends to infinity because the sum rate of \( R_J + R_C > I(Y; U) \) and \( \Pr\{E_3|W = i\} \) goes to zero by the law of large numbers. For \( \Pr\{E_3|W = i\}, \) it can be evaluated as follows. Here, we newly define the event \( E_3(i'): \hat{u}^n \in B(J(i'), C) \) and \((Z^n, \hat{u}^n) \in A^n(Z U) \) for some \( \hat{u}^n. \)

Then,

\[
\Pr\{E_3|W = i\} = \Pr\Big\{\bigcup_{i' \neq i} E_3(i')|W = i\Big\} \\
\leq M_I \Pr\{E_3(1)|W = i\} \\
= M_I \sum_{z^n} P_{Z^n}(z^n) \\
\cdot \Pr\{\hat{u}^n \in B(J(1), C)\} \\
= M_I \sum_{z^n} P_{Z^n}(z^n) \\
\cdot \Pr\{\hat{u}^n \in B(J(1), C)|Z^n = z^n\} \\
\overset{(a)}{=} M_I \cdot |A^n(U|z^n)| \\
\cdot \Pr\{\hat{u}^n \in B(J(1), C)|Z^n = z^n\} \\
\leq 2^{n(R_I + H(U|Z) + \delta)} \cdot 2^{-n(R_J + R_C + \epsilon)} \tag{15}
\]

for large enough \( n, \) where \( a \) follows because the events \{\( \hat{u}^n \in B(J(i), C) \)\} and \{\( Z^n \) \} are mutually independent. Since \( R_J + R_C > H(U|Z) + R_I \), \( \Pr\{E_3|W = i\} \) can be made arbitrarily small. By using [19] Lemma 1, \( \Pr\{E_3|W = i\} \) goes to zero as well because \( R_J + R_C > H(U|Z) \). Therefore, it can be made that

\[
\Pr\{(\hat{W}, \hat{s}(\hat{W})) \neq (W, S(W))|W = i\} \leq 4\delta \tag{16}
\]

for all sufficiently large \( n. \)
Next, we check the bounds of identification, secrecy, template, secrecy-leakage and privacy-leakage rates averaged over randomly chosen bin assignments $E_n$, which is an RV corresponding to $e$.

**Intermediate Step:**

We consider a system with a virtual decoder $g_i$ for deriving the upper bound on the privacy-leakage rate. In this system, knowing index $i$ (leads to obtain $J(i)$) and seeing $Z^n_i$ (defined as the output sequence of $X^n_i$ via $P_{Z|X}$), the virtual decoder $g_i$ estimates $U^n_i$ as $U^n_i = g_i(Z^n_i, J(i), C)$. Note that this system is just for analysis, and the virtual decoder is not actually used during the decoding process.

For any given $i \in I$, $g_i$ operates as follows: observing $z^n_i$, $j(i)$ and $c$, it looks for $u^n$ such that $(z^n_i, u^n) \in A^c_e(ZU)$ and $u^n \in B(j(i), c)$. If there are multiple such $u^n$, it declares error. The potential events of error probability for this case are $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_4$. Letting $P_e(i)$ be the error probability of $g_i$, we readily see that

$$P_e(i) \leq Pr\{(\hat{W}, \hat{S}(\hat{W})) \neq (W, S(W))|W = i\} \leq 4\delta,$$

where the middle term in (17) denotes the error probability of $g$ (the real decoder) for individual $W = i$.

The function of this virtual decoder enables us to bound the following conditional entropy

$$H(U^n_i | Z^n_i, J(i), C, E_n) \leq H(U^n_i | \hat{U}^n_i) \leq n\delta_n,$$

where

(b) follows because $\hat{U}^n_i$ is a function of $Z^n_i$ and $J(i)$ given $C$ and conditioning reduces entropy,

(c) follows because Fano’s inequality and (17) are applied, and we set $\delta_n = \frac{1}{2} (1 + 4n\delta \log |I|)$.

**Lemma 1.** For any fixed $\delta > 0$, it holds that

$$\frac{1}{n} H(Y^n_i | X^n_i, U^n_i, C, E_n) \leq H(Y | X, U) + \delta$$

for all large enough $n$.

(Proof) The proof is given in Appendix A-A.

**Lemma 2.** It holds that

$$\frac{1}{n} I(X^n_i; C|E_n) \leq \delta'_n,$$

$$\frac{1}{n} I(Y^n_i; X^n_i, C|E_n) \leq \delta'_n,$$

where $\delta'_n$ is some positive number satisfying $\delta'_n \downarrow 0$ as $n \to \infty$.

(Proof) The proofs are given in Appendix A-B.

**Analyses of Identification and Template Rates:**

Equations (7) and (8) follow obviously from the parameter settings.

**Analyses of Secrecy Rate and Secrecy-Leakage:**

By invoking [19] Theorem 1 and [23] Lemma 1 (alternative proof), it can be proved that the total variation $\mathbb{V}(P_{S(i),J(i),C}, P_{S(i),J(i),C}')$ converges to zero exponentially with $n$ for $R_S + R_I + R_C < H(U)$. This implies that (9) and (11) in Definition 1 hold.

**Analysis of Privacy-Leakage:**

We start by expanding the following mutual information as

$$I(X^n_i; J(i), C|E_n) = I(X^n_i; C|E_n) + I(X^n_i; J(i)|C, E_n)$$

(d) follows because conditioning reduces entropy, (e) follows because conditioning reduces entropy, (f) follows due to (20) in Lemma 2, (g) follows because $J(i)$ is a function of $(Y^n_i, C, E_n)$, (h) follows because conditioning reduces entropy, (i) follows because $H(J(i)) \leq nR_I$ and (18) is applied, (j) holds because $I(Y; U|X) = H(U|X) - H(U|Y) \geq I(Y; U) - I(X; U)$.

Thus, from (22), we have

$$\frac{1}{n} I(X^n_i; J(i)|C, E_n) \leq I(X; U) - I(Z; U) + R_I + 3\delta$$

$$= R_L + \delta$$

for large enough $n$.

Lastly, by applying the selection lemma [24] Lemma 2.2, there exists a good instance of binning $e$ and common randomness $c$ satisfying all conditions in Definition 1 for large enough $n$. 

$\square$
B. Converse Part

Notice that the capacity region under the strict secrecy criterion is contained in the one under the weak secrecy criterion. It has been shown in [22] that the capacity region of the GS-BIS model under the weak secrecy criterion is contained in the right-hand side of (13). Here, we write the detailed proof of converse part of the GS-BIS model under the weak secrecy criterion for reader’s sake.

A more relaxed case where $W$ is uniform is considered and the condition (4) in Definition 1 is replaced with the average error criterion $\Pr\{(W, S(W)) \neq (W, S(W))\} \leq \delta$.

We assume that a rate tuple $(R_t, R_S, R_J, R_L)$ is achievable. Here, we provide useful lemmas. For $t \in [1, n]$, we define an auxiliary RV $U_t = (Z_{t-1}, J(W), S(W))$. We denote strings of RVs by $X^n_W = (X_1(W), \ldots, X_n(W))$ and $Y^n_W = (Y_1(W), \ldots, Y_n(W))$.

**Lemma 3.** The following Markov chains hold

\[
Z_{t-1} - (Y_{t-1}(W), J(W), S(W)) - Y_t(W),
\]

\[
Z_{t-1} - (X_{t-1}(W), J(W), S(W)) - X_t(W).
\]

(Proof) The proofs are given in [10] Appendix C-A.

**Lemma 4.** There exists an RV $U$ satisfying $Z - X - Y - U$ and

\[
\sum_{i=1}^n I(Z_t; U_t) = n I(Z; U),
\]

\[
\sum_{i=1}^n I(Y_t(W); U_t) = n I(Y; U),
\]

\[
\sum_{i=1}^n I(X_t(W); U_t) = n I(X; U).
\]

(Proof) The proof is provided in [10] Appendix B.

**Analysis of the Sum of Identification and Secrecy Rates:**

We can show that

\[
R_I + R_S \leq I(Z; U) + 3\delta + 2\delta_n,
\]

where $\delta_n = \frac{1}{n}(1 + \log M_1 M_S)$ and $\delta_n \downarrow 0$ as $n \to \infty$.

(Proof) The proof is provided in Appendix B-A.

**Analysis of Template Rate:**

It is shown that

\[
R_J \geq I(Y; U) - I(Z; U) + R_I - (2\delta + \delta_n).
\]

(Proof) The proof is provided in Appendix B-B.

**Analysis of Privacy-Leakage Rate:**

The privacy-leakage rate can be bounded as

\[
R_L \geq I(X; U) - I(Z; U) + R_I - (2\delta + \delta_n).
\]

(Proof) The proof is provided in Appendix B-C.

Finally, by letting $n \to \infty$ and $\delta \downarrow 0$, it follows that the capacity region of the GS-BIS model is contained in the right-hand side of (12) from (29)–(31).

To complete the proofs of Theorem 1 and Theorem 2 we discuss the bounds on the cardinality of auxiliary RV $U$. For proving the bound on the cardinality of alphabet $U$ in the region $R_G$ (cf. (12)) and $R_C$ (cf. (13)), we use the support lemma in [21] Appendix C to show that the auxiliary RV $U$ should have $|U| \leq 1$ elements to preserve $P_U$ and add three more elements to preserve $H(Z(U), H(Y(U), H(X(U). This implies that it suffices to take $|U| \leq |U| + 2$ for preserving both $R_G$ and $R_C$.

V. CONCLUSION AND FUTURE WORK

In this study, the fundamental limits of BIS with exponentially many users under a strong secrecy criterion were analyzed. We characterized the capacity regions of identification, secrecy, template, and privacy-leakage rates in both the GS-BIS and CS-BIS models under the strong secrecy criterion in regard to secrecy-leakage. The method developed in [16] for characterizing the capacity region of the BIS where there exists only a single user is extended to deal with the one with multiple users. As a result, it was shown that the characterizations of the capacity regions remain the same even the security criterion of secrecy-leakage is shifted from weak to strong secrecy. For future work, we plan to clarify the fundamental limits of the BIS with the presence of adversary at the decoder.

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Appendix A

Supplementary Proofs for Direct Part

A. Proof of Lemma 7

We define a new random variable $T$ as follows:

$$T = \begin{cases} 1 \text{ if } (Y_i^n, X_i^n, U_i^n) \in A^n_i(YXU), \\ 0 \text{ otherwise.} \end{cases} \quad (32)$$

Notice that it has been shown that $P_T(0)$ or the probability which $(Y_i^n, X_i^n, U_i^n)$ are not jointly typical is smaller than $4\delta$ for large enough $n$ in the analysis of error probability.

We begin with expanding the entropy in the left-hand side of (19) as

$$H(Y_i^n | X_i^n, U_i^n, C, E_n) \leq 1 + 4n\delta \log |\mathcal{Y}| + \sum_{(x_i^n, u_i^n) \in A_i^n(YXU)} \{P_{X_i^n, U_i^n | T}(x_i^n, u_i^n | 1) \cdot \log |A_i^n(Y | x_i^n, u_i^n)| \}$$

(b) follows because the typical average lemma [21] and $\delta(\epsilon) = \epsilon H(Y | X, U)$ with small enough positive $\epsilon$.

Thus,

$$\frac{1}{n} H(Y_i^n | X_i^n, U_i^n, C, E_n) \leq H(Y_i^n | X_i^n, U_i^n) \leq H(Y_i^n | X_i^n, U_i^n, C, E_n) \leq H(Y_i^n | X_i^n)$$

where

(a) follows because $C - Y_i^n - X_i^n$ holds for given $E_n$, (b) follows because conditioning reduces entropy, (c) follows because [25] Theorem 6 is applied.

Hence,

$$\frac{1}{n} I(Y_i^n; C | X_i^n, E_n) \leq \delta'$$

for large enough $n$. \hfill \square
Appendix B
Supplementary Proofs for Converse Part

A. Analysis of the Sum of Identification and Secrecy Rates

We show that the claim of Remark 1 can be checked below. Again note that we are considering the case where \( W \) is uniformly distributed in this part. First, we start by evaluating the identification rate

\[
\log M_I = H(W) = H(W|\mathcal{J}_M, Z^n) + I(W; \mathcal{J}_M, Z^n)
\]

(a) follows because the pair \((\hat{W}, \hat{S}(W))\) is a function of \((Z^n, \mathcal{J}_M)\) (cf. (5)).

(b) follows because conditioning reduces entropy.

(c) follows due to Fano’s inequality and \( \delta_n = \frac{1}{n} (1 + \log M_I M_J) \) and \( \delta_n \downarrow 0 \) as \( n \to \infty \).

(d) follows because \( W \) is independent of other RVs.

(e) follows because only \( J(W) \) is possibly dependent on \( Z^n \) due to statistical independent among bio-data sequences.

Next, we consider the entropy of secrecy key as

\[
H(S(W)) = H(S(W)|\mathcal{J}_M, Z^n) + I(S(W); \mathcal{J}_M, Z^n)
\]

(i) holds because only \( Z^n, S(W), \) and \( J(W) \) are possibly dependent on each other.

(j) follows due to (11).

Combining (37) and (38), and from (7) and (9), we obtain

\[
R_I + R_S \leq \frac{1}{n} \left\{ H(Z^n) - H(Z^n|J(W), S(W)) \right\} + 3\delta + 2\delta_n
\]

where

(k) holds because \( \frac{1}{n} H(S(W)) \geq \min_{w \in \mathcal{Z}} \frac{1}{n} H(S(w)) \).

(l) is due to (26) in Lemma 4.

B. Analysis of Template Rate

The template rate can be evaluated as follows:

\[
n(R_I + \delta) \geq \log M_J \geq H(J(W)) = I(Y^n_W; J(W)|W) = I(Y^n_W; J(W)|\mathcal{J}_M)
\]

(a) follows because \( W \) is independent of other RVs.

(b) holds because \( S(W) \) is a function of \( Y^n_W \).

For the third term in (44), it can be evaluated further as

\[
H(Y^n_W, W, S(W)|\mathcal{J}_M)
\]

(c) follows because conditioning reduces entropy.

\[
\leq H(Y^n_W|S(W), J(W)) + H(Z^n) - H(Z^n|S(W), J(W))
\]

(d) follows due to Fano’s inequality.

\[
+ H(W, S(W)|\mathcal{J}_M, Z^n)
\]

(e) follows because only \( Z^n, S(W), \) and \( J(W) \) are possibly dependent on each other.

\[
\leq H(Y^n_W|S(W), J(W)) + H(Z^n)
\]
where
(c) follows because conditioning reduces entropy and \( \frac{4}{5} \) is taken into account,
(d) follows because conditioning reduces entropy,
(e) follows because Fano’s inequality.

Substituting (45) into (44), we obtain

\[
n(R_L + \delta) \geq H(W) + H(Y^n_W) - H(Y^n_W|S(W), J(W)) - H(Z^n) + H(Z^n|S(W), J(W)) - n\delta_n
\]

\[= \log M_I + \sum_{t=1}^{n} \left\{ H(Y_t(W)) - H(Y_t(W)|Y_t^{t-1}(W), S(W), J(W)) \right\}
- \sum_{t=1}^{n} I(Z_t; U_t) - n\delta_n
\]

\[= H(Y_t(W)|Z_t^{t-1}, Y_t^{t-1}, S(W), J(W)) - n\delta_n
\]

\[= \log M_I + \sum_{t=1}^{n} \left\{ H(Y_t(W)) \right\}
- H(Y_t(W)|Z_t^{t-1}, Y_t^{t-1}, S(W), J(W))
- \sum_{t=1}^{n} I(Z_t; U_t) - n\delta_n
\]

\[\geq H(W^n, S(W)|J(M_I)) + H(W, S(W)|J(W))
+ H(W, S(W)|J_M, Z^n)
\]

\[\geq nI(Y; U) - nI(Z; U) + \log M_I - n\delta_n
\]

\[\geq nI(Y; U) - nI(Z; U) + nR_I - n(\delta + \delta_n)
\]

Thus,

\[R_J \geq I(Y; U) - I(Z; U) + R_I - 2\delta - \delta_n. \tag{43}\]

\[\Box\]

C. Analysis of Privacy-Leakage Rate

The entire flow is similar to the arguments seen in the previous appendix. However, we need to pay a little bit more attention during the development of formulas since

\[(J(W), S(W)) \text{ is not a function of } X^n_W \] as the analysis of template rate. From (10), we have

\[n(R_L + \delta) \geq \max_{i \in \mathcal{L}} I(X^n_i; J(i)) \geq \frac{1}{M_I} \sum_{i=1}^{M_I} I(X^n_i; J(i))
= I(X^n_W; J(W)) = I(X^n_W, J(M_I))
= I(X^n_W, W; J(M_I))
= I(X^n_W, W, S(W); J(M_I)) - I(S(W); J(M_I))
\geq H(X^n_W, W, S(W)) - H(X^n_W, W, S(W)|J(M_I))
- H(S(W))
\]

\[(a) \leq H(W^n, S(W)|J(M_I))
- H(X^n_W, W, S(W)|J(M_I)) + H(S(W))
- H(W^n, S(W)|J(M_I) - H(S(W)|J(W))
- H(W) + H(X^n_W, S(W)|J(M_I)) - H(S(W|J(W))
= H(W) + H(X^n_W, S(W)|J(M_I)) - H(S(W)|J(W))
\]

\[\geq \log M_I + \sum_{i=1}^{n} I(X^n_i; J(i))
- nI(Y; U) - nI(Z; U) + \log M_I - n\delta_n
\]

\[= \log M_I + \sum_{i=1}^{n} I(X^n_i; J(i))
- nI(Y; U) - nI(Z; U) + \log M_I - n\delta_n
\]

\[= \log M_I + \sum_{i=1}^{n} I(X^n_i; J(i))
- nI(Y; U) - nI(Z; U) + \log M_I - n\delta_n
\]

\[\geq \log M_I + \sum_{i=1}^{n} I(X^n_i; J(i))
- nI(Y; U) - nI(Z; U) + \log M_I - n\delta_n
\]

\[\geq \log M_I + \sum_{i=1}^{n} I(X^n_i; J(i))
- nI(Y; U) - nI(Z; U) + \log M_I - n\delta_n
\]
\begin{equation}
\geq nI(X;U) - nI(Z;U) + nR_I - n(\delta + \delta_n), \tag{46}
\end{equation}

where

(f) follows because (25) in Lemma 3 is applied,
(g) holds due to (26) and (28) in Lemma 4,
(h) follows due to (7).

Thus,

\begin{equation}
R_L \geq I(X;U) - I(Z;U) + R_I - 2\delta - \delta_n. \tag{47}
\end{equation}