Abstract

The assessment of water discharge in open channel flow is one of the most crucial issues for hydraulic engineers in the fields of water resources management, river dynamics, eco-hydraulics, irrigation, hydraulic structure design, etc. Recent studies state that the entropy velocity law allows expeditive methodology for discharge estimation and rating curve development due to the simple mathematical formulation and implementation. A lot of works have been developed based on the entropy velocity profile supporting measurements in lab for rating curve assessment in regular ditch flows showing a good performance. The present work deals with the use of entropy velocity profile approach in order to give a general framework of threats and opportunities related to robust operational application of such laws in the field of rating curve assessment. The analysis has been carried on a laboratory flume with regular roughness under controlled boundary conditions and different stages generating an exhaustive dashboard for the better appraisal of the approaches. Finally, entropy model may represent a robust and useful tool for the water discharge assessment in rough ditches.

Keywords: entropy velocity ratio, relative submergence, aspect ratio, water discharge

1. Introduction

Water discharge assessment in open channel still represents a fundamental aspect for hydraulic engineer in several operative and technical fields like water resources management, ecological flow assessment and control, drainage and irrigation system as well as runoff and flood routing model calibration and implementation. Nevertheless, the water discharge evaluation in generic open channel is heavily affected by local fluid dynamics and geometric conditions, which well arise once flow velocity measurements and morphological boundaries are available.
at the same site. On the other hand, the drainage and irrigation channel present a regular cross section which might provide facilities in water discharge assessment and control, inducing also reduction in time and operative costs. That is, the implementation of operative procedures enabling operative charges simplifying the commitment of field activities, indeed, plays a fundamental role in channel monitoring for natural flow and manmade hydraulic structures. The main idea is related to the definition of expeditive procedures for flow field assessment and water discharge evaluation capable to optimize the surveying resources in time and efforts. Thus, the opportunity to manage with a simple and straightforward velocity law, different from the classical logarithm formulation but capable to provide suitable results is all the more technically fruitful. That is, an operative tool for expeditive velocity distribution assessment basing on simple and immediate parameters.

Recent theoretical and experimental studies endorse the informational content hold into the distributed velocity measurements following an entropy-probabilistic approach. That is, Chiu [1, 2] drew the correlation between the mean flow velocity and maximum flow velocity defining the entropy parameter, M introducing the velocity ratio $\Phi(M)$. Considering the important implication that this finding could have for monitoring of high flows in rivers, many authors investigated the reliability of this relationship using field data [3–7]. Overall, they found M as a river site depending and not influenced by the flood intensity both in terms of amount and duration. Thus, M should be considered a specific factor of the gauged cross section as outlined by Moramarco and Singh [7] exploring the dependence of M on the hydraulics and geometries of the river cross sections.

The study was able to explain the constancy of M value on the ground that M is not depending on the dynamic of flood, such as expressed by the energy or water surface slope, $S_f$ and to identify a formula expressing M as a function of the hydraulic radius, Manning’s roughness and the location, $y_0$, where the horizontal velocity is hypothetically equal to zero. For the latter, it was preliminarily found that if $y_0$ was assessed by distinguishing low flows from high flows, then a better estimation of M would have been obtained across a gauged river site. However, considering that the $y_0$ location is not of simple assessment and then might have high uncertainty, the assessment of M should be addressed, mainly for ungauged river sites, using hydraulic and geometric variables easy to acquire. Such a thought might be discussed introducing the relative submergence $D/d$ (in which, $D =$ average water depth and $d =$ roughness dimension). That is, the velocity distribution in natural rivers depends on several variables like channel geometry, bed and bank roughness, and the vertical velocity distribution generally increases monotonically from 0 at the channel bed, to the maximum at the water surface and can be assumed 1-D flow dominant. Moreover, whenever the channel cannot be considered “wide”, that is the aspect ratio (B/D with B channel width and D water depth) is less than 6, besides the presence of the boundary, the velocity varies even transversely and a two-dimension distribution occurs, leading G as the 2D entropy parameter. The maximum velocity places below the water surface inducing dip-phenomenon and the position of maximum velocity is also influenced by the aspect ratio [8], which is of simple assessment once channel cross-section geometry is known. Thus, investigating the influence of bed roughness and cross section geometry on medium and maximum velocity ratio at the global scale assumes a relevant interest in the field of open channel flow.
Therefore, M might represent an intrinsic parameter of the gauged site and this insight led several authors to explore the dependence of M on hydraulic and geometric characteristics of the flow site [3, 7]. In the case of river flows, Greco [9] enlightened a different behavior of $\Phi(M)$ depending on the roughness dimension: the velocity ratio is heavily influenced by the magnitude of relative submergence if large or intermediate scale [10]. Finally, the results support and validate a robust and fruitful operative chain to be implemented for expeditive water discharge assessment in rough and smooth irrigation ditch.

2. Entropy velocity profiles in open channels

The concept of informational entropy as a measure of uncertainty associated to a probability distribution was formulated for the first time in the field of hydraulics by Shannon [11]. The principle of maximum entropy introduces the least-biased probability distribution of a random variable constrained by defined information system as well as the theorem of the concentration for hypothesis testing, introducing the informational entropy theory [12]. A direct evaluation of uncertainty related to the probability distribution of a continuous random variable expressed in terms of entropy, $H$, is defined as follows

$$H = - \int_{-\infty}^{+\infty} p(x) \log p(x) \, dx \quad (1)$$

where, $p(x)$ is the continuous probability density function of random variable $x$.

Using POME, entropy can be maximized through the method of Lagrange multiplier as follows:

$$L = - \frac{1}{m-1} \int_{-\infty}^{+\infty} p(x) \left\{ 1 - [p(x)]^{m-1} \right\} \, dx + \sum_{i=1}^{N} \lambda_i g_i(x) \quad (2)$$

in which, $m > 0$, $g_i(x)$ is the ith constraint function and $\lambda_i$ is the constrain Lagrange multiplier as a weight in the maximization of entropy.

Chiu [1, 2] applied the concept of entropy to open-channel analysis to model velocity and shear stress distribution as well as sediment concentration. In such a way, the velocity distribution in the probability domain allows to obtain the cross-sectional mean velocity and the momentum and energy coefficients disregarding the geometrical shape of cross sections, which is generally complex in natural channels [2, 13].

Further, an assumption on the probability distribution in the space domain is needed to relate the entropy-based probability distribution to the spatial distribution. Therefore, defining $u$ by the time-averaged velocity placed on an isovelocity curve with the assigned value $\xi$, the value of $u$ is almost 0 at $\xi_0$, which corresponds to the channel boundary, while $u$ reaches $U_{\text{max}}$ at $\xi_{\text{max}}$, which generally occurs at or below the water surface, depending on the dip-phenomenon. Thus, the velocity $u$ monotonically increases from $\xi_0$ to $\xi_{\text{max}}$ and for each value of the spatial coordinate...
greater than $\xi$, the velocity is greater than $u$, and the cumulative distribution function can be written as

$$F(u) = \frac{\xi - \xi_0}{\xi_{\text{max}} - \xi_0}$$  \hspace{1cm} (3)$$

Thus, the Shannon entropy of velocity distribution can be written as:

$$H = -\int_0^{U_{\text{max}}} p(u) \log p(u) du$$  \hspace{1cm} (4)$$

Through a similar procedure, the probability density function of the velocity distribution is obtained by maximizing the Shannon entropy equation

$$L = \int_0^{U_{\text{max}}} \frac{f(u)}{m - 1} \{1 - [f(u)]^{m-1}\} du + \lambda_0 \left[ \int_0^{U_{\text{max}}} f(u) du - 1 \right] + \lambda_1 \left[ \int_0^{U_{\text{max}}} uf(u) du - \bar{u} \right]$$  \hspace{1cm} (5)$$

in which, $\lambda_0$ and $\lambda_1$ are the Lagrange multipliers and the following constraint equations

$$C_1 = \int_0^{U_{\text{max}}} f(u) du = 1$$  \hspace{1cm} (6)$$

$$C_2 = \int_0^{U_{\text{max}}} uf(u) du = \bar{u}$$  \hspace{1cm} (7)$$

$$f(u) = \exp(\lambda_0 - 1 + \lambda_1 u)$$  \hspace{1cm} (8)$$

Thus, Chiu’s 1D velocity distribution results as:

$$u = \frac{U_{\text{max}}}{M} \ln \left[ 1 + (e^M - 1) F(u) \right] = \frac{U_{\text{max}}}{M} \ln \left[ 1 + (e^M - 1) \frac{\xi - \xi_0}{\xi_{\text{max}} - \xi_0} \right]$$  \hspace{1cm} (9)$$

where $M$ is the dimensionless entropy parameter introduced in the entropy-based derivation [14, 15]. Hence, $M$ can be used as a measure of uniformity of probability and velocity distributions. The value of $M$ can be determined by the mean, $U_m$, and the maximum velocity values are derived from the following equation:

$$\Phi(M) = \frac{U_m}{U_{\text{max}}} = \left( \frac{e^M - 1}{M} \right)$$  \hspace{1cm} (10)$$

$\Phi(M)$ is a relevant parameter which contains relevant information about the flow field asset: the mean velocity value, the location of the mean velocity [14–16], and the energy coefficient [14, 16] can be obtained from $M$. That is, once known the mean velocity, the flow discharge, sediment transport, and pollutant transport can be derived. Furthermore, mean vs. maximum velocity assumes linear relationship as discovered by Xia collecting velocity data in several cross-sections of the Mississippi River [17].
Eq. (10), in fact, represents the fundamental relationship, from an applied point of view, of the entropy velocity distribution and the assessment of the entropy parameter passing through the knowledge of the ratio between mean and maximum velocities, \( \Phi(M) \).

In order to identify the dependence of \( M \) from the hydraulic and geometric characteristics of channels, that is, the relative submergence and aspect ratio, respectively, the formulation proposed by Greco [9] for \( U_m \) is considered:

\[
\frac{U_m}{u_*} = \frac{1}{k} \ln \frac{D}{d} + \frac{1}{k} \ln C_0
\]  

(11)

where \( u_* \) is the shear velocity, \( d \) is the bed roughness height (i.e., \( d_{50} \)), \( k \) is the Von Karman constant, and \( C_0 \) is the dimensionless coefficient.

Even the maximum velocity plays an important role in the flow dynamics, and more than its magnitude, a relevant aspect is related to the position of the maximum velocity inside the flow domain. That is, the location of maximum velocity from the channel bottom, \( y_{max} \), does not always occur at water surface, but a “velocity-dip” may occur as an indicator of secondary currents [18], which represents the circulation in a transverse channel cross section, while the longitudinal flow component is called the primary flow.

In this context, Moramarco and Singh [7] identified the ratio between \( U_{max} \) and \( u_* \) as:

\[
\frac{U_{max}}{u_*} = \frac{1}{k} \ln \left( \frac{D}{y_0(1+\alpha)} \right) + \frac{\alpha}{k} \ln \left( \frac{\alpha}{1+\alpha} \right)
\]  

(12)

with \( \alpha = (D/y_{max}-1) \).

\( y_0 \) can be assumed proportional to the characteristic bottom roughness height, \( d \), as suggested by Rouse [19] through the experimental parameter \( C_\xi = y_0/d \). Therefore, Eq. (12) turns into:

\[
\frac{U_{max}}{u_*} = \frac{1}{k} \ln \left( \frac{D}{d} \right) + \frac{1}{k} \ln \left( \frac{\alpha^\alpha}{C_\xi(1+\alpha)^{1+\alpha}} \right)
\]  

(13)

Unlike Moramarco and Singh [7], here the ratio between Eq. (11) and Eq. (13), based on logarithm properties, explicitly proposes \( \Phi(M) \) as a function of the relative submergence \( D/d \):

\[
\Phi(M) = \frac{U_m}{U_{max}} = \frac{\ln \left( \frac{C_0 D}{d} \right)}{\ln \left( \frac{D}{d C_\xi(1+\alpha)^{1+\alpha}} \right)} \approx A_\Phi \ln \frac{D}{d} + B_\Phi
\]  

(14)

where \( A_\Phi \) and \( B_\Phi \) are the numerical coefficients. Eq. (14) follows under the hypothesis of linear interpolation between the pairs \( \left[ \ln \left( \frac{C_0 D}{d} \right) / \ln \left( \frac{D}{d C_\xi(1+\alpha)^{1+\alpha}} \right) ; \ln \left( \frac{D}{d} \right) \right] \) [13].

Eq. (14) highlights, indeed, a possible effect of bed roughness on the entropy velocity distribution in open channel flows, which depends on the roughness scale according to [1]. The dependence between the ratio \( \Phi(M) \) and the relative submergence, \( D/d \), has been widely
3. Laboratory measurements in rectangular smooth and rough ditch

The experimental tests were carried out in the Hydraulics Laboratory of Basilicata University, on two free surface rectangular flumes of 9 m length and with a cross section of $0.5 \times 0.5$ and $1 \times 1$ m, whose slope can vary from 0 up to 1%. Figure 1 shows pictures about the flume, one of the bed configuration and the flow-meters.

The bed roughness (d) has been modulated between smooth surfaces, with 0.0005 m roughness height, and a rough bottom, obtained with both a sand bed, with a characteristic diameter of 0.002 m and standard deviation $\sqrt{d_{84}/d_{16}} = 1.67$, and a set of wood spheres of 0.035 m in diameter.

The measurement reaches were placed at the distance of 4 m from the beginning of the flumes, in order to damp large-scale disturbances and allow a quasi-uniform water depth. In the end section of the flume, a grid was installed to regulate the water depth for each assigned discharge or rather to obtain a small longitudinal variation of the flow depth. The experiments were performed in steady flow conditions for different values of discharge (0.015–0.100 m$^3$/s) and slope (0.05–1%). The measurement cross section was located in the middle of the rough reach in order to observe a fully developed flow, avoiding edge effects. The flow depth was measured by two hydrometers placed at both the beginning and the end.

Figure 1. The experimental apparatus for laboratory measures.
of the measurement reach, and the water depth, D, was assumed as the average value. The velocity was acquired through a micro current-meter with a measuring head diameter of 0.01 m, while the water discharge was measured with a concentric orifice plate installed in the feed pipe and on a laboratory weir placed at the end of the flumes, and compared to the value calculated according to the velocity-area method [23], with a maximum error of around 1–2%. In particular, the adopted velocity-area method must be applied dividing the cross section into a fixed number of verticals and thus, on each vertical, a fixed measurement points are selected. In each point along the vertical, the velocity is acquired in order to compute the mean velocity of the flow along each vertical. Furthermore, the number of measures on each vertical was chosen with respect to the criterion that the difference in velocity between two consecutive points was less than 20%, of the higher measured velocity value, and the points close to the channel bottom and the water surface was fixed according to the size of the micro-current meter.

In such a way, two roughness configurations were enabled:

- RRF: rough rectangular flume, with relative submergence ranging in between 1.89 and 6.43; and
- SRF: smooth rectangular flume, with relative submergence greater than 50.

Table 1 synthetically reports the ranges of variation of the main parameters observed during the experiments for the RRF and SRF configuration, while Q is the water discharge, D is the water depth, D/d is the relative submergence, B/D is the aspect ratio, and \( \Phi(M) \) is the ratio between the mean and maximum velocities.

For each configuration and for all the stages explored, a relevant bulk of velocity measurements was collected in order to provide a detailed reconstruction of the flow field allowing to obtain mean, \( \bar{U} \), and maximum, \( U_{\text{max}} \), cross section velocities.

Figure 2 shows the linear relationship existing between the pairs \( (U_{\text{max}}; \bar{U}) \) for the two configurations investigated, RRF and SRF.

From Figure 2, some useful issues arise. Even if the correlation among homogeneous data is very strong in both cases with \( R^2 \) greater than 0.95, it is immediately realized a slight different behavior between rough and smooth channels. That is, for the smooth rectangular flow, \( \Phi(M) \) assumes the value 0.9, while for the rough condition, the value decreases to 0.67. That is, in other terms, it seems to be evident and sufficiently confirmed, the dependence of the velocity ratio on the roughness here represented by the relative submergence \( D/d \) as discussed in the previous section for Eq. (14).

| Type | Q (mc/sec) | D (m) | D/d | B/D | \( \Phi(M) \) |
|------|-----------|-------|-----|-----|------------|
| RRF  | 0.007–0.076 | 0.07–0.23 | 1.89–6.43 | 2.22–7.58 | 0.52–0.73 |
| SRF  | 0.025–0.100 | 0.06–0.40 | 50–298  | 2.50–10  | 0.7–0.93  |

Table 1. Range of variation for the main parameters of the laboratory experiments.
Figure 2. Average vs. maximum velocities observed for rough and smooth channel.

Figure 3. Velocity ratio vs. relative submergence.
Figure 3 clearly outlines such an outcome, showing how the velocity ratio is austerely dependent on relative submergence in case of rough flows, while it is sufficiently uniform for values of D/d > 20. Furthermore, the same picture proposes several literature data collected by other authors during experimental laboratory campaigns carried on smooth and rough flumes [22, 24–27], plotted and compared to those arising from the here presented research activity. The same Figure 4 immediately deals with the robust correspondence between data sets related to the low rough/smooth flow conditions for which the hypothesis of the constant value of mean-to-maximum velocities ratio might be assumed consistent, at least from an operative point of view for D/d > 20. At the same time, Eq. (14) still remains compelling for D/d < 20, but it needs to be recalibrated and the coefficients $A_{φ}$ and $B_{φ}$ can be assumed 0.136 and 0.468, respectively ($R^2 = 0.95$).

Such a result can be immediately implemented in the operative chain of water discharge assessment, in order to derive the rating curve in a ditch or artificial channel. Furthermore, such knowledge allows us to assess the level of integrity of the channel in terms of sensitive changes in the bottom roughness, may be due to the local deposition of sediment or vegetation.

Furthermore, in case of D/d > 20, typical of concrete channels, the setting of rating curve is quite direct collecting few measures of velocity, in a little volume of the flow field mainly located in the center of the upper part of the cross section where is generally located at the maximum

![Figure 4. Comparison between the computed (Qcalc) and observed (Qobs) discharges.](image-url)
velocity. Thus, assuming the value of $\Phi(M)$ equal to 0.9, the mean velocity can be computed and the water discharge as well. The benefit even deals with the reduction of measurement time and costs. On the other side, once performed velocity measurements in a cross section following the above mentioned procedure, the observed value of $\Phi(M)$ can suggest whether or not some changes in bed roughness occurred.

Finally, the use of the entropy velocity profile gives a robust feedback in terms of operative assessment of water discharge, due to the easy and immediate evaluation of the $M$ parameter.

4. Entropy velocity profile approach for rating curve assessment

The wide bulk of measurements obtained through the laboratory experiments allows us to perform a robust analysis in order to obtain suitable information for the use in the operative chain of water discharge assessment as well as in numerical flow dynamics modeling in regular open channel flow.

In Eq. (10), the mean velocity can be evaluated using Manning’s formula:

$$U_m = \frac{1}{n} R^{2/3} \sqrt{S_f}$$

(15)

where $n$ is the Manning’s roughness, $R$ is the hydraulic radius, and $S_f$ is the energy slope.

To determine the maximum velocity of the cross-section, $U_{max}$ along the $y$-axis assumed perpendicular to the bottom, the dip-modified logarithmic law for the velocity distribution in a smooth uniform open channel flow, proposed by Yang et al. [8], is considered:

$$u(y) = u_* \left[ \frac{1}{k} \ln \frac{y}{y_0} + \frac{\alpha}{k} \ln \left( 1 - \frac{y}{D} \right) \right]$$

(16)

where $u^* = \sqrt{g R S_f}$ is the shear velocity ($g =$ gravity acceleration); $k$ is the von Karman constant equal to 0.41; $y_0$ is the distance at which the velocity is hypothetically equal to zero; $\alpha$ is the dip-correction factor, depending only on the ratio between the relative distance of the maximum velocity location from the river bed, $y_{max}$, and the water depth, $D$, along the $y$-axis, where $U_{max}$ is sampled.

The location of the maximum velocity, supporting the dip-phenomenon hypothesis, can be obtained by differentiating Eq. (16) and equating $du/dy = 0$, which gives:

$$\frac{y_{max}}{D} = \frac{1}{1 + \alpha}$$

(17)

Experimental studies [2–9] have shown that, for channels at different shapes of the cross-section, the velocity maximum is below the free surface around the 20–25% of the maximum depth. Thus, considering $y_{max}$ equal to $\frac{3}{4}$ of the maximum depth, $D$, according to Eq. (17), $\alpha$ becomes equal to $1/3$. Replacing the value of $\alpha$ in Eq. (16), and after a few algebraic manipulation, the maximum flow velocity can be expressed as:
\[ U_{\text{max}} = \frac{u^*}{k} \left[ \ln \left( \frac{3D}{4y_0} \right) - 0.4621 \right] \]  

(18)

Therefore, inserting Eqs. (15) and (18) in Eq. (10), \( \Phi(M) \) can be expressed in terms of hydraulic and geometric characteristics of a river:

\[ \Phi(M) = \frac{1}{k} \frac{R^{2/3}}{\sqrt{S_f}} \left[ \ln \left( \frac{3D}{4y_0} \right) - 0.4621 \right] \]  

(19)

From this latter equation, a new formulation of Manning’s roughness, \( n_e \), based on \( \Phi(M) \) is derived:

\[ n_e = \frac{R^{1/6}}{\sqrt{S_f}} \frac{\Phi(M)}{k} \left[ \ln \left( \frac{3D}{4y_0} \right) - 0.4621 \right] \]  

(20)

Therefore, if \( \Phi(M) \) is available, then Eq. (20) allows us to estimate the \( n \) value in the cross-section. Replacing Eq. (20) in Eq. (15), the modified form of the Manning’s equation is obtained:

\[ U_m = \frac{\Phi(M)}{k} \left[ \ln \left( \frac{3D}{4y_0} \right) - 0.4621 \right] \sqrt{gRS_f} \]  

(21)

which takes into account the variation of a flow hydraulic and geometric characteristics following the change of the water discharge. Eq. (20) computes Manning’s roughness once the values of \( \Phi(M) \) are known and the values of \( y_0 \) are calibrated. Once the Manning’s coefficient, \( n_e \), was evaluated, the mean velocity was recalculated according to Eq. (21).

Figure 4 shows the correspondence between \( Q_{\text{calc}} \), computed through the Eq. (21), and those observed \( Q_{\text{obs}} \), for both cases RRF and SRF. The result shows the perfect correlation between the observed and computed values and enforces the use of the proposed Manning’s Eq. (20), derived by the entropy velocity theory and the assumption of a constant value of the dip velocity. The approach leads to get water discharge assessment by integrating the information about hydraulic and geometric characteristics of the flow.

Finally, the following Figures 5 and 6 report the theoretical rating curves obtained by the modified Manning’s equation and the experimental data collected for both cases rough and smooth channel.

Defining the standard error, \( S_e \), as suggested by the ISO 1100-2 [28], through the following relationship:

\[ S_e = \left[ \sum \frac{(\ln Q_{\text{obs}} - \ln Q_{\text{calc}})^2}{N - 2} \right]^{0.5} \]  

(22)

where \( N \) is the number of available measures, the computed \( S_e \) is permanently less than 5% for the rectangular rough flow (RRF), while increases up to 15%, with a generalized
overestimation, in case of smooth rectangular flow. In both cases, the results support the use of this expeditive methodology in the chain of operative procedures leading a good assessment of the rating curve.

5. Conclusion

The use of a rating curve formulation derived from the entropy velocity theory complained to the assumption of a constant value of the dip velocity and taking into account the variables
describing the geometric and hydraulic characteristics of a rectangular ditch, should allow us the improvement of water discharge assessment.

This approach was tested, in a first phase, on a suitable data set of water discharge measures collected in the laboratory on both rough and smooth rectangular cross section proposing practical and common flow conditions.

The rating curve evaluation, derived for the rough rectangular flow, underlines a standard error less than 5%, generally, favoring an expeditive assessment of the flow stage with a sufficient level of reliability, while such an error increase up to 15% in case of smooth cross section.

Author details

Greco Michele

Address all correspondence to: michele.greco@unibas.it

1 Engineering School, University of Basilicata, Potenza, Italy
2 Regional Environmental Observatory Research Foundation of Basilicata, Italy

References

[1] Chiu C-L. Entropy and probability concepts in hydraulics. Journal of Hydraulic Engineering, ASCE. 1987;113(5):583-600
[2] Chiu C-L. Velocity distribution in open channel flow. Journal of Hydraulic Engineering, ASCE. 1989;115(5):576-594
[3] Greco M, Mirauda D, Volpe Plantamura A. Manning’s roughness through the entropy parameter for steady open channel flows in low submergence. Procedia Engineering, ISSN 1877-7058, Published by Elsevier Ltd. 2014;70:773-780. DOI: 10.1016/j.proeng.2014.02.084. 2-s2.0-84899680421
[4] Greco M, Mirauda D. An entropy based velocity profile for steady flows with large-scale roughness. In: Lollino G, Arattano M, Rinaldi M, Giustolisi O, Marechal JC, Grant G, editors. Engineering Geology for Society and Territory. Vol. 3. Cham: Springer International Publishing; 2015. pp. 641-645. Print; ISBN: 9 978-331909054-2; 978-331909053-5, DOI: 10.1007/978-3-319-09054-2_128, WOS: 000358990300128, EID: 2-s2.0-84944599725
[5] Mirauda D, Greco M, Moscarelli P. Practical method for flow velocity measurements in fluvial sections. WIT Transactions on Ecology and the Environment. 2011;146:355-366. ISBN:978-1-84564-516-8 e ISSN 17433541. DOI: 10.2495/RM110301
[6] Moramarco T, Saltalippi C, Singh VP. Estimation of mean velocity in natural channels based on Chiu’s velocity distribution equation. Journal of Hydrologic Engineering, ASCE. 2004;9(1):42-50
[7] Moramarco T, Singh VP. Formulation of the entropy parameter based on hydraulic and geometric characteristics of river cross sections. Journal of Hydrologic Engineering. 2010; 15(10):852-858

[8] Yang SQ, Tan SK, Lim SY. Velocity distribution and dip-phenomenon in smooth uniform open channel flows. Journal of Hydrologic Engineering, ASCE. 2004;130(12):1179-1186

[9] Greco M. Effect of bed roughness on 1-D entropy velocity distribution in open channel flow. Hydrology Research. 2015;46(1):1-10. ISSN 1998-9563, EID: 2-s2.0-84926637541. DOI: 10.2166/nh.2013.122

[10] Bathurst JC, Li RM, Simons DB. Resistance equation for large-scale roughness. Journal of the Hydraulics Division. 1981;107(HY12):1593-1613

[11] Shannon CE. A mathematical theory of communication. The Bell System Technical Journal. 1948;27:623-656

[12] Tsallis C. Possible generalization of Boltzmann-Gibbs statistics. Journal of Statistical Physics. 1988;52(1–2):479-487

[13] Greco M, Moramarco T. Influence of bed roughness and cross section geometry on medium and maximum velocity ratio in open-channel flow. Journal of Hydraulic Engineering. 2016;142(1):06015015. DOI: 10.1061/(ASCE)HY.1943-7900.0001064. Article number 0601501 EID: 2-s2.0-84950267443

[14] Chiu C-L, Said CA. Maximum and mean velocity and entropy in open channel flow. Journal of Hydraulic Engineering, ASCE. 1995;121(1):26-35

[15] Luo H, Singh V. Entropy theory for two-dimensional velocity distribution. Journal of Hydrologic Engineering. 2011;167(4):303-315

[16] Chiu CL, Tung NC. Maximum velocity and regularities in open-channel flow. Journal of Hydraulic Engineering. 2002;128(8):803-803

[17] Xia R. Relation between mean and maximum velocities in a natural river. Journal of Hydrologic Engineering, ASCE. 1997;123(8):720-723

[18] Nezu I, Nakagawa H. Turbulence in open-channel flows. In: Balkema. The Netherlands: Rotterdam; 1993

[19] Rouse H. Critical analysis of open-channel resistance. Journal of the Hydraulics Division. 1965;91(4):1-23

[20] Greco M, Mirauda D. Entropy parameter estimation in large-scale roughness open channel. Journal of Hydrologic Engineering. 2015;20(1–2). DOI: 10.1061/(ASCE)HE.1943-5584.0000109. ISSN: 1084-0699 eISSN: 1943-5584, Article number 04014047, EID 2-s2.0-84921442470

[21] Greco M. Entropy-based approach for rating curve assessment in rough and smooth irrigation ditches. Journal of Irrigation and Drainage Engineering. 2016;142(3):04015062. DOI: 10.1061/(ASCE)IR.1943-4774.0000986. EID 2-s2.0-84958758546
[22] Nikmehr S, Farhoudi J. Estimation of velocity profile based on Chiu’s equation in width of channels. Research Journal of Applied Sciences, Engineering and Technology. 2010;2(5):476-479

[23] ISO 748. 2007 Hydrometry—Measurement of liquid flow in open channels using currentmeters or floats-ISO/TC 113/SC 1 Velocity area methods

[24] Guy HP, Simons DB, Richardson EV. Summary of Alluvial Channel data from flume experiments, 1956–1961, Geological Survey Professional Paper, 1966, 462-I. US Government Printing Office, Washington, DC

[25] Steffler PM, Rajaratnam N, Peterson AW. LDA Measurements of Mean Velocity and Turbulence Distribution in a Smooth Rectangular Open Channel. Edmonton: Department of Civil Engineering., University of Alberta; 1983

[26] Bortz K. Parameter estimation for velocity distribution in open channel flow [M.S. Thesis]. Department of Civil Engineering, University of Pittsburgh: Pittsburgh, PA; 1989

[27] Guo ZR. Personal communication in Chiu and Hsu, 2006. In: Southeast China Environmental Science Institute. China: Yuancun; 1990

[28] ISO 1100-2: 2010 Hydrometry—Measurement of liquid flow in open channels—Part 2: Determination of the stage-discharge relation-ISO/TC 113/SC 1 Velocity area methods
