Neutralino Pair Production and 3–Body Decays at $e^+e^-$ Linear Colliders as Probes of CP Violation in the Neutralino System

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Abstract

In the CP–invariant supersymmetric theories, the steep S–wave (slow P–wave) rise of the cross section for any non–diagonal neutralino pair production in $e^+e^-$ annihilation, $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ ($i \neq j$), near threshold is accompanied by the slow P–wave (steep S–wave) decrease of the fermion invariant mass distribution of the 3–body neutralino decay, $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 f\bar{f}$ ($f = l$ or $q$), near the end point. These selection rules, unique to the neutralino system due to its Majorana nature, guarantee that the observation of simultaneous sharp S–wave excitations of the production cross section near threshold and the lepton or quark invariant mass distribution near the end point is a qualitative, unambiguous evidence for CP violation in the neutralino system.
1 Introduction

Most supersymmetric extensions of the Standard Model (SM) based on some soft supersymmetry (SUSY) breaking mechanism contain several CP phases, whose large values tend to render lepton and quark electric dipole moments (EDM) too large to satisfy stringent experimental constraints [1]. Such CP crises are generic in supersymmetric theories, but may be resolved by pushing the masses of some sparticles, especially the first and second generation sfermions, above a few TeV, by arranging for internal cancellations, or by simply setting phases to be extremely small [2]. On the other hand, new sources of CP violation beyond the SM are required to explain the non-zero baryon asymmetry in the universe in the standard Big Bang framework [3]. Therefore, it is crucial to look for new signatures for CP violation in such SUSY scenarios with some large phases, as long as they are consistent with the stringent EDM and other low–energy constraints. In this light, detailed analyses of the neutralino sector at future $e^+e^-$ linear collider experiments [4] can prove particularly fruitful [5, 6, 7, 8, 9], because in most supersymmetric theories neutralinos belong to the class of the lighter supersymmetric particles [10] and the neutralino system contains two non–trivial CP violating phases.

There are many different ways for probing CP violation in the neutralino system. The imaginary parts of the complex parameters in the neutralino mass matrix could most directly and unambiguously be determined by measuring suitable CP violating observables by exploiting initial beam polarization and angular correlations between neutralino production and decay at future high–energy colliders [6, 7, 8, 9]. But, their experimental measurements will be quite difficult. The presence of the CP violating phases can also be identified through by their impact on CP–even quantities such as neutralino masses, branching ratios and so on. However, since these quantities are already non–zero in the CP conserving case, the detection of the presence of non–trivial CP phases will require a careful quantitative analysis of a number of physical observables, especially for small CP–odd phases giving rise to very small deviations from the CP–conserving values [1]. On the other hand, the rise of excitation curves near threshold for non–diagonal neutralino pair production in $e^+e^-$ collision is altered qualitatively in CP–noninvariant theories [5, 6], by allowing the steep S–wave increase of all pairs simultaneously. Thus, as demonstrated in Ref. [6], precise measurements of the threshold behavior of the non–diagonal neutralino pair production processes may give clear indications of non–zero CP violating phases in the neutralino sector, if at least three different neutralino states are accessible kinematic-
In the present note we provide a new powerful method for probing CP violation in the neutralino system, which is based on a combined analysis of the threshold excitations of neutralino pair production in $e^+e^-$ annihilation and the fermion invariant mass distribution near the end point of the 3–body neutralino fermionic decays:

$$e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0 \quad (i \neq j) \quad \text{and} \quad \tilde{\chi}_i^0 \to \tilde{\chi}_j^0 f \bar{f} \quad (f = l, q).$$

[The 3–body decay process includes clean $\mu^+\mu^-$ and $e^+e^-$ decay channels with little background, which allow a clear reconstruction of the kinematical configuration with good precision.] This method relies on selection rules, unique to the neutralino system due to its Majorana nature in CP–invariant theories, and it can work effectively if the branching ratios of the 3–body neutralino fermionic decays are not suppressed. [Once two–body decays of the neutralino $\tilde{\chi}_i^0$ into $Z$, Higgs bosons or sfermions are open, the new method is ineffective.]

Before demonstrating the new method for probing CP violation in the neutralino system in detail, we describe briefly the mixing for the neutral gauginos and higgsinos in CP–noninvariant theories with non–vanishing phases in Sec. 2. In Sec. 3 we introduce the selection rules for the production of neutralino pairs and the neutralino to neutralino transition via a (virtual) vector boson or sfermion exchange. Then, we prove that in any CP–invariant SUSY theory, if the production cross section for any non–diagonal neutralino pair in $e^+e^-$ annihilation increases steeply in S–waves (slowly in P–waves) near threshold, the lepton or quark invariant mass distribution of the decay $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 f \bar{f} \quad (f = l \text{ or } q)$ decreases slowly in P–waves (steeply in S–waves) near the end point. Thus, the observation of simultaneous sharp S–wave excitations of both the production of any non–diagonal neutralino pair $\tilde{\chi}_i^0\tilde{\chi}_j^0$ near threshold and the fermion invariant mass distribution of the decay $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 f \bar{f}$ near the end point will be a qualitative, unambiguous evidence for CP violation in the neutralino system. A quantitative demonstration of the method based on a specific set of the relevant supersymmetry parameters is given in the last part of Sec. 3. Finally, conclusions are drawn in Sec. 4.

## 2 Neutralino Mixing

In the minimal supersymmetric extension of the Standard Model (MSSM), the mass matrix of the spin-1/2 partners of the neutral gauge bosons, $\tilde{B}$ and $\tilde{W}^3$, and of the
neutral Higgs bosons, $\tilde{H}_1^0$ and $\tilde{H}_2^0$, takes the form

$$M = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix},$$

(1)

in the $\{\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0\}$ basis. Here $M_1$ and $M_2$ are the fundamental supersymmetry breaking $U(1)$ and $SU(2)$ gaugino mass parameters, and $\mu$ is the higgsino mass parameter. As a result of electroweak symmetry breaking by the vacuum expectation values of the two neutral Higgs fields $v_1$ and $v_2$ ($s_\beta = \sin \beta$, $c_\beta = \cos \beta$ where $\tan \beta = v_2/v_1$), non–diagonal terms proportional to the $Z$–boson mass $m_Z$ appear and the gauginos and higgsinos mix to form the four neutralino mass eigenstates $\tilde{\chi}_i^0$ ($i = 1–4$). In general the mass parameters $M_1$, $M_2$ and $\mu$ in the neutralino mass matrix (1) can be complex. By re–parameterization of the fields, $M_2$ can be taken real and positive, while the $U(1)$ mass parameter $M_1$ is assigned the phase $\Phi_1$ and the higgsino mass parameter $\mu$ the phase $\Phi_\mu$.

The neutralino mass eigenvalues $m_i \equiv m_{\tilde{\chi}^0_i}$ ($i = 1, 2, 3, 4$) can be chosen positive by a suitable definition of the mixing matrix $N$, rotating the gauge eigenstate basis $\{\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0\}$ to the mass eigenstate basis of the Majorana fields $\tilde{\chi}_i^0$ ($i = 1–4$). In general the matrix $N$ involves 6 angles and 10 phases, and can be written as

$$N = \text{diag} \left\{ e^{i \alpha_1}, e^{i \alpha_2}, e^{i \alpha_3}, e^{i \alpha_4} \right\} R_{34} R_{24} R_{14} R_{23} R_{13} R_{12},$$

(2)

where $R_{jk}$ are rotations in the complex $[jk]$ plane characterized by a mixing angle $\theta_{jk}$ and a (Dirac) phase $\beta_{jk}$. One of (Majorana) phases $\alpha_i$ is nonphysical and, for example, $\alpha_1$ may be chosen to vanish. None of the remaining 9 phases can be removed by rotating the fields since neutralinos are Majorana fermions. The neutralino sector is CP conserving if $\mu$ and $M_1$ are real, which is equivalent to $\beta_{ij} = 0$ (mod $\pi$) and $\alpha_i = 0$ (mod $\pi/2$). Majorana phases $\alpha_i = \pm \pi/2$ do not signal CP violation but merely indicate different intrinsic CP parities of the neutralino states in CP–invariant theories [12].

3 Neutralino Pair Production and 3–Body Decays

Both the production processes, $e^+ e^- \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0$ ($i, j = 1–4$), and the 3–body neutralino decays, $\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_j^0 f\bar{f}$, are generated by the five mechanisms: $s$–channel $Z$ exchange, and $t$–
and $u$–channel $\tilde{f}_{L,R}$ exchanges with $\tilde{f} = \tilde{e}$ for the production processes. After appropriate Fierz transformations of the sfermion exchange amplitudes and with the fermion masses neglected, the transition matrix element of the production process $e^+ e^- \rightarrow \tilde{\chi}^0_i \tilde{\chi}^0_j$ and that of the 3–body fermionic neutralino decays $\tilde{\chi}^0_i \rightarrow \tilde{\chi}^0_j f \bar{f}$ can be written as

$$T(e^+ e^- \rightarrow \tilde{\chi}^0_i \tilde{\chi}^0_j) = \sum_{\alpha,\beta = L,R} Q_{\alpha\beta} \left[ \bar{u}(e^+) \gamma_\mu P_\alpha u(e^-) \right] \left[ \bar{u}(\tilde{\chi}^0_i) \gamma_\mu P_\beta v(\tilde{\chi}^0_j) \right],$$

(3)

$$D(\tilde{\chi}^0_i \rightarrow \tilde{\chi}^0_j f \bar{f}) = \sum_{\alpha,\beta = L,R} Q'_{\alpha\beta} \left[ \bar{u}(f) \gamma_\mu P_\alpha v(\tilde{f}) \right] \left[ \bar{u}(\tilde{\chi}^0_i) \gamma_\mu P_\beta u(\tilde{\chi}^0_j) \right],$$

(4)

that is to say, as a sum of the products of a $\tilde{\chi}^0$ vector or axial vector current and a fermion vector or axial vector current, respectively. We refer to Ref. [6] and Ref. [8] for the expressions of the generalized bilinear charges $Q_{\alpha\beta}$ and $Q'_{\alpha\beta}$, just mentioning that the bilinear charges become independent of the kinematical variables when two neutralinos are at rest. Therefore, in this static limit, both the production and the decays can be considered to proceed via a static vector boson exchange.

Some general properties of the bilinear charges $Q_{\alpha\beta}$ and $Q'_{\alpha\beta}$ in Eqs. (3) and (4) can be derived in CP–invariant theories by applying CP invariance and the Majorana condition for neutralinos to the transition matrix elements. In CP–invariant theories, the production of a neutralino pair through a vector or axial vector current with positive intrinsic CP parity satisfies the CP relation [9, 13]

$$1 = \eta^i \eta^j (-1)^L,$$

(5)

in the non–relativistic limit of two neutralinos, where $\eta^i = \pm i$ is the intrinsic CP parity of $\tilde{\chi}^0_i$ and $L$ is the orbital angular momentum of the neutralino pair. The selection rule [5] reflects the fact that if two neutralinos $\tilde{\chi}^0_i$ and $\tilde{\chi}^0_j$ have the same or opposite CP parity, the current for the neutralino pair production must be pure axial–vector or pure vector form, respectively, cf. [13]. Because the axial–vector current and the vector current involve the combination of $u$ and $v$ spinors for the two Majorana particles, the axial vector corresponds to the P–wave ($L = 1$) and the vector to the S–wave ($L = 0$).

On the other hand, the neutralino decay, $\tilde{\chi}^0_i \rightarrow \tilde{\chi}^0_j + V$, where $V$ stands for the final fermion current in Eq. (4), satisfies the CP relation

$$\eta^i = \eta^j (-1)^L \quad \text{or equivalently} \quad 1 = -\eta^i \eta^j (-1)^L,$$

(6)

in the non–relativistic limit of two neutralinos, where $L$ is the orbital angular momentum of the final state of $\tilde{\chi}^0_j$ and $V$. We emphasize first that the neutralino to neutralino
transition current is pure axial–vector or pure vector form for the two neutralinos of the same or opposite CP parity, respectively, as in the production case. However, because two u–spinors are associated with the currents in the neutralino to neutralino transition, the axial–vector corresponds to S–wave excitation while the vector corresponds to P–wave excitation, giving rise to the relative minus sign between (5) and (6).

One immediate consequence of the selection rules (5) and (6) is that, in CP–invariant theories, if the production of a pair of neutralinos with the same (opposite) CP parity through a vector or axial vector current is excited slowly in P–waves (steeply in S–waves) [12], then the neutralino to neutralino transition via such a vector or axial vector current is excited sharply in S–waves (slowly in P–waves). More explicitly, the power of the selection rules (5) and (6) can clearly be seen by inspecting the expressions for the S–wave excitations of the total cross section \( \sigma \) and of the fermion invariant mass distribution of the 3–body neutralino decay \( \tilde{\chi}_i^0 \rightarrow \tilde{\chi}_j^0 f \bar{f} \) (with the fermion masses neglected) near the point:

\[
\sigma_{ij} \approx \frac{4\pi\alpha^2 m_i m_j}{(m_i + m_j)^4} \beta \left\{ |\text{Im} G_R|^2 + |\text{Im} G_L|^2 \right\} + O(\beta^3),
\]

\[
\frac{d\Gamma_{ij}}{dz_{ff}} \approx \frac{2\alpha^2}{\pi} \left( \frac{m_j}{m_i} \right)^{3/2} (m_i - m_j) \beta' \left\{ |\text{Re} G_R'|^2 + |\text{Re} G_L'|^2 \right\} + O(\beta'^3),
\]

where \( \beta = \sqrt{1 - (m_i + m_j)^2/s} \) and \( \beta' = \sqrt{1 - z_{ff}^2/m_{ff}^2} \) with the dimensionless variable \( z_{ff} = m_{ff}/m_{ff}^{\text{max}} \), the ratio of the fermion invariant mass \( m_{ff} \) to its maximal value \( m_{ff}^{\text{max}} = m_i - m_j \). Here, the coupling dependent parts, each of which is connected with the chirality of the neutralino current, are given by

\[
G^{(n)}_{R} = -\frac{Q_f}{2c_W^2} D^{(n)}(N_{i3}N_{j3}^* - N_{i4}N_{j4}^*) - \frac{Q_f^2}{c_W^2} F^{(n)}_R N_{i1}N_{j1}^*,
\]

\[
G^{(n)}_{L} = \frac{(I_3^f - Q_f s_W^2)}{2c_W s_W^2} D^{(n)}(N_{i3}N_{j3}^* - N_{i4}N_{j4}^*) + \frac{1}{s_W^2 c_W^2} F^{(n)}_L N_{i2}N_{j2}^*,
\]

with \( N_{i2}^* = (I_3^f - Q_f) s_W N_{i1} - I_3^f c_W N_{i2} \), and the kinematic functions are given by

\[
D = (m_i + m_j)^2/((m_i + m_j)^2 - m_Z^2),
\]

\[
F_{L,R} = (m_i + m_j)^2/(m_{e_{L,R}}^2 + m_i m_j),
\]

\[
D' = (m_i - m_j)^2/((m_i - m_j)^2 - m_Z^2),
\]

\[
F'_{L,R} = (m_i - m_j)^2/(m_{e_{L,R}}^2 - m_i m_j).
\]
In CP–invariant theories, all the (complex) rotation matrices $R_{jk}$ in Eq. (2) become real and orthogonal. Therefore, if the neutralinos $\tilde{\chi}^0_i$ and $\tilde{\chi}^0_j$ have the same CP parity, then the Majorana phase difference, $\alpha_i - \alpha_j$, is 0 or $\pi$, and so $N_{ik}N_{jl}^*$ is real. On the contrary, if the neutralino pair have the opposite CP parity, the phase difference $\alpha_i - \alpha_j$ is $\pm \pi/2$ and so $N_{ik}N_{jl}^*$ is purely imaginary. Consequently, in CP–invariant theories the cross section of a non–diagonal neutralino pair rises steeply in S–waves only when the produced neutralinos have the opposite parity, as dictated by the first CP relation (5) and as clearly indicated by Eq. (7). One important implication of the selection rule is that, even if the $\{ij\}$ and $\{ik\}$ pairs are excited steeply in S–waves, the pair $\{jk\}$ must be excited slowly in P–waves characterized by the slow rise $\sim \beta^3$ of the cross section [5, 6]. In contrast to the production case, the characteristic sharp S–wave decrease of the fermion invariant mass distribution near the end point is possible only if the neutralinos have the same CP parity, as dictated by the second CP relation (6) and as clearly indicated by Eq. (8).

However, in the CP–noninvariant theories the orbital angular momentum is no longer restricted by the selection rules (5) and (6). The production of all non–diagonal pairs can simultaneously be excited steeply in S–waves near threshold, and the corresponding neutralino to neutralino transition can be excited steeply in S–waves even if the production cross section of the same non–diagonal neutralino pair is excited steeply in S–waves. Consequently, CP violation in the neutralino system can clearly be signalled by (i) the sharp S–wave excitations of the production of three non–diagonal $\{ij\}$, $\{ik\}$ and $\{jk\}$ pairs near threshold [6] or by (ii) the simultaneous S–wave excitations of the production of any non–diagonal $\{ij\}$ neutralino pair in $e^+e^-$ annihilation, $e^+e^- \rightarrow \tilde{\chi}^0_i\tilde{\chi}^0_j$, near threshold and of the fermion invariant mass distribution of the neutralino 3–body decays, $\tilde{\chi}^0_i \rightarrow \tilde{\chi}^0_j f\bar{f}$, near the end point.

It is noteworthy that only the light neutralinos $\tilde{\chi}^0_{1,2}$ among the four neutralino states, which are expected to be lighter than sfermions and gluino in many scenarios, may be kinematically accessible in the initial phase of $e^+e^-$ linear colliders. In this situation, the method based on the threshold behaviors of the production of three different non–diagonal neutralino pairs for probing CP violation is not available. On the contrary, the combined analysis of the threshold excitation of the production process, $e^+e^- \rightarrow \tilde{\chi}^0_i\tilde{\chi}^0_j$, and the fermion invariant mass distribution of the decay, $\tilde{\chi}^0_2 \rightarrow \tilde{\chi}^0_1 f\bar{f}$, near the end point can still serve as one of the most powerful probes of CP violation in the neutralino system even in the initial phase of $e^+e^-$ linear colliders.
In order to illustrate the method for probing CP violation numerically, we take a parameter set for the fundamental SUSY parameters
\[\tan \beta = 10; \quad |M_1| = 100 \text{ GeV}, \quad M_2 = 150 \text{ GeV}, \quad |\mu| = 400 \text{ GeV}; \quad \Phi_\mu = 0 \]
and we choose two different values, \(\{0, \pi\}\) for the phase \(\Phi_1\), in the CP–invariant case and one value, \(\pi/2\), in the CP non–invariant case. [The parameter point with such a large phase \(\Phi_1 = \pi/2\) might already have been excluded by the stringent EDM constraints. Nevertheless, this point is taken just for illustrative purpose in the present work; the indirect EDM limits depend also on many parameters of the theory outside the neutralino sector.] We take the slepton masses, \(m_{\tilde{\ell}_L} = 250 \text{ GeV}\) and \(m_{\tilde{\ell}_R} = 200 \text{ GeV}\) and consider the 3–body leptonic decay \(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 l^+ l^-\), especially with \(l = e, \mu\), for the illustration. We note that the neutralinos \(\tilde{\chi}_1^0\) and \(\tilde{\chi}_2^0\) have the same (opposite) CP parity for \(\Phi_1 = 0\) (\(\Phi_1 = \pi\)). As expected from the selection rules (5) and (6) in the CP–invariant case, Figure 1 clearly shows that if the production of the neutralino pair \(\tilde{\chi}_1^0 \tilde{\chi}_2^0\) in \(e^+ e^-\) annihilation increases slowly in P–waves (steeply in S–waves) near threshold, then the lepton invariant mass distribution of the decay \(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 l^+ l^-\) decreases steeply in S–waves (slowly in P–waves).

\[\text{Figure 1: (a) The threshold behavior of the neutralino production cross–sections } \sigma\{12\} \text{ near the threshold and (b) the lepton invariant mass distribution of the decay } \tilde{\chi}_2^0 \to \tilde{\chi}_1^0 l^+ l^- \text{ near the end point, illustrated for the parameter set: } \tan \beta = 10, \quad |M_1| = 100 \text{ GeV}, \quad M_2 = 150 \text{ GeV}, \quad |\mu| = 400 \text{ GeV and } \Phi_\mu = 0 \text{ as well as the slepton masses, } m_{\tilde{\ell}_L} = 250 \text{ GeV and } m_{\tilde{\ell}_R} = 200 \text{ GeV.}\]

\[1\text{Analyses of electric dipole moments strongly suggest that CP violation in the higgsino sector will be very small in the MSSM if this sector is non–invariant at all.}\]

8
near the end point for the neutralino pair of the same (opposite) CP parity with $\Phi_1 = 0$ ($\Phi_1 = \pi$). On the contrary, in the CP–noninvariant case ($\Phi_1 = \pi/2$) the production and decay are excited steeply both in S–waves.

4 Conclusions

We have shown that only in CP–noninvariant theories the production of any non–diagonal neutralino pair $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ ($i \neq j$) in $e^+ e^-$ annihilation near threshold and the fermion invariant mass distribution of the 3–body neutralino fermionic decay $\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_j^0 f \bar{f}$ near the end point can simultaneously be excited steeply in S–waves.

In light of the possibility that only the two light neutralinos $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ among the four neutralinos can be accessed kinematically in the initial phase of $e^+ e^-$ linear colliders, the combined analysis of the production of the neutralino pair $\tilde{\chi}_1^0 \tilde{\chi}_2^0$ in $e^+ e^-$ annihilation near threshold and the neutralino decay $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 f \bar{f}$ near the end point of its fermion invariant mass could provide a first qualitative indication of the CP violation in the neutralino system.

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