Hadronic Weak Decay $B_b(\frac{1}{2}^+) \rightarrow B(\frac{1}{2}^+, \frac{3}{2}^+) + V$

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Abstract

It is shown that for the effective Lagrangian with factorization ansatz considered here, the two body hadronic decay $B_b(\frac{1}{2}^+) \rightarrow B(\frac{1}{2}^+, \frac{3}{2}^+) + V$, for $B_b(\frac{1}{2}^+)$ belonging to the representation 3, only allowed decay channel is $B_b(\frac{1}{2}^+) \rightarrow B(\frac{1}{2}^+) + V$, where $B(\frac{1}{2}^+)$ belongs to the representation 8 of $SU(3)$. However, for $B_b(\frac{1}{2}^+)$ belonging to the sextet representation 6, the allowed decay channels are $B_b(\frac{1}{2}^+) \rightarrow B(\frac{1}{2}^+, \frac{3}{2}^+) + V$, where $B(\frac{1}{2}^+)$ and $B(\frac{3}{2}^+)$ belongs to the octet representation $8'$ and the decuplet 10 of $SU(3)$, respectively. The decay channel $B_b(\frac{1}{2}^+) \rightarrow B(\frac{1}{2}^+) + V$ is analyzed in detail. The decay rate ($\Gamma$) and the asymmetry parameters $\alpha, \alpha', \beta, \gamma$ and $\gamma'$ are expressed in terms of four amplitudes. In particular for the decay $\Lambda_b \rightarrow \Lambda + J/\psi$ it is shown that within the factorization framework, using heavy quark spin symmetry, the decay rate and the asymmetry parameters can be expressed in terms of two form factors $F_1$ and $F_2/F_1$, which are to be evaluated in some model. By using the values of these form factors calculated in a quark model, the branching ratio and the asymmetry parameters $\alpha$ and $\alpha'$ are calculated numerically. For other heavy quarks belonging to the triplet and sextet representation, the results can be easily obtained by using $SU(3)$ symmetry and phase space factor. Finally, the decay $\Omega_b^- \rightarrow \Omega^- + J/\psi$ is analyzed within the factorization framework. It is shown that the asymmetry parameter $\alpha$ in this particular decay is zero. The branching ratio obtained in the first approximation is compared with the experimental value.
I. INTRODUCTION

Heavy flavor physics is of topical interest. New data for decays of $b$–hadrons will be coming out from the LHCb. In 2013 the LHCb Collaboration has performed an angular analysis of the decay $\Lambda_b \rightarrow \Lambda + J/\Psi$ where $\Lambda_b$'s are produced in proton-proton (pp) collisions at the centre of mass energy $\sqrt{s} = 7$ TeV at the LHC. By fitting several asymmetry parameters in the cascade decay distribution of $\Lambda_b \rightarrow \Lambda(\rightarrow p\pi^-) + J/\Psi(\rightarrow \ell^+\ell^-)$, the collaboration has reported the relative magnitude of helicity amplitudes in $\Lambda_b \rightarrow \Lambda + J/\Psi$ decay and also the transverse polarization of $\Lambda_b$ relative to the production plane.

Theoretically, the nonleptonic decay $\Lambda_b \rightarrow \Lambda + J/\Psi$ is quite attractive because only factorizable tree diagram contributes to the decay and there is no contribution due to $W$–exchange diagrams. In the $b$–baryon sector, the decay $\Lambda_b \rightarrow \Lambda + J/\Psi$ has been studied theoretically in quark model by using the factorization hypothesis and the results of some of these calculations have been compared to the new experimental results by the LHCb Collaboration.

The results of the branching fraction of $\Lambda_b \rightarrow \Lambda + J/\Psi$ decay given in the PDG $Br(\Lambda_b \rightarrow \Lambda + J/\Psi) \times Br(b \rightarrow \Lambda_b^0) = (5.8 \pm 0.8) \times 10^{-5}$ is deduced from the measurements by the CDF and the D0 collaborations. The result of branching fraction from the LHCb is still missing for this decay. In the present study, we give a general formalism for $B_b(\frac{1}{2}^+) \rightarrow B(\frac{1}{2}^+) + V$, especially with $V = J/\psi$. Using this formalism, we analyze $\Lambda_b \rightarrow \Lambda + J/\Psi$ decay in detail.

Heavy baryons with $J^P = \frac{1}{2}^+$, belong to either representation 3 or the sextet 6, whereas $J^P = \frac{3}{2}^+$ belongs only to the the sextet representation of the $SU(3)$.

\[
\begin{align*}
3 : & \quad A_{ij} = \frac{1}{\sqrt{2}}(q_i q_j - q_j q_i)Q_{\chi MA}, \\
6 : & \quad S_{ij} = \frac{1}{\sqrt{2}}(q_i q_j + q_j q_i)Q_{\chi MS}, \\
6^* : & \quad S^*_{ij} = \frac{1}{\sqrt{2}}(q_i q_j + q_j q_i)Q_{\chi S},
\end{align*}
\]

(1)

where $q_i, q_j$ are $u, d, s$; $Q = b$ or $c$ and $\chi$'s are the spin wave functions. In Eq. (1) $A_{ij}, S_{ij}$ and $S^*_{ij}$ correspond to $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$, respectively. The triplet of heavy baryons are

\[
(A_{12}, A_{13}, A_{23}) : (\Lambda_{b, c}^0, +, \Xi_{b, c}^0, +, \Xi_{b, c}^-, 0),
\]

(2)

whereas the sextet are

\[
(S_{11}, S_{12}, S_{22}) : (\sqrt{2}\Sigma_{b, c}^{++}, \Sigma_{b, c}^0, +, \sqrt{2}\Sigma_{b, c}^-, 0), \\
(S_{13}, S_{23}) : (\Xi_{b, c}^0, +, \Xi_{b, c}^-, 0), \\
S_{33} : \sqrt{2}\Omega_{b, c}^-, 0.
\]

(3)

In the Standard Model (SM) two body hadronic decays of heavy flavor mesons and baryons are analyzed in terms of the effective Lagrangian or Hamiltonian. Here, we take the Hamiltonian

\[
H_{\text{eff}} = V_{cb}V_{cs}^* [a_1(\bar{s}c)_V(\bar{c}b)_V - A(\bar{c}b)_V - A(\bar{s}b)_V - A],
\]

(4)

where $a_1 = C_1 + \zeta C_2$ and $a_2 = C_2 + \zeta C_1$, with $\zeta$ being the parameter for the possible number of colors. In terms of the diagrams, $a_1$ and $a_2$ correspond to the contribution from tree and color suppressed tree diagrams, respectively.

In the factorization ansatz, for the tree diagram and color suppressed tree diagram, the relevant matrix elements are $\langle B_c | (\bar{c}b)_V | B_b \rangle$ and $\langle B_s | (\bar{s}b)_V | B_b \rangle$, respectively. First, one can notice that that $\bar{c}b$ is $SU(3)$ singlet, whereas $\bar{s}b$
is $SU(3)$ triplet. Now

$$3 \times 3 = 8 + 1,$$

$$3 \times 6 = 10 + 8'.$$

Hence the possible decay modes for $B_b(\frac{1}{2}^+)\,$, for the first term in Eq. (4) are

$$3 : \quad (\Lambda_b^0, \Xi_b^0, \Xi_b^-) \to (\Lambda_c^+, \Xi_c^+, \Xi_c^0)(D_s^-)^*,$$

$$6 : \quad (\Sigma_b^0, \Sigma_b^-, \Sigma_b^+) \to (\Sigma_c^{++}, \Sigma_c^+, \Sigma_c^0)(D_s^-)^*,$$

$$(\Xi_b^0, \Xi_b^-) \to (\Xi_c^+, \Xi_c^0)(D_s^-)^*,$$

$$\Omega_b^- \to \Omega_c^0(\Omega_c^0)(D_s^-)^*. \quad (6)$$

Some of these decays given in Eq. (6) have been studied in ref. [16]. The main focus of the present study is the heavy to light decays of $b$-baryons.

For the color suppressed tree diagram, as noted in Eq. (5), for $B_b(\frac{1}{2}^+)\,$ belonging to the representation $\bar{3}$ the possible decay mode is

$$B_b(\frac{1}{2}^+) \to B(\frac{1}{2}^+)J/\Psi, \quad (7)$$

with $B(\frac{1}{2}^+)\,$ belongs to the octet representation 8 of $SU(3)$. However, for $B_b(\frac{1}{2}^+)\,$ belonging to the sextet representation, we have two possible decay modes:

$$B_b(\frac{1}{2}^+) \to B(\frac{1}{2}^+)J/\Psi, \quad (8)$$

For this case, $B(\frac{1}{2}^+)\,$ and $B(\frac{3}{2}^+)\,$ belong to the octet representation $8'$ and decuplet representation 10 of $SU(3)$, respectively. For the decay $B_b(\frac{1}{2}^+) \to B(\frac{1}{2}^+)J/\Psi$, the decay channels are

$$3 : \quad (\Lambda_b, \Xi_b^0, \Xi_b^-) \to (\Lambda, \Xi^0, \Xi^-)J/\Psi,$$

$$6 : \quad (\Sigma_b^0, \Sigma_b^-, \Sigma_b^+) \to (\Sigma^0, \Sigma^-, \Sigma^+)J/\Psi,$$

$$(\Xi_b^0, \Xi_b^-) \to (\Xi^0, \Xi^-)J/\Psi, \quad (9)$$

where $\Lambda, \Xi^0, \Xi^-$ are members of the octet representation 8 and $\Sigma^0, \Sigma^-, \Sigma^+, \Xi^0, \Xi^-$ are members of the octet representation $8'$. This study focus on the analysis of $B_b(\frac{1}{2}^+) \to B(\frac{1}{2}^+)V$ decays.

For the decay $B_b(\frac{1}{2}^+) \to B(\frac{3}{2}^+)J/\Psi$, where $B_b(\frac{1}{2}^+)\,$ belong the representation 6, the decay channels are

$$(\Sigma_b^0, \Sigma_b^-, \Sigma_b^+) \to (\Sigma^0, \Sigma^-, \Sigma^+)J/\Psi,$$

$$(\Xi_b^0, \Xi_b^-) \to (\Xi^0, \Xi^-)J/\Psi, \quad (10)$$

where the last decay is most interesting in this category.

\section{II. Hadronic Weak Decay of Baryon $B_b(\frac{1}{2}^+) \to B(\frac{1}{2}^+)V$: A General Formalism}

For the decay

$$B_b(\frac{1}{2}^+) (p) \to B(\frac{1}{2}^+) (p') + V(k, \epsilon) \quad (11)$$
where \( p = p' + k \) and \( k \cdot \epsilon = 0 \), the Lorentz structure of the \( T^- \) matrix is given by

\[
T = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2k_0}} \sqrt{m_{m'}} \bar{u}(p')[\gamma \cdot \epsilon (A(s) + B(s)\gamma_5) + i\epsilon^\mu \sigma_{\mu\nu} k^\nu (C(s) + D(s)\gamma_5)]u(p). \tag{12}
\]

In Eq. (12) the amplitudes \( A, B, C \) and \( D \) are the function of the square of momentum transfer, i.e., \( s = (p - p')^2 \). In the rest frame of baryon \( B_0 \)

\[
m = p'_0 + k_0,
\]

\[
\bar{p}' = -\bar{k} = -|\bar{k}|\bar{n}.
\tag{13}
\]

In this particular frame, one can write

\[
T = \chi^i_j M \chi_i
\tag{14}
\]

where

\[
M = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2k_0}} |\bar{k}| [f_1 \sigma \cdot (\bar{n} \times \bar{\epsilon}) + g_1 \bar{\sigma} \cdot \bar{\epsilon} + f_2 \bar{n} \cdot \bar{\epsilon} + g_2 (\bar{n} \cdot \bar{\epsilon}) (\bar{\sigma} \cdot \bar{n})],
\tag{15}
\]

with \( \sigma \) are the Pauli matrices. The amplitudes \( f_{1,2}, g_{1,2} \) and \( h \) can be written in terms of \( A, B, C, D \):

\[
f_1 = \frac{|\bar{k}|}{\sqrt{2p'_0(p'_0 + m')}} [A(s) - C(s)(m + m')],
\tag{16}
\]

\[
g_1 = -\frac{1}{\sqrt{2p'_0(p'_0 + m')}} [B(s)(p'_0 + m') + D(s)(k_0(m + m') - m_V^2)],
\tag{17}
\]

\[
f_2 = \frac{|\bar{k}|}{\sqrt{2p'_0(p'_0 + m')}} [A(s)(m + m') - C(s)m_V^2],
\tag{18}
\]

\[
g_2 = \frac{1}{\sqrt{2p'_0(p'_0 + m')}} [\bar{k}^2] [B(s) + D(s)(m + m')],
\tag{19}
\]

\[
h = g_1 + g_2 = \frac{1}{\sqrt{2p'_0(p'_0 + m')}} [B(s)((m + m')k_0 - m_V^2) + D(s)m_V^2(p'_0 + m')].
\tag{20}
\]

Under space reflection \( \sigma \rightarrow \bar{\sigma}, \bar{n} \rightarrow -\bar{n} \) and \( \bar{\epsilon} \rightarrow -\bar{\epsilon} \), thus \( f_1 \) and \( f_2 \) are the parity conserving i.e., \( p \)-wave amplitudes whereas \( g_1 \) and \( g_2 \) are the parity violating \( s \)-wave amplitudes. We also note that for the transverse polarization of \( V \) meson, only \( f_1 \) and \( g_1 \) are relevant, whereas, for the longitudinal polarization the relevant amplitudes are \( f_2 \) and \( h \).

The decay width of the above mode is given by

\[
d\Gamma = (2\pi)^7 \delta^4(p - p' - k) \frac{1}{2} Tr(M M^\dagger) d^3p' d^3k
\tag{21}
\]

which gives

\[
\Gamma = \frac{|\bar{k}|p'_0}{2\pi n} [(2|f_1|^2 + |g_1|^2) + \frac{k_0^2}{m_V^2}|f_2|^2 + |h|^2].
\tag{22}
\]

The first term on the left hand side of Eq. (22) corresponds to the transverse polarization and the second term to the longitudinal one.

Let \( \vec{S} \) and \( \vec{s} \) be the polarizations (spins) of \( B_0 \) and \( B \), respectively. The decay probability in terms of these polarization vectors is given by

\[
dW = (2\pi)^7 \delta^4(p - p' - k) \frac{1}{2} Tr[(1 + \sigma \cdot \vec{s})(1 + \bar{\sigma} \cdot \vec{S})M M^\dagger] d^3p' d^3k.
\tag{23}
\]
Hence, the transition rate is:

\[
\frac{dW}{\Gamma} = \frac{d\Omega_S d\Omega_s}{(4\pi)^2} [1 + \alpha \vec{S} \cdot \vec{n} + \alpha' \vec{s} \cdot \vec{n} + \beta \vec{s} \cdot (\vec{S} \times \vec{n}) + ((\vec{s} \cdot \vec{n})(\vec{S} \cdot \vec{n}))(\gamma - 1 + \gamma') + \gamma \vec{s} \cdot (\vec{n} \times (\vec{S} \times \vec{n}))],
\]

where

\[
\alpha = 2Re[-2f_1^* g_1 + (\frac{k_0}{m_V})^2 f_2^* h]p'_0 \frac{|\vec{k}|}{2\pi m \Gamma},
\]

\[
\alpha' = 2Re[2f_1^* g_1 + (\frac{k_0}{m_V})^2 f_2^* h]p'_0 \frac{|\vec{k}|}{2\pi m \Gamma},
\]

\[
\beta = 2Im[f_2^* h]p'_0 \frac{|\vec{k}|}{2\pi m \Gamma},
\]

\[
\gamma = (\frac{k_0}{m_V})^2 [2|f_2|^2 - |h|^2]p'_0 \frac{|\vec{k}|}{2\pi m \Gamma},
\]

\[
\gamma' = (\frac{k_0}{m_V})^2 [2|f_2|^2 + |h|^2]p'_0 \frac{|\vec{k}|}{2\pi m \Gamma}.
\]

Following comments are in order. For the transverse polarization, the asymmetry parameters are

\[
\alpha = -\frac{4Re[f_1^* g_1]p'_0 k_0}{2\pi m \Gamma} = -\alpha',
\]

whereas in case of the longitudinal polarization

\[
\alpha = (\frac{k_0}{m_V})^2 \frac{2Re[f_2^* h]p'_0 k_0}{2\pi m \Gamma} = \alpha'.
\]

It is clear from Eqs. (27, 28) and Eq. (29), that \(\beta, \gamma\) and \(\gamma'\) are non-zero only for the longitudinal polarization. For the longitudinal polarization, we get exactly the same result as that in the non-leptonic decay of \(B\) baryon, when \(V\) is replaced by pseudo-scalar meson \(P\).

**III. FACTORIZATION: BARYON FORM FACTORS**

In the factorization framework, the effective Hamiltonian for the decay \(B_6(\frac{1}{2}^+) \to B(\frac{1}{2}^+) + J/\psi\) is given by

\[
H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 (0) \bar{c} \gamma^\mu (1 - \gamma_5) c |J/\Psi\rangle \langle B | \bar{s} \gamma^\mu (1 - \gamma_5) b |B_6\rangle.
\]

The relevant matrix elements are

\[
\langle 0 | \bar{c} \gamma^\mu (1 - \gamma_5) c |J/\Psi\rangle = \left( \frac{1}{2\pi} \right)^{3/2} \frac{1}{\sqrt{2k_0}} F_{J/\psi m_J/\psi} \epsilon^\mu,
\]

\[
\langle B | \bar{s} \gamma^\mu (1 - \gamma_5) b |B_6\rangle = \left( \frac{1}{2\pi} \right)^3 \sqrt{\frac{mm'}{mk_0^2}} \bar{u}(p') [(g_V(k^2) - g_A(k^2)\gamma_5)\gamma^\mu - i\epsilon^\nu \sigma_{\nu\mu} k^\nu - (h_V(k^2) - f_A(k^2)\gamma_5)k_\mu u(p)]
\]

where \(f_V(k^2), g_V(k^2), f_A(k^2), g_A(k^2), h_V(k^2)\) and \(h_A(k^2)\) are the form factors. Now using Eqs. (33) and (34) in Eq. (32), the \(T\)-matrix can be written as

\[
T = \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{mm'}{2k_0^2 p_0 p'_0}} \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 F_{J/\psi m_J/\psi} \bar{u}(p') \gamma \cdot (g_V(k^2) + g_A(k^2)\gamma_5) - i\epsilon^\nu \sigma_{\nu\mu} k^\nu (f_V(k^2) + h_A(k^2)\gamma_5)]u(p).
\]
Hence, comparing Eq. (12) and Eq. (35), one gets
\[ A = G' m_{J/\psi} F_{J/\psi} g_V(k^2), \]
\[ B = G' m_{J/\psi} F_{J/\psi} g_A(k^2), \]
\[ C = -G' m_{J/\psi} F_{J/\psi} f_V(k^2), \]
\[ D = -G' m_{J/\psi} F_{J/\psi} h_A(k^2), \] (36)
where
\[ G' = V_{cb} V_{c}\frac{G_F}{\sqrt{2}}(C_2 + \zeta C_1). \] (37)
The short distance QCD effects are taken care of in the Wilson Coefficients \( C_1 \) and \( C_2 \). The long distance interactions are shifted to the form factors \( g_V, g_A, f_V \) and \( h_A \) which are needed to be evaluated in some model. Using the heavy quark spin symmetry, one can relate the different form factors [18] for which there are two choices:
\[(i) \ : \ g_V(k^2) = g_A(k^2) = F_1(k^2) + \frac{m_b}{m} F_2(k^2), \]
\[ f_V(k^2) = h_A(k^2) = \frac{1}{m} F_2(k^2) \]
\[(ii) \ : \ g_V(k^2) = -g_A(k^2) = F_1(k^2) + \frac{m_b}{m} F_2(k^2), \]
\[ f_V(k^2) = -h_A(k^2) = \frac{1}{m} F_2(k^2), \] (38)
where \( m = m_{b} \) and \( m_b \) is the mass of \( b \)-quark which in this work is taken to be to be 4.65GeV. Thus in terms of the form factors \( F_1(k^2) \) and \( F_2(k^2) \), we can write
\[ A = G' m_{J/\psi} F_{J/\psi} F_1(k^2) \left( 1 + \frac{m_b}{m} F_2(k^2) \right) = \pm B, \]
\[ C = -G' m_{J/\psi} F_{J/\psi} F_1(k^2) \frac{F_2(k^2)}{F_1(k^2)} = \pm D. \] (39)
where \( \pm \) sign in Eq. (39) corresponds to the choices \((i)\) and \((ii)\), respectively. We need the form factors at \( k^2 = m_{J/\psi}^2 \):
\[ F_1(m_{J/\psi}^2) = F_1, \quad F_2(m_{J/\psi}^2) = \frac{F_2}{F_1}. \] (40)
From Eqs. (16 - 19) by using Eq. (39) and Eq. (40), we can express the amplitudes \( f_1, g_1, f_2 \) and \( h \) in terms of form factors \( F_1 \) and \( F_2/F_1 \) as
\[ f_1 = R \frac{|\vec{k}|}{\sqrt{2p_0'(p_0' + m')}} F_1 \left[ 1 + (m_b + (m + m')) \frac{1}{m} F_2 \right], \] (41)
\[ g_1 = R \frac{p_0' + m'}{\sqrt{2p_0'(p_0' + m')}} F_1 \left[ \mp 1 + \left( \frac{m_b (p_0' + m') \pm k_0 (m + m') - m_{J/\psi}^2}{(p_0' + m')} \right) \frac{1}{m} F_2 \right], \] (42)
\[ f_2 = R \frac{|\vec{k}|}{k_0} \frac{m + m'}{\sqrt{2p_0'(p_0' + m')}} F_1 \left[ 1 + \frac{m_b (m + m') + m_{J/\psi}^2}{m + m'} \frac{1}{m} F_2 \right], \] (43)
\[ h = R \frac{(m + m')k_0 - m_{J/\psi}^2}{\sqrt{2p_0'(p_0' + m')}} F_1 \left[ \mp 1 + \left( \frac{m_b (k_0 (m + m') \pm m_{J/\psi}^2 (p_0' + m'))}{(k_0 (m + m') - m_{J/\psi}^2)} \right) \frac{1}{m} F_2 \right], \] (44)
with \( R = G' m_{J/\psi} F_{J/\psi} \) which is a dimensionless parameter.

We now consider the decay \( \Lambda_b \to \Lambda + J/\psi \) which is of experimental interest. In order to calculate this decay, various models to evaluate the form factors have been considered in the literature [9-11]. In reference [9] form factors were
TABLE I. Numerical values of the amplitudes for $\Lambda_b \to \Lambda + J/\psi$ for, $F_2/F_1 \approx 0.169$. Here $R^2 = (V_{cb}V_{cb}^{*}/\sqrt{2})(C_2 + \zeta C_1)m_{J/\psi}F_{J/\psi}$ \approx 18.97 \times 10^{-14}(C_2 + \zeta C_1)^2$. The values of the masses are used from \[19\]. Here ‘–’ and ‘+’ signs are for the choices (i) and (ii), respectively.

| Amplitudes | Numerical Values |
|------------|------------------|
| $f_1$      | $R F_1(0.644)$   |
| $g_1$      | $R F_1(\mp0.880)$ |
| $f_2$      | $R F_1(1.075)$   |
| $h$        | $R F_1(\mp1.197)$ |

evaluated in a quark model, and their values are $F_1 \approx -0.219$ and $F_2/F_1 \approx 0.169$. After putting $F_2/F_1 \approx 0.169$ and other input parameters in Eqs. \[11\] \[12\] \[13\] and Eq. \[14\], the numerical values of the amplitudes are given in Table I. These results can be extended for other baryons, by using physical masses for relevant parameters and $SU(3)$ symmetry.

Making use of the values of amplitudes outlines in Table I, the value of branching ratio for $\Lambda_b \to \Lambda + J/\psi$ decay is obtained to be (c.f. Eq. \[22\])

$$B_r \approx 1.18 \times 10^{-2}(C_2 + \zeta C_1)^2,$$

where $\zeta = \frac{1}{N_c}$, where $N_c$ is the effective number of colors. As noted in \[19\], there are two regime, viz $N_c < 1/3$ (Eq. \[46\]) and large $N_c$ limit. Using the values of Wilson coefficients $C_2 = -0.257$, $C_1 = 1.009$ \[23\] and for different values of $\zeta$ that correspond to the large $N_c$ limit, the values are given in Table II. One can see that for $\zeta = 0$, our results of branching ratio is compared with the $8.9 \times 10^{-4}$ that is obtained in ref. \[11\].

TABLE II. The values of Branching ratio for $\Lambda_b \to \Lambda + J/\psi$ for different values of $\zeta$ with large $N_c$ limit.

| $\Lambda_b \to \Lambda + J/\psi$ | $\zeta = 0$ | $\zeta = 0.01$ | $\zeta = 0.05$ |
|-------------------------------|------------|---------------|---------------|
| $B_r$                         | $7.8 \times 10^{-4}$ | $6.1 \times 10^{-4}$ | $5.0 \times 10^{-4}$ |

Similarly, for the value of $\zeta$ that correspond to small $N_c$ limit, the values of branching ratios are given in Table III. The experimental value of the branching ratio \[19\] is

$$B_r(\Lambda_b \to \Lambda J/\psi) \times B_r(b \to \Lambda_b^0) = (5.8 \pm 0.8) \times 10^{-5}.$$

Using $B(b \to \text{baryon}) \approx 9.29 \times 10^{-3}$, the experimental value of branching ratio for $\Lambda_b \to \Lambda J/\psi$ is $B_r = (6.2 \pm 0.8) \times 10^{-4}$ and it is comparable to our value $6.1 \times 10^{-4}$ when $\zeta = 0.48$ as well as for $\zeta = 0.01$.

TABLE III. The values of Branching ratio for $\Lambda_b \to \Lambda + J/\psi$ for different values of $\zeta$ that correspond to small $N_c$ limit.

| $\Lambda_b \to \Lambda + J/\psi$ | $\zeta = 1/3$ | $\zeta = 0.40$ | $\zeta = 0.45$ | $\zeta = 0.48$ | $\zeta = 0.50$ |
|-------------------------------|--------------|---------------|---------------|---------------|---------------|
| $B_r$                         | $0.74 \times 10^{-4}$ | $2.6 \times 10^{-4}$ | $4.6 \times 10^{-4}$ | $6.1 \times 10^{-4}$ | $7.2 \times 10^{-4}$ |

The values of asymmetry parameters for $\Lambda_b \to \Lambda + J/\psi$ decay are obtained from Eqs. \[30\] and \[31\] and these are

$$\alpha \approx \mp0.19 \quad \alpha_T \approx \pm0.39 \quad \alpha_L \approx \mp0.58$$

$$\alpha' \approx \mp0.98 \quad \alpha'_T \approx \pm0.39 \quad \alpha'_L \approx \mp0.58$$

(46)
The experimental value of the asymmetry parameter $\alpha = 0.18 \pm 0.13$. With our choice (i) of the form factors given in Eq. (38), the value of asymmetry $\alpha = -0.19$ is comparable to the values obtained in refs. (3–9). However, for choice (ii) of the form factors, the value of asymmetry parameter $\alpha = 0.19$ is comparable to the experimental value $\alpha = 0.18 \pm 0.13$.

We have discussed $\Lambda_b \to \Lambda + J/\Psi$ decay in detail and with this in hand, for heavy baryon belonging to the representation 3 and 6, the branching ratio can be easily obtained by using $SU(3)$ symmetry, taking into account the phase space for each baryon decay. For the decays $B_b(\frac{1}{2}^+) \to B(\frac{1}{2}^+) J/\Psi$, $SU(3)$ gives the relation

$$\sum \Lambda_b \to (3, \Xi^-) \to (3, \Xi^-) J/\Psi : \sqrt{3}(1, 1, 1),$$

for $B_b(\frac{1}{2}^+)$ belonging to representation 3 and $B(\frac{1}{2}^+)$ belonging to the octet representation. In case of $B_b(\frac{1}{2}^+)$ belonging to the sextet representation and $B(\frac{1}{2}^+)$ belonging to the representation $8'$, $SU(3)$ gives

$$\sum \Xi^- \to (\Xi^-) J/\Psi : \sqrt{2}(1, 1).$$

IV. THE DECAY $\Omega_b \to \Omega^- + J/\Psi$

In the factorization ansatz, corresponding to the effective Hamiltonian given in Eq. (41) the matrix element for $\Omega_b \to \Omega^- + J/\Psi$ decay is

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cb}^* (C_2 + \zeta C_1) e_{\gamma^\mu} (1 - \gamma^5) c |J/\Psi\rangle \langle \Omega^- | s_{\mu 1}(1 - \gamma^5) b |\Omega^-\rangle^*.$$  

We can write

$$\langle \Omega^- | s_{\mu 1}(1 - \gamma^5) b |\Omega^-\rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_m}{p_0 p_0'}} \left[ (F_1^V - \gamma^5 F_2^A)(\bar{u}_1(p') u(p)) + \right] .$$

where dots denote the contribution from other form factors which are suppressed by a factor of $\frac{1}{m_{\Omega b}}$ compared to $F_1^V$ and $F_2^A$ and hence will be neglected. From Eq. (49) and Eq. (50) along with Eq. (38), we get

$$|\mathcal{M}|^2 = G' F' J/F_{J/\Psi} m_J^{2 J/\Psi} e^\mu e^\nu [u_\nu(p') u_\mu(p') (F_1^V - \gamma^5 F_2^A) u(p') u(p)] ,$$

where $G' = \frac{G_F}{\sqrt{2}} V_{cb} V_{cb}^* (C_2 + \zeta C_1)$. Now

$$\sum_{\text{Polarization}} \epsilon^\nu(k) \epsilon^\nu(k) = \left( - \eta^\mu \eta^\nu + \frac{k^\mu k^\nu}{m^2_{J/\Psi}} \right) ,$$

$$\sum_{\text{Spin}} u_\nu(p') u_\mu(p') = - \frac{\gamma \cdot p'}{2m} \left[ \eta_{\nu \mu} - \gamma^\nu \gamma_\mu + \frac{i}{3m} (\gamma^\nu p'_\mu - p'_\nu \gamma_\mu) - \frac{2}{3m^2} \bar{p}_\nu p_{\nu'} \right] ,$$

$$\sum_{\text{Spin}} u(p) u(p) = \frac{1}{2} \frac{\gamma \cdot p + m}{2m} .$$

Using above equations, the decay rate is given by

$$\Gamma = \frac{1}{2\pi m} |\bar{k} (G' F_{J/\Psi} m_J^{J/\Psi})^2 (1 + \frac{m}{2m^2}) |^2 \left[ |F_1^V|^2 (p_0 + m') + |F_2^A|^2 (p_0 - m') \right] (C_2 + \zeta C_1)^2 .$$

In particular, for $\Omega_b \to \Omega^- + J/\Psi$, we have $m = m_{\Omega b}$ and $m' = m_{\Omega}$. Now

$$|\Omega^-\rangle = \frac{1}{\sqrt{3}} (s^a \bar{s}^a + s^b \bar{s}^b + s^c \bar{s}^c)$$

$$|\Omega_b^-\rangle = - \frac{1}{\sqrt{6}} (s^a \bar{s}^b + s^b \bar{s}^c + 2s^c \bar{s}^a)$$

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In NRQM, relevant operators to $O(v^2/c^2)$ are (for details see [20]) $\beta$ and $\beta \sigma_i$ with $i = z$. Using $\beta|b⟩ = |s⟩$, we have

$$\beta|\Omega_b^−, 1/2⟩ = -\frac{1}{\sqrt{6}}((s^\dagger s^\dagger s^\dagger + s^\dagger s^\dagger s^\dagger - 2s^\dagger s^\dagger s^\dagger))$$

and

$$\beta \sigma_z |\Omega_b^−, 1/2⟩ = -\frac{1}{\sqrt{6}}((s^\dagger s^\dagger s^\dagger + s^\dagger s^\dagger s^\dagger + 2s^\dagger s^\dagger s^\dagger)).$$

This gives $F_Y^1 = 0$ and $F_Y^A(0) = -\frac{2\sqrt{2}}{3}$ [21]. Thus

$$\Gamma(Ω_b^− → Ω^− + J/Ψ) ≈ 1.76 \times 10^{-14}|F_Y^A|^2(C_2 + ζC_1)^2 GeV .$$

(54)

Hence the branching ratio

$$Br(Ω_b^− → Ω^− + J/Ψ) = \frac{\tau_{Ω_b}}{h} \Gamma(Ω_b^− → Ω^− + J/Ψ) = 2.94 \times 10^{-2}|F_Y^A|^2(C_2 + ζC_1)^2$$

(55)

where $F_Y^A = F_Y^A(m_{J/Ψ}^2)$ and $\tau_{Ω_b}$ is the decay time of $Ω_b$. Now using

$$F_Y^A = \frac{1}{m_{J/Ψ}} \frac{m_b m_s}{m_b + m_s} F_Y^A(0) ≈ \frac{m_s}{m_{J/Ψ}} F_Y^A(0) ≈ 0.152 ,$$

(56)

The form factor $F_Y^A$ at $m_{J/Ψ}$ is expected to be smaller than $F_Y^A(0)$. For this purpose, we have introduced a dimensionless phenomenological factor $\left(\frac{1}{m_{J/Ψ}}\right)\left(\frac{m_b m_s}{m_b + m_s}\right)$, where the second factor is the reduced mass of the constituents of $Ω_b^−$. Using $F_Y^A ≈ 0.152$, the branching ratio is

$$Br ≈ 6.8 \times 10^{-4}(C_2 + ζC_1)^2.$$

Corresponding to the different values of $ζ$, the value of branching ratio is given in the Table [IV]

| $Ω_b → Ω + J/Ψ$ | $ζ = 0$ | $ζ = 0.01$ | $ζ = 0.05$ | $ζ = 1/3$ | $ζ = 0.40$ | $ζ = 0.44$ |
|-------------------|--------|---------|---------|---------|---------|---------|
| $Br$              | $4.5 \times 10^{-5}$ | $4.1 \times 10^{-5}$ | $2.9 \times 10^{-5}$ | $0.8 \times 10^{-5}$ | $1.8 \times 10^{-5}$ | $3.0 \times 10^{-5}$ |

Experimental $Br(Ω_b^− → Ω^− + J/Ψ) × Br(r → Ω_b) = (2.9^{+1.1}_{−0.8}) × 10^{-6}$ with $B(b → \text{baryon}) ≈ 9.29 \times 10^{-3}$ [19] gives

$$Br(Ω_b^− → Ω^− J/Ψ) = (3.12^{+1.1}_{−0.8}) \times 10^{-5} .$$

(57)

Finally, in this model, the asymmetry parameter is

$$α(Ω_b^− → Ω^− J/Ψ) = 0 .$$

(58)

To conclude: using the effective Lagrangian together with factorization ansatz the two body hadronic decay $B_b(\frac{1}{2}^+) → B(\frac{1}{2}^+, \frac{3}{2}^+) + V$ is calculated. In case of the $B_b(\frac{1}{2}^+) \rightarrow B(\frac{1}{2}^+, \frac{3}{2}^+) + V$, where $B(\frac{1}{2}^+)$ belong to the representation 8 of $SU(3)$. However, if $B_b(\frac{1}{2}^+) \rightarrow B(\frac{1}{2}^+, \frac{3}{2}^+) + V$, the allowed decay channels are $B_b(\frac{1}{2}^+) → B(\frac{1}{2}^+, \frac{3}{2}^+) + V$ where $B(\frac{1}{2}^+)$ and $B(\frac{3}{2}^+)$ belong to the octet representation 8′ and the decuplet 10 of $SU(3)$, respectively. We have analyzed the decay channel $B_b(\frac{1}{2}^+) → B(\frac{1}{2}^+) + V$ in detail, where the decay rate $Γ$ and the asymmetry parameters $α, α’, β, γ$ and $γ’$ are expressed in terms of four amplitudes. These amplitudes are written in terms of the transverse and the longitudinal polarization of $V$. This general formalism is then applied to the decay $Λ_b → Λ J/ψ$. It is shown that
within the factorization framework, using heavy quark spin symmetry, the decay rate and asymmetry parameters can be expressed in terms of two form factors $F_1$ and $F_2/F_1$, which being the non-perturbative quantities needed to be evaluated in some model. Here, by taking the values of these form factors calculated in the quark model the branching ratio and asymmetry parameters $\alpha$ and $\alpha'$ are obtained numerically. By taking the color factor $\zeta = 0.01$ or $\zeta = 0.48$, the branching ratio for the decay $\Lambda_b \rightarrow \Lambda + J/\psi$ is matchable to the corresponding experimental value. Having worked out $\Lambda_b \rightarrow \Lambda + J/\psi$ decay, this formalism can be easily applied to other heavy quarks belonging to triplet and the sextet representation, by using $SU(3)$ symmetry and the phase space factor. Finally, the decay $\Omega^-_b \rightarrow \Omega^- + J/\psi$ is analyzed within the factorization framework. It is shown that the asymmetry parameter $\alpha$ in this particular decay is zero. The branching ratio obtained in the first approximation is compared with the experimental value.

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