Large Microresonator-Assisted Optical-Parametric Amplification at Ultra-Low Pump Powers

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Chip-based optical amplifiers can significantly expand the functionalities of photonic devices. In particular, optical-parametric amplifiers (OPAs), with engineerable gain-spectra, are well-suited for a broad array of applications. Chip-based OPAs typically require high pump powers and long waveguides that occupy a large footprint to achieve sufficient gain. We propose and demonstrate a new microresonator-assisted OPA regime that benefits from the large nonlinearity enhancement of microresonators and yields a high gain in a small footprint. We achieve 30-dB parametric gain with only 9 mW of cw-pump power and show that the gain spectrum can be engineered to cover telecom channels inaccessible with Er-based amplifiers. We further demonstrate the amplification of Kerr-soliton comb lines and the preservation of their phase properties. Novel dispersion engineering techniques such as coupled cavity and higher-order-dispersion phase matching can further extend the tunability and spectral coverage of our OPA design. The combination of high gain, small footprint, low pump power, and flexible gain-spectra engineering of our OPA can lead to important applications in on-chip optical- and microwave-frequency synthesis and precise timekeeping.

Optical amplifiers are essential components in many optical systems and applications. Amplification is critical in many optical-frequency-comb-related precision metrology tasks, such as f-2f locking [1], optical-frequency synthesis [2], and microwave generation [3, 4], for achieving the desired accuracy. Furthermore, gas sensing and optical communications allow limited power in the transmitter due to safety and cost concerns, leading to low signal-to-noise ratio (SNR) due to detector noises, and low-noise optical amplifiers are often needed before the receiver to improve the SNR [5]. Typically, optical gain is produced via electronic population inversion, which depends critically on material properties. Most gain materials have largely fixed gain spectra and limited bandwidths. Modification of the gain spectra, when possible, requires significant engineering efforts and results in high production costs. On-chip implementation of optical amplifiers also faces challenges of compatibility issues between different materials. Notable demonstrations of on-chip gain materials include III-V semiconductors integrated on III-V chips [6, 8] or bonded to silicon chips [9] and Er doping in silicon dioxide, silicon nitride, or lithium niobate [10]. However, to date, these approaches only cover limited spectral ranges and have not been adopted by standard silicon-based foundries.

Alternatively, optical gain can be produced via non-linear parametric processes, and can cover a broad spectral range. While optical parametric amplifiers (OPAs) have been routinely used for amplifying signals where conventional material gains are unavailable, due to the limited nonlinearities of most materials, OPAs require long waveguides [11, 12] or synchronous-ultrashort-pulse pumping [13, 14]. Optical cavities and resonators have been shown to enhance nonlinearities, with applications including broad-band Kerr-comb generations [15–18], low-power frequency conversions [19, 20], and photon-level nonlinear interactions [21]. Multipass pseudocavities [22] and singly resonant cavities [23] have been used to develop bulk pulse-pumped OPA systems. However, such approaches still require high peak powers and are not easily adaptable to chip-based systems.

We propose a new regime of microresonator-assisted optical parametric amplification on a chip that is easily attainable with standard microresonators and allows for high gain with low pump powers. It is well-known that when the round-trip gain $G_{rt}$ exceeds the round-trip loss $L_{rt}$ inside a cavity, the net gain results in optical oscillation [Fig. 1], instead of amplification. We show that the field in the bus waveguide can be amplified when round-trip loss is larger than the intrinsic loss $L_i$ of the cavity. Such conditions can be readily realized in microresonators with parametric processes. Moreover, signal amplification can be achieved with a pump power below the oscillation threshold, which has been shown to be at sub-milliwatt levels in state-of-the-art high-Q devices [24]. Importantly, the gain spectra of this approach can be readily adjusted via dispersion engineering without changing the fabrication process. Furthermore, in this regime the signal effectively experiences multi-pass gain, which far exceeds that in a singly-resonant configuration, allowing for high gain coefficients. We demonstrate up to 30-dB gain outside the Er-gain band with 9 mW of pump power in a silicon-nitride microresonator, where the amplified signal preserves the phase properties of the input signal. We note that the same configuration has previously been used for frequency conversion with output idler powers being much less than the input signal power [30, 31], but the high-gain regime has not been ex-
FIG. 1. Schematic of the microresonator-assisted OPA. a, Different operation regimes of cw-pumped microresonators, where $G_{rt}$ is the roundtrip parametric gain, $L_i$ is the intrinsic roundtrip loss, and $L_{rt}$ is the total roundtrip loss. The microresonator functions as a notch filter, parametric oscillator, and parametric amplifier for $G_{rt} < L_i$, $G_{rt} > L_{rt}$, and $L_i < G_{rt} < L_{rt}$, respectively. b, Optical microscope image of the experimental device. The footprint can be further reduced by replacing the ring resonator with a spiral resonator. c, Schematic of the experiment. We use our OPA to amplify a seed laser or lines of a Kerr comb. The output is analyzed with an oscilloscope (Osc), a phase-noise-analysis (PNA) system, and an optical spectrum analyzer (OSA). d, Experimental input and output spectra of the OPA system. The green-shaded region corresponds to the spectral range covered by Er-doped fiber amplifiers.

Alternatively, it has been shown that cw signals can be amplified via injection locking to lasers. Similarly, we demonstrate that optical parametric oscillators (OPOs) can be injection-locked to an external signal, and the signal’s phase-noise properties are inherited up to certain offset frequencies. This approach allows for higher pump-to-signal power efficiency at the cost of losing the amplitude information.

Our scheme is based on below-threshold degenerately-pumped four-wave mixing (DFWM) in a microresonator, and the same principle can be applied to all parametric gain processes. For simplicity, we represent the nonlinear interaction of the signal mode as a roundtrip power gain term $G_{rt}$ and a detuning term $\Delta_{NL}$ (see Methods). The small-signal power gain for the output field can be expressed as,

$$G = 1 + \frac{4(L_{rt} - L_i)(G_{rt} - L_i)F^2}{(L_{rt} - G_{rt})^2F^2 + 4(\Delta - 2\Gamma|A|^2 - \Delta_{NL})^2},$$

where $L_i$ is the intrinsic roundtrip loss, $L_{rt}$ is total roundtrip loss, $\Gamma$ is the cavity-enhanced nonlinear coefficient, $\Delta$ is the signal detuning, $F$ is the FSR, and $A$ is the intracavity pump amplitude such that $|A|^2$ is normalized to power. Inspection of Eq. (1) shows that the system has a net gain $G > 1$ if $G_{rt} > L_i$, that is, the nonlinear gain is larger than the intrinsic loss. The second terms in the numerator and denominator correspond to the detuning from the cavity resonance, which impact the maximum gain and bandwidth of the gain profile. As $G_{rt}$ approaches $L_{rt}$, the system corresponds to an ultrahigh-$Q$ cavity with a high extraction rate, a regime that is unattainable with passive cavities. The gain process can be attributed to the extraction of the strong cavity-enhanced field at a high rate. An ideal phase-insensitive optical amplifier has a noise figure $NF = 2$ in the high-gain regime, where the NF is defined as the ratio between the SNRs of the output and input signals, and SNR is defined as the ratio between the square of the optical power and the variance of the optical power (see Methods). In addition, the NF of 2 can be realized only when the bandwidth of the noise is filtered to match that of the signal, which is then limited by the quantum noise. Similar to many cavity-based quantum systems, the quantum-noise-limited behavior of the microresonator-assisted OPA depends on the coupling condition of the cavity. As detailed in Methods, NF of 2 can be achieved in our OPA configuration when the cavity is strongly over coupled, that is, $L_{rt} \gg L_i$. For a critically coupled cavity ($L_{rt} = 2L_i$), which minimizes the input pump power, the lowest achievable NF is 4, which is due to the additional fluctuations induced by the scattering losses. In practical systems, the NF is also affected by the pump intensity and phase noise, and the thermal noise of the microresonator, which can be mitigated by using lower-$Q$ cavities and higher pump powers.

In our experimental demonstration, we design our
FIG. 2. Characterization of the microresonator-assisted OPA gain. a, Experimental gain and saturated output power of the OPA system, where dashed lines are fits to an exponential function. b, c the gain spectral response for 125 nW input power and 19 µW input power, respectively. d, The OPA gain coefficient as a function of pump power. The shaded region on the left corresponds to pump power below the OPO threshold. The central region corresponds to pump power being higher than the threshold power. However, the detuning is chosen such that the cavity does not oscillate. The shaded region on the right corresponds to the first OPO sideband being different from the signal wavelength being amplified due to increased pump power.

![Graphs showing OPA gain characteristics](image)

FIG. 3. Simulations of the microresonator-assisted OPA system. a, Simulated gain with close-to-threshold pump power, showing a small-signal gain of 45 dB and an onset of gain saturation at 100 pW. b, Simulated gain bandwidth for varying seed powers. All input levels have a constant GBW of 1.9 GHz. c, Simulated frequency-noise transfer, showing large pump-noise isolation at lower offset frequencies, which diminishes at higher offset frequencies.

![Graphs showing OPA simulation results](image)

microresonator-based OPA to operate near 1522 nm, which is outside the Er-gain window. The microresonator cross-section is 730 x 1500 nm with a radius of 100 µm and has a loaded and intrinsic Q of 1.4 x 10^6 and 3.6 x 10^6, respectively. We use a pump at 1540.8 nm, corresponding to a peak gain at 1521.8 nm and 1560.3 nm. As shown in Fig. 1d, the orange traces correspond to the input pump and signal fields, and the blue traces correspond to the output fields in the OPA regime. With 9 mW of pump power, the 125-nW input signal is amplified by a factor of ≈ 1000. An almost equally strong idler field is generated due to energy conservation of the DFWM process. We further observe cascaded lines due to the high effective nonlinearity of our system. Since high gain requires the system to exhibit high Q, there is a trade-off between the gain and the bandwidth, which is typical for many types of amplifiers. Figures 2b and 2c show the frequency response of the 125 nW and 19 µW input signals with a gain coefficient of 30 dB and 13 dB, respectively. The gain-bandwidth product (GBW) can be further improved by increasing the bus-ring coupling coefficient and pump power. Due to the high gain of the microresonator-assisted OPA, it operates in the saturation regime even for a 125-nW input signal, which is the current limit of our photodetector sensitivity. We measure the output powers for varying input powers and find that \( P_o \propto P_i^{0.26} \) where \( P_o \) and \( P_i \) are the output and input powers, respectively [Fig. 2d]. Figure 2d shows the gain as a function of pump power. The gain quickly rises as the pump power approaches the OPO threshold and increases slowly after the pump power crosses the threshold. At high pump powers, the first OPO sideband shifts farther from the pump, leading to a reduced gain at the original OPA resonance. We also numerically investigate the gain for a larger range of input powers as shown in Fig. 3a (see Methods). We observe a small-signal-gain of 45 dB and the onset of gain saturation at 100 pW (Fig. 3b). The gain reduces for higher seed powers due to saturation. However, the GBW remains constant at 1.9 GHz (Fig. 3c).
We characterize the phase-noise properties of the microresonator-assisted OPA system, which is important for applications including \( f - 2f \) stabilization \([1]\), optical frequency division \([3]\), and optical frequency synthesis \([2]\). We use a signal laser at 1560.3 nm that is lower noise than the pump laser at 1540.8 nm, and characterize the frequency noise of amplified signals using a delayed self-heterodyne system \([39]\). The phase noise of the seed laser is characterized at a power level of 5 mW, and the seed is strongly attenuated before being sent into the OPA. Figure 4a shows the phase noise of the amplified signals with varying input power levels. The amplified signals have the same noise characteristics as the original signal at lower offset frequencies. The deviation at the high offset frequencies is largely due to the noise floor of the photodetector. For comparison, we pump the microresonator above the OPO threshold and characterize the corresponding OPO noise. The OPO noise largely follows that of the pump laser (see Supplementary Material), which is generally much higher than the seed laser and has distinctive high-noise sidebands. As shown in Fig 4a, the phase noise of the amplified signal is not affected by the pump noise, where the suppression of the noise feature at 22 kHz corresponds to a minimum of 31-dB pump-noise isolation. This pump-noise isolation is also verified with our numerical model. As shown in Fig. 4b (see Methods), larger pump-noise suppression is achieved at lower offset frequencies and higher seed powers.

Kerr combs allow for the on-chip generation of broadband modelocked combs with a cw pump laser and have shown great promise for frequency-synthesis applications \([18, 20]\). However, in the soliton-modelocking regime, most Kerr combs have low comb line powers, which could require off-chip amplification for many applications \([2, 3]\). We experimentally show that our microresonator-assisted OPA system can achieve strong on-chip amplification at specified soliton comb lines. We generate the Kerr soliton using a microresonator with a cross-section of \( 730 \times 1700 \) nm and a free-spectral range (FSR) of 77 GHz. Figures 4b and 4c show the sech\(^2\) soliton spectral shape where the center part is filtered due to the OPA-pump combination using a wavelength-division multiplexer (WDM). The WDM can also be replaced by an on-chip add-drop ring filter that does not affect any comb lines. We use an OPA pump power of 8.6 mW and a soliton pump power of 340 mW. We jointly tune the OPA pump wavelength and the OPA ring resonance (via an on-chip heater) to achieve amplification of different comb lines. As shown in Figs. 4b and 4c, traces (i) and (ii) correspond to amplified comb lines of the 35\(^{th}\) (1522.0 nm) and 24\(^{th}\) modes (1528.7 nm), with a gain factor of 22 dB and 21 dB, respectively. The corresponding idler modes are shown as trace (v), which do not overlap with any comb modes. The amplification allows us to characterize the phase noise of the high-order comb lines, which is not detectable with our measurement system at their original power levels. As shown in Fig. 4d, the comb-line noise increases with the mode order, which is due to the cascading of FSR fluctuations. Consequently, the FSR fluctuations can be significantly reduced if high-order comb lines are measured and stabilized, which is the critical part of frequency synthesis. Our OPA provides an approach for high SNR measurement of the noise at the low-power comb lines, which is otherwise strongly limited by detec-
tor noise.

For applications only requiring phase information, optical amplification can also be performed via injection locking. In such a process, a weak laser within a certain frequency range of a free-running optical tone is injected onto the cavity, where the free-running tone is subsequently phase-locked to the injecting laser. The capturing range (allowable frequency separation between the seed and the free-running tone) depends on the strength of the seed laser and the cavity-coupling condition. This procedure has been demonstrated for laser cavities, with applications in phase-encoded telecommunications [34] and interferometric gravitational-wave sensors [40]. Here, we show that it can also be realized in the OPO regime, where the free-running tone is the signal of the OPO.

The experiment setup of injection locking is identical to that of the OPA (Fig. 1c). However, in the OPA regime, the pump is tuned below the oscillation threshold. In contrast, in the injection-locking regime, the OPO is generated before we send in the seed laser. To better demonstrate the frequency noise characteristics, we use a probe laser at 1560 nm, which has lower noise than the pump laser. However, dispersion engineering can also be applied to amplify fields at other frequencies. With a pump power of 11 mW, we generate an OPO with 2.1 mW of signal/idler power, corresponding to a -7-dB power-conversion efficiency for each wavelength. The free-running phase noise is shown as the green trace in Fig. 5, which largely follows the pump noise (see Supplementary Material). We vary the injection power at 1560.8 nm and observe the change of phase noise. Experimentally, a minimum seed power of 15 µW is required to lock the OPO signal to the seed laser, which corresponds to a maximum power gain of 21 dB (Fig. 5a). As shown by the red trace in Fig. 5b, the injection forces the phase noise of the OPO signal to follow that of the seed laser at offset frequencies below 300 kHz, where the original noise features are completely removed. Notably, the removal of the free-running-OPO noise peak at 23 kHz corresponds to a > 30 dB pump-noise isolation. At higher offset frequencies, the noise of the locked signal approaches that of the free-running OPO, which can be mitigated with higher seed-laser powers (Fig. 5b).

In conclusion, we have proposed and demonstrated a novel approach to achieve high on-chip optical-parametric amplification, which allows for spectral coverage not possible with active gain materials. Benefiting from cavity enhancement, our scheme allows for higher gain at much lower pump powers and smaller footprints than previous on-chip OPA demonstrations [11–14], and the footprint of our device is also significantly smaller than most devices with comparable material gain [41]. Our method also applies to other nonlinear gain processes, such as χ(2) processes. Novel dispersion engineering techniques can be applied to expand the gain region. For example, coupled-ring structures can be used to locally tune the dispersion [42, 43], and we observe active tuning of the peak gain by ≈ 90 nm on each side of the pump (Supplementary Material). Higher-order-dispersion phase matching can allow for peak gain far away from the pump [23, 44, 46], which can provide high-gain amplifiers for mid-infrared or ultraviolet regimes. We observe that the phase noise of the amplified signal is well-isolated from those of the pump. We have shown that our OPA scheme is suitable for the amplification of Kerr-comb lines, which have intrinsically low powers, and this technology can help expand the scope of fully integrable Kerr-comb applications, including atomic clocks [3], optical frequency synthesis [2], and ultra-low-noise microwave generation [4]. Data-communication applications can also be implemented with low-Q microresonators and materials with higher intrinsic nonlinearities.

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**Disclosures.** The authors declare no competing interests.

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METHODS

Classical Model of OPA. The classical dynamics of our OPA system are governed by

\[
\frac{d A}{dt} = -\frac{\alpha}{2} A - i \Delta_A A + i \Gamma (|A|^2 + 2|B|^2 + 2|C|^2) A + i 2 \Gamma A^* B C + \sqrt{\alpha F} A_i, \tag{2}
\]

\[
\frac{d B}{dt} = -\frac{\alpha}{2} B - i \Delta_B B + i \Gamma (|A|^2 + |B|^2 + |C|^2) B + i \Gamma A^2 C^* + \sqrt{\alpha F} B_i, \tag{3}
\]

\[
\frac{d C}{dt} = -\frac{\alpha}{2} C - i \Delta_C C + i \Gamma (|A|^2 + |B|^2 + |C|^2) C + i \Gamma A^2 B^*, \tag{4}
\]

where \( A, B, \) and \( C \) are the intracavity field amplitudes of the pump, signal, and idler fields, respectively, \( \alpha \) is the loss rate of the cavity, \( \Delta_A, \Delta_B, \) and \( \Delta_C \) are the detunings of the pump, signal, and idler fields, respectively, \( \Gamma \) is the cavity-enhanced nonlinear coefficient, \( \alpha_0 \) is the bus-ring coupling rate, \( \mathcal{F} \) is the FSR, and \( A_i \) and \( B_i \) are the input fields of the pump and signal, respectively. We normalize the fields such that the square norm of the amplitudes corresponds to power. The cavity-enhanced nonlinear coefficient \( \Gamma = \gamma L \mathcal{F} \), where \( \gamma \) is the nonlinear coefficient, and \( L \) is the cavity length. We also note that the idler detuning \( \Delta_C \) is fully determined by \( \Delta_A, \Delta_B, \) and the cavity dispersion.

We first look at the gain of a weak input signal. We limit the amplitude \( B_i \) such that \( |B|^2, |C|^2 \ll |A|^2 \) at steady state, and ignore the thermal effect (\( \eta = 0 \)). The nonlinear term in Eq. (3) can be written as

\[
i \Gamma A^2 C^* = \frac{\alpha_0}{2} \Gamma |A|^4 \frac{2 \Gamma |A|^2}{4 + |A|^2} \\frac{|B|^2}{2} + \frac{2 \Gamma |A|^2}{4} (|C|^2 - 2 |A|^2) \Gamma |A|^4 \frac{|B|^2}{2} + \frac{2 \Gamma |A|^2}{4} (|C|^2 - 2 |A|^2) \frac{|B|^2}{2}
\]

\[
= G_{nt} \mathcal{F} B + i \Delta_{NL} B, \tag{5}
\]

where \( G_{nt} \) is the induced roundtrip power gain and \( \Delta_{NL} \) is the induced detuning. The output field can be found
as

\[ B_o = B_i - \sqrt{\frac{\alpha_1}{F}} B \]

\[ = \frac{\alpha - 2\alpha_1 - G_{rt} F + i 2 (\Delta B - 2\Gamma |A|^2 - \Delta_{NL})}{\alpha - G_{rt} F + i 2 (\Delta B - 2\Gamma |A|^2 - \Delta_{NL})} B_i. \]

(7)

With \( L_{rt} = \alpha / F \) and \( L_i = (\alpha - \alpha_1) / F \), we get the net power gain as shown in Eq. (11).

In general, Eqs. \( 2, 3 \) can be solved numerically to fully incorporate gain saturation. The input fields can also be treated as stochastic variables to simulate noise properties. We numerically simulate a systems with \( F = 200 \text{ GHz}, \alpha = 2\pi \times 100 \text{ MHz}, \alpha_1 = 2\pi \times 100 \text{ MHz}, \Gamma = 1.38 \times 10^8 \text{ W}^{-1}s^{-1}, \Delta_A = 2\pi \times 28 \text{ MHz}. \) We assume a group-velocity dispersion of -50 ps²/km which corresponds to a detuning relation \( \Delta_B + \Delta_C = 2\Delta_A + 1.3 \times 10^9 \text{ s}^{-1}. \) To explore the gain saturation at a pump power of 9 mW, we keep \( \Delta_B = \Delta_C, \) which corresponds to the peak gain detuning, and we vary the input signal power. The corresponding result is shown in Fig. 3b. To simulate the phase noise properties, we keep \( \Delta_B = \Delta_C \) and let \( A_1 \) be a random process corresponding to a Lorentzian spectral shape with a full-width-at-half-maximum linewidth of 10 kHz. The corresponding result is shown in Fig. 3b.

Quantum Model of OPA. For a weak signal input, that is, without gain saturation, the quantum dynamics of the system can be modeled as

\[ \frac{d\hat{b}}{dt} = -\frac{\alpha_1 + \alpha_2}{2} \hat{b} - i\Delta_B \hat{b} + i 2\Gamma |A|^2 \hat{b} + i\Gamma A^2 \hat{c}^\dagger \\
+ \sqrt{\alpha_1 F \hbar \omega \hat{b}_i} + \sqrt{\alpha_2 F \hbar \omega \hat{l}}, \]

(8)

\[ \frac{d\hat{c}^\dagger}{dt} = -\frac{\alpha_1 + \alpha_2}{2} \hat{c}^\dagger + i\Delta_C \hat{c}^\dagger - i 2\Gamma |A|^2 \hat{c}^\dagger - i\Gamma A^2 \hat{b} \\
+ \sqrt{\alpha_1 + \alpha_2 F \hbar \omega \hat{g}}, \]

(9)

where \( |A|^2 \) is the intracavity pump power, \( \hat{b} \) and \( \hat{c} \) are the annihilation operators of the cavity modes, \( \alpha_1 \) is the coupling rate, \( \alpha_2 \) is the loss rate, \( \Delta_B \) and \( \Delta_C \) are the detunings of the signal and idler, \( \Gamma \) is the cavity-enhanced nonlinear coefficient, \( F \) is the FSR, \( \omega \) is the pump frequency, \( \hat{b}_i \) is the annihilation operator of the input field, \( \hat{l} \) is the annihilation operator of the reservoir modes coupled to \( \hat{b} \) via scattering, and \( \hat{g} \) is the annihilation operator corresponding to all dissipation mechanisms of mode \( \hat{c} \). In our treatment, the intracavity fields serve as intermediate steps to connect the input and output fields via \( \hat{b}_o = \hat{b}_i - \sqrt{\alpha_1 (F \hbar \omega)} \hat{b}, \) where \( \hat{b}_o \) is the annihilation operator of the output field. We choose the convention such that the intracavity fields are normalized to power, which is consistent with the classical equations. Equations \( 5 \) and \( 6 \) can be solved in the frequency domain, where we define

\[ \tilde{\hat{o}}(\Omega) = \frac{1}{\sqrt{2\pi}} \int \hat{o}(t)e^{i\Omega t} dt, \]

(10)

with \( \hat{o} \) being any annihilation operator. For simplicity, we let \( \Delta_B = \Delta_C = 2\Gamma |A|^2, \) which corresponds to the peak gain. With some calculation, we can find that

\[ \tilde{\hat{o}}(\Omega) = G_1(\Omega)\tilde{\hat{o}}_i(\Omega) + G_2(\Omega)\tilde{\hat{l}}(\Omega) + G_3(\Omega)\tilde{\hat{g}}^\dagger(-\Omega), \]

(11)

where

\[ G_1(\Omega) = \frac{\Gamma^2 |A|^2 + \frac{\alpha_1 + \alpha_2}{2} - i\Omega (\frac{\alpha_1 - \alpha_2}{2} + i\Omega)}{(\frac{\alpha_1 + \alpha_2}{2} - i\Omega)^2 - \Gamma^2 |A|^2}, \]

(12)

\[ G_2(\Omega) = \frac{\sqrt{\alpha_1 \alpha_2 (\frac{\alpha_1 + \alpha_2}{2} - \Omega)^2 - \Gamma^2 |A|^2}}{(\frac{\alpha_1 + \alpha_2}{2} - \Omega)^2 - \Gamma^2 |A|^2}, \]

(13)

\[ G_3(\Omega) = \frac{\Gamma |A| \sqrt{\alpha_1 (\alpha_1 + \alpha_2)}}{(\frac{\alpha_1 + \alpha_2}{2} - \Omega)^2 - \Gamma^2 |A|^2}. \]

(14)

We have absorbed extra phase terms into the operators, which does not change their commutation relations. We consider an optical pulse with a spectral envelope \( s(\Omega) \), where \( \int s(\Omega)d\Omega = 1. \) For simplicity, we let the bandwidth of \( s \) be much smaller than the gain bandwidth. The input-output relation for this pulse mode can be written as

\[ \tilde{\hat{o}}_{os}(\Omega) = G_1(\Omega)\tilde{\hat{o}}_{i}(\Omega) + G_2(\Omega)\tilde{\hat{l}}(\Omega) + G_3(\Omega)\tilde{\hat{g}}^\dagger(-\Omega), \]

(15)

where \( \hat{o}_s = \int s(\Omega)\hat{o}(\Omega)d\Omega, \) \( \hat{o} \) being any frequency-domain annihilation operator. We use \( |\beta| \) to represent a coherent state input in mode \( \hat{b}_i. \) Thus, the input SNR is

\[ \text{SNR}_i = \frac{\langle \hat{b}_i \hat{b}_i^\dagger \rangle^2}{\langle \hat{b}_i^\dagger \hat{b}_i \rangle^2} = |\beta|^2. \]

(16)

Similarly, we can find the output SNR as

\[ \text{SNR}_o = \frac{G_1^2(0)|\beta|^4}{2G_2^2(0)G_3^2(0)|\beta|^2 + G_1^2(0)|\beta|^2 + G_3^2(0)}. \]

(17)

When the scattering loss of the cavity is negligible, we have \( G_3(0) = \sqrt{G_1^2(0) - 1}. \) In the high-gain and high-input regime, \( i.e., G_1(0), |\beta| \gg 1, \) the noise figure is

\[ \text{NF} = \frac{\text{SNR}_i}{\text{SNR}_o} \approx 2. \]

(18)

In a critically coupled cavity, we have \( G_3(0) = \sqrt{2G_1(0)(G_1(0) + 1)}. \) In the high-gain and high-input regime, we have

\[ \text{NF} = \frac{\text{SNR}_i}{\text{SNR}_o} \approx 4. \]

(19)

Phase Noise Analysis System. We measure the phase noise of lasers using a coherent delayed self-heterodyne system [33]. As shown in Fig. 4, the system consists of an arm-length imbalanced Mach-Zehnder interferometer. The fiber delay is 100 m which is shorter than the coherence length of the lasers we measure. We
FIG. 6. Phase noise analysis system. PC is polarization controller, and AOM is acousto-optic modulator.

drive the acousto-optic modulator (AOM) at 80 MHz, which has a much lower phase noise than the lasers. Thus, the phase noise of the resulting beat signal is

\[ S_{\text{rf}}(\omega) = 4 \sin^2 \left( \frac{\omega \tau}{2} \right) S_{\text{opt}}(\omega), \]  

where \( \tau \) is the time delay corresponding to the arm-length difference.
PEAK-GAIN TUNING WITH COUPLED CAVITIES

Resonances from two cavities with similar frequencies strongly interact when coupling is introduced between them, leading to shifts of their resonance frequencies. For cavities with different free-spectral ranges (FSRs), such strong mode interactions occur locally, which can allow phase matching of optical-parametric-oscillation (OPO) processes at the mode interaction region. Since the strength of the localized resonance shift depends on the separation of resonance frequencies, they can be actively tuned via on-chip heaters, which is equivalent to tuning the peak optical-parametric-amplifier (OPA) gain location. We experimentally show this using coupled microresonators with a device similar to that in [S1]. We use a cross-section of $640 \times 1800$ nm, a main-ring FSR of 200 GHz, and an auxiliary-ring FSR of 201.6 GHz (Fig. S1b). The pump power used in the experiment is 100 mW, which is due to the effective loss introduced by mode coupling. Such loss can be mitigated by adjusting the coupling strength, and frequency difference of the rings [S2]. We generate OPOs at disparate wavelengths by tuning the auxiliary-ring heater (Fig. S1b) while keeping the main-ring heater at a constant voltage. The peak gain on the short-wavelength side shifts from 1460.6 nm to 1540.7 nm, and the peak gain on the long-wavelength side shifts form 1652 nm to 1560.5 nm, corresponding to a 10.7 THz tuning on each side of the pump.

GAIN COEFFICIENT AT DIFFERENT RESONANCES

We characterize the OPA gain for different resonances under the same pumping condition. We use a pump power of 9 mW and a seed power of 650 nW. The peak gain is at 1521.8 nm, which corresponds to the OPO wavelength. Due to the limit of our seed-laser wavelength-tuning range, we probe the gain for wavelengths larger than the peak gain wavelength. Figure S1c shows the different gain factors for different cavity resonances. We see that the gain is significantly lower for resonances other than the peak-gain resonance. This is because these resonances are farther away from oscillation. Nonetheless, we observe a 7-dB gain for the resonance 5-nm away from the peak-gain location.

FIG. S1. Peak gain tuning and gain at non-peak resonances. a, The change of peak-gain location by coupled-ring tuning. b, The coupled-ring structure. c, Comparison of gain at peak and non-peak resonances.
FIG. S2. Comparison of pump and OPO frequency noise. The OPO noise is dominated by thermal noise at low offset frequency, and dominated by pump noise at high frequencies and around 23 kHz where pump noise is high.

**PUMP AND OPO NOISE**

The phase noise analysis system shown in Methods is used to characterize the frequency noise of the pump and OPO. As shown in Fig. S2, The OPO noise agrees with pump noise at frequencies above 100 kHz where thermorefractive noise is low. Additionally, the OPO noise is dominated by pump noise around 23 kHz where the pump noise has a strong peak. At lower offset frequencies, the thermorefractive noise dominates the OPO noise. Finally, the spurious noise peak of the OPO at 65 kHz is due to the voltage supplier connected to the integrated heater.

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