Dynamic response of interacting
one-dimensional fermions in the harmonic
atom trap: Phase response and the
inhomogeneous mobility

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Abstract

The problem of the Kohn mode in bosonized theories of one-dimensional interacting
fermions in the harmonic trap is investigated and a suitable modification of the
interaction is proposed which preserves the Kohn mode. The modified theory is
used to calculate exactly the inhomogeneous linear mobility $\mu(z, z_0; \omega)$ at position $z$
in response to a spatial force pulse at position $z_0$. It is found that the inhomogeneous
particle mobility exhibits resonances not only at the trap frequency $\omega_t$ but also at
multiples $m\tilde{\epsilon}$, $m = 2, 3, \ldots$ of a new renormalized collective mode frequency which
depends on the strength of the interaction. In contrast, the homogeneous response
obtained by an average over $z_0$ remains that of the non-interacting system.

Key words: One-dimensional ultracold fermions, harmonic trap, Kohn’s theorem,
inhomogeneous mobility

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1 Introduction

Under suitable conditions, a gas of interacting fermions in one spatial dimension can be asymptotically described by a bosonic phase operator obeying a simple equation of motion. Conditions for this correspondence between fermionic and bosonic descriptions are a linear dispersion of free particle states allowing bosonization via Kronig’s identity \([1,2]\) and the addition of an anomalous vacuum which requires large fermion numbers not to cause harm. Furthermore the bosonic Hamiltonian should be bilinear in the field operator to facilitate diagonalization. The best known example is the Luttinger model \([3,4,5,6]\). One-dimensional systems are interesting because even weak interactions transform the quasi-particles of Fermi liquid theory into collective excitations of density wave type, and correlation functions are characterized by non-universal power laws determined by one basic coupling constant \(K\) (in case of one component, i.e. spin-polarized fermions). Similar techniques can be applied directly to interacting Bose gases as has been done recently with ultracold quantum gases \([7,8,9]\) based on pioneering work by Haldane \([10]\) and reviewed in \([11]\). Applications to mixtures of one-dimensional bosons and fermions were given in \([12]\). Luttinger methods (for reviews cf \([13,14,15,16]\)) were extended to include a trapping potential \([17,18]\) by means of the local density approximation. In \([19,20,21,23,24]\) attempts were made to include exactly - as in the Calgero Sutherland model \([25,26]\) - a harmonic trapping potential into the Luttinger approach utilizing the linear dispersion of free oscillator states, a concept recently adopted to quantum dissipation \([27]\). In the case of interacting quantum gases the penalty are truncated interactions in order to render the model exactly solvable. They then are no more faithful representa-
tions of real interactions. Furthermore, such type of approximation may violate essential symmetries of the original problem. For harmonically trapped quantum gases Kohn’s theorem [28,29,30] is such a symmetry. It states that the response to a homogeneous external field with dipole coupling to the system displays a resonance exactly at the trap frequency irrespective of any translation invariant particle-particle interactions. This raises the question which many-body approximations respect the Kohn theorem. This problem was intensively studied in the case of ultracold quantum gases and mainly in the context of bosons. In [31] it was found that the local density approximation at zero temperature is conserving. At finite temperature $T$ the random phase approximations with exchange as well as the Hartree-Fock-Bogoliubov approximation are also conserving as argued in [32]. The problem was taken up in [33] and the Bogoliubov approximation was proven to be conserving at zero temperature. In [34] it was pointed out that the Gross-Pitaevskii equation is non-conserving at $T > 0$ because the thermal cloud of the Bose-Einstein condensate is neglected. The work [35] showed that kinetic equations extending the Hartree-Fock-Popov method are conserving. The general dielectric approach was proven in [36] to be conserving too. Recently it was found [37] that superfluid fermions with a Feshbach resonance described by the generalized random phase approximation respect the Kohn theorem. The method of bosonization is intrinsically approximate and raises the same question. The phase formulation of a “Tomonaga-Luttinger model with harmonic confinement” was given in [24]. In studying its linear response it turned out that the model in its original form violates Kohn’s theorem. Here, we propose a method to remedy this deficiency. We then go on to calculate explicitly the inhomogeneous particle mobility in response to a spatially localized time varying force for a one-component gas. To this order we set up a linear but non-local differ-
ential equation for a phase operator adopted to the problem of harmonically trapped fermions. We solve it by means of its Green’s function.

2 The Kohn mode and bosonization

In a strictly harmonic trap the trapping potential raises the translation mode at zero frequency to the trap frequency $\omega_\ell$ independent of inter-particle interactions provided these are translation invariant. In a microwave experiment, when the electric field is dipole coupled to harmonically trapped charges, the resonance occurs exactly at this frequency. This was first noted by Kohn [28] in the context of magneto-resonance and later generalized to solid state systems [29]. It was also discussed for ultracold quantum gases [30]. In the latter case of neutral atomic gases the effect is described in terms of a "sloshing mode": A homogeneous mechanical force alternating in time leads to a resonance exactly at the frequency $\omega_\ell$. A simple theoretical route to discuss the implications of the Kohn theorem is to consider the operator $\hat{z}_S$ of the center of mass (c.m.) position. In second quantization it is

$$\hat{z}_S = \frac{1}{N} \int_{-\infty}^{\infty} dz \, \hat{\psi}^+(z) z \hat{\psi}(z). \quad (1)$$

Transforming from local creation and annihilation operators to the harmonic oscillator representation gives

$$\hat{z}_S = \frac{1}{N\alpha\sqrt{2}} \left( \sum_{n=0}^{\infty} \sqrt{n+1} \hat{c}_{n+1}^+ \hat{c}_n + \sum_{n=1}^{\infty} \sqrt{n} \hat{c}_{n-1}^+ \hat{c}_n \right). \quad (2)$$

Here, $\alpha$ is the inverse oscillator length and $N$ the fermion number. For large particle numbers $N$, a condition implicit in the method of bosonization, the
square roots can be replaced by $\sqrt{N}$ [27]. The bosonization prescription for density fluctuation operators $\hat{\rho}(p) \equiv \sum_q \hat{c}_{q+p}^\dagger \hat{c}_q$ (cf. e.g. [16,15]) according to

$$\hat{\rho}(p) = \begin{cases} 
\sqrt{|p|} \hat{d}_{|p|}, & p < 0, \\
\sqrt{p} \hat{d}_p^+, & p > 0,
\end{cases}$$  \hspace{1cm} (3)$$

where the operators $\hat{d}$ and $\hat{d}^+$ destroy and create collective bosonic excitations [6], then leads to

$$\dot{z}_S = \frac{1}{k_F} (\hat{d}_1 + \hat{d}_1^+) = \frac{2}{\pi k_F} \int_{-\pi}^{0} du \sin u \hat{\phi}_{\text{odd}}(u).$$  \hspace{1cm} (4)$$

The Fermi wave number $k_F$ is given by $k_F = \alpha \sqrt{2N}$. The phase operator used to bosonize fermionic destruction and creation operators in the formulation [2] is

$$\hat{\phi}(u) = -i \sum_{m=1}^{\infty} \frac{1}{\sqrt{m}} e^{im(u+i\eta)} \hat{d}_m, \quad -\pi \leq u \leq \pi,$$  \hspace{1cm} (5)$$

with a positive infinitesimal $\eta$. In [23] we found that the physical phase operator determining the particle density is, however,

$$\hat{\phi}_{\text{odd}}(u) = \frac{1}{2} (\hat{\phi}(u) + \hat{\phi}^+(u) - \hat{\phi}(-u) - \hat{\phi}^+(-u))$$

$$\equiv \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} e^{i\eta n} \sin(nu) \left( \hat{d}_n + \hat{d}_n^+ \right).$$  \hspace{1cm} (6)$$

The Kohn mode requires a total Hamiltonian which gives the same Heisenberg equation of motion for the c.m. coordinate

$$d_t^2 \dot{z}_S(t) = -\frac{1}{\hbar^2} \left[ [\hat{z}_S, \hat{H}], \hat{H} \right] = -\omega^2 \dot{z}_S(t)$$  \hspace{1cm} (7)$$
as the free Hamiltonian

$$\hat{H}_0 = \frac{1}{2} \hbar \omega_\ell \sum_{m=0}^{\infty} (\hat{d}_m^+ \hat{d}_m + \hat{d}_m \hat{d}_m^+).$$

We have introduced earlier [19,21] a Luttinger-like model of interacting one-dimensional fermions in the harmonic trap where the interaction is given by

$$\hat{V}_c = -\frac{1}{2} V_c \sum_{m=1}^{\infty} m (\hat{d}_m^2 + \hat{d}_m^{2+}).$$

This interaction approximates p-wave scattering in the one-component gas [22,23] in the simplest possible way. It is obvious that the condition (7) is violated because the part

$$\hat{V}_{c1} \equiv -\frac{1}{2} V_c \hat{d}_1^2 (\hat{d}_1^2 + \hat{d}_1^{2+}) \equiv \frac{1}{2} \hbar \omega_\ell \hat{V}_c (\hat{d}_1^2 + \hat{d}_1^{2+})$$

in the interaction results in $\hat{d}_1^2 \hat{z}_s(t) = -\tilde{\epsilon}^2 \hat{z}_s(t)$ with

$$\tilde{\epsilon}^2 = \omega_\ell^2 (1 - \tilde{V}_c^2) \neq \omega_\ell^2$$

for the interacting system. Quantum mechanically, the Kohn mode is a coherent state which would be turned into a squeezed state with renormalized frequency by the interaction equation (10). This is not admissible and one must subtract out that part of the interaction. We are mainly interested in the equation of motion for $\hat{\phi}_{odd}$ because it determines the response to external perturbations in the most direct way. One then must work out the consequences of the subtraction prescription for the phase formulation of the theory. In the first step one introduces the phase operator form of $\hat{V}_{c1}$ using

$$\hat{d}_1 = \int_{-\pi}^{\pi} du e^{i u} \left\{ \frac{1}{2\pi} \partial_u \hat{\phi}_{odd}(u) + \hat{\Pi}(u) \right\}.$$
The momentum density $\hat{\Pi}$ is

$$\hat{\Pi}(u) = -\frac{i}{2\pi} \sum_{n=1}^{\infty} \sqrt{n} e^{-n\eta} \sin nu \left( \hat{d}_n - \hat{d}_n^\dagger \right),$$

(13)

with the commutator

$$[\hat{\phi}_{\text{odd}}(u), \hat{\Pi}(v)] = \frac{i}{2} \delta(u-v)$$

(14)

in the reduced interval $I_{\pi} \equiv (-\pi < u < 0)$. This shows that $2\hbar \hat{\phi}_{\text{odd}}$ and $\hat{\Pi}$ are canonically conjugate on this interval. In applications the auxiliary variable $u$ is related to the physical position $z$ inside the harmonic trap by

$$u \rightarrow u_0(z) = \arcsin \left( \frac{z}{L_F} \right) - \frac{\pi}{2}, \quad z = L_F \cos(u_0),$$

(15)

where $L_F = \sqrt{\hbar 2N/\alpha}$ is half the quasi-classical extension of the Fermi sea.

The non-linear relation between the formal variable $u$ and physical position $z$ expresses the trap topology. It is also obvious that the reduced interval $I_{\pi}$ is sufficient to describe the physics.

The incriminating part in the original interaction becomes

$$\hat{V}_{c1} = -V_c \int_{-\pi}^{\pi} du \int_{-\pi}^{u} e^{iu+iu'} \left\{ \frac{1}{4\pi^2} \partial_u \hat{\phi}_{\text{odd}}(u) \partial_{u'} \hat{\phi}_{\text{odd}}(u') + \hat{\Pi}(u) \hat{\Pi}(u') \right\}$$

(16)

$$= \hbar \omega_F \int_{-\pi}^{0} du \int_{-\pi}^{u} e^{iu+iu'} \left\{ \frac{1}{4\pi^2} \partial_u \hat{\phi}_{\text{odd}}(u) \partial_{u'} \hat{\phi}_{\text{odd}}(u') \right\}.$$

The phase form of the reduced Hamiltonian (neglecting the zero-mode which would cancel in subsequent calculations) is

$$\hat{H}_{\text{red}} = \frac{\epsilon}{2} \int_{-\pi}^{\pi} du \left[ 2\pi K \hat{\Pi}^2(u) + \frac{1}{2\pi K} \left( \partial_u \hat{\phi}_{\text{odd}}(u) \right)^2 \right] - \hat{V}_{c1},$$

(17)
with
\[ K = \sqrt{\frac{1 + \tilde{V}_c}{1 - \tilde{V}_c}}, \quad \epsilon = \hbar \tilde{\epsilon} = \hbar \omega_\ell \frac{2K}{K^2 + 1} \equiv \hbar \omega_\ell \sqrt{1 - \tilde{V}_c^2}. \] (18)

The first part of the reduced Hamiltonian is diagonal with renormalized excitation energies \( m\epsilon, m = 1, 2, 3 \ldots \), while the second part is a non-local but still bilinear correction. The same results can be obtained by a selective renormalization of the trap frequency similar to the well known procedure in coupling a linear bath to a harmonic oscillator:
\[ \hat{H}_0 \to \hat{H}_0 + \frac{1}{2} \hbar (\tilde{\omega}_\ell - \omega_\ell)(\hat{d}_1^+ \hat{d}_1 + \hat{d}_1^+ \hat{d}_1^+), \quad \tilde{\omega}_\ell \equiv \omega_\ell \sqrt{1 + \tilde{V}_c^2}. \] (19)

3 Inhomogeneous linear mobility

The inhomogeneous mobility is the particle current response at position \( z \) to a \( \delta \)-function force \( f \)
\[ f(z, z_0; t) = F(t) \delta(z - z_0) \] (20)
at a different position \( z_0 \) inside the trap. Quantitatively, the inhomogeneous linear mobility \( \mu(z, z_0; \omega) \) relates the expectation value of the local particle current density operator \( \hat{j}_p(z) \) to the force by
\[
\langle \hat{j}_p(z) \rangle = \int_{-\infty}^{\infty} dt \, e^{i\omega t} \langle \hat{j}_p(z) \rangle_t
\] (21)
\[ = \int dz' \mu(z, z'; \omega) f_\omega(z', z_0) = \mu(z, z_0; \omega) F_\omega. \]
The Hamiltonian for a time-dependent, spatially localized external force \( f(z, z_0; t) \)
affecting \( N \) fermions on the \( z \)-axis is

\[
\hat{H}_{\text{ext}}(z_0, t) = -F(t) \sum_{n=1}^{N} \Theta(\hat{z}_n - z_0).
\]  

(22)

In second quantized form it becomes

\[
\hat{H}_{\text{ext}}(z_0, t) = -F(t) \int_{z_0}^{\infty} dz \, \hat{\psi}^+(z) \hat{\psi}(z).
\]  

(23)

For simplicity, a one-component gas of spin polarized fermions is assumed. Without enhancement of the interactions by Feshbach resonances [38] this is an admittedly academic case when regarding the role of interactions. The total particle density operator contains a part \( \delta\hat{\rho}_{\text{slow}} \) which varies slowly in space.

In an appropriate WKB expansion (for large \( N \)), it is given by [23]

\[
\delta\hat{\rho}_{\text{slow}}(z) = \frac{1}{\pi} \partial_z \hat{\phi}_{\text{odd}}(u_0(z)).
\]  

(24)

Again this has an analogous expression in the theory of Luttinger liquids and its bosonic equivalent [10,11]. However, in the present case, there is a non-linear relation between the argument \( u_0 \) of the phase operator and the actual spatial position \( z \) according to equation (15). There is also a rapidly varying Friedel part in the particle density which results from the confinement. Away from the classical boundaries \( z = \pm L_F \) it gives a small correction to the mobility if we average the latter over a small length \( L \) around \( z_0 \) with \( L_F \gg L \gg 1/k_F \). It will be neglected here. Utilizing the continuity equation leads to

\[
\partial_t \delta\hat{\rho}_{\text{slow}}(z, t) = \frac{1}{\pi} \partial_z \left( \partial_t \hat{\phi}_{\text{odd}}(u_0(z), t) \right) = -\partial_z \hat{j}_{\text{slow}}(z, t),
\]  

(25)

and shows that the slow current density is given by
\[ \hat{j}_{\text{slow}}(z, t) = -\frac{1}{\pi} \partial_t \hat{\phi}_{\text{odd}}(u_0(z), t). \] (26)

We make the identification \( \hat{j}_p \equiv \hat{j}_{\text{slow}} \) which according to equation (21) leads to the generic form

\[ \mu(\omega) = \frac{i\omega \langle \hat{\phi}_{\text{odd}} \rangle_\omega}{\pi F_\omega}. \] (27)

for the mobility. Coupling the external force to the slowly varying density amounts to the replacement

\[ \hat{\psi}^+(z) \hat{\psi}(z) \to \delta \hat{\rho}_{\text{slow}}(z) = \frac{1}{\pi} \partial_z \hat{\phi}_{\text{odd}}(u_0(z)) \] (28)

in equation (23) and leads to the external phase Hamiltonian

\[ \hat{H}_{\text{ext}} = F(t) \frac{1}{\pi} \hat{\phi}_{\text{odd}}(u_0(z_0)). \] (29)

4 Equations of motion

The equations of motion are most easily obtained via functional derivatives using

\[ 2\hbar \partial_t \hat{\phi}_{\text{odd}}(u, t) = \frac{\delta \hat{H}_{\text{red}}}{\delta \hat{\Pi}(u, t)} , \quad 2\hbar \partial_t \hat{\Pi}(u, t) = -\frac{\delta \hat{H}_{\text{red}}}{\delta \hat{\phi}_{\text{odd}}(u, t)}. \] (30)

One obtains:

\[ \partial_t \hat{\phi}_{\text{odd}} = 2\pi \varepsilon \hat{\Pi} - 4\omega \hat{V}_c \sin u \int_{-\pi}^{0} du' \sin u' \hat{\Pi}(u') \] (31)

and
\[ \frac{\partial \hat{\Pi}}{\partial t} = \frac{\tilde{\epsilon}}{2\pi K} \partial_u^2 \hat{\phi}_{\text{odd}} \]

\[ -\frac{1}{\pi^2} \omega_v \hat{V}_c \sin u \int_{-\pi}^{0} du' \sin u' \hat{\phi}_{\text{odd}}(u') - F(t) \frac{1}{2\pi \hbar} \delta(u_0(z_0) - u). \]

The equation of motion for \( \hat{\phi}_{\text{odd}} \) closes at the level of the second time derivative:

\[ \partial_t^2 \hat{\phi}_{\text{odd}} = \tilde{\epsilon}^2 \partial_u^2 \hat{\phi}_{\text{odd}} - \omega_v^2 \hat{V}_c^2 \sin u \left[ \frac{2}{\pi} \int_{-\pi}^{0} du' \sin u' \hat{\phi}_{\text{odd}}(u') \right] \]

\[ - F(t) \frac{\tilde{\epsilon} K}{\hbar} \delta(u_0(z_0) - u) + F(t) \frac{2\omega_v \hat{V}_c}{\pi \hbar} \sin u_0(z_0) \sin u. \]

Here, essential use has been made of the relation

\[ \tilde{\epsilon} \left( K - \frac{1}{K} \right) = 2\omega_v \hat{V}_c. \]

The non-local operator results from the subtraction procedure and projects onto the subspace of the Kohn mode: This is best seen by considering excitations \( \varphi(u,t) = \langle \hat{\phi}_{\text{odd}}(u) \rangle_t \) i.e. averaging the equation of motion over a non-equilibrium state. For \( F = 0 \) one gets

\[ \partial_t^2 \varphi = \tilde{\epsilon}^2 \partial_u^2 \varphi - \omega_v^2 \hat{V}_c^2 \sin u \left[ \frac{2}{\pi} \int_{-\pi}^{0} du' \sin u' \varphi(u',t) \right]. \]

The Kohn mode is a density (dipole) oscillation with \( \varphi_K \propto \sin u_0(z) \) and associated real space density modulation \( \delta \rho(z) \propto \partial_z \varphi_K \propto z/\sqrt{1-z^2/L^2} \).

It solves equation (35): \( \partial_t^2 \varphi_K = -\tilde{\epsilon}^2 \varphi_K - \omega_v^2 \hat{V}_c^2 \varphi_K \) with the correct frequency because of \( \tilde{\epsilon}^2 = \omega_v^2 (1 - \hat{V}_c^2) \). The non-local projection operator has no influence on modes orthogonal to the Kohn mode. Their excitation energies scale with \( \tilde{\epsilon} \), the renormalized frequency of the model.
Calculation of inhomogeneous linear mobility

The eigenvalue problem associated with the phase equation

\[ -\omega^2 \varphi_\omega = \tilde{\epsilon}^2 \partial_u^2 \varphi_\omega - \omega^2 \tilde{V}_c^2 \sin u \left[ \frac{2}{\pi} \int_{-\pi}^{0} du' \sin u' \varphi_\omega(u') \right] \]

(36)
can be cast into a convenient form by introducing the function

\[ \varphi_1(u) \equiv \sqrt{\frac{2}{\pi}} \sin u, \]

(37)
which is normalized in \( I_\pi = (-\pi, 0) \) and satisfies the boundary conditions implied by \( \hat{\phi}_{\text{odd}}(u) \), namely \( \varphi_1(-\pi) = 0 = \varphi_1(0) \). Introducing the linear symmetric operator \( L_\omega \)

\[ L_\omega \equiv \frac{\tilde{\epsilon}^2}{\omega^2} \partial_u^2 + \frac{\omega^2}{\omega^2} - \tilde{V}_c^2 \varphi_1(u) \int_{-\pi}^{0} \sin u' \varphi_1(u') \left( \ldots \right), \]

(38)
the eigenvalue problem becomes

\[ L_\omega \varphi_n = -\lambda_n^2(\omega) \varphi_n. \]

(39)
Because of the relation \( \tilde{\epsilon}^2 \equiv \omega^2 \left(1 - \tilde{V}_c^2\right) \) it is easy to see that \( \varphi_1(u) \) is an eigenfunction with eigenvalue

\[ \lambda_1^2(\omega) = 1 - \frac{\omega^2}{\omega^2}. \]

(40)
Furthermore,

\[ \varphi_n(u) = \sqrt{\frac{2}{\pi}} \sin nu, \quad n = 1, 2, \ldots \]

(41)
is an orthonormal basis on \( I_\pi \). It is thus clear that for \( n \geq 2 \)
\[
\mathbf{L}_\omega \varphi_n = \left( -n^2 \frac{\bar{\epsilon}^2}{\omega^2} + \frac{\omega^2}{\omega_c^2} \right) \varphi_n \equiv -\lambda_n^2(\omega) \varphi_n, \quad \lambda_n^2(\omega) = n^2 \frac{\bar{\epsilon}^2}{\omega^2} - \omega^2 \tag{42} \]

holds. The relevant Green’s function is

\[
G_\omega(u, u') = -\sum_{n=1}^{\infty} \frac{\varphi_n(u) \varphi_n(u')}{\lambda_n^2(\omega)}. \tag{43} \]

This allows to solve for the inhomogeneous local mobility according to equation (27) with \( \langle \hat{\phi}_{\text{odd}} \rangle_\omega \equiv \varphi_\omega(u_0(z), u_0(z_0)) \). Utilizing the identity \( \bar{\epsilon}K \equiv \omega_c(1 + \bar{V}_c) \) leads to

\[
\varphi_\omega = -\frac{F_\omega \omega_c}{\hbar} \frac{\varphi_1(u_0(z)) \varphi_1(u_0(z_0))}{\omega_c^2 - \omega^2} - \frac{F_\omega \bar{\epsilon}K}{\hbar} \sum_{n=2}^{\infty} \frac{\varphi_n(u_0(z)) \varphi_n(u_0(z_0))}{n^2 \bar{\epsilon}^2 - \omega^2}. \tag{44} \]

Using \( \sin u_0(z) = -\sqrt{1 - z^2/L_c^2} \equiv -Z(z) \) finally gives

\[
\mu(z, z_0; \omega) = -\frac{2i \omega \omega_c}{\pi^2 \hbar} \frac{Z(z)Z(z_0)}{\omega_c^2 - \omega^2} - \frac{2i \omega \bar{\epsilon}K}{\pi^2 \hbar} \sum_{n=2}^{\infty} \frac{\sin nu_0(z) \sin nu_0(z_0)}{n^2 \bar{\epsilon}^2 - \omega^2}. \tag{45} \]

Analyticity of the response function for \( \Im \omega \geq 0 \) requires \( \omega \to \omega + i\eta \) with a positive infinitesimal \( \eta \). It is seen that the inhomogeneous mobility has resonances at all excitation frequencies including \( \omega_c \). This applies also to the non-interacting case. It is only the homogeneous response which is trivialized by Kohn’s theorem. In order to see this, we calculate the homogeneous mobility via

\[
\mu(z, \omega) = -L_F \int_{-\pi}^{0} du_0 \sin u_0 \mu(z, z_0(u_0); \omega), \tag{46} \]

and find

\[
\mu(z, \omega) = \frac{L_F}{\pi \hbar} \frac{i \omega \omega_c}{\omega^2 - \omega_c^2} Z(z). \tag{47} \]
This is identical to the local response of non-interacting fermions in the harmonic trap to a homogeneous force. The Kohn theorem is clearly fulfilled.

Returning to the inhomogeneous mobility, the sum can be evaluated analytically. By including the $n = 1$ term in the summation one obtains

$$\mu(z, z_0; \omega) = -\frac{iK}{2\pi \hbar \sin(\pi a)}$$

$$\times \{ \cos[a(\pi + u_0(z) + u_0(z_0))] - \cos[a(\pi - |u_0(z) - u_0(z_0)|)] \}$$

$$+ \frac{2\tilde{V}_c Z(z) Z(z_0)}{\pi^2 \hbar} \left\{ \frac{i\omega \omega_k ((1 + \tilde{V}_c)\omega_f^2 - \omega^2)}{(\omega_f^2 - \omega^2)((1 - \tilde{V}_c^2)\omega_f^2 - \omega^2)} \right\}.$$

with $a = \omega/\tilde{\epsilon}$. Here, the first term also includes a contribution from the Kohn mode $\varphi_1$ which removes all renormalizations due to the interaction from the second term in going over to the homogeneous response. By this mechanism Kohn’s theorem is restored.

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