Path Ranking with Attention to Type Hierarchies

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Abstract
The knowledge base completion problem is the problem of inferring missing information from existing facts in knowledge bases. Path-ranking based methods use sequences of relations as general patterns of paths for prediction. However, these patterns usually lack accuracy because they are generic and can often apply to widely varying scenarios. We leverage type hierarchies of entities to create a new class of path patterns that are both discriminative and generalizable. Then we propose an attention-based RNN model, which can be trained end-to-end, to discover the new path patterns most suitable for the data. Experiments conducted on two benchmark knowledge base completion datasets demonstrate that the proposed model outperforms existing methods by a statistically significant margin. Our quantitative analysis of the path patterns show they balance between generalization and discrimination.

1 Introduction
Knowledge bases (KBs), such as WordNet [Miller, 1995] and Freebase [Bollacker et al., 2008], have been used to provide background knowledge for various tasks such as recommendation [Wang et al., 2018], and visual question answering [Aditya et al., 2018]. Such KBs typically contain facts stored in the form of (source entity, relation, target entity) triples, such as (fork, in, kitchen).

Despite containing millions of facts, existing KBs still have a large amount of missing information [Min et al., 2013]. As a result, robustly reasoning about missing information is important not only for improving the quality of KBs, but also for providing more reliable information for applications relying on the contained data. The knowledge base completion problem is the problem of inferring missing information from existing facts in KBs. More specifically, fact prediction is the problem of predicting whether a relation holds between two entities given the existing information in a KB.

Prior work on fact prediction can be separated into two categories. The first leverages latent feature models such as tensor factorization or neural networks [Nickel et al., 2015]. The second discovers observable patterns in graphs, which can be constructed from triples by representing entities as nodes and relations as edges. One common graph-based approach is the Path Ranking Algorithm (PRA), which uses sequences of relations as generic patterns of paths in graphs.

However, contradictions often arise because this type of patterns are generic and can often apply to widely varying scenarios. Figure 1 shows two examples having the same sequence of relations $\langle \text{in}, -\text{in}, \text{used}_\text{for} \rangle$ (the minus sign indicates reversed direction for the relation) but the prediction is only correct in the first example. A way to solve this problem is to leverage the information gained by considering the entities involved in paths. [Das et al., 2016] incorporate entity information by concatenating vector representations of relations with that of entities. The use of entity information effectively increases the accuracy of fact prediction but the generality of path patterns have been lost.

We propose a new class of path patterns, which in addition to relations, includes a type for each entity in a path. This
type is selected from the type hierarchy for each entity, where types are ordered based on their levels of abstraction. For example, types from the most specific to the most abstract in the type hierarchy of the entity fork are cutlery, tableware, article, and object. Based on the distributional informativeness hypothesis proposed in {Santus et al., 2014}, a generic type can occur in more general contexts in place of the specific types that entail it. Therefore, this new class of path patterns can balance between generalization and discrimination if the types selected for entities have the right levels of abstraction.

Because the possible assignments of types for entities in all paths are too many for exhaustive search, we use the attention mechanism from {Bahdanau et al., 2014} to learn the assignments of types from data. Specifically, we use an Recurrent Neural Network (RNN) based encoder to encode relations in paths to create contexts that are used by attention modules to assign a type for each entity. The contexts also evolve in a second RNN encoder based on types that have been selected. Finally, the prediction is made based on the information contextually combined from all paths. Because the whole model is trained to maximize the likelihood of the provided data, the attention mechanism finds the levels of abstraction for entities that make the fact prediction most accurate.

The contribution of this paper are the following: We introduce a new class of generalizable path patterns, which leverage type hierarchies of entities to both avoid ambiguity and maintain generalization. We propose an attention-based RNN model, which can be trained end-to-end, to assign types for entities in order to discover the proposed path patterns most suitable for the data. We also propose a new pooling method based on attention to jointly reason about contextually important information from multiple paths. We then quantitatively show our model finds path patterns that balance between generalization and discrimination. Finally, we quantitatively validate the usefulness of the new path patterns and the model with statistically significant improvement on current state of the art performance on two benchmark datasets: WN18RR and FB15k-237.

2 Related Work

In this section, we summarize the related work and discuss the connections to our approach.

2.1 Knowledge Based Reasoning

In prior work, there exist multiple approaches to knowledge based reasoning. Knowledge graph embedding methods map relations and entities to vector representations in continuous spaces {Nickel et al., 2015}. Methods based on inductive logic programming discover general rules from examples {Quinlan and Cameron-Jones, 1993}. Statistical relational learning (SRL) methods combine logics and graphical models to probabilistically reason about entities and relations {Getoor and Taskar, 2007}. Reinforcement learning based models treat link prediction (predicting target entities given a source entity and a relation) as Markov decision processes {Xiong et al., 2017; Das et al., 2017}. Path-ranking based models use supervised learning to discover generalizable path patterns from graphs {Lao et al., 2011; Gardner and Mitchell, 2015; Neelakantan et al., 2015; Das et al., 2016}.

Our work focuses on the knowledge base completion problem, specifically on fact prediction. We focus on path-ranking based models because they have the ability to model complex reasoning patterns. Path Ranking Algorithm (PRA) {Lao et al., 2011} first propose to use random walk to discover generalizable sequences of relations between two entities and treat each sequence as a unique feature with a weight equal to the random walk probability. Subgraph Feature Extraction (SFE) {Gardner and Mitchell, 2015} uses bi-directional breadth-first search (BFS) to exhaustively search for sequences of relations and additional patterns in graphs. They treat each pattern as a binary feature because calculating the weights is computationally intensive and not profitable with respect to the performance. To generalize semantically similar path patterns, {Neelakantan et al., 2015} use recurrent neural networks (RNNs) to sequentially encode relations in path patterns to create their vector representations, which are then used as multidimensional features for prediction. {Das et al., 2016} improve the accuracy of the RNN method by making use of the additional information in entities and entity types. We also use entity types but we focus on using types at different levels of abstraction for different entities.

2.2 Representing Hierarchical Structures

Learning representations of hierarchical structures in natural data such as text and images has been shown to be effective for tasks such as hypernym classification and textual entailment. {Vendrov et al., 2016} order text and images by mapping them to a non-negative space in which entities that are closer to the origin are more general than entities that are further away. {Athiwaratkun and Wilson, 2018} use density order embeddings where more specific entities have smaller, concentrated probabilistic distributions and are encapsulated in broader distributions of general entities. In this work, we do not explicitly learn the representation of hierarchical types. Instead, we leverage the fact that types in type hierarchies have different levels of abstraction to create path patterns that balance generalization and discrimination.

2.3 Attention

Attention was first introduced in {Bahdanau et al., 2014} for machine translation. The attention mechanism helps the encoder-decoder model to condition generation of translated words on different parts of the original sentence. Later, cross-modality attention was shown to be effective at image captioning {Xu et al., 2015} and speech recognition {Chan et al., 2015}. Our approach uses attention to focus on contextually important information from multiple paths much like the above methods. In addition, we use attention in an original way to efficiently discover the correct levels of abstraction for entities from a large search space.

3 Approach

Our objective is, given a KB that is incomplete, to predict whether a missing triple holds.

3.1 Problem Definition

A KB is formally defined as a set of triples, also called relation instances, \( \mathcal{X} = \{(e_i, r_j, e_k) | e_i, e_k \in \mathcal{E} \land r_j \in \mathcal{R}\}, \)
where $\mathcal{E}$ denotes the entity set, and $\mathcal{R}$ denotes the relation set. We make the closed world assumption that all triples in the KB are true and any triple that is not in the KB is false. The knowledge graph $\mathcal{G}$ then can be constructed from $\mathcal{X}$, where nodes are entities and edges are relations. A directed edge from $e_i$ to $e_k$ with label $r_j$ exists for each triple $(e_i, r_j, e_k)$ in $\mathcal{X}$. A path between $e_i$ and $e_k$ is defined as $p = (e_1, r_1, ..., r_M, e_{M+1})$, where $e_1 = e_i$ and $e_{M+1} = e_k$. The length of a path is defined as the number of relations in the path, $M$ in this case. For all pairs of entities $e_i$ and $e_k$ in the graph $\mathcal{G}$, we can discover a set of $N$ paths up to a fixed length, $\mathcal{P}_{ik} = \{p_1, ..., p_N\}$.

Our model is a path-ranking based method, which uses the path set $\mathcal{P}_{ik}$ to predict whether the entity pair $e_i$ and $e_k$ can be linked by $r_j$ or not.

### 3.2 Attentive Path Ranking

We present details of Attentive Path Ranking (APR). Our proposed model consists of three components: a relation encoder, an entity type encoder, and an attention-based pooling method. We first describe how the model creates the proposed new path pattern $\hat{p}$ from a path $p = (e_1, r_1, ..., r_M, e_{M+1})$. We then discuss the type of the data and the loss function for training the model end-to-end.

#### Relation Encoder

This part of the model has a RNN that sequentially encodes vector representations of relations $v_δ(r_t)$ for all relations in a path $p$. Here, $v_δ(r_t)$ can be obtained using a look up table, which maps each unique relation to a vector. The last state of the RNN carries information from all the relations, therefore, is used as a vector representation of them, denoted as $v_δ(\hat{p})$. We use LSTM [Hochreiter and Schmidhuber, 1997] instead of simple RNN for its ability to better model long-term dependencies.

#### Type Encoder

This part of the model is consisting of another LSTM and attention modules applied to entities. Together, they are responsible for selecting types for entities in the path $p$ to create the proposed path pattern $\hat{p} = (l_1, r_1, ..., l_t, ..., r_M, l_{M+1})$, where $l_t$ represents the selected type for the entity $e_t$ from its type hierarchy. Formally, the type hierarchy for each entity $e_t$ in the path $p$ is:

$$ L_t = \{l_{t,1}, ..., l_{t,C}\} $$

where the lowest level $l_{t,1}$ represents the most specific type and the highest level $l_{t,C}$ represents the most abstract type. $C$ is the height of the hierarchy.

Our assumption is that there exists a best type $l^*_t$ for each entity $e_t$ in the path $p$, which helps create the best path pattern that is both discriminative and generalizable, and using the best path patterns leads to maximized accuracy for prediction. However, the substantial number of combinations when considering possible types for entities in all path patterns makes exhaustively searching for all $l^*_t$’s impossible.

Instead, we use the deterministic “soft” attention introduced in [Bahdanau et al., 2014] to create an approximated vector representation of $l^*_t$ from the set of type vectors $\{v_\tau(l_{t,1}), ..., v_\tau(l_{t,C})\}$, where $v_\tau(l_{t,i})$ can be obtained using a look up table that maps each unique type to a vector. We name this approximated vector representation of $l^*_t$ as the type context vector $\hat{a}_t$. For each type vector $v_\tau(l_{t,i})$ of entity $e_t$, a weight $a_{t,i}$ is computed by a feed-forward network $f_{\text{att,type}}$ conditioned on an evolving context $\hat{c}_t$. This weight can be interpreted as the probability that $l_{t,i}$ is the right level of abstraction or the relative importance for level $i$ to combine $v_\tau(l_{t,i})$’s together. Formally, $\hat{a}_t$ can be calculated as:

$$ e_{t,i} = f_{\text{att,type}}(v_\tau(l_{t,i}), \hat{c}_t) $$

$$ a_{t,i} = \frac{\text{exp}(e_{t,i})}{\sum_{k=1}^{C} \text{exp}(e_{t,k})} $$

$$ \hat{a}_t = \sum_{i=1}^{C} a_{t,i} v_\tau(l_{t,i}) $$

With the type context vector $\hat{a}_t$ computed for each entity $e_t$, the LSTM sequentially encodes the corresponding $\hat{a}_t$ for all entities $e_t$ in the path $p$.

Now we describe how we obtain the context $\hat{c}_t$, which provides context information for the model to better approximate $l^*_t$. Specifically, we use the previous hidden state of the LSTM as the context of the current step $\hat{c}_t = h_{t-1}$. We also
use $v_e(\hat{p})$, the last state of the relation encoder, to compute the initial memory state and hidden state of the LSTM:

$$c_0 = f_{init,c}(v_e(\hat{p}))$$  \hspace{1cm} (5)

$$h_0 = f_{init,h}(v_e(\hat{p}))$$  \hspace{1cm} (6)

where $f_{init,c}$ and $f_{init,h}$ are two separate feed-forward networks. Initializing $c_i$ based on $v_e(\hat{p})$ first gives the model a broad context to select the best types for the first few entities. As types for more entities are selected, the context refines based on all previous selections.

The last hidden state of the LSTM carries all information from entities’ types, therefore is used as a vector representation of them $v_c(\hat{p})$. We then concatenate $v_e(\hat{p})$ and $v_c(\hat{p})$ together to get the final representation of the path pattern $v_p(\hat{p}) = [v_e(\hat{p}); v_c(\hat{p})]$.

### Attention Pooling

After the representations of path patterns between two entities are obtained using the above models, it’s important to jointly reason about these representations to make prediction. Previous neural network models [Neelakantan et al., 2015] and [Das et al., 2016] use a feed-forward network to condense path representation for each path $p_i$ to a single value $s_i$. One of the following pooling methods $f_{pool}$: Max, Average, Top-K, or LogSumExp, is then used to combine these values together

$$P(r|e_i, e_k) = \sigma(f_{pool}(s_i)), \forall s_i$$  \hspace{1cm} (7)

Compressing vector representations of paths to single values hinders the model’s ability to jointly reason about the rich contextual information in the vectors. We propose to leverage another similar attention mechanism as that used to compute the type context vector, for integrating information from all paths. We use a trainable context vector $u$ to represent the relation we are trying to predict. Following similar steps for computing $a_i$ from $\{v_e(l_{i,1}), ..., v_e(l_{i,C})\}$, here we compute a vector representation of all path patterns $\tilde{p}$ from $\{v_p(p_i), ..., v_p(p_N)\}$ conditioned on the context vector $u$:

$$e_i = f_{att,path}(v_p(p_i), u)$$  \hspace{1cm} (8)

$$\alpha_i = \frac{\exp(e_i)}{\sum_{k=1}^{N} \exp(e_k)}$$  \hspace{1cm} (9)

$$\tilde{p} = \sum_{i=1}^{N} \alpha_i v_p(p_i)$$  \hspace{1cm} (10)

Because $\tilde{p}$ represents all the path patterns carrying information from both relations and entity types with correct level of abstraction, the probability that $r$ exists between $e_i$ and $e_k$ can be more accurately predicted using a feed-forward network $f_{pred}$ along with a sigmoid function $\sigma$:

$$P(r|e_i, e_k) = \sigma(f_{pred}(\tilde{p}))$$  \hspace{1cm} (11)

### 3.3 Training Procedure

For predicting each relation $r$, we train a model using true triples in a KB as positive examples and an equal amount of false triples not in the KB as negative examples, requiring all triples having relation $r$. Because KBs only contain true triples, a practical issue of path-ranking based methods is sampling negative examples. We follow [Gardner and Mitchell, 2015] to generate negative examples using the method based on personalized page rank. Let $D^+\tau$ be the set of true triples for relation $r$, and $D^-\tau$ be the set of false triples for relation $r$, our training objective is to minimize the negative log-likelihood:

$$L = - \sum_{(e_i, r, e_k) \in D^+\tau} \log P(r|e_i, e_k)$$

$$+ \sum_{(e_i, r, e_k) \in D^-\tau} \log P(r|e_i, e_k)$$  \hspace{1cm} (12)

We use backpropagation to update the learnable model parameters, which are the relation embedding $v_r$, type embedding $v_e$, 2 LSTMs for relation encoder and type encoder, and feedforward networks $f_{init,c}$, $f_{init,h}$, $f_{att,type}$, $f_{att,path}$, and $f_{pred}$.

### 4 Experiments

#### 4.1 Data

We evaluated our method and baseline methods on two standard datasets for knowledge base completion, FB15k-237 [Toutanova et al., 2015] and WN18RR [Dettmers et al., 2018]. FB15k-237 is a subset of the commonsense knowledge graph Freebase [Bollacker et al., 2008], and WN18RR is a subset of the English lexical database WordNet [Miller, 1995]. Table 1 shows the statistics of these two datasets. We split the data into 80% training and 20% testing. For testing, we used all 11 relations in WN18RR and randomly selected 10 semantically different relations in FB15k-237. We used the SFE [Gardner and Mitchell, 2015] code to sample negative examples for each query relation.

To extract paths between entities, we first constructed graphs from true triples in the datasets. We augmented the graphs with reverse relations following existing methods [Lao et al., 2011]. We extracted paths for pairs of entities using bi-directional BFS. We set the maximum length of paths to 6 for

|               | WN18RR | FB15k-237 |
|---------------|--------|-----------|
| # Relations   | 11     | 235       |
| # Entities    | 40,943 | 13,545    |
| # Relation instances | 134,720 | 508,580 |
| # Relations tested | 11     | 10        |
| Avg. # train inst/relation | 7,748   | 4,713     |
| Avg. # testing inst/relation | 2,422   | 1,744     |
| Avg. path length | 5.2    | 3.5       |
| Max path length | 6      | 4         |
| Avg. # paths per instance | 84     | 162       |
| # Types       | 8,092  | 1,029     |
| Max height of type hierarchy | 14    | 7         |
| Avg. height of type hierarchy | 4.6   | 6.4       |

Table 1: Four sections show the statistics of the datasets, the training/testing examples, the paths extracted, the type hierarchies.
WN18RR and 4 for more densely connected FB15k-237. We sampled 200 paths for each pair if there are more paths. Limiting the length of paths and sub-sampling paths are both due to computational concerns.

To create type hierarchies, we extracted inherited hypernyms available in WordNet [Miller, 1995] for WN18RR entities and used type data released in [Xie et al., 2016] for FB15K-237 entities. We used all available types for WN18RR. We ordered Freebase types based on their numbers of occurrences in ascending order because types for Freebase entities are not strictly hierarchical. We then followed [Das et al., 2016] to select up to 7 most frequently occurring types for each entity. For WN18RR, we mapped types to their vector representations using a pre-trained Google News word2vec model. Because we did not find a suitable pre-trained embedding for types of FB15K-237 entities, we trained an embedding with the whole model end-to-end.

### 4.2 Experimental Setup

To train the neural network model, the dimension of relation embedding, type embedding, and hidden states of both relation encoder and type encoder are set to 50, 150, and 200 respectively. We used Adam [Kingma and Ba, 2014] for optimization with default parameters (learning rate=$1e^{-3}$, $\beta_1=0.9$, $\beta_2=0.999$, $\epsilon=1e^{-8}$). For all neural network models including baselines, we trained the models fully to 50 epochs. Then we used early stopping on validation accuracy as regularization.

For evaluation, we used both average precision (AP) and accuracy. AP is used to score the ranked list of entity pairs. We reported mean average precision (MAP), the mean of APs across all tested relations. To understand the significance of improvements, we performed a paired t-test considering each relation as paired data.

We compared the performance of following methods:

**PRA** from [Lao et al., 2011]. This method uses random walks to find paths between two entities. Each unique sequence of relations is treated as a singleton feature, with the feature weight equal to the random walk probability. This method uses a support vector machine for predictions.

**SFE** from [Gardner and Mitchell, 2015]. This method uses bi-directional BFS to enumerate all paths. Each unique sequence of relations is treated as a binary feature. This method uses support vector machine for prediction.

|                         | WN18RR |          | FB15k-237 |          |
|-------------------------|--------|----------|-----------|----------|
|                         | Accuracy% | MAP%  | Accuracy% | MAP%   |
| **PRA**                 | 87.82  | 45.23   | 56.14     | 57.28   |
| **SFE**                 | 78.29  | 47.88   | 66.73     | 55.48   |
| **Path-RNN (relation)** | 89.51  | 91.77   | 68.91     | 69.48   |
| **Path-RNN (relation + type)** | 76.58 | 77.15  | 70.38     | 72.95   |
| **Path-RNN (relation + type + entity)** | 78.40 | 79.22  | 68.72     | 71.11   |
| **Attention (LogSumExp pooling)** | 91.29 | 93.25  | 70.87     | 73.92   |
| **Attention (Average pooling)** | 89.88 | 90.70  | 73.31     | 77.04   |
| **Attention (Attention pooling)** | **93.52** | **95.42** | **73.66** | **77.33** |

Table 2: Comparison of performance of our proposed model with baseline methods.

Path-RNN (relation) from [Neelakantan et al., 2015]. This method uses an RNN to encode relations sequentially to create vector representations of paths, which are then used for prediction.

Path-RNN (relation+type) from [Das et al., 2016]. This model uses an RNN to encode concatenated vector representations of entities and relations.

Path-RNN (relation+type+entity) from [Das et al., 2016]. This model use RNN to encode concatenated vector representations of entities, relations, and types.

### 5 Results

In Table 2, we provide a summary of the experiment. Our model **Attention (attention pooling)** performs statistically significantly better than state of the art performance on both datasets. This result shows that adding types, with correct balance between being generalizable and discriminative, helps create better path patterns that allows for more accurate prediction. **Path-RNN (relation+type)** achieving higher scores for Freebase data compared to **Path-RNN (relation)** and **Path-RNN (relation+type)** is consistent with the result in [Das et al., 2016]. However, one surprising result is that **Path-RNN (relation+type+entity)** and **Path-RNN (relation+type)** have lower scores than **Path-RNN (relation)** on WN18RR. We suspect that the large number of entities and types in WN18RR and simple concatenation of vector representations of these features cause learning from these paths to not generalize even for highly adaptable neural network models. The use of abstraction helps our model avoid varying information from individual entities and types, and helps focus on the commonalities for more robust prediction.

We also compare the performance of different pooling methods. **Attention (attention pooling)** performs better than the other two methods in all categories. The best performance is likely coming from this pooling method’s ability to collectively reason about the rich contextual information in vector representations of paths. **Attention (Average pooling)** and **Attention (LogSumExp pooling)** lose a lot of contextual information when they compress path representations to single values. The performance of the other two methods also depends on the data: **Attention (LogSumExp pooling)** performs better than **Attention (Average pooling)** on WN18RR while the reverse is true on FB15k-237. One possible justification is...
For each path pattern $\pi$ only appears in positive or negative examples, but not both. We define that a path pattern (consisting of both relations and types in this case) is discriminative if this path pattern $\pi$ will appear in positive examples, denote $o_i^+$, and in negative examples, denote $o_i^-$. Then we calculate a ratio $r_i = o_i^+ / (o_i^- + o_i^+)$, which implies a path pattern’s distribution in positive and negative examples. Then we calculate the variance $d$ in the ratios $r_i$ for all path patterns. $d$ summarizes the discriminativeness of a set of path patterns. For example, if most path patterns appear in both positive and negative examples, $d$ will be small.

We define that a path pattern is generalizable if it appears both in training and testing. Quantitatively we define completeness as following: we assign path patterns that appear in training to set $\Pi_{train}$ and path types in testing to set $\Pi_{test}$. Then we calculate value $g$:

$$g = 1 - \frac{|\Pi_{test} \setminus \Pi_{train}|}{|\Pi_{test}|}$$

where the nominator of the fraction calculates the number of path patterns in testing but not in training. We use $g$ to measure the generalizability of a set of path patterns.

Table 3 shows discriminativeness $d$ and generalizability $g$ of path patterns used by attention, specific, and abstract. Train $d$ and Test $d$ columns show that discriminativeness $d$ is the smallest when using the most abstract types and the largest when using the most specific types for both training and testing. $g$ column shows that generalizability $g$ is the highest when using the most abstract types and the lowest when using the most specific types. Path patterns with the most specific types are too distinct making them hard to generalize. Our model learn path patterns that balance between generalizability and discriminativeness.

Table 4 shows the effect of different levels of abstraction on performance. The difference in performance for WN18RR are very small. However the differences for FB15k-237 are statistically significant. Table 3 shows using specific types increase $d$ more and decreases $g$ less for FB15k-237 compared to WN18RR. This may explain why the benefit of selecting the right types for FB15k-237 is obvious. Shown in Table 2, Path-RNN (relation+type) and Path-RNN (relation+type+entity) have significantly dropped performance compared Path-RNN (relation), which reflects that WN18RR has highly varying entities and types. This irregularity may also limit the improvement when selecting the correct level of abstraction.

### 5.1 Correct Level of Abstraction

We further investigated whether the attention over types learns the levels of abstraction for entities that help create path patterns that are both generic and discriminative. For comparison, we modified the best model Attention (attention pooling) by fixing the selection of types to either the most specific types or the most abstract types. To create representatives of paths used by the attention model, we extracted the type with the highest weight for each entity. Then we compared the different path patterns that are used by these three models, which we denote as attention, specific, and abstract. To quantitatively evaluate whether paths are discriminative and generalizable, we define these two characteristics below.

We define that a path pattern (consisting of both relations and types in this case) is discriminative if this path pattern only appears in positive or negative examples, but not both. For each path pattern $\pi_i$, we count its number of occurrences in positive examples, denote $o_i^+$, and in negative examples, denote $o_i^-$. Then we calculate a ratio $r_i = o_i^+ / (o_i^- + o_i^+)$, which implies a path pattern’s distribution in positive and negative examples. Then we calculate the variance $d$ in the ratios $r_i$ for all path patterns. $d$ summarizes the discriminativeness of a set of path patterns. For example, if most path patterns appear in both positive and negative examples, $d$ will be small.

We define that a path pattern is generalizable if it appears both in training and testing. Quantitatively we define completeness as following: we assign path patterns that appear in training to set $\Pi_{train}$ and path types in testing to set $\Pi_{test}$. Then we calculate value $g$:

$$g = 1 - \frac{|\Pi_{test} \setminus \Pi_{train}|}{|\Pi_{test}|}$$

### 6 Conclusion

For the problem of knowledge base completion, we propose a new class of generalizable path patterns leveraging type hierarchies of entities. We develop an attention-based RNN model to discover the new path patterns from data. On two benchmark datasets WN18RR and FB15k-237, our model outperforms existing methods by a statistically significant margin. Quantitative analysis of the discovered path patterns also show they achieve a balance of generalization and discrimination.
