Student Algebraic Reasoning to Solve Quadratic Equation Problem

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Abstract. The reasoning is closely related to students’ mathematical abilities. Reasoning becomes a key element in mathematics learning. The student’s reasoning habits need to be developed. Competence in algebra is important. Algebra is one branch of mathematics that an essentials role in mathematics education. Algebra was prerequisite for higher studies mathematics and science courses, and a lack of algebra skills have a chance results in poor performance in courses. The purpose of this research is to describe students' algebraic reasoning in solving quadratic equation problem. The method used in this research is descriptive qualitative. Research subjects were three students of class X. The results in the study showed that students use a different strategy in solving the problem of quadratic equations. This can make a description of the existing schema. There are also students who misunderstand the concept of understanding operations in algebra so that this can affect their reasoning process and makes their answer incorrect.

1. Introduction

Reasoning is closely related to students' mathematical abilities. Mathematics is a collection of activities, something people do [1]. Reasoning becomes a clue element in mathematics learning. Reasoning can be classified from various points of view. This depends on the focus being studied [2]. Competence in algebra is essential. Algebra was prerequisite for higher studies mathematics and science courses, and a lack of algebra skills have a chance results in poor performance in courses [3]. Algebra is a tool that is useful for generalizing arithmetic, representing patterns, explaining order and consistency of problems and giving students opportunity to generalize [4]. Algebra is a branch of mathematics that deals with properties of operations and the structures which these operations are defined on [1] and [5]. One type of reasoning is algebraic reasoning. Algebraic reasoning is also called algebraic thinking [4]. Essentials key role of algebraic reasoning are abilities in representing (using variables/symbols) and solving word problems, recognizing and generalizing patterns, and analytic thinking (which involves solving equations) [6],[7]. Algebraic reasoning is a main activity of mind for constructing conceptual knowledge [8]. Success in algebra learning is strongly influenced by students’ algebraic reasoning [9]. Algebraic reasoning is used to generalize arithmetic, to notice patterns that hold true in algorithms and with
properties, and to reason quantitatively about such things as whether expressions are equivalent or not. Algebra must be shown in a way that student see it is a useful tool for making sense of all areas of mathematics and real-world situations. Algebraic thinking involves forming generalizations from experiences with number and computation and formalizing these ideas with the use of a meaningful symbol system. Far from being a topic with little real-world use, algebraic thinking pervades all of mathematics and is important for making mathematics useful in daily life [4].

Algebraic reasoning was started in kindergarten as young students which represent the sum of objects with fingers, mental images, drawing, sounds and acting out condition [10]. Middle school students’ prerequisite algebra skills progress from learning about patterns through diagrams and number sequences in elementary school to learning about patterns that represent functions, exploring proportional relationships, and making connections between properties of arithmetic and algebra [11-13]. Algebraic reasoning is heart of what it means to do mathematics [14]-[16]. Students can make conjectures about properties as early as first or second grade, and this must be encouraged by the teacher. Researchers suggest three strands of algebraic reasoning, all infusing the central notions of generalization and symbolization [1],[11] (a) the study of structures in the number system, including those arising in arithmetic; (b) the study of patterns, relations, and functions; (c) the process of mathematical modeling, including the meaningful use of symbols.

2. Method
This type of research is qualitative research. All participants are male. Research subjects were three students of class X. This study describes algebraic reasoning that occurs in students in solving quadratic equation problems. The study was conducted on senior high school students as much as three students. The data in this study is obtained from the student work. Research steps undertaken include (1) designing learning activities (2) preparing the test to be used (3) preparing research subject (4) observing students (5) data reduction (6) data analysis. Data analysis in this research is qualitative data analysis. Students are given questions and they solve the problem. In this research also observed the learning process to find out how students solve quadratic equation problems.

3. Result and Discussion
Students used several forms of reasoning as they presented and justified the arguments. Quadratic equation problem to be solved is as follow

\[
\frac{3(x-1)^2 + 2(x+1)^2}{4} = (x - 3)
\]

Subjects in this study were S1, S2 and S3. S1 work results are on Figure 1 as follows.

![Figure 1. work results S1](image)

S1 manipulated the algebraic form \((x - 1)^2\) became \((x^2 - 2x + 1)\) dan \((x + 1)^2\) became \((x^2 + 2x + 1)\). then S1 multiplied the results \((x^2 - 2x + 1)\) with 3 and \((x^2 + 2x + 1)\) with 2 as on Figure 2 below.

![Figure 2. work results S1](image)

And then S1 manipulated the denominator with 4 as on Figure 3 below.
And then S1 operate the algebraic form \((3x^2 - 6x + 3) + (4x^2 + 8x + 4)\) So he wrote the result as \(7x^2 + 2x + 7\) as on Figure 4 below.

S1 multiply 4 with \(x - 3\) then manipulated so the equation become \(7x^2 + 2x + 7 - 4x + 12 = 0\) as on Figure 5 below.

Then actually S1 found the general term of quadratic equation \(ax^2 + bx + c = 0\) and then he identified \(a\), \(b\) and \(c\) from the equation as on Figure 6 below.

S2 has different ways found the solution. First he multiply by 3 and 2 directly without writing the squares results from \((x - 1)^2\) and \((x + 1)^2\). S2 work results are on Figure 7 as follows.

In this ways below, it was same way with S1 work. S2 equated the denominator from algebraic form with 4 as on Figure 8 below.
In this ways below, it was same way with S1 work. S2 operate the algebraic form \((3x^2 − 6x + 3) + (4x^2 + 8x + 4)\) So he wrote the result as \(7x^2 + 2x + 7\) as on Figure 9 below.

**Figure 9.** work results S2

Without writing the process of multiplying 4 with \(x − 3\), S2 found the general term of quadratic equation \(ax^2 + bx + c = 0\) directly and then he identified \(a\), \(b\) and \(c\) from the equation as on Figure 10 below.

**Figure 10.** work results S2

S3 immediately write the denominator to 4 without describing the algebraic form \((x − 1)^2\) dan \((x + 1)^2\). S3 work results are on Figure 11 as follows.

**Figure 11.** work results S3

Then S3 found the result \(7x^2 − 4x + 13 = 0\) which is different with S1 and S2. And he havent already found the general for quadratic equation. S3 work results are on Figure 12 as follows.

**Figure 12.** work results S3

Generally, the steps to solve the quadratic equation \(\frac{3(x-1)^2}{4} + \frac{2(x+1)^2}{2} = (x − 3)\) are illustrated in Chart 1.1
To develop algebraic reasoning, teacher role was very important. Teacher-focused to allows for students to engage in algebraic reasoning, a process that is supported by the following teacher practices as follow.

- helping students learn to use a variety of representations, to understand how these representations are connected, and to be systematic and organized when representing their ideas;

- listening to student’s thinking and using this to find ways to build more algebraic reasoning into instruction; and

- helping students build generalizations through exploring, conjecturing, and testing mathematical relationships [11]. Furthermore, teacher can help their student improved algebraic reasoning by asking questions such as. “Do you see a pattern?” “Can you explain what to do next?” How is this number (picture, shape) related to the one that came before it and the one that comes after it [3]. [17] suggests there are 5 stages of growth point of algebraic reasoning as explained in the Table 1.

**Table 1. A framework of growth points in algebraic reasoning adopted from [17].**

| Growth Point        | Characteristics                                                                                                                                 |
|---------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| **GP 0: Pre-formal pattern** | Children do not have a formal understanding of “pattern.”  
Children cannot identify a repeating term in a pattern.                                                                                      |
| **GP 1: Informal pattern** | Children can identify a commonality and demonstrate understanding of pattern by copying, extending, inputting missing term, in visual spatial, numeric, repeating and growing patterns. |
| **GP 2: Formal pattern** | Children can describe a pattern verbally.  
Children can offer a possible near (not next) term with reasoning.                                                                                |
| **GP 3: Generalisation** | Children can correctly identify a near term.  
Children can describe a pattern explicitly.  
Children can offer a possible far term with reasoning.                                                                                            |
Growth Point | Characteristics
--- | ---
GP 4: Abstract generalisation | Children can describe a pattern explicitly, describe the rule as an expression in symbolic notation and utilise the expression in order to generate a far term.

The teacher played an important role in the introduction and development in classrooms. Students improved algebraic reasoning by focusing on patterns. They found the pattern by observing the pattern and relationship. Three Strands of Algebra [1] are showed below.

Strand 1. Building generalizations from arithmetic and quantitative reasoning is taken by many educators and researchers as the primary route into algebra.

Strand 2. The second strand involves generalizing of a fairly particular kind, basically toward the idea of function, where expressing the generalization can be thought of as describing systematic variation of instances across some domain.

Strand 3. Modeling as an algebraic activity seems to be of three basic types based on how the two core aspects of algebra are employed.

4. Conclusions
The first step in doing algebraic reasoning is that students identify patterns in problem. Students describe one by one problem items, then look for solutions for each item. In the realm of this research students have not yet been at the stage of finding the value of x. However, their reasoning process is seen to describe per item problem. Students use different strategy in solving the problem of quadratic equations. This can make a description of the existing schema. There are also students who misunderstand the concept of understanding operations in algebra so that this can affect their reasoning process and makes their answer incorrect.

References
[1] Kaput, J. J. 2008 What is Algebra? What is Algebraic Reasoning? In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), Algebra in the early grades. Reston, VA: NCTM.
[2] Uszkoreit, M. P. H., Wahlster, M. V. W., Wooldridge, M. J., Buchanan, B. G., Hayes, P. J., Hendler, J. A., … Oviatt, S. 2009 Neural-Symbolic Cognitive Reasoning. https://doi.org/10.1007/978-3-540-7346-4
[3] Jones, J. C. 2011 Visualizing Elementary and Middle School Mathematics Methods. Wiley. https://doi.org/10.1360/zd-2013-43-6-1064
[4] Walle, J Van 2007 Elementary and Middle School Mathematics. Toronto: Pearson.
[5] Hall W 2005 The Teaching of Algebra pp 1–8
[6] Lew, H. C. 2004 Developing Algebraic Thinking in The Earlier Grades: A Case Study of The South Korean Elementary School Mathematics Curriculum.—In: The Mathematics Educator (Singapore) 8(No. 1), p. 88–106
[7] Tolar TD, Lederberg AR, Fletcher JM 2009 A Structural Model of Algebra Achievement: Computational Fluency and Spatial Visualization as Mediators Of The Effect Of Working Memory on Algebra Achievement. Educational Psychology. 2009;29:239–266. doi:10.1080/01443410802708903.
[8] Glassmeyer, D., & Edwards, B. 2016 How Middle Grade Teachers Think about Algebraic Reasoning. Mathematics Teacher Education and Development, 182, 92–106.
[9] Andriani, P. 2015 Penalaran Aljabar dalam Pembelajaran Matematika. Beta, 8(2), 1–13. Retrieved from http://jurnalbeta.ac.id
[10] CCSSO (Council of Chief State School Officers) 2010 Common core state standards. Retrieved from http://corestandards.org
[11] Blanton, M. L. 2008 Algebra and The Elementary Classroom: Transforming Thinking, Transforming Practice. Portsmouth, NH: Heinemann
[12] Blanton, M., Levi, L., Crites, T. W., & Dougherty, B. J. 2011 Developing essential understanding
of algebraic thinking for teaching mathematics in grades 3–5. Essential Understanding Series. Reston, VA: NCTM.

[13] Bush, S. B., & Karp, K. S. 2013 Prerequisite Algebra Skills and Associated Misconceptions of Middle Grade Students: A Review. *Journal of Mathematical Behavior*, 32(3), 613–632.

[14] Ball, D. L., & Bass, H. 2003. Making Mathematics Reasonable in School. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics*. Reston, VA: NCTM.

[15] Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Portsmouth, NH: Heinemann.

[16] Schifter, D., Monk, G. S., Russell, S. J., & Bastable, V 2007 Early Algebra: What does Understanding The Laws of Arithmetic Mean in The Elementary Grades? In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades*. Mahwah, NJ: Erlbaum.

[17] Twohill, A. 2013 Algebraic Reasoning in Primary School: Developing a Framework of Growth Points. Smith, C. (Ed.) *Proceedings of the British Society for Research into Learning Mathematics* 33(2).

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