Low Barrier Nanomagnets as p-bits for Spin Logic

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Abstract: It has recently been shown that a suitably interconnected network of tunable telegraphic noise generators or “p-bits” can be used to perform even precise arithmetic functions like a 32-bit adder. In this paper we use simulations based on the stochastic Landau-Lifshitz-Gilbert (sLLG) equation to demonstrate that similar impressive functions can be performed using unstable nanomagnets with energy barriers as low as a fraction of a kT. This is surprising since the magnetization of low barrier nanomagnets is not telegraphic with discrete values of ±1. Rather it fluctuates randomly among all values between −1 and +1, and the output magnets are read with a thresholding device that translates all positive values to 1 and all negative values to zero. We present sLLG-based simulations demonstrating the operation of a 32-bit adder with a network of several hundred nanomagnets, exhibiting a remarkably precise correlation: The input magnets \(\{A, B\}\) as well as the output magnets \(\{S\}\) all fluctuate randomly and yet the sum \(A+B-S\) is sharply peaked around zero! If we fix \(A\) and \(B\), the sum magnets \(\{S\}\) rapidly converge to a unique state with \(S=A+B\) so that the system acts as an adder. But unlike standard adders, the operation is invertible. If we fix \(\{S\}\) and \(\{B\}\), the remaining magnets \(\{A\}\) converge to the difference \(A=S-B\). These examples emphasize a new direction for the field of nanomagnetics away from stable high barrier magnets towards stochastic low barrier magnets which not only operate with lower currents, but are also more promising for continued downscaling.

Index Terms: Spintronic memory and logic, nanomagnetics, Landau-Lifshitz-Gilbert equation, arithmetic functions.

I. INTRODUCTION

The developments in spintronics and nanomagnetics are having enormous influence on the field of storage and memory devices and it has been shown that the WRITE (W) and READ (R) elements can also be integrated into units that implement Boolean as well as non-Boolean logic [1–8]. These applications, however, usually make use of stable magnets with energy barriers \(\sim 40 \text{kT}\) which require relatively large currents for their operation. The critical spin current needed to switch a magnet with a thermal energy barrier of \(\Delta = H_K M_s V/2\) is given by [9]

\[
I_c = I_0 \frac{\Delta}{kT} \quad I_0 = \frac{4 q \alpha}{h} kT \left( 1 + f_t \frac{H_d}{2 H_K} \right)
\]

where \(q\) is the electronic charge, \(M_s\) is the saturation magnetization, \(H_K\) is the anisotropy field, \(H_d\) is the demagnetization field, \(V\) is the volume, \(\alpha\) is the Gilbert damping coefficient and the factor \(f_t\) is equal to zero for perpendicular anisotropy magnets (PMA) and one for in-plane anisotropy magnets (IMA). With \(\Delta \sim 40 \text{kT}\) and \(\alpha \sim 0.01\), the critical switching spin current for a PMA magnet is \(4 q \alpha \Delta / h \approx 10 \mu A\). Magnets with lower barriers could operate with lower currents but their application in conventional memory or logic is severely limited due to their stochastic nature. However, their possible use in unconventional applications has been discussed both theoretically and experimentally [10–18]. The implementation of logic operations based on an ensemble average over stable nanomagnets has been explored in [19–21] while [23] describes an approach to the traveling salesman problem based on a time average over unstable nanomagnets that cycle through millions of collective correlated states potentially at GHz rates. Note that for such nanomagnets (\(\Delta \ll 25 \text{kT}\) [24]), the Arrhenius model that predicts a telegraphic change between two magnetizations is no longer applicable, and the magnetization becomes a continuous variable. The present paper describes the application of the latter approach (time average) to implement precise Boolean logic operations like a 32-bit adder that provides the sum \(S\) for given inputs \(A\) and \(B\). Remarkably the adder also evaluates the inverse function, cycling through all combinations of \(A\) and \(B\) that add up to a given sum \(S\). We have recently shown [25] that a suitably interconnected network of tunable telegraphic noise generators or telegraphic “p-bits” can be used to perform even precise arithmetic functions like a 32-bit adder. However, it is not clear whether such p-bits can be implemented with real physical systems, especially if the noise in these systems are not telegraphic but continuous. The objective of this paper is to demonstrate that p-bits can be implemented using unstable nanomagnets with energy barriers as low as a fraction of a kT, even though their magnetization is not telegraphic and fluctuate among all values from −1 to +1. We assume that the magnets can be read with a thresholding device that translates all positive values to +1 and all negative values to zero. But this thresholding is applied only to the output nodes when we need to read a magnet at the end of an operation and not to the internal nodes or during device operation.

We start in Section 2 by showing that low barrier magnets, both PMA and IMA, exhibit the key property

\[
\Delta \approx \frac{H_d}{2 H_K} \ll 1
\]

and therefore behave like a telegraphic noise source with only a fraction of a kT energy barrier, while Section 3 demonstrates that similar functions can be performed using unstable nanomagnets with energy barriers as low as a fraction of a kT.
random number generators (RNG). Their magnetization \( m(t) \) fluctuates randomly in time, and the time-averaged \( \langle m \rangle \) can be tuned from \(-1\) to \(+1\) with a spin current. IMA magnets require a larger current to tune, but this is offset by a more rapid fluctuation rate, allowing a faster evaluation of the time average, and hence faster operation (Fig. 1). Note also that the PMA magnetization is relatively continuous compared to IMA magnetization which is more telegraphic in nature.

To harness either for logic applications, they have to be interconnected such that the spin current \( I_d \) driving magnet ‘k’ has to be derived from the magnetization of other magnets.

\[
\frac{2I_{sk}}{I_{c0}} = -I_0 \left( h_k + \sum_j J_{kj}m_j \right) \tag{2}
\]

where \( I_{c0} \) is normalization constant defined as the critical current (Eq. 1) for a magnet with a barrier \( \Delta = 1 \) kT and \( I_0 \) determines the overall strength of the interconnections. The bias \( \{ h \} \) and interconnection \( \{ J \} \) matrices have to be designed appropriately in implementing specific operations. We will not go into the implementation of these matrices since there are many options requiring careful discussion [3],[6],[27],[28]. We will assume that a network of stochastic nanomagnets (PMA and IMA) has been interconnected according to Eq. 2 and simulate their behavior using the stochastic Landau-

FIG. 1. Low-barrier stochastic Nanomagnet as a p-bit: (a) Time-averaged magnetization of low barrier IMA and PMA magnets (\( \Delta = 1 \) kT, \( H_K = 60 \) mT, \( \alpha = 0.01, H_d = 1.5 \) T for IMA) as a function of the bias spin current which is normalized to \( I_{c0} \) (Eq. 1). Average magnetization of PMA magnets obtained from sLLG which agrees well with the analytical solution from the FPE, Eq. 6. Inset shows a physical structure using a giant spin Hall effect (GSHE) material that could be used to convert a charge current into a spin current with the correct polarization to bias an IMA. (b) The magnetization \( m(t) \) for IMA as a function of time for three different bias currents obtained from a numerical solution of sLLG equation. (c) Same plot for PMA with the same barrier height. Note that the fluctuations are much faster and more telegraphic for IMA than for PMA. (d) A connection scheme for two p-bits is shown where the magnetization of a p-bit is implicitly converted into the bias current/voltage for the next p-bit (Eq. 2). A possible hardware implementation to turn the magnetization \( m \) into a voltage \( V \), could combine a GSHE layer with MTJs as in [2], replacing the stable write magnets by low barrier nanomagnets that are discussed here.

Lifshitz-Gilbert (sLLG) equation to demonstrate useful functionalities. We assume that the currents specified by Eq. 2 are applied to each magnet on a time scale that is much shorter than the magnet dynamics, and new features could arise if delays associated with these interconnections are comparable to magnet dynamics. These issues are beyond the scope of this paper. All numerical examples are presented for IMA with parameters shown in Fig. 1 but similar results are obtained with PMA as well.

In Section 3 we describe how simple logic gates can be implemented by suitably designing the \( \{ h \} \) and \( \{ J \} \) matrices so that the magnet configurations corresponding to the desired truth table represent ‘low energy’ states where the network spends most of its time according to the Boltzmann law of equilibrium probabilities: \( P(\{ m \}) \sim \exp(-E(\{ m \})/kT) \). Although the use of spin currents does not in general permit us to write an energy functional [29], for symmetrically interconnected PMA magnets we can use a functional of the form [22, 23]:

\[
-\frac{E(\{ m \})}{kT} = \sum_i \frac{\Delta_i}{kT} m_i^2 + I_0 \left( \sum_i h_i m_i + \frac{1}{2} \sum_{i,j} J_{ij}m_i m_j \right) \tag{3}
\]

to describe the network of interconnected magnets. This can be seen by noting that from the Boltzmann law and Eq. 3

\[
\frac{\partial \ln P}{\partial m_k} = \frac{2\Delta_k}{kT} m_k + I_0 \left( h_k + \frac{1}{2} \sum_j (J_{kj} + J_{jk})m_j \right)
\]
so that for a symmetric $[J]$ matrix, from Eq. 2

$$P(m_k) \sim \exp \left( \frac{\Delta k}{kT} m_k^2 - \frac{2 I_{sk}}{I_{0}} m_k \right)$$

which is exactly the steady-state condition for magnet ‘k’ that we would obtain from the Fokker-Planck equation (FPE) ([30] Eq. (3.9)) for PMA. Moreover, our “empirical” results show that the energy functional shows good agreement even when magnets have an additional shape anisotropy. Note that even though $I_{0}$ is size-independent, the distribution of the nanomagnet depends on size through $\Delta$: for higher $\Delta$ magnets, more spin current is required to pin the magnetization. We will refer to Eq. 4 as the FPE probability.

The probability distributions obtained from the numerical solution of the sLLG equation for both PMA and IMA magnets follow the FPE result quite well (Fig. 2). The highest probabilities correspond to the lowest energy states, which correspond to the desired truth table relating the input magnets A and B to the output magnet C. If we force the inputs A and B to specific values by using appropriate values for $h_A$ and $h_B$, C would take on the specific value required by the truth table, just like standard digital gates. But unlike standard gates, these gates are invertible, similar to those discussed in the context of memcomputing [31]. They can be operated in reverse: if we clamp the output C to a specific value, the inputs A and B will spend most of its time in those configurations $\{AB\}$ that produce that output. We also illustrate this reversible operation with a more complex logic gate, namely a full adder treating it as a Boltzmann machine (BM) and using the same principle of energy minimization to design the $[h]$ and $[J]$ matrices.

Finally in Section 4 we demonstrate the operation of a 32-bit adder obtained from 31 full adders and one half adder with the output carry from each bit connected to the carry in of the next higher bit through the appropriate element of the overall $[J]$ matrix. Note that these are unidirectional connections so that the overall $[J]$ matrix is not symmetric, though the $[J]$ matrix for each full adder is symmetric. We show that this network of nearly five hundred nanomagnets exhibits a remarkably precise correlation that provides the exact sum S of any two given inputs, A and B (Fig.4). What is even more remarkable is that if we do not fix either the inputs or the outputs, the quantities A, B and S all fluctuate randomly and yet the quantity A+B−S is sharply peaked around zero, so that the network can be used to extract either A, B or S, if the other two are fixed, which is similar to the NP-complete “subset sum” problem (Fig.5) [32, 33].

### II. STOCHASTIC NANOMAGNET MODEL

Fig.1(b,c) shows the time response of the magnetization $m_z$ along the easy axis calculated using the sLLG equation (integrated by Heun’s method within the Stratonovich calculus [36]) with $\Delta t = 0.95$ ps for IMA and $\Delta t = 11.8$ ps for PMA.

$$\frac{1 + \alpha^2}{q N_t} \frac{d\hat{m}_i}{dt} = -|\gamma| \hat{m}_i \times \hat{H}_i - \alpha |\gamma| (\hat{m}_i \times \hat{m}_i \times \hat{H}_i)$$

$$+ \frac{1}{q N_t} (\hat{m}_i \times \hat{I}_S \times \hat{m}_i) + \left( \frac{\alpha}{q N_t} (\hat{m}_i \times \hat{I}_S \times \hat{m}_i) \right)$$

where $\hat{H}_i$ is the effective field including the uniaxial and shape anisotropy terms, as well as the thermally fluctuating magnetic field due to three dimensional uncorrelated thermal noise $\hat{H}_n$ having Gaussian distribution with mean $(\langle H_n \rangle = 0$ and standard deviation $(\langle H_n^2 \rangle = 2kT/|\gamma| M_s V$ along each direction [34–38], $\gamma$ is the gyromagnetic ratio and $N_t = M_s V/\mu_B$ is the total number of Bohr magnetons comprising the magnet. Our simulations are based on the macrospin approximation, as is common in the literature [24, 39, 40]. This approx-
E(\{s\}, m) = \left( \sum_i \frac{\Delta_i}{kT} + \frac{1}{2} I_0 \sum_{i,j} J_{ij} s_i s_j \right) m^2 + \left( I_0 \sum_i h_i s_i \right) m

\tag{7}

The peaks corresponding to the preferred states in Fig. 2c do not have equal probability, even at steady state as predicted by Eq. (7). This skew is due to the continuous nature of magnetization with small \( \Delta \) magnets that affect the thresholded results.

Note that the probabilities are strongly affected by the choice of \( I_0 \) as we might expect from the exponential dependence of the Boltzmann function. If we use a much smaller value of \( I_0 \) we obtain a uniform probability across all eight states as we would expect for three uncorrelated magnets. If we use a much larger value of \( I_0 \) the Boltzmann law predicts all states with equal energy to be equally occupied, but in a numerical simulation, the system tends to get stuck for long periods in one of the preferred states, instead of moving freely among them.

Consider now a full adder having three inputs \( A, B, C_{in} \) and two outputs \( S, C_{out} \), \( S \) being the sum bit, and \( C_{in}, C_{out} \) being the incoming carry and the outgoing carry bits. Fig. 3 shows a full adder constructed out of 14 p-bits treating it as a BM with a symmetric J-matrix ([26]) which is obtained by a suitable extension of the principles developed in the context of Hopfield networks ([42], Eq. 4.20) and extended in [25]. This design not only gives the correct output for a given input, but also the correct set of inputs for a given output.

IV. 32-BIT ADDER/SUBTRACTOR

Finally we demonstrate the operation of a 32-bit adder obtained from 31 full adders and one half adder with a single directed connection from the \( C_{out} \) of one bit to the \( C_{in} \) of the next bit, in accordance with the standard design of ripple carry adders (RCA). Here, we treat the RCA as a standalone block without any peripheral read-out circuitry to simply demonstrate how the nanomagnet network can operate as a directed combinational logic unit. If we provide two input numbers \( A \) and \( B \), and look at the sum \( S \), which includes all the sum bits along with the carry-out from the last bit, \( C_{out} \) (32) we find numerically that the system relaxes to the correct sum with occasional jumps from the correct state. It is really quite surprising that a network of \( 14 \times 31 + 6 = 440 \) nanomagnets fluctuating continuously over the range \(-1 < m < +1 \) get correlated precisely enough to point to the correct answer out of \( 2^{33} \approx 8 \text{ billion} \) possibilities without getting stuck in metastable states [25]. Interestingly it also works as a subtractor: if we fix the sum and one of the inputs \( B \), the remaining input gives the correct difference \( A = S - B \) (Fig. 4). Even more surprisingly, the overall system seems to act like a BM when all magnets are allowed to fluctuate. Each set of magnets \( A, B \) and \( S \) fluctuates randomly over a wide range of values. But the quantity \( A + B - S \) shows a sharp peak around zero (Fig. 5), showing that the interconnected network reflects the desired truth table.
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