Space-time counterfactuals

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Abstract

A definition is proposed to give precise meaning to the counterfactual statements that often appear in discussions of the implications of quantum mechanics. Of particular interest are counterfactual statements which involve events occurring at space-like separated points, which do not have an absolute time ordering. Some consequences of this definition are discussed.

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1 Introduction

Discussions of the interpretation or the implications of quantum mechanics often use the language of counterfactuals (see for example refs. [1]–[14]); however, there does not seem to be agreement on exactly what these counterfactuals are to be taken to mean. In this paper I will propose a precise definition for counterfactual statements which are used in discussions of quantum theory.

Consider this simple situation: I have two particles called $A$ and $B$, each of spin $\frac{1}{2}$, initially prepared in a state of total spin zero. First I measure $S_x$ (the x-component of the spin) of $A$, and find the value $+1$ (expressed in units of $\frac{1}{2}\hbar$); then I measure the value of $S_y$ of $B$, and also find the value $+1$. I suspect that most of us would consider, in this situation, the statement “If I had measured $S_x$, instead of $S_y$, for $B$, I would have obtained the value -1” to be true, and likewise the statement “If I had measured $S_x$, instead of $S_y$, for $B$, I would have obtained the value +1” to be false. But what do these statements, about a situation that does not exist, really mean? By the antecedent “If I had measured $S_x$, instead of $S_y$, for $B$” I am imagining a situation which is exactly like the actual one up until the time of the measurement of the spin of $B$ (in particular, a situation in which the particles did indeed have initial total spin zero, and in which I did indeed first measure $S_x$ for $A$ and obtained the value $+1$), and in which the value of $S_x$ of $B$ is then measured. The correct conclusion “I would have obtained the value -1” can be understood as simply being the implication of quantum theory for this imagined situation.

In the next section I will propose a definition of counterfactual which will formalize this understanding of this simple example, and which will permit an extension to a class of counterfactuals which I call space-time counterfactuals. This class will include statements about situations more complicated than the one discussed above, for example situations in which several choices are made; most importantly, it will include counterfactual statements about events at different locations, for which a Lorentz-invariant notion of temporal precedence does not exist. I believe that most of the counterfactuals used in discussions of quantum theory fall into this class, but of course many counterfactuals which are used in other contexts do not. I am certainly not presenting a new general theory of counterfactuals; the definition I will present is essentially the application of the theory of counterfactuals of Lewis [15] to space-time counterfactuals, but with a modification of that theory to accommodate the lack of a relativistically-invariant time
ordering.

I will restrict the discussion to counterfactuals which concern localized macroscopic phenomena; I will refer to such phenomena generically as “events”, and will employ the idealization that an event occupies a single point of space-time. Of course I will allow statements about combinations of events; the restriction is that each event has a definite location. In the simple example given above, the event in the antecedent was the choice of which measurement to make; the class of counterfactuals I consider also includes those whose antecedent event is, for example, the result of a measurement performed on a quantum system. I will certainly want to discuss situations in which particles at different locations are in a quantum-entangled state, which therefore cannot be represented by any combination of properties each of which refers only to a single location. Even in these situations, I choose to consider only those statements which refer to macroscopic, and hence localizable, properties. Strictly speaking, the simple example discussed above, in which I specified the quantum state of the two-particle system, does not respect this restriction. However, I can consider the specification of the quantum state as merely an abbreviation for a description of the macroscopic procedure by which it was prepared. Saying this does not imply a commitment to any interpretation (e.g., an instrumentalist interpretation) of quantum theory; it is merely a restriction on what I choose, for the present purposes, to talk about.

I will be using, starting in the next section, a language involving the assessment of the similarity of a “possible world” to the actual world. As a generalization of the statement in the simple example that “everything is the same up to the time of the measurement on B”, I will base the assessment on the region of space-time over which the two compared worlds agree, without introducing any notion of a “degree” of disagreement at individual space-time points. Of course such a notion could be important for other applications of counterfactual statements, but I limit the discussion here to cases in which it is not. So the “space-time counterfactuals” I consider are those which involve only localizable (in fact, macroscopic) properties, and whose truth can be assessed by considering the regions of space-time in which those properties are, or are not, obeyed.

In the next section I will define counterfactuals in a way which is appropriate for this case. In the final section I will present some applications of this definition, and point out some problems with some alternative definitions.
2 Definition of counterfactual statements

I want to adapt the analysis of Lewis [15] to space-time counterfactuals. That analysis uses the semantics of “possible worlds.” We can take possible world to be any which is consistent with the laws of physics (in practice, it is usually the laws of quantum physics with which we will be concerned); no commitment to the existence of any world other than the one actual world is required.

Lewis’ analysis requires a judgment as to the comparative similarity of various possible worlds to the actual world. Call a world in which a proposition \( \phi \) is true a “\( \phi \)-world”. Then, in a sense which will be made precise below, the counterfactual which is written \( \phi \rightarrow \psi \), and which is read “If \( \phi \) were true, then \( \psi \) would be true” is taken to mean that \( \psi \) is true in the \( \phi \)-worlds most similar to the actual world; it is not required that \( \psi \) be true in all possible \( \phi \)-worlds. In the simple example discussed in the previous section, \( \phi \) was “\( S_x \) of \( B \) was measured” and \( \psi \) was “Value -1 was obtained”. Letting \( t \) be the time of the measurement of \( B \), and not yet worrying about the relativistic absence of an unambiguous time ordering, we said in effect that \( \phi \rightarrow \psi \) was true because \( \psi \) was true in all \( \phi \)-worlds which agreed with the actual world at all times earlier than \( t \). To express this in Lewis’ language, we would say that all worlds which agree with the actual world at all times earlier than \( t \) are equally similar to the actual world, and are more similar to it than is any world which differs from the actual world at any time earlier than \( t \).

To see how to generalize this judgment to the relativistic case, consider three space-time points called \( A \), \( B \), and \( C \), such that \( B \) is unambiguously later than \( A \), and that \( C \) is space-like separated from both \( A \) and \( B \). This situation is depicted, using the coordinates of some particular Lorentz frame, in figure 1, which also indicates the forward light-cones of these three points. Consider also three possible worlds: world \( W_A \) first differs from the actual world at point \( A \); world \( W_B \) first differs from the actual world at \( B \); likewise world \( W_C \) first differs at \( C \). From the preceding discussion, we would say that \( W_B \) is more similar to the actual world than is \( W_A \), since it first differs from the actual world at an (unambiguously) later time. Comparing \( W_A \) with \( W_C \), there is no way to judge between them; in fact in the frame shown they each differ from the actual world at the same time. We might therefore be tempted to judge that \( W_A \) and \( W_C \) are equally similar to the actual world. But wait! Since \( C \) and \( B \) are space-like separated, there is a frame in which \( W_C \) and \( W_B \) first differ from the actual world at the same time,
so we should be just as tempted to judge that $W_C$ and $W_B$ are equally similar to the actual world. However, if we judge that $W_A$ and $W_C$ are equally similar, and also that $W_C$ and $W_B$ are equally similar, we would be forced to conclude that $W_A$ and $W_B$ are equally similar, in conflict with the judgment that $W_B$ is more similar than is $W_A$.

What to do? We could arbitrarily pick a Lorentz frame, and declare that the possible world which, in that frame, deviates from the actual world at the latest time is the most similar; we could then use this judgment of similarity to define counterfactuals as in the analysis of Lewis. However, this definition will lead to the situation that the truth of certain counterfactual statements will depend on which frame we happened to pick (an example of this will be presented in the next section). It is not inconsistent to say that the truth of a statement may depend on the choice of a frame (the statement “the earth is at rest” is true in some frames and not in others), but we should be able to do better. Suppose we say that a counterfactual statement $\phi \square \rightarrow \psi$ is “verified” in a given frame if $\psi$ is true in the $\phi$-world which deviates at the latest time in that frame, and then say that the statement is true if there is any frame in which it is verified. This would certainly give us a frame-independent criterion for truth of a counterfactual, but at the price of, for example, permitting situations in which two counterfactuals $\phi \square \rightarrow \psi_1$ and

Figure 1: Three space time points $A$, $B$, and $C$, and their forward light cones. $B$ is in the unambiguous future of $A$, and $C$ is space-like separated from both $A$ and $B$. 


φ □→ ψ₂, with ψ₁ and ψ₂ contradictory, would both be true. We will see an example of this in the next section.

Without further ado, here is my proposal for a definition of space-time counterfactual statements. For any space-time point r, let \( F(r) \) be the set consisting of r itself and of all points which are unambiguously at a later time than r (that is, points on or within the forward light-cone of r). For any set of points D, let \( \overline{D} \) be the union of \( F(r) \) over all \( D \); that is,

\[
\overline{D} := \{ r' | r' \in F(r) \text{ for some } r \in D \}.
\]

Call \( \overline{D} \) the “future closure” of \( D \), and note that for any \( D, \overline{D} = \overline{D} \). Now let \( W_a \) denote the actual world, and \( W_p \) denote some other possible world. Let \( D_p \) be the set of space-time points at which \( W_p \) differs from \( W_a \), that is, the set of points at which there is an event which is different in \( W_p \) than in \( W_a \). The definition of a space-time counterfactual which I propose, and which I will refer to as DSTC, is:

The statement \( \phi \square \rightarrow \psi \) is true when either

1. There are no possible \( \phi \)-worlds, or

2. For any \( \phi \)-world \( W_q \) in which \( \psi \) is not true, there is a \( \phi \)-world \( W_p \) in which \( \psi \) is true, with \( \overline{D}_p \subset \overline{D}_q \).  

In the second condition above, I am taking \( \overline{D}_p \subset \overline{D}_q \) to require that \( \overline{D}_p \) be a proper subset of \( \overline{D}_q \). This definition DSTC indicates that I regard \( \overline{D}_p \) as a measure of the similarity of \( W_p \) to \( W_a \); the condition \( \overline{D}_p \subset \overline{D}_q \) could be read as “\( W_p \) is more similar to \( W_a \) than is \( W_q \).” In fact, DSTC closely follows the definition given by Lewis [15], except that the use of set inclusion here means that we have a partial (pre-)order on the set of possible worlds (not every pair of possible worlds can be compared), while Lewis requires that the set of possible worlds have a total (pre-)order. We have a partial order because we want similarity to reflect the earliest time at which a possible world differs from the actual world, and in a relativistic context temporal precedence is a partial (rather than a total) order on the set of space-time points. The existence of a partial order for possible worlds has also been suggested by Pollock[16], on very different grounds.

An alternative to DSTC, “\( \phi \square \rightarrow \psi \) when there exists a \( \phi \)-world \( W_p \) in which \( \psi \) is true such that, for every \( \phi \)-world \( W_q \) in which \( \psi \) is not true, \( \overline{D}_p \subset \overline{D}_q \),” which follows even more closely the formulation given by Lewis, does not work with a partial order. Also, it should be understood that, although it is not indicated here explicitly, the truth of a counterfactual depends on the actual world from which it is issued (as is stated by Lewis).
There is a (pathological) situation in which DSTC would yield an unreasonable answer: suppose that the possible $\phi$-worlds could be arranged in an infinite sequence, say $W_i$ for $i = 1, 2, 3, ...$ with $D_{i+1} \subset D_i$, and with $\psi$ true in $W_i$ iff $i$ is even. Then both $\phi \Box \rightarrow \psi$ and $\phi \Box \rightarrow (\sim \psi)$ would be true. This situation obviously would not arise if there were only a finite number of possible worlds. Since in essentially all applications the number of worlds considered is in fact finite (and often is quite small) we would not lose much generality if we were to restrict ourselves to cases of finite numbers of worlds. However, an even weaker condition will suffice. For a possible $\phi$-world $W_p$, say that $W_p$ is a “primary” $\phi$-world if there is no possible $\phi$-world $W_q$ with $D_q \subset D_p$. Now say that a proposition $\phi$ is “closed” if, for every possible $\phi$-world $W_q$, either $W_q$ is itself primary, or else there is a primary $\phi$-world $W_p$ with $D_p \subset D_q$. Then DSTC should be understood as applying only to antecedents $\phi$ which are closed. This restriction is related to the Limit Assumption discussed (but neither needed nor adopted) by Lewis. Essentially any space-time proposition $\phi$ which we might reasonably want to consider will turn out to be closed, so that DSTC will apply; in the next section we will see examples of propositions which can be proven to be closed.

The definition DSTC can now be restated as follows: For a closed proposition $\phi$, the statement “$\phi \Box \rightarrow \psi$” means that either 1) there are no possible $\phi$-worlds, or 2) $\psi$ is true in every primary $\phi$-world. With this definition, some but not all of the usual inferences for conditionals are valid. Some examples are: $(\phi \Box \rightarrow \psi_a)$ and $(\phi \Box \rightarrow \psi_b)$ implies $\phi \Box \rightarrow (\psi_a \land \psi_b)$; $\phi \Box \rightarrow \psi_a$ implies $\phi \Box \rightarrow (\psi_a \lor \psi_b)$; but $\psi_a \Box \rightarrow \psi$ does not imply $(\phi_a \land \phi_b) \Box \rightarrow \psi$ (as it also does not in Lewis’ theory). Let me record here a proof of the first example: Assume that $\phi \Box \rightarrow \psi_a$ and that $\phi \Box \rightarrow \psi_b$. This means that both $\psi_a$ and $\psi_b$ are true in every primary $\phi$-world; hence $\psi_a \land \psi_b$ is true in every primary $\phi$-world, and hence the statement $\phi \Box \rightarrow (\psi_a \land \psi_b)$ is true.

Since $D_p$ is the set of points at which $W_p$ differs from $W_a$, it might be thought that $D_p$, rather than $D_p$, should be taken as the measure of similarity between $W_p$ and $W_a$. But even in the simple, non-relativistic example discussed in the first section, for which (I believe) the interpretation of the counterfactual was uncontroversial, that is not what happened. I expect that my reader agreed with me that what the counterfactual meant in that example was that the conclusion should hold in any situation (i.e. any world) which is exactly like the actual situation up until the time of the measurement of $B$. If we simply translate this understanding into the language of Lewis, we see that, at least in this example, we must judge as equally similar any two possible worlds which first differ from the actual
world at a given time \( t \); we do not even have to inquire whether they differ from the actual world at times later than \( t \).

But why do we not have to inquire? Let me re-phrase (in a perhaps slightly whimsical manner) the answer that Lewis gave to this question in the non-relativistic context [17]. Consider a possible world \( W_p \) in which a certain experiment, performed at space-time point \( r \), goes differently than in the actual world \( W_a \). Then in \( W_p \) the journal article reporting the result of that experiment is written differently than in \( W_a \), and so perhaps the experimenter receives tenure in \( W_p \) but not in \( W_a \), and so... Or to extend our whimsy in a slightly different direction, we know that any event will necessarily produce an infinite number of (very low energy) photons; if an event at \( r \) is different in the two worlds, the distribution of photons will be different also. Of course, left to themselves, the photons will travel along the (outside of the) forward light-cone of \( r \); however, because of scattering (e. g. from microwave-background photons) there will also be effects from these photons throughout the interior of that light-cone. The bottom line is that, if \( W_p \) differs from \( W_a \) at \( r \), and if it obeys all the laws of physics (including the arrow of time which we are certainly not explaining here) it will also differ (at least) at all points in \( F(r) \).

This argument indicates that, for any possible world \( W_p \), the set \( D_p \) will in fact coincide with the set \( D_p \). Of course, in practice we want to specify a possible world in as economical a way as is possible, without having to describe in detail who receives tenure in that world, or in even more detail the effects of an infinite number of infrared photons. And so we specify a possible world by saying that it differs from the actual world at a small number of points, while we expect that it differs (at least) in the future closure of those points. Lewis [17] argued that, for the non-relativistic case (i. e. with absolute time ordering), if \( W_p \) and \( W_a \) differ at time \( t \), then we expect that they will also differ at times later than \( t \), but not necessarily at times before \( t \). We now see that the generalization of this conclusion to the relativistic case is that, if they differ at space-time point \( r \), we expect them also to differ at all points in \( F(r) \) (of course they may differ at points outside \( F(r) \) as well).

The definition of counterfactual I have given does allow the trivial case in which there is no possible \( \phi \)-world. However, most applications of counterfactuals in discussions of quantum theory do assume that, due to quantum indeterminism, \( \phi \) is in fact compatible with the past of the actual world. That is, in the case that \( \phi \) specifies a difference from the actual world at a single point \( r \), it will usually be expected that there will be a possible
\(\phi\)-world which agrees with the actual world at all points which are unambiguously earlier than \(r\). This expectation is in addition to the expectation mentioned previously that there will not be agreement at points which are unambiguously later than \(r\).

3 Some applications

Let \(\Delta\) be a region of space-time; I say that \(\Delta\) is a “\(\phi\)-region” if \(\Delta = \overline{D_q}\) for some \(\phi\)-world \(W_q\). Let \(\Sigma\) be a set of \(\phi\)-regions; I write \(\Sigma = \{\Delta_\alpha\}\), and say that \(\Sigma\) “supports” \(\phi\) if for any \(\phi\)-world \(W_q\) there is a \(\Delta_\alpha \in \Sigma\) with \(\Delta_\alpha \subseteq \overline{D_q}\). (A set \(\Sigma\) which supports \(\phi\) is also said to be a “coinitial” subset of the (partially) ordered set of \(\phi\)-regions.\(^{[18]}\)) Then if \(\Sigma\) supports \(\phi\): If \(\Sigma\) contains only a single element \(\Delta\), then any \(\phi\)-world \(W_p\) with \(\overline{D_p} = \Delta\) is primary; if \(\Sigma\) contains only a finite number of elements, then \(\phi\) is necessarily closed; if \(W_p\) is a primary \(\phi\)-world, then \(\overline{D_p} \in \Sigma\), hence to evaluate the truth of a statement \(\phi \square \rightarrow \psi\) we need only to consider \(\phi\)-worlds \(W_q\) with \(\overline{D_q} \in \Sigma\). All of the examples considered in this section will have antecedents which are supported by a set \(\Sigma\) containing only a finite number of elements; therefore these antecedents are closed, and DSTC does indeed apply.

In many cases in which counterfactual statements are made about quantum systems (including the example in the first section) the counterfactual antecedent represents a free choice. This is sometimes described by saying that a person can exercise free will to choose between alternatives, but can just as well be modeled as being controlled by the flip of a quantum coin: a device prepares an auxiliary particle with spin aligned along the \(z\) direction, and then measures the \(x\)-component of spin; the outcome of the measurement then determines the “choice”. Say that \(\hat{\chi}_a\) is a choice actually made at a space-time point \(r\), and that \(\hat{\chi}\) is an alternative choice that could have been made. To say that the choice between \(\hat{\chi}_a\) and \(\hat{\chi}\) was a free choice is to say both that \(\hat{\chi}\) would have been possible, and that the choice is not correlated with anything which is space-like separated from \(r\). We can express this meaning by saying that, if \(\hat{\chi}\) is a free choice, there must exist a possible \(\chi\)-world \(W_p\) which agrees with the actual world everywhere outside \(F(r)\); thus \(\overline{D_p} = F(r)\).

Let \(\hat{\chi}\) be a free choice at \(r\), and let \(\chi\) be the proposition that the choice \(\hat{\chi}\) is made; then \(\chi\) is supported by the set consisting of the single element \(F(r)\). Therefore the primary \(\chi\)-worlds are those worlds \(W_p\) with \(\overline{D_p} = F(r)\). Now let \(\Psi\) denote the statement “\(\chi \square \rightarrow \psi\)” (This \(\Psi\) represents the type
of counterfactual statement considered in ref. [4]). Then \( \Psi \) is equivalent to the statement that \( \psi \) is true in each world which agrees with the actual world everywhere outside \( F(r) \). As a special case of this, suppose that \( \psi \) refers only to events located outside \( F(r) \) (that is, events which are either unambiguously before or else are space-like separated from \( r \)); then \( \Psi \) is true iff \( \psi \) is true in the actual world. This result has been shown to follow from the definition of counterfactual given in the preceding section. Even if a \( \psi \) which refers only to events located outside \( F(r) \) is true in the actual world, there may well be possible \( \chi \)-worlds in which \( \psi \) is not true; if we had defined, for example, the statement \( \Psi \) to mean that \( \psi \) was true in all possible \( \chi \)-worlds, we would not have concluded that \( \Psi \) was true.

Suppose that we have several independent free choices, say \( \hat{\chi}_i \) located at \( r_i \), for \( 1 \leq i \leq n \). The proposition \( (\chi_1 \land \chi_2 \land \cdots \land \chi_n) \) is supported by the set consisting of the single element \( \bigcup_{i=1}^{n} F(r_i) \), hence the statement \( (\chi_1 \land \chi_2 \land \cdots \land \chi_n) \rightarrow \psi \) is equivalent to the statement that \( \psi \) is true in every \( (\chi_1 \land \chi_2 \land \cdots \land \chi_n) \)-world which agrees with the actual world everywhere outside \( \bigcup_{i=1}^{n} F(r_i) \) (that is, at all points which are not at nor in the future of any of the \( r_i \)). Also, the proposition \( (\chi_1 \lor \chi_2 \lor \cdots \lor \chi_n) \) is supported by the set \( \Sigma = \{ F(r_i) \} \); if the choices are mutually space-like separated, then any \( \chi_i \)-world \( W_p \) with \( D_p = F(r_i) \) is primary, and so the statement \( (\chi_1 \lor \chi_2 \lor \cdots \lor \chi_n) \rightarrow \psi \) is equivalent to the statement that, for each \( i \), \( \psi \) is true in any \( \chi_i \)-world which agrees with the actual world everywhere outside \( F(r_i) \), that is, \( \chi_i \rightarrow \psi \) for each \( i \).

The definition DSTC can also be applied to counterfactual statements of the type discussed in ref. [12]. Consider two spin-\( \frac{1}{2} \) particles \( A \) and \( B \), and measurements performed at a given location (so that the time-ordering is unambiguous). Say this is the actual world: at time \( t = 0 \) the particles are prepared in a state of total spin zero; at time \( t = 1 \) nothing is done; at time \( t = 2 \) the value of \( S_x \) of \( B \) is measured, with result -1. It follows, from the fact that the total spin is zero, that the product of the values of \( S_x \) of \( B \) and \( S_x \) of \( A \) must be -1; nevertheless, the following statement is, on my analysis, not true: “If \( S_x \) of \( A \) had been measured at \( t = 1 \), the result +1 would have been obtained.” The reason this is not true is that, among worlds which already differ from the actual world at \( t = 1 \), a world with result +1 at \( t = 2 \) and a world with result -1 at \( t = 2 \) count as equally similar to the actual world (even though the actual result at \( t = 2 \) was -1). However, the following statement is true: “If \( S_x \) of \( A \) had been measured at \( t = 1 \), and \( S_x \) of \( B \) measured at \( t = 2 \) with result -1, then the result of \( S_x \) for \( A \) would have been +1”.

Given the actual world as described above,
this could also be written “If $S_x$ of $A$ had been measured at $t = 1$, and the measurement and result at $t = 2$ were the same as in the actual world, then the result of $S_x$ for $A$ would have been +1.”) The reason this is true is that a world with result +1 for $B$ at $t = 2$ (and hence -1 for $A$ at $t = 1$) is not a possible $\phi$-world, if $\phi$ includes the specification that the result for $B$ was -1.

As an example of a counterfactual statement whose antecedent does not involve choices, consider the case of three particles, each of spin $\frac{1}{2}$, located at space-time points $A$, $B$, and $C$. The $x$-component of the spin of each is measured; denote the possible results by $a = \pm 1$, $b = \pm 1$, and $c = \pm 1$. However, the particles are in an entangled state (for example, the GHZ state $|\Psi^+\rangle$) for which the product $abc$ surely has the value -1. Say that in the actual world, $a = -1$, $b = +1$, $c = +1$, and consider a statement $\Psi_1$, defined by

$$\Psi_1 := (a = +1) \rightarrow (c = +1).$$

The proposition $(a = +1)$ is supported by the set

$$\Sigma := \{F(A) \cup F(B), F(A) \cup F(C)\}.$$ 

Denote by $W_1$ any $(a = +1)$-world with $\overline{D}_1 = F(A) \cup F(B)$; $W_1$ has $a = +1$, $b = -1$, $c = +1$. Denote by $W_2$ any $(a = +1)$-world with $\overline{D}_2 = F(A) \cup F(C)$; $W_2$ has $a = +1$, $b = +1$, $c = -1$. If $A$, $B$, and $C$ are mutually space-like separated, as shown in figure 2, then both $W_1$ and $W_2$ are primary, and since $c \neq +1$ in $W_2$, statement $\Psi_1$ is false. On the other hand, if $B$ is in the unambiguous future of $A$, as shown back in figure 1, then since in this case $\overline{D}_1 \subset \overline{D}_2$, only $W_1$ is primary, and then $\Psi_1$ is true.

We could also analyze this same GHZ example using the frame-dependent definition of counterfactual mentioned in the previous section. For that, we pick a Lorentz frame, and then say that the possible $(a = +1)$-world which deviates from the actual world at the later time is more similar; then say that the statement $\Psi_1$ defined above is true if $c = +1$ in the more-similar world. Take the case in which the three events are mutually space-like separated, as shown in figure 2. If we pick a frame in which the times of the events satisfy $t_C < t_A < t_B$ (call that frame $\alpha$), then world $W_1$ first deviates at time $t_A$, and world $W_2$ first deviates at time $t_C$; thus $W_1$ is more similar to the actual world than is $W_2$, and the statement $\Psi_1$ is true. On the other hand, suppose we pick a frame in which $t_B < t_A < t_C$ (frame $\beta$); then $W_1$ first deviates at time $t_B$, while $W_2$ first deviates at time $t_A$, and so $\Psi_1$ is false. Thus in this example the truth of $\Psi_1$ depends on which frame we pick. Finally, we can use this same GHZ example to see what would happen if we
said that a counterfactual statement is true if we can find any Lorentz frame in which it can be “verified” by the above analysis. We would then say that $\Psi_1$ can be verified in frame $\alpha$, and hence it is true. Also, since $c = -1$ in world $W_2$, and since in frame $\beta$ world $W_2$ is more similar than is world $W_1$, the statement $(a = +1) \rightarrow (c = -1)$ can be verified in frame $\beta$, and hence it also is true. So we wind up saying that statements $(a = +1) \rightarrow (c = +1)$ and $(a = +1) \rightarrow (c = -1)$ are both true.

In summary, this paper has proposed a definition for space-time counterfactuals. Applications are not limited to those in which the antecedent represents free choices, but the implications of this definition are most simply stated for those cases. Some of these implications are: if $\hat{\chi}$ represents a free choice, the counterfactual $\chi \square \rightarrow \psi$ is true whenever $\psi$ is true in all possible $\chi$-worlds which agree with the actual world everywhere outside the unambiguous future of the choice; $(\chi_1 \land \chi_2) \square \rightarrow \psi$ means that $\psi$ is true in all possible $(\chi_1 \land \chi_2)$-worlds which agree with the actual world everywhere outside the unambiguous future of either choice; and when the two choices are space-like separated, $(\chi_1 \lor \chi_2) \square \rightarrow \psi$ means that both $\chi_1 \square \rightarrow \psi$ and $\chi_2 \square \rightarrow \psi$ are true.
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