Making High $T_c$ Higher: A Theoretical Proposal

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There is considerable evidence that the highest $T_c$ obtainable in a copper-oxide plane is limited by the competition between two effects: On the one hand, as the concentration of doped-holes, $x$, is increased, the pairing scale, which is related to the properties of a doped Mott insulator, decreases. On the other hand, the superfluid density, which controls the stiffness of the system to phase fluctuations, vanishes as $x \to 0$, and increases with increasing $x$. Optimal $T_c$ is obtained at a crossover from a phase ordering dominated regime at small $x$ to a pairing dominated regime at large $x$. If this description is valid, then higher $T_c$'s can be obtained in an array of coupled planes with different doped hole concentrations, such that a high pairing scale is derived from the underdoped planes and a large phase stiffness from the optimally or overdoped ones.

This paper was prepared for a volume honoring the contributions of Zach Fisk to materials physics on the occasion of his 60th birthday. At the associated “Future of Materials Physics Workshop” there were many quips (none by Fisk) concerning the question of whether theoretical studies really are an essential part of this future. This paper is, in part, a response to those quips. If we believe that we have progressed in our theoretical understanding of the physics of high temperature superconductivity, then we should be able to use that understanding to provide guidance in the search for new, and possibly higher temperature superconductors. Given our incomplete understanding of the physics of highly correlated electronic systems, it is unreasonable to expect theory to point to specific materials, or make quantitative predictions of superconducting transition temperatures. However, I feel that there are aspects of the theory of high temperature superconductivity that are sufficiently crisp that they can provide qualitative information - what sort of materials are likely to be high temperature superconductors, and what sorts of modifications of the existing materials might lead to higher transition temperatures. In the final section of this paper, I make a specific proposal for raising the transition temperature of the cuprate superconductors which is based on the theoretical notion that $T_c$ in the underdoped materials is substantially suppressed below its mean-field value by phase fluctuations. At the very least, the success or failure of such suggestions provides a non-trivial test of the usefulness of some theoretical ideas.

I. BACKGROUND

Superconductivity in metals is the result of two distinct quantum phenomena, pairing and long-range phase coherence. In conventional homogeneous superconductors the phase stiffness is so great that these two phenomena occur simultaneously. On the other hand, in granular superconductors and Josephson junction arrays, pairing occurs at the bulk transition temperature of the constituent metal, while long-range phase coherence occurs, if at all, at a much lower temperature characteristic of the Josephson coupling between superconducting grains. High temperature superconductivity is hard to achieve, even in theory, because it requires that both scales be elevated simultaneously–yet they are usually incompatible. Consider, for example, the strong-coupling limit of the negative $U$ Hubbard or Holstein models. Pairs have a large binding energy but, typically, they condense at a very low temperature because of the large effective mass of a tightly bound pair - the effective mass is proportional to $|U|$ in the Hubbard model and is exponentially large in the Holstein model. (This also makes the system susceptible to other types of order, such as charge-density wave order, which compete with superconductivity.)

It was pointed out some time ago by Vic Emery and me that the small superfluid densities in the cuprate high temperature superconductors implies that the competition between pairing and condensation plays a significant role in the physics of these materials. The superfluid density in these materials is low, in part, because they are “doped Mott insulators” in the sense that the superfluid density is roughly proportional to the density of “doped holes,” $x$, rather than to the full density of holes, $1 + x$.

To make these considerations explicit, imagine we start from a knowledge of the ground-state properties of a superconductor and ask the question, “At what temperature would a given class of thermal fluctuations destroy the superconducting long-range order?” If the superfluid density is large, so condensation and pairing occur roughly simultaneously, as in the BCS theory, then $T_c \approx T_{pair}$ where $T_{pair}$ is the temperature at which most of the pairs fall apart; $T_{pair} \equiv \Delta_0/2$ is proportional to a typical value of the zero temperature superconducting gap, $\Delta_0$. If, on the other hand, the superfluid density is small, the effects of fluctuations of the phase of the superconducting order determine $T_c \sim T_\theta$ where, at least in
the case of layered superconductors, the phase ordering temperature, \( T_\theta \equiv \hbar^2 n_\theta / 2m^* \), can be deduced directly from the superfluid stiffness per layer, \( \hbar^2 n_\alpha c / m^* \) and the mean spacing between layers, \( a_\alpha \). In conventional metals, even in thin homogeneous films of conventional metals, \( T_\theta \) is much greater than \( T_c \), which implies that phase fluctuations hardly effect the value of \( T_c \).

However, in the cuprate superconductors, \( T_\theta \) is comparable to \( T_c \).

**A)** This conclusion can be reached on a priori theoretical grounds - in a doped antiferromagnet, we expect the effective bandwidth to be renormalized to be of order the antiferromagnetic exchange energy, \( J \), and the density of pairs, \( x / 2 \), to be determined by the density of doped holes. The resulting estimate \( T_\theta \sim Jx/2 \) is in the same ballpark as \( T_c \) for \( J \approx 1500K \) and \( x \sim 0.1 - 0.2 \).

**B)** More importantly, this conclusion is supported on empirical grounds, if we use measured values of the London penetration depth to deduce \( T_\theta \). Indeed, not only is \( T_\theta \) comparable in magnitude to \( T_c \), it often exhibits similar trends with doping and changes in other material properties, especially in the “underdoped region” where \( x \) is small; this important correlation was originally noticed by Umura et al \[3\] and is called the “Umura relation.” On the other hand, in “overdoped” materials, \( T_\theta \) overestimates \( T_c \) by as much as a factor of three or four. While this is still a number of order 1, it suggests that “optimal doping” (the value of \( x \) at which \( T_c \) reaches a maximum value) may be loosely viewed as a crossover from an underdoped regime in which \( T_c \) is limited by \( T_\theta \) to an overdoped regime where it is limited by \( T_{pair} \).

This same conclusion is reinforced by a variety of other empirical observations \[4\]. In the first place, there is evidence that in optimally and underdoped cuprates, fluctuation effects are observed \[4\] in a rather broad range of temperatures (±10% or greater) about \( T_c \), with a character and symmetry suggestive of the fluctuations in a classical XY model - this is the expected behavior of a system in which phase fluctuations dominate the physics in the neighborhood of \( T_c \) and is quite different than the fluctuation effects seen in conventional metallic superconductors. (Further evidence that superconducting fluctuations of some sort persist well above \( T_c \) is presented in Refs. \[1\] and \[2\]). At the same time, various measures of the zero temperature superconducting gap \[1\] \[4\] suggest that \( T_{pair} \) is a monotonically decreasing function of increasing \( x \) with a magnitude large compared to \( T_c \), especially in underdoped materials. Moreover, the same sort of measurements invariably find that, for small enough \( x \), the gap does not vanish at \( T_c \), but rather evolves into a smeared and broadened “pseudo-gap.” It is presently not unambiguous whether this large zero temperature gap and accompanying pseudo-gap can actually be interpreted as a superconducting (or pairing) gap \[4\]. However, if this interpretation is taken at face value, then the fact that \( T_{pair} \) varies in the opposite sense as \( T_c \) in the underdoped regime, and in the same sense in overdoped materials supports the identification of optimal doping as a crossover phenomenon.

**II. THEORETICAL CONSIDERATIONS**

The idea we pursue here is that we can raise the optimal \( T_c \) in a two-component system by optimizing the pairing correlations in one subsystem and the superfluid stiffness in the other \[16\] \[18\]. We have previously carried out extensive theoretical studies of this idea in model one-dimensional systems, and shown that it works \[10\] \[15\]. In particular, we demonstrated that strong superconducting correlations can be obtained by coupling a one dimensional electron gas with a large spin-gap (a “Luther-Emery liquid”) but small or vanishing superfluid density to a gapless one-dimensional electron gas (a “Luttinger liquid”) with little or no spin-gap and a small superconducting susceptibility, but with a large Drude weight. That is, it is possible to retain the best aspects of each separate subsystem in the coupled system.

The theory of the one dimensional electron gas is well developed, and both weak and strong coupling limits are well understood theoretically, so these previous model studies are a sound basis for developing an intuition about real systems. However, the theoretical analysis necessary to solve these models is somewhat technical, and aspects of the theory are special to one dimension. Here, we discuss the qualitative considerations that affect the ordering temperature of such a two-component system in higher dimensions.

Specifically, we will discuss a system which consists of alternating layers of "A" and "B" type. In the absence of interlayer coupling, the A-layers have a large zero-temperature superconducting gap, \( \Delta_A \), and consequently a high mean-field transition temperature \( T_{pair} \sim \Delta_A / 2 \), but a very small superfluid density so that their actual ordering temperature, \( T_A \ll \Delta_A / 2 \). Conversely, at \( T = 0 \), the B-layers have a very small (or vanishing) gap, but a large superfluid density (or, if not superconducting, a large Drude weight in the optical conductivity). We then ask the question: What happens when we turn on moderately weak interlayer couplings?

A. The proximity effect and the amplitude of the order parameter

One effect of single-particle tunnelling \( t_\perp \) between planes is to produce the usual proximity effect. What this means is that the electrons on plane A tend to leak onto plane B. This typically has the combined effect of decreasing the pair-field magnitude on plane A and increasing (or inducing) pairing on plane B. In the weak coupling
limit, where $\Delta_A \ll t_\perp$, this effect can be easily studied in the context of BCS mean-field theory. Here, one first diagonalizes the single-particle Hamiltonian, including $t_\perp$, and then evaluates the BCS gap equation using the exact single-particle eigenstates. Because the electron wave-functions are not localized on a single-plane, the amplitude for finding two-particles near each other in plane A (where they experience a relatively strong effective attraction) is reduced for those electrons that were originally localized on plane A, and increased for those that were originally on plane B.

If the Fermi surfaces of planes A and B coincide, so that the exact eigenstates have equal amplitudes on each plane, this effect is very significant, even for relatively small $t_\perp$ - roughly, each plane develops a gap whose magnitude is proportional to $\Delta_A/W$, where $W$ is the in-plane bandwidth.

If the Fermi surfaces of the two planes lie at substantially different locations in $k$ space, then the effect of small $t_\perp$ is small and can be computed in perturbation theory. Indeed, by explicit calculation it is possible to see that in this limit, $\Delta_A \rightarrow \Delta_A[1 + \mathcal{O}(t_\perp/W)^2]$ and $\Delta_B \rightarrow \Delta_B + \Delta_A \times \mathcal{O}(t_\perp/W)^2$. Put another way, we are considering here the case in which single-particle tunnelling processes between plains are high energy, virtual processes which need not be considered explicitly. Rather, the most important interactions for superconductivity are pair-tunnelling processes, in which a zero-momentum pair with energy near zero, tunnels between planes.

B. Effects of pair-tunnelling on pairing

While for weakly correlated systems, correlated pair-tunnelling interactions $J_\perp$ between layers are always weak, so the proximity effect dominates the interlayer physics, in strongly interacting systems, such pair-tunnelling interactions are induced at low energies, and need not be weak compared to the single-particle effects. Pair-tunnelling can affect the pairing scale (as discussed here) and can enhance phase ordering, as discussed below.

Pair tunnelling between plains can actually lead to an enhancement of the basic pairing scale. The driving force for this effect is that, by pairing, the electrons can more easily delocalize between neighboring planes, and so lower their zero point kinetic energy. This effect can, by itself, be a principle mechanism of pairing if, for either kinematic or dynamical reasons, single-particle tunnelling processes are greatly suppressed while pair-tunnelling remains strong. This same physical idea underlies the interlayer tunnelling mechanism of high temperature superconductivity, and is also closely related to various ideas of multi-band superconductivity. Its power as a pairing mechanism in the one-dimensional context is well established - it gives rise to what we have called the spin-gap proximity effect mechanism of pairing.

C. Effects of pair-tunnelling on phase ordering

Josephson coupling between planar superconductors increases the stiffness of the system to phase twists, and so raises the phase ordering temperature. To elucidate this effect, we consider the phase ordering in a set of coupled XY planes. Clearly, the more strongly coupled the planes, the higher the phase ordering temperatures.

For identical planes, this effect is more rapidly saturated than is commonly supposed. We addressed this issue in a recent paper by studying the properties of layered classical XY models with nearest-neighbor coupling $J$ in a layer and $J_\perp$ between layers - in terms of measurable quantities at $T = 0$: $J = (hc)^2/(16\pi e^2\lambda_{ab}^2)$ and $J_\perp/J \sim (\lambda_{ab}/\lambda_c)^2(\xi_{ab}/a_c)^2$, where $\lambda_{ab}$ and $\lambda_c$ are the in-plane and interplane components of the London penetration depth, $\xi_{ab}$ is the in-plane coherence length, and it is assumed that $a_c \geq \xi_c$. We solved these models using Monte-Carlo methods. For $J_\perp = 0$, $T_c = 0.9J$; it rises rapidly to $T_c = 1.1J$ for the relatively small value of $J_\perp = 0.01J$. (This is an impressive 20% enhancement from a 1% perturbation, and is the reason that two dimensional critical phenomena are so difficult to see in layered XY models.) However, $T_c$ only rises to $T_c = 1.3J$ for a further order of magnitude increase to $J_\perp = 0.1J$. Similarly, we modelled the case of multilayer materials by taking $J_\perp = 0.1J$ between the $n$ layers in a multiplet and $J_\perp = 0.01J$ between multilayers; here, for $n = 1, 2, 3, 4, \infty$, we found that $T_c = 1.09J, 1.20J, 1.24J, 1.26J$, and 1.32$J$, respectively.

While the general trend for $T_c$ to increase as identical layers are increasingly coupled together is encouraging, it is clear from the above that this will not lead to very large $T_c$ enhancements. To get large enhancements of the phase ordering temperature, it is necessary to increase the superfluid density per plane. This can be achieved in a two-component system, in which one of the components has a large phase stiffness. For a broad range of interlayer couplings, a layered XY model with a large coupling, $J_\text{large}$, in one set, and a small coupling, $J_\text{small}$, in the other, will have a phase ordering transition at a critical temperature $T_c \sim J_\text{large}$.

There is one subtlety here, that bears mentioning. At least in conventional BCS superconductors, at temperatures near $T_c$, the Josephson coupling between two planes is proportional to the product of the order parameters on the two planes. In expressing the physics of phase
ordering in terms of an XY model with temperature independent couplings, I have ignored all interference between the physics of pairing and that of phase ordering. While for temperatures well less than $\Delta_A/2$ it may be reasonable to neglect the temperature dependence of the superfluid stiffness in the A planes, this approximation is certainly not valid for the interplane and intra B-plane couplings at temperatures above the mean-field ordering temperature of the B-planes. To properly treat this full problem is beyond the scope of any theory I know how to do.

III. MATERIALS CONSIDERATIONS

Clearly, to make a two component system superconduct at high temperatures, it is better to use as constituents as good superconductors as possible. The new idea, here, is that a higher transition temperature than is possible with either constituent alone can be obtained by achieving a particularly high pairing scale in one constituent and a sufficiently large phase stiffness in the other. This is a very general strategy, and can be applied to the design of many materials. There are a few obvious, but important aspects of this strategy, indicated in the theoretical discussions above, which bear summarizing:

Firstly, in choosing the material from which the pairing scale derives, the first goal is to achieve a high pairing scale (superconducting gap), but for given gap magnitude, the higher the superfluid density, the better. Conversely, the material with the high superfluid density will be more effective if it has a substantial pair-field susceptibility at elevated temperatures. In short, we may not want to go to extremes.

Secondly, in coupling the neighboring planes, we want to suppress low energy single-particle tunnelling (which will tend to decrease the maximum pairing scale) but not at the expense of pair-tunnelling, which is necessary to couple the phase fluctuations in the two components. This can be the result of kinematics (i.e. a mismatch in Fermi momenta in the two systems) or dynamics. (See, aslo, \[24\])

A. A Model System

An interesting study has been carried out on a model system by Merchant et al, in which the crossover from phase ordering to pairing can be observed directly. The starting material for this study is a film of granular lead with sufficiently small lead coverage that, resistively, it is an insulator at low temperatures. However, transverse tunnelling into this starting film reveals the presence, locally, of a well developed superconducting gap below the bulk $T_c$ of lead; the insulating behavior is unambiguously the result of quantum and thermal phase fluctuations in this film, and is reflective of the fact that the superfluid density (in this case, determined by the typical Josephson coupling between grains) is small.

From this starting point, a sequence of films were made by adding increasing amounts of silver. Silver is thought to be deposited as a homogeneous covering film, even at very low coverages. Two effects of increasing silver coverage are observed. Firstly, the silver enhances the superfluid stiffness, resulting, at small coverage, in a resistive $T_c$ (defined as the point at which the resistance drops to a suitably small fraction of its normal state value) which is an increasing function of the amount of silver deposited. However, the proximity effect results in a monotonic decrease of the superconducting gap observed in tunnelling, which is slow at first, but then more rapid with increasing silver coverage. Eventually, this results in a reversal of the trend observed in the resistivity, so that $T_c$ is found to reach a maximum (at an “optimum silver coverage”) and then drop rapidly with increasing silver coverage.

There are many subtleties of these experiments that are still not well understood. For instance, although in the “overdoped” regime, the superconducting transition is sharp and the superconducting state itself well defined, on the underdoped side the transition is rounded and it appears, for a range of silver coverage, that a zero resistance state is never achieved, even in the limit $T \to 0$. However, the basic trends observed in these experiments serve as a proof in a real material that the basic strategy for $T_c$ enhancement proposed here can work!

B. Considerations specific to the cuprates

In a crystal with one layer per unit cell, unless the crystalline symmetry is spontaneously broken, each layer will have the same doped hole concentration. This remains true in a bilayer material which possesses a reflection plane or screw symmetry which links the planes. This situation applies, to the best of my knowledge, to all the bilayer cuprate superconductors that have been studied to date. One could imagine, however, bilayer materials of lower symmetry designed so that the doped hole concentration in each layer can be varied separately. All trilayer and four-layer materials have two crystalllographically inequivalent copper-oxide planes, which should therefore be expected, a priori, to have different doped hole concentrations. Thus, there already exist materials that realize, to some extent, the scenario envisaged here.

This simple observation leads to the suggestion that the enhanced $T_c$’s observed in three and four layer materials, for which various other mechanisms have already been suggested, may be explained by the present ideas. To test this idea, one should examine, with various local probes, the differences in the doped hole concentrations in the different layers. One striking observation is...
that in some of these materials, notably in the three and four layer materials \[ \text{HgBa}_2\text{Ca}_{n-1}\text{Cu}_n\text{O}_{2n+2+y} \text{ and } \text{Tl}_2\text{Ba}_2\text{Ca}_{n-1}\text{Cu}_n\text{O}_{2n+4+y} \text{ with } n = 3 \text{ and } 4 \], \( T_c \) is remarkably pressure dependent. A potentially testable prediction based on the present analysis is that this dependence is a consequence of a pressure dependent charge transfer between planes \[ \text{[22]} \]. If so, it should lead to a charge distribution which looks more and more optimal (in the sense described below) as \( T_c \) rises.

What then is the optimal distribution of doped holes between two coupled layers. From the observation that the superfluid density in overdoped cuprates does not increase with increasing hole concentration, although the gap size decreases markedly, we deduce that the “high superfluid density layer” should be optimally, or at most very slightly overdoped. On the otherhand, the gap scale appears to rise rapidly with underdoping, at least according to some measures. Thus, the “large pairing scale layer” should ideally be substantially underdoped. However, the empirical trends are less clear with extreme underdoping. For one thing, in this range, screening becomes very poor and the effects of disorder appear to be much enhanced. Thus, at least at first, extreme underdoping should probably be avoided as well.

This work is an outgrowth of collaborative research with many colleagues, but especially with V. J. Emery, O. Zachar, E. W. Carlson, D. Orgad, and E. Fradkin.

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[1] J. G. Bednorz and K. A. Müller, Z. Phys. B 64, 189 (1986).
[2] V. J. Emery and S. A. Kivelson, Nature 374, 434 (1995).
[3] In one way or another, this point has been appreciated implicitly in many theoretical and experimental studies. The earliest of which I am aware are S.A.Kivelson, D.Rokhsar, and J.P.Sethna, Phys. Rev. B35, 8865 (1987), D.Rokhsar and S.A.Kivelson, Phys. Rev. Lett.61, 2376 (1988), and G.Kotliar and Liu, Phys. Rev. B38, 5142-5145 (1988). The same basic idea that pairing derives from the Mott insulating state, but that the superfluid density rises continuously from zero with doping, is implicit in the earliest work of Anderson on the subject, Science 235, 1169 (1987), but without however, the notion of a gap associated with pairing correlations.
[4] Y. J. Uemura et al, Phys. Rev. Lett.62, 2317 (1989); 66, 2665 (1991).
[5] For a recent discussion of reasons to believe that the gap and pseudo-gap in underdoped materials are associated with a competing order, rather than with superconducting pairing, see Ref. [2]. Still more recently, dramatic STM studies have uncovered spatial inhomogeneities which are suggestive of nano-scale phase coexistence of a superconducting and a non-superconducting (or more weakly-superconducting) phase in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ - see Refs. \[ \text{[8]} \] and \[ \text{[9]} \] and references therein. This observation complicates the identification of a single, average gap measured in ARPES or lower resolution STM as being the superconducting gap. However, it also means that, within a single plane or bilayer, some of the two-component physics envisaged in this paper may already be operating - the “non-superconducting” regions are also the regions of largest gap.
[6] S. Chakravarty, R. B. Laughlin, D. K. Morr, and C. Nayak, Phys. Rev. B63, 94503 (2001).
[7] C. Howald, I. Dasgupta, T. Saha-Dasgupta, O. Jepsen, O.K. Andersen, cond-mat/0012051.
[8] K. M. Lang, V. Madhavan, J. E. Hoffman, E. W. Hudson, H. Eisaki, S. Uchida, and J. C. Davis, unpublished.
[9] D-H. Lee, private communication.
[10] See, for example, D.A.Bonn et al, Czech. J. Phys. 46, 3195 (1996), J. Corson et al Nature 398, 221 (1999), C. Meingast et al, Phys. Rev. Lett.86, 1606 (2001), and V. Pasler, Phys. Rev. Lett.81, 1094 (1998).
[11] I. Iguchi, T. Yamaguchi, and A. Sugimoto, Nature, 412, 420 (2001).
[12] Z. A. Xu, N. P. Ong, Y. Wang, T. Kageshita, and S. Uchida, Nature 406, 486 (2000), and Y. Wang, Z. A. Xu, and N. P. Ong, unpublished.
[13] Early evidence of this from ARPES was presented by J. Harris et al, Nature (London) 382, 51 (1996) and D. S. Marshall et al, Phys. Rev. Lett.76, 4841 (1996).
[14] A recent study of this issue using scanning tunneling spectroscopy appears in M. Kugler et al, Phys. Rev. Lett.86, 4911 (2001).
[15] For a recent overview of the evidence concerning the importance of phase fluctuations in the cuprate high temperature superconductors, see the insightful article by J. Orenstein and A. J. Millis, Science 288, 468-74 (2000).
[16] V. J. Emery, S. A. Kivelson, and O. Zachar, Phys. Rev. B56, 6120-6147 (1997).
[17] V. J. Emery, S. A. Kivelson, and O. Zachar, Phys. Rev. B59, 15641 (1999).
[18] T. H. Geballe and B. Y. Moyzhes, Physica C 347-348, 1821 (2000). For a further discussion, see [19].
[19] V. Oganesyan et al., unpublished.
[20] H. Suhl, B.T. Matthias, L.R. Walker, Phys. Rev. Lett.3, 552 (1959).
[21] D.H. Lee and G.T. Zimanyi, Phys. Rev. B40, 9404 (1989).
[22] E.W.Carlson et al, Phys. Rev. Lett.83, 612 (2000).
[23] S. Chakravarty et al, Science 261, 337 (1993).
[24] S. Chakravarty, Eur. Phys. Jour. B 5, 337-343 (1998).
[25] L. M. Merchant et al, Phys. Rev. B63, 134508 (2001).
[26] M. Di Stasio, K. A. Müller, and L. Pietronero, Phys. Rev. Lett.64, 2827 (1990).
[27] See, for example, Refs. [24, 28] and [29].
[28] A. Leggett, Phys. Rev. Lett.83, 392-3 (1999).
[29] E. Pavarini, I. Dasgupta, T. Saha-Dasgupta, O. Jepsen, O.K. Andersen, cond-mat/0012051.
[30] Pressure dependence of \( T_c \) of \( \text{HgBa}_2\text{Ca}_{n-1}\text{Cu}_n\text{O}_{2n+2+y} \) can be found in D. T. Jover et al, Phys. Rev. B54, 4265 (1996).
[31] Pressure dependence of \( T_c \) of \( \text{Tl}_2\text{Ba}_2\text{Ca}_{n-1}\text{Cu}_n\text{O}_{2n+4+y} \) can be found in D. T. Jover et al, Phys. Rev. B54, 10175.
The idea that the pressure dependence of $T_c$ could be a consequence of a pressure dependent charge-transfer between inequivalent planes is already present in [31]. However, in this work, $T_c$ was considered to result from purely single-plane considerations, and to be governed by the $T_c$ of whichever plane, left to its own devices, would have the higher transition temperature. The idea that interplane interactions in an optimally doped trilayer material, in this case Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_y$, can lead to both a higher superfluid density and a higher pairing scale than is possible from single layer considerations is implicit in the recent work of Feng et al [33].

[32] The idea that the pressure dependence of $T_c$ could be a consequence of a pressure dependent charge-transfer between inequivalent planes is already present in [31]. However, in this work, $T_c$ was considered to result from purely single-plane considerations, and to be governed by the $T_c$ of whichever plane, left to its own devices, would have the higher transition temperature. The idea that interplane interactions in an optimally doped trilayer material, in this case Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_y$, can lead to both a higher superfluid density and a higher pairing scale than is possible from single layer considerations is implicit in the recent work of Feng et al [33].

[33] D. L. Feng, A. Damascelli, K. M. Shen, N. Motoyama, D. H. Lu, H. Eisaki, K. Shimizu, J.-I. Shimoyama, K. Kishio, N. Kaneko, M. Greven, G. D. Gu, X. J. Zhou, C. Kim, F. Ronning, N. P. Armitage, Z.-X. Shen, cond-mat/0108386.