Primordial Isocurvature Perturbations: Testing the Adiabaticity of the CMB Anisotropy*

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Prospects for testing the adiabaticity of the primordial cosmological perturbations using MAP and PLANCK are evaluated. The most general cosmological perturbation in a universe with just baryons, photons, neutrinos, and a cold dark matter (CDM) component is described. In addition to the familiar adiabatic mode, there are four nonsingular isocurvature modes: a baryon isocurvature mode, a CDM isocurvature mode, a neutrino density isocurvature mode, and a neutrino velocity isocurvature mode. The most general perturbation is described by a $5 \times 5$, positive-definite, symmetric, matrix-valued function of wave number whose off-diagonal elements represent the correlations between the above mentioned modes. We found that when three modes and their correlations are admitted, the fractional uncertainties in the cosmological parameters and amplitudes of the isocurvature modes and their correlations become of order unity. These degeneracies, however, can be broken with the polarization information provided by PLANCK, reducing the uncertainties to below the ten percent level. Polarization is thus crucial to testing the adiabaticity of the primordial fluctuations.

One of the fundamental challenges of the new cosmology is to establish the underlying character of the primordial perturbations. Long before inflation was proposed, it was pointed out that in their simplest form the primordial perturbations would be adiabatic and Gaussian. It was also pointed out that a scale-free power spectrum, the so-called Harrison-Zeldovich-Peebles power spectrum [1], was an aesthetically preferred form and also seemed to account for the then available data quite well.

The subject of this contribution is to propose a test of the first of these two hypotheses: that the primordial perturbations were adiabatic in character. In this context adiabatic means that the primordial stress-energy of the universe was governed by a single, spatially uniform equation of state — in other words, that on surfaces of constant temperature the densities of the various components (e.g. baryons, CDM, neutrinos, etc.) are uniform and that these components share a common velocity field.

In order to test the hypothesis of adiabaticity, it is necessary to contemplate models with non-adiabatic perturbations and then place constraints on the possible amplitudes of these non-adiabatic modes by comparing with the data. Most of the work on how to interpret the future CMB data has assumed a simple one-field inflationary model, for which the fluctuations are adiabatic absent new physics, and a simple power law for the power spectrum [2]. A host of cosmological parameters (such as $H_0, \Omega_\Lambda, \Omega_b, \Omega_k, n_s$ and $\tau_{\text{reion}}$) are allowed to vary,

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and Bayesian statistical analysis is applied to determine the values of these undetermined cosmological parameters and their corresponding uncertainties. In many of the papers of this genre that discuss how to analyse the future CMB data, the primary emphasis is placed on how well one will be able to measure the various undetermined cosmological parameters and very little discussion is devoted to testing the underlying assumptions. Rather it is assumed that should the real data not be described by a template taken from the assumed class, this error would naturally become self-evident.

Perhaps more interesting and of more profound significance to discovering how the perturbations from homogeneity and isotropy were first generated in the early universe is to determine the fundamental character of the primordial perturbations. Some examples of questions that one would wish to resolve include the following: Were the primordial perturbations entirely adiabatic in character, or were isocurvature modes excited as well? Were the primordial perturbations exactly Gaussian? Were they ‘scalar,’ or were ‘vector’ and ‘tensor’ modes excited as well? Is an ‘acausal’ theory such as inflation required, or could the perturbations have been imprinted as a continual process? Were ‘decaying’ modes excited as well?

To answer these questions will require new approaches to analysing the forthcoming CMB data. The work reported in this proceeding constitutes a modest step in this direction. In our approach we make two simplifying assumptions: First, we assume a universe with no new physics: a universe with only photons, baryons (and their associated electrons), neutrinos, and a cold dark matter component. Second, we assume that the perturbations were ‘primordial’, meaning that they were initially excited well before recombination so that any decaying modes have had ample opportunity to die away. The first hypothesis excludes, for example, theories of structure formation based on field ordering such as cosmic strings where the dynamics of some scalar or other order parameter field evolves thus perturbing the other components [3, 4].

Bucher et al. [5] analysed the most general cosmological perturbation possible under these assumptions using synchronous gauge and found in addition to the two gauge modes, five regular modes and two decaying modes for each wavenumber $k$:

1. An adiabatic growing mode.
2. An adiabatic decaying mode.
3. A baryon isocurvature mode.
4. A baryon velocity decaying mode.
5. A CDM isocurvature mode.
6. A neutrino density isocurvature mode.
7. A neutrino velocity isocurvature mode.

The leading order terms for all perturbation variables in the series expansion solution, which is valid at early times and on superhorizon scales, is presented in Table I for the regular modes listed above. Here $R_\gamma$ and $R_\nu$ represent the fractional contribution of photons and neutrinos to the total density at early times deep within the radiation dominated epoch,
respectively, while $\Omega_{b,0}$ and $\Omega_{c,0}$ represent the present day fractional contribution to the total density of baryons and cold dark matter, respectively. The first four modes have already been discussed extensively in the literature. The observational consequences of the neutrino density and neutrino velocity modes, although implicit in the work of Rebhan and Schwarz [8] and of Challinor and Lasenby [7], however, have only recently been explored [5].

We next provide a brief qualitative discussion of these modes. For more details the reader is referred to [5]. For the adiabatic mode, as previously mentioned, all components obey a common equation of state and share a common velocity field. The ratios of baryons to photons, CDM to photons, and neutrinos to photons do not vary spatially. For the adiabatic ‘decaying’ mode the perturbation becomes singular as $t \to 0$. In the CDM [8] and baryon [9–11] isocurvature modes, the ratios of CDM to photons and of baryons to photons do not vary spatially. Initially, far into the radiation dominated era, these components, idealized to be non-relativistic particles whose density scales as $\rho \sim a^{-4}$ for radiation) contribute negligibly to the total stress-energy. However, as the universe becomes matter dominated, these variations translate into variations in the equation of state that generate metric perturbations which in turn generate further perturbations in the densities and velocities of all the components. For the baryon velocity ‘decaying’ mode, the velocity of the baryons relative to the photons is rapidly damped by the Thomson scattering of photons off the baryons.

For the neutrino isocurvature modes the neutrino sector is perturbed relative to the radiation sector (i.e., the photons and other components that are strongly coupled to them at early times through Thomson scattering). The perturbations are initially of equal magnitude but in opposite directions, so that the total energy density and momentum density vanishes.
FIG. 1. CMB spectra for adiabatic and isocurvature modes with cosmological parameters $\Omega_b = 0.06$, $\Omega_\Lambda = 0.69$, $\Omega_{cdm} = 0.25$, $h = 0.65$, $\tau_{reion} = 0.1$ and $n_s = 1$.

Let us first discuss the neutrino density mode. If neutrinos and photons evolved identically, no metric perturbation would subsequently result. But as a mode enters the horizon, the neutrinos free stream while the photons behave as a perfect fluid because of Thomson scattering off electrons. This differential behavior leads to perturbations in the total stress-energy, which generate metric perturbations, which in turn generate perturbations in all components. The case of the neutrino velocity mode is similar. Initially, the momentum densities cancel. However, because of the differential dynamics upon horizon crossing, stress-energy perturbations arise, which then source metric perturbations, which in turn generate perturbations in the other components. One may have suspected that this mode would be singular because the ‘velocity’ mode in the adiabatic sector is decaying and thus singular. But because of the cancellation initially this too is a nonsingular mode.

In Figure 1 the CMB anisotropies predicted for the various modes are indicated. The CDM and baryon isocurvature modes predict CMB spectra of the same shape, to within a fraction of a percent, so only the baryon isocurvature mode is shown and studied below. Of the isocurvature modes, the baryon isocurvature and CDM isocurvature modes have greatly suppressed power on small scales relative to large scales while the neutrino density isocurvature mode exhibits a rise at $\ell \approx 75$ leading to a plateau before the first Doppler peak. However, interestingly, the neutrino velocity isocurvature mode lacks these features, rather having the same qualitative behavior as the adiabatic growing mode.

Now that we have described the five regular modes, we turn to describing the most general primordial cosmological perturbation in a universe with the matter content given above. We momentarily assume Gaussianity, an assumption that we will soon be able to relax somewhat. For a single adiabatic mode describing a Gaussian random process that is homogeneous and isotropic, the statistical properties are completely described by the power spectrum $P(k)$, a real valued function of wavenumber. In the case here of five modes, whose amplitudes we indicate as $A_a(k)$, ($a = 1, ..., 5$), the power spectrum generalizes to a $5 \times 5$, positive-definite, real, symmetric, matrix-valued function of the wavenumber $P_{ab}(k)$ where
Table II. Percentage errors on the cosmological parameters and the amplitudes of isocurvature auto-correlation and cross-correlation modes as measured by MAP and PLANCK. Here ‘T’ signifies that only temperature information is used while ‘TP’ signifies that temperature, polarization and cross-correlation information is used.

\[
\langle A_n(k) A_b(-k') \rangle = P_{ab}(k) \delta^3(k-k').
\]

If the fluctuations are non-Gaussian, there are nontrivial higher-order correlations to consider as well, but when observables that are quadratic in the small perturbations are considered, \(P_{ab}(k)\) suffices to characterize their expectation values completely.

The off-diagonal elements of \(P_{ab}(k)\) represent correlations between the various modes. In multi-field inflationary models these generically occur. In an inflationary model with five or more fields (or a single inflaton with the equivalent number of components), the five fields control the amplitudes of five principal components comprising linear combinations of the five modes. But there is no reason why the directions of these principal components should align with those given here for the individual modes. Therefore, correlations generically occur.

Because CMB data of the quality required to place reasonably stringent constraints on the amplitudes of such modes (or to detect their presence) is not yet available, it is not possible to anticipate the broad range of possibilities for what one might discover in the new data. But a rough estimate of the ability of new experiments (in particular MAP and PLANCK) to constrain or detect these modes may be obtained by assuming an underlying cosmological model with only an adiabatic mode to produce the CMB sky (with cosmic variance, of course) and an anticipated error model for the detector noise of these experiments. We then determine the expected likelihood function approximated to
quadratic order about its maximum and use this information to compute the errors on the various mode amplitudes and the cosmological parameters when various combinations of isocurvature modes and their correlations are admitted. A more detailed account of this work is given in [15]. Some related work can be found in [16].

Our results are summarized in Table II. Both the MAP and PLANCK experiments are included, with and without polarization information, and with just the adiabatic mode and with the three isocurvature modes and their correlations. The amplitudes of the isocurvature modes are normalized to give the same total CMB power from $\ell = 2$ through $\ell = 1500$ as the adiabatic mode. In all cases a scale-invariant spectrum (i.e., $P(k) \sim k^{-3}$) was assumed. Tensor modes have not been included. These would only increase the computed uncertainties. With just the adiabatic mode allowed, the errors on the cosmological parameters are as indicated elsewhere in the literature. With just one extra isocurvature mode and its correlation with the adiabatic mode added, these errors increase modestly, in all cases by less than a factor of two. But when three isocurvature modes and their correlations are allowed, the uncertainties become enormous. Errors of order unity imply a breakdown of the quadratic approximation. The above results demonstrate that polarization information will play a crucial role in testing adiabaticity.

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