Shape optimization of SG6043 airfoil for small wind turbine blades

K S Abdelwahed and A I Abd El-Rahman
The Department of Mechanical Power Engineering, Faculty of Engineering, Cairo University Rd., Giza 12613, Egypt
E-mail: karim.sa.ahmed@cu.edu.eg

Abstract. The geometry of airfoil sections in small wind turbine blades affect the boundary layer separation, promote the flow transition to turbulence and consequently influence the performance of many types of small wind turbines. However, few optimization results are available for typical small wind turbine blades. Here, we report a particular shape optimization study of the standard SG6043 airfoil that applies the genetic algorithm along with the low-Re XFOIL solver. The numerical model is first validated against available experimental values for the SG6043 airfoil at a Reynolds number of 100,000. The predicted lift and drag profiles are qualitatively found in good agreement with the experimental values, particularly in the pre-stall regime, provided that the XFOIL’s transition exponent is adjusted to 11. The simulation is then extended to a lower Reynolds number of 60,000 that is consistent with an average tip-speed-ratio $\lambda = 4.0$. Four specific scenarios seeking different objective functions are proposed for optimization. While the first two scenarios focus on typical optimum configurations at which the maximum lift-to-drag ratios are realized, the objective function of the latter scenarios considers the maximum arithmetic-mean over multiple consecutive degrees of the corresponding angle of attacks. The preliminary results show remarkable improvement in the range from 24% to 10% in the predicted lift-to-drag ratios in comparison with the original configuration at the expense of the corresponding airfoils’ stiffness. This is followed by an elaboration on the entire blade design along with the specification of the resulting power coefficient and annual energy production.

1. Introduction
The increasing global demand of renewable energies call for innovative design and optimization methodologies of wind turbine industry that provide suitable solutions for remote areas seeking sustainable energy generation apart from the grid. Because of that, horizontal-axis small wind turbines (SWTs) with a power capacity in the range from 1 to 50 kW are becoming promising alternative of energy generation for several residential and mini-grid applications. In contrary with large wind-turbine (LWT) applications, SWTs are simple turbo-machines that can be easily installed and controlled to operate under appropriate starting characteristics. Nevertheless, to date, horizontal-axis small wind turbines are still lacking effective improvements in the airfoil shapes, blade materials, and aerodynamic and structural designs that lead to more efficient turbine operations.

Among the many available optimization methods [1], the genetic algorithm has recently received remarkable attention by several researchers [2, 3, 4, 5, 6] due to its capability of achieving the required global optimum without referring to the relationships between the...
optimization variables and the objective parameters in the process of many single and multi-objective optimization studies. Ribeiro and his collaborators [3] were able to develop an airfoil optimization technique for large wind turbines (LWTs) that coupled the genetic algorithm with a computational fluid dynamics (CFD) model to evaluate the aerodynamic performance of the optimized airfoil. Their results showed limited agreement for the lift-to-drag ratio with the corresponding experimental values beyond the stall limit, partly because of the incorporation of the one-equation Spalart–Allmaras turbulence model. In a similar fashion, He and Agarwal [4] reported a shape optimization study of NREL S809 airfoil, typical of LWTs, through the application of a multi-objective genetic algorithm within the framework of the finite volume solver of ANSYS FLUENT. They however replaced the one-equation turbulence model with the SST $k$-$\omega$ model and their results showed significant improvement in both lift coefficient and lift-to-drag ratio of the optimized airfoil with respect to the original airfoil. Recently, Hansen [1] developed an optimization method for LWT blades that is based on the class-shape transformation technique for the airfoil parametrization along with an adjusted version of the panel code XFOIL.

A different approach was presented by Singh et al. [7] who carried out a parametric study that considered few existing low-Reynolds-number airfoils while seeking the development of more appropriate airfoil sections for small wind turbine (SWT) blades. Particle image velocimetry (PIV) measurements was further conducted to visualize the possible boundary-layer separation at different Reynolds numbers. It is worth noting that no separation was captured in their study up to an angle of attack $\alpha = 8$ when operating at $Re = 56,000$ under a turbulence intensity of 1%. Moreover, their XFOIL-predicted maximum lift-coefficients at Reynolds numbers in the range from 38,000 to 205,000 were found in good agreement with wind-tunnel measurements. Other documented research efforts [8, 9] focused on the application of the genetic algorithm along with the blade element momentum (BEM) theory in their multi-dimensional optimization of SWT blades. They were able to solve for the optimum chord and twist-angle distributions while respecting the starting characteristics, structural requirements and concerns over the aerodynamic impact on the turbine performance of the blade materials. Building on the existing efforts, Ram et al. [6] numerically investigated the performance of few optimized SG6043 airfoils during operation under soiled conditions using the NREL-developed HARP Opt code. A corresponding annual energy production (AEP) of $4.787 \times 10^4$ kWh was reported for their 20-kW wind turbine with a cut-in speed of 2 m/s and a rated wind speed of 9 m/s.

The objective of the present research work is to conduct a particular optimization study that applies the meta-heuristic genetic algorithm and employs the low-Re XFOIL solver to provide accurate prediction of their resulting aerodynamics performance. The SG6043 airfoil is typically used for small wind turbine blades operating at low Reynolds numbers $Re$, and is thus considered here as the reference base profile. Bézier curves are used for airfoil parametrization. Shape optimization of the SG6043 airfoil proceeds through accurate specification of the optimum lift-to-drag $L/D$ ratios and the corresponding angles of attack $\alpha$, as detailed in the next sections. This is followed by the optimum design of the entire turbine blades using the BEM theory. The present work demonstrates remarkable improvement in the overall turbine performance and the estimated annual energy production (AEP).

2. Optimization Method

2.1. Airfoil parametrization and genetic algorithm

In the present work, the airfoil geometry is mathematically described using Bézier curves [10] that is usually expressed as:

$$B(t) = \sum_{i=1}^{n} \frac{n!}{i!(n-1)!} t^i (1-t)^{n-i} P_i$$  \hspace{1cm} (1)
The chord length, \( c \)

0.2 0.4 0.6 0.8 1

-0.2 0
0.2 0.4

\( y_3 \) \( y_4 \) \( y_5 \) \( y_6 \) \( y_7 \) \( y_8 \) \( y_{10} \) \( y_9 \) \( y_1 \) \( y_2 \)

\( B(t) \) represents a position vector of any point on the curve, wherein \( t \) is a parameter whose value ranges from 0 to 1. Here, \( n \) is the degree of Bézier curves, corresponding to \( n + 1 \) control points, while \( P_i \) refers to a certain control point \( i \) with random \( x \) and \( y \) coordinates. These curves allow us to mathematically express the airfoil surface geometries as a function of few control points, coordinates of which are iteratively updated to form new shapes. In the present work, the \( x \)-coordinates of the control points remain unchanged while the \( y \)-coordinates are allowed to vary except for those points located at both leading and trailing edges. This helps reduce the number of variables and thus consume less computational effort. The \( x \)-coordinates of the control points are chosen at chord distances 0, 0.01, \( \cdots \), 0.9, 1 with equally-spaced mid-points, following the polynomial order of Bézier curves.

Figure 1 illustrates the parametrization of the SG6043 airfoil using two separate 6th-order curves for both sides. The open squares represent the free control points at the upper surface while the open circles refer to the corresponding points at the lower surface, and the full circles indicate the fixed control points at both the leading and trailing edges. In this particular illustration, the airfoil geometry is drawn such that its angle of attack \( \alpha = 0 \). This produces 10 control points (\( y_1 \) to \( y_{10} \)). The number of surface points is chosen through the specification of the number of divisions of the parameter \( t \). For instance, 200 divisions are assumed for each side of the airfoil. In the present analysis, the maximum deviations between the original and the parametrized upper and lower surfaces are found less than 0.2\% of the chord length, which closely match the allowable limits [11].

This step is then followed by the generation of the initial population through the re-position of the respective control points by displacing them in the vertical direction with random distances falling within specified limits. Additionally, each individual within the population is randomly rotated by different degrees provided that the resulting angle of attack \( 0 \leq \alpha \leq 10 \). The genetic algorithm begins with random generation of an initial population of 100 individuals. Each individual is represented by a set of variables, as shown in Fig. 1, which are sufficient to define the shapes and the angles of attack of the designated airfoils. The genetic algorithm keeps on improving the fitness values in successive generations while retaining their diversity in the process of searching for their global optimum by means of specific operators (namely, the selection, crossover, mutation and elitism), whose details and effect on the optimization process are found elsewhere [12, 13, 14].
2.2. XFOIL computational model

The XFOIL solver [15] is a panel-based viscous/inviscid computational scheme whose inviscid analysis is based on linear-vorticity streamfunction formulation of the involved potential flow equations. Its laminar/turbulent viscous formulation typically consists of the compressible integral momentum and shape parameter equations with the replacement of the shear stress formula by an expression for the spatial growth rates of the amplitude $N$ of the largest Tollmien-Schlichting wave, as determined from the solution of the linear viscous instability ‘Orr-Sommerfeld’ equation. It is not until the wave amplitude $N$ reaches an adjustable critical value $N_{\text{crit}}$ that transition to turbulence is allowed to take place [16]. Such critical limit is representative of the disturbance levels, such that the lower the disturbance levels, the larger the laminar separation bubbles and the higher the drag.

To demonstrate the usefulness of the XFOIL computational tool, particularly at low-Re flows, the lift and drag characteristics are first predicted for the SG6043 airfoil at $Re = 99,681$ and compared with the reported test data of Lyon et al. [17] at a turbulence intensity of 0.1%. The airfoil surface is divided into 240 panels and the simulation proceeds for 300 iterations. In the present analysis, it is found that a critical value $N_{\text{crit}}$ of 11 yields sufficient overall agreement with the measured values, as indicated in Fig. 2. The lift-to-drag ratio is shown to rise with an increasing rate to its maximum value in the pre-stall region while a smooth decay is captured afterwards. Besides, the measured peak value of the $L/D$ is noticed to be over-predicted by 5% and slightly shifted to lower angle of attack, thus, indicating earlier separation of the predicted laminar boundary layer.

2.3. Shape optimization of SG6043 airfoil

In the present work, four different scenarios are proposed for the optimization process to achieve the optimum airfoil shape, giving the SG6043 airfoil geometry as the base profile. The optimization process is carried out at $Re = 59,315$, as estimated for a 0.8-m-length wind turbine blades running under an average wind speed of 4 m/s. In all four computational runs, the optimization process is constrained with a positive non-zero airfoil thickness while a population size of 100 is equally applied. The angle of attack is allowed to vary in the range from $0^\circ$ to $10^\circ$. The process is terminated when the variation in the optimum lift-to-drag ratio in the last 50 generations falls below $10^{-4}$. The optimization parameters are summarized in Table 1.

The first two scenarios share the same objective function (maximum $L/D$ ratio), but having
Table 1. Optimization parameters of the four scenarios.

| Scenarios | A   | B   | C   | D   |
|-----------|-----|-----|-----|-----|
| Number of generations | 910 | 662 | 465 | 150 |
| Number of control points | 7   | 9   | 7   | 7   |
| Top boundary limits (±%)  | 50  | 50  | 50  | 50  |
| Bottom boundary limits (±%) | 50  | 80  | 80  | 80  |
| Crossover index, \( \eta_c \) | 10  | 5   | 5   | 10  |

different number of generations and control points beside different limits for both top and bottom surfaces. Furthermore, the effect on the objective value of varying both crossover and mutation distributive indices is investigated. Outputs from these two runs allow us to explore the influence of these specific optimization parameters and choose the most appropriate ones in further simulations. For instance, the present simulation runs reveal insignificant change in the predicted \( L/D \) ratio with the increase of the number of control points from 7 to 9. Crossover and mutation indices of 5 – 10 and 20, respectively, are found appropriate in this particular optimization study and are therefore pursued in the latter runs, as indicated in Table 1. The probabilities of the crossover and mutation are 0.9 and 0.1, respectively, whereas the elitism ratio is 0.1. Although the next section shall elaborate on the optimization results for all cases, it is worth noting here that the first two scenarios are unexpectedly put at a serious disadvantage because of the impractical sharp peaks in their \( L/D \) profiles, as shown in Fig. 4.

Therefore, differently from the first two approaches, the objective functions in scenarios \((C \text{ and } D)\) are here extended to account for the arithmetic-mean values of the predicted lift-to-drag ratios considering every two (Scenario \( C \)) and every five (Scenario \( D \)) consecutive degrees of the respective angles of attack, as described mathematically in Eqn. 2. This new algorithm allows us to search for those shapes at which the maximum corresponding values of \((L/D)_{\text{max}}\) are achieved within specific bins of the respective angles of attack rather than at particular points. To our knowledge, this has not been reported before. The following proposed fitness value \((L/D)_{\alpha}\) is developed such that \( \xi \) corresponds to the bin width, while \( M \) refers to the number of divisions in each bin. For instance, \( \xi = 2 \) and 5, whereas \( M = 5 \) and 26 for scenarios \( C \) and \( D \), respectively.

\[
(L/D)_{\alpha} = \frac{1}{M} \sum_{i=\alpha-\xi/2}^{\alpha+\xi/2} (L/D)_{i}
\]

The flowchart of the optimization process is presented in Fig. 3. The reference airfoil SG6043 is first parametrized using Bèzier curves to create the limits of the search space. This is followed by random generation of the initial population within the specified limits using the MATLAB’s genetic algorithm. In the present work, a MATLAB program is particularly developed for this purpose. The lift-to-drag ratio that represents the objective ‘fitness’ value to be optimized is calculated for each individual in the population using XFOIL. To further proceed, the predicted values are then fed back to the genetic algorithm to create a new generation of airfoils with improved aerodynamics shapes. This process keeps on going until the algorithm fails to improve the fitness of the best individual in new generations further.

3. Results and discussion
The variations of the XFOIL-predicted lift-to-drag ratios as a function of the angle of attack are presented in Fig. 4 for all four scenarios. The present numerical results indicate sharp
The angle of attack, $\alpha$
The lift-to-drag ratio, $L/D$

**Figure 3.** A flowchart illustrating steps of the optimization process.

**Figure 4.** The variations of the predicted $L/D$-profiles with the angle of attack $\alpha$ at $Re = 59,315$ for the four scenarios are plotted in comparison with the base profile.

**Figure 5.** Shapes of the optimized airfoil sections in comparison with the original configuration (The vertical axis is stretched to display the different thicknesses).

peaks of 55.59 and 57.86 at corresponding angles of attack of 5.0 and 5.12 in scenarios A and B, respectively. Here, the optimum $L/D$ in B is found to be slightly higher than that of A possibly due to the extended limits of the lower surface. Both aerodynamic responses are however discouraging for typical SWT operations because of the expected non-trivial intermittent operational conditions that keep on shifting the $L/D$ ratios from their optimum values. Interestingly, the optimization results in C and D reveal smoother $L/D$ profiles with lower, yet broader, peaks of 53.87 and 49.56, respectively, that make them more suitable candidates for wind turbine operations at off-design conditions. The $L/D$-profile in C is shown to be consistently similar with that of the original airfoil, yet shifted to lower angles of attack and higher lift-to-drag ratios. The simulation in D however shows a much broader ‘dome-like’ profile with nearly flat zone at $5 \leq \alpha \leq 8$.

Furthermore, the predicted improvements in the lift-to-drag ratios are 23.86%, 28.92%, 20.03% and 10.43% for scenarios A, B, C and D, respectively. To help understand the present optimization results, Fig. 5 displays the optimized shapes of the SG6043 airfoil. The figure clearly shows the variation of each airfoil’s thickness along its chord in comparison with the
Table 2. Aerodynamics performance of the optimum profiles at Re = 59,315.

| Scenarios | SG6043 | A    | B    | C    | D    |
|-----------|--------|------|------|------|------|
| (L/D)$_{\text{max}}$ | 44.88  | 55.59 | 57.86 | 53.87 | 49.56 |
| $\alpha_{\text{opt}}$ | 8.30   | 5.00  | 5.12  | 5.50  | 5.60  |
| t/c (%)  | 10.0   | 5.2   | 5.4   | 6.3   | 7.0   |
| AEP, kWh | 554.81 | 583.07 | 580.95 | 578.80 | 573.24 |

Figure 6. The lift characteristics at Re = 59,315 of the four scenarios in comparison with the original lift profile.

Figure 7. The drag characteristics at Re = 59,315 of the four scenarios in comparison with the original drag profile.

original geometry. The optimized shapes in A and B are remarkably thinner and thus expected to be less stiff than that of the original SG6043 airfoil. Nevertheless, moderate airfoil thicknesses are found in the latter scenarios. Table 2 summarizes the present aerodynamics results for the four scenarios along with their corresponding airfoil thicknesses in terms of their maximum thickness-to-chord ratios t/c. It is here concluded that the thinner the airfoil the higher the lift-to-drag ratio. Therefore, the present work suggests a trade-off between both the aerodynamics performance and the structural behavior of typical airfoil shapes. Such result qualitatively supports the discussion of Giguère and Selig [18].

In addition, considering the multi-dimensional optimization results of SWT blades of Wood et al. [19, 8], it becomes obvious that the starting characteristics of SWTs essentially depends on the blade inertia and the consequent aerodynamic torque, such that the lower the blade inertia the shorter the starting time for the turbine rotor. The blade’s moment of inertia is highly affected by the blade geometry and the material density, besides the manufacturing process. The present optimization study reveals new thinner, and thus light-weighted, airfoil sections with at least 30% reduction in the corresponding thickness-to-chord ratio with respect to the original SG6043. Interestingly, such reduction in the blade weight helps reduce the corresponding moment of inertia, and therefore leading to rapid starting rotor blades.

Figure 6 displays the lift characteristics of the new airfoils in comparison with the original profile. The lift coefficients of the optimized airfoils are remarkably higher than that of the basic profile as $\alpha$ varies in the range from 0 to approximately 8. Nevertheless, the airfoil-stalling
characteristic becomes evident as the angle of attack $\alpha$ exceeds 8.5; a sharp decrease in $C_L$ suddenly appears in A, while smooth decays in $C_L$ slightly emerge using airfoils C and D within the range of studied angles of attack $0 \leq \alpha \leq 10$. SWTs are designed to operate in the pre-stall regime, specifically around their optimum angle of attack. In the present analysis and within such range of operation, the optimized airfoils are mostly free of potential stalling. In the particular scenario ‘C’, $C_L$ progressively increases to reach a value of 1.32 at the optimum angle of attack $\alpha_{\text{opt}} \approx 5.5$.

Figure 7 plots the variations of the corresponding drag coefficients of the four scenarios in comparison with the base profile. In contrary with the lift profiles, the resulting drag coefficients are noticeably lower than the corresponding original profile. Here, the drag coefficients in the four scenarios are shown to gradually increase until reaching 0.04 at $\alpha \approx 4$. The drag profiles then experience sudden drops to nearly 0.02 at $\alpha \approx 5.0$, which are followed by sharp rises in the drag characteristics. In the particular scenario ‘C’, the drag coefficient has a minimum of 0.0245 at $\alpha = 5.5$. Consequently, in the present study and based on the above discussion, the third airfoil ‘C’ offers the most appropriate aerodynamic performance and is thus recommended as far as an optimized solution is sought.

3.1. Annual energy production
The present work is here extended to provide the entire blade design that enables the estimate of the overall turbine performance in terms of the power coefficient and the annual energy production (AEP). Here, the four optimized airfoils together with the original SG6043 airfoil are considered in the following optimum blade design. A three-bladed wind turbine with a rotor radius $R = 0.8 \text{ m}$ is assumed. The optimum chord and twist angle distributions are calculated following a similar procedure to that presented by Wood [19]. The present analysis essentially respects the drag-to-lift ratio $\epsilon$ and accounts for the Prandtl’s tip loss factor $F$ while allowing the tip speed ratio $\lambda$ to vary in the range from 3 to 10 with a step size of 0.5 to investigate its influence on the resulting aerodynamics performance. At each value of the tip speed ratio, the variations along the span ($0.15 \leq r/R \leq 0.95$) of both the axial $a$ and angular $a'$ flow induction factors are estimated and the corresponding power coefficient $C_p$ is calculated.

Next, the variations with $\lambda$ of the resulting power coefficients are shown in Fig. 8. Obviously, the calculated $C_p$’s peak at $\lambda \approx 4.0$, in consistency with the optimum lift-to-drag ratio. A maximum value of approximately 0.515 is indicated for scenario C that fulfills a 2% improvement compared to the original configuration. Though such improvement is much less than that achieved with the corresponding $L/D$, it is surprisingly extended to include the entire range of $\lambda$, as indicated in Fig. 8. With the turbine’s rotational speed being held nearly constant as typically found in permanent-magnet constant-speed generators, it is worth noting here that the above result suggests that the present optimal profiles are expected to perform equally well for different wind speeds $U$ and thus different Reynolds numbers.

To further proceed with the AEP calculations, the standard BEM theory is applied together with the XFOIL code to evaluate the energy production per year using each of the optimized airfoils with the tip speed ratio being held constant at its optimum value $\lambda \approx 4.0$. A flowchart that describes the entire process is here developed and presented in Appendix A, where a Weibull distribution with a shape factor $k = 2$ and a scale factor $A = 4.5 \text{ m/s}$ is assumed for the wind speed variation throughout the year. According to this particular approach, the resulting annual energy productions are found to be 583.07, 580.95, 578.80 and 573.24 kWh for scenarios A, B, C and D, respectively, while the AEP of the original SG6043 airfoil is 554.81 kWh, as also indicated in Table 2.
4. Conclusions
A new process of shape optimization was presented for the SG6043 airfoil, in which the XFOIL program was coupled with the genetic algorithm. The present analysis enabled better prediction of the optimum lift-to-drag ratios at low Reynolds numbers, typically experienced by SWTs. The numerical results for the SG6043 airfoil were first validated against experimental results at $Re \approx 100,000$, and found in good agreement with the test data at a transition exponent $N_{crit} = 11$. Next, four different scenarios were here suggested for the SG6043 airfoil optimization at $Re = 59,315$. In contrary with the sharp peaks developed in both scenarios $A$ and $B$, the optimization results in $C$ and $D$ revealed smoother $L/D$ profiles with lower, yet broader, peaks of 53.87 and 49.56, respectively, that make them more suitable candidates for wind turbine operations at off-design conditions. Furthermore, the resulting optimum airfoil sections in $A$ and $B$ were noted to be thinner, and thus more compliant, particularly nearby the trailing edges. The present study was further extended to account for the entire blade design and the assessment of the turbine’s aerodynamics performance. In this particular study, a maximum power coefficient of 0.51 along with an approximate improvement in the AEP of 5% with respect to the original airfoil were predicted at a tip speed ratio of 4.0.

Appendix A.
Figure A1 elaborates on the developed iterative procedure that couples the BEM theory with the XFOIL program to evaluate the AEP. The optimum tip speed ratio $\lambda = 4.0$ is considered. Here, each blade is divided into 12 radial segments. The analysis proceeds for each incremental wind speed in the range from 3 to 10 m/s with a total number of increments $N$. The axial $a$ and angular $a'$ induction factors distributions are first initialized then re-calculated using the presented iterative procedure. Next, both the lift and drag coefficients are updated consequently using the XFOIL solver through the specification of the operational angles of attack $\alpha = \phi - \theta$, where $\theta$ refers to the optimum twist angles. This is followed by the calculation of the power coefficient $C_p$ at the relevant wind speed. The AEP is then estimated (in kWh) as follows [20]:

$$AEP = 8760 \sum_{i=1}^{N} \left[ \exp \left( - \left( \frac{U_i}{A} \right)^k \right) - \exp \left( - \left( \frac{U_{i+1}}{A} \right)^k \right) \right] \times 0.5 \left( P_w(U_i) + P_w(U_{i+1}) \right) \quad (A.1)$$

In Eqn. A.1, $U$ is the wind speed in m/s, $A$ and $k$ refer the Weibull distribution scale and shape factors respectively, and $P_w$ is the produced power in kW.
Figure A1. A flowchart illustrating the iterative process for the estimation of the variation of the $C_p$ and the corresponding AEP with the wind speed for each of the four scenarios. Here, $\phi$ represents the relative flow angle of the inlet relative velocity $W$.

References

[1] Hansen T H 2018 Wind Energy 21 502–514
[2] Gardner B and Selig M 2003 Airfoil Design Using a Genetic Algorithm and an Inverse Method (USA: American Institute of Aeronautics and Astronautics) pp 1–12 AIAA 2003-43
[3] Ribeiro A, Awruch A and Gomes H 2012 Applied Mathematical Modelling 36 4898 – 4907
[4] He Y and Agarwal R K 2014 International Journal of Aerospace Engineering 2014 1–13
[5] Chan C, Bai H and He D 2018 Applied Energy 213 148 – 157
[6] Ram K R, Lal S P and Ahmed M R 2019 Renewable Energy 144 56 – 67
[7] Singh R K, Ahmed M R, Zullah M A and Lee Y H 2012 Renewable Energy 42 66 – 76
[8] Sessarego M and Wood D 2015 Renewables: Wind, Water, and Solar 2(1) 87
[9] Pourrajabian A, Afshar P A N, Ahmadizadeh M and Wood D 2016 Renewable Energy 87 837 – 48
[10] Bieri H P and Prautzsch H 1999 Computer Aided Geometric Design 16 579 – 581
[11] Qi LUO X, Jun ZHU G and Jun FENG J 2014 Journal of Hydrodynamics, Ser. B 26 807 – 817
[12] Deb K 2001 Multi-Objective Optimization Using Evolutionary Algorithms (USA: John Wiley & Sons, Inc.)
[13] Deb K and Agrawal R B 1995 Complex Systems 9 115–148
[14] Deb K and Deb D 2014 International Journal of Artificial Intelligence and Soft Computing 4 1–28
[15] Drela M 1989 Low Reynolds Number Aerodynamics ed Mueller T J (Berlin, Heidelberg: Springer) pp 1–12
[16] Selig M S, Lyon C A, Giguère P, Ninham C P and Guglielmo J J 1996 Summary of Low Speed Airfoil Data vol 2 (Virginia Beach, Virginia: SoarTech Publications)
[17] Lyon C A, Broeren A P, Giguère P, Gopalarathnam A and Selig M S 1998 Summary of Low Speed Airfoil Data vol 3 (Virginia Beach, Virginia: SoarTech Publications)
[18] Giguère P and Selig M S 1998 Journal of Solar Energy Engineering 120 108–114
[19] Wood D 2011 Small Wind Turbines 1st ed Green Energy and Technology (London: Springer-Verlag)
[20] Hansen M O L 2008 Aerodynamics of Wind Turbines 2nd ed (New York: Taylor & Francis)