Full-field optical metrology in polar and cylindrical coordinates

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Abstract
There are a large number of engineering measurement demands that can be very well met with optical methods. Among them, there are some that deal with circular or cylindrical geometries, for which Cartesian coordinates are not the ideal. Measurements in cylindrical and polar coordinates are much more natural choices. This paper presents an overview of various principles and configurations to measure textures, wear, geometry and deformations of external and internal cylindrical parts, displacements, deformations, stresses and residual stresses in cylindrical or polar coordinates. Optical measurement techniques such as triangulation, photogrammetry, deflectometry, white light interferometry, speckle interferometry, shearography in cylindrical or polar coordinates, are addressed. Most of the configurations presented here involve internal or external conical mirrors. These special optical components have several very interesting properties capable of performing transformations between coordinate systems. Examples of real engineering demands are presented and briefly discussed. The results obtained met the demands for which they were developed. Our expectation is that the configurations presented here can serve as a reference for already established applications and as a source of inspiration for new demands and possibilities.

1. Introduction

Cartesian coordinate systems are widely used to model a large number of natural phenomena as well as systems developed by mankind. Following this trend, the vast majority of full-field optical measurement systems also present results in Cartesian coordinates. However, polar or cylindrical coordinate systems are more natural for solving problems and describing situations in which circular and cylindrical geometries, or boundary conditions, are involved. For these cases, it is desirable to measure and express the results directly in these coordinates.

Some optical components can be used to transform cylindrical coordinates so that cylindrical surfaces can be imaged by a single camera. This is the basis for several full-field measurement systems reported in the literature. For example, Gilbert et al have published several papers on applications of full-field optical methods in cylindrical coordinates using a panoramic annular lens [1–3]. Genovese used an optical configuration in cylindrical coordinates to measure all way around 360° the geometry of an artery using digital image correlation [4]. There are several other applications in the field of autonomous navigation using hyperbolic mirrors [5, 6]. Weckenmann et al [7] and Nuerge and Schwider [8] developed a very interesting and accurate interferometric system in cylindrical coordinates using a pair of axicon diffractive elements in a grazing incidence configuration. These are just a few examples. As this is a promising field, in which there are more demands than solutions, certainly new developments will emerge in the next years.

The main focus of this paper is to bring an overview of various full-field measurements systems in cylindrical or polar coordinates for measuring seven different quantities. A special emphasis is focused on based conical mirror configurations.
2. Conical mirrors

Forty-five degrees conical mirrors are key elements to enable optical measurements in cylindrical coordinates. This section presents and discusses the most relevant optical properties of this type of mirrors.

2.1. Optical transformation

Figure 1 represents an external 45° conical mirror and a properly centered cylindrical part. The surface of the conical mirror reflects the rays toward the surface of the cylindrical part in a radial way. Therefore, an optical transformation occurs changing parallel rays into radial rays, namely, changing Cartesian to cylindrical coordinates. This transformation is valid for both observation and illumination paths of the cylindrical part.

When viewed through a 45° conical mirror, the cylindrical surface of a perfectly centered part becomes an annular region of a flat disc. This optical transformation maps points A and B on the cylindrical surface to \(A'\) and \(B'\) on the flat disk, as well as C and D into \(C'\) and \(D'\).

Figure 2 shows the equivalent optical transformation through an external 45° conical mirror located inside a cylindrical cavity and well aligned with the cylinder axis. Again, there is a transformation from Cartesian to cylindrical coordinates. When viewed through the conical mirror, the internal cylindrical surface also becomes a flat disk.

The maximum height that theoretically can be observed, or illuminated, by a 45° conical mirror inside a cylinder corresponds to the mirror’s radius. However, the image reflected in the vicinity of the nose of the conical mirror is very compact, with limited lateral resolution. Therefore, the practical limit for measurement is about 2/3 of the mirror radius. The first third, near the nose of the conical mirror, is generally not used. Figure 3 shows a way to extend the maximum height measured, or observed, from \(h_c\) to \(h_s\) using a stepped conical mirror. In this case, a continuous region is no longer measured or observed, but only few rings.

The alignment between the axis of the conical mirror, the measured part and the direction of observation or illumination is a critical issue. Small misalignments, in the range of \(\pm 2^\circ\), have little influence and are analytically compensable. Expressive angular or lateral misalignments produce image distortions or measurement sensitivity variations that are difficult to efficiently compensate for.
Images reflected by conical mirrors produce, to a lesser or greater extent, astigmatism. Therefore, it is not possible to achieve a sharp focus of the radial and circumferential features of the image at the same time. This effect is a natural consequence of the reflection in the conical mirror, whose geometry has different radii of curvature, being minimal in the circumferential direction and infinite along the generatrix. Consequently, there is a degradation of the sharpness of the reflected image, which has a greater impact on texture mapping and less influence on interferometric techniques.

### 2.2. Digital image mapping

Figure 4 shows a real external conical mirror, machined in high quality aluminum by a precision lathe using a diamond tool.

Figure 5 shows the top view of a small gas compressor piston located in the center of a conical mirror. The upper surface of the piston is visible in the central part of the image. The cylindrical surface of the piston is reflected by the conical mirror and becomes an annular region of a flat disc. The two circular holes of the piston bearings are also visible, but they are distorted by reflection in the conical mirror.

Mapping the image reflected by the conical mirror on the cylindrical surface is straightforward. If the reflected image is described using polar coordinates and the origin being coincident with the center of the conical mirror, the relationship between polar and cylindrical coordinates is immediate. Figure 6 shows that the angle $\theta$ remains unchanged. The radius in polar coordinates is linearly related to the height in the cylinder. For external mirrors, the largest radius coincides with the highest point on the cylinder, as shown in the figure.

The lateral resolution in the flattened image is constant for the radial polar coordinate and highly variable for the circumferential coordinate. In the cylindrical part, the lateral resolution is constant along the axis of the cylinder and variable in the circumferential coordinate. The best circumferential lateral resolution is obtained for the largest radius of the flattened image.
3. Cylindrical texture measurement

For some engineering applications it is very important to analyze the texture of inner cylindrical surfaces. For example, combustion engine cylinders have a grooved texture to retain lubricating oil. The angles that the grooves form with the cylinder axis is a very important feature that needs to be well controlled during the manufacturing process. Figure 7 shows an example. The left part is an enlarged view of inner texture of a real engine motor. The right part is the same image after a Fourier angular band pass filtering. The groove angles are easily measured in the Fourier plane.

The optical configuration of figure 8 is suitable for mapping the texture of the inner surface of cylinders all over $360^\circ$. The mapped region is a ring with a height of about $2/3$ of the conical mirror radius. A ring of LEDs illuminates the surface at an appropriate angle to highlight the surface's features. The internal image is obtained by reflection in the conical mirror.

The left part of figure 9 shows the reconstruction of the texture of the internal cylindrical surface in the measured ring. The right part represents, in a color scale, the mapping of the groove angles along the entire length of the measured ring. This principle is today the basis of a system already available on the market [9].
The alignment between the part to be measured and the measurement system is not critical issue due to the aid of a self-centering device. This device ensures a $\pm1^\circ$ alignment tolerance. There are no special stability requirements. Groove angles between $-180^\circ$ and $+180^\circ$ are measured with a typical uncertainty of $\pm1.5^\circ$. A 50 mm cylindrical region is captured at a time with a lateral resolution of 0.2 mm. It is possible to measure longer cylindrical surfaces by stitching a set of sequential measurements with an overlapping of approximately 20% of measured region.

4. Cylindrical geometry measurement

There are a large number of engineering demands for measuring the geometry of cylindrical parts. Applications range from assessing the degradation of corroded surfaces to shape deviations or wear of high-precision parts.

4.1. Triangulation

It is possible to perform triangulation in cylindrical coordinates with the aid of a conical mirror. Figure 10 shows the basic configuration [10–14]. The collimated beam of a laser is directed towards the nose of a 45° conical mirror where it is deflected forming a planar radial light sheet, which intercepts the cylindrical surface, forming a bright light ring. A camera forms the image of the light ring from an oblique angle. If the alignment of the optical system with the axis of a cylinder is perfect, the shape of the ring of light will be a perfect circle. Anomalies on the inner surface geometry will cause the image of the ring of light to deviate from a perfect circle. An appropriate calibration allows to determine the radius of the measured surface from the radius of the image of the light ring.

To measure the internal cylindrical surface, it is necessary to scan the region of interest. Figure 11 shows a three-wheel self-centering device where the optical system is installed [13]. Although misalignments can be corrected by software, it is convenient to keep the optical system relatively well centered in relation to the measured part (a centering error less than 2 mm). While the system is moved inside the cylindrical cavity an odometer triggers the acquisition of images by the camera, which makes the distance between the sampling...
rings constant and independent of the scanning speed. Typically, 1440 points are acquired from each ring spaced by 1.0 mm. This combination results in 1.44 million points per meter. A graphics processing unit (GPU) based algorithm makes possible to acquire and process about 50 images per second. For this particular configuration, the radial measurement range is from 70 to 80 mm, with a measurement uncertainty (95%) of about 0.2 mm. The typical lateral resolution is 0.6 mm in the circumferential direction and can be 1.0 mm in the longitudinal direction.

Figure 12 shows a practical application. The left part shows an image of a region of a tube after being used in the oil industry for several years, which shows severe corrosion damage. The right part of the figure shows the results of the measurement by the triangulation system in cylindrical coordinates. The color scale is related to the radial coordinate.

4.2. Active photogrammetry
Both fringe projection and active photogrammetry techniques are feasible in cylindrical coordinates [15–17]. Figure 13 shows a successful optical configuration. It consists of two cameras and a fringe projection system, which are mounted on a transparent tube. The fringe projection system contains a lamp inside and a second transparent tube on which helical fringe patterns is printed. A step motor, not shown in the figure, rotates the helical fringe pattern in a controlled manner, phase shifting the shadow of the fringes projected on the internal surface of the measured cylinder.

As in classical photogrammetry, each camera sees the central region of the cylindrical surface from a different angle. The identification of homologous points is made with the aid of the projected fringe phase. A trick is needed to correctly extract the phase value from the projected fringes. The left image of figure 14 shows the typical spiral aspect of a phase map calculated from a sequence of phase-shifted images. A philosophical question arises: how to properly unwrap spiral fringes? To solve this issue, a synthetic phase ramp, with phase values computed as the module $2\pi$ of minus twice the polar angle, shown in the central part of the figure, is subtracted point to point, resulting in the phase map of the right part of the figure, which can be processed by conventional phase unwrapping algorithms.

Once the phase unwrapped images are known for each camera, the mathematical model represented in figure 15 is used to find the cylindrical coordinates of the measured cylindrical surface.

Pinhole cameras $A$ and $B$ inherit the intrinsic and extrinsic parameters of real cameras. In this way, it is straight-forward to project a point described in real world coordinates and determine its coordinates in each image. A regular mesh in cylindrical coordinates is established and the goal is to determine the radius values at each node. A numerical scan is performed along the radius, represented as a blue line in the figure, and, for
Figure 12. Top: a corroded pipe surface. Bottom: measurements results. The color scale is related to the radius values ranging from 68.0 to 80.0 mm.

Figure 13. An optical configuration for both fringe projection and active photogrammetry in cylindrical coordinates.

Figure 14. Phase retrieval using a synthetic phase ramp.

each radial position, the corresponding projections are calculated on each image. The phase values of the corresponding points are then compared. The correct radius value is the one that results in equal phase values in both images [15].

Figure 16 shows an actual view of the central part of a prototype of this optical system. It was assembled in a three wheels self-centering mechanical device, not shown in the figure, similar to the one shown in figure 11.

Finally, figure 17 shows the results of a weld seam at the junction between two steel tubes measured from inside. The quantities of greatest interest are the height of the weld bead and the misalignment between the two tubes, which are successfully accessed. This particular prototype was designed to measure internal annular regions with a radius range from 74 to 80 mm and a length of 30 mm. The lateral resolution was 0.10 mm. The typical measurement uncertainty (95%) was ±0.02 mm.
4.3. White light interferometry

White light interferometry is an already established technique to measure the shape of small objects with excellent uncertainty levels [18]. It combines the high sensitivity of interferometers and the ability to perform absolute height measurements [19–22]. Parts with lateral sizes starting from few micrometers up to over 100 mm can be measured. It is possible to achieve height resolution better than one nanometer and measurement ranges up to some millimeters, which makes this technique excellent for industrial applications involving geometric quality control of precision parts. Several commercial systems using this measurement principle are already available on the market [23, 24].

A possible arrangement for a white light interferometer is a Michelson configuration. The light from a light source with limited coherent length is collimated and propagates to a beam splitter (BS). About 50% of the light propagates through the BS and reaches a flat reference surface—a high quality mirror—that reflected back the light toward the BS and the camera. About 50% of the incident light is deflected by the BS.
toward the surface to be measured which, in turn, reflects back the light to the imaging device. The two light components are recombined on the camera’s sensor. Since the light has a very short coherence length, the interference figure is only detectable for regions where the difference in optical path (OPD) between the two arms of the interferometer is almost zero. To measure the geometry of a surface, the reference mirror is scanned in a controlled manner. The regions of the part where the interference figures are visible correspond to a certain height, as if they were contour lines. Specialized algorithms are used to accurately find the maximum contrast position of the interference figure for each pixel in the image and assign a height value to it [24].

4.3.1. Internal cylinders

According to the optical transformation shown in figure 2, a traditional Michelson white light interferometer can be used to measure inner cylindrical surfaces by scanning the geometry of the generated virtual flat disc. In the same way, an inverse conical mirror (see figure 1) can be used to measure external cylindrical surface.

The white light interferometer was modified according to schemes showed in figures 18 and 19 to measure in cylindrical coordinates [25–27]. For both cases, the light source is a near infrared LED, (\(\lambda = 880\) nm), with a coherence length of the order of 20 \(\mu\)m.

Figure 18 shows the appropriate white light interferometer configuration for internal cylinder measurements. The expanded LED light is collimated by the plane-convex lens (\(L_1\)). Then, it reaches a 50% BS and is divided into two parts. The first part—the reference beam—goes through BS, reaches a flat mirror \(M_1\), used as a reference flat surface, and it is reflected back to the BS. The second part—the active beam—is
Figure 20. Different kinds of cylindrical surfaces: (a) short inner, (b) long internal, (c) short external, and (d) long external.

reflected by the BS and directed to a conical mirror. The conical mirror reflects the collimated beam to the internal surface of the cylinder to be measured. The light reflected by the cylinder follows the same path back when it is reflected by the conical mirror and reaches the BS. Both reference and active beams are combined by the BS and imaged through a telecentric lens on the image plane of a \((1300 \times 1030)\) digital camera.

The reference mirror is progressively moved in a controlled manner, scanning the volume where the inner cylinder is. During scanning, for each pixel in the image the contrast of the interference figure passes through the maximum contrast peak, which occurs when the OPD is zero. The software monitors this interference pattern contrast simultaneously for each pixel in the image. The radius of each point on the inner surface of the cylinder is directly related to the scanning position of the reference mirror in which the contrast of the interference figure is maximized.

4.3.2. External cylinders
For the external white light interferometer (figure 19), the partial mirror is positioned in the region where the illumination beam expands. Two twin collimation lenses are placed in the vicinity of the reference mirror and the internal conical mirror respectively. This arrangement allows a clear aperture large enough to cover the \(45^\circ\) conical mirror, which has an outside diameter of 80 mm. It is important that the collimation lenses are similar to compensate for any optical aberrations present in the two arms of the interferometer.

4.3.3. Measurement strategies
There are two different measurement strategies: (a) for short cylinders, which are measured in a single positioning, and (b) for long cylinders, when the measurement is made in stages and the results are stitched together.

Figure 20(a) represents a short inner cylinder (ring) aligned and positioned close to a \(45^\circ\) conical mirror and ready to be measured. The cylinder is said to be short when the length of the cylinder \(h\) is smaller than 2/3 of the conical mirror radius \((h < 2r_m/3)\). Only one positioning and scanning are sufficient to measure short cylinders, since the entire cylindrical surface is visible through reflection in the conical mirror. The cylinder represented in figure 20(b) is considered long, since the cylinder length is much greater than 2/3 of the radius of the conical mirror \((h > 2r_m/3)\). It is not possible to measure the entire surface from a single positioning. The measurement has to be done in stages.

The same logic applies to measure short and long external cylinders using internal conical mirrors, as shown in figures 20(c) and (d) respectively. In such cases, the length \(h\) of the cylinder is compared with the depth of the mirror \(w\). It will be said long if \(h > w\).

To measure long cylinders, the measurement is done in steps involving measurements of adjacent areas, always maintaining about 30%–40% of overlap. The reconstruction of the long cylinder is done by stitching the point clouds of adjacent areas. The cylinder to be measured is moved by means of a motorized translation stage. The conical mirror is held in place (see figure 20). The measured cylinder is fixed in an alignment device composed of two stages of translation and two of inclination to correct possible misalignments between the measured cylinder and the optical axis of the conical mirror. The stitching process is described in details in [26].

Figure 21 shows a 3D plot, in false color scale, of a 19 mm diameter gas compressor piston measured using an interferometer similar to figure 19. Left: the 3D plot is in natural scale. Right: the plot scale was enlarged by a factor of about 1000×.
This interferometer measures external cylinders with radii between 2 and 12 mm and lengths up to 20 mm per stage. The lateral resolution changes with the measured diameter and the position in the image between 0.01 and 0.06 mm. The typical measurement uncertainty (95%) is in the order of 0.35 \( \mu m \).

Figure 22 shows an example of internal geometry of two journal bearings of the part seen in (a). The smallest journal bearing shown in part (b) was measured piecewise through three positioning with a 6.5 mm diameter conical mirror. Then, the cloud of points was stitched according to the described procedure. It is possible to observe that the resulting surfaces did not show any lack of continuity between the stitched areas. The bigger bearing was measured by a single positioning.

The developed and evaluated prototype was able to successfully measure internal cylinders with radii between 7 and 50 mm and lengths from 2 to 6 mm per stage. As before, the lateral resolution also changes, with the measured diameter and the position in the image, between 0.02 and 0.12 mm. The typical measurement uncertainty (95%) is also \( \sim 0.35 \mu m \).

For the external measurements as well as for the internal one, the stitching process degraded the measurement uncertainty by about 20%–40%.

4.3.4. Wear measurement
The interferometer has been successfully applied to identify areas with wearing in gas compressor pistons and to measure the wore volume. These measurements were performed in the following way: (a) first at all, the pistons were measured with a white light interferometer similar to that presented in figure 19. (b) Then, for testing purposes, the piston was subjected to artificial wear in some parts. Finally, the wore pistons were repositioned and measured with the interferometer. A careful repositioning ensured the matching between the correspondent points of both point clouds before wearing and after wearing.

Figures 23(a)–(c) show the measured cloud of points of a pin used to connect the piston and the connecting rod. Part (a) shows the measured cloud of points of the pin before wear. Part (b) shows the same pin but after wear. Part (c) shows a 3D image of a longitudinal cross-section of both cloud of points overlapped. In this image the difference between both cloud of points is clear. The knowledge of the
Figure 23. 3D plots in color scales of the measurement of (a) pin before wear, (b) after wear and (c) a longitudinal cross-section of both cloud of points showing the wore volume.

Figure 24. Basic configuration for deflectometry in cylindrical coordinates.

dimensions of the part and the radial difference between both cloud of points allows computing the wore volume.

The diameter range, length and lateral resolution are similar to the interferometer presented for external cylinders. The achieved measurement uncertainty (95%) was estimated as ±0.70 µm.

4.4. Deflectometry

Deflectometry indirectly measures the shape of reflective surfaces by analyzing the distortions in the images reflected of a structured pattern on the surface. Perfect geometries produce predictable distortions. Shape errors departure the reflected image away from predicted one. From the analysis of this distortions using the reflection law, it becomes possible to indirectly measure the surface geometry. With an appropriate configuration it is possible to measure form errors with measurement uncertainty of the order of a few nanometers [28, 29].

Deflectometry can be extended for cylindrical coordinates using a conical mirror [30–32]. Figure 24 shows the basic configuration. A reflective cylinder is aligned with the internal 45° conical mirror axis. The camera acquires the image of the screen reflected by the partial mirror, then the conical mirror, then by the cylindrical part and again by the conical mirror. The red and blue lines represent the OPDs of two different rays.

If all the geometries are perfect, as well as the alignment between the parts, the screen image is reflected in an undistorted way. To measure in cylindrical coordinates, sequences of phase shifted radial and circumferential fringe images are acquired, as shown in figure 25. As a result, both radial and circumferential phase values are acquired and integrated to reconstruct the cylindrical geometry. The systematic errors resulting from the non-idealities of the optical components and imperfections in the alignment were quantified using a precision pin as a reference. The determined systematic errors were corrected in all subsequent measurements. Small misalignment errors can be detected and corrected as well [32].

Figure 26 shows the results of a comparison of the cross section of a same reflective 5.05 mm diameter rod measured by the deflectometer (red line) and by a tactile system (blue line). Both results are quite close.

When compared to white light interferometry, the measurement of cylindrical surfaces by deflectometry is faster and involves cheaper components. However it is limited to mirrored surfaces and parts with radius variations up to 20 µm. The difficulties in aligning the parts to be measured are comparable. The stability requirements and the dimensions of the parts that can be measured are also comparable.

The achieved measurement uncertainty (95%) is estimated as 20 µm. In the current state of the art, white light interferometry presents better measurement uncertainty. Nevertheless, as developments in deflectometry in cylindrical coordinates are still in the early stages, there is great potential for improvement.
5. Cylindrical deformation measurement

Cylindrical surfaces are widely used in the construction of machines to guide movements and for sealing purposes. For applications that require high precision, it is important that the cylindrical shape is ensured. The action of assembly or operating loads, or even non-homogenous thermal expansion, can make the cylindrical shape departure from the ideal. Therefore, there is a large demand to measure cylinder deformation accurately. This section deals with different configurations suitable for measuring deformations of internal and external cylindrical surfaces using speckle interferometry.

5.1. Speckle interferometry

Figure 27 shows a basic configuration of a speckle interferometer suitable for measuring out-of-plane deformations [33–36]. The laser light is expanded and directed to a BS. Part of the light is deflected to the right and reaches the rough deformable surface. Part of the light passes through the BS and reaches a second rough surface, the reference surface. Since both surfaces are rough, a random pattern of light and dark spots are visible on the surface, the speckle pattern. The image resulting from the overlapping of the two speckle patterns is formed in the plane of the camera’s sensor and mutually interfere. The resulting intensity on each pixel of the resulting interference speckle pattern image depends on the corresponding light intensity and phase differences.

Usually a piezoelectric actuator, not shown in the figure, is used to move the reference surface in a parallel way by small amounts, which results in a homogeneous phase shift in all points of the image. Deformation measurement is done in two stages: before and after applying a load. For each stage a phase value is computed for each pixel on the image and, afterwards, the phase differences are computed. The phase difference shows a wrapped fringe pattern. After phase unwrapping, the phase value is related to the out-of-plane displacement for each image pixel. For this particular configuration, when the directions of illumination and observation coincide, $2\pi$ phase difference corresponds to a displacement of half of the laser wavelength ($\lambda/2$) along the illumination/viewing direction.
The basic configuration of figure 27 can be modified by means of conical mirrors to measure in cylindrical coordinates, as shown in the following sections [37]. In all cases the resolution in the radial out-of-plane displacement is of the order of 0.001 \( \mu m \) with a typical measurement uncertainty of 0.015 \( \mu m \). The remaining measurement performance parameters will be discussed in section 7.

5.1.1. External cylinder

The speckle interferometer of figure 28 is suitable for measuring radial deformation of external cylinders in cylindrical coordinates. A large external conical mirror is the key element, within which the part to be measured is positioned and aligned. Laser light is split by a partial mirror. Only a small portion of the laser power is deflected, reflected on a movable mirror connected to a piezoelectric actuator for phase shifting, expanded and illuminates a ground glass, which will serve as a reference illumination. Most of the laser light’s power goes on through the partial mirror, is expanded by a lens and goes through a hole in the central part of a flat mirror positioned at 45° and illuminates the cylindrical part to be measured through the conical mirror. A quasi-radial illumination direction is achieved. The cylindrical surface to be measured is imaged in the camera’s sensor through the reflection on the conical mirror and on the 45° flat mirror. The small hole in the later does not disturb the periphery of the image where the cylindrical surface is reflected by the conical mirror. The partial mirror just in front of the camera is used to interfere the image of the cylinder with the reference illumination. Both illumination and viewing directions are quasi-radial. Therefore, this interferometer is almost only sensitive to the radial out-of-plane displacement component.

The measurement of thermal deformations of an aluminum automotive engine piston is given as a practical application example. Figure 29 shows how the engine piston looks like when reflected by the conical mirror. Although the bearings on the piston side surface are circular, they appear distorted by the reflection in the conical mirror. A few turns of electrical wires were accommodated inside the piston grooves to promote heating through electric current. An image of the reference phase was acquired before heating. After applying a current for few seconds, the piston temperature was increased by about 2 K. Afterwards, another phase image was acquired. The right part of the figure 29 shows the phase difference pattern. The analysis of the fringes reveals that the thermal deformations are not uniform. The particular piston model has steel inserts inside the aluminum body. They are intentionally designed to control the way the piston thermal expansion occurs under the working conditions inside a running engine.
Figure 29. Left: actual view of the cylindrical surface of the automotive engine piston reflected on the conical mirror surface. Right: phase difference pattern after thermal loading showing the influence of steel inserts.

Figure 30. Polar diagrams of the radial thermal deformations of two different sections of the engine piston after heating. The radial scale division in both cases is only 0.1 μm.

Figure 31. Basic configuration of a speckle interferometer for radial deformation of short internal cylinders.

Figure 30 shows two polar diagrams that represent the radial deformations for two sections located at different axial positions of the piston. The differences between the deformations in the two sections are quite significant, indicating that an elliptical shape is obtained in the deformed piston geometry. This behavior is intended to improve the lubrication performance of the running engine.

5.1.2. Short internal cylinder

The configuration shown in figure 31 is suitable for measuring deformations of short inner cylindrical surface in the radial direction. To illuminate and observe the internal surface of the cylinder, a 45° conical mirror is positioned and aligned with the axis of the internal cylindrical surface to be measured. Since the reflection of the internal cylindrical surface in a 45° conical mirror is optically transformed into a flat disk, measuring the out-of-plane deformations of that flat disk is equivalent to measuring the radial deformations of the cylindrical surface.

The expanded laser beam is collimated and divided into two parts: the reference beam and the active beam. The latter, after being reflected by the conical mirror, is directed at an angle perpendicular to the normal of the internal cylindrical surface to be measured. The light reflected from the internal cylindrical surface follows the same path back to the partial mirror. Part of the light hits the camera and the image of the inside of the cylinder is formed. The reference beam is directed towards a rough flat surface, which serves as a reference. It is reflected back, hits the partial mirror and it is reflected to the camera. The camera records the interference figure resulting from the superposition of the two coherent images: the internal cylindrical
surface and the reference surface. The reference surface is moved in a controlled manner by a small amount to apply phase shifts to better quantify the interference signal through image processing algorithms.

The length along the cylinder axis that can be measured from a single measurement is about $2/3$ of the conical mirror radius. The next section presents an appropriate setting for measuring longer lengths.

5.1.3. Long internal cylinder

The use of stepped conical mirrors, such as the one on the right part of figure 3, extending measurement high using an internal stepped conical mirror, allows to extend the measured axial length, but limits the measured region to a few rings. The basic configuration is presented in figure 32, which is similar to that used for short cylinders, except for the stepped conical mirror. The stepped conical mirror allows to measure greater extensions of the internal cylindrical surface. However, the price to be paid is that it is not possible to obtain a continuous measurement, but on only in a few rings, which correspond with the conical mirror sections. The interferometer is blind to the regions in between the conical mirror sections.

Figure 33 shows a seven stages stepped conical mirror machined with a diamond tool in an ultra-precision lathe. The maximum radius dimension is 10 mm and the length is about 34 mm. It was not possible to use the first stage for measurement due to the limited spatial resolution.

Figure 34 shows a typical phase difference pattern. It is possible to notice a small phase step between consecutive rings. Since the jumps were small, they did not cause any problems in the phase unwrapping procedure.

The radial deformation of a long cylinder was measured on the body of the small gas compressor used for cooling purposes shown at the top of figure 35. The tightening of the four body fixing screws on the base was sufficient to cause the measured deformations. A phase image was acquired when a preload had been applied to the four screws. Another phase image was acquired shortly after the full load was applied to the four screws. Figure 34 shows the corresponding phase difference map. The deformations measured in six rings gave rise to the 3D representation of figure 35 where the deformed cylindrical surface is represented on a very exaggerated scale.
The high noise level in speckle interferometry measurements and the high number of fringes generally limit the largest displacement that can be measured at once from the difference between two phase maps to about $\pm 2 \, \mu m$. This limitation can be easily overcome if loading is done in steps and measurements are made between each step of the loading. At the end, after applying low-pass filters, the measurements are added. Therefore, the largest measurable displacement is practically unlimited.

5.2. Shearography

Shearography is a short name for shear speckle pattern interferometry. It is a special kind of speckle interferometry where the interference occurs by the superposition of two laterally displaced images of the measured surface illuminated by laser light. Shearography measures the differences between the relative displacements between the two laterally displaced images. Consequently, the measurements are not very sensitive to rigid body displacements. This feature makes shearography very robust for in-field applications.

Figure 36 shows a classical configuration for shearography. A BS and two flat mirrors form the double image of the measured region. To achieve this effect, one of the mirrors must be slightly tilted. The magnitude and direction of the lateral displacement are controlled by the angle and axis of rotation of the tilted mirror. Macedo et al developed a shearography configuration suitable for measuring in cylindrical coordinates [38–40]. Two conical mirrors are used: one to promote lateral displacement of the images in a radial way and another to observe the inside of the cylindrical surface, as shows figure 37.

The main conical mirror is positioned inside the cylindrical surface to be measured using a mechanical structure not shown in the figure [40]. Its function is to promote the visualization from inside of the cylindrical surface from a viewing angle almost perpendicular to the cylindrical surface. The expanded beam of a laser positioned behind the main conical mirror obliquely illuminates the surface to be measured. The interferometer, shown on the right of the figure, in addition to the camera and the lens system, contains the BS and two mirrors. The flat mirror is driven by a piezoelectric element for phase shifting. The secondary conical mirror has a very shallow angle, of only $1^\circ$ related to a flat surface, or $89^\circ$ related to the cylinder axis.

The function of the secondary conical mirror is to promote a lateral displacement of the image radially. This effect can be seen through figure 38. Radial shear resulting from the action of the secondary conical mirror. Part (a) shows the image of a target with concentric circles obtained through reflection in the flat mirror only. Part (b) shows the reflection through the secondary conical mirror only. Note that there was a
homogenous radial shift of the entire image of about one scale mark. The first label is now 20 instead of 10. Part (c) shows the superposition of both images reflected by the two mirrors simultaneously.

Figure 39 shows an example of application of this system in non-destructive testing. A carbon fiber-reinforced plastic tube with artificial defects was subjected to a thermally loaded test. A sequence of phase-shifted images was acquired before loading and another after heating the tube from the outside with a hairdryer. The phase difference map shows the presence of the three artificial defects (surrounded by ellipses) and, in addition, two unintended defects (surrounded by rectangles).

The image in figure 39 is in polar coordinates. The polar angle corresponds to the cylinder angle. The radius is related to the axial coordinate of the cylinder. With this analysis it is clear that a uniform radial lateral displacement corresponds to a uniform lateral displacement in the axial direction of the cylinder. To facilitate the location of the defects, the image in figure 39 was unrolled and mapped to cylindrical coordinates, the horizontal axis being related to the angle and the vertical coordinate, the axial position. The result is in figure 40.

Although in this application shearography has been used qualitatively to detect the presence of defects, it is possible to achieve a measurement range of about ±4 µm, with a measurement uncertainty of ±0.05 µm for each loading stage. It is also possible to combine data from different loading stages.
6. In-plane deformation measurement

Speckle interferometry can be applied for the measurement of in-plane displacements, strains, stresses and residual stresses. This section presents speckle interferometry configurations for these purposes.

Figure 41 shows a classical double illumination configuration, which is suitable for measuring in-plane displacement components. The measured surface is illuminated with two beams coming from the same laser from different angles. A speckle pattern is developed for each illumination. When both illuminations are active at the same time, the camera image records the interference between the two speckle patterns. The mirror on the right is coupled to a piezoelectric actuator for phase shift purposes. If the unitary illumination vectors are symmetrical to the normal vector of the surface, only in-plane measurement sensitivity is obtained. The sensitivity vector is resulting from the difference between unitary observation and illumination vectors [33–36].

Let us assume that the axis coincident with the direction $Z$ is perpendicular to the surface. The lighting directions can be adjusted so that the sensitivity vector coincides with the $X$ axis. Thus, only the $X$ axis displacement component in the direction is assessed. To measure component $Y$, another pair of lighting is required. The lighting with each pair has to be sequential, which makes measuring components in the plane not very much practical.
The next subsections describe alternatives for measuring the radial polar coordinates, that is, radial in-plane sensitivity. Radial in-plane sensitivity is extremely attractive for engineering measurement and more practical since a pair of double illumination is uniquely required.

6.1. The radial speckle interferometer

Figure 42 represents the cross-section of a conical mirror used for achieving in-plane radial sensitivity in speckle interferometry [41–43]. This conical mirror is placed close to the specimen surface for the double illumination.

Figure 42 also shows a pair of collimated light rays coming from the same illumination source. Each ray impinges the surface of the conical mirror surface being reflected towards a point $P$ located over the specimen surface. Both rays illuminate the surface forming the same angle of incidence in relation to the surface normal. The unitary vectors $\mathbf{n}_A$ and $\mathbf{n}_B$ are coincident with the illumination directions. The difference between these two unit vectors defines the sensitivity vector $\mathbf{k}$. Due to the symmetry of the unitary vectors, the sensitivity vector is parallel to the surface and defines in-plane sensitivity at point $P$. It should be noted that no other pair of rays reaches point $P$. For any other point on the specimen's surface there is always only a pair of collimated rays which results in in-plane sensitivity aligned with the diameter direction. The only exception is the central point, which is a singular point.

This geometrical description if figure 42 is valid for any other cross-section of the conical mirror. Thus, with the exception of the center point, every other point on the specimen's surface will be illuminated by one, and only one, pair of light rays. As the pairs of rays will be contained in a same cross section, radial and in-plane sensitivity will be obtained throughout the circular region on the specimen surface. In other words, the measurement will be made in the radial polar coordinate.

Figure 43 shows a practical configuration of the radial in-plane interferometer. The source light is a diode laser which is expanded and collimated with a couple of convergent lenses. The light is reflected to the conical mirror by a mirror tilted an angle of $45^\circ$ respected from the axis of the conical mirror. The $45^\circ$ mirror has a central hole in order to avoid that the laser beam directly reaches the sample surface having triple illumination. Additionally, it gives a viewing window for the camera.

The light intensity is not uniform for the whole circular illuminated area. Moreover, the central point receives light from all cross sections and consequently it is a bright spot, with particularly high intensity. This bright spot can affect and reduce the fringe quality. To overcome this effect, the conical mirror is made by two parts. There is a small gap between them. The distance between the mirrors is selected to block the light rays reflected to the center. Consequently, a small circular shadowed region is created in the center of the illuminated area instead of a bright spot.

As before stated, every point over the specimen is illuminated for only two rays coming from the upper and lower part of the conical mirror. A piezoelectric actuator is glued in the upper part of the conical mirror. For this reason, this part is a mobile mirror. On the other hand, the lower part is fixed. The piezoelectric actuator moves and increases the gap between both mirrors. Consequently, the OPD for the reflected interfering rays is changed allowing the application of a phase shift to compute the wrapped phase by using phase shifting algorithms.

Two disadvantages can be found in the optical setup in figure 43: (a) the first one is use of a high quality machined conical mirror made in aluminum which usually is quite expensive; (b) the second one is the need of a laser source with wavelength stabilization. Compact laser diodes usually do not have good wavelength stability, which makes it difficult or even unfeasible for applications outside a controlled laboratory.
Another alternative to obtain radial in-plane sensitivity is to use diffraction gratings. Diffractive gratings are capable of directing light according to predefined angles. Modern technologies make it possible to produce custom diffractive gratings to meet specific demands [44].

Developments in microlithography manufacturing enabled the production of customized diffractive optical elements (DOEs). Additionally, the development of a new and flexible family of optical elements was allowed by the ability to manufacture a large variety of geometries and configurations for diffraction gratings. BSs diffractive lenses and diffractive shaping optics are clear examples for these devices. Following this same trend, a specific DOE was designed for achieving radial in-plane sensitivity with speckle interferometry. In this case, it is a circular diffraction grating with a binary profile and a constant period $p$, as it is shown in figure 44.

When an axis-symmetric circular binary DOE (see figure 44) is illuminated by collimated light, a double illuminated circular area with radial in-plane sensitivity is achieved. This region has similar properties to those obtained by conical mirrors [45, 46]. The diffracted orders $-1$ and $+1$ create double illumination with symmetrical angles generating radial in-plane sensitivity.
The first order diffraction angle $\xi$ of a diffraction grid is related to the pitch of the grid $p_r$ and the light wavelength $\lambda$ of the light source by equation (1):

$$\sin \xi = \frac{\lambda}{p_r}.$$  

(1)

The relationship between the radial in-plane displacement $u_r(r, \theta)$ and the phase difference $\Delta \varphi$, the light wavelength $\lambda$ and the illumination angle $\gamma$ is established by equation (2):

$$u_r(r, \theta) = \frac{\lambda}{4\pi \sin \gamma} \Delta \varphi(r, \theta).$$  

(2)

According to equation (2), the sensitivity of the method changes when the angle $\gamma$ or the wavelength of the light $\lambda$ source are altered.

Figure 44 makes clearly that the diffraction angle $\xi$ and the illumination direction angle $\gamma$ are the same. Therefore, $\sin \xi = \sin \gamma$. Combining equations (1) and (2), the phase to displacement relationship becomes:

$$u_r(r, \theta) = \frac{\lambda}{4\pi} \Delta \varphi(r, \theta) = \frac{p_r}{4\pi} \Delta \varphi(r, \theta).$$  

(3)

Equation (3) relates the radial displacement field $u_r$ and the optical phase distribution $\Delta \varphi$ using the period of the grating $p$. The wavelength of the laser $\lambda$ is cancelled in the equation making this relationship achromatic. This particular and curious effect is understood with the following clarification. When the wavelength of the laser increases/decreases, the sine function of the diffraction angle decreases/increases in the same amount. Therefore, an achromatic speckle interferometer is obtained.

A similar optical arrangement to the layout of figure 43 can be designed to use the DOE. This practical setup of the radial in-plane interferometer is sketched in figure 45. The diverging beam of a diode laser (L) is expanded by the lens (E). After that, the beam passes through the elliptical hole of the 45° mirror $M_1$, impinging mirrors $M_2$ and $M_3$. The light posteriorly is reflected back to the mirror $M_1$. It is clear from the figure that the central hole at $M_1$ allows that the light coming from the laser reaches mirrors $M_2$ and $M_3$. As before, this hole has additional functions. The first one is to prevent direct illumination of the specimen surface having triple illumination. The second one to deliver a viewing window for the CCD (charge-coupled device) camera. At this point of the expansion light, mirror $M_1$ bends the expanded laser light to the collimating lens (CL). Finally, the light is diffracted by the DOE mainly in the 1st diffraction order towards the specimen surface.

In this configuration, $M_2$ and $M_3$ are two concentric circular mirrors. $M_2$ is glued to a piezoelectric actuator. On the other hand, $M_3$ has a central hole with a diameter slightly larger than diameter of $M_2$. According to this setup, $M_2$ is mobile and $M_3$ is fixed. The piezoelectric actuator moves the mirror $M_2$ along its axial direction generating a relative phase difference between the reflected beam coming from $M_2$ (central beam) and $M_3$ (external beam). The border line between both beams is indicated in figure 45 with dashed
lines. Every point over the illuminated area receives only one ray coming from $M_2$ and only one from $M_3$. As consequence the piezoelectric actuator enables the application of phase shifting along the double illuminated area.

As before, the intensity of the light is not constant over the complete illuminated area on the specimen surface. The central point is saturated because it receives light contribution from all cross sections. Consequently, the diameter of mirror $M_2$ and the diameter of central hole at $M_3$ are designed to have a gap of $\sim 1.0$ mm in order to block the light rays reflected to the center of the measurement area.

The optical layout is presented in figure 45 which allows a measurement area of approximately 10 mm in diameter.

6.2. In-plane translation measurement

The radial in-plane interferometer can measure absolute in-plane translations. Assume that the $Z$ axis is perpendicular to the measured surface. Let $T$ be rigid body translation on the $XY$ plane and in the direction forming an angle $\beta$ with the $X$ axis. Equation (4) establishes the relationship between the radial component of the displacement field $u_r(r, \theta)$ and the rigid body translation component. Figure 46 shows a typical wrapped phase for a rigid body translation. Note that the fringes do not depend on the radius, which is in accordance with equation (4):

$$u_r(r, \theta) = T \cos(\theta + \beta).$$  

The analysis of equation (4) and of figure 46 allows the conclusion that in the directions where $\cos(\theta + \beta) = \pm 1$, which corresponds to the translation direction, there is maximum displacement and, consequently, the greatest number of fringe orders. On the other hand, when $\cos(\theta + \beta) = 0$, which corresponds to the direction perpendicular to the translation, the sensitivity is zero, which means that the zero order fringe is always present. Thus, it is always possible to measure rigid body translations in an absolute way.

Translations up to 5.0 $\mu$m can be measured from a single phase difference map. Above this limit the density of fringes becomes too high. To measure greater displacements, it is possible to measure in stages, that is, divide the translation into several stages, measure the translations between each stage and then add them up. Repeatability tests led to standard deviations ranging from 3 to 6 nm [46–48].

6.3. Mechanical stress measurement

When the measured region is subjected to a stress state a displacement field is formed. The relationship between the radial component of the displacements field and the stress state can be obtained from the displacement-stress relationships according to the theory of elasticity. In this case, polar coordinates are used in function of the main stress 1 and 2, which are located in the angles to which the strains and stresses have the maximum and minimum magnitudes respectively. Let $\beta$ be the angle between the main axis 1 and the $X$
Figure 47. Phase map obtained for a uniaxial stress field applied in the vertical direction.

The radial displacement field in the plane is related to the main deformations and stresses through the following equations [48, 49]:

\[ u_r(r, \theta) = \frac{r}{2} \left[ (\varepsilon_1 + \varepsilon_2) + (\varepsilon_1 - \varepsilon_2) \cos(2\theta - 2\beta) \right] \]  

\[ u_r(r, \theta) = \frac{r}{2E} \left[ (1 - \nu) (\sigma_1 + \sigma_2) + (1 + \nu) (\sigma_1 - \sigma_2) \cos(2\theta - 2\beta) \right] \]  

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are the principal strains, \( \sigma_1 \) and \( \sigma_2 \) are the principal stresses, \( \beta \) is the principal angle and \( E \) and \( \nu \) are the material’s Young modulus and Poisson ratio respectively.

Figure 47 presents an example of a phase difference pattern measured with a speckle interferometer with radial in-plane for a uniaxial stress field applied in the vertical direction.

Figure 48 shows a sketch of the experimental setup. This device allows obtaining several levels of mechanical stresses. The working range of the mechanical strains is between 50 to 800 \( \mu \)m m\(^{-1}\). According to this scheme, a compressive load is applied to the arms of the U-body, opening them. Therefore, a four-point bending configuration is achieved generating a uniform bending moment over the base of the U-body (inset I1 shows the stress distribution). A tensile stress is acting on the bottom surface which is monitored by means of a couple of two-element strain gauge rosettes symmetrically placed from the center of the U-body. On the other hand, a compressive stress is applied on the top of the base which is measured by the optical strain sensor. Because of the bending configuration, both sensors, namely the optical device and the strain gage (SG) rosettes measure the same strain magnitude but with opposite signs. According to the figures, it can be seen that SGs are in the bottom part of the U-body to avoid interfering during the setting of the optical strain sensor.

This experimental procedure is repeated for many strains levels. The stress field is computed for the reference strain value of 50 \( \mu \)m m\(^{-1}\). They are also computed for the other strain levels up to 800 \( \mu \)m m\(^{-1}\) with increments of 50 \( \mu \)m m\(^{-1}\). For larger reference strain values, relative deviations deviate between 1.4% and 4.3% [49]. Repeatability tests led to standard deviations ranging from 2 to 6 \( \mu \)m m\(^{-1}\) [47].

6.4. Residual stress measurement

Residual stresses are stress fields that develop in mechanical components without the action of external loads. They are always in self-equilibrium [50, 51]. When residual stresses are combined with the service stresses of components in use, the resulting values can exceed the material strength limits and give rise to failures.

There are different methods to qualitatively and quantitatively characterize residual stresses in engineering materials. In general, these methods are classified as destructive or non-destructive.

The former is based on the local relief of the state of the residual stresses in equilibrium. Thus, the residual stress is quantified by the measurement of the effects generated by the local stress relieving (deformations or displacements).

The procedure used to measure can be briefly described as follows:
A new local stress state is created because of the machining or removing a layer of the stressed material. The local stress variation is detected by the measurement of deformation or displacement fields. The residual stresses are computed as a function of the deformation or the displacement through the use of a model based on the theory of elasticity.

The most important semi-destructive methods is the hole drilling method. It is the most currently and widely used for industrial and laboratorial applications [52–54].

6.4.1. Residual stress measurement with hole-drilling
The measurement of residual stress fields with unknown principal directions is an extremely attractive application for radial in-plane speckle interferometry systems, based on the optical layout shown figure 45. The measurement is also allowed due to the combination of the interferometer with a hole drilling device. To achieve this, a portable measurement device can be obtained throughout modular configuration.

This modular configuration is shown in figure 49. It has two modules: (a) the measurement module (MM) and (b) the hole drilling module (DM). Both modules are attached to a same rotational base (RB) and assembled on a supporting base (SB). The automatic rotation of RB is performed by using the combination of a small DC motor (M) and two thin cross section ball bearings (between SB and RB). This combined system enables a fast and easy interchanging between both modules (for additional details, please see inset). Moreover, high accuracy and a smooth performance is achieved during the rotation. Both modules are correctly positioned the acquisition and drilling operations, with repeatability better than 0.5 µm. An axis-symmetric binary DOE is the most essential optical part inside the MM [45, 55]. Considering [45], the speckle interferometer system computes residual stress fields with an uncertainty of 7%.

The usual experimental procedure for the measurement of residual stresses is the following. The portable device (figure 49), is positioned on the measurement region as the first procedure step. Four magnetic legs and three feet with sharp conical tips clamp rigidly the system on the surface. As second step, the MM is located on the measured area and the reference phase is acquired. In the third step, the hole DM is
automatically positioned by rotating the plate (RB) to drill the first hole step (up to a depth of $\sim 0.05$ mm). After that, the MM is relocated over the measurement region to acquire a new phase corresponding to the hole increment. Figure 50 shows the portable device during its application in in-field measurements. The wrapped and unwrapped phase difference maps as well as the radial in-plane displacement field are computed for this hole step. The previous procedure (phase acquisition, module rotation, drilling, module rotation) is repeated 20 consecutive times to compute the discrete profile of stresses in along 1.0 mm hole depth. The usual diameter of the end mill is $\sim 1.6$ mm. As good practice, the end mill is replaced for a new one before to start a new residual stress measurement.

As an example figure 51 shows a residual stress comparison measurement performed in an oil transportation pipe sample applying the hole-drilling technique with the portable digital speckle pattern interferometer system (DSPI) and with a classical SG device. According to this figure, readers can observe that both techniques identified similar longitudinal stresses ($\sigma_L$) and circumferential stresses ($\sigma_C$) distributions into the pipe sample.

6.4.2. Residual stress measurement with indentation

An alternative possibility for measuring residual stress is combining indentation with the radial in-plane speckle interferometer. This possibility was explored and evaluated in [56, 57]. The measurement principle involves the application of a controlled indentation to the surface of the specimen by a spherical tip indenter. Therefore, a local yielding is created, and the superficial material of the specimen displaces away from the indentation print. Contrariwise to the drilling method, the indentation adds stresses into the material by creating a local plastic zone without the release of residual stresses. Additionally, it introduces a compressive stress field and affects a small region of the material since its depth is about 0.3 mm being smaller than the hole depth obtained in a hole drilling measurement. Figure 52 exemplifies the wrapped difference maps...
Figure 52. Difference phase maps obtained after indentation (a) stress-free material (b) with a residual stress of 200 MPa. (c) Difference between (a) and (b) (from [56]).

Figure 53. Left: the indentation module with spherical indenter. Right: radial in-plane speckle interferometer.

Figure 54. MM and indentation one under a measurement situation.

measured in (a) a stress-free steel plate, (b) a 200 MPa loaded plate and (c) the difference between the previous phase patterns. The basic shape of the fringe pattern is not far from the fringes obtained by the hole-drilling method.

Posteriorly [57], presented an instrumented indentation module which is shown in the left part of figure 53. This indenter enables a maximum indentation loading and penetration depth of ~2000 N and ~0.3 mm, respectively. To have a compact device compatible with the MM, the indenter uses a hydraulic amplifier to produce high loads with small mechanical parts and drivers. A load cell and an inductive sensor make possible to measure indentation load and indentation penetration. The control software allows force driven or penetration driven modes.

The right part of figure 53 shows the MM. It is based on the laser expansion shown in figure 45. The universal base has a kinematic interface allowing a fast and accurate interchanging of the modules. Moreover, the interface ensures positioning repeatability better than 0.5 µm.

Figure 54 shows the two possible configurations. On the left side of figure 54 readers can observe the clamping base and the MM mounted on it. Two fixing straps ensure rigid clamping of the basis to a steel pipe where residual stresses are measured. The right side of the figure shows the indentation module installed on the base in another measurement position (upside down).

The 200 mm diameter steel tube shown in figure 54 is part of a 12 m long test bench where the tube is subjected to known bending stress. Eight cross-sections, named as S1 to S8, are considered for measurement in test pipe. Two strain gauges are installed in opposite positions along the perimeter to control and measure the level of bending stress applied in each cross-section.
When the bending load applied the test bench, the eight cross-sections were evaluated summing 160 measurement points in the pipe. As an example, figure 55 shows wrapped maps obtained in different angular positions for the perimeter of the cross-section under the effect of the largest bending moment. The figure also shows that the neutral line lays along the vertical direction. The phase maps, for the angular positions labeled as 0° and 180°, show circular fringe patterns which are mainly related to the indentation effect and to the manufacturing residual stresses of the pipe. Therefore, these points are clearly not influenced by the bending stresses.

The circular fringes are progressively deformed according to the right-hand side of figure 55(a). The map for the angular position of 90° has the largest deformation along the horizontal direction. This effect shows the presence of tractive stresses, which reasonably agrees with the bending condition applied in right-hand side of the pipe. On the other hand, the left-hand side of figure 55(a) present fringe maps that are lengthened along the vertical direction of the phase images. This fact corresponds to a compressive stress state. This effect is emphasized in figure 55(b) by the selected point A. Arrows show the displacement direction and the point on the material under the effect of the bending stress.

### 6.4.3. Residual stress measurement with thermal relaxation

Regardless of the excellent performance of hole-drilling technique to measure residual stresses has been firmly proven, this experimental technique is semi-destructive due to the final presence of a small hole in the material under evaluation. Consequently, alternative non-destructive methods for measuring residual stresses have been investigated during the last years. Among them, local heating technique is a highlighted stress relaxation technique that was first introduced by Pechersky et al \[58\] and posteriorly developed by Vikram et al \[59\] and Viotti et al \[42\]. These authors proved the viability for the measurement of residual stresses by the combination of digital speckle pattern interferometry (DSPI) and local annealing.

If the hole-drilling technique is applied, the magnitude and direction of the residual stresses is computed by using an analytic solution developed by Kirsch. This computation is performed from the displacement field generated around the hole \[50\]. Unfortunately, if local heating is used, there is a lack of a close solution relating the residual stresses and the relieved displacement field. Alternatively, the heating process can be simulated using finite element analysis. Thus, the thermomechanical displacement field around the heating spot can be simulated. With the numerical solution in hands, a modified approach can be achieved from the hole-drilling solution to compute the residual stresses.

The portable measurement device used in \[42\] is a modular device composed of three parts: a measurement (MM) and a heating (HM) module and universal base (UB) \[41\]. In order to reduce relative
movements between the base (UB) and the specimen surface, this base is rigidly clamped throughout four magnetic legs and three feet with sharp conical tips.

The local heating technique uses a contact resistance. Thus, an electrical current is applied by means of a soft graphite tip. For this application, the heating module uses a carbon electrode (2 mm in diameter) connected to a controlled power supply.

As the previous systems, the measurement and the heating modules located in the base by using a kinematic interface.

The next experimental procedure was used to make the measurements. As a first step, the specimen was mounted in the loading device. The universal base was placed in the interest region and rigidly clamped. After that, the MM was placed in the universal base with the kinematic device. Next, the load was applied in the test specimen in a gradual way. Thus, the applied stress was computed from the axial displacement measured with the interferometer.

The next step is the acquisition of a set of phase-shifted specklegrams to compute the reference phase distribution by the Carré algorithm. Then, the MM was replaced by the heating one. Thus, the specimen was locally heated during a period of 5 s. Posteriorly, the heating module was replaced again by MM. After waiting time of 120 s, a new set of specklegrams was acquired and the after-heating phase distribution was calculated. Finally, the wrapped phase difference was evaluated.

The temperature achieved for the heating process was monitored using a thermocouple located at the back side of the specimen. The measurement allowed to adjust the process to the numerical simulation. The instrumentation with the thermocouple was carefully performed to ensure that the contact pressure of did not influence the measurements obtained with the interferometer. The average measured temperature was $\sim 190^\circ C$.

As a usual example, figure 56 shows the wrapped map measured for an applied stress equivalent to the 45% of the yield stress of the material (67.5 MPa).

Several preloaded specimens subjected to different stress values were analyzed to evaluate the performance of the combined interferometer for the measurement of residual stresses.

The measurements executed with the radial in-plane interferometer were processed and the result was compared with the applied tensile stress. This comparison showed that the DSPI-local-heating combined system allowed the measurement of residual stresses with an average relative uncertainty of $\sim 7\%$ for uniaxial stresses ranging from 25% to 75% of the yield stress of the material.

7. Summary chart

Table 1 presents typical values for the main performance parameters for each of the 16 full-field optical configurations for measurement in cylindrical or polar coordinates. Although it is not pertinent to make a full comparison of the numerical parameters, since different physical quantities are often measured, some characteristics will be discussed below.

Columns 2 and 3 contain, respectively, a short description and the section number where the referred configuration is discussed. The measured quantity is in column 4. There are eight different physical quantities.

Column 5 shows the typical measurement ranges. For some configurations, these limits can be extended with the use of different optical components. In the configurations marked with (b), the measurement range can be easily extended when loading in progressive stages is possible. Measurements are sequentially made.
Table 1. Summary chart with performance parameters for all 16 configurations.

| Configuration          | Section | Measured quantity | Measurement range | Uncertainty (95%) | Measured field | Pixel size | Alignment effort | Stability requirements |
|------------------------|---------|-------------------|-------------------|-------------------|----------------|------------|-----------------|-----------------------|
| 1                      | Texture mapping | 3      Grove angles | $-180^\circ$ to $+180^\circ$ | $\pm 1.5^\circ$ | $360^\circ \times 50 \text{ mm}^2$ | 0.20 mm | Self-centering | Not critical            |
| 2                      | Triangulation   | 4.1    Inner radius | 70–80 mm          | $\pm 200 \mu \text{ m}$ | $360^\circ \times 30 \text{ mm}$ | 0.6–1.0 mm | Self-centering | Not critical            |
| 3                      | Active photogrammetry | 4.2    Inner radius | 74–80 mm          | $\pm 20 \mu \text{ m}$ | $360^\circ \times 2–6 \text{ mm}$ | 0.02–0.12 mm | Difficult | Low stability          |
| 4                      | White light interf. Internal | 4.3.1  Inner radius | 7–50 mm           | $\pm 0.35 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.01–0.06 mm | Difficult | High stability        |
| 5                      | White light interf. external | 4.3.2 Internal radius | 2–12 mm           | $\pm 0.35 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.01–0.06 mm | Difficult | High stability        |
| 6                      | White light interf. wear | 4.3.3  External radius | 2–12 mm           | $\pm 0.70 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.01–0.06 mm | Difficult | High stability        |
| 7                      | Deflectometry   | 4.4    Radius deviation | $\pm 0.20 \mu \text{ m}$ | $\pm 0.015 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.01–0.06 mm | Difficult | High stability        |
| 8                      | DSPI external   | 5.1.1  Out-of-plane radius displacement | $\pm 2.0 \mu \text{ m}$ | $\pm 0.015 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.01–0.06 mm | Difficult | Medium              |
| 9                      | DSPI internal short | 5.1.2  Out-of-plane radius displacement | $\pm 2.0 \mu \text{ m}$ | $\pm 0.015 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.01–0.06 mm | Difficult | Medium              |
| 10                     | DSPI internal long | 5.1.3  Out-of-plane radius displacement | $\pm 4.0 \mu \text{ m}$ | $\pm 0.05 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.01–0.06 mm | Difficult | Medium              |
| 11                     | Shearography    | 5.2    Out-of-plane radius displacement | $\pm 2.0 \mu \text{ m}$ | $\pm 0.015 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 10 mm | Medium | Medium              |
| 12                     | DSPI in-plane translation | 6.2     In-plane deformation | $\pm 50 \mu \text{ m}$ | $\pm 0.06 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.015 mm | Medium | Medium              |
| 13                     | DSPI hole-drilling | 6.3     In-plane residual stress | $\pm 150 \mu \text{ m}$ | $\pm 0.05 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.015 mm | Medium | Medium              |
| 14                     | DSPI indentation | 6.4.1  In-plane residual stress | $\pm 400 \mu \text{ m}$ | $\pm 0.10 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.015 mm | Medium | Medium              |
| 15                     | DSPI thermal relaxation | 6.4.2  In-plane residual stress | $\pm 300 \mu \text{ m}$ | $\pm 0.15 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.015 mm | Medium | Medium              |
| 16                     | DSPI indentation | 6.4.3  In-plane residual stress | $\pm 300 \mu \text{ m}$ | $\pm 0.15 \mu \text{ m}$ | $360^\circ \times 20 \text{ mm}$ | 0.015 mm | Medium | Medium              |

Note: Measurement field for one positioning only. Can be extended by stitching multiple adjacent measurements.

b Measurement range for a single-stage loading. Can be extended by dividing the loading into stages.
for each loading stage, processed and then added together. The typical measurement uncertainty, for the 95% confidence level, is in column 6. Under very favorable or very unfavorable conditions, these values can vary between 50% to 200% of typical estimated values respectively.

The measurement field refer to the full field region that can be measured by the configuration. All cases involving cylindrical coordinates (1–11) cover the 360° angle. The cylindrical region longitudinal lengths are typical. They can be enlarged with the use of special optical components or, for the cases marked with (a), stitching together sequential measurements of adjacent regions, always maintaining 20%–30% of overlapping. For cases involving polar coordinates (12–16), the value shown corresponds to the typical diameter of the measured circular area. These values can be extended by modifying the configuration using different optical components.

The lateral resolution is naturally variable throughout the image since there is a mapping of the cylindrical, or polar, coordinates to the rectangular coordinates of the image. It is not very easy to estimate the lateral resolution in the presence of effects such as speckles and astigmatism resulting from imaging through conical mirrors. Instead of the lateral resolution, column 8 refer to the pixel size projected on the measured surface for typical configurations. A camera with a 1600 × 1200 pixel sensor was considered.

Column 9 evaluates subjectively the effort required to align the part to be measured with the optical measurement system. At the current stage of development, systems using white light interferometry and deflectometry are more difficult to align, which, depending on experience, can involve 5–15 min of work. There are some promising results with automated procedures for alignment [27]. The alignment of speckle interferometry systems to measure out-of-plane radial displacements requires a medium alignment effort. In general, it should be sufficient to obtain uniform illumination on the measured surface. Angular misalignment errors are cosine errors, which become small for misalignments less than 1° and can be corrected by software. Configurations 1–3 have self-centering mechanisms, which make the alignment very practical. Configurations 12–16 have leveling foot that make the alignment as simpler as possible.

Finally, the stability requirements presented in column 10 are related to the sensitivity of the configuration to disturbances, mainly caused by mechanical vibrations and air movements. Some configurations (3, 4–10 and 12) are very sensitive and need to be operated on an isolated optical table in a lab environment. Other configurations, although sensitive, have robust measurement principles or robust constructive aspects, which make them more tolerant for operation outside the laboratory (3, 11, 13–16). There are others that do not involve high sensitivity and can operate in environments without special stability requirements.

Although configurations 2, 3 and 4 measure inner radius, they present very different measurement uncertainties, stability requirements and user skill requirements. Configuration 2 has been used successfully on cylinders of a few tens of meters with a measurement speed of 50 mm s⁻¹ and a coarser lateral resolution. System 3 has intermediate lateral resolution and measurement uncertainty and it is robust enough for use in the field. Configuration 4 can only be used in the laboratory and has excellent measurement uncertainty as well as lateral resolution.

Deflectometry is not suitable for measuring absolute radius values. It is only possible to measure variations in radius in cross sections. It is a much simpler and cheaper configuration than the white light interferometer. Its current performance is not far from white light interferometry for cylindrical parts. However, as it is a new configuration, great progress is expected in the coming years. The extension of this configuration to measure internal cylinders is still an open possibility.

Configurations 14–16 for measuring residual stresses differ by the way stress relief is produced: indentation, drilling or thermal relaxation. They result in different amounts of local damage to the measured part. In each case there are different mathematical models involved that lead to different measurement ranges and measurement uncertainties. Typical values are presented. They can vary depending on the condition of the surface, the type of material and the measurement conditions.

8. Conclusions

The paper brings together several techniques and configurations for full-field optical measurement in cylindrical coordinates or with polar sensitivity. Sixteen different configurations are described, mostly using conical mirrors, involving eight different measurement techniques and 13 different applications.

Most of systems and configurations presented here were developed with a focus on meeting concrete measurement demands, mostly motivated by the industry. The examples presented here make this relationship clear.

We hope that the configurations, solutions and properties presented here can serve as a reference or as a source of inspiration for new configurations and applications that are to come in an area that has great demands not yet fully met.
Data availability statement

No new data were created or analyzed in this study.

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