Chaos and quantum interference effect in semiconductor ballistic micro-structures

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We study the quantum-interference effect in the ballistic Aharonov-Bohm (AB) billiard. The wave-number averaged conductance and the correlation function of the non-averaged conductance are calculated by use of semiclassical theory. Chaotic and regular AB billiards have turned out to lead to qualitatively different semiclassical formulas for the conductance with their behavior determined only by knowledge regarding the underlying classical scattering.

1. INTRODUCTION

Recently a growing number of works accumulated around the subject of an interplay between quantum chaos and ballistic quantum transport in mesoscopic systems \cite{1,2}. Particularly, an important role of quantum chaos is addressed in quantum interference in Aharonov-Bohm (AB) billiards. In this paper, we shall focus our attention to the latest issue of our research on the $\hbar/2e$ Altshuler-Aronov-Spivak (AAS) oscillation \cite{3,4} and $\hbar/e$ AB oscillation \cite{5}. We shall present the analysis of semiclassical theory of AAS and AB oscillation in an open chaotic system, e.g., the Sinai billiard. Comparative study on other regular billiards will also be presented.

2. $\hbar/2e$ AAS OSCILLATION

In this section, we shall consider an open single AB billiard in uniform normal magnetic field $B$ penetrating only through the hollow. The ballistic weak localization (BWL) correction \cite{6} is most easily discussed in terms of the reflection coefficient $R = \sum_{n,m=1}^{N_M} |r_{n,m}|^2$, where $N_M$ and $r_{n,m}$ are the mode number and the reflection amplitude, respectively. Therefore, our starting point is the reflection amplitude \cite{6}

$$r_{n,m} = \delta_{n,m} - i\hbar \nu_m \int dy \int dy' \psi_n^*(y') \psi_m(y) G(y', y, E_F), \quad (1)$$

where $\nu_m (\nu_n)$ and $\psi_n (\psi_m)$ are the longitudinal velocity and transverse wave function for the mode $m(n)$ at a pair of lead wires attached to the dot. $G$ is the retarded Green’s function connecting points $(x, y)$ and $(x', y')$ on the left and right leads, respectively.

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In order to carry out the semi-classical approximation, we replace $G$ by its semiclassical Feynman path-integral expression \[8\],

\[
G^{sc}(y', y, E) = \frac{2\pi}{(2\pi i \hbar)^{3/2}} \sum_{s(y, y')} \sqrt{D_s} \exp \left[ i/h S_s(y', y, E) - i\frac{\pi}{2} \mu_s \right],
\]

where $S_s$ is the action integral along classical path $s$, $D_s = (v_F \cos \theta')^{-1} |(\partial \theta/\partial y')_y |$, $\theta$ ($\theta'$) is the incoming (outgoing) angle, and $\mu_s$ is the Maslov index.

Substituting eq.(2) into eq.(4) and carrying out the double integrals by the saddle-point approximation, we obtain

\[
r_{n,m} = -\frac{\sqrt{2\pi i \hbar}}{2W} \sum_{s(\tilde{n}, \tilde{m})} \text{sgn}(\tilde{n}) \text{sgn}(\tilde{m}) \sqrt{D_s} \exp \left[ \frac{i}{\hbar} \tilde{S}_s(\tilde{n}, \tilde{m}; E) - i\frac{\pi}{2} \tilde{\mu}_s \right],
\]

where $W$ is the width of leads and $\tilde{m} = \pm m$. The summation is over trajectories between the cross sections at $x$ and $x'$ with angles $\sin \theta = \tilde{m}\pi/kW$ and $\sin \theta' = \tilde{n}\pi/kW$. In eq.(3), $\tilde{S}_s(\tilde{n}, \tilde{m}; E) = S_s(y_0', y_0; E) + h\pi(\tilde{m}y_0 - \tilde{n}y_0')/W$, $\tilde{D}_s = (m_n v_F \cos \theta')^{-1} |(\partial y/\partial \theta')_\theta |$ and $\tilde{\mu}_s = \mu_s + H (- (\partial \theta'/\partial y')_y ) + H (- (\partial \theta'/\partial y')_\theta )$, respectively, where $H$ is the Heaviside step function.

Taking the diagonal approximation, there is a natural procedure for finding the average of $\delta R_D = \sum_{n=1}^{N_M} \delta R_{n,n}$ over wave-number $k$, which is denoted by $\delta \mathcal{R}_D$. Therefore the contribution to the BWL correction term $\delta \mathcal{R}_D(\phi)$ is just given by the pair of time reversal paths. With use of the extended semiclassical theory \[3\], we can take account of the off-diagonal part and the influence of the small-angle diffraction as $\delta \mathcal{R}(\phi) = \delta \mathcal{R}_D(\phi)/2$, for the case that the width of the lead wires are equal. Then we obtain the full quantum correction of the conductance for chaotic AB billiards as

\[
\delta g(\phi) = -e^2 \frac{\cosh \eta - 1}{\pi \hbar} \left\{ 1 + 2 \sum_{n=1}^{\infty} \exp(-\eta n) \cos \left( 4\pi n \frac{\phi}{\phi_0} \right) \right\},
\]

where $\eta = \sqrt{2T_0/\alpha}$. \[3\] System-dependent constants $\alpha$, $T_0$ and $\gamma$ correspond to the variance of the winding number distribution \[10\], the dwelling time for the shortest classical orbit and the escape rate, i.e., $N(T) \sim \exp(-\gamma T)$, \[3\] respectively. In eq. (4), the period of the conductance oscillation is $h/2e$, analogous to the AAS oscillation in diffusive systems. \[12\] In chaotic case, the oscillation amplitude decays exponentially with increasing the rank of higher harmonics $n$, so that the main contribution to the conductance oscillation comes from $n = 1$ component which oscillates with the period of $h/2e$.

On the other hand, for regular AB billiards, we obtain

\[
\delta g(\phi) = \frac{1 + 2 \sum_{n=1}^{\infty} \Gamma \left( \beta - \frac{1}{2}, \beta + \frac{1}{2}, \frac{n^2}{2\alpha} \right) \cos \left( 4\pi n \frac{\phi}{\phi_0} \right)}{1 + 2 \sum_{n=1}^{\infty} \Gamma \left( \beta - \frac{1}{2}, \beta + \frac{1}{2}, \frac{n^2}{2\alpha} \right)},
\]

where $\Gamma$ and $\alpha$ are the hypergeometric function of confluent type and the exponent of dwelling time distribution $N(T) \sim T^{-\alpha}$ \[3\], respectively. In eq.(5) the oscillation amplitude decays algebraically for large $n$, therefore the higher-harmonics components give
noticeable contribution to magneto-conductance oscillation. These discoveries indicate that the $h/2e$ AAS oscillation occurs in both ballistic and diffusive systems forming AB geometry and the behavior of higher harmonics components reflects a difference between chaotic and non-chaotic classical dynamics.

In real experiments [13], magnetic field would be applied to all region (both the hollow and annulus) in the billiard. In this situation, we will observe $h/2e$ oscillation together with the negative magneto-resistance and dampening of the oscillation amplitude with increasing magnetic field as in the case of diffusive AB rings. [14]

3. $h/e$ AB OSCILLATION

In previous section, we have investigated the $h/2e$ AAS oscillation for energy-averaged magneto-conductance. The result of quantum-mechanical calculations [4] indicated that the period of the energy averaged conductance,

$$<g(\phi)>_E = \frac{1}{\Delta E} \int_{E_{F} - \Delta E/2}^{E_{F} + \Delta E/2} g(E, \phi) dE,$$

(6)

changed from $h/2e$ to $h/e$, when the range of energy average $\Delta E$ is decreased. In this section, we shall calculate the correlation function $C(\Delta \phi)$ of the non-averaged conductance by using the semiclassical theory and show that $C(\Delta \phi)$ is qualitatively different between chaotic and regular AB billiards. [5]

The fluctuations of the conductance $g = (e^2/\pi\hbar)T(k) = (e^2/\pi\hbar)\sum_{n,m} |t_{n,m}|^2$ are defined by their deviation from the classical value; $\delta g \equiv g - g_{cl}$. In this equation $g_{cl} = (e^2/\pi\hbar)T_{cl}$, where $T_{cl}$ is the classical total transmitted intensity. In order to characterize the $h/e$ AB oscillation, we define the correlation function of the oscillation in magnetic field $\phi$ by the average over $\phi$, $C(\Delta \phi) \equiv \langle \delta g(\phi)\delta g(\phi + \Delta \phi)\rangle_{\phi}$. With use of the ergodic hypothesis, $\phi$ averaging can be replaced by the $k$ averaging, i.e., $C(\Delta \phi) = <\delta g(k, \phi)\delta g(k, \phi + \Delta \phi)>_k$. The semiclassical correlation function of conductance is given by

$$C(\Delta \phi) = \left(\frac{e^2}{\pi\hbar}\right)^2 \frac{1}{8} \left(\cosh\eta - 1 \over \sinh\eta\right)^2 \cos\left(2\pi\frac{\Delta \phi}{\phi_0}\right) \left\{1 + 2\sum_{n=1}^{\infty} e^{-\eta n} \cos\left(2\pi n \frac{\Delta \phi}{\phi_0}\right)\right\}^2,$$

(7)

where $\eta = \sqrt{2T_0^2\gamma}/\alpha$. In deriving eq. (6) we have used the exponential dwelling time distribution and the Gaussian winding number distribution.

On the other hand, for the regular cases, we use $N(T) \sim T^{-\beta}$. [6] Assuming as well the Gaussian winding number distribution, we get

$$C(\Delta \phi) = C(0) \cos\left(2\pi\frac{\Delta \phi}{\phi_0}\right) \left\{1 + 2\sum_{n=1}^{\infty} F\left(\beta - \frac{1}{2}, \beta + \frac{1}{2}; -\frac{n^2}{2\alpha}\right) \cos\left(2\pi n \frac{\Delta \phi}{\phi_0}\right)\right\}^2.$$

(8)

Therefore, these results indicate that the difference of $C(\Delta \phi)$ of these ballistic AB billiards can be attributed to the difference between chaotic and regular classical scattering dynamics.
4. SUMMARY

The statistical aspect of classical open AB billiards is characterized by the dwelling time distributions $N(T)$ which reflects the integrability of the system. For chaotic billiards, $N(T)$ obeys the exponential distribution. On the other hand, for regular AB billiards, $N(T)$ obeys the power-law distribution. The difference of the distributions affects the AAS and AB oscillation in the ballistic regimes.

In conclusion, we indicated a way to observe a quantum signature of chaos through the novel quantum interference phenomena in ballistic AB billiards.

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