Dynamics of Charged Radiating Collapse in Modified Gauss-Bonnet Gravity

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Abstract

This paper deals with the dynamics of a shearfree charged radiating collapse in modified Gauss-Bonnet gravity. The field equations for shearfree spherical interior geometry of a charged dissipative star are formulated. To study the dynamical behavior of collapsing matter, we derive the dynamical as well as transport equations. We conclude that the gravitational force in modified Gauss-Bonnet gravity is much stronger as compared to general relativity which implies the increase in the rate of collapse. Finally, we study the effect of charge on the dynamics of collapse.

Keywords: Modified Gauss-Bonnet gravity; Charged Dissipative fluid; Gravitational collapse.

1 Introduction

During the last few decades, there has been a growing interest to study fate of universe in alternative theories of gravity. These theories provide the
recognition of dark energy (DE) which might be the major agent for the rapid expansion of the universe [1]-[4]. The consistency of such theories has been confirmed with the observations and experiments [5]-[7]. The most simple modification to general relativity (GR) is $f(R)$ theory of gravity in which $f$ is an arbitrary function of the Ricci scalar $R$. Although, it is the simplest generalized form of GR but one can formulate such general models of $f(R)$ that are consistent with the gravitational experiments. The validity of $f(R)$ models requires some additional restrictions due to which the theory loses its original features [8]-[10].

The modified Gauss-Bonnet theory, so called $f(G)$ gravity has been proposed by several authors [11]-[13] as an alternative theory of gravity. The mathematical structure of this theory can be obtained by introducing some arbitrary function $f(G)$ in Einstein-Hilbert action of standard GR, where $G = R^2 - 4R^\mu\nu R_{\mu\nu} + R^\mu\nu\gamma\delta R_{\mu\nu\gamma\delta}$ is the Gauss-Bonnet invariant. This theory is consistent with observational constraints and might be helpful for reproducing the cosmic history [14]. It has been found that transition of the universe from matter dominated era to accelerated phase would be explained in the framework of $f(G)$ theory of gravity [15]. Myrzakulov et al. [16] have investigated that in this theory ΛCDM model can be well explained without cosmological constant term.

The dynamics of the radiating gravitational collapse is an important issue in GR. Initially this problem was formulated by Misner and Sharp [17] for non-radiating collapse and by Misner [18] for radiating collapse. The radiating process during the formation of neutron star or black hole [19] is important due to the energy loss. It has been investigated [20, 21] that gravitational collapse is a radiating process, so the effects of dissipation must be studied for the collapse of massive star. Herrera and Santos [22] discussed the dynamics of spherically symmetric shearfree anisotropic dissipative fluid. Chan [23] formulated the realistic model of a dissipative star with shear viscosity.

Herrera [24] examined the role heat flux during dynamics of radiating matter collapse. Herrera et al. [25] also formulated the dynamical equations and causal heat transport equations for viscous and non-adiabatic collapse. They proved that the shearfree slowly evolving and non-dissipative self-gravitating shearfree in Newtonian limit leads to homologous collapse. Di Prisco et al. [26] investigated the dynamics of charged viscous non-adiabatic gravitational collapse. Sharif and his collaborators [27]-[34] have investigated the dynamics of charged and uncharged dissipative gravitational collapse in
GR as well as in modified theories of gravity.

In this paper, we extend our previous work \cite{34}, to the charged case. We use the field equations of modified Gauss-Bonnet gravity \cite{35} derived by using the covariant gauge invariant (CGI) perturbation approach with 3 + 1 formalism. The collapsing matter under consideration has been taken as charged radiating in the interior of a star. The interior geometry of the star is matched with the charged Vaidya geometry by using Darmois junction conditions \cite{36}. This plane of the paper is as follows: In the next section, we present the field equations in modified Gauss-Bonnet gravity for charged radiating shearfree collapse and discuss the matching conditions. We derive the dynamical and transport equations and then couple them in section 3. The last section provides the summary of the results.

2 Field Equations in Modified Gauss-Bonnet Gravity

The Einstein-Hilbert action for the modified Gauss-Bonnet gravity is

\[ S = \int d^4x \sqrt{-g} \left( \frac{R + f(G)}{2\kappa} + \mathcal{L}_m \right), \]  

(1)

where \( g \) is the determinant of the metric tensor \( g_{\mu\nu} \). \( R \) is the Ricci scalar, \( f(G) \) is an arbitrary function of Gauss Bonnet invariant \( G \), \( \mathcal{L}_m \) is the matter Lagrangian and \( \kappa \) is the coupling constant. The variation of this action with respect to the metric tensor gives the field equations

\[
G_{\mu\nu} = \kappa T_{\mu\nu} + \frac{1}{2}g_{\mu\nu} f - 2fGR_{\mu\nu} + 4fGR^\gamma_\mu R^\gamma_\nu - 2fGR^\gamma_\mu\gamma^\delta\omega R^\delta\omega_\nu - 4fGR^\gamma_\mu\gamma^\delta\omega R^\delta\omega_\nu - 2fGR_{\mu\nu}\nabla^2fG - 4fGR^\gamma_\mu\gamma^\delta\omega R^\delta\omega_\nu - 4fGR_{\mu\nu}\nabla^2fG - 4fGR_{\mu\nu}\nabla^\gamma fG - 4fGR_{\mu\nu}\nabla^\gamma fG - 4fGR_{\mu\nu}\nabla^\gamma fG - 4fGR_{\mu\nu}\nabla^\gamma fG - 4fGR_{\mu\nu}\nabla^\gamma fG - 4fGR_{\mu\nu}\nabla^\gamma fG,
\]  

(2)

where \( f_G = \partial f(G)/\partial G \). The metric interior to \( \Sigma \) is assumed to be comoving and shearfree in the following form

\[ ds^2 = A^2dt^2 - B^2(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2), \]  

(3)
where \(A\) and \(B\) are functions of time \(t\) and radial coordinate \(r\). The corresponding four velocity, heat flux and radial vector take the form

\[
V^\mu = A^{-1} \delta^\mu_0, \quad q^\mu = q \delta^\mu_1, \quad \chi^\mu = B^{-1} \delta^\mu_1, \quad (4)
\]

where \(V^\mu q_\mu = 0\). The expansion scalar for the fluid sphere is \(\Theta = \frac{3 \dot{B}}{AB}\).

The energy-momentum tensor for charged radiating fluid is

\[
T_{\mu\nu} = (\rho + p) V_\mu V_\nu - pg_{\mu\nu} + q_\mu V_\nu + q_\nu V_\mu + \frac{1}{4\pi} \left( -F^\gamma_\mu F^\gamma_\nu + \frac{1}{4} F^\gamma_\delta F^\gamma_\delta g_{\mu\nu} \right), \quad (5)
\]

where \(\rho\), \(p\), \(V_\mu\), \(q_\mu\) and \(F_{\mu\nu}\) are density, pressure, four velocity, radial heat flux and anti-symmetric Maxwell field tensor, respectively. The Maxwell equations are given by

\[
F_{\mu\nu} = \Phi_{\nu,\mu} - \Phi_{\mu,\nu}, \quad F^{\mu\nu;\nu} = 4\pi J^\mu, \quad (6)
\]

where \(\Phi_\mu\) is the four potential and \(J_\mu\) is the four current. We assume that charge is at rest with respect to comoving coordinate system, so the magnetic field is zero. Consequently, the four potential and the four current can be chosen as

\[
\Phi_\alpha = \Phi_\delta^0_\alpha, \quad J^\alpha = \sigma V^\alpha, \quad (7)
\]

where \(\Phi = \Phi(t, r)\) is an arbitrary function and \(\sigma = \sigma(t, r)\) is the charge density. For the interior spacetime, the Maxwell field equations take the form

\[
\dot{\Phi}' = \frac{sA}{Br^2}, \quad (10)
\]

where \(s\) \((r) = 4\pi \int_0^r \sigma B^3 r^2 dr\) is the total charge inside the spherical symmetry.

In general, it is very difficult to deal with the field equations \(2\). We use a simplified form of the field equations derived by Li et al. \(35\). For this
purpose, they used the CGI perturbation approach with 3 + 1 formalism. The highly nonlinear terms in $R_{\mu\nu}$ and $R_{\mu\nu;\lambda\sigma}$ can be expressed in the form of dynamical quantities. Consequently, the field equations as well as other quantities are simplified as follows

$$G_{\mu\nu} = \kappa (T_{\mu\nu} + T_{\mu\nu}^{G}),$$

where $T_{\mu\nu}^{G}$ is the Gauss-Bonnet correction term. Using CGI approach, the components of $T_{\mu\nu}^{G}$ are evaluated in terms of the dynamical quantities as

$$\rho_{\mu}^{G} = \frac{1}{\kappa} \left( \frac{1}{2} (f - f_{G} G) + \frac{2}{3} (\Omega - 3 \Psi) (f_{G} \Theta + \tilde{\nabla}^{2} f_{G}) \right),$$

$$-p_{\mu}^{G} = \frac{1}{\kappa} \left( \frac{1}{2} (f - f_{G} G) + \frac{2}{3} (\Omega - 3 \Psi) f_{G} - \frac{8}{9} \Omega (f_{G}^{2} + \tilde{\nabla}^{2} f_{G}) \right),$$

$$q_{\mu}^{G} = \frac{1}{\kappa} \left( -\frac{2}{3} (\Omega - 3 \Psi) (\tilde{\nabla}_{\mu} f_{G} - \frac{1}{3} \Theta \tilde{\nabla}_{\mu} f_{G}) + \frac{4}{3} f_{G} \Theta \zeta_{\mu} \right).$$

Here, $\tilde{\nabla}$ is spatial covariant derivative, $\Theta = V_{;\mu}^{\mu}$ is the expansion scalar and the quantities $\Omega, \Psi, \zeta_{\mu}$ are expressed in terms of dynamical quantities as

$$\Omega = -(\dot{\Theta} + \frac{1}{3} \Theta^{2} - \tilde{\nabla}^{\mu} A_{\mu}),$$

$$\Psi = -\frac{1}{3} (\dot{\Theta} + \Theta^{2} + \tilde{R} - \tilde{\nabla}^{\mu} A_{\mu}),$$

$$\zeta_{\mu} = -\frac{2 \tilde{\nabla}_{\mu} \Theta}{3} + \tilde{\nabla}^{\nu} \sigma_{\mu \nu} + \tilde{\nabla}^{\nu} \omega_{\mu \nu},$$

where $\tilde{R}$ is the Ricci scalar of 3D spatial spherical surface, $\sigma_{\mu \nu} = V_{(\mu \nu)} - A_{(\mu} V_{\nu)} - \frac{1}{3} \Theta h_{\mu \nu}$ (where $h_{\mu \nu} = g_{\mu \nu} - V_{\mu} V_{\nu}$) is the shear tensor, $A_{\mu} = V_{\mu} V_{\nu}$ is four acceleration and $\omega_{\mu \nu} = V_{[\mu \nu]} + \tilde{\nabla}_{[\mu} V_{\nu]}$ is the vorticity tensor.

The Gauss-Bonnet invariant $G$ in CGI approach is

$$G = 2 \left( \frac{1}{3} R^{2} - R^{\mu \nu} R_{\mu \nu} \right),$$

where

$$R = -2 \dot{\Theta} - \frac{4}{3} \Theta^{2} + 2 \tilde{\nabla}^{\mu} A_{\mu} - \tilde{R},$$

$$R^{\mu \nu} R_{\mu \nu} = \frac{4}{3} (\dot{\Theta}^{2} + \dot{\Theta} \Theta^{2} + \frac{1}{3} \Theta^{4}) + \frac{2}{3} (\dot{\Theta} + \Theta^{2}) \tilde{R} - \frac{8}{3} (\dot{\Theta} + \frac{1}{2} \Theta^{2}) \tilde{\nabla}^{\mu} A_{\mu}. $$
Using Eqs. (3), (11), (3) and (4), the field equations yield

\[
8\pi (\rho + \rho^G) A^2 + \frac{(sA)^2}{(rB)^4} = 3 \left( \frac{\dot{B}}{B} \right)^2 - \left( \frac{A}{B} \right)^2 \left( 2 B'' + \frac{B'}{B} \right) + \frac{4B'_{,r}}{Br},
\]

(19)

\[
8\pi (q + q^G) AB^2 = 2 \left( \frac{\dot{B}'}{B} - \frac{\dot{B}B'}{B^2} - \frac{\dot{B}A'}{BA} \right),
\]

(20)

\[
8\pi (p + p^G) B^2 - \frac{(sB)^2}{(rB)^4} = \left( \frac{B'}{B} \right)^2 + \frac{2}{r} \left( \frac{B'}{B} + \frac{A'}{A} \right) + 2 \frac{AB'}{AB} - \left( \frac{B}{A} \right)^2
\]

\times \left( 2 \frac{\ddot{B}}{B} + \left( \frac{\dot{B}}{B} \right)^2 - 2 \frac{\dot{A}B}{AB} \right),
\]

(21)

\[
8\pi (p + p^G) r^2 B^2 + \frac{s^2}{(rB)^2} = r^2 \left( \frac{B''}{B} - \left( \frac{B'}{B} \right)^2 + \frac{1}{r} \left( \frac{B'}{B} + \frac{A'}{A} \right) + \frac{A''}{A} \right)
\]

\[\quad - r^2 \left( \frac{B}{A} \right)^2 \left( 2 \frac{\ddot{B}}{B} + \left( \frac{\dot{B}}{B} \right)^2 - 2 \frac{\dot{A}B}{AB} \right),
\]

(22)

where \( \rho^G \), \( p^G \) and \( q^G \) correspond to Gauss-Bonnet contribution to GR. Making use of Eqs. (12)-(14), these turn out to be

\[
\kappa \rho^G = \frac{1}{2} (f - Gf_G) + \frac{4}{3B^2} \left( 3 \frac{\dot{B}^2}{A^2} - \left( \frac{B'}{B} \right)^2 + \frac{4B'_{,r}}{Br} + 2 \frac{B''}{B} \right)
\]

\times \left( 3 \frac{f_G \dot{B}}{AB} + \nabla^2 f_G \right),
\]

(23)

\[
\kappa p^G = \frac{1}{2} (Gf_G - f) - \frac{4}{3B^2} \left( 3 \frac{\dot{B}^2}{A^2} - \left( \frac{B'}{B} \right)^2 + \frac{4B'_{,r}}{Br} + 2 \frac{B''}{B} \right) \ddot{f}_G
\]

\[\quad - \frac{8}{3} \left( \frac{\dot{B}}{AB} - \frac{\dot{A}B}{A^2 B} - \frac{\ddot{B}^2}{AB^2} + \frac{\ddot{B}^2}{A^2 B^2} - \frac{1}{3} \frac{A''}{A} - \frac{A'^2}{A^2} + \frac{2}{r} \frac{A'}{A} + \frac{A'B''}{AB} \right)
\]

\times \left( 3 \frac{f_G \dot{B}}{AB} + \nabla^2 f_G \right),
\]

(24)
\[ \kappa q^G = \frac{4}{3B^2} \left( \frac{3\dot{B}^2}{A^2} - \left( \frac{B'}{B} \right)^2 + \frac{4B'}{Br} + 2\frac{B''}{B} \right) \left( \frac{\dot{B}f_G}{BA} - \dot{f}_G \right) + 8f_G \hat{A} \left( \frac{\dot{B}B'}{AB^2} + \frac{\dot{B}A'}{A^2B} - \frac{\dot{B}'}{AB} \right). \] (25)

The Gauss-Bonnet invariant is found from Eq.(18) as

\[ G = \frac{2}{3} \left( -6 \left( \frac{\dot{B}}{AB} \right)' - 12 \frac{\dot{B}^2}{(AB)^2} - 2 \frac{A'B'}{AB} - \left( \frac{A'}{A} \right)' \right) + 4\frac{A'(Br)^'}{AB^3r} - 2 \left( \frac{4B'}{B^3r} + 2\frac{B''}{B^3} - \frac{B'^2}{B^3} \right)^2 - 24 \left( \frac{\dot{B}}{(AB)} \right)^2 \]
\[ - \frac{8}{3} \left( \frac{3\dot{B}}{AB} \right)^2 \left( \frac{3\dot{B}}{AB} \right)' + \frac{1}{3} \left( \frac{3\dot{B}}{AB} \right)^2 - \frac{8}{3} \left( \frac{3\dot{B}}{AB} \right)' \left( \frac{3\dot{B}}{AB} \right)^2 \]
\[ \times \left( \frac{4B'}{B^3r} + 2\frac{B''}{B^3} - \frac{B'^2}{B^3} \right) + 16 \left( \frac{3\dot{B}}{AB} \right)' + \frac{1}{2} \left( \frac{3\dot{B}}{AB} \right)^2 \]
\[ \times \left( -\frac{1}{B^2} \left( \frac{A'B'}{AB} \right) - \left( \frac{A'}{A} \right)' + \frac{2A'(Br)^'}{AB^3r} \right). \] (26)

It follows from Eq.(20) that

\[ 8\pi(q + q^G)B^2 = \frac{2\Theta'}{3}. \] (27)

The Misner-Sharp (1964) mass becomes

\[ m(r, t) = \frac{r^3}{2} \left( \frac{BB^2}{A^2} - \frac{B'^2}{B} - \frac{2B'}{r} \right) + \frac{s^2}{2rB}. \] (28)

We assume that the exterior region of the charged radiating star is described by the charged Vaidya spacetime in a single null coordinate

\[ ds^2 = \left( 1 - \frac{M(\nu)}{R} + \frac{Q^2}{R^2} \right) d\nu^2 + 2d\nu d\tilde{R} - \tilde{R}^2(d\theta^2 + \sin^2 \theta d\phi^2), \] (29)
where $M(\nu), Q^2$ and $\nu$ represent total mass, charge and retarded time, respectively. For the matching of the interior and exterior regions, we follow the procedure of Sharif and his collaborators [27]-[34] and get

$$m(r, t) \equiv M(\nu), \quad p + p^G \equiv (q + q^G)B \iff s \equiv Q.$$  

(30)

These are the necessary and sufficient conditions for the matching of two regions of a collapsing star.

### 3 Dynamical and Transport Equations

In this section, we establish the dynamical and heat transport equations for the collapsing charged radiating fluid. For this purpose, we use the Misner and Sharp formalism [17]. In this case, the velocity of the collapsing matter is given by

$$U = r D_t B,$$  

(31)

where $U < 0$ for the collapsing fluid and $D_t = \frac{1}{A} \frac{\partial}{\partial t}$. The Misner-Sharp mass given by (28) can be written as

$$\frac{(Br)'}{B} = \left( 1 + U^2 - \frac{2m(r, t)}{rB} + \left( \frac{s}{rB} \right)^2 \right)^{\frac{3}{2}} = E.$$  

(32)

This is known as energy of the collapsing fluid. The proper time derivative of the mass function (28) leads to

$$D_t m = r^3 \frac{\dot{B}\dot{B}B}{A^3} + \frac{r^3}{2} \left( \frac{\dot{B}}{A} \right)^3 - \frac{r^3 \dot{B}\dot{B}^2 A}{A^4} + \frac{r^3 \dot{B}B^2}{2 AB^2} - \frac{r^3 B'\dot{B}'}{BA} - \frac{r^2 \dot{B}'}{A} - \frac{s^2 \dot{B}}{2rAB^2}.$$  

(33)

Using Eqs. (20), (21), (29) and (31), this can be written as

$$D_t m = -4\pi \left[ (p + p^G)U + (q + q^G)BE \right] (rB)^2 - \frac{s^2 \dot{B}}{2rAB^2}.$$  

(34)

This equation gives the variation of the energy inside a gravitating sphere of radius $R = rB$. As $U < 0$, so the first term on right side of the above
equation increases the energy of a system while the negative sign with second term implies out flow of energy.

In Misner-Sharp approach, the proper radial derivative is defined by $D_R = \frac{1}{r} \frac{\partial}{\partial r}$. The proper radial derivative of Eq.(28) with Eqs.(19), (20), (29) and (31) leads to

$$D_R m = 4\pi \left( (\rho + \rho^G) + (q + q^G) \frac{B U}{E} \right) R^2 + \frac{s}{R} D_R s - \frac{s^2}{2R^2}. \quad (35)$$

The variation in energy between two adjacent layers of the fluid inside the spherical boundary can be described by the above equation. The contribution of electromagnetic field increases the variation of energy inside the collapsing sphere. In this case, the Gauss-Bonnet term affects the matter density and outward directed heat flux. Since $q$ is directed outward due to $U < 0$, so the Gauss-Bonnet term causes the system to radiate away effectively.

The gravitational acceleration of the collapsing spherical boundary can be obtained by using Eqs.(21), (28) and (31), which is given by

$$D_t U = - \left( \frac{m}{(rB)^2} + 4\pi (p + p^G)(rB) \right) + \frac{A'E}{AB} + \frac{s^2}{(rB)^3}. \quad (36)$$

The simplification of $\left( T^\alpha{}_{\beta} + T^G{}^\alpha{}_{\beta} \right)_{\beta} \chi_a = 0$, for the given shearfree spherical boundary gives

$$\frac{1}{B} (p + p^G)' + \frac{A'}{BA} (\rho + \rho^G + p + p^G) + (q + q^G) \frac{B}{A} + 5(q + q^G) \frac{\dot{B}}{A}$$

$$- \frac{s\dot{s}}{4\pi BR^4} = 0. \quad (37)$$

Using the value of $\frac{A'}{A}$ from this equation in Eq.(36) with (28) and (31), we get the dynamical equation

$$(\rho + \rho^G + p + p^G) D_t U = - (\rho + \rho^G + p + p^G) \left[ m + 4\pi (p + p^G) R^2 - \frac{s^2}{R} \right] \frac{1}{R^2}$$

$$- E^2 \left[ D_R (p + p^G) - \frac{s}{4\pi R^2} D_R s \right] - E [5B(q + q^G) \frac{U}{R} + BD_t (q + q^G)]. \quad (38)$$

This equation has form force = mass density × acceleration, which is called "Newtonian" form of dynamical system, where mass density = $\rho + \rho^G + p + p^G$. 
According to this equation, the Gauss-Bonnet term affects the mass density due to higher curvature terms. In this equation, the first square bracket on right side is the gravitational force whose Newtonian contribution is $m$ and relativistic contribution is $p + p^G$. Thus the Gauss-Bonnet term affects gravitational force of the gravitating sphere. The second term is the hydrodynamical force which is affected by the electromagnetic field, overall this term prevents collapse because $D_R(p + p^G) < 0$ and $\frac{1}{4\pi R^2} D_R s > 0$. The last square bracket describes the role of heat flux during the dynamics of collapsing sphere. Also, in this case, the first term is positive $(U < 0, (q + q^G) > 0)$, indicating that outflow of heat flux reduces the rate of collapse by producing radiations zone in the exterior region of the collapsing sphere. The effects of $D_t(q + q^G)$ can be explained by introducing the heat transport equation as follows.

Here, we discuss the transportation of heat during the charged shearfree collapse of radiative fluid in modified Gauss-Bonnet gravity. To this end, we use Muller-Israel-Stewart heat transport equation for heat conducting fluids [37, 38]. It is well-known that Maxwell-Fourier law [39] for the heat flux results to heat equation which indicates perturbation at very high speed. For the relativistic heat conducting charged fluids, the heat transportation can be explained by Eckart-Landau theory [40, 41], but results obtained on the basis of this theory leads to some inconsistent consequences. To resolve this problem, many relativistic theories have been proposed. The common point of all these theories is that these provide heat transport equation which is a hyperbolic equation.

The heat transport equation in this case reads [42]

$$
\tau h^\mu{}^\nu V^\lambda \tilde{q}_\nu;\lambda + \tilde{q}^\mu = \kappa h^\mu{}^\nu (T^\nu - TA_\nu) - \frac{1}{2} \kappa T^2 \left( \frac{\tau V^\nu}{\kappa T^2} \right) \tilde{q}^\mu, 
$$

(39)

where $h^\mu{}^\nu$ is the projection tensor, $\tilde{q} = q + q^G$, $\tau$ is relaxation time, $T$ is temperature and $\kappa$ is thermal conductivity. For the under consideration spacetime, this equation reduces to

$$
B\tau \frac{\partial}{\partial t} [(q + q^G)B] + (q + q^G)AB^2 = -\kappa(TA)' - \frac{\kappa T^2(q + q^G)B^2}{2} \left( \frac{\tau}{\kappa T^2} \right),
$$

$$
- 3\tau \dot{B}B(q + q^G),
$$

(40)
Using Eqs. (31), (32) and (36), this implies that

\[ BD_t(q + q^G) = -\kappa T \frac{D_t U}{\tau E} - \frac{\kappa T}{\tau B} - \frac{B(q + q^G)}{\tau} \left( 1 + \frac{\tau U}{r B} \right) \]

\[ - \frac{\kappa T}{\tau E} \left[ m + 4\pi(p + p^G)R^3 - \frac{s^2}{R} \right] R^{-2} - \frac{\kappa T^2(q + q^G)B}{2\tau A} \frac{\partial}{\partial t} \left( \frac{\tau}{\kappa T^2} \right) \]

\[ - \frac{3UB(q + q^G)}{2R}. \]  

(41)

In order to study the effects of heat flux on the dynamics of collapsing sphere in the modified Gauss-Bonnet gravity, we couple the dynamical and heat transport equations given by Eqs. (38) and (41). This coupling leads to

\[ (\rho + \rho^G + p + p^G)(1 - \alpha)D_t U = F_{grav}(1 - \alpha) + F_{hyd} + \frac{E\kappa T'}{\tau B} \]

\[ + \frac{E(q + q^G)B}{\tau} + \frac{\kappa T^2(q + q^G)B}{2\tau A} \frac{\partial}{\partial t} \left( \frac{\tau}{\kappa T^2} \right) - \frac{5UB(q + q^G)E}{2R}. \]  

(42)

Here

\[ F_{grav} = -(\rho + \rho^G + p + p^G) \left( m + 4\pi(p + p^G)R^3 - \frac{s^2}{R} \right) \frac{1}{R^2}, \]

\[ F_{hyd} = -E^2 \left[ D_R(p + p^G) - \frac{s}{4\pi R^4} D_R s \right], \]

\[ \alpha = \frac{\kappa T}{\tau} (\rho + \rho^G + p + p^G)^{-1}. \]

Now we explain the effects of the Gauss-Bonnet term and electromagnetic field on the dynamics of the radiating collapsing fluid. Since the Gauss-Bonnet term in the thermal coefficient would affect the value of \( \alpha \) and the factor \((1 - \alpha)\) being the the multiple of \( F_{grav} \) would affect the value of \( F_{grav} \). For \( \rho^G > 0(< 0) \) and \( p^G > 0(< 0) \), the value of \( \alpha \) in this case will be less (more) as compared to GR and the value of \((1 - \alpha)\) will be larger (smaller), consequently gravitational force will be stronger. This fact can be explained by choosing a particular form of \( f(G) \) model, if \( f(G) = f_0 \), a positive (negative) constant, Eqs. (23)-(25) yield \( \rho^G = -p^G = \frac{f_0}{2} \), \( q^G = 0 \), giving \( \rho^G = \frac{f_0}{2} > 0(< 0) \) and the rate of collapse will be increased (decreased). In both cases, the electromagnetic field affects the gravitational and hydrodynamical forces of the system.
This fact can be verified for the more generalized $f(G)$ models. When $\alpha \to 1$, the system will be in hydrostatic equilibrium as inertial and gravitational forces become zero in this limit of $\alpha$. The amount of temperature $T$ for which $\alpha \to 1$ is equivalent to the expected amount of temperature that can be achieved during the supernova explosion. When $\alpha$ crosses the critical value, the gravitational force plays the role of antigravity and the reversal of collapse would occur.

4 Summary

In this paper, we have studied the dynamics of charged radiating collapse in modified Gauss-Bonnet gravity. For this purpose, an arbitrary function of Gauss-Bonnet invariant is introduced in the Einstein-Hilbert. Generally, it is very difficult to deal with the field equations in modified Gauss-Bonnet gravity. We use a simplified form of the field equations formulated by Li et al. [35] using the CGI perturbation formalism with $3 + 1$ formalism. In this approach, the nonlinear terms in $R_{\mu\nu}$ and $R_{\mu\nu\lambda\sigma}$ can be expressed in the form of dynamical quantities. The simplified field equations are used to study the evolution of charged radiating star. The Darmois junction conditions have been used to discuss the matching of the interior region of collapsing star to the exterior charged Vaidya geometry. This matching implies that on the boundary of collapsing star, the effective pressure is balanced by the effective radial heat flux provided the amount of charges in both regions of star is same. We have formulated the dynamical equation which shows that Gauss-Bonnet term affects the rate of collapse.

To discuss the transportation of heat during the charged radiative fluid collapse, we have developed the heat transport equation for the system under consideration. Further, to study the effects of radial heat flux on the dynamics of charged collapsing fluid sphere, we have coupled the dynamical equation with the heat transport equation. The coupling of these equations leads to a single equation (42), which describes complete dynamics of the system. We have investigated that when $\alpha$ exceeds the critical value, the gravitational force plays the role of antigravity and the bouncing would occur in the system.

The importance of charge term in the dynamics of dissipative collapse has been discussed. In particular, it is worth mentioning that unlike pressure "charge term" does not act as regeneration source. In other words,
electric charge does not increase the active gravitational mass. On the basis of this fact, we can conclude that presence of charge is important for the dynamics of dissipative collapse. A model with bouncing behavior has been presented numerically by Herrera et al. [43]. This work can be extended for the generalized $f(G)$ model [16] like $f(G) = AG + BG^\frac{1+\beta}{1+\beta}$, $A$, $B$ and $\beta$ are constants.

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