Energy Conservation at the Gravitational Collapse

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July 4, 2018

Abstract

We apply the principle of energy conservation to the motion of the test particle in gravitational field by requiring that its energy, gained by gravitation, has to be balanced by decrease of its rest mass. Due to the change of mass in gravitational field Newton’s force law between gravitating bodies is modified, too. With this modified force law we build up the the classical field theory of gravitation in which all relevant field quantities are in the definition domain \( r \in [0, \infty) \) finite and positive. We show that under such circumstances, the energy release at any gravitational collapse is finite. On the other side, the energy conservation leads to an equation which relates the mass change of the test particle due to gravitation and the metric of the corresponding gravitational field. The mass change in Newton’s gravitational field lead to a remarkable simple metric which shifts, in contrast to the Schwarzschild metric, the horizon of events to the gravity center of the gravitational collapse.

KEYWORDS: energy conservation, naked singularity, black hole, dark matter

1 Introduction

It is generally accepted that if a star-like object with a sufficient high mass is collapsing over a certain limit radius, it becomes unstable and due to stronger and stronger gravitation forces it shrinks to a mathematical point. In Newtonian physics the final state of such gravitational collapse - the naked singularity - is a state of infinite mass density not separated from the outside word by a horizon of events and its formation is accompanied by radiating an infinite amount of heat. The general relativity offers the
possibility of stable end state of collapsing mass body called black hole representing a mass object from which light radiation can not escape. Beside the general feeling of physicists who have a psychological resistance against the existence of any singular state of matter in both cases one also faces difficulties with the physical interpretation of the final product of collapsing star. The classical naked singularity represents a highly unphysical state of matter and the existence of black hole in its general form is conditioned with the cosmic censorship hypothesis which appears not generally prove so far. Therefore, the endeavor to find alternatives to the classical naked singularity as well as to the black holes seems still to be justified. In what follows, we will investigate the possibility of creation of the naked singularity and the classical black hole by strict application principle of energy conservation to the gravitational interaction.

As is well-known, the energy is one of the most fundamental concepts of physics and its conservation is evident in all its subdisciplines. This is why we apply the principle of energy conservation to the motion of the test particle in gravitational field by requiring that its energy, gained by gravitation, has to be balanced by the decrease of its mass. Consider, for simplicity, the static spherically symmetrical gravitational field of a large mass \( M \) in which a test particle \( m \) occurs at distance \( r \) from \( M \). We demand that the sum of the inertial as well as the gravitational mass of the test particle at a spatial point \( r \) and the work being done by its adiabatic translation (\( v \approx 0 \)) from infinity to \( r \) must be equal to its rest mass \( m_\infty \) in infinity, i.e.

\[
m_\infty = m(r) + \frac{1}{c^2} \int_{\infty}^{r} F(x) dx, \tag{a}
\]

where \( F(r) \) is the force acting on the test particle in gravitational field. Inserting Newton’s law into Eq.(a) one obtains

\[
m(r) = m_\infty \gamma(r) = m_\infty \exp\left(-\frac{\lambda}{r}\right); \quad \lambda = \frac{GM}{c^2} \tag{b}
\]

and the new modified Newton’s force law reads

\[
F_N(r) = -\frac{GMm_\infty \exp\left(-\frac{\lambda}{r}\right)}{r^2}. \tag{c}
\]

The factor \( \exp\left(-\frac{\lambda}{r}\right) \), appearing in the Newton-like force law, has a remarkable consequence, namely when the test particle is approaching sufficiently near to a (point-like) gravitating mass, the force acting on it becomes weaker and weaker till it completely ceases at the gravity center. On the other side,
\( F_N(r) \) coincides asymptotically with common Newton’s law in the regions where the field is weak (\( \lambda \ll 1 \)).

In our approach, the particle mass in gravitational field apparently depends on its position. On the other side, it is generally assumed that particle rest mass is, in classical gravity as well as in general relativity, constant. This assumption, as stressed by Dicke [1], is very partially supported by experiment. Especially, this constancy has never been tested in strong gravitational fields. The concept of variable rest mass, motivated by different reasons, is not new. It appears, e.g. in Dicke reformulation [2] of Brans-Dicke theory [3], in Hoyle and Narlikar’s theory of gravitation [4], in Malin [5] cosmological theory of variable rest masses, in Beckenstein theory of rest mass field [7] and in Vondrácik [6] Mach’s theory of gravitation, to mention only a few.

Since at each spatial point of gravitational field the force acting on the test particle is given by Eq.(c) one can consider the neighborhood of \( M \) as a classical force field, i.e. one can determine its intensity, its potential, its field energy density, and its total field energy. For the Newton-like field, all these field quantities assume everywhere finite and positive values. As a consequence, the energy conservation, the total energy released by any gravitational collapse can not be larger than \( n m_{\infty} c^2 \), \( n \) being the number of particles participating on the gravitational collapse. Since the energy released by a gravitational collapse is restricted no naked singularity, in classical sense, can be formed in nature. Moreover, a star-like objects that is larger than approximately \( 2M_{\text{sun}} \)-Chandrasekhar limit- might exist in equilibrium due to weakening of gravitation toward the gravity center (see [14]).

There is a link between the Newton-like gravitation theory and the geometrical structure of spacetime. To show this we consider a spherically symmetrical, asymptotically flat gravitation field with local Lorentz reference frame co-moving with the freely falling elevator. The velocity of this system is changing and, accordingly, the corresponding relativistic length contraction and time dilatation. Therefore, one can ascribe to each spatial point of gravitational field the special-relativistic factor \( \gamma = \sqrt{1 - v^2/c^2} \).

Knowing this factor as a function of position \( r \) the line element in the reference frame of rest observer turns out to be

\[
ds^2 = \gamma^{-2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) - \gamma^2 c^2 dt^2,
\]

\( \gamma = \sqrt{1 - \frac{v^2}{c^2}} \) \hspace{1cm} (d)

The special-relativistic energy conservation for a freely falling particle leads
to the formula
\[ m_\infty c^2 \gamma(r) \left( \frac{1}{\sqrt{1 - v^2/c^2}} \right) = m_\infty c^2, \tag{e} \]
where \( m_\infty \) is the mass of particle in infinity. This formula, which tell us that the total energy at each spatial point of gravitational field is constant equal to \( m_\infty c^2 \), represents a bridge between mass decrease function (MDF, for short) \( \gamma(r) \) of a particle in gravitational field and its geometrical structure. Inserting MDF of the Newton-like field \( \gamma(r) = \exp(-\lambda/r) \) into Eq.(e) then the line element (d) becomes the form
\[ ds^2 = \exp \left( 2 \frac{\lambda}{r} \right) dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) - \exp \left( - 2 \frac{\lambda}{r} \right) c^2 dt^2. \tag{f} \]

Apparently, this line element coincides for weak field \( (\lambda/r \ll 1) \) asymptotically with the Schwarzschild one.

The metric, associated with the Newton-like field, represents a non-vacuum solution of Einstein’s equations. In order to determine the source terms assigned to this metric we solved the inverse problem of general relativity, i.e. to find the source terms given the metric. The (tt)-component of energy-momentum, assigned to the metric (f) represents a regular function of \( r \) assuming at \( r = 0 \) a finite value.

As is common knowledge, gravitationally collapsing object of sufficient mass are doomed to form black holes defined by an event horizon within which resides the singularity of general relativistic equations. Characteristic feature of the metric associated with the Newton-like field is that, in the classical black hole model, the surface of events and the infinite red shift are shifted into center of a collapsing mass, which implies the non-existence of common black-hole. Hence, strict application of energy conservation has at least to important consequences: the impossibility of forming the naked singularity and the non-existence of black holes.

It appears that the ‘field’ and ‘geometrical’ aspects of gravitation are mutual related. Given MDF one can determine the geometry of spacetime where the gravitational field occurs, and given the geometry one can determine the corresponding MDF and so its field quantities.

The organization of this article is as follows. In Section 2, we derive MDF and the modified force law from the requirement of the constancy of the energy content of the test particle during its motion in gravitational field. In Section 3, we describe properties of the Newton-like field which arises if one inserts into Eq.(a) the common Newton force law. In Section 4, we determine the metric associated with the Newton-like field and calculate the source terms of Einstein’s equations by inserting components of metric
tensor into left-hand sides of these equations. In Section 5, we study the actual gravitational fields and metrics associated with them. In section 6, we put forward the hypothesis concerning gravitation as a force field embedded into metric determined by the given mass decrease function.

2 The Mass as a Function of Gravitational Field

In electrostatics, one derives an expression for the energy density of the electrostatic field by calculating the work done in assembling a charge distribution from elements of charge that are initially in a dispersed state. A similar situation in classical Newton theory leads to a strange conclusion that the energy density of gravitational field is negative definite (see, e.g. [12]). In the Newton theory, the gravitational energy stored in a system of masses can be found by calculating the work done in bringing these masses from infinity to the final position. For a system of two masses $m_1$ and $m_2$ we get the gravitational potential energy by the familiar formula $U(r) = -\frac{Gm_1m_2}{r}$, where $r$ is the distance between the masses. An interesting feature of classical gravity is the possibility that in certain situations the total energy of a gravitating system can become negative supposing the constancy of its masses. As an example, consider a sphere of gas in convective equilibrium which radiates away its excess thermal energy as it slowly contracts. Its total energy is [8]

$$E = Mc^2 - \frac{6GM^2}{7r}.$$  

We see that up a certain $r$ the total energy of this system becomes negative. In the Maxwell-like field theory of gravitation, it is assumed that the total energy content of gravitating mass system consists of positive energy of its masses and negative field energy so that the total energy appears to be constant (for details see, e.g. [12]). However, since the energy is by definition positive the gravitating system with the total negative energy is in principle inadmissible.

The simplest way how to avoid this situation and to conserve the total energy of a gravitating mass system, without introducing the negative field energy, is to assume that the work done by a moving test particle in the gravitational field is going at the expense of its internal energy, i.e.

$$m(r) = m_\infty - \frac{1}{c^2} \int_\infty^r F(x)dx = m_\infty - \frac{1}{c^2} \int_\infty^r m(x)\chi(x)dr, \quad (1)$$

where $m_\infty$ is the mass of particle in infinity and $\chi(r)$ is the the so-called force function. Writing $m(r)$ and $F(r)$ in the form $m_\infty\gamma(r)$ and $F(r) = m(r)\chi(r)$,
and differentiating Eq. (1) with respect to \( r \), we get the following differential equation (a prime means differentiation with respect to \( r \))

\[
\gamma(r)' = -\frac{\gamma(r)\chi(r)}{c^2}
\]

whose solution

\[
\gamma(r) = \exp \left( -\frac{1}{c^2} \int_{\infty}^{r} \chi(x)dx \right)
\]

gives the relation between \( \chi(r) \) and \( \gamma(r) \). We remark that the inertial and the gravitational mass are in gravitational field equally changed so that its ratio remains constant and the principle of equivalence is not violated.

Given \( \gamma(r) \) or \( \chi(r) \) one can derive several important field and characteristics of gravitational field. Using the familiar formula, \( F(r) = \frac{dE(r)}{dr} \), and taking into account that the energy content of a particle is \( m(r)c^2 \), the force acting on this particle in gravitational field is

\[
F_g(r) = -\frac{d(m(r)c^2)}{dr} = -m_\infty \chi(r) \exp \left( -\int_{\infty}^{r} \frac{\chi(x)dx}{c^2} \right) = -m_0 \gamma(r)'c^2.
\]

The potential energy of \( m_\infty \) reads as \( (\gamma(\infty) = 1) \)

\[
E_{pot} = -\int_{\infty}^{r} F(x)dx = m_\infty c^2 (\gamma(r) - 1).
\]

The spatial point \( r_0 \) at which the rest energy of \( m_\infty \) is totally exhausted is given by the equation

\[
\frac{1}{c^2} \int_{\infty}^{r_0} F_g(x)dx = m_\infty.
\]

In the next section we will apply the above formulas to a type of gravitational field called the Newton-like force field.

3 Field Properties of the Newton-like Gravitational Field

The field that arises if one inserts into Eq. (1) the Newton force law we obtain the Newton-like field. By means of Eq. (1), one gets for MDF of this field the expression \( \gamma_N(r) = \exp \left( -\frac{\lambda}{r} \right) \), so that it holds

\[
m_N(r) = m_\infty \gamma_N(r) = m_\infty \exp \left( -\frac{\lambda}{r} \right), \quad \lambda = GM/c^2.
\]
The Newton-like force law follows from Eq.(3)

\[ F_N(r) = -\frac{GMm_\infty}{r^2} \exp \left( -\frac{\lambda}{r} \right) = -\frac{c^2\lambda m_\infty \exp (-\lambda/r)}{r^2}. \] (5)

The corresponding potential \( U_N(r) \) reads as

\[ U(r) = -\int_r^\infty F(r)dr = m_\infty c^2(\exp (-\lambda/r) - 1). \] (6)

The graph of \( F_N(r) \) as a function of \( r \) is depicted in Fig. 1.

Figure 1: The Newton-like force \( F_N \) as a function of \( r \) \((\lambda = G = c = m_\infty = 1)\).

\( F_N(r) \) represents everywhere finite function assuming at \( r = 0 \) and \( r = \infty \) zero value. The maximal value of \( F_N(r) \) is reached at \( r_{max} = GM/2c^2 \) where it assumes the finite value

\[ F_N^{(\text{max})} = \left( \frac{c^4}{G} \right) \left( \frac{m_\infty}{M} \right) (4 \exp (-2)). \]

\( F_N^{(\text{max})} \) depends on the quantity \( c^4/G \), the ratio \( m_\infty/M \) and a numerical factor \( 4 \exp (-2) \approx 1 \). The quantity \( c^4/G \) is related to the other Planck relativistic constants (Planck mass \( m_P \), Planck-Wheeler’s length \( l_P \) and Planck time \( t_p \)) through the equation

\[ \frac{c^4}{G} = \frac{\hbar c}{l_P^2} = m_P l_P (t_p)^{-2}. \]
Note that $c^4/G$, though being one of Planck's relativistic constants arisen by the combination of $G$, $c$ and $h$, does not contain $h$.

Eq.(6) we rewrite in a symmetrical form

$$F_{(N)}(r) = -\frac{\sqrt{Gm_\infty}\sqrt{GM}}{r^2} \exp\left(-\frac{\lambda}{r}\right),$$  \hspace{1cm} (7)

where $\sqrt{Gm_\infty}$ and $\sqrt{GM}$ are the so-called gravitational charges of $m_\infty$ and $M$ \cite{10}. The force acting on one unit of gravitational charge $\sqrt{Gm_\infty}$ becomes field intensity of the Newton-like gravitational field

$$I_{(N)}(r) = -\frac{\sqrt{GM}}{r^2} \exp\left(-\frac{\lambda}{r}\right).$$  \hspace{1cm} (8)

For the field energy/mass density of the Newton-like field we get the expression

$$E_N(r) = \frac{I_N(r)^2}{8\pi} = \frac{GM^2}{8\pi r^4} \exp\left(-\frac{2\lambda}{r}\right) \quad \text{and} \quad \rho_N(r) = \frac{GM^2}{8\pi r^4c^2} \exp\left(-\frac{2\lambda}{r}\right),$$

\hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (9)

respectively. The graph of $E_N(r)$ as a function of $r$ is depicted in Fig. 2.

![Figure 2: The energy density of the Newton-like field $E_N$ as a function of $r$ ($\lambda = G = M = 1$).](image)

Again, $E_N(r)$ represents an everywhere positive and finite function. It is interesting that the total energy of the Newton-like gravitational field of a point-like $M$ is a finite value.

$$E_N = \frac{1}{2} \int_{\infty}^{0} \frac{GM^2}{r^4} \exp\left(-\frac{2\lambda}{r}\right) r^2 dr = \frac{Mc^2}{4}.$$  

8
The work done by adiabatic translation a test particle from infinity to the gravity center in the Newton-like field is just equal to its whole internal energy content
\[ \int_{\infty}^{0} F_N(r)dr = m_\infty c^2. \]

Next we show that MDF and the space-time geometry of a static central symmetric gravitational field are related.

4 Metric in a Freely Falling Elevator

Consider a spherically symmetrical and asymptotically flat gravitational field with the local Lorentz reference frame comoving with the freely falling elevator from infinity toward the central body \( M \). In this elevator, one feels no gravitation field, because it carries the Euclidean metric supposed to be in infinity. In distance \( r \) from the central body the elevator moves with velocity \( v \). Relativistic length contraction and time dilatation in the reference frame of the central body is changing in accord with the velocity of the elevator. The line element in the reference frame of elevator (\( \Sigma_1 \)) is
\[ ds^2 = dr^2 + r^2(d\theta^2 + \sin^2(\theta)d\phi^2) - c^2dt^2, \]
while in that of the central body (\( \Sigma_2 \)), due to the Lorentz transformation, becomes
\[ ds^2 = \gamma^2 dr^2 + r^2(d\theta^2 + \sin^2(\theta)d\phi^2) - \gamma^2 c^2dt^2, \quad \gamma = \sqrt{1 - \frac{v^2}{c^2}}. \]

The principle of the energy conservation applied to a freely falling particle in gravitational field, supposing that \( E_{tot} = 0 \) in infinity, leads to the equation
\[ E_{tot} = E_{kin} + E_{pot} = m_\infty c^2 \gamma(r) \sqrt{1 - \frac{v^2}{c^2}} - m_\infty \gamma(r) + m_\infty c^2(\gamma(r) - 1) = 0 \quad (10) \]
From Eq. (10) follows
\[ \frac{m_\infty c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = m_\infty c^2, \quad \gamma(r) = \sqrt{1 - \frac{v^2}{c^2}}, \quad (11) \]
i.e. the energy content of a test particle during its moving in gravitational field remains constant equal to \( m_\infty c^2 \).
This is an important equation that expresses energy conservation of a particle in gravitational field. It represents also a bridge between the spacetime geometry and field properties of a gravitational field. Given $\gamma(r)$ we can determine the metric of every static spherically symmetrical field. Likewise, given the metric one can determine the corresponding MDF for every static spherically symmetrical field. $\gamma_N(r)$ of the Newton-like field implies the metric

$$g_{rr} = \gamma(r)^2 = \exp(2\lambda/r) \quad g_{tt} = -(\gamma)^{-2} = -\exp(-2\lambda/r).$$

The line element associated with the Newtonian MDF is

$$ds^2 = -\exp(-\frac{2\lambda}{r})dt^2 + \exp(\frac{2\lambda}{r})dr^2 + r^2(d\theta^2 + \sin^2 \theta d\psi^2). \quad (12)$$

This line element asymptotically coincides with the Schwarzschild one for weak field $\lambda \ll 1$. The metric (12) describes a non-vacuum solution of Einstein’s equations with non-zero source terms.

As is well-known, the line element (metric) of a static, spherically symmetric spacetime can be written in the form

$$ds^2 = -\exp(2B(r))dt^2 + \exp(2A(r))dr^2 + r^2(d\theta^2 + \sin^2 \theta d\psi^2), \quad (13)$$

where the standard spherical coordinates of a distant observer are used (we implicitly suppose the asymptotic flatness of the spacetime). The non-null components of the standard Einstein tensor $G_{\nu\mu}$ read as $(G=c=1)$

$$G_{tt} = \frac{\exp(2B)}{r^2} \left[ r[1 - \exp(-2A)] \right] \quad (14)$$

$$G_{rr} = -\frac{\exp(2A)}{r^2}[1 - \exp(-2A)] + \frac{2dB}{r dr} \quad (15)$$

$$G_{\theta\theta} = r^2 \exp(-2A) \left[ \frac{d^2B}{dr^2} + \left( \frac{dB}{dr} \right)^2 + \frac{1}{r} \frac{dB}{dr} - \frac{1}{r} \frac{dA}{dr} - \frac{dA}{dr} \frac{dB}{dr} \right] \quad (16)$$

$$G_{\phi\phi} = \sin^2 \theta G_{\theta\theta} \quad (17)$$

Comparing Eq. (13) and Eq. (12) we find that $A(r) = \lambda/r$ and $B(r) = -\lambda/r$. With $A(r)$ and $B(r)$, Eqs. (14), (15), (16) and (17) turn out to be

$$G_{tt} = \frac{\exp(-2\lambda/r)}{r^2}Q(r); \quad G_{rr} = -\frac{\exp(2\lambda/r)}{r^2}Q(r),$$

where

$$Q(r) = 1 - \exp(-2\lambda/r)(1 + \frac{2\lambda}{r})$$
and
\[ G_{\theta\theta} = \frac{2\lambda^2}{r^2} \exp\left(-\frac{2\lambda}{r}\right) \quad G_{\phi\phi} = \frac{2\lambda^2}{r^2} \exp\left(-\frac{2\lambda}{r}\right) \sin^2 \theta. \]

\(Q(r)\) represents a monotonous decreasing function of \(r\) that for \(r \to 0\) and \(r \to \infty\) assumes the limit value \(Q(r) \to 1\) and \(Q(r) \to 0\), respectively. \(Q(r)\) as a function of \(r\) is depicted in Fig.3.

![Figure 3: The quantity \(Q(r)\) as a function of \(r\).](image)

For weak field \(\lambda/r \ll 1\), one can set \(\exp\left(-\frac{2\lambda}{r}\right) \approx 1 - \frac{2\lambda}{r}\) which inserting in \(Q(r)\) yields \(Q(r) = -\frac{4\lambda^2}{r^2}\) and we have

\[ G_{tt} \approx -\frac{4\lambda^2 \exp\left(-\frac{2\lambda}{r}\right)}{r^4} = 8\pi G \left( \frac{4GM^2 \exp\left(-\frac{2\lambda}{r}\right)}{8\pi r^4} \right) \]

\[ = 8\pi G(4\rho_M(r)). \] (18)

The results concerning the non-vacuum spacetime associated with the Newton-like field can be summed up as follows:

(i) \(G_{tt}(r)\) represents everywhere regular function assuming at the center of gravity zero value. The graphical representation of \(G_{tt}(r)\) as a function of \(r\) (\(\lambda = 1\)) is shown in Fig.4.

The graph despises a one-hump curve which reaches its maximal value at \(r \approx 0.7\) and then asymptotically decreases to zero.

(ii) The integrals of \(G_{tt}(r),G_{\theta\theta}(r)\) and \(G_{\phi\phi}(r)\) over the interval \(r \in [0, \infty]\) assume finite values \((G_{tt}(r) \approx 1.2\lambda, G_{\theta\theta}(r) = G_{\phi\phi}(r) = 1\lambda)\).

(iii) Comparing graphs of \(G_{tt}(r)\) and \(\rho_N(r)\) (Fig.5) we see that they have practically the same shapes which point out that \(G_{tt}\) is essentially given by the energy density of the Newton-like field.
The components of the Ricci tensor for the metric $g_{rr} = \exp\left(2\lambda/r\right)$ and $g_{tt} = \exp\left(-2\lambda/r\right)$ assume the forms

$$R_{rr} = \frac{2\lambda^2}{r^4} \quad R_{tt} = -\frac{2\lambda^2}{r^4} \exp\left(-4\lambda/r\right) \quad R_{\theta\theta} = -1 + \frac{\exp\left(-2\lambda/r\right)(2\lambda + r)}{r}$$

and

$$R_{\theta\theta} = R_{\phi\phi} \sin^2\theta.$$

It is straightforward to verify that

$$R_{tt}g_{rr} = -R_{rr}g_{tt} = 8\pi G \left(\frac{GM^2}{8\pi r^4} \exp\left(-\frac{2\lambda}{r}\right)\right) = 8\pi G \rho_N(r),$$

where $\rho_M(r)$ is exactly the field energy density of the Newton-like field.

In an almost empty flat space, where the gravitational field is relatively weak, the source of gravitational field is its own field energy density $E_g(r) = I_g^2(r)/8\pi$. Inserting $E_g(r)$ into the familiar differential equation we get

$$\frac{dI_g(r)}{dr} + \frac{r}{2} I_g(r) = -\frac{1}{2\kappa} E_g(r) = -\frac{1}{2\kappa} I_g^2(r),$$

where $\kappa$ is a phenomenological constant. ¹ This differential equation has for a point-like mass $M$ a simple general solution

$$I_g(r) = -C \frac{2\kappa}{r^2} - \frac{2\kappa}{r},$$

¹Strictly speaking the energy density of gravitational field is, in analogy to electrostatics, given as $E_g(r) = D_g(r)I_g(r)/8\pi$, where $I_g(r)$ is the intensity and $D_g(r)$ the displace-
Figure 5: $G_{tt}$ (down) and $\rho$ (up) as a function of $r$ ($\lambda = 1$).

Setting $C = GM$ we get

$$I_g(r) = T_1(r) + T_2(r); \quad T_1 = \frac{GM}{r^2}, \quad T_2 = \frac{-2\kappa}{r}. \quad (19)$$

In the region where the first term $T_1(r)$ prevails ($r \ll 2\kappa$) the intensity of gravitational field follows practically Newton’s law while in the region where the second term $T_2(r)$ prevails ($r \gg 2\kappa$) we get a modified force law proportional to $1/r$. Taking the simplest model of a spiral galaxy as a point-like mass concentrating in the galactic core then, according to Eq. (19), the most part of gravitational energy is concentrated in its halo. Rotation curves in the vicinity of the galactic core are Keplerian while those in the region where the second term $T_2 \propto 1/r$ is prevailing become asymptotically flat.

It is noteworthy that when setting $\kappa = \sqrt{GMa_0}$, $a_0$ being the Milgrom acceleration constant, we get similar formula for gravitation force as in the successful formalism of MOND which describes regularity in the formulation and evolution of galaxies (see, e.g. [15]). Therefore, it is tempting to interpreted the gravitational energy as one of the components of dark matter.

The intensity and displacement are related by the equation $$D_g(r) = \epsilon_g E_g(r)$$ where $\epsilon_g$ is gravitational ‘dielectric’ constant. Therefore, the energy density of gravitational field can be written as $E_g(r) = \epsilon_g I_g(r)^2$ and $1/\kappa$ may be interpreted as the gravitational ‘dielectric’ constant for the considered medium.
5 Force fields and their metrics

As is well known, the Reissner-Nordström metric represents a non-vacuum solution of Einstein’s equations. The spacetime around the mass with charge \( Q \) (in e.s.u.), is, in static spherically symmetrical case, described by the metric
\[
g_{tt}(r) = -(1 - 2GM(r)/r + Q^2/c^2r^2)\quad \text{and}\quad g_{rr}(r) = (1 - 2GM(r)/r + Q^2/c^2r^2)^{-1}.
\]

\( M(r) \) is the mass interior to radius \( r \). The third term of the metric stems from the energy density of the electrostatic field. If we take, instead of the energy density of the electrostatic field (\( E(r) = Q^2/r^4 \)), the energy density of the Newtonian field as the first approximation to the energy density of gravitational field, i.e. \( E(r) = GM^2(r)/r^4 \), then we obtain a modified Reissner-Nordström metric for a neutral gravitating mass \( M \):
\[
g'_{tt}(r) = -(1 - 2GM(r)/r + GM(r)^2/c^2r^2)\quad \text{and}\quad g'_{rr}(r) = (1 - 2GM(r)/r + GM^2/c^2r^2)^{-1}.
\]

Here, the third term in \( g'_{tt}(r) \) and \( g'_{rr}(r) \) stems from the energy density of the Newtonian force field. If the gravitating mass \( M \) is isolated then the modified Reissner-Nordström metric becomes a simple form
\[
g''_{tt}(r) = -(1 - \lambda/r)^2\quad \text{and}\quad g''_{rr}(r) = (1 - \lambda/r)^{-2}.
\]

MDF assigned to the modified Reissner-Nordström field is \( \gamma_{RN}(r) = -(1 - \lambda/r) \) which implies Newton’s law. The intensity and field energy density of Newton’s field are shown in Table 1 (the second colon). Neglecting \( \lambda^2/r^2 \) in \( g''_{tt}(r) \) and \( g''_{rr}(r) \), i.e. neglecting any energy density of gravitational field, the Reissner-Nordström field reduces to Schwarzschild one whose MDF is \( \gamma_S(r) = \sqrt{1 - 2\lambda/r}^2 \) and the corresponding force law have the form
\[
F_S(r) = \frac{GMm_\infty}{r^2\sqrt{1 - 2\lambda/r}}.
\]

Table 1 shows properties of various fields derived from the corresponding MDFs. The intensity and energy density for Schwarzschild’s field are shown in Table 1 (third colon). Last field we considered is that whose MDR is identically equal to 1 (or generally to a constant). This means that the mass in this field does not change at all and, as a consequence of it, its force is identical equal to zero (see Table 1 fourth colon). In Table 2, we present the metrics assigned to the above force fields.

\(^{2}\gamma_S(r)\) is identical with the 'Bardeen' potential introduced by the investigation of black holes [7].
field intensity

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
field & MDF & field intensity & energy density & total energy \\
\hline
Newton-like & \( -\exp (-\lambda/r) \) & \(-\lambda \exp (-\lambda/r)/r^2 \) & \(\lambda^2 \exp (-2\lambda/r)/(8\pi r^4) \) & \(M/4\) \\
Mod. R-N & \(-1 - \lambda/r\) & \(-\lambda/r^2\) & \(\lambda^2/(8\pi r^4(1 - 2\lambda/r))\) & \(\infty\) \\
Schwarzschild & \(-\sqrt{1 - 2\lambda/r}\) & 0 & 0 & 0 \\
flat & 1 & 0 & 0 & 0 \\
\hline
\end{tabular}
\caption{Table 1}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
 & \( g_{tt} \) & \( g_{rr} \) & spatial point \( r_0 \) & \( G_{tt} \) \\
\hline
Newton-like & \(-\exp (-2\lambda/r)\) & \(\exp (2\lambda/r)\) & 0 & \((\exp (-2\lambda)Q_{tt})/r^2\) \\
Mod. R-N & \(-\lambda/r^2\) & \((1 - \lambda/r)^{-2}\) & \(\lambda\) & \(\lambda^2/(1 - \lambda/r)^2/r^4\) \\
Schwarzschild & \(-\lambda/r^2\) & \((1 - 2\lambda/r)^{-1}\) & \(2\lambda\) & 0 \\
flat & 1 & 1 & - & 0 \\
\hline
\end{tabular}
\caption{Table 2}
\end{table}

6 Concluding Remarks

Taking into account what has been said so far we put forward the following hypothesis concerning gravitation. We assume that the neighborhood of a gravitating object is the space where the rest mass of a particle is transformed into its kinetic energy, according to Einstein’s equivalence principle between mass and energy, whereby its total energy remains conserved. Due to transformation of mass into energy the velocity of particles in gravitational field are changed and with this change also the local Lorentz factor and the metric. Spacetimes in regions, where gravitation occurs, appears to rest observer generally as curved. Their metrics are determined by the density of the ordinary matter as well as by the energy density of the force fields. Gravitational field appears as a force field embedded into metric determined by its mass decrease function.

The presented picture of gravitation has at least two important consequences:

(i) In classical gravitation theory. When a star run out of nuclear fuel, the only force left to sustain it against gravity is the pressure associated with the zero-point oscillation of its constituent fermions. This is valid if the gravitational force obeys Newton’s law. If one take, instead of Newton’s, the Newton-like force law then the force is weakening near the gravity center as a consequence of the exhausting of particle rest mass. Due to
principle of the energy conservation, therefore the total energy released by the gravitational collapse can not be larger than the sum of all masses participating at it. The strict application of energy conservation prevent so the forming of the naked singularity. The energy balance for an isolated collapse (nothing is put into or taken out of it), i.e. energy release plus the internal energy of masses has to be during the whole process constant. On the other side, the weakening of force near to the gravity center in the Newton-like field makes it possible the existence of star-like objects in equilibrium which are more collapsed than neutron stars (see [14]. The work done by translation of a test particle from infinity to the center of gravity is, in contrast to the Schwarzschild field, for the Newton-like field just equal to its rest mass in infinity.

(ii) In the general relativity. The strict application of energy conservation yields an important relation between a gravitational force field and the associated spacetime geometry. The metric associated with the Newton-like field is remarkable simple \( g_{tt} = \exp(-2\lambda/r) \) and \( g_{rr} = \exp(2\lambda/r) \). It coincides for weak field \( (\lambda/r \ll 1) \) asymptotically with the Schwarzschild metric and represents a non-vacuum solution of Einstein’s equations. The (tt)-component of energy-momentum represents assigned to this metric represents a regular function of \( r \) assuming at \( r = 0 \) a finite value. Its integral over the whole range of radius \( r \) is likewise a finite number. In the common black hole model, the metric associated with the Newton-like field shifts the surface of an infinite red shift and the horizon of events into the center of gravity, which implies the non-existence of the ‘classical’ black holes.

It is noteworthy that the dependence of mass on position satisfies one requirement of Mach’s principle, namely that that mass of particle should depend on neighboring mass distribution (see, e.g. [9],[10]).

The application of the principle of energy conservation to the motion of particle in gravitational field leads to a modified force law and implies a remarkable simple metric with the important consequences. It prevents the naked singularity and leads to a metric preventing also the forming of classical black holes. The energy of gravitational field can be interpreted as one component of dark matter.

The aim of this article was only to outline the basic ideas concerning the strict application of energy conservation in the gravitation theory, therefore everything is simplified and many important issues remained open which will be subject of subsequent works.
References

[1] R. H. Dicke, in *Gravitation and Relativity*, edited by H.Y.Chiu and W. F. Hoffmann (Benjamin, New York, 1964).

[2] R. H. Dicke, Phys. Rev. 125, 925 (1962).

[3] C. Brans and R. H. Dicke, Phys. Rev 124, 925 (1961).

[4] F. Hoyle and R. V. Narlikar in *Cosmology, Function and Other Matters*, edited by F. Reines (Colorado associated Universities Press, Boulder, 1972).

[5] S. Malin, Phys. Rev. D 9, 3228 (1974).

[6] P. Voráček, Astroph. Space Sci. 65, 397 (1979).

[7] J. D. Bekenstein, Phys. Rev. D 15, 1458 (1977).

[8] W. Israel, Can. J. Phys. 63, 34 (1985).

[9] J. D. Nightingale, Amer. J. Phys. 45, 376 (1977).

[10] A. Einstein, The Meaning of relativity, 5th ed. Princeton U. P., Princeton, 1956.

[11] BA J. M. Bardeen, W. H. Press and S. A. Teukolsky, Astrophys. J. 178, 374, (1972).

[12] S. Ulrych, Phys. Lett. B 633, 631 (2006).

[13] D. Iwanenko and A. Sokolov, Klassische Feldtheorie. Akademie-Verlag, Berlin (1953).

[14] J. Gembarovič and V. Majerník, Astrophys. Space Science 32, 265 (1990).

[15] R. H. Sanders and Stacy S. McGaugh, Annual Rep. of Astron. and Astrophys. 40, 1 (2002).