Dynamical phase transition due to preferential cluster growth of collective emotions in online communities

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We consider a preferential cluster growth in a one-dimensional stochastic model describing the dynamics of a binary chain with long-range memory. The model is driven by data corresponding to emotional patterns observed during online communities’ discussions. The system undergoes a dynamical phase transition. For low values of the preference exponent, both states are observed during the string evolution in the majority of simulated discussion threads. When the exponent crosses a critical value, in the majority of threads an ordered phase emerges, i.e. from a certain time moment only one state is represented. The transition becomes discontinuous in the thermodynamical limit when the discussions are infinitely long and even an infinitely small preference exponent leads to the ordering behavior in every discussion thread. Numerical simulations are in a good agreement with approximated analytical formula.

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I. INTRODUCTION

It is well known (see e.g. [1]) that a one-dimensional (1D) system with short-range forces cannot undergo a phase transition at a nonzero temperature. The situation changes when the interaction range increases, e.g., the Ising chain displays a second order phase transition when spin interactions decay with the distance $r$ as $r^{-(1+\sigma)}$ for $\sigma < 1$ and non-standard critical exponents are observed for $0.5 < \sigma < 1$ [2]. Another example is the 1D long-range $q$-states Potts model in which, depending on the $\sigma$ exponent and $q$-parameter, a first-order or a second-order phase transition is possible [3].

Some properties of 1D spatial systems with long-range interactions can be mapped to $N$-step (long memory) Markov chains where transitional probabilities depend on a system history. Analytical and numerical solutions for the resulting time-dependent probability distributions were presented in [4, 5] for fixed values of the time horizon $N$. The formalism was extended in [6, 7] to an infinite-range memory that covers the whole history of a 1D random walker. In such a case, a dynamical phase transition takes place from the normal diffusion to a super-diffusive behavior. When the parameter describing the memory influence is small enough, the variance $D_L$ of a walker position scales with the walking time $L$ as $D_L \sim L$. It increases however as $D_L \sim L^\kappa$, $\kappa > 1$ when the memory influence parameter crosses a critical value. The results can explain the long-term behavior of coarse-grained DNA sequences, written texts and financial data [6].

In this work, we consider a stochastic 1D model of preferential cluster growth where a special form of long-memory dynamics follows from recent observations of emotional patterns in online communities discussions [8–13]. In fact, complex phenomena taking place during the information search and communication exchange over the Internet have been investigated by several authors using diverse methods of statistical physics, see e.g. [14–20]. The studies are facilitated by an easy access to massive data sources [21, 22]. Information and opinion diffusion in online communities is frequently compared to epidemiological phenomena [23–29]. Both processes, however, need separate approaches, what was shown e.g. in recent investigations [30, 31] of social contagion in online social networks that emerged during a political protest in Spain.

Our model is based on a special collective phenomenon of emotional interactions reported in [11]. Consecutive comments posted on blogs, the BBC Forum, IRC channels and the Digg website when represented by binary variables corresponding to posts’ emotional valencies [32–34] tend to group in clusters of a similar valence and the cluster growth rate can be well described by a sub-linear preferential rule [11]. It follows a negative comment is more likely posted after a sequence of five negative messages than after four such posts. The persistent dynamics of this system has been confirmed by the Hurst exponent analysis in [10]. The aim of this paper is to study the global behavior of this system from the point of view of dynamical phase transitions. We will investigate when during the course of time the process of preferential cluster growth leads to the emergence of a critical cluster that is followed by posts displaying always the same valence and what a fraction is of such an ordered phase in all posts.

This paper is organized as follows. In Sec. II we describe observations of emotional clusters in massive data sets, in Sec. III we define a data-driven model for posts appearance and in Sec. IV we present numerical simulations showing a transition between a mostly disordered (hetero-emotional) and a mostly ordered (mono-emotional) phase in a two-state case of such a model.
The model extension to a three-state system is studied in Sec. VI and in Sec. VII we compare critical model parameters to data from selected online communities.

II. PREFERENTIAL GROWTH OF EMOTIONAL CLUSTERS

According to the behavior found in several online communities (BBC Forum [35, 36], Digg, IRC, blog data) and presented in [11, 13], the preferential growth mechanism is the main process responsible for forming emotional clusters. It is manifested by the power-law formula for conditional probability \( p(e|n) \) that after \( n \) comments with the same emotion \( e \) the next comment will express a similar sentiment. The data (see Fig. 1) reveals the relation \( p(e|n) = p(e)n^{\alpha} \) where \( p(e) \) is the conditional probability that two consecutive messages have the same emotion \( e = -1, 0, 1 \) (negative, neutral, positive). For the description of automatic sentiment analysis applied for the data retrieval see [11, 37, 39]. The characteristic exponent \( \alpha \) represents the strength of the preferential process leading to the long-range attraction between posts of the same emotion. The probability of finding the cluster of size \( n \) is proportional to the factor \( C = p(e)p(e)^{n-1}|(n-1)!|^{\alpha} \) responsible for appearance of the sequence of \( n \) consecutive messages. It should be also taken into account that the cluster of size \( n \) is defined as exactly \( n \) posts with mono-emotional expressions. Thus, to get the cluster distribution function one multiplies the factor \( C \) by probabilities \( 1 - p(e) \), \( 1 - p(e)n^{\alpha} \) corresponding to events that before and after the cluster users write comments with emotional states different from \( e \). The analytical form of the normalization factor can be obtained only as an approximation. As a result, the distribution of the emotional clusters is represented by the function:

\[
P^e(n) \approx p(e)n^{\alpha-1}|(n-1)!|^{\alpha}[1 - p(e)n^{\alpha}]
\]  

\[\text{(1)}\]
dependent on only two parameters \( \alpha \) and \( p(e) \).

III. MODEL DESCRIPTION

Here we try to simulate the process of preferential cluster growth in an artificial environment. To make the problem simpler, we consider a two-state system, so only positive \( e = 1 \) or negative \( e = -1 \) messages can appear in this artificial discussion. Each thread has the same length \( L \), not as in real data, where the thread distribution was close to a power-law relation (see Supporting Material in [11] and [13]).

The evolution rules of this two-state system are as follows:

- the emotion in the first message is randomly chosen with even probabilities \( p(e = 1) = p(e = -1) = 1/2 \)
- the probability of emotion \( e \) in the next message is dependent on the discussion history. Information about this history is coded in size \( n \) of the recently observed emotional cluster. The cluster of size \( n \) is defined as a sub-chain of the length \( n \) of consecutive states with the same values as the valencies \( e \).
- the probability of emotion \( e \) in the next comment occurring with the same emotion \( e \) for \( Digg, BBC, blogs \) and IRC data [11, 13]. Symbols are data (blue triangles, red circles and white squares, for negative, positive, and neutral clusters, respectively), and lines represent the fit to the preferential attraction relation \( p(e|n) = p(e)n^{\alpha} \).

The process of the cluster growth is based on the behavior observed in real data. The conditional probability that the cluster containing \( n \) consecutive messages with the same valency \( e \) increases its length to \( n + 1 \) is given by the equation:

\[
p(e|n) = x_{e}n^{\alpha_{e}}
\]  

\[\text{(2)}\]

where \( x_{e} \) is a constant dependent on the cluster valency \( e \) (it amplifies the cluster growth, and is equivalent to \( p(e|e) \)) while the exponent \( 0 < \alpha_{e} < 1 \) describes a strength of interactions for the emotion \( e \). In the numerical simulation in each time step we randomly choose a value between \([0, 0.1]\). If it is smaller than \( p_{e}(n) \), then the cluster of the emotion \( e \) is continued; otherwise, the cluster is terminated, and the opposite emotion \((-e)\) appears.

- if \( p_{e}(n) = 1 \), then the cluster reaches its critical size \( n_{c} \), which means that starting from this moment the discussion will be permanently ordered and all next messages in this thread will possess the same emotion \( e \).

One can define \( T_{c} \) as the time when the cluster of the critical size \( n_{c} \) appears. The \( \langle T_{c} \rangle \) is the average over \( R \) realizations (threads); in almost all cases we use \( R = 10^{4} \).
Since for some threads the critical cluster is not observed at all, \(<T_c>\) is not an appropriate observable, and a more convenient variable is a mean inverse of the critical time

\[
\langle \lambda \rangle = \frac{1}{\langle T_c \rangle} \quad (3)
\]

where \(\tilde{R}\) is the number of threads that were ordered during the simulation, which means that their critical times were smaller than the thread length. In Fig. 4 we present a relation between \(\langle \lambda \rangle\) and \(\alpha\). The left plot is in the linear scale and clearly displays the staircase shape of this dependence that follows from the integer values of \(T_c\) (compare Fig. 3). The right plot presents in the log-linear scale a rapid decrease in \(\langle \lambda \rangle\) for \(\alpha \approx 0.15\). The multi-steps shape for \(\alpha > 0.3\) and a rapid decrease observed for \(0.13 < \alpha < 0.2\) are only weakly dependent on the system size \(L\). We tested this behavior for different values of \(L\); for clarity, we show only representative simulations for \(L = 10^6\), \(L = 2 \times 10^5\) and \(L = 5 \times 10^5\). Of course, the length of the thread \(L\) influences the value \(\alpha\) when the order is observed for the first time. It is \(\alpha = 0.13\) for a system of the size \(L = 5 \times 10^7\) and \(\alpha = 0.15\) when \(L = 10^3\).

Probability \(P_c\), that a certain post starts a critical cluster can be estimated under the assumption that in a single discussion thread only one critical cluster can appear

\[
P_c = \langle \lambda \rangle = \frac{1}{T_c}. \quad (4)
\]

However, the probability of finding a cluster with the critical size can be described by a relation similar to one presented in [11]:

\[
P_c = \tilde{P}(n_c) = A(x, \alpha)x^{n_c - 1}[(n_c - 1)!]^{\alpha}, \quad (5)
\]

where \(n_c = 2^{x - \alpha}\) is the size of the critical cluster. There is a difference between Eq. 5 and an analytic calculation presented in [11] (see also remarks in Sec. III) since here we consider the beginning and not the end of the critical cluster.

The normalization constant in Eq. 5

\[
A(x, \alpha) = \sum_{n=1}^{n=n_c} x^n [(n - 1)!]^{\alpha} \quad (6)
\]

was calculated numerically and is presented in Fig. 5. Since the upper limit in the above sum is \(n_c\), this normalization constant is different from that in Eq. 4. For \(\alpha \ll 1\) we get

\[
A(x, \alpha) \approx x^x/(1 - x). \quad (7)
\]

Combining Eqs. 6-9, together we receive

\[
\langle A(x, \alpha) \rangle = A(x, \alpha)x^{-1/\alpha} \left[ (x^{-1/\alpha} - 1) \right]^{\alpha} \quad (8)
\]
that well fits to the behavior of \( \langle \lambda(\alpha) \rangle \) received from the numerical simulations (see the right panel in Fig. 4). The value of \( \langle \lambda(\alpha = 1) \rangle \) is not obtained from Eq. \( \text{8} \) but may be easily calculated from a simple branching process as:

\[
\lambda(x = 0.5, \alpha) = 2 \sum_{n=n_c}^{\infty} \left( \frac{1}{2} \right)^n \frac{1}{n} = 2 \ln 2 - 1 = 0.386294
\]

In the limit \( \alpha \ll 1 \) Eq. \( \text{8} \) reduces to

\[
\langle \lambda(x, \alpha) \rangle \approx \frac{x^2}{1-x} \exp \left( -\alpha x^{-1/\alpha} \right)
\]

and we get \( \langle \lambda(x, 0) \rangle = 0 \)

Let us consider a discussion in thread of length \( L \) with affective interactions described by the characteristic exponent \( \alpha \) and let us define a fraction of discussions that are mono-emotional ordered (MET) from a certain moment in such a thread as \( r(\alpha, L) = \frac{R}{R} \). This value is also a probability of the MET occurrence before time \( t = L \). It follows the value of \( r \) can be written as

\[
r(\alpha, x, L) = 1 - \left[ 1 - \lambda(\alpha, x) \right]^L
\]

where the explicit form can be received by inserting into Eqs. \( \text{7} \) and \( \text{8} \). In the limit \( \alpha \ll 1 \) we get from \( \text{10} \)

\[
r(\alpha, x, L) = 1 - \left[ 1 - \frac{x^2}{1-x} \exp \left( -\alpha x^{(1/\alpha)} \right) \right]^L
\]

Results of numerical simulations and theory from Eq. \( \text{12} \) are presented in Fig. 5. As one could expect a fraction \( r \) of the MET phase in all threads increases with the \( \alpha \) exponent and with the thread length \( L \). Moreover for longer threads the agreement between Eq. \( \text{12} \) and numerical simulations is better and the transition between the states \( r \approx 0 \) and \( r \approx 1 \) becomes steeper. In the thermodynamical limit \( L \to \infty \) this transition is discontinuous since

\[
\lim_{L \to \infty} r(\alpha = 0, x, L) = 0
\]

and

\[
\lim_{L \to \infty} r(\alpha > 0, x, L) = 1
\]

Let us define the critical value of the interaction strength as \( \alpha_c = \alpha(r = 0.5) \). After a short algebra we get from \( \text{12} \)

\[
1 - \frac{x^2}{1-x} \exp \left[ -\alpha_c x^{(-1/\alpha_c)} \right] = 2^{-(1/L)}
\]

For the symmetrical case \( x = 1/2 \) and \( L \gg 1 \) (if it is not otherwise written we shall use these assumptions further) we get a simpler relation

\[
\alpha_c 2^{(1/\alpha_c)} \approx \ln(L) - \ln[2 \ln(2)]
\]

that can be disentangled as:

\[
\alpha_c \approx -\frac{\ln(2)}{W_{-1}(\ln(2)/\ln(L/\ln(4)))}
\]
where $W_{-1}(.)$ is the lower branch of Lambert $W$ function [40]. A quantitative measure of the system behavior near the transition point $\alpha_c$ is the slope

$$\tan \phi = \left( \frac{\partial r(\alpha, x, L)}{\partial \alpha} \right)_{\alpha_c}$$

that can be expressed as:

$$\tan \phi \approx -\frac{\ln(2)}{2} x^{-1/\alpha_c} \left[ 1 + \frac{\ln(x)}{\alpha_c} \right].$$

For $x = 1/2$ Eq.19 can be written as an explicit function of the length $L$ using the result [17]. Relations [17] and [19] are presented at Fig. 7 where we see good fit to corresponding numerical simulations. In the limit $L \to \infty$ the critical value $\alpha_c(L)$ tends to zero while the slope $\phi(L)$ diverges to infinity what is a sign of a discontinuous transition in the thermodynamical limit. It should be stressed that for $\alpha = 0$ the MET phase does not exist, what is shown by Eq.18.

V. THREE-STATE SYSTEM

A natural extension of the two-state system is to add one more state, i.e., $e \in \{-1, 0, 1\}$. To compare properties of such systems with our previous results, we considered a symmetrical three-state model where $x_{-1} = x_0 = x_1 = 0.5$ and $\alpha_{-1} = \alpha_0 = \alpha_1$ with a symmetrical two-state model where $x_{-1} = x_1 = 0.5$ and $\alpha_{-1} = \alpha_1$. Values of the inverse of critical time ($\lambda$) as a function of the exponent $\alpha$ are presented in Fig. 8. Since results for both systems lie on the same line, we can state that the number of possible emotional states does not influence a critical time needed for the emergence of MET. This observation can be explained as follows. The occurrence of MET needs a growth of a critical cluster of any emotion $e$. The growth process is dependent only on the conditional probability of cluster growth (Eq.2) that is insensitive to the number of possible emotional states. If initial probabilities $p(e)$ of a spontaneous occurrence of every emotional state $e$ are equal and clusters of every emotion possess the same growth parameters $\alpha_e$ and $x_e$ then an average time needed for the emergence of any critical cluster should be independent from the number of possible emotional states.

Fig. 7 shows the results for an asymmetrical three-state system where $x_{-1} = x_0 = x_1 = 0.33$. We investigated models when one or two emotional states are random ($\alpha_{-1} = 0 \lor \alpha_0 = 0$) and the preferential process appears only for the remaining emotional state. We observe that for a small value of $\alpha < 0.25$ all three considered curves collapsed.

FIG. 8. (Color online) Relation between the observable $<\lambda>$ and the exponent $\alpha$; black points: $x = 0.5$ and $\alpha_{-1} = \alpha_1$ (two-state system), red diamonds: $x = 0.5$ and $\alpha_{-1} = \alpha_1 = \alpha_0$ (three-state system), orange, violet and green points are for $x = 0.33$ (three-state system) and different values of $\alpha$ $L = 2 \times 10^6$. 

FIG. 7. (Color online) Dependence of the critical value $\alpha_c$ (violet rombs) and the slope of tan $\phi$ (red circles) on the system size $L$. Solid line corresponds to Eq. [17] and a dashed one to Eq. [18] where the value of $\alpha_c$ was taken from the Eq. [17].

FIG. 6. (Color online) Fraction of ordered threads as a function of the exponent $\alpha$ for various thread lengths $L$. Lines correspond to Eq. [12].
VI. REAL-WORLD DATA

Let us consider the behavior of the proposed model for parameter values corresponding to a real exchange of messages. For the BBC Forum, the parameters are $\alpha_1 = 0.051$, $\alpha_0 = 0.38$, $\alpha_0 = 0.45$ (see Ref. [11]). In a numerical simulation, the first messages were randomly chosen according to values of the emotional probabilities $p(1) = 0.16$, $p(-1) = 0.65$, $p(0) = 0.19$ calculated for this data set. Also the parameters $x_0 = 0.2$, $x_0 = 0.27$ and $x_0 = 0.69$ were taken from the BBC Forum as conditional probabilities $p(e|c)$.

It follows that the average time corresponding to the ordering phenomenon can be estimated as $\langle T_{c,BBC} \rangle \approx 57000$. This value is much larger than the average thread length observed in the BBC data. However since the BBC dataset contains in total $N_{BBC} = 2,474,781$ comments [11], on average there were $M_{BBC} = N_{BBC} * \lambda_{BBC} \approx 43$ cases where the MET phase could appear and discussion participants were not able to present another emotion. A similar situation took place for the Digg data, where $\lambda_{Diag} = 9.9 \times 10^{-6}$ which corresponds to $\langle T_{c,Diag} \rangle \approx 101,000$. Since $N_{Diag} = 1,646,153$ [11], $M_{Diag} \approx 16$. Both values $M_{BBC}$ and $M_{Diag}$ are much lower than the total numbers of the observed threads in both communities that were correspondingly [11] $N_{thread,BBC} = 97,946$ and $N_{thread,Diag} = 129,998$. Thus although there are collective emotional interactions in above online communities, the majority of discussions threads are not pinned to a given emotion.

VII. CONCLUSIONS

We studied a specific long-memory stochastic process that represents a data driven binary model of emotional online discussions. Analytical and numerical calculations show that in the course of time persistent mono-emotional threads can emerge from the clusters of a critical size. Such threads exist as a majority phase above a critical value of the emotional interactions exponent $\alpha_c$ that value decays to zero when the discussion length tends to infinity. In this thermodynamical limit there is a discontinuous transition between a phase without mono-emotional threads and a phase when every thread is emotional ordered from a certain time moment $T_c$. The value of $T_c$ is independent from the system size however there are discontinuous changes of $T_c$ for $\alpha \geq 0.3$. We received analytical forms for values of $T_c$, $\alpha_c$ and a fraction $r$ of the ordered threads.

The extension of the model to a three-state dynamics does not change its main properties, e.g. the critical time $T_c$ depends in the same way on the emotional interaction exponent $\alpha$. Applying the results of our model to the BBC and Digg data provides an evidence that the mono-emotional state could be present in a very small fraction of the observed discussion threads.

Comparing our results to long memory Markov chains studied in [3–7] we see that the preferential cluster growth process described by Eq.2 leads to a phase transition only in the thermodynamical limit $L \rightarrow \infty$. For finite systems we observe a continuous increase of the MET phase with the strength of interactions (see Eq. 11 and Fig 6) even for $\alpha \rightarrow 0$. Thus our model behaves differently as compared to the N-step Markov model [4–7] where finite size effects do not preclude a dynamical phase transition. On the other hand in the thermodynamical limit our system displays a first order phase transition that was not observed in quoted studies.

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Emotional valence is probably the most important of emotion components. It stands usually at the center of emotion experience since the relevant aspect of any object that elicits emotions is whether we like it or not, or whether it is good for us or not. For a broader description of emotional valence see [33, 34].

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