String-Inspired Special Grand Unification

Naoki Yamatsu *

Maskawa Institute for Science and Culture, Kyoto Sangyo University, Kyoto 603-8555, Japan

December 24, 2021

Abstract

We discuss a grand unified theory (GUT) based on an $SO(32)$ GUT gauge group broken to its subgroups including a special subgroup. In the $SO(32)$ GUT on six-dimensional (6D) orbifold space $M^4 \times T^2 / \mathbb{Z}_2$, one generation of the SM fermions can be embedded into a 6D bulk Weyl fermion in the $SO(32)$ vector representation. We show that for a three generation model, all the 6D and 4D gauge anomalies in the bulk and on the fixed points are canceled out without exotic chiral fermions at low energies.

1 Introduction

Grand unification [1] is one of the most attractive idea to construct unified theories beyond the standard model (SM). It is known in e.g., Refs. [2,3] that the candidates for grand unified theory (GUT) gauge groups in four-dimensional (4D) theories are only $SU(n)(n \geq 5)$, $SO(4n + 2)(n \geq 2)$, and $E_6$ because of rank and type of representations, while the candidates for GUT gauge groups in higher dimensional theories are $SU(n)(n \geq 5)$, $SO(n)(n \geq 9)$, $USp(2n)(n \geq 4)$, $E_6(n = 6, 7, 8)$, and $F_4$. Many GUTs have been already discussed in e.g., Refs. [1,4-10] for 4D GUTs; Refs. [11-21] for 5D GUTs; Ref. [22] for 6D GUTs.

Recently, a new-type GUT has been proposed by the author in Ref. [23]. In usual GUTs, GUT gauge groups are broken to their regular subgroups; e.g., $E_6 \supset SO(10) \times U(1)$, $SU(5) \times U(1) \times U(1) \supset G_{SM} \times U(1) \times U(1)$. In the new GUT called a special GUT, GUT gauge groups are broken down to special subgroups. (For Lie groups and their regular and special subgroups, see e.g., Refs. [3,24-26].)

In Ref. [23], the author proposed an $SU(16)$ special GUT whose gauge group $SU(16)$ is broken to a special subgroup $SO(10)$. The results are summarized as follows. In a 4D $SU(16)$ special GUT, one generation of quarks and leptons can be embedded into a 4D $SU(16)$ 16 Weyl fermion; the 4D $SU(16)$ gauge anomaly restricts the minimal number of generations. Unfortunately, the minimal number is 12 in 4D framework. In a 6D $SU(16)$ special GUT on 6D orbifold space $M^4 \times T^2 / \mathbb{Z}_2$, one generation of quarks and leptons can be embedded into a 6D $SU(16)$ 16 Weyl fermion; the 6D $SU(16)$ gauge anomaly and the 4D $SU(16)$ gauge anomaly on the fixed points restricts the minimal number of generations; three generation of quarks and leptons is allowed without 4D exotic chiral fermions.

Superstring theory [27,28] has been considered as a candidate for unified theory to describe all the interaction including gravity. There are a lot of attempts to construct the SM from string theories. One of its trials is based on $E_8 \times E_8$ and $SO(32)$ heterotic string theories [29,40]. Usually, the $E_8 \times E_8$ heterotic string model building is much more popular than $SO(32)$ one. One of the biggest reason seems that when we only consider regular embeddings, for the branching rules of $SO(32) \supset SO(10)(\times U(1)^3)$, the $SO(32)$ vector and adjoint representations 32 and 496 do not contain $SO(10)$ spinor representations 16 and 16, while for the branching rules of $E_8 \supset SO(10)(\times U(1)^3)$, the $E_8$ adjoint representation 248 contain $SO(10)$ spinor representations 16 and 16.

*Electronic address: yamatsu@cc.kyoto-su.ac.jp
16 and $\overline{16}$. However, for a special embedding, on the other hand, the branching rules of $SO(32) \supset SU(16) \times U(1)_Z \supset SO(10) \times U(1)_Z$ for $SO(32)$ vector and adjoint representations $32$ and $496$ are given by

$$32 = (16)(2) \oplus (\overline{16})(-2),$$

$$496 = (210)(0) \oplus (45)(0) \oplus (120)(4) \oplus (120)(-4) \oplus (1)(0),$$

where we follow the convention of their $U(1)$ normalization in Ref. $[3]$. Obviously, an $SO(32)$ vector representation is decomposed into $SO(10)$ spinor representations, and an $SO(32)$ adjoint representation contains an $SO(10)$ bi-spinor representation $210$. When we take into account the special embedding $SU(16) \supset SO(10)$, $SO(32)$ gauge theories contain $SO(10)$ spinors, easily. In the following discussion, we will not consider how to realize models from string theories.

There are several good features of special GUTs pointed out in Ref. $[23]$. First, almost all unnecessary $U(1)$s can be eliminated; e.g., $SO(32) \supset G_{SM} \times U(1)^{12}$ by using only regular embeddings, while $SO(32) \supset SU(16) \times U(1)_Z \supset SO(10) \times U(1)_Z \supset G_{SM} \times U(1)^2$ by using regular and special embeddings. Second, by using only regular embeddings, the SM fermions cannot be embedded into an $SO(32)$ vector representation $32$, while by using regular and special embeddings $SO(32) \supset SU(16) \times U(1)_Z \supset SO(10) \times U(1)_Z \supset G_{SM} \times U(1)^2$, the SM fermions can be embedded into an $SO(32)$ vector representation. It is known in e.g., Refs. $[2,3]$ that any 4D $SO(32)$ gauge theory is a vectorlike theory since an $SO(32)$ group has only real representations. To realize the SM, i.e., a 4D chiral gauge theory, we take orbifold space construction $[11,12]$. It allows us to realize 4D Weyl fermions from 5D Dirac fermions, 6D Weyl fermions, etc. In the 6D $SU(16)$ special GUT $[23]$, the nonvanishing VEV of a 5D $SU(16)$ $\mathbf{5440}$ brane scalar is responsible to break the $SU(16)$ GUT gauge group to its special subgroup $SO(10)$ via the Higgs mechanism $[13,14]$. For $SO(32)$ special GUTs, the $SO(32)$ GUT gauge group can be broken to $SO(10)$ by using the nonvanishing VEV of a scalar in an appropriate representation of $SO(32)$; the lowest dimensional representation is $\mathbf{86768}$. (The spontaneous symmetry breaking of $SU(n)$ to its special subgroups has been discussed in e.g., Refs. $[15,16]$.)

In this paper, we will discuss an $SO(32)$ special GUT on 6D orbifold spacetime $M^4 \times T^2/\mathbb{Z}_2$. As in 6D $SU(16)$ special GUTs, we need to take into account 6D and 4D gauge anomalies. As the same as the 6D $SU(16)$ gauge anomaly in the 6D $SU(16)$ special GUT $[23]$, the 6D $SO(32)$ gauge anomaly can be canceled out by introducing 6D positive and negative Weyl fermions in the same representation of $SO(32)$ gauge group. Unlike an $SU(16)$ gauge group, an $SO(32)$ gauge group itself has no 4D gauge anomaly for any fermion in any representation of $SO(32)$, but there can be 4D gauge anomalies for its subgroups. We will see it in Sec. $3$ in detail.

The main purpose of this paper is to show that in a 6D $SO(32)$ special GUT on $M^4 \times T^2/\mathbb{Z}_2$ we can realize three generations of the 4D SM Weyl fermions from six 6D $SO(32)$ $32$ bulk Weyl fermions without 4D exotic chiral fermions at low energies, and without any 6D and 4D gauge anomaly.

This paper is organized as follows. In Sec. $2$ before we discuss a special GUT based on an $SO(32)$ gauge group, we quickly review basic properties of $SO(32)$ and its subgroups shown in Ref. $[3]$. In Sec. $3$ we construct a 6D $SO(32)$ special GUT on $M^4 \times T^2/\mathbb{Z}_2$. Section $4$ is devoted to a summary and discussion.

## 2 Basics for $SO(32)$ and its subgroups

First, we check how to embed the SM Weyl fermions into $SO(32)$ vector multiplets. For regular and special embeddings $SO(32) \supset SU(16) \times U(1)_Z \supset SO(10) \times U(1)_Z$, an $SO(32)$ vector representation $32$ is decomposed into $SO(10)$ spinor representations $16$ and $\overline{16}$. Further, the $SO(10)$ spinor representation $16$ is decomposed into $G_{SM} \times U(1)_X = SU(3)_C \times SU(2)_L \times U(1)_Y \times$
$U(1)_X$ representations:

\[
16 = (3, 2)(-1)(1) \oplus (\overline{3}, 1)(-2)(-3) \oplus (3, 1)(4)(1) \\
\oplus (1, 2)(3)(-3) \oplus (1, 1)(-6)(1) \oplus (1, 1)(0)(5). \tag{2.1}
\]

Since the $SO(32)$ vector representation $32$ is real, a 4D Weyl fermion in $SO(32)$ $32$ representation includes not only 4D SM Weyl fermions but also their conjugate fermions. To realize chiral fermions, we take orbifold symmetry breaking mechanism \[11\][12]. After taking into account orbifold effects, we can regard the zero modes of an $SO(32)$ $32$ fermion as one generation of the SM fermions plus a right-handed neutrino. Note that there is no 4D pure $SO(32)$ gauge anomalies of any representation of $SO(32)$ gauge group, while there can be 4D $SU(16)$ and $U(1)$ anomalies generated by 4D Weyl fermions in complex representations of $SU(16)$ and $U(1)$, respectively. Then, after orbifolding, a maximal regular subgroup $SU(16) \times U(1)_Z$ of $SO(32)$ may be anomalous. We will discuss how to cancel out 4D pure $SU(16)$, pure $U(1)_Z$, mixed $SU(16) - SU(16) - U(1)_Z$ and mixed grav. − grav. − $U(1)_Z$ generated by 6D bulk fermions in the next section.

We consider a symmetry breaking pattern from $SO(32)$ to $G_{SM}$. One way of achieving it is to use orbifold symmetry breaking boundary conditions (BCs) and several GUT breaking Higgses. One example is to choose orbifold BCs breaking $SO(32)$ to $SU(16) \times U(1)$ and to introduce three $SO(32)$ 86768, 496, 32 scalar fields, where we assume their proper components get non-vanishing VEVs. First, the following orbifold BC for the $SO(32)$ vector representation $32$ breaks $SO(32)$ to $SU(16) \times U(1)_Z$:

\[
P_{32} = \sigma_2 \otimes I_2 \otimes I_2 \otimes I_2 \otimes I_2, \tag{2.2}
\]

where the projection matrix $P_{32}$ is proportional to the $U(1)_Z$ generator and satisfies $(P_{32})^2 = I_{32}$. (The matrix form of $P_{32}$ depends on basis.) Next, the non-vanishing VEV of the $SO(32)$ 86768 scalar field is responsible for breaking $SO(32) \supset SU(16) \times U(1)_Z$ to $SO(10)$ or $SO(10) \times U(1)_Z$, where its branching rule of $SO(32) \supset SU(16) \times U(1)_Z$ is given by

\[
86768 = (18240)(0) \oplus (14144)(0) \oplus (5440)(8) \oplus (5440)(-8) \oplus (255)(0) \oplus (1)(0) \\
\oplus (21504)(4) \oplus (21504)(-4) \oplus (120)(4) \oplus (120)(-4). \tag{2.3}
\]

$SU(16)$ 18240 and 5440 (5440) contain singlet under its $SO(10)$ special subgroup. Their nonvanishing VEV can break $SU(16)$ to its special subgroup $SO(10)$ \[23\], where their $SO(10)$ decompositions are given in Ref. \[3\] by

\[
18240 = (8910) \oplus (5940) \oplus (770) \oplus (1050) \oplus (1050) \oplus (54) \oplus (2)(210) \oplus (45) \oplus (1), \tag{2.4}
\]

\[
5440 = (1215) \oplus (1050) \oplus (210) \oplus (54) \oplus (1). \tag{2.5}
\]

The VEV of the $SO(32)$ 32 scalar, then, breaks $SO(32) \supset SO(10) \times U(1)_Z$ to $SU(5)$, where its branching rule of $SO(32) \supset SU(16) \times U(1)_Z$ is given by

\[
32 = (16)(2) \oplus (16)(-2). \tag{2.6}
\]

The VEV of the $SO(32)$ 496 scalar further reduces $SO(32) \supset SU(16) \supset SO(10) \supset SU(5)$ to $G_{SM}$, where its branching rule of $SO(32) \supset SU(16) \times U(1)_Z$ is given by

\[
496 = (255)(0) \oplus (120)(4) \oplus (120)(-4) \oplus (1)(0), \tag{2.7}
\]

and the $SU(16)$ adjoint representation 255 is decomposed into $SO(10)$ representations

\[
255 = 210 \oplus 45. \tag{2.8}
\]

(For further information, see e.g., Ref. \[3\].)
Table 1: The table shows the matter content in the SO(32) special GUT on $M^4 \times T^2 / \mathbb{Z}_2$. The representations of SO(32) and 6D, 5D, 4D Lorentz group, the orbifold BCs of 6D bulk fields and 5D brane fields, and the spacetime location of 5D and 4D brane fields are shown.

### 3 SO(32) special grand unification

We start to discuss an SO(32) special GUT on 6D orbifold spacetime $M^4 \times T^2 / \mathbb{Z}_2$ with the Randall-Sundrum (RS) type metric [22, 47] given by

$$ds^2 = e^{-2\sigma(y)}(\eta_{\mu\nu}dx^\mu dx^\nu + dv^2) + dy^2,$$

where $y$ is the coordinate of RS warped space, $v$ is the coordinate of $S^1$, $\eta_{\mu\nu} = \text{diag}(-1,+1,+1,+1)$, $\sigma(y) = \sigma(-y) = \sigma(y + 2\pi R_5)$, $\sigma(y) = k|y|$ for $|y| \leq \pi R_5$, and $v \sim v + 2\pi R_6$. There are four fixed points on $T^2 / \mathbb{Z}_2$ at $(y_0, v_0) = (0, 0)$, $(y_1, v_1) = (\pi R_5, 0)$, $(y_2, v_2) = (0, \pi R_6)$, and $(y_3, v_3) = (\pi R_5, \pi R_6)$. For each fixed point, the $\mathbb{Z}_2$ parity reflection is described by

$$P_j : (x_\mu, y_j + y, v_j + v) \rightarrow (x_\mu, y_j - y, v_j - v),$$

where $j = 0, 1, 2, 3$, and $P_3 = P_1 P_0 P_2 = P_2 P_0 P_1$. 5th and 6th dimensional translation $U_5 : (x_\mu, y, v) \rightarrow (x_\mu, y, v + 2\pi R_5, v)$ and $U_6 : (x_\mu, y, v) \rightarrow (x_\mu, y, v + 2\pi R_6)$ satisfy $U_5 = P_1 P_0$ and $U_6 = P_2 P_0$, respectively.

We consider the matter content in the SO(32) special GUT that consists of a 6D SO(32) bulk gauge boson $A_M$; three 6D SO(32) 32 positive Weyl fermions with the orbifold BCs $(\eta_0, \eta_1, \eta_2, \eta_3) = (16, -1, -1, -1)$ and three 6D negative one with $(-1, +1, +1, +1)$ $\Psi_{32}^{(b)} (b = 1, 2, 3)$; 5D SO(32) $86768, 496$ and 16 brane scalar bosons at $y = 0$ $\Phi_{86768}, \Phi_{496}$, $\Phi_{32}$; a 4D SU(16) $\times U(1)$ $\{120\}(0)$ Weyl brane fermion, twelve 4D $SU(16) \times U(1)$ $(16)(0) \oplus (\overline{16})(-2)$ Weyl brane fermions at the fixed point $(y_0, v_0) = (0, 0)$ $\psi_{120}$, $\psi_{16}$, and $\psi^{(c,d)} (c,d = 1, 2, ..., 12)$. The matter content of the SO(32) special GUT is summarized in Table 1. We will see what kind of roles each field has in detail in the followings.

First, a 6D SO(32) bulk gauge boson $A_M$ is decomposed into a 4D gauge field $A_y$ and 5th and 6th dimensional gauge fields $A_y$ and $A_v$. The orbifold BCs of the 6D SO(32) gauge field
are given by
\[
\begin{pmatrix}
A_\mu \\
A_y \\
A_v
\end{pmatrix}
(x, y_j - y, v_j - v) = P_{32} \begin{pmatrix}
A_\mu \\
-A_y \\
-A_v
\end{pmatrix}
(x, y_j + y, v_j + v) P_{32}^{-1},
\]
where $P_{32}$ is a projection matrix satisfying $(P_{32})^2 = I_{32}$. We consider the orbifold BCs $P_0$ and $P_1$ preserving $SO(32)$ symmetry, while the orbifold BCs $P_2$ and $P_3$ reduce $SO(32)$ to its regular subgroup $SU(16) \times U(1)_Z$. We take $P_{32}$ as
\[
P_{32} = \begin{cases}
\sigma_2 \otimes I_2 \otimes I_2 \otimes I_2 & \text{for } j = 2, 3 \\
I_{32} & \text{for } j = 0, 1
\end{cases}.
\]

In this case, the 4D $SO(32)$ 496 gauge field $A_\mu$ have Neumann BCs at the fixed points $(y_0, v_0)$ and $(y_1, v_1)$, while the 5th and 6th dimensional gauge fields $A_y$ and $A_v$ have Dirichlet BCs because of the negative sign in Eq. (3.3). On the other hand, since $SO(32)$ symmetry is broken to $SU(16) \times U(1)_Z$ at the fixed points $(y_2, v_2)$ and $(y_3, v_3)$, by using the branching rules of the $SO(32)$ adjoint representation 496 given in Eq. (2.7), the $SU(16) \times U(1)_Z (255, 0) \oplus (1, 0)$ and $((120) \otimes (120)) \otimes (496)$ components of the 4D gauge field $A_\mu$ have Neumann and Dirichlet BCs at the fixed points $(y_2, v_2)$ and $(y_3, v_3)$, respectively; the $SU(16) \times U(1)_Z (255, 0) \oplus (1, 0)$ and $((120) \otimes (120)) \otimes (496)$ components of the 5th and 6th dimensional gauge fields $A_y$ and $A_v$ have Dirichlet and Neumann BCs, respectively. Thus, since the $SU(16) \times U(1)_Z (255, 0) \oplus (1, 0)$ components of the 4D gauge field $A_\mu$ have four Neumann BCs at the four fixed points $(y_j, v_j) (j = 0, 1, 2, 3)$, they have zero modes corresponding to 4D $SU(16)$ and $U(1)_Z$ gauge fields; since the other components of $A_\mu$ and any component of $A_y$ and $A_v$ have four Dirichlet BCs or two Neumann and two Dirichlet BCs at the four fixed points, they do not have zero modes. The orbifold BCs reduce $SO(32)$ to $SU(16) \times U(1)_Z$. (Since there are no zero modes of the extra dimensional gauge field $A_y$ and $A_v$, we cannot rely on symmetry breaking known as the Hosotani mechanism in this setup.)

To achieve the SM gauge symmetry $G_{SM}$ at low energies, we consider the symmetry breaking sector via spontaneous symmetry breaking. We introduce 5D $SO(32)$ 86768, 496 and 32 brane scalar fields $\Phi_{86768}$, $\Phi_{496}$ and $\Phi_{32}$ on the 5D brane $(y = 0)$. Their orbifold BCs are given by
\[
\Phi_x(x, v_\ell - v) = \eta_{\ell x} P_{\ell x} \Phi_x(x, v_\ell + v),
\]
where $\ell = 0, 2$, $x$ stands for 86768, 496 and 32, $\eta_{\ell x}$ is a positive or negative sign, and $P_{\ell x}$ is a projection matrix. We take $\eta_{86768} = -\eta_{86768} = -1$ and $\eta_{496} = \eta_{32} = 1$. The tensor products of $SO(32)$ for 496 and 32 are given in Ref. [3] by
\[
496 \otimes 496 = (86768)_S \oplus (35960)_S \oplus (527)_S \oplus (1)_S \oplus (122264)_A \oplus (496)_A,
\]
\[
32 \otimes 32 = (527)_S \oplus (1)_S \oplus (496)_A.
\]

The branching rules of $SO(32) \supset SU(16) \times U(1)$ for 496 and 86768 are given in Eqs. (2.7) and (2.8), respectively; for 527, 35960, and 122264 are listed in Ref. [3]. For $\Phi_{86768}$, the $SU(16) \times U(1) (18240, 0) \oplus (14144), (0) \oplus (5440), (0) \oplus (5440), (0) \oplus (255), (0) \oplus (1, 0)$ components have zero modes; for $\Phi_{496}$, the $SU(16) \times U(1) (255, 0) \oplus (1, 0)$ components have zero modes; and for $\Phi_{32}$, the $SU(16) \times U(1) (16, 0)$ components have zero modes. We assume that the nonvanishing VEV of the scalar field $\Phi_{86768}$ is responsible for breaking $(SO(32) \supset SU(16) \times U(1)_Z$ to $SO(10)$; the nonvanishing VEV of the scalar field $\Phi_{32}$ breaks $(SO(32) \supset SU(16)$ to $SU(5)$; the nonvanishing VEV of $\Phi_{496}$ breaks $(SO(32) \supset SU(5)$ to $G_{SM}$.

The SM Weyl fermions are identified with zero modes of 6D $SO(32)$ 32 Weyl bulk fermions. The orbifold BCs of a 6D $SO(32)$ 32 positive or negative Weyl bulk fermion can be written by
\[
\Psi_{32 \pm} (x, y_j - y, v_j - v) = \eta_j (-i \Gamma^5 \Gamma^6) P_{32 \pm} \Psi_{32 \pm} (x, y_j + y, v_j + v),
\]
where the subscript of $\Psi \pm$ stands for 6D chirality, $\eta_j$ is a positive or negative sign, $\prod_{j=0}^{3} \eta_j = 1$, $\Gamma^M \ (M = 1, 2, \cdots, 7)$ is a 6D gamma matrix, and $P^M_{32}$ is give in Eq. (3.3). Here, we check zero modes of, e.g., a 6D $SO(32)$ 32 positive Weyl fermion with orbifold BCs $(\eta_0, \eta_1, \eta_2, \eta_3) = (−, −, −, −)$. At fixed points $(y_0, v_0)$ and $(y_1, v_1)$, the 4D $SO(32)$ 32 left-handed Weyl fermion components have Neumann BCs, while the 4D $SO(32)$ 32 right-handed Weyl fermion components have Neumann BCs. At fixed points $(y_0, v_0)$ and $(y_1, v_1)$, the 4D $SU(16) \times U(1)_Z (16)(2)$ and $(\overline{16})(−2)$ left-handed Weyl fermion components have Neumann and Dirichlet BCs, respectively, while the 4D $SU(16) \times U(1)_Z (16)(2)$ and $(\overline{16})(−2)$ right-handed Weyl fermion components have Dirichlet and Neumann BCs, respectively. In this case, only the 4D $SU(16) \times U(1)_Z (16)(2)$ left-handed Weyl fermion has zero modes. Note that only 6D $SO(32)$ 32 positive Weyl fermions suffer from 6D gauge anomalies.

From the above, to realize three generations of the SM chiral fermions, we introduce three sets of the pair of 6D $SO(32)$ 32 positive and negative Weyl fermions to cancel out the 6D gauge anomalies. More explicitly, each set of 6D Weyl fermions consists of a 6D $SO(32)$ 32 positive Weyl fermion with orbifold BCs $(\eta_0, \eta_1, \eta_2, \eta_3) = (−, −, −, −)$ and a 6D negative one with orbifold BCs $(+, +, +, +)$. Only the $SU(16) \times U(1)_Z (16)(2)$ components of the positive Weyl fermion have zero modes for its 4D left-handed Weyl fermion components because its 4D left-handed Weyl fermion components have Neumann BCs at all the fixed points. The corresponding 4D right-handed Weyl fermion components have Dirichlet BCs at all the fixed points. The other components of the positive Weyl fermion and all the components of the negative Weyl fermion have two Neumann and two Dirichlet BCs at four fixed points $(y_j, v_j)$.

Here, we check the contribution to 6D bulk and 4D brane anomalies from the above 6D Weyl fermion sets. The fermion set does not contribute to 6D $SO(32)$ gauge anomaly because of the same number of 6D $SO(32)$ 32 positive and negative Weyl fermions. We need to check 4D gauge anomaly cancellation at four fixed points $(y_j, v_j)(j = 0, 1, 2, 3)$ by using 4D anomaly coefficients listed in Ref. [3]. At two fixed points $(y_j, v_j)(j = 0, 1)$, there is no 4D pure $SO(32)$ gauge anomaly because any 4D anomaly coefficient of $SO(32)$ is zero. At the other two fixed points $(y_j, v_j)(j = 2, 3)$, there can be 4D pure $SU(16)$, pure $U(1)_Z$, mixed $SU(16) − SU(16) − U(1)_Z$ and mixed grav. – grav. – $U(1)_Z$ anomalies. At a fixed point $(y_3, v_3)$, the anomalies generated from the 6D $SO(32)$ 32 positive and negative Weyl fermions are canceled each other; at the other fixed point $(y_2, v_2)$, the 6D $SO(32)$ 32 positive and negative Weyl fermions generate 4D pure $SU(16)$, pure $U(1)_Z$, mixed $SU(16) − SU(16) − U(1)_Z$ and mixed grav. – grav. – $U(1)_Z$ anomalies. We focus on how to cancel the 4D anomalies at the fixed point $(y_2, v_2)$ below.

To achieve 4D gauge anomaly cancellation at the fixed point $(y_2, v_2)$, we introduce 4D brane Weyl fermions in appropriate representations of $SU(16) \times U(1)$. First, we consider the pure $SU(16)$ gauge anomaly cancellation. The 4D $SU(16)$ gauge anomaly of twelve 4D $SU(16)$ 16 left-handed Weyl fermions is canceled out by the anomaly of a 4D $SU(16)$ $\mathbf{120}$ Weyl fermion $[23]$. (It can be checked by using 4D $SU(16)$ anomaly coefficients listed in Ref. [3].) When we introduce a 4D $SU(16) \times U(1)$ $(\mathbf{120})(0)$ Weyl fermion at $(y_2, v_2)$, its 4D $SU(16)$ anomaly cancels one generated from the 6D bulk $SO(32)$ 32 Weyl fermions. Next, 4D pure $U(1)_Z$, mixed $SU(16) − SU(16) − U(1)_Z$, and mixed grav. – grav. – $U(1)_Z$ anomalies can be canceled out by introducing twelve 4D $SU(16) \times U(1)$ $(\mathbf{(16)}(0) \oplus (\overline{16})(−2))$ left-handed Weyl fermions. This is because the matter content is vectorlike from the view of the $U(1)_Z$ gauge theory. More explicitly, at the fixed point $(y_2, v_2)$, there are twelve $SU(16) \times U(1)_Z (\mathbf{16})(2)$ left-handed Weyl fermions from the 6D $SO(32)$ 32 Weyl fermions and we introduced twelve $SU(16) \times U(1)_Z (\overline{16})(−2)$ left-handed Weyl brane fermions. Also, the 4D $SU(16) \times U(1)$ $((\mathbf{16})(0) \oplus (\overline{16})(−2))$ Weyl fermions do not generate 4D pure $SU(16)$ anomaly. Note that the above brane fermions become vectorlike when $SU(16) \times U(1)$ symmetry is broken to $SO(10)$, so there is no exotic chiral fermions at low energies, where the branching rules of $SU(16) \supset SO(10)$ for an $SU(16)$ complex representation $\mathbf{120}$ is identified with an $SO(10)$ real representation $\mathbf{120}$:

\[ \mathbf{120} = 120 \quad (120 = 120). \]  

\[ (3.9) \]
4 Summary and discussion

In this paper, we constructed an \(SO(32)\) special GUT by using a special breaking \(SU(16)\) to \(SO(10)\). In this framework, the zero modes of the 6D \(SO(32)\) \(32\) Weyl fermion can be identified with one generation of quarks and leptons; the 6D \(SO(32)\) and the 4D \(SU(16) \times U(1)\) gauge anomalies on the fixed points allow a three generation model of quarks and leptons in 6D framework; as in the \(SU(16)\) special GUT \[23\], exotic chiral fermions do not exist due to a special feature of the \(SU(16)\) complex representation \(T_{20}\) once we take into account the symmetry breaking of \(SO(32)\) to \(SO(10)\).

In this paper, we simply assumed that the nonvanishing VEV of a scalar field \(\Phi\) breaks \((SO(32) \supset SU(16) \times U(1))Z\) to \(SO(10)\). Instead, we may consider dynamical symmetry breaking scenario \[50–58\] to realize the special breaking \(SU(16)\) to \(SO(10)\). Its breaking can be realized by using the pair condensation of a fermion in the \(SO(32)\) adjoint representation \(496\) or the \(SU(16)\) second-rank anti-symmetric tensor \(120\) (\(T_{20}\)). The analysis will be reported in a separate paper \[59\]. (The dynamical symmetry breaking of \(SU(16)\) to its special subgroup \(SO(10)\) is essentially the same as one of \(E_{6}\) to its special subgroups \(F_{4}\) and \(USp(8)\) or \(G_{2}\) or \(SU(3)\) discussed in Ref. \[57\].)

To cancel 4D pure \(SU(16)\), pure \(U(1)\), mixed \(SU(16)\)–\(SU(16)\)–\(U(1)\), and mixed grav.–grav. – \(U(1)\) anomalies on a fixed point, we introduced several brane Weyl fermions. For the mixed anomalies, one may rely on Green-Schwarz (GS) anomaly cancellation mechanism \[29\] for 4D version \[10\] by introducing a pseudo-scalar field that transforms non-linearly under the anomalous \(U(1)\) symmetry.

Acknowledgments

The author would like to thank Yutaka Hosotani, Kentaro Kojima, Taichiro Kugo, Shogo Kuwakino, Kenji Nishiwaki, and Shohei Uemura for valuable comments.

References

[1] H. Georgi and S. L. Glashow, “Unity of All Elementary Particle Forces,” Phys. Rev. Lett. 32 (1974) 438–441.
[2] R. Slansky, “Group Theory for Unified Model Building,” Phys. Rept. 79 (1981) 1–128.
[3] N. Yamatsu, “Finite-Dimensional Lie Algebras and Their Representations for Unified Model Building,” arXiv:1511.08771 [hep-ph].
[4] K. Inoue, A. Kakuto, and Y. Nakano, “Unification of the Lepton-Quark World by the Gauge Group SU(6),” Prog. Theor. Phys. 58 (1977) 630.
[5] H. Fritzsch and P. Minkowski, “Unified Interactions of Leptons and Hadrons,” Ann. Phys. 93 (1975) 193–266.
[6] M. Ida, Y. Kayama, and T. Kitazoe, “Inclusion of Generations in SO(14),” Prog. Theor. Phys. 64 (1980) 1745.
[7] Y. Fujimoto, “SO(18) Unification,” Phys. Rev. D26 (1982) 3183.
[8] F. Gursey, P. Ramond, and P. Sikivie, “A Universal Gauge Theory Model Based on \(E_{6}\),” Phys. Lett. B60 (1976) 177.
[9] Y. Kawamura and T. Miura, “Classification of Standard Model Particles in \(E_{6}\) Orbifold Grand Unified Theories,” Int. J. Mod. Phys. A28 (2013) 1350055, arXiv:1301.7469 [hep-ph].
[10] K. Kojima, K. Takenaga, and T. Yamashita, “The Standard Model Gauge Symmetry from Higher-Rank Unified Groups in Grand Gauge-Higgs Unification Models,” JHEP 06 (2017) 018, arXiv:1704.04840 [hep-ph].

[11] K. Kojima, K. Takenaga, and T. Yamashita, “Grand Gauge-Higgs Unification,” Phys. Rev. D84 (2011) 051701 arXiv:1103.1234 [hep-ph].

[12] K. Kojima, K. Takenaga, and T. Yamashita, “Gauge Symmetry Breaking Patterns in an SU(5) Grand Gauge-Higgs Unification Model,” Phys. Rev. D95 no. 1, (2017) 015021, arXiv:1608.05496 [hep-ph].

[13] G. Burdman and Y. Nomura, “Unification of Higgs and Gauge Fields in Five-Dimensions,” Nucl. Phys. B656 (2003) 3–22 arXiv:hep-ph/0210257 [hep-ph].

[14] C. Lim and N. Maru, “Towards a Realistic Grand Gauge-Higgs Unification,” Phys.Lett. B653 (2007) 320–324 arXiv:0706.1397 [hep-ph].

[15] H. D. Kim and S. Raby, “Unification in 5-D SO(10),” JHEP 01 (2003) 056 arXiv:hep-ph/0212348 [hep-ph].

[16] T. Fukuyama and N. Okada, “A Simple SO(10) GUT in Five Dimensions,” Phys. Rev. D78 (2008) 015005 arXiv:0803.1758 [hep-ph].

[17] Y. Hosotani and N. Yamatsu, “Gauge-Higgs Grand Unification,” Prog. Theor. Exp. Phys. 2015 (2015) 11B01 arXiv:1504.03817 [hep-ph].

[18] Y. Hosotani and N. Yamatsu, “Gauge-Higgs Grand Unification,” PoS PLANCK2015 (2015) 058, arXiv:1511.01674 [hep-ph].

[19] N. Yamatsu, “Gauge Coupling Unification in Gauge-Higgs Grand Unification,” Prog. Theor. Exp. Phys. 2016 (2016) 043B02 arXiv:1512.05559 [hep-ph].

[20] A. Furui, Y. Hosotani, and N. Yamatsu, “Toward Realistic Gauge-Higgs Grand Unification,” Prog. Theor. Exp. Phys. 2016 (2016) 093B01 arXiv:1606.07222 [hep-ph].

[21] Y. Hosotani, “Gauge-Higgs EW and Grand Unification,” Int. J. Mod. Phys. A31 no. 20n21, (2016) 1630031 arXiv:1606.08108 [hep-ph].

[22] Y. Hosotani and N. Yamatsu, “Gauge-Higgs Seesaw Mechanism in Six-Dimensional Grand Unification,” arXiv:1706.03503 [hep-ph].

[23] N. Yamatsu, “Special Grand Unification,” Prog. Theor. Exp. Phys. 2017 (2017) 061B01 arXiv:1704.08827 [hep-ph].

[24] E. Dynkin, “Maximal Subgroups of the Classical Groups,” Amer. Math. Soc. Transl. 6 (1957) 245.

[25] E. Dynkin, “Semisimple Subalgebras of Semisimple Lie Algebras,” Amer. Math. Soc. Transl. 6 (1957) 111.

[26] R. Cahn, Semi-Simple Lie Algebras and Their Representations. Benjamin-Cummings Publishing Company, 1985.

[27] J. Polchinski, String Theory I -An Introduction to Bosonic String-. Cambridge University Press, 1998.

[28] J. Polchinski, String Theory II -Superstring Theory and Beyond. Cambridge University Press, 1998.
M. B. Green and J. H. Schwarz, “Anomaly Cancellation in Supersymmetric D=10 Gauge Theory and Superstring Theory,” Phys. Lett. B149 (1984) 117–122.

M. B. Green and J. H. Schwarz, “Infinity Cancellations in SO(32) Superstring Theory,” Phys. Lett. B151B (1985) 21–25.

J. P. Derendinger, L. E. Ibanez, and H. P. Nilles, “On the Low-Energy d = 4, N=1 Supergravity Theory Extracted from the d = 10, N=1 Superstring,” Phys. Lett. B155B (1985) 65–70.

J. Giedt, “Z(3) Orbifolds of the SO(32) Heterotic String. 1. Wilson Line Embeddings,” Nucl. Phys. B671 (2003) 133–147, arXiv:hep-th/0301232 [hep-th].

K.-S. Choi, S. Groot Nibbelink, and M. Trapletti, “Heterotic SO(32) Model Building in Four Dimensions,” JHEP 12 (2004) 063, arXiv:hep-th/0410232 [hep-th].

R. Blumenhagen, G. Honecker, and T. Weigand, “Supersymmetric (Non-)Abelian Bundles in the Type I and SO(32) Heterotic String,” JHEP 08 (2005) 009, arXiv:hep-th/0507041 [hep-th].

H. P. Nilles, S. Ramos-Sanchez, P. K. S. Vaudrevange, and A. Wingerter, “Exploring the SO(32) Heterotic String,” JHEP 04 (2006) 050, arXiv:hep-th/0603086 [hep-th].

M. Ito, S. Kuwakino, N. Maekawa, S. Moriyama, K. Takahashi, K. Takei, S. Teraguchi, and T. Yamashita, “E_6 Grand Unified Theory with Three Generations from Heterotic String,” Phys. Rev. D83 (2011) 091703, arXiv:1012.1690 [hep-ph].

H. Abe, T. Kobayashi, H. Otsuka, Y. Takano, and T. H. Tatsuishi, “Gauge Coupling Unification in SO(32) Heterotic String Theory with Magnetic Fluxes,” PTEP 2016 no. 5, (2016) 053B01, arXiv:1507.04127 [hep-ph].

H. Abe, T. Kobayashi, H. Otsuka, Y. Takano, and T. H. Tatsuishi, “Flavor Structure in SO(32) Heterotic String Theory,” Phys. Rev. D94 no. 12, (2016) 126020, arXiv:1605.00898 [hep-ph].

Y. Kawamura, “Gauge Symmetry Breaking from Extra Space S^1/Z_2,” Prog. Theor. Phys. 103 (2000) 613–619, arXiv:hep-ph/9902423 [hep-ph].

Y. Kawamura, “Triplet-Doublet Splitting, Proton Stability and Extra Dimension,” Prog. Theor. Phys. 105 (2001) 999–1006, arXiv:hep-ph/0012125.

P. W. Higgs, “Broken Symmetries, Massless Particles and Gauge Fields,” Phys. Lett. 12 (1964) 132–133.

P. W. Higgs, “Broken Symmetries and the Masses of Gauge Bosons,” Phys. Rev. Lett. 13 (1964) 508–509.

L.-F. Li, “Group Theory of the Spontaneously Broken Gauge Symmetries,” Phys. Rev. D9 (1974) 1723–1739.
[46] S. Meljanac, M. Milosevic, and S. Pallua, “Extrema of Higgs Potential and Higher Representations,” Phys. Rev. D26 (1982) 2936–2939.

[47] L. Randall and R. Sundrum, “A Large Mass Hierarchy from a Small Extra Dimension,” Phys. Rev. Lett. 83 (1999) 3370–3373, arXiv:hep-ph/9905221.

[48] Y. Hosotani, “Dynamical Mass Generation by Compact Extra Dimensions,” Phys. Lett. B126 (1983) 309.

[49] Y. Hosotani, “Dynamics of Nonintegrable Phases and Gauge Symmetry Breaking,” Annals Phys. 190 (1989) 233.

[50] S. Raby, S. Dimopoulos, and L. Susskind, “Tumbling Gauge Theories,” Nucl. Phys. B169 (1980) 373.

[51] S. Dimopoulos and L. Susskind, “Mass Without Scalars,” Nucl. Phys. B155 (1979) 237–252 [2,930(1979)].

[52] E. Farhi and L. Susskind, “Technicolor,” Phys. Rept. 74 (1981) 277.

[53] M. E. Peskin, “The Alignment of the Vacuum in Theories of Technicolor,” Nucl. Phys. B175 (1980) 197–233.

[54] V. A. Miransky, M. Tanabashi, and K. Yamawaki, “Dynamical Electroweak Symmetry Breaking with Large Anomalous Dimension and t Quark Condensate,” Phys. Lett. B221 (1989) 177–183.

[55] V. A. Miransky, M. Tanabashi, and K. Yamawaki, “Is the t Quark Responsible for the Mass of W and Z Bosons?,” Mod. Phys. Lett. A4 (1989) 1043.

[56] W. A. Bardeen, C. T. Hill, and M. Lindner, “Minimal Dynamical Symmetry Breaking of the Standard Model,” Phys. Rev. D41 (1990) 1647.

[57] T. Kugo and J. Sato, “Dynamical Symmetry Breaking in an E6 GUT Model,” Prog. Theor. Phys. 91 (1994) 1217–1238, arXiv:hep-ph/9402357 [hep-ph].

[58] Y. Nambu and G. Jona-Lasinio, “Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I,” Phys. Rev. 122 (1961) 345–358.

[59] T. Kugo and N. Yamatsu, in preparation.

[60] P. Binetruy and E. Dudas, “Gaugino Condensation and the Anomalous U(1),” Phys. Lett. B389 (1996) 503–509, arXiv:hep-th/9607172 [hep-th].