NEW TRENDS IN PROJECT PORTFOLIO MANAGEMENT

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Abstract: The use of project portfolio management is increasingly becoming a tool for promoting the strategy of the organization. Using sophisticated quantitative tools becomes a significant competitive advantage for project portfolio management. Project portfolio management is a dynamic multi-criteria decision-making problem under risk. The paper presents new approaches for analyzing the problem. A dynamic version of the Analytic Network Process (ANP) captures the network, multicriteria and dynamic structure of the problem. Multicriteria decision trees analyze risk of project portfolios. Possible projects are characterized by sets of inputs and outputs, where inputs are resources for project realization and outputs measure multiple criteria of goals of the organization. The Data Envelopment Analysis (DEA) is an appropriate approach to select efficient project portfolios.

Keywords: project portfolio, multiple criteria, dynamics, risk, efficiency

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INTRODUCTION

Projects are in accelerating world rhythm the right option of solving problems of lot of organizations. Nothing is permanent, everything is temporary, and that makes pressure on companies to finish new products or services faster, cheaper and definitely not to fail. Risk is a very important factor in project management. Most project organizations exist in a multi-project environment. This environment creates the problems of project interdependency and the need to share resources. Strategic alignment of projects is of major importance to effective use of organization resources. Selection criteria need to ensure each project is prioritized and contributes to strategic goals. There is a very extensive literature on the management of individual projects and project portfolios (Fiala, 2003; Larson & Gray, 2013). Management of the project portfolio ensures that only the most valuable projects are approved and managed. All of the projects selected become part of a project portfolio that balances the total risk for the organization. The key to success in project portfolio management is to select the right projects at the right time (Levine, 2005). Portfolio management is a process evaluated by multiple criteria. This process must improve over time.

To select a portfolio of projects are basically two approaches, one is based on standard methods used in practice, the second approach is based on searching and applying new sophisticated methods based on quantitative analysis. Lot of professionals tried to find sophisticated way to improve techniques for project portfolio management in different ways.

The paper focuses on the of project portfolio management problem solved by applying sophisticated models. The aim is to develop a general model, which would be completed for the specific needs of problems. This paper aims to verify the ability to model and solve the problem of dynamic project portfolio using the Analytic Network Process (ANP) model. The organization must decide under risk whether to assign all available resources to present proposals or to reserve a portion of the funds unused for some time and wait for better alternatives that may occur later. We propose to complete our ANP model by a decision tree with multiple criteria and interactive multi-criteria analysis for solving this problem.
Efficiency of project portfolio is measured by proposed method based on Data Envelopment Analysis (DEA) approach.

1. PROJECT PORTFOLIO

Project portfolio is set of all projects that are implemented in the organization at that time. The basic objectives of the project portfolio management include:
- Optimize the results of the entire project portfolio and not individual projects
- The selection of projects to start
- Interruption or discontinuation of projects
- Defining priorities for projects
- Coordinate internal and external sources

Project opportunities come in time and it is necessary to decide which will be accepted for creating a dynamic portfolio of projects and which will be rejected (Fig. 1).

Fig. 1 Dynamic flow of projects

![Dynamic flow of projects](source: Authors)

It is generally expected that the portfolio should be designed in such a way as to maximize the possibility of achieving the strategic goals of the company. This is consistent with the notion that portfolio selection problem is a multi-criteria decision making. The main goal of each project is to increase the value of the organization, so most managers prefer financial criteria for project evaluation. The most commonly used indicators include net present value, internal rate of return, payback period, rate of return. In addition to these financial indicators, however, in selecting a portfolio of projects should be considered other characteristics.

The portfolio management domain encompasses project management oversight at the organization level through the project level. Full insight of all components of the organization is crucial for aligning internal business resources with the requirements of the changing environment. Project portfolios are frequently managed by a project office that serves as a bridge between senior management and project managers and project teams.

2. ANP MODEL

The Analytic Network Process (ANP) is the multi-criteria method (Saaty, 2001) that makes it possible to deal systematically with all kinds of dependence and feedback in the performance system. The ANP approach seems to be very appropriate instrument for project portfolio management. Another issue is the appropriate selection of clusters, which would be the basis of the basic model and their fulfilment by system elements. Another specific problem is the creation of sub-networks in the ANP model characterizing the specific important circumstances of the model. The current economic environment is characterized by significant changes. An important problem of the model will be to capture the dynamics that would represent appropriate changes. Time dependent priorities play an increasingly important role in a rapidly changing environment of network systems. Long-term priorities can be based on time dependent comparisons of system elements.
2.1 Elements of ANP model

The structure of the ANP model for dynamic project portfolio (DPP) is described by clusters of elements connected by their dependence on one another. A cluster groups elements (projects, resources, criteria, time) that share a set of attributes. At least one element in each of these clusters is connected to some element in another cluster. These connections indicate the flow of influence between the elements (see Fig. 2).

Fig. 2 Flows of influence between the elements

Source: Authors

The ANP model consists of four basic clusters with their elements and influences:

Projects: This cluster consists of potential alternatives of projects of which will be selected a dynamic portfolio. There are priorities among projects for inclusion in the portfolio. The cluster has connections to all other clusters.

Resources: Resources are necessary for the implementation of projects. Main resources are human resources between which are relations important for creating project teams. The cluster has connections to all other clusters.

Criteria: Projects are evaluated according to criteria which include benefits, opportunities, costs, and risks (BOCR). The cluster has connections to all other clusters.

Time: Time is measured in discrete units. Elements of other clusters vary in time and their values depend on the values in previous time periods.

The basic ANP model is completed by specific sub-networks. The sub-networks are used to model important features of the DPP problem. The most important features in our ANP-based framework for DPP management are captured in sub-networks: dynamic flow of projects, time dependent resources.

Dynamic flow of projects: Project opportunities come in time and it is necessary to decide which will be accepted for creating a dynamic portfolio of projects and which will be rejected. The sub-network connects clusters: time and projects.

Time dependent resources: A specific sub-network is devoted to model time dependent amounts of resources. The time dependent amount of resources is given by. The sub-network connects clusters: time, resources and projects.

2.2 Dynamics of ANP model

An important characteristic of project portfolio management is dynamics. Time dependent priorities in the ANP model can be expressed by forecasting using pairwise comparison functions (Fiala, 2006; Saaty, 2007). Dynamic extensions of ANP method can work with time-dependent priorities in a networked system.

Judgment matrix in dynamic form

\[
(t) = \begin{bmatrix}
    a_{11}(t) & a_{12}(t) & \ldots & a_{1k}(t) \\
    a_{21}(t) & a_{22}(t) & \ldots & a_{2k}(t) \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{k1}(t) & a_{k2}(t) & \ldots & a_{kk}(t)
\end{bmatrix}
\]

There are two approaches for time-dependent pairwise comparisons: structural, by including scenarios, functional by explicitly involving time in the judgment process. For the functional dynamics there are
analytic or numerical solutions. The problem leads to a time-dependent algebraic equation, the solution of which formally gives the time-dependent eigenvalues of the dynamic judgment matrix $A(t)$. Another problem is maintaining the reciprocity and transitivity of the elements of the time-dependent matrix $A(t)$. The basic idea with the numerical approach is to obtain the time-dependent principal eigenvector by simulation (Saaty, 2007).

3. RISK OF PROJECT PORTFOLIOS
In each period, the project portfolio is reviewed in line with the strategic objectives of the organization. Management may decide to initiate new projects, but also to end some others that are currently being implemented. Even if the organization has available funds, it is sometimes better to decide not initiate a new project and wait for better one. The organization must decide under risk whether to assign all available resources to present proposals or to reserve a portion of the funds unused for some time and wait for better alternatives that may occur later. We propose to use a decision tree with multiple criteria and interactive multi-criteria analysis for solving this problem (Fiala & Majovska, 2019).

3.1 Decision trees
Sequences of partial decisions which follow one another frequently occur in assessing potential projects. They are multi-stage decision processes. The task of the decision maker is to select one of the possible sequences that leads to the best final goal solution. Decision-making takes place in periods $t=1,2,...,T$. The decision trees are used to solve these problems successfully.

Solution of multi-stage decision problems proceed in two phases. The first phase is the construction of a decision tree and the second phase is its evaluation. The graph tree structure is used by the construction of decision trees that appropriately models the branching options. The decision-maker creates and evaluates its parts in order to find the optimal sequence of decisions. Two types of nodes are considered, decision and chance nodes. The edges of the tree represent branching of decision and chance possibilities. We start with the decision node from which they emanate lines that represent the possible decisions $a_i$. The ends of these edges are chance nodes on which they rely edges representing $s_j$ possible situations that may occur with conditional probabilities $p_j$. These edges can be followed by another decision nodes with possible decisions, as well as chance nodes with possible situations, etc. Large decision trees may arise by combining these basic elements. End edges, which are not followed by further decision and chance nodes, represent the possible end sequences of partial decisions that are evaluated.

Evaluation of the decision trees proceed in the opposite direction from the end edges back to the starting node of the decision. The decision-maker selects the decision that cannot affect the occurrence of situations and must consider all situations with their conditional probabilities of occurrence. The decision from possible decisions is always chosen that delivers a better evaluation. Principle of maximizing the expected value is used in the selection. The optimal sequence of decisions is obtained in this manner.

3.2 Multi-criteria analysis
Multi-criteria decision trees (Haimes & Tulsiani, 1990, Nowak & Nowak, 2013) are used to select the most suitable strategy for a dynamic project portfolio management. We use standard methods of multi-criteria decision-making for their analysis. We will seek a final compromise strategy for dynamic project portfolio selection. This strategy should be called effective. Effective multi-criteria strategy is one to which no exists other alternative strategy that would be better at least under one criterion, and not worse under other criteria Fiala, 2008). Multi-criteria analysis is at two levels: identification of all effective strategies for dynamic portfolio selection, interactive procedure for determining the final compromise strategy for dynamic portfolio selection.

The following simple procedure can be applied for the identification of effective strategies:
Step 1: Starting from the last period \( t = T \), identify sub-effective strategy for all decision nodes of the period \( T \).

Step 2: Go to the previous period \( t = t-1 \).

Step 3: Identify strategies that meet the conditions of effectiveness for each decision node of the period \( t \).

Step 4: If \( t > 1 \), go to step 2, otherwise the procedure stops.

Number of effective strategies can be large. It is possible to use a simple interactive process between the decision maker and solver for the selection of the final compromise strategy from the set of all effective strategies. In each iteration \( q \), a set of strategies \( S(q) \) is analysed and the ideal alternative \( H(q) \) (vector of best values according to each criterion) and the anti-ideal alternative \( D(q) \) (vector of worst values according to each criterion) are determined. The decision maker compares between such values may vary criteria values. The decision maker is asked about the aspiration levels of criteria \( A(q) \), which he would accept as a compromise strategy. If the decision-maker is satisfied with the proposed strategy, the process stops.

Interactive process to determine the final compromise strategy has the following steps:

Step 1: Iteration \( q = 1 \), the set of all analysed strategies \( S(1) \) is equal to the set of all effective strategies.

Step 2: Determine the ideal alternative \( H(q) \) and the anti-ideal alternative \( D(q) \).

Step 3: Decision-maker is asked to accept anti-ideal values. If yes, go to Step 8.

Step 4: The decision-maker is asked to propose the aspiration levels \( A(q) \). If not, go to step 6.

Step 5: The decision-maker enters aspiration levels \( A(q) \) and he determines the corresponding set of acceptable strategies \( S(q+1) \). If \( S(q+1) = \emptyset \), go to step 4, otherwise to step 7.

Step 6: The decision-maker is asked which anti-ideal value is unacceptable for him. A new set of strategies is defined \( S(q+1) \) which exceed the unacceptable anti-ideal value.

Step 7: Set \( q = q+1 \), go to step 2.

Step 8: The decision-maker is asked which criterion should reach the ideal value. The strategy that maximizes this criterion is the resulting compromise one.

4. EFFICIENCY OF PROJECT PORTFOLIOS

We propose a new approach for efficient project portfolio designing based on a Data Envelopment Analysis (DEA). Possible projects are characterized by sets of inputs and outputs. The DEA is an appropriate approach to select efficient projects (Fiala, 2018). Inputs are resources for project realization and outputs are criteria. Charnes, Cooper, and Rhodes developed the first DEA model (Charnes, Cooper & Rhodes, 1978). The DEA model is based on the reduction of the multiple inputs and multiple outputs to that of a single "virtual" input and a single "virtual" output using weights. The model searches for the set of weights which maximize the efficiency of the project. The DEA may be characterized as a method of objective weight assessment. The DEA includes a number of models and methods to evaluating performance (Charnes, Cooper, Lewin & Seiford, 2013).

4.1 Efficient individual projects

For our problem, there is supposed a set \( P = \{P_1, P_2, \ldots, P_n\} \) of \( n \) projects each consuming \( r \) inputs and producing \( s \) outputs; \((r, n)\)-matrix and \((s, n)\)-matrix are observed input and output measures. The CCR model with supposed constant return to scale was used for project evaluations (Charnes, Cooper & Rhodes, 1978). Constant return to scale means that changing the amounts of inputs results in similar changes in the amounts of outputs. For a particular project, the ratio of the single output to the single input provides a measure of efficiency that is a function of the weight multipliers \((u, v)\). The relative efficiency \( e_k \) of the project \( P_k \) is maximised to the condition that the relative efficiency of each project is less than or equal to one.

A DEA-based approach allows each project to evaluate itself, relative to all the projects under consideration. The formulation leads to a linear fractional programming problem.
The problem is then formulated as follows:

\[
e'_k = \sum_{i=1}^{s} u_i y_{ik} \rightarrow \max, \quad k = 1, 2, \ldots, n
\]

(1)

\[
\frac{\sum_{i=1}^{s} u_i y_{ih}}{\sum_{j=1}^{r} v_j x_{jh}} \leq 1, \quad h = 1, 2, \ldots, n
\]

(2)

\[
u_i, v_j \geq 0, \quad i = 1, 2, \ldots, s, \quad j = 1, 2, \ldots, r
\]

(3)

If it is possible to find a set of weights for which the efficiency ratio of the project \( P_k \) is equal to one, the project \( P_k \) will be considered as efficient otherwise it will be considered as inefficient. The set of efficient projects is designed in this way.

Solving this nonlinear nonconvex problem directly is not an efficient approach. The following linear programming problem with new variable weights \((u, v)\) that results from the Charnes - Cooper transformation gives optimal values that will also be optimal for the fractional programming problem.

\[
e'_k = \sum_{i=1}^{s} u_i y_{ik} \rightarrow \max, \quad k = 1, 2, \ldots, n
\]

(4)

\[
\frac{\sum_{i=1}^{s} v_j x_{jk}}{\sum_{j=1}^{r} v_j x_{jh}} = 1
\]

(5)

\[
\sum_{i=1}^{s} u_i y_{ih} - \sum_{j=1}^{r} v_j x_{jh} \leq 0, \quad h = 1, 2, \ldots, n
\]

(6)

\[
u_i, v_j \geq 0, \quad i = 1, 2, \ldots, s, \quad j = 1, 2, \ldots, r
\]

(7)

The efficiency scores \( e'_k \) might be used to rank the projects. Implementing the most effective projects until resources are consumed will not always lead to the most effective portfolio. The reason is the same as for the knapsack problem.

### 4.2 Efficient portfolios

A portfolio as a subset \( C \) of the set of possible projects \( P (C \subseteq P) \) can be taken as a single combined project. The combined project is defined by combinations of outputs and combinations of inputs. The combination vector is \( \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n) \) where \( \lambda_i = 1 \) (the individual project \( P_i \) is included in the portfolio) or \( \lambda_i = 0 \) (the individual project \( P_i \) is not included in the portfolio). Total inputs of the combined project denoted as \( x_i(C) = \sum_{h=1}^{n} \lambda_h x_{jh} = 1, 2, \ldots, r \), and total outputs denoted as \( y_i(C) = \sum_{h=1}^{n} \lambda_h y_{ih} \), \( i = 1, 2, \ldots, s \), are determined by the combination vector \( \lambda \). The set of all combined projects is the so-called power set of \( P \) and the set denoted as \( R(P) \) where the number of elements in \( R(P) \) is \( 2^n \).

DEA-approach can be used for evaluation of each combined project relative to the power set \( R(P) \).

\[
e_C = \sum_{i=1}^{s} u_i \sum_{h=1}^{n} \lambda_h y_{ih} \rightarrow \max
\]

(8)

\[
\sum_{j=1}^{r} v_j \sum_{h=1}^{n} \lambda_h x_{jh} = 1
\]

(9)

\[
\sum_{i=1}^{s} u_i \sum_{h=1}^{n} \lambda_h y_{ih} - \sum_{j=1}^{r} v_j \sum_{h=1}^{n} \lambda_h x_{jh} \leq 0, \quad C \in R(P)
\]

(10)

\[
\lambda_h \in \{0, 1\}, h = 1, 2, \ldots, n
\]

(11)

\[
u_i, v_j \geq 0, \quad i = 1, 2, \ldots, s, \quad j = 1, 2, \ldots, r
\]

(12)

The model (8)-(12) is a non-linear one with variables \( \lambda_h, u_i, v_j \) where \( \lambda_h \) are elements of an unknown project combination vector and \( u_i, v_j \) are weights of outputs and inputs. Due to the large number of constraints (10) it is difficult to solve.

Introducing new variables

\[
c_{ih} = u_i \lambda_h, \quad d_{jh} = v_j \lambda_h, \quad i = 1, 2, \ldots, s, \quad j = 1, 2, \ldots, r, \quad h = 1, 2, \ldots, n
\]

(13)

linearizes this problem. The portfolio total inputs and outputs are compared against the set of all portfolios \( R(P) \) but it is easy to see that the general constraints (10) are additive combination of constraints for individual projects and it is sufficient to compare them with individual projects from the set \( P \) given by the constraint (16) (Cook and Green, 2000). Constraints for combined projects are redundant. The constraints (19) and (20) link new variables \( c_{ih}, d_{jh} \) and old variables \( u_i, v_j, \lambda_h \), where \( M \) is a large number. The constraint (19) links the variables \( c_{ih}, u_i, \lambda_h \). If the binary variable \( \lambda_h = 1 \), then \( 0 \leq c_{ih} \leq M, u_i = c_{ih} \) and if the binary variable \( \lambda_h = 0 \), then \( 0 \leq u_i \leq M, c_{ih} = 0 \). The constraint (20) analogically links the variables \( d_{jh}, v_j, \lambda_h \). The problem is then formulated as follows:

\[
e_C = \sum_{i=1}^{s} \sum_{h=1}^{n} c_{ih} y_{ih} \rightarrow \max
\]

(14)
\[
\sum_{j=1}^{r} \sum_{h=1}^{n} d_{jh} x_{jh} = 1 \quad (15)
\]
\[
\sum_{i=1}^{s} c_{ih} y_{ih} - \sum_{j=1}^{r} d_{jh} x_{jh} \leq 0, h = 1, 2, \ldots, n \quad (16)
\]
\[
\lambda_h \in \{0, 1\}, h = 1, 2, \ldots, n \quad (17)
\]
\[
u_i, v_j \geq 0, \quad i = 1, 2, \ldots, s, \quad j = 1, 2, \ldots, r, \quad (18)
\]
\[
c_{ih} \geq 0, c_{ih} \leq M \lambda_h, \quad u_i \geq c_{ih}, u_i \leq c_{ih} + M(1 - \lambda_h), i = 1, 2, \ldots, s, h = 1, 2, \ldots, n \quad (19)
\]
\[
d_{jh} \geq 0, d_{jh} \leq M \lambda_h, \quad v_j \geq d_{jh}, v_j \leq d_{jh} + M(1 - \lambda_h), j = 1, 2, \ldots, r, h = 1, 2, \ldots, n \quad (20)
\]

### 4.3 Illustrative example

The proposed analysis will be illustrated on a simple example to make the procedure and calculations easy to understand and to make detailed analyses of all portfolio structures. An organisation considers 5 potential projects \(P_1, P_2, \ldots, P_5\) that are characterized by two inputs \(I1, I2\) and two outputs \(O1, O2\).

The parameters of potential projects are given in Tab. 1.

| \(P_i\) | \(P_1\) | \(P_2\) | \(P_3\) | \(P_4\) | \(P_5\) |
|----------|------------|------------|------------|------------|------------|
| \(I1\)   | 6          | 3          | 8          | 9          | 5          |
| \(I2\)   | 8          | 4          | 2          | 4          | 6          |
| \(O1\)   | 9          | 7          | 6          | 10         | 8          |
| \(O2\)   | 12         | 10         | 15         | 8          | 12         |
| \(c_i\)  | 0.643      | 1          | 1          | 1          | 0.761      |

**Source:** Authors

The efficiency ratios \(e_i\) of projects were computed using the model (4)-(7). The set of efficient projects consists of projects \(P_2, P_3, P_4\).

There are 31 possible project portfolios from the 1-project to the 5-projects structure. The model (14)-(20) was used for efficiency evaluation of portfolios with different structures. Tab. 2 captures portfolio structures, a number of portfolios in structures, maximal efficiency ratios \(e_C\) in the structure, and the portfolios with maximal efficiency ratios.

| Structure | Number | Max \(e_C\) | Portfolios |
|-----------|--------|-------------|------------|
| 1-project | 5      | 1           | \(P_2, P_3, P_4\) |
| 2-projects| 10     | 1           | \(P_2, P_3\), \(P_2, P_4\), \(P_3, P_4\) |
| 3-projects| 10     | 1           | \(P_2, P_3, P_4\) |
| 4-projects| 5      | 0.920       | \(P_2, P_3, P_4, P_5\) |
| 5-projects| 1      | 0.838       | \(P_1, P_2, P_3, P_4, P_5\) |

**Source:** Authors

### CONCLUSION

The aim of the paper was to capture new trends in project portfolio management. These new trends are analyzed using sophisticated quantitative approaches. Popular approaches such as Analytic Network Process (ANP), multicriteria analysis, decision trees and Data Envelopment Analysis (DEA) were used for the analysis. These procedures have been adapted specifically for project portfolio management.

The proposed portfolio management procedure respects the characteristics of the problem:
- Network structure.
- Multi-criteria evaluation.
- Dynamics.
- Risk.
- Efficiency.

The procedure is flexible and can be modified and generalized. For example, other options for capturing the network structure, other methods of multi-criteria analysis, other principles for measuring risk and capturing dynamics, other ways of measuring the performance of project portfolios may be used. Experimental results will affect the specification, completing and extending the model.

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