Statistics of Entropy Production in Linearized Stochastic System

K. Turitsyn a,b, M. Chertkov b, V.Y. Chernyak b,c, A. Puliafito b,d,

a Landau Institute for Theoretical Physics, Moscow, Kosygina 2, 119334, Russia
b Theoretical Division and Center for Nonlinear Studies, LANL, Los Alamos, NM 87545, USA
c Department of Chemistry, Wayne State University, 5101 Cass Ave, Detroit, MI 48202, USA
d INLN, 1361 route des Lucioles, F-06560, Valbonne, France

(Dated: March 30, 2022)

PACS numbers: 83.80.Rs, 05.70.Ln, 05.10.Gg

We consider a wide class of linear stochastic problems driven off the equilibrium by a multiplicative asymmetric force. The force breaks detailed balance, maintained otherwise, thus producing entropy. The large deviation function of the entropy production in the system is calculated explicitly. The general result is illustrated using an example of a polymer immersed in a gradient flow and subject to thermal fluctuations.

PACS numbers: 83.80.Rs, 05.70.Ln, 05.10.Gg

The Gibbs distribution, \( \mathcal{P}_{eq}(x) \sim \exp[-U(x)/T] \), describes the probability for a system characterized by the microscopic potential \( U(x) \) and maintained at equilibrium at temperature \( T \) to be observed in the state \( x \). In particular, in our model case of a polymer the elastic potential \( U(x) \) depends on the end-to-end position vector \( x \). The system at equilibrium maintains the detailed balance, which is the most fundamental principle of equilibrium statistical mechanics [1, 2, 3, 4]. Formally, the detailed balance means that the probabilities \( \mathcal{P}\{x\} \) and \( \mathcal{P}\{x^{*}\} \) for a stochastic trajectory, \( \{x\} \equiv \{x(t')\}; 0 < t' < t \) and its “conjugated twin” \( \{x^{*}\} \equiv \{x^{*}(t') = x(t - t'); 0 < t' < t \) are related by

\[
\text{in balance: } \ln \frac{\mathcal{P}\{x\}}{\mathcal{P}\{x^{*}\}} = \frac{U(x(t)) - U(x(0))}{T}, \tag{1}
\]

Asymmetric external force breaks down the detailed balance. For example, a shearing flow forces the polymer to tumble and results in steady entropy production [6,7]. In general, configurational entropy is naturally defined as a mismatch between the left and right hand sides of Eq. (1):

\[
\text{off balance: } S = \ln \frac{\mathcal{P}\{x^{*}\}}{\mathcal{P}\{x\}} + \frac{U(x(t)) - U(x(0))}{T} \neq 0. \tag{2}
\]

For a wide class of thermalized systems, driven out of equilibrium by external non-conservative forces the entropy has also a standard thermodynamic interpretation: It determines the total heat produced by the system over time \( t \). In the off-detailed balance case entropy is a fluctuating function of the entire configurational trajectory \( \{x\} \). Therefore, in the statistically steady non-equilibrium case fluctuations occur on the top of a steady mean growth of the entropy and one can argue that at sufficiently large observation time the distribution function of the produced entropy \( S \) takes a large deviation form [8,9]:

\[
\mathcal{P}(S|t) \sim \exp[-t\mathcal{L}(S\tau/t)/\tau], \tag{3}
\]

where \( \tau \) is the typical correlation (turnover) time of the system and \( \mathcal{L}(\omega) \) is referred to as the large deviation function. Description of the large deviation function for a truly non-equilibrium problem is a difficult task, and only a few successful results have been reported so far [10]. To clarify the difficulty let us also mention that a simpler problem of finding an off-detailed balance analog of the Gibbs distribution, posed in a classical work of Onsager [11], has been solved for a few examples only. (See [12, 13, 14] for discussion of some difficulties, progress and results achieved on this thorny path.) It is worth to note that the large deviation function of entropy production was computed and verified experimentally for a number of other physical situation, e.g. optically dragged Brownian particles, electrical circuits and forced harmonic oscillators [15]. Although these works are ideologically similar, technically they study different non-equilibrium systems, which are either non-steady or do not have the detailed balance broken.

In this letter we present a solution of this challenging task for a wide class of linear problems driven by multiplicative asymmetric “force” and also connected to a Langevin reservoir. Such problems arise whenever a statistically steady non-equilibrium state is externally driven by space \( x \)-dependent non-conservative external forces. In this context our main physical example is of a Hookean polymer stretched in terms of an one-dimensional integral Eq. (16). For the linear non-equilibrium setting we report an explicit expression for the large deviation function of the entropy production. For the most general case our result is given as a solution of a well-defined system of algebraic equations or, alternatively, in terms of an one-dimensional integral Eq. (16). For the linear polymer in a constant gradient flow the large deviation function is presented in terms of elementary functions, see e.g. Eq. (13). The most important features of the results for the large deviation function derived in this Letter are:

- 1. The steady state solutions are consistent with the fluctuation theorem [16,17,18]:

\[
\mathcal{L}(\omega) - \mathcal{L}(-\omega) = -\omega. \tag{4}
\]

- 2. The large deviation function is found to be very different from the Gaussian shape. Extreme tails of the entropy PDF are exponential, that are the steepest tails allowed by the large deviation form [13].

- 3. One observes reduction in the number of parameters affecting the shape of the large deviation function. For example, the

- 4. The theoretical predictions are verified by a number of specific examples.
large deviation function is completely insensitive to the symmetric part of the velocity gradient in the case of the linear polymer immersed in a 2d flow.

The letter is organized as follows. After introducing the polymer in a flow example, we focus on this model calculating the entropy production and defining the corresponding generating function. We further derive a Fokker-Planck equation for the generating function and solve it using a Gaussian ansatz for the related eigenvalue problem. This results in a system of matrix algebraic equations that define the principle part of the long-time asymptotic of the generating function.

The solution for the most general case of two-dimensional flow is expressed in terms of elementary functions. This explicit description is also extended to a special three dimensional case. Then, we switch to a general linear model with a constant multiplicative force. The general model, defined in Eq. (14), is analyzed using a direct solution of the linear stochastic equations followed by averaging the resulting expression for the generating function over the Langevin noise.

This method is complementary to the aforementioned Fokker-Planck approach. The final result for the large time asymptotic of the generating function is presented in terms of the one-dimensional integral over frequency.

The polymer’s end-to-end vector, \(x_i\), satisfies a stochastic equation of motion [19]

\[
\dot{x}_i - \sigma_{ij} x_j = -\partial_x U(x) + \xi_i,
\]

where the traceless matrix \(\hat{\sigma}\) describes the local value of the velocity gradient in the generic incompressible flow (\(i = 1, \ldots, d\)). The smoothness of velocity on the scale of an even very extended polymer is justified by many experimental observations, see e.g. [19]. Eq. (5) describes the balance of forces: the second term on the rhs of Eq. (5) account for the polymer elasticity and for the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise.

The letter is organized as follows. After introducing the polymer in a flow example, we focus on this model calculating the entropy production and defining the corresponding generating function. We further derive a Fokker-Planck equation for the generating function and solve it using a Gaussian ansatz for the related eigenvalue problem. This results in a system of matrix algebraic equations that define the principle part of the long-time asymptotic of the generating function.

The solution for the most general case of two-dimensional flow is expressed in terms of elementary functions. This explicit description is also extended to a special three dimensional case. Then, we switch to a general linear model with a constant multiplicative force. The general model, defined in Eq. (14), is analyzed using a direct solution of the linear stochastic equations followed by averaging the resulting expression for the generating function over the Langevin noise.

This method is complementary to the aforementioned Fokker-Planck approach. The final result for the large time asymptotic of the generating function is presented in terms of the one-dimensional integral over frequency.

The polymer’s end-to-end vector, \(x_i\), satisfies a stochastic equation of motion [19]

\[
\dot{x}_i - \sigma_{ij} x_j = -\partial_x U(x) + \xi_i,
\]

where the traceless matrix \(\hat{\sigma}\) describes the local value of the velocity gradient in the generic incompressible flow (\(i = 1, \ldots, d\)). The smoothness of velocity on the scale of an even very extended polymer is justified by many experimental observations, see e.g. [19]. Eq. (5) describes the balance of forces: the second term on the rhs of Eq. (5) account for the polymer elasticity and for the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise.

The letter is organized as follows. After introducing the polymer in a flow example, we focus on this model calculating the entropy production and defining the corresponding generating function. We further derive a Fokker-Planck equation for the generating function and solve it using a Gaussian ansatz for the related eigenvalue problem. This results in a system of matrix algebraic equations that define the principle part of the long-time asymptotic of the generating function.

The solution for the most general case of two-dimensional flow is expressed in terms of elementary functions. This explicit description is also extended to a special three dimensional case. Then, we switch to a general linear model with a constant multiplicative force. The general model, defined in Eq. (14), is analyzed using a direct solution of the linear stochastic equations followed by averaging the resulting expression for the generating function over the Langevin noise.

This method is complementary to the aforementioned Fokker-Planck approach. The final result for the large time asymptotic of the generating function is presented in terms of the one-dimensional integral over frequency.

The polymer’s end-to-end vector, \(x_i\), satisfies a stochastic equation of motion [19]

\[
\dot{x}_i - \sigma_{ij} x_j = -\partial_x U(x) + \xi_i,
\]

where the traceless matrix \(\hat{\sigma}\) describes the local value of the velocity gradient in the generic incompressible flow (\(i = 1, \ldots, d\)). The smoothness of velocity on the scale of an even very extended polymer is justified by many experimental observations, see e.g. [19]. Eq. (5) describes the balance of forces: the second term on the rhs of Eq. (5) account for the polymer elasticity and for the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise.

The letter is organized as follows. After introducing the polymer in a flow example, we focus on this model calculating the entropy production and defining the corresponding generating function. We further derive a Fokker-Planck equation for the generating function and solve it using a Gaussian ansatz for the related eigenvalue problem. This results in a system of matrix algebraic equations that define the principle part of the long-time asymptotic of the generating function.

The solution for the most general case of two-dimensional flow is expressed in terms of elementary functions. This explicit description is also extended to a special three dimensional case. Then, we switch to a general linear model with a constant multiplicative force. The general model, defined in Eq. (14), is analyzed using a direct solution of the linear stochastic equations followed by averaging the resulting expression for the generating function over the Langevin noise.

This method is complementary to the aforementioned Fokker-Planck approach. The final result for the large time asymptotic of the generating function is presented in terms of the one-dimensional integral over frequency.

The polymer’s end-to-end vector, \(x_i\), satisfies a stochastic equation of motion [19]

\[
\dot{x}_i - \sigma_{ij} x_j = -\partial_x U(x) + \xi_i,
\]

where the traceless matrix \(\hat{\sigma}\) describes the local value of the velocity gradient in the generic incompressible flow (\(i = 1, \ldots, d\)). The smoothness of velocity on the scale of an even very extended polymer is justified by many experimental observations, see e.g. [19]. Eq. (5) describes the balance of forces: the second term on the rhs of Eq. (5) account for the polymer elasticity and for the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise.

The letter is organized as follows. After introducing the polymer in a flow example, we focus on this model calculating the entropy production and defining the corresponding generating function. We further derive a Fokker-Planck equation for the generating function and solve it using a Gaussian ansatz for the related eigenvalue problem. This results in a system of matrix algebraic equations that define the principle part of the long-time asymptotic of the generating function.

The solution for the most general case of two-dimensional flow is expressed in terms of elementary functions. This explicit description is also extended to a special three dimensional case. Then, we switch to a general linear model with a constant multiplicative force. The general model, defined in Eq. (14), is analyzed using a direct solution of the linear stochastic equations followed by averaging the resulting expression for the generating function over the Langevin noise.

This method is complementary to the aforementioned Fokker-Planck approach. The final result for the large time asymptotic of the generating function is presented in terms of the one-dimensional integral over frequency.

The polymer’s end-to-end vector, \(x_i\), satisfies a stochastic equation of motion [19]

\[
\dot{x}_i - \sigma_{ij} x_j = -\partial_x U(x) + \xi_i,
\]

where the traceless matrix \(\hat{\sigma}\) describes the local value of the velocity gradient in the generic incompressible flow (\(i = 1, \ldots, d\)). The smoothness of velocity on the scale of an even very extended polymer is justified by many experimental observations, see e.g. [19]. Eq. (5) describes the balance of forces: the second term on the rhs of Eq. (5) account for the polymer elasticity and for the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise. In this Letter we consider a deterministic constant gradient flow, the Langevin term in Eq. (5) is modeled by the Langevin thermal noise.
where the generating function and the eigenvalue of the ground state are well defined only within a finite interval, \( q \in [q_-; q_+] \), where \( q_\pm = 1/2 \pm \sqrt{1/4 - 1/(c^2 \tau^2)} \). Notice that the eigen-value \( \lambda_q \) given by Eq. (12) does not depend on the symmetric part of \( \sigma \), even though the resulting \( \hat{B}_q \) function does depend explicitly on both symmetric and anti-symmetric components. Formally this reflects the invariance of the Fokker-Planck operator with respect to a family of iso-spectral transformations that keep the spectra (or at least its ground state) invariant. We do not know however a physically intuitive explanation for this remarkable symmetry/reduction in the model. A similar and equally surprising phenomenon of reduction in the degrees-of-freedom number that control the large deviation function of a current has been recently reported for a different non-equilibrium system that models a contact between two thermostats kept at different temperatures \([10]\). Combining Eqs. (8,12) leads to

\[
\mathcal{L}(\omega) = \sqrt{(1 + c^2 \tau^2)}(4c^2 \tau^2 + \omega^2) - 1 - \omega/2. \tag{13}
\]

Note that, first, \( \mathcal{L}(\omega) \) satisfies the fluctuation theorem \([4]\); and, second, the asymptotics of \( \mathcal{L}(\omega) \) at \( |\omega| \gg c \tau \) are both linear in \( \omega \), making the extreme deviation asymptotics of the entire distribution function of \( S \) exponential and time independent.

Tracking the origin of the exponential tails back to a special form of the generating function \([12]\), one finds that these extreme asymptotics correspond to the square root singularity of \( \lambda_q \) at \( q_\pm \). Time-independence of the \( P(S|t) \) asymptotics means that typical trajectories contributing the PDF tail are correlated at some finite times, so that the observational time \( t \) increase does not change corresponding probabilities. Figure 1 shows the large deviation function given by Eq. (13) verified versus Brownian dynamics simulations. In the case of a \( d = 3 \) gradient flow the algebraic system of Eqs. (11) is too complicated to allow a solution in terms of elementary functions for an arbitrary form of the velocity gradient matrix \( \tilde{\sigma} \). Thus we mention only a special example of a 3d flow with the following nonzero elements of \( \tilde{\sigma} \): \( \sigma_{11} = -a_1, \sigma_{22} = a_1 + a_2, \sigma_{33} = -a_2, \sigma_{13} = c, \sigma_{31} = -c \). In this case the generating function and the large deviation function is given by Eqs. (13) modified according to the following simple renormalization of \( \tau \rightarrow \tau/|1 + at| \) (and \( \omega \) respectively).

An alternative derivation of the large deviation function starts with considering a general linear problem

\[
\dot{x}_i = \Phi_{ij} x_j + \Upsilon_{ij} \xi_j, \tag{14}
\]

where \( \hat{\Phi} \) and \( \hat{\Upsilon} \) are arbitrary constant matrices. Eq. (14) describes linear stochastic dynamics around a fixed point, that can be stable or unstable. We discuss here a truly non-equilibrium (off-detailed balance) steady state maintained if the fluctuations do not exceed a threshold so that nonlinear effects can be ignored (see \([14]\) and references therein). A flux state observed in diffusive system \([22]\) is a popular example that involves an infinite-dimensional configurational variable \( x \). Many examples of the off-detailed balance steady systems, e.g. vesicles or red-blood cells in external flows \([23]\) and macromolecular biological devices, such as enzyme motors \([24]\), come from biology and soft-matter physics. Obviously, the \( \tilde{\Upsilon} = 1 \) version of Eq. (14) describes the aforementioned two beads Hookean polymer model as well, however it is worth mentioning that the full version of Eq. (14) also appears naturally in a more general polymer context, where the \( \tilde{\Upsilon} \neq 1 \) case models hydrodynamic interactions between the different parts of the polymer chain \([19]\). Eq. (14) also describes fluctuations around a stretched state above the coil-stretch transition \([25, 26, 27]\) in a strong gradient flow \([28]\).

According to Eq. (2) the entropy production in the system described by Eq. (14) is given by \( S = \int_0^t dt' \dot{x}^T (t') (\dot{K} \dot{\Phi} - \dot{\Phi}^+ \dot{K}) x(t')/(2T) \), where \( \dot{K} \equiv (\tilde{\Upsilon} \dot{\Upsilon}^+)^{-1} \). To discuss dynamics at large but finite time \( t \) it is convenient to invoke a discrete (a-la Matsubara) frequency representation

\[
\dot{x}(t') = \sum_k (c_k \exp(i \omega_k t') + c_k^* \exp(-i \omega_k t')) / \sqrt{T},
\]

\[
Z_q = \int \prod_k Dc_k Dc_k^* \exp \left( - \sum_k c_k^* A_q(\omega_k)c_k/(2T) \right), \tag{15}
\]

\[
A_q(\omega) = \omega^2 \dot{K} + \dot{\Phi}^+ \dot{K} \dot{\Phi} + i \omega (1 - 2q)(\dot{K} \dot{\Phi} - \dot{\Phi}^+ \dot{K}),
\]

where \( \omega_k = 2\pi k/t, \ k = 1, 2, \ldots \). Straightforward Gaussian integration in Eq. (15) yields: \( \lambda_q t = \sum_k \log(\det A_q(\omega_k)/\det A_0(\omega_k)) \). In the \( t \rightarrow \infty \) limit one replaces summation over \( k \) by integration and arrives at

\[
\lambda_q = \int_0^\infty \frac{d\omega}{2\pi} \log \left( \frac{\det A_q(\omega)}{\det A_0(\omega)} \right), \tag{16}
\]

which is the most general long-time asymptotic result reported in this letter. It is straightforward to verify that in the special case of linear polymer advected by \( d = 2 \) gradient flow, \( \dot{\Phi} = \dot{\sigma} - 1/t, \) and Langevin driving, \( \tilde{\Upsilon} = 1 \), the integral representation (15) for \( \lambda_q \) turns into Eq. (12) derived earlier using the spectral method.

![FIG. 1: Large deviation function in d = 2 for three values of the governing parameter cτ = 1, 2, 4. (red;green;blue).](image-url)
Note that Eq. (16) also suggests a convenient way to determine the values of \( q_\pm \) for a general linear system (14). One finds that \( q_+ \) is a minimal positive value of \( q \) for which a solution of the equation \( \det A_q(\omega) = 0 \) does exist. The value of \( \omega = \omega_+ \) which solves this equation is related to the characteristic time-scale of an optimal fluctuation which controls the exponential tail of the entropy PDF. For the two-dimensional polymer problem this leads to the following explicit expression for \( \omega_+ = \sqrt{1 + c^2 - a^2 - b^2}/T. \) The limit of \( \omega_+ \to 0 \) corresponds to the coil-stretch transition.

We conclude compiling an incomplete list of future challenges related to the approach and results reported in this Letter. First, our analysis of the entropy production in a polymer system extends to the case of chaotic flows, e.g. realized in the recently discovered elastic turbulence (21). Of a particular interest here is to check sensitivity of the large deviation function to the coil-stretch transition observed in the chaotic problem (25, 26, 27). Second, introducing a finite time protocol for a controlled parameter (e.g. shear in the polymer solution experiments) one may be interested to go beyond the analysis of the stationary problem, in particular discussing an off-detailed description of the entropy production in turbulence by experimental, numerical and theoretical means may have a tremendous impact on understanding of this most challenging problem in nonequilibrium physics.

This work was carried out under the auspices of the National Nuclear Security Administration of the U.S. Department of Energy at Los Alamos National Laboratory under Contract No. DE-AC52-06NA25396. VYC also acknowledges the support through WSU. KT acknowledges the support by ENS and INTAS.

[1] R.C. Tolman, Phys.Rev. 23, 693 (1924).
[2] P.W. Bridgman, Phys.Rev. 31, 90 (1928).
[3] H. Nyqist, Phys.Rev. 32, 110 (1928).
[4] Strictly speaking Eq. (1) should be called “microscopic reversibility” while the “detailed balance” is referred to a probability of changing states without a reference to a particular temporal path. See e.g. [5] for modern discussion of the terminological nuance.
[5] G.E. Crooks, Phys.Rev.E. 60, 2721-2726 (1999).
[6] E. J. Hinch, Phys. Fluids, 20, S22 (1977).
[7] M. Chertkov, I. Kolokolov, V. Lebedev and K. Turitsyn, Journal of Fluid Mechanics 531, 251-260 (2005).
[8] R. Ellis, Entropy, Large Deviations and Statistical Mechanics, Springer-Verlag, Berlin, 1985.
[9] J.L. Lebowitz, H. Spohn, J. Stat.Phys. 95, 333 (1999).
[10] B. Derrida, Pramana J.of Physics 64, 695 (2005).
[11] L. Onsager, Phys.Rev. 37, 405 (1931).
[12] N.G. van Kampen, Stochastic Processes in Physics and Chemistry, Elsevier, Amsterdam, 1992.
[13] S. Tanase-Nicola, J. Kurchan, J.Stat.Phys. 116, 1201 (2004).
[14] C. Kwon, P. Ao, D.J. Thouless, PNAS 102, 13029 (2005).
[15] R. Van Zon, E. G. D. Cohen, Phys. Rev. E. 67, 461021 (2003); R. Van Zon, S. Ciliberto, E. G. D. Cohen, Phys.Rev Lett 92, 130601 (2004); N. Garnier, S. Ciliberto, Phys. Rev. E, 71, 060101 (2005); F. Dourache, S. Joubaud, N. B. Garnier, A. Petrosyan, S. Ciliberto, Phys Rev Lett. 97, 140603 (2006).
[16] D.J. Evans, E.G.D. Cohen, G.P. Morris, Phys.Rev.Lett. 71, 2401 (1993).
[17] G. Gallavotti, E.G.D. Cohen, Phys.Rev.Lett. 74, 2694 (1995).
[18] J. Kurchan, J.Phys.A 31, 3719 (1998).
[19] R.B. Bird, C.F. Curtiss, R.C. Armstrong, O. Hassager, Dynamics of polymeric liquids, vol.2, Wiley & Sons, 1987.
[20] V. Chernyak, M. Chertkov and C. Jarzynski, J. Stat. Mech. P08001 (2006).
[21] A. Groisman and V. Steinberg, Nature 405, 53 (2000).
[22] B. Derrida, J. L. Lebowitz, E. R. Speer, Phys Rev Lett 87, (2001); T. Bodineau, B. Derrida, Phys. Rev. E 72,066110 (2005).
[23] V. Kantsler, V. Steinberg, Phys Rev Lett 95, 258101 (2005); M. Kraus, W. Wintz, U. Seifert, R. Lipowsky, Phys Rev Lett 77, 3685 (1996).
[24] G. Oster, H. Wang, Trends Cell Biol. 13, 114 (2003); U. Seifert, Europhys. Lett. 70, 36 (2005).
[25] J. L. Lumley, Annu. Rev. Fluid Mech. 1, 367 (1969); J. Polymer Sci.: Macromolecular Reviews 7, 263 (1973).
[26] E. Balkovsky, A. Fouxon, V. Lebedev, Phys. Rev. Lett. 84, 4765 (2000); Phys. Rev. E 64, 056301 (2001).
[27] M. Chertkov,Phys. Rev. Lett. 84, 4761 (2000).
[28] Nonlinear polymer in a strong gradient flow is described by Eq. 5 with \( U(x) = \gamma(x)x^2/2 \), where the nonlinear relaxation rate, \( \gamma(x) \), is a monotonic function of the polymer length with \( \gamma(0) = T^{-1} < \gamma(x_{max}) = +\infty \), and the largest eigenvalue \( \mu \) of \( \sigma \) satisfies \( \mu^\tau > 1 \). Deterministic polymer dynamics has a fixed point \( X: \sigma X = \gamma(X)X \). Thermal fluctuations around the stable fixed point, \( y = x - X \), are described by the linear equation \( \dot{y} = \sigma y - \gamma(X)y \). The equation is valid only for small fluctuations near the fixed points \( X \) or \( -X \), present in the problem. It does not explain thermally driven tunneling between the fixed points. However, for sufficiently small temperature the tunneling is weak and its contribution to the total entropy production is negligible.
[29] C. Jarzynski, Phys. Rev. Lett. 78, 2690 (1997); Phys.Rev. E 56, 5018 (1997).
[30] T. Hatano, Phys.Rev.E. 60, R5017 (1999).
[31] V. Chernyak, M. Chertkov and C. Jarzynski, Phys.Rev. E 71, 025102 (2005).
[32] D. Collin, F. Ritort, C. Jarzynski, S.B. Smith, I. Tinoco, C. Bustamante, Nature 437, 231 (2005).
[33] S. Chu, Phil. Trans. R. Soc. Lond. A 361, 689 (2003).
[34] S. Gerashchenko, C. Chevallard, and V. Steinberg, Europhys.Lett. 71 221 (2005).
[35] U. Frisch, Turbulence. The legacy of A.N. Kolmogorov, Cambridge U. Press, 1995.