Spin-$\frac{3}{2}$ doubly charmed baryon contribution to the magnetic moments of the spin-$\frac{1}{2}$ doubly charmed baryons

Hao-Song Li$^{1,2,3,*}$ and Wen-Li Yang$^{1,2,3}$

$^1$Institute of Modern Physics and School of Physics, Northwest University, Xian 710127, China
$^2$Shaanxi Key Laboratory for Theoretical Physics Frontiers, Xian 710127, China
$^3$Peng Huanwu Center for Fundamental Theory, Xian 710127, China

We have systematically investigated the magnetic moments of spin-$\frac{1}{2}$ doubly charmed baryons in the framework of the heavy baryon chiral perturbation theory. In this paper, one loop corrections with intermediate spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ doubly charmed baryon states are considered. The numerical results are calculated to next-to-leading order: $\mu_{\Xi^{++}cc} = 0.35\mu_N$, $\mu_{\Xi^{+}cc} = 0.62\mu_N$, $\mu_{\Omega^{++}cc} = 0.41\mu_N$. Our results may be useful for future experiment and chiral extrapolation of the lattice QCD.

PACS numbers:
Keywords:

I. INTRODUCTION

The doubly charmed baryon was first claimed by SELEX Collaboration in the decay mode $\Xi^{++}_{cc} \to \Lambda^{+}_{cc}K^-\pi^+$ with the mass $M_{\Xi^{++}_{cc}} = 3519 \pm 1$ MeV [1]. Since then, many experimental collaborations have shown great interest to the doubly charmed baryons. Although other experimental collaborations like FOCUS [2], BABAR [3] and Belle [4] did not find any evidence for doubly charmed baryons, researchers have never stopped studying on the doubly charmed baryons. In 2017, LHCb collaboration reported the discovery of $\Xi^{++}_{cc}$ in the mass spectrum of $\Lambda^{+}_{cc}K^-\pi^+\pi^+$ with the mass $M_{\Xi^{++}_{cc}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ MeV [5].

In the past decade, the masses and decay properties of double charmed baryons have been studied extensively in literature [6–44]. It is very important to investigate the baryon electromagnetic form factors, especially the magnetic moments as the electromagnetic properties give information about the internal structures and shape deformations, which provide valuable insight in describing the inner structures of hadrons and understanding the mechanism of strong interactions at low-energy. With nonrelativistic quark model, Lichtenberg first investigated the doubly charmed baryons magnetic moments in Ref. [45]. With relativistic quark model, the magnetic moments have also been evaluated in Refs. [46, 47]. Besides the quark models, magnetic moments have been studied in different theoretical models and approaches [48–54], however it is difficult to include the chiral corrections. In fact, Chiral perturbation theory (ChPT) [55] and heavy baryon chiral perturbation theory (HBChPT) [56–59] are quite helpful to analyze the low-energy interactions order by order. To consider the chiral corrections, the magnetic moments of spin-$\frac{1}{2}$ doubly charmed baryons have been investigated with HBChPT in Ref. [60]. In Ref. [61], the electromagnetic form factors of the doubly charmed baryons have been studied in covariant chiral perturbation theory within the extended on-mass-shell (EOMS) scheme. However, the spin-$\frac{3}{2}$ doubly charmed baryon was not included explicitly in the analysis of Refs. [60, 61].

In this work, we will investigate the magnetic moments of spin-$\frac{1}{2}$ doubly charmed baryons in HBChPT. We explicitly consider both the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ doubly charmed baryon intermediate states in the loop calculation because the mass splitting between the octet and decuplet baryons is small. Moreover, the spin-$\frac{3}{2}$ doubly charmed baryon generally strongly couple to the spin-$\frac{1}{2}$ doubly charmed baryon. We use quark model to determine the corresponding low energy constants (LECs) and calculate the magnetic moments order by order. The analytical and numerical results are calculated to next-to-leading order.

This paper is organized as follows. In Section II we discuss the electromagnetic form factors of spin-$\frac{1}{2}$ doubly charmed baryons. We introduce the effective chiral Lagrangians of spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ doubly charmed baryons in Section III. We calculate the magnetic moments of the spin-$\frac{3}{2}$ doubly charmed baryons order by order in Section IV and present our numerical results in Section V. A short summary is given in Section VI. We collect some useful formulae in the Appendix A.

*Electronic address: haosongli@nwu.edu.cn
II. ELECTROMAGNETIC FORM FACTORS

For spin-$\frac{1}{2}$ doubly charmed baryons, the matrix elements of the electromagnetic current read,

$$<\Psi(p')|J_\mu|\Psi(p)> = e\bar{u}(p')\mathcal{O}_\mu(p', p)u(p),$$

with

$$\mathcal{O}_\mu(p', p) = \frac{1}{M_H}[P_\mu G_E(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2}G_M(q^2)].$$

where $P = \frac{1}{2}(p' + p)$, $q = p' - p$, $M_H$ is doubly charmed baryon mass.

In the heavy baryon limit, the doubly charmed baryon field $B$ can be decomposed into the large component $H$ and the small component $L$.

$$B = e^{-iM_Hv\cdot x}(H + L),$$

$$H = e^{iM_Hv\cdot x}\frac{1 + \gamma}{2}B, \quad L = e^{iM_Hv\cdot x}\frac{1 - \gamma}{2}B,$$

where $M_H$ is the spin-$\frac{1}{2}$ doubly charmed baryon mass and $v^\mu = (1, \vec{0})$ is the velocity of the baryon. Now the doubly charmed baryon matrix elements of the electromagnetic current $J_\mu$ can be parametrized as

$$<H(p')|J_\mu|H(p)> = e\bar{u}(p')\mathcal{O}_\mu(p', p)u(p).$$

The tensor $\mathcal{O}_\mu$ can be parameterized in terms of electric and magnetic form factors.

$$\mathcal{O}_\mu(p', p) = v_\mu G_E(q^2) + \frac{[S_\mu, S^\nu]q^\nu}{M}G_M(q^2),$$

where $G_E(q^2)$ is the electric form factor and $G_M(q^2)$ is the magnetic form factor. When $q^2 = 0$, we obtain the charge $(Q)$ and magnetic moment $(\mu_H)$,

$$Q = G_E(0), \quad \mu_H = \frac{e}{2M_H}G_M(0).$$

III. CHIRAL LAGRANGIANS

A. The strong interaction chiral Lagrangians

To calculate the chiral corrections to the magnetic moment of doubly charmed baryon, we follow the basic definitions of the pseudoscalar mesons and the spin-$\frac{1}{2}$ baryon chiral effective Lagrangians in Refs. [59, 65] to construct the relevant chiral Lagrangians. The pseudoscalar meson fields read

$$\phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}K^0 & \frac{1}{\sqrt{2}}\eta \end{pmatrix}. \quad \text{(8)}$$

The chiral connection and axial vector field read [55, 56],

$$\Gamma_\mu = \frac{1}{2} \left[ u^\dagger(\partial_\mu - i\gamma_5)u + u(\partial_\mu - i\gamma_5)u^\dagger \right], \quad \text{(9)}$$

$$u_\mu \equiv \frac{1}{2} i \left[ u^\dagger(\partial_\mu - i\gamma_5)u - u(\partial_\mu - i\gamma_5)u^\dagger \right], \quad \text{(10)}$$
where

\[ u^2 = U = \exp(i\phi/f_0). \]  

(11)

We use the pseudoscalar meson decay constants \( f_\pi \approx 92.4 \text{ MeV}, f_K \approx 113 \text{ MeV} \) and \( f_\eta \approx 116 \text{ MeV} \).

The doubly charmed baryon fields read,

\[
\Psi = \begin{pmatrix} \Xi_{cc}^{+++} \\ \Xi_{cc}^{++} \\ \Omega_{cc}^{++} \end{pmatrix}, \quad \Psi^\mu = \begin{pmatrix} \Xi_{cc}^{+++} \\ \Xi_{cc}^{++} \\ \Omega_{cc}^{++} \end{pmatrix},
\]

\[
(12)
\]

where \( \Psi \) and \( \Psi^\mu \) are spin-\( \frac{1}{2} \) and spin-\( \frac{3}{2} \) doubly charmed baryon fields, respectively. The leading order pseudoscalar meson and baryon interaction Lagrangians read

\[
\tilde{\mathcal{L}}_0^{(1)} = \bar{\Psi}(i\partial - M_H)\Psi + \bar{\Psi}\gamma^\mu[\gamma_\mu, \Psi] + i(\gamma_\mu D_\mu + \gamma_\nu D_\nu) - \gamma_\mu(i\partial + M_T)\gamma_\nu \bar{\Psi}^\mu,
\]

\[
(13)
\]

\[
\tilde{\mathcal{L}}_{\text{int}}^{(1)} = \frac{\tilde{g}_A}{2} \bar{\Psi} \gamma_5 \Psi + \frac{\tilde{g}_C}{2} \bar{\Psi}^\mu \gamma_5 u_\mu \Psi + \bar{\Psi} u_\mu \Psi^\mu,
\]

\[
(14)
\]

where \( M_H \) is the spin-\( \frac{1}{2} \) doubly charmed baryon mass, \( M_T \) is the spin-\( \frac{3}{2} \) doubly charmed baryon mass,

\[
D_\mu \Psi = \partial_\mu \Psi + \Gamma_\mu \Psi,
\]

\[
D^\nu \Psi^\mu = \partial^\nu \Psi^\mu + \Gamma^\nu \Psi^\mu.
\]

(15)

In the framework of HBCChPT, we denote the large component of the spin-\( \frac{3}{2} \) doubly charmed baryon as \( T_\mu \). The leading order nonrelativistic pseudoscalar meson and baryon Lagrangians read

\[
\mathcal{L}_0^{(1)} = \bar{H}(iv \cdot D)H - iT_\mu(v \cdot D - \delta)T_\mu,
\]

\[
(16)
\]

\[
\mathcal{L}_{\text{int}}^{(1)} = \tilde{g}_A \bar{H} S_\mu u^\mu H + \frac{\tilde{g}_C}{2} \{[\bar{\Psi}^\mu u_\mu H + \bar{H} u_\mu T^\mu],
\]

\[
(17)
\]

where \( S_\mu \) is the covariant spin-operator, \( \delta = M_T - M_H \) is the spin-\( \frac{1}{2} \) and spin-\( \frac{3}{2} \) doubly charmed baryon mass splitting. The \( \phi HH \) coupling \( \tilde{g}_A \) and \( \phi TT \) coupling \( \tilde{g}_C \) can be estimated in the quark model in Section \( \Box \). For the pseudoscalar mesons masses, we use \( m_\pi = 0.140 \text{ GeV}, m_K = 0.494 \text{ GeV}, \) and \( m_\eta = 0.550 \text{ GeV} \). We use the nucleon masses \( M_B = 0.938 \text{ GeV} \) and the spin-\( \frac{1}{2} \) doubly charmed baryon mass \( M_H = 3.62 \text{ GeV} \).

B. The electromagnetic chiral Lagrangians at \( \mathcal{O}(p^2) \)

The lowest order \( \mathcal{O}(p^2) \) Lagrangian contributes to the magnetic moments of the doubly charmed baryons at the tree level

\[
\mathcal{L}_{\mu\nu}^{(2)} = a_1 \frac{-i}{4 M_B} \bar{H}[S^\mu, S^\nu] \hat{F}^+_{\mu\nu} H + a_2 \frac{-i}{4 M_B} \bar{H}[S^\mu, S^\nu] H \text{Tr}(F^+_{\mu\nu}),
\]

(18)

where the coefficients \( a_{1,2} \) are the LECs. The chirally covariant QED field strength tensor \( F^\pm_{\mu\nu} \) is defined as

\[
F^+_{\mu\nu} = \begin{pmatrix} u^1 F_{\mu\nu}^R \pm u F_{\mu\nu}^L \end{pmatrix},
\]

\[
F_{\mu\nu}^R = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu],
\]

\[
F_{\mu\nu}^L = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu],
\]

(19)

(20)

where \( r_\mu = l_\mu = -e Q_H A_\mu \) and \( Q_H = \text{diag}(2,1,1) \). The operator \( \hat{F}^+_{\mu\nu} = F^+_{\mu\nu} - \frac{1}{2} \text{Tr}(F^+_{\mu\nu}) \) is traceless and transforms as the adjoint representation. Recall that the direct product \( 3 \otimes 3 = 1 \oplus 8 \). Therefore, there are two independent interaction terms in the \( \mathcal{O}(p^2) \) Lagrangians for the magnetic moments of the doubly charmed baryons.
FIG. 1: The $\mathcal{O}(p^2)$ tree level diagrams where the doubly charmed baryon is denoted by the solid line. The dot represents second-order coupling.

IV. FORMALISM UP TO ONE-LOOP LEVEL

We follow the standard power counting scheme as in Ref. 60, the chiral order $D_\chi$ of a given diagram is given by

$$D_\chi = 4N_L - 2I_M - I_B + \sum nN_n,$$  \hspace{1cm} (21)

where $N_L$ is the number of loops, $I_M$ is the number of internal pion lines, $I_B$ is the number of internal baryon lines and $N_n$ is the number of the vertices from the $n$th order Lagrangians. The chiral order of magnetic moments $\mu_H$ is $(D_\chi - 1)$ based on Eq. (7).

We assume the exact isospin symmetry with $m_u = m_d$ throughout this work. The tree-level Lagrangians in Eq. (18) contribute to the doubly charmed baryon magnetic moments at $\mathcal{O}(p^1)$ as shown in Fig. 1. The Clebsch-Gordan coefficients for the various doubly charmed baryons are collected in Table II. All tree level doubly charmed baryon magnetic moments are given in terms of $a_1$, $a_2$.

FIG. 2: The one-loop diagrams where spin-$\frac{1}{2}$ (spin-$\frac{3}{2}$) doubly charmed baryon is denoted by the single (double) solid line. The dashed and wiggly lines represent the pseudoscalar meson and photon respectively.

There are two Feynman diagrams contribute to the doubly charmed baryon magnetic moments to next-to-leading order at one-loop level as shown in Fig. 2. The intermediate states in diagrams (a) and (b) are spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ doubly charmed baryons respectively. The photon vertex is from the meson photon interaction term while the meson vertex is from the interaction terms in Eq. (17).

We collect our numerical results of the doubly charmed baryon magnetic moments to the next-to-leading order in Table II. We also compare the numerical results of the magnetic moments when the chiral expansions are truncated at $\mathcal{O}(p^1)$ and $\mathcal{O}(p^2)$ respectively in Table III.

Summing the one-loop level contributions to the doubly charmed baryon magnetic moments in Fig. 2 considering both the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ doubly charmed baryon intermediate states, the loop corrections to the doubly charmed baryon magnetic moments can be expressed as

$$\mu_H^{(2,\text{loop})} = \sum_{\phi = \pi, K} \left( -\frac{g_A^2 m_\phi M_N \beta_\phi^a}{64\pi f_\phi^2} \right)$$

$$\left[ \beta_\phi^b \frac{g_\phi^2}{f_\phi^2} \left( \delta \log \left( \frac{m_\phi^2}{\lambda^2} \right) - 1 \right) + \frac{2 \sqrt{m_\phi^2 - \delta^2 \arccos \left( \frac{\delta}{m_\phi} \right)}}{192\pi^2 f_\phi^2} \right]$$  \hspace{1cm} (22)

where $\lambda = 4\pi f_\pi$ is the renormalization scale. Here, the coefficients $\beta_{a-b}^\phi$ arise from the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ doubly charmed baryon intermediate states respectively in Fig. 2. We collect their explicit expressions in Tables II.

With the low energy counter terms and loop contributions (22), we obtain the magnetic moments,

$$\mu_H = \left\{ \mu_1^{(1)} \right\} + \left\{ \mu_1^{(2,\text{loop})} \right\}$$  \hspace{1cm} (23)
TABLE I: The coefficients of the loop corrections to the doubly charmed baryon magnetic moments from Figs. 2(a) and 2(b).

| Baryons | $\beta_a$ | $\beta_K$ | $\beta_a$ | $\beta_K$ |
|---------|-----------|-----------|-----------|-----------|
| $\Xi_{cc}^{++}$ | 2 | 2 | 2 | 2 |
| $\Xi_{cc}^+$ | -2 | 0 | -2 | 0 |
| $\Omega_{cc}^+$ | 0 | -2 | 0 | -2 |

TABLE II: The doubly charmed baryon magnetic moments to the next-to-leading order (in unit of $\mu_N$).

| Baryons | $O(p^1)$ tree | $O(p^2)$ loop (a) | $O(p^2)$ loop (b) |
|---------|---------------|-------------------|-------------------|
| $\Xi_{cc}^{++}$ | $\frac{2}{3}a_1 + 4a_2$ | $-0.51\tilde{g}_A^2$ | $0.20\tilde{g}_C^2$ |
| $\Xi_{cc}^+$ | $-\frac{1}{3}a_1 + 4a_2$ | $0.15\tilde{g}_A^2$ | $-0.07\tilde{g}_C^2$ |
| $\Omega_{cc}^+$ | $-\frac{1}{3}a_1 + 4a_2$ | $0.36\tilde{g}_A^2$ | $-0.12\tilde{g}_C^2$ |

where $\mu_H^{(1)}$ is the tree-level magnetic moments from Eqs. (18).

V. NUMERICAL RESULTS AND DISCUSSIONS

As there are not any experimental data of the doubly charmed baryon magnetic moments so far, in this paper, we adopt the same strategy as in Ref. [62]. We use quark model to determine the leading-order tree level magnetic moments. In other words, our calculation is actually the chiral corrections to the quark model.

We collect our numerical results of the doubly charmed baryon magnetic moments to the next-to-leading order in Table III. We also compare the numerical results of the magnetic moments when the chiral expansions are truncated at $O(p^1)$ and $O(p^2)$ respectively in Table III.

At the leading order $O(p^1)$, as the charge matrix $Q_H$ is not traceless, there are two unknown LECs $a_{1,2}$ in Eq. (18). Notice the second column in Table II, the $a_1$ parts are proportional to the light quark charge within the doubly charmed baryon and the $a_2$ parts are the same for all three doubly charmed baryons and arise solely from the two charm quarks.

At the quark level, the flavor and spin wave functions of the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ doubly charmed baryons $\Xi_{ccq}$ and $\Xi^{*}_{ccq}$ read:

$$|\Xi_{ccq}; s_3 = \frac{1}{2} \rangle = \frac{1}{3\sqrt{2}} [2c \uparrow c \uparrow q \downarrow -c \uparrow c \downarrow q \uparrow -c \downarrow c \uparrow q \downarrow +2c \uparrow q \downarrow c \uparrow -c \downarrow q \uparrow c \uparrow$$
$$-c \downarrow q \downarrow c \uparrow +2q \downarrow c \uparrow c \uparrow -q \downarrow c \downarrow c \uparrow -q \uparrow c \downarrow c \uparrow], \quad (24)$$

$$|\Xi^{*}_{ccq}; s_3 = \frac{1}{2} \rangle = \frac{1}{\sqrt{3}} [c \uparrow c \uparrow q \downarrow +c \uparrow c \downarrow q \uparrow +c \downarrow c \uparrow q \uparrow], \quad (25)$$

where the arrows denote the third-components of the spin. $q$ can be $u, d$ and $s$ quark. The magnetic moments in the quark model are the matrix elements of the following operator:

$$\vec{\mu} = \sum_i \mu_i \vec{\sigma}^i, \quad (26)$$

where $\mu_i$ is the magnetic moment of the quark:

$$\mu_i = \frac{e_i}{2m_i}, \quad i = u, d, s, c. \quad (27)$$

We adopt the $m_u = m_d = 336$ MeV, $m_s = 540$ MeV, $m_c = 1660$ MeV as the constituent quark masses and give the results in the second column in Table III.
TABLE III: The doubly charmed baryon magnetic moments when the chiral expansion is truncated at $O(p^3)$ and $O(p^2)$, respectively (in unit of $\mu_N$).

| Baryons($\delta = 0.1$ GeV) | $O(p^3)$ | $O(p^2)$ loop | $O(p^2)$ total |
|-----------------------------|----------|----------------|-----------------|
| $\Xi_{cc}^{++}$            | $\frac{4}{3}\mu_c - \frac{1}{3}\mu_u = -0.12$ | 0.47            | 0.35            |
| $\Xi_{cc}^{+}$             | $\frac{4}{3}\mu_c - \frac{1}{3}\mu_d = 0.81$  | -0.19           | 0.62            |
| $\Omega_{cc}^{+}$          | $\frac{2}{3}\mu_c - \frac{1}{3}\mu_s = 0.69$  | -0.28           | 0.41            |

Up to $O(p^2)$, we need to include both the tree-level magnetic moments and the $O(p^2)$ loop corrections. We use the quark model to estimate the leading-order tree level transition magnetic moments. Thus, there exist only two LECs: $\tilde{g}_A$, and $\tilde{g}_C$ at this order. $\tilde{g}_A = -0.5$ has been estimated in Ref. [60]. Similarly, we can also obtain the $\phi HT$ coupling $\tilde{g}_C$.

At quark level, the pion-quark interaction reads

$$L_{\text{quark}} = \frac{g_0}{2} (\bar{u} \gamma^\mu \gamma_5 \partial_\mu \pi^0 u - \bar{d} \gamma^\mu \gamma_5 \partial_\mu \pi^0 d)$$

where $g_0$ is the quark level coupling constant. The matrix elements both at hadron level and quark level,

$$\langle H_1 | i L_{H1} H_2 | H_2 \rangle = \langle H_1 | i L_{\text{quark}} H_2 | \pi^0 \rangle,$$

where $H_{1,2}$ are the hadrons. Considering the $\pi^0$ coupling at hadron level

$$L_{\Xi_{cc}^{++}, \Xi_{cc}^{++}, \pi^0} = -\frac{1}{2F_0} \tilde{g}_C \sqrt{\frac{2}{3}} \Xi_{cc}^{++} \partial_\mu \pi^0 \Xi_{cc}^{++},$$

we obtain the $\Xi_{cc}^{++}, \Xi_{cc}^{++}, \pi^0$ matrix elements at the hadron level,

$$\langle \Xi_{cc}^{++} | i L_{\Xi_{cc}^{++}, \Xi_{cc}^{++}, \pi^0} | \Xi_{cc}^{++} + \pi^0 \rangle \sim -\frac{1}{2F_0} \tilde{g}_C \sqrt{\frac{2}{3}} q_3,$$

and at the quark level,

$$\langle \Xi_{cc}^{++} | i L_{\text{quark}} | \Xi_{cc}^{++} + \pi^0 \rangle \sim -\frac{2\sqrt{2}}{6} q_3.$$

Compare with the axial charge of the nucleon,

$$\frac{\frac{4}{3}q_A}{-\frac{2\sqrt{2}}{3} q_3} = \frac{\frac{4}{3}g_A}{\frac{2\sqrt{2}}{6}}.$$

We obtain the $\phi TH$ coupling $\tilde{g}_C = -\frac{4\sqrt{2}}{3} g_A = -1.75$.

With $\tilde{g}_A$ and $\tilde{g}_C$, we also need the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ doubly charmed baryon mass splitting $\delta = M_T - M_H$ to obtain the numerical results of the $O(p^2)$ doubly charmed baryon magnetic moments. As the spin-$\frac{3}{2}$ doubly charmed baryons have not been observed in the experiments, the masses of the spin-$\frac{3}{2}$ doubly charmed baryons remain unknown. There are independent determinations of mass splittings in nonrelativistic lattice QCD [14], lattice QCD [15, 17, 18] and pNRQCD [19], the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ doubly charmed baryon mass splitting $\delta$ vary from 40 MeV to 120 MeV.

We adopt $\delta = 0.1$ GeV approximatively in Table III and show the variations of doubly charmed baryon magnetic moments with the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ doubly charmed baryon mass splitting $\delta$ in Fig. 3. Compared to the results in Ref. [60], it is interesting to notice that after taking the spin-$\frac{1}{2}$ doubly charmed baryon contribution into consideration, the $O(p^2)$ loop corrections for the $\Xi_{cc}^{++}$ magnetic moment change from -0.13 $\mu_N$ to 0.47 $\mu_N$. The reason for this is that the $\phi TH$ coupling $\tilde{g}_C = -1.75$ is much bigger than the $\phi HH$ coupling $\tilde{g}_A = -0.5$. Thus, the spin-$\frac{3}{2}$ doubly charmed baryon contribution is magnified. The total $\Xi_{cc}^{++}$ magnetic moment changes from -0.25 $\mu_N$ to 0.35 $\mu_N$. We are looking forward to further progresses in experiment.

In Table IV, we compare our results obtained in the HBChPT with those from other model calculations such as quark model (QM) [49], relativistic three-quark model (RTQM) [40], relativistic quark model (RQM) [47], skyrmion description [51], confining logarithmic potential (CLP) [50], MIT bag model [48], nonrelativistic quark model (NQM) [52], lattice QCD(LQCD) [54] and chiral perturbation theory (ChPT) [60].
FIG. 3: The variations of doubly charmed baryon magnetic moments $\mu$ (in unit of $\mu_N$) with $\delta$ (in unit of GeV).

| Baryons       | $\Xi^{++}_{cc}$ | $\Xi^{+}_{cc}$ | $\Omega^{+}_{cc}$ |
|---------------|----------------|----------------|-------------------|
| QM [45]       | -0.124         | 0.806          | 0.688             |
| RTQM [46]     | 0.13           | 0.72           | 0.67              |
| RQM [47]      | -0.10          | 0.86           | 0.72              |
| Skyrmion [51] | -0.47          | 0.98           | 0.59              |
| CLP [50]      | -0.154         | 0.778          | 0.657             |
| MIT bag model [48] | 0.17   | 0.86           | 0.84              |
| NQM [52]      | -0.208         | 0.785          | 0.635             |
| LQCD [54]     | —              | 0.425          | 0.413             |
| ChPT [60]     | -0.25          | 0.85           | 0.78              |
| This work     | 0.35           | 0.62           | 0.41              |

TABLE IV: Comparison of the decuplet to octet baryon transition magnetic moments in literature including quark model (QM) [45], relativistic three-quark model (RTQM) [46], relativistic quark model (RQM) [47], skyrmion description [51], confining logarithmic potential (CLP) [50], MIT bag model [48], nonrelativistic quark model (NQM) [52], lattice QCD(LQCD) [54] and chiral perturbation theory (ChPT) [60] (in unit of $\mu_N$).

VI. CONCLUSIONS

In short summary, we have investigated the magnetic moments for the spin-$\frac{1}{2}$ doubly charmed baryons to the next-to-leading order in the framework of HBChPT. The spin-$\frac{3}{2}$ doubly charmed baryons were included as an explicit degree of freedom and its contribution to the spin-$\frac{1}{2}$ doubly charmed baryon magnetic moments evaluated. We use quark model to determine the leading-order magnetic moments and obtain the numerical results to next-to-leading
order. Our calculation shows that $\mu_{\Xi^{++}} = 0.35\mu_N$, $\mu_{\Xi^+} = 0.62\mu_N$, $\mu_{\Omega^{++}} = 0.41\mu_N$.

Our analysis indicates that after taking the spin-$\frac{3}{2}$ doubly charmed baryon contribution into consideration, the $\Xi^{++}_{cc}$ magnetic moment changes a lot from -0.25 $\mu_N$ to 0.35 $\mu_N$. We are looking forward to further progresses and hope our results may be useful for future experimental measurement of the doubly charmed baryons magnetic moments. Our analytical results may be useful to the possible chiral extrapolation of the lattice simulations.

ACKNOWLEDGMENTS

H. S. Li is very grateful to Shi-Lin Zhu and Zhan-Wei Liu for very helpful discussions. This project is supported by the National Natural Science Foundation of China under Grants 11905171 and 12047502. This work is also supported by the Double First-class University Construction Project of Northwest University.

Appendix A: Integrals and loop functions

We collect some common integrals and loop functions in this appendix.

$$
\Delta = i \int \frac{d^4 \lambda^{4-d}}{(2\pi)^d} \frac{1}{l^2 - m^2} = 2m^2(L(\lambda) + \frac{1}{32\pi^2 \lambda^2}), \\
L(\lambda) = \frac{\lambda^{d-4}}{16\pi^2} \Big[ \frac{1}{d-4} - \frac{1}{2} \ln(4\pi + 1 + 1), \\
I_0(q^2) = i \int \frac{d^4 \lambda^{4-d}}{(2\pi)^d} \frac{1}{(l^2 - m^2 + i\epsilon)((l + q)^2 - m^2 + i\epsilon)}, \quad r = \sqrt{1 - 4m^2/q^2}, \\
= \begin{cases} 
- \frac{1}{16\pi^2} \left( 1 - \ln \frac{m^2}{\lambda^2} + 2L(\lambda) \right) & (q^2 < 0) \\
- \frac{1}{16\pi^2} \left( 1 - \ln \frac{m^2}{\lambda^2} - 2r \arctan \frac{1}{r} + 2L(\lambda) \right) & (0 < q^2 < 4m^2), \\
- \frac{1}{16\pi^2} \left( 1 - \ln \frac{m^2}{\lambda^2} - r \ln \frac{1 + r}{1 - r} + i\pi r + 2L(\lambda) \right) & (q^2 > 4m^2) 
\end{cases}
$$

$$
\int \frac{d^d l}{(2\pi)^d} \frac{[1, l_\alpha, l_\beta, l_\delta]}{(l^2 - m^2 + i\epsilon)(\omega + v \cdot l + i\epsilon)} = [J_0(\omega), v_\alpha J_1(\omega), g_{\alpha\beta} J_2(\omega) + v_\alpha v_\beta J_3(\omega)], \quad \omega = v \cdot r + \delta
$$

$$
J_0(\omega) = \begin{cases} 
- \frac{\omega}{8\pi^2} \left( 1 - \ln \frac{m^2}{\lambda^2} \right) + \frac{\sqrt{\omega^2 - m^2}}{4\pi^2} \arccosh \frac{\omega}{m} - i\pi + 4\omega L(\lambda) & (\omega > m) \\
- \frac{\omega}{8\pi^2} \left( 1 - \ln \frac{m^2}{\lambda^2} \right) + \frac{\sqrt{\omega^2 - m^2}}{4\pi^2} \arccos \frac{\omega}{m} + 4\omega L(\lambda) & (\omega < m) \\
- \frac{\omega}{8\pi^2} \left( 1 - \ln \frac{m^2}{\lambda^2} \right) - \frac{\sqrt{\omega^2 - m^2}}{4\pi^2} \arccosh \frac{\omega}{m} + 4\omega L(\lambda) & (\omega < m) 
\end{cases}
$$

$$
J_1(\omega) = -\omega J_0(\omega) + \Delta, \\
J_2(\omega) = \frac{1}{d-1}[(m^2 - \omega^2)J_0(\omega) + \omega \Delta], \\
J_3(\omega) = -\omega J_1(\omega) - J_2(\omega), \\
L_0(\omega) = i \int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2 - m^2 + i\epsilon)((l + q)^2 - m^2 + i\epsilon)(\omega + v \cdot l + i\epsilon)}, \quad v \cdot q = 0
$$
\[
\begin{cases}
\frac{-1}{8\pi^2} \frac{1}{\sqrt{\omega^2 - m^2}} \left( \text{arccosh} \frac{\omega}{m} - i\pi \right) & (\omega > m) \\
\frac{1}{8\pi^2} \frac{1}{\sqrt{\omega^2 - m^2}} \left( \text{arccos} \frac{-\omega}{m} \right) & (\omega^2 < m^2) \\
\frac{1}{8\pi^2} \frac{1}{\sqrt{\omega^2 - m^2}} \left( \text{arccosh} \frac{-\omega}{m} \right) & (\omega < -m)
\end{cases}
\] (A9)
[58] V. Bernard, N. Kaiser, J. Kambor and U. G. Meissner, Nucl. Phys. B 388, 315 (1992).
[59] V. Bernard, N. Kaiser and U. G. Meissner, Int. J. Mod. Phys. E 4, 193 (1995).
[60] H. S. Li, L. Meng, Z. W. Liu and S. L. Zhu, Phys. Rev. D 96 (2017) no.7, 076011.
[61] A. N. Hiller Blin, Z. F. Sun and M. J. Vicente Vacas, Phys. Rev. D 98, no.5, 054025 (2018).
[62] H. S. Li, L. Meng, Z. W. Liu and S. L. Zhu, Phys. Lett. B 777 (2018), 169-176.
[63] S. Scherer, Adv. Nucl. Phys. 27, 277 (2003).
[64] G. Ecker, Prog. Part. Nucl. Phys. 35, 1 (1995).
[65] H. S. Li, Z. W. Liu, X. L. Chen, W. Z. Deng and S. L. Zhu, Eur. Phys. J. C 79 (2019) no.1, 66.