Analysis of the possibility of deterministic chaos occurrence during moving of an Earth remote sensing satellite with gyro dampers

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Abstract. The purpose of the work is the mathematical modeling and analysis of pitch oscillations of an Earth remote sensing satellite, in the point of view of the possibility of occurrence of deterministic chaos during the process of calming the natural vibrations relative to the satellite's center of mass. A nonlinear mathematical model of oscillations of an Earth remote sensing satellite with floated gyro dampers with an integrated thermal control system based on Peltier thermopiles is constructed. According to the results of computer simulation by using the developed library of temperature perturbation models, it is shown that for some values of the parameters of the system under study, deterministic chaos can occur. Phase portraits of the transition of the satellite in pitch oscillation from regular to chaotic motions were constructed. Areas of deterministic chaos are created as functions of the parameters of systems with gyro dampers and the characteristics of external temperature and mechanical disturbances.

1. Introduction
Nowadays, the studying and analysis of various physical processes in modern aerospace devices are still significant problems. For example, temperature disturbances, along with other factors, have a substantial impact on the efficiency, durability, and lifetime of devices and their electronic components [1-10]. Various kinds of thermal control systems are used to reduce the influence of temperature influence on the device provides. The thermal control systems can be both passive and active, for example, based on Peltier thermopiles. However, usage of the thermal control system can lead to unexpected effects, such as the phenomenon of deterministic chaos, which can lead to loss of working capacity complex devices like, for example, Earth's satellites.

The purpose of this work is to study the pitch motion of a system "orbiting spacecraft - with float gyro dampers" (SC-GD) on the example of an earth remote sensing satellite and to find parameters when the chaotic motion of the satellite can occur.

The lifecycle of any complex precision navigation, orientation, or stabilization system can include different stages - design, testing, certification, exploitation. Maintenance of the stability of the dissipative properties of gyro dampers on each stage of the "SC-GD" lifecycle is necessary, for example, by minimizing the influence of external perturbing factors on their operation. The temperature influence is one of the most significant external perturbation factors for "SC-GD" because of the viscous damping properties of the working fluids of dampers depend on the temperature. One of
the ways to solve this problem is the usage of thermal control systems with the Peltier thermoelements in the mutual damping nodes.

However, the possible occurrence of irregular, unstable modes in the system "SC-GD" should be studied due to the properties of active thermal control systems, and temperature-perturbed equations of the dynamics of this system are essentially nonlinear. Moreover, the study of such irregular mode as the deterministic chaos phenomenon is of particular interest for the space vehicles on long term orbital missions.

The following problems were stated and resolved to achieve the purpose:
- the experience of authors and other scientists[1,6,7,9,13,18,19] in mathematical modeling of sensors and their components, as well as complex mechanical and electronic systems, are summarized;
- the implementation of a nonlinear mathematical model of "damper – spacecraft" with temperature-perturbed hydro dampers in specialized software is performed;
- the phenomenon of the possible occurrence of deterministic chaos when a spacecraft with gyro-damping is in a circular orbit around the Earth under conditions of temperature and mechanical disturbances is investigated.

The stated problems were resolved using the following methods: methods of analysis of nonlinear dynamic systems; methods of analytical and numerical investigation of deterministic chaos; method of elementary balances for complex gyroscopic systems with inertial, dissipative and potential forces; nonlinear dynamics of components of navigation and orientation systems for spacecraft.

2. Mathematical model
Several studies (for example [1,19]) have shown that relatively "simple" systems described, for example, by a small number of deterministic (not contained randomly changing parameters or effects) ordinary but nonlinear differential equations can have irregular (chaotic) behavior.

The scheme of spacecraft (SC) with a damper on floating gyroscopes "V - roll" is shown in Figure 1 [20].

![Figure 1. Scheme of the "SC-GD" system with a mutual damping node: $X_0, Y_0, Z_0$ – the orbital coordinate system; $X, Y, Z$ – the coordinate system associated with the SA; 1, 2 – floats of gyrodampers; 3 – Peltier thermopiles; 4 – the mutual damping node; $H_1, H_2$ – the kinetic moments of the gyroscope rotors.](image)

Let us consider, without loss of generality, the channel for calming the eigenvalues of the SC along with the pitch angle $\theta$[7].
The nonlinear equations of perturbed motion of "SC-GD" system in this case [21] have the form:

$$\dot{\theta} + a_1 \sin \theta - a_2 \beta = B \cos \alpha \theta,$$
$$\eta \ddot{\beta} + a_3 \sin \beta + a_4 \dot{\theta} = 0,$$

where: $a_1 = 3 \omega_0^2 (J_y - J_z) / J_y$ - parameter, which defines the configuration of SC; $a_2 = HD / J_y$; $a_3 = 2HD$; $a_4 = H \omega_0 S$; $H$ - the kinetic moment of gyroscopes rotors; $J_x, J_y, J_z$ - inertia moments of SC; $\omega_0$ - angular orbital velocity; $d = \cos \epsilon$, $S = \sin \epsilon$ - parameters of the relative angular position of the kinetic moment vectors of gyroscop rotors regarding the coordinate system that is rigidly connected to the object; $b = b_1 - b_2$; $h_r = h + 2 h(T_i)$; $h$ - coefficient of damping of hydro dampers in the gaps between the float and the body; $h_i(T_i)$ - additional temperature-dependent damping coefficient in the mutual damping node; $B$, $\omega$ - amplitude and frequency of external mechanical action on the SA.

The gyro-damper subsystem can be represented (due to the presence of liquid in the mutual damping node and based on the method of elementary balances) by heat exchange between two elementary volumes. One of these volumes corresponds to the inertial junction of the thermopile, which contacts the mutual damping node. Another one corresponds to the external junction of the thermopile in contact with the environment.

Then the system of heat transfer equations obtained using the method of elementary balances can be written as:

$$c T_1 + q(T_1 - T_2) = Q_1,$$
$$c T_2 + q(T_2 - T_1) + q(T_2 - T_1) = Q_2,$$

where $Q_1, Q_2$ - power sources of heat (cooling) generation on the joints of Peltier thermal containers.

If the geometric parameters of the mutual damping node are unchangeable, the dependence of the damping coefficient on the temperature is determined by the dependence of the dynamic viscosity coefficient of the working fluid on the temperature.

This dependence can be represented as:

$$h_i(T_i) = h_i^0 \frac{m(T_i)}{m^0},$$

where: $h_i^0, m^0$ - nominal values of the damping coefficient and dynamic viscosity coefficient of the working fluid.

The relations (2)-(4) define the relationships between the damping ratio, temperature, and viscosity of the working fluid in the gaps between the float of gyrodamper and its case.

The law of temperature control with an ideal temperature sensor located on the inertial junction 1 of the Peltier's thermopile (Figure 1):

$$J = \begin{cases} J_{\text{max}}, & \text{with } T_1 - T_i \geq T_L, \\ \frac{\tan \alpha (T_1 - T_i)}{J_{\text{max}}}, & \text{with } -T_L \leq T_1 - T_i \leq T_L, \\ -J_{\text{max}}, & \text{with } T_1 - T_i \leq -T_L. \end{cases}$$

Significant nonlinearity of the mathematical model (1)-(5) is formed by the working principle of Peltier’s thermopile, temperature control law (5), the nonlinearity of the equations of motion system "SC-GD" (1) and the relations (2)-(5).

For numerical simulation, the software was created (Figure 2) based on the mathematical model (1)-(5) and using the library of temperature perturbation models developed by authors [16].

The following parameter of “SC-GD” system were used for for analytical and numerical studies [20]: $J_x = 25$ kN⋅m⋅s$^{-2}$, $J_y = 23.5$ kN⋅m⋅s$^{-2}$, $J_z = 9$ kN⋅m⋅s$^{-2}$, $H = 0.01$ kN⋅m⋅s, $\omega_0 = 0.00102$ s$^{-1}$, $\varepsilon = 39.8^0$. 

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The areas of the possible occurrence of deterministic chaos in the plane “frequency of the driving force - temperature” are shown in Figure 3. These areas were constructed based on the results of analytical research using the methods described in [19], [22].

Fluctuations and evolution of the phase portrait during the transition from the regular area (point D in Figure 3 with parameters, $T = 13^\circ C$, $w = 0.7\omega_0$) to the area of deterministic chaos (point H on Figure 3 with coordinates $T = 19^\circ C$, $w = 0.7\omega_0$) are shown in Figure 4.

Thus, at a particular critical value of the amplitude of the driving force ($B \approx 2 \cdot 10^6$ s$^{-2}$), specific frequencies (for example, $\omega = 0.7\omega_0$) and temperatures (for example, $T = 19^\circ C$) in the mutual damping node, the appearance of the deterministic chaos in the movement of the spacecraft along the pitch is possible.

All qualitative and quantitative criteria of chaos are fulfilled. The correlation dimension of the strange attractor is $\approx 2.3$; the Kolmogorov entropy $K = \text{const} \approx 0.15$. 

Figure 2. Some screens of the developed software.

Figure 3. Areas of the possible occurrence of deterministic chaos H and the areas of harmonic character of oscillations D in the forced pitch motion of the SC ($B \approx a_1$).
Figure 4. Evolution of the phase portrait during the transition of "SC-GD" system from the regular area (a) \(T = 13^\circ\text{C}\) to chaos area (b) \(T = 19^\circ\text{C}\) in the mode of forced pitch motion of SC. 1 - angle \(\theta\), 2 - angular speed \(\dot{\theta}\).

3. Conclusions
As a result of the work, a nonlinear mathematical model of motion along with the pitch of an Earth remote sensing satellite with gyrodamper and a thermal control system on Peltier thermopiles in mutual damping node is constructed and studied.

As a result of the work specialized software was developed, the main advantages of that software include the absence of necessity to purchase expansive software packages for automatic design and modeling (Simulink, ANSYS, etc.) Besides, the features of the developed software fully meet the needs of the researcher of the studied problem. For example, such software provides an opportunity to observe transient physical processes occurring in the investigated objects (mechanical movement, thermal processes, etc.). The software has features for constructing three-dimensional topograms of temperature fields in structural elements, such as printed circuit board, and graphs of the temperature at the points of the object, etc.

According to the results of computer experiments, it was found that the occurrence of deterministic chaos is possible at specific combinations of the parameters of the considered nonlinear dynamic system, parameters of external temperature, and mechanical disturbances. The necessary condition for the deterministic chaos occurrence is the presence of at least one unstable equilibrium state of a dynamic system.

Areas of determinate chaos are constructed as functions of the parameters of the considered spacecraft for remote sensing of the Earth's surface under conditions of external temperature and mechanical disturbances. These disturbances have a harmonic nature due to the movement of spacecraft in circular orbit around the Earth. Phase portraits were obtained during the transition of spacecraft movements in pitch from regular to chaotic oscillations. Qualitative and quantitative parameters of the chaotic regime of vibrations of the spacecraft were also obtained.
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