On the jets emitted by driven Bose–Einstein condensates

Miguel Arratia

Department of Physics, University of California, Berkeley, CA 94720, United States of America
E-mail: marratia@berkeley.edu

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Abstract
Features of the emission of jets by driven Bose–Einstein condensates, discovered by Clark et al. (2017 Nature 551 356–9), can be understood by drawing analogies with particle and nuclear physics. In particular, the widening of the $\Delta f = \pi$ peak in the angular correlation function is due to a dijet acollinearity, which I estimate to be about $5^\circ$ rms. I also propose new correlation studies using observables commonly used in studies of the quark-gluon plasma.

Keywords: Bose–Einstein condensate (BEC), driven BEC, atom jets, strongly interacting system

(Some figures may appear in colour only in the online journal)

1. Introduction
Clark et al [1] recently discovered a new phenomenon in which a stimulated Bose–Einstein condensate (BEC) emits a burst of collimated jets of atoms. This non-equilibrium process was achieved by controlling the atomic interactions in the condensate using the Feshbach resonance technique. The jets forms due to a runaway process of stimulated inelastic scattering that occurs when the modulation amplitude surpasses a certain threshold.

The data analyzed by means of a second-order angular correlation function show two peaks at $\Delta \phi = 0$ and $\Delta \phi = \pi$. These were attributed to back-to-back emission of jets, reflecting momentum conservation in the primary atom–atom scattering that triggers the dijet runaway formation.

However, their data deviates from the predicted correlation, specially in the region of the peak at $\Delta \phi = \pi$ that is much wider than the peak at $\Delta \phi = 0$ and has only about $70\%–85\%$ of its integral. This fact is not totally understood, and the authors claimed in [1] that ‘further investigation into the differences between the two peaks is required’. Here, I offer an explanation for the broadening of the peak at $\Delta \phi = \pi$ by drawing analogies with observations in high-energy particle and nuclear physics. I also propose new measurements on this jet phenomenon inspired in studies of the quark-gluon plasma.

This article is organized as follows: section 2 shows an estimate of the dijet acollinearity present in the Clark et al. experiment by drawing an analogy with proton–proton collisions with ‘intrinsic parton $k_T$’; section 3 discusses the vertical dimension of the (BEC); section 4 shows proposals of new observables; and section 5 describes the conclusions.

2. Dijet acollinearity
2.1. Parton–parton scattering
In the mid seventies, the production of roughly back-to-back sprays of collimated hadrons (jets) in proton collisions was attributed to collinear parton–parton scattering with large momentum transfer. However, this model did fail to describe the data from experiments at the CERN Intersecting Storage Rings—the world’s first hadron collider.

In 1977, Feynman et al [2] modified the collinear parton–parton scattering by introducing an ‘extra kick’ to the partons, intrinsic parton $k_T$, that yielded a dijet acollinearity. This allowed them to explain, among other things, the data from two-particle correlations that showed a peak at $\Delta \phi = \pi$ that was broader than the peak at $\Delta \phi = 0$.

2.2. Atom–atom scattering
Clark et al compared their measured second-order angular correlation function, $g^2$, with an analytically calculation
given by:

\[ g^2(\Delta \phi) = 1 + \left| \frac{2J_1(k_f R \Delta \phi)}{k_f R \Delta \phi} \right|^2 + \left| \frac{2J_1(k_f R [\Delta \phi - \pi])}{k_f R [\Delta \phi - \pi]} \right|^2, \]

(1)

where \( J_1 \) is the first Bessel function (resulting from the Fourier transform of the density of a two-dimensional uniform disk), \( k_f \) is the wavenumber of the ejected atoms, and \( R \) is the radius of the BEC.

This function shows two identical peaks at \( \Delta \phi = 0 \) and \( \Delta \phi = \pi \), reflecting the assumption of exactly back-to-back emission of jets that is based on ‘conservation of momentum in the underlying pair-scattering process’ [1]. I suggest that the deviation of data from equation (1) arises from a small dijet acollinearity.

2.3. Estimate of dijet acollinearity

The dijet acollinearity can be estimated from the widths of \( \Delta \phi = 0 \) and \( \Delta \phi = \pi \) peaks in the measured \( g^2(\Delta \phi) \), following a method first used in particle physics by the CCOR collaboration [3] about 40 years ago, and more recently by the PHENIX collaboration [4]. This method relies on a Gaussian approximation for the jet transverse spread and basic trigonometry to obtain an average angle of dijet acollinearity.

For the cases of the jets observed in the Clark et al experiment, the equations involved are simplified because all the jet constituents (atoms) have roughly the same angular density and therefore the same momentum relative to the jet axis is the same for all jets, i.e. they have the same angular density width. This is because they are the result of a bosonic enhancement and their angular spread reflects the size of the source (i.e. a Handbudy Brown and Twiss bunching). The average transverse momentum can be estimated from the \( \Delta \phi = 0 \) peak of the correlation function, \( \sigma_A \). This can be combined with the width of the \( \Delta \phi = \pi \) peak, \( \sigma_0 \), to extract the average dijet acollinearity:

\[ \langle \phi \rangle \approx \frac{k_f}{k_f} = \frac{1}{\sqrt{2}} \sqrt{\sin^2 \left( \frac{\sqrt{2} \sigma_A}{\sqrt{\pi}} \right) - \left( \frac{\sigma_0}{\sqrt{\pi}} \right)^2}. \]

(2)

From [1], we know that the half-maximum half-width of the \( \Delta \phi = 0 \) peak is about 2°, for \( R = 8.5 \mu \text{m} \) and \( f = 2 \text{ kHz} \), and the width of the \( \Delta \phi = \pi \) peak is about three times larger. It follows from equation (2) that \( \langle \phi \rangle \) is about of about 5°.

Note that this small angle has a big effect in the width of the \( \Delta \phi = \pi \) peak (that measures inter-jet correlations) but no effect in the \( \Delta \phi = 0 \) peak (that measures intra-jet correlations).

2.4. Numerical calculation of \( g^2 \) with dijet acollinearity

To illustrate the effect of a dijet acollinearity on the measured \( g^2 \) function, I used a simple numerical simulation in which the azimuthal angle between the jet centers, \( \phi \), is drawn from a Gaussian with standard deviation of 5°; the jets angular density is approximated by a Gaussian with a standard deviation of 1.5°. Thus the di-jet angular density is given by \( n = N(0, 1.5^\circ) + N(180 + \phi, 1.5^\circ) \). The \( g^2 \) function is calculated as:

\[ g^2(\Delta \phi) = \frac{\left\langle \frac{\int d\theta n(\theta) n(\theta + \Delta \phi)}{\int d\theta n(\theta)^2} \right\rangle}{\text{rms}}, \]

where the average is taken over 1000 draws of different acollinearity angles, minimizing the statistical uncertainty on the calculation. Note that by construction, the jets have the same angular density and therefore the same momentum magnitude (which is proportional to the integral of the angular density). Therefore, the asymmetry studied here is in real space but not in momentum space.

Figure 1 shows the result of this calculation with data from the Clark et al experiment. The calculated \( g^2 \) is multiplied by a constant factor such that the height of the \( \Delta \phi = 0 \) peak roughly matches the data. The width of the \( \Delta \phi = 0 \) peak in the calculation matches the width of the measured peak by construction; the discrepancy in the tails arise due to the Gaussian approximation of the jet densities. The width of the \( \Delta \phi = \phi \) peak of the calculation matches the data well.

This result is based on general assumptions and it simply states that the data in [1] can be explained with a dijet acollinearity of 5° rms. The explanation of the origin of such dijet acollinearity lies beyond the scope of this work. In [5], this was attributed to ‘the destructive interference between atoms with different angular momenta’. Another possible origin was proposed in a recent work by the Chicago group [6], which revealed that the jet emission is preceded by the emergence of density waves in the driven condensate. Within this new microscopic picture, they postulate that ‘the asymmetry arises from the interference between overlapping matter-wave
3. Vertical direction

The dimensions of the BEC described in [1] are given by a typical value of 8.5 μm, and vertical extend of 0.5 μm (root mean square). So, most of the atoms scattered with a polar angle smaller than \( \arcsin(0.5/8.5) \approx 3^\circ \) will traverse most of the BEC, thus forming an observable dijet.

As noted by Clark et al, some atoms within a jet might lie outside the field-of-view of the experiment (in particle jargon, this is an acceptance loss due to limited pseudorapidity coverage). This can explain part of the 15%-30% difference of the integral of the peaks at \( \Delta \phi = 0 \) and \( \Delta \phi = \pi \), and the discrepancy between equation (1) and data at the \( \Delta \phi = 0 \) peak.

Here, I suggest that this loss could be corrected, or at least be considered when calculating the predicted correlation function. This should consider the vertical structure of the BEC. The improved calculation might describe data better and serve as a baseline to search for anomalous effects, such as those described in [7] and section 4.

4. Proposed new studies

Clark et al suggested that ‘one could probe excitations that are present in more exotic states of matter by amplifying them to form detectable jets’ [1]. That would not be the first time that ‘jets’ are used as ‘probes’ of exotic states of matter. Here I suggest measurements inspired in angular correlations and jet studies that probe the quark-gluon plasma, which is also a strongly interacting system.

The events shown in [1] have multiple dijets. The authors claim that the dijet directions are random. However, I note that the spacing between dijets looks suspiciously uniform. Given that driven Bose–Einstein condensates are a quantum many-body system, it is not unreasonable to expect an overall pattern caused by a collective behaviour. This might be even more evident when probing the appearance of vortices, solitons, and other exotic particles alluded in [1].

To further study this and search for more complex correlations than what is caused by momentum conservation and HBT bunching, I suggest to perform a multi-particle correlation study like the ones described in [8–10]. These ‘cumulants’ techniques were designed to study the collective behaviour caused by hydrodynamical flow of the quark-gluon plasma, which manifests as an anisotropy in the particle emission. These techniques consider correlations among all particles in the event at once and not just between pairs, to suppress ‘non-flow’ correlations that arise due to momentum conservation, jets, and HBT correlations.

While in principle these sources of correlations can be suppressed using higher-order correlation functions, \( g^n \) with large \( n \), in practice the calculations get cumbersome quickly. In contrast, these techniques correlate all particles \( (n \to \infty) \) in the event in an efficient way. More importantly, they can reveal true collective behaviour that might be obscured by strong correlations among a small number of atoms [2].

5. Conclusions

In conclusion, this work explains features of the novel phenomenon of atom jets emitted by driven Bose–Einstein condensates. The broader peak in the angular correlation function at \( \Delta \phi = \pi \) can be explained by a dijet acollinearity of about 5°. I also have suggested novel observables for the study of driven Bose–Einstein condensates inspired by studies of the quark-gluon plasma. This is among the first papers on the phenomenology of jets in driven Bose–Einstein condensates.

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ORCID iDs

Miguel Arratia  @ https://orcid.org/0000-0001-6877-3315

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1 Note that contrary to what is suggested in [6], the analysis presented in this work does not require an asymmetry in momentum space; in fact, the momentum magnitude (angular density) of the jets is assumed to be the same, so the asymmetry is only in real space.

2 After the release of the first e-print draft of this manuscript, the Chicago group independently released a e-print with a measurement of higher-order correlations [11]; they indeed observed higher-order correlations indicating a more complex pattern than reported in [1].
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