Bubble, Critical Zone
and the Crash of Royal Ahold

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Abstract

Our analysis of financial data, in terms of super-exponential growth, suggests that the seed of the 2002/03 crisis of the Dutch supermarket giant AHOHL was planted in 1996. It became quite visible in 1999 when the post-bubble destabilization regime was well-developed and acted as the precursor of an inevitable collapse fueled by raising expectations of investors to maintain strong herding pressures. We have adapted Weidlich’s theory of opinion formation to describe the formation of buy or sell decisions among investors, based on a competition between the mechanisms of herding and of personal opinion opposing the herd. Among four typical patterns of stock price evolution, we have identified a “critical zone” in the model characterized by a strong sensitivity of the price trajectory on the herding and personal inclination parameters. The critical zone describes the maturation of a systemic instability forewarning of an inevitable crash. Classification and recognition of the spontaneous emergence of patterns of stock market evolution based on Weidlich’s theory of complex systems, and in particular our discovery of the post-bubble destabilization regime which acts as a precursor to a subsequent crash or antibubble, not only presents the possibility of developing early warning signals but also suggests to top management ways of dealing with the coming crisis.

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1 Introduction

The 21st century opened with a deep confidence crisis in the financial markets caused by unprecedented corporate scandals both in the USA (Enron, Worldcom) and in Europe (Ahold, Parmalat). Financial authorities and investors would greatly benefit from a systematic analysis of publicly available corporate financial performance data such as sales and earnings and, particularly, stock price dynamics of high-growth companies that would enable them to detect early warning signals of impending problems in bullish times. It is our purpose to present a study showing the relationship between these variables and a model to understand the origin of crises in individual companies.

In the wake of the worldwide stock markets bubbles followed by crashes or ‘antibubbles’ of the second half of the 1990s and early 2000s, the analysis of such critical phenomena in aggregate stock markets has been intensified; see for instance (Abreu, 2001; Richardson and Ofek, 2001; Visano, 2002; Caballero and Hammour, 2002; Bohl, 2003; Brooks and Katsaris, 2003; Siegel, 2003; Scheinkman and Xiong, 2003; Griffin et al., 2003; Brunnermeier and Nagel, 2003; Chari and Kehoe, 2003; Kaizoji, 2004; Engsted and Tanggaard, 2004). A series of synthesis papers (including Kindleberger (2000), Shefrin (2000), Shiller (2000), Sornette (2003)) have pointed out the role of collective behaviors, such as herding and optimism feedback on itself, in the development of bubbles in aggregate markets. Far less attention has been paid to the fate of individual companies during these bubbles (see however (Johansen and Sornette, 2000) and (Lamdin, 2002)) and particularly those high-growth companies that couldn’t resist to manipulate their earnings to influence investors’ behavior eventually ending up with cutting their own flesh. That a propensity of earnings manipulation during bullish times is not unusual was recently emphasized from research in the Chinese stock market (Jiao, 2003).

The present work has two main goals. First, we study the recent bubble and its aftermath of the Ahold company. Our purpose is to extend previous works on bubbles and crashes that were essentially performed on aggregates, such as indices or major currencies. We particularly refer to (Johansen et al., 1999; 2000; Sornette and Johansen, 2001; Sornette and Zhou, 2002; Zhou and Sornette, 2003; Sornette, 2003; Johansen and Sornette, 2004), where a quantitative framework has been developed to test for the presence of speculative bubbles and to predict their termination, often in the form of crashes. In a nutshell, these works propose that speculative bubbles reflect the interplay between preponderant positive feedbacks modulated by intermittent negative feedbacks, leading to characteristic “log-periodic power law” (LPPL) signatures. These works have tested the LPPL signatures on major financial aggregates. Johansen and Sornette (2000) have previously performed a rapid analy-
sis of the price of the shares of IBM and of Procter & Gamble and showed the presence of a speculative bubble preceding the crash in both cases. Professionals have used this methodology to develop trading techniques on individual companies (private communications) but we are not aware of other published works on other individual companies. To attain sufficient depth, we focus in this article on one particular case, Royal Ahold, which is of interest to both sides of the Atlantic, because a relatively large share of the sales of one of the world’s largest supermarket chains occurs in both Europe, and the Netherlands in particular, and in the USA. We address the questions to what degree bubbles can occur on individual companies, how they are linked to objective variables characterizing the company (such as sales and earnings), how they are coupled with the dynamics of related indices.

Our second goal is to introduce the new concept of a “critical zone” characterizing the termination of a bubble and the transition to another regime, be it via a crash, a correction or simply a plateau. Johansen et al. (1999; 2000) have developed a rational expectation (RE) model of bubbles and crashes incorporating herding behavior, which shows that the bubble termination time is the instant at which a crash is the most probable (but is still not certain). In this class of RE models, a crash can occur at any time but is more and more probable as the bubble develops. In addition, there is a finite probability that no crash occurs, that is, that the bubble ends smoothly. However, the question remains open as to how the bubble transitions to a different phase after its demise. Sornette and Zhou (2004) have shown that the LPPL theory can be used in a general pattern recognition method with efficient generalizing properties to detect times of changes of regime which are not necessarily or immediately associated with a crash (that is a sharp price drop of say more than 15% occurring over a short time interval of say less than 2-3 weeks). In addition, most of the LPPL studies of bubbles previously performed (see (Sornette, 2003) and references therein) have shown that the maximum of a bubble is not immediately followed by a crash but that a rather complex behavior (lasting a few days to a few weeks) develops before the crash occurs. Here, we propose a simple model of the interplay between prevailing opinions and personal preference of investors to buy or sell shares which provides a useful classification of evolutionary patterns of stock price dynamics including the so-called “critical zone.”

2 A short history of Ahold

Albert Heijn started in 1887 with a small grocery store in Zaandam, just north of Amsterdam. The Netherlands-based holding company Ahold was listed on the AEX, the Amsterdam Stock Exchange, in 1948 and opened its first self-service supermarket in Rotterdam in 1955. The operating company of Albert
Heijn became the supermarket leader in the Netherlands and a household word for quality and value-for-money. Shortly after the Queen awarded the company the designation “Royal,” the last Heijn retired in 1989 as CEO from the company. Under his leadership, Royal Ahold had started already in the 1980s an expansion program in the USA. Ahold is the largest food provider in the Netherlands and one of the largest in the United States. At the end of 2002, Ahold operated some 5600 stores and employed approximately 280,000 people. Its major operations are in Europe and the United States, but it has also expanded into retail operations in Latin America and Asia as well. In the latter regions the company is now in the process of divesting all its operations.

In 1993, Cees van der Hoeven became the new CEO. He had the at first widely applauded ambition to make Ahold one of the largest food providers in the world. At that time, Ahold’s sales were about 10 billion euro’s. When he was forced to resign ten years later, in 2003, the sales amounted to almost 63 billion euro’s, a six-fold increase. But then, the company almost collapsed under its debt burden of 13 billion euro’s, fraud and mismanagement. It is generally assumed that the acquisition in 2000 of U.S. Foodservice, a non-core activity, which in itself can be viewed as a strategic mistake, was the start of a host of problems including the bookkeeping fraud at this same company. On 24 February 2003, the Ahold bubble collapsed and, while the stock price on the AEX had been coming down under the threat of profit warnings during the whole of 2002 from a high of 35 euro in 2001 to about 10 euro at the end of the year, it fell on that day with a bang of 63% down to 3.6 euro. The crisis was deep and prolonged for customers, investors and employees alike.

![Fig. 1. Annual ln(sales) (billion euro) of Ahold fitted with two linear regression lines indicating an acceleration of the exponential growth rate at the intersection of the lines in 1996. The lower line was fitted to the data from 1983 to 1995 and corresponds to ln(sale) = 0.103(t – 1900) – 7.16. The upper line is a fit of the data from 1995 to 1999 and corresponds to ln(sales) = 0.223(t – 1900) – 18.58. The sales data of 2000 and 2001 indicate another growth acceleration. The corrected 2000 and 2001 sales data and the 2002 sales, all reported in October 2003 by Ahold, are shown to coincide with the upper regression line.](image-url)
Inspection of a simple diagram (Figure 1), which shows the natural logarithm of the annual sales of Ahold from 1983 onwards, ln(sales), is already quite revealing. A straight line in this plot indicates exponential growth. The comparison of the two lines in Figure 1 is suggestive of an acceleration of the growth rate of net sales in 1996. From the regression lines, it can be seen that the average growth rate has more than doubled from the first (1983-1995) to the second period (1995-1999). The beginning of the second period corresponds approximately to the time when Ahold started to consolidate the results of their latest acquisition, the US supermarket chain Stop & Shop. Also in 1996, Royal Ahold became, in more than one way, an “American company” rather than a Dutch one because, for the first time, Ahold’s sales in the USA exceeded 50% of total sales bypassing the sales in The Netherlands (41%), which in the previous year still represented the largest share of 48%. Furthermore, in the years 2000 and 2001, the growth appeared to accelerate again (The corrections on the reported sales made in October 2003 indicate that the sales figures were somewhat “blown up” since they better fit the extension of the 1995-1999 regression line). Though revealing, this simple description in terms of two, three or more periods with different growth rates falls short of capturing adequately the behavior of the Ahold sales as we discuss in the next section.

3 “Super-exponential” growth of Ahold’s sales, earnings and stock market prices

3.1 Super-exponential growth due to positive feedback

Figure 1 actually shows a super-exponential growth in sales. Such super-exponential behavior can be explained by the concept of “positive feedbacks,” that is, conditioned on the observation that the sales or market have recently moved up (respectively, down), this makes them more probable to keep them moving up (respectively, down), so that a large cumulative move ensues. The concept of “positive feedbacks” has a long history in economics and is related to the idea of “increasing returns,” which says that goods become cheaper the more of them are produced (and the closely related idea that some products, like fax machines, become more useful the more people use them). Positive feedback is the opposite of negative feedback which is well-known in population dynamics: the larger the population of rabbits in a valley, the less grass there is per rabbit. If the population grows too much, the rabbits will eventually starve, slowing down their reproduction rate, which thus reduces their population at a later time. Thus negative feedback means that the higher the population, the slower the growth rate, leading to a spontaneous regulation of the population size. In finance, value investing for instance leads to negative
feedbacks: if the observed price of a company is larger than the estimated fundamental value, a value investor will tend to sell, expecting the price to converge back in the future to the true value. But by selling, he also tends to push the price down, the very expected move that led to the investment decision in the first place.

Positive feedback is the opposite phenomenon. In the context of population dynamics, positive feedbacks have characterized the evolution of human population growth over most of the last two thousand years (see (Johansen and Sornette, 2001) and references therein). The difference between humans and rabbits in this context is that humans modify the carrying capacity of the planet by various agriculture and technological innovations: the larger the human population, the more probable the occurrence of improvements and the exploitation of new habitats and resources, etc., leading to a positive feedback of population on food availability, providing a positive feedback on the growth rate and so on. The consequence is that, until recently, the growth rate of the human population has grown itself, leading to a super-exponential growth of the population (one could say that Malthus was an optimist!). For stock markets, when positive feedbacks dominate, the higher the price or the price return in the recent past, the higher will be the price growth in the future. Apart from technical mechanisms for positive feedback such as derivative hedging and portfolio insurance strategies, behavioral traits of investors such as imitation and herding akin to self-fulfilling prophecies play an important role (see Chap. 4 of (Sornette, 2003) for a detailed discussion and references therein). Positive feedbacks, when unchecked, can produce runaways until the deviation from equilibrium is so large that the growth becomes unstable so that any disturbance/news may lead to ruptures or crashes (Sornette, 2003). See (Sornette et al., 2003) for examples of run-away hyperinflation due to positive feedbacks.

To illustrate the concept of positive feedback in a mathematical form, consider the variable \( X \) (net sales, or earnings, or logarithm of prices). The familiar picture of a healthy growth of \( X \) is related to an exponential\[ \frac{dX}{dt} = rX, \tag{1} \]

where \( r \) is the instantaneous growth rate. For \( r = r_0 \) constant, \( X \) grows exponentially with the constant growth rate \( r \). We interpret \( r_0 \) as the natural and normal growth rate for instance associated with population growth and gains of productivity. Since gains of productivity are usually small (Fair, 2002; van Biesebroeck, 2004) and population growth is very small for a developed country, \( r_0 \) is small for a company that has already a substantial part of the local market as is the case of Ahold.

However, top managers are usually not satisfied with such small growth rates and attempt to increase growth for example by making acquisitions in foreign
countries, which is partly due to the fact that they can perceive more positive benefits from high growth rates (Erickson, Hanlon & Maydew, 2004). Then $r$ itself becomes a function of time. There are several ways to describe this phenomenon. For instance, one can argue that $r$ increases with $X$ due to expansion through acquisitions (like in Ahold’s case) according to

$$r = r_0 + aX^k,$$  \tag{2}

where $k > 0$ and $a$ a positive constant. This expression embodies the concept of positive feedback of the sales on its growth rate. Putting this dependence (2) in (1) yields

$$dX/dt = r_0X + (X/X_0)^{k+1},$$  \tag{3}

where $X_0 = a^{1/(k+1)}$ is a constant. Since $k > 0$, the positive feedback in (3) is initially small for small $X < X_0$ and then accelerates progressively as $X$ reaches and overpasses the characteristic value $X_0$. For $X > X_0$, the solution of (3) tends to the asymptotic power law

$$X(t) \propto \left(\frac{t}{t_c - t}\right)^{1/k},$$  \tag{4}

where $t_c$ is a constant of integration. The solution (3) leads to what is called in mathematical terms a “movable finite-time singularity”: “singularity” because of the divergence in finite time as $t \to t_c$ and “movable” because the critical time $t_c$ is not fixed a priori by the structure of the feedback and of the dynamics but is sensitively dependent upon (and fixed by) the initial conditions.

The acceleration of growth can also take the form of a feedback of the velocity of change of the sales on the growth rate, such that

$$r = b(dX/dt)^q$$  \tag{5}

with $0 \leq q < 1$. In words, the larger the slope of variation of the sales, the larger the number of acquisitions and therefore the larger the growth rate and so on. Placing (5) in (1) leads to an equation of the form (3) (without the linear term in the r.h.s.) with $1 < k + 1$ replaced by $1 < 1/(1 - q)$. This leads to $X(t) \propto 1/(t_c - t)^{1-q}$. Note that the limit $q \to 0$ recovers the standard constant growth rate, with an exponential growth formally retrieved as the limit of a power law with an infinite exponent. These two models (2) and (5) are end-members of more general feedback mechanisms that can combine both effects.

The important point is that a rather broad class of positive feedback mechanisms give rise to a power law acceleration, which can be written generally as

$$X(t) = A + B(t_c - t)^m,$$  \tag{6}
Fig. 2. Super-exponential growth of the quarterly net sales (Billion Euro). The lines are respectively the fits to a pure power law model (6) and to a periodically oscillatory power law model (7). The power-law fit gives $t_c = 2002/05/20$, $m = -0.79$, $A = -0.975$, and $B = 2185$ with a r.m.s of fit residual equal to 0.609. The oscillatory power-law fit gives $t_c = 2002/04/29$, $m = -0.65$, $f_s = 1.0006/365$, $\phi = 4.99$, $A = -2.24$, $B = 983.5$, and $C = -54.4$ with a r.m.s. of fit residuals equal to 0.5186.

where $m = -\frac{1}{k} = -(1 - q)/q < 0$ and $A$ and $B > 0$ are two constants. In particular, the constant $A$ has been added to represent the first correction to the leading power law behavior valid for $t$ close to $t_c$. Note that model (6) is much more parsimonious than a model such as suggested in Figure 1 consisting in several distinct regimes of exponential growth.

The form of the solution (6) illustrates the concept that super-exponential growth hides an inherent danger. Though theoretically, it may appear to go to infinity, what everybody knows is that in practice it can not continue indefinitely. The finite-time singular structure of (6) is actually the mathematical translation of a change of regime, a cross-over from an unsustainable pace to a crash or maybe a smooth landing of the accelerating bubble regime (Johansen et al., 1999; 2000).

3.2 Super-exponential growth in net sales and earnings

Figures 2 and 3 expand on Figure 1 and show respectively the quarterly net sales (from 1991 to 2001) and net earnings (from 1991 to 2001) of Royal Ahold. The fits by the power law (6) of the quarterly net sales (Billion Euro), net earnings (Million Euro) and logarithm of stock prices of Ahold from 1991 to their historical highs at $t_{max}$ are shown in figures 2 and 3. For the quarterly net sales, $t_{max} = 2001/03/31$. For the net earnings, $t_{max} = 2000/12/31$. These three time series exhibit a clear super-exponential growth. This occurred at a time when the company was aggressively increasing its share of the world market, transforming itself into an ever faster growing entity.
Fig. 3. Super-exponential growth of the quarterly net earnings (Million Euro). The lines are respectively the fits to a pure power law model (6) and to a periodically oscillatory power law model (7). The power-law fit gives $t_c = 2002/09/28$, $m = -0.95$, $A = -39.8$, and $B = 17882$ with a r.m.s of the fit residuals equal to 13.7. The oscillatory power-law fit gives $t_c = 2004/08/02$, $m = -1.77$, $f_s = 0.9989/365$, $\phi = 1.75$, $A = -8.43$, $B = 1.1 \times 10^8$, and $C = 1.3 \times 10^7$ with a r.m.s. of fit residuals equal to 8.26.

Inspection of the figures 2 and 3 shows that the last quarter is always by far the highest sales and earnings of the year. We can take into account this yearly periodicity by the improved model

$$X(t) = A + B(t_c - t)^m + C(t_c - t)^m \cos(2\pi f_s t + \phi), \tag{7}$$

where $f_s$ is the fundamental frequency of the seasonal cycle. The theoretical value of $1/365$ (for $t$ in units of day) of a yearly cycle is confirmed by fitting. The fits of the sales and earnings with (7) are also shown in Figures 2 and 3. A standard Wilks test (Rao, 1966) for the statistical significance of the added term $C(t_c - t)^m \cos(2\pi f_s t + \phi)$ gives a log-likelihood ratio between the models (6) and (7) equal to 13.5, so that the probability that the added explanatory power provided by (7) over (6) results from chance is 0.12%. The statistical significance of model (7) is thus established at the 99.9% confidence level. Similarly, the corresponding log-likelihood ratio between the models (6) and (7) for the net earnings is equal to 41.5, so that the probability that the explanatory power of (7) results from chance is essentially zero.

3.3 Bubble and post-bubble regimes in stock prices

For the stock prices of Royal Ahold, the super-exponential bubble culminated on $t_{\text{max}} = 1997/07/23$. Figure 4 shows the super-exponential growth of the logarithm of Ahold stock prices.

The post-bubble regime since the local high on 1997/07/23 has also been
Fig. 4. Ahold stock price as a function of time. The vertical axis is in logarithmic scale while the horizontal axis uses a linear scale. The power-law fit to the data from 1991/01/10 to \( t_{\text{max}} = 1997/07/23 \) gives \( t_c = 2004/06/19, m = -4.62, A = 1.71, B = 8.11 \times 10^{15} \) with a r.m.s of the fit residuals equal to 0.062. The log-periodic power-law fit to the data from 1997/04/09 to 2002/08/09 gives \( t_c = 1996/07/24, \alpha = 0.48, \omega = 12.31, \phi = 3.61, A = 3.14, B = 0.0078, C = 0.0055, \) and the r.m.s. of the fit residuals is \( \chi = 0.103. \)

fitted with a log-periodic post-bubble power law (Johansen and Sornette, 1999; Sornette, 2003)

\[
\ln [P(t)] = A + B(t - t_c)^m + C(t - t_c)^m \cos [\omega \ln(t - t_c) + \phi],
\]

showing an oscillatory pattern of increasing amplitude. The fit is also illustrated in Fig. 4. It is striking to note that \( t_c \) is also located in 1996, the point in time where according to the simpler analysis in Fig. 1 the growth acceleration took place. This reinforces the conclusion that 1996 is the year where a change of regime took place and the seed of the 2002/03 crisis was planted initiating a “critical zone” of destabilization. This pattern will be discussed more thoroughly later in the paper.

These analyses suggest that the crisis was not due to a proximate event but was seeded a long time ago when the policy of accelerating growth started to be implemented in 1996. Sornette (2003) has synthesized a large body of the academic literature showing that the general reason for a crash or crisis is rarely due to proximate causes but results from the progressive maturation and emergence of an unstable phase. When the unstable phase is ripe, basically any perturbation may trigger the crisis. This concept allows to understand why it is often so difficult to explain or understand the origin of the crisis or why so many different explanations are sometimes advanced, when one insists in searching for a proximate origin. The fundamental origin based on a growing instability could be called “emergent” and “systemic” in the jargon of the theory of complex systems. For these, one needs to develop a description of the collective bottom-up behavior rather than attempting to find a smoking
An important question concerns the predictive power of such fits. We should stress that no precise timing can be claimed here, for instance by using the fitted $t_c$ as a prediction for an impending crash. Our analysis performed jointly on the sales, earnings and prices can only be used to point to the approach of a change of regime which is indicated by the critical times. Results from fitting procedures using truncated time series on general markets like the S&P 500 have shown that, as a rule of thumb, the critical time $t_c$ can be “rather robust to approximately one year prior to a crash” (Sornette, 2003, p.330). The determination of the precise timing seems more complex for individual companies than for the overall market bubbles ending in crashes documented in (Sornette, 2003), probably due to several factors: the interference with the global market mood, the coupling with the policy of acquisitions and the manipulations of the corporate performance data as well as other idiosyncratic factors. Nonetheless it is striking to note that the above rule of thumb also appears to apply to the prediction of the critical times in 2002 from the sales data (Fig. 2) when fitted by (6) and (7). Using a truncated fitting procedure this result of a $t_c$ in 2002 appears to be rather robust. Although (6) also predicts a $t_c$ in 2002 for the earnings data, this is not a robust result. This comes as no surprise since quarterly earnings data are much more subject to manipulation showing no consistent results.

So, whereas the super-exponential growth of sales and earnings can not be expected to give precise warnings of an impending crash within at most one year, combining these data with the stock market price evolution opens new possibilities for advanced warnings. As shown in Fig. 4, 4-5 years before the crash in 2002 the stock price has changed from super-exponential growth to a different oscillatory regime. It is also clear that the transition from the super-exponential growth to the crash did not proceed directly but through an intermediate regime punctuating the evolution. The model we discuss below suggests an interpretation in terms of a variation with time of the opinion formation process among investors after the peak of the bubble, during the critical zone and then during the crash. The important point of our analysis is to focus on medium-term evolutionary patterns rather than on short-term vagaries. This strategy leads to the possibility of recognizing years before the actual crash occurred that a company, while its sales and earnings are racing upward, is really moving through a critical zone which, if allowed to continue, makes a crash inevitable.
4 Weidlich’s model of opinion formation applied to the stock market

4.1 Motivation

To summarize, we have argued that the evidence of positive feedbacks on the sales, earnings and stock market price of Ahold can be characterized by the occurrence of an apparent finite-time singularity. This singularity must be understood as announcing a change of regime. Usually, this change of regime takes the form of a break in the super-exponential growth which transitions into what we propose to define as a “critical zone.” The maturation of the critical zone may last weeks, months to years (rarely), and is usually followed by a catastrophic crash. Our motivation for introducing the concept of a “critical zone” is based on the following.

• Previous related works have found that the crash does not immediately follow the top of the stock market bubble but there is a transitional period (Sornette, 2003 and references therein).
• Models of rational expectation (RE) bubbles incorporating the mechanism of positive feedbacks (Johansen et al., 1999; 2000; Sornette and Andersen, 2002) predict a very large crash hazard rate in the vicinity of the end of the bubble; in such models, there is an effective “critical zone” characterized by a very large risk of a crash, without the need for the price to continue to accelerate. Since RE models of bubbles are only providing coarse-grained descriptions, it would be interesting to develop finer models of this regime.
• There are general arguments to expect that a finite-time singularity is rounded by finite-size effects (Cardy, 1988) and by the triggering of negative feedback mechanisms that were sub-dominant but come progressively into play as the dynamics approach the singularity. This rounding may be part of the “critical zone.”

We now turn to the development of a model of the “critical zone” and other patterns as well, based on Weidlich’s approach to social models using concepts from synergetics, which we will use to describe and interpret the evolution of Ahold stock price.

4.2 Background of the Weidlich model

Synergetics was originally developed by the German physicist Hermann Haken (1983) to study the behavior of complex systems of any kind. Composed of many interacting parts, these systems, whether physical, biological or social, are known to be able to spontaneously form analogous spatial, temporal or
functional structures or patterns through self-organization. Since its focus is particularly on what happens in those situations where complex system change their behavior qualitatively, Haken (2000) also considered synergetics as a theory of emergence of new qualities at a macroscopic level. The mechanisms underlying the observed processes at non-equilibrium phase transitions and bifurcations are studied with concepts of instability, order parameters and slaving. The slaving principle yields the important insight that close to instability points complex systems are governed by a low-dimensional, though noisy, dynamics (Haken, 2000).

Haken’s colleagues, Weidlich and Haag (1983), focused synergetics on social science applications such as the dynamics of the political opinion formation process and, in economics, on the non-equilibrium theory of investment behavior, also known as ‘Schumpeter’s clock.’ Studying the dynamics of two types of investment projects, expansionary and rationalizing through a model that formally resembles the opinion formation model, Weidlich and Haag (1983) were able to faithfully reproduce the evolutionary pattern of industrial strategic investment in Germany between 1956 and 1978. Under the title Sociodynamics, Weidlich (2000) published an elaborated version of the formal opinion formation model and included further examples of patterns of political phase transition and destabilization which are accompanied by unpredictable critical fluctuations where ‘anything can happen.’ The formal model, though relatively simple, is very powerful and has also been applied to the phenomenon of lock-in of a dominant technology in a situation of competing technologies (Arthur, 1994) and the inverse phenomenon of the lock-out of a dominant technology or product through disruptive innovations (Broekstra, 2002).

We will here closely follow Weidlich’s (2000) extended formal model of opinion formation and political phase transitions which has a strong resemblance to the dynamics of investors’ opinions or decisions as to ‘buying’ or ‘selling’ shares in the stock market. Particularly, the discontinuity that occurs in the transition from a liberal democratic to a totalitarian political system or vice versa (the lib-tot phase transition) is argued to have a close analogy to the development of bubbles and crashes in the financial markets. Political phenomena like public pressure to conform to the ruling ideology on the one hand and dissident behavior on the other can equally be transferred to the economic domain where the combination of imitation pressures and herding on the one hand and idiosyncratic or contrarian behavior on the other may be responsible for potentially unstable situations in stock markets (Sornette, 2003).

Simple models of dynamic behavior of investors in the stock market consider investors to be in one of only two possible states, such as ‘buy’ or ‘sell,’ ‘optimistic’ or ‘pessimistic,’ and ‘bullish’ or ‘bearish.’ In addition, several models have considered at least two types of investors such as ‘trend followers’ and ‘value investors’ (e.g. Farmer, 1998; Lux and Marchesi, 1999; Levy et al., 2000;
Ide and Sornette, 2002). Here, we will not assume from the outset that there are two rigid types of investors who basically account for positive and negative feedback processes, but we will introduce parameters for a population of investors holding ‘buy’ or ‘sell’ opinions. These control parameters can co-evolve with the order parameters to determine the degree of public conformity with the prevailing opinion, which influences the degree of imitation and herding, and the personal inclination to approve or disapprove of the prevailing opinion, which governs the degree of affirmation or dissidence with it. This allows us to investigate all possible patterns of evolution of share prices and, particularly, the complete patterns of destabilization of the price trajectories other than bubbles and crashes alone. We will show that, because of the flexible psychology of investors, the dynamics of one of the parameter of the model controlling trends will play an important if not decisive role. Describing the dynamics of such control parameters in an endogenous manner massively complicates the structure of the model. Furthermore, it will not be simple to find plausible systems of equations of motion for them. Such equations would also have to depend on the global economical and political situation, i.e., not only on the endogenous variables described below following Weidlich (2000). Therefore, we choose the simpler but efficient method (also used in the Schumpeter clock model (Weidlich and Haag, 1983)) to distinguish different time periods, each with plausibly chosen trends to evaluate the stock price evolution of Ahold. This requires a careful interpretation based on detailed information on the historical development of the Ahold drama.

Naturally, a lot of attention has focused on the development of bubbles and crashes, but hardly to what happens in the time period right after a super-exponential growth spurs, when the bubble has run out of steam, and before a crash may occur. This intermediate stage preceding a crash turns out to be a period of increasing destabilization, which may take weeks, months or years. Furthermore, a lot more attention has been given to stock market indices than to the fate of individual companies.

4.3 The evolution equations of majority and personal opinions

We first consider the evolution of the so-called “investors’ configuration”, \( \{ n_B(t), n_S(t) \} \), where it is assumed that at a given point in time the sum of the time-varying numbers of buyers \( n_B(t) \) and sellers \( n_S(t) \) is a constant and equal to \( 2N \). The investors’ configuration can then be defined by the normalized variable \( y(t) \) defined by

\[
y(t) = \frac{n_B(t) - n_S(t)}{2N}, \quad \text{where} \quad -1 \leq y(t) \leq 1.
\]
When \( y(t) = 0 \), the two types of investors holding opinions ‘buy’ or ‘sell’ are in balance: \( n_B(t) = n_S(t) \). Supply and demand are equal and the share price remains constant. If the number of buyers is larger (smaller) than the number of sellers, \( y(t) > 0 \) (resp. \( y(t) < 0 \)), then demand exceeds (resp. is lower than) supply and the price will go up (resp. down).

Investors may hold a personal inclination or preference concerning buying or selling, which may approve or disapprove of the prevailing opinion. For example, during a bubble formation when the psychology of herding and imitation takes over, traders may be forced to conform to pressures from their clients to buy even when they privately disapprove. This effect is taken into account by a personal preference variable \( m_i(t) \) where \( i \) is either the ‘buy’ or ‘sell’ state, such that a value \( m_i(t) > 0 \) describes an individual’s personal preference for the \( i \)-th state, a value \( m_i(t) = 0 \) describes inner neutrality to the state \( i \); a value \( m_i(t) < 0 \) describes personal disapproval of the \( i \)-th state. Assuming that \( m_B = -m_S = m(t) \) varies between limits, \( -M \leq m(t) \leq M \), a normalized variable (the personal preference index) is introduced

\[
  x(t) = m(t)/M, \quad \text{where} \quad -1 \leq x(t) \leq 1.
\]

If \( x(t) = 0 \), personal preferences towards buying and selling are neutral; if \( x(t) > 0 \) (resp. \( x(t) < 0 \)), personal inclinations are to approve of the prevailing opinion to buy (sell) and disapprove of selling (buying). The more positive or negative \( x(t) \) is, the stronger the growth of the personal preference towards one or the other position.

The quantities that govern the dynamics of investment decision making are the individual transition probabilities per unit time period of a ‘buy’ to a ‘sell’ state or opinion and vice versa, and the individual transition rates of the personal preference towards higher (lower) values of approval or disapproval. An individual changes its opinion randomly depending on the aggregated average opinion found in the population and on his own opinion. Thus, individuals do not interact directly at the micro level, but indirectly via the macro level (see for instance Helbing, 1992 for a generalization including pair interactions).

In Weidlich’s (1983, 2000) theory, an ensemble of configurations is considered where each is a homogeneous population of \( 2N \) investors. The probability distribution over these configurations \( p(x, y; t) \) is then defined as the probability that one sample configuration has the configuration \( \{x, y\} \) at time \( t \). A non-linear differential equation is then defined, the master equation, which describes the changes of the probability distribution over time. For further details the reader is referred to Weidlich (2000; see also Broekstra, 2002).

Approximate solutions to the probabilistic master equation can be obtained by considering the quasi-meanvalue equations for the mean evolution of the order parameters \( x \) and \( y \). This is the approach followed here. However, for a fuller understanding of the origin of the quasi-meanvalue equations to follow,
it is noted that Weidlich (2000) proved without loss of generality that the transition rates must be of the form $m \exp(u(\text{final}) - u(\text{initial}))$, where $m$ is a mobility factor and $u(\text{initial})$ and $u(\text{final})$ are utility functions depending on the system variables and personal state before and after the transition. The actual form of the utility function still has to be determined and, for reasons of simplicity, was based by Weidlich on a linear dependence on the number of investors holding the ‘buy’ or ‘sell’ opinion (Weidlich, 2004, private communication). The quasi-mean value equations consist of a system of two nonlinear differential equations for the normalized personal preference $x(t)$ and investors configuration $y(t)$ (Weidlich, 2000):

$$\frac{dy}{dt} = \sinh(\kappa y + \gamma x) - y \cosh(\kappa y + \gamma x), \tag{11}$$

$$\frac{dx}{dt} = \mu [\sinh(\beta y) - x \cosh(\beta y)]. \tag{12}$$

To simplify the notations, $x$ and $y$ are now to be read as mean values and the time $t$ is also a scaled variable. Equations (11) and (12) have four parameters $\kappa, \gamma, \beta$ and $\mu$, which describe behavioral trends and express the investors’ psychology. We will treat them as constant in each relevant regime.

$\kappa$ is a parameter coupling individuals to the majority. We refer to it as the coupling or conformity parameter. If positive, it strengthens the investors’ personal readiness to conform to the majority opinion. If it is high, then the influence of a ‘buy’ majority on an individual transition from ‘sell’ to ‘buy’ is high. Inspection of Eq.(11) shows that if $y > 0$, the existing majority opinion to ‘buy’ further favors transitions from ‘sell’ to ‘buy’, and similarly discourages the inverse transitions. As we will see, if $\kappa$ attains a critical value, a herding process starts which can lead to a bubble. Conversely, if $\kappa > 0$ and $y < 0$, a majority opinion to ‘sell’ will put pressure on individual investors to conform by changing their position from ‘buy’ to ‘sell’. As Eq.(11) shows, the evolution of the majority also depends on the investors’ personal preference $x$. Clearly, with $\gamma > 0$ and personal preference $x$ is positive, these effects will be reinforced. If however $x < 0$ and investors tend to personally disapprove of the majority ‘buy’ position, this will diminish the effect of the majority pressure to conform. In fact, this idiosyncratic or contrarian inclination, which Weidlich (2000) called a ‘dissidence propensity’ in the case of a political opinion formation process, may eventually lead to the destabilization of the system.

The inclination parameter $\beta$ governs the evolution of the personal preference. If positive, it measures an approving tendency and, as Eq.(12) shows, when the majority $y > 0$ favors the ‘buy’ decision, the evolution of a positive $x$ will be reinforced stabilizing the majority opinion. However, if $\beta$ is sufficiently negative, that is, attains a critical value, the dissident propensity may lead to a strong tendency to personally disapprove of the existing situation, eventually
leading to a negative $x$, which in turn may lead to a destabilization of the majority position.

The preference influence parameter $\gamma$ determines the strength of the influence of the personal preference $x$ on the evolution of the majority opinion, and $\mu$ is a parameter which determines the speed of evolution of the personal preference relative the speed of evolution of the majority opinion $y$. In other words, $\mu$ is the ratio of the characteristic time scales of the dynamics of the majority opinion to the dynamics of personal preference. A small (large) $\mu$ corresponds to a slow (fast) adjustment of personal preference to majority opinion.

We complement this system (11) and (12) by an equation for the price. We make the simplified assumption that the mean individual order sizes $O_B$ for buyers and $O_S$ sellers are equal and constant, such that $O_B = -O_S = O$. Then, the net order size of the population is $\Omega(t) = n_BO_B + n_SO_S = O(n_B - n_S) = 2NOy(t)$, using (9). Dividing both sides by the size $2N$ of the population, we obtain the scaled linear equation for the net order size per agent

$$\omega(t) = \alpha y(t)$$

where $\alpha$ is a positive constant. If supply and demand balance each other, $y(t) = 0$, and the net order size equals zero. If buyers prevail over sellers (or vice versa), $y(t) > 0$ ($y(t) < 0$), the net order size is positive (negative). To make the theory simple, we assume following many previous workers that the market impact function which relates the logarithm of the stock price $P$ to the net order size $\Omega$ reads

$$\frac{d \ln P(t)}{dt} = \frac{\Omega(t)}{L},$$

where $L$ is some ‘market depth’ assumed to be constant. Scaling this equation, and henceforth for reasons of convenience using the notation $p(t)$ for $\ln P(t)$ for the mean value of the stock price, in the numerical integration we used the simple equation for the logarithmic stock price evolution

$$\frac{dp(t)}{dt} = ay(t),$$

where $a$ is a positive constant. This expression (15) implies that the logarithm $p(t)$ of the price is proportional to the integral of the majority opinion order parameter $y(t)$ and reciprocally $y(t)$ is proportional to the market return $dp/dt$. The system we study below is thus made of (11), (12) and (15).

Using expression (15) to eliminate $y(t)$ in the r.h.s. of (11) and (12) offers an interesting interpretation of the opinion formation process in stock markets: the proxy for the majority opinion $y(t)$ is indeed the stock market return $dp/dt$ which is the major factor influencing the dynamics of the opinions.
following quip “in the long run, markets are weighing machines, but in the short term, they are voting machines” is often attributed to B. Graham and W. Buffet. This quote exemplifies why the theory of Weidlich developed to model opinion formation processes can indeed be a useful model of stock market price evolution at short and intermediate times. At long times, economic factors and fundamental valuations have to be introduced, giving a competition between the “voting” and “weighing” properties of the stock market.

4.4 Stability analysis

The analysis of Eqs. (11) and (12) reveals that the evolution equations exhibit five singular points, that is, points where the left-hand side derivatives become equal to zero (Weidlich and Haag, 1983; Weidlich 2000). Stability analysis of the behavior of $x(t)$ and $y(t)$ in the vicinity of the singular point $(x, y) = (0, 0)$ is of particular interest because it deals with the possibility of deviation from a balanced situation where the number of buyers and sellers is equal and their personal preference towards either decision is neutral. When adding noise (stochastically), this stationary point $(0, 0)$ corresponds to the random walk situation and the question whether it is a stable or unstable point becomes important to detect the emergence of a runaway bubble or crash. Without going into further detail we will just state the following stability conditions (see Weidlich, 2000).

The stationary point $(x, y) = (0, 0)$ is stable if and only if

$$\kappa < 1 + \mu \quad \text{and} \quad \beta < \frac{(1 - \kappa)}{\gamma} .$$

(16)

The stationary state $(0, 0)$ is thus unstable if at least one of the two inequalities in (16) is broken. These conditions (16) express that a balanced situation can continue to exist only if the pressure $\kappa$ to conform to the majority opinion is relatively small, while simultaneously the personal inclination $\beta$ to approve or disapprove also remains small. Starting from a small degree of conformity, if $\kappa > 0$ starts to grow meaning that the tendency to herd starts to increase, condition (16) shows that the value of $\beta$ has to become increasingly smaller and even negative, indicating that the degree of dissidence among investors needs to increase in order for the point $(0, 0)$ to remain stable. However, if $\kappa$ exceeds the critical threshold value $1 + \mu$, whatever the amount of dissidence, $(0, 0)$ becomes an unstable stationary point, and a runaway situation, where the market locks in on either a bull market or a crash, may result. As (16) also shows, if $\beta$ exceeds a critical value even if $\kappa$ has not, $(0, 0)$ becomes unstable. This is reasonable to find that too much dissidence can equally destabilize the neutral point.

Finally, a limit cycle solution or attractor of the mean value equations exists
if at least one of inequalities in (16) is broken and if in addition

\[ \beta < -\frac{(\mu + \kappa - 1)^2}{4\mu \gamma} < 0 \]  

holds.

### 4.5 Classification of evolution patterns

The system of Eqs. (11) and (12) was numerically integrated using a 4th order Runge-Kutta scheme, for a number of different values of the parameters. In the numerical integrations presented below, we have used a scaled time step of 0.01 throughout. The numerical integrations have allowed us to classify four basic patterns of evolution of the mean values of the investors configuration \( y(t) \), of the personal preference \( x(t) \) and of the price \( p(t) \), for different parameter values. First, to attain some basic insights, we will keep the trend parameters \( \kappa \) and \( \beta \) constant. However, during transitions, it is more reasonable to assume that particularly the trend parameters \( \kappa \) as a measure of herding pressure and \( \beta \) as a measure of the degree of dissidence will co-evolve with the system variables. This will be dealt with in the next section when we discuss the evolution of the stock price value of Ahold.

#### 4.5.1 Pattern A: \((0,0)\) stability and ‘random walk’

This pattern is exemplified in Figure 5, where a large negative initial value of \( y(0) \), a sudden preponderance of sellers over buyers, is quickly restored to the balanced situation. Due to the large negative, disapproving \( \beta \) used in this example, the personal preference index \( x \) becomes highly positive, driving the out-of-balance situation quickly back to normal. Furthermore, since \( \beta = -4 < \frac{-(\mu + \kappa - 1)^2}{4\mu \gamma} = -0.8 \), condition (17) indicates that we may expect that the return will occur in an oscillatory manner. In contrast, for example with \( \beta = -1 \), the \( y \)-trajectory will approach the \((0,0)\) state somewhat more slowly, but without oscillations.

As a response to the sudden strong ‘sell’ mood in the market, the stock price evolution drops sharply and, with diminishing selling pressure, approaches in an oscillatory manner a new stable value. This is reminiscent of a sudden large sell-off of some company stock which unexpectedly publishes some bad news but, where due to insufficient conformity or herding pressure and an immediate reaction opposing the irrational sell-off, the damage is limited to a price drop to a new stable though lower level representing the new price discounting the novel piece of news. This behavior of a sharp drop followed by damped oscillations converging to a well-defined lower plateau is also observed.
Fig. 5. Evolution Pattern A when \((x(t), y(t)) = (0, 0)\) is a stable stationary state. This simulation corresponds to \(\mu = 2, \gamma = 1, \kappa = 1.5, \beta = -4,\) and \(a = 1.\) Since \(\kappa = 1.5 < 1 + \mu = 3\) and \(\beta = -4 < (1 - \kappa)/\gamma = -0.5,\) according to condition (16), \((x, y) = (0, 0)\) is a stable stationary state and any deviation from it will quickly return to that state. For reasons of presentation, in this and the following figures, the logarithm of the stock price \(p(t)\) starts at an arbitrary initial point \(p(0)\). Very clearly for the crash of October 1987, for instance in the US (see figure 3 of (Sornette et al., 1996) reproduced here as figure 6). Symmetrically, if the initial condition for \(y(0)\) is positive, the average opinion \(y(t)\) decreases sharply to 0 and there is an increase of the stock price level.

To compare with empirical data, one needs to re-introduce a stochastic component, for instance in the form of an additive white noise to the right-hand-side of expression (15) (Weidlich, 2000). In this case, the fact that \((0, 0)\) is stable corresponds to the log-price \(p(t)\) following a random walk.

4.5.2 Pattern B: “the CEO’s dream”

As shown in Figure 7, this is an interesting situation because the opposing tendency \(\beta\) is too weak to prevent a runaway situation to occur. In the stock market, this represents the bullish formation of a bubble as the price trajectory indicates. In terms of investors’ behavior, the value analysts lose to the trend followers. Eventually, this leads to a new stable state for \(y\) where the buyers form a large majority, while simultaneously personal preference settles at a negative stationary value which is not big enough to defuse or destabilize the bubble. Weidlich (2000) discussed this scenario in the context of the transition of a liberal to a totalitarian state. He called it quite appropriately ‘the dictator’s dream,’ because while the pressure to conform is moderate, due to a too small opposition of dissident opinion, it is enough to transform a liberal situation into a completely totalitarian state. Likewise, in the stock market, this situation of a bullish market could be called the CEO’s dream.
Fig. 6. Time evolution of the S&P 500 index over a time window of a few weeks during and after the crash of October 19, 1987. The fit with an exponentially decaying sinusoidal function suggests that a good model for the short-time response of the US market is close to single dissipative harmonic oscillator. Reproduced from (Sornette et al., 1996).

A more precise analytical understanding of this regime can be obtained by neglecting $x$ in expression (11). Starting with a small initial value $y(0)$, the initial growth of the global opinion follows

$$\frac{dy}{dt} \approx \kappa y + \frac{1}{6} \kappa^3 y^3 - y \left( 1 + \frac{1}{2} \kappa^2 y^2 \right) + O(y^5) = (\kappa - 1) y + \left( \frac{\kappa}{3} - 1 \right) \frac{\kappa^2}{2} y^3 + O(y^5),$$

up to third order in $y$. As long as $y$ is small, it first grows exponentially (corresponding to the first linear term $(\kappa - 1)y$) for $\kappa > 1$. This first regime
Fig. 7. Evolution Pattern B: Illustration with $\mu = 2; \gamma = 1; \kappa = 1.5; \beta = -0.25; a = 0.5$. Since $\kappa = 1.5 < 1 + \mu = 3$ and $\beta = -0.25 > (1 - \kappa)/\gamma = -0.5$, condition (16) is broken and $(x, y) = (0, 0)$ is unstable. The limit cycle condition (17) does not apply since $b = -0.25 > -(\mu + \kappa - 1)^2/(4\mu\gamma) = -0.8$. Note the upward curvature of the log-price $p(t)$ exemplifying the super-exponential growth discussed in the text.

translates also into an initial exponential growth of the log-price $p(t)$ and thus to an exponential of an exponential growth of the price. This is a clear super-exponential growth characterizing a bubble (Sornette and Johansen, 2001; Sornette, 2003; Johansen and Sornette, 2004). Such exponential of an exponential law, proposed to describe phases of accelerated hyperinflation, has been shown to be essentially indistinguishable from power law growth leading to an apparent finite-time singularity of the form (6); actually, the discretized version of an accelerated growth leading to (6) can been shown to be an exponential of an exponential growth (Sornette et al., 2003).

Then, as $y$ grows, the nonlinear term starts to dominate and leads to an even faster transient super-exponential growth if $\kappa > 3$. For instance, if the cubic nonlinearity held throughout the dynamics with $\kappa > 3$, this would give rise eventually to a solution at later times of the form $y(t) \sim 1/(t_c - t)^{1/2}$ clearly expressing the super-exponential behavior characteristic of bubbles. In this case, the log-price would becomes of the form (6) with $m = 1/2$ by integration of (15). However, as $y(t)$ increases and becomes a finite fraction of 1, the approximation (18) becomes inaccurate as $y$ saturates and converges to the stable fixed point $y^* < 1$, solution of $\sinh(\kappa y) - y \cosh(\kappa y) = 0$. A constant asymptotic value of $y$ then translates into a linear growth of the log-price $p(t)$ and thus to an exponential growth of the price. This suggests that the super-exponential characteristic of a bubble is, according to Weidlich’s theory, only a transient regime.

There are additional factors to consider. In addition to the saturation mechanism captured by the second term $y \cosh(\kappa y + \gamma x)$ in the r.h.s. of (11), the super-exponential growth of the stock price may be halted in many cases before
the new stationary state of $y$ is reached where a large majority of investors are buyers. As shown below, at some time during the formation of the stock price bubble, although initially supportive, some investors will personally become increasingly worried despite the herding pressures, and the contrarian inclination $\beta$ is likely to become more strongly negative. Likewise, the totalitarians will become more bullish and $\kappa$ may also increase. This emerging battle between the degree of personal opposition $\beta$ and the degree of public conformity pressures $\kappa$, between order and disorder, will eventually induce a critical phase of destabilization of the bubble. Note that this reasoning amounts to add positive feedbacks of the opinion and price dynamics on the control parameters of the contrarian inclination $\beta$ and public conformity pressures $\kappa$.

Finally, note that pattern B reproduces the well-known lock-in through fluctuation situation in cases where, rather than two opposing opinions, two competing new products or technologies through some random fluctuations in market share declare one winner, who takes it all, and one loser (Arthur, 1994; Broekstra, 2002). In other words, if in Figure 7 the initial condition would have been slightly negative rather than positive, the symmetrical situation of a stock market crash would have been developed resulting in a new stationary state of a high majority of sellers. This sensitivity to initial conditions is also captured by Ising-like models of opinion formation (Sornette, 2003, Fig. 4.10) associated with the presence of a critical point around which the collapse into one of two states bifurcates essentially randomly.

4.5.3 Pattern C (“critical zone”): the destabilization regime ending in a collapse

![Fig. 8. Evolution Pattern C: Illustration with $\mu = 2$, $\gamma = 1$, $\kappa = 3.5$, $\beta = -4$, and $a = 0.5$. Since $\kappa = 3.5 > 1 + \mu = 3$ and $\beta = -4 < (1 - \kappa)/\gamma = -2.5$, condition (16) is broken and $(x, y) = (0, 0)$ is unstable. The limit cycle condition (17) applies since $b = -4 < - (\mu + \kappa - 1)^2/(4\mu\gamma) = -2.5$. Rather than an abrupt transition point, there exists a transition range between]
relatively small negative values of $\beta$ below the limit cycle condition ($-2.5 < \beta < 0$), where $y$ swiftly approaches one of two opposing new stationary states, and relatively high negative values of $\beta < -4.1$, where a true limit cycle and a stable attractor appears (see pattern D below). This transition range reflects the dependence of the exact pattern of evolution on the initial conditions. Within this transition range $-4.1 < \beta < -2.5$, starting from low values of $|\beta|$ and small initial conditions for $y$, with increasing $|\beta|$, one observes an increasing number of oscillations of increasing amplitude which all end in a breakdown characterized, depending on the initial conditions, by one of two opposing stationary states of large positive or negative $y$.

Figure 8 shows a typical Pattern C in which, starting from small initial conditions, both $x$ and $y$ show increasingly larger and opposing oscillations until a sudden breakdown occurs. This typical pattern where the stationary state $(0,0)$ has become unstable and whose unstable oscillatory growth ends in a collapse deserves the name “destabilization regime.” The stock price shows a similar pattern of increasing oscillations ending in a crash. This destabilization regime which acts a precursor to an inevitable collapse only occurs under the condition of strong herding pressures, $\kappa = 3.5 > 1 + \mu$. This is in qualitative correspondence with previous observations of the ubiquitous existence of oscillations of increasing amplitudes preceding crashes, which have also been interpreted as the signature of strong herding pressure (Sornette et al., 1996; Sornette and Johansen, 2001; Sornette, 2003). The tendency to strongly conform with the prevailing majority is fueled by exogenous factors like the overall market mood, as happened during the bull market of the late 1990s, but also endogenous factors like announcements of acquisitions and management promises of significantly increased earnings. As we will see below, Ahold is a case in point.

Fig. 9. Phase portrait in the $x(t); y(t)$ phase space, corresponding to Figure 8. The stable limit cycle for $\beta = -4.2$ is shown as the elliptic-like attractor. The spiraling trajectory is for $\beta = -4$, which falls in the destabilization regime. It starts near the origin and escapes the basin of attraction at the lower right end, corresponding to the collapse discussed in the text.
A better understanding of Pattern C can be obtained from its phase portrait in the \( \{x(t), y(t)\} \) phase space. Figure 9 shows the stable elliptic-like attractor for a slightly higher negative value \( \beta = -4.2 \), other parameters being equal. When started from conditions close to the unstable stationary state \((0, 0)\), all trajectories stay within the domain of attraction and converge towards it. This leads to a stationary limit cycle (see Pattern D below). Note that the attractor is somewhat deformed at its far ends, tipping slightly upward on the far left side, and downward on the far right side. Increasing \( \beta \) slightly to \(-4\), as shown above, enters the transition range characterized by trajectories starting close to \((0, 0)\) as shown in Figure 9: after a number of increasing oscillations, the trajectory escapes at the far end from the attractor to collapse in the lower right bottom of the \((x, y)\) phase space. Under different initial conditions, the trajectory may also escape from the upper far end on the left side to the upper right corner of the phase space.

4.5.4 Pattern D: ‘Roller coaster’

![Fig. 10. Evolution Pattern D: the Roller Coaster. Starting from small initial conditions near (0, 0), the trajectories develop into stable sinusoidal patterns. The simulation shown corresponds to \( \mu = 2, \gamma = 1, \kappa = 3.5, \beta = -5, a = 1 \).](https://example.com/figure10)

This regimes corresponds to large negative \( \kappa \) and \( \beta \), such that \((0, 0)\) is unstable but a stationary limit cycle exists, as shown in Figure 10. The phase portrait of this limit cycle corresponding to Figure 10 is similar to that shown in Figure 9 for \( \beta = -4.2 \) but with more rounded edges, which explains the smoothed regular sinusoidal movement shown in Figure 10.

It is somewhat unlikely that such a pattern of big upswings and downswings from a majority of buyers to a majority of sellers and vice versa and corresponding ‘roller coaster’ swings of the stock price will be sustainable for a long time. It indicates extreme uncertainty among investors about the company’s predicament. As shown, it can only occur when the inclination \( \beta \) to disapprove of the prevailing majority remains extremely strong, causing big oscillations...
in personal preference. However, the slightest drop of $|\beta|$ will cause the system to fall back into the destabilization regime ending in a collapse (Pattern C).

### 4.5.5 Summary of the classification of patterns

Let us now summarize the obtained classification in four fundamental patterns of evolution. As long as the conformity parameter $\kappa$ remains sufficiently small and less than $1 + \mu$, indicating little propensity among investors to herd, and as long as the inclination parameter $\beta$ remains also below its threshold $(1 - \kappa)/\gamma$, counteracting as it were any inclination to herd, the stability condition (16) for $(0,0)$ applies and nothing much interesting happens. This is Pattern A. However, if the stability condition (16) for $(0,0)$ breaks down, either the herding pressure exceeds the critical value $1 + \mu$, or if it does not but the counteracting disapproval parameter $\beta$ is insufficiently strong, the investors’ system locks into a runaway situation. Whether this is a bubble or a crash sensitively depends on the initial conditions. This is evolution Pattern B.

If the stability condition (16) breaks down, and for a given herding pressure $\kappa$, when the inclination parameter $\beta$ becomes sufficiently negative so that condition (17) applies, a broad transition range of $\beta$ values appears between unstable growing oscillations ending in a run-away (Pattern C) for smaller values of $|\beta|$ and a stable limit cycle for sufficiently large $|\beta|$ (Pattern D). Pattern C is the more interesting pattern particularly when the bullish rise of the stock price is temporarily halted in a moment of hesitation, and according to Eq.(15), $y(t)$ becomes equal to zero. Within the transition range, it appears as if a climate of escalation between trend followers and value investors induces a destabilization regime which acts as a precursor to an inevitable collapse (Sornette et al., 1996; Sornette and Johansen, 2001; Sornette, 2003).

The specifics of the basic evolution patterns are also governed by the speed and influence parameters $\mu$ and $\gamma$. To simplify the discussion, we have kept them constant throughout.

As already indicated, in actual practice, the conformity and personal inclination parameters $\kappa$ and $\beta$ will not remain constant but will change under the influence of or co-evolve with changes of $x(t)$ and $y(t)$. This would require the introduction of evolution equations for these parameters, thus complicating the model considerably not in the least because it will also introduce aspects of chaos dynamics. A simpler approach, which has the advantage of bringing out the essence of what is involved, is to change parameters discontinuously. Weidlich and Haag (1983) has successfully applied this method in reproducing the strategic investment evolution in Germany (see also Weidlich, 2000). We have followed this approach to reproduce the qualitative and main features of the Ahold stock price evolution. In this way, we propose an interpretation and
understanding of what happened.

5 Interpretation of the stock price evolution of Ahold

Figure 11 shows the result of a simulation using Weidlich’s equations (11) and (12) performed to reproduce the basic qualitative features of the Ahold stock price evolution. For this, four phases are distinguished, each characterized with a different pair \((\kappa, \beta)\).

![Figure 11](image)

This figure shows a simulation of the log-price using Weidlich’s equations (11) and (12) performed to reproduce the basic qualitative features of the Ahold stock price evolution shown in Figure 4. To obtain the corresponding log-price trajectory, four stages can be distinguished in which each time just one parameter was changed discontinuously. The parameters \(\mu = 2\), \(\gamma = 1\), and \(a = 0.5\) are kept constant throughout the simulation. Only \(\kappa\) and \(\beta\) are changed.

5.1 Phase 1 (time period: \(0 \leq t \leq 12\), i.e. 1993-mid 1997): \(\kappa = 1.5, \beta = -0.25\) (Pattern B)

This regime corresponds to Pattern B, in which a bullish super-exponential price rise occurs because of relatively moderate herding pressures which are not counteracted by a sufficiently strong personal disapproval. This is the CEO’s dream. From roughly 1995/1996 onwards, the stock market as a whole (the AEX) had become increasingly bullish. So Ahold was riding the bull when it accelerated its growth strategy during that period. In the third quarter of 1996, due the consolidation of the large American supermarket chain Stop & Shop, the quarterly net earnings increase, which during the preceding period in the 1990s usually remained well below 20%, discontinuously rose to 54%, initiating a new period of (promises of) considerably higher increases in both sales and earnings.
5.2 Phase 2 (time period: $12 < t \leq 14$, i.e., mid 1997-end 1997): $\kappa = 1.5, \beta = -4$ (Pattern A)

At some point in time for some reason, investors hesitate and the stock price reaches a first peak. Perhaps fear of heights and/or a decrease in the stock market as a whole (the AEX showed a peak followed by a temporary decline in mid 1997) have increased investors’ wariness and fueled the contrarians. Just by decreasing $\beta$ from a weak ($\beta = -0.25$) to a more strongly disapproving propensity ($\beta = -4$), the herding pressure is temporarily counteracted such that the stability condition for $(0,0)$ applies and Pattern A occurs (actually the inverse of the example given in Fig.5). After a peak in the stock price, when $y(t) = 0$ (see Eq.(15), the bullishness quickly returns with a vengeance leading to phase 3.

5.3 Phase 3 (time period: $t > 14$ until $p(t)$ drops below $p(0)$, i.e., 1998- end 2002): $\kappa = 3.5, \beta = -4$ (Pattern C)

Keeping the degree of dissidence high, $\beta = -4$, but now increasing the herding pressures from $\kappa = 1.5$ to 3.5, indicating both the general bullish market tendencies and investors’ expectations about the company, Pattern C comes into play. By the end of 1997, Ahold had completed its first peak and a new upward trend occurred. This point in time could be regarded as the start of the destabilization regime which became increasingly clear over time when further oscillations occurred. This was the precursor for the collapse of the stock price in 2002 followed by a final blow due to the publication of fraud in February 2003.

In general, it is not unlikely that top management fuels the dynamics of Pattern C, by raising expectations of increased earnings in the investors’ community. For example, before 1996, the year the growth rate accelerated, the announcement that ‘further increases’ in earnings (per share) were expected was the standard statement in press releases of Ahold. After the report of the unprecedented 54% earnings increase in the third quarter of 1996, announcements were upgraded in terms of ‘significantly’ or ‘considerably higher earnings’ that could be expected. Finally, the company became daringly precise and bullish by announcing as of March 1999 that its outlook for the whole year foresaw an expected earnings per share growth of at least 15%.

However, 1999 was also the year that Ahold’s stock price declined 44% from an all time high of 38.5 euro in April to a low of 21.2 euro in February 2000, while in the same period the Dutch stock market index AEX showed an overall upward trend and appreciated by more than 20%. The start of this downward
trend could have made Ahold’s top management somewhat nervous, and in an unprecedented statement, the company included an upbeat quote of its CEO in its 10 June 1999 press release, covering the Q1 1999 results and outlook for 1999, saying that the prospects were excellent and that the company was well on course. A statement like this could, of course, fuel the conformity pressures towards bullishness.

On the other hand, a growing skepticism may have developed among investors who were concerned by the overvaluation of the stock and disapproved of a change in management strategy. This happened in 1999 to Ahold, after the failed takeover of yet another supermarket chain (Pathmark in the USA), when it announced the acquisition of US Foodservice at the end of 1999. Some analysts (including Standard & Poor’s) disapproved of this change of course to the unfamiliar non-core activities of the wholesale food service industry. Also, early 2001, the dollar started its decline, which may have increased investors’ degree of disapproval throughout the last part of the destabilization phase. This situation of increasing opposing tendencies is reflected in the relatively high values of the conformity parameter \( \kappa \) and the negative inclination parameter \( \beta \), which are responsible for the destabilization as a precursor to the unavoidable collapse of the stock price as shown above in Pattern C and Figure 11.

The year 1999 appears to have been a critical year. With the destabilization well on its way (according to our classification), Ahold’s stock price continued its overall decline in 1999 until the higher increase of net earnings of 37% in 1999 (compared with 29% in 1998) was reported in March 2000. This together with the future consolidation of US Foodservice and other acquisitions in 2000 (other food service companies and PYA/Monarch in the USA among others) in 2000 were expected to further enhance growth. By then, the AEX was also on a final growth spurt which ended early September 2000 when the market started its long decline. The Ahold stock price then started its last mini-bubble, undoubtedly fueled by these expectations of higher earnings. As became evident in 2003, however, these turned out to be misplaced, and the company performance results had to be corrected downwards for 2000 and 2001.

Qualitatively, the overall oscillating pattern of stock price evolution after the first peak is strikingly similar to that of Pattern C. Although Ahold price shows a somewhat increasing trend in this regime, it should be noted that, for reasons of clarity, longer-term trend lines have been left out of the above simple model of patterns of evolution (see also Weidlich’s example of Schumpeter’s clock, 1983).

Pattern C shows that the increasing oscillations are a part of the destabilization of the bubble, which, if not defused in time, is the precursor for an
impending collapse as happened with Ahold. Pattern C is really the expression of the age-old adage: What goes up, must come down.

5.4 Phase 4 (Beyond the time where $p(t)$ drops below $p(0)$: 2003- mid 2003):

\[ \kappa = 2.5, \beta = -4 \]  (Pattern A)

Some time during the collapse of the stock price, here somewhat arbitrarily located at the point in time when $p(t)$ drops below $p(0)$, the herding pressures will deflate, and we are back to Pattern A. In the process, the inclination parameter $\beta$ will also return to smaller values, but to keep to the essentials, for a return to the stability of $(0, 0)$, it is already sufficient for the conformity parameter to drop below $1 + \mu$.

With phase 4, we have completed the description of a complete bubble using a very simple synergetics model and Ahold as an example to demonstrate the power of the method. The discovery of the destabilization regime after a bullish rise of the market enhances the possibility of developing early warning signals by focusing on pattern recognition rather than on short-term gains. Would this knowledge have been available, in Ahold’s case, having spotted its growth acceleration in 1996 to super-exponential proportions, as early as the end of 1997 when the first oscillation was completed and the next one started, investors could have become somewhat concerned about the consequences of their own bullishness by realizing that they were really riding the waves of a self-created destabilization regime which, if nothing changed, is the precursor of an inevitable collapse, sooner or later. But certainly by the end of 1999, the destabilization pattern C was quite evident and an eventual collapse could only have been prevented by defusing overheated expectations. This evidently did not happen, on the contrary, and the destabilization regime ran its course ending in a dramatic collapse of the stock price.

The paradoxical conclusion is that, particularly at times of overheated post-bubble stock markets where strong herding pressures are counteracted by equally strong contrarian propensities, companies are advised not to throw oil on the fire by overly raising or even manipulating expectations of higher earnings. Although it may go against the grain for ambitious CEOs, given the humans’ latent animal instinct to herd, cooling the stampeding crowd of investors would be a far better policy in an attempt to defuse a dangerous destabilization regime which acts as the precursor of a future collapse.

Destabilization is not an abstract phenomenon that is without consequences for the internal functioning of a company. Although this aspect requires further research, it is highly likely that in the case of Ahold starting in 1999 a certain loss of control became internally visible. For example, newspapers have re-
ported that the bookkeeping problems at US Foodservice were already known in 2000. Dogmatic preoccupation with the growth strategy by top management, lack of adequate supervision by and short-term orientations of directors and accountants alike prevented the internal consequences of the destabilization to surface in time.

5.5 Another corporate example: Aegon

The Dutch insurance group Aegon is one of the largest in the world operating in three major markets of North-America, The Netherlands and the United Kingdom. Its net income for the year 2003 was 1.8 billion Euros of which, like Ahold, the largest share is in the USA. Figure 12 shows another remarkable illustration of a similar sequence of phases as discussed for Ahold. The first regime is characterized by an upward curvature of the log-price of Aegon as a function of time, which is again a signature of super-exponential growth, that culminated mid-1998. Then, a plateau decorated with oscillations from mid-1998 to the beginning of 2001 can be represented rather faithfully by the “roller-coaster” pattern D (fitted with a regular sinusoidal oscillation \( \ln[P(t)] = A + B \cos(\omega t + \phi) \)). This second phase contrasts with the enthusiasm of investors of the first phase, as being a time of large uncertainty among investors about the company’s future. Recall that this regime only occurs when the personal inclination \( \beta \) to disapprove of the prevailing majority remains extremely strong. Then, the price seems to transition from pattern D to the critical zone C ending in a crash. The subsequent behavior is reminiscent of pattern A in which oscillations are progressively dampened towards an equilibrium point.

Figure 13 shows a schematic synthesis of the regimes after the bubble phase (Phase 1 and 2) which is basically similar to the Ahold simulation.

**Phase 3 (time period \( 0 \leq t \leq 12 \)):** \( \kappa = 3.5 \) and \( \beta = -5 \). This is pattern D, where after the bubble there are strong herding pressures and such strong dissident inclinations that \( \beta \) is beyond the transition range, and a stable limit cycle results. However, as suggested when we discussed Pattern D above, if the disapproval starts to relax somewhat so that \( \beta \) falls back into the transition range, the stable wave quickly destabilizes and collapses. Although due to very strong herding and dissidence pressures a stable limit cycle is produced, the danger lures that, with the slightest relaxing of the pressures, the system is likely to fall back into the destabilization regime causing it to collapse.

**Phase 4 (time period \( 12 < t \leq 15 \)):** \( \kappa = 3.5, \beta = -4.1 \) (Pattern C). With \( \beta \) now in the transition range, the limit cycle destabilizes and, as can be seen from Fig. 13, the \( y \) trajectory starts to collapse causing a stock price crash.
Fig. 12. Aegon stock price as a function of time. The vertical axis uses a logarithmic scale while the horizontal axis uses a linear scale. The power-law fit to the data from 1990/01/02 to \( t_{\text{max}} = 1998/07/21 \) gives \( t_c = 2015/06/15, m = -5.40, A = 0.51, \) and \( B = 9.48 \times 10^{20} \) with a r.m.s. of the fit residuals equal to \( \chi = 0.078 \). The sinusoidal fit to the data from 1998/07/21 to 2001/03/20 gives \( \omega = 0.0204, \phi = 3.73, A = 3.69, B = -0.0959 \) with a r.m.s. of the fit residuals equal to \( \chi = 0.0755 \).

Fig. 13. Simulation of the stock price pattern of Aegon after the bubble with \( \mu = 2, \gamma = 1, \) and \( a = 1. \)

This would continue indefinitely, so like in the Ahold case, when investors become dizzy from falling so fast, the herding pressures will evaporate, and the balance between buyers and sellers will return.

**Phase 5 (time period \( t > 15 \)):** \( \kappa = 1.5, \beta = -4.1 \) (Pattern A). This is the return to the \((0, 0)\) stability zone, as exemplified in Figure 6.
6 Concluding remarks

We have documented the concept of super-exponential growth characterizing corporate sales, earnings and stock market prices of companies such as Ahold and Aegon which developed a furious race for growth via aggressive acquisitions as well as creative accounting. These behaviors allowed these companies to create extraordinary positive or bullish sentiments among investors, and to manipulate these sentiments for the sake of growth feeding positively on itself. It is well-known that high stock prices help a company grow and reciprocally the company’s growth fuels the stock prices: a high stock market price makes it easier to raise money, to acquire other companies, to attract high level collaborators and employees and so on. As pointed out by Krugman (2002), a high stock price facilitates aggressive plans for growth as well as accounting tricks, making the growth similar to a Ponzi scheme. The innovation of our paper has been to propose a simple quantification of these unsustainable growth phases often reported at a qualitative or descriptive level.

We have adapted Weidlich’s theory of opinion formation to describe the formation of buy or sell decisions among investors, based on a competition between the mechanisms of herding and of personal opinion opposing the herd. A study of this model has provided a classification of four different regimes/patterns for the price, which have then be used to explain the price evolution of Ahold as well as Aegon, from the bubble phase to the final crash. This study complements previous studies of bubbles holding that a crash is the most probable close to the end of a bubble by introducing the concept of a “critical zone” corresponding to a transition from the super-exponential bubble to the crash. The critical zone is characterized by (i) herding and (ii) strong shift of sentiments (or in other words, instabilities in the shift of sentiments). The critical zone is characterized by a strong sensitivity on these herding and inclination parameters and describes the maturation of an instability, that is, the coming crash. These ingredients have allowed us to explain the trajectories of Ahold and Aegon by suitable choices of the herding and inclination parameters, according to scenarios and a succession of phases which are essentially identical. While we have focused on just these two cases in order to get sufficient depth, we conjecture that more studies on other cases will allow to strengthen and refine our model.

Concerning the potential for prediction, the important point of our analysis is to focus on medium-term evolutionary patterns which leads to the possibility of recognizing years in advance that a critical zone is unfolding following a super-exponential bubble phase, which foreshadows a coming crash. We have been able to show from the power law fits of sales and earnings of Ahold that about a year in advance a $t_c$, which indicates a change of regime, was predicted, i.e. a crash in 2002. About five years earlier, in 1997, while sales...
and earnings were still in the super-exponential growth regime, the stock price evolutionary pattern entered a critical zone characterized by a destabilization regime that over the years became increasingly apparent and was bound to end in a crash as the remarkable Pattern C of the critical zone has shown. Our findings that the combination of these patterns, continuing super-exponential growth of sales and earnings of a company that resembles an unguided missile, and a stock market which exhibits the collective intelligence and transparency to indicate, at least as early as 1998, that the missile is out of control and moving through a dangerous destabilization zone entails an important lesson for top managers and investors alike.

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