THE $M_*$–$\sigma_*$ RELATION DERIVED FROM SPHERE OF INFLUENCE ARGUMENTS

D. Batcheldor

Department of Physics and Space Sciences, Florida Institute of Technology, 150 West University Blvd., Melbourne, FL 32901, USA; dbatcheldor@fit.edu

Received 2009 November 9; accepted 2010 February 5; published 2010 February 22

ABSTRACT

The observed relation between supermassive black hole (SMBH) mass ($M_*$) and bulge stellar velocity dispersion ($\sigma_*$) is described by $\log M_* = \alpha + \beta \log (\sigma_*/200 \text{ km s}^{-1})$. As this relation has important implications for models of galaxy and SMBH formation and evolution, there continues to be great interest in adding to the $M_*$ catalog. The “sphere of influence” ($r_i$) argument uses spatial resolution to exclude some $M_*$ estimates and pre-select additional galaxies for further SMBH studies. This Letter quantifies the effects of applying the $r_i$ argument to a population of galaxies and SMBHs that do not follow the $M_*$–$\sigma_*$ relation. All galaxies with known values of $\sigma_*$, closer than 100 Mpc, are given a random $M_*$ and selected when $r_i$ is spatially resolved. These random SMBHs produce a $M_*$–$\sigma_*$ relation of $\alpha = 8.3 \pm 0.2$, $\beta = 4.0 \pm 0.3$, consistent with observed values. Consequently, future proposed $M_*$ estimates should not be justified solely on the basis of resolving $r_i$. This Letter shows that the observed $M_*$–$\sigma_*$ relation may simply be a result of available spatial resolution. However, it also implies that the observed $M_*$–$\sigma_*$ relation defines an upper limit. This potentially provides valuable new insight into the processes of galaxy and SMBH formation and evolution.

Key words: black hole physics – galaxies: bulges – galaxies: fundamental parameters

1. INTRODUCTION

There has been a growing database of direct supermassive black hole (SMBH) mass ($M_*$) estimates from the centers of nearby galactic bulges (e.g., Graham 2008b). While the limits of our current abilities to significantly expand this database may have been reached (Batcheldor & Koekemoer 2009), the last decade has seen a wealth of $M_*$ estimates that have increased the SMBH catalog from 13 or 26 (Ferrarese & Merritt 2000; Gebhardt et al. 2000) to ~70 (Graham 2008b; Hu 2008; Gültağ et al. 2009). An intense interest in populating the SMBH database was sparked by observed correlations between $M_*$ and fundamental properties of their host bulges (e.g., Kormendy & Richstone 1995; Magorrian et al. 1998; Ferrarese & Merritt 2000; Gebhardt et al. 2000; Graham et al. 2001; Ferrarese 2001; Marconi & Hunt 2003; Baes et al. 2003; Häring & Rix 2004; Pizzella et al. 2005). These $M_*$ scaling relations have generated numerous theoretical investigations (e.g., Ciotti & van Albada 2001; Adams et al. 2003; Cattaneo et al. 2005; Robertson et al. 2006) and have possibly added valuable limits to evolutionary models (e.g., Heckman et al. 2004; Wyithe & Loeb 2005; Treu et al. 2007; Ciotti 2009).

The degree to which an SMBH’s sphere of influence, $r_i$, is resolved has been used as a quality measure for $M_*$ estimates (Ferrarese 2002; Marconi & Hunt 2003; Valluri et al. 2004). The only method available to a priori determine if a $M_*$ estimate can be made is to assume $r_i = GM_*/\sigma_*^2$ (Peebles 1972). However, to calculate $r_i$ in galaxies with known $\sigma_*$, $M_*$ is estimated using the $M_*$–$\sigma_*$ relation given by $\log M_* = \alpha + \beta \log (\sigma_*/200 \text{ km s}^{-1})$, where $\alpha = 8.1–8.2$ and $\beta = 3.7–4.9$. The observed scatter, $\epsilon$, is 0.4 dex (Novak et al. 2006; Graham & Li 2009). Following this, Figure 1 demonstrates where $r_i$ will be resolved, given a spatial resolution of $\mathcal{R} = 0’.1$. A value of $\mathcal{R} = 0’.1$ is used here as that is the typical FWHM of the Hubble Space Telescope (HST) point-spread function (PSF). To date, HST has been responsible for most $M_*$ estimates.

Ferrarese & Ford (2005, FF05) discussed the limited abilities of HST to resolve $r_i$, and Gültağ et al. (2009, G09) found the $M_*$–$\sigma_*$ relation to be biased when applying the $r_i$ argument. The influence of $r_i$ cuts on the $M_*$–$\sigma_*$ relation is continued in this Letter with two significant advances. First, a sample of all galaxies with a known $\sigma_*$ (<100 Mpc) is used. It is important to only use these galaxies as there are no $\sigma_*$ = 400 km s$^{-1}$ galaxies at 1 Mpc, for example. Second, random $M_*$ estimates are applied to each galaxy, i.e., no galaxy is assumed to intrinsically fall on the $M_*$–$\sigma_*$ relation. This ensures no pre-selection of galaxies that lie on the $M_*$–$\sigma_*$ relation. Throughout, a distinction is made between the observed $M_*$–$\sigma_*$ relation (published values) and the observable $M_*$–$\sigma_*$ relation (the relation that can be fitted using the data simulated here).

2. METHODS AND RESULTS

In short, these simulations take a galaxy with a known $\sigma_*$ and distance, assign a $M_*$, then calculate $r_i$. The $r_i$ argument is then applied; if $r_i$ is unresolved the galaxy is removed from the sample (the low-mass cut). If the assigned $M_*$ generates a galaxy that lies above the observed $M_*$–$\sigma_*$ relation, it is also removed from the sample (the high-mass cut). The observable $M_*$–$\sigma_*$ relation is then fit to the remaining galaxies using a Levenberg–Marquardt least-squares add-on to IDL.

A SQL search of the Hyperleda catalog was performed (Paturel et al. 2003). All galaxies with a kinematic distance modulus less than 35.0 (100 Mpc, assuming $H_0 = 70 \text{ km s}^{-1}\text{ Mpc}^{-1}$) were found. Of the 49,740 galaxies returned, 2518 have published values of $\sigma_*$. The incompleteness of this sample, due to difficulties in measuring $\sigma_*$ in faint galaxies, is noted. Values of $r_i$ (in arcsec) were then calculated for each galaxy by assigning a $M_*$. In the first cases, the low-mass cut was made by applying $\mathcal{R} = 0’.1$ to represent the HST PSF. However, several values of $\mathcal{R}$ are applied later. The high-mass cuts were then made by considering the observed distribution of galaxies in the $M_*$–$\sigma_*$ plane; there is a clear absence of over-massive SMBHs, i.e., there are no $\sigma_* = 50 \text{ km s}^{-1}$ dwarf spheroids in the Local Group with $M_* \sim 10^6M_\odot$. Such a nearby over-massive SMBH population would have been detected, and therefore it is assumed that these objects do not exist. Galaxies selected for the high-mass cuts were determined by assigning a scatter ($\epsilon = 0.4 \text{ dex}$).
In this case, the low-mass cut was made using \( M \) and upper limits to the \( \alpha \) and \( \sigma \) of the observed region of the \( M_\bullet - \sigma \) plane, which is resolved if the galaxy with the given \( \sigma \) hosts the given SMBH.

Each galaxy was first assigned 90 separate values of \( \alpha \) as the previously used value of \( \alpha \) from \( 7.8 \) to \( 8.7 \) in \( 0.1 \) steps and \( \beta \) from \( 5.0 \) to \( 5.8 \) in \( 0.1 \) steps. In this case, the high-mass cut was made using the \( (M_\bullet - \sigma_\star)_h \) relation of G09. All present estimates of \( M_\bullet \) and \( \sigma \) are shown as filled black circles, with uncertainties. The fit to the observable galaxies (red circles and line) is \( \alpha = 8.3, \beta = 4.1 \).

| \( \% \) (Zero Point) | \( \beta \) (Slope) | \( \epsilon \) (Scatter) |
|---------------------|-------------------|---------------------|
| Variable Upper Cutoffs         |                  |                    |
| 0'05                | 8.25 ± 0.17       | 3.80 ± 0.33        | 0.41 dex          |
| 0'10                | 8.25 ± 0.21       | 3.90 ± 0.34        | 0.27 dex          |
| 0'15                | 8.25 ± 0.23       | 3.93 ± 0.34        | 0.22 dex          |
| 0'20                | 8.25 ± 0.27       | 4.01 ± 0.38        | 0.18 dex          |
| Fixed Upper Cutoffs         |                  |                    |
| 0'05                | 8.44 ± 0.03       | 3.93 ± 0.23        | 1.82 dex          |
| 0'10                | 8.57 ± 0.02       | 4.02 ± 0.21        | 1.22 dex          |
| 0'15                | 8.64 ± 0.02       | 4.03 ± 0.25        | 0.97 dex          |
| 0'20                | 8.68 ± 0.03       | 4.07 ± 0.32        | 0.85 dex          |

Notes: Average results from applying variable and fixed \( M_\bullet - \sigma \) upper limits to 50 random \( M_\bullet \) samples. A fixed upper limit of \( \alpha \) and \( \beta \) was used based on the upper-limit fit in Figure 4. If there was no intrinsic \( M_\bullet - \sigma \) relation, these are the values of the \( M_\bullet - \sigma \) relation that would be observed using the different spatial resolutions listed.

The similarity between the \( M_\bullet - \sigma \) distribution of observable random mass SMBHs and observed SMBHs masses is striking. However, as this result derives from a single random sampling of \( M_\bullet \), it represents a single possible observable \( M_\bullet - \sigma \) relation. If galaxies intrinsically have random \( M_\bullet \) values, then the range of observable SMBHs could have a distribution given by the distribution in Figure 2. Therefore, 50 separate random \( M_\bullet \) samplings were then made for each galaxy, and in each case a fit to the observable \( M_\bullet - \sigma \) relation was performed. In addition, as the previously used value of \( \% = 0'1 \) only applies to \( HST \) observations, the analysis was repeated for a range of \( r_i \) lower cutoffs. Finally, as the high-mass cuts may not actually be defined by the G09 and FF05 fits, values of \( \alpha \) from \( 7.8 \) to \( 8.7 \) (in 0.1 steps) and \( \beta \) from \( 5.0 \) to \( 5.8 \) (in 0.2 steps) were used to create an addition 100 individual \( (M_\bullet - \sigma_\star)_h \) cutoffs. The mean values of \( \alpha \), \( \beta \), and \( \epsilon \) derived by applying these different \( \% \) criteria, and from using these variable upper limits, are presented in Table 1.

Table 1 has several notable features. First, as \( \% \) rises \( \epsilon \) falls. This is expected because the range of observable SMBHs...
decreases with larger values of $\tilde{\epsilon}$. Second, the slope ($\beta$) remains consistent. Finally, in the case of the variable upper limits, the zero point ($\alpha$) remains constant and is consistent with the mid-value of the ($M_\ast$-$\sigma_\ast$) limits imposed on the sample. To test whether these values of $\alpha$ are a consequence of the high-mass cut conditions, the analysis was repeated using $\alpha_u$ limits of 7.0 and 8.8 (in 0.2 steps). Applying a $\tilde{\epsilon} = 0.1$ low-mass cut, an observable relation with $\alpha = 8.0 \pm 0.5$ was found. Again, the values of $\alpha$ are consistent with the mid-value of the imposed limits. However, an estimate of the observed ($M_\ast$-$\sigma_\ast$) relation can be made. The observed SMBH sample in Figure 3 was used to define an upper limit by fitting all observed SMBHs that fall above the FF05 and G09 relations, and above $\epsilon = 0.4$. The fit to these over-massive SMBHs produce an upper limit of $\alpha_u = 8.7$, $\beta_u = 5.0$ (Figure 4). This ($M_\ast$-$\sigma_\ast$) fit was then applied to 50 random values of $M_\ast$ in each galaxy, and the observable $M_\ast$-$\sigma_\ast$ relations were then fitted in each case. The average of these fits are also presented in Table 1. In this case, the fitted values of $\alpha$ and $\beta$ are consistent with the variable upper limits, but the values of $\epsilon$ are significantly larger. This is expected, as these fits were applied to data that had the maximum range in the $M_\ast$-$\sigma_\ast$ plane due to the high observed values of $\alpha_u$ and $\beta_u$.

3. DISCUSSIONS

In summary, as a result of applying the $r_i$ argument, a $M_\ast$-$\sigma_\ast$ relation consistent with observed values ($\alpha \approx 8$, $\beta \approx 4$, $\epsilon \approx 0.3$ dex) can be fitted to a sample of galaxies that contain random mass SMBHs, and as a consequence do not follow a $M_\ast$-$\sigma_\ast$ relation. The $r_i$ argument removes low-mass SMBHs where $r_i$ would not be resolved, and high-mass SMBHs where $r_i$ should have been resolved if such a population were present. Therefore, for scaling relations to be of value in constraining galaxy evolution models, $M_\ast$ estimates must not be solely proposed on the basis of critically resolving values of $r_i$ derived from $\sigma$.

It is unlikely that an intrinsic $M_\ast$-$\sigma_\ast$ relation has been observed as a result of critically sampling $r_i$ in the $M_\ast$-$\sigma_\ast$ plane, and it will be some time until we are able to sample a significant area rightward of the observed $M_\ast$-$\sigma_\ast$ relation (e.g., FF05; Batcheldor & Koekemoer 2009). Therefore, it is clear that caution must be used in the application of the observed $M_\ast$-$\sigma_\ast$ relation until this issue can be resolved.

The possibility of there being no tight intrinsic $M_\ast$-$\sigma_\ast$ relation, and that the distribution of observed galaxies in the $M_\ast$-$\sigma_\ast$ plane determine an upper limit only, is now discussed. As the same $M_\ast$ estimates are used to define the other $M_\ast$ scaling relations, this also implies that all observed scaling relations are upper limits. There are a number of questions arising from this possibility, beginning with the most fundamental. Why do observed SMBHs fall on the $M_\ast$-$\sigma_\ast$ relation? There are several SMBHs with such high quality $M_\ast$ estimates (Sgr A*, NGC 4258, M87), that there clearly is a relation between $M_\ast$ and $\sigma_\ast$ in some galaxies. However, the $M_\ast$-$\sigma_\ast$ plane has been increasingly populated with less certain $M_\ast$ estimates that have potentially been included on the assumption that $r_i$ is spatially resolved, i.e., that they follow a previously estimated $M_\ast$-$\sigma_\ast$ relation defined by higher quality $M_\ast$ estimates. In these cases, as the simulations presented here show, an observed $M_\ast$-$\sigma_\ast$ relation will arise simply as a result of the $r_i$ selection effect, even if $M_\ast$ is randomly distributed within galaxies.

Does a population of under-massive SMBHs exist, and have they been detected? If the $M_\ast$-$\sigma_\ast$ relation is an upper limit, then there should be galaxies that host low-mass SMBHs, as suggested by simulations (Vittorini et al. 2005; Volonteri 2007). Indeed, both the simulations presented here, and the observations plotted in Figure 3, show that under-massive SMBHs can be, and have been, detected. Some notable cases are that of NGC 4435 (Coccato et al. 2006), a sample of barred galaxies (Graham 2008a), and narrow-line Seyfert 1 galaxies (Mathur & Grupe 2005). In fact, the evidence to suggest the presence of under-massive SMBHs is not matched by any evidence for a significant population of over-massive SMBHs leftward of the observed $M_\ast$-$\sigma_\ast$ relation.

It is important to note some intricacies with the two dominant techniques for measuring $M_\ast$ (stellar and gas dynamics). In the case of stellar dynamics, there are systematics to the models that may allow a large range of $M_\ast$ in a given bulge (Valluri et al. 2004). In the case of gas dynamics, it is unclear what the inclination of the nuclear gas disk may be (e.g., Marconi et al. 2003). Both methods allow the potential for many of the current $M_\ast$ estimates to in fact be upper limits, i.e., the true $M_\ast$-$\sigma_\ast$ plane may have a large distribution of under-massive SMBHs. Therefore, under-massive SMBHs may have been observed, their mass overestimated, and their impact overlooked.

All SMBH models allow stringent upper limits to be placed on $M_\ast$ (e.g., Sarzi et al. 2002; Beifiori et al. 2009). However, these $M_\ast$ upper limits will also be dependent on the available spatial resolution. Any kinematical data will have two velocity points spatially separated on a scale of $\tilde{\epsilon}$. An upper limit to $M_\ast$ is estimated by including an increasing dark mass until the derived model becomes inconsistent with the data, i.e., a higher upper limit to $M_\ast$ will be estimated using a lower $\tilde{\epsilon}$. There are still important constraints that can be added to the $M_\ast$-$\sigma_\ast$ plane from estimates of $M_\ast$ upper limits, however, as upper limits that fall below the observed $M_\ast$-$\sigma_\ast$ relation provide the same evidence for under-massive SMBHs as would a tightly constrained low-mass $M_\ast$. At present, most $M_\ast$ upper limits are based on data derived from gas dynamics. Consequently, these limits generally fall above the observed $M_\ast$-$\sigma_\ast$ relation due to unknown amounts of line broadening from non-gravitational processes, and uncertainty in the inclination of the nuclear gas disks.

If the $M_\ast$-$\sigma_\ast$ relation represents an upper limit in the $M_\ast$-$\sigma_\ast$ plane, then what is this limit? In Section 2, an upper limit of $\alpha_u = 8.7$, $\beta_u = 5.0$ was found based on the distribution of...
observed SMBHs leftward of the observed $M_\bullet-\sigma_*$ relation. This
is likely a good approximation to the $(M_\bullet-\sigma_*)_0$ limit as there
are no reports of a steeper relation. However, this estimate does
not include the potential over-massive SMBHs from the upper
limits calculated by Beifiori et al. (2009). Including these limits
to the upper limit sample gives values of $\alpha_u = 8.8$, $\beta_u = 3.9$
to $(M_\bullet-\sigma_*)_u$. As expected, due to the addition of SMBH limits
at lower $M_\bullet$, this $(M_\bullet-\sigma_*)_u$ limit is more shallow with a higher
zero point. Including these limits at the lower $M_\bullet$ end of the
$M_\bullet-\sigma_*$ plane addresses, in part, a limitation of the sample
used here. As already noted, the $\sigma_*$ catalog used here is likely
incomplete due to the difficulty of measuring $\sigma_*$ in faint galaxies
at greater distances. In addition, the $\sigma_*$ catalog likely contains
inhomogeneous measurements that may not translate from bulge
to bulge.

What are the consequences to galaxy evolution models if
there is only a $(M_\bullet-\sigma_*)_u$ relation? First, galaxies will no longer
be required to obey the $M_\bullet-\sigma_*$ relation, and could host a SMBH
with any $M_\bullet$ below $(M_\bullet-\sigma_*)_u$. Models that include feedback
from the SMBH to the galaxy will then need to be carefully
reconsidered. While the SMBH will undoubtedly have some
influence on a portion of the host galaxy, it would not need to
affect large-scale properties; evolution of the SMBH would be
a result of host galaxy evolution. An upper limit in the $M_\bullet-\sigma_*$
plane would also represent the pinnacle of SMBH evolution as a
function of $\sigma_*$, in which galaxies evolve up to the $(M_\bullet-\sigma_*)_u$ limit.
A signature of such a scenario could be a cosmic variation in $\epsilon$
(the scatter would increase with redshift) and an observed $M_\bullet-\sigma_*$
relation that does not exceed $(M_\bullet-\sigma_*)_u$. If the distribution of $M_\bullet$
is random within bulges, then when compared with the local
observed $M_\bullet-\sigma_*$ relation, $M_\bullet$ estimates from higher redshift
could fall to the left or the right. Treu et al. (2007) find
a population of $z = 0.36$ Seyfert 1 galaxies that lie above
the local observed $M_\bullet-\sigma_*$ relation by $\Delta \log \sigma = 0.13$, $\Delta \log
M_\bullet = 0.54$, but this population still lies below the $(M_\bullet-\sigma_*)_u$ limit
estimated here. Finally, if SMBHs can reside anywhere below
the $(M_\bullet-\sigma_*)_u$ limit, then the local black hole mass function may
have been overestimated. This would relax the observation that
merging is not important and that SMBH growth is dominated
by accretion (e.g., Marconi et al. 2004). This potentially allows
anti-hierarchical SMBH growth to no longer present problems
for hierarchical galaxy formation models.

The use of the HyperLeda database (http://leda.univ-lyon1.fr)
is acknowledged. D.B. thanks Alessandro Marconi, David
Merritt, and Andy Robinson for useful discussions. Support for
this work was provided by proposal number HST-AR-10935.01
awarded by NASA through a grant from the Space Telescope
Science Institute, which is operated by the Association of
Universities for Research in Astronomy, Inc., under NASA
contract NAS5-26555.

REFERENCES

Adams, F. C., Graff, D. S., Mbonye, M., & Richstone, D. O. 2003, ApJ, 591, 125
Baes, M., Bulylev, P., Hau, K. G. T., & Dejonghe, H. 2003, MNRAS, 341, L44
Batchelor, D., & Koekemoer, A. M. 2009, PASP, 121, 1245
Beifiori, A., Sarzi, M., Corsini, E. M., Dalla Bontà, E., Pizzella, A., Coccato,
L., & Bertola, F. 2009, ApJ, 692, 856
Cattaneo, A., Blaizot, J., Devriendt, J., & Guiderdoni, B. 2005, MNRAS, 364, 407
Cioffi, L. 2009, Rivista del Nuovo Cimento, 32, 1
Cioffi, L., & van Albada, T. S. 2001, ApJ, 552, L13
Coccato, L., Sarzi, M., Pizzella, A., Corsini, E. M., Dalla Bontà, E., & Bertola,
F. 2006, MNRAS, 366, 1050
Ferrarese, L. 2002, ApJ, 578, 90
Ferrarese, L., & Ford, H. 2005, Space Sci. Rev., 116, 523
Ferrarese, L., & Merritt, D. 2000, ApJ, 539, L9
Gebhardt, K., et al. 2000, ApJ, 539, L13
Graham, A. W. 2008a, ApJ, 680, 143
Graham, A. W. 2008b, PASA, 25, 167
Graham, A. W., Erwin, P., Caon, N., & Trujillo, I. 2001, ApJ, 563, L11
Graham, A. W., & Li, I. 2009, ApJ, 698, 812
Gültekin, K., et al. 2009, ApJ, 698, 198
Haring, N., & Rix, H.-W. 2004, ApJ, 604, L89
Heckman, T. M., Kauffmann, G., Brinchmann, J., Charlot, S., Tremonti, C., &
White, S. D. M. 2004, ApJ, 613, 109
Hu, J. 2008, MNRAS, 386, 2242
Kormendy, J., & Richstone, D. 1995, ARA&A, 33, 581
Magorrian, J., et al. 1998, AJ, 115, 2285
Marconi, A., & Hunt, L. K. 2003, ApJ, 589, L21
Marconi, A., Risaliti, G., Gilli, R., Hunt, L. K., Maiolino, R., & Salvati, M.
2004, MNRAS, 351, 169
Marconi, A., et al. 2003, ApJ, 586, 868
Mathur, S., & Grupe, D. 2005, ApJ, 633, 688
Novak, G. S., Faber, S. M., & Dekel, A. 2006, ApJ, 637, 96
Paturel, G., Petit, C., Prugniel, P., Theureau, G., Rousseau, J., Brouty, M.,
Dubois, P., & Cambresy, L. 2003, A&A, 412, 45
Peebles, P. J. E. 1972, ApJ, 178, 371
Pizzella, A., Corsini, E. M., Dalla Bontà, E., Sarzi, M., Coccato, L., & Bertola,
F. 2005, ApJ, 631, 785
Robertson, B., Hernquist, L., Cox, T. J., Di Matteo, T., Hopkins, P. F., Martini,
P., & Springel, V. 2006, ApJ, 641, 90
Sarzi, M., et al. 2002, ApJ, 567, 237
Treu, T., Woo, J.-H., Malkan, M. A., & Blandford, R. D. 2007, ApJ, 667, 117
Valluri, M., Merritt, D., & Emsellem, E. 2004, ApJ, 602, 66
Vittorini, V., Shankar, F., & Cavaliere, A. 2005, MNRAS, 363, 1376
Volonteri, M. 2007, ApJ, 663, L5
Wyithe, J. S. B., & Loeb, A. 2005, ApJ, 634, 910