Bulk-Brane Interaction and Holographic Dark Energy

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Abstract

In this paper we consider the bulk-brane interaction to obtain the equation of state for the holographic energy density in non-flat universe enclosed by the event horizon measured from the sphere of horizon named $L$. We assumes that the cold dark matter energy density on the brane is conserved, but the holographic dark energy density on the brane is not conserved due to brane-bulk energy exchange. Our calculation show, taking $\Omega_\Lambda = 0.73$ for the present time, the lower bound of $w^{\text{eff}}_\Lambda$ is $-0.9$. This implies that one can not generate phantom-like equation of state from an interacting holographic dark energy model in non-flat universe.

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1 Introduction

One of the most important problems of cosmology, is the problem of so-called dark energy (DE). The type Ia supernova observations suggests that the universe is dominated by dark energy with negative pressure which provides the dynamical mechanism of the accelerating expansion of the universe [1, 2, 3]. The strength of this acceleration is presently matter of debate, mainly because it depends on the theoretical model implied when interpreting the data. Most of these models are based on dynamics of a scalar or multi-scalar fields (e.g. quintessence [4, 5] and quintom model of dark energy, respectively).

An approach to the problem of DE arises from holographic Principle that states that the number of degrees of freedom related directly to entropy scales with the enclosing area of the system. It was shown by 'tHooft and Susskind [6] that effective local quantum field theories greatly overcount degrees of freedom because the entropy scales extensively for an effective quantum field theory in a box of size $L$ with UV cut-off $\Lambda$. As pointed out by [7], attempting to solve this problem, Cohen et al. showed [8] that in quantum field theory, short distance cut-off $\Lambda$ is related to long distance cut-off $L$ due to the limit set by forming a black hole. In other words the total energy of the system with size $L$ should not exceed the mass of the same size black hole i.e. $L^3 \rho_\Lambda \leq L M_p^2$ where $\rho_\Lambda$ is the quantum zero-point energy density caused by UV cutoff $\Lambda$ and $M_p$ denotes Planck mass ($M_p^2 = 1/8\pi G$). The largest $L$ is required to saturate this inequality. Then its holographic energy density is given by $\rho_\Lambda = 3 c^2 M_p^2 / 8\pi L^2$ in which $c$ is free dimensionless parameter and coefficient 3 is for convenience.

As an application of Holographic principle in cosmology, it was studied by [9] that consequence of excluding those degrees of freedom of the system which will never be observed by that effective field theory gives rise to IR cut-off $L$ at the future event horizon. Thus in a universe dominated by DE, the future event horizon will tend to constant of the order $H_0^{-1}$, i.e. the present Hubble radius. The consequences of such a cut-off could be visible at the largest observable scales and particulary in the low CMB multipoles where we deal with discrete wave numbers. Considering the power spectrum in finite universe as a consequence of holographic constraint, with different boundary conditions, and fitting it with LSS, CMB and supernova data, a cosmic duality between dark energy equation of state and power spectrum is obtained that can describe the low $l$ features extremely well.

Based on cosmological state of holographic principle, proposed by Fischler and Susskind [10], the Holographic Model of Dark Energy (HDE) has been proposed and studied widely in the literature [11, 12]. In [13] using the type Ia supernova data, the model of HDE is constrained once when $c$ is unity and another time when $c$ is taken as free parameter. It is concluded that the HDE is consistent with recent observations, but future observations are needed to constrain this model more precisely. In another paper [14], the anthropic principle for HDE is discussed. It is found that, provided that the amplitude of fluctuation are variable the anthropic consideration favors the HDE over the cosmological constant.

In HDE, in order to determine the proper and well-behaved system’s IR cut-off, there are some difficulties that must be studied carefully to get results adapted with experiments that claim our universe has accelerated expansion. For instance, in the model proposed by [11], it is discussed that considering particle horizon, as the IR cut-off, the HDE density reads to be

$$\rho_\Lambda \propto a^{-2(1 + \frac{c}{3})},$$

that implies $w > -1/3$ which does not lead to accelerated universe. Also it is shown in
that for the case of closed universe, it violates the holographic bound. The problem of taking apparent horizon (Hubble horizon) - the outermost surface defined by the null rays which instantaneously are not expanding, $R_A = 1/H$ - as the IR cut-off in the flat universe, was discussed by Hsu [16]. According to Hsu’s argument, employing Friedman equation $\rho = 3M^2_P H^2$ where $\rho$ is the total energy density and taking $L = H^{-1}$ we will find $\rho_m = 3(1-c^2)M^2_P H^2$. Thus either $\rho_m$ and $\rho_{\Lambda}$ behave as $H^2$. So the DE results pressureless, since $\rho_{\Lambda}$ scales as like as matter energy density $\rho_m$ with the scale factor $a$ as $a^{-3}$. Also, taking apparent horizon as the IR cut-off may result the constant parameter of state $w$, which is in contradiction with recent observations implying variable $w$ [17]. On the other hand taking the event horizon, as the IR cut-off, gives the results compatible with observations for flat universe.

It is fair to claim that simplicity and reasonability of HDE provides more reliable frame to investigate the problem of DE rather than other models proposed in the literature [18, 19, 20]. For instance the coincidence or ”why now” problem is easily solved in some models of HDE based on this fundamental assumption that matter and holographic dark energy do not conserve separately, but the matter energy density decays into the holographic energy density [21]. Some experimental data has implied that our universe is not a perfectly flat universe and recent papers have favored the universe with spatial curvature [22]. As a matter of fact, we want to remark that although it is believed that our universe is flat, a contribution to the Friedmann equation from spatial curvature is still possible if the number of e-foldings is not very large [23]. Defining the appropriate distance, for the case of non-flat universe has another story. Some aspects of the problem has been discussed in [23, 24]. In this case, the event horizon can not be considered as the system’s IR cut-off, because for instance, when the dark energy is dominated and $c = 1$, where $c$ is a positive constant, $\Omega_{\Lambda} = 1 + \Omega_k$, we find $\dot{R}_h < 0$, while we know that in this situation we must be in de Sitter space with constant EoS. To solve this problem, another distance is considered- radial size of the event horizon measured on the sphere of the horizon, denoted by $L$- and the evolution of holographic model of dark energy in non-flat universe is investigated.

Motivated by string/M theory, the AdS/CFT correspondence, and the hierarchy problem of particle physics, braneworld models were studied actively in recent years [25]-[28]. In this models, our universe is realized as a boundary of a higher dimensional spacetime. In this paper we turn our attention to the interaction between the bulk and the brane, which is non-trivial aspect of brane world theories. In particular, we will discuss the flow of energy onto or away from the brane–universe. Then, using the holographic model of dark energy in non-flat universe, we obtain equation of state for interacting holographic dark energy density in a universe enveloped by $L$ as the system’s IR cut-off. At first we review the formalism of bulk-brane energy exchange, then we applies this material to a brane-world cosmology containing cold dark matter (CDM), DE, and spatial curvature. The DE is assumed to be of HDE variety. We assumes that the CDM energy density on the brane is conserved, but the the holographic dark energy density on the brane is not conserved due to brane-bulk energy exchange.
2 Bulk-Brane Energy Exchange

We consider the following gravitational brane-bulk action

\[ S = \int d^5x \sqrt{-G} \left( \frac{R_5}{2\kappa_5^2} - \Lambda_5 + L_B^m \right) + \int d^4x \sqrt{-g} \left( -\sigma + L_b^m \right), \]

where \( R_5 \) is the curvature scalar of the five-dimensional metric, \( \Lambda_5 \) is the bulk cosmological constant and \( \sigma \) is the brane tension, \( L_B^m \) and \( L_b^m \) are the matter Lagrangian in the bulk and on the brane respectively. We consider an ansatz for the metric of the form

\[ ds^2 = -n^2(t, y) dt^2 + a^2(t, y) \gamma_{ij} dx^i dx^j + b^2(t, y) dy^2, \]

where \( \gamma_{ij} \) is the metric for the maximally symmetric three-dimensional space. The non-zero components of Einstein tensor can be written as \[29, 30]\n
\[ G_{00} = 3 \left[ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{n^2}{b^2} \left( \frac{a''}{a} + \frac{a'}{a} \left( \frac{a'}{a} - \frac{b'}{b} \right) \right) + k \frac{n^2}{b^2} \right] \]

\[ G_{ij} = \frac{a^2}{b^2} \gamma_{ij} \left[ \frac{a'}{a} \left( \frac{a'}{a} + 2 \frac{n'}{n} \right) - \frac{b'}{b} \left( \frac{n'}{n} + 2 \frac{a'}{a} \right) + 2 \frac{a''}{a} + \frac{n''}{n} \right] + \frac{a^2}{n^2} \gamma_{ij} \left[ \frac{\dot{a}}{a} \left( -\frac{\dot{a}}{a} + 2 \frac{\dot{n}}{n} \right) - 2 \frac{\dot{b}}{b} \left( -2 \frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) - \frac{\dot{b}}{b} \right] - k \gamma_{ij}, \]

\[ G_{05} = 3 \left( \frac{n' \dot{a}}{na} + \frac{a' \dot{b}}{ab} - \ddot{\dot{a}} \right), \]

\[ G_{55} = 3 \left[ \frac{a'}{a} \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{b^2}{n^2} \left( \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \ddot{a} \right) - k \frac{b^2}{a^2} \right], \]

where \( k \) denotes the curvature of space \( k=0,1,-1 \) for flat, closed and open universe respectively. The three dimensional brane is assumed at \( y = 0 \). The Einstein equations are \( G_{\mu\nu} = \kappa_5^2 T_{\mu\nu} \), where the stress-energy momentum tensor has bulk and brane components and can be written as

\[ T^\mu_\nu = T^\mu_\nu|_{\sigma, b} + T^\mu_\nu|m, b + T^\mu_\nu|_{\Lambda, B} + T^\mu_\nu|m, B, \]

where

\[ T^\mu_\nu|_{\sigma, b} = \frac{\delta(y)}{b} \text{diag}(-\sigma, -\sigma, -\sigma, -\sigma, 0), \]

\[ T^\mu_\nu|_{\Lambda, B} = \text{diag}(-\Lambda_5, -\Lambda_5, -\Lambda_5, -\Lambda_5, -\Lambda_5), \]

\[ T^\mu_\nu|m, b = \frac{\delta(y)}{b} \text{diag}(-\rho, p, p, 0, 0), \]

\( \rho \) and \( p \) are energy density and pressure on the brane, respectively. Integrating eqs.(4, 5) with respect to \( y \) around \( y = 0 \) give the following jump conditions

\[ a'_+ = -a'_- = -\frac{\kappa_5^2}{6} a_0 b_0 (\sigma + \rho), \]

\[ n'_+ = -n'_- = \frac{\kappa_5^2}{6} b_0 n_0 (-\sigma + 2\rho + 3p), \]
Employing Eqs. (12) and (13), we can derive[30]
\begin{align}
\dot{\rho} + 3 \frac{\ddot{a}_0}{a_0} (\rho + p) &= -\frac{2n_0^2}{b_0} T_5^0, \\
\frac{1}{n_0^2} \left[ \frac{\ddot{a}_0}{a_0} + \left( \frac{\dot{a}_0}{a_0} \right)^2 - \frac{\dot{a}_0 \dot{n}_0}{a_0 n_0} \right] + \frac{k}{a_0^2} = \frac{\kappa_5^2}{3} \left( \Lambda_5 + \frac{\kappa_5^2 \sigma^2}{6} \right) \\
&- \frac{\kappa_5^2}{36} [\sigma(3p - \rho) + \rho(3p + \rho)] - \frac{\kappa_5^2}{3} T_5^5.
\end{align}

(14)

(15)

where $T_{05}$ and $T_{55}$ are the 05 and 55 components of $T_{\mu\nu}|_{m,B}$ evaluated on the brane. In order to derive a solution that is largely independent of the bulk dynamics, we can neglect $T_5^5$ term by assuming that the bulk matter relative to the bulk vacuum energy is much less than the ratio of the brane matter to the brane vacuum energy [31]. Considering this approximation and concentrating on the low-energy region with $\rho/\sigma \ll 1$, Eqs. (14,15) can be simplified into [32]
\begin{align}
\dot{\rho} + 3H(1 + w)\rho &= -2T_5^0 = T \\
H^2 &= \frac{8\pi G}{3} (\rho + \chi) - \frac{k}{a^2} + \lambda \\
\dot{\chi} + 4H\chi &\approx 2T_5^0 = -T.
\end{align}

(16)

(17)

(18)

Thus with the energy exchange $T$ between the bulk and brane, the usual energy conservation is broken down. In the following we will consider that there are two dark components in the universe, dark matter and dark energy, $\rho = \rho_m + \rho_\Lambda$. Here we assume that the adiabatic equation for the dark matter is satisfied while it is violated for the dark energy due to the energy exchange between the brane and the bulk,
\begin{align}
\dot{\rho}_m + 3H\rho_m &= 0, \\
\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda &= T.
\end{align}

(19)

(20)

The interaction between bulk and brane is given by the quantity $T = \Gamma \rho_\Lambda$, where $\Gamma$ is the rate of interaction. Taking a ratio of two energy densities as $u = \chi/\rho_\Lambda$, the above equations lead to
\begin{equation}
\dot{u} = 3Hu\left[ w_\Lambda - \frac{1}{3} - \frac{1 + u}{u} \frac{\Gamma}{3H} \right]
\end{equation}

(21)

If we define
\begin{equation}
w_\Lambda^{\text{eff}} = w_\Lambda - \frac{\Gamma}{3H},
\end{equation}

(22)

Then, the continuity eq.(20) can be written in following standard form
\begin{equation}
\dot{\rho}_\Lambda + 3H(1 + w_\Lambda^{\text{eff}})\rho_\Lambda = 0.
\end{equation}

(23)

We study the case of a universe with zero effective cosmological constant, but with spatial curvature. A closed universe with a small positive curvature ($\Omega_k \sim 0.01$) is compatible with observations [22]. Define as usual
\begin{align}
\Omega_m &= \frac{\rho_m}{\rho_{cr}}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}, \quad \Omega_\chi = \frac{\chi}{\rho_{cr}}, \quad \Omega_k = \frac{k}{a^2 H^2}
\end{align}

(24)
where \( \rho_{cr} = \frac{3H^2}{8\pi G_4} \). Now we can rewrite the first Friedmann equation (17) as

\[
\Omega_m + \Omega_\Lambda + \Omega_\chi = 1 + \Omega_k. 
\]

(25)

Using Eqs.(24,25) we obtain following relation for ratio of energy densities \( r \) as

\[
u = 1 + \Omega_k - \Omega_\Lambda - \Omega_m
\]

(26)

In non-flat universe, our choice for holographic dark energy density is

\[
\rho_\Lambda = \frac{3c^2}{8\pi G_4L^2}
\]

(27)

As it was mentioned, \( c \) is a positive constant in holographic model of dark energy \((c \geq 1)\) and the coefficient 3 is for convenient. \( L \) is defined as the following form:

\[
L = ar(t),
\]

(28)

here, \( a \), is scale factor and \( r(t) \) can be obtained from the following equation

\[
\int_0^{r(t)} \frac{dr}{\sqrt{1-kr^2}} = \int_t^\infty \frac{dt}{a} = \frac{R_h}{a},
\]

(29)

where \( R_h \) is event horizon. For closed universe we have (same calculation is valid for open universe by transformation)

\[
r(t) = \frac{1}{\sqrt{k}} \sin y.
\]

(30)

where \( y \equiv \sqrt{k} R_h/a \). Using definitions \( \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} \), we get

\[
HL = \frac{c}{\sqrt{\Omega_\Lambda}}
\]

(31)

Now using Eqs.(28, 29, 30, 31), we obtain

\[
\dot{L} = HL + ar(t) = \frac{c}{\sqrt{\Omega_\Lambda}} - \cos y,
\]

(32)

By considering the definition of holographic energy density \( \rho_\Lambda \), and using Eqs.(31, 32) one can find:

\[
\dot{\rho}_\Lambda = -2H(1 - \frac{\sqrt{\Omega_\Lambda}}{c} \cos y)\rho_\Lambda
\]

(33)

Substitute this relation into Eq.(20) and using definition \( T = \Gamma \rho_\Lambda \), we obtain

\[
\bar{w}_\Lambda = -\left( \frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} \cos y - \frac{\Gamma}{3H} \right).
\]

(34)

Here we choose the following ansatz [33]

\[
\Gamma = 3b^2(1 + u)H
\]

(35)

with the coupling constant \( b^2 \). Using Eq.(26), the above equation take following form

\[
\Gamma = 3b^2H \frac{(1 + \Omega_k - \Omega_m)}{\Omega_\Lambda}
\]

(36)
Substitute this relation into Eq.(34), one finds the holographic energy equation of state

\[ w_\Lambda = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c}\cos y + \frac{b^2(1 + \Omega_k - \Omega_m)}{\Omega_\Lambda}. \]  

(37)

From Eqs.(22, 34), we have the effective equation of state as

\[ w^{\text{eff}}_\Lambda = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c}\cos y. \]  

(38)

If we take \( c = 1 \), then \( w^{\text{eff}}_\Lambda \) is bounded from below by

\[ w^{\text{eff}}_\Lambda = -\frac{1}{3}(1 + 2\sqrt{\Omega_\Lambda}) \]  

(39)

taking \( \Omega_\Lambda = 0.73 \) for the present time, the lower bound of \( w^{\text{eff}}_\Lambda \) is \(-0.9\).\(^1\) Therefore it is impossible to have \( w^{\text{eff}}_\Lambda \) crossing \(-1\). This implies that one can not generate phantom-like equation of state from an interacting holographic dark energy model in non-flat universe. As it was mentioned in introduction, \( c \) is a positive constant in holographic model of dark energy, and \( (c \geq 1) \). However, if \( c < 1 \), the holographic dark energy will behave like a Quintom model of DE [34], the amazing feature of which is that the equation of state of dark energy component \( w_\Lambda \) crosses \(-1\). Hence, we see, the determining of the value of \( c \) is a key point to the feature of the holographic dark energy and the ultimate fate of the universe as well. However, in the recent fit studies, different groups gave different values to \( c \). A direct fit of the present available SNe Ia data with this holographic model indicates that the best fit result is \( c = 0.21 \) [13]. Recently, by calculating the average equation of state of the dark energy and the angular scale of the acoustic oscillation from the BOOMERANG and WMAP data on the CMB to constrain the holographic dark energy model, the authors show that the reasonable result is \( c \sim 0.7 \) [35]. In the other hand, in the study of the constraints on the dark energy from the holographic connection to the small \( l \) CMB suppression, an opposite result is derived, i.e. it implies the best fit result is \( c = 2.1 \) [36].

### 3 Conclusions

It is of interest to remark that in the literature, the different scenarios of DE has never been studied via considering special similar horizon, as in [37] the apparent horizon, \( 1/H \),

\[^1\] The differential equation for \( \Omega_\Lambda \) is as

\[ \frac{d\Omega_\Lambda}{dx} = 2\Omega_\Lambda(\frac{\sqrt{\Omega_\Lambda}}{c}\cos y + q) \]  

(40)

where \( x = \ln a \), and

\[ q = -1 - \frac{\dot{H}}{H^2} \]  

(41)

Differentiating Eq.(17) with respect to cosmic time \( t \) and then using Eqs.(16,18) one have

\[ \dot{H} = \frac{-4\pi G}{3}[3\rho_\Lambda(1 + w_\Lambda) + 3\rho_m + 4\chi] - \frac{k}{a^2}. \]  

(42)
determines our universe. As we discussed in introduction for flat universe the convenient horizon looks to be event horizon, while in non flat universe we define $L$ because of the problems that arise if we consider event horizon or particle horizon (these problems arise if we consider them as the system’s IR cut-off). Thus it looks that we need to define a horizon that satisfies all of our accepted principles; in [38] a linear combination of event and apparent horizon, as IR cut-off has been considered. In present paper, we studied $L$, as the horizon measured from the sphere of the horizon as system’s IR cut-off. Then by considering the effects of the interaction between a brane universe and the bulk we have obtained the equation of state for the interacting holographic energy density in the non-flat universe. Our calculation show, taking $\Omega_\Lambda = 0.73$ for the present time, the lower bound of $w_{\Lambda}^{\text{eff}}$ is $-0.9$. Therefore it is impossible to have $w_{\Lambda}^{\text{eff}}$ crossing $-1$. This implies that one can not generate phantom-like equation of state from an interacting holographic dark energy model in non-flat universe.

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