Research article

Optimal manpower recruitment and promotion policies for the finitely graded systems using dynamic programming

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ABSTRACT

Background: At the heart in the development of any organization or nation is human resource. Over the years the world over, a sharp increase in too many numbers of qualified persons is being experienced yearly. This has impacted on recruitment and promotion costs to increase immensely, thereby affecting negatively manpower system costs. Dynamic Programming (DP) approach to optimal manpower recruitment and promotion policies for the two grade system has been proposed.

Methods: Considering the fact that contemporary manpower systems are not limited to just two grades - a kind of “switch-approach” to manpower systems, we first establish the link between a manpower planning problem and a dynamic decision-making process. This linkage resulted to a multistage real-life decision-making problem whose solution demands that decisions be made sequentially at different levels and at different points in time and space. Dynamic Programming is a mathematical technique well appropriate for the optimization of multistage decision problems. This allows us to give a generalization to manpower systems by modifying the model to finite grades which came out to be more robust and actionable, a constrained deterministic Dynamic Programming (CDDP) found to function computationally as the very well-known Wagner-Whitin Model in inventory/production management. Five cost variables associated with manpower planning were identified and used as inputs to the modified deterministic DP model.

Results/Relevance: The data shows yearly recruitments and promotions totaling 507 and 266 staff respectively for a ten-period (year) planning horizon. Total manpower system cost (in oo’s of Nigerian Naira) occasioned by yearly recruitments and promotions exercises for the period is 11334 (7092 for recruiting, 4100 for promoting, and 142 for overstaffing). Our DP model minimizes the manpower system cost to 9462 making a significant reduction of 1872. The optimal policy for the planning period calls for recruiting and promoting respectively 79 and 41 in period 1 only, 86 and 24 in period 2 for periods 2 and 3, 86 and 46 in period 4 for periods 4 and 5, 89 and 29 in period 6 only, 85 and 70 in period 7 for periods 7 and 8, and 82 and 56 in period 9 for periods 9 and 10.

The study will contribute to the growing literature on applications of OR models to problems in manpower planning. The model outcomes would provide the basis for evaluating decision policies aimed at conducting recruitment promptly and to eliminate over-stagnated promotions.

Conclusion/Further research: We formulate decisions making in a finitely-graded manpower system as a multistage decision-making optimization problem which are best handled by dynamic programming. Five cost variables associated with manpower planning were identified and used as inputs to the modified deterministic DP model. Our model is resolute for minimizing manpower system costs occasioned by recruitments and promotions exercises in a wide range of multi-graded manpower systems instead of just two grades. The study’s limitations and scope for future research work are presented in the end.

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1. Introduction

At the heart in the development of any organization or nation is human resource. In today’s competitive world, described by a large volume of eligible applicants, coordinating for manpower in the face of biting constraints on diverse options remains a general concern for Operations Research (OR) scientists and mathematical modelers. According to [1] manpower planning is the process of forecasting and planning for the right number, the right kind of people in the right places and at the right time to carry out activities that will be beneficial to both the organization and the individuals working in it. The analytical study of manpower systems is a key component of strategic economic development of any nation or organization, as efficiency in management has a direct bearing on productivity.

Determining policies for manpower planning is among the most sensitive issues of an organization, because planning resulting from an incorrect analysis can eventually lead to either oversupply or undersupply of manpower causing inefficiencies. Planning for manpower must accommodate the extremely varying human actions and the volatile social atmosphere within which the system operates. The manpower system structure in any organization is generally designed to be finitely graded so that an employee in the organization can only belong to one and only one of the many incompatible grades determined by qualifications, skills, experience etc. Also, [2] assert that every graded manpower system has a dynamic behaviour and controllability elements. For every organization, the expected staff strength is always maintained through fresh recruitments. In principles, a graded manpower system may be controlled by recruitment, and by promotion policies. Optimal operation of manpower is a dominant challenge to a manager of a dynamic management system.

A major problem to any manager managing a changing system is optimal utilization of manpower. Solutions to manpower problems are best approached using mathematical theories, [3, 4, 5] acknowledged that manpower planning is an important and complicated process and that although it differs among organizations or nations, using mathematical optimization methods would result in more effective and rational manpower planning decisions than intuitive ones. Different mathematical models like: Markov chains, Computer simulation, Supply chains (system dynamics), and optimization have been applied to analyze graded manpower systems. Our focus in this paper is on deterministic optimization models, whose techniques may be Linear Programming (LP), Goal Programming (GP), Integer Programming (IP), Stochastic Programming (SP), or Dynamic Programming (DP). The study by [6] suggests that, optimization models always identify specific management actions while dismissing variability.

Over the years the world over, a sharp increase in too many numbers of qualified persons is being experienced yearly [7]. Consequently, competitions have multiplied resulting in cumbrous recruitment and promotion procedures in corporations and organizations. Further, it has caused recruitment and promotion costs to increase immensely, which have negative effects on manpower system costs. The high cost is one of the major reasons why recruitment and promotion are important to managers and employers. Knowing the cost is representing important information for managing an organization and for controlling its administration [8]. The result of this development has made managers of organizations and corporations to shy away from prompt recruitment and promotion exercises. The thrust of the model framework in this paper is the result of [9] who proposed a dynamic programming model to determine optimal manpower recruitment and promotion policies for the two grade manpower system.

Considering the fact that contemporary manpower systems are not limited to just two grades - a kind of graduate or Non-graduate, Academics or Non-academics, Commissioned or Non-commissioned; switch-approach to manpower systems, we first establish the link between a manpower planning problem, a dynamic decision-making process, and a multistage decision-making optimization problem. This link-age resulted to a multistage real-life decision-making problem whose solution demands that decisions be made sequentially at different levels and at different points in time and space. Dynamic Programming is the mathematical technique well appropriate for the optimization of multistage decision problems since this class of optimization problems are modifiable to the type of solution procedures that DP prescribes and/or can be summarized down to an optimality (recursive) functional equation [10]. This allows us to give a generalization to manpower systems by modifying both the objective and the constraints functions of the model to \( n \geq 2 \) finite grades based on deterministic manpower system discrete costs components. The formulated model came out to be more robust and actionable, a Constrained Deterministic Dynamic Programming found to function computationally as the well-known Wagner-Whitin Model (W-MM) [11] in the production or inventory management.

Five cost variables associated with manpower planning were identified, vis-a-vis: Total number recruited, \( R(t) \); Fixed recruitment cost, \( S(t) \); Total number promoted, \( P(t) \); Fixed cost of promotion, \( Q(t) \) and \( K(t) \), the overstaffing cost per recruit or promote. These form a hypothesized recruitment and promotion data set for a ten-year (from 2001 to 2010) planning horizon as inputs to the modified deterministic DP model to minimize manpower system costs occasioned by recruitments and promotions exercises. All the related data were retrospectively reviewed based on past recruitments and promotions histories to guarantee their accuracy, completeness and validity. The use of synthetic data set was informed by [12] who opined that comparatively data on recruiting and promotion costs are rare and are difficult to come by, especially when specific workplace information is involved. Our model on one hand, like the W-MM recursive algorithm is forward execution, and on the other hand is resolute so that planning for recruitment and promotion costs can be applied to a wide range of graded manpower systems instead of just two grades.

The remainder of this paper is organized as follows. Section 2, reviews the relevant literature on manpower planning optimization-based models. In Section 3, we present a model of an n-graded manpower system, and show that planning for such system is a dynamic and multistage problem. The study methodology is outlined in section 4, while section 5 presents the modified manpower system model, the DP model based on this research design, and the planning horizon theorem for manpower planning. We illustrate the application of the model in section 6. Section 7 concludes the study, with directions for future research.

2. Literature review

There are many already published applications of operations research techniques to strategic modeling of manpower systems. Various applications of Operations Research in the setting up of workforce and basic modeling of military grounding were reviewed in [13]. System Dynamics (SDs) is use to decompose the models majorly into four categories: Optimization, Markov chains, Computer simulations, and Supply chains management. Every category was reviewed to understand the elemental mathematical formulations and concepts, benefits as well as possible limitations. An explicit review of all the approaches and techniques in manpower planning, concentrating on their assumptions and applications are given in [1] and [14]. Broadly, these models can be divided into two supplementary formulations: the stochastic optimization or random demands for manpower, and the deterministic optimization or deterministic demands for manpower. These approaches draw attention to separate areas of manpower systems by adopting different application procedures. Purely stochastic models are commonly applied to predict the future size of manpower and the transitions (caused by death, firing, retirement, resignation etc.) that will occur within a given time interval in future. Markov Chain Models have been applied to different aspects of manpower systems control measures - Recruitment, Promotion and
Wastage. An exponentiated exponential distribution to solve stochastic models with grading system was applied in [15]. The problem of control (attainability and maintainability) of manpower system structures has received a good deal of attention over the past decades from works like: [11, 16, 17]. A stochastic study of the aspects of time to recruitment for an organization is studied in [18], while [19] reviewed other stochastic optimization models of manpower systems with constant demands. Nevertheless, [20] argue that, Stochastic Programs are much more likely to yield superior solutions than their deterministic counterparts, but their solutions procedures are much more complex.

A major approach to manpower planning is based on the theory of optimization. Optimization models usually are very important standard models, capable of computing optimal sets of organizational personnel decisions on recruitment, promotion, wastage, training etc. as opposed to goals formulated as objective functions. These optimization models can assume the demands for manpower to be both deterministic and probabilistic. The general principles of the optimization methods in manpower planning originated from the work of [21], where these theoretical principles where presented in a form of quadratic cost functions comprising both the maintenance costs and the costs of replacing the workforce. Mathematical approaches to manpower planning were extensively discussed in [22] and [23]. They used the concept of LP to develop a graded personnel’s structure to assist management control both the rates of recruitment and transfer between the different grades in a manpower system. A GP model was developed by [24] to find optimal promotion policies for officers distributed among the ranks within the Canadian forces. Their objective was to minimize the loaded sum of deviations in manpower Manning, promotion, financial, and accounting constraints. An optimization model to determine the recruitment policies for a manpower system was developed in [25], while [26] formulated a Markov chain-preemptive GP model using sequential approach to solve large scope manpower planning problems existing under competitive objectives and different constraints.

A special Goal Programming model – “The Accession Supply Costing and Requirement” (ASCR) was developed by [27] to enable easy evaluation of the US annual supplies of recruits to attain strength and personnel levels requirements for the All-Volunteer Armed Forces. The “Army Training Mixed Model” in [28] was formulated as an IP program to find the optimal training requirements that will minimize training costs under major proficiency constraint, while [29] constructed a model to determine optimal size of manpower to be engaged in workforce. A Mixed-Integer Goal Programming model was proposed in [30] to address the multidimensional manpower planning problems in a hierarchical organization. The multiobjective manpower planning problem model with annualized hours flexibility in a fuzzy scenario was presented in [31]. To solve problems of these sorts, they applied a modified GP approach to minimize the capacity shortage involving multitasking, overtime cost, and under time preference hierarchies among the goals. On improving operations of recruitment and placement decisions, [32] developed a comprehensive analytics framework- a decision support tool for Human Resource (HR) recruiters. The first stage was an application of Machine Learning (ML) to a large recruitment dataset to determine a local prediction plan for recruitment success. The second uses these predictions to provide a global recruitment optimization model for the entire organization.

A tractable Risk-based manpower model is presented in [33]. The concept of pipeline invariance under multi-period optimization was applied to minimize a decision criterion’s risk function expressed in employees’ time-in-grade. A new manpower planning model as a continuous-time optimal control problem where promotions take place inside the system was proposed by [34]. This formulation demands linear system transition equations with quadratic cost functional to assist managers to attain certain decision targets. To minimize workforce costs, [35] developed two mixed-integer deterministic and two-stage stochastic programming models. By Bender’s decomposition algorithm these models give reliable results in the real-life contexts with individual and hierarchical skills. An integration of the concept of fuzzy uncertainty into the Annualized Hours Planning Problem (AHPP) was presented by [36]. A flexible fuzzy mathematical programming (FFMP) approach was used to determine and minimize the relative workforce capacity shortage involving multitasking workers in respect to demand. Markov models were developed by [37] to quantify and measure the performance of promotion and recruitment policies in the context of the career growth facilitation accorded to the employees in the organization. This measure of effectiveness provides a numerical basis for comparing the performance of different policies.

DP has been applied to solve specific planning problems in manpower systems. For example, [38] developed a DP model to determine effective recruitment and transition policies for manpower systems and [39] proposes a constrained Multi-stage Decision Process (MDP) manpower planning model to solve decision-making problems in labour-intensive organizations where demands for manpower suffer dynamic fluctuations. Stochastic programming model was formulated in [6] to optimize recruitment of manpower into the various grades in organizations. A manpower capacity-planning problem with dynamic demands for manpower was formulated in [40] as a DP problem. The model took into account factors such as: recruitment and firing, setup costs (fixed cost of recruitment), and the possibility of staff substitution by breaking down the parent function into four sub-functions. Applying multimodularity and supermodularity analyses they obtained a monotonically characterized optimal policy of the model. Again, a DP approach is illustrated in [10] to determine optimal manpower planning for recruitment policies and [9] developed a DP model to minimize costs during recruitment and promotions exercises for the 2-grade manpower system.

Generally speaking, while the manpower planning models developed in previous studies have considered financial and labour costs, and various recruitment policy objectives as indicated above, surprisingly none took into account the objective of minimizing the manpower system costs where the manpower system has $n > 2$ finite grades. This work addresses this gap and differs from previous works by explicitly formulating an actionable deterministic DP model in discrete time for manpower systems with $n$ grades. The study gives decision policies to carryout recruitment and promotions in a typical manpower system.

3. Manpower planning as a dynamic multistage problem

3.1. Model of manpower system

The manpower system structure in any organization is generally designed to be finitely graded so that an employee in the organization can only belong to one and only one of the many incompatible grades determined by qualifications, skills, experience etc. In every organization staff circulates in various flows or movements. In a typical manpower system ([see Fig. 1]), the rectangles represent “employees” and the arrows stand for “movements” or “flows” across the different levels of the system. Every such model should consider the main factors: Recruitment, Promotion, and Wastages which are dominant determinants of any manpower system’s behaviour. Generally, in any system recruitment levels are linked to wastages and promotions.

3.2. Manpower planning as an optimization problem

Within the OR literature, Dynamic Decision Making (DDM) is absolutely regarded as Multi-Stage or Sequential Decision Making, [41]. A Dynamic Decision Making (DDM) process consists of decisions patterns made in succession over a given time interval. Below, Fig. 2 establishes the link between manpower planning, a dynamic decision process, and a multistage optimization problem.
4. Methodology

4.1. Multistage decision processes

As applied to DP, and as can be seen in Fig. 2 (showing the schematic description of the dynamics of a decision making process), a multistage decision problem is one in which a number of single-stage processes are connected head-to-tail (in series) without recycling so that the output of one stage is the input of the succeeding stage. Multistage decision problems arise in many types of practical problems. Solutions to most of these real-life problems demand that decisions be made sequentially at different levels and at different points in time and space. Because these decisions are to be made at a number of stages, they are commonly referred to as sequential or multistage decision problems. [42].

An obvious prominent part of great significance, also considered the distinct hallmark of these decision-making processes is the fact that a decision made at any given time \( t \) is influenced by previously made decisions and almost always influences the future decisions to be made. Consequently, the optimization models representing this specific group of problems are named multistage or sequential decision models. Different conditions of sophistication of these models are formulated using the under listed fundamental components:

- **Stage/Period** - a succession of elements indicating smaller sub-problems in the decision process.
- **State** - a condition of the decision process at a stage.
- **Stage-State Pair** - a finite or infinite collection specifying the condition or status of the decision process at any stage.
- **Transition function** - showing the dynamics of the states as a result of the policy decision at each stage.
- **Objective function** - the overall worth or cost produced by any sequence of decisions.

Analytically, a multistage decision model is characterized by the combination \( (T, \Omega, D, G, \Omega_1, f) \) where:

- \( T \) - a positive integer such that \( t := \{1, 2, \ldots, T\} \) specifying the time horizon or the number of decision stages in the decision-making process. \( T \) is finite if the process is shortened and infinite otherwise. Curtailed processes have \( T \) decision stages, each at which a decision is made. Since the process terminates at the final stage \( t = T + 1 \), no decision is made at the associated final state, \( \omega_{T+1} \). There is no final stage for un-shortened processes. Nevertheless, if the sequence of states \( \omega_1, \omega_2, \omega_3, \ldots \) converges to some state \( \omega \in \Omega \), then this state can be taken as the process’s final state.
- \( \Omega \) - a non-empty set called the ‘state space’ whose elements are referred to as ‘states’ \( \omega_1, \omega_2, \omega_3, \ldots \).
\*D - a function denoting the stage-state pair \((t, \omega) \in T \times \Omega\) to each of which it assigns a subset of the decision space, i.e. \(\delta \in \Delta\). The notation \(D(t, \omega)\) denotes the set of feasible decisions taken in state \(\omega\) at stage \(t\).

\*G - a transition function on \(T \times \Omega \times \Delta\) with values in \(\Omega\). In the triplet \((t, \omega, \delta)\) where \(t \in T, \omega \in \Omega\), and \(\delta \in D(t, \omega)\), the element \(\omega' = G(t, \omega, \delta)\) represents the process state at stage \(t + 1\) for taking decision \(\delta\) on state \(\omega\) at stage \(t\).

\*\(\Omega_1\) - a non-empty subset of \(\Omega\) whose members are initial states only.

\*f - a real-valued function on \(\Omega_1 \times \Delta^T\) called the objective function. Given that the initial state of the process is \(\omega_1 \in \Omega_1\), the value of \(f\) produced by \(f(\omega, \delta_1, \delta_2, \ldots, \delta_T)\) is the total return, in either cost or profit, and is the measure of the effectiveness of taking the decisions \((\delta_1, \delta_2, \ldots, \delta_T)\) at stages 1, 2, 3, …, \(T\) respectively [10].

For a single-stage decision process (which is a component of a multistage problem) the output is related to the input by a stage transformation function (designed equations) denoted by:

\[ G = g(\Omega, \Delta) \]  

(1)

Because the decisions made are influenced by the inputs into the system, the return function is represented as:

\[ H = h(\Omega, \Delta) \]  

(2)

Where:

\[ \Delta := \text{the decision space vector} \]

\[ \Omega := \text{the state space vector} \]

For a \(t\)-stage (multistage) decision process, the forward and backward \(t^{th}\) stage input state vectors are denoted by \(\Omega_{t+1}\) and \(\Omega_{t-1}\), and the output state vectors by \(\omega_t\) respectively (see Fig. 2).

Since the system is sequential, the output from stage \(t + 1\) must be equal to the input into stage \(t\), (forward), and the output from stage \(i\) must be equal to the input into stage \(t - 1\) (backward). Their states transformations and returns functions are represented as:

\[ \omega_{t+1} = g_t(\omega_t, \delta_t), \text{ or } \omega_t = g_t(\omega_{t+1}, \delta_t) \]

\[ H_{t+1} = h_t(\omega_t, \delta_t), \text{ or } H_t = h_t(\omega_{t+1}, \delta_t) \]  \text{Forward Recursion} \]

\[ \omega_{t-1} = g_t(\omega_t, \delta_t), \text{ or } \omega_t = g_t(\omega_{t-1}, \delta_t) \]

\[ H_{t+1} = h_t(\omega_t, \delta_t), \text{ or } H_t = h_t(\omega_{t-1}, \delta_t) \]  \text{Backward Recursion} \]

(3)

Where:

\( t \) = the time horizon or decision stages.

\( \delta_t \in \Delta \) denotes the control vector of decision variables at stage \( t, t = 1, 2, \ldots, T \).

\( \omega_t \in \Omega \) := the state variable at stage \( t \).

Normally this sequence of decisions is made in order to reach a certain goal that is subject to some explained or explicitly stated constraints. The objective of a multistage decision problem is to find \( \delta_t \in \Delta \) so as to optimize a function, \( f \) expressed in terms of the individual state returns, \( f(H_t), t = 1, 2, \ldots, T \). By the separability and monotonicity of the objective function [42], we have:

\[ f(H_t) = \sum_{i=1}^{T} f_t(H_i) = \sum_{i=1}^{T} H_t(\omega_{t+1}, \delta_t) \]

(4)

Thus, the general mathematical program of a multistage decision problem is given by:

\[ \text{Minimize } Z = \sum_{i=1}^{T} f_t(\delta_t) \]

Subject to:

\[ \sum_{i=1}^{T} \delta_t \leq b \]

\( \delta_t \in \Delta, \delta_t \geq 0, t = 1, 2, \ldots, T \) (stages).

(5)

Where:

\( \delta_t \) = decisions variables, each with a resulting contribution to the total return.

\( b \) = represent possible values of the number of resources available for allocation (example, applicants or employees available for recruitment and promotion) [43].

4.2. Dynamic programming (DP)

DP deals with sequential or multistage decision problems, which are models of dynamic systems under the direct influence of a decision maker. A multistage decision problem is one that can always be broken down into sub-problems in order to yield the optimality functional equation. Any optimization problem that can be summarized down to an optimality (recursive) functional equation and/or modifiable to the type of solution procedures that DP prescribes is a DP problem, and are therefore best handled by DP technique [10].

The DP technique, when applicable adopts either the Top-down or Bottom-up approach to decompose a \( T \)-variable multistage decision problem to a sequence of \( T \) single-variable decision problems to be solved successively. The decomposition to \( N \) sub-problems is done in such a manner that the optimal solution of the original \( N \)-variable problem can be obtained from the optimal solutions of the \( N \) one-dimensional problems. This idea of using sub-optimization in solving problems is based on the argument that \( N \) sub-problems are easier solved compared to the original problem [42].

This process of sub-optimization usually is expressed theoretically as the Bellman’s principle of optimality and mathematically as the Bellman’s equation (or DP optimality equation and/or the DP functional equation).

Bellman’s Principle of Optimality: “An optimal policy (a sequence of actions or a set of decisions) has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regards to the state resulting from the first decision” [10].

The general deterministic DP in discrete time (stage) for program (5) can be written as:

\[ f_T(\omega_T, \delta_T) = \text{Minimize } \sum_{i=1}^{T} f_t(\omega_t, \delta_t) + f_{t+1}(\omega_{t+1}) \]

Subject to:

\[ \sum_{i=1}^{T} \delta_t \leq b, \omega_0 > 0 \]

\( \omega_{t+1} = f_t(\omega_t, \delta_t), \omega_{t-1} = f_t(\omega_t, \delta_t) \]

\( \omega_{t+1} = \omega_t \delta_t, f_t(\omega_t, \delta_t) = \omega_t - \delta_t \)

\( \omega_t \in \Omega, \delta_t \in \Delta, \delta_t \geq 0, t = 1, 2, \ldots, T \).

(6)

Since at any point in time the set of possible actions always depends on the current state of the system, the decision maker’s objective would be to choose an appropriate set of control vectors or decision variables, \((\delta_1, \delta_2, \ldots, \delta_T)\) within some feasible action space, (i.e. \( \delta_t \in \Delta \)) that optimizes the performance of the system over the planning horizon (\( T \) stages). This sequence of actions or decisions is referred to as a policy, and a policy that satisfies the constraints of (6) and minimizes the objective function is called an optimal policy [44].

The required Bellman’s (DP recursive) equation representing the Bellman principle of optimality upon which the solution method of the DP program in (6) is based is presented as

\[ f_t^*(\omega_t, \delta_t) = \text{Min } f_t(\omega_t, \delta_t) + f_{t+1}(\omega_{t+1}) \]

\[ = \text{Min } \{ f_t(\omega_t, \delta_t) + f_{t+1}(\omega_{t+1}) \} \]

\text{(Backward recursion)}

\[ f_t^*(\omega_t, \delta_t) = \text{Min } f_t(\omega_t, \delta_t) + f_{t+1}(\omega_{t+1}) \]

\[ = \text{Min } \{ f_t(\omega_t, \delta_t) + f_{t+1}(\omega_{t+1}) \} \]

\text{(Forward recursion)}

(7)
However, in DP, every recursive relation depends on the nature of the given problem, where the computation procedure can start at the end (backward recursion) or from the beginning (forward recursion).

5. The manpower planning model formulation

5.1. Manpower system costs

Planning decisions for manpower system costs are influenced by the under listed cost variables:

1. “Costs of Recruitment and Promotion”
2. “Costs of Overstaffing”
3. “Costs of Attrition or Wastage”
4. “Costs of Attrition or Wastage”
5. “Costs of Retention”.

5.1.1. Costs of recruitment and promotion

Normally, the costs of recruitment and promotion come from:

(a) Advertising (b) Application processing (c) Administrative (d) Aptitude test (e) Information processing (f) Interview committee Wages (g) Candidates transportation (optional) (h) Medical screening (i) Personnel training (j) Miscellaneous like, e-mails, recharged cards etc.

Decision makers are exposed to these costs during the actual recruitment and promotion exercises because of:

i. A demand to expand the organization
ii. A demand to Promote from Grade i to i + 1 at time t, i = 1, 2, ..., n.
iii. A Separation or Wastage occurring in Grade i, i = 1, 2, ..., n - 1.
iv. A combination of (i), (ii), and (iii).

In a wider scope:

Total Recruitment and Promotion Costs,

\[ C_{R,P} = \text{Total Fixed Cost of Recruitment and Promotion} + \text{Total Variable Cost of Recruitment and Promotion} \]

which is a direct function of the volume of individuals actually recruited and promoted.

Note [7] that the cost variables above are merely denotative because what truly constitutes the recruitment and promotion costs for an organization depends largely on the recruitment and promotion styles adopted by that organization. However, in many situations these charges are billed on applicants through applications processing, even though the generated expected revenue does not equate to the true recruitment and promotion costs. The costs variables (b), (d), (e), (h) and (i) above certainly will constitute both Fixed and Variable components per recruiter or promoter. Usually, the fixed component of these costs is more if group-selection process is adopted as in the usual recruitment exercise for the military.

5.1.2. Costs of overstaffing

These costs result from unused workforce or manpower. They correspond to the Holding or the Carrying costs in an inventory or production system.

5.1.3. Costs of understaffing

In profit-oriented organizations, shortage in the workforce brings about a fall in production and eventual loss of market favour or advantage. A cost resulting from this condition is termed “Understaffing” and corresponds to “Stock-out costs” in Production or Inventory scenarios.

5.1.4. Attrition (wastage) costs

These costs occur as a result of death, firing, retirement, health, accident, or dissatisfaction etc. of the employees and can occur anytime. The duty of an organization’s committee responsible for recruiting usually begins immediately a vacant position occurred. Generally, the committee quickly senses Separations of employees as warnings of impending unproductive dangers worthwhile to immediately put precautionary plans in place.

5.1.5. Retention costs

Apart from the different costs already identified, certain costs still exist when an employee is kept in the organization during:

a. Probation or Adaptation or Integration: – costs liable because employees have to learn during the interval of probation.

b. “Training and development”: – costs resulting from developmental programs to improve employee’s efficiencies during their period of service. These are separate from those already discovered as components of recruitment and promotion costs.

c. “Internal mobility”: – costs that result because of employee’s transfer, or relegation within the manpower system.

5.2. Model notations

- \( R(t) \): A demand for Recruitment in period t
- \( S(t) \): A Fixed Recruitment cost in period t
- \( P(t) \): A demand for Promotion in period t
- \( Q(t) \): Fixed cost of Promotion per year t
- \( K(t) \): Overstaffing Cost per recruit or promotee per year
- \( I(t) \): Present workforce, or those affected by last promotion i.e. Number of people previously recruited or promoted for the requirements of year t
- \( X_i(t) \): Number of people recruited in period t into grade i, i = 1, 2, ..., n
- \( Y_i(t) \): Number of people promoted in period t from grade i to i + 1, i = 1, 2, ..., n
- \( V_i \): Variable cost of recruitment at grade i per employee recruited, i, i = 1, 2, ..., n
- \( U_i \): Variable cost of promotion from grade i to i + 1, i = 1, 2, ..., n
- \( C_{R}(t) \): Cost of recruitment in period t
- \( C_{P}(t) \): Cost of promotion in period t
- \( C_{O}(t) \): Cost of overstaffing in period t
- \( C_{R}(T) \): Total cost of recruitment for the T-period
- \( C_{P}(T) \): Total cost of promotion for the T-period
- \( C_{R,P}(T) \): Total cost of Recruitment, plus Promotion for the T-period

5.3. Model assumptions

To formulate the manpower planning program that will determine the most effective recruitment and promotion policies, we make these assumptions:

i. Recruitment and Promotion sizes are specified and determinate

\[ i.e. \ X_i(t), Y_i(t) \geq 0 \]

ii. Only recruitment and promotion into a specified grade-level is looked at.

iii. Recruitment, Promotion and Overstaffing costs are specified and determinate,

\[ i.e. \ C_{R}(t), C_{P}(t), C_{O}(t), S(t), Q(t), V_i, U_i, K(t) \geq 0 \]

iv. Understaffing is not permitted in any grade i, i = 1, 2, ..., n.

v. Overstaffing cost is not permitted for the highest grade-level n, since not desired

\[ i.e. \ K(t) = 0, \ for \ i = n, \forall t \]

vi. Only next level promotion is permitted, no higher levels promotion.

vii. All Recruitment and Promotions demands must be satisfied timely.

Under these assumptions, the “cost structure” in period t will comprise of these major functions:

(i) Recruitment cost in period t can be described with the concave function
We seek the minimization of (22), the aggregate of the total costs, subject to the constraints that all recruitments and promotions demands must be satisfied timely. That is, to find a policy such that \( X_i(t) \geq 0 \) and \( Y_i(t) \geq 0 \); \( \forall i = 1, 2, \ldots, n \); and \( \forall t = 1, 2, \ldots, T \) so that all recruitments and promotions can be satisfied at a minimum total cost.

As a constrained mathematical optimization program (22) becomes:

\[
\begin{align*}
\text{Minimize} \sum_{i=1}^{T} \left[ S(t_i) \cdot a_i[X_i(t_i)] + Q(t_i) \cdot \beta_i[Y_i(t_i)] + K(t_i) \cdot I(t_i) \right] \\
\text{Subject to:} \\
\sum_{i=1}^{T} R_i(t_i) = \sum_{i=1}^{T} \sum_{i=1}^{n} X_i(t_i) \\
\sum_{i=1}^{T} P_i(t_i) = \sum_{i=1}^{T} \sum_{i=1}^{n} Y_i(t_i), \quad i = 1, 2, \ldots, n; \quad t = 1, 2, \ldots, T. 
\end{align*}
\]

5.4. The manpower planning dynamic programming formulation

In order to formulate the DP version of (22), we use the widely known Wagner-Whitin model (W-WM) characterized in [11]. Here, the fixed recruitment and promotion cost corresponds to the Setup Cost, while the cost of overstaffing corresponds to the cost of holding or carrying inventory under Production or Inventory situation. The intended formulation is necessitated by the fact that, just as it is possible to search for an optimal solution by checking only the validity of the set of basic variables in Linear Programming models, the optimal program can really be obtained without actually exhausting the \( T \) periods data, that is, it may be possible without loss of optimality to concentrate our policy to a shorter “planning horizon” than the \( T \)-periods given data. To facilitate the DP formulation, we first present the fundamental lemmas underlying recruitment and promotion in manpower systems. Particularly, Lemma 1 suggests that it is feasible to consider policies in which at year/period \( t \) management may not carry out recruitment or a promotion.

Lemma 1. An optimal policy or program exists in the organization such that:

\[ I(t)X_i(t) = I(t)Y_i(t) = 0, \forall t = 1, 2, \ldots, T; \quad i = 1, 2, \ldots, n. \]

(I.e. before a demand for recruitment \( X_i(t) = 0 \) and a demand for promotion \( Y_i(t) = 0 \).)

**Proof.** Given that, in period \( t \) an optimal policy or program demands that \( I(t)X_i(t) > 0 \) and \( I(t)Y_i(t) > 0 \). This means it is more less costly paying the existing workforce \( I(t) \) by subsuming them among the recruits \( X_i(t) \), or the eligible to promote \( Y_i(t) \). Changes of these kinds are not liable to fresh recruitment \( S(t) \), or promotion \( Q(t) \) costs, and do save the previous costs:

\[ S(t-1) \cdot I(t-1) \geq 0 \quad \text{and} \quad Q(t-1) \cdot I(t-1) \geq 0 \quad [11] \]

Lemma 2. A basic property of a Minimum Cost Policy or Program is that the number of people recruited in period \( t \) into grade \( i \), \( X_i(t) \) will always assume some values in this order:

\[ 0, R(t), R(t) + R(t + 1), \ldots, R(t) + R(t + 1) + \ldots + R(T), \]

Likewise, the number of people promoted in period \( t \) from grade \( i \) to \( i + 1, i = 1, 2, \ldots, n \), \( Y_i(t) \) will always assume the values:

\[ 0, P(t), P(t) + P(t + 1), \ldots, P(t) + P(t + 1) + \ldots + P(T). \]

**Proof.** Since all demands (i.e. \( R(t) \) and \( P(t) \)) need to be satisfied, every different value of \( X_i(t) \) and \( Y_i(t) \) indicates the existence of a period \( t \geq r \) such that \( I(t)X_i(t) > 0 \) and \( I(t)Y_i(t) > 0 \). But according to Theorem 1, it is quite enough to think of policies or programs where this kind of situation does not emerge. Therefore, Lemma 2 strongly suggests...
that it is possible to restrict all the $I(t)$ values in (17) from period $t$ to zero including all additive aggregates of the demands $(R(t)$ and $P(t)$ for periods $t$ till the last $T$. If initially i.e. at $t = 1$, $I(1) = 0$, then in total, values of $I(t)$ across the entire $T$ periods that need be considered is only $T(T + 1)/2$ different entries [11].

**Lemma 3.** There exists an optimal program such that if $R(i)$ is satisfied by some $X_i(i)$ and $P(i)$ is satisfied by some $Y_i(i)$, then $R(i) \land P(i)$, $i = t + 1,\ldots,i - 1$ is also satisfied by $X_i(i)$ and $Y_i(i)$.

**Proof.** Any strategy/program not satisfying this lemma will imply that either $I(\hat{i})$ for period $\hat{i}$ is positive or $I(\tilde{i})$ for period $\tilde{i}$ is brought into some period $\tilde{t}$, $\hat{i} < \tilde{t} < \tilde{i}$, for $X_i(t') > 0$. Again, by the Lemma 1, it is enough to consider strategies/programs in which such conditions do not exist.

The following lemma guaranteed the condition under which we may divide our problem into smaller sub-problems.

**Lemma 4.** Suppose $I(t) = 0$ for period $t$, then examining periods $1$ through $t - 1$ by themselves will still make up an optimal policy.

**Proof.** By all amounts of claims, replacing the $i$ in (22) by $t - 1$ over the entire $T$ period/year, yields the model:

$$\begin{align*}
\text{Minimize} & \sum_{j=1}^{T-1} [S(t-1) \cdot \eta_j, [X_j(t-1)] + Q(t-1) \cdot \beta_j, [Y_j(t-1)] \\
& + K(t - 1) \cdot I(t - 1)]
\end{align*}$$

Subject to:

$$\begin{align*}
\sum_{i=1}^{T-1} R(t - 1) &= \sum_{i=1}^{T-1} \sum_{j=1}^{n} X_j(t - 1) \\
\sum_{i=1}^{T-1} P(t - 1) &= \sum_{i=1}^{T-1} \sum_{j=1}^{n} Y_j(t - 1), i = 1, 2, \ldots, n \text{ grades; } \\
& i = 1, 2, \ldots, T - 1 \text{ periods/year }
\end{align*}$$

Comparing (22) and (23) shows the functional relations only change in the objective function’s summation index, implying that what is optimal for (22) will also be optimal for (23).

This optimality holds for all the earlier periods because of the recursive nature of the model.

If we defined $W(t)$ as the minimal cost strategy/program for periods 1 through $t$. Then

$$W(t) = \min \left[ \sum_{t=1}^{T} [S(t) + Q(t) + \sum_{x=1}^{n} K(x)[R(x) + P(x)] + W(\theta - 1)] \\
+ S(t) + Q(t) + W(t - 1) \right]$$

Subject to:

$$\begin{align*}
\sum_{i=1}^{T} R(t) &= \sum_{i=1}^{T} \sum_{j=1}^{n} X_j(t) \\
\sum_{i=1}^{T} P(t) &= \sum_{i=1}^{T} \sum_{j=1}^{n} Y_j(t), i = 1, 2, \ldots, n \text{ grades; } \\
& i = 1, 2, \ldots, T \text{ periods/year }
\end{align*}$$

The constrained recursion (24) is the DP formulation of (22) indicating that the minimum cost for the first $t$ periods consists of a fixed recruitment and promotion costs in period $\theta$, plus the charges for satisfying recruitments $R(x)$ and promotion $P(x)$, $x = \theta + 1, \ldots, t$ by recruiting and promoting manpower in period $\theta$, which results in overstaffing cost, plus cost of accepting an optimal policy in periods 1 through $\theta - 1$ considered separately.

To help make the resolution of optimal policies a lot easier, we present the driving force of the model; the planning horizon theorem for manpower which is comparable to the famous Wagner-Whitin planning horizon theorem.

**Theorem 1** (The Manpower Planning Horizon). If the minimum in (24) at period $i$ exists for $\theta = i \leq 0$ then for periods $t > i$, it will be necessary to regard only those $i \leq \theta \leq t$. In particular, it can be inferred that if $i = \tilde{t}$, then it will only be necessary to regard only programs that satisfy the conditions $X_i(\tilde{t}) > 0$ and $Y_i(\tilde{t}) > 0$.

**Proof.** Observe that the path to optimality will not degenerate if we limit our focus to policies of the kinds defined in the Lemmas 1 through 4. Suppose a policy demands $R(i)$ and $P(i)$ to satisfy respectively $X_i(i)$ and $Y_i(i)$ where $\tilde{i} < \tilde{t} \leq t < i$. From Lemma 3, the demands $R(i)$ and $P(i)$ are also satisfied respectively by $X_i(\tilde{t})$ and $Y_i(\tilde{t})$. Again, from basic assumptions, re-arranging the program so that $R(\tilde{t})$ and $P(\tilde{t})$ may be guaranteed by the conditions $X_i(\tilde{t}) > 0$ and $Y_i(\tilde{t}) > 0$ respectively, cannot merely increase costs.

The planning horizon theorem for manpower planning partially states that if optimality is not lost for incurring a recruitment and promotion costs in period $\tilde{t}$ while periods 1 through $\tilde{t}$ are determined separately, then allowing the conditions $X_i(\tilde{t}) > 0$ and $Y_i(\tilde{t}) > 0$ in the $T$-period model will not lose optimality. Looking at Lemmas 1 through 4, it is convincing that it is possible to select a best policy for periods 1 through $\tilde{t} - 1$ provided they are separately considered.

**5.5. The DP algorithm**

We present the manpower planning version of the Wagner-Whitin Algorithm (W-WA) to determine the optimal recruitment and promotion policies. Thus, starting at period $\tilde{t}$, $i = 1, 2, \ldots, T$ the steps of the algorithm can generally be outlined as:

**Step 1:** Focus on policies to recruit and promote in period $\tilde{t}, \tilde{t} = 1, 2, \ldots, \tilde{t}$ to fill recruits, $R(t)$ and promoting, $P(t)$, $t = i, 1, \tilde{t} + 1, \ldots, \tilde{t}$ according to these recruitments and promotions.

**Step 2:** Obtain the total cost for these $\tilde{t}$ various decision by summing the $S(t), Q(t)$ and $K(t)$ concerned with making recruitment and promotion at period $\tilde{t}$, including the cost of making periods 1 through $\tilde{t} - 1$ optimal each separately. Note that the cost of making periods 1 through $\tilde{t} - 1$ optimal has been previously determined during the calculations for the periods $t = 1, 2, \ldots, \tilde{t} - 1$.

**Step 3:** Among the $\tilde{t}$ choices, select the minimum-cost Program for the periods 1 through $\tilde{t}$ being considered separately.

**Step 4:** Continue this process to period $\tilde{t} + 1$ and stop when $\tilde{t} = T$. The denotation $\{1, 2, \ldots, \tilde{t}\}$ as presented in Table 1 signifies that recruitment and promotion is carried out in year $\tilde{t} + 1$ to make for the demands $X_i(t)$ and $Y_i(t)$, $t = \tilde{t} + 1, \tilde{t} + 2, \ldots, i = \tilde{t} + 1, \tilde{t}, \ldots, n$, and the optimal policy is taken for years 1 through $T$ considered independently. The “Minimum Cost Plan” for years 1 through $\tilde{t}$ is recorded at the base of the table.

This will lead in general to the necessity of testing $T$ different decision policies at the $T^{th}$ period, meaning that a table of $T(T + 1)/2$ entries with regards to (24) versus $2^{T-1}$ with regards to (22) for all possibilities. Thus the forward/backward algorithm of (24) is at least as sufficient as (22) recursively. But as a matter of necessity, we will observe that the actual sample space (entries) generally come to be smaller than $T(T + 1)/2$, if we make complete use of the manpower planning horizon theorem.
Table 1. A Symbolic Scheme for the DP model Algorithm analogous to W-WA.

| Year/Period | 1      | 2      | 3      | 4      | …… | T      |
|-------------|--------|--------|--------|--------|-----|--------|
| Recruitment Cost, $S(t)$ | $K(t)$ | $Q(t)$ | $K(t)$ | $Q(t)$ | …… | $K(t)$ |
| Promotion Cost, $Q(t)$ | $K(t)$ | $Q(t)$ | $K(t)$ | $Q(t)$ | …… | $K(t)$ |
| Overstaffing Cost, $K(t)$ | $K(t)$ | $Q(t)$ | $K(t)$ | $Q(t)$ | …… | $K(t)$ |
| Demand for Recruitment, $R(t)$ | $R(t)$ | $R(t)$ | $R(t)$ | $R(t)$ | …… | $R(t)$ |
| Demand for Promotions, $P(t)$ | $P(t)$ | $P(t)$ | $P(t)$ | $P(t)$ | …… | $P(t)$ |

(1,2,⋯,j-1) R(1,2,⋯,j-1) = 1 (1,2,⋯,j-2) R(1,2,⋯,j-2) = 1 (1,2,⋯,j-3) R(1,2,⋯,j-3) = 1 (1,2,⋯,j-4) R(1,2,⋯,j-4) = 1

Minimum Cost
Optimal Policy *(1,2,⋯,j)*

(1) (1) (2,3) (2,3,4) …… (2,3,4,5)

Table 2. The Sample set of Hypothesized data for a 10-year planning period.

| Year/Period (i) | 1 | 2 | 3 | 4 | …… | T |
|-----------------|---|---|---|---|-----|---|
| Recruitment Cost, $S(i)$ | 79 | 52 | 61 | 25 | 56 | 24 |
| Period (i) | 1 | 2 | 3 | 4 | 5 | 6 |
| P(i) | 728 | 705 | 714 | 708 | 739 | 729 |
| Min in (ooo's) | 540 | 220 | 385 | 398 | 462 | 216 |
| Q(i) in (ooo's) | 540 | 220 | 385 | 398 | 462 | 216 |
| K(i) in (ooo's) | 540 | 220 | 385 | 398 | 462 | 216 |

Thus, in 2002, the optimal policy is to recruit in 2001 for both years (periods) since 1,928 < 2,193.

Period 3: In 2003, there are three possibilities to assess their values and these are: perform recruitment and promotion in year 2003 for only the year 2003 and rely on the optimal decisions obtained from the previous years 2001 and 2002 considered alone. That is:

\[ S(3) + Q(3) + r^*(2) \Rightarrow 698 + 385 + (1,928) = 3011; \]

or recruit and promote in 2002 for the years 2002 and 2003, and follow the optimal policy of year 2001, that is:

\[ (S(2) + Q(2)) + R(3) + P(3) + r^*(1) \Rightarrow (705 + 220) + 12(52 + 14) + 1268 = 2,985; \]

or recruit and promote in 2001 for the years 2001, 2002 and 2003 i.e.:

\[ (S(1) + Q(1) + R(2) + P(3)) \Rightarrow (728 + 540) + 15 (34 + 10) + 15 (52 + 14) + 12 (52 + 14) + 1268 = 3,704. \]

Thus, in 2003, the best policy is to recruit and promote in 2002 to cover for the years 2002 and 2003, since 2,985 < 3,011 < 3,704.

Period 4: From the Planning-Horizon theorem for manpower, it may be observed that in years $t = 2004, \ldots, 2010$; or periods $t = 4, \ldots, 10$, it is normal without doing harm to the optimality to compute at most three possible decision alternatives. This is always so because, it is irrational (as it would never pay) to recruit and promote in periods 1 or 2 to meet the demands in periods $t = 4, \ldots, 10$, since the overstaffing cost would out sum both the recruitment and promotion costs in those periods, (see Lemma 3). Consequently, for $t = 4$, (i.e. in year 2004) one option is to recruit and promote in 2004 for the year 2004 only but recognizing the optimal policy of 2003, i.e.:

\[ (S(2) + Q(2)) + r^*(3) \Rightarrow (714 + 412) + 2985 = 4,111; \]

or recruit and promote in 2003 for the years 2003 and 2004, and follow the optimal policy of 2002, i.e.:

\[ (S(1) + Q(1) + R(2) + P(3)) \Rightarrow (728 + 540) + 15(34 + 10) + 15(52 + 14) + 12(52 + 14) + 1268 = 3,704. \]
(S(3) + Q(3)) + K(3)[R(4) + P(4)] + r^*(2)
\Rightarrow (698 + 385) + 16(61 + 38) + 1928 = 4595;

or recruit and promote in 2002 for the years 2002, 2003, and 2004 and use the optimal policy of 2001, i.e.:

\[(S(2) + Q(2)) + K(2)(R(3) + P(3)) + K(2)[R(4) + P(4)] + K(3)[R(4) + P(4)] + r^*(1)\]
\Rightarrow (705 + 220) + 12(52 + 14) + 12(61 + 38) + 16(61 + 38) + 1268 = 6,757.

The optimal policy is thus, to recruit and promote in 2004 only for 2004 only since;

4,111 < 4,595 < 6,757.

**Period 5:** In 2005 the available options are, to recruit and promote only in 2005 and keep the best policy of year 2004, i.e.:

\[(S(5) + Q(5)) + r^*(4) \Rightarrow (708 + 398) + 4111 = 5,217;\]

or recruit and promote in 2004 for the years 2004 and 2005, and use the optimal policy of 2003, i.e.:

\[(S(4) + Q(4)) + K(4)[R(5) + P(5)] + r^*(3)\]
\Rightarrow (714 + 412) + 14(25 + 8) + 2985 = 4,573.

Since 4,573 < 5,217, the optimal policy is to recruit and promote in 2004 for both 2004 and 2005.

### 6.2. Interpretation of model results

The sample set of hypothesized data as it stands in Table 2 represents the inputs (Row 2 of Table 1) whose accuracy, completeness and validity were retrospectively reviewed based on past recruitments and promotions histories. The data shows yearly recruitments and promotions, and their related costs for a ten-period (year) planning horizon. This means our T here is equals ten (10). Columns 1 and 2 give the actual number recruited and promoted for each of the ten (10) years or periods, totaling 507 and 266 respectively. Total manpower system cost in thousands of Nigerian Naira (NGN 000's) occasioned by yearly recruitments and promotions exercises for the planning interval is 11334, that is 7092 for recruiting, 4100 for promoting, and 142 for overstaffing as given in Columns 3, 4, and 5 respectively. Table 3 gives the computations summary matrix (output) of the dynamic programming model (2a) with Table 2 as inputs. Row 3 is the actual cost computation results of the model corresponding to each planning year/period. Row 4, Line 1 contains the “minimum cost” values compared to other alternatives in each of this planning year/period. The last value in this line (9462) represents the minimum of the total manpower planning cost for recruiting 507 new staff into the n grades and promoting 266 employees within the remaining n-1 grades in the manpower system. Line 2 gives the “optimal policy” which is strictly dependent on the positions of the minimum cost values. The optimal policy for this forward recursion is:

1. Recruit and promote in 2009 (period 9) to cover 2010 (period 10) which yields:

\[X(9) + Y(9) = (48 + 34) + (26 + 30) = 138;\]

and apply the best policy for the years 2001 through 2008 (periods 1 through 8) meaning;

2. to recruit and promote in 2007 to include 2008, that yields:

\[X(7) + Y(7) = (56 + 29) + (36 + 34) = 155;\]

and apply the best policy for the years 2001 through 2006 (periods 1 – through 6) meaning,

3. recruit and promote in 2006 (period 6) only which yields:

\[X(6) + Y(6) = (89 + 29) = 118;\]

and apply the best policy for the years 2001 through 2005 (periods 1 – through 5) meaning,

4. recruit and promote in 2004 (period 4) to cover 2005 which yields:

\[X(4) + Y(4) = (61 + 25) + (38 + 8) = 132;\]

and apply the best policy for the years 2001 through 2003 (periods 1 through 3) meaning,

5. recruit and promote in 2002 (period 2) to include 2003 which yields:

\[X(2) + Y(2) = (34 + 52) + (10 + 14) = 110;\]

and apply the best policy for the years 2001 (period 1) meaning,

6. recruit and promote” in 2001 (period 1) which yields:

\[X(1) + Y(1) = 79 + 41 = 120.\]

In other words, the optimal policy for the ten-year planning period calls for recruiting and promoting respectively 79 and 41 in period 1 only, 86 and 24 in period 2 for periods 2 and 3, 86 and 46 in period 4 for periods 4 and 5, 89 and 29 in period 6 only, 85 and 70 in period 7 for periods 7 and 8, and 82 and 56 in period 9 for periods 9 and 10. That is, total staff;

120 + 110 + 132 + 118 + 155 + 138 = 773 = 507 recruited + 266 promoted

These figures affirm in principles [34] that, a graded manpower system is controlled by recruitment and by promotion policies. Also, the value 773 is high enough to agree with [7] that over the years, a yearly sharp increase in too many numbers of qualified persons is being observed around the world. This induces yawning increments in recruitment and promotion costs.

According to our DP model this optimal number of recruits and promoted will result to a minimum manpower system cost of 9462 as against the hitherto 11334, a significant cut of 1872. These results are more effective and rational for manpower planning decisions than intuitive ones confirming the opinions of [3, 4, 5] who acknowledged that Solutions to manpower problems are best approached using mathematical optimization methods. Knowing the cost is representing important
information for managing an organization and for controlling its administration [8].

Our model on one hand, like the W-WM recursive algorithm is forward execution, and on the other hand is resolute so that planning for recruitment and promotion costs can be applied to a wide range of graded manpower systems instead of just two grades. While W-WM considers varying inventories, our model allows recruitment and promotions sizes to vary over the $T$ planning horizon. However, managing inventory or production is quite different from planning for manpower. The result of both models minimizes their respective objective function with the former focusing on inventory and the later on manpower system costs.

The study will contribute to the growing literature on applications of OR models to problems in manpower planning. The optimized outcomes are complete encouragement to decision makers in organizations in terms of minimum budget requirements during recruitment and promotion operations. They will also constitute an immense relevance for decision making policies aimed at conducting recruitment promptly to narrow down unemployment, and over stagnated promotions and the general organizational stability.

7. Conclusion

In this study, we first give a model of an n-graded manpower system, established the link between a manpower planning problem, a dynamic decision-making process and a multistage decision-making optimization problem. This linkage resulted to a multistage real-life decision-making problem which we formulated in a DP structure. The formulated model came out to be more robust and actionable, a Constrained Deterministic Dynamic Programming (CDDP) found to function computationally as the well-known Wagner-Whitin Model [11] in the production or inventory management. To achieve the study goals that aim to reduce organizations’ recruitment and promotion costs, we hypothesized data that focuses on five identified variables pertinent to organization’s manpower planning. The accuracy, completeness and validity of these data were retrospectively reviewed based on past recruitment and promotions histories. We used this data as inputs into the modified DP model for validation. The model results show manpower system costs are minimized.

Our model on one hand, like the W-WM recursive algorithm is forward execution, and on the other hand is resolute for minimizing manpower system costs occasioned by recruitments and promotions exercises in a wide range of multi-graded manpower systems instead of just two grades. This model stands to contribute to the growing literature on the applications of OR models to problems in manpower planning. The outcomes could be of immense relevance for decision making policies aimed at conducting recruitment promptly to narrow down unemployment, and reducing over-stagnated promotions.

Manpower planning is among the wide range of the classes of DP applications areas. Often, the public routinely use DP without even realizing it. Our model has ignored different restrictions and functional principles present in a real world manpower system. A very important continuation would be to consider these model limitations including relaxing some of the assumptions. Also, time and spatial location often cause data to vary, and very often affect some of the model results. A possible future approach to handle these potential challenges coming from the volatility in data is to bring in uncertainty explicitly into the model parameters which can be achieved easily by formulating the model from a stochastic optimization perspective.

Declarations

Author contribution statement

Peter Nga Assi: Conceived and designed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper. Efanga Okon Effanga: Conceived and designed the experiments; Contributed reagents, materials, analysis tools or data.

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