A Design Method of Missile Acceleration Autopilot Based on Active Disturbance Rejection Control

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Abstract. In order to solve the problem that the performance of the traditional two-loop acceleration autopilot is relatively large affected by the missile aerodynamic, an improved design method based on active disturbance rejection control (ADRC) is proposed. The ADRC acceleration autopilot adopts the extended state observer (ESO) to estimate the disturbance and eliminate the impact of the model uncertainties and external disturbances, which achieve good interference suppression and accurate tracking of command. Then the pole placement method is used for the ADRC autopilot parameter design. Finally, the ADRC autopilot is compared to traditional two-loop autopilot and two-loop autopilot with PI correction under parameters uncertainty and external disturbance, the simulation results demonstrate that the impact of these two factors on the performance of the ADRC autopilot is relatively small. Therefore, the ADRC autopilot is proved to have fast response speed, good transient process, strong anti-interference ability and robustness.

1. Introduction

In the field of tactical missiles,autopilots have been used successfully for many years. The autopilot can increase the damping and improve the stability and response speed of the missile system. Its primary task is to ensure that the missile accurately and robustly tracks the input command generated by the guidance system, so that the missile can fly as expected. A high-performance autopilot design has always been a challenging problem [1]. Harald [2] proposed a full flight envelop missile autopilot design using gain scheduled control. Reichert [3] designed a missile autopilot using the H(infinity) control theory while ignoring the uncertainty of the system. Huang [4] used sliding mode control in the design of the autopilot, and achieved good control effects, but the feedback linearization process often requires an existing precise nonlinear model. Considering the aerodynamic parameter perturbation and external disturbance, Uang [5] proposed a robust adaptive tracking controller. Based on the missile linearization model, LinCF [6] designed three kinds of autopilots by using LQR, linear quadratic singularity and H(infinity) control theory respectively, these autopilots were further proved effective from the simulation results.

In this paper, an acceleration autopilot design method based on ADRC is proposed. ADRC is a control technology proposed by HAN [7] that can estimate and compensate the uncertain dynamics in the system in real time. On the basis of traditional two-loop acceleration autopilot, the ESO is added to the
inner damping loop to estimate the disturbance of missile body and compensate. Therefore, the steady state error of the autopilot can be eliminated and the system robustness is improved. The pole placement method is further adopted to design the parameters of the ADRC acceleration autopilot. Finally, the performance of the two-loop acceleration autopilot, the two-loop acceleration autopilot with Proportional-Integral (PI) correction and the ADRC acceleration autopilot is compared by simulation and the superiority of the ADRC acceleration autopilot is verified.

2. Two-loop acceleration autopilot

2.1. Dynamic model

Taking the axisymmetric missile as example, ignoring the influence of servo dynamics lag and gravity, a linearization equation in pitching direction is established as [2]:

\[ x = Ax + Bu \]
\[ y = Cx + Du \]

where,

\[ x = \begin{bmatrix} \alpha \\ \dot{\alpha} \end{bmatrix}; y = \begin{bmatrix} a_y \\ \dot{\omega} \end{bmatrix}; u = \delta \]

\[ A = \begin{bmatrix} -b_\alpha & 1 \\ -a_y & -a_\omega \end{bmatrix}; B = \begin{bmatrix} -b_\dot{\alpha} \\ -a_\dot{\omega} \end{bmatrix}; C = \begin{bmatrix} Vb_\alpha - ca_\omega & -ca_\omega \\ 0 & 1 \end{bmatrix}; D = \begin{bmatrix} Vb_\dot{\alpha} - ca_\dot{\omega} \end{bmatrix} \]

\( \alpha \) is the angle of attack, \( \dot{\theta} \) is the pitch angle, \( a_y \) is the pitch acceleration, \( \delta \) is the elevator deflection angle, \( V \) is missile velocity. \( a_\alpha \), \( a_\delta \) and \( a_\omega \) represent the pitch moment coefficients corresponding to angle of attack, elevator deflection angle and pitch rate, respectively. \( b_\alpha \) and \( b_\delta \) represent pitch force coefficients corresponding to angle of attack and rudder angle, respectively.

From Eq.1, we can get the transfer function from the elevator deflection angle to the pitch rate and the transfer function from the pitch rate to the normal acceleration:

\[ \frac{\dot{\delta}(s)}{\delta(s)} = \frac{k_\delta(T_\delta s + 1)}{T_\omega s^2 + 2\mu T_\omega s + 1} \]  

\[ \frac{a_y(s)}{\dot{\delta}(s)} = V\frac{A_y s^2 + A_\delta s + 1}{T_\alpha s + 1} \]

where,

\[ k_\delta = \frac{-a_\delta b_\alpha - a_\alpha b_\delta}{a_\alpha b_\delta + a_\delta}; \quad T_\omega = \frac{1}{\sqrt{a_\omega b_\alpha + a_\alpha}}; \quad \mu_\omega = \frac{(a_\omega + b_\omega)}{2\sqrt{a_\omega b_\alpha + a_\alpha}} \]

\[ A_1 = \frac{-a_\delta b_\delta}{a_\alpha b_\delta - a_\omega b_\delta}; \quad A_2 = \frac{-b_\dot{\alpha}}{a_\delta b_\alpha - a_\omega b_\delta}; \quad T_\alpha = \frac{a_\delta}{(a_\delta b_\alpha - a_\omega b_\delta)} \]

The inner loop of the traditional two-loop acceleration autopilot is a damping loop, which uses angular rate feedback to improve the system damping. The outer loop is an acceleration feedback loop to realize the tracking of the acceleration command \( a_{yc} \). The structure diagram of the autopilot is shown in Fig 1.
The two-loop acceleration autopilot usually adopts the input command gain compensation method to eliminate the steady state error existing in the tracking of the system. However, when the aerodynamic parameters of the missile change, the closed-loop gain of the system also changes, and then the tracking steady state error cannot be eliminated. Therefore, the two-loop acceleration autopilot with PI correction is widely used in the engineering design, which can eliminate the tracking steady state error of the autopilot. The structure diagram of the two-loop acceleration autopilot with PI correction is shown in Fig. 2.

The two-loop acceleration autopilot with PI correction provides a large gain at low frequency, which makes the closed-loop system gain of the autopilot close to 1, eliminates the steady state error of the original system, but also slows down the response of the original system, thus deteriorating the robustness.

3. ADRC acceleration autopilot design

3.1. Extended state observer design

The pitch angular acceleration model of the missile can be obtained from Eq. 1:

\[ \dot{\alpha} = -a_a \cdot \dot{\alpha} - a_a \cdot \alpha - a_\delta \cdot \delta \]  

According to ADRC theory, the control model can be rewritten. Define \( f(t) = -a_a \cdot \dot{\alpha} - a_a \cdot \alpha \), \( f(t) \) represents angular acceleration of disturbing motion without elevator influence, if \( b = -a_\delta \), \( b \) represents the control gain coefficient, then the Eq. 4 can be described as follows:

\[ \dot{\alpha} = f(t) + bu \]  

Define the state variable \( x_1 = \dot{\alpha} \), \( x_2 = f(t) \) and measured variable \( y = \dot{\alpha} \), then the state equation and the measurement equation of the damping inner loop are:

\[
\begin{align*}
\dot{x}_1 &= x_1 + bu \\
y &= x_1
\end{align*}
\]  

The second-order ESO of the damping loop is designed as follows:
\[
\begin{cases}
z_j = z_2 + b_0 u - l_1 (z_1 - y) \\
z_2 = -l_2 (z_1 - y)
\end{cases}
\] (7)

Where \( z_1 \) represents an estimate of state \( x_1 \), \( z_2 \) represents an estimate of \( f(t) \), and \( b_0 \) is an estimate of the control gain \( b \). If the bandwidth coefficient is \( \omega_0[8] \), adopting the pole placement method to place the poles of the observer to the same \(-\omega_0 \), then the observer parameters can be designed as \( l_1 = 2\omega_0 \), \( l_2 = \omega_0^2 \).

The traditional two-loop acceleration autopilot is improved through adopting ADRC in the inner damping loop, using the ESO to estimate the internal and external disturbances of the system, then utilizing the error feedback control law to compensate. Its outer loop remains the acceleration feedback loop. The diagram of ADRC acceleration autopilot is shown in Fig. 3.

The control law of the two-loop acceleration autopilot is proportional-differential (PD) control. The control law form is as follows:

\[
u_0 = k_w (k_c a_{cc} - a_c) - k_k \dot{y} = r - k_y y
\] (8)

Where \( r \) represents the damping loop input command. After using the ESO estimation to obtain the internal and external disturbances \( z_2 \), we can get control law as follows:

\[u = \frac{u_0 - z_2}{b_0}\] (9)

Then, the Eq.5 can be written as:

\[\dot{y} = f(t) + bu = f(t) + b \cdot \frac{u_0 - z_2}{b_0} \approx u_0\] (10)

Therefore the forward channel of inner loop becomes the first-order integral and the closed-loop system transfer function is:

\[
a_y = \frac{k_w V_k w_c \left(A_2 s^2 + A_s + 1\right)}{a_c \left(T_a + A_2 V k_w\right) s^2 + \left(T_a k_g + A_V k_w + 1\right) s + k_g + V k_w}
\] (11)

If the bandwidth coefficient of the system is \( \omega_c \), then the control parameters are:

\[
k_g = \frac{2\omega T_a + \omega^2 A_2 - A_4 \omega A_2 - 1}{T_a \left(1 - \omega^2 A_2\right) + 2\omega A_2 - A_4}
\] (12)

\[
k_{ac} = \frac{\omega^2 T_a - k_g}{V - \omega^2 A_2 V} = \frac{\omega^2 T_a - k_g}{\left(1 - \omega^2 A_2\right) V}
\] (13)
When the system reaches steady state, i.e., $s \to 0$, the closed-loop steady-state gain $K$ of the ADRC acceleration autopilot is:

$$
K = \frac{k V_{ac}}{k_g + V k_{ac}}
$$

(14)

According to Eq. 14, the $K$ is determined by $k_c$, $k_{ac}$, $k_g$ and $V$, which means it relates to the autopilot design parameters and the missile speed, and is independent of the aerodynamic parameters of the missile. Even if the aerodynamic parameters of the missile change and external disturbances exist, the gain of the autopilot is constant. If $K=1$, then the forward channel compensation gain coefficient can be obtained:

$$
k_c = \frac{k_c + V k_{ac}}{V k_{ac}}
$$

(15)

4. Simulation analysis

A set of typical missile dynamics parameters is selected for simulation [2], which are shown in Table 1. Considering the servo is a typical second-order system, based on the current hardware technology level, the servo frequency and damping are set as $\omega_s = 220$ rad/s and $\xi_s = 0.65$, respectively.

| $V/(m/s)$ | $a_a/s^2$ | $a_\delta/s^2$ | $a_a/s^2$ | $b_a/s^2$ | $b_\delta/s^2$ |
|-----------|-----------|----------------|-----------|-----------|----------------|
| 914.4     | 250       | 280            | 1.5       | 1.6       | 0.23           |

Considering the control bandwidth coefficient is set as 20 rad/s for the two-loop acceleration autopilot, the two-loop acceleration autopilot with PI correction and the ADRC acceleration autopilot. The observer bandwidth coefficient of the ADRC autopilot is set as 200 rad/s and the corrective parameter of the two-loop acceleration autopilot with PI correction is designed as $T_i = 0.06$.

4.1. Dynamic parameter deviation

The dynamic coefficients $a_a$ and $a_\delta$ are the main factors affecting the pitch channel. These two parameters are biased by 30% respectively and the step response results for different autopilots are shown in Fig 4 and Fig 5.

![Figure 4. Step response when $a_a$ changes](image-url)
It can be seen from Fig. 4 and Fig. 5, when $a_\alpha$ or $a_\delta$ changes, the closed-loop gain of the two-loop acceleration autopilot is no longer 1, and the steady state error is obvious. In terms of the two-loop acceleration autopilot with PI correction, the closed-loop gain is guaranteed to be 1 when $a_\alpha$ or $a_\delta$ changes, however there is an overshoot, while the response speed is reduced and the steady state time is greatly increased. Nevertheless, the $a_\alpha$ or $a_\delta$ change has little effect on the step response of the ADRC acceleration autopilot, the closed-loop gain of the autopilot remains 1, the steady state time is relatively short and the response speed is similar to the two-loop acceleration autopilot.

4.2. External disturbance analysis

The impacts of external disturbances on the performance of the three autopilots are taken into further consideration. Considering the external constant disturbance angular acceleration $\Delta f=0.5\text{rad/s}^2$, the step response results are shown in Fig. 6. Considering the servo zero error $\Delta \delta=0.2^\circ/\text{s}$, the response results of these autopilots’ zero command input are shown in Fig. 7.
Figure 7. Autopilots response with a constant servo zero error

It can be seen from the simulation results that when there are external disturbances, the closed-loop gain of the two-loop autopilot has a large deviation; the two-loop autopilot with PI correction can correct the closed-loop gain to the desired state, but the overshoot is relatively large and the transient process is time-consuming. Compared with these autopilots, the ADRC acceleration autopilot can eliminate disturbance quickly and shows a fast response and good transient process by estimating the external disturbance and compensating it.

5. Conclusion

In this paper, an improved autopilot design method based on ADRC is proposed. Based on the traditional two-loop acceleration autopilot, the ADRC autopilot adopts ESO to estimate the disturbance and eliminate the impact of the model uncertainties and external disturbances. The pole placement method is also used for the autopilot parameter design. After comparing with the other two kinds of autopilots, the ADRC autopilot achieves zero steady state error tracking under various deviations and shows its fast response speed, good transient process, strong robustness and prospects in engineering application.

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