Antenna Array Aperture Resource Management of Opportunistic Array Radar for Multiple Target Tracking

QINGHUA HAN1, YUANSHI ZHANG2, ZHEN YANG1, WEIJUN LONG2,3, AND ZHIHENG LIANG4

1College of Artificial Intelligence, Zaozhuang University, Zaozhuang 277160, China
2College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China
3The 14th Research Institute of China Electronics Technology Group Corporation, Nanjing 210039, China
4Department of Precision Instrument, School of Mechanical Engineering, Tsinghua University, Beijing 100084, China

Corresponding author: Qinghua Han (hanqinghua123@163.com)

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ABSTRACT
In this paper, an antenna array aperture resource management scheme of opportunistic array radar (OAR) based on chance-constraint programming (CCP) is proposed for multiple target tracking (MTT). In the multi-target tracking scenario, multiple beams need to be synthesized to illuminate the targets. According to the predicted distance of targets, the OAR can intelligently segment the antenna array aperture, and select the optimal combination of array elements to synthesize the satisfactory beams with the element-level antenna steering. Owing to the unique array arrangement and work mode of opportunistic array, the element number in working modes is uncertain. To represent the uncertainty, a fuzzy CCP model is introduced. The element number in working modes is viewed as a fuzzy variable. The maximum tracking error, which is denoted by Bayesian Cramér-Rao lower bound (BCRLB), of the targets is regarded as the objective function. The fuzzy simulation algorithm is embedded into genetic algorithm (GA) to compose a hybrid intelligent optimization algorithm to solve the CCP problem. The simulation results verify the validity and practicability of the antenna array aperture resource allocation scheme.

INDEX TERMS
Antenna array aperture resource management, Bayesian Cramér-Rao lower bound (BCRLB), chance-constraint programming (CCP), opportunistic array radar (OAR).

I. INTRODUCTION
A. BACKGROUND AND MOTIVATION
The problem of multiple target tracking (MTT) has always been receiving considerable attention in military field [1]–[4]. Through generating multiple orthogonal beams simultaneously, many modern avionics systems have the ability to track multiple targets with different working modes. According to the traditional method of pattern synthesis, in order to synthesize multiple beams, multiple antenna arrays are needed [5]. However the antenna array is limited in mobile platforms operating over prolonged time periods. Nonetheless as a new system radar assigning the array elements arbitrarily and aperiodically over the available open areas of entire 3-D space on the platform [6], [7], the opportunistic array radar (OAR) can segment the antenna aperture and control the antenna elements at the element-level to synthesize the satisfactory pattern on the condition that the specified confidence level is satisfied [8]–[10]. Different from the node selection [11], the resource management of antenna aperture is for one antenna array, and we segment the antenna array and allocate the antenna elements. Whereas the node selection of radars is for multiple radars.

Up to now, there exists some work involving the antenna array aperture resource management. The advanced multifunction radio frequency concept (AMRFC) is proposed [12]. Through segmenting the antenna aperture, the architecture can transmit and receive multiple independent beams simultaneously for radar, electronic countermeasures and communication. Aiming at the radar antenna array dynamic
segmentation, the multiple task parallel earliest deadline first (MTPEDF) algorithm is proposed to achieve adaptive scheduling of the system [13]. In [14], the characteristic of the dynamic aperture segmentation task is first analyzed, and the resource management problem is presented conditional on two-dimensional time and aperture by introducing the rectangular layout idea in mechanical engineering. Although all the aforementioned papers touch on the antenna array aperture segmentation, they do not consider the optimal selection of antenna elements in working modes. Under some conditions, the sparse array can achieve or exceed the performance of full antenna array with the equal aperture length. The genetic algorithm (GA) is adopted to optimize antenna array to minimize the peak side lobe level. This algorithm can obtain the side lobe level less than −20 dB in the case of thinning antenna array [15]. An improved real-code GA, which includes the multiple constrains of elements number, aperture and minimal element space, is proposed for the sparse liner array [16]. Different from the genetic variables and the fixed corresponding relationship between codes in standard GA, this algorithm uses the code reset of genetic variables to avoid the infeasible solutions and reduce the search area of GA. In [17], [18], the problem of pattern synthesis of antenna array is equivalently transformed into the recovery problem of sparse signal under linear constraint. This algorithm can determine the minimum of elements number intelligently conditional on satisfying the antenna constraints. The above algorithms optimize and decrease the number of working elements. However, in the papers, the aperture length of antenna array is restricted by the antenna array arrangement, and the antenna array aperture segmentation is not considered either. Besides, all the above aperture resource management algorithms only stop at pattern synthesis, but generating the satisfactory beams is not the final purpose. In the actual working process, the synthesized beams should be used to execute various tasks such as search, track, verification, identification friend or foe and fire control. Based on these, this paper studies the antenna array aperture resource management of OAR for MTT.

In the traditional resource allocation process, the radar cross section (RCS) is viewed as a fixed value [19], [20]. However, owing to the complex environments and unknown target information, the RCS is impacted by many factors, such as the clutter, the attribute and attitude of targets, observation angle, etc. The RCS is an uncertain value within a certain range [21]. Some scholars treat the uncertainty of RCS as a random variable [22]. But the random distribution law is based on a large number of statistical data, and the historical data may be biased for the lack of data, leading to inaccuracy results. Combined with historical data and relevant experience of experts, it is easy to determine the most likely value and the range of possible distribution. Therefore, we use fuzzy variables to represent the uncertainty. To build the resource management model including the fuzzy variable, the fuzzy chance-constraint programming (CCP) is brought in. The fuzzy constraints holds at a desired confidence level [23]. Due to the environments, resource and desired tracking performance, the confidence level could be adjusted flexibly. That is to say, the CCP can balance the radar resource and tracking error.

This work is a further research of the previous work of authors [24], [25]. In reference [24], the authors studied the antenna array aperture resource management of OAR for pattern synthesis. But generating beams is not our ultimate goal. On the basis of [24], we use the synthesized beams for MTT. Through the predicted target information, we can reasonably allocate the antenna array aperture resource. Reference [25] only researches the allocation of power and beams, and it does not involve the pattern synthesis.

Through the aforementioned analysis, we propose an antenna array aperture resource management scheme based on CCP for MTT in OAR system. The predicted distance of each target is considered as a criterion to segment the antenna array aperture. Then we utilize pattern synthesis to generate the optimal beams. All the beams are used to track the targets. In the tracking process, the Bayesian Cramér-Rao lower bound (BCRLB) is calculated to measure the error for target state estimation, and it provides us a criterion to preallocate radar resource. The CCP encapsulates the determined resource management model as an uncertain model. A hybrid intelligent optimization algorithm (HIOA) is formed to solve the fuzzy programming problem.

B. MAIN CONTRIBUTIONS

The main contributions of this paper are as follows:

1) The resource management of antenna array aperture is studied. Due to the limitation of traditional antenna, in the previous papers the research contents of resource management usually consist of power, time, beams, waveform, etc. The resource management of antenna array aperture is seldom involved. However, the optimal allocation of antenna elements can result in the optimal allocation of power, beams, waveform, etc. Owing to the unique array arrangement and work mode of the opportunistic array, the element number and work mode can be optimized in pattern synthesis.

2) The pattern synthesis and multiple target tracking are researched together. The pattern synthesis is embedded into the process of target tracking. The synthesized beams are used for multiple target tracking. Through optimizing the tracking performance of targets, the antenna aperture and element number are optimized. The optimal allocation of antenna aperture improves the tracking performance further in turn.

3) The fuzzy CCP is introduced to deal with the uncertainty of target information in tracking process. We treat the target RCS as a fuzzy variable. Through adjusting the confidence level of CCP model, the antenna resource and tracking performance can be balanced in different scenarios.

The remainder of this paper is structured as follows. The system mode is provided in Section II. In Section III, the CCP model of antenna aperture resource management is formulated. Section IV gives the resource allocation process. To show the superiority of this algorithm, the simulation results
According to the target states, the following equation of each target.

In order to reasonably segment the antenna array aperture before optimizing each subarray. The antenna array aperture length that each task occupies is different. In general, it can be obtained according to the distance of each target. In order to reasonably segment the antenna aperture according to the target states, the following equation can be selected approximately.

\[
\frac{d_{i,k}}{d_{j,k}} = \left(\frac{R_{i,k}}{R_{j,k}}\right)^{\frac{2}{T}}
\]

where at the \(k\)th sampling instant, \(R_{i,k}\) and \(R_{j,k}\) are the predicted distance of the \(i\)th target and the \(j\)th target, and the length of the antenna array aperture are \(d_{i,k}\) and \(d_{j,k}\), respectively, \(i, j = 1, 2, \ldots, Q\).

At the \(k\)th sampling instant, the pattern are synthesized in accordance with the optimal allocation vector \(D_{k,\text{opt}}\) of the antenna array aperture resource of OAR. The decision vector \(D_{k}\) is shown as follows:

\[
D_{k} = \left[D_{1,k}^T, D_{2,k}^T, \ldots, D_{Q,k}^T\right]^T
\]

\[
= \left[d_{1}^{1}, \ldots, d_{N_{1,k}}^{1}, d_{2}^1, \ldots, d_{N_{2,k}}^2, \ldots, d_{Q}^Q, \ldots, d_{N_{Q,k}}^Q\right]^T
\]

where \(D_{q,k} = \left[d_{1}^{q}, \ldots, d_{N_{q,k}}^{q}\right]^T\) is the antenna subarray vector of the \(q\)th target at the \(k\)th sampling instant; the total number of the elements is \(N_{q,k}\), and \(\sum_{q=1}^{Q} N_{q,k} = N\); \([\cdot]^T\) denotes matrix transposition; \(d_{q}^{q}\) denotes the \(q\)th array element which belongs to the \(q\)th antenna subarray: \(d_{q}^{1} = 1\) denotes the working state and \(d_{q}^{0} = 0\) denotes the closed state.

For a subarray, the coordinates of the array elements are \((x_{i}^{q}, y_{i}^{q}, z_{i}^{q})\), where \(i = 1, 2, \ldots, N_{q,k}\). Suppose that all the array elements are isotropous. The pattern function of the \(q\)th subarray is

\[
p_{q}^{q}(\theta^{q}, \varphi^{q}) = \sum_{i=1}^{N_{q,k}} d_{i}^{q} I_{i}^{q} \exp \left(i2\pi \frac{c r_{i}^{q} (\theta^{q}, \varphi^{q})}{\lambda} \right) \exp \left(i\psi_{i}^{q} \right)
\]

where \(\theta^{q}\) and \(\varphi^{q}\) denote the elevation and azimuth of the \(q\)th target; \(I_{i}^{q}\) and \(\psi_{i}^{q}\) denote the current amplitude and current phase of the \(i\)th element in the \(q\)th subarray, respectively; \(c\) denotes the speed of light; \(\lambda\) denotes the wavelength; compared with phase reference point, the expression of time delay \(\tau_{i}^{q}(\theta^{q}, \varphi^{q})\) of the \(i\)th elements is

\[
\tau_{i}^{q}(\theta^{q}, \varphi^{q}) = \frac{x_{i}^{q} \sin \theta^{q} \cos \varphi^{q} + y_{i}^{q} \sin \theta^{q} \sin \varphi^{q} + z_{i}^{q} \cos \theta^{q}}{c}
\]

\[
A. PATTERN SYNTHESIS
\]

The desired beams must be synthesized before the targets are illuminated. Suppose that the array elements are distributed randomly. We should determine that how to segment the antenna array aperture before optimizing each subarray. The antenna array aperture length that each task occupies is different. In general, it can be obtained according to the distance of each target. In order to reasonably segment the antenna aperture according to the target states, the following equation

\[
\frac{d_{i,k}}{d_{j,k}} = \left(\frac{R_{i,k}}{R_{j,k}}\right)^{\frac{2}{T}}
\]

and corresponding analysis are provided in Section V. Finally, we conclude this paper in Section VI.

\section*{II. SYSTEM MODEL}

Consider a collocated OAR system in a 2-dimensional multiple targets tracking scenario. The radar is located at \((x_{0}, y_{0})\).

Set \(T_{0}\) to denote the time interval of successive frames, which is also viewed as the tracking interval. The \(q\)th \((q = 1, 2, \ldots, Q\), \(Q \geq 2)\) target at time \(kT_{0}\) is located at \((x_{q,k}, y_{q,k})\) \((k = 0, 1, 2, \ldots)\) with an the speed of \((\dot{x}_{q,k}, \dot{y}_{q,k})\). Since the whole antenna array can be segmented into several subarrays according to the tracking scenario, the OAR has the ability to simultaneously generate multiple beams to cover all the targets. In order to ensure that the beams and the targets are one-to-one, the targets are widely distributed in the surveillance region. The sketch map of the antenna array aperture resource management is shown in Fig. 1.

Fig. 2. gives the block diagram of antenna resource allocation schematic corresponding to Fig. 1.

\section*{A. PATTERN SYNTHESIS}

The target moves uniformly in \(xy\) plane:

\[
x_{q,k} = F_{q}x_{q,k-1} + u_{q,k-1}
\]

where \(x_{q,k} = [x_{q,k}, \dot{x}_{q,k}, y_{q,k}, \dot{y}_{q,k}]^T\) denotes the state vector. The state transition matrix \(F_{q}\) is:

\[
F_{q} = I_{2} \otimes \begin{bmatrix} 1 & T_{0} \\ 0 & 1 \end{bmatrix}
\]
where $\mathbf{I}_2$ denotes an identity matrix of order 2; $\otimes$ is the Kronecker operator; $T_0$ is the sampling interval. $u_{q,k-1}$ represents a process noise, and it is supposed to be a zero-mean Gaussian noise with a known covariance:

$$Q_{q,k-1} = \varrho_q \cdot \mathbf{I}_2 \otimes \begin{bmatrix} \frac{1}{2} T_0^3 & \frac{1}{2} T_0^2 \\ \frac{1}{2} T_0^2 & T_0 \end{bmatrix}$$  \hspace{1cm} (7)

where $\varrho_q$ denotes the process noise intensity [26].

The state transition for the $q$th target channel is assumed to be a first order Markovian process, and it is described by the following equation[27]:

$$h_{q,k} = h_{q,k-1} + u_{q,k-1}^h$$  \hspace{1cm} (8)

where $u_{q,k-1}^h$ is a white Gaussian with a known covariance matrix $Q_{q,k-1}. h_{q,k} = [h^R_{q,k}, h^I_{q,k}]^T$ denotes the channel state vector. We form an extended state vector by concatenating the target state vector and the channel state vector into a single vector of dimension $n_q+2$ ($n_q$ is the dimension of state vector) defined as $\xi_{q,k} = [x_T^q, h_T^q]^T$. The state transition equation for $\xi_{q,k}$ is given as

$$\xi_{q,k} = \mathbf{F}_q^x \xi_{q,k-1} + \zeta_{q,k-1}$$  \hspace{1cm} (9)

where the overall state transition matrix is given as

$$\mathbf{F}_q^x = \begin{bmatrix} \mathbf{F}_q & 0_{2 \times n_q} \\ 0_{n_q \times 2} & \mathbf{I}_2 \end{bmatrix}$$  \hspace{1cm} (10)

and $\zeta_{q,k-1}$ is the additive white Gaussian noise with covariance matrix.

$$Q_{q,k-1}^x = \text{blkdiag} \left\{ Q_{q,k-1}, Q_{q,k-1}^h \right\}$$  \hspace{1cm} (11)

where blkdiag(•) denotes the block diagonal matrix. Henceforth, when we say state vector, we refer to the extended state vector formed by concatenating the target state and the channel state.

### C. MEASUREMENT MODEL

The received signal is the attenuation of the transmitted signal. The baseband representation of the echo signal for the $q$th target at the $k$th sample interval is:

$$r_{q,k} (t) = h_{q,k} \sqrt{\alpha_{q,k}} p_{q,k} S_{q,k} (t - \tau_{q,k}) \exp (-j 2 \pi f_{q,k} t)$$

$$+ \sigma_{q,k} (t)$$  \hspace{1cm} (12)

where $h_{q,k}$ denotes the target RCS, which is a fuzzy variable[28]. $\alpha_{q,k} \propto 1/R_{q,k}^4$ denotes the variation of the signal strength due to path loss along the path of OAR - target $q$ - OAR. $p_{q,k}$ denotes the transmitted power to the $q$th target at the $k$th sample interval. $S_{q,k} (t)$ denotes the complex envelope of the transmitted signal. Its effective bandwidth and effective time duration can be calculated in [26]. The time delay is $\tau_{q,k}$ and the Doppler frequency is $f_{q,k}$. $\sigma_{q,k} (t)$ is a zero-mean, complex Gaussian white noise.

According to (12), the relationship between the azimuth and the state vector can be described as:

$$\theta_{q,k} = \arctan \left( \frac{y_{q,k} - y_0}{x_{q,k} - x_0} \right)$$  \hspace{1cm} (13)

The size of the antenna is neglected relative to the distance between the antenna and the targets. Meanwhile considering the RCS as a measurement, the measurement equation of the $q$th target is represented as:

$$z_{q,k} = h (\xi_{q,k}) + w_{q,k}$$  \hspace{1cm} (14)

where

$$h_{q,k} (\xi_{q,k}) = \begin{bmatrix} \theta_{q,k}, h^R_{q,k}, h^I_{q,k} \end{bmatrix}^T$$  \hspace{1cm} (15)

where $h^R_{q,k}$ and $h^I_{q,k}$ are denoted as:

$$h^R_{q,k} = \begin{bmatrix} e_{q,i+1}^T \xi_{q,k} \\ e_{q,i+2}^T \xi_{q,k} \end{bmatrix}$$  \hspace{1cm} (16)

and $e_i$ is a zero vector of length $i$ with the $j$th element to be one.

$w_{q,k}$ is a zero-mean Gaussian white noise with a covariance

$$\Sigma_{q,k} = \text{blkdiag} \left( \sigma_{q,k}^2, \sigma_{q,k}^2, \sigma_{q,k}^2 \right)$$  \hspace{1cm} (17)

where blkdiag denotes a block diagonal matrix. $\sigma_{q,k}^2$, $\sigma_{q,k}^2$, and $\sigma_{q,k}^2$ are the BCRLBs of the azimuth and RCS at high signal-to-noise ratio (SNR) [19], [29].

$$\begin{cases} \sigma_{\theta_{q,k}}^2 = 3 B_{q,k}^{\text{NN}} \left/ \left( 8 \pi^2 \cdot \text{SNR}_{q,k} \right) \right. \\
\sigma_{h_{q,k}^R}^2 = \sigma_{h_{q,k}^I}^2 = \sigma_{\theta_{q,k}}^2 \left/ \left( 2 \alpha_{q,k} p_{q,k} \right) \right. \end{cases}$$  \hspace{1cm} (18)

where $B_{q,k}^{\text{NN}}$ is the null-to-null beamwidth of the receiver antenna. The shorter the working wavelength and the larger the antenna aperture, the higher the measuring accuracy of angle [26]. The longer the coherent integration time, the higher the SNR [29]. The SNR of the echo signal from the $q$th target at the $k$th sample instant[30]:

$$\text{SNR}_{q,k} \propto \alpha_{q,k} \left| h_{q,k} \right|^2 p_{q,k} \propto \left| h_{q,k} \right|^2 p_{q,k} / R_{q,k}^4$$  \hspace{1cm} (19)

On the whole, the probabilistic density function (PDF) of each moment can be iteratively calculated through (9) and (14). Then the state $\hat{\xi}_{q,k}$ at each sampling instant is estimated.

### III. CHANCE-CONSTRAINT PROGRAMMING MODEL FOR RESOURCE MANAGEMENT

#### A. BAYESIAN CRAMÉR–RAO LOWER BOUND

Let $\hat{\xi}_{q,k}$ be an estimate of $\xi_{q,k}$, which is a function of measurement vector $Z_{q,k}$. The Bayesian Cramér-Rao inequality shows that the MSE of any estimator is always greater than the BCRLB $C_{q,k}^{\text{BCRLB}}$ [32]

$$E_{\xi_{q,k}} \left[ (\hat{\xi}_{q,k} - \xi_{q,k}) (\hat{\xi}_{q,k} - \xi_{q,k})^T \right]$$

$$\geq C_{q,k}^{\text{BCRLB}} (\hat{\xi}_{q,k}) = J^{-1} (\hat{\xi}_{q,k})$$  \hspace{1cm} (20)
where $E_{\xi,q_k,z_{q,k}}$ denotes the mathematical expectation over $\xi_{q,k}$ and $Z_{q,k}$.

The BIM $J(\xi_{q,k})$ is shown as follows:
\[
J(\xi_{q,k}) = E_{\xi_{q,k},z_{q,k}} \left( \begin{pmatrix} \frac{\partial \ln p(z_{q,k},\xi_{q,k})}{\partial \xi_{q,k}} \\ \frac{\partial \ln p(z_{q,k},\xi_{q,k})}{\partial \xi_{q,k}} \end{pmatrix}^T \right)
\]  

(21)

The $p(Z_{q,k},\xi_{q,k})$ is a joint PDF, which can be factorized into the following PDF $p(\xi_{q,k})$ and conditional PDF $p(z_{q,k}|\xi_{q,k})$:
\[
p(z_{q,k},\xi_{q,k}) = p(\xi_{q,k}) p(z_{q,k}|\xi_{q,k})
\]  

(22)

An excellent recursive algorithm is introduced to calculate the BIM $J(\xi_{q,k})$ [31]. This algorithm avoids managing the large matrices at each sampling instant. The expression of BIM $J(\xi_{q,k})$ is:
\[
J(\xi_{q,k}) = E_{\xi_{q,k}} \left( \begin{pmatrix} \frac{\partial \ln p(\xi_{q,k})}{\partial \xi_{q,k}} \\ \frac{\partial \ln p(\xi_{q,k})}{\partial \xi_{q,k}} \end{pmatrix}^T \right)
\]
\[
+ E_{\xi_{q,k},z_{q,k}} \left( \begin{pmatrix} \frac{\partial \ln p(z_{q,k}|\xi_{q,k})}{\partial \xi_{q,k}} \\ \frac{\partial \ln p(z_{q,k}|\xi_{q,k})}{\partial \xi_{q,k}} \end{pmatrix}^T \right)
\]  

(23)

where $J_P(\xi_{q,k})$ is the prior information matrix and $J_D(\xi_{q,k})$ is the data information matrix.

1) PRIOR INFORMATION MATRIX $J_P(\xi_{q,k})$

In accordance with [31], the prior information matrix $J_P(\xi_{q,k})$ can be described as:
\[
J_P(\xi_{q,k}) = D_{k-1}^{22} - D_{k-1}^{21} \left( J(\xi_{q,k-1}) + D_{k-1}^{11} \right)^{-1} D_{k-1}^{12}
\]  

(24)

Since the motion equation (9) is linear and Gaussian, the matrices $D_{k-1}^{11}, D_{k-1}^{12}$ and $D_{k-1}^{22}$ can be computed.
\[
\begin{pmatrix}
D_{k-1}^{11} &=& \left( F_{q,k-1}^T \right)^T \\
D_{k-1}^{12} &=& - \left( F_{q,k-1}^T \right)^T \left( Q_{q,k-1}^T \right)^{-1} \\
D_{k-1}^{22} &=& \left( Q_{q,k-1}^T \right)^{-1}
\end{pmatrix}
\]  

(25)

Bring (25) into (24), and $J_P(\xi_{q,k})$ can be derived in light of the matrix inversion lemma[33]:
\[
J_P(\xi_{q,k}) = \left[ Q_{q,k-1}^T + F_{q}^T J^{-1} (\xi_{q,k-1}) \left( F_{q}^T \right)^T \right]^{-1}
\]  

(26)

where the $J(\xi_{q,k})$ can be derived according to the prior information of targets.

2) DATA INFORMATION MATRIX $J_D(\xi_{q,k})$

According to (23), the $J_D(\xi_{q,k})$ can be rewritten as[32]:
\[
J_D(\xi_{q,k}) = E_{z_{q,k}} \left\{ \left( \begin{pmatrix} \frac{\partial \ln p(z_{q,k}|\xi_{q,k})}{\partial \xi_{q,k}} \\ \frac{\partial \ln p(z_{q,k}|\xi_{q,k})}{\partial \xi_{q,k}} \end{pmatrix}^T \right) \times \left( \begin{pmatrix} \frac{\partial \ln p(z_{q,k}|\xi_{q,k})}{\partial \xi_{q,k}} \\ \frac{\partial \ln p(z_{q,k}|\xi_{q,k})}{\partial \xi_{q,k}} \end{pmatrix}^T \right) \right\}
\]  

(27)

According to the derivative rule of the composite function, the above expectation can be transformed into:
\[
E_{z_{q,k}} \left\{ \left( \begin{pmatrix} \frac{\partial h_{q,k}^T}{\partial \xi_{q,k}} \\ \frac{\partial h_{q,k}^T}{\partial \xi_{q,k}} \end{pmatrix} \times \left( \begin{pmatrix} \frac{\partial \ln p(z_{q,k}|\xi_{q,k})}{\partial \xi_{q,k}} \\ \frac{\partial \ln p(z_{q,k}|\xi_{q,k})}{\partial \xi_{q,k}} \end{pmatrix}^T \right) \right) \right\}
\]  

(28)

where $h_{q,k}$ is the measurement.

Extract $\frac{\partial h_{q,k}^T}{\partial \xi_{q,k}}$ out of $E_{z_{q,k}}$|${\xi_{q,k}}$. The new $E_{z_{q,k}}$|${\xi_{q,k}}$ is a classic Fisher information matrix (FIM) whose purpose is to estimate $h_{q,k}$ with measurement $z_{q,k}$. According to [32], we get:
\[
\Sigma_{q,k}^{-1} = E_{z_{q,k}} \left\{ \left( \begin{pmatrix} \frac{\partial \ln p(z_{q,k}|h_{q,k})}{\partial h_{q,k}} \\ \frac{\partial \ln p(z_{q,k}|h_{q,k})}{\partial h_{q,k}} \end{pmatrix}^T \right) \times \left( \begin{pmatrix} \frac{\partial \ln p(z_{q,k}|h_{q,k})}{\partial h_{q,k}} \\ \frac{\partial \ln p(z_{q,k}|h_{q,k})}{\partial h_{q,k}} \end{pmatrix}^T \right) \right\}
\]  

(29)

$H_{q,k} = \left[ \frac{\partial h_{q,k}^T}{\partial \xi_{q,k}} \right]^{T} T$ is a matrix of $n_x \times (n_x + 2)$. Then (27) can be rewritten as:
\[
J_D(\xi_{q,k}) = E_{\xi_{q,k}} \left\{ H_{q,k}^T \Sigma_{q,k}^{-1} H_{q,k} \right\}
\]  

(30)

The completed BIM $J(\xi_{q,k})$ can be denoted as[34]:
\[
J(\xi_{q,k}) = J_P(\xi_{q,k}) + J_D(\xi_{q,k})
\]  

\[
= \left[ Q_{q,k-1}^T + F_{q}^T J^{-1} (\xi_{q,k-1}) \left( F_{q}^T \right)^T \right]^{-1}
\]
\[
+ H_{q,k}^T \Sigma_{q,k}^{-1} H_{q,k}\right|_{\xi_{q,k,k-1}}
\]  

(31)

B. MODELING OF FUZZY CCP FOR ANTENNA ARRAY APERTURE RESOURCE

Through the derived $J(\xi_{q,k})$ in section IIIA, the BCRB $C_{q,k}^{BCRLB}$ can be calculated to measure the tracking performance for the resource management of antenna array aperture. At each sampling instant, the antenna array aperture $D_k$ needs to be allocated optimally. Firstly, the antenna array $D_k$ is divided into $Q$ subarrays as $D_k = [D_1^k, D_2^k, \ldots, D_Q^k]^T$ according to the predicted target state. Secondly, owing to the random state and diverse combination of the array elements, the element number in working state is within $[\xi_1, \xi_2]$ ($\xi_1$ and $\xi_2$ are independent fuzzy variables). Under the condition of chance constraints, the corresponding beam is synthesized to
illuminate each target. Finally, the optimal element combination of each subarray in working state is selected in terms of the predicted BCRLB of the corresponding target tracking error. Suppose that the transmitting power and dwell time of each beam are equal in the target tracking process, and the number of elements in working state corresponding to the $q$th target at the $k$th sampling interval is $n_{q,k}$. In order to accurately measure the tracking performance of each target and conveniently build a resource management model, the objective function of the antenna array aperture resource allocation is given conditioned on the previous analysis.

$$F(B_{NN}(D_{q,k}, n_{q,k})) = \text{trace} \left( \left[ C_{q,k}^{BCRLB}(B_{NN}(D_{q,k}, n_{q,k})) \right]_{4 \times 4} \right)$$

(32)

where $BCRBBC_{q,k}^{BCRLB}$ is denoted as:

$$C_{q,k}^{BCRLB}(B_{NN}(D_{q,k}, n_{q,k})) = J^{-1}(B_{NN}(D_{q,k}, n_{q,k}))$$

(33)

The influence of the beamwidth on the target tracking accuracy is considered in this paper, i.e. the influence of the antenna array allocation is considered. The CCP of the antenna array aperture resource management is built conditioned on satisfying the constraints of the elements number and the pattern.

$$\begin{align*}
& \min \max_q F(B_{NN}(D_{q,k}, n_{q,k})) \\
& \text{s.t.} \\
& Cr \left\{ \xi_1 \leq \sum_{q=1}^{Q} n_{q,k} \leq \xi_2 \right\} \geq \alpha \\
& P_{NPSL}(D_{q,k}, n_{q,k}) - P_{NPSL}^q \leq 0 \\
& D_k = \begin{bmatrix} D_{1,k}^T, D_{2,k}^T, \ldots, D_{Q,k}^T \end{bmatrix}^T = \\
& \begin{bmatrix} d_{1}^1 \cdots d_{N_{1,k}}^1, d_{2}^1 \cdots d_{N_{2,k}}^1, \ldots, d_{Q}^1 \cdots d_{N_{Q,k}}^1 \end{bmatrix}^T \\
& \sum_{i=1}^{N_{q,k}} d_{i}^q = n_{q,k}, \quad d_{i}^q = 0 \text{ or } 1, \quad \|q = 1, 2, \ldots, Q \right\}
\end{align*}$$

(34)

where $Cr$ denotes the credibility measure; $n_{q,k}$ is the number of the elements in working mode; $\xi_1$ and $\xi_2$ are the threshold of the elements number in working mode, and these two fuzzy variables are mutually independent; $P_{NPSL}^q$ is the normalized peak sidelobe level (NPSL) of the desired pattern; $P_{NPSL}(D_{q,k}, n_{q,k})$ and $B_{NN}(D_{q,k}, n_{q,k})$ denote the computed NPSL and the computed beamwidth between first nulls (BWFN); $\alpha$ is the predetermined confidence level.

Here we adopt the maximal target tracking error as the objective function. This object function can make sure all the targets have the approximately equal target tracking error. The excessive large or small error will not be obtained. In addition, due to the uncertain RCS, unique array arrangement and work mode of array elements, the number of working elements is uncertain. Hence we adopt a fuzzy CCP to confine the bounds of the number of working elements. In addition, the selection of element position is implicit in the algorithm of pattern synthesis. The constraints on the element position need not be specified.

### IV. RESOURCE ALLOCATION PROCESSING PROCEDURE

The fuzzy simulation algorithm is embedded into the genetic algorithm (GA) to constitute HIOA for solving the fuzzy CCP model. The maximal target tracking error is minimized conditioned on satisfying the constraints of elements number and antenna pattern.

#### A. FUZZY SIMULATION ALGORITHM

$\xi_1$ and $\xi_2$ are both fuzzy variables, which are the functions mapping from the credibility space ($\{\theta_1, \theta_2, \theta_3\}$) to the set of real numbers. Since $\xi_1$ and $\xi_2$ are mutually independent, the constraint in (34) can be equally transformed into two parts.

$$\begin{align*}
& \text{Cr} \left\{ \xi_1 \leq \sum_{q=1}^{Q} n_{q,k} \right\} \geq \alpha \Leftrightarrow \\
& \text{Cr} \left\{ \xi_1 \leq \sum_{q=1}^{Q} n_{q,k} \right\} \geq \alpha \\
& \text{Cr} \left\{ \sum_{q=1}^{Q} n_{q,k} \leq \xi_2 \right\} \geq \alpha \\
& \text{Cr} \left\{ \sum_{q=1}^{Q} n_{q,k} \leq \xi_2 \right\} \geq \alpha
\end{align*}$$

(35)

Take

$$\text{Cr} \left\{ \xi_1 \leq \sum_{q=1}^{Q} n_{q,k} \right\} \geq \alpha$$

(36)

as an example. The process of fuzzy simulation is described as follows[23].

1. For fuzzy variable $\xi_1$, sample $\theta_1(1), \theta_1(2), \ldots, \theta_1(M)$ (according to Strong Law of Large Numbers, $M$ is big enough) uniformly from the credibility space with $v_1(j) = \text{Pos}(\theta_1(j)) \geq \varepsilon (j = 1, 2, \ldots, M)$, where $\text{Pos}$ denotes possibility measure and it is equal to the membership function of the fuzzy variable $\xi_1$, and $\varepsilon$ is a sufficiently small positive number.

2. For each sampling values $\theta_1(j)$, let $f(\theta_1(j)) = \theta_1(j)$. Get the calculated $L(r)$ according to the credibility inversion theorem that

$$L(r) = \frac{1}{2} \left( \max_{1 \leq j \leq M} \left\{ v_1(j) \left| f(\theta_1(j)) \leq r \right\} + \min_{1 \leq j \leq M} \left\{ 1 - v_1(j) \left| f(\theta_1(j)) > r \right\} \right\} \right)$$

(37)

$L(r)$ is a monotone function about $r$. Therefore, we can adopt the bisection search to find the optimal $r$, which satisfies the inequation.
The aforementioned process is the lower bound of the elements number \( \Sigma Q q_n, k \), satisfying the constraint.

The fuzzy simulation process of

\[
\{ C_r \left( \sum_{q=1}^{Q} n_{q,k} \leq \xi_2 \right) \} \geq \alpha \tag{38}
\]

is similar as (36). Exchanging the inequality signs of (37) and repeating steps (1) and (2), we can get the upper bound of the elements number \( \Sigma Q q_n, k \), satisfying the constraint.

**B. HYBRID INTELLIGENT OPTIMIZATION ALGORITHM**

At the \( k \)th sampling instant, the fuzzy simulation is introduced and embedded into GA to constitute HIOA for solving the fuzzy CCP model[23]. Thus the optimal antenna array allocation \( D_{k+1, opt} \) satisfying the constraints at the \((k + 1)\)th sampling instant is obtained. The corresponding antenna array aperture \( D_k = [D_1, D_2, \ldots, D_k] \) of all the targets are viewed as the optimized variables in the tracking process. The block diagram of HIOA is described in Fig. 3.

**C. TARGET STATE ESTIMATION**

The optimal resource allocation \( D_{k+1, opt} \) of antenna array aperture is calculated through HIOA. When the \((k + 1)\)th sampling instant is coming, the predicted \( D_{k+1, opt} \) can be used for pattern synthesis so as to generate the beams for illuminating the targets. The extend Kalman filter (EKF) is common utilized for solving the nonlinear filtering problem[33]. Combining the resource allocation algorithm proposed in this paper, the detailed steps are given as follows.

**Step 1**: Let \( k = 1 \), and for the \( q \)th target, initialize \( \xi_{q,k-1|k-1} = J_{-1}^{-1}(\xi_{q,k-1|k-1}) \), and \( D_{0, opt} = D_0 \) (\( D_0 \) denotes the uniform allocation of antenna array aperture) where \( q = 1, 2, \ldots, Q \);

**Step 2**: Synthesize the beams in terms of the allocation vector \( D_{k, opt} \), and illuminate each targets to get the observed value \( z_{q,k} \), and calculate the variance \( \Sigma_{q,k} \) according to (17).

**Step 3**: Predict the target state, observed value and covariance:

\[
\begin{align*}
E_{q,k} & = F_{q}^{T} E_{q,k-1} F_{q} + Q_{q,k-1} \\
Z_{q,k} & = H_{q,k} E_{q,k} \tag{39}
\end{align*}
\]

**Step 4**: Calculate the gain matrix \( K_{q,k} \)

\[
K_{q,k} = C_{q,k} H_{q,k}^{T} \left( H_{q,k} C_{q,k} H_{q,k}^{T} + \Sigma_{q,k} \right)^{-1} \tag{40}
\]

**Step 5**: Update the target state and covariance

\[
\begin{align*}
\hat{E}_{q,k} & = \hat{E}_{q,k} + K_{q,k} (z_{q,k} - z_{q,k}|k-1) \\
C_{q,k} & = C_{q,k}|k-1 - K_{q,k} H_{q,k} C_{q,k}|k-1 \tag{41}
\end{align*}
\]

**Step 6**: Allocate the antenna array aperture according to HIOA in section 4.2, and employ optimal \( D_{k+1, opt} \) for the target tracking in next moment.

**Step 7**: Let \( k = k + 1 \), and go to Step 2.

**V. SIMULATION RESULTS AND ANALYSIS**

Some simulation results are provided to verify the effectiveness of the antenna array management scheme proposed in this paper.

333 elements are distributed arbitrarily and densely within \([-34.5\lambda, 34.5\lambda]\). The initial distribution of elements is shown in Fig. 4. For a clearer display, the whole linear array is displayed in 3 sections.

Supposed that the OAR is located at \((0, 0)\) km, and three beams can be generated simultaneously. The carrier wavelength is \( \lambda = 0.03 \) m. Without restricting generality, the transmitting power of each beam is equal. The sampling interval of tracking process is \( T_0 = 3 \) s. The number of the coherent pulse is 64. The number of the targets is 3, and their parameters are in TABLE 1. We utilize 30 frames of data for the simulation.

The deployment of all the targets and OAR is in Fig. 5. The antenna array is divided into three subarrays uniformly, and all the elements take part in pattern synthesis. The three synthesized beams are used to track three targets. The tracking error of each target is shown in Fig. 6.

| Target | 1 | 2 | 3 |
|--------|---|---|---|
| Position (/km) | (-43.5,68.5) | (48.5,80.15) | (112.5,60) |
| Distance (/km) | 81.14 | 93.68 | 127.50 |
| Velocity (/m/s) | (150,-200) | (-100,-200) | (-300,-100) |

**TABLE 1. The parameters of all the targets.**

Next, the antenna array aperture is allocated optimally. According to the experience and some experimental data, the number of excitation elements of the whole antenna array is within \([\xi_1, \xi_2]\), and the values of trapezoidal fuzzy variables are \( \xi_1 = (72, 80, 90, 106) \) and \( \xi_2 = (114, 124, 130, 138) \), respectively.
A. OPTIMAL ALLOCATION OF ANTENNA ARRAY

Without loss of generality, let the credibility measure of the fuzzy CCP model be $\alpha = 0.8$. The antenna aperture is firstly segmented according to the distance.

As seen in Fig. 7, the antenna aperture is segmented into three parts intelligently according to the distance of targets. The further the distance is, the larger the aperture length is. In addition, it can be seen that each horizontal line has a slope, i.e., in the target tracking process, the antenna aperture is segmented dynamically.

$N_{\text{sum}}$, $N_{\text{total}}$ and $N_i$ are the total number of working elements, the total number of elements and the number of working elements of each subarray.

The antenna array aperture allocation algorithm proposed in this case can not only intelligently segment the antenna aperture among the targets, but reduce the number of working elements strikingly. The corresponding simulation results are shown in Fig. 8. Owing to the arbitrary and dense distribution of antenna elements, not all elements need to be in working mode in pattern synthesis. This algorithm selects the optimal array element combination under the condition that the tracking accuracy is guaranteed.

To highlight the validity of saving array elements, Fig. 9 and Fig. 10 give the distribution of working elements corresponding to the 15th and the 30th sampling in the tracking process.
Compared with Fig. 4, it can be seen that the number of working elements are cut off strikingly. Nevertheless the length of the antenna aperture does not decrease. The beamwidth does not widen. Thus the target tracking accuracy is not affected.

The objective function is to minimize the maximal tracking error of the targets in the model of allocating the antenna array aperture resource optimally. The optimized tracking error of each target is shown in Fig. 11. In Fig. 6, when the aperture resource is allocated uniformly, target 3 has the
largest tracking error for the farthest distance, and target 1 has the least tracking error. Compared with Fig. 6, this algorithm segments the antenna aperture intelligently according to the predicted distance of targets. Thus the tracking error of the worst target could be also improved.

B. FUZZY CHANCE-CONSTRAINT PROGRAMMING

Owing to the unique array arrangement and the time-varying environment, the number of working elements is uncertain. Compared with the random variable, the fuzzy variable is obtained through the expert experience and historical data, and it is more realistic. Through changing the confidence levels of the chance constraints, a sequence of simulation results can be obtained as shown in TABLE 2.

With the changing of the confidence level and the desired NPSL, the total number \( n_T \) of working elements has little changes. Besides the number of working elements, the position of working elements also affects the performance of beams. Hence, the number of working elements is not a monotone function of the confidence level and the desired NPSL, i.e., there is no absolute relationship between the pattern parameters and the elements number. In addition, combining Fig. 8(b) and Table 2, the farther the target is, the longer the aperture length is, and the narrower the corresponding beams width is. The number of working elements of each target is impacted by the length of aperture. The longer the length of aperture is, the larger the number of working elements is.

VI. CONCLUSION

This paper presents an antenna aperture resource management scheme based on CCP for MTT of OAR. Initially, we segment the antenna aperture according to the number and predicted distance of targets. Owing to the unique array arrangement and the time-varying environment, the number of working elements of each target is uncertain in each sub-array. A fuzzy variable is introduced to represent the uncertainty. The BCRLB is brought in to measure the tracking accuracy. Conditioned on satisfying the desired number of elements and the desired NPSL, the optimal element combination is selected to minimize the maximal tracking error among all targets. We package the deterministic resource management model as an uncertain model through CCP conditioned on a specified confidence level. Finally the fuzzy simulation is embedded into GA to produce HIOA to obtain the optimal element combination. This algorithm can not only segment the antenna aperture intelligently, but reduce the number of working elements strikingly. The uncertainty introduced by the arbitrary distribution and operating modes of array elements is solved by fuzzy CCP. The pattern parameters and the tracking accuracy are the contributions of both the number and distribution of array elements.
In future work, besides the predicted distance of the target, the velocity of the target and RCS should be also considered for the better predictive effect.

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WEIJUN LONG was born in Yunnan, China, in 1979. He received the B.S. and M.S. degrees in electronic engineering and optical engineering from the Harbin Institute of Technology, Harbin, China, in 2001 and 2003, respectively, and the Ph.D. degree in communication and information system from the Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2011. He is currently a Research Fellow with the 14th Research Institute of China Electronics Technology Group Corporation, Nanjing. His research interests include radar signal processing and new system radar.

ZHIHENG LIANG received the B.S., M.S., and Ph.D. degrees in instruments and apparatuses from the Beijing University of Aeronautics and Astronautics, in 1991, 1997, and 2002, respectively. He is currently an Associate Professor with the School of Mechanical Engineering, Tsinghua University. His research interests include RF simulation and radar signal processing.