Suppressing quantum fluctuations in classicalization

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Abstract – We study vacuum quantum fluctuations of simple Nambu-Goldstone bosons —derivatively coupled single scalar-field theories possessing shift symmetry in field space. We argue that quantum fluctuations of the interacting field can be drastically suppressed with respect to the free-field case. Moreover, the power spectrum of these fluctuations can soften to become red for sufficiently small scales. In quasiclassical approximation, we demonstrate that this suppression can only occur for those theories that admit such classical static backgrounds around which small perturbations propagate faster than light. Thus, a quasiclassical softening of quantum fluctuations is only possible for theories which classicalize instead of having a usual Lorentz invariant and local Wilsonian UV completion. We illustrate our analysis by estimating the quantum fluctuations for the DBI-like theories.

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Introduction. – Quantum mechanics (QM) of a finite number of degrees of freedom is a paradigm which should a priori be applicable to arbitrary Hamiltonian systems including strongly coupled models. In particular, QM describes systems with noncanonical and even highly nonlinear kinetic terms as well as it works for those with the highly nonlinear potentials. However, currently, in the case of QM for a continuum number of degrees of freedom (i.e., for quantum field theory (QFT)) all models well defined at all scales (renormalizable) are just quadratic in canonical momentum. For QFT a noncanonical structure of the kinetic term usually implies the existence of a strongly coupled regime where the usual perturbative renormalization procedure breaks down and where one has to use nonperturbative methods, e.g., lattice computation. However, lattices of different size correspond to the approximation of QFT by finite-dimensional QM systems with different number of degrees of freedom and the convergence of this procedure is not obvious.

It is a common belief that Nature is such that it can be described in terms of fields which are different in the weak- and strong-coupling regimes. The best examples for this behavior are given within the Standard Model by QCD and Higgs field in the electroweak sector. One tends to think that in UV the number of degrees of freedom —types of particles— should increase, see, e.g., most recent works [1,2]. New particles should be integrated in to provide a UV complete theory —this is the basis of the Wilsonian UV completion. This is the case for String Theory where in the UV there is a continuum of fields. In this regard natural questions arise: is continuum QM —QFT— infinitely more restrictive than QM for a finite number of degrees of freedom? Can noncanonical or perturbatively nonrenormalizable QFT’s make sense in a continuum limit, i.e., for all scales? If it is the case, can one understand the mechanism used by Nature in these theories in terms of some weakly coupled fields/particles? These questions could be crucial for the understanding of quantum gravity which has a noncanonical Hamiltonian with the strong-coupling scale given by the Planck mass.

Recently it was proposed [3–6] that gravity may be self-UV-completed via classicalization —the softening of quantum fluctuations, i.e., loops in Feynman diagrams at the transplanckian transferred momenta, because of the formation of intermediate black holes. Classicalization is caused by the high level of nonlinearity and corresponding self-sourcing. Moreover, it was suggested that this mechanism may also work for a quite generic class of the Nambu-Goldstone bosons [5,8] and even more general scalar fields including nonlinear sigma models [11], where the role of black holes is played by

1For an earlier similar discussion of the quasiclassical high-energy scattering in gravity, see, e.g., [7].
2For criticism of this interesting proposal, see, e.g., refs. [9,10].

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classicalons — extended and long-lived classical configurations. However, this phenomenon was argued [12,13] to occur for those models for which the usual local and Lorentz-invariant Wilsonian UV completion is absent, because they allow for a superluminal propagation of perturbations around nontrivial backgrounds [14].

In this paper we analyse the self-consistency of classicalization for general noncanonical Nambu-Goldstone bosons in \( d + 1 \) spacetime dimensions. We did not restrict our attention to \( d = 3 \), because in lower spacial dimensions there are much higher chances to find theories for which our estimations can be checked by numerical simulations or by other nonperturbative methods. We use the Heisenberg uncertainty relation for canonically conjugated field and momentum operators averaged on a given scale to estimate vacuum quantum fluctuations of the field on this scale. In our analysis we follow the ideas of classicalization and work in the quasiclassical approximation — neglecting higher-order correlators. On top of that, we use analytic continuation which is a standard tool for interacting QFT with the energy functional bounded from below. We show that vacuum quantum fluctuations, \( \delta \phi \), can be suppressed with respect to the free-field case only if the theory admits static classical backgrounds which are either i) absolutely unstable (have gradient instability) or ii) such that perturbations around them propagate faster than light. Option i) is definitely even more worrisome than ii) and should be excluded. Moreover, if the suppression is such that the spectrum of perturbations is red in the UV, then the speed of perturbations \( v^2 > 1 \). We illustrate our finding by the analysis of quantum fluctuations for Dirac-Born-Infeld (DBI)—like theories.

Heisenberg uncertainty relation for averaged fields. — We are interested in the characteristic quantum fluctuation \( \delta \phi (\ell) \) on scale \( \ell \). In four spacetime dimensions, and weakly coupled canonical field theories the standard result is \( \delta \phi (\ell) \simeq \sqrt{\hbar / \ell} \), for scales much shorter than the possible Compton wavelengths in the theory. Following the classicalization idea, we assume that even for the lengths smaller than the strong-coupling scale \( M^2 \), the field operator, \( \hat{\phi} (x) \), and the canonically conjugated momentum \( \hat{p} (x) \), can still be defined\(^3\). Also we assume that one can define states \( | \Psi \rangle \) and in particular a vacuum state, \( | 0 \rangle \), applicable for all scales for the interacting theory.

We define the operators averaged\(^5\) on scale \( \ell \) as

\[
\hat{\phi}_\ell (x,t) = \int d^d x' W^\phi_\ell (x-x') \hat{\phi} (x',t),
\]

where \( W^\phi_\ell (x-x') \) is a window function centered at \( x \). It is convenient and common to assume that a device measuring the scalar field can operate with adjustable (but always finite!) resolution (averaging on different scales \( \ell \) in such a way that a family of window functions characterising this device scales as

\[
W^\phi_\ell (x) = \ell^{-d} \cdot w^\phi \left( \frac{X}{\ell} \right),
\]

see, e.g., [16], where the shape function \( w^\phi \) is positive with \( \int d^d r w^\phi (r) = 1 \). For simplicity one can assume that \( w^\phi (r) = w^\phi (-r) \). Every single measurement of the scalar field by a device located at position \( x \) gives an eigenvalue of the operator (1). In the state \( | \Psi \rangle \) the field fluctuation on length \( \ell \) is defined as

\[
\delta \phi_\ell^2 \equiv \langle \Psi | \hat{\phi}_\ell^2 | \Psi \rangle - \langle \Psi | \hat{\phi}_\ell | \Psi \rangle^2.
\]

A device measuring the canonical momentum \( \hat{p} (x) \) will operate with a different window function family \( W^p_\ell (x) \) possessing the same natural scaling property (2) with some shape function \( w^p (r) \). The canonical commutation relation is

\[
\{ \hat{\phi}_\ell (t,x) , \hat{p}_\ell (t,y) \} = i \hbar \delta (x-y),
\]

which, for the operators averaged on the same scale \( \ell \), gives

\[
\{ \hat{\phi}_\ell (t,x) , \hat{p}_\ell (t,y) \} = i \hbar \cdot \ell^{-d} \cdot \mathcal{D} \left( \frac{X-Y}{\ell} \right),
\]

where the function \( \mathcal{D} (r) \) is given by a convolution of the shape functions

\[
\mathcal{D} (r) = \int d^d r' w^\phi (r-r') w^p (r').
\]

Now we can use the uncertainty relation in the form derived by Robertson [17] to obtain

\[
\delta \phi_\ell (t,x) \cdot \delta p_\ell (t,y) \geq \frac{\hbar}{2} \cdot \mathcal{D} \left( \frac{X-Y}{\ell} \right) \cdot \ell^{-d}.
\]

To measure the same set of degrees of freedom one would like to measure the field and conjugated momentum possibly at the same point in space with the precision better than \( \ell \) so that at the same point we have

\[
\delta \phi_\ell (t,x) \cdot \delta p_\ell (t,x) \geq \frac{\hbar}{2} \cdot \mathcal{D}_0 \cdot \ell^{-d},
\]

\(^3\)Note that an averaging in time is not needed, because one can equally well consider the system in the Schrödinger picture where all operators are time independent. Moreover, for any energy eigenstate \( | E \rangle \) the expectation values for all observables are time independent.

\(^5\)Here it is interesting to note that already in 1952 Heisenberg argued that derivative self-interaction is important for the correct description of high multiplicity in scattering in strongly interacting theories. See [15] where the usual subluminal DBI is used to analyse this problem.
where the number $\mathcal{D}_0 \equiv \mathcal{D}(0) = \mathcal{O}(1) > 0$ depends on the exact properties of the window functions (corresponding to the measuring devices) but does not depend on the scale of averaging or on the quantum state $|\Psi\rangle$. An estimation for the product of fluctuations for averaged fields can be found in earlier works, e.g., in [18].

The uncertainty relation (8) unambiguously shows that the shorter the scale of averaging is, the more quantum the field theory is. However, if the theory is noncanonical then the field can have a “fake quasiclassical” property, i.e., $\delta\phi t \times \delta\phi x - 0$. Further, one can remark that in standard canonical theories for a multiparticle state $|N\rangle$ which is an eigenstate of energy the product of fluctuations is larger than that for the vacuum $|0\rangle$, while $\langle N|\phi t|N\rangle = \langle N|\phi t|N\rangle = 0$, see, e.g., [16]. Thus, a large number of particles is not a sufficient condition for a truly quasiclassical behaviour.

In the next section we use the uncertainty relation for a scalar field in the Lorentz-invariant vacuum state $|0\rangle$ to estimate quantum fluctuations $\delta\phi t (x)$. In the Lorentz-invariant vacuum the fluctuations are the same at all points of the space, so that $\delta\phi t (x) = \delta\phi (\ell)$. An attractive interaction reduces $\delta\phi (\ell)$ with respect to the free-field case, while a repulsive interaction increases $\delta\phi (\ell)$. It is well known (see, e.g., [16]) that $\delta\phi^2 (\ell)$ corresponds to the normalised power spectrum for the two-point correlation function, $\langle 0| \phi (t, x) \phi (t, y)|0\rangle$, where $\ell = |x - y|$. In the UV, $\delta\phi^2 (\ell) \equiv O(0) \equiv \gamma = \mathcal{O}(1) > 0$; however it can always be absorbed into the normalisation of $\delta\phi$ and $\mathcal{D}_0$. Here we have also assumed Lorentz invariance of the vacuum: $\delta\phi t = \delta\phi (x) = \delta\phi (\ell)$, and used the Wick rotation to evaluate the expectation value in (12), so that $\Omega (\ell) < 0$. This is the second of our assumptions. This assumption can also be motivated by dimensional regularization. Indeed, $\langle 0| X |0\rangle \neq 0$ but to calculate this quantity one has to use some regularization which is compatible with the Lorentz symmetry of $|0\rangle$. Usually the Wick rotation works for interacting theories with energies bounded from below. Moreover, light-like $X_\ell = 0$, would not change the fluctuations at all, whereas time-like $X_\ell = \langle \delta\phi t / \ell \rangle^2$ intuitively implies some flow or evolution in time which should not be the case in a static situation.

Further, we assume that the vacuum $|0\rangle$ is standard so that $\langle 0| \phi t |0\rangle = \langle 0| \phi t |0\rangle = 0$. In vacuum $|0\rangle$ the product of uncertainties is smaller than in any other energy eigenstate $|E\rangle$. We can express the product of fluctuations as

$$\delta\phi (\ell) \cdot \delta\phi (\ell) = h^{\mathcal{D}_0} / 2 \cdot |\Omega (\ell)| \cdot \ell^{-d} > h^{\mathcal{D}_0} / 2 \cdot \ell^{-d}.$$  

where $\Omega (\ell)$ is a function, $\Omega (\ell) > 1$, fixed by properties of the vacuum state which, in turn, are fixed by the theory. If $\Omega (\ell) = 1$ the vacuum saturates the Heisenberg uncertainty relation by absolute minimization of the product of fluctuations. Now it is easy to see that $d\Omega / d\ell$ should be negative in UV or asymptotically approach zero. Indeed, if $\Omega (\ell)$ were increasing, $d\Omega / d\ell > 0$, it would imply that in deep UV the system would violate the uncertainty relation $\Omega (\ell) \geq 1$. Otherwise, one has to assume that there is another scale different from $M_u^{-1}$ where $d\Omega / d\ell$ changes the sign. If $d\Omega / d\ell < 0$ in UV, it can change the sign and approach zero in IR for $\ell \geq M_u^{-1}$.

Further, by plugging the characteristic momentum into the Heisenberg uncertainty relation (15) we obtain

$$\mathcal{L}_X (X_\ell) \delta\phi^2 (\ell) \equiv h^{\mathcal{D}_0} / 2 \cdot |\Omega (\ell)| \cdot \ell^{-1-d}.$$  

This yields an implicit function $\delta\phi (\ell)$ for general theories with Lagrangians $\mathcal{L} (X)$ for which our assumptions
(classicalization) are fulfilled. Below in this letter, we find $\delta \phi (\ell)$ analytically for DBI-like theories, assuming that $\Omega (\ell)$ is a slowly varying function of the scale $\ell$, so that $d \ln \Omega / d \ln \ell \ll 1$. In most cases it is not possible to solve this algebraic equation (16) exactly even for a given $\Omega (\ell)$. Nevertheless, we can obtain important information, e.g., the slope of the fluctuations by differentiating this equation (16) with respect to the length scale $\ell$ to obtain

$$\frac{d \delta \phi}{d \ell} = \frac{\delta \phi}{\ell} \left[ 2 \ell^2 \frac{d \Omega / d \ln \ell}{\sqrt{\Omega}} \right], \quad (17)$$

where we have introduced the notation

$$v^2 = v^2 (x) = 1 + \frac{2X \mathcal{L} X}{\mathcal{L}^2} \bigg|_{x = x_i}. \quad (18)$$

It is important to note that any dependence on the window function parameter $\mathcal{D}_0$ disappeared from eq. (17), which can be also considered as a first-order differential equation for $\delta \phi (\ell)$. In that case, $\mathcal{D}_0$ is the integration constant. The physical meaning of the function $v(X)$ is the (radial) speed of propagation for small perturbations around a classical static background $\phi (x)$ at a point where $X = -\frac{1}{4} \partial_i \partial_i \phi$ coincides with the characteristic $\ell_\phi$ given by (14) for quantum fluctuations. Indeed, it is well known (see ref. [19]) that for the space-like classical backgrounds the (radial) sound speed is given by the formula (18) which is the inverse of the one derived in ref. [20] for cosmological backgrounds in the context of general $k$-essence theories [21–23].

It is worth mentioning that for $d \ln \Omega / d \ln \ell = 0$ the differential equation (17) is invariant under simultaneous rescaling $\delta \phi \to s \delta \phi$ and $\ell \to s \ell$, while for a constant $v$ the rescaling parameters are independent. However, any solution of this equation given by (16) with a fixed $\mathcal{D}_0$ is not invariant under this rescaling.

It is also convenient to write eq. (17) as

$$\frac{d \delta \phi}{d \ell} = \frac{\delta \phi}{\ell} \left[ 1 - d + \frac{d \ln \Omega / d \ln \ell}{2 (v^2 + 1)} \right]. \quad (19)$$

Thus, quantum fluctuations can be suppressed with respect to the weekly coupled case, provided either

$$v^2 > 1 - \frac{2}{1 + d} \frac{d \ln \Omega}{d \ln \ell} \geq 1, \quad (20)$$

or $v^2 < -1$. While for an absolute suppression, with red tilt, $d \delta \phi / d \ell > 0$, one has to require either a stronger condition

$$v^2 > d - \frac{d \ln \Omega}{d \ln \ell} \geq d, \quad (21)$$

or again that $v^2 < -1$.

The first case, $v^2 > 1$, implies that the theory possesses classical static backgrounds $\phi (x)$ around which small perturbations propagate faster than light. Thus, a quasiclassical attractive interaction is only possible for Nambu-Goldstone bosons allowing for superluminality. The second case, $v^2 < -1$, implies that there are backgrounds $\phi (x)$ which are absolutely unstable — they possess gradient instabilities. Faster-than-light propagation around nontrivial backgrounds $\phi (x)$ does not imply an immediate inconsistency, see [19, 24–27], but rather signals the absence of the usual Lorentz invariant and local Wilsonian UV completion by other fields appearing in UV [14]. However, exactly such theories may be able to classicalize, as was argued in [12, 13]. Another well-known problem associated with such theories is that the relation between $p$ and $\dot{\phi}$ can be multivalued [28], for a recent interesting proposal of how to deal with this problem see [29, 30].

Do the fluctuations calculated above correspond to the minimal energy, i.e., vacuum? Similarly to the well-known estimation for the lowest energy level of hydrogen we can estimate the size of fluctuations minimising the Hamiltonian density $\mathcal{H}(p, (\nabla \phi)^2) = p \dot{\phi} - \mathcal{L}$ where the characteristic momentum is $p = \delta p$ and given by the Heisenberg uncertainty relation (8). Further, we will assume that the level of self-interaction for each mode is much stronger than the coupling to other modes. The condition for the extremum is

$$\frac{\partial \mathcal{H}}{\partial \delta \phi} = \dot{\phi} \frac{\partial p}{\partial \delta \phi} + \frac{1}{2} \mathcal{L} \frac{\partial (\nabla \phi)^2}{\partial \delta \phi} \approx 0. \quad (22)$$

One can check that, under our assumptions, (16) with some $\mathcal{O}$ is a solution of this equation.

### Lagrange multiplier renormalises $\hbar$ – It is more intuitive to deal with systems quadratic in derivatives. We reformulate the theory using a Lagrange multiplier field $\lambda$ and the Legendre transformation in the following way:

$$S = \int d^{1+d} x \left( \lambda \cdot X - V (\lambda) \right). \quad (23)$$

The equations of motion for $\lambda$ would give us $X = V_\lambda$, providing an implicit function $\lambda (X)$. Therefore, the “potential”, $V (\lambda)$, describing the original theory (10) is given by the relation

$$V (\lambda (X)) = \lambda (X) \cdot X - \mathcal{L} (X). \quad (24)$$

Differeniating the relation (24) with respect to $X$ one obtains $\lambda = \mathcal{L}_X$. If we assume a negligibly weak correlation between $\lambda$ and $\phi$, then

$$\delta p^2 (\ell) \approx \langle (\lambda \dot{\phi})^2 \rangle \approx \langle \lambda^2 \rangle \langle \dot{\phi}^2 \rangle \approx \langle \lambda^2 \rangle \langle \dot{\phi}^2 \rangle \ell. \quad (25)$$

Note that $\langle \lambda \rangle \ell \neq 0$. If we assume weak coupling, or the quasiclassical approximation, i.e., negligible dispersion of $\lambda$, then $\langle \lambda^2 \rangle \ell \approx \langle \lambda^2 \rangle$. Further, we use the notation $\langle \lambda \rangle \ell = \lambda (\ell)$ omitting sometimes the dependence on $\ell$, and writing just $\lambda$. The standard Heisenberg uncertainty relation (8) takes the form

$$\delta \phi (\ell) \cdot \delta \dot{\phi} (\ell) \geq \frac{\mathcal{D}_0}{2} \left( \frac{\hbar}{\lambda (\ell)} \right) \cdot \ell^{-d}. \quad (26)$$
If we assume that the Lorentz-invariant vacuum saturates the inequality and drop the unimportant numerical factor $\mathcal{R}_0/2$ the quantum fluctuations are given by

$$\delta \phi (\ell) \simeq \left( \frac{h}{\lambda(\ell)} \right)^{1/2} \cdot \ell^{(1-d)/2}. \quad (27)$$

Thus, quantum fluctuations $\delta \phi (\ell)$ are suppressed with respect to the standard result, provided $\lambda(\ell) = \langle \mathcal{L}_X \rangle > 1$ parametrically. If, moreover, $\lambda(\ell)$ evolves with $\ell$ so that, the shorter $\ell$ is, the larger $\lambda(\ell)$ is, then the shorter $\ell$ is, the more suppressed quantum fluctuations are. Thus, classicalization occurs when $d \lambda/d \ell < 0$.

Further, just from dimensional reasons and again assuming weak coupling, or the quasiclassical approximation, we obtain\(^8\)

$$\lambda(\ell) V_\lambda(\lambda(\ell)) \approx \langle V_\lambda(\ell) \rangle \simeq \ell^{-(1+d)}. \quad (28)$$

Differentiating this expression with respect to $\ell$ yields

$$\frac{d \lambda}{d \ell} (V_\lambda + \lambda V_\lambda) = -(1 + d) \frac{\lambda V_\lambda}{\ell}, \quad (29)$$

so that, noting that $V_\lambda = 1/\mathcal{L}_X$, we can recast this equation in the form

$$\frac{d \lambda}{d \ell} = -(1 + d) \cdot \lambda \cdot \left( \frac{v^2 - 1}{v^2 + 1} \right), \quad (30)$$

where the velocity $v$ is given by the formula (18). From this equation we infer that to have larger $\lambda$ for smaller scales it is necessary that either $v^2 < -1$ or $v^2 > 1$. But it is exactly this behavior of $\lambda(\ell)$ which is required to suppress $\delta \phi (\ell)$ relatively to the standard case, see eq. (27). Thus, we confirm in that way our previous analysis. Finally differentiating (27) with respect to $\ell$ and using (30) we reproduce the result (17) for slowly varying $\Omega$.

(a)DBI. – Here, to illustrate our analysis, we consider theories described by the Lagrangian

$$\mathcal{L} = \sigma M_S^{d+1} \left[ \frac{2X}{\sigma M_S^{d+1}} - 1 \right], \quad (31)$$

where $M_s$ is a strong-coupling scale, and $\sigma = \pm 1$. The case $\sigma = -1$, corresponds to the standard Dirac-Born-Infeld (DBI) theory, where small perturbations always propagate with the speed less than or equal to the speed of light, i.e., they are never superluminal. Whereas the case $\sigma = +1$ corresponds to the so-called anti-DBI (aDBI) theory which was introduced in [31–33] and for which small perturbations are never \textit{subluminal}. aDBI was extensively studied as an example of a theory which can classicalize, see [4,13,34–36]. For small derivatives the system approaches a free massless scalar field. Another useful property of (a)DBI is that the real equations of motion are always hyperbolic (i.e., there is no gradient instability). Moreover, (a)DBI satisfy the null energy condition—they do not have ghosts and possess non-negative energy density, see, e.g., [35]. The unphysical region of phase space corresponding to imaginary observables is separated by a barrier of infinite positive energy density.

For this theory the saturated uncertainty relation (16) takes the form

$$\frac{(\ell^{(d-1)/2} \delta \phi)^2}{\sqrt{1 - 2\sigma (\ell^{(d-1)/2} \delta \phi)^2 / (\ell M_S)^{d+1}}} \simeq 1, \quad (32)$$

where we have omitted the irrelevant window function factor $\mathcal{R}_0/2$ which would also only change the normalisation of $\delta \phi (\ell)$ and $M_s$ by a factor $\mathcal{O}(1)$ but not the scaling with $\ell$. The above equation has the following solution:

$$\delta \phi^2 (\ell) = \frac{\ell^{1-d} \ell^{(d-1)/2}}{(\ell M_S)^{d+1}} \left[ \sqrt{1 + (M_S\ell)^{2(d+1)} - \sigma} \right]. \quad (33)$$

In IR, $\ell \gg M_s^{-1}$, one obtains

$$\delta \phi (\ell) \simeq \ell^{(1-d)/2} \cdot \left( \frac{1 - \sigma}{2} (\ell M_S)^{-\frac{3}{2}} \right). \quad (34)$$

The leading term reproduces the standard result. However, the next-to-leading term reveals that for aDBI the fluctuations are slightly suppressed, whereas for the DBI the fluctuations are slightly enhanced. In the UV, $\ell \ll M^{-1}$, quantum fluctuations already in the leading order depend on the sign $\sigma$. Namely, for superluminal aDBI, $(\sigma = +1)$ the quantum fluctuations have a red spectrum:

$$\delta \phi_{aDBI} (\ell) \sim M_s^{(d+1)/2} \cdot \ell, \quad (35)$$

whereas for the standard DBI $(\sigma = -1)$ the spectrum is blue:

$$\delta \phi_{DBI} (\ell) \sim M_s^{-(d+1)/2} \cdot \ell^{-d}. \quad (36)$$

Thus, for scales shorter than the strong-coupling scale $M_s^{-1}$, quantum fluctuations in aDBI become softer—the behavior which one would expect for those systems which classicalize. Here it should be mentioned that a classical \textit{on} scale $\ell$ would have the maximal field value given by (35).

For the corresponding conjugate momenta in the UV, the Heisenberg uncertainty relation (8) yields

$$\delta p_{aDBI} (\ell) \sim M_s^{-(d+1)/2} \cdot \ell^{-(d+1)}, \quad \delta p_{DBI} (\ell) \sim M_s^{(d+1)/2}. \quad (37)$$

Thus, there is a duality between these two theories in the sense of

$$\delta p_{aDBI} (\ell) \sim \frac{\delta \phi_{aDBI} (\ell)}{\ell}, \quad \delta p_{DBI} (\ell) \sim \frac{\delta \phi_{DBI} (\ell)}{\ell}. \quad (38)$$

\(^8\)Here we could also estimate that $V(\lambda(\ell)) \simeq \langle V \rangle > \ell^{-(1+d)}$. However, this estimation leads to the dependence of the final result not only on the derivatives of the Lagrangian $\mathcal{L}$, but also on its value and in particular on the possible cosmological constant. Clearly vacuum fluctuations of the scalar field should not depend on the value of the cosmological constant provided we ignore the effects related to spacetime curvature.
Moreover, the vacuum state is squeezed in orthogonal directions for these theories:
\[
\frac{\delta p_{\text{DBI}}}{\delta \phi_{\text{DBI}}} / \ell \sim (\ell M_{\text{Pl}})^{-(d+1)} \sim \frac{\delta \phi_{\text{DBI}} / \ell}{\delta p_{\text{DBI}}} \gg 1 . \tag{39}
\]

It is also interesting to note that one does not obtain these estimations for the spectrum \(\delta \phi (\ell)\) by assuming the Euclidian action to be of order unity, \(i.e., h\). Presumably this happens, because classically the canonical momentum is restricted for the aDBI, see, \(e.g., [34,35]\).

To conclude this section a cautionary remark is necessary. The usual DBI can be UV-completed in a standard Wilsonian way. Therefore, the validity of the quasiclassical approximation is rather questionable in this case and we put much less value on the estimations obtained for the usual DBI in this way.

Conclusions. – We have demonstrated that the quasiclassical suppression of quantum fluctuations of Nambu-Goldstone bosons is only possible for theories which either possess catastrophically unstable backgrounds or allow for superluminality. This finding supports the analysis performed before in [12,13].

It would be very interesting to study whether these results can be generalised to other systems and quantum states.

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