Modeling of multidimensional light bullets in Fermi liquid and ADS/CFT correspondence

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Abstract. This paper discusses two-dimensional and three-dimensional solitary electromagnetic waves propagation in Fermi liquid in the framework of ADS/CFT correspondence. The electromagnetic field is classically considered within the framework of Maxwell's equations. The obtained effective equation is numerically analyzed and the state of the electromagnetic field that is localized in two / three spatial dimensions is revealed.

1. Introduction
The study of the Fermi liquid properties led to investigate various kinds of nonlinear optical phenomena in these media. So recently, the dynamics of few cycle optical pulses (FCP) with a plane wave front in the Fermi liquid was studied [1]. However, there are a number of questions connected with going beyond the approximation of a plane wave front and studying the dynamics of propagation with taking into account the transverse dispersion.

Another interesting challenge is investigation of the dynamics of three-dimensional optical pulse localized in all spatial directions. This confirms the fact that the study of such pulses, called "light bullets"[2,3], has recently become very popular due to the many practical applications [5,6].

In the last few years, there has been a growing interest in ideas from both superstring theory and quantum gravitation [7,8], and they find application in the physics of condensed matter. And, namely, ADS/CFT correspondence allows us to get the Green's function [13,14] for both Fermi liquid and non-Fermi liquid, and for marginal Fermi liquid. The ideas of ADS/CFT correspondence (the anti-de Sitter/conformal field theory correspondence), which is closely related to the conformal field theory, which has the applications in various fields of solid state theory and works particularly fruitfully in the region of phase transitions.

The following approach corresponds to the ideology of the so-called "semi-holography" [15], where the dispersion law of strongly interacting subsystems is taken based on the ADS/CFT correspondence, and further consideration is "classically" based on the developed approaches based on the Maxwell and Boltzmann equations.

2. Basic equations
Let us begin with the dispersion law (here k is measured from the Fermi level):

\[ \varepsilon = \varepsilon \left( |\vec{k}|, |\vec{k}^2 + k_z^2|^{3/2} \right) \]  

Or it is more convenient to present in the form:
\[ e = e\left(k_x, k_y^2\right) \]  

We have the following expression for the density of current [16]:  

\[ j = n_0 \int dk \nu_x \left(k_x - A, k_y^2\right) f\left(k_x, k_y^2\right) \]  

where \( f \) is the distribution Fermi function, \( \nu_x = \frac{\partial e}{\partial k_x} \). Here we consider that electric field has only one component: \( E_x = -\partial A_c / \partial t \), then the speed of light \( c \) is assumed equal to unity.

In the case of low temperatures, we replace \( f \) with 1 in a layer near the Fermi surface. The depth of the layer will select for reasons of particles conservation:

\[ \int_A^\Delta dk = \int_{2\theta} dk f \]  

Then

\[ j = n_0 \int_A^\Delta \frac{dk}{k_x} \int_{k_x}^{2\theta} dk_y \int_{0}^{2\pi} \frac{\partial e}{\partial k_x} \left(k_x - A, k_y^2\right) = \pi n_0 \int_0^{2\pi} dx \left( e\left(\Delta - A, x\right) - e\left(-\Delta - A, x\right) \right) \]  

We specify the dispersion law using ADS/CFT correspondence ideas and following the work [17]. Let us consider two- and one-dimensional boundary theories, corresponding to a gravity theory in asymptotically ADS4. Lagrangian can be chosen in standard type for both gravitational field and gauge field with U(1) symmetry [17]:  

\[ L = \frac{1}{2} g^2 \left(R + \frac{6}{\Lambda^2}\right) - \frac{1}{4} e^2 F_{\mu \nu} F^{\mu \nu} \]  

where \( g \) is gravitational interaction constant, \( e \) is a charge, \( F_{\mu \nu} \) is stress tensor for the gauge field. The given choice of the Lagrangian is in accordance with the solution as the Reissner-Nordstrom-AdS black hole. As it is shown in [17] the Green's function has the form:

\[ G_k\left(w, \vec{k}\right) = \frac{\hbar}{k - k_x - w / \nu_f - \sum (w, k_y)} \]  

where \( k = |\vec{k}| \), \( k_x \) is Fermi momentum, \( \nu_f \) is Fermi velocity. The main difference of the given Green's function which poles determine the dispersion law of quasiparticles lies in the fact that the quasiparticles dissipation highly depends on parameter: \( \nu_k = \left(m^2 L^2 \mu^2 + 3k^2 - \gamma^2 \mu^2\right) / 6 \).

We note that \( \nu_k \) (hereinafter simply) is closely related to the critical exponent.

The dispersion law in the case of low temperatures is given by the following expression:

\[ e\left(k\right) = c\left(k_x^2 + k_y^2\right)^{\nu / 2} \]  

Then, we obtain the current density in the following form:

\[ j\left(A\right) = \frac{\pi n_0 e}{0.5 \nu + 1} \left(\left(\Delta + A\right)^{\nu + 2} + \left(\Delta - A\right)^{\nu + 2} + \left(\left(\Delta - A\right)^2 + 2\Delta^2\right)^{0.5 \nu + 1} - \left((\Delta + A)^2 + 2\Delta^2\right)^{0.5 \nu + 1}\right) \]  

Let us write a wave equation in 2D (10a) and 3D (10b) cases:

\[ \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} - \frac{1}{c^2} A_n + \frac{4\pi}{c} j\left(A\right) = 0 \]  

\[ A_n = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A}{\partial r} \right) + \frac{\partial^2 A}{\partial z^2} + \frac{1}{r^2} \frac{\partial^2 A}{\partial \varphi^2} + 4\pi j\left(A\right) \]  

Further, we consider in the cylindrical symmetry:

\[ \frac{\partial}{\partial \varphi} \to 0 \]
An important point that needs to pay special attention is associated with a cylindrically symmetric case. Due to the inhomogeneity of the field along an axis (for definiteness, let field is to be specific along the x axis), and the current is not uniform. In consequence of the current inhomogeneity we observe accumulation of a charge at some region. The easiest way to assess this accumulation from the law of the charge conservation:

\[
\frac{d\rho}{dt} + \frac{dj}{dx} = 0 \quad \text{(11a)}
\]

\[
\rho \approx \tau \frac{j}{l_x} \quad \text{(11b)}
\]

\(\rho\) is the charge density, \(j\) is the density of current along x axis, \(\tau\) is the electromagnetic field duration, \(l_x\) is the characteristic length on which the electric field changes.

(11b) is shown, that accumulated charge, which generates an additional electric field interferes with the field of few cycle pulses, essentially depends on the FCP duration. Estimates show that the FCP duration is \(\tau \approx 10^{-15}\text{s}\), and the current density is \(j \approx 10^{12}\, \text{A/m}^2\), \(\varepsilon \approx 2\). Then we have for additional field the following value: \(E \approx 10^{-3}\, \text{V/m}\). It should be noted, that the FCP field is about \(10^6\, \text{V/m}\), therefore we can neglect with accumulation of a charge for femtosecond pulses. According to calculation results in Ref. [18] for carbon nanotubes case and pulse with \(\tau \approx 10^{-20}\, \text{to} 10^{-15}\text{s}\), the accumulated charge is about 1-2% of the charge contributing to the current (10).

3. Results and conclusion

The effective equation (12) was solved numerically by using direct difference scheme of the “cross” type [19]. The time and coordinate steps were determined from the standard stability conditions. Difference scheme steps where iteratively decreased twice until the solution became unchanged in the eighth decimal place. The initial pulse profile is supposed to have a following form:

\[
A(z,0) = Q \exp \left\{-\left(\frac{z-z_0}{\gamma_z}\right)^2\right\} \exp \left\{-\left(\frac{y-y_0}{\gamma_y}\right)^2\right\},
\]

\[
\frac{dA(z,0)}{dt} = 2Q(z-z_0)\nu_z \left\{-\left(\frac{z-z_0}{\gamma_z}\right)^2\right\} \exp \left\{-\left(\frac{y-y_0}{\gamma_y}\right)^2\right\} \quad \text{(12)}
\]

here \(Q\) is the initial amplitude, its initial velocity along z-axis, \(\gamma_z, \gamma_y\) are the pulse width along \(z\) and \(y\), \(z_0, y_0\) are the initial coordinate of the pulse center.

Figure 1 demonstrates an example of electromagnetic wave propagation in Fermi liquid. It shows the dependence of the electric field strength \(E(z,y,t)\) from the coordinate at different times (\(\nu=0.8\)). We can observe a spreading of the light bullet with a loss of amplitude at the propagation in the sample (it is less by a factor of 5 than for figure d in comparison with figure a). Moreover, the pulse is not destroyed. It only changes the form, which indicates the possibility of fairly stable pulse propagation in Fermi liquid.
Figure 1. Dependence of the strength $E(z,y,t)$ for two-dimensional electromagnetic pulse (1 r.u.=$10^6$ V/m) on coordinates: a) the initial profile of the pulse; b) the profile of the pulse at $t=5.0\cdot10^{-13}$s; c) $1.5\cdot10^{-12}$s ($\nu=0.8$). Coordinate $y$ corresponds to 30 nm and $z = 20$ nm.

Figure 2 shows the pulse intensity at the different time points in the case when $\nu=0.6$.

Figure 2. Dependence of the strength $E(z,y,t)$ for two-dimensional electromagnetic pulse (1 r.u.=$10^6$ V/m) on coordinates: a) $t=5.0\cdot10^{-13}$s; b) $1.5\cdot10^{-12}$s ($\nu=0.6$). Coordinate $y$ corresponds to 30 nm and $z = 20$ nm.

As can be seen from Figures 1 and 2, an influence of the parameter $\nu$ is manifested in the following: as the value of $\nu$ decreases, an increase in the pulse amplitude is observed, and the smaller spreading in the transverse direction does not change the character of the propagation of the pulse.
Further we consider the pulse in the cylindrically symmetric coordinate system at the $v=0.8$ (figure 3). The initial conditions in 3D case have the following form:

$$A(z,0) = Q\exp\left[-\frac{(z-z_0)^2}{\gamma_z^2}\right]\exp\left[-\frac{r^2}{\gamma_r^2}\right],$$

$$\frac{dA(z,0)}{dt} = \frac{2Q(z-z_0)v_z}{\gamma_z^2}\exp\left[-\frac{(z-z_0)^2}{\gamma_z^2}\right]\exp\left[-\frac{r^2}{\gamma_r^2}\right]$$

(13)

Here $r$ is the radius, $\gamma_r$ determines the pulse width in the radial direction.

Figure 3. Dependence of the strength $E(z,r,t)$ for 3D electromagnetic pulse (1 r.u.=$10^6$ V/m) on coordinates: a) the initial profile of the pulse; b) $t=10^{-13}$s; c) $t=5.0\cdot10^{-13}$s; d) $1.5\cdot10^{-12}$s ($v=0.8$). Coordinate $r$ corresponds to 30 nm and $z=20$ nm.

In 3D case, there is a greater spreading of the pulse compared to the 2D consideration. Note that the pulse structure is changed. The pulse is divided into several peaks along the propagation axis, that breaking into the transverse direction, continue to spread with conservation its structure. Also, we investigate the influence of parameter $v$ at the pulse propagation in Fermi liquid in the cylindrical coordinate system. We obtain that a decrease in the parameter $v$ also leads, as in the previous case, to an increase in the pulse amplitude, while it has a stabilizing effect, manifested in a smaller diffraction spreading. That is, using the value of $v$, one can influence the process of the pulse propagation in the Fermi liquid.
An important question is also the stability of the obtained solutions with respect to perturbations that depend on the angle $\phi$. The stability analysis can be carried out on the basis of the linearized equation (10b), which for small perturbations $\delta A$ will have the form:

$$
\delta A_n = \frac{1}{r} \left( r \frac{\partial \delta A}{\partial r} \right) + \frac{\partial^2 \delta A}{\partial z^2} + \frac{1}{r^2} \frac{\partial^2 \delta A}{\partial \phi^2} + 4\pi \frac{\partial j(A)}{\partial A} \delta A
$$

(14)

We note that the last term in (14) is calculated on the solutions $A(z,r,t)$ of equation (10b). Because of the linearity of (14) $\delta A$, one can search as:

$$
\delta A \propto \delta A(z,r,t) \exp(i m \phi)
$$

(15)

Next, we can calculate the corresponding corrections to the electric field: $\delta E = -c^{-1} \partial \delta A/\partial t$

The corresponding results for the electric field are presented in Figure 4 and Figure 5. These figures show the maximum magnitude modulus $\delta E$ (throughout the calculation area) as a function of time and number $m$.

Figure 4. Dependence of maximum modulus $E(r,z,t)$ on time $t$ (1 r.u. = $10^{-12}$ s).

Figure 5. Dependence of maximum $\delta E(r,z,t)$ on $m$

Figure 5 (for $m=5$) shows that the perturbations decrease with time. According to Figure 6 (calculations are made for one instant of time), the perturbation decreases the faster, the larger the number $m$. These results allows us to conclude that the obtained solutions are stable to the angular perturbations.

The above dependences are typical, and it follows that in Fermi liquid a stable propagation of pulses localized in two/three dimensions is possible, which are often called "light bullets" in the literature. Calculations showed that, although there is a diffraction spreading of the pulse in the transverse direction to the direction of propagation, the pulse as a whole retains its shape. It should also be noted that there is a partial curvature of the pulse front, which arises from diffraction.

Key results of this work may be summarized as follows:

1. We proposed a model describing the dynamics of 2D and 3D laser pulse in a Fermi liquid in the framework of the ADS/CFT correspondence. The approximations used in constructing the model are indicated.
2. Numerical calculations have shown that in the 2D case stable nonlinear waves, light pulses localized in two directions, which are analogs of "light bullets" are possible.
3. In the propagation of a "bullet" in a Fermi liquid, its spreading in the transverse direction is rather weak, and the energy is mainly concentrated in the central region of the pulse.
4. It is shown that the value of the parameter $\nu$ has a significant effect on the pulse form.
5. The results of the calculations allow us to conclude that the obtained solutions are stable with respect to the perturbations in the angle $\varphi$.

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