Research Article

Research on the Models of Coupling Dynamics and Damage Classification for Vehicle-Engine Vibration

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Many researchers have designed the dynamic models to study the vehicle-engine vibration. However, the existing mechanical models are relatively simple, and the analysis of engine vibration damage is discussed rarely. In this paper, we proposed the models of coupling dynamics and damage classification of vehicle-engine vibration. The key advantages of these proposed models are (1) the finite elements method is adopted for the rotor and casing system, and the complex structure with multirotor and multicasing is modeled by defining support system and linking methods; (2) the hybrid numerical integral method is used to obtain the inherent frequency of the nonlinear dynamic system; and (3) the algorithms based on backpropagation (BP) neural network and radial basis function (RBF) neural network are chosen to construct the damage classification model of rotors. Experimental results based on the engine rotor tester prove that the proposed models are not only more robust than the existing works but also show that the classification algorithms can support engine damage analysis effectively.

1. Introduction

Currently, the main methods of calculating the critical speed and imbalance steady state response of the rotor-support system are the conventional transfer matrix, finite element (FEM), and substructure modal synthesis [1]. Sun and Dai used the substructure transfer matrix to calculate and analyze the vibration of the engine [2]. Zheng et al. applied the overall transfer matrix to discuss the critical speed and strain energy distribution of a certain engine rotor-support-casing installation system [3]. In [4], the model synthesis method and FEM were combined to study the vibration characteristics of the engine rotor-support-casing system. The influence of rotor stiffness on the dynamic characteristics was analyzed [5], and the static stiffness, dynamic stiffness, and FEM model were used to calculate the characteristics. A flexible double-rotor FEM model with the nonlinearity of rolling bearing and squeeze film damper was proposed [6], and then the transient response simulation of blade loss was implemented. Sun et al. proposed a FEM model of two-rotor gas turbine engine [7]. In the work, the numerical integration method was used to calculate the sudden unbalance response generated by the blade loss. Meanwhile, the touch nonlinearity of the rolling bearing, the nonlinearity of the squeezed oil film force, and the thermal growth effect of bearing components during blade loss were also discussed. Hai et al. built two-rotor dynamic model with nonlinear squeeze film damper bearing [8]. They utilized NASTRAN FEM software to obtain the model parameters of the linear undamped system, and then the nonlinear numerical simulation analysis was performed by MATLAB software. Finally, the calculation speed was greatly improved compared with the traditional method. Finley et al. provided an analytical approach for expeditiously understanding and solving the vibration problems [9]. They
utilized the proper data collection and analysis techniques, and then the true source of the vibration can be discovered. In [10], the work considered the free vibration of a Jeffcott rotor whose shaft has a strong nonlinear elastic property. Wang et al. focused on the dynamic responses of whole aeroengine with blade-casing rubbing. They proposed several methods [11–15] to improve energy efficiency and compact structure. These results can provide important theoretical and engineering references for the safe operation of dual-rotor system and the exact identification of coupling faults.

However, the methods of modal synthesis, the FEM, and the impedance coupling are limited in practical applications due to the model complexity and the large computational workload. For example, the transfer matrix method can only handle simple boundary conditions and the calculation accuracy of the substructure transfer method is related to the model simplification error. In addition, the existing methods focus on the analysis of linear systems. Actually, there are many nonlinear factors, such as rolling bearing clearance, nonlinear contact force, variable flexibility (VC) vibration, and squeeze film damper. While the rotor produces rubbing, looseness, misalignment, and bearing failure, the rotor-support-casing coupling system will show strong nonlinearity, which can make the existing methods noneffective. In [16], a new rotor-rolling bearing-mechanical coupling dynamics model was established. Considering the squeeze film damping effect, the rolling bearing nonlinearity, and the rubbing fault, the rotor was seen as the equal section free Euler beam. Then, the model was analyzed by the modal truncation method, and the numerical response method was used to obtain the system response. Herisanu et al. analyzed the nonlinear dynamic behavior of a rotating electrical machine rotor-bearing system using the optimal auxiliary functions method [17]. The approximate analytical solution is in very good agreement with the numerical simulation results, which prove the reliability of this procedure.

In order to solve the complicated structure of the engine, this paper describes a general model of rotor-support-casing coupling dynamics for engine vibration, which uses the finite element method to model the rotor and casing system firstly. Secondly, this work takes the support system as a lumped parameter model, and then the various support and connection types were defined for the complex rotor-support-casing system modeling. Thirdly, the system nonlinear response is obtained by the numerical value method based on the nonlinear factors. In order to verify the method, the experiments have been implemented on the rotor testers.

Besides the analysis of model coupling dynamics, this paper also studied the rotor damage detection. There are several intelligent methods that have been used, such as the BP neural network method [18–21], the radial basis function (RBF) neural network method [22–24], genetic algorithm [25], the decision tree method [26], the k-nearest neighbour method [27], and the support vector machine [28]. Among the above methods, the BP method is a kind of multilayered feed-forward neural network, which includes the signal forward transfer and the error backward propagation. With abilities of the distributed storage, nonlinear mapping, high accuracy, and parallel computing, it is adaptive for the dynamic damage detection of the rotor. Compared to BP, RBF neural network is an efficient feed-forward network with the best approximation performance and global optimal characteristics that other forward networks do not have, and the structure is simple and the training speed is fast. According to the advantages of the two algorithms, BP and RBF are selected to construct the detection models. Numerical analysis indicates that both the two classification models are feasible to build damage classification model, and RBF performs better than BP.

The paper is organized as follows. Section 2 presents the proposed coupled dynamics model. In Section 3, the rotor damage detection model is introduced. Section 4 shows the experimental results. In Section 5, we conclude this paper.

2. Complex Rotor-Support-Casing Dynamic Model

2.1. Rotor Model. Figure 1 shows the proposed FEM rotor dynamic model. In the model, the rotor includes a number of supports and turntables, which are divided into common beam elements by the finite element method. Meanwhile, the shear deformation, gyro moment, and moment of inertia of the rotor system are also considered. Any rotor is coupled with other rotors, casings, and supports by nonlinear forces and moment of force. In addition, the rotor node will also be subjected to external excitation.

The rotor contains distributed mass, elastic shaft, and stiff plate, which have N nodes and M plates. For each shaft unit, $E$, $I$, $G$, $\mu$, $L$, $p$, and $A$ represent the elastic modulus, area moment of inertia, shear modulus, Poisson ratio, shaft length, shaft density, and shaft cross-sectional area, respectively. For disc $P_r$, $m_{d, r}$, $J_{d, r}$, and $J_{p, r}$ are disc mass, equatorial moment of inertia, and pole moment of inertia; $F_{xi}$ and $F_{yi}$ are the force of the $i$th node; $M_{xi}$ and $M_{yi}$ are the moment of force of the $i$th node of the rotor.

In the system, $x$, $y$, and $z$ are the fixed coordinate system. In the deformed state, the position of any section can be determined as follows: the position of the elastic center line can be obtained by the displacements of $x(s, t)$ and $y(s, t)$, the orientation of the cross-sectional is decided by wrapping $(s, t)$ angles of $x$ and $y$, and the cross section also rotates its center line.

2.1.1. Rigid Plate Element Equation of Motion. The plate mass, moment of inertia of the equator, the moment of inertia of the pole, and rotational angular velocity of plate are
set as \( m_p, J_{dd}, J_{pd}, \) and \( \omega, \) respectively. According to the Lagrangian equation, the equation of motion of rigid plate with fixed coordinate system can be set as

\[
(M_{td} + M_{rd})\ddot{q}_d - \omega G_d \dot{q}_d = Q_d,
\]

where \( Q_d \) is generalized external force vector; \( M_{td} \) and \( M_{rd} \) are mass matrix and mass inertia matrix; \( G_d \) is gyro-matrix; and \( q_d = [x, y, \phi, \psi] \) is generalized displacement vector. They can be described as follows:

\[
M_{td} = \begin{bmatrix}
    m_p & 0 & 0 & 0 \\
    0 & m_p & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
M_{rd} = \begin{bmatrix}
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & J_{dd} & 0 \\
    0 & 0 & 0 & J_{dd}
\end{bmatrix},
\]

\[
G_d = \begin{bmatrix}
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & -J_{pd} \\
    0 & 0 & J_{pd} & 0
\end{bmatrix}.
\]

2.1.2. Ordinary Beam Element Equation of Motion. Let the element modulus of elasticity, the shear modulus, the Poisson ratio, the inner diameter, the outer diameter, and the length be \( E, G, \mu, d, D, \) and \( L; \) then, \( I \) can be calculated:

\[
I = \frac{\pi}{64} (D^4 - d^4).
\]

Cross-sectional area \( A: \)

\[
A = \frac{\pi}{4} (D^2 - d^2).
\]

Effective shear area \( A_s: \)

\[
A_s = \frac{A}{10/9 (1 + (1.6D \times d/D^2 + d^2)^2)}
\]

Shear deformation coefficient \( \phi_s: \)

\[
\phi_s = \frac{12EI}{GA_sL^2}.
\]

Each beam element has 2 nodes and 8 degrees of freedom. Each node has 4 degrees of freedom, i.e., the coordinate system of \( x \) and \( y, \) and the angles around them. The cross-sectional displacement of the unit as a function of time is also a function of the position along the axis of the unit. The generalized displacement of the unit endpoint with time change is

\[
q_e(t) = [q_{1e} q_{2e} q_{3e} q_{4e} q_{5e} q_{6e} q_{7e} q_{8e}]^T.
\]

According to Lagrange equation, the equation of motion of the beam element relative to the fixed coordinate is obtained:

\[
(M_{te} + M_{re})\ddot{q}_e + (-\omega G_e)\dot{q}_e + (K_{te} - K_{re})q_e = Q_e,
\]

where \( Q_e \) is the generalized external force vector, \( M_{te} \) and \( M_{re} \) are mass matrix and mass inertia matrix, \( G_e \) is gyro-matrix, \( K_{te} \) is shearing stiffness matrix, and \( K_{re} \) is the unit tensile stiffness matrix. So, they can be calculated as follows:

\[
\begin{figure}
\centering
\includegraphics[width=\textwidth]{finite_element_rotor_dynamic_model.png}
\caption{Finite element rotor dynamic model.}
\end{figure}
\begin{equation}
\mathbf{M}_{te} = \frac{\rho L}{(1 + \phi_s)^2} \begin{bmatrix}
M_{t_1} & 0 & 0 & M_{t_4} & M_{t_3} & 0 & 0 & -M_{t_5} \\
0 & M_{t_1} & -M_{t_4} & 0 & 0 & M_{t_3} & M_{t_4} & 0 \\
0 & -M_{t_4} & M_{t_2} & 0 & 0 & -M_{t_5} & M_{t_6} & 0 \\
M_{t_4} & 0 & 0 & M_{t_2} & M_{t_5} & 0 & 0 & -M_{t_4} \\
M_{t_3} & 0 & 0 & M_{t_5} & M_{t_1} & 0 & 0 & M_{t_4} \\
0 & M_{t_5} & M_{t_6} & 0 & 0 & M_{t_4} & M_{t_2} & 0 \\
-M_{t_5} & 0 & 0 & M_{t_6} & -M_{t_4} & 0 & 0 & M_{t_2} 
\end{bmatrix},
\end{equation}

\begin{align}
M_{t_1} &= \frac{13}{15} + \frac{7}{10} \phi_s + \frac{1}{3} \phi_s^2, \\
M_{t_2} &= \left(\frac{1}{105} + \frac{1}{60} \phi_s + \frac{1}{120} \phi_s^2\right)L^2, \\
M_{t_3} &= \frac{9}{70} + \frac{3}{10} \phi_s^2 + \frac{1}{6} \phi_s, \\
M_{t_4} &= \left(\frac{11}{210} + \frac{1}{120} \phi_s + \frac{1}{24} \phi_s^2\right)L, \\
M_{t_5} &= \left(\frac{13}{420} + \frac{3}{40} \phi_s + \frac{1}{24} \phi_s^2\right)L, \\
M_{t_6} &= \left(\frac{1}{140} + \frac{1}{60} \phi_s + \frac{1}{120} \phi_s^2\right)L^2,
\end{align}

\begin{equation}
\mathbf{M}_{re} = \frac{\rho L}{(1 + \phi_s)^2} \left(\frac{r_p}{T}\right)^2 \begin{bmatrix}
M_{r_1} & 0 & 0 & M_{r_4} & -M_{r_1} & 0 & 0 & M_{r_4} \\
0 & M_{r_1} & -M_{r_4} & 0 & 0 & -M_{r_1} & -M_{r_4} & 0 \\
0 & -M_{r_4} & M_{r_2} & 0 & 0 & M_{r_4} & M_{r_3} & 0 \\
M_{r_4} & 0 & 0 & M_{r_2} & -M_{r_4} & 0 & 0 & M_{r_3} \\
-M_{r_1} & 0 & 0 & -M_{r_2} & M_{r_1} & 0 & 0 & -M_{r_4} \\
0 & -M_{r_1} & M_{r_4} & 0 & 0 & M_{r_1} & M_{r_4} & 0 \\
0 & -M_{r_4} & M_{r_3} & 0 & 0 & M_{r_4} & M_{r_2} & 0 \\
M_{r_4} & 0 & 0 & M_{r_3} & -M_{r_4} & 0 & 0 & M_{r_2} 
\end{bmatrix},
\end{equation}

\begin{align}
M_{r_1} &= \frac{6}{5}, \\
M_{r_2} &= \left(\frac{2}{15} + \frac{1}{6} \phi_s + \frac{1}{3} \phi_s^2\right)L^2, \\
M_{r_3} &= \left(\frac{1}{30} - \frac{1}{6} \phi_s + \frac{1}{6} \phi_s^2\right)L^2, \\
M_{r_4} &= \left(\frac{1}{10} - \frac{1}{2} \phi_s\right)L, \\
r_p &= \sqrt{\frac{I}{A}},
\end{align}

\begin{equation}
G_e = \frac{\rho}{15L} \left(\frac{r_p}{1 + \phi_s}\right)^2 \begin{bmatrix}
0 & -G_1 & G_2 & 0 & 0 & G_1 & G_2 & 0 \\
G_1 & 0 & 0 & G_2 & -G_1 & 0 & 0 & G_2 \\
-G_2 & 0 & 0 & -G_4 & G_2 & 0 & 0 & G_3 \\
0 & -G_2 & G_4 & 0 & 0 & G_2 & -G_4 & 0 \\
0 & G_1 & -G_2 & 0 & 0 & -G_1 & -G_2 & 0 \\
-G_1 & 0 & 0 & -G_2 & G_1 & 0 & 0 & -G_4 \\
-G_2 & 0 & 0 & G_3 & G_2 & 0 & 0 & -G_4 \\
0 & -G_2 & -G_3 & 0 & 0 & G_2 & G_4 & 0 
\end{bmatrix},
\end{equation}
In equation (13),

\[ G_1 = 36, \]
\[ G_2 = 3L - 15L\phi_s, \]
\[ G_3 = L^2 + 5L^2\phi_s - 5L^2\phi_s^2, \]
\[ G_4 = 4L^2 + 5L^2\phi_s + 10L^2\phi_s^2, \]

\[
K_{be} = \frac{EI}{L^3} \begin{bmatrix}
K_{b_4} & 0 & 0 & K_{b_1} & 0 & 0 & K_{b_4} \\
0 & K_{b_4} & 0 & 0 & K_{b_1} & 0 & K_{b_4} \\
0 & 0 & K_{b_1} & 0 & 0 & K_{b_4} & K_{b_3} \\
K_{b_4} & 0 & 0 & K_{b_2} & 0 & 0 & K_{b_3} \\
-K_{b_1} & 0 & 0 & -K_{b_2} & K_{b_1} & 0 & 0 & -K_{b_3} \\
0 & -K_{b_4} & 0 & 0 & K_{b_1} & K_{b_4} & 0 \\
0 & 0 & K_{b_4} & 0 & 0 & K_{b_4} & K_{b_2} & 0 \\
K_{b_4} & 0 & 0 & K_{b_3} & -K_{b_4} & 0 & 0 & K_{b_2}
\end{bmatrix},
\]

\[
K_{b_1} = \frac{12}{1 + \phi_s};
\]
\[
K_{b_2} = \frac{4 + \phi_s L^2}{1 + \phi_s};
\]
\[
K_{b_3} = \frac{2 - \phi_s L^2}{1 + \phi_s};
\]
\[
K_{b_4} = \frac{6}{1 + \phi_s^2}.
\]
2.1.3. Motion Equation of Rotor System. By assembling the equation of motion of the element, the motion equation of the rotor system can be obtained:

\[ M_\varepsilon \ddot{q}_e + \left( C_s - \omega G_s \right) \dot{q}_e + K_s q_e = Q_e, \]  

(15)

where \( Q_e \) is the system generalized external force vector, \( M_\varepsilon \) is the system mass matrix, \( G_s \) is the system gyromatrix, \( K_s \) is the system stiffness matrix, and \( C_s \) is the system damping matrix.

2.2. Casing Model. There are three different treatment methods available for the casing: beam elements, cone shell elements, and curved shell elements. Although the casing is a kind of shell structure, its vibration mode is in various forms, including a mode with a circumferential wave number of 0, 1, 2, ..., \( \omega / \pi \). However, when it is coupled with the rotor, it can only be one circumferential wave number. A bending moment is generated at a coupling point with the rotor and is coupled to the rotor. As for the vibration modes of other circumferential wave numbers, the torque generated at the coupling node with the rotor is self-balancing and is not coupled with the rotor bending. Therefore, in the rotor dynamics analysis, the vibration mode of the casing with a circumferential wave number of 1 is usually considered. At this time, the cross section of the casing is not deformed, and its axial direction is a curved mode. Accordingly, the current general processing method is to treat the casing as a beam unit, which is equivalent to the nonrotating shaft (beam unit structure) and also considers the shear effect and moment of inertia [1].

Therefore, this paper treats the casing as a nonrotating beam, which is the same as the rotor model. The FEM can be used to obtain the motion differential equation of the casing:

\[ M_c \ddot{q}_c + C_c \dot{q}_c + K_c q_c = Q_c, \]  

(16)

where \( Q_c \) is the generalized external force vector of the casing system, \( M_c \) is the stiffness matrix of the casing system, \( K_c \) is the stiffness matrix of the casing system, and \( C_c \) is the damping matrix of the casing system and is assumed to be proportional damping.

2.3. Discrete Support Model. In order to model the coupling system for multirotor and multimachine of the actual engine, it is necessary to fully consider the connection and support between the rotor and the casing. Therefore, a variety of support and connection for rotor and casing are, namely, the rotor-casing support connection, the intermediate support connection between the rotors and the coupling between the rotors, the support connection between the casings, the elastic support between the casing and the foundation. The combination of these supports and connections can be used to model the multirotor and multicasing of engine.

2.3.1. Supporting Connection between Rotor and Casing. For \( RC_i (i = 1, 2, \ldots) \) between the rotor and the casing, rolling bearings, squeeze film dampers, bearing housings, and other components are included. This research assumes that \( m_{rai} \) is the quality of the outer ring of the rolling bearing, \( m_{bi} \) is the mass of the bearing seat, \( k_t \) is the elastic bearing stiffness between the outer ring of the rolling bearing and the bearing seat, \( c_t \) is the damping coefficient between the outer ring of the rolling bearing and the bearing housing and it is nonlinear damping if there is a squeeze film damper, \( k_b \) and \( c_b \) are the support stiffness and damping between the casing and the bearing chock, \( F_{xri} \) and \( F_{yri} \) are the force acting on the support for the rotor, and \( F_{xci} \) and \( F_{yci} \) are the force acting on the support for the casing.

In this paper, the outer ring of the bearing is fixed on the bearing chock and the inner ring is fixed on the rotating shaft. The displacement of the \( m \) node of the rotor is set to \( x_{ri} \) and \( y_{ri} \) where \( x = x_{ri} - x_{wi} \) and \( y = y_{ri} - y_{wi} \). According to [29, 30], the bearing force of the rotor acting on the \( i \) support can be obtained:

\[
\begin{align*}
F_{xri} &= \sum_{j=1}^{N} C_{bi}(x \cos \theta_j + y \sin \theta_j - r_0)^{3/2} H(x \cos \theta_j + y \sin \theta_j - r_0) \cos \theta_j, \\
F_{yri} &= \sum_{j=1}^{N} C_{bi}(x \cos \theta_j + y \sin \theta_j - r_0)^{3/2} H(x \cos \theta_j + y \sin \theta_j - r_0) \sin \theta_j,
\end{align*}
\]  

(17)

where \( C_{bi} \) is the Hertz contact stiffness obtained by Hertzian elastic analysis between the inner and outer of ring and the ball. \( H(\ast) \) is the Heaviside function. When the function variable is greater than 0, the function value is 1; otherwise it is 0. \( \theta_j \) is the angular position of the first ball and \( \theta_j = \omega_{coge} \times t + (2\pi / N_p) (j - 1), j = 1, 2, \ldots, N_p, N_p \) is the number of balls. \( \omega_{coge} \) is the rotation speed of the frame, \( R \) is the radius of the outer raceway, \( r \) is the radius of the inner raceway, and \( \omega_{coge} = (\omega \times r / R + r) \), where \( \omega \) is the angular velocity of the rotary shaft. Therefore, the differential equation of motion of the outer ring is

\[
\begin{align*}
m_{bi} \ddot{x}_{bi} + k_{ti}(x_{bi} - x_{wi}) + c_{ti}(\dot{x}_{bi} - \dot{x}_{wi}) &= F_{xroi}, \\
m_{bi} \ddot{y}_{bi} + k_{ti}(y_{bi} - y_{wi}) + c_{ti}(\dot{y}_{bi} - \dot{y}_{wi}) &= F_{yroi} - m_{bi} \ddot{g},
\end{align*}
\]  

(18)

where \( F_{dxi} \) and \( F_{dyi} \) are the damping forces.
2.3.2. Intermediary Bearing Support between Rotors. For the intermediary bearing support between the rotors, RRM (i = 1, 2, ..., N) includes rolling bearings, bearing housings, and other components. Here, the outer ring mass of the rolling bearing is set to \( m_{x_{bi}} \) and the mass of the bearing seat, \( k_{bi} \) and \( c_{bi} \), are the elastic bearing stiffness and damping coefficient between the outer ring of the bearing and the bearing chock, and \( k_{bi} \) and \( c_{bi} \) are the bearing stiffness and damping between the outer rotor and the bearing chock. We suppose \( F_{x_{ri}} \) and \( F_{y_{ri}} \) are the force acting on the inner rotor and the support and \( F_{x_{roi}} \) and \( F_{y_{roi}} \) are the force acting on the outer rotor on the support. RRM (i = 1, 2, ..., N) is connected to the inner rotor and the outer rotor.

The outer ring of the intermediate bearing is fixed on the outer rotor and rotates with the outer rotor, and the inner ring is fixed on the inner rotor and rotates with the inner rotor. The rotation speeds are set to \( \omega_{out} \) and \( \omega_{in} \). The calculation formula of the bearing force is the same as formula (8). Since both the inner and outer rings of the bearing rotate, the rotation frequency of the cage is calculated:

\[
\omega_{cage} = \omega_{in} \times r + \omega_{out} \times \frac{R}{R + r}.
\]

Set the displacement of the rolling bearing to \( x_{rm} \) and \( y_{rm} \), where \( x = x_{m} - x_{bi} \) and \( y = y_{m} - y_{bi} \). According to formula (17), the bearing force of the rotor acting on the support can be obtained as \( F_{x_{yi}} \) and \( F_{y_{yi}} \). Therefore, the differential motion equation of the outer ring of the rolling bearing is

\[
\begin{align*}
m_{x_{bi}} \ddot{x}_{bi} + k_{bi} (x_{bi} - x_{ri}) + c_{bi} (\dot{x}_{bi} - \dot{x}_{ri}) &= F_{x_{roi}}, \\
m_{y_{bi}} \ddot{y}_{bi} + k_{bi} (y_{bi} - y_{ri}) + c_{bi} (\dot{y}_{bi} - \dot{y}_{ri}) &= F_{y_{roi}}, \\
&(i = 1, 2, ..., N).
\end{align*}
\]

Furthermore, the displacement of the outer rotor is set as \( x_{ron} \) and \( y_{ron} \), and the displacement of the bearing chock for RRM is \( x_{bi} \) and \( y_{bi} \). Then, the force acting on the outer rotor is

\[
\begin{align*}
F_{x_{roi}} &= k_{fi} (x_{ron} - x_{ri}) + c_{fi} (\dot{x}_{ron} - \dot{x}_{ri}), \\
F_{y_{roi}} &= k_{fi} (y_{ron} - y_{ri}) + c_{fi} (\dot{y}_{ron} - \dot{y}_{ri}), \\
&(i = 1, 2, ..., N).
\end{align*}
\]

Therefore, the differential equation of motion of the bearing chock for is

\[
\begin{align*}
m_{x_{bi}} \ddot{x}_{bi} + k_{bi} (x_{bi} - x_{ri}) + c_{bi} (\dot{x}_{bi} - \dot{x}_{ri}) &= F_{x_{roi}}, \\
m_{y_{bi}} \ddot{y}_{bi} + k_{bi} (y_{bi} - y_{ri}) + c_{bi} (\dot{y}_{bi} - \dot{y}_{ri}) &= F_{y_{roi}}, \\
&(i = 1, 2, ..., N).
\end{align*}
\]

2.3.3. Rotor-Rotor Coupling Connection. For the coupling connection RRC (k = 1, 2, ..., N) between the rotors, let the \( i \)th node of the left rotor connect the \( j \)th node of the right rotor, \( k_{rr} \) be the radial stiffness of the coupling, \( k_{ra} \) be the angular stiffness, \( c_{rr} \) be the radial damping, and \( c_{ra} \) be the angular damping. Meanwhile, \( x_{ri}, y_{ri}, \phi_{ri}, \psi_{ri} \) are set to the displacement of the \( i \)th node of the left rotor and \( x_{r ji}, y_{r ji}, \phi_{r ji}, \psi_{r ji} \) are the velocity; \( x_{r ji}, y_{r ji}, \phi_{r ji}, \psi_{r ji} \) are set to the displacement of the \( j \)th node of the right rotor and \( x_{m ji}, y_{m ji}, \phi_{m ji}, \psi_{m ji} \) are the velocity. Then, the forces and moments acting on the left rotor node \( i \) and the right rotor node \( j \) are \( F_{x_{ri}}, F_{y_{ri}}, M_{x_{ri}}, M_{y_{ri}}, M_{r xi}, M_{r yi} \) can be obtained:

\[
\begin{align*}
F_{x_{ri}} &= k_{rr} (x_{r ji} - x_{ri}) + c_{rr} (\dot{x}_{r ji} - \dot{x}_{ri}), \\
F_{y_{ri}} &= k_{rr} (y_{r ji} - y_{ri}) + c_{rr} (\dot{y}_{r ji} - \dot{y}_{ri}), \\
M_{x_{ri}} &= k_{ra} (\phi_{r ji} - \phi_{ri}) + c_{ra} (\dot{\phi}_{r ji} - \dot{\phi}_{ri}), \\
M_{y_{ri}} &= k_{ra} (\psi_{r ji} - \psi_{ri}) + c_{ra} (\dot{\psi}_{r ji} - \dot{\psi}_{ri}), \\
F_{x_{yj}} &= -F_{y_{yi}}, \\
M_{x_{yi}} &= -M_{x_{yi}}, \\
M_{y_{yi}} &= -M_{y_{yi}}.
\end{align*}
\]

2.3.4. Elastic Connection between the Casing. For the elastic connection between the casings CC (k = 1, 2, ..., N), let the \( i \)th node of casing 1 connect the \( j \)th node of casing 2 by bolts, \( k_{c} \) be the radial stiffness of the bolt, \( k_{ca} \) be the angular stiffness, \( c_{c} \) be the radial damping, and \( c_{ca} \) be the angular damping. Meanwhile, let \( x_{c i}, y_{c i}, \phi_{c i}, \psi_{c i} \) be the displacement of the \( i \)th node of casing 1 and \( x_{c ji}, y_{c ji}, \phi_{c ji}, \psi_{c ji} \) be the velocity; let \( x_{c 2 j}, y_{c 2 j}, \phi_{c 2 j}, \psi_{c 2 j} \) be the displacement of the \( j \)th node of casing 2 and \( x_{c 2 j}, y_{c 2 j}, \phi_{c 2 j}, \psi_{c 2 j} \) be the velocity. Then, the forces and moments acting on the node \( i \) of casing 1 are \( F_{c_{i}}, F_{y_{c i}}, M_{x_{c i}}, M_{y_{c i}} \) and the forces and moments acting on the node \( j \) of the casing 2 are \( F_{c_{2 j}}, F_{y_{c 2 j}}, M_{x_{c 2 j}}, M_{y_{c 2 j}} \). The corresponding formulas can be obtained:

\[
\begin{align*}
F_{c_{xi}} &= k_{ca} (x_{c 2 j} - x_{c i}) + c_{ca} (\dot{x}_{c 2 j} - \dot{x}_{c i}), \\
F_{c_{yi}} &= k_{ca} (y_{c 2 j} - y_{c i}) + c_{ca} (\dot{y}_{c 2 j} - \dot{y}_{c i}), \\
M_{c_{xi}} &= k_{ca} (\phi_{c 2 j} - \phi_{c i}) + c_{ca} (\dot{\phi}_{c 2 j} - \dot{\phi}_{c i}), \\
M_{c_{yi}} &= k_{ca} (\psi_{c 2 j} - \psi_{c i}) + c_{ca} (\dot{\psi}_{c 2 j} - \dot{\psi}_{c i}), \\
F_{c_{2 xi}} &= -F_{c_{xi}}, \\
F_{c_{2 yi}} &= -F_{c_{yi}}, \\
M_{c_{2 xi}} &= -M_{c_{xi}}, \\
M_{c_{2 yi}} &= -M_{c_{yi}}.
\end{align*}
\]

2.4. Numerical Simulation Solution Method. Since the dynamic model of the complex rotor-support-case coupling system has many free numbers and a lot of nonlinear factors, the effective method is nonlinear iteration. In this paper, the differential equations are solved by the combination of the
Newmark method and an improved method. As shown in Figure 2, the whole process is described. The Newmark method is used to solve the formed matrix of the rotor and the finite element model of casing. The support members that do not need to form matrix are solved by hybrid integral method. The method is characterized by the fact that only the dynamic matrix of a single rotor or casing component needs to be assembled, without forming a bulky matrix of the entire system.

Let the system’s dynamic equation be the following form:

\[ [M] \{ \ddot{X} \} + [K] \{ X \} = \{ P \}, \quad (25) \]

where \([M], [K]\) are system inertia and stiffness matrix; \(\{ X \}\) is the generalized displacement vector of the system; \(\ddot{X}\) is the generalized acceleration vector of the system; and \(\{ P \}\) is the generalized load vector of the system.

The steps of the proposed method are as follows:

(i) Initial calculation:

(1) Generate stiffness matrix \([K]\) and mass matrix \([M]\).
(2) Obtain \(\{ X_0 \}, \{ V_0 \}, \{ A_0 \}\).
(3) Choose step \(\Delta t\) and parameters \(\alpha, \beta\) and then calculate constant:

\[
\begin{align*}
    &a_0 = \frac{1}{\alpha \Delta t^2}, \\
    &a_1 = \frac{\beta}{\alpha \Delta t}, \\
    &a_2 = \frac{1}{\alpha \Delta t}, \\
    &a_3 = \frac{1}{2\alpha} - 1, \\
    &a_4 = \frac{\beta}{\alpha} - 1, \\
    &a_5 = \frac{\Delta t}{2} \left( \frac{\beta}{\alpha} - 2 \right), \\
    &a_6 = \Delta t (1 - \beta), \\
    &a_7 = \beta \Delta t.
\end{align*}
\]

(21)

(4) Generate effective stiffness matrix:

\[ [\tilde{K}] = [K] + a_0[M] + a_1[C]. \quad (27) \]

(5) Obtain inverse matrix \([\tilde{K}]^{-1}\).

(ii) Calculate each time step:

(1) The load vector at \(n + 1\):

\[
\begin{align*}
\{ \tilde{P}_{n+1} \} &= \{ P_n \} + [M] (a_0 \{ X_n \} + a_2 \{ V_n \} + a_3 \{ A_n \}) \\
&\quad + [C] (a_1 \{ X_n \} + a_4 \{ V_n \} + a_5 \{ A_n \}).
\end{align*}
\]

(28)

(2) Get the displacement, velocity, and acceleration at \(n + 1\):

\[
\begin{align*}
\{ X_{n+1} \} &= [\tilde{K}]^{-1} \{ \tilde{P}_{n+1} \}, \\
\{ A_{n+1} \} &= a_0 \{ X_{n+1} \} - X_n - a_2 \{ V_n \} - a_3 \{ A_n \}, \\
\{ V_{n+1} \} &= \{ V_n \} + a_6 \{ A_n \} + a_7 \{ A_{n+1} \}. \quad (29)
\end{align*}
\]

For equation (25), the integral of hybrid method is

\[
\begin{align*}
\{ X \}_{n+1} &= \{ X \}_n + \{ V \}_n \Delta t + \left( \frac{1}{2} + \psi \right) \{ A \}_{n\Delta t^2} - \psi \{ A \}_{n-1\Delta t^2}, \\
\{ V \}_{n+1} &= \{ V \}_n + \{ \phi \} \{ A \}_{n\Delta t} - \phi \{ A \}_{n-1\Delta t}, \quad (30)
\end{align*}
\]

where \(\Delta t\) is the time step, \(n\) represents \(t = n \Delta t\) instant, \(n + 1\) describes \(t = (n + 1) \Delta t\) instant, \(n - 1\) is the instant of \(t = (n - 1) \Delta t\), \(\psi, \phi\) are instant parameters, and \(\psi = \phi = 1/3\). Take \(\phi\), for example, the reason for choosing parameters as follows:

\[
\begin{align*}
\{ A \}_{n+1/2} &= \frac{3}{2} \{ A \}_n - \frac{1}{2} \{ A \}_{n-1}, \quad \{ A \}_{n+1} = 2 \{ A \}_n - \{ A \}_{n-1}, \\
\{ V \}_{n+1} &= \{ V \}_n + \Delta t (\{ A \}_{n+1} + 4 \{ A \}_{n+1/2} + \{ A \}_n) / 6 \\
&= \{ V \}_n + \frac{4}{3} \{ A \}_{n\Delta t} - \frac{1}{3} \{ A \}_{n-1\Delta t}.
\end{align*}
\]

(31)

For the two methods, we can find that the Newmark-\(\beta\) method needs to form the dynamic matrix, but does not require the diagonalization of the mass matrix. The improved hybrid integral method (31) does not require the formation of the dynamic matrix, and it can be solved directly from the differential equation. But the mass matrix is required to be the angular array, so this paper combines the implicit integral Newmark-\(\beta\) and explicit hybrid integral method to avoid the formation of large dynamic matrix, which could improve the efficiency of system modeling and the speed of solution.

3. Rotor Damage Detection Model

This section discusses how to build a classification model for rotor damage detection. According to the previous work’s experience, this paper applies the neural network methods of BP and RBF to train the frequency data obtained by the built couple dynamics model in Section 2, and then the classification models are generated. The details are described in the following sections.

3.1. The Model Based on BP Neural Network. The neural network has various forms, and the backpropagation (BP) model is the most widely used. It is a one-way propagation multilayer forward neural network. Its typical structure is shown in Figure 3. BP has one or more hidden nodes.
Because there is no coupling on the same node, the output of each node only affects the output of the next node. Meanwhile, BP network is a highly nonlinear mapping from input to output, which is based on the gradient descent method. The learning of BP network is decomposed into two processes in this paper: in Phase 1 (forward propagation process), it gives input information and calculates the actual output value of each unit layer by layer through the hidden layer and the input layer. When the output layer fails to produce the desired output, it will switch to the backpropagation. In Phase 2 (backpropagation process), the difference between the actual output and the expected output is recursively calculated layer by layer, and then the weight is adjusted based on the difference. Repeat the above two processes until the error is adjusted to the error tolerance and stop learning.

The neural network needs to be trained during the processes of function approximation and pattern recognition. The data sample is first required before training, including the input vector $p$ and the corresponding expected output vector $D$. Then, the weight and threshold are constantly adjusted during the training in order to minimize the error function of the neural network. Normally, the BP neural network error function is defined as the mean square error of the expected output vector $D$ and the network output $A$.

The hidden layer output is

$$y_j = f \left( \sum_i w_{ji} x_i - \theta_j \right) = f \left( \text{net}_j \right). \quad (32)$$

The actual output of output layer is

$$z_k = f \left( \sum_i w_{kj} y_j - \theta_l \right) = f \left( \text{net}_k \right), \quad (33)$$

where $x_i, y_j, z_k$ are input nodes, hidden layer nodes, and output nodes, respectively. $W_{ji}$ is the network weight of the input node and the hidden layer node, and $W_{kj}$ is the network weight of the hidden layer node and the output node.

If the expected output is $t_k$, the error is

$$E = \frac{1}{2} \sum_k \left( t_k - z_k \right)^2 \quad (34)$$

Next, it is implemented by correcting the weights and thresholds and deriving the hidden layer and output layer weights, respectively.

$$\begin{align*}
\frac{\partial E}{\partial W_{ji}} &= \sum_k \frac{\partial E}{\partial Z_k} \frac{\partial Z_k}{\partial W_{ji}} = \frac{\partial E}{\partial Z_k} \frac{\partial Z_k}{\partial W_{ji}} \\
\frac{\partial E}{\partial W_{kj}} &= \sum_j \frac{\partial E}{\partial Z_j} \frac{\partial Z_j}{\partial W_{kj}} \frac{\partial W_{kj}}{\partial W_{kj}} \\
\frac{\partial E}{\partial \theta_l} &= \sum_j \frac{\partial E}{\partial Z_j} \frac{\partial Z_j}{\partial \theta_l} \\
\frac{\partial E}{\partial \theta_j} &= \sum_k \frac{\partial E}{\partial Z_k} \frac{\partial Z_k}{\partial \theta_j} \frac{\partial \theta_j}{\partial \theta_j}
\end{align*} \quad (35)$$

Since the correction of the weights $\Delta W_{ji}$ and $\Delta W_{kj}$ are proportional to the error function falling along the gradient,

$$\begin{align*}
\Delta w_{kj} &= -\eta \frac{\partial E}{\partial w_{kj}} \\
\Delta w_{ji} &= -\eta \frac{\partial E}{\partial w_{ji}} \quad (36)
\end{align*}$$

where $\eta$ is the learning rate, and then the weight correction formula is

$$\begin{align*}
w_{kj}(k+1) &= w_{kj}(k) + \Delta w_{kj}, \\
w_{ji}(k+1) &= w_{ji}(k) + \Delta w_{ji} \quad (37)
\end{align*}$$

The above learning process reflects that the correction of the network weight and threshold should follow the direction in which the performance function declines the fastest—the negative gradient direction.

3.2. The Model Based on RBF Neural Network. There are three layers including the input layer, the hidden layer with non-linear neurons, and the output layer with linear neurons in
RBF neural network [22–24]. It uses the radial basis function (RBF) as the basis of the hidden layer unit to form the hidden layer space. The hidden layer projects the input vector of the low-dimensional data into the high-dimensional space, which makes it linearly separable in high-dimensional space.

The RBF network and the BP network are basically the same in structure, but the difference is that the RBF network hidden layer node transfer function is a radial basis function. The radial basis function appears in many forms, and the common one is a Gaussian function:

$$\varphi_i(x) = \exp\left(-\frac{||x - c_i||^2}{2\sigma_i^2}\right),$$

where $c_i$ is the center of $i$, $\sigma_i$ is the parameter that controls the size of the receiving domain, and $||\cdot||$ is European norm.

The $k$th output node of the network completes the linear combination of the output of the hidden layer node:

$$Y_k = \sum_i \omega_{k,i} \varphi_i - b_k,$$

where $b_k$ is the domain value of the $k$th output node and $\omega_{k,i}$ is the output weight of $\varphi_i$ to $Y_k$.

Next, this paper carries out the RBF model by two stages, including the learning of the hidden layer and the learning of the output layer. Firstly, the hidden layer learning is performed on the determination of the basis function, the center, and width. It is usually unsupervised learning. This paper selects the center vector $C_i$ and the normalized parameter $\sigma_i$ with reference to the input sequence. Secondly, the learning of the output layer is reflected in the study of weights. The connection weight of the neurons between the hidden layer and the output layer can be directly calculated by the least square method, namely, the partial derivative of $w$ is solved for the loss function.

3.3. The Damage Detection Process. With the support of BP and RBF, the damage detection process is shown in Figure 4. The whole process is generally divided into three parts. Firstly, the training data need to be preprocessed by considering the wear range, which will be introduced in the next section. Secondly, the learning algorithms of BP and RBF are applied to build the damage detection models. Finally, the testing data are used to evaluate the models, and the better model can also be chosen in comparison with the results.

3.4. Wear Range of Rotor Setting. Actually, the rotor damage detection can be recognized as two classification problems. We need to find whether the rotor is damaged. In this paper, the damaged situation is seen as the positive samples and vice versa, and it falls into negative samples. However, rotor wear is normal when the it is running in the engine. Hence, the wear range of the rotor needs to be set in order to ensure the positive samples and the negative samples in this paper. Considering the practical situation, the research designs the wear range of the rotor artificially. Generally speaking, the wear range of the rotor is between 3% and 5%, and the rotor ends are easy to wear. Here, the paper sets the lower limits of the loss of the rotor ends and other parts are 95% and 98%, respectively.
4. Model Verification

4.1. Coupled Dynamics Model Verification

4.1.1. Rotor-Support Bearing Chock Fault Tester. The rotor-rolling bearing fault tester designed and manufactured can effectively simulate the rotor imbalance and the common faults of rolling bearings. The rotor-rolling bearing fault tester physical map and dynamics model are shown in Figure 5. The rotor fault tester mainly comprises a rotating shaft, a rotor disc, a flange connecting plate, a bearing seat, a speed regulating motor, a gear speed increasing device, and the sensors of vibration displacement, rotational speed, and acceleration which can be flexibly mounted on the experimental device in order to conduct the comprehensive vibration test. The experimental device is a single-span and double-disc rotor. The vibration displacement of the turntable is measured by eddy current displacement sensors in both horizontal and vertical directions, and the rotational speed is measured by an eddy current sensor installed at the connection between the speed increaser and the rotating shaft. The sensor senses the displacement pulse caused by the rotation and obtains the rotation speed by counting. In the rotor-support coupling system model, there are two turntables, which include rotor disk $P_1$ and flange disk $P_2$ connected by gear output shaft. $S_1$, $S_2$ are the supports and $L_1$, $L_2$, $L_3$, $L_4$ are the position of the supports on the rotating shaft.

4.1.2. Analysis and Verification

(i) Experimental model analysis: in this paper, the modal analysis of the tester under natural support state is carried out by the hammering method. The main instruments include NI9234 dynamic signal acquisition module of NI (National Instrument) company, 30927 Hammer of ENDEVCO Company, and 4508 type ICP accelerometer from B&K Company. A single measurement point and multipoint incentive program is applied. The tester shaft is evenly tapped from the support to the support by 20 points, recorded as $1, \ldots, 20$. The acceleration sensor is placed at the fourth point, and the acceleration response signal is measured. Finally, the force signal and the acceleration signal are put into the collector, and then the modal parameters of the tester are obtained through the modal analysis software.

(ii) Computational model analysis: for the FEM rotor-support coupling dynamics model, the rotation axis is divided into 48 units from the left end to the right end, and then 49 nodes are obtained, where support is at node 7, support is at node 43, and the hammer point is fixed at node 13. Then, the measurement will include the change from node 1 to node 49. In the calculation, the tapping method of the simulation experiment is to apply a pulse force on node 13 in order to obtain the acceleration response of each node of the rotor through simulation, and then the response values are input into the model analysis software to obtain the intrinsic frequency and the intrinsic modal of the system.

(iii) The analysis results of calculation modal and experimental modal are compared as shown in Tables 1–3. Table 1 shows rotor calculation parameters, Table 2 shows rolling bearing parameters, and Table 3 shows the support parameters.

In Figure 6, the blue point presents the experimental results obtained by the slapping method, the red line presents the simulation results obtained by the proposed coupled dynamics model, and the number values present the mean results of twenty groups’ experiment. The $x$-axis presents the order of the frequency and the $y$-axis presents the value of the frequencies. The comparative analysis of parameters for experimental modal and calculated modal shows that the simulation values of the coupled system are close to the experimental values, and the calculated vibration modes of the rotor are closer to the experimental vibration modes. The simulated values are similar to the experimental values. It proves the correct validity of the modeling method in this paper.
4.2. Rotor Damage Detection Model Verification. In this section, the numerical results are given to compare accuracies of the BP neural network and the RBF neural network. The two models were implemented in Matlab 2014a environment, and experiments were conducted on an Intel(R) Core(TM) i7-6700HQ 8-core CPU (main frequency: 2.60 GHz for each core) and 16 GB RAM.

By the above mentioned couple dynamics model, we firstly gather the frequencies that can be divided into qualified samples and unqualified samples. Then, 10000 sets of data are used in the experiment. Next, 100 times’ ten-fold cross validation (70% of data are randomly selected as the training data and 30% of data as the test data) is used to train the models. Finally, the model performance is mainly validated by accuracy that presents the ratio of the number of correctly predicted samples to the total number of predicted samples, which can be calculated as follows:

\[
\text{accuracy} = \frac{TP + TN}{TP + TN + FP + FN}
\]  

where \( TP \) presents the prediction result is damaged (positive) and the practical result is damaged (positive), FP

### Table 1: Main calculation parameters of the rotor.

| Parameters                      | Value |
|---------------------------------|-------|
| Elastic modulus \( E \) (10^{11} \text{ Pa}) | 2.1   |
| Shaft diameter \( D \) (m)       | 0.019 |
| Density \( \rho/10^3 \) (kg/m^3) | 7.8   |
| Poisson’s ratio \( \mu \)        | 0.3   |
| Proportional damping coefficient \( \alpha_0 \) | 5     |
| Proportional damping coefficient \( \alpha/10^{-8} \) | 1.35  |
| \( L_1 \) (mm)                  | 100   |
| \( L_2 \) (mm)                  | 342   |
| \( L_3 \) (mm)                  | 370   |
| \( L_4 \) (mm)                  | 130   |
| Plate \( P_1 \) mass \( m_p \) (kg) | 2.4   |
| Plate \( P_1 \) polar inertia moment \( J_{dp1} \) (kg·m^2) | 0.0125 |
| Plate \( P_1 \) equatorial inertia moment \( J_{dd1} \) (kg·m^2) | 0.00625 |
| Plate \( P_2 \) mass \( m_p \) (kg) | 0.45  |
| Plate \( P_2 \) polar inertia moment \( J_{dp1} \) (kg·m^2) | 0.00025 |
| Plate \( P_2 \) equatorial inertia moment \( J_{dd1} \) (kg·m^2) | 0.000125 |

### Table 2: Rolling bearing parameters.

| Parameters                     | Value |
|--------------------------------|-------|
| Pitch diameter \( D_m \) (mm)  | 36    |
| Ball diameter \( d \) (mm)    | 9.6   |
| Outer radius \( r \) (mm)     | 22.8  |
| Inner radius \( r \) (mm)     | 13.2  |
| Numbers of balls \( N_b \)    | 7     |
| Stiffness \( C_p/10^9 \) (N/m^3/2) | 11.67 |
| Bearing gap \( r_0 \) (μm)    | 0     |
| Outer ring mass \( m_w \) (kg) | 0.08  |
| Axle-bearing mass \( m_b \) (kg) | 76    |

### Table 3: Support parameters.

| Parameters                     | Value |
|--------------------------------|-------|
| \( k_t/10^8 \) (N/m)          | 1.0   |
| \( c_t \) (N·s/m)             | 500   |
| \( k_f/10^7 \) (N/m)           | 3.0   |
| \( c_f \) (N·s/m)             | 2000  |

![Figure 5: Rotor-rolling bearing tester.](image-url)
indicates the prediction result is damaged (positive) and the practical result is undamaged (negative), TN describes the prediction result is undamaged (negative) and the practical result is undamaged (negative), and FN shows the prediction result is undamaged (negative) and the practical result is damaged (positive).

In practical work, the actual metrical error is less than 1%, and this paper adds 5% white noise for evaluating the robustness of the problem. In order to implement these models, we apply functions newrb() and sim() in the Matlab toolbox to create an approximate BP and RBF neural networks and predict outputs of network, respectively.

In Table 4, the results show the accuracy comparisons of RBF neural network and BP neural network under five kinds of white noise interference, and the RBF neural network performs slightly better detection than BP neural network when the noise data increase.

5. Conclusions

This paper studied the coupling dynamics model of the engine vibration and damage detection model of rotor. For the coupling dynamics model, the FEM is used to model the rotor and the casing system firstly, and then a complex structure with multiorotor and multi-causing is proposed by defining support system and linking methods. Next, the numerical integral method is used to obtain the inherent frequency of the nonlinear dynamic system. Finally, the model experiment is carried out by the rotor tester, and the results prove the correctness and effectiveness of the couple dynamics model.

After the coupled dynamics model is established, two damage detection models of the rotor by BP and RBF neural network are built. The experimental results show that the damage detection models have good robustness and RBF method is better while the data noise increases.

In the future, we will optimize the initialized connection weights and threshold values and apply the deep learning techniques to solve the fault diagnosis of practical engine.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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