THE PHOTON-PION TRANSITION FORM FACTOR FOR VIRTUAL PHOTONS *

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Abstract
We discuss the photon to meson transition form factor for virtual photons, which can be measured in $e^+e^-$ collisions. We demonstrate that this form factor is independent of the shape of the meson distribution amplitude over a wide kinematical range. This leads to a parameter-free prediction of perturbative QCD to leading twist accuracy, which has a status comparable to the famous leading-twist prediction for real photons has been used to constrain the distribution amplitudes being close to their asymptotic form under evolution.

1 INTRODUCTION
Exclusive reactions in QCD involving a large momentum scale are amenable to a perturbative treatment. A particular perturbative approach is the so-called hard scattering formalism [1], where the transition amplitude of a process is written in factorized form as the convolution of a hard scattering amplitude, specifying a partonic subprocess at large scale, and a universal, i.e., process independent, hadronic distribution amplitude. While the hard scattering amplitude is perturbatively calculable, distribution amplitudes describe the soft transition from partons to hadrons and thus cannot be calculated from first principles as yet. Therefore, in order to make reliable predictions for exclusive reactions, it is crucial to obtain information about the shape of distribution amplitudes from other sources.

Figure 1: Sketch of the transition form factor as measured in $e^+e^- \rightarrow e^+e^- P$.

The simplest exclusive observable is the form factor for transitions from a real or virtual photon to a pseudoscalar meson $P$, measurable in electron-positron scattering, $e^+e^- \rightarrow e^+e^- P$, shown in Fig. 1. The CLEO data [2] for real photons has been used to constrain the distribution amplitudes for the pion, the $\eta$, and the $\eta'$, see for instance Refs. [3]–[8], and is compatible with the distribution amplitudes being close to their asymptotic form under evolution.

Generically, the distribution amplitude $\Phi_P$ of a pseudoscalar meson can be expanded in terms of Gegenbauer polynomials $C_n^{3/2}$, the eigenfunctions of the leading-order evolution kernel:

$$\Phi_P(\xi, \mu_F) = \Phi_{AS}(\xi) \left[ 1 + \sum_{n=2,4,\ldots} \infty B_n^{P}(\mu_F) C_n^{3/2}(\xi) \right], \quad (1)$$

where $\Phi_{AS}$ denotes the asymptotic meson distribution amplitude,

$$\Phi_{AS}(\xi) = \frac{3}{2} (1 - \xi^2). \quad (2)$$

$\xi$ is related to the usual longitudinal momentum fraction $x$ of the quark with respect to the meson by $\xi = 2x - 1$. The Gegenbauer coefficients $B_n^{P}$ depend on a factorization scale $\mu_F$ in the following way:

$$B_n^{P}(\mu_F) = B_n^{P}(\mu_0) \left( \frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)} \right)^{\gamma_n}, \quad (3)$$

where $\mu_0$ is the starting point of evolution and typically chosen as a hadronic scale of order 1 GeV. Since the anomalous dimensions $\gamma_n$ are positive fractional numbers, any distribution amplitude evolves into $\Phi_{AS}$ at large scales. The Gegenbauer coefficients contain non-perturbative information and are largely unknown.

The topic of this talk is an investigation of the photon-to-meson transition form factor for virtual photons. In particular, we address the question whether we can obtain additional information on the Gegenbauer coefficients $B_n^{P}$ of $\Phi_P$ from the measurement of the form factor at current and planned $e^+e^-$ colliders. We will limit ourselves to the case of a pion and only briefly comment on $\eta, \eta'$ towards the end of the talk. A more detailed account of the analysis will be presented in Ref. [9].

2 THE $\gamma^*\pi$ TRANSITION FORM FACTOR

The $\gamma^*\pi$ transition form factor $F_{\pi\gamma^*}$ is formally defined through the $\gamma^*\gamma^*\pi$ vertex:

$$\Gamma_{\mu\nu} = -i e^2 F_{\pi\gamma^*}(Q^2, Q'^2) \epsilon_{\mu\nu\alpha\beta} q^\alpha q'^\beta, \quad (4)$$

where $q$ and $q'$ denote the photon momenta with respective spacelike virtualities $Q^2 = -q^2, Q'^2 = -q'^2$. For the following discussion it is convenient to express $F_{\pi\gamma^*}$ in terms of the average photon virtuality $Q_f^2$ and a dimensionless parameter $\omega$:

$$Q_f^2 = \frac{1}{2} (Q^2 + Q'^2), \quad \omega = \frac{Q^2 - Q'^2}{Q_f^2}, \quad (5)$$

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with $-1 \leq \omega \leq 1$. The two photons cannot be distinguished so that the transition form factor is symmetric under $\omega \leftrightarrow -\omega$.

Since we are interested in the behavior of $F_{\pi \gamma^*}$ at large $Q^2$ we only take into account the lowest, i.e., valence Fock state of the pion and employ the collinear approximation, i.e., we neglect partonic transverse momenta in the hard scattering. Power corrections arising from transverse momenta will be estimated later on. The leading-twist expression at next-to-leading order (NLO) in $\alpha_s$ reads [10]

$$F_{\pi \gamma^*}(Q, \omega) = \frac{1}{3\sqrt{2}} \frac{f_\pi}{Q^2} \int_0^1 d\xi \frac{\Phi_\pi(\xi, \mu_F)}{1 - \xi^2 \omega^2} \times \left[ 1 + \frac{\alpha_s(\mu_R)}{\pi} K(\omega, Q/\mu_F) \right] .$$

(6)

The function $K(\omega, Q/\mu_F)$ parameterizes the $O(\alpha_s)$ corrections, which have been calculated in Refs. [10, 11] within the $\overline{\text{MS}}$ scheme. The factorization scale $\mu_F$ and the renormalization scale $\mu_R$ are both of order $Q$. $f_\pi \approx 131$ MeV is the well-known pion decay constant. The Born graphs contributing to the transition form factor become sensitive to the end-point regions $\omega \rightarrow \pm 1$. This corresponds to the situation of the quark or antiquark in the pion having small momentum fraction and the internal quark between the photon vertices going on-shell. Large power corrections arising from, e.g., transverse momentum effects, soft overlap contributions, or the non-perturbative behavior of $\alpha_s$ in the infrared region, may spoil the accuracy of a leading-twist data analysis.

Before we proceed to a discussion of the region away from the limit $\omega \rightarrow 1$, we have to comment on possible power corrections in the large $\omega$ region, where the transition form factor becomes sensitive to the end-point regions $\xi \rightarrow \pm 1$. This means that the transition form factor is sensitive to the Gegenbauer coefficients only for $\omega \rightarrow 1$. Up to $O(\alpha_s)$ corrections the transition form factor in this limit measures the $(1 + \xi)^{-1}$-moment of the pion distribution amplitude, which is given by the sum over all Gegenbauer coefficients,

$$\int_{-1}^1 d\xi (1 + \xi)^{-1} \Phi_\pi = \frac{3}{2} \left[ 1 + \sum_n B_n^\pi(\mu_F) \right] .$$

(8)

The phenomenological analysis [8] of the CLEO data led to the constraint $\int d\xi (1 + \xi)^{-1} \Phi_\pi = 1.37$ at $Q^2 = 8$ GeV$^2$ [8]. If one assumes that $B_n^\pi = 0$ for $n \geq 4$ this constraint translates into $B_2^\pi(\mu_0 = 1$ GeV) $= -0.15$, which implies the distribution amplitude being close to its asymptotic form, as already mentioned in the introduction.

In order to estimate the effects of partonic transverse momentum we employ the modified hard scattering approach [12], where the expression (6) is replaced by

$$F_{\pi \gamma^*}(Q, \omega) = \frac{1}{4\sqrt{2} \pi} \int d\xi d^2b \bar{\Psi}_\pi^*(\xi, b, \mu_F) \times K_0(S(\xi, b, Q^2)) \exp \left[ -S(\xi, b, Q^2) \right] .$$

(9)

The modified Bessel function $K_0$ appears as the Fourier transform of the hard scattering kernel in leading order $\alpha_s$. The transverse quark-antiquark separation $b$ is Fourier conjugated to the partonic transverse momentum $k_\perp$, and $\bar{\Psi}_\pi$ is the Fourier transform of the wave function for the outgoing pion. The exponential is the Sudakov form factor, which describes gluonic radiative corrections at scales intermediate between the confinement region and the hard scattering

![Figure 2: Born graphs contributing to the $P\gamma^*$ transition form factor.](image)

![Figure 3: The coefficients $c_n(\omega)$ in the expansion (7) of the $\pi\gamma^*$ form factor. NLO corrections are included with $\mu_F = \mu_R = Q$, which is taken as $Q = 2$ GeV.](image)
region; for details see Ref. [12]. The most important feature of the Sudakov form factor is its damping of large quark-antiquark separations. Asymptotically, only configurations with vanishing transverse separations survive. Since $b$ acts as an infrared cut-off, the factorization scale $\mu_F$ is to be taken as 1/$b$. The renormalization scale is chosen according to the max-prescription [12] as $\mu_R = \max \{1/b, \sqrt{1 + \xi \omega Q}, \sqrt{1 - \xi \omega Q}\}$. Following [8, 13] we assume for the light-cone wave function in $b$-space the Gaussian ansatz

$$\hat{\psi}_\pi(\xi, b) = \frac{2\pi f_\pi}{\sqrt{6}} \Phi_\pi(\xi) \exp\left[-\frac{(1 - \xi^2) b^2}{16 a_\pi^2}\right]$$

with a transverse size parameter given by $a_\pi^2 = 8\pi^2 f_\pi^2 (1 + B_\pi^2 + B_\pi^+ + \ldots)$. The $\gamma \to \pi$ form factor calculated in the modified perturbative approach using this wave function with $\Phi_\pi = \Phi_{AS}$ is in very good agreement with the CLEO data [8].

In order to see in which kinematical region the transverse momentum corrections are important, we show in Fig. 4 the ratio between the form factor evaluated in the modified hard scattering approach, Eq. (8), and the leading-twist approximation at LO in $\alpha_s$, i.e., neglecting the contributions from $\mathcal{K}(\omega, \xi, Q/\mu_F)$ in Eq. (6). In both schemes we use the asymptotic pion distribution amplitude $\Phi_{AS}$. It is interesting to note that the dominant effects come from $k$-corrections to the hard scattering amplitude; the Sudakov corrections amount to less than about 1.5% in the kinematics considered here. We see that the transverse momentum corrections rapidly decrease as one goes away from $\omega = 1$. The sensitivity to the Gegenbauer coefficients decreases however at the same time, as shown in Fig. 5. While it appears difficult to pin down the individual values for the coefficients $B_n^\pi$, one should at least be able to discriminate between the wide range of theoretical results for the lowest $B_n^\pi$, ranging from a QCD sum rule analysis [14] which predicted $B_2^\pi(1\text{ GeV}) = 0.44$ and $B_3^\pi(1\text{ GeV}) = 0.25$, to a preliminary result from lattice QCD [15] providing $B_2^\pi = -0.41 \pm 0.06$ at a low scale.

We now turn to a discussion of the kinematical region away from the real-photon limit $\omega \to 1$. In particular, we investigate the limit $\omega \to 0$, where the two photons approximately have the same virtualities, $Q^2 \sim Q^\prime^2$. The fast decrease of the functions $c_n$ appearing in Eq. (7) can be understood by expanding the hard scattering kernel in Eq. (8) in powers of $\omega$. Using the properties of the Gegenbauer polynomials we find

$$F_{\gamma\gamma}^\pi(Q, \omega) = \frac{\sqrt{2} f_\pi}{3 Q} \left[1 - \frac{\alpha_s(Q)}{\pi} \right] + \omega^2 \left[1 - \frac{5}{3} \frac{\alpha_s(Q)}{\pi} \right] + \frac{12}{35} \omega^2 B_2^\pi(\mu_F) \left[1 + \frac{5}{12} \frac{\alpha_s(Q)}{\pi} \left[1 - \frac{10}{3} \ln \frac{Q^2}{\mu_F^2}\right]\right] + O(\omega^4, \alpha_s^2),$$

where for definiteness we have taken $\mu_R = Q$. While the above result clearly shows the insensitivity of the transition form factor to the Gegenbauer coefficients $B_n^\pi$ as soon as $\omega$ departs from the limit $\omega \to 1$, it provides us with a parameter-free prediction from QCD to leading-twist accuracy in the small-$\omega$ region:

$$F_{\gamma\gamma}^\pi(Q, \omega) = \frac{\sqrt{2} f_\pi}{3 Q} \left[1 - \frac{\alpha_s(Q)}{\pi} \right] + O(\omega^2, \alpha_s^2).$$

To leading order in $\alpha_s$, this result has been derived a long time ago [14]. The $\alpha_s$-corrections can be found in Ref. [14] and have been rederived in [8] for the real-photon case on the basis of the conformal operator product expansion. In Fig. 5 we compare the approximations (11) and (12) with the full result (8). As we can see, the leading expression (12) provides a very good approximation not only for $\omega \to 0$, but in fact over a wide range of $\omega$, up to about $\omega \simeq 0.5$, where $\omega^2$ corrections start to become important. Any clear deviation from the leading-twist prediction would signal large power corrections, and therefore this prediction well deserves experimental verification. It has a status comparable to the famous leading-twist expression of the ratio $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ and to certain sum rules in inclusive deep inelastic scattering.

A completely analogous discussion with essentially the same conclusions can be pursued for $\gamma \to \eta, \eta'$ transitions. The analysis is, however, complicated through the mixing of $\eta$ and $\eta'$ and through contributions from the gluon distribution amplitude at $O(\alpha_s)$. The gluon contributions come with a factor of $\omega^2$, and we find again that the transition form factors for the $\eta$ and $\eta'$ are hardly sensitive to the Gegenbauer coefficients over a wide range of kinematics.
3 CONCLUSIONS

We have investigated the possibility to exploit the $\gamma^* \to \pi$ transition form factor in order to determine the Gegenbauer coefficients $B_n^\pi$ of the pion distribution amplitude. Performing an expansion in terms of the dimensionless kinematical parameter $\omega$, which is the ratio of the difference and the sum of the two photon virtualities, we have shown that the form factor is independent of the shape of the pion distribution amplitude over a wide range of $\omega$. As a consequence, one has a parameter-free prediction from QCD to leading-twist accuracy, which is valid in a large kinematical region, and which deserves experimental verification. Any observable deviation from this prediction would be a signal for power corrections or for unexpectedly large Gegenbauer coefficients in the pion distribution amplitude.

While the data for the real-photon case $\gamma \to \pi$, where $|\omega| \approx 1$, approximately fixes the sum of the Gegenbauer coefficients, data for values of $|\omega|$ around 0.9, say, may allow for a discrimination of the wide range of theoretical predictions for the lowest $B_n^\pi$. Similar conclusions hold for $\gamma^* \to \eta, \eta'$ transitions.

Concerning the accessibility of the transition form factor at the running experiments BaBar, Belle and CLEO, our studies have revealed that it seems possible, although challenging, to measure the form factor for $Q^2 \lesssim 3$ GeV$^2$, both in regions of moderate $\omega$ and for $|\omega| \approx 1$. The planned asymmetric low-energy $e^+ e^-$ collider at SLAC may be suitable for studies of the form factor after an upgrade to larger center of mass energies and luminosities.

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