The impact parameter dependent gluon distribution of the proton \( xG(x, Q^2, |\vec{b}_\perp|) \) is investigated in a loop-loop correlation model that respects the \( S \)-matrix unitarity condition in impact parameter space. We find low-\( x \) saturation of \( xG(x, Q^2, |\vec{b}_\perp|) \) as a manifestation of \( S \)-matrix unitarity. The integrated gluon distribution \( xG(x, Q^2) \) does not saturate because of the growth of the effective proton radius with decreasing \( x \).

1 Introduction

The steep rise of the gluon distribution \( xG(x, Q^2) \) and structure function \( F_2(x, Q^2) \) of the proton towards small \( x = Q^2/s \) is one of the most exciting results of the HERA experiments. As \( F_2(x, Q^2) \) is equivalent to the total \( \gamma^*p \) cross section, \( \sigma_{\gamma^*p}^{tot}(s, Q^2) \) with increasing c.m. energy \( \sqrt{s} \) which becomes stronger with increasing photon virtuality \( Q^2 \). In hadronic interactions, the rise of the total cross sections is limited by the Froissart bound, which is a direct consequence of \( S \)-matrix unitarity, \( SS^\dagger = S^\dagger S = 1 \), and allows at most a logarithmic energy dependence at asymptotic energies. Analogously, the rise of \( \sigma_{\gamma^*p}^{tot}(s, Q^2) \) is expected to slow down. The microscopic picture behind this slow-down is the concept of gluon saturation: Since the gluon density in the proton becomes large at high energies \( \sqrt{s} \) (small \( x \)), gluon fusion processes are expected to tame the growth of \( \sigma_{\gamma^*p}^{tot}(s, Q^2) \), and it is a key issue to determine the energy at which these processes become significant.

In this talk, gluon saturation is considered in an effective loop-loop correlation model (LLCM) that respects the \( S \)-matrix unitarity condition in impact parameter space and allows a unified description of \( pp, \gamma^*p, \) and \( \gamma\gamma \) reactions. Concentrating on \( \gamma_L^*p \) reactions, the impact parameter dependent gluon distribution of the proton \( xG(x, Q^2, |\vec{b}_\perp|) \) is computed and found to saturate in accordance with \( S \)-matrix unitarity. The presented results are extracted from Ref. 2 where more details can be found.
2 The Loop-Loop Correlation Model

Recently, we have developed a loop-loop correlation model (LLCM) to compute high-energy hadron-hadron, photon-hadron, and photon-photon reactions involving real and virtual photons as well. Based on the functional integral approach to high-energy scattering, the T-matrix element for elastic $\gamma^*p$ reactions with transverse momentum transfer $q_\perp (t = -q_\perp^2)$, c.m. energy squared $s$, and photon virtuality $Q^2$ reads

$$ T_{\gamma^*p}(s, t, Q^2) = 2is \int d^2b_1 e^{i\vec{q}_\perp \cdot \vec{b}_1} J_{\gamma^*p}(s, |\vec{b}_1|, Q^2) $$

where the correlation of two light-like Wegner-Wilson loops, the loop-loop correlation function,

$$ S_{DD}(\vec{b}_1, z_1, \vec{r}_1, z_2, \vec{r}_2) = \left< W[C_1]W[C_2] \right>_G $$ with $W[C_i] = \frac{1}{3} \text{Tr} P \exp \left[ -ig \int d\mu \gamma_\mu G_\mu(z) \right]$

describes the elastic scattering of two light-like color-dipoles (DD) with transverse size and orientation $\vec{r}_i$ and longitudinal quark momentum fraction $z_i$ at impact parameter $\vec{b}_1$, i.e., the loops $C_i$ represent the trajectories of the scattering color-dipoles. For elastic $\gamma^*p$ scattering, the color-dipoles are given by the quark and antiquark in the photon and in a simplified picture by a quark and diquark in the proton. The $\vec{r}_i$ and $z_i$ distributions of these color-dipoles are given respectively by the perturbatively derived longitudinal photon wave function $|\psi_{\gamma^*}(z_1, \vec{r}_1, Q^2)|^2$ and the simple phenomenological Gaussian wave function $|\psi_p(z_2, \vec{r}_2)|^2$. To account for the non-perturbative region of low $Q^2$ in the photon wave function, quark masses $m_f(Q^2)$ are used that interpolate between the current quarks at large $Q^2$ and the constituent quarks at small $Q^2$.

In contrast to the wave functions, the loop-loop correlation function $S_{DD}$ is universal for $pp$, $\gamma^*p$, and $\gamma\gamma$ reactions. We have computed $S_{DD}$ in the Berger-Nachtmann approach in which the S-matrix unitarity condition is respected as a consequence of a matrix cumulant expansion and the Gaussian approximation of the functional integrals. To describe QCD interaction of the color-dipoles, we have used the non-perturbative stochastic vacuum model supplemented by perturbative gluon exchange. This combination allows us to describe long and short distance correlations in agreement with lattice computations of the gluon field strength correlator and leads to the static quark-antiquark potential with color-Coulomb behavior at short distances and confining linear rise at long distances. Two components are obtained of which the perturbative ($P$) component, $(\chi^P)^2$, describes two-gluon exchange and the non-perturbative (NP) component, $(\chi^{NP})^2$, the corresponding non-perturbative two-point interaction Ascribing a weak ($e^{NP} = 0.125$) and strong ($e^P = 0.73$) powerlike energy dependence to the non-perturbative and perturbative component, respectively,

$$ (\chi^{NP}(s))^2 = (\chi^P)^2 \left( \frac{s}{s_0} \right) \left( \frac{r_1^2 r_2^2}{R_0^4} \right)^{e^{NP}} \quad \text{and} \quad (\chi^P(s))^2 = (\chi^P)^2 \left( \frac{s}{s_0} \right) \left( \frac{r_1^2 r_2^2}{R_0^4} \right)^{e^P}, $$

our final result for $S_{DD}$ reads

$$ S_{DD} = \frac{2}{3} \cos \left( \frac{1}{3} \chi^{NP}(s) \right) \cos \left( \frac{1}{3} \chi^P(s) \right) + \frac{1}{3} \cos \left( \frac{2}{3} \chi^{NP}(s) \right) \cos \left( \frac{2}{3} \chi^P(s) \right) \quad . $$

The cosine functions ensure the unitarity condition in impact parameter space as they average to zero in the integration over the dipole orientations at very high energies. In fact, the higher order terms in the expansion of the cosine functions describe multiple gluonic interactions and are crucial for the saturation of the impact parameter dependent gluon distribution $xG(x, Q^2, |\vec{b}_1|)$. Moreover, they tame the growth of the cross sections at ultra-high energies.
virtuality interacting particles become blacker and larger with increasing c.m. energy elastic amplitudes, i.e., obtained directly from the normalization of the longitudinal photon wave function.

Geometrical picture for the energy dependence of $\gamma_s^p$ a measure for the blackness or opacity of the interacting particles and, thus, gives an intuitive picture for the energy dependence of $\gamma_s^p$ reactions: As shown in Fig. 1, the interacting particles become blacker and larger with increasing c.m. energy $\sqrt{s}$. At ultra-high energies, $\sqrt{s} \gtrsim 10^8$ GeV for $Q^2 = 1$ GeV$^2$, the opacity saturates at the black disc limit first for zero impact parameter while the transverse expansion of the scattered particles continues.

In the LLCM with a proton wave function normalized to one, the black disc limit is given by the normalization of the longitudinal photon wave function

$$J_{\gamma_L^p}^{\text{max}}(Q^2) = \int dz d^2 r |\psi_{\gamma_L^p}(z, \vec{r}, Q^2)|^2$$

(7)

and is reached at very high energies when the cosine functions in $S_{DD}$ average to zero in the $z_i$ and $\vec{r}_i$ integrations. Accordingly, $J_{\gamma_L^p}^{\text{max}}(Q^2)$ induces an upper bound on $xG(x, Q^2, |\vec{b}_\perp|)$, the low-$x$ saturation value

$$xG(x, Q^2, |\vec{b}_\perp|) \lesssim xG^{\text{max}}(Q^2) \approx 1.305 \frac{Q^2}{\pi^2 \alpha_s} \frac{\pi}{\alpha} J_{\gamma_L^p}^{\text{max}}(Q^2) \approx \frac{Q^2}{\pi^2 \alpha_s},$$

(8)

which is consistent with complementary investigations and indicates strong color-field strengths $g_{\mu\nu} \sim 1/\sqrt{\alpha_s}$ as well. Since the black disc limit is a rigid unitarity bound for purely imaginary elastic amplitudes, i.e., obtained directly from the $S$-matrix unitarity condition in impact parameter space, we conclude that the low-$x$ saturation of the impact parameter dependent gluon distribution $xG(x, Q^2, |\vec{b}_\perp|)$ is a manifestation of $S$-matrix unitarity.

In Fig. 1, the small-$x$ saturation of the gluon distribution at zero impact parameter $xG(x, Q^2, |\vec{b}_\perp| = 0)$ is illustrated for $Q^2 = 1, 10$, and $100$ GeV$^2$ as obtained in the LLCM.
saturation occurs at very low values of $x \lesssim 10^{-10}$ for $Q^2 \gtrsim 1$ GeV$^2$, where the photon virtuality $Q^2$ determines the saturation value $\langle 0 \rangle$ and the Bjorken-$x$ at which it is reached. For larger $Q^2$, the low-$x$ saturation value is larger and is reached at smaller values of $x$. Moreover, the growth of $xG(x, Q^2, |\vec{b}_\perp| = 0)$ with decreasing $x$ becomes stronger with increasing $Q^2$. This results from the stronger energy increase of the perturbative component in the LLCM, $\epsilon_P = 0.73$, that becomes more important with decreasing dipole size. In conclusion, our approach predicts the onset of the $xG(x, Q^2, |\vec{b}_\perp|)$-saturation for $Q^2 \gtrsim 1$ GeV$^2$ at $x \lesssim 10^{-10}$ which is far below the $x$-regions accessible at HERA ($x \gtrsim 10^{-6}$) and THERA ($x \gtrsim 10^{-7}$).

Note that the $S$-matrix unitarity condition together with relation $\langle 0 \rangle$ requires the saturation of the impact parameter dependent gluon distribution $xG(x, Q^2, |\vec{b}_\perp|)$ but not the saturation of the integrated gluon distribution $xG(x, Q^2)$. Indeed, approximating $xG(x, Q^2, |\vec{b}_\perp|)$ in the saturation regime by a step-function, $xG(x, Q^2, |\vec{b}_\perp|) \approx xG^\text{max}(Q^2) \Theta(R(x, Q^2) - |\vec{b}_\perp|)$, where $R(x, Q^2)$ denotes the full width at half maximum of the profile function, one obtains

$$xG(x, Q^2) \approx 1.305 \frac{Q^2 R^2(x, Q^2)}{\pi \alpha_s} \pi \int_{|\vec{b}_\perp|}^{\max} (Q^2) \approx \frac{Q^2 R^2(x, Q^2)}{\pi \alpha_s}, \tag{9}$$

which does not saturate because of the increase of the effective proton radius $R(x, Q^2)$ with decreasing $x$. Nevertheless, although $xG(x, Q^2)$ does not saturate, the saturation of $xG(x, Q^2, |\vec{b}_\perp|)$ leads to a slow-down in its growth towards small $x$.

4 Conclusion

We have computed the impact parameter dependent gluon distribution $xG(x, Q^2, |\vec{b}_\perp|)$ from the profile function for $\gamma^*p$ reactions in a loop-loop correlation model (LLCM). The LLCM combines perturbative and non-perturbative QCD in agreement with lattice investigations, provides a unified description of $pp$, $\gamma^*p$, and $\gamma\gamma$ reactions, and respects the $S$-matrix unitarity condition in impact parameter space. As a manifestation of $S$-matrix unitarity, we have found low-$x$ saturation of $xG(x, Q^2, |\vec{b}_\perp|)$ for $Q^2 \gtrsim 1$ GeV$^2$ at $x \lesssim 10^{-10}$ but only a slow-down of the integrated gluon distribution $xG(x, Q^2)$ since the effective proton radius grows with decreasing $x$.

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