Engineering method for assessing the strength of reinforced concrete beams

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Abstract. A calculation method is proposed that combines two existing methods for calculating the strength of reinforced concrete beams. The author of the article considers the possibility of presenting the work of a reinforced concrete beam in the form of an arch with a curvilinear shape with a puff, which is a statically definable system. The operation of a reinforced concrete beam with a relative span of a slice from 1 to 3 is modeled as an arched system with a horizontal stretched reinforcing belt and a compressed concrete strip of curvilinear shape. The purpose of the tightening in a beam with an average relative cut span is to cancel the bending moment inherent in a simple beam. Thus, this model (arch with tightening) most fully reflects the work of reinforced concrete beams with an average relative cut span compared to the model adopted in regulatory documents.

1. Introduction
When designing reinforced concrete beams in the Russian Federation according to the current Code of Practice, it is recommended to use two calculation methods. Evaluation of the strength of conventional beams is made on the basis of the traditional regulatory approach on the inclined (most dangerous) and normal sections [1]. The calculation of the strength of short beams (with a small relative span of cut $a/h_0$) is recommended to be made on the basis of an arched model consisting of compressed straight oblique strips of concrete and a stretched reinforcing belt [1]. The calculation of short beams based on the arch model is limited by the conditional boundaries of the relative cut span $a/h_0 \leq 1$. The calculation based on the regulatory approach shows significant discrepancies in assessing the strength of an inclined section [2, 3, 4]. The most noticeable discrepancies between the experimental and calculated values in the beams with a relative cut span of $a/h_0 \leq 3$. Thus, a whole class of beams (with an average relative cut span of $1 \leq a/h_0 \leq 3$) does not have a calculation methodology reflecting the physical work of these structures. Below is a calculation method that combines two existing methods for calculating the strength of reinforced concrete beams. We consider the modeling of the work of a reinforced concrete beam in the form of an arch with a curvilinear outline with a puff, which is a statically definable system.

2. Results and research methods
On the basis of experimental studies, it was established that the tendency to reduce the breaking load with increasing $a/h_0$ from 1 to 3, as in the beams with $a/h_0 \leq 1$, is preserved. At the same time, the pattern of change in destructive efforts in reinforced concrete beams with a relative cut span of
$1 \leq \frac{a}{h_0} \leq 3$ harmoniously continues the corresponding patterns for beams with $\frac{a}{h_0} \leq 1$. In addition, the nature of the formation of cracks and types of destruction does not change: along an inclined concrete strip and along a stretched reinforcing belt. Given that the calculated frame-rod model reflects the physical work of short reinforced concrete elements, the modernization of this model is considered within $\frac{a}{h_0}$ from 1 to 3. In a bent reinforced concrete element, under the action of a breaking load, destruction is possible both along the section normal to the element axis and over inclined section (beams with a regular cut span are considered).

With the destruction of the normal section, there are two cases of the work section. The first case. In normally reinforced sections, destruction begins with the element's stretched zone. When the stresses in the reinforcement reach the yield strength (when reinforced with mild steel), local plastic deformations develop. Under the influence of a significant deflection of the element, the concrete of the compressed zone of the section is destroyed (plastic failure). When reinforced with solid steel (with a small elongation at break), simultaneously with the break of the stretched zone, the concrete of the compressed zone collapses (brittle fracture). The second case. In the re-reinforced sections, the destruction begins in the concrete of the compressed zone of the element, the stresses in the tensioned reinforcement do not reach their ultimate resistance. This type of destruction is always fragile.

According to [1], the calculation of reinforced concrete bent reinforced concrete beams in the normal section is made of

$$M \leq R_p b x (h_0 - 0.5 x).$$

(1)

The height of the compressed zone of concrete $x$ is determined from the equilibrium condition at the stage of destruction (equality of internal forces in concrete and reinforcement, figure 1):

$$R_x A_x = R_p b x.$$

(2)

**Figure 1.** The scheme of efforts in the calculation of a reinforced concrete element along a normal section for the action of moments.

In case of fracture along an inclined section, three cases of section operation are observed, figure 2. The first case is destruction along an inclined strip between cracks due to the action of main compressive stresses. The strength of the sloping strip is checked by empirical formula

$$Q \leq 0.3 R_p b h_0.$$

(3)

The second case is the mutual shift of two parts of a bent element, separated by an inclined crack. The shear is caused by the transverse force $Q$, while the shear and reinforced reinforcement and the concrete of the compressed zone, which works for shear, resist it. With such a scheme of destruction, an oblique section is considered for the effect of a transverse force. ($Q$ is no more than accepted $2.5 R_s b h_0$ and no less $0.5 R_s b h_0^2$):

$$Q < Q_o + Q_s + Q_{s, inc},$$

(4)

where $Q_o$ – is the transverse force perceived by concrete, is determined by the formula:
\[ Q_b \leq 1.5R_b b h_0^2 c^{-2} , \]  

(5)

where \( c \) – is the length of the projection of the most dangerous inclined section on the longitudinal axis of the element.

The transverse forces \( Q_{sw} \) and \( Q_{s,inc} \) are defined as the sum of the projections on the normal to the longitudinal axis of the element of the longitudinal forces, respectively, in clamps and bends.

The third case – is the reciprocal rotation of two parts of a bent element separated by an inclined crack, which is caused by the action of a bending moment.

\[ M \leq M_s + M_{sw}, \]  

(6)

where \( M \) – is the moment in the oblique section with the length of the projection \( c \) on the longitudinal axis of the element, determined from external forces;

\( M_s \) – is the moment perceived by the longitudinal reinforcement crossing the inclined section;

\( M_{sw} \) – is the moment perceived by transverse reinforcement crossing an inclined section.

The following types of damage were determined in reinforced concrete beams with small and medium span: destruction along a section normal to the axis of the element; destruction by oblique section [5–8].

The destruction of the normal section in short beams is similar to the destruction in ordinary beams. Destruction of an inclined section in short reinforced concrete beams occurs in one of the following three schemes. The first case - the destruction of the compressed concrete strip is characterized by the appearance of a series of small parallel cracks, which is characteristic of the destruction of concrete prisms during compression. The second case is the formation of a trunk crack, whose trajectory roughly repeats the diagonal of a compressed concrete strip. The third case is the destruction of compressed inclined concrete strips accompanied by the development of cracks, which produce compressed inclined strips (observed in most experimental samples).

To assess the strength of short reinforced concrete beams (and their varieties) by Professor T.I. Baranova (Penza State University of Architecture and Construction) proposed a new method of calculation based on a frame - core model consisting of oblique compressed strips and a stretched reinforcement belt [5, 6]. The strength calculation method is based on the design scheme shown in figure 3. It is recommended to calculate short beams using compressed and stretched zones. According to the computational model, the limiting state in the compressed zone occurs when the tensile strength is reached in concrete \( R_b \), and when it is stretched, when the tensile strength in reinforcement \( R_s \) is reached. On the basis of the design scheme for estimating the strength of reinforced concrete beams with a cut span of \( a/h_0 \leq 1 \), the following calculated dependences are proposed.

The calculation of the strength of the compressed zone of short beams (including transverse reinforcement) is made according to the formula

\[ F \leq 2\varphi B R_b b l_b \sin \alpha + F_s, \]  

(7)

where \( F \) – is the external force;

\( \varphi \) – coefficient taking into account the effect of concrete surrounding the calculated strip;

\( B \) – is the coefficient obtained on the basis of experiments;

\( l_b \) – is the width of the calculated concrete strip;

\( \alpha \) – is the angle of inclination of the compressed concrete strip;

\( F_s \) – is the force perceived by transverse reinforcement.

The calculation of the strength of the stretched zone is determined by the formula

\[ F \leq \gamma_s R_s A_s \tan \alpha, \]  

(8)

where \( \gamma_s \) – is the experimental coefficient;

\( A_s \) – is the area of the stretched longitudinal reinforcement.
The calculation of the strength of the compressed inclined and stretched zones are produced independently of each other. In the development of the previously developed frame – rod model for calculating short reinforced concrete elements [5–8], it is proposed to calculate reinforced concrete beams with a relative cut from 1 to 3 based on the arch system in the form of a three-hinged arch with a tie, figure 4. The model in question has two hinges located directly on the supports at the level of the longitudinal reinforcement, two hemispheres that are hingedly supported on the supports are connected to each other with the help of the third hinge. Thus, the work of a reinforced concrete beam with a span of a slice from 1 to 3 is modeled as a spacer system with a horizontal stretched belt represented by a longitudinal reinforcement and a curved contour compressed by a concrete arch.

In the considered system, the span of the arch \( l \) is the distance between the support verticals, and the arrow of the arch lift \( f \) – is the distance from the most distant point of the axis of the arch (key hinge) to the line connecting the centers of the supports.

For an analytical calculation of the system under consideration, we limit ourselves to the case of a vertical load. It is easy to make sure that the values of the support reactions in the arch are no different from the vertical reactions of the left and right supports, which would have been obtained from the load under consideration if the arch had been replaced by a simple beam supported at points \( A \) and \( B \).

When drawing up the bending moment expression for section C (relative to any point of the plane) we get

\[
M_c^0 - Hf = 0,
\]

where \( M_c^0 \) is the bending moment from the given forces in the cross section of a simple beam and

\[
H = M_c^0 f^{-1}.
\]

The resulting formula shows that at any vertical load, the thrust \( H \):

- For a given arrangement of the hinges \( A, B \) and \( C \) does not depend on the shape of the axis of the arch;
- At a given load depends only on the location of the hinges \( A, B \) and \( C \);
- Equals the bending moment in a simple beam.

Thus, the thrust increases with increasing bending moment (the appointment of thrust in the arch consists precisely in redeeming the bending moment inherent in a simple beam). Thus, this model (arch with tightening) most fully reflects the work of reinforced concrete beams with small and medium span of the cut compared with the adopted model [1]. When the shape of the arch axis changes (provided that the heel hinges \( A \) and \( B \) remain in place, and the key hinge moves vertically up or down), the spacing changes inversely proportional to the size of the arch boom \( f \). Thus, a flatter arch (system) has a greater spread than the arch with a large value of \( f \).
We define the rational axis of the arch. The arch line reaches its optimal position in the case when the total bending moment along the entire curvilinear axis of the system vanishes. To fulfill this condition, it is sufficient that the vertical ordinates $y$ of all points of the axis, counted from the straight line $AB$, be proportional to the corresponding ordinates of the beam section $M_0$. Based on this condition, for any given vertical load, at any position of the hinges $A$, $B$ and $C$, it is possible to choose a rational arch section ensuring the absence of bending. So, in the case of loading with a uniformly distributed load $q$ over the span $l$, the rational axis of the arch is described by the equation of a square parabola (relative to the coordinate system with the beginning at point $A$)

$$y = 4fx(l-x)l^{-2}.$$ (11)

Then, for an arbitrary section of the arch (whose supports are located on the same level), the shear force from the vertical load

$$Q = Q^0 \cos \alpha - H \sin \alpha,$$ (12)

where $Q^0$ is the transverse force in the corresponding cross section of a conventional beam with a horizontal axis for a given loading is determined by the expression

$$Q^0 = V_A - \sum P_i.$$ (13)

Under the sign of the sum are the forces located to the left of the section under consideration, figure 5.

![Figure 5](image)

**Figure 5.** The scheme of efforts in the calculation of reinforced concrete beams with an average relative cut span.

Expression (12) shows that in the three-hinged arch, not only is the curve of bending moments, but also the chart of shear forces has smaller values than in a simple beam.

Projecting all the forces on the tangent to the axis of the arch in section $K$, we define the longitudinal force as

$$N = -(Q^0 \sin \alpha - H \cos \alpha).$$ (14)

This formula shows that the reduction of moments and shear forces in the cross section entails the appearance of an effort that characterizes a compressed inclined concrete strut.

In the computational model under consideration, it is proposed to estimate the limiting state in the compressed zone in the same way as short beams: when the compressive stress reaches its compressive strength $\phi_b R_b$, and if it is stretched when tensile stresses are reached $\gamma_s R_s$. In this case, the width of the compressed concrete strut is recommended to be determined from the equilibrium condition in the destruction stage [1]:

$$l_b = R_s A_y (R_p b)^{-1}.$$ (15)

Calculate the strength of the compressed zone of beams with $1 \leq a / h_0 \leq 3$ (taking into account transverse reinforcement) to produce by the longitudinal force along the most dangerous section:
where $N$ – is the longitudinal force in a compressed concrete strip, determined from external forces according to (14);

$N_b$ – the longitudinal force perceived by concrete is determined by the formula:

$$N_b = \varphi_b \gamma_b R_0 b_l b,$$

$N_s$ – transverse reinforcement force;

$\varphi_b$ – coefficient taking into account the effect of concrete surrounding the settlement strip;

$\gamma_b$ – coefficient obtained on the basis of experiments.

The calculation of the strength of the stretched zone is determined by the condition

$$H \leq \gamma_s R_s A_s.$$

where $H$ – is the longitudinal force in the horizontal tensioned reinforcement belt (thrust), determined from external forces according to (10);

$\gamma_s$ – is the experimental coefficient;

$A_s$ – is the area of the stretched longitudinal reinforcement.

The calculation of the strength of the compressed inclined and stretched zones are produced independently of each other.

3. Conclusion

An engineering method for assessing the strength of reinforced concrete beams with a mid-span shear based on an arched model that reflects the physical work of the structures under consideration is proposed. Calculation of strength is recommended to be made on the compressed and stretched zones. According to the computational model, the limiting state in the compressed zone occurs when the tensile strength of the compressed inclined concrete strip is reached, and the stretched one - when the reinforcement strength is reached in horizontal tightening. The considered method of calculation combines two existing approaches to assess the strength of reinforced concrete beams.

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