Controller Design for Three-Mass Resonant System Based on Polynomial Method

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Abstract: The mechanical system in the industrial electrical drives can be modelled by a multi-mass system. Torsional vibration suppression and attainment of robustness in motion control systems is a requisite in industry applications. In this paper, the system that is three-mass system is considered. A speed control method based on polynomial method introduced to implementation of torsional vibration is proposed. The effectiveness of the proposed controller is demonstrated by transfer function analysis and simulation results. The simulation results show that the proposed controller improves dynamic performance and suppresses torsional vibration of three-mass resonant system.

Keywords: Three-Mass System, Speed Control, Polynomial Method, Transfer Function

1. Introduction

In industrial motor drive systems, a shaft torsional vibration is often generated in which a motor and load are connected via a flexible shaft [1-3]. A mechanical system composed of some masses connected with flexible shafts such as steel rolling mills, variable-speed wind turbine, industrial robot, elevators systems and so on, is called multi-mass resonant system [4, 5]. This simplified approach has been used in different industrial applications [6-8]. Vibration suppression and high speed response control in motion control system is an important problem in industry applications [9-11]. Control problems of a multi-mass system is especially difficult when not all system variables are measurable, which happens very often in industrial applications [12, 13].

Several references in technical literature can be found on the application of control methods to improvement performance of the multi-mass [14-16]. A vibration-suppression control method for both two- and three-mass system is proposed in [17], which uses controller with three elements: the disturbance observer, the imperfect derivative filter, and the feedback gain. In order to reduce the computational complexity, the model predictive control for high-performance speed control and torsional vibration suppression in the elastic three-mass drive system is described in [18]. An algorithm to design a speed control strategy of a two-mass resonant system by a integral–proportional-derivative (PID) controller based on the response frequency and the response step is developed in [19]. In order to have sufficient damping, in [20] three different controller base on PID controller are designed for the speed control of a two-mass system using a normalized model and polynomial method. The PID, PID-P and modify PID controllers in order to control torque of a two-mass resonant system is presented in [21], which the pole-placement controller such as coefficient diagram method and integral of time multiplied by absolute error is used to assign closed-loop poles of the system characteristic equation.

Control loop strategies of multi-mass system to reduce the effects of resonance are very different [22, 23]. In this paper the speed control of three-mass system based on the polynomial method is proposed. This paper is organized as follows. Section II describes modeling of the three-mass system for simulation. The open-loop system is represented in the matrix form in section III. The proposed controller structures and problem formulation are described in Section IV. The simulation result is included in Section V to verify good performance obtained the proposed controller, and conclusions are provided in Section VI.
2. Mathematical Model of Three-Mass System

A motor drive system is composed of a motor connected to a load machine through a flexible shaft. Simplified model of the three-inertia torsional system (motor, load and gear) is shown in Figure 1 [24]. Here $\omega_M$ is motor speed, $\omega_L$ is load speed, $T_S$ is shaft torsional torque, $T_G$ is gear torque and $\omega_G$ is gear speed. The control input is motor torque ($T_M$) and the input disturbance is load torque ($T_L$). The speed motor is the measurable, but we have no sensor to measure load speed. The system parameters used for study and simulation are listed in Table I [25, 26].

The block diagram of the three-mass system is shown in Figure 2. The gear backlash is simplified as a nonlinear element with backlash angle band $[-\delta, +\delta]$ and elastic coefficient $K_G$ [27, 28]. The state equation of the three-mass torsional system is expressed as [29, 30]:

\[
\frac{d\omega_L}{dt} = -\left(\frac{B_L}{J_L} + \frac{B_S}{J_S}\right)\omega_L + \frac{B_S}{J_L}T_S - \frac{1}{J_L}T_M
\]  
\[
\frac{d\omega_M}{dt} = -\left(\frac{B_G}{J_M} + \frac{B_G}{J_M}\right)\omega_M + \frac{B_G}{J_M}T_G - \frac{1}{J_M}T_M + \frac{1}{J_M}T_M
\]  
\[
\frac{d\omega_G}{dt} = -\left(\frac{1}{J_C}\right)T_G - \frac{1}{J_C}T_G' + (B_G + B_C)\omega_G + B_S\omega_L + B_G\omega_M
\]  
\[
\frac{dT_S}{dt} = K_S(-\omega_G + \omega_L)
\]  
\[
\frac{dT_G}{dt} = K_G(\omega_M - \omega_G)
\]

where $J_M, J_L, J_G, B_M, B_L, B_G, K_G, K_S$ are the motor inertia, the load inertia, the gear inertia, the motor viscous damping coefficient, the load viscous damping coefficient, the gear damping coefficient, the gear stiffness, and the shaft stiffness, respectively.

| Parameters       | Symbol | Value       |
|------------------|--------|-------------|
| motor inertia    | $J_M$  | 0.0641 Kgm$^2$ |
| motor damping coefficient | $B_M$  | 0           |
| load inertia     | $J_L$  | 0.0523 Kgm$^2$ |
| load damping coefficient | $B_L$  | 0           |
| shaft stiffness  | $K_S$  | 242 Nm/rad  |
| shaft damping coefficient | $B_S$  | 0.1 Nm-s/rad |
| gear stiffness   | $K_G$  | 2000 Nm/rad |
| gear damping coefficient | $B_G$  | 0.2 Nm-s/rad |
| gear inertia     | $J_G$  | 0.0868      |
| resonance frequencies | $\omega_1, \omega_2$ | 77.98, 235.75 rad/s |
| anti-resonance frequencies | $\omega_3, \omega_4$ | 63.52, 162.55 rad/s |

Figure 1. Simplified model of three-mass torsional system

Figure 2. Block diagram of the nonlinear three-mass system.

Also we will have:
The characteristic equation of open loop system are given by:
\[
\Delta_s(s) = s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0
\]  
(7)

By varying the operating point, the coefficient parameter values \( p_4 \) through \( p_0 \) also vary. A necessary condition for stability of the system is that all the roots in characteristic equation have a negative real part. By forbear from the value of the damping coefficients, \( p_0 \), \( p_2 \) and \( p_4 \) are zero. The \( p_1 \) and \( p_3 \) are given by:
\[
p_1 = \frac{K_S K_G}{J_L J_M} (1 + \frac{J_L + J_M}{J_C})
\]  
(8)

\[
p_3 = \frac{K_S}{J_L} (1 + \frac{J_L}{J_C}) + \frac{K_G}{J_M} (1 + \frac{J_L + J_M}{J_C})
\]  
(9)

3. Open-Loop Three-Mass System

Let a two-input two-output process be represented by the block diagram shown in Figure 3. The three-mass system can be represented in the following matrix form [31, 32]:
\[
\begin{bmatrix}
\omega_L \\
\omega_M \\
\end{bmatrix} = \begin{bmatrix}
-H_{LL}(s) & H_{LM}(s) \\
-H_{ML}(s) & H_{MM}(s) \\
\end{bmatrix} \begin{bmatrix}
T_L \\
T_M \\
\end{bmatrix}
\]  
(10)

The transfer function of the open-loop three-mass system are given by:
\[
H_{LL}(s) = \frac{a}{b^2 + ac}
\]  
(12)

\[
H_{LM}(s) = H_{ML}(s) = \frac{b}{b^2 + ac}
\]  
(13)

where the coefficient \( a, b \) and \( c \) are given by:
\[
a = -1 + \frac{G_G(s)[G_G(s) - J_C s]}{G_L(s) J_C s - G_S(s) - G_G(s)}
\]
\[
b = \frac{G_G(s) G_G(s)}{J_C s - G_S(s) - G_G(s)}
\]
\[
c = \frac{1}{G_M(s) \left( J_C s - G_S(s) - G_G(s) \right)}
\]  
(14)

The transfer functions of the motor, load, shaft and gear are \( G_M(s), G_L(s), G_S(s) \) and \( G_G(s) \).

The plot of the open-loop frequency response from the \( H_{MM}(s) \) is for two-mass and three-mass are shown in Figs. 4 and 5, respectively. Two high peaks arise for three-mass system in the gain characteristic plot at resonance frequencies \( \omega_{r1} \) and \( \omega_{r2} \) and two down peak arise at anti-resonance frequency \( \omega_{a1} \) and \( \omega_{a2} \). The amount of the attenuation coefficients are deleted, because the values are very small. Open-loop transfer function between the motor torque and the motor speed, which is most important in the closed-loop design can be derived as:
\[
H_{MM}(s) = \frac{\omega_M}{T_M} = \frac{1}{\Delta_s(s)} \frac{1}{J_M s^4} + \frac{K_S + K_G}{J_M} \frac{J_L + J_M}{b_c} \frac{1}{b_c} \left( \frac{J_S}{b_c} \frac{J_M}{b_c} \frac{J_L J_C J_M}{b_c} \right)
\]  
(15)

The transfer function can be represented in the form below:
\[
H_{MM}(s) = \frac{\left( s^2 + \omega_{n1}^2 \right) \left( s^2 + \omega_{n2}^2 \right)}{J_M s^2 (s^2 + \omega_{r1}^2)(s^2 + \omega_{r2}^2)}
\]  
(16)

where \( \omega_{n1} \) and \( \omega_{n2} \) are the natural resonance frequencies respectively corresponding to the shaft resonance and the gear-resonance.

4. Applied Speed Control Structure

The direct feedbacks from the load speed and the shaft torque are very difficult in the mechanical system in the industrial application. The block diagram of the controlled plant for the three-mass system is presented in Figure 6. The transfer function of the PID damping controller for the three-mass system in s domain is given by:
\[ G_C(s) = \left( K_P + \frac{K_I}{s} + K_Ds \right) \frac{T_W s}{T_W s + 1} \]  \hspace{1cm} (17)

where \( K_W \) and \( T_W \) are the gain and time constant of the washout term while \( K_P, K_I \) and \( K_D \) are the proportional gain, integral gain, and derivative gain of the damping controller, respectively. \( K_F \) is the forward gain of the controller. The transfer function of the first-order low-pass filter is \( \frac{K_W}{s(T_W + 1)} \) in the speed controller. The speed command is \( \omega_C \).

In this I-PD controller, proportional and differential action are worked only on the detected quantity \( \omega_M \) [33]. The close-loop state space equation can be written as:

\[
\begin{aligned}
\frac{d}{dt} X &= A X + B U + D W \\
Y &= C X 
\end{aligned}
\]  \hspace{1cm} (18)

where \( X \) is the state vector, \( Y \) is the output vector, \( U \) is the external or compensated input vector, \( W \) is the external disturbance input vector while \( A, B, C \) and \( D \) are all constant matrices of appropriate dimensions.

With speed controller, the state vector contained seven state variables and the control vector contained command speed as shown below:

![Motor speed transfer function performance](image)

\[
\begin{aligned}
X &= \begin{bmatrix} \omega_I & \omega_M & \omega_C & T_S & T'_S & \omega_T & T_X \end{bmatrix}^T \\
U &= [\omega_C] \\
W &= [T_L] 
\end{aligned}
\]  \hspace{1cm} (19)

The state variable \( T_M \) is:

\[ T_M = -T_X + (K_F + \frac{K_W}{T_W})(\omega_I - K_P \omega_M - K_D \frac{d\omega_M}{dt}) \]  \hspace{1cm} (20)

In addition to the five open-loop equation, there are two following equations in closed-loop system:

\[
\begin{aligned}
\frac{d\omega_I}{dt} &= -K_I \omega_M + K_I \omega_C \\
\frac{dT_X}{dt} &= -\frac{1}{T_W} T_X + \frac{K_W}{T_W^2} \omega_I - \frac{K_W K_P}{T_W^2} \omega_M - \frac{K_W K_D}{T_W^2} \frac{d\omega_M}{dt} 
\end{aligned}
\]  \hspace{1cm} (21)

Closed-loop transfer functions from speed command to load speed is given by:

\[ H(s) = \frac{\omega_M}{\omega_C} = \frac{K_c T_M H_M(s)}{K_W \left( K_D s^2 + K_P s + K_I \right) H_M(s) + T_W s + 1} \]  \hspace{1cm} (23)
Figure 4. Bode diagram of magnitude and phase for two-mass model.
The characteristic equation of the close loop three-mass system equipped with controller is given by:

\[ \Delta_c(s) = h_7 s^7 + h_6 s^6 + h_5 s^5 + h_4 s^4 + h_3 s^3 + h_2 s^2 + h_1 s + h_0 \]  

where the coefficients \( h_0 \) through \( h_7 \) are given by:

\[ \text{Figure 5. Bode diagram of magnitude and phase for three-mass model.} \]

\[ \text{Figure 6. Controlled plant block diagram.} \]
\[
\begin{align*}
h_0 &= K_I K_F b_0 \\
h_1 &= K_p K_F b_0 + K_I b_0 (K_w + K_p T_w) \\
h_2 &= p_1 + K_p b_2 (K_w + K_p T_w) + K_p b_2 K_I + K_F b_2 K_D \\
h_3 &= T_w p_1 + K_F K_p b_2 + (K_I b_2 + K_D b_2) (K_w + K_F T_w) \\
h_4 &= p_3 + \frac{K_I K_F}{J_M} + K_p b_2 (K_w + K_p T_w) + K_D K_F b_2 \\
h_5 &= p_3 T_w + (\frac{K_p}{J_M} + K_D b_2) (K_w + K_F T_w) + \frac{K_p K_F}{J_M} \\
h_6 &= 1 + \frac{K_D K_F}{J_M} + \frac{K_p}{J_M} (K_w + K_F T_w) \\
h_7 &= T_w + \frac{K_D}{J_M} (K_w + K_F T_w)
\end{align*}
\]

A good set of controller parameters \(K_I, K_F, T_w, K_p, K_D\) and \(K_I\) will yield a good response and results in the vibration suppression in time domain.

5. Simulation Results

The simulation results of the speed control of the three-mass using the proposed controller will be shown in this section in order to demonstrate the efficiency of the controller. The Simulink model of the system is show in Figure 7. In Figures 8 and 9, the frequency responses considering three-mass system equipped with a speed controller is compared with considering three-mass system without controller. The solid lines show the plots of the closed-loop frequency response from the speed command to motor speed and to shaft torque.

The gain plot of transfer function of motor speed to command speed in three-mass system for two different forward gain is shown in Figure 10. Figures 11 and 12 show the load speed and shaft torsional of the three-mass system with controller. The reference input of 1 rad/s is commanded at \(t=0\), and the 1 Nm disturbance torque is given at \(t=1.5s\). The change of the load speed for three-mass system is show in Figures 13.

![Simulink model of the three-mass system with controller.](image-url)
Figures 8. Comparative frequency response of motor speed transfer function.
Figure 9. Comparative frequency response of shaft torque transfer function.
Figure 10. Gain-phase plot for two different forward gain and without controller.

Figure 11. Load speed with controller for three-mass system.
Figure 12. Shaft torsional with controller for three-mass system.

Figure 13. Load speed for change in the input of the three-mass system.
6. Conclusion

Vibration suppression and disturbance rejection control in torsional system is an important problem in the future motion control. In this paper a simple speed control method of three-mass torsional system is proposed. The mathematical model of the open-loop and closs-loop was analyzed. The controller selected is an I-PD controller with the gains of which were select base on polynomial method. With control system, a fast response without oscillation can be achieved. The simulation results show that the proposed controller improves dynamic performance and suppresses torsional vibration of three-mass resonant system.

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