CP Violation for Leptons at Higher Energy Scales

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Abstract

The phase convention independent measure of CP violation for three generations of leptons is evaluated at different energy scales. Unlike in the quark sector, this quantity does not vary much between the weak and the grand unification scales. The behavior of the measure of CP violation in the Standard Model is found to be different from that in the extensions of the Standard Model.
with the discovery of neutrino mixing, which indicates nonzero neutrino masses [1], there recently has been a lot of interest in neutrino physics, including measuring possible CP violation in the leptonic sector of the three-generation Standard Model [2]. CP nonconservation in the leptonic sector has important implication in leptogenesis in the early universe, as it can feed into a baryon asymmetry through the sphaleron mechanism [3]. One of the necessary conditions to have baryogenesis or leptogenesis is CP violation [4]. It is thus natural to discuss CP nonconservation in the leptonic sector at higher energy scales.

In analogy with the quark sector, the Maki-Nakagawa-Sakata (MNS) matrix describes the mixing among the flavor eigenstates $\nu_\alpha$ and mass eigenstates $\nu_i$ of neutrinos with $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$ [5]

$$V_{MNS} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & c_{13} s_{23} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - c_{23} s_{12} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix},$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ ($0 \leq \theta_{ij} \leq \pi/2$), and $\delta$ is a phase. Instead of referring to the above phase-dependent matrix, it has been proven that with three generations of fermions CP is not conserved if and only if the single parametrization-independent quantity

$$\det C = \det[NN^\dagger, EE^\dagger]$$

is nonvanishing [6], where $N$ and $E$ denote the $3 \times 3$ Yukawa coupling matrices for the neutrinos and charged leptons, respectively. In terms of $V_{MNS}$ matrix elements,

$$\det C = 2(m_\tau^2 - m_\mu^2)(m_\mu^2 - m_e^2)(m_e^2 - m_\tau^2)(m_{\nu_\tau}^2 - m_{\nu_\mu}^2)(m_{\nu_\mu}^2 - m_{\nu_e}^2)(m_{\nu_e}^2 - m_{\nu_\tau}^2)J,$$

where $m_{\nu_\tau, \nu_\mu, \nu_e}$ are eigenvalues of $N$ and $E$, respectively, and $J = s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 s_{13}^2$ is an invariant proposed by Jarlskog in the quark sector [6].

As pointed out in Ref. [4], the definition in Eq. (2) implies that the neutrinos are Dirac-type particles. If one has Majorana neutrinos, the definition should be $\det C = \det[N^\dagger N, EE^\dagger]$ which, however, does not affect later analyses.
quantity $\det C$ vanishes if (i) any two leptons with the same charge have the same mass; (ii) any of the angles $\theta_{ij}$ has the value $0$ or $\pi/2$, or (iii) $\delta = 0$ or $\pi$.

To study the scale dependence of $\det C$, it is natural to solve its renormalization group equation (RGE) given a set of initial values for the relevant quantities such as the Yukawa and gauge couplings at the weak scale. Unlike the situation in the quark sector where the analogous quantity drops by 4 to 5 orders of magnitude and may provide insufficient $CP$ violation for baryogenesis [9], we will show that this quantity for leptons does not change very much between the weak and grand unification theory (GUT) scales, which we take to be $\sim 10^{16}$ GeV. Since $CP$ violation is one of the ingredients for baryogenesis or leptogenesis [9], this may have an important consequence for the evolution of the early universe.

We will consider two methods for neutrinos to acquire mass: (1) the neutrinos have the Dirac masses generated through the the same mechanism as the other fermions; and (2) the neutrinos obtain Majorana masses through the seesaw mechanism [10]. We will study the situation in the Standard Model (SM), the two Higgs doublet model (THDM) and the minimal supersymmetric model (MSSM).

As shown in Ref. [9] for quarks, $\det C$ obeys a rather simple equation under the renormalization group. In the leptonic sector, this equation can be derived from the renormalization group equations for $N$ and $E$. They can be written in the following form:

$$\frac{dN}{dt} = \left[-G_N + T_N + c_{11}NN^\dagger + c_{12}EE^\dagger\right]N, \quad (4a)$$

$$\frac{dE}{dt} = \left[-G_E + T_E + c_{21}EE^\dagger + c_{22}NN^\dagger\right]E, \quad (4b)$$

where $t \equiv (1/16\pi^2)\ln(\mu/M_Z)$, and $G_N, G_E, T_N, T_E$ and $c_{ij}$ are listed in Table I with $U$ and $D$ being the $3 \times 3$ up-type and down-type Yukawa coupling matrices, respectively[2]. Using Eqs. (2) and (3), one obtains

2These conditions are not independent of one another; see, for example, Ref. [8].

3The trace part of radiative corrections comes from the fermion loops on the Higgs fields where quark contributions enter.
$$\frac{dC}{dt} = \{C, A\},$$

(5)

where $A = -(G_N + G_E) + (T_N + T_E) + (2c_{11} + c_{12})N^\dagger N + (2c_{21} + c_{22})E^\dagger E$. Since

$$\frac{d \ln \det C}{dt} = 2 \text{Tr} A,$$

we obtain a simple evolution equation

$$\frac{(\det C)_\mu}{(\det C)_{M_Z}} = \exp \left[ \int_0^\tau (2 \text{Tr} A) dt \right].$$

(6)

Assuming a hierarchy structure in the Yukawa couplings, one can neglect contributions from $D, N$ and $E$ as unimportant because at the leading order of the perturbative correction the top quark loop in the Higgs field renormalization dominates. Some comments are in order:

- Assuming that all contributions to the Yukawa coupling evolution are due to a heavy top quark, we find that all lepton Yukawa couplings scale in the same way $\{C, A\}$. Therefore, the ratio $(\Delta M^2)_\mu/(\Delta M^2)_{M_Z}$ is independent of the lepton masses, where

$$\Delta M^2 \equiv (m_\tau^2 - m_\mu^2)(m_\mu^2 - m_e^2)(m_e^2 - m_\tau^2)(m_{\nu_e}^2 - m_{\nu_\mu}^2)(m_{\nu_\mu}^2 - m_{\nu_\tau}^2)(m_{\nu_\tau}^2 - m_{\nu_e}^2)(m_{\nu_e}^2 - m_{\nu_\tau}^2).$$

- Most of the running of $\det C$ is due to the running of masses (instead of that of the Jarlskog invariant $J$), which is mainly a result of the dominant top quark Yukawa...
coupling. By comparing the running of $\text{det } C$ and $\Delta M^2$, we find that, as expected, $J$ does not change very much over the energy scale range considered.

When running $\text{det } C$ in the three models considered, the vacuum expectation energy of the Higgs field is taken to be $v = 174$ GeV in the SM and $v = \sqrt{v_u^2 + v_d^2}$ with $\tan \beta \equiv v_u/v_d = 10^4$ in both the THDM and MSSM. With $y_t$ being the Yukawa coupling of the top quark, one has \[ (7) \]

\[
\text{Tr} A \simeq \begin{cases} 
-3 \left( \frac{54}{20} g_1^2 + \frac{9}{2} g_2^2 \right) + 9y_t^2 & (\text{SM}), \\
-3 \left( \frac{54}{20} g_1^2 + \frac{9}{2} g_2^2 \right) + 9y_t^2 & (\text{THDM}), \\
-3 \left( \frac{12}{5} g_1^2 + 6g_2^2 \right) + 9y_t^2 & (\text{MSSM}).
\end{cases}
\]

As one can see from Eqs. (7), the evolution of $\text{det } C$ for leptons differs from that for quarks because it does not involve the QCD coupling, which significantly drags down the running quark masses.

The evolution of the ratio $(\text{det } C)_{\mu}/(\text{det } C)_{M_Z}$ as a function of the energy scale $\mu$ for Dirac neutrinos is shown in FIG. 1. It is seen that $\text{det } C$ has a very distinct behavior in the SM that is different from that in the THDM and MSSM. It is the result of a stronger dependence on $y_t$ for $\text{Tr} A$ in the SM in Eq. (7) and the evolution of $y_t$ itself. A convex curve for the SM follows from $y_t$ becoming smaller at larger scales, thus decreasing $\text{Tr} A$. The minor difference between the THDM and MSSM is due to the differences between the coefficients of $g_1$ and $g_2$. Therefore, the dependence on $y_t$ is the dominant factor in determining the evolution of $(\text{det } C)_{\mu}/(\text{det } C)_{M_Z}$. Within the SM, $\text{det } C$ becomes larger as the energy scale goes up; whereas in both the THDM and MSSM, $\text{det } C$ evolves to a smaller value. At the GUT scale, $\text{det } C$ in the SM is $\sim 6$ times bigger than in the THDM and $\sim 12$ times than in the MSSM. The curves for the scale dependence of $\Delta M^2$ are almost identical to those for $\text{det } C$.

Numerically, $(\text{det } C, \Delta M^2)$ at the GUT scale is $(2.42, 2.42)$, $(0.40, 0.39)$, and $(0.21, 0.19)$ in

\[4\]The value of $\tan \beta$ cannot be too large if the top quark effect is considered to be dominant. If $\tan \beta$ is very large, bottom quark contributions to the evolution should be taken into account too.
FIG. 1. The evolution of \((\det C)_\mu / (\det C)_{M_Z}\) with the scale \(\mu\). We assume Dirac masses for neutrinos. The solid curve is for the Standard Model, the dotted curve is for the two Higgs model, and the dashed curve is for the minimal supersymmetric model.

the SM, THDM and MSSM, respectively.

If a smaller value of \(\tan \beta\) is used, the ratio \((\det C)_\mu / (\det C)_{M_Z}\) in the THDM and MSSM will behave more like that in the SM. This is because in this case \(y_t\) is larger at the electroweak scale compared to the case with a larger \(\tan \beta\). One should also note that in some circumstances the evolution of \(y_t\) blows up because within the dominant top Yukawa coupling assumption,

\[
y_t(\tau) = \frac{y_t(0) E(\tau)}{1 - c y_t(0) \int_0^\tau E(t) dt},
\]

where \(E(\tau) = \exp \left[- \int_0^\tau a_i g_i^2 dt\right]\) and \(c > 0\). The numerical values of \(a_i\) with \(i = 1, 2, 3\) and \(c\) depend upon the model. Eq. (8) tells us that either one has to include other fermion contributions or that higher order perturbative corrections [13] should be considered when the denominator gets close to zero.

The other interesting case occurs when the neutrino masses are acquired through the seesaw mechanism. In this case, we have the effective interaction \(N \bar{\psi}_{\nu_L} \psi_{\nu_L} H H\). This will introduce two extra loop contributions on the Higgs legs compared to the previous case. Therefore, in the seesaw model both \(G_N\) and \(T_N\) are twice as big as for Dirac neutrinos.
FIG. 2. The evolution of $(\det C)_{\mu}/(\det C)_{M_Z}$ with the scale $\mu$. The labeling convention is the same as FIG. 1, but neutrino masses are generated through the See-Saw mechanism.

We see in Fig. 2 that the quantity $\det C$ at the GUT scale is roughly twice as big as in Fig. 1. $(\det C, \Delta M^2)$ at the GUT scale is $(5.05, 5.05)$, $(0.84, 0.79)$, and $(0.49, 0.42)$ in the SM, THDM and MSSM, respectively.

Regardless of their different behavior, we see that for all models considered the evolution of $\det C$ is, unlike the quark sector [9], of order $10^{-1}$ to 10 in going from the electroweak scale to the GUT scale. The actual factors that enter a $CP$-violating quantity are not necessary only $\det C$, but may involve other mass scales relevant to the process. The $CP$-violating mechanism responsible for leptogenesis may have a different origin. As long as the physics that gives rise to lepton asymmetry effects at a higher energy scale is in the same multiplet of the Standard Model particles within the unified theory, as an effective theory, the quantity $\det C$ will still be a good indicator of how much $CP$ nonconservation would be at higher energy scales, though the exact relation depends upon models.

As delineated in Ref. [14] with a toy model containing two very heavy SU(2)-doublet scalars, if the neutrinos acquire small Dirac masses, the $CP$-violating processes due to the heavy scalars at the large energy scale can produce a neutrino asymmetry. Both the heavy scalars and the light Dirac neutrinos are out of equilibrium at the weak scale where
sphalerons come into play to produce baryon and lepton number violation. Similarly, if
the neutrino masses are generated through the seesaw mechanism with very heavy ($\sim 10^{16}$
GeV) singlet right-handed neutrinos, then both the out-of-equilibrium and $CP$ violation
conditions can be satisfied around the GUT scale. One then can have lepton asymmetry
leading to baryogenesis [3].

We conclude that hierarchical neutrino masses suggest that the possible leptonic $CP$
violation strength at the GUT scale differs from that at the electroweak scale by no more
than one order of magnitude. Since this quantity is reparametrization invariant, models of
grand unified theories should give consistent predictions for it. The fact that the measure of
the leptonic $CP$ nonconservation does not have a significant decrease at higher scales may
help leptogenesis in the early Universe.

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