On the origin of families of fermions and their mass matrices -
Approximate analyses of properties of four families within
approach unifying spins and charges

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Abstract

The approach unifying all the internal degrees of freedom - proposed by one of us[1, 2, 3, 4, 5, 6, 7, 8, 9, 10] - is offering a new way of understanding families of quarks and leptons. Spinors, namely, living in \(d = 1 + 13\) -dimensional space, manifest in the observed \(d = 1 + 3\)-dimensional space (at "physical energies") all the known charges of quarks and leptons (with the mass protection property of the Standard model - only the left handed quarks and leptons carry the weak charge while the right handed ones are weak chargeless - included), while a part of the starting Lagrange density in \(d = 1 + 13\) transforms the right handed quarks and leptons into the left handed ones, manifesting a mass term in \(d = 1 + 3\). Since a spinor carries two kinds of spins and interacts accordingly with two kinds of the spin connection fields, the approach predicts families and the corresponding Yukawa couplings. In the paper[11] the appearance of families of quarks and leptons within this approach was investigated and the explicit expressions for the corresponding Yukawa couplings, following from the approach after some approximations and simplifications, presented. In this paper we continue investigations of this new way of presenting families of quarks and leptons by further analyzing properties of mass matrices, treating quarks and leptons in an equivalent way. We connect free parameters of the approach with the known experimental data and investigate a possibility that the fourth family of quarks and leptons appears at low enough energies to be observable with the new generation of accelerators.
I. INTRODUCTION

The Standard model of the electroweak and strong interactions (extended by assuming nonzero masses of the neutrinos) fits with around 25 parameters and constraints all the existing experimental data. However, it leaves unanswered many open questions, among which are also the questions about the origin of the families, the Yukawa couplings of quarks and leptons and the corresponding Higgs mechanism. Understanding the mechanism for generating families, their masses and mixing matrices might be one of the most promising ways to physics beyond the Standard model.

The approach, unifying spins and charges\([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]\), might by offering a new way of describing families, give an explanation about the origin of the Yukawa couplings.

It was demonstrated in the references\([6, 8, 9, 10]\) that a left handed \(SO(1, 13)\) Weyl spinor multiplet includes, if the representation is analyzed in terms of the subgroups \(SO(1, 3), SU(2), SU(3)\) and the sum of the two \(U(1)\)'s, all the spinors of the Standard model - that is the left handed \(SU(2)\) doublets and the right handed \(SU(2)\) singlets of (with the group \(SU(3)\) charged) quarks and (chargeless) leptons. There are the (two kinds of) spin connection fields and the vielbein field in \(d = (1 + 13)\)–dimensional space, which might manifest - after some appropriate compactifications (or some other kind of making the rest of \(d - 4\) space unobservable at low energies) - in the four dimensional space as all the gauge fields of the known charges, as well as the Yukawa couplings.

The paper\([11]\) analyzes, how do terms, which lead to masses of quarks and leptons, appear in the approach unifying spins and charges as a part of the spin connection and vielbein fields. No Higgs is needed in this approach to "dress" right handed spinors with the weak charge, since the terms of the starting Lagrangean, which include \(\gamma^0 \gamma^s\), with \(s = 7, 8\), do the job of a Higgs field.

Since we have done no analyses (yet) about the way of breaking symmetries of the starting group \(SO(1, 13)\) to \(SO(1, 7) \times U(1) \times SU(3)\) and further within our approach (except some very rough estimations in ref.\([24]\)), we do not know how might symmetry breaking in the ordinary space influence the fields, which determine the Yukawa couplings. We can accordingly in this investigation, by connecting Yukawa couplings with the experimental data, only discuss about the appearance of the "vacuum expectation values" of the spin connection fields which enter into the Yukawa couplings.
We also have no explanation yet why the second kind of the Clifford algebra objects do not manifest in $d = 1 + 3$ any charges, which could appear in addition to the known ones.

Since the generators of the Lorentz transformations and the generators of families commute, and since only the generators of families contribute to nondiagonal elements of mass matrices (which means, that the off diagonal matrix elements of quarks and leptons are strongly correlated), the question arises, what makes leptons so different from quarks in the proposed approach. Can it be that at some energy level they are very alike and that there are some kinds of boundary conditions together with the nonperturbative effects which lead to observable properties, or might it be that Majorana like objects, not taken into account in these investigations, are responsible for the observed differences?

In this paper we try to understand properties of quarks and leptons within the approach unifying spins and charges treating quarks and leptons equivalently. Within this approach we discuss also a possibility, that the fourth family of quarks and leptons appears at low enough energies to be observable with new accelerators.

In Sect III of this paper we present the action for a Weyl spinor in $(1 + 13)$-dimensional space and the part of the Lagrangean, which manifests at ”physical energies” as an effective Lagrangean, with the Yukawa mass term included. This section is a brief repetition of the derivations presented in the ref. [11].

Sect III repeats the explicit expressions for the four mass matrices of four families of quarks and leptons as derived in the paper [11] under several assumptions and simplifications from the starting action of the approach unifying spins and charges. We study properties of the mass matrices in this assumed approximation.

In Sect IV we discuss the problem of the appearance of negative masses in connection with the internal parity, defined within the presented approach.

In Sect V we relax some of the assumptions and evaluate approximately the improved properties of quarks and leptons.

In Sect VII we comment on the approximate predictions of our approach.
II. WEYL SPINORS IN $d = (1+13)$ MANIFESTING AT "PHYSICAL ENERGIES" FAMILIES OF QUARKS AND LEPTONS

We assume a left handed Weyl spinor in $(1 + 13)$-dimensional space. A spinor carries only the spin (no charges) and interacts accordingly with only the gauge gravitational fields - with spin connections and vielbeins. We assume two kinds of the Clifford algebra objects and allow accordingly two kinds of gauge fields\[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\]. One kind is the ordinary gauge field (gauging the Poincaré symmetry in $d = 1 + 13$). The corresponding spin connection field appears for spinors as a gauge field of $\mathcal{S}^{ab} = \frac{1}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$, where $\gamma^a$ are the ordinary Dirac operators. The contribution of these fields to the mass matrices manifests in only the diagonal terms - connecting the right handed weak chargeless quarks or leptons to the left handed weak charged partners within one family of spinors.

The second kind of gauge fields is in our approach responsible for families of spinors and couplings among families of spinors - contributing to diagonal matrix elements as well - and might explain the appearance of families of quarks and leptons and the Yukawa couplings of the Standard model of the electroweak and colour interactions. The corresponding spin connection fields appear for spinors as gauge fields of $\tilde{\mathcal{S}}^{ab}$ ($\tilde{\mathcal{S}}^{ab} = \frac{1}{2}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$) with $\tilde{\gamma}^a$, which are the Clifford algebra objects\[2, 16\], like $\gamma^a$, but anticommute with $\gamma^a$.

Following the ref.\[11\] we write the action for a Weyl (massless) spinor in $d(= 1 + 13)$ - dimensional space as follows\[29\]

$$S = \int d^dx \mathcal{L}$$

$$\mathcal{L} = \frac{1}{2}(E \bar{\psi} \gamma^a p_0 \psi) + h.c. = \frac{1}{2}(E \bar{\psi} \gamma^a f^a \alpha \rho_{\alpha} \psi) + h.c.$$

$$p_{\alpha} = p_\alpha - \frac{1}{2} S^{ab} \omega_{aba} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{aba}. \quad (1)$$

Here $f^a \alpha$ are vielbeins (inverted to the gauge field of the generators of translations $e^a \alpha$, $e^a \alpha f^a \beta = \delta^\alpha_\beta$, $e^a \alpha f^\beta \alpha = \delta^\alpha_\beta$), with $E = \det(e^a \alpha)$, while $\omega_{aba}$ and $\tilde{\omega}_{aba}$ are the two kinds of the spin connection fields, the gauge fields of $S^{ab}$ and $\tilde{S}^{ab}$, respectively, corresponding to the two kinds of the Clifford algebra objects\[10, 15\], namely $\gamma^a$ and $\tilde{\gamma}^a$, with the properties

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+, \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0, \quad (2)$$

leading to $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$. We kindly ask the reader to learn about the properties of these two kinds of the Clifford algebra objects - $\gamma^a$ and $\tilde{\gamma}^a$ and of the corresponding $S^{ab}$ and $\tilde{S}^{ab}$.
- and about our technique in the ref. [11] or the refs. [15, 16].

One Weyl spinor representation in $d = (1+13)$ with the spin as the only internal degree of freedom, manifests, if analyzed in terms of the subgroups $SO(1, 3) \times U(1) \times SU(2) \times SU(3)$ in four-dimensional "physical" space as the ordinary ($SO(1, 3)$) spinor with all the known charges of one family of the left handed weak charged and the right handed weak chargeless quarks and leptons of the Standard model. The reader can see this analyses in the paper [11] (as well as in several references, like the one [10]).

We may rewrite the Lagrangean of Eq. (1) so that it manifests the usual $(1+3)$-dimensional spinor Lagrangean part and the term manifesting as a mass term [11]

$$\mathcal{L} = \bar{\psi} \gamma^m (p_m - \sum_{A,i} g^A \tau^{Ai} A^A_m) \psi + \sum_{s=7,8} \bar{\psi} \gamma^s p_0 \psi + \text{the rest.}$$

(3)

Index $A$ determines the charge groups ($SU(3), SU(2)$ and the two $U(1)$’s), index $i$ determines the generators within one charge group. $\tau^{Ai}$ denote the generators of the charge groups

$$\tau^{Ai} = \sum_{s,t} C^{A}_{st} S^{st},$$

$$\{ \tau^{Ai}, \tau^{Bj} \} = i\delta^{AB} f^{Aijk} \tau^{Ak}. (4)$$

with $s, t \in 5, 6, ..., 14$, while $A^A_m, m = 0, 1, 2, 3$, denote the corresponding gauge fields (expressible in terms of $\omega_{stm}$).

We have $Y = \tau^{41} + \tau^{21}$, $Y' = \tau^{41} - \tau^{21}$, with $\tau^{11} := \frac{1}{2}(S^{58} - S^{67}), \tau^{12} := \frac{1}{2}(S^{57} + S^{68}), \tau^{13} := \frac{1}{2}(S^{56} - S^{78}), \tau^{21} := \frac{1}{2}(S^{56} + S^{78}), \tau^{31} := \frac{1}{2}(S^{9,12} - S^{10,11}), \tau^{32} := \frac{1}{2}(S^{9,11} + S^{10,12}), \tau^{33} := \frac{1}{2}(S^{9,10} - S^{11,12}), \tau^{34} := \frac{1}{2}(S^{9,14} - S^{10,13}), \tau^{35} := \frac{1}{2}(S^{9,13} + S^{10,14}), \tau^{36} := \frac{1}{2}(S^{11,14} - S^{12,13}), \tau^{37} := \frac{1}{2}(S^{11,13} + S^{12,14}), \tau^{38} := \frac{1}{2\sqrt{3}}(S^{9,10} + S^{11,12} - 2S^{13,14}), \tau^{41} := -\frac{1}{3}(S^{9,10} + S^{11,12} + S^{13,14}).$

The subgroups are chosen so that the gauge fields in the "physical" region agree with the known gauge fields. If the break of symmetries in the $\bar{\tau}^{ab}$ sector demonstrates the same symmetry after the break as in the $S^{ab}$ sector, then also the corresponding operators with $\bar{\tau}^{Ai}$ should be defined.

Making several assumptions, explained in details in the ref. [11] - we shall repeat them bellow - needed to manifest the observable phenomena (and can not yet be derived, since we do not yet know how the break of symmetries influences the starting Lagrangean), we are
able to rewrite the mass term of spinors (fermions) from Eq. (3) ($\sum_{s=7,8} \bar{\psi} \gamma^s p_0 \psi$ (neglecting the rest)) by assuming that they are small in comparison with what we keep at “physical energies”) as $L_Y$, demonstrating the Yukawa couplings of the Standard model

$$L_Y = \psi^+ \gamma^0 \left\{ \begin{array}{l}
(+) \left( \sum_{y=Y,Y'} y A_y^+ + \bar{y} \bar{A}_y^+ \right) + \\
(-) \left( \sum_{y=Y,Y'} y A_y^- + \bar{y} \bar{A}_y^- \right) + \\
(+) \sum_{\{(ac)(bd)\},k,l}^{78} ac bd \bar{A}^{kl}_{+}((ac),(bd)) + \\
(-) \sum_{\{(ac)(bd)\},k,l}^{78} ac bd \bar{A}^{kl}_{-}((ac),(bd)) \right\} \psi, \quad (5)
\end{array} \right.$$ 

with

$$ab(k) := \frac{1}{2}(\gamma^a + \eta^{ab} \gamma^8), \quad ab(\bar{k}) := \frac{1}{2}(\bar{\gamma}^a + \eta^{ab} \bar{\gamma}^b) \quad (6)$$

and with $k = \pm 1$, if $\eta^{aa} \eta^{bb} = 1$ and $\pm i$, if $\eta^{aa} \eta^{bb} = -1$. While $ab(k)$ are expressible in terms of ordinary $\gamma^a$ and $\gamma^b$, $(\bar{k})$ are expressible in terms of the second kind of the Clifford algebra objects, namely in terms of $\bar{\gamma}^a$ and $\bar{\gamma}^b$.

The Yukawa part of the starting Lagrangean (Eq. (5)) has the diagonal terms, that is the terms manifesting the Yukawa couplings within each family, and the off diagonal terms, determining the Yukawa couplings among families.

The operators, which contribute to the non diagonal terms in mass matrices, are superposition of $\bar{S}^{ab}$ (times the corresponding fields $\bar{\omega}_{abc}$) and can be represented as factors of nilpotents

$$ab cd \quad (\bar{k})(\bar{l}), \quad (7)$$

with indices $(ab)$ and $(cd)$ which belong to the Cartan subalgebra indices and the superposition of the fields $\bar{\omega}_{abc}$. We may write accordingly

$$\sum_{(a,b)} \frac{-1}{2} \bar{S}^{ab} \bar{\omega}_{ab\pm} = - \sum_{\{(ac),(bd)\},k,l}^{78} ac bd \bar{A}^{kl}_{\pm}((ac),(bd)), \quad (8)$$

where the pair $(a,b)$ in the first sum runs over all the indices, which do not characterize the Cartan subalgebra, with $a, b = 0, \ldots, 8$, while the two pairs $(ac)$ and $(bd)$ denote only the
Cartan subalgebra pairs (for SO(1,7) we only have the pairs (03), (12); (03), (56) ;(03), (78); (12), (56); (12), (78); (56), (78)) ; k and l run over four possible values so that k = ±i, if
(ac) = (03) and k = ±1 in all other cases, while l = ±1. The fields \( \tilde{A}_{kl}^{\pm}((ac),(bd)) \) can then be expressed by \( \tilde{\omega}_{ab\pm} \) as follows

\[
\begin{align*}
\tilde{A}_{++}^{\pm}((ab),(cd)) &= -\frac{i}{2}(\tilde{\omega}_{ac\pm} - i r \tilde{\omega}_{bc\pm} - i \tilde{\omega}_{ad\pm} - \frac{1}{r} \tilde{\omega}_{bd\pm}), \\
\tilde{A}_{-+}^{\pm}((ab),(cd)) &= -\frac{i}{2}(\tilde{\omega}_{ac\pm} + i r \tilde{\omega}_{bc\pm} + i \tilde{\omega}_{ad\pm} - \frac{1}{r} \tilde{\omega}_{bd\pm}), \\
\tilde{A}_{-}^{-\pm}((ab),(cd)) &= -\frac{i}{2}(\tilde{\omega}_{ac\pm} + i r \tilde{\omega}_{bc\pm} - i \tilde{\omega}_{ad\pm} + \frac{1}{r} \tilde{\omega}_{bd\pm}), \\
\tilde{A}_{+}^{\pm}((ab),(cd)) &= -\frac{i}{2}(\tilde{\omega}_{ac\pm} - i r \tilde{\omega}_{bc\pm} + i \tilde{\omega}_{ad\pm} + \frac{1}{r} \tilde{\omega}_{bd\pm}),
\end{align*}
\]

with \( r = i \), if \((ab) = (03)\) and \( r = 1 \) otherwise. We simplify the index \( kl \) in the exponent of the fields \( \tilde{A}_{kl}^{\pm}((ac),(bd)) \) to ±, omitting \( i \).

We must point out that a way of breaking any of the two symmetries - the Poincaré one and the symmetry determined by the generators \( \tilde{S}_{ab} \) in \( d = 1 + 13 \) - strongly influences the Yukawa couplings of Eq.(5), relating the parameters \( \tilde{\omega}_{abc} \). Not necessarily any break of the Poincaré symmetry influences the break of the other symmetry and opposite. Although we expect that it does. Accordingly the coefficients \( c^{Ai}_{ab} \) determining the operators \( \tau^{Ai} \) in Eq.(4) and the coefficients \( \tilde{c}^{\tilde{A}i}_{ab} \) determining the operators \( \tilde{\tau}^{\tilde{A}i} \) in the relations

\[
\begin{align*}
\tilde{\tau}^{\tilde{A}i} &= \sum_{a,b} \tilde{c}^{\tilde{A}i}_{ab} \tilde{S}_{ab}, \\
\{ \tilde{\tau}^{\tilde{A}i}, \tilde{\tau}^{\tilde{B}j} \} &= i \delta^{\tilde{A}\tilde{B}} \tilde{f}^{\tilde{A}ijk} \tilde{\tau}^{\tilde{A}k}
\end{align*}
\]

might even not be correlated. If correlated (through boundary conditions, for example) the break of symmetries might cause that off diagonal matrix elements of Yukawa couplings distinguish between quarks and leptons.

We made, when deriving the mass matrices of quarks and leptons from the approach unifying spins and charges, several assumptions, approximations and simplifications in order to be able to make at the end some rough predictions about the properties of the families of quarks and leptons:

i. The break of symmetries of the group \( SO(1,13) \) (the Poincaré group in \( d = 1 + 13 \)) into \( SO(1,7) \times SU(3) \times U(1) \) occurs in a way that only massless spinors in \( d = 1 + 7 \) with the charge \( SU(3) \times U(1) \) survive, and yet the two \( U(1) \) charges, following from \( SO(6) \) and \( SO(1,7) \), respectively, are related. (Our work on the compactification of a massless spinor
in \( d = 1 + 5 \) into \( d = 1 + 3 \) and a finite disk gives us some hope that such an assumption might be justified [14]. The requirement that the terms with \( S_5^{a\omega_{5ab}} \) and \( S_6^{a\omega_{6ab}} \) do not contribute to the mass term, assures that the charge \( Q = \tau^4 + S_5^{56} \) is conserved at low energies.

ii. The break of symmetries influences both, the (Poincaré) symmetry described by \( S^{ab} \) and the symmetries described by \( \tilde{S}^{ab} \), and in a way that there are no terms, which would transform \((+)\) into \([+]\). This assumption was made that at "low energies" only four families have to be treated and can be explained by a break of the symmetry \( SO(1,7) \) into \( SO(1+5) \times U(1) \) in the \( \tilde{S}^{ab} \) sector so that all the contributions of the type \( \tilde{S}_5^{a\omega_{5ab}} \) and \( \tilde{S}_6^{a\omega_{6ab}} \) are equal to zero. We also assume that the terms which include components \( p_s, s = 5, \ldots, 14, \) of the momentum \( p^a \) do not contribute to the mass matrices. We keep in mind that any further break of symmetries strongly influences the relations among \( \tilde{\omega}_{abc} \), appearing in the paper [11] as "vacuum expectation values" in mass matrices, so that predictions in Sect. VII strongly depend on the way of breaking.

iv. We make estimations on a "tree level".

v. We assume the mass matrices to be real and symmetric (expecting that complexity and nonsymmetric properties will not influence considerably masses and mixing matrices of quarks and leptons).

### III. FOUR FAMILIES OF QUARKS AND LEPTONS

Taking into account the assumptions, presented in Sect. III we end up with four families of quarks and leptons

\[
\begin{align*}
I. & \quad (+#)(+) \mid (+)(+)] \mid \ldots \\
II. & \quad [+][+] \mid (+)(+) \mid \ldots \\
III. & \quad [+][+] \mid (+)(+) \mid \ldots \\
IV. & \quad (+)[+] \mid (+)[+] \mid \ldots
\end{align*}
\]

The Yukawa couplings for these four families are for \( u \)-quarks and neutrinos presented on Table II where \( \alpha \) stays for \( u \)-quarks and neutrinos.

The corresponding mass matrix for the \( d \)-quarks and the electrons is presented on Table III where \( \beta \) stays for \( d \)-quarks and electrons.
and II there are 13 free parameters, expressed in terms of the fields unifying spins and charges under the assumptions i.-v. in ref.\cite{11}. According to Eq.(12) and Table I terms of $\omega$
quarks and the two types of leptons. determine (due to assumptions i.-v.) all the properties of the four families of the two types of $\nu$-quarks and neutrinos, obtained within the approach unifying spins and charges under the assumptions i.-v. in ref.\cite{11}. According to Eq.(12) and Table II and III there are 13 free parameters, expressed in terms of the fields $A_\alpha^I$ and $\tilde{\omega}_{abc}$, which accordingly determine (due to assumptions i.-v.) all the properties of the four families of the two types of quarks and the two types of leptons.

The explicit form of the diagonal matrix elements for the above choice of assumptions in terms of $\omega_{abc}$’s and $A_\pm^Y, y = Y$ and $Y', \tilde{\omega}_{abc}$ and $\tilde{A}_\pm^{III}$ is as follows

| $\alpha$ | $I_R$ | $II_R$ | $III_R$ | $IV_R$ |
|----------|-------|-------|-------|-------|
| $I_L$    | $A_\alpha^I$ | $A_\alpha^I$ | $A_\alpha^I$ | $A_\alpha^I$ |
|          | $\frac{1}{2}(\tilde{\omega}_{327} + \tilde{\omega}_{018})$ | $\frac{1}{2}(\tilde{\omega}_{387} + \tilde{\omega}_{078})$ | $\frac{1}{2}(\tilde{\omega}_{277} + \tilde{\omega}_{187})$ | $\frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$ |
| $II_L$   | $\tilde{A}_\alpha^-(03), (12))$ | $A_\alpha^{II} = A_\alpha^I + (\tilde{\omega}_{127} - \tilde{\omega}_{038})$ | $A_\alpha^{II} = A_\alpha^I + (\tilde{\omega}_{787} - \tilde{\omega}_{038})$ | $A_\alpha^{II} = \frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018})$ |
|          | $\frac{1}{2}(\tilde{\omega}_{387} + \tilde{\omega}_{078})$ | $-\frac{1}{2}(\tilde{\omega}_{277} - \tilde{\omega}_{187})$ | $\frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018})$ | $\frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$ |
| $III_L$  | $\tilde{A}_\alpha^-((03), (78))$ | $-\tilde{A}_\alpha^-((12), (78)) = A_\alpha^{III} = A_\alpha^I + (\tilde{\omega}_{787} - \tilde{\omega}_{038})$ | $\tilde{A}_\alpha^+(03), (12)) = A_\alpha^{IV} = \frac{1}{2}(\tilde{\omega}_{327} + \tilde{\omega}_{018})$ | $\tilde{A}_\alpha^+(03), (12)) = A_\alpha^{IV} = \frac{1}{2}(\tilde{\omega}_{387} + \tilde{\omega}_{078})$ |
|          | $\frac{1}{2}(\tilde{\omega}_{277} + \tilde{\omega}_{187})$ | $\frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$ | $\frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018})$ | $\frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$ |
| $IV_L$   | $\tilde{A}_\alpha^-(12), (78) = A_\alpha^I + (\tilde{\omega}_{127} - \tilde{\omega}_{038})$ | $\tilde{A}_\alpha^+(03), (12)) = A_\alpha^{IV} = \frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018})$ | $\tilde{A}_\alpha^-((03), (12)) = A_\alpha^{IV} = \frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$ | $\tilde{A}_\alpha^-((03), (12)) = A_\alpha^{IV} = \frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$ |
|          | $\frac{1}{2}(\tilde{\omega}_{277} + \tilde{\omega}_{187})$ | $\frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$ | $\frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018})$ | $\frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$ |

TABLE I: The mass matrix of four families of $u$-quarks and neutrinos, obtained within the approach unifying spins and charges under the assumptions i.-v. in ref.\cite{11}. According to Eq.(12) and Table II and III there are 13 free parameters, expressed in terms of the fields $A_\alpha^I$ and $\tilde{\omega}_{abc}$, which accordingly determine (due to assumptions i.-v.) all the properties of the four families of the two types of quarks and the two types of leptons.
TABLE II: The mass matrix of four families of the $d$-quarks and electrons, $\beta$ stays for the $d$-quarks and the electrons. Comments are the same as on Table I.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\beta & I_R & II_R & III_R & IV_R \\
\hline
I_L & A^I_\beta & \tilde{A}_\beta^+((03), (12)) = \frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018}) & -\tilde{A}_\beta^+((03), (78)) = \frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078}) & \tilde{A}_\beta^+((12), (78)) = -\frac{1}{2}(\tilde{\omega}_{277} + \tilde{\omega}_{187}) \\
\hline
II_L & \tilde{A}_\beta^-((03), (12)) = \frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018}) & \tilde{A}_\beta^H = A^I_\beta + (\tilde{\omega}_{127} + \tilde{\omega}_{038}) & -\tilde{A}_\beta^+((12), (78)) = \frac{1}{2}(\tilde{\omega}_{277} - \tilde{\omega}_{187}) & -\frac{1}{2}(\tilde{\omega}_{387} + \tilde{\omega}_{078}) \\
III_L & -\tilde{A}_\beta^-((03), (78)) = \tilde{A}_\beta^H = A^I_\beta + (\tilde{\omega}_{787} + \tilde{\omega}_{038}) & -\tilde{A}_\beta^+((12), (78)) = \frac{1}{2}(\tilde{\omega}_{277} - \tilde{\omega}_{187}) & -\frac{1}{2}(\tilde{\omega}_{018} + \tilde{\omega}_{327}) \\
IV_L & -\tilde{A}_\beta^-((12), (78)) = \tilde{A}_\beta^H = A^I_\beta + (\tilde{\omega}_{787} + \tilde{\omega}_{038}) & -\tilde{A}_\beta^+((03), (78)) = \frac{1}{2}(\tilde{\omega}_{018} + \tilde{\omega}_{327}) & -\frac{1}{2}(\tilde{\omega}_{127} + \tilde{\omega}_{787}) \\
\hline
\end{array}
\]

with $\alpha = u, \nu, \beta = d, e$ and $-\tilde{\omega}^I_\pm = \frac{1}{2}(i\tilde{\omega}_{03\pm} + \tilde{\omega}_{12\pm} + \tilde{\omega}_{56\pm} + \tilde{\omega}_{78\pm} + \frac{1}{3}A^H_\beta)$. The assumption that all the matrix elements are real relates $\tilde{\omega}^I_+ = \frac{1}{2}\tilde{\omega}_{038} + \tilde{\omega}, \tilde{\omega}^I_- = -\frac{1}{2}\tilde{\omega}_{038} + \tilde{\omega}$, where $\tilde{\omega}$ is (in case that breaking of symmetries does not influence quarks and leptons differently) one common parameter.

If the break of symmetries does not influence the quarks and the leptons in a different way, then under the assumptions i.-v. the off diagonal matrix elements of mass matrices for quarks are the same as for the corresponding leptons (the off diagonal matrix elements of the $u$-quarks and the neutrinos are the same, and the off diagonal matrix elements for the $d$-quarks and the electrons are the same) and since the diagonal matrix elements differ only in a constant times a unit matrix, the predicted mixing matrices of the quarks and the
leptons would be the same.

We must ask ourselves at this stage: Can we find any way of breaking symmetries - allowing any very special boundary conditions - which would lead to so different properties of quarks and leptons as observed or we must take the Majorana like degrees of freedom into account?

In this paper, we are not yet able to answer this question. We can only make some estimates trying to learn from the approach unifying spins and charges about possible explanations for the properties of quarks and leptons.

We proceed by relating the experimental data and the mass matrices from the approach. Knowing from the experimental data that the first two families of quarks and leptons are much lighter than the third one, while in the refs.\[21, 22, 23\] the authors, analyzing the experimental data, conclude that the experimental data do not forbid masses of the fourth family of quarks to be between 200 GeV and 300 GeV, of the fourth electron to be around 100 GeV and of the fourth neutrino to be at around 50 GeV we make one more assumption, which seems quite reasonable also from the point of view of the measured matrix elements of the mixing matrix for quarks. Namely, we assume that the mass matrices of the four families of quarks and leptons are diagonalizable in two steps, so that the first diagonalization transforms them into block-diagonal matrices with two $2 \times 2$ sub-matrices. This assumption, which means, that a real and symmetric $4 \times 4$ matrix is diagonalizable by only three rather than six angles, simplifies considerably further studies, making conclusions very transparent. Such a property of mass matrices could be a consequence of an approximate break of symmetry in the $\tilde{S}^{ab}$ sector from $SO(1,5)$ to $SU(2) \times SU(2) \times U(1)$, which makes, for example, all the terms $\tilde{S}^{7a} \tilde{\omega}^{7ab}$ and $\tilde{S}^{8a} \tilde{\omega}^{8ab}$ contributing small terms to mass matrices. The exact break of this type makes that the lower two families completely decouple from the higher two. (Similarly we have required, in order to end up with only four rather than eight families, that $SO(1,7)$ breaks to $SO(1,5) \times U(1)$ so that all $\tilde{S}^{5a} \tilde{\omega}^{5a \pm}$ and $\tilde{S}^{6a} \tilde{\omega}^{6a \pm}$ contribute nothing to mass matrices.) While the exact break of $SO(1,5)$ to $SU(3) \times U(1)$ makes that the fourth family decouples from the first three.

It is easy to prove that a $4 \times 4$ matrix is diagonalizable in two steps only if it has a structure

\[
\begin{pmatrix}
A & B \\
B & C = A + kB
\end{pmatrix}
\] (13)
Since $A$ and $C$ are, as assumed on Table I and Table III symmetric $2 \times 2$ matrices, so must then be also $B$. The parameter $k$ is assumed to be an unknown number.

The assumption (13) requires: i. $\omega_{277} = 0$, $\omega_{327} = -\frac{k}{2} \omega_{187}$, $\omega_{787} = \frac{k}{2} \omega_{387}$, $\omega_{038} = -\frac{k}{2} \omega_{078}$, and ii. $k_u = -k_d$ and $k_{\nu} = -k_e$, where $k_u$ and $k_{\nu}$ will be taken as two independent parameters. (If $k = 0$ in Eq. (13), the angle of rotation is $45^\circ$ - then, if also all the $2 \times 2$ matrices would have the same structure (namely equal diagonal and equal nondiagonal elements), the corresponding mixing matrices for quarks and leptons would be the identity.)

We shall present in what follows some simple relations which demonstrate transparently properties of mass matrices. After the one step diagonalization determined by the angle of rotation

$$
\tan \varphi_\alpha = \pm \left( \sqrt{1 + \left( \frac{k_\alpha}{k} \right)^2} \pm \frac{k_\alpha}{k} \right), \quad \tan \varphi_\beta = \pm \left( \sqrt{1 + \left( \frac{k_\beta}{k} \right)^2} \pm \frac{k_\beta}{k} \right),
$$

with \( \tan(\varphi_\alpha - \varphi_\beta) = \pm \frac{k_\alpha}{k} \) (or $\pm \frac{2}{k}$)

we end up with two by diagonal matrices, with $k = k_u$ for quarks and $k = k_{\nu}$ for leptons, while $\alpha$ concerns the $u$-quarks and $\nu$, and $\beta$ the $d$-quarks and electrons.

The first by diagonal mass matrix of the $u$-quarks ($\alpha = u$) and neutrinos ($\alpha = \nu$) is as follows

$$
A^a_\alpha = \begin{pmatrix}
a_\alpha, & \frac{1}{2}(\omega_{018} - \sqrt{1 + \left( \frac{k_\alpha}{k} \right)^2} \omega_{187}) \\
\frac{1}{2}(\omega_{018} - \sqrt{1 + \left( \frac{k_\alpha}{k} \right)^2} \omega_{187}), & a_\alpha + \omega_{127} + \sqrt{1 + \left( \frac{k_\alpha}{k} \right)^2} \omega_{078}
\end{pmatrix},
$$

with $a_u = \frac{2}{3}A Y - \frac{1}{3}A Y' + \bar{\omega} - \frac{1}{2}\bar{\omega}_{038} + \frac{1}{2}\left( \frac{k_\alpha}{k} \right) - \sqrt{1 + \left( \frac{k_\alpha}{k} \right)^2} (\omega_{078} + \omega_{387})$ and $a_\nu = -A Y + \bar{\omega} - \frac{1}{2}\bar{\omega}_{038} + \frac{1}{2}\left( \frac{k_\nu}{k} \right) - \sqrt{1 + \left( \frac{k_\nu}{k} \right)^2} (\omega_{078} + \omega_{387})$. The mass matrix for the second two families of $u$-quarks ($\alpha = u$) and neutrinos ($\alpha = \nu$) is equal to

$$
A^b_\alpha = \begin{pmatrix}
a_\alpha + \sqrt{1 + \left( \frac{k_\alpha}{k} \right)^2} (\omega_{078} + \omega_{387}), & \frac{1}{2}(\omega_{018} + \sqrt{1 + \left( \frac{k_\alpha}{k} \right)^2} \omega_{187}) \\
\frac{1}{2}(\omega_{018} + \sqrt{1 + \left( \frac{k_\alpha}{k} \right)^2} \omega_{187}), & a_\alpha + \omega_{127} + \sqrt{1 + \left( \frac{k_\alpha}{k} \right)^2} \omega_{078}
\end{pmatrix}.
$$

Accordingly we find for the first two families of $d$-quarks ($\beta = d$) and electrons ($\beta = e$)

$$
A^a_\beta = \begin{pmatrix}
a_\beta, & -\frac{1}{2}(\omega_{018} - \sqrt{1 + \left( \frac{k_\beta}{k} \right)^2} \omega_{187}) \\
-\frac{1}{2}(\omega_{018} - \sqrt{1 + \left( \frac{k_\beta}{k} \right)^2} \omega_{187}), & a_\beta + \omega_{127} + \sqrt{1 + \left( \frac{k_\beta}{k} \right)^2} \omega_{078}
\end{pmatrix},
$$

with $a_d = -\frac{1}{3}A Y + \frac{2}{3}A Y' + \bar{\omega} + \frac{1}{2}\bar{\omega}_{038} - \frac{1}{2}\left( \frac{k_\beta}{k} \right) + \sqrt{1 + \left( \frac{k_\beta}{k} \right)^2} (\omega_{078} - \omega_{387})$ and $a_e = -A Y + \bar{\omega} + \frac{1}{2}\bar{\omega}_{038} - \frac{1}{2}\left( \frac{k_\beta}{k} \right) + \sqrt{1 + \left( \frac{k_\beta}{k} \right)^2} (\omega_{078} - \omega_{387})$. $k_\alpha$ in (17) is $k_\alpha$ for $d$-quarks and $k_{\nu}$ for electrons.
For the second two families of $d$-quarks ($\beta = d$) and electrons ($\beta = e$) it follows

\[
A_\beta^b = \begin{pmatrix}
  a_\beta + \sqrt{1 + \left(\frac{k_\alpha}{2}\right)^2 (\tilde{\omega}_{078} - \tilde{\omega}_{387})}, & -\frac{1}{2}(\tilde{\omega}_{018} + \sqrt{1 + \left(\frac{k_\alpha}{2}\right)^2 \tilde{\omega}_{187}}) \\
  -\frac{1}{2}(\tilde{\omega}_{018} + \sqrt{1 + \left(\frac{k_\alpha}{2}\right)^2 \tilde{\omega}_{187}}), & a_\beta + \tilde{\omega}_{127} - \sqrt{1 + \left(\frac{k_\alpha}{2}\right)^2 \tilde{\omega}_{387}}
\end{pmatrix}.
\]  

(18)

Again, $k_\alpha$ in (18) is $k_u$ for $d$-quarks and $k_\nu$ for electrons.

There are three angles, which in the two step orthogonal transformations rotate each mass matrix into a diagonal one. The angles of rotations for $u$–quarks and $d$–quarks, and accordingly for neutrinos and electrons, are related as seen from Eq. (14) for the first step rotation. It follows namely that $\tan \varphi_\alpha = \tan^{-1} \varphi_\beta$ and accordingly

\[
\varphi_\alpha = \frac{\pi}{2} - \varphi_\beta, \quad \varphi_\alpha = \frac{\pi}{4} - \frac{\varphi}{2}, \quad \varphi_\beta = \frac{\pi}{4} + \frac{\varphi}{2},
\]

with $\varphi = \varphi_\alpha - \varphi_\beta$.  

(19)

Similarly also the two angles of rotations of the two by two diagonal matrices are related. Reminding the reader that in the unitary transformations ($S^\dagger S = I$) the trace and the determinant are among the invariants, while the angle of rotation, which diagonalizes 2 by 2 matrices (of the type (13)), and the values of the diagonal matrices are related as follows

\[
\tan \Phi = (\sqrt{1 + \left(\frac{C - A}{2B}\right)^2} \mp \frac{C - A}{2B}),
\]

\[
\lambda_{1,2} = \frac{1}{2}((C + A) \pm \sqrt{(C - A)^2 + (2B)^2}),
\]

(20)

where for $A, B, C$ the corresponding matrix elements from Eqs. (15, 16, 17, 18) must be taken, one easily finds that

\[
^a_\alpha \eta = -^a_\beta \eta, \quad ^b_\alpha \eta = -^b_\beta \eta, \quad \alpha = u, \nu, \beta = d, e,
\]

(21)

where index $a$ denotes the first two families and $b$ the second two families of either quarks ($\alpha = u, \beta = d$) and leptons ($\alpha = \nu, \beta = e$) and $\eta = \frac{C - A}{2B}$. One then finds the relations, equivalent to those of Eq. (19)

\[
^a_\alpha \varphi = \frac{\pi}{2} - ^a_\beta \varphi, \quad ^a_\beta \varphi = \frac{\pi}{4} - \frac{^a_\beta \varphi}{2}, \quad ^b_\beta \varphi = \frac{\pi}{4} + \frac{^b_\beta \varphi}{2},
\]

with $^a_\beta \varphi = ^a_\beta \varphi - ^a_\beta \varphi$.  

(22)

We further find

\[
|m_{u_2} - m_{u_1}| = |m_{d_2} - m_{d_1}|, \quad |m_{u_4} - m_{u_1}| = |m_{d_4} - m_{d_1}|,
\]
\[ |m_{\nu_2} - m_{\nu_1}| = |m_{e_2} - m_{e_1}|, \quad |m_{\nu_4} - m_{\nu_3}| = |m_{e_4} - m_{e_3}|, \]
\[ |(m_{u_4} + m_{u_3}) - (m_{u_2} + m_{u_1})| = |(m_{d_4} + m_{d_3}) - (m_{d_2} + m_{d_1})|, \]
\[ |(m_{\nu_4} + m_{\nu_3}) - (m_{\nu_2} + m_{\nu_1})| = |(m_{e_4} + m_{e_3}) - (m_{e_2} + m_{e_1})|, \]
\[ |m_{u_4} + m_{u_3}| \approx 2 \sqrt{1 + \left(\frac{k_u}{2}\right)^2 \tilde{\omega}_{387}} \approx |m_{d_4} + m_{d_3}|, \]
\[ |m_{\nu_4} + m_{\nu_3}| \approx 2 \sqrt{1 + \left(\frac{k_\nu}{2}\right)^2 \tilde{\omega}_{387}} \approx |m_{e_4} + m_{e_3}|. \]  

We take the absolute values of the sums and the differences, since whenever an eigenvalue \( \lambda_{1,2} \) (Eq.20) appears to be negative, an appropriate change of a phase of the corresponding state transforms the negative value into the positive one by changing simultaneously the internal parity of the particular state, as it will be discussed in Sect. IV.

The above relations (23) do not agree with the experimental data. We can only accept them as a very rough estimation - after making so many assumptions and approximations - in the limit if masses of the fourth family are much higher than the mass \( m_t \), knowing that \( m_t \) is more than 30 times larger than the mass \( m_b \).

It is obvious that we made on the way of deriving properties of quarks and leptons from the starting action (Eq.1) (for a spinor carrying only two kinds of the spin degrees of freedom - no charge - and interacting with only the gauge fields of the corresponding groups, which leads to the Yukawa mass matrices (Eq.3) without assuming Higgs) so many assumptions, simplifications and approximations, that we lose the predictive power.

We did not (could not yet - to do this is a huge project, which we do have in mind) take into account influences of possible breaks of symmetries, which would certainly bring relations among \( \tilde{\omega}_{abc} \) fields, but could also - due to some boundary conditions or some other effects - change the relations among \( \tilde{\omega}_{abc} \) fields for quarks and leptons. One could also expect that possible nonperturbative effects might be a very strong reason for the differences among properties of observed fermions. In sect. V we shall try to simulate these effects in a very rough way - just by assuming that the off diagonal matrix elements might be different for different species of fermions while keeping the relations among the angles of the orthogonal rotations from Eq.(14,19,21,22).
IV. NEGATIVE MASSES AND PARITY OF STATES

We have mentioned in the previous section that after the diagonalization of mass matrices of quarks and leptons, masses of either positive or negative sign can appear.

Let us first recognize that while the starting Lagrange density for spinors (Eq.11) commutes with the operator of handedness in \( d = 1+3 \)-dimensional space \( \Gamma^{(1,13)} \) \((\Gamma^{(1,13)} = i 2^7 S^{03} S^{12} S^{56} \ldots S^{13,14})\), it does not commute with the operator of handedness in \( d = 1+3 \)-dimensional space \( \Gamma^{(1,3)} \) \((\Gamma^{(1,3)} = -i 2^2 S^{03} S^{12})\). Accordingly also the term, which manifests at "physical energies" as the mass term \( \hat{m} \) \((\gamma^0 \hat{m} = \gamma^0 \{ (+) \left( \sum_{y=Y',Y} y A^y_+ + \sum_y \vec{\n}_{3,3}, \vec{\n}_{3,13}, \vec{\gamma}, \vec{\gamma}, \vec{A} \vec{y}_+ \right) + (-) \left( \sum_{y=Y,Y} y A^y_- + \sum_y \vec{\n}_{3,3}, \vec{\n}_{3,13}, \vec{\gamma}, \vec{\gamma}, \vec{A} \vec{y}_- \right) \} + (+) \sum_{\{(ac),(bd)\}, k, l} \vec{A}_{(+)}(\{ab\}, \{cd\}) + (-) \sum_{\{(ac),(bd)\}, k, l} \vec{A}_{(-)}(\{ab\}, \{cd\}) \} \), (Eq.13), does not commute with the \( \Gamma^{(1,3)} \), they instead anticommute \((\{ \Gamma^{(1,3)}, \gamma^0 \hat{m} \}_+ = 0)\). But the rest of the "effective" Lagrangean (Eq.13) commutes with the operator of handedness in \( d = 1+3 \)-dimensional space: \( \{ \gamma^0 \gamma^m(p_m - \sum_{A,i} g A^A \gamma A_m^A), \Gamma^{(1,3)} \}_- = 0 \).

It then follows that the Lagrange density

\[
\mathcal{L} = (\Gamma^{(1,3)} \psi)^\dagger \left[ \gamma^0 \gamma^m(p_m - \sum_{A,i} g A^A \gamma A_m^A) - \Gamma^{(1,3)} \gamma^0 \hat{m} \Gamma^{(1,3)} \right] (\Gamma^{(1,3)} \psi)
\]

(24)

for the Dirac spinor \( \Gamma^{(1,3)} \psi \) differs from the one from Eq.(13) in the sign of the mass term, while the function \( \Gamma^{(1,3)} \psi \) differs from \( \psi \) in the internal parity, if \( \psi \) is the solution for the Dirac equation. Since the internal parity is just the convention, the negative mass changes sign if the internal parity of the spinor changes. The same argument was used in ref.(24), while ref.(27) uses the equivalent argument, namely, that the choice of the phase of either the right or the left handed spinors can always be changed and that accordingly also the signs of particular mass terms change.

Let us demonstrate now on Table III how does the operator of parity \( \mathcal{P} \), if postulated as

\[
\mathcal{P} = \gamma^0 \gamma^8 I_x, \text{with } I_x x^m(I_x)^{(-1)} = x_m,
\]

(25)

transform a right handed \( u \)-quark into the left handed \( u \)-quark: \( \mathcal{P} u_R = \alpha u_L \), where \( \alpha \) is the proportionality factor.

One notices that, if the operator \( \mathcal{P} \) is applied on a state, which represents right handed weak chargeless \((\tau^{13} = 0)\) \( u \)-quark of one of the three colours and is presented in terms of nilpotents in the first row of Table III it transforms this state into the state, which can be
found in the seventh row of the same table and represents the left handed \( u \)-quark of the same colour and spin and it is weak charged. Taking into account Eq. (25) and Eq. (12,16) from ref. [11] one finds \( \mathcal{P} u_R = i u_L \), while \( \mathcal{P} \mathcal{P} = I \). By changing appropriately the phases of this two basic states (\( u_R \) and \( u_L \)) we can easily achieve that \( \mathcal{P} u_R = u_L, \mathcal{P} u_L = u_R \). We should in addition keep in mind that \( \mathcal{P} \) must take into account also the appearance of families. We shall study discrete symmetries of our approach in the "low energy region" in a separate paper.

TABLE III: The 8-plet of quarks - the members of \( SO(1,7) \) subgroup, belonging to one Weyl left handed \( (\Gamma^{(1,13)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}) \) spinor representation of \( SO(1,13) \). It contains the left handed weak charged quarks and the right handed weak chargeless quarks of a particular colour \((1/2,1/(2\sqrt{3}))\). Here \( \Gamma^{(1,3)} \) defines the handedness in \((1 + 3)\) space, \( S^{12} \) defines the ordinary spin (which can also be read directly from the basic vector, since \( S^{ab} \ (k) = \frac{k}{2} \ [k] \) if \( S^{ab} \) belong to the Cartan subalgebra set), \( \tau^{13} \) defines the weak charge, \( \tau^{21} \) defines the \( U(1) \) charge, \( \tau^{33} \) and \( \tau^{38} \) define the colour charge and \( \tau^{41} \) another \( U(1) \) charge, which together with the first one defines \( Y \) and \( Y' \).
V. PREDICTIONS WITH RELAXED ASSUMPTIONS

The mass matrices for quarks and leptons, following from the approach unifying spins and charges lead, after making several assumptions and simplifications (presented in Sects. II and III), to mass matrices with elements, which obviously very strongly correlate properties of the \( u \)-quarks, the \( d \)-quarks, the neutrinos and the electrons. The rough estimation only makes sense, if the masses of the first three families are small in comparison with the masses of the fourth family and loses accordingly the predictive power.

We shall keep in this section some of the assumptions made in Sects. II and III - namely the relations among the angles of rotations (Eqs. (14,19,21,22)) - and assume that either breaks of symmetries with some peculiar boundary conditions together with nonperturbative effects might lead to the Yukawa couplings which distinguish stronger among quarks and leptons than seen above.

We keep the following assumptions: i) **matrices are symmetric and real**, ii) **diagonalization in two steps** (Eq. 13) is possible, and iii) the relations among the angles of rotations for the \( u \)-quarks and the \( d \)-quarks matrices, as well as among the angles of rotations for the matrices of the neutrinos and the electrons, which determine the first and the second step diagonalizations, stay related as presented in (Sect. III) Eqs. (19,22).

We assume that fields \( \tilde{\omega}_{abc} \) on Table I and II (Sect. II) carry an (additional) index \( \alpha \) \((\tilde{\omega}_{abc\alpha})\), which distinguishes among \( u \)-quarks, \( d \)-quarks, neutrinos and electrons. Any break of symmetries would further relate \( \tilde{\omega}_{abc\alpha} \) but might also make differences in the properties of the members of one family more expressed.

We hope that when trying to reproduce the experimental data, the ratios among the fields \( \tilde{\omega}_{abc\alpha} \) will tell us something about the break of symmetries or about other possible reasons for so different properties of quarks and leptons even on this very preliminary stage of studying the predictions of the approach unifying spins and charges.

It follows then that in Eqs. (15,16,17,18) all the fields carry an additional index \( \alpha = u,\nu,d,e \), while we keep from the previous study the relations \( k_\alpha = -k_\beta \), \( \alpha = u,\nu \) and \( \beta = d,e \), where \( k_{\alpha,\beta} \) define the first step orthogonal transformations leading to 2 by 2 by diagonal mass matrices and the relations among the angles of rotations in the second step of orthogonal transformations determined by \( a^a b^b \eta^c_\alpha = a^a b^b \left( \frac{2B}{C-A} \right)_\alpha \), requiring that (Eq. 22)

\[
a^a b^b \eta^c_\alpha = -a^a b^b \eta^c_\beta,
\]

where \( C, A, B \) are replaced by the corresponding matrix elements for the
first two families determined by the matrix $A^\alpha_{\alpha,\beta}$ (Eqs. (13) (17)) and the second two families determined by the matrix $A^b_{\alpha,\beta}$ (Eqs. (16) (18)).

It then follows

\[
\begin{align*}
\alpha \xi_\alpha (\tilde{\omega}_{018\beta} - \sqrt{1 + \frac{k_\alpha}{2}} \tilde{\omega}_{187\beta}) &= (\tilde{\omega}_{018\alpha} - \sqrt{1 + \frac{k_\alpha}{2}} \tilde{\omega}_{187\alpha}), \\
\beta \xi_\alpha (\tilde{\omega}_{018\beta} + \sqrt{1 + \frac{k_\alpha}{2}} \tilde{\omega}_{187\beta}) &= (\tilde{\omega}_{018\alpha} + \sqrt{1 + \frac{k_\alpha}{2}} \tilde{\omega}_{187\alpha}), \\
\alpha \xi_\alpha (\tilde{\omega}_{127\beta} + \sqrt{1 + \frac{k_\alpha}{2}} \tilde{\omega}_{078\beta}) &= (\tilde{\omega}_{127\alpha} + \sqrt{1 + \frac{k_\alpha}{2}} \tilde{\omega}_{078\alpha}), \\
\beta \xi_\alpha (\tilde{\omega}_{127\beta} - \sqrt{1 + \frac{k_\alpha}{2}} \tilde{\omega}_{078\beta}) &= (\tilde{\omega}_{127\alpha} - \sqrt{1 + \frac{k_\alpha}{2}} \tilde{\omega}_{078\alpha}),
\end{align*}
\]  

(26)

where index $a$ and $b$ distinguish between the two by two matrices for the first two and the second two families, correspondingly, while $\alpha = u, \nu$ and $\beta = d, e$.

The question is whether or not the three quantities $k_\alpha, a \eta_\alpha, b \eta_\alpha$, $\alpha = u, \nu$, for the quarks and the three for the leptons are enough to fit the existing experimental data for the quark and the lepton mixing matrices. Both have namely the form

\[
V_{\alpha\beta} = \begin{pmatrix}
  c(\varphi)c(a\varphi) & -c(\varphi)s(a\varphi) & -s(\varphi)c(a\varphi) & s(\varphi)s(a\varphi) \\
  c(\varphi)s(a\varphi) & c(\varphi)c(a\varphi) & -s(\varphi)s(a\varphi) & -s(\varphi)c(a\varphi) \\
  s(\varphi)c(a\varphi) & -s(\varphi)s(a\varphi) & c(\varphi)c(b\varphi) & -c(\varphi)s(b\varphi) \\
  s(\varphi)s(a\varphi) & s(\varphi)c(a\varphi) & c(\varphi)s(b\varphi) & c(\varphi)c(b\varphi)
\end{pmatrix},
\]  

(27)

with the angles (Eq. (19) (22)) described by the three parameters $k_\alpha, a \eta_\alpha, b \eta_\alpha$ as follows

\[
\varphi = \varphi_\alpha - \varphi_\beta, \quad a\varphi = a\varphi_\alpha - a\varphi_\beta, \quad a\varphi = -\frac{a\varphi + b\varphi}{2}.
\]  

(28)

If the mixing matrix for either quarks or leptons is described by the three parameters (each) $k_\alpha, a \eta_\alpha, b \eta_\alpha; \alpha = u, \nu$, then we have just enough free parameters to make any choice for the masses of the fourth family of quarks and leptons. To see this we just express $A, B, C$ in any of the two 2 by 2 matrices in terms of the corresponding diagonal values that is in terms of the masses $m_{ai}, m_{bi}; i = 1, 2, 3, 4; \alpha = u, \nu, \beta = d, e$, and the parameters $k_\alpha = -k_\beta, a \eta_\alpha = -a \eta_\beta, b \eta_\alpha = -b \eta_\beta; \alpha = u, \nu$. The matrix elements of $A^a_\alpha$ ($a a_\alpha, b a_\alpha, c a_\alpha$), for $u$-quarks and neutrinos are expressible with the masses $m_{a1}, m_{a2}$ of the first two families of $u$-quarks or neutrinos and the corresponding angles of rotations as follows

\[
A^a_\alpha = \begin{pmatrix}
  \frac{1}{2}(m_{a1} + m_{a2} - \frac{a \eta_\alpha(m_{a2} - m_{a1})}{\sqrt{1 + (a \eta_\alpha)^2}}), & \frac{m_{a2} - m_{a1}}{2\sqrt{1 + (a \eta_\alpha)^2}} \\
  \frac{m_{a2} - m_{a1}}{2\sqrt{1 + (a \eta_\alpha)^2}}, & \frac{1}{2}(m_{a1} + m_{a2} + \frac{a \eta_\alpha(m_{a2} - m_{a1})}{\sqrt{1 + (a \eta_\alpha)^2}})
\end{pmatrix},
\]  

(29)
while the expressions for the matrix $A_{a}^{b}$ with matrix elements $a_{\alpha}, b_{\alpha}, c_{\alpha}, d_{\alpha}$ follow, if we replace $m_{\alpha 1}$ with $m_{\alpha 3}$ and $m_{\alpha 2}$ with $m_{\alpha 4}$. Equivalently we obtain the mass matrices for the $d$-quarks and the electrons by replacing $\alpha$ by $\beta$ in all expressions. Eq.(29) below demonstrates that if once the three parameters $k_{\alpha}, a_{\eta_{\alpha}}, b_{\eta_{\alpha}}$ are chosen to fit the experimental data, any four masses for the fourth family of quarks and leptons agree with the proposed requirements.

The starting mass matrices $M_{\alpha, \beta}$ - the Yukawa couplings - for quarks and leptons, which are 4 by 4 matrices and are the generalized versions of Table I and Table II are expressible with the matrices $A_{a,\beta}^{a,b}$ of Eq.(29) as follows

$$
\begin{pmatrix}
\frac{1}{2}[A_{a,\beta}^{a,b} + A_{a,\beta}^{b,a}] - (A_{a,\beta}^{b,a} - A_{a,\beta}^{a,b}) \frac{k_{a,b}}{2 \sqrt{1 + (k_{a,b})^2}}),
(A_{a,\beta}^{b,a} - A_{a,\beta}^{a,b}) \frac{1}{2 \sqrt{1 + (k_{a,b})^2}}),
\end{pmatrix}
$$

One easily sees that the matrix $M_{\alpha, \beta}$ is equal to a democratic matrix with all the elements equal to $m_{\alpha i}/4$, with $\alpha = u, \nu, d, e$, if all the angles of rotations are equal to $\pi/4$ (that is for $k_{\alpha} = 0, a_{b,\eta_{\alpha}} = 0$), while $m_{\alpha, \beta i} = 0, i = 1, 3$, and that the mixing matrices are then the identity.

Once knowing the matrices $M_{\alpha, \beta}$, one easily derives the parameters $\tilde{\omega}_{abc,a,\beta}$, with $(abc)$ equal to (018), (078), (127), (187), (387), which (in our generalized version) enter into Table I and Table II. One namely finds

$$
\tilde{\omega}_{018a} = \frac{1}{2} \left[ \frac{m_{\alpha 2} - m_{\alpha 1}}{\sqrt{1 + (a_{\eta_{\alpha}})^2}} + \frac{m_{\alpha 4} - m_{\alpha 3}}{\sqrt{1 + (b_{\eta_{\alpha}})^2}} \right],
\tilde{\omega}_{078a} = \frac{1}{2} \left[ \frac{a_{\eta_{\alpha}} (m_{\alpha 2} - m_{\alpha 1})}{\sqrt{1 + (a_{\eta_{\alpha}})^2}} - \frac{b_{\eta_{\alpha}} (m_{\alpha 4} - m_{\alpha 3})}{\sqrt{1 + (b_{\eta_{\alpha}})^2}} \right],
\tilde{\omega}_{127a} = \frac{1}{2} \left[ \frac{a_{\eta_{\alpha}} (m_{\alpha 2} - m_{\alpha 1})}{\sqrt{1 + (a_{\eta_{\alpha}})^2}} + \frac{b_{\eta_{\alpha}} (m_{\alpha 4} - m_{\alpha 3})}{\sqrt{1 + (b_{\eta_{\alpha}})^2}} \right],
\tilde{\omega}_{187a} = \frac{1}{2} \left[ \frac{-m_{\alpha 2} - m_{\alpha 1}}{\sqrt{1 + (a_{\eta_{\alpha}})^2}} + \frac{m_{\alpha 4} - m_{\alpha 3}}{\sqrt{1 + (b_{\eta_{\alpha}})^2}} \right],
\tilde{\omega}_{387a} = \frac{1}{2} \left[ \frac{(m_{\alpha 4} + m_{\alpha 3}) - (m_{\alpha 2} + m_{\alpha 1})}{\sqrt{1 + (a_{\eta_{\alpha}})^2}} \right],
\alpha_{a} = \frac{1}{2} (m_{\alpha 1} + m_{\alpha 2} - \frac{a_{\eta_{\alpha}} (m_{\alpha 2} - m_{\alpha 1})}{\sqrt{1 + (a_{\eta_{\alpha}})^2}}),
$$

(29)
VI. NUMERICAL RESULTS

In this section we connect parameters $\bar{\omega}_{abc}$ of the Yukawa couplings following from our approach unifying spins and charges (after several assumptions, approximations and simplifications) with the experimental data. We investigate, how the parameters of the approach reflect the known data. We also investigate a possibility of making some predictions.

A. Experimental data for quarks and leptons

We present in this subsection those experimental data, which are relevant for our study: that is the measured values for the masses of the three families of quarks and leptons and the measured mixing matrices.

We take in our calculations the experimental masses for the known three families from the ref. 19.

$$m_u/GeV = (0.0015 - 0.004, 1.15 - 1.35, 174.3 - 178.1),$$
$$m_d/GeV = (0.004 - 0.008, 0.08 - 0.13, 4.1 - 4.9),$$
$$m_{\nu}/GeV = (1 \times 10^{-12}, 1 \times 10^{-11}, 5 \times 10^{-11}),$$
$$m_{e_i}/GeV = (0.0005, 0.105, 1.8).$$ (30)

Predicting four families of quarks and leptons at "physical" energies, we require the unitarity condition for the mixing matrices for four rather than three measured families of quarks 19

$$\begin{pmatrix}
0.9730 - 0.9746 & 0.2174 - 0.2241 & 0.0030 - 0.0044 \\
0.213 - 0.226 & 0.968 - 0.975 & 0.039 - 0.044 \\
0.0 - 0.08 & 0.0 - 0.11 & 0.07 - 0.9993
\end{pmatrix}. \quad (31)$$

The experimental data are for the mixing matrix for leptons known very weakly 20

$$\begin{pmatrix}
0.79 - 0.88 & 0.47 - 0.61 & < 0.20 \\
0.19 - 0.52 & 0.42 - 0.73 & 0.58 - 0.82 \\
0.20 - 0.53 & 0.44 - 0.74 & 0.56 - 0.81
\end{pmatrix}. \quad (32)$$

We see that within the experimental accuracy both mixing matrices - for quarks and leptons - may be assumed to be symmetric up to a sign. We then fit with these two matrices the six parameters $k_\alpha, a_\eta_\alpha, b_\eta_\alpha$, $\alpha = u, \nu$. 

20
B. Results

We started with the explicit expressions for the Yukawa couplings suggested by the approach unifying spins and charges and made several assumptions and approximations, also simplifications, in order to be able to make some approximate predictions. Since we do not know the way of breaking symmetries for either the Poincaré group or for the group defining families and how breaking of these two kinds of symmetries is connected and to which properties of the system would they lead - and whether they would or not accordingly support the assumptions and approximations we made - we proceeded in two steps. The first step brought us to strongly related mass matrices for quarks and leptons, suggesting that the fourth family of quarks and leptons lies very high. In the second step we keep those symmetries of the mass matrices suggested by the first step, which lead to correlated rotations for the $u$-quarks and the $d$-quarks on one side and for the neutrinos and the electrons on the other side, but allow that $\tilde{\omega}_{abc}$ fields might not be the same for the $u$ and $\nu$ and $d$ and $e$, hoping that a kind of breaking symmetries with special boundary conditions and nonperturbative effects might effect these fields in the assumed way.

We can now connect the parameters of the approach (left after several approximations and assumptions) with the experimental data and try to find out what can we learn from the corresponding results. As we have said: Any choice for the masses of the fourth family fits the experimental data, once (twice) the three angles of the orthonormal transformations, determining the (two) mixing matrices are chosen.

We fit (twice) the three angles of Eqs. (14, 22) with the Monte-Carlo method under the requirement that the ratios of the parameters $\tilde{\omega}_{abc}$, entering into mass matrices, are so close to a rational number as possible. This requirement is made in order to see whether some kind of symmetry might be responsible for the difference in properties of quarks and leptons.

We allow the masses of the fourth family as follows: The two quark masses must lie in the range from 200 GeV to 1 TeV, the fourth neutrino mass must be within the interval $50 - 100$ GeV and of the fourth electron mass within $50 - 200$ GeV.

Fig. 1 shows the results of the Monte-Carlo simulation for the three angles determining the mixing matrix for quarks. There are the experimental inaccuracies, which determine the allowed regions for the three angles.

The results for the quarks are presented on Table IV and V (together with the corre-
FIG. 1: Figure shows the Monte-Carlo fit of the experimental mixing matrix for quarks (Eq. 31) with the three angles of Eq. (29). The three angles define the three parameters $k_u$, $a_\eta_u$ and $b_\eta_u$ (Eqs. 14, 20). We make a choice among those values for the best fit, which makes the ratios $\tilde{\omega}_{abcu}/\tilde{\omega}_{abcd}$ as close to rational numbers as possible while assuring that the masses of the three known families stay within the acceptable values from Eq. (30), with no constraints on $a_\alpha$ and the two quark masses of the fourth family lie in the range 200 – 1000 MeV.

Fig. 2 shows the Monte-Carlo fit for the three angles determining the mixing matrix for leptons. There are the experimental inaccuracies which limit the values of the three angles. Again we make a choice among those values for the best fit, which make the ratios $\tilde{\omega}_{abcu}/\tilde{\omega}_{abcd}$ as close to rational numbers as possible. Since in the lepton case the mixing matrix for the three known families as well as the masses for the three neutrinos are weakly known, the calculations for four families bring much less information than in the quark case.

The results for the leptons are presented together with the results for the quarks on Table IV and V.

In Eq. (33) we present masses for the four families of quarks and leptons as obtained after the Monte-Carlo fit

$$m_{ui}/GeV = (0.0034, 1.15, 176.5, 285.2),$$
FIG. 2: Figure shows the Monte-Carlo fit of the experimental data for the mixing matrix for leptons (Eq. (32)). The three angles define the three parameters $k_\nu$, $a_\eta$, and $b_\eta$ (Eqs. (14), (20)). Again we make a choice among those values for the best fit, which make the ratios $\tilde{\omega}_{abcu}/\tilde{\omega}_{abcd}$ as close to rational numbers as possible.

TABLE IV: The Monte-Carlo fit to the experimental data [19, 20] for the three parameters $k$, $a_\eta$, and $b_\eta$ determining the mixing matrices for the four families of quarks and leptons are presented.

|     | u   | d   | $\nu$ | e   |
|-----|-----|-----|-------|-----|
| $k$ | -0.085 | 0.085 | -1.254 | 1.254 |
| $a_\eta$ | -0.229 | 0.229 | 1.584 | -1.584 |
| $b_\eta$ | 0.420 | -0.440 | -0.162 | 0.162 |

The results of the Monte-Carlo fit shows that the requirement, that the ratios of the corresponding parameters of $\tilde{\omega}_{abc}$ for the quarks and the leptons should be as close to the rational numbers as possible, makes that the fourth family lies within the experimentally allowed values as evaluated by the refs. [21, 22, 23]. Eq. (29), however, tells us, that it is the...
| $\tilde{\omega}$ | $u$  | $d$  | $u/d$ | $\nu$ | $e$  | $\nu/e$ |
|----------|-----|-----|------|-----|-----|--------|
| $\tilde{\omega}_{018}$ | 21205 | 42547 | 0.498 | 10729 | 21343 | 0.503 |
| $\tilde{\omega}_{078}$ | 49536 | 101042 | 0.490 | 31846 | 63201 | 0.504 |
| $\tilde{\omega}_{127}$ | 50700 | 101239 | 0.501 | 37489 | 74461 | 0.503 |
| $\tilde{\omega}_{187}$ | 20930 | 42485 | 0.493 | 9113 | 18075 | 0.505 |
| $\tilde{\omega}_{387}$ | 230055 | 114042 | 2.017 | 33124 | 67229 | 0.493 |
| $a^a$ | 94174 | 6237 | 1149 | 1142 |

TABLE V: Values for the parameters $\tilde{\omega}_{abc}$ (entering into the mass matrices for the $u$–quarks, the $d$–quarks, the neutrinos and the electrons, as suggested by the approach) as following after the Monte-Carlo fit, relating the parameters and the experimental data.

top mass and the masses of the fourth family which mostly (not entirely) determine these ratios. But integer or half integer ratios could still be a sign that some group properties or even nonperturbative effects connected with the charges of quarks and leptons determine the masses of fermions, since if we move the masses from those allowed by the refs. [21, 22, 23], the ratios go to one only when all the masses of the fourth family are equal and are high in comparison with the top mass.

The Monte-Carlo fit leads to the following mixing matrix for the quarks

$$
\begin{pmatrix}
0.974 & 0.223 & 0.004 & 0.042 \\
0.223 & 0.974 & 0.042 & 0.004 \\
0.004 & 0.042 & 0.921 & 0.387 \\
0.042 & 0.004 & 0.387 & 0.921 \\
\end{pmatrix}
$$

(34)

and for the leptons

$$
\begin{pmatrix}
0.697 & 0.486 & 0.177 & 0.497 \\
0.486 & 0.697 & 0.497 & 0.177 \\
0.177 & 0.497 & 0.817 & 0.234 \\
0.497 & 0.177 & 0.234 & 0.817 \\
\end{pmatrix}
$$

(35)

The estimated mixing matrix for the four families of quarks predicts quite a strong couplings between the fourth and the other three families, limiting (due to the assumptions and approximations we made, which manifest in the symmetric mixing matrices) some of the matrix elements of the three families as well.
The estimated mixing matrix for the four families of leptons predicts very probably far too strong couplings between the known three and the fourth family (although they are not in contradiction with the report in [19]).

Let us end up this section by repeating that all the predictions must be taken as a very rough estimation, since they follow from the approach unifying spins and charges after many approximations and assumptions, which we made to be able to come in quite a short way to simple and transparent predictions.

VII. DISCUSSIONS AND CONCLUSIONS

In this paper and in the previous one [11] we study a possibility that the approach of one of us [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], unifying spins and charges, might be a new right way for answering those of the open questions of the Standard model of the electroweak and colour interaction, which are connected with the appearance of families of fermions, of the Yukawa couplings and of the weak scale: Why do only the left handed spinors carry the weak charge, while the right handed are weak chargeless? Where do the families of the quarks and the leptons come from? What does determine the strenghts of the Yukawa couplings and the weak scale?

Within the approach unifying spins and charges the answer to the question, why only the left handed spinors carry the weak charge, while the right handed are weak chargeless, does exist: The representation of one Weyl spinor of the group SO(1,13), analyzed with respect to the properties of the subgroups SO(1,7)x SU(3)xU(1) of this group and further with respect to SU(2) and the second U(1), manifests the left handed weak charged quarks and leptons and the right handed weak chargeless quarks and leptons.

The approach answers as well the question about a possible origin of the ”dressing” of the right handed quarks and leptons in the Standard model: The approach proposes the Lagrange density for fermions in $d(= 1 + 13)$-dimensional space in which the gauge field of the Poincaré group is the only interaction through spin connections and vielbeins. It is a part of the spin connection field, which connects the right handed weak chargeless spinors with the left handed weak charged ones, playing the role of the Higgs field (and the Yukawa couplings within a family) of the Standard model.

The approach is answering also the question about the origin of the families of quarks and
leptons: Two kinds of the Clifford algebra objects gauging two kinds of the spin connection fields, are assumed. One kind takes care of the spin and the charges and of connecting right handed weak chargeless fermions with left handed weak charged fermions. The other kind takes care of the families of fermions and consequently of the Yukawa couplings among the families contributing also to the diagonal elements. In the previous paper we derived from the approach - after making several approximations, assumptions and simplifications - the expressions for the Yukawa couplings for four families of quarks and leptons. Approximations, assumptions and simplifications lead to very simple expressions for the mass matrices for the four families of quarks and leptons in terms of the spin connection fields of the two kinds.

The approximate break of the symmetry - from $SO(1 + 5)$ to $SU(2) \times SU(2) \times U(1)$ in the $\tilde{S}$ sector - suggests that three angles might in quite a good approximation determine the mixing matrices for the four families of quarks and leptons. We use this suggestion to simplify further estimations. We must, however, add that an approximate break of the symmetry from $SO(1 + 5)$ to $SU(3) \times U(1)$ instead would suggest that the fourth family is very weakly coupled to the first three and would accordingly strongly change our - very preliminary - results. (While such a break seems to be even acceptable when describing properties of leptons, it would predict for quarks much too strong couplings between the third and the first two families than they measured.)

Not knowing the way of breaking symmetries from $SO(1,13)$ to the observed ones for any of the two types of the symmetries (the Poincaré one and the one connected with the generators, $\tilde{S}^{ab}$), we could only guess it through assumptions which do not contradict the experimental data and by treating the breaking in both sectors ($S^{ab}$ and $\tilde{S}^{ab}$) equivalently as much as possible. We assume that effects like the breaking of symmetries or nonperturbative effects might be responsible for the difference in the nondiagonal matrix elements of the Yukawa couplings, while in the diagonal ones the difference in matrix elements originates also in the difference in the quantum numbers carried by quarks and leptons.

We treat quarks and leptons equivalently and did not take into account a possible existence of the Majorana neutrinos: all the masses are the Dirac masses.

We make in this paper a rough prediction of the properties of the fourth family for quarks and leptons by connecting the parameter of our approach with the experimental data. We fix the masses of the fourth family by requiring that the ratios of the corresponding parameters
of the approach for quarks and leptons are as close to the rational numbers as possible. We get numbers like $\frac{1}{2}$ or 1 for these ratios and let for further studies to better understand the influence of the way of breaking symmetries and of the nonperturbative (or perturbative) effects on the properties of families at ”physical energies”.

Our rough estimation of the properties of the fourth family agrees with the analyses of refs. [21, 22, 23] and it predicts the fourth family masses $m_{u4} = 285 \text{ GeV}$, $m_{d4} = 224 \text{ GeV}$, $m_{\nu_4} = 65 \text{ GeV}$, $m_{e4} = 129 \text{ GeV}$. The mixing matrices are in our rough prediction symmetric since mass matrices are assumed to be symmetric and real. Predictions for the couplings between the fourth and the other three families seem reasonable for quarks, while for leptons the corresponding mixing matrix elements might suggest that either different break of symmetries in the $\tilde{S}^{ab}$ sector from the assumed one, or the Majorana neutrinos, or both effects should at least be further studied.

To try to answer within the approach unifying spins and charges the open question of the Standard model: Why the weak scale appears as it does? a more detailed study of the breaks of symmetries in both sectors is needed.

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[29] Latin indices $a, b, ..., m, n, ..., s, t, ...$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, ..., \mu, \nu, ..., \sigma, \tau, ...$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ...$ and $\alpha, \beta, \gamma, ...$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ...$ and $\mu, \nu, ...$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ...$ and $\sigma, \tau, ...$). We assume the signature $\eta^{ab} = \text{diag}\{1, -1, -1, \cdots, -1\}$. 