Simulation of long waves in a layered Sabang waters

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Abstract. Long waves in the waters of Sabang are simulated with a 1-dimensional layered model. The model is discretized with Δx = 3 seconds or approximately 92.5 meters and Δz = 15 meters. The time interval (Δt) is determined based on the CFL criteria. Depth data is obtained from SRTM15 data. The motivation of this research is to study the dynamics of long wave and the dynamics of seawater layer interface which has various density. As the input, it is given the sinusoidal wave with amplitude 2 m and 12 hour period in the open boundary. From the simulation results it is known that the displacement of the interface is about -11 meters to 1.1 meters, while the horizontal current speed is about -1 cm/s to 3 cm/s.

1. Introduction
Sabang waters are at coordinates 95° 13’ 02”- 95° 22’ 36” E, 05° 46’ 28”- 05° 54’ - 28” N which is directly adjacent to the Indian Ocean, Andaman Sea, Malacca Strait and Sumatra Island [1, 2, 3, 4, 5]. Thus the dynamics of Sabang waters are influenced by the dynamics of the Indian Ocean, Andaman Sea, Malacca Strait. There are interesting things that need to be studied in the Sabang waters, namely the internal waves. Internal waves are formed as a result of encounters between layers of seawater that have different densities with the force of the generator that can come from wind, tides or even the movements of ships.

Internal waves can only be detected by observing or measuring directly on the pycnocline (the layer where the density of seawater changes rapidly with thickness) or thermocline (the layer at which sea water temperature changes rapidly with thickness) using temperature measurement sensors and seawater salinity, and current velocity. Visually, internal waves can be seen if carried out in a laboratory experiment or observed from air or space using remote sensing technology. There are similarities between diagnostics calculated from numerical models and linear ray search, the use of linear theory can support to understand the fundamental dynamics of scattering internal waves on canyon [6].

The development of the analytical model using perturbation theory method to be applied in studying internal waves and waves by using the Boussinesq equation that relies on the amplitude and growth rate and internal wave attenuation on important parameters in a two-fluid system, namely the angle of direction of the internal wave ratio, depth, density and viscosity of the fluid layer [7].

This research is oriented to physical processes, with the aim of modeling long waves in Sabang waters which are directly adjacent to the Andaman Sea and the Indian Ocean. The hydrodynamic model used is a 1-D model of seawater dynamics [8]. In this study, the wave profile component will be simulated so that the wave dynamics that occurs can be observed and studied.
2. Material and Methods

The bathymetric data sampled from the SRTM15 data (Shuttle Radar Topography Mission) were interpolated according to the discrete model [9]. The model is discretized with $\Delta x = 3$ seconds or 92.5 meters, $\Delta z = 15$ meters, and $\Delta t = 1$ second. This model is derived from the finite difference of long wave equations in layered fluids [8]:

$$\frac{q}{\partial t} = -\frac{1}{\rho_i} \frac{\partial P_i}{\partial x}$$

where the horizontal speed, density, and dynamic pressure of each layer ($i = 1, 2, 3, ..., $ the last layer) are marked with $u_i$, $\rho_i$, and $P_i$, respectively.

The dynamic pressure of each layer is determined by the sum of the following hydrostatic pressures:

$$P_i = P_{i-1} + (\rho_i - \rho_{i-1}) g \eta_i$$

where the pressure in the first layer is $P_{i=1} = \rho_1 g \eta_1$, while in the atmospheric layer it is zero ($P_{i=0} = 0$). The $P_i$ pressure moves or increases from the first layer to the bottom ($i = 1, 2, 3, ..., $ the last layer). Displacement of the free interface or sea level is indicated by $\eta_i$ and gravitational acceleration with $g$.

The volume conservation in each layer is stated as follows:

$$\frac{\partial h_i}{\partial t} = -\frac{\partial (u_i h_i)}{\partial x}$$

while the thickness of each layer is as follows:

$$h_i = h_{i,0} + \eta_i - \eta_{i+1}$$

where $h_{i,0}$ is the thickness of the undisturbed fluid in the initial conditions. In the seafloor layer $\eta_{n+1} = 0$.

This equation can be written in general form:

$$\frac{\partial \eta_i}{\partial t} = -\frac{\partial (u_i h_i)}{\partial x} + \frac{\partial \eta_{i+1}}{\partial t}$$
Figure 1. (a) Research location and Sabang Bay bathymetry (meters). The black line indicates the model transect, (b) The vertical transect (meter). The transect location of the model 5.75°N - 5.82°N and 95.3833°E - 95.3458°E.

To observe current dynamics and interface displacements, fluid models are given a disturbance of a sinusoidal wave (amplitude = 2 meters and period = 12 hours) and the stable frequency of \( N = 5 \times 10^{-5} \) [10, 11]. The total of time simulation is 120 hours, and the results are stored for the analysis every 3 hours, 60 hours, and 120 hours.

3. Results and Discussion
Figures 2 a, b, c shows the current velocity after 3 hours, 60 hours, and 120 hours of simulation. We can observe that the maximum current velocity is around sandbar (at point x = 15-20). Because the wave first hits the sandbar, a maximum shoaling and current velocity near the base is formed. In general, the current speed continues to increase up to 6 hours of simulation time. Then the current speed is stable again until the simulation is complete (120 hours). The direction of the current can move east or west but at 3 hours, 60 hours, and 120 hours the direction of the dominant current moves toward the east. To show shoaling forming in the sandbar, we have sampled the depth value \( h_i + \eta_i \) every 3 hours, 6 hours, and 120 hours. Table 1 is a description of depth interface thickness changes and its velocity. A quite striking change is seen in layers 4-6 or (60-90 meters) where sandbar is located. We make an agreement that the velocity of the current towards the land is positive (+) and velocity towards the waters or left of domain is negative value (-) in the southeast of Sabang waters. The highest current velocity at a depth of 75 m with a value of 3.7 cm/s occurs at the 5th layer, and the lowest current velocity is at the first layer at a depth of 15 m and at the 11th, 13th and 15th layers located at a depth of 150 m, 180 m, and 210 m, respectively (Figure 2).

Table 1 shows the displacement of interface and current velocity for a 12 hour period with a sinusoidal amplitude of 2 meters. The maximum current velocity occurs at the 5th layer. Changes of maximum layer thickness occur at 3 hours, namely at the 5th layer, with a value of 0.07 m, while at 60 hours the highest value is at the 5th layer with a value of 2.1 m, while at 120 hours the highest value is at the 5th layer, namely 2.09 m.
Figure 2. Horizontal current velocity after (a) 3 hours, (b) 60 hours, and (c) 120 hours of the time of simulation (cm/s).
Table 1. Value of interface thickness changes \( (h_i + \eta_i) \) during numerical simulations around sandbar (point \( x = 15 \) or 1387.5 meters from the land)

| Layers | Change of thickness (m) | Displacement of interface (m) | Current velocity (cm/s) |
|--------|-------------------------|-------------------------------|-------------------------|
|        | 3 hours                 | 60 hours                      | 120 hours               |
|        | 3 hours                 | 60 hours                      | 120 hours               |
|        | 3 hours                 | 60 hours                      | 120 hours               |
| 1      | 15.00                   | 14.88                         | 15.04                   |
|        | 0.00                    | 0.08                          | 0.08                    |
|        | 0.21                    | 0.48                          | 0.46                    |
| 2      | 15.00                   | 14.70                         | 15.13                   |
|        | -0.01                   | 0.33                          | 0.32                    |
|        | 0.44                    | 1.00                          | 1.00                    |
| 3      | 14.99                   | 14.95                         | 14.80                   |
|        | -0.03                   | -0.30                         | -0.30                   |
|        | 0.70                    | 1.75                          | 1.78                    |
| 4      | 14.97                   | 15.21                         | 14.29                   |
|        | -0.05                   | -1.12                         | -1.13                   |
|        | 1.01                    | 2.78                          | 2.78                    |
| 5      | 14.93                   | 16.05                         | 13.75                   |
|        | -0.07                   | -2.10                         | -2.09                   |
|        | 1.40                    | 3.67                          | 3.72                    |
| 6      | 14.32                   | 14.51                         | 13.70                   |
|        | 1.96                    | 1.13                          | 1.13                    |
|        | 1.93                    | 0.28                          | 0.18                    |

4. Conclusion

Based on the model simulation results, the maximum current velocity is around the sandbar. This is consistent with the theory, i.e., when the wave approaches the sandbar, the wave height can increase and break.

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