A Comparative Investigation of Magnetoelectric Characteristics of Nano Bi-Layer Bar and Plate Structures

T Saengow and R Silapunt

Department of Electronic and Telecommunication Engineering, King Mongkut’s University of Technology Thonburi Bangkok, Thailand
Email: treetep.marksaengow@mail.kmutt.ac.th

Abstract. In this paper, the magnetoelectric (ME) characteristics of bar and plate structures of the nano bi-layer ME materials are investigated and compared. The mathematical models of ME coefficients of the nano bi-layer bar and plate structures operated in the longitudinal-transverse (L-T) mode are developed using the constitutive equations and Newton’s second law. The nano bi-layer of interest consists of Terfenol-D and Lead Zirconate Titanate (PZT) as ferromagnetic (FM) and ferroelectric (FE) phases, respectively. In the low frequency regime, the optimal thickness ratios of bar and plate structures that yield maximum ME coefficients are around 0.43 and 0.33, respectively. The optimal thickness ratios are applied in the high frequency regime to obtain the conditions where the maximum ME coefficients can be achieved. It is found that the resonant frequencies of the bar and plate structures decrease exponentially with the bi-layer length and thickness, respectively. The results also indicate that at the same resonant frequency, the dimension of the bar structure is significantly smaller than the plate structure, it thus becomes more preferable for nano scale sensing applications.

1. Introduction

Magnetoelectric (ME) material is one of multiferroic (MF) composites that combine ferromagnetism and ferroelectricity phenomena and exhibits the ME effect, the reversible coupling between the magnetic and electric properties of a material [1-2]. The magnetoelectric multiferroics have been extensively researched and proved that they have tremendous potential for electric and magnetic sensing applications [2-3]. The sensing capability is typically measured by the ME coefficient, which is the ratio of the electric (magnetic) field output to the electric (magnetic) field input. In order to achieve the optimal ME coefficient, types of sensing materials and their properties, operating frequency, and sensor geometries are all important factors. Most of the bi-layer, tri-layer and multilayer ME materials are studied in the bar structure, which defines the strain deformation only in the longitudinal direction [4-7]. Until [8], the first report of the bi-layer plate structure where the strain deformation in the thickness direction was published. However, the comparison between the two structures has never been reported.

This paper aims to investigate and compare the ME characteristics of the nano bi-layer bar and plate structures. Note that, the nano bi-layer ME materials are proposed here as they have become increasingly attractive for nano scale sensing applications. It is also important to point out that a desirable film sensor such as ones depicted in Figures 1-2, is commonly designed by following the geometric rule that is $l >> t, w$, where $l$ is length, $t$ is thickness, and $w$ is width [4]. The mathematical model of ME coefficient
for the longitudinal-transverse (L-T) mode bar structure will first be developed based on the constitutive equations and Newton’s second law. The mathematical model of ME coefficient for the L-T mode plate structure is obtained from the previous report [8]. Then, in the demonstration of ME effects, Terfenol-D and PZT are selected as the nano bi-layer ME material, owing to their good magnetostrictive and piezoelectric properties. The ME coefficient profiles as a function of thickness ratios will be determined in the low frequency regime for both structures. The optimal thickness ratios will then be used as a baseline to determine the conditions to achieve maximum ME coefficients in a typical high frequency operation. Finally, the relation between resonant frequency and bi-layer geometry will be evaluated.

2. ME Coefficients Derivation

The magnetostrictive constitutive equations are used for the FM layer and piezoelectric constitutive equations are used for the FE layer [7-8]. Both constitutive equations are linked by the Newton’s second law via mechanical coupling. The L-T mode bar and plate structures are shown in Figures 1-2, respectively.

![Figure 1. The L-T mode bar structure.](image1)

![Figure 2. The L-T mode plate structure.](image2)

2.1. Coupling between the FM and FE phases using the Newton’s second law

In order to combine the FM phase and FE phase, the Newton’s second law with the elastic displacement, $\xi_j$, is applied. For bar and plate structures, the displacements occur in the longitudinal direction, $\xi_1$, and thickness direction, $\xi_3$, respectively. The results are shown in equations (1) and (2) for bar and plate structures, respectively.

$$\eta \frac{d\sigma_1^e}{dx} + (1-\eta) \frac{d\sigma_1^m}{dx} = \left[ \eta \rho_e + (1-\eta) \rho_m \right] \frac{\partial^2 \xi_1}{\partial t^2}$$

(1)

$$\frac{d\sigma_3^e}{dz} + \frac{d\sigma_3^m}{dz} = \left[ \eta \rho_e + (1-\eta) \rho_m \right] \frac{\partial^2 \xi_3}{\partial t^2}$$

(2)

where $\sigma$ is strain, $\rho$ is the material density, and $\eta$ is the thickness ratio and is defined by $\eta = \frac{t_e}{t_e + t_m}$.

$t_e$ and $t_m$ are the thickness of FE layer and FM layer respectively. Note that, the superscripts ‘$m$’ and ‘$e$’ are referred to FM layer and FE layer, respectively. The subscripts ‘1’ is the longitudinal axis, and the subscript ‘3’ is the thickness direction.

Equations (1) and (2) can be rewritten in the time domain as

$$\frac{\partial^2 \xi_1}{\partial x^2} + k^2 \xi_1 = 0$$

(3)

$$\frac{\partial^2 \xi_3}{\partial z^2} + k^2 \xi_3 = 0$$

(4)
where \( k^2 = \omega^2 \frac{\eta \rho_e - (1-\eta)\rho_m}{s_{11}^m (1-\eta) + s_{11}^e \eta} \) and \( k^2 = \omega^2 \frac{\eta \rho_e + (1-\eta)\rho_m}{s_{33}^m (1-\eta) + s_{33}^e \eta} \) for equations (3) and (4), respectively. \( \xi_1 = A \cos(kx) + B \sin(kx) \), \( \xi_3 = A \cos(kz) + B \sin(kz) \), and \( \omega \) is the angular frequency.

2.2. ME coefficient expressions

2.2.1. Bar structure

From the constitutive equations, the stresses can be determined as shown in equations (5) and (6).

\[
\sigma_i^m = \frac{[Ak \cos(kx) - Bk \sin(kx)]}{s_{11}^m} - \frac{d_{11}^m H_1}{s_{33}^m} \tag{5}
\]

\[
\sigma_i^e = \frac{[Ak \cos(kx) - Bk \sin(kx)]}{s_{11}^e} - \frac{d_{31}^e E_3}{s_{33}^e} \tag{6}
\]

Using the boundary condition at \( x = \frac{l}{2} \) and \( x = -\frac{l}{2} \), with no external force, the coefficients \( A \) and \( B \) can be solved and obtained as \( A = \frac{(1-\eta) s_{11}^e d_{11}^e H_1 + \eta s_{33}^m d_{31}^e E_3}{k \left[ (1-\eta) s_{11}^e + \eta s_{33}^m \right] \cos(kt)} \) and \( B = 0 \).

Equation (6) is then applied to the constitutive equations to calculate the current \( I \) flow through the sensor from \( I = \frac{dQ}{dt} \) and \( D = \frac{dQ}{DA} \), where \( dA \) is the differential area element through which the current flows.

In order to find the ME coefficient, the relation of electric field across the FE layer and magnetic field along the FM layer are solved by applying the open-circuit condition \( (I = 0) \). The ME coefficient of the bar structure is shown in equation (7).

\[
\alpha_{31} = \frac{(1-\eta) s_{11}^e d_{31}^e d_{11}^m}{d_{31}^e \left[ (1-\eta) s_{11}^e + \eta s_{33}^m \right] - \frac{\tan(kl/2)}{kl/2}} \tag{7}
\]

It can be seen that the ME coefficient of the bar structure is varied by \( \eta \) and \( kl \) that are a function of thickness, sensor length, and operating frequency.

2.2.2. Plate structure

Referred to the previous report [8], the ME coefficient of the plate structure is shown in equation (8).

\[
\alpha_{31} = \frac{\eta s_{33}^m d_{33}^e d_{31}^m \sin(kt)}{d_{33}^e \sin(k\eta - kt) - \left[ d_{33}^e - s_{33}^e \right] \sin(kt)} \left[ \eta s_{33}^m - (1-\eta) s_{33}^e \right] + \eta s_{33}^m d_{33}^e \sin(kt) \tag{8}
\]

It can be seen that the ME coefficient of the plate structure is varied by \( \eta \) and \( kt \) that are a function of thickness and operating frequency.

3. Results and Discussion
3.1. Optimal thickness ratio
In the low frequency regime, the operating frequency is set close to zero or $k \rightarrow 0$. Applying equations (7) and (8) yields the ME coefficients for difference thickness ratios. Figure 3 shows the ME coefficients of the bar structure and Figure 4 shows the ME coefficients of the plate structure.

![Figure 3. ME coefficient vs. thickness ratio for the bar structure.](image1)

![Figure 4. ME coefficient vs thickness ratio for the plate structure.](image2)

The optimal thickness ratios for both bar and plate structures are equal to 0.43 and 0.33, respectively. However, the overall ME coefficient of plate structure is significantly higher than that of the bar structure since the denominator of equation (8), the plate structure model, converges to zero while that of equation (7), the bar structure model, remains constant.

3.2. Geometry dependent frequency characteristics
By applying the optimal thickness ratios to equations (7) and (8), the conditions where the maximum ME coefficients can be achieved are $kl = \pi$ and $kt = 0.854747\pi$ for bar and plate structures, respectively. It is clear that the maximum ME coefficients occur at the resonant frequencies for both bar and plate structures and they are influenced by length $l$ for the bar structure and thickness $t$ for the plate structure. The resonant frequency profiles of the bar and plate structures are plotted as a function of bi-layer geometry as shown in Figures 5 and 6, respectively. It can be seen that the resonant frequencies decrease exponentially with length for the bar structure and with thickness for the plate structure. Although the overall ME coefficient of the plate structure is higher than the bar structure, it is not suitable for current nano sensing applications due to dimension constraint. The results confirm that at the same resonant frequency, the dimension of the bar structure is always smaller than the plate structure [4].

![Figure 5. Resonant frequency vs bi-layer length for the bar structure](image3)

![Figure 6. Resonant frequency vs bi-layer length for the plate structure](image4)

4. Conclusions
The ME coefficient mathematical models are derived in order to investigate and compare the ME characteristics of the nano bi-layer bar and plate structures. The nano bi-layer of interest consists of Terfenol-D and PZT. The calculation in the low frequency regime yields the optimal thickness ratios of 0.43 and 0.33 for bar and plate structures, respectively. Applying the optimal thickness ratios yields the conditions for maximum ME coefficients in high frequency operation that are $k_l = \pi$ and $k_t = 0.854747\pi$ for bar and plate structures, respectively. The relations of resonant frequency as a function of bi-layer lengths and thicknesses of the bar and plate structures respectively, are determined. At the same resonant frequency, the dimension of the bar structure is always smaller, suggesting it as preferable choice for nano sensing applications. Future work includes the study of the nano bi-layer bar structure as the magnetic read head sensing element and its performance when integrated with the additional bias layer and other complementary layers.

5. References

[1] Wang Y, Hu J, Lin Y and Nan Ce W 2010 Multiferroic magnetoelectric composite nanostructures NPG Asia Mater. 2(2) 61-68
[2] Palneedi H, Annapureddy V, Priya S and Ryu J 2016 Status and Perspectives of Multiferroic Magnetoelectric Composite Materials and Applications Actuators 2016 1-31
[3] Kambale R, Jeong D and Ryu J 2012 Current Status of Magnetoelectric Composite Thin/Thick Films Advances in Condensed Matter Physics 2012 15pages
[4] Vopsaroiu M, Blackburn J, Muniz-Piniella A and Cain Markys G 2008 Multiferroic magnetic recording read head technology for 1 Tbit/ in² and beyond J. Appl. Phys. 103 07F506 1-3
[5] Choowitsakunlert S, Satitchantrakul T and Silapunt R 2014 A 1D Analysis of Nano Multiferroic Composites for the Novel Read Head Technology Advanced Materials Research 1052 149-54
[6] Vopsaroiu M, Blackburn J and Cain Markys G 2007 A new magnetic recording read head technology based on the magneto-electric effect J. Phys. D: Appl. Phys. 40 5027-33
[7] Dong S, Li Jie F and Viehland D Theory Analysis on Magnetoelectric Voltage Coefficients of the Terfeneol-D/PZT Composite Transducer Materials Science & Engineering, Virginia Tech, Blacksburg, VA24061
[8] Saengow T, Satitchantrakul T and Silapunt R 2018 Investigation of Magnetoelectric Effect in the Bi-Layer Plate Structure Solid State Phenomena 280 9-14

6. Acknowledgments

This work has been supported by Thailand Research Fund under Research and Researchers for Industries-RRI program with the contract number PHD/58I0004. The authors would like to thank King Mongkut’s University of Technology Thonburi, the University of Portsmouth, and Seagate Technology, Thailand for their equipment, software and any valuable advices.