A Fast Solving Method for Curve Profile of Suspension Bridge Catwalk System in Plane

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Abstract. With increasing demands of in plane suspension bridge for mountainous area of P. R. China in recent year, rapid erection of catwalk system has become the researching hotspot for constructors, wherein curve profile is one of the key factors that matter. In this paper, a nonlinear optimization method is introduced for fast solving curve profile of catwalk system. To achieve the goal, statics fundaments and compliant characteristics of general catenary is firstly analyzed. Structural detailing of catwalk system is then described so that segmentation prior to computation can be made as per loads types or requirements of obtaining finer curve coordinates. In addition, nominal curve profile of catwalk system is derived according to segmentation of main cable, based on which the actual catwalk curve profile solving problem is afterwards formulated as an unconstrained nonlinear programming in terms of external penalty parameter and solved by using Levenberg-Marquardt (L-M) algorithm. Finally, a study case is shown to illustrate the effectiveness of proposed method.

1. Introduction

Catwalk, an indispensable structure for modern construction of suspension bridge, works as the temporary detours or footbridge for builders or other equipment to easily proceed wire wrapping, cable traction and installation into the saddle, etc. on both upstream and downstream sides of the bridge, and will be finally dismantled after completion of construction [1,2]. For accurate erection of main cable, catwalks shall be adaptively controlled as a similar parallel curve to main cables with certain clearance when main cables are in load-free condition before being fixed or anchored. As the transformation of catwalk curve profiles is executed by winch, pulley blocks and locking mechanism amid cable erection period [3], a fast guidance on dragging force, stress-free length, rope selection verification and its combination design, etc. are required [4].

Some researchers have been dedicated for decades on catenary analysis and its applications to in-plane suspension bridges based on cable elasticity theory[5-9], most of their works focus on solution of main cable curve profile and erection force prediction[10,11], wherein iteration algorithm like Newton-Raphson[2,7,12] and linearized computation techniques have been used[13-16]. With the development of computing tools, some other researches in terms of finite element method and indoor experiment [17-21] have been carried out to worked as evaluation measures not only for suspension bridge but all other cable supported ones. Whereas those methods are either accurate or ready-to-hand to some extent for design or verification of catwalk system, practicability for builders is relative less as they also
have time-consuming demerit both for modeling and computation, and most importantly catwalk is a complex catenary structural system that identify itself to simply suspended cables. Compared to researches of main cable, majority works regarding catwalk system are focusing on formulating its erection technology and evaluation of wind resistance capability [22-26] rather than prediction curve profile, for which previous researches either regard rope combinations of catwalk platform and the same hanged on doorframes as a whole bearing structure but actually are separate controlled, or do not take handrails poles, wood transoms and other individually set facilities as point loads but transform them to uniformly distributed loads for computational convenience, or simply considering the objective catwalk curve profile as simply the translation of main cable curve profile that clearance in curve normal direction may vary Therefore, in this paper, we give general descriptions and design requirements of in-plane suspension bridge catwalk system, which is capable of considering all possible loader components as per required computation scale and propose a fast nonlinear optimization method for solving the curve profile by using L-M algorithm[27-29] for builder’s better reference in practice.

2. Analysis of General Catenary

2.1 Fundaments of Catenary.

As main cables and bearing rope combinations of catwalk platform or doorframes are either suspended between tower tops or ground anchoring structural components, the curve profile of which can be regarded as catenaries if only perfectly flexible assumption of cable is applied that action of any part of the line upon its neighbor is purely tangential, we can consider an arbitrary selected part of catenary at \((x, z)\), show in Figure.1, and formulate its static equilibrium equations as:

\[
\frac{d}{ds} \left( T \frac{dz}{ds} \right) = q
\]

(1)

\[
\frac{d}{ds} \left( T \frac{dx}{ds} \right) = 0
\]

(2)

where \(T\) is the tension of cable, \(s\) is the length of cable, \(q\) is the cable weight per unit length. \(dz / ds\) and \(dx / ds\) are the sine and cosine of inclination angle.

As equation (2) may be integrated to

\[
T \frac{dx}{ds} = H
\]

(3)

Equation (1) can be further rewritten as

\[
H \frac{d^2 z}{dx^2} = -q \frac{ds}{dx}
\]

(4)
where $H$ is the constant horizontal component of cable tension $T$, since there is no longitudinal force acting on rope along its curve.

Given that

$$\left( \frac{dx}{ds} \right)^2 + \left( \frac{dz}{ds} \right)^2 = 1$$

then differential equation of rope element may be finalized as

$$H \frac{d^2z}{dx^2} = -q \left\{ 1 + \left( \frac{dz}{ds} \right)^2 \right\}^{1/2}$$

the solution of which can be described in terms of Lagrange coordinate $s$ as follow

$$x(s) = \frac{H}{q} \left\{ sh^{-1} \left( \frac{V}{H} \right) - sh^{-1} \left( \frac{V - qs}{H} \right) \right\}$$

$$z(s) = \frac{H}{q} \left\{ \sqrt{1 + \left( \frac{V}{H} \right)^2} - \sqrt{1 + \left( \frac{V - qs}{H} \right)^2} \right\}$$

where $V$ are the vertical component of rope tension.

In addition, we may regard this arbitrary part as the $i$th segment of cable, $i = 1, 2, \ldots, m$, $m$ is numbers of segments as per requirements of modeling and computation complexity required, we obtain distance components $l_i$ and $h_i$ between the two ends of the $i$th segment in horizontal and vertical directions,

$$l_i = \frac{H_i}{q} \left\{ sh^{-1} \left( \frac{V_i}{H_i} \right) - sh^{-1} \left( \frac{V_i - qs_i}{H_i} \right) \right\}$$

$$h_i = \frac{H_i}{q} \left\{ \sqrt{1 + \left( \frac{V_i}{H_i} \right)^2} - \sqrt{1 + \left( \frac{V_i - qs_i}{H_i} \right)^2} \right\}$$

and further have cable length $s_i$ by rewriting equation (9)

$$s_i = \frac{\left\{ V_i - sh \left[ sh^{-1} \left( \frac{V_i}{H_i} \right) - q \left( \frac{1}{H_i} \right) \right] \right\} H_i}{q}$$

When $|T(s)\cdot EA| < 1$, stress-free length of arbitrary segment of cable can be also described as

$$s_{i0} = s_i - \frac{H_i}{2E(A_0 \epsilon_s)} \left[ \epsilon_s l_i + sh(2\epsilon_s l_i/2) - sh(2\epsilon_s) \right]$$

where $\epsilon_s = q/H_i$, $\epsilon_s = sh^{-1} \left[ h_i \epsilon_s / 2h(\epsilon_s l_i / 2) \right] - \epsilon_s l_i / 2$. $E$ is the elastic modulus, $A_0$ is the equivalent section area of cable. Further, we consider a vertical point load $P_i$ exerted at the end of the $i$th segment, the second consecutive segment $(i+1)$ may have

$$H_{i+1} = H_i = H$$

$$V_{i+1} = V_i - (P_i + q s_i)$$

2.2 Compliant characteristics of catenary.
In order to solve the curve profile of catenary segment, constant updated linearized compliant relationship between tension and curve profile is better required for solution of nonlinear optimization programming.

From equation (9) and equation (10), we have,

$$de_i = \left[ \frac{dl_i}{dh_i} \right] = J_{i,i,H} \left[ \frac{dH_i}{dV_i} \right] = J_i dT_i$$

Where $J_i$ is the Jacobian matrix that represent the linear compliance characteristics of each cable segment, components of which can be described as
3. Detailing of Catwalk System and Computation Preparation of Curve Profile

3.1. Structural Composition.

As a very important bearing structure and transportation platform for main cables erection, see Figure 2, catwalks system generally consists of manipulation platform, doorframes, cross passageways, and is often supported in terms of bearing ropes combinations suspended on tower top or fixed to ground anchoring structures on both ends of bridge span. There are two main bearing rope combinations for erection of catwalk system, one is usually set underneath main cable curve profile with a constant 1.2-1.5m clearance and is together with uniformly arranged bottom and side net surface plates, wood transoms, handrails and its supporting poles to form a workable bench for builders manipulation convenience and passage of wrapping machine, the other is set mainly for force distribution of catwalk system and proper erection of discretely arranged doorframes and cross passageways. Generally, catwalk platform width is accordingly designed as per requirements of work space where 4-5m width is enough for most of the cases unless otherwise required to increase to obtain more stability capacity to counteract the heavy wind effects, in condition of which measures using extra wind cable combinations or increasing numbers of cross passageways shall be taken when wind load prevails.

3.2 Segmentation of Catwalk System.

Unlike main cable in load-free condition, which can be simply regarded as a catenary, two rope combinations of catwalk system bear impacts of uniformly distributed loads from various kind of functioning ropes assembled on and regularly arranged point loads from structural components or

\[ J_{s,0} = \frac{1}{q} \left[ \sinh^{-1} \left( \frac{V_c}{H_c} \right) - \sinh^{-1} \left( \frac{V_r - H_r}{H_r} \right) \right] + \frac{1}{q} \left[ \frac{V_r / H_r}{1 + (V_r / H_r)^2} \right] \left[ \frac{V_r - q_s}{H_r} \right] \frac{1}{\left[ 1 + \left( \frac{V_r - q_s}{H_r} \right)^2 \right]^{1/2}} \right]^{1/2} \]

\[ J_{s,1} = \frac{1}{q} \left[ 1 + \left( \frac{V_r}{H_r} \right)^{1/2} \right]^{1/2} \left[ 1 + \left( \frac{V_r - q_s}{H_r} \right)^{1/2} \right]^{1/2} \right] + \frac{1}{q} \left[ \frac{V_r / H_r}{1 + (V_r / H_r)^2} \right] \left[ \frac{(V_r - q_s) / H_r}{1 + ((V_r - q_s) / H_r)^2} \right]^{1/2} \]

\[ J_{s,2} = \frac{1}{q} \left[ 1 + \left( \frac{V_r}{H_r} \right)^{1/2} \right]^{1/2} \left[ 1 + \left( \frac{V_r - q_s}{H_r} \right)^{1/2} \right]^{1/2} \right] + \frac{1}{q} \left[ \frac{V_r / H_r}{1 + (V_r / H_r)^2} \right] \left[ \frac{(V_r - q_s) / H_r}{1 + ((V_r - q_s) / H_r)^2} \right]^{1/2} \]

\[ J_{s,3} = \frac{1}{q} \left[ 1 + \left( \frac{V_r}{H_r} \right)^{1/2} \right]^{1/2} \left[ 1 + \left( \frac{V_r - q_s}{H_r} \right)^{1/2} \right]^{1/2} \right] + \frac{1}{q} \left[ \frac{V_r / H_r}{1 + (V_r / H_r)^2} \right] \left[ \frac{(V_r - q_s) / H_r}{1 + ((V_r - q_s) / H_r)^2} \right]^{1/2} \]
equipment in a certain form of force distribution at locations of doorframes. Thus, to decouple this complex relationship and to have a clear guidance for iterated solving curve profile, we reciprocally divide main cable and catwalk system along longitudinal direction into \( m \) segments, as per above description of structural compositions, requirements computational complexity or obtaining more refined curve coordinates.

4. Nominal Curve Profile of Catwalk System

4.1 Formation of Main Cable Curve Profile Solving Problem.
As to acquire the full curve profile of catwalk system is the target of this paper, main cable curve profile shall be obtained in prior by using aforementioned catenary analysis as per identical segmentation in bridge longitudinal direction so that each nominal coordinates of catwalk platform can be available by then as computation reference of actual curve profile.

\[
\text{Figure.3 Design of Main Cable in Load-Free Condition.}
\]

As shown in Figure.3, we general consider the middle span for instance, wherein the span length \( l_s \), level difference \( \Delta h = z_{ip1} - z_{ip2} \) between main cable intersection points (IP) on tower saddles, sag values \( \Delta h_{sag} \) at middle point span are given as the design inputs, and assume the segmentation of main cable as \( l_{w0} = [l_1, \ldots, l_m]^{T} \), \( \text{sum}(l_{w0}) = l_s \), where \( l_{w1} \) and \( l_n \) are the horizontal lengths of left and right segments beside middle point respectively. Generally speaking, \( n = m/2 \). Then, we can sequentially compute every curve profile in terms of recursive using equations (9), (10) and equation (13), (14) if tension vector \( T_{ip} = [H_{ip}, V_{ip}]^{T} \) at the staring IP is regarded as the design variables to be solved. Tension at the ending IP \( T_{ip} = [H_{ip}, V_{ip}]^{T} \) may be carried out as well by using equation (13) and (14) one more time. Thus, we can formulate main cable curve profile solving problem as following nonlinear programming problem:

\[
\text{minimize } \Psi(\Omega) \quad (20)
\]

where \( \Psi(\Omega) = [\varphi_1, \varphi_2]^{T} \) is the objective function vector, \( \varphi_1 = \sum_{i=1}^{n} h_i - \Delta h_{sag}, \varphi_2 = \sum_{i=1}^{n} h_i - \Delta h \).

4.2 Solution of nominal catwalk system curve profile.
In case of the main cable curve profile is known, as shown in Figure.4, we may consider the \( i \)th cable segment start from point \( p_i(x_i, z_i) \) for an assumed horizontal length \( l_i \), the vertical clearance to point \( p_i \) on catwalk system shall be

\[
h_i = h_i + C \cos \beta_{i+1} \quad (21)
\]

where \( h_i, \beta_{i+1} \) are corresponding vertical lengths of the \( i \)th segment and inclination angle of the \((i+1)\)th segment which can be solved in terms of given clearance \( C \), previously defined equations (9), (10), (13), (14) and following equations (22) and (23).

\[
\beta_{i+1} = \tan^{-1}\left(\frac{V_{i+1}}{H_{i+1}}\right) \quad (22)
\]

\[
l_i = C \sin \beta_{i+1} \quad (23)
\]
Then, we can recursively obtain all nominal curve profile parameters $\mathbf{l}_{\text{ncat}}$ and $\mathbf{h}_{\text{ncat}}$ of catwalk platform, as well as the rope combinations of doorframes $\mathbf{l}_{\text{ndf}}$ and $\mathbf{h}_{\text{ndf}}$, at corresponding segment point $\mathbf{P}_i\left(\mathbf{x}_i,\mathbf{z}_i\right)$, which are all described in the identical Cartesian coordination system.

$$P_i(\mathbf{x}_i,\mathbf{z}_i)$$

**Figure 4** Solution of Nominal Catwalk Curve Profile. **Figure 5** Sketch of the $j$th Force Distribution

4.3 Force distribution on doorframes. As rope combinations underneath catwalk platform and the same hanged on doorframes simultaneously bear whole loads of catwalk system in an uncertain distributed manner through doorframe structure, please see Figure 5, we assume there are $u$ numbers of doorframes wherein the $j$th interaction between two bearing rope combinations located at the $j$th doorframe to be

$$P_j = P_{pd} + P_{pl} = \left(1 - \lambda_j\right) p_j + \lambda_j p_j$$

(24)

where $p_j$ is sum of the point loads exerted by doorframe or cross passageway at location of the $j$th doorframe, $P_{pd} = \left(1 - \lambda_j\right) p_j$ and $P_{pl} = \lambda_j p_j$ are the force components respectively supported by bearing rope combinations underneath manipulation platform and hanged on the $j$th doorframe, $\lambda_j \in [0,1]$ is the $j$th force distribution factor to be solved.

5. Optimal Solution of Catwalk System Curve Profile

5.1 Formulation of Catwalk System Curve Profile Solving Problem. Using catenary statics, curve profile solutions of main cable and nominal catwalk system, force distributions relationship at each doorframe, we can formulate the catwalk system curve profile solving problem as the following constrained nonlinear programming problem

$$\minimize \Xi(\Omega), \text{ subject to } D: \begin{cases} g_j(\Omega) = 0, & f = 1, \ldots, u + 1 \\ h_k(\Omega) \leq 0, & k = 1, \ldots, 2u \end{cases}$$

(25)

where $\Omega = [H_i, V_i, H_d, V_d, \lambda_1, \ldots, \lambda_n]^T$ is the design variables, $H_i, V_i, H_d, V_d$ are the tension components of bearing rope combinations for catwalk system and doorframe at starting point, $\Xi(\Omega) = [\delta_1(\Omega), \ldots, \delta_n(\Omega)]^T, g_j(\Omega) = [g_1(\Omega), \ldots, g_{u+1}(\Omega)]^T_{(u+1) \times 1}$ and $h_k(\Omega) = [-\lambda_1, \ldots, -\lambda_n, -1, \ldots, -1]^T_{n \times 1}$ are the objective function and boundary conditions. $\delta_i(\Omega) = \sum (h_i - h_{ni})$ and $g_j(\Omega) = \sum (h_{pl} - h_{ppl})$ represent the $i$th and $j$th accumulative deviations of nominal curve profile to actual values of catwalk platform and doorframe rope combinations at segment points, respectively. $h_k(\Omega)$ represents all inequality constraints of force distribution control at location of each doorframe.
Accordingly, we can transform this constrained nonlinear programming into an unconstrained nonlinear programming problem in terms of introducing a very large penalty parameter \( r \) that the minimum of the problem (26) corresponds to the solution of problem (25).

\[
\text{minimize } \Pi(\Omega) = \Psi^T(\Omega)\Psi(\Omega)
\]

(26)

where \( \Psi(\Omega) = \left[ \Xi^T(\Omega), r^T h^T(\Omega), r^T h^T(\Omega) \right]^T = \left[ \psi_1^T(\Omega), \ldots, \psi_{n+1}(\Omega) \right]^T \), the gradient vector of which can be given as \( \nabla \Pi(\Omega) = 2J^T(\Omega)\Psi(\Omega) \). Then, Hessian matrix \( \nabla^2 \Pi(\Omega) = 2J^T(\Omega)J(\Omega) + 2G(\Omega) \) can be obtained, wherein \( G(\Omega) = \sum_{i=1}^{n+1} \psi_i(\Omega)\nabla^2 \psi_i(\Omega) \).

5.2 Application of Levenberg-Marquardt Algorithm.

Levenberg-Marquardt algorithm is an efficient and effective method that providing numerical solutions for nonlinear minimization. By modifying parameters during iterations, this algorithm can improve the disadvantages of Gauss-Newton algorithm and settle problems when initial value is too far from the local minimum solution. Thus, in terms of using L-M algorithm, we can firstly determine the search direction \( \theta_i(\Omega) \) for the \( i \)th iterative numerical computation through following equation (27)

\[
\left( J^T(\Omega)J(\Omega) + \nu_i(\Omega)I \right) \theta_i(\Omega) = -J^T(\Omega)\psi_i(\Omega)
\]

(27)

where \( J(\Omega), \psi_i(\Omega) \) are the Jacobian matrix and function vector at the \( i \)th iteration. \( \nu_i(\Omega) \geq 0 \) and \( I \) are scalar member and identical matrix, respectively.

Function \( \psi_i(\Omega) \) and its design variables \( \Omega \) can be further formulated as

\[
\Pi_{i+1}(\Omega) \approx \Pi_i(\Omega) + \alpha_i(\Omega) \theta_i(\Omega) \nabla \Pi(\Omega_i) \]

(28)

\[
\Omega_{i+1}(\Omega) = \Omega_i(\Omega) + \alpha_i(\Omega) \theta_i(\Omega)
\]

(29)

where \( \alpha_i(\Omega) \geq 0 \) is the step length of the \( i \)th iteration. Further, we rewrite equation (27) into equation (30) as

\[
\theta_i(\Omega) \nabla \Pi(\Omega_i) = -2\alpha_i(\Omega) \left[ J^T(\Omega)J(\Omega) + \nu_i(\Omega)I \right] \theta_i(\Omega)
\]

(30)

when \( \nu_i(\Omega) \) is sufficiently enough, \( \theta_i(\Omega) \nabla \Pi(\Omega_i) \) is positive definite that \( \theta_i(\Omega) \nabla \Pi(\Omega_i) < 0 \), which draws the conclusion that

\[
\Pi_i(\Omega) > \Pi_{i+1}(\Omega)
\]

(31)

So that, we can finally obtain the optimal solution of cable curve profile as per specified convergent indexes.

6. Case Study

In this section, an engineering practice of catwalk under construction is shown to verify the effectiveness of this method.

Figure.6 Design of Main Cable in Load-Free Condition for an Under-Construction Suspension Bridge.

As shown in Figure.6, the span layout of suspension bridge is 330m+1386m+205m, curve profile of main cable, with 21.7kg/m per rope in weight, in load-free condition, wherein sags for each span is designed as 6.755m, 128.542m and 2.599m. The corresponding catwalk platform, controlled to be 1.5m/1.7m below main cable curve profile in middle span and side spans, uses 10-Ø54mm and 2-Ø54mm
steel wired ropes, each one of which is 12.2 kg/m in weight and with 1960 MPa tensile strength grade, to form bearing rope combinations respectively for itself and proper erection of doorframes of 7m in height. Doorframes and cross passageways are set along bridge direction every 57.75m and 173.25m for middle span, and about every 50m/55m and 100m/150m for side spans, based on which segmentations vectors along longitudinal direction are carried out as:

\[ l_l = \left[ 52.461, 55, 55, 50, 50, 67.539 \right]_l, \quad l_m = \left[ 57.75, ... , 57.75 \right]_m, \quad \text{and} \quad l_r = \left[ 51.564, 50, 50, 53.436 \right]_r \]

where \( l_l, l_m \) and \( l_r \) are in meters, the design of which may be finer as per user required computation scale. In addition, loads applying of catwalk system are listed in Table 1, wherein, uniformly distributed loads (U.D.L.) and point loads (P.L.) are comprehensively considered.

| Table 1. Loads Applying of Catwalk System | Load/Unit | Quan. |
|------------------------------------------|-----------|-------|
| **Category**                            | **Items** | **Load/Unit** | **Quan.** |
| **U D L.**                              | Catwalk platform rope | 12.2 kg/m | 10 |
|                                          | Handrail rope | 3.81 kg/m | 2 |
|                                          | Doorframe rope | 12.2 kg/m | 2 |
|                                          | Traction rope | 5.42 kg/m | 2 |
|                                          | Surfaces | 58.51 kg/m | — |
|                                          | Ancillary | 10 kg/m | — |
|                                          | Pedestrian | 5 kg/m | — |
|                                          | Doorframe | 250 kg | 23 |
| **P. L.**                               | Passageway | 6348.2 kg | 7 |

By adopting computation procedure, we first obtain the curve profile of main cable, the same of nominal catwalk system and doorframe rope combination, which is shown in Figure.7 with data aspect ratio of 1:1, as the computation reference. Then, we can correspondingly figure out the curve profile deviations, point load distribution factors, tension forces, free of stress rope length of both bearing rope combinations in each span in terms of using our proposed method, which are respectively shown in Figure.8, Figure.9, Figure.10 and Table 2.

Figure.7 Curve Profile of Main Cable, Catwalk Platform and Doorframe Rope Combinations.

Figure.8 Curve Profile Deviations of Catwalk Platform and Doorframe Rope Combinations.

Figure.9 Force Distribution at Location of Each Doorframe.
Figure 10: Tensions of Single Bearing Rope for Catwalk and Doorframe.

| Items  | Span | Left    | Middle  | Right   | Sum     |
|--------|------|---------|---------|---------|---------|
| Main Cable | m    | 357.41  | 1416.24 | 217.53  | 1991.18 |
| Catwalk rope | m    | 356.72  | 1413.61 | 217.08  | 1987.41 |
| Doorframe rope | m    | 357.75  | 1417.10 | 217.72  | 1991.57 |

The computation results show that the curve profile deviation of catwalk platform is controlled within 0.2 m, which is a proper result for builder’s manipulation in practice, while the same of doorframe rope is relatively less than 0.05 m that most of the loads are supported by catwalk platform bearing rope, maximum tension of which is 5.1e4 kg and occurred at right IP with safety factor of 4.06. Moreover, free of stress rope lengths are also available through this computation for builder’s proper preparation and implementation.

7. Conclusion
To conclude, this paper mainly focused on providing a fast solving method for prediction of catwalk system curve profile amid main cable erection in terms of using L-M algorithm, which may work as a replacement of the complex and time-consuming finite element methodology for builders and designers. To achieve that, we firstly analyzed the compliant characteristics of general suspended cable. In addition, by description of catwalk system, we correspondingly divided the main cable and catwalk system for convenience of computation. Then, we formulated main cable curve profile solving problem and gave the solution of nominal curve profile of catwalk system, based on which the numerical solution of actual curve profile can be derived by using the L-M algorithm. Moreover, a study case of catwalk design for a under construction suspension bridge was shown for further illustration of proposed method, the results of which have showed the effectiveness of this proposed method.

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