Multiplicity fluctuations in heavy-ion collisions using canonical and grand-canonical ensemble

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Abstract. We report the higher-order cumulants and their ratios for baryon, charge and strangeness multiplicity in canonical and grand-canonical ensembles in ideal thermal model including all the resonances. When the number of conserved quanta is small, an explicit treatment of these conserved charges is required, which leads to a canonical description of the system and the fluctuations are significantly different from the grand-canonical ensemble. Cumulant ratios of total-charge and net-charge multiplicity as a function of collision energies are also compared in grand-canonical ensemble.

1 Introduction

Event-by-event fluctuations of particle multiplicities and transverse energy have been of great interest and are measured at AGS, SPS, RHIC and LHC energies in relativistic heavy-ion collision experiments [1–8]. The main motivation of these studies is to explore the phase transition and/or a critical end point (CEP), which is believed to exist somewhere between the hadronic phase and the quark-gluon phase of the QCD phase diagram [9–13]. Most commonly measured event-by-event fluctuations in heavy-ion collision experiments are particle ratios (\(K/\pi\), \(p/\pi\), etc.), transverse energy (\((E_T)\)), transverse momentum (\((p_T)\)) and multiplicity (\((N)\)) fluctuations [14–17]. Recently, the higher moments of conserved numbers measured in beam energy scan (BES) program at RHIC have attracted further attention towards the usefulness of fluctuation studies in heavy-ion collisions [6,18,19]. In addition, it has been proposed that the kurtosis of the order parameter becomes negative when the critical point is approached and as a result, the kurtosis of a fluctuating observable [11], e.g., proton multiplicity, may become smaller than the value given by independent Poisson statistics. In the present work we explore the fact that the entropy is closely related to the particle multiplicity, and it is expected to be approximately conserved during the evolution of the matter created at the early stage of the collision. Therefore, the higher-order fluctuations of entropy would also be an interesting observable to look for the possibility of phase transition and critical point [20,21]. The entropy fluctuations are not directly observed but can be inferred from the experimentally measured quantities. The system’s entropy is related to the mean particle multiplicity, as the final-state mean multiplicity is proportional to the entropy of the initial state (\((N) \sim S\) [22]). The particle multiplicity can be measured on an event-by-event basis, whereas the entropy is defined by averaging the particle multiplicities in the ensemble of events. Thus, the dynamical entropy fluctuations can be measured experimentally by measuring the fluctuations in the mean multiplicity. Recently measured higher-order fluctuations of net-proton multiplicity distributions show that, at lower energies, net-proton fluctuation is mostly dominated by proton, as the anti-proton production is very small [6]. Hence, measuring only proton fluctuation will also provide the similar conclusion as measuring net-proton fluctuation.

Assuming a thermal system, formed in heavy-ion collisions, the grand-canonical ensemble (GCE) is the most appropriate description as only part of the particles from the system around mid-rapidity are measured by experiments. In heavy-ion collisions, if one could make measurements with full phase-space coverage, no conserved number fluctuation would be seen, as baryon number (\(B\)), electric charge (\(Q\)) and strangeness (\(S\)) are strictly conserved. In thermal models, the magnitude of multiplicity fluctuations and correlations in limited phase-space crucially depends on the choice of the statistical ensemble that imposes different conservation laws [23]. The micro-canonical ensemble (MCE) considers all the microstates where energy, momentum and charge are conserved. The canonical ensemble (CE) relaxes the energy conservation by introducing an infinite heat bath which can exchange energy but con-
serves the charge. The GCE introduces chemical potential and the requirement of charge conservation is also dropped. The multiplicity fluctuation patterns in full or finite momentum space are very different in MCE and CE as both impose different conservation conditions [24–26]. The fluctuations of the energy (⟨E⟩) are identical between CE and GCE, but fluctuations of particles which carry the conserved charge are affected. Since in GCE, the energy and conserved numbers may be exchanged with the rest of the system, therefore, they may fluctuate on an event-by-event basis. The experimentally measured multiplicity and transverse energy fluctuations can be related to the number susceptibilities and the heat capacity of the system, respectively [22].

If the number of conserved quanta is small, the grand-canonical approach is not adequate [22]. Instead, the description needs to ensure that the quantum number is conserved explicitly in each event. The CE has been used to describe the system formed in p+p collisions and e+e− collisions where the particle production is small [27]. In such systems, the deposited energy is still large and distributed over many degrees of freedom, hence the canonical treatment is the appropriate ensemble. Further, at lower energies, due to production threshold for strange particles or anti-baryons, one can apply the CE prescription to describe system formed in the heavy-ion collisions. Fluctuation results obtained in CERN SPS with lower energies and different collision species [28] motivate us to study the fluctuation variables in CE and compare the results obtained from those in GCE.

The paper is organized as follows: Section 2 describes the formalism for CE and GCE partition functions and the cumulants for the total-charge fluctuations. In sect. 3, the comparison of cumulant ratios for baryons, charge and strangeness number obtained in CE and GCE are discussed. Comparison of total-charge and net-charge multiplicity fluctuations using GCE are discussed in sect. 4. We summarize the present work in sect. 5.

### 2 Canonical and grand-canonical partition functions and their corresponding cumulants

Let us consider a system of particles and their corresponding anti-particles. In Boltzmann approximation, the grand-canonical partition function can be written as [25]

\[ Z_{\text{gce}}(V,T,\mu) = \sum_{N_{1+},N_{1-},N_{2+},N_{2-}} \cdots \sum_{N_{j+},N_{j-},N_{k+},N_{k-}} \frac{(\lambda_{1+} z_1)^{N_{1+}}}{N_{1+}!} \frac{(\lambda_{1-} z_1)^{N_{1-}}}{N_{1-}!} \cdots \frac{(\lambda_{j+} z_j)^{N_{j+}}}{N_{j+}!} \frac{(\lambda_{j-} z_j)^{N_{j-}}}{N_{j-}!} \cdots \times \exp \left[ 2z \cosh \left( \frac{\mu}{T} \right) \right], \]  

Here \( \lambda_{1\pm} = \exp(\pm \mu/T) \) corresponds to the fugacity of the j-th particle and \( \mu \) is the chemical potential. The “+” and “−” signs correspond to the particle and anti-particle, respectively. And, \( z_j \equiv \sum_j z_j \), where \( z_j \) is the single-particle partition function defined as follows:

\[ z_j = \frac{g_j V}{2\pi^2} \int_0^{\infty} p^2 dp \exp \left[ -\frac{(p^2 + m_j^2)^{1/2}}{T} \right] \]

\[ = \frac{g_j V}{2\pi^2} T m_j^2 K_2 \left( \frac{m_j}{T} \right), \]  

where \( m_j \) is the mass of the j-th particle, \( K_2 \) is the modified Hankel function, \( T \) and \( V \) are temperature and volume of the system, respectively.

In the canonical ensemble, the number of particles is strictly conserved and only the energy can be exchanged with the system’s surrounding, hence the chemical potential is zero, which leads to charge conservation constraint \( \langle Q \rangle = \langle N_+ \rangle - \langle N_- \rangle = 0 \) and the partition function reads as follows [25,29]:

\[ Z_{\text{ce}}(V,T,Q) = \sum_{N_{1+},N_{1-},N_{2+},N_{2-}} \cdots \sum_{N_{j+},N_{j-},N_{k+},N_{k-}} \frac{(\lambda_{1+} z_1)^{N_{1+}}}{N_{1+}!} \frac{(\lambda_{1-} z_1)^{N_{1-}}}{N_{1-}!} \cdots \frac{(\lambda_{j+} z_j)^{N_{j+}}}{N_{j+}!} \frac{(\lambda_{j-} z_j)^{N_{j-}}}{N_{j-}!} \cdots \times \exp \left[ i (N_{j+} + \cdots + N_{j+} + \cdots - N_{j-} - \cdots - N_{j-} - \cdots ) - Q \right] \]

\[ = \int_0^{2\pi} \frac{d\phi}{2\pi} \prod_j \sum_{N_{j+},N_{j-},N_{k+},N_{k-}} \frac{(\lambda_{j+} z_j)^{N_{j+}}}{N_{j+}!} \frac{(\lambda_{j-} z_j)^{N_{j-}}}{N_{j-}!} \cdots \times \exp \left[ i Q \phi + \sum_j (\lambda_{j+} e^{i\phi} + \lambda_{j-} e^{-i\phi}) \right] \]

\[ = I_Q(2z). \]  

Further, the CE partition function can be modified for an explicit charge conservation constraint, i.e., \( \sum_j (N_{j+} - N_{j-}) = Q \), for each microscopic state of the system [25],

\[ Z_{\text{ce}}(V,T,Q) = \sum_{N_{1+},N_{1-},N_{2+},N_{2-}} \cdots \sum_{N_{j+},N_{j-},N_{k+},N_{k-}} \frac{(\lambda_{1+} z_1)^{N_{1+}}}{N_{1+}!} \frac{(\lambda_{1-} z_1)^{N_{1-}}}{N_{1-}!} \cdots \frac{(\lambda_{j+} z_j)^{N_{j+}}}{N_{j+}!} \frac{(\lambda_{j-} z_j)^{N_{j-}}}{N_{j-}!} \cdots \times \exp \left[ i (Q \phi + \sum_j z_j (\lambda_{j+} e^{i\phi} + \lambda_{j-} e^{-i\phi}) \right] \]

\[ = \int_0^{2\pi} \frac{d\phi}{2\pi} \prod_j \sum_{N_{j+},N_{j-}} \frac{(\lambda_{j+} z_j)^{N_{j+}}}{N_{j+}!} \frac{(\lambda_{j-} z_j)^{N_{j-}}}{N_{j-}!} \cdots \times \exp \left[ i Q \phi + \sum_j z_j (\lambda_{j+} e^{i\phi} + \lambda_{j-} e^{-i\phi}) \right] \]

\[ = I_Q(2z). \]  

In eq. (4), the integral representations of the δ-Kronecker symbol and the modified Bessel function are defined as [30]:

\[ \delta(n) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \exp(i n \phi), \]

\[ I_Q(2z) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \exp[-i Q \phi + 2z \cos \phi]. \]
It is to be noted that, in eq. (4), \( \lambda_{j+} \) and \( \lambda_{j-} \) are not fugacities but just auxiliary parameters, only to calculate the mean number and the fluctuations of positively and negatively charged particles. They are set to one in the final formula.

Using the above partition functions for CE, one can derive the other thermodynamic properties of the system at freeze-out. Commonly, the mean multiplicity and variance of the particle number distributions are derived using the partition functions. The cumulants of multiplicity distribution in GCE can be derived as follows:

\[
\langle N_{\pm} \rangle_{\text{gce}} = \left( \frac{\partial}{\partial \lambda_{\pm}} \ln Z_{\text{gce}} \right) \lambda_{\pm},
\]

\[
\langle N_{\pm}^2 \rangle_{\text{gce}} = \frac{1}{Z_{\text{gce}}} \left( \frac{\partial}{\partial \lambda_{\pm}} \right)^2 Z_{\text{gce}} = z \lambda_{\pm} + z^2 \lambda_{\pm}^2.
\]

In the present work, we have extended these studies to the higher-order cumulants. Hence, third and fourth cumulants of the particle and anti-particle multiplicities are derived as

\[
\langle N_{\pm}^3 \rangle_{\text{gce}} = \frac{1}{Z_{\text{gce}}} \left( \frac{\partial}{\partial \lambda_{\pm}} \right)^3 Z_{\text{gce}} = z \lambda_{\pm} + 3z^2 \lambda_{\pm}^2 + 3z^3 \lambda_{\pm}^3,
\]

\[
\langle N_{\pm}^4 \rangle_{\text{gce}} = \frac{1}{Z_{\text{gce}}} \left( \frac{\partial}{\partial \lambda_{\pm}} \right)^4 Z_{\text{gce}} = z \lambda_{\pm} + 7z^2 \lambda_{\pm}^2 + 6z^3 \lambda_{\pm}^3 + 3z^4 \lambda_{\pm}^4.
\]

Similarly, in CE, the cumulants can be easily derived using the CE partition function as defined in eq. (4),

\[
\langle N_{\pm} \rangle_{\text{ce}} = z \frac{I_{Q+}(2z)}{I_Q(2z)},
\]

\[
\langle N_{\pm}^2 \rangle_{\text{ce}} = z \frac{I_{Q+}(2z)}{I_Q(2z)} + z^2 \frac{I_{QZ}(2z)}{I_Q(2z)},
\]

\[
\langle N_{\pm}^3 \rangle_{\text{ce}} = z \frac{I_{Q+}(2z)}{I_Q(2z)} + 3z^2 \frac{I_{QZ}(2z)}{I_Q(2z)} + z^3 \frac{I_{QZ}(2z)}{I_Q(2z)},
\]

\[
\langle N_{\pm}^4 \rangle_{\text{ce}} = z \frac{I_{Q+}(2z)}{I_Q(2z)} + 7z^2 \frac{I_{QZ}(2z)}{I_Q(2z)} + 3z^3 \frac{I_{QZ}(2z)}{I_Q(2z)} + 6z^4 \frac{I_{QZ}(2z)}{I_Q(2z)},
\]

and the correlation between particles and their anti-particles can be estimated using the following generalized relation:

\[
\langle N_{\pm}^{n_1} N_{\mp}^{n_2} \rangle = \frac{1}{Z} \left( \frac{\partial}{\partial \lambda_{\pm}} \right)^{n_1} \left( \frac{\partial}{\partial \lambda_{\mp}} \right)^{n_2} Z.
\]

Using the above relations cumulants of the charge multiplicity in both GCE and CE can be obtained as follows:

\[
C_1 = (N_+ + N_-) = (N_+) + (N_-),
\]

\[
C_2 = ((\delta N)^2) = ((N_+ + N_-)^2) - (N_+ + N_-)^2,
\]

\[
C_3 = ((\delta N)^3) = ((N_+ + N_-)^3) - 3((N_+ + N_-)^2) ((N_+ + N_-)) + 2 ((N_+ + N_-))^3,
\]

\[
C_4 = ((\delta N)^4) - 3((\delta N)^2)^2 = ((N_+ + N_-)^4) - 4((N_+ + N_-)^3) C_1 + 6((N_+ + N_-)^2) C_1^2 - 3C_1^4 - 3C_2^2.
\]

The properties of distribution functions are characterized by the various moments, such as mean \((M)\), variance \((\sigma)\), skewness \((S)\) and kurtosis \((K)\). These moments are the alternative methods to characterize a distribution besides the cumulants. Various moments and cumulants are related as \(M = C_1, \sigma^2 = C_2, S = C_3/C_2^{3/2}\) and \(K = C_4/C_3^2\) and hence their ratios and products can be written in term of cumulants as: \(\sigma^2/M = C_2/C_1, S\sigma = C_3/C_2\) and \(\kappa\sigma^2 = C_4/C_2\). Experimentally, one measures the multiplicity distributions of particles (both \(N_+\) and \(N_-\)) on an event-by-event basis and construct the \((N_+ + N_-)\) for total-charge and \((N_+ - N_-)\) for net-charge multiplicity distribution. Recently, net-baryon (proton), net-electric charge and net-strangeness (kaon) fluctuations measured in BES at RHIC have further attracted attention towards the event-by-event fluctuation studies using their higher moments [6,18,19]. Ratios and products of the moments of total multiplicity distributions can also be experimentally measured and it will be interesting to see their dependences on the collision energy \((\sqrt{s_{NN}})\).

### 3 Results and discussion

The cumulants of the total-charge multiplicities and their ratios are calculated in CE and GCE. Figure 1 shows comparison of the ratios of cumulants for total-charge multiplicity as a function of \(z\) by considering both CE and GCE. As pointed out in [25], \(C_2/C_1\) calculated in GCE and CE becomes equivalent in the large volume limit \((i.e., z \to \infty)\), it is constructive to look for the ratios of higher-order fluctuations in two different ensembles, which might be more sensitive to the fluctuations. In GCE with Boltzmann approximation, the total-charge multiplicities are strictly Poissonian, as the ratios are unity for all \(z\) values, while it is not true in case of CE. It can be seen from fig. 1 that \(C_3/C_2\) and \(C_4/C_2^2\) ratios in CE approaches to GCE for higher \(z\) values. However, the cumulant ratios are quite different at lower \(z\) values. The ratios of higher-order cumulants approach the corresponding GCE.
values faster than the lower-order ratio \((C_2/C_1)\). In the canonical ensemble, the particle fugacity is zero in order to maintain the charge conservation. To study the non-zero value of conserved number fluctuation and its effect on different \(z\) values, one can explicitly introduce the net charge of the system as \(\Delta Q = 1\) and 2 as discussed in the previous section. Figure 2 shows the \(z\) dependence of the ratio of cumulants for total charge with explicit net charge \((\Delta Q = 0, 1 \text{ and } 2)\) of the system. One notices that all the cumulant ratios at large \(z\) (in thermodynamic limit) for different net-charge conservation approaches 1, but the behavior at small \(z\) is quite different.

As discussed in [25], for \(\Delta Q \geq 1\) the \(C_2/C_1\) ratios of total charged particles decreases at smaller \(z\). In the case of small systems \((z \to 0)\), the average number of positive particles is comparable to the \(Q\) and the fluctuations of \(N_+\) are small. On the other hand, at small \(z\) and fixed \(Q\) the average number of negatively charged particles is much smaller than \(Q\) and the fluctuations of \(N_-\) are not affected by the conservation law. Hence, the total-charge fluctuation is mostly driven by fluctuation of the positively charge particles. As can be seen, the \(C_3/C_2\) and \(C_4/C_2\) ratios approach to asymptotic value faster for \(\Delta Q = 0\) compared to non-zero \(\Delta Q\) values of the system. Further, the ratios of higher-order cumulants \((C_3/C_2, C_4/C_2)\) approach to their asymptotic values at smaller \(z\) values compared to \(C_2/C_1\). All the ratios of cumulants converges at both extremes except for \(\Delta Q = 0\) in \(C_2/C_1\). Fluctuation of other conserved quantities such as baryon number, electric charge or strangeness as a function of \(\sqrt{s_{NN}}\) can be studied in the GCE and CE framework. The cumulants of total baryon, electric charge, strangeness and their ratios are calculated in CE and GCE. Figure 3 shows the ratios of cumulants \(C_2/C_1, C_3/C_2, \text{ and } C_4/C_2\) in GCE as a function of collision energies \(\sqrt{s_{NN}}\) for total baryon, charge and strangeness calculated in the thermal model approach with quantum statistics, in which all resonances are included to incorporate the particle interactions. The freeze-out parameters (baryon chemical potential \(\mu_B\) and freeze-out temperature \(T\)) as a function of \(\sqrt{s_{NN}}\) are parametrized as [31]: 

\[
\mu_B(\sqrt{s_{NN}}) = a - b \sqrt{s_{NN}} - c \mu_B^3
\]

with \(a = 0.166 \pm 0.002 \text{ GeV}\), \(b = 0.139 \pm 0.016 \text{ GeV}^{-1}\), and \(c = 0.053 \pm 0.021 \text{ GeV}^{-3}\). The energy dependence of \(\mu_B\) is given as \(\mu_B(\sqrt{s_{NN}}) = d/(1 + e \sqrt{s_{NN}})\) with \(d = 1.308 \pm 0.028 \text{ GeV}\) and \(e = 0.273 \pm 0.008 \text{ GeV}^{-1}\). It is to be noted that, in the case of total baryons, the ratios of cumulants follow the Poisson expectation of individual baryons and anti-baryons and hence the ratios of cumulants are at unity. For heavier mass particles (when \(m_i \gg \mu\)), the momentum distributions can be approximated by the classical Boltzmann functions, hence, the

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**Fig. 1.** (Color online) Comparison of \(z\) dependence of the ratio of cumulants \(C_2/C_1, C_3/C_2\) and \(C_4/C_2\) for total charge in GCE (dotted line) and CE (solid line) for \(\Delta Q = 0\).

**Fig. 2.** (Color online) The \(z\) dependence of the ratio of cumulants for total electric charge \((C_2/C_1, C_3/C_2 \text{ and } C_4/C_2)\) in the canonical ensemble assuming the explicit net charge of the system \(\Delta Q = 0, 1 \text{ and } 2\).
particle multiplicity in HRG model will be Poissionian. Whereas, in the case of total charge and total strangeness, the ratios of cumulants do not follow the Poisson expectations in quantum statistics because of the higher charge and strangeness, $|Q|$ and $|S| > 1$, of the particles. For total baryon and total charge all the three ratios of cumulants $(C_2/C_1, C_3/C_2, C_4/C_2)$ remain constant with collision energies. However, in the case of total strangeness, $C_2/C_1$ decreases with increasing energies. Although CE should be applied to the lower collision energies where the particle multiplicities are small, we have carried out a similar study for total multiplicities using CE. Figure 4 shows the cumulant ratios of total multiplicity as a function of $\sqrt{s_{NN}}$ in CE. At lower energies which correspond to smaller $z$ values, the cumulant ratios increase and higher $\sqrt{s_{NN}}$ cumulant ratios of total baryon, charge and strangeness approach similar values.

4 Comparison between total-charge and net-charge fluctuations in GCE

Recent results from RHIC BES program for net-proton, net-charge and net-strangeness fluctuations have been proposed to extract the freeze-out parameters and to explore the CEP in the QCD phase diagram [10,32]. Furthermore, it is proposed that deviation of these quantities from thermal baseline would indicate the presence of CEP. At lower energies, anti-baryon and strangeness production is small. Hence, fluctuations such as net baryons or net strangeness are mostly dominated by proton or kaon production, respectively. For example, net-proton fluctuations reported in [6,33] are dominated by fluctuation of protons. Therefore, it is intuitive to look for the fluctuations of total as well as net multiplicities of different conserved quantities. Total-charge cumulants are calculated using eqs. (14)–(17), similarly one can calculate the cumulants for net-charge fluctuations. Figure 5 shows the $C_3/C_2$ and $C_4/C_2$ ratios as a function of $\sqrt{s_{NN}}$ for total- and net-conserved quantities. In the case of net baryon and net charge, $C_3/C_2$ strongly depends on collision energies, whereas for total baryon and total charge, $C_3/C_2$ ratios are almost constant at all energies. For $C_4/C_2$, both total charge and net charge are exactly the same as a function of $\sqrt{s_{NN}}$. If there is no correlation between different particles, the various order ($n = 1, 2, 3$ and 4) of cumulants for net-charge multiplicity can be written as $C^{net}_n = C_n(N^+) + (−1)^n C_n(N^-)$, whereas as cumulants for total-charge multiplicity can be written as: $C^{tot}_n = C_n(N^+) + C_n(N^-)$. The reason for this equivalence for $C_4/C_2$ in net-charge and total-charge multip-
near the CEP, if it exists. Since the CEP is expected to show large deviation from the baseline values, it is necessary to explicitly introduce the net charge as a function of \( \sqrt{s_{NN}} \) for both total baryon (\( B \)) and total charge (\( Q \)) to look for the non-monotonic behavior as a function of collision energies for the total-charge and net-charge cases, while \( C_4/C_2 \) ratios are the same in both cases. We argue that it would be constructive to look for the fluctuations of total-charge distributions measured experimentally for different energies and can be compared with the thermal baseline as discussed in the present work to look for the ratios of cumulants in CE and GCE for \( z = 0, 1 \) and 2 conservation, significant differences are observed for all the three cases at lower \( z \) values. Comparing the ratios of cumulants in CE and GCE for total charge suggests noticeable difference for lower \( z \) values. When the number of conserved quanta is small, an explicit treatment of these conserved charges is required, which leads to a canonical description of the system and the fluctuations are significantly different from the grand-canonical ensemble. Significant differences are observed for \( C_4/C_2 \) ratios as a function of collision energies for the total-charge and net-charge cases, while \( C_4/C_2 \) ratios are the same in both cases. We argue that it would be constructive to look for the fluctuations of total-charge distributions measured experimentally for different energies and can be compared with the thermal baseline as discussed in the present work to look for the non-monotonic behavior. Further, it will be exciting to check the conserved number fluctuations at other lower energies of heavy-ion collision data. Since the number of conserved quanta will be very small, it will be interesting to check whether the system follows the canonical prescription at these energies or not. If it follows, then fluctuations in CE should be used as a thermal model baseline to check any deviation due to dynamical origins. In the present work we have not incorporated various other phenomena, for example, experimental acceptance, the effect of hydrodynamic flow and resonance decay. Therefore, while comparing the experimental measurements with our calculations, one has to take care of the above-mentioned effects.

5 Summary

In summary, we have calculated the higher-order cumulants and their ratios for total baryon, charge and strangeness multiplicity in the canonical and grand-canonical ensembles. These fluctuations in CE are further extended by explicitly introducing the net charge as a function of \( \sqrt{s_{NN}} \).
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