ADDENDUM TO SHARPLY 2-TRANSITIVE GROUPS OF CHARACTERISTIC 0

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Abstract. In this short note we show how to modify the construction of non-split sharply 2-transitive groups of characteristic 0 given in [RT] to allow for arbitrary fields of characteristic 0.

1. Introduction

The first sharply non-split sharply 2-transitive groups in characteristic 0 were constructed in [RT]. However, the construction given there only works when starting from the group $AGL(1, \mathbb{Q}) = \mathbb{Q}_+ \rtimes \mathbb{Q}^*$. We modify the construction given there in order to prove:

Theorem 1.1. For any field $K$ of characteristic 0 the group $AGL(1, K) = K_+ \rtimes K^*$ can be embedded into a sharply 2-transitive group of characteristic 0 not containing any regular normal subgroup.

To prove Theorem 1.1 we introduce the following equivalence relation (replacing the equivalence relation given in [RT]): for any group $G$ and involution $j \in G$ we say that involutions $s, t \in G$ are equivalent relative to $j$ (and write $s \approx_j t$) if $Cen(js) = Cen(jt)$.

The following proposition replaces Proposition 1.3 of [RT] and provides the induction step for the proof of Theorem 1.1.

Proposition 1.2. Let $G$ be a group containing involutions $j, t$ and $t'$ with $jt' = t$ and $A = Cen_G(j)$. Assume that $G, j, t, t'$ and $A$ satisfy assumptions (1) – (3) of Theorem 1.1. in [RT] and furthermore:

(4') for any involution $s$ with $s \approx_j t$ there is some $a \in A$ such that $s = t^a$.
(5') for any involution $s \neq j$, we have $Cen(js) = \{1\} \cup \{js': s' \approx_j s\}$.
(6') for any involution $s \notin AtA, s \neq j$, there is an involution $s' \in G$ with $s' \approx_j s$ such that $Cen(js) = \langle js' \rangle$.

Then for any involution $v \in G$ with $v \neq j$ there exists an extension $G_1$ of $G$ such that for $A_1 = Cen_{G_1}(j)$ there exists some $f \in A_1$ with $t'^f = v$ and conditions (1) – (3) and (4') – (6') continue to hold with $G_1$ and $A_1$ in place of $G, A$.

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Note that for the group $AGL(1, \mathbb{Q})$ the new equivalence relation agrees with the one given in [RT]. It is easy to see exactly as in [RT] that for any field $\mathbb{K}$ of characteristic 0, the group $AGL(1, \mathbb{K})$ satisfies properties (1) – (3) and (4’) – (6’).

Using the following lemma, the proof of Proposition 1.3 in [RT] carries over verbatim to this setting.

**Lemma 1.3.** In the situation of Proposition 1.2 we have $Cen(js) = Cen((js)^n)$ for any involution $s \in G$. In particular, for involutions $s, s' \in G$ and $n, m \in \mathbb{Z}$ such that $(js)^n = (js')^m$ we have $s \approx j s'$.

**Proof.** Since $js$ centralizes $(js)^n$, assumption (5’) implies $s \approx jj (js)^n$. The second part follows directly from this. □

As in [RT] we also see that for any field $\mathbb{K}$ of characteristic 0, the group $AGL(1, \mathbb{K}) \ast \mathbb{Z}$ satisfies properties (1) – (3) and (4’) – (6’). Now the proof of Theorem 1.1 follows exactly as in [RT].

**References**

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