INTRODUCTION

A neutrino factory, with decays of $\mu^+$ or $\mu^-$ beams in the straight sections of a storage ring, produces a spectrum of electron and muon neutrinos via $\mu^+ \rightarrow \nu_\mu e^+ \nu_e$ and $\mu^- \rightarrow \nu_\mu e^- \nu_e$. Charged-current (CC) interactions of the muon anti-neutrinos (neutrinos) in a detector lead to the production of $\mu^+$ ($\mu^-$) or the “right-sign” (RS) muons. The electron neutrinos (anti-neutrinos), on the other hand, can oscillate into muon neutrinos (anti-neutrinos) producing muons with charge opposite to that of the unoscillated case and get detected as “wrong sign” (WS) muons.

Most studies with neutrino factories[1] have focused on the use of WS events to pin down the unknown reactor angle $\theta_{13}$, the CP violating phase $\delta_{CP}$ and the mass hierarchy. RS muons are useful for precision measurements of the atmospheric mixing angle $\theta_{23}$ and the mass squared difference $\Delta m^2$, apart from understanding cross-section and flux uncertainties. In particular, they are sensitive probes of whether $\nu_\mu \leftrightarrow \nu_\tau$ mixing is maximal (i.e. $\theta_{23} = \pi/4$, referred to as maximal $\theta_{23}$). Measurement of deviation from maximality is of significance in developing models for neutrino masses and mixings.

We address the little-studied issue of contamination of the (RS or WS) muon events sample from oscillations of the muon or electron neutrinos (anti-neutrinos) to tau neutrinos (anti-neutrinos), which, through CC interactions, result in tau leptons that decay to muons. We focus on how this contribution affects a precision measurement of the atmospheric mixing parameters and the deviation from maximal mixing.

The oscillation probability $P_{\mu\tau}$ is large even if $\theta_{13}$ vanishes, being driven by a nearly maximal $\theta_{23}$. In spite of CC cross-section suppression for the massive tau production, there is still a sizeable production rate and tau decay (with rate into muons of $\sim 17\%$) enhances the RS muon event rates, especially at small muon energies. Since the $\theta_{23}$ dependent terms in $P_{\mu\mu}$ and $P_{\mu\tau}$ come with opposite sign, the combination of muons from direct production and from tau decays marginally decreases the sensitivity of the event rates to this angle. Cuts imposed to remove the tau events drastically reduce the direct muon events as well, worsening the sensitivity to the oscillation parameters. Thus, the total muon events (direct muons and from tau decay) are less sensitive to the deviation from maximality. Neglect of the tau contribution will lead to an incorrect conclusion about the precision possible for the deviation from maximality.

THE INPUTS: FLUXES, KINEMATICS, CROSS-SECTIONS

The neutrino factory fluxes

We assume a basic muon storage ring configuration [2] with muon beam energy $E_b = 25$ GeV, with $n_\mu = 5 \times 10^{20}$ useful decays per year. We integrate the neutrino fluxes over the muon beam angle $\alpha$, assuming a gaussian angular divergence of the muon beam around the $z$-axis with standard deviation [3] $\sigma = 0.1/\gamma$, where $\gamma = E_b/m_\mu$. This is then averaged over a small neutrino opening angle, $\theta' < \epsilon = 0.3\sigma$ or roughly 0.1 mr. The resulting neutrino (or antineutrino) spectrum with gaussian angular spread averaged over $(\theta' < \epsilon, 0 \leq \phi' \leq 2\pi)$ at the baseline distance $L$ from source to detector is given by,

$$\frac{dN_\mu}{dE_\nu} = \frac{1}{E_b} \frac{1}{d\Omega} \int d\Omega' \left( \frac{dN_\nu}{d\Omega d\gamma} \right)_{G}$$

$$= \frac{4n_\mu \gamma^2}{\pi L^2 E_b} \left( 3 - 4 \gamma^2 - \frac{\beta}{2} \right) \frac{1}{(1 + c_\epsilon) e^{-\sigma^2/2}}$$
\[
-\frac{1}{3} y (y^2 - 1) \left[ 4 + c \varepsilon + c_2^2 + e^{-2\sigma} 3 c \varepsilon (1 + c \varepsilon) \right],
\]
\[
\frac{dN_\nu}{dE_\nu} = \frac{1}{E_\nu} \int d\Omega \frac{dN_\nu}{d\Omega dE_\nu} G
= \frac{24m_\nu y^2}{\pi L^2 E_\nu} \left\{ -2y^2 - \frac{\beta}{2} (1 + c_2 y^2) e^{-\sigma^2/2} 
\right. \\
\left. - \frac{1}{3} y (y^2 - 1) \left[ 4 + c_\varepsilon + c_2^2 + e^{-2\sigma} 3 c \varepsilon (1 + c \varepsilon) \right] \right\},
\] where \( y = E_\nu/E_\beta \), \( c \varepsilon \), \( s \varepsilon \) refer to \( \cos \varepsilon \), \( \sin \varepsilon \).

The kinematics

We focus on the spectrum of the final state muons and hence require the detailed kinematics of the CC interactions, in which either muons or taus are produced, with the latter decaying into muons, where again we use the differential decay rates (see Ref. [4] for details). In the laboratory frame, a neutrino of flavor \( l = \mu \) or \( \tau \) interacts with a nucleon and produces the corresponding charged lepton \( l \) at an angle \( \theta_l \) w.r.t. the incident neutrino direction. In the case of \( \tau \) interactions, the tau is produced at a very forward angle while the azimuthal angle of the muon from tau decay \( \phi_\mu \), is restricted by the decay kinematics. The available phase space is restricted in both direct muon and tau production due to the constraint on the available energy for the lepton: \( E_\nu < E_l < E_\tau \); see Ref. [4] for the detailed expressions on the constraints. The effect of this pinching in available energy, for the case of a \( \tau \) lepton being produced, can be seen in Fig. 1 where the final hadronic mass \( m_X \) is plotted as a function of \( E_\tau \). The notation is standard: \( m_X^2 = W^2 = (p + q)^2 \), where \( p, q \) are the nucleon and intermediate gauge boson 4-momenta in the laboratory frame. For a typical neutrino energy \( E_\nu = 10 \text{ GeV} \), allowed energy range for \( \cos \theta_\tau = 0.91 \) is tiny; tau leptons are hence produced in a very forward direction while the direct muons, due to their lighter mass, are less restricted.

The cross-sections

Since the energies of interest range from a few GeV to 25 GeV, the CC interactions include quasi-elastic (QE), resonance (Res) and deep inelastic (DIS) processes. We consider the double differential cross-sections,

\[
\frac{d\sigma}{dE_l d\cos \theta_l} = \left\{ \frac{G_F^2 \kappa^2 p_l}{2\pi M} \sum_{i=1}^{S} a_i W_i \right\}^2,
\]

where \( G_F \) is the Fermi constant, \( \kappa = M_W^2/(Q^2 + M_W^2) \) is the propagator factor with the W boson mass, \( M_W \), \( p_l \) is the magnitude of three-momentum of the charged lepton produced and \( W_i \) are structure functions corresponding to the general decomposition of the hadronic tensor. \( W_{\text{dis}} \) appear only for massive final leptons. We have,

\[
\sum_i \frac{n_i W_i}{M} = \left( \frac{2W_1 + m_l^2}{M^2} W_4 \right) (E_l - p_l \cos \theta_l) + W_2 (E_l + p_l \cos \theta_l)

\]

\[
= \frac{W_3}{M} (E_l + p_l^2 - (E_\mu + E_l) p_l \cos \theta_l) - \frac{m_l^2}{M} W_5.
\]

The detailed expressions for \( W_i \) are taken from Ref. [5] where the specific structure functions are listed for QE, Res and DIS leading order (LO) processes.

**EVENT RATES IN A FAR-DETECTOR**

Preliminaries

We assume that the neutrinos interact with a 50 kton iron detector such as the proposed INO/ICAL or MIND. Both \( \mu^+ \) and \( \mu^- \) beams with equal exposure are considered. While RS events are sensitive to deviations of \( \theta_{23} \) from maximality, inclusion of WS events may only marginally worsen the results; however, the advantage in being “charge-blind” is significant, hence, all muon events are simply added. The generic number of muon events at a distance \( L \), as a function of the observed muon energy \( E \), is

\[
\mathcal{R}^{\text{b}_{\mu}}(E) = K \int_{E_{\mu}}^{E_{\mu}} dE_\nu \frac{dN_{\nu}(E_\nu, L)}{dE_\nu} \cdot P_{\mu}(E_\nu, L) \int_0^1 d\cos \theta_\mu

\]

\[
\int_{E_\mu}^{E_\nu} E_\mu d\mu \frac{d\sigma_{\mu}(E_\nu, E_\mu, \theta_\mu)}{d\mu d\cos \theta_\mu} \cdot R(E_\mu, E),
\]

\[
\mathcal{R}^{\text{b}_{\tau}}(E) = K \int_{E_{\tau}}^{E_{\tau}} dE_\nu \frac{dN_{\nu}(E_\nu, L)}{dE_\nu} \cdot P_{\tau}(E_\nu, L) \int_{c_{\tau}}^1 d\cos \theta_{\tau}

\]

\[
\int_{E_\tau}^{E_\nu} E_\tau d\tau \frac{d\sigma_{\tau}(E_\nu, E_\tau, \theta_\tau)}{d\tau d\cos \theta_\tau} \int_{\cos \theta_{\tau}}^1 d\cos \theta_\mu d\phi_\mu

\]

\[
\frac{1}{\Gamma} \int \frac{d\Gamma(E_\nu, E_\tau, \theta_\tau, \theta_\mu, \phi_\mu)}{d\cos \theta_{\tau} d\cos \theta_\mu d\phi_\mu} \cdot R(E_\mu, E).
\]

FIGURE 1. Kinematics of \( \nu_\tau \)-nucleon CC interactions. The allowed parabolas of constant \( \cos \theta_\tau \) in the \( m_X-E_\tau \) plane are shown for \( E_\nu = 10 \text{ GeV} \). The ends of the parabolas (at \( m_X = M \)) give the limits of the tau energy.
Here the sum is the Gaussian energy resolution function of width $R$ summed isoscalar), Tau contribution and precision measurement of to the deviation of the mixing angle $\theta$ are studied. The limits of integration and the restriction from an angular constraint, in case of tau oscillation parameters, are as given in text.

Typical event rates at $L=7400$ Km (magic baseline) for oscillation parameters, $\Delta m^2 = 2.4 \times 10^{-3}$ eV$^2$, $\theta_{23} = 42^\circ$, $\theta_{13} = 1^\circ$, $\sin^2 \theta_{12} = 0.304$ and $\Delta_{21} = 7.65 \times 10^{-5}$ eV$^2$, are shown as a function of the observed lepton energy in Fig. 2. The panels show the direct muon production and tau decay contributions to the RS events from tau decay into muons. The tau contribution is with an angular cut. Also, since the tau contribution is substantial at small observed muon energies where the tau decay rate is large, a muon energy cut can also be contemplated. The effect of cuts on the event rates is seen in Fig. 3 – the only cut effective in removing the tau contribution is one ($\theta > 25^\circ$) that removes the signal itself! Alternately a muon energy cut of $E > 10 - 15$ GeV can substantially remove the tau contribution, still leaving sufficient direct muons. However, such a large energy cut will worsen the measured precision of the mixing parameters as sensitivity is higher in the lower energy bins where matter effects are large. In short, it is not feasible to cut out the tau contribution and still make a precision measurement, in this case, of the deviation of $\theta_{23}$ from maximality.

Cuts on tau contribution

Since tau production in neutrino-nucleon interactions is extremely forward-peaked, one obvious way to remove the tau contribution is with an angular cut. Also, since the tau contribution is substantial at small observed muon energies where the tau decay rate is large, a muon energy cut can also be contemplated. The effect of cuts on the event rates is seen in Fig. 3 – the only cut effective in removing the tau contribution is one ($\theta > 25^\circ$) that removes the signal itself! Alternately a muon energy cut of $E > 10 - 15$ GeV can substantially remove the tau contribution, still leaving sufficient direct muons. However, such a large energy cut will worsen the measured precision of the mixing parameters as sensitivity is higher in the lower energy bins where matter effects are large. In short, it is not feasible to cut out the tau contribution and still make a precision measurement, in this case, of the deviation of $\theta_{23}$ from maximality.

Effect of the tau contribution

As stated earlier, the tau contribution alters the dependence on the mixing parameters, altering the precision to which we can determine them. While the tau events have less sensitivity to $\theta_{23}$, the rate increases while the direct event rate decreases, as $\theta_{23}$ increases towards max-
imal $\theta_{23} = \pi/4$. The inclusion of muons from tau events also alters the uncertainties considerably. A near detector sensitive to muons, measures the combination of flux times cross-section of the muons. This also appears in the RS event rate for direct muon production and is therefore well constrained. However, for tau production and decay, the RS event rate has the combination of muon flux and the tau production cross-section. The heavy tau cross-sections have larger uncertainties, where mass corrections are large. Furthermore, since these contributions result from oscillations, no near detector can help reduce the uncertainties. Hence overall uncertainties are much larger for the tau contribution than for direct muons.

Hence, in our numerical calculations we use an overall normalization error of 0.1% for direct, while a modest 2% is used for the total (direct-tau), muon events. We use typical input values of $(\Delta m^2, \theta_{23}, \theta_{13})$ to estimate how well the generated “data” can be fitted, and calculate the resulting precision on the parameters. We keep the solar parameters fixed at their best-fit values of Ref. [6] and set $\delta_{CP}$ to zero. The best fits (and regions of confidence levels in parameter space) are obtained by minimizing the chi-squared with a pull corresponding to the normalization uncertainties specified.

We present results for a typical sample set of input parameters, $(\Delta m^2, \theta_{23}, \theta_{13} = 2.4 \times 10^{-3} \, \text{eV}^2, 41.9^\circ, 1^\circ)$. We minimize first over the pull, and then over $\Delta m^2$ and $\theta_{23}$, keeping $\theta_{13}$ fixed. Fig. 4 shows the allowed $\Delta m^2 - \theta_{23}$ parameter space at 99% CL. The solid and dashed lines correspond to considering direct and total (including those from tau decay) muon events respectively. Note that the 99% CL contour is much more constrained with direct than for total muons. In particular, it is the $\Delta m^2$ values that are smaller than the input value, that broaden the contour and limit the discrimination. The largest true value of $\theta_{23}$ that can be discriminated from maximal is shown in Fig. 5, as a function of $\Delta m^2$ again, for $\theta_{13} = 1^\circ$. It is seen that tau contamination worsens the ability to discriminate $\theta_{23}$ from maximal, thus making this measurement harder than originally expected.

**CONCLUSION**

The oscillations of the muon or electron neutrinos (anti-neutrinos) to tau neutrinos (anti-neutrinos) results in tau leptons produced through CC interactions in the detector which on leptonic decay add to the right as well as wrong sign muon events obtained directly. This tau contamination worsens the ability to discriminate against maximal $v_\mu \leftrightarrow v_\tau$ mixing. It is practically impossible to devise satisfactory cuts to remove this tau contamination. Uncertainties from this tau background must be brought under control before making precision parameter measurements at neutrino factories.

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**REFERENCES**

1. A. Bandyopadhyay et al. [ISS Physics Working Group], Rept. Prog. Phys. 72, 106201 (2009) and references therein.
2. IDS-NF baseline study, IDS-NF-Baseline-2007/1.0, prepared by the IDS-NF Steering Group, Nov. 2008.
3. C. Crisan and S. Geer, Fermilab-TM-2101; A. Broncano and O. Mena, Eur. Phys. J. C 29, 197 (2003) and Ref. [1].
4. D. Indumathi and N. Sinha arXiv:0910.2020 [hep-ph].
5. K. Hagiwara, K. Mawatari and H. Yokoya, Nucl. Phys. B 668, 364 (2003) [Erratum-ibid. B 701, 405 (2004)].
6. T. Schwetz et al., New J. Phys. 10 113011 (2008).