Aharonov-Bohm oscillations in a mesoscopic ring with a quantum dot

A. Levy Yeyati

Abstract

We present an analysis of the Aharonov-Bohm oscillations for a mesoscopic ring with a quantum dot inserted in one of its arms. It is shown that microreversibility demands that the phase of the Aharonov-Bohm oscillations changes abruptly when a resonant level crosses the Fermi energy. We use the Friedel sum rule to discuss the conservation of the parity of the oscillations at different conductance peaks. Our predictions are illustrated with the help of a simple one channel model that permits the variation of the potential landscape along the ring.

PACS numbers: 72.15.Gd, 73.20.Dx

A recent experiment by Yacoby et al. [1] investigated the Aharonov-Bohm (AB) oscillations in a ring with a quantum dot (see Fig. 1). This experiment is of fundamental interest since it depends not only on the total transmission through the quantum dot but also on the phase accumulated by carriers traversing the dot. The experiment thus gives a direct demonstration that coherent resonant tunneling and sequential tunneling are not equivalent [2-4]. Yacoby et al. emphasize two features of the Aharonov-Bohm oscillations: First, it was found that the phase of the AB oscillations changes abruptly whenever transmission through the quantum dot reaches a peak. Second it was found that the AB oscillations at consecutive conductance peaks are in phase. Here we discuss these two observations,
invoking only basic physical principles, and illustrate them with a simple model calculation.

First, consider the phase jump of $\pi$ in the AB-oscillations, which is observed each time a resonant condition is achieved. In a two terminal conductance experiment the measured conductance is necessarily an even function of the AB flux through the ring $G(\Phi) = G(-\Phi)$. In a Fourier representation of the conductance

$$G(\Phi) = G_0 + \Delta \cos(2\pi \Phi/\Phi_0 + \delta) + \ldots,$$

this implies that the phase $\delta$ can only be either zero or $\pi$ but nothing in between. In the experiment the phase $\delta$ is a function of gate voltage. If a phase change occurs as function of gate voltage it must, therefore, be a sharp jump of zero width. We call the two possibilities $\delta = 0$ and $\delta = \pi$ the parity of the AB-oscillations. In contrast Yacoby et al. compare the sharp phase jump with an analysis that leads to a broad transition of the phase, and violates the symmetry demanded by microreversibility. This leads Yacoby et al. to argue that a sharp phase jump is in contradiction with a non-interacting electron-transport picture. Early work on the transmission through one-channel loops does indeed show a symmetry breaking term. Closer inspection of this result shows that the transmission probability is an even function of flux in a two terminal geometry. Below we show that the abrupt phase change is a consequence of microreversibility only. It is a phenomena that occurs independently of whether interactions are significant or not. Moreover the phase jump is abrupt even if there exists inelastic scattering. We conclude that any deviations from a sharp jump must be a consequence of fluctuations in the external control parameters.

We analyze the second feature, the conservation of parity of the AB oscillations at consecutive peaks, with the help of the Friedel sum rule, which remains valid in the presence of electron-electron interactions. The Friedel sum rule relates the phase $\Delta \eta$ accumulated by a carrier traversing a region $\Omega$ to the electronic charge in this volume. The increment of phase and charge are related by

$$dQ = e d\eta/\pi.$$
If the volume $\Omega$ is chosen to include only the quantum dot then each addition of an electron to the dot requires an increase of $\eta$ by $\pi$ (see Fig. 2). Associated with this phase jump there is a parity change of the AB-oscillations at each conductance peak. Consequently the AB-oscillations at consecutive conductance peaks would not be in phase. This is in contrast with the experimental observation of Ref. [1]. However, what counts is not the phase of the quantum dot alone. The ring structure is connected to leads which are in turn connected to reservoirs. As will be shown below, it is the phase accumulated in the entire coherence volume which counts. As a consequence, the relative parity on contiguous resonances might change if the addition of an electronic charge to the quantum dot is accompanied by the addition of a charge $\alpha e$ to the leads of the ring. Over large distances, the arms of the ring can be expected to remain in a charge neutral state. The additional charge is most likely accumulated at the barriers which separate the arms of the ring from the quantum dot. The physical reason is that the gate used to regulate the charge on the dot couples capacitively also to the gates used to form the barriers between dot and ring. A strict conservation of parity of the AB-oscillations occurs if the total charge $(1 + \alpha)e$ added is zero or an even multiple of $2e$. Interestingly, because the phase observed in the transmission coefficient can only be 0 or $\pi$ a “phase-locking” occurs. Even if the additional charge $\alpha$ is not exactly an odd integer the parity of the AB oscillations at a number of consecutive conductance peaks will be the same. We expect that the parity of the AB-oscillations is conserved only over a limited number of peaks and that this number depends on the geometry and electrostatic properties of the sample.

In order to understand the behavior of the AB oscillations in a device like that of Fig. 1a we start by analyzing a single channel noninteracting model. Our aim is to investigate both the influence of inelastic scattering within the dot and of the effective potential landscape along the ring. We use a tight-binding representation of the electron states (the corresponding lattice model is represented in Fig. 1b) which allows for a qualitative description of any potential profile. The effect of the magnetic flux $\Phi$ is taken into account by a phase factor affecting the hopping matrix elements $V_{i,j}$. We denote by $L$, $R$, $D$ and $F$ the left
and right leads, the arm with the dot and the free arm. The effective electrostatic potential on the dot arm is parametrized by the quantities $\epsilon_D$ (dot potential), $\epsilon_B$ (barriers height) and $\epsilon_0$ (potential outside the dot) which are schematically represented in Fig. 1b. Inelastic scattering is simulated by a third lead [2,10–12] (denoted by $I$) coupled to the dot arm by a hopping element $V_I$.

The transmission properties of this model can be easily obtained in terms of Green functions [11–13]. In the absence of inelastic scattering ($V_I = 0$), the two terminal conductance is proportional to the transmission coefficient $T_{LR}$, which can be written in terms of the retarded Green functions as [14]

$$T_{LR} = 4V_L^2V_R^2|G_{0,N+1}(E_F)|^2\text{Im}g_L(E_F)\text{Im}g_R(E_F),$$

where $g_{L,R}(E_F)$ denote the local Green functions on the uncoupled leads at the Fermi energy and $V_{L,R}$ are the hopping elements connecting the ring to the leads. One can establish a correspondence between $2V_LV_R\sqrt{\text{Im}g_L(E_F)\text{Im}g_R(E_F)}G_{0,N+1}(E_F)$ and the elastic transmission amplitude $t$ for this single channel case. The phase $\eta$ of $t$ is, therefore, equal to that of $G_{0,N+1}(E_F)$.

Taking the case where $V_L = V_R = 0$ as the unperturbed case for which the isolated ring Green functions are denoted by $g_{i,j}$, $G_{0,N+1}$ can be written as

$$G_{0,N+1} = \frac{g_{0,N+1}}{(1 - g_{0,0}\Sigma_L)(1 - g_{N+1,N+1}\Sigma_R) - g_{0,N+1}\Sigma_R g_{N+1,0}\Sigma_L},$$

where $\Sigma_{L,R} = V_{L,R}^2g_{L,R}$. For a ring without inelastic scattering the functions $g_{i,j}$ behave as $\exp[\iota\phi(i - j)/(N + 1)]f_{i,j}(\phi)$ where $2\phi = \pi\Phi/\Phi_0$ is the phase associated to the magnetic flux and $f_{i,j}$ is a real even function of $\phi$. The transmission coefficient, therefore, satisfies the symmetry relation $T_{LR}(\Phi) = T_{RL}(-\Phi)$ [5]. We can analyze the flux dependence of the two terminal conductance in this case by coupling the
ring to the third lead. The condition of no net current flow through this lead yields a two terminal conductance proportional to the total transmission probability, given by

\[ T_{\text{total}} = T_{LR} + \frac{T_{LI} T_{IR}}{1 - R_{II}}, \quad (5) \]

where \( R_{II} \) is the reflection probability on the third lead. Taking into account the property \( \sum_j T_{ij} = 1 - R_{ii} \) one can easily show that

\[ T_{\text{total}} = 1 - R_{LL} + \frac{T_{LI} T_{IL}}{1 - R_{II}}, \quad (6) \]

and therefore \( T_{\alpha\beta}(\Phi) = T_{\beta\alpha}(-\Phi) \) implies that \( T_{\text{total}} \) is an even function of the magnetic flux. This simple calculation shows that even in the presence of inelastic scattering the only possible phases for the AB oscillations are 0 and \( \pi \) and thus the transition from one to the other should always be abrupt. The only effect of inelastic scattering is to reduce the amplitude of the AB oscillations by decreasing the direct elastic transmission \( T_{LR} \).

Notice that this result is also true at finite temperatures: thermal averaging can degrade the amplitude of the AB oscillations but cannot introduce additional phases between 0 and \( \pi \). The only possible sources of phase smearing in the experiments should be traced to fluctuations in the gate voltages.

We can thus study the parity of the AB oscillations by computing \( \Delta_2 = \partial^2 T_{LR}/\partial\Phi^2 \big|_{\Phi=0} \) which tells us whether \( \delta = 0 \) (\( \Delta_2 < 0 \)) or \( \delta = \pi \) (\( \Delta_2 > 0 \)). We now show how the parity change in the AB effect is related to the parity effect of the isolated ring. It is well known \[15\] that, for the case of spinless electrons, the ring with an odd number of particles has a diamagnetic response whereas for even number the response is paramagnetic. In a noninteracting model the parity is determined mainly by the uppermost occupied state. Near a resonant level, we can approximate \( G_{0,N+1} \) as

\[ G_{0,N+1} \sim \frac{\psi_{n_0}^* \psi_{n_{N+1}}^*}{(E_F - \epsilon_n - \Delta_n) + i\Gamma_n}, \quad (7) \]

where \( \epsilon_n \) is the isolated ring eigenvalue closest to \( E_F \), \( \psi_{n_j} \) denote the components of the corresponding wavefunction, and \( \Delta_n \) and \( \Gamma_n \) are the real and imaginary parts of the electron
self-energy due to coupling with the leads \((\Delta_n + i\Gamma_n = |\psi_{n0}|^2 V_L^2 g_L + |\psi_{nN+1}|^2 V_R^2 g_R)\). The only flux sensitive quantities in this expression are \(\epsilon_n\) and \(\psi_n\). In particular, \(\psi_n(\phi) = \exp\left[i\phi j/(N + 1)\right]|\psi_n(0)\), and one has

\[
\frac{\partial G_{0,N+1}}{\partial \phi}\big|_{\phi=0} \sim -iG_{0,N+1},
\]

\[
\frac{\partial^2 G_{0,N+1}}{\partial \phi^2}\big|_{\phi=0} \sim G_{0,N+1}\left[-1 + \frac{1}{(E_F - \epsilon_n - \Delta_n) + i\Gamma_n} \left(\frac{\partial^2 \epsilon_n}{\partial \phi^2}\right)_{\phi=0}\right],
\]

where we have used that \(\epsilon_n(\phi) = \epsilon_n(-\phi)\). The behavior of \(\Delta_2\) near a resonance is thus given by:

\[
\Delta_2 \sim T_{LR} \frac{E_F - \epsilon_n - \Delta_n}{(E_F - \epsilon_n - \Delta_n)^2 + \Gamma_n^2} \left(\frac{\partial^2 \epsilon_n}{\partial \phi^2}\right)_{\phi=0}.
\]

We see that when the resonance corresponds to a paramagnetic state of the isolated ring (i.e. \(\frac{\partial^2 \epsilon_n}{\partial \phi^2}\big|_{\phi=0} < 0\)) \(\Delta_2\) changes from positive to negative as the state crosses the Fermi energy, while the opposite behavior is found when \(\frac{\partial^2 \epsilon_n}{\partial \phi^2}\big|_{\phi=0} > 0\).

Next, let us investigate why the phase of the AB oscillations on contiguous dot resonances appears to be the same. Within the spinless electrons model and assuming that the effect of the dot gate is to modify the value of \(\epsilon_D\) alone, Eq. (9) predicts that the AB oscillations on contiguous resonances should be out of phase. This is illustrated in Fig. 2 where \(T_{LR}(\Phi = 0)\) is plotted as a function of \(\epsilon_D\). The full and dotted lines indicate the regions where \(\Delta_2\) is positive or negative respectively. We also show the phase of the transmission amplitude which, as commented above, is proportional to the electronic charge accumulated within the sample as \(\epsilon_0 - \epsilon_D\) increases. As can be observed, this rigid model for the potential landscape variation leads to an increase in the charge of one electron each time a resonance is crossed.

In a real situation one expects the potential in the regions close to the QD (not only within the dot) to vary as the gate voltage is modified. This effect can be included in our model by allowing \(\epsilon_0\) to vary together with \(\epsilon_D\). Let us assume that this variation can be described by \(\delta \epsilon_0 = a\delta \epsilon_D/(\rho_0 \Delta E)\) where \(\rho_0\) is the mean density of states for the ring regions where the potential equals \(\epsilon_0\) and \(\Delta E\) is the mean separation between dot resonances. The actual relationship between \(\epsilon_0\) and \(\epsilon_D\) should depend on the mutual capacitances between
the ring and the gate electrodes. The effect of this self-consistency condition is simply to add a fractional charge $\alpha e \sim ae$ to the ring between two resonances. Note that $\alpha$ and $a$ are in general not equal since the charge added depends on the actual density of states and not the average density of states $\rho_0$.

Fig. 3 illustrates the effect of increasing the parameter $a$. Notice that the calculated transmission exhibits now a varying background in addition to the dot resonances, which reflects the level structure of the ring. In case (a) the extra charge added to the system is $ae \sim 0.30e$ per cycle. It can be observed that an additional phase jump appears close to the third resonance. Notice that the second and third resonances exhibit now the same parity. For increasing $a$ new phase jumps appear between resonances. In this way, when $\alpha \sim 1$ (Fig. 3b) several peaks with the same parity may be found.

Since the phase $\delta$ of the AB oscillations can only be 0 or $\pi$, it is not necessary to add exactly a multiple of $2e$ to find the same phase at consecutive peaks. Instead, the parity of the AB-oscillations at the $n$-th resonance will be determined by the integer multiple of charge $en_{eff}$ where $n_{eff}$ is the integer that is closest to the charge $n(1 + \alpha)$ added after $n$ cycles. For $-1 \leq \alpha \leq -0.5$, (if the ring and dot remain approximately charge neutral) this will create a sequence of effective charge states $en_{eff}$ with $n_{eff} = 0$ for a number of cycles $k$. The parity will change after the first $k$ cycles which add half an electronic charge and cause the effective charge state to jump to $en_{eff} = e$. Hence for this case the number of parity conserving cycles is $k(1 + \alpha) = 1/2$ or $k = (1/2)(1 + \alpha)^{-1}$. For $0.5 \leq \alpha \leq 1$ (if we add nearly two electrons) we will still obtain an effective charge sequence $en_{eff}$ with $en_{eff}$ equal to an even multiple of $e$ but only for a finite sequence of cycles. The parity will change after $k$ cycles for which a deficit of half an electronic charge occurs. For this case the number of parity conserving cycles is $k(|\alpha - 1|) = 1/2$ or $k = (1/2)(|\alpha - 1|)^{-1}$. If $\alpha$ is in the interval $-0.5 < \alpha < 0.5$, then the parity will change at every peak except, occasionally, when the effective charge state jumps by $2e$. For $\alpha$ in this interval we can at most observe two consecutive peaks which are in phase. Thus we find that it is possible to observe many consecutive conductance peaks at which the parity of the AB-oscillations is
conserved if $\alpha \sim -1$ or if $\alpha \sim 1$. Which of the two cases, the approximate preservation of overall charge neutrality, or the addition of nearly two electrons (or another even multiple) per cycle is realized in the experiment cannot be answered without a detailed determination of the relevant capacitance matrix for the structure.

We therefore conclude that within the spinless electron model the conservation of parity of the AB phase on contiguous resonances is indicating that either zero or an \textit{even} number of electrons are added to the system per cycle. We expect that the inclusion of spin degrees of freedom do not change our conclusions: The charging energy of the dot will ensure that in each cycle at most one electronic charge can be added to the dot. The important conclusion of our analysis is that the phase of AB oscillations is not related to the dot charge \textit{alone} but to the total charge of the system. It is the charge of the ring and the dot that counts.

ACKNOWLEDGMENTS

This work was supported by the Swiss National Science Foundation. One of us (A.L.Y.) also acknowledges support by the European Community under contract No.CI1*CT93-0247.
REFERENCES

* Present address: Departamento de Física de la Materia Condensada C-XII, Facultad de Ciencias. Universidad Autónoma de Madrid, E-28049 Madrid, Spain.

[1] A. Yacoby, M. Heiblum, D. Mahalu, and H. Shtrikman, Phys. Rev. Lett. 74, 4047 (1995).

[2] M. Büttiker, IBM J. Res. Develop. 32, 63 (1988).

[3] N. Wingreen et al., Phys. Rev. B 40, 11834 (1989).

[4] H. van Houten, C.W.J. Beenakker and A.A.M. Staring, in Single Charge Tunneling; Edited by H. Grabert and M.H. Devoret, Plenum Press, New York (1991).

[5] M. Büttiker, IBM J. Res. Develop. 32, 317 (1988).

[6] M. Büttiker, Y. Imry and M. Ya. Azbel, Phys. Rev. A30, 1982 (1984).

[7] Y. Gefen, Y. Imry and M. Ya. Azbel, Phys. Rev. Lett. 52, 139 (1983).

[8] J.S. Langer and V. Ambegaokar, Phys. Rev. 121, 1090 (1961).

[9] A. Levy Yeyati, A. Martín-Rodero and F. Flores, Phys. Rev. Lett. 71, 2991 (1993).

[10] S. Datta, Phys. Rev. B 40, 8169 (1989).

[11] J.L. D’amato and H.M. Pastawski, Phys. Rev. B 41, 7411 (1990).

[12] S. Hershfield, Phys. Rev. B 43, 11586 (1991).

[13] A. Levy Yeyati, Phys. Rev. B 45, 14189 (1992).

[14] D.S. Fisher and P.A. Lee, Phys. Rev. B 23, 6851 (1981).

[15] D. Loss and P. Goldbart, Phys. Rev. B43, 13762 (1991); H.F. Cheung, Y. Gefen, E.K. Riedel and W.H. Shih, Phys. Rev. B 37, 6050 (1988).

[16] This feature, which is not observed experimentally, disappears in the numerical calcu-
lations when the ring level separation is much smaller than the self-energy introduced by the leads.
FIGURES

FIG. 1. (a) Schematic representation of a mesoscopic ring threaded by a magnetic flux $\Phi$ with a quantum dot included in one of its arms. (b) Lattice model for this system.

FIG. 2. Transmission probability as a function of dot potential $\epsilon_D$ for fixed potential on the rest of the ring (fixed $\epsilon_0$). The full and dotted lines indicate the regions of positive and negative parity respectively (see text). The dashed line corresponds to the phase of the transmission amplitude.

FIG. 3. Same as in Fig. 2 but allowing $\epsilon_0$ to vary together with $\epsilon_D$. Case (a) corresponds to parameter $a \sim 0.3$ and case (b) to $a \sim 1$. 