During the LEP2 period the $e^+e^-$ collider increased its center of mass energy from 161 GeV to 209 GeV and a total integrated luminosity of approximately 700 pb$^{-1}$ was recorded per experiment. Pairs of W bosons are produced and allow the study of gauge boson couplings involving W, Z and photon. The coupling of the W boson to the neutral gauge bosons have been measured to be $g_1^{Z} = 0.998^{+0.023}_{-0.025}$, $\kappa_\gamma = 0.943^{+0.053}_{-0.033}$, and $\lambda_\gamma = -0.020^{+0.044}_{-0.024}$ and are in agreement with the Standard Model expectation. Limits are set on CP-violating couplings by a Spin Density Matrix analysis of the W decay products. No evidence has been found for couplings of three neutral gauge bosons, parametrized by $f_{1,5}^{Z,\gamma}$ and $h_{1,2,3,4}^{Z,\gamma}$. Limits are derived on couplings of four gauge bosons, parametrized by $a_0^{Z,W}/\Lambda^2$, $a_n^{W}/\Lambda^2$ and $a_{e}^{Z,W}/\Lambda^2$ where $\Lambda$ represents the energy scale for new physics.

1 Gauge Boson Couplings

At LEP2, data are collected at center of mass energies ranging from 161 GeV up to 209 GeV. Massive W bosons are produced in pairs via $e^+e^-$ interactions and gauge couplings involving W, Z and photon are studied by the four LEP experiments ALEPH, DELPHI, L3 and OPAL$^1$. The non-Abelian $SU(2)_L \otimes U(1)_Y$ gauge symmetry of the Standard Model$^2$ predicts tree level interactions between three or four charged and neutral gauge bosons called triple and quartic gauge couplings. At LEP2 energies, triple gauge couplings are directly observed while quartic gauge couplings are negligible. Interactions between neutral gauge bosons do not exist in the Standard Model.
2 Triple Gauge Boson Couplings

The most general Lorentz invariant Lagrangian involving \( W W V \) (\( V = \gamma, Z \)) vertices can be parametrized by 14 real parameters\(^3\)

\[
i \mathcal{L}^{WWV} / g_{WWV} = g_1^V V^\mu (W_{\mu \nu} W^{+\nu} - W^{+\mu} W^{-\nu}) + \kappa_V W^{+\mu} W^{-\nu} V^{\mu\nu} \\
+ \frac{\lambda_V}{M_W^2} V^{\mu\nu} W^{+\nu} W^{-\mu} + ig_5^V \epsilon_{\mu\nu\rho\sigma} ([\partial^\rho W^{+\mu}) W^{+\nu} - W^{-\mu} (\partial^\rho W^{+\nu})] V^\sigma
\]

\[
+ ig_4^V W^{+\mu} W^{-\nu} (\partial^{\mu} V^{\nu} - \partial^{\nu} V^{\mu}) - \frac{\tilde{\lambda}_V}{2 M_W^2} W^{\mu\nu} W^{+\rho} e^{\mu\nu\rho\sigma} V_{\rho\sigma} - \frac{\tilde{\kappa}_V}{2 M_W^2} W^{+\mu} W^{-\nu} e^{\mu\nu\rho\alpha} V_{\rho\alpha\beta}
\]

where \( g_1^V, \kappa_V, \lambda_V \) and \( g_5^V \) are CP-conserving couplings while \( g_4^V, \tilde{\kappa}_V \) and \( \tilde{\lambda}_V \) are CP-violating. Assuming CP-conservation and electromagnetic gauge invariance, five parameters are left : \( g_1^Z, \kappa_Z, \kappa_\gamma, \lambda_Z \) and \( \lambda_\gamma \). The custodial \( SU(2) \) symmetry of the Lagrangian imposes the constraints

\[
\kappa_Z = g_1^Z - (\kappa_\gamma - 1) \tan^2 \theta_W \\
\lambda_\gamma = \lambda_Z
\]

where \( \theta_W \) is the weak mixing angle. Three free parameters are left : \( g_1^Z, \kappa_\gamma \) and \( \lambda_\gamma \). They are related to the magnetic dipole and electric quadrupole moment of the W. The Standard Model predicts their values to be \( g_1^Z = \kappa_\gamma = 1 \) and \( \lambda_\gamma = 0 \) at tree level.

The triple gauge couplings are studied in W-pair production, sensitive to all three parameters, and single W and single photon production which are sensitive to \( \kappa_\gamma \) and \( \lambda_\gamma \) only. The corresponding Feynman diagrams are presented in Figure 1. Only results from W-pair and single W production are used at present for LEP combination.
The third parameter is at its fitted value.

A deviation from a coupling value predicted by the Standard Model would modify the total cross section, the shape of the W production angle $\theta_W$, the polar angle $\theta^*_{W}$ and azimuthal angle, $\phi^*_{W}$ of the W-decay fermions in the corresponding W rest frame. These angles are presented in Figure 2 together with the distribution of the W production angle as measured by the ALEPH experiment in fully hadronic W-pair events. The expected distribution in presence of an anomalous coupling $\lambda_\gamma = \pm 0.5$ is also indicated.

The couplings are extracted by a maximum likelihood fit to the angular distributions (DELPHI, L3) or by a $\chi^2$-fit to Optimal Observables distributions (ALEPH, OPAL). The results from each LEP experiment are then combined using a log-likelihood method.\textsuperscript{1,4}

The result from the three-parameter fit, including LEP2 data from DELPHI, L3 and OPAL, is presented in Figure 3. The LEP combined one-parameter fit results\textsuperscript{1}

\[ g_1^Z = 0.998^{+0.023}_{-0.025}, \quad \lambda_\gamma = 0.943^{+0.055}_{-0.055}, \quad \kappa_\gamma = -0.020^{+0.024}_{-0.024}. \]

are in agreement with the Standard Model prediction. The quoted errors include both statistical and systematic uncertainties. In the LEP combination all W decay channels were used except the semi-leptonic channel for L3 and LEP2 data from $\sqrt{s} = 189$ GeV on for DELPHI.

The correlated systematic uncertainties in the combined fit results are presented in Table 1. The largest contribution comes from the $O(\alpha)$ radiative corrections, mainly due to virtual radiative corrections between the W bosons and the initial and final state particles. The effect is taken into account by Monte Carlo generators as YFSWW3\textsuperscript{5} in the Leading Pole Approximation (LPA) and RacoonWW\textsuperscript{6} in the Double Pole Approximation (DPA) and introduces a 0.7% uncertainty on the slope of the $\cos \theta_W$ distribution. Up to now the full difference between the Monte Carlo prediction with and without $O(\alpha)$ radiative corrections is taken as a systematic.

| Source                        | $g_1^Z$ | $\lambda_\gamma$ | $\kappa_\gamma$ |
|-------------------------------|---------|-------------------|------------------|
| $O(\alpha)$ corrections       | 0.015   | 0.015             | 0.039            |
| Hadronisation                 | 0.004   | 0.002             | 0.004            |
| Bose-Einstein correlation     | 0.005   | 0.004             | 0.009            |
| Colour Reconnection           | 0.005   | 0.004             | 0.010            |
| $\sigma_{WW}$ prediction     | 0.003   | 0.005             | 0.014            |
| $\sigma_{single \ W}$ prediction | 0.011   |                   |                  |
Table 2: The LEP combined one-dimensional limits at 95 % confidence level for the neutral triple gauge couplings.

| 95% CL | $V = Z$ | $V = \gamma$ |
|--------|--------|-------------|
| $f_Y^4$ | [-.31 ; .28] | [-.17 ; .19] |
| $f_Y^5$ | [-.36 ; .39] | [-.36 ; .40] |
| $h_Y^1$ | [-.13 ; .13] | [-.06 ; .06] |
| $h_Y^2$ | [-.08 ; .07] | [-.05 ; .03] |
| $h_Y^3$ | [-.20 ; .07] | [-.05 ; -.01] |
| $h_Y^4$ | [-.05 ; .12] | [-.002 ; .034] |

Neutral triple gauge couplings do not exist in the Standard Model. The most general Lorentz invariant Lagrangian \[^3\,^8\] for the $VVZ$ ($V = \gamma, Z$) vertex is described by 12 parameters. The couplings $h_Y^1, h_Y^2, h_Y^3$ and $h_Y^4$ are studied at LEP in the $e^+e^- \rightarrow Z\gamma$ production, while $f_Y^4$ and $f_Y^5$ are accessible in $e^+e^- \rightarrow ZZ$ production. Electromagnetic gauge invariance and Bose symmetry for final states with identical bosons are imposed. The couplings are determined from the angular distributions of the decay products and the total cross section. No evidence for anomalous $h$- and $f$- couplings has been found. The LEP combined one-dimensional limits at 95 % confidence level are\[^1\,^4\] summarized in Table 2. Both statistical and systematic uncertainties are included.

3 W Spin Density Matrix

The Spin Density Matrix (SDM) method\[^7\] has been introduced to study the W polarisation and is also used to set direct limits on CP-violating couplings, absent in the Standard Model.

Considering the helicity, the W-pair production process is written as

$$e^+(\lambda') e^-(\lambda) \rightarrow W^+(\tau_2) W^-(\tau_1) ,$$  

(3)

where $\lambda (\lambda') = \pm 1/2$ represents the helicity of the electron (positron). The helicities of the $W^-$ and the $W^+$, denoted by $\tau_1$ and $\tau_2$ respectively, take the value $\tau = \pm 1$ for transversely polarised W bosons and the value $\tau = 0$ for W bosons with a longitudinal polarisation.

The two-particle joint SDM elements are then defined as\[^9\,^10\]

$$\rho_{\tau_1\tau_1'\tau_2\tau_2'}(s, \cos \theta_W) = \frac{\sum_{\lambda} F^\lambda_{\tau_1\tau_2} (F^\lambda_{\tau_1'\tau_2'})^*}{\sum_{\lambda, \tau_1, \tau_2} |F^\lambda_{\tau_1\tau_2}|^2} ,$$  

(4)

where $s$ is the center of mass energy and $F^\lambda_{\tau_1\tau_2}$ is the helicity amplitude for the production of a W pair with helicities $\tau_1$ and $\tau_2$. The single particle SDM elements are obtained by summation over all possible helicities of one of the W’s

$$\rho^W_{\tau_1\tau_1'}(s, \cos \theta_W-) \equiv \sum_{\tau_2} \rho_{\tau_1\tau_1'\tau_2\tau_2}(s, \cos \theta_W-) .$$  

(5)

The SDM elements are constrained by Hermiticity and their diagonal terms are normalised to unity $\sum_{\tau} \rho^W_{\tau\tau} = 1$. The diagonal elements of the SDM matrix are real and express the probability to produce a $W^-$ with helicity $\tau_1$. The off-diagonal elements are complex and provide a test of CP-violation.
Figure 4: The nine single SDM elements, $\rho_{\tau\tau^{'}}$, as a function of $\cos \theta_{W^-}$. The errors are statistical only.

The SDM elements are calculated in bins of $\cos \theta_{W^-}$ using a projection operator method assuming a V-A decay of the W boson into fermions

$$\rho^W_{\tau\tau^{'}}(k) = \frac{1}{N_k} \sum_{i=1}^{N_k} \Lambda^W_{\tau\tau^{'}}(\theta^*_f, \phi^*_f)_i ,$$

where $N_k$ is the number of events in the $k$-th bin and where the projection operator $\Lambda^W_{\tau\tau^{'}}$ is applied event by event. The reconstructed SDM elements need to be corrected for detector acceptance, resolution effects and background contamination for a direct comparison with the theoretical expectation.

The SDM elements measured by L3 in the decay channels $q\bar{q}e\nu_e$ and the $q\bar{q}\mu\nu_\mu$ at $\sqrt{s} = 189 - 209$ GeV are combined and presented in Figure 4. The measurements for the leptonically decaying $W^+$ and $W^-$ are combined assuming CPT-invariance. A good agreement is found with the Standard Model prediction represented by the solid line. The expected distributions in presence of an anomalous CP-conserving coupling $\Delta\kappa_\gamma = +0.5$ (dotted line) and the CP-violating coupling $\tilde{\lambda}_Z = -0.5$ (dashed line) are also shown.

The imaginary parts of the off-diagonal elements are insensitive to CP-conserving couplings and only contribute in presence of tree level CP-violation. This makes the SDM method particularly suitable to measure CP-violating couplings which are extracted by a $\chi^2$-fit to the nine SDM-element distributions. As the $\cos \theta_{W^-}$ information is averaged out in the definition of the SDM elements, the shape of the $W$ production angle is incorporated in the fit to increase the sensitivity. The following results are obtained by the OPAL experiment with the $q\bar{q}\nu\ell$ events selected at 189 GeV

$$g_4^Z = -0.01^{+0.32}_{-0.33} \quad \tilde{\kappa}_\gamma = -0.18^{+0.24}_{-0.16} \quad \tilde{\lambda}_\gamma = -0.20^{+0.10}_{-0.07} .$$

All couplings are set to their Standard Model value except the measured one and these related to it by custodial $SU(2)$ symmetry. Both statistical and systematic uncertainties are included.
In the Standard Model W-pair production is assumed to be a CPT- and CP-invariant process, hence the following relations are satisfied

\[ \text{CPT} - \text{invariance} : \Im(\rho^W_{\tau\tau'}) + \Im(\rho^W_{-\tau-\tau'}) = 0 \]  
\[ \text{CP} - \text{invariance} : \Im(\rho^W_{\tau\tau'}) - \Im(\rho^W_{-\tau-\tau'}) = 0 \]

The imaginary part of all SDM elements has to be zero and deviations from equation (8) provide an unambiguous signature for CP-violation at tree level. Deviations from equation (7) would arise from loop effects beyond tree level or CPT-violation. Figure 5 shows the test of CPT-invariance (right) and CP-invariance (left) measured with $q\bar{q}\mu\nu$ events selected by DELPHI at $\sqrt{s} = 189$ GeV. Within the statistical error, the sum, as well as the differences, of the imaginary parts are compatible with zero and confirm the absence of CPT- and CP-violation at tree level as predicted by the Standard Model (solid line). This is confirmed by the L3 and OPAL results.

In DELPHI and OPAL, the SDM analysis is also used to measure the cross sections for the production of transversely and longitudinally polarised W bosons which are a consequence of the spontaneous symmetry breaking mechanism in the Standard Model. The fraction of longitudinally polarised W bosons measured by OPAL at 189 GeV in the $q\bar{q}\nu\ell$ channel is $\sigma_L/\sigma_{\text{total}} = 21.0 \pm 3.3 \pm 1.6 \%$ where the first error is statistical and the second systematic. This is in agreement with the Standard Model expectation of 25.7%.

In L3, the W polarisation is measured by a direct fit of analytical helicity distributions to the shape of the polar angle $\theta^*_W$ of the W decay products in semi-leptonic W pair events. The W helicity fractions are presented in Figure 6 in four different bins of $\cos \theta_W$. The fraction of longitudinally polarised W bosons measured with the L3 detector using $q\bar{q}\ell\nu$ events at $\sqrt{s} = 183 - 209$ GeV is $21.8 \pm 2.7 \pm 1.6\%$ and in agreement with the Standard Model expectation of 24.1%. Separate analyses of the $W^+$ and $W^-$ events are consistent with CP-conservation.
Figure 6: The W helicity fractions in four different bins of \(\cos\theta_W\) measured with the L3 detector using \(q\bar{q}l\nu\) events at \(\sqrt{s} = 183 - 209\) GeV compared to the Standard Model predictions from the KORALW Monte Carlo\(^{15}\).

Table 3: The one-dimensional limits on quartic gauge couplings set by DELPHI, L3 and OPAL at 95 % CL.

| 95 % CL ( GeV\(^{-2}\)) | \(a_W^0/\Lambda^2\) | \(a_W^c/\Lambda^2\) | \(a_W^n/\Lambda^2\) |
|---------------------------|------------------|------------------|------------------|
| DELPHI                    | [-.018 ; +.018]  | [-.057 ; +.030]  | [.16 ; +.12]     |
| L3                        | [-.015 ; +.015]  | [-.048 ; +.026]  | [.14 ; +.13]     |
| OPAL                      | [.054 ; +.052]   | [.15 ; +.14]     | [.61 ; +.57]     |

4 Quartic Gauge Couplings

The Standard Model quartic gauge couplings contribution are too small to be seen at LEP and any deviation is therefore a hint for new physics. Deviations are introduced into the Lagrangian\(^{16,17}\) as effective couplings at a new physics scale \(\Lambda\).

Starting from electromagnetic gauge invariance and custodial \(SU(2)\) symmetry, the most general Lorentz invariant Lagrangian has 5 parameters. The quartic gauge couplings \(a_W^0/\Lambda^2\), \(a_W^n/\Lambda^2\) and \(a_W^c/\Lambda^2\) are studied in the \(e^+e^- \rightarrow W^+W^-\gamma\) process and in W fusion into a final state with two photons and missing energy due to the emission of two neutrino’s, while the neutral quartic gauge couplings \(a_Z^0/\Lambda^2\) and \(a_Z^c/\Lambda^2\), not existent in the Standard Model, are searched for in the \(e^+e^- \rightarrow ZZ\gamma\) process. Both are mainly determined from the photon energy spectrum and the total cross section.

No evidence for anomalous quartic gauge couplings has been found. The one-dimensional limits on quartic gauge couplings set by DELPHI\(^{18}\), L3\(^{19}\) and OPAL\(^{20}\) at 95 % confidence level are summarized in Table 3. Both statistical and systematic uncertainties are included.

The one-dimensional limits on neutral quartic gauge couplings set by L3 and OPAL at 95 % confidence level are\(^{1}\)

\[-.009 < a_Z^0/\Lambda^2 < .026\]

\[-.033 < a_Z^c/\Lambda^2 < .046\]
New results from ALEPH\textsuperscript{21} yield

\[-0.11 < a_\theta^2 / \Lambda^2 \cdot \text{GeV}^2 < 0.17 \quad -0.37 < a_\xi^2 / \Lambda^2 \cdot \text{GeV}^2 < 0.40\]

at 95 \% confidence. Both statistical and systematic uncertainties are included. A LEP combination of the quartic gauge couplings is expected soon.

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