Reflectance of Dirac electrons in organic conductor

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Abstract. We examine the reflectance $R$ of Dirac electrons of organic conductor $\alpha$-(BEDT-TTF)$_2$I$_3$ with a tilted cone which is characterized by a degree of the tilting $\eta (= 0.8)$ and the angle $\alpha$ measured from the applied electric field. The reflectance is calculated using the complex dielectric constant, $\varepsilon_1 + i\varepsilon_2$, which is evaluated from the in-plane dynamical conductivity with a constant damping $\Gamma$. The bulk reflectance on the two-dimensional plane is estimated by taking account of an interlayer distance of organic conductor $\alpha$-(BEDT-TTF)$_2$I$_3$. We compare a difference of $R$ between $\alpha = 0$ (i) and $\pi/2$ (ii), which correspond to the tilting being parallel and perpendicular to the electric field, respectively. The frequency $\omega$ dependence of $R$ is examined for several magnitudes of the chemical potential $\mu$ to find a crossover from the zero doping to a finite doping. With increasing $\omega$, $R$ decreases from unity. For the small doping with $\mu < \Gamma$, a difference of $R$ between (i) and (ii) is small, while, for the large doping $\Gamma < \mu$, a noticeable difference of $R$ between (i) and (ii) exists in a certain region of $\omega$ around $2\mu$. The behavior that $R$ of (ii) is larger than that of (i) is enhanced by the doping. We discuss the case of the clean limit and the role of both the intraband excitation and the interband excitation in the presence of $\Gamma$.

1. Introduction

The Dirac electrons in organic conductors have been studied extensively since the discovery in organic conductor $\alpha$-(BEDT-TTF)$_2$I$_3$ [1] based on the X-ray Diffraction experiment [2]. There are several characteristics of Dirac electron in organic conductor, e.g., the presence of the tilting of Dirac cone and four sites per unit cell [3] compared with that of graphene [4].

Using an effective Hamiltonian which describes the electrons close to the Dirac point [5], the transport properties such as Nernst effect [6] and electric conductivity [7, 8] have been examined where the effect of tilting gives rise to a noticeable anisotropy of the conductivity for both the zero doping and finite but small doping in contrast to the isotropic Dirac cone [9]. The effect of tilting on the dynamical conductivity appears even for large frequency where the chemical potential is close to the onset of interband excitation [10].

In the present paper, we examine the reflectance based on the previous calculation of the in-plane dynamical conductivity [8]. By comparing with the isotropic case[11, 12], we show how the effect of tilting appears in the crossover from the intraband excitation to the interband excitation with increasing a frequency $\omega$.

In section 2, the formulation is given where the reflectance $R$ on the plane is estimated from the dielectric constant based on the dynamical conductivity with a frequency $\omega$. In section
3. \( R \) as a function of \( \omega \) is examined to show the crossover from the intraband excitation to the interband excitation which depends on the magnitude of the chemical potential, i.e., the degree of doping. The section 4 is devoted to summary and discussion on the relevance to the reflectance in organic conductors.

2. Formulation

We consider an effective Hamiltonian for the organic conductor expressed as

\[
H = v_0 \cdot p + v_x p_x \sigma_x + v_y p_y \sigma_y ,
\]

where \( p = (p_x, p_y) \) is a two-dimensional momentum and \( \sigma = (\sigma_x, \sigma_y) \) are Pauli matrices. The velocities \( v = (v_x, v_y) \) and \( v_0 = (v_{0x}, v_{0y}) \) determine anisotropy of the Dirac cone and the tilting of the cone axis, respectively. The eigenvalues of the Hamiltonian in Eq. (1) are given by

\[
E_p^{(\pm)} = v_0 \cdot p \pm \sqrt{v_x^2 p_x^2 + v_y^2 p_y^2}.
\]

Defining \( \alpha \) as the angle between the projection of the tilted Dirac cone axis on the \( x-y \)-plane and the \( x \)-axis, and using the parametrization \( v_{0x} = \eta v_x \cos \alpha \), \( v_{0y} = \eta v_y \sin \alpha \) with \( \eta = \sqrt{(v_{0x}/v_x)^2 + (v_{0y}/v_y)^2} \), \( v_x p_x = k \cos \theta_p \), and \( v_y p_y = k \sin \theta_p \), we can rewrite the eigenvalues in Eq. (2) as

\[
E_p^{(\pm)} = \eta k \cos(\theta_p - \alpha) \pm k,
\]

with the corresponding eigenfunctions given by \( \psi_p^{(+)} = [\! -e^{-i\theta_p}, 1 \!]^T / \sqrt{2} \) and \( \psi_p^{(-)} = [1, e^{i\theta_p}]^T / \sqrt{2} \). The parameter \( \eta \) \((0 \leq \eta < 1)\) represents degree of the tilting. The electron current density operators \( j = -e \partial H / \partial p \) in the same parametrization can be expressed as

\[
\begin{align*}
  j_x &= -e v_x (\eta \cos \alpha + \sigma_x), \\
  j_y &= -e v_y (\eta \sin \alpha + \sigma_y),
\end{align*}
\]

where \(-e\) denotes the charge of the electron.

Within the framework of the Kubo formula, the dynamical conductivity with a frequency \( \omega \) can be written in the well-known form: [6, 7, 8, 13, 14]

\[
\sigma_{\mu\nu}(\omega) = \frac{i}{\omega} \int_{-\infty}^{\infty} d\varepsilon f(\varepsilon) \times \text{Tr} \left\{ j_\mu \left[ G^{(+)}(\varepsilon + i\omega) - G^{(+)}(\varepsilon) \right] j_\nu A(\varepsilon) - j_\mu A(\varepsilon) j_\nu \left[ G^{(-)}(\varepsilon) - G^{(-)}(\varepsilon - i\omega) \right] \right\},
\]

where \( f(\varepsilon) \) is the Fermi distribution function with a chemical potential \( \mu \) and a temperature \( T(\rightarrow 0) \). \( A = \frac{1}{\pi} \left[ G^{(+)} - G^{(-)} \right] \) and Green’s functions are defined by \( G^{\pm}(\varepsilon) = (\varepsilon - H \pm i\Gamma)^{-1} \) with a damping constant \( \Gamma \). We treat \( \Gamma \) as a constant parameter while the dynamical effect on \( \Gamma \) within the self-consistent Born approximation [15] is discussed in Ref. [8]. In Eq. (5) the conductivities per spin and per valley are considered, implying that the final result should be multiplied by 4.

The conductivity \( \sigma_{xx} \) is expressed as \( (v_x = v_y) \)

\[
\sigma(\omega, \eta, \alpha) = \sigma_{\text{intra}} + \sigma_{\text{inter}} = \sigma_1(\omega) + i \sigma_2(\omega),
\]

where \( \sigma_1 \) and \( \sigma_2 \) denote the real part and the imaginary part. Quantities \( \sigma_{\text{intra}} \) and \( \sigma_{\text{inter}} \) denote the intraband and interband contributions coming from the excitation within the same energy
band and the excitation from $E_{p}^{(-)}$ to $E_{p}^{(+)}$, respectively. The real part $\sigma_{1}(\omega)$ of $\sigma(\omega, \eta, \alpha)$ is calculated as $\sigma_{1} = \text{Re}\{\sigma_{\text{intra}}^{(+)}\} + \text{Re}\{\sigma_{\text{intra}}^{(-)}\} + \text{Re}\{\sigma_{\text{inter}}\}$ where

$$\text{Re}\{\sigma_{\text{intra}}^{(\pm)}\} = \left. \int_{0}^{\infty} \frac{dy}{y} \left( \eta \cos \alpha \pm \cos \theta \right)^{2} f(E^{(\pm)}_{\text{intra}}) \right|_{\theta} ,$$

$$\text{Re}\{\sigma_{\text{inter}}\} = \left. \int_{0}^{\infty} \frac{dy}{y} \left( \sin \theta \right)^{2} \left[ f(E^{(+)_{\text{inter}}}, E^{(-)_{\text{inter}}}) + f(E^{(-)_{\text{inter}}}, E^{(+)_{\text{inter}}}) \right] \right|_{\theta} ,$$

where $\theta = \theta_{p}$, and $\langle \cdots \rangle_{\theta}$ denotes $(2\pi)^{-1} \int_{0}^{2\pi} d\theta \cdots$. The quantity $f(a, b)$ is defined by

$$f(a, b) = \frac{e^{2}}{\hbar^{2} \omega} \int_{-\infty}^{\infty} dz \left\{ \frac{\Gamma}{(z-a)^{2} + \Gamma^{2}} \times \frac{\Gamma}{(z + \hbar \omega - b)^{2} + \Gamma^{2}} - (\omega \rightarrow -\omega) \right\} .$$

The real part and imaginary parts of $\sigma(\omega, \eta, \alpha)$ satisfy the relations $\sigma_{1}(\omega) = \sigma_{1}(-\omega)$ and $\sigma_{2}(\omega) = -\sigma_{2}(-\omega)$, respectively. The imaginary part $\sigma_{2}(\omega)$ is calculated from the real part using the Kramers-Kronig relation,

$$\sigma_{2}(\omega) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dz}{z - \omega} \sigma_{1}(z) ,$$

where the numerical integration is performed for $\omega_{e} = 30$. The limiting behavior of large $\mu$ gives the conductivity in the clean limit $\Gamma \rightarrow +0$ (Appendix A) [9, 10, 8] in which the real part of $\sigma_{\text{intra}} + \sigma_{\text{inter}}$ is expressed as

$$\text{Re}\{\sigma_{\text{intra}}\} = \frac{e^{2}}{\pi \hbar} \frac{\pi^{2} \mu \delta(\hbar \omega)}{2} \left\{ \frac{1 - \sqrt{1 - \eta^{2}}}{\eta^{2}} \cos^{2} \alpha + \frac{1 - \sqrt{1 - \eta^{2}}}{\eta^{2} \sqrt{1 - \eta^{2}}} \sin^{2} \alpha \right\} ,$$

$$\text{Re}\{\sigma_{\text{inter}}\} = \frac{e^{2}}{\pi \hbar} \frac{\pi^{2}}{8} F_{c}(\alpha) .$$

$F_{c}(\alpha) = 0$ for $\omega < 2\mu/(1 + \eta)$, $F_{c}(\alpha) = 1$ for $2\mu/(1 - \eta) < \omega$, and $F_{c}(\alpha) = \pi^{-1}(\varphi_{0} - \sin \varphi_{0} \cos \varphi_{0} \cos(2\alpha)) < 1$ with $\cos \varphi_{0} = (2\mu/\omega - 1)/\eta$ for $2\mu/(1 + \eta) < \omega < 2\mu/(1 - \eta)$. Note that $F_{c}(\alpha)$ with a fixed $\omega$ takes a maximum (minimum) at $\alpha = \pi/2$ and a minimum (maximum) at $\alpha = 0$ for $2\mu/(1 + \eta) < \omega < 2\mu(2\mu < \omega < 2\mu/(1 - \eta))$.

In terms of Eq. (6), the dielectric constant is calculated from

$$\varepsilon(\omega, \eta, \alpha) = 1 + \frac{4\pi \tilde{\sigma}(\omega, \eta, \alpha)}{\omega} = \varepsilon_{1}(\omega, \eta, \alpha) + i \varepsilon_{2}(\omega, \eta, \alpha) ,$$

where $\tilde{\sigma}(\omega, \eta, \alpha) = C \sigma(\omega, \eta, \alpha)/d$ with an interlayer distance $d$ and $C = 4$ for the freedom of spin and valley. Hereafter we introduce normalized quantities as

$$\frac{\hbar \omega}{\Gamma} \rightarrow \overline{\omega} , \quad \frac{\sigma}{e^{2}/\pi \hbar} \rightarrow \overline{\sigma} , \quad \frac{\mu}{\Gamma} \rightarrow \overline{\mu} ,$$

where $h$ denotes the Plank constant. Using the dynamical conductivity $\overline{\sigma}(\overline{\omega}, \eta, \alpha)$ for both zero and finite doping[8], the real part and imaginary parts $\varepsilon_{1}$ and $\varepsilon_{2}$ in Eq. (13) are obtained as

$$\varepsilon_{1}(\overline{\omega}, \eta, \alpha) = 1 - 4\pi F \overline{\sigma}_{2}(\overline{\omega}, \eta, \alpha)/\overline{\omega} ,$$

$$\varepsilon_{2}(\overline{\omega}, \eta, \alpha) = 4\pi F \overline{\sigma}_{1}(\overline{\omega}, \eta, \alpha)/\overline{\omega} ,$$

where

$$F = \frac{2e^{2}}{\pi^{2} d \Gamma} .$$
with \( d \) being an interlayer distance. The quantity \( F \) is estimated as \( F \sim 660 \) when \( \Gamma/k_B \simeq 3 \) K [6] and \( d \simeq 1.7 \times 10^{-9} \) m [16]. Finally the reflectance \( R \) is calculated as

\[
R(\omega, \eta, \alpha) = \left| \frac{\sqrt{\varepsilon_1 + \varepsilon_2} - 1}{\sqrt{\varepsilon_1 + \varepsilon_2} + 1} \right|^2. \tag{17}
\]

Note that \( F \) corresponds to the plasma frequency where the reflectance decreases for \( \omega > F \) owing to \( \varepsilon_2 \).

The characteristics of Dirac electrons of organic conductors is examined by choosing \( \eta = 0.8 \) with two limiting angles \( \alpha = \pi/2 \) and \( \alpha = 0 \) which correspond to the tilting direction being perpendicular to the electric field and parallel to the electric field, respectively. The case of \( \eta = 0 \) without tilting of the isotropic Dirac cone is shown for the comparison.

3. Reflectance

In Fig. 1, the conductivity as a function of \( \omega \) is shown for both the real part \( \sigma_1 \) and the imaginary part \( \sigma_2 \) with four cases of \( \mu = 0 \) (a), \( 2 \) (b), \( 4 \) (c) and \( 8 \) (d). The solid and dashed lines represent the case for \( \eta = 0.8 \) with \( \alpha = \pi/2 \) and \( \alpha = 0 \), while the dotted line represents that for \( \eta = 0 \). The asymptotic value of \( \sigma_1(\eta) \) for large \( \omega \) is given by \( \pi^2/8 \) (0). For \( \mu = 0 \), \( \sigma_1 \) takes a maximum while that of \( \eta = 0 \) increases monotonously. The imaginary part shows \( \sigma_2 > 0 \) for \( \alpha = \pi/2 \) and both \( \sigma_2 < 0 \) and \( \sigma_2 > 0 \) for \( \alpha = 0 \) in contrast to that of \( \eta = 0 \) showing \( \sigma_2 < 0 \) with a minimum.

With increasing \( \mu = 2, 4, \) and \( 8 \), \( \sigma_1(\alpha, \eta) = \sigma_1(\varepsilon_0, \eta, \alpha) \) ) at \( \sigma_0 = 0 \) increases owing to the increase of the density of states at \( \omega = \mu \). There is a characteristic frequency \( \omega_0 \) showing a crossover where \( \sigma_1(\pi/2, 0.8) > \sigma_1(0, 0.8) > \sigma_1(0, 0) \) for small \( \omega \) and \( \sigma_1(0, 0) > \sigma_1(0, 0.8) > \sigma_1(\pi/2, 0.8) \) for the large \( \omega \). The behavior around the characteristic frequency suggests the noticeable effect of the interband excitation as shown in Eq. (12), i.e., in the clean limit, where the characteristic frequency is given by \( \omega = 2\pi F \) for \( \mu >> 1 \). As for the imaginary part \( \sigma_2 \) of \( \eta = 0.8 \), there is a maximum which increases with increasing \( \mu \) due to the increase of \( \sigma_1 \) as seen from Eq. (10). For \( \eta = 0 \), a change from \( \sigma_2(0, 0) < 0 \) into \( \sigma_2((0, 0) > 0 \) occurs with increasing \( \mu \) where \( \sigma_2 \) for the large \( \mu \) takes both maximum and minimum. The effect of the interband excitation becomes prominent for the large \( \mu \) (i.e., \( \mu >> \Gamma \)).

In Fig. 2, the \( \omega \) dependence of the dielectric constant \( \varepsilon(\alpha, \eta) = \varepsilon(\varepsilon_0, \eta, \alpha) \) is shown corresponding to Fig. 1. It is found that \( \varepsilon_2(\alpha, \eta) > 0 \) from Eq. (15) and \( \sigma_1 > 0 \). For \( \mu = 0 \), the difference of these three lines of \( \varepsilon_2 \) reduces rapidly since \( \sigma_1 \) is reduced to \( \pi^2/8 \) rapidly with increasing \( \omega \). The difference of three lines of \( \varepsilon_1 \) comes from \( \sigma_2 \) as seen from Eq. (14) and Fig 1 (a). The introduction of \( \mu \) gives the enhancement of the difference among these lines for both \( \sigma_1 \) and \( \sigma_2 \). For \( \mu = 2 \), the behavior of \( \sigma_1 \) gives \( \varepsilon_2(\pi/2, 0.8) > \varepsilon_2(0, 0.8) \) for small \( \omega \) (not shown in Fig. 2 ) and \( \varepsilon_2(0, 0.8) < \varepsilon_2(\pi/2, 0.8) \) for large \( \omega \). The maximum of \( \varepsilon_1 \) for \( \alpha = 0 \) is reduced owing to the increase of \( \sigma_2 \). For \( \mu = 4 \) and \( 8 \), \( \varepsilon_2 \) exhibits a crossover where \( \varepsilon_2(\pi/2, 0.8) > \varepsilon_2(0, 0.8) > \varepsilon_2(0, 0) \) for small \( \omega \) and \( \varepsilon_2(0, 0) > \varepsilon_2(0, 0.8) > \varepsilon_2(\pi/2, 0.8) \) for large \( \omega \). Such a relation is the same as that of \( \sigma_1 \) in Fig. 1 as seen from Eqs. (15) and (12). The real part of \( \varepsilon(\alpha, \eta) \) shows \( \varepsilon_1(0, 0) > \varepsilon_1(0, 0.8) > \varepsilon_1(\pi/2, 0.8) \) in the most region of \( \omega \) where the negative value of \( \varepsilon_1 \) is enhanced by \( \mu \) suggesting a metallic behavior.
Figure 1. Frequency dependence of the conductivity $\sigma(\alpha, \eta) = \sigma(\omega, \eta, \alpha)$ for the real part $\sigma_1$ and imaginary part $\sigma_2$ for $\mu = 0$ (a), 2 (b), 4 (c), and 8 (d). The horizontal and vertical axes are expressed by normalized quantities $\omega (= \bar{\omega} \omega / \Gamma)$, and $\sigma (= \sigma / (e^2 / \pi \hbar))$. The solid curve and dashed curve denote $\alpha = \pi / 2$ and $\alpha = 0$, respectively, with $\eta = 0.8$. The dotted curve corresponds to $\sigma_1$ and $\sigma_2$ for $\eta = 0$. The dashed line is always located between the solid line and the dotted line.
Figure 2. \( \varpi \) dependence of the real part \( \varepsilon_1(\varpi, \eta, \alpha) \) and the imaginary part \( \varepsilon_1(\varpi, \eta, \alpha) \) of the dielectric constant for \( F = 660 \) with \( \mu = 0 \) (a), 1 (b), 4 (c), and 8 (d). The horizontal axis denotes \( \varpi (= \hbar \omega / \Gamma) \). The solid line and the dashed line denote \( \alpha = \pi / 2 \) and 0 for \( \eta = 0.8 \), respectively. The dotted line corresponds to \( \eta = 0 \).

In Fig. 3, the \( \varpi \) dependence of the reflectance \( R(\alpha, \eta) (= R(\varpi, \eta, \alpha)) \) corresponding to Fig. 1 is shown for \( \mu = 0 \) (a), 2 (b), 4 (c), and 8 (d) where \( F = 660 \). The solid line and the dashed line denote \( R(\pi / 2, 0.8) \) and \( R(0, 0.8) \) while the dotted line denotes \( R(0, 0) \), i.e., without tilting. For \( \mu = 0 \), the monotonous decrease of \( R \) is seen with a relation \( R(\pi / 2, 0.8) > R(0, 0.8) > R(0, 0) \). The fact that \( R(\pi / 2, 0.8) \) is slightly larger than \( R(0, 0.8) \) and \( R(0, 0) \) comes from \( \varepsilon_1(\pi / 2, 0.8) < 0 \) suggesting the effect of \( \sigma_2 \) being larger than that of \( \sigma_1 \). For \( \mu = 2 \), \( R(\pi / 2, 0.8) \) is enhanced owing to the presence of the Fermi surface. There is a crossover from \( R(0, 0.8) > R(0, 0) \) to \( R(0, 0) > R(0, 0.8) \) around \( \varpi \sim 4 \) with increasing \( \varpi \). The effect of \( \mu \) is enhanced for \( \mu = 4 \).
Figure 3. \( \varpi \) dependence of reflectance \( R(\varpi, \eta, \alpha) \) for \( F = 660 \) with \( \varpi = 0 \) (a), 2 (b), 4 (c), and 8 (d). The horizontal axis denotes \( \varpi (\equiv \hbar \omega / \Gamma) \). The solid line and the dashed line denote \( \alpha = \pi / 2 \) and 0 for \( \eta = 0.8 \), respectively. The dotted line corresponds to \( \eta = 0 \).

where the increase of \( \varpi \) gives the variation of such as \( R(\pi / 2, 0.8) > R(0, 0.8) > R(0, 0) \), \( R(\pi / 2, 0.8) > R(0, 0) > R(0, 0.8) \), and \( R(0, 0) > R(0, 0.8) > R(\pi / 2, 0.8) \). When \( \varpi \) is large, a metallic behavior is found in the region where \( R(\pi / 2, 0.8) \) as a function of \( \varpi \) becomes convex upward. A shoulder is seen in \( R(0, 0) \). For \( \varpi = 8 \), the difference among these three lines becomes large and the behavior in Fig. 3(c) is enhanced, i.e., the noticeable crossover from \( R(\pi / 2, 0.8) > R(0, 0.8) > R(0, 0) \) to \( R(0, 0) > R(0, 0.8) > R(\pi / 2, 0.8) \). The dip exists in \( R(0, 0) \) as already found in the isotropic case \([12]\). This behavior comes from a crossover from intraband excitation to the interband excitation, i.e., the former (the latter) comes from \( \varpi < 2\varpi \) (\( \varpi > 2\varpi \)). The dip of the dotted line (\( \eta = 0 \)) disappears for small \( \varpi \) owing to the strong effect of \( \Gamma \).
Finally, we examine the role of the interband excitation. Figure 4 shows $R$ which is calculated only in the presence of the interband excitation, i.e., Eq. (8). Compared with Fig. 3. $R$ in the absence of intraband excitation is reduced. The most region for $R(\pi/2, 0.8) > R(0, 0.8)$ in Fig. 3(a) is replaced by $R(0, 0.8) > R(\pi/2, 0.8)$ in Fig. 4(a) suggesting that the former comes from the intraband excitation. The behavior of $R$ in Figs. 4(b), (c) and (d) clearly shows the property of $\sigma_1$ in Eq. (12), which gives a crossover from $R(\pi/2, 0.8) > R(0, 0.8)$ to $R(0, 0.8) > R(\pi/2, 0.8)$ with increasing $\varpi$. In the case of $\varpi = 2$ the effect of the interband excitation is still large owing to the presence of $\Gamma$. With increasing $\varpi$ (=4 and 8), the region of $\varpi$ for the contribution of the interband excitation emerges gradually where the effect of intraband excitation is still large.
We note the crossover from $R(\pi/2, 0.8) > R(0, 0.8) > R(0, 0.8) > R(\pi/2, 0.8)$ appears for large $\varpi$ being much larger than $2\pi$. The sign of $\varpi_2$ of the intraband excitation is opposite to that of Fig. 4 suggesting that these two excitations are incompatible for the small $\varpi$. However, in the presence of large $\varpi$, both contributions to $R$ become compatible owing to the increase of $\varpi_1$ where the intraband contribution is large for small $\varpi$ and the interband contribution becomes large for large $\varpi$.

4. Summary
We examined the reflectance of Dirac electrons with a tilted cone using a parameter of organic conductor $\alpha$-(BEDT-TTF)$_2$I$_3$, $\eta = 0.8$ and $\alpha = \pi/2$ and 0.

The reflectance on the plane is calculated from the dielectric constant which is estimated from the in-plane dynamical conductivity of the previous calculation. In addition to the chemical potential, the characteristic energy of the reflectance is the plasma frequency which is estimated with the interlayer distance for organic conductor $\alpha$-(BEDT-TTF)$_2$I$_3$. We compared a difference of the reflectance and the dielectric constant between $\alpha = 0$ and $\pi/2$, where the former (the latter) corresponds to the tilting being parallel and perpendicular to the electric field. The $\varpi$ dependence of the reflectance is examined by choosing several magnitudes of the chemical potential $\mu$ to find a crossover from the zero doping to a finite doping. With increasing $\varpi$, the reflectance decreases monotonously from unity. For the small doping with $\mu < \Gamma$, the reflectance shows a visible difference in a finite region of $\omega(> \mu)$. In this case, the reflectance of $\alpha = \pi/2$ is larger than that of $\alpha = 0$ for a small frequency, while the relation is reversed for a large frequency. Such a crossover originates from the interplay the intraband excitation and the interband excitation.

We comment on the doping in organic conductor. Our result shows that such a crossover occurs for $\omega/\Gamma = 10 \sim 30$ for $\mu/\Gamma \sim 5$. These are reasonable parameters since $\Gamma/k_B \simeq 3$ K, and $|\mu| < \sim 40$ K is obtained for the hole doping by the PEN substrate[3].

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Appendix A. Dynamical conductivity in the clean limit
In the clean limit, the dynamical conductivity $\sigma(\omega)$ is obtained as ($\delta \rightarrow +0$) [9, 8]

$$\sigma_{\text{intra}}^{\alpha} = \frac{e^2 i \mu \pi}{\pi \hbar \hbar \omega + i \delta} \left\langle \frac{(\eta \cos \alpha + \theta)^2}{(1 + g)^2} \right\rangle_{\theta},$$  \hspace{1cm} (A.1)

$$\sigma_{\text{inter}} = \frac{e^2}{\pi \hbar} 4 \left\langle \sin \theta \right\rangle^2 \left(1 + \frac{i}{\pi} \ln \frac{\hbar \omega - 2\mu/(1 + g) + i \delta}{\hbar \omega + 2\mu/(1 + g) + i \delta} \right)_{\theta},$$  \hspace{1cm} (A.2)

where $g = \eta \cos(\theta - \alpha)$. The intra part is calculated as

$$\sigma_{\text{intra}} = \frac{e^2 i \mu \pi}{\pi \hbar \hbar \omega + i \delta} \left\{ \frac{1 - \sqrt{1 - \eta^2}}{\eta^2} \cos^2 \alpha + \frac{1 - \sqrt{1 - \eta^2}}{\eta^2 \sqrt{1 - \eta^2}} \sin^2 \alpha \right\}.$$  \hspace{1cm} (A.3)
Noting that the real part of the bracket in Eq. (A.2) vanishes for $\hbar \omega < 2\mu/(1+g)$, the real part of the interband contribution is calculated as [10]

$$\text{Re}\{\sigma_{\text{inter}}\} = \frac{e^2}{\pi \hbar} \frac{\pi^2}{8} F_c(\alpha),$$  \hspace{1cm} (A.4)

where $F_c(\alpha) = 0$ for $\omega < 2\mu/(1+\eta)$ and $F_c(\alpha) = 1$ for $2\mu/(1-\eta) < \omega$. For $2\mu/(1+\eta) < \omega < 2\mu/(1-\eta)$,

$$F_c(\alpha) = \frac{1}{\pi} \left( \varphi_0 - \frac{1}{2} \sin(2\varphi_0) \cos(2\alpha) \right),$$  \hspace{1cm} (A.5)

with $\cos \varphi_0 = (2\mu/\hbar \omega - 1)/\eta$.

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