Large Scale Suppression of Scalar Power on a Spatial Condensation

Seyen Kouwn\textsuperscript{1,2}, O-Kab Kwon\textsuperscript{2,3}, and Phillial Oh\textsuperscript{4}

\textsuperscript{1}Korea Astronomy and Space Science Institute, Daejeon 305-348, Republic of Korea
\textsuperscript{2}Institute for the Early Universe and Department of Physics, Ewha Womans University, Seoul 120-750, South Korea
\textsuperscript{3}Department of Physics, Kyungpook National University, Taegu 702-701, Korea
\textsuperscript{4}Department of Physics, BK21 Physics Research Division, Institute of Basic Science, Sungkyunkwan University, Suwon 440-746, Korea

seyenkouwn@kasi.re.kr, okabkwon@ewha.ac.kr, ploh@skku.edu

Abstract

Suppression of the scalar power spectrum on large scales is one way to reconcile the tension between Planck and BICEP2 data. This suppression can occur by introducing a phase transition from the fast-roll phase to the slow-roll phase in a single field inflation model. In this paper we consider a deformed single field inflation model in terms of three SO(3) symmetric moduli fields. We find that spatially linear solutions for the moduli fields induces a phase transition during the early stage of the inflation and the suppression of scalar power spectrum at large scale perturbation modes.
1 Introduction

The recent discovery of B-mode polarization in the Cosmic Microwave Background (CMB) by the BICEP2 collaboration \[1\] has attracted a great deal of interests. One important property of the BICEP2 result is the large value of the tensor-to-scalar ratio, \( r = 0.2^{+0.07}_{-0.05} \) (or \( r = 0.16^{+0.06}_{-0.05} \) after foreground subtraction) within 1σ CL\[1\]. Then large field inflation models are favored according to the Lyth bound \[5\], \( \Delta \phi \sim N \sqrt{r}/M_P \), where \( \Delta \phi \) is the minimal variation of the inflaton field and \( N \) the e-folding number, and \( M_P \) the Planck mass. Therefore, the result of the BICEP2 collaboration is in tension with those of the Planck collaboration \[6\], where the tensor-to-scalar ratio is \( r \lesssim 0.11 \) at pivot scale \( k_* = 0.002 \text{ Mpc}^{-1} \) (2σ CL) without running of the scalar spectral index \( n_s \) and then the large field inflation models are disfavored. To resolve the tension, the BICEP2 collaboration introduced the large negative value of the running of \( n_s \), \( \alpha_s \equiv dn_s/d \ln k = -0.022 \pm 0.010 \) (1σ CL) \[1\]. Once this amount of \( \alpha_s \) is taken into account, the bound of \( r \) in the Planck data rises to \( r \lesssim 0.23 \) at \( k_* = 0.05 \text{ Mpc}^{-1} \) (2σ CL) and the result of the BICEP2 can be compatible with that of the Planck. However, this large value of \( \alpha_s \) is not easily achieved in single field models of slow-roll inflation, where the running is given by \( \alpha_s \simeq -(1-n_s)^2 + r(1-n_s)/8 \sim \mathcal{O}(10^{-3}) \). Therefore, large field inflation models

\[1\]It has been pointed out that the BICEP2 result can be consistent with \( r = 0 \) due to the foreground effect \[2,4\].
need some modifications to include that amount of the running of \( n_s \) in the context of reconciling the results of Planck and BICEP2 collaborations.

The detection of the \( B \)-mode power in the BICEP2 collaboration was implemented in large scale perturbations with the multipole range \( 30 \lesssim l \lesssim 100 \) \([1]\). Then the inclusion of the large negative running means that the spectral index \( n_s \) on large scales becomes large by increasing the scale in the analyses of the Planck and BICEP2 data. On the other hand, a large running does not seem to fit observational data at small scales \([6,7]\), which implies that usual single field slow-roll models can be good models to explain current cosmological data at small scales. One simple way to explain the running behaviour is to introduce suppression of the scalar power spectra on large scales \([8–12]\).

In this paper we consider a modification of the canonical single field inflation model, which induces large negative running of \( n_s \) and results in a suppression of scalar power spectrum on large scales. As a specific model, we consider a deformation of a single field inflation model by adding kinetic terms for a number of scalar moduli fields,

\[
\frac{1}{2} \int d^4x \sqrt{-g} \sum_{m=1}^{\tilde{N}} \partial_\mu \sigma^m \partial^\mu \sigma^m.
\] (1.1)

We also consider a background solution with spatially linear configurations, \( \sigma^a \sim x^a \), \((a = 1, 2, 3)\), and \( \sigma^i = 0 \), \((i = 4, \cdots, \tilde{N})\). Then the usual cosmological evolution for the single field under the FRW metric with the background solution for \( \sigma^a \) guarantees the homogeneity and isotropy of the cosmological principle \([13–16]\). In the perturbation level, fluctuations for \( \sigma^i \), \((i = 4, \cdots, \tilde{N})\), are decoupled and have no influence to cosmological observables \([16]\). For this reason, we consider the \( \tilde{N} = 3 \) case for simplicity. This model corresponds to the case with \( f(\varphi) = 1 \) in the work \([16]\). On the other hand, without the usual single inflaton field contribution, inflation is also possible when one uses higher order combination of \( X = \partial_\mu \sigma^a \partial^\mu \sigma^a \), \((a = 1, 2, 3)\), with spatially linear configuration of \( \sigma^a \). This inflation model is known as the solid inflation \([15]\). See also \([17]\).

In our model the background evolution is the same with that of the single field model with the curvature term of the open universe. That is, the solution \( \sigma^a \sim x^a \) induces the curvature term of the open universe in the Friedmann equation, though we start from the flat FRW metric\(^2\). The curvature term is proportional to inverse square of the scale factor, and so the effect of the spatial condensation appears during the very early stage of the inflation and disappears quickly as the scale factor grows up. Since we start from the phase where the curvature term is much more dominant than the potential term of

\(^2\)We call the remnant of the solution \( \sigma^a \sim x^a \) in the background evolution as spatial condensation.
the single field, there appears a phase transition from the curvature term dominant phase to the potential term dominant phase. Due to the phase transition in the early stage of the background evolution, there appears the suppression of the scalar power spectrum. This situation has some resemblance to that of inflation models referred as ‘whipped inflation’ [9,10] and ‘open inflation’ [11,12,18], in which there exist phase transitions from the fast-roll phase to the slow-roll phase of the single scalar field model. These phase transitions during the early stage of inflation induces the suppression of the scalar power spectrum on large scales, though the detailed suppression mechanisms are different from that in our model.

The organisation of this paper is as follows. In the next section, we explain the properties of the background evolution under the spatial condensation. In section 3, we investigate the effects of the spatial condensation in linear perturbation level. We find the suppression of the scalar power spectrum and large value of the running of the scalar spectral index on large scales. We conclude in section 4.

2 Background Evolution on a Spatial Condensation

We start from the action for the single field inflation model with an additional triad of moduli scalar fields,

$$S = \int d^4 x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) - \frac{1}{2} \partial_\mu \sigma^a \partial^\mu \sigma^a \right],$$

(2.2)

where $a = 1, 2, 3$ and $M_P$ denotes the Planck mass, $M_P \equiv (8\pi G)^{-1/2}$. The SO(3)-symmetric fields $\sigma^a$ have no potential. Then equations of motion of the scalar fields $\sigma^a$ are read as

$$\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \sigma^a \right) = 0.$$  

(2.3)

Under the background FRW metric, $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$ with the scale factor $a(t)$, the spatially linear configuration

$$\sigma^a = M_P^2 \alpha x^a$$

(2.4)

satisfies the equations (2.3). Here the constant gradient $\alpha$ is an arbitrary dimensionless parameter. As usual inflation models which are not compatible with the cosmological principle of homogeneity and isotropy in the background evolution, we assume that the
field $\varphi$ depends on time only. Then the remaining equations of motion of $g_{\mu\nu}$ and $\varphi$ in (2.2) are given by

$$
H^2 = \frac{1}{3M_p^2} \left( \frac{1}{2} \dot{\varphi}^2 + \frac{3M_p^4 \alpha^2}{2a^2} + V \right),
$$

$$
\dot{H} = -\frac{1}{2M_p^2} \left( \dot{\varphi}^2 + \frac{M_p^4 \alpha^2}{a^2} \right), 
$$

$$
\ddot{\varphi} + 3H \dot{\varphi} + V_\varphi = 0, \tag{2.5}
$$

where $H \equiv \dot{a}/a$ and $V_\varphi \equiv dV/d\varphi$. As was discussed in [16], the $\alpha$-terms in (2.5) correspond to the curvature terms by identifying the curvature constant $K$ as $K = -M_p^2 \alpha^2/2$. Since the curvature constant is negative in this case, the equations representing the background evolution in (2.5) are the same with those of the open universe in the single field inflation model. Though the single field model in the open universe is the same with our model in the background level, they are different in the perturbation level due to the contribution of fluctuation modes of $\sigma^a$. In our case three degrees of freedom (one scalar mode and two vector modes) originated from the triad of scalar fields appear in the perturbation level, while there is no additional perturbation degree of freedom in the usual single field inflation model with negative curvature constant.

Now we investigate some characteristic properties of the background evolution of our model. The effect of $\alpha$-terms in (2.5) is decreasing rapidly during the inflation and has some influence on the early time of the inflation period. Especially as we see in the first line of (2.5) the Hubble horizon $r_H \equiv 1/H$ starts from a small value when we introduce a large value of the $\alpha$-dependent term at the initial state, increases during the early stage of the inflation, and approaches the value of the single field inflation model at late time.

As we discussed previously, to reconcile the results of Planck and BICEP2 collaborations one needs large running of the scalar spectral index through a strong suppression of scalar power spectrum on large scales. In our model it can be achieved by introducing a large value of $\alpha$-term at the early time of inflation. For that purpose, we consider the case that the $\alpha$-term in the first equation of (2.5) is much larger than the potential term of the inflaton field, i.e.,

$$
\frac{3M_p^4 \alpha^2}{2a^2} \gg V(\varphi). \tag{2.6}
$$

Here we consider the potential $V(\varphi)$ corresponding to a large field inflation model, which matches the recent observational data of BICEP2 [1]. Obtaining analytic solution for the equations in (2.5) is a formidable task for the potentials of the large field inflation
models. So we rely on a semi-analytic way to figure out the behaviour of the background evolution governed by the equations in (2.5), based on numerical method. By employing the simplest scalar potential $V(\varphi) = \frac{1}{2}m^2\varphi^2$, for concreteness, we find that the scalar field $\varphi$ remains almost constant until the e-folding number $N = 10 \sim 20$. See Fig.1. Then there appears a stage that the value of $\alpha$-term is comparable to that of the potential, i.e.,

$$\frac{3M_P^4\alpha^2}{2a^2} \simeq V(\varphi).$$  \hspace{1cm} (2.7)

After the universe passes through this stage, the scalar field starts to decrease and follows the behaviour of the canonical slow-roll inflation. The behaviours of the background scalar field and the Hubble horizon with respect to the e-folding number $N$ are plotted in Fig.1.

![Figure 1](image)

Figure 1: The graphs of inflaton field $\varphi(N)$ (left) and Hubble radius $1/H(N)$ (right). We set the pivot scale $k_0 = 0.05\text{Mpc}^{-1}$ and $N_e = 60$. We choose the initial condition as $a_i = 1$, $\varphi_i = 16.5M_P$, $\dot{\varphi}_i = 0$, $m = 5.85 \times 10^{-6}M_P$, and $\alpha = 10^2$.

As we see in Fig.1, there is a sharp transition point near

$$N_{eq} \simeq \ln \sqrt{\frac{3\alpha^2M_P^4}{2V(\varphi(N_{eq}))}},$$  \hspace{1cm} (2.8)

where $N_{eq} \equiv \log a_{eq}$ represents the e-folding number when the $\alpha$-term is the same with the potential of the scalar field. After the transition point the background evolution rapidly follows the behaviour of the usual slow-roll inflation by rolling down the potential slop. For this reason, we can approximate the background equations in (2.5) under the
assumption of the slow-rolling of the scalar field as follows:

\begin{align*}
\text{early time} & : \quad 3H^2 \simeq \frac{3\alpha^2 M_P^2}{2a^2} + \frac{\Lambda}{M_P^4}, \quad (2.9) \\
\text{late time} & : \quad 3H^2 \simeq \frac{3\alpha^2 M_P^2}{2a^2} + \frac{V(\varphi)}{M_P^2}, \quad (2.10)
\end{align*}

where \( \Lambda \equiv V(\varphi_i) \) with an initial value of the scalar field \( \varphi_i \). From the relation (2.8) we also have the relation

\[ N_{eq} \simeq \ln \sqrt{\frac{3\alpha^2 M_P^4}{2\Lambda}}. \quad (2.11) \]

In the early time, the background equation (2.9) has a solution \[20,21\],

\[ a(t) \simeq a_{eq} \sinh \left( \sqrt{\frac{\Lambda}{3M_P^2}} (t + t_0) \right), \quad (2.12) \]

where \( t_0 \) is the initial time with the scale factor \( a_0 \). On the other hand, in the late time for a given value of \( \alpha \), the scale factor ‘\( a \)’ is already very large, and then the \( \alpha \)-term in (2.10) becomes much smaller than the potential term. Based on the numerical result during the late time in Fig.1, we see that the scalar field starts to roll down the potential slope matching the behaviour of the usual slow-roll approximation. Since the behaviour of the background evolution for the case \( \frac{3\alpha^2 M_P^2}{2a^2} \ll \frac{V}{M_P^2} \) was already investigated in \[16\], we omit the detailed background behaviours in this paper.

### 3 Suppression of Large Scale Scalar Power Spectrum

#### 3.1 Generality

We consider the linear scalar perturbation of the FRW metric,

\[ ds^2 = -(1 + 2A) dt^2 + 2a \partial_i B dt dx^i + a^2 \left[ (1 - 2\psi) \delta_{ij} + 2\partial_i \partial_j E \right] dx^i dx^j, \quad (3.13) \]

where \( A, B, \psi, \) and \( E \) are four scalar modes. In the linear perturbation, there is also a contribution from fluctuations of scalar fields,

\[ \varphi(t, x) = \varphi(t) + \delta \varphi(t, x), \]

\[ \sigma^a(t, x) = \sigma^a(x) + \delta \sigma^a(t, x). \quad (3.14) \]
The three perturbation modes $\delta \sigma^a$ are decomposed into one scalar and two vector modes,

$$\delta \sigma^a = \delta \sigma^a_{\|} + \delta \sigma^a_{\perp}. \quad (3.15)$$

In this work, we focus on the scalar mode $\delta \sigma^a_{\|}$ in perturbation level. We express the longitudinal mode $\delta \sigma^a_{\|}$ in terms of a scalar mode $u$ with a normalization $13, 16$,

$$\delta \sigma^a_{\|} = \frac{1}{k} \partial_a u, \quad (3.16)$$

where $k$ is the comoving wave number. So there are two physical scalar degrees of freedom after gauge fixing.

Employing the spatially flat gauge ($\psi = 0$ & $E = 0$), the perturbed scalar equations are reduced to

$$\ddot{Q}_\varphi + 3H \dot{Q}_\varphi + \left( \frac{k^2}{a^2} + \frac{\dot{\varphi}V_\varphi}{M_p^2 H} + V_{\varphi\varphi} \right) Q_\varphi + 2 \left( \frac{\dot{H} \dot{\varphi}}{H} - \ddot{\varphi} \right) A = 0,$$

$$\ddot{Q}_u + 3H \dot{Q}_u + \left( \frac{k^2}{a^2} + \frac{2\alpha^2 M_p^2}{a^2} \right) Q_u = 0, \quad (3.17)$$

where $Q_\varphi \equiv \delta \varphi - \frac{\dot{\varphi}}{\dot{H}} \psi$ and $Q_u = u - \alpha k M_p^2 E$ are gauge invariant quantities $16$. Scalar modes $A$ and $B$ satisfy the constraints,

$$3AH^2 - \frac{k^2 BH}{a} = \frac{1}{2M_p^2} \left( A\dot{\varphi}^2 - \dot{\varphi} \dot{Q}_\varphi - V_{\varphi\varphi} \right) + \frac{\alpha k}{2a^2} Q_u,$$

$$2AH = \frac{\dot{\varphi} Q_\varphi}{M_p^2} + \frac{\alpha}{k} Q_u - \frac{\alpha^2 M_p^2 B}{a}. \quad (3.18)$$

Using these constraints one can express the modes $A$ and $B$ in terms of $Q_\varphi$ and $Q_u$. In this multifield perturbation system, the comoving curvature perturbation is written as $16$,

$$R = H \left[ \frac{\dot{\varphi} Q_\varphi - \alpha M_p^2 \left( \frac{\alpha M_p^2 B}{a} - \frac{\dot{Q}_u}{k} \right)}{\dot{\varphi}^2 + \frac{\alpha^2 M_p^4}{a^2}} \right]. \quad (3.19)$$

Differently from the single field inflation model, there is also non-vanishing isocurvature perturbation $16$. However, here we only concentrate on the adiabatic curvature perturbation.

### 3.2 Suppression of the scalar power spectrum

As we discussed in section 2, there are two inflation phases in the background evolution, the $\alpha$-term dominant phase and the scalar potential dominant phase. Due to the phase
transition during the inflation, the computation of the power spectrum is different from that of the usual single scalar field model. In order to calculate power spectrum and related observational quantities, such as the scalar spectral index $n_s$ and the running of the spectral index $\alpha_s$, we use the method developed in [19], where the authors calculated the power spectrum of the single scalar field model with the potential having a step transition. Due to the shape of the scalar potential, there are two inflationary phases, fast-roll phase and slow-roll phase. The origin of the phase transition in [19] is different from ours, but there is a robust similarity between these two cases in the sense that there is a transition during the inflation and the background evolution approaches the usual slow-roll inflation phase of the single field model at late time. For this reason, we follow the method developed in [19] to compute the power spectrum and related perturbation quantities. Due to the phase transition in the background level, there is also a phase transition to the perturbed equations in (3.17). We try to solve the perturbed equations for the $\alpha$-term dominant phase and the scalar potential dominant phase separately and apply the matching condition at the transition point.

3.2.1 Early time

In the early time having the limit $\frac{3M_p^4\alpha^2}{2a^2} \gg V$, we obtain $A, B$ from (3.18) in the leading order of the limit as

$$A \approx \frac{\alpha}{k} M_p Q_u \left( 2 + 3 \frac{\alpha^2 M_p^2}{k^2} \right), \quad B \approx -\frac{\sqrt{V_0}}{z M_p^2 \left( 2 + 3 \frac{\alpha^2 M_p^2}{k^2} \right)} Q_u.$$  (3.20)

Using the relation (3.20) and the fact that $\varphi$ is almost a constant during the $\alpha$-term dominant phase, we conclude that the $A$-dependent term in (3.17) is negligible. One can also neglect $V_\varphi$ and $V_{\varphi\varphi}$-dependent terms in (3.17) since $V(\varphi)$ is almost constant in the early time phase. Introducing the Sasaki-Mukhanov variables,

$$V \equiv a Q_\varphi, \quad U \equiv a Q_u,$$  (3.21)

and the conformal time coordinate $\tau = \int dt/a$, we obtain the decoupled differential equations for $V$ and $U$ as

$$V'' + \left( k_{e1}^2 - \alpha^2 M_p^2 \cosh \left( \frac{\alpha(\tau)}{\sqrt{2}} \right) \right) V = 0,$$

$$U'' + \left( k_{e2}^2 - \alpha^2 M_p^2 \cosh \left( \frac{\alpha(\tau)}{\sqrt{2}} \right) \right) U = 0,$$  (3.22)
where the prime represents the differentiation with respect to the conformal time, the subscript \( 'e' \) denotes the early time phase, and
\[
\begin{align*}
k_{e1} &\equiv \sqrt{k^2 - \frac{\alpha^2 M_P^2}{2}}, \\
k_{e2} &\equiv \sqrt{k^2 + \frac{3\alpha^2 M_P^2}{2}}.
\end{align*}
\] (3.23)

Using general solutions for \( V_e \) and \( U_e \), we obtain normalized solutions \[20\],
\[
\begin{align*}
V_e(\tau) &= \frac{M_P^\frac{3}{2}}{\sqrt{2k_{e1}}} \left( -\frac{\alpha M_P}{\sqrt{2}} \coth \left( \frac{\alpha(-\tau)}{\sqrt{2}} \right) + ik_{e1} \right) e^{-ik_{e1}\tau}, \\
U_e(\tau) &= \frac{M_P^\frac{3}{2}}{\sqrt{2k_{e2}}} \left( -\frac{\alpha M_P}{\sqrt{2}} \coth \left( \frac{\alpha(-\tau)}{\sqrt{2}} \right) + ik_{e2} \right) e^{-ik_{e2}\tau}.
\end{align*}
\] (3.24)

Here we adjusted the integration constants for the solutions \( V_e \) and \( U_e \) to get the Bunch-Davis vacua in \( \tau \to -\infty \) limit,
\[
\begin{align*}
V_e(\tau) &= \frac{M_P^\frac{3}{2}}{\sqrt{2k_{e1}}} e^{-ik_{e1}\tau}, \\
U_e(\tau) &= \frac{M_P^\frac{3}{2}}{\sqrt{2k_{e2}}} e^{-ik_{e2}\tau}.
\end{align*}
\] (3.25)

As we see in \( (3.25) \), effective wave numbers for oscillation modes \( V_e \) and \( U_e \) at early time are \( k_{e1} \) and \( k_{e2} \) which are deformed from the wave number \( k \) due to the non-vanishing value of \( \alpha \). Then we find that there is minimum value of the comoving wave number \( k \). The modes below the minimum value always stay in super horizon scale and never cross the horizon, so those modes do not contribute to current observable quantities. Now we try to obtain the minimum value of the wave number. As we will see later, the leading contribution to the power spectrum comes from \( V_e \) mode in the limit we are considering. So we focus on the mode \( V_e \). Horizon crossing condition for the mode \( V_e \) at the early stage of inflation is given by
\[
k_{e1} = a_* H_*,
\] (3.26)
and the corresponding conformal time \( \tau_* \) is
\[
\tau_* = -\frac{\sqrt{2}}{\alpha M_P} \tanh^{-1} \left( \frac{\alpha M_P}{\sqrt{2}k_{e1}} \right).
\] (3.27)

From this relation, we notice that at the early stage of inflation, the Hubble crossing occurs only when perturbation modes satisfy the condition to give a real value of \( \tau_* \),
\[
\frac{\alpha M_P}{\sqrt{2}k_{e1}} < 1.
\] (3.28)

This condition determines the minimum value of the comoving wave number,
\[
k_{\text{min}} = \alpha M_P.
\] (3.29)
3.2.2 Late time

On the other hand, in the late time satisfying the condition \( \frac{3\alpha^2 M_P^2}{2a^2} \ll V(\varphi) \), one can express the scalar modes \( A \) and \( B \) in terms of the gauge invariant variables, \( Q_\varphi \) and \( Q_u \) from the constrain (3.18),

\[
A \simeq \frac{\dot{\varphi}}{2HM^2_P} Q_\varphi + \frac{\alpha}{2kH} Q_u,
\]
\[
B \simeq \frac{\alpha}{2k} \left( \frac{3a}{k^2} - \frac{1}{aH} \right) Q_u + \frac{a\dot{\varphi}}{2Hk^2 M^2_P} \dot{Q}_\varphi.
\] (3.30)

Using (3.30), one can easily see that the coefficient of the \( A \)-dependent term in the first line of (3.17) is belonged to higher order for slow-roll parameters. For this reason, the differential equations for \( Q_\varphi \) and \( Q_u \) are decoupled in the leading contribution of slow-roll parameters. Then we obtain differential equations for \( V_l \) and \( U_l \) in the conformal time coordinate as

\[
V_l'' + \left( k_{l1}^2 - \frac{\mu_1^2 - \frac{1}{4}}{\tau^2} \right) V_l = 0,
\]
\[
U_l'' + \left( k_{l2}^2 - \frac{\mu_2^2 - \frac{1}{4}}{\tau^2} \right) U_l = 0,
\] (3.31)

where the subscript ‘\( l \)’ denotes the late time phase and

\[
k_{l1}^2 \equiv k^2 - \frac{\alpha^2 M^2_P}{6}, \quad k_{l2}^2 \equiv k^2 + \frac{11\alpha^2 M^2_P}{6},
\]
\[
\mu_1 \simeq \frac{3}{2} + 3\epsilon - \eta, \quad \mu_2 \simeq \frac{3}{2} + \epsilon.
\] (3.32)

Here the slow-roll parameters, \( \epsilon \) and \( \eta \), are defined as

\[
\epsilon = \frac{\dot{\varphi}^2}{2M^2_P H^2}, \quad \eta = \frac{V_{\varphi\varphi}}{3H^2}.
\] (3.33)

General solutions of \( V_l \) and \( U_l \) modes for differential equations in (3.31) are given by

\[
V_l(\tau) = M^\frac{3}{2} P^{\frac{1}{2}} \left( C_1 H_{\mu_1}^{(1)}(-k_{l1}\tau) + C_2 H_{\mu_1}^{(2)}(-k_{l1}\tau) \right),
\]
\[
U_l(\tau) = M^\frac{3}{2} P^{\frac{1}{2}} \left( D_1 H_{\mu_2}^{(1)}(-k_{l2}\tau) + D_2 H_{\mu_2}^{(2)}(-k_{l2}\tau) \right),
\] (3.34)

where \( H_{\mu}^{(i)}(x) \) \((i = 1, 2)\) are the first and second kinds of the Hankel functions and \( C_{1,2}, D_{1,2} \) are integration constants.
3.2.3 Matching condition

As we discussed in the previous section, there are two phases in our model and we obtained perturbation modes for each phase separately. Then all perturbed modes should satisfy matching conditions at the transition point $\tau_{eq}$ in conformal time,

$$
V_e(\tau)|_{\tau=\tau_{eq}} = V_l(\tau)|_{\tau=\tau_{eq}}, \quad V'_e(\tau)|_{\tau=\tau_{eq}} = V'_l(\tau)|_{\tau=\tau_{eq}},
$$
$$
U_e(\tau)|_{\tau=\tau_{eq}} = U_l(\tau)|_{\tau=\tau_{eq}}, \quad U'_e(\tau)|_{\tau=\tau_{eq}} = U'_l(\tau)|_{\tau=\tau_{eq}}.
$$

(3.35)

Here we notice that the perturbed modes $V$ and $U$ satisfy the same type of differential equations with different parameters. So in what follows, we only consider the matching condition for the mode $V$. Then the results can be extended to the case of the mode $U$ as well. From the matching condition in (3.35) we obtain the corresponding integration constants,

$$
C_1 - C_2 = \frac{e^{-i\beta \tau_{eq}} \sqrt{\pi}}{2k\sqrt{-k_{e1}\tau_{eq}}} \left[ (k_{e1} + i\alpha M_P)k_{l1}\tau_{eq} J_{\mu-1} + \left( (k_{e1} + i\alpha M_P) \left( \mu - \frac{1}{2} \right) + (i\alpha^2 M_P^2 + 2\alpha k_{e1} M_P - 2i k_{e1}^2) \tau_{eq} \right) J_{\mu} \right],
$$

$$
C_1 + C_2 = \frac{-ie^{-i\beta \tau_{eq}} \sqrt{\pi}}{2k\sqrt{-k_{e1}\tau_{eq}}} \left[ (k_{e1} + i\alpha M_P)k_{l1}\tau_{eq} Y_{\mu-1} + \left( (k_{e1} + i\alpha M_P) \left( \mu - \frac{1}{2} \right) + (i\alpha^2 M_P^2 + 2\alpha k_{e1} M_P - 2i k_{e1}^2) \tau_{eq} \right) Y_{\mu} \right],
$$

(3.36)

where we used the relations between the Hankel functions and Bessel functions, $H^{(1,2)}_\mu(x) \equiv J_\mu(x) \pm iY_\mu(x)$, and defined the quantities at the transition point as

$$
J_\mu = J_\mu(-k_{l1}\tau_{eq}) , \quad Y_\mu = Y_\mu(-k_{l1}\tau_{eq}) , \quad \tau_{eq} = -\frac{\sqrt{2} \coth^{-1}(\sqrt{2})}{\alpha M_P}.
$$

(3.37)

3.2.4 Power spectrum

Now we try to obtain the power spectrum for the curvature perturbation $\mathcal{R}$ in (3.19) and related observational quantities. For a single scalar model, one usually reads the power spectrum at the horizon crossing point since it is guaranteed in the absence of the transition point that curvature perturbations of perturbed modes are frozen after the horizon crossing. In our case with two inflationary phases, however, reading the power spectrum at the horizon crossing point can cause some possible errors for large scale modes
which are deformed due to the presence of the nonvanishing \( \alpha \)-term. That is, one can not guarantee the freezing of the curvature perturbation after the horizon crossing for large scale modes. For this reason, we read the power spectrum at the limit \( \tau \to 0 \) for all values in the region \( k > \alpha M_p \).

Using the relation (3.30) in the limit \( \frac{3\alpha^2 M_p^4}{2a^4} \ll V(\varphi) \), we obtain leading contributions for the curvature perturbation [16],

\[
\mathcal{R} \simeq \frac{1}{2 + \frac{\alpha^2 M_p^2}{a^2 H^2 \epsilon}} \left( -\frac{\sqrt{2}}{M_p \sqrt{\epsilon}} Q_\varphi + \frac{\alpha}{k H \epsilon} \dot{Q}_u \right).
\]

(3.38)

As we see in (3.29), the comoving wave number has a minimum value, and so we notice that all comoving wave numbers relevant to the observation are in the range \( \frac{\alpha M_p}{k} < 1 \). Due to this fact, from now on we neglect the contribution of \( \dot{Q}_u \) in (3.38) by keeping the leading contribution of \( \frac{\alpha M_p}{k} \) since the \( \dot{Q}_u \)-term in (3.38) gives \( \mathcal{O}\left(\frac{\alpha M_p}{k}\right)^4 \) contribution to the the resulting power spectrum [16]. Then the power spectrum of the curvature perturbation is given by

\[
\mathcal{P}_R(k) \equiv \frac{k^3}{2\pi^2} \langle \mathcal{R} \mathcal{R}^* \rangle_*
\]

\[
\simeq \left(1 + \frac{\alpha^2 M_p^2}{2\epsilon_* k^2}\right)^{-2} \frac{H_*^2}{2\epsilon_* M_p^2} \lim_{\tau \to 0} k_l (V_l(\tau) V_l(\tau)^*) ,
\]

(3.39)

where the subscripted asterisk indicates the value at the horizon crossing point \( k_l = a H \) and we take the late time limit \( \tau \to 0 \) to read the power spectrum. Plugging the first line of (3.34) into (3.39), we obtain

\[
\mathcal{P}_R \simeq \mathcal{P}^{(0)}_R \frac{\left| C_1 - C_2 \right|^2}{\left(1 + \frac{\alpha^2 M_p^2}{2\epsilon k^2}\right)^2} ,
\]

(3.40)

where \( \mathcal{P}^{(0)}_R \) denotes the power spectrum of the canonical single inflation model,

\[
\mathcal{P}^{(0)}_R = \frac{H_*^2}{8\pi^2 M_p^2 \epsilon} \left(1 + (2 - 2C) \eta + (6C - 8) \epsilon \right).
\]

(3.41)
Here, \( C = 2 - \ln 2 - \gamma \) with the Euler-Mascheroni constant \( \gamma \approx 0.5772 \) and

\[
|C_1 - C_2|^2 = -\frac{\pi \alpha M_P}{X k_{e1}} \left[ \beta^2 \left( 1 + \frac{\alpha^2 M_P^2}{2k^2} \right) \frac{k_{l1}^2}{\alpha^2 M_P^2} \mu_{\mu_{1}}^{2} - \right.
\]

\[
+ 2\beta \left\{ \left( 1 + \frac{\alpha^2 M_P^2}{2k^2} \right) \left( \mu - \frac{1}{2} \right) + \frac{\beta \alpha^2 M_P^2}{k^2} \right\} \frac{k_{l1}}{\alpha M_P} J_{\mu_{1}} J_{\mu_{1} - 1}
\]

\[
+ \left\{ \left( 1 + \frac{\alpha^2 M_P^2}{2k^2} \right) \left( \mu - \frac{1}{2} \right) + \frac{\beta \alpha^2 M_P^2}{k^2} \left( \mu - \frac{1}{2} \right) \right. 
\]

\[
+ \beta^2 \left( 1 + \frac{3\alpha^2 M_P^2}{4k^2} \right) \left\{ J_{\mu_{1}}^2 \right. \right. \right. \right. \right. \right. \right. 
\]

\[ \tag{3.42} \]

where \( \beta \equiv \alpha \tau_{eq} = -\sqrt{2} \coth^{-1}(\sqrt{2}). \)

![Figure 2](image)

Figure 2: The primordial power spectrum of curvature perturbation for the usual single field model (dashed blue line) and the inflation model on the spatial condensation (solid red line).

As discussed previously, we notice that during the early time phase, modes satisfying the condition \( k > \alpha M_P \) can only cross the Hubble horizon. In other words, modes in the range \( k < \alpha M_P \) stay outside the Hubble horizon and never cross the horizon. Therefore, those super horizon modes are causally disconnected to our universe and irrelevant to observational quantities. As we see the plot of power spectrum in Fig.2, the power spectrum in our model asymptotically approaches that of the single field model (dashed blue line) by increasing the comoving wave number \( k \), while it is strongly suppressed by decreasing the value \( k \).

In this paper, we investigate the behaviour of power spectrum in terms of the value \( k \). To do that, we divide the values of \( k \) into two regions, \( k \gg \alpha M_P \) and \( k \gtrsim \alpha M_P \). At
first in the region $k \gg \alpha M_P$, from (3.40) we have the following asymptotic form of power spectrum

$$P_R \simeq P_R^{(0)} \left[ 1 - \frac{\alpha^2 M_P^2}{\epsilon k^2} + \frac{\alpha^2 M_P^2}{\beta^2 k^2} \sin^2 \left( \frac{\beta k}{\alpha M_P} \right) \right].$$

(3.43)

The power spectrum is almost scale invariant but modulated with small oscillation. This oscillation behaviour of power spectrum was also reported in different inflation model with phase transition [10, 19]. The corresponding spectral index $n_R$ and the running of the spectral index $\alpha_R$ at the pivot scale $k_0$ in the leading order of $\alpha/k$ with small slow-roll parameters are given by

$$n_R \simeq n_R^{(0)} + \frac{2\alpha^2 M_P^2}{\epsilon k_0^2}, \quad \alpha_R \simeq -\frac{4\alpha^2 M_P^2}{\epsilon k_0^2},$$

(3.44)

where $n_R^{(0)}$ denotes the spectral index at the pivot scale $k_0$ for the single field inflation model. We notice that the spectral index is slightly increasing and the running is negative and slightly decreasing by decreasing the value $k$ due to the spatial condensation. On the other hand for the region $k \gtrsim \alpha M_P$, we obtain the behaviour of the power spectrum as

$$P_R \sim \frac{k^4}{\epsilon^4 M_P^4} P_R^{(0)}.$$

(3.45)

This behaviour was plotted in Fig.2 in terms of the red line in logarithmic scale of the wave number. Then the corresponding spectral index and its running are given by

$$n_R \simeq 5, \quad \alpha_R \simeq 0.$$  

(3.46)

From this behaviour of the power spectrum in the spatial condensation, one can clearly notice a strong suppression of the scalar power spectrum on large scales. Since the spectral index is approaching a constant on these large scale limit, the running of the spectral index becomes almost zero.

We analysed the behaviour of the power spectrum in terms of semi-analytic methods for large scales $k \gtrsim \alpha M_P$ and small scales $k \gg \alpha M_P$ in the previous paragraph. However, as we see the numerical result shown in Fig.2, there is a sudden transition of the power spectrum for intermediate scales between $k \gtrsim \alpha M_P$ and $k \gg \alpha M_P$. Therefore, for this region, the spectral index is growing suddenly by decreasing the comoving wave number, i.e., the scalar power spectrum starts to suppressed strongly, and then one has large negative running of the spectral index in the intermediate region, which can reconcile the tension between the Planck and BICEP2 data.
4 Conclusion

To reconcile the results of Planck and BICEP2 collaborations, one needs large running of the scalar spectral index. One way to obtain the large running is to introduce a suppression of the scalar power spectrum on large scales. There are several models to accomplish the suppression, such as the whipped inflation \cite{9} and open inflation \cite{11} models. One common property of these models is that there exists a phase transition of the background evolution and it is connected to the slow-roll phase of the single field inflation model at late time. In this paper, we showed that a deformed single field inflation model in terms of the spatial condensation has a phase transition which is similar to that of models in \cite{9,11} and the suppression of scalar power spectrum on large scales.

We deformed a single field inflation model in terms of three SO(3) symmetric moduli fields $\sigma^a$. On the solution with constant gradient $\sigma^a = \alpha x^a$, the background evolution is equivalent to that of the single inflation model with curvature term of the open universe. During very early time, the background evolution is governed by the curvature term but soon after the curvature term is rapidly decreased. Then at the late time, the evolution is governed by the potential term of the single scalar field and asymptotically approaches that of a single field inflation model. This means that there exists a phase transition of the background evolution, and so, for an analytic approach we divided the background evolution into two phases, the $\alpha$-term dominant phase and the potential term dominant phase.

During the $\alpha$-term dominant phase, we assumed that the inflation started with a very large value of the $\alpha$-term (curvature term) and then the e-folding could be accumulated very rapidly. Therefore, during the early time phase with a short cosmic time process, the single scalar field remains almost constant. Assuming the scalar potential is a constant, there is an exact solution governing the background evolution. On the other hand, in the late time phase, the $\alpha$-term becomes very small and the evolution is governed by the potential term and asymptotically approaches that of the single field inflation. Under the slow-roll assumption, the system is governed by slow-roll parameters and small contribution of the $\alpha$-term.

Under the above circumstance of the background evolution, we investigated the behaviour scalar modes in linear perturbation level. We considered the perturbation modes in the early and late time phases separately. For perturbed modes in the two phases, we applied the junction condition at the transition point. Then we obtained the power spectrum, the spectral index, and the running of the spectral index for scalar modes. We found that the power spectrum is apparently suppressed by decreasing the comoving wave
number, while it approaches the value of the single field inflation model for large value of the comoving wave number. Therefore, one can obtain large negative running of the scalar spectral index on large scales. We also found a oscillation behaviour of the power spectrum at late time.

We focused on the suppression of the scalar power spectrum on large scales, since this behaviour can reconcile the results of the Planck and the BICEP2 collaborations. However, we also expect that there will be a nontrivial contribution to the isocurvature perturbation since our model has two perturbed scalar modes. Actually our model introduces a free gradient parameter $\alpha$ to a single field inflation model in isotropic and homogeneous way. Therefore, in order to accommodate observational data, similar analysis as we did in this paper can be applied to various inflation models by adjusting the free parameter $\alpha$.

Acknowledgments

This work was supported by the Korea Research Foundation Grant funded by the World Class University grant no. R32-10130 (S.K.,O.K.), the National Research Foundation of Korea(NRF) grant funded by the Korea government(MEST) through the Center for Quantum Space- time(CQUeST) of Sogang University with grant number 2005-0049409 (P.O.), the BSRP through the NRF of Korea funded by the MEST (No. 2010-210021996) (P.O.), and the Mid-career Researcher Program through the NRF grant funded by the Korean government (MEST) (No. 2014-051185) (O.K.).

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