Mathematical model and calculation of safety and stability evaluation of underground space engineering by TOPSIS and analytic hierarchy process

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Abstract. The development of economy promotes the development of urbanization, and the power of urban space engineering, so the stability research of urban underground space engineering is very important. Although the existing evaluation models can consider a single form of underground space safety problems, they seldom consider the uncertainty of cross-blending of multiple information types within a certain interval, and cannot continuously express the quantitative results of the suitability assessment of urban underground space. This paper takes an underground tunnel in China as the research background, calculates the weight of the selected construction stability index through the analytic hierarchy process, and finally evaluates the stability of different sections of the project by TOPSIS method, and draws the corresponding conclusions.

1. Introduction
Recent years, scholars at home and abroad have applied mathematical models such as neural network method, fuzzy comprehensive evaluation method and extension method to the suitability assessment of urban underground space, which has improved the rationality of model screening and selection. Durmisevic et al. analyzed underground stations and processed data with neural network method, and finally established an evaluation model that integrated multiple influencing factors [1]. In terms of numerical simulation, Shen [2] fully considered the anti-floating effect between the segment connecting bolt of the shield tunnel and the surrounding soil, and derived the calculation formula of the minimum overlaying thickness of the Qiantang River tunnel based on the numerical calculation. On this basis, Peng et al. [3] took into account the arching effect of shield over the tunnel in composite stratum construction, and established a prediction model of reasonable overburden thickness of underwater tunnel based on BP neural network algorithm, and successfully applied the research results to Shiziyang tunnel project. Combined with numerical simulation and quantitative calculation, Li Shucai [4, 5] et al. put forward a method to determine the minimum thickness of overburden of the submarine tunnel, and calculated the minimum thickness of overburden of the submarine tunnel of Xiangshan Port. Li Xin et
al. used data mining method to study the stability of urban underground space engineering, and successfully applied it to urban underground space engineering [6].

As mentioned above. This paper takes an underground tunnel in China as the research background, calculates the weight of the selected construction stability index through the analytic hierarchy process, and finally evaluates the stability of different sections of the project by TOPSIS method, and draws the corresponding conclusions.

2. Calculation method and principle

AHP method is used to calculate the subjective weight of the evaluation index of the stability state of urban underground space based on the degree index of the influence of different decision indexes on the stability state of tunnel.

(1) Judgment matrix

Choose different representative in the field of experts and scholars to 1 ~ 9 as indicators of decision-making importance evaluation standard, importance of indexes overall ranking. Taking index \( I_i \) and \( I_j \) as an example, when the relative importance degree of index \( I_i \) to index \( I_j \) is 1, 3, 5, 7 and 9, it means that \( I_i \) → \( I_j \) is the same, slightly, obviously, strongly and extremely important than \( I_j \), respectively. When the values of \( I_i \) → \( I_j \) are 2, 4 and 6, it means that the importance of index \( I_i \) relative to \( I_j \) is between the strength corresponding to the above odd numbers

(2) Solution of judgment matrix

By using AHP to calculate the weight, the maximum eigenvalue method can be used to obtain the maximum eigenvalue and corresponding eigenvector \( u \) of the judgment matrix, which can be calculated by formulas (1) ~ (3) to obtain the ranking of the importance of evaluation indexes.

\[
G \cdot u = \lambda_{\text{max}} \cdot u
\]  

(1)

\[
CI = \frac{\lambda_{\text{max}} - n}{n-1}
\]  

(2)

\[
CR = \frac{((\lambda_{\text{max}} - n)(n-1))}{RI}
\]  

(3)

Where \( 0 \leq w_i \leq 1 \), and \( \sum_{i=1}^{n} w_i = 1 \).

Finally, the consistency of the judgment matrix is judged to be reasonable or not. When, the judgment matrix \( G \) is considered to have acceptable consistency. On the contrary, it is considered that the degree of deviation from the consistency of the judgment matrix is too large, so it is necessary to modify the matrix element value in.

It is assumed that UI is the set of excavation sections to be evaluated in the process of evaluating the stability of the tunnel in water-rich and soft strata using TOPSIS theory, and its matrix form is shown in Eq.(4):

\[
U = \begin{pmatrix}
   u_{1,1} & u_{1,2} & \ldots & u_{1,n} \\
n_{2,1} & u_{2,2} & \ldots & u_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
   u_{m,1} & u_{m,2} & \ldots & u_{m,n}
\end{pmatrix}
\]  

(4)
The set to be evaluated contains M types of evaluation indicators, namely \( Q(q_1,...q_m) \). For the expression of the stability state attribute of the tunnel, there are L stability grades, i.e. \( C(l = 1,2,...,L) \), that is, the stability grade can be divided into L grades. Accordingly, according to the attribute and value range of each index, the stability of tunnel is divided into \( L \) intervals under different stability grades, which constitutes a hierarchical evaluation system for the stability of tunnel.

3. TOPSIS method decision

In TOPSIS theory, the nature of decision index should be defined first, and the decision index should be divided into positive index and negative index on the basis of this. The larger the index value is, the worse the stability will be defined as a negative index. Similarly, the smaller the index value is, the worse the stability will be defined as a positive index. For the set of positive indicators \( Q^+ (q_1,...q_k) \), and the determination criteria of positive ideal point \( A^+ \) and negative ideal point \( A^- \) are shown in the Eqs. (5) and (6).

\[
\begin{align*}
A^+_i &= \{q^+_1,q^+_2,...,q^+_k\} = \{\max(q^+_j) | j = 1,2,...,k\} \\
A^-_i &= \{q^-_1,q^-_2,...,q^-_k\} = \{\min(q^-_j) | j = 1,2,...,k\} \\
A^+_i &= \{q^+_1,q^+_2,...,q^+_{m-k}\} = \{\min(q^+_j) | j = 1,2,...,m-k\} \\
A^-_i &= \{q^-_1,q^-_2,...,q^-_{m-k}\} = \{\max(q^-_j) | j = 1,2,...,m-k\}
\end{align*}
\] (5)

Similarly, according to the definition of negative indices, for the set of negative indices \( Q^- (q_1,...q_k) \) to establish the determination criteria of positive and negative ideal points.

4. Approximation calculation

According to the TOPSIS theory, the criterion for determining the optimal target is the relative distance between the attribute value of the index and the corresponding positive ideal point \( A^+ \) and negative ideal point \( A^- \). The closer the distance between the attribute value and the corresponding grade \( A^+ \) and \( D^- \), the better the index is under the condition of this grade. In order to obtain the value of the relative distance \( D \), the closeness distance function \( D(x) \) is introduced. As shown in Equations (6) and (7).

\[
\begin{align*}
D^+_{i-k} &= \sum_{i=1}^{m}W_i \left( \frac{q_i - A^-_{i-k}}{q_{i-k \text{max}} - q_{i-k \text{min}}} \right)^2 \left(1 + \frac{1}{2} \right) \\
D^-_{i-k} &= \sum_{i=1}^{m}W_i \left( \frac{q_i - A^+_{i-k}}{q_{i-k \text{max}} - q_{i-k \text{min}}} \right)^2 \left(1 + \frac{1}{2} \right)
\end{align*}
\] (6) and (7)

Where, \( q_{i-k \text{max}} \) and \( q_{i-k \text{min}} \) are the upper and lower limits of the index value range of the ith index of the evaluation object under the K-level stability grade.

5. Results of evaluation

After obtaining the relative distance \( D \) between all index attribute values and the ideal point A, the ideal point closeness degree \( T_i \) \( \{T_1, T_2, ..., T_l\} \) \( (l = 1,2,3,...,l) \).

\[
T_i = \frac{D^+_{i-k}}{D^+_{i-k} + D^-_{i-k}}
\] (8)
According to the calculation formula, the range of $T_i$ is the interval $[0,1]$, and the stability level corresponding to the maximum closeness of the calculated results is taken as the evaluation result. If $T_k = \max\{T_1, T_2, \ldots, T_l| i = 1,2,\ldots, l\}$, then the stability level of the tunnel is determined to be $K$.

6. Engineering application

To evaluate the stability of urban underground engineering, and the stability grade is divided into five grades. i.e. I (extremely stability) II (medium stability) III (very stability) IV (modest stability) (medium stability) V (modest stability).

| Table 1. Grading system of the tunnel stability |
|---|---|---|---|---|
| Indicator | Extremely stability (I) | Very stability (II) | Medium stability (III) | Stability (IV) | Slightly stability (V) |
| $I_1$ | 0~30 | 30~60 | 60~90 | 90~150 | 150~200 |
| $I_2$ | 1.0 | 0.8 | 0.6 | 0.4 | 0.2 |
| $I_3$ | 0~30 | 30~50 | 50~80 | 80~100 | 100~120 |
| $I_4$ | 0.8~1.0 | 0.6~0.8 | 0.4~0.6 | 0.2~0.4 | 0~0.2 |
| $I_5$ | 40~50 | 30~40 | 20~30 | 10~20 | 0~10 |
| $I_6$ | 0–4 | 4–8 | 8–12 | 12–14 | 14–16 |

Further, an expert and scholar of Marine geology, geotechnical engineering and Marine engineering were invited to score the selected indicators. The scoring table was established as follows, and the weight of each index was obtained according to the Eqs.(6)-(8).

| Table 2. Comparison matrix |
|---|---|---|---|---|
| 1/2 | 1 | 2 | 3 | 3 |
| 1/2 | 1 | 1 | 2 | 2 |
| 1/2 | 1 | 1 | 2 | 2 |
| 1/3 | 1/2 | 1/2 | 1 | 1 |
| 1/3 | 1/2 | 1/2 | 1 | 1 |
| 1/4 | 1/3 | 1/3 | 1/2 | 1/2 |

According to the calculation principle of AHP, the decision table was substituted into the formula to obtain the leading weight set of indexes $W_i \{0.3042, 0.1839, 0.1839, 0.1340, 0.1340, 0.060\}$, and the index consistency test parameter was obtained, and the index ranking process met the requirements of consistency. As the index value increases, it also reduces the stability index of the project to some extent, so it belongs to the negative index. Relatively speaking, when its index value increases within a certain range, it will increase the index of engineering stability, which is a positive index. According to the definition of ideal point in TOPSIS theory, the positive ideal point matrix of the parameters of the tunnel to be evaluated is determined by Eq. (9) and Eq. (10) respectively.

$$A^+(a) = \begin{pmatrix}
5.0 & 200 & 0 & 0 & 0 & 55 \\
4.0 & 120 & 10 & 10 & 20 & 40 \\
3.4 & 90 & 20 & 20 & 40 & 30 \\
2.6 & 60 & 30 & 30 & 60 & 20 \\
1.8 & 30 & 40 & 40 & 80 & 10
\end{pmatrix}$$ (9)
The parameters of 6 groups of actual indicators were selected and the TOPSIS theoretical algorithm was adopted to evaluate the stability grade of the tunnel during construction. The results are shown in Table.

**Table 3. Sample data of tunnel parameters**

| Number | Ideal point proximity | Results |
|--------|-----------------------|---------|
|        | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ |          |
| 1      | 0.4110 | 0.8125 | 0.4693 | 0.2830 | 0.3853 | II       |
| 2      | 0.6194 | 0.6195 | 0.4556 | 0.2967 | 0.3810 | II       |
| 3      | 0.5790 | 0.2617 | 0.6544 | 0.6575 | 0.3465 | II       |
| 4      | 0.5721 | 0.5779 | 0.5304 | 0.4248 | 0.4174 | II       |
| 5      | 0.5662 | 0.6052 | 0.5953 | 0.4200 | 0.3883 | II       |

Based on the tunnel stability evaluation system, the evaluation results of the model were compared with the actual grade, as shown in the figure. The TOPSIS evaluation model can basically accurately evaluate the stability of the tunnel in an objective and reasonable manner.

**7. Conclusion**

Due to the extremely importance to the stability of underground engineering construction, this paper chose six indexes, including the maximum depth, maximum deformation of surrounding rock, degree of construction disturbance, construction level, rockmass elastic modulus, and the analytic hierarchy process (AHP) is adopted to the six indexes sensitivity analysis and the weight calculation, the calculation results show that: \{0.3042, 0.1839, 0.1839, 0.1340, 0.1340, 0.060\}, where the maximum buried depth has the largest weight, that is, 0.3042, and the minimum weight of elastic modulus is 0.06.

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