Gravitational quantization of exoplanet orbits in HD 10180, Kepler-32, Kepler-33, Kepler-102, and Kepler-186

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Abstract

The so-called “global polytropic model” is applied to the numerical study of the exoplanet systems HD 10180, Kepler-32, Kepler-33, Kepler-102, and Kepler-186. We compare computed distances of planets from their host stars with corresponding observations and discuss some further orbit predictions made by the model.

Keywords: exoplanets; global polytropic model; planets: orbits; quantized orbits; stars: individual (HD 10180, Kepler-32, Kepler-33, Kepler-102, Kepler-186)

1 Introduction

This work is continuation of two previous papers regarding exoplanet systems ([1], [2]). We do not intend to repeat here issues developed in these papers; interested readers can find a detailed account of the so-called “global polytropic model” — which assumes hydrostatic equilibrium for a planetary system and proceeds to relevant computations — in [1] (Secs. 2, 3, and references therein). Here, we study numerically the exoplanet systems HD 10180, Kepler-32, Kepler-33, Kepler-102, and Kepler-186.

In the following tables, the first root $\xi_1$ of the Lane–Emden function $\theta$, coinciding with the radius of the host star, is expressed in both “classical polytropic units” (cpu) — in such units, the length unit is equal to the polytropic parameter $\alpha$ ([3], Eq. (3b)) — and solar radii $R_\odot$. All other orbit radii are expressed in AU.
2 The HD 10180 System

For the HD 10180 system ([4], [5], [6]; evidence for 9 planets in this system is discussed in [5]), our results are given in Table 1. The minimum sum of absolute percent errors is

\[ \Delta_{\text{min}} \left( n_{\text{opt}}(\text{HD 10180}) = 3.060; q_b = 2, q_c = 3, \right. \]
\[ \left. q_d = 4, q_e = 5, q_f = 6, q_g = 8, q_h = 11 \right) \approx 83.4. \]  

(1)

Smaller error is that for f’s distance, \( \approx 0.05\% \), and larger one is that for b’s distance, \( \approx 43.6\% \). The average error for the computed orbit radii of the 7 planets in HD 10180 is \( \approx 11.9\% \).

Regarding the large error involved in the distance of b (the larger one among the systems examined here), it may arise due to the proximity of the shell No 2, accommodating the planet b, with the host star. In fact, HD 10180 is the only system examined here with a planet hosted in the innermost shell. This situation is similar to that revealed for the planet e of the 55 Cnc system ([1], Sec. 4 and Table 1) with its computed distance differing \( \sim 32\% \) from the observed one.

Alternatively, an interesting conjecture — made firstly for the planet f of the HD 40307 system ([2], Eq. (2) and Sec. 3.1) — is to associate this distance with the right average-density orbit \( \alpha_{R2} = 0.0234 \text{ AU} \), provided that the maximum-density orbit \( \alpha_2 \) is already occupied by another planet not yet observed. If so, then the error for b’s distance drops to \( \approx 5.4\% \), the minimum sum of absolute percent errors drops to \( \approx 45.2\% \), and the average error for the HD 10187 system drops to \( \approx 6.5\% \).

For convenience, we will use hereafter the abbreviations LADC and RADC for the “left average-density orbit conjecture” and the “right average-density orbit conjecture”, respectively.

Next, regarding the evidence given in Table 3 of [5] for the existence of two more planets i and j (9-planet solution for the system HD 10180) at distances 0.09 AU and 0.33 AU from the host star, we find that the distance of i can be associated with the left average-density orbit \( \alpha_{L4} = 0.1040 \text{ AU} \), so differing \( \approx 15.6\% \) from that. The distance of j can be associated in turn with the right average-density orbit \( \alpha_{R5} = 0.3292 \text{ AU} \), thus differing \( \approx 0.2\% \) from that. In both shells No 4 and No 5, the maximum-density orbits \( \alpha_4 \) and \( \alpha_5 \) are occupied by the already observed planets d and e. Thus the condition: “provided that the maximum-density orbit is already occupied by another planet” in our conjecture becomes a fact for these shells.

3 The Kepler-32 System

For the Kepler-32 system ([7]; observational data used in the comparisons are from this paper), the optimum case found by the global polytropic model (Ta-
ble (2) has minimum sum of absolute percent errors

\[ \Delta_{\min}\left(n_{\text{opt}}(\text{Kepler} - 32) = 2.608; q_f = 3, q_e = 5, q_b = 5, q_c = 6, q_d = 8\right) \simeq 45.2. \]

(2)

Smaller error is that for f’s distance, \( \simeq 2.2\% \); while larger one is that for b’s distance, \( \simeq 23.9\% \). The average error for the computed distances of the 5 planets in Kepler-32 is \( \simeq 9.1\% \).

Regarding the large error involved in b’s distance, it seems interesting to invoke for this case both LADC and RADC. In particular, provided that the maximum-density orbit \( \alpha_5 = 0.0395 \) AU is already occupied by a planet not yet observed, the planet e is hosted on the left average-density orbit \( \alpha_{L5} = 0.0337 \) AU (as in Table 1), and the planet b occupies the right average-density orbit \( \alpha_{R5} = 0.0457 \) AU. Thus we need to employ all three available hosting orbits within the shell No 5; a similar situation has been revealed in the discussion on certain orbit predictions regarding the Kepler-275 system (2, Sec. 3.5). Accordingly, the difference for b’s distance drops to \( \simeq 11.9\% \), the minimum sum of absolute percent errors drops to \( \simeq 33.2\% \), and the average error for the computed distances of the planets in Kepler-32 drops to \( \simeq 6.6\% \).

4 The Kepler-33 System

Concerning the Kepler-33 system (8; observational data used in the comparisons are from this paper), the optimum case computed by the global polytropic model (Table 3) gives minimum sum of absolute percent errors

\[ \Delta_{\min}\left(n_{\text{opt}}(\text{Kepler} - 33) = 2.592; q_b = 4, q_c = 5, q_d = 6, q_e = 6, q_f = 7\right) \simeq 30.8. \]

(3)

Smaller error is that for e’s distance, \( \simeq 0.2\% \), while larger error is that for d’s distance, \( \simeq 15.7\% \). The average error for the computed distances of the 5 planets in Kepler-33 is \( \simeq 6.2\% \).

5 The Kepler-102 System

The optimum case for the 5-planet Kepler-102 system (Table 4) gives minimum sum of absolute percent errors

\[ \Delta_{\min}\left(n_{\text{opt}}(\text{Kepler} - 102) = 2.605; q_b = 5, q_c = 5, q_d = 6, q_e = 7, q_f = 8\right) \simeq 23.1. \]

(4)
6 The Kepler-186 System

The optimum case for the 5-planet Kepler-186 system ([9], [10]) gives minimum sum of absolute percent errors (Table 5)

\[ \Delta_{\text{min}} \left( n_{\text{opt}}(\text{Kepler - 186}) = 2.530; \ q_b = 6, \ q_c = 7, \ q_d = 8, \ q_e = 9, \ q_f = 15 \right) \approx 34.3. \]  

Smaller error is that for b’s distance, \( \approx 1.9\% \), while larger error is that for c’s distance, \( \approx 13.2\% \). The average error for the computed distances of the 5 planets in Kepler-102 is \( \approx 4.6\% \).
Table 1: The HD 10180 system: central body $S_1$, i.e. the host star HD 10180, and polytropic spherical shells of the planets b, c, d, e, f, g. For successive shells $S_j$ and $S_{j+1}$, inner radius of $S_{j+1}$ is the outer radius of $S_j$. All radii are expressed in AU, except for the host’s radius $\xi_1$. Percent errors $\%E_j$ in the computed orbit radii $\alpha_j$ are given with respect to the corresponding observed radii $A_j$, $\%E_j = 100 \times |(A_j - \alpha_j)|/A_j$. Parenthesized signed integers following numerical values denote powers of 10.

| Host star HD 10180 – Shell No | 1         | 2         | 3         | 4         | 5         | 6         | 7         | 8         | 9         | 10        | 11        |
|--------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $n_{\text{opt}}$              | 3.060 (+00)|          |           |           |           |           |           |           |           |           |           |
| $\xi_1$ (cpu)                  | 7.1385 (+00)|          |           |           |           |           |           |           |           |           |           |
| $\xi_1$ ($R_\odot$)           | 1.20 (+00) |          |           |           |           |           |           |           |           |           |           |
| $\%E_j$                        |           |           |           |           |           |           |           |           |           |           |           |
| $A$                            |           |           |           |           |           |           |           |           |           |           |           |
| $\alpha_j$                     |           |           |           |           |           |           |           |           |           |           |           |

| b – Shell No                   | 2         | 2         | 3         | 4         | 5         | 6         | 7         | 8         | 9         | 10        | 11        |
| Inner radius, $\xi_1$          | 5.5827 (−03)|          |           |           |           |           |           |           |           |           |           |
| Outer radius, $\xi_2$          | 3.0524 (−02)|          |           |           |           |           |           |           |           |           |           |
| Orbit radius, $\alpha_b = \alpha_2$ | 1.2513 (−02)| 2.22 (−02) | 6.41 (−02) | 1.83 (+01)|           |           |           |           |           |           |           |
| c – Shell No                   | 3         | 3         | 4         | 5         | 6         | 7         | 8         | 9         | 10        | 11        |           |
| Outer radius, $\xi_3$          | 9.0232 (−02)|          |           |           |           |           |           |           |           |           |           |
| Orbit radius, $\alpha_c = \alpha_3$ | 5.2398 (−02)| 6.41 (−02) | 1.83 (+01)|           |           |           |           |           |           |           |           |
| d – Shell No                   | 4         | 4         | 5         | 6         | 7         | 8         | 9         | 10        | 11        |           |           |
| Outer radius, $\xi_4$          | 2.0074 (−01)|          |           |           |           |           |           |           |           |           |           |
| Orbit radius, $\alpha_d = \alpha_4$ | 1.3373 (−01)| 1.286 (−01) | 3.99 (+00)|           |           |           |           |           |           |           |           |
| e – Shell No                   | 5         | 5         | 6         | 7         | 8         | 9         | 10        | 11        |           |           |           |
| Outer radius, $\xi_5$          | 3.7905 (−01)|          |           |           |           |           |           |           |           |           |           |
| Orbit radius, $\alpha_e = \alpha_5$ | 2.7529 (−01)| 2.699 (−01) | 1.99 (+00)|           |           |           |           |           |           |           |           |
| f – Shell No                   | 6         | 6         | 7         | 8         | 9         | 10        | 11        |           |           |           |           |
| Outer radius, $\xi_6$          | 6.4201 (−01)|          |           |           |           |           |           |           |           |           |           |
| Orbit radius, $\alpha_f = \alpha_6$ | 4.9270 (−01)| 4.929 (−01) | 4.08 (−02)|           |           |           |           |           |           |           |           |
| g – Shell No                   | 8         | 8         | 9         | 10        | 11        |           |           |           |           |           |           |
| Inner radius, $\xi_7$          | 1.0079 (+00)|          |           |           |           |           |           |           |           |           |           |
| Outer radius, $\xi_8$          | 1.4938 (+00)|          |           |           |           |           |           |           |           |           |           |
| Orbit radius, $\alpha_g = \alpha_8$ | 1.2255 (+00)| 1.422 (+00) | 1.38 (+01)|           |           |           |           |           |           |           |           |
| h – Shell No                   | 11        |           |           |           |           |           |           |           |           |           |           |
| Inner radius, $\xi_{10}$       | 2.9012 (+00)|          |           |           |           |           |           |           |           |           |           |
| Outer radius, $\xi_{11}$       | 3.8603 (+00)|          |           |           |           |           |           |           |           |           |           |
| Orbit radius, $\alpha_h = \alpha_{11}$ | 3.3449 (+00)| 3.4 (+00) | 1.62 (+00)|           |           |           |           |           |           |           |           |
Table 2: The Kepler-32 system: central body $S_1$, i.e. the host star Kepler-32, and polytropic spherical shells of the planets f, e, b, c, d. Other details as in Table 1.

| Host star Kepler-32 – Shell No | 1 |
|---------------------------------|---|
| $n_{opt}$                       | 2.608 (+00) |
| $\xi_1$ (cpu)                   | 5.6307 (+00) |
| $\xi_1$ ($R_\odot$)            | 5.3 (−01) |

|              | A (Å) | %E  |
|--------------|-------|-----|
| f – Shell No | 3     |     |
| Inner radius, $\xi_2$ | 8.9187 (−03) |     |
| Outer radius, $\xi_3$ | 1.6317 (−02) |     |
| Orbit radius, $\alpha_f = \alpha_3$ | 1.3290 (−02) 1.3 (−02) 2.23 (+00) |     |
| c – Shell No | 5     |     |
| Inner radius, $\xi_4$ | 2.9566 (−02) |     |
| Outer radius, $\xi_5$ | 5.2173 (−02) |     |
| Orbit radius, $\alpha_c = \alpha_{1.5}$ | 3.3697 (−02) 3.23 (−02) 4.33 (+00) |     |
| b – Shell No | 5     |     |
| Outer radius, $\xi_6$ | 7.5932 (−02) |     |
| Orbit radius, $\alpha_b = \alpha_5$ | 3.9499 (−02) 5.19 (−02) 2.39 (+01) |     |
| c – Shell No | 6     |     |
| Outer radius, $\xi_7$ | 9.8619 (−02) |     |
| Orbit radius, $\alpha_c = \alpha_6$ | 6.9474 (−02) 6.7 (−02) 3.69 (+00) |     |
| d – Shell No | 8     |     |
| Inner radius, $\xi_8$ | 1.3745 (−01) |     |
| Outer radius, $\xi_8$ | 1.1388 (−01) |     |
| Orbit radius, $\alpha_d = \alpha_8$ | 1.28 (−01) 1.10 (+01) |     |
Table 3: The Kepler-33 system: central body $S_1$, i.e. the host star Kepler-33, and polytropic spherical shells of the planets b, c, d, e, f. Other details as in Table 2.

| Host star Kepler-33 – Shell No | 1 |
|-------------------------------|---|
| $n_{opt}$ | 2.592 (+00) |
| $\xi_1$ (cpu) | 5.5882 (+00) |
| $\xi_1$ ($R_\odot$) | 1.615 (+00) |

| Shell No | b | c | d | e | f |
|-----------|---|---|---|---|---|
| Inner radius, $\xi_3$ | 4.8624 (−02) | 4.8624 (−02) | 4.8624 (−02) | 4.8624 (−02) | 4.8624 (−02) |
| Outer radius, $\xi_4$ | 8.8875 (−02) | 1.5606 (−01) | 2.2816 (−01) | 2.9495 (−01) | 2.9495 (−01) |
| Orbit radius, $\alpha_b = \alpha_4$ | 5.8352 (−02) | 6.544 (−02) | 6.544 (−02) | 6.544 (−02) | 6.544 (−02) |
| $\%E$ | 1.08 (+01) | 2.97 (+00) | 1.84 (−01) | 1.57 (+01) | 1.15 (+00) |

| Shell No | | | | | |
|-----------|---|---|---|---|---|
| Outer radius, $\xi_5$ | 1.1825 (−01) | 1.5606 (−01) | 2.2816 (−01) | 2.9495 (−01) | 2.9495 (−01) |
| Orbit radius, $\alpha_c = \alpha_5$ | 1.1484 (−01) | 1.6053 (−01) | 2.0656 (−01) | 2.0656 (−01) | 2.0656 (−01) |
| $\%E$ | 2.97 (+00) | 1.84 (−01) | 1.15 (+00) | 1.15 (+00) | 1.15 (+00) |

| Shell No | | | | | |
|-----------|---|---|---|---|---|
| Outer radius, $\xi_6$ | 2.0618 (−01) | 2.0618 (−01) | 2.0618 (−01) | 2.0618 (−01) | 2.0618 (−01) |
| Orbit radius, $\alpha_d = \alpha_6$ | 2.0656 (−01) | 2.0656 (−01) | 2.0656 (−01) | 2.0656 (−01) | 2.0656 (−01) |
| $\%E$ | 1.84 (−01) | 1.84 (−01) | 1.84 (−01) | 1.84 (−01) | 1.84 (−01) |

| Shell No | | | | | |
|-----------|---|---|---|---|---|
| Outer radius, $\xi_7$ | 2.4773 (−01) | 2.4773 (−01) | 2.4773 (−01) | 2.4773 (−01) | 2.4773 (−01) |
| Orbit radius, $\alpha_f = \alpha_7$ | 2.449 (−01) | 2.449 (−01) | 2.449 (−01) | 2.449 (−01) | 2.449 (−01) |
Table 4: The Kepler-102 system: central body $S_1$, i.e. the host star Kepler-102, and polytropic spherical shells of the planets b, c, d, e, f. Other details as in Table III.

| Host star Kepler-102 – Shell No | 1 |
|---------------------------------|---|
| $n_{\text{opt}}$                | 2.605 (+00) |
| $\xi_1$ (cpu)                   | 5.6227(+00) |
| $\xi_1$ ($R_\odot$)            | 7.4 (-01) |

| b – Shell No | 5 |
|--------------|---|
| Inner radius, $\xi_4$          | 4.1175(-02) |
| Outer radius, $\xi_5$          | 7.2596(-02) |
| Orbit radius, $\alpha_b = \alpha_5$ | 5.4983(-02) 5.5(-02) 3.05(-02) |

| c – Shell No | 5 |
|--------------|---|
| Orbit radius, $\alpha_c = \alpha_{R5}$ | 6.3616(-02) 6.7(-02) 5.05(+00) |

| d – Shell No | 6 |
|--------------|---|
| Outer radius, $\xi_6$          | 1.0576(-01) |
| Orbit radius, $\alpha_d = \alpha_6$ | 9.6527(-02) 8.6(-02) 1.22(+01) |

| e – Shell No | 7 |
|--------------|---|
| Outer radius, $\xi_7$          | 1.3722(-01) |
| Orbit radius, $\alpha_e = \alpha_7$ | 1.1395(-01) 1.16(-01) 1.77(+00) |

| f – Shell No | 8 |
|--------------|---|
| Outer radius, $\xi_8$          | 1.9100(-01) |
| Orbit radius, $\alpha_f = \alpha_8$ | 1.5828(-01) 1.65(-01) 4.08(+00) |
Table 5: The Kepler-186 system: central body $S_1$, i.e. the host star Kepler-186, and polytropic spherical shells of the planets b, c, d, e, f. Other details as in Table 1.

| Host star Kepler-186 – Shell No | 1 |
|---------------------------------|--|
| $n_{\text{opt}}$               | 2.530 (+00) |
| $\xi_1$ (cpu)                  | 5.4292(+00) |
| $\xi_1$ ($R_\odot$)            | 4.7 (−01) |

| b – Shell No | 6 |
|-------------|--|
| Inner radius, $\xi_5$        | 3.7778(−02) |
| Outer radius, $\xi_6$        | 5.3777(−02) |
| Orbit radius, $\alpha_b = \alpha_6$ | 3.8502(−02) | 3.78(−02) | 1.86(+00) |

| c – Shell No | 7 |
|-------------|--|
| Outer radius, $\xi_7$        | 7.9508(−02) |
| Orbit radius, $\alpha_c = \alpha_7$ | 6.4962(−02) | 5.74(−02) | 1.32(+01) |

| d – Shell No | 8 |
|-------------|--|
| Outer radius, $\xi_8$        | 1.0746(−01) |
| Orbit radius, $\alpha_d = \alpha_8$ | 9.6356(−02) | 8.61(−02) | 1.19(+01) |

| e – Shell No | 9 |
|-------------|--|
| Outer radius, $\xi_9$        | 1.3021(−01) |
| Orbit radius, $\alpha_e = \alpha_9$ | 1.1837(−01) | 1.216(−01) | 2.66(+00) |

| f – Shell No | 15 |
|-------------|--|
| Inner radius, $\xi_{14}$      | 3.5064(−01) |
| Outer radius, $\xi_{15}$      | 3.9583(−01) |
| Orbit radius, $\alpha_f = \alpha_{15}$ | 3.7423(−01) | 3.926(−01) | 4.68(+00) |
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