Abstract—The rapid development of autonomous vehicles (AVs) holds vast potential for transportation systems through improved safety, efficiency, and access to mobility. However, the progression of these impacts, as AVs are adopted, is not well understood. Numerous technical challenges arise from the goal of analyzing the partial adoption of autonomy: partial control and observation, multi-vehicle interactions, and the sheer variety of scenarios represented by real-world networks. To shed light into near-term AV impacts, this article studies the suitability of deep reinforcement learning (RL) for overcoming these challenges in a low AV- adoption regime. A modular learning framework is presented, which leverages deep RL to address complex traffic dynamics. Modules are composed to capture common traffic phenomena (stop-and-go traffic jams, lane changing, intersections). Learned control laws are found to improve upon human driving performance, in terms of system-level velocity, by up to 57% with only 4-7% adoption of AVs. Furthermore, in single-lane traffic, a small neural network control law with only local observation is found to eliminate stop-and-go traffic—surpassing all known model-based controllers to achieve near-optimal performance—and generalize to out-of-distribution traffic densities.

Index Terms—Automation technologies for smart cities, deep learning in robotics and automation, deep reinforcement learning, intelligent transportation systems

I. INTRODUCTION

Autonomous vehicles (AVs) are projected to enter society in the very near future, with full adoption in select areas expected as early as 2050 [1]. However, the uncertainty in potential impacts is vast. A recent study estimated that fuel consumption in the U.S. could decrease as much as 40% or increase as much as 100% once autonomous fleets of vehicles are rolled out [1], potentially exacerbating the 28% of energy consumption that is attributed to transportation in the US [2]. As such, computational tools are needed for the design, study, and control of these complex, large-scale robotic systems.

Existing tools are largely limited to two commonly studied regimes: where AVs are few enough as to not affect the surrounding traffic dynamics [3], [4], [5], or so ubiquitous as to become a coordination problem [6], [7], [44]. For clarity, we refer to these as the isolated autonomy and full autonomy cases, respectively. At the same time, the intermediate regime, which is the long and arduous transition from no (or few) AVs to full adoption, is poorly understood. We term this intermediate regime mixed autonomy. The understanding of mixed autonomy is crucial for the design of suitable vehicle controllers, efficient transportation systems, sustainable urban planning, and public policy in the advent of AVs. This article focuses on autonomous vehicles, which we expect to be among the first robotic systems to enter and widely affect existing societal systems. Additional highly anticipated robotic systems, which may benefit from similar techniques as presented in this article, include aerial vehicles, household robotics, automated personal assistants, and additional infrastructure systems.

The mixed autonomy setting exposes heightened complexities due to the interactions of numerous human and robotic agents in highly varied contexts, for which the predominant analytical approaches of the traffic community are largely unsuitable. Instead, we observe that model-free deep reinforcement learning (RL) permits the decoupling of the mathematical modeling of the system dynamics from the control law design, thereby side-stepping limitations of classical approaches. Specifically, we propose a modular learning framework, in which environments representing complex control scenarios are comprised of reusable components, analogous to “LEGO” blocks. We validate the proposed methodology on a classic traffic control scenario exhibiting backward-propagating traffic shockwaves in a partially-observed environment, and we subsequently produce a learned control law which far exceeds all previous methods, generalizes to unseen traffic densities, and closely matches theoretical performance bounds. Finally, we demonstrate the efficacy of the framework by studying more complex traffic scenarios for which control-theoretic results are not known. By appropriately composing the reusable components, we further demonstrate the effectiveness of the methodology with preliminary results on
novel multi-lane, multi-AV, and intersection control scenarios. To facilitate future research in mixed autonomy traffic, we developed and open-sourced the modular learning framework as Flow, which exposes design primitives for composing traffic control scenarios. Our contributions aim to enable the community to study not only mixed autonomy scenarios which are composites of analytical mathematical frameworks, but also arbitrary networks or even full-blown traffic microsimulators, designed to simulate hundreds of thousands of agents in complex environments (see examples in Figure 1).

The contributions of Flow to the research community are multi-faceted. For the robotics community, Flow seeks to enable characterizations and empirical study of complex, large-scale, and realistic multi-robot control scenarios. For the machine learning community, Flow seeks to expose to modern RL algorithms to a class of challenging control scenarios derived from an important real-world domain. For the control community, Flow seeks to provide intuition, through successful learned-control laws, for new provable control techniques for traffic-related control scenarios. Finally, for the transportation community, Flow seeks to provide a new methodological pathway, through reusable traffic modules and modern RL methods, that addresses new challenges concerning AVs and long-standing challenges concerning traffic control.

The rest of the article is organized as follows: Section II introduces the problem of mixed autonomy. Section III presents related work to place this article in the broader context of automated vehicles, traffic flow modeling, and deep RL. Section IV summarizes requisite concepts from RL and traffic. Section V describes the modular learning framework for the scalable design and experimentation of traffic control scenarios. This is followed by two experimental sections: Section VI in which we validate the modular learning framework on a canonical traffic control scenario; and Section VII which presents more sophisticated applications of the framework to more complex traffic control scenarios.

II. Mixed Autonomy

To shed light into the progression of impacts that AVs may have as they are adopted into transportation systems, this article introduces the problem of mixed autonomy: that is, given a traffic system with a fraction $p$ of AVs, what level of performance is achievable under utility function $U_p(\cdot)$? More generally, for a particular type of autonomy, or advanced automation, (e.g. AVs, traffic signals, roadway pricing), mixed autonomy is the intermediate regime between a system with no adoption of autonomy and a system where autonomy is fully
employed. For example, in the context of AVs, full autonomy corresponds to 100% of vehicles being autonomously driven, no autonomy corresponds to 0% AVs, and mixed autonomy corresponds to some fraction \( p \) of vehicles being AVs. In this article, we take \( U_p(\cdot) \) to be the average velocity of the system.

| Uncertainty in system dynamics | Uncertainty in objective |
|-------------------------------|-------------------------|
| Low                           | Low                     |
| High                          | High                    |
| Isolated autonomy             | Full autonomy            |
| Mixed autonomy                |                         |

**TABLE I: Uncertainties arising in autonomy settings.**

Mixed autonomy differs from the earlier introduced cases in two nuanced but important ways, summarized in Table I. First, the evaluation criteria of interest in mixed autonomy settings may exhibit more uncertainty than in isolated autonomy. Isolated autonomy is often evaluated with respect to a known outcome, such as human or expert performance for a similar task. The canonical example is the performance of a single AV, compared to a human driver [9], [10]. Similarly, expert demonstrations are often considered a gold standard in robotic learning for a variety of tasks, including locomotion, grasping, and manipulation [11], [12], [13]. In these cases, we assume both knowledge of a good control law and that it is feasible to attain. However, evaluating with respect to a known outcome makes implicit assumptions about the capabilities of the autonomous system and the optimality of the known outcome, both of which may be incorrect. For example, in the context of traffic networks, evaluating with respect to the known human performance is restrictive; it is well known that human driving behavior induces (suboptimal) stop-and-go traffic in a wide regime of traffic scenarios [14], [15]. In mixed autonomy settings, we are instead interested in evaluating with respect to a broader performance measures, such as traffic congestion, to understand potential effects on the system as a whole; the measures may be system-level, may be unattainable, and may be only partially controllable.

Second, the degree of uncertainty in the system dynamics is greater in mixed autonomy than in full autonomy. In contrast to isolated autonomy, full autonomy is indeed often evaluated with respect to a system-level objective, such as average travel time. However, its system dynamics often exhibit low uncertainty due to the simple fact that much of the system’s state evolution is directly determined by autonomy. That is, all autonomous components are known, as are the effects of their actions; and uncertainty from human behavior, common to isolated and mixed autonomy, is largely eliminated. The low uncertainty in system dynamics permits direct modeling and analysis within a number of powerful mathematical frameworks, including partial differential equations [16], [17], [18], [19], ordinary differential equations [20], [21], [22], [23], and queuing systems [24], [25], [26]. In a few cases, control-theoretic performance bounds or optimal controllers can even be analytically derived. Even so, full autonomy is far from solved. In contrast, mixed autonomy suffers from additional challenges including interactions with humans of unknown or complex dynamics, partial observability from sensing limitations, and partial controllability due to the lack of full autonomy. Therefore, a strict coupling between the mathematical modeling and the system evaluation is not sensible for studying mixed autonomy; in this article, we thus take a model-free approach.

Mixed autonomy thus inherits the challenges from both isolated autonomy and full autonomy (see Table I). In this work, we propose and validate a new methodology for addressing mixed autonomy in the context of traffic networks. We posit that sampling-based optimization allows us to decouple mathematical modeling of the system dynamics and control-law design for arbitrary evaluation objectives, thereby overcoming the limitations of studies in both isolated and full autonomy. In particular, we propose that model-free deep RL is a compelling and suitable framework for the study of mixed autonomy. The decoupling allows the designer to specify arbitrary control objectives and system dynamics to explore the effects of autonomy on complex systems. For the system dynamics, the designer may model a system of interest in whichever mathematical or computational framework they wish, and we require only that the model is consistent with a (Partially Observed) Markov Decision Process ((PO)MDP) interface. For control law design, deep neural network architectures may be used for representing large and expressive control law (also called policy) classes. Finally, the resulting framework employs model-free deep RL to enable the designer to explore the effects of autonomy on a complex system, up to local optimality with respect to the control law parameterization.

**III. RELATED WORK**

**Control of automated vehicles.** Automated and autonomous vehicles have been studied in a myriad of contexts; here, we describe prior work in isolated, full, and mixed autonomy.

**Isolated autonomy.** Spurred by the US DARPA challenges in autonomous driving in 2005 and 2007 [27], [3], countless efforts have demonstrated the increasing ability of vehicles to operate autonomously on real roads and traffic conditions, without custom traffic infrastructure. These vehicles instead rely largely on on-board sensors (LIDAR, radar, camera, GPS), computer vision, motion planning, mapping, and behavior prediction, and are designed to obey traffic rules. Robotics has continued to demonstrate tremendous potential in improving transportation systems through AV research; highly related problems include localization [28], [29], [30], path planning [31], [32], collision avoidance [33], and perception [34]. Considerable progress has also been made in recent decades in vehicle automation, including anti-lock braking systems (ABS), adaptive cruise control (ACC), lane keeping, lane changing, parking, overtaking, etc. [35], [36], [37], [38], [39], [40], [41], [42], [43], which also have great potential to improve safety and efficiency in traffic. The development of these technologies is currently focused on the performance of the individual vehicle, rather than its interactions or effects on other parts of the transportation system.

**Full autonomy.** At the other end of the autonomy spectrum, all vehicles are automated and operate efficiently with collaborative control. Model-based approaches to such full autonomy have permitted the reservation system design and derivation of
vehicle weaving control for fully automated intersections \[6\], \[44\] and straight roads \[7\].

**Mixed autonomy.** A widely deployed form of vehicle automation is adaptive cruise control (ACC), which adjusts the longitudinal dynamics for vehicle pacing and driver comfort, and is grounded in classical control techniques \[45\], \[46\], \[47\]. Similarly, cooperative ACC (CACC) uses control theory to simultaneously optimize the performance of several adjacent vehicles, such as for minimizing the fuel consumption of a vehicle platoon \[48\], \[49\], \[50\], \[51\], \[56\]. Partial (C)ACC adoption is a form of mixed autonomy. This article similarly studies longitudinal vehicle controllers, with a key difference being our focus on system-level rather than local objectives, such as the average velocity of all vehicles in the system, which is important for system operations and planning.

A few studies have started to use formal techniques to design controllers for system-level evaluation of mixed autonomy traffic, including state-space \[52\] and frequency domain analysis \[53\]. There are also several modeling- and simulation-based evaluations of mixed autonomy systems \[54\], \[55\], \[56\] and model-based approaches to mixed autonomy intersection management \[57\]. Despite these advances in controller design, these approaches are generally limited to simplified models, such as homogeneous, non-delayed, deterministic driver models, or restricted network configurations. This article proposes to overcome these barriers through model-agnostic sampling-based methods. Impressively, concurrent work by Stern, et al. \[58\] demonstrated, in field operational tests, a reduction in fuel consumption of 40% by the insertion of an autonomous vehicle in traffic in a circular track to dampen the famous instabilities displayed by Sugiyama, et al. \[14\].

On a large-scale network, fleets of AVs have been studied for shared-mobility systems, such as autonomous mobility-on-demand \[59\], \[5\], \[60\], which abstracts out the low-level vehicle dynamics and considers a queuing theoretic model. Low-level vehicle dynamics, however, are crucial \[61\] because many low-level traffic phenomena affect energy consumption, safety, and travel time \[14\], \[62\], \[63\], \[64\].

**Single-lane traffic.** Although single-lane traffic has been studied for decades, the focus has been on modeling and control for local performance (e.g. comfort), rather than system-level performance (e.g. traffic congestion). Therefore, we take this to be our starting point. Various modeling approaches include closed networks \[14\], \[15\], \[52\], \[65\], open networks \[46\], \[66\], \[53\], different human driving models \[67\], and different objectives \[54\]. To the best of the authors’ knowledge, work has achieved an optimal controller in the mixed autonomy setting for single-lane traffic congestion. While studies in eco-driving practices provide heuristic guidance to drivers to ease traffic congestion \[68\], \[69\], the characterization of optimality of these practices has received limited attention.

The most closely related work includes Horn, et al. \[7\] and Stern, et al. \[58\]. Horn, et al. \[7\] presents a near-optimal controller for the full autonomy setting. Stern, et al. \[58\] presents two hand-designed control laws for the mixed autonomy setting, which incorporate knowledge of the environment and thus we refer to them as model-based control laws. These control laws are included in our experiments as baseline methods and are detailed in Appendix C. In contrast, the approach proposed by this work requires significantly less “design supervision” in the form of a reward function, which avoids explicitly employing domain knowledge or mathematical analysis.

**Modeling and control of traffic.** Mathematical modeling and analysis of traffic dynamics is notoriously complex and yet is a prerequisite for traffic control \[67\], \[70\], regardless of whether the control input is a vehicle, traffic light, ramp meter, or toll. Such mathematical frameworks include partial differential equations, ordinary differential equations (ODEs), queuing systems, and stochastic jump systems; for the modeling of highway traffic, longitudinal dynamics, intersections, and lateral dynamics, respectively. Researchers trade the complexity of the model (and thus its realism) for the tractability of analysis, with the ultimate goal of designing optimal and practical controllers. Consequently, results in traffic control can largely be classified as simulation-based numerical analysis with rich models but minimal theoretical guarantees \[66\], \[71\], \[72\], \[73\], or theoretical analysis on simplified models such as assuming non-oscillatory responses \[74\] or focusing on a single-lane circular track \[15\], \[75\], \[76\], \[7\], \[77\], \[53\].

Analysis techniques are often tightly coupled with the mathematical framework. For instance, linear systems theory techniques may be paired with ODEs and stochastic processes may be paired with queuing systems, but they may be incompatible with other mathematical frameworks. Thus, there are virtually no theoretical works that simultaneously study lateral dynamics, intersection dynamics, longitudinal dynamics, etc., for the reason that they are typically all modeled using different mathematical frameworks. As a notable exception, the work of Miculescu and Karaman \[44\] takes a model-based approach which considers both intersections and longitudinal dynamics for a two-way fully-automated intersection with simplified dynamics. This article seeks to take a step towards decoupling the reliance of control from the mathematical modeling of the problem, by proposing abstractions which permit the composition of reusable modules to specify problems and optimize for locally optimal controllers.

**Deep reinforcement learning (RL).** Deep RL is a powerful methodology for sequential decision making \[78\], \[79\], which inherits from machine learning and optimal control, and has demonstrated success in complex, data-rich problems such as Atari games \[80\], 3D locomotion and manipulation \[81\], \[82\], \[83\], and navigation of stratospheric balloons \[84\]. Deep RL is the key workhorse in our framework. The advances in deep RL provide a promising alternative to model-based controller design. More broadly, analyzing mixed autonomy traffic with RL is an intermediate step towards eventual deployment of autonomous fleets, providing insight to the system designer about a variety of complex performance metrics.

**Deep RL and Traffic.** Deep RL has been used for traffic prediction \[85\], \[86\] and control \[87\], \[88\], \[89\]. A deep RL architecture was used by Polson, et al. \[85\] to predict traffic flows, demonstrating success even during special events with nonlinear features. To learn features to represent states involving both space and time, Lv, et al. \[86\] additionally used
hierarchical autoencoding for traffic flow prediction. Deep Q Networks (DQN) were employed for learning traffic signal timings in Li, et al.\cite{87}. A multi-agent deep RL algorithm was introduced in Belletti, et al.\cite{88} to learn a control law for ramp metering. Wei, et al.\cite{90,89} employs RL and graph attention networks for control of traffic signals. For additional uses of deep learning in traffic, we refer the reader to Karlaftis, et al.\cite{91}, which presents an overview comparing non-neural statistical methods and neural networks in transportation research. These results demonstrate the promise of deep RL for traffic problems. This article is the first to employ deep RL to design controllers for AVs and assess their impacts on traffic flow. An early prototype of Flow is published\cite{92} and an earlier version of this manuscript is available\cite{93}. In comparison, this article provides a substantive presentation of the mixed autonomy problem, the learning framework, case studies, and experimental findings which contribute to the understanding of AVs and traffic dynamics.

IV. PRELIMINARIES

We now define notation and key concepts used subsequently.

A. MARKOV DECISION PROCESSES

The framework described in this article tackles scenarios which conform to the standard interface of an episodic finite-horizon discounted Markov decision process (MDP)\cite{94,95}, defined by the tuple \((S, A, P, r, \rho_0, \gamma, T)\), where \(S\) is a (possibly infinite) set of states, \(A\) is a (possibly infinite) set of actions, \(P : S \times A \times S \to \mathbb{R}_{\geq 0}\) is the transition probability distribution, \(r : S \times A \to \mathbb{R}\) is the reward function, \(\rho_0 : S \to \mathbb{R}_{\geq 0}\) is the initial state distribution, \(\gamma \in (0, 1]\) is the discount factor, and \(T\) is the time horizon for an episode. Partially observable scenarios conform to the interface of a partially observable Markov decision process (POMDP), and two more components are required: \(\Omega\), a set of observations, and \(O : S \times \Omega \to \mathbb{R}_{\geq 0}\), the observation probability distribution.

A note on terminology: We use the term scenario to describe a full traffic setting, including all learning and non-learning components. It conforms to a (PO)MDP interface. In robotics, the term “task” is commonly used; we prefer the word “scenario” to also capture settings with no learning components (e.g., situations with only human driver models).

Although traffic may be most naturally formulated as an infinite horizon problem, traffic phenomena such as traffic jams are ephemeral or even periodic. Thus, we formulate finite horizon MDPs. More generally, traffic has periodic patterns on a daily or weekly basis. Thus, the finite horizon problem can be a suitable approximation of the infinite horizon problem, so long as the horizon is sufficiently long to capture the transient or periodic behavior. For example, in the single-lane track scenario (Section V), the periods are around 40 sec. To enable the formation of the periodic behavior, we select a fairly long horizon length of 300 seconds (or 3000 simulation steps). Furthermore, we will select a high discount factor (close to 1) to approximate a non-discounted problem.

B. REINFORCEMENT LEARNING

RL studies the problem of how agents can learn to take actions in its environment, often formulated as an (PO)MDP, to maximize its cumulative reward\cite{78,79}. This article uses policy gradient methods\cite{96}, a class of RL algorithms which optimize a stochastic policy \(\pi_\theta : S \times A \to \mathbb{R}_{\geq 0}\), e.g., deep neural networks. Although commonly called a policy, we will generally refer to \(\pi\) as a controller or control law in this article, to be consistent with traffic control terminology. Three classes of control laws are considered: Linear network, Multilayer Perceptron (MLP), and Gated Recurrent Unit (GRU). The linear network is a parameterized linear function. The MLP is a classical artificial neural network with one or more hidden layers\cite{97}, consisting of linear weights and nonlinear activation functions (e.g. tanh, ReLU). The GRU is a recurrent neural network that makes use of parameterized update and reset gates, which enable decision making based on both current and past inputs\cite{98}.

C. VEHICLE DYNAMICS MODELS

The environments studied in this article are traffic systems. Basic traffic dynamics on single-lane roads can be represented by ordinary differential equation (ODE) models known as car following models (CFMs). These models describe the longitudinal dynamics of human-driven vehicles, given only observations about itself and the vehicle preceding it. CFMs vary in terms of model complexity, interpretability, and their ability to reproduce prevalent traffic phenomena, including stop-and-go traffic waves. For modeling of more complex traffic dynamics, including lane changing, merging, driving near traffic lights, and city driving, we refer the reader to the text of Treiber and Kesting\cite{67} dedicated to this topic.

Standard CFMs are of the form:

\[ a_i = \bar{v}_i = f(h_i, \dot{h}_i, v_i), \]  

where the acceleration \(a_i\) of car \(i\) is some typically non-linear function of \(h_i, \dot{h}_i, v_i\), which are the headway, relative velocity, and velocity for vehicle \(i\), respectively. Though a general model may include time delays, we will consider a non-delayed system, where all signals are measured at the same time instant. Example CFMs include the Intelligent Driver Model (IDM)\cite{21} and the Optimal Velocity Model (OVM)\cite{21,22}. IDM is used in the experiments of this article to model human driving (see Appendix B).

V. FLOW: A MODULAR LEARNING FRAMEWORK

While RL testbeds have enabled significant progress for algorithm development\cite{99,100,101}, testbeds which enable RL to shed light into important real-world problem domains remain limited. Moreover, in multi-agent systems such as traffic, the flexibility to study a wide range of scenarios is important due to their varied and complex nature, including different numbers and types of vehicles, heterogeneity of agents, network configurations, regional behaviors and regulations, etc. To this end, this article contributes an approach that decomposes a scenario into modules which can be configured and composed to create new scenarios of interest. Flow is
the resulting modular learning framework for enabling the creation, study, and control of complex traffic scenarios.

A. Scenario modules

Flow is comprised of the following modules, which can be assembled to form traffic scenarios of interest (see Figure 2).

Network: The network specifies the physical road layout, in terms of roads, lanes, length, shape, roadway connections, and additional attributes. Examples include a two-lane circular track with circumference 200m, or the structure described by importing a map from OpenStreetMap (see Figure 1 for examples of supported networks). Several of these examples will be used in demonstrative experiments in Sections VI and VII. More details about the specific networks are in Appendix A.

Actors: The actors describe the physical entities in the environment which issue control signals to the environment. In contrast to isolated autonomy settings, in a traffic setting, there are typically many interacting physical entities. Due to the focus on mixed autonomy, this article specifically studies vehicles as its physical entities. Other possible actors may include pedestrians, bicyclists, traffic lights, roadway signs, toll booths, as well as other transit modes and infrastructure.

Observer: The observer describes the mapping \( S \rightarrow O \) and yields the function of the state that is observed by the actor(s). The output of the observer is taken as input to the control law, described below. For example, while the state may include the position, lane, velocity, and accelerations of all vehicles in the system, the observer may restrict access to only information about local vehicles and aggregate statistics, such as average speed or queue length at an intersection.

Control laws: Control laws dictate the behaviors of the actors and are functions mapping observations to control inputs \( O \rightarrow A \). All actors require a control law, which may be pre-specified or learned. For instance, a control law may represent a human driver, an autonomous vehicle, or even a set of vehicles. That is, a single control law may be used to control multiple vehicles in a centralized control setting. Alternatively, a single control law may be used by multiple actors in a shared parameter control setting.

Dynamics: The dynamics module consists of additional sub-modules which describe different aspects of the system evolution, including vehicle routes, demands, stochasticity, traffic rules (e.g., right-of-way), and safety constraints.

Metrics: The metrics describe pertinent aggregated statistics of the environment. The reward signal for the learning agent is a function of these metrics. Examples include the average velocity of all vehicles and the number of hard braking events.

Initialization: The initialization describes the initial configuration of the environment at the start of an episode. Examples include setting the position and velocity of vehicles according to different probability distributions.

Sections VI and VII demonstrate the potential of the framework. Whereas in a model-based framing, many modules are simply not re-configurable due to differences in the mathematical descriptions (e.g. discrete versus continuous control inputs, such as in the case of longitudinal and lateral control), in this model-agnostic framework, disparate dynamics may be captured in the same scenario and effectively studied using sampling-based optimization techniques such as deep RL.

B. Architecture and implementation

The implementation of Flow is open source and builds upon open source software to promote access and extension. The project aims to support the development of custom modules and thereby permit the study of richer and more complex environments, agents, metrics, and algorithms. The implementation builds upon SUMO (Simulation of Urban MOBility) for traffic modeling, Ray RLlib for RL methods, and OpenAI gym for the MDP interface. SUMO is a microscopic traffic simulator, which explicitly models individual vehicles, pedestrians, traffic lights, and public transportation. It supports urban-scale road networks. Flow utilizes SUMO’s Python API, TraCI (Traffic Control Interface). Ray RLlib is a distributed framework for training and evaluating of RL algorithms. OpenAI gym is an MDP interface for RL tasks.

Flow is implemented as a lightweight architecture to connect the modules described in the previous section and permit experimentation. As typical in RL, an environment encodes
the MDP (scenario). The environment facilitates the composition of dynamics and other modules, stepping through the simulation, retrieving the observations, sampling and applying actions, computing the metrics and reward, and resetting the simulation at the end of an episode. A generator produces network configuration files compatible with SUMO according to the network description. The generator is invoked by the experiment upon initialization and, optionally, upon reset of the environment, allowing for a variety of initialization conditions, such as sampling from a distribution of vehicle densities. Flow then assigns control inputs from the different control laws to the corresponding actors, according to an action assigner, and uses the TraCI library to apply actions for each actor. Actions specified as accelerations are converted into velocities, using numerical integration and based on the timestep and the current state.

Finally, Flow is designed to be inter-operable with classical model-based methods for evaluation purposes. In other words, the learning component of Flow is optional, and this permits the fair comparison of diverse methods for traffic control.

VI. CONFIGURABLE MODULES FOR MIXED AUTONOMY

This section demonstrates that deep RL can solve a classic yet challenging traffic scenario. Specifically, the canonical setup of Sugiyama, et al. [14] is studied, which consists of 22 human-driven vehicles on a circular track with a circumference of 230 m. This seminal experiment shows that human driving causes backward propagating traffic waves, resulting in some vehicles to come to a complete stop (see left side of Figures 4 and 5). Remarkably, this occurs even in the absence of typical sources of traffic perturbations, such as lane changes, merges or stop lights. To analyze the impact of AVs, we adapt the setup to mixed autonomy using the framework presented in Section V-A

A. Experiment Modules

We design the following experiment, in which one human driver is replaced by an AV, by composing modules proposed in Section V-A. The network and simulation-specific parameters of the numerical experiments are summarized in Table I

| Experiment parameters | Value     |
|-----------------------|-----------|
| simulation step       | 0.1 s/step|
| circular track range (train) | [220, 270] m |
| circular track range (test)  | [210, 290] m |
| warmup time           | 75 s      |
| time horizon          | 300 s     |
| total number of vehicles | 22      |
| number of AVs         | 1         |

TABLE II: Network and simulation parameters for mixed autonomy circular track (single-lane) experiment.

differential \( \dot{h}_i = v_{i-1} - v_i \). The observation can be viewed as normalized inputs to a car-following model.

Control laws: Of the 22 actors, 21 are modeled as human drivers according to the Intelligent Driver Model (IDM) [67] (see Appendix B and Table IV). For the single autonomous vehicle actor, we compare the following control laws. Recall that actors partially observe the environment.

Learned control laws:

- GRU (memory): hidden layer (5), tanh non-linearity.
- MLP (no memory): diagonal Gaussian MLP, two-layer network with hidden layers (3,3), tanh non-linearity.
- Linear network (no memory).

Model-based control laws:

- FollowerStopper [58] (see Appendix C-1), with desired velocity parameter fixed at 4.15 m/s, calibrated for a track length of 260 m; this is further discussed in the results.
- Proportional Integral (PI) control with saturation, given in Stern, et al. [58] and is detailed in Appendix C-2.
- IDM: For a no autonomy baseline.

Dynamics: The overall system dynamics consists of a cascade of nonlinear dynamics models from \( n - 1 \) (homogeneous) actors and 1 autonomous vehicle actor (learning agent). The \( n - 1 \) IDM dynamics models are additionally perturbed by Gaussian acceleration noise of \( \mathcal{N}(0, 0.2) \), calibrated to match measures of stochasticity to the IDM model presented by Treiber, et al. [104]. The traffic simulator enforces safety through built-in failsafe mechanisms. Before starting the 300 second episode, there is a warmup period of 75 seconds, in which the acceleration of the AV is overridden by the IDM model to allow for randomization of the initial state and for the formation of stop-and-go waves.

Metrics: We consider two natural metrics: the average velocity of all vehicles in the network and a control cost, which penalizes acceleration. The reward function supplied to the learning agent is a weighted combination of the two metrics.

\[
    r(s, a) = \frac{1}{n} \sum_{i} v_i - \alpha |a| \tag{2}
\]

where \( \alpha = 0.1 \).

Initialization: The vehicles are evenly spaced around the circular track, with an initial velocity of 0 m/s.
B. Learning setup

The AVs execute parameterized control laws, trained using policy gradient methods. We use the Trust Region Policy Optimization (TRPO) [82] method, with linear feature baselines as described in Duan, et al. [105], discount factor $\gamma = 0.999$, and step size 0.01. The numerical experiments were conducted on three Intel(R) Core(TM) i7-6600U CPU @ 2.60GHz processors for six hours. A total of 6,000,000 samples (167 driving hours) were simulated during the training procedure.

C. Performance bounds

Before presenting the results, we discuss performance bounds, which provide a reference for evaluating the learned controllers. Specifically, because we are concerned with steady-state performance of the traffic system, we take the limit cycles of the system to be the performance bounds. Limit cycles are closed curves that trajectories tend towards, if stable (or away, if unstable). Limit cycles generalize the notion of a system equilibria from a point to a trajectory.

Uniform flow describes the situations where all vehicles move at some constant velocity $v^*$ and constant headway $h^*$, and corresponds to one of the limit cycles of the traffic system. For a general car following model $f$, the relationship between the equilibrium headway $h^*$ and equilibrium velocity $v^*$ is written as:

$$a_i = 0 = f(h^*, 0, v^*). \quad (3)$$

The specific relationship for IDM is displayed in Figure 5 (dotted green curve). These equilibria have high velocities, which is desirable, but they are unstable due to properties of human driving behavior [106], and thus do not naturally occur. We take this to be the performance upper bound.

On the other hand, a stable limit cycle of the system corresponds to traffic waves (also called stop-and-go waves) [15]. In other words, the traffic system tends towards traffic jams (see Figure 3, dotted red curve, for the resulting average velocity under IDM). This is a practical performance lower bound because any AV control law yielding worse performance could be replaced by a human driver model for a better outcome.

We note that, because we are analyzing the no autonomy scenario, but evaluating in a mixed autonomy scenario, these performance bounds should be viewed as close approximations to the true bounds. For more detailed performance bounds, we refer the reader to related work [15], [107], [65].

D. Results

By studying the mixed autonomy track, we demonstrate 1) that Flow enables composing modules to study an open problem in traffic control and 2) that reliable controllers for complex problems can be efficiently learned, which surpass the performance of all known model-based controllers. This section details our findings. Videos and additional results are also available.

1) Performance: First, Figure 3 evaluates the AV controllers across a wide range of traffic conditions (210 to 290 m circumference tracks). We observe that GRU and MLP control laws match the optimal velocity closely for traffic densities, thereby practically eliminating congestion. The PI with Saturation and FollowerStopper control, on the other hand, only dissipate stop-and-go traffic at densities less than or equal to their calibration density (less congested settings). The Linear control law performs well but not as well as the MLP/GRU. This indicates that a linear function may be unable to express the equilibrium flow velocity, whereas a two-layer neural network can. Our learned controllers outperform all the model-based controllers, with the exception of the PI with saturation controller outperforming the Linear controller in low density traffic.

![Figure 3: Performance of AV control laws for the single-lane mixed autonomy track. The overall system velocity of learned (GRU, MLP, and Linear) and model-based (FollowerStopper and PI Saturation) control laws are averaged for the final 100 s of simulation time over ten runs at each evaluated density. Also displayed are the performance upper and lower bounds, derived from the unstable and stable system limit cycles, respectively. The white and gray regions indicate the training-time and testing-time densities, respectively.](https://sites.google.com/view/ieee-tro-flow)

Figure 4 shows velocity profiles for the learned and model-based AV control laws on a 260 m track. Although both types of controllers eventually bring the system to uniform flow, the GRU control law reaches the equilibrium velocity fastest. The GRU and MLP control laws mitigate congestion with less oscillatory behavior than the FollowerStopper and PI with Saturation control laws. The FollowerStopper control law is the least performant; it settles at a steady-state speed of 4.15 m/s, well below the 4.82 m/s equilibrium velocity.

![Figure 4](https://sites.google.com/view/ieee-tro-flow)

Figure 5 shows space-time curves for all vehicles, for different AV control laws. We observe that the PI with Saturation and FollowerStopper control laws leave much smaller gaps (headways) than the MLP and GRU control laws. The MLP control law exhibits the largest gaps, as can be seen by the large white portion of the MLP plot within Figure 5. If this had been a multi-lane scenario, then the smaller gaps would have the benefit of preventing opportunistic lane changes, so this observation can lead to improved reward design for more complex mixed autonomy traffic studies.

2) Robustness: A strength of learned control laws is that they do not rely on external calibration of parameters that are specific to a traffic setting, such as traffic density. On the
other hand, in our experience, model-based controller baselines often exhibit considerable sensitivity to the traffic setting. We found the performance of the PI with Saturation control law to be sensitive to parameters and initial conditions, even though in principle it adjusts to different densities with a moving average filter. Using parameters calibrated for the 260 m track (as described in Stern, et al. [58]), the control law performs decently at 260 m; however, its performance quickly degrades at higher densities (more congested settings), dropping close to the performance lower bound (Figure 3). Additionally, even for a fixed track of length 260 m, it is inconsistent in mitigating the traffic waves. A successful episode is shown in Figures 4 and 5; however, unsuccessful episodes bring down its average performance (Figure 3).

Similarly, the FollowerStopper control law requires careful tuning before usage, which is beyond the scope of this work. Specifically, the desired velocity must be provided beforehand. Interestingly, we found experimentally that this control law is often ineffective if provided too high of a desired velocity, even if it is well below the uniform flow equilibrium velocity.

3) Generalization of the learned control law: By training with a range of vehicle densities, we found the learned control laws to generalize even to densities outside of the training regime, leading to a more robust control law. Figure 3 shows the learned control laws closely tracking the performance upper bound in the testing regime. Additionally, even training in the absence of noise in the human driver models, learned control laws still successfully stabilized settings with human model noise during test time (not shown).

4) Partial observability eases controller learning: At this early stage of autonomous vehicle development, we do not yet have a clear picture of what manufacturers will choose in terms of sensing infrastructure for the vehicles, what regulators will require, or what technology will enable (e.g. communication technologies). Furthermore, we do not know how the observation landscape of autonomous vehicles will change over time, as AVs are gradually adopted. Therefore, a framework which is modular and provides flexibility for the study of AVs is crucial. By invoking the composable observation components, we can readily study a variety of possible scenarios.

As such, this study considers a partially-observed setting for several reasons: 1) it is the more realistic setting for near-term deployments of autonomous vehicles, and 2) it permits a fair comparison with previously studied model-based controllers, which typically utilize partial observation. Finally, since we found the learned control laws to achieve the optimal velocity curve, we do not extensively explore the use of fuller observations. However, in Section VII, we do explore a variety
of additional settings, ranging from partially observed to fully observed settings.

Our partially observed experiments uncover several surprising findings which warrant further investigation. First, contrary to the classical view on partially observed control (e.g. POMDPs), these experiments suggest that partial observability may ease training instead of making it more difficult; as compared to full observations, we found that partial observations decreased the training time required from around 24 hours to 6 hours. Second, as seen in Figure 3, the results demonstrate that a near global optimum is achievable even under partial observation. Finally, the MLP control law closely mirrors the GRU control law and the optimal velocity curve; despite the partially observed setting, this suggests that memory is not necessary to achieve near optimal velocity across the full range of vehicle densities with a single learned controller.

A possible explanation is that a neural network with fewer weights may require fewer samples and iterations to converge to a local optimum, thus contributing to faster training. A more rigorous understanding of this phenomenon is left as a topic of future study, as well as questions concerning the situations under which partial observations still lead to a globally optimal solution in a learning framework. These early results suggest that deep RL methods may more efficiently utilize partial observations when they are provided appropriately, avoiding the need to learn to discount extraneous inputs.

5) Interpreting the controllers: Yet another advantage of the partially observed setting is that the low dimensionality of the observation space lends itself to interpretation. In Figure 6, we illustrate differences between the learned controllers and IDM. We use a heatmap to show 2-dimensional slices of the controllers of 3-dimensional inputs, and the color of the heatmap represents the output (acceleration). From the heatmap, we can see that the MLP controller (left subfigure) generally speeds up when it is slower than its leader and slows down when it is faster. However, it will also increase its speed when faster if it is far away. Notable for the MLP controller is the “0.0” entry in the left heatmap; it corresponds closely to the uniform flow equilibrium density. The controller can be interpreted as regulating its speed and headway such that its speed (4.2 m/s) matches the speed of its leader (4.2 m/s) at a specific density (corresponding to headway of 12 m).

On the other hand, the Linear controller is minimally reactive (middle subfigure); across the board of speeds and headways, the control law issues very small accelerations (and decelerations). However, note that the Linear controller is still nonlinear due to the failsafe mechanisms built into the traffic simulator, which prevent vehicle crashes. That is, the controller is overridden by the simulator whenever the AV’s headway is too small. Without enabling failsafe mechanisms, we found in separate experiments (not shown) that the Linear controller typically converges to a controller with frequent collisions or significantly lower performance. Additionally, there may be further sources of nonlinearity that are introduced through the learning algorithm (e.g. observation and action clipping), and this warrants further investigation. Our experiments indicate that a Linear model, even with nonlinearities introduced by failsafes and otherwise, may not be able to achieve the optimal velocity for the mixed autonomy circular track. The MLP controller (left), is similarly prone to exploiting the simulator
fail-safes, and thus for ease of interpretation, the heat map is displaying a controller trained without fail-safes enabled.

In comparison to both learned controllers, IDM (right subfigure) is visibly more aggressive in its acceleration and deceleration. Even when the vehicle is faster than its leader, it continues to accelerate until its headway is very small. This behavior, sensibly, results in stop-and-go traffic.

VII. REUSABLE MODULES FOR MIXED AUTONOMY

The previous section showed that the modules presented in Section VII-A can be composed to study open problems in traffic control and to rigorously evaluate RL and model-based approaches. Additionally, the capacity of RL to optimally solve the idealized circular track scenario in Section VI provides motivation to build more complex traffic scenarios and study the mixed autonomy performance with RL. This section goes beyond commonly studied scenarios and demonstrates that the modules can be configured to create new scenarios with important traffic characteristics, such as multiple AVs interacting, lane changes, and intersections. While larger scale scenarios can also be composed, they are out of scope of this article, due to the sample efficiency limitations of current deep RL methods; this is an important direction of future work. Instead, we present several scenarios to demonstrate the richness of composing simple modules and the insights that can be derived from training controllers therein.

Notably, in contrast to typical model-based control approaches to traffic control, RL requires limited domain knowledge or analysis for the study of these more complex traffic control problems. In contrast to sophisticated mathematical analysis, we instead need to design a suitable reward function, which does require some degree of trial-and-error. Ultimately, we selected a sensible reward function as before, focusing on system velocity and a secondary control cost term. The control cost will vary depending on the type of control action.

For brevity, we describe only the differences relative to the modules described in Section VI. All methods are compared against a baseline of human performance (IDM), as there are no known AV control laws for the following scenarios. Results are summarized in Table III. Based on the findings of Section VI, these experiments use a memory-less diagonal Gaussian MLP control law, with hidden layers (100, 50, 25) and tanh non-linearity.

A. Single-lane track with multiple autonomous vehicles

In this section, a simple extension to the previous experiment shows that many variants to the same problem may be analyzed numerically, bypassing the need to adhere to strict mathematical frameworks for analysis. The following shows additionally that even simple extensions can yield interesting and significant performance improvements. Here we describe the experimental modules, as they differ from the previous experiment.

Networks: A single-lane track with fixed length \( L = 230 \) m.

Observer: All vehicle positions and velocities, that is \( o = s = (x_1, v_1, x_2, v_2, \ldots, x_n, v_n) \).

Control laws: Three to eleven of the actors are dictated by a single (centralized) learned control law; that is, \( A = \mathbb{R}^m \) where \( m \in \{3, \ldots, 11\} \) and \( \pi_\theta : \mathcal{O} \times A \rightarrow \Delta^m \). The remaining actors are modeled as human drivers according to IDM.

Metrics: The reward function is a weighted combination of the average velocity of all vehicles and a control cost.

\[
    r(s,a) = \frac{1}{n} \sum_i v_i - \alpha \sum_{j \in [m]} |a_j| \tag{4}
\]

where \( \alpha = 0.1 \).

Result: A string of consecutive AVs learns to proceed with a smaller headway than the human driver models (platooning),

| Network | # vehicles | # AVs | improvement vs human | improvement vs uniform flow |
|--------|------------|-------|----------------------|-----------------------------|
| VI     | 22         | 1     | 36.46-41.02%         | -2.92-2.40%                 |
| VII-A  | 22         | 3     | 54.19%               | 4.07%                       |
| VII-A  | 22         | 11    | 85.03%               | 27.07%                      |
| VII-B  | 44         | 6     | 52.53%               | 6.09%                       |
| VII-C  | 14         | 1     | 57.45%               | -                           |
| VII-C  | 14         | 14    | 150.49%              | -                           |

Fig. 6: Visualization of vehicle control laws. The heatmaps are 2-dimensional slices of the controllers (3-dimensional), and the color depicts the output (acceleration). The x-axis is a representative range of headways seen by vehicles during training. The y-axis is a representative range of AV speeds. Displayed is the slice of acceleration values of the model when the leader vehicle speed is fixed at 4.2 m/s (a typical speed for the 250 m track). The single colorbar is shared by all plots. Left: Learned MLP model, with fail-safes disabled. Middle: Learned Linear model, with fail-safes enabled. Right: IDM.
resulting in greater roadway utilization, thereby surpassing the upper bound from Section VI, as can be seen in Figure 7.

![Velocity profile for single-lane circular track with multiple AVs.](image)

Fig. 7: Velocity profile for single-lane circular track with multiple AVs. With additional AVs, the average velocity exceeds the uniform flow equilibrium velocity, and continues to increases as the number of AVs increase. At three AVs, the average velocity settles at 3.70 m/s; at 11 AVs, the average velocity settles at 4.44 m/s.

**B. Multi-lane track with multiple autonomous vehicles**

Multi-lane settings are challenging to study from a model-based control-theoretic perspective due to the discontinuity in model dynamics from lane changes, as well as due to the complexity of mathematically modeling lane changes accurately. The experimental modules are described here:

**Networks:** The network is a two-lane circular track of \( L = 230 \) m, as displayed in Figure 1 (top center).

**Actors:** There are \( n = 44 \) vehicles.

**Observer:** All vehicle positions, velocities, and lanes, that is \( o = s = (x_1, v_1, l_1, x_2, v_2, l_2, \ldots, x_n, v_n, l_n) \).

**Control laws:** Six of the actors are dictated by a single learned control law for both acceleration and lane changes (continuous action representation); that is, \( A = \mathbb{R} \) or \( A = \mathbb{R}^m \). The rest are dictated by IDM for longitudinal control and the lane changing model of SUMO for lateral control.

**Initialization:** The vehicles are evenly spaced in the track, using both lanes, and six AVs are placed together in sequence, all in the outer lane.

**Metrics:** The reward function is a weighted combination of the average velocity of all vehicles and the a control cost.

\[
    r(s, a) = \frac{1}{n} \sum_i v_i - \alpha \frac{1}{m} \sum_{j \in [m]} |a_j| \tag{6}
\]

where \( \alpha = 0.1 \). Note that both excessive accelerations and lane changes are penalized.

**Result:** The learned control law yields AVs balancing across the two lanes, with three in each lane, and avoiding stop-and-go waves in both lanes. The resulting average velocity is 3.66 m/s, an improvement over the 3.45 m/s uniform flow equilibrium velocity. Even though the control inputs are a mix of inherently continuous and discrete signals (acceleration and lane change, respectively), a well-performing control law was learned using only a continuous representation, which is a testament to the flexibility of the approach.

**C. Intersection with mixed and full autonomy**

We now consider a simplified intersection scenario, which demonstrates the ease of considering different network topologies and traffic rules, such as right-of-way rules at intersections. The authors are not aware of any model-based mixed autonomy results for intersections, with which to compare. In the absence of autonomous vehicles, human drivers queue at the intersection, leading to significant delays; this serves as our baseline. We now describe the traffic control scenario:

**Networks:** The network resembles a figure eight, as displayed in Figure 1 (top right), with an intersection in the middle, two circular tracks of radius 30 m, and total length of 402 m.

**Actors:** There are \( n = 14 \) vehicles.

**Observer:** All vehicle positions and velocities, that is \( o = s = (x_1, v_1, x_2, v_2, \ldots, x_n, v_n) \).

**Control laws:** One or all of the actors are dictated by a single learned control law for acceleration control inputs; that is, \( A = \mathbb{R} \) or \( A = \mathbb{R}^m \). The rest are dictated by IDM for longitudinal control and intersection rules from SUMO.

**Dynamics:** There is no traffic light at the intersection; instead vehicles crossing the intersection follow SUMO’s right-of-way model to enforce traffic rules and to prevent crashes.

**Metrics:** The reward function is a weighted combination of the average velocity of all vehicles and the a control cost.

\[
    r(s, a) = \frac{1}{n} \sum_i v_i - \alpha \frac{1}{m} \sum_{j \in [m]} |a_j| \tag{6}
\]

Further study is needed to understand, interpret, and analyze the above learned behaviors and control laws, and thereby take steps towards a real-world deployment and policy analysis. Some preliminary investigations can be found in [107], [108].
VIII. CONCLUSION

The complex integration of autonomy into existing systems introduces new technical challenges beyond studying autonomy in isolation or in full. This article aims to make progress on these upcoming challenges by studying how modern deep RL can be leveraged to gain insights into complex mixed autonomy systems. In particular, we focus on the integration of autonomous vehicles into urban systems, as we expect these to be among the first robotic systems to enter and affect existing societal systems. The article introduces Flow, a modular learning framework which eases the composition of modules, to enable learning control laws for AVs in complex traffic scenarios. Several experiments yielded controllers which far exceeded state-of-the-art performance (in fact, achieving near-optimal performance) and demonstrated the generality of the methodology for disparate traffic dynamics. Since an early version of this manuscript was made available online in 2017 [93], several works have employed this framework to achieve results in the directions of the discovery of emergent behaviors in traffic, transfer learning, bottleneck control, the design of traffic control benchmarks, and sim2real [107], [108], [109], [110], [111].

Open directions of research include the study of mixed autonomy in larger and more complex traffic networks; studying different, more realistic, and regional objective functions; studying delayed vehicle models; devising new RL techniques for large-scale networked systems; studying scenarios with variable numbers of actors; incorporating advances in safe RL; and incorporating multi-agent RL for the study of variable numbers of autonomous vehicles. Another open direction is to study fundamental limitations of this methodology; in particular, 1) establishing when guarantees of global convergence, stability, robustness, and safety of classical approaches can not be achieved with deep RL, and 2) quantifying the effects of simulation model error or misspecification on training outcomes. We additionally plan to extend Flow with modules suitable for the study of other forms of automation in traffic, such as problems concerning traffic lights, road directionality, signage, roadway pricing, and infrastructure communication. Another interesting direction is whether an analogous design of reusable modules may be used for other robotics and transportation-related scenarios, such as for motion planning, navigation, ridesharing, and land use planning.

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APPENDIX A

NETWORKS

Flow supports learning policies on arbitrary (user-defined) networks. In this section, we include a basic set of networks, which we have designed or adapted from the literature to capture important traffic phenomena. These include closed networks, such as single and multi-lane circular tracks, figure-eight networks, and loops with merge, as well as open networks, such as intersections, merge networks and highway networks. In contrast to closed networks, open networks require pre-defined in-flows of vehicles into the traffic system. See Figure [I] for various example networks supported by Flow. In each of these networks, Flow can be used to study the design or learning of controllers which optimize the system-level velocity or other objectives, in the presence of different types of vehicles, model noise, etc.

Single-lane circular tracks: This network consists of a circular lane with a specified length, inspired by the 230m track studied by Sugiyama et al. [13]. This canonical benchmark has been extensively studied.

Multi-lane circular tracks: Multi-lane circular tracks are a natural extension to the single-lane track. The inclusion of lane-changing behavior in this setting makes studying such problems exceedingly difficult from an analytical perspective, thereby constraining most classical control techniques to the single-lane case. Many multi-lane models forgo longitudinal dynamics in order to encourage tractable analysis [112], [113], [114], [115]. Recent strides have been made in developing simple stochastic models that retain longitudinal dynamics while capturing lane-changing dynamics in a single lane setting [116]. Modern machine learning methods, however, do not require a simplification of the dynamics for the problem to become tractable, as explored in Section VII-B.

Figure-eight network: The figure-eight network is a simple closed network with an intersection. Two circular tracks, placed at opposite ends of the network, are connected by two perpendicular roads that cross at an intersection. Vehicles that try to cross this intersection from opposite ends are constrained by SUMO’s right-of-way model to prevent crashes.

Loops with merge network: This network permits the study of merging behavior within a closed network. This network consists of two circular tracks which are connected together. Vehicles in the smaller track stay within this track, while vehicles in the larger track try to merge into the smaller track and then back out to the larger track. This typically results in congestion at the merge points.

Intersections: This network permits the study of intersection management in an open network. Vehicles arrive in the control zone of the intersection according to a Poisson distribution. At the control zone, the system speeds or slows down vehicles to either maximize average velocity or minimize experienced delay. This building block can be used to build a general schema for arbitrary maps such as the one shown in Figure [I] (bottom right).
APPENDIX B
INTELLIGENT DRIVER MODEL

The Intelligent Driver Model (IDM) is a car following model capable of accurately representing realistic driver behavior [23] and reproducing traffic waves, and is commonly used in the transportation research community. We employ IDM in the numerical experiments of this article, and therefore analyze this specific model to compute the theoretical performance bounds of the overall traffic system. The acceleration for a vehicle modeled by IDM is defined by:

\[ a_{IDM} = \frac{dv}{dt} = a \left[ 1 - \left( \frac{v}{v_0} \right)^\delta - \left( \frac{H(v, \bar{h})}{\bar{h}} \right)^2 \right], \tag{7} \]

where \( H(\cdot) \) is the desired headway of the vehicle, denoted by:

\[ H(v, \bar{h}) = s_0 + \max\left(0, vT + \frac{v\dot{\bar{h}}}{2\sqrt{ab}}\right), \tag{8} \]

where \( h_0, v_0, T, \delta, a, b \) are given parameters. Table [IV] provides typical parameters for highway driving [67].

| Parameter | Value |
|-----------|-------|
| \( v_0 \) | 30 m/s |
| \( T \) | 1 s |
| \( a \) | 1 m/s² |
| \( b \) | 1.5 m/s² |
| \( \delta \) | 4 |
| \( h_0 \) | 2 m |
| \( \text{noise} \) | \( \mathcal{N}(0, 0.2) \) |

TABLE IV: Parameters for car-following control law

APPENDIX C
MODEL-BASED LONGITUDINAL CONTROL LAWS

In this section, we detail the two state-of-the-art control laws for the mixed autonomy circular track, against which we benchmark our learned policies generated using Flow.

1) FollowerStopper: Recent work by [58] presented two control models that may be used by autonomous vehicles to attenuate the emergence of stop-and-go waves in a traffic network. The first of these models is the FollowerStopper. This model commands the AVs to maintain a desired velocity \( U \), while ensuring that the vehicle does not crash into the vehicle behind it. Following this model, the command velocity \( v^{\text{cmd}} \) of the autonomous vehicle is:

\[ v^{\text{cmd}} = \begin{cases} 0 & \text{if } \Delta x \leq \Delta x_1 \\ v \frac{\Delta x - \Delta x_1}{\Delta x_2 - \Delta x_1} & \text{if } \Delta x_1 < \Delta x \leq \Delta x_2 \\ v + (U - v) \frac{\Delta x - \Delta x_2}{\Delta x_3 - \Delta x_2} & \text{if } \Delta x_2 < \Delta x \leq \Delta x_3 \\ U & \text{if } \Delta x_3 < \Delta x \end{cases} \tag{9} \]

where \( v = \min(\max(v^{\text{lead}}, 0), U) \), \( v^{\text{lead}} \) is the speed of the leading vehicles, \( \Delta x \) is the headway of the autonomous vehicle, subject to boundaries defined as:

\[ \Delta x_k = \Delta x_0 + \frac{1}{2\delta_k} (\Delta v_-)^2, \quad k = 1, 2, 3 \tag{10} \]

The parameters of this model can be found in [58] and are also provided in Table [V].

2) PI with Saturation: In addition to the FollowerStopper control law, [58] presents a model called the PI with Saturation control law that attempts to estimate the average equilibrium velocity \( U \) for vehicles on the network, and then drives at that speed. This average is computed as a temporal average from its own history: \( U = \frac{1}{m} \sum_{j=1}^{m} v^j \). The target velocity at any given time is then defined as:

\[ v^{\text{target}} = U + v^{\text{catch}} \times \min \left( \max \left( \frac{\Delta x - g_l}{g_u - g_l}, 0 \right), 1 \right) \tag{11} \]

Finally, the command velocity for the vehicle at time \( j + 1 \), which also ensures that the vehicle does not crash, is:

\[ v_{j+1}^{\text{cmd}} = \beta_j (\alpha_j v_j^{\text{target}} + (1 - \alpha_j) v_j^{\text{lead}}) + (1 - \beta_j) v_j^{\text{cmd}} \tag{12} \]

The values for all parameters in the model can be found in [58] and are also provided in Table [V].

REFERENCES

[1] Z. Wadud, D. MacKenzie, and P. Leiby, “Help or hindrance? the travel, energy and carbon impacts of highly automated vehicles,” Transportation Research Part A: Policy and Practice, vol. 86, pp. 1–18, 2016.

[2] U. DOT, “National transportation statistics,” Bureau of Transportation Statistics, Washington, DC, 2016.

[3] S. Thrun, M. Montemerlo, H. Dahlkamp, D. Stavens, A. Aron, J. Diebel, P. Fong, J. Gale, M. Halpenny, G. Hoffmann et al., “Stanley: The robot that won the darpa grand challenge,” Journal of field robotics, vol. 23, no. 9, pp. 661–692, 2006.

[4] M. Buehler, K. Iagnemma, and S. Singh, The DARPA urban challenge: autonomous vehicles in city traffic., Springer, 2009, vol. 56.

[5] K. Spieser, K. Trelleven, R. Zhang, E. Frazzoli, D. Morton, and M. Pavone, “Toward a systematic approach to the design and evaluation of automated mobility-on-demand systems: A case study in singapore,” in Road Vehicle Automation. Springer, 2014, pp. 229–245.

[6] K. Dresner and P. Stone, “A multiagent approach to autonomous intersection management,” Journal of artificial intelligence research, vol. 31, pp. 591–656, 2008.

[7] B. K. Horn, “Suppressing traffic flow instabilities,” in Intelligent Transportation Systems-ITSC, 2015 16th International IEEE Conference on. IEEE, 2013, pp. 13–20.

[8] D. Miculescu and S. Karaman, “Polling-systems-based autonomous vehicle coordination in traffic intersections with no traffic signals,” arXiv preprint arXiv:1607.07896, 2016.

[9] M. Bojarski, D. Del Testa, D. Dworakowski, B. Firner, B. Flepp, P. Goyal, L. D. Jackel, M. Monfort, U. Muller, J. Zhang et al., “End to end learning for self-driving cars,” arXiv preprint arXiv:1604.07316, 2016.

[10] S. Shalev-Shwartz, S. Shammah, and A. Shashua, “Safe, multi-agent, reinforcement learning for autonomous driving.” CoRR, vol. abs/1610.03295, 2016. [Online]. Available: http://arxiv.org/abs/1610.03295

[11] C. G. Atkeson and S. Schaal, “Robot learning from demonstration,” in ICML, vol. 97. Citeseer, 1997, pp. 12–20.

[12] P. Abbeel and A. Y. Ng, “Apprenticeship learning via inverse reinforcement learning,” in Proceedings of the twenty-first international conference on Machine learning. ACM, 2004, p. 1.

[13] S. Levine, C. Finn, T. Darrell, and P. Abbeel, “End-to-end training of deep visuomotor policies,” Journal of Machine Learning Research, vol. 17, no. 39, pp. 1–40, 2016.

[14] Y. Sugiyama, M. Fukui, M. Kikuchi, K. Hasebe, A. Nakayama, K. Nishinari, S.-i. Tadaki, and S. Yukawa, “Traffic jams without bottlenecks–experimental evidence for the physical mechanism of the formation of a jam,” New Journal of Physics, vol. 10, no. 3, p. 033001, 2008.

[15] G. Orosz, R. E. Wilson, and G. Stépán, “Traffic jams: dynamics and control,” Philosophical Trans. of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, vol. 368, no. 1928, pp. 4455–4479, 2010.
TABLE V: Parameters for model-based controllers

| Parameters | $\Delta v^1_3$ | $\Delta x^0_2$ | $\Delta v^3_3$ | $d_1$ | $d_2$ | $d_3$ | $\Delta v$ | $\Delta x_1$ | $\Delta x_2$ | $\Delta x_3$ | $U$ |
|------------|----------------|----------------|----------------|--------|--------|--------|------------|------------|------------|------------|-------|
| Values     | 4.5 m          | 5.25 m         | 6.0 m          | 1.5 m/s² | 1.0 m/s² | 0.5 m/s² | -3 m/s     | 7.5 m/s    | 9.75 m/s    | 15 m/s     | 4.15 m/s |

[16] M. J. Lighthill and G. B. Whitham, “On Kinematic Waves II: A Theory of Traffic Flow on Long Crowded Roads,” Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, vol. 229, pp. 317–345, 1955.

[17] P. I. Richards, “Shock Waves on the Highway,” Operations Research, vol. 4, pp. 42–51, 1956.

[18] H. J. Payne, “Frelo: A macroscopic simulation model of freeway traffic,” Transportation Research Record, no. 722, 1979.

[19] A. W. and M. Rascel, “Resurrection of second order” models of traffic flow,” SIAM journal on applied mathematics, vol. 60, no. 3, pp. 916–938, 2000.

[20] P. G. Gipps, “A behavioural car-following model for computer simulation,” Transportation Research Part B: Methodological, vol. 15, no. 2, pp. 105–111, 1981.

[21] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, “Structure stability of congestion in traffic dynamics,” Japan Journal of Industrial and Applied Mathematics, vol. 11, no. 2, pp. 203–223, 1994.

[22] ——, “Dynamical model of traffic congestion and numerical simulation,” Physical review E, vol. 51, no. 2, p. 1035–1042, 1995.

[23] M. Treiber, A. Hennecke, and D. Helbing, “Congested traffic states in empirical observations and microscopic simulations,” Physical review E, vol. 62, no. 2, p. 1805–1824, 2000.

[24] A. J. Miller, “A queueing model for road traffic flow,” Journal of the Royal Statistical Society: Series B (Methodological), vol. 23, no. 1, pp. 64–75, 1961.

[25] D. Heidemann, “A queueing theory approach to speed-flow-density relationships,” in Transportation and traffic theory. Proceedings of the 15th International Symposium on Transportation and Traffic Theory, Lyon, France, 24-26 July 1996, 1996, pp. 103–18.

[26] Z. Shiller and Y. R. Gwo, “Dynamic motion planning of autonomous vehicles,” Asia-Pacific Journal of Operational Research, vol. 24, no. 04, pp. 435–461, 2007.

[27] S. Drakunov, U. Ozguner, P. Dix, and B. Ashrafi, “Abs control using optimum search via sliding modes,” IEEE Transactions on Control Systems Technology, vol. 3, no. 1, pp. 79–85, March 1995.

[28] M. W. M. G. Dissanayake, P. Newman, S. Clark, H. F. Durrant-Whyte, and M. Csorba, “A solution to the simultaneous localization and mapping (slam) problem,” IEEE Transactions on Robotics and Automation, vol. 17, no. 3, pp. 229–241, Jun 2001.

[29] S. Thrun, W. Burgard, D. Fox et al., Probabilistic robotics. MIT press Cambridge, 2005, vol. 1.

[30] S. Sukkarieh, E. M. Nebot, and H. F. Durrant-Whyte, “A high integrity imu/gps navigation loop for autonomous land vehicle applications,” IEEE Transactions on Robotics and Automation, vol. 15, no. 3, pp. 572–578, Jun 1999.

[31] M. W. M. G. Dissanayake, P. Newman, S. Clark, H. F. Durrant-Whyte, and M. Csorba, “A solution to the simultaneous localization and map building (slam) problem,” IEEE Transactions on Robotics and Automation, vol. 17, no. 3, pp. 229–241, Jun 2001.

[32] Y. Cui and S. S. Ge, “Autonomous vehicle positioning with gps in urban canyon environments,” IEEE Transactions on Robotics and Automation, vol. 19, no. 1, pp. 15–25, Feb 2003.

[33] Z. Shiller and Y. R. Gwo, “Dynamic motion planning of autonomous vehicles,” IEEE Transactions on Robotics and Automation, vol. 7, no. 2, pp. 241–249, Apr 1991.

[34] P. A. Ioannou and C.-C. Chien, “Autonomous intelligent cruise control,” IEEE Trans. on Vehicular Technology, vol. 42, no. 4, pp. 657–672, 1993.

[35] A. Vahidi and A. Eskandarian, “Research advances in intelligent collision avoidance and adaptive cruise control,” IEEE transactions on intelligent transportation systems, vol. 4, no. 3, pp. 143–153, 2003.

[36] E. E. Paromitich and C. Laugier, “Motion generation and control for parking an autonomous vehicle,” in Robotics and Automation, 1996. Proceedings., 1996 IEEE International Conference on, vol. 4. IEEE, 1996, pp. 3117–3122.

[37] V. Mil´an, D. F. Lorca, J. Villagr´a, J. P´erez, C. Fern´andez, I. Parra, C. Gonz´alez, and M. A. Sotelo, “Intelligent automatic overtaking system using vision for vehicle detection,” Expert Systems with Applications, vol. 39, no. 3, pp. 3362–3373, 2012. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0957417411013339

[38] D. Nicas, C. Gonz ´alez, and M. A. Sotelo, “Intelligent automatic overtaking in traffic intersections with no traffic signals,” IEEE Transactions on Automatic Control, vol. 65, no. 2, pp. 680–694, 2019.

[39] Technical Committee ISO/TC 204 Intelligent transport systems, Intelligent transport systems – Adaptive Cruise Control systems – Performance requirements and test procedures, ISO 15 622:2010, 2010.

[40] P. A. Ioannou and C.-C. Chien, “Autonomous intelligent cruise control,” IEEE Trans. on Vehicular Technology, vol. 42, no. 4, pp. 657–672, 1993.

[41] A. Vahidi and A. Eskandarian, “Research advances in intelligent collision avoidance and adaptive cruise control,” IEEE transactions on intelligent transportation systems, vol. 4, no. 3, pp. 143–153, 2003.

[42] S. E. Shladover, “Automated vehicles for highway operations (autonomous highway systems),” Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, vol. 219, no. 1, pp. 53–75, 2005.

[43] R. Rajamani and C. Zhu, “Semi-autonomous adaptive cruise control systems,” IEEE Trans. on Vehicular Technology, vol. 51, no. 5, pp. 1186–1192, 2002.

[44] X. Lu, S. Shladover, and J. Hedrick, “Heavy-duty truck control: Short inter-vehicle distance following,” in American Control Conference, 2004. Proceedings of the 2004, vol. 5. IEEE, 2004, pp. 4722–4727.

[45] S. Sheikholeslam and C. A. Desoer, “A system level study of the longitudinal control of a platoon of vehicles,” Journal of dynamic systems, measurement, and control, vol. 114, no. 2, pp. 286–292, 1992.

[46] S. Cui, B. Seibold, R. Stern, and D. B. Work, “Stabilizing traffic flow via a single autonomous vehicle: Possibilities and limitations,” in Intelligent Vehicles Symposium (IV), 2017 IEEE. IEEE, 2017, pp. 447–453.

[47] C. Wu, A. M. Bayen, and A. Mehta, “Stabilizing traffic with autonomous vehicles,” in 2018 IEEE International Conference on Robotics and Automation (ICRA). IEEE, 2018, pp. 1–7.
M. Papageorgiou, C. Diakaki, V. Dinopoulou, A. Kotsialos, and A. Bose and P. A. Ioannou, “Analysis of traffic flow with mixed manual and automated vehicles: A hybrid modelling approach,” Physica A: Statistical Mechanics and its Applications, vol. 388, no. 12, pp. 2483–2491, 2009.

T.-C. Au, S. Zhang, and P. Stone, “Semi-autonomous intersection management,” in Proceedings of the 2014 international conference on Autonomous agents and multi-agent systems. International Foundation for Autonomous Agents and Multiagent Systems, 2014, pp. 1451–1452.

G. Sharon and P. Stone, “A protocol for mixed autonomous and human-operated vehicles at intersections,” in International Conference on Autonomous Agents and Multiagent Systems. Springer, 2017, pp. 151–167.

R. E. Stern, S. Cui, M. L. Delle Monache, R. Bhdani, M. Bunting, M. Churchill, N. Hamilton, H. Pohlmann, F. Wu, B. Piccoli et al., “Dissipation of stop-and-go waves via control of autonomous vehicles: Field experiments,” Transportation Research Part C: Emerging Technologies, vol. 89, pp. 205–221, 2018.

M. Pavone, S. L. Smith, E. Frazzoli, and D. Rus, “Robotic load balancing for mobility-on-demand systems,” The International Journal of Robotics Research, vol. 31, no. 7, pp. 839–854, 2012.

R. Zhang and M. Pavone, “Control of robotic mobility-on-demand systems: a queueing-theoretical perspective,” 2014.

D. Sadig, S. Sastry, S. A. Seshia, and A. D. Dragan, “Planning for autonomous cars that leverage human actions,” vol. 2, pp. 1–9, 2016.

J. Lee, M. Park, and H. Yeo, “A probability model for discretionary lane changes in highways,” KSCE Journal of Civil Engineering, vol. 20, no. 7, pp. 2938–2946, 2016.

J. Rios-Torres and A. A. Malikopoulos, “A survey on the coordination of connected and automated vehicles at intersections and merging at highway on-ramps,” IEEE Transactions on Intelligent Transportation Systems, vol. 18, no. 5, pp. 1066–1077, 2017.

—, “Automated and cooperative vehicle merging at highway on-ramps,” IEEE Transactions on Intelligent Transportation Systems, vol. 18, no. 4, pp. 780–789, 2017.

Y. Zheng, J. Wang, and K. Li, “Smoothing traffic flow via control of autonomous emergency vehicles,” arXiv preprint arXiv:1812.09544, 2018.

C.-Y. Liang and P. Hui, “String stability analysis of adaptive cruise control systems,” ISME International Journal Series C Mechanical Systems, Machine Elements and Manufacturing, vol. 43, no. 3, pp. 671–677, 2000.

M. Treiber and A. Kesting, “Traffic flow dynamics,” Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg, 2013.

CIECA, “Internal project on eco-driving in category b driver training and the driving test,” 2007.

M. Barth and K. Boriboonsomsin, “Energy and emissions impacts of a freeway-based dynamic eco-driving system,” Transportation Research Part D: Transport and Environment, vol. 14, no. 6, pp. 400 – 410, 2009, the interaction of environmental and traffic safety policies. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1361920909000121

M. Papageorgiou, C. Diakaki, V. Dinopoulou, A. Kotsialos, and Y. Wang, “Review of road traffic control strategies,” Proceedings of the IEEE, vol. 91, no. 12, pp. 2043–2067, 2003.

A. Bose and P. A. Ioannou, “Analysis of traffic flow with mixed manual and semiautomated vehicles,” IEEE Trans. on Intelligent Transportation Systems, vol. 4, no. 4, pp. 173–188, 2003.

P. A. Ioannou and M. Stefanovic, “Evaluation of acc vehicles in mixed traffic: Lane change effects and sensitivity analysis,” IEEE Transactions on Intelligent Transportation Systems, vol. 6, no. 1, pp. 79–89, 2005.

M. A. S. Kamal, J.-i. Imura, T. Hayakawa, A. Ohata, and K. Aihara, “Smart driving of a vehicle using model predictive control for improving traffic flow,” IEEE Transactions on Intelligent Transportation Systems, vol. 15, no. 2, pp. 878–884, 2014.

D. Swaroop, “String stability of interconnected systems: An application to platooning in automated highway systems,” Ph.D. dissertation, 1994.

G. B. Carlson, J. M. Pasco, and F. Bullo, “Progression for human and robotic drivers,” in ASME 2011 International Design Engineering Technical Conference and Computers and Information in Engineering Conference. American Society of Mechanical Engineers, 2011, pp. 529–538.

I. G. Jin and G. Orosz, “Dynamics of connected vehicle systems with delayed acceleration feedback,” Transportation Research Part C: Emerging Technologies, vol. 46, pp. 46–64, 2014.

L. Wang, B. K. Horn, and G. Strang, “Eigenvalue and eigenvector analysis of stability for a line of traffic,” Studies in Applied Mathematics, 2016.

D. P. Bertsekas and J. N. Tsitsiklis, Neuro-dynamic programming. Athena Scientific Belmont, MA, 1996, vol. 5.

R. S. Sutton and A. G. Barto, Reinforcement learning: An introduction. MIT press Cambridge, 1998, vol. 1, no. 1.

Y. M. Minh, K. Kavacek, Su. Silver, A. A. Rusu, J. Veness, M. G. Bellemare, A. Graves, M. Riedmiller, A. K. Fidjeland, G. Ostrovski et al., “Human-level control through deep reinforcement learning,” Nature, vol. 518, no. 7540, pp. 529–533, 2015.

J. Schulman, P. Moritz, S. Levine, M. Jordan, and P. Abbeel, “High-dimensional continuous control using generalized advantage estimation,” in Proceedings of the 33rd Conference on Learning Representations (ICLR), 2016.

J. Schulman, S. Levine, P. Abbeel, M. I. Jordan, and P. Moritz, “Trust region policy optimization,” in ICML, 2015, pp. 1889–1897.

N. Heess, G. Wayne, D. Silver, T. Lillicrap, T. Erez, and Y. Tassa, “Learning continuous control policies by stochastic value gradients,” in Advances in Neural Information Processing Systems, 2015, pp. 2944–2952.

M. G. Bellemare, S. Candiolo, P. S. Castro, J. Gong, M. C. Machado, S. Moitra, S. S. Ponda, and Z. Wang, “Autonomous navigation of stratospheric balloons using reinforcement learning,” Nature, vol. 588, no. 7836, pp. 77–82, Dec 2020.

N. G. Polson and V. O. Sokolov, “Deep learning for short-term traffic flow prediction,” Transportation Research Part C: Emerging Technologies, vol. 79, pp. 1–17, 2017.

Y. Lv, Y. Duan, W. Kang, Z. Li, and F.-Y. Wang, “Traffic flow prediction with big data: a deep learning approach,” IEEE Transactions on Intelligent Transportation Systems, vol. 16, no. 2, pp. 865–873, 2015.

L. Li, Y. Lv, and F. Wang, “Traffic signal timing via deep reinforcement learning,” IEEE/CIAA Journal of Automatica Sinica, vol. 3, no. 3, pp. 247–254, July 2016.

F. Belletti, D. Haizha, G. Gomes, and A. M. Bayen, “Expert level control of ramp metering based on multi-task deep reinforcement learning,” IEEE Transactions on Intelligent Transportation Systems, vol. 19, no. 4, pp. 1198–1207, 2018.

H. Wei, N. Xu, H. Zhang, G. Zheng, X. Zang, C. Chen, W. Zhang, Y. Zhu, K. Xu, and Z. Li, “Colight: Learning network-level cooperation for traffic signal control,” in Proceedings of the 28th ACM International Conference on Information and Knowledge Management, 2019, p. 1913–1922, arXiv preprint arXiv:1905.05717.

H. Wei, G. Zheng, H. Yao, and Z. Li, “Intellilight: A reinforcement learning approach for intelligent traffic light control,” in Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining. ACM, 2018, pp. 2496–2505.

M. G. Karlaftis and E. I. Vlahogianni, “Statistical methods versus neural networks in transportation research: Differences, similarities and some insights,” Transportation Research Part C: Emerging Technologies, vol. 19, no. 3, pp. 387–399, 2011.

C. Wu, K. Parvate, N. Ketzerpal, L. Dickstein, A. Mehta, E. Vinitsky, and A. M. Bayen, “Framework for control and deep reinforcement learning in traffic,” in Intelligent Transportation Systems (ITSC), 2017 IEEE 20th International Conference on, 2017.

C. Wu, A. Kreidieh, K. Parvate, E. Vinitsky, and A. M. Bayen, “Flow: Architecture and benchmarking for reinforcement learning in traffic control,” arXiv preprint arXiv:1710.05465v1, 2017.

R. Bellman, “A markovian decision process,” vol. 6, no. 5, 1957.

R. A. Howard, Dynamic programming and Markov processes. MIT Press, 1960.

R. S. Sutton, D. A. McAllester, S. P. Singh, and Y. Mansour, “Policy gradient methods for reinforcement learning with function approximation,” in Advances in neural information processing systems, 2000, pp. 1057–1063.

S. Haykin, Neural networks: a comprehensive foundation. Prentice Hall PTR, 1994.

J. Chung, C. Gulcehre, K. Cho, and Y. Bengio, “Gated feedback recurrent neural networks,” in International Conference on Learning Representations (ICLR), 2016.

M. G. Bellemare, Y. Naddaf, J. Veness, and M. Bowling, “The arcade learning environment: An evaluation platform for general agents,” J. Artif. Intell. Res.(JAIR), vol. 47, pp. 253–279, 2013.
