Heat exchanger design based on minimum entropy generation

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Abstract. The paper deals with the rating of the performance of heat exchangers. Each attempt in augmenting the overall heat transfer coefficient is accompanied by an increase in the pressure drop. Based on this observation the paper points out that both processes – heat transfer across a finite temperature difference and friction have a common destructive effect that can be measured by the entropy generation. The paper discusses the possibility of designing or simulating the behaviour of an existing heat exchanger based on the number of entropy generation unit NSU. The correlation between the thermal efficiency of the heat exchanger and its constructive characteristics is based on the NTU-ε method. The paper develops a correlation for calculating the entropy generation unit NSU function on the NTU number. The paper presents diagrams and methods for designing a heat exchanger for a specific number NSU or for simulating an existing design or physical heat exchanger.

1. Introduction

Any attempt in augmenting the heat transfer performance of a heat exchanger is accompanied by an increase in the pressure drop.

For a given thermal load and a specific heat transfer area

\[ \dot{Q} = U \cdot A \cdot \Delta t_{ML} \]  \hspace{1cm} (1)

The increase in the overall heat transfer coefficient leads to a decrease in the temperature difference between the hot and cold currents but in the same time the friction factor increases.

Apparently the temperature difference seems to be a thermal decisional parameter while the pressure drop has a mechanical effect. In fact both heat transfer across a finite temperature difference and pressure drop (friction) are irreversible processes and have a common effect – they destroy (consume) mechanical work.

The magnitude of the entropy generation that accompanies these irreversible processes can be used as a measure of the heat exchanger performance. The less is the entropy generation the more performing is the heat exchanger.

Scientists and engineers are preoccupied to design more performing heat exchangers but this overall performance is sometimes difficult to define.

Uttam Roy and Mrinmoy Majumder [1] used a Neural Network to evaluate and optimize the performance of a shell and tube heat exchanger looking for the maximization of the exergetic plant efficiency and to minimize the cost of the heat exchanger.
Bahmanyar and Talebi [2] used the entropy generation in the analysis of a shell and tube steam generator. They proposed a number for the generation of entropy to analyze the effect of the irreversibilities connected to heat transfer, friction and mixing. The conclusion of the work is that the decrease in the entropy generation of the heat exchanger leads to a higher performance for the overall system.

Deng Shengxiang et al. [3] enhanced the heat transfer by embedding fins in a phase change material. They constructed a numerical model based on the method of finite volume to estimate the efficiency of the heat exchanger for different fins arrangements.

Hara Shumpei et al. [4] performed an exergy analysis where water in which a surfactant is added is used as cooling medium.

An analysis of the techniques used for heat transfer enhancement in plate heat exchangers is presented by Zhang Ji et al. [5].

Navid Mahdavi and Shahram Khalilarye [6] introduced a new method for determining the pinch temperature of heat exchangers for refrigeration purposes.

In all these papers after a comprehensive analysis of the techniques of estimating the performance of heat exchangers the main interest is focused on defining the objective function for the extreme (optimum) search. The minimization of the overall entropy generation in the heat exchanger represents a valuable concept and strategy for finding the best heat transfer configuration.

2. Useful energy destruction in a heat exchanger

For a system in stationary regime the destruction of useful energy (exergy) is due to the irreversibility of the working processes:

\[ \dot{I} = \dot{W}_{\text{reversible}} - \dot{W}_{\text{real}} \] (2)

For power systems \( \dot{I} \) represents the amount of power that cannot be produced due to irreversibility.

For refrigeration systems, \( \dot{I} \) is the additional power supplementary to the minimum ideally required to accomplish the cooling task.

Quantitatively the useful energy destruction (exergy destruction) is equal to the product between the absolute temperature of the environment \( T_0 \) and the system overall entropy generation [7]

\[ \dot{I} = T_0 \sum \dot{S}_{\text{gen}} \] (3)

By estimating the entropy generation in a specific component of the system, i.e. a heat exchanger, one can determine the degree in which this component contributes to the destruction of the system useful energy (exergy).

The analyses focuses on the recuperative type gas to gas heat exchanger used in cryogenic refrigeration or liquefaction systems.

2.1. Thermal efficiency of a heat exchanger

The temperature – heat transfer surface diagram of the gas to gas heat exchanger is presented in figure1.

The thermal efficiency is defined as the ratio between the actual thermal load \( \dot{Q} \) and the maximum possible one, \( \dot{Q}_{\text{max}} \).

\[ \varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} = \frac{C_h (t_i^h - t_i^c)}{C_i (t_o^h - t_o^c)} = \frac{C_h (t_o^c - t_i^c)}{C_i (t_i^h - t_i^c)} \] (4)

where \( C_{\text{min}} \) is the lowest heat capacity between the hot stream and the cold one.
To respect the energy balance equation, the fluid experiencing the largest temperature difference should have the minimum thermal capacity. The maximum thermal load $Q_{\text{max}}$ could be obtained through an infinite heat transfer area.

Depending on the value of $C_{\text{min}}$, one gets:

$$
\varepsilon = \frac{t_i^h - t_o^h}{t_i^h - t_i^c}, \quad (C_{\text{min}} = C_h)
$$

(5)

$$
\varepsilon = \frac{t_i^c - t_i^h}{t_i^h - t_i^c}, \quad (C_{\text{min}} = C_c)
$$

(6)

2.2. Number of thermal units (NTU)

By definition [8] the number of thermal units is calculated in the following way:

$$
NTU = \frac{U \cdot A}{C_{\text{min}}} = \frac{1}{C_{\text{min}}} \int U(A) dA
$$

(7)

Let us consider a counterflow heat exchanger (Figure 2).

Accounting for Eq. (3) the efficiency of the heat exchanger can be written such that:

$$
\varepsilon = \frac{t_i^h - t_o^h}{\Delta t_0} = \frac{C_c}{C_h} \cdot \frac{t_i^c - t_i^h}{\Delta t_0}
$$

(8)

Figure 1. Variation of temperature across the heat transfer area. (a) $C_c < C_h$; (b) $C_h < C_c$

Figure 2. Counterflow heat exchanger. (a) Pipe diagram; (b) t-A diagram
If the inlet temperatures \( t_{i}^{h} \) and \( t_{i}^{c} \) are known and one considers that \( C_{\text{min}} = C_c \) and consequently, \( C_{\text{max}} = C_{h} \), equation (8) becomes:

\[
\varepsilon = \frac{t_{i}^{h} - t_{i}^{c}}{\Delta t_{0}} = \frac{C_{\text{min}}}{C_{\text{max}}} \frac{t_{i}^{c} - t_{i}^{e}}{\Delta t_{0}}
\]  
(9)

From Figure 2 it follows that:

\[
\Delta t_{\text{max}} = t_{i}^{h} - t_{i}^{c} = -(t_{i}^{h} - t_{o}^{h}) + (t_{i}^{c} - t_{o}^{c}) + (t_{i}^{h} - t_{i}^{c}) = -\varepsilon \frac{C_{\text{min}}}{C_{\text{max}}} \Delta t_{0} + \varepsilon \Delta t_{0} + (\Delta t_{0} - (t_{i}^{c} - t_{i}^{e})) = \]

\[
= \left(1 - \varepsilon \frac{C_{\text{min}}}{C_{\text{max}}} \right) \Delta t_{0}
\]  
(10)

\[
\Delta t_{\text{min}} = t_{i}^{c} - t_{o}^{c} = \Delta t_{0} - (t_{o}^{c} - t_{i}^{c}) = \Delta t_{0} - \varepsilon \cdot \Delta t_{0} = (1 - \varepsilon) \Delta t_{0}
\]  
(11)

Accounting for Eqns. (10) and (11) one gets:

\[
\Delta t_{\text{ML}} = \frac{\Delta t_{\text{max}} - \Delta t_{\text{min}}}{\ln \frac{\Delta t_{\text{max}}}{\Delta t_{\text{min}}}} = \frac{\Delta t_{0} \cdot \varepsilon \left(1 - \frac{C_{\text{min}}}{C_{\text{max}}} \right)}{\ln \frac{1 - \varepsilon \frac{C_{\text{min}}}{C_{\text{max}}}}{1 - \varepsilon}}
\]  
(12)

Observing that for \( C_{\text{min}} = C_c \)

\[
\text{NTU} = \frac{U \cdot A}{C_c}
\]  
(13)

\[
\dot{Q} = C_c (t_{o}^{c} - t_{i}^{c}) = U \cdot A \cdot \Delta t_{\text{ML}}
\]  
(14)

the NTU relationship becomes:

\[
\text{NTU} = \frac{t_{i}^{c} - t_{i}^{e}}{\Delta t_{\text{ML}}}
\]  
(15)

Equation (8) gives:

\[
t_{i}^{c} - t_{i}^{e} = \varepsilon \cdot \Delta t_{0}
\]  
(16)

By replacing Eqs. (16) and (12) into Eq. (15) gives:

\[
\text{NTU} = \frac{1 - \varepsilon \frac{C_{\text{min}}}{C_{\text{max}}}}{\ln \frac{1 - \varepsilon \frac{C_{\text{min}}}{C_{\text{max}}}}{1 - \varepsilon}}
\]  
(17)

It follows that:

\[
\text{NTU} \left(1 - \frac{C_{\text{min}}}{C_{\text{max}}} \right) = \ln \frac{1 - \varepsilon \frac{C_{\text{min}}}{C_{\text{max}}}}{1 - \varepsilon}
\]  
(18)
(19)

\[
\frac{NTU}{\varepsilon} \left( \frac{C_{\text{min}}}{C_{\text{max}}} \right) = \frac{1 - \varepsilon}{C_{\text{max}}} \frac{C_{\text{min}}}{1 - \varepsilon}
\]

By solving Eq. (18) for \( \varepsilon \) one gets:

\[
\varepsilon = \frac{\exp \left[ NTU \left( 1 - \frac{C_{\text{min}}}{C_{\text{max}}} \right) \right] - 1}{\exp \left[ NTU \left( 1 - \frac{C_{\text{min}}}{C_{\text{max}}} \right) \right] - \frac{C_{\text{min}}}{C_{\text{max}}}}
\] (20)

2.3. Entropy generation in the heat exchanger

For the control volume of the heat exchanger (Figure 3) the entropy balance equation becomes:

\[
\dot{S}_{\text{gen}} = \sum \dot{S}_i - \sum \dot{S}_o - \sum \dot{\mathcal{Q}} / T
\] (21)

Considering that the heat exchanger is very well insulated \((\dot{Q} = 0)\), Eq. (21) becomes:

\[
\dot{S}_{\text{gen}} = \dot{m}_h (s_o - s_i)_h + \dot{m}_c (s_o - s_i)_c
\] (22)

The cryogenic recuperative heat exchanger is of a gas to gas type.

Accounting for the entropy variation relationship for an ideal gas, Eq. (22) becomes:

\[
\dot{S}_{\text{gen}} = C_{\text{max}} \left( \ln \frac{T_o^h}{T_i^c} - \frac{R}{c_p} \ln \frac{p_o^h}{p_i^c} \right) + C_{\text{min}} \left( \ln \frac{T_o^c}{T_i^c} - \frac{R}{c_p} \ln \frac{p_o^c}{p_i^c} \right)
\] (23)

The mathematical model can be completed with the energy balance equation and the heat exchanger efficiency (Eq. (5))

\[
C_{\text{min}} \left( T_o^c - T_i^c \right) + C_{\text{max}} \left( T_o^h - T_i^h \right) = 0
\] (24)

\[
\varepsilon = \frac{T_o^c - T_i^c}{T_o^h - T_i^h}
\] (5)

The outlet temperatures \(T_o^h\) and \(T_o^c\) can be calculated from Eqns. (5) and (24) function of the given inlet temperatures:

\[
T_o^c = T_i^c + \varepsilon \left( T_i^h - T_i^c \right)
\] (25)

\[
T_o^h = T_i^h - \frac{C_{\text{min}}}{C_{\text{max}}} \varepsilon \left( T_i^h - T_i^c \right)
\] (26)

Accounting for Eqs. (23), (25) and (26) and denoting by \( \text{NSU} = \frac{\dot{S}_{\text{gen}}}{C_{\text{max}}} \) [9] the number of entropy generation units becomes:

\[
\text{NSU} = \frac{\dot{S}_{\text{gen}}}{C_{\text{max}}} = C_{\text{min}} \left[ \ln \left( 1 + \varepsilon \left( \frac{T_i^h}{T_i^c} - 1 \right) \right) - \left( \frac{R}{c_p} \right) \ln \left( 1 - \frac{\Delta p_c}{p_i^c} \right) \right] + \ln \left[ 1 - \frac{C_{\text{min}}}{C_{\text{max}}} \varepsilon \left( 1 - \frac{T_i^c}{T_i^c} \right) \right] - \left( \frac{R}{c_p} \right) \ln \left( 1 - \frac{\Delta p_h}{p_i^h} \right)
\] (27)
Based on Eqs. (7), (20) and (27) one can design a heat exchanger for a specific number of entropy generation units.

3. **Design a heat exchanger based on the NSU**

Figures (3) and (4) show the variation of the Number of Entropy Generation Units (NSU) (Figure 3) and Number of Thermal Units (NTU) (Figure 4) with the heat exchanger efficiency.

![Figure 3. Variation of the NSU number with the heat exchanger efficiency $\varepsilon$](image3)

![Figure 4. Variation of the NTU number with the heat exchanger efficiency $\varepsilon$](image4)
One can observe that:

\[ NSU = \frac{\dot{S}_{\text{gen}}}{C_{\text{max}}} = \frac{T_0 \cdot \dot{S}_{\text{gen}}}{T_0 \cdot C_{\text{max}}} = \frac{\dot{I}}{T_0 \cdot C_{\text{max}}} = \text{NEU} \]  

(28)

where \( \dot{I} = T_0 \cdot \dot{S}_{\text{gen}} \) is the exergy destruction caused by the irreversible processes in the heat exchanger.

Equation (28) defines the Number of Exergy Destruction Units (NEU) which is equal to the Number of Entropy Generation Units (NSU).

The operating and constructive optimum design of a thermal or cryogenic system may state a specific exergy destruction coefficient for a heat exchanger

\[ \Psi_{\text{HX}} = \frac{\dot{I}_{\text{HX}}}{\text{Fuel}} \times 100 \]  

(29)

where the Fuel represents the system exergy input.

Based on equations (28) and (29) one can calculate the NSU that corresponds to the recommended exergy destruction in the specified heat exchanger.

From figure (3), for a specific \( C_r \), one can determine the efficiency \( \varepsilon_0 \) for a desired NSU and use it in figure (4) to determine the NTU number and finally the necessary heat transfer area required to reach the specified magnitude of exergy destruction.

4. Conclusions

For rating both irreversible processes of heat transfer across a finite temperature difference and pressure drop due to friction, the Entropy Generation (Exergy Destruction) number is defined and used.

A method of designing a heat exchanger based on a stated NSU is presented.

The design method is based on the NTU-\( \varepsilon \) and NSU-\( \varepsilon \) correlations.

**Nomenclature**

- \( A \): heat transfer area [m²]
- \( C \): caloric capacity [J/kgK]
- \( \Delta t \): temperature difference [K]
- \( p \): pressure [bar]
- \( T \): temperature [K]
- \( t \): temperature [°C]
- \( U \): overall heat transfer coefficient [W/m²K]

**Indices**

- \( HX \): heat exchanger
- \( \text{gen} \): generated
- \( i \): inlet
- \( \text{min} \): minimum
- \( \text{max} \): maximum
- \( o \): outlet
- \( \emptyset \): equilibrium with the environment

**Superscripts**

- \( c \): cold
- \( h \): hot

**Abbreviations**

- NTU: Number of Thermal Units
- NSU: Number of Entropy Generation Units
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