A hybrid numerical scheme for a new formulation of delayed detached-eddy simulation (DDES) based on elliptic relaxation

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Abstract. A new formulation of Delayed Detached-Eddy Simulation (DDES) based on the $\varphi - f$ elliptic relaxation model has been derived and calibrated. A suitable correction function $\Psi$ and a model constant $C_{DDES}$ have been formulated and validated using decaying isotropic turbulence (DIT). The model has been applied to a 2D wall-mounted hump at high-Reynolds number where it has shown promising improvements over the common SST-DDES model. The improved RANS modelling of the near-wall physics results in an improved prediction of the levels of turbulent Reynolds stresses which leads to a better approximation of the recirculation zone and flow physics. A hybrid numerical scheme has also been developed for use with DDES; this blends an upwind based scheme and a centred scheme based explicitly on the blending function of the DDES.

1. Introduction

Hybrid RANS-LES approaches have seen a surge in popularity in the past decade as academia and industry are increasingly interested in instantaneous information of high-Reynolds number turbulent flows. For these flows, Large-Eddy Simulation (LES) is still not a practical option for most industrial applications in the light of currently available computational resources. Reynolds-Averaged Navier-Stokes (RANS) models are well established as numerical tools for industrial applications, but when applied to highly unsteady separated flows, their limitations are well-known. For many cases their inability to capture the large scale unsteadiness and a tendency to under-predict the shear-stress in the separated layer (which leads to a longer separation length) mean they are unsuitable for these types of flows (Haase et al., 2007).

Hybrid RANS-LES methods attempt to merge these two modelling approaches and apply them either zonally or non-zonally. Hybrid methods aim to relax grid resolution requirements relative to original LES, by providing improved sub-grid scale RANS modelling, in particular to capture the near wall turbulence (therefore enabling a coarser near-wall grid to be applied). In addition, applying LES in the regions of highly unsteady flow allows the large scale instabilities to be resolved, which in turn can provide more information about the flow structure.
2. Hybrid RANS-LES

One of the more common hybrid RANS-LES methods in usage is the Delayed Detached-Eddy Simulation (DDES) approach (Spalart \textit{et al.}, 2006) which is an improved version of the original Detached-Eddy Simulation (DES) method of Spalart \textit{et al.} (1997). DDES can be seen as a three-dimensional, unsteady model based on an underlying ‘off-the-shelf’ RANS model. It seamlessly joins a sub-grid scale model in regions where the grid is fine and outside of the attached boundary layer to a RANS model in all other regions. The principle of DDES is to modify the RANS length scale, $L_{RANS}$, in such a way that it is now based on the grid size and a blending function, $f_d$:

$$L_{DDES} = L_{RANS} - f_d \max(0, L_{RANS} - L_{LES}), \quad f_d = 1 - \tanh \left( \frac{\left( \frac{\nu_t + \nu}{\sqrt{\bar{U}_{ij} \bar{U}_{ij} \kappa^2 y^2}} \right)^3}{8} \right) \quad (1)$$

where $L_{LES} = \Psi C_{DDES} \Delta$ and $C_{DDES}$ is an empirical parameter which needs to be calibrated. $\Psi$ is a correction term to ensure the model returns to the classical Smagorinsky form when in LES mode. Finally $\Delta$ is the filter width for LES, commonly taken to be the cell volume for unstructured grids. In the expression for $f_d$, $\kappa$ is the von Kármán constant, and $y$ is the distance to the wall. The function $f_d$ takes the value 1 in the LES region and 0 elsewhere.

The first version of DES was based on the Spalart-Allmaras (SA) model, as this was seen as ‘the most convenient length scale to inject $\Delta$ and turn a RANS model into a SGS model’ (Travin \textit{et al.}, 2000a). Since then, many RANS models have been applied to DES & DDES, however the most popular models for commercial codes are still the SA and $k - \omega$ Shear-Stress Transport (SST) DDES models.

3. Novel DDES model

An EU funded research project (DESider, Haase \textit{et al.} (2007)) looked into the issue of model sensitivity within DDES. An important conclusion from this research was that for flows where separation occurs due to the presence of a sharp change in geometry, the solution is only weakly dependant on the underlying RANS model. However for flows where the separation occurs through an adverse pressure gradient or complex near wall physics, then the choice of the underlying RANS model is important to the accuracy of the solution. The majority of the RANS models tested in this project however were variants of the SA model and did not include Reynolds stress models or models based on elliptic relaxation.

It is in this light that a new DDES model is formulated in the current paper, based on the principle of elliptic relaxation. The $\varphi - f$ (Laurence \textit{et al.}, 2004) model provides a new DDES formulation with improved modelling of the near-wall physics which is also numerically robust for industrial applications.

This novel variant of DDES has been calibrated using the standard Decaying Isotropic Turbulence (DIT) test case in order to obtain the correct values of the model parameter $C_{DDES}$ and $\Psi$. To demonstrate this new formulation, it has been applied to a 2D-wall mounted hump flow at a high-Reynolds number.

4. $\varphi - f$ RANS model

The $\varphi - f$ model is a numerically robust version of the ‘Code Friendly’ $\bar{u}^2 - f$ model developed by Lien & Durbin (Lien & Durbin, 1996). In this version, $\bar{u}^2$ is replaced by $\varphi = \frac{\bar{u}^2}{\kappa}$, which results in a more robust model that converges more easily and allows the use of larger time steps compared to the model of Durbin when used with an uncoupled solver. The model uses the
same transport equation for the turbulent kinetic energy $k$, as the high Reynolds number $k - \varepsilon$ model (Jones & Launder, 1972), but uses a modified equation for the turbulent dissipation $\varepsilon$, that incorporates a modified $\varphi$ dependent $C_{\varepsilon_1}$ term (denoted $C'_{\varepsilon_1}$) to allow for wall modifications. A transport equation is also solved for $\varphi$ and the elliptic operator $f$.

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_i} = P_k - \varepsilon + \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_t \right) \frac{\partial k}{\partial x_j} \right]$$ \hspace{1cm} (2)

$$\frac{\partial \varepsilon}{\partial t} + U_j \frac{\partial \varepsilon}{\partial x_i} = \frac{C'_{\varepsilon_1} P_k - C_{\varepsilon_2} \varepsilon}{T} + \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_t \right) \frac{\partial \varepsilon}{\partial x_j} \right]$$ \hspace{1cm} (3)

$$\frac{\partial \varphi}{\partial t} + U_j \frac{\partial \varphi}{\partial x_i} = f - P_k \frac{\varphi}{k} + \frac{2}{k} \left( \frac{\nu_t}{\sigma_k} \right) \frac{\partial \varphi}{\partial x_j} \frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j} \left[ \left( \frac{\nu_t}{\sigma_k} \right) \frac{\partial \varphi}{\partial x_j} \right]$$ \hspace{1cm} (4)

$$L^2 \frac{\partial^2 f}{\partial x_j^2} = \frac{1}{T} (C_1 - 1) \left[ \varphi - \frac{2}{3} \right] - C_2 \frac{P_k}{k} - 2 \left( \frac{\nu_t}{k} \right) \frac{\partial \varphi}{\partial x_j} \frac{\partial k}{\partial x_j} - \nu \frac{\partial^2 \varphi}{\partial x_j^2}$$ \hspace{1cm} (5)

$$\nu_t = C_\mu \varphi k T$$ \hspace{1cm} (6)

| $\sigma_k$ | $\sigma_\varepsilon$ | $C_{\varepsilon_2}$ | $C_1$ | $C_2$ | $C_L$ | $C_\mu$ | $C_\eta$ | $C_T$ |
|---|---|---|---|---|---|---|---|---|
| 1.0 | 1.3 | 1.9 | 1.4 | 0.3 | 0.25 | 0.22 | 110 | 6 |

Table 1. Model coefficients for the $\varphi - f$ model

where the time scale $T$, and the length scale $L$, are bounded by the Kolmogorov scales in order to avoid a singularity occurring in Equations 3, 5 and 6:

$$T = \max \left( \frac{k}{\varepsilon}, C_T \sqrt{\frac{\nu}{\varepsilon}} \right) \hspace{1cm} L = C_L \max \left[ \frac{k^{3/2}}{\varepsilon}, C_\eta \varepsilon^{3/4} \right].$$ \hspace{1cm} (7)

The term $C'_{\varepsilon_1}$ in Equation 3 is changed from the original value of 1.44 used in the standard $k - \varepsilon$ model to:

$$C'_{\varepsilon_1} = 1.4 \left( 1 + 0.05 \sqrt{\frac{T}{\varphi}} \right)$$ \hspace{1cm} (8)

4.1. $\varphi - f$ DDES

For the $\varphi - f$ model, the standard DDES modification is made to the turbulent kinetic energy equation; thus equation 2 becomes:

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_i} = P_k - \varphi \frac{k^{3/2}}{L_{DDES}} + \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_t \right) \frac{\partial k}{\partial x_j} \right]$$ \hspace{1cm} (9) Modified

In the current formulation, the DDES length scale modification appears only in the turbulent kinetic energy equation and not in equation 4 for $\varphi$. This was chosen to be consistent with the principle of the original DES formulation. The study of Yan et al. (2005) investigated
the differing results that may be obtained when using an alternative length scale substitution (through $\nu_t$, which is comparable to also using $\varphi$), however in the scope of the present study, it was decided to establish a baseline version of the $\varphi - f$ DDES model before investigating alternative substitutions.

One of the primary verification tests for a DDES formulation is it’s simplification under local equilibrium conditions. The turbulent viscosity should return to a sub-grid scale Smagorinsky-like form when using the DDES length scale $L_{DDES} = \Psi C_{DDES} \Delta$, i.e $\nu_t = (C \Delta)^2 S$, where $C$ is a constant and $\Psi$ is the correction function. Under local equilibrium conditions (where production, $P_k = \nu S^2$, is equal to dissipation) the $k$ equation (in LES mode) and $\varepsilon$ equation become:

$$\nu_t S^2 = \frac{\varphi k^{3/2}}{\Psi \varphi f C_{DDES} \Delta}$$  \hspace{1cm} (10)$$

$$\nu_t S^2 = \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}} \varepsilon$$  \hspace{1cm} (11)$$

From which it is straightforward to show that the Smagorinsky form of the $\varphi - f$ sub-grid scale model is given by:

$$\nu_t = A_{\varphi f} (\Psi \varphi f C_{DDES} \Delta)^2 S, \hspace{1cm} A_{\varphi f} = \left(\frac{C_{\varepsilon 2}}{C_{\varepsilon 1}}\right)^{3/2} \left(\frac{1}{\varphi}\right)^{1/2}$$  \hspace{1cm} (12)$$

The correction term, $\Psi \varphi f$ should be of the form $A_{\varphi f} \Psi_{\varphi f}^2 = const$. Unlike the SST-DDES model $A_{\varphi f}$ is not a constant and is dependant on $\varphi$ both directly and via the $C'_{\varepsilon 1}$ parameter (Equation 8). This effectively results in a dynamic $\varphi$-dependant LES (Figure 1). The $\varphi$-dependance is removed by setting:

$$\Psi_{\varphi f} = \left(\frac{C'_{\varepsilon 1}}{C'_{\varepsilon 2}}\right)^{3/4} (\varphi)^{1/4}$$  \hspace{1cm} (13)$$

This cancels out the terms in front of $C_{DDES} \Delta$ and returns the model to the standard LES mode for DDES.

![Figure 1](image)

**Figure 1.** Demonstration of the functionality of the $\Psi$ correction term for the DIT case using a $64^3$ grid at $t=2s$ and $C_{DDES} = 0.60$
5. Hybrid Numerical Scheme

Undertaking a DDES (calculation) poses a dilemma to the end-user when deciding upon the correct numerical convection scheme. An explicit LES requires a low dissipative numerical scheme as the sub-grid scale model provides the dissipation of the small turbulent scales. 2nd order or higher order central differencing schemes (CDS) are frequently used to facilitate this requirement as upwind differencing schemes (UDS) typically add too much numerical dissipation.

Applying a CDS scheme to a RANS model can lead to excessive numerical oscillations when the grid is too coarse or the cell Péclet number is large. To overcome this, UDS schemes or schemes based on a blend of CDS and UDS are frequently used.

In a seamless hybrid RANS-LES approach such as DDES, there should also be a seamless switch between the numerical schemes to fulfill the numerical requirements of each mode. The hybrid numerical scheme of Travin et al. (2000b) is one such approach. It allows a switch between an upwind based scheme and a central differencing scheme depending on a blending function. This hybrid numerical scheme found widespread use with the standard version of DES, and was automatically adopted for use with the newer DDES formulation. However, since the switching between LES and RANS in DDES is based on the blending function \( f_d \), we contend that the numerical scheme should also be blended using the same function, and not by means of a separate function, developed originally for standard DES. Thus, in the current work we employ \( f_d \) to control the numerical scheme as follows:

\[
\phi_f = \phi_{f,SOLU} \quad \text{if } L_{RANS} < L_{LES} \tag{14}
\]

\[
\phi_f = (1 - f_d) \phi_{f,SOLU} + f_d \phi_{f,CDS} \quad \text{if } L_{RANS} > L_{LES} \tag{15}
\]

Where SOLU and CDS represent a second-order upwind based scheme and a centred scheme, respectively.

6. Computational method

All calculations were performed using the open-source code Code_Saturne (Archambeau et al., 2004) developed by EDF Energy. It is a 3D code based on a finite-volume numerical method capable of solving steady or transient laminar or turbulent flows on structured or unstructured grids. A range of RANS turbulence models is available, including high and low-Reynolds number second moment closure and eddy-viscosity based models. The Smagorinsky and Dynamic LES models are also available. All other hybrid RANS-LES models referred to in this work are implemented by the author unless otherwise stated.

7. Test case description

7.1. Decaying Isotropic Turbulence

The natural evolution of homogeneous decaying isotropic turbulence (DIT) provides a fundamental test case in assessing the numerical performance of the LES branch of DDES. This test case allows the following to be evaluated:

- The capability of the turbulence model to predict the energy cascade and to resolve turbulent structures.
- Calibration of the \( C_{DDES} \) parameter in DDES.
- The performance of the \( \Psi \) correction term.
- Assessment of the solver’s level of numerical dissipation, e.g verification of the use of a centred convection scheme against an upwind scheme in the LES branch of DDES.
DIT is therefore seen as an essential verification and validation test case for any DDES formulation. The solution domain of the DIT calculation is cubic with length $2\pi$. The solution domain is meshed with two grids consisting of $32^3$ and $64^3$ cubic and equidistant cells. Periodic boundary conditions are imposed throughout. The initial velocity field is set with a suitable instance of isotropic turbulence by the use of an inverse Fourier transform using the experimental data of Comte-Bellot & Corrsin (Comte-Bellot & Corrsin, 1971). In order to obtain the initial values for other variables such as the pressure and turbulence quantities ($k$ & $\varepsilon$ etc), a frozen velocity field simulation was conducted, where the velocity field is frozen and only the other variables are solved, which once converged was used as initial conditions for the unsteady decay of turbulence simulation.

7.2. 2D wall-mounted hump

The turbulent flow over a wall-mounted, 2-D hump (Figure 2) at a high-Reynolds number of $Re_c = 9.36 \times 10^5$ (based on a chord length of $c = 0.42$ m and a free stream velocity of $U_\infty = 34.6$ m/s) was examined experimentally at the NASA Langley Research Centre (Greenblatt et al., 2004) as part of the CFDVAL workshop on computational methods and turbulence models validation. This test case has also been studied at the 11th ERCOFTAC workshop on refined turbulence modelling (Johansson and Davidson, 2005) and the 12th ERCOFTAC/IAHR workshop on refined turbulence modelling (Thiele and Jakirlic, 2006). It is also being investigated as part of the European ATAAC project (Advanced Turbulence Simulation for Aerodynamic Application Challenges).

Two structured meshes were investigated, which were provided by Chalmers University and New Technologies and Services (NTS) for the ATAAC project. The first mesh has a solution domain of $(L_x \times L_y \times L_z = 6.14c \times 0.904c \times 0.2c)$ which is meshed with 1638400 cells ($400 \times 128 \times 32$). The second mesh that has a greater concentration of cells close to the wall and has a solution domain of $(L_x \times L_y \times L_z = 6.14c \times 0.909c \times 0.4c)$ which is meshed with 2934976 cells ($379 \times 121 \times 64$).

Although the experimental domain started at -6.39c compared to the current -2.14c, work by Saric et al. (2006) shows that there is no significant difference between the choice of the inlet position and thus it was decided to reduce the solution domain to the smaller size. The dimensionless wall distance of the nodes closest to the wall were $y^+ < 1$ for the lower wall and $y^+ = 30$ for the upper wall.

The oncoming flow is characterised by a zero-pressure-gradient turbulent boundary layer, whose thickness $\delta$ is approximately 57% of the hump height measured at the upstream extent of the domain (-2.14c), this corresponds to a momentum-thickness-based Reynolds number.
Reθ=7200 (Greenblatt et al., 2004). The mean profiles of the velocity and turbulent quantities which were used as boundary conditions at the inlet were taken from a precursor computation. A no-slip boundary condition was applied at the bottom wall and a slip wall was applied to the top wall. Periodic boundary conditions were used along the span-wise direction. All cases ran using a non-dimensional time step of 0.001 (based on the hump chord and reference velocity).

8. \textit{C}\textsubscript{DDES} calibration

The \textit{C}\textsubscript{DDES} parameter for the new DDES formulation is calibrated using the DIT case. \textit{C}\textsubscript{DDES} is similar to the Smagorinsky constant and must necessarily be calibrated for each DDES formulation. For each model, three different values for \textit{C}\textsubscript{DDES} were investigated using two grids of 32\textsuperscript{3} and 64\textsuperscript{3} in order to account for the effect of mesh refinement. The temporal discretization is 2\textsuperscript{nd} order, and the hybrid numerical scheme from Section 5 is used to spatially discretise the momentum convective terms while a 1\textsuperscript{st} order upwind scheme is applied to the turbulent quantities. All results are presented for a non-dimensional time of \(t = 2.0\) and are compared to the experimental data of Comte-Bellot & Corrsin (1971). To ensure that the LES mode of DDES is used, the length scale is set to that of a LES:

\[
L_{\text{DDES}} = L_{\text{LES}} = \Psi C_{\text{DDES}} \Delta.
\]  

(16)

The results for three values of \(C_{\text{DDES}}\) using the SST-DDES model (Figure 3(a)) show that increasing the value of \(C_{\text{DDES}}\) corresponds to increased high-wave number damping, and can be seen as adding more numerical dissipation. Although DIT is a much simplified test case, a value of \(C_{\text{DDES}} = 0.65\) seems optimal considering both grids. This is in agreement with most other implementations summarised in the DESider project (Haase et al., 2007).

For the \(\varphi - f\) DDES model shown in Figure 3(b), the value of \(C_{\text{DDES}}\) varies in a similar fashion to the SST-DDES formulation and shows the same sensitivity to the mesh refinement. A value of \(C_{\text{DDES}} = 0.60\) was seen to be optimal although it is not possible to satisfy both grids with the same value of \(C_{\text{DDES}}\) and any value between 0.55 and 0.65 could be justified. This emphasises that the calibration of the \(C_{\text{DDES}}\) must be conducted for each DDES formulation.

Both models show a weaker dependency on \(C_{\text{DDES}}\) as the grid is refined. This can be explained by the fact that as \(\Delta\) becomes smaller, the modelled turbulence should become smaller and there will be an increasing proportion of resolved turbulence.

![Figure 3](image.png)

\textbf{Figure 3.} Energy spectra versus wavenumber (\(\kappa\)) for the velocity field, using varying values of \(C_{\text{DDES}}\) for the 32\textsuperscript{3} and 64\textsuperscript{3} grids (a) SST DDES (b) \(\varphi - f\) DDES
9. Numerical scheme sensitivity study

As DDES switches from RANS mode to LES-like mode it is important to investigate the effect of the numerical scheme on the LES branch of DDES. The simplest test case to evaluate the effect of the numerical scheme is again Decaying Isotropic Turbulence (DIT).

For this investigation, several numerical schemes for the convective terms are used including the hybrid numerical scheme developed in this paper. For the DIT test case all results are presented for a non-dimensional time of $t = 2.0$ and are again compared to the experimental data of Comte-Bellot & Corrsin (1971). A summary of the numerical schemes evaluated is given in Table 2.

| Convective schemes                           |
|---------------------------------------------|
| First order upwind scheme (UDS)             |
| Second order linear upwind scheme (SOLU)    |
| Centred difference scheme (CDS)             |
| Hybrid numerical scheme (Hybrid)            |

Figures 4(a) & 4(b) show the results for the DIT case using the SST-DDES and $\varphi - f$ DDES models. The effect of different numerical schemes is a substantial shift in the prediction of the energy spectrum. Anything other than a fully CDS scheme results in additional damping at high wave numbers and an over-prediction of the lower wave numbers (corresponding to the energy-containing range). This energy-containing range depends on the velocity gradients so the manner in which these gradients are calculated will effect the prediction of this portion of the energy cascade and therefore may explain the spread of results in this area for different numerical schemes.

The hybrid numerical scheme introduced in Section 5 performs well, employing a fully CDS scheme and therefore matching the spectrum predicted by the CDS scheme applied without the hybrid numerical scheme. There is considerable difference between the 2\textsuperscript{nd} order SOLU scheme and CDS, with the SOLU scheme grossly over-predicting the lower wave numbers. Therefore, in order to correctly capture the energy decay, a low-dissipative scheme such as the CDS scheme must be used to avoid any additional high-wave number damping.

Figures 5(a) & 5(b) show the results for the mean span-wise averaged skin-friction coefficient results for the 2D wall-mounted hump (400×128×32 mesh) using the SST DDES and $\varphi - f$ DDES models. For both models using the SOLU scheme (i.e a numerical scheme commonly used for RANS models) produces a strongly delayed separation region that is common to both DDES models. While the length of the recirculation zone is similar to the experiment, the delay means that it is unphysical and strongly suggests that an upwind based scheme should not be used for a DDES (calculation). This agrees with the results from the DIT case where the SOLU scheme produced excessive numerical dissipation. While using a CDS scheme gives much improved results compared to the SOLU scheme, applying it across the whole domain in both RANS and LES modes resulted in some numerical instabilities. In such cases it became necessary to reduce the time step and increase under-relaxation, although when using the 379×121×64 mesh, the simulation could be run with reasonable time steps. It is not yet clear if these limitations stem from the modelling scheme or the underlying numerics of the code itself.

The hybrid numerical scheme behaves much like the CDS scheme in terms of accuracy but without the numerical instabilities. As it is directly based on the DDES blending function,
the choice of SOLU or CDS is explicitly linked to the $f_D$ function. As the recirculation region behind the hump is in LES mode (apart from the boundary layer) (Figures 6(a) & 6(b)) then it is guaranteed that the numerical scheme will be CDS provided that the LES and RANS zone are correctly meshed, as this ultimately decides the accuracy of the simulation.

**Figure 4.** Energy spectra versus wavenumber ($\kappa$) for the velocity field, with different numerical schemes using the $64^3$ grid (a) SST DDES (b) $\varphi - f$ DDES

**Figure 5.** Mean span-wise averaged skin friction coefficient ($C_f$) using different numerical schemes for the 2D-wall mounted hump (a) SST DDES (b) $\varphi - f$ DDES
Figure 6. The location of the ddes zones (0=RANS 1=LES) and the blending function $f_d$.

Figure 7. Mean span-wise averaged skin friction coefficient ($C_f$) for the two meshes using (a) SST DDES (b) $\varphi - f$ DDES for the 2D-wall mounted hump.

Figure 8. Mean streamlines for the $\varphi - f$ DDES model (top) and the SST DDES model (bottom) for the 2D wall-mounted hump.
10. DDES model sensitivity

The two meshes are compared for both SST & $\varphi - f$ DDES models in Figures 7(a) & 7(b). For both models the second mesh (379×121×64) produces better results, and this is most noticeable with the $\varphi - f$ model, where the strength and size of the recirculation zone is closer to the experiment. The following analysis relates to the second mesh.

Figures 8-9(b) highlight the differences between the two DDES models. The $\varphi - f$ DDES model predicts the correct recirculation length and matches the reattachment point ($x/c = 1.1$) more closely to the experiment than the SST DDES model. This can also be seen in the streamline plots in Figure 8 where the SST DDES predicts too large a recirculation zone. The improved performance of the $\varphi - f$ DDES model over the SST DDES model can be attributed to the level of modelled and resolved turbulence levels in the initial part of the recirculation zone ($x/c=0.66-0.9$). Figures 10(a)-10(d) show the evolution of the velocity for both models, they show that while the oncoming velocity field is the same for the both DDES models prior to the separation point ($x/c=0.65$), by $x/c=0.9$, the $\varphi - f$ DDES model predicts more closely the correct strength of the flow reversal compared to the SST DDES and overall a velocity profile closer to the experimental data.

The reason for this improved performance appears to be linked to the level of the shear stress ($u'v'$) and the Reynolds Stresses as seen in Figures 11(a)-12(d). The $\varphi - f$ DDES model predicts a larger resolved shear stress component than the SST DDES model and matches the experimental curve more closely. This larger shear stress value coupled with larger values for the Reynolds stresses results in a shorter recirculation region.

The underlying $\varphi - f$ model solves an additional transport equation for $\phi (=\overline{u^2}/k)$, which means the model is able to model some of the anisotropy of the flow (as well as using $\varphi$ to damp the turbulent viscosity near the wall). It is worth noting when observing the over-prediction of the Reynolds stresses by the $\varphi - f$ DDES that the error of the experimental PIV data (for the turbulent quantities) is up to 20%.

While the SST- DDES model suffers from the well documented trait of the ‘grey area’ problem (Spalart et al., 1997) (when the modelled turbulence levels drop but the resolved component cannot immediately replace it, which leads to a drop in the overall turbulence levels), the $\varphi - f$ DDES model, at least for the 2D hump case, appears to be less sensitive to this.

![Figure 9](image-url)  
**Figure 9.** Mean span-wise averaged (a) skin friction coefficient ($C_f$) (b) pressure coefficient ($C_p$) for the 2D-wall mounted hump
Figure 10. Mean span-wise averaged velocity profiles at (a) x/c = 0.65 (b) x/c = 0.80 (c) x/c=0.90 (d) x/c = 1.0 for the 2D wall-mounted hump

Figure 11. Mean span-wise averaged modelled and resolved turbulent shear stress profiles at (a) x/c=0.65 (b) x/c=0.80 (c) x/c=0.90 (d) x/c=1.0 for the 2D wall-mounted hump

Figure 12. Mean span-wise averaged modelled and resolved turbulent shear stress profiles at (a) x/c=0.65 (b) x/c=0.80 (c) x/c=0.90 (d) x/c= 1.0 for the 2D wall-mounted hump
11. Conclusion

The $\phi-f$ DDES model has been derived and calibrated using DIT and a 2D wall-mounted hump test case. A suitable formulation of the correction function ($\Psi$) and $C_{\text{DDES}}$ have also been derived and demonstrated using the DIT case. The results from the 2D wall-mounted hump case are promising and suggest that the underlying RANS model can play a role in helping to improve future DDES formulations, even for a case with a largely fixed separation point. The improved modelling of the near-wall physics appear to improve the prediction of the shear stress which in turn leads to a better prediction of the strength of the recirculation region.

The importance of using the correct numerical scheme for each DDES mode has been highlighted for both a fundamental case like DIT and also for the 2D wall-mounted hump. The hybrid numerical scheme has performed as expected and linked the accuracy of a centred differencing scheme with the stability of an upwind scheme. The explicit link between the $f_d$ function of DDES and the choice of a suitable numerical scheme ensures that within the boundary layer the numerical scheme is correct for the RANS mode and in all areas where the LES mode is active, a fully centred scheme is used. Further tests will be conducted on different test cases to fully validate this new scheme and the new $\phi-f$ DDES model.
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References

ARCHAMBEAU, F, MECHITOUA, N & SAKIZ, M 2004 A finite volume method for the computation of turbulent incompressible flows - industrial applications. *Int. J. Finite Volumes* 1, 1–62.

COMTE-BELLOT, G & CORRSIN, S 1971 Simple Eulerian Time Correlation of Full and Narrow-Band Signals in Grid-Generated Isotropic Turbulence. *J. Fluid Mech* 48.

GREENBLATT, D, PASCHAL, K B, YAO, C S, HARRIS, J, SCHAEFFLER, N W & WASHBURN, A E 2004 A separation control CFD validation test case, Part 1: baseline and steady suction. *AIAA paper* 2004-2220.

HAASE, W, BRAZA, M & REVELL, A, ed. 2007 DESIDER - A European Effort on Hybrid RANS-LES Modelling, Notes on Numerical Fluid Mechanics and Multidisciplinary Design, vol. 103. Springer-Verlag.

JONES, W P & LAUNDER, B E 1972 The prediction of laminarization with a two-equation model of turbulence. *Int. J. Heat. Mass. Transfer* 15, 301–314.

LAURENCE, D L, URIBE, J C & UTYUZHINKOV, S V 2004 A robust formulation of the $v^2 - f$ model. *Flow Turbul. and Combust.* 73, 169–185.

LIEN, P S & DURBIN, P A 1996 Non linear $\kappa \epsilon v^2$ modelling with application to high-lift. In *Proceedings of the Summer Program 1996*, pp. 5–22. Stanford University.

SARIC, S, JAKIRLIC, S, DJUGUM, A & TROPEA, C 2006 Computational analysis of locally forced flow over a wall-mounted hump at high-Re number. *Int. J. Heat Fluid Flow* 27, 707–720.

SPALART, P. R., DECK, S., SHUR, M. L., SQUIRES, K. D., STRELETS, M. KH. & TRAVIN, A. 2006 A New Version of Detached-eddy Simulation, Resistant to Ambiguous Grid Densities. *Theor. Comput. Fluid Dynam.* 20 (3), 181–195.

SPALART, P R, JOU, W H, STRELETS, M & ALLMARAS, S R 1997 Comments on the feasibility of LES for wings and on a hybrid, RANS/ES approach. *Advances in DNS/LES, Proceedings of 1st AFOSR International Conference on DNS/LES* 1, 137–147.

TRAVIN, A, SHUR, M, STRELETS, M & SPALART, P R 2000a Detached-eddy simulations past a circular cylinder. *Flow Turbul. Combust.* 63, 293–313.

TRAVIN, A, SHUR, M, STRELETS, M & SPALART, P R 2000b Physical and numerical upgrades in the detached-eddy simulation of complex turbulent flows. In *Proceedings of the 412th Euromech Colloquium on LES and Complex Transitional and Turbulent Flows*, Munich.

YAN, J., MOCKETT, C. & THEILE, F. 2005 Investigation of Alternative Length Scale Substitutions in Detached-Eddy Simulation. *Flow Turbul. Combust.* 74 (1), 85–102.