Cloud Matrix Machine for Julia and Implicit Parallelization for Matrix Languages

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ABSTRACT

Matrix computations are widely used in increasing sizes and complexity in the fields of scientific computing and engineering. But with current matrix language implementations it is a challenging task to fully utilize Cloud compute capacities. We present a new framework called cloud matrix machine, which extends the Julia high-performance compute language to automatically parallelize matrix computations for the cloud. With this framework, users are shielded from the complexity of explicitly parallel computations. Instead, users employ a novel matrix data type with lazy evaluation semantics to facilitate implicit parallelization of matrix operations. A combination of offline profiling, dynamic simulation, and scheduling are utilized to select optimal tile sizes, schedule, and execute matrix operations. All computations occur in the Cloud, with minimal user intervention. We conducted an extensive experimental evaluation on a set of eight benchmarks using up to eight nodes (288 vCPUs) in the AWS public cloud. Our framework achieved speedups of up to a factor of $3.49 \times$, within 20.5% of the theoretically possible maximum speedup.

CCS CONCEPTS

• Computing methodologies → Shared memory algorithms; Parallel computing methodologies.

KEYWORDS

Matrix computations, Julia, HEFT, Distributed systems, Simulation, Parallel computing

1 INTRODUCTION

As a fundamental form of computing found in both linear algebra and scientific computing, matrix multiplication has been the subject of many investigations into parallel computing [15, 16]. Overcoming the physical limits of single-core processors, parallel computing takes advantage of the processing power of multi-core processors to improve computing performance, particularly in solving larger problems. It can involve utilizing multiple computers at once, as offered by Amazon’s EC2 service allowing remote connections of computers in a wide area network. However, it is notably more challenging to fully take advantage of such parallelism as opposed to sequential computing, particularly when heterogeneous distributed systems are involved.

There is a difficulty in reconciling multiple multicore nodes, given that the bottleneck is often the network in the distributed system as opposed to memory bandwidth in a single node. We thereby utilize a hierarchical approach to the management of the distributed system, along with analyzing dependencies to effectively run tasks in parallel, to reduce dependency on the network bottleneck. We implement this in Julia, a dynamic language for high performance computing using JIT compilation and equipped with a robust parallel programming environment [2].

Our contributions are summarized as follows:

• The cloud matrix machine (CMM) framework that extends the Julia high-performance compute language with implicitly-parallel matrix routines through an extension of the regular matrix data type offered in the Julia language.

• An offline profiling tool for predicting execution time using a time model and regression analysis with online simulation. We utilize this in conjunction with automatic matrix parallelization to predict the most efficient method of parallelization.

• A modification of the HEFT algorithm that takes a node-level cache into account to reduce the communication bottleneck incurred by the network.

• An extensive experimental evaluation on the AWS public cloud to show the validity of our approach.

2 RELATED WORK

Level 1 BLAS achieves vector-vector computations [9], level 2 BLAS achieves vector-processing machines [5], and level 3 BLAS utilizes caches as part of a multi-memory hierarchy to obtain decent performance. To take advantage of the latter, matrices are partitioned into smaller matrices to maximize the cache hit ratios [4]. We utilize
3 CMM FRAMEWORK

Figure 1 depicts the overall structure of our CMM framework for Julia. Users cast their matrices into the ClusteredMatrix type to interact with the framework, which breaks the matrix equations down into ASTs then task-dependency graphs. The graphs are tiled according to input tile sizes to generate divided subtasks based on tile dimensions. Separately, offline profiling of the execution of tasks generates operand sizes and times for use in interpolation equations subject to polynomial regression to create predictions of the time model. This is used with the tiled dependency graphs under the HEFT algorithm to find a schedule of the tasks that best optimizes the earliest finish times in accordance with expected times for each tiled task. The generated schedule is then run under a simulation with the predicted time model to generate simulated task schedule. Finally, execution of the work is carried out.

3.1 Task Classification

There are several task classifications involved with the matrix computations. The memory allocation task, malloc, is scheduled first on each node followed by a fillzero task to initialize the data. The communication tasks are separated into send, where a node initiates a request to send data to another node, and recv, where a node indicates that it has received data from another node. The computation tasks primarily consist of addmul tasks, where matrix addition and multiplication take place. The sub! tasks denote where matrix subtraction occurs. Finally, the takecopy task type is involved in copying data from worker nodes into the master node for the final matrix computation.

3.2 Dependency graphs

Figure 2 uses a Markov benchmark example to demonstrate the conversion from ClusteredMatrix in our CMM framework for Julia to tiled dependency graphs. P and u point to a ClusteredMatrix object that generates a random matrix according to the input dimensions. The input matrix P is multiplied by P three times before the final multiplication with matrix u. It automatically generates a dependency graph based on the expression tree. The ClusteredMatrix type allows accessing objects by their unique ID, where each object represents nodes and corresponding edges in the graph.

We specify tile sizes, which the framework uses to automatically tile the matrix in accordance with the dimensions. In Figure 2, the input matrix is of size 10k, and the specified tile size is 5k. The automatic tiling framework thereby generates four partitioned tiles of dimensions 5k by 5k from matrix P, and two tiles of dimensions 1 by 5k from matrix u. The dependency graph changes accordingly with the tiles while preserving the task dependencies, introducing additional operators consistent with the tile dependencies. The algorithm we use for tiling is described in Listing 1.

Listing 1: Implementation of tile function

```julia
function tile(P::ClusteredMatrix, tile_size::Tuple)
    mP, nP = size(P)
    mtile, ntile = tile_size
    mTiledP, nTiledP = cld(mP, mtile), cld(nP, nTile)
    tiledP = Matrix(ClusteredMatrix)(mTiledP, nTiledP)
    for j=1:nTiledP
        colStart = ntile * (j-1)+1
        colEnd = min(ntile * j, nP)
```

Parallel BLAS (PBLAS) which provides multi-process parallelization on a distributed system [5].

Apart from our work, there have been other research into automatic parallelization of matrix-related operations. Sato and Iwasaki implemented matrix multiplication parallelization into compiler technology [14], and Miyasaka and Goda propose a method for automatically generating an optimal parallel algorithm for matrix-vector multiplication in a neural network though the correctness is still uncertain [11]. Nguyen and Park worked on incorporating parallel matrix multiplication routines while accounting for communication costs, though they focused primarily on modern Intel computers [12]. Hemeida and Hassan developed a matrix multiplication optimization for Intel’s multicore processors [6]. However, in both cases, the parallelism is not implicitly derived. Benson and Ballard developed a hybrid version of BFS and DFS algorithms for task-based parallelism to compensate for load imbalance issues [1], and Li and Hu developed an automatic tuning of sparse Matrix-Vector multiplications on multicore clusters equipped with a thread and process-based communication layer [10].

There are other implementations of the HEFT algorithm that address issues with load imbalance in a cloud network the same way ours is modified to address the issue. Of note, throughput is decreased due to creating idle resources from processing parent and child tasks with different input instances in parallel [18]. EHEFT addresses this by reducing the number of communication tasks. The scheduler defines the task distribution across the nodes and a simulation is run [13]. We are already utilizing the node-level cache to reduce the communication tasks and have already implemented a separate scheduler and simulation mentioned in the paper. Conversely, EHEFT-R utilizes remapping resource allocation rules to optimize optimal machines for the ranked tasks [17].
We employ tile sizes that determine the granularity of partitioned matrix operations. Smaller tiles will result in a larger number of suboperations and subsequently higher level of parallelism. However, it will increase internode communication overhead due to the higher number of dependent suboperations. Thus, there is a need to adequately balance the tile size with load balancing and communication. We utilize time simulation with time models when searching for an optimal tile size instead of using actual execution times. In our experiments, we found that a tile size of 5k works best for most benchmarks when working with a matrix size of 10k.

In our experiments, we found that there is a total tile order with hard dependencies among tasks, where resources would be idle while waiting for their dependencies to resolve. In particular, there is an initial phase of allocating and filling data across all the nodes before the rest of the schedule begins. To help reduce this hard dependency, we initialize the tiles when they are allocated to the respective nodes and schedule the data fill only right before the first tasks are executed (by identifying the assigned id of the first task) rather than only at the beginning of the schedule. Additionally, the takecopy task type was originally serialized in waiting for all jobs in the network to complete first before proceeding. As part of the balancing optimization, we modified it such that it only waits for its dependencies in the tree to resolve before starting.

In converting the memory allocation task to calloc (a single task handling both the data allocation and initialization), we make it asynchronous to further reduce the hard schedule dependency. We implement a queue inside the engine to spawn asynchronous tasks and acknowledgements of receiving the tasks is sent to the master node. We found that due to the heap actively developing over time, calculating a static formula for the memory allocation task is not possible in the simulation. In our analysis of the framework we implement an abstract syntax tree (AST) with recursive operation series detection to better aid in examining the flow of tasks in a benchmark.

### 3.4 Profiling

We utilize profiling with regression analysis to predict the execution time of the time model. It is crucial to accurately predict the execution time of each task such that the task allocation will correctly minimize the total execution. The execution times for each task is profiled offline in our CMM framework. The time model is constructed using multivariate polynomial regression analysis through the ordinary least square (OLS) method[8]. The amount of data is proportional to the size of the matrix, as is the number of floating point operations in relation to the size of the operands. We thus make use of polynomial equations in predicting the execution time.

Table 1 show the classification for interpolation equations used in constructing the time predictions. The number of floating point operations to transmit is represented by a multivariate polynomial represented by the matrix size. The constants are obtained by regression analysis, and differ between each computation unit and node relationship.

### Table 1: Interpolation equation based on the size of the operands and operator. The first two columns are the left and right operands respectively. The third column are the corresponding operators. The fourth column is the interpolation equation.

| L. Operand | R. Operand | Operator | Interpolation Eqn. |
|------------|------------|----------|--------------------|
| (n, 1)     | (1, 1)     | `+`, `-`, `*`, `sin`, `cos` | $a_0 + a_1 n + a_2 m + a_3 n m + a_4 m n$ |
| (m, x)     | (n, 1)     | `+`, `-`, `*`, `sin`, `cos` | $a_0 + a_1 m + a_2 n + a_3 n m + a_4 m n + a_5 m n$ |

Figure 2: Conversion from our Julia CMM Markov benchmark code to tiled task dependency graphs. A task dependency graph is a directed acyclic graph (DAG).
In our initial experiments, we found that the framework consistently preferred the master node and a single worker node for scheduling their tasks, thereby completely disregarding all other worker nodes, even when the network is established to have consistent connection speeds with all nodes. We discovered that it was an issue with the communication profiling, which overestimated the communication overhead concerning new worker nodes, instead preferring to stick with already established nodes where node-level caches are present. Communication profiling was thereby modified to more accurately reflect the network capabilities by additionally taking the connection speeds between two nodes into account to better predict the communication time and increasing the number of dedicated communication processes to reduce any stalled communication tasks.

3.5 Node-level Cache
As communication between different nodes in a shared network operates as the bottleneck, we implemented a node-level cache in each node’s main memory. The rationale is that when parallelizing a matrix operation, it is partitioned into suboperations with its own set of dependencies that oftentimes lead to multiple successors. As such, results from a given suboperation may be used multiple times in following suboperations, resulting in redundant data should they all be communicated to other nodes. With the node-level cache, data transferred from another node is instead duplicated in shared main memory on the cache, reducing excess internode communication.

3.6 HEFT Scheduler
We employ a modified HEFT algorithm in our scheduler to serve as an underlying layer of low complexity to help effectively select optimal schedules. The original HEFT algorithm is split into two phases. As covered in Section 3.2, we generated a dependency graph for the tasks and automatically tiled them. In the first phase, the tasks are recursively ranked based on the average communication and computation costs, thereby ordering them based on priority. We make use of the time predictions covered in Section 3.4 as the estimated times of each task. The accuracy and reliability of the generated schedules is proportional to the accuracy of the cost predictions. To take advantage of the node-level cache, the HEFT algorithm utilizes the average internode communication costs and dynamically caches information in memory, which is preserved to estimate communication costs during scheduling. The second phase assigns the tasks to processors such that the earliest finish time is selected.

The framework utilizes a distributed system over a network instead of shared memory. As such, data communication overhead is noticeable where nodes send and receive data from one another. We make use of a master node in the system that propagates the matrix multiplication tasks to worker nodes. There is always a small amount of latency at the beginning of benchmarks between the master node and the worker nodes due to this overhead, where the worker nodes must first wait to receive data from the master. The scheduling, which prioritizes earliest finish time, thus initially prefers the master node over worker nodes due to having no communication requirements. However, as the capacity of the master node is limited, worker nodes are taken into consideration for scheduling tasks. Communication time is taken into account when scheduling tasks, including the time spent sending and receiving data between nodes, and will reject scheduling on a node if the communication time is predicted to take longer than the current best candidate node. On account of the master communicating with all the workers, the master has more dedicated communication processes than the workers to process communications tasks earlier, thereby reducing communication overhead. In doing so, it would reduce load imbalance that arise from stalling caused by communication tasks waiting to be scheduled.

4 EVALUATION

4.1 Benchmarks and Environment
In our evaluation of the effectiveness of the CMM for Julia framework, we primarily utilized a set from a Cell Octave benchmark set [7], which is known to be highly parallelizable and therefore a prime candidate for evaluation. We specifically took the Markov, K-Means, Hill, Leontief, Neural-Network, DFT, Synth, Reachability, and Hits benchmarks from the set and rewrote them in the Julia language. We briefly worked with a separate benchmark based on Grover’s algorithm, but discarded it for having poor parallelizable performance due to having a lack of matrix multiplication.

The environment we used are from Amazon Web Services (AWS)’ EC2 service, with instance type c5.xlarge. This instance type was selected for having a guaranteed 10 Gbps network bandwidth in the shared network, and each instance has 36 advertised vCPUs (18 physical cores). The framework was developed on version 1.0 of the Julia language.

| Num_threads | GFLOPS Avg | GFLOPS |
|-------------|------------|--------|
| 4           | 1.77167631 | 1.771553 |
| 8           | 3.57179533 | 3.57191 |
| 12          | 4.864344   | 4.86441 |
| 14          | 5.548473   | 5.54839 |
| 16          | 6.36412505 | 6.36538 |
| 32          | 6.35824703 | 6.35792 |
| 64          | 6.362186   | 6.35969 |

4.2 Findings
In our experiments, we found that the ideal configuration for most benchmarks in our CMM for Julia uses three worker processes, two communication processes, and four threads per worker process. One worker process will be comprised of the number of threads configured for BLAS, while one communication process will contain only one thread. For example, in the ideal configuration above, there will be a total of 16 threads operating in the node. Table 2 shows the GFLOPS numbers for our CMM for Julia based on the number of threads configured for an AWS c5.xlarge instance. We find that oversubscription of resources in a given node occurs at 16 threads or more. One c5.xlarge instance has 36 vCPUs and 18 physical cores. Thus, adding more worker threads than the number of available resources in a node will cause oversubscription as threads compete to utilize the available resources. This contention outweighs any
potential benefit that may arise with introducing more worker processes.

We found that adding more communication processes tends to oversaturate the network with more communication tasks being sent at one point in time, leading to a degradation in the framework performance. As such, with additional worker processes, the framework will often prefer utilizing the node-level cache in “seeded” nodes over sending matrix suboperations to new nodes due to increased communication overhead. There is a roughly 300 GFLOPS difference between baseline Julia without our CMM optimizations and the CMM framework in favor of the latter.

Table 3: Comparison of execution and simulation times in our Julia CMM for benchmarks of the Cell benchmark set, at a matrix size of 10k. The left-most column depicts the execution times, the “Sim. Accuracy” column depicts the degree of accuracy to which simulation predicts the time, “Speedup” depicts the relative speedup from 1–8 nodes, and “Sim. Time” depicts the execution time predicted by the simulation for the corresponding benchmark. Each benchmark was run with 1k, 3k, and 5k tile sizes, as well as an additional 7k tile size for simulation only.

| Benchmark | Node | Exec. Time (s) | Sim. Accuracy | Speedup (×) | Sim. Time (s) |
|-----------|------|----------------|---------------|-------------|---------------|
| Kmeans    | 1    | 12.18 10.71 7.77 119% 122% 126% 100% 1.00 1.00 1.00 10.27 8.81 6.15 21.76 | 2.76 6.29 5.98 130% 128% 125% 1.59 1.70 1.30 5.91 4.91 4.80 13.69 | 4.12 5.47 4.86 142% 142% 142% 1.99 1.96 1.60 4.31 3.86 3.17 12.35 | 5.59 4.24 3.39 123% 143% 136% 2.18 2.53 2.29 4.54 2.97 2.50 12.14 | 4.62 3.73 2.86 133% 140% 121% 2.64 2.87 2.92 3.48 2.67 2.20 12.23 |
| Hits      | 1    | 13.83 11.45 8.23 121% 106% 105% 100% 1.00 1.00 1.00 12.53 10.77 7.64 20.43 | 8.29 7.02 6.33 108% 123% 109% 1.47 1.63 1.30 7.68 5.71 5.78 13.17 | 7.44 6.10 5.95 117% 125% 117% 1.86 1.88 1.38 6.34 4.87 5.08 11.58 | 6.10 5.12 4.75 118% 131% 123% 2.27 2.24 1.73 5.17 4.51 3.86 9.68 | 5.48 4.52 3.82 132% 133% 132% 2.52 2.53 2.48 4.10 3.41 2.51 9.43 |
| Markov    | 1    | 12.90 10.78 7.52 113% 110% 115% 100% 1.00 1.00 1.00 11.44 9.78 6.89 18.86 | 7.63 6.24 5.56 117% 116% 114% 1.69 1.73 1.35 6.50 5.38 3.79 12.80 | 6.93 5.97 4.23 140% 140% 122% 1.86 1.81 1.78 4.95 4.27 3.48 10.42 | 5.82 4.96 3.63 139% 139% 130% 2.22 2.17 2.07 4.18 3.65 2.46 10.39 | 5.48 4.52 3.82 132% 133% 132% 2.52 2.53 2.48 4.10 3.41 2.51 9.43 |

5 DISCUSSION

Our CMM for Julia framework allows us to automatically parallelize large matrix operations with minimal intervention on the user side. We find that the performance of this parallelization improves with the addition of more nodes to the network, particularly when going from one node up to six nodes. However, as Table 3 shows, we find that the speedup degrades at eight nodes; the performance is worse than if six nodes were in the network. We found that this performance degradation results from the framework not taking advantage of all the nodes in the network. It discriminates against certain nodes in favor of utilizing the node-level cache to greater effect, effectively limiting all communication to additional nodes. It finds that the cost with additional internode communication is greater than if the node-level cache were used instead.

Table 3 demonstrates that the accuracy of the simulation is comparable with the execution of the benchmark, with predicted values usually comprising a 10–20 % difference from the actual values. The simulation thereby is at a point where its prediction of the time model of the framework is a useful tool in accurately scheduling the tasks such that the earliest finish time is achieved, particularly with the average simulation time being less than 0.2 s acting as a marginal overhead. There is an advantage in the point at which the task dependencies, causing the node-level cache to be slightly underestimated in the simulation, thereby leading to a slight deviation in the simulated times from execution down.

In all benchmarks, there is a common trend of the time performance with increasing tile sizes. This reflects the understanding that with too small tile sizes, the communication overhead would be large enough to slow the benchmark performance down. However, the performance degrades from 5k to 7k size tiles for all benchmarks, reflecting the opposite spectrum where not enough parallelism takes place in the framework.

Figure 3 demonstrates the results of scheduling tasks for the Markov benchmark with an increasing number of nodes in the network. The tasks presented in the schedules are according to the classification in Section 3.1. Figure 3a depicts a network with just simulations is generally reliable, having a tendency to be within 5 % to 20 % of the actual execution, though typically is always slightly worse than the actual execution. This is due to the simulated communication being a reflection of profiled communication and the ordinary least square regression time model, and not the actual communication that takes place during active network usage. When running the simulations, it takes an average of 0.1 s to complete for each benchmark in all size parameters. The overhead in running a simulation is therefore marginal, and allows for repeat simulations with minimal cost.

The results in Table 3 for running on one node are from using the CMM framework. The results of running a benchmark with one node in the CMM framework compared to native Julia (no CMM framework) are similar. Using the Markov benchmark as an example with a matrix size of 10k and a tile size of 5k on a c.5xlarge AWS instance, the execution time on the CMM framework was 7.77 s. By comparison, the execution of the same benchmark without the CMM framework was 7.46 s. Due to being similar in value, we use the CMM framework when running on one node for consistency.

Table 3 shows the execution and simulated predicted times for all benchmarks in the Cell benchmark set. The accuracy of the
Figure 3: Task schedules for a Markov benchmark of matrix size 10k and tile size 3k with AWS c5.9xlarge instances.
the master and one worker node. The depicted schedule contains a total of 421 CMM tasks. Communication is sparse with the low number of nodes, being present only at the beginning (for sending data to the worker node) and at the end (for receiving data from the worker node). Therefore, only two communication processes in the master is sufficient to ensure good load balance. When more nodes are present in the network, as depicted in Figure 3b, two communication processes in the master is insufficient to ensure a balanced workload across the network, as processes would already be busy when waiting to schedule new communication requests. There is also a higher number of tasks to schedule, as there are 579 CMM tasks in Figure 3b and 644 CMM tasks in Figure 3c. Additional communication processes in the master thereby improves the load balance by allowing communication requests to worker nodes to be finished earlier. However, it is insufficient to add an arbitrary number of communicators, as too many will cause network contention by saturating the network with excess requests.

A feature common with all schedules is that the first workloads on worker nodes are always scheduled some time after the first workloads in the master node. This is a result of the worker nodes waiting to receive data from the master node, whereas no communication is needed to schedule tasks on the master node. It also

| Matrix Size | 1k | 3k | 5k |
|-------------|----|----|----|
| Relative Speedup | 1.0 | 1.5 | 2.0 |
| vCPUs (AWS Instances) | 036 (1) | 072 (2) | 144 (4) | 216 (6) | 288 (8) |

Figure 4: Diagrams depicting the relative speedups of eight benchmarks run on c5.9xlarge AWS instances with our CMM framework for Julia. The speedups span from one node (36 vCPUs) to eight nodes (288 total vCPUs). The matrix sizes denote tile sizes used in the matrix multiplication, ranging from tile sizes of 1k to 5k. All benchmarks used a matrix size of 10k.
reflects on the small gaps of stalling present between some tasks in the worker nodes, made more prominent when more nodes are present in the network, as they are waiting for communication to resolve to receive their next data.

5.1 Speedup

Table 4: The speedups are based on a configuration using eight nodes, a matrix size of 10k, and a tile size of 5k. Observed speedups are calculated by comparing the time obtained with eight nodes to that with one node, as seen in Table 3. Theoretical speedups are calculated with the assumption that there is no communication time.

| Benchmark | Observed Speedup | Theoretical Speedup |
|-----------|------------------|----------------------|
| Markov    | 2.92             | 4.02                 |
| Kmeans    | 2.48             | 3.88                 |
| Hll       | 3.02             | 4.19                 |
| Leastf    | 2.14             | 3.99                 |
| DFT       | 1.79             | 3.83                 |
| Synth     | 3.49             | 4.23                 |
| Reachability | 1.74 | 3.96 |
| Hits      | 1.91             | 3.95                 |

Figure 4 showcases the relative speedups of eight different benchmarks from one node (36 vCPUs) to eight nodes (288 vCPUs). Most of the relative speedups exceed 2×, with some reaching 3×. As shown in Table 3, the Markov benchmark reaches a speedup of 2.92× with a tilesize of 5k, and the Synth benchmark reaches 3.49× relative speedup with a tilesize of 5k.

We calculate the maximum theoretical speedup for each benchmark by setting the communication speed to zero. All the tasks are already parallelizable and asynchronous, so communication overhead acts as the bottleneck to achieving full parallelism in a distributed system. In the simulation, we set the condition that there is zero latency between nodes in the network, meaning that communication is instantaneous. The other parameters of the benchmark remain the same: eight nodes with a matrix size of 10k and tile size of 5k. As the values in Table 4 show, most benchmarks do not significantly deviate from the theoretical speedups, being in the range of 55–80% from the maximum speedup. It is not feasible to achieve the theoretical speedup in reality at this point in time due to necessitating instantaneous communication, so some deviation as depicted is expected.

6 CONCLUSIONS AND FUTURE WORK

Matrix computations are widely used in the fields of scientific computing and engineering. But with current matrix language implementations it is a challenging task to fully utilize Cloud compute capacities. To overcome this problem, we proposed the CMM framework which extends the Julia high-performance compute language to automatically parallelize matrix computations for the cloud. CMM automatically converts matrices into a tiled task dependency graph with minimal intervention from the user. The framework employs offline profiling to generate accurate time prediction models using polynomial regression. Both the tiled dependencies and time predictions are utilized by the modified HEFT scheduler to allocate tasks to nodes such that overall execution time is minimized.

Modifications are made to reduce the effect of communication overhead on the scheduling by introducing additional communication processes, communication speeds, and the node-level cache. We conducted an extensive experimental evaluation on a set of eight benchmarks using up to eight nodes (288 vCPUs) in the AWS public cloud. Our framework achieved speedups of up to a factor of 3.49× within 20.5% of the theoretically possible maximum speedup.

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