Stability of Magnetic Bearings on Lorentz Forces

F R Ismagilov, V Ye Vavilov, and V V Ayguzina

Ufa State Aviation Technical University, 12, K. Markxa St. Ufa, 450008, Russia
e-mail: vtipy@mail.ru

Abstract. In this paper, the rotor stability of the ultra-high-speed electrical rotating machines on the magnetic bearings on Lorentz forces was proved. For this system, the Lyapunov function was obtained. Its study allowed determining the sustainable movement of the rotor on the magnetic bearings on Lorentz forces and making recommendations of the controller coefficient selection. The stability analysis was carried out both for magnetic bearings with a rectangular and cylindrical air gaps. The obtained results were verified on an experimental unit and compared with the results of other authors, which shows the convergence of theoretical and experimental data in both cases.

1. Introduction
The development of high-precision laser systems, high-speed photography, medical equipment and robotics requires an increase in the speed of Electrical Rotating machines (ERMs) with a simultaneous trend towards their miniaturization. For some industries, the slotless 100–500 W ERMs with a rotational speed from 500,000 rpm to 1,000,000 rpm and a rotor diameter of 5-10 mm are perspective. World corporations are actively working to create and improve the efficiency of ERM. In particular, the IHI Corporation (Japan) develops a slotless 400 W microgenerator with a rotational speed of 400,000 rpm for robotics. The Celeroton Corporation (Switzerland) serially develops slotless 100 W electric motors with a rotational speed of 500,000 rpm. These motors are designed for high-precision optical systems, machine tools and medical equipment. Similar works are being conducted in the USA, Belgium and South Korea in the interests of space agencies and the army of these countries [1–3]. As a long-term goal of all these companies are aiming to exceed 1 000 000 rpm, as this will open up entirely new perspectives in different industries.
One of the main problems of Ultra-High Speed Electrical Machine (UHSEM) design is a problem of bearing unit reliability. Miniature, instrumental mechanical bearings can reliably operate at the rotational speed below 400 000 – 500 000 rpm.
Various types of non-contact bearings, such as active magnetic bearings and gas-dynamic bearings, also do not provide reliable operation of such ERMs.
At high rotational speeds, eddy currents induces in a rotor on the active magnetic bearings, which are the reason of additional rotor heating. Moreover, a rotor cooling is extremely difficult at high rotational speeds. Therefore, the active magnetic bearings are not used at frequencies above 250,000 rpm.
Gas-dynamic bearings ensure a non-contact rotation only at a certain frequency (frequency of ascent). Before reaching a given frequency, they work as a mechanical bearing, which significantly reduces their reliability. In addition, gas-dynamic bearings are not able to operate in a vacuum, which limits their use.
The solution of the bearing reliability problems of ultra-high-speed ERMs can be non-contact bearings on the Lorentz forces [4].
The advantages of these bearings are the lack of the stator core, which significantly improves their performance by reducing the electromagnetic time constant. In addition, it is minimum weight and size parameters, a possibility of using at rotor rotational speeds above 500,000 rpm. The main disadvantage of magnetic bearings on the Lorentz forces are the low bearing capacity. However, since the rotor of ultra-high-speed ERM weighs 5–50 grams, this drawback does not prevent the use of magnetic bearings on Lorentz forces (MBLF) in ultra-high-speed ERMs.

2. Statement of research tasks
With all MBLF advantages for ultra-high-speed ERMs, it began to develop relatively recently [4–7]. The research works of this bearing type are aimed to determination of energetic characteristics of MBLF. In particular, it is studied of their forces [4, 5], new design [6] or the calculation methods [7]. At the same time, in addition to energy characteristics, the control system parameters and the rotor stability of the ultra-high-speed ERM have a significant influence on the efficiency of the MBLF operation.

The rotor stability analysis and the control system parameter selection is presented in [8, 9] for active magnetic bearings, in [10] for hybrid magnetic bearings and in [11] for diamagnetic bearings. However, a significant MBLF differences from another bearing types and their higher performance do not allow a full use of the [8–11] work results to assess the MBLF rotor stability. Moreover, these works are performed for the system with PD controller. And as it is shown in [12], PID controllers are more efficient to use in MBLF.

Thus, the objective of this work is the MBLF rotor stability analysis, the study of the processes in the MBLF control system and the experimental verification of the theoretical results.

3. The research object
The research object if this work is the ultra-high-speed ERM with MBLF (Fig. 1). The structural control scheme of the research object is represented in Fig. 2.

![Figure 1](image1.png)

**Figure 1.** The ultra-high-speed ERM with MBLF: 1 – stator core; 2 – ERM windings; 3 – windings of the left MBLF L; 4 – rotor with permanent magnets; 5 – shaft; 6 – windings of the right MBLF.

![Figure 2](image2.png)

**Figure 2.** The structural control scheme of the research object: 1 – the rotor position sensor; 2 – the rotor displacement sensor along the x axis; 3 – the rotor displacement sensor along the y axis.
The description of the MBLF operation given on the example of the ERM in the motor mode. The processes occurring in ERM in the generator mode are similar to the processes in the motor mode. The MBLF windings NSL are performed as ordinary polyphase ERM windings. Herewith, the following condition is fulfilled: if the ERM rotor has \( p \) pole pairs, the MBLF winding will be carried out for the poles number of \( p = p \pm 1 \).

In Fig. 1, the ERM rotor has a 1 poles pair, the ERM windings consist of two coils. The MBLF windings consist of 4 coils, i.e. it is made for two poles pairs.

The MBLF contains two windings (right and left), which are installed in the air gap above the rotor with permanent magnets, the rotor position sensors and the control system. To provide a mechanical moment, a current is fed to the motor windings and switched by an ERM inverter. The switching law is determined by the rotor position sensor, the current value is switched in the "current controller". Also current is fed to the MBLF windings and switched by an MBLF inverter. The frequency of switching current is the same with the frequency in the ERM windings.

In each moment of time, between a certain magnet pole and current flowing through the windings, the Lorentz force is arises, which ensures the rotor levitation. And since the MBLF windings are made with the \( p = p \pm 1 \) poles number, no mechanical moment is created for the ERM. The rotor displacement, as well as in conventional bearings, is estimated by rotor displacement sensors. The signal is fed to the MBLF control system. Based on this signal, a current value is formed and applied to the MBLF.

For generality of this work results, it is considered a two-phase rotor suspension with a plane-parallel air gap (Fig. 3a) and with a cylindrical air gap (Fig. 3b).

![Figure 3. Two-phase rotor suspension of the MBLF with a plane-parallel air gap (a), with a cylindrical air gap (b).](image)

The following assumptions are used to solve the research problems.

– Permanent magnets of the same brand, the same thickness and length are applied. The magnetic flux density on the permanent magnet surface does not change under the external factor influence.

– The environment and winding temperatures are constant throughout the entire working process. Therefore, the MBLF winding resistance is assumed as constant.

– The change in inductance from the current is negligible.
4. Mathematical description of the MBLF

The main characteristic describing the processes in MBLF is the Lorentz force, which act on the rotor with permanent magnets.

For ERM with cylindrical air gap, the resulting Lorentz force can be presented in the matrix form [4]:

$$F_{LF} = \frac{3}{2} l k_w (I + I_0) B_M \left( \frac{D_1}{2} \right)^2 \ln \left( \frac{D_3}{D_2} \right) \begin{bmatrix} \sin((\phi - \phi_M) \cdot \sigma) \\ -\cos((\phi - \phi_M) \cdot \sigma) \\ 0 \end{bmatrix},$$

(1)

where $l$ is the MBLF active length; $k_w$ is the winding factor; $I_0$ is the current flowing in the winding; $I$ is the control current; $B_M$ is the amplitude of the radial component of the magnetic flux density in the air gap; $D_1 - D_3$ are the geometric dimensions, according to Fig. 3; $\phi$ is the phase angle of the MBLF winding; $\phi_M$ is the rotor rotation angle.

For the MBLF with the plane-parallel air gap (Fig. 3a), the winding phase angle and the rotor rotation angle are equal. And the Lorentz force is determined as following:

$$F_{LF} = \frac{3}{2} l k_w (I + I_0) B_M \left( \frac{D_1}{2} \right)^2 \ln \left( \frac{D_3}{D_2} \right),$$

(2)

In [13], it is presented equations to determine the magnetic flux density in the air gap, which are functions of the permanent magnet properties and geometry and allow the accurate numerical calculation of the permanent magnet parameters. Since these equations are cumbersome, the more simplified equations for the magnetic flux density calculations were used in this work:

$$B_M = \frac{1}{1 + \frac{(\delta + x) B_r}{h \mu_0 H_{cB}}} \cdot \frac{B_r}{\sigma},$$

(3)

where $B_r$ is the residual flux density of the permanent magnet; $\delta$ is the air gap of the ERM; $H_{cB}$ is the coercive force of the permanent magnet; $h$ is the height of the permanent magnet; $\mu_0$ is the magnetic permeability of vacuum; $\sigma$ is the leakage coefficient of the rotor magnetic system; $x$ is the rotor displacement.

The appearance of rotor displacement in the x-axis direction leads to the following forces in the MBLF with cylindrical air gap:

$$F_{LF1} = \frac{3}{2} l k_w (I_0 - I) \cdot \frac{B_r}{\sigma} \left( \frac{D_1}{2} \right)^2 \cdot \ln \left( \frac{D_3}{D_2} \right) \begin{bmatrix} \sin((\phi - \phi_M) \cdot \sigma) \\ -\cos((\phi - \phi_M) \cdot \sigma) \\ 0 \end{bmatrix},$$

(4)

$$F_{LF2} = \frac{3}{2} l k_w (I + I_0) \cdot \frac{B_r}{\sigma} \left( \frac{D_1}{2} \right)^2 \cdot \ln \left( \frac{D_3}{D_2} \right) \begin{bmatrix} -\sin((\phi - \phi_M) \cdot \sigma) \\ \cos((\phi - \phi_M) \cdot \sigma) \\ 0 \end{bmatrix},$$

(5)

In system with the plane-parallel gap, it is followings:

$$F_{LF1} = \frac{3}{2} l k_w \left( \frac{I + I_0}{\delta - x} \right) \cdot \frac{B_r}{\sigma} \left( \frac{D_1}{2} \right)^2 \cdot \ln \left( \frac{D_3}{D_2} \right),$$

(6)
\[ F_{LF2} = \frac{3}{2} \cdot \frac{l k_w(I + I_0)}{1 + \frac{(\delta + x)B_r}{h \mu_0 H_{c,\delta} \sigma}} \cdot B_r \cdot \left( \frac{D_1}{2} \right)^2 \cdot \ln \left( \frac{D_1}{D_2} \right), \]  

(7)

The obtained equations (2.4) – (2.7) show that the forces in the MBLF system linearly dependent on the current unlike the active or hybrid magnetic bearings due to active magnetic bearing and hybrid magnetic bearings have a quadratic dependence. A linear dependence greatly simplifies the control processes.

Next, for simplification of the calculations, a system with plane-parallel gap was considered. With above observations, the resulting solution can be used for systems with cylindrical gap.

To analyze the stability, it is necessary to take into account the MBLF winding inertia:

\[ U_{inc} = R_0 I + \frac{d}{dt}(L_0 I + \varphi_{pm}), \]

(8)

where \( U_{inc} \) is the voltage increment in the MBLF windings; \( R_0 \) is the resistance of the MBLF windings; \( L_0 \) is the inductance in the MBLF windings; \( \varphi_{pm} \) is the flux linkage of the permanent magnets.

In addition, it is necessary to consider the inertia of the measurement sensors [9]:

\[ \gamma \frac{dx_{rp}}{dt} = x - x_{rp}, \]

(9)

where \( \gamma \) is the time constant; \( x_{rp} \) is the rotor position coordinate.

5. The mathematical description of the rotor position control system in the MBLF

As a logical control device for MBLF, the PID controller is used. The regulation current is determined depending on the controller coefficients and the measured error \( e \) of the rotor position:

\[ I = -(K_1 e + K_2 \frac{de}{dt} + K_3 \int_0^t edt), \]

(10)

where \( e = q - r \) is the difference between the actual and the desired values of the generalized coordinates; \( K_1, K_2, K_3 \) are diagonal matrices of respective amplification factors:

\[
K_1 = \text{diag}(K_{11}, K_{12}, K_{12}), \\
K_2 = \text{diag}(K_{21}, K_{21}, K_{22}, K_{22}), \\
K_3 = \text{diag}(K_{31}, K_{31}, K_{32}, K_{32}).
\]

The estimated amplification values are determined by equations:

\[
K_{11} = K_{12} = \frac{-2k_{s1,2}}{k_{11,2}}, \\
K_{21} = K_{22} = \frac{-Mk_{s1,2}}{2k_{11,2}}, \\
K_{31} = K_{32} = \frac{Mp}{k_{11,2}},
\]

where \( M \) is the rotor mass, \( p \) is a positive number.
The control purpose is to stabilize of the rotor with permanent magnets or the magnet at the equilibrium position, then \( r = 0 \). Measurement of the magnet position relative to the air gap is carried out using motion sensors such as linear Hall sensors, inductive sensors or eddy-current sensors. Therefore, the control law takes the form:

\[
I = K_1 q_s + K_2 \frac{dq_s}{dt} + K_3 \int_{t_0}^{t} q_s dt,
\]

(11)

where \( q_s = (x_{rp}, x_{wp})^T \) is the matrix of magnet displacement in the sensor measurement area.

A correlation between the generalized coordinates of the levitating magnet mass centre or of the rotor and the matrix \( q_s \) is determined by the ratio \( q_s = T_q q \), where \( T_q \) is the transformation matrix, which is taken in the form:

\[
T_q = \begin{bmatrix}
1 & 0 & 0 & -a_{s1} \\
0 & 1 & a_{s1} & 0 \\
1 & 0 & 0 & a_{s2} \\
0 & 1 & -a_{s2} & 0
\end{bmatrix}.
\]

(12)

6. Stability analysis of the ERM rotor with MBLF

To analyse the stability of the rotor on the MBLF, it is necessary to formulate the equation of the perturbed motion of the rotor on the MBLF, which, taking into account the time constant \( \gamma \) of the position sensors along the x axis, is:

\[
m \left( \frac{d^3 x_{rp}}{dt^3} + \frac{d^2 x_{rp}}{dt^2} \right) = -\frac{3}{2} I_k w (I + I_0) \frac{k_2}{1 + (\delta - x)k_1} \left( \frac{D_L}{2} \right)^2 \ln \left( \frac{D_3}{D_2} \right) + \\
+ \frac{3}{2} I_k w (I - I_0) \frac{k_2}{1 + (\delta + x)k_1} \left( \frac{D_L}{2} \right)^2 \ln \left( \frac{D_3}{D_2} \right),
\]

(13)

where \( k_1 = \frac{(\delta - x)B_L}{h \mu_0 H_{ci} \sigma} \); \( k_2 = \frac{B_L}{\sigma} \); \( m \) is the rotor mass.

To account the inertia of the rotor position and winding sensors, expressions (2.7) and (2.8) are introduced in (4.1):

\[
m \left( \frac{d^3 x_{rp}}{dt^3} + \frac{d^2 x_{rp}}{dt^2} \right) = -\frac{3}{2} I_k w \left( \frac{D_L}{2} \right)^2 \ln \left( \frac{D_3}{D_2} \right) k_2 \times \\
\left[ \frac{U_{inc} - dI_0}{R_0} \frac{dt}{dt} - \frac{d \phi_{pm}}{dt} \right] \left[ I_0 - \frac{U_{inc} + dI_0}{R_0} \frac{dt}{dt} + \frac{d \phi_{pm}}{dt} \right] \left( \frac{U_{inc} - dI_0}{R_0} \frac{dt}{dt} - \frac{d \phi_{pm}}{dt} \right) \left( I_0 - \frac{U_{inc} + dI_0}{R_0} \frac{dt}{dt} + \frac{d \phi_{pm}}{dt} \right).
\]

(14)

Both parts of expression (4.2) are integrated:
The right part is represented as a time derivative:
\[
\frac{d}{dt} \left[ \gamma K_2 \left( \frac{dx_{lp}}{dt} \right)^2 + K_1 x_{lp} + \gamma K_1 \left( x_{lp} \frac{dx_{lp}}{dt} \right) + m\gamma K_3 \frac{dx_{lp}}{dt} \int x_{lp} dt \right] = \\
\left[ \int \left( \frac{U_{inc}}{R_0} + \frac{dH_0}{dt} + \frac{d\phi_{pm}}{dt} \right) - \int \left( \frac{U_{inc}}{R_0} - \frac{dH_0}{dt} - \frac{d\phi_{pm}}{dt} \right) \\
1 + \left( \delta + \frac{\gamma}{\delta} \frac{dx_{lp}}{dt} + x_{lp} \right) \right] \times Q_1 \cdot \left( K_1 x_{lp} + K_2 \frac{dx_{lp}}{dt} + K_3 \int x_{lp} dt \right) + m\gamma K_3 \left( \frac{dx_{lp}}{dt} \right)^2 - m K_2 \frac{dx_{lp}}{dt} - m K_3 \frac{dx_{lp}}{dt} \int x_{lp} dt.
\]

It is obvious that the right side is Lyapunov's function, and the left side is its derivative. The left part is a sign-definite negative function provided that \( \gamma K_1 > K_2 + K_3 \). Herewith, the right side is a sign-definite positive function, which is the evidence of the asymptotic system stability according to the Lyapunov theorem. Then, the general rule for the asymptotic rotor stability of the ultra-high-speed ERM is the regulator setting rule: \( \gamma K_1 < K_2 + K_3 \).

7. Theoretical result verification

To verify the obtained results, an experimental unit with MBFL was developed. It is presented in Fig. 4.

Figure 4. Experimental unit with MBFL: 1 – coil; 2 – control system with PID controller; 3 – Hall sensor

Two Hall sensors with a time constant of 3 ms were used as a rotor displacement sensor. A permanent magnet NdFeB weighing 15 grams was used for suspension. The current control through the coil was carried out by the pulse-width modulation. The PID regulator coefficients were chosen according to the received inequality and accounted for \( K_1 = 1, K_2 = 5, K_3 = 1 \). These coefficients allow to achieve a steady permanent magnet suspension during 5-7 seconds. After this time, an error was accumulated in the integrating link and the system emerged from the stability state. To solve this problem, it is necessary to use the cleaning of the integrating link.

When the coefficients were changed by \( K_1 = 1000, K_2 = 0.5, K_3 = 0.1 \), the magnet levitation was absent, the magnet adhered to the coil, i.e. the system immediately lost its stability.

It should be noticed that similar experiments were carried out in [11], where the stable rotor suspension in the MBFL was obtained with the coefficients \( K_1 = 4000, K_2 = 15, K_3 = 200 \) and the sensor time constant of 0.3 ms. These coefficients also satisfy the obtained inequality \( 0.00003-4000 < 215 \).

Thus, the experimental results of [12] confirm theoretical and experimental results if this research.
8. Conclusion
In this paper, the rotor stability of the ultra-high-speed ERM on MBFL was proved. For this system, the Lyapunov function was obtained. Its study allowed determining the sustainable movement of the rotor on the MBFL and making recommendations of the controller coefficient selection. The stability analysis was carried out both for magnetic bearings with a rectangular and cylindrical air gaps. The obtained results were verified on an experimental unit and compared with the results of other authors, which shows the convergence of theoretical and experimental data in both cases.

Acknowledgments
This work was supported by the Russian Foundation for Basic Research (project 16-38-60001).

References
[1] Zwyssig C, Kolar J W and Round S D 2009 Mega-Speed Drive Systems: Pushing Beyond 1 Million RPM IEEE/ASME Transactions on Mechatronics 14 (5) pp 564–574
[2] Zwyssig C, Kolar J W and Round S D 2006 Power Electronics Interface for a 100 W, 500000 rpm Gas Turbine Portable Power Unit USA Applied Power Electronics Conference pp 283–289
[3] Isomura K, Murayama M and Teramoto S 2006 Experimental Verification of the Feasibility of a 100W Class Micro-scale Gas Turbine at an Impeller Diameter of 10 mm J. Micromech. Microeng 16 pp 254–261
[4] Looser A, Baumgartner T, Kolar J W and Zwyssig C 2010 Novel highspeed, Lorentz-type, slotless self-bearing motor IEEE Energy Conversion Congress and Exposition (ECCE 2010) pp 3971–3977
[5] Looser A, Baumgartner T, Kolar J W and Zwyssig C 2012 Analysis and Measurement of Three-Dimensional Torque and Forces for Slotless Permanent-Magnet Motors IEEE Transactions on Industry Applications 48 (4) pp 1258–1266
[6] Looser A, Baumgartner T, Kolar J W and Zwyssig C 2014 Analysis and Design of a 300-W 500 000-r/min Slotless Self-Bearing Permanent-Magnet Motor IEEE Trans. Industrial Electronics 61 pp 4326–4336
[7] Baumgartner T and Kolar J W 2013 Multivariable state feedback control of a 500 000 rpm self-bearing motor Electric Machines & Drives Conference (IEMDC), 2013 IEEE International pp 347–353
[8] Balandin D V and Kogan M M 2011 Motion control for a vertical rigid rotor rotating in electromagnetic bearings (in Russian) Izvestiya Rossiyskoy Akademii Nauk. Teoriya i sistemy upravleniya 5 pp 3–17
[9] Vostokov V S, Lebedeva S V, 2011 Stability of electromagnetic suspension of a rotor (in Russian) Izvestiya Rossiyskoy Akademii Nauk. Teoriya i sistemy upravleniya 2 pp 3–37
[10] Ismagilov F, Gerasin A, Khayrullin I and Vavilov V 2014 Stability analysis of hybrid magnetic bearings Journal of Computer and Systems Sciences International 53 (1) pp 130–136
[11] Jie-Yu Chen and Jian-Bin Zhou 2008 Diamagnetic bearings for MEMS: Performance and stability analysis Mechanics Research Communications 35 (8) pp 546–552
[12] Ueno S and Kato T 2009 A novel design of a Lorentz-force-type small self-bearing motor Taipei International Conference on Power Electronics and Drive Systems (PEDS) pp 926–931
[13] Ismagilov F, Khayrullin I, Vavilov V and Yakupov A 2016 Mathematical Model of High-frequency Electromechanical Energy Transducer with High-coercive Permanent Magnets IAENG International Journal of Applied Mathematics 46 (3) pp 1–9