THREE FLAVOR NEUTRINO OSCILLATION CONSTRAINTS FROM ACCELERATOR, REACTOR, ATMOSPHERIC AND SOLAR NEUTRINO EXPERIMENTS

Osamu Yasuda\textsuperscript{1} and Hisakazu Minakata\textsuperscript{2}

Department of Physics, Tokyo Metropolitan University
1-1 Minami-Osawa Hachioji, Tokyo 192-03, Japan

Abstract

We discuss constraints on three flavor neutrino mixings from the accelerator and reactor experiments, the Kamiokande multi-GeV data, and the solar neutrino observations. The LSND result is excluded at 90\%CL by the constraints imposed by all the data of reactor and accelerator experiments and the Kamiokande multi-GeV data if the mass scale required for the solution to the solar neutrino problem is hierarchically small. The region of a set of the effective two-flavor mixing parameters ($\Delta m^2$, $\sin^2 2\theta$) is given for the

\textsuperscript{1}Email: yasuda@phys.metro-u.ac.jp
\textsuperscript{2}Email: minakata@phys.metro-u.ac.jp
channel $\nu_\mu \to \nu_e$ which is allowed at 90%CL by the multi-GeV Kamiokande data alone.
Recently the LSND collaboration has claimed that they have found candidate events for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation [1] (See also [2]). If their result turns out to be correct, it gives us an important information for masses and mixing angles of neutrinos. In this paper a possibility is explored that all the experimental data, solar, atmospheric, reactor and accelerator data including LSND can be explained within a framework of three flavor neutrino mixing. It turns out that the LSND result is excluded at 90%CL by the constraint imposed by all the data of reactor and accelerator experiments and the Kamiokande multi-GeV data. The statement is true even without invoking actual solar neutrino data as far as the mass scale required for the solution is hierarchically small.

Among the atmospheric neutrino data [3] [4] [5] [6] [7] we discuss only the Kamiokande multi-GeV data [4] throughout this paper. The reason for this restriction is three-fold; (1) It is the only experiment that gives us a nontrivial zenith-angle dependence which leads to the upper and lower bounds for the mass-squared difference of neutrinos. (2) The result of Monte-Carlo simulation for neutrino energy spectrum is published only for the the Kamiokande multi-GeV data. (3) It seems difficult to reconcile it with NU-SEX [6] and Frejus [7] data.

The Dirac equation for three flavors of neutrinos in vacuum is given by

$$i \frac{d}{dx} \Psi(x) = U \text{diag}(E_1, E_2, E_3) U^{-1} \Psi(x), \quad (1)$$

which is easily solved:

$$\Psi_\alpha(x) = \sum_{j=1}^{3} U_{\alpha j}^* e^{iE_j x} \Psi_\beta(x), \quad (2)$$

where $E_j \equiv \sqrt{p^2 + m_j^2}$ ($j = 1, 2, 3$) is the energy of neutrinos in the mass basis, $\Psi_\alpha(x) = (\nu_e(x), \nu_\mu(x), \nu_\tau(x))^T$ ($\alpha = e, \mu, \tau$) is the wave function of
neutrinos in the flavor basis, and

$$U \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$\equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$ (3)

with $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ is the orthogonal mixing matrix of neutrinos.

We will not discuss the CP violating phase of the mixing matrix here for simplicity. The probability of $\nu_\alpha \to \nu_\beta$ transition is given by

$$P(\nu_\alpha \to \nu_\beta; E, L) \equiv \begin{cases} 1 - 4 \sum_{i<j} \sin^2 \left( \frac{\Delta E_{ij}L}{2} \right) U_{ai}^2 U_{aj}^2 & \text{for } \alpha = \beta \\ -4 \sum_{i<j} \sin^2 \left( \frac{\Delta E_{ij}L}{2} \right) U_{ai}U_{aj}U_{\beta i}U_{\beta j} & \text{for } \alpha \neq \beta \end{cases}$$ (4)

where $\Delta E_{ij} \equiv E_i - E_j \simeq (m_i^2 - m_j^2)/2E \equiv \Delta m_{ij}^2/2E$ is the difference of the energy of two mass eigenstates.

The number of neutrinos $\nu_\beta$ $(\beta = e, \mu, \tau)$ is measured in terms of charged leptons $\ell_\beta$ which come out from a scattering $\nu_\beta X \to \ell_\beta X'$. In appearance experiments of $\nu_\alpha \to \nu_\beta$ the expected number of the charged leptons $\ell_\beta$ is given by

$$N(\nu_\alpha \to \ell_\beta; L) = n_T \int_0^\infty dE \int_0^{q_{\text{max}}} dq \ \epsilon(q) F_\alpha(E) \frac{d\sigma_\beta(E, q)}{dq} P(\nu_\alpha \to \nu_\beta; E, L),$$ (5)

whereas in disappearance experiments we measure attenuation of beam

$$N_{\alpha\alpha}(L) - N(\nu_\alpha \to \ell_\alpha; L) = n_T \int_0^\infty dE \int_0^{q_{\text{max}}} dq \ \epsilon(q) F_\alpha(E) \frac{d\sigma_\alpha(E, q)}{dq} (1 - P(\nu_\alpha \to \nu_\alpha; E, L)) \ \cdot$$ (6)
where

\[
N_{\alpha\beta}(L) \equiv n_T \int_0^\infty dE \int_0^{q_{\text{max}}} dq \; \epsilon(q) F_\alpha(E) \frac{d\sigma_\beta(E, q)}{dq}.
\]

(7)

\(F_\alpha(E)\) is the flux of neutrino \(\nu_\alpha\) with energy \(E\), \(n_T\) is the number of target nucleons, \(\epsilon(q)\) is the detection efficiency function for charged leptons \(\ell_\beta\) of energy \(q\), \(d\sigma_\beta(E, q)/dq\) is the differential cross section of the interaction \(\nu_\beta X \rightarrow \ell_\beta X'\), \(P(\nu_\alpha \rightarrow \nu_\beta; E) \equiv |\langle \nu_\alpha(0) | \nu_\beta(L) \rangle|^2\) is the probability of \(\nu_\alpha \rightarrow \nu_\beta\) transitions with energy \(E\) after traveling a distance \(L\). The results of experiments are usually expressed in terms of a set of the oscillation parameters \((\Delta m^2, \sin^2 2\theta)\) in the two-flavor analysis. The probability in the two-flavor mixing is given by

\[
P(\nu_\alpha \rightarrow \nu_\beta; E, L) \equiv \begin{cases} 
1 - \sin^2 2\theta_\alpha \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) & \text{for } \alpha = \beta \\
\sin^2 2\theta_{\alpha\beta} \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) & \text{for } \alpha \neq \beta.
\end{cases}
\]

(8)

Introducing the notation

\[
\left\langle \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \right\rangle_{\alpha \rightarrow \beta} \equiv \frac{1}{N_{\alpha\beta}(L)} n_T \int_0^\infty dE \int_0^{q_{\text{max}}} dq \; \epsilon(q) F_\alpha(E) \frac{d\sigma_\beta(E, q)}{dq} \sin^2 \left( \frac{\Delta m^2 L}{4E} \right),
\]

(9)

we have the expected number of charged leptons in the two-flavor scenario

\[
1 - \frac{N^{(2)}(\nu_\alpha \rightarrow \ell_\alpha; L)}{N_{\alpha\alpha}(L)} = \sin^2 2\theta_\alpha (\Delta m^2) \left\langle \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \right\rangle_{\alpha \rightarrow \alpha} \quad \text{(disappearance experiments)}
\]

\[
\frac{N^{(2)}(\nu_\alpha \rightarrow \ell_\beta; L)}{N_{\alpha\beta}(L)} = \sin^2 2\theta_{\alpha\beta} (\Delta m^2) \left\langle \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \right\rangle_{\alpha \rightarrow \beta} \quad \text{(appearance experiments)}
\]

(10)
where we have denoted the $\Delta m^2$ dependence of the mixing angle $\theta(\Delta m^2)$ explicitly to indicate that $\theta(\Delta m^2)$ is a function of $\Delta m^2$ in the two-flavor analysis.

The boundary of the excluded region (for negative results) in the $(\Delta m^2, \sin^2 2\theta)$ plot is determined by

$$\epsilon = \frac{N^{(2)}(\nu_\alpha \to \ell_\beta; L)}{N_{\alpha\beta}(L)} = \sin^2 2\theta_{\alpha\beta}(\Delta m^2) \left\langle \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \right\rangle_{\alpha\to\beta}, \quad (11)$$

for appearance experiments where the charged leptons are detected at one point at a distance $L$, or

$$\epsilon = \frac{N^{(2)}(\nu_\alpha \to \ell_\alpha; L_1)}{N_{\alpha\alpha}(L_1)} - \frac{N^{(2)}(\nu_\alpha \to \ell_\alpha; L_2)}{N_{\alpha\alpha}(L_2)} = \sin^2 2\theta_\alpha(\Delta m^2) \left[ \left\langle \sin^2 \left( \frac{\Delta m^2 L_2}{4E} \right) \right\rangle_{\alpha\to\alpha} - \left\langle \sin^2 \left( \frac{\Delta m^2 L_1}{4E} \right) \right\rangle_{\alpha\to\alpha} \right], \quad (12)$$

for disappearance experiments where the charged leptons are detected at two points at distances $L_1$ and $L_2$ ($L_1 < L_2$). In (11) and (12) $\epsilon$ denotes the largest fraction of the appearance events allowed by a given confidence level, i.e., $N^{(2)}(\nu_\alpha \to \ell_\beta; L)/N_{\alpha\beta}(L) < \epsilon$ for appearance experiments, and the largest fraction of beam attenuation, $|N^{(2)}(\nu_\alpha \to \ell_\alpha; L_1)/N_{\alpha\alpha}(L_1) - N^{(2)}(\nu_\alpha \to \ell_\alpha; L_2)/N_{\alpha\alpha}(L_2)| < \epsilon$ for disappearance experiments.

From (11) and (12), we can read off the value of $\left\langle \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \right\rangle$ for arbitrary $\Delta m^2$ from the figure of the two-flavor mixing parameters $(\Delta m^2, \sin^2 2\theta)$ given in each experimental paper as long as $\sin^2 2\theta(\Delta m^2) \leq 1^3$:

$$\left\langle \sin^2 \left( \frac{\Delta m^2 L_2}{4E} \right) \right\rangle_{\alpha\to\alpha} - \left\langle \sin^2 \left( \frac{\Delta m^2 L_1}{4E} \right) \right\rangle_{\alpha\to\alpha} = \frac{\epsilon}{\sin^2 2\theta_\alpha(\Delta m^2)} \quad \text{(disappearance experiments)}$$

3In case of the CP violating phase, which will not be discussed in the present paper, this is not the case. To discuss the CP violating effect, one needs the information of $\left\langle \sin \left( \frac{\Delta m^2 L}{2E} \right) \right\rangle$, which cannot be obtained from the information in published papers only.
\begin{equation}
\left\langle \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \right\rangle_{\alpha \rightarrow \beta} = \frac{\epsilon}{\sin^2 2\theta_{\alpha\beta}(\Delta m^2)}
\end{equation}

(apparatus experiments).  \hspace{1cm} (13)

In (13) and in the following, $\sin^2 2\theta_{\alpha}(\Delta m^2)$ or $\sin^2 2\theta_{\alpha\beta}(\Delta m^2)$ stands for the value of $\sin^2 2\theta(\Delta m^2)$ on the boundary of the allowed region in the $(\Delta m^2, \sin^2 2\theta)$ plot in the two-flavor analysis.

Similarly we can express the number of the expected charged lepton s in the three flavor mixing:

\begin{equation}
1 - \frac{N^{(3)}(\nu_\alpha \rightarrow \ell_\alpha; L)}{N_{\alpha\alpha}(L)} = 4 \sum_{i<j} U_{\alpha i}^2 U_{\alpha j}^2 \left\langle \sin^2 \left( \frac{\Delta m^2_{ij} L}{4E} \right) \right\rangle_{\alpha \rightarrow \alpha}
\end{equation}

for $\nu_\alpha \rightarrow \nu_\alpha$ (disappearance experiments) \hspace{1cm} (14)

\begin{equation}
N^{(3)}(\nu_\alpha \rightarrow \ell_\beta; L) - \frac{N^{(3)}(\nu_\alpha \rightarrow \ell_\alpha; L_1)}{N_{\alpha\alpha}(L)} - \frac{N^{(3)}(\nu_\alpha \rightarrow \ell_\alpha; L_2)}{N_{\alpha\alpha}(L)} = \sum_{i<j} \frac{\epsilon \sin^2 2\theta_{\alpha}(\Delta m^2_{ij})}{\sin^2 2\theta_{\alpha\beta}(\Delta m^2_{ij})} U_{\alpha i}^2 U_{\alpha j}^2
\end{equation}

for $\nu_\alpha \rightarrow \nu_\beta$ (appearance experiments). \hspace{1cm} (15)

From (13) we observe that the quantity $\langle \sin^2 (\Delta m^2 L/4E) \rangle / \epsilon$ is equal to $\sin^2 2\theta(\Delta m^2)$ which can be read off from the published literatures, and we can express the conditions for the three flavor mixing parameters in case of negative results:

\begin{equation}
\epsilon > \frac{N^{(3)}(\nu_\alpha \rightarrow \ell_\alpha; L_1)}{N_{\alpha\alpha}(L_1)} - \frac{N^{(3)}(\nu_\alpha \rightarrow \ell_\alpha; L_2)}{N_{\alpha\alpha}(L_2)} = \sum_{i<j} \frac{\epsilon \sin^2 2\theta_{\alpha}(\Delta m^2_{ij})}{\sin^2 2\theta_{\alpha\beta}(\Delta m^2_{ij})} U_{\alpha i}^2 U_{\alpha j}^2
\end{equation}

(disappearance experiments)

\begin{equation}
\epsilon > \frac{N^{(3)}(\nu_\alpha \rightarrow \ell_\beta; L)}{N_{\alpha\beta}(L)} = -4 \sum_{i<j} \frac{\epsilon \sin^2 2\theta_{\alpha\beta}(\Delta m^2_{ij})}{\sin^2 2\theta_{\alpha\beta}(\Delta m^2_{ij})} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j}
\end{equation}

(apparatus experiments). \hspace{1cm} (15)

Notice that the left-hand side of (13) is defined independent of the number of flavors of neutrinos.

Throughout this paper we assume that a single mass scale is involved in the solution of the solar neutrino problem, which is hierarchically small compared to others. Namely, we assume that

\begin{equation}
\Delta m_{21} \ll \Delta m^2_{32} < \Delta m_{31}^2,
\end{equation}

7
where we have assumed $m_1^2 < m_2^2 < m_3^2$ without loss of generality. In the case where $\Delta m_{32} \ll \Delta m_{21}^2 < \Delta m_{31}^2$, we can show that we obtain the same conclusions, although we will not give the calculation here.

The hierarchy (16) is satisfied in the two-flavor mixing solution to the solar neutrino problem [9] [10] which requires,

\[
(\Delta m^2, \sin^2 2\theta) \approx (\Delta m_{21}^2, \sin^2 2\theta_{12}) \circ \equiv \begin{cases} 
(O(10^{-11}eV^2), O(1)), & \text{vaccum solution} \\
(O(10^{-5}eV^2), O(10^{-2})), & \text{small angle MSW solution} \\
(O(10^{-5}eV^2), O(1)), & \text{large angle MSW solution}.
\end{cases}
\]  

These mass scales are much smaller than those which appear in the atmospheric neutrino observations [4], or in the LSND experiment [1]. In fact the only condition on $(\Delta m_{21}^2, \sin^2 2\theta_{12})$ which is essential to the discussions below is (16), and our conclusions remain unchanged irrespective of the value of $\theta_{12}$ as long as (16) holds.

One can show [12] [13] that under the mass hierarchy (16) the relation

\[
U_{e3}^2 \ll 1,
\]

must hold in order to have solar neutrino deficit under the constraints from the accelerator and the reactor experiments. In this setting it can also be demonstrated that the solar neutrino problem is indeed solved by a two-flavor framework in the MSW and the vacuum solutions [12].

In the present case the formulas in (14) become much simpler [11]:

\[
1 - P(\nu_\alpha \to \nu_\alpha) = 4 \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E} \right) U_{\alpha 1}^2 U_{\alpha 2}^2 + 4 \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) U_{\alpha 3}^2 (1 - U_{\alpha 3}^2)
\]

\[
P(\nu_\alpha \to \nu_\beta) = -4 \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E} \right) U_{\alpha 1} U_{\alpha 2} U_{\beta 1} U_{\beta 2} + 4 \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) U_{\alpha 3} U_{\beta 3}.
\]  

(19)
For the cases discussed below, we have $|\Delta m_{31}^2 L/4E| \ll 1$ and the first term in each formula in (13) can be ignored. Hence we get

$$
4U_{\alpha 3}^2 (1 - U_{\alpha 3}^2) \leq \sin^2 2\theta_\alpha (\Delta m_{31}^2) \quad \text{(disappearance experiments)}
$$
$$
4U_{\alpha 3}^2 U_{\beta 3}^2 \leq \sin^2 2\theta_{\alpha\beta} (\Delta m_{31}^2) \quad \text{(appearance experiments)}
$$
(20)

for negative results, respectively.

The data by the LSND group suggests that the allowed region for the mass-squared difference $\Delta m_{31}^2$ is approximately $\Delta m_{31}^2 \gtrsim 5 \times 10^{-2} \text{eV}^2$. If we combine the data on the same channel $\nu_\mu \rightarrow \nu_e$ (and $\nu_\mu \rightarrow \nu_e$) by the E776 group [14], however, the region $\Delta m_{31}^2 \gtrsim 2.5 \text{eV}^2$ seems to be almost excluded, and we have $5 \times 10^{-2} \text{eV}^2 \lesssim \Delta m_{31}^2 \lesssim 2.5 \text{eV}^2$ at 90% confidence level.

As has been pointed out in Ref. [11], strong constraints on the mixing angle come from the reactor experiment [15]. Using (20) we have the constraint from the reactor experiment [15]

$$
\sin^2 2\theta_{13} = 4U_{e3}^2 (1 - U_{e3}^2) \leq \sin^2 2\theta_{\text{Bugey}} (\Delta m_{31}^2),
$$
(21)

where $\sin^2 2\theta_{\text{Bugey}} (\Delta m_{31}^2)$ stands for the value of $\sin^2 2\theta$ on the boundary of the allowed region in the $(\Delta m^2, \sin^2 2\theta)$ plot in [13]. For the entire region $5 \times 10^{-2} \text{eV}^2 \lesssim \Delta m_{31}^2 \lesssim 2.5 \text{eV}^2$, $\sin^2 2\theta_{\text{Bugey}} (\Delta m_{31}^2)$ is small [15]:

$$
\frac{1}{50} \lesssim \sin^2 2\theta_{\text{Bugey}} (\Delta m_{31}^2) \lesssim \frac{1}{10}.
$$
(22)

From (21) we have either small $U_{e3}^2$ or large $U_{e3}^2$, but the latter possibility is excluded by (18). Therefore we have

$$
s_{13}^2 = U_{e3}^2 \lesssim \frac{1}{4} \sin^2 2\theta_{\text{Bugey}} (\Delta m_{31}^2).
$$
(23)

On the other hand, we have another constraint from the disappearance experiment of $\nu_\mu$ [16]

$$
4s_{23}^2 c_{13}^2 (1 - s_{23}^2 c_{13}^2) = 4U_{\mu 3}^2 (1 - U_{\mu 3}^2) \leq \sin^2 2\theta_{\text{CDHSW}} (\Delta m_{31}^2),
$$
(24)
where $\sin^2 2\theta_{\text{CDHSW}}(\Delta m^2_{31})$ stands for the value of $\sin^2 2\theta$ on the boundary of the allowed region in the $(\Delta m^2, \sin^2 2\theta)$ plot in [10]. The mixing angle in this case is constrained for $0.7\text{eV}^2 \lesssim \Delta m^2_{31} \lesssim 13\text{eV}^2$ as

$$\sin^2 2\theta_{\text{CDHSW}}(\Delta m^2_{31}) \lesssim 0.2. \quad (25)$$

If we consider the probability $P(\nu_\mu \to \nu_\mu)$ in the atmospheric neutrino experiments for the mass region above, we have small deviation of $P(\nu_\mu \to \nu_\mu)$ from unity

$$1 - P(\nu_\mu \to \nu_\mu) \simeq 2U_{\mu 3}^2(1 - U_{\mu 3}^2) \lesssim 0.1, \quad (26)$$

where we have averaged over rapid oscillations. (26) is obviously inconsistent with the atmospheric neutrino observations [3] [4] [5] [8], since we cannot have gross deficit of $\nu_\mu$ in this case. In fact we have verified explicitly, by the same calculation as in Ref. [17], that the region $\Delta m^2_{31} \gtrsim 0.47\text{eV}^2$ with the constraints (21) and (24) is excluded at 95% confidence level by the Kamiokande multi-GeV data ($\chi^2 \geq 17.6$ for 3 degrees of freedom, where $\chi^2_{\text{min}} = 3.2$, $(\chi^2 - \chi^2_{\text{min}})/7 \simeq 2.1$ implies $2\sigma$).

So we are left with the region $5 \times 10^{-2}\text{eV}^2 \lesssim \Delta m^2_{31} \lesssim 0.47\text{eV}^2$. Applying the formula (19) to the case of LSND, we have

$$s^2_{23} \sin^2 2\theta_{13} = 4U_{e3}^2U_{\mu 3}^2 = \sin^2 2\theta_{\text{LSND}}(\Delta m^2_{31}), \quad (27)$$

where $\sin^2 2\theta_{\text{LSND}}(\Delta m^2_{31})$ stands for the value of $\sin^2 2\theta$ within the allowed region in the $(\Delta m^2, \sin^2 2\theta)$ plot in [1], and the LSND data indicates

$$1.5 \times 10^{-3} \lesssim \sin^2 2\theta_{\text{LSND}}(\Delta m^2_{31}) \leq 1. \quad (28)$$

From (21) and the constraint $s^2_{23} \leq 1$, it follows that

$$\sin^2 2\theta_{\text{LSND}}(\Delta m^2_{31}) \leq \sin^2 2\theta_{\text{Bugey}}(\Delta m^2_{31}), \quad (29)$$
so that we have

\[ \Delta m_{31}^2 \gtrsim 0.25\text{eV}^2, \]  

(30)

with

\[ 1.5 \times 10^{-3} \lesssim \sin^2 2\theta_{\text{LSND}}(\Delta m_{31}^2) \lesssim 3.8 \times 10^{-2}, \]  

(31)

where we have used the \((\Delta m^2, \sin^2 2\theta)\) plots in [15] and [1]. However, we have verified explicitly again that the region \(\Delta m^2 \gtrsim 0.25\text{eV}^2\) is excluded at 90% confidence level by the Kamiokande multi-GeV data \(\chi^2 \geq 15\) for three degrees of freedom, \((\chi^2 - \chi^2_{\text{min}})/7 \simeq 1.7\) implies 1.6\(\sigma\). Therefore, we conclude that the LSND data cannot be explained by neutrino oscillations among three flavors, if all the accelerator and reactor data as well as the Kamiokande multi-GeV data are taken for granted.

The present LSND data allows conflicting interpretations either as a possible evidence for neutrino oscillation [1], or a stringent bound for the mixing parameters [2]. It might be possible that the allowed region of the set of the parameters \((\Delta m^2, \sin^2 2\theta)\) implied by the LSND data changes in the future.

We have looked for the region of \((\Delta m^2, \sin^2 2\theta)\) for \(\nu_\mu \rightarrow \nu_e\) (or \(\bar{\nu}_\mu \rightarrow \bar{\nu}_e\)), in which \(\nu_\mu \rightarrow \nu_e\) oscillation is consistent with all the experiments (except LSND), including the Kamiokande multi-GeV data. To obtain the \((\Delta m^2, \sin^2 2\theta_{\nu_e})\) plot of the two-flavor analysis for general \(\nu_\mu \rightarrow \nu_e\) oscillations with a mixing angle \(\theta_{\nu_e}(\Delta m^2_{31})\), we use the following correspondence between the rates for the \(\nu_\mu \rightarrow e\) process in the three and the two-flavor frameworks:

\[ \frac{N(\nu_\mu \rightarrow e; L)}{N_{\nu_e}(L)} \simeq 4U_{e3}^2U_{\mu3}^2 \left\langle \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) \right\rangle_{\mu \rightarrow e} \]

\[ = s_{23}^2 \sin^2 2\theta_{13} \left\langle \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) \right\rangle_{\mu \rightarrow e} \]

\[ \equiv \sin^2 2\theta_{\nu_e}(\Delta m_{31}^2) \left\langle \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) \right\rangle_{\mu \rightarrow e}, \]  

(32)
where we have used the hierarchical condition (16). From (21) and (32) we have

\[
\sin^2 2\theta_{\mu e} (\Delta m^2_{31}) \simeq s_{23}^2 \sin^2 2\theta_{13}
\leq s_{23}^2 \sin^2 2\theta_{\text{Bugey}} (\Delta m^2_{31}).
\]  

(33)

The Kamiokande multi-GeV data has better fit for larger values of \(\sin^2 2\theta_{23}\), so (33) shows that \(\sin^2 2\theta_{\mu e} (\Delta m^2_{31})\) cannot be larger than \(\sin^2 2\theta_{\text{Bugey}} (\Delta m^2_{31})/2\), if \(\sin^2 2\theta_{13} \ll 1\]. The result is shown in Fig.1.

(Insert Fig.1 here.)

The region suggested by the LSND experiment [1] is close to the 90%CL region obtained in our analysis. If the 20% systematic uncertainty mentioned in Ref. [1] shifts the allowed region of LSND in the direction of smaller mixing, there may be a chance that all the neutrino anomalies are explained by three flavor neutrino oscillations. Hopefully further data from the LSND group will clarify the situation.

**Acknowledgement**

One of the authors (O.Y.) would like to thank K.S. Babu, P.I. Krastev, C.N. Leung, and A. Smirnov for discussions. We would like to thank K. Iida for collaboration in the early stages of this work and members of the Physics Department of Yale University for their hospitality during our stay while a part of this work was done. This research was supported in part by a Grant-in-Aid for Scientific Research, Ministry of Education, Science and Culture, #05302016, #05640355, and #07044092.

---

4This is not the case for \(\Delta m^2_{31}<3\times10^{-2}\text{eV}^2\), where \(1 - P(\nu_\mu \to \nu_\mu) = 4s_{23}^2 c_{13}^2 (1 - s_{23}^2 c_{13}^2) \sim \mathcal{O}(1)\) becomes possible even for \(\theta_{23} < \pi/4\).
References

[1] LSND Collaboration, C. Athanassopoulos et al., Phys. Rev. Lett. 75 (1995) 2650.

[2] J.E. Hill, Phys. Rev. Lett. 75 (1995) 2654.

[3] Kamiokande Collaboration, K.S. Hirata et al., Phys. Lett. B205 (1988) 416; ibid. B280 (1992) 146.

[4] Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. B335 (1994) 237.

[5] IMB Collaboration, D. Casper et al., Phys. Rev. Lett. 66 (1989) 2561; R. Becker-Szendy et al., Phys. Rev. D46 (1992) 3720.

[6] NUSEX Collaboration, M. Aglietta et al., Europhys. Lett. 8 (1989) 611.

[7] Frejus Collaboration, Ch. Berger et al., Phys. Lett. B227 (1989) 489; ibid. B245 (1990) 305; K. Daum et al, Z. Phys. C66 (1995) 417.

[8] Soudan 2 Collaboration, M. Goodman et al., Nucl. Phys. B (Proc. Suppl.) 38 (1995) 337.

[9] See, e.g., J.N. Bahcall and R.K. Ulrich, Rev. Mod. Phys. 60 (1988) 297; J.N. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. 64 (1992) 885; J.N. Bahcall, R. Davis, Jr., P. Parker, A. Smirnov, R. Ulrich eds., SOLAR NEUTRINOS: the first thirty years Reading, Mass., Addison-Wesley, 1994 and references therein.

[10] S. P. Mikheyev and A. Smirnov, Nuovo Cim. 9C (1986) 17; L. Wolfenstein, Phys. Rev. D17 (1978) 2369.
[11] G.L. Fogli, E. Lisi, D. Montanino, Phys. Rev. D49 (1994) 3626; H. Minakata, Phys. Lett. B356 (1995) 61; Phys. Rev. D52 (1995) 6630; S.M. Bilenky, A. Bottino, C. Giunti, C.W. Kim, Phys. Lett. B356 (1995) 273; preprint DFTT 2/96 (hep-ph/9602216); S.M. Bilenky, C. Giunti, C.W. Kim, preprint DFTT-30-95 (hep-ph/9505301); K.S. Babu, J.C. Pati and F. Wilczek, Phys. Lett. B359 (1995) 351, B364 (1995) 251 (E); M. Narayan, M.V.N. Murthy, G. Rajasekaran, S. Uma Sankar, preprint IMSC-95-96-001 (hep-ph/9505281); G.L. Fogli, E. Lisi and G. Scioscia, Phys. Rev. D52 (1995) 5334; S. Goswami, K. Kar and A. Raychaudhuri, preprint CUPP-95-3 (hep-ph/9505395).

[12] The first paper of H. Minakata in [11].

[13] The first paper of S.M. Bilenky et al. in [11].

[14] L. Borodovsky et al., Phys. Rev. Lett. 68 (1992) 274.

[15] B. Ackar et al., Nucl. Phys. B434, (1995) 503.

[16] CDHSW Collaboration, F. Dydak et al., Phys. Lett. 134B (1984) 281.

[17] O. Yasuda, preprint TMUP-HEL-9603 (hep-ph/9602342).
Figures

**Fig.1** Below the solid and dashed lines are the regions for $\nu_\mu \rightarrow \nu_e$ (or $\nu_\mu \rightarrow \nu_\tau$) oscillation which are compatible with all the experiments (except LSND), including the atmospheric multi-GeV data of Kamiokande at 68%CL (solid) and 90%CL (dashed), respectively. To get this plot of the two-flavor mixing parameters, we have used the correspondence $\sin^2 2\theta \equiv \sin^2 2\theta_{\mu e} (\Delta m_{31}^2) = s_{23}^2 \sin^2 2\theta_{13}^2$ (cf. (32)). The shadowed area stands for the region allowed by all the accelerator and reactor experiments including LSND (The dotted, dashed and dot-dashed lines stand for the LSND [1], E776 [14], and Bugey [15] experiments, respectively).
Fig. 1

$P(\nu_\mu \rightarrow \nu_e)$

$\Delta m^2$ [eV$^2$]

$\sin^2 2\theta$