The analysis of the FM/FM/1 queue with single working vacation and impatience of customers

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Abstract  This paper deals with an M/M/1 queueing system with a single working vacation and reneging of impatient customers and fuzzy parameters. For this fuzzy queueing model, researches fetch performance measure of interest such as the mean number of customer in the system during working vacation, the mean number of customers in the system during regular busy period and the mean of reneging rate of the system. Finally numerical results are presented to show the effects of system parameter.

Keywords :: FM/FM/1 queue, single working vacation, Reneging, Membership values pentagonal fuzzy numbers.

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1 Introduction

The application of queueing models with server vacations have been examined by several researchers. Queueing system with customer impatience and server vacation could easily represent congestion problems in different fields such as traffic systems, communication systems, production systems and telecommunication systems etc. The study of queueing system must be needed in customer’s impatience behaviour to model real conditions exactly. If the probability of customer losses due to balking and reneging then the server remaining not working for periods of time might gain. This model has proposed real life application in production and order system to improve the performance of the production and to prevent the production from computer system and medical service system. Ke J C, Wu C H and Zhang Z G introduced the queues in vacation period have been inspected extensively.

Tian N, Zhao X and Wang K extended study of M/M/1 queue single working vacation. Yue et al. presented an M/M/1/N queueing system with server vacations, balking and reneging. During the working vacation period there are several situations where the server stays active and the server can provide service at a lower speed. Instead of stopping service completely. Altman and Yechiali considered the impatience of customers only when the unavailable for service and the servers are on vacation. Perel and Yechiali analysed in a slow and fast Markovian random environment for the impatient customers. R. Kalayanaraman et al.(2009) presented a single server vacation queue with fuzzy service time and vacation time distributions with some performance measure.

Fuzzy logic contained the construction of an almost acumen mechanism to gear the uncertainty related with human activities. Many behavior and approaches have been planned to prevent and reduce the uncertainty, that generated by decision making; however there are few embracing theories available in the literature. Zadeh explained a new idea, which is known as fuzzy sets theory. The basic theory of ambiguity has been used with huge success in different fields. Chang and Zadeh invented the main idea of fuzzy sets and numbers. T. Pathinathan and K. Ponnivalavan introduced a pentagonal fuzzy number and they explained the important concepts of pentagonal fuzzy numbers. Dubois and Prade explained fuzzy numbers as a fuzzy subset of the real line and the fuzzy number can be thought of as a
In this paper we observe the \( FM/FM/1 \) queue with single working vacation and impatience of customers in the queue during working vacation period. In crisp model, we describe the queue model. In fuzzy environment and performance of measure, we discuss the fuzzy model with the mean number of customers in the system during working vacation, regular busy period and the mean of reneging rate of the system are studied in fuzzy environment respectively. In section 5 includes numerical study about the performance measures.

2 The crisp model

We consider an \( M/M/1 \) queuing model with single working vacation. During working vacation the impatient of customers waiting in queue. Arrivals to a poisson distribution with parameter \( \lambda \). The server provides service to an exponential distribution with parameter \( \beta \) and \( \gamma \) during regular busy period and working vacation periods \( \gamma < \beta \). when the system is empty, the server goes for a single working vacation and the vacation time according to an exponential distribution with parameter \( \mu \). A customer waiting for service may get impatient due to delay and decide to renege from the queue. Reneging times are exponential distribution with parameter \( \eta \). The average reneging rate is \( (n-1)\eta \) for The arrival times, service times, the reneging times and the vacation times are all identically and independently distributed. The queue discipline is first come first served. Let \( M \) denote the number of customers in the system at time \( t \) and \( I \) denotes the state of the system at time \( t \).

In this section, the arrival rate, Service for regular busy period, service for working vacation periods, working vacation time and reneging are assume to be fuzzy numbers \( \tilde{\lambda}, \tilde{\beta}, \tilde{\gamma}, \tilde{\theta}, \tilde{\eta} \) respectively. Now

\[
\tilde{\lambda} = \{(x, \mu_{\lambda}(x)) ; x \in S(\tilde{\lambda})\},
\tilde{\beta} = \{(y, \mu_{\beta}(y)) ; y \in S(\tilde{\beta})\},
\tilde{\gamma} = \{(z, \mu_{\gamma}(z)) ; z \in S(\tilde{\gamma})\},
\tilde{\theta} = \{(u, \mu_{\theta}(u)) ; u \in S(\tilde{\theta})\},
\tilde{\eta} = \{(v, \mu_{\eta}(v)) ; v \in S(\tilde{\eta})\}.
\]

where \( S(\tilde{\lambda}) \), \( S(\tilde{\beta}) \), \( S(\tilde{\gamma}) \), \( S(\tilde{\theta}) \), \( S(\tilde{\eta}) \) are the universal set’s of the arrival rate, Service for regular busy period, service for working vacation periods, working vacation time and reneging respectively. Define \( f(x, y, z, u, v) \) as the system performance measure related to the above defined fuzzy queuing model, which depends on the fuzzy membership function \( S(\tilde{\lambda}), S(\tilde{\beta}), S(\tilde{\gamma}), S(\tilde{\theta}), S(\tilde{\eta}) \). Applying Zadeh’s extension principle (1978) the membership function of the performance measure \( S(\lambda), S(\beta), S(\gamma), S(\theta), S(\eta) \) can be defined as

\[
\mu_{f(\tilde{\lambda}, \tilde{\beta}, \tilde{\gamma}, \tilde{\theta}, \tilde{\eta})}(H) = \sup_{x \in S(\tilde{\lambda})} \sup_{y \in S(\tilde{\beta})} \sup_{z \in S(\tilde{\gamma})} \sup_{u \in S(\tilde{\theta})} \sup_{v \in S(\tilde{\eta})} \{ \mu_{\lambda}(x), \mu_{\beta}(y), \mu_{\gamma}(z), \mu_{\theta}(u), \mu_{\eta}(v) / H = f(x, y, z, u, v) \} \tag{1}
\]

If the \( \alpha \)-cuts of \( f(\tilde{\lambda}, \tilde{\beta}, \tilde{\gamma}, \tilde{\theta}, \tilde{\eta}) \) degenerate to some fixed value, then the system performance is a crisp number, otherwise it is a fuzzy number.

The mean number of customers in the system during working vacation

\[
E[N_{wv}] = \frac{\lambda(\beta - \lambda)(\gamma - \eta)(\lambda - \gamma + \eta + \theta)}{\lambda(\gamma - \eta)[(\eta + \theta)(\beta - \lambda) + \theta(\lambda - \gamma + \eta) + \beta\eta^2 + \beta\eta + \beta^2\eta^2](\beta - \lambda)}
\]

Here

\[
G_0(1) = \frac{(\gamma - \eta)}{\theta}^{p_{0,0}}
\]
The expected reneing rate of the system is

\[ F_1(1) = F_2(1) = 1 \]

and

\[ p_{0.0} = \frac{\theta \lambda (\eta + \theta) (\beta - \lambda)}{\lambda (\gamma - \eta) \left[ (\gamma - \eta)(\beta - \lambda) + \theta (\lambda - \gamma + \eta) + \theta^2 \right] + \beta \theta^2 (\eta + \theta) (\beta - \lambda)} \]

The mean number of customers in the system during regular busy period

\[ E[N_B] = \frac{(\theta \lambda)(\gamma - \eta)(\lambda - \gamma + \eta + \theta)}{\lambda (\gamma - \eta)(\beta - \lambda)(\beta - \lambda) + \theta (\lambda - \gamma + \eta) + \theta^2 + \beta \theta^2 (\eta + \theta)(\beta - \lambda)^2} \]

\[ + \left[ \frac{\lambda (\gamma - \eta)(\beta - \lambda)(\gamma - \eta + \beta \theta(\eta + \theta))}{\lambda (\gamma - \eta)(\beta - \lambda)(\gamma - \eta) + \theta (\lambda - \gamma + \eta) + \theta^2 + \beta \theta^2 (\eta + \theta)(\beta - \lambda)^2} \right] \]

where

\[ G_0'(1) = \frac{\lambda (\beta - \lambda)(\gamma - \eta)(\lambda - \gamma + \eta + \theta)}{\lambda (\gamma - \eta)(\beta - \lambda)(\beta - \lambda) + \theta (\lambda - \gamma + \eta) + \theta^2 + \beta \theta^2 (\eta + \theta)(\beta - \lambda)} \]

\[ G_0''(1) = \frac{p_{0.0}(\gamma - \eta)[2(\lambda - \theta - \gamma)(\lambda - \gamma + \eta + 2\lambda \eta + \theta)]}{\theta (\gamma + \theta)(2\eta + \theta)} \]

The mean of reneing rate of the system

The expected reneing rate of the system is \( \eta (G_0'(1) - G_0(1) + p_{0.0}) \)

\[ E[RR] = \frac{\eta \lambda (\beta - \lambda)(\gamma - \eta)(\lambda - \gamma + \eta + \theta)}{\lambda (\gamma - \eta)(\beta - \lambda)(\beta - \lambda) + \theta (\lambda - \gamma + \eta) + \theta^2 + \beta \theta^2 (\eta + \theta)(\beta - \lambda)} \]

\[ E[N_{sw}] = \frac{\lambda (\beta - \lambda)(\gamma - \eta)(\lambda - \gamma + \eta + \theta)}{\lambda (\gamma - \eta)(\beta - \lambda)(\beta - \lambda) + \theta (\lambda - \gamma + \eta) + \theta^2 + \beta \theta^2 (\eta + \theta)(\beta - \lambda)} \]

we acquire the membership function for some performance measures, namely the mean number of customers in the system during working vacation \( E[N_{sw}] \), the mean number of customers in the system during regular busy period \( E[N_B] \), the mean of reneing rate of the system \( E[RR] \). For the system in terms of this membership function, we as follows

\[ \mu_{P|\Pi_1}(P) = \sup_{x \in S(\lambda), y \in S(\beta)} \left\{ \mu_\lambda(x), \mu_\beta(y), \mu_\gamma(z), \mu_\phi(u), \mu_\eta(v) | P = f(x, y, z, u, v) \right\} \]

(2)

where

\[ P = \frac{x(y - x)(z - v)(x - z + v + u)}{x(z - v)(z - v)(x - z + v + u)} \]

\[ \mu_{P|\Pi_1}(Q) = \sup_{x \in S(\lambda), y \in S(\beta)} \left\{ \mu_\lambda(x), \mu_\beta(y), \mu_\gamma(z), \mu_\phi(u), \mu_\eta(v) | Q = f(x, y, z, u, v) \right\} \]

(3)

where

\[ Q = \frac{(ux)(z - v)(x - z + v + u)}{x(z - v)(2v + u)(y - x)} \]

\[ \mu_{P|\Pi_1}(S) = \sup_{x \in S(\lambda), y \in S(\beta), z \in S(\gamma)} \left\{ \mu_\lambda(x), \mu_\beta(y), \mu_\gamma(z), \mu_\phi(u), \mu_\eta(v) | S = f(x, y, z, u, v) \right\} \]

(4)
where
\[ S = \frac{vx(y - x)[(z - v)(x - z) + u(v + u)]}{x(z - v)[(v + u)(y - x) + u(x - z + v) + u^2] + yu^2(v + u)(y - x)} \]

Using the fuzzy analysis technique describe, we can find the membership of \( \mu_{\bar{\omega}(N_{wv})}, \mu_{\bar{\omega}(R_{wv})} \) as a function of the parameter \( \alpha \). Thus the \( \alpha \)-cut approach can be used to develop the membership function of \( \mu_{\bar{\omega}(N_{wv})}, \mu_{\bar{\omega}(R_{wv})} \).

### 4 Performance of measure

The following performance measure are studied for this model in fuzzy environment.

#### The mean number of customers in the system during working vacation

Based on Zadeh’s extension principle \( \mu_{\bar{\omega}(N_{wv})}(P) \) is the supremum of minimum over \( \mu_{\bar{\omega}}(x), \mu_{\bar{\omega}}(y), \mu_{\bar{\omega}}(z), \mu_{\bar{\omega}}(u), \mu_{\bar{\omega}}(v) \)

\[
P = \frac{x(y - x)[(z - v)(x - z + v + u)]}{x(z - v)[(v + u)(y - x) + u(x - z + v) + u^2] + yu^2(v + u)(y - x)}
\]

to satisfying \( \mu_{\bar{\omega}(N_{wv})}(P) = \alpha, \ 0 < \alpha \leq 1 \).

We consider the following four cases:

Case(i): \( \mu_{\bar{\omega}}(x) = \alpha, \mu_{\bar{\omega}}(y) \geq \alpha, \mu_{\bar{\omega}}(z) \geq \alpha, \mu_{\bar{\omega}}(u) \geq \alpha, \mu_{\bar{\omega}}(v) \geq \alpha, \)

Case(ii): \( \mu_{\bar{\omega}}(x) \geq \alpha, \mu_{\bar{\omega}}(y) = \alpha, \mu_{\bar{\omega}}(z) \geq \alpha, \mu_{\bar{\omega}}(u) \geq \alpha, \mu_{\bar{\omega}}(v) \geq \alpha, \)

Case(iii): \( \mu_{\bar{\omega}}(x) \geq \alpha, \mu_{\bar{\omega}}(y) = \alpha, \mu_{\bar{\omega}}(z) = \alpha, \mu_{\bar{\omega}}(u) \geq \alpha, \mu_{\bar{\omega}}(v) \geq \alpha, \)

Case(iv): \( \mu_{\bar{\omega}}(x) \geq \alpha, \mu_{\bar{\omega}}(y) \geq \alpha, \mu_{\bar{\omega}}(z) \geq \alpha, \mu_{\bar{\omega}}(u) = \alpha, \mu_{\bar{\omega}}(v) = \alpha, \)

For case (i) the lower and upper bound of \( \alpha \)- cuts of \( \mu_{\bar{\omega}(N_{wv})} \) can be acquire through the corresponding parametric non-linear programs,

\[ E[N_{wv}]_{\alpha L}^{L} = \min \{ [P] \} \quad \text{and} \quad E[N_{wv}]_{\alpha U}^{U} = \max \{ [P] \} \]

Similarly, we can calculate the lower and upper bounds of the \( \alpha \)-cuts of \( E[N_{wv}] \) for the case (ii), (iii), (iv) and (v).

By considering all the cases simultaneously the lower and upper bounds of the \( \alpha \)-cuts of \( E[N_{wv}] \) can be written as

\[ E[N_{wv}]_{\alpha L}^{L} = \min \{ [P] \} \quad \text{and} \quad E[N_{wv}]_{\alpha U}^{U} = \max \{ [P] \} \]

such that

\[ x_{\alpha L}^{L} \leq x \leq x_{\alpha U}^{U}, \quad y_{\alpha L}^{L} \leq y \leq y_{\alpha U}^{U}, \quad z_{\alpha L}^{L} \leq z \leq z_{\alpha U}^{U}, \quad u_{\alpha L}^{L} \leq u \leq u_{\alpha U}^{U}, \quad v_{\alpha L}^{L} \leq v \leq v_{\alpha U}^{U} \]

If both \( E[N_{wv}]_{\alpha L}^{L} \) and \( E[N_{wv}]_{\alpha U}^{U} \) are invertible with respect to \( \alpha \), the left and right shape function, \( L(P) = [E[N_{wv}]_{\alpha L}^{L}]^{-1} \) and \( R(P) = [E[N_{wv}]_{\alpha U}^{U}]^{-1} \) can be derived from which the membership function \( \mu_{\bar{\omega}(N_{wv})}(P) \) can be constructed as

\[
\mu_{\bar{\omega}(N_{wv})}(P) = \begin{cases} 
L(P), & \text{if } E[N_{wv}]_{\alpha L}^{L} \leq P \leq E[N_{wv}]_{\alpha U}^{U} \\
1, & \text{if } E[N_{wv}]_{\alpha U}^{U} \leq P \leq E[N_{wv}]_{\alpha L}^{L}^{-1} \\
R(P), & \text{if } E[N_{wv}]_{\alpha L}^{L}^{-1} \leq P \leq E[N_{wv}]_{\alpha U}^{U} \end{cases}
\]

(5)

In the same way we get the following results.

#### The mean number of customers in the system during regular busy period

\[
\mu_{\bar{\omega}(NB)}(Q) = \begin{cases} 
L(Q), & \text{if } E[\bar{\omega}(NB)]_{\alpha L}^{L} \leq Q \leq E[\bar{\omega}(NB)]_{\alpha U}^{U} \\
1, & \text{if } E[\bar{\omega}(NB)]_{\alpha U}^{U} \leq Q \leq E[\bar{\omega}(NB)]_{\alpha L}^{L}^{-1} \\
R(Q), & \text{if } E[\bar{\omega}(NB)]_{\alpha L}^{L}^{-1} \leq Q \leq E[\bar{\omega}(NB)]_{\alpha U}^{U} \end{cases}
\]

(6)
The mean system reneging rate of the system

\[ \mu_{RR}(S) = \begin{cases} 
L(S), & E[RR]^L_{\alpha=0} \leq S \leq E[RR]^U_{\alpha=0} \\
1, & E[RR]^L_{\alpha=1} \leq S \leq E[RR]^U_{\alpha=1} \\
R(S), & E[RR]^L_{\alpha=1} \leq S \leq E[RR]^U_{\alpha=0} 
\end{cases} \] (7)

5 Numerical study

The mean number of customers in the system during working vacation

Suppose the fuzzy arrival rate \( \lambda \), service for regular busy period \( \beta \), service for working vacation period \( \gamma \), working vacation time \( \eta \), and reneging rate \( \eta \) are assumed to be pentagonal fuzzy numbers described by:

\( \lambda = [16,17,18,19,20] \)
\( \beta = [26,27,28,29,30] \)
\( \gamma = [41,42,43,44,45] \)
\( \eta = [31,32,33,34,35] \)

and per hour respectively then \( \lambda(x) = \min \{ x \in s(\lambda), G(x) \geq \alpha \} \), \max \{ x \in s(\lambda), G(x) \geq \alpha \}, \alpha \) where

\[ G(x) = \begin{cases} 
0, & if \ x \leq x_a \\
1 - (1 - r) \frac{x - a_2}{a_3 - a_2}, & if \ a_2 \leq x \leq a_3 \\
1, & if \ x = a_3 \\
1 - (1 - r) \frac{a_4 - x}{a_4 - a_3}, & if \ a_3 \leq x \leq a_4 \\
\frac{a_5 - x}{a_5 - a_4}, & if \ a_4 \leq x \leq a_5 \\
0, & if \ x \geq a_5 
\end{cases} \]

That is, \( \lambda(x) = [16 + \alpha, 20 - \alpha], \beta(x) = [26 + \alpha, 30 - \alpha], \gamma(x) = [41 + \alpha, 45 - \alpha], \theta(x) = [51 + \alpha, 55 - \alpha], \eta(x) = [31 + \alpha, 35 - \alpha] \)

It is clear that, when \( x = x_a, y = y, z = z_a, u = u_a \) and \( v = v_a \) \( M \) attains its maximum value and when \( x = x_a, y = y, z = z_a, u = u_a \) and \( v = v_a \) \( M \) attains its minimum value.

From the generated for the given input values of \( \lambda, \beta, \gamma, \eta, \gamma \),

i) For fixed values of \( x, y, z \) and \( u \), \( M \) decreases as \( v \) increase.
ii) For fixed values of \( y, z, u \) and \( v \), \( M \) decreases as \( x \) increase.
iii) For fixed values of \( z, u, v \) and \( x \), \( M \) decreases as \( y \) increase.
iv) For fixed values of \( u, v, x \) and \( y \), \( M \) decreases as \( z \) increase.
v) For Fixed value of \( x, y, z \) and \( z \), \( M \) decreases as \( u \) increase.

The smallest value of occurs when \( x \) takes its lower bound. i.e., \( x = 16 + \alpha \) and \( y, z, u, \) and \( v \) take their upper bounds given by \( y = 30 - \alpha, \ z = 45 - \alpha, \ u = 55 - \alpha, \) and \( v = 35 - \alpha \) respectively. And maximum value of \( E[N_{ew}] \) occurs when \( x = 20 - \alpha, \ y = 26 + \alpha, \ z = 41 + \alpha, \ u = 51 + \alpha, \) and \( v = 31 + \alpha \). If both \( E[N_{ew}] \) and \( E[N_{ew}] \) are invertible with respect to ‘\( \alpha \)’ then, the left shape function \( L(P) = [E(N_{ew})]^L_{\alpha=1} \) and right shape function \( R(P) = [E(N_{ew})]^U_{\alpha=1} \) can be acquired and from which the membership function \( \mu_{E(N_{ew})}(\alpha) \) can be constructed as:

\[ \mu_{E(N_{ew})}(\alpha) = \begin{cases} 
0, & if \ P \leq P_1 \\
0.3(x - 17), & if \ P_1 \leq P \leq P_2 \\
0.3(19 - x), & if \ P_2 \leq P \leq P_3 \\
0.7(20 - x), & if \ P_3 \leq P \leq P_4 \\
0, & if \ P \leq P_5 
\end{cases} \] (8)

The values of \( P_1, P_2, P_3, P_4, \) and \( P_5 \) as acquired from (8) are:

\[ \begin{align*}
\mu_{E(N_{ew})}(\alpha) & = \begin{cases} 
0, & if \ P \leq P_1 \\
0.3(x - 17), & if \ P_1 \leq P \leq P_2 \\
0.3(19 - x), & if \ P_2 \leq P \leq P_3 \\
0.7(20 - x), & if \ P_3 \leq P \leq P_4 \\
0, & if \ P \leq P_5 
\end{cases} \\
\end{align*} \]
\[ \mu_{E[N_{nv}]}(P) = \begin{cases} 
0, & \text{if } P \leq 0.0000 \\
0.3(x - 17), & \text{if } 0.0000 \leq P \leq 0.03684, \\
1, & \text{if } x = 1 \\
0.3(19 - x), & \text{if } 0.03684 \leq P \leq 0.08512, \\
0.7(20 - x), & \text{if } 0.08512 \leq P \leq 0.04102, \\
0, & \text{if } P \geq 0.0000 
\end{cases} \]

In the same way we get the following results.

The mean number of customers in the system during regular busy period

\[ \mu_{E[N_{nb}]}(Q) = \begin{cases} 
0, & \text{if } Q \leq Q_1 \\
0.5(x - 17), & \text{if } Q_1 \leq Q \leq Q_2, \\
0.5(19 - x), & \text{if } Q_2 \leq Q \leq Q_3, \\
0.5(20 - x), & \text{if } Q_3 \leq Q \leq Q_4, \\
0, & \text{if } Q \geq Q_5 
\end{cases} \] (9)

The values of \( Q_1, Q_2, Q_3, Q_4 \) and \( Q_5 \) as acquired from (9) are:

\[ \mu_{E[N_{rr}]}(Q) = \begin{cases} 
0, & \text{if } Q \leq 0.0000 \\
0.4(x - 17), & \text{if } 0.0000 \leq Q \leq 0.8831, \\
0.4(19 - x), & \text{if } 0.8831 \leq Q \leq 1.1209, \\
0.6(20 - x), & \text{if } 1.1209 \leq Q \leq 0.9810, \\
0, & \text{if } Q \geq 0.0000 
\end{cases} \]

The mean system reneging rate of the system

\[ \mu_{E[R_{rr}]}(S) = \begin{cases} 
0, & \text{if } S \leq S_1 \\
0.2(x - 17), & \text{if } S_1 \leq S \leq S_2, \\
0.2(19 - x), & \text{if } S_2 \leq S \leq S_3, \\
0.8(20 - x), & \text{if } S_3 \leq S \leq S_4, \\
0, & \text{if } S \geq S_5 
\end{cases} \] (10)

The values of \( S_1, S_2, S_3, S_4 \) and \( S_5 \) as acquired from (10) are:

\[ \mu_{E[R_{rr}]}(S) = \begin{cases} 
0, & \text{if } S \leq 0.0000 \\
0.2(x - 17), & \text{if } 0.0000 \leq S \leq 2.3578, \\
0.2(19 - x), & \text{if } 2.3578 \leq S \leq 5.7273, \\
0.8(20 - x), & \text{if } 5.7273 \leq S \leq 2.9474, \\
0, & \text{if } S \geq 0.0000 
\end{cases} \]

CONCLUSION

In this paper we have studied the analysis of \( M/M/1 \) queueing system with customers impatience and single working vacation using pentagonal fuzzy numbers. We have acquired the performance measure such as the mean number of customers in the system during working vacation, the mean number of customers in the system during regular busy period and the mean of reneging rate of the system. We have acquired the numerical results to all the performance measures for this fuzzy queues. The application of the fuzzy queues many day today life situations such as manufacturing/inventory system, digital communication, telecommunication and computer networks.

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