A geographical information system approach for evaluating the optimum location of point-like facilities in a hierarchical network

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This paper discusses a methodology for solving the problem of finding an optimum location for a number of facilities entering an existing hierarchical network. The locational optimization problem is approached with a continuous model. The facilities are considered as point-like. The proposed methodology solves these problems by using Voronoi diagrams to define the catchment area and minimizes the average distance traveled by the users. The approach is implemented by loose coupling between the Directed Tabu Search algorithm for continuous nonlinear optimization with constraints and the Geographical Information System where the cost function is evaluated. In the case study example, facilities operate in a hierarchical network consisting of bank branches and Automated Teller Machines (ATM) of a private Greek bank. A new bank branch location and the insertion of an ATM were examined and compared, providing an improvement of 25.6 and 12.8%, respectively. The proposed approach resulted in visualizing the optimum location on the map, providing the ability to study the modifications that occurred. This could be a useful tool in the hands of practitioners and provide new options in the decision-making process.

Keywords: GISs; spatial analysis; nonlinear programing; locational planning

1. Introduction

Locational optimization problems have been studied for more than 100 years. In their more general form, they require as an input the study area, the metric used to evaluate distances, and the density of users in the area of interest (1). The calculation of the optimum location of a number of facilities providing specific services is based on a cost function (2). The variety of applications is wide, starting from location for fire stations, school units, and bus stops up to location of distribution centers, retail market, and sport fields.

The methodology presented in the literature for solving locational optimization problems can be classified into four major classes based on the way the region of interest (1–3). The first class captures the analytic models based on a number of simplifications of the problem; such as the hypothesis that demand rises homogeneously over the region. This assumption leads to closed form solutions that are function of the problem parameters. The second class includes network models, where the facilities location problem is treated in the context of a network consisting of links and nodes. Another class is the discrete location model, where demand and supply of a service appear as discrete points in the study region. Their mathematical formulation leads to N-P hard problems for integer programing. These models are those most commonly found in the literature with different formulations such as the p-median model, the location set covering model, and the center model (1, 3).

The last class is continuous models which assume that facilities can be located anywhere in the study region; demand arises usually in a discrete number of points. These problems are treated with methods of nonlinear programming and algorithms to find the global optimum. Despite the criticism related to its inability to model real life, it has been applied to a number of problems such as the location of mail boxes, in transportation, and in logistics (4).

The continuous models, with the use of Voronoi diagrams, solve three different kinds of problems, point-like, line, or space-time problems (5). In the first case where point-like facilities are treated, the goal is to find the optimum location in order to minimize the average cost. The point-like facilities can be facilities that are used by independent users or by groups of users, hierarchical services, observation points, or service points of mobile facilities.

In the literature discussed, it is evident that surveys using a connection between the location models, a Geographical Information System (GIS), and efficient algorithms able to solve these problems are rare and problem specific (6, 7). Example of such research is a GIS framework, proposed by Bozkaya et al. (6) integrating location-routing model and a hybrid heuristic solution.
methodology for a competitive multifacility location problem in the Instanbul area.

The research reported here tries to fill the gap by loosely coupling the GIS with a continuous location model based on Voronoi diagrams and a global optimization algorithm named Directed Tabu Search (DTS).

2. Description of the problem

In the literature, location decision problems have mostly been studied as single-level systems. In reality, though facilities can provide customers different kinds of services and thus can be considered as hierarchical facility systems. These systems can include two or more levels of facilities. Examples of such systems are health care services, composed of clinics and hospitals; educational services consisting of different levels of schools; and distribution systems capturing factories and warehouses and solid waste management systems composed by waste generation points, transfer stations, and disposal sites (8, 9). According to Sahin and Sural’s (8) classification scheme, the hierarchical systems differentiate in flow patterns (single or multi flow), in service varieties (nested or not), in spatial configuration (coherent systems), and in objective function used (median, covering or fixed charged).

Most of these models described in the literature are based on models developed on networks and are solved as integer programming problems. In contrast, in the approach adopted here, the model is formulated as a continuous plane (5, 10). In the continuous location model, facilities can be located anywhere in the service area and demand arises at discrete points.

The mathematical formulation will follow Okabe et al.’s model (11). Consider n locations of a service in region S that are ranked from 1 to m. Assume that n1 of them have rank i (\( \sum_{j=i+1}^{m} n_j = n \)) and \( x_1, x_2, \ldots, x_n \) are the location of n facilities. Further, suppose that the first n1 has a rank of 1, the next n2 has a rank of 2, etc.

Assume that n facilities supply k different services \( s_1, s_2, \ldots, s_k \) and facilities of rank i supply \( s_1, s_{i+1}, \ldots, s_m \) services. In other words, the service of its rank and all the services of its lower rank are \( i+1, i+2, \ldots, m \). This is known as a nested hierarchy. In addition, in this model, two assumptions are made. First, every user goes to the nearest facility that provides the required service. So the catchment areas are given by dividing the region for every rank into Voronoi polygons. Second, every user consumes \( a_j \) time units of service \( s_j \). Finally, it is assumed that \( \sum_{j=1}^{k} a_j = 1 \).

The cost function can be defined in terms of the distance that users travel in order to move into their nearest service area. As a distance function, the Euclidean distance is used, which is close to reality as the comparisons have shown (12). So the cost function is:

\[
F(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{k} a_j \sum_{m=n+1}^{n} \int_{x_j}^{x_{j+1}} \left( \frac{\phi(x)}{x} \right) dx
\]

In this formulation, only the user cost that arises is modeled and comparable to the p-median hierarchical location–allocation model.

3. Methodology

In order to solve the optimum location problem for facilities as described in the previous section, software that incorporates two evaluating mechanisms is needed. The first is a GIS, where using the relating spatial information, the cost function can be evaluated by using specifically developed code. The second is an optimization algorithm, allowing us to solve nonlinear problems with constraints. In the absence of an integrated solution in the GIS and due to the cost involved into building it, a loose coupling approach was used to integrate these two mechanisms (13, 14). In our case, the first part is based on Mapbasic scripting language of MapInfo (15), while the optimization algorithm was run on Matlab. The framework is modeling-centric meaning that the optimization algorithm is dominant in the system and the GIS functions are called from the Matlab code in the form of a script when needed. Further, the loose coupling involves a bidirectional integration with data exchanged between these two mechanisms through files.

Optimization problems are not new in the GIS environment (16–18). The problems of map labeling, generalization, line simplification, or shortest path algorithms for traffic networks and land use planning have already been treated by global optimization techniques such as genetic algorithms and modern heuristic search techniques. These methods are in demand especially because they do not have to incorporate problem assumptions and find fast near-optimal global solutions. In our case, the method of optimization adopted is a variant of Tabu search (19, 20) for continuous fields (DTS) developed by Hedar and Fukushima (21). The method DTS used is based on the metaheuristics Tabu search method that has been enhanced so that it can treat problems of continuous functions. The DTS uses direct search methods to provide the direction of search for the Tabu method and used to stabilize the search in the vicinity of the local optimum. These techniques are based on the method of Nelder–Mead (22) enriched by an adaptive pattern search. Further, in the DTS three search procedures are used, exploration, diversification, and intensification. The exploration search generates trial moves and diversification drives the search into the unvisited areas. Finally, intensification is used to refine the collected elite solutions.

In the framework developed the DTS code, provided by Hedar and Fukushima (23), runs in Matlab and requires an evaluation of the cost function. For each
function evaluation, the relevant problem parameters are defined and passed to the GIS through data files. These are the number of facilities that need to be inserted in the study region as well as their position. Also for each facility, there are two problem variables, one for each coordinate. So, the GIS script called was developed in the language MapBasic (15). The problem parameters are read and the value of the cost function is returned into the optimization code by using the data files. The cost function is described by Equation (1) and is discretized as follows:

\[ F(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{k} \sum_{j=i}^{n} a_{ij} + \sum_{l=1}^{n} \frac{b(l)}{\text{Area}_{i}^{\text{total}}} \cdot \frac{\text{Population}_{j}^{\text{total}}}{\text{Population}_{i}^{\text{total}}} \cdot \|x - x_l\| \]  

(2)

where \(x_1, x_2, \ldots, x_n\) is the location of the \(n\) point-like facilities; \(i\) refers to the different services offered; \(j\) refers to the Voronoi polygons that the study region is divided; and \(l\) refers to the blocks within each Voronoi polygon. As already noted earlier, Euclidean distance is used.

As it can be seen from Equation (2), the cost function is evaluated by summing up, for each different service, for each Voronoi, and for each block within, the distance that the customers have to travel to the facility, weighted by the percentage of the area of this block to the total area and by the percentage of the population of this block to the total population.

Another important point is the constraints, having the aim to keep the optimum locations within the study area. These are implemented at first as inequality constraints in the DTS, equal to min-max box of the study region, and in a second phase as penalty functions.

4. Results and discussion

Based on the methodology described in Section 2, and on the tool developed for that purpose, the problem of locating either a new bank branch or a new ATM on an existing hierarchical network of a private Greek bank is considered. The study area is a subregion of Athens consisting by five municipalities has a total area of 21.4 km\(^2\) and total population of 375,000. Also, the population is known and used for each block (Figure 1).

Although there various studies investigating the location of bank branches in a competitive environment (24), and even merging or closure of bank branches (25, 26), or on the location of ATMs (27), there is no approach considering their interaction in a single model.

In our case, the problem of finding optimum locations for bank branches and ATMs is modeled as a problem in a hierarchical network. For this purpose, a Greek bank was chosen and 14 branches and 9 ATMs were geocoded in the study area (Figure 2). The bank branches were ranked as 1 and the ATMs as rank 2, based on the approach that all the services of ATMs, and not limited to those, are provided by branches as well. In this application the cost function used Equation (1) and the time units of usage are considered equal for each service, i.e. \(a_1 = a_2 = 0.5\).

This implementation provides us answers to questions such as “where is the optimum location of \(n\) new bank branches and \(m\) ATMs in a network of services under use.”

Figure 3 illustrates the solution of two problems. The first one refers to finding the optimum location of a new bank branch, while the second one refers to finding the location of an ATM in a network already in use. The insertion of the bank branch yielded an improvement of 25.6% in the cost function and the insertion of an ATM...
yielded a 12.8% improvement. Figure 3 shows that these two points are close to each other. For the solution of these two problems, 263 and 269 function evaluations were made, respectively, and each function evaluation took about 5.4 s on an AMD Athlon x2250.

In order to assess the validity of the solution, the covering polygons (service areas), for the facilities considered, are calculated, with the assumptions that the potential customers are within the walking distance of 700 m (network distance used) and that service areas are not overlapping (Figure 4). In Figure 4, the covering area of both bank branches and ATMs is illustrated but a similar result can be generated for bank branches only. As expected, the locations provided by the proposed method were not covered by already existing facilities. Finally, it should be pointed out that the solution provided is limited by the model assumptions, as economic and traffic data, distance to shopping center and barriers are not included in the model.
5. Conclusion

The purpose of this paper was to develop a loose coupling methodology between the GIS and the optimization algorithm to solve hierarchical location problems. In this perspective, a Voronoi diagram approach was adopted and space is treated as continuous. This model is simple, robust, and useful when planning a new service or the expansion of a new one (4). At the same time, the method bypasses the difficulties present in discrete models such as the size of the problem (28). Another important issue considered in this model is that the objective functions used in location problems are neither concave nor convex and usually contain several local minima. In this respect, modern heuristics are attractive despite the accompanied difficulty of slow convergence close to the optimum. This can be solved by using hybrid methods such as the DTS used in this paper. This algorithm has been chosen among other algorithms of global optimization because it outperformed the Genocop III (genetic algorithm-based) and scatter search method, giving high-quality solutions (21).

Using this methodology in an application, the insertion of a bank branch and of an ATM, alternatively, on a hierarchical network in use, was examined. The solution provided an improvement of 25.6 and 12.8%, respectively. This approach can display the consequences of the user input on the map and most importantly, the changes that could occur from these modifications. This could be a useful in the hands of practitioners and provide new options in the decision-making process (13).

It should be noted that real world problems are far more complex than the models presented. There are many other factors to be taken into account such as availability of suitable sites, cost of sites, convenience, and regulatory issues (28). Despite that, the location models can provide a helpful tool in the decision-making process and can provide comparative alternative solutions.

Notes on contributor

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References

(1) ReVelle, C.S.; Eiselt, H.A. Location Analysis: A Synthesis and Survey. Eur. J. Oper. Res. 2005, 165, 1–19.
(2) Plastria, F. Static Competitive Facility Location: An Overview of Optimization Approaches. Eur. J. Oper. Res. 2001, 129, 461–470.
(3) ReVelle, C.S.; Eiselt, H.A.; Daskin, M.S. A Bibliography for some Fundamental Problem Categories in Discrete Location Science. Eur. J. Oper. Res. 2008, 184, 817–848.
(4) Novaes, A.G.N.; Souza de Cursi, J.E.; da Silva, A.C.L.; Souza, J.C. Solving Continuous Location-Districting Problems with Voronoi Diagrams. Comp. Oper. Res. 2009, 36 (4), 40–59.
(5) Okabe, A.; Boots, B.; Sugihara, K.; Chiu, N.S. Spatial Tessellations: Concepts and Applications of Voronoi Diagrams; Wiley: New York, NY, 2000.
Bozkaya, B.; Yanik, S.; Balcisoy, S. A GIS-based Optimization Framework for Competitive for Multi-facility Location-routing Problem. *Networks Spatial Econ.* 2010, 10, 297–320.

Stratiff, S.L.; Cromley, R.G. Using GIS and \( K = 3 \) Central Place Lattices for Efficient Solutions to the Location Set-covering Problem in a Bounded Plane. *Trans. GIS* 2010, 14 (3), 331–349.

Saïhin, G.; Sural, H. A Review of Hierarchical Facility Location Models. *Comp. Oper. Res.* 2007, 34, 2310–2331.

Teixeira, J.C.; Antunes, A.P. A Hierarchical Location Model for Public Facility Planning. *Eur. J. Oper. Res.* 2008, 185, 92–104.

Okabe, A.; Suzuki, A. Locational Optimization Problems Solved through Voronoi Diagrams. *Eur. J. Oper. Res.* 1997, 98, 445–456.

Okabe, A.; Okunuki, K.I.; Suzuki, T. A Computational Method for Optimizing the Hierarchy and Spatial Configuration of Successively Inclusive Facilities on a Continuous Plane. *Locat. Sci.* 1997, 5 (4), 255–268.

Apparicio, P.; Abdelmajid, M.; Riva, M.; Shearmur, R. Comparing Alternative Approaches to Measuring the Geographical Accessibility of Urban Health Services: Distance Types and Aggregation-error Issues. *Int. J. Health Geogr.* 2008, 7 (7). DOI: 10.1186/1476-072X-7-7.

de Smith, J.M.; Goodchild, M.F.; Longley, P.A. *Geospatial Analysis: A Comprehensive Guide to Principles, Techniques and Software Tools*, 3rd ed.; SPLINT: Leicester, 2009.

Anselin, L.; Dodson, R.F.; Hudak, S. Linking GIS and Spatial Data Analysis in Practice. *Geogr. Syst.* 1993, 1, 3–23.

Mapinfo. *MapBasic Version 9.5. Reference Guide*; Pitney Bowes Software Inc.: New York, NY, 2008.

van Dijk, S.; Thierens, D.; de Berg, M. Using Genetic Algorithms for Solving Hard Problems in GIS. *Geoinformatica* 2002, 6 (4), 381–413.

Feng, L.U.; Chenghu, Z.; Qing, W. An Optimum Vehicular Path Algorithm for Traffic Network based on Hierarchical Spatial Reasoning. *Geo-spatial Inform. Sci.* 2002, 3 (4), 36–42.

Yangfanc, L.; Liang, H. Land use Structure Optimization under Systematic Framework. *Geo-spatial Inform. Sci.* 2002, 5 (3), 46–52.

Glover, F. Tabu search – Part I. *ORSA J. Comput.* 1989, 1 (3), 190–206.

Glover, F. Tabu search – Part II. *ORSA J. Comput.* 1990, 2 (1), 4–32.

Hedar, A.; Fukushima, M. Tabu search Directed by Direct search Methods for Nonlinear Global Optimization. *Eur. J. Oper. Res.* 2006, 170 (2), 329–349.

Kelly, C.T. *Iterative Methods for Optimization*; SIAM: Philadelphia, PA, 1999.

Global Optimization. Directed Tabu Search (DTS) Method, 2011. http://www-optima.amp.i.kyoto-u.ac.jp/member/student/hedar/Hedar_files/go_files/Page777.htm

Miliotis, P.; Dimopoulou, M.; Giannikos, I. A Hierarchical Location Model for Locating Bank Branches in a Competitive Environment. *Int. Trans. Oper. Res.* 2002, 9, 549–565.

McDonald, E.H. GIS in Banking: Evaluation of Canadian Bank Mergers. *Can. J. Regional Sci.* 2001, 3, 419–442.

McMorris, P.S.; O’Brien, R. Bank Branch Closures in New Zealand: The Application of a Spatial Interaction Model. *Appl. Geogr.* 2001, 21, 301–330.

Aldajani, M.A.; Alfares, H.K. Location of Banking Automatic Teller Machines Based on Convolution. *Comp. Ind. Eng.* 2009, 57, 1194–1201.

Neema, M.N.; Maniruzzaman, K.M.; Ohgai, A. New Genetic Algorithms based Approaches to Continuous p-median Problem. *Networks Spatial Econ.* 2011, 11, 83–99.