Hybrid Inflation and Particle Physics

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Abstract

The prototype hybrid SUSY SU(5) inflation models, while well motivated from particle physics, and while allowing an acceptable inflationary phase with little or no fine tuning, are shown to have two fundamental phenomenological problems. (1) They inevitably result in the wrong vacuum after inflation is over; and (2) they do not solve the monopole problem. In order to get around the first problem the level of complexity of these models must be increased. One can also avoid the second problem in this way. We also demonstrate another possibility by proposing a new general mechanism to avoid the monopole problem with, or without inflation.

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Over the past 20 years, the interface between particle physics and cosmology has blossomed remarkably, spurred on by the development of Grand Unified Theories (GUTs) in the 1970’s. These promised, for the first time, an understanding of microphysics which would determine the equation of state for the FRW expansion at very early times. It was in this context that the most influential modern theoretical development in early universe cosmology occurred. We refer of course to inflation [1].

Nevertheless, in spite of this close connection between Guth’s original idea and the reigning particle theory of the day, many recent so-called particle physics inspired models for early universe cosmology are often nothing of the sort. Inflationary models are generally designed and tuned not in response to particle physics issues, but rather cosmological ones. The constraints which govern the model parameters involve such issues as the necessity for sufficient inflation, and the generation of subsequent density perturbations. Often the motivation from known low energy physics questions is not immediately apparent.

In this paper, we address what seems a very promising set of inflationary models which have evolved from considerations of supersymmetry and supergravity, ideas which are central to current particle physics model building. These models, involving what has become known as hybrid inflation [2], are based on a generic characteristic of supersymmetric model building: pseudo-flat directions in scalar field potentials. Moreover, they turn out to allow a novel graceful exit from inflation which results in an acceptable scale for primordial density perturbations without an apparent fine tuning of parameters, the bane of most inflationary models. Finally, they have the very attractive feature that the inflationary phase transition is related to the GUT transition, which is again well motivated by particle physics.

Nevertheless, in spite of their origins in GUTs and supersymmetry, a number of fundamental particle physics issues associated with hybrid inflation have not received attention in the literature. We demonstrate that a consideration of such issues implies that the GUT prototypical hybrid inflationary models are generally not viable, and must be supplemented by new complications.

Two generic problems face GUT models, and inflation was developed in some sense in
the effort to cure both of them. First and foremost, GUT symmetry breaking tends to inevitably produce stable magnetic monopoles, whose abundance, in a cosmological context, is unacceptable. Inflation was first proposed in the context of resolving this problem. Next, GUT models such as $SU(5)$ can break in many different ways. Since low-energy physics is described by an $SU(3) \times SU(2) \times U(1)$ model, it is clearly a potential phenomenological disaster if $SU(5)$ breaks to $SU(4) \times U(1)$ preserving vacuum instead of the known low energy vacuum configuration. Considerations of metastability of the “false” $SU(4) \times U(1)$ vacuum in part led to the development of techniques which would later be applied to inflationary models.

It may seem surprising, therefore, that simplest GUT prototype of hybrid inflationary models, which come closest perhaps to be inspired by current particle theory, in fact generically do not resolve either of these issues. If we preserve the attractive feature that the inflationary transition is tied to the GUT transition, then, as we show here, the prototype hybrid inflationary models are generically not phenomenologically viable without either additional structures in the superpotential, or changing the nature of the inflaton field.

First, let us review a minimal SUSY GUT version of the canonical hybrid model \[2\]. It is given by a superpotential, with two fields \[3\], the inflaton field, and the Higgs field in the present context to be associated with GUT symmetry breaking \[4,5\]. The simplest superpotential that leads to hybrid inflation in the minimal supersymmetric $SU(5)$ is

\[
W = \frac{g}{2} S \text{Tr} \Sigma^2 - SM^2 + \frac{h}{3} \text{Tr} \Sigma^3
\]

where $S$ is a gauge singlet field (the inflaton) and $\Sigma$ is an $SU(5)$ adjoint Higgs. From a particle physics perspective, the role of $S$ is to remove the origin $\Sigma = 0$ from the vacuum manifold and break $SU(5)$. The role of the cubic invariant is to remove the unwanted continuous degeneracy and fix the orientation of the $\Sigma$ VEV. In a globally supersymmetric limit the system admits the two degenerated minima with unbroken $G_{3-2-1} = SU(3) \otimes SU(2) \otimes U(1)$

\[
\Sigma = M \sqrt{\frac{1}{15g}} (2, 2, 2, -3, -3),
\]

(2)
and $G_{4-1} = SU(4) \otimes U(1)$ symmetries respectively

$$\Sigma = M \sqrt{\frac{1}{10g}} (1, 1, 1, 1, -4).$$  \hfill (3)

Supersymmetry breaking effects (e.g. in gravity-mediated scenarios) remove the degeneracy, but the energy splitting is tiny (suppressed by a factor $\sim (\frac{M_{GUT}}{M_p})^2$ with respect to the potential barrier $\sim M_{GUT}^4$).\footnote{As shown by Weinberg \cite{weinberg}, in the minimal $SU(5)$, provided the cosmological constant in one of the vacua is fine-tuned to zero by adding an explicit constant in the superpotential, the others in general appear to have negative energy density. Since the energy splitting is small, the gravitational effects can prevent the zero energy vacuum from decaying \cite{footnote1}.} As a result tunneling is negligible and the minima are stable for all practical purposes \cite{footnote2}. Which minima will be chosen by the theory is thus determined in a cosmological context.

This system can lead to hybrid inflation for large values of the $S$ field. The scenario proceeds as follows. For large values $S >> \frac{M}{\sqrt{g}}$, the $\Sigma$ field becomes very heavy, has a vanishing VEV and can be integrated out. The effective superpotential for the remaining light $S$ field is just linear

$$W_{inflation} = SM^2$$ \hfill (4)

and the classical potential is exactly flat: giving a constant vacuum energy density that breaks supersymmetry

$$V = M^4.$$ \hfill (5)

The one-loop corrected Kähler potential (for $S >> \frac{M}{\sqrt{g}}$) is of the form

$$K = SS^+ \left( 1 - \frac{g^2 3}{2\pi^2} \ln \frac{SS^+}{m^2} \right)$$ \hfill (6)

Since for large $S$, supersymmetry is broken, this simply translates into the following effective potential for $S$ \footnote{As shown by Weinberg \cite{weinberg}, in the minimal $SU(5)$, provided the cosmological constant in one of the vacua is fine-tuned to zero by adding an explicit constant in the superpotential, the others in general appear to have negative energy density. Since the energy splitting is small, the gravitational effects can prevent the zero energy vacuum from decaying \cite{footnote1}.}
\[ V(s) \simeq \Lambda^4 \left( 1 + \frac{g^2 3}{2 \pi^2} \ln \frac{SS^*}{m^2} \right) \]  

(7)

which drives inflation. When \( S \) drops below \( S_c = M/\sqrt{g} \), the point \( \Sigma = 0 \) becomes a local maximum. \( \Sigma \) picks up a nonzero VEV which grows to cancel the vacuum energy density. After this moment the system rapidly relaxes to one of the minima and oscillates about it. Note that the end of inflation does not necessarily coincide with this transition but can happen much earlier when the slow roll conditions break down.

We now demonstrate that the minimum which the system chooses right after the phase transition is \( G_{4-1} \) symmetric. This conclusion is independent of the detailed structure of the superpotential for \( \Sigma \).

To see this let us consider the potential for \( \Sigma \) in the background of the \( S \) field. We will be concerned by the structure of this potential at the moment when \( S \) drops to its critical value, after which \( \Sigma \) gets tachionic and begins to roll away from zero. The effective potential for \( \Sigma \) is

\[
V = h \sqrt{g} M \text{Tr} \Sigma^* \Sigma^2 + h.c. + \left( \frac{g^2}{4} - \frac{3}{5} \right) |\text{Tr} \Sigma^2|^2 + h^2 \text{Tr} \Sigma^* \Sigma^2 \\
+ \frac{g^2_{\text{gauge}}}{2} \text{Tr}[\Sigma \Sigma^*] 
\]

(8)

The last \((D)\) term is automatically minimized for a diagonal \( \Sigma \). For the remaining terms only the first is phase-dependent. Thus irrespective of the sign of the parameters we can set \( \Sigma \) real (the \( S \) and \( \Sigma \) phases will automatically adjust in such a way that to make the overall sign of the first term negative). Then, for a fixed \( S \), the potential is effectively a function of three quantities: the absolute value \( \sigma^2 = \text{Tr} \Sigma^2 \) and the two angles (orbit parameters) \[ \theta = \text{Tr} \Sigma^3 / \sigma^3 \] and \( \phi = \text{Tr} \Sigma^4 / \sigma^4 \).

\[ \text{In a supergravity context, for the generic Kähler metric, the above potential can receive non-trivial corrections that may affect details of the inflationary dynamics (slow-roll etc.). Our conclusions, however, are independent of the precise form of the inflaton potential, subject to a general assumption that the GUT transition is triggered by } S \text{ at the end of inflation.} \]
\[ V = 2h\sqrt{g\sigma^3}\theta + \left(\frac{g^2}{4} - \frac{h^2}{5}\right)\sigma^4 + h^2\sigma^4\phi \]  

(9)

The orientation of \( \Sigma \) is entirely determined through these orbit parameters, which take different values for the different breaking patterns. It is well known \[9\] that the minima of (8) (as well as of an arbitrary quartic or lower order gauge-invariant polynomial of (real) \( \sigma \)) can only correspond to one of the maximal unbroken subgroups: \( G_{3-2-1} \) or \( G_{4-1} \). The corresponding values of the orbit parameters are \( \theta_{3-2-1} = 1/\sqrt{30} \), \( \phi_{3-2-1} = -7/30 \) and \( \theta_{4-1} = 3/\sqrt{20} \), \( \phi_{4-1} = -13/20 \). As we have argued, the sign of \( \theta \) is irrelevant since the overall sign of the \( \theta \)-term will be negative. Thus at the moment when \( \sigma \) starts rolling away from zero, the cubic term dominates and the preferred orientation is the one that gives the largest value of \( |\theta| \), that is the \( G_{4-1} \) direction. One may wonder whether this orientation can change for larger values of \( \sigma \) when the quartic term becomes significant. We can easily see that this is not the case all way until the nearest minimum, which therefore corresponds to a \( G_{4-1} \) unbroken subgroup. First we note that the change from \( G_{4-1} \) to \( G_{3-2-1} \) can not happen gradually, since this are the only possible minima independently of the absolute value of \( \sigma \). So \( G_{4-1} \) will stay as the preferred direction until \( \sigma \) grows to a certain critical value \( \sigma_c \) at which time the \( \phi \) term will dominate and \( G_{3-2-1} \) will become preferred. If \( \sigma_c \) is larger than the value in the closest minimum, \( \sigma_0 \), \( \sigma \) will be trapped before reaching \( \sigma_c \). By the definition of \( \sigma_c \), \( \sigma_0 \) corresponds to \( G_{4-1} \) minimum. Thus, the system will automatically relax to the \( G_{4-1} \) minimum, if \( \sigma_c > \sigma_{4-1} \). For \( S = S_c \) this is indeed the case:

\[
\frac{\sigma_{4-1}}{\sigma_c} = \frac{15\sqrt{3}}{16 \left.(3\sqrt{3} - \sqrt{2})(\frac{g^2}{h^2} + 9/20)\right)^{-1} < 1
\]

(10)

The fact that the steepest direction for \( S = S_c \) is \( G_{4-1} \) symmetric is independent of the detailed structure of the superpotential. To see this add to the superpotential an arbitrary number of self-interaction terms. These are the all possible polynomials made from independent holomorphic invariants \( A_n = \text{Tr}\sigma^n \)

\[
W = \frac{g}{2} S\text{Tr}\Sigma^2 - SM^2 + h_{n_1...n_m}A_1^{n_1}...A_m^{n_m}
\]

(11)
The key point is that the lowest non-trivial invariant in the potential is non-hermitian and has a form

\[ S^* \theta^n \phi^m \chi^l \sigma^* \sigma^k + h.c. \]

where \( n, m, k, l \) are integers and \( \chi = \text{Tr} \Sigma^5 / \sigma^5 \). Thus, for the small values of \( \sigma \) it dominates and its phase automatically will be adjusted to negative. So the energetically most attractive orientation will be the one that maximizes this term’s absolute value. This is the \( 4 - 1 \) direction, for which all orbit parameters take larger values \(^3\).

Neglecting the possibility of ‘overshooting’ in the \( 3 - 2 - 1 \) vacuum, there thus appears to be no away around this phenomenological problem in the context of the standard field content and superpotential given above. In order to avoid this phenomenological disaster then one is driven to two possibilities. Either the field content of the theory must be made more complicated, introducing extra SU(5) fields for example (see below), or the canonical normalization of the kinetic term in the Lagrangian must be altered, as one might expect would result if some strongly coupled sector is responsible for the inflationary potential \(^4\).

Ending up in the wrong vacuum is a severe enough problem. However, it is clear from the scenario described above, which we reiterate is the standard hybrid inflation scenario, that the GUT phase transition occurs after inflation has ended. This of course implies that one is left with the standard GUT monopole problem.

In order to resolve the monopole problem in the context of GUT hybrid inflation, we can think of two alternatives \(^4\). First, the \( S \) field need not be an SU(5) singlet. In this case,

\(^3\)If the dependence on \( \chi \) is included, in principle, there can exist other phenomenologically unacceptable local minima, e.g. with unbroken \( SU(2) \otimes SU(2) \otimes U(1) \otimes U(1) \).

\(^4\)Once again we want to stress that this discussion is essential for the \( SU(5) \) GUT, in which there is only one GUT phase transition. In extended GUTs such as \( SO(10) \) \(^6\) or \( SU(6) \) \(^7\), which assume more stages of symmetry breaking, it is possible to separate the inflationary and GUT phase transitions.
SU(5) is broken before inflation, and one can imagine producing monopoles before inflation, which then get inflated away as in the standard picture. An example of a superpotential which exhibits this behavior is:

\[ W = S(g \text{Tr} \Sigma^2/2 - M^2) + h \text{Tr} \Sigma' \Sigma^2 + W_1(\Sigma' \Sigma) \]  

(13)

where \( \Sigma' \) is another adjoint and \( W_1 \) is an arbitrary gauge invariant superpotential such that 1) it allows for the \( 3-2-1 \) vacuum; and 2) \( \partial_{\Sigma'} W_1 = \partial_\Sigma W_1 = 0 \) for \( \Sigma = 0 \). In this theory hybrid inflation can be driven by \( \Sigma' = (2, 2, 2, -3, -3) \sigma' \), since for \( \sigma' \pm S >> M \), \( \Sigma \) will vanish and the tree-level potential for \( \sigma' \) and \( S \) is essentially flat. Thus some combination of these fields can play the role of the inflaton field.

Another alternative, which to our knowledge has not been discussed in the literature before, is to suppose that an Affleck-Dine type mechanism [15] is exploited, even when the inflaton field \( S \) is a singlet and the GUT transition happens after the inflation, or even if inflation never happens. This mechanism, was originally invented for generating the baryon asymmetry, and is based on the assumption that some of the squark and slepton VEVs that parameterize the flat directions of the SUSY vacua, are left far away from the origin during inflation and later perform coherent oscillations about it, generating the desired baryon to entropy ratio. Such initial conditions are probable in supergravity theories, since the flat direction fields usually get large curvature (of order the Hubble parameter) during inflation [16], which may have either sign depending on the precise structure of the Kähler potential. We propose that this same mechanism can cure the monopole problem, since the flat direction fields can (generically do) break electromagnetism [17]. Therefore the monopoles may never form. As a simple example (for further details see [17] consider the flat direction parameterized by the invariant \( \Phi^3 = 10^a \bar{5}^c \bar{5}^b \) where \( 10^a \), \( \bar{5}^c \), \( \bar{5}^b \) are matter superfields and \( a, b, c \) are generation indexes (obviously \( b \neq c \) due to the antisymmetric properties). In the component form this flat direction reads \( 10^a_{ik} = (\delta_{i1} \delta_{k2} - \delta_{i2} \delta_{k1}) \Phi \), \( \bar{5}^a_i = \delta_{i1} \Phi \), \( \bar{5}^b_k = \delta_{k2} \Phi \) and breaks \( SU(5) \) to \( SU(3)_c \) during inflation. Thus, the phase transition at the end of inflation will not lead to the monopole formation. It is worth noting that the
problem can be solved even if the $\Phi$ direction gets a nonzero VEV only after the GUT phase transition, e.g. as a result of energy transfer from the oscillating inflaton field. During this time, the $U(1)$ breaking will confine monopoles, causing them to eventually efficiently annihilate. (Note that the time during which $U(1)$ must be broken must be fairly long if the reheating temperature is sufficiently below the GUT scale \[^{18}\].) It is clear that this manifestation of this mechanism (i.e. the $\Phi$ direction getting a nonzero VEV after the GUT transition) can resolve the monopole problem even if the GUT transition does not involve inflation.

As we have shown, while simple hybrid inflation models are well motivated in principle on particle physics grounds and provide natural inflation models with little or no fine tuning, the simplest GUT prototype models are in general phenomenological disasters. In order to avoid these difficulties, significant additional complexity must be introduced into these models, or the inflationary phase transition and the GUT transition must be decoupled. (In this regard we note that in order to produce acceptable density perturbations, the GUT scale in even these hybrid models comes out close to, but not exactly the standard SUSY SU(5) GUT scale. Some extra field content, or threshold corrections would then be needed to truly tie the GUT and inflationary scales together.) Finally, our considerations have led us to recognize a new possible solution to the monopole problem, either in the context of a hybrid GUT inflation model, or even in a model with no inflation. This mechanism may thus be of interest well beyond the issue of the viability of hybrid inflation.
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