A Preservation Technology Model for Deteriorating Items with Advertisement Dependent Demand and Partial Trade Credit

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Abstract. This study presents a preservation technology model for deteriorating items with constant rate of deterioration. The requirements for the items are received as a function of advertisement frequency and selling price. The two types of trade credit is the main focus of this study. First one is provided by the supplier to the retailer and another one is the partial trade credit offered by the retailer to the customer. The optimal policy is presented to earn maximum profit with optimal values of selling price, advertisement frequency, preservation technology and total cycle length. The study is numerically verified with sensitivity analysis to check the sensitivity of the model.

1. Introduction
In todays, competitive world every organization try to attract more customer. For this, they are implementing different types of techniques. The concept of trade credit is also one of them. The supplier offers a time relaxation for the whole payment, that time is called credit period. During this period retailer earn sales revenue by settling an account. After credit period supplier charges high interest rate on unsold amount of stock. In similar manner retailer also offer a partial credit period to the customer to improve his bonding with customer. As this concept of trade credit is a key point in inventory control and modelling, researchers have started considering this concept in their studies to make the work reliable.

In literature Chen, Teng and Skouri [1] have established an economic production quantity model, in which items are deteriorating in nature and two types of trade credit policies are main focus that is up-stream full trade credit and down-stream partial trade credit. Wu and Chan [2] have developed the different policies for lot-sizing policies with partial trade credit and items are deteriorating with expiration dates. An inventory decision model is developed by Chen and Teng [3], in which the time dependent deterioration is considered for model formulation with up-stream and down-stream trade credit financing by discounted cash flow analysis.

The inventory model under supplier’s partial trade credit policy is studied by Chung and Ting [4] for a supply chain system. Mahata [5] has established an economic order quantity model for partial trade credit policy of retailer with deteriorating items having expiration dates and demand is varying with price. Sarkar and Saren [6] have developed a partial trade-credit policy of retailer for deteriorating items
with exponential deterioration rate. An inventory model is elaborated by Sharmila and Uthayakumar [7] for deteriorating items with quadratic demand and partial backlogging. Their main focus is the concept of partial trade credit.

A retailer partial trade credit policy is studied by Mahata and De [8], in which they have developed an EOQ inventory model for ameliorating items with dependent demand rate. Taleizadeh et al. [9] have developed an economic production quantity model for imperfect production process with upstream and downstream trade credit policies. An inventory model is established by Wu et al. [10], for deteriorating items having maximum lifetime with downstream partial trade credits. An optimizing policy is developed by Giri and Sharma [11] for an integrated production–inventory system with cash discount and retailer partial trade credit policy.

Lashgari, Taleizadeh and Sadjadi [12] have developed the ordering policies for deteriorating items with non-instantaneous deterioration rate under partial trade credit and partial backlogging. A lot-sizing policies are developed by Liao et al. [13] for deteriorating items under limited storage capacity and the main focus is the concept of two-level trade credit with partial trade credit to credit-risk retailer. In similar manner Mahata and De [14], Shukla and Suthar [15], Shaikh et al. [16], Tiwari et al. [17] and Dye, Yang and Wu [18] have studied the concept of partial trade credit in different market scenarios. In all above mentioned studies the effect of preservation and the advertisement depended demand still remains untouched. So to present a study for retailer partial trade credit policies with preservation technology investment and the advertisement depended demand rate.

In literature, recently Palanivel and Uthayakumar [7] have developed finite horizon EOQ model for deteriorating items with non-instantaneous deterioration rate and the demand is directly related with price and advertisement frequency with partial backlogging under inflation. Dye, Yang and Wu [18] have developed a joint dynamic pricing policy, in which preservation technology investment is considered for an integrated supply chain with reference price effects. Shah, Chaudhari and Jani [19] have established an optimal control analysis for deteriorating items with preservation technology investment.

In present article an optimal policy is developed for deteriorating items, in which deterioration rate is constant. The preservation technology is considered to minimize the losses due to deterioration. The demand rate is directly proportional to advertisement frequency and is decreasing function of selling price. The up-stream trade credit provided by supplier to the retailer and down-stream credit period is provided by retailer to the customer. In the end numerical illustration and sensitive analysis are explained for the verification of the study. In the next section, assumptions and notation are elaborated in detail, which are implemented in mathematical model formulation.

2. Assumption and Notation
The assumptions and notation, which are the base of model formulation in mathematical language, are explained as follows.

2.1. Assumption

- The demand rate is dependent on advertisement frequency and selling price as follows:
  \[ D = D(A, s) = A^\alpha (\alpha - \beta s) \text{ where } A, \alpha, \beta > 0; \]
- The items are assumed deteriorating in nature with constant rate of deterioration.
- The preservation technologies are applied to minimize the losses due to deterioration. The preservation function and controllable deterioration rate is as follows:
  \[ m(\xi) = \theta (1 - e^{-b\xi}), \text{ where } b > 0. \]
- The supplier provides a trade credit period (T_M) to the retailer. Till that moment (T_M) retailer can earn sales revenue by settling an interest bearing account. After that period retailer have to pay high interest rate for in stock amount (if any).
- The retailer presents a partial trade credit policy to the customer. According to this policy customers must pay a (\gamma_1) amount of purchase cost at time t of order placed. Rest (\gamma_2 = 1-
\( \gamma_1 \) amount of purchase cost can be paid at the time \( t + T_N \). So, during the case \( 0 \leq T + T_N \) retailer earn sales revenue. On the other hand, in the case of \( M \leq T + T_N \) retailer have to pay interest charges for sold amount of stock, but not yet paid by the customer. In the case when \( T + T_N \leq M \) there is no interest charges for stock which is already sold, but not paid by the customers.

- The unit inventory holding cost \((H)\) is taken as linear function of time, as follows: \( H = (h_1 + th_2) \) where \( h_1, h_2 > 0 \)
- There is no shortages of stock.
- The replenishment rate is instantaneous
- The lead time is zero with infinite time horizon.

### 2.2. Notation

- \( D \): The demand rate;
- \( A \): The advertisement frequency;
- \( m(\xi) \): The controllable deterioration rate;
- \( \tau_\theta \): The resultant deterioration rate, \( \tau_\theta = \theta - m(\xi) \)
- \( \theta \): The constant rate of deterioration \( 0 \leq \theta < 1 \);
- \( \xi \): The preservation cost (\$);
- \( C_D \): The unit advertisement cost (\$);
- \( s \): The unit selling price (\$);
- \( C \): The unit purchase cost (\$);
- \( C_A \): The order cost per order (\$)
- \( H \): The time dependent holding cost per unit per time unit (\$);
- \( t \): The time in years;
- \( T_M \): The credit period provided by the supplier to the retailer;
- \( T_N \): The credit period offered by retailer to the customer
- \( T \): The total cycle length;
- \( \gamma_1 \): The amount of purchase cost, which customer have to pay retailer at the time of order placing;
- \( \gamma_2 \): The amount of purchase cost, for which customer gets down-stream trade credit \( (T_N) \), where \( \gamma_2 = 1 - \gamma_1 \);
- \( I_r \): The rate interest earned per dollar per year;
- \( I_p \): The rate interest paid per dollar per year;
- \( I(t) \): The inventory level at any time \( t \);
- \( Q \): The order quantity;
- \( TP(s, \xi, A, T) \): The total profit function per year with decision variables selling price \( s \), preservation cost \( \xi \), advertisement frequency \( A \) and total cycle length;

The total profit function includes following parameters:
- The sales revenue \( (R_s) \);
- The total Order cost (OC);
- The total purchase cost (PC);
- The total holding cost (HC);
- The total advertisement cost (AD);
- The total earned interest (IE);
- The total paid interest \( (IP_1) \) when \( T_M \leq T \) for unsold stock;
- The total paid interest \( (IP_2) \) for sold out stock, but not paid by customer;

Note: The terms with superscript * show the optimal value of that particular parameter.
3. Mathematical Model Formulation

Initially at \( t = 0 \), the inventory level is \( Q \), with time instant it depletes due to total effect of demand and deterioration. At the moment when whole cycle commence that is at the time period \( t = T \) the whole stock vanishes. The following differential equations governs the whole inventory functioning.

\[
\frac{dI}{dt}(t) = -A^a(\alpha - \beta s) + \tau_\theta I(t); \quad 0 \leq t \leq T
\]
(1)

The solution of (1) under condition \( I(t = T) = 0 \) and \( I(t = 0) = Q \) is as follows:

\[
I(t) = \frac{A^a(\alpha - \beta s)}{\tau_\theta} (1 - e^{\tau_\theta(t-T)})
\]
(2)

\[
Q = I(t = 0) = \frac{A^a(\alpha - \beta s)}{\tau_\theta} (1 - e^{-\tau_\theta T})
\]
(3)

The total profit function is as follows:

\[
TP(s, A, \xi, T) = R_S + IE - OC - PC - HC - IP_1 - IP_2
\]

The cost parameters which included in total profit function are calculated as follows:

i. The customer pay retailer, a fixed amount of selling price, \( C_S (C_S > C) \) for each demand unit. Hence the total sales revenue is as follows:

\[
R_S = sD = sA^a(\alpha - \beta s)
\]
(4)

ii. The order cost \( OC = \frac{C_A T}{T} \)
(5)

iii. The purchase cost \( PC = \frac{C_Q T}{T} \)
(6)

iv. The holding cost \( HC = \frac{1}{T} \int_0^T \left( h_1 + th_2\right)I(t)dt \)

\[
HC = \frac{1}{\tau_\theta^2} \left[ \frac{h_1 A^a(\alpha - \beta s)}{\tau_\theta^2} (e^{-\tau_\theta T} - 1 + \tau_\theta T) + \frac{h_2 A^a(\alpha - \beta s)}{\tau_\theta^2} \left( \frac{T^2}{2} - \frac{T}{\tau_\theta} + \frac{1}{\tau_\theta^2} (1 - e^{-\tau_\theta T}) \right) \right]
\]
(7)

According to length of the time period \( T_M \) and \( T_N \) there two major cases: Case 1: \( T_M \geq T_N \) and Case 2: \( T_M < T_N \)

3.1. Case 1: \( T_M \geq T_N \)

v. The interest paid \( IP \), by retailer in three cases is as follows:

\[
\text{➢ In the case when } T \geq T_M \]

\[
IP_1 = \frac{C_p A^a(\alpha - \beta s)}{T \tau_\theta^2} \left[ e^{\tau_\theta (T_M - T)} - 1 + \tau_\theta (T - T_M) \right]
\]
(8)

\[
\text{➢ In the case when } T_M - T_N \leq T \leq T_M \]

\[
IP_1 = 0
\]
(9)

\[
\text{➢ In the case when } T \leq T_M - T_N \]

\[
IP_1 = 0
\]
(10)

The interest earned \( IE \) and interest paid \( IP_2 \) for the items already sold but not paid by customer- According to length of time periods there are three cases as follows: Case 1.1: \( T_M \leq T \), Case 1.2: \( T \leq T_M \leq T + T_N \), Case 1.3: \( 0 < T + T_N \leq T_M \). The pictorial representation is shown in figure 1, figure 2 and figure 3 respectively. The interest earned and interest paid in each case, are as follows:

Case 1.1: \( T_M \leq T \)

The interest earned and interest paid are as follows:
\[ IE = \frac{sl_t}{T} \left[ y_1 T_N A^a(\alpha - \beta s) \right. \right. \]
\[ + \left. \left. \frac{(T_M - T_N)}{2} \left( y_1 T_N A^a(\alpha - \beta s) + T_M A^a(\alpha - \beta s) - (1 - y_1) T_N A^a(\alpha - \beta s) \right) \right] \]
\[ IP_2 = \frac{cl_{s,\xi} (1 - y_1) A^a(\alpha - \beta s)}{2T} \left[ T_{\xi} + T_N (T - T_M) \right] \]
\[ Case \ 1.2: \ T \leq T_M \leq T + T_N \]
\[ IE = \frac{sl_t}{T} \left[ y_1 T_N A^a(\alpha - \beta s) \right. \right. \]
\[ + \left. \left. \frac{(T - T_N)}{2} \left( y_1 T_N A^a(\alpha - \beta s) + T A^a(\alpha - \beta s) - (1 - y_1) T_N A^a(\alpha - \beta s) \right) \right] \]
\[ + \left. \left. \frac{(T_M - T)}{2} \left( (1 + y_1) T A^a(\alpha - \beta s) - (1 - y_1) T_N A^a(\alpha - \beta s) (T_M - 2T_N) \right) \right] \]
\[ IP_2 = \frac{cl_{s,\xi} (1 - y_1) A^a(\alpha - \beta s)}{T} \left[ (T + T_N - T_M) \right] \]
\[ Case \ 1.3: \ 0 < T + T_N \leq T_M \]
\[ IE = \frac{sa^a(\alpha - \beta s)}{2T} [ y_1 T (2T_N - T) + T (y_1 + T_M - T_N) ] \]
\[ IP_2 = 0 \]

Now the total profit function is as follows:
\[ TP(s, \xi, A, T) = \left\{ \begin{array}{ll}
TP_1(s, \xi, A, T) & \text{if } T \geq T_M \\
TP_2(s, \xi, A, T) & \text{if } T \leq T_M \leq T + T_N \\
TP_3(s, \xi, A, T) & \text{if } 0 < T + T_N \leq T_M 
\end{array} \right. \]

For fixed selling price \( s \), \( TP_1(s, T_M) = TP_2(s, T_M) \) and \( TP_2(s, T_M - T_N) = TP_3(s, T_M - T_N) \). Therefore \( TP(s, \xi, A, T) \) is a continuous function of \( T > 0 \).

3.2. Case 2: \( T_N \geq T_M \)

In similar way, there are two main cases as follows: Case 2.1: \( T_M \leq T \) and Case 2.2: \( T \leq T_M \). The pictorial representation is shown in figure 4 and figure 5 respectively. The values of interest earned and paid, in both cases are as follows:

\[ Case \ 2.1: \ T_M \leq T \]
\[ IE = \gamma s l_{s,\xi} T M_a^a(\alpha - \beta s) \]
\[ IP_2 = \frac{cl_{s,\xi} (1 - y_1) A^a(\alpha - \beta s)}{2T} \left[ (T + T_M - T_N) + (T_N - T_M) \right] \]

\[ Case \ 2.2: \ T \leq T_M \]
\[ IE = \gamma s l_{s,\xi} T A^a(\alpha - \beta s) (T_M - T) + T_M \]
\[ IP_2 = \frac{cl_{s,\xi} (1 - y_1) A^a(\alpha - \beta s)}{2T} \left[ (T - T_M) + (T + T_N - T_M) \right] \]

Now the total profit function is as follows:
\[ TP(s, \xi, A, T) = \left\{ \begin{array}{ll}
TP_4(s, \xi, A, T) & \text{if } T \geq T_M \\
TP_5(s, \xi, A, T) & \text{if } T \leq T_M 
\end{array} \right. \]

For fixed selling price \( s \), \( P_4(s, T_M) = TP_5(s, T_M) \). Therefore \( TP(s, \xi, A, T) \) is a continuous function of \( T > 0 \).

Refer figure 1, 2, 3, 4 and 5.
Figure 1. The Inventory Functioning During the Period When $T_N \leq T_M \leq T$

Figure 2. The Inventory Functioning During the Period When $T_N \leq T_M$ and $T_M - T_N \leq T \leq T_M$

Figure 3. The Inventory Functioning during the period when $T_N \leq T_M$ and $T \leq T_M - T_N$
The sole aim of the system in both the cases is to find the optimum value of the total profit function.

For the optimization the necessary and sufficient conditions for \( i = 1, 2, 3, 4, 5 \) are as follows:

\[
\frac{\partial TP(T, s, A, \xi)}{\partial T} = 0; \quad \frac{\partial TP(T, s, A, \xi)}{\partial A} = 0; \\
\frac{\partial TP(T, s, A, \xi)}{\partial \xi} = 0; \quad \frac{\partial TP(T, s, A, \xi)}{\partial s} = 0;
\]

Provided \( \text{DET} (H1) < 0, \text{DET} (H2) < 0, \text{DET} (H3) < 0 \) where \( H1, H2, H3 \) are the principal minor of the Hessian matrix. Here is the Hessian Matrix of the total cost function for \( i = 1, 2, 3, 4, 5 \):
4. Numerical Illustration
To show the numerical verification, following data is used:

\[ C = $12 \text{/order}, \ C_A = $50/\text{unit}, \ C_D = $10, \ I_p = $0.12/\text{s/year}, \ I_c = $0.15/\text{year}, \gamma_1 = 0.4, \Theta = 0.05, \alpha = 151, \beta = 3.5, \ a = 3, \mu = 0.2, \ h_1 = $15/\text{unit/year}, \ h_2 = $0.5/\text{unit/year}. \]

**Case 1.** \( T_N \leq T_M: T_M = 0.15 \text{ year}, \ T_N = 0.1 \text{years}. \)

The optimal values of the parameters are as follows:

\[ TP(s^*, A^*, \xi^*, T^*) = $93830.23, \ s^* = $32.3652, \ A^* = 0.067, \xi = $12.79, \ T^* = 0.0535, \ Q^* = 156.65. \]

**Case 2.** \( T_M < T_N: T_M = 0.05 \text{ year}, \ T_N = 0.1 \text{years}. \)

The optimal values of the parameters are as follows:

\[ TP(s^*, A^*, \xi^*, T^*) = $57991.87, \ s^* = $31.1265, \ A^* = 0.065, \xi = $12.74, \ T^* = 0.035, \ Q^* = 150.65. \]

5. Sensitivity Analysis
To sensitize this study, a sensitivity analysis is done by varying values of some important parameters like \( C_A, \ C, \gamma_1 \) etc. The variations in decision variable and the profit function is mentioned in table 1.

| Change in parameter | Selling price \((s^*)\), $ | Preservation cost \((\xi^*)\), $ | Advertisement frequency \((A^*)\) | Order quantity \((Q^*)\) | Total Cycle length \((T^*)\), year | Total Profit function \((TP^*)\), $ |
|---------------------|---------------------------|-------------------------------|-----------------------------|------------------------|-------------------------------|-------------------------------|
| Case 1. when \( T_M \leq T_N \) |
| \( C_A \) | 50 | 31.1265 | 12.74 | 0.065 | 150.65 | 0.035 | 57991.87 |
| 55 | 31.6741 | 12.74 | 0.065 | 152.73 | 0.043 | 57782.67 |
| 60 | 32.1521 | 12.74 | 0.065 | 153.12 | 0.056 | 57512.32 |
| \( C \) | 15 | 32.7261 | 12.74 | 0.065 | 182.67 | 0.045 | 7127.91 |
| 20 | 33.1231 | 12.74 | 0.065 | 183.21 | 0.046 | 67213.61 |
| 25 | 34.8361 | 12.74 | 0.065 | 185.54 | 0.048 | 46889.71 |
| \( \gamma_1 \) | 0.3 | 31.8711 | 12.81 | 0.067 | 151.67 | 0.042 | 97281.17 |
| 0.5 | 31.1178 | 12.29 | 0.064 | 151.56 | 0.033 | 97280.12 |
| 0.8 | 31.0234 | 12.10 | 0.063 | 151.43 | 0.029 | 97279.01 |
| Case 2. when \( T_M < T_N \) |
| \( C_A \) | 52 | 31.2265 | 12.74 | 0.065 | 151.65 | 0.037 | 57999.87 |
| 56 | 31.8741 | 12.74 | 0.065 | 153.73 | 0.044 | 57789.67 |
| 62 | 32.4521 | 12.74 | 0.065 | 155.12 | 0.059 | 57518.32 |
| \( C \) | 15 | 32.8261 | 12.74 | 0.065 | 183.67 | 0.046 | 78128.91 |
| 20 | 33.6231 | 12.74 | 0.065 | 184.21 | 0.047 | 67215.61 |
| 25 | 34.9361 | 12.74 | 0.065 | 186.54 | 0.049 | 46888.71 |
| \( \gamma_1 \) | 0.3 | 31.9711 | 12.82 | 0.068 | 151.89 | 0.045 | 97282.17 |
| 0.5 | 31.2178 | 12.27 | 0.065 | 151.78 | 0.037 | 97279.12 |
| 0.8 | 31.0434 | 12.09 | 0.062 | 151.56 | 0.026 | 97276.01 |
The variation in the respective parameters reflects change in the values of total profit function the decision variables. This shows that this model is very sensitive with respect to its optimal values. The concavity of the total profit function is well presented through the graph in figure 6, figure 7 and figure 8.

**Figure 6.** The Concave Curve of Total Profit function (\(TP^*\)) with Respect to \(T^*\) and \(s^*\)

**Figure 7.** The Concave Curve of Total Profit Function (\(TP^*\)) with Respect to \((A^*)\) and \((\xi^*)\)
6. Conclusion
In the present study the optimal policies are established, to find the optimal profit function, order quantity, selling price, advertisement frequency, preservation technology cost and the total cycle length. The optimal values are calculated by using mathematical software Mathematica9. The concavity of the profit function is also presented with respect to the different decision variables like in figure 6 we have shown the optimality of profit with respect to total cycle length and selling price. In figure 7 concavity of profit function with variation in advertisement frequency and preservation technology cost. The optimality of profit function is well presented in figure 8 with respect to total cycle length and selling price. It is also observed that when a slight change made in the parameters related to this model the optimal value of profit function is also fluctuate. Which show the sensitivity of the study. The model can be further modified by inculcating other parameters of the inventory system.

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