Locality & Entanglement in Table-Top Testing of the Quantum Nature of Linearized Gravity

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This paper highlights the importance of the assumption of locality of physical interactions, and the concomitant necessity of the off-shell propagation of quanta between two non-relativistic test masses in probing the quantum nature of linearized gravity in the laboratory. At the outset, we will argue that observing the quantum nature of a system is not limited to evidencing $O(t)$ corrections to a classical theory: it instead hinges upon verifying tasks that a classical system cannot accomplish, which is the method adopted in the aforementioned tabletop experiments. We explain the background concepts needed from quantum field theory, namely forces arising through the exchange of virtual (off-shell) quanta, as well as the background exploited from quantum information theory, such as Local Operations and Classical Communication (LOCC) and entanglement witnesses. We clarify the key assumption inherent in our evidencing experiment, namely the locality of physical interactions, which is a generic feature of interacting systems of quantum fields around us, and naturally incorporates micro-causality in the description of our experiment. We also present the types of states the matter field must inhabit, putting the experiment on firm relativistic quantum field theoretic grounds. At the end we use a non-local (but not complete action at a distance) theory of gravity to illustrate how our mechanism may still be used to detect the qualitatively quantum nature of a force when the scale of non-locality is finite. We find that the scale of non-locality, including the entanglement entropy production in local/ non-local gravity, may be revealed from the results of our experiment.

I. INTRODUCTION

Recently, there have been two papers [1, 2] which have discussed the possibility of detecting quantum behavior of a linearized gravitational field in a table top experiment. The proposal crucially relies on local quantum interactions between matter and the gravitational field leading to the generation of entanglement between the two non-relativistic test masses, each initially prepared in a superposition of distinct spatial states. This entanglement is a proof of the quantum-ness of the mediator of the interaction is an off-shell (virtual) graviton. This is exchanged between the test masses, and through a tree-level diagram leads to the Newtonian interaction. Fundamentally, according to quantum field theory, forces between two sources (say two static charges) can be understood from the exchange of virtual particles between them – photons, $W^\pm$, $Z$ bosons and gluons – which are uncontroversially (by definition) quantum mechanical [13]. Similarly, in the low curvature regime, gravity can be regarded as perturbations on a background, and these perturbations can be regarded as a field. Within this setting, the Newtonian interaction between two masses can be considered as originating from the exchange of virtual gravitons [14], which puts gravity, at least in this regime, in exactly the same quantum footing as the other known fundamental forces of nature.

For the conclusion about the quantum nature of gravity to follow from the aforementioned entanglement, it is very important that “something” is exchanged between the test masses when they interact mutually through their Newtonian interaction. This point is often unclear when the proposed entanglement generation experiment is presented in terms of a direct Newtonian interaction between the test masses resulting in appropriate phase evolutions in their states which entangle the masses. In fact, that approach is adopted purely for convenience and we highlight here that there is a very well defined quantum mechanism for the Newtonian interaction where the entity which acts as a mediator of the interaction is an off-shell (virtual) graviton. This is exchanged between the test masses, and through a tree-level diagram leads to the Newtonian interaction. Fundamentally, according to quantum field theory, forces between two sources (say two static charges) can be understood from the exchange of virtual particles between them – photons, $W^\pm$, $Z$ bosons and gluons – which are uncontroversially (by definition) quantum mechanical [13]. Similarly, in the low curvature regime, gravity can be regarded as perturbations on a background, and these perturbations can be regarded as a field. Within this setting, the Newtonian interaction between two masses can be considered as originating from the exchange of virtual gravitons [14], which puts gravity, at least in this regime, in exactly the same quantum footing as the other known fundamental forces of nature.

However, the mere theoretical existence of a quantum mechanism for the origin of the Newtonian force does not prove that it is indeed that quantum mechanism that nature has decided to adopt – only experiments can do that. As far as current experimental evidences are concerned, it could equally well be a classical field generated by a source mass which affects a probe mass placed in that field – indeed there are several proposed classical and semiclassical mechanisms to generate a force with the same features as the Newtonian force [15–24]. How do we know whether any of these other mechanisms are adopted by nature or whether it is indeed the exchange of quantum off-shell gravitons?
Detecting the quantum nature of an entity has historically been through radical "qualitative" departures (such as the photoelectric effect detecting energy quantization, or matter wave interferometry detecting quantum superpositions or spin correlation measurements detecting the entanglement of two particles [25]) or through "quantitative" \( O(\hbar) \) quantum corrections to energies and interaction potentials. Among all the aforementioned strategies, hardly any seem to be adaptable readily to the case of a laboratory test for the quantum nature of gravity. Its on-shell quantum wavepackets, the gravitons (say, of a gravitational wave), carry too little energy, while any \( O(\hbar) \) quantum modifications of the Newtonian potential are too small to witness for currently available masses. The underlying quantum nature of the Newtonian force is completely hidden in a subset of experiments which look solely at the classical effects of the force field such as displacement of an object or the phase development of a wavefunction of a quantum object in that classical field. Furthermore, previous suggestions regarding observation of gravitational effects cannot unambiguously falsify quantum gravity [26]. Thus, the recent papers have had to propose an indirect strategy [1, 2]. If an agent entangles two quantum entities, the agent must be performing quantum communication between them, i.e., it must itself be a quantum entity. Through this idea, the generation of entanglement between two masses is used to witness the quantum nature of the agent acting between them. Several viewpoints have been presented regarding the interpretation and applications of this idea [1] (supplementary material), [2, 21, 27–33], noise analysis [34], and there have also been related independent suggestions [35, 36] and paradox resolutions [37, 38] which point towards the necessity of gravity to be quantum in nature.

In this paper, we seek to clarify the crucial assumptions underlying the claim that the witnessing of entanglement in the laboratory demonstrates the quantum nature of gravity. Moreover, we will show that it all works consistently within a quantum field theory context using a fully relativistically covariant formalism for the propagator. This also naturally clarifies how relativistic causality can be respected in the treatment of the above experiments. We start by laying down all our assumptions, the most important being the locality of physical interactions, in the above evidencing of quantum-ness. We clarify the manner in which the gravitational field would entangle the spins via the energy momentum tensor of the non relativistic mesoscopic superpositions. We will further clarify the necessity of the interaction to be through a quantum entity to allow such entanglement to form, by clarifying why local operations and classical communications (LOCC) cannot entangle the masses in our scenario. Specifically, as the term "communication" may sound somewhat cryptic to the physicist who thinks about interactions between fields, we show the impossibility of a classical gravitational field to create entanglement. The notion of classical field here is kept very general and automatically includes situations such as semi classical gravity (quantum matter sourcing a classical gravitational field) as well as were the matter is not strictly quantum mechanical in the usual sense - i.e., it has stochastic evolutions beyond standard quantum mechanics (e.g., when they are subject to fundamental collapse models) so that the gravitational field generated is also stochastic. As far as the experimental aspects are concerned, we emphasize why we seek the simplest statistical procedure to witness the entanglement rather than trying to estimate an entanglement measure. Given the fundamentally quantum field theoretic nature of all systems, one should also treat the test masses as described by quantum fields. In this context we present the type of states the matter field must be assumed to inhabit for a simple "bipartite" witnessing of the entanglement. Finally, adopting the example of a "non-local" theory of gravity, where there is a valid quantum propagator, we provide an example where our method can still be used to witness the underlying quantum nature of the field even though the theory is fundamentally non-local at some scale. In fact, this example illustrates that as long as the length scale of non-locality is "finite", our mechanism is a valid approach as there is the need for entities to propagate from point to point to convey an interaction. Interestingly enough, our experiment can also be used to reveal the length scale of non-locality, if present.

II. UNDERLYING ASSUMPTIONS

To begin with, it is worth highlighting the key assumption underlying the inference of the quantum nature of gravity from a tabletop experiment on gravitationally mediated entanglement

- **Locality of physical interactions**: One of the pillars of quantum field theory is the assumption of locality. All the interactions are assumed to be local at both classical and at a quantum level. Locality also ensures micro-causality \(^1\). In the context of gravity, the local interaction is given by:

\[
\kappa^2 h_{\mu\nu}(\vec{r}, t) T^{\mu\nu}(\vec{r}, t)
\]

where \(\kappa^2 = (8\pi G)^{-1}\), \(G = \hbar/M_p^2\) is Newton’s constant, \(M_p \sim 10^{19}\)GeV, \(\mu, \nu = 0, 1, 2, 3\) and we are working with signature \((-+, +, +, +)\). The energy momentum tensor of matter is given by \(T_{\mu\nu}\). The

\(^1\)Specifically, the field operators for two masses \(\phi_1(x_a)\) and \(\phi_2(x_b)\), where \(x_i\) are the four vectors for masses, minimally coupled through the gravitational field can be considered. When the two masses are space-like separated \(\Delta x^2(x_a - x_b) > 0\) and \(\{\phi_1(x_a), \phi_2(x_b)\} = 0\) and as such we have no faster than light signalling. Now of course when \(\Delta x^2(x_a - x_b) < 0\) and \(\{\phi_1(x_a), \phi_2(x_b)\} \neq 0\) and so all causal relationships will behave as expected.
metric perturbation around Minkowski background is

\[ g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \]

where \( \eta_{\mu\nu} \) is the Minkowski metric, and \( |\kappa h_{\mu\nu}| \ll 1 \), in order to maintain the linearity. \textit{A priori} \( h_{\mu\nu} \) need not be quantum at all. Though the matter part of the energy momentum tensor could be a quantum entity.

The concept of locality is also an important criteria from the perspective of quantum information and quantum entanglement. In particular, under LOCC, two particles exchanging only classical energy momentum will not lead to enhancement in entanglement. Note that while LOCC is used as a principle to define mixed state entanglement [39, 40], it can be easily proved when we start from an unentangled state of two objects as in the case of the experiments described in Refs.[1, 2]. In fact in these experiments, the two applications of locality, i.e., in defining local quantum field theories and in prohibiting entanglement generation at a distance without quantum communication, are brought together. It is very important to note that the locality is not proven through our experiment – that is not its purpose – locality is assumed from our knowledge of physical interactions in the observed regimes. It is the quantum aspect which we prove after assuming the locality.

Of course, as opposed to a local quantum theory, non-local field theories have also been developed since the days of Yukawa [41], and Pais and Uhlenbeck [42]. There has been a recent resurgence in understanding them as well in the context of field theory [43–46], and in quantum mechanics [47–49]. Note these are however not “action at a distance” theories. One of the features of a non-local theory is that it does not have a point support [45, 50, 51], therefore it is very helpful towards ameliorating some of the singularities in nature, such as point singularity due to gravitational \( 1/r \) potential [2, 3].

In this paper, as an alternative to local gravitational interaction, we will also study a non-local theory of gravity [46], and show, although nonlocal, its quantum nature can still be evidenced. This means that the type of locality assumption we require in our experiment is not prohibitively restrictive and depends on the scale of non-locality of the theory.

• \textbf{Linearized gravity:} Note that we are always working in a regime of weak field gravity, linearized around the Minkowski background. This also means that the gravitational potential is always bounded below unity. In fact, below the micrometre scale we have no direct constraint on Newtonian \( 1/r \) potential [63]. We are working in a regime of roughly > 100 microns, and for the masses under consideration, the gravitational interaction is indeed weak and justifies the treatment of linearized gravity. At distances > 100 microns the Casimir interaction is weaker than that of the gravitational interaction, see [1]. We have also outlined in Ref.[1] how to get rid of all the competing electromagnetic forces, so that the only force is gravitational, we have to also ensure that no as yet unknown “fifth force” acts here which essentially can also entangle the masses as a Newtonian force would do. This again, is easy to ensure for separations > 100 micron for which Newtonian gravity has been very well tested. Similarly, the velocities of the masses are firmly in the non-relativistic regime so that the physics is well described by the Newtonian regime.

• \textbf{A reasonable definition of a classical field:} We also clarify at the outset what we mean by a “classical” field, which we think is a very natural definition. Disagreements on this definition can, of course, result in a disagreement as to whether our experiment demonstrates a non-classicality. We simply define a classical field to be an entity which with general probabilities \( P_j \) has fixed (unique) values \( h^j_{\mu\nu} \) at each point of space-time (here we have used a tensor field in the definition, but it could

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2 Infinite derivatives acting on delta Dirac source does not have a point support. Let us consider a one-dimensional problem,

\[ e^{\alpha x^2} \delta(x) = \frac{1}{\sqrt{2\pi}} \int dk e^{-\alpha k^2} e^{ikx} = \frac{1}{\sqrt{2\alpha}} e^{-x^2/4\alpha}, \]

Note that the left hand side is a non-local operator acting on a delta Dirac source, with a scale of non-locality given by \( \alpha^{-1} \). The result is a Gaussian distribution. In a very similar fashion one can also resolve the singularity present in a rotating metric in general relativity [52], and the singularity due to a charged electron [53].

3 The non-local theories arise in many contexts in quantum gravity, in string theory, the notion of point objects are replaced by strings and branes [54], Dynamical triangulation [55] and loop quantum gravity [56] exploits Wilson operators which are inherently non-local. The string field theory introduces non-locality at the string scale, for a review [57], and infinite derivative ghost free theory of gravity (IDG) [46], which does not introduce any instability around a given background, is motivated from string field theory [58–60]. Especially, in string field theory and in IDG the non-locality appears only at the level of interactions. Note that loss of locality will also give rise to violation of micro-causality. However, it has been shown that for a specific class of non-local theories we are interested in here, the violation of causality is limited to the scale of non-locality [43, 44, 61, 62].

4 Recently, the bound on short distance gravitational potential has been improved but the constraints are for the Yukawa type gravitational potential, which depends also on the strength of the Yukawa interaction [63–65].
be scalar, spinor etc.). Of course a special case of that is when there are no probabilities at all – the field just has a value $h_{\mu\nu}$. There is a reason that we are using a much broader definition than simply a unique value – namely we are allowing also the probability of the field statistically having different values with different probabilities. This is just to carefully emphasize that the statistical nature of something does not make it quantum (think of a classical dice) – quantum comes with the possibility to go beyond statistical mixtures of field configurations to coherent superpositions of field configurations. Additionally, we demand that a classical field means that we are not even allowed to think of a Hilbert space for the field, i.e., even joint quantum states of fields with other (say, matter) systems is disallowed, i.e., states of the form $\sum_j \sqrt{P_j} |j, h_{\mu\nu}^j\rangle$ are not allowed. Only allowed joint states of quantized matter and classical field are the Probability distributions $P_j$ of configurations $\{ |j\rangle, h_{\mu\nu}^j \}$, where $h_{\mu\nu}^j$ is a tensor for each point in space time, but not an operator valued quantity. Here we are defining a classical field as, for example, used by Feynman during his 1957 debate with other researchers on the quantum nature of gravity [66] “... if I have an amplitude for a field, that’s what I would define as a quantized field.” So a classical field is one which has probabilities for various field configurations rather than amplitudes for various field configurations.

III. LINEARIZED QUANTUM GRAVITY

\[ S_{EH} = \frac{1}{4} \int d^4x \, h_{\mu\nu} \, \mathcal{O}^{\mu\nu\rho\sigma} \, h_{\rho\sigma} + \mathcal{O}(\kappa h^3), \quad (4) \]

where $O(\kappa h^3)$ takes into account of higher order terms in the perturbation, while the four-rank operator $\mathcal{O}^{\mu\nu\rho\sigma}$ is totally symmetric in all its indices and defined as

\[
\mathcal{O}^{\mu\nu\rho\sigma} := \frac{1}{4} \left( \eta^{\mu\nu} \eta^{\rho\sigma} + \eta^{\mu\rho} \eta^{\nu\sigma} \right) \Box - \frac{1}{2} \eta^{\mu\nu} \eta^{\rho\sigma} \Box \\
+ \frac{1}{2} \left( \eta^{\mu\rho} \partial^\nu \partial^\sigma + \eta^{\rho\sigma} \partial^\mu \partial^\nu - \eta^{\mu\nu} \partial^\rho \partial^\sigma - \eta^{\mu\sigma} \partial^\nu \partial^\rho \right),
\]

for $\Box = \eta_{\mu\nu} \nabla^\mu \nabla^\nu$. By inverting the kinetic operator we obtain the graviton propagator around the Minkowski background, and its saturated between two conserved currents (in our case these are energy momentum tensors, see below Eq.(7)) and gauge independent part is given by [67, 68]

\[
\Pi_{\mu\nu\rho\sigma}(k) = \left( \frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{k^2} - \frac{\mathcal{P}^0_{\mu\nu\rho\sigma}}{2k^2} \right), \quad (6)
\]

where $\mathcal{P}^2$ and $\mathcal{P}^0$ are two spin projection operators projecting along the spin-2 and spin-0 components, respectively; see Refs.[67, 68] for further details. The Newtonian potential between the two masses, $T_{\mu\nu} \sim m \delta^0_0 \delta^{(3)}(\vec{r})$ and the unit mass $T_{\mu\nu} \sim \delta^0_0 \delta^{(3)}(0)$ can be computed. For a non-relativistic setup, we are only interested in 00 components. The potential can be given by integrating all the momenta of the off-shell graviton

\[
\Phi(r) = -\kappa^2 \int \frac{d^3|k|}{(2\pi)^3} T^{00}_1(k) \Pi_{0000}(k) T^{00}_2(-k) e^{i\vec{k} \cdot \vec{r}} = -\frac{\kappa^2 m}{2} \int \frac{d^3|k|}{(2\pi)^3} \frac{1}{k^2} e^{i\vec{k} \cdot \vec{r}} = -\frac{G m}{r}, \quad (7)
\]

which recovers the Newtonian potential. Note that we are integrating over the off-shell massless graviton, which is the crucial aspect of quantum-ness of the graviton in this scattering amplitude. The exchange here is inherently quantum through the propagator, there is nothing classical about this exchange. The matter components along with the graviton all obeys the quantum principle, through the interaction, see Eq. 1. If gravity were treated classically, we would expect momentum transfer, but there would be no propagator at all to mediate the momentum transfer. This will be reminiscent to action at a distance principle.

For different modifications of the graviton propagator the potential will be different, for instance if there is an extra scalar degree of freedom propagating, then potential will be Yukawa type. We will consider one such modification in the non-local setup. The analysis of this section, however, provides the quantum mechanism necessary to make sense of the result that the observation of entanglement generation mediated by gravity implies the quantum nature of linearized gravity.

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5 By definition, off-shell graviton does not obey the classical equations of motion. The graviton propagator in general relativity can be recast as: $\Pi_{\mu\nu\rho\sigma}(\vec{k}) = \frac{1}{2k^2} (\eta_{\mu\rho} \eta_{\nu\sigma} + \eta_{\mu\sigma} \eta_{\nu\rho} - \eta_{\mu\nu} \eta_{\rho\sigma})$. 
To highlight the role of a quantum channel in the formation of entanglement we will consider the possibility of entanglement mediated by a purely classical gravitational field and show that it is impossible. At first, for simplicity, we will consider the case of a deterministic gravitational field \( h_{\mu\nu} \) which has a fixed value at each point of space-time. The matter states consist of two quantized systems \( A \) and \( B \) at separate locations (we will consider the description of these matter states as states of matter fields in a later section). The gravity-matter coupling is given by \( h_{\mu\nu}T^{\mu\nu} \) for a quantum operator \( T^{\mu\nu} \). This will give rise to solely unitary evolutions of the quantized matter systems. To clarify this, we consider three time steps, \( t < 0 \) before the spatial superposition is created at time \( t = 0 \), \( 0 \leq t \leq \delta t \), after the superposition has been created but before any information has had time to propagate between \( A \) and \( B \), and finally \( t > \delta t \) when both \( A \) and \( B \) are within the light cone of the other mass’ past when its in a superposition (Here \( t \) is the time in the laboratory frame, assuming very low curvatures of the background, such as for an experiment conducted on earth).

When \( t < 0 \), the mass states will be in a product (unentangled) state \( |\psi(t)\rangle_A \otimes |\phi(t)\rangle_B \) (because this is how they are prepared at the start in the experiments under consideration) and the gravitational field will be \( h_{\mu\nu}(\vec{r}, t < 0) \). Trivially any evolution occurring in the systems \( A \) and \( B \) will not induce entanglement as the superposition states to be entangled do not exist (cf section V for the outline of the protocol which starts with the preparation of pure superpositions of spatial states of each system at a time \( t = 0 \)). For \( 0 \leq t \leq \delta t \), the mass states will evolve according to the unitary operator \( U_A(t, |\psi(t)\rangle_A \otimes |\phi(t)\rangle_B) \otimes U_B(t, |\psi(t)\rangle_B \otimes |\phi(t)\rangle_A) \), with the gravitational field in the regions of the two masses will be of the form \( h_{\mu\nu}(\vec{r}_A, t) = f(|\psi(t)\rangle_A \otimes |\phi(t)\rangle_B) \) and \( h_{\mu\nu}(\vec{r}_B, t) = f(|\psi(t)\rangle_B \otimes |\phi(t)\rangle_A) \). At later times, once both superposition states have interacted through the gravitational field \( t > \delta t \), the gravitational field will be of the form \( h_{\mu\nu}(\vec{r}, t) = f(|\psi(t)\rangle_A \otimes |\phi(t)\rangle_B) \), and consequently, the evolution operator will be of the form \( U_A(t, |\psi(t)\rangle_A \otimes |\phi(t)\rangle_B) \otimes U_B(t, |\psi(t)\rangle_B \otimes |\phi(t)\rangle_A) \). In other words, each system can evolve in a time dependent fashion, and depending on the state of both systems, however the evolution occurs in a separable manner.

The above description can be expanded to allow a probabilistic gravitational field \( h_{\mu\nu}(\vec{r}, t, j) \) whereby the field value depends probabilistically on, for example, it ‘measuring’ the location of the matter, giving the \( j \)th result with probability \( P(j) \). In such a case however the result is still qualitatively the same, with the state’s evolution operators being of the form \( U_A(t, |\psi(t)\rangle_A \otimes |\phi(t)\rangle_B) \), again taking separable states to separable states. As such it can clearly be seen that at no stage can entanglement between the two masses form via a purely classical intermediary.

Now we will consider the situation in a even more general context. We will allow arbitrary sets of local operations, \( \{A_j\} \) and \( \{B_j\} \) that can act on systems \( A \) and \( B \) respectively. These include unitary operations, more generally completely positive maps, as well as the action of general measurement operators (as given by Kraus operators). These operations can be thought of as enacted by experimentalists Alice or Bob, or, alternatively, occurs due to natural evolution of the system in isolation or due to the interaction with a classical field (unitary) or even through fundamental nonlinear modifications of quantum dynamics leading to intrinsic measurement operations (as in collapse models). Alice (or nature) can then act on system \( A \) to produce the state

\[
\rho_{A,j} = \hat{A}_j|\psi\rangle\langle\psi|\hat{A}_j^\dagger, \tag{8}
\]

where we have ignored normalization. In the above, the symbol \( j \) comes in only if the operation on \( A \) was a generalized measurement (by Alice or due to some dynamical collapse model) with \( \rho_{A,j} \) corresponding to the \( j \)th outcome. This leads to the gravitational field \( h_{\mu\nu}(t, \vec{r}, j) \) derived as a function of \( \rho_{A,j} \) (the dependence of the classical field \( h_{\mu\nu}(t, \vec{r}, j) \) on \( \rho_{A,j} \) can be according to any rule). Now depending on this \( h_{\mu\nu}(\vec{r}, t, j) \), system \( B \) will evolve as

\[
\rho_B^{(j)} = \hat{B}_{k}^{(j)}|\phi\rangle\langle\phi|\hat{B}_{k}^{(j)} \tag{9}
\]

which again manipulates the field, now giving \( h_{\mu\nu}(t, \vec{r}, j, k) \). Taking an initial state as a product state (again, this is by virtue of the protocols, cf V), the states at times \( t = 0 \) and \( t = \delta t \) will be

\[
\rho(t = 0) = \rho_A \otimes \rho_B, \quad \rho(t = \delta t) = \sum_j \hat{A}_j|\psi\rangle\langle\psi|\hat{A}_j^\dagger \otimes \hat{B}_{k}^{(j)}|\phi\rangle\langle\phi|\hat{B}_{k}^{(j)}, \tag{10}
\]

which is a product state. At this state we can consider the continued swapping of information between \( A \) and \( B \) as time increases. Furthermore we have here treated the gravitational field as if it only transmits information in a single direction at a time rather. Realistically the information originating at \( A \) and \( B \) travels to \( A \) and \( B \) respectively at the same time, however this yields qualitatively similar results, only with more subscripts and so was left off for clarity. To extend this to a completely general case we must allow for an arbitrary separable initial state, which can be a probabilistic summation over product states, giving the state

\[
\rho(t = \delta t) = \sum_l \sum_j p(l) \hat{A}_j|\psi\rangle\langle\psi|\hat{A}_j^\dagger \otimes \hat{B}_{k}^{(j)}|\phi\rangle\langle\phi|\hat{B}_{k}^{(j)}. \tag{11}
\]
Because of the generality of the above treatment, it follows that starting from a separable state of two systems $A$ and $B$, quantum entanglement cannot be generated by any model in which gravity is a classical field (classical according to the very reasonable definition given in section II). This automatically encompasses all specific models such as the Møller-Rosenfeld semiclassical gravity model or models where the matter field undergoes collapses and sources a stochastic classical gravitational field [69, 70].

V. ENTANGLEMENT IN GRAVITATIONALLY INTERACTING INTERFEROMETER

For completeness, in this section will provide an overview of the experiment being discussed throughout this paper. The set-up, shown in Fig. 2, consists of two mesoscopic mass ($\sim 10^{-14}$kg) microspheres with embedded spins traversing two Stern-Gerlach interferometers in close proximity to one another. The two masses become entangled due to the varying gravitational interaction between them due to the differing separations of the interferometer arms. The interferometric process is completed by bringing together the two spatial wavepackets which leads to the path phase differences being imprinted into the particles spin state, with any entanglement measured by their spin correlations (cf Sec. VI).

Each mass will initially be in a spatial superposition of being both ‘left’ and ‘right’ with the two particle joint state as a function of the form $\langle ab \rangle$ where $a \in \{L, r\}$, $b \in \{L, R\}$ as shown in Fig. 2. The two masses are treated as non-relativistic (stationary) point particles, both with mass $m$ such that the only non-zero component of the stress energy tensor will be

$$T^{00} = m\delta^3(\vec{x} - \vec{x}_a) + m\delta^3(\vec{x} - \vec{x}_b)$$

(12)

It is perhaps worth clarifying that given the spatial superpositions, we do not have $T^{00}$ but instead in Eq. 12, it is $\vec{x}_a$ and $\vec{x}_b$ which are in superpositions.

To model the results of interactions between two particles, both in superposition states, we employ the Feynman style logic treating the resulting total state as the sum of four individual amplitudes, each belonging to the separate field configurations created by each possible joint state for the matter, with each component evolving as

$$\langle ab \rangle \to e^{-i\frac{\phi_{ab}}{\hbar c}} \langle ab \rangle$$

(13)

where this evolution is derived from Eq. 7 and $\tau$ is the interaction time, implicitly assuming each mass is within the light cone of the other situated with its origin at the point in which the superposition is created ($t = 0$). Using Eq. 13, and considering the four interactions shown as solid grey lines in Fig 2 gives

$$|\psi(0 \leq t < \delta t)\rangle = \frac{1}{\sqrt{2}} (|l\rangle + |r\rangle) \otimes \frac{1}{\sqrt{2}} (|L\rangle + |R\rangle),$$

(14)

$$|\psi(t = \tau + \delta t)\rangle = \frac{1}{2} \left( e^{-i\frac{Gm^2}{\hbar c^2 L}} |L\rangle + e^{-i\frac{Gm^2}{\hbar c^2 R}} |R\rangle \right)$$

(15)

giving the entanglement between the two masses as found in [1], where $r_{ab} = |\vec{x}_a - \vec{x}_b|$ is the distance between the two masses. As such we have the standard newtonian potential appearing to mediate the interaction between the two masses.

Quantum Field Theoretic Notation for the Matter States: Strictly speaking, as presented in Eq. 12, $T^{00}$ is not a recognisable object within RQFT, instead the masses should be considered as excitations of a quantum field [71], such as a fock state of fields $\hat{\phi}^1$ and $\hat{\phi}^2$ where

$$|\langle ab \rangle \rangle = \left( \hat{\phi}^1(a) \hat{\phi}^2(b) \right) |0\rangle.$$  

(16)
Here $\hat{\phi}_l^i(x)$ is the creation operator which creates a mass centred at $x$ and where each object is in a spatial superposition

$$\frac{1}{\sqrt{V}} \left( \hat{\phi}_l^i(l) + \hat{\phi}_r^i(r) \right) |0\rangle$$  \hspace{1cm} (17)

such that there cannot be any interaction between the two arms within an interferometer of the form shown in Fig 2 by $U_{lr}$. For each mass, the mass field must be in a state qualitatively similar to Eq. 17. In the proposals the mass states are taken to be in this exact state. One can also identify each mass as, rather than a single object, a collection of fundamental particles in a NOON state $(|n,0\rangle + |0,n\rangle)$, where $n$ is the number of fundamental particles (nucleons, electrons etc.). Note that here the crystalline nature of the masses will ensure that they are indeed in a NOON state. If on the other hand, for example, a Bose-Einstein Condensate (BEC) is used one might expect something more akin to coherently mixing states in each arm of the interferometer of the form $e^{-i\omega t} e^{-i\alpha_l\hat{\phi}_l^i(l)\hat{\phi}_r^i(r)} |0\rangle$. This would create interactions of the form $U_{lr}$ which will not result in entanglement of the form necessary to demonstrate the quantum nature of gravity. Furthermore it is desirable to use highly localised states such that $U_{lr}$ type interactions as shown in Fig. 2. If a continuous stream of particles were used, such interactions ($U_{aur/lbr}$ and $U_{lr/lbr}$) could overwhelm any signal from the inter-arm interactions ($U_{ab}$), effectively overwhelming the entangling signal in the noise of these other interactions. For this reason NOON states of BECs would have to be used [72]. For example, consider if the mass state employed was $|\psi(0 \leq t < \delta t)\rangle = \hat{\phi}_l^i(l) \hat{\phi}_r^i(r) \hat{\phi}_k^i(L) \hat{\phi}_k^i(R) |0\rangle$, then interactions of the form $U_{lr}$ would be allowed, and Eq. 15 would become

$$|\psi(t = \tau + \delta t)\rangle = e^{-iS(a_{\alpha l}^a + \tau + i\alpha L + \tau + i\alpha R)} \times \hat{\phi}_l^i(l) \hat{\phi}_r^i(r) \hat{\phi}_k^i(L) \hat{\phi}_k^i(R) |0\rangle$$  \hspace{1cm} (18)

which is clearly not an entangled state. Thus it is necessary to prepare the matter states in NOON states of a quantum field during the initialisation of the experiment.

VI. WITNESSING ENTANGLEMENT THROUGH MEASUREMENT STATISTICS

The experimental proposal [1] will result in an output state consisting of two entangled spin qubits (that is of course assuming gravity is quantum). To understand how such entanglement is verified, it is worth discussing what quantum entanglement is. For a bipartite state to be entangled the state cannot be written as the tensor product of the states of each particles, that is, a state is not entangled (it is separable) if it can be written

$$\rho = \sum_j p(j) \rho_{A,j} \otimes \rho_{B,j}$$  \hspace{1cm} (19)

where $|\phi\rangle_A$ and $|\chi\rangle_B$ are arbitrary states belonging to the Hilbert space of particles A and B respectively and $\sum_j p(j) = 1$. If we restrict ourselves to bipartite, pure qubit states, then we can understand and quantify entanglement by the Von Neumann entropy of the reduced density matrix, defined as

$$S(\hat{\rho}_A) = -Tr[\hat{\rho}_A \log(\hat{\rho}_A)].$$  \hspace{1cm} (20)

Take for example the maximally entangled, product state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$  \hspace{1cm} (21)

then we have a corresponding density matrix $\hat{\rho} = |\psi\rangle \langle \psi |$. Tracing out one of the particles leaves a reduced density matrix

$$\hat{\rho}_A = Tr_B(\hat{\rho}) \propto I_A$$  \hspace{1cm} (22)

which corresponds to a maximal entropy state, where $S(\hat{\rho}_A) = 1$. This can be understood as fully entangled particles will contain information about the other particle too, by throwing away the information held by only one of the particles (tracing it out), the result contains no useful information. If the initial state was instead separable, then the reduced density matrix would correspond to that for a completely ordered state. In view of the above, one might expect witnessing the masses initially in pure states (low entropy) evolving into mixed states (high entropy) would prove entanglement, however this is not the case. In a realistic experiment, decoherence (such as entanglement with the environment) which create mixed state, and so also maximise entropy, cannot be ruled out. As such no conclusion could be drawn from actual measurements of the entropy.

Alternatively entanglement measures which are compatible with mixed states can be used, for example concurrence or an entanglement witness can be used. The concurrence can be calculated for a general (pure or mixed) two qubit state, which the two spin states can be thought of as, using

$$C(\rho) = \max \left\{ 0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 \right\}$$  \hspace{1cm} (23)

where $\lambda_i$ is the square root of the eigenvalues of the matrix $\rho^T$ arranged in decreasing order, for $\rho = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$. Again, this is maximised by maximally entangled states such as Eq. 21, which gives $C(\rho) = 1$. However, to calculate the concurrence the entire states density operator is needed which requires full state tomography, a measurement intensive process (requires 6 expectation and 9 correlation measurements). To avoid this entanglement witnesses can be used, which looks at correlations between the two particles, in this way any measured entanglement is confirmed to be between the two particles and not one particle and its environment. Such an entanglement witness $W(\hat{\rho})$ is defined
such that it has the property that it evaluates to greater than 1 only if $\hat{\rho}$ is entangled. It is important to note that the converse is not true, that is, if it is not greater than 1, it does not imply anything about $\hat{\rho}$. Furthermore such witnesses need to be created to detect the specific entangled state which can be difficult in general, and will necessarily detect different states as entangled, even if it is maximally entangled. However, due to the simple nature of the final state, a suitable witness was found to be

$$\mathcal{W} = \left| \left\langle \sigma_x^{(1)} \otimes \sigma_z^{(2)} \right| - \left\langle \sigma_y^{(1)} \otimes \sigma_y^{(2)} \right\rangle \right|,$$

(24)

which is sufficient for discriminating the entanglement as it is expected to develop in the tabletop experiment. It also only requires two sets of measurements for each particle.

VII. NON-LOCAL GRAVITY

The entanglement experiment protocol is also not limited to probing the quantum nature of local gravitational models, it could also be used to probe the quantum nature of gravity which is non-local over a microscopic scale as well as modifications to the gravitational potential at short distances. For instance, modifications of gravity in the ultraviolet. The most general quadratic action in 4 dimensions, which is invariant under parity and also torsion-free is given by [46]

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left\{ \mathcal{R} + \beta (\mathcal{R} \mathcal{F}_1(\Box_s)) \mathcal{R} + \mathcal{R}_{\mu\nu} \mathcal{F}_2(\Box_s) \mathcal{R}_{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma} \mathcal{F}_3(\Box_s) \mathcal{R}_{\mu\nu\rho\sigma} \right\},$$

(25)

where $\Box_s = \Box/M_s^2$ and $M_s$ is considered as the fundamental scale of non-locality, which in the context of string theory corresponds to the string scale. Within $M_s^{-1}$ the micro-causality is violated [43, 44, 61, 62]. For $\Box \ll M_s^2$, the theory becomes that of a local theory with a low energy limit given purely by the Einstein-Hilbert action [46]. Furthermore, by considering such a modified gravity we are also demonstrating that our local gravity assumption is not as strict as it might appear provided the locality is violated at a microscopic level and the time and length scale of our experimental setup is larger than $M_s^{-1}$. The three gravitational form-factors $\mathcal{F}_i(\Box_s)$ are covariant functions of the d’Alembertian and can be uniquely determined around the Minkowski background [46, 73]. We can set $\mathcal{F}_3(\Box_s) = 0$, without loss of generality up to quadratic order in the metric perturbation around the flat background, we can keep the massless spin-2 graviton as the only dynamical degree of freedom by imposing the following condition [6]: $2\mathcal{F}_1(\Box_s) = -\mathcal{F}_2(\Box_s)$ as shown in Ref. [46] around the Minkowski background. By expanding around Minkowski, $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, we obtain

$$S = \frac{1}{4} \int d^4x h_{\mu\nu} (1 - \mathcal{F}_1(\Box_s)) \mathcal{O}^{\mu\nu\rho\sigma} h_{\rho\sigma} + \mathcal{O}(\kappa^3),$$

(26)

and the saturated and gauge independent part of the propagator is given by [46, 68]

$$\Pi_{\mu\nu\rho\sigma}(k) = \frac{1}{1 + \mathcal{F}_1(k^2/M_s^2)} \left( \frac{\mathcal{P}_{\mu\nu\rho\sigma}}{k^2} - \mathcal{P}^0_{\mu\nu\rho\sigma} \right),$$

(27)

where $\mathcal{P}^2/k^2 - \mathcal{P}^0/2k^2$ is the graviton propagator of Einstein’s GR, see Eq.(6). Note that in order not to introduce any extra dynamical degrees of freedom other than the massless spin-2 graviton, we need to require that the function $1 + \mathcal{F}_1(k^2/M_s^2)$ does not have any zeros, i.e. that it is an exponential of an entire function [46]:

$$1 + \mathcal{F}_1(k^2/M_s^2) = e^{\gamma(k^2/M_s^2)},$$

(28)

where the $\gamma(k^2/M_s^2)$ is an entire function. We will mainly work with the simplest choice $\gamma(k^2) = k^2/M_s^2$, see also Ref. [77, 78] for other examples of entire functions. In all these examples the short distance behaviour becomes soft and in the IR the gravitational potential matches that of Newtonian prediction. Now we can compute the scattering diagram. The key difference from a local gravitational theory is that now the existence of a new scale, $M_s$, which determines the interaction at short distances. For $k^2 \ll M_s^2$, the non-local contribution becomes exponentially small, or in length scale $r > M_s^{-1}$, the theory predicts the results of local Einstein-Hilbert action.

We can now compute the gravitational potential by integrating all the momenta of the off-shell graviton, assuming the two vertices are non-relativistic. Essentially, taking the $T_{00}$ components only, and with modified graviton propagator, we obtain:

$$\Phi_{IDG}(r) = -\kappa^2 \int \frac{d^3|\vec{k}|}{(2\pi)^3} T_{00}(k) \Pi_{0000}(k) T_{22}(-k) e^{i\vec{k} \cdot \vec{r}}$$

$$= -\kappa^2 \frac{m}{2} \int \frac{d^3|\vec{k}|}{(2\pi)^3} e^{-\vec{k}^2/M_s^2} \frac{M_s^2}{k^2} e^{i\vec{k} \cdot \vec{r}},$$

$$= -\frac{Gm}{r} \text{Erf} \left( \frac{M_s r}{2} \right).$$

(29)

Note that the gravitational potential is now modified.

In particular when $r < 2/M_s$, the error function increases linearly with $r$, which cancels the denominator. Therefore at short distances, for $r < 2/M_s$, the gravitational potential becomes constant and given by:

$$\Phi_{IDG}(r) \sim \frac{GmM_s}{\sqrt{\pi}},$$

(30)

6 In this paper we will only consider analytic form-factors. However, it is worth mentioning that non-local models with non-analytic differential operators have been investigated by many authors; see, for example, Refs. [74–76].
while for \( r > 2/M_s \), the error function approaches \( \pm 1 \), and therefore the potential recovers the standard newtonian potential, \(-Gm/r\), as seen in Fig. 3.

One can compute various gravitational invariances, including the Kretschmann invariant, which remains constant as \( r \to 0 \), see [79]. Indeed, note that this computation has been performed in the linear theory. To be consistent here, the gravitational singularity is ameliorated when the gravitational potential is still within the linear regime.

\[
2|\Phi_{IDG}(r)| < 1, \quad m M_s < M_p^2, \tag{31}
\]

FIG. 3: Potential energy per unit test mass as generated by a \( m = 10^{-14} \) kg source mass for both the standard newtonian potential (\( \Phi_N \)) and the modified infinite derivative gravity potential (\( \Phi_{IDG} \)). The non-local parameter for \( \Phi_{IDG} \) was set to \( M_s = 0.004 \) which corresponds to a non-local range \( \lambda = 5 \times 10^{-5} \) m, see [78].

Since the entanglement phase depends on the potential, at short distances \( r < 2/M_s \) the gravitational potential approaches constant as long as the inter separation distance is well within the non-local region. It has also been shown that non-locality never exceeds beyond the non-local scale of \( M_s \), see for instance [42, 43, 61, 62]. Therefore, if all superposition components of the two masses are well inside the radius of \( r = 2/M_s \), the entanglement phase, which is dependent on the potential varying for different spin components, will linearly go to zero. This has indeed very intriguing repercussions for the entanglement phase, despite the fact that the treatment of the linearized graviton remains quantum. The non-local interaction weakens the gravitational potential by smoothening out the spacetime. This serves as an interesting example how non-local interactions can alter the quantum behaviour of the many body system. However, for \( r > 2/M_s \), the entanglement phase is the same as that of general relativity, which is similar to our previous local case.

The entanglement witness experiment results can be quantified by the two parameters \( \Delta \Phi_{LR} \) and \( \Delta \Phi_{RL} \) which we can compare for the two gravitational potentials considered here. For an experimental set-up involving \( 10^{-14} \) kg masses, \( 2.5 \times 10^{-4} \) m superpositions and a minimum separation of \( 2 \times 10^{-4} \) m, assuming standard Newtonian gravity, \( \Delta \Phi_{LR} = -0.125 \) rad and \( \Delta \Phi_{RL} = 0.439 \) rad, whereas for IDG \( \Delta \Phi_{LR} = -0.125 \) rad and \( \Delta \Phi_{RL} = 0.435 \) rad, for \( M_s = 0.004 \) eV, which corresponds to \( 5 \times 10^{-5} \) m. This translates to an expected entanglement witness value \( \mathcal{W} = 1.223 \) with IDG compared to \( \mathcal{W} = 1.224 \) for standard Newtonian gravity.

Given the power of entanglement entropy in fully quantifying the amount of entanglement in pure states, and its current importance in quantifying entanglement in quantum field theories [80], it can also be insightful to consider. Furthermore, although incredibly difficult, if we could ensure that the two-mass state, and eventually the two spin state to which the entanglement is mapped, remains pure, we can measure the full density matrix for one of the qubits with only 3 spin measurement settings and from that calculate the entanglement entropy given by Eq. 20. The entanglement entropy for the experiment, given by

\[
\mathcal{S} (\hat{\rho}_A) = - (\lambda_- \log_2 (\lambda_-) + \lambda_+ \log_2 (\lambda_+)) \tag{32}
\]

where

\[
\lambda_\pm = 1 \pm \frac{1}{2} \left[ 1 + \cos \left( \frac{m\tau}{\hbar} (\Phi (r_0 - \Delta x) + \Phi (r_0 + \Delta x) - 2\Phi (r_0)) \right) \right]^{1/2}, \tag{33}
\]

\( r_0 \) and \( \Delta x \) are the distance between the centre of the interferometers and superposition size respectively, is shown in Fig. 4 for both gravitational potentials. See Appendix A for more detail. In the experimental proposal, using time \( \tau \approx 2.5 \) we can see, although it is small, there is a quantitative difference between the two gravitational potentials with \( \mathcal{S} (\hat{\rho}_A) = 0.054 \) for a Newtonian potential and \( \mathcal{S} (\hat{\rho}_A) = 0.053 \) for IDG. The figure also shows that there is very little entanglement in the output state, which tends to zero for increasing separations, which is a result of the weakness and spatial dependence of gravity. As such there would be slight changes (revealing the scale \( M_s \)) in the result however, as the experiment is conducted outside the non-local region, all conclusions still hold, even in presence of non-local gravitational interaction.

VIII. CONCLUSION

In this paper we have highlighted the key assumptions made in the paper [1], in order to clarify what is meant by the statement that witnessing entanglement in the proposed experiment verifies the quantum nature of the gravitational field. First, we have presented the manner
in which General Relativity lends itself to be quantized in a linearized limit. Doing so predicts the existence of gravitons and its minimal coupling to matter, including the off-shell gravitons, the exchange of which leads to the Newtonian gravitational force. Such an interaction is not by itself sufficient to mediate the formulation of entanglement. Specifically entanglement will require the interaction to be combined with the linearity of superpositions as highlighted in Eq. 15. It is only with the off-shell graviton (quantum) source for the newtonian potential interaction and the potential itself in a superposition that entanglement can be generated. Furthermore through the premise of LOCC (as clarified in Sec. IV, for quantum masses sourcing a classical mediating field), we know that the mediating channel, i.e. the gravitational field, must be quantum for the formation of entanglement. The gravitational field is then also evidenced to be not just quantum but to exist in a coherent quantum superposition. Further, the fact that the matter states can be described as a quantized field has been clarified, including the fact that in this case these are in superpositions of Fock states and, more appropriately, when one considers the microscopic constituents, in NOON states. As microcausality (cf. footnote 1 in Sec. II) is built into the standard relativistic quantisation, within which the virtual graviton exchange process acts, here the question of whether the Newtonian force is fundamentally action at a distance does not arise. As long as the masses are within each other’s light cone, the potential is given by Eq. 7 and vanishes outside it.

We have also provided an example of non-local ghost free theory of gravity, where the gravitational potential is modified drastically to resolve the $1/r$ singularity. In this scenario, the gravitational interaction with matter becomes non-local, and provides a different prediction for the entanglement phase inside the non-local regime. Since, the experiment is always conducted outside the non-local region no significant change would be expected and this highlights that all our conclusions can still hold, even after breaking the local gravity assumption. Also of interest is the fact that the entanglement entropy, arising from local/non-local gravity, can be determined by the proposed experiment through measurements of the final spin states.

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Appendix A: Entropy Calculations

To calculate the entropy we begin by considering the entangled state

$$|\psi\rangle = \frac{1}{2} (|\downarrow\downarrow\rangle + e^{i\Delta\theta} |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle + e^{i\Delta\phi} |\uparrow\uparrow\rangle)$$  \hspace{1cm} (A1)

where $\Delta\theta = \frac{mG}{\hbar} (\Phi (r_0 - \Delta x) - \Phi (r_0))$ and $\Delta\phi = \frac{mG}{\hbar} (\Phi (r_0 + \Delta x) - \Phi (r_0))$ for an average distance between the interferometer arms $r_0$ and superposition size $\Delta x$.

The density matrix defined as

$$\hat{\rho} = |\psi\rangle \langle \psi|$$  \hspace{1cm} (A2)

and the reduced density matrix is then

$$\hat{\rho}_A = Tr_B [\hat{\rho}] = \left[ e^{-i\Delta\theta} + e^{i\Delta\phi} \frac{1}{2} \right].$$  \hspace{1cm} (A3)

Now the eigenvalues of $\hat{\rho}_A$ can be used to calculate the entropy, using the standard eigenvalue equation we have

$$0 = |\hat{\rho}_A - \lambda I|$$  \hspace{1cm} (A4)

$$0 = \left( \frac{1}{2} - \lambda \right)^2 - \left( e^{i\Delta\theta} + e^{-i\Delta\phi} \right) \left( e^{-i\Delta\theta} + e^{i\Delta\phi} \right)$$  \hspace{1cm} (A5)

$$\Rightarrow \lambda_{\pm} = \frac{1}{2} \pm \frac{1}{2} \left[ 1 + \cos \left( \frac{mG}{\hbar} (\Phi (r_0 - \Delta x) + \Phi (r_0 + \Delta x) - 2\Phi (r_0)) \right) \right]^{1/2}$$  \hspace{1cm} (A6)

and so finally giving the entropy as

$$S(\hat{\rho}_A) = - (\lambda_- \log_2 (\lambda_-) + \lambda_+ \log_2 (\lambda_+)).$$  \hspace{1cm} (A7)