Model-independent test of $T$ violation in neutrino oscillations

Alejandro Segarra

Based (mostly) on
T. Schwetz and AS, arXiv:2106.16099
Neutrino oscillations

\[
\begin{bmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{bmatrix} = 
\begin{bmatrix}
U_{PMNS} \\
(\theta_{12}, \theta_{13}, \theta_{23}, e^{i\delta})
\end{bmatrix}
\begin{bmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{bmatrix}
\]

★ Precision era:

\[
\begin{align*}
\Delta m^2_{21} &= 7.55(20) \times 10^{-5} \text{ eV}^2, \\
|\Delta m^2_{31}| &= 2.50(3) \times 10^{-3} \text{ eV}^2,
\end{align*}
\]

\[
\begin{align*}
s^2_{12} &= 3.20(20) \times 10^{-1}, \\
s^2_{23} &= 5.51(30) \times 10^{-1}, \\
s^2_{13} &= 2.160(83) \times 10^{-2}.
\end{align*}
\]

[P.F. de Salas et al. Phys. Lett. B782 (2018)]
Open questions: CP violation

• CPT in vacuum: CPV = TV

• Appearance required: P(\nu_\alpha \to \nu_\alpha) is T-invariant
Open questions: CP violation

- CPT in vacuum: CPV = TV

- Appearance required: P(να → να) is T-invariant

- > 2 generations required: U_{2x2} is a rotation

  \[ \alpha \equiv \frac{\Delta m_{21}^2}{|\Delta m_{31}^2|} \approx 0.03 \]

  - 1\textsuperscript{st} osc. max: \( \Delta_{31} = \pi/2 \) \( \rightarrow \) \( L/E \sim 500 \text{ km/GeV} \), \( \Delta_{21} \sim 0.05 \)
  - 2\textsuperscript{nd} osc. max: \( \Delta_{31} = 3\pi/2 \) \( \rightarrow \) \( L/E \sim 1500 \text{ km/GeV} \), \( \Delta_{21} \sim 0.15 \)

- Accelerator ν_{μ} to ν_{e} transition
Open questions: CP violation

• CPT in vacuum: CPV = TV

• Appearance required: P(ν_α → ν_α) is T-invariant

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  - 2^{\text{nd}} \text{ osc. max: } \Delta_{31} = 3\pi/2 \quad \rightarrow \quad L/E \sim 1500 \text{ km/GeV}, \quad \Delta_{21} \sim 0.15

  ➢ Accelerator ν_μ to ν_e transition

• Matter effects induce environmental CPV

\[ \alpha \equiv \frac{\Delta m_{21}^2}{|\Delta m_{31}^2|} \approx 0.03 \]
Next-gen experiments: “CPV” sensitivity

Fit data assuming SM production, detection and propagation

[DUNE Design Report, 1512.06148]

[HK Design Report, 1805.04136]
Model-independent $P_{\alpha\beta}$: assumptions

1. Propagation of the three SM neutrino states is described by a hermitian Hamiltonian $H(E; x)$
Model-independent $P_{\alpha\beta}$ : assumptions

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2. For the experiments of interest, medium effects can be described to sufficient accuracy by a constant matter density which is approximately the same for all considered experiment

- Matter effects induce no TV
- $H(E) = W \lambda W^\dagger$
Model-independent $P_{\alpha\beta}$: assumptions

3. We allow for arbitrary (non-unitary) mixing of the energy eigenstates $\nu_i$ with the flavour states $\nu_\alpha$ relevant for detection and production

$$ |\nu_\alpha\rangle = \sum_{i=1}^{3} N_{\alpha i}^{\text{prod, det}} |\nu_i\rangle $$

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3. We allow for arbitrary (non-unitary) mixing of the energy eigenstates $\nu_i$ with the flavour states $\nu_\alpha$ relevant for detection and production

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} N_{\alpha i}^{\text{prod, det}} |\nu_i\rangle$$

$P_{\mu\alpha} = \left| \sum_{i=1}^{3} c_i^\alpha e^{-i\lambda_i L} \right|^2$

$$P_{\mu\alpha} = \sum_i |c_i^\alpha|^2 + 2 \sum_{j<i} \text{Re}(c_i^\alpha c_j^{\alpha*}) \cos(\omega_{ij} L) - 2 \sum_{j<i} \text{Im}(c_i^\alpha c_j^{\alpha*}) \sin(\omega_{ij} L)$$
The proposed TV test: are there L-odd terms?

\[ P_{\mu\alpha}^{\text{even}}(L, E; \theta) = \sum_{i} (c_i^\alpha)^2 + 2 \sum_{j < i} c_i^\alpha c_j^\alpha \cos(\omega_{ij} L) \]

- Data \( P(L) \) at the same energy: same parameters
- Separate neutrino and antineutrino fits: different \( \omega_{ij} \) and \( c_i^\alpha \)

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- Data P(L) at the same energy: same parameters
- Separate neutrino and antineutrino fits: different \( \omega_{ij} \) and \( c_i^\alpha \)
- Consider both \( \nu_\mu \rightarrow \nu_\mu \) and \( \nu_\mu \rightarrow \nu_e \) (same \( \omega_{ij} \), different \( c_i^\alpha \))
  - Disappearance is T invariant, fixes the frequencies
  - Appearance provides sensitivity to TV

\[ \chi^2_{\text{even}}(E; \theta) = \sum_{b=1}^{N_L} \left[ \frac{P_{\mu\mu}^{\text{even}}(L_b, E; \theta) - p_b^{\text{dis}}}{\sigma_b^{\text{dis}}} \right]^2 + \sum_{b=1}^{N_L} \left[ \frac{P_{\mu e}^{\text{even}}(L_b, E; \theta) - p_b^{\text{app}}}{\sigma_b^{\text{app}}} \right]^2 \]
Necessary data points

\[
\chi^2_{\text{even}}(E; \theta) = \sum_{b=1}^{N_L} \left[ \frac{P^{\text{even}}(L_b, E; \theta) - p_b^{\text{dis}}}{\sigma_b^{\text{dis}}} \right]^2 + \sum_{b=1}^{N_L} \left[ \frac{P^{\text{even}}(L_b, E; \theta) - p_b^{\text{app}}}{\sigma_b^{\text{app}}} \right]^2
\]

- 8 parameters: 6 real coefficients \( c_i^\mu, c_i^e \) (\( i = 1, 2, 3 \))
  2 independent frequencies, \( \omega_{21} \) and \( \omega_{31} \)

- 2 data points per baseline (\( > 4 \) experiments)
Necessary data points

\[ \chi^2_{\text{even}}(E; \theta) = \sum_{b=1}^{N_L} \left[ \frac{P^{\text{even}}_{\mu\mu}(L_b, E; \theta) - p^\text{dis}_b}{\sigma^\text{dis}_b} \right]^2 + \sum_{b=1}^{N_L} \left[ \frac{P^{\text{even}}_{\mu\epsilon}(L_b, E; \theta) - p^\text{app}_b}{\sigma^\text{app}_b} \right]^2 \]

- 8 parameters: 6 real coefficients \( c^\mu_i, c^\epsilon_i \ (i = 1, 2, 3) \)
  2 independent frequencies, \( \omega_{21} \) and \( \omega_{31} \)
- 2 data points per baseline ( > 4 experiments)
- Zero-distance effects: near detectors provide \( L \approx 0 \) ( > 3 exps.)

\[ P^{\text{even}}_{\mu\alpha}(L, E; \theta) = \sum_i (c^\alpha_i)^2 + 2 \sum_{j<i} c^\alpha_i c^\alpha_j \cos(\omega_{ij} L) \]
How it works

1. The plots show data points for different values of $\delta = 0^\circ$ and $\delta = 90^\circ$.

2. For $\delta = 0^\circ$, the best-fit curve is given by $\omega_{21} = 7.7\omega_{21}$.

3. For $\delta = 90^\circ$, the best-fit curve is given by $\omega_{31} = 1.2\omega_{31}$.

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A 4\textsuperscript{th} assumption

4. We impose that the oscillation frequencies $\omega_{ij}$ deviate only weakly from the ones corresponding to the standard three-flavour oscillation case.

We include the prior term

$$
\chi_{\text{even}}^2(E; \theta) \rightarrow \chi_{\text{even}}^2(E; \theta) + \left[ \frac{\Delta \tilde{m}_{21}^2(E) - 2E\omega_{21}}{\sigma_{21}} \right]^2
$$

with $\sigma_{21} = 0.1 \Delta \tilde{m}_{21}^2(E)$
TV fit without vs. with prior term

\[ \delta = 90^\circ \]

- Prior term
  - b.f. \( \omega_{21} = 7.7 \omega_{21} \)
  - b.f. \( \omega_{31} = 1.2 \omega_{31} \)
Dependence on the model

Fit result depends on the true values of the parameters
- Oscillation probabilities depend on E
- Masses/mixings depend on the model
Realistic data: energy overlap

| Experiment | Baseline | Energy range |
|------------|----------|--------------|
| T2HK       | 295 km   | 0.2 – 1.1 GeV| [1805.04163] |

**Disappearance ν mode**

**Appearance ν mode**

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Realistic data: energy overlap

| Experiment | Baseline | Energy range   | Reference     |
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| T2HK       | 295 km   | 0.2 – 1.1 GeV  | [1805.04163]  |
| DUNE       | 1300 km  | 0.5 – 6 GeV    | [1607.00293]  |

This work

- $\nu_e + \bar{\nu}_e$ CC Signal
- $\nu_\tau + \bar{\nu}_\tau$ CC
- $\nu + \bar{\nu}$ NC
- $\nu_\mu + \bar{\nu}_\mu$ CC
- Beam $\nu_\mu + \bar{\nu}_\mu$ CC
- $\nu_\tau + \bar{\nu}_\tau$ CC
- $\nu + \bar{\nu}$ NC
Realistic data: energy overlap

| Experiment | Baseline | Energy range | Reference       |
|------------|----------|--------------|-----------------|
| T2HK       | 295 km   | 0.2 – 1.1 GeV | [1805.04163]    |
| DUNE       | 1300 km  | 0.5 – 6 GeV  | [1607.00293]    |
| NOνA       | 810 km   | 1 – 5 GeV    | [Neutrino2020]  |

![Graph showing energy distribution and ratio for muon neutrinos](image)
Realistic data: energy overlap

| Experiment | Baseline | Energy range   | Reference        |
|------------|----------|----------------|------------------|
| T2HK       | 295 km   | 0.2 – 1.1 GeV  | [1805.04163]     |
| DUNE       | 1300 km  | 0.5 – 6 GeV    | [1607.00293]     |
| NOvA       | 810 km   | 1 – 5 GeV      | Neutrino2020     |
| T2HKK      | 1100 km  | 0.2 – 1.1 GeV  | [1611.06118]     |
## Realistic data: energy overlap

| Experiment | Baseline | Energy range            | Reference               |
|------------|----------|-------------------------|-------------------------|
| T2HK       | 295 km   | 0.2 – 1.1 GeV           | [1805.04163]            |
| DUNE       | 1300 km  | 0.5 – 6 GeV             | [1607.00293]            |
| NO$_\nu$A  | 810 km   | 1 – 5 GeV               | [Neutrino2020]          |
| T2HKKK     | 1100 km  | 0.2 – 1.1 GeV           | [1611.06118]            |
| ESS$_\nu$SB| 540 km   | 0.2 – 0.8 GeV           | [1912.04309]            |

![Energy distribution graph](image)
Estimation of statistical uncertainties

Number of signal events $N_{br}$

$$S_{br} = N_{br} \times \frac{P_{\text{even}}(L_b, E_r; \theta)}{P^{3\nu}(L_b, E_r)}$$

$$\frac{\sigma_{br}}{P_{\text{even}}(L_b, E_r)} = \frac{\sqrt{S_{br} + B_{br}}}{S_{br}}$$

Near detector: $P_{\alpha\beta}(L \rightarrow 0) = \delta_{\alpha\beta}$ with $\sigma = 0.01$
Data points ($\delta = 90^\circ$)
Energy resolution assumptions

• Energy resolution $\Delta E = 10\% E_0$ around the central $E_0$

• Data smearing:
  - Convolute $P(E)$ with gaussian ($\mu = E_0$, $\sigma = \Delta E$)

• Smearing $P_{\text{even}}(L, E; \theta)$ in the fit:
  - $E$ dependence of $c_i^\alpha$ is slow enough to be neglected
  - Oscillation frequencies: $\omega_{21}$ irrelevant
    \[ \omega_{31} \propto \frac{1}{E} \]
10% energy resolution

DUNE + T2HK + T2HKK + ESSvSB

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E = 0.75 GeV

E = (0.75 ± 10%) GeV

\[ P_{\mu\mu} \]

\[ P_{\mu e} \]

L [km]
**χ² values for δ = 90°**

Perfect [10%] energy resolution

| E (GeV) | 3L (ESS) | 3L (HKK) | 4L     |
|---------|----------|----------|--------|
| 0.65    | 0.07 [0.03] | 0.04 [0.27] | 0.83 [0.47] |
| 0.75    | 0.04 [0.04] | 7.04 [3.82]  | 7.40 [3.84]  |
| 0.85    | 0.54 [0.53] | 2.10 [1.97]  | 2.92 [2.05]  |
| 0.95    | -         | 0.21 [0.77]  | -         |
| Total   | 0.65 [0.60] | 9.39 [6.83]  | 11.15 [6.36] |

- Most sensitive bins: 0.75 and 0.85 GeV
- Korean detector is necessary
- Energy resolution is crucial
Dependence on $\delta$
What happens at 270°?

\[ \delta = 90° \]

\[ \delta = 270° \]
What happens at $270^\circ$?

$$P_{\text{even}}(\Delta') \approx a \sin^2 \Delta' + b \Delta \sin 2\Delta' + c \Delta'^2$$

Disappearance: $|\Delta'| \approx |\Delta|$

$a, c \geq 0 \quad \Rightarrow \quad P_{\text{even}}(3\pi/2) \geq P_{\text{even}}(\pi/2)$

\[\text{Graphs for } \delta = 90^\circ \text{ and } \delta = 270^\circ\]
Antineutrino data

\[ P_{3\nu}(\delta = \pm 90^\circ) \approx 2s_{13} \sin^2 \Delta [s_{13} \mp (\pm)\tilde{\alpha} \Delta] + \frac{1}{2}\tilde{\alpha}^2 \Delta^2 \]

\[ \Delta = \Delta m_{31}^2 L / 4E \]

\[ \tilde{\alpha} = \sin 2\theta_{12} \Delta m_{21}^2 / \Delta m_{31}^2 \]

Vacuum Invariance
\[ \nu \rightarrow \bar{\nu} \]
\[ \delta \rightarrow 2\pi - \delta \]

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Effect of the mass ordering

\[ P_{3\nu}(\delta = \pm 90^\circ) \approx 2s_{13} \sin^2 \Delta [s_{13} \mp \tilde{\alpha} \Delta] + \frac{1}{2} \tilde{\alpha}^2 \Delta^2 \]

\[ \Delta = \Delta m_{31}^2 L / 4E \]

\[ \tilde{\alpha} = \sin 2\theta_{12} \Delta m_{21}^2 / \Delta m_{31}^2 \]

Vacuum Invariance

\[ \text{NH} \rightarrow \text{IH} \]

\[ \delta \rightarrow \pi - \delta \]
Non-constant density effects

- $\rho(x)$ not constant
- Different mean densities

HyperK - Korea [1107.5857]

DUNE [1707.02322]
Non-constant density effects

Perturbative study: \( H(x) = H_{\text{vac}} + V_0 + V(t) = H_0 + V(t) \)

- 0\(^{\text{th}}\) order

\[
A^{(0)}_{\alpha\beta} = \sum_i N^{s*}_{\alpha i} N^d_{\beta i} e^{-i\lambda_i(t_d-t_s)} \quad \text{T} A^{(0)}_{\alpha\beta} = A^{(0)}(\nu^d_\beta \rightarrow \nu^s_\alpha)^*
\]

➢ The usual \( TP_{\alpha\beta} = P_{\beta\alpha} \) requires \( N^s = N^d \)!
Non-constant density effects

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- **0\(^{\text{th}}\) order**

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A_{\alpha\beta}^{(0)} = \sum_i N_{\alpha i}^{s*} N_{\beta i}^d e^{-i\lambda_i (t_d - t_s)} \quad \text{T} A_{\alpha\beta}^{(0)} = A^{(0)}(\nu_\beta^d \rightarrow \nu_\alpha^s)^*
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➢ The usual \( TP_{\alpha\beta} = P_{\beta\alpha} \) requires \( N^s = N^d ! \)

- **1\(^{\text{st}}\) order**

\[
A_{\alpha\beta}^{(1)} = -i \sum_{ij} N_{\alpha i}^{s*} N_{\beta j}^d e^{-i(\lambda_j + \lambda_i)L/2} \int_{-L/2}^{L/2} dt V_{ij}(t) e^{i(\lambda_j - \lambda_i)t},
\]

\[
T A_{\alpha\beta}^{(1)} = i \sum_{ij} N_{\alpha i}^{s*} N_{\beta j}^d e^{i(\lambda_j + \lambda_i)L/2} \int_{-L/2}^{L/2} dt V_{ij}(-t) e^{-i(\lambda_j - \lambda_i)t}
\]

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Non-constant density effects

Perturbative study: \( H(x) = H_{\text{vac}} + V_0 + V(t) = H_0 + V(t) \)

\begin{itemize}
  \item 0\textsuperscript{th} order
    \[ A_{\alpha\beta}^{(0)} = \sum_i N_{\alpha i}^{s*} N_{\beta i}^{d} e^{-i\lambda_i(t_d-t_s)} \]
    \[ T A_{\alpha\beta}^{(0)} = A^{(0)}(\nu_\beta^d \rightarrow \nu_\alpha^s)^* \]
    ➢ The usual TP\(_{\alpha\beta} = P_{\beta\alpha}\) requires \(N^s = N^d\)!

  \item 1\textsuperscript{st} order
    \[ A_{\alpha\beta}^{(1)} = -i \sum_{ij} N_{\alpha i}^{s*} N_{\beta j}^{d} e^{-i(\lambda_j+\lambda_i)\frac{L}{2}} \int_{-L/2}^{L/2} dt V_{ij}(t) e^{i(\lambda_j-\lambda_i)t} \]
    \[ T A_{\alpha\beta}^{(1)} = i \sum_{ij} N_{\alpha i}^{s*} N_{\beta j}^{d} e^{i(\lambda_j+\lambda_i)\frac{L}{2}} \int_{-L/2}^{L/2} dt V_{ij}(-t) e^{-i(\lambda_j-\lambda_i)t} \]
    \textcolor{blue}{1 \% correction}
Different constant densities

\[
\bar{\rho}_{\text{HK}} = 2.5 \text{ g/cm}^3 \\
\bar{\rho}_{\text{HKK}} = 3.0 \text{ g/cm}^3 \\
\bar{\rho}_{\text{DUNE}} = 2.85 \text{ g/cm}^3 \\
\]

\[
c_i^b = \bar{c}_i + \delta c_i^b \\
\omega_{ij}^b = \bar{\omega}_{ij} + \delta \omega_{ij}^b
\]
Different constant densities

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\[ c_i^b = \bar{c}_i + \delta c_i^b \]
\[ \omega_i^b = \bar{\omega}_i + \delta \omega_i^b \]

\[
P_{\alpha\beta}^{\text{even}} = \sum_i \bar{c}_i^2 + 2 \sum_{j<i} \bar{c}_i \bar{c}_j \cos(\bar{\omega}_{ij} L) + 2 \sum_i \bar{c}_i \delta c_i + 2 \sum_{j<i} (\bar{c}_i \delta c_j + \bar{c}_j \delta c_i) \cos(\bar{\omega}_{ij} L) - 2 \sum_{j<i} \bar{c}_i \bar{c}_j \delta \omega_{ij} L \sin(\bar{\omega}_{ij} L). \]

1% correction
Summary

• 

T violation induces L-odd terms in the oscillation probability.

• Can data be described by

\[
P_{\mu \alpha}^{\text{even}}(L, E; \theta) = \sum_{i} (c_{i}^{\alpha})^2 + 2 \sum_{j<i} c_{i}^{\alpha} c_{j}^{\alpha} \cos(\omega_{ij}L)
\]
Summary

- T violation induces L-odd terms in the osc. Probability

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\[ P_{\mu\alpha}^{\text{even}}(L, E; \theta) = \sum_i (c_i^\alpha)^2 + 2 \sum_{j<i} c_i^\alpha c_j^\alpha \cos(\omega_{ij} L) \]

- General parametrization:
  - P(L) measurements at the same E with good \( \Delta E \)
  - At least 3 different baselines: Korea is crucial!
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• Actual \( \rho(x) \) known: effect is small (1%) and can be subtracted
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• Neutrinos probe \(\sin \delta > 0\), antineutrinos probe \(\sin \delta < 0\) regardless of mass ordering
Thank you for your attention!