Superdeformation of Ar hypernuclei

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We investigate the differences in the Λ separation energies (SΛ) of the ground and superdeformed (SD) states in 37ΛAr, 39ΛAr, and 41ΛAr within the framework of antisymmetrized molecular dynamics (AMD). In this study, we find that the calculated SΛ values in the SD states are much smaller than those in the ground states, unlike the result using the relativistic mean-field (RMF) calculation [B.-N. Lu et al., Phys. Rev. C, 89, 044307 (2014)]. One of the reasons for this difference between the present work and the RMF calculation is the difference in the density profile of the SD states in the core nuclei. We also find that the property of the ΛN odd-parity interaction affects the SΛ trend between the ground and SD states.

Subject Index D14

1. Introduction

One of the most important subjects in hypernuclear physics is the study of the dynamical changes in core nuclei by the addition of a Λ particle. For this purpose, many authors have studied the p-shell [2–18], sd-shell [1,12,14–16,19–24], and pf-shell [1,12,24] Λ hypernuclei. For example, in the p-shell Λ hypernuclei 10ΛBe [17,18] and 13ΛC [7], it is discussed that the Λ particle significantly reduces the r.m.s. radii of the well developed cluster states 9Be(1/2+) ⊗ Λ and 12C(02+) ⊗ Λ by about 20% compared to the compact ground states 9Be(3/2−) ⊗ Λ and 12C(01+) ⊗ Λ, respectively. In the sd-shell hypernucleus 21ΛNe [22], it is predicted that the 20ΛNe(1−) ⊗ Λ state with a well developed α +16O cluster structure will exhibit significantly different changes of r.m.s. radii compared with the ground state 20Ne(0+) ⊗ Λ. In addition to the dynamical core modification, it is also discussed that the Λ separation energy SΛ is sensitive to the core nucleus structure. For example, in 13ΛC, SΛ in the 12C(02+) ⊗ Λ state is predicted to be smaller by approximately 3 MeV than that in the ground state [7,25]. It is considered that this comes from the difference in nuclear density. Namely, the density of the well developed cluster state, 12C(02+), is more dilute than that of the ground state, resulting in a weaker attraction between Λ and the nucleons.

In the case of the sd- and pf-shell hypernuclei, it has been discussed theoretically that SΛ will be dependent on the core deformation. In A ~ 40 core nuclei, many authors have discussed the existence of superdeformed (SD) states experimentally and theoretically [26–39]. In Ref. [24], we investigated the SD states in 41ΛCa, 46ΛSc, and 48ΛSc for the first time within the framework of the antisymmetrized molecular dynamics (AMD). As a result, it was pointed out that the SΛ in the SD states were smaller by about 1 MeV than those in the ground states in 41ΛCa, 46ΛSc, and 48ΛSc, while the r.m.s. radii of the ground and SD states were almost unchanged by the addition of a Λ particle. This is caused by the
decrease of the overlap between the Λ and core nucleus. Since the overlap in the SD state is smaller than that in the ground state, the $S_\Lambda$ in the SD state becomes smaller with a small change of the core structure.

Recently, the relativistic mean-field (RMF) method was applied to study $S_\Lambda$ in SD states in the hypernuclei with mass number 30–60 [1]. A controversial result was pointed out in the theoretical study of $S_\Lambda$. They found that the $S_\Lambda$ in the SD states were smaller than those in the ground states in $^{33}\Lambda S$, $^{57}\Lambda Ni$, and $^{61}\Lambda Zn$, which is consistent with our prediction [24]. On the other hand, in $^{41}\Lambda Ca$, $^{37}\Lambda Ar$, $^{39}\Lambda Ar$, and $^{41}\Lambda Ar$, the $S_\Lambda$ in the SD states were larger than those in the ground states, which contradicts our calculation [24]. They mentioned that this was caused by the characteristic density profiles of the SD states, which showed a strong localization of nucleons in a ring shape [1]. In Ref. [1], to investigate the relation between $S_\Lambda$ and the ring-shape localization, they calculated the overlap of the densities between Λ and the core nuclei, $I_{\text{overlap}}$, defined by

$$ I_{\text{overlap}} = \int d^3 r \rho_N(r) \rho_\Lambda(r), $$(1)

where $\rho_N(r)$ and $\rho_\Lambda(r)$ denoted the density of the nucleons and the Λ particle, respectively. They mentioned that the ring-shape localization made $I_{\text{overlap}}$ larger, and, as a result, a larger $I_{\text{overlap}}$ led to a larger $S_\Lambda$ in the SD states. In this sense, in the AMD calculation for $^{41}\Lambda Ca$, since the SD states of $^{40}\Lambda Ca$ and $^{41}\Lambda Ca$ do not have the ring-shape localization [24,37], it is reasonable that the $S_\Lambda$ in the SD state is smaller than that in the ground state. In Ar hypernuclei, it is quite interesting to investigate the difference in $S_\Lambda$ between the ground and SD states and whether or not the SD states have the ring-shape localization in the AMD calculation (cf. in the RMF calculation, the $S_\Lambda$ in the SD states are larger than those in the ground states). It is also quite important to reveal the correlation between the $S_\Lambda$ and $I_{\text{overlap}}$ in the ground and SD states, which is pointed out by the RMF calculation [1].

For this purpose, we perform an AMD calculation for $^{37}\Lambda Ar$, $^{39}\Lambda Ar$, and $^{41}\Lambda Ar$ in the present study. It is found that the SD states in Ar hypernuclei do not have the ring-shape localization and $S_\Lambda$ are smaller in the SD states, as in $^{41}\Lambda Ca$. On the other hand, we find that the $S_\Lambda$ are not always correlated with $I_{\text{overlap}}$ in Ar hypernuclei. The properties of the $\Lambda N$ interaction are also important for the difference of $S_\Lambda$. In this paper, we discuss the relation between $S_\Lambda$ and the properties of $\Lambda N$ interaction as well as that between $S_\Lambda$ and the density distribution of SD states in Ar hypernuclei.

This paper is organized as follows. In the next section, we explain the theoretical framework of HyperAMD. In Sect. 3, the difference in $S_\Lambda$ between the ground and SD states is discussed, together with the comparison between the present and RMF calculations [1]. The final section summarizes this work.

2. Theoretical framework

In this study, we perform the energy variation to obtain the energy curve as a function of nuclear quadrupole deformation. The Hamiltonian is

$$ \hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{V}_C + \hat{T}_\Lambda + \hat{V}_{\Lambda N} - \hat{T}_g, $$(2)

where $\hat{T}_N$, $\hat{T}_\Lambda$, and $\hat{T}_g$ are the kinetic energies of the nucleons, Λ particle, and center-of-mass motion, respectively. We use the Gogny D1S interaction [40] as the effective nucleon–nucleon interaction $\hat{V}_{NN}$, and the Coulomb interaction $\hat{V}_C$ is approximated by the sum of seven Gaussians. As the $\Lambda N$ interaction $\hat{V}_{\Lambda N}$, we adopt YNG-ESC08c (see the tables in the appendix in Ref. [24]), which depends
on the nuclear Fermi momentum $k_F$. In the present study, the $k_F$ values are determined by the averaged density approximation [41], and the resulting values are $k_F = 1.26, 1.28,$ and $1.26$ fm$^{-1}$ for $^{37}_\Lambda$Ar, $^{39}_\Lambda$Ar, and $^{41}_\Lambda$Ar, respectively. The variational wave function of a single $\Lambda$ hypernucleus is described by the parity-projected wave function,

$$\Psi^\pi = \hat{\beta}^\pi \{ A(\varphi_1, \ldots, \varphi_A) \otimes \varphi_{\Lambda} \},$$ (3)

$$\varphi_i \propto e^{-\sum_\sigma v_{0} (r_{\sigma} - z_{m})^2} \otimes (u_i \chi_\uparrow + v_i \chi_\downarrow) \otimes (p \text{ or } n),$$ (4)

$$\varphi_{\Lambda} \propto \sum_{m=1}^{M} c_m e^{-\sum_\sigma v_{0} (r_{\sigma} - z_{m})^2} \otimes (a_m \chi_\uparrow + b_m \chi_\downarrow),$$ (5)

where the single-particle wave packet of nucleon $\varphi_i$ is described by a single Gaussian, while that of $\Lambda$, $\varphi_{\Lambda}$, is represented by a superposition of Gaussian wave packets. The variational parameters $\hat{Z}_i$, $z_m$, $v_{0}$, $u_i$, $v_i$, $a_m$, $b_m$, and $c_m$ are determined to minimize the total energy under the constraint on the nuclear quadrupole deformation $\beta_2$, defined as

$$\beta_2 = \frac{4\pi}{3AR^2} \langle \hat{Q}_{20} \rangle, \quad R = 1.2 \times A^{1/3}. $$ (6)

By the energy variation, we obtain the optimized wave function $\Psi^\pi (\beta_2)$ for each given value of $\beta_2$.

To discuss the relation between the $\Lambda$ separation energy and the density profile, we calculate the $\Lambda$ separation energy $S_{\Lambda}$ and the overlap between $\Lambda$ and the nucleons $I_{\text{overlap}}$, defined in Ref. [1]. Namely, $S_{\Lambda}$ at the ground and SD minima are defined as

$$S_{\Lambda} = E_{N}^{\text{min}} - E_{\Lambda}^{\text{min}}, $$ (7)

where $E_{N}^{\text{min}}$ and $E_{\Lambda}^{\text{min}}$ are energies at the minima of the normal nuclei and corresponding hypernuclei, respectively. The definition of $I_{\text{overlap}}$ is given in Eq. (1).

We also calculate excitation spectra by performing the generator coordinate method (GCM) to provide a quantitative prediction of the $\Lambda$ separation energy.

3. Results and discussions

In Figs. 1(a)–(c), we illustrate the energy curves of $^{36}$Ar, $^{38}$Ar, and $^{40}$Ar and those of the corresponding $\Lambda$ hypernuclei as a function of $\beta_2$. It is seen that several intrinsic energy minima are obtained in the core nuclei: two in $^{36}$Ar and $^{40}$Ar, and four in $^{38}$Ar. Among these minima, we find that the local minima shown by filled circles have single-particle configurations corresponding to the SD states. It is found that $^{37}_\Lambda$Ar, $^{39}_\Lambda$Ar, and $^{41}_\Lambda$Ar also have corresponding SD minima. In these hypernuclei, we see no significant changes in $\beta_2$ by the addition of a $\Lambda$ particle at each minimum.

Let us discuss the energy gain by the addition of a $\Lambda$ particle to the core nuclei, i.e., the $\Lambda$ separation energy $S_{\Lambda}$ defined by Eq. (7). In Table 1, the calculated values of $S_{\Lambda}$ are listed as well as the energies $E$ and deformations $\beta_2$ at the lowest (GS in Fig. 1) and SD minima. It is found that the $S_{\Lambda}$ at the SD minima are smaller than those at the GS minima in all of the calculated Ar hypernuclei. For example, in $^{37}_\Lambda$Ar, the $S_{\Lambda}$ at the SD minimum is 18.04 MeV, while that of the GS minimum is 18.59 MeV. This trend of $S_{\Lambda}$ is different from the RMF calculation [1], in which the $S_{\Lambda}$ at the SD minima are larger than those in the ground states. In Ref. [1], they pointed out that the increase of $S_{\Lambda}$ at the SD minima was caused by the characteristic density distribution having a strong ring-shape localization in $^{37}_\Lambda$Ar, $^{39}_\Lambda$Ar, and $^{41}_\Lambda$Ar. In Fig. 2, we show the density distribution at the SD minima. It is found that
the SD minima do not show ring-shape localization in $^{36}$Ar, $^{38}$Ar, and $^{40}$Ar and the corresponding hypernuclei. It is considered that the disappearance of the localization in the SD states is mainly due to the difference in the framework between the present work and Ref. [1], since similar ring-shape localization was predicted in the ground state of $^{20}$Ne with relativistic energy density functionals.

![Energy curve as a function of the nuclear quadrupole deformation $\beta_2$ for (a) $^{36}$Ar and $^{37}$Ar, (b) $^{38}$Ar and $^{39}$Ar, and (c) $^{40}$Ar and $^{41}$Ar. The dashed curve shows the energy curve for the core nuclei, while the solid curve corresponds to the $\Lambda$ hypernuclei. Open circles represent the positions of the energy (local) minima, and filled circles correspond to the SD minima. Particle–hole ($p-h$) configurations at the SD minima are also shown.](https://academic.oup.com/ptep/article-abstract/2015/10/103D02/2461029)
Fig. 2. (a) Nuclear density distributions ($\rho_N$) at the GS and SD minima of $^{36}$Ar (top) and $^{37}\Lambda$Ar (bottom). (b), (c) Same as (a) for $^{38}$Ar and $^{40}$Ar and the corresponding hypernuclei, respectively.

(EDFs) [43,44], whereas it did not appear in the AMD calculation [22,45]. It is known that the relativistic mean-field potential is constructed by the large cancellation between the scalar and vector potentials so that the radial dependence of the mean-field potential often has a large fluctuation. Therefore, the ring-shape localization would be caused by this fluctuation of the mean-field potential. Moreover, the different treatment of the nucleon–nucleon spin–orbit force between the present and RMF calculations could also contribute to forming the localization. Thus, since the localization does not appear in the SD states with AMD, it seems to be reasonable to have smaller $S_{\Lambda}$ at the SD minima in the present calculation. To reveal the relation between $S_{\Lambda}$ and the density distributions, we calculate $I_{\text{overlap}}$, defined by Eq. (1), as discussed in Ref. [1], in which $S_{\Lambda}$ and $I_{\text{overlap}}$ are correlated with each other at the GS and SD minima.

In Table 1, the calculated $I_{\text{overlap}}$ are summarized together with those with RMF [1]. In Ref. [1], the authors pointed out that the larger values of $S_{\Lambda}$ at the SD minima were caused by the larger overlap $I_{\text{overlap}}$, the $I_{\text{overlap}}$ at the SD minima were larger than those at the GS minima in $^{37}\Lambda$Ar, $^{39}\Lambda$Ar, and $^{41}\Lambda$Ar. In $^{37}\Lambda$Ar and $^{39}\Lambda$Ar, it is found that the $I_{\text{overlap}}$ are smaller at the SD minima compared with the GS minima, which is opposite behavior to that of the RMF calculation [1]. On the other hand, in $^{41}\Lambda$Ar, the $I_{\text{overlap}}$ at the SD minimum is larger than that at the GS minimum. Therefore, it is not likely that the $S_{\Lambda}$ and $I_{\text{overlap}}$ are correlated with each other in $^{41}\Lambda$Ar in the present calculation, which is inconsistent with the RMF calculation [1].

Let us discuss the reason for the uncorrelated behavior of $S_{\Lambda}$ and $I_{\text{overlap}}$ in the present calculation. It should be noted that the $\Lambda N$ interaction, especially the odd-parity force, employed in the
Table 2. Comparison of $S_\Lambda$ (MeV) and $I_{\text{overlap}}$ (fm$^{-3}$) at the GS and SD (shown by asterisks) minima. Case (i) refers to $S_\Lambda$ and $I_{\text{overlap}}$ with the odd-parity $\Lambda N$ force switched off, case (ii) to those with the attractive odd-parity force, which is the same as the even-parity force. YNG-ESC08c shows the results with the original odd-parity force of the YNG-ESC08c interaction.

|                  | YNG-ESC08c | case (i) | case (ii) |
|------------------|------------|----------|-----------|
|                  | $S_\Lambda$| $I_{\text{overlap}}$ | $S_\Lambda$| $I_{\text{overlap}}$ | $S_\Lambda$| $I_{\text{overlap}}$ |
| $^{37}_{\Lambda} \text{Ar}$ | 18.59      | 0.1338   | 21.12     | 0.1385   | 30.05        | 0.1482   |
| $^{37}_{\Lambda} \text{Ar}^*$ | 18.04      | 0.1310   | 20.53     | 0.1361   | 29.40        | 0.1469   |
| $^{39}_{\Lambda} \text{Ar}$ | 18.55      | 0.1416   | 21.44     | 0.1423   | 30.14        | 0.1534   |
| $^{39}_{\Lambda} \text{Ar}^*$ | 17.70      | 0.1351   | 20.45     | 0.1388   | 29.93        | 0.1521   |
| $^{41}_{\Lambda} \text{Ar}$ | 19.27      | 0.1353   | 21.81     | 0.1355   | 30.11        | 0.1388   |
| $^{41}_{\Lambda} \text{Ar}^*$ | 18.99      | 0.1370   | 21.63     | 0.1385   | 30.26        | 0.1454   |

The present calculation is different to that in Ref. [1]. The strength and character of the odd-parity $\Lambda N$ force are still under debate, and various models of the odd-parity force with different properties have been proposed. In the present work, we employ the latest version of the YNG $G$-matrix interaction (YNG-ESC08c [24]) derived from ESC08c developed by the Nijmegen group [46,47], in which the odd-parity force is weakly repulsive. However, in Ref. [1], they used the parameter set of the PK1-Y1 as the $\Lambda N$ interaction, which was adjusted to reproduce the observed data of $S_\Lambda$ from light to heavy mass regions [48,49]. In the RMF calculation [1], the treatment of the $\Lambda N$ interaction is quite different from the present calculation. Since RMF is a Hartree theory, in which the exchange terms of the $\Lambda N$ interaction are discarded, the attractive even-parity force works irrespective of the relative angular momenta between $\Lambda$ and the nucleon. Therefore, the odd-parity force is regarded as having the same character as the even-parity force and is attractive in nature.

In order to see the contributions of the even-parity and odd-parity parts of the $\Lambda N$ interaction in detail, we modify the odd-parity force of YNG-ESC08c. First, we switch off the odd-parity force in the calculation of the Ar hypernuclei (case (i)). Next, we use the attractive odd-parity $\Lambda N$ force, which is the same as the even-parity force of YNG-ESC08c (case (ii)). Case (ii) would be similar to the RMF calculation [1]. In Table 2, we illustrate $S_\Lambda$ and $I_{\text{overlap}}$ in cases (i) and (ii). In case (i), we see that the $S_\Lambda$ at the SD minima are smaller than those at the GS minima, which is the same behavior as in the YNG-ESC08c result. Therefore, the difference of $S_\Lambda$ between the GS and SD minima in the YNG-ESC08c result is mainly caused by the even-parity $\Lambda N$ force. It is also seen that the trend of $I_{\text{overlap}}$ is the same as that in the YNG-ESC08c result. Therefore, the $S_\Lambda$ and $I_{\text{overlap}}$ are not correlated when the odd-parity $\Lambda N$ force is switched off. In case (ii), as shown in Table 2, the $S_\Lambda$ at the SD minimum of $^{41}_{\Lambda} \text{Ar}$ becomes larger than that at the GS minimum, when the odd-parity force is the same as the attractive even-parity force. In $^{39}_{\Lambda} \text{Ar}$, we also find that the difference in $S_\Lambda$ between the GS and SD minima becomes much smaller. Therefore, it could be said that the attractive odd-parity force changes the difference in $S_\Lambda$ between the GS and SD minima. In Table 2, it is also seen that $S_\Lambda$ is correlated with $I_{\text{overlap}}$ in case (ii). Therefore, it is likely that the uncorrelated behavior of $S_\Lambda$ and $I_{\text{overlap}}$ in the YNG-ESC08c result is due to the property of the odd-parity $\Lambda N$ force. Thus, the odd-parity force could be one of the possible reasons for the opposite trend of $S_\Lambda$ between the present and RMF [1] calculations as well as the difference in the density profile of the SD states.

Finally, we discuss how the difference in $S_\Lambda$ at the intrinsic GS and SD minima will appear as an energy difference in the corresponding $J^\pi$ states after the GCM calculations. By performing the
In summary, the differences in the $\Lambda$ separation energy ($S_\Lambda$) between the ground and superdeformed (SD) states have been investigated in $^{37}_{\Lambda}Ar$, $^{39}_{\Lambda}Ar$, and $^{41}_{\Lambda}Ar$ with AMD. It was found that the $S_\Lambda$ in the SD states were smaller than those in the ground states. This result was consistent with the AMD calculation for $^{41}_{\Lambda}Ca$, $^{46}_{\Lambda}Sc$, and $^{48}_{\Lambda}Sc$ [24], but contradicted the RMF calculations [1]. One of the reasons for the different trend of $S_\Lambda$ between the present and RMF [1] calculations was the density profile of the core nucleus. In the SD states of the Ar isotopes, in the AMD calculation, we found no ring-shape localization of the nucleons, which was predicted by the RMF calculation [1]. However, $S_\Lambda$ was not correlated with the overlap of the densities between $\Lambda$ and the nucleons. It was found

\begin{table}[h]
\centering
\caption{Table 3. Total ($E$) and correlation ($\Delta E$) energies and the $\Lambda$ separation energy ($S_\Lambda$) of the ground and SD (shown by asterisks) states in $^{36}_{\Lambda}Ar$, $^{37}_{\Lambda}Ar$, $^{38}_{\Lambda}Ar$, and $^{40}_{\Lambda}Ar$, and the corresponding hypernuclei obtained after the GCM calculation. The correlation energy ($\Delta E$) is explained in the text.}
\begin{tabular}{|c|c|c|c|c|}
\hline
$^{\text{X}}_{\Lambda}\text{Ar}$ & $J^\pi$ & $E$ [MeV] & $\Delta E$ [MeV] & $S_\Lambda$ [MeV] \\
\hline
$^{36}_{\Lambda}\text{Ar}$ & $0^+$ & $-304.36$ & $-3.30$ & \\
$^{37}_{\Lambda}\text{Ar}$ & $1/2^+$ & $-322.87$ & $-3.23$ & $18.51$ \\
$^{36}_{\Lambda}\text{Ar}$ & $0^+$ & $-297.39$ & $-5.62$ & \\
$^{37}_{\Lambda}\text{Ar}$ & $1/2^+$ & $-315.52$ & $-5.71$ & $18.13$ \\
$^{38}_{\Lambda}\text{Ar}$ & $0^+$ & $-324.03$ & $-1.26$ & \\
$^{39}_{\Lambda}\text{Ar}$ & $1/2^+$ & $-342.52$ & $-1.20$ & $18.49$ \\
$^{38}_{\Lambda}\text{Ar}$ & $0^+$ & $-317.67$ & $-7.11$ & \\
$^{39}_{\Lambda}\text{Ar}$ & $1/2^+$ & $-335.63$ & $-7.37$ & $17.96$ \\
$^{40}_{\Lambda}\text{Ar}$ & $0^+$ & $-340.11$ & $-1.96$ & \\
$^{41}_{\Lambda}\text{Ar}$ & $1/2^+$ & $-359.47$ & $-2.05$ & $19.36$ \\
$^{40}_{\Lambda}\text{Ar}$ & $0^+$ & $-337.83$ & $-6.46$ & \\
$^{41}_{\Lambda}\text{Ar}$ & $1/2^+$ & $-356.91$ & $-6.55$ & $19.08$ \\
\hline
\end{tabular}
\end{table}

GCM calculation, the rotational motions, configuration mixings, and shape fluctuations are taken into account as the correlation energies $\Delta E$, which potentially change the trend of $S_\Lambda$ discussed above. In Table 3, we summarize the total energies $E(0^+)$ and $E(1/2^+)$, the $\Lambda$ separation ($S_\Lambda$), and correlation ($\Delta E$) energies calculated with the GCM. The $0^+$ and $1/2^+$ states correspond to the core nuclei and the hypernuclei, respectively, and the SD states are denoted by asterisks. Here, we define the correlation energies $\Delta E$ as the differences between the energies calculated by the GCM and the corresponding intrinsic energies at the minima, i.e., $\Delta E = E(0^+) - E_{\text{min}}^{N}$ for each $0^+$ state in the core, and $\Delta E = E(1/2^+) - E_{\text{min}}^{\Lambda}$ for the $1/2^+$ states in the corresponding hypernuclei. In Table 3, in $^{36}_{\Lambda}Ar$, it is seen that the correlation energies $\Delta E$ are quite different between the ground and SD states. This is because the energy gain due to rotational motion is larger in the SD state. In $^{37}_{\Lambda}Ar$, it is found that the trend of $\Delta E$ is similar to that in $^{36}_{\Lambda}Ar$. Namely, the $\Delta E$ in the ground state of $^{37}_{\Lambda}Ar$ is almost the same as that in the ground state of $^{36}_{\Lambda}Ar$, and the $\Delta E$ in the SD states are close to each other in $^{36}_{\Lambda}Ar$ and $^{37}_{\Lambda}Ar$. As a result, the $S_\Lambda$ calculated by GCM are almost the same as those calculated by the intrinsic minimum energies in $^{37}_{\Lambda}Ar$. In Table 3, we see the similar behavior of $\Delta E$ in $^{39}_{\Lambda}Ar$ and $^{41}_{\Lambda}Ar$. Therefore, the differences of $S_\Lambda$ between the intrinsic GS and SD minima are almost the same as those in the GCM results; this is also the case for $^{39}_{\Lambda}Ar$ and $^{41}_{\Lambda}Ar$. 

4. Summary

In summary, the differences in the $\Lambda$ separation energy ($S_\Lambda$) between the ground and superdeformed (SD) states have been investigated in $^{37}_{\Lambda}Ar$, $^{39}_{\Lambda}Ar$, and $^{41}_{\Lambda}Ar$ with AMD. It was found that the $S_\Lambda$ in the SD states were smaller than those in the ground states. This result was consistent with the AMD calculation for $^{41}_{\Lambda}Ca$, $^{46}_{\Lambda}Sc$, and $^{48}_{\Lambda}Sc$ [24], but contradicted the RMF calculations [1]. One of the reasons for the different trend of $S_\Lambda$ between the present and RMF [1] calculations was the density profile of the core nucleus. In the SD states of the Ar isotopes, in the AMD calculation, we found no ring-shape localization of the nucleons, which was predicted by the RMF calculation [1]. However, $S_\Lambda$ was not correlated with the overlap of the densities between $\Lambda$ and the nucleons. It was found
that the $S_{\Lambda}$ was correlated with $I_{\text{overlap}}$ when the odd-parity force was changed to be the same as the attractive even-parity force, as was the case in the RMF model, since it was a Hartree theory and the exchange terms had no contributions to the $\Lambda N$ interaction. Therefore, not only the differences in the density distributions but also the properties of the $\Lambda N$ interaction employed could cause the inconsistency of $S_{\Lambda}$ between the present and RMF [1] calculations.

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