IMPACTS OF HORIZONTAL MERGERS ON DUAL-CHANNEL SUPPLY CHAIN

XI ZHAO
College of Management and Economics
Tianjin University
Nankai District, Tianjin 300072, China

TENG NIU*
College of Management and Economics
Tianjin University
Nankai District, Tianjin 300072, China

(Communicated by Ada Che)

Abstract. This paper investigates the impacts of horizontal mergers on a dual-channel supply chain given the rapid development of e-commerce. Three types of horizontal mergers are considered in a dual-channel supply chain consisting of three firm types: suppliers, single-channel retailer, and dual-channel retailer. A comparison with a benchmark pre-merger scenario underscores the impacts of each horizontal merger on firms in the dual-channel supply chain. First, where horizontal mergers occur (i.e., at upstream or downstream tier) has an impact on firms in the dual-channel supply chain. Second, synergy costs trigger the domination of the synergy effect. Third, the degree of consumer preference for channels affects the trigger due to which the synergy effect outweighs the competitive effect. Although dual channels prevail in supply chain management, few studies pay attention to horizontal mergers in this context. Unlike literature on horizontal mergers in single-channel supply chains, we suggest that the impacts of horizontal mergers in dual-channel supply chains have unique features, and channel preference plays an important role in such impacts.

1. Introduction. Mergers and acquisitions (M&As) are important for rapid company expansion. Firms can accumulate long-term competitive advantages by obtaining stronger market power and saving costs through synergy effects—for example, the acquisition of Volvo by Geely Automobile in 2010 helped open up European markets and expand sales channels, as well as strengthen the local market position of Volvo as its premium brand [24].

Horizontal mergers, which combine firms in the same line of business into a single entity, occur in many industries—for example, the home appliance retail industry witnessed a merger between GOME and DaZhong Electronics, and the supermarket industry witnessed a merger between Vanguard and TESCO. The network information service and e-commerce industry has also seen mergers such as the one between

2020 Mathematics Subject Classification. Primary: 91B42, 91B54; Secondary: 91A06.
Key words and phrases. Horizontal mergers, dual-channel supply chain, channel preference, synergy effect, competitive effect.

* Corresponding author: Teng Niu.
Meilishuo and Mocu Street. With the rapid development of e-commerce, online sales channels play varied and important roles. An increasing number of retailers operate both bricks-and-mortar (B&M) stores and online shops, called a dual-channel [4]. Adopting a dual-channel model enables retailers to fulfill different consumer needs. In a dual-channel supply chain, a horizontal merger can be employed by a dual-channel firm for business expansion. For example, Suning, which owns more than 1,600 B&M stores and an online shop, merged with Redbaby, a famous online shop for mother and baby care products, in 2012.

This study considers a dual-channel supply chain consisting of two tiers. The upstream tier contains $m$ suppliers, while the downstream tier contains a dual-channel retailer and $n$ single-channel retailers. Consumers have different degrees of preferences for the B&M store and the online shop. Firms in the supply chain engage in quantity competition with each other at their own tier. The competition between upstream and downstream tiers also constitutes quantity competition. Thus, the authors study the effect of a horizontal merger in a dual-channel supply chain on different firms. Given three types of firms in a dual-channel supply chain, a series of models of horizontal mergers including one upstream merger and two downstream mergers is developed. To evaluate the effect of a horizontal merger, a benchmark where firms in the supply chain compete with each other before the merger occurs is also developed, allowing the authors to analyze the effect of a horizontal merger by comparing the equilibrium solutions between the merger model and the benchmark.

Since all firms in a supply chain form a system, a merger influences each member in the supply chain, and hence systematic changes caused by a horizontal merger are addressed based on three research questions. First, how does a merger at one tier affect a firm at its own tier and the other tier? Second, what is the difference in mergers occurring at different tiers of a dual-channel supply chain? Third, how does the degree of consumers’ channel preference affect the results of a horizontal merger?

To answer the above questions, the effect of a horizontal merger is separated into two aspects: the synergy effect and the competitive effect [5]. The former focuses on cost savings from the synergy of the merger, while the latter on the decreasing number of competitors at the tier where the merger occurs.

The key findings from the current study are as follows. First, the merger position could lead to different impacts on prices and outputs for both suppliers and retailers. In mergers between firms at the same tier (i.e. downstream), whether the dual-channel retailer acts as one of the merging firms also plays a role. Second, synergy cost can trigger the outweighing of competitive effect by synergy effect, which manifests only after exceeding a certain value after a horizontal merger. Third, consumer preference for channels affects the trigger due to which the synergy effect outweighs the competitive effect.

The remainder of the paper is organized as follows. Section 2 presents a brief overview of the literature; Section 3 focuses on a benchmark model of a two-tier dual-channel supply chain; Section 4 presents an upstream merger model and Section 5, two downstream merger models. Section 6 presents the numerical analysis of the profits of different mergers. Section 7 concludes the study.

2. Literature review. The authors now review two streams of literature focusing on (1) horizontal mergers in supply chains, and (2) the effects of a dual-channel on supply chain management.
2.1. **Horizontal mergers in supply chains.** According to the structures of supply chains, existing studies about horizontal mergers in supply chains can be classified into two categories, that is, horizontal mergers occur in the merging industries or in multitier supply chains.

First, the literature on horizontal mergers in the merging industries is reviewed. Early literature focusing on the incentive for horizontal mergers proposed analysis of the tradeoff between reduced competition and efficiency gains in the Cournot framework [11, 26, 27]. Since then, researchers had extended the analysis of horizontal mergers with the tradeoff. Inderst and Wey [18] showed that a horizontal merger was likely to occur under a Cournot competition industry if the products were complementary. Froeb et al. [12] investigated the relationship between the price effect of horizontal mergers and the pass-through rate: both changed consistently, and the direction of changes depended on the demand condition. In the free entry and exit industry, Davidson and Mukherjee [9] showed that a horizontal merger never impacts the industry output but benefited both the post-merging firm and society because of lower cost. Escrihuela-Villar and Fauli-Oller [10] showed that a merger between ineffective followers is profitable because of the output deduction of the leaders, and the merger resulted in price increase. Heywood and McGinty [16] focused on a horizontal merger between a leader and followers, suggesting that a profitable merger hurt non-participant firms but benefited society under convex costs. Unlike Escrihuela-Villar and Fauli-Oller [10], Cunha and Vasconcelos [8] allowed for increased efficiency caused by mergers and found that horizontal mergers could cause the price to fall if cost synergies generated by the merger were sufficiently high. Liu and Wang [21] noted that the effect of a horizontal merger gave the post-merger firm a strategic advantage, making it an industry leader. These studies lacked for attention on supply chain structures. Cho and Wang [6] analyzed the horizontal merger between newsvendors in an oligopolistic market. They argued that the merger benefited the post-merger entity through the reduction of the demand uncertainty. Xiao [30] also studied the horizontal merger in an oligopolistic market, and firms competed on production quantities with uncertainty. Xiao [30] modeled the effects of horizontal mergers in three aspects: reduced competition, cost synergy, and supply diversification. Similarly, Yuan et al. [32] also used those three effects to investigate the implications of uncertainty on horizontal mergers in a Cournot oligopolistic market.

Second, the literature on horizontal mergers in upstream or downstream tiers is reviewed. Compared effects of horizontal mergers in different tiers, Fumagalli and Motta [13] argued that a downstream merger was more detrimental to social welfare than an upstream merger in an industry with secret vertical contracts. Lommerud et al. [23] considered a downstream merger in an oligopolist industry comprising three retailers. They suggested that the downstream merger always decreases the input prices of the post-merger retailer, whereas the input price of the non-participant retailer changed with the product differentiation and the preferences of supplier. Inderst and Shaffer [17] examined downstream mergers in a supply chain where two retailers procured products from two suppliers. The results showed that the post-merger retailer could enhance its power by applying a single-sourcing purchasing strategy that resulted in less product variety. Symeonidis [28] considered oligopolist retailers that sold differentiated products and found that the downstream merger may increase consumer surplus and total welfare. Milliou and Pavlou [25] considered an upstream merger and found that the upstream merger could increase research and
development investment and decrease wholesale price. Cho [5] studied the impacts of horizontal mergers in both upstream and downstream tiers in the supply chain. Cho [5] showed that the upstream merger was less likely to increase consumer price than the downstream merger would. Zhu et al. [34] allowed for market power and operational synergy benefit generated by horizontal mergers. They suggested that upstream and downstream mergers led to similar consequences for non-participant firms. Lan et al. [20] considered a supply chain comprising one supplier and two asymmetric retailers. After a downstream merger, order quantities and profits of retailers decreased, meanwhile wholesale price dropped if the downstream merger effect was weak. Although these studies considered the structures of supply chains, they still lacked for attention on horizontal mergers in a dual-channel supply chain. Bimpikis et al. [2] provided the equilibrium analysis of horizontal mergers in the network structure with the Cournot competition.

2.2. Effects of a dual-channel on supply chain management. With the development of e-commerce, dual-channels play more important roles in supply chains. In this subsection, literature on the effects of a dual-channel on supply chain management is examined. Xu et al. [31] studied the effects of a two-way revenue sharing contract on pricing decisions in a dual-channel supply chain. The two-way revenue sharing contract could coordinate the supply chain in the presence of risk-averse attitudes of a manufacturer and retailer. Wang et al. [29] analyzed the channel selection decisions in a supply chain that involves a retailer and a manufacturer in a supply chain. Both the manufacturer and retailer had dual channels. The results showed that the retailer’s operating cost between both channels played an important role in the decision making. Cao et al. [3] considered a retailer with both offline and online channels for selling products. The retailer’s pricing decisions were associated with the parameters when both channels were available. Gao and Su [15] studied online and offline information regarding delivery to consumers. Three different information mechanisms were considered. The results suggested that the mechanisms may be not complementary, since one may hurt the profits of the others. Gao and Su [14] later studied the impacts of the option to buy-online-and-pickup-in-store on a dual-channel supply chain. The option was found to be not always profitable, especially for best-selling products. Liu et al. [22] found that when the manufacturer operated a dual-channel, the retailers may form an alliance only if the cost of online channel was sufficiently low, and thus decreased the profit of the manufacturer. Arya and Mittendorf [1] studied the condition for a retailer to operate a dual-channel in the presence of consumer sales taxes. Although the dual-channel allowed the retailer to meet more consumer needs, the taxes decrease the willingness of both consumers and retailers to pay for products, thereby affecting clearing price.

Considering the remarkable impacts in economy, horizontal mergers have been an important topic of research across fields, such as economics, strategy, and finance, for decades. However, few studies have dealt with operations and supply chain management with respect to horizontal mergers. Even though some scholars addressed the effects of horizontal mergers in supply chains, such as Symeonidis [28] and Cho [5], they did not pay attention to mergers in a dual-channel supply chain. Thus, this study analyzes the effects of horizontal mergers at both upstream and downstream tiers in a dual-channel supply chain.
3. Benchmark: the pre-merger model. In this section, we consider a supply chain comprising two tiers. The upstream tier includes \( m \) suppliers and the downstream tier includes \( n + 1 \) retailers. As the merger resulting in monopoly would be refused by the antitrust agencies [5], we assume that each tier of the dual-channel supply chain at least consists of three firms, i.e., \( m \geq 3 \) and \( n \geq 2 \) [33, 34]. The suppliers produce homogeneous products, while the retailers procure these products for selling them to consumers using the given channels [30]. The structure of the dual-channel supply chain is shown in Figure 1. Both the suppliers and the retailers engage in Cournot quantity competition with their peers at the horizontal level [5, 7, 19, 30, 33]. Moreover, firms engage in a Stackelberg competition at the vertical level [5]. The suppliers, as the Stackelberg leaders, first determine their output quantities. The retailers, as the Stackelberg followers, then determine their sale quantities. Finally, the retail prices of the two sales channels are realized [1]. Without loss of generality, we assume that the number of productions sold by the retailers equals the number of productions produced by the suppliers [5, 7, 19].

![Figure 1. The structure of a dual-channel supply chain](image)

In the upstream tier, supplier \( j (= 1, 2, \ldots, m) \) produces \( y_j \) productions at the unit production cost of \( c_s \). The wholesale price is \( w \). In the downstream tier, there are two different kinds of retailers. Retailer 0 has dual channels comprising an online shop and a B&M store. Retailer 0 sells \( q_0^o \) products through the online shop and \( q_0^s \) products through the offline store.

The unit sales costs of the online and offline channels are \( c_o^r \) and \( c_s^r \), respectively, \( c_o^r < c_s^r \). Retailers \( i (= 1, 2, \ldots, n) \) only have a B&M store. Retailer \( i \) sells \( q_i^s \) products at the unit sales cost \( c_s^r \). Assume that all the B&M stores are identical. Therefore, each retailer sets the same offline sales price. The online and the offline sales prices are denoted by \( p^o \) and \( p^s \).

Similar to Arya and Mittendorf [1], given that consumers have differentiated preference for the online shop against the B&M store, the inverse demand functions are as follows:

\[
p^o = a - q_0^o - \gamma \sum_{i=0,\ldots,n} q_i^s, \tag{1}
\]

\[
p^s = a - \sum_{i=0,\ldots,n} q_i^s - \gamma q_0^o, \quad i = 0, \ldots, n. \tag{2}
\]

In the inverse demand functions, \( \gamma \in (0, 1) \) represents the degree of consumers’ channel preference, and there is very little preference difference between the two channels when \( \gamma \) approaches one; \( a \) represents the highest sales price when \( q_0^o = q_0^s = 0 (i = 0, 1, 2, \ldots, n) \), which is considered the retail market potential.
With this formulation, the decisions problems can be described as follows:

For the suppliers,

$$\max_{y_j} (w - c_s)y_j, \quad j = 1, 2, \ldots, m.$$ (3)

For the retailers,

$$\max_{q_0, q_0^*} (p_0^* - c_s^* - w)q_0^* + (p_s^* - c_s^* - w)q_0^*, \quad i = 0, \ldots, n.$$ (4)

$$\max_{q_i} (p_s^* - c_s^* - w)q_i^*,$$ (5)

Through backward induction, we solve the equilibrium sales of the downstream tier when the wholes price is $w$, and the solutions are shown in Lemma 3.1.

**Lemma 3.1.** Given the wholesale price $w$, the sales of the retailers are as follows:

(a) For the dual-channel retailer ($i = 0$), the sales are

$$q_0^* = \frac{a - w}{2(1 + \gamma)} + \frac{\gamma c_s^* - c_r^*}{2(1 - \gamma^2)}$$ (6)

$$q_0^* = \frac{a - w - c_r^*}{2 + n} - \gamma \frac{a - w}{2(1 + \gamma)} - \gamma \frac{\gamma c_s^* - c_r^*}{2(1 - \gamma^2)}$$ (7)

(b) For each single-channel retailer ($i = 1, 2, \ldots, n$), the sales are

$$q_i^* = \frac{a - w - c_r^*}{2 + n}$$ (8)

The proof is in the Appendix.

Then, we deal with the outputs of the upstream tier. There is an equivalent relationship between the total sale quantity and the total supply quantity, that is, $Q = Y$, where $Q$ and $Y$ separately present the total downstream sales and the total upstream outputs. By rewriting the equation, we can obtain the expression of the wholesale price $w$ as shown in Lemma 3.2.

**Lemma 3.2.** $w = A - BY$,

where $A \equiv a - \frac{2 + n(2 + \gamma)c_s^* + (2 + n)c_r^*}{4 + n(3 + \gamma)}$ and $B \equiv \frac{2 + n(1 + \gamma)}{4 + n(3 + \gamma)}$.

The proof is in the Appendix.

For the sake of convenience, let $G \equiv a - c_r^* - c_s^*$ and $H \equiv c_s^* - c_r^*$. By combining Lemma 3.1 and Lemma 3.2 together, we can obtain the equilibrium solutions of the model without a merger as shown in Proposition 1.

**Proposition 1.** The equilibrium solutions of the model without a merger are as follows:

(a) For the dual-channel retailer ($i = 0$),

$$q_0^* = \frac{m((1 - \gamma)G + H)}{2(1 + m)(1 - \gamma^2)} + \frac{(1 + n)H}{(1 + m)(1 - \gamma)(4 + n(3 + \gamma))}$$ (9)

$$q_0^* = \frac{m(2 - n\gamma)G}{2(1 + m)(2 + n)(1 + \gamma)} - \frac{n\gamma H}{(1 + m)(1 - \gamma)(4 + n(3 + \gamma))} - \frac{m\gamma H}{(1 + m)(1 - \gamma)(4 + n(3 + \gamma))}$$ (10)

(b) For each single-channel retailer ($i = 1, 2, \ldots, n$), the sales are

$$q_i^* = \frac{a - w - c_r^*}{2 + n}$$ (8)

The proof is in the Appendix.
Horizontal mergers, including the upstream and the downstream, have systematic influences on the dual-channel supply chain. Proposition 1 shows that all the equilibrium solutions in the pre-merger scenario are affected by five parameters: (1) the number of the suppliers $m$; (2) the number of the single-channel retailers $n$; (3) the production cost $c_s$; (4) the off-line retail cost $c_{sr}$; (5) the on-line retail cost $c_{or}$. However, horizontal mergers can affect not only the number of members in the supply chain but also the cost of the merged entities. That is, horizontal mergers can alter at least one of those five parameters, which means that the horizontal merge has an influence on not only the level where the merger occurs but also the other level in the supply chain.

Then, the authors consider scenarios in which a merger occurs and analyze its influence. In sections 4 and section 5, the effects of an upstream merger and two different kinds of downstream mergers are respectively analyzed.

4. **An upstream merger.** The effects of an upstream merger in the dual-channel supply chain are examined here. Suppose a merger occurs between two suppliers, namely, the *merging suppliers*. Without loss of generality, assume that Supplier 1 and Supplier 2 merge. The merged supplier is the *post-merger supplier*, indexed by $j = 1$. The indexes of other suppliers are same with the benchmark $j = 3, 4, \ldots, m$, and they are called *non-participate suppliers*. The retailers match their indexes to the benchmark.

To illustrate clearly, Figure 2 shows the structure of the supply chain with an upstream merger. The post-merger supplier ($j = 1$) replaces the merging suppliers (Supplier 1 and Supplier 2 in the pre-merger scenario) after the merger. The rectangle frame is used to describe the upstream merger in Figure 2.

![Figure 2. The structure of an upstream merger](image)

According to the common method in Cho [5] and Xiao [30], the aggregate effect of a horizontal merger is broken down into two individual effects: the *synergy effect*
and the competitive effect. (a) A horizontal merger generally creates the synergy effect by reducing the marginal cost of the merged entity; (b) Each horizontal merger produces the competitive effect by reducing the competition intensity in the market where the merger occurs because there are fewer firms in the market after the merger. However, either of those two effects is only a partial result of a horizontal merger. The aggregate effect of a horizontal merger can be thereafter obtained by combining the synergy effect and the competitive effect.

4.1. The synergy effect. To isolate the synergy effect, a scenario is considered where the post-merger suppliers \( (j = 1, 2) \) maintain their independence but share the saved production costs resulting from synergies. The saving cost is denoted by \( \Delta U \equiv c_s - c_s^U \geq 0 \). The superscript \( U \) denotes the upstream merger scenario. Other non-participant suppliers keep their cost unchanged as the benchmark.

Given the wholesale price \( \bar{w}^U \), retailers compete in the same way as the benchmark. Following a similar process, the inverse demand function of the supply market is evidently \( \bar{w}^U = A - BY^U \). Then, the profit of supplier \( j \) is

\[
\bar{\pi}_{sj}^U = \begin{cases} (\bar{w}^U - c_s^U)\bar{y}_j^U, & j = 1, 2 \\ (\bar{w}^U - c_s)\bar{y}_j^U, & j = 3, 4, \ldots, m \end{cases}
\]  

(13)

Conducting similar analytical procedures to that for the benchmark, equilibrium solutions of the upstream merger model can be derived. Then, by comparing them with the benchmark, the synergy effect can be obtained as shown in Proposition 2.

**Proposition 2.** The synergy effect of an upstream merger on firms is as follows:

(a) (Upstream tier) For the wholesale price, \( \bar{w}^U < w \). For the outputs of each post-merger supplier \( (j = 1, 2) \), \( \bar{y}_1^U = \bar{y}_2^U > y_j \). For the outputs of each non-participant suppliers \( (j = 3, 4, \ldots, m) \), \( \bar{y}_j^U < y_j \).

(b) (Downstream tier) For the online and offline sales prices, \( \bar{p}^{SU} < p^o \), and \( \bar{p}^{SU} < p^s \). For the sales of the dual-channel retailer \( (i = 0) \), \( \bar{q}_0^{SU} > q_0^o \) and \( \bar{q}_0^{SU} > q_0^s \). For the sales of each single-channel retailer \( (i = 1, 2, \ldots, n) \), \( \bar{q}_i^{SU} > q_i^s \).

The proof is in the Appendix.

First, the synergy effect of an upstream merger on suppliers is analyzed. Because of the synergy, the marginal cost of each post-merger supplier decreases, namely, \( c_s^U < c_s \). Hence, each post-merger supplier increases the output. Owing to a cost disadvantage compared with the post-merger suppliers, each non-participant supplier suffers an output loss. Therefore, the synergy effect procures a negative externality for the non-participant suppliers. Overviewing the whole supply market, the wholesale price \( w \) decreases because of the average marginal cost decreases, that is, \( [(m - 2)c_s + 2c_s^U]/m < c_s \).

Then, for the retailers, the synergy of the upstream merger affects retailers by the wholesale price. As the wholesale price decreases, retailers enjoy the reduced purchase cost, and thereby drop their sales prices, while the sales of two different kinds of retailers improve.

4.2. The competitive effect. To analyze the competitive effect, the scenario in which supplier 1 and supplier 2 merge and turn into a single entity is examined. Hence, the total number of suppliers reduces from \( m \) to \( m - 1 \). To isolate the competitive effect, suppose there is no merger synergy here. Since all the suppliers are homogeneous, the equilibrium solutions of the upstream merger considering only the competitive effect can be obtained by replacing \( m \) with \( m - 1 \) in the solutions
of the benchmark. By conducting a comparison between the post-merger solutions and the benchmark, the competitive effect of the upstream merger is derived as shown in Proposition 3.

**Proposition 3.** The competitive effect of upstream merger on firms is as follows:

(a) (Upstream tier) For the wholesale price, \( \hat{w}^U > w \). For the outputs of the post-merger supplier \((j = 1)\), \( \tilde{y}_j^U < y_1 + y_2 \). For the outputs of each non-participant supplier \((j = 3, 4, \ldots, m)\), \( \tilde{y}_j^U > y_j \).

(b) (Downstream tier) For the online and offline sales prices, \( \tilde{p}^{oU} > p^o \) and \( \tilde{p}^{U} > p^s \). For the sales of the dual-channel retailer \((i = 0)\), \( \tilde{q}_0^U < q_0^o \) and \( \tilde{q}_0^U < q_0^s \). For the sales of each single-channel retailer \((i = 1, 2, \ldots, n)\), \( \tilde{q}_i^U < q_i^s \).

The proof is in the Appendix.

First, the competitive effect of the upstream merger on suppliers is analyzed. *Ceteris paribus*, each supplier makes a greater output than the benchmark. That is, the competitive effect procures a positive externality to the non-participant suppliers. The post-merger supplier’s output does not exceed the aggregate outputs of the two merging suppliers in the pre-merger scenario. Given the reduced intensity of competition within suppliers, the total output of the upstream tier thereby decreases, which results in an increase in the wholesale price.

Next, the competitive effect of the upstream merger on retailers is discussed. Since the wholesale price rises, the sales price of both the online and the offline channels increase as well. Moreover, two kinds of retailers’ sales decrease.

### 4.3. The aggregate effect

In this section, the authors examine the aggregate effect of the upstream merger. Considering both the cost synergy shared by the merging suppliers and the reduced intensity of competition, how the merger affects the supply chain can be illustrated. Let \( C_1^U \equiv \frac{C_d^U}{1 + m} \), \( C_2^U \equiv G + \frac{(2+n)H}{4 + n(4 + r)} \); it is easy to see that \( 0 < C_1^U < C_2^U \). Assume \( y_j^U > 0 \) for all \( j \) is justified, and then derive \( 0 < \Delta^U < C_2^U \). Within this range of the cost synergy, the authors compare the post-merger solutions with the benchmark.

**Proposition 4.** The aggregate effect of the upstream merger on firms is as follows:

All the following results hold if and only if \( \Delta^U > C_1^U \).

(a) (Upstream tier) For the wholesale price, \( w^U < w \). For the outputs of the post-merger supplier \((j = 1)\), \( y_j^U > y_1 + y_2 \). For the outputs of each non-participant supplier \((j = 3, 4, \ldots, m)\), \( y_j^U < y_j \).

(b) (Downstream tier) For the online and offline sales prices, \( p^{oU} < p^o \) and \( p^U < p^s \). For the sales of the dual-channel retailer \((i = 0)\), \( q^U > q^o \) and \( q^U > q^s \). For the sales of each single-channel retailer \((i = 1, 2, \ldots, n)\), \( q_i^U > q_i^s \).

The proof is in the Appendix.

Evidently, when the cost synergy \( \Delta^U \) exceeds \( C_1^U \), the aggregate effect performs the same as the synergy effect. Hence, \( C_1^U \) is the trigger for the synergy effect. That is, the synergy effect could outweigh the competitive effect as long as \( \Delta^U > C_1^U \). Because \( C_1^U \in (0, C_2^U) \), either the synergy effect or the competitive effect can dominate in an upstream merger. Since \( \partial C_1^U / \partial \gamma < 0 \), \( C_1^U \) would increase if the preference between channels decreases. When \( C_1^U \) increases, it is harder for the post-merger supplier to trigger the synergy effect. As mentioned in Section 4.1, the synergy effect increases the wholesale and the sales prices. Therefore, a higher \( C_1^U \) may indicate that all the members of the supply chain are less likely to increase their prices after the merger occurs.
5. A downstream merger. In this section, the effects of a downstream merger in a dual-channel supply chain are analyzed. Two models of downstream merger are developed based on the retailer type: one is downstream merger between two single-channel retailers (the \textit{SS model}) and the other is downstream merger between dual-channel and single-channel retailers (the \textit{DS model}). In the \textit{SS model}, the merging firms are two single-channel retailers (denoted by the superscript \text{SS}); in the \textit{DS model}, the merging firms are a dual-channel retailer and a single-channel retailer (denoted by the superscript \text{DS}).

5.1. The \textit{SS model}. In this subsection, the authors assume that the merging firms are retailer 1 and retailer 2. After the merger occurs, two merging retailers turn into a single entity, namely, the \textit{post-merger} retailer. Besides, the \textit{post-merger} retailer is indexed as retailer 1. Other non-participant retailers ($i = 0, 3, 4, \ldots, n$) maintain their index at the benchmark. The \textit{SS model} stands for all the mergers that occur between any two single-channel retailers. Figure 3 illustrates the structure of the SS merger.

![Figure 3. The structure of a SS merger](image)

5.1.1. The synergy effect. Assume that the post-merger retailers ($i = 1, 2$) only share a cost reduction because of the synergy, but still make their decision independently. Their retail cost reduces from $c^r_i$ to $c^{SS}_i$, and the reduced cost is denoted by $\Delta^{SS} ≡ c^r_i - c^{SS}_i$. The cost of retailer $i (i = 0, 3, 4, \ldots, n)$ is the same with the benchmark.

The inverse demand functions are similar to the benchmark. Given the wholesale price $\bar{w}^{SS}$, the profit of retailer $i$ is

$$\bar{\pi}^{SS}_{ri} = \begin{cases} (\bar{p}^{SS} - \bar{w}^{SS} - c^r_i)q_0^{SS} + (\bar{p}^{SS} - \bar{w}^{SS} - c^{SS}_i)q_i^{SS}, & i = 0 \\ (\bar{p}^{SS} - \bar{w}^{SS} - c^{SS}_i)q_i^{SS}, & i = 1, 2 \\ (\bar{p}^{SS} - \bar{w}^{SS} - c^r_i)q_0^{SS}, & i = 3, 4, \ldots, n \end{cases}. \quad (14)$$

Then, the total sales for the downstream tier in equilibrium can be derived:

$$\bar{Q}^{SS} = \frac{(a - \bar{w}^{SS})(4 + n(3 + \gamma))}{2(2 + n)(1 + \gamma)} + \frac{4\Delta^{SS} + nc^r_i}{2(2 + n)} - \frac{c^r_i + c^{SS}_i}{2(2 + \gamma)}. \quad (15)$$

Based on the equation between sale and production, the inverse demand function for the upstream tier can be derived, which is shown in Lemma 5.1.

\textbf{Lemma 5.1.} $\bar{w}^{SS} = \bar{A}^{SS} - B^{SS}\bar{Q}^{SS}$,
where $\bar{A}^{SS} = a - \frac{(2 + n)c^r_i + (2 + n(2 + \gamma)c^r_i - 4(3 + \gamma)\Delta^{SS})}{4 + n(3 + \gamma)}$ and $\bar{B}^{SS} = \frac{2(2 + n)(1 + \gamma)}{4 + n(3 + \gamma)}$. 
Comparing the equilibrium solutions between the SS model and the pre-merger model helps obtain the synergy effect on the supply chain as shown in Proposition 5. Let $\tilde{\gamma} = \sqrt{4mn(2+n) + (2+n+3mn)^2 - (2+3mn)^2}/2mn$.

**Proposition 5.** The synergy effect of the SS merger on firms is as follows:

(a) (Upstream tier) For the wholesale price, $\tilde{w}^{SS} > w$. For the outputs of each supplier ($i = 1, 2, \ldots, m$), $\tilde{y}_i^{SS} > y_i$.

(b) (Downstream tier) For the online sales price, $\tilde{p}^{oSS} > p^o$ if $0 < \gamma < \tilde{\gamma}$, and $\tilde{p}^{oSS} < p^o$ if $\tilde{\gamma} < \gamma < \tilde{\gamma}/n$. For the offline sales price, $\tilde{p}^{SS} < p^o$. For the sales of each post-merger retailer ($i = 1, 2$), $\tilde{q}_i^{SS} = \tilde{q}_2^{SS} > q_i^*$. For the sales of the dual-channel non-participant retailer ($i = 0$), $\tilde{q}_0^{oSS} < q_0^o$ and $\tilde{q}_0^{SS} < q_0^s$. For the sales of each single-channel non-participant retailer ($i = 3, 4, \ldots, n$), $\tilde{q}_i^{SS} < q_i^*$. The proof is in the Appendix.

First, the synergy effect of the SS merger on the upstream tier is analyzed. With the improved market potential of the upstream tier, $\tilde{A}^{SS} > A$, each supplier increases its outputs. That is, the synergy effect of the SS merger brings positive externalities to the other tier in the supply chain. The wholesale price is always higher than the benchmark. This result is contrary to the synergy effect of the upstream merger since the synergy effect of the upstream merger reduces the price of the other tier.

Then, the synergy effect of the SS merger on the downstream tier is discussed. The post-merger retailers share a reduced retail cost and bring a cost advantage. Therefore, their sales improve. For the non-participant retailers, the sales decrease because of the relative disadvantage of their retail cost. The total sales increase because of the equation between the total sales and the total outputs. Therefore, the offline sales price falls following fiercer quantity competition. Besides, channel competition also exists in addition to the quantity competition in the downstream tier. A higher (lower) $\gamma$ indicates more (less) intense channel competition. The online sales price decreases (increases) if the channel competition is high (low).

**5.1.2. The competitive effect.** The competitive effect of the SS merger is analyzed by considering that the post-merger retailer ($i = 1$) does not enjoy the synergy from the merger. Similar to the upstream competitive effect, the equilibrium solutions of the SS model can be derived by replacing $n$ with $n-1$ in the equilibrium solutions of the benchmark. Thereby, the inverse demand function of the upstream tier can be deduced as follows:

**Lemma 5.2.** $\tilde{w}^{SS} = \tilde{A}^{SS} - \tilde{B}^{SS} \tilde{Y}^{SS}$, where $\tilde{A}^{SS} = a - \frac{(1+n)c_o^e + (n(2+\gamma) - \gamma)c_r^e}{1-\gamma+n(3+\gamma)}$ and $\tilde{B}^{SS} = \frac{2(2+n)(1+\gamma)}{1-\gamma+n(3+\gamma)}$.

The downstream competitive effect on the supply chain is shown in Proposition 6.

**Proposition 6.** The competitive effect of SS merger on firms is as follows:

(a) (Upstream tier) For the wholesale price, $\tilde{w}^{SS} > w$. For the outputs of each supplier ($i = 1, 2, \ldots, m$), $\tilde{y}_i^{SS} < y_i$.

(b) (Downstream tier) For the online and offline sales price, $\tilde{p}^{oSS} > p^o$ and $\tilde{p}^{SS} > p^o$. For the sales of each post-merger retailer ($i = 1$), $\tilde{q}_i^{SS} < q_i^* + q_2^*$. For the sales of the dual-channel non-participant retailer ($i = 0$), $\tilde{q}_0^{oSS} < q_0^o$ and $\tilde{q}_0^{SS} < q_0^s$. For the sales of each single-channel non-participant retailer ($i = 3, 4, \ldots, n$), $\tilde{q}_i^{SS} > q_i^*$. 


The proof is in the Appendix.

First, the competitive effect of the SS merger on the upstream tier is discussed. Suppliers decrease outputs after the SS merger, indicating that the SS merger’s competitive effect brings negative externalities to the other tier in the supply chain. This phenomenon is analogous to the upstream competitive effect on the downstream tier. Thus, the competitive effect of one tier generates negative externalities to the other tier in the supply chain. As the outputs of the upstream tier decreases, the wholesale price thereby increases. Combining with Proposition 3, the results show that the competitive effect of one tier always increases the price in the other tier.

For the competitive effect of the SS merger on the downstream tier, all single-channel non-participant retailers increase their sales. Thus, the downstream merger’s competitive effect brings positive externalities to the non-participant retailers. The post-merger retailer’s sales are less than the two merging retailers’ sales in the benchmark. The offline sales of the non-participant dual-channel retailer (retailer 0) decrease, while its online sales increase. As the total sales decrease, both the online and the offline prices increase.

5.1.3. The aggregate effect. This subsection investigates the aggregate effect of the SS merger. The aggregate effect’s model is built by combining the synergy effect and the competitive effect.

Following a similar process, the inverse demand function of the upstream tier is derived in Lemma 5.3.

Lemma 5.3. \( w^{SS} = A^{SS} - B^{SS} \psi^{SS} \), where \( A^{SS} = a - \frac{(n(2+\gamma)-\gamma)c_{i}^{s}+(1+n)c_{i}^{o}-2(1+\gamma)\Delta^{SS}}{1-\gamma+n(3+\gamma)} \) and \( B^{SS} \equiv \frac{2(1+\gamma)(1+n)}{1-\gamma+n(3+\gamma)} \).

Then, the aggregate effect of the SS merger is discussed, as presented in Proposition 7.

Proposition 7. The aggregate effect of the SS merger on firms is as follows:

(a) (Upstream tier) For the wholesale price, \( w^{SS} > w \). For the outputs of each supplier \((j = 1, 2, \ldots, m)\), \( y_{j}^{SS} > y_{j} \) if and only if \( \Delta^{SS} > C_{SS}^{1} \).

(b) (Downstream tier) For the online and offline sales price, \( \{p_{o}^{SS} < p_{o}^{0} \) and \( p_{s}^{SS} < p_{s}^{0} \} \) if and only if \( \Delta^{SS} > C_{SS}^{2} \). For the sales of the post-merger retailer \((i = 1)\), \( q_{i}^{SS} > q_{i}^{0} + q_{i}^{2} \) if and only if \( \Delta^{SS} > C_{SS}^{3} \). For the online sales of the dual-channel non-participant retailer \((i = 0)\), \( q_{0}^{SS} < q_{0}^{0} \). For the offline sales of the dual-channel non-participant retailer \((i = 0)\), \( q_{0}^{SS} < q_{0}^{0} \) if and only if \( \Delta^{SS} > C_{SS}^{4} \). For the sales of each single-channel non-participant retailer \((i = 3, 3, \ldots, n)\), \( q_{i}^{SS} < q_{i}^{0} \) if and only if \( \Delta^{SS} > C_{SS}^{5} \).

If and only if \( \Delta^{SS} > C_{SS}^{6} \),

\[
\begin{align*}
C_{SS}^{2} & = \frac{m(4+n(3+\gamma))((1-\gamma+n(3+\gamma))g-(1+n)(2+n)(1-\gamma)h)}{(2+n)(4+n(3+\gamma))(1+n)(1-\gamma+n(3+\gamma))}, \\
C_{SS}^{3} & = \frac{mn(1-\gamma+n(3+\gamma))(4+n(3+\gamma))G+(1+n)(2+n)(2-3n(-2-2n)\gamma)H}{(2+n)(4+n(3+\gamma))(1+n)(1-\gamma+n(3+\gamma))}, \\
C_{SS}^{4} & = \frac{m(4+n(3+\gamma))((1-\gamma+n(3+\gamma))G-(1+n)(2+n)(1-\gamma)h)}{(2+n)(4+n(3+\gamma))(1+n)(1-\gamma+n(3+\gamma))}, \\
C_{SS}^{5} & = \frac{m(4+n(3+\gamma))((1-\gamma+n(3+\gamma))G-(1+n)(2+n)(3+\gamma)H)}{(2+n)(4+n(3+\gamma))(1+n)(1-\gamma+n(3+\gamma))}.
\end{align*}
\]

The proof is in the Appendix.

While Proposition 4 illustrates only one threshold for the reduced cost in the upstream merger, Proposition 7 shows that the reduced cost in the SS merger has
five different thresholds, namely, $C_{1SS}^1$, $C_{2SS}^2$, $C_{3SS}^3$, $C_{4SS}^4$, and $C_{5SS}^5$, where $C_{1SS}^1 > C_{2SS}^2 > C_{3SS}^3 > C_{4SS}^4 > C_{5SS}^5$. A higher threshold places a higher requirement for the reduced cost enjoyed by the post-merger retailer.

Proposition 7 shows the aggregate effect on both outputs and prices in the dual-channel supply chain. First, the aggregate effect of the SS merger on the upstream tier is analyzed. As the synergy effect and the competitive effect on the outputs of each supplier are opposite, $C_{1SS}^1$ exists for the outputs. The outputs increase if and only if $\Delta^{SS} > C_{1SS}^1$. The wholesale price is always lower than the benchmark.

Next, the aggregate effect of the SS merger on the downstream tier is discussed. For the post-merger retailer, the sales increase only if $\Delta^{SS} > C_{3SS}^3$. The sales of the non-participant single retailers are always higher than the benchmark. The online sales of the dual-channel retailer always decrease, while its offline sales decrease only if $\Delta^{SS} > C_{4SS}^4$. Both the online and the offline prices decrease only if $\Delta^{SS} > C_{2SS}^2$.

As $\partial C_{SS}^1/\partial \gamma > 0$, $\partial C_{SS}^3/\partial \gamma > 0$, and $\partial C_{SS}^4/\partial \gamma > 0$, all the thresholds for the SS merger increase if $\gamma$ increases. If the channel competition is less intense, the offline sales of retailer 0 would be less likely to decrease, the sales of the post-merger retailer would be less likely to increase, and both the online and the offline prices would be less likely to decrease.

5.2. The DS model. In this subsection, the DS model is analyzed. Assume that the merging firms are retailer 0 and retailer 1. After the merger, the two merging retailers merge into one firm, retailer 0, the post-merger retailer. All the non-participant retailers ($i = 2, 3, \ldots, n$) keep their indexes unchanged, and so do the suppliers. The DS model stands for the merger that occurs between the dual-channel retailer and any single-channel retailer. Figure 4 illustrates the structure of DS merger.

![Figure 4. The structure of a DS merger](image)

5.2.1. The synergy effect. Assume that the post-merger retailers ($i = 0, 1$) only share a cost reduction because of the synergy. Their online retail cost reduces from $c_r^o$ to $c_r^{DS}$, and their offline retail cost reduces from $c_r^s$ to $c_r^{DS}$; the reduced cost is denoted by $\Delta^{DS} \equiv c_r^o - c_r^{DS} = c_r^s - c_r^{DS} > 0$.

The inverse demand functions are similar to the benchmark. Given the wholesale price $w^{DS}$, the profit of retailer $i$ is

$$
\pi_i^{DS} = \begin{cases} 
(p^{oDS} - \bar{w}^{DS} - c_r^{oDS})q_0^{DS} + (p^{sDS} - \bar{w}^{DS} - c_r^{sDS})q_0^{DS}, & i = 0 \\
(p^{oDS} - \bar{w}^{DS} - c_r^{oDS})q_i^{DS}, & i = 1 \\
(p^{sDS} - \bar{w}^{DS} - c_r^{sDS})q_0^{DS}, & i = 2, 3, \ldots, n
\end{cases}
$$
Then, the total sales for the downstream tier in equilibrium can be derived:

\[
\bar{Q}^{DS} = \frac{(a - \bar{w}^{DS})(4 + n(3 + \gamma)) + (6 + n + 2\gamma - n\gamma)\Delta^{DS}}{2(2 + n)(1 + \gamma)} - \frac{c^o + c^s}{2(1 + \gamma)} - \frac{nc^s}{2(2 + n)}.
\]  

(17)

Based on the equation between sale and production, the inverse demand function for the upstream tier can be obtained as follows.

Lemma 5.4. \(\hat{w}^{DS} = \bar{A}^{DS} - \bar{B}^{DS}\bar{Y}^{DS}\),

where \(\bar{A}^{DS} = a - \frac{(2 + n)c^o + (2 + n(2 + \gamma))c^s - (6 + n + 2\gamma - n\gamma)\Delta^{DS}}{4 + n(3 + \gamma)}\) and \(\bar{B}^{DS} = \frac{2(1 + \gamma)(2 + n)}{4 + n(3 + \gamma)}\).

Comparing the equilibrium solutions between the DS model and the pre-merger model, Proposition 8 is derived.

Proposition 8. The synergy effect of the DS merger on firms is as follows:

(a) (Upstream tier) For the wholesale price, \(\hat{w}^{DS} > \hat{w}\). For the outputs of each supplier \((j = 1, 2, \ldots, m)\), \(\bar{y}^{DS}_j > y_j\).

(b) (Downstream tier) For the online and offline prices, \(\hat{p}^{DS} < p^o\) and \(\hat{p}^{DS} > p^s\). For the sales of the post-merger retailers \((i = 0, 1), \bar{q}^{0DS}_i > q^o_0\) and \(\bar{q}^{0DS}_i > q^o_0\), and \(\bar{q}^{1DS}_i > q^o_1\). For the sales of each non-participant retailer \((i = 2, 3, \ldots, n)\), \(\bar{q}^{1DS}_i < q^o_1\).

The proof is in the Appendix.

First, the authors analyze the synergy effect of the DS merger on the upstream tier. After the DS merger, suppliers increase their outputs due to the improved market potential \((\bar{A}^{DS} > A)\). The total outputs for the upstream tier thereby increase. That is, the synergy effect of the DS merger also brings positive externalities to the other tier in the supply chain. Combining the synergy effects of the SS merger and the DS merger shows that the synergy effect of a downstream merger always brings positive externalities to the other tier in the dual-channel supply chain. The wholesale price increases after the DS merger.

For the synergy effect of the DS merger on the downstream tier, the sales of the post-merger retailers increase since they share a cost saving. The sales of the non-participant retailers decrease.

The competitive effect of the DS merger reflects that of the SS merger; hence, it is omitted.

5.2.2. The aggregate effect. Similar to the processes for the aggregate effects, the inverse demand function of the upstream tier is shown in Lemma 5.5.

Lemma 5.5. \(w^{DS} = A^{DS} - B^{DS}\bar{Y}^{DS}\),

where \(A^{DS} \equiv a - \frac{(n(2 + \gamma) - \gamma)c^o + (1 + n)c^s - (3 + n + \gamma - n\gamma)\Delta^{DS}}{1 - \gamma + n(3 + \gamma)}\) and \(B^{DS} \equiv \frac{2(1 + \gamma)(1 + n)}{1 - \gamma + n(3 + \gamma)}\).

Proposition 9. The aggregate effect of the DS merger on firms is as follows:

(a) (Upstream tier) For the wholesale price, \(w^{DS} > w\). For the outputs of each supplier \((j = 1, 2, \ldots, m), \bar{y}^{DS}_j > y_j\) if and only if \(\Delta^{DS} > C^1_{DS}\).

(b) (Downstream tier) For the online sales price, \(p^{oDS} < p^o\) if and only if \(\Delta^{DS} > C^2_{DS}\). For the offline sales price, \(p^{sDS} > p^s\). For the online sales of the post-merger retailer \((i = 0), q^o_0 > q^o_0\) if and only if \(\Delta^{DS} > C^3_{DS}\). For the offline sales of the post-merger retailer \((i = 0), q^{oDS}_0 > q^o_0\). For the total sales of the post-merger
retailer \( (i = 0) \), \( q_{0}^{DS} + q_{s}^{DS} > q_{0}^{o} + q_{1}^{o} \) if and only if \( \Delta^{DS} > C_{DS}^{1} \). For the sales of each non-participant retailer \( (i = 2, 3, \ldots, n) \), \( q_{i}^{DS} < q_{i}^{o} \) if and only if \( \Delta^{DS} > C_{DS}^{5} \).

\[
\begin{align*}
C_{DS}^{1} & = \frac{2(1+\gamma)G}{(2+n)(3+n)(1+\gamma)^{2}}, \\
C_{DS}^{2} & = \frac{2m\gamma(1-\gamma+n(3+n))}{(2+n)(2n+1)((1-\gamma)n+n(3+n))}, \\
C_{DS}^{3} & = \frac{2m\gamma(1/n-1)((1-\gamma+n(3+n))n(3+n)+m(1-\gamma+n(3+n))^2)}{2(1+\gamma)^{2}H}, \\
C_{DS}^{4} & = \frac{2mn(4+n(3+n))(1+\gamma)(1-\gamma+n(3+n))G-(2n^2-1)(2+n)(1+\gamma)(3+n)H}{(4+n(3+n))(2n)((n+1)(1+\gamma)+m(1-\gamma+n(3+n))^2)}, \\
C_{DS}^{5} & = \frac{m(1-\gamma+n(3+n))(4+n(3+n))(1+\gamma)(2+n)(3+n)H}{(4+n(3+n))(2n)((4+m+4n+3mn+n-1)\gamma^n)}.
\end{align*}
\]

The proof is in the Appendix.

First, the authors discuss the aggregate effect on the upstream tier. Since both the synergy effect and the competitive effect cause the wholesale price to increase, the aggregate effect results in an increasing wholesale price. As for the influence on the outputs of each supplier, there is a threshold for the cost synergy, \( C_{DS}^{1} \). If and only if \( \Delta^{DS} > C_{DS}^{1} \), the outputs increase.

Then, for the aggregate effect of the DS merger on the downstream tier is discussed, the online sales of the post-merger retailer (retailer 0) increase if and only if \( \Delta^{DS} > C_{DS}^{3} \); while its offline sales always increase. Only if \( \Delta^{DS} > C_{DS}^{4} \), the sales of the post-merger retailer are higher than the two merging retailers. The sales of the non-participant retailers decrease if and only if \( \Delta^{DS} > C_{DS}^{5} \). The online and the offline prices are affected by the DS merger. The online price decreases if and only if \( \Delta^{DS} > C_{DS}^{2} \). The offline price always increases.

In addition, \( \partial C_{DS}^{k} / \partial \gamma > 0 \) \( (k = 1, 2, \ldots, 5) \) indicates that when \( \gamma \) increases, the suppliers are less likely to increase their outputs, the dual-channel retailer is less likely to increase its online sales, and the single-channel retailers are less likely to decrease their sales. Meanwhile, the online sales price is less likely to decrease.

6. Numerical analysis. In this section, the authors discuss the roles of different horizontal mergers in the dual-channel supply chain by using numerical examples given in Figures 5, 6, and 7. Let \( m = 4, n = 3, a = 20, c_{r} = 2, c_{s} = 1, c_{o} = 3, \) and \( \gamma = 0.5 \).

In general, when the synergy cost equals zero, the aggregate effect performs as the competitive effect; as the synergy cost increases, the synergy effect begins to take effect. When the synergy cost is sufficiently high, the aggregate effect is taken over by the synergy effect. Therefore, the vertical intercept is considered to be the competitive effect, whereas the solid lines are the synergy effect in Figures 5, 6, and 7.

Figure 5 illustrates the profits affected by the upstream merger. Let \( \Delta^{U} \in (0, 3) \). Figure 5(a) shows the profits of the tier where the upstream merger occurs. The vertical intercepts of the post-merger supplier and each non-participant supplier are same. That is, there are no differences in profits between the post-merger supplier and each non-participant supplier when only the competitive effect exists. However, the intercept of the post-merger supplier is below its benchmark, and each non-participant supplier’s intercept is higher than its benchmark. Focusing on the profits of the upstream tier shows that it is higher than the benchmark. Thus, the competitive effect hurts the post-merger supplier, but benefits the non-participant supplier and the whole upstream tier.
As the synergy cost increases, the profits of the post-merger supplier go up and the profits of each non-participant go down. Only when the cost synergy is sufficiently high can the profits of the post-merger supplier be higher than the benchmark; the profits of each non-participant supplier are always higher than the benchmark. That is, there is a minimum requirement of the synergy cost for the upstream merger to occur. Besides, the total profits of the upstream tier increase as the synergy cost increases.

Figure 5(b) shows the profits of the other tier. The intercepts of the dual-channel retailer and each single-channel retailer are lower than the benchmark, indicating that the upstream merger have a negative effect on the profits of the retailers. As the synergy cost increases, the profits increase, but none are higher than the benchmark. That is, even the synergy cost can benefit the profits of the retailers, but it cannot offset the impact of the competitive effect of the upstream merger. Hence, the total profits of the whole downstream tier are always lower than the benchmark.

Figure 6 illustrates the profits affected by the SS merger. Let $\Delta^{SS} \in (0, 2)$. Figure 6(a) shows the profits of the tier where the SS merger occurs. The vertical intercepts of the post-merger retailer and each single-channel retailer are the same. Moreover, in the presence of a competitive effect, the single-channel retailer, the dual-channel retailer and the downstream tier are expected to reap above-benchmark profits except the post-merger retailers. That is, the competitive effect of the SS merger hurts the post-merger retailer but benefits each non-participant retailer and the whole downstream tier.
Along with the synergy cost increasing, the profits of the post-merger retailer increase while the profits of each non-participant retailer go down. Until the synergy cost is sufficiently high, the profits of the post-merger retailer are higher than its benchmark. Moreover, the profits of the non-participant retailers are always higher than their benchmark. Similar to the upstream merger, there is a minimum requirement of the synergy cost for the SS merger to occur.

Figure 6(b) shows the profits of the other tier. The intercept of each supplier is below its benchmark. That is the competitive effect of the SS merger hurts suppliers. As the synergy cost increases, the profits of each supplier increase as well. However, there is a threshold for each supplier to exceed its benchmark profits; this is different for the upstream merger.

![Figure 6(a): Profits of Retailers](image1)

![Figure 6(b): Profits of Suppliers](image2)

**Figure 7.** Aggregate effect of the DS merger on the profits

Figure 7 illustrates the profits affected by the DS merger. Let $\Delta \in (0, 2)$. The trends of the firms when the DS merger occurs are similar to that of the SS merger. A minimum requirement of the synergy cost also exists for the DS merger to occur, as well as a threshold for each supplier to exceed its benchmark profits. However, the requirement and the threshold in the DS merger scenario are lower than those in the SS merger scenario. That is, they are easier to achieve in the DS merger than in the SS merger.

7. **Conclusion.** This study examined the impacts of horizontal mergers on firms in a dual-channel supply chain. Considering three types of firms in the supply chain, the authors built three models for horizontal mergers: the upstream merger model, the SS merger model, and the DS merger model. To illustrate the impacts of horizontal mergers clearly, the impacts are separated into the synergy effect and the competitive effect.

Comparing the scenarios of horizontal mergers with the benchmark scenario yielded some conclusions. First, upstream and downstream mergers differ in outputs and prices. That is, the position where the merger occurs influences outputs and prices, with different impacts on firms involved in the upstream and downstream mergers. Moreover, downstream mergers can be further specified into two types, each with its own characteristics. Second, the synergy cost is the trigger between the synergy effect and the competitive effect. Most of the impacts of horizontal mergers on firms vary with the level of the synergy cost. By comparing the aggregate effect with the two single effects, the study reveals an overall synergy effect in scenarios where the synergy cost is sufficiently high. That is, the synergy cost is the trigger between the two single effects. Moreover, upstream and downstream
mergers have their own triggers. Third, the degree of consumer preference for channels plays an important role in the triggers. Almost all the triggers increase as the degree increases. Therefore, with an increasing degree of channel preference, the impacts of the horizontal mergers are more likely to perform as synergy effects.

This study contributes to the literature on both horizontal mergers and dual-channel supply chains. By examining the influence where a merger occurs would have on merging firms, the study demonstrates the crucial role of synergy cost for post-merger firms. It highlights the need for post-merger firms to take care of their positions in dual-channel supply chains and to beware of hardly achievable synergy costs. In addition, a positive relationship is established between the degree of channel preference and the trigger of synergy effect. This suggests retailers are recommended to maintain their customer relationships for relative advantages in horizontal mergers.

Although the authors investigated the impacts of horizontal mergers on the dual-channel supply chain, there are still numerous factors that need to be considered. For example, the study assumed symmetric information scenarios where the demand is common knowledge for each firm, whereas information asymmetries prevail in reality. Therefore, future studies can examine horizontal mergers under asymmetric information.

Appendix.

Proof of Lemma 3.1. In the pre-merger scenario, the dual-channel retailer \((i = 0)\) solves Eq.(4). The first order conditions of Eq.(4) are

\[
a - w - c^o_r - 2q^o_0 - 2\gamma q^o_0 - \gamma q^s_i - \gamma \sum_{k=1,2,\ldots,n \ k \neq i} q^k = 0 \quad (A.1)
\]

\[
a - w - c^s_r - 2\gamma q^o_0 - 2q^o_0 - q^s_i - \sum_{k=1,2,\ldots,n \ k \neq i} q^k = 0 \quad (A.2)
\]

Each single-channel retailer \((i = 1,2,\ldots,n)\) solves Eq.(5). The first order conditions of Eq.(5) are

\[
a - w - c^s_r - \gamma q^o_0 - q^s_i - 2q^s_i - \sum_{k=1,2,\ldots,n \ k \neq i} q^k, \quad i = 1,2,\ldots,n \quad (A.3)
\]

By adding the \(n\) first order conditions in Eq.(A.3) together, \(n(a - w - c^s_r - \gamma q^o_0 - q^s_i) - 2\sum_{k=1,2,\ldots,n} q^k - (n - 1) \sum_{k=1,2,\ldots,n} q^k = 0\) is obtained. Therefore,

\[
\sum_{k=1,2,\ldots,n \ k \neq i} q^k = \frac{n(a - w - c^s_r - \gamma q^o_0 - q^s_i)}{n + 1} - q^s_i \quad (A.4)
\]

By Substituting Eq.(A.4) into Eq.(A.1), Eq.(A.2), and Eq.(A.3) and solving for \(q^o_0\), \(q^s_0\), and \(q^s_i\), Lemma 3.1 is obtained.

Proof of Lemma 3.2. According to Lemma 3.1, the total sale quantity \(Q\) for all the retailers can be written as follows:

\[
Q = q^o_0 + q^s_0 + \sum_{i=1}^{n} q^s_i \quad (A.5)
\]
Following Cho [5] and Kyparisis and Koulamas [19], there is an equation between the total sale quantity $Q$ and the total output quantity $Y$, i.e., $Y = Q$. After substituting the results of Lemma 3.1 and replacing $Q$ with $Y$, Eq.(A.5) can be written as follows:

$$Y = \frac{(a - w)(4 + n(3 + \gamma)) - (2 + n)c_r^s - (2 + n(2 + \gamma))c_r^s}{2(2 + n)(1 + \gamma)}$$  \hspace{1cm} (A.6)

Solving Eq.(A.6) for $w$, Lemma 3.2 is obtained. \hfill \Box

Proof of Proposition 1. According to the reverse demand function of suppliers in Lemma 3.2, the wholesale price $w$ can be substituted into the objective functions for each of $m$ suppliers Eq.(3). In addition, there is $Y = \sum_{k=1,2,...,m, k \neq j} y_k + y_j$.

Hence, the decision problem of each supplier Eq.(3) can be rewritten as follows:

$$(A - B \sum_{k=1,2,...,m, k \neq j} y_k - By_j - c_s)y_j$$  \hspace{1cm} (A.7)

The first-order condition of supplier $j$ is

$$A - B \sum_{k=1,2,...,m, k \neq j} y_k - 2By_j - c_s = 0$$  \hspace{1cm} (A.8)

By adding these $m$ first-order conditions together, the following equation can be obtained:

$$mA - (m - 1)BY - mc_s - 2BY = 0$$ \hspace{1cm} (A.9)

Solving Eq.(A.9) for $Y$, the total output quantity of the upstream tier in equilibrium can be derived.

$$Y = \frac{m(A - c_s)}{B + Bm}$$ \hspace{1cm} (A.10)

By the symmetry of suppliers, the equilibrium solution of each supplier’s output is as follows:

$$y_j = \frac{Y}{m} = \frac{A - c_s}{B + Bm}$$ \hspace{1cm} (A.11)

In addition, since obtaining the total supply quantity in equilibrium, the equilibrium wholesale price can be rewritten as follows according to Lemma 3.2.

$$w = A - BY$$

$$= A - \frac{mB(A - c_s)}{B + Bm}$$ \hspace{1cm} (A.12)

$$= \frac{(4 + n(3 + \gamma))(a + mc_s) - (2 + n)c_r^s - (2 + n(2 + \gamma))c_r^s}{(1 + m)(4 + n(3 + \gamma))}$$

Substitute Eq.(A.12) into Lemma 3.1 can we obtain the equilibrium sales for each retailer. Therefore, Proposition 1 is obtained. \hfill \Box
Proof of Proposition 2. Following a similar process to the benchmark, the equilibrium solutions for the upstream merger model can be obtained. The results of comparing these solutions with the benchmark are shown as follows.

\[ \bar{w}^U - w = -\frac{4m\Delta^U}{2 + 3m + m^2} \]
\[ \bar{y}_1^U - y_1 = \bar{y}_2^U - y_2 = \frac{(m(m - 1) + 2)(4 + n(3 + \gamma))\Delta^U}{2(m + 1)(m + 2)(n + 2)(1 + \gamma)} \]
\[ \bar{y}_j^U - y_j = -\frac{(m - 1)(4 + n(3 + \gamma))\Delta^U}{(m + 1)(m + 2)(n + 2)(1 + \gamma)} \]
\[ \bar{p}^{oU} - p^o = -\frac{2m(2 + n(1 + \gamma))\Delta^U}{(m + 1)(m + 2)(n + 2)} \]
\[ \bar{p}^{sU} - p^s = -\frac{4m(1 + n)\Delta^U}{(m + 1)(m + 2)(n + 2)} \]
\[ \bar{q}_0^{oU} - q_0^o = \frac{2m\Delta^U}{(2 + 3m + m^2)(1 + \gamma)} \]
\[ \bar{q}_0^{sU} - q_0^s = \frac{2m(2 - n\gamma)\Delta^U}{(m + 1)(m + 2)(n + 2)(1 + \gamma)} \]
\[ \bar{q}_1^{sU} - q_1^s = \frac{4m\Delta^U}{(2 + 3m + m^2)(n + 2)} \]

As mentioned, \( m \geq 3, n \geq 2, 0 < \gamma < \frac{2}{n}, \) and \( \Delta^U > 0. \) Thus, the results of \( \bar{w}^U - w, \bar{y}_j^U - y_j, \bar{p}^{oU} - p^o, \) and \( \bar{p}^{sU} - p^s \) are negative, while the rest results are positive. \( \square \)

Proof of Proposition 3. The equilibrium solutions of upstream merger competitive effect can be solved by substituting \( m - 1 \) for \( m \) in the benchmark solutions. Then, the results are shown as follows.

\[ \tilde{w}^U - w = \frac{(4 + n(3 + \gamma))G + (2 + n)H}{m(1 + m)(4 + n(3 + \gamma))} \]
\[ \tilde{y}_1^U - (y_1 + y_2) = -\frac{(m - 1)((4 + n(3 + \gamma))G + (2 + n)H)}{2m(m + 1)(n + 2)(1 + \gamma)} \]
\[ \tilde{y}_j^U - y_j = \frac{(4 + n(3 + \gamma))G + (2 + n)H}{2m(m + 1)(n + 2)(1 + \gamma)} \]
\[ \tilde{p}^{oU} - p^o = \frac{2m(2 + n(1 + \gamma))((4 + n(3 + \gamma))G + (2 + n)H)}{2m(1 + m)(2 + n)(4 + n(3 + \gamma))} \]
\[ \tilde{p}^{sU} - p^s = \frac{(1 + n)((4 + n(3 + \gamma))G + (2 + n)H)}{m(1 + m)(2 + n)(4 + n(3 + \gamma))} \]
\[ \tilde{q}_0^{oU} - q_0^o = -\frac{(4 + n(3 + \gamma))G + (2 + n)H}{2m(1 + m)(1 + \gamma)(4 + n(3 + \gamma))} \]
\[ \tilde{q}_0^{sU} - q_0^s = -\frac{2m(2 - n\gamma)((4 + n(3 + \gamma))G + (2 + n)H)}{2m(1 + m)(2 + n)(1 + \gamma)(4 + n(3 + \gamma))} \]
\[ \tilde{q}_1^{sU} - q_1^s = -\frac{(4 + n(3 + \gamma))G + (2 + n)H}{m(1 + m)(2 + n)(4 + n(3 + \gamma))} \]
As mentioned in section 3, $G \equiv a - c_s - c_r^s > 0$, $H \equiv c_r^s - c_r^o > 0$, $m \geq 3$, $n \geq 2$, and $0 < \gamma < \frac{2}{m}$. Therefore, the results of $\tilde{y}_1 - (y_1 + y_2)$, $\tilde{q}_0^U - q_0^o$, $\tilde{q}_0^U - \alpha i$, and $\tilde{q}_0^o - \alpha i$ are negative, while the rests of the results are positive.

Proof of Proposition 4. Considering both the synergy effect and the competitive effect in the upstream merger, the equilibrium solutions of the aggregate effect of the upstream merger can be obtained. That is, the cost of the post-merger supplier reduces from $c_s$ to $c_s - \Delta U$ and the number of suppliers reduces from $m$ to $m-1$. The results are shown as follows.

$$w^U - w = \frac{(4 + n (3 + \gamma)) (G - (1 + m) \Delta U) + (2 + n) H}{m (1 + m)(4 + n (3 + \gamma))}$$

$$y_U^1 - (y_1 + y_2) = \frac{(m - 1)(4 + n (3 + \gamma)) (G - (1 + m) \Delta U) + (2 + n) H}{2m (1 + m)(2 + n)(1 + \gamma)}$$

$$y_U^2 - y_2 = \frac{(4 + n (3 + \gamma)) (G - (1 + m) \Delta U) + (2 + n) H}{2m (1 + m)(2 + n)(1 + \gamma)}$$

$$p^oU - p^o = \frac{(2 + n (1 + m)) (4 + n (3 + \gamma)) (G - (1 + m) \Delta U) + (2 + n) H}{2m (1 + m)(2 + n)(4 + n (3 + \gamma))}$$

$$p^sU - p^o = \frac{(1 + n)(4 + n (3 + \gamma)) (G - (1 + m) \Delta U) + (2 + n) H}{m (1 + m)(2 + n)(4 + n (3 + \gamma))}$$

$$q_0^oU - q_0^o = \frac{(4 + n (3 + \gamma)) (G - (1 + m) \Delta U) + (2 + n) H}{2m (1 + m)(1 + \gamma)(4 + n (3 + \gamma))}$$

Notice that all the results have a same factor term, and $\Delta U = C_U^1$ is the solution of $(4 + n (3 + \gamma)) (G - (1 + m) \Delta) + (2 + n) H = 0$. When $\Delta U > C_U^1$, $(4 + n (3 + \gamma)) (G - (1 + m) \Delta) + (2 + n) H < 0$.

Proof of Proposition 5. The results of comparisons for the synergy effect of the SS merger are shown as follows.

$$\bar{w}^{SS} - w = \frac{4 (1 + \gamma) \Delta^{SS}}{(1 + m)(4 + n (3 + \gamma))}$$

$$\bar{y}_j^{SS} - y_j = \frac{2 \Delta^{SS}}{2 + 2m + n + mn}$$

$$\bar{p}^{oSS} - p^o = \frac{2 (m n \gamma^2 + (2 + 4m + n + 3mn) \gamma - (n + 2)) \Delta^{SS}}{(1 + m)(2 + n)(4 + n (3 + \gamma))}$$

$$\bar{p}^{sSS} - p^o = \frac{2 (m (4 + n (3 + \gamma)) + (1 - \gamma) (2 + n)) \Delta^{SS}}{(1 + m)(2 + n)(4 + n (3 + \gamma))}$$

$$\bar{q}_1^{iSS} - q_1^i = \frac{n (1 + m)(4 + n (3 + \gamma)) - 4 (1 + \gamma) \Delta^{SS}}{(1 + m)(2 + n)(4 + n (3 + \gamma))}$$

$$\bar{q}_0^{SS} - q_0^o = \frac{2 \Delta^{SS}}{(1 + m)(4 + n (3 + \gamma))}$$
\[ \bar{q}_0^{SS} - q_0^* = -\frac{2(m(4+n(3+\gamma)) + 3(2+n))\Delta^{SS}}{(1+m)(2+n)(4+n(3+\gamma))} \]
\[ \bar{q}_i^{SS} - q_i^* = -\frac{2(m(4+n(3+\gamma)) + (3+\gamma)(2+n))\Delta^{SS}}{(1+m)(2+n)(4+n(3+\gamma))} \]

It is easy to see that \( \bar{w}^{SS} - w, \bar{q}_j^{SS} - y_j, \) and \( \bar{q}_i^{SS} - q_i^* \) are positive, while \( \bar{p}^{SS} - p^s, \bar{q}_0^{SS} - q_0^o, \bar{q}_0^{SS} - q_0^*, \) and \( \bar{q}_i^{SS} - q_i^* \) are negative. Besides, \( \bar{p}^{SS} - p^o = 0 \) indicates that

\[ mn\gamma^2 + (2 + 4m + n + 3mn)\gamma - (n + 2) = 0 \quad \text{(A.13)} \]

Moreover, \( \bar{\gamma} = \frac{\sqrt{4mn(2+n)+(2+n+m(4+3n))(2+4m+n+3mn)}}{2mn} \) is the positive solution of Eq. (A.13). Hence, \( \bar{p}^{SS} > p^o \) if \( 0 < \gamma < \bar{\gamma} \), and \( \bar{p}^{SS} < p^o \) if \( \bar{\gamma} < \gamma < \frac{2}{n} \).

**Proof of Proposition 6.** The competitive effect of the SS merger is shown as follows.

\[ \bar{w}^{SS} - w = \frac{2(1+\gamma)H}{(1+m)(4+n(3+\gamma))(1-\gamma+n(3+\gamma))} \]
\[ \bar{y}_j^{SS} - y_j = -\frac{G}{(1+m)(2+3n+n^2)} \]
\[ \bar{p}^{SS} - p^o = \frac{m\gamma G}{(1+m)(1+n)(2+n)} \]
\[ + \frac{(1-\gamma)H}{(1+m)(4+n(3+\gamma))(1-\gamma+n(3+\gamma))} \]
\[ \bar{p}^{SS} - p^s = \frac{mG}{(1+m)(1+n)(2+n)} \]
\[ - \frac{(1-\gamma)H}{(1+m)(4+n(3+\gamma))(1-\gamma+n(3+\gamma))} \]
\[ \bar{q}_i^{SS} - (q_1^* + q_2^*) = -\frac{mnG}{(1+m)(1+n)(2+n)} \]
\[ + \frac{(3n-2 + (n-2)\gamma)H}{(1+m)(4+n(3+\gamma))(1-\gamma+n(3+\gamma))} \]
\[ \bar{q}_0^{SS} - q_0^o = -\frac{H}{(1+m)(4+n(3+\gamma))(1-\gamma+n(3+\gamma))} \]
\[ \bar{q}_0^{SS} - q_0^* = \frac{mG}{(1+m)(1+n)(2+n)} \]
\[ - \frac{3H}{(1+m)(4+n(3+\gamma))(1-\gamma+n(3+\gamma))} \]
\[ \bar{q}_i^{SS} - q_i^* = \frac{mG}{(1+m)(1+n)(2+n)} \]
\[ - \frac{(3+\gamma)H}{(1+m)(4+n(3+\gamma))(1-\gamma+n(3+\gamma))} \]

As mentioned before, \( \frac{G}{H} > \frac{(2+n)(2(1+n)(1+n\gamma)+m\gamma(4+n(3+\gamma)))}{m(1-\gamma)(2-n\gamma)(4+n(3+\gamma))} \). Moreover, as \( m \geq 3, n \geq 2, \) and \( 0 < \gamma < \frac{2}{n} \), it is not difficult to verify that all the results are positive except \( \bar{y}_j^{SS} - y_j, \bar{q}_i^{SS} - (q_1^* + q_2^*), \) and \( q_0^{SS} - q_0^o \). \( \square \)
Proof of Proposition 7. The synergy effect and the competitive effect are combined, giving the aggregate effect of the SS merger, which is shown as follows.

\[ w^{SS} - w = \frac{2(1 + \gamma) H^{SS}}{(1 + m)(4 + n(3 + \gamma))(1 - \gamma + n(3 + \gamma))} \]

\[ y_j^{SS} - y_j = -\frac{G^{SS}}{(1 + m)(1 + n)(2 + n)} \]

\[ p^{oSS} - p^o = -\frac{(1 - \gamma) H^{SS}}{(1 + m)(4 + n(3 + \gamma))(1 - \gamma + n(3 + \gamma))} \]

\[ p^{SS} - p^* = \frac{mG^{SS}}{(1 + m)(1 + n)(2 + n)} \]

\[ q_1^{sSS} - (q_1^s + q_2^s) = -\frac{mnG^{SS}}{(1 + m)(1 + n)(2 + n)} \]

\[ q_0^{oSS} - q_0^o = -\frac{H^{SS}}{(1 + m)(4 + n(3 + \gamma))(1 - \gamma + n(3 + \gamma))} \]

\[ q_0^{sSS} - q_0^s = \frac{3H^{SS}}{(1 + m)(1 + n)(2 + n)} \]

\[ q_i^{sSS} - q_i^s = \frac{mG^{SS}}{(1 + m)(1 + n)(2 + n)} \]

Note that \( G^{SS} = G - (2 + n) \Delta^{SS} \), \( H^{SS} = H + (4 + n(3 + \gamma)) \Delta^{SS} \), \( \Delta^{SS} = C_{SS}^{1} \), \( \Delta^{SS} = C_{SS}^{2} \), \( \Delta^{SS} = C_{SS}^{3} \), \( \Delta^{SS} = C_{SS}^{4} \), and \( \Delta^{SS} = C_{SS}^{5} \) are the solutions of \( y_j^{SS} - y_j = 0 \), \( p_0^{oSS} - p_0^o = 0 \) or \( p_0^{sSS} - p_0^s = 0 \), \( q_1^{sSS} - (q_1^s + q_2^s) = 0 \), \( q_0^{sSS} - q_0^s = 0 \), and \( q_i^{sSS} - q_i^s = 0 \).

Proof of Proposition 8. Considering the cost synergy in the DS merger, the results of the comparison are shown as follows.

\[ \hat{w}^{DS} - w = \frac{(6 + n + 2\gamma - n\gamma) \Delta^{DS}}{(1 + m)(4 + n(3 + \gamma))} \]

\[ \hat{y}^{DS}_j - y_j = \frac{(6 + n + 2\gamma - n\gamma) \Delta^{DS}}{2(1 + m)(2 + n)(1 + \gamma)} \]

\[ \hat{p}^{oDS} - p^o = -\frac{\Delta^{DS}}{2(1 + m)(2 + n)(4 + n(3 + \gamma))} \cdot [2(n^2 + n - 2)(1 - \gamma)
\]  

\[ + m(n(1 - \gamma) + 2(1 + \gamma))(4 + n(3 + \gamma))] \]
\[
\bar{p}^{*DS} - p^* = \frac{((n^2 + n - 2)(1 - \gamma) - 2m(4 + n(3 + \gamma)))\Delta^{DS}}{(1 + m)(2 + n)(4 + n(3 + \gamma))}
\]
\[
\bar{q}_0^{*DS} - q_0^* = \frac{(m(4 + n(3 + \gamma)) + 2(n - 1)(1 + \gamma))\Delta^{DS}}{2(1 + m)(1 + \gamma)(4 + n(3 + \gamma))}
\]
\[
\bar{q}_0^{sDS} - q_0^* = \frac{\Delta^{DS}}{(1 + m)(2 + n)(4 + n(3 + \gamma))} (6n(1 + \gamma) + mn^2(2 + \gamma)(3 + \gamma) - 2m(\gamma^2 + \gamma - 4) - 4(3 + (3 + 2m)\gamma)]
\]
\[
\bar{q}_1^{sDS} - q_1^* = \frac{p}{(1 + m)(2 + n)(4 + n(3 + \gamma))} (1 + m)(2 + n) + (1 - \gamma)(H - (n - 1)\Delta^{DS})
\]
\[
\bar{q}_1^{sDS} - q_1^* = \frac{(1 + m)(2 + n)(1 + n) + (7 + \gamma)(2 + n))\Delta^{DS}}{2(1 + m)(1 + \gamma)(2 + n)}
\]
\[
\bar{q}_i^{DS} - q_i^* = \frac{(2m(4 + n(3 + \gamma)) + (7 + \gamma)(2 + n))\Delta^{DS}}{2(1 + m)(1 + n)(2 + n)(4 + n(3 + \gamma))}
\]

It is not difficult to verify that the results are positive except \(\bar{p}^{oDS} - p^o\) and \(\bar{q}_i^{sDS} - q_i^*\).

**Proof of Proposition 9.** The results of the aggregate effect of the DS merger are shown as follows.

\[
w^{DS} - w = \frac{2(1 + \gamma)H + (3 + \gamma + n(1 - \gamma))\Delta^{DS}}{(1 + m)EF}
\]
\[
y^{DS} - y = \frac{2(1 + \gamma)G - (2 + n)(3 + \gamma + n(1 - \gamma))\Delta^{DS}}{2(1 + m)(1 + n)(2 + n)(1 + \gamma)}
\]
\[
p^{oDS} - p^o = \frac{m(2\gamma G - (2 + n)(1 + \gamma + n(1 - \gamma))\Delta^{DS}}{2(1 + m)(1 + n)(2 + n)} + \frac{(1 - \gamma)(H - (n - 1)G\Delta^{DS})}{(1 + m)EF}
\]
\[
p^{sDS} - p^s = \frac{m(G - (2 + n)\Delta^{DS}}{2(1 + m)(1 + n)(2 + n)} - \frac{(1 - \gamma)(H - (n - 1)G\Delta^{DS})}{(1 + m)EF}
\]
\[
q^{oDS} - q_0^o = \frac{2(1 + \gamma)H - (mF + (2 + n)(1 + \gamma))\Delta^{DS}}{2(1 + m)(1 + \gamma)EF}
\]
\[
q^{sDS} - q_0^s = \frac{m(2 + n)(1 + \gamma)G + (2 + n)(2n + (n - 1)\gamma)\Delta^{DS}}{2(1 + m)(1 + n)(2 + n)(1 + \gamma)} - \frac{3(H - (n - 1)\Delta^{DS})}{(1 + m)EF}
\]
\[
Q^{oDS} - (Q_0 + q_1^o) = \frac{-m(2n(1 + \gamma)G - F(2 + n)\Delta^{DS})}{2(1 + m)(1 + n)(2 + n)(1 + \gamma)} + \frac{(n - 1)(3 + \gamma)H + 4E\Delta^{DS}}{(1 + m)EF}
\]
\[
q^{sDS} - q_1^* = \frac{m(G - (2 + n)\Delta^{DS}}{2(1 + m)(1 + n)(2 + n)} - \frac{(3 + \gamma)H + 4E\Delta^{DS}}{(1 + m)EF}
\]

where \(Q^{oDS}_o = q^{oDS}_o + q^{oDS}_o, Q_0 = q^{oDS}_0 + q^{oDS}_0, E = 4 + n(3 + \gamma)\) and \(F = 1 - \gamma + n(3 + \gamma)\).

It is not difficult to verify \(w^{DS} - w > 0, p^{sDS} - p^s > 0,\) and \(q^{sDS}_o - q_0^s > 0.\) Besides, \(\Delta^{2DS} = C_{DS}^2, \Delta^{DS} = C_{DS}^1, \Delta^{3DS} = C_{DS}^3, \Delta^{4DS} = C_{DS}^4,\) and
\[ \Delta^{DS} = C_{DS} \] are solutions of \( y_j^{DS} - y_j = 0, \ p^{oDS} - y^o = 0, \ q_0^{oDS} - q_0^o = 0, \)
\( (q_0^{oDS} + q_0^{sDS}) - (q_0^o + q_0^s + q_1^s) = 0, \) and \( q_1^{sDS} - q_1^s = 0, \) respectively. \( \square \)

REFERENCES

[1] A. Arya and B. Mittendorf, Bricks-and-mortar entry by online retailers in the presence of consumer sales taxes, Management Science, 64 (2018), 5220–5233.

[2] K. Bimpikis, S. Ehsani and R. Ilkilic, Cournot competition in networked markets, Association for Computing Machinery, (2014), 733.

[3] J. Farrell and C. Shapiro, Horizontal mergers: An equilibrium analysis, European Journal of Operational Research, 248 (2016), 234–245.

[4] W.-Y. K. Chiang, D. Chhajed and J. D. Hess, Direct-marketing, indirect profits: A strategic analysis of dual-channel supply-chain design, Management Science, 49 (2003), 1–20.

[5] S.-H. Cho, Horizontal mergers in multitier decentralized supply chains, Management Science, 60 (2014), 356–379.

[6] S. H. Cho and X. Wang, Newsvendor mergers, Management Science, 63 (2017), 298–316.

[7] C. J. Corbett and U. S. Karmarkar, Competition and structure in serial supply chains with deterministic demand, Management Science, 47 (2001), 966–978.

[8] M. Cunha and H. Vasconcelos, Mergers in stackelberg markets with efficiency gains, Journal of Industry Competition & Trade, 15 (2015), 105–134.

[9] C. Davidson and A. Mukherjee, Horizontal mergers with free entry, International Journal of Industrial Organization, 25 (2007), 157–172.

[10] M. Escrichuela-Villar and R. Faúl-Oller, Mergers in asymmetric stackelberg markets, Spanish Economic Review, 10 (2008), 279–288.

[11] J. Farrell and C. Shapiro, Horizontal mergers: An equilibrium analysis, The American Economic Review, 80 (1990), 107–126.

[12] L. Froeb, S. Tschantz and G. J. Werden, Pass-through rates and the price effects of mergers, International Journal of Industrial Organization, 23 (2005), 703–715.

[13] C. Fumagalli and M. Motta, Upstream mergers, downstream mergers, and secret vertical contracts, Research in Economics, 55 (2001), 275–289.

[14] F. Gao and X. Su, Omnichannel retail operations with buy-online-and-pick-up-in-store, Management Science, 63 (2017), 2478–2492.

[15] F. Gao and X. M. Su, Online and offline information for omnichannel retailing, M & Som-Manufacturing & Service Operations Management, 19 (2017), 84–98.

[16] J. S. Heywood and M. McGinty, Leading and merging: Convex costs, stackelberg, and the merger paradox, Southern Economic Journal, 74 (2008), 879–893.

[17] R. Inderst and G. Shaffer, Retail mergers, buyer power and product variety, Economic Journal, 117 (2007), 45–67.

[18] R. Inderst and C. Wey, The incentives for takeover in oligopoly, International Journal of Industrial Organization, 22 (2004), 1067–1089.

[19] G. J. Kyparissis and C. Koulamas, Competition in two-tier serial and assembly supply chains with general consumer utility functions, International Journal of Production Research, 56 (2018), 5854–5865.

[20] Y. F. Lan, H. Yan, D. Ren and R. Guo, Merger strategies in a supply chain with asymmetric capital-constrained retailers upon market power dependent trade credit, Omega-International Journal of Management Science, 83 (2019), 299–318.

[21] C.-C. Liu and L. F. S. Wang, Leading merger in a stackelberg oligopoly: Profitability and consumer welfare, Economics Letters, 129 (2015), 1–3.

[22] H. Liu, S. Sun, M. Lei, H. Deng and G. K. Leong, The impact of retailers’ alliance on manufacturer’s profit in a dual-channel structure, International Journal of Production Research, 55 (2017), 6592–6607.

[23] K. E. Lommerud, O. R. Straume and L. Sorgard, Downstream merger with upstream market power, European Economic Review, 49 (2005), 717–743.

[24] K. Meyer, What is “strategic asset seeking fdi”? Multinational Business Review, 23 (2015), 57–66.

[25] C. Milliou and A. Pavlou, Upstream mergers, downstream competition, and r&d investments, Journal of Economics & Management Strategy, 22 (2013), 787–809.

[26] M. K. Perry and R. H. Porter, Oligopoly and the incentive for horizontal merger, The American Economic Review, 75 (1985), 219–227.
[27] S. W. Salant, S. Switzer and R. J. Reynolds, *Losses from Horizontal Merger: The Effects of An Exogenous Change in Industry Structure on Cournot-Nash Equilibrium*, Cambridge: Cambridge University Press, 1989.

[28] G. Symeonidis, *Downstream merger and welfare in a bilateral oligopoly*, *International Journal of Industrial Organization*, 28 (2010), 230–243.

[29] W. Wang, G. Li and T. C. E. Cheng, *Channel selection in a supply chain with a multi-channel retailer: The role of channel operating costs*, *International Journal of Production Economics*, 173 (2016), 54–65.

[30] Y. Xiao, *Horizontal mergers under yield uncertainty*, *Production and Operations Management*, 29 (2020), 24–34.

[31] G. Y. Xu, B. Dan, X. M. Zhang and C. Liu, *Coordinating a dual-channel supply chain with risk-averse under a two-way revenue sharing contract*, *International Journal of Production Economics*, 147 (2014), 171–179.

[32] Z. N. Yuan, F. Y. Chen, X. M. Yan and Y. G. Yu, *Operational implications of yield uncertainty in mergers and acquisitions*, *International Journal of Production Economics*, 219 (2020), 248–258.

[33] W. Zhou, *Endogenous horizontal mergers under cost uncertainty*, *International Journal of Industrial Organization*, 26 (2008), 903–912.

[34] J. Zhu, T. Boyaci and S. Ray, *Effects of upstream and downstream mergers on supply chain profitability*, *European Journal of Operational Research*, 249 (2016), 131–143.

Received January 2020; revised August 2020.

E-mail address: tjesmile@163.com
E-mail address: teng.niu@foxmail.com