On Second–Quantized Open Superstring Theory

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Abstract

The $SO(32)$ theory, in the limit where it is an open superstring theory, is completely specified in the light–cone gauge as a second–quantized string theory in terms of a “matrix string” model. The theory is defined by the neighbourhood of a 1+1 dimensional fixed point theory, characterized by an Abelian gauge theory with type IB Green–Schwarz form. Non–orientability and $SO(32)$ gauge symmetry arise naturally, and the theory effectively constructs an orientifold projection of the (weakly coupled) matrix type IIB theory (also discussed herein). The fixed point theory is a conformal field theory with boundary, defining the free string theory. Interactions involving the interior of open and closed strings are governed by a twist operator in the bulk, while string end–points are created and destroyed by a boundary twist operator.
1. Opening Remarks

1.1. Motivations

In the earliest searches for a better understanding of the non-perturbative structure of string theory, there was considerable attention given to finding a second-quantized description of the theory\cite{1,2}. Considering the logic of the development of quantum field theory, this was a plausible next step.

As it turned out, the powerful tools which may be exploited in the presence of extended supersymmetry led to the discovery that “strong/weak coupling duality” was a compelling hint about the non-perturbative nature of string theory\cite{3,4}. This eventually led to the simplest possible statement about the five ten dimensional string theories and the meaning of their duality relationships: The string theories are all “weak coupling” limits of a single parent theory, “M–Theory”, and the duality operations connecting them are consequences of geometrical relationships between the limits.

The search for a full definition of M–theory continues. A partial one is given in terms of “Matrix Theory”\cite{5}, which has been shown to have many of the exciting and peculiar properties required by a theory which must play such a distinguished role\cite{6}.

Among these properties are (of course) the recovery of the five weakly coupled superstring theories in the appropriate limits. These theories are all described in terms of the neighbourhood of a 1+1 dimensional orbifold conformal field theory. The strings are free in the conformal field theory limit, and interactions are turned on by turning on a twist operator in the theory. The operator is irrelevant, and therefore deforms the theory away from the fixed point.

The conformal field theories are characterized in Lagrangian form by a matrix–valued version of the appropriate Green–Schwarz action for the string. The matrices are diagonal, and the permutations of diagonal elements is a gauge symmetry of the model. The orbifold target space of the conformal field theory is the gauge invariant vacuum moduli space of this theory, and long strings arise in the twisted sectors of the orbifold.

Away from the conformal field theory limit, where interactions have been turned on, the theories cease to have such similar descriptions, as required by duality. Best understood are the type IIA\cite{8,10,12} and $E_8 \times E_8$ heterotic\cite{14,13,15} string theories, because the interaction operator takes one back along the IR flow to a 1+1 dimensional non–Abelian string theories, as described in the previous paragraph (the matrices are no longer diagonal). Furthermore, for extreme values of the string coupling, the theory returns to a 0+1 dimensional matrix quantum mechanics.

By contrast, the type IIB and the two $SO(32)$ theories do not become 1+1 dimensional non–Abelian matrix Green–Schwarz models for arbitrary values of the string coupling, but

\footnote{As argued in ref.\cite{7}, by organizing and extending results of refs.\cite{3,4,10,12,14,15}}
instead become (at intermediate values) 2+1 dimensional fixed point theories, which are so far not very well studied. At strong coupling, they become 1+1 dimensional again, and ultimately give the appropriate weakly coupled dual string theory.

A pleasant and perhaps ironic feature of the neighbourhood of weak coupling in all five cases is that the description is intrinsically a second–quantized theory of strings in the light–cone gauge. In this way, we see just how the early studies of light–cone string field theory fit into the modern scheme of things. From the previous two paragraphs, it is clear from this perspective just how the physics encoded in weak coupling (light–cone) string field theory is completed into non–perturbative information consistent with the duality results.

(For a few more comments on the role of string field theory in a modern context, see the closing remarks (section 5).)

The purpose of this paper is to make explicit the matrix realization of the second–quantized description of the $SO(32)$ “type IB” string theory. This stringy description is a novel 1+1 dimensional theory which was discovered in ref. by taking the appropriate limits of the matrix description of M–theory on a cylinder. It is necessarily different from the description of the type II and heterotic theories, and therefore perhaps deserves some pedagogy.

1.2. Summary of Results

- The free strings are described by a conformal field theory with boundary. This conformal field theory is characterized by an $N \times N$ matrix Green–Schwarz action for the type IB string. The action is simply the matrix Green–Schwarz action for the type IIB string, with reflecting boundary conditions inserted.

- The second–quantized Fock space contains both open and closed strings with momentum in the light–cone direction proportional to their length. These strings are built out of winding type IA strings glued together in the twisted sectors of the orbifold to make long strings which survive the large $N$ limit, defining the infinite momentum light–cone frame.

- The long strings are unoriented. By examining the strings which arise in the twisted sectors, it is clear that the strings are non–oriented. The construction ensures that each sequence of constituent type IA string bits of a given orientation is accompanied by the same structure with the opposite orientation.

- The open strings carry an $SO(32)$ gauge symmetry. This follows from the non–perturbative origins of the model: The model is built on a type IA fundamental string.

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2 Aspects of the description of the free theory were actually found earlier in ref. in the context of a description of the $E_8 \times E_8$ heterotic and type IA strings. I thank L. Motl for pointing this out to me.
background, which has eight D8–branes and an O8–plane at each end of the interval. The required $SO(32)$ quantum numbers are explicitly assembled out of $SO(16) \times SO(16)$ quantum numbers: For example, the adjoint of $SO(32)$ is filled out by open strings of even length, which transform as $(1, 120) \oplus (120, 1)$, together with open strings of odd length, which fill out $(16, 16)$.

- Interactions involving the interior of open and closed strings are governed by a vertex operator analogous to that presented in ref. [12]. The coefficient of the operator is proportional to the closed string coupling $g_{IB}$.

- Interactions involving only string end–points are also governed by a vertex operator, but this operator lives on the boundary of the conformal field theory. The operator also decorates the ends of strings with the Chan–Paton factors which live on the boundary.

The strength of this open–open operator is proportional to the square root of the closed string coupling, as is appropriate for a Yang–Mills coupling: $g_{YM} \sim g_{IB}^{1/2}$. This is of course the string field theory statement that type IB strings end on D9–branes.

2. Second–Quantized Type IIB Strings

We begin with an explicit description of the matrix theory of the weakly coupled type IIB string, emphasizing its similarities to and differences from its type IIA counterpart.

Consider the following action:

$$S = \frac{1}{2\pi} \int d^2\sigma \text{Tr} \left( (\partial_a X^i)^2 - i \Theta^T \gamma^a \partial_a \Theta \right).$$

The $X^i$ are eight scalar fields, $(i=1, \ldots, 8)$, and $\Theta$ contains two Majorana–Weyl fermionic fields $\theta_L^\alpha$ and $\theta_R^\alpha$ which respectively transform in the $8_v$, $8_s$ and $\bar{8}_s$ (vector and spinor; the conjugate spinor is denoted $\bar{8}_c$) representations of the $SO(8)$ R–symmetry of the model. They are all diagonal $N \times N$ hermitian matrices. The world–volume coordinates are $\tau \equiv \sigma^0$, $\sigma \equiv \sigma^1$, with $0 \leq \sigma \leq 2\pi$.

The action (2.1) is invariant under the 16 supersymmetries:

$$\delta X^i = \frac{2}{\sqrt{2p^+}} \epsilon \Gamma^i \Theta$$

$$\delta \Theta = \frac{i}{\sqrt{2p^+}} \Gamma^{-\gamma^a \partial_a X^i} \epsilon$$

where $\epsilon$ contains $\epsilon_L^\alpha$ and $\epsilon_R^\alpha$. The $\Gamma$ and $\gamma$ are ten dimensional and two dimensional gamma matrices, respectively.

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The constant $p^+$ has the interpretation of light–cone (IMF) momentum once we recall the identification of the remaining spacetime directions $X^\pm=(X^0 \pm X^9)/\sqrt{2}$ with the world–sheet time: $X^+(\sigma, \tau)=x^++p^+\tau$.

This model is thus far simply $N$ copies of the type IIB Green–Schwarz action[19]. (We may also think of it as $N$ copies of the type IIA string in a static gauge, aligned along a space–like direction which we will call $X^9$, with radius $R_9$.)

There are two supercharges $Q^\hat{\alpha}_{L(R)}$:

$$Q^\hat{\alpha}_{L(R)} = \frac{i}{\sqrt{2p^+}} \int_0^{2\pi} d\sigma \text{Tr} \left[ \Gamma^i_{\alpha\hat{\alpha}}(\gamma^a \partial_a X^i(\sigma))\gamma^0 \theta^\alpha_{L(R)}(\sigma) \right]$$ (2.3)

representing the 16 non–trivial supersymmetries of the model. The other 16 are trivially realized as:

$$\delta X^i=0, \quad \delta \Theta = \sqrt{2p^+\eta}. \quad (2.4)$$

This model has a discrete gauge symmetry $S_N$, corresponding to the freedom to permute the $N$ eigenvalues of the matrices. The moduli space of this theory is characterized by the manifold of gauge inequivalent values of the bosonic fields as simply:

$$\mathcal{M}_{\text{IIB}} \equiv (\mathbb{R}^8)^N/S_N. \quad (2.5)$$

This space is of course the same as that for the type IIA string, but here, the fermions transform appropriately for the type IIB string.

We present this as the target space of a free orbifold conformal field theory. The untwisted sectors of this orbifold correspond to $N$ independent (light–cone gauge) type IIB strings with a single unit of momentum in $X^+$. Explicitly, this represents $N$ (static gauge) type IIA strings with 1 unit of winding around $X^9$. The twisted sectors of the orbifold are classified in terms of the conjugacy classes of $S_N$, which are labeled by the number $n$ of eigenvalues which are permuted by a given representation. The twisted sector $n$ defines a type IIA string of length $n$, i.e., it is wound $n$ times around the circle. This in turn defines a type IIB string with $n$ units of momentum in the $X^+$ direction.

Explicitly, a configuration representing a string of length $n$ is described by (sub–) matrices of the form:

$$X^i(\sigma) = \begin{pmatrix} x^i_1(\sigma) & 0 & 0 & \ldots & 0 \\ 0 & x^i_2(\sigma) & 0 & \ldots & 0 \\ 0 & 0 & x^i_3(\sigma) & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & x^i_n(\sigma) \end{pmatrix}, \quad (2.6)$$

with

$$X^i(\sigma + 2\pi) = S_nX^i(\sigma)S_n^{-1}, \quad (2.7)$$

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where

\[ S_n = \begin{pmatrix}
0 & 0 & 0 & \ldots & 1 \\
1 & 0 & 0 & \ldots & 0 \\
0 & 1 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & 1 & 0
\end{pmatrix} \] (2.8)

is an \( n \times n \) (sub–) matrix which permutes the eigenvalues. In other words:

\[ X^i(\sigma + 2n\pi) = X^i(\sigma). \] (2.9)

A diagram of a closed string of length 3 is depicted in fig.1:

![Diagram of a closed string of length 3](image)

Figure 1.

The eigenvalues \( x_j^i(\sigma) \), (which sort of play the role of “string bits” [20]) are otherwise arbitrary functions of \( \sigma \) and therefore the complete Fock space of the two dimensional field theory contains type IIB strings of arbitrary shape in \( \mathbb{R}^8 \), in the various discrete momentum sectors. This is why this is a second–quantized description of the strings.

Ultimately, we want the large \( N \) limit. We make long strings of length \( n \) by keeping the ratio \( n/N \) fixed as we send \( N \to \infty \). This defines for us the light–cone gauge (IMF) type IIB string, where \( n/N \) is the finite fraction of the total \( p^+ \) which the long string possesses. As shown in ref. [12], the correct string tension and level matching for the long strings emerges correctly. We will not repeat this here.

If this was the type IIA string, we could think of the Lagrangian above as the strong coupling limit of a non–Abelian matrix Green–Schwarz theory (such as would be derived from the world–volume of \( N \) coincident D1–branes). This is the 1+1 dimensional gauge theory from which the free type IIA matrix theory flows [12,21]:

\[ S = \frac{1}{2\pi} \int d^2\sigma \text{Tr} \left( (D_\mu X^i)^2 - i\Theta^T \gamma^\mu D_\mu \Theta + g_{\text{IIA}}^2 F_{\mu\nu}^2 - \frac{1}{g_{\text{IIA}}^2} [X^i, X^j]^2 + \frac{1}{g_{\text{IIA}}} \Theta^T \Gamma_i [X^i, \Theta] \right). \] (2.10)

The dimensionless coupling \( g_{\text{IIA}} \) is the matrix type IIA string coupling inversely proportional to the 1+1 dimensional Yang–Mills coupling.
In the present case, the situation is more complicated. The free type IIB matrix Green–Schwarz Lagrangian does not flow from such an interacting 1+1 gauge theory, but arises as a reduction from a 2+1 dimensional fixed point theory. (This is consistent with the fact that type IIA does not contain D1–branes.)

We do not have a gauge theory definition of the theory analogous to eqn.(2.10) even slightly away from the 2D fixed point but for weak string coupling, where we are close to the free conformal field theory limit, we can describe non–zero string coupling in terms of giving an expectation value to a certain operator in the conformal field theory.

The operator is very similar to the one presented in ref.[12], for the type IIA case. There, the joining of the closed strings is described by the exchange of eigenvalues between configurations representing two different strings (see fig. 6). (The reverse procedure will give the splitting of a closed string into two closed strings.) This can be written as a $\mathbb{Z}_2$ twist operator in the field theory. The string interaction vertex is built out of twist fields in the conformal field theory.

A similar construction will give the interaction vertex for the type IIB case, the only difference being that the left– and right–moving parts of it (and structures which descend from them) will have identical individual $SO(8)$ transformations. We will return to this operator later in section 4.1, as it will appear in the construction of the interactions of unoriented closed and open strings.

3. Second–Quantized Type IB Strings

The construction of ref.[8] showed that the matrix string description of the type IB string is given in terms of winding type IA strings.

The type IA model is characterized by the orientifold group \{1, $\Omega R_9$\}, where $R_9$ is the reflection $X^9 \rightarrow -X^9$ and $\Omega$ exchanges left– and right–moving fields. In other words, given that $\sigma$ is identified with $X^9$ (static gauge) we must study a model similar to the above, but with the $\sigma$ circle $S^1$ replaced by the orbifold $S^1/\mathbb{Z}_2$.

The fundamental domain of $\sigma$ is now $[0, \pi]$ instead of $[0, 2\pi]$, and there are fixed points of $\Omega R_9$ at $\sigma=0$ and $\sigma=\pi$. There is an O8–plane and 8 D8–branes at each end of the interval. Essentially the Lagrangian we seek will be associated with a macroscopic type IA string stretched along the interval.

Eventually, just as we worked on the covering space of the circle to make the long type IIB strings, we will work on the covering space of the orbifold which has a description as follows: Break the infinite line up into segments of length $2\pi$. Half of that segment is a copy of the fundamental domain described above, while the other half is also a copy, but

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3 I thank P. Pouliot for a discussion on this point.
mirror reflected. This unit then repeats periodically. String bits will live on this covering space, subject to the reflection conditions (see figs. 2 and 5).

We define the Lagrangian of the model by imposing conditions upon the Lagrangian (2.1) consistent with the above description of the interval.

We (initially) impose that the fields satisfy:

\[ X^i(2\pi - \sigma) = X^i(\sigma) \]
\[ \theta^{\alpha}_{L(R)}(2\pi - \sigma) = \theta^{\alpha}_{R(L)}(\sigma). \]  

(3.1)

3.1. Short Strings

Condition (3.1) defines two types of short string.

(i) One type of string is an open string stretched along the interval \((0, \pi)\), with boundary conditions on a single eigenvalue function (and its superpartner):

\[ \partial x^i(0, \tau)/\partial \sigma = 0 \quad \text{and} \quad \partial x^i(\pi, \tau)/\partial \sigma = 0 \]
\[ \theta^{\alpha}_L(0, \tau) = \theta^{\alpha}_R(0, \tau) \quad \text{and} \quad \theta^{\alpha}_L(\pi, \tau) = \theta^{\alpha}_R(\pi, \tau). \]  

(3.2)

This immediately tells us that the strings which survive the projection of the last section are open strings (of length 1), with endpoints on the ends of the interval. This is as we might expect, as we have simply imposed type IB boundary conditions on the Green–Schwarz action.

(ii) Another type of string can be seen by working on the doubled cell \(0 < \sigma \leq 2\pi\). Now we see that we can define a closed string of length 1 by covering the type IA interval twice, using one eigenvalue:

\[ x^i(2\pi - \sigma) = x^i(\sigma) \]
\[ \theta^{\alpha}_L(2\pi - \sigma) = \theta^{\alpha}_R(\sigma). \]  

(3.3)

![Figure 2.](attachment:image.png)

We may think of the illustration in fig. 2 in two ways, therefore. In both ways the two halves of the diagram may be thought of as two parts of a string. The first way depicts
the left and right moving parts of a string, which are identical, and are put together to
make the open string. The second way of thinking describes both halves as sections of
a closed string. Because they are identical in shape, except for a reflection, the resulting
closed string is unorientable.

Notice that these strings have only $\mathcal{N}=1$ ten dimensional spacetime supersymmetry be-
cause we have reduced the number of supercharges by one half by identifying the $\theta_L$ and
$\theta_R$ as we cross $\sigma=\pi$. The remaining supercharge is:

$$Q^\alpha = \frac{1}{2} (Q^\alpha_L + Q^\alpha_R). \quad (3.4)$$

So we see that two types of string descend from our oriented type IIB string: the unoriented
open and closed strings. They both come from identifying left and right, the former with
two fixed points on the strings —giving end-points— and the latter without.

One can think of all open and unoriented string diagrams in this way. For example, at tree
level the sphere $S^2$ can be projected to give the disc $D^2$ and the real projective plane $\mathbb{RP}^2$
(See fig. 3).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{Figure 3.}
\end{figure}

Meanwhile, at one loop, the torus $T^2$ projects to give the cylinder $C^2$, the Möbius strip
MS and the Klien bottle KB (see fig. 4):

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.png}
\caption{Figure 4.}
\end{figure}
So altogether we have $N$ independent static gauge type IA strings of length 1, or $N$ light–cone type IB strings each with a single unit of momentum in the $X^+$ direction.

3.2. Long Strings

Of course, the magic of this construction is that we can exploit the discrete gauge symmetry $S_N$ to construct long strings from the twisted sectors of the orbifold. This translates into considering twisted versions of the conditions (3.1).

To make a long string of length $n$, we again exploit a permutation of $n$ eigenvalues, $S_n$, every time we translate by $2\pi$. In addition, however, we must obey the requirement of $\mathbb{Z}_2$ reflection symmetry (3.1) within each closed string unit cell.

In general, the matrix $X^i$ will be made of $n$ different eigenvalue functions $x^i_1, \ldots, x^i_n$, which each decorate the covering space in a way which is $2\pi$ periodic and $\mathbb{Z}_2$ symmetric about every multiple of $\pi$. The $S_n$ permutation makes the matrix $X^i(\sigma)$ a $2n\pi$ periodic matrix, defining either a long closed string of length $n$, or a long open string of length $n$ and its mirror. For example, a closed (or open) string of length 3 is shown in fig. 5:

![Figure 5.](image)

Notice that by construction any sequence of string bits of a given shape is accompanied by an identical sequence of the mirror reflected shape somewhere along the string in such a way that the resulting closed strings which are made by this procedure are guaranteed to be unorientable.

Another way to characterize the twisted sectors we have found is as follows: We must recall that there is a left–right exchange symmetry $\Omega_n$ of all long closed strings which we can make in the type IIB matrix model. We may think of it as the $\Omega$ which already exists for each short type IIB string (i.e., it acts on one eigenvalue) combined with a permutation of $n$ eigenvalues $S_n$ in such a way as to define $\Omega_n$, the exchange of left and right–moving parts of the length $n$ string which is periodic over $0 < \sigma \leq 2n\pi$.

In this way, we see that long unoriented closed and open strings arise as twisted sectors of the orbifold theory. Again, asking that we keep finite fractions $n/N$ fixed in the large $N$ limit, we construct the complete Fock space of the free type IIB strings in light–cone gauge.
3.3. Spacetime $SO(32)$ Gauge Symmetry

One last property of our second quantized Fock space is that the open string end–points carry $SO(32)$ gauge symmetry. This is easy to show.

First of all, the limits which led[7] to the construction of this model from a non–perturbative framework very specifically took us to a type IA background, which has 8 D8–branes and 1 O8–plane at each end of the interval. We have therefore a manifest $SO(16)\times SO(16)$ gauge symmetry as a background, but the Fock space will assemble $SO(32)$ out of this as follows. There are two classes of long open string:

(i) Those which have endpoints only at one or other end of the interval. These are strings with $n$ even. This class will give states in the $(120, 1)\oplus(1, 120)$, the adjoint of $SO(16)\times SO(16)$.

(ii) Those which have one end on each interval. These are strings with $n$ odd. These will give the bi–fundamental $(16, 16)$ of $SO(16)\times SO(16)$.

The presence of all of these long strings in the large $N$ Fock space means that we can assemble the 496 dimensional adjoint of $SO(32)$, showing that our light–cone open string spectrum indeed has the correct gauge symmetry.

It is amusing to see $SO(32)$ arise in this way, as it is the precise analogue of what took place in realizing $E_8\times E_8$ gauge symmetry for the matrix heterotic string.

In the 0+1 dimensional picture, the discussion was phrased[13] in terms of bound states of D0–branes. Bound states of even numbers of D0–branes in the neighbourhood of either (1 O8–plane+8 D8–brane) unit gave the adjoint $((120, 1)\oplus(1, 120))$ of $SO(16)\times SO(16)$ while bound states of odd numbers of D0–branes gave the spinor $((128, 1)\oplus(1, 128))$ filling out the adjoint $(248\oplus248)$ of $E_8\times E_8$.

In the 1+1 dimensional picture, this was translated[11] into periodic (P) and anti–periodic (A) boundary conditions on the heterotic fermions (arising from D1–D9 strings in the presence of an $SO(32)\rightarrow SO(16)\times SO(16)$ Wilson line). In constructing long heterotic strings, the standard heterotic GSO projection assembled $E_8\times E_8$ out of the adjoints (AA) (even length strings) and the spinors (AP) (odd length strings).

4. Interacting Second–Quantized Type IB Strings

4.1. String Bulk Interactions

As previously discussed in the case of the type IIA and $E_8\times E_8$ heterotic strings[12][16], the joining of closed strings is controlled by the exchange of eigenvalues between two configurations representing separate closed strings. This is a simple $\mathbb{Z}_2$ operation on the coordinates $x_-\equiv x_1–x_2$ and $\theta_-\equiv \theta_1–\theta_2$: $(x_-, \theta_-)\rightarrow(-x_-, -\theta_-)$, which can occur for any
value of $\sigma$, i.e. anywhere on the closed string (see fig. 6). (Obviously, this also controls the reverse, splitting, process.)

In the conformal field theory, the interaction is introduced by adding an operator which is constructed out the standard $\mathbb{Z}_2$ conformal field theory twist fields.

Let us recall the discussion of ref.[12]. The following operator product:

$$x^i(z) \cdot \sigma(w) \sim (z - w)^{-\frac{1}{2}} \tau^i(w)$$

(4.1)

defines the standard twist field $\sigma(z)$ and its conjugate partner $\tau^i(z)$, which have conformal dimension $8 \times \frac{1}{16} = \frac{1}{2}$ and 1, respectively. The field $\tau^i(z)$ transforms in the vector $\mathbf{8}_v$ of $SO(8)$. (We use complex world sheet coordinate $z = \exp(\tau + i\sigma)$, defining the complex plane. Our light–cone strings are matrices wound on circles centered on the origin. Time runs radially.)

The operators products:

$$\theta^\alpha(z) \cdot \Sigma^i(w) \sim (z - w)^{-\frac{1}{2}} \Gamma_{\alpha\dot{\alpha}}^i \Sigma^\dot{\alpha}(w)$$

$$\theta^\alpha(z) \cdot \Sigma^\dot{\alpha}(w) \sim (z - w)^{-\frac{1}{2}} \Gamma_{\alpha\dot{\alpha}}^i \Sigma^i(w)$$

(4.2)

define the spin fields $\Sigma^i$ and $\Sigma^\dot{\alpha}$, which transform in the vector $\mathbf{8}_v$ and the conjugate spinor $\mathbf{8}_c$ respectively.

The interaction vertex is constructed out of the field

$$\mathcal{O}(z) = [\tau^i \Sigma^i](z),$$

(4.3)

which is manifestly $SO(8)$ invariant.

For the type IIA theory, the full operator is constructed by performing the same construction on the right hand side, but with the occurrences of fields in $\mathbf{8}_v$ and $\mathbf{8}_c$ $SO(8)$ representations exchanged.

In the case in hand, we wish to construct first the type IIB operator, and so the same $SO(8)$ spinor representations appear for the fermions and spin fields on the right hand side as above. At the end of the day, the actual vertex is constructed out of vectors $\mathbf{8}_v$, and so it is particularly simple to write the $SO(8)$ invariant supersymmetric operator.
For completeness, we must sum over all the possible eigenvalues (labeled $I, J$) which the operator will permute (as was done for type IIA), to give the interaction:

$$\lambda \int d^2 z \sum_{I<J} (\tau^i \Sigma^i \otimes \bar{\tau}^j \bar{\Sigma}^j)_{IJ}.$$  \hfill (4.4)

As this is a weight $(\frac{3}{2}, \frac{3}{2})$ field, the coupling $\lambda$ has conformal dimension $-1$, and is linear in the matrix string coupling $g_{\text{IIB}}$:

$$\lambda \sim g_{\text{IIB}} \ell_s.$$  \hfill (4.5)

As the interaction operator $\mathcal{O}(z, \bar{z}) = \mathcal{O}(z)\bar{\mathcal{O}}(\bar{z})$ (and its possible descendants) has exactly the same structure on the left and right, we readily see that we can construct a symmetrized version of it:

$$\tilde{\mathcal{O}}(z, \bar{z}) = \frac{1}{2} (\mathcal{O}(z)\bar{\mathcal{O}}(\bar{z}) + \mathcal{O}(\bar{z})\bar{\mathcal{O}}(z))$$  \hfill (4.6)

to give a bulk operator which controls the splitting and joining of unoriented strings somewhere along their length (see fig 7(b)). This operator naturally lives on $\mathbb{RP}^2$, and the complex $z$–plane we are working on is a double cover of it.

This operator $\tilde{\mathcal{O}}$ also has dimension $(\frac{3}{2}, \frac{3}{2})$, and so its coupling $\lambda_1$ will also be proportional to the string coupling:

$$\lambda_1 \sim g_{\text{IIB}} \ell_s.$$  \hfill (4.7)

Furthermore, the same operator will control the emission of closed strings by an open string (see fig. 7(c)), because away from the open string end–points, the operator is not sensitive to whether it is splitting or joining a closed or an open string.

The operator $\tilde{\mathcal{O}}$ will also control the open–open string interaction which does not involve the end–points. This is the case where two open string segments split and join somewhere along their length to give two final open strings (see fig. 7(a)). This interaction is again independent of whether the strings have end–points.

\begin{center}
\includegraphics[width=0.7\textwidth]{fig7.png}
\end{center}

\textit{Figure 7.}

Another type of interaction which the operator will control is the orientation reversal of a string somewhere along its bulk (See fig. 8). This is because of its symmetrical form.
4.2. String End–Point Interactions

The other string interaction we must consider is the case where the end–points of open strings can join to form a longer open string, or a closed string; or when a string (open or closed) can split somewhere along its length to produce new free end–points (see fig. 9).

As string end–points can only exist on the boundaries $\sigma=0, \pi$, we can anticipate that the relevant conformal field theory operator responsible for this interaction must only operate there.

It is easy to see where this boundary operator comes from. We may include an operator which obeys the reflection conditions in the previous section by requiring that $\bar{O}(\bar{z})$ is the same function of $\bar{z}$ as $O(z)$ is of $z$. We have effectively inserted a mirror on the boundary, selecting only a single functional dependence for our operator (i.e. only the holomorphic or anti–holomorphic part of $O(z, \bar{z})$). So this operator is naturally defined on the upper half plane (disc, $D^2$), the full plane being a double cover.

We may infer the existence of boundary operators in the standard way, examining the operator product expansion of $O(z)$ with its mirror image:

$$O(z)O(\bar{z}) \sim \sum_i \frac{1}{(z-\bar{z})^{3-h_i}} B^{(i)}(\tau) + \ldots$$  \hspace{1cm} (4.8)

where $h_i$ are the weights of the operators $B^{(i)}$. A natural guess for the operator we need is the part of $O(z)$ living on the boundary:

$$B_{ab}(\tau) = \lambda_{ab} [\tau \bar{\Sigma}^{i}] (z)|_{\sigma=0, \pi}.$$  \hspace{1cm} (4.9)

(We have endowed the operator with a Chan–Paton matrix $\lambda_{ab}$ labeling which D8–branes lie on the boundary. This will in turn be carried by the string end–points which are involved in the interaction.)
This operator is added into the theory as follows (again summing over all possible eigenvalues):

$$\lambda_2 \int d\tau \sum_{I<J} B_{IJ}(\tau).$$

(4.10)

A quick way to visualize that this is the correct operator is to think of this interaction as a descendent of the oriented closed–closed interaction, where we have subsequently identified the two halves of the closed string to make open strings (see fig. 10). The splitting–joining interactions of the parent type II string also controls the splitting joining interaction of end–points. The required operator $B(\tau)$ is simply the diagonal action of $\tilde{O}$ at the fixed points of the projection which descends open strings from closed ones. (In some sense, one can think of the boundary operator as arising in the twisted sector of the $\sigma \rightarrow -\sigma$ reflection operation, restricted to live at the fixed points, $\sigma=0, \pi$. This is reminiscent of the observations made in ref.[23].)

![Figure 10](image)

As $B(\tau)$ has conformal dimension $\frac{3}{2}$ and we are integrating over the boundary, the coupling $\lambda_2$ must have dimension $\frac{1}{2}$. Evidently, it must be the square root of $\lambda_1$, and so

$$\lambda_2 \sim g_{\text{IB}}^2 \ell_s^{\frac{3}{2}}.$$  

(4.11)

This is exactly what we want, for the splitting–joining interaction of string end–points should have the Yang–Mills interaction strength, which in turn should be the square root of the string bulk interaction strengths, controlling gravity.

In other words, we have recovered the well–known relation

$$g_{\text{YM}}^2 \sim g_{\text{IB}},$$

(4.12)

familiar from D–brane physics[24,25]. Here, of course, the D–branes in question are type IB’s D9–branes, and the Yang–Mills theory is $SO(32)$.

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4 See ref.[22] for another example of a twist operator on the boundary introducing open string sectors.
5. Closing Remarks

In summary, we have constructed the matrix theory (first arrived at in ref. [7]) of the weakly coupled second–quantized type IB string. The Fock space is built by “winding” type IA strings.

The theory of the free second–quantized type IB string theory is a 1+1 dimensional orbifold conformal field theory which is characterized by a matrix type IB Green–Schwarz action, which has a discrete Abelian gauge symmetry, the permutations of the eigenvalues.

The twisted sectors of the new theory contain long strings, which are both open and closed. String end–points exist at the fixed points of the type IA orientifold group \( \{1, \Omega \mathcal{R}_9\} \), which define the type IA interval. The D8–branes and orientifold O8–planes there supply the Chan–Paton factors of the open strings, and \( SO(32) \) gauge symmetry arises by the correct assembly of the \( SO(16) \times SO(16) \) charged strings from the different twisted sectors.

There are two classes of string interactions in this open–closed string theory, and we constructed them as operators in the orbifold conformal field theory.

The first class, which involves splitting and joining in the interior of open and closed strings, has the strength of the closed string coupling \( g_{IB} \), and is the non–orientable descendant of the type IIB interaction vertex, which is left–right symmetric by construction.

The second class involves the creation or destruction of string end–points, and is controlled by a boundary operator which is a “descendant” of the bulk interaction twist operator, in the sense that it is the part of it restricted to the boundary. Appropriately, it has the strength of the square root of the string coupling.

This completes the demonstration that all\([7]\) the five ten dimensional superstring theories near weak coupling have a string field theory description in terms of winding strings of the T–dual species, where an economical description is given in terms of a matrix Green–Schwarz Lagrangian.

Let us continue the discussion begun at the end of section 1.1. It is not clear to the author whether the natural appearance of a light–cone second–quantized field theory of strings from matrix theory represents more than a delicious irony.

Perhaps it is possible that such a second–quantized string description will always appear naturally in the stringy limit of any definition of M–theory.

This begs the question as to whether a possible route to a covariant definition of M–theory may be sought by studying a (possibly discretized) version of covariant string field theory. It may be that there are some important lessons to be learned from covariant string field theory after all.

\[5\] There is a different proposal in the literature\([26]\).
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