Nucleon matrix elements with domain wall fermions
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We present the status of our calculation of the first few moments of the nucleon structure functions. Our calculations are done using domain wall fermions in the quenched approximation with the DBW2 gauge action at 1.3GeV inverse lattice spacing.

The structure of the nucleon is one of the fundamental problems that lattice QCD can address. In the last few years, substantial efforts have been made by several groups in calculating the non-perturbative matrix elements relevant to nucleon structure. Up to now only Wilson fermions, improved and unimproved, have been used in both the quenched approximation and in full QCD. In this report we examine the feasibility of studying nucleon matrix elements with domain wall fermions in the quenched approximation. Domain wall fermions have only \mathcal{O}(a^2) lattice artifacts, non-perturbative renormalization works very well, and have no problem with exceptional configurations. Furthermore, the chiral symmetry they preserve on the lattice eliminates mixings with lower dimensional operators, rendering the renormalization of certain matrix elements significantly simpler. For the above reasons, a study of the nucleon structure with domain wall fermions is definitely important.

We study the nucleon matrix elements relevant to the leading twist contributions to the moments of the nucleon structure functions. The leading twist matrix elements are:

\[
\begin{align*}
\frac{1}{2} \sum_s \langle p, s | \mathcal{O}_{q\mu_1...\mu_n}^{\mu_1...\mu_n} | p, s \rangle & = 2 \langle \alpha^{n-1} \rangle_q (\mu) \times \\
& \times [p_{\mu_1} p_{\mu_2} \cdots p_{\mu_n} + \cdots - tr] \\
- \langle p, s | \mathcal{O}_{5q}^{\mu_1...\mu_n} | p, s \rangle & = \frac{2}{n+1} \langle \alpha^n \rangle_5 q_{\mu_1 \mu_2...\mu_n} (\mu) \times \\
& \times [s_{\sigma} p_{\mu_1} p_{\mu_2} \cdots p_{\mu_n} + \cdots - tr] \\
\langle p, s | \mathcal{O}_{\sigma}^{\mu_1...\mu_n} | p, s \rangle & = \frac{1}{n+1} \delta_{\mu_1 \mu_2...\mu_n} (\mu) \times \\
& \times [(s_{\sigma} p_{\mu_1} - s_{\mu_1} p_{\sigma}) p_{\mu_2} \cdots p_{\mu_n} + \cdots - tr]
\end{align*}
\]

where \( p_\mu \) and \( s_\mu \) are the nucleon momentum and spin vectors, \( m_N \) the nucleon mass, and

\[
\begin{align*}
\mathcal{O}_{q\mu_1...\mu_n}^{\mu_1...\mu_n} & = \left( \frac{i}{2} \right)^{n-1} \bar{q} \gamma_{\mu_1} D_{\mu_2} \cdots D_{\mu_n} q - tr \\
\mathcal{O}_{5q}^{\mu_1...\mu_n} & = \left( \frac{i}{2} \right)^n \bar{q} \gamma_{\sigma} \gamma_5 \bar{D}_{\mu_2} \cdots \bar{D}_{\mu_n} q - tr \\
\mathcal{O}_{\sigma}^{\mu_1...\mu_n} & = \left( \frac{i}{2} \right)^n \bar{q} \gamma_5 \sigma_{\rho\nu} \bar{D}_{\mu_1} \cdots \bar{D}_{\mu_n} q - tr
\end{align*}
\]

\{\} implies symmetrization and [\] implies antisymmetrization. For the conventions used see [1].

Our current results are restricted only to those matrix elements that can be computed with zero momentum nucleon states. We use the DBW2 gauge action which is known to improve the domain wall fermion chiral properties. These results are from 204 lattices of size \( 16^3 \times 32 \) at \( \beta = 0.870 \) with lattice spacing \( a^{-1} = 1.3\)GeV. This provides us with a physical volume \((\sim (2.4fm)^3)\) large enough to reduce finite size effects known to affect some nucleon matrix elements such as \( g_A \). Using fifth dimension length \( L_5 = 16 \) we achieve a residual mass \( m_{\text{res}} \sim 0.8\)MeV [16]. The quark masses used are \( m_q a = 0.02, 0.04, 0.06, 0.08, \) and 0.10 which give pion masses ranging from 390MeV to 850MeV.

We used Coulomb gauge fixed box sources with size \( 8^3 \) which have been shown to couple very well
Figure 1. Quark density $\langle x \rangle$ vs. the pion mass squared. (a) The connected up (octagons) and down (diamonds) quark contributions. (b) The flavor non-singlet $\langle x \rangle_{u-d}$.

to the nucleon ground state [8]. The source - sink separation was set to 10 time slices or $\sim 1.5 \text{fm}$ in physical units. Finally in order to construct the three point functions we used sequential propagators [11,12] with point sinks. With these choices the signal to noise ratio for the three point functions in the plateau region is about 10 for the lightest quark mass. A part of our code tests was a small quenched run with Wilson fermions and Wilson gauge action at $\beta = 6$.0. We were able to reproduce the results in [3,4].

Fig. 1 presents our results for the quark density distribution $\langle x \rangle_q$. This is related to the lowest moment of the unpolarized structure functions $F_1$ and $F_2$. We plot the unrenormalized result for $\langle x \rangle_u$, $\langle x \rangle_d$ and the flavor non-singlet $\langle x \rangle_{u-d}$. The latter exhibits a noticeable curvature for the two lighter quark masses indicating that the quenched result for $\langle x \rangle_{u-d}$ may be closer to the phenomenological expectations than previously thought [8]. This curvature may be the first indication of the chiral log behaviour that has to set in at sufficiently small quark masses [13]. The ratio $\langle x \rangle_{u-d}$ is 2.37(6) at the chiral limit in agreement the quenched Wilson fermion result [3].

We measure also the helicity distributions $\langle x \rangle_{\Delta q}$.

Figure 2. Helicity $\langle x \rangle_{\Delta q}$ vs. the pion mass squared. (a) The connected up (octagons) and down (diamonds) quark contributions. (b) The flavor non-singlet $\langle x \rangle_{\Delta u-\Delta d}$.

$\langle 1 \rangle_{\Delta q}$ and $\langle x \rangle_{\Delta q}$. A detailed discussion of our results for $\langle 1 \rangle_{\Delta q}$ can be found in [13]. In Fig. 2 we present our unrenormalized data for $\langle x \rangle_{\Delta q}$. Unlike the quark density distributions we do not see a significant dependence on the quark mass. On the basis of chiral perturbation theory arguments this matrix element is indeed expected to show the chiral log behavior at smaller quark masses than the quark density distributions [13]. The ratio $\langle x \rangle_{\Delta u}/\langle x \rangle_{\Delta d}$ is roughly $-4$, consistent with other lattice results [2,4].

The lowest moment of the transversity $\langle 1 \rangle_{\delta q}$ is also measured. In Fig. 3 we plot the unrenormalized contributions for both the up and down quark, and the flavor non-singlet combination $\langle 1 \rangle_{\delta u-\delta d}$. Again the quark mass dependence is very mild and there is no sign of a chiral log behavior. The ratio $\langle 1 \rangle_{\delta u}/\langle 1 \rangle_{\delta d}$ is also roughly $-4$.

Finally we computed the $d_1$ matrix element which is a twist 3 contribution to the first moment of $g_2$. If chiral symmetry is broken the operator

$$O_{34}^{\delta q} = \frac{1}{4} \bar{q} \gamma_5 \left[ \gamma_3 \bar{D}_4 - \gamma_4 \bar{D}_3 \right] q$$

which is used to measure $d_1$ mixes with the lower dimensional operator $O_{34} = \bar{q} \gamma_5 \sigma_{34} q$. Hence in
Wilson fermion calculations a non perturbative subtraction has to be performed \[2,1\]. With domain wall fermions this kind of mixing is proportional to the residual mass which in our case is negligible. Thus, we expect that a straightforward computation of \(d_1\) with domain wall fermions provides directly the physically interesting result. In Fig. 4 we present our unrenormalized results for \(d_1\) as a function of the quark mass. For comparison we also plot the unsubtracted quenched Wilson results for \(\beta = 6.0\) from \[4\]. The fact that our result almost vanishes at the chiral limit is an indication that the power divergent mixing is absent for domain wall fermions. The behavior we find for the \(d_1\) matrix element is consistent with that of the subtracted \(d_2\) measured by QCDSF \[2,1\] with Wilson fermions. In conclusion, we have started the computation of moments of nucleon structure functions with domain wall fermions. Our current results are unrenormalized and restricted to those matrix elements that can be computed with zero momentum nucleon states. Yet we already have hints of a couple of potentially interesting results. First we have an indication of the possible onset of chiral log behavior for \(\langle x \rangle_{u-d}\). Also it is very encouraging, although expected, to see the

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