SENSITIVITY FOR 21CM BISPECTRUM FROM EPOCH OF REIONIZATION

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ABSTRACT

The 21cm line brightness temperature brings rich information about Epoch of Reionization (EoR) and high-z universe (Cosmic Dawn and Dark Age). While the power spectrum is a useful tool to investigate the EoR signal statistically, higher-order statistics such as bispectrum are also valuable because the EoR signal is expected to be highly non-Gaussian. In this paper, we develop a formalism to calculate the bispectrum contributed from the thermal noise taking array configuration of telescopes into account, by extending a formalism for the power spectrum [McQuinn et al. 2006]. We apply our formalism to the ongoing and future telescopes such as expanded Murchison Widefield Array (MWA), Low Frequency Array (LOFAR) and Square Kilometre Array (SKA). We find that expanded MWA does not have enough sensitivity to detect the bispectrum signal. On the other hand, LOFAR has better sensitivity and will be able to detect the peaks of the bispectrum as a function of redshift at large scales with comoving wavenumber $k \lesssim 0.03\,\text{Mpc}^{-1}$ and redshift $z \lesssim 20$.

Subject headings: cosmology: Sensitivity — Square Kilometre Array — Epoch of Reionization — 21cm line — bispectrum

1. INTRODUCTION

The redshifted 21cm line emission from neutral hydrogens is a promising way to probe Epoch of Reionization (EoR), Cosmic Dawn and Dark Age [Pritchard & Furlanetto 2007, Pober et al. 2014] because it reflects the physical state of intergalactic gas. Actually, the brightness temperature depends on quantities crucial for the understanding of these epochs, such as the neutral hydrogen fraction, spin temperature and baryon density. However, the observation of the redshifted 21cm signal is very challenging due to the presence Galactic and extragalactic foreground emissions, Low-frequency radio telescopes such as Murchison Widefield Array (MWA) [Beardsley et al. 2013], Low Frequency Array (LOFAR) [Jensen et al. 2013] and PAPER [Parsons et al. 2014] have started their observations and set upper bounds on the brightness temperature. The upper bounds will improve further as our understanding of the foreground proceeds and the subtraction techniques become more sophisticated. Ultimately, the Square Kilometre Array (SKA) [Dewdney et al. 2013, Mellema et al. 2013] will perform precise observations and will reveal the physical process of EoR and Cosmic Dawn.

One of the useful tools to extract information from observed data is to take the power spectrum of fluctuations in brightness temperature at a fixed redshift (frequency). This is effective even for relatively low S/N data, which could be obtained by ongoing telescopes, while making a map of brightness temperature through imaging requires much higher sensitivity the SKA is expected to have. Actually, the power spectrum of brightness temperature has been studied by many authors [Pritchard & Furlanetto 2007, Pober et al. 2014, Shimabukuro et al. 2014, Furlanetto et al. 2006, Baek et al. 2010, Mesinger et al. 2014, Santos et al. 2008].

When fluctuations follow Gaussian probability distribution, they can be well characterized by the power spectrum and higher-order statistics such as bispectrum and trispectrum have no further independent information. However, since reionization is a highly non-Gaussian process which involves non-linear density fluctuations, star formation and expansion of HII bubbles, the brightness temperature fluctuations are also expected to be strongly non-Gaussian [Shimabukuro et al. 2014]. In this case, the power spectrum does not have sufficient information to describe the fluctuations and higher-order statistics have independent and complimentary information [Cooray 2005, Pillepich et al. 2007].

In this paper, we develop a formalism to calculate the errors in bispectrum measurement contributed from thermal noise. Noise estimation has been studied by many authors in case of power spectrum [Morales & Hewitt 2004, Morales 2004, McQuinn et al. 2006], and we extend the formalism given in McQuinn et al. 2006. Starting from the error in visibility obtained by a single baseline, we consider its summation over the baseline distribution in $uv$ plane. A striking feature of thermal-noise bispectrum is that its ensemble average vanishes because thermal noise is Gaussian. Nevertheless, thermal noise contributes to the bispectrum error through its variance. Considering the variance of thermal noise error is the main extension to the previous formalism.

The structure of this paper is the following. In section 2, we define the brightness temperature, it’s power spectrum and bispectrum. In section 3, we review the formalism of calculation of thermal-noise power spectrum given by [McQuinn et al. 2006]. Then, we develop a formalism for bispectrum and estimate thermal-noise bispectrum for several specific configuration of the wave number in section 4. The summary and discussion will be given in section 5.
5. Throughout this paper, we assume ΛCDM cosmology with \((\Omega_m, \Omega_\Lambda, \Omega_b, H_0) = (0.27, 0.73, 0.046, 70 \text{ km/s/Mpc})\) [Komatsu et al. 2011].

2. 21CM LINE SIGNAL

In this section, we define basic quantities concerning the 21cm signal. The brightness temperature \(\delta T_b\) is defined by spin temperature offsetting from CMB temperature,\[\delta T_b(z) = \frac{T_e - T_{\gamma}}{1 + z} (1 - e^{-\tau(v)})\]

\[\approx 27x_H(1 + \delta_m)\left(\frac{H}{dv_r/\Delta v + H}\right) \left(1 - \frac{T_e}{T_s}\right) \left(\frac{1 + z}{10 \Omega_m h^2} \right)^{1/2} \left[\frac{\Omega_b h^2}{0.023}\right] \text{ [mK]}, \tag{1}\]

where \(x_H\) is the neutral fraction of hydrogen, \(\delta_m\) is the matter over density, \(H\) is the Hubble parameter and \(dv_r/\Delta v\) is the velocity gradient along the line of sight. Then we introduce fluctuation of \(\delta T_b(x)\),

\[\delta_{21}(x) = \delta T_b(x) - \bar{T}_b, \tag{2}\]

where \(\bar{T}_b\) is the average value of brightness temperature, \(x\) is spatial position. The power spectrum of brightness temperature is defined from its Fourier transform, \(\delta_{21}(k)\), as,

\[\langle \delta_{21}(k_1) \delta_{21}(k_2) \rangle = \delta(k_1 + k_2)P_{21}(k_1), \tag{3}\]

where \(\langle \rangle\) represents the ensemble average, \(k\) is position in Fourier space. The bispectrum \(B_{21}\) can be defined in a similar way:

\[\langle \delta_{21}(k_1) \delta_{21}(k_2) \delta_{21}(k_3) \rangle = \delta(k_1 + k_2 + k_3)B_{21}(k_1, k_2, k_3). \tag{4}\]

Here the delta function forces the three wave vectors to make a triangle and \(B_{21}\) is dependent on only two of the three vectors (chosen \(k_1\) and \(k_2\) here) due to this triangle condition.

3. POWER SPECTRUM SENSITIVITY

In this section, we summarize a formalism to estimate the thermal noise for power spectrum, following McQuinn et al. (2006). First, we define visibility \(V(u, v, \nu)\) for a pair of antennae as,

\[V(u, v, \nu) = \int d\hat{n} T_N(\hat{n}, \nu) W(\hat{n}, \nu) e^{2\pi i (\nu \cdot \hat{n})}, \tag{5}\]

where \(T_N\) is the thermal-noise temperature, \(\hat{n}\) is the direction of primely beam, \(\nu\) is observed frequency and \(W(\hat{n}, \nu)\) is a product of the window functions concerning the field of view and bandwidth. The rms thermal-noise fluctuation per visibility is given by,

\[V_N = \frac{\lambda^2 T_{sys}}{A_e \sqrt{\Delta \nu \tau_0}} \text{ [K]}, \tag{6}\]

where \(\lambda\) is the observed wavelength, \(T_{sys}\) is the total system temperature, \(A_e\) is the effective area of antenna, \(\Delta \nu\) is the width of the frequency channel and \(\tau_0\) is total observing time. By Fourier transforming the visibility in the frequency direction, we obtain,

\[I(u, v, \eta) = \int dv V_N(u, v, \nu) \exp(2\pi i \nu \eta) \]

\[= \sum_{i=1}^{B/\Delta \nu} V_N(u, v, \nu_i) \exp(2\pi i \nu_i \eta) \Delta \nu \text{ [K \cdot Hz]}, \tag{7}\]

where \(B(\gg \Delta \nu)\) is the bandwidth, \(\nu_i\) is the \(i\)-th frequency channel and we define \(u = (u, v, \eta)\). The covariance matrix of detector noise for a single baseline is given by,

\[C_N(u_i, u_j) = \langle I_N(u_i) \overline{I_N(u_j)} \rangle \]

\[= \int du' \int du'' \langle \overline{T_N(u') \overline{T_N(u'')}} \rangle \overline{W(u_i - u')} W(u_j - u'') \]

\[= \int du' \int du'' P_N(u') \delta_3^B(u' - u'') \overline{W(u_i - u')} W(u_j - u'') \]

\[= \int du' \int du'' P_N(u') \overline{W(u_i - u')} W(u_j - u') \]

\[\approx \delta_{ij} P_N(u_i) \int d^3u' |\overline{W(u_i - u')}|^2, \tag{8}\]

where, we used the definition of power spectrum for noise temperature (Eq. 3) in third equality and assumed that the covariance vanishes when \(u_i \neq u_j\) in the last equality. Further, we assumed that the power spectrum
is constant for a range where the window function have non-zero value and we pulled \( P_N \) out of the integration.

Then the integration of window functions can be evaluated as follows:

\[
\int d^3u' |W(u-u')|^2 = \int d^3u' \int d^3r \int d^3r' |W(r)||W(r')|e^{2\pi i (u-u') \cdot (r+r')}
\]

\[
= \int d^3r_0 \int d^3r' \delta_D(r_0) |W(r_0 - r')||W(r')|e^{2\pi i u \cdot (r_0)}
\]

\[
= \int d^3r' |W(r')||W(-r')| \approx \Omega B \approx \frac{\lambda^2 B}{A_e},
\]

where \( \Omega \) is the field of view. Thus we obtain,

\[
C_N(u_i, u_j) \approx \frac{\lambda^2 B}{A_e} P_N(u_i) \delta_{ij}.
\]

On the other hand, the covariance matrix for a single baseline can be evaluated from Eq. (10),

\[
C_{N,bb}(u_i, u_j) = \langle \tilde{I}(u_i) \tilde{I}^*(u_j) \rangle_{bb} = \sum_l \sum_m |V_N(u_i, v_i, \nu_l)|^2 \delta_{ij} \delta_{lm}
\]

\[
= \sum_l \sum_m |V_N(u_i, v_i, \nu_l)|^2 \Delta \nu \delta_{ij} = \frac{B}{\Delta \nu} (\Delta \nu)^2 (V_N(u_i, v_i, \nu))^2 \delta_{ij}
\]

\[
= \left( \frac{\lambda^2 B \nu \nu}{A_e} \right) \delta_{ij} B t_0.
\]

Again, we assumed that there is no correlation between the thermal noise with different \( u, v \) and \( \nu \). If multiple baselines contribute to the same pixel, the observing time is effectively increased. Here we assume that the number density of the baselines in uv-plane is constant under rotation with respect to \( \eta \)-axis, that is, depends only on \( |u_\perp| = |u| \sin \theta \) where \( \theta \) is the angle between \( u \) and \( \eta \)-axis. Therefore, the effective observing time \( t_u \) can be written as,

\[
t_u \approx \frac{A_e}{\lambda^2 n(|u| \sin \theta) t_0}.
\]

Here \( A_e / \lambda^2 \) represents area per pixel on uv-plane which reflects the resolution on uv-plane and \( n(|u| \sin \theta) \) is the number density of baselines on uv-plane. Thus, we obtain the covariance matrix for a pixel in uv\( \eta \)-space, replacing \( t_0 \) with \( t_k \), as,

\[
C_N(u_i, u_j) = \left( \frac{\lambda^2 B \nu \nu}{A_e} \right) \delta_{ij} \frac{B t_k}{t_0}.
\]

Thus, comparing with Eq. (10) and substituting Eq. (12), we obtain,

\[
P_N(u) = \frac{\lambda^4 T^2_{\text{sys}}}{A_e^2 n(|u| \sin \theta) t_0}.
\]

Now we convert the noise power spectrum of \( u \) space to the one of cosmological Fourier space \( k \). Using the following relations

\[
|u_\perp| = \frac{D_M(z)}{2\pi} k_\perp \approx \frac{x}{2\pi} k_\perp,
\]

\[
\eta \approx \frac{c(1+z)^2}{2\pi H_0 f_{21} E(z)} k_\perp \approx \frac{y}{2\pi} k_z,
\]

where, \( H_0 \) is the Hubble constant, \( f_{21} \) is the frequency of 21cm radiation and

\[
D_M(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}\]

\[
E(z) = \sqrt{\Omega_M (1+z)^3 + 1 - \Omega_M}
\]

where \( \Omega_M \) is the density parameter of matter and we assumed the flat universe. Thus, we obtain,

\[
P_N(k) = \frac{x^2 y}{B} P_N(u) = \frac{x^2 y \lambda^4 T^2_{\text{sys}}}{B A_e^2 n(|u| \sin \theta) t_0}.
\]

Because the power spectrum of 21cm signal is dependent only on the length of the wave vector, we take a sum of the above noise power spectrum over a spherical shell which corresponds to the same \( k \). First, we consider
an annulus with radial width $\Delta k$ and angular width $\Delta \theta$. Noting that the baseline distribution is assumed to be uniform in an annulus, the number of pixels in the annulus is,

$$N_a = 2\pi k^2 \sin \theta \, \Delta \theta \, \Delta k \frac{V}{(2\pi)^3}, \quad (20)$$

where $V = \lambda^2 x^2 y/A_e$ is the observed volume in real space, $(2\pi)^3/V$ is the resolution in Fourier space and the other factor, $2\pi k^2 \sin \theta \Delta \theta \Delta k$, is the annulus volume in Fourier space. Then the noise power spectrum reduces by a factor of $1/\sqrt{N_a}$. Next, we consider a sum over $\theta$. Taking $\Delta k = \epsilon k$, where $\epsilon$ is a constant factor, the spherically averaged sensitivity is given by,

$$\delta P_N(k) = \sum_\theta \left( \frac{1}{P_N(k,\theta)/\sqrt{N_a}} \right)^{2} \approx \left( k^3 \int_{\arcsin[\min(\frac{k\epsilon}{2\pi},1)]} d\theta \sin \theta \epsilon(n(k \sin \theta))^2 A_e^3 B^2 t_0^2 \right)^{-1/2}, \quad (21)$$

where $k_\epsilon$ is the longest transverse wave vector, which corresponds to the maximum baseline length. The lower limit of the integral corresponds to the pixel size.

4. BISPECTRUM SENSITIVITY

In this section, we estimate the bispectrum from the thermal noise in a similar way in the previous section. However, we should notice that, because the thermal noise is Gaussian, its bispectrum is actually zero. Nonetheless, its statistical fluctuation, that is, its variance is non zero and contributes to the noise to the bispectrum signal. Thus, the calculation in this case is more subtle than that of the power spectrum, although we can use similar techniques as we see below. In [Saivad Ali et al. 2006], an order estimation for the thermal noise bispectrum has been done without considering this fact and also the baseline distribution.

4.1. covariance of bispectrum

Remembering the definition of the bispectrum in Eq. 4, the covariance of the bispectrum can be defined by,

$$\text{Cov}(B_N(u_1, u_2, u_3), B_N(u_4, u_5, u_6)) = \langle (\tilde{T}_N(u_1) \tilde{T}_N(u_2) \tilde{T}_N(u_3) - (\tilde{T}_N(u_1) \tilde{T}_N(u_2)) \tilde{T}_N(u_3)) \rangle \tilde{T}_N(u_4) \tilde{T}_N(u_5) \tilde{T}_N(u_6) \rangle$$

$$= \int du_1' \int du_2' \int du_3' \int du_4' \int du_5' \int du_6' \langle \tilde{T}(u_1') \tilde{T}(u_2') \tilde{T}(u_3') \rangle \tilde{T}(u_4') \tilde{T}(u_5') \tilde{T}(u_6')$$

$$\times \tilde{W}(u_1 - u_1') \tilde{W}(u_2 - u_2') \tilde{W}(u_3 - u_3') \tilde{W}(u_4 - u_4') \tilde{W}(u_5 - u_5') \tilde{W}(u_6 - u_6'). \quad (24)$$

To proceed further, we substitute Eq. [31] and consider the first term in Eq. [23].

$$\text{Cov}(B_N(u_1, u_2, u_3), B_N(u_4, u_5, u_6)) = \int du_1' \int du_2' \int du_3' \int du_4' \int du_5' \int du_6'$$

$$\times \text{Cov}(B_N(u_1', u_2', u_3') B_N(u_4', u_5', u_6')) \delta(u_1' - u_4') \delta(u_2' - u_5') \delta(u_3' - u_6')$$

$$\times \tilde{W}(u_1 - u_1') \tilde{W}(u_2 - u_2') \tilde{W}(u_3 - u_3') \tilde{W}(u_4 - u_4') \tilde{W}(u_5 - u_5') \tilde{W}(u_6 - u_6')$$

$$= \int du_1' \int du_2' \int du_3' \int du_4' \int du_5' \int du_6' \text{Cov}(B_N(u_1', u_2', u_3') B_N(u_4', u_5', u_6'))$$

$$\times \tilde{W}(u_1 - u_1') \tilde{W}(u_2 - u_2') \tilde{W}(u_3 - u_3') \tilde{W}(u_4 - u_4') \tilde{W}(u_5 - u_5') \tilde{W}(u_6 - u_6') \quad (25)$$

This is non-zero only when $u_1 \approx u_4$ and $u_2 \approx u_5$ (and then $u_3 \approx u_6$ from the triangular conditions). If these conditions are satisfied,

$$\text{Cov}(B_N(u_1, u_2, u_3, u_4, u_5, u_6)) \approx \text{Cov}(B_N(u_1, u_2, u_3) B_N(u_4, u_5, u_6)) \int du_1' \int du_2' \int du_3' \int du_4' \int du_5' \int du_6'$$

$$\times \tilde{W}(u_1 - u_1') \tilde{W}(u_2 - u_2') \tilde{W}(u_3 - u_3') \tilde{W}(u_4 - u_4') \tilde{W}(u_5 - u_5') \tilde{W}(u_6 - u_6')$$

$$= \left( \frac{\lambda^2 B}{A_e} \right)^2 \text{Cov}(B_N(u_1, u_2, u_3) B_N(u_4, u_5, u_6)) \quad (26)$$

where we used Eq. [9] and

$$\int du' \tilde{W}(u - u') = 1, \quad (27)$$
and assumed \( \text{Cov}(B_N(u_1, u_2, u_3)B_N(u_4, u_5, u_6)) \) is approximately constant within the window function. Thus, taking other terms in Eq. (24) into account, we have,

\[
C_B(u_1, u_2, u_3, u_4, u_5, u_6) = D\left(\frac{\lambda^2 B}{Ae}\right)^2 \text{Cov}(B_N(u_1, u_2, u_3)B_N(u_4, u_2, u_3)).
\]  

(28)

On the other hand, the product of six noise intensities can also be calculated as follows.

\[
C_B(u_1, u_2, u_3, u_4, u_5, u_6) = \langle \hat{I}(u_1)\hat{I}(u_2)\hat{I}(u_3)\hat{I}(u_4)\hat{I}(u_5)\hat{I}(u_6)\rangle = \langle \hat{I}(u_1)\hat{I}(u_2)\rangle\langle \hat{I}(u_3)\hat{I}(u_6)\rangle + (5 \text{ permutations})
\]

\[
= D\left(\frac{\lambda^2 BT_{\text{sys}}}{Ae}\right)^6 \frac{1}{B^3t_u t_u^2 t_u^3},
\]  

(29)

where we used Wick theorem (Joachimi et al. 2009) in the second equality and Eq. (14) in the last equality.

Thus, from Eqs. (28) and (29), we obtain,

\[
\text{Cov}(B_N(u_1, u_2, u_3)B_N(u_1, u_2, u_3)) = \left(\frac{Ae}{\lambda^2 B}\right)^2 \left(\frac{\lambda^2 BT_{\text{sys}}}{Ae}\right)^6 \frac{1}{B^3t_u t_u^2 t_u^3}.
\]  

(30)

Converting the argument from \( u \) to \( k \), we finally obtain,

\[
\text{Cov}(B_N(k_1, k_2, k_3)B_N(k_1, k_2, k_3)) = \left(\frac{x^2 y}{B}\right)^4 \text{Cov}(B_N(u_1, u_2, u_3)B_N(u_4, u_5, u_6))
\]

\[
= \left(\frac{x^2 y \lambda^2}{Ae}\right)^4 \frac{T_{\text{sys}}^6}{B^3t_{k_1} t_{k_2} t_{k_3}}.
\]  

(31)

This equation corresponds to Eq. (18) for the power spectrum, if we substitute Eq. (12).

4.2. spherical average

In this subsection, we take a sum of the noise bispectrum over spherical shell as we did for the power spectrum in the previous section. However, the situation is much more complicated in the case of bispectrum, because \( |k_1|, |k_2| \) and \( |k_3| \) can be all different with each other in general so that we must consider two spherical shells with the radius \( |k_1| \) and \( |k_2| \), while \( |k_3| \) is determined by the triangular condition, \( k_1 + k_2 + k_3 = 0 \). In this paper, we calculate the noise bispectrum for equilateral type (\( |k_1| = |k_2| = |k_3| \)) and isosceles type (\( |k_2| = |k_3| \)) and define \( K \equiv |k_1| \) and \( k \equiv |k_2| = |k_3| \).

First, as in the case of the power spectrum, \( k_1 \) can run over a spherical shell with radius \( k \) which can be parametrized by two of the spherical coordinate of \( k_1 \), \( (\theta_1, \phi_1) \). Further, for a fixed \( k_1 \), there is a rotational degree of freedom for \( k_2 \) with respect to \( k_1 \), which is denoted by an angle \( \alpha \) with \( 0 \leq \alpha < 2\pi \). Thus, we need to integrate the covariance matrix in Eq. (31) with respect to \( \theta_1, \phi_1 \) and \( \alpha \). Noting that the covariance matrix does not depend on \( \phi_1 \), the weight of the integration, which corresponds to Eq. (26), is given by

\[
N_{\alpha} = \left[ 2\pi \sin \theta_1 K^2 \Delta K \Delta \theta_1 \frac{V}{(2\pi)^3} \right] \times \left[ k^2 \sin \theta_2 \sin \gamma \Delta k \Delta \theta_2 \Delta \alpha \frac{V}{(2\pi)^3} \right] .
\]  

(32)

where the first factor comes from the sum for \( k_1 \) over the spherical shell and the second factor takes the rotational degree of freedom of \( k_3 \) for each \( k_1 \) into account. Here \( \theta_2 \) is the polar angle of \( k_2 \) and \( \gamma \) is the angle \( \partial k_2 / \partial \alpha \) and \( \partial k_2 / \partial \theta_2 \). \( \Delta \theta_2 \) is the width of the annulus of \( k_2 \) when \( k_1 \) is fixed, which we set equal to the resolution in Fourier space, \( 2\pi / \sqrt{1/3} \).

It is convenient to express \( \theta_2 \) by \( \theta_1, \alpha \) and the angle between \( k_1 \) and \( k_2 \) denoted as \( \beta \). Noting \( k_2 \) can be express as

\[
k_2 = k(\cos \theta_1 \cos \alpha \sin \beta + \sin \theta_1 \cos \beta, \sin \alpha \sin \beta, -\sin \theta_1 \cos \alpha \sin \beta + \cos \theta_1 \cos \beta),
\]  

(33)

we obtain,

\[
\cos \theta_2 = -\sin \theta_1 \cos \alpha \sin \beta + \cos \theta_1 \cos \beta
\]  

(34)

Then, setting \( \Delta k = \epsilon k \) and \( \Delta K = \epsilon K \), the bispectrum variance due to the thermal noise is written by an integrate with respect to \( \theta_1 \) and \( \alpha \),

\[
\delta B_N(k, K, \beta) = \left[ \sum_{\theta} \sum_{\alpha} \left( \frac{1}{\sqrt{N_{\alpha}}} \frac{1}{\sqrt{\text{Cov}(B_1 B_2)(k, K, \theta_1, \alpha)}} \right)^{-2} \right]^{-1/2}
\]

\[
= \frac{(2\pi)^{3/2}}{\sqrt{\Delta \theta_2 k K^{3/2}}} \left( \frac{x^2 y \lambda^2}{Ae} \right) \left( \frac{T_{\text{sys}}^2 \lambda^2}{Ae B_0} \right)^{3/2}
\]

\[
\times \left[ \int d\theta_1 \int d\alpha \sin \theta_1 \sin \theta_2 \sin \gamma(\theta_1, \alpha) n(k_1)n(k_2)n(k_3) \right]^{-1/2}.
\]  

(35)
This is a general expression for isosceles-type bispectrum. For equilateral type, we just set $K = k$ and $\beta = 2\pi/3$.

### 4.3. estimation of noise bispectrum

To calculate the bispectrum sensitivity, we need the number density of baselines on uv-plane. In this paper, we consider expanded MWA, LOFAR and SKA. The expanded MWA will have 500 antennae within a radius of 750 m with $r^{-2}$ distribution (Bowman et al. 2000). LOFAR has 24 antennae within a radius of 2000 m with $r^{-2}$ distribution (van Haarlem et al. 2013). SKA will have 466 antennae within 600 m with $r^{-2}$ distribution, 670 antennae within 1000 m, 866 antennae within 3000 m (Dewdney et al. 2013). For simplicity, we assume that the antennae density is constant between 600 m to 1000 m and 1000 m to 3000 m, respectively. We list parameters in Table 1. Further, we assume $t_0 = 1000$ hour for the total observing time and 6 MHz bandwidth.

For comparison, we show the bispectrum of 21cm signal from the epoch of reionization, using a public code, 21cmFAST (Mesinger et al. 2011). This is based on a semi-analytic model of reionization and we can obtain 3D brightness temperature maps at arbitrary redshifts. We set the simulation box to (200 Mpc)$^3$ with 300$^3$ grids and take a set of model parameters as $(\zeta, \xi_X, T_{\text{vir}}, R_{\text{mfp}}) = (31.5, 10^{56}/M_\odot, 10^4 \, \text{K}, 30 \, \text{Mpc})$. Here, $\zeta$ is the ionizing efficiency, $\xi_X$ is the number of X-ray photons per solar mass, $T_{\text{vir}}$ is the minimum virial temperature of halos which host stars and $R_{\text{mfp}}$ is the mean free path of ionizing photons.

In Fig. 1 we compare the equilateral-type bispectrum signal with thermal noise at $z = 8, 10, 12$ and 17. Generally, the noise increases toward smaller scales which reflects the deficiency of longer baselines. On the other hand, the sensitivity for larger scales are limited by the survey volume. We see the signals are larger than SKA noise for $k \lesssim 0.3 \, \text{Mpc}^{-1}$ at all redshifts. However, the thermal noise dominates over the signal for the expanded MWA at almost all scales and redshifts, while the bispectrum may be observable for large scales $k \lesssim 0.05 \, \text{Mpc}^{-1}$ at $z = 10$. LOFAR has better sensitivity over the signal and the signal will be observable at scales with $k \lesssim 0.1 \, \text{Mpc}^{-1}$ at $z = 10$ and 17. Here it should be noted that the bispectrum signal has several peaks as a function of redshift and they are at $z = 10$ and 17 (Shimabukuro et al. 2011). The peak redshifts depend on the specific values of the model parameters and observation of them will give us information on the process of reionization. Thus, it is expected that LOFAR is enough sensitive to detect the bispectrum at the peak redshifts for large scales.

The isosceles-type bispectrum with $K = 0.06 \, \text{Mpc}^{-1}$ bispectra are plotted in Fig. 2. The behavior and relative amplitudes of the signal and noise are very similar to the case of the equilateral type but SKA is more sensitive at smaller scales.

### 5. SUMMARY AND DISCUSSION

In this paper, we estimated the bispectrum of thermal noise for redshifted 21cm signal observation for Epoch of Reionization by extending the formalism of the noise power spectrum estimation given by McQuinn et al. (2006). Because thermal noise was assumed to be Gaussian, the ensemble average of the bispectrum vanishes and its variance contributes to the noise to the bispectral signal. We developed a formalism to calculate the noise bispectrum for an arbitrary triangle, taking the array configuration into account. We applied it to the cases with equilateral and isosceles triangles and estimated the noise bispectrum for expanded MWA, LOFAR and the SKA. Consequently, it was found that the SKA has enough sensitivity for $k \lesssim 0.3 \, \text{Mpc}^{-1}$ for both types of triangles. On the other hand, LOFAR will have sensitivity for the peaks of the bispectrum as a function of redshift. The expanded MWA has even less sensitivity but it will be possible to put a meaningful constraints on model parameters which induce larger signals than those with the parameters used in this paper.

Not only the thermal noise but signal of bispectrum depend on the configuration of the triangle of three wave numbers. It is possible that the signal bispectrum has a large amplitude for a specific configuration of the triangle and observation may become easier in that case. An investigation of the details of the bispectrum signal and comparison with noise bispectrum will be presented elsewhere (Shimabukuro et al. 2014).

Actually, thermal noise is just one of many obstacles for the observation of 21cm signal. Other serious sources of noise are Galactic and extragalactic foreground and sample variance, and the foreground emission has not been well understood even for power spectrum. However, the observation of the bispectrum is very important because 21cm signal from Epoch of Reionization is highly non-Gaussian so that the bispectrum will give us enormous information complementary to the power spectrum.

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| redshift | 6 | 8 | 10 | 12 | 17 | $N_{\text{station}}$ |
|----------|---|---|----|----|----|-------------------|
| frequency [MHz] | 203 | 158 | 129 | 109 | 79 | |
| $T_{\text{sys}}$ [K] | 250 | 440 | 600 | 1000 | 1900 | |
| $A_e$ [m$^2$] (MWA) | 9 | 14 | 18 | 18 | 18 | 500 |
| $A_e$ [m$^2$] (LOFAR) | 288 | 512 | 600 | 900 | 900 | 24 |
| $A_e$ [m$^2$] (SKA) | 245 | 462 | 725 | 962 | 962 | 866 |

**TABLE 1**

Parameters for telescopes: $T_{\text{sys}}$ is system temperature, $A_e$ is effective area of a station and $N_{\text{station}}$ is the number of stations.
Fig. 1.— Comparison of equilateral-type bispectrum of 21cm signal (dotted line) and thermal noise of MWA (red), LOFAR (blue) and SKA (magenta) at $z = 8, 10, 12$ and $17$.

Fig. 2.— Comparison of isosceles-type bispectrum of 21cm signal (dotted line) and thermal noise of MWA (red), LOFAR (blue) and SKA (magenta) at $z = 8, 10, 12$ and $17$. Here $k$ is fixed to $0.06$ Mpc$^{-1}$. 
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