Hořava-Lifshitz cosmology in light of new data

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Abstract

We present new observational constraints on Lorentz violating Hořava-Lifshitz cosmological scenarios using an updated cosmological data set from Cosmic Microwave Background (Planck CMB), expansion rates of elliptical and lenticular galaxies, JLA compilation (Joint Light-Curve Analysis) data for Type Ia supernovae (SneIa), Baryon Acoustic Oscillations (BAO) and priors on the Hubble parameter with an alternative parametrisation of the equations. Unlike in other approaches we consider the curvature parameter $\Omega_k$ as a free parameter in the analysis we considered the parameters $\Omega_k$ and $\Delta N_\nu$ as completely free, which helped to place new, updated bounds on several of the theory parameters. Remarkably, the detailed balance scenario exhibits positive spatial curvature to more than $3\sigma$, whereas for further theory generalizations we found evidence for positive spatial curvature at $1\sigma$. This could create circumstantial evidence from observations and could be used to single out distinct formulations and scenarios.

Keywords: Lorentz violation, Hořava-Lifshitz

1. Introduction

One of many challenging problems in modern theoretical physics is how to merge quantum field theory with general relativity. Attempts at formulating a quantum theory for gravitation have been made for many decades, but there is still no theory which clearly stands out from its competitors. One of the main obstacles one encounters when trying to quantise general relativity using standard techniques is that it is not a perturbatively renormalisable theory. This is indeed a serious problem as general relativity breaks down...
at small scales, and alternative formulations for a quantum theory of gravity have been developed in order to deal with this issue. For example, string theory and loop quantum gravity have been developed for this reason, and although these theories resolve some problems (such as singularities) there are few phenomenological channels through which to test these theories \[1, 2\]. Although no theory of quantum gravity is known it is possible to predict some of its features. For example, it is likely to contain the three constants $c$ (speed of light, relativity), $G$ (gravitational constant, gravitation), and $\hbar$ (Planck’s constant, quantum mechanics). Using these constants, we can construct a characteristic energy scale for quantum gravity as $E_{Pl} = \sqrt{\hbar c^5/G}$. This is to be interpreted at the “frontier” beyond which we can expect quantum gravity effects to manifest themselves. If this is indeed the case, we can only expect to find quantum gravity effects in regions with very high energy, strong gravity, and high velocities. The phenomenological avenues to probe these regions are sparse, and do not in general allow themselves to be recreated in the laboratory, so we are forced to rely on astrophysical observations. One possibility is that of TeV particle astrophysics, as observed with the next generation of telescopes. There are several scenarios in which quantum gravity effects could affect the opacity of the Universe to such high-energy particles \[3, 4\]. These effects would multiply with the distance, and could in principle be detectable with (for example) the CTA \[5\].

An area of observation which has recently opened up a new window to the Universe is that of gravitational wave astronomy. Recently, LIGO found evidence of a binary black hole merger \[6\], which was consistent with the prediction from general relativity for such events. Moreover, LIGO and VIRGO have also observed a binary neutron star merger \[7\] which was accompanied by a short $\gamma$-ray burst picked up by the FERMI and INTEGRAL $\gamma$-ray telescopes \[8\]. This put strong constraints on the speed of tensorial gravitational waves, as the difference $\Delta t$ from the speed of light $c$ was found to be $-3 \cdot 10^{-15} < \Delta t/c < 7 \cdot 10^{-16}$ \[9\]. As opposed to general relativity, which permits only tensor metric perturbations, many alternative models of gravity have more degrees of freedom. For example, scalar-tensor theories such as Brans-Dicke theory permits a transverse-scalar mode, and Hořava-Lifshitz gravity allows both vector and tensor modes \[10, 11\]. This has also been discussed in the context of Lorentz-violating theories, for example in \[12\] (Hořava gravity) and in \[13\] (Einstein-Æther theory). In general, gravitational wave data places strong constraints on the coupling constants of the theories, as well as the speed of the tensor mode. Moreover, these observa-
tions also place constraints on the Lorentz breaking scale (often denoted $M_*$) in Hořava gravity [12].

Recently, the LIGO and VIRGO team reported the first ever direct limits of the strain of scalar and vector modes at $< 1.5 \cdot 10^{-26}$ at 95% [14, 15]. Prior to this, the magnitude of scalar modes were completely unconstrained, and this new discovery can place tight bounds on alternative theories of gravity, such as Hořava-Lifshitz. This new field of multimessenger astronomy has the potential to teach us more about strong gravity events and will be some of the most important ways of probing new physics in the future.

The fact that general relativity has been tested and found consistent with virtually all observations tells us that it is a very good model for the IR behaviour of quantum gravity. Thus, any candidate quantum gravity model proposed must have general relativity as its low energy limit. It therefore makes sense to look at proposals for UV-completions of general relativity, for example [16, 17]. In general these include a cutoff energy scale beyond which general relativity breaks down. If this energy scale is low enough (in the TeV range) there may be a lot of accessible phenomenology in these models. Another interesting proposal in this category is Hořava gravity, which is a concrete proposal of a UV complete theory of gravity, and possibly of quantum gravity [18, 19]. It is also referred to as Hořava-Lifshitz (HL) gravity since it contains a fixed point in the UV with an anisotropic Lifshitz scaling between space and time, explicitly breaking Lorentz invariance in the UV. A lot of work has gone into this model, including studies on cosmological solutions [20, 21, 22], dark radiation and braneworlds [22, 23], and the resolution of the initial singularity [21, 25, 26]. Several papers have placed bounds on different regions of the Hořava-Lifshitz framework, for example using cosmological data [27], binary pulsars [28, 29], and in the context of dark energy [30]. This has also been done [31] in the effective field theory formalism of the the extended version of Hořava theory [32]. Other authors have placed bounds on more general Lorentz violation in the context of dark matter and dark energy, for example [33, 34].

The breaking of Lorentz invariance mentioned above may seem counterintuitive, as it is one of the fundamental principles of modern physics and is strongly supported by experiment. In fact, every single experiment so far has been consistent with Lorentz invariance [35, 36]. This means that breaking Lorentz symmetry has to be done with great care. However, there is no a priori reason to expect that a theory of quantum gravity should uphold Lorentz invariance, as according to our understanding, spacetime takes on
a quantum nature in the Planck regime, and as we have mentioned before, continuous classical spacetime emerges as a low energy limit of this quantum theory. Then, as Lorentz symmetry is a *continuous symmetry* of nature, it bears to reason that it might not exist at all in the quantum regime, but rather emerge as an IR symmetry of nature. Once Lorentz symmetry is no longer a concern, one may include higher-order derivatives in the Lagrangian in order to cure divergencies in the UV \[37\].

There is an open discussion about instabilities and pathologies in different formulations of HL-type theories (see e.g. \[38,32,39\] or a recent review \[37\]). The original HL theory suffers from many inconsistencies and shortcomings, the most significant of which being the existence of a parity violating term \[38\], problems with ghost instabilities and strong coupling at very low energies \[40,41\], wrong sign and very large value of cosmological constant \[42,43\], problems with power counting renormalisation of the scalar mode propagation \[44,45\]. Some of those issues (ghost instabilities and the sign of the cosmological constant) are discussed in more detail later on in this paper and we discuss alternative solutions to them, like carrying out an analytic continuation of the constant parameters (\[46\]). Other problems arise from the conditions imposed in the original Hořava formulation of the theory; the detailed balance condition, where it is assumed that potential part of the action is given from the superpotential, thus limiting the number of its terms and independent couplings. There is also the projectability condition which assumes that the lapse function \(\mathcal{N}\) depends only on time \(\mathcal{N} = \mathcal{N}(t)\). Some authors (\[41\], end note) claim that the non-projectable version leads to serious strong coupling problem and does not have a viable GR limit at any energy scale. On the other hand more recent considerations, e.g. \[47,42\], claim the opposite. These authors state that most of the shortcomings (like parity violation, problems with renormalisation of the scalar mode and its IR behaviour) can be cured by adding additional terms to the superpotential and relaxing projectability condition, while still keeping detailed balance (or eventually softly breaking it). However later works provide evidence for problems with power-counting renormalizability of the scalar mode (spin-0 graviton) in the detailed balance version of the theory \[44\]. Recent formulation of the so called "mixed-derivative" Hořava gravity \[45,48\] claim that this formulation is actually power counting renormalizable.

There is still an open issue within HL-type theories, namely the large value of the cosmological constant, as there is a descrepancy of at least 60 orders of magnitude between the value demanded by detailed balance \[42\] and
the observed one. It is discussed in [43] that adding the effects of very large vacuum energy of quantum matter fields may cancel out the negative sign and magnitude of the bare cosmological constant leaving the tiny observed residual. There is also problem of a huge predicted value for the quantum vacuum (review in [49]), as there is a large difference between the magnitude of the vacuum energy expected from zero-point fluctuations and scalar potentials and the observed value, and maybe those issues have common solution.

Taking into account various problems and contradicting statements in the works studying different HL-type theories from the analytical side, it is worth to investigate how different formulations fit the observational data. Currently Hořava gravity and its extensions are not ruled out observationally (although the recent binary neutron star merger GW170817 [9] provides tight bounds on some parameters [12]); thus further observational constraints should either rule out its specific scenarios or the whole model, or in case of agreement with observations provide a better justification for deeper theoretical research. It is still believed that HL theory might provide a promising cosmological scenario solving some shortcomings of classical GR, like non-renormalisability. We are actually interested in fitting cosmological scenarios predicted by different version of HL theories to existing observational data. We believe that it is a separate issue from theoretical analysis, as observational constraints may provide valuable input for theoretical analysis.

Studies providing bounds on HL theory constants and parameters based on observational data already exist in the literature. Some are based on the two simplest HL scenarios and quite old cosmological data, like [27] (non-projectable version with and without detailed balance). Others like [31] fit data to the more general extension of Horava theory [32], and also consider linear perturbations around that background. However, this research is based on a flat model (in our case the curvature density parameter is left as a free parameter). As a part of a bigger project we would like to place new bounds on parameters appearing in Hořava-Lifshitz cosmology based on the more recent data. As a first step we would like to fit the data to the two simplest cosmological models. We will look at scenarios based on the original Hořava formulation [18] [19] with detailed balance condition imposed, as well as one with this condition relaxed and with the generalised form of the gravitational action (Sotiriou-Visser-Weinfurtner (SVW) generalisation) [38]. This way we can see the effect of applying newer observational data and different parameterisation on the values of HL cosmological parameters. In the
next step we are going to include bigger sets of data, like PANTHEON SN1a catalogue \[50\] and gamma-ray bursts \[51\] and/or apply it to a more general HL formulation, like \[32\] - the so called "healthy" or "consistent" extension of the original formulation. There are works \[31\] putting constraints on this extension written in the effective field theory framework, but the latter is based on a flat background, limiting the number of parameters. In this work we have focused on a simpler cosmological model and a smaller data set in order to better capture the influence of new data on the model as compared to earlier works. We also used the least number of free parameters and more natural parameterisation of the other ones. By increasing the size of the data sets incrementally we gain a better understanding as to the sensitivity of the model to data with higher resolution and range.

In this paper, being the first one in a planned series, we provide improved observational constraints on two basic HL scenarios, which are based on the most recent cosmological data set from Cosmic Microwave Background (Planck CMB) \[52\], expansion rates of elliptical and lenticular galaxies \[53\], JLA compilation (Joint Light-Curve Analysis) data for Type Ia supernovae (SneIa) \[54\], Baryon Acoustic Oscillations (BAO) \[55, 56, 57\] and priors on the Hubble parameter \[58\]. These updated observational results, together with an alternative parameterisation of the Friedmann equations, provided much tighter constraints on parameters of the HL-type theories, more prone to further observational verification.

The paper is organised as follows: in Section 2 we go through brief review of Hořava-Lifshitz gravity and describe setup for observational constraints in two cases, the original one and the SVW generalisation \[38\]. In Section 3 we present the new bounds on the theory parameters, based on a large updated cosmological dataset, and we conclude in Section 4. The details of our numerical analysis are presented in the Appendix.

2. Hořava-Lifshitz Cosmology

2.1. Basics

As we mentioned previously, one of the main obstacles on the road to quantum gravity is the fact that general relativity is non-renormalisable. This is easy to see if we notice that the expansion of any quantity $\mathcal{F}$ in terms
of the gravitational constant can be written as [37]:

\[ F = \sum_{n=0}^{\infty} a_n \left( G_N E^2 \right)^n, \] (1)

where \( E \) is the energy of the system, \( a_n \) is a numerical coefficient and \( G_N \) is the gravitational coupling constant. From this we see that then \( E^2 \geq G^{-1} \), this expansion diverges, and it is expected that general relativity loses perturbative renormalisability at such high energies.

It is possible to improve the ultraviolet behaviour of general relativity by introducing higher-order derivative terms to the Einstein-Hilbert Lagrangian. For example, we can write it as: \( S = \int d^4x \sqrt{-g} (R + R_{\mu\nu}R^{\mu\nu}) \), which changes the graviton propagator from \( 1/k^2 \) into \( 1/(k^2 - G_N k^4) \). The new term proportional to \( k^{-4} \) will indeed cancel the ultraviolet divergence, but the theory is now non-unitary and equipped with a massive spin-2 ghost [37]. In fact, this ghost comes from the fact that this higher-order gravity theory has time derivatives of \( O > 2 \). In the example above, the resulting field equations are fourth order. In order to address this problem, Hořava decided to evade the ghost by constructing a higher-order theory of gravity where only the spatial derivatives are of \( O > 2 \), while keeping second-order time derivatives only [18]. This requires an explicit breaking of Lorentz invariance in the ultraviolet, but to be consistent with all current experiments, which have so far failed to detect any significant signals of Lorentz invariance violation, this symmetry has to be restored in the infrared limit. The way Hořava overcame this hurdle was to introduce an anisotropic scaling of space and time in the ultraviolet (also known as Lifshitz scaling). This scaling takes the form (in a 4-dimensional spacetime):

\[ t \to b^{-z} t, \; x^i \to b^{-1} x^i, \; (i = 1, 2, 3), \] (2)

where \( z \) is a critical exponent. To restore Lorentz invariance we need to set \( z = 1 \), but to have power-counting renormalisability we need to have \( z \geq 4 \) [37]. The most common choice is to set \( z = 3 \). Thus, ultraviolet Lorentz violation lies at the core of Hořava-Lifshitz gravity and cosmology. Hořava assumed that it is broken down to \( t \to \xi_0(t), \; x^i \to \xi^i(t, x^k) \), which preserves the spatial diffeomorphisms. This theory acquires a symmetry group denoted \( \text{Diff}[M,F] \), which is known as foliation-preserving diffeomorphisms [18, 37]. This allows for arbitrary changes of the spatial coordinates on each constant
time slice, and also picks out a preferred time foliation of spacetime, which breaks Lorentz invariance.

Because of the anisotropic scaling between space and time, it is convenient to introduce the ADM decomposition of the spacetime metric in a preferred foliation:

\[ ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \]

(3)

where the dynamical variables of the theory now are the lapse function \( N \), the shift vector \( N^i \), and the spatial metric \( g_{ij} \) \((i, j = 1, 2, 3)\). Given this, we can write down the most general action for the theory as:

\[ S = \int d^3 x dt N \sqrt{g} \left[ K^{ij} K_{ij} - \lambda K^2 - \mathcal{V}(g_{ij}) \right], \]

(4)

where \( g \) is the determinant of the spatial metric \( g_{ij} \), \( \lambda \) is a running coupling (dimensionless), and \( \mathcal{V} \) is a potential term. Moreover, \( K_{ij} \) is the extrinsic curvature:

\[ K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \]

(5)

where an overdot denotes a full derivative with respect to the time \( t \). The trace of \( K_{ij} \) is denoted \( K \). The potential \( \mathcal{V} \) depends only on the spatial metric and its spatial derivatives, and is also invariant under three-dimensional diffeomorphisms \[32\]. It contains only operators of dimension 4 and 6 which can be constructed from the spatial metric \( g_{ij} \).

### 2.2. Detailed Balance

A way to simplify the action \[4\] is to impose the so-called detailed balance condition, in which we assume that it should be possible to derive \( \mathcal{V} \) from a superpotential \( W \) \[18\] \[39\] \[42\]:

\[ \mathcal{V} = E^{ij} G_{ijkl} E_{kl}, \quad E^{ij} = \frac{1}{\sqrt{g}} \frac{\delta W}{\delta g_{ij}}, \]

(6)

and

\[ G^{ijkl} = \frac{1}{2} (g^{ik} g^{jl} + g^{il} g^{jk}) - \lambda g^{ij} g^{kl}. \]

(7)
From this requirement, the most general action for Hořava-Lifshitz gravity is given by [39]:

\[
S_{\text{db}} = \int dt d^3x \sqrt{g} N \left[ \frac{2}{\kappa^2} (K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2}{2\omega^4} C_{ij}C^{ij} - \frac{\kappa^2\mu}{2\omega^2} \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R_k ight. \\
+ \left. \frac{\kappa^2\mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2\mu^2}{8(1 - 3\lambda)} \left( \frac{1 - 4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right) \right],
\]

(8)

where

\[
C^{ij} = \epsilon^{ikl} \nabla_k \left( R^l_j - \frac{1}{4} R \delta^l_j \right)
\]

(9)
is the Cotton tensor, \( \epsilon^{ikl} \) is the totally antisymmetric tensor, and the parameters \( \kappa, \omega, \) and \( \mu \) have mass dimension \(-1, 0, \) and \( 1, \) respectively. The action (8) for HL gravity has been obtained from the original one [18] by carrying out an analytic continuation (e.g. [46]) of constant parameters: \( \mu \mapsto i\mu \) and \( \omega^2 \mapsto -i\omega^2. \) This enabled to obtain positive values of the cosmological constant \( \Lambda \) (that correspond to current observational results) in the low energy limit of the theory, unlike in the original formulation.

The coupling constant \( \lambda \) is dimensionless. In general, it runs (logarithmically in the high energy limit – in UV) and may eventually reach one the three infrared (IR) fixed points ([18]): \( \lambda = 1/3, \lambda = 1 \) or \( \lambda = \infty. \) The range \( 1 > \lambda > 1/3 \) leads to ghost instabilities in the IR limit of the theory [59]. However, this range of \( \lambda \) is exactly the flow-interval between the UV and IR regimes. The only physically interesting case that remains, allowing for a possible flow towards GR – at \( \lambda = 1 \) – is the regime \( \lambda \geq 1. \) Region \( \lambda \leq 1/3 \) is disconnected from \( \lambda = 1 \) and cannot be included in realistic considerations.

We expect that the action (8) corresponds to the Einstein-Hilbert near the IR limit of the theory. This happens for the speed of light \( c \) and gravitational constant \( G \) expressed in terms of HL parameters as follows:

\[
G = \frac{\kappa^2}{32\pi c}, \quad c = \frac{\kappa^4\mu^2\Lambda}{8(3\lambda - 1)^2}.
\]

(10)

In order to study cosmology in this model it is necessary to populate the Universe with matter and radiation. Since we are interested in the phenomenological bounds on such a theory we will introduce a cosmological energy-momentum tensor into the modified Einstein field equations with the
simple demand that the standard general relativistic expression is recovered in the infrared limit. Therefore, as it is described in one of our previous papers [26], we equip our Universe with a hydrodynamic approximation where \( p_m \) and \( \rho_m \) (pressure and energy density of the dark plus baryonic matter) are parameters subject to the continuity equation \( \dot{\rho}_m + 3H(\rho_m + p_m) = 0 \). We also include a standard model radiation component through \( p_r \) and \( \rho_r \) (note that we already have a cosmological constant \( \Lambda \) in \( S_{db} \)), which are also subject to the evolution equation \( \dot{\rho}_m + 3H(\rho_m + p_m) = 0 \).

Moreover, we use the projectability condition \( N = N(t) \) [18] and the standard FLRW line element: \( g_{ij} = a^2(t)\gamma_{ij}, N_i = 0 \), where \( \gamma_{ij} \) is a maximally symmetric constant curvature metric:

\[
\gamma_{ij} dx^i dx^j = \frac{dr^2}{1 - K r^2} + r^2 d\Omega^2, \tag{11}
\]

values \( K = \{-1, 0, 1\} \) corresponds to closed, flat, and open Universe, respectively. On this background

\[
K_{ij} = \frac{H}{N} g_{ij}, \quad R_{ij} = \frac{2K}{a^2} g_{ij}, \quad C_{ij} = 0, \tag{12}
\]

where \( H \equiv \dot{a}/a \) is the Hubble parameter.

The equations of motion are obtained by varying the action (8) written in the FLRW metrics with respect to \( N \) and \( a \). After that lapse is set to one: \( N = 1 \) (if it was set so at the beginning, standard Friedmann equations would have been obtained) and terms with density \( \rho \) and pressure \( p \) are added). This leads to the Friedmann equations for the projectable Hořava-Lifshitz universe under detailed-balance condition:

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{\kappa^2}{6(3\lambda - 1)} [\rho_m + \rho_r] + \frac{\kappa^2}{6(3\lambda - 1)} \left[ \frac{3\kappa^2 \mu^2 K^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \frac{\kappa^4 \mu^2 \Lambda K}{8(3\lambda - 1)^2 a^2}, \tag{13}
\]

\[
\frac{d}{dt} \left( \frac{\dot{a}}{a} \right) + \frac{3}{2} \left( \frac{\dot{a}}{a} \right)^2 = -\frac{\kappa^2}{4(3\lambda - 1)} [\rho_m + \rho_r] - \frac{\kappa^2}{4(3\lambda - 1)} \left[ \frac{\kappa^2 \mu^2 K^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \frac{\kappa^4 \mu^2 \Lambda K}{16(3\lambda - 1)^2 a^2}. \tag{14}
\]

We can introduce in the above equations the dark energy parameters, namely energy density \( \rho_{DE} \) and pressure density \( p_{DE} \), defined as follows:

\[
\rho_{DE|db} := \frac{3\kappa^2 \mu^2 K^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)}, \tag{15}
\]

\[
p_{DE|db} := \frac{\kappa^2 \mu^2 K^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)}. \tag{16}
\]
2.2.1. Setup for Observational Constraints

Here we describe the theoretical setup which made it possible to constrain the parameters of the theory. Introducing natural units $8\pi G = 1 = c$ and taking the IR limit $\lambda = 1$ reduces relations (10) to the following ones:

$$\kappa^2 = 4, \quad \mu^2 \Lambda = 2.$$  

(17)

Substituting these values to the the Friedmann equation (13) leads to:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3} (\rho_m + \rho_r) + \frac{1}{3} \left( \frac{3K^2}{2\Lambda a^4} + \frac{3\Lambda}{2} \right) - \frac{K}{a^2},$$

(18)

We also define in the IR limit the canonical density parameters for the current universe $a_0 = 1$ (subscript 0 indicates the value as measured today) as follows

$$\Omega_m^0 = \frac{\rho_m}{3H_0^2}, \quad \Omega_r^0 = \frac{\rho_r}{3H_0^2}, \quad \Omega_k^0 = -\frac{K}{H_0^2 a_0^2},$$

(19)

where $H_0$ is the Hubble parameter. Using these parameter we rewrite equation (18) as follows:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[ \Omega_m^0 a^{-3} + \Omega_r^0 a^{-4} + \Omega_k^0 a^{-2} + \frac{(\Omega_k^0)^2 H_0^2}{2\Lambda} a^{-4} + \frac{\Lambda}{2H_0^2} \right],$$

(20)

where the last two term correspond to dark energy density parameter obtained from equations (15) and (19):

$$\Omega_{DE} = \frac{\rho_{DE}}{3H_0^2} = \frac{(\Omega_k^0)^2 H_0^2}{2\Lambda} a^{-4} + \frac{\Lambda}{2H_0^2}.$$  

(21)

In the above Friedmann equation (20) we encounter the term $\Omega_k^0 H_0^2/2\Lambda a^4$, which is the coefficient of dark radiation in Hořava-Lifshitz. We can conveniently express this in terms of the effective number of neutrino species present in the BBN épouque. This is because the dark radiation must have been present during that time and is thus subject to BBN constraints from other experiments. As such, we can obtain a constraint equation for the dark radiation component [60, 61]:

$$\frac{(\Omega_k^0)^2 H_0^2}{2\Lambda} = 0.135 \Delta N_\nu \Omega_r^0,$$

(22)
where $\Delta N_\nu$ is the deviation of the effective number of massless neutrino species from the standard $\Lambda$CDM value ($N_\nu = 3 + \Delta N_\nu$). There are already several bounds on $\Delta N_\nu$ from different experiments. The limits from [62, 61]:

$-1.7 \leq \Delta N_\nu \leq 2.0$ originate from a BBN analysis, and in [63], the authors use data from BBN and CMB (Planck) to arrive at an upper limit of $\Delta N_\nu < 0.2$. Moreover, using Planck CMB data, the authors of [64] arrive at the limits $-0.32 \leq \Delta N_\nu \leq 0.44$ (TTTEEE+lensing). Other approaches and bounds can be found in [65, 66], for example. Through the relation (22) we see that $\Delta N_\nu = 0$ leads to the possibility of a flat Universe. This is concerning, since it has been established that a flat Hořava-Lifshitz coincide with the standard $\Lambda$CDM model [22, 21]. Therefore the constraints on the curvature parameter $\Omega_k$ and the BBN neutrino parameter $\Delta N_\nu$ are of utmost importance, and they have been left as free parameters in our analysis. That is, we have not imposed the BBN limit on $\Delta N_\nu$, but the limits we found on this parameter automatically satisfied the BBN constraint. Now, since all the density parameters have to add up to unity we have:

$$
\Omega_m^0 + \Omega_r^0 + \Omega_k^0 + \frac{(\Omega_k^0)^2}{4 \cdot 0.135\Delta N_\nu \Omega_r^0} + 0.135\Delta N_\nu \Omega_r^0 = 1, \quad (23)
$$

and we can rewrite the Friedmann equation (20) as:

$$
\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[ \Omega_m^0 a^{-3} + \Omega_r^0 a^{-4} + \Omega_k^0 a^{-2} + \frac{(\Omega_k^0)^2}{4 \cdot 0.135\Delta N_\nu \Omega_r^0} + 0.135\Delta N_\nu \Omega_r^0 a^{-4} \right],
$$

and this is the equation that we have used in our MCMC analysis of detailed balance.

2.3. Beyond Detailed Balance

Since there has been a discussion in the literature about whether detailed balance is too restrictive ([22, 21, 37]) it is important to investigate the case when we do not impose this condition. When we relax the detailed balance condition we open up for including more terms into the potential $V$. In this case, the Friedmann equations can be expressed as [41, 39]:

$$
\left(\frac{\dot{a}}{a}\right)^2 = \frac{2\sigma_0}{3\lambda - 1}(\rho_m + \rho_r) + \frac{2}{3\lambda - 1} \left[ \frac{\Lambda}{2} + \frac{\sigma_3 K^2}{6a^4} + \frac{\sigma_4 K}{6a^6} \right] + \frac{\sigma_2 K}{3(3\lambda - 1) a^2}, \quad (25)
$$

$$
\frac{d}{dt} \left(\frac{\dot{a}}{a}\right) + \frac{3}{2} \left(\frac{\dot{a}}{a}\right)^2 = - \frac{3\sigma_0}{3\lambda - 1} \rho_r - \frac{3}{3\lambda - 1} \left[ - \frac{\Lambda}{2} + \frac{\sigma_3 K^2}{18a^4} + \frac{\sigma_4 K}{6a^6} \right] + \frac{\sigma_2 K}{6(3\lambda - 1) a^2}, \quad (26)
$$
where \( \sigma_i \) are arbitrary constants \( (\sigma_2 < 0) \) and in analogy with the detailed balance scenario:

\[
G = \frac{6\sigma_0}{16\pi}, \quad \frac{\sigma_2}{3(3\lambda - 1)} = -1. \tag{27}
\]

Dark energy and pressure densities may be defined as follows:

\[
\rho_{DE}|_{bdbh} := 3 \left( \frac{\Lambda}{2} + \frac{\sigma_3 K^2}{6a^4} + \frac{\sigma_4 K}{6a^6} \right), \tag{28}
\]

\[
p_{DE}|_{bdbh} := 3 \left( -\frac{\Lambda}{2} + \frac{\sigma_3 K^2}{6a^4} + \frac{\sigma_4 K}{6a^6} \right). \tag{29}
\]

### 2.3.1. Setup for Observational Constraints

Here, we follow the setup for the detailed balance scenario, with \( 8\pi G = 1 = c \), and the same definition of the density parameters \( (\Omega_X^0) \). Following the notation in [27] we introduce similar parameters in the IR limit \( \lambda = 1 \):

\[
\omega_1 = \frac{\Lambda}{2H_0^2}, \quad \omega_3 = \frac{\sigma_3 H_0^2 \Omega_k^0}{6}, \quad \omega_4 = -\frac{\sigma_4 \Omega_k^0}{6}, \tag{30}
\]

and introducing the following Bing Bang nucleosynthesis (BBN) constraint [60]:

\[
\omega_3 + \omega_4 (1 + z_{BBN})^2 = 0.135 \Delta N_\nu \Omega_r^0, \tag{31}
\]

where \( z_{BBN} \) is the redshift at BBN \( (\sim 4 \times 10^8) \). Moreover, we have the following constraint on the density parameters:

\[
\Omega_m^0 + \Omega_r^0 + \Omega_k^0 + \omega_1 + \omega_3 + \omega_4 = 1. \tag{32}
\]

Using the above two equations to eliminate \( \omega_4 \) along with the \( \sigma \)-parameters, we can rewrite the Friedmann equation (25) for the beyond detailed balance scenario as:

\[
\left( \frac{\dot{a}}{a} \right)^2 = H_0^2 \left[ \Omega_m^0 a^{-3} + \left( \Omega_r^0 + \omega_3 \right) a^{-4} + \Omega_k^0 a^{-2} + \omega_1 + \frac{0.135 \Delta N_\nu \Omega_r^0}{(1 + z_{BBN})^2} a^{-6} \right]. \tag{33}
\]

This is the equation we have used in our MCMC analysis of the beyond detailed balance scenario. Additionally, the dark energy density parameter can be written as:

\[
\Omega_{DE}^0 = \omega_1 + \omega_3 + \omega_4 = \omega_1 + \omega_3 + \frac{0.135 \Delta N_\nu \Omega_r^0 - \omega_3}{(1 + z_{BBN})^2}. \tag{34}
\]
3. Results and Discussion

Using the equations (13) and (25) as a starting point, we now want to find the parameter set which best fits the data. To do this we use a large updated data set with CMB (Planck), SN1a, BAO and more (see Appendix for details). In order to do this, we used a Markov-Chain Monte Carlo (MCMC) method, which was evaluated on the CIŚ computer cluster. The parameters are completely unconstrained but are given initial guesses, which speed up computation. We introduced a Gaussian prior on one parameter: \( H_0 \), derived from the Hubble constant value in [58], \( H_0 = (69.6 \pm 0.7) \) km s\(^{-1}\) Mpc\(^{-1}\). That was the only prior imposed on any of the parameters. During every step in the computation, the MCMC method calculates the \( \chi^2 \), and in the end returns the parameter set which minimised the \( \chi^2 \) function. This way, we are able to obtain information about the posterior probability distribution without knowing it explicitly. For full details about the data and method, see Appendix.

3.1. Detailed Balance

To derive the constraints on the detailed balance scenario, we used Eq. (24) and (24) in our MCMC method. As the detailed balance equations are relatively simple there was no need to introduce many constraints, and the parameters \( \Delta N_\nu \) has been left completely free in this section. One interesting result is that we find significant evidence for positive spatial curvature at 3\( \sigma \) confidence (see Figure 1). This is an encouraging prospective observational signal, since any measurement of positive spatial curvature would be a step towards validating this model. Parameters with 1\( \sigma \) limits are presented in Table 1.

In our analysis, the parameters \( H_0, \Omega_m^0, \Omega_r^0 \) all have familiar and reasonable bounds. The parameters which stand out are \( \Omega_k^0, \Delta N_\nu \) and \( \Lambda \). Our initial simulations revealed that the spatial curvature is actually significantly different from zero, and, instead of expressing the parameters \( \omega \) in terms of the above parameters as in [27], we have chosen to leave it as \( \omega = \Lambda/2H_0^2 \). Therefore there was no need to split the analysis in \( \Omega_k^0 > 0 \) and \( \Omega_k^0 < 0 \). Through Eq. (22), a non-zero spatial curvature automatically leads to a non-zero \( \Delta N_\nu \). Previous works, like [27], have also used \( \Delta N_\nu \) as a free parameter, but with this updated data set and different parametrisation we arrive at different results and parameter bounds (compared, for example, to [27]). In this
scenario the value of $\Lambda \sim 0.676 \cdot 10^{-35} \text{ s}^{-2} \sim 0.836 \cdot 10^{-52} \text{ m}^{-2}$ and the cosmological constant in the $\Lambda$CDM model is of the same order $\Lambda_{\text{CDM}} \sim 1.11 \cdot 10^{-52} \text{ m}^{-2}$ \cite{52}. There is a marginal difference but the overall magnitude is still much smaller than the lower bound on $\Lambda_{\text{HL}}$ estimated by the energy scales at which Lorentz invariance breaking remains undetected in sub-mm precision tests. Authors of \cite{10,47,42} provide a rough estimation for the lower bound of $\Lambda_{\text{HL}}$ to be somewhere in the range from 1–100 meV$^2$ (at least) in natural units, which converts to $10^7$–$10^9$ m$^{-2}$ in metric units. The issue of the huge bare value of the cosmological constant demanded by the need for suppression at IR limit of fourth-order operators appearing in the HL action is discussed in the literature, eg. \cite{42,43} and references therein, that is why the obtained numerical value should be viewed from that perspective. Additionally, through $\mu^2 \Lambda = 2$, $\mu$ acquires finite values, unlike the results in \cite{27}, where $\Lambda$ is bound by zero from below, and $\mu$ is unbounded from above.

Moreover, using Eq. (21) we arrive to the bounds $\Omega^0_{\text{DE}} = 0.686 \pm 0.015$ at 3$\sigma$. This is very close to that from Planck 2015 \cite{52} where the tabled value is $\Omega^0_{\text{DE}} = 0.6911 \pm 0.0062$ at 1$\sigma$ and to the result from \cite{31}. This is interesting as the authors of the latter use effective field theory and a “healthy extension” of HL theory \cite{32} to constrain Hořava-Lifshitz cosmology against data. They arrive at the value $\Omega^0_{\text{DE}} = 0.69 \pm 0.02$ at 3$\sigma$, which is in close agreement to our findings, although they used not only background Friedmann equation as we do, but also considered linear perturbations on this background. We also use a different HL model, so our background equation (20) is slightly different from their Friedmann equation (46). Except different parameters of the theory, here contains also curvature term, which arrived to be non-zero, but it does not include massive neutrinos explicitly, as it is contained in eq. (46) of \cite{20}. In contrast, our results differ from those in \cite{27} which uses older data, and this strengthens the case that our results are more accurate, as we use similar background equation.

3.2. Beyond Detailed Balance

To obtain constraints on parameters in the beyond detailed balance scenario we used equations (31), (32), and (33) in the same MCMC method as for detailed balance. Here, we introduced another constraint equation in order to make sure that the Hubble parameter is always completely real. From
Figure 1: 1, 2, and 3σ contours of the curvature parameter $\Omega_0^k$ and the dimensionless Hubble parameter $h$ under detailed balance. Solid (blue) colour corresponds to the 1σ limit. Spatial flatness ($\Omega_0^k = 0$) is excluded at more than 3σ.

Using the equations and constraints mentioned above we carried out an MCMC analysis of the beyond detailed balance model. All the results are shown in Table 1. Here, it is interesting to note that in this scenario we find evidence for positive spatial curvature at 1σ: $\Omega_k^0 = -0.0033 \pm 0.0020$. The contours showing this is plotted against $h$ in Figure 3. Here, the marginalised probability for $\Omega_k^0$ in detailed balance is completely contained inside the beyond detailed balance scenario, away from the best-fit value. This can be seen in Figure 2.
Moreover, the parameter $\sigma_3 H_0^2$ seems to acquire a log-normal distribution, and the two-sided confidence intervals for this parameter must be considered unreliable. Because of the log-normal distribution, however, it was possible to find limits on $\log \sigma_3 H_0^2$, which is shown in Figure 4. The errors on this parameter are very large, which is an artefact of the log-normal distribution which has long tails on both ends. We express the two $\sigma$ parameters as $\sigma_1/H_0^2$ and $\sigma_3 H_0^2$ (see Eq. (30)) as there is no need to specify units or value for $H_0$. It is also worth noting that it is possible to differentiate between the two scenarios by looking at the neutrino species parameter $\Delta N_\nu$. As can be seen in Figure 5, only some of the marginalised probability of the detailed balance model is contained within the range of beyond detailed balance, and a large part of the curve lies outside. Thus, $\Delta N_\nu$ is a promising parameter to help differentiate between the two scenarios. Remarkably, the lower bound of $\Delta N_\nu$...
Figure 2: Marginalised constraints of the curvature parameter $\Omega^0_\kappa$. The dashed (blue) curve represents the detailed balance scenario while the solid (red) curve shows beyond detailed balance. The detailed balance scenario is completely contained within beyond detailed balance.

from Table 1 is in conflict with the bound ($\Delta N_\nu < 0.2$) derived from BBN and Planck CMB (mentioned in Section 2.2.1). In fact, looking at the 2$\sigma$ and 3$\sigma$ bounds on $\Delta N_\nu$ (0.21, 0.17) we see that our analysis yields a significantly different result at more than 2$\sigma$ confidence.

Using Eq. (34) we find the bounds $\Omega^0_{DE} = 0.678^{+0.017}_{-0.019}$ at 3$\sigma$ confidence level, which is close to the value from Planck 2015 ($\Omega^0_{DE} = 0.6911 \pm 0.0062$ at 1$\sigma$), but not as close as for the detailed balance scenario. This is mainly due to different estimated values of $\Lambda/2H_0^2$, makes up the main part of $\Omega^0_{DE}$ in both scenarios. Beyond detailed balance parameters at 3$\sigma$ are $\omega_1 = 0.679^{+0.017}_{-0.019}$ (the leading term in $\Omega^0_{DE}$), $\omega_3 = (2.05^{+4.38}_{-2.04}) \cdot 10^{-6}$ and $\omega_4 (1 + z_{BBN})^2 = (4.18 \cdot 10^{-6})^{+0.0056}_{-0.0064}$ (the errors on $\omega_3$ and $\omega_4$ are very large, so only order is relevant here).

4. Conclusions

Using an updated cosmological data set we calculated new observational constraints on cosmological parameters of different formulations of Hořava-
Figure 3: 1, 2, and 3σ contours of the curvature parameter $\Omega_k^0$ and the dimensionless Hubble parameter $h$ in the beyond detailed balance scenario. Solid (blue) colour corresponds to the 1σ limit. Spatial flatness is excluded at 1σ.

Lifshitz gravity. Unlike in the previous approaches [27] we use a different parametrisation of our parameters, the most important one is treating $\Omega_k^0$ (together with $\Delta N_{\nu}$) as completely free parameter. This allowed us to avoid the splitting of considerations and calculations into two separate cases of negative and positive curvature parameters that was used before.

We investigated two basic Hořava-Lifshitz scenarios, the one with detailed balance condition imposed in the action and the other one, the so called Sotiriou-Visser-Weinfurtner (SVW) generalization [38], which relaxes this condition and assumes higher order, i.e. cubic terms in the action. We found very remarkable results, namely detailed balance scenario exhibits positive spatial curvature to more than 3σ whereas for the SVW generalization there is evidence for positive spatial curvature at 1σ, which could be a smoking gun for observations. Some of our parameters differ quite a lot from previous works, but as we have some different assumptions (for example, in [27] the parameter $\omega_3$ is assumed to be positive for convenience), different values are to be expected.

The best fit value of the parameter $\Delta N_{\nu}$ (kept as a completely free parameter of the theory) at 1σ is definitely positive and within BBN limits, as
Figure 4: Marginalised constraints of the parameter $\log(\sigma_3 H_0^2)$. The slightly elongated tail on the right hand side shifts the mean away from 0.

well as within some of the bounds from CMB. Since there is a variety of papers with different bounds on $\Delta N_\nu$ available, it is unclear which one should compare to. In general, more data is needed to make a final verdict on this issue, and we aim to address this in future work as it becomes available. Our analysis, however, excludes from both scenarios the zero-curvature flat universe and corresponds to the non-zero positive cosmological constant $\Lambda$.

Most values of the parameters of the theory are similar as in $\Lambda$CDM model. For example, our values for $\Omega_m = 0.316 \pm 0.0054(0.324 \pm 0.0068)$ for detailed balance (beyond detailed balance) are both within 1$\sigma$ of $\Lambda$CDM [52]. We are also close to the dark energy density parameter, as discussed in Section 3.2. However, what sets the HL model apart is the non-zero curvature parameter $\Omega_k$. We found it to be different from zero to 1$\sigma$ (3$\sigma$). According to Planck 2015 (when including BAO’s) this parameter is a Gaussian centered around zero [52]. As our analysis also includes BAO’s, further investigation of this parameter may be the way to finally exclude one of these models. Naively, one must still prefer the $\Lambda$CDM model, as it has fewer parameters than HL and fits the data well. However, bearing in mind the theoretical basis of HL cosmology (candidate for a UV-complete theory of gravity) there are, in our opinion, plenty of reasons to keep investigating this model.
Figure 5: Marginalised constraint of the parameter $\Delta N_{\nu}$. The dashed (blue) curve represents the detailed balance scenario, and the solid (red) shows beyond detailed balance. The two scenarios only shows a small amount of overlap.

It is also worth mentioning that different experiments measuring the Hubble constant $H_0$ provide different values (the so-called $H_0$ tension). Our results: $68.5 \pm 0.4$ and $69.6 \pm 0.6$ km s$^{-1}$ Mpc$^{-1}$ for detailed balance and beyond detailed balance, respectively, lie close to the Planck 2018 result $67.27 \pm 0.6$ km s$^{-1}$ Mpc$^{-1}$ in mean, but our 1$\sigma$ confidence levels overlap with those from e.g. strong gravitational lensing, $71.9^{+2.4}_{-3}$ [67] and lies close to those of cosmic ladder experiments like the Sh0es project, where the Hubble constant was found to be $73.8 \pm 2.4$ km s$^{-1}$ Mpc$^{-1}$ [68]. However, this experiment used a Gaussian prior on $H_0$ which we have also done, so this agreement might be an artifact of this choice. In short, $H_0$ is not a good parameter for differentiating between HL models or to cast any light on the $H_0$ tension.

In this paper we were mostly interested in the IR regime of the HL theory and wanted to know if the values of cosmological observables predicted within the considered models fit observational data, and also whether they may be used to distinguish HL cosmology from GR, even in the low-energy limit. Therefore we set the value of the running coupling constant $\lambda$ to its IR value: $\lambda = 1$. There are works [69] where authors estimate the value of $\lambda$ using
similar low-energy data sources (however, our data is more recent), providing that this value is restricted to $|\lambda - 1| \leq 0.02$ at 1\sigma. At the moment we are performing calculations within more general formulation of the theory and wider observational data, with more high-energy sources, where we also investigate the impact of running of $\lambda$ on other results.

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**Appendix: The Method**

In order to estimate the values of the parameters present in Hořava-Lifshitz we used a large updated cosmological data set. The data used includes expansion rates of elliptical and lenticular galaxies, Type Ia Supernovae, Baryon Acoustic Oscillations, Cosmic Microwave Background and priors on the Hubble parameter. We used the parallelised Markov-Chain Monte Carlo (MCMC) code developed in Mathematica and used in many papers, for example. There are many cosmological MCMC codes readily available, for example CosmoMC and Monte Python, but the advantages of our code of choice is that it is easy to add new data, and it is also simple to modify. Things such as the cosmological model, statistical method, and parameters used can easily be changed. Moreover, thanks to the flexible way in which the code is written, introducing more exotic models with varying $c$ or $G$ is also straightforward. Even though Mathematica code is generally slower than C or FORTRAN, for example, the simplicity and transparency of the code makes up for this small drawback.

All expressions below are written in the flat case ($\Omega_k^0 = 0$) for simplicity. However, $\Omega_k^0$ is left as a free parameter in the MCMC analysis, and thus all equations extend to the more general case of $\Omega_k^0$.
With this in mind, the expression for the comoving distance is:

\[
D_M(z) = \begin{cases} 
\frac{D_H}{\sqrt{\Omega_k}} \sinh \left( \sqrt{\Omega_k} \frac{D_C(z)}{D_H} \right) & \text{for } \Omega_k^0 > 0 \\
D_C(z) & \text{for } \Omega_k^0 = 0 \\
\frac{D_H}{\sqrt{|\Omega_k|}} \sin \left( \sqrt{|\Omega_k|} \frac{D_C(z)}{D_H} \right) & \text{for } \Omega_k^0 < 0 ,
\end{cases}
\] (36)

where \( D_H = c_0/H_0 \) is the Hubble distance, \( D_C(z) = D_H \int_0^z dz' / \mathcal{E}(z') \) is the line-of-sight comoving distance, and \( \mathcal{E}(z) = H(z)/H_0 \). Also, luminosity distance \( (D_L(z)) \) and angular diameter distance \( (D_A(z)) \) are given by:

\[
D_L(z) = (1 + z)D_M(z) ,
\] (37)

\[
D_A(z) = \frac{D_M(z)}{1 + z} .
\] (38)

**Hubble data**

For Hubble parameter data, we use the compilation from [53], which is derived from the evolution of elliptical and lenticular galaxies at redshifts \( 0 < z < 1.97 \). The expression for \( \chi^2_H \) is:

\[
\chi^2_H = \sum_{i=1}^{24} \frac{(H(z_i, \theta) - H_{\text{obs}}(z_i))^2}{\sigma_{H_i}^2} ,
\] (39)

where \( \theta \) is a vector containing the parameters of the model, \( H_{\text{obs}}(z_i) \) are the measured values of the Hubble constant and \( \sigma_H(z_i) \) are the corresponding observational errors. We will also add a prior on the Hubble constant from [58], \( H_0 = 69.6 \pm 0.7 \) km s\(^{-1}\) Mpc\(^{-1}\).

**Type Ia Supernovae**

We made use of the updated JLA compilation (Joint Light-Curve Analysis) data for Type Ia supernovae (SneIa) [54] at redshifts \( 0 < z < 1.39 \). In this case, the \( \chi^2_{SN} \) is:

\[
\chi^2_{SN} = \Delta \mu \cdot C_{SN}^{-1} \cdot \Delta \mu ,
\] (40)

where \( \Delta \mu = \mu_{\text{theo}} - \mu_{\text{obs}} \) is the difference between theoretical and observational values of the distance modulus \( \mu \). \( C_{SN} \) is the total covariance matrix. The distance modulus is written as:

\[
\mu(z, \theta) = 5 \log_{10}[D_L(z, \theta)] - \alpha X_1 + \beta C + M_B .
\] (41)
Here, $X_1$ parametrises the shape of the supernova light-curve, $C$ is the colour, and $\mathcal{M}_B$ is a nuisance parameter \[54\], which together with the weighting parameters $\alpha$ and $\beta$ are included in $\theta$. $D_L$ is the luminosity distance which we write as:

$$D_L(z, \theta) = \frac{1 + z}{H_0} \int_0^z \frac{dz'}{dE(z', \theta)}.$$  \hspace{1cm} (42)

Here, and only in this Supernova analysis, we do specify $H_0 = 70 \text{ km/s Mpc}^{-1} \[54\].

**Baryon Acoustic Oscillations**

The total $\chi^2$ for Baryon Acoustic Oscillations is given by:

$$\chi^2_{BAO} = \Delta \mathcal{F}^{BAO} \cdot C_{BAO}^{-1} \cdot \Delta \mathcal{F}^{BAO},$$  \hspace{1cm} (43)

where $\mathcal{F}^{BAO}$ is different from survey to survey. In this work, we used the WigleZ Dark Energy Survey with redshifts $z = \{0.44, 0.6, 0.73\} \[55\]. For this analysis the acoustic parameter and the Alcock-Paczynski distortion parameter are of interest. The acoustic parameter is defined as follows:

$$A(z, \theta) = 100 \sqrt{\Omega_m h^2 D_V(z, \theta)} z,$$  \hspace{1cm} (44)

and the Alcock-Paczynski parameter is:

$$F(z, \theta) = (1 + z) D_A(z, \theta) H(z, \theta),$$  \hspace{1cm} (45)

where $D_A$ is the angular diameter distance, which is Eq. (38) in the case of $\Omega_k^0 = 0$:

$$D_A(z, \theta) = \frac{1}{H_0} \frac{1}{1 + z} \int_0^z \frac{dz'}{dE(z', \theta)},$$  \hspace{1cm} (46)

and $D_V$ is the geometric mean of the physical angular diameter distance $D_A$ and the Hubble function $H(z)$. It reads as:

$$D_V(z, \theta) = \left[ (1 + z)^2 D_A^2(z, \theta) \frac{z}{H(z, \theta)} \right]^{1/3}. $$  \hspace{1cm} (47)

Included in the Baryon Acoustic Oscillation analysis is also data from Sloan Digital Sky Survey (SDSS-III) Baryon Oscillation Spectroscopic Survey (BOSS) DR12 \[56\]. It can be written as:

$$D_M(z) \frac{r^s_{mod}(z_d)}{r^s(z_d)} \quad \text{and} \quad H(z) \frac{r^s_{mod}(z_d)}{r^s(z_d)} $$  \hspace{1cm} (48)
Here, $r_s(z_d)$ represents the sound horizon at the \textit{dragging redshift} $z_d$. $r_s^{\text{mod}}(z_d)$ is the same horizon, but evaluated for the given cosmological model. Here, it is used that $r_s^{\text{mod}}(z_d) = 147.78$ Mpc as in \cite{56}. A good approximation of the sound horizon can be found in \cite{75}:

$$
z_d = \frac{1291(\Omega_m^0 h^2)^{0.251}}{1 + 0.659(\Omega_m^0 h^2)^{0.828}} \left[ 1 + b_1(\Omega_b^0 h^2)^{b_2} \right], \quad (49)
$$

where

$$
b_1 = 0.313(\Omega_m^0 h^2)^{-0.419} \left[ 1 + 0.607(\Omega_m^0 h^2)^{0.6748} \right],
$$

$$
b_2 = 0.238(\Omega_m^0 h^2)^{0.223}. \quad (50)
$$

The sound horizon $r_s$ can then be defined as:

$$
r_s(z, \theta) = \int_z^\infty \frac{c_s(z')}{H(z', \theta)} \, dz', \quad (51)
$$

and the sound speed is given by:

$$
c_s(z) = \frac{1}{\sqrt{3(1 + \bar{R}_b (1 + z)^{-1})}}, \quad (52)
$$

and

$$
\bar{R}_b = 31500 \Omega_b^0 h^2 \left( T_{CMB}/2.7 \right)^{-4}, \quad (53)
$$

with $T_{CMB} = 2.726$ K.

Ending the Baryon Acoustic Oscillation analysis, considered data from the Quasar-Lyman $\alpha$ Forest from Sloan Digital Sky Survey - Baryon Oscillation Spectroscopic Survey DR11 \cite{57}:

$$
\frac{D_A(z = 2.36)}{r_s(z_d)} = 10.8 \pm 0.4, \quad (54)
$$

$$
\frac{1}{H(z = 2.36) r_s(z_d)} = 9.0 \pm 0.3. \quad (55)
$$

With the contributions mentioned throughout this section, the total $\chi^2$ for Baryon Acoustic Oscillations will be $\chi^2_{BAO} = \chi^2_{\text{WiggleZ}} + \chi^2_{\text{BOSS}} + \chi^2_{\text{Lyman}}$. 

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Cosmic Microwave Background

In this analysis, we write the \( \chi^2 \) for the Cosmic Microwave Background (CMB) in the following way:

\[
\chi^2_{\text{CMB}} = \Delta \mathcal{F}^{\text{CMB}} \cdot C^{-1}_{\text{CMB}} \cdot \Delta \mathcal{F}^{\text{CMB}}.
\]

Here, \( \mathcal{F}^{\text{CMB}} \) is a vector quantity given in [76], which summarises the information available in the full power spectrum of the Cosmic Microwave Background from the 2015 Planck data release [52]. \( \mathcal{F}^{\text{CMB}} \) contains the Cosmic Microwave Background shift parameters and the baryonic density parameter. The shift parameters are:

\[
R(\theta) \equiv \sqrt{\Omega_m^0 H_0^2 r(z_*, \theta)}
\]

\[
l_a(\theta) \equiv \pi \frac{r(z_*, \theta)}{r_s(z_*, \theta)} ,
\]

whereas the baryonic density parameter is simply \( \Omega_b^0 h^2 \). As mentioned before, \( r_s \) is the comoving sound horizon at the photon-decoupling redshift \( z_* \), which is given by [77]:

\[
z_* = 1048 \left[ 1 + 0.00124(\Omega_b^0 h^2)^{-0.738} \right] \left( 1 + g_1(\Omega_m^0 h^2)^{g_2} \right) ,
\]

with:

\[
g_1 = \frac{0.0783(\Omega_b^0 h^2)^{-0.238}}{1 + 39.5(\Omega_b^0 h^2)^{-0.763}} ,
\]

\[
g_2 = \frac{0.560}{1 + 21.1(\Omega_b^0 h^2)^{1.81}} ; \]

and \( r \) is the comoving distance:

\[
r(z, \theta_b) = \frac{1}{H_0} \int_0^z \frac{dz'}{\mathcal{E}(z', \theta)} dz' .
\]

All the methods mentioned above all contribute to the total \( \chi^2 \), and the function to minimise is: \( \chi^2_{\text{tot}} = \chi^2_{\text{H0}} + \chi^2_{H} + \chi^2_{\text{SN}} + \chi^2_{\text{WiggleZ}} + \chi^2_{\text{BOSS}} + \chi^2_{\text{Lyman}} + \chi^2_{\text{CMB}} \).
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