Power-law entropy-corrected Ricci dark energy and dynamics of scalar fields

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Abstract

Motivated by the holographic principle, it has previously been suggested that the dark energy (DE) density can be inversely proportional to the area $A$ of the event horizon of the Universe. However, this kind of model would have a causality problem. In this work, we study the power-law entropy-corrected holographic DE (PLECHDE) model in the non-flat Friedmann–Robertson–Walker universe, with the future event horizon replaced by the average radius of the Ricci scalar curvature. We derive the equation of state parameter $\omega$, the deceleration parameter $q$ and the evolution of energy density parameter $\Omega_{D}$ in the presence of interaction between DE and dark matter. We consider the correspondence between our Ricci-PLECHDE model and the modified Chaplygin gas and the tachyon, K-essence, dilaton and quintessence scalar fields. The potential and dynamics of the scalar field models have been reconstructed according to the evolutionary behaviour of the interacting entropy-corrected holographic DE model.

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1. Introduction

Cosmological observations such as type Ia supernovae, the cosmic microwave background (CMB) radiation anisotropies, the large-scale structure and x-ray experiments support the evidence for an accelerated expansion of our Universe [1]. A missing energy component with negative pressure is considered by astrophysicists and cosmologists as responsible for this accelerated expansion. This missing component is also known as dark energy (DE). A recent analysis of cosmological observations indicates that two-thirds of the total energy of the Universe is occupied by DE, whereas dark matter (DM) occupies most of the remaining part (the baryonic matter we observe represents only a few per cent of the total mass of the Universe) [2]. The contribution of the radiation is negligible.

The nature of DE is still unknown and many candidates have been proposed to describe it [3]. The simplest candidate for DE is a tiny positive cosmological constant, with a negative constant equation of state (EoS) parameter $\omega$, i.e. $\omega = -1$. However, cosmologists know that the cosmological constant suffers from two well-known difficulties, the fine-tuning and the cosmic coincidence problems: the former asks why the vacuum energy density is so small (of the order of $10^{-123}$ smaller than what we observe) and the latter says why vacuum energy and DM are nearly equal today (which represents an incredible coincidence if internal connections between them do not exist) [4].

As a possible alternative to the cosmological constant, dynamical scalar field models have been proposed, among which are quintessence [5], phantom [6], f-essence [7] and K-essence [8].

An important advance in the studies of black hole theory and string theory is the suggestion of the so-called holographic principle which was proposed by Fischler and Susskind in 1998 [9]. According to the holographic principle, the number of degrees of freedom of a physical system should be finite.
and should scale with its bounding area rather than with its volume [10], and it should be constrained by an infrared (IR) cut-off [11]. The holographic DE (HDE), based on the holographic principle, is one of the most widely studied models of DE [12]. HDE models have also been constrained and tested by various astronomical observations [13] and by the anthropic principle [14].

Applying the holographic principle to cosmology, the upper bound of the entropy contained in the Universe can be obtained. Following this line, Li [15] suggested as a constraint on the energy density of the Universe $\rho_{\Lambda} \leq 3\gamma M_P^2 L^{-2}$, where $\gamma$ is a numerical constant, $L$ is the IR cut-off radius and $M_P = (8\pi G)^{-1/2} \approx 10^{19}$ GeV is the reduced Planck mass. The equality sign holds when the holographic bound is saturated.

The holographic principle, is one of the most widely studied holographic models of DE [20], which studied the casual entropy bound in the holographic framework, the Ricci model gets an appropriate reason for which it could be motivated, providing an appropriate physical motivation for the holographic Ricci DE (RDE).

RDE has been widely studied in the literature in various different ways: the state-finder diagnostic of RDE [21], reconstruction of $f (R)$ [22], quintom [23], contributions of viscosity to RDE [24] and related observational constraints [25].

Replacing $L$ with $R^{-1/2}$ in equation (3), we obtain the energy density of R-PLECHDE as

$$\rho_{\Lambda} = 3\gamma M_P^2 R - \beta M_P^2 R^2,$$

which is an extension of the RDE model proposed by Gao.

This paper is organized as follows. In section 2, we describe the physical context we are working in and the R-PLECHDE model is described; moreover, we derive the EoS parameter $\omega_{\Lambda}$, the deceleration parameter $q$ and the evolution of the energy density parameter $\Omega_{\Lambda}$. In section 3, we establish the correspondence between the R-PLECHDE model and the modified Chaplygin gas (MCG) and the tachyon, K-essence, dilaton and quintessence scalar fields. In section 4, the conclusions are presented.

2. The model of R-PLECHDE

Since cosmological observations show that our Universe is not perfectly flat but has a small positive curvature which implies a closed universe, in this paper we consider a non-flat universe and we work in the FRW universe background. The tendency for a closed universe is obtained in different independent cosmological experiments [26]. The line element for the non-flat FRW universe is given by

$$ds^2 = - dt^2 + a^2 (t) \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right),$$

where $t$ is the cosmic time, $r$ refers to the radial component and $(\theta, \phi)$ are the angular coordinates.

The Friedmann equation for the non-flat FRW universe dominated by DE and DM takes the form

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_P^2} (\rho_{\Lambda} + \rho_m),$$

where $\rho_{\Lambda}$ and $\rho_m$ are the energy densities of DE and DM, respectively.

We also define the fractional energy densities for DM, curvature and DE, respectively, as

$$\Omega_m = \frac{\rho_m}{\rho_{\text{crit}}} = \frac{\rho_m}{3M_P^2 H^2},$$

$$\Omega_k = \frac{\rho_k}{\rho_{\text{crit}}} = \frac{k}{H^2 a^2},$$

$$\Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{\text{crit}}} = \frac{\rho_{\Lambda}}{3M_P^2 H^2},$$

where $\rho_{\text{crit}} = 3M_P^2 H^2$ is the critical energy density. $\Omega_k$ is the contribution to the total density from the spatial curvature.
Recent observations reveal that $\Omega_k \cong 0.02$ [27], which supports a closed universe with a small positive curvature.

Using the Friedmann equation given in equation (7), equations (8)–(10) yield

$$\Omega_m + \Omega_\Lambda = 1 + \Omega_k.$$  \hfill (11)

In order to preserve the Bianchi identity or the local energy-momentum conservation law, i.e. $\nabla_\mu T^{\mu\nu} = 0$, the total energy density $\rho_{tot} = \rho_p + \rho_m$ must satisfy the following relation:

$$\rho_{tot} + 3H (1 + \omega) \rho_{tot} = 0,$$  \hfill (12)

where $\omega \equiv \rho_{tot}/\rho_m$ is the total EoS parameter. Since we are considering the interaction between DE and DM, the two energy densities $\rho_\Lambda$ and $\rho_m$ are preserved separately and the equations of conservation assume the forms

$$\dot{\rho}_\Lambda + 3H \rho_\Lambda (1 + \omega_\Lambda) = -Q,$$  \hfill (13)

$$\dot{\rho}_m + 3H \rho_m = Q,$$  \hfill (14)

where $Q$ represents an interaction term which can be an arbitrary function of cosmological parameters, such as the Hubble parameter $H$ and energy densities $\rho_m$ and $\rho_\Lambda$, i.e. $Q(\dot{H},\rho_\Lambda, H, \dot{H}_\Lambda)$. The simplest and most widely used expression for $Q$ is given by

$$Q = 3b^2 H (\rho_m + \rho_\Lambda),$$  \hfill (15)

where $b^2$ represents a coupling parameter between DE and DM [28]. If $b^2 > 0$ we have a transition from DE to DM, whereas $b^2 < 0$ implies a transition from DM to DE. The case corresponding to $b^2 = 0$ represents the non-interacting FRW model, whereas $b^2 = 1$ yields a complete transfer from DE to DM. Recently, it was reported that this interaction is observed in the Abell cluster A586 showing a transition from DE into DM and vice versa [29]. However, the strength of this interaction is not yet clearly identified [30].

Observations of CMB and galactic clusters show that the coupling parameter is $b^2 < 0.025$, i.e. a small positive constant of the order of unity [31]. A negative coupling constant results in the violation of thermodynamic laws and so it is avoided. We must also note that the ideal interaction term must be motivated by the quantum gravity theory; otherwise it is avoided. We must also note that the ideal interaction term

$$\omega_\Lambda = -1 - \frac{\rho_\Lambda}{3H \rho_\Lambda} - \frac{Q}{3H \rho_\Lambda},$$  \hfill (22)

which represents the EoS parameter of the R-PLECHDE model.

We now want to derive the expression for the evolution of energy density parameter $\Omega_\Lambda$. From equation (13), we can obtain the following expression for the EoS parameter $\omega_\Lambda$:

$$\omega_\Lambda = \frac{\rho_\Lambda}{3H \rho_\Lambda} - \frac{Q}{3H \rho_\Lambda},$$  \hfill (21)

Using the expression for $Q$ given in equation (15), the derivative of the DE energy density $\rho_\Lambda$ with respect to the cosmic time can be written as

$$\dot{\rho}_\Lambda = 3H \left[ -\rho_\Lambda - (\rho_m + \rho_\Lambda) \left( b^2 + \frac{1}{3} \right) + \frac{RM_\Lambda^2}{3} \right].$$  \hfill (23)

Dividing equation (23) by the critical density $\rho_c = 3H^2 M_p^2$, we obtain

$$\frac{\dot{\rho}_\Lambda}{\rho_c} = 3H \left[ -\Omega_\Lambda - (1 + \Omega_k) \left( b^2 + \frac{1}{3} \right) + \frac{R}{9H^2} \right] = \Omega_\Lambda + 2\Omega_\Lambda \frac{H}{H}. \hfill (24)

From equation (4), we can derive

$$\frac{R}{9H^2} = \frac{2}{3} \left( \frac{H}{H} + 2 + 3\Omega_\Lambda \right).$$  \hfill (25)

Substituting equation (25) into (24), it is possible to obtain the derivative of $\Omega_\Lambda$ with respect to the cosmic time $t$:

$$\Omega_\Lambda = \frac{2}{3} \frac{H}{H} (1 - \Omega_\Lambda) + 3H \left[ -\Omega_\Lambda - (1 + \Omega_k) \left( b^2 + \frac{1}{3} \right) + \frac{2}{3} \right].$$  \hfill (26)

Since $\Omega_\Lambda = \frac{d\Omega_\Lambda}{dt} = \frac{1}{H} \dot{\Omega}_\Lambda$ (where $x = \ln a$), we derive

$$H\Omega_\Lambda' = 2H(1 - \Omega_\Lambda) + 3H \left[ -\Omega_\Lambda - (1 + \Omega_k) \left( b^2 + \frac{1}{3} \right) + \frac{2}{3} \right].$$  \hfill (27)
which yields
\[
\Omega_\Lambda' = \frac{2}{H} (1 - \Omega_\Lambda^2) + 3 \left[ -\Omega_\Lambda - (1 + \Omega_\Lambda) \left( \frac{b^2}{3} - \frac{1}{3} \right) + \frac{2}{3} \right].
\] (28)

In equation (28) we used the fact that
\[
H' = \frac{a'}{a} = 1.
\] (29)

For completeness, we now derive the expression for the deceleration parameter \( q \), which is defined as
\[
q = -\frac{\ddot{a}a}{aH^2} = -\frac{\ddot{H}}{aH^2} = -1 - \frac{\dot{H}}{H^2}.
\] (30)

The deceleration parameter, combined with the Hubble parameter \( H \) and the dimensionless density parameters, forms a set of parameters very useful for the description of the astrophysical observations. Taking the time derivative of the Friedmann equation given in equation (7) and using equations (11) and (14), it is possible to write the deceleration parameter \( q \) as
\[
q = \frac{1}{2} \left[ 1 + \Omega_k + 3\Omega_\Lambda \omega_\Lambda \right].
\] (31)

Substituting into equation (31) the expression for the EoS parameter \( \omega_\Lambda \) of the R-PLECHDE given in equation (21), we obtain that
\[
q = 1 - \frac{\Omega_\Lambda}{2(3\gamma - \beta R^{-1})} + \Omega_k.
\] (32)

We can now derive important quantities of the R-PLECHDE model in the limiting case, for a flat dark-dominated universe, i.e. when \( \beta = 0 \), \( \Omega_\Lambda = 1 \) and \( \Omega_k = 0 \).

The energy density \( \rho_\Lambda \) given in equation (5) reduces to
\[
\rho_\Lambda = 3\gamma M_p^2 R.
\] (33)

From the Friedmann equation given in equation (7), we can derive the following expressions for the Hubble parameter \( H \) and the Ricci scalar curvature \( R \):
\[
H = \frac{6\gamma}{12\gamma - 1} \left( \frac{1}{r} \right),
\] (34)
\[
R = \frac{36\gamma}{(12\gamma - 1)^2} \left( \frac{1}{r^2} \right).
\] (35)

Finally, the EoS parameter \( \omega_\Lambda \) and the deceleration parameter \( q \) reduce, respectively, to
\[
\omega_\Lambda = \frac{1}{3} - \frac{1}{9\gamma},
\] (36)
\[
q = 1 - \frac{1}{6\gamma}.
\] (37)

From equation (36), we see that in this limit, the EoS parameter of DE becomes a constant value and for \( \gamma < 1/12 \) we have \( \omega_\Lambda < -1 \), where the phantom divide can be crossed. Since the Ricci scalar \( R \) given in equation (35) diverges at \( \gamma = 1/12 \), this value cannot be taken into account. From equation (37), we obtain that the acceleration starts at \( \gamma \leq 1/6 \), where the quintessence regime is started \( (\omega_\Lambda \leq -1/3) \).

The result obtained is very similar to the power-law expansion of scale factor found by Granda and Oliveros in 2008 [32], in which \( a(t) = t^{6\gamma/(12\gamma - 1)} \).

### 3. Correspondence between R-PLECHDE and scalar fields

In this section, we establish the correspondence between the interacting Ricci power-law corrected model and the tachyon, K-essence, dilaton and quintessence scalar field models and the MCG. The importance of this correspondence is that the scalar field models are an effective description of an underlying theory of DE. Therefore, it is worthwhile reconstruct the potential and dynamics of scalar fields according to the evolutionary form of the Ricci scalar model. For this purpose, first we compare the energy density of the Ricci scale model given in equation (5) with the energy density of the corresponding scalar field model. Then, we equate the EoS parameters of scalar field models with the EoS parameter of the Ricci scalar model given in equation (21).

#### 3.1. Interacting tachyon model

In recent years, much interest has been devoted to the study of the inflationary model with the help of the tachyon field, since it is believed that the tachyon can be assumed to be a possible source of DE [33]. The tachyon is an unstable field that can be used in string theory through its role in the Dirac–Born–Infeld action to describe the D-brane action [34]. The tachyon might be responsible for cosmological inflation in the early evolutionary stage of the universe, due to tachyon condensation near the top of the effective scalar potential. A rolling tachyon has an interesting EoS whose parameter smoothly interpolates in the range \([-1,0]\). This discovery motivated us to take DE as a dynamical quantity, i.e. a variable cosmological constant and model inflation using tachyons.

The effective Lagrangian for the tachyon field is given by
\[
L = -V(\phi)\sqrt{1 - \dot{\phi}^2} \partial_\mu \phi \partial^\mu \phi,
\] (38)
where \( V(\phi) \) represents the potential of the tachyon and \( g^{\mu\nu} \) is the metric tensor. The energy density \( \rho_\phi \) and pressure \( p_\phi \) for the tachyon field are given, respectively, by
\[
\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}},
\] (39)
\[
p_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2}.
\] (40)

The EoS parameter \( \omega_\phi \) of the tachyon scalar field can be obtained from the following expression:
\[
\omega_\phi = \frac{p_\phi}{\rho_\phi} = \dot{\phi}^2 - 1.
\] (41)

In order to have a real energy density for a tachyon field, it is required that \( -1 < \dot{\phi} < 1 \). Consequently, from equation (41), the EoS parameter of the tachyon is constrained in the range \( -1 < \omega_\phi < 0 \). Hence, the tachyon field can interpret the accelerated expansion of the universe, but it cannot enter the phantom regime which has \( \omega_\Lambda < -1 \).

Comparing equations (5) and (39), we obtain the following expression for the potential \( V(\phi) \) of the tachyon field:
\[
V(\phi) = \rho_\Lambda \sqrt{1 - \dot{\phi}^2}.
\] (42)
Instead, equating equations (21) and (41), we derive the expression for the kinetic energy term $\dot{\phi}^2$ as follows:

$$\dot{\phi}^2 = 1 + \omega_{\Lambda} = 1 - \frac{1}{3(3\gamma - \beta R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_\Lambda}.$$  (43)

Moreover, inserting equation (43) into (42), it is possible to write the potential $V(\phi)$ of the tachyon as follows:

$$V(\phi) = \rho_\Lambda \sqrt{\frac{1}{3(3\gamma - \beta R^2 - 1)} - \frac{(1 + \Omega_k)}{3\Omega_\Lambda}}.$$  (44)

Since $\dot{\phi} = \phi' H$, from equation (43) it follows that

$$\phi' = \frac{1}{H} \sqrt{\frac{1}{3(3\gamma - \beta R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_\Lambda}}.$$  (45)

It is now possible to derive the evolutionary form of the tachyon scalar field integrating equation (45) with respect to the scale factor $a$:

$$\phi(a) - \phi(a_0) = \int_{a_0}^a \frac{da}{aH} \left[ 1 - \frac{1}{3(3\gamma - \beta R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_\Lambda} \right],$$  (46)

where $a_0$ is the present value of the scale factor.

In the limiting case for a flat dark-dominated universe, i.e. when $\beta = 0$, $\Omega_\Lambda = 1$ and $\Omega_k = 0$, the scalar field and potential of the tachyon become, respectively,

$$\phi(t) = \sqrt{\frac{12\gamma - 1}{9\gamma}} \phi,$$  (47)

$$V(\phi) = \frac{4M_p^2 \sqrt{\gamma} (1 - 3\gamma)}{(12\gamma - 1)} \left( \frac{1}{\phi^2} \right).$$  (48)

In this correspondence, the scalar field exists when $\gamma > 1/12$, which shows that the phantom divide cannot be achieved.

### 3.2. Interacting K-essence model

A model in which the kinetic term of the scalar field appears in the Lagrangian in a non-canonical way is known as the K-essence model. The idea of the K-essence scalar field was motivated by the Born–Infeld action of string theory and is used to explain the late time acceleration of the universe [35]. The general scalar field action $S_K$ for the K-essence field as a function of $\phi$ and $\chi = \phi/2$ is given as [36]

$$S_K = \int d^4x \sqrt{-g} \ p(\phi, \chi).$$  (49)

The Lagrangian density $p(\phi, \chi)$ corresponds to a pressure density. According to equation (49), the pressure $p(\phi, \chi)$ and the energy density $\rho_\Lambda(\phi, \chi)$ of the K-essence can be written, respectively, as

$$p(\phi, \chi) = f(\phi) (-\chi + \chi^2),$$  (50)

$$\rho_\Lambda(\phi, \chi) = f(\phi) (-\chi + 3\chi^2).$$  (51)

The EoS parameter $\omega_K$ of the K-essence scalar field is then given by

$$\omega_K = \frac{p(\phi, \chi)}{\rho(\phi, \chi)} = \frac{-\chi}{3\chi - 1},$$  (52)

From equation (52), we can see that the phantom behaviour of K-essence scalar field ($\omega_K < -1$) is obtained when the parameter $\chi$ lies in the range $1/3 < \chi < 1/2$.

In order to consider the K-essence field as a description of the interacting R-PLECHDE model, we establish the correspondence between the K-essence EoS parameter $\omega_K$ and the R-PLECHDE EoS parameter $\omega_\alpha$ given in equation (21).

The expression for $\chi$ can be found by equating equations (21) and (52), which yields

$$\chi = \frac{\omega_\alpha - 1}{3\omega_\alpha - 1} = -1 - \frac{1}{3(3\gamma - \beta R^2 + 1)} + \frac{(1 + \Omega_k)}{3\Omega_\Lambda}.$$  (53)

Moreover, equating equations (5) and (51), we derive

$$f(\phi) = \frac{\rho_\Lambda}{\chi(3\chi - 1)}.$$  (54)

Using $\dot{\phi}^2 = 2\chi$ and the relation $\phi = \phi' H$, we derive from equation (53) that

$$\phi' = \sqrt{\frac{12\gamma - 1}{9\gamma}} \phi.$$  (55)

We can now find the evolutionary form of the K-essence scalar field integrating equation (55) with respect to the scale factor $a$:

$$\phi(a) - \phi(a_0) = \sqrt{\frac{12\gamma - 1}{9\gamma}} \int_{a_0}^a \frac{da}{aH} \left[ -\frac{1}{3(3\gamma - \beta R^2 + 1)} + \frac{(1 + \Omega_k)}{3\Omega_\Lambda} \right],$$  (56)

where $a_0$ is the present value of the scale factor.

In the limiting case for a flat dark-dominated universe, i.e. when $\beta = 0$, $\Omega_\Lambda = 1$ and $\Omega_k = 0$, the scalar field and potential of the K-essence field reduce, respectively, to

$$\phi(t) = \left( \sqrt{\frac{12\gamma + 2}{3}} \right) t,$$  (57)

$$f(\phi) = \frac{36\gamma M_p^2}{(12\gamma - 1)^2} \left( \frac{1}{\phi^2} \right),$$  (58)

which are a result of power-law expansion.

We see that the universe may behave in all the accelerated regimes (i.e. phantom and quintessence), since all values of $\gamma$ are possible. We also note that the results of this subsection can be extended to g-essence as well as f-essence [37]. This task is left for future investigations.
3.3. Interacting dilaton model

A dilaton scalar field, originating from the lower-energy limit of string theory [38], can also be assumed to be a source of DE.

The process of compactification of the string theory from higher to four dimensions introduces the scalar dilaton field which is coupled to curvature invariants. The coefficient of the kinetic term of the dilaton can be negative in the Einstein frame, which means that the dilaton behaves as a phantom-like scalar field. The pressure (Lagrangian) density and the energy density of the dilaton DE model are given, respectively, as [39]

\[ p_D = -\chi + c \, e^{\phi} \chi^2 , \quad (59) \]
\[ \rho_D = -\chi + 3c \, e^{\phi} \chi^2 . \quad (60) \]

\( c \) and \( \lambda \) are two positive constants and \( 2\chi = \dot{\phi}^2 \). The EoS parameter \( \omega_D \) for the dilaton scalar field can be obtained from

\[ \omega_D = \frac{p_D}{\rho_D} = -1 + c \, e^{\phi} \chi + 3c \, e^{\phi} \chi , \quad (61) \]

In order to consider the dilaton field as a description of DE, we now establish the correspondence between the dilaton EoS parameter \( \omega_D \) and the EoS parameter \( \omega_{\Lambda} \) of the R-PLECHDE model given in equation (21). By equating equations (21) and (61), we find that

\[ c \, e^{\phi} \chi = \frac{\omega_{\Lambda} - 1}{3\omega_{\Lambda} - 1} = -1 - \frac{1}{3(3\gamma - 2R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_k} - 1 - \frac{1}{3(3\gamma - 2R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_k} . \quad (62) \]

Since \( \dot{\phi}^2 = 2\chi \), equation (62) can be rewritten as

\[ e^{\phi/2} \phi = \sqrt{\frac{2}{c}} \left[ -1 - \frac{1}{3(3\gamma - 2R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_k} - 1 - \frac{1}{3(3\gamma - 2R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_k} \right] . \quad (63) \]

Integrating equation (63) with respect to the scale factor \( a \), we obtain

\[ e^{\phi(a)/2} = e^{\phi(a_0)/2} + \frac{\lambda}{\sqrt{2c}} \int_{a_0}^a \frac{da}{aH} \left[ -1 - \frac{1}{3(3\gamma - 2R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_k} - 1 - \frac{1}{3(3\gamma - 2R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_k} \right] , \quad (64) \]

where \( a_0 \) is the present value of the scale factor.

The evolutionary form of the dilaton scalar field is given by

\[ \phi (a) = 2 \frac{\lambda}{\sqrt{2c}} \int_{a_0}^a \frac{da}{aH} \left[ -1 - \frac{1}{3(3\gamma - 2R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_k} - 1 - \frac{1}{3(3\gamma - 2R^2 - 1)} + \frac{(1 + \Omega_k)}{3\Omega_k} \right] , \quad (65) \]

In the limiting case for a flat dark-dominated universe, i.e. when \( \beta = 0, \Omega_{\Lambda} = 1 \) and \( \Omega_k = 0 \), the scalar field of the dilaton field reduces to

\[ \phi (t) = \frac{2}{\lambda} \ln \left[ \lambda t \sqrt{\frac{1 + 6\gamma}{6c}} \right] . \quad (66) \]

We see from equation (66) that \( \gamma \) can assume all possible values. Therefore, by this correspondence, the universe may behave in both the phantom and quintessence regimes.

3.4. Quintessence

Quintessence is described by an ordinary time-dependent and homogeneous scalar field \( \phi \) which is minimally coupled to gravity, but with a particular potential \( V (\phi) \) that leads to the accelerated universe. The action for quintessence is given as [40]

\[ S = \int d^4x \sqrt{-g} \left[ \frac{-1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V (\phi) \right] . \quad (67) \]

The energy momentum tensor \( T_{\mu\nu} \) of the quintessence field is derived by varying the action \( S \) given in equation (67) with respect to the metric tensor \( g^{\mu\nu} \):

\[ T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} . \quad (68) \]

which yields, using equation (67),

\[ T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[ \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V (\phi) \right] . \quad (69) \]

Considering an FRW background, the energy density \( \rho_Q \) and pressure \( p_Q \) of the quintessence scalar field \( \phi \) are given, respectively, by

\[ \rho_Q = -T_0^0 = \frac{1}{2} \dot{\phi}^2 + V (\phi) , \quad (70) \]
\[ p_Q = T_i^i = \frac{1}{2} \dot{\phi}^2 - V (\phi) . \quad (71) \]

Moreover, the EoS parameter \( \omega_Q \) for the quintessence scalar field is given by

\[ \omega_Q = \frac{p_Q}{\rho_Q} = \frac{\dot{\phi}^2 - 2V (\phi)}{\dot{\phi}^2 + 2V (\phi)} . \quad (72) \]

We now derive from equation (72) that, when \( \omega_Q < -1/3 \), the universe accelerates if the condition \( \dot{\phi}^2 < V (\phi) \) is satisfied. Then, the scalar potential needs to be shallow enough in order to have that the field evolves slowly along the potential.

The variation with respect to \( \phi \) of the quintessence action given in equation (67) yields

\[ \dot{\phi} + 3H \dot{\phi} + V_{,\phi} = 0 , \quad (73) \]

where the two dots indicate the second derivative with respect to the cosmic time \( t \), the single dot indicates the derivative with respect to the cosmic time \( t \) and \( V_{,\phi} \equiv dV/d\phi \).

Many quintessence potentials have already been proposed. They have been usually classified into (i) freezing models and (ii) thawing models [41]. In the freezing models,
the field was rolling along the potential in the past, but the movement gradually slows down after the system enters the phase of cosmic acceleration. In the thawing models, the field (with mass \( m_0 \)) has been frozen by Hubble friction (i.e. the term \( H \dot{\phi} \)) until recently and then begins to evolve once \( H \) drops below \( m_0 \). The EOS of DE is \( \omega_0 \equiv 1 \) at early times, which is followed by the growth of \( \omega_0 \).

Here we establish the correspondence between the interacting scenario and the quintessence DE model: equating equation (72) with the EoS parameter (21) (i.e. \( \omega_0 = \omega_A \)) and equation (70) with equation (5) (i.e. \( \rho_0 = \rho_A \)), we obtain the following expressions for the kinetic energy term \( \dot{\phi}^2 \) and the quintessence potential energy \( V (\phi) \):

\[
\dot{\phi}^2 = (1 + \omega_A) \rho_A, \quad (74)
\]
\[
V (\phi) = \frac{1}{2} (1 - \omega_A) \rho_A. \quad (75)
\]

Substituting the EoS parameter given in equation (21) into equations (74) and (75), we obtain

\[
\dot{\phi}^2 = \rho_A \left( 1 - \frac{1}{3} (3 \gamma - \beta R^2) + \frac{(1 + \Omega_b)}{3 \Omega_A} \right), \quad (76)
\]
\[
V (\phi) = \frac{\rho_A}{2} \left( 1 + \frac{1}{3} (3 \gamma - \beta R^2) - \frac{(1 + \Omega_b)}{3 \Omega_A} \right). \quad (77)
\]

We can now obtain the evolutionary form of the quintessence scalar field integrating equation (76) with respect to the scale factor \( a \) and using the relation \( \dot{\phi} = \dot{\phi}' H \):

\[
\phi (a) - \phi (a_0) = \int_{a_0}^{a} \frac{da}{a} 3 \Omega_A M_p^2 \left( 1 - \frac{1}{3 (3 \gamma - \beta R^2)} + \frac{(1 + \Omega_b)}{3 \Omega_A} \right), \quad (78)
\]

where \( a_0 \) is the present value of the scale factor.

In the limiting case of a flat dark-dominated universe, i.e. when \( \beta = 0, \Omega_A = 1 \) and \( \Omega_b = 0 \), the scalar field and potential of quintessence reduce, respectively, to

\[
\phi (t) = \frac{6 \gamma M_p}{\sqrt{3(12 \gamma - 1)}} \ln (t), \quad (79)
\]
\[
V (\phi) = \frac{6 \gamma (6 \gamma + 1)}{(12 \gamma - 1)^{\gamma}} M_p^2 \exp \left[ - \frac{\gamma}{3 \gamma (12 \gamma - 1)} \phi \right]. \quad (80)
\]

The potential exists for all values of \( \gamma > 1/12 \) (which correspond to the quintessence regime). The potential has also been obtained by power-law expansion of the scale factor.

3.5. Modified Chaplygin gas

In this section, we want to obtain the correspondence between the MCG and the R-PLECHDE model.

One of the suggested candidates for DE is the generalized Chaplygin gas (GCG), which represents the generalization of the Chaplygin gas [42]. GCG has the favourable property of interpolating the evolution of the universe from the dust to the accelerated phase; hence it better fits the observational data [43]. The GCG and its further generalization have been widely studied in the literature [44].

The GCG is defined as [45]

\[
\rho_A = \frac{D}{\rho_A^D}, \quad (81)
\]

where \( D \) and \( \theta \) are two constants \( (D \) is also positive defined). The Chaplygin gas is obtained in the limiting case \( \theta = 1 \).

The MCG represents a generalization of the GCG with the addition of a barotropic term. The MCG seems to be consistent with the 5-year Wilkinson Microwave anisotropy probe (WMAP) data and henceforth supports the unified model with DE and matter based on generalized Chaplygin gas.

The MCG is defined as [46]

\[
\rho_A = A \rho_A - \frac{D}{\rho_A^D}, \quad (82)
\]

where \( A \) and \( D \) are two positive constants and \( 0 \leq \theta \leq 1 \).

The density evolution of the MCG, calculated using the density conservation equation, is given by

\[
\rho_A = \left[ \frac{D}{A + 1} + \frac{B}{a^{(\theta + 1)(A + 1)}} \right] \rho_A^D, \quad (83)
\]

where \( B \) represents a constant of integration.

We now want to reconstruct the potential and dynamics of the scalar field \( \Phi \) in the light of the R-PLECHDE. For a homogeneous and time-dependent scalar field \( \Phi \), the energy density and pressure are defined, respectively, by

\[
\rho_A = \frac{\sigma}{2} \phi^2 + V (\Phi), \quad (84)
\]
\[
\rho_A = \frac{\sigma}{2} \phi^2 - V (\Phi). \quad (85)
\]

The case with \( \sigma = -1 \) corresponds to the phantom, whereas the case with \( \sigma = +1 \) corresponds to the standard scalar field which represents the quintessence field. Moreover, \( V (\phi) \) represents the scalar potential of the field.

The EoS parameter \( \omega_A \) of the MCG is given by

\[
\omega_A = \frac{\rho_A}{\rho_A} = \frac{\sigma \Phi^2 - 2 V (\Phi)}{\sigma \Phi^2 + 2 V (\Phi)}. \quad (86)
\]

Using equations (84)–(86), we obtain the kinetic energy \( \dot{\phi}^2 \) and the scalar potential \( V (\Phi) \) terms, respectively, as

\[
\dot{\phi}^2 = \frac{1}{\sigma} (1 + \omega_A) \rho_A, \quad (87)
\]
\[
V (\Phi) = \frac{1}{2} (1 - \omega_A) \rho_A. \quad (88)
\]

We also know that the EoS parameter \( \omega_A \) can be written as

\[
\omega_A = A - \frac{D}{\rho_A^D}. \quad (89)
\]

From equation (83), we can easily derive that

\[
B = a^{2(\theta + 1)(A + 1)} \left( \rho_A^D - \frac{D}{A + 1} \right). \quad (90)
\]
Moreover, from equation (89), we obtain the following relation for $D$:

$$D = \rho_0^{\phi+1} (A - \omega_0).$$

(91)

Substituting into equation (90) the expression of $D$ given in equation (91), we obtain that $B$ can be rewritten as

$$B = \left[ a^{3(\Lambda + 1)} \rho_{A}^{\phi+1} \left( 1 + \frac{1 + \omega_0}{1 + A} \right) \right].$$

(92)

Inserting the EoS parameter $\omega_0$ of the R-PLECHDE given in equation (21) into equation (91), we derive

$$D = \left[ \rho_{A}^{\phi+1} \left( A + \frac{1}{3 (3 \gamma - \beta R^2)} - \frac{1}{3} \left( \frac{1 + \Omega_k}{\Omega_{\Lambda}} \right) \right) \right].$$

In the limiting case of a flat dark-dominated universe, i.e. $\beta = 0$, $\Omega_{\Lambda} = 1$ and $\Omega_k = 0$, the phantom divide may be crossed, i.e. $\omega_0 < 1$, and the present acceleration expansion ($q > 0$) is achieved where the quintessence is started. Moreover, we established the correspondence between the interacting R-PLECHDE model and the MCG and the tachyon, K-essence, dilaton and quintessence scalar fields in the hypothesis of a non-flat FRW universe.

These correspondences are important for understanding how various candidates for DE are mutually related to each other. The limiting case of a flat dark-dominated universe without entropy correction as studied in each scalar field and we see that the EoS parameter is constant in this case, and we calculate the scalar field and its potential that can be obtained by the idea of power-law expansion of the scalar field.

To make a comparison between the R-PLECHDE and other works in the PLECHDE-scalar field model, we concentrate on two recently written papers, one by Granda and Oliveros in 2009 [47] and the other by Khodam-Mohammadi in 2011 [48]. Granda and Oliveros introduced an IR cut-off which is a function of the Hubble parameter $H$ and the derivative of the Hubble parameter $H$ with respect to the cosmic time, i.e. $L_{GO} = (\alpha H^2 + 2 H^2)^{-1/2}$, where $\alpha$ and $\beta$ are two constants. In the limiting case of $\alpha = 2$ and $\beta = 1$, $L_{GO}$ is equal to the Ricci scalar in the case when the curvature parameter $k$ is equal to zero. Khodam-Mohammadi studied the power-law entropy-corrected HDE model using as the IR cut-off $L_{GO}$. The results obtained in this work are in good agreement with those obtained in both works in the limiting case of a flat dark-dominated universe.

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