Quadrupole dominance in light Cd and Sn isotopes

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Shell model calculations with the neutron effective charge as single free parameter describe well the $B(E2 : 2^+ \rightarrow 0^+)$ and $B(E2 : 4^+ \rightarrow 2^+)$ rates for $N \leq 64$ in the Cd and Sn isotopes. The former exhibit weak permanent deformation corroborating the prediction of a Pseudo SU3 symmetry, which remains of heuristic value in the latter, where the pairing force erodes the quadrupole dominance. Calculations in $10^7$ and $10^{10}$-dimensional spaces exhibit almost identical patterns: A vindication of the shell model.

All nuclear species are equal, but some are more equal than others. The tin isotopes deserve pride of place, because $Z = 50$ is the most resilient of the magic numbers, because they are very numerous, and many of them stable, starting at $A = 112$. For these, accurate data have been available for a long time. As seen in Fig. 1 a parabola accounts very well for the $B(E2 : 2^+ \rightarrow 0^+)$ trend, except at $112-114$. That these early results (Jonsson et al. [2]) truly signal a change of regime became evident through work on the unstable isomers, starting with the measure in $^{108}$Sn by Banu et al. [3]. A flurry of activity followed [4–10], from which a new trend emerged in which the parabola—characteristic of a seniority scheme—gives way to a platform, predicted by a Pseudo SU3 scheme (the squares). Here we are going a bit fast to follow the injunction of Montaigne: start by the end (“Je veux qu’on commence par le dernier point” Essais II 10). To slow down, we go back to the origin of this study, the Cadmium isotopes, where things are simpler.

The basic idea is inspired by Elliott’s SU3 scheme [11, 12] and consists in building intrinsic determinantal states that maximize the quadrupole operator $\langle 2q_{20} \rangle$. Fig. 2 implements the idea for $^{104}$Cd ($Z = 48, N = 56$). The single shell (S) contribution of the $g_{9/2} \equiv g$ proton orbit is given by $\langle 012 \rangle$ (with changed sign for hole states). For the neutron orbits, the Pseudo SU3 scheme (P generically, $Pr_p$ for specific cases) amounts to assimilate all the orbits of a major oscillator shell of principal quantum number $p$, except the largest (the $r_p$ set) to orbits in the $p – 1$ major shell. In our case the $sdg$ shell has $p = 4$, and $r_4$ is assimilated to a $pf$ shell. As the $(2q_{20})$ operator is diagonal in the oscillator quanta representation, maximum $\langle 2q_{20} \rangle$ is obtained by orderly filling states $(n_z n_y n_x = (300), (210), (201) \ldots (012), (003)$, with $\langle 2q_{20} \rangle = 2n_z - n_y - n_x = 6, 3, 3, 0, 0, -2, -2$, as in Fig. 2. Using $q(n)$ for the cumulated $q_0$ value (e.g. 24 for $^{104}$Cd in Fig. 2), the intrinsic quadrupole moment then follows as a sum of the single shell (S) and pseudo SU3.
(P) contributions

\[
q_0(S) = 2(r^2 C_{20}) = \sum_{m} (p + 3/2) \frac{j(j+1) - 3m^2}{2j(j+1)} \tag{1}
\]

\[
q_0(P) = q(n), \quad Q_0(SP) = [(8e_\pi + q(n)e_\nu)b^2] \text{fm}^2 \tag{2}
\]

where we have introduced effective charges and recovered dimensions through \( Q = b^2 q \) with \( b^2 \approx 41.4/\hbar \omega \text{fm}^2 \), \( \hbar \omega = 45A^{-1/3} - 25A^{-2/3} \).

To qualify as a Bohr Mottelson rotor, \( Q_0(SP) \) must coincide with the intrinsic spectroscopic \( Q_0 \), and transition \( Q_{0\alpha} \) quadrupole moments, defined through (as e.g in \([13]\))

\[
Q_{\text{spec}}(J) = < J|3z^2 - y^2|J >
\]

\[
Q_0 = (J + 1)(J + 3) \text{spec}(J), \quad K \neq 1 \tag{3}
\]

\[
B(E2, J \rightarrow J - 2) = \frac{5}{16\pi} e^2 (|J K 20|J, J - 2, K)|^2 Q_0^2 \tag{4}
\]

\[
K \neq 1/2, 1, B(E2 : 2^+ \rightarrow 0^+) = Q_{0SP}^2/50.3 \text{e}^2\text{fm}^4 \tag{5}
\]

To speak of deformed nuclei two conditions must be met \( B(E2 : 4^+ \rightarrow 2^+) / B(E2 : 2^+ \rightarrow 0^+) = 1.43 \) (the Alaga rule from Eq. \([4]\)), and the “quadrupole quotient” rule, \( Q_0 \) which follows from Eqs.\([5]\) and \([4]\) and equating \( Q_0 \approx Q_0: \)

\[
50.27B(E2 : 2^+ \rightarrow 0^+) / (3.5Q_{\text{spec}}^2) = (Q_0)^2 \approx 1 \tag{6}
\]

Full verification demands calculations but Eq. \([5]\) can be checked directly by inspecting Fig. \(2\) as done in Table \(4\).

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**TABLE I.** \( B20 = B(E2 : 2^+ \rightarrow 0^+) \) estimates for \(^{98}\text{Cd}\) in \( e^2\text{fm}^4 \) from Eq. \([3]\) (sp) using (naive) \( q(n)_h \) from diagonalization of \( Q_0 \) in the \( pf \) shell i.e., strict SU3, with \( (e_\pi, e_\nu) = (1.1, 1.7) \). The \( B20SP \) numbers use (full) \( q(n)_f \) from diagonalization of \( Q_0 \) in the \( pf \) space, \( (e_\pi, e_\nu) = (1.0, 1.5) \). The \( b^2 \) values range from 4.78 fm\(^2\) for \( A = 98 \) to 4.94 fm\(^2\) for \( A = 110 \). Experimental values \((B20e)\) for \(^{102-104}\text{Cd}\) are taken from \([18]\) and \([14]\) and from www.nndc.bnl.gov/be2 compilations \([1]\) for \(^{106-110}\text{Cd}\).

| \( A \) | 100 | 102 | 104 | 106 | 108 | 110 |
| \( n \) | 2 | 4 | 6 | 8 | 10 | 12 |
| \( q(n)_h \) | 12 | 18 | 24 | 24 | 24 | 24 |
| \( q(n)_f \) | 14.8 | 22.6 | 29.5 | 30.0 | 29.6 | 29.3 |
| B20e | <560(4) | 562(46) | 779(80) | 814(24) | 838(28) | 852(42) |
| B20sp | 330 | 517 | 751 | 756 | 770 | 776 |
| B20SP | 330 | 555 | 809 | 838 | 833 | 827 |

Note that the naive form of P used so far (in \( q(n)_h \) and \( B20sp \)) is supplemented by the more accurate \( q(n)_f \) and \( B20SP \) using fully diagonalized values of \( 2q_0n \). The remarkable property of the \( r^2q_\pi^2 \) space that produces four identical \( q(n)_h \) values for \( m = 6 - 12 \) has already been put to good use in \([13]\) and \([14]\). FIG. 38, TABLE VII. In the present case it is seen to do equally well.

Now for the shell model diagonalizations in spaces defined by \((y^{10-17}r^4_\pi)(y^{10-14}r^4_\nu)^{10-4}, X = 8 \) for \( \text{Cd} \) and 10 for Sn. The proton \( (u) \) and neutron \( (t) \) excitations are restricted to have \( u + t \leq M \). The calculations were done for \( utM = 000 \) (the case in Fig. \( 2\)), 111, 101 and 202 using \( V_{\text{low-k}} \) variants \([21]\) of the precision interaction N3LO \([22]\) (I in what follows) with oscillator parameter \( \hbar \omega = 8.4 \text{ MeV} \) and cutoff \( \lambda = 2 \text{ fm}^{-1} \). As a first step the monopole part of \( I \) is removed and replaced by single-particle energies for \(^{100}\text{Sn}\) from Ref. \([23]\) (GEMO), consistent with the analysis of Ref. \([24]\). The \( I \) interaction is then subject to an overall 1.1 scaling and renormalized by increasing the \( \lambda_\mu = 20 \) quadrupole and \( JTI = 01 \) pairing components by \( q \times 10\% \) and \( p \times 10\% \), respectively. The resulting interactions are called \( I.q.p \). According to Ref. \([25]\) the quadrupole renormalization (due to \( 2\hbar \omega \) perturbative couplings) amounts to 30\%, a theoretically sound result empirically validated by the best phenomenological interactions in the \( sd \) and \( pf \) shells. By the same token the effective charges in \( 0\hbar \omega \) spaces are estimated as \( (e_\pi, e_\nu) = (0.46, 1.31) \), as confirmed in Refs. \([26, 27]\). For the pairing component, perturbation theory is not a good guide, but comparison with the phenomenological interactions demands a 40\% increase \([14, 25]\). It follows that \( 3.4 \) and \( (e_\pi, e_\nu) = (0.46, 1.31) \) should be taken as standard for full \( 0\hbar \omega \) spaces.

As we will be working in very truncated ones, renormalizations should be implemented. A hint comes from the need to reduce the very large effective charges invoked in Table \(4\) through polarization mechanisms that involve excitations to the \( g \) shell. Proton jumps will contribute to \( e_\pi \) and are expected to have greater impact than the corresponding neutron jumps, rapidly blocked by the \( (r^4_\pi)^{10-4} \) particles. As a consequence we set \( e_\pi = 1.4 \), a guess close to the standard value, and let \( e_\nu \) vary, thus becoming the only adjustable parameter in the calculations. A choice validated later and illustrated in Fig. \(3\).

In Figure \(3\) it is seen that \( utM = 000 \) and 101 give the same results provided \( e_\nu \) is properly chosen. There is little difference between \( utM = 111 \) and \( utM = 101 \).
because as soon as neutrons are added they block the corresponding jumps, as mentioned above.

The calculation yields near perfect agreement with the Alaga rule: \( B(E2 : 4^+ \rightarrow 2^+) / B(E2 : 2^+ \rightarrow 0^+) \approx 1.43 B(E2 : 2^+ \rightarrow 0^+) \). In the figure it is shown for \( uM = 101 \) but it holds as well for 000 and 111. The more stringent quadrupole quotient rule Eq. (2) yields an average \( qQ = 0.96 \) for \(^{106-110}\)Cd, corroborating the existence of a deformed region.

![Graph](image)

FIG. 4. Upper panel: Experimental and calculated \( B(E2 : 2^+ \rightarrow 0^+) \) values for Sn isotopes. 1.3.4 interaction. In parenthesis \( (e_u, e_o = 1.40) \) is fixed. Experimental values are averages from [3–10]. Except for \(^{104-106}\)Sn, the experimental points are consistent with those in FIG. 3. Lower panel \( B(E2 : 4^+ \rightarrow 2^+) \) data from Jonsson et al. [2] for \(^{112-114}\)Sn, and from Siciliano et al. [28] for \(^{106-108}\)Sn. 1.3.2 and 1.3.4 calculations. The latter with the \( s_{\frac{1}{2}} \) single particle energy displaced by 0.0, 0.4 and 0.8 MeV with respect to GEMO value (0.8 MeV).

In moving to the tin isotopes the \( Pr^4 \) part of the SP scheme becomes isolated and sensitive to details of the interaction. Bäck and coworkers [29, FIG. 3] suggest that a parabolic trend as found in Banu et al. [3], or schematically in FIG. 4 can be modified in a \( utM = 000 \) context, by changes in the single particle behavior, thus leading to the first tentative explanation of the plateau. The more complete calculations of Togashi et al. [30, FIG. 2] demand \( g \) excitations to achieve a satisfactory result, very close to ours in the upper FIG. 4 in spite of huge differences in the \( g \) proton occupancies (spin and mass

dependent in their case, nearly constant in ours.

The \( B(E2 : 4^+ \rightarrow 2^+) \) rates at the bottom of FIG. 4 turn out to be strongly dependent on the \(^{101}\)Sn single particle spectrum. In [28, Fig. 3.2.1] it is seen that our choice, DZ(GEMO) [23], differs from the extrapolated value (EX) in the position of the \( s_{\frac{1}{2}} \) orbit: 0.8 and 1.6 MeV respectively. Accordingly, in FIG. 4 the effect of displacing the DZ orbit to bring it closer to EX is seen to be significant. Thanks to the recent \(^{108}\)Sn \( B(E2 : 4^+ \rightarrow 2^+) \) measure of Siciliano et al. [28], the DZ choice is clearly favored.

The \( B(E2 : 4^+ \rightarrow 2^+) / B(E2 : 2^+ \rightarrow 0^+) \approx 1 \) anomaly had been detected in \(^{114}\)Xe [33], in \(^{114}\)Te [34], and more recently in \(^{172}\)Pt, [35], where it is stressed that no theoretical explanation is available. The difference our work makes is that a calculation can produce it. A tentative interpretation will be sketched at the end.

In Table II the naive \( Pr^4 \) dimensionless intrinsic quadrupole moments for prolate (\( q_0p \)) and oblate (\( q_0o \)) states. Calculated spectroscopic Quadrupole moments and g-factor, \( Q^2, Q^4, g \) for 1.3.4 \( (e_u, e_o) = 0.72, 1.40; g_{lm}=-2.869, g_{lπ}=0.070 \) [31], \( g_{νπ}=4.189, g_{2π}=1.00. \) Experimental \( Q^2^* \) and \( g^* \) from Allmond et al. [32], \( g_{2π} \) quenched by 0.75 with respect to bare values [14, FIG. 28]

| N   | \( q_0p \) | \( q_0o \) | \( Q^2 \) | \( Q^4 \) | \( Q^2^* \) | \( g^* \) | \( g \) |
|-----|-----------|-----------|---------|---------|----------|---------|-----|
| 52  | 12        | 6         | 18      | 24      | 101      | 0.150   | 0.157 |
| 54  | 18        | 12        | 17      | 21      | 101      | 0.012   |       |
| 56  | 24        | 18        | 17      | 17      | 101      | 0.103   |       |
| 58  | 24        | 24        | 14      | 14      | 101      | 0.142   |       |
| 60  | 24        | 24        | 10      | 10      | 101      | 0.142   |       |
| 62  | 24        | 24        | 26      | 26      | 101      | 0.150(43)| 0.135 |
| 64  | 18        | 24        | 10      | 10      | 101      | 0.138(63)| 0.106 |

TABLE II. Intrinsic adimensional \( q^0 \) for prolate (\( q_0p \)), and oblate (\( q_0o \)) states.
are quite different.

In the case of Sn, the interaction, when in doubt, favors oblate. In particular at \( N = 64 \) when it becomes the only option (consistent with data in Allmond et al. [32]). At \( N = 66 \) the \( \Pr_4 \) scheme fails as, according to Eq. 5, adding a pair leads to \( q(n) = -18 \) and \( BE(2 \to 0) \approx 200 e^2 fm^4 \), too small compared to the observed \( \approx 400 e^2 fm^4 \). The alternative is to add a \( Sh \) pair \((q_0 = -10\text{, from Eq. 1})\), for a total \((q(n) = 24 + 10\text{ and some, too strong, } 600 e^2 fm^4)\). Certainly of help in providing the necessary boost thorough some mixing mechanisms that we shall not try to discover here.

For \( N = 64 \) the calculations square with the experimental trend at \( ^{114} \text{Sn} \), while they badly underestimate it at \( ^{112} \text{Cd} \) (about 600 \( e^2 fm^4 \) against the observed 900). Promoting an oblate \( h \) pair is of no help. What we suggest is a Quasi SU3 mechanism (Q) illustrated in Fig. 6. It is seen that promoting one or two Q\((hfp)\) pairs brings in \( q_0 = 19 \) or 32, which added to the corresponding \( q(n) \) in Eq. 2 lead, through Eq. 3, to \( BE(2 \to 0^+) \approx 1500-2500 e^2 fm^4 \) (equivalent to some 50-80 W.u.) Experientially there is a strong intruder transition \((BE2 : 2_3 \to 0_2 = 120(50) \text{ W.u.) and in turn a substantial } BE2 : 0_2 \to 2_1 = 51(14) \text{ W.u. indicating strong mixing between the intruder and the ground state } (BE2 : 2_1 \to 0_1 = 30(3) \text{ W.u.) bands. Such a mechanism can be expected to provide the needed boost in the latter.}

The core argument of this study is that variants of Elliott’s SU3 symmetry provide an interpretable background to shell model work. For the Cd isotopes, SP mechanisms are quantitatively valid, for Sn, P is a good guide though it does not hold strictly, and the digression in the previous paragraph calls upon Q as a plausible actor: SPQR (R for representation) is or will be useful in other regions. For similar argumenta see [28]

The preceding paragraph amounts to closing remarks but two things are missing: 1) a proof that working in a restricted space is equivalent to working in the full one i.e., validating the Shell Model. II) The precise status of \( \Pr_4 \) in the Sn isotopes.

There are no calculations in the full space, but very large ones, \( utM = 444 \) (m-dimensional \( 10^{10} \)) are available for \( ^{106-108} \text{Sn} \) using the same interaction (called B in what follows) as in Bam et al. [3] i.e., CDB [29], renormalized following Ref. [40]. In Figure 6 it is shown as B444 (open circles) and compared with B202 (squares, the same interaction in our standard space). The agreement is very good for the two points in \( B(E2 : 2^+ \to 0^+) B(E2 : 4^+ \to 2^+) \). The result amounts to a splendid vindication of the shell model.

For much of the region discrepancies between I.3.4 and B can be traced to poor monopole behavior of the latter. If the interaction is made monopole free and supplemented by the GEMO single particle field used in our I.p.q forces, the resulting BG202 in Fig. 6 produces \( B(E2 : 2^+ \to 0^+) \) patterns identical to our I.3.4-202, while for \( B(E2 : 4^+ \to 2^+)/B(E2 : 2^+ \to 0^+) \) the pattern is close to I.3.0 (not shown, but it can be guessed by extrapolation in Fig. 6 and from the analysis in [25] revealing the same \( q-q \) content in I.3.4 and B, and a much weaker pairing for the latter. So weak in fact that the B results become close to the Alaga rule.

It follows that for I.3.0, say, the \( \Pr_4 \) symmetry will hold. As the pairing force is switched on, the \( J = 0_1, 2_1 \) states are not affected, while \( J = 4_1 \) is. Obviously be-
cause some mixing takes place with states that are very sensitive to pairing. This is the crucial point: coexistence and mixing of deformed bands are common but mixing between a deformed state and a pairing dominated one is not common. Work is in progress to identify these pairing states fully to explain the $B(E2 : 4^+ \rightarrow 2^+)/B(E2 : 2^+ \rightarrow 0^+) < 1$ anomaly.

The Sn isotopes were an example of pairing, then of quadrupole, then of pairing again. The world is but a perennial swing (Le monde n’est qu’une branloire perenne. Essais III 2).

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