Hole doping dependences of the magnetic penetration depth and vortex core size in YBa$_2$Cu$_3$O$_y$: Evidence for stripe correlations near 1/8 hole doping

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(Dated: February 1, 2008)

We report on muon spin rotation ($\mu$SR) measurements of the internal magnetic field distribution $n(B)$ in the vortex solid phase of YBa$_2$Cu$_3$O$_y$ (YBCO) single crystals, from which we have simultaneously determined the hole doping dependences of the in-plane Ginzburg-Landau (GL) length scales in the underdoped regime. We find that $T_c$ has a sublinear dependence on $1/\lambda_{ab}^2$, where $\lambda_{ab}$ is the in-plane magnetic penetration depth in the extrapolated limits $T \to 0$ and $H \to 0$. The power coefficient of the sublinear dependence is close to that determined in severely underdoped YBCO thin films, indicating that the same relationship between $T_c$ and the superfluid density is maintained throughout the underdoped regime. The GL coherence length $\xi_{ab}$ (vortex core size) is found to increase with decreasing hole doping concentration, and exhibit a field dependence that is explained by proximity-induced superconductivity on the CuO chains. Both $\lambda_{ab}$ and $\xi_{ab}$ are enhanced near 1/8 hole doping, supporting the belief by some that stripe correlations are a universal property of high-$T_c$ cuprates.

I. INTRODUCTION

Abrikosov vortices in a superconductor are governed by two characteristic length scales. The core of a vortex has a size dependent on the Ginzburg-Landau (GL) coherence length $\xi$, while the supercurrents circulating around the core decay on the scale of the GL penetration depth $\lambda$. In the early days of high-$T_c$ superconductivity it was common practice to infer the behavior of the in-plane magnetic penetration depth $\lambda_{ab}$ from measurements of the muon depolarization rate $\sigma$ in the vortex state of polycrystalline samples. The temperature dependence of $\sigma$ was found to be consistent with $s$-wave pairing symmetry and a universal linear scaling of $T_c$ with $\sigma$ was observed in the underdoped regime (the so-called ‘Uemura plot’), indicating that $T_c \propto 1/\lambda_{ab}^2 \propto \rho_s$, where $\rho_s$ is the superfluid density. Later, microwave and $\mu$SR measurements on YBa$_2$Cu$_3$O$_y$ (YBCO) single crystals in the Meissner and vortex phases established a limiting low-temperature linear $T$ dependence of $\lambda_{ab}$ that is consistent with $d$-wave pairing. More recently, non-$\mu$SR studies of YBCO in the Meissner phase have revealed that $T_c$ has a sublinear dependence on $1/\lambda_{ab}^2$. On the other hand, the relation $T_c \propto \rho_s$ inferred from the Uemura plot is supported by a recent study of electric-field induced superfluid density modulations in a single underdoped ultra-thin film of La$_2-\delta$Sr$_\delta$CuO$_4$. The problem with assuming $\sigma \propto 1/\lambda_{ab}^2$ is that there are additional inseparable contributions to $\sigma$ from electronic magnetic moments and flux-line lattice (FLL) disorder, which may vary with doping. To circumvent this difficulty we have studied YBCO single crystals. In a single crystal the FLL contribution to the $\mu$SR line shape $n(B)$ is asymmetric and distinct from the other sources of field inhomogeneity. Not only can the behavior of $\lambda_{ab}$ be isolated, but because the finite size of the vortex cores is apparent in a single-crystal measurement of $n(B)$, $\xi_{ab}$ can be simultaneously determined. While $\xi_{ab}$ may be accurately determined in conventional superconductors from measurements of the upper critical field $H_{c2}$, in high-$T_c$ cuprates $H_{c2}$ is generally a very high magnetic field marking the transition from a vortex liquid to the normal phase. Here we present $\mu$SR measurements that probe $\lambda_{ab}$ and $\xi_{ab}$ in the bulk of YBCO single crystals deep in the superconducting state. The accuracy of our method was demonstrated in previous studies of conventional superconductors and is reinforced here through comparisons with the results from other techniques.

II. EXPERIMENTAL DETAILS

YBCO single crystals with purity of 99.995 % were grown by a self-flux method in fabricated BaZrO$_3$ crucibles at the University of British Columbia. An exception are $y = 6.60$ single crystals that were grown in yttria-stabilized-zirconia crucibles and characterized by a purity greater than 99.5 %. Single crystals of Ca-doped YBCO were also prepared in BaZrO$_3$ crucibles. Typical sample sizes consisted of 3 to 5 single crystals from the same growth batch arranged in a mosaic to form a total $\bar{a}$-$\bar{b}$ surface area of 20-30 mm$^2$. The thickness of the crystals are on the order of $\sim 0.1$ mm. The superconducting transition temperatures of the single crystals were measured using a SQUID magnetometer. Twin boundaries
TABLE I: Characteristics of the YBa$_2$Cu$_3$O$_y$ and (Y, Ca)Ba$_2$Cu$_3$O$_{6.98}$ single crystals. The hole concentration $p$ per CuO$_2$ layer is determined from the dependence of $T_c$ on $p$ presented in Ref. 19 for similar YBCO single crystals.

| $y$     | $p$  | $T_c$ (K) | Detwinned |
|---------|------|-----------|-----------|
| 6.60    | 0.103| 62.5      |           |
| 6.57    | 0.110| 59.0      |           |
| 6.67    | 0.120| 66.0      |           |
| 6.75    | 0.132| 74.6      | ✓         |
| 6.80    | 0.140| 84.5      | ✓         |
| 6.95    | 0.172| 93.2      | ✓         |
| (Y,Ca)6.98| 0.192| 86.0      |           |

were removed from some of the higher-doped single crystals by applying pressure along the $\hat{a}$ or $\hat{b}$ directions at elevated temperature. These basic sample characteristics are summarized in Table I.

The $\mu$SR experiments were performed over a 3 year period on the M15 and M20B surface muon beam lines at TRIUMF, Vancouver, Canada. The $\mu$SR spectra were recorded in a transverse-field (TF) geometry with the applied magnetic field $\mathbf{H}$ perpendicular to the initial muon spin polarization direction, and perpendicular to the $c$-axis of the single crystals. A TF-$\mu$SR spectrum comprised of 20 to 30 million muon decay events was taken at each temperature and magnetic field.

In a transverse-field, the muon spin precesses in a plane perpendicular to the field direction (which we define here as the $z$-direction). The time evolution of the muon spin polarization $P(t)$ is determined from the $\mu$SR “asymmetry” spectrum formed from the muon decay events detected in opposing positron counters

$$A(t) = \frac{N_1(t) - N_2(t)}{N_1(t) + N_2(t)} = a_0 P(t), \quad (1)$$

where $N_i(t)$ is the time histogram of the temporal dependence of decay positron count rate in the $i^{th}$ detector, and $a_0$ is the asymmetry maximum. In our experiments, four positron counters were used to completely cover the 360° solid angle in the $x$-$y$ plane (see Fig. 1). The muon spin polarization function for the “Left”-“Right” pair of detectors is defined as

$$P_x(t) = \int_0^\infty n(B) \cos(\gamma_\mu Bt + \phi) dB, \quad (2)$$

and for the “Up”-“Down” pair as

$$P_y(t) = \int_0^\infty n(B) \cos(\gamma_\mu Bt + \phi - \pi/2) dB, \quad (3)$$

where $\gamma_\mu$ is the muon gyromagnetic ratio, $\phi$ is a phase constant, and

$$n(B') = \langle \delta(B' - B(x)) \rangle, \quad (4)$$

is the probability of finding a local magnetic field $B$ in the $z$-direction at an arbitrary position $r$ in the $x$-$y$ plane. (Color online) Schematic of the positron counter arrangement used in the present study. The muon beam axis and the applied magnetic field are perpendicular to the page (i.e. parallel to the $z$-axis). The muon spin precesses in the $x$-$y$ plane about the local $z$-component of the magnetic field before undergoing the decay $\mu^+ \rightarrow e^+ + \nu_e + \nu_\mu$. In our study, the muon spin polarization $P(t)$ is formed from approximately 20 to 30 million muon decay events. The decay events detected in the “Left” and “Right” positron counters form the $x$-component of $P(t)$, while decay events detected in the “Up” and “Down” positron counters form the $y$-component.

### III. DATA ANALYSIS METHOD

The TF-$\mu$SR time spectra for each sample were fit assuming the following analytical solution of the GL equations

$$B(r) = B_0 \sum_G \frac{e^{-iG \cdot r}}{\lambda_{ab}^2 G^2} F(G), \quad (5)$$

where $G$ are the reciprocal lattice vectors, $B_0$ is the average internal magnetic field, $F(G) = uK_1(u)$ is a cutoff function for the $G$ sum, $K_1(u)$ is a modified Bessel function, and $u = \sqrt{\xi_{ab}G}$. The cutoff function $F(G)$ depends on the spatial profile of the superconducting order parameter at the center of the vortex core. Consequently, $\xi_{ab}$ is a measure of the vortex core size. The FLL in all samples was assumed to be hexagonal. Neutron scattering experiments on fully-doped YBCO indicate that the FLL below $H \approx 40$ kOe is only slightly distorted from hexagonal symmetry due to $a/b$ anisotropy. We find that accounting for this small distortion changes the values of $\xi_{ab}$ and $\lambda_{ab}$ by less than 5%. Consequently, the FLL was assumed to be hexagonal for all samples studied.

To avoid the difficulty of modelling the contribution of electronic magnetic moments to the $\mu$SR line shape, we restricted our study to YBCO crystals free of static or quasistatic spins. For the applied fields considered in this study, this has been determined to be the case for...
FIG. 2: (Color online) μSR line shapes in single-crystal YBCO. Fourier transforms of the TF-μSR signal from $y = 6.75$ and $y = 6.95$ samples at $T = 2.5$ K and $H = 4.92$ kOe (green circles). The righthand peak for $y = 6.95$ is a background signal coming from muons stopping outside the sample (Note, the background and sample peaks nearly coincide in the μSR line shape for $y = 6.75$). The red curves through the data are the Fourier transforms of the fits in the time domain. The contour plots show the corresponding spatial dependence of the supercurrent density $j(x, y) = |\nabla \times B(x, y)|$, providing a visual illustration of the change in core size with hole doping. Oxygen content $y > 6.50$.22

To properly account for disorder of the FLL, the dimensionality of the vortices must be considered. Josephson plasma resonance measurements on YBa$_2$Cu$_3$O$_{6.50}$ ortho-II single crystals grown by Liang, Bonn and Hardy indicate that the vortices are 3D-like at low temperatures, while mutual inductance measurements on thin films by Zuev et al show that even severely underdoped YBCO is quasi-2D only near $T_c$.18 Since the focus in the present study is on the variation of $\lambda_{ab}$ and $\xi_{ab}$ in higher doped samples at low temperatures, the vortices are assumed to be rigid 3D lines of flux. This assumption is also consistent with the observation of highly asymmetric μSR line shapes for all of our samples at low $T$ (see Fig. 2). For rigid flux lines, random displacements of the vortices from their positions in the ideal hexagonal FLL are accounted for by convoluting the theoretical line shape $n(B)$ by a Gaussian distribution of fields.23 A Gaussian function also describes the local distribution of dipolar fields originating from static nuclear moments.23 Taking into account both sources of line broadening, the corresponding theoretical polarization functions are

$$P_x(t) = e^{-\sigma_{x}^2 t^2/2} \int_0^\infty n(B) \cos(\gamma_B t + \phi) dB,$$  

$$P_y(t) = e^{-\sigma_{y}^2 t^2/2} \int_0^\infty n(B) \cos(\gamma_B t + \phi - \pi/2) dB,$$

where,

$$\sigma_{eff}^2 = \sigma_{dip}^2 + \sigma_{FLL}^2,$$

is an effective depolarization rate due to nuclear dipole moments ($\sigma_{dip}$) and FLL disorder ($\sigma_{FLL}$). Values for $\sigma_{dip}$ were obtained by fitting the TF-μSR signal above $T_c$ to the theoretical polarization function

$$P(t) = e^{-\sigma_{dip}^2 t^2/2} \cos(\gamma_B t + \phi).$$

To account for the background signal from muons that did not stop in the sample, an additional term of the form $(1-f)e^{-\sigma_{dip}^2 t^2/2} \cos(\gamma_B t + \phi_B)$ was added to Eq. (7) and to Eq. (8), where $f$ is the fraction of muons that stopped inside the sample. Values of $f$ for the different samples ranged from 0.8 to 0.9.

In the present work there are two marked improvements over the analysis done in our previous studies of YBCO single crystals.26,27,28 (i) The earlier works used the asymptotic limit $K_1(u) = \sqrt{\pi/2u} \exp\left(-u\right) (u \gg 1)$ for the Bessel function that appears in the cutoff $F(G)$, whereas here $K_1(u)$ was evaluated numerically. (ii) The second improvement is that due to increased computer speed, $B(\mathbf{r})$ was calculated at 15,132 equally-spaced locations in the rhombic unit cell of the hexagonal FLL, compared to 5,628 locations in previous works. Further increasing the number of real-space points sampled in the FLL unit cell did not result in appreciable changes in the fitted parameters. We note that both improvements in our data analysis method influence the absolute values of $\lambda_{ab}$ and $\xi_{ab}$, but the temperature and magnetic field dependences of these parameters remain qualitatively similar to that determined in our previous studies.

IV. RESULTS FOR $\lambda_{ab}$

A. Temperature dependence

Figure 3 shows the temperature dependence of $\lambda_{ab}$ at low $T$ determined at two different values of the applied
magnetic field. The solid curves are fits to \( \lambda_{ab}(T, H) = \lambda_{ab}(0, H) + \alpha T^n \), where \( \alpha \) and \( n \) are field-dependent coefficients. The dependence of \( 1/\lambda_{ab}^2 \) on \( T \) is shown in Fig. 4 for selected values of the applied field. An inflection point at \( T \approx 20 \) K is visible in some of the lower field data. This feature was also apparent in our previous measurements of YBa\(_2\)Cu\(_3\)O\(_{6.95}\)\(\lambda\) and Harshman et al have argued that the inflection point is caused by thermal depinning of vortices\(^{29}\) although an invalid treatment of the data was used to support this assertion.\(^{29}\) Recently, Khasanov et al have ruled out depinning as the source of a similar inflection point in the temperature dependence of \( 1/\lambda_{ab}^2 \) measured in La\(_{1.85}\)Sr\(_{0.17}\)CuO\(_4\) by TF-\(\mu\)SR.\(\text{[30]}\) Instead they attribute this feature to the occurrence of both a large \( d \)-wave and a small \( s \)-wave superconducting gap. As we will show later, the anomalous magnetic field dependence of the vortex core size in YBCO can be explained by an induced superconducting energy gap in the CuO chains that run along the \( b \) direction. Theoretical calculations by Atkinson and Carbotte\(\text{[30]}\) for a \( d \)-wave superconductor with proximity-induced superconductivity in the CuO chains, show that \( 1/\lambda_{ab}^2(T) \) exhibits an inflection point caused by an upturn of \( 1/\lambda_{ab}^2(T) \) at low \( T \) (where \( \lambda_0 \) is the penetration depth in the \( b \) direction).

In Fig. 3 it is shown that \( \lambda_{ab}^2(T \to 0)/\lambda_{ab}^2(T) \) exhibits a near universal linear temperature dependence at low \( T \). We attribute deviations from universal behavior near \( T_c \) to softening of the FLL, which narrows the \( \mu\)SR line shape and enhances the fitted value of \( \lambda_{ab} \). The universal scaling implies that \( \lambda_{ab}^2(T \to 0)/\lambda_{ab}^2(T) \) is a constant\(\text{[32]}\) where \( v_F \) is the Fermi velocity, \( v_\Delta \) is a velocity corresponding to the slope of the gap at the nodes, and \( Z^c \) is a charge renormalization parameterizing the coupling of the quasiparticles to phase fluctuations. Using values of \( v_\Delta \) from thermal conductivity measurements\(\text{[33]}\) we find that \( Z^c \) is basically doping independent.

### B. Magnetic field dependence

Figure 4 shows the magnetic field dependences of \( \lambda_{ab} \). Here we stress that the observed behaviors do not imply that the magnetic penetration depth or superfluid density depend on field in this way. The sublinear dependence of \( \lambda_{ab} \) on \( H \) is primarily due to the failure of Eq. 8 to account for all field-dependent contributions to the internal magnetic field distribution. In Refs.\(\text{28,34} \) the strong field dependence of \( \lambda_{ab} \) in YBa\(_2\)Cu\(_3\)O\(_{6.95} \) de-
A nonlocal supercurrent response to the applied field in the vicinity of the vortex cores stemming from the divergence of the coherence length at the gap nodes modifies the spatial distribution of field. With increasing $H$, the increased overlap of the regions around the vortex cores reduces the width of the $\mu$SR line shape. The gap anisotropy also results in a nonlinear supercurrent response to the applied field, resulting from a quasiclassical ‘Doppler shift’ of the quasiparticle energy spectrum by the flow of superfluid around a vortex. When the Doppler shift exceeds the energy gap, Cooper pairs are broken, and $\lambda_{ab}$ increases.

These effects are not restricted to $d$-wave superconductors. Sizeable nonlinear and/or nonlocal effects can also occur in $s$-wave superconductors with a smaller energy gap on one of the Fermi sheets and/or a highly anisotropic Fermi surface. Moreover, these anisotropies result in a rapid delocalization of quasiparticle core states with increasing $H$ that modify $n(B)$. Indeed, strong field dependences of $\lambda_{ab}$ from Eq. (5) have been observed in the multi-band superconductor NbSe$_2$ and the marginal type-II superconductor V. It has been experimentally established for a variety of materials including YBCO that the $H \to 0$ extrapolated value of $\lambda_{ab}$ agrees with the magnetic penetration depth measured by other techniques in the Meissner phase. Consequently, we stress that only $\lambda_{ab}(H \to 0)$ can be considered a “true” measure of the magnetic penetration depth.

**C. Hole doping dependence**

In Fig. 7 we show $T_c$ as a function of $1/\lambda_{ab}^2(T \to 0)$ at two different fields. The more inclusive data set at $H = 5$ kOe is described by $T_c \propto (1/\lambda_{ab}^2)^{0.38}$, which deviates substantially from the linear scaling in the Uemura plot. The power 0.38 is surprisingly close to 0.43 determined by Zuev et al in a Meissner phase study of severely underdoped YBCO thin films. It is surprising because these thin films have a superfluid density that is significantly lower than in single crystals. It is known from microwave studies of YBCO that the doping dependences of $\lambda_a$ and $\lambda_b$ are not the same due to the conductivity of the CuO chains. Thus there is no reason to expect the power $\sim 0.4$ to be universal for the cuprates. While a sublinear dependence of $T_c$ on $1/\lambda_{ab}^2$ has also been inferred from more recent $\mu$SR measurements of the muon depolarization rate $\sigma$ in other high-$T_c$ superconductors, the contributions of magnetism and FLL disorder to the $\mu$SR line shape were not factored out.

Figure 8 shows $1/\lambda_{ab}^2$ as a function of hole doping, where the values of $p$ are determined from the depen-
FIG. 7: (Color online) Dependence of $T_c$ on $1/\lambda_{ab}^2(T \rightarrow 0)$. The two data points on the far right are for the overdoped sample $(Y, Ca)Ba_2Cu_3O_{6.98}$. The solid curves are fits to the data for $YBa_2Cu_3O_y$, yielding $T_c = (19.3 \pm 1.7 \text{ K})/\lambda_{ab}^2$ and $T_c = (25.5 \pm 1.9 \text{ K})/\lambda_{ab}^2$ at $H = 5 \text{ kOe}$ and $H = 15 \text{ kOe}$, respectively.

FIG. 8: (Color online) Dependence of $1/\lambda_{ab}^2(T \rightarrow 0)$ on hole doping concentration $p$ at $H = 5 \text{ kOe}$. The dashed curve is the function $1/\lambda_{ab}^2 \propto (p - 0.05)^{1.42}$, where $p = 0.5$ is the critical hole doping concentration for the onset of superconductivity.

FIG. 9: (Color online) (a) Hole doping dependences of the Gaussian depolarization rates $\sigma_{\text{FLL}}$ and $\sigma_{\text{dip}}$ at $T \rightarrow 0 \text{ K}$ and $H = 5 \text{ kOe}$. (b) Hole doping dependence of the root-mean-square displacement $\langle s^2 \rangle^{1/2}$ of the vortices from their positions in the perfect hexagonal FLL at $T \rightarrow 0 \text{ K}$ and $H = 5 \text{ kOe}$ plotted as a percentage of the intervortex spacing $L$.

The hole doping dependences of $\sigma_{\text{dip}}$ and $\sigma_{\text{FLL}}$ at $H = 5 \text{ kOe}$ are shown in Fig. 9(a). While $\sigma_{\text{dip}}$ is independent of $p$, $\sigma_{\text{FLL}}$ basically tracks $1/\lambda_{ab}^2$. Using the fitted values of $\sigma_{\text{FLL}}$ and $\lambda_{ab}$ we have calculated the hole doping dependence of the root-mean-square displacement $\langle s^2 \rangle^{1/2}$ of the vortices from their positions in the perfect hexagonal FLL. As shown in Fig. 9(b), the degree of FLL disorder is small and as expected highest in the Ca-doped sample.
the core size by the presence of the CuO chains. The calculations of in particular the upturn at low field, can be explained considerably stronger than predicted for a pure model representing a superconducting CuO approximation (see Appendix) for either a single-layer explanation. Consequently, we consider an alternative core size is predicted to shrink with increasing field. Hence the vortex particles between vortices at low field, which is further enhanced by an increase in vortex density. This allows for a large transfer of low-energy quasiparticles between vortices at low field, which is further enhanced by an increase in vortex density. The vortex core size is predicted to shrink with increasing field. However, the field dependence of \( \xi_{ab} \) in YBCO is considerably stronger than predicted for a pure \( d \)-wave superconductor. Consequently, we consider an alternative explanation.

In Fig. 11 we show that the field-dependence of \( \xi_{ab} \), in particular the upturn at low field, can be explained by the presence of the CuO chains. The calculations of the core size \( R \) are based on a semiclassical Doppler-shift approximation (see Appendix) for either a single-layer model representing a superconducting CuO\(_2\) plane or a proximity-coupled model representing a CuO\(_2\)-CuO bilayer. In the bilayer model there are two distinct energy scales for pair breaking: the energy gap associated with Cooper pairs in the CuO\(_2\) planes, and a smaller proximity-induced gap associated with the chains. It is the latter scale which is responsible for the expansion of the vortex cores at low field.

**V. RESULTS FOR \( \xi_{ab} \)**

**A. Magnetic field dependence**

The field dependences of \( \xi_{ab} \) are shown in Fig. 11. The increase in \( \xi_{ab} \) at low field, which corresponds to an expansion of the vortex cores, was previously reported for YBa\(_2\)Cu\(_3\)O\(_{6.60}\) (Ref. 26) and YBa\(_2\)Cu\(_3\)O\(_{6.95}\) (Refs. 27, 28). In all samples we find that \( \xi_{ab} \) scales as \( 1/\sqrt{H} \), which is proportional to the intervortex spacing. In \( s \)-wave superconductors the field dependence of \( \xi_{ab} \) has been shown to originate from the delocalization of bound quasiparticle core states.14,15 This is because a change in the spatial dependence of the pair potential accompanies the change in electronic structure of the vortex cores. In YBCO the low-energy quasiparticle core states should be extended along the nodal directions of the \( d \)-wave gap function.29 This allows for a large transfer of low-energy quasiparticles between vortices at low field. However, the field dependence of \( \xi_{ab} \) in YBCO is considerably stronger than predicted for a pure \( d \)-wave superconductor. Consequently, we consider an alternative explanation.

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**B. Hole doping dependence**

Figure 12 shows the hole doping dependences of \( \xi_{ab}(T \to 0) \) at \( H = 5 \) kOe and \( H = 15 \) kOe. Qualitatively, the doping dependence of \( \xi_{ab} \) is similar to that reported by Ando et al from magnetoconductance measurements on dwinned YBCO single crystals.33 This result is also shown in Fig. 12 but plotted as \( \xi_{ab} \) versus \( \delta \). Note that our data must be plotted as \( \xi_{ab} \) versus \( \delta \), because the hole doping concentration of our \( y = 6.60 \) single crystals (grown in a different kind of crucible than the other samples), is smaller than that of \( y = 6.57 \) (see Table I). The general trend of all data sets is an increase of \( \xi_{ab} \) with decreasing \( \delta \). Such behavior has also been observed in the underdoped regime of La\(_{2−x}\)Sr\(_x\)CuO\(_4\) (Ref. 26) and Bi\(_2\)Sr\(_2\)CuO\(_6+x\).37 With increasing magnetic field, our values for \( \xi_{ab} \) approach those determined by Ando et al. Note that based on our proximity-induced model for the field dependence of \( \xi_{ab} \) (see Appendix), it is the high-field values of \( \xi_{ab} \) that reflect the intrinsic superconductivity of the CuO\(_2\) planes.

Since the doping dependences of \( \xi_{ab} \) at \( H = 5 \) kOe and
measurements showing a refinement of the Uemura plot of conventional superconductors. This definition was established in previous studies of con- 

We ourselves find no evidence for static stripes in zero-field μSR or TF-μSR measurements on our samples. However, the suppression of superconductivity near \( p = 1/8 \) could be caused by fluctuating stripes, recently argued to be relevant in YBCO and other cuprates. Experimental evidence for dynamic stripes in YBCO includes the detection of low-energy one-dimensional incommensurate modulations in \( \text{YBa}_2\text{Cu}_3\text{O}_y \) by inelastic neutron scattering. Furthermore, we found no evidence for the behavior of the GL coherence length, \( \xi_{ab} \), which characterizes the size of the vortex cores. While it mimics the spatial dependence of the superconducting order parameter, enhancement of the GL coherence length or vortex core size near \( 1/8 \) hole doping, has also been observed in \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) calculations by Mierzejewski and Máša. They show that static or quasi-static stripes actually intensify \( H_c^\ast \) by reducing diamagnetic pair breaking, and hence cannot explain the growth of \( \xi_{ab} \) near \( p = 1/8 \). On the other hand, these measurements on our samples. However, the suppression of superconductivity near \( p = 1/8 \) could be caused by fluctuating stripes, recently argued to be relevant in YBCO and other cuprates. Experimental evidence for dynamic stripes in YBCO includes the detection of low-energy one-dimensional incommensurate modulations in \( \text{YBa}_2\text{Cu}_3\text{O}_y \) by inelastic neutron scattering. Furthermore, we found no evidence for the behavior of the GL coherence length, \( \xi_{ab} \), which characterizes the size of the vortex cores. 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the other hand, Kadono et al. have shown that an expansion of the vortex cores with decreasing hole doping can result from a strengthening of antiferromagnetic correlations competing with superconductivity. Thus, dynamic stripes are a viable explanation for the increased size of the vortex cores near 1/8 hole doping.

**Acknowledgements**

We gratefully acknowledge D. Broun, I. Vekhter and A. J. Millis for helpful and informative discussions. We also thank Y. Ando for allowing us to reproduce his data here. This work was supported by the Canadian Institute for Advanced Research and the Natural Sciences and Engineering Research Council (NSERC) of Canada.

**Appendix: Semiclassical Calculation of the Vortex Core Size**

Calculations of the vortex core size are based on a generalisation of the so-called “doppler-shift” approximation for the vortex structure to the case of YBCO, which is a multiband superconductor. In the case of YBCO, there is strong evidence that both the two-dimensional CuO$_2$ planes and one-dimensional CuO chains superconduct. Furthermore, it is likely that the chains are intrinsically normal, but are driven superconducting by the proximity effect. Proximity models for YBCO have been extensively described elsewhere. The essential idea is that the superconductivity originates from a pairing interaction which is confined to the two-dimensional CuO$_2$ planes, and that the mixing of chain and plane wavefunctions induces superconductivity in the one-dimensional chain layers.

We adopt a simplified bilayer model consisting of a single plane and a single chain, with one Wannier orbital retained per unit cell for each layer. For comparison purposes, calculations are also performed for a single-layer model of an isolated superconducting plane. The Bogoliubov-de Gennes Hamiltonian for the bilayer is

\[ \hat{H} = \sum_{ij} \begin{bmatrix} \hat{t}_{1,ij} & \Delta_{ij} & t_{\perp} \delta_{i,j} & 0 \\ \Delta_{ij}^* & -\hat{t}_{1,ij}^* & 0 & -t_{\perp} \delta_{i,j} \\ t_{\perp} \delta_{i,j} & 0 & t_{2,ij} & 0 \\ 0 & -t_{\perp} \delta_{i,j} & 0 & -\hat{t}_{2,ij}^* \end{bmatrix} \hat{\Psi}_j \]  

where \( \hat{\Psi}_i = [\psi_{1i}^\dagger \psi_{2i}^\dagger \psi_{1i} \psi_{2i}] \) and \( \psi_{ni}^\dagger \) (\( \psi_{ni} \)) are creation operators for quasiparticles (quasiholes) at lattice site \( i \) in layer \( n \). Here, we take \( n = 1 \) for the plane layer and \( n = 2 \) for the chain layer. The parameters \( t_{n,ij} \) and \( t_{\perp} \delta_{i,j} \) are the single-electron hopping matrix elements between sites \( i \) and \( j \) within and between layers respectively, while \( \Delta_{ij} \) is the superconducting order parameter along bonds connecting nearest neighbour sites \( i \) and \( j \). From the form of Eq. (10), it is apparent that \( \Delta_{ij} \) only couples quasiparticles belonging to the plane layer. The single-layer Hamiltonian is obtained by setting \( t_{\perp} = 0 \).

A magnetic field \( H \) applied perpendicular to the layers induces circulating currents in the superfluid. The superfluid velocity is given by \( \mathbf{v}_s = \mathbf{M}^{-1} \cdot \mathbf{p}_s \) where \( M \) is the effective mass tensor, \( \mathbf{p}_s(r) = (2\pi \hbar/c) \mathbf{A}(r) + \hbar \nabla \phi(r) \) is the superfluid momentum, \( \mathbf{A}(r) \) is the magnetic vector potential and \( \phi(r) \) is the local phase of the order parameter. In the limit that \( \phi(r) \) and \( \mathbf{A}(r) \) are slowly varying functions, one can treat the superflow as uniform in the neighbourhood of \( r \). Then, one can make a local gauge transformation such that the phase is removed from the order parameter and appears instead in the hopping matrix elements \( t_{n,ij} \):

\[ \hat{t}_{n,ij} = t_{n,ij} e^{-i \mathbf{p}_s(r) \cdot (r_i - r_j) / 2 \hbar} \approx t_{n,ij} + \frac{1}{2} \mathbf{v}_{ij} \cdot \mathbf{p}_s(r) \]  

where \( t_{n,ij} \) are the hopping matrix elements in zero-field and \( \mathbf{v}_{ij} = -i t_{n,ij} (r_i - r_j) / \hbar \) are the matrix elements of the zero-field quasiparticle velocity. Equation (1) follows from Eq. (10) in the limit that \( \mathbf{p}_s \) is small. Then, the order parameter takes on the simple \( d \)-wave form \( \Delta_{ij} = \frac{1}{2} \Delta (-1)^{n_i - n_j} \) which, in reciprocal space, corresponds to \( \Delta_k = \Delta \cos(k_x a) - \cos(k_y a) \), where \( a \) is the lattice constant. The local gauge transformation leads to a doppler-shifted spectrum and is exact in the limit of slowly varying superfluid velocity. This procedure has been shown, in many circumstances, to provide a reasonable description of the vortex lattice.

We take band structures which are appropriate for YBCO and adopt

\[ t_{1,ij} = \begin{cases} t_0 & i = j \\ t_1 & i, j \text{ are nearest neighbours} \\ t_2 & i, j \text{ are next-nearest neighbours} \end{cases} \]

and

\[ t_{2,ij} = \begin{cases} t_3 & i = j \\ t_4 & i, j \text{ are nearest neighbours along } \hat{e}_y \end{cases} \]

For this work, we measure energies in units of \( |t_1| \) and take \( \{t_0, \ldots, t_4\} = \{-1, 0, 0.45, 2, -4\} \). In reciprocal space, the dispersions of the isolated plane and chain layers are then \( \epsilon_{1k} = t_0 + 2t_1 \cos(k_x a) + \cos(k_y a) \) + \( 4t_2 \cos(k_x a) \cos(k_y a) \) and \( \epsilon_{2k} = t_3 + 2t_4 \cos(k_y a) \), respectively. The chain-plane hopping matrix element \( t_{1,4} \) is not well known in YBCO and is taken to be \( t_{\perp} = 0.75 \).
For a slowly varying \( \mathbf{p}_s(\mathbf{r}) \) we can locally Fourier transform the Hamiltonian in the neighbourhood of \( \mathbf{r} \) to give

\[
\hat{H}(\mathbf{r}) = \sum_k \hat{\Psi}_k^\dagger \begin{bmatrix}
\epsilon_{1k} + \frac{i}{2} \mathbf{v}_{1k} \cdot \mathbf{p}_s(\mathbf{r}) & \frac{\Delta_k(\mathbf{r})}{t_\perp} & 0 \\
\frac{\Delta_k(\mathbf{r})}{t_\perp} & -\epsilon_{1k} + \frac{i}{2} \mathbf{v}_{1k} \cdot \mathbf{p}_s(\mathbf{r}) & 0 \\
\frac{1}{2} \mathbf{v}_{2k} \cdot \mathbf{p}_s(\mathbf{r}) & 0 & -\epsilon_{2k} + \frac{i}{2} \mathbf{v}_{2k} \cdot \mathbf{p}_s(\mathbf{r})
\end{bmatrix} \hat{\Psi}_k
\]  

(15)

where \( \mathbf{v}_{nk} = h^{-1} \partial \epsilon_{nk}/\partial \mathbf{k} \) and \( \hat{\Psi}_k = N^{-1/2} \sum_i \hat{\phi}_i \) where \( N \) is the number of \( k \)-points in the sum in Eq. (15).

We need to make an ansatz for \( \mathbf{p}_s(\mathbf{r}) \). For a single vortex in an isotropic medium, one has \( \mathbf{p}_s(\mathbf{r}) = (2\pi \hbar/r) \hat{\mathbf{e}} \), where \( \hat{\mathbf{e}} \) is the azimuthal unit vector and the radius \( r \) is measured relative to the centre of the vortex.\(^{32}\) For the bilayer model, however, \( \mathbf{p}_s(\mathbf{r}) \) is not isotropic: the chains provide a conduction channel along the \( \hat{y} \) direction which is in parallel with the isotropic plane conduction channel. We mimic this anisotropy by assuming that the superfluid momentum will be similar to that of a single-layer superconductor with an anisotropic (diagonal) effective mass tensor \( \mathbf{M} \) with \( M_{yy} < M_{xx} \). (For the single-layer model, we take \( M_{xx} = M_{yy} \).) We then have two requirements which must be satisfied:

\[
\nabla \times \mathbf{p}_s = 2\pi \hbar \sum \mathbf{G} \delta^2(\mathbf{r} - \mathbf{R}) \quad \text{and} \quad \nabla \cdot \mathbf{v}_s = \nabla \cdot \mathbf{M}^{-1} \mathbf{p}_s = 0.
\]

The first requirement introduces vortex cores at the vortex lattice sites \( \mathbf{R} \), while the latter incompressibility requirement is strictly true in regions where \( \Delta(\mathbf{r}) \) is uniform. This pair of equations is solved by

\[
\mathbf{p}_s(\mathbf{r}) = \frac{2\pi \hbar}{L^2} \sum' \mathbf{G} \mathbf{r} \circ \mathbf{G} \cdot (\mathbf{M} \times \hat{\mathbf{z}}) / \left( M_{xx} G_y^2 + M_{yy} G_x^2 \right),
\]

(16)

where \( \sum' \) indicates that \( \mathbf{G} = 0 \) is excluded from the sum, \( \mathbf{G} \) are reciprocal lattice vectors of the magnetic unit cell (we assume a square lattice here) with area \( L^2 \) and magnetic length \( L \). The results do not depend strongly on the ratio \( M_{yy}/M_{xx} \), which we take to be 0.6 for the parameters chosen above. This choice minimizes \( \nabla \cdot \mathbf{j}(\mathbf{r}) \), where \( \mathbf{j}(\mathbf{r}) \) is the total (plane and chain) current in the bilayer.

\[
\mathbf{j}(\mathbf{r}) = \frac{1}{N} \sum_k \sum_{n=1}^2 \langle \mathbf{v}_{1k} + \mathbf{v}_{2k} \rangle \mathbf{r},
\]

(17)

and \( \langle \ldots \rangle_r \) indicates the expectation value with respect to \( \hat{H}(\mathbf{r}) \), Eq. (15). In principle, one could improve on the approximation of Eq. (16) by determining \( \mathbf{p}_s(\mathbf{r}) \) self-consistently from \( \mathbf{j}(\mathbf{r}) \); however this will not change the qualitative physics of the vortex core expansion.

We then solve self-consistently for the order parameter

\[
\Delta(\mathbf{r}) = -\frac{V}{N} \sum_k [\cos(k_x a) - \cos(k_y a)] \langle \psi_{1-\mathbf{k}}^\dagger \psi_{1\mathbf{k}} \rangle r,
\]

(18)

with \( V = 1.7 \). Self-consistent solutions find that \( \Delta(\mathbf{r}) \) vanishes near the vortex core centre and obtains an asymptotic value \( \Delta_{\text{max}} = 0.35 \) far from the vortex core. In order to measure the vortex core size, we define a quantity \( \delta \Delta(\mathbf{r}) = \Delta_{\text{max}} - \Delta(\mathbf{r}) \). The vortex core size is then defined by the first moment of the radial coordinate \( r \) with respect to \( \delta \Delta(\mathbf{r}) \):

\[
R = \frac{\sum r \delta \Delta(\mathbf{r})}{\sum \delta \Delta(\mathbf{r})},
\]

(19)

where \( r = 0 \) corresponds to the vortex core centre. For presentation purposes, \( R \) is shown relative to the BCS coherence length \( \xi_0 \equiv \hbar v_F / \pi \Delta_{\text{max}} \), where \( v_F \) is the average of the Fermi velocity on the Fermi surface. The magnetic field is related to the magnetic length by \( H = \Phi_0 / L^2 \) where \( \Phi_0 \) is the superconducting flux quantum. For presentation purposes, \( H \) is scaled by the upper critical field, \( H_{c2} \equiv \Phi_0 / 2\pi \xi_0^2 \), so that \( H / H_{c2} = 2\pi \xi_0^2 / L^2 \).

1 D.R. Harshman, G. Aeppli, E.J. Ansaldo, B. Batlogg, J.H. Brewer, J.F. Carolan, R.J. Cava, M. Celio, A.C.D. Chaklader, W.N. Hardy, S.R. Kreitzman, G.M. Luke, D.R. Noakes, and M. Senba, Phys. Rev. B 36, 2386 (1987).

2 Y.J. Uemura, V.J. Emery, A.R. Moodenbaugh, M. Suegawa, D.C. Johnston, A.J. Jacobson, J.T. Lewandowski, J.H. Brewer, R.F. Kim, S.R. Kreitzman, G.M. Luke, T. Riseman, C.E. Stromach, W.J. Kossler, J.R. Kempton, X.H. Yu, D. Opie, and H.E. Schone, Phys. Rev. B 38 909 (1988).

3 D.R. Harshman, L.F. Schneemeyer, J.V. Waszczak, G. Aeppli, R.J. Cava, B. Batlogg, L.W. Rupp, E.J. Ansaldo, and D.L. Williams, Phys. Rev. B 39 851 (1989).

4 B. Pümpin, H. Keller, W. Kündig, W. Odermatt, I.M. Savić, J.W. Schneider, H. Simmler, P. Zimmermann, E. Kaldis, S. Rusiecki, Y. Maeno, and C. Rossel, Phys. Rev. B 42 8019 (1990).

5 Y.J. Uemura, G.M. Luke, B.J. Sternlieb, J.H. Brewer, J.F. Carolan, W.N. Hardy, R. Kadono, J.R. Kempton, R.F. Kiefl, S.R. Kreitzman, P. Mulhern, T.M. Rise-
W.A. Atkinson, Phys. Rev. B 59, 3377 (1999).