Radiative decays of $f_1(1285)$ as the $K^* \bar{K}$ molecular state

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Within a picture of the $f_1(1285)$ being a dynamically generated resonance from the $K^* \bar{K}$ interactions, we estimate the rates for the radiative transitions of the $f_1(1285)$ meson to the vector mesons $\rho^0$, $\omega$ and $\phi$. These radiative decays proceed via the kaon loop diagrams. The calculated results are in fair agreement with the experimental measurements. Some predictions can be tested by experiments and their implementation and comparison with these predictions will be valuable to decode the nature of the $f_1(1285)$ state.

I. INTRODUCTION

The radiative decay mode of the $f_1(1285)$ resonance is interesting because it is the basic element in the description of the $f_1(1285)$ photoproduction data $[1,2]$. It is also advocated as one of the observables most suited to learn about the nature of the $f_1(1285)$ state $[3,8]$. By means of a chiral unitary approach, the $f_1(1285)$ appears as a pole in the complex plane of the scattering amplitude of the $K^* \bar{K}$ $c.c.$ interaction in the isospin $I = 0$ and $J^{PC} = 1^{++}$ channel $[9]$. Or in another word, the axial-vector meson $f_1(1285)$ can be taken as a $K^* \bar{K}$ molecular state. For brevity, we use $K^* \bar{K}$ to represent the positive parity combination of $K^* \bar{K}$ and $K^0 \bar{K}$ in the following parts. An extension of the work of Ref. $[9]$, including higher order terms in the Lagrangian, has shown that the effect of the higher order terms is negligible $[10]$. Using these theoretical tools, predictions for lattice simulations in finite volume have been done in Ref. $[11]$.

The experimental decay width of the $f_1(1285)$ is $22.7 \pm 1.1$ MeV $[8]$, quite small for its mass, and naturally explained within the molecular state picture $[9]$. The dominant decay modes contributing to the width are peculiar. For example, the $\eta \pi \pi$ channel accounts for 52% of the width, and the branching ratio of $\pi a_0(980)$ channel is 38%. The $\pi a_0(980)$ channel has been well reproduced in Ref. $[12]$ within the $K^* \bar{K}$ molecular state picture for the $f_1(1285)$, since the $a_0(980)$ strongly couples to $K \bar{K}$. In Ref. $[12]$ the $\pi f_0(980)$ decay mode was also studied, and the decay rate and the invariant $\pi^+ \pi^-$ mass distribution were predicted. These predictions have been confirmed in a recent BESIII experiment $[13]$. There is another important decay channel, i.e. the $K \bar{K} \pi$, of which the branching ratio is $(9.1 \pm 0.4)\%$ $[8]$. This channel has even been investigated in Ref. $[14]$ with the same picture as in Ref. $[12]$, and the theoretical calculations are compatible with the experimental measurements. As a matter of fact the success of $f_1(1285)$ as a $K^* \bar{K}$ molecular state, being guided by the chiral unitary approach $[3]$, has become more remarkable than before especially for its hadronic decay models. Yet, all the above test have been done in the hadronic decay modes and not in the radiative decays. This offers us the first opportunity to do this new test, which we conduct here.

On the experimental side, the Particle Data Group (PDG) averaged values on the radiative decays of $f_1(1285)$ are $[8]$

$$\begin{align*}
Br(f_1(1285) \rightarrow \gamma \rho^0) &= (5.3 \pm 1.2)\%,
Br(f_1(1285) \rightarrow \gamma \phi) &= (7.5 \pm 2.7) \times 10^{-4},
\end{align*}$$

which lead to the partial decay width $\Gamma_{f_1(1285)\rightarrow\gamma\rho^0} = 1.2 \pm 0.3$ MeV and a ratio $R_{\gamma} = Br(f_1(1285) \rightarrow \gamma\rho^0)/Br(f_1(1285) \rightarrow \gamma\phi) = 71 \pm 30$. There is currently no experimental data about the $f_1(1285) \rightarrow \gamma\omega$ decay. While the recent value of $\Gamma_{f_1(1285)\rightarrow\gamma\omega}$ obtained by the CLAS Collaboration at Jefferson Lab from the analysis of the $f_1(1285)$ photoproduction off a proton target is much smaller, which is $0.453 \pm 0.177$ MeV $[11]$. On the theoretical side, the authors in Ref. $[2]$ give $\Gamma_{f_1(1285)\rightarrow\gamma\rho^0} = 0.311$ MeV and $\Gamma_{f_1(1285)\rightarrow\gamma\omega} = 0.0343$ MeV under the assumption that $f_1(1285)$ has a quark-antiquark nature. This $\Gamma_{f_1(1285)\rightarrow\gamma\rho^0}$ value is compatible with that of CLAS Collaboration within errors, but much smaller than the above PDG averaged value. Within the picture of $f_1(1285)$ being a quark-antiquark state, another theoretical prediction for the $f_1(1285)$ radiative decay is done in Ref. $[15]$ using a covariant oscillator quark model. It predicts the $\Gamma_{f_1(1285)\rightarrow\gamma\rho^0}$ is in the range of $0.509 \sim 0.565$ MeV, and $\Gamma_{f_1(1285)\rightarrow\gamma\omega}$ in the range of $0.024 \sim 0.057$ MeV, which depend on a particular mixing angle.

In this work, we extend the works of Refs. $[12,14]$ for the hadronic decays of $f_1(1285)$ to the case of the radiative decays. In the molecular state scenario, the $f_1(1285)$ decays into $\gamma V (V = \rho^0, \omega, \phi)$ via the kaon loop diagrams, and we can evaluate simultaneously these processes. We show that the numerical results are in good agreement with the experiment, hence supporting the molecular nature of the $f_1(1285)$ state.

The present paper is organized as follows: In sec. $[11]$ we discuss the formalism and the main ingredients of the model;
In sec. II, we present our numerical results and conclusions; A short summary is given in the last section.

II. FORMALISM

We study the decay of $f_1(1285) \to \gamma V$ with the assumption that the $f_1(1285)$ is dynamically generated from the $K^*\bar{K} + c.c.$ interaction, thus this decay can proceed via $f_1(1285) \to K^*\bar{K} \to \gamma V$ through the triangle loop diagrams, which are shown in Fig. 1. In this mechanism, the $f_1(1285)$ first decays into $K^*\bar{K}$, then the $K^*$ decays into $K\gamma$, and the $K\bar{K}$ interact to produce the vector meson in the final state. We use $p$, $k$, and $q$ for the momentum of $f_1(1285)$, and $K$ and $K^0$ in Figs. 1(A) and B), respectively. Then one can easily get the momentum of final vector meson is $p-k$, and the momenta for $K^*$ and $K$ are $p-q$ and $p-q-k$, respectively.

FIG. 1: Triangle loop diagrams representing the process $f_1(1285) \to \gamma V$ with $V$ being the $\rho^0$, $\omega$, or $\phi$ meson.

In order to evaluate the partial decay width of $f_1(1285) \to \gamma V$, we need the decay amplitudes of these diagrams shown in Fig. 1. As mentioned above, the $f_1(1285)$ resonance is dynamically generated from the interaction of $K^*\bar{K}$. For the charge conjugate transformation, we take the phase conventions $CK^* = -K^*$ and $C\bar{K} = \bar{K}$, which are consistent with the standard chiral Lagrangians, and write

$$|f_1(1285) > = \frac{1}{\sqrt{2}}(K^*\bar{K} - \bar{K}^*K)$$

$$= -\frac{1}{2}(K^{+}K^{-} + K^{0}\bar{K}^{0} - K^{+}\bar{K}^{-} - K^{0}\bar{K}^{0}) \; (3)$$

Then we can easily obtain the factors $C_1$ of $f_1\bar{K}K^*$ vertex for each diagram shown in Fig. 1

$$C_1^{A,B} = -\frac{1}{2}; \; C_1^{C,D} = \frac{1}{2}. \; (4)$$

For the $K\bar{K}V$ vertices, the effective Lagrangian describing the vector-pseudoscalar-pseudoscalar ($VPP$) interaction reads [16-19],

$$\mathcal{L}_{VPP} = -ig < V^\mu [P, \partial_\mu P] > \; , \; (5)$$

where $g = M/2f = 4.2$ with $M \approx (m_\rho + m_\omega)/2$ and $f = 93\text{ MeV}$ the pion decay constant. The pseudoscalar- and vector-nonet are collected in the $P$ and $V$ matrices, respectively. The symbol $<>$ stands for the trace.

According to the Lagrangian of Eq. (5), the $\phi \to K\bar{K}$ decay width is given by

$$\Gamma_{\phi \to K\bar{K}} = \frac{g^2m_\phi}{48\pi} \left(1 - \frac{4m_\phi^2}{m_\omega^2}\right)^{3/2},$$

and we can obtain the coupling $g \approx 4.5$ with the averaged experimental value of $\Gamma_{\phi \to K\bar{K}}$ in PDG [8]. We use $g = 4.2$ in our calculations.

Thus, the vertex of $K\bar{K}V$ can be written as

$$-it_{K\bar{K}V} = igC_2(2q + k - p)\varepsilon_\mu(p - k, \lambda_\nu), \; (6)$$

where $\varepsilon_\mu(p - k, \lambda_\nu)$ is the polarization vector of the vector meson. From Eq. (5) and from the explicit expressions of the $P$ and $V$ matrices, the factors $C_2$ for each diagram shown in Fig. 1 can be obtained,

$$C_2^{A,C} = \frac{1}{\sqrt{2}}; \; C_2^{B,D} = \frac{1}{\sqrt{2}} \; \text{ for } \rho \text{ production},$$

$$C_2^{A,C} = -\frac{1}{\sqrt{2}}; \; C_2^{B,D} = -\frac{1}{\sqrt{2}} \; \text{ for } \omega \text{ production},$$

$$C_2^{A,C} = 1; \; C_2^{B,D} = 1 \; \text{ for } \phi \text{ production}. \; (7)$$

In terms of Eqs. (4) and (7), it is easy to know that Figs. 1(A) and C) give the same contribution and Figs. 1(B) and D) also give the same contribution. We hence only consider Figs. 1(A) and B) in the following calculation.

For the electromagnetic vertex $K^*\gamma$, the interaction takes the form [20, 23]

$$\mathcal{L}_{K^*\gamma} = \frac{egK^*K^\gamma}{m_{K^*}^{\gamma}}\epsilon^{\mu\nu\alpha\beta}\partial_\mu K^*\partial_\nu A_{\alpha\beta}, \; (8)$$

where $K^\gamma$, $A_{\mu\nu}$ and $K$ denote the $K^*$ vector meson, photon, and the $K$ pseudoscalar meson, respectively. The partial decay width of $K^* \to K\gamma$ is given by

$$\Gamma_{K^* \to K\gamma} = \frac{eg^2K^*K^\gamma}{96\pi} \left(\frac{m_{K^*}^2 - m_K^2}{m_{K^*}^2}\right)^3. \; (9)$$

The values of the coupling constants $gK^*K^\gamma$ can be determined from the experimental data [8], which lead to

$$g_{K^*\gamma} = 0.75, \; g_{K^*\gamma} = -1.14. \; (10)$$

Here we fix the relative phase between the above two couplings taking into account the quark model expectation [24].

Here we give explicitly the decay amplitude of Fig. 1(A) for $\rho^0$ production,

$$M_A = -\frac{egg_{\rho}}{2\sqrt{2}m_{K^*}} \int d^4q \frac{1}{(2\pi)^4} \frac{1}{q^2 - m_{K^*}^2 + i\epsilon} \times \frac{1}{2\omega^2(q)} \frac{M_{f_1} - q^2 - \omega^2(q) + i\Gamma_{K^*}}{D_1} \times \frac{1}{(p - q - k)^2 - m_{K^*}^2 + i\epsilon}, \; (11)$$

$$\times \frac{D_2}{p^2 - m_{\rho^0}^2 + i\epsilon}.$$

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Here we give explicitly the decay amplitude of Fig. 1(A) for $\rho^0$ production,
where $\omega^*(q) = \sqrt{\vec{q}^2 + m_{K^0}^2}$ is the $K^{*0}$ energy, and we have taken the positive energy part of the $K^*$ propagator into account, which is a good approximation given the large mass of the $K^*$ (see more details in Ref. [12]). In Eq. (11), the factors $D_1$ and $D_2$ read

$$D_1 = \varepsilon_{\rho\alpha\beta}(p - q)\varepsilon^{\alpha\beta}(p, \lambda_f)k^\rho \varepsilon^{*\rho}(k, \lambda_\gamma),$$

$$D_2 = (2q + k - p)^\rho \varepsilon^{*\rho}(p - k, \lambda_\rho),$$

with $\lambda_f, \lambda_\gamma, \text{and} \lambda_\rho$ the spin polarizations of $f_1(1285)$, photon and $\rho^0$ meson, respectively. The amplitude $M_B$ corresponding to Fig. 1A can be easily obtained through the substitutions $m_{K^{*+}} \rightarrow m_{K^{*0}}, m_{K^+} \rightarrow m_{K^0}, \text{and} m_{K^-} \rightarrow m_{K^0}$ in $M_A$. The decay amplitudes for $f_1(1285) \rightarrow \gamma\phi$ and $f_1(1285) \rightarrow \gamma\rho$ share the similar formalism as Eq. (11).

The partial decay width of the $f_1(1285) \rightarrow \gamma\rho^0$ decay is given by

$$\Gamma_{f_1(1285)\rightarrow \gamma\rho^0} = \frac{E_\gamma}{12\pi M_{f_1}^2} \sum_{\lambda_f, \lambda_\gamma, \lambda_\rho} |M_A + M_B|^2.$$  

The cases for $\omega$ and $\phi$ production can be obtained straightforwardly.

To calculate $M_A$ in Eq. (11), we first integrate over $\rho^0$ using Cauchy's theorem. For doing this, we take the rest frame of $f_1(1285)$, in which one can write

$$p = (M_{f_1}, 0, 0, 0), \quad k = (E_\gamma, 0, 0, E_\gamma),$$

$$q = (q^0, |q^0| \sin \theta \cos \phi, |q^0| \sin \theta \sin \phi, |q^0| \cos \theta),$$

with $\theta$ and $\phi$ the polar and azimuthal angles of $q^0$ along the $\vec{k}$ direction, and the energy of photon $E_\gamma = |\vec{k}| = (M_{f_1}^2 - m_{\rho^0}^2)/2M_{f_1}$. The energy of final vector meson is $E_V = M_{f_1} - E_\gamma$. Then we have

$$V_1 = D_1D_2 = \mp iE_\gamma |q^0|^2 \sin^2 \theta,$$

for $\lambda_f = 0, \lambda_\gamma = \pm 1, \text{and} \lambda_\rho = \mp 1$,

$$V_2 = D_1D_2 = \pm \frac{2E_\gamma^2}{m_{\rho^0}} (q^0 - M_{f_1} - |q^0| \cos \theta) \times (q^0 \mp \frac{E_V}{E_\gamma} |q^0| \cos \theta),$$

for $\lambda_f = \pm 1, \lambda_\gamma = \pm 1, \text{and} \lambda_\rho = 0$. Notice that we have dropped those terms containing $\sin \phi$ or $\cos \phi$, because after the integration over $\phi$, they do not give contributions.

After integrating over $q^0$ in Eq. (11), we have

$$F_1^A = \frac{\hat{q}^4 (1 - \cos^2 \theta)}{\omega^* \omega^*} (X_1^A + X_2^A + X_3^A),$$

$$F_2^A = \frac{\hat{q}^2}{\omega^* \omega^*} \left[ \left( \frac{M_{f_1} - \omega^* + \frac{E_V}{E_\gamma} |q^0| \cos \theta}{-\omega^* - |q^0| \cos \theta} \right) X_1^A + \frac{1}{M_{f_1} - \omega^* - \omega + i\Gamma_{K^{*+}}/2} \left( E_V + \omega - |q^0| \cos \theta \right) X_2^A \right],$$

for $\omega = \sqrt{\hat{q}^2 + m_{K^+}^2}$ and $\omega' = \sqrt{|\vec{q}^2 + 2E_\gamma |q^0| \cos \theta + m_{K^+}^2}$ the energies of $K^-$ and $K^+$ in the diagram of Fig. 1A. $F_1^B$ and $F_2^B$ will be obtained just applying the substitution to $F_1^A$ and $F_2^A$ with $m_{K^{*+}} \rightarrow m_{K^{*0}}, m_{K^-} \rightarrow m_{K^0}$, and $m_{K^+} \rightarrow m_{K^0}$.

The partial decay width takes the form

$$\Gamma_{f_1(1285)\rightarrow \gamma V} = \frac{e^2 q^2 g_{f_1}^2 g_V^2}{192\pi M_{f_1}^2 m_V^5} \sum_{i=1,2} \int_0^1 d|q^0| \int_0^1 d\cos \theta \left( C_A F_i^A + C_B F_i^B \right)^2,$$

with

$$C_A = -\frac{\sqrt{2} g_{K^+ K^+ \gamma}}{4 M_{K^+} m_{K^+}}, \text{ for } V = \rho^0, \omega,'$$

$$C_A = \frac{g_{K^+ K^+ \gamma}}{2 M_{K^+}}, \text{ for } V = \phi,$$

$$C_B = \frac{\sqrt{2} g_{K^{*0} K^0 \gamma}}{4 M_{K^{*0}}}, \text{ for } V = \rho^0$$

$$C_B = -\frac{\sqrt{2} g_{K^{*0} K^0 \gamma}}{4 M_{K^{*0}}}, \text{ for } V = \omega,$$

$$C_B = -\frac{g_{K^{*0} K^0 \gamma}}{2 M_{K^{*0}}}, \text{ for } V = \phi.$$
For $\rho^0$ production, the relative minus sign between $C_A$ and $C_B$ combined with the minus sign between the coupling $g_{K^+K^+\gamma}$ and $g_{K^0K^0\gamma}$ is positive, and hence the interference of the two diagrams $A$ and $B$ shown in Fig. 1 is constructive. However, it is destructive for $\omega$ and $\phi$ production, which will make the $\Gamma_f(1285) \rightarrow \gamma\rho^0$ is much larger than the other two partial decay widths.

III. NUMERICAL RESULTS AND DISCUSSION

![Graph](image)

FIG. 2: Partial decay width of $f_1(1285) \rightarrow \gamma\rho^0$ decay as a function of the cutoff parameter $\Lambda$.

A momentum cutoff $\Lambda$ is introduced in Eq. (24), and the partial decay width of $f_1(1285) \rightarrow \gamma\rho^0$ decay as a function of the $\Lambda$ from 900 to 2500 MeV is illustrated in Fig. 2. We can see that, in the range of cutoff we consider, the $\Gamma_{f_1(1285) \rightarrow \gamma\rho^0}$ varies from 0.5 to 1.4 MeV, which is consistent with the experimental result within errors $[1, 8]$. In Table I, we show explicitly the numerical results of the $f_1(1285) \rightarrow \gamma\nu$ decays with some particular cutoff parameters.

TABLE I: Partial decay width for $f_1(1285) \rightarrow \gamma\nu$. All units are in MeV.

| $\Lambda$ (MeV) | $f_1 \rightarrow \gamma\rho^0$ | $f_1 \rightarrow \gamma\omega$ | $f_1 \rightarrow \gamma\phi$ |
|----------------|-----------------|-----------------|-----------------|
| 1000           | 0.86            | 1.87            | 0.93            |
| 1500           | 0.88            | 3.01            | 1.40            |
| 2000           | 1.14            | 4.01            | 1.78            |
| 2500           | 1.36            | 4.87            | 2.09            |
| Exp. [8]       | 1.18 ± 0.27     | —               | 1.70 ± 0.61     |

In general, we cannot provide the value of the cutoff parameter, however, if we divide $\Gamma_{f_1(1285) \rightarrow \gamma\rho^0}$ by $\Gamma_{f_1(1285) \rightarrow \gamma\omega}$ or $\Gamma_{f_1(1285) \rightarrow \gamma\phi}$, the dependence of these ratios on the cutoff will be smoothed. Two ratios are defined as:

$$
R_1 = \frac{\Gamma_{f_1(1285) \rightarrow \gamma\rho^0}}{\Gamma_{f_1(1285) \rightarrow \gamma\omega}}
$$

$$
R_2 = \frac{\Gamma_{f_1(1285) \rightarrow \gamma\phi}}{\Gamma_{f_1(1285) \rightarrow \gamma\omega}}.
$$

These two ratios are correlated with each other. With $R_1$ measured by the experiment, one can fix the cutoff in the model and predict the ratio $R_2$.

![Graph](image)

FIG. 3: The $\Lambda$ dependence of the ratios $R_1$ (solid line) and $R_2$ (dashed line) defined in Eq. (30). The error band correspond to the experimental result for $R_1$.

In Fig. 3, we show the numerical results for the above ratios, where the solid line stands for the results for $R_1$, while the dashed line stands for the results for $R_2$. Indeed, one sees that the dependence of both ratios on the cutoff is rather weak. The ratio $R_1 \approx 60$ is in agreement with the experimental result $71 \pm 30 [8]$. On the other hand, the result of $R_2$ is about 30. It is a firm conclusion that the partial decay width of $f_1(1285) \rightarrow \gamma\rho^0$ is much larger than the ones to $\gamma\omega$ and $\gamma\phi$ channels. This is because the destructive interference between Fig. 1A and 1B for $\omega$ and $\phi$ production. Our conclusion here is different with these quark model calculations $[2, 15]$. We hope that the future experimental measurements can clarify this issue. Further theoretical research considering both the molecular and quark components for the $f_1(1285)$ state would be most welcome after the discussion made here.

IV. SUMMARY

In this work, we evaluate the partial decay width of the radiative decays $f_1(1285) \rightarrow \gamma\nu$ with the assumption that the $f_1(1285)$ is dynamically generated from the $K^+K^-$ interaction. The results we obtained for the partial widths are compatible with experimental data within errors. Furthermore, we find some relevant features of our model calculations, which turn out to be very different from other theoretical predictions using quark models. The precise experimental observations of
those radiative decays would then provide very valuable information on the relevance of components in the $f_1(1285)$ wave function.

Acknowledgments

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