Lattice QCD at finite isospin density and/or temperature

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We simulate two-flavour lattice QCD with a finite chemical potential $\mu_I$ for isospin, and finite temperature. At small $\mu_I$, we determine the position of the crossover from hadronic matter to a quark-gluon plasma as a function of $\mu_I$. At larger $\mu_I$ we observe the phase transition from the superfluid pion-condensed phase to a quark-gluon plasma, noting its change from second order to first order as $\mu_I$ is increased. We also simulate two-flavour lattice QCD at zero quark mass, using an action which includes an additional 4-fermion interaction, at temperatures close to the chiral transition on $N_t = 8$ lattices.

1. Introduction

QCD at finite temperature and/or densities is relevant to the physics of the early universe, neutron stars and relativistic heavy-ion collisions — RHIC and the CERN heavy-ion program.

Because of the difficulties involved with simulating QCD at finite chemical potential $\mu$ for quark number with its complex fermion determinant, we are simulating $N_f = 2$ lattice QCD with a finite chemical potential $\mu_I$ for isospin ($I_3$), which has a real positive fermion determinant [1]. We are currently studying the $\mu_I$ dependence of the finite temperature transition [2]. At small $\mu$, the Bielefeld-Swansea collaboration has observed that the phase of the fermion determinant is sufficiently well behaved that the $\mu$ and $\mu_I$ dependence of the transition are identical [3]. Our predictions are in good agreement with those of de Forcrand and Philipsen [4].

At large values of $\mu_I$ ($\mu_I > m_\pi$), the low temperature phase is characterized by a pion condensate. The finite temperature transition is now a true phase transition, which appears to be second order for lower $\mu_I$ values and first order at higher $\mu_I$ values.

We also simulate lattice QCD with an irrelevant chiral 4-fermion term which allows us to simulate at zero quark mass, giving us direct access to the critical exponents at the finite temperature transition [5]. We are using $16^3 \times 8$ and $24^3 \times 8$ lattices for these simulations.

In section 2 we present preliminary results of our finite $\mu_I$ and temperature simulations. Section 3 gives some preliminary graphs from our $N_t = 8$ finite-temperature simulations using our modified action. Finally we present our conclusions and indicate further avenues of research in section 4.

2. QCD at finite $\mu_I$ and temperature

The quark part of our lattice action is

$$ S_f = \sum_{\text{sites}} \left[ \bar{x} \left( [\partial^2 - \tau_3 \mu_I] / 2 \right) \chi + i \lambda \epsilon \bar{x} \tau_2 \chi \right]. \quad (1) $$

For simulations at $\mu_I < m_\pi$, we set the symmetry breaking parameter $\lambda = 0$.

We perform simulations on an $8^3 \times 4$ lattice with $m = 0.05$, $\lambda = 0$ and $0 \leq \mu_I \leq 0.55$, for a set of $\beta$s covering the crossover region for each $\mu_I$. We measure the chiral condensate, the plaquette, the Wilson Line and the isospin density and their corresponding susceptibilities for each set of parameters. The position of the crossover was obtained as the peak of the susceptibilities. This was determined using Ferrenberg-Swendsen reweighting techniques [6]. Figure 1 shows the chiral susceptibilities obtained from such reweightings for 3 of the 7 $\mu_I$ values we use. The multiple values for...
each $\mu_I$ are the results of using distributions from several $\beta$s close to the peak.

The crossover $\beta$ values, $\beta_c$, obtained from the peaks in the susceptibilities for each of the 4 observables, are plotted in figure 2 as functions of $\mu_I^2$, since the leading term is expected to be quadratic in $\mu_I$. We note that the 4 predictions for each $\mu_I$ appear consistent within errors, indicating that this is a reasonable definition of the position of the crossover. The straight line

$$\beta_c = 5.322 - 0.143\mu_I^2$$

in this figure is only meant as a rough guide. Using the relation $\beta_c(\mu) = \beta_c(\mu_I = 2\mu)$, which should hold for small $\mu$, our results are consistent with those of de Forcrand and Philipsen [4].

We are repeating these simulations for $m = 0.05$ and $m = 0.2$ where the critical $\mu_I$s, $\mu_c = m_\pi$, are larger. In no case does there appear to be a critical end point, beyond which the transition becomes first order, for $\mu_I < m_\pi$.

We are extending our $m = 0.05$ simulations to $\mu_I > m_\pi$. While the thermal phase transition where the pion condensate evaporates appears to be second order for $\mu_I = 0.6$, simulations on $16^3 \times 4$ lattices show indications of the metastability expected for a first order transition, at $\mu_I = 0.8$.

3. \textit{“QCD” at finite temperature}

We simulate lattice QCD with 2 flavours of massless staggered quarks and an irrelevant chiral 4-fermion interaction which allows us to run at zero quark mass, at finite temperature on $16^3 \times 8$ and $24^3 \times 8$ lattices. Using $N_t = 8$ should free us from the lattice artifacts which were present for $N_t = 4, 6$ [5].

Having massless quarks should enable us to extract the critical exponent $\delta$, which describes the vanishing of the chiral condensate as the chiral transition is approached from below, and the critical $\beta$ ($\beta_c$). Running at $\beta_c$ with small quark masses will give us the critical exponent $\delta$.

Preliminary results for the chiral condensate and Wilson Line are given in figure 3. These clearly indicate that the transition occurs in the
range $5.530 < \beta_c < 5.545$ and appears sufficiently smooth to be second order.

![Diagram](image)

Figure 3. Chiral condensate and Wilson line of "$\chi$QCD" as functions of $\beta$ on $N_t = 8$ lattices.

4. Conclusions

We have determined the $\mu_I$ dependence of the finite temperature transition for 2-flavour lattice QCD. For small $\mu$, the fluctuations of the phase of the fermion determinant are small enough that the $\mu$ and $\mu_I$ dependence of this transition are the same [3]. The $\mu_I$ dependence we measure predicts a $\mu$ dependence consistent with that obtained from simulations with imaginary $\mu$ [4]. We will check if it is also consistent with series expansions around $\mu = 0$ [3]. Our simulations at 3 different quark masses indicate that the transition is a crossover for $\mu_I < m_\pi$. At higher $\mu_I$, where the the pion condensate evaporates at the transition, we see evidence for a change to first order behaviour, but it is unclear if this is related to the critical endpoint expected at finite $\mu$.

Others have estimated the position of the critical endpoint at finite $\mu$ for the more physical 2+1-flavour QCD [7], the closely related 3-flavour QCD [10] and 4-flavour QCD [11]. We are extending our simulations to 3-flavours, where one can tune the quark mass to make the critical endpoint as close to $\mu = 0$ as desired. If the endpoint is at small enough $\mu$, we expect a critical endpoint at the corresponding $\mu_I$, giving another estimate for the position of this endpoint.

We are using the "$\chi$QCD" action to determine the critical exponents for the chiral transition of finite temperature QCD, on $N_t = 8$ lattices.

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