Abstract

The N=2 supersymmetric gauge theory with gauge group SU(2) is considered. The instanton field is calculated explicitly using the superfield formalism. The instanton-induced effects are encoded in the effective vertex in the Lagrangian. This vertex produces the large distance expansion of the low energy effective Lagrangian in derivatives of fields. The leading term in this expansion coincides with the one-instanton-induced term of the Seiberg-Witten exact solution of the model. All orders of corrections in derivatives of fields are also calculated.
1 Introduction

A lot of progress has been made last time in the study of supersymmetric
gauge theories in four dimensions. The use of electromagnetic duality [1] and
holomorphy allows Seiberg and Witten to solve exactly for the low energy
effective Lagrangian of the N=2 SUSY SU(2) gauge theory with [2] and with-
out matter hypermultiplets [3]. The long standing problems of confinement
and chiral symmetry breaking get its natural explanation via condensation
of dual fields [3, 2]. The extrapolation of these ideas to N=1 SUSY and
non-abelian gauge groups is under intensive study [4, 5, 6].

The exact solution for the low energy effective Lagrangian was obtained in
[3, 2] using the symmetries of the theory as well as certain assumptions about
the nature of singularities on the modular space of vacua. It is a challenging
problem to reobtain these results as well as to calculate corrections to them
performing explicit calculations in microscopic theory. In this paper we are
making another step in this direction.

To be more specific we consider N=2 SUSY SU(2) gauge theory without
matter hypermultiplets. The theory has a flat direction along which the com-
plex scalar field can develop a vacuum expectation value (vev). The gauge
group is broken to U(1). Thus, low energy theory contains an abelian N=2
supermultiplet. If we ignore high derivative/many fermions terms then the
low energy effective Lagrangian is strongly constrained by supersymmetry.
The most general form of the action [3]

$$S_{SW} = \int d^4x \left[ \int d^4 \theta \frac{1}{2} \frac{1}{\partial A} F(A) \bar{A} + \int d^2 \theta \frac{1}{2} \frac{1}{\partial A^2} F(A) W_\alpha W^\alpha + c.c. \right]$$

(1.1)

depends on the one holomorphic function $F(A)$ on the moduli space of vacua.
This function determines the effective coupling constant as a function of
vev. Here we use N=1 superfield notations, $W$ is the field strength of the
N=1 vector supermultiplet, while $A$ is the N=1 chiral supermultiplet of U(1)
theory ($a$ is the scalar component of $A$, which develop vev).

The function $F(A)$ was determined exactly in [3]. Its expansion in inverse
powers of $A$ is given by

$$F(A) = \frac{1}{8\pi^2} \left[ A^2 \log \frac{2A^2}{\Lambda^2} - \frac{\Lambda^4}{A^2} + \cdots \right],$$

(1.2)

where $\Lambda$ is the scale of the theory.
At large $a$ the microscopic theory is in the weak coupling and could be studied semiclassically. In particular, several terms in the expansion of the function $F(A)$ in the weak coupling limit were reobtained in a straightforward manner making explicit calculations in the microscopic theory.

Namely, the logarithmic term in (1.2) comes from the one loop contribution to the $\beta$-function. There are no higher order perturbative corrections in $\text{N}=2$ supersymmetry [7]. The second term in (1.2) comes from one instanton while ellipses stands for high order multi-instanton corrections. The one-instanton contribution to (1.1) was first studied in [7] (see ref. [8] for a review of the instanton calculus in the SUSY theories). The numerical coefficient in front of it was calculated in [8] and was shown to coincide with the prediction of the Seiberg-Witten solution (1.2) [7]. In ref. [10] the two-instanton effect was studied and the coefficient in front of $\Lambda^8/A^6$ in the expansion (1.2) was calculated in accordance with Seiberg-Witten result.

In this paper we continue a detailed study of instanton effects in the low energy effective theory making a step in another direction. We restrict ourselves to the one-instanton effects considering weak coupling limit of the theory and study large distance expansion in the low energy Lagrangian. Namely, we go beyond the leading at low energies order approximation (1.1) and present a general method to calculate corrections to (1.1) in higher derivatives of fields.

In sect. 2 we calculate the instanton fields in terms of $\text{N}=1$ supermultiplets generalizing the method of ref. [11, 12] developed for $\text{N}=1$ SUSY to $\text{N}=2$ case. Then in sect. 3 we derive the effective instanton-induced vertex following the general method of [13, 14, 15] (see also [16]). This vertex encodes instanton-induced effects in the microscopic theory. In sect. 4 we rewrite it in $\text{N}=2$ superfields and then consider it in the low energy limit in sect. 5. The leading term appears to be the Seiberg-Witten effective action (1.1) with function $F(A)$ given by the second term in (1.2). We also calculate all orders of corrections to (1.1) in the long-distance expansion. Sect. 6 contains our conclusions.

\footnote{This calculation requires fixing of the regularization scheme, both in perturbative theory and in the instanton sector. This defines the particular scale parameter $\Lambda$. Equation (1.2) is written down in the Pauli-Villars regularization scheme [9]. We use the same scheme in this paper.}
2 N=2 superinstanton.

Consider N=2 SU(2) gauge theory. It can be described in terms of N=1 superfields by the chiral scalar multiplet

$$\Phi = \phi + \sqrt{2} \theta^a \psi^a + \theta^2 F$$

(2.1)

and by the field strength chiral multiplet

$$W^a = -\lambda^a + \theta^a D + \frac{1}{2}(\sigma_{\mu} \sigma_{\nu} \theta)^a F_{\mu \nu} - \theta^2 i D^{\alpha \bar{\alpha}} \bar{\lambda}_{\bar{\alpha}}$$

(2.2)

in Wess-Zumino gauge. Both fields in (2.1) and (2.2) are in the adjoint representation of SU(2) and carry the colour index $a = 1, 2, 3$. Sigma matrices in (2.2) are defined as follows $\sigma_{\mu \bar{\nu}}^a = (1, -i \tau^a)$, $\sigma_{\mu \bar{\nu}}^{a \bar{\nu}} = (1, i \tau^a)$, $\tau^a$ being the Pauli matrices. The action of the model reads

$$S = \frac{1}{g^2} \int d^4x \left[ \int d^2 \theta d^2 \bar{\theta} \Phi^a (e^{-2V_\theta} \Phi)^a + \frac{1}{4g^2} \int d^2 \theta W^a_{\alpha} W^a_{\bar{\alpha}} \right].$$

(2.3)

N=2 SUSY assumes the existence of global $SU(2)_R$ group. Fermions $\lambda, \psi$ form a doublets under the action of this group, while gauge and scalar fields are singlets.

As we already mentioned above, this theory has a flat direction in the scalar potential. Namely, the condition of vanishing of the D-term is $[\bar{\phi}, \phi] = 0$ (throughout the paper we use both the matrix as well as the vector notations for fields in the adjoint representation of the colour group, say $\phi = \phi^a \tau^a / 2$). As a solution for the vacuum moduli space we use $\phi = v \tau^3 / 2$ following [3]. Thus, the colour group is broken down to U(1) and light fields which appear in the low energy theory (1.1) are

$$A = \Phi^3, \ W = W^3$$

(2.4)

For large $v \gg \Lambda$ the effective coupling is small and the theory can be studied in the semiclassical approximation. As we mentioned in the Introduction the perturbative contribution to the effective coupling

$$\frac{1}{g^2_{\text{eff}}} = \frac{\partial^2 F}{\partial A^2}$$

(2.5)
is exactly given by one loop result (first term in (1.2)) so we are left only
with instanton effects.

In the reminder of this section we present different components of the
instanton fields in the theory (2.1) and arrange them into N=1 supermulti-
plets.

Let us start with the boson fields first. The gauge potential is the original
BPST instanton [17] written down in the singular gauge

\[ A_\mu^I = \eta^a_{\mu\nu} \rho^2 u^\nu \bar{y} y^\mu / y^4 H, \]  

where \( y = x - x_0 \) and

\[ H = 1 + \frac{\rho^2}{y^2}. \]  

Here \( x_0, \rho \) and \( u \) are the instanton center, its size and orientation matrix
\( u^\alpha = \sigma_\mu^\alpha u_\mu, \ u_\mu^2 = 1 \). In order to construct the chiral field \( W \) in (2.2) we
need the expression for the gauge field strength

\[ \frac{1}{2} \sigma_\mu^\alpha \sigma_\nu^\beta F_{\mu\nu}^{Ia} = -8i \rho^2 \frac{\left( y\bar{u} \tau^a u \bar{y} \right)^\alpha}{y^6 H^2}, \]  

where matrix \( y^\alpha = \sigma_\mu^\alpha y_\mu \).

The scalar component of the instanton field in the Higgs vacuum is given
by [18]

\[ \phi_I = \frac{\tau^3 v}{2 H} \]  

Strictly speaking the configuration (2.6), (2.9) is not an exact solution of
equations of motion in the Higgs vacuum. The scalar field (2.9) satisfies the
equation \( D^2 \phi_I = 0 \), but the gauge field in (2.6) do not satisfy the equation
of motion with the source term induced by scalars. This term becomes
important at large distances \( y^2 \geq M_{\bar{W}}^2 \), where \( M_{\bar{W}}^2 \sim v^2 \) is the mass of the
W boson. The solution (2.6), (2.9) can be viewed as a constraind instanton
[13]. It is given by (2.6), (2.9) at short distances, whereas has an exponential
fall off at large distances for heavy components. The action on fields (2.6),
(2.9) is

\[ S_{Ibos}^I = \frac{8\pi^2}{g^2} + \frac{4\pi^2}{g^2} |v|^2 \rho^2. \]  

5
This means that the integral over the instanton size is convergent at large \( \rho \) due to the second term in (2.10) and dominated at

\[
\rho^2 \sim \frac{g^2}{|v|^2}.
\]  

(2.11)

We can look at bosonic instanton fields as given by (2.6), (2.9) at all distances up to \( y^2 \leq \mu^2 \) where scale \( \mu \) is within the bounds \( \rho^2 \ll \mu^2 \ll M_W^{-2} \). Because of (2.11) we have some range of validity of eqs. (2.6), (2.9) even in the large distance limit \( y^2 \gg \rho^2 \) we are interested in in this paper. To go beyond that at very large distances \( y^2 \gg v^{-2} \) (which is essential to derive the low energy Lagrangian (1.1)) we have to take into account the mass term for heavy fields in (2.3), while light components are still given by the leading terms of large distance expansion in eqs. (2.6), (2.9). We make this extrapolation in sect. 5.

Let us now discuss fermion zero modes of instanton. There are eight of them, four for \( \lambda \) and four for \( \psi \), if \( v = 0 \). Gaugino ones are given by \[20, 11\]

\[
\lambda^{a\alpha} = 8i\rho^2 \frac{(y\bar{u}\tau^a u\bar{y}(\alpha - y\bar{\beta}))^\alpha}{y^6 H^2},
\]  

(2.12)

Here we introduce two Grassmann parameters \( \alpha^\alpha \) which parametrize supersymmetric zero modes as well as \( \bar{\beta}_\alpha \) which parametrize superconformal ones. To get the \( \psi \) modes we use the \( SU(2)_R \) group which interchanges \( \lambda \) and \( \psi \). We have

\[
\psi^{a\alpha} = -8i\rho^2 \frac{(y\bar{u}\tau^a u\bar{y}(\zeta - y\bar{\omega}))^\alpha}{y^6 H^2},
\]  

(2.13)

where we introduce another set of Grassmann parameters \( \zeta^\alpha \) and \( \bar{\omega}_\alpha \).

Once \( v \neq 0 \) superconformal modes in (2.12), (2.13) are lifted due to the conformal symmetry breaking. Nevertheless we still keep the integration over corresponding parameters \( \bar{\beta} \) and \( \bar{\omega} \) in the instanton measure following \[20, 11\]. These are fermionic counterpart of the integration over the instanton size \( \rho \).

Now let us rewrite components of instanton field in terms of N=1 supermultiplets using the general method of ref \[11, 12\]. Note first, that under the N=1 SUSY transformation components of vector multiplet (2.6) and (2.12) transform into the same expressions up to a gauge transformations with collective coordinates redefined as follows:

\[
x_{0\mu}' = x_{0\mu} - 2i(\bar{\epsilon}\sigma_\mu\alpha),
\]
\[ \alpha' = \alpha - \epsilon, \]
\[ \rho'^2 = \rho^2 (1 + 4i\bar{\beta} \bar{\epsilon}), \]
\[ \bar{\beta}' = \bar{\beta} (1 + 4i\bar{\beta} \bar{\epsilon}), \]
\[ u'^{\alpha \dot{\alpha}} = u^{\alpha \dot{\beta}} [1 - \delta^{\alpha \beta} 2i \bar{\beta} \bar{\epsilon} - 4i \bar{\beta} \bar{\beta} \bar{\epsilon} \dot{\alpha}], \quad (2.14) \]

where \( \epsilon \) and \( \bar{\epsilon} \) are parameters of the N=1 SUSY transformation.

These expressions were obtained in [11, 21, 22] for N=1 SUSY gauge theory and stay untouched for the vector supermultiplet in N=2 SUSY theory. To derive transformation laws for \( \zeta \) and \( \bar{\omega} \) let us rewrite the supersymmetric mode in (2.13) as

\[ \psi_{SS}^\alpha = \frac{1}{2} (\sigma_\mu \bar{\sigma}^\nu \zeta) F_{\mu \nu}^I \quad (2.15) \]

and superconformal one as

\[ \psi_{SC}^\alpha = -i \sqrt{2} D^{\alpha \bar{\alpha}} \phi^I \bar{\eta}_{\bar{\alpha}}, \quad (2.16) \]

where \( F_{\mu \nu}^I \) and \( \phi^I \) are given by (2.8), (2.9). Here we introduce a new collective coordinate \( \bar{\eta} \) instead of \( \bar{\omega} \):

\[ \bar{\omega}_{\dot{\alpha}} = \frac{v}{2\sqrt{2}} (\bar{u} \tau^3 u \bar{\eta})_{\dot{\alpha}} \quad (2.17) \]

Using (2.15), (2.16) it is easy to see that N=1 SUSY transformations lead to the following transformation law for \( \zeta, \bar{\eta} \):

\[ \zeta' = \zeta, \quad \bar{\eta}' = \bar{\eta} - \bar{\epsilon} + 4i \bar{\beta} (\bar{\eta} \bar{\epsilon}) \quad (2.18) \]

Now we are ready to construct instanton supermultiplets. Fields \( \Phi \) and \( W \) are chiral superfields, therefore, they should be invariant (up to a gauge transformation) under the SUSY transformation of coordinates

\[ x'_{L\mu} = x_{L\mu} - 2i (\bar{\epsilon} \bar{\sigma}_\mu \theta) \]
\[ \theta' = \theta - \epsilon \quad (2.19) \]

followed by transformation of the collective coordinates (2.14), (2.18). Here \( x_{L\mu} \) and \( \theta \) are arguments of chiral field

\[ x_{L\mu} = x_\mu + i (\bar{\theta} \bar{\sigma}_\mu \theta) \quad (2.20) \]
Comparing (2.14) with (2.19) we see that instanton center transforms like a left-handed coordinate, while α transforms like θ. To complete this analogy showing that instanton collective coordinates can be interpreted as the usual superspace coordinates, let us introduce

\[ \tilde{\eta} = \frac{\eta}{1 + 4i\bar{\beta}\eta}, \]  

(2.21)
which gets shifted

\[ \tilde{\eta}' = \tilde{\eta} - \tilde{\epsilon} \]  

(2.22)
under the SUSY transformations \[\text{[22]}\]. Eq. (2.22) shows that \( \tilde{\eta} \) transforms like \( \tilde{\theta} \).

The invariance of fields \( \Phi \) and \( W \) means, that they could depend only on certain SUSY invariant combinations of coordinates \( x_L, \theta \) and collective coordinates (cf. \[\text{[11]}\]). Using (2.19) as well as (2.14) and (2.18) we can construct this invariants. The invariant distance is

\[ y_{L\mu}^{\text{inv}} = x_{L\mu} - x_{0\mu} + 2i(\tilde{\eta}_L\bar{\sigma}_\mu[\theta + \alpha]), \]  

(2.23)
where \( \theta + \alpha \) is invariant by itself. The invariant size of instanton is \[\text{[11]}\]

\[ \rho_{\text{inv}}^2 = \rho^2(1 + 4i\bar{\beta}\eta), \]  

(2.24)
while invariant \( \bar{\beta} \) is \[\text{[22]}\]

\[ \bar{\beta}_{\text{inv}} = \bar{\beta}(1 + 4i\bar{\beta}\eta). \]  

(2.25)

The choice of the invariant distance in (2.23) is slightly different from those in \[\text{[11]}\]. The one in (2.23) looks like invariant distance between ordinary left-handed coordinates. We used this analogy in the next section. To construct fields \( \Phi^a, W^{\alpha a} \) (rather than gauge invariant combinations \( W^aW^a \) and \( \Phi^a\Phi^a \)) we also need a notion of the invariant orientation. It is given by

\[ u_{\text{inv}}^{\alpha \dot{\alpha}} = u^{\alpha \dot{\alpha}} \exp[-4i\bar{\beta}\beta\eta + 2i\delta_{\dot{\alpha}}^\dot{\beta}(\eta\beta)] \]  

(2.26)

Now we can write down the answer for vector field (cf. \[\text{[11]}\])

\[ W^\alpha = \frac{1}{2}(\sigma_\mu\sigma_\nu[\theta + \alpha - y_{L\mu}^{\text{inv}}\bar{\beta}_{\text{inv}}])^\alpha F^I_{\mu\nu}(y_{L\mu}^{\text{inv}}, \rho_{\text{inv}}, u_{\text{inv}}), \]  

(2.27)
where \( F^I_{\mu\nu} \) is given by (2.8). It is a straightforward calculation to check that components of (2.27) are given by (2.8), (2.12) up to a gauge transformation.
Note, that extra terms which comes from expansion of $F_{\mu\nu}^I(y_L^{inv})$ in the deviation of $y_L^{inv}$ from $y_L$ (see (2.23)) are zero by the use of equation of motion. A useful formula to make this check is

$$\frac{y_L^{inv}}{\rho_{inv}} = \frac{y_L - 4i[\theta + \alpha - y_L\beta]\bar{\eta}}{\rho} \bar{u}_{inv}.$$ 

The similar procedure gives for the scalar chiral field

$$\Phi = \frac{1}{\sqrt{2}}(\theta + \alpha - y_L^{inv}\beta_{inv})\sigma_{\mu}\sigma_{\nu}\zeta F_{\mu\nu}^I(y_L^{inv}, \rho_{inv}, u_{inv})$$

$$+ U_1\phi_I(y_L^{inv}, \rho_{inv})U_1^{-1}, \quad (2.28)$$

where $U_1$ is the unitary rotation.

$$U_1 = \exp[2A_{\mu}(\bar{\eta}\sigma_{\mu}[\theta + \alpha - y_L\beta])] \quad (2.29)$$

This rotation could be ignored in large distance limit $x^2 \gg \rho^2$. Again one can check that the components of (2.28) are given by (2.9), (2.13).

It is important to note that supersymmetric zero mode of $\psi$ (2.15) comes from the first term in (2.28) while the superconformal one comes from the expansion of the second one. This is the way N=2 SUSY works. Roughly speaking, the first term in (2.28) is an N=2 supersymmetrization of the original BPST instanton in (2.8) (action of N=1 SUSY gives $\lambda_{SS}$ in (2.12), then the action of $SU(2)_R$ gives $\psi_{SS}$ in (2.13). The second term in (2.28) is the supersymmetrization of the t’Hooft solution (2.9) which seems unrelated to (2.8). They “meet together” in (2.28) as the first term produces supersymmetric zero mode (2.15), while the second one produces the superconformal mode (2.16). To put it another way, the coefficient $\bar{\omega}$ in front of $\psi_{SC}$ in (2.13) is related to N=2 superconformal transformation of BPST instanton (2.8), while parameter $\bar{\eta}$ is related to N=1 SUSY transformation of the t’Hooft solution (2.9). Eq. (2.17) is the equivalence between both.

Now let us turn to anti-chiral fields

$$\bar{\Phi} = \bar{\phi} + \sqrt{2}\bar{\theta}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}} + \bar{\theta}^2 F^2 \quad (2.30)$$

and

$$\bar{W}_{\dot{\alpha}} = -\bar{\lambda}_{\dot{\alpha}} + \bar{\theta}_{\dot{\alpha}} D - \frac{1}{2}(\sigma_{\mu}\sigma_{\nu}\bar{\theta})_{\dot{\alpha}} F_{\mu\nu} - \bar{\theta}^2 (i\bar{D}\lambda)_{\dot{\alpha}} \quad (2.31)$$
which depend on \( \bar{\theta} \) and right-handed coordinate

\[
x_{R\mu} = x_\mu - i(\bar{\theta}\bar{\sigma}_\mu\bar{\theta})
\]

(2.32)

If the vev of the scalar field were zero there would be no anti-chiral fields. Instanton would be self-dual, chiral object. Once \( v \neq 0 \) this is no longer true \[20\]. The \( \bar{\phi} \) component is given by complex conjugate of the t’Hooft solution (2.9)

\[
\bar{\phi}_I = \frac{\tau_3 \bar{v}}{2 H}
\]

(2.33)

Then N=1 SUSY transformation generates

\[
\delta \bar{\psi}_{\dot{\alpha}} = i\sqrt{2} \bar{\sigma}_{\dot{\mu}\dot{\alpha}} \epsilon^\alpha D_\mu \bar{\phi}_I
\]

(2.34)

This field becomes a nontrivial solution of equations of motion

\[
iD\bar{\psi} - i\sqrt{2}[\lambda_{SS}, \bar{\phi}_I] = 0
\]

(2.35)

once \( \bar{\phi} \neq 0 \). From (2.34),(2.35)we have

\[
\bar{\psi}_{\dot{\alpha}} = i\sqrt{2} \bar{\sigma}_{\dot{\mu}\dot{\alpha}} \epsilon^\alpha D_\mu \bar{\phi}_I
\]

(2.36)

Note, that according to (2.35) we parametrize \( \bar{\psi} \) mode in (2.36) by parameter \( \alpha \), which appears in the expression for supersymmetric gaugino zero mode (2.12). The \( SU(2)_R \) symmetry then gives for \( \bar{\lambda} \)

\[
\bar{\lambda}_{\dot{\alpha}} = -i\sqrt{2} \bar{D}_{\dot{\alpha}\dot{\alpha}} \phi_I \xi^\alpha
\]

(2.37)

Now let us construct anti-chiral supermultiplets. To do this we should introduce the right-handed invariant distance. Recalling that

\[
x'_{R\mu} = x_{R\mu} + 2i(\bar{\theta}\bar{\sigma}_\mu\epsilon)
\]

\[
\bar{\theta}' = \bar{\theta} - \bar{\epsilon}
\]

(2.38)

and taking into account (2.14) one finds

\[
y_{R\mu}^{\text{inv}} = x_{R\mu} - x_{0\mu} - 2i(\bar{\theta}\bar{\sigma}_\mu\alpha)
\]

(2.39)

Using this invariant distance as well as \( \rho_{\text{inv}} \) and \( u_{\text{inv}} \) (2.24), (2.26) we make an obvious guess

\[
\bar{\Phi} = U\bar{\phi}_I(y_{R\mu}^{\text{inv}}, \rho_{\text{inv}}) U^{-1}
\]

(2.40)
for scalar multiplet and
\[ W_\alpha = i\sqrt{2}U \bar{D}_{\bar{\alpha}} \phi_I(y_R^{inv}, \rho_{inv}, u_{inv})U^{-1} \zeta^\alpha \] (2.41)
for the vector one. Here
\[ U = \exp[-2A_I^I(\bar{\theta}_\mu \alpha)] \] (2.42)
is the unitary rotation which transforms ordinary derivatives into covariant ones in the expansion of these expressions in the deviation \((y_R^{inv} - y_R)\) (see (2.39)). This rotation is inessential in the large distance limit.

Eqs.(2.27) and (2.28), as well as (2.40) and (2.41), are our manifestly \(N=1\) supersymmetric results for instanton supermultiplets.

## 3 Instanton-induced effective vertex

In this section we are going to derive the effective vertex which once added to the tree level Lagrangian (2.1) reproduces instanton contributions to all correlation functions in the framework of a perturbation theory.

Let us first recall the notion of this vertex in non-supersymmetric theories. Consider pure gauge theory.

The instanton induced effective vertex suggested by Callan, Dashen and Gross a long time ago [13] (see also [14]) has the form
\[ V_{I_{CDG}} = -c \int d^4x \frac{d\rho}{\rho^5} \frac{d^3u}{2\pi^2} (\rho \Lambda)^b e^{\frac{\pi^2}{g^2} i \rho^2 Tr(\bar{\sigma}_\mu \sigma_\nu \bar{u} \tau^a u) F^{a}_{\mu \nu}] (3.1) \]
where \(b\) is the first coefficient of the \(\beta\)-function, \(c\) is a number. To derive it let us consider the \(n\)-point correlation function
\[ < A_{\mu_1}(x_1), \ldots, A_{\mu_n}(x_n) > \] (3.2)
in the instanton background in the large distance limit \((x_i - x_0)^2 \gg \rho^2, i = 1, \ldots, n\). Here the double index \(\mu i\) denotes the set of indices \(\mu_1 \ldots \mu_n\) On one hand, in the leading semiclassical approximation \((g^2 \ll 1)\) the correlation function (3.2) is given by the product of the classical expressions (2.6) for each \(A_{\mu_i}(x_i)\)
\[ \prod_{i=1}^{n} \eta_{\mu_i \nu_i} \rho^2 \frac{u \tau^{ai} \bar{u}}{(x_i - x_0)^4}(x_i - x_0)_{\nu_i} [1 + O(\frac{\rho^2}{(x_i - x_0)^2})] \] (3.3)
Note, that we expand (2.6) in the limit $(x_i - x_0) \gg \rho^2$.

On the other hand, to get the same expression using (3.1), let us add (3.1) to the classical action and then expand $e^{-V} = \sum_k \frac{1}{k!}(-V^k)$. Each term in this expansion corresponds to the $k$-instanton contribution. Now we concentrate on $k=1$. We have for (3.2)

$$< A_{\mu_1}(x_1), \ldots, A_{\mu_n}(x_n)V^CDG_I >_{pert} \quad (3.4)$$

where average means the integration over quantum fluctuations around perturbative vacuum. Expanding the exponent in (3.1) $n$ times and using

$$< A^a(x), F^b_{\alpha\beta}(x_0) >_{pert} = \frac{2g^2}{(2\pi)^2} \delta^{ab} \frac{(\delta_{\mu\beta}y_\alpha - \delta_{\mu\alpha}y_\beta)}{y^4} \quad (3.5)$$

in the large distance limit we arrive at (3.3).

Actually in this paper we will use the effective vertex method outlined above only in the large distance limit $(x_i - x_0)^2 \gg \rho^2$. However, it is worth noting that it is much more powerful. Namely, the effective vertex in (3.1) reproduces not only large distance behaviour of the classical instanton field in (2.6) but $\rho^2/y^2$ corrections as well \[23\]. These corrections comes from loop graphs. We can write symbolically the 1-point correlation function in (3.4)

$$< A(x), V^CDG_I >_{pert} = \frac{1}{g^2} < A, F >_{pert} + \frac{1}{2! g^4} < A, F, F >_{pert} + \cdots \quad (3.6)$$

The first term here gives the leading order contribution (3.5) we considered above. The others are nonzero provided interaction vertices from the classical Lagrangian are inserted. For example, the second term in the r.h.s. of (3.6) requires the insertion of the three-gluon vertex. However, these corrections are not suppressed by $g^2$ \[23\]. The extra powers of $g^2$ are canceled against powers $1/g^2$ appeared in the expansion (3.6). In fact, these higher order loop graphs give proper expansion of instanton field (2.6) in powers of $\rho^2/y^2$ \[23\].

It is worth note also that instanton-induced vertices of type (3.1) describe not only single instanton effects but multi-instanton interactions as well. In particular, instanton-anti-instanton interaction comes from

$$< V_I, V_I^\bar{\sigma} >_{pert} \quad (3.7)$$

where $V_I$ for anti-instanton is given by the similar to (3.1) formula with the replacement $\sigma \leftrightarrow \bar{\sigma}, u \leftrightarrow \bar{u}$. The above statement was checked in the large
distance limit for pure gauge theory \[13\] as well as for gauge-Higgs theory \[15\] and N=1 SUSY QCD \[22\].

Let us now make our theory more complicated adding Higgs fields in the adjoint representation. The effective vertex takes the form

$$V_{GH} = c_{GH} \int d^4 x \frac{d\rho}{\rho} (\rho \Lambda)^b d^3 u [\frac{-4\pi^2}{g^2} \rho^2 \bar{\phi}^a \phi^a + \frac{\pi^2}{g^2} \rho^2 Tr(\bar{\sigma}_ \mu \sigma_ \nu \bar{u} r^a u) F_{\mu \nu}^a],$$

where $c_{GH}$ is another calculable constant.

The similar vertex for Higgs fields in the fundamental representation was obtained in \[15\]. To derive (3.8) let us expand $\phi$ around its vev. We have

$$\bar{\phi}^a \phi^a = |v|^2 + \bar{v} \delta \phi^3 + \delta \bar{\phi}^3 v + \delta \phi^a \delta \phi^a. \quad (3.9)$$

It is sufficient to consider only correlation functions of Higgs fields

$$<\phi^{a_1}(x_1), \ldots, \phi^{a_n}(x_n), \bar{\phi}^{b_1}(z_1), \ldots, \bar{\phi}^{b_k}(z_k)>, \quad (3.10)$$

because the factor associated with gauge fields in (3.8) stays untouched as compared with (3.1) and produces the same correlation functions (3.3) of gauge fields.

First note, that $|v|^2$ term in (3.9) when inserted in (3.8) gives correctly the piece of the instanton action (2.10) associated with scalars. Consider now the second and the third terms in r.h.s. of (3.9) linear in quantum fluctuations. Expanding $exp - V_{GH}^I$ in $\delta \bar{\phi}^a \delta \phi^a$ and $\bar{v} \delta \phi$ and using propagation function for scalar field

$$<\phi^a(x), \delta \bar{\phi}^3(x_0)> = \delta^{a_3} \frac{g^2}{(2\pi)^2} \frac{1}{y^2} \quad (3.11)$$

one gets for the correlation function (3.10)

$$\prod_{i=1}^n \delta^{a_3} v [1 - \frac{\rho^2}{(x_i - x_0)^2}] \prod_{j=1}^k \delta^{b_3} \bar{v} [1 - \frac{\rho^2}{(z_j - x_0)^2}]. \quad (3.12)$$

This is exactly the same expression one gets in large distance limit substituting classical t’Hooft solutions (2.9), (2.33) into (3.10).

As for the last term in r.h.s. of (3.9), quadratic in fluctuations, it corresponds to quantum corrections and could be restored only if quantum propagators in the external instanton field are taken into account. An explicit
check on this term was made in [24] for Higgs fields in the fundamental representation. Note, that the promotion
\[ |v|^2 + \bar{v} \delta \phi^3 + \delta \bar{\phi}^3 v \rightarrow \bar{\phi} \phi \]  
(3.13)
which comes in a non-trivial way from quantum corrections in the language of correlation functions, appears as an obvious guess in the effective vertex approach. The reason is that effective Lagrangian cannot depend on vev of a given field (that would be signal for an explicit symmetry breaking rather than spontaneous). Effective Lagrangian “does not know” whether the spontaneous symmetry breaking occurs or not, and could depends only on fields. Therefore, the promotion (3.13) we made in (3.8) looks as a natural generalization compatible with symmetries of the theory. We use this rather powerful idea throughout the paper.

Now let us consider our N=2 SUSY gauge model. In fact, the instanton-induced vertex in this model is a straightforward N=2 supersymmetrization of (3.8). However, we will derive it first in terms of N=1 supermultiplets using results of the previous section. First we write down the result and then check it using the same method as for pure gauge and gauge-Higgs theories. Our result is
\[ V_{I}^{N=2} = -c_{N=2} \Lambda^4 \int d^4x_0 \frac{d\rho}{\rho} d^5u_{inv} d^2\alpha d^2\zeta d^2\rho_{inv}^a d^2\bar{\eta}_1 \frac{1}{\Phi^a} \]
\[ \exp \left[ -\frac{4\pi^2}{g^2} \rho_{inv}^2 \bar{\Phi}^a \Phi^a - \frac{4\sqrt{2\pi^2}}{g^2} \rho_{inv}^2 \Phi^a W^2_{a} \Upsilon^a - \frac{4\sqrt{2\pi^2}}{g^2} \rho_{inv}^2 \Phi^a W^2_{a} \Upsilon^a \right] \]
\[ + \frac{2\pi^2}{g^2} \rho_{inv}^2 \nabla_{\beta}(W^2 \tilde{u}_{inv} \tau^a_{inv} u_{inv})^\beta - \frac{16\pi^2}{g^2} \rho_{inv}^2 (W^2 \tilde{u}_{inv} \tau^a_{inv} u_{inv})^\beta \]
\[ + \frac{2\sqrt{2\pi^2}}{g^2} \rho_{inv}^2 (\zeta D u_{inv} \tau^a_{inv} u_{inv} \nabla_{\Phi}^a) \]
\[ - \frac{16\sqrt{2\pi^2}}{g^2} i \rho_{inv}^2 (\zeta D ^\bar{u}_{inv} \tau^a_{inv} u_{inv} \bar{\beta}_{inv} \Phi^a) \]
(3.14)
where supercovariant derivatives defined in the usual way
\[ \nabla_{\hat{\alpha}} = \frac{\partial}{\partial \hat{\theta}_{\hat{\alpha}}} \]
\[ \nabla^a = \frac{\partial}{\partial \alpha} - 2i \hat{\theta}^{\alpha\hat{\alpha}} \hat{\theta}_{\hat{\alpha}} \]
(3.15)
when acting on functions of left-handed coordinate \(x_0\).

Here N=1 superfields are understood as the following chiral functions
\[
\Phi = \Phi(x_0, \theta = -\alpha)
\]
\[
W = W(x_0, \theta = -\alpha),
\]
(3.16)
as well as anti-chiral functions
\[
\bar{\Phi} = \bar{\Phi}(x^R_0, \bar{\theta} = -\bar{\eta}_1)
\]
\[
\bar{W} = \bar{W}(x^R_0, \bar{\theta} = -\bar{\eta}_1),
\]
(3.17)
where right-handed instanton center is
\[
x^R_{0\mu} = x_{0\mu} - 2i(\bar{\eta}_1 \bar{\sigma}_\mu \alpha).
\]
(3.18)
The numerical constant \(c_{N=2}\) will be fixed below.

Now let us check the result (3.14). First note that deriving the vertex (3.1) for gauge theory in the semiclassical approximation we could restrict ourselves to checking only 1-point correlation function instead of (3.2). The reason is that exponential nature of (3.1) will reproduce then the result (3.3) for n-point correlation function with correct combinatorics automatically. In a similar way, for the gauge-Higgs theory we could make a check for the correlation function (3.10) with \(n=1, k=1\) only due to the same reason.

Therefore, to check the exponential factor in (3.14) it is sufficient to consider the following correlation function
\[
< \Phi^a(x^L_1, \theta_1), \bar{\Phi}^b(x^R_2, \bar{\theta}_2), \bar{W}^d_\alpha(x^R_3, \bar{\theta}_3), W^\alpha c(x^L_4, \theta_4) >
\]
(3.19)
in the instanton background. Let us plug \(exp(-V^N_{I=2})\) into (3.19) and keep only the one-instanton contribution \((-V^N_{I=2})\). Now consider the first term in the square brackets in (3.14). Expanding \(\bar{\Phi}\) and \(\Phi\) around their expectation values, we have
\[
\bar{\Phi}^a \Phi^a = |v|^2 + \bar{v} \delta \Phi^3 + \delta \bar{\Phi}^3 v + O(\delta \Phi \delta \Phi),
\]
(3.20)
where we dropped out the term quadratic in quantum fluctuations.

Expanding the exponent in (3.14) in powers of \(\delta \Phi^3 v\) and contracting the field \(\Phi^a(x^L_1, \theta_1)\) with \(\delta \bar{\Phi}^3(x^R_0, \bar{\theta} = -\bar{\eta}_1)\), using the propagation function for chiral fields
\[
< \Phi^a(x^L_1, \theta_1), \delta \bar{\Phi}^b(x^R_0, \bar{\theta}) > = \frac{\delta^{ab} g^2}{(2\pi)^2} e^{-2i \bar{\theta} \theta_1} \frac{1}{(x^L_1 - x^R_0)^2}
\]
(3.21)
one gets for the first factor in (3.19)

\[
\delta^{a3} \bar{v} [1 - \frac{\rho^2_{\text{inv}}}{(x^L - x_0 + 2i\bar{\eta}_1 \bar{\sigma}_\mu (\theta_1 + \alpha))}]
\] (3.22)

which is nothing else then the second term in the expression (2.28) for scalar supermultiplet \(\Phi(x^L, \theta_1)\) in the large distance limit. The first term we will recover later on. The invariant distance in the (3.22) comes as follows. The shift in \(2i\bar{\eta}_1 \bar{\sigma}_\mu \theta_1\) comes from the exponential in (3.21) at \(\bar{\theta} = -\bar{\eta}_1\) (see (3.17)), while the shift \(2i\bar{\eta}_1 \bar{\sigma}_\mu \alpha\) appears when we substitute in (3.21) the right-handed instanton center (3.18).

Now consider \(\bar{v} \delta \Phi^3\) term in the exponent in (3.14). Contracting it with the field \(\Phi(x^R, \theta_2)\) (3.19) we get

\[
\delta^{\bar{b}3} \bar{v} [1 - \frac{\rho^2_{\text{inv}}}{(x^R - x_0 - 2i\bar{\theta}_2 \bar{\sigma}_\mu \alpha)}].
\] (3.23)

This is the expression (2.40) for \(\Phi(x^R, \theta_2)\) in the large distance limit. Now to get the first term in the exponent in (3.14) we promote (3.20) to the full expression \(\Phi^a \Phi^a\). Although this term respects \(N=1\) SUSY it is not completely gauge invariant due to the shift (3.18). Therefore, we would expect that the correct promotion of (3.20) in (3.14) is \(\Phi^a (e^{-2V_g} \Phi)^a\), instead of \(\Phi^a \Phi^a\). This effect, however, is beyond our control in the large distance limit.

The next step is to look at the second term in the exponent in (3.14). As usual we are going to recover it only in semiclassical approximation

\[
\Phi^a W^a = \bar{v} W^3 + O(\delta \Phi^a W^a)
\] (3.24)

Contracting \(\bar{v} W^3\) term in the exponent in (3.14) with field \(\bar{W}(x^R, \theta_3)\) and making use of the propagation function of the W-field at large distances

\[
< W^{\alpha \alpha}(x^L, \theta), \bar{W}^\bar{b}\alpha (x^R, \bar{\theta}) > = \frac{ig^2}{2\pi^2} \delta^{ab} e^{-2i\bar{\theta}_3 \bar{\sigma}_\mu \alpha} (x^L - x^R)^{\alpha \bar{\alpha}} (x^L - x^R)^4
\] (3.25)

we get

\[
4\sqrt{2i\rho^2_{\text{inv}} \bar{v}} \frac{[(\bar{x}^R - \bar{x}_0)_{\alpha \alpha} - 4i\bar{\theta}_3 \bar{\sigma}_\mu \alpha]}{(x^R - x_0 - 2i\bar{\theta}_3 \bar{\sigma}_\mu \alpha)^4}
\] (3.26)

This is exactly what we need for the field \(W^a_\alpha(x^R, \theta_3)\) (2.41) in the large distance limit.
Continuing this process one can check in the same way that the third and the fourth terms in the exponent in (3.14) give us large distance limit of the expression (2.27) for the last term $W(x_1^4, \theta_4)$ in (3.19). Finally two last terms in the exponent in (3.14) give the missing first term in (2.28) for the field $\Phi(x_1, \theta_1)$ in (3.19). Again note, that to get completely gauge invariant effective vertex we should insert $e^{-2Vg}$ in all terms containing anti-chiral field as a function of a shifted argument (3.18).

Now let us discuss the preexponential factor in (3.14). The instanton measure in SUSY theories is well-known (see, for example [25, 9]). The factor associated with the integral over bosonic collective coordinates reads

$$2^{10} \pi^6 M^8 \rho^8 e^{-\frac{8\pi^2}{g^2} d^4x \frac{d\rho}{\rho^5} \frac{d^3u}{2\pi^2}}. \quad (3.27)$$

Here $M$ is the UV cutoff. The eighth power of $M\rho$ corresponds to the eight boson zero modes. To get the full instanton measure we have to take into account the normalization of fermion zero modes. Using eqs. (2.12), (2.15) and (2.16) we get

$$\frac{1}{16\pi^2 M} d^2\alpha \frac{1}{16\pi^2 M} d^2\zeta \frac{1}{32\pi^2 \rho^2 M} d^2\bar{\beta} \frac{1}{4\pi^2 \rho^2 \bar{\beta}^2 M} d^2\bar{\eta} \quad (3.28)$$

Here the numerical factor in front of each Grassmann integration accounts for the normalization of the corresponding fermion zero mode. Combining (3.27) and (3.28) together we have

$$\frac{1}{32\pi^2 \bar{\beta}^2} d^4x \frac{d\rho}{\rho} \frac{d^3u_{\text{inv}}}{\rho^5} d^2\alpha d^2\zeta d^2\bar{\beta}_{\text{inv}} d^2\bar{\eta}_{\text{inv}} \quad (3.29)$$

where $\Lambda^4 = M^4 e^\nu - 8\pi^2 / g^2$ in the Pauli-Villars regularization scheme. We also change variables from $u$ to $u_{\text{inv}}$ using (2.26) and from $\bar{\beta}$, $\bar{\eta}$ to $\bar{\beta}_{\text{inv}}$, $\bar{\eta}_{\text{inv}}$ using $d^2\bar{\beta} d^2\bar{\eta} = d^2\bar{\beta}_{\text{inv}} d^2\bar{\eta}_{\text{inv}}$ (see(2.21),(2.25)). The instanton measure in (3.29) is manifestly N=1 SUSY invariant.

Now we need only one final step to get the prefactor in (3.14). As we explained above the effective Lagrangian could not depend on the vev $\nu$ of

\footnote{This is clear because we can change variables $d\rho/\rho \rightarrow d\rho_{\text{inv}}/\rho_{\text{inv}}$ in (3.29) (or (3.14)). We have not done this explicitly here because the integral over $\rho$ is logarithmically divergent at small $\rho$ and will be performed below in a more careful way (see also [11]).}
the field $\Phi$. Therefore, we make the obvious promotion

$$\frac{1}{v^2} \rightarrow \frac{1}{\Phi^2}$$

(3.30)

in the instanton measure (3.29) similar to what we made in the exponent. This accounts for quantum corrections which are beyond our control in the semiclassical approximation.

Thus we recover the effective vertex (3.14). The numerical coefficient $c_{N=2}$ can be read off from (3.29):

$$c_{N=2} = \frac{1}{32\pi^2}. \quad (3.31)$$

Let us note in the conclusion of this section that the promotion

$$v \rightarrow \Phi^3$$

(3.32)

we use deriving (3.14) in the preexponential factor as well as in quadratic terms in the exponent should be considered as an approximation. Terms with derivatives of fields could be added to r.h.s. of (3.32). However, possible corrections, for example,

$$\rho \nabla^2 \Phi^a$$

(3.33)

are down by extra factor of $g$ because $\rho^2 \sim g^2 / \Phi^2$, see (2.11). In other words corrections (3.33) produce expansion in powers $\rho^2 / x^2 \ll 1$. However, there are also other corrections to (3.32) like

$$\zeta_\alpha W^{\alpha a}, \quad (3.34)$$

which are compatible with N=1 SUSY. These are not suppressed by powers of coupling constant. These corrections could be recovered using N=2 SUSY. We will do it in the next section.

4 Effective vertex and N=2 SUSY

In the last section we derived the effective instanton-induced vertex (3.14) making use of the classical expressions for superinstanton fields of section 2. However, certain terms in (3.14) (preexponential factor as well as terms
quadratic in quantum fluctuations in the exponent) are beyond our control
in the leading order of the semiclassical approximation. We fixed them using
$N=1$ supersymmetric promotion rule (3.32). Now we are going to check
if our vertex (3.14) is $N=2$ supersymmetric. Of course, classical terms
should be. But what about “quantum pieces”? We will show that the rule
(3.32) should be modified to be compatible with $N=2$ SUSY. This modification
fixes possible corrections of (3.34) type which remains undetermined in
the previous section.

To made a check on $N=2$ SUSY of (3.14) we use $N=2$ superfield formalism.
The $N=2$ chiral superfield is a subject to certain constrains [26]. Its low
components can be written in the form

$$\bar{\Psi}^a(x, \theta_1, \theta_2) = \Phi^a(x, \theta_1) + \sqrt{2} \theta_2 W^{\alpha a}(x, \theta_1) + \cdots,$$

(4.1)
where $x$ is assumed to be a left-handed coordinate and $\theta_1, \theta_2$ are two $\theta$-
parameters associated with the first and the second supersymmetry respectively. The anti-chiral field has the form

$$\bar{\Psi}^a(x_N^{R=2}, \bar{\theta}_1, \bar{\theta}_2) = \bar{\Phi}^a(x_N^{R=2}, \bar{\theta}_1) + \sqrt{2} \bar{\theta}_2 \bar{W}^a_{\dot{\alpha}}(x_N^{R=2}, \bar{\theta}_1) + \cdots,$$

(4.2)
where $N=2$ righthanded coordinate is related to $x$ in (4.1) as

$$(x_N^{R=2})_{\mu} = x_{\mu} - 2i \bar{\theta}_1 \bar{\sigma}_\mu \theta_1 - 2i \bar{\theta}_2 \bar{\sigma}_\mu \theta_2,$$

(4.3)
while $\bar{\theta}_1, \bar{\theta}_2$ are conjugate $\theta$-parameters.

Let us check that exponent in (3.14) can be rewritten in terms of superfields (4.1), (4.2), at least classically. The identification (3.16), (3.17) (as well as transformation laws (2.14), (2.22)) shows that $-\alpha$ and $-\bar{\eta}_1$ plays the role of $N=1$ SUSY $\theta$-parameters $\theta_1$ and $\bar{\theta}_1$. The symmetry between $\lambda$ and $\psi$ suggests that parameter $\zeta$ should be identified with $\theta_2$. What about $\bar{\theta}_2$? The symmetry between $\bar{\beta}$ and $\bar{\psi}$ (see (2.12), (2.13)) suggests that if we introduce a new integration variable $\bar{\eta}_2$ in (3.14) instead of $\bar{\beta}_{inv}$ as

$$\bar{\beta}_{inv} = \frac{\nu}{2\sqrt{2}} (\bar{u}_{inv} \tau_3 u_{inv} \bar{\eta}_2)_{\dot{\alpha}},$$

(4.4)
(this should be compared with (2.17)) we can try to identify it with $\bar{\theta}_2$.

It is easy to see that the first, the second, the forth and the last terms in
the exponent in (3.14) can be combined into the simple expression

$$- \frac{4\pi^2}{g^2} \rho_{inv}^2 \bar{\Psi}^a \Psi^a,$$

(4.5)
where

$$\Psi^a = \Psi^a(x_0, \theta_1 = -\alpha, \theta_2 = \zeta) \quad (4.6)$$

and

$$\bar{\Psi}^a = \bar{\Psi}^a(x_{0\mu} - 2i\bar{\eta}_1\sigma_\mu\alpha - 2i\bar{\eta}_2\sigma_\mu\zeta, \bar{\theta}_1 = -\bar{\eta}_1, \bar{\theta}_2 = \bar{\eta}_2) \quad (4.7)$$

Let us explain (4.5) in more detail. The first term in the exponent in (3.14) is reproduced when we combine fields $\Phi$ and $\bar{\Phi}$ in expansions (4.1) and (4.2) in (4.5). The second term comes from the $W$ component of $\Psi$ times the $\bar{\Phi}$ component of $\bar{\Psi}$ in (4.5). The forth term corresponds to the $\bar{W}$ component of $\bar{\Psi}$ times the $\Phi$ component of $\Psi$. Here we make use of definition (4.4) and drop out the non-abelian commutator term proportional to $\epsilon^{abc}\bar{\Psi}^a\Psi^b$ which is unessential for our future purposes. Strictly speaking we get $\bar{W}^3v$ instead of $\bar{W}^a\Phi^a$ in (4.5) from the forth term in the exponent in (3.14). To get actually $\bar{W}^a\Phi^a$ we use the promotion rule (3.32) as well as the gauge invariance. The last term in the exponent in (3.14) corresponds to the expansion of $\bar{\Psi}$ in (4.5) in powers of the shift $2i\bar{\eta}_2\bar{\sigma}_\mu\zeta$ of the N=2 right-handed coordinate, see (4.7). Note, that $\bar{W}W$ term in (4.5) cannot be checked at the classical level as it is quadratic in quantum fluctuations.

In the similar way the third and the fifth terms in (3.14) can be combined into expression

$$-\frac{\sqrt{2}\pi^2}{g^2} i\rho_{inv}^2 (\bar{\nabla}_2 \bar{\Psi}_{inv}^a \tau^a u_{inv} \bar{\nabla}_1) \Psi^a, \quad (4.8)$$

where

$$\bar{\nabla}_{2\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}_2^{\dot{\alpha}}} \quad (4.9)$$

is the covariant derivative with respect to the second supersymmetry, while $\bar{\nabla}_1$ is given by (3.15).

Combining (4.5) and (4.6) we arrive finally to the N=2 supersymmetric vertex

$$V_i^{N=2} = -\frac{1}{2\pi^2 \Lambda^4} \int d^4x \frac{dp}{\rho} d^3u_{inv} \frac{1}{\Psi_{a4}} d^2\alpha d^2\zeta d^2\bar{\eta}_1 d^2\bar{\eta}_2$$

$$\exp \left[ -\frac{4\pi^2}{g^2} \rho_{inv}^2 \Psi^a \bar{\Psi}^a - \frac{\pi^2}{\sqrt{2}g^2} \rho_{inv}^2 \bar{\Psi}^a \bar{\Psi}^a \bar{\nabla}_i \bar{\nabla}_j \tau^a u_{inv} \tau^a \bar{\nabla}_i u_{inv} \tau^a \bar{\nabla}_j \right] \quad (4.10)$$

where $i=1,2$ denotes the first and the second supersymmetry and

$$(\bar{\nabla}_i \bar{\nabla}^a u_{inv} \tau^a u_{inv} \bar{\nabla}^i) = 2(\bar{\nabla}_2 \bar{\nabla}^a u_{inv} \tau^a u_{inv} \bar{\nabla}_1).$$
The arguments of superfields here are given by (4.6), (4.7). To get the factor $1/\Psi^4$ in the exponent in (4.10) we change our promotion rule (3.32) to

$$v \to \Psi^3,$$  \hspace{1cm} (4.11)

which is obviously compatible with N=2 SUSY. The SUSY invariant size $\rho_{\text{inv}}$ should be understood in (4.10) as follows (see change of variables (4.4))

$$\rho_{\text{inv}}^2 = \rho^2 \left[ 1 + \sqrt{2i\Psi^a(\bar{\eta}_1 \bar{u}_{\text{inv}} \tau^a u_{\text{inv}} \bar{\eta}_2) \right] \hspace{1cm} (4.12)$$

Here we use (4.11) once again.

Eq. (4.10) is our final result for the instanton-induced effective vertex in N=2 SUSY gauge theory. Note, that we derived (4.10) from (3.14) using only low components of superfields $\Psi, \bar{\Psi}$ (see (4.1), (4.2)). However, the expression (4.10) is more general. It contains also higher components which cannot be restored in the large distance limit. The reason is that they contain terms like $\nabla^2 \Phi$ which give $\delta$-functional contributions to the correlation function (3.19) and therefore are invisible in the large distance limit.

In the conclusion of this section let us summarize approximations we use to obtain (4.10). First, we assumed the weak coupling regime and pick up the leading effect in $g^2$. Second, we assumed the large distance limit $x^2 \gg \rho^2$ because the way we derived the effective vertex assumes checking the large distance behaviour of instanton field (see section 3). Also our promotion rule (4.11) could contain corrections (3.33) which gives powers of $\rho^2/x^2$.

Besides this, for the sake of simplicity we do not include in (4.10) certain terms related to the non-abelian structure of our theory. These effects play no role in the low-energy effective Lagrangian we are aiming to derive using (4.10) in the next section. Namely, we do not write explicitly gauge factors $e^{-2V_g}$ in (3.14) and (4.10) as well. These factors can be easily restored using gauge invariance. We also drop out the term proportional to $e^{abc} \bar{\Psi}^a \Psi^b$ in the exponent in (4.10). This term can be extracted from (3.14) in a straightforward manner.

Finally, we assumed condition $x^2 \ll v^2$, in order not to deal with exponential tails of instanton field. This condition can be easily relaxed. In the next section we go to the limit $x^2 \gg v^2$ and derive low-energy vertex for light fields using (4.10).

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3This condition probably could be relaxed, see the discussion on page 12. We come back to this point in the conclusion.
Let us note, that our result in eq. (4.10) could be obtained in a much more simple way by N=2 supersymmetrization of the effective vertex (3.8) for the gauge-Higgs theory. To see this observe, that term (4.8) is the N=2 supersymmetrization of the Callan-Dashen-Gross effective vertex, while term (4.5) is a direct N=2 supersymmetrization of the first term in the exponent (3.8).

5 Low energy effective Lagrangian

In this section we are going to derive the low energy effective Lagrangian starting with our effective vertex (4.10) for microscopic theory. In particular, we recover Seiberg-Witten result (1.1) in which the one-instanton contribution to the prepotential is

$$ F_I(A) = -\frac{1}{8\pi^2} \frac{\Lambda^4}{A^2}, \quad (5.1) $$

as a leading order effect in the large distance expansion.

So far we considered large but not very large distance limit $\rho^2 \ll x^2 \ll 1/v^2 \sim 1/M_W^2$. Now we are going to relax the condition $x^2 \ll 1/v^2$. It is clear that to do so we have to add the instanton-induced vertex (4.10) to the tree-level Lagrangian (2.3) and consider gauge symmetry breaking in this theory expanding field $\Psi^a$ around its vev

$$ \Psi^a = \delta^{a3} v + \delta\Psi^a \quad (5.2) $$

Then $\Psi_1$ and $\Psi_2$ components become massive and do not propagate at large distances. To write down the low energy effective Lagrangian for light fields in $\Psi_3$ supermultiplet at distances $x^2 \gg 1/v^2$ we have to integrate out heavy fields in (2.3) with (4.10) added.

For one-instanton induced effects this boils down to dropping out heavy fields in (4.10). We have for the low energy effective vertex

$$ V_I^{LE} = -\frac{1}{4\pi^2} \Lambda^4 \int d^4x \frac{d\rho \, d^3 u_{inv}}{\rho} \, \frac{1}{2\pi^2} \frac{d^2 \theta_1 d^2 \bar{\theta}_1 d^2 \theta_2 d^2 \bar{\theta}_2}{\Psi_3^4} \exp \left[ -\frac{4\pi^2}{g^2} \rho_{inv}^2 \bar{\Psi}_3 \Psi_3 - \frac{\pi^2}{\sqrt{2} g^2} \rho_{inv}^2 \bar{u}_{inv} \tau^3 u_{inv} \nabla^i \Psi_3 \right] \quad (5.3) $$

22
Grassmann parameters here are redefined to their conventional notations using (4.6), (4.7), while

\[ \rho_{\text{inv}}^2 = \rho^2 [1 - \sqrt{2} i \Psi_3 (\bar{\theta}_1 \bar{u}_{\text{inv}} \tau_3 u_{\text{inv}} \bar{\theta}_2)] \]

\[ = \rho^2 [1 + \frac{1}{\sqrt{2}} i \Psi_3 (\bar{\theta}_1 \bar{u}_{\text{inv}} \tau_3 u_{\text{inv}} \bar{\theta}_2)] \] (5.4)

Corrections to (5.3) involve loop graphs with vertex (4.10) inserted as well as perturbative interactions from (2.3). In these graphs light fields are in external legs while heavy fields propagate in loops. As we explained in section 3 these graphs will give corrections \((\rho^2/x^2)^n\) to the leading large distance behaviour of correlation functions. These effects can be encoded in (5.3) as an insertions of powers of the operator (see(3.33))

\[ \rho \nabla^2 \sim g \frac{\nabla^2}{\Psi_3} \] (5.5)

We ignore these effects as they are suppressed in coupling constant. Thus, to the leading order in \(g^2\) we are left with the low energy vertex (5.3).

Let us now note, that our result in (5.3) make a bridge between two approaches to instanton calculations in SUSY theories. The first one developed in [11, 12] use \(N=1\) superfield formalism to calculate certain correlation functions in the instanton background. Instanton superfields in this approach are written down in manifestly supersymmetric way in terms of SUSY invariant combinations. We follow this method in section 2. In refs. [11, 12] certain correlation functions of chiral superfields were calculated. However, the naive extrapolation of this method to correlation functions of anti-chiral fields would give wrong results.

Now we know the reason for this. Besides classical effects our effective vertex (5.3) (or (4.10)) contains certain “quantum” effects which appear when we use the promotion rule (4.11). In particular, the promotion of \(1/v^4\) to \(1/\Psi^4\) in the preexponent of (4.10), (5.3) has no effects on expressions for chiral fields. However, in correlation functions of anti-chiral fields this give an additional “quantum” contributions. Surprisingly, they appear to be of the same order as classical ones after integrating over instanton size. The reason is that classical expressions of fields are proportional to \(\rho^2\), however typical \(\rho^2 \sim g^2/v^2\). Thus, they are of the same order. The effective vertex
(4.10), (5.3) incorporates both classical and “quantum” effects in manifestly supersymmetric way.

Another approach originated by Affleck-Dine-Seiberg [20] (see also [1, 11]) deals with instanton fields in components. In particular, in order to extract the Seiberg-Witten term (5.1) the correlation function of anti-chiral fermions \( \langle \bar{\lambda} \lambda \bar{\psi} \psi \rangle \) has been considered [7, 9]. Although components of chiral fields can be arranged in supermultiplets (2.27), (2.28) components of anti-chiral fields in this approach differ from the expansion of our formulas (2.40), (2.41) in components. The difference is that they are proportional to \( \rho^2 \), rather then \( \rho^2 \) inv as in our approach. Thus, the supersymmetry is spoiled. In fact, it should be restored after the integration over collective coordinates.

We can mimic this approach in our effective Lagrangian method writing down the vertex

\[
V' = -\frac{\Lambda^4}{4\pi^2} \int d^4x \frac{d\rho}{\rho} \frac{d^3u}{2\pi^2} d^2\bar{\theta}_1 d^2\theta_1 d^2\bar{\theta}_2 d^2\theta_2 \frac{1}{v^4} \exp \left[ \pi \rho^2 \left( \Psi_3 \right) \right]
\]

where \( \Psi_3 \) is expanded as in (5.2).

The vertex (5.6) differs from our vertex in (5.3) in two ways. First, \( \Psi_3 \delta \Psi_3 \) comes multiplied by \( \rho^2 \) in (5.6), hence the expressions for anti-chiral fields produced by (5.6) are proportional to \( \rho^2 \), rather then \( \rho^2 \) inv as we discussed above. Second, no promotion rule is used in the preexponent, as well as in definition of \( \rho^2 \) inv in front of \( \Psi_3 v \). Thus (5.6) contains only classical effects.

Now we show that (5.6) reduces to (5.3) at least at the Seiberg-Witten order (the leading order in derivatives of fields). To do this rewrite the exponent in (5.6) as

\[
\exp \left[ \frac{4\pi^2}{g^2} \rho^2 \left( \Psi_3 \Psi_3 - \sqrt{2i} (\bar{\theta}_1 \tilde{u} \tau_3 u \bar{\theta}_2) \Psi_3 v^2 \right) \right]
\]  

(5.7)

Then make a change of variables

\[
\bar{\theta}_2' = \bar{\theta}_2 \frac{v^2}{\Psi_3}
\]

(5.8)

We get

\[
V' = -\frac{\Lambda^4}{4\pi^2} \int d^4x \frac{d\rho}{\rho} \frac{d^3u}{2\pi^2} d^2\bar{\theta}_1 d^2\theta_1 d^2\bar{\theta}_2 d^2\theta_2 \bar{\theta}_2'
\]
\[ \frac{1}{\Psi_3^4} \exp \left[ -\frac{4\pi^2}{g^2 \rho_{inv}} \bar{\Psi}_3 \Psi_3 \right], \quad (5.9) \]

with \( \rho_{inv} \) from (5.4). This is nothing else then our vertex (5.3) with Callan - Dashen - Gross term omitted (it contains higher derivatives of fields). In fact, the discussion above can be viewed as a purely “classical derivation” of our promotion rule (4.11). Note, that the change of variables (5.8) is possible only in the leading order of large distance limit when the dependence of \( \bar{\Psi}_3 \) on its argument \( \bar{\theta}_2 \) is ignored.

Thus we have shown that our effective vertex (4.10), (5.3) combine the superfield formalism of refs. [11, 12] with the component approach in refs. [20].

Now let us integrate over the size instanton \( \rho \) in (5.3). The dependence on \( \bar{\theta}_1, \bar{\theta}_2 \) in (5.3) comes from explicit dependence of \( \rho_{inv}^2 \) on these parameters as well as the dependence of \( \bar{\Psi}_3 \) on its arguments. According to this, there are three ways to saturate the integrals over \( \bar{\theta}_1, \bar{\theta}_2 \).

First, ignore the \( \theta \)-dependence of \( \bar{\Psi} \). This will give us the leading Seiberg - Witten term. Once the dependence of \( \bar{\theta}_1, \bar{\theta}_2 \) comes only via \( \rho_{inv}^2 \) we can write down

\[ \int d^2 \bar{\theta}_1 d^2 \bar{\theta}_2 f(\rho_{inv}^2) = -\frac{1}{2} \Psi_3^2 \left[ (\rho^2 \frac{\partial}{\partial \rho^2})^2 - \rho^2 \frac{\partial}{\partial \rho^2} \right] f(\rho^2) \quad (5.10) \]

for any function \( f \).

We see that the integral over \( \rho \) reduces to a total derivative. Moreover, infinite size instanton do not contribute in (5.3) and we are left with zero size instanton. This important phenomenon was first noticed in [11]. Substituting (5.10) into (5.3) we have

\[ V_{LE}^{F} = \frac{\Lambda^4}{16\pi^2} \int d^4 x d^2 \bar{\theta}_1 d^2 \bar{\theta}_2 \frac{1}{\Psi_3^4}. \quad (5.11) \]

Note that (5.11) is N=2 F-term.

This is nothing else then Seiberg - Witten one-instanton vertex. To see this rewrite (5.11) in N=1 supermultiplets. It reads

\[ V_{LE}^{F} = \frac{\Lambda^4}{8\pi^2} \int d^4 x d^2 \bar{\theta}_1 d^2 \bar{\theta}_2 A \frac{\bar{A} A}{A^4} - \frac{3\Lambda^4}{16\pi^2} \int d^4 x d^2 \bar{\theta}_1 W_3^2 \frac{W_3 A^2}{A^4}. \quad (5.12) \]

where \( A \equiv \Phi_3 \). This coincides with (1.1) where function \( F_I \) is given by (5.1).
Now let us work out other contributions in (5.3). The second possibility is to extract one power of \( \bar{\theta}_1 \bar{\theta}_2 \) from \( \bar{\Psi}_3 \) and another power from \( \rho_{\text{inv}}^2 \). It is easy to see that integral over \( \rho \) again reduces to the total derivative. However, in this case the function \( f \) is zero in both limits \( \rho = 0 \) and \( \rho = \infty \).

The last possibility is to ignore \( \bar{\theta} \)-dependence in \( \rho_{\text{inv}}^2 \) and saturate integrals using \( \bar{\theta} \)-dependence of \( \bar{\Psi}_3 \) in (5.3). Then the integral over \( \rho \) gives

\[
V_D^{LE} = \frac{\Lambda^4}{8\pi^2} \int d^4xd^2\theta_1 d^2\bar{\theta}_1 d^2\theta_2 d^2\bar{\theta}_2 \frac{d^3u}{2\pi^2} \frac{1}{\Psi_3^4} \log[\Psi_3\bar{\Psi}_3 + i\sqrt{2}(\bar{\nabla}_i u_{3\tau} u_{\bar{i}})\bar{\Psi}_3]
\]

(5.13)

This is \( N=2 \) D-term in contrast to the leading Seiberg-Witten effect (5.11), which is F-term. The factor \( 1/\Psi_3^4 \) in front of logarithm account correctly for the anomalous chiral symmetry violation induced by instanton. Note, that the dependence on the orientation vector \( u_\mu \) enters only in the Callan-Dashen-Gross piece.

Eqs. (5.11) and (5.13) is our final result for the one-instanton low-energy effective Lagrangian in the theory at hand. Anti-instanton generate the complex conjugate to (5.11), (5.13).

The D-term (5.13) is the correction in derivatives of fields to the leading Seiberg-Witten F-term (5.11). In general the next-to-leading correction to the Seiberg-Witten effective theory (1.1) is given by \( N=2 \) D-term [27]

\[
S_{\text{next-to-leading}}^{LE} = \int d^4xd^2\theta_1 d^2\bar{\theta}_1 d^2\theta_2 d^2\bar{\theta}_2 K(\Psi, \bar{\Psi}),
\]

(5.14)

where \( K \) is a real function of its arguments. In \( N=1 \) superfields (5.14) reads

\[
S_{\text{next-to-leading}}^{LE} = \frac{1}{16} \int d^4xd^2\theta_1 d^2\bar{\theta}_1 \left( K_{\alpha A} \nabla^2 A \bar{\nabla}^2 \bar{A} + 2 \bar{\nabla}^\alpha \nabla_\alpha A \bar{\nabla}^\alpha \bar{\nabla}_\alpha \bar{A} \right.

+ 4 \nabla_\alpha W_3^\alpha \bar{\nabla}^\alpha \bar{W}_{3\alpha} - 4 \nabla_\alpha (W_3^\alpha \bar{W}_{3\alpha}) \nabla^\alpha W_3^{\alpha \beta} - 4 \nabla^\alpha (\bar{W}_{3\alpha} W_3^{\alpha \beta}) \nabla_\alpha \bar{W}_{3\beta} -

2 \nabla^2 W_3^2 - 2 \nabla^2 \bar{W}_{3\alpha}^2 \right) - 2 K_{\alpha A A} W_3^2 \nabla^2 A - 2 K_{\alpha A A} \bar{W}_{3\alpha} \nabla^2 \bar{A} +

K_{\alpha A A A} \left[ -8 W_3^\alpha \nabla^\alpha A W_3^{\alpha \beta} \bar{\nabla}_{\alpha} \bar{A} + 4 W_3^2 \bar{W}_{3\alpha} \right].
\]

(5.15)

The subscripts on \( K \) here denotes derivatives with respect to its arguments.
Comparing (5.15) with (1.1) we see that the expansion goes in powers of operators
\[ \frac{\nabla^2}{A}, \frac{W^2_3}{A^3}. \] (5.16)
If we ignore for a moment the Callan-Dashen-Gross term in (5.13) our result implies that the one-instanton contribution to the function \( K(A, \bar{A}) \) is given by
\[ K_I(A, \bar{A}) = \frac{\Lambda^4}{8\pi^2 A^4} \log \bar{A}A + \text{c.c.} \] (5.17)

The Callan-Dashen-Gross term in (5.13) represents further corrections to (5.15) in the same parameters (5.16).

To conclude let us sum up our approximations. Our effective Lagrangian in (5.13) controls all powers of corrections in derivatives (5.16) to the leading Seiberg-Witten term (5.11). Integrating over orientations in (5.13) will produce the infinite series in powers of this parameters. Instead, the corrections of type (5.5) which involved extra powers of coupling constant are beyond our control. The later effects becomes important at distances \( x^2 \sim \rho^2 \), which is much less then the limit \( x^2 \gg 1/v^2 \sim 1/M_W^2 \) assumed for a low energy effective theory to describe the physics of light degrees of freedom correctly.

6 Conclusions

The Seiberg-Witten exact solution for the low energy effective Lagrangian of the N=2 SUSY gauge theory accounts for all possible powers of coupling constant \( g^2 \) and instanton factor \( \Lambda^4/A^4 \) in the leading order of the large distance expansion. Instead in this paper we limit ourselves in the weak coupling region of the moduli space taking into account only one-instanton-induced effects in the leading order in \( g^2 \) and study the large distance expansion of low-energy theory. Our effective Lagrangian (5.13) contains all orders of corrections in derivatives of fields to the Seiberg-Witten term (5.11). We ignore only corrections in powers of \( \rho^2/x^2 \), which are really corrections (5.5) in \( g^2 \), provided large distance limit \( x^2 \gg 1/v^2 \) is considered.

In particularly, the next-to-leading order correction to the Seiberg-Witten exact solution in the large distance expansion is given by eq.(5.14), (5.15) [27]. We have calculated here the one instanton contribution to the function \( K(A, \bar{A}) \) (5.17). The general form of the perturbative contribution to this
function has been studied in recent paper [28]. For the abelian low energy theory at hand we have

\[ K(A, \bar{A}) = c_{00} \log \bar{A} \log A, \]  

(6.1)

where \( c_{00} \) is unknown constant. Comparing (6.1) and (5.17) one might expect that in general function \( K(A, \bar{A}) \) is given by the expansion

\[ K(A, \bar{A}) = \sum_{n,m=0}^{\infty} L_n c_{nm} L_m, \]  

(6.2)

modulo the real part of the holomorphic function which does not contribute to (5.15). Here Coulomb vector \( L_n \) is defined as follows

\[ L_0 = \log A, \quad L_n = \frac{\Lambda^4}{A^4}, n = 1, 2 \ldots \]  

(6.3)

and coefficients \( c_{nm} \) satisfy the reality condition \( \bar{c}_{nm} = c_{mn} \). The expansion in (6.2) is invariant under \( Z_8 \) group (up to Kahler transformation). Each \( L_n \) with \( n \geq 1 \) comes from the instanton with the topological charge \( n \), while \( \bar{L} \)'s come from anti-instantons. In particular, our result (5.17) gives \( c_{01} = 1/8\pi^2 \). In principle, coefficients \( c_{nm} \) could receive further logarithmic corrections due to higher loops.

It would be interesting to study the behaviour of function \( K(A, \bar{A}) \) in the strong coupling region of the modular space. Of particular interest is to find the singularities of this function. The reason is that the low energy theory is not well defined in these singular points due to the absence of the proper kinetic term. The duality transformation transforms (5.15) into the same expression with new function \( K_D \) defined as [27]

\[ K_D(A_D, \bar{A}_D) = K(A, \bar{A}), \]  

(6.4)

where dual field is \( A_D = \mathcal{F}'(A) \). Particularly dangerous effect could occur if \( K_D \) has a singularity at \( a_D = 0 \) or \( a_D = a \) where monopole or dyon becomes massless. This would spoil the nice physical description [3] of the low energy theory as a dual massless QED near these points.

Let us now say a few words about \( \rho^2/x^2 \) corrections to (5.3). In fact, the natural conjecture is that our effective vertex (4.10) for the microscopic
theory provides the systematic method to calculate all terms in the long-distance expansion of low-energy effective Lagrangian, including $\rho^2/x^2$ corrections. The reason for that conjecture is that, as we explained in section 3 [23], $\rho^2/x^2$ corrections to the leading at large distances behaviour of instanton field appears in instanton-induced effective Lagrangian method as loop diagrams. Still one may doubt in the above conjecture because actually we derived our effective vertex (4.10) using not only classical considerations but the promotion rule (4.11) as well which could provide additional $\rho^2/x^2$ corrections. However, we showed in the last section that the promotion rule (4.11) does not account for true quantum corrections. It can be understood as the result of a change of variables within the purely classical analysis.

In particularly, we can argue now that the Seiberg–Witten N=2 F-term (5.11) does not get any $\rho^2/x^2$ corrections at all. To see this recall that F-term appears when we integrate in (5.3) over $\bar{\theta}_1$ and $\bar{\theta}_2$ using the explicit dependence of $\rho_{inv}$ on this parameters. This insures that the integral over $\rho$ reduces to the integral of total derivative. Hence, the result for F-term comes from zero size instanton $\rho^2 \rightarrow 0$, so the corrections in $\rho^2/x^2$ are zero.

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