Study of the semileptonic decays $B \rightarrow \pi$, $D \rightarrow \pi$ and $D \rightarrow K$

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Abstract. The semileptonic decay $B \rightarrow \pi$ is studied starting from a simple quark model that takes into account the effect of the $B^*$ resonance. A novel, multiply subtracted, Omnès dispersion relation has been implemented to extend the predictions of the quark model to all $q^2$ values accessible in the physical decay. By comparison to the experimental data, we extract $|V_{ub}| = 0.0034 \pm 0.0003(\text{exp}) \pm 0.0007(\text{theory})$. As a further test of the model, we have also studied $D \rightarrow \pi$ and $D \rightarrow K$ decays for which we get good agreement with experiment.

PACS. 12.15.Hh – 11.55.Fv – 12.38.Jh – 13.20.He

1 Introduction

The exclusive semileptonic decay $B \rightarrow \pi l^+ \nu_l$ provides an important alternative to inclusive reactions $B \rightarrow X_u l^+ \nu_l$ in the determination of de Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$.

This reaction has been studied in different approaches like lattice-QCD (both in the quenched and unquenched approximations), light-cone sum rules (LCSR) and constituent quark models (CQM), each of them having a limited range of applicability: LCSR are suitable for describing the low momentum transfer square ($q^2$) region, while lattice-QCD provides results only in the high $q^2$ region. CQM can in principle provide form factors in the whole $q^2$ range but they are not directly connected to QCD. A combination of different methods seems to be the best strategy.

The use of Watson’s theorem for the $B \rightarrow \pi l^+ \nu_l$ process allows one to write a dispersion relation for each of the form factors entering in the hadronic matrix element. This procedure leads to the so-called Omnès representation, which can be used to constrain the $q^2$ dependence of the form factors from the elastic $\pi B \rightarrow \pi B$ scattering amplitudes. The problem posed by the unknown $\pi B \rightarrow \pi B$ scattering amplitudes at high energies can be dealt with by using a multiply subtracted dispersion relation. The latter will allow for the combination of predictions from various methods in different $q^2$ regions.

In this work we study the semileptonic $B \rightarrow \pi l^+ \nu_l$ decay. The use of a multiply subtracted Omnès representation of the form factors will allow us to use the predictions of LCSR calculations at $q^2 = 0$ in order to extend the results of a simple nonrelativistic constituent quark model (NRCQM) from its region of applicability, near the zero recoil point, to the whole physically accessible $q^2$ range. To test our model we shall also study the $D \rightarrow \pi$ and $D \rightarrow K$ semileptonic decays for which the relevant CKM matrix elements are well known and there is precise experimental data.

2 $B \rightarrow \pi l^+ \bar{\nu}$

The matrix element for the semileptonic $B^0 \rightarrow \pi^- l^+ \nu_l$ decay can be parametrized in terms of two dimensionless form factors

$$
\langle \pi(p_\pi) | V^{\mu} | B(p_B) \rangle = \left(p_B + p_\pi - q \frac{m_B^2 - m_\pi^2}{q^2}\right) f^+(q^2) + q^2 \frac{m_B^2 - m_\pi^2}{q^2} f^0(q^2)
$$

(1)

where $q^\mu = p_B - p_\pi$ is the four momentum transfer and $m_B = 5279.4$ MeV and $m_\pi = 139.57$ MeV are the $B^0$ and $\pi^-$ masses. For massless leptons, the total decay width is given by

$$
\Gamma(B^0 \rightarrow \pi^- l^+ \nu_l) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B} \int_0^{q^2_{\text{max}}} dq^2 |\lambda(q^2)|^2 |f^+(q^2)|^2
$$

(2)

with $q^2_{\text{max}} = (m_B - m_\pi)^2$, $G_F = 1.16637 \times 10^{-5}$ GeV$^{-2}$ and $\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2 = 4m_B^2 |p_\pi|^2$, with $p_\pi$ the pion three-momentum in the $B$ rest frame.

2.1 Nonrelativistic constituent quark model: Valence quark and $B^*$ resonance contributions

Figure 1 shows how the naive NRCQM valence quark description of the $f^+$ form factor fails in the whole $q^2$ range.
In the region close to \( q^2_{\text{max}} \), where a nonrelativistic model should work best, the influence of the \( B^* \) resonance pole is evident. Close to \( q^2 = 0 \) the pion is ultra relativistic, and thus predictions from a nonrelativistic model are unreliable.

As first pointed out in Ref. [5], the effects of the \( B^* \) resonance pole dominate the \( B \to \pi l^+\nu_l \) decay near the zero recoil point \( (q^2_{\text{max}}) \). Those effects must be added coherently as a distinct contribution to the valence result. The hadronic amplitude from the \( B^* \)-pole contribution is given by

\[
-iT^\mu = -i\hat{g}_{B^*B^\pi}(q^2)p^\mu_{\pi^0}(i\frac{-q^2_B + q^2_{\pi^0}/m^2_B}{q^2 - m^2_B}) i\sqrt{q^2}\hat{f}_{B^*}(q^2)\]

with \( m_{B^*} = 5325 \text{ MeV} \). \( \hat{f}_{B^*} \) and \( \hat{g}_{B^*B^\pi} \) are respectively the off-shell \( B^* \) decay constant and off-shell strong \( B^*B^\pi \) coupling constant. See Ref. [11] and references therein for details on their calculation. From the above equation one can easily obtain the \( B^* \)-pole contribution to \( f^+ \) which is given by

\[
f^+_{\text{pole}}(q^2) = \frac{1}{2} \hat{g}_{B^*B^\pi}(q^2)\frac{\sqrt{q^2\hat{f}_{B^*}(q^2)}}{m^2_B - q^2} \]

The inclusion of the \( B^* \) resonance contribution to the form factor improves the simple valence quark prediction down to \( q^2 \) values around 15 GeV\(^2\). Below that the description is still poor.

### 2.2 Omnès representation

Now one can use the Omnès representation to combine the NRCQM predictions at high \( q^2 \) with the LCSR at \( q^2 = 0 \). This representation requires as an input the elastic \( B\pi \to B\pi \) phase shift \( \delta(s) \) in the \( f^P = 1^- \) and isospin \( I = 1/2 \) channel, plus the form factor at different \( q^2 \) values below the \( \pi B \) threshold where the subtractions will be performed. For a large enough number of subtractions, only the phase shift at or near threshold is needed. In that case one can approximate \( \delta(s) \approx \pi \), arriving at the result that

\[
f^+(q^2) \approx \frac{1}{s_{th} - q^2} \prod_{j=0}^{n}(f^+(q^2_j)(s_{th} - q^2_j))^{-\alpha_j(q^2)}, \ n \gg 1 \quad (5)
\]

with \( s_{th} = m_B + m_\pi \) and \( \alpha_j(q^2) = \prod_{j\neq k=0} Q^2_j q^2_j - q^2_k \)

Figure 2 shows with a solid line the form factor obtained using the Omnès representation with six subtraction points: we take five \( q^2 \) values between 18 GeV\(^2\) and \( q^2_{\text{max}} \) for which we use the \( f^+ \) NRCQM predictions (valence + \( B^* \) pole), plus the LCSR prediction at \( q^2 = 0 \). The \( \pm \sigma \) lines enclose a 68% confidence level region that we have obtained from an estimation of the theoretical uncertainties. The latter have two origins: (i) uncertainties in the quark–antiquark nonrelativistic interaction and (ii) uncertainties on the product \( g_{B^*B^\pi}f_{B^*} \), and on the input to the multiply subtracted Omnès representation. See Ref. [11] for details.

By comparison with the experimental value for the decay width, we obtain

\[
|V_{ub}| = 0.0034 \pm 0.0003(\text{exp.}) \pm 0.0007(\text{theo.}) \quad (6)
\]

in very good agreement with the value found by the CLEO Collaboration [4].

### 3 D → πlν_l and D → Klν_l

Our results for the \( f^+ \) form factor are depicted in Figures 8 and 11. As before we have considered valence quark plus resonant pole contributions (\( D^* \) and \( D^*_e \) respectively). In both cases, we obtain a good description in the physical region of the experimental data [7] and previous lattice...
We compare with experimental data by the FOCUS Collaboration [10]. Besides we have found for the decay widths $\Gamma(D^0 \rightarrow K^- e^+ \nu_e)$ of the $D \rightarrow \pi$ and $D \rightarrow K$ decays and $q^2$ in the physical region we have found good agreement with experimental and lattice data.

4 Concluding remarks

We have shown the limitations of a pure valence quark model to describe the $B \rightarrow \pi, D \rightarrow \pi$ and $D \rightarrow K$ semileptonic decays. As a first correction, we have included vector resonance pole contributions which dominate the relevant $f^+$ form factor at high $q^2$ transfers. Subsequently, for the $B \rightarrow \pi$ decay, we have applied a multiply subtracted Omnès dispersion relation. This has allowed us to extend the results of the NRCQM model to the whole $q^2$ range. Our result for $|V_{ub}|$ is in good agreement with recent experimental data by the CLEO Collaboration. For $f^+(q^2)$ of the $D \rightarrow \pi$ and $D \rightarrow K$ decays and $q^2$ in the physical region we have found good agreement with experimental and lattice data.

5 Acknowledgments

This work was supported by DGI and FEDER funds, under Contracts No. FIS2005-00810, BFM2003-00856 and FPA2004-05616, by the Junta de Andalucía and Junta de Castilla y León under Contracts No. FQM0225 and No. SA104/04, and it is a part of the EU integrated infrastructure initiative Hadron Physics Project under Contract No. RI3-CT-2004-506078. J.M.V.-V. acknowledges an E.P.LF contract with the University of Salamanca. C. A. acknowledges a research contract with the University of Granada.

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