Purifying two–bit quantum gates and joint measurements in cavity QED

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Using a cavity QED setup we show how to implement a particular joint measurement on two atoms in a fault tolerant way. Based on this scheme, we illustrate how to realize quantum communication over a noisy channel when local operations are subject to errors. We also present a scheme to perform and purify a universal two–bit gate.

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One of the most intriguing features of quantum mechanics is the possibility of entangling physical systems, which has both practical and fundamental implications. On the one hand, Bell’s theorem \[1\] states that quantum mechanics and any local realist theory are incompatible based on the peculiar properties of entanglement. On the other, quantum communication and computation exploit these properties to guarantee secure communication and to construct algorithms that allow fast computations \[2\]. In that physical scenario the only remaining problem is local errors. Although one could in principle use standard error correction schemes to solve this problem, this would again require infinite resources.

In this work we will give a physical implementation that allows to perform local operations and measurements ideally using finite resources. The scheme is based on cavity QED and therefore can be easily connected to the previous proposal for quantum communication \[7,8\]. We will assume that operations acting on a single atom are error free, whereas any other operation is not. This is motivated by the experimental fact that single bit operations are much simpler than multiple bit operations \[6\].

First we will show how to perform a particularly useful joint measurement which is fault tolerant \[13\] in the sense that it operates even in the presence of errors occurring during this measurement. An essential element for this measurement is the introduction of a “red light atom” \(R\) \[14\] which reveals the occurrence of errors. We will also show how to implement a universal two–bit operation \(14\) which also involves measurements that indicate whether an error took place or not. In the former case, one has to start the procedure again, whereas in the latter case, one knows one has succeeded. Our schemes can be regarded as purification protocols \[11\] since with certain probability they are successful, while sometimes the information is lost. We emphasize that in applications in quantum communication the loss of information is not central, whereas the knowledge that one has reliably transmitted the quantum information is indispensable.

We start by discussing the physical details of our setup. We consider two atoms, 1 and 2 inside a single cavity. The internal structure of the atoms is displayed in Fig. 1; the qubit is stored in the states \(|0\rangle\) and \(|1\rangle\), and there is an auxiliary state \(|r\rangle\). The states \(|1\rangle\) and \(|r\rangle\) are coupled by a far–off–resonance Raman transition induced by an external laser field and the cavity mode, whereas the state \(|0\rangle\) is not coupled by either the laser or the cavity field. The Hamiltonian describing the interaction between the atoms and the cavity mode is given, in a rotating frame at the cavity mode frequency, by

\[
H = \frac{g_1}{2} |1\rangle_1 \langle 1|_1 |r\rangle + \frac{g_2}{2} |1\rangle_2 \langle 2|_2 |r\rangle + h.c. \tag{1}
\]

where \(a\) is the annihilation operator for the cavity mode, and \(g_{1,2}\) are the effective coupling constants of the Raman transition. In the following, we will consider that a laser pulse of duration \(\Delta t_1 = \pi/g_1\) is applied to atom 1 and then another laser pulse of duration \(\Delta t_2 = \pi/g_2\) is applied to atom 2 \[15\]. Denoting by \(|0\rangle_{cav}\) and \(|1\rangle_{cav}\) the
where we have considered only the cases in which the first atom is in $|0\rangle_1$ or $|1\rangle_1$ and the second atom is in $|0\rangle_2$ or $|1\rangle_2$, since this will be sufficient for our purposes. Note that if the first atom is in the state $|0\rangle_1$, nothing will change. However, if it is in $|1\rangle_1$, then it will be transferred to $-i|1\rangle_1$. Then, if the second atom is in $|1\rangle_2$, then it will be transferred to the state $-i|1\rangle_2$, whereas if it is in $|0\rangle_2$, it will not change its state and a cavity photon will remain in the cavity. In reality there will be errors. Since we are considering a far–off resonance Raman transition, the most important ones will be photon losses either at the mirrors or by leaking out of the cavity. As in our previous work [8] we will also consider systematic errors in the detuning, timing, laser pulses, phase shifts, etc. It is straightforward to account for these errors in Eq. (2) by including the state of the environment and different operators acting on it, as well as adding new terms in the last two lines which describe the effect of photon loss (see below). On the other hand, we will also need single–atom operations involving the three atomic levels. As mentioned in the introduction, we will concentrate here on errors occurring in processes involving two bits.

In the first part of this Letter, we will be interested in the following situation: atom 2 is initially in state $|0\rangle_2$, and is transferred to state $|1\rangle_2$; then the process takes place, followed by two single–atom operations, namely $-|r\rangle_1 \leftrightarrow |1\rangle_1$ and $|r\rangle_2 \leftrightarrow |0\rangle_2$ in the first and second atom, respectively. Hence, ideally we have

$$ |0\rangle_1 |0\rangle_2 \rightarrow |0\rangle_1 |0\rangle_2, \quad |1\rangle_1 |0\rangle_2 \rightarrow |1\rangle_1 |1\rangle_2. \quad (3) $$

In the presence of the errors mentioned above,

$$ |0\rangle_1 |0\rangle_2 |1\rangle \rightarrow |0\rangle_1 |0\rangle_2 \mathcal{L}_0 |1\rangle \quad (4a) $$

$$ |1\rangle_1 |0\rangle_2 |1\rangle \rightarrow |1\rangle_1 |1\rangle_2 \mathcal{L}_1 |1\rangle + |1\rangle_1 |0\rangle_2 \mathcal{L}_a |1\rangle \quad (4b) $$

where $|E\rangle$ denotes the initial state of the environment (including the cavity mode), and the operators $\mathcal{L}$ act on this state. We used that one can optically pump the state $|r\rangle_1$ to the state $|1\rangle_1$ after the whole procedure. Note that with this notation this process is formally equivalent to the photonic channel introduced in Ref. [13].

In the following we will assume the environment operators $\mathcal{L}_{0,1}$ fulfill the stationary property for two consecutive operations

$$ \mathcal{L}_2(\mathcal{L}_1^{(2)} |E\rangle) = \mathcal{L}_0(\mathcal{L}_1^{(1)} |E\rangle), \quad (5) $$

starting at times $t_{1,2}$, of duration $\Delta t_{1,2}$, respectively. Here we have used the short hand notation $\mathcal{L}_i^{(j)} \equiv \mathcal{L}_i(t_j, \Delta t_j)$, where $i = 0, 1$ and $j = 1, 2$. In Ref. [8], the validity of (3) has been demonstrated for the present model using the quantum trajectories approach. Here, as a simple example, we illustrate this stationarity property in the context of photon absorption: we consider a cavity mode coupled to a bath of oscillators in the vacuum state $|E\rangle \equiv |0\rangle$ (i.e., at zero temperature). We assume a linear coupling Hamiltonian

$$ H = \omega a^\dagger a + \sum_k \omega_k b_k^\dagger b_k + \sum_k g_k (a^\dagger b_k + h.c.), \quad (6) $$

where $b_k$, $b_k^\dagger$ are creation and annihilation operators for the bath oscillators, and $\omega_k$ and $g_k$ the corresponding frequencies and coupling constants. Denoting by $t$ the initial time, after a time $\Delta t$ we will have

$$ |0\rangle_2 |E\rangle \rightarrow |0\rangle_2 |E\rangle, \quad \equiv |0\rangle_2 \mathcal{L}_0(t, \Delta t) |E\rangle, $$

$$ |1\rangle_2 |E\rangle \rightarrow c(\Delta t)|1\rangle_2 |E\rangle + |0\rangle_2 \sum_k c_k(\Delta t)b_k^\dagger |E\rangle, \quad \equiv |1\rangle_2 \mathcal{L}_1(t, \Delta t) |E\rangle + |0\rangle_2 \mathcal{L}_a(t, \Delta t) |E\rangle. $$

where $c$ and $c_k$ are $c$–numbers. Note that $\mathcal{L}_{0,1}$ only depend on $\Delta t$ but not on the initial time $t$. Moreover, they commute and therefore they satisfy (3). The stationary property is related to the zero temperature of the reservoir, which for optical frequencies is a good approximation even at room temperature. On the other hand, one can verify that systematic errors also fulfill (3) since the corresponding $\mathcal{L}_{0,1}$ will be $c$–numbers only depending on $\Delta t$ but not on $t$.

Our goal is to use (3) to perform ideal joint measurements and entanglement operations as are required in quantum communication via a photonic channel [8,10]. In this scheme, one has to perform a local joint measurement on two atoms to check whether they are in the state $|0\rangle_1 |0\rangle_2$ or not. It must be implemented such that an error occurring during this measurement will be detected by the measurement itself. To be specific, let us consider two atoms in a state $|\Psi\rangle = |\Psi_e\rangle |E_e\rangle + |0\rangle_1 |0\rangle_2 |E_a\rangle$, where $|E_{e,a}\rangle$ denote unnormalized states of the environment, and $|\Psi_e\rangle = \alpha|0\rangle_1 |1\rangle_2 + \beta|1\rangle_1 |0\rangle_2$ with $\alpha$ and $\beta$ arbitrary coefficients. The goal is to make a filtering measurement of the state $|0\rangle_1 |0\rangle_2$, so that with certain probability the state of the atoms is projected onto the $|\Psi_e\rangle$ which is the one we want to keep intact. In order to perform the joint measurement we need the red light atom, $R$, initially prepared in the state $|0\rangle_R$. We use (3) between atoms 1 and $R$, and then between atoms 2 and $R$ [See Fig. 2(a)]. This gives the transformation

$$ |0\rangle_1 |1\rangle_2 |0\rangle_R \rightarrow |0\rangle_1 |1\rangle_2 |1\rangle_R \mathcal{L}^{(2)}_1 |E_e\rangle + |0\rangle_1 |1\rangle_2 |0\rangle_R \mathcal{L}^{(2)}_0 |E_e\rangle $$

$$ |1\rangle_1 |0\rangle_2 |0\rangle_R \rightarrow |1\rangle_1 |0\rangle_2 |1\rangle_R \mathcal{L}^{(2)}_1 |E_e\rangle + |1\rangle_1 |0\rangle_2 |0\rangle_R \mathcal{L}^{(2)}_0 |E_e\rangle $$

$$ |0\rangle_1 |0\rangle_2 |0\rangle_R \rightarrow |0\rangle_1 |0\rangle_2 |0\rangle_R \mathcal{L}^{(2)}_0 |E_e\rangle $$

where we have left out the state of the environment. Now a single–atom measurement on atom $R$ in the state $|1\rangle_R$
The last two terms in (10) arise from photon loss errors, and can be detected by performing a joint measurement on atoms 2 and a, namely checking whether they are in the state $|0\rangle R_c$ or $|0\rangle R_a$. If they are not found in this state, a single ion measurement on atom $a$ (in the basis $|0\rangle \pm |1\rangle$) leaves atoms 1 and 2 in a maximally entangled state. The joint measurement requires entanglement, and therefore is susceptible to errors. However, we can use instead our implementation of this joint measurement using the red light ion in cavity 2 [see Fig. 2(b)]. Repeating the transmission $|0\rangle$ and the subsequent measurement $|0\rangle$ until no photon loss was detected (the red light ion is found in the state $|1\rangle$), yields, after having measured atom $a$ in the basis $|0\rangle \pm |1\rangle$, the state $|\psi\rangle_1 = |0\rangle|0\rangle \pm |1\rangle|1\rangle$. With this EPR state one can already distribute a random secret key using the Ekert protocol [87] for quantum cryptography [99].

For certain applications in quantum communication and quantum computing a two–bit universal gate is required, since when combined with one–bit operations this is sufficient for any unitary operation [83]. This gate cannot be implemented using Eq. (4) since there the state $|1\rangle|1\rangle$ is absent as input state, whereas in the gate this state has to be present. We show now how to perform the universal gate

$$|0\rangle|1\rangle|0\rangle|2\rangle \rightarrow |0\rangle|1\rangle|0\rangle|2\rangle - |1\rangle|0\rangle|0\rangle|2\rangle; \quad (12a)$$

$$|0\rangle|1\rangle|0\rangle|2\rangle \rightarrow |0\rangle|1\rangle|0\rangle|2\rangle - |1\rangle|1\rangle|2\rangle, \quad (12b)$$

with the present implementation in the presence of errors. The gate consists of three steps: (i) A single atom operation on atom 2 exchanges $|1\rangle|2\rangle \leftrightarrow |r\rangle|2\rangle$ while leaving the state $|0\rangle|2\rangle$ unchanged; (ii) we perform a conditional operation using the cavity mode such that the state $|1\rangle|0\rangle|2\rangle \rightarrow -|1\rangle|0\rangle|2\rangle$ by applying (2) twice; (iii) we apply the inverse of step (i). Note that, according to the evolution given by (8), if the initial state is $|1\rangle|0\rangle|2\rangle$ the cavity photon produced the first time will be absorbed again by atom 1 the second time, yielding a minus sign, as desired.

In reality there will be errors due to photon losses, phase shifts of the states involved, and imperfect state transfer. After applying the gate one obtains, including these errors,

$$|0\rangle|1\rangle|0\rangle|2\rangle \rightarrow |0\rangle|1\rangle|0\rangle|2\rangle \mathcal{L}_{10} \mathcal{E}_{0} \quad (13a)$$

$$|0\rangle|1\rangle|0\rangle|2\rangle \rightarrow |0\rangle|1\rangle|0\rangle|2\rangle \mathcal{L}_{01} \quad (13b)$$

$$|0\rangle|1\rangle|0\rangle|2\rangle \rightarrow -|1\rangle|1\rangle|2\rangle \mathcal{L}_{10} + |r\rangle|1\rangle|0\rangle|2\rangle \mathcal{L}_{r0} \quad (13c)$$

$$|1\rangle|1\rangle|0\rangle|2\rangle \rightarrow |1\rangle|1\rangle|2\rangle \mathcal{L}_{11} + |r\rangle|1\rangle|2\rangle \mathcal{L}_{r1} + |r\rangle|1\rangle|2\rangle \mathcal{L}_{rr}. \quad (13d)$$

The “photon loss” errors $\mathcal{L}_{r0,r1,rr}$ can be detected by measuring if the first atom is in state $|r\rangle$. In order to perform the gate in the presence of all these errors we apply (13) four times but changing $(0) \leftrightarrow |1\rangle$ first in atom 1, then in atom 2 and again in atom 1, after subsequent applications. Moreover, in the last one we change the phase of the laser field acting on atom 2 by $\pi$ in the second part of step (ii) so that no extra minus sign is added to the state $|1\rangle|1\rangle|0\rangle|2\rangle$; therefore, this fourth application performs just the (noisy) identity operation in order to
symmetrize the errors]. If no error is found during the whole procedure (i.e. population in state $|r\rangle_1$) we obtain

$$
|0\rangle_1 \rightarrow |0\rangle_1 L_{10}^2 L_{00}^2 L_{11}^2 L_{01}^2 (14a)
$$

$$
|0\rangle_1 \rightarrow |0\rangle_1 L_{10}^2 L_{00}^2 L_{11}^2 L_{01}^2 (14b)
$$

$$
|1\rangle_1 \rightarrow |1\rangle_1 L_{10}^2 L_{00}^2 L_{11}^2 L_{01}^2 (14c)
$$

$$
|1\rangle_1 \rightarrow |1\rangle_1 L_{10}^2 L_{00}^2 L_{11}^2 L_{01}^2 (14d)
$$

Using the same arguments as in (12), one can check that all these operators are identical. Thus, once no error was found the gate worked perfectly.

So far, we used the stationary properties (11) and (5) for transmission and local operations. It is important to realize that, even if the former one (11) does not hold, one can still establish a perfect EPR pair, since we have shown here how to purify all local operations (including the gate) needed for the procedure developed in [16]. On the other hand, if also (5) would not hold, one can establish an entangled state whose degree of entanglement is limited by the degree to which (3) is satisfied.

In summary, we have shown how perform joint measurements in the presence of errors in a cavity QED implementation. The scheme works even if errors occur during the measurement itself. We have shown how to apply this proposal in quantum communication to achieve perfect transmission over a noisy channel including local errors. Using the same implementation, we have also presented a universal two–bit gate that operates perfectly in the presence of errors.

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