Research Article

Homogeneously Mixed Memory Charts with Application in the Substrate Production Process

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Abstract

The cumulative sum (CUSUM) and the exponentially weighted moving average (EWMA) charts are renowned classical memory charts used to monitor small and moderate shifts in the process(s). Mixed memory charts like mixed EWMA-CUSUM (MEC) and mixed CUSUM-EWMA (MCE) are the advanced forms of classical memory charts used to identify shifts quickly in process parameters (location and/or dispersion). Similarly, the homogeneously weighted moving average (HWMA) chart is used for improved process monitoring. It will be worthwhile to combine the HWMA chart features with the existing mixed memory (MCE and MEC) charts to enhance the effectiveness of the mixed memory charts. Therefore, we proposed new charts: mixed HWMA-homogeneously CUSUM (MHWHC) and mixed homogeneously CUSUM-HWMA (MHCHW) charts. The Monte Carlo simulations are used to evaluate the proposed charts’ effectiveness. The average run length (ARL) is utilized to compare the proposed MHWHC and MHCHW charts’ performance with existing charts such as classical CUSUM and EWMA, MEC, MCE, and HWMA charts. The comparison revealed that the proposed mixed charts are superior to the existing counterparts, specifically monitoring small and moderate shifts. Finally, a real-life application using the manufacturing process’s data set is also provided from a practical point of view.

1. Introduction

Natural and unnatural variations are part of every manufacturing/nonmanufacturing process. Natural variations regularly occur and are essential part of any stable process, whereas unnatural variations damage the quality process caused by process deficiencies. Statistical process control (SPC) is a valuable toolkit that allows one to monitor unnatural variations. The charts are popular SPC tools to monitor shifts in the process parameters. Generally, charts are divided into two groups: memoryless and memory-type charts. Shewhart [1] proposed the memoryless charts used to monitor large shifts in the process parameters. On the other hand, the classical memory charts like the cumulative sum (CUSUM) chart designed by Page [2] and the exponentially weighted moving average (EWMA) chart offered by Roberts [3] are used for the efficient handling of small and moderate shifts.

Different modifications and improvements in SPC literature are continuously used in the classical memory charts to enhance process monitoring. One modification is mixed memory charts, which improve conventional memory-type charts’ performance. Abbas [4] and Abbas et al. [5] proposed mixed EWMA-CUSUM (MEC) charts for the process location and dispersion shift monitoring, respectively. The MEC chart is effective in monitoring shifts in the process location. Similarly, Aslam et al. [6] designed a mixed chart based on a variable sampling scheme that outperforms the traditional nonparametric chart in terms of reduced sample size. Likewise, Aslam [7] offered the mixed EWMA-CUSUM chart for Weibull distribution. After that, Aslam et al. [8] proposed a mixed hybrid EWMA-CUSUM control chart for
the monitoring of the normal process. Subsequently, Zaman et al. [9] suggested a reverse version of the MEC chart, named the mixed CUSUM-EWMA (MCE) chart. Furthermore, Zaman et al. [10] suggested the MEC chart for monitoring process location and dispersion. In addition, Ajadi and Riaz [10] presented the mixed multivariate EWMA-CUSUM chart for efficient process monitoring. Osei-Aning et al. [12] constructed the MEC and MCE charts to monitor the first-order autoregressive processes. Later on, Ali and Haq [13] extended the work of Abbas et al. [14] and introduced a mixed generally weighted moving average CUSUM chart to monitor process mean. Also, Rao et al. [15] constructed a new mixed EWMA-CUSUM chart for monitoring small shifts relying on the COM-Poisson distribution. Anwar et al. [16] and Anwar et al. [17] recently introduced mixed charts through single auxiliary information labeled as MEC_AIB, MCE_AIB, and combined MEC_AIB (CMECAIB) charts, respectively. The MEC_AIB, MCE_AIB, and CMECAIB charts perform better than the existing counterparts. Zaman et al. [18] proposed a multivariate mixed memory chart for improved process monitoring. Likewise, Aslam and Anwar [19] and Anwar et al. [20] proposed Bayesian modified-EWMA and auxiliary information-based modified-EWMA charts to boost process location monitoring, respectively.

The limitation of the classical EWMA chart is that the EWMA plotting statistic assigned more weights to current observations than the previous observations [21]. Abbas [22] suggests the homogeneous weighted moving average (HWMA) chart to overcome this limitation. The HWMA chart assigns specific weights to the current observation and remaining weights to the previous observations homogeneously. The distribution of weights in such a way improves the HWMA chart’s performance ability compared to its competitor’s charts. Later on, Adegoke et al. [23] generalized the idea of Abbas [22] and introduced an auxiliary-based HWMA (AHWMA) chart. Similarly, Abid et al. [24] extended the work of Abbas [22] and advocated a double HWMA (DHWMAC) chart. Adeoti and Koleoso [25] offered the hybrid HWMA (HHWMA) chart for efficient monitoring shift in the process location. After that, Adeoti et al. [26] introduced the HWMA chart for count data. This newly introduced structure is based on a COM-Poisson distribution and named as CMP-HWMA chart. Moreover, Abid et al. [27] proposed an MHC chart for monitoring shifts in the process location. Also, Riaz et al. [28] presented a triple HWMA (THWMA) chart for monitoring the process location parameter. Recently, Anwar et al. [29] introduced auxiliary information-based DHWMA chart for enhanced process location monitoring.

As mentioned earlier, the mixed memory charts (MEC and MCE) are practiced to monitor small and moderate shifts. Similarly, the homogeneous memory charts are the enhanced versions of classical memory charts for efficient monitoring of process parameters. To the best of our knowledge, no single chart combining the features of mixed memory chart along with homogeneously memory charts has been investigated previously. It is a clear research gap and needs to be explored. So, to address this research gap, this study introduces mixed memory charts based on homogeneous features for monitoring process location. These charts are named the mixed HWMA-homogeneously CUSUM and mixed homogeneously CUSUM-HWMA charts, symbolized as MHWHC and MHCHW charts. It is expected that combining features of mixed memory charts along with homogeneously charts will further enhance the detection ability of the ultimate charts. Average run length (ARL), extra quadratic Loss (EQL), performance comparison index (PCI), and relative ARL (RARL) measurements are used to test the effectiveness of the suggested charts. Besides, an algorithm is designed in R software using the Monte Carlo simulations technique to obtain numerical results. A variety of existing charts, including the classical EWMA, CUSUM, MEC, MCE, and HWMA charts, are compared with the proposed charts. Moreover, the proposed MHWHC and MHCHW charts are executed in a real-life application to ensure their practicability.

The remainder of the paper is organized as follows: Section 2 provides the variable of interest and methodologies of the existing charts. Similarly, the design structures for HCUSUM, MHWHC, and MHCHW charts are presented in Section 3. Likewise, Section 4 defines the various performance evaluation measures. Also, the performance comparisons of the proposed charts to the existing CUSUM, EWMA, MEC, MCE, and HWMA are given in Section 5. Furthermore, Section 6 contains real-life data analysis for practical implementation of the proposed charts. Lastly, the summary, conclusion, and recommendation are set out in Section 7.

2. Existing Methods

Here, Section 2.1 explains the variable of interest. The classical CUSUM chart is explained in Section 2.2. In Section 2.3, the classical EWMA chart is demonstrated. The MEC and MCE charts’ design structures are presented in Sections 2.4 and 2.5, respectively, whereas Section 2.6 contains the HWMA chart’s methodology.

2.1. Variable of Interest. Suppose that $X$ is the process variable that follows a normal distribution, that is, $X \sim N(\mu_X + \delta \sigma_X, \sigma_X)$. If the process is in control (IC), then $\delta = 0$; otherwise, if the process is out of control (OOC), $\delta \neq 0$. Let $X_i = \sum_{i=1}^{n} X_n / n$ and $S_i^2 = \sum_{i=1}^{n} (X_n - X_i)^2 / (n - 1)$ be the mean and variance of $X$, respectively, of $i^{th}$ observation of $i^{th}$ sample of size $n$. For the IC scenario, $X_i$ and $S_i^2$ are mutually independent identically distributed, that is, $X_i \sim N(\mu_X, \sigma_X^2 / n)$ and $S_i^2 \sim \chi^2_i / (n - 1) / (\chi^2_{n-1})$.

2.2. Classical CUSUM Chart. Page [2] introduced the classical CUSUM chart and it is used to monitor small and moderate shifts in the process location. The classical CUSUM chart’s plotting statistics are defined as follows:

$$C_i^+ = \max[0, (X_i - \mu_0) - K + C_{i-1}^+],$$
$$C_i^- = \max[0, -(X_i - \mu_0) - K + C_{i-1}^-].$$
The $C^+_0$ and $C^-_0$ statistics are equal to zero. Also, $K = k(\sigma/\sqrt{n})$ is a slack value and is usually taken as $k = \delta/2$. Here $\delta$ denotes shift, and it is defined as $\delta = |\mu_1 - \mu_0|/\sigma/\sqrt{n}$, where $\mu_1$ is deviated value of $\mu_0$. The $C^+_t$ and $C^-_t$ statistics are plotted against $H = h(\sigma/\sqrt{n})$ and $h$ is a control limit coefficient. If $C^+_t > H$ or $C^-_t > H$, the process is considered OOC; otherwise, it is considered IC.

2.3. Classical EWMA Chart. The classical EWMA chart was introduced by Roberts [3]. It is also used to monitor small and moderate shifts in the process location. For the classical EWMA chart, the plotting statistic is given by

$$Z_t = \lambda \bar{X}_t + (1 - \lambda)Z_{t-1},$$

(2)

where $Z_0 = 0$ and $\lambda$ is a smoothing constant such that $0 < \lambda < 1$. For the conventional EWMA chart, the upper control limit (UCL) and lower control limit (LCL) are as follows:

$$\text{LCL}_{(\text{EWMA})} = \mu_0 - L_{\text{EWMA}}\sigma_\bar{X} \sqrt{\frac{1}{1 - \lambda}} (1 - (1 - \lambda)^2t),$$

$$\text{UCL}_{(\text{EWMA})} = \mu_0 + L_{\text{EWMA}}\sigma_\bar{X} \sqrt{\frac{1}{1 - \lambda}} (1 - (1 - \lambda)^2t).$$

(3)

Here $L_{\text{EWMA}}$ is the control limit coefficient. The $Z_t$ statistic is plotted against $\text{LCL}_{(\text{EWMA})}$ and $\text{UCL}_{(\text{EWMA})}$. The process is considered OOC whenever $Z_t < \text{LCL}_{(\text{EWMA})}$ or $Z_t > \text{UCL}_{(\text{EWMA})}$; otherwise, it is considered to be IC.

2.4. MEC Chart. Abbas, Riaz, and Dose [14] proposed the MEC chart by combining the classical EWMA and CUSUM charts’ properties. The classical EWMA statistic is used as an element in the classical CUSUM statistic to construct the MEC chart. The plotting statistics of the MEC chart are as follows:

$$\text{MEC}^+_t = \max \{0, (Z_t - \mu_0) - K_{Z_t} + \text{MEC}^-_{t-1} \},$$

$$\text{MEC}^-_t = \max \{0, -(Z_t - \mu_0) - K_{Z_t} + \text{MEC}^+_{t-1} \},$$

(4)

where $K_{Z_t}$ is defined as

$$K_{Z_t} = k \sqrt{\text{Var}(Z_t)} = k\sigma \sqrt{\frac{1}{n(2 - \lambda)} (1 - (1 - \lambda)^2t)},$$

(5)

The MEC$^+_t$ and MEC$^-_t$ statistics for the MEC chart are plotted against $H_{Z_t}$, which is defined as

$$H_{Z_t} = h \sqrt{\text{Var}(Z_t)} = h\sigma \sqrt{\frac{1}{n(2 - \lambda)} (1 - (1 - \lambda)^2t)},$$

(6)

Here, $h$ is the control limit coefficient. The MEC chart will provide an OOC signal if $\text{MEC}^+_t > H_{Z_t}$ or $\text{MEC}^-_t > H_{Z_t}$.

2.5. MCE Chart. Zaman et al. [30] proposed a reverse of the MEC chart. Unlike the MEC chart, the design structure of the MCE chart offers the classical CUSUM statistics as input for the EWMA chart given as

$$\text{MCE}^+_t = \lambda C^+_t + (1 - \lambda)\text{MCE}^+_t,$$

$$\text{MCE}^-_t = \lambda C^-_t + (1 - \lambda)\text{MCE}^-_t,$$

(7)

where $C^+_t$ and $C^-_t$ are the classical CUSUM statistics, $\lambda (0 < \lambda \leq 1)$ is the smoothing parameter of the proposed chart, and $\text{MCE}^+_0 = \text{MCE}^-_0 = \mu$. The mean and variance of the plotting statistics are given as Mean ($C^+_t$) = Mean ($C^-_t$) = $\mu_t$ and $\text{Var}(C^+_t) = \text{Var}(C^-_t) = \sigma_t^2$, respectively. The control limit for the MCE chart is as follows:

$$\text{UCL}_{(\text{MCE})} = \mu_t + L_{\text{MCE}} \frac{\lambda}{n(2 - \lambda)} (1 - (1 - \lambda)^2t),$$

(8)

where $L_{\text{MCE}}$ represents the control limit coefficient. If $\text{MCE}^+_t > \text{UCL}_{(\text{MCE})}$ or $\text{MCE}^-_t > \text{UCL}_{(\text{MCE})}$, the process is not IC; otherwise, it remains as IC.

2.6. HWMA Chart. Abbas [22] introduced the HWMA chart to enhance process location monitoring. Unlike the classical EWMA chart, the HWMA chart equally allocates weights to all the previous observations. The plotting statistic of the HWMA chart is given as

$$E_t = \lambda \bar{X}_t + (1 - \lambda)\overline{X}_{t-1},$$

(9)

where $\bar{X}_t$ is the sample average of the $t^{th}$ group and $\lambda (0 < \lambda \leq 1)$ is the smoothing constant and $\overline{X}_{t-1} = \sum_{i=1}^{t-1} \bar{X}_i / t - 1$ is the mean of sample average of previous $t - 1$ samples and $\bar{X}_0 = \mu_0$. The UCL and LCL of the HWMA chart are given as

$$\text{LCL}_{(\text{HWMA})} = \mu_0 - L_{(\text{HWMA})} \frac{\lambda}{n},$$

if $t = 1,$

$$\mu_0 - L_{(\text{HWMA})} \frac{1}{n} \left( \frac{\lambda^2 + (1 - \lambda)^2}{(t - 1)} \right),$$

if $t > 1,$

$$\mu_0 + L_{(\text{HWMA})} \frac{\lambda}{n},$$

if $t = 1,$

$$\mu_0 + L_{(\text{HWMA})} \frac{1}{n} \left( \frac{\lambda^2 + (1 - \lambda)^2}{(t - 1)} \right),$$

if $t > 1,$

(10)

where $L_{\text{HWMA}}$ is the chart’s coefficient, and its value is based on the IC ARL. The process is to be OOC if $E_t < \text{LCL}_{(\text{HWMA})}$ or $E_t > \text{UCL}_{(\text{HWMA})}$.

3. Design Structure of the Proposed Charts

Section 3.1 sets out the structure of the HCUSUM chart. The MHWHC chart’s methodology is defined in Section 3.2, while Section 3.3 defines the MHCWH chart’s methodology.

3.1. Proposed HCUSUM Chart. Following Abbas [22], the plotting statistics of the HCUSUM are defined as
\[ HC^+_t = \max\{0, (\bar{X}_t - \mu_0) - K + \bar{X}_{t-1} \}, \quad (11) \]
\[ HC^-_t = \max\{0, -(\bar{X}_t - \mu_0) - K + \bar{X}_{t-1} \}, \quad (12) \]

where \( \bar{X}_0 = \mu_0 \) and \( K = k(\sigma/\sqrt{n}) \) is a slack value as mentioned in CUSUM chart. The HCUSUM statistics \( HC^+_t \) and \( HC^-_t \) are plotted against \( H_{(HCUSUM)} = h(\sigma/\sqrt{n}) \), and the process is considered to be OOC if \( HC^+_t > H_{(HCUSUM)} \) or \( HC^-_t > H_{(HCUSUM)} \); otherwise, it is considered to be IC.

3.2. Proposed MHWHC Chart. The proposed MHWHC chart is constructed by mixing HWMA and HCUSUM charts. The plotting statistics of the proposed MHWHC chart are obtained by using the HWMA statistic in equation (9) as input for the HCUSUM statistics in equations (11) and (12).

\[ MHWHC^+_t = \max\{0, (E_t - \mu_0) - K_{(MHWHC)} + \bar{X}_{t-1} \}, \quad (13) \]
\[ MHWHC^-_t = \max\{0, (E_t - \mu_0) - K_{(MHWHC)} + \bar{X}_{t-1} \}, \quad (14) \]

where the reference value \( K_{(MHWHC)} \) is defined as follows:

\[ K_{(MHWHC)} = \begin{cases} 
  k\sigma \sqrt{\frac{\lambda^2}{n}}, & \text{if } t = 1, \\
  k\sigma \frac{1}{n} \left[ \frac{\lambda^2 + (1-\lambda)^2}{(t-1)} \right], & \text{if } t > 1.
\end{cases} \quad (15) \]

MHWHC^+_t and MHWHC^-_t are the statistics of MHWHC chart. These statistics are plotted against the control limit \( H_{(MHWHC)} \), which is defined as

\[ UCL_{(MHWHC)} = \mu_{(MHWHC)} + L_{MHWHC}\sigma_{(MHWHC)} \sqrt{\frac{\lambda}{2-\lambda} \left( 1 - (1-\lambda)^2 \right)}, \quad (20) \]

where \( L_{MHWHC} \) is control limit coefficient, which decides the predefined false alarm rate. The MHWHC chart will detect OOC signal if \( MHWHC^+_t > UCL_{(MHWHC)} \) or \( MHWHC^-_t > UCL_{(MHWHC)} \).

4. Performance Evaluation Measures

This section consists of the performance evaluation measures of the proposed charts. ARL is explained in Section 4.1, and the overall performance evaluation measures, that is, EQL, RARL, and PCI, are outlined in Section 4.2. Likewise, the Monte Carlo simulation is defined in Section 4.3. Parameters' roles and choices and special cases of the proposed

\[ H_{(MHWHC)} = \begin{cases} 
  h\sigma \sqrt{\frac{\lambda^2}{n}}, & \text{if } t = 1, \\
  h\sigma \frac{1}{n} \left[ \frac{\lambda^2 + (1-\lambda)^2}{(t-1)} \right], & \text{if } t > 1,
\end{cases} \quad (16) \]

where \( h \) is the predicted false alarm risk coefficient. If \( MHWHC^+_t < H_{(MHWHC)} \) or \( MHWHC^-_t < H_{(MHWHC)} \), the process is IC; otherwise, it is OOC.

3.3. Proposed MHCHW Chart. The proposed MHCHW chart is constructed by merging the HCUSUM and HWMA charts. The plotting statistics are obtained for the MHCHW chart by using the plotting statistics of equations (11) and (12) as input statistics in equation (9). Consequently, the plotting statistics of the proposed MHCHW chart are given as

\[ MHCHW^+_t = \lambda HC^+_t + (1-\lambda)MHCHW^-_t, \quad (17) \]
\[ MHCHW^-_t = \lambda HC^-_t + (1-\lambda)MHCHW^+_t, \quad (18) \]

where the MHCHW^+_t and MHCHW^-_t statistics are the lower and upper statistics of the proposed MHCHW chart and \( \lambda (0 < \lambda < 1) \) is the smoothing parameter of the proposed MHCHW chart. The mean and variance of the MHCHW statistics are given as

\[ \text{Mean} (MHCHW^+_t) = \text{Mean} (MHCHW^-_t) = \mu_{(MHCHW)}; \]
\[ \text{Var} (MHCHW^+_t) = \text{Var} (MHCHW^-_t) = \sigma^2_{(MHCHW)}. \quad (19) \]

The UCL for the proposed MHCHW chart is given as

\[ UCL_{(MHCHW)} = \mu_{(MHCHW)} + L_{MHCHW}\sigma_{(MHCHW)} \sqrt{\frac{\lambda}{2-\lambda} \left( 1 - (1-\lambda)^2 \right)}. \]

MHWHC and MHCHW charts are given in Sections 4.4 and 4.5, respectively. Finally, a sensitivity analysis of the proposed MHWHC and MHCHW charts is given in Section 4.6.

4.1. Average Run Length. Usually, the ARL measures are used to assess the performance of charts. The ARL can be classified into two types, that is, ARL_0 and ARL_1. ARL_0 is an IC ARL and ARL_1 is an OOC ARL. If a process is working IC scenario, the desired ARL_0 should be large enough to avoid frequent false alarms. However, ARL_1 should be small enough; it quickly detects the shift. A chart is preferred when it has smaller ARL_1 than that of its competitors.
4.2. Overall Performance Assessment Measures. The ARL is used for measuring the performance of a chart at a specific shift, but some other performance measurements, like EQL, RARL, and PCI, are preferred over a specific range of shifts. The specifics of these approaches are outlined in the subsequent subsections.

4.2.1. Extra Quadratic Loss. The EQL is the weighted average of ARL over its entire shift domain, and the square shift is used as a weight (Wu et al. [31]). It is written as follows:

\[ \text{EQL} = (\delta_{\text{max}} - \delta_{\text{min}})^{-1} \int_{\delta_{\text{min}}}^{\delta_{\text{max}}} \delta^2 \text{ARL}(\delta) \, d\delta, \]  

(21)

where \( \delta_{\text{min}} \) and \( \delta_{\text{max}} \) are the minimum and maximum shift values, respectively. ARL(\( \delta \)) is the ARL value at specific \( \delta \). The chart with higher EQL values is assumed to be less effective than the other charts.

4.2.2. Relative Average Run Length. Mathematically, RARL can be defined as

\[ \text{RARL} = (\delta_{\text{max}} - \delta_{\text{min}})^{-1} \int_{\delta_{\text{min}}}^{\delta_{\text{max}}} \frac{\text{ARL}(\delta)}{\text{ARL}_{\text{benchmark}}(\delta)} \, d\delta, \]  

(22)

where ARL(\( \delta \)) is the ARL of competing chart and ARL\(_{\text{benchmark}}(\delta)\) is the ARL of the benchmark chart at shift \( \delta \). The benchmark chart’s RARL measure is equal to one but it is higher than one for other contending charts [16].

4.2.3. Performance Comparison Index. Ou et al. [32] introduced the PCI measure for a chart’s ultimate performance evaluation. PCI is the ratio of the EQL of the competing chart with the EQL of the benchmark chart. Symbolically, it can be defined as follows:

\[ \text{PCI} = \frac{\text{EQL}}{\text{EQL}_{\text{benchmark}}}, \]  

(23)

The PCI of the benchmark chart should be 1. If the PCI > 1, the benchmark chart is superior to the competing charts.

4.3. Monte Carlo Simulation. Observations \( X_i \) (where \( i = 1, 2, \ldots, n \) and \( t > 1 \)) are normally distributed of different parameters with increasing shifts in the process location. The nature of the shift is as follows: \( \mu \) to \( \mu + \delta \alpha \), where \( \delta = 0.25, 0.50, 0.75, 1.00, 1.50, \) and 2.00. The Monte Carlo simulation methods are used for the computation of the numerical results. The simulation work is done in the R software that consists of 50,000 replications at shift \( \delta \). The numerical results are presented in Tables 1–6. The simulation algorithm to develop the proposed MHWHC chart follows the following steps:

(i) Generate a random sample from a normal distribution with IC parameters
(ii) Choose a specific value of \( \lambda \)
(iii) Assume the value of \( h \)
(iv) Calculate the plotting statistics of the MHWHC chart as given in equations (13) and (14)
(v) Calculate the control limit \( (H_{\text{MHWHC}}^\pm) \) as mentioned in equation (16)
(vi) Plot the MHWHC\(_t^\pm\) statistics against the control limit \( (H_{\text{MHWHC}}^\pm) \)
(vii) If the plotting statistic MHWHC\(_t^\pm > H_{\text{MHWHC}}^\pm\) or MHWHC\(_t^- > H_{\text{MHWHC}}^-\), then note this sample number as an RL
(viii) Repeat this process from step (i) to step (vii) \( 10^5 \) times to obtain ARL\(_q\)
(ix) To compute the ARL\(_1\) measures, again draw a sample from a normal distribution using shifted location parameter and repeat this process from step (ii) to step (viii)

Similarly, the procedure to construct the proposed MHCHW chart is given in the following steps:

(i) Draw a sample from a normal distribution using IC parameters
(ii) Select specific values of design parameters \( \lambda \) and \( L_{\text{MHCHW}} \)
(iii) Determine plotting statistics of the MHCHW chart from equations (17) and (18)
(iv) Find out the upper control limit \( UCL_{\text{MHCHW}} \) from equation (20)
(v) Plot the statistics MHCHW\(_t^\pm\) against the upper control limit \( UCL_{\text{MHCHW}} \)
(vi) Record the sample number as an RL whenever MHCHW\(_h^\pm > UCL_{\text{MHCHW}}^\pm\) or MHCHW\(_h^- > UCL_{\text{MHCHW}}^-\)
(vii) Continue this process \( 10^5 \) times and record RLs
(viii) Find the average of \( 10^5 \) times noted as RLs, called ARL\(_q\)
(ix) For ARL\(_1\) values, take a random sample from a normal distribution with shifted process location and repeat the steps from (ii) to (viii)

4.4. Effect of Parameter Choices. Parameters \( \lambda, h, \) and \( k \) of the MHWHC and \( \lambda, k, \) and \( L_{\text{MHCHW}} \) of the MHCHW charts have a certain impact on the detection ability. In this regard, various settings of the parameters are used and hence corresponding ARLs are calculated. Parameter \( \lambda \) is taken as 0.10, 0.25, 0.50, and 0.75, and \( k \) is chosen as 0.50, 1.00, and 1.50. Similarly, \( h \) and \( L_{\text{MHCHW}} \) are chosen in such a way to obtain prespecified ARL\(_q\) as 370 and 500. Numerical results of the proposed MHWHC and MHCHW charts are given in Tables 1–6.

4.5. Special Cases of the Proposed MHWHC and MHCHW Charts. The proposed MHWHC and MHCHW charts are reduced to the HCUSUM chart for the specific parameters’ values. Exceptional cases are presented here with their testimonies.
4.5.1. A Special Case of the Proposed MHWHC Chart.
The proposed MHWHC chart is reduced to the HCUSUM chart for \( \lambda \leq 1 \).

Proof. The plotting statistics of the proposed MHWHC chart are

\[
\begin{align*}
\text{MHWHC}_t^+ &= \max \left[ 0, (E_t - \mu_0) - K_{(\text{MHWHC})_t} + \overline{X}_{t-1}, \right], \\
\text{MHWHC}_t^- &= \max \left[ 0, (E_t - \mu_0) - K_{(\text{MHWHC})_t} + \overline{X}_{t-1}, \right]
\end{align*}
\]

and \( E_t \), defined in equation (9), is given as

\[
E_t = \lambda \overline{X}_t + (1 - \lambda) \overline{X}_{t-1}. \tag{25}
\]

When \( \lambda = 1 \), \( E_t \) can be written as

\[
E_t^{(*)} = \overline{X}_t. \tag{26}
\]

Replacing the resulting value of \( E_t^{(*)} \) in statistics of the proposed MHWHC chart, it will become

\[
\begin{align*}
\text{MHWHC}_t^{+ \text{(1)}} &= \max \left[ 0, \left( E_t^{(*)} - \mu_0 \right) - K_{(\text{MHWHC})_t} + \overline{X}_{t-1}, \right], \\
\text{MHWHC}_t^{- \text{(1)}} &= \max \left[ 0, \left( E_t^{(*)} - \mu_0 \right) - K_{(\text{MHWHC})_t} + \overline{X}_{t-1}, \right]
\end{align*}
\]

and equations (27) and (28) show that the proposed MHWHC chart approaches the HCUSUM chart when \( \lambda = 1 \). \qed

4.5.2. A Special Case of the Proposed MHCHW Chart.
The proposed MHCHW chart reduces to the HCUSUM chart for \( \lambda \leq 1 \).

Proof. The plotting statistics of the proposed MHCHW chart are

\[
\begin{align*}
\text{MHCHW}_t^+ &= \max \left[ 0, (E_t - \mu_0) - \lambda HC + (1 - \lambda) HC_{t-1}, \right] \\
\text{MHCHW}_t^- &= \max \left[ 0, (E_t - \mu_0) - \lambda HC + (1 - \lambda) HC_{t-1}, \right]
\end{align*}
\]

and equations (30) and (31) are similar to those in equations (11) and (12), except their notations, showing that the proposed MHCHW chart is reduced to the HCUSUM chart.

As both proposed, the MHWHC and MHCHW charts reduce to the HCUSUM chart when \( \lambda = 1 \). Thus, if someone is interested in obtaining ARL values of the HCUSUM chart, he will directly use the MHWHC or MHCHW chart with \( \lambda = 1 \). \qed

4.6. Sensitivity Analysis of the Parameters. A sensitivity analysis is used to examine the impact of different parameter values on the performance of the proposed MHWHC and MHCHW charts. The following are the important findings of the proposed charts:

(i) The proposed MHWHC and MHCHW charts are very sensitive to \( \lambda \) values. A small change in the \( \lambda \) values significantly affects the ARL performance.
5. Evaluation and Performance Comparison

This section contains a detailed performance comparison of the proposed MHWHC and MHCHW charts with the existing charts. These include the classical CUSUM [2], the classical EWMA [3], MEC [14], MCE [9], and HWMA charts (Sections 5.1–5.5).

5.1. Proposed versus Classical CUSUM Charts. It is observed that the proposed MHWHC and MHCHW charts work well, unlike the classical CUSUM chart. To illustrate at \( \lambda = 0.50, k = 0.50, \) and \( \delta = 0.50, \) the ARL values of the proposed MHWHC and MHCHW charts are 16.69 and 25.99, respectively, while the ARL\(_1\) value of the classical CUSUM chart is 34.09 (see Table 4 versus Table 8). Figures 1 and 2 also show that the MHWHC and MHCHW charts are superior to the classical CUSUM chart. Similarly, the proposed MHWHC and MHCHW charts also have a better overall performance than the classical CUSUM chart. As an illustration, at \( \lambda = 0.05, \) the RARL values of the proposed MHWHC and MHCHW charts are 1.00 and 1.31, whereas the RARL measure of the classical CUSUM chart is 2.08 (see Table 7).

5.2. Proposed versus Classical EWMA Chart. The proposed MHWHC and MHCHW charts are more responsive than the EWMA chart to detect a shift in the process location. For all combinations of shift (\( \delta \)) and \( \lambda, \) the ARL\(_1\) values for the proposed charts are lower than the ARL\(_1\) values of classical EWMA chart. For example, for \( \lambda = 0.10, \delta = 0.25, \) and \( k = 0.50, \) ARL\(_1\) value of the classical EWMA chart is 103.32, whereas ARL\(_1\) of the proposed MHWHC and MHCHW charts are 24.69 and 19.37, respectively (see Tables 4 versus 8). Visual presentation in Figures 1 and 2 also confirms that the proposed MHWHC and MHCHW charts are superior to the classical EWMA chart. Similarly, the EQL, RARL, and PCI values provided in Table 7 suggested the better performance of the proposed MHWHC and MHCHW charts compared to the classical EWMA chart. For example, at \( \lambda = 0.75, \) the EQL values of MHWHC, MHCHW, and EWMA charts are 6.57, 8.11, and 24.74, respectively, showing the inferiority of the classical EWMA charts (see Table 7).

5.3. Proposed versus MEC Chart. The comparison of the proposed MHWHC and MHCHW charts to the MEC chart shows that the proposed MHWHC and MHCHW charts are superior. For example, at \( \lambda = 0.25, \delta = 0.25, \) and \( k = 0.50, \) the ARL\(_1\) values for the MEC chart are 83, 31, 19, 14, 10, and 8, whereas, for MHWHC and MHCHW charts, the ARL\(_1\) values are 34.53, 10.25, 5.47, 3.7, 2.51, and 2.07 and 52.34, 15.7, 7.77, 4.93, 2.86, and 2.19, respectively (see Table 4 versus Table 8). Likewise, the proposed MHWHC and MHCHW charts’ efficiency against the MEC chart is demonstrated in Figures 1 and 2. Similarly, in overall performance measures, the proposed MHWHC and MHCHW charts also show better performance compared to the MEC chart. As an illustration, at \( \lambda = 0.10, \) and \( k = 0.50, \) the EQL, PCI, and RARL values of MHWHC, MHCHW, and MEC charts are (3.98, 1.13, 1.17), (3.52, 1.00, 1.00), and (20.34, 5.78, 5.33), respectively (see Table 7).

5.4. Proposed versus MCE Chart. The proposed MHWHC and MHCHW charts have smaller ARL\(_1\) values for all \( \lambda \) in comparison with the MCE chart. As illustration at \( \lambda = 0.10 \) and \( \delta = 0.25, \) the ARL\(_1\) for the proposed charts are 24.69, 8.66, and 4.95 and 19.37, 6.81, and 4.05, respectively, whereas ARL\(_1\) for the MCE chart are 127.63, 36.03, and 17.61 (see Tables 4 versus 8). Similarly, Figures 2 and 3 also highlight the MHWHC and MHCHW charts’ superiority over the MCE chart. The proposed MHWHC and MHCHW charts also show better performance compared to the MCE chart based on the EQL, PCI, and RARL measures. To illustrate, at \( \lambda = 0.50 \) and \( k = 0.50, \) the EQL, PCI, and RARL values of the MHWHC and MHCHW charts are 4.79, 1.00, 1.00, and 6.17, 1.29, 1.31, respectively, whereas for the MCE chart they are 10.11, 2.13, and 2.09 (see Table 7).

5.5. Proposed versus HWMA Chart. The proposed MHWHC and MHCHW charts have smaller ARL\(_1\) values than the HWMA chart for \( \lambda. \) For example, if \( \delta = 0.25 \) and \( \lambda = 0.10, \) ARL\(_1\) of the proposed MHWHC and MHCHW charts are 24.69 and 19.37, respectively, while the HWMA chart has ARL\(_1\) value of 81.48 (see Tables 4 versus 8). Additionally, Figure 4 shows the proposed charts’ dominance over the HWMA chart. In terms of overall performance, the MHWHC and MHCHW charts perform better than the HWMA chart. The RARL values of
### Table 2: ARL measures of the proposed MHWHC and MHCHW charts for $k = 1.00$ at $ARL_0 = 370.$

| Shift | MHWHC | MHCHW |
|-------|-------|-------|
|       | $\lambda$ | $\lambda$ |
| 0.00  | 370.35 | 369.74 |
| 0.05  | 155.94 | 290.77 |
| 0.10  | 74.15  | 222.19 |
| 0.20  | 27.04  | 124.26 |
| 0.25  | 19.74  | 92.37  |
| 0.50  | 7.20   | 6.2    |
| 0.75  | 4.27   | 3.84   |
| 1.00  | 3.22   | 5.60   |
| 1.50  | 2.37   | 5.26   |
| 2.00  | 2.01   | 4.05   |
| 2.50  | 1.76   | 1.74   |
| 3.00  | 1.52   | 1.52   |
| 5.00  | 1.02   | 1.03   |

$h$ | 2.019 | 2.0832 | 2.0561 | 2.031 | $L_{MHCHW}$ | 1.554 | 4.1708 | 5.15 | 5.475

### Table 3: ARL measures of the proposed MHWHC and MHCHW charts for $k = 1.50$ at $ARL_0 = 370.$

| Shift | MHWHC | MHCHW |
|-------|-------|-------|
|       | $\lambda$ | $\lambda$ |
| 0.00  | 371.57 | 370.54 |
| 0.05  | 144.82 | 291.18 |
| 0.10  | 65.49  | 132.27 |
| 0.20  | 23.44  | 124.10 |
| 0.25  | 16.76  | 92.36  |
| 0.50  | 6.04   | 59.86  |
| 0.75  | 3.83   | 14.16  |
| 1.00  | 2.95   | 7.15   |
| 1.50  | 2.28   | 2.25   |
| 2.00  | 2.00   | 2.05   |
| 2.50  | 1.77   | 1.74   |
| 3.00  | 1.55   | 1.52   |
| 5.00  | 1.03   | 1.03   |

$h$ | 1.606 | 1.6108 | 1.562 | 1.5324 | $L_{MHCHW}$ | 1.518 | 4.1154 | 5.2205 | 5.442

### Table 4: ARL measures of the proposed MHWHC and MHCHW charts for $k = 0.50$ at $ARL_0 = 500.$

| Shift | MHWHC | MHCHW |
|-------|-------|-------|
|       | $\lambda$ | $\lambda$ |
| 0.00  | 500.19 | 499.35 |
| 0.05  | 204.54 | 391.52 |
| 0.10  | 92.05  | 292.30 |
| 0.20  | 35.11  | 159.47 |
| 0.25  | 24.69  | 118.71 |
| 0.50  | 8.66   | 31.38  |
| 0.75  | 4.95   | 11.88  |
| 1.00  | 3.62   | 6.10   |
| 1.50  | 2.55   | 2.90   |
| 2.00  | 2.10   | 2.03   |
| 2.50  | 1.80   | 1.79   |
| 3.00  | 1.56   | 1.56   |
| 5.00  | 1.03   | 1.03   |

$h$ | 2.563 | 2.655 | 2.639 | 2.6195 | $L_{MHCHW}$ | 1.783 | 4.443 | 5.428 | 5.7288
Table 5: ARL measures of the proposed MHWHC and MHCHW charts for $k = 1.00$ at $ARL_0 \approx 500$.

| Shift | MHWHC $\lambda$ | MHCHW $\lambda$ |
|-------|-----------------|-----------------|
|       | 0.10 | 0.25 | 0.50 | 0.75 | 0.10 | 0.25 | 0.50 | 0.75 |
| 0.00  | 499.87 | 501.18 | 500.67 | 501.18 | 500.37 | 501.06 | 500.88 | 503.33 |
| 0.05  | 191.60 | 262.10 | 344.30 | 392.73 | 193.17 | 282.39 | 360.03 | 412.47 |
| 0.10  | 83.05 | 48.01 | 97.65 | 159.98 | 37.76 | 71.95 | 254.38 | 322.71 |
| 0.20  | 21.20 | 32.40 | 65.23 | 118.77 | 26.74 | 51.73 | 95.08 | 141.02 |
| 0.25  | 5.01 | 5.98 | 15.99 | 31.12 | 9.38 | 15.45 | 25.58 | 41.43 |

Table 6: ARL measures of the proposed MHWHC and MHCHW charts for $k = 1.50$ at $ARL_0 \approx 500$.

| Shift | MHWHC $\lambda$ | MHCHW $\lambda$ |
|-------|-----------------|-----------------|
|       | 0.10 | 0.25 | 0.50 | 0.75 | 0.10 | 0.25 | 0.50 | 0.75 |
| 0.00  | 499.76 | 501.00 | 499.90 | 499.43 | 502.83 | 499.48 | 500.09 | 508.39 |
| 0.05  | 177.17 | 255.11 | 342.56 | 391.34 | 194.25 | 278.94 | 358.87 | 382.76 |
| 0.10  | 74.05 | 45.63 | 96.00 | 159.22 | 37.45 | 68.46 | 127.62 | 173.76 |
| 0.20  | 26.32 | 29.55 | 64.30 | 118.21 | 26.49 | 48.92 | 94.05 | 135.59 |
| 0.25  | 18.47 | 15.50 | 31.01 | 9.15 | 14.49 | 24.85 | 39.68 |
| 0.50  | 6.52 | 6.77 | 11.71 | 5.16 | 7.07 | 10.20 | 15.02 |
| 1.00  | 3.06 | 3.25 | 4.03 | 6.05 | 3.71 | 4.51 | 5.58 | 7.61 |
| 1.50  | 2.33 | 2.33 | 2.45 | 2.86 | 2.57 | 2.72 | 2.87 | 3.26 |
| 2.00  | 2.02 | 2.00 | 2.00 | 2.10 | 2.12 | 2.12 | 2.20 |
| 2.50  | 1.81 | 1.80 | 1.77 | 1.79 | 1.81 | 1.79 | 1.80 |
| 3.00  | 1.59 | 1.59 | 1.57 | 1.56 | 1.58 | 1.57 | 1.56 |
| 5.00  | 1.03 | 1.03 | 1.03 | 1.03 | 1.03 | 1.03 | 1.03 |

Table 7: EQL, PCI, and RARL measures of the proposed versus existing charts.

| Measures | MHWHC ($k = 0.50$) | MHCHW ($k = 0.50$) | HWMA | MEC ($k = 0.50$) | MCE ($k = 0.50$) | EWMA | CUSUM $k = 0.50$ |
|----------|---------------------|---------------------|------|-----------------|-----------------|------|-----------------|
| EQL      | 3.98                | 3.52                | 8.79 | 20.34           | 12.06           | 7.88 | 10.27           |
| PCI      | 1.13                | 1.00                | 2.50 | 5.78            | 3.43            | 2.24 | 2.92            |
| RARL     | 1.17                | 1.00                | 2.88 | 5.33            | 3.90            | 2.80 | 3.41            |

| Measures | MHWHC ($k = 0.50$) | MHCHW ($k = 0.50$) | HWMA | MEC ($k = 0.50$) | MCE ($k = 0.50$) | EWMA | CUSUM $k = 0.50$ |
|----------|---------------------|---------------------|------|-----------------|-----------------|------|-----------------|
| EQL      | 4.12                | 5.07                | 9.28 | 15.20           | 10.67           | 10.27 | 10.27           |
| PCI      | 1.00                | 1.23                | 2.26 | 3.69            | 2.59            | 2.50 | 2.50            |
| RARL     | 1.00                | 1.29                | 2.43 | 3.36            | 2.74            | 2.88 | 2.59            |

| Measures | MHWHC ($k = 0.50$) | MHCHW ($k = 0.50$) | HWMA | MEC ($k = 0.50$) | MCE ($k = 0.50$) | EWMA | CUSUM $k = 0.50$ |
|----------|---------------------|---------------------|------|-----------------|-----------------|------|-----------------|
| EQL      | 4.79                | 6.17                | 13.25| 12.10           | 10.20           | 15.70 | 10.27           |
| PCI      | 1.00                | 1.29                | 2.77 | 2.53            | 2.13            | 3.28 | 2.14            |
| RARL     | 1.00                | 1.31                | 2.85 | 2.36            | 2.09            | 3.40 | 2.08            |

| Measures | MHWHC ($k = 0.50$) | MHCHW ($k = 0.50$) | HWMA | MEC ($k = 0.50$) | MCE ($k = 0.50$) | EWMA | CUSUM $k = 0.50$ |
|----------|---------------------|---------------------|------|-----------------|-----------------|------|-----------------|
| EQL      | 6.57                | 8.11                | 23.25| 11.13           | 9.41            | 24.74 | 10.27           |
| PCI      | 1.00                | 1.24                | 3.54 | 1.70            | 1.43            | 3.77 | 1.56            |
| RARL     | 1.00                | 1.22                | 3.44 | 1.66            | 1.42            | 3.66 | 1.54            |
MHWHC, MHCHW, and HWMA charts at $\lambda = 0.10$ are 1.17, 1.00 and 2.90, respectively (see Table 7). They indicate the inferiority of the HWMA chart.

### 6. Real-Life Application

The application of the proposed charts is primarily associated with manufacturing processes and finished goods, but they can also be applied to a variety of other fields like health, planning, accounts, neutrosophic statistics, banking, and so forth. This section offers the application of the proposed MHWHC and MHCHW charts versus the existing HWMA chart using a semiconductor manufacturing data set in the production process [33]. In the phase-I study, the variable of interest is taken to measure the “flow width of the resist” from 25 subgroups of size five each. In the phase-II study, there are 20 more subgroups. These subgroups are used for real-life applications to compare existing HWMA and

| $\lambda$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.50 | 2.00 |
|-----------|------|------|------|------|------|------|------|
| HWMA      |      |      |      |      |      |      |      |
| 0.10      | 499.78 | 81.48 | 28.61 | 14.85 | 9.35 | 4.98 | 3.32 |
| 0.25      | 498.06 | 113.34 | 33.79 | 16.19 | 9.71 | 4.94 | 3.20 |
| 0.50      | 498.56 | 218.06 | 69.04 | 27.87 | 14.08 | 5.67 | 3.20 |
| 0.75      | 499.89 | 311.99 | 132.52 | 58.15 | 28.33 | 9.15 | 4.25 |

| MEC ($k = 0.50$) |      |      |      |      |      |      |      |
| 0.10 | 501.00 | 79.00 | 35.00 | 24.00 | 19.00 | 14.00 | 11.00 |
| 0.25 | 502.00 | 83.00 | 31.00 | 19.00 | 14.00 | 10.00 | 8.00 |
| 0.50 | 503.00 | 100.00 | 30.00 | 17.00 | 12.00 | 7.00 | 6.00 |
| 0.75 | 499.00 | 120.00 | 34.00 | 17.00 | 11.00 | 6.00 | 5.00 |

| MCE ($k = 0.50$) |      |      |      |      |      |      |      |
| 0.10 | 499.00 | 126.00 | 36.00 | 18.00 | 12.00 | 7.00 | 5.00 |
| 0.25 | 500.00 | 137.00 | 38.00 | 17.00 | 10.00 | 6.00 | 4.00 |
| 0.50 | 503.00 | 144.00 | 38.00 | 17.00 | 10.00 | 5.00 | 4.00 |
| 0.75 | 499.00 | 144.00 | 37.00 | 16.00 | 9.00 | 5.00 | 3.00 |

| CUSUM ($k = 0.50$) |      |      |      |      |      |      |      |
| 0.50 | 500.09 | 108.97 | 34.09 | 16.40 | 10.27 | 5.79 | 4.04 |

| EWMA      |      |      |      |      |      |      |      |
| 0.10      | 500.00 | 103.32 | 28.81 | 13.61 | 8.21 | 4.17 | 2.66 |
| 0.25      | 500.00 | 169.34 | 47.36 | 19.31 | 10.41 | 4.78 | 2.94 |
| 0.50      | 500.00 | 254.53 | 88.43 | 35.57 | 17.18 | 6.27 | 3.39 |
| 0.75      | 500.00 | 320.92 | 140.17 | 62.40 | 30.50 | 9.81 | 4.46 |

Figure 1: ARL comparison of HWMA, MEC, MCE, EWMA, and CUSUM charts with the proposed MHWHC chart at $\lambda = 0.10$, $k = 0.50$, and $\text{ARL} \geq 500$. 
proposed MHWHC and MHCHW charts. For monitoring phase-II, the population mean and standard deviation values are taken as $\mu = 1.5056$ and $s_p = 0.13943$, respectively [22].

The parameters of the proposed MHWHC and MHCHW charts are taken as $\lambda = 0.25$, $h = 2.655$, $k = 0.50$ and $\lambda = 0.50$, $L_{MHCHW} = 3.191$, $h = 6$, $k = 0.50$, respectively, to obtain $ARL_0 = 500$, while the parameters of the existing HWMA chart are taken as $\lambda = 0.25$, $L = 3.075$, at $ARL_0 = 500$. Tables 9 and 10 and Figures 5–7 show that the HWMA chart detects shifts at the 20th sample, whereas the proposed MHWHC and MHCHW charts detect shifts at the 18th sample. Moreover, the proposed MHWHC and MHCHW charts detect three OOC points, whereas the HWMA chart detects only one OOC point. Hence, the performances of the proposed MHWHC and MHCHW charts are superior to that of the existing HWMA chart. Similarly, the proposed charts can also be compared with other charts.
Table 9: Application of the proposed MHWHC and MHCHW charts versus HWMA chart.

|   | $X_1$   | $X_2$   | $X_3$   | $X_4$   | $X_5$   |
|---|---------|---------|---------|---------|---------|
| 1 | 1.448   | 1.546   | 1.454   | 1.430   | 1.621   |
| 2 | 1.544   | 1.690   | 1.583   | 1.336   | 1.419   |
| 3 | 1.518   | 1.345   | 1.472   | 1.666   | 1.666   |
| 4 | 1.545   | 1.093   | 1.407   | 1.504   | 1.526   |
| 5 | 1.442   | 1.506   | 1.512   | 1.462   | 1.626   |
| 6 | 1.430   | 1.273   | 1.595   | 1.540   | 1.525   |
| 7 | 1.498   | 1.451   | 1.617   | 1.584   | 1.496   |
| 8 | 1.301   | 1.506   | 1.623   | 1.583   | 1.645   |
| 9 | 1.413   | 1.460   | 1.581   | 1.711   | 1.731   |
|10 | 1.382   | 1.314   | 1.495   | 1.489   | 1.460   |
|11 | 1.577   | 1.701   | 1.403   | 1.277   | 1.454   |
|12 | 1.494   | 1.437   | 1.514   | 1.481   | 1.529   |
|13 | 1.573   | 1.674   | 1.505   | 1.565   | 1.747   |
|14 | 1.809   | 1.551   | 1.825   | 1.439   | 1.656   |
|15 | 1.624   | 1.539   | 1.674   | 1.870   | 1.504   |
|16 | 1.412   | 1.793   | 1.735   | 1.639   | 1.779   |
|17 | 1.737   | 1.566   | 1.491   | 1.781   | 1.550   |
|18 | 1.597   | 1.739   | 1.683   | 1.668   | 1.797   |
|19 | 1.430   | 1.654   | 1.913   | 1.727   | 1.437   |
|20 | 1.622   | 1.822   | 1.792   | 1.674   | 1.940   |

Table 10: Application of the proposed MHWHC and MHCHW charts versus HWMA chart.

|   | MHWHC1 | MHWHC2 | $H_{\text{MHWHC}}$ | MHCHW1 | MHCHW2 | UCL_{MHCHW1} | UCL_{MHCHW2} | HWMA | UCL_{HWMA} | LCL_{HWMA} |
|---|--------|--------|--------------------|--------|--------|---------------|---------------|------|------------|------------|
| 1 | 0.00   | 0.00   | 0.11              | 1.51   | 1.51   | 1.56          | 1.50          | 1.46 | 1.55       |            |
| 2 | 0.00   | 0.00   | 0.34              | 1.49   | 1.49   | 1.66          | 1.50          | 1.35 | 1.66       |            |
| 3 | 0.00   | 0.00   | 0.24              | 1.51   | 1.49   | 1.62          | 1.51          | 1.39 | 1.62       |            |
| 4 | 0.00   | 0.00   | 0.20              | 1.49   | 1.53   | 1.61          | 1.49          | 1.41 | 1.60       |            |
| 5 | 0.00   | 0.00   | 0.17              | 1.48   | 1.48   | 1.60          | 1.50          | 1.42 | 1.59       |            |
| 6 | 0.00   | 0.00   | 0.15              | 1.48   | 1.49   | 1.59          | 1.49          | 1.43 | 1.59       |            |
| 7 | 0.00   | 0.00   | 0.13              | 1.49   | 1.48   | 1.58          | 1.50          | 1.43 | 1.58       |            |
| 8 | 0.00   | 0.00   | 0.12              | 1.51   | 1.47   | 1.58          | 1.52          | 1.44 | 1.58       |            |
| 9 | 0.00   | 0.00   | 0.12              | 1.48   | 1.52   | 1.58          | 1.49          | 1.44 | 1.57       |            |

Figure 4: ARL measures of the proposed MHCHW chart at different values of $\lambda$. 
Table 10: Continued.

| Sample number | MHWHC1 | MHWHC2 | $H_{(MHWHC)}$ | MHCHW1 | MHCHW2 | UCL$_{(MHCHW)}$ | HWMA1 | HWMA2 | UCL$_{(HWMA)}$ | LCL$_{(HWMA)}$ |
|---------------|--------|--------|----------------|--------|--------|-----------------|--------|--------|----------------|----------------|
| 11            | 0.00   | 0.00   | 0.11           | 1.49   | 1.50   | 1.57            | 1.50   | 1.44   | 1.57           | 1.57           |
| 12            | 0.00   | 0.00   | 0.11           | 1.49   | 1.50   | 1.57            | 1.50   | 1.44   | 1.57           | 1.57           |
| 13            | 0.00   | 0.00   | 0.10           | 1.52   | 1.46   | 1.57            | 1.53   | 1.44   | 1.57           | 1.57           |
| 14            | 0.01   | 0.00   | 0.10           | 1.54   | 1.46   | 1.57            | 1.54   | 1.44   | 1.57           | 1.57           |
| 15            | 0.02   | 0.00   | 0.10           | 1.54   | 1.48   | 1.57            | 1.55   | 1.44   | 1.57           | 1.57           |
| 16            | 0.05   | 0.00   | 0.10           | 1.56   | 1.48   | 1.57            | 1.56   | 1.45   | 1.57           | 1.57           |
| 17            | 0.07   | 0.00   | 0.09           | 1.56   | 1.50   | 1.57            | 1.56   | 1.45   | 1.57           | 1.57           |
| 18            | 0.11   | 0.00   | 0.09           | 1.58   | 1.49   | 1.57            | 1.56   | 1.45   | 1.57           | 1.57           |
| 19            | 0.14   | 0.00   | 0.09           | 1.57   | 1.51   | 1.57            | 1.56   | 1.45   | 1.57           | 1.57           |
| 20            | 0.21   | 0.00   | 0.09           | 1.61   | 1.48   | 1.57            | 1.61   | 1.45   | 1.57           | 1.57           |

Figure 5: The proposed MHWHC chart limits for real-life data.

Figure 6: The proposed MHCHW chart limits for real-life data.
7. Summary, Conclusions, and Recommendations

Memory charts are used to monitor small and moderate shifts in the process. Likewise, the homogeneous memory charts represent the updated version of the conventional memory-type chart for efficiently monitoring process parameters. This study presents the design structures of new charts, that is, the mixed homogeneously weighted moving average-homogeneously cumulative sum (MHWHC) and the mixed homogeneously cumulative sum-homogeneously weighted moving average (MHCHW) charts for the monitoring of the process location. The MHWHC chart is formed using the homogeneously weighted moving average (HWMA) chart statistic to input the homogeneously cumulative sum (HCUSUM) statistic. In contrast, the MHCHW chart is developed using the HCUSUM statistics as input to the HWMA statistic. Different performance measures like the average run length, extra quadratic loss, relative average run length, and performance comparison index are used to assess the proposed MHWHC and MHCHW charts’ performance. The study shows that, for small and moderate shifts, the proposed MHWHC and MHCHW charts provide better performance compared to various existing charts; these include the classical CUSUM and EWMA, mixed EWMA-CUSUM, mixed CUSUM-EWMA, and HWMA charts. A real-life data analysis is also given to demonstrate the proposed chart’s practical implementation. This study is carried out where the process variable follows the univariate normal distribution. So the proposed MHWHC and MHCHW charting schemes can be used to enhance the monitoring of high-quality processes, time between events [34], nonnormal processes, and multivariate processes and scenarios.

Data Availability

The real-life data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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