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Light axial vector mesons

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Inspired by the abundant experimental observation of axial vector states, we study whether the observed axial vector states can be categorized into the conventional axial vector meson family. In this paper we carry out analysis based on the mass spectra and two-body Okubo-Zweig-Iizuka-allowed decays. Besides testing the possible axial vector meson assignments, we also predict abundant information for their decays and the properties of some missing axial vector mesons, which are valuable to further experimental exploration of the observed and predicted axial vector mesons.

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I. INTRODUCTION

Among the light unflavored mesons listed in Particle Data Group (PDG) [1], there are abundant light axial vector mesons with a spin-parity quantum number $J^P = 1^+$, which form a $P$-wave meson family. Usually, we adopt $h_1$, $b_1$, $f_1$, and $a_1$ to express the corresponding states with the quantum numbers, $F^G(J^{PC}) = 0^-(1^{++}), 1^+(1^{--}), 0^+(1^{++})$, and $1^-(1^{++})$, respectively. In Table I, we collect the experimental information of the observed $h_1$, $b_1$, $f_1$, and $a_1$ states, which includes the corresponding resonance parameters and these observed decay channels.

Facing so many axial vector states in PDG, we need to examine whether all these states can be categorized into the axial vector meson family, which is crucial to reveal their underlying structures. We also notice that most axial vector states are either omitted by PDG or are recent findings needing confirmation. Just because of unclear experimental status of light axial vector states, we need to carry out a quantitative investigation of them, which is helpful to further experimental study, especially for these axial vector states either omitted by PDG or unconfirmed by other experiments.

In this work, we carry out a systematic study of the axial vector states by analyzing mass spectra and Okubo-Zweig-Iizuka (OZI)-allowed two-body strong decay behaviors. Our investigations are based on the assumption that all the axial mesons can be explained within the conventional $q\bar{q}$ picture. Comparing our numerical results with the experimental data, we can further test the possible assignments of the states in the axial vector meson family. In addition, information of the predicted decays of the axial vector states observed or still missing in experiment is valuable to further experimental study of axial vector meson.

This paper is organized as follows. After Introduction, we present the phenomenological analysis by combining our theoretical results with the corresponding experimental data in Sec. II, where the Regge trajectory analysis is adopted to study mass spectra of the axial vector meson family and the quark pair creation (QPC) model is applied to calculate their OZI-allowed strong decay behavior. Finally, the discussion and conclusion are given in Sec. III.

II. PHENOMENOLOGICAL STUDY OF OBSERVED AXIAL VECTOR STATES

A Regge trajectory analysis is an effective approach to study a meson spectrum [31], especially to a light meson spectrum. Masses and radial quantum numbers of light mesons with the same quantum number satisfy the following relation

$$M_n^2 = M_0^2 + (n - 1)\mu^2,$$

where $M_0$ and $M$ are the masses of a ground state and the corresponding radial excitation with a radial quantum number $n$, respectively. $\mu$ denotes a slope of a trajectory with a universal $\mu^2 = 1.25 \pm 0.15$ GeV$^2$ [31].

In Fig. 1, we present the Regge trajectory analysis, in which we consider all the axial vector states listed in PDG as shown in Table I. Besides the observed ones, we also predict some missing states and show them in Fig. 1. Additionally, we notice that there are two possible candidates for the $a_1$ meson with quantum number $n=1^{+}$, i.e., the $a_1(1930)$ and $a_1(1095)$. On the other hand, both the $f_1(1420)$ and $f_1(1510)$ can be an $s\bar{s}$ partner of the $f_1(1285)$ by analyzing only the Regge trajectory. Thus, a further study of their strong decay behaviors is helpful to test these possible assignments to the observed axial vector states and can provide more predictions of the observed and still missing axial vector mesons, which are valuable to future experimental exploration of axial vector mesons.

To obtain the decay behaviors of the axial vector mesons, we adopt the quark pair creation (QPC) model, which was first proposed by Micu [32], and further developed by the Orsay group [33–37]. This model was widely applied to study the OZI-allowed two-body strong decay of hadrons [38–59]. In the following, we briefly introduce the QPC model.

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TABLE I: Resonance parameters and strong decay channels of the axial vector states collected in PDG [1]. The mass and width are average values taken from PDG. The states omitted from PDG summary table are marked by a superscript *, while the states listed as further states in PDG are marked by a superscript b.

| JPC (J^P_C) | State      | Mass (MeV)       | Width (MeV) | The observed decay channels                                                                 |
|-------------|------------|------------------|-------------|-------------------------------------------------------------------------------------------|
|            | a_1(1260) | 1230 ± 40        | 250 − 600   | 3π [2], πρ [3], σπ [4]                                                                     |
| 1' (+++)   | a_1(1640) | 1647 ± 22        | 254 ± 27    | 3π [5], πρ [4, 6], σπ [5], f_2(1270)π [5]                                                   |
|            | a_1(1930) | 1930 ± 30        | 155 ± 45    | 3ρ_0 [7]                                                                                 |
|            | a_1(2095) | 2096 ± 17 ± 121  | 451 ± 41 ± 81 | π'π' − π' [8]                                                                         |
| 1' (+++)   | a_1(2270) | 2270 ± 56        | 305 ± 40    | 3ν_0 [7]                                                                               |
|            | b_1(1235) | 1229.5 ± 3.2     | 142 ± 9     | ωπ [9–11]                                                                              |
| 1' (+++)   | b_1(1960) | 1960 ± 35        | 345 ± 75    | ωπ_0 [12]                                                                              |
|            | b_1(2240) | 2240 ± 35        | 320 ± 85    | ωπ_0 [12]                                                                              |
|            | f_1(1285) | 1282.1 ± 0.6     | 24.2 ± 1.1  | ρ_0ρ_0 [13], ηπτ [14–16], a_0π [15–17], KΚπ [15, 16, 18]                                |
|            | f_1(1420) | 1426.4 ± 0.9     | 54.9 ± 2.6  | KΚπ [19, 20], KΚ*(892) + c.c. [18–20]                                                     |
| 0' (+++)   | f_1(1510) | 1518 ± 5         | 73 ± 25     | KΚ*(892) + c.c. [21, 22], π'π' − η' [23]                                                   |
|            | f_1(1970) | 1971 ± 15        | 240 ± 45    | ηπ_0π_0 [24]                                                                           |
|            | f_1(2310) | 2310 ± 60        | 255 ± 70    | ηπ_0π_0 [24]                                                                           |
|            | h_1(1170) | 1170 ± 20        | 360 ± 40    | πρ [25–27]                                                                              |
|            | h_1(1380) | 1386 ± 19        | 91 ± 30     | KΚ*(892) + c.c. [21, 28]                                                                |
| 0' (+++)   | h_1(1595) | 1594 ± 15 ± 10   | 384 ± 60 ± 70 | ωη [29]                                                                               |
|            | h_1(1965) | 1965 ± 45        | 345 ± 75    | ωη [30]                                                                               |
|            | h_1(2215) | 2215 ± 40        | 325 ± 55    | ωη [30]                                                                               |

For a decay process \( A \rightarrow B + C \), we can write out

\[
\langle BC | T | A \rangle = \delta^3(P_B + P_C)M_{M_A M_B M_C},
\]

(2)

where \( P_{B(C)} \) is a three-momentum of a meson \( B(C) \) in the rest frame of a meson \( A \). A subscript \( M_i \) (\( i = A, B, C \)) denotes an orbital magnetic momentum. The transition operator \( T \) is introduced to describe a quark-antiquark pair creation from vacuum, which has a quantum number \( J^{PC} = 0^{++} \), i.e., \( T \) can be expressed as

\[
T = -3\gamma \sum_{m} \langle 1m; 1 - m|00 \rangle \int d\mathbf{p}_2 d\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4)
\times \chi_{1m} (\mathbf{p}_3 - \mathbf{p}_4) \chi_{1m} \phi^4(\mathbf{p}_3) \phi^4(\mathbf{p}_4),
\]

(3)

which is constructed via a completely phenomenological way to reflect a creation of a pair of quark and antiquark from vacuum, where quark and antiquark are denoted by indices 3 and 4, respectively. As a dimensionless parameter, \( \gamma \) depicts the strength of a creation of \( q\bar{q} \) from vacuum, where \( \gamma \) is 8.7 and \( 8.7/\sqrt{3} \) [51] corresponding to the \( \bar{u}\bar{d} \) and \( s\bar{s} \) creations, respectively. \( \chi_{1m} (\mathbf{p}) = |\mathbf{p}|^2 \chi_{1m} (\mathbf{p}) \) is the solid harmonic. \( \chi, \phi, \) and \( \omega \) denote the spin, flavor, and color wave functions, which can be treated separately. In addition, \( i \) and \( j \) denote the color indices of a \( q\bar{q} \) pair.

By the Jacob-wick formula [60], a decay amplitude is expressed as

\[
M^{IL}(\mathbf{p}) = \frac{\sqrt{4\pi(2L + 1)}}{2A_1 + 1} \sum_{M_B M_C} \langle LO; J_M J_A | J_M J_A \rangle
\times \langle J_B M_B; J_C M_C | J_A M_A \rangle M_{M_A M_B M_C},
\]

(4)

and a general decay width reads as

\[
\Gamma = \frac{\pi}{4} \frac{|\mathbf{p}|}{m_A} \sum_{J L} |M^{IL}(\mathbf{p})|^2.
\]

(5)

where \( m_A \) is the mass of an initial state \( A \). We use the simple harmonic oscillator (SHO) wave function to describe a space wave function of mesons, which has the following expression

\[
\Psi_{ilm}(\mathbf{R}, \mathbf{p}) = R_{lm}(\mathbf{R}, \mathbf{p}) \chi_{ilm}(\mathbf{p}),
\]

(6)

where the concrete values of a parameter \( R \) involved in our calculation are given in Ref. [61] for the ground states. However, its value is to be fixed for each excited state.

With the above preparation, we further discuss the OZI-allowed decay behaviors of the axial vector mesons, where the allowed decay modes are listed in Tables II-III.
denote theoretical and experimental values, respectively. When there has been no evidence of \( a_1(1260) \rightarrow \pi \rho \) with the partial decay width 1.82 MeV, which explains why there has been no evidence of the \( a_1(1260) \rightarrow \pi \rho \) is the dominant channel. In Fig. 2, we give the partial decay widths of \( a_1(1260) \rightarrow \pi \rho \) from the \( S \)-wave and \( D \)-wave contributions. Here, the \( D \)-wave/\( S \)-wave amplitude ratio in the decay \( a_1(1260) \rightarrow \pi \rho \) is \(-0.248\) with a typical value of \( R = 3.846 \text{ GeV}^{-1} \) [61] in our calculation, which is comparable with the B852 data \((-0.14 \pm 0.04 \pm 0.07)\) [4]. Our result also shows that \( a_1(1260) \rightarrow f_0 \pi \) is a subordinate decay mode with the partial decay width 1.82 MeV, which explains why there has been no evidence of \( a_1(1260) \rightarrow f_0 \pi \) in experiment [3]. As shown in Fig. 2, the calculated total width can reproduce the CMD2 data given by Ref. [62]. In addition, we also give some typical ratios relevant to the partial decay and total widths together with the corresponding experimental data in Table IV. In summary, our results are comparable with the experimental values and support \( a_1(1260) \) as a ground state in the \( a_1 \) meson family.

If the \( a_1(1640) \) is the first radial excitation of \( a_1(1260) \), its decay behavior depending on the \( R \) value is shown in Fig. 3. We use the experimental total width [5] and the ratio \( \Gamma(f_2(1270)\pi)/\Gamma(\sigma \pi) = 0.24 \pm 0.07 \) [5] to get \( R = (4.30 \sim 4.64) \).
GeV^{-1}. The main decay modes of \( a_1(1640) \) are \( \pi \rho, \rho \pi(1450), \pi f_2(1270), \pi f_1(1285), \) and \( \rho \omega \). Additionally, we further provide information of the typical ratios of the \( a_1(1640) \) decays in Table V.

There are two possibilities of a candidate of the second radial excitation of the \( a_1(1260) \). In the following, we discuss the decay behaviors of the \( a_1(1300) \) and \( a_1(2095) \) combining the corresponding experimental data. In Figures 4 and 5, we present the \( R \) dependence of the decay behaviors of these \( a_1 \)'s, respectively. That is, the obtained total width of the \( a_1(1300) \) can be fitted with the data in Ref. [7] when \( R = 4.58 \sim 4.92 \text{ GeV}^{-1} \), while that of \( a_1(2095) \) can overlap with the experimental data [7] when \( R = 4.78 \sim 5.16 \text{ GeV}^{-1} \). Thus, it is difficult to distinguish which \( a_1 \) is more

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1 By the experimental total width [5], we find that there exists overlap between our theoretical and experimental results when taking \( R = 4.26 \sim 4.92 \text{ GeV}^{-1} \). Then, we can further constrain the \( R \) values by the ratio \( \Gamma(f_2(1270)/\gamma(\omega)) = 0.24 \pm 0.07 [5] \), where the constrained \( R = (4.30 \sim 4.64) \text{ GeV}^{-1} \), which is adopted to present other typical ratios of \( a_1(1640) \).
TABLE III: OZI-allowed two-body decay channels for $b_1$ and $f_1$ states marked by $\checkmark$. Here, $\rho$, $\omega$, and $\eta$ denote $\rho(770)$, $\omega(782)$, and $\eta(548)$, respectively. The axial vector states predicted by the Regge trajectory analysis are marked by a superscript $\sharp$.

| Channel | $b_1(1235)\, b_1(1640)\, b_1(1960)\, b_1(2240)$ | Channel | $f_1(1285)\, f_1(1420)\, f_1(1510)\, f_1(1800)\, f_1(1970)\, f_1(2110)\, f_1(2210)\, f_1(2310)$ |
|---------|-----------------------------------------------|---------|-----------------------------------------------|
| $\pi\omega$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\pi a_0(980)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\pi a_1(1260)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\pi a_2(1230)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\pi a_2(1420)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\pi a_2(1450)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\eta\rho$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\rho\rho$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $KK^*$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\sigma\rho$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\eta'(958)\rho$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\rho f_0$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\omega a_0(980)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $KK_1(1270)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $KK_2(1410)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $KK_3(1430)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\sigma b_1(1235)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $K^* K^*$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\omega a_2(1320)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\omega a_3(1300)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $K^* K_1(1270)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\eta f_1(1285)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $KK^*(1680)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\sigma f_1(1270)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\rho f_2(1270)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\rho f_1(1285)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\rho f_1(1420)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\rho b_1(1235)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\rho a_0(980)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\omega a_1(1260)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |

suitable for a candidate of the second radial excitation of the $a_1(1260)$ by studying only the total decay widths. Besides, we can learn from Regge trajectory analysis that there is only one state for $3^3 P_1$ state, and it is doubtful that both $a_1(1930)$ and $a_1(2095)$ are exist as mentioned in Ref. [7]. However, there exist different behaviors of the partial decay widths of these $a_1$’s. The $a_1(1930)$ mainly decays into final states $\eta\rho$, $\eta f_0$, and $\eta f_1$, while $a_1(2095)$ is a new state that were not listed in Fig. 4. As for the $a_1(2095)$, its dominant decay channels are $\pi b_1(1235)$, $\pi\rho$, and $\pi\rho(1450)$ and are shown in Fig. 5. The other decay channels like $\rho a_0(980)$, $\pi f_0(1700)$, $\pi f_1(1285)$, $\pi f_0$, and $\sigma\pi$ also have...
TABLE IV: Some typical ratios of decay widths of the $a_1(1260)$. The $\Gamma(\pi p)_{(D)}$ represent the $S(D)$-wave decay width of $a_1(1260) \rightarrow \pi p$.

| Our work | Experimental data |
|----------|------------------|
| $\Gamma((\pi p)_{l})/\Gamma_{Total}$ | 0.86 | 0.60 [3] |
| $\Gamma((\pi p)_{D})/\Gamma_{Total}$ | $5.3 \times 10^{-2}$ | $(1.30 \pm 0.60 \pm 0.22) \times 10^{-2}$ [3] |
| $\Gamma_{\sigma l}/\Gamma_{Total}$ | $8.2 \times 10^{-2}$ | $(18.76 \pm 4.29 \pm 1.48) \times 10^{-2}$ [3] |
| $\Gamma_{\sigma l}/\Gamma_{\gamma\pi\pi}$ | 0.09 | 0.06 $\pm$ 0.05 [1] |

FIG. 3: (color online). $R$ dependence of the decay behaviors of the $a_1(1640)$. Here, the dot-dashed line with band is the experimental total width in Ref. [5]. All results are in units of MeV.

considerable contributions to the total decay width. In Table VI, we also list some typical ratios relevant to their decays. We still need to emphasize one point. At present, $a_1(1930)$ and $a_1(2095)$ are not well established in experiment. The authors of Ref. [7] indicated that $a_2(1950)$ and $a_1(1930)$ are not securely identified in mass and width, though some such contributions are definitely required [7]. However, If considering the Regge trajectory analysis, one finds that a $3^3P_1$ state in the $a_1$ meson family has the mass around 2000 MeV. Two unconfirmed $a_1(1930)$ and $a_1(2095)$ can be as the candidate of $3^3P_1$ state in the $a_1$ meson family, since their masses are close to that of $3^3P_1$ state in the $a_1$ meson family. Thus, experimental study of the partial decay widths of $a_1(1930)$ and $a_1(2095)$ will helpful to pin down two possible candidates, $a_1(1930)$ and $a_1(2095)$, of the second radial excitation of the $a_1(1260)$ to one. In the following, experimental confirmation of $a_1(1930)$ and $a_1(2095)$ will be crucial for identifying the candidate of a $3^3P_1$ state in the $a_1$ meson family. If $a_1(1930)$ and $a_1(2095)$ cannot be established in experiment, we suggest experimental search for $a_1(3^3P_1)$, where the present results of $a_1(3^3P_1)$ predicted in this work are helpful to further exploration of it.

TABLE VI: Typical ratios for the $a_1(1930)$ and $a_1(2095)$. The $R$ ranges are $(4.58 \sim 4.92) \text{ GeV}^{-1}$ and $(4.78 \sim 5.16) \text{ GeV}^{-1}$ for the $a_1(1930)$ and $a_1(2095)$, respectively.

| Ratio | $a_1(1930)$ | $a_1(2095)$ |
|-------|------------|-------------|
| $\Gamma_{\pi l}/\Gamma_{Total}$ | 0.151 $\sim$ 0.162 | 0.139 $\sim$ 0.176 |
| $\Gamma_{\pi f(1235)}/\Gamma_{Total}$ | 0.092 $\sim$ 0.160 | 0.206 $\sim$ 0.254 |
| $\Gamma_{\pi f(1700)}/\Gamma_{Total}$ | 0.005 $\sim$ 0.024 | 0.039 $\sim$ 0.0529 |
| $\Gamma_{\pi l}/\Gamma_{Total}$ | 0.088 $\sim$ 0.097 | 0.058 $\sim$ 0.073 |
| $\Gamma_{\pi f(1450)}/\Gamma_{Total}$ | 0.339 $\sim$ 0.347 | 0.189 $\sim$ 0.253 |
| $\Gamma_{\pi f(1235)}/\Gamma_{\pi f(1450)}$ | 0.271 $\sim$ 0.462 | 0.348 $\sim$ 1.813 |
| $\Gamma_{\pi f(1260)}/\Gamma_{KK^{*}(892)}$ | 0.629 $\sim$ 0.719 | 1.141 $\sim$ 1.742 |
| $\Gamma_{\pi f(1270)}/\Gamma_{\pi f(1270)}$ | 0.705 $\sim$ 0.850 | 0.188 $\sim$ 0.451 |
| $\Gamma_{\pi f(1450)}/\Gamma_{\rho\omega}$ | 0.508 $\sim$ 0.553 | 1.685 $\sim$ 4.846 |
| $\Gamma_{KK^{*}(1400)}/\Gamma_{KK^{*}(1400)}$ | 0.145 $\sim$ 0.184 |
| $\Gamma_{\pi f(1450)}/\Gamma_{KK^{*}(1400)}$ | 0.206 $\sim$ 0.838 |

FIG. 4: (color online). $R$ dependence of the calculated partial and total decay widths of the $a_1(1930)$. Here, the dot-dashed line with band is the experimental total width in Ref. [7]. All results are in units of MeV.

In Fig. 6, we discuss the decay behavior of the $a_1(2270)$ as the third radial excitation of the $a_1(1260)$. We find that the main decay mode includes decay channels, $\pi b_1(1235)$, $\pi p$, $\pi(1450)$, and $\rho(1700)$. In addition, $KK^{*}(1410)$, $\rho b_1(1170)$, $KK^{*}(1680)$, $\pi\sigma$, and $\gamma\pi\pi(1260)$ have important contributions to the total decay width. The $\rho(1220)$, $\eta(958)\eta(980)$, and $K^{*}K^{*}(1270)$ are subordinate decay modes, which are not shown in Fig. 6. In Table VII, we also list the typical ratios of decays of the $a_1(2270)$.
b Dependence of the calculated partial and total decay widths of the $a_1(2095)$. Here, the dot-dashed line with band is the experimental total width in Ref. [7]. All results are in units of MeV.

FIG. 6: (color online). $R$ dependence of the calculated partial and total decay widths of the $a_1(2270)$. Here, the dot-dashed line with band is the experimental total width in Ref. [7]. All results are in units of MeV.

B. $b_1$ states

The Regge trajectory analysis indicates that the $b_1(1235)$, $b_1(1960)$, and $b_1(2240)$ are the ground state, the second radial excitation, and the third radial excitation in the $b_1$ meson family, respectively. In addition, we also predict a missing $b_1(1640)$ as the first radial excitation. In the following, we study their decays.

As for the $b_1(1235)$, there are two allowed decay channels, $\pi \omega$ and $\pi a_0(980)$. The result shown in Fig. 7 shows that the obtained total width overlaps with experimental data in Ref. [63]. Since $b_1 \rightarrow \omega \pi$ occurs via $S$ and $D$ waves, we obtain the $D$-wave/$S$-wave amplitude ratio of $b_1 \rightarrow \omega \pi$ process, which is 0.465 in our work which is consistent with the Crystal Barrel data $(0.45 \pm 0.04)$ [10]. On the other hand, the decay channel $\pi f_0$ has a partial decay width less than 1 MeV.

As a predicted $b_1$ state, $b_1(1640)$ has the decay behavior listed in Fig. 8, where we take the same $R$ range as that
TABLE VII: Calculated ratios of decays of the $a_1(2270)$. Here, all the results correspond to the $R$ range (5.12 ~ 5.32 GeV$^{-1}$).

| Ratio | Value | Ratio | Value |
|-------|-------|-------|-------|
| $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.164 ~ 0.184 | $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.313 ~ 0.435 |
| $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.247 ~ 0.264 | $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.313 ~ 0.487 |
| $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.052 ~ 0.056 | $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.404 ~ 0.469 |
| $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.064 ~ 0.070 | $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.532 ~ 0.612 |
| $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.134 ~ 0.157 | $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.999 ~ 1.131 |
| $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.789 ~ 0.926 | $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.236 ~ 0.273 |
| $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.352 ~ 0.446 | $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.256 ~ 0.297 |
| $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.573 ~ 0.639 | $\Gamma_{s}/\Gamma_{\text{Total}}$ | 0.633 ~ 0.638 |

FIG. 7: (color online). $R$ dependence of the calculated total decay width of the $b_1(1235)$. Here, the dot-dashed line with band is the experimental total width in Ref. [63]. The total decay width is in units of MeV.

for $a_1(1640)^2$. Its main decay channel is $\pi a_0(980)$, while $\pi a_0(1320), \rho\rho, \pi\omega(1420), KK^*$ and $\omega\pi$ also have considerable contributions to the total decay width. The total decay width is predicted to be $200 \sim 232$ MeV. Table VIII shows some ratios relevant to the decays of $b_1(1640)$, which is valuable for further experimental search for this axial vector state.

Assuming the $b_1(1600)$ as the second radial excitation of the $b_1(1235)$, we present its total and partial decay widths in Fig. 9. Our calculated total width can cover the experimental data given in Ref. [12]. Its main decay channels are $\pi a_0(1450), \pi\omega, \pi a_0(980)$ and $\pi a_0(1420)$, while the partial decay widths of the decay modes $\pi a_0(1260), \rho\pi$, and $\pi a_0(1320)$ are also considerable. We also obtain some ratios of partial decay widths of the $b_1(1600)$ in Table IX.

In Fig. 10, We show the decay behavior of the $b_1(2240)$ as the third radial excitation of the $b_1(1235)$. Additionally, its main decay modes are $\omega\pi, \pi\omega(1420), \pi a_0(980)$,

2 Since $b_1(1640)$ is as a predicted state, we take the same $R$ range as that of $a_1(1640)$ to predict the decay behavior of $b_1(1640)$. This treatment is due to $b_1(1640)$ as the isospin partner of $a_1(1640)$, which have similar $R$ range.
πσ(1450). Of course, the decay modes ρρ, ρb(1235), πσ(1320), πσ(1260) also have obvious contributions to the total decay width. For convenience of further experimental study of this state, we provide information of typical ratios of the partial width of the b(1(2240)) in Table X.

### Table X: Calculated ratio for the b(1(2240)) corresponding to R = 5.20 × 5.54 GeV⁻¹

| Ratio | Value | Ratio | Value |
|-------|-------|-------|-------|
| Γσ(980)/Γ_{Total} | 0.097 ± 0.128 | Γσ(1450)/Γ_{Total} | 0.131 ± 0.232 |
| Γσ(1235)/Γ_{Total} | 0.068 ± 0.071 | Γσ(1260)/Γ_{Total} | 0.179 ± 0.199 |
| Γσ(1270)/Γ_{Total} | 0.075 ± 0.102 | Γσ(1260)/Γ_{Total} | 0.057 ± 0.066 |
| Γσ(1450)/Γ_{Total} | 0.055 ± 0.064 | Γσ(1260)/Γ_{Total} | 0.050 ± 0.062 |
| Γπ/Γ_{Total} | 0.042 ± 0.057 Γσ/Γ_{Total} | 0.067 ± 0.081 |
| Γπ/Γ_{Total} | 0.112 ± 0.173 Γπ/Γ_{Total} | 0.254 ± 0.317 |
| Γπ/Γ_{Total} | 0.233 ± 0.383 Γπ/Γ_{Total} | 0.364 ± 0.381 |
| Γπ/Γ_{Total} | 0.903 ± 0.950 Γπ/Γ_{Total} | 0.158 ± 0.204 |

#### C. f₁ states

When discussing f₁ states, we need to consider the admixtures of the flavor wave functions |n̅n⟩ = (|u̅u⟩ + |d̅d⟩)/√2 and |s̅s⟩. The f₁(1285) and f₁(1420)/f₁(1510) satisfy

\[ \begin{pmatrix} |f₁(1285)⟩ \\ |f₁(1420)/f₁(1510)⟩ \end{pmatrix} = \begin{pmatrix} \cos \phi - \sin \phi \\ \sin \phi \cos \phi \end{pmatrix} \begin{pmatrix} |n̅n⟩ \\ |s̅s⟩ \end{pmatrix}, \]

where both the f₁(1420) and f₁(1510) are partners of the f₁(1285). Later, we present their decay behaviors. φ denotes a mixing angle. This mixing angle was determined in a phenomenological way [64] and is given by φ = (20° ± 30°) which is consistent with φ = (24° ± 3°) reported by the LHCb Collaboration [65] and φ = (21° ± 5°) from the updated Lattice QCD analysis [66]. When calculating the decays of the f₁(1285) and f₁(1420)/f₁(1510), we take the LHCb value φ = 24°.

In Fig. 1, we have predicted the f₁(1640) as the first radial excitation of the f₁(1285), while the f₁(1800) as a partner of the f₁(1640) is also predicted, where these two predicted axial vector mesons have relations

\[ \begin{pmatrix} |f₁(1640)⟩ \\ |f₁(1800)⟩ \end{pmatrix} = \begin{pmatrix} \cos \phi₁ - \sin \phi₁ \\ \sin \phi₁ \cos \phi₁ \end{pmatrix} \begin{pmatrix} |n̅n⟩ \\ |s̅s⟩ \end{pmatrix}, \]

In addition, there exist relations among the f₁(1970), the predicted f₁(2110), f₁(2210), and f₁(2310), i.e.,

\[ \begin{pmatrix} |f₁(1970)⟩ \\ |f₁(2110)⟩ \end{pmatrix} = \begin{pmatrix} \cos \phi₂ - \sin \phi₂ \\ \sin \phi₂ \cos \phi₂ \end{pmatrix} \begin{pmatrix} |n̅n⟩ \\ |s̅s⟩ \end{pmatrix}, \]

and

\[ \begin{pmatrix} |f₁(2210)⟩ \\ |f₁(2310)⟩ \end{pmatrix} = \begin{pmatrix} \cos \phi₃ - \sin \phi₃ \\ \sin \phi₃ \cos \phi₃ \end{pmatrix} \begin{pmatrix} |n̅n⟩ \\ |s̅s⟩ \end{pmatrix}. \]

Here, the mixing angles φᵢ (i = 1, 2, 3) cannot be constrained by our analysis. In the following discussions, we take a typical value φᵢ = φ = 24° to give the quantitative results.

As for the f₁(1285), we show its partial and total decay widths in Fig. 11, where the calculated total decay width is in agreement with the experimental data in Ref. [67]. However, we notice that the calculated branching ratio for Γπ/Γ_{Total} = 0.67 ± 0.68 corresponding to R = 3.00 × 4.00 GeV⁻¹, which is a little bit larger than (36 ± 7)% listed in PDG [1]. The PDG data also shows that the branching ratio of its decay ρπ can reach up to (52.4 ± 1.2)% [1], which is the main contribution to the total decay width of the f₁(1285). In this work, we study processes f₁(1285) → ηπ → ηππ and f₁(1285) → πσ(980) → ηπ, which can be calculated by the QPC model. Thus, the decay width of f₁(1285) → ηππ can be written as [44]

\[ \Gamma(f₁ → η + π → η + ππ) = \frac{1}{\pi} \int_{4m_{π}^2}^{(m_{π}^2)^2} dr r \sqrt{Γ_{f₁ → ηπ}(r) · Γ_{σ → ππ}(r)} \frac{(r - (2m_{π}^2)r)^{1/2}}{2 \sqrt{r}}. \]

The coupling constant g₃ = 2.12 × 2.81 GeV is determined by the total width Γ₃ = 400 ~ 700 MeV [1], and the decay width reads as

\[ \Gamma_{σ → ππ}(r) = \frac{g₃^2λ^2}{8πr} [(r - 2m_{π}^2)r]^{1/2}, \]

where λ is √2 and 1 for π⁺π⁻ and π⁰π⁰, respectively.

The process f₁(1285) → πσ(980) → ηππ is calculated in a similar way and the equation is given by

\[ \Gamma(f₁ → a₀ + π → η + ππ) = \frac{1}{\pi} \int_{m_{π}^2}^{(m_{π}^2)^2 + 2m_{π}^2} dr r \sqrt{Γ_{f₁ → πσ(980)}(r) · Γ_{a₀ → ππ}(r)} \frac{(r - (m_{π}^2)^2 + (a₀r)^2)}{2 \sqrt{r}}, \]

where the decay width for a₀(980) → ηπ is

\[ \Gamma_{a₀(980) → ηπ}(r) = \frac{g_{a₀}^2}{8πr} [(r - (m_{π}^2 + m_{a₀}^2)(r - (m_{η} - m_{π}^2))^2)]^{1/2}, \]

where the coupling constant g_{a₀} = 1.262 ~ 2.524 GeV is determined by the total width of a₀(980) (Γ_{a₀(980)} =
FIG. 10: (color online). $R$ dependence of the calculated partial and total decay widths of the $b_1(2240)$. Here, the dot-dashed line with band is the experimental total width in Ref. [12]. We do not present the $K^+K_1(1270)$ contribution since this decay has tiny width. All results are in units of MeV.

FIG. 11: (color online). $R$ dependence of the total and partial decay widths of the $f_1(1285)$. We also present the decay width of $f_1(1285) \rightarrow \eta\pi\pi$ via the intermediate channels $\eta\sigma$ and $\pi\alpha_0(980)$ (green band), and only from the intermediate channel $\eta\sigma$ (pink band). Here, the experimental total width from Ref. [67] is denoted by the dot-dashed line with band. All results are in units of MeV.

DM2 result [68] as shown in Fig. 12. Its main decay channel is $KK^*$. Thus, the present study of decay of the $f_1(1420)$ supports the $f_1(1420)$ as a partner of the $f_1(1285)$. As for the $f_1(1510)$, its partial and total decay widths are listed in Fig. 13, which shows that the calculated total decay width is larger than the experimental data [68]. Thus, the $f_1(1510)$ as a partner of the $f_1(1285)$ can be excluded.

FIG. 12: (color online). $R$ dependence of the total and partial decay widths of the $f_1(1420)$. Here, the experimental total width in Ref. [68] is shown by the dot-dashed line with band. All results are in units of MeV.

In Figs. 14–15, we further illustrate the decay properties of two predicted states $f_1(1640)$ and $f_1(1800)$. In addition, we also list some of their typical ratios, which are weakly dependent on the $R$ value (see Table XI), where we take $R = (3.60 \sim 4.40) \text{GeV}^{-1}$. From Figs. 14–15 and Table XI, we can obtain information of the main decay modes and the
its total and partial decay widths are calculated (see Fig. 18).

In this work, we predict the \( f_1(1285) \) (the first row) and as the first radial excitation of the \( f_1(1285) \) (the second row). The experimental total width in Ref. [69] is denoted by the dot-dashed line with band. All results are in units of MeV.

TABLE XI: Some obtained ratios relevant to decays of the \( f_1(1640) \) and \( f_1(1800) \). All values correspond to the \( R \) range (3.60 ~ 4.40) GeV\(^{-1}\).

| \( f_1(1640) \) | \( f_1(1800) \) |
|-----------------|-----------------|
| \( \Gamma_{\pi^0(1260)}/\Gamma_{\text{Total}} \) | 0.400 ~ 0.440 | 0.102 ~ 0.290 |
| \( \Gamma_{\pi^0(1320)}/\Gamma_{\text{Total}} \) | 0.114 ~ 0.312 | 0.254 ~ 0.665 |
| \( \Gamma_{\omega}/\Gamma_{\rho} \) | 0.244 ~ 0.254 | 0.088 ~ 0.141 |
| \( \Gamma_{KK^*}/\Gamma_{\text{Total}} \) | 0.026 ~ 0.284 | 0.043 ~ 0.162 |

resonance parameters of two predicted \( f_1 \) mesons.

As for the \( f_1(1510) \), there also exists another possible assignment, i.e., the \( f_1(1510) \) can be as a radial excitation of the \( f_1(1285) \) since the mass of \( f_1(1510) \) is close to that of the predicted \( f_1(1640) \). Here, we use the mixing angle expression

\[
|f_1(1510)\rangle = \cos \phi_1 |\bar{m}n\rangle - \sin \phi_1 |ss\rangle,
\]

which is the same as \( f_1(1640) \). Thus, we also further illustrate the decay behavior of \( f_1(1510) \) as a radial excitation of the \( f_1(1285) \) (see Fig. 13). Under this assignment, the obtained total decay width can be fitted with the LASS data [69]. The \( KK^* \) mode also has a large contribution to the total decay width. These facts indicate that the \( f_1(1510) \) as a radial excitation of the \( f_1(1285) \) is possible.

In Fig. 16, we show the \( R \) dependence of decay behavior of the \( f_1(1970) \) as the second radial excitation of the \( f_1(1285) \). Its main decay channels are \( KK^*(1410), \pi a_0(980), \pi a_1(1260) \), and \( KK^* \). As a partner of the \( f_1(1970) \), the predicted \( f_1(2110) \) mainly decays into \( KK^*(1270), KK^*(1410) \), and \( KK^* \) and has a large total decay width as shown in Fig. 17.

The third radial excitation of the \( f_1(1285) \) is still missing in experiment. In this work, we predict the \( f_1(2210) \), for which its total and partial decay widths are calculated (see Fig. 18).

As a partner of this predicted \( f_1(2210) \), the \( f_1(2310) \) has the decay properties listed in Fig. 19, in which the experimental width [24] is depicted by our calculation when taking \( R = (4.58 \sim 5.10) \) GeV\(^{-1}\). Its main decay channels are \( KK^*(1270), KK^*(1680), KK^*(1410) \), and \( KK^* \).

D. \( h_1 \) states

Similar to the \( f_1 \) mesons, the following study of \( h_1 \) states is relevant to the admixtures of flavor wave functions \( m\bar{n} \) and \( s\bar{s} \). As the ground states in the \( h_1 \) meson family, the \( h_1(1170) \) and
\[ h_1(1380) \text{ satisfy} \]
\[
\begin{pmatrix}
|h_1(1170))| \\
-|h_1(1380))|
\end{pmatrix} = 
\begin{pmatrix}
\sin \theta_1 & \cos \theta_1 \\
-\cos \theta_1 & \sin \theta_1
\end{pmatrix} 
\begin{pmatrix}
|\pi n\rangle \\
|s\bar{s}\rangle
\end{pmatrix},
\]
(17)
where the mixing angle \( \theta_1 \) is introduced, the first line of this equation is adopted in this paper, and the second line is used in Ref. [70]. In Ref. [70], Cheng obtained \( \theta_1 \sim 82.7^\circ \). The calculation of Lattice QCD indicates \( \theta_1 = 86.8^\circ \) [71]. In addition, \( \theta_1 = 85.6^\circ \) was obtained in Ref. [72]. In our calculation, we present our result as \( \theta_1 = 85.6^\circ \).

The obtained partial and total decay widths of the \( h_1(1170) \) and \( h_1(1380) \) are shown in Fig. 20. Our results indicate that the \( h_1(1170) \) and \( h_1(1380) \) as the ground states in the \( h_1 \) meson family is suitable. Our result that the \( h_1(1380) \) mainly decays into \( K\bar{K}^* \) is consistent with the experimental fact that the \( h_1(1380) \) has a dominant \( s\bar{s} \) component [21, 28].

According to the Regge trajectory analysis in Fig. 1, the \( h_1(1595) \), \( h_1(1965) \), and \( h_1(2215) \) are the first, the second and the third radial excitations of \( h_1(1170) \). Here, the \( h_1(1595) \), \( h_1(1965) \), and \( h_1(2215) \) have the same flavor wave functions as that of the \( h_1(1170) \) in Eq. (17). The mixing angle \( \theta_1 \) in Eq. (17) is replaced by \( \theta_2 \), \( \theta_3 \), and \( \theta_4 \) for the corresponding \( h_1 \) states. As for these higher radial excitations, the mixing angles \( \theta_i \) \( (i = 2, 3, 4) \) were not well determined. Thus, we take a typical mixing angle \( \theta_1 \sim 85.6^\circ \) to discuss the decay behaviors of \( h_1(1595) \), \( h_1(1965) \), and \( h_1(2215) \).

As for the \( h_1(1595) \), we find that the obtained total decay width is much smaller than 384 \( \pm 60^{+70}_{-100} \) MeV measured by the BNL-E852 Collaboration [29]. Thus, we suggest to do the precise measurement of resonance parameters of the \( h_1(1595) \), which is helpful to clarify this discrepancy. The result shown in Fig. 21 indicates that \( \pi \rho \) is a dominant decay mode of the \( h_1(1595) \). In addition, \( h_1(1595) \rightarrow \omega \eta \) has a sizable contribution to the total decay width, which explains why the \( \omega \eta \) mode was found in Ref. [73]. As the predicted partner of \( h_1(1595) \), the \( h_1(1780) \) dominantly decays into \( K\bar{K}^* \) as presented in Fig. 21.

Figure 22 presents the results of the \( h_1(1665) \), where the calculated total decay width can overlap with the Crystal Barrel data [30] when \( R = (5.02 \sim 5.28) \text{ GeV}^{-1} \). Its main decay channels are \( \pi \rho \), \( \pi \rho (1450) \), and \( \pi \rho (1700) \), while \( \sigma h_1(1170) \) also provides a considerable value. As a partner of \( h_1(1665) \), the \( h_1(2120) \) is predicted in this work, where its main decay modes are \( K\bar{K}^* \), \( K\bar{K}^* (1410) \), and \( K\bar{K}^0 (1430) \) (see Fig. 23 for more details of its decay properties).

The total and partial decay widths of the \( h_1(2215) \) and its partner \( h_1(2340) \) predicted in this work are listed in Figs. 24 and 25, respectively. The main decay modes of the \( h_1(2215) \) and \( h_1(2340) \) can be found in Figures 24 and 25.

### III. DISCUSSION AND CONCLUSION

Although there are abundant axial vector states in PDG [1], the properties of the observed axial vector states are still unclear till now. The present unsatisfactory research status of the observed axial vector states stimulates us to systematically study them, which will be helpful for revealing their underlying structures. As a crucial step, we have studied whether the observed axial vector states can be categorized into the axial vector meson family.

In this work, we have discussed the observed axial vector states by assigning them as conventional states in the axial vector meson family, where both analysis of the mass spectra and calculation of their two-body OZI-allowed strong decays have been performed.

In our calculation via the QPC model, we take different \( R \) range for reproducing the total width of discussed axial vector states. With the discussed \( a_1 \) and \( b_1 \) states as an example, we listed the obtained \( R \) values for different states (see Table XII for more details). We find that the corresponding \( R \) values become more and more larger with increasing the radial quantum number, which is consistent with our understanding, i.e., the size of higher radial excitation is larger than that of lower radial excitation. Thus, our calculation can reflect this phenomenon, which provide a test of the reliability of our calculation. In addition, we also notice that the states with the same radial quantum number in the \( a_1 \) and \( b_1 \) families have similar \( R \) range, which reflects the fact that the \( a_1 \) state is as the isospin partner of the corresponding \( b_1 \) state.

When we discuss the decay behaviors of higher radial excitations in the \( f_1 \) and \( h_1 \) meson families, we fix the corresponding mixing angle to present the numerical results, which is due to the absence of theoretical study of these mixing angles. And these mixing angles cannot be determined by the present experimental data [1]. However, for the ground states of \( f_1 \) and \( h_1 \), the situation is totally different, where the corresponding mixing angles are fixed by experimental data. Thus, in this work we adopt a very simple and crude approach, i.e., we take the same value of mixing angle for ground and the corresponding radial excitations. We expect more experimen-
tal data of radial excitations in the $f_1$ and $h_1$ meson families. Then we can carry out further theoretical study by considering the effect of the mixing angle.

In summary, this phenomenological analysis not only tests possible assignments of the axial vector states, but also predicts abundant information of their partial decays, which is valuable for further experimental study of the observed states. In addition, we have also predicted some missing axial vector mesons, where their rough mass values and decay behaviors have been given. We have also suggested an experimental search for the missing states, where the BESIII and COMPASS experiments will be a good platform to carry out the study of light hadron spectra.

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FIG. 19: (color online). $R$ dependence of the total and partial decay widths of $f_1(2310)$. The experimental total width in Ref. [24] is denoted by the dot-dashed line with band. All results are in units of MeV.
FIG. 20: (color online). $R$ dependence of the total decay width of the $h_1(1170)$ (up) and the $h_1(1380)$ (down). Here, the dot-dashed lines with band are from Refs. [25, 28]. All results are in units of MeV.

FIG. 21: (color online). $R$ dependence of the total and partial decay widths of the $h_1(1595)$ (up) and the $h_1(1780)$ (down). All results are in units of MeV.

FIG. 22: (color online). $R$ dependence of the total and partial decay widths of the $h_1(1650)$. The experimental total width from Ref. [30] is denoted by the dot-dashed line with band. All results are in units of MeV.

FIG. 23: (color online). $R$ dependence of the total and partial decay widths of $h_1(2120)$. All results are in units of MeV.

| state | $n^{3S+1}L_J$ | $R$ (GeV$^{-1}$) |
|-------|---------------|-----------------|
| $a_1(1260)$ | $1^3P_1$ | 3.846 |
| $a_1(1640)$ | $2^3P_1$ | 4.30 $\sim$ 4.64 |
| $a_1(1930)$ | $3^3P_1$ | 4.58 $\sim$ 4.92 |
| $a_1(2095)$ | $3^3P_1$ | 4.78 $\sim$ 5.16 |
| $a_1(2270)$ | $4^3P_1$ | 5.12 $\sim$ 5.32 |
| $b_1(1235)$ | $1^3P_1$ | 3.704 |
| $b_1(1640)$ | $2^3P_1$ | |
| $b_1(1960)$ | $3^3P_1$ | 4.66 $\sim$ 5.16 |
| $b_1(2240)$ | $4^3P_1$ | 5.20 $\sim$ 5.54 |
FIG. 24: (color online). $R$ dependence of the total decay width of the $h_1(2215)$. The experimental total width in Ref. [30] is denoted by the dot-dashed line with band. All results are in units of MeV.
FIG. 25: (color online). $R$ dependence of the total decay width of the $h_1(2340)$. All results are in units of MeV.
