FLAVOR STRUCTURE OF PENTAQUARK BARYONS IN QUARK MODEL

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The flavor SU(3) group structure of pentaquark baryons which form 1, 8, 10, \(\bar{10}\), 27, and 35 multiplets is investigated in quark model. The flavor wave functions of all the pentaquark baryons are constructed in SU(3) quark model and their Yukawa interactions with meson octet are obtained in general and in the special case of the octet-antidecuplet ideal mixing with the OZI rule. The mass sum rules of pentaquark baryons are also discussed.

1. Introduction

Great interests in exotic baryons in hadron physics have been initiated by the discovery of \(\Theta^+(1540)\) state by the LEPS Collaboration and subsequent experiments \(^1,^2\). The observation of \(\Xi^{--}(1862)\) by NA49 Collaboration \(^3\) may suggest that \(\Xi(1862)\) forms pentaquark antidecuplet with \(\Theta^+(1540)\) as anticipated by the soliton model study \(^4\). Later, the H1 Collaboration reported the existence of anti-charmed pentaquark state \(^5\), which revives the interests in the heavy pentaquark system \(^6,^7,^8\). However, the existence of pentaquark baryons are not fully confirmed by experiments yet as some high energy experiments report null results for those states \(^9,^{10}\). A summary for the experimental situation and perspectives can be found, e.g., in Refs. 11. Theoretically, many ideas have been suggested and developed to study the exotic pentaquark states in various approaches and models \(^12,^{13}\), but more detailed studies are required to understand the properties and formation of pentaquark states.
As the pentaquark baryons may be produced in photon-hadron or hadron-hadron reactions, it is important to understand their production mechanisms and decay channels in order to confirm the existence of the pentaquark states and to study their properties. The present studies on the production reactions are limited by the lack of experimental and phenomenological inputs on some couplings \(^{14,15,16,17,18}\). In particular, those studies could not include the contributions from the intermediate pentaquark states in production mechanisms. Therefore, it is strongly desired to understand the interactions of pentaquark baryons with other hadrons.

On the other hand, many theoretical speculations suggest that the physical pentaquark states would be mixtures of various multiplets \(^{19,20,21}\). Thus it is necessary to construct the wavefunctions of pentaquark baryons in terms of quark and antiquark for understanding the structure of pentaquark states. In this talk, we discuss a way to construct the wavefunctions of all the pentaquark baryons in quark model and obtain their SU(3) symmetric interactions with other baryons. Then several mass relations among the pentaquark baryons are discussed. In addition, we explore the couplings in the special case of the antidecuplet-octet ideal mixing with the OZI rule. The topics presented here are discussed in more detail in Refs. 22, 23, 24.

2. Wavefunctions and interactions of pentaquark baryons

We start with the representations for quark and antiquark. We denote a quark by \(q_i\) and an antiquark by \(q^i\) with \(i = 1, 2, 3\), so that \(q_1, q_2,\) and \(q_3\) are \(u, d,\) and \(s\) quark, respectively. The inner products of the quark and antiquark operators are normalized as

\[
(q_i, q_j) = \delta_{ij}, \quad (q^i, q^j) = \delta^{ij}, \quad (q_i, q^j) = 0. \tag{1}
\]

Then the diquark state is decomposed as

\[
q_j q_k = \frac{1}{\sqrt{2}} S_{jk} + \frac{1}{2\sqrt{2}} \epsilon_{ijk} \epsilon^{ij} T^i, \tag{2}
\]

where

\[
\begin{pmatrix}
S_{jk} \\
A_{jk}
\end{pmatrix}
= \frac{1}{\sqrt{2}} (q_j q_k \pm q_k q_j), \tag{3}
\]

and

\[
T^i = \epsilon^{ijk} A_{jk}, \tag{4}
\]

so that \(S_{jk}\) and \(T^i\) represent \(\mathbf{6}\) and \(\mathbf{3}\), respectively. This shows that \(\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \mathbf{3}\).
The product of two diquarks can be written as

\[(q_j q_k)(q_l q_m) = \frac{1}{2\sqrt{6}} T_{ijkl} + \frac{1}{4\sqrt{2}} \left( \varepsilon_{ijkl} \delta^b_{m} \delta^c_{n} + \varepsilon_{aijkl} \delta^d_{m} \delta^e_{n} \right) S_{bc}^a \]

\[+ \frac{1}{\sqrt{6}} (\varepsilon_{ajl} \varepsilon_{bkm} + \varepsilon_{akl} \varepsilon_{bjm}) T^{ab} \]

\[+ \frac{1}{4} \varepsilon_{ijk} \left\{ T_{ij}^m + \frac{1}{\sqrt{2}} (\delta^4_{m} \delta^5_{n} + \delta^5_{m} \delta^4_{n}) Q_a \right\} \]

\[+ \frac{1}{4} \varepsilon_{ijk} \left\{ \bar{T}_{ij}^m + \frac{1}{\sqrt{2}} (\delta^5_{m} \delta^4_{n} + \delta^4_{n} \delta^5_{m}) \bar{Q}_a \right\} \]

\[+ \frac{1}{4} \varepsilon_{ijk} \varepsilon_{nml} \left( 2S^{in} + \epsilon^i a T_a \right). \tag{5}\]

Here, \(T_a, Q_a,\) and \(\bar{Q}_a\) are (1, 0) type and represent 3. \(T^{ij}\) and \(S^{ij}\) are (0, 2) type of \(\bar{S}\). Also \(T_{jk}^i, \bar{T}_{jk}^i,\) and \(S_{jk}^i\) are (2, 1) type of \(15\) and \(T_{ijkl}\) are (4, 0) type of \(15\). Their explicit forms can be found in Ref. 24.

Then it is straightforward to obtain the wavefunctions of pentaquark baryons. Since the product of two diquarks can form 3, \(\bar{S},\) and 15, the pentaquark states can have 1, 8, 10, 10, 27, and 35, while the normal three-quark baryons have 8 and 10. So the pentaquark states have much richer spectrum than the normal baryons. Their flavor wavefunctions are obtained by direct product of \((q_j q_k)(q_l q_m)\) and an antiquark \(\bar{q}^a\). For example, the 35-plet tensors \(T_{ijkl}^a\) and the 27-plet tensors \(T_{kl}^i\) can be constructed as

\[T_{ijkl}^a = T_{ijkl} q^a - \frac{1}{6} \left( \delta^a_{ij} T_{kl} q^m + \delta^a_{ij} T_{kl} q^m + \delta^a_{ij} T_{kl} q^m + \delta^a_{ij} T_{kl} q^m \right), \]

\[T_{kl}^i = c_1 \left\{ \frac{1}{2\sqrt{2}} \left( T_{kl}^i q^j + T_{kl}^j q^i \right) \right\} \]

\[- \frac{1}{10\sqrt{30}} \left\{ \delta_i^j T_{km} q^m + \delta^j_k T_{km} q^m + \delta^i_k T_{lm} q^m + \delta^j_k T_{lm} q^m + \delta^i_k T_{tm} q^m + \delta^j_k T_{tm} q^m \right\} \right\} \]

\[+ c_2 \left\{ \frac{1}{2\sqrt{2}} \left( \bar{T}_{kl}^i q^j + \bar{T}_{kl}^j q^i \right) \right\} \]

\[- \frac{1}{10\sqrt{30}} \left\{ \delta^i_k \bar{T}_{km} q^m + \delta^j_k \bar{T}_{km} q^m + \delta^i_k \bar{T}_{lm} q^m + \delta^j_k \bar{T}_{lm} q^m + \delta^i_k \bar{T}_{tm} q^m + \delta^j_k \bar{T}_{tm} q^m \right\} \right\} \]

\[+ c_3 \left\{ \frac{1}{2\sqrt{2}} \left( S_{kl}^i q^j + S_{kl}^j q^i \right) \right\} \]

\[- \frac{1}{10\sqrt{30}} \left\{ \delta^i_k S_{km}^j q^m + \delta^j_k S_{km}^i q^m + \delta^i_k S_{lm}^j q^m + \delta^j_k S_{lm}^i q^m + \delta^i_k S_{tm}^j q^m + \delta^j_k S_{tm}^i q^m \right\} \right\}. \tag{6}\]
The pentaquark octet $P_j^j$ and antidecuplet $T^{ijk}$ read

$$P_j^j = \frac{c_1}{\sqrt{2}} \left( T_{j\ell}^\ell - \frac{1}{3} \delta_j^\ell T_m^\ell \right) + \frac{c_2}{\sqrt{2}} \left( Q_j^\ell - \frac{1}{3} \delta_j^\ell Q_m^\ell \right) + \frac{c_3}{\sqrt{2}} \left( \tilde{Q}_j^\ell - \frac{1}{3} \delta_j^\ell \tilde{Q}_m^\ell \right)$$

and the pentaquark octet $D_{ijk}$ and the singlet $S$ are

$$D_{ijk} = \frac{c_1}{\sqrt{6}} T_{ijk}^\ell + \frac{c_2}{\sqrt{24}} \left( \epsilon_{iab} T_{jk\ell}^a + \epsilon_{jab} T_{ki\ell}^a + \epsilon_{kab} T_{ij\ell}^a \right)$$

$$+ \frac{c_3}{\sqrt{24}} \left( \epsilon_{iab} \tilde{T}_{jk\ell}^a + \epsilon_{jab} \tilde{T}_{ki\ell}^a + \epsilon_{kab} \tilde{T}_{ij\ell}^a \right) + \frac{c_4}{\sqrt{24}} \left( \epsilon_{iab} S_{jk\ell}^a + \epsilon_{jab} S_{ki\ell}^a + \epsilon_{kab} S_{ij\ell}^a \right)$$

$$S = -\frac{c_1}{\sqrt{6}} T_{m}^\ell - \frac{c_2}{\sqrt{6}} Q_{m}^\ell - \frac{c_3}{\sqrt{6}} \tilde{Q}_{m}^\ell.$$  

(8)

Therefore, by constructing all possible pentaquark tensors we can verify

$$3 \otimes 3 \otimes 3 \otimes \bar{3} = 35 \oplus (3) 27 \oplus (2) \bar{10} \oplus (4) 10 \oplus (8) 8 \oplus (3) 1,$$  

(9)

where the numbers in parentheses are the number of multiplicity. The inner products of the multiplets are given in Ref. 24. With those informations at hand, one can identify the tensor representations with the baryon states of definite isospin and hypercharge. (See Ref. 24 for details.)

The SU(3) symmetric interactions of pentaquarks can be constructed by fully contracting the upper and lower indices of the three tensors representing two baryon multiplets and the meson octet. When the number of upper indices does not match that of lower indices, the Levi-Civita tensors $\epsilon_{ijk}$ are introduced to make the interactions fully contracted. The SU(3) symmetric interactions constructed in this way give several constraints or selection rules to the pentaquark interactions, which should be useful to identify the pentaquark states. Since

$$8 \otimes 8 = 27 \oplus 10 \oplus \bar{10} \oplus 8_1 \oplus 8_2 \oplus 1,$$

$$10 \otimes 8 = 35 \oplus 27 \oplus 10 \oplus 8,$$

$$\bar{10} \otimes 8 = 35 \oplus 27 \oplus \bar{10} \oplus 8,$$

$$27 \otimes 8 = 64 \oplus 35 \oplus \bar{35} \oplus 27_1 \oplus 27_2 \oplus 10 \oplus \bar{10} \oplus 8,$$

$$35 \otimes 8 = 81 \oplus 64 \oplus 35_1 \oplus 35_2 \oplus 28 \oplus 27 \oplus 10,$$  

(10)
we find the followings. First, the pentaquark singlet can couple to pentaquark octet only. Second, the 27-27 and 35-35 interactions have two types (f and d types) like 8-8 interaction. Third, the interactions including 10-10, 35-8, and 35-10 are not allowed as they cannot form SU(3)-invariant interactions. Thus, 35-plet couplings are limited to the interactions with 27-plet and decuplet.

We refer to Refs. 22, 24 for the explicit relations for pentaquark interactions. The SU(3) symmetry breaking terms can be included in a standard way 23,25,26.

3. Mass sum rules
Since all the particles belonging to an irreducible representation of SU(3) are degenerate in the SU(3) symmetry limit, it is required to include SU(3) symmetry breaking to obtain the mass splittings. It is well-known that the Hamiltonian which breaks SU(3) symmetry but still preserves the isospin symmetry and hypercharge is proportional to the Gell-Mann matrix $\lambda_8$, from which we introduce the hypercharge tensor as $Y = \text{diag}(1, 1, -2)$. Then the baryon masses can be obtained by constructing all possible contractions among irreducible tensors and the hypercharge tensor. As the mass formulas contain several parameters which take different values depending on the multiplet in general, we can obtain only the mass relations. The Hamiltonian constructed in this way reads

$$H_8 = a T^i_j P^j_i + b T^i_j Y^j_i P^j_i + c T^j_i Y^i_j P^j_i,$$
$$H_{10} = a T^{ijk} D_{ijk} + b T^{ijk} Y^j_k D_{ijk},$$
$$H_{27} = a T^{ijkl} T_{ijkl} + b T^{ijkl} Y^j_k T_{ijkl},$$
$$H_{35} = a T^{ijklm} T_{ijklm} + b T^{ijklm} Y^j_k T_{ijklm} + c T^{ijklm} Y^j_k T_{ijklm},$$

where $a$, $b$, and $c$ are mass parameters. Then, in addition to the well-known Gell-Mann–Okubo mass relation for the baryon octet and the decuplet equal-spacing rule, we have some interesting mass sum rules for antidecuplet, 27-plet, and 35-plet. In antidecuplet, we have the equal spacing rule 4,

$$\Xi_{\pi, 3/2} - \Sigma_{\pi} = \Sigma_{\pi} - N_{\pi} = N_{\pi} - \Theta. \tag{12}$$

In the 27-plet, we find the analog of the Gell-Mann–Okubo mass relation,

$$2(N_{27} + \Xi_{27}) = 3\Lambda_{27} + \Sigma_{27}. \tag{13}$$
In addition, we find that some of the $27$-plet members, i.e., $\Theta_1$, $\Delta_{27}$, $\Sigma_{27,2}$, $\Xi_{27,3/2}$, and $\Omega_{27,1}$, satisfy two independent equal-spacing rules,

$$\begin{align*}
\Omega_{27,1} - \Xi_{27,3/2} &= \Xi_{27,3/2} - \Sigma_{27,2}, \\
\Sigma_{27,2} - \Delta_{27} &= \Delta_{27} - \Theta_1.
\end{align*}$$

(14)

Note that they are the states with maximum isospin for a given hypercharge and the equal-spacing rule holds independently for the upper half of the $27$-plet weight diagram and for the lower half of that weight diagram.

For the $35$-plet baryons, we observe that there are two sets of baryons which satisfy the equal-spacing rule separately, namely,

$$\begin{align*}
\Omega_{35} - \Xi_{35} &= \Xi_{35} - \Sigma_{35} = \Sigma_{35} - \Delta_{35} = \Delta_{35} - \Theta_2, \\
X - \Omega_{35,1} &= \Omega_{35,1} - \Xi_{35,3/2} = \Xi_{35,3/2} - \Sigma_{35,2} = \Sigma_{35,2} - \Delta_{5/2}.
\end{align*}$$

(15)

4. Ideal mixing of antidecuplet and octet with the OZI rule

In the diquark-diquark-antiquark model for pentaquarks, Jaffe and Wilczek advocated the ideal mixing of the antidecuplet with the octet. By referring the detailed discussion on the ideal mixing and the OZI rule to Ref. 23, here we discuss the consequence of the OZI rule in the interactions of pentaquark octet and antidecuplet. As can be seen from Eq. (10), the pentaquark octet interaction with normal baryon octet and meson octet has two couplings, $f$ and $d$. A relation between the two couplings can be found by imposing the OZI rule or the fall-apart mechanism. To see this, we go back to Eq. (5) and note that the pentaquark octet and antidecuplet come together from the $\mathcal{S}$ of two diquarks and $\mathcal{F}$ of one antiquark, i.e., $\mathcal{S} \otimes \mathcal{F} = \mathbf{10} \oplus \mathbf{8}$. This follows from

$$S^{ij} \otimes q^k = T^{ijk} \oplus S^{[ij,k]}.$$

(16)

Obviously, the last part, being an octet representation, can be replaced by a two-index field $P^j_i$ such as

$$S^{[ij,k]} = \epsilon^{ijk} P^i_j + \epsilon^{ijk} P^j_i.$$

(17)

In this scheme, the pentaquark antidecuplet and pentaquark octet have the same universal coupling constant. It is now clear to see that the index $k$ in Eq. (17), the index for the antiquark, should be contracted with the antiquark index of the meson field to represent the fall-apart mechanism or the OZI rule, as the usual baryon $B$ does not contain an antiquark in the OZI limit. Hence, the interaction should follow the form as

$$\mathcal{L}_{\text{int}} = g_8 \epsilon^{ilm} S^{[ij,k]} B^j_i M^k_m + (\text{H.c.})�.$$  

(18)
Substituting Eq. (17) into Eq. (18), one has
\[ \mathcal{L}_{\text{int}} = 2g_8 T_i^m B_i^l M_i^l m + g_8 T_i^m M_i^l B_i^l m + \text{(H.c.)}. \]  
(19)

Comparison with the standard expression for the octet baryon interactions of \( f \) and \( d \) types leads to \( f = 1/2 \) and \( d = 3/2 \). Therefore, one can find that the OZI rule makes a special choice on the \( f/d \) ratio as \( f/d = 1/3 \). \(^{20,23}\)

5. Summary

We have obtained the flavor wavefunctions of all the pentaquark baryons in quark model. Then the SU(3) symmetric interactions of the pentaquark baryons as well as their mass sum rules are derived. This will help to identify not only exotic baryons but also crypto-exotic states. At this stage, we notice that there are several recent reports about the existence of crypto-exotic pentaquark states \(^{27}\), whose existence, however, should be clarified by further experiments \(^{28}\).

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