Impact of melt flow on the process of glass melting

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ABSTRACT
The glass melting process was mathematically modeled in the designed space to establish the controlled melt flow and study its effect on the process character and melting performance. The conversion region of the space with combined heating was intended for batch conversion, and the homogenization region heated by the longitudinal energy barrier guaranteed bubble removal and sand dissolution. The theoretical background was formulated to define the melt flow conditions in a space with batch blanket. High batch conversion rates were acquired under conditions of structured heating, and the values increased almost linearly with the growing fraction of combustion heat delivered in the space conversion region. The increased fraction of total heat in the conversion region and cooling effect of the flue gases adjusted the effective helical flow in the space homogenization region, increased the space utilization and, consequently, the melting performance. The effects of energy distribution and position of the batch borderline on the sand dissolution and bubble removal kinetics were clarified, and the competence of modeling results for advanced melting was discussed.

KEYWORDS
Mathematical modeling; glass melting; controlled flow; T-space; melting performance

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1. Introduction
The crucial role of glass melting kinetics is both experimentally and theoretically considered by examining glass batch conversion to glass melt [1], melt dissolution phenomena [2–4] bubble removal [5–8], and the homogenization history of the phenomena in the continuous space [9–12]. Another aspect of the process should, however, be respected as well, which can be called the “path of the process”. Here, the character of the melt flow through the space determines the way by which the melting phenomena are accomplished. Cooper [13] was one of the first to refer to the significance of the natural longitudinal and transversal melt circulations for the glass melting process, and later on mathematical modeling described and identified the character of the melt flow. Despite these detailed descriptions, it was still difficult to evaluate the melt flow significance for the glass melting process.

Recently, the quantity utilization of the space has been introduced [14,15] which provided the classification of different types of flow with respect to principal melting phenomena, bubble removal, and sand dissolution. By using the quantity space utilization, the relevant melting path was quantified. The uniform flow and the spiral flow provided the best results showing space utilization values between 0.5 and 0.8 [16,17], whereas the space utilization in the industrial melting furnace was assessed as being around 0.05.

The energetic model predicted the character of the melt flow in the melting space [18]. The function of the melt flow rate under balanced energy distribution in the horizontal space, $\dot{M}_{\text{bal}}$ was defined which indicated a favorable melt flow character there. The maximum values of the space utilization $\eta$ and the critical melting performance $\dot{M}_{\text{crit}}$ were examined in detail as a function of the relevant parameters in the melting channel without a glass batch blanket [18–22]. Nevertheless, adequate information has yet to be attained about the conditions of the optimized melt flow in the real melting space with a batch blanket.

This work addresses the study of the effect of melt flow character on the melting process and the achievement of the maximum melting performance in the completed glass melting space, having the T-shape and divided into the conversion and homogenization regions. The conversion region is heated from both sides; the batch blanket is located either in the conversion or partially also in the homogenization region. The position of the batch borderline influences the average temperatures in both regions, the kinetics of sand dissolution and bubble removal, and the volume part of the homogenization region with controlled melt flow; all effects will be investigated. The impact of the ratio between Joule and combustion heat in the conversion region on the specific batch conversion rate will be followed, too. The effect of flue gases and batch borderline position on the melt flow character in the homogenization region will be clarified together with...
the theoretical background. The new equations describing the character and position of the \( M_{\text{bal}} \) curve will be derived, and a supplemented definition of the space utilization will be presented for the two-region melting space.

The mathematical model of glass melting, the quantity space utilization and predictions of the energetic balance will be applied in the suggested melting space with two functional regions. The role of melting kinetics and model competence for melting predictions will be discussed.

2. Theoretical

The melting performance of the space involving the space utilization takes the form of the equation:

\[
\dot{M} = \frac{\rho V}{\tau_{\text{ref}}} u_H
\]

(1)

where \( u_H \) is the space utilization for the controlling (less effective) homogenization phenomenon, \( \tau_{\text{ref}} \) the reference time of the controlling phenomenon and \( V \) is the entire space volume \([14,15]\). The value of \( M_{\text{crit}} \) is thus determined not only by the reference time of the controlling phenomenon but also by the space utilization. As the quantity space utilization \( u_H = \frac{\tau_{\text{ref}}}{\tau_G} \) is the mentioned ratio between the reference time of the controlling phenomenon and the mean residence time of the melt in the continual space (\( \tau_G = \bar{V}/\bar{V} \), where \( \bar{V} \) is the volume flow rate of the melt). In fact, it expresses the divergence between the times needed for melting in a crucible and in a continuous space. \( M_{\text{crit}} \) increases linearly with the space utilization, whereas it is inversely proportional to the reference melting time of the controlling phenomenon. The crucial factors of the continual process – the kinetics and space utilization in equation (1) are thus expressed separately and can also be separately examined and evaluated. The potential controlling phenomenon is either sand dissolution or bubble removal, and the role of the controlling phenomenon is affected by the melting conditions in both the batch and melt. The reference melting time of the controlling phenomenon in equation (1) is designated by quantity \( \tau_{\text{Dav}} \) for sand dissolution and \( \tau_{\text{Dav}} \) for bubble removal (fining) \([14,15]\).

The longitudinal distribution of energy in the space is the crucial factor of the melt flow character because the resulting longitudinal melt circulations control the value of the space utilization. The optimal case is represented by the mass flow rate at the balanced energy distribution in the space. This is described for the space without a batch blanket by the following equation \([18]\):

\[
\dot{M}_{\text{bal}}(k_1) = \frac{k_1^2}{H_M^2(1-k_1)}
\]

(2)

where \( \dot{F}_T \) means the overall flux of heat losses through the boundaries, \( k_1 \) is the fraction of the overall energy delivered in the input part of the space, \( \xi \) is the fraction of the heat losses in the input part of the space and \( H_M \) is the theoretical heat. The quantities of the \( M_{\text{bal}}(k_1) \) curve may be assessed from parameters known in advance.

If the calculated values of the critical melting performance fall on the \( M_{\text{bal}} \) curve, the space utilization and consequently the melting performance are maximal because the uniform or helical melt flow sets up in the space. Thus, the \( M_{\text{bal}}(k_1) \) dependence indicates the correct heading of the modeling calculations.

If the batch blanket is present on the melt level, the melting space is naturally separated into two regions: the conversion region with the batch blanket and the homogenization region with a free level and adjustable melt flow. The batch blanket affects the kinetics of the homogenization phenomena, sand dissolution, and bubble removal (see below). Anytime the combustion heat is used, the remaining flue gas stream above the free level of glass in the homogenization region of the space, and they either heat it or cool it. In addition, the position of the batch borderline plays its role, as Figure 1 demonstrates.

At high values of \( k_1 \) and consequently, at high melting performances, the advancing batch blanket is not able to absorb all the heat intended for the conversion region, i.e., the heat delivered by burners \( \dot{Q}_{\text{batch}} \) (combustion heat) and especially the heat delivered by electrodes \( \dot{Q}_{\text{elec}} \) (Joule heat). In the case of combustion heat, a part of the heat is absorbed by batch blanket in the conversion region and a part leaves this region as \( \dot{Q}_{\text{loss}} \). In the case of
Joule heat delivered into the melt, the limited absorption of heat arises owing to slow heat transfer from the melt to the shifting batch. The delivered but unabsorbed heat flux $H_{\text{overheat}}$ warms up the melt in the conversion region to a temperature higher than the average value in the space; the part of the unconverted batch advances to the homogenization region. Subsequently, the majority of the heat flux $H_{\text{overheat}}$, needed for the conversion of the advanced batch, is appropriated from the melting of the homogenization region. This leads to a melt temperature decrease in the homogenization region $T_{G2}$ distinctly below the average melt temperature in the space $T_G$.

Both interactions affect the energy balance in the area with the free level and the character of the $M_{\text{bal}}(k_1)$ dependence, as well as the overall character of the melt flow. To derive the $M_{\text{bal}}(k_1)$ dependence for the double-sided heated melting space with the batch blanket, the expression for the characteristic velocity of the longitudinal circulations $v_\text{lc}$ [18] is defined as follows:

$$v_\text{lc} = -C_1 \left[ (1 - k_1) \left( \frac{H_\text{gas}^M + \dot{H}_{\text{gas}} \dot{M} + \dot{H}_G}{H_\text{gas}^M + \dot{H}_G + \dot{H}_{\text{overheat}}} \right) - \frac{\dot{H}_G^L \left( 1 - \xi \right) + \dot{H}_\text{gas}^M + \dot{H}_{\text{overheat}}}{H_\text{gas}^M + \dot{H}_G + \dot{H}_{\text{overheat}}} \right]$$

(3)

where $C_1$ is a constant and three new quantities measure: the specific energy necessary to heat the combustion gas and oxidizer to the melting temperature $H_{\text{gas}^M}$ (related to 1 kg of produced glass), the abovementioned heat flux exchanged between the flue gases and melt in the homogenization region $\dot{H}_\text{gas}^M$, ($\dot{H}_G^L$ is positive when the melt delivers heat to the flue gases and negative at the opposite heat flux) and $\dot{H}_{\text{overheat}}$, the heat flux needed to heat the glass in the homogenization region from temperature $T_{G2}$ to $T_G$ ($T_{G2} < T_G$), $\dot{H}_{\text{overheat}} > 0$.

The first term in the square bracket of equation (3) represents the energy actually delivered in the second region, which is always positive, whereas the following term expresses the energy that is theoretically needed to achieve the energetically balanced state. The difference between the two terms forms the driving force of the longitudinal circulations. According to the value of the second term, two cases are distinguished:

- **a1)** $(1 - \xi) \dot{H}_G^L + \dot{H}_\text{gas}^M + \dot{H}_{\text{overheat}} \geq 0$: Heat should be delivered in the homogenization region to compensate for all the energy needs of the region (sign $>$) or the heat losses, and potential heating of glass in the homogenization region ($\dot{H}_{\text{overheat}}$) are fully compensated by flue gases (equal sign): $\dot{H}_\text{gas}^M = (\xi - 1) \dot{H}_G^L - \dot{H}_{\text{overheat}}$.

In agreement with [18], anti-clockwise longitudinal circulations are established in the homogenization region with the left input when the first term in equation (3) is larger than the second one ($v_\text{lc} < 0$), a uniform flow is achieved when both terms are equivalent ($v_\text{lc} = 0$) and clockwise circulations are established when the first term is smaller than the second ($v_\text{lc} > 0$). For $v_\text{lc} = 0$ (the energetically balanced state), the dependence $M_{\text{bal}}(k_1)$ is to be calculated from equation (3):

$$M_{\text{bal}} = \frac{\dot{H}_G^L (k_1 - \xi) + \dot{H}_\text{gas}^M + \dot{H}_{\text{overheat}}}{(H_\text{gas}^M + \dot{H}_G + \dot{H}_{\text{overheat}}) (1 - k_1)}$$

(4)

and for $k_1$ at $M_{\text{bal}} = 0$ (the start of the dependence):

$$k_1 (M_{\text{bal}} = 0) = \frac{\dot{H}_G^L (k_1 - \xi) + \dot{H}_\text{gas}^M + \dot{H}_{\text{overheat}}}{\dot{H}_G^L}$$

(5)

The values of $M_{\text{bal}}$ and $k_1 (M_{\text{bal}} = 0)$ cannot be directly calculated from the input data since the quantities $\dot{H}_\text{gas}^M$, $H_{\text{gas}^M}$, and $\dot{H}_{\text{overheat}}$ involved in equations (4–5) come only from the solution of the numerical model. Nevertheless, equations (4) and particularly (5) can be used for predictions and assessments of the behavior of function $M_{\text{bal}}(k_1)$ and values of $k_1 (M_{\text{bal}} = 0)$ when selected values of quantities are applied. This facilitates understanding of the numerical results and predictions of favorable melt flow characters.

In equation (5), the values of $\dot{H}_G^L$ and $\xi$ remain almost constant and the value of $k_1 (M_{\text{bal}} = 0)$, i.e. the start of the $M_{\text{bal}}(k_1)$ curve (and its general position), depend particularly on both the value of $\dot{H}_\text{gas}^M$ and $\dot{H}_{\text{overheat}}$.

Three cases arise at different important values of $\dot{H}_\text{gas}^M$:

- **a1)** If no energy is exchanged between the flue gases and melt, $\dot{H}_\text{gas}^M = 0$ and as well $\dot{H}_{\text{overheat}} = 0$ ($T_{G2} = T_G$, $k_1 (M_{\text{bal}} = 0) = \xi$ according to equation (5). The $M_{\text{bal}}(k_1)$ curve starts at $k_1 = \xi$ and both anti-clockwise and clockwise circulations may establish themselves in the homogenization region. It is the case of the melting module without a batch blanket and with electric heating (equation (2) is valid) or the case with no temperature gradient between the flue gas and melt.

If the batch border line occurs in the homogenization region, $\dot{H}_{\text{overheat}} > 0$ and according to equation (5), the point $k_1 (M_{\text{bal}} = 0)$ shifts slightly to the left from $\xi$.

- **a2)** If $\dot{H}_\text{gas}^M = (\xi - 1) \dot{H}_G^L - \dot{H}_{\text{overheat}}$, i.e. the flue gases are cold and the batch border line is in the homogenization region, according to equation (5), $k_1 (M_{\text{bal}} = 0) = 0$, the $M_{\text{bal}}(k_1)$ curve starts at the zero point of $k_1$. Both types of longitudinal circulations may occur too, but their interface is generally shifted to low values of $k_1$. An energy balanced state with uniform flow is achieved at low values of $k_1$. The case of the space heated from both sides comes close to this case.

- **a3)** If $\dot{H}_\text{gas}^M = (\xi - 1) \dot{H}_G^L - \dot{H}_{\text{overheat}}$ according to the abovementioned case with equal sign, the energy needed for heat losses of the homogenization region
and glass heating from $T_{G2}$ to $T_G$ are fully compensated by hot flue gases, then \( \dot{H}_{\text{gas}} \) should be $< 0$ and the value of $k_1 (M_{\text{bal}} = 0) = 1$. The $M_{\text{bal}}(k_1)$ curve starts at $k_1 = 1$ and is identical with the vertical at $k_1 = 1$. No additional energy is needed in the homogenization region. The energy balanced state at $k_1 = 1$ is achieved at any mass flow rate of the melt, but the anti-clockwise circulations characterized by $v_{zc} < 0$ arise at any value of $k_1 < 1$. The double-sided heating with strong participation of combustion heat represents this case.

Thus, the $M_{\text{bal}}(k_1)$ curve shifts from $k_1 (M_{\text{bal}} = 0) = 0$ at relatively high positive values of \( \dot{H}_{\text{gas}} = \xi \hat{H} - H_{\text{overheat}} \) through $k_1 (M_{\text{bal}} = 0) = \xi$ at \( \dot{H}_{\text{gas}} = 0, H_{\text{overheat}} = 0 \) and further to $k_1 (M_{\text{bal}} = 0) = 1$ at a negative value of \( \dot{H}_{\text{gas}} = (\xi - 1) \hat{H} - H_{\text{overheat}} \).

The calculation of $M_{\text{bal}}$ values is instructive when searching for the maximal melting performance. If the value of $M_{\text{crit}}$, calculated by mathematical modeling, differs only slightly from the value of $M_{\text{bal}}$ at the same value of $k_2$, the calculated value of $M_{\text{crit}}$ approaches the maximal melting performance.

The interaction between the flue gas and glass melt in the homogenization region also affects the transversal energy distribution in the homogenization region. If the values of \( \dot{H}_{\text{gas}} \) are positive (the flue gases are relatively cool) and sufficiently high, the transversal circulations of the melt in the homogenization region are strengthened and a helical flow occurs in the region.

b) \( (1 - \xi) \hat{H} + \dot{H}_{\text{gas}} + H_{\text{overheat}} < 0 \): The homogenization region is overheated by flue gases. The value of \( \dot{H}_{\text{gas}} < (\xi - 1) \hat{H} - H_{\text{overheat}} \) is obviously highly negative. The case is unfavorable for the controlled flow.

Consequently, both terms on the right side of equation (3) contribute to the negative value of $v_{zc}$, and no $M_{\text{bal}}$ curve exists in the entire interval of $k_1$. Then only anticlockwise longitudinal circulations occur in the melt, and the value of the space utilization achieves only a low value. Heating with high participation of combustion heat (U-flame heating, e.g.) may represent such a case.

Two practical energetic factors essentially affect the melting in the space: the fraction of the overall energy supplied into the input (batch conversion) region of the space $k_1$, and the fraction of the Joule energy delivered to the batch conversion region $k_{fJ}$. The factors are defined under the condition of no combustion energy delivered to the homogenization region:

$$k_1 = \frac{\dot{H}_{\text{Joule1}} + \dot{H}_{\text{comb}}}{\dot{H}_{\text{Joule1}} + \dot{H}_{\text{Joule2}} + \dot{H}_{\text{comb}}} = \frac{\dot{H}_{\text{Joule1}} + \dot{H}_{\text{comb}}}{\hat{H}_{\text{tot}}}$$  \hspace{1cm} (6)

where $\dot{H}_{\text{Joule1}}$ and $\dot{H}_{\text{Joule2}}$ are the heat fluxes coming from electrodes in the conversion region and homogenization region and $\dot{H}_{\text{comb}}$ is the heat flux in the combustion space.

\[ k_{fJ} = \frac{\dot{H}_{\text{Joule1}}}{\dot{H}_{\text{Joule1}} + \dot{H}_{\text{comb}}} \]  \hspace{1cm} (7)

The total flux of delivered energy $\dot{H}_{\text{tot}}$ given by the expression in the denominator of equation (6) is obtained from the theoretical heat, assumed melting performance and total heat losses:

$$\dot{H}_{\text{tot}} = (H_{M} + \dot{H}_{\text{gasM}}) M + \hat{H}$$  \hspace{1cm} (8)

The values of delivered heat fluxes are then calculated from equations (6–7) for the pre-set values of $k_1$ and $k_{fJ}$. The specific rate of batch-to-glass conversion $M_{\text{batch}}$ is given by the relation:

$$M_{\text{batch}} = \frac{M}{S_{\text{batch}}}$$  \hspace{1cm} (9)

where $M$ is the mass flow rate of the melt and $S_{\text{batch}}$ is the area of the batch blanket covering the melt surface. The specific conversion rate is determined as the mass flux of arisen glass through the batch-melt interface, with a unit of kg/(m²s).

3. Calculation procedure and conditions

An intention to link the earlier examined homogenization space (melting module) [18–20] with an appropriate batch conversion space led to the proposed space having a T-shape, where the 1st region was primarily designed for batch conversion use and the 2nd region for glass homogenization.

Three similar T-melters were proposed and examined: the smaller T-space I, the smaller T-space II, and the larger T-space (III); the axonometric representative picture of the smaller T-space II is shown in Figure 2. All the proposed spaces have the homogenization region with inner dimensions: $X = 6.77$ m, $Y = 2$ m, $Z = 1$ m. The batch conversion region has the inner dimension of $X = 2$ m for two smaller T-spaces and 2.75 m for the larger T-space, the equivalent

**Figure 2.** The axonometric view of the smaller T-space II revealing its construction and the positions of the electrodes and burners.
remaining inner dimensions are $Y = 6 \text{ m}$ and $Z = 1 \text{ m}$. The heating arrangement of both smaller T-spaces involved 8 vertical $\text{CH}_4/\text{O}_2$ burners in the crown of the combustion chamber, whereas the larger T-space contained 10 vertical burners. The smaller T-space I is heated by a set of 36 vertical electrodes of diameter of 76 mm and length of 0.8 m in the batch conversion region and by the 2 central longitudinal rows of 32 vertical electrodes 0.3 m long in the homogenization region. The smaller T-space II is provided by a set of 42 vertical bottom electrodes with a diameter 100 mm and length of 0.8 m in the conversion region and by 2 central longitudinal rows of 32 vertical bottom electrodes with a diameter of 100 mm and length of 0.6 m in the homogenization region. The conversion region of the larger T-space (III) is heated by a set of 62 vertical bottom electrodes with a diameter 100 mm and length of 0.8 m and the homogenization region also by 2 central longitudinal rows of 32 vertical bottom electrodes with a diameter of 100 mm and length of 0.6 m. The inputs of the batch were 1 m wide in smaller T-spaces and 1.375 m wide in larger T-space and 0.2 m high, being central on the side or shifted to the front wall; the batch layer thickness was 0.05 m.

The quantity $k_1$, moved in the interval 0.25 up to almost 1, the values of $k_{1j}$ in the entire interval of validity 0–1. The defined constant heat fluxes were used in several simulations.

The Glass Furnace Model [23] was used as a tool for mathematical modeling. The properties of refractory and other materials and energetic properties of batch, glass, and natural gas were taken from the model's data bank. The float glass as the typical soda-lime-silica glass was chosen to be the model glass.

The batch conversion model is based on the controlling role of heat transfer from either flue gases or glass melt. The batch model contains the experimental dependence of the batch conversion degree on temperature. The batch with 50% of glass cullets was used in the experiment. The related combustion model is a part of the entire Glass Furnace Model. The effect of bubbles and potential foam on the heat transfer between melt and batch layer was ignored. The batch feeding temperature amounted to 30°C, the average melt exit temperature moved around 1440°C, the exit temperature in optimal cases was 1415°C. The validity limits of the main parameters of the batch conversion model are the following ones: the degree of the batch conversion in the range of 70–1036°C, the effective thermal conductivity in the range of 0–1900°C. The heat from feeding temperature to melt down (the theoretical heat) was 2050 kJ/kg of glass and the chemical heat (the batch reaction heat) amounted to 290 kJ/kg. The overall heat flux from electrodes moved from 4500 to 10,500 kW, and the overall heat flux from the burners was in the range of 1100–6500 kW. The detailed energy distribution among electrodes or burners was substantially proportional.

GFM also includes sub-models simulating homogenization processes, as bubble removal and sand dissolution. Here, the sub-models represent the final quality of glass. The sand dissolution model started from laboratory crucible melts with the aim to assess the sand dissolution times in the temperature region 1200–1450°C. Assuming the linear course of sand particle dissolution at a given temperature, the average rate of the sand dissolution was determined in the given temperature range [24]. The values were extrapolated to 1500°C and to 1100°C. The values of the average bubble growth rates were experimentally determined in the interval 1123–1500°C [16]. The average concentration of the fining gas in the bubbles was experimentally determined between 1380°C and 1520°C and fitted by the empirical S-shape function. The extrapolated values were applied in the region of lower temperatures close to the batch layer.

The fining model assumed the principal role of fining gases; hence, the diffusion of the fining gas determined the bubble composition and, consequently, the bubble size. The model is competent to describe the development of bubble size at both growing and decreasing temperature [25]. The tracing of sand particles of maximum size obtained by the sand sieve analysis and bubbles of minimum size (the assessment from the crucible melts) in the calculated temperature and velocity fields of the melt was the modeling method to calculate the space utilization values. The method neglects the mutual collisions of particles and the potential effect of particles on the melt properties.

The typical simulation case involved the calculation of the temperature and velocity fields in the space under the given value of $k_1$ and subsequent tracing of the dissolution history of maximal sand particles and fining history of bubbles of the estimated minimum size. The model’s complete structure and practically verified partial models by the authors of the model and by others give a chance to solve satisfactorily the problem of the space utilization.

The above-mentioned temperature dependence of the average rate of sand dissolution and the semi-empirical model of bubble behavior were applied for particle tracing in the float type of glass. The sand particles started at the inputs of the batch; bubbles arose over the entire interface between the batch and melt.

The principal quantities have been obtained from the mathematical modeling of batch and melt heating, melt flow, and particle tracing in the space: the rate of the melt flow $\dot{m}$, the rate of the melt flow at the critical melting performance $\dot{m}_{crt}$, the mean residence time of
glass in the space \( T_G \), and the reference melting times of either critical bubble \( T_{ref} \) or sand particle \( T_{dave} \) to calculate the space utilization \( U \), the area of batch \( S_{batch} \), to calculate the specific conversion rate, the theoretical heat \( H^\text{th} \), the specific heat consumption of melting \( H^\text{m} \), the flux of heat losses through boundaries \( \dot{H}^\text{b} \), the fraction of heat losses relating to the conversion region \( \xi \), the average temperature of the melt in the conversion region \( T_{G1} \), the average temperature of the melt in the homogenization region \( T_{G2} \), the average temperature of flue gases in the conversion region \( T_{C1} \), the average distance of the batch borderline from the inner side of the front wall \( B \), the specific energy necessary to heat the combustion gas and oxidizer to the melting temperature \( H^\text{path} \) (related to 1 kg of produced glass), the heat flux exchanged between the flue gases and melt in the homogenization region \( \dot{H}^\text{g} \), the unabsorbed heat flux from electrodes in the conversion region causing overheating of the region \( H^\text{overheat} \) and the volume of the melt in the homogenization region with the free melt level (convenient for controlled flow) \( V^\text{hom} \).

The critical melting performance represented the mass flow rate of the melt at which a critical particle of the controlling phenomenon – either sand particle or bubble – was removed above the refractory barrier before the output of the space (the barrier before the output is drawn in Figure 2). The value of the space utilization was calculated from equation (1) and the value of \( M_{\text{bat}} \) from equation (4); the value of the specific rate of batch conversion is prescribed by equation (9). The average temperature of the melt in the entire space was always kept at 1420°C and the thickness of batch layer 0.05 m.

4. Results of calculations

4.1. The batch conversion intensity

Two sets of simulations were executed to evaluate the batch conversion intensity of electric heating by electrodes and combustion heating by burners in the conversion region.

In the first series of simulations, the value of \( M_{\text{batch}} \) was examined in the small T-space II as a function of the \( k_1 \) value, which determined the intensity of heating and consequent melt convection established under the batch blanket. The constant energy supply of 2500 kW to the burners and the constant melt flow rate of 3.5 kg/s were applied in simulations to keep the effect of both factors unchanged, while the value of \( k_1 \) moved between 0.25 and 0.9. The convection cells characterized by intensive melt mixing developed below the batch blanket at a growing value of \( k_1 \). Figure 3 shows the results of simulations as the dependence of \( M_{\text{batch}} \) on the value of \( k_1 \).

Two effects work when the batch blanket is heated from below: the heat transport by vertical electrodes joined with the intensive melt convection under batch, which supports the batch conversion at high values of \( k_1 \), and the heat transport by the hot melt backflow close-to-level, with low mixing efficiency, which is effective at low \( k_1 \) values. Figure 3 shows that both effects supporting the batch conversion rate are comparable and only moderate (the maximum difference of the \( M_{\text{batch}} \) values is less than 20 rel. %). Consequently, the expected mixing effect of the melt appears to be less significant factor.

The impact of the heating from above by burners on the value of \( M_{\text{batch}} \) is then worth being studied. The simulations of the \( k_1 \) effect on the batch conversion rate in single simulation sets were realized at constant values of \( k_1 \) around 0.9, corresponding to the area of the maximum critical performance, and at the constant melt flow rate in each of the three T-spaces at the given constant temperature of 1420°C. In order to complete the obtained data file, further simulations were conducted at marginal values of \( k_1 \) and lower values of \( k_1 \). Table 1 provides values of the specific conversion rate \( M_{\text{batch}} \), the average temperature of the combustion chamber \( T_{c1} \) in the conversion region and other relevant values. The dependence of \( M_{\text{batch}} \) on the decreasing value of \( k_1 \) between 0.85 and 0.5 and the constant flow rate of 4.65 kg/s was acquired in the smaller T-space I and further values at lower and marginal values of \( k_1 \) were obtained in the smaller T-space II. Generally, the results show a considerable increase of \( M_{\text{batch}} \) with decreasing value of \( k_1 \), i.e. with the increasing portion of combustion heat in the conversion region.

The effect of \( k_1 \) on the value of \( M_{\text{batch}} \) in the larger T-space provides Table 2. The first run of calculations involves the cases with central batch inputs in the T-space and the second one, designated by \( S \), was
realized with batch inputs shifted to the front side of the space to avoid creation of free-level areas in the conversion region (see hereinafter). The flow rate was unified at a constant value of 5.33 kg/s.

4.2. The critical melting performances

Four sets of simulations were realized to determine the critical melting performance as a function of increasing \(k_{ij}\), i.e. the fraction of energy supplied to the conversion region of the T-space. Two sizes of the conversion space and two levels of the factor \(k_{ij}\) (the fraction of the Joule heat delivered to the conversion region) were chosen. The first two sets are calculated at constant values of the combustion heat (Tables 3 and 4) and reveal the impact of the value of \(k_1\), whereas the remaining two sets of calculations (Tables 5 and 6) also include the contribution of the value of \(k_{ij}\) (the specific rate of batch conversion).

The results of the simulations involving the values of the space utilization \(u_m\), the critical melting performance \(M_{cr}\), and other relevant quantities of the smaller T-space II with thicker electrodes are shown in Table 3.

In order to include the favorable effect of \(k_{ij}\) on the value of \(M_{batch}\) and consequently, the maximum critical melting performance, the lower value of \(k_{ij}\) was chosen for the third and fourth set of simulations. Whereas the value of \(k_{ij}\) at the maximum critical melting performance in the first two sets of simulation was around 0.75, the values of \(k_{ij} = 0.675\) and 0.6 were tested for the following calculation of \(M_{cr\text{max}}\) (cases I-23 and I-24 in Table 5 and cases III-16S and III-11 in Table 6). The value of \(k_{ij} = 0.6\) was selected for both sets of calculations and applied at any value of \(k_1\). The important values are assembled in the following Table 5 for the small T-space I and Table 6 for the larger T-space.

The values of temperatures \(T_G\) and \(T_G\) in Tables 3–6 generally show that the temperatures in the homogenization region \(T_G\) are higher and the temperatures in the conversion region \(T_G\) are lower than the average temperature in the space, which is the source of the longitudinal melt circulations. However, the sudden decrease of \(T_G\) and increase of \(T_G\) occurs near the maximum critical melting performance with the balanced energy distribution; the sinking and rising trend leads to temperature \(T_G\) being lower and temperature \(T_G\) higher than the expected average temperature. The final temperatures are determined by the progress of the batch borderline in the homogenization region (see Theoretical, Figure 1). The values of \(T_G\) show an overall increase with increasing value of \(k_1\) (Tables 5–6) and with decreasing value of \(k_{ij}\) (Tables 3–4 at lower \(k_1\) values). This is induced by the increasing amount of combustion energy delivered in the conversion region.

5. Discussion of results

5.1. The impact of fraction of Joule and combustion heat delivered to the conversion region on the specific rate of batch conversion \(M_{batch}\)

Figure 4 shows the values of \(M_{batch}\) from Tables 1–2 as a function of \(k_{ij}\) for both small T-spaces and the large T-space at the constant batch input in the typically used interval of \(k_{ij}\) between 0.5 and 0.85, additional values at \(k_{ij} = 1\) and at different melt flow rates extend the original dependence of \(M_{batch}(k_{ij})\).

The values of \(M_{batch}\) for double-sided heating are generally high and substantially higher than found for the conventional glass melting furnace with electric boosting where the values of \(M_{batch}\) moved between 0.08 and 0.11 kg/(m²s) at the average melting temperature of 1387°C [26]. It speaks for the fact that the vertical transfer of heating energy from electrodes and burners in the T-space enhances the batch conversion rate. Taking into account the information competences of the contemporary batch models, however, the
tendency of modeling results in Figure 4 should be considered more significant than the concrete values. The results presented in Figure 4 show an obvious growth of the $M_{\text{batch}}$ values with increasing fraction of combustion heat delivered in the conversion region, i.e. with decreasing $k_{I,2}$. The increased heat flux from flue gases into batch is reflected by the growth of the average temperature in the combustion chamber (see the average temperatures in the combustion space $T_{C1}$ in Tables 1–2 and in Figure 5). No positive impact of $k_{I,2}$
values on the values of $\dot{M}_{\text{batch}}$ can on the contrary be expected from the side of the glass melt in the conversion region where the significant temperature $T_{\text{C1}}$ generally decreases with decreasing $k_{ij}$ (Tables 1–2) and could rather slow down the batch conversion. The results in Figure 3 do not confirm an adequate impact of the melt convection, as well.

The marginal values of $\dot{M}_{\text{batch}}$ at $k_{ij} = 1$ (all-electric heating) in Figure 4 may be understood as a link to the presented dependences in the interval $k_{ij} = 0.5–0.85$. Only small scattering of $\dot{M}_{\text{batch}}$ values are found when comparing both types of T-spaces, the different thicknesses of electrodes, their number, and arrangement in the small T-space. The impact of $k_{ij}$ on the specific batch conversion rate (at similar values of $k_{i}$) appears therefore principal.

To summarize, the substitution of the Joule heat delivered in the conversion region by the combustion heat is the principal reason for the growth of the batch conversion rate and consequently, the growth of the critical melting performance.

### Table 6. The larger T-space (III) working at average melting temperature 1420°C and $k_{ij} = 0.6$. The optimum cases are highlighted.

| Case   | $k_1$ | $k_2$ | $u_i$ | $t_{\text{refl}}$ (s) | $M_{\text{tot}}$(kg/s) | $M_{\text{pre}}$(kg/s) | $S_{\text{batch}}$ (m²) | $M_{\text{batch}}$(kg/m² s) | $H_{\text{P}}$(kJ/kg) | $T_{\text{C1}}$ (°C) | $T_{\text{C2}}$ (°C) | $T_{\text{C3}}$ (°C) |
|--------|-------|-------|-------|------------------------|------------------------|------------------------|--------------------------|--------------------------|----------------------|-------------------|-------------------|-------------------|
| III-16S | 0.94  | 0.675 | 0.323 | 2733                   | 5.96                   | -                      | 18.82                    | 0.317                    | 2578                 | 1386              | 1433              | 1403              |
| III-11 | 0.921 | 0.60  | 0.199 | 2750                   | 5.06                   | -                      | 15.76                    | 0.321                    | 2672                 | 1501              | 1436              | 1402              |
| III-21S | 0.50  | 0.60  | 0.067 | 2739                   | 1.70                   | 0.04                   | 6.69                     | 0.254                    | 1271                 | 1377              | 1399              | 1443              |
| III-20S | 0.60  | 0.60  | 0.083 | 2647                   | 2.20                   | 0.24                   | 8.84                     | 0.249                    | 3100                 | 1363              | 1397              | 1446              |
| III-19S | 0.75  | 0.60  | 0.118 | 2668                   | 3.10                   | 0.67                   | 11.79                    | 0.263                    | 2942                 | 1362              | 1395              | 1449              |
| III-18S | 0.85  | 0.60  | 0.098 | 1269                   | 5.40                   | 1.85                   | 17.94                    | 0.301                    | 2744                 | 1404              | 1401              | 1442              |
| III-17S | 0.90  | 0.60  | 0.160 | 1919                   | 5.81                   | 2.68                   | 17.99                    | 0.323                    | 2687                 | 1406              | 1419              | 1421              |
| III-11S | 0.945 | 0.60  | 0.198 | 2160                   | 6.40                   | 5.01                   | 18.34                    | 0.349                    | 2640                 | 1477              | 1428              | 1410              |
| III-22S | 0.96  | 0.60  | 0.191 | 2067                   | 6.45                   | 6.12                   | 18.22                    | 0.354                    | 2631                 | 1488              | 1430              | 1407              |
| III-23S | 0.975 | 0.60  | 0.171 | 1966                   | 6.08                   | 9.13                   | 17.17                    | 0.354                    | 2662                 | 1483              | 1427              | 1411              |

#### 5.2. The distribution of the batch conversion effect between the Joule and combustion heat

The apparent effect of combustion heat (burners) on batch conversion, readable from Figure 4, can be expressed by fractions of batch converted to glass owing to the effect of either electrodes or burners. The batch conversion model used in this work [23] is one of the class of models with a controlling role of the heat transfer [1]. Consequently, the fractions of batch converted to glass were calculated from heat absorbed by either upper (combustion heat) or lower (Joule heat) surface of the batch. The fractions of converted batch are comprehensibly visible for the relevant dependence plotted over the entire region of $k_{ij}$ as Figure 5 presents for both smaller T-spaces (the values of $M_{\text{batch}}$ come from Table 1).

The permanent increase in the values of $M_{\text{batchC}}$ occurs in Figure 5 which is in correlation with the increasing average temperature $T_{\text{C1}}$ in the combustion space. As expected, the values of $M_{\text{batchC}}$ show only a small dependence on $k_{ij}$, decreasing slowly between the values of $k_{ij} = 1–0.4$ and showing slow increase

![Figure 4](#) The dependence between the average value of the specific conversion rate of batch $M_{\text{batch}}$ and the fraction of Joule energy delivered to the first region $k_{ij}$. $M_{\text{batchC}}$ – the smaller T-space I with thinner electrodes (light blue), $M_{\text{batchC}}$ – the larger T-space II with thicker electrodes at $k_{ij} = 1$ (dark blue), $M_{\text{batch}}$ – the larger T-space (III) with the central side batch inputs (green), $M_{\text{batchC}}$ – the larger T-space with the inputs shifted to the front wall (Orange).

![Figure 5](#) The fractions of converted batch relevant to either Joule or combustion heat as a function of $k_{ij}$. The total specific conversion rate of the batch $M_{\text{batch}}$ (gray), the fraction of $M_{\text{batch}}$ converted to glass by Joule heat $M_{\text{batch}}$ (blue), the fraction of $M_{\text{batch}}$ converted to glass by the combustion heat $M_{\text{batchC}}$ (Orange), and the relevant average temperatures in the combustion chamber of the conversion region $T_{\text{C1}}$ (red). The smaller T-spaces I and II. The $k_{ij}$ values correspond to the state close to the $M_{\text{refl}}(k_{i})$ dependence between $k_{ij} = 0.4$ and 1.
between 0.4 and 0. The presented \( \dot{M}_{\text{batch}} \) development is in correlation with the course of melt temperatures in the conversion region \( T_{G1} \) which shows a minimum also at \( k_{1j} = 0.4 \) (Table 1). The initial temperature decrease in the conversion region is brought about by the batch drawing back from the homogenization region (see the values \( S_{\text{batch}} \) in Table 1), whereas the following temperature increase at two lowest values of \( k_{1j} \) is caused by heating of the melt owing to the arising free level of the conversion region. Consequently, the value of \( \dot{M}_{\text{batch}} \) varies only slightly with decreasing \( k_{1j} \), whereas the value of \( \dot{M}_{\text{batch}} \) steadily grows with increasing \( T_{C1} \) temperature. The sum of both values grows thus according to Figure 5 when \( k_{1j} \) decreases.

5.3. The impact of the value of \( \dot{M}_{\text{batch}} \) and the extent of the batch blanket on the space utilization and critical melting performance

If the batch borderline is established in the homogenization region, the increase in the specific conversion rate \( \dot{M}_{\text{batch}} \) at declining values of \( k_{1j} \) reduces the extent of the batch blanket in the region at the same flow rate and the larger free-level part of the homogenization region \( V_{\text{thom}} \) is at disposal for controlled flow. The total utilization of the space \( u_{H1} \), expressed by the weighted average value, grows then with the increasing value of \( V_{\text{thom}} \):

\[
u_{H1} = \frac{(V - V_{\text{thom}})u_{H1\text{conv}} + V_{\text{thom}}u_{\text{thom}}}{V}; \quad u_{\text{thom}} \gg u_{H1\text{conv}} \tag{10}
\]

where \( u_{H1\text{conv}} \) is the local low value of the space utilization in the volume covered by the batch \( (V - V_{\text{thom}}) \), and \( u_{\text{thom}} \) is the local utilization value corresponding to the volume \( V_{\text{thom}} \) with controlled melt flow. Note that the numerical solution provides the total value \( u_{H1} \) and both local utilization quantities were introduced only to concretize the effect of batch extent on the space utilization and, consequently, melting performance.

The phenomenon of batch drawing back in the larger T-space after decreasing the value of \( k_{1j} \) from 0.85 to 0.60 is obvious in Figure 6 and the permanent decrease in the average distance \( B \) between the batch borderline in the homogenization region and the inner side of the front side of the space is provided by Table 2 (values of \( B \) in cases III-13S, III-4S, III-16S, III-11S, III-12S).

If the sand dissolution would be the controlling phenomenon at the increasing value of \( \dot{M}_{\text{batch}} \) (at decreasing value of \( k_{1j} \)), the value of \( T_{\text{ave}} \) in equation (1) would slightly sink owing to enhanced dissolution inside the batch, but the following dissolution in the melt is slowed down by the decrease in the temperature \( T_{G1} \) in a broad range of \( k_{1j} \) (the sand dissolution realizes particularly in the conversion region, see the chapter 5.5). The decreasing values of \( T_{G1} \) between \( k_{1j} = 0.4-1 \) are seen in Table 1. The resulting effect for sand dissolution tends to slow down. The growth of the melting performance owing to increasing \( V_{\text{thom}} \) is then reduced by the slow sand dissolution. If the bubble removal is the controlling phenomenon, the increase of \( \dot{M}_{\text{batch}} \) and subsequent decrease of both \( S_{\text{batch}} \) and \( B \) keeps the sufficiently high temperature in the homogenization region and accelerates the kinetics of the bubble removal. The melting performance is then supported by accelerated bubble removal. The increase of the average temperature in the homogenization region \( T_{G2} \) and the decrease of the relevant reference time \( T_{\text{ref}} \) at descending values of \( k_{1j} \) is presented in Table 1 (cases II-17, I-22, I-15, I-23, I-20, I-21, II-40S) and in Table 2.

The shift of the side batch inputs from the central position to the space front wall diminishes the areas of the free level in the conversion region and, consequently, draws back the batch borderline \( B \), too. The example in Figure 7 shows the effect in the larger T-space for two cases III-11 M and III-11SM from Table 2 where both cases differ only in the batch input positions. The shift of the batch inputs to the front wall, applied particularly at lower values of \( k_{1j} = 0.5-0.6 \), appropriately converges the melting performance.

- **Figure 6.** The view from above as the batch border line \( B \) moves back when the value of \( k_{1j} \) decreases from 0.85 to 0.60. The larger T-space (III), \( k_{1} = 0.921 \). a) \( k_{1j} = 0.85 \), b) \( k_{1j} = 0.60 \).
shifted
Figure
potentials of batch pathways in the conversion region and by drawing back the batch borderline increases the disposability of the homogenization region for the controlled melt flow. The resulting higher average temperature in the homogenization \( T_G \) as well accelerates the bubble removal by decreasing the values of \( T_{ref} \) as shown for cases of different batch input positions in Table 2.

The batch moving back by shifting the batch inputs opened the way to an increase in the critical melting performance from 5.06 kg/s to 6.40 kg/s in the larger T-space (Table 6, cases III-11 and III-115) and from 5.3 to 5.5 kg/s in the smaller T-space I (Table 5, cases I-24 and I-25).

Both referred consequences of reduced batch blanket are obvious from Figure 8, valid for larger T-spaces and controlling the phenomenon of bubble removal. The values of \( B \) and \( T_{ref} \) generally decrease with decreasing \( k_{1,b} \), the values being clearly lower for the cases with batch inputs shifted to the front wall. The contraction of the distance \( B \) increases the space utilization owing to increase of \( V_{thom} \) (equation (10)), whereas the decrease of \( T_{ref} \) enhances the fining kinetics, provided bubble removal is the controlling phenomenon; both effects contribute to the increase in the critical melting performance in equation (1).

Another modeled effect of \( k_{1,j} \) has to be repeated to complete its competence in the melting process. The flue gases arising at \( k_{1,j} \) values between 0.6 and 1 and leaving the conversion region with relatively low temperature, cool the hot melt in the homogenization region and noticeably support the helical flow, which is characterized by high space utilization and by tendency to the only small longitudinal circulations [21,24,26,27].

Summarizing the role of \( k_{1,j} \) in the glass melting process, its decreasing value produces the very hot flue gases which increase substantially the temperature in the combustion space and execute more intensive contact with the upper side of the batch blanket. Consequently, the value of the specific batch conversion rate grows owing to faster batch conversion kinetics. Under condition of batch borderline present in the homogenization region, the volume of the free-level part of the homogenization region \( V_{thom} \) grows with batch drawing back and the total value of the space utilization increases (equation (10)). If the controlling role belongs to the sand dissolution, the growth of the melting performance is reduced by the decreasing rate of the sand dissolution owing to decreasing temperature in the conversion region. If the more frequent bubble removal is the controlling phenomenon, the value of the temperature in the homogenization region increases with decreasing \( k_{1,j} \) and the reference bubble removal time decreases. The effect of increasing \( V_{thom} \) is thus supported by the faster bubble removal and the critical melting performance inescapably grows (equation (1) and (10)). The shifting of batch inputs draws the batch borderline back as well and shows the same favorable effects as \( k_{1,j} \) decreases. The high values of \( k_{1,b} \) on the contrary, support the helical flow in the homogenization region, increase the space utilization and, consequently, the critical melting performance.

### 5.4. The development of critical melting performance as a function of \( k_{1,j} \) in the T-spaces

The following results of simulations in Figures 9a–d bring the values of the critical melting performance \( M_{crit} \) and the values of mass flow rate at the balanced energy distribution \( M_{bal} \) as functions of the fraction of energy delivered in the conversion region \( k_{1} \). The results of both spaces working either at the constant value of combustion heat with the characteristic values of \( k_{1,j} = 0.755 \) and 0.742 at the maximum critical melting performances or at \( k_{1,j} = 0.60 \) are presented. The development of the average temperature in the homogenization regions \( T_{G} \) are also added. Figure 9c provides the comparison of \( M_{crit} \) values at both examined levels of \( k_{1,b} \) as well as the position of

![Figure 7](image_url)

**Figure 7.** The view from above as the batch border line moves back from the homogenization region when the batch inputs were shifted from the central position to the position near the front wall. a) the central position of batch inputs, b) the batch inputs shifted to the front wall of the space. The larger T-space (III), \( k_{1} = 0.921, k_{1,j} = 0.60 \).
The curves of the critical melting performances $\dot{M}_{\text{crit}}$ in Figures 9a–d show a steady increase with growing $k_j$ up to the maximum value. It occurs thanks to energy transfer from the homogenization to the conversion region and leads to the controlled melt flow in the homogenization region, which is characterized by a high value of the space utilization. The melting performances in the respective cases were multiplied by 2.3 times up to 3.8 times in the $k_j$ interval 0.5–0.96. In agreement with the effect of $k_{ij}$ on the specific rate of batch conversion $\dot{M}_{\text{batch}}$, the values of $\dot{M}_{\text{critmax}}$ are always higher in the case of the lower $k_{ij}$ value as shown by the comparison $\dot{M}_{\text{crit}}$ dependences in Figure 9c (the maxima of solid red and dashed red curves) and as provides comparison of the relevant maximal values of the critical melting performance in Tables 4 and 6. The dependences in Figure 9c nevertheless show that the values of $\dot{M}_{\text{crit}}$ at the lower value of $k_{ij}$ are higher than the relevant values in the set with the constant input of combustion energy even in the region of low $k_j$ values. This is caused by the more intensive longitudinal melt circulations in the second case and is indicated by lower values of the space $T$-space $\dot{M}_{\text{crit}}$.

Figure 8. The reference bubble removal times $T_{\text{ref}}$ corresponding to temperature $T_G2$ in the homogenization region and the average distances of the batch border line from the inner side of the front wall $B$ as the function of $k_{ij}$ in the larger $T$-space (III) with the central or to the front wall shifted (index 5) batch inputs.

The batch borderline $B$, and Figure 9d moreover involves the development of average temperatures in the conversion region $T_{G1}$.

Figure 9. a–d The critical melting performances $\dot{M}_{\text{crit}}$ of T-spaces as a function of the fraction of heat delivered to the conversion region $k_j$. The $\dot{M}_{\text{crit}}(k_j)$ dependence (red), the relevant $\dot{M}_{\text{batch}}(k_j)$ dependence (blue), the average melt temperature in the conversion region $T_G2$ (Orange), the average melt temperature in the homogenization region $T_G1$ (green), the batch border line $B$ (black). The average melt temperature in the entire space is 1420°C. a) The smaller T-space II, the constant input of energy from burners of 2586 kW, $k_{ij} = 0.755$ at the maximal melting performance, b) the larger T-space (III), the constant input of energy from burners of 3233 kW, $k_{ij} = 0.742$ at the maximal melting performance, c) the smaller T-space I, $k_{ij} = 0.60$. The $\dot{M}_{\text{batch}}(k_j)$ dependence for the smaller T-space II is taken from Figure 9a for comparison (dashed red), d) the larger T-space (III), $k_{ij} = 0.60$. 
utilization in Table 3, compared to the values in Table 5. The respective $\dot{M}_{\text{bal}}$ dependences were calculated according to equation (4), their reliability is only qualitative in the region of low $k_1$ but adequate in the important interval of maximum critical melting performance at high $k_1$ values.

All the Figures 9a–d show a mutual approach of $M_{\text{crit}}(k_1)$ and $\dot{M}_{\text{bal}}(k_1)$ dependences where the maximum values $M_{\text{critmax}}$ almost fit on the crossing of both dependences. The position of the $M_{\text{critmax}}$ value on the $\dot{M}_{\text{bal}}$ curve gives evidence about the principal role of the space utilization not only in the melting module [18–22] but also in the space with the batch blanket. The dependences of the average temperatures in the homogenization region $T_{\text{G0}}$ show an obvious decrease near the value of $k_1$ corresponding to the maximum melting performance. The reason for that decrease is the extension of the batch blanket into the homogenization region, which is depicted in Figure 1 and leads to overheating of the conversion and undercooling of the homogenization region as described by the accompanying text of the Theoretical part: Not all the heat determined for batch conversion can be used directly in the conversion region. A part of the batch escapes to the homogenization region and takes the necessary conversion heat there. Consequently, the melt in the homogenization region undercools and the unabsorbed heat in the conversion region overheats the melt there. The batch borderline shifted to the homogenization region is demonstrated by the example of the smaller T-space I in Figure 10. The bubble removal became the final controlling phenomenon in the cases described by Figures 9a–c; however, the sand dissolution is established in the larger T-space at $k_1 = 0.60$ (Figure 9d). All the cases belong to the class of the melt flows in which both directions of longitudinal circulations and uniform flow may exist according to the value of $k_1$ as presents paragraph 3a) and sign $>$ in the Theoretical.

The character of the melt flow always showed the helical melt flow in the homogenization region as predicted from the role of the flue gases in the Theoretical and presented in Figure 10. The following Figure 11b introduces as an example the helical trajectory of the critical bubble in the larger T-space at the maximum melting performance. Helical courses of the critical trajectories at the maximum melting performance were found in all the simulation sets. For the sake of comparison, Figure 11a shows the critical trajectories of sand particles at low values of $k_1$ which map the longitudinal melt circulations in the homogenization region.

The maximum specific melting performances of T-spaces at controlled melt flow was achieved between about 15–19 tons/(m$^3$·day) and the specific energy consumption moved between 2520 and 2630 kJ/kg. The melting performances result very high, reflecting also the found high values of the specific conversion rate. When assessing the competence of different model constituents, the part of the model describing the process in the melt appears credible and results showing the effect of the space utilization seem to be realistic (see results of the

![Temperature (°C)](image)

Figure 10. The view from above on level streamlines of the glass melt for the critical (optimal) case at $k_1 = 0.96$, close to the $\dot{M}_{\text{bal}}$ curve. The smaller T-space I with thinner electrodes. The average melt temperature is 1420°C, $k_T = 0.6$. 
melting module in [18–22]. The calculated values of the specific rate of batch conversion are only qualified approximations, probably higher than the real ones. This fact should be taken into account when appreciating the values of the critical melting performances; here the found favorable trends should be preferred to absolute values when making predictions. Under this condition, the mathematical model retains its cognitive role.

5.5. The role of homogenization phenomena in the glass melting process

The kinetics of homogenization phenomena, sand dissolution and bubble removal, affects the values of the critical melting performance as is presented by equation (1).

The phenomenon of sand dissolution realizes primarily in the conversion region owing to sand particle history starting in the batch and continuing particularly in the melt under batch blanket, whereas bubble removal mostly occurs in the homogenization region because the starting points of critical bubbles predominantly come from the area around the batch borderline. Consequently, the time-temperature history in the conversion region is more important for the sand dissolution, and the history in the homogenization region is of cardinal importance for the bubble removal. The energy transfer under the \( k_f \) variations naturally affects the temperatures in the regions and, consequently, the establishment and kinetics of the controlling phenomena (sand dissolution or bubble removal) inside regions. Another effect comes from the position of the batch borderline in the homogenization region, which affects the region temperatures too, this effect being dependent on the kinetics of batch decomposition (see Figure 1 and chapter 5.3.). The following overall changes in the melt flow character caused by \( k_f \) variations feed back to the established final temperature distribution in the regions. The actual effect of temperature distribution on the kinetic of homogenization phenomena can then be approximately read from the course of the average temperature of the melt in the homogenization region \( T_G2 \) in Figures 9a–d. The average temperature of the conversion region \( T_G1 \) is symmetrically distributed with respect to \( T_G2 \) as is shown in Figure 9d.

The described role of the phenomena also depends on the degree by which the batch invades the homogenization region at the critical melting performance, the sizes of regions can play a role as well. If the batch borderline in the homogenization region at the maximum melting performance only slightly overcomes the interface between the regions (the conversion region is relatively large, the value of \( k_f \) relatively low, see Figure 12), the temperature \( T_G2 \) sets slightly higher and \( T_G1 \) slightly lower than in the above-mentioned Figures 9a–c, as Table 6 and Figure 9d show. The sand dissolution establishes as the controlling phenomenon even in the region of the maximum critical melting performance (Table 6, cases III-115, III-225). The kinetics of the sand dissolution speeds up in this interval owing to growing \( T_G1 \) temperature, the value of \( t_{Dav1} \) decreases and supports the growth of the critical melting performance (equation (1)). The basic effect of the growing space utilization supported by decreasing value of \( t_{Dav1} \) stops after achievement of the \( M_{bal} \) dependence; subsequently, the clockwise circulations are established. The value of the maximum critical melting performance \( M_{crmax} \) lies again on the \( M_{bal} \) dependence or can be found slightly behind it. The achieved melting performance is nevertheless higher than in the case of bubble removal control.

Summarizing the role of \( k_f \), the quantity appeared to be the principal factor of the melting performance increase. The growth of \( k_f \), i.e. the increase in the fraction of total heat delivered to the conversion region, caused the establishment of the uniform forward flow in the homogenization region, ensured the high value of the space utilization there and substantially enlarged the melting performance. The changes in \( k_f \) affected the temperature distribution in the space.
and through the melting kinetics also influenced the melting performance. The maximum melting performance depended on the position of the batch borderline. If the position is established far in the homogenization region, the slow and controlling bubble removal limited the growth of the critical melting performance. The decreasing active volume of the homogenization region also limited the critical melting performance. However, if the borderline is established near the interface of regions, the accelerated controlling sand dissolution supported the performance growth. The value of the maximum critical melting performance results therefore higher for the batch borderline established near the region interface and for the sand dissolution as the controlling phenomenon. The values of the maximum melting performance always lay on the \( \dot{M}_{\text{bat}} \) dependence or slightly behind it owing to the decisive role of the space utilization for the process. The growing value of \( k_1 \) only insignificantly affected the value of \( \dot{M}_{\text{bat}} \) by increasing the intensity of the melt convection.

6. Conclusion

The presented modeling installation of the uniform melt flow in the melting module [18–20] envisaged applications, in which the achievement of high homogenization capacity caused by the controlled melt flow, could be adapted for the glass melting process and space with batch blanket. To accomplish it, new knowledge had to be introduced in the problem: The comprehension of the impact of the batch border on the temperature development and kinetics of homogenization phenomena in both regions and on the volume part of the homogenization region with the controlled melt flow. The effect of the Joule and combustion heat in the conversion region on the specific batch conversion rate was distinguished. The necessary theoretical background had to be provided and the new expressions derived for the development of the \( \dot{M}_{\text{bat}} \) curve and definition of the space utilization in the two-region melting space with batch blanket.

Applying the space with two regions and both sided batch heating in this work, the efficient helical flow, unattainable in the melting module [19], was allowed in the homogenization region of the space at the energy balanced state and under the effect of the flue gases. The values of the batch conversion rate \( \dot{M}_{\text{bat}} \) indicated an increase with the growing fraction of combustion heat delivered in the conversion region and projected into the calculated values of the critical melting performances. The values of the critical melting performance \( \dot{M}_{\text{bat}} \) obtained by four runs of simulations showed even multiple growth with the increasing fraction of heat delivered to the conversion region \( k_1 \). The kinetics of homogenization phenomena, sand dissolution, and removal of bubbles, either supported or inhibited the growth of the melting performance but the role of the space utilization, i.e., the character of the melt flow, was always substantial. The obtained result trends are indicative for practical applications. They encourage further search and predictions of trends and improved arrangements of the process.

The presented simulations suggest that should be more reference to the role of both energetic factors, the fraction of overall heat, and the fraction of combustion heat delivered to the conversion region \( (k_1, k_2) \),

![Figure 12](image-url)  

Figure 12. The view from above on level streamlines of the glass melt for the critical case at \( k_1 = 0.945 \) and maximal space utilization – near the \( \dot{M}_{\text{bat}} \) curve (case III-115 in Table 6). The larger T-space. The average melt temperature is 1420°C, \( k_{11} = 0.6 \).
in a broader area of their values, particularly in the area of lower \( k_{1,2} \). The effect of considerably faster or slower melting kinetics should also be clarified. The important role of the position of the batch borderline draws a link to the optimal relation between the kinetics of the melting phenomena and the proper sizes of the regions.

**List of symbols**

- \( B \) - the average distance of the batch borderline from the inner side of the front wall (m)
- \( C_1 \) - the proportionality constant of the characteristic velocity of longitudinal circulations (m\( \cdot \)kJ\(^{-1} \))
- \( \dot{h}_F \) - the flux of heat losses flux through boundaries (kJ/kg\(^{-1} \))
- \( \dot{h}_{\text{BatchC}} \) - the specific heat consumption of melting (kJ/kg\(^{-1} \))
- \( \dot{h}_{\text{Batch}} \) - the heat flux from burners absorbed by the glass batch in the conversion region (kJ/kg\(^{-1} \))
- \( \dot{h}_{\text{Comb}} \) - the heat flux in the combustion space (kJ/kg\(^{-1} \))
- \( \dot{h}_{\text{Gas}} \) - the specific energy necessary to heat the combustion gas and oxidizer to the melting temperature (related to 1 kg of produced glass) (kJ/kg\(^{-1} \))
- \( \dot{h}_{\text{Gas}} \) - the heat flux exchanged between the flue gases and melt in the homogenization region (kJ/kg\(^{-1} \))
- \( \dot{h}_{\text{Joule}} \) - the heat flux coming from electrodes in the conversion region (kJ/kg\(^{-1} \))
- \( \dot{h}_{\text{JouleC}} \) - the heat flux coming from electrodes in the homogenization region (kJ/kg\(^{-1} \))
- \( \dot{h}_{\text{Unheated}} \) - the unabsorbed heat flux from electrodes in the conversion region causing overheating of the region (kJ/kg\(^{-1} \))
- \( \dot{h}_{\text{tot}} \) - the total flux of delivered energy (kJ/kg\(^{-1} \))
- \( \dot{h}_{\text{th}} \) - the theoretical heat (kJ/kg\(^{-1} \))
- \( k_{1,2} \) - the fraction of the overall energy supplied into the input (batch conversion) region of the space
- \( k_{1,3} \) - the fraction of the Joule energy from overall energy delivered to the batch conversion region
- \( M \) - the mass flow rate of the melt (kg/s\(^{-1} \))
- \( M_{\text{Bal}} \) - the mass flow rate of the melt at balanced energy distribution (kg/s\(^{-1} \))
- \( M_{\text{tot}} \) - the mass flow rate of the melt at the critical melting performance (the critical melting performance) (kg/s\(^{-1} \))
- \( M_{\text{max}} \) - the maximum critical melting performance (kg/s\(^{-1} \))
- \( M_{\text{batch}} \) - the specific conversion rate of the batch (in kg of glass) (g/mm\(^{-2} \))
- \( M_{\text{batchC}} \) - the fraction of \( M_{\text{batch}} \) converted to glass by the combustion heat (g/mm\(^{-2} \))
- \( M_{\text{batchH}} \) - the fraction of \( M_{\text{batch}} \) converted to glass by Joule heat (g/mm\(^{-2} \))
- \( S_{\text{batch}} \) - the area of batch blanket (m\(^2 \))
- \( T_{C1} \) - the average temperature of flue gases in the conversion region (°C)
- \( T_{C2} \) - the average temperature of the melt in the entire space (°C)
- \( T_{G1} \) - the average temperature of the melt in the conversion region (°C)
- \( T_{G2} \) - the average temperature of the melt in the homogenization region (°C)
- \( \eta_{\text{H}} \) - the space utilization under either sand dissolution or bubble removal (fining) control
- \( \eta_{\text{G}} \) - the space utilization under control of sand dissolution
- \( \eta_{\text{F}} \) - the space utilization under control of bubble removal
- \( \eta_{\text{Hconv}} \) - the value of the space utilization in the volume of melt covered by the batch \((V-V_{\text{thom}})\)
- \( \eta_{\text{thom}} \) - the value of the space utilization in the volume \( V_{\text{thom}} \) with controlled melt flow
- \( \nu_{\text{C}} \) - the characteristic velocity of longitudinal circulations (m/s\(^{-1} \))
- \( V \) - the volume of glass melt in the entire space (m\(^3 \))
- \( V_{\text{conv}} \) - the volume of melt in the conversion region (m\(^3 \))
- \( V_{\text{thom}} \) - the volume of melt in the homogenization region (m\(^3 \))
- \( \xi \) - the fraction of heat losses relating to the conversion region
- \( \rho \) - the density of glass melt (kg/m\(^3 \))
- \( \tau_{\text{Dove}} \) - the reference melting time of sand dissolution (the average melting time of maximum sand particles) (s)
- \( \tau_{\text{G}} \) - the mean residence time of glass in the space \((\tau_{\text{G}}=V/V)\) (s)
- \( \tau_{\text{Heet}} \) - the reference melting time of either sand dissolution or bubble removal(s)
- \( \tau_{\text{Ref}} \) - the reference melting time of bubble removal(s)

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