Supergravity Loop Contributions
to Brane World Supersymmetry Breaking

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Abstract

We compute the supergravity loop contributions to the visible sector scalar masses in the simplest 5D ‘brane-world’ model. Supersymmetry is assumed to be broken away from the visible brane and the contributions are UV finite due to 5D locality. We perform the calculation with $\mathcal{N} = 1$ supergraphs, using a formulation of 5D supergravity in terms of $\mathcal{N} = 1$ superfields. We compute contributions to the 4D effective action that determine the visible scalar masses, and we find that the mass-squared terms are negative.
1 Introduction

In this paper, we study supersymmetry (SUSY) breaking in the simplest 5D ‘brane world’ scenario. In brane world scenarios, some or all of the visible sector fields are assumed to be localized on a brane, and SUSY is broken away from the visible brane. In this case, bulk fields transmit the message of SUSY breaking to the visible sector. We consider the minimal case where the bulk fields are the 5D supergravity (SUGRA) multiplet. Thus, supergravity plays the role of the messenger for SUSY breaking. Previously, Ref. [1] showed that the leading contribution to visible sector SUSY breaking for large radius, comes from anomaly-mediated SUSY breaking (see also Ref. [2]). If the visible sector consists only of the minimal supersymmetric standard model, the slepton mass-squared terms are negative. Thus, for these brane-world models to be realistic we require other contributions to SUSY breaking in the visible sector. With the hope of getting positive mass-squared terms, we will calculate the leading contributions to SUSY breaking by SUGRA loops.

The simplest 5D brane-world scenario can be described as follows. The 5D space-time is flat and compactified on an $S^1/Z_2$ orbifold. There is one 3-brane at each of the $Z_2$ fixed points. These 3-branes can be regarded as the boundaries of the extra dimension of length $\ell = \pi r$, where $r$ is the radius of the $S^1$. We assume that SUSY is broken by the vacuum expectation value of a chiral superfield $X$ localized on the hidden brane. The visible chiral superfields $Q$ are assumed to be localized on the other brane. In this 5D effective theory, contact terms between $Q$ and $X$ are forbidden by 5D locality.\(^1\) The effects of SUGRA mediated SUSY breaking can be analyzed systematically using the 4D effective Lagrangian that describes the physics below the compactification scale $1/r$. The effective theory contains the chiral superfields $Q$ and $X$, the 4D SUGRA multiplet, and the chiral radion multiplet

$$T = \pi r + \cdots + \theta^2 F_T.$$ \hspace{1cm} (1.1)

Expanding the 4D effective action in $Q$ and $X$, the leading terms involving $Q$ that cannot be forbidden by symmetries are

$$\Delta \mathcal{L}_{4\text{eff}} = \int d^4 \theta \left[ c_1(T) Q^\dagger Q + c_2(T) X^\dagger X Q^\dagger Q + \cdots \right]. \hspace{1cm} (1.2)$$

At tree level, $c_1$ is independent of $T$, and Ref. [3] showed that $c_2$ vanishes. Therefore, we must consider loop effects.

\(^1\)In a more fundamental theory with additional states with masses $M \gg 1/r$, contact terms between $Q$ and $X$ will be suppressed by $e^{-Mr}$.\[1]
At 1-loop level, there are contributions to $c_1$ from the diagrams in Fig. 1. These contributions are of order
\[ c_1 \sim \frac{1}{M_5^3 (T + T^\dagger)^2}. \]
(1.3)
The dependence on $T$ is fixed by dimensional analysis and the observation that $c_1$ cannot depend on the fifth component of the graviphoton of 5D SUGRA, which is contained in $T - T^\dagger$. Loop corrections to $c_1$ are finite because they are sensitive to the size of the extra dimension, while all divergent effects are local. These corrections can give important contributions to the scalar masses of $Q$ if $\langle F_T \rangle \neq 0$:
\[ \Delta m_Q^2 = -3\langle c_1 \rangle \langle \frac{F_T}{T} \rangle^2. \]
(1.4)
Here we have neglected 1-loop operators of the form
\[ \Delta L_4^{\text{eff}} = \int d^4 \theta \frac{|D^2 T|^2}{M_5^3 (T + T^\dagger)^2} Q^\dagger Q, \]
(1.5)
which give contributions to the scalar masses proportional to $\langle F_T \rangle^4$. Thus, the contribution from $c_1$ in Eq. (1.3) dominates only if $\langle F_T \rangle \ll 1$. A nonzero value for $\langle F_T \rangle$ is equivalent to the Scherk–Schwarz mechanism for SUSY breaking, as discussed in Ref. [4]. The SUGRA loop effect proportional to $c_1$ was computed in Ref. [6] using the off-shell formulation of supergravity due to Zucker. It was found that the resulting scalar mass-squared terms are negative.

There are 1-loop contributions to $c_2$ from the diagrams in Fig 2. These diagrams are UV finite because the loop cannot shrink to zero size. By dimensional analysis, these give
\[ c_2 \sim \frac{1}{M_5^6 (T + T^\dagger)^4}. \]
(1.6)
This is suppressed by extra powers of $M_5$ compared to $c_1$. This contribution may be important if $\langle F_T \rangle$ is sufficiently small. In this case, it gives a contribution to the $Q$ scalar mass
\[ \Delta m_Q^2 = -\langle c_2 \rangle |\langle F_X \rangle|^2. \]
(1.7)
Although $c_1$ is known in the literature, $c_2$ has never been calculated. In this paper, we will present explicit calculations of both $c_1$ and $c_2$. We perform quantum computations using supergraphs (see e.g. [8], [9]) applied to the formulation of 5D SUGRA in $\mathcal{N} = 1$ superspace developed in Ref. [10]. This formalism has several
advantages over component calculations. First, higher powers of Dirac delta functions from brane-bulk interactions do not arise in this formulation. Higher powers of Dirac delta functions occur only after integrating out auxiliary fields [12], and therefore are absent in supergraph calculations. Furthermore, the gauge can be fixed so that the superspace supergravity propagator has the following trivial tensor structure:

$$\langle V_m V_n \rangle \sim \eta_{mn}$$

(1.8)

where \( m, n = 0, \ldots, 3 \) are 4D Lorentz indices and \( V_m \) is the SUGRA superfield pre-potential. The simple form of this propagator makes quantum calculations straightforward. Another advantage of this approach is that we only need to calculate five super Feynman graphs. In a direct component formulation, this number would grow by an order of magnitude.

This paper is organized as follows. Section 2 reviews 5D SUGRA in \( \mathcal{N} = 1 \) superspace [10], and proves the existence of the remarkably simple gauge fixing noted above. Section 3 gives the supergraph Feynman rules for the theory. Sections 4 and 5 contain the calculations of \( c_1 \) and \( c_2 \), respectively. We find that both \( c_1 \) and \( c_2 \) give negative scalar mass-squared terms in the visible sector. The result for \( c_1 \) agrees with Ref. [6], while the result for \( c_2 \) is new.

2 Lagrangian and Gauge-fixing

The Lagrangian for linearized minimal 5D SUGRA was written in terms of \( \mathcal{N} = 1 \) superfields in Ref. [10]. Here, we describe the component field embedding and state the superfield action. We then prove the existence of the gauge choice Eq. (1.8).

The formulation of Ref. [10] contains two real superfields \( V_m \) and \( P \), a chiral superfield \( T \), and an unconstrained superfield \( \Psi_\alpha \).\(^2\) The embedding of the 5D propagating fields into these superfields is accomplished as follows. The graviton, graviphoton and gravitino are first dimensionally reduced:

\[
\begin{align*}
 h_{MN} &\to h_{mn},\ h_{5m}, h_{55}, \\
 B_M &\to B_m, B_5, \\
 \psi_{M\dot{\alpha}} &\to \psi_{m\dot{\alpha}},
\end{align*}
\]

(2.1)

Here the 5D gravitino is decomposed into components with parity \( \pm 1 \) under the \( Z_2 \)

\(^2\)The field \( \Psi_\alpha \) corresponds to what was called \( \hat{\Psi}_\alpha \) in Ref. [10].
transformation $x^5 \mapsto -x^5$. These reduced fields are embedded in superfields as

$$ V_m \sim \theta \sigma^\alpha \bar{h}_{mn} + \bar{\theta}^2 \theta^n \psi_{m\alpha}^+ + \cdots, $$

$$ \Psi_\alpha \sim \bar{\theta}^\alpha (B_{\alpha\dot{\alpha}} + i h_{5\alpha\dot{\alpha}}) + \theta \sigma^m \bar{\theta} \psi_{m\alpha}^+ + \bar{\theta}^2 \psi_{5\alpha}^+ + \cdots, \tag{2.2} $$

$$ T \sim h_{55} + i B_5 + \theta^\alpha \psi_{5\alpha}^+ + \cdots. $$

In this formulation, when the $\mathbb{Z}_2$ even superfields $V_m$ and $P$ are evaluated on either boundary they are the usual 4D $\mathcal{N} = 1$ SUGRA multiplet. (The real field $P$ is the prepotential for the usual conformal compensator: $\Sigma = -\frac{i}{4} \bar{D}^2 P$.) This makes coupling 5D SUGRA to fields localized on the boundaries particularly simple. For details, see Ref. [10].

The Lagrangian for linearized 5D SUGRA is

$$ \mathcal{L}_{5D \text{SUGRA}} = \mathcal{L}_{\mathcal{N}=1} + \Delta \mathcal{L}_5, \tag{2.3} $$

where $\mathcal{L}_{\mathcal{N}=1}$ is the linearized $\mathcal{N} = 1$ SUGRA Lagrangian (see e.g. [9]):

$$ \mathcal{L}_{\mathcal{N}=1} = M_5^3 \int d^4 \theta \left[ \frac{1}{8} V^m D^\alpha \bar{D} D_\alpha V_m + \frac{1}{48} \left( [D^\alpha, \bar{D}^\alpha] V_{\alpha\dot{\alpha}} \right)^2 - (\partial^n V_m)^2 ight. $$

$$ \left. - \frac{1}{2} \Sigma^\dagger \Sigma + \frac{2}{3} (\Sigma - \Sigma^\dagger) \partial^n V_m \right], \tag{2.4} $$

and

$$ \Delta \mathcal{L}_5 = -\frac{1}{2} M_5^3 \int d^4 \theta \left\{ [T^\dagger (\Sigma - i \partial_{\alpha\dot{\alpha}} V^{\dot{\alpha}\alpha}) + \text{h.c.}] ight. $$

$$ \left. - \frac{1}{2} \left[ D^\alpha \Psi_\alpha + \bar{D}_{\dot{\alpha}} \bar{\Psi}_{\dot{\alpha}} - \partial_5 P \right]^2 ight. $$

$$ \left. + \left[ \partial_5 V_{a\dot{\alpha}} - (D_{\alpha} \Psi_\alpha - D_{\dot{\alpha}} \bar{\Psi}_{\dot{\alpha}}) \right]^2 \right\}. \tag{2.5} $$

In this normalization, $M_P^2 = \frac{1}{2} \pi r M_5^3$, where $M_P = 2 \times 10^{18}$ GeV is the 4D Planck scale.

The terms in the Lagrangian involving the brane-localized superfields $X$ and $Q$ are

$$ \Delta \mathcal{L}_{\text{brane}} = \delta(x^5) \mathcal{L}_{4,\text{kin}}(Q) + \delta(x^5 - \ell) \mathcal{L}_{4,\text{kin}}(X), \tag{2.6} $$

where $\mathcal{L}_{4,\text{kin}}(\Phi)$ is the kinetic term for a 4D chiral superfield $\Phi$ coupled to 4D SUGRA:

$$ \mathcal{L}_{4,\text{kin}}(\Phi) = \int d^4 \theta \left[ \Phi^\dagger \Phi + \frac{2}{3} V^m \Phi^\dagger \bar{D}_m \Phi - \frac{1}{6} V^m K_{mn} V^n \Phi^\dagger \Phi + \cdots \right]. \tag{2.7} $$

$^3$We use the conventions of Wess and Bagger [11].
Here we have absorbed the conformal compensator $\Sigma$ into $\Phi$. We have omitted terms $O(V^3)$ and higher, as well as $O(V^2)$ with derivatives acting on the chiral fields, since these do not contribute to the terms in Eq. (1.2). Finally, $K_{mn}$ represents the quadratic terms in Eq. (2.4) and is given explicitly by:

$$K_{mn} := \frac{1}{4} \eta_{mn} D^\alpha \bar{D}^2 D_\alpha + \frac{1}{24} \sigma^\alpha_m \sigma^\beta_n [D_\alpha, \bar{D}_\beta][D_\beta, \bar{D}_\alpha] + 2 \partial_n \partial_m$$

(2.8)

To define the propagator for quantum calculations, we must first fix the gauge. We require just the $V_m V_n$ propagator, because the vertices from Eq. (2.7) involve only $V_m$. We now show that there exists a gauge fixing term that cancels the mixing between $V_m$ and the other bulk superfields $P, \Psi_\alpha, T$, and simultaneously reduces the $V_m$ kinetic term to the simplest possible form $V_m^m \Box_5 V_m$. To do this, we rewrite the quadratic terms in $V$ as

$$\mathcal{L}_{5D \text{SUGRA}} = M_5^3 \int d^4 \theta \left[ - \frac{1}{2} V_m^m \Box_5 V_m + Q(\bar{D}_\alpha V^{\hat{\alpha}}) + \ldots \right],$$

(2.9)

where

$$Q(\chi^\alpha) = \frac{1}{24} \chi^2 - \frac{1}{3} \chi^\alpha (\bar{D}_\alpha D_\alpha - \frac{1}{3} D_\alpha \bar{D}_\alpha) \chi^\alpha.$$  

(2.10)

We then add the gauge fixing term

$$\Delta \mathcal{L}_{gf} = - M_5^3 \int d^4 \theta \chi Q(\mathcal{G}^\alpha),$$

(2.11)

where the gauge fixing function takes the form

$$\mathcal{G}^\alpha = D_\alpha V^{\hat{\alpha}} + \frac{\partial_5}{\Box_5} \left( D^2 \bar{\Psi}^\alpha - \frac{6i}{3} \sigma^{\hat{\alpha} \alpha} \bar{\Psi}_\alpha - \frac{2}{3} [D^\alpha, \bar{D}^{\hat{\alpha}}] \bar{\Psi}_\alpha \right) - \frac{4}{3} \sigma^{\hat{\alpha} \alpha} \bar{D}_\alpha (T^\dagger - \frac{1}{3} \Sigma^\dagger).$$

(2.12)

With this addition, we have

$$\mathcal{L}_{5D \text{SUGRA}} + \Delta \mathcal{L}_{gf} = M_5^3 \int d^4 \theta \left[ - \frac{1}{2} V_m^m \Box_5 V_m \right] + \mathcal{L}(P, T, \Psi_\alpha).$$

(2.13)

Note that we do not need the ghost action, since we are not computing loops involving SUGRA self-couplings. Hence the ghosts decouple and do not contribute to the quantities under consideration.

### 3 Superpropagators on the Orbifold

The perturbative theory for the model with the superfield Lagrangian given by Eq. (2.3) can be completely formulated in terms of superfields with the help of supergraphs (see e.g. [8], [9]). Although we will not review these techniques, we will
describe the relevant modifications to describe the brane-world scenario. In this brane-world scenario we have an $S^1/Z_2$ orbifold. Thus, it is convenient to write the Feynman rules in mixed 4D momentum space and 5D position space. Further, for supergraph calculations, we use the following abbreviations

$$\int_{1,\ldots,n} = \int d^4 \theta_1 \cdots \int d^4 \theta_n, \quad \delta_{12} = \delta^4(\theta_1 - \theta_2), \quad \mathcal{D}_1(p) = -\frac{1}{4} D^2_1(p),$$

(3.1)

where $D_\alpha(p)$ is the SUSY covariant derivative in momentum space. We omit the $p$ argument when this leads to no ambiguity. We also note the following identities used for manipulating covariant derivatives and delta functions under superspace integrals:

$$\mathcal{D}_1 \delta_{12} = \mathcal{D}_2 \delta_{12},$$

$$\delta_{12} \left[ \mathcal{O}(D^a_1 \overline{D}_1^m) \delta_{12} \right] = 0 \text{ for } n < 2 \text{ or } m < 2,$$

(3.2)

$$\delta_{12} \left[ \mathcal{D}_1 \mathcal{D}_1 \mathcal{D}_1 \delta_{12} \right] = \delta_{12} \left[ \overline{\mathcal{D}}_1 \mathcal{D}_1 \mathcal{D}_1 \delta_{12} \right] = -p^2 \delta_{12}.$$

The $V_m$ propagator with one endpoint fixed on the visible brane is

$$\langle V_m(1, x^5 = 0)V_n(2, x^5 = y) \rangle = i \eta_{mn} \delta_{12} \Delta(p, y),$$

(3.3)

where the Green function $\Delta(p, y)$ satisfies the equation

$$M_5^3 (\partial_y^2 - p^2) \Delta(p, y) = -\delta(y).$$

(3.4)

Since $V_m$ is an even field, the boundary conditions are $\partial_y \Delta = 0$ at the branes. In the domain $-\ell < y < \ell$, we have

$$\Delta(p, y) = \frac{1}{M_5^3 \cosh(p(|y| - \ell))} \frac{1}{2p \sinh(p\ell)},$$

(3.5)

where $p = +\sqrt{p^m p_m}$. The propagator from the visible brane to the visible brane is given by the limit $y \to 0$:

$$\Delta_{\text{vis,vis}}(p) = \frac{1}{M_5^3} \frac{1}{2p \tanh(p\ell)}.$$

(3.6)

The propagator between the visible and hidden branes is given by the limit $y \to \ell$:

$$\Delta_{\text{vis,hidden}}(p) = \frac{1}{M_5^3} \frac{1}{2p \sinh(p\ell)}.$$

(3.7)
The chiral propagators localized on the brane are given by the standard 4D expression (see e.g. \[8\], \[9\])

\[ \langle \Phi^\dagger(1)\Phi(2) \rangle = -\frac{i}{p^2}D_1\bar{D}_2\delta_{12}. \] (3.8)

The vertices between chiral fields and \(V_m\) are read off from Eq. (2.7).

We now have all of the necessary ingredients to compute the coefficients \(c_1\) and \(c_2\). We neglect contributions due to the derivatives of \(Q, Q^\dagger\) and \(X, X^\dagger\), because we are only interested in corrections to scalar masses. Since \(c_1\) and \(c_2\) are gauge invariant, they can be computed using the simple gauge choice described above.

A few comments about the supergraph technique are in order. The standard procedure of supergraph calculations is reviewed in Refs. \[8\] and \[9\]. The main feature is that SUSY is manifest at every step. Another feature of the supergraph technique is the presence of SUSY covariant derivatives and Grassman \(\delta\)-functions. The covariant derivative algebra is what simplifies the calculations in comparison to component formulations of SUSY theories. In an arbitrary supergraph, one can transfer all covariant derivatives onto one Grassman \(\delta\)-function using integration by parts. This removes all integrals over anticommuting variables except one. Then one can transform the last integral over superspace to a standard Feynman integral over conventional momentum space. This is accomplished by using the rules given in Eq. (3.2). This procedure avoids calculating large numbers of diagrams that would appear in a component formulation of a SUSY theory. Furthermore, it automatically accounts the cancelations of conventional diagrams stipulated by \(\mathcal{N}=1\) SUSY.

### 4 Radion-mediated Contribution

We now compute the coefficient \(c_1\) in the effective lagrangian Eq. (1.2). The 1-loop supergraphs that contribute are shown in Fig. 1.

The diagram in Fig. 1a is given by

\[ \text{Fig. 1a} = -\left(\frac{2}{3}\right)^2 \int \frac{d^4p}{(2\pi)^4} \Delta_{\text{vis,vis}}(p) \frac{P_mP_n}{p^2} I_{1a}^{mn}, \] (4.1)

The superspace integral \(I_{1a}^{mn}\) is

\[ I_{1a}^{mn} = \eta^{mn} \int_{1,2} \delta_{12} D_1 \bar{D}_2 \delta_{12} = \eta^{mn} \int_1, \] (4.2)

and leads to

\[ \text{Fig. 1a} = -\left(\frac{2}{3}\right)^2 \int d^4\theta \int \frac{d^4p}{(2\pi)^4} \Delta_{\text{vis,vis}}(p). \] (4.3)
Fig. 1. Supergraphs contributing to the coefficient $c_1$, the radion mediated corrections to SUSY breaking.

The second diagram Fig. 1b gives

\[
\text{Fig. 1b} = \frac{1}{6} \int \frac{d^4 p}{(2\pi)^4} \Delta_{\text{vis,vis}}(p) I_{1b}. \quad (4.4)
\]

For this diagram, the $V_m$ propagator must be evaluated in the limit that $\theta_1$ goes to $\theta_2$. This is equivalent to inserting one more delta-function and integrating over $\theta_2$. Thus, the superspace integral $I_{1b}$ is

\[
I_{1b} = \int_{1,2} \delta_{12} \eta^{mn} K_{mn} \delta_{12} = \frac{32}{3} \int_1,
\]

which yields

\[
\text{Fig. 1b} = \frac{16}{9} \int d^4 \theta \int \frac{d^4 p}{(2\pi)^4} \Delta_{\text{vis,vis}}(p). \quad (4.6)
\]

The momentum integral is UV divergent, but its divergent part is independent of $\ell$. This is easily seen from the leading behavior of the propagator at large $p$, which is $\Delta \to 1/(2p)$. Physically, this UV divergent contribution renormalizes the $Q$ kinetic term on the visible brane, which is insensitive to the size of the extra dimension. For radion-mediated SUSY breaking we are interested in the $\ell$ dependent contribution, so we write

\[
\int \frac{d^4 p}{(2\pi)^4} \Delta_{\text{vis,vis}}(p) = \frac{1}{M_5^3} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{2p \tanh(p\ell)}
\]

\[
= \frac{1}{M_5^3} \int \frac{d^4 p}{(2\pi)^4} \frac{e^{-p\ell}}{2p \sinh(p\ell)} + \text{independent of } \ell
\]

\[
= \frac{1}{4\pi^2 M_5^3 (2\ell)^3} + \text{independent of } \ell. \quad (4.7)
\]
Fig. 2. Supergraphs contributing to the coefficient $c_2$, the brane-to-brane corrections to SUSY breaking.

where $\zeta(3) \approx 1.202$ is the Riemann zeta function. Combining Eqs. (4.3), (4.6), and (4.7) the total result for $c_1$ is

$$c_1 = \frac{1}{3\pi^2 M_5^2} \zeta(3).$$

(4.8)

5 Brane-to-Brane Contribution

We now compute the coefficient $c_2$ in the effective lagrangian Eq. (1.2). The 1-loop supergraphs that contribute are shown in Fig. 2. We first consider the diagram of Fig. 2a, consisting of four 4-point interactions. There are two possible contractions for this diagram. One of them vanishes due to the SUSY covariant derivative algebra, and the other yields

$$\text{Fig. 2a} = \left(\frac{2}{3}\right)^4 \int \frac{d^4 p}{(2\pi)^4} \left[\Delta_{\text{vis, hid}}(p)\right]^2 I_{2a}. \quad (5.1)$$

The superspace integral $I_{2a}$ is

$$I_{2a} = \int_{1,\ldots,4} \left(\mathcal{D}_1 \mathcal{D}_3 \delta_{13}\right) \left(\mathcal{D}_4 \mathcal{D}_2 \delta_{24}\right) \delta_{12} \delta_{34} = - \int p^2, \quad (5.2)$$

and gives

$$\text{Fig. 2a} = - \left(\frac{2}{3}\right)^4 \int d^4 \theta \int \frac{d^4 p}{(2\pi)^4} p^2 \left[\Delta_{\text{vis, hid}}(p)\right]^2. \quad (5.3)$$

The diagram of Fig. 2b contains two 4-point interactions

$$\text{Fig. 2b} = 2 \left(\frac{1}{6}\right)^2 \int \frac{d^4 p}{(2\pi)^4} \left[\Delta_{\text{vis, hid}}(p)\right]^2 I_{2b}. \quad (5.4)$$
where the superspace integral $I_{2b}$ is

$$I_{2b} = \int_{1,2} \delta_{12} (K_{1}^{mn} K_{nm,1} \delta_{12}) = \frac{112}{9} \int_{1} p^2, \quad (5.5)$$

and leads to

$$\text{Fig. 2b} = \frac{224}{9} \left( \frac{1}{6} \right)^2 \int d^4 \theta \int \frac{d^4 p}{(2\pi)^4} \left[ \Delta_{\text{vis, hid}}(p) \right]^2 p^2. \quad (5.6)$$

The diagram Fig. 2c contains two 3-point and one 4-point interaction. There are two contractions, each giving the same contribution. We obtain

$$\text{Fig. 2c} = 2 \times \left( -\frac{1}{3} \right) \left( \frac{2}{3} \right)^2 \int d^4 p \left( \frac{2\pi}{4} \right)^4 \int \frac{d^4 p}{(2\pi)^4} \frac{p_m p_n}{p^2} \left[ \Delta_{\text{vis, hid}}(p) \right]^2 I_{2c}^{mn}, \quad (5.7)$$

where the superspace integral $I_{2c}^{mn}$ is

$$I_{2c}^{mn} = \int_{1,2,3} \left( D_{1} \bar{D}_{2} \delta_{12} \right) (K_{3}^{mn} \delta_{13}) \delta_{23} = -\frac{4}{3} \int_{1} p^m p^n, \quad (5.8)$$

and yields

$$\text{Fig. 2c} = \frac{8}{9} \left( \frac{2}{3} \right)^2 \int d^4 p \left( \frac{2\pi}{4} \right)^4 p^2 \left[ \Delta_{\text{vis, hid}}(p) \right]^2. \quad (5.9)$$

The momentum integral is UV finite and can be evaluated directly. Physically, the UV finiteness is due to the fact that the SUGRA propagator cannot shrink to zero size because the endpoints are fixed on different branes. The integral is

$$\int \frac{d^4 p}{(2\pi)^4} p^2 \left[ \Delta_{\text{vis, hid}}(p) \right]^2 = \frac{1}{4 M_5^2} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{\sinh^2(p \ell)} = \frac{3}{4 \pi^2 M_5^2} \left( \frac{\zeta(3)}{2 \ell} \right) \quad (5.10)$$

Combining Eqs. (5.3), (5.6), (5.9), and (5.10) the final result is

$$c_2 = \frac{2}{3\pi^2 M_5^2} \left( \frac{\zeta(3)}{2 \ell} \right) \quad (5.11)$$

This completes our calculation. The coefficients $c_1$ and $c_2$ in the 4D effective lagrangian defined in Eq. (1.2) are given by Eqs. (5.8) and (5.11), respectively.
6 Conclusion

We have formulated an $\mathcal{N} = 1$ supergraph approach to 5D supergravity (SUGRA) loop calculations, using the formulation of 5D SUGRA in terms of $\mathcal{N} = 1$ superfields of Ref. [10]. This formalism makes $\mathcal{N} = 1$ SUSY manifest, and makes couplings between bulk and brane fields particularly simple. In particular, there are no terms with higher powers of delta functions appearing in the calculation, as in component approaches.

We applied this formalism to compute the leading SUGRA loop contributions to visible sector scalar masses in the simplest ‘brane world’ scenario based on a flat 5D space compactified on a $S^1/Z_2$ orbifold. The terms in the effective lagrangian are defined in Eq. (1.2) and our results are given in Eqs. (4.8) and (5.11). The calculation requires the calculation of only five supergraphs.

The same effective lagrangian terms have been calculated by R. Rattazzi, C.A. Scrucca, and A. Strumia using the component formulation of 5D supergravity. Our results agree [13].

There are a number of directions to extend the present results. Warped compactifications may give positive loop contributions to scalar masses. It would also be interesting to extend the present results to higher dimensions, possibly to make direct contact with string theory, and also to construct the fully nonlinear theory. We leave these questions to future work.

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