Estimation of return period of the destructive earthquake in aceh using the maximum likelihood method

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Abstract. Aceh is one of the provinces in Sumatra Island which coincides with the meeting of the Eurasian plate and the Indo-Australian plate. The negative impact of the plate meeting is the occurrence of an earthquake. The earthquakes that occurred in this region have caused various losses including casualties. Therefore, estimates of earthquake events are required for disaster mitigation caused by earthquakes. In this research the period of earthquake re-occurring in Aceh and Surrounding region using the relation between frequency and magnitude of earthquake known as Gutenberg-Richter relationship. The maximum likelihood method is used to determine the seismic parameters obtained from the relationship. The data used ware sourced from the USGS earthquake catalogue (1907-2008) and the BMKG earthquake catalogue (2009-2017) with minimum depth is 80 Km that coincides at 92 ° - 98.5 ° East Longitude and 0 ° - 8 ° North latitude. The obtained magnitude data were converted to the moment magnitude (Mw), then selected data that have a minimum Magnitude (Mw) of 5 which is a destructive earthquake. The results obtained from this study are the period of the destructive earthquake (Mw ≥ 5) is expected to re-occur in 19.7 years in the sea region between Aceh Barat Daya and Simeulue.

1. Introduction
The province of Aceh is one region that coincides with the meeting of the Eurasian plate with the Indo-Australian plate along the Indian Ocean. In this zone, there have been some major earthquakes that have caused severe damage in some areas around the Indian Ocean, among them the earthquake and tsunami of December 26, 2004 which has a magnitude of 9.1 SR and the earthquake that occurred in Pidie Jaya on December 7, 2016 measuring 6.5 SR.

Research on seismotectonic potential as a precursor of earthquake level in Aceh has been done, among them, analysing the level of seismicity and the period of earthquake re-occurring in Aceh region using the relationship between the frequency with earthquake magnitude, using the Maximum Likelihood method. The study concludes that between 2 to 5 years there will be an earthquake for the 6 Mw scale in Simeulue Island, Northwest Aceh and North Aceh (Andaman)[1]. An earthquake with a greater magnitude generally concentrated in the district of Simeulue, the western part of Aceh Besar and northwest of Sabang[2]. Based on the above background, spatially and temporally, the researcher wants to calculate the period of the earthquake in Aceh region as the initial information for mitigation, so that the impact of the earthquake that will happen can be minimized.

2. Materials and Method
Tectonic earthquakes occur due to the encounter or movement between plates, the active tectonic plates with other tectonic plates have three types of movement, i.e. the two plates are spreading, collision, and mutually (transform) [3].

Aceh Province is one of the areas on the island of Sumatra located at 2° – 6° North Latitude and 92° - 98° East Longitude. The island of Sumatra is an island that coincides at the meeting of the Eurasian Plate and the Indo-Australian Plate. The collisions between the plates resulted in a series of non-volcanic island arcs (such as Simeulue Island, Pulau Banyak, Nias Island to Enggano Island) and a
series of hillside mountains with volcanic trails in the centre[4]. In addition to being in two active plate meetings, the Aceh region also belongs to the active fault zone of Sumatra. The existence of this fault resulted in frequent earthquakes in the Aceh region with great power and small, both on land and at sea.

2.1 Earthquake Frequency (N) dan Magnitude (M) Relationship

Earthquake frequency-magnitude relationships can be used to see the number of earthquakes occurring in a region, one of them using the Gutenberg-Richter law [5].

\[
\log N(M) = a - bM
\]

where:
- \(N\) = the number of earthquakes
- \(a\) = a determination that depends on the period, total area and area of observation activity
- \(b\) = seismotectonic parameter of an area in which in an earthquake occur and depending on the local rock.
- \(M\) = magnitude

[6]

The value of the tectonic parameters in the research area can be seen by looking at b-value. b-value can show the stress level of the rock. If b-value is low then the stress condition in the area is high (Nuannin, 2006). According to Kulhanek, et al. (2018), b-value is said to be low if b-value has a value smaller than the average value of b-value in the entire study area. b-value approximation is correct if the standard deviation of b-value is less than one [7].

2.2 Maximum Likelihood Method

Maximum Likelihood aims to estimate statistics of random variables by maximizing probabilities with existing samples. This method is often used in different applications and studies in various situations[8]. Likelihood's maximum method is better for calculating b-value. Magnitude \(M\) is considered as a random variable with the Probability Density Function (PDF) for \(M\) is as follows [9]:

\[
p(M) = b \ln(10) \frac{10^{-bM}}{10^{-bM_{min}} - 10^{-bM_{max}}}
\]

Where \(M_{min}\) is the minimum magnitude and \(M_{max}\) is the maximum magnitude, if \(M_{max} > M_{min}\) then equation (2) becomes:

\[
p(M) = b \ln(10) 10^{-b(M-M_{min})}
\]

the maximum etic of likelihood in equation (3) has b-value by maximizing the likelihood function, where the equation becomes:

\[
b = \frac{1}{\ln(10)(\bar{M} - M_{min})}
\]

With \(1/\ln(10)\) equal to \(\log e\), then the b-value obtained from the maximum of Likelihood is expressed as follows:

\[
b = \frac{\log e}{\bar{M} - M_{0}}
\]

Where:
- \(\bar{M}\) = Average magnitude
\( M_0 = \) Minimum magnitude  
\( \log e = \log 2,71828 = 0.3434 \)

The error rate or standard deviation of the b-value calculation with the magnitude can be estimated using the equation:

\[
\sigma_b = 2.30 b^2 \frac{\sum_{i=1}^{N}\left(M_i - \mu\right)^2}{N(N-1)}
\]  

(7)

In addition, there is also an a-value that states the level of seismicity in a region. The smaller the a-value, the smaller the seismic activity [10].

For a-value can be calculated from the cumulative frequency relationship that is:

\[
a = \log N(M \geq M_0) + \log(b \ln 10) + M_0 b
\]

(8)

In the calculation is used observation value of, which then obtained a-value[1].

2.3 Earthquake return period

To get the average value of earthquake return periods in a region can be calculated using the following equation:

\[
P(\Theta) = \frac{1}{N_1(M \geq M_0)}
\]

(9)

Where \( \Theta \) is the period of earthquake repeat for magnitude \( M \geq M_0 \).

2.4 The probability of the occurrence of earthquakes

The probability of earthquakes occurring is the possibility of destructive earthquakes in one area over a period. Assuming the time interval follows the exponential form \( e^{-NT} \), the probability of earthquake occurrence is as follows:

\[
P(M, T) = 1 - e^{-N_1(M \geq M_0)T}
\]

(10)

Where \( P(M, T) \) is the probability of an earthquake with magnitude \( M \) and period \( T \).

3. Methodology

The data used is earthquake occurrence data from January 1907 to December 2017. Because the data obtained from Badan Meteorologi Klimatologi and Geofisika (BMKG) only from 2009, then the data from 1907-2008 accessed from the catalogue of earthquakes United States Geological Survey (USGS). The selected data is earthquake data located on longitude 92° - 98.5° East Longitude and latitude 0° - 8° North latitude which coincides in Aceh province and at maximum depth 80 Km, research area was divided into the grid so obtained 832 grids with earthquake incidence as much 9202. The distribution of earthquakes in the region of Aceh and Surrounding in the period 1907 to 2017 can be seen in [11]. The cumulative frequency of earthquake events in the region of Aceh shows that the earthquake increased significantly after the year 2000 where in 2004 there has been a major earthquake that caused the tsunami and there have been many subsequent earthquakes after that. For recalculation periods, the earthquake distribution was grouped by the division of the study area [11].

4. Result

The spatial b-value analysis is done in the research area with the number of earthquakes as much as the occurrence, the value of b-value is obtained and the standard deviation is obtained which means less than one then the b-value approximation is correct. Spatial variation of b-value is determined by dividing the research area located at 92° - 98.5° East Longitude and 0° - 8° West Longitude into 832 parts with grid 0.25° × 0.25°. The result of the analysis shows that b-value ranges from 0.132 - 3.434 with average 0.924. There is 347 grid that has a low b-value that is a region with b-value smaller than average, the grid that has a high b-value is an area with b-value more than average. In addition, 331 grids cannot be analyzed because 4 grids have an average magnitude equal to the
minimum magnitude and 327 grids with no earthquake data. However, for mapping, the area with an earthquake frequency of more than 10 was used, resulting in 73 areas with an average b-value of 0.751. The variation of b-value can be seen in Figure 2.

Figure 1 shows that the blue region is a low b-value area, the region is a region with high levels of stress and rare earthquakes, but in case of earthquakes, there will be earthquakes with a large magnitude. It may be due to the buildup of energy in the region and when energy is released will result in large-scale earthquakes. While the red area is a region with high b-value, it indicates that the area is often earthquake with small magnitude due to the earthquake immediate energy released. The level of accuracy of the calculation can be seen based on the standard deviation value of b-value. Overall the standard deviation of b-value is less than 1 which means the calculation of b-value is close to true.

The temporal b-value analysis is done by distributing earthquake occurrence time. Temporarily in the event of a b-value earthquake decreases until a major earthquake occurs. In 2012 and 2016, there has been a major earthquake in Simeulue and Pidie Jaya. In 2011, b-value increased after the earthquake in Simeulue in 2010. In addition, b-value also increased in 2017 after the earthquake in Pidie Jaya at the end of 2016.
4.1 Return period and Probability of Earthquake.

The re-occurrence period of earthquakes in Aceh and surrounding areas with $M_w \geq 5$ is illustrated in the following contour maps.

![Map of contours of earthquake repeating period](image)

Figure 2. Map of contours of earthquake repeating period

Figure 2. shows that red areas are areas with periods of re-earthquakes in the shortest period of 19 to 25 years, then the occurrence of repeated periods of earthquakes continues increase every change in red, yellow, green to blue. While the white colour indicates the area that there is an earthquake frequency of less than 10. The type of return period the earthquake can be seen in Table 1.

| Grid | Period (years) | Year estimates of return period | Occurrence probability |
|------|----------------|---------------------------------|------------------------|
| 356  | 19.7           | 2037                            | 0.632122               |
| 151  | 21.5           | 2039                            | 0.632652               |
| 125  | 22.8           | 2040                            | 0.632452               |

Table 1 shows that the most rapidly estimated earthquake will occur in the 356th grid that coincides in the southwestern seas of Southwest Aceh District or between Aceh Barat Daya and Simeulue with a repeat period of 19.7 years. The 151 and 125 grids coincide in the southernmost seas of Pulau Banyak island of Aceh Singkil with a return period of approximately 21.5 years and 22.8 years.
The probability of earthquake occurrences with on the grid to 356, 151 and 125 is about 63 percent. The estimated year the earthquake repeat period is calculated after 2017 where the last earthquake event data is used, so earthquakes are expected to occur in the 356th grid by 2037, the 151st grid by 2039 and the 125th grid by 2040. As for the validity of return period analysis, an analysis of the probability of earthquake occurrence in the near-estimated period will occur the period of the earthquake. The probability of earthquake occurrence can be seen in Table 2.

**Table 2.** The probability of the occurrence of destructive earthquakes

| Grid | 5 years | 10 years | 30 years | 50 years | 100 years |
|------|---------|----------|----------|----------|-----------|
| 356  | 0.224   | 0.398    | 0.782    | 0.921    | 0.994     |
| 151  | 0.208   | 0.372    | 0.753    | 0.903    | 0.991     |
| 125  | 0.197   | 0.355    | 0.732    | 0.889    | 0.988     |

Table 2 shows that the probability of an earthquake re-occurrence increases with increasing period. It is known that in the 356th grid is estimated in the next 10 years has a probability of earthquake 39.8 percent and in the next 30 years has a probability of 78.2 percent. The results are by the estimated repeat period of the earthquake that occurred within 19.7 years with a probability of occurrence of 63.2 percent (Table 1).

In addition, in Table 2 it can be seen that the greatest probability of earthquakes with an estimated 356th grid within 50 years will occur with a probability of 92.1 percent and within 100 years with a probability of 99.4 percent.

**5. Conclusion**

Based on estimation of return period of the destructive earthquake in Aceh area using the maximum likelihood method, the fastest destructive earthquake is expected to occur in the sea area between Aceh Barat Daya and Simeulue at 96° - 96.25° East Longitude and 35°-35.25° North Latitude in 19.7 years, it means that the earthquake expected to occur in 2037.

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