STACKING CLUSTERS IN THE ROSAT ALL-SKY SURVEY

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ABSTRACT

Ongoing and planned wide-area surveys at optical and infrared wavelengths should detect a few times $10^5$ galaxy clusters, roughly 10% of which are expected to be at redshifts $\geq 0.8$. We investigate what can be learned about the X-ray emission of these clusters from the ROSAT All-Sky Survey. While individual clusters at redshifts $\geq 0.5$ contribute at most a few photons to the survey, a significant measurement of the mean flux of cluster subsamples can be obtained by stacking cluster fields. We show that the mean X-ray luminosity of clusters with mass $M \geq 2 \times 10^{14} h^{-1} M_\odot$ selected from the Sloan Digital Sky Survey should be measurable out to redshift unity with signal-to-noise $\gtrsim 10$, even if clusters are binned with $\Delta z = 0.1$ and $\Delta \ln M \sim 0.3$. For such bins, suitably chosen hardness ratio allows the mean temperature of clusters to be determined out to $z \sim 0.7$ with a relative accuracy of $\Delta T/T \lesssim 0.15$ for $M > 10^{14} h^{-1} M_\odot$.

1. INTRODUCTION

With moderately deep, wide-area imaging surveys in the optical or near infrared it is now possible to detect large samples of galaxy clusters. Dalcanton (1996) proposed that clusters could be detected as surface brightness enhancements even when all but a few of their galaxies are too faint to be detected individually. Her suggested procedure consists of identifying and removing stars and galaxies from carefully flat-fielded images, smoothing the residual with a kernel matched to the core size of clusters, and searching for significant peaks in the resulting smoothed map. Gonzalez et al. (2001) successfully constructed the Las Campanas Distant Cluster Survey (LCDCS) by applying this technique to drift-scan data taken with the Las Campanas Great Circle Camera (Zaritsky, Schectman & Bredthauer 1996). They mapped well over 100 square degrees and constructed a catalog of 1073 groups and clusters. The estimated redshift limits of the catalog range from $\sim 0.3$ for groups to $\sim 0.9$ for massive galaxy clusters.

The high intrinsic uniformity of drift-scan surveys like the LCDCS makes them ideal for applying Dalcanton’s cluster-detection technique. In a theoretical study, Bartelmann & White (2002) showed that massive galaxy clusters should be detectable in the Sloan Digital Sky Survey (SDSS) out to redshifts of $\sim 1.2$ if data in the $r'$, $i'$ and $z'$ bands are summed. For the final projected SDSS survey area of $10^4$ square degrees, $\gtrsim 10^5$ galaxy clusters should be detectable at the 5-σ level, and $\sim 10$% of those are expected to be at redshifts $\gtrsim 0.8$.

Relatively little is known about the X-ray emission of clusters at redshifts beyond $\sim 0.5$ despite numerous cluster surveys based on X-ray data. The main reason for this is the steep decrease with redshift of the observed X-ray flux, which implies that at $z > 0.5$ individual massive clusters produce at most a few photons in surveys like the ROSAT All-Sky Survey (RASS; Snowden & Schmitt 1990). Detections of distant galaxy clusters from the X-ray data alone are limited to very luminous systems which can be detected in the restricted areas where deeper observations are available.

The upcoming availability of large cluster surveys in wavebands other than the X-ray regime allows a reversal of the traditional X-ray survey strategy. Rather than identifying clusters in the X-ray data, it becomes possible to stack X-ray survey data for a large number of fields where clusters are already known from other surveys. The low background count rate at X-ray wavelengths makes this an efficient technique for detecting the summed emission from a large stack of clusters.

In this paper we investigate the prospects for using the RASS to detect X-rays from suitable samples of clusters identified in the SDSS data. In Sect. 2 we describe our model for the cluster population. This is based closely on the properties of nearby clusters and specifies the cluster distribution in mass, redshift, X-ray temperature and luminosity. In Sect. 3 we convert cluster X-ray luminosities to count distributions expected in the ROSAT All-Sky Survey. Based on these, we calculate in Sect. 4 the expected signal-to-noise both for the detection of mean cluster emission and for estimates of mean cluster temperature. Sect. 5 summarises and discusses our conclusions.

2. MODEL SPECIFICATIONS

2.1. Cosmology

Much evidence suggests that the universe is spatially flat with low nonrelativistic matter density $\Omega_0$. Baryons make up only a small fraction of this matter; the rest is dark, presumably consisting of some massive, weakly interacting particle. A cosmological constant $\Omega_\Lambda$ or an equivalent “quintessence” field contributes the remaining energy density. For definiteness, we assume $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$ and $h = 0.7$.

We assume structure to form from an initially Gaussian density fluctuation field $\delta$ with statistical properties specified by its linear power spectrum, for which we adopt the CDM form given by Bardeen et al. (1986) with primordial spectral index $n = 1$. The only remaining free parameter is then the normalisation of the initial fluctuation field which we take as $\sigma_8 = 0.9$. This value was originally estimated based on the observed local abundance of galaxy clusters (White, Efstathiou & Frenk 1993; Eke, Cole & Frenk 1996; Viana & Liddle 1996; Pierpaoli, Scott & White 2001; Evrard et al 2002) but some recent analyses favour smaller values (Reiprich & Böhringer 2002; Viana, Nichol & Liddle 2002; Lahav et al. 2002). We will show results for $\sigma_8 = 0.9 \pm 0.1$.

2.2. Cluster population

Haloes form from Gaussian primordial density fluctuations through gravitational collapse. Press & Schechter (1974) first derived an approximate formula for the mass distribution of
haloes as a function of redshift \( z \). This has recently been modified by Sheth, Mo & Tormen (2001) and Sheth & Tormen (2002) based on an ellipsoidal rather than a spherical model for collapse. They give the differential comoving number density of haloes as

\[
n(M,z) \, dM = A \sqrt{\frac{2}{\pi}} \left( 1 + \frac{1}{\sqrt{2}a} \right) \frac{\Delta}{M} \frac{dv}{dM} \exp \left( -\frac{v^2}{2} \right) \, dM ,
\]

where \( v = \sqrt{\Delta_0 \sigma^{-1}(M,z)} \) defines the linear amplitude required for collapse of a density fluctuation and \( \Delta \) is the mean cosmic density today. \( \sigma(M,z) \) in this definition is equal to \( \sigma_0(M)D_w(z) \), where \( \sigma_0(M) \) is the present rms fluctuation in the dark matter density contrast within spheres containing the mean mass \( M \), and \( D_w(z) \) (with \( D_w(0) = 1 \)) is the growth factor for the linear growing mode (cf. Carroll, Press & Turner 1992). The linear density contrast required for collapse \( \Delta \) depends weakly on cosmology; for the \( \Lambda \)CDM model we have chosen \( \Delta \) = 1.673 (e.g. Lokas & Hoffman 2001). The parameters \( A, a \) and \( q \) are constants; the original Press-Schechter formula is obtained from \( 1 \) by putting \( A = 0.5, a = 1 \) and \( q = 0 \). This mass function, with \( A = 0.322, a = 0.707 \) and \( q = 0.3 \), has been shown to fit high resolution numerical simulations of structure growth in a wide range of cosmologies, provided the halo mass is defined at fixed density contrast relative to the cosmic mean density (Jenkins et al. 2001).

Next, we need to know the X-ray luminosity of a cluster of mass \( M \). We adopt the observed relation between cluster temperature \( T \) and bolometric X-ray luminosity \( L_X \)

\[
L_X = 10^{44} \text{erg s}^{-1} \left( \frac{kT}{1.66 \text{keV}} \right)^{2.331} ,
\]

as derived by Allen & Fabian (1998). Observations suggest that there is little evolution in the \( L_X - T \) relation out to redshifts \( z \sim 0.4 \) (e.g. Mushotzky & Scharf 1997; Allen & Fabian 1998; Reichart, Castander & Nichol 1999). Lacking any reliable information about evolution to higher redshifts, we assume \( 2 \) to hold at all redshifts. This, of course, is a major uncertainty of our study.

According to the virial theorem, a halo of mass \( M \) in equilibrium at redshift \( z \) with a structure similar to observed clusters should have a mean temperature given by

\[
kT = 4.88 \text{keV} \left( \frac{M \, h(z)}{10^{15} M_\odot} \right)^{2/3} ,
\]

where \( h(z) \) is the Hubble constant at redshift \( z \) in units of \( 100 \text{km s}^{-1} \text{Mpc}^{-1} \) and, in contrast to Eq. \( 1 \), \( M \) is here defined as the mass interior to a sphere with mean overdensity 200 times the critical value at redshift \( z \). Recall that we assume \( h(0) = 0.7 \) throughout our analysis. The constant in this relation is taken from the cluster simulations of Mathiesen & Evrard (2001; their Table 1) and is appropriate for specifying the temperature of the best fit single temperature model for the X-ray spectrum over the mass and redshift ranges of interest. When necessary, we use an NFW model of concentration parameter 5 to convert between cluster masses defined at different overdensities.

### 2.3 X-ray emission

We assume that clusters emit X-rays through thermal radiation according to a Raymond-Smith plasma model (Raymond & Smith 1977). Apart from cluster temperature and redshift, the model has two free parameters, the metal abundance and an overall normalisation corresponding to the total X-ray luminosity. We fix the metal abundance to \( Z = 0.3 Z_\odot \) at all \( z \) in agreement with the observed abundances of local clusters (e.g. Fukazawa et al. 1998). The results of Schindler (1999) suggest little evolution towards higher redshift and the final count rates we derive depend only very weakly on metallicity. Thus neglecting any dependence on redshift does not induce significant uncertainty.

Let \( F_v(T,z) \, dv \) be the total X-ray luminosity emitted in the spectral interval \([v,v + dv]\) by a cluster of temperature \( T \) at redshift \( z \). If the cluster is observed in an energy band bounded by \( E_1 \) and \( E_2 > E_1 \), only a fraction \( f \) of its bolometric flux is included in the bandpass, where

\[
f = \int_{E_1(1+z)}^{E_2(1+z)} F_v(T,z) \, dv \left[ \int_0^{\infty} F_v(T,z) \, dv \right]^{-1} .
\]

Thus the band-limited flux \( S_X \) is related to the bolometric X-ray luminosity through

\[
S_X = \frac{f L_X}{4 \pi D_L(z)^2} ,
\]

where \( D_L(z) \) is the luminosity distance from the observer to redshift \( z \). Note that this flux must still be modified to account for foreground absorption.

We use version 11.1 of the xspec software package (Arnaud 1996) to tabulate \( f \) for an observing band between 0.5 and 2.4 keV for cluster temperatures between 0.5 and 12 keV, and for redshifts between 0 and 2. Interpolating within this table and using Eqs. \( 2 \), \( 3 \) and \( 4 \), we can convert cluster masses to cluster temperatures, X-ray luminosities, and finally to unabsorbed fluxes in the observed energy range.

The azimuthally averaged X-ray surface brightness profile \( \Sigma(\theta) \) of galaxy clusters is often modelled using the so-called beta profile (Cavaliere & Fusco-Femiano 1978),

\[
\Sigma(\theta) = \Sigma_0 \left[ 1 + \left( \frac{\theta}{\theta_c} \right)^2 \right]^{-\left( 3\beta - 1/2 \right)} ,
\]

where \( \theta_c \) is an angular core radius, and the amplitude \( \Sigma_0 \) is chosen to produce the required X-ray flux \( S_X \). Based on observation, we choose \( \beta = 2/3 \) (e.g. Mohr, Mathiesen & Evrard 1999). For the linear core radius \( r_c \), we adopt the relation

\[
r_c = 125 \text{kpc} h^{-1} \left( \frac{L_X}{5 \times 10^{44} \text{erg s}^{-1}} \right)^{0.2} ,
\]

where \( L_X \) is the X-ray luminosity between 0.5 and 2.4 keV. This relation is a fair representation of at least some clusters with luminosities within \( 10^{43-45} \text{erg s}^{-1} \) (Jones et al. 1998). Following Vikhlinin et al. (1998), we assume that \( 6 \) does not evolve with redshift. The angular core radius is then \( \theta_c = r_c D^{-1}(z) \), where \( D(z) \) is the angular-diameter distance. In fact, Eq. \( 6 \) is a poor fit to the profiles of many clusters, particularly those with strong apparent cooling flows. This is not, however, of any great consequence for our modelling since the RASS does not, in any case, resolve the inner regions of most clusters.

Having fixed \( \beta, S_X \) and the angular core radius \( \theta_c \), the beta profile is normalised by

\[
\Sigma_0 = \frac{S_X}{2 \pi \theta_c^2} .
\]
3. HALO DETECTION

3.1. Point-spread function

The point-spread function \( f(\theta; E, \phi) \) of the ROSAT-PSPC had three components, a Gaussian kernel, Lorentzian wings, and a component which falls off exponentially with the angular separation \( \theta \) from the centre of the image (Hasinger et al. 1995). The parameters for these components generally depend not only on photon energy \( E \), but also on \( \phi \), the off-axis angle of the source.

The width of the PSPC point-spread function can be characterised by the effective solid angle \( \delta \Omega(E, \phi) \) covered,

\[
\delta \Omega(E, \phi) = 2\pi \int_0^{\infty} \theta d\theta f(\theta, E, \phi)
\]

and we can define an effective radius \( \theta_{\text{eff}}(E, \phi) \) by

\[
\theta_{\text{eff}}(E, \phi) = \left( \frac{\delta \Omega(E, \phi)}{\pi} \right)^{1/2}.
\]

The effective radii for six different off-axis angles between 10′ and 60′ are shown as functions of photon energy in Fig. 1.

![Effective radii of the PSPC point-spread function as functions of photon energy, for off-axis angles between 10′ and 60′](image)

**Fig. 1.** Effective radii of the PSPC point-spread function as functions of photon energy, for off-axis angles between 10′ and 60′ (from bottom to top).

The field-of-view of the PSPC was large, with a radius of approximately 60′. Since a given point on the sky was scanned at many different off-axes angles during the All-Sky Survey, the appropriate point-spread function for the ROSAT All-Sky Survey at a given photon energy is an area-weighted average of \( f(\theta; E, \phi) \) over the field-of-view,

\[
\bar{f}(\theta, E) = \frac{2}{(60')^2} \int_0^{60'} \theta d\theta f(\theta, E, \phi).
\]

Figure 2 shows the result for four different photon energies between 0.5 and 2.0 keV.

![The ROSAT-PSPC point-spread function, averaged over off-axis angles within the PSPC field-of-view.](image)

**Fig. 2.** The ROSAT-PSPC point-spread function, averaged over off-axis angles within the PSPC field-of-view. The different curves show the point-spread function for four different photon energies, as indicated.

The effective exposure time in the All-Sky Survey varies across the sky because of the ROSAT scanning strategy. It is highest near the ecliptic poles and lowest close to the ecliptic plane (cf. Snowden et al. 1995). Maps for the exposure time and the flux normalisation and for cluster temperatures between 0.5 and 12 keV and redshifts between 0 and 2. Fluxes determined from Raymond-Smith spectra of a fixed absorbed column of 4 × 10^20 cm\(^{-2}\) for absorbed Raymond-Smith spectra of a fixed unabsorbed flux normalisation and for cluster temperatures between 0.5 and 12 keV and redshifts between 0 and 2 can then be converted to absorbed count rates by interpolating within this table.

3.2. Converting fluxes to count rates

We now need to estimate the signal expected in the All-Sky Survey from a cluster with unabsorbed flux \( S_X \) and temperature \( T \) at redshift \( z \). To do this, we first modify the fluxes \( S_X \) calculated using Eq. 5 to allow for absorption by foreground neutral hydrogen. We assume a constant hydrogen column of 4 × 10^20 cm\(^{-2}\), which is typical for the high galactic latitudes covered by the SDSS (e.g. Dickey & Lockman 1990). We convert the absorbed fluxes to PSPC count rates, using the `fakeit` task of the `xspec` package with the PSPC response matrix.

In practice, we compute a two-dimensional table containing PSPC count rates in the energy range between 0.5 and 2 keV for absorbed Raymond-Smith spectra of a fixed unabsorbed flux normalisation and for cluster temperatures between 0.5 and 12 keV and redshifts between 0 and 2. Fluxes determined from Eq. 5 can then be converted to absorbed count rates by interpolating within this table.

3.3. Exposure times; background level

The effective exposure time in the All-Sky Survey varies across the sky because of the ROSAT scanning strategy. It is highest near the ecliptic poles and lowest close to the ecliptic plane (cf. Snowden et al. 1995). Maps for the exposure time and the background count rates were downloaded from the ROSAT All-Sky Survey web page. The left panel in Fig. 3 shows the cumulative exposure-time distribution for the complete All-Sky Survey (dashed curve), and for the area around the Northern Galactic cap covered by the Sloan Digital Sky Survey. The median exposure times are marked by vertical lines.

The effective exposure times on the whole sphere and on the SDSS area are only marginally different. For the SDSS area, we find a median value \( t_{\text{exp}} = 414 \text{ s} \).

\[t_{\text{exp}} = 414 \text{ s}.
\]
Similarly, the background level is anisotropic across the sky. The right panel in Fig. 3 shows the cumulative distributions of the background count rate in the All-Sky Survey for the whole sky (dashed line) and for the SDSS area (solid line).

The background count rate within the SDSS area is noticeably lower than on the whole sky; its median value is

\[ B = 0.94 \, \text{s}^{-1} \, \text{deg}^{-2} = 2.61 \times 10^{-4} \, \text{s}^{-1} \, \text{arcmin}^{-2}. \]  

4. Results

Figure 3 shows photon-count contours in the plane spanned by cluster mass and redshift. On a grid covering that plane, we compute temperature, luminosity, flux, and count rate as described in the previous section. We then multiply the count rate by the median exposure time in the SDSS area, averaged over photon energies in the 0.5–2.4 keV band.

The contours are logarithmically spaced by 0.25 dex between 0.1 and 100 counts (upper and lower solid curves, respectively). They appear jagged because a substantial fraction of the X-ray flux is contributed by metal lines which move in and out of the observed energy band as the redshift changes. The contours become smooth if the metal abundance is set to zero. From this plot one can see, for example, that the redshift limit below which individual clusters contribute more than ten photons to the All-Sky Survey increases from \( z_{\text{max}} \approx 0.1 \) at \( M = 10^{14} h^{-1} M_\odot \) to \( z_{\text{max}} \approx 0.8 \) at \( M = 10^{15} h^{-1} M_\odot \). The dashed curve shows the redshift limit for detection of clusters as 5-σ surface brightness enhancements in the combined \( r', i' \) and \( z' \) bands of the SDSS (Bartelmann & White 2002). Clearly the SDSS should produce cluster catalogues which are much deeper at all masses than those that can be made from the RASS.

Figure 3 illustrates that only 0.3 photons per cluster are expected for clusters of \( M \sim 10^{14} h^{-1} M_\odot \) at redshift \( z \sim 0.8 \). The number of such clusters expected in the SDSS is so large, however, that it should be possible to determine their mean X-ray properties by stacking data for many fields. This is true even if the mass-redshift plane is divided into relatively narrow bins. We now investigate this in more detail.

The background level of the All-Sky Survey is quite low, of order 1 s\(^{-1}\) deg\(^{-2}\) which translates to approximately 0.8 total counts per resolution element within the median exposure time of the survey. The background will nevertheless dominate the noise in a stacked image of distant clusters. Let \( B \) be the mean surface density of background photons in a single image, and \( C(M, z) \) be the expected number of photons from a single cluster of mass \( M \) at redshift \( z \). Let \( p(\theta) \) be the expected surface density of these cluster photons as a function of angular distance \( \theta \) from cluster centre. \( p(\theta) \) is given by a convolution of the mean cluster surface brightness profile [Eq. (3)] with the point-spread-function of the survey (Fig. 3) and we normalise it so that \( \int p(\theta) 2\pi \theta d\theta = 1 \). In practice for distant clusters the p.s.f. is much broader than the cluster image so that \( p(\theta) \) is proportional to the p.s.f. itself.

For a stack of \( N \) cluster fields the surface density of the background is \( NB \) and the expected surface density profile is \( NCP(\theta) \). Assuming Poisson photon statistics, the optimal estimator of the cluster signal is then:

\[ \tilde{N}C = \int w(\theta) |O(\theta) - B| 2\pi \theta d\theta, \]  

where \( 2\pi O(\theta)d\theta \) is the observed photon count in an annulus width \( d\theta \), and the filter function \( w \), normalised so that \( \int w(\theta)p(\theta)2\pi\theta d\theta = 1 \), is given by

\[ w(\theta) = \frac{p(\theta)}{p(\theta) + B/C} \left[ \int \frac{p^2(\theta)2\pi\theta d\theta}{p + B/C} \right]^{-1}. \]

Clearly the expectation value of the estimator of equation (14) is just \( NC \) while its variance is

\[ \text{Var}(\tilde{N}C) = NC \int w^2(\theta)p(\theta)2\pi\theta d\theta. \]

Thus the expected signal-to-noise for detecting the stacked cluster is

\[ \left( \frac{S}{N} \right) = \left( \frac{NC}{\sqrt{\int \frac{p^2(\theta)2\pi\theta d\theta}{p(\theta) + B/C}} \right)^{1/2}. \]
If clusters are individually well above background \((C_p(\theta) \gg B)\) over most of the broadened image) this gives the obvious result, \((S/N) \approx (NC)^{1/2}\) for the stack. When background dominates \((C_p(0) \ll B)\) the corresponding result is \((S/N) \approx NC/[N^2(\Delta p^2(\theta)2\pi d\theta)]^{1/2}\). In both cases the signal-to-noise of the detection grows as \(N^{1/2}\) for the stacked image. Figure 5 shows the number of cluster fields required for a 5-\(\sigma\) detection in the stacked image as a function of cluster mass and redshift.

\[
\Delta N_{ij} = \int_{z_i}^{z_i+\Delta z} dz \int_{M_j}^{M_{j+1}} dM n(M,z) \left| \frac{dV}{dz} \right| (1+z)^3. \tag{18}
\]

The volume per unit redshift is

\[
\left| \frac{dV}{dz} \right| = \pi D^2(z) \frac{dD_{\text{prop}}}{dz}, \tag{19}
\]

where \(D\) is the angular diameter distance and \(D_{\text{prop}}\) the proper distance. The factor \(\pi\) instead of \(4\pi\) accounts for the fact that the SDSS only covers a quarter of the sky. Figure 6 shows the resulting cluster numbers \(\Delta N_{ij}\) and the total photon numbers \(\Delta C_{ij}\) expected from these clusters. The solid and dotted curves show results for the lower and upper redshift bins, respectively. In order to illustrate the sensitivity of the results to \(\sigma_8\), the error bars mark the range obtained for \(\sigma_8 = 0.9 \pm 0.1\). The curves showing the total photon counts received in each mass bin are flatter than those showing the total cluster number because clusters with higher mass are more X-ray luminous.

The figure shows that, even with relatively fine mass binning, more than \(10^4\) clusters should be detectable per mass bin below \(10^{14} h^{-1} M_\odot\) in the lower redshift interval \(0.6 \leq z \leq 0.7\) (solid curve) and \(0.9 \leq z \leq 1.0\) (dotted curve) in mass bins of logarithmic width \(\Delta \ln M \sim 0.3\) between the SDSS completeness limit in the respective redshift interval and \(10^{15} h^{-1} M_\odot\). The total counts received from all clusters per mass bin drop much less steeply than the cluster number because the number of counts received per cluster increases strongly with cluster mass. The error bars bracket results obtained by changing \(\sigma_8\) by \(\pm 0.1\) and illustrate the sensitivity to the power-spectrum normalisation.

At the lower redshift the signal-to-noise ratio starts above 40 near \(4 \times 10^{13} h^{-1} M_\odot\), where the contribution of metal lines to
the flux is high. With increasing mass, the line contribution decreases and $S/N$ has a shallow minimum near $10^{14} h^{-1} M_\odot$. Increasing continuum emission causes a broad peak at $\geq 40$ centred on $3 \times 10^{14} h^{-1} M_\odot$. It then decreases slowly towards higher masses. The drop-off results from from the low cluster number at the high-mass end. If we set the metal abundance to zero, the low X-ray flux at the low-mass end makes the signal-to-noise ratio drop to $\sim 20$ near $4 \times 10^{13} h^{-1} M_\odot$. Even in the upper redshift interval, the signal-to-noise ratio is above 10, rising to $\geq 20$ in the lowest mass bin. These results are, however, very sensitive to $\sigma_8$. Near $10^{15} h^{-1} M_\odot$ in the upper redshift interval, the signal-to-noise ratio varies between $\sim 5$ and $\sim 20$ as $\sigma_8$ is increased from 0.8 to 1.0.

The high signal-to-noise ratio even for high-redshift clusters encourages us to investigate whether it will be possible to estimate cluster temperatures from hardness ratios. We introduce two energy bands, one with $0.5 \leq E/\text{keV} < 1$ and the second with $1 \leq E/\text{keV} \leq 2$. The counts $C_{1,2}$ in these two bands determine the hardness ratio

$$R = \frac{\text{hard counts}}{\text{soft counts}} = \frac{C_2}{C_1}. \quad (20)$$

We use xspec to compute the hardness ratio $R(T,z)$ expected for RASS data for clusters with temperature $T$ at redshift $z$. For clusters of mass $M$ at redshift $z$, the uncertainty in the temperature measurement is then

$$\Delta T(M,z) = \left( \frac{\partial R}{\partial T}(T(M),z) \right)^{-1} \Delta R,$$

where the uncertainty $\Delta R$ of the measured hardness ratio (20) is determined by the count statistics. The boundaries of the energy bands were chosen so that $R$ is typically of order unity in the mass and redshift ranges considered here. The signal-to-noise ratio of the hardness ratio $R/\Delta R$ is $\gtrsim 10$ for all cluster mass bins in the redshift interval $0.6 \leq z \leq 0.7$, and is $\gtrsim 8$ for the bins in the redshift interval $0.7 \leq z \leq 1.0$. The derivative of $R$ with respect to $T$ is $\sim 0.8$ for $T \sim 1 \text{ keV}$ and falls to $\sim 0.1$ the highest temperatures. As a result temperature determinations should be most accurate for clusters with $M \sim 10^{14} h^{-1} M_\odot$; at lower masses, line emission in the low-energy band dominates and the uncertainty $\Delta R$ increases because of poor photon statistics in the high-energy band. We show $T$ and $\Delta T/T$ in Fig. 8 for the same mass bins and redshift intervals used previously. For comparison, the plot also gives the mean cluster temperature expected as a function of mass in each redshift interval. Note that both $T$ and $\Delta T/T$ are emitted rather than observed values.

**5. Discussion**

Ongoing and planned wide-area surveys will detect tens of thousands of galaxy clusters out to redshifts near and above unity. For example, searching for surface-brightness enhancements in a smoothed stack of the $i$-, $i^-$- and $z^\prime$-band data of the Sloan Digital Sky Survey should allow clusters of $5 \times 10^{13} h^{-1} M_\odot$ to be detected out to $z \sim 0.7$, while $z > 1$ is reached for masses above $\sim 3 \times 10^{14} h^{-1} M_\odot$ (Bartelmann & White 2002).

![Figure 7](image1.png)

**Figure 7.**—Signal-to-noise ratios in stacked cluster fields in the given mass bins for the two redshift intervals $0.6 \leq z \leq 0.7$ (solid curve) and $0.9 \leq z \leq 1.0$ (dotted curve). As in Fig. 6, the error bars show the range obtained by varying $\sigma_8$ by $\pm 0.1$. The signal-to-noise ratio in the lower redshift interval reaches $\sim 40$ near $3 \times 10^{14} h^{-1} M_\odot$. Even near $10^{15} h^{-1} M_\odot$ in the upper redshift interval, the signal-to-noise ratio is $\sim 10$.

![Figure 8](image2.png)

**Figure 8.**—The curves with open squares show the relative uncertainty $\Delta T/T$ of cluster temperatures determined from the hardness ratio between a soft ($E \in [0.5,1] \text{ keV}$) and a hard ($E \in [1,2] \text{ keV}$) band. Clusters are stacked in mass bins in the two redshift intervals $0.6 \leq z \leq 0.7$ (solid curve) and $0.9 \leq z \leq 1.0$ (dotted curve). As in Figs. 6 and 7, the error bars indicate the range obtained by varying $\sigma_8$ by $\pm 0.1$. While the temperature uncertainty is very large for the low-mass clusters, it drops near 10% for moderate-redshift clusters with $M \gtrsim 10^{14} h^{-1} M_\odot$, and is $\lesssim 20\%$ for the high-redshift clusters with masses $\lesssim 6 \times 10^{13} h^{-1} M_\odot$. The curves with filled circles show the cluster temperature in keV for the given mass bins and redshift intervals. Figure 8 shows that the relative uncertainty in the mean temperature of the clusters in each mass bin is remarkably small for cluster masses $M \gtrsim 10^{14} h^{-1} M_\odot$. While the temperature uncertainty is very large for the low-mass clusters, it drops near 10% for moderate-redshift clusters with $M \gtrsim 10^{14} h^{-1} M_\odot$, and is $\lesssim 20\%$ for the high-redshift clusters with masses $\lesssim 6 \times 10^{13} h^{-1} M_\odot$. The error bars in Fig. 8 indicate the range obtained by varying the power-spectrum normalisation $\sigma_8$ by $\pm 0.1$. For clusters in the high-redshift band, $0.9 \leq z \leq 1.0$, the relative temperature uncertainty increases both because of count statistics and because of decreasing sensitivity of $R$ to $T$. Despite this, temperature measurements at $3 \to 10\sigma$ should be possible. Note that a careful maximum likelihood measurement of $T$ would give results with somewhat higher significance than the simple hardness ratio approach we have adopted here.
in mass and redshift as given by the numerical results of Jenkins et al. (2001). Their temperatures are taken to be proportional to \(M h(z)^{2/3}\), with the normalisation taken from the N-body/SPH simulations of Mathiesen & Evrard (2001). We adopt the observed low-redshift relation between bolometric X-ray luminosity and temperature, and we assume that it holds at all redshifts. We model cluster X-ray surface-brightness profiles by a beta profile, although this has little effect on our results because most distant clusters are not resolved in the RASS. We convert the bolometric X-ray luminosity into a count rate using the xspec software, assuming a Raymond-Smith plasma model with a metallicity of 0.3 solar and a foreground neutral-hydrogen column of \(4 \times 10^{22}\) cm\(^{-2}\).

The only suitable survey of the X-ray sky is the ROSAT All-Sky Survey (RASS). With its median exposure time of approximately 415 seconds and its effective detector area of \(\sim 230\) cm\(^2\), it detected \(\sim 10\) photons from a cluster of mass \(10^{14} h^{-1} M_\odot\) at \(z \sim 0.1\), and only about one photon from a similar cluster at \(z \sim 0.5\). Since the effective angular resolution of the RASS is \(\sim 2\arcmin\), cluster emission is typically spread over an effective solid angle of \(\sim 14\) square arcminutes. Due to the low background of the PSPC detector, only \(\sim 1.5\) background photons are expected within this solid angle during the median RASS exposure time. This corresponds to the number of photons expected from a cluster with mass \(M \sim 10^{14} h^{-1} M_\odot\) at redshift \(z \sim 0.35\), or with mass \(M \sim 4 \times 10^{15} h^{-1} M_\odot\) at \(z \sim 1\). Thus stacked cluster fields are background dominated at lower mass or higher redshift than this.

If we bin the clusters by mass into logarithmic bins with width \(\Delta M\), the detection increases to \(2\) at \(z \sim 0.5\), or with \(M \sim 10^{14} h^{-1} M_\odot\) at \(z \sim 1\). A stack of ten cluster fields should give a 5\(\sigma\) detection of massive clusters with \(M \sim 10^{15} h^{-1} M_\odot\) at \(z \sim 1\). Should be possible to detect of order 30000 galaxy clusters on the sky outside the Galactic plane, approximately 10% of which will be at redshifts beyond 0.5.

Wide-area surveys in the microwave regime will be carried out in the near future which will detect of order one cluster per square degree trough the thermal Sunyaev-Zel'dovich effect. The Planck satellite, for instance, due for launch in early 2007, is expected to detect of order 30000 galaxy clusters on the sky outside the Galactic plane, approximately 10% of which will be at redshifts beyond 0.5. Stacking these clusters in the same way as described here, and combining their total integrated Compton-y parameter with their X-ray emission, will allow their total baryonic mass and perhaps their temperatures to be constrained.

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