Methodological foundations of scaling in modern Measurement Theory. Classification of measurement scales and their application under uncertainty based on Bayesian Intelligent Technologies

S V Prokopchina

1 Department of System Analysis in Economics, Financial University under the Government of the Russian Federation, 49 Leningradsky Prospekt, 125993, Moscow, Russia

E-mail: svprokopchina@mail.ru

Abstract. Measurement theory is currently being actively developed due to its relevance in modern information technologies that implement the principles and algorithms of DATA SCIENCE, BIG DATA, Internet of Things, Business Intelligence, and DATA MINING. One of the most important applications of measurement theory methods and tools is implemented in artificial intelligence technologies. This is due to the need to use a variety of measurement data in these technologies. The conditions for obtaining such measurement information are associated with uncertainty, which necessitates the use of special methods and tools for obtaining measurement data. This, in turn, determines the need to develop and apply new types of scales focused on uncertainty conditions. The article offers a classification of measurement scales for the implementation of classical and intelligent measurement algorithms. The principles of implementing scales with dynamic constraints for measurement intellectualization based on Bayesian intelligent technologies are considered. Examples of using smart measurement scales are given.

1. Introduction
Modern information technologies widely use measurement information. It is basic for all types of intelligent technologies, Internet of Things (IoT) systems, decision support systems, management and management systems.

Measurement information is traditionally understood as information received from various measuring devices and complexes, usually presented in numerical form.

However, at the same time, active scientific work is constantly being done to create new types of scales for measuring non-quantitative values, complex properties, and expert assessments.

During the intellectualization of measurement processes, it became clear, as shown earlier in the author's works [4, 5, 7], that depending on the type of information situation and measurement conditions, classical measurement scales are not enough to use various knowledge flows in measurement processes.

To this end, in the early 90's of the last century, the author of this article [2-7] proposed new types of scales that allow using both quantitative and non-numerical information of objective (obtained from sensors and other information devices) and subjective (obtained from subjects-information carriers) types for obtaining measurement solutions. This type of scale is called dynamic constraint scales
(SDC) because of their ability to rearrange the scale structure depending on changes in measurement conditions, requirements, and constraints. Such functionality of the SDC allowed to make measurements in conditions of significant uncertainty about the model of the measurement object, its properties, relationships with the external environment, in the absence of enough a priori information.

The development of methods for intellectualizing measurement processes based on the regularizing Bayesian approach (RBA) [2-7] has led to the creation of a number of new types of scales, such as linguistic scales, conjugate (convolutional) scales, subjective scales, criterion scales, sensitive and cognitive scales, entropy scales, and other types.

For practical applications, all types of scales should be provided with a methodological basis, formalization and systematization of their types and properties.

This article is devoted to these issues.

2. Model for measuring the properties of complex objects

The implementation of the measurement approach in information processing tasks, including computerization tasks, is associated with the transition from the functional space of the object model to the metric spaces of controlled properties. This can be achieved by introducing measurement scales that reflect the gradation of object properties. At the same time, a fundamentally important point is the correspondence of the properties of the measured values and the scales used for their measurement. In an ideal situation, the scale adequately reflects all the properties of the measured object. However, as you know, even when modeling object properties, there are several restrictions that lead to homomorphic display of object properties at the model level. In information processing tasks, the process of reflecting object properties on a scale has a three-level structure consisting of the stages of displaying the properties of the object O in the properties of its MO model, displaying the properties of the MO model in the properties of the measurement object model (MOM), and displaying the MOM properties on the measurement scale as a measurement result (MR).

\[
O \rightarrow MO \rightarrow MOM \rightarrow MR
\]  

All these mappings are homomorphic and introduce additional limitations and inadequacies in the model, as well as uncertainty in the measurement result. Within these constraints, the resulting measurement solution is always conditional. In this regard, the measurement results on such scales are always conditional and can only be recommended for use under these conditions. In addition, they should be accompanied by a set of metrological characteristics, including indicators of their accuracy, reliability, reliability, and other indicators.

For recommendations on choosing a scale that can best reflect the properties of a complex object, it is necessary to consider in more detail the properties of scales currently used in measurement practice.

For the classical scheme of quantitative measurements, the concept of an abstract measurement scale defines the scale as a set of benchmarks that allow reflecting the values of the measured property, and the measurement method as a set of rules that allow translating the model of the object system into a signed scale system (in particular, numeric), which is a relational system \(G^{(O)}\) with properties \(O^{(O)}_i, i = 1, I\) and relations \(W^{(O)}_j, j = 1, J\).

\[
G^{(O)} = \{ * i = 1, I \{ Q^{(O)} \}; * j = 1, J \{ W^{(O)}_j \} \}
\]  

In the conceptual equation (2), the set of scale properties \(* i = 1, I \{ Q^{(O)} \}\) includes the type of scale structure, the type of reference points, the type of distance in the metric space of the scale, and the type of distance metric. The set of relations \(* \{ W^{(O)}_j \}, (j = 1, J)\) contains a set of functions that can be implemented on the scale; an algorithm or measurement method on the scale that defines the rules for comparing model elements (benchmarks) with the source information.

In measuring practice, various scales are used for measuring power, the theoretical justification and applications of which are devoted to numerous scientific works. Their most complete classification is given in the works of some scientists on the representative measurement theory, which is being successfully developed at the present time. In accordance with it, the whole variety of scales consists
of several groups, which include scales of order names, intervals, relations, absolute, functional, etc. Depending on their metrological properties, they are divided into metric and non-metric. Metric scales allow you to determine the distance between neighboring reference points of the scale carrier and are organized in the metric space of measurement results – scale reference points. Non-metric scales are organized on a set (ordered or unordered) of scale carriers, for which the concept of a distance metric is usually not defined. These scales include scales of names and order, they have defined equivalence relations (equivalence and order), and the carriers of the scale can be represented as linguistic sets. These scales are used in classification, recognition, and taxonomy tasks for making decisions.

Non-metric scales, which do not have computational power, are strong in semantic content, while metric scales, which allow performing various computational actions and implementing powerful computational algorithms, are rather "poor" in terms of interpreting the results obtained on them.

It will be shown that combining these two types of scales provides a unique opportunity to use all their qualities, which leads to a significant increase in the power of scales and implemented measurement algorithms, as well as to the intellectualization of measurement processes.

The model of a scale of type (2) when filling its components with the specific properties and relations specified above, conceptually corresponds to the above-mentioned classical types of measuring scales.

The classical scheme of the measurement scales of the model (2) corresponds to the situation a priori certainty of knowledge about the model of the object of measurement (which is assumed constant for all measurements at this scale), the variation ranges of measured properties on the plans the measurement of the experiment, reflected by the measurement technique, in the absence of the influence of environmental factors and the persistence of restrictions and requirements.

However, for measuring the properties of complex systems and objects that actively interact with the environment and change depending on it, this scale model is not appropriate for this information situation.

3. Intellectualization of measurements based on scales with dynamic constraints

When implementing a measurement scale for measuring the properties of complex objects, there are restrictions specific to specific types of scales \( \{C_j \mid 1 \leq j \leq J(S)\} \) which are associated with the gradation of the functional space of the measured properties, limitations of the measurement range, linearization, and other factors.

In addition, due to the lack of information about the elements of the scale, a variety of a priori information should be used. Thus, the model of the implemented scale of measurement of MO properties can be represented as a composition of its properties and restrictions that determine the measurement conditions, and these components should be reflected in the scale model:

\[
S^{(O)} = \left\{ * i = 1, I \left\{ Q^{(O)} \right\}; * j = 1, J \left\{ W_j^{(O)} \right\}; * k = 1, K \left\{ C_k^{(S)} \right\} \right\}
\]  

(3)

At the level of mathematical abstractions, the set of scale constraints can be divided into a priori known and unknown. In this case, the \( S^{(O)} \) system can be represented as a subsystem of systems of the measured property itself and its model. With the accumulation of information in the measurement process and the use of this scale in intelligent measurements, the scale structure develops with a gradual “immersion” of the scale system into the system of the measurement object. So, in an ideal situation, with complete “immersion”, it is possible to adequately display the properties of an object or its models on a given measuring scale, which corresponds to an isomorphic transformation of the form (1). In this situation, the scale system is essentially the same as the object system. Then the spaces of the object’s properties or its models are covered by the space of the scale incompletely, and at the corresponding levels of the measurement process there are errors in the inadequacy of the measurement results. It is obvious that to meet the metrological requirements for measuring the properties of complex objects to measuring scales, it is necessary to implement the principle of "dynamic immersion" of the scale system in the object system. This is possible if there are properties of the scale that allow you to remove restrictions when new information is received during the measurement process, which will cause a number of restrictions to pass into the category of properties.
inherent in the scale, as well as translate a number of restrictions from the set of unknown to the set of a priori known and will allow the development of the measuring scale. Obtaining such additional information in the measurement process reduces the error of inadequate results and increases the accuracy of measurements.

To ensure these properties of the scale, the set of relations of the scale system must contain the following relations: \( W_1 \) - equivalence and similarity (for implementing the verification scheme on the scale); \( W_2 \) - order (for implementing the scale carrier); \( W_3 \) - algebraic type (for implementing information transformations); \( W_4 \) - structurization, which allows the synthesis of multidimensional hierarchical scales for measuring complex properties of objects based on their characteristics; \( W_5 \) - generalization of measurement information (for enabling the implementation of self-learning processes, reorganization and development of the scale).

A measurement scale model with "dynamic immersion" in the system of the modeling object can be written for time \( t \) as:

\[
S^{(O)}_t = \{ j = 1, L \{ Q^{(O)}_t \}; \ast \ j = 1, J \left\{ W^{(O)}_{j t} \right\}; \ast \ k = 1, K \left\{ C^{(S)}_k \right\} \} 
\]  

(4)

Earlier in the work of the author [5], this type of measuring scale, which has the properties of the system (4), is defined as a measuring dynamic scale, or a scale with dynamic restrictions (SDC). If the measurement scale is created to display complex properties based on \( q \) attributes, then the SDC must have a hierarchical structure, the abstract system of which has the form:

\[
S^{(O)}_t = \{ j = 1, L \{ Q^{(O)}_t \}; \ast \ j = 1, J \left\{ W^{(O)}_{j t} \right\}; \ast \ k = 1, K \left\{ C^{(S)}_k \right\} \} 
\]  

(5)

The properties of the SDC make it possible to effectively study complex objects using a measurement approach, which makes it possible to focus on this type of scale when organizing monitoring of complex objects. This recommendation is based on the fact that over time, the knowledge obtained in the process of measurement or monitoring leads to a scale structure in which for specific classes of measurement tasks it is achievable to meet the condition of transformation isomorphism (1), when knowledge about the object of measurement can be obtained sufficiently complete for practical purposes.

If you have a limited time to observe the object, the information is usually inaccurate. Obviously, to reflect the probabilistic properties of the result, it would be advisable to identify several alternative results that are possible with varying degrees of probability in this measurement situation. This would result in not just one element of the scale carrier, but a subset of the set of these elements.

This scale of the "intelligent" type is understood in the works as a scale for measuring the properties of objects of the SDC type, capable of:

- obtaining a result in the form of a value and (or) a linguistic message, a functional dependency, a system of functions, and other mathematical and semantic models of the properties and States of the measurement object;
- metrological justification of measurement results on the scale in the form of accuracy and reliability indicators;
- determination of a complete and objective result in the form of a set of all possible alternative solutions – elements of the scale carrier in conditions of incomplete or unclear information;
- organization structure of the scale, hierarchical and mixed in the form of the result (together with linguistic numeric) types;
- dynamic restructuring of the carrier structure and the set of relationships of the scale system (expanding the scale range or connecting new branches of the hierarchical structure of the scale in accordance with the knowledge obtained in the measurement process).

Obviously, additional properties of metric scales can be attached to the organization of the measurement process itself by selecting the appropriate conversion function implemented in the form of measurements, which determines the need to consider existing measurement technologies that fundamentally allow the implementation of a scale of the SDC type in the measurement process.
Information technologies of measurement processes, in fact, implement the measurement methodology. The variety of types and interconnectedness of information flows in modern measurement systems necessitates measurement technologies that optimize the functioning of measuring instruments in order to achieve the required quality of the obtained measurement results based on the specifics of the properties of measurement objects, measurement conditions and the external environment. Among the many types of information, the most important is experimental information obtained directly when measuring the properties of an object and in the process of converting measurement results, otherwise called measurement information. A specific feature of this type of information is its metrological justification, which reflects the quality of measurements. Information technology is understood as a set of knowledge, concepts and ways to implement the processes of circulation and processing of all types of information. In information and measurement system (IMS), one can distinguish the part of it that is related to the processing of measurement information, defining it as a measurement information technology.

Thus, by measuring it we will understand the technology of implementing the measurement process, including the measurement concept, the formation of models of the measurement object and operating environment, measurement scales of controlled properties of the measurement object and influencing environmental factors, mathematical, information and technical measurement tools, issues of organizing and optimizing a measurement experiment in specific measurement situations, metrological requirements and technological limitations of specific IMS for the purpose of collecting, transmitting, converting, storage and interpretation of measurement information about the properties of the measurement object. The main requirements for modern measurement technologies include the following:

- ensuring quality control of the results obtained and determining their main metrological indicators in the form of accuracy, reliability and reliability values that form the fundamental basis of metrological support;
- considering the specifics of the measurement object, the type and degree of certainty of a priori and obtained experimental information about it when choosing measurement methods and tools;
- the ability to optimize the planning of a measurement experiment in accordance with metrological requirements and restrictions;
- ensuring the completeness and sufficiency of the received solutions;
- the ability to dynamically adapt the technology to changing conditions and properties of the measurement object during the experiment;
- possibility of self-study and based on this reorganization of SDC;
- creation of developing information technologies and SDC.

Obviously, these requirements determine the trend towards the intellectualization of its dimensions.

4. The measurement scale for non-quantitative measurements
Typification and classification of measurement scales can be carried out based on several main features:

- objective and subjective measurements and scales;
- quantitative and non-quantitative measurements and scales.

Quantitative scales are divided into five main types:

- nominal scales;
- ordinal scales;
- interval scales;
- relationship scales;
- absolute scale.

The following classification system for non-metric scales is proposed.

Non-quantitative scales can be divided into sensitive and cognitive scales. The first type of scales is associated with the measurement of emotional properties, sensations, and sensory perceptions; the second – with the representation of knowledge, experience, and skills of subjects and society, declarative attitudes, norms, and rules in measurement processes.

Sensitive scales are divided into:
scales of maxims (sensations);
- intuitive.

The scale of sensations is divided into:
- sound of the scale;
- visual scales;
- organoleptic(aromatic) scales;
- tactile scales;
- scales of emotions.

Cognitive scales are divided into:
- descriptive;
- predictive;
- prescriptive scales that reflect the existing knowledge and experience of subjects, the predictive process, as well as the predictive process with explanations.

All these scales are based on the gradation of properties. For their construction, the concept of an ordinal universal scale is used, for which the reference points are linguistic variables that reflect this gradation of properties.

To measure complex integral properties, emergent scales are used, which allow you to measure the emergent properties of complex objects based on measurements of simple properties.

4.1. **Criterion conjugate scales**

Linguistic scales can be used to construct criteria scales of a multi-level structure. The concept and technologies for creating such scales are proposed and implemented in applied problems within the framework of the regularizing Bayesian approach methodology.

Criteria scales can be objective when they are based on standards, regulations, and technical guidelines.

They can also be fiducial (subjective) when norms and principles of a subjective type are laid down in the basis.

Criteria scales in the BIT methodology are interfaced with the main linguistic SDC for multi-criteria scaling. At the same time, a multi-criteria audit of the measured characteristics is carried out together with the measurement process.

The results of measurement on such scales are fuzzy alternative estimates with a full metrological justification of the BIT scales.

4.2. **Integral (emergent) of the scale**

Scales for measuring the emergent properties of complex objects and systems. This type of scale is designed to measure the integral properties of complex objects. The measurement method provides a functional possibility to implement convolution (convolution) of intermediate measurements of simple properties included in the integral property.

These scales are used to measure the indices, indicators of a hierarchical type, the integral characteristics and indicators.

5. **Conclusion**

In General, this typification of scales allows you to cover all available types of measuring objects – both real and virtual, which is especially important when using measurement methods and tools in artificial intelligence systems and when intellectualizing measurement processes.

**References**

[1] Knorring V G and Solopchenko G N 2003 Measurement theory as an independent area of knowledge: characterization goals and objectives Measuring technology 6 13–17 (in Russian)

[2] Sapozhnikova K, Hussein S, Taymanov R and Baksheyeva Iu 2019 Music and growl of a lion: anything in common? Measurement model optimized with the help of artificial intelligence will answer Journal of Physics: Conference Series 1379 012055
[3] Nedosekin D D, Prokopchina S V and Chernyavsky E A 1995 *Information technologies for intellectualization of measuring processes* (SPb.: Energoatomizdat) 185 (in Russian)

[4] Prokopchina S V 1997 The concept of Bayesian intellectualization of measurements in the problems of monitoring complex objects *News of Artificial Intelligence* 3 7–56 (in Russian)

[5] Prokopchina S V 2005 Methodological aspects of the theory of soft measurements *Collection of reports of the international conference “SCM-2005”* (SPb.: Saint Petersburg Electrotechnical University “LETI”) 49–63 (in Russian)

[6] Prokopchina S V 2017 Methodological foundations of the theory of Bayesian intellectual measurements *Soft measurements and computing* 1 395 (in Russian)

[7] Prokopchina S V 2018 Principles and methodological aspects of constructing a scale with dynamic constraints for measurement under uncertainty *Soft measurements and computing* 3 4–15 (in Russian)